Time Since the Beginning

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Abstract. While there is no consensus about the history of time since the beginning, in this paper I will discuss some possibilities. We have a pretty clear picture of cosmic history from the electroweak phase transition through the time of recombination, a period which includes the QCD phase transition and big bang nucleosynthesis. This paper includes a quantitative discussion of the age of the universe, of the radiation-matter transition, and of hydrogen recombination. There is much evidence that at earlier times the universe underwent inflation, but the details of how and when inflation happened are still far from certain. There is even more uncertainty about what happened before inflation, and how inflation began. I will describe the possibility of “eternal” inflation, which proposes that our universe evolved from an infinite tree of inflationary spacetime. Most likely, however, inflation can be eternal only into the future, but still must have a beginning.

1. Introduction

In this paper I will attempt to discuss the history of time from the beginning, even though no complete description exists. In Sec. 2 I will lay out the basic equations, and in Sec. 3 I will discuss the time period from about $10^{-12}$ s to 300,000 years. In Sec. 4 I will discuss what happened earlier, suggesting that inflation is the answer. Sec. 5 will deal with the question of what happened before inflation, to which I will argue that the answer is more inflation—i.e., eternal inflation. In the final section I will summarize.

2. Fundamentals of Early Universe Physics

The time-evolution of the early universe seems to be well-described by a remarkably simple theory, known alternatively as the hot big bang theory or the standard cosmological model. The model assumes that the universe is well-approximated as being homogeneous and isotropic, which implies that the metric can be written in the Robertson-Walker form,

$$ds^2 = -dt^2 + a^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 \left[ d\theta^2 + \sin^2 \theta \, d\phi^2 \right] \right\},$$

(1)
where $k$ denotes a constant which indicates whether the universe is open ($k < 0$), closed ($k > 0$), or flat ($k = 0$), and throughout this article I will use units for which $\hbar \equiv c \equiv k_B \equiv 1$. The Einstein equations imply that the scale factor $a(t)$ evolves according to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G \rho - \frac{k}{a^2}$$  \hspace{2cm} (2)

$$\ddot{a} = -\frac{4\pi}{3} G (\rho + 3p) a$$  \hspace{2cm} (3)

$$\frac{d}{dt} \left( a^3 \rho \right) = -p \frac{d}{dt} \left( a^3 \right) ,$$  \hspace{2cm} (4)

where $\rho$ is the mass density, $p$ is the pressure, $G$ is Newton’s constant, and the overdot denotes a derivative with respect to $t$. These equations are not independent, since any one of them can be derived from the other two. Assuming that the mass density consists of matter ($\rho_m \propto a^{-3}$), radiation ($\rho_r \propto a^{-4}$), and vacuum mass density ($\rho_{\text{vac}} = \text{constant}$), then Eq. (2) can be integrated to give the relationship between the scale factor $a$ and the time $t$. Denoting the present value of the Hubble parameter $H \equiv \dot{a}/a$ by $H_0$, and normalizing the scale factor so that its present value is 1, one finds:

$$t = H_0^{-1} \int_0^a \frac{a' \, da'}{\sqrt{\Omega_m a' + \Omega_r + \Omega_{\text{vac}} a'^4 + \Omega_k a'^2}} ,$$  \hspace{2cm} (5)

where $\Omega_X \equiv \rho_X/\rho_c$, the subscript 0 denotes the present time, $\rho_c$ denotes the critical density $3H_0^2/(8\pi G)$, and $\Omega_k = 1 - \Omega_m - \Omega_r - \Omega_{\text{vac}}$. 

There is now much evidence that the universe is flat, coming predominately from studies of the cosmic microwave background (CMB). Wang, Tegmark, & Zaldarriaga (2001) have carried out a comprehensive study in which they combined the measurements of the CMB (most importantly the results from BOOMERaNG (Netterfield et al. 2001), DASI (Halverson et al. 2001), Maxima (Lee et al. 2001), and CBI (Padin et al. 2001)) with measurements of large scale structure (IRAS PSCz survey (Saunders et al. 2000; Hamilton, Tegmark, & Padmanabhan 2000)) and the Hubble parameter (Freedman et al. 2000) to find that $\Omega_k = 0.0 \pm 0.06$ at the 95% confidence level. Since this result is also in agreement with the prediction of the simplest inflationary models, for the remainder of this paper I will consider only models that are exactly flat ($k = 0$).

