On the Three-Phase Mass Transfer with Solid Particles Adhered to the Gas-Liquid Interface

Endre Nagy*

Research Institute of Chemical and Process Engineering,
Kaposvar University,
H-8201 Veszprém, P.O. Box 125, Hungary

Received 16 January 2003; accepted 7 April 2003

Abstract: A heterogeneous, multi-layer mass transfer model is proposed for prediction of the effect of multi-layer packing of catalyst particles adhered to the gas-liquid interface. The behavior of the mass transfer rate with respect to the multi-layer packing, to the particle size and mass transfer coefficient without particles is discussed. It is shown that enhancement can be considerably increased by multi-layer packing compared to that of mono-layer packing, depending on the values of particle size and mass transfer coefficient. The predicted mass transfer rates using the proposed model was verified with experimental data taken from the literature. The model should be superior to that of published in the literature.

Keywords: three-phase absorption, catalyst particles, enhancement, multi-layer mass transfer model, surface renewal theory.

1 Introduction

The effects of fine solid particles on the gas-absorption rate, where the particles are adhered to the gas-liquid interface, have not been previously described by suitable mathematical expressions. Primarily, real behaviors of particles on a flat gas-liquid interface or on a moving bubble interface (their contact time, the amount adhered to the interface, the thickness of the packing of the adhered particles, etc.) are unknown.

Holstvoogd et al. [6] analyzed theoretically the enhancement of the gas-absorption rate in the presence of fine catalyst particles. They demonstrated that the experimental enhancement factor could be much higher than that predicted. Unrealistically high solid concentrations were calculated to be necessary to reach the measured [1], [2] enhance-

* E-mail: nagy@mukki.richem.hu
ment. As has been shown, the solid concentration in the boundary layer at the gas-liquid interface should be much higher (it may also be several hundred times higher) than that in the bulk phase.

Vinke et al. [20],[19] measured the amount of solid particles at the bubble interface by a flotation method where they showed that the solid particles could effectively adhere to the bubble interface. As a result, their interface concentration was found to be about 100 times higher than that in the bulk concentration when considering a mono-layer at the interface (for example interface coverage can reach 0.07 at suspension concentration of 1 kg/m³). The authors initially developed a heterogeneous model for the absorption into slurry with adhered particles using the film theory. Their model is simplified since they assumed that the backside of the particles, forming a mono-layer on the interface, is closed for mass transport. As a consequence, the predicted absorption rates are rather low.

Later Demmink et al. [5] developed the ”surface renewal particle to interface adhesion model” for gas absorption into slurry with adhered particles into the gas-liquid interface. Their unsteady-state model also allows for mass transport through to the backside of the particles. The internal diffusion was taken to be instantaneous, thus, the role of the internal mass transport in the absorption rate is not taken into account in their model. In this respect Nagy’ models for three-phase mass transfer [11], [12] should also be cited. These models were worked out for a mass transfer process in the presence of liquid droplets in the continuous phase, assuming the same contact times for both the droplets and liquid elements in the boundary layer. That is a more general model than the aforementioned which also takes into account the internal (inside the particles) mass transport and even the internal and external chemical reactions as well. The first model of Nagy [14] gives the absorption rate for a single cubic particle in the boundary layer. The second paper [12] gives the absorption rate for spherical particles with number of particles behind each other in the diffusion path. These models are more general than that of Vinke et al. [19] and Demmink et al. [5], essentially they involve these latter ones as limiting cases. Demmink et al. [5] compared the predicted data with the experimental data obtained during absorption of acetylene into carbon particle slurries with flat gas-liquid interface. They measured enhancement at various physical mass transfer coefficient \((\beta_o = 0.156 \times 10^{-4} \text{ to } 1.1 \times 10^{-4} \text{ m/s})\) and solid phase concentration \((\gamma_s = 0.02 \text{ to } 0.56 \text{ kg/m}^3)\). They determined that the data predicted by their model was much lower than the experimental even when they assumed total coverage of the gas-liquid interface \((\alpha = 1)\) by the adhered particles.

