Flavour leptogenesis with tribimaximal mixings and beyond

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Abstract

We compute and compare the baryon asymmetry of the universe in thermal leptogenesis scenario with and without flavour effects for different neutrino mass models namely degenerate, inverted hierarchical and normal hierarchical models, with tribimaximal mixings and beyond. Considering three possible diagonal forms of Dirac neutrino mass matrices $m_{LR}$, the right-handed Majorana mass matrices $M_{RR}$ are constructed from the light neutrino mass matrices $m_{LL}$ through the inverse seesaw formula. The normal hierarchical model is found to give the best predictions of the baryon asymmetry for both cases. This analysis serves as an additional information in the discrimination of the presently available neutrino mass models. Moreover, the flavour effects is found to give enhancement of the baryon asymmetry in thermal leptogenesis.

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1 Introduction

The existence of heavy right-handed Majorana neutrinos in some of the left-right symmetric GUT models, not only gives small but non-vanishing neutrino masses through the celebrated seesaw mechanism[1], it also plays an important role in explaining the baryon asymmetry of the universe [2, 3]. Such an asymmetry can be dynamically generated if the particle interaction rate and the expansion rate of the universe satisfy Sakharov’s three famous conditions [4]. Majorana right-handed neutrinos satisfy the second condition i.e., C and CP violation as they can have an asymmetric decay to leptons and Higgs particles, and the process occurs at different rates for particles and antiparticles. The lepton asymmetry is then partially converted to baryon asymmetry by electroweak sphaleron process [5, 6, 7, 8, 9, 10].

In order to calculate the baryon asymmetry from a given neutrino mass model, one usually starts with the light neutrino mass matrices $m_{LL}$ and then relates it with the heavy Majorana neutrino mass matrices $M_{RR}$ and the Dirac neutrino mass matrix $m_{LR}$ through the inverse seesaw mechanism in an elegant way. We consider the Dirac neutrino mass matrix $m_{LR}$ as either the charged lepton mass matrix, down-quark mass matrix or up-quark mass matrix for phenomenological analysis. The complex CP violating phases necessary for lepton asymmetry are usually derived from the MNS leptonic mixing matrix. In the present work we are interested to consider the complex Majorana phases which are derived from the right-handed Majorana mass matrix $M_{RR}$, in the estimation of baryon asymmetry of the universe. We consider the left-handed light Majorana neutrino mass matrices $m_{LL}$ which obey the $\mu - \tau$ symmetry, where tribimaximal mixings and below are realised [11, 12], for all possible patterns of neutrino masses, viz, degenerate, inverted hierarchical and normal hierarchical mass patterns. We first parametrise the light left-handed Majorana neutrino mass matrices which are subjected to correct predictions of neutrino mass parameters and mixing angles. The calculation of baryon asymmetry may serve as an additional information to further discriminate the correct pattern of neutrino mass models and also shed light on the structure of Dirac neutrino mass matrix.

In section 2 we briefly mention the formalism for estimating the lepton asymmetry in flavoured thermal leptogenesis through the “out-of-equilibrium” decay of the heavy right-handed Majorana neutrinos and also discuss briefly
on $\mu - \tau$ symmetry with Tribimaximal mixings (TBM) as a special case. Section 3 is devoted to the numerical calculation and results. Finally in section 4 we conclude with a summary and discussions. Important expressions related to $m_{LL}$ which obey $\mu - \tau$ symmetry for three neutrino mass models, are relegated to Appendix A.

\section{Flavoured Thermal leptogenesis}

The canonical seesaw formula \cite{1} relates the left-handed Majorana neutrino mass matrix $m_{LL}$ and heavy right handed Majorana mass matrix $M_{RR}$ in a simple way

\begin{equation}
    m_{LL} = -m_{LR}M_{RR}^{-1}m_{LR}^{T}
\end{equation}

where $m_{LR}$ is the Dirac neutrino mass matrix. For our calculation of lepton asymmetry, we consider the model\cite{5, 6, 7} where the asymmetric decay of the lightest of the heavy right-handed Majorana neutrinos, is assumed. The physical Majorana neutrino $N_R$ decays into two modes:

\begin{equation*}
    N_R \rightarrow l_L + \phi^\dagger \\
    \rightarrow \bar{l}_L + \phi
\end{equation*}

where $l_L$ is the lepton and $\bar{l}_L$ is the antilepton and the branching ratio for these two decay modes is likely to be different. The CP-asymmetry which is caused by the interference of tree level with one-loop corrections for the decays of lightest of heavy right-handed Majorana neutrino $N_1$, is defined by \cite{6, 8}

\begin{equation*}
    \epsilon = \frac{\Gamma(N_1 \rightarrow l_L\phi^\dagger) - \Gamma(N_1 \rightarrow \bar{l}_L\phi)}{\Gamma(N_1 \rightarrow l_L\phi^\dagger) + \Gamma(N_1 \rightarrow \bar{l}_L\phi)}
\end{equation*}

where $\Gamma = \Gamma(N_1 \rightarrow l_L\phi^\dagger)$ and $\bar{\Gamma} = \Gamma(N_1 \rightarrow \bar{l}_L\phi)$ are the decay rates.

