On the Possibility of Combustion of Neutrons into Strange Quark Matter

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Abstract

It is shown that a conversion of neutrons into strange quark matter in neutron stars is possible by means of a combustion process with a well-defined front. Conditions for the realization of a specific combustion mode, whether deflagration, detonation, or fast combustion, are discussed for several forms of the equations of state of neutron and strange matters.

Key words: Neutron stars, dense matter, combustion

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1 Introduction

Recently, the possibility of emergence and existence of strange quark stars has been intensively discussed in the literature \cite{1,2}. A likely origin for strange stars is the conversion of neutrons into strange quark matter in neutron stars \cite{3,1}. The process is accompanied by a powerful release of energy (of about $10^{53}$ erg) since nascent light quarks acquire relativistic temperature \cite{1,2}. Cooling of high-temperature strange stars has been proposed as a mechanism of intensive neutrino and $\gamma$-ray emission (see \cite{4,5}, and references therein). There is mounting, albeit controversial, observational evidence for the existence of strange stars. Likely candidates to be strange stars are the the compact objects 3C58 and RX J1856.5-3754.\cite{6,7}. The possibility to detect indirectly strange quark stars (that may be formed after SNe explosions) by their effect on the SN-ejecta and connections between the formation of strange stars and GRBs were discussed in \cite{8,9}. Recently, the authors of paper \cite{10} have pointed out that a soft gamma ray precursor, like GRB980425/SN1998bw, detected by Swift could provide evidence for the formation of a quark star.

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Hydrodynamical considerations lead to the conclusion that the conversion is likely to proceed in the form of combustion with a well defined front [11,12,13]. General conditions under which combustion in neutron stars may occur have been discussed in paper [14]. Based on the conservation laws of hydrodynamics the authors of that paper have investigated the necessary conditions for combustion for a wide ranges of values for bag constant $B$ of the quark matter equation of state and the density of the neutron matter (NM). They have argued that neither slow combustion (deflagration) nor detonation may be realized. Their analysis, however, assumed that the energy per baryon must be smaller in the quark matter after the conversion. This assumption is correct only if one ignores the kinetic and internal energies gained by the quark at the combustion front.

In this paper we show that all combustion modes are possible and present the relevant constraints on the parameters of the neutron and quark matter equations of state.

2 Conservation laws

Our strategy is to assume the existence of a combustion front and then assess the necessary conditions for its preservation and the realization of specific combustion modes. Let $v_i$, $n_i$, $p_i$, and, $\varepsilon_i$, be, respectively, the velocity relative to the front, the baryon number density, the pressure, and energy density, where the subscript “$i$” is either “$n$” or “$s$” referring to NM or strange quark matter (SM), respectively. In a frame of reference moving with the combustion front, the state of NM and SM at both sides of the front are related by the equations of baryon flux conservation,

$$n_s v_s W_s = n_n v_n W_n,$$  \hspace{1cm} (1)

the momentum flow conservation,

$$\omega_s v_s W_s^2 = \omega_n v_n W_n^2;$$  \hspace{1cm} (2)

and energy flow conservation,

$$\omega_s v_s^2 W_s^2 + p_s = \omega_n v_n^2 W_n^2 + p_n,$$  \hspace{1cm} (3)

where $\omega_i = p_i + \varepsilon_i$ is the specific enthalpy and $W_i = 1/\sqrt{1-v_i^2}$ is Lorentz factor [15]. These conservation laws allow us to express the velocities $v_n$ and $v_s$ in terms of the energy density and pressure of NM and SM as follows,

$$v_s^2 = \frac{(p_s - p_n)(\varepsilon_n + p_s)}{(\varepsilon_s - \varepsilon_n)(\varepsilon_s + p_n)},$$  \hspace{1cm} (4)

$$v_n^2 = \frac{(p_s - p_n)(\varepsilon_s + p_n)}{(\varepsilon_s - \varepsilon_n)(\varepsilon_n + p_s)}.$$  \hspace{1cm} (5)

The conservation laws also yield,

$$n_s^2 = n_n^2 \frac{(\varepsilon_s + p_s)(\varepsilon_s + p_n)}{(\varepsilon_n + p_n)(\varepsilon_n + p_s)}.$$  \hspace{1cm} (6)

3 Equations of State

To make progress, we have to adopt specific forms for the equations of state of NM and SM. We will assume the ideal gas approximation
for the quark SM with temperature $T$ and chemical potential $\mu$, with vanishing quark masses $m_q = 0$ and a strong coupling constant $\alpha_s = 0$. The MIT bag model equation of state (EOS) [16] for this matter is expressed as follows\footnote{For the SM, $T$, $\mu$ are measured in fm$^{-1}$ and $\varepsilon$ and $P$ in fm$^{-4}$, while for the NM $\varepsilon$ and $p$ are in Mev fm$^{-3}$.},

\[
\begin{align*}
\varepsilon_s &= \frac{19}{12} \pi^2 T^4 + \frac{9}{2} T^2 \mu^2 + \frac{9}{4 \pi^2} \mu^4 + B, \\
p_s &= \frac{19}{36} \pi^2 T^4 + \frac{3}{2} T^2 \mu^2 + \frac{3}{4 \pi^2} \mu^4 - B; \\
n_s &= T^2 \mu + \frac{1}{\pi^2} \mu^3, \\
s_s &= \frac{19}{9} \pi^2 T^3 + 3T \mu^2,
\end{align*}
\]

where $B$ is the MIT bag constant. The sound speed in such matter is defined as $(\partial p/\partial \varepsilon)_s \equiv c_s^2$. It is easy to see that the sound speed is $c_s = 1/\sqrt{3}$.