The question arises in how the high absorption rate measured can be best explained. Several reasons maybe considered such as: the irregular shape of the particles, likelihood of formation of multi-layers of particles at the gas-liquid interface, the possibility of direct gas-particle mass transfer [5], the non-linearity of the absorption isotherm in the solid particles (see [13]), etc. The focus of attention for this paper shall be on the effect of a multi-layer packing of particles at the gas-liquid interface. Vinke et al. [20], [19] assumed mono-layer packing at the interface, whilst Zon et al. [21] investigated the
gas-solid adhesion and solid-solid agglomeration of carbon particles in the slurry reactor. They reported that multi-layer packing of particle can indeed be formed at the gas-liquid interface. The thickness of this layer, whilst not given in the paper can be predicted as $2-3 \, d_p$ (formed by 2-3 layers of particles), from the results. Our model for a multi-layer of particles [12] allows us to adapt it for cases where the catalyst particles are adhered to the gas-liquid interface either forming a mono- or multi-layers of packing and also to predict the absorption rates as well. Another important factor is the fraction of the gas-liquid interface, $\alpha$, covered by the adhering particles. According to Vinke’s measurements and evaluations [20], [19] it has a maximum of about 0.07 to 0.13 in the solid phase concentration range of $\gamma_s = 0$ to 2 kg/m$^3$ depending on the stirrer rate. Similar results were predicted by Demmink et al. [5]. Zon et al. [21] found that the maximum values of the coverage range between 0.1 and 0.25 in the case of flotation experiments under non-stagnant conditions. A further important factor for consideration maybe the non-linearity of the solubility (partition) curve between the liquid and solid phases. Li and Deckwer [8] measured the adsorption of O$_2$, CO$_2$ and ethylene by the active carbon slurry. They estimated that the adsorption isotherms of CO$_2$ and ethylene deviate essentially from linear. Nagy [13], using the pseudo-homogeneous model developed by Nagy and Moser [14], showed that the non-linearity could significantly enhance the absorption rate at higher values of the partition coefficient. Finally, it should be noted that the role of the lateral diffusion into the particles in the boundary layer can also be of importance, especially, in the case of the high solubility coefficient, $H$. This was demonstrated by Lin and Zhou [9] and later by Brilman et al. [3] using two- and three-dimensional models, respectively. The lateral diffusion can act, however, when there is enough liquid volume surrounding a particle in the absence of other particles. This seems not to be the case when particles are adhered to the interface forming a packing layer were they are as close as possible to each other [20]. Two kinds of multi-layer packing are illustrated in Fig. 1. As shown, there is, practically, no place for lateral diffusion among the particles.

The aim of this paper is to analyze the absorption rate giving consideration to the above aspects in order to find at least, theoretical explanations, on the odd behavior of the absorption rate in the presence of gas-adsorbing particles adhered to the gas-liquid interface and to show how a multi-layer packing can alter the absorption rate.

2 Mass transfer rates

Two models shall be analyzed here, a model given by Demmink et al. [5] for a mono-layer packing and that developed by Nagy [12] for multi-layers of particles in the diffusion pathway. The heterogeneous model based on a number of particles in the boundary layer [12] was developed for gas absorption into liquid containing a dispersed third phase using the film-penetration theory. This model also takes into account the chemical reaction in both the continuous and dispersed phases. Based on the surface renewal theory, the mass transfer in the presence of micro-particles was also discussed by Mehra [10]. The author gave a numerical solution for the absorption rate using a multi-layer model. Sim-
plifying the Nagy’s equation [12] according to the surface renewal theory, this well known Danckwerts’s mass transfer theory [4] is included in the model as a limiting case, namely where \( \delta \to \infty \), the mass transfer rate without chemical reactions in any of the phases. The heterogeneous part of the interface (portion of the gas-liquid interface covered by adhering particles), can be given as follows:

\[
j = \beta (A^* - A^o)
\]

where

\[
\beta = \beta^o \frac{1}{2 \cosh \lambda_1} \exp (\lambda_1) n_{2,N} - \exp (-\lambda_1) n_{1,N}
\]

The values of \( n_{t,N} \) can be obtained by the next two equations (Eqs. 2a, 2b) calculating the \( n_{t,i} \) and \( m_{t,i} \) values from \( i = 1 \) up to \( i = N \) in a series, respectively.