To apply Eq. (5) for times near the present, it is sufficient to neglect $\Omega_r \approx 10^{-4}$, in which case Eq. (5) can integrated analytically:

$$t = \frac{2H_0^{-1}}{3\sqrt{\Omega_{\text{vac}}}} \tanh^{-1} \sqrt{\frac{\Omega_{\text{vac}} a^4}{a^4\Omega_{\text{vac}} + a(1 - \Omega_{\text{vac}})}} .$$  \hspace{2cm} (6)

To determine the present age we set $a = 1$ in the above equation, finding $t_0 = (2H_0^{-1}/(3\sqrt{\Omega_{\text{vac}}}) \tanh^{-1} \sqrt{\Omega_{\text{vac}}}$. Numerical evaluations of this formula are shown in Figure 1. The final value for the Hubble parameter obtained by the Hubble Key Project (Freedman et al. 2000) was $H_0 = 72 \pm 8 \text{ km-s}^{-1}\cdot\text{Mpc}^{-1}$, so I take $H_0 = 72$ as the central value in Figure 1. There is more uncertainty in $\Omega_{\text{vac}}$, but I will take $\Omega_{\text{vac}} \approx 0.65$ as the central value for the graphs.
The calculated age of the universe is shown as a function of $\Omega_{\text{vac}}$, for various values of the Hubble parameter $H_0$, measured in km-s$^{-1}$-Mpc$^{-1}$. The models are flat, and assumed to have negligible radiation density. The circled dot shows the currently popular model with $H_0 = 72$, $\Omega_{\text{vac}} = 0.65$, and an age of 12.5 Gyr.

While radiation is a negligible contribution to the total mass density today, the universe is believed to have been radiation-dominated from just after inflation until some tens of thousands of years after the big bang. The energy density of such thermal radiation is given by

$$\rho = g \frac{\pi^2}{30} T^4,$$

where $g$ denotes the number of effectively massless bosonic spin states, plus 7/8 times the number of effectively massless fermionic spin states. The entropy density is given by

$$s = g \frac{2\pi^2}{45} T^3.$$

Except for inflation the entropy of the early universe is believed to have remained essentially constant, so that the relationship between time $t$ and temperature $T$ can be found from $a^3 s = \text{constant}$ and the dynamical equations (2)–(4). If $g$ can be treated as a constant, as it can for various time intervals, this relation becomes

$$T = \left( \frac{45}{16\pi^3 g G} \right)^{1/4} \frac{1}{\sqrt{t}}.$$

After the disappearance of the muons at about $10^{-4}$ s, the contributions to $g$ consist of photons ($g = 2$), electron-positron pairs ($g = 7/2$), and three species of neutrinos ($g = 21/4$), for a total of $g = 10^{\frac{3}{2}}$. During this interval Eq. (9) reduces to

$$T = \frac{0.8592 \text{ MeV}}{\sqrt{t/(1 \text{ sec})}} = \frac{9.971 \times 10^9 \text{ K}}{\sqrt{t/(1 \text{ sec})}}.$$
As an order of magnitude estimate, one can use the above formula for all times between about $10^{-12}$ s and $10^3$ years. As a precise formula, Eq. (10) begins to fail at about $t = 1$ s, when the $e^+e^-$ pairs start to disappear from the thermal equilibrium mix. By this time the neutrinos have effectively decoupled, so all the entropy of the $e^+e^-$ pairs (with $g = 7/2$) is given to the photons (with $g = 2$) and not the neutrinos. As a result, the entropy density of photons is increased by a factor of $(7/2 + 2)/2 = 11/4$, so the temperature of the photons relative to the neutrinos is increased by a factor of $(11/4)^{1/3}$. This ratio is believed to persist to the present.

