Nonlinear superluminality in cold atoms confined in hollow core photonic crystal fiber

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Abstract. We propose a scheme for superluminal propagation of a linearly polarized smooth light pulse in cold atoms trapped inside a hollow-core fiber with intensity-dependent negative group velocity. The tight confinement of the atoms inside the fiber core prevents atom-wall collisions, while the Doppler width is negligible as compared to atomic decay rate. We show that with initially prepared Zeeman coherence, the absorption of light is smaller enough to be ignored, while due to nonlinear dependence on laser intensity more intense parts of the pulse travel slower leading to the formation of the extremely sharp trailing edge and to the lengthening of the leading edge of the pulse. Such a behavior is contrast to the case of nonlinear propagation of the pulse in a medium with normal dispersion where a shock wave is generated. It is highly important that with $10^4$ atoms inside the fiber the large nonlinearities in the medium refractive index can be achieved at extremely low light levels.

1. Introduction

Propagation of light pulses through the media with a normal dispersion is adequately described by the group velocity, which in this case is the velocity of signal transfer. In the region of anomalous dispersion, for example near an absorption line or nonlinear gain [1], the group velocity of light becomes faster than vacuum speed of light $c$ or even negative, thus losing its role of signal velocity. However, it has been shown in Ref. [2] that the group velocity is a useful mathematical concept to describe the propagation of sufficiently smooths pulses, such as Gaussian wave packets, in media with anomalous dispersion. Since then a number of theoretical [3-8] and experimental [9-11] works have been published reporting superluminal propagation of a probe field in different schemes of anomalous dispersion in gaseous and solid media.

The results of these papers are based on linear response of the medium. Similar results have been obtained, when a weak probe beam propagates with an effective index of refraction induced by a nonlinear interaction of the medium with a strong pump field. Under these conditions both the probe light ultraslow propagation [12-14] at the electromagnetically induced transparency (EIT) [15] and the negative “group-delay” propagation associated with electromagnetically induced opacity [16, 17] have been recently observed experimentally. However, from the point of view of the dependence of medium optical properties on probe field intensity, these systems may be clearly considered as effectively linear optical media with an extraordinary large dispersion.

Intriguing questions arise whether the nonlinear superluminality occurs in a medium, the refractive index of which is modified by the propagating light pulse itself. Here the main requirement is to observe the pulse superluminal propagation in a medium with an intensity-dependent refractive index, but without accompanying other nonlinear processes, which lead to pulse reshaping as well. The simplest system displaying these properties is a three-level Λ atom with ground-state Zeeman sublevels, which interacts with the right (RCP) and left (LCP) circularly polarized components of the optical light pulse in the presence of weak magnetic field applied along the direction of light-pulse propagation (Fig1). This system has been first analyzed for room-temperature Rb vapor [18] and it has been shown that the relaxations due to Doppler broadening and the collisions between warm alkali-
metal atoms impose severe limitations on the field intensity, which represents a fundamental obstacle for observation of the proposed effect. In this paper, we propose to use hollow core photonic-crystal fibers [19], where these obstacles are overcome. The advantage of hollow core fibers is that the strong confinement of cold atoms prevents atom-wall collisions inside the fiber core, while the Doppler broadening is negligibly small as compared to the linewidth of atomic transitions, so that the effect of the nonlinear superluminality can be observed for small probe fields thus making the hollow core scheme feasible for experimental realization.

2. Basic equations
We consider a near-resonant interaction of a linearly polarized probe field $E$ of frequency $\omega$ and wave vector $k$ parallel to the static longitudinal magnetic field $B$. Its two circularly left- and right-polarized components $E_{1,2}$ act on the atomic transitions $1 \rightarrow 3$ and $2 \rightarrow 3$, with the detuning $\Delta_{1,2} = \omega - \omega_0 \mp \Delta$, where $\omega_0$ is the frequency of the unshifted atomic resonance and $\Delta = \mu_0 B/\hbar$ is the Zeeman shift, which is induced by the magnetic field for the ground-state sublevels 2 and 1 with magnetic quantum numbers $\pm 1$. Here $g$ is the ground-state gyromagnetic factor and $\mu_0$ is Bohr’s magneton.