In this section we study the flavour effects in leptogenesis \cite{2} in the context of our neutrino mass models in \cite{11, 12} (see Appendix A for details). Earlier leptogenesis calculations were done by studying Boltzman Equations (BE) for the B-L asymmetry. But later \cite{13} studied flavour $B - L_\alpha$ asymmetries where the results were significantly different from the "single flavour approximation". Subsequently many authors \cite{14, 15, 16} have included flavour effects to enhance the baryon asymmetry in particular models. In thermal
leptogenesis the importance of flavour effects comes from the wash-out effects, where scattering produces $N_1$ population of neutrinos at temperature $T \simeq M_1$. When $T$ drops below $M_1$, this $N_1$ population decays to leptons and if these decays are CP violating, it can produce asymmetries in all the lepton flavours. If the interactions are "out-of-equilibrium", then the above asymmetries would survive.

In thermal leptogenesis [2] the Yukawa coupling constant related to the production of $N_1$ also controls the decay of $N_1$. Initially it seems that both the CP asymmetries will be washed out leaving no lepton asymmetry. However, a net asymmetry survives after the potential cancellation of CP asymmetry between processes with $N_1$ and $l_{\alpha}(\alpha = e, \mu, \tau)$ in the final state such as $X \rightarrow N l_{\alpha}$ scattering and $N$ in the initial state and $l_{\alpha}$ in the final state, such as $N \rightarrow \phi l_{\alpha}$. Only processes with $l_{\alpha}$ in the final state can produce the asymmetry. There is no cancellation between asymmetries produced in the decays and inverse decays. Any initial asymmetry produced with the $N$ population is depleted by scattering, decays and inverse decays. This depletion is called washout. The initial state of washout contains a lepton, so it is important to know which leptons are distinguishable. It is always assumed that interactions whose timescale is very different from the leptogenesis scale are dropped out from the Boltzman Equations(BE) [17, 18, 19, 20].

In the interaction Lagrangian the different flavours are distinguished by their Yukawa couplings $h_{\alpha}$. Thus if the $h_{\alpha}$ mediated interactions are fast compared to the leptogenesis scale and the universe expansion rate, these distinguishable $h_{\alpha}$ will have induced differences in the thermal masses of different leptons as each of $h_{e, \mu, \tau}$ has different strengths. Thus when the charge lepton Yukawa interactions are fast then flavour basis is the correct basis for the BE, otherwise leptogenesis has no knowledge of the lepton flavour for 'slow interactions'.

In the flavour basis the equation for the lepton asymmetry in $N_1$ decay becomes ($\alpha = e, \mu, \tau$),

$$\epsilon_{\alpha\alpha} = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{11}} \left[ \sum_{j=2,3} Im \left[ h_{\alpha1}^*(h^\dagger h)_{1j} h_{\alpha j} \right] g(x_j) + \sum_j Im \left[ h_{\alpha1}^*(h^\dagger h)_{j1} h_{\alpha j} \right] \frac{1}{(1-x_j)} \right]$$  \hspace{1cm} (2)
The efficiency factor for “out-of-equilibrium” situation i.e., $\Gamma_{ID} < H = 1.66 \sqrt{g_* T^2/m_p}$ is given by
\begin{equation}
\eta_\alpha \equiv \frac{m_*}{\bar{m}_{\alpha\alpha}}
\end{equation}
where $m_* = 8\pi \frac{v^2}{M_1} H \sim 1.1 \times 10^{-3} eV$ [2] and
\begin{equation}
\bar{m} = \frac{h_\alpha^\dagger h_\alpha v^2}{M_1}.
\end{equation}

For SM we have
\begin{equation}
v = 174 GeV, \quad g_* = 106.75.
\end{equation}

The second term in Eq. [2] violates the single lepton flavours but conserves the total lepton number. It vanishes when summed over flavours. Thus,
\begin{equation}
\epsilon \equiv \sum_\alpha \epsilon_{\alpha\alpha} = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{11}} \sum_j Im \left[ (h^\dagger h)_{1j}^2 \right] g(x_j).
\end{equation}