\[
n_{t,i} = n_{t,i-1} [1 + p \tanh \lambda_p \tanh \lambda_d] + m_{t,i-1} \left[ \tanh \lambda_p + \frac{1}{p} \tanh \lambda_d \right]
\]

\[
m_{t,i} = n_{t,i-1} [\tanh \lambda_p + p \tanh \lambda_d] + m_{t,i-1} \left[ 1 + \frac{1}{p} \tanh \lambda_p \tanh \lambda_d \right]
\]

with

\[
p = H \sqrt{D_d \over D}; \quad \lambda_1 = \sqrt{s \delta_1^2 \over D}; \quad \lambda_p = \sqrt{s \delta_p^2 \over D}; \quad \lambda_d = \sqrt{s \delta_d^2 \over D}
\]

For the case of \( i = N \) Eqs. (2) to (2b) have to be modified by the following manner since there are no more particles behind the \( N^{th} \) particle, thus \( \lambda_p = \infty \):

If \( i = N \) then \( \tanh \lambda_p = 1 \).

For the calculation of the values of \( n_{t,i} \) and \( m_{t,i} \), in the case of \( i = 1 \) (the values of \( n_{t,1} \) and \( m_{t,1} \)), the values of \( n_{t,1} = n_{t,0} \) and \( m_{t,1} = m_{t,0} \) have to be known. They are as follows:

\[
n_{0,0} = \tanh \lambda_1; \quad m_{0,0} = 1; \quad n_{1,0} = -m_{1,0} = -1; \quad n_{2,0} = m_{2,0} = 1;
\]

The mass transfer rate (Eqs. 1-2) was obtained by the solution of the differential mass balance equations given for the continuous phase and for the particles located in the diffusion pathway behind each other and perpendicular to the gas-liquid interface. The number of the differential mass balance equations given for the boundary layer is equal to \( 2N + 1 \) and there belong 2 times more boundary conditions, \( 2(2N + 1) \) to these equations [12], [10]. This algebraic equation system was solved by traditional mathematical methods in order for the mass transfer rate to be expressed by Eqs. (1-2). From Eqs. (1-2), one can easily obtain the mass transfer coefficient for a monolayer packing of the adhering particles at the gas-liquid interface (here again the value of \( \tanh \lambda_p \) was chosen to be unit: \( \tanh \lambda_p = 1 \)):

\[
\beta = \beta^o \frac{\tanh \lambda_1 (\tanh \lambda_d + p) + p (1 + p \tanh \lambda_d)}{\tanh \lambda_d + p + p \tanh \lambda_1 (1 + p \tanh \lambda_d)}
\]
The mass transfer rate for finite thickness of the boundary layer is also of interest. In this case the thickness of the layer (δ) may be taken to be equal to the penetration depth [10]. For that particular case the mass transfer coefficient for a mono-layer packing at the interface can be expressed:

\[
\beta = \beta^o \frac{\tanh \lambda_1 (\tanh \lambda_d + p \tanh \lambda_p) + p (1 + p \tanh \lambda_p \tanh \lambda_d)}{\tanh \lambda_d + p \tanh \lambda_p + p \tanh \lambda_1 (1 + \tanh \lambda_p \tanh \lambda_d)}
\]  \(3b\)

with

\[
\lambda_p = \sqrt{\frac{s (\delta - \delta_1 - d)^2}{D}}
\]

From Eqs. (1) to (3), the absorption rate for cubic or prismatic particles (rectangular slabs) with constant values of δ₁, δₚ and d can be directly predicted. In the case of a spherical interface, these values continuously change at the heterogeneous section of the interface, as can be seen in Fig. 1 (see the right, lower edge of this figure where it can be “shown” how the values of δ₁, δₚ and d can vary as a function of \(R^*\)). In the packing of A and B (pack-A and pack-B, Fig. 1) the spherical, uniform particles are modeled as prismatic particles with the same volume as the spherical particles and with a length of \(d_p\), while the third drawing in this figure represents real spheres at the interface. Values can be calculated: for the sizes of the prisms from \(d = d_p \sqrt{\pi/6}\); the distance from the gas-liquid interface from \(\delta_1 = (d_p - d)/2\); the distances between the prismatic particles from \(\delta_p = 2\delta_1\) and \(\delta_p = \delta_1 - 0.134d_p\) for pack A and Pack B, respectively. The data given here can very easily be obtained by geometric considerations. The average value of the absorption rate, taking into account their spherical interface with changing of the values of δ₁, δₚ and d can be obtained by the following integration [12]:

\[
j^{ave} = \frac{1}{R^2 \pi} \int_0^R 2R^* \pi j(R^*) dR^*
\]  \(4\)

The value of \(j(R^*)\) takes into account the dependency of the values of δ₁, δₚ and d as a function of \(R^*\) [12]. The specific absorption rate related to the total gas-liquid interface can be expressed as follows [19]:

\[
E = \frac{j^{ave}}{J} + (1 - \alpha)
\]  \(5\)

\(J\) denotes the absorption rate without particles. The value of \(j^{ave} = j\) in the case of a cubic- and prismatic particles.

