A unified Scenario for modeling the Galactic and Cosmological Dark Matter Components

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In this work we analyze the viability of use a particular models of scalar fields in the context of the galactic dark matter problem. These models are based on a single scalar field, minimally coupled to the gravity in a asymptotically flat or asymptotically de Sitter spacetime. We discuss the opening possibility of constructing a unified model for both the cosmological and the galactic dark matter.

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I. INTRODUCTION

The problem of the galactic dark matter lies in the fact that the rotational curves of the spiral galaxies (away from the galactic center) are flat while the expected behavior is a keplerian decay, and that the total gravitational pull inferred, indicates an amount of gravitating matter which exceeds by almost an order of magnitude the luminous matter within the galaxy. A second dark matter problem is the cosmological dark matter problem, the fact that the data about the cosmological evolution seems to require the mean density of the universe to be quite close to the critical density while ordinary matter can only account for a small fraction thereof [1]. The evidence in this regard comes from recent observation of distant type Ia supernovas [2], and from the latest data about the CMB anisotropies [3], all of which is in accordance with the standard inflationary prediction that $\Omega_{\text{total}} = 1$ [4]. The latter problem has been considered as a completely separated issue and has lead even to its remaining as the dark energy problem. The approach to this has centered in the introduction of a new type of scalar field dubbed the “Quintessence”.

We will briefly consider the possibility of connecting the two problems through scenarios that hope to treat both simultaneously. Regarding the galactic dark matter problem two kinds of models are usually considered: The first one consists in exotic dark matter in the form of very weakly interacting particles, while the second one consists in classical field configurations. The former enjoy a widespread popularity, specially amongst astronomers who are used to regard particles as the natural constituents of almost all kinds of matter, while many physicist are more attracted by the second type due to the fact that the particle concept is regarded as secondary, and available only under certain conditions [5]. However we know that both the classical field and the particle models have a fundamental underlying quantum field model, and whether the field or the particle descriptions are the most appropriate depends on the circumstances. In fact particles and classical fields represent two distinct classical limits of a quantum field.

It is well known that the two descriptions are rather analogous to the momentum and position descriptions of elementary quantum mechanics as the two relevant operators do not commute. In fact if we consider a scalar field $\hat{\phi}(x) = \sum_s \{a_s e^{-ik \cdot x} + a_s^\dagger e^{ik \cdot x}\}$ and $\hat{N} = \sum_r a_r^\dagger a_r$ the total number operator for the corresponding particles, we easily see that they do not commute: $[\hat{N}, \hat{\phi}(x)] = \sum_r \{a_r^\dagger e^{ik \cdot x} - a_r e^{-ik \cdot x}\} \neq 0$. Thus when considering the first types of models, astronomers have to introduce not only the explicit—and not quite self evident—hypothesis regarding the evolution of the gas of dark matter particles, but also, implicitly a series of assumptions that are often inadvertently made regarding the mechanisms by which the dark matter field underlying the dark matter came to be in the type of incoherent configuration that justifies the particle description.

Regarding the second type of models, we must point out that a major difficulty that has to be overcome is the increasing evidence regarding the presence, in most galaxies, of large central black holes. A model based on fields must therefore, be able to represent a static stable configuration of the fields, with no sources (to avoid even more assumptions about additional new types of matter that would represent such sources, given the fact that there are very stringent limits—associated with test of the equivalence principle—on ordinary matter couplings to long range fields [6]) and it must admit black holes at the center of the galaxies in order to be a viable model of the galactic dark matter.

In the single scalar field arena, it had not been considered possible to obtain models with black holes and static

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stable configurations of scalar fields, due to the no-hair theorems that ensure the nonexistence of such configurations \[7\]. Even the so-called boson star configurations cease to exist when black holes are introduced \[8\].

Recently, however, scalar hairy solutions in asymptotically Anti de-Sitter spacetimes have been found \[9\]. In a recent analysis \[10\], we proposed the existence of solutions of this kind in asymptotically flat regime, which later found in \[11\]. Such solutions escape the no-hair theorems due to the fact the energy conditions considered in the theorems do not hold in the corresponding models. In this essay we consider the question of whether such type of models could be a good candidate for the unified treatment of the clustered dark matter and the cosmological dark energy problems. At this point we should mention previous attempts to construct such unifying scenarios, which are generically dubbed “Quartessence” models \[12\]. The first kind is based on a purely phenomenological fluid model known as the “Chaplygin Gas” which is capable of exhibiting under different conditions the types of equations of state associated with both the dark energy and dark matter. The second approach exemplified by the works \[13\] deals with a string inspired tachyonic field, again capable of exhibiting the two different types of equations of state required, but which as far as we know, have not dealt at all with the serious issue that a black hole at the galactic centers would pose for the existence of standard scalar field configurations as mentioned above.

In \[II\] we describe the model and in \[III\] we discuss its viability as well as some of the aspects that need to be further explored.

II. THE MODEL

We will consider a model of a scalar field minimally coupled to gravity and with a potential having the generic features shown in Fig. \[I\]. The model is thus based on the following Lagrangian:

\[
\mathcal{L} = \sqrt{-g} \left[ \frac{1}{16\pi} R - \frac{1}{2} \nabla_{\alpha} \phi \nabla^{\alpha} \phi - V(\phi) \right].
\]  

(2.1)

An example of the kind of potential requires is given by:

\[
V(\phi) = V_0((\phi - s)^2 - a)e^{-\beta \phi},
\]  

(2.2)

where \(V_0, a, s\) and \(\beta\) are constants.