Due to the nearly maximal Zeeman coherence induced by the field, large nonlinear susceptibilities are created at the resonance frequency, analogous to those observed in EIT experiments (for relationship between EIT and nonlinear magneto-optics see Ref. [14]). The initial preparation of Zeeman coherence is necessary to avoid the pulse distortion due to the energy loss needed to initiate the EIT. The Zeeman coherence suppresses the absorption of the medium, while it creates a highly anomalous and intensity-dependent dispersion in the vicinity of the resonance line center. This results in a negative nonlinear group velocity at the values of light intensity, which are many times smaller than the saturation intensity of the corresponding atomic transitions.

The Hamiltonian of the system is written as

$$H_{\text{int}} = \hbar \Delta_1 \sigma_{11} + \hbar \Delta_2 \sigma_{22} - \hbar \left( \Omega_1 \sigma_{31} + \Omega_2 \sigma_{32} + H.c. \right)$$

(1)

Here $\sigma_{ij}=|i><j|$ are atomic operators. The Rabi frequencies of field polarization components are defined as $\Omega_{1,2} = \mu E_{1,2}/\hbar$, where the dipole electrical moments of the transitions $|1> \rightarrow |3>$ and $|2> \rightarrow |3>$ are taken equal: $\mu_{13} = \mu_{23} = \mu$.

Figure 1. Open Λ configuration for studying the nonlinear superluminal propagation.
\[ \frac{\partial}{\partial t} \rho = -\frac{i}{\hbar} [H_{\text{int}}, \rho] + \Lambda \rho \quad (2) \]

The matrix \( \Lambda \rho \) accounts for all atomic relaxations

\[ \Lambda \rho = \begin{pmatrix} \gamma \rho_{33} & -\gamma \rho_{32} & -\Gamma \rho_{33} \\ -\gamma \rho_{23} & \gamma \rho_{33} & -\Gamma \rho_{23} \\ -\Gamma \rho_{31} & -\Gamma \rho_{32} & -(\gamma_0 + 2\gamma) \rho_{33} \end{pmatrix} \quad (3) \]

where \( \gamma \) and \( \gamma_0 \) are spontaneous decay rates of the upper state 3 on the transitions \(|1> \rightarrow |3>\), \(|2> \rightarrow |3>\) and \(|0> \rightarrow |3>\), respectively, \( \gamma_c \) is a ground-state decoherence rate and \( \Gamma = (2\gamma + \gamma_0)/2 \).

The evolution of the slowly varying field amplitudes \( E_{1,2} \) along the \( z \) axis is determined by Maxwell’s equations

\[ \left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) E_{1,2}(z,t) = 2\pi i \frac{\omega}{c} P_{1,2}(z,t) \quad (4) \]

where \( P_{1,2}(z,t) \) are the field induced polarizations

\[ P_i(z,t) = -N \left( \frac{\partial H_{\text{int}}}{\partial E_i} \right) = N \mu \rho_{3i} \quad (5) \]

with \( N \) the atomic number density. As has been shown in [18], \( P_i(z,t) \) is expressed through field amplitudes \( E_i(z,t) \) via Taylor series

\[ P_i(z,t) = \chi_i(\omega, z) E_i(z,t) + i \frac{\partial \chi_i}{\partial \omega} \frac{\partial E_i}{\partial t} - \frac{1}{2} \frac{\partial^2 \chi_i}{\partial \omega^2} \left( \frac{\partial E_i}{\partial t} \right)^2 + \ldots \quad (6) \]

where

\[ \chi_i(\omega, z) = N \mu^2 \rho_{3i} / \hbar \Omega_i \quad (7) \]

The third term in Eq.(6) describes the group velocity dispersion, which is responsible for the pulse spreading during its propagation through the medium. In what follows we neglect the pulse distortion due to this and higher dispersion terms in order to observe the pure pulse self-steepening without pulse spreading. This approximation constrains the sample length to a value for which the pulse broadening caused by the self-steepening does not exceed the medium absorption line width \( \Gamma \), more detailed discussion of this point can be found in [18], where it is shown that even in the presence of Doppler broadening the upper bound of sample length, for reasonable values of the parameters, is few hundreds of cm, while in our case \( L \) does not exceed several centimeters.

