Modelling of Transduction Heaters Using Transformer Equivalent Circuit and Finite Element Analysis

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Abstract: This paper presents the modelling of transduction heaters using the TEC (transformer equivalent circuit) model and FEA (finite element analysis). Each model was used to simulate a set of transduction heating experiments and the results compared. Analysis of the TEC calculated results suggested modification of three parameters: the secondary resistance, the core tube eddy current resistance and the core tube magnetizing reactance. The improved TEC model was then used to design, build and test a 6 kW transduction heater. The measured results are compared with calculated results from the TEC and FEA models. The TEC model accurately predicts the performance of the heater.

Key words: Induction heating, transduction heating, transformer equivalent circuit, finite element analysis.

1. Introduction

Consumer demand for safe, efficient, and less energy consuming products is rising. Induction heating has been widely used for different applications such as heating of metals to heating of dairy fluids [1-3], because of its preciseness of application which satisfy these consumer demands. Electric water heaters have been the focus of several previous studies because of their pervasiveness in power systems and their consequential potential importance when considering conservation of energy through more efficient design and operation [4].

Heating fluid using induction heating can be achieved by passing the fluid through a hollow ferromagnetic workpiece as shown in Fig. 1. The removal of the centre of a workpiece in Fig. 1 has little effect on the magnetics and hence the current drawn from the supply, as long as the thickness of the tube is greater than the skin depth of the magnetic field and hence the steel is not in saturation.

Alternatively, the heating can be achieved by passing the fluid over the workpiece, on the outside, or between two tubes of different diameters such that they form a jacket through which the fluid flows.

An alternative to induction heating of fluids is transformer heating, as shown in Fig. 2. The fluid gains heat as it passes through the hollow tube of the shorted secondary, or through a cooling channel adhered to the back of a solid conductor.

The technologies of a transformer heater and an induction heater can be combined as illustrated in Fig. 3. An alternative concept is to combine these technologies to form a single device, the “transduction fluid heater” as shown in Fig. 4. The conceptual arrangement is a modification of an early commercial manifestation of such a device, the “Transflux” heater [5].

The arrangement is to have a primary winding that does not produce much heat, a core tube that produces
heat, and a shorted secondary winding that produces most of the heat. By making the secondary conductor hollow, or by adding a fluid channel to the back of its conductor, the real power losses in the secondary tube can be transferred by convection to the fluid, thus heating it up.

This paper presents the modelling of transduction heaters using both a TEC model and FEA. Results from these two models are compared with performance testing of an actual transduction heater with various core and secondary tubes. The measured performances were then used to improve the performance prediction of the TEC model. A 6 kW transduction heater was then designed using the improved TEC model. The calculated TEC and FEA results are compared with the measured results of the built device.

2. Transduction Heater

A manifestation of the conceptual design of Fig. 4 can be achieved by placing a highly conductive tube (such as copper or aluminium) on the outside of the core tube and passing the fluid through the core tube. If a suitable size tube is not available, such a conductor can be rolled from sheet metal and bonded to the core tube. In this case the secondary is effectively a short circuited single turn. This does not alter the characteristics of the heater and the same equivalent circuit can be used to evaluate the performance of the heater.

One option is for the fluid to make a single pass up through the centre of the workpiece. Thus the heat from the secondary and the workpiece tubes conducts radially inwards to the fluid. However, since the primary winding is electrically and hence thermally insulated from the core and the secondary tubes, not much of the heat generated in the primary winding is captured by the fluid. Most is radiated and convected outwards to ambient surroundings.

Alternatively, the fluid can be passed between the core and secondary tubes, and the primary winding former. This helps capture some of the primary winding heat. However, again some heat goes to the
ambient surroundings. Thus the transduction fluid heater is less than 100% efficient. While the natural cooling of the primary winding by the ambient surroundings may be sufficient for continuous operation, some form of forced cooling may be required if higher temperatures are involved.

A cut-away image of the transduction heater, where the fluid passes between the bonded secondary and core tubes and the primary winding former, is shown in Fig. 5.

Stainless steel liners inside the primary winding former and outside (or instead of) the secondary winding, allow heating of corrosive fluids, or fluids where non-contamination is critical, without significantly affecting the electrical to thermal performance of the device. This is because the relative permeability of stainless steel is effectively unity and the depth of penetration of the magnetic field is large allowing the magnetic fields essentially to pass straight through and into the ferromagnetic workpiece.

Because of the strength of the materials involved and the robustness of the manufacture, with this technology, relative high fluid pressures and temperatures can be achieved.

