Friction Coefficient Inversion Calculation Based on Quasi-newton Method and Particle Swarm Optimization

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Abstract. Establishing the pipeline network model of oilfield water injection under known partial pressure through fuzzy clustering method. In order to solve the inversion of pipe element friction coefficient in the case that the joint pressure of water injection pipe network is partly known, the simulation mathematical model of network node pressure and the optimal mathematical model of pipe element friction coefficient inversion are established respectively. By combining the simulation of nodal pressure by quasi-newton method and the inversion of friction coefficient by particle swarm optimizati on, the ideal pipe element friction coefficient of the pipe network with the partly unknown part of the node pressure can be inverted to achieve the partial accurate solution and partial estimation solution of the pipe element friction coefficient.

1. Introduction

With the development of domestic oil fields entering the middle and late stage, water injection has become the main extraction mode of oil fields at the present stage. It effectively replenishes the energy of the formation and plays an active role in ensuring high and stable production and enhancing oil recovery. At the same time, it is also a big energy user for oil fields, accounting for 30-40% or more of the total electricity consumption of oil fields. Therefore, it is necessary to carry out optimal dispatch calculation to reduce water injection consumption.

The optimal dispatching of water injection system is based on the modeling and simulation of water injection piping system. Whether the pipe network model data is consistent with the actual data is related to the correctness of simulation calculation and optimal dispatching calculation results. However, after decades of water injection extraction, corrosion, precipitation, scale formation and other phenomena have occurred in the buried water injection pipeline in the oilfield, which increases the actual friction coefficient of the pipeline. But the hydraulic calculation of the pipeline network still adopts the original friction coefficient, leading to the inconsistency between the calculated results and the measured results [1]. The inverse solution of friction coefficient of water injection pipeline network is to find out the actual friction coefficient (or equivalent friction coefficient) of pipe section through appropriate algorithm processing under the condition that the test pressure of some pipe network nodes is known. In essence, it is a problem to correct the structural parameters of large fluid network system. The actual structural parameters of pipe network can be estimated through inversion calculation, and
then these structural parameters which are close to the real condition can be put into the hydraulic calculation of pipe network to get the calculation results which are in line with the reality.

2. Hydraulic Model of Water Injection Pipe Network in Oilfield

The establishment of hydraulic calculation model of oilfield water injection pipe network is mainly based on the continuity equation of nodes:

\[ U_j - Q_j - \sum_{k \in I_j} q_{jk} = 0 \quad j = 1, 2, ..., n - 1 \]  

Type: \( U_j \) is the water supply when the node \( j \) is the water source; \( Q_j \) is the injection amount of the node \( j \); \( I_j \) is the set of all nodes adjacent to the node \( j \); \( q_{jk} \) is the flow rate of the pipe segment connected to the node \( j \) from the node \( k \); \( n \) is the total number of pipe network nodes; \( n \) is the total number of pipe network nodes; \( q_{jk} \) is the flow rate of the pipe segment connected to the node \( j \).

Considering the relationship between node flow rate and pressure drop of pipe section in the pipe network unit model, equation (1) can be converted into equation (2):

\[ U_j - Q_j - \sum_{k \in I_j} \phi (L_i, d_i, s_i, H_j - H_k, H_k - H_i) = U_j - Q_j - \sum_{k \in I_j} K^i (H_j - H_i) = 0 \]  

Type: \( \phi \) is a complex function composed of pipe length, pipe diameter, friction coefficient and pressure drop. \( L_i \) is the length of the pipe segment \( i \); \( d_i \) is the diameter of the pipe segment \( i \); \( s_i \) is the friction coefficient of the pipe segment \( i \); \( H_j - H_k \) is the pressure drop of the pipe segment between node \( j \) and node \( k \), the pipe segments between node \( j \) and node \( k \) are pipe segment \( i \); Let's substitute \( K^i \) for \( \phi \). \( K^i \) is a complex function related to the pressure drop, length, friction coefficient and diameter of pipe segment \( i \).