For the period after the disappearance of the $e^+e^-$ pairs, one conventionally uses $T$ to denote the temperature of the photons, while the neutrinos have a temperature $T_\nu = (4/11)^{1/3} T$. The COBE FIRAS measurements (Mather et al. 1999) determined that $T_0 = 2.725 \pm 0.002$ K, which implies a present mass density in photons and neutrinos of $7.804 \times 10^{-34}$ g/cm$^3$. Using $H_0 = 72$ km-s$^{-1}$-Mpc$^{-1}$, one finds $\Omega_r = 8.013 \times 10^{-5}$.

3. Cosmic Events from $10^{-12}$ Second to 300,000 Years

The first key event of this period is the electroweak phase transition, at which the $SU(2) \times U(1)$ symmetry of the Glashow-Weinberg-Salam electroweak theory is broken to the familiar $U(1)$ symmetry of quantum electrodynamics. At this phase transition a Higgs field is believed to acquire a nonzero expectation value. The interaction of the Higgs field with other fields is then responsible for masses of the corresponding particles, which include the $W$, the $Z$, the leptons ($e$, $\mu$, and $\tau$), and the quarks. It is worth noting, however, that the masses acquired by the $u$ and $d$ quarks through the electroweak symmetry-breaking are called the “current-quark” masses, and have values under 10 MeV. They have very little influence on the masses of protons and neutrons, which are associated with the “constituent” quark masses that arise from the strong interactions of the quarks.

The details of the electroweak phase transition remain unknown, since the Higgs particle and its detailed properties remain out of reach. The lack of attractive alternatives has convinced most particle physicists that the Higgs particle almost certainly exists, but it remains possible that nature is more complicated than the simple models with a single Higgs field. The energy scale of electroweak symmetry breaking is certainly, however, on the order of 1 TeV, so the time of the electroweak phase transition can be estimated from Eq. (10) at about $10^{-12}$ s.

The next important event was the quantum chromodynamics (QCD) phase transition, which has an energy scale of approximately 1 GeV, and therefore took place at about $10^{-6}$ s. At this phase transition the quark-gluon plasma, with its essentially free quarks, disappeared in favor of a phase in which the quarks are permanently bound inside mesons and baryons. At about the same time the overwhelming majority of quarks and antiquarks annihilated in pairs. A tiny excess of quarks over antiquarks, of about one part in $10^9$, resulted in the survival of a tiny fraction of the hadronic matter, and this tiny excess is responsible for the existence of the protons and neutrons that populate the current universe. We believe that the excess was generated by a process, known as baryogenesis,
which may have occurred anytime from the grand unified theory era through the electroweak phase transition.

At \( t \approx 1 \) s, when the temperature fell to about 1 MeV, the processes that led to big bang nucleosynthesis began. The first step was the decoupling of the neutrinos, which cut off the reactions that had until this time maintained a thermal equilibrium balance between protons and neutrons. Since the neutron mass exceeds the proton mass by 1.29 MeV, the number of neutrons was suppressed relative to the number of protons, but not by a large amount.

It is often pointed out that it appears to be an important coincidence that the temperature at which the neutrinos decouple, determined by the strength of the weak interactions and various cosmological parameters, is very nearly equal to the neutron-proton mass difference, which is presumably the result of an interplay between the strong and electromagnetic interactions. If the neutrinos remained coupled for much longer, the thermal equilibrium between protons and neutrons would be maintained down to lower temperatures, resulting in the almost complete disappearance of neutrons from the universe. If the neutrinos decoupled earlier, then the universe would be left with a nearly 50%/50% mix of protons and neutrons, which would result in an almost total conversion to \( \text{He}_4 \) in big bang nucleosynthesis.

After the neutrinos decoupled at \( t \approx 1 \) s, the only relevant reaction that could interchange protons and neutrons was the free decay of the neutron, with a mean life of about 15 minutes. After about 3 to 4 minutes, however, the temperature fell to \( T \approx 0.1 \) MeV, which is cool enough for the deuteron to become stable. At this point nuclear reactions proceeded quickly, converting almost all the neutrons that remained into \( \text{He}_4 \), which today has an abundance of 23%-25% by mass (see, for example, Burles, Nollett, & Turner 2001b). In addition, detectable amounts of deuterium, \( \text{He}_3 \), and \( \text{Li}_7 \) were produced. Note that 0.1 MeV is far below the deuteron binding energy of 2.2 MeV, but that such low temperatures are needed for stability because of the huge ratio of photons to baryons, about \( 10^9 : 1 \). Thus each deuteron that formed must have survived a huge number of photon collisions before it had the chance to proceed with further nuclear reactions.