3 Results and discussion

Initially presented are theoretical results using Eqs. 1-2, (for Pack A and B with prismatic particles or Eq. 3 for spherical particles with pack A), as well as the Demmink’s model [5]. The effect of two important parameters, that of the particle size, \(d_p\), and the physical mass transfer coefficient, \(\beta^o\) (mass transfer coefficient without solid particles), are demonstrated
here. Both parameters have a key role in the enhancement of the absorption process. Enhancement of the gas absorption rate as a function of particle size is illustrated in Fig. 2 for a mono-layer packing, $N = 1$, with total coverage of particles on the gas-liquid interface, $\alpha = 1$. The values for the parameters used for calculations are listed in the legend and are the same as those obtained by Demmink et al. [5]. The model developed from them is compared to that of Nagy. Demmink et al. modeled the spherical particles as prisms with the distance from the interface of $\delta_1 = d_p/4$ and with thickness of $d = d_p/2$. Thus, Demmink’s model (dotted curve) gave practically the same results as those of the Nagy model using prismatic particles with the same values of $\delta_1$ and $d$ as those used by Demmink et al. (curve 2). The slight difference between them is partially caused by the special conditions used for them in the differential mass balance equation (Eq. 9 in [5]). These conditions are the factor $3/d_p$ for the specific area of mass transfer and the zone values which give lesser particle volume than that of the original particle. On the other hand, Demmink et al. [5] assumed that the diffusion is instantaneous inside the particle, that is $D_d \to \infty$. This condition is involved in our model as a limiting case, namely if $D_d \to \infty$. Curve 1 gives enhancement for a mono-layer formed by prismatic particles with $\delta_1 = 0.138d_p$ and $d = 0.723d_p$. In the case of the spherical particle layer the average values for the mass transfer rate of the heterogeneous part of the interface was calculated by Eq. (4). This is the only model that regards the particles as spheres. Enhancement is much higher in these two latter cases than that of Demmink et al. [5]. This is caused mainly by the large difference between the values of $\delta_1$ of the models. The particles modeled by spheres gave a somewhat higher enhancement than that modeled by prismatic particles at the higher particle size range. It is worth noting that $E$ versus $d_p$ curves have a maxima. The position of this maxima strongly depends on the value of the $\beta^\circ$ mass transfer coefficient. This behavior of the $E$ values towards $d_p$ is connected with the penetration depth of the absorbed component [12]. It is about equal to $\delta_1 + d$ at the maximum value. Decreasing the particle size lowers the role of the particle on the mass transfer rate, thus, lowering the enhancement. At higher particle size range, when $d_p$ is larger than the diffusion depth, the distance of the particle from the interface, $\delta_1$, increases with the particle size, and consequently decreases the role of the particle in the absorption process. This fact leads to a decrease in the enhancement.

The effect of the number of particle layers in multi-layer packing is illustrated in Fig. 3 as a function of the particle size. The continuous lines give enhancement obtained for packing built by spheres at different values of $N$. This figure illustrates the effect of size on enhancement. In lower particle size regime, the $N$ value greatly alters the absorption rate. The particles size range is less than about 20$\mu$m at the parameter values applied for the simulation. Enhancement gradually increases with the increasing number of particle layers, i.e. with $N$. It can generally be said that, in this size range, the penetration depth of the absorbed component is larger than the thickness of the packing of the adhered particles and further increases in the layer’s numbers can reflect an increase in the absorption rate. This is true to the point when the thickness of packing reaches the penetration depth. In the larger particle size range, where $d_p >$ about 20$\mu$m,
enhancement is independent on the values of $N$. The additional layers, the second-, third layers, etc. have no effect on the absorption rate in this size range. Enhancement is also given for pack $A$ and $B$ with prismatic particles at $N = 10$. Pack $B$ gives the highest enhancement due to its lower $\delta_p$ values in the packing.