Set up a large complex oilfield water injection pipeline network with \( n \) nodes and \( m \) pipe sections. \( s_i \) is the friction coefficient of the pipe segment \( i \), \( H_j \) is the pressure of the node \( j \), \( d_i \) is the diameter of the pipe segment \( i \), \( L_i \) is the length of the pipe segment \( i \), and \( Q_j \) is the measured flow rate of the node \( j \); 

\[ s = (s_1, s_2, ..., s_m)^T, s \text{ is the friction coefficient vector; } H = (H_1, H_2, ..., H_n)^T, H \text{ is the node pressure vector; } \\ d = (d_1, d_2, ..., d_m)^T, d \text{ is the diameter vector of pipe segment; } \\ L = (L_1, L_2, ..., L_m)^T, L \text{ is the length vector of pipe section; } Q = (Q_1, Q_2, ..., Q_n)^T, Q \text{ is the node flow vector.} \]

For this pipe network, the continuity equation of \( n \) nodes can be established. Among them, the equation of any node can be expressed by the other \( n - 1 \) node equations, so this pipe network has \( n - 1 \) independent equations.

The simulation mathematical model of oilfield water injection pipe network can be written as matrix vector [2-3] :

\[ JH = Q \]  

Type: \( J \) is the matrix formed by \( K^i \); \( H \) is the node pressure vector; \( Q \) is the node flow vector.

The equations are high-dimensional nonlinear equations and can be solved by quasi-Newton method.

3. Establishment of Friction Coefficient Inversion Model of Water Injection Pipeline Network in Oilfield

Ideally, if other pipe network parameters are known, friction coefficient can be inversely solved by setting pressure sensors at each node to measure the pressure difference between the two ends of each pipe section. However, due to the restriction of investment cost and actual situation on site, only the
outlet of the pumping station and the inlet of the injection well are equipped with pressure and flow testing devices in the oilfield water injection system. Most of the nodes in the pipeline network are not equipped with test devices. However, pressure sensors can be set at key nodes according to the situation, so as to invert and solve the actual friction coefficient of pipe network. In order to maintain generality, assuming that there are $y$ nodes with test value pressure, setting as $\{p_1, p_2, ..., p_y\}$, and $y < n$. Taking the friction coefficient as the research object, the inverse mathematical model of water injection pipeline network is established. In the establishment of the model, it is necessary to consider the unknown pressure of some nodes, so the friction coefficient of the connected pipe segment cannot guarantee its accuracy\(^{[4-6]}\). Therefore, unknown node pressure should be avoided in the establishment of the inversion model. At the same time, considering the joint action of the simulated mathematical model structure and the actual working condition, flow rate, pressure, friction coefficient and other constraints of the pipe network system, Taking the flow rate as the breakthrough point and the minimum absolute sum of the difference between the calculated flow rate and the measured flow rate at the node with known node pressure as the target\(^{[7-8]}\), the friction coefficient inversion model of oil field water injection pipe network is established as follows:

$$\min \phi(s) = \sum_{t=1}^{\|J(s)H_{pt} - Q_{pt}\|}, s.t. s_k^{\min} \leq s_k \leq s_k^{\max}, k = 1, 2, ..., m$$

(4)

Type: $J(s)$ is the matrix related to $s$; $Q_{pt}$ is the node flow corresponding to the node with known test value pressure; $m$ is the total number of pipe segments; $s_k^{\min}$ is the minimum value of friction coefficient within the range of experience; $s_k^{\max}$ is the maximum friction coefficient within the range of experience; $k$ is the pipe element number related to the node with known pressure; $m$ is the total number of pipe elements related to the inversion model.

The mathematical model is an optimal mathematical model, which can be solved by improved particle swarm optimization algorithm.

4. Improved Particle Swarm Optimization

Oilfield water injection pipeline network is usually large and complex, and its mathematical model is a high-dimensional nonlinear equation group. Compared with genetic algorithm, particle swarm optimization (PSO) has a fast convergence speed and is more suitable for high-dimensional system solution without selection, crossover, mutation and other operators\(^{[9-11]}\). Particle swarm optimization (PSO) algorithm is called PSO, which seeks the global optimal value by tracking the current optimal value. PSO initializes a group of random solutions in the search space. These random solutions are called random particles, and each random particle is measured by velocity, position and fitness value. Random particles seek the optimal solution through iteration. After each iteration, the fitness value of random particles will be updated. By comparing the updated fitness value, individual extreme value and global extreme value, the individual extreme value and global extreme value will be updated, and the velocity and position of particles will be updated.