Thus in the strong washout case for all flavours, we obtain the baryon asymmetry i.e; baryon-to-entropy [2] ratio as,
\begin{equation}
Y_{3B} \sim 10^{-3} \sum_\alpha \eta_\alpha \epsilon_{\alpha\alpha} \sim 10^{-3} m_* \sum_\alpha \frac{\epsilon_{\alpha\alpha}}{\bar{m}_{\alpha\alpha}}.
\end{equation}

For single flavoured case, one can consider the direction in flavour space into which $N_1$ decays. In single flavour case the baryon asymmetry is given by
\begin{equation}
Y_{1B} \sim 10^{-3} m_* \frac{\epsilon}{\bar{m}},
\end{equation}
where
\begin{equation}
\epsilon = \sum_\alpha \epsilon_{\alpha\alpha}, \quad \bar{m} = \sum_\alpha \bar{m}_{\alpha\alpha}.
\end{equation}

The entropy density $s$ of the universe can be related to the photon number density $n_\gamma$ as $s = 7.04 n_\gamma$. So baryon-to-entropy ratio is estimated to be around $\sim 8.74 \times 10^{-11}$ [3].
2.1 Neutrino mass models with $\mu - \tau$ symmetry: Tribimaximal mixings

The recent global $3\nu$ oscillation analysis [21] indicates towards a specific form of leptonic mixing - Tribimaximal mixing and a slight deviation from tribimaximal mixing pattern which is a special case of $\mu - \tau$ symmetry. The $\mu - \tau$ reflection symmetry in the neutrino mass matrix, implies an invariance under the simultaneous permutation of the second and third rows as well as the second and third columns in neutrino mass matrices [22, 23, 24, 25, 26, 27, 28].

$$m_{LL} = \begin{pmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{pmatrix}.$$  \hspace{1cm} (11)

This has the permutation symmetry, $Pm_{LL}P = m_{LL}$, where

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \hspace{1cm} (12)$$

Neutrino mass matrix in eq.(11) predicts the maximal atmospheric mixing angle, $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. However the prediction on solar mixing angle $\theta_{12}$ is arbitrary, and it can be fixed by the input values of the parameters present in the mass matrix. Thus

$$\tan 2\theta_{12} = \left| \frac{2\sqrt{2}Y}{(X - Z - W)} \right| \hspace{1cm} (13)$$

which depends on four input parameters $X, Y, Z$ and $W$. This makes us difficult to choose the values of these free parameters for a solution consistent with neutrino oscillation data. This point is addressed in [11, 12] where the solar angle is made dependent only on the ratio of two parameters, $\eta/\epsilon$. Such parametrization of the mass matrix enables us to analyse the neutrino mass matrix in a systematic and economical way [29]. The actual values of these two new parameters will be fixed by the data on neutrino mass squared differences.

The MNS leptonic mixing matrix $U_{MNS}$ which diagonalises $m_{LL}$ is defined by $m_{LL} = U_{MNS}D U_{MNS}^\dagger$ where $D = \text{diag.}(m_1, m_2, m_3)$, and

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}. \hspace{1cm} (14)$$
From the consideration of $\mu$-$\tau$ reflection symmetry, $U_{MNS}$ has the following general properties [22], $|U_{\mu i}| = |U_{\tau i}|$, $|U_{\mu i}|^2 = (1 - |U_{ei}|^2)/2$ where $i = 1, 2, 3$. For $i = 3$, $|U_{\mu 3}|^2 = (1 - |U_{e 3}|^2)/2$. For $|U_{e 3}| = 0$, we have $|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2}$. The MNS mixing matrix is generally parametrised by three rotations ($\theta_{23} = \pi/4$, $\theta_{13} = 0$):

$$U_{MNS} = O_{23}O_{13}O_{12} = O_{23}O_{12} = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \\ s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

(15)

where $c_{12} = \cos \theta_{12}$, $s_{12} = \sin \theta_{12}$. Tri-bimaximal mixing (TBM) is a special case with $c_{12} = \sqrt{2}/3$ and $s_{12} = \sqrt{1/3}$ [27], [28],

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

(16)

where

$$O_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix},$$

(17)

and

$$O_{12} = \begin{pmatrix} \sqrt{2/3} & -1/\sqrt{3} & 0 \\ 1/\sqrt{3} & \sqrt{2/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

(18)

For completeness we also give the three neutrino mass eigenvalues [25] corresponding to the neutrino mass matrix in eq.(11),

$$-m_1 = \frac{1}{2}[Z + W + X - \sqrt{8Y^2 + (Z + W - X)^2}],$$

(19)

$$m_2 = \frac{1}{2}[Z + W + X + \sqrt{8Y^2 + (Z + W - X)^2}],$$

(20)

$$m_3 = (Z - W).$$

(21)