For the NM we will work with the following three forms for the EOS,

- the zero-temperature ideal Fermi-Dirac (FD) neutron gas EOS [17]
  \[
  \varepsilon_n = m_n n_n + 119 n_n^{5/3}, \tag{8}
  \]
  \[
  p_n = 79.3 n_n^{5/3}; \tag{9}
  \]

- the zero-temperature Bethe-Johnson (BJ) EOS [17]
  \[
  \varepsilon_n = 236 n_n^{2.54} + m_n n_n, \tag{10}
  \]
  \[
  p_n = 364 n_n^{2.54}; \tag{11}
  \]
  where $m_n$ is neutron mass in MeV.

- finite temperature Lattimer - Ravenhall (LR) EOS [18]
  \[
  \varepsilon_n = \left( \frac{2m^*}{h^2 \beta} \right)^{3/2} F_{1/2}(y), \tag{12}
  \]
  \[
  \tau = \frac{1}{2 \pi^2} \left( \frac{2m^*}{h^2 \beta} \right)^{5/2} F_{3/2}(y), \tag{13}
  \]
  \[
  m^* \text{ is effective mass of the neutron given by}
  \]
  \[
  h^2/2m^* = d_1 + d_2 n_n, \tag{14}
  \]
  and $F_i(y)$ is Fermi integral,
  \[
  F_i(y) = \int_0^\infty \frac{u^i du}{\exp(u+y) + 1}. \tag{15}
  \]
The coefficients have the values $d_1 = 20.75 \text{ Mev fm}^2$, $d_2 = -8 \text{ Mev fm}^5$, $d_3 = -752.27 \text{ Mev fm}^3$, $d_4 = 466.6 \text{ Mev fm}^6$; $\beta = 1/k_B T$.

Fig. 1 is an illustration of these three choices for the NM EOS.

4 Constraints

For a given set of the the parameters, $T_n$, $T_s$, $n_n$, and $B$, the EOS of the NM uniquely determines $\varepsilon_n$ and $p_n$, while the SM EOS can be used to obtain $\varepsilon_s$ and $p_s$ as a function of $\mu$. Further, by substitution in the relation (6) the value of strange matter chemical potential $\mu$ may be derived. Typically, there are several roots for (6), but only one satisfying the physical condition $\mu > 0$. Once $\mu$ is obtained the relations (4) and (5) yield $v_n^2$ and $v_s^2$, respectively. For any given $T_n$, $T_s$, $n_n$, and $B$ combustion is possible whenever $v_n^2$ and $v_s^2$ are positive and less than unity. Once this occurs three modes of combustion are identified as follows:

- detonation: $p_s > p_n$, $c_n \leq v_n \leq 1$ and $v_s \leq c_s$;
- deflagration: $v_n < c_n$ and $p_s < p_n$;
- “super-sonic” combustion $p_s > p_n$, $c_n \leq v_n \leq 1$ and $c_s < v_s < 1$

Figs. (2)-(4) illustrate the regions in the $n_n - B$ plane in which the various combustion modes occur, for a few representative values of $T_s$. Fig.2 shows results for the FD EOS. We can see that all modes of combustion may be realized for a wide range of the parameters. Regions which are not explicitly labeled represent physically unacceptable solutions having any of the following cases: imaginary baryon flux; $v_n < c_n$, $v_q > c_n$ (absolutely unstable combustion); $p_s < p_n$ and $v_n > c_n$; $p_s > p_n$ and $v_n < c_n$ (see [15]).

Fig. 3 presents the combustion regions for BJ EOS for different temperatures. We can see again conditions for deflagration, detonation and “super-sonic” combustion. Results for LR EOS are shown in Fig.4. They only slightly differ from those of BJ EOS.

5 Discussion

We have shown that conversion of neutrons into strange quark matter in a combustion mode with a well defined front is possible for a wide range of the parameters, $T_s$, $n_n$, $T_n$, and $B$. The concrete type of the process depends on the values of these parameters and the equation of state of neutron. The results of the current work differ from paper [14] which assume that the energy per baryon in the strange matter $(\varepsilon_s/n_s)$ must be less than the energy per baryon $(\varepsilon_n/n_n)$ in neutron matter. However, the energies $\varepsilon_s$ and $\varepsilon_n$ contain thermal contributions in addition to the rest energies of the baryons and therefore the condition, $\varepsilon_n/n_n > \varepsilon_s/n_s$, does not necessarily hold in general.

In the current paper we have treated $T_s$ as a free parameter. Physically, the value of $T_s$ is affected by the details of the initial conditions for the conver-
Fig. 2. The different regions of combustion of FD NM into SM with $T_s = 10$ and $T_s = 60$ MeV as indicated in the panels. The regions of the detonation, “super-sonic” combustion (from the viewpoint $v_s > c_s$), and deflagrations are bounded by solid lines, dotted line, and dashed lines, respectively. Regions which are not explicitly labeled represent physically unacceptable solutions.

Fig. 3. Combustion regions for BJ EOS NM $T_s = 47$ (panel to the left) and $T_s = 60$ MeV (to the right). The notation of the lines is the same as in the previous figure.

sion process. In a subsequent paper, we will discuss in detail, possible constraints on this parameter assuming that seed strange matter is initially generated by means of quantum nucleation [21,22].

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Fig. 4. The combustion regions for the LR EOS with $T_s = 60$ Mev and $T_n = 0$ (panel to the left) and $T_n = 30$ Mev (to the right). The notation of the line is the same as in the two previous figures.

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