Inertia weights indicate that a particle maintains its previous velocity level. In basic particle swarm optimization (PSO), the inertia weight is fixed, and the improved PSO adopts linear decreasing inertia weight. At first, a large inertia weight is given to particles for global search, and as time goes by, the inertia weight is reduced for local search. By increasing the weight of linear diminishing inertia, the PSO is improved to balance the ability of local search and global search, and improve the performance of the algorithm. Linear decline inertia weight formula is as follows:

$$w(x) = w_c - (w_c - w_o)(M - x) / M$$

(5)

Type: $w_c$ is the initial inertia weight, and 0.85 is the best result after multiple tests; $w_o$ is the inertia weight of the maximum number of iterations, and 0.35 is the best result after multiple tests. $x$ is the current iteration number; $M$ is the maximum number of iterations.
Assume that the search space is $A$ dimension and the community size is $B$. After each iteration, with the update of individual extremum and global extremum, the velocity and position of particles are also updated. The updating formula of the velocity and position of particles is as follows:

$$V_{id}^{t+1} = w(x) * V_{id}^t + c_1 r_1 (P_i^t - S_i^t) + c_2 r_2 (P_g^t - S_i^t) \quad (6)$$

$$S_{id}^{t+1} = S_{id}^t + V_{id}^{t+1} \quad (7)$$

Type: $c_1$ and $c_2$ is the acceleration constant, and usually is $1.4945$; $I = 1, 2, ..., B$; $D = 1, 2, ..., A$; $r_1$ and $r_2$ is a random number between 0 and 1.

5. Algorithm Flow

The solution strategy of friction coefficient inversion of water injection pipe network with unknown partial node pressure is to use quasi-newton method to solve pipe section pressure through simulation. The improved particle swarm optimization algorithm is used to inverse the pipe network and solve the friction coefficient [12-15]. The specific process is as follows:

1. Define the node number of the pressure of the test value of the known node, make the set of its node number as set $G$, and the set of the remaining node number of the pressure of the unknown node as set $F$, and establish the coefficient matrix according to the network topology $J$.
2. The simulated pressure of each pipe section is solved by using quasi-newton method with the empirical friction coefficient and known pipe network parameters.
3. The node pressure of the node corresponding to set $F$ and the test value pressure of the node corresponding to set $G$ solved in step (2) constitute the pressure vector of all nodes.
4. Improved particle swarm optimization was used to invert and solve the friction coefficient vector $s$.
5. Select the friction coefficient vector $s$ solved in step (4).
6. Simulate and solve the pressure of each pipe section by quasi-newton method.
7. In the selection step (6), the calculated pressure of the node corresponding to set $G$ is compared with the test value pressure corresponding to set $G$.
8. If the calculated pressure and tested pressure of each node in the set meet the precision requirements, then output the friction coefficient $s$ in step (4); If the accuracy does not meet the requirements, then, the calculated pressure of the node corresponding to set $F$ and the tested pressure of the node corresponding to set $G$ in step (6) are selected to form the pressure vector of all nodes. Then, steps (4), (5), (6), (7) and (8) are repeated.

The quasi-newton method iteration process in step (2) is as follows:

1. $z_{kk+1} = z_{kk} - E \psi(z_{kk})$
2. $u_{kk} = z_{kk+1} - z_{kk}$
3. $W_{kk} = \psi(z_{kk+1}) - \psi(z_{kk})$
4. $E_{kk+1} = E_{kk} + \frac{(W_{kk} - E_{kk}u_{kk})^T u_{kk}}{\|u_{kk}\|_2}$

Type: $z_{kk+1}$ is the value of the $kk + 1$ superimposed variable $z$; $E$ is a relatively simple matrix to replace $\psi'(z)$; $u_{kk}$ is the difference between the generation and the previous generation of independent variables; $W_{kk}$ is the difference between two functions; $T$ is the transpose of the matrix; $\|u_{kk}\|_2$ is the two norm of $u_{kk}$.