The solar mixing angle is given by $\cos \theta_{12} = \sqrt{m_2 + X/m_1 + m_2}$, $\sin \theta_{12} = \sqrt{m_1 - X/m_1 + m_2}$. If $X = 0$, then we have a simple relation, $\tan^2 \theta_{12} = m_1/m_2$. For Tribimaximal mixing we get $\tan^2 \theta_{12} = 0.5$, $\tan^2 \theta_{23} = 1$ and $\tan^2 \theta_{13} = 0$ for particular ‘flavor twister’ term as mentioned in details in our earlier works [11], [29].
3 Numerical estimation of baryon asymmetry

For numerical calculation we first choose the light left-handed Majorana neutrino mass matrix $m_{LL}$ proposed in Appendix A \[11, 12\]. These mass matrices obey the $\mu - \tau$ symmetry which guarantees the tribimaximal mixings.

The Dirac neutrino mass matrix $m_{LR}$ appeared in seesaw formula can have any arbitrary structure which either is diagonal or non-diagonal. In the seesaw mechanism, for a specific structure of $m_{LL}^I$, we can have three possible combinations of $m_{LR}$ and $M_{RR}$: (a) both $m_{LR}$ and $M_{RR}$ are non-diagonal, (b) $m_{LR}$ diagonal and $M_{RR}$ non-diagonal, (c) $m_{LR}$ non-diagonal and $M_{RR}$ diagonal. These three combinations can be realised in different physical situations. For example, when one calculate lepton asymmetry, one needs to consider the diagonal basis of heavy right-handed neutrinos, and combination (c) becomes relevant. In the present calculations, the diagonal form of $m_{LR}$ is chosen for different neutrino mass matrices. To see this let us consider the seesaw relation $m_{LL}^I = -m_{LR}M_{RR}^{-1}m_{LR}^T$, where both $m_{LR}$ and $M_{RR}$ are non-diagonal. Using some left and right handed rotations, the Dirac neutrino mass matrix can be diagonalised \[30\] as

$$m_{LR}^{diag} = U_L m_{LR} U_R^\dagger. \quad (22)$$

In terms of diagonal basis of $m_{LR}$, the seesaw relation reduces to

$$m_{LL}^I = -U_L^\dagger m_{LR}^{diag} M_{RR}^{-1} m_{LR}^{diag} U_R^\dagger, \quad (23)$$

where, $M_{RR}^{-1} = U_R M_{RR}^{-1} U_R^T$. It is assumed that eigenvalues of $m_{LR}^{diag}$ are hierarchical (similar to quarks or charged leptons). In absence of Dirac left handed rotations \[30\], we can set $U_L \sim 1$. For slight deviation from unity we can assume $U_L \approx U_{CKM}$, where $U_{CKM}$ is the quark mixing matrix. Again this can be set to unity as quarks mixings are very small. This type of approximations do not produce significant change in numerical calculations. For $U_L \sim 1$ the eq. (23) reduces to $m_{LL}^I = -m_{LR}^{diag} M_{RR}^{-1} m_{LR}^{diag}$ where $M_{RR}$ is in the diagonal basis of $m_{LR}$. We follow this representation in the present calculation.

In some Grand Unified Theory such as $SO(10)$ GUT, the possible structure of $m_{LR}$ \[31\] can be $m_{LR} = diag(\lambda^m, \lambda^n, 1)v$, where $v$ is the overall scale factor representing electroweak vacuum expectation values. In the present calculation we take $\lambda = 0.3$ and $v = 174\text{GeV}$. We consider three choices of...
(m, n) pair: case(i) \((m, n) \equiv (6, 2)\) for charged lepton, (ii) \((m, n) \equiv (8, 4)\) for up-quark mass matrices and (iii) \((m, n) \equiv (4, 2)\) for down-quark mass matrices representing the Dirac neutrino mass matrix.

For our calculation we choose a basis \(U_R\) where \(M_{RR}^{\text{diag}} = U_R^T M_{RR} U_R\) \(= \text{diag}(M_1, M_2, M_3)\) with real and positive eigenvalues \([30, 32]\) which enters in the expression of CP-asymmetry \(\epsilon_{\alpha\alpha}\) in Eq.\([2]\). The term \(h^\dagger h\) is in the basis where the \(M_R\) is diagonal with real and positive eigenvalues. Using the relation \(h = m_{LR} / v\) and Eq.\([22]\) we get

\[
h^\dagger h = \frac{1}{v^2} U_R^T (m_{LR}^{\text{diag}})^2 U_R, \tag{24}\]

where \(v\) is the electroweak vacuum expectation value(174 GeV).