The physical mass transfer coefficient also shows a strong effect on enhancement (Fig. 4) for a mono-layer packing at the gas-liquid interface. The curves were calculated for packing formed by spheres and by prismatic particles and by use of Demmink et al.’s model [5], for comparison. The shape of the curves are similar to that obtained as a function of the particle size. The explanation of this behavior can again be given with comparison of the penetration depth and the value of $\delta_1 + d$.

How the number of particle layers alters enhancement as a function of $\beta^o$ is illustrated in Fig. 5. At lower values of $\beta^o$, the role of $N$ increases with the decrease of the $\beta^o$ values. With a decreasing value of $\beta^o$, the contact time of the liquid elements at the gas-liquid interface increases, thus, the penetration depth also increases. This fact increases the role of multi-layer packing on the absorption rate. If the value of $\beta^o$ is enough low, even the fiftieth layer in the packing can theoretically enhance the absorption rate. In comparison of the different packing systems in Fig. 5., $B$ gave somewhat higher results than the other as a consequence of the lower value for $\delta_p$.

Unfortunately, insufficient experimental data limits the verification of the theoretical data. Vinke et al. [19] measured the amount of particles adhered to the bubble interface by flotation using demineralized water for their experiments. These are the only available experimental data for the $\alpha$ values. For the evaluation of the experiments they assumed a monolayer pack of particles on the surface. There are no data to show how the turbulence of the suspension or the physical mass transfer coefficient can affect the $\alpha$ values. Fig. 6 illustrates the experimental enhancements by Vinke et al. ([19], Fig. 8) obtained for a plane interface (with stirrer speed of $51/s$). The experiments were carried out with demineralized water (points * with $H = 945$) and with electrolyte solution (points x with $H = 915$). The theoretical data are represented by continuous lines (our model using Eqs. 3 to 5) and by dotted lines obtained by Demmink’s model [5]. An important factor is the covered portion of the gas-liquid interface. For this we used the measured data of Vinke et al. [19] and chose the following expression: $\alpha = \alpha_{\text{max}}6.13\gamma/[1+6.13\gamma]$ with $\alpha_{\text{max}} = 0.076$. The authors suggested this equation in Fig. 9 of their publication [19]. The mono-layer packing of the spheres used for the simulation gives very good agreement with the experimental data for the electrolyte solution. The theoretical data for the demineralized water are somewhat higher than that of the experimental. According to Eq. 5 the enhancement is practically proportional to the value of $\alpha$ at its lower values. Thus, the change in its values directly alters the value of the enhancement. To know its real value is of primary importance for a good prediction. The calculated data using Demmink’s model are somewhat lower than the experimental.

The next figure (Fig. 7) shows the verification results using the experimental data of Demmink et al. [5]. They measured the absorption of acetylene into carbon particle slurries at various $\gamma_s$ concentrations ($\gamma_s = 0.005$ to $0.6 \text{ kg/m}^3$), and physical mass transfer
coefficients \( \beta^o = 0.15 \times 10^{-4} \) to \( 1.1 \times 10^{-4} \) m/s). Fig. 7 shows the experimental data measured at \( \beta^o = 1.115 \times 10^{-4} \) m/s in the concentration range of the dispersed phase (see Table 4 in [5]). For prediction of the absorption rate, the portion of gas-liquid interface, \( \alpha \), was calculated as a function of \( \gamma_s \) according to the experimental results of Vinke et al. [19]. Taking into consideration the shape of \( \alpha_e \) vs. \( \gamma_s \) curves obtained by Vinke et al. [19], the following relationship has been used for \( \alpha \) where \( \alpha = \alpha_{max} 15 \gamma_s /(1 + 15 \gamma_s) \) with \( \alpha_{max} = 0.25 \). A higher factor of \( \gamma_s \) also results in higher values of \( \alpha \) at low values of \( \gamma_s \).