At \( t \approx 30,000 \) years, the mass density of the universe gradually changed from radiation-dominated to matter-dominated, where “matter” refers to both dark matter and baryonic matter. This change is described by the equations presented in Sec. 2, and is actually a very gradual transition. The results of a numerical integration of these equations is shown in Figure 2.

Finally, the last important event of this period is known as hydrogen “recombination,” although “combination” would be a more accurate term. In the context of the standard cosmological model, the electrons and protons had never been combined at any point in the past. Recombination is often said to take place at a temperature of 4000 K and at a time of 300,000 years. These numbers are in fact reasonable estimates, but the actual process of recombination, like that of matter-domination, is gradual. Note that 4000 K \( \approx 0.34 \) eV, so like the deuteron during nucleosynthesis, atomic hydrogen in the early universe did not become stable until \( k_B T \) was far below its binding energy.
Figure 2. The left graph shows the time of matter-radiation equality as a function of the present value of $\Omega_{\text{vac}}$, for various values of $H_0$ (in km-s$^{-1}$-Mpc$^{-1}$). The circled dot shows the currently popular model with $H_0 = 72$ and $\Omega_{\text{vac}} = 0.65$, with $t_{\text{equality}} = 31,070$ yr. The graph on the right shows the fraction of the total mass density of the universe in radiation as a function of time, for various values of $\Omega_{\text{vac}}$. Both graphs represent the same flat cosmological models, which are assumed to have three species of massless neutrinos and a present radiation temperature of 2.725 K. The “dark energy” component is taken to be a cosmological constant, with a fixed vacuum mass density.

If one assumes thermal equilibrium, then the fraction $x$ of protons or electrons that remain ionized is given by the Saha equation,

$$\frac{x^2}{1-x} = \frac{(2\pi m_e k_B T)^{3/2}}{(2\pi \hbar)^3 n} e^{-B/k_B T}.$$  \hspace{1cm} (11)

Here $m_e$ is the electron mass, $k_B$ is the Boltzmann constant, and $B$ is the binding energy of hydrogen, 13.60 eV. (For a pedagogical treatment of the Saha equation, see Peebles 1993, pp. 165–167.) The Saha equation provides a reasonable approximation for the onset of recombination, but the process soon departs significantly from thermal equilibrium, as was shown by Peebles (1968). The reasons for the departure from thermal equilibrium are a bit subtle, since the reaction rates for ionization and recombination are much faster than the expansion rate of the universe. The problem, however, is that almost every decay to the ground state of hydrogen emits a Lyman alpha photon which then has a high probability of ionizing another hydrogen atom. Thus, the sum of the number of ground state hydrogen atoms plus the number of Lyman alpha photons changes slowly, and lags behind thermal equilibrium as the universe expands.
The dominant mechanisms for changing this sum are the rare two-photon decay of the 2s level of hydrogen to the ground state, and the gradual redshifting of the Lyman alpha photons out of the relevant range of frequencies.

Numerical results for recombination are shown in Figure 3, using the currently indicated values of the parameters. In particular, the calculations use a flat model with $T_0 = 2.725 \text{ K}$, $\Omega_{\text{vac}} = 0.65$, and $\Omega_B h^2 = 0.020$, following Burles, Nollett, & Turner (2000a & b), who found $\Omega_B h^2 = 0.020 \pm 0.002$ (95% confidence level). (Note that $h$ is defined by $H_0 = 100 h \text{ km-s}^{-1}\cdot\text{Mpc}^{-1}$.) The results were obtained by numerical integrations carried out by me, using the equations of Peebles (1968) and Peebles (1993), pp. 165–173.