By this we allow the non-zero elements \(M_i\) of the diagonalised RH neutrino mass matrix \(M_R^{\text{dia}}\) to be complex. The unitary matrix \(U_R\) is defined in such a way that it relates the basis where \(m_{LR}\) is diagonal to the basis where \(M_R\) is diagonal with real and positive non-zero elements.i.e, the phases of \(M_i\) should be included in the definition of \(U_R\).

So, we transform \(m_{LR} = \text{diag}(\lambda_m, \lambda_n, 1)v\) to the \(U_R\) basis by \(m_{LR} \rightarrow m_{LR} U_R\).

The Yukawa coupling matrix \(h = \frac{m_{LR}}{v}\) so constructed, also becomes complex, and hence the term \(\text{Im}(h^\dagger h)_{ij}\) appearing in lepton asymmetry \(\epsilon_{\alpha\alpha}\) gives a non-zero contribution. In our numerical estimation of lepton asymmetry, we choose some arbitrary values of \(\alpha\) and \(\beta\) other than \(\pi/2\) and 0. For example, light neutrino masses \((m_1, -m_2, m_3)\) lead to \(M_{RR}^{\text{diag}} = \text{diag}(M_1, -M_2, M_3)\), and we thus fix the Majorana phase \(Q = \text{diag}(1, e^{i\alpha}, e^{i\beta}) = \text{diag}(1, e^{i(\pi/2+\pi/4)}, e^{i\pi/4})\) for \(\alpha = (\pi/4 + \pi/2)\) and \(\beta = \pi/4\). The extra phase \(\pi/2\) in \(\alpha\) absorbs the negative sign before heavy Majorana mass \(M_2\). In our search programme such choice of the phases leads to highest numerical estimations of lepton CP asymmetry. The corresponding light left-handed neutrino mass matrix obeying \(\mu - \tau\) symmetry, is collected from Appendix A as mentioned.

4 Results and Discussion

The numerical predictions on \(\Delta m_{21}^2\) and \(\Delta m_{23}^2\) of these seven neutrino mass models \(m_{LL}\) under consideration in Appendix A, are presented in Table 1.
Table 1: Predicted values of the solar and atmospheric neutrino mass-squared differences for $\tan^2 \theta_{12} = 0.50$, using $m_{LL}$ given in the Appendix A.

| Type   | $\Delta m^2_{21}[10^{-5}eV^2]$ | $\Delta m^2_{23}[10^{-3}eV^2]$ | $\tan^2 \theta_{12}$ | $\sin^2 2\theta_{23}$ | $\sin \theta_{13}$ |
|--------|---------------------------------|---------------------------------|-----------------------|------------------------|---------------------|
| Deg. (IA) | 7.8                             | 2.6                             | 0.5                   | 1.0                    | 0.0                 |
| Deg. (IB) | 7.9                             | 2.5                             | 0.5                   | 1.0                    | 0.0                 |
| Deg. (IC) | 7.9                             | 2.5                             | 0.5                   | 1.0                    | 0.0                 |
| IH. (IIA) | 7.3                             | 2.5                             | 0.5                   | 1.0                    | 0.0                 |
| IH. (IIB) | 8.5                             | 2.3                             | 0.5                   | 1.0                    | 0.0                 |
| NH. (IIIA) | 7.1                             | 2.1                             | 0.5                   | 1.0                    | 0.0                 |
| NH. (IIIB) | 7.5                             | 2.4                             | 0.5                   | 1.0                    | 0.0                 |

Table 2: Predicted values of the solar and atmospheric neutrino mass-squared differences for $\tan^2 \theta_{12} = 0.45$, using $m_{LL}$ given in the Appendix A.