Since in this work we restrict our attention to the superluminal propagation of the pulse, so we will consider equation only for the dimensionless intensities defined as: \( I_i = \Omega_i^2 / \Gamma \Delta \). By substituting Eqs.(5-7) into Eq.(4), we obtain the following equation for the intensities.

\[ \left( \frac{\partial}{\partial z} + \frac{1}{\nu_i} \frac{\partial}{\partial t} \right) I_i(z,t) = -\alpha_i I_i(z,t) \quad (8) \]
where the nonlinear absorption coefficients $\alpha_i$ and group velocities $v_i$ are given by

$$\alpha_i = \frac{4\pi\omega}{c} \text{Im} \chi_i$$

(9)

$$v_i = \frac{c}{n_{gi}} = \frac{c}{c} \left[ 1 + 2\pi\omega \text{Re} \frac{\partial \chi}{\partial \omega} \right]^{-1}$$

(10)

Below we omit the subscripts 1 and 2, as the absorption and group velocities of the two components are the same.

We solve Eq. (8) assuming that the initial laser beam is a Gaussian pulse with duration $T$

$$I(0,t) = I_0 \exp \left[ \left( -t/T \right)^2 \right]$$

(11)

We suppose that the incident pulse does not lose energy for preparation of EIT and its reshaping in the medium is caused solely by the nonlinear dispersion. Besides, as has been mentioned above, the tight confinement of cold atoms in hollow core fiber prevents atom-wall collisions and, hence the ground states decoherence rate $\gamma_c$ is taken zero. Then the steady-state solution of Eq.(2) is given as

$$\text{Im} \frac{\rho_{10}}{\Omega_1} = \Gamma \Delta^2 \delta \rho / D$$

$$\text{Re} \frac{\rho_{11}}{\Omega_1} = -\Delta (\varepsilon \Delta + \Omega^2) \delta \rho / D$$

$$\text{Re} \frac{\rho_{22}}{\Omega_2} = -\Delta (\varepsilon \Delta - \Omega^2) \delta \rho / D$$

$$D = \Delta^2 \left( \varepsilon^2 + \Gamma^2 \right) + \Omega^2$$

(12)

Where $\Omega^2 = |\Omega_1|^2 = |\Omega_2|^2$. This solution is obtained under the assumptions $\Delta << \Gamma$ and $\Gamma^2 >> \Omega_0^2$. We suppose that the ground atomic states are equally populated and are not altered during the interaction with the field, so that the population difference $\delta \rho$ between the ground Zeeman sublevels and the upper state $|3\rangle$ is equal 1/3.

From the Eqs(9), (10) and (12) for the group index and the absorption coefficient we obtain

$$n_g = 1 - \frac{2\pi\omega N \mu^2 \delta \rho}{\hbar \Gamma^2} \left( \frac{1}{1 + I^2} \right)$$

(13)

$$\alpha = \frac{4\pi\omega N \mu^2 \delta \rho}{\hbar c \Gamma} \left( \frac{1}{1 + I^2} \right)$$

(14)

In our simulations for the model of the atomic system we have chosen parameters corresponding to the D1 absorption line of $^{87}\text{Rb}$. The total decay rate of the upper level is $\Gamma = \pi \times 5 \times 10^6$ s$^{-1}$, the wavelength of optical transitions $\lambda = 800$ nm, for the hollow core fiber length $L = 3$ cm, diameter $\sim 10 \mu m$ and number of atoms $\sim 10^4$ the atomic density is $\sim 10^9$ cm$^{-3}$. For these parameters, $\alpha L < 0.3$ at $I = 0$, so that the absorption is negligible, while the group index $n_g \sim 40/(1+I^2)$ ensuring sensitive dependence of group velocity on
light intensity. The external magnetic field is taken $B=30\text{mG}$, which corresponds to $\Delta=0.002\Gamma$ for the ground state of rubidium.

A negative velocity can be understood by comparing the times it would take for identical pulses of light to cover the distance $L$ in a vacuum (traveling at velocity $c$) and in a superluminal medium (traveling at velocity $v$). The difference in transit times $\Delta T = L/v_g - L/c$ is a negative quantity, if the velocity is superluminal. If $v_g$ has a negative value, then $\Delta T$ can become sufficiently negative that the peak of the pulse emerges from the medium at an instant earlier than when the peak of the pulse enters. Moreover, with nonlinear dependence of $v_g$ on laser intensity, as is the case here, the more intense parts of the pulse travel slower leading to controllable shaping of output pulse.