Laminated steel placed inside the core tube improves the power factor of this device if the core tube steel is in saturation. This can also help counter the decreased power factor due to the increased spacing between the primary and the secondary windings, because of the stainless steel insertions and the fluid channel.

3. Measured Performance

A coil of a small transduction heater was constructed with the following parameter dimensions are shown in Table 1.

A number of mild steel, stainless steel, aluminium, and copper tubes of various diameters and thicknesses were cut to provide workpieces and secondary tubes that fitted inside the coil. The dimensions of these are listed in Table 2.

Various combinations of the tubes were assembled inside the coil. Fig. 6 shows a photo of the transduction heater with combination of 2 + 5 tubes inside the coil.
Table 3  Measured performance at 100 V supply.

| Tubes | Voltage (V) | Current (A) | Real power (W) | Apparent power (VA) | Power factor |
|-------|-------------|-------------|----------------|---------------------|-------------|
| 1 + 4 | 100         | 2.4         | 110            | 239                 | 0.46        |
| 2 + 5 | 100         | 5.1         | 370            | 470                 | 0.78        |
| 3 + 6 | 100         | 5.2         | 410            | 490                 | 0.82        |
| 3 + 9 | 100         | 4.0         | 320            | 380                 | 0.83        |
| 7 + 9 | 100         | 3.8         | 320            | 370                 | 0.87        |
| 8 + 9 | 100         | 3.8         | 320            | 370                 | 0.88        |

Their performances are given in Table 3. These values were used as a benchmark to compare calculated performance values against.

4. Equivalent Circuit

The transduction heater can be modelled as a transformer with a single shorted turn secondary winding, and the core tube as a partial core of a transformer [6]. The core tube has the function of acting like the core of a transformer, but it is incomplete in that the return path of the magnetic flux is through the ambient medium. The equivalent circuit that is used to model the transduction heater is shown in Fig. 7, where $R_1$, $R_2$ are the primary and secondary resistances, $X_{mc}$ is the core tube magnetizing reactance, $R_{ce}$ is the core tube eddy current resistance, and $X_1$, $X_2$ are the primary and secondary leakage reactances. Hysteresis losses, usually the dominant part of core losses in a transformer, are ignored in this model as they are insignificant compared to the eddy current losses.

The parameters of the equivalent circuit in Fig. 7 are calculated using the basic dimensions and physical characteristics of materials used, as per the reverse design method [6, 7]. These are entered as known data and then the individual circuit components are calculated using magnetic and electric circuit component models.

4.1 Winding Resistance

The primary winding resistance is given by:

$$R_1 = \rho_{1w} \left( \frac{I_{1w}}{A_{1w}} \right)$$

where, $\rho_{1w}$ is the resistivity, $I_{1w}$ is the length and $A_{1w}$ is the cross sectional area of the primary winding conductor.

The secondary winding resistance referred to the primary winding is:

$$R_2 = \rho_{2w} \left( \frac{I_{2w}}{A_{2w}} \right) a^2$$

where, $\rho_{2w}$ is the resistivity, $I_{2w}$ is the length of the secondary tube, $A_{2w}$ is the cross sectional area of the secondary tube, and $a$ is the primary/secondary turns ratio.

The variation of resistivity with temperature of the conductor materials are accounted for. In general, the operating resistivity of a material is:

$$\rho_c = \left( 1 + \Delta \rho (T_e - 20) \right) \rho_{20°C}$$

where, $\rho_{20°C}$ is the resistivity at 20 °C, $T_e$ is the material temperature, and $\Delta \rho$ is the thermal resistivity coefficient.

4.2 Magnetizing Reactance

The magnetizing reactance is:

$$X_{mc} = \frac{\epsilon_0 N_t^2 \mu_{rel} A_c}{l_c}$$

where, $N_t$ is the number of turns on the primary winding, $A_c$ is the effective cross sectional area of the core tube, calculated using the depth of penetration, $\delta_c$ and $l_c$ is the flux path length. Also, $\mu_{rel}$ is the effective value based on the composite air/iron magnetic flux
path, as for the transformer model of an induction heater [8].

If the magnetic flux density in the core tube exceeds the saturation value, the relative permeability reduces, forcing the flux skin depth to increase and the density to reduce. If the calculated skin depth is greater than the thickness of the core tube, then the flux flows through both the core tube metal and inside fluid space. This is also modelled as for the transformer model of an induction heater [8].