For prediction of enhancement, a multi-layer packing of adhering particles was assumed. The question is the number of the possible layers in the packing. Zon et al. [21] measured that the agglomeration of carbon particles and determined that the adhered packing at the gas-liquid interface might consist of up to 2-3 layers. The continuous lines in Fig. 7 represent the simulated results of a mono-layer \( (N = 1) \) and a triple-layer packing, \( (N = 3) \), using pack \( B \) with prismatic particles. Even the multi-layer packing simulation gives somewhat less enhancement than that measured experimentally. The predicted results, in the case of \( N = 3 \), are about 10-25 % less than the experimental. This difference is acceptable from an engineering point of view. These theoretical results are much closer to the experimental than that obtained by Demmink’s model. [5]. Demmink et al. needed a much higher distribution coefficient for the absorbed component, i.e. \( H = 400 \), to match the experimental data in their simulation. We applied \( H = 15 \), measured by Demmink et al. [5], for our simulation. This confirms that a multi-layer model can be very useful and realistic to describe this mass transfer process. Despite these good results, the question arises as to why the theoretical results are still somewhat lower than the experimental. This could possibly be explained by indicating such influences as the covered part of the interface, the number of layers in the multi-layer packing, or the shape of the particle maybe closer to a rectangular slab (prism) than that of a sphere, etc.

If one considers the latter’s influence on the absorption rate, where the adhered particles are prismatic particles that form a block consisting of multi-layer particles, accordingly, the distance between the adhered particles will be zero, that is \( \delta_p = 0 \). In Fig. 7 the predicted results using block packing at the gas-liquid interface are plotted with \( N = 4 \) (dotted line). Agreement between the predicted and experimental results is even much better than that represented by the continuous line at \( N = 3 \) with pack \( B \). These latter results, however, must be carefully treated, since we do not have any experimental proof as to whether or not a multi-layer packing, at the gas-liquid interface, could consist of four layers of adhered particles. Nor is there any evidence with regard to the shape of the particles.

A further possible effect of the non-linearity of the partition coefficient should also be noted as a property which can increase the mass transfer rate. Its value is probably linear for permanent gases as Li and Deckwer [8] obtained for oxygen. While for acetylene, non-linearity should occur according to the data of Li and Deckwer measured for \( \text{CO}_2 \) and ethylene. Assuming the same shape for the equilibrium curve for acetylene, then that obtained for ethylene and taking into account the theoretical data of Nagy ([13], Fig.3) as well as the relatively low \( H \) value, the enhancement can be estimated to be about 1.0 to
1.2 caused by non-linearity. This effect can only moderately modify the estimated data.

4 Conclusions

The heterogeneous, multi-layer mass transfer model proposed here seems to be suitable for prediction of the absorption rate in the presence of catalyst particles adhered to the gas-liquid interface. This model clearly shows how the particle layers behind each other in the packing at the gas-liquid interface can increase enhancement, depending on the particle size and the physical mass transfer coefficient. This effect greatly depends on the diffusion depth of the absorbed component, and consequently, on the mass transfer coefficient. The theoretical results obtained confirm that multi-layer packing of particles should be formed at the interface. The agreement between the predicted and the experimental data can essentially be improved using the multi-layer mass transfer model. Further experiments are needed to obtain more accurate data about the portion of interface covered by the particle, on the thickness of the multi-layer packing at the gas-liquid interface, and on the shape of the particles in order to get more accurate predictive data.

Acknowledgement

This work was supported by the Hungarian Research Foundation under Grant No. OTKA T 29272 and by the Research and Development Programmes for High Schools (FKFP) Grant No. 0313/2000.

Notation

$A^o$ concentration of the absorbed component in the bulk phase, mol/m$^3$

$A^*$ concentration of the absorbed component at the gas-liquid interface, at $x = 0$, mol/m$^3$

$d$ diffusion distance within particles (for a prism $d = d_p \sqrt{\pi/6}$), m

$d_p$ particle size, $d_p$

$D$ diffusion coefficient, m$^2$/s

$D_d$ diffusion coefficient in particles, m$^2$/s

$D_e = D/D_d$

$E$ enhancement of the gas absorption rate (given by Eq. 5), -

$H$ solubility (partition) coefficient of the adsorbed component between continuous phase and particles, -

$i$ $i$th layer in a multi-layer pack of particles

$j$ mass transfer rate for the heterogeneous part of interface, i.e. portion of the gas-liquid interface covered by adhered particles, mol/m$^2$s

$j^{\text{ave}}$ average value of $j$ in the case of spherical particle interface, mol/m$^2$s