| Type   | $\Delta m^2_{21}[10^{-5}eV^2]$ | $\Delta m^2_{23}[10^{-3}eV^2]$ | $\tan^2 \theta_{12}$ | $\sin^2 2\theta_{23}$ | $\sin \theta_{13}$ |
|--------|---------------------------------|---------------------------------|-----------------------|------------------------|---------------------|
| Deg. (IA) | 7.6                             | 2.6                             | 0.45                  | 1.0                    | 0.0                 |
| Deg. (IB) | 7.9                             | 2.8                             | 0.45                  | 1.0                    | 0.0                 |
| Deg. (IC) | 7.9                             | 2.5                             | 0.45                  | 1.0                    | 0.0                 |
| Inh. (IIA) | 7.6                             | 2.5                             | 0.45                  | 1.0                    | 0.0                 |
| Inh. (IIB) | 8.4                             | 2.0                             | 0.45                  | 1.0                    | 0.0                 |
| Nh. (IIIA) | 7.7                             | 2.6                             | 0.45                  | 1.0                    | 0.0                 |
| Nh. (IIIB) | 8.0                             | 2.6                             | 0.45                  | 1.0                    | 0.0                 |
| Type | (m,n) | $M_1$GeV | $M_2$GeV | $M_3$GeV |
|------|-------|----------|----------|----------|
| IA   | (4,2) | $1.46 \times 10^{10}$ | $-6.20 \times 10^{11}$ | $2.59 \times 10^{13}$ |
| IA   | (6,2) | $1.22 \times 10^{8}$ | $-6.01 \times 10^{11}$ | $2.59 \times 10^{13}$ |
| IA   | (8,4) | $9.86 \times 10^{5}$ | $-5.03 \times 10^{9}$ | $2.51 \times 10^{13}$ |
| IB   | (4,2) | $5.01 \times 10^{9}$ | $6.16 \times 10^{11}$ | $7.60 \times 10^{13}$ |
| IB   | (6,2) | $4.05 \times 10^{7}$ | $6.16 \times 10^{11}$ | $7.60 \times 10^{13}$ |
| IB   | (8,4) | $3.28 \times 10^{5}$ | $4.99 \times 10^{9}$ | $7.60 \times 10^{13}$ |
| IC   | (4,2) | $5.01 \times 10^{9}$ | $-6.69 \times 10^{12}$ | $6.99 \times 10^{12}$ |
| IC   | (6,2) | $4.05 \times 10^{7}$ | $-6.69 \times 10^{12}$ | $6.99 \times 10^{12}$ |
| IC   | (8,4) | $3.28 \times 10^{5}$ | $-4.83 \times 10^{11}$ | $7.84 \times 10^{11}$ |
| IIA  | (4,2) | $4.01 \times 10^{10}$ | $9.73 \times 10^{12}$ | $6.25 \times 10^{16}$ |
| IIA  | (6,2) | $3.29 \times 10^{8}$ | $9.73 \times 10^{12}$ | $6.25 \times 10^{16}$ |
| IIA  | (8,4) | $2.63 \times 10^{6}$ | $7.94 \times 10^{10}$ | $6.21 \times 10^{16}$ |
| IIB  | (4,2) | $-1.19 \times 10^{11}$ | $2.71 \times 10^{12}$ | $5.59 \times 10^{14}$ |
| IIB  | (6,2) | $-9.97 \times 10^{8}$ | $2.63 \times 10^{12}$ | $5.59 \times 10^{14}$ |
| IIB  | (8,4) | $-8.10 \times 10^{6}$ | $2.14 \times 10^{10}$ | $5.57 \times 10^{14}$ |
| IIIA | (4,2) | $3.59 \times 10^{12}$ | $-5.48 \times 10^{12}$ | $2.89 \times 10^{14}$ |
| IIIA | (6,2) | $3.93 \times 10^{11}$ | $-4.09 \times 10^{11}$ | $2.87 \times 10^{14}$ |
| IIIA | (8,4) | $3.19 \times 10^{9}$ | $-3.22 \times 10^{9}$ | $2.85 \times 10^{14}$ |
| IIIIB | (4,2) | $3.57 \times 10^{12}$ | $-5.29 \times 10^{12}$ | $3.01 \times 10^{14}$ |
| IIIIB | (6,2) | $3.85 \times 10^{11}$ | $-3.99 \times 10^{11}$ | $2.99 \times 10^{14}$ |
| IIIIB | (8,4) | $3.13 \times 10^{9}$ | $-3.25 \times 10^{9}$ | $2.97 \times 10^{14}$ |