4.3 Core Tube Eddy Current Resistance

In this model the eddy current losses in the core tube are accounted for by the resistance $R_{ce}$ across $\mathcal{X}_{mc}$. $R_{ce}$ is calculated from:

$$R_{ce} = N_1^2 \rho_c \frac{l_e}{A_{ce}} \tag{5}$$

where, $\rho_c$ is the core tube resistivity, $l_e$ is the length of the eddy current path around the core tube, and $A_{ce}$ is the cross sectional area of the eddy current path, the product of $l_e$ and $\delta_c$. Again, the variation of resistivity with temperature of the conductor material is accounted for.

4.4 Leakage Reactance

The primary and secondary leakage reactances of a transformer are calculated from the total leakage reactance which embodies the effects of both windings together. The leakage reactances $X_1$ and $X_2$ are calculated from:

$$X_1 = X_2 = \frac{\mu_0 N_1^2 \pi \tau_{12}}{2l_e} \tag{6}$$

where, $\tau_{12}$ is the winding thickness factor, related to the thickness of the primary winding and the secondary tube [8].

5. Calculated Performance

The equivalent circuit of the transduction heater can be solved to calculate the performance of the device.

5.1 Material Properties

The physical characteristics of the copper, stainless steel, aluminium and mild steel tubes were taken as given in Table 4. The same combinations of tubes as assembled for the measurements were used and the parameters of the equivalent circuits calculated from their material characteristics and dimensions. The performance of the heater was calculated by solving the equivalent circuit equations and determining the current, power and power factor for a given applied voltage. The equivalent circuit also allowed separation of the real power losses into individual component values. In addition, as required, efficiencies, voltage gradients, voltages per turn and current densities can be calculated.

The calculated results are given in Table 5.

In general, the calculated real powers are significantly higher than those measured. This is reflected in the higher current values and apparent power. This suggested that the overall calculated impedance was low compared to the measured value. However, the power factors were similar.

Within the detail of the calculated results, one specific anomaly is that the losses in the core tube are significantly higher than those in the secondary tube for the $(1 + 4)$ combination. However for the rest of the combinations, the losses in the secondary tube are significantly higher than those in the core tube. Also, the primary winding losses are significantly lower compared to the secondary tube losses for all the combinations except for $(1 + 4)$. This anomaly may be due to an incorrect assumed resistivity of the stainless steel.

Analysis of the TEC calculated results in comparison to the measured values suggested empirical modification of the parameters to bring the equivalent circuit into alignment. The inclusion of scaling factors has been successfully applied in similar related work [9, 10].

The secondary resistance $R_2$, the core tube eddy current resistance $R_{ce}$, and the core tube magnetizing reactance $X_{mc}$ were all scaled by a factor of 1.8. The secondary resistance $R_2$ is modified as the magnetic coupling between the primary and the secondary is not
Table 4  Physical properties.

| Material          | Copper | Mild steel | Stainless steel | Aluminium |
|-------------------|--------|------------|-----------------|-----------|
| Resistivity ($\Omega m \times 10^{-8}$) | 1.72   | 16         | 70              | 2.65      |
| Relative permeability | 1      | 750        | 1               | 1         |
| Skin depth at 50 Hz (mm) | 9.3    | 1.0        | 60              | 11.6      |
| Density (kg/m³)    | 8,960  | 7,870      | 8,000           | 2,700     |

Table 5  Calculated performance using TEC model for a 100 V supply.

| Workpiece | Voltage (V) | Current (A) | Primary loss (W) | Secondary loss (W) | Core loss (W) | Real power (W) | Apparent power (VA) | Power factor |
|-----------|-------------|-------------|------------------|-------------------|---------------|----------------|---------------------|--------------|
| 1 + 4     | 100         | 3.9         | 36.7             | 18.1              | 165.3         | 220.1          | 398.0               | 0.55         |
| 2 + 5     | 100         | 7.6         | 135.2            | 313.1             | 109.2         | 557.5          | 765.0               | 0.73         |
| 3 + 6     | 100         | 7.2         | 118.8            | 308.9             | 133.8         | 561.5          | 717.0               | 0.78         |
| 3 + 9     | 100         | 6.9         | 111.1            | 310.3             | 157.3         | 578.7          | 693.0               | 0.83         |
| 7 + 9     | 100         | 5.8         | 77.4             | 337.9             | 64.2          | 479.5          | 579.0               | 0.83         |
| 8 + 9     | 100         | 5.7         | 75.8             | 339.4             | 59.1          | 474.3          | 573.0               | 0.83         |

The remaining parameters $R_1$, $X_1$, and $X_2$ are left unchanged. The calculation was close to the measured value so it was not modified. The leakage reactances $X_1$, and $X_2$ are left unchanged as the magnetic flux is through the ambient medium and therefore not affected by the core tube.