The improved particle swarm optimization algorithm in step (4) flows as follows:

1. Receive the pressure parameters, initialize the particle swarm position within the known range, determine the population size, evolution number, random particle speed and position, individual extreme value and global extreme value of the population.
② Call the target function to calculate the fitness value of the population.
③ The fitness value of the population is compared with the extremum of the individual. If the fitness value of the population is less than the extremum of the individual, the extremum of the individual is replaced by the fitness value of the population.
④ The population fitness value and the global extreme value are compared. If the population fitness value is less than the global extreme value, the global extreme value is replaced by the population fitness value.
⑤ Update the position and velocity of the population particle.
⑥ If the maximum number of iterations is reached, then the end, otherwise return to step 2.

6. Practical Calculation

Figure 1 shows the water injection pipeline network of an oil field[16], which is composed of 3 water supply pump stations, 15 nodes and 20 pipe segments, among which nodes 4, 9 and 15 are the location of the pump station and the rest are the water injection nodes. 2, 3, 6, 10, 12 and 13 pressure test points are not installed.

![Simplified water injection pipeline network of an oilfield](image1)

Figure 1. Simplified water injection pipeline network of an oilfield

According to the algorithm flow of this paper, quasi-newton method and improved particle swarm optimization program is used. The population size is determined to be 20 and the population evolution (maximum number of iterations) is 1000. \( c_1 \) is 1.4945; \( c_2 \) is 1.4945; \( w_{e1} \) is 0.85; \( w_{mu} \) is 0.35. The optimal fitness of the population is shown in figure 4. It can be seen from figure 4 that the optimal fitness of the population is about 0.549, and the population tends to be stable after 800 generations and has excellent fitness value.

![Optimal individual fitness](image2)
Table 3. The parameters of water injection pipe network segment in oilfield

| The pipe segment number | The pipe length(m) | The pipe Diameter(m) | Original friction coefficient | Inverse friction coefficient |
|------------------------|--------------------|----------------------|------------------------------|-----------------------------|
| 1                      | 1730               | 0.40                 | 0.013                        | 0.0140                      |
| 2                      | 1500               | 0.30                 | 0.013                        | 0.0140                      |
| 3                      | 480                | 0.45                 | 0.013                        | 0.0120                      |
| 4                      | 620                | 0.45                 | 0.013                        | 0.0140                      |
| 5                      | 1270               | 0.35                 | 0.013                        | 0.0140                      |
| 6                      | 1150               | 0.30                 | 0.013                        | 0.0140                      |
| 7                      | 1380               | 0.40                 | 0.013                        | 0.0140                      |
| 8                      | 1390               | 0.30                 | 0.013                        | 0.0140                      |
| 9                      | 1130               | 0.30                 | 0.013                        | 0.0120                      |
| 10                     | 1020               | 0.30                 | 0.013                        | 0.0120                      |
| 11                     | 1140               | 0.20                 | 0.013                        | 0.0139                      |
| 12                     | 760                | 0.20                 | 0.013                        | 0.0120                      |
| 13                     | 1510               | 0.20                 | 0.013                        | 0.0136                      |
| 14                     | 1040               | 0.25                 | 0.013                        | 0.0120                      |
| 15                     | 1670               | 0.35                 | 0.013                        | 0.0140                      |
| 16                     | 650                | 0.45                 | 0.013                        | 0.0140                      |
| 17                     | 225                | 0.50                 | 0.013                        | 0.0120                      |
| 18                     | 200                | 0.30                 | 0.013                        | 0.0131                      |
| 19                     | 150                | 0.35                 | 0.013                        | 0.0131                      |

Table 4. Pipeline network node parameters of oilfield water injection

| Node number | The node flow(m³/s) | The node pressure(m) | Calculation of pressure(m) |
|-------------|---------------------|----------------------|----------------------------|
| 1           | 0.0506              | 53.967               | 52.063                     |
| 2           | 0.0487              | No check points installed | 55.912                     |
| 3           | 0.0432              | No check points installed | 51.138                     |
| 4           | 0.4000              | 58.327               | 58.788                     |
| 5           | 0.0362              | 57.696               | 57.896                     |
| 6           | 0.0815              | No check points installed | 55.047                     |
| 7           | 0.1058              | 50.000               | 49.506                     |
| 8           | 0.0355              | 64.589               | 62.768                     |
| 9           | 0.3150              | 60.961               | 58.364                     |
| 10          | 0.0368              | No check points installed | 51.912                     |
| 11          | 0.1987              | 51.417               | 50.051                     |
| 12          | 0.0661              | No check points installed | 51.415                     |
| 13          | 0.0825              | No check points installed | 54.769                     |
| 14          | 0.0364              | 59.574               | 63.995                     |
| 15          | 0.1070              | 65.395               | 67.460                     |