$j(R^*)$ value of $j$ depending on $R^*$, mol/m$^2$s

$J$ mass transfer rate without dispersed phase, mol/m$^2$s
\( N \) number of the particle layer in multi-layer packing
\( p = H/(D_r)^{0.5} \)
\( R \) radius of a particle, m
\( R^* \) parameter varying between 0 and R (see Fig. 1), m
\( s \) surface renewal frequency, l/s

**Greek letters**

\( \alpha \) actual interface fractional coverage by adhering particles, -
\( \alpha_{max} \) maximum value of the covered part of the interface,-
\( \beta^o \) physical mass transfer coefficient, \((=\sqrt{D_s})\), m/s
\( \beta \) mass transfer coefficient in presence of particles, m/s
\( \delta \) thickness of the boundary layer or the penetration depth, m
\( \delta_1 \) distance of the particle from the gas-liquid interface (for a prism \( \delta_1 = 0.5d_p[1-d] \)), m
\( \delta_p \) distance between the prism particles (for pack A: \( \delta_p = 2\delta_1 \);
for Pack B: \( \delta_p = 2\delta_1 - 0.134d_p \), m
\( \gamma_s \) amount of the solid particle in the suspension. kg/m\(^3\)

**References**

[1] E. Alper, B. Wichtendahl, W.-D. Deckwer: “Gas absorption mechanism in catalytic slurry reactors”, *Chem., Engng. Sci.*, Vol. 35, (1980), pp. 217–222.
[2] E. Alper and W.-D. Deckwer: “Comments on gas absorption with catalytic reaction”, *Chem., Engng. Sci.*, Vol. 36, (1981), pp. 1097–1099.
[3] D.W.F. Brilman, M.J.V. Goldschmidt, G.F. Versteeg, W.P.M. van Swaaij: “Heterogeneous mass transfer models for gas absorption in multiphase systems”, *Chem., Engng. Sci.*, Vol. 55, (2000), pp. 2793–2812.
[4] P.W. Danckwerts: *Gas-Liquid Reactions*, McGraw-Hill, New York, 1970.
[5] J.P. Demmink, A. Mehra, A.A.C.M. Beenackers: “Gas absorption in the presence of particles showing interfacial affinity: case of fine sulphur precipitates”, *Chem. Engng. Sci.*, Vol. 53, (1998), pp. 2885–2902.
[6] R.D. Holstvoogd, W.P.M. van Swaaij, L.L. van Dierendonck: “The absorption of gases in aqueous activated carbon slurries enhanced by adsorbing on catalytic particles”, *Chem. Engng. Sci.*, Vol. 43, (1988), pp. 2181–2187.
[7] A. Karve and V.A. Juvekar: “Gas absorption into slurries containing fine catalyst particles”, *Chem. Engng. Sci.*, Vol. 45, (1990), pp. 587–594.
[8] Li, Zuohu and W.-D. Deckwer: “Adsorption von Gasen an suspendierter Aktivkohle”, *Chem.-Ing.-Tech.*, Vol. 54, (1982), pp. 669–671.
[9] C. Lin, M. Zhou, C.J. Xu,: “Axysimmetrical two-dimensional heterogeneous mass transfer model for the absorption of gas into liquid-liquid dispersion”, *Chem. Engng. Sci.*, Vol. 54, (1999), pp. 389–399.
[10] A. Mehra: “Heterogeneous modelling of gas absorption in emulsions”, *Ind. Eng. Chem. Res.*, Vol. 38, (1999), pp. 2460–2468.

[11] E. Nagy: “Three-phase mass transfer: one-dimensional heterogeneous model”, *Chem. Engng. Sci.*, Vol. 50, (1995), pp. 827–836.

[12] E. Nagy: “Heterogeneous model for a number of particles in the diffusion path”, *Hung. J. Industrial Chemistry*, Vol. 26, (1998), pp. 229–240.

[13] E. Nagy: “Three-phase mass transfer with non-linear equilibrium”, *Chem. Engng. J.*, Vol. 72, (1999), pp. 43–51.

[14] E. Nagy and A. Moster: “Three-phase mass transfer: improved pseudo-homogeneous model”, *A.I.Ch.E.J.*, Vol. 41, (1995), pp. 23–34.

[15] E. Nagy and A. Ujhidy: “Model of the effect of chemical reaction on bulk-phase concentration”, *A.I.Ch.E.J.*, Vol. 35, (1989), pp. 1564–1568.