Table 6 lists the calculated performances using the TEC model with the scale factor applied. The current, primary winding loss, secondary winding loss, core loss, and apparent power are significantly lower than those in Table 5. The power factors have, slightly, increased.

The calculated results are now much closer to the measured values.

6. Finite Element Analysis

MagNet software [11] was used to create a 2D FEA model of the actual heater. Fig. 8 shows examples of the flux plots for the workpiece combinations of (1 + 4) and (2 + 5). The coil is energised with a 100 V supply. The maximum flux density in the workpiece is 1.7 T. A small amount of fringing can be seen at both ends of the workpiece. Most of the flux is confined to the region near the surface of the mild steel workpiece. There is very little field in the interior of the workpiece as the radius is several times greater than the skin depth of the workpiece.

Table 7 shows the performance of the experimental heater at a 100V supply voltage, calculated using FEA. All the workpiece maximum flux densities are at the
results. The coil and workpiece dimensions of the designed heater are given in Tables 8 and 9.

Fig. 9 shows a photo of the designed heater with the mild steel and secondary copper tubes inside the coil. The physical properties of the mild steel and copper tubes are given in Table 6.

Table 8 Coi dimensions.

| Material             | Copper   | Unit          |
|----------------------|----------|---------------|
| Resistivity          | $1.72 \times 10^{-5}$ | $\Omega$ m |
| Length               | 420      | mm            |
| Outside diameter     | 88       | mm            |
| Inside diameter      | 83       | mm            |
| Wire diameter        | 3.4      | mm            |
| Wire cross-sectional area | $9.08 \times 10^{-6}$ | m$^2$ |
| Number of layers     | 7        | -             |
| Turns per layer      | 140      | -             |
| Number of turns      | 980      | -             |
| Length of wire       | 289      | m             |
| Calculated resistance| 0.55     | $\Omega$      |
| Measured resistance  | 0.54     | $\Omega$      |

Table 9 Workpiece dimensions.

| Workpiece            | Length (mm) | Diameter (mm) | Thickness (mm) |
|----------------------|-------------|---------------|----------------|
| 1 Mild steel core tube | 500        | 60.0          | 4.6            |
| 2 Secondary copper tube | 500       | 62.6          | 1.3            |

Fig. 9 6 kW transduction fluid heater.
### Table 10  Component values for the TEC model.

| Circuit component | Value (Ω) |
|-------------------|-----------|
| $R_1$             | 0.55      |
| $R_2$             | 8.74      |
| $X_1$             | 1.24      |
| $X_2$             | 1.24      |
| $X_{nc}$          | 35.8      |
| $R_{ee}$          | 27.8      |

### Table 11  Calculated and measured results of the designed transduction heater.

|                | TEC   | FEA   | Measured |
|----------------|-------|-------|----------|
| Voltage (V)    | 230   | 230   | 230      |
| Current (A)    | 30.9  | 43.9  | 31.0     |
| Apparent power (kVA) | 7.13  | 10.1  | 7.13     |
| Real power (kW) | 6.5   | 8.4   | 6.3      |
| Power factor   | 0.91  | 0.83  | 0.91     |

The various components of the TEC circuit are given in Table 10.

The calculated results of the transduction heater using the TEC and FEA models are given in Table 11, along with the measured performance.

The calculated results from the TEC match the measured results well, reinforcing the usefulness of the TEC model to predict the performance of transduction heaters. The calculated results from the FEA model show significantly higher values in terms of the current, apparent power, and real power compared to the results from the TEC. However, the power factor is significantly lower than the TEC and measured results.

### 8. Conclusions

This paper presents the modelling of transduction heaters using TEC and FEA models. The performances of the transduction heaters from these models were compared to the measured performances of an experimental heater for a variety of sizes of mild steel, copper, aluminium and stainless steel tubes. The calculated performances of the TEC model showed significantly lower values in terms of the current and workpiece losses compared to the FEA model. The TEC model was improved by applying an empirically derived scale factor to selected components to give much closer values to those measured.

The improved TEC model was then used to design a new transduction heater. The calculated and measured performances were very close. The TEC model gave a better prediction than the FEA model, suggesting that the TEC model can be a useful tool to design transduction heaters.

The research will now move to the design of an industrially useful device.

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