Fig.2 shows the calculated output pulse (red) traversed through a distance $z/c\Gamma\sim0.2$ in the rubidium vapor. For comparison we depict also the input Gaussian pulse (in blue), which propagates the same distance in vacuum with light speed $c$. It is seen that nonlinear propagation in an anomalous dispersive medium leads to the strong self-steepening of trailing edge of the pulse.

![Figure 2](image_url)

**Figure 2.** The output pulse (red) as a function of time (in units of $\Gamma^{-1}$) at the distance $z/(c\Gamma^{-1})=0.7$ for $I_0=5$ corresponding to the intensity $\sim0.2\text{mW/cm}^2$. The input pulse propagating at speed $c$ through the same distance in vacuum is shown in blue.

Meanwhile, the parameters for numerical simulation are chosen such that the group velocity at maximal intensity is $c$, so that the peak points of real and unperturbed pulses coincide.

### 3. Summary

In this paper we have analyzed the superluminal propagation of a light pulse in a nonlinear medium and showed that necessary conditions for these effects to be observable are realized in a three-level $\lambda$-system interacting with a linearly polarized laser beam in the presence of a static magnetic field. We have demonstrated that similar to the linear case the group velocity for smooth pulses is physically meaningful even when it is negative. But, it cannot be interpreted as a velocity of real signal, which could propagate no faster than light without violating the Einstein causality [21]. It is highly important that in our case the nonlinearity of the refractive index of the medium arises in a very low power regime, when all other nonlinear processes are negligible. We have shown that this regime is realized in cold atoms confined in hollow-core photonic crystal fiber, where a light pulse experiences a strong pulse shaping with formation of an extremely sharp trailing edge and lengthening of the leading edge of the pulse. Such a behavior is opposite to the case of nonlinear propagation of the pulse in a medium with normal dispersion [21], where a shock wave is generated.
Acknowledgments

This research has been conducted in the scope of the International Associated Laboratory (CNRS-France SCS-Armenia) IRMAS. We also acknowledge the support from the Scientific Research Foundation of the Government of the Republic of Armenia.

References

[1] Marangos J 2000 Nature 406 243-244.
[2] Garrett C G B and Mc Cumber D E 1970 Phys. Rev. A 1 305.
[3] Chu S and Wong S 1982 Phys. Rev. Lett. 48 738
[4] Segard B and Macke B 1985 Phys. Rev. Lett. A 109 213
[5] Steinberg A M, Kwiat P G and Chiao R Y 1993 Phys. Rev. Lett. 71 708
[6] Steinberg A M and Chiao R Y 1994 Phys. Rev. A 49 2071
[7] Bolda E L, Garrison J C and Chiao R Y 1994 Phys. Rev. A 49 2938
[8] Dogariu A, Kuzmich A and Wang L J 2001 Phys. Rev. A 63 053806
[9] Wang L J, Kuzmich A and Dogariu A 2000 Nature (London) 406 277
[10] Bigelow M S, Lepeshkin N N, Boyd R W 2003 Science 301 200
[11] Gehring G M et al, 2006 Science 312 895
[12] Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 Nature London 397 594
[13] Kash M M et al. 1999 Phys. Rev. Lett. 82 5229
[14] Budker D, Kimball D F, Pochester S M and Yashchuk V V 1999 Phys. Rev. Lett. 83 1767
[15] Harris S E 1997 Phys. Today 50 (7) 36
[16] Akulshin A M, Barreiro S and Lezama A 1998 Phys. Rev. A 57 2996
[17] Lezama A, Barreiro S and Akulshin A M 1999 Phys. Rev. A 59 4732
[18] Gulghazaryan R and Malakyan Yu 2003 Phys. Rev. A 67 063806
[19] Russel P St J 2006 J.Lightwave Technol. 24, 479
[20] Budker D, Kimball D F, Pochester S M, and Yashchuk V V. 1999 Phys. Rev. Lett. 83, 1767
[21] Brillouin L, Wave Propagation and Group Velocity, Academic 1960, New York
[22] Grischkowsky D, Courtens E, and Armstrong J A 1973 Phys. Rev.Lett. 31 422