Table 3: Heavy right-handed Majorana neutrino masses $M_j$ for degenerate models (IA,IB,IC), inverted models (IIA,IIB) and normal hierarchical models (IIIA, IIIB), with $\tan^2 \theta_{12}=0.5$, using light neutrino mass matrices $m_{LL}$ given in Appendix A. The entry $(m,n)$ indicates the type of Dirac neutrino mass matrix $m_{LR} = (\lambda^m, \lambda^n, 1)v$, as down quark mass matrix (4,2), charged lepton mass matrix (6,2) and up quark mass matrix (8,4), as explained in the text.
| Type     | (m,n) | $M_1$       | $M_2$       | $M_3$       |
|----------|-------|-------------|-------------|-------------|
| IA       | (4,2) | $5.43 \times 10^{10}$ | $-3.34 \times 10^{12}$ | $8.42 \times 10^{13}$ |
| IA       | (6,2) | $4.51 \times 10^8$   | $-3.26 \times 10^{12}$ | $8.42 \times 10^{13}$ |
| IA       | (8,4) | $3.65 \times 10^6$   | $-2.77 \times 10^{10}$ | $8.03 \times 10^{13}$ |
| IB       | (4,2) | $5.01 \times 10^9$   | $6.16 \times 10^{11}$  | $7.60 \times 10^{13}$  |
| IB       | (6,2) | $4.05 \times 10^7$   | $6.16 \times 10^{11}$  | $7.60 \times 10^{13}$  |
| IB       | (8,4) | $3.28 \times 10^5$   | $4.99 \times 10^9$     | $7.60 \times 10^{13}$  |
| IC       | (4,2) | $5.01 \times 10^9$   | $-6.69 \times 10^{12}$ | $6.99 \times 10^{12}$  |
| IC       | (6,2) | $4.05 \times 10^7$   | $-6.69 \times 10^{12}$ | $6.99 \times 10^{12}$  |
| IC       | (8,4) | $3.28 \times 10^5$   | $-4.81 \times 10^{11}$ | $7.60 \times 10^{11}$  |
| IIA      | (4,2) | $4.02 \times 10^{10}$| $9.73 \times 10^{12}$  | $6.59 \times 10^{16}$  |
| IIA      | (6,2) | $3.25 \times 10^8$   | $9.73 \times 10^{12}$  | $6.59 \times 10^{16}$  |
| IIA      | (8,4) | $2.63 \times 10^6$   | $7.94 \times 10^{10}$  | $6.54 \times 10^{16}$  |
| IIB      | (4,2) | $-9.76 \times 10^{10}$| $2.89 \times 10^{12}$  | $6.23 \times 10^{14}$  |
| IIB      | (6,2) | $-8.10 \times 10^8$  | $2.83 \times 10^{12}$  | $6.23 \times 10^{14}$  |
| IIB      | (8,4) | $-6.56 \times 10^6$  | $2.29 \times 10^{10}$  | $6.21 \times 10^{14}$  |
| IIIA     | (4,2) | $1.74 \times 10^{12}$| $-2.28 \times 10^{13}$ | $2.96 \times 10^{14}$  |
| IIIA     | (6,2) | $1.83 \times 10^{10}$| $-1.82 \times 10^{13}$ | $1.04 \times 10^{14}$  |
| IIIA     | (8,4) | $1.48 \times 10^8$   | $-1.79 \times 10^{11}$ | $8.56 \times 10^{13}$  |
| IIIB     | (4,2) | $3.73 \times 10^{12}$| $-5.68 \times 10^{12}$ | $2.96 \times 10^{14}$  |
| IIIB     | (6,2) | $4.08 \times 10^{11}$| $-4.24 \times 10^{11}$ | $2.93 \times 10^{14}$  |
| IIIB     | (8,4) | $3.31 \times 10^9$   | $-3.45 \times 10^9$    | $2.91 \times 10^{14}$  |

Table 4: Heavy right-handed Majorana neutrino masses $M_j$ for degenerate models (IA,IB,IC), inverted models (IIA,IIB) and normal hierarchical models (IIIA, IIIB), with $\tan^2 \theta_{12}=0.45$, using neutrino mass matrices given in Appendix A. The entry $(m,n)$ indicates the type of Dirac neutrino mass matrix as down quark mass matrix (4,2), charged lepton mass matrix (6,2) and up quark mass matrix (8,4) as explained in the text.
respectively, using light neutrino mass matrices given in Appendix A. The entry for degenerate models (IA, IB, IC) with \( \tan^2 \theta_{12} = 0.50 \) without and with flavour effects respectively, using light neutrino mass matrices given in Appendix A. The entry \((m, n)\) indicates the type of Dirac mass matrix as explained in the text.