[16] E. Nagy: “Three-phase oxygen absorption and its effect on fermentation”, *Advances in Biochemical Engineering/Biotechnology*, Vol. 75, (2002), pp. 51–81.

[17] H. Yagi and H. Hikita: “Gas absorption into a slurry accompanied by chemical reaction with solute from sparingly soluble particles”, *Chem. Engng J.*, Vol. 36, (1987), pp. 169–174.

[18] H.L. Toor and J. Marcello: “Film-penetration model mass and heat transfer”, *A.I.Ch.E.J.*, Vol. 4, (1958), pp. 97.

[19] H. Vinke, P.J. Hamersma, J.M.H. Fortuin: “Enhancement of the gas-absorption rate in agitated slurry reactors by gas-adsorbing particles adhering to gas bubbles”, *Chem. Engng. Sci.*, Vol. 48, (1993), pp. 2197–2210.

[20] H. Vinke, G. Bierman, P.J. Hamersma, J.M.H. Fortuin: “Adhesion of small catalyst particles to gas bubbles: determination of small effective solid-liquid-gas contact angles”, *Chem. Engng. Sci.*, Vol. 46, (1991), pp. 2497–2506.

[21] M. van der Zon, P.J. Hamersma, E.K. Poels, A. Blick: “Gas-solid adhesion and solid-solid agglomeration of carbon supported catalysts in three phase reactors”, *Catalysis Today*, Vol. 48, (1999), pp. 131–138.
Fig. 1 Geometrical approximations of multi-layer packings of particles at the gas-liquid interface.
Fig. 2 The effect of the particle size on enhancement using various models ($\beta^0 = 1 \times 10^{-4} \text{ m/s}$, $D = 1.4 \times 10^{-9} \text{ m}^2/\text{s}$, $H = 20$, $D_r = 1$, $N = 1$, $\alpha = 1$, volume of a prismatic particle = volume of a sphere; length of prism = $d_p$).
Fig. 3 Enhancement as a function of the particle size at the different number of layers of a multi-layer packing at the gas-liquid interface ($\beta^o = 1 \times 10^{-4}$ m/s, $D = 1.4 \times 10^{-9}$ m$^2$/s, $H = 20$, $D_r = 1$, $N = 1$, $\alpha = 1$; pack $A$ and $B$ are built by prismatic particles).
Fig. 4 Enhancement as a function of the physical mass transfer coefficient (that without particles in the continuous phase) using mono-layer packing of particles ($d_p = 2.6 \times 10^{-6}$ m, $D = 1.4 \times 10^{-9}$ m$^2$/s, $H = 20$, $D_r = 1$, $N = 1$, $\alpha = 1$).
Fig. 5 Enhancement as a function of the physical mass transfer coefficient (that without particles in the continuous phase) using mono- and different multi-layer packing of particles ($d_p = 2.6 \times 10^{-6}$ m, $D = 1.4 \times 10^{-9}$ m²/s, $H = 20$, $D_r = 1$, $\alpha = 1$).
Fig. 6 Verification of experimental data ($x_*,*$) of Vinke et al. ([19], in Table 8 at stirrer speed of 5 l/s; data for demineralized water: $*: \beta_o = 1.66 \times 10^{-4}$ m/s, $D = 3.6 \times 10^{-9}$ m$^2$/s, $H = 945$, $d_p = 6 \times 10^{-6}$ m, $D_r = 1$; as well as for the electrolyte solution: $x$: $\beta_o = 1.3 \times 10^{-4}$ m/s, $D = 3.6 \times 10^{-9}$ m$^2$/s, $H = 915$, $d_p = 6 \times 10^{-6}$ m, $D_r = 1$: with $\alpha = \alpha_{max} 6.13 \gamma /[1+6.13 \gamma]$, and $\alpha_{max} = 0.076$; $N=1$).
Fig. 7 The predicted enhancements by pack B (continuous lines) and with a block of prismatic particles without distance between them ($\delta_p = 0$; dotted line) as well as several experimental data ($x$ : [5]; $D = 1.4 \times 10^{-9}$ m$^2$/s, $H = 15$, $D_r = 1$, $d_p = 2.6 \times 10^{-6}$ m; with $\alpha = \alpha_{\text{max}} 15\gamma_s / [1 + 15\gamma_s]$ and $\alpha_{\text{max}} = 0.25$).