According to Table 3 and Table 4, the average error of inversion friction coefficient and original friction coefficient is 6.73%, and the maximum error of test pressure and calculation pressure is 7.42%, which meets the accuracy requirements, and further proves the correctness of the pipe network mathematical model and the feasibility of the algorithm.
7. The Conclusion

According to the topological structure of water injection pipeline network, the inverse mathematical model of friction coefficient of water injection pipeline network with known partial node pressure is established. This method can not only control the accuracy of the pressure calculation value of the known pressure node, but also solve the approximate value of the unknown pressure node, and realize the partial accurate solution of the friction coefficient of the pipe element and partial prediction solution. The calculation results satisfy the practical application of engineering, and prove the correctness of the mathematical model of pipe network and the feasibility of the algorithm.

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References

[1] Chang Y L, Ren Y L, et al. Research on inversion method of inner diameter of oil field water supply pipe network [J]. Mathematical practice and cognition, 2006,03 (20): 93-98.
[2] Chang Y L Gao S, et al. Optimal control mathematical model and system design of oilfield water injection system [J]. Petroleum machinery, 2001,12 (25): 24-26.
[3] Ren Y L, Chang Y L, et al. Establishment and solution of hydraulic model of water injection string in oilfield [J]. Journal of system simulation, 2007,19 (07): 1468-1470.
[4] YU Zhongchen, MA Dong, WANG Song, et al. Friction factors of oilfield water injection network--calculating cases analysis[J]. Advanced Materials Research.2013, 765:908-911.
[5] WANG Song, MA Dong, YU Zhongchen, et al. Friction factors of oilfield water injection network--research on solving approach[J]. Advanced Materials Research.2013, 765:920-923.
[6] Chen Mingxi, CHENG Guojian, QIANG Xinjian. Building a high efficient and intelligent digital oilfield water injection system[J]. Proceedings - 2014 5th International Conference on Intelligent Systems Design and Engineering Applications[C].Institute of Electrical and Electronics Engineers Inc, 2014.
[7] Zhang D F.MATLAB R2015b mathematical modeling [M]. Beijing: tsinghua university press, 2016:198-199.
[8] Liang Y T, Zhou X Y, et al. Optimization of water injection pipeline system in large ring dendritic oilfield [J]. Journal of China university of petroleum (natural science edition). 2012, 06(42):122-131.
[9] Cui Z M, Wang Y F. Study on optimization of oilfield water injection system based on genetic algorithm [J]. Science, technology and engineering, 2012,03 (08): 1657-1658.
[10] MAO Z Y, Huang L P, et al. Application of multidimensional coding genetic algorithm in optimal operation of urban water supply network [J]. Journal of Hunan university of science and technology (natural science edition). 2007,01 (22):73-76.
[11] Zhang Y X, Wang H F, et al. Optimization algorithm of urban water supply allocation model based on particle swarm optimization [J]. Journal of Beijing university of technology. 2016,03 (42):468-472.
[12] Ren W J, Huang J, et al. Optimal design of oilfield water injection pipeline network based on improved particle swarm optimization algorithm [J]. Science, technology and engineering. 2009,9 (11) : 2930-2933.
[13] Wang Y X, Guo L T, et al. Friction factor inversion of water injection pipeline network based on expected value model [J]. Journal of Daqing petroleum institute, 2012,02 (15): 92-94.
[14] Wang Y X, Liu C L, et al. Optimization of oilfield water injection system based on ant colony particle swarm optimization [J]. Journal of Daqing petroleum university, 2010,04 (15): 73-76.
[15] Wang Y X, Wei S H, et al. Inversion of friction factor of water injection pipeline network based on least square method [J]. Journal of Daqing petroleum institute, 2011,12 (15): 64-66.
[16] Ren Y L. Research on inversion method of structural parameters of oilfield water system model[D]. Daqing : Northeast Petroleum University, 2003.