![Figure 3](attachment:figure3.png)

Figure 3. Both graphs show the process of recombination as a function of time, for a model described in the text. The ionization fraction is the fraction of all protons or electrons that are ionized at any given time. The left graph is logarithmic, and the right graph is linear. The line labeled “Equilibrium” shows the result of solving the Saha equation, while the line labeled “Actual” shows the result of integrating the rate equations, showing the nonequilibrium effects. For reference, the temperature is also shown, keyed to the scale on the right which applies to both graphs.

4. Before $10^{-12}$ Second: Inflation

At times before $10^{-12}$ second, some of us believe that the universe almost certainly underwent a period of inflation. The reason is that cosmic inflation can explain a number of features of our universe that would otherwise be unexplained. In particular, inflation can explain:
1. **How the universe acquired \( > 10^{90} \) particles**

Starting from the general and then moving toward the specific, one salient feature of the universe is its enormous size. The visible part of the universe contains about \( 10^{90} \) particles. It is easy to take this for granted, and many cosmologists are not bothered by the fact that the “standard” FRW cosmology, without inflation, simply postulates that about \( 10^{90} \) or more particles were here from the start. However, in the present context many of us hope that even the creation of the universe can be described in scientific terms, and thus the number of particles would have to be the result of some calculation. The easiest way by far to get a huge number, with presumably only modest numbers as input, is for the calculation to involve an exponential. The exponential expansion of inflation reduces the problem of explaining \( 10^{90} \) particles to the problem of explaining 60 to 70 e-foldings of inflation. In fact it is easy to construct underlying particle theories that will give far more than 70 e-foldings of inflation, so inflationary cosmology suggests that the observed universe is only an infinitesimal fraction of the entire universe.

2. **Why the universe is uniformly expanding**

The Hubble expansion is also easy to take for granted, and in the standard FRW cosmology the Hubble expansion is accepted as a postulate about the initial conditions. But inflation offers the possibility of actually explaining how the Hubble expansion began. The repulsive gravity associated with the inflaton field—the scalar field that drives the inflation—is exactly the kind of force needed to propel the universe into a pattern of motion in which each pair of particles is moving apart with a velocity proportional to their separation.

3. **How the CMB can be uniform to 1 part in \( 10^5 \)**

The degree of uniformity in the universe is startling. The intensity of the cosmic background radiation is the same in all directions, after it is corrected for the motion of the Earth, to the incredible precision of one part in 100,000.

The cosmic background radiation was released at the time of recombination, about 300,000 years after the big bang, when the universe cooled enough so that the opaque plasma neutralized into a transparent gas. The cosmic background radiation photons have mostly been traveling on straight lines since then, so they provide an image of what the universe looked like at 300,000 years after the big bang. The observed uniformity of the radiation therefore implies that the observed universe had become uniform in temperature by that time. In standard FRW cosmology, a simple calculation shows that the uniformity could be established so quickly only if signals could propagate at 100 times the speed of light, a proposition clearly in contradiction with the known laws of physics. In inflationary cosmology, however, the uniformity is easily explained. The uniformity is created initially on microscopic scales, by normal thermal-equilibrium processes, and then inflation takes over and stretches the regions of uniformity to become large enough to encompass the observed universe.
4. Why the early universe was so close to critical density

I find this issue particularly impressive, because of the extraordinary numbers that it involves. This “flatness problem” concerns the value of the ratio

$$\Omega_{\text{tot}} \equiv \frac{\rho_{\text{tot}}}{\rho_c},$$

(12)

where $\rho_{\text{tot}}$ is the average total mass density of the universe and $\rho_c = 3H^2/(8\pi G)$ is the critical density, the density that would make the universe spatially flat. (In the definition of “total mass density,” I am including the vacuum energy $\rho_{\text{vac}} = \Lambda/(8\pi G)$ associated with the cosmological constant $\Lambda$, if it is nonzero.)