The gravitational and scalar field equations following from the Lagrangian \[2.1\] can be written as

\[
G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad \Box \phi = \frac{\partial V(\phi)}{\partial \phi},
\]  

(2.3)

where
\[ T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left[ \frac{1}{2} \nabla_\alpha \phi \nabla^\alpha \phi + V(\phi) \right] \] (2.4)

When considering the galactic dark matter we focus on the static spherically symmetric metric parametrized as

\[ ds^2 = -e^{2\delta} \mu dt^2 + \mu^{-1} dr^2 + r^2 d\Omega^2 , \] (2.5)

Where \( \mu(r) = 1 - \frac{2m(r)}{r} \). Thus Einstein’s equations take the form:

\[ \partial_r m = 4\pi r^2 \left[ \frac{\mu(\partial_r \phi)^2}{2} + V(\phi) \right] , \quad \partial_r \delta = 4\pi r (\partial_r \phi)^2 , \] (2.6)

and the scalar field equation can be written as

\[ \partial_r^2 \phi = - \left[ \frac{2}{r} + 2 \left( \frac{m}{r^2} - 4\pi r V(\phi) \right) \right] \partial_r \phi + \frac{1}{\mu} \frac{\partial V(\phi)}{\partial \phi} , \] (2.7)

this type of potentials are known to lead, as all ready mentioned to non trivial field configurations containing central black holes in the asymptotically flat and asymptotically Anti de Sitter case and our heuristic analysis indicates that the solutions should also exist in the asymptotically de Sitter case (when the asymptotic value of the potential is positive). Moreover a similar solution can be expected when the scalar field at infinity is not exactly in the false vacuum but is close enough, including a situation where in the asymptotic region it is slowly rolling down towards that value!

In the cosmological case one focus on a \( k = 0 \) Robertson Walker 4 spacetime

\[ ds^2 = -dt^2 + a^2 (dr^2 + r^2 d\Omega^2) , \] (2.8)

that Einstein’s equations take the well known form

\[ \left( \frac{\partial a}{a} \right)^2 = \frac{4\pi}{3} \left[ (\partial_\phi)^2 + 2V(\phi) \right] , \quad \frac{\partial^2 a}{a} = - \frac{8\pi}{3} (\partial_\phi)^2 - V(\phi) \] . (2.9)

Thus in this regard we could be essentially in the so called quintessence models based on a scalar field slowly rolling down to a minima of a potential (the one corresponding to \( \phi = +\infty \)). Recall that in these schemes the scalar field is spatially homogeneous and it is evolving in time, rolling down ever slowly under the combined influence of the cosmic expansion and its self interaction potential because the potential can considered as an exponentially decaying one such as \( V_{\text{Quintessence}} = V_0 e^{-\phi} \). The unifying scenario we are proposing is then the following: At early enough times in cosmic history, when the universe was hot enough, the scalar field would be in its false vacuum \( \phi = 0 \) (as in the scenarios of spontaneous symmetry breaking) and as the universe cools the scalar field will start to move to the minima of the potential. In those regions where the field started to move towards the negative values we would have formation of solitons and eventually black holes [14] that would account for the early structure formation leading to the attraction of barionic matter after its decoupling from radiation, and to the present day galaxies, while those regions where the field started to move towards the positive values would evolve into the intergalactic regions where the cosmic expansion of the quintessence type would take place. The equality in the order of magnitude of the contributions to the total energy density of the universe in the form of clumped “dark matter”, and the essentially homogeneously distributed “dark energy”, would be a result of the approximate symmetry between positive and negative valued fluctuations of the scalar field about its initial mean value \( \phi = 0 \).

### III. DISCUSSION

We have considered the possible use of single scalar field model for the galactic dark matter which are consistent with the existence of large black holes in the center of the galaxies, as well as the cosmological dark energy components of the universe. The reluctance of considering field models in the first context should be tempered by the observation
that magnetic fields are unquestionable components (of relative small quantitative value) of the galactic matter content, and the only fundamental reason behind the difference in the way they are considered seems to be related to familiarity, a criteria that can hardly be justified when dealing with the nature of the dark matter. We have argued that in fact the field models should be considered superior from the point of view of number of hypothesis involved (in the spirit of Occam’s razor), to the standard models base on gases of very weakly interacting particles, due to the large number of unmentioned underlying assumptions that are involved in the latter. We have seen that single scalar field models open the possibility of regarding the dark matter (the clumped component) and the dark energy (the cosmological component) within a unified context. Most astronomers tend to regard this two problem as completely different issues, and even to apply very different criteria of naturalness when considering the two. However we must also point out the estimates of the contribution of both types of matter to the total cosmic mean density are of the same order of magnitude, making a unified approach all the more desirable. Moreover we would have at the same time a new approach to understanding the origin of the super massive black hole at the galactic centers. There are of course further issues to investigate, such as the precise form of the potential of the model that would account for the different rotation curves in the different galaxies, the correlations between the rotation curves and the central black holes (15), (we must mention that the introduction of a non minimal coupling for the scalar field holds some promise regarding these last two issues as has been argued in a study of such questions in a different approach (16) the interaction between the configurations associated with different galaxies, the behavior of these solutions under the effects general cosmic expansion, and in fact their cosmic evolution starting from the primordial density perturbations. In many of these regards the scenario looks very promising at this very early point, however it is clear that all of them should be the focus of further research.

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