There is now strong evidence that $\Omega$ is very near to 1, but the flatness problem is much older and does not require us to believe the most recent results. We have believed for a long time that

$$0.1 \lesssim \Omega_0 \lesssim 2,$$

(13)

and this is all that is needed to motivate the flatness problem. Despite the breadth of this range, the value of $\Omega$ at early times is highly constrained, since $\Omega = 1$ is an unstable equilibrium point of the standard model evolution. Thus, if $\Omega$ was ever exactly equal to one, it would remain exactly one forever. However, if $\Omega$ differed slightly from one in the early universe, that difference—whether positive or negative—would be amplified with time. In particular, it can be shown that $\Omega - 1$ grows as

$$\Omega - 1 \propto \begin{cases} \frac{t}{\sqrt{t}} & \text{(during the radiation-dominated era)} \\ \frac{t^{2/3}}{\sqrt{t}} & \text{(during the matter-dominated era)} \end{cases}.$$  

(14)

It was shown by Dicke and Peebles (1979), for example, that as the processes of big bang nucleosynthesis were just beginning at $t = 1$ sec, $\Omega$ must have equaled one to an accuracy of one part in $10^{15}$. Classical cosmology provides no explanation for this fact—it is simply assumed as part of the initial conditions. In the context of modern particle theory, where we try to push things all the way back to the Planck time, $10^{-43}$ sec, the problem becomes even more extreme. If one specifies the value of $\Omega$ at the Planck time, it has to equal one to 58 decimal places in order to be anywhere in the allowed range today.

While this extraordinary flatness of the early universe has no explanation in classical FRW cosmology, it is a natural prediction for inflationary cosmology. During the inflationary period, instead of $\Omega$ being driven away from one as described by Eq. (14), $\Omega$ is driven towards one with exponential swiftness:

$$\Omega - 1 \propto e^{-2H_{\text{inf}}t},$$

(15)

where $H_{\text{inf}}$ is the Hubble parameter during inflation. Thus, as long as there is a long enough period of inflation, $\Omega$ can start at almost any value, and it will be driven to one by the exponential expansion.
5. Why the inhomogeneities have a nearly flat (Harrison-Zeldovich) spectrum

The process of inflation smooths the universe essentially completely, but density fluctuations are generated as inflation ends by the quantum fluctuations of the inflaton field, the scalar field that drives the inflationary expansion. Generically these are adiabatic Gaussian fluctuations with a nearly scale-invariant spectrum (Starobinsky 1982; Guth & Pi 1982; Hawking 1982; Bardeen, Steinhardt, & Turner 1983; Mukhanov, Feldman, & Brandenberger 1992). New data is arriving quickly, but so far the observations are in excellent agreement with the predictions of the simplest inflationary models. For a review, see for example Bond and Jaffe (1999), who find that the combined data give a slope of the primordial power spectrum within 5% of the preferred scale-invariant value. See also Wang, Tegmark, & Zaldarriaga (2001), which includes a review of the most current data.

Since the theme here is time and time scales, it is natural to ask when inflation occurred. The answer is that we do not really know. Originally inflation was proposed to take place at the scale of grand unified theories, at a characteristic energy scale of $10^{16}$ GeV (Guth 1981; Linde 1982; Albrecht & Steinhardt 1982). Applying Eq. (5) with $g \approx 200$, typical of grand unified theories, one finds a starting time for inflation of about $10^{-39}$ s. This is extraordinarily early, but it is still late compared to the Planck time, $\sqrt{\overline{G} h/c^3} \approx 5 \times 10^{-44}$ s, the time scale at which quantum gravity is believed to become important. Thus, it is plausible that the field theoretic formalism that is used to describe inflation is valid at the appropriate energy scale.

It is possible that inflation did occur at the grand unified theory scale, but it might very well have occurred later. The only known restriction on the lateness of inflation is the requirement that baryogenesis occur after inflation, since any net density of baryon number generated before inflation would be diluted to a negligible level. It is now believed that baryogenesis might happen as late as the electroweak scale, operating through the mechanism of electroweak current conservation anomalies (Kuzmin, Rubakov, & Shaposhnikov 1985).

Observationally it is difficult to determine the energy scale and hence the time scale of inflation, since the consequences are very insensitive. The only known way to determine the energy scale of inflation is to directly or indirectly measure the gravitational wave background, which is more intense if the energy scale of inflation was high. In fact the energy scale could not have been significantly higher than the grand unified scale, or else the gravity waves would be so strong that they should have already been detected.

5. Before Inflation: (Eternal) Inflation

The question of what happened before inflation is an open one, and different cosmologists would venture different ideas. In my opinion, the most plausible answer to what happened before inflation is — more inflation.

Specifically, it appears that essentially all working models of inflation are eternal, in the sense that once inflation starts, it never stops. Instead inflation goes on forever, with pieces of the inflating region breaking off and producing
a never-ending stream of “pocket universes” (Vilenkin 1983; Steinhardt 1983; Linde 1986a&b; Goncharov, Linde, & Mukhanov 1987).

The mechanism that leads to eternal inflation is rather straightforward to understand. Normally one expects inflation to end because the “false vacuum”—the state of the inflaton field that is responsible for the repulsive gravity driving the inflation—is unstable, so it decays like a radioactive substance. As with familiar radioactive materials, the decay of the false vacuum is generally exponential: during any period of one half-life, on average half of it will decay. This case is nonetheless very different from familiar radioactive decays, however, because the false vacuum is also expanding exponentially. Furthermore, it turns out that the expansion is generally much faster than the decay. Thus, if one waits for one half-life of the decay, half of the false vacuum region would on average convert to ordinary matter. But meanwhile the part that remains would have undergone many doublings, so it would be much larger than the region was at the start. Even though the false vacuum is decaying, the volume of the false vacuum would actually grow with time. The volume of the false vacuum would continue to grow, without limit and without end. Meanwhile pieces of the false vacuum region decay, producing an infinite number of what I call pocket universes.

![Diagram of eternal inflation](image)

Figure 4. An illustration of eternal inflation, as described in the text.

In Figure 4 I show a schematic illustration of how this works. The top row shows a region of false vacuum, shown very schematically as a horizontal bar. After a certain length of time, a little less than a half-life, the situation looks like the second bar, in which about a third of the region has decayed. The energy released by that decay produces a pocket universe, which will inflate to become much larger than the presently observed universe.

On the second bar, in addition to the pocket universe, there are two regions of false vacuum. On the diagram I have not tried to show the expansion, so the diagram can fit on the page. So, you are expected to remember that each bar is actually bigger than the previous bar, but drawn on a smaller scale so that it looks the same size. To discuss a definite example, let us assume that each bar represents three times the volume of the previous bar. In that case, each region of false vacuum on the second bar is just as big as the entire bar on the top line.

The process can then repeat. If we wait the same length of time again, the situation will be as illustrated on the third bar of the diagram, which represents a region 3 times larger than the second bar, and 9 times larger than the top bar. For each region of false vacuum on the second bar, about a third of the
region decays and becomes a pocket universe, leaving regions of false vacuum in between. Each region of false vacuum shown on the diagram is as large as the original region in the top bar. The process goes on literally forever, producing pocket universes and regions of false vacuum between them, ad infinitum. The universe on the very large scale acquires a fractal structure.

The illustration of Figure 4 is of course oversimplified in a number of ways: it is one-dimensional instead of three-dimensional, and the decays are shown as if they were very systematic, while in fact they are random. But the qualitative nature of the evolution is nonetheless accurate: eternal inflation really leads to a fractal structure of the universe, and once inflation begins, an infinite number of pocket universes are produced.

Since inflation is eternal into the future, it is natural to ask if it might also be eternal into the past. The explicit models that have been constructed are eternal only into the future and not into the past, but that does not show whether or not is possible for inflation to be eternal into the past. Borde & Vilenkin (1994) presented a proof that an eternally inflating spacetimes must start from an initial singularity, and hence must have a beginning, but later they pointed out (1997) that their proof assumed a condition that is true classically but is violated by quantum field theories. Today the issue is undecided. My own suspicion is that eternally inflating spacetimes must have initial singularities, because it seems significant that no one has been able to construct a model which does not.

6. Summary

For the period between about $10^{-12}$ s and 300,000 years, we have a rather detailed description of cosmology that I believe has a good chance of being correct. I believe that inflation played a very significant role at earlier times, but the details are unclear. As one might expect, our view of the earliest moments of the universe is still clouded with uncertainties.

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