| Type | \((m,n)\) | \(m_{\alpha\alpha}\)(eV) | \(m_{1}\)(eV) | \(\epsilon_{\alpha\alpha}\) | \(\epsilon\) | \(Y_{B1}\) | \(Y_{B3}\) |
|------|----------|-----------------|-------------|-----------------|------|----------|----------|
| IA   | (4,2)    | \(1.13 \times 10^{-1}\) | 1.12        | \(9.13 \times 10^{-10}\) |      | \(1.39 \times 10^{-5}\) | \(1.37 \times 10^{-11}\) | \(1.42 \times 10^{-10}\) |
| IA   | (6,2)    | \(5.28 \times 10^{-1}\) | 1.19        | \(6.09 \times 10^{-14}\) |      | \(1.21 \times 10^{-7}\) | \(1.12 \times 10^{-13}\) | \(2.58 \times 10^{-13}\) |
| IA   | (8,4)    | \(5.28 \times 10^{-1}\) | 1.19        | \(1.37 \times 10^{-13}\) |      | \(1.04 \times 10^{-9}\) | \(9.62 \times 10^{-16}\) | \(2.16 \times 10^{-15}\) |

| IA   | (4,2)    | \(3.97 \times 10^{-1}\) | 0.3968      | \(1.82 \times 10^{-18}\) |      | \(2.76 \times 10^{-14}\) | \(7.66 \times 10^{-20}\) | \(1.09 \times 10^{-11}\) |
| IA   | (6,2)    | \(2.79 \times 10^{-9}\) | 0.3968      | \(1.19 \times 10^{-22}\) |      | \(2.24 \times 10^{-16}\) | \(6.20 \times 10^{-22}\) | \(8.30 \times 10^{-14}\) |
| IA   | (8,4)    | \(2.79 \times 10^{-9}\) | 0.3968      | \(4.72 \times 10^{-25}\) |      | \(1.81 \times 10^{-18}\) | \(5.02 \times 10^{-24}\) | \(7.15 \times 10^{-16}\) |

Table 5: Values of CP asymmetry \(\epsilon\) and \(\epsilon_{\alpha\alpha}\) and the baryon asymmetry \(Y_{B1}\) and \(Y_{B3}\) for degenerate models (IA, IB, IC) with \( \tan^2 \theta_{12} = 0.50 \) without and with flavour effects respectively, using light neutrino mass matrices given in Appendix A. The entry \((m, n)\) indicates the type of Dirac mass matrix as explained in the text.
| Type (m,n) | $m_{aa}(eV)$ | $\tilde{m}_1(eV)$ | $\epsilon_{aa}$ | $\epsilon$ | $Y_{B1}$ | $Y_{B3}$ |
|-----------|--------------|-----------------|----------------|--------|--------|--------|
| IA (4,2)  | $3.57 \times 10^{-2}$ | $2.38 \times 10^{-1}$ | $1.43 \times 10^{-9}$ | $1.49 \times 10^{-5}$ | $7.03 \times 10^{-10}$ | $2.16 \times 10^{-9}$ |
| IA 9.92 $\times 10^{-2}$ | $2.98 \times 10^{-7}$ | $1.50 \times 10^{-5}$ | |
| IA (6,2)  | $3.57 \times 10^{-2}$ | $2.50 \times 10^{-1}$ | $9.58 \times 10^{-14}$ | $1.31 \times 10^{-7}$ | $5.76 \times 10^{-12}$ | $1.34 \times 10^{-11}$ |
| IA 1.07 $\times 10^{-1}$ | $2.50 \times 10^{-9}$ | $1.28 \times 10^{-7}$ | |
| IA (8,4)  | $3.57 \times 10^{-2}$ | $2.50 \times 10^{-1}$ | $7.52 \times 10^{-18}$ | $1.16 \times 10^{-9}$ | $5.12 \times 10^{-14}$ | $1.19 \times 10^{-13}$ |
| IA 1.07 $\times 10^{-1}$ | $1.16 \times 10^{-9}$ | |

| Type (m,n) | $m_{aa}(eV)$ | $\tilde{m}_1(eV)$ | $\epsilon_{aa}$ | $\epsilon$ | $Y_{B1}$ | $Y_{B3}$ |
|-----------|--------------|-----------------|----------------|--------|--------|--------|
| IB (4,2)  | $3.96 \times 10^{-1}$ | $0.3964$ | $1.68 \times 10^{-18}$ | $2.56 \times 10^{-14}$ | $7.15 \times 10^{-19}$ | $1.09 \times 10^{-10}$ |
| IB 2.65 $\times 10^{-9}$ | $923 \times 10^{-19}$ | $2.06 \times 10^{-16}$ | $5.76 \times 10^{-21}$ | $8.84 \times 10^{-13}$ |
| IB 2.58 $\times 10^{-9}$ | $2.56 \times 10^{-14}$ | $2.06 \times 10^{-16}$ | $5.76 \times 10^{-21}$ | $8.84 \times 10^{-13}$ |
| IB (6,2)  | $3.96 \times 10^{-1}$ | $0.3964$ | $7.27 \times 10^{-27}$ | $4.88 \times 10^{-25}$ | $4.67 \times 10^{-23}$ | $7.16 \times 10^{-15}$ |
| IB 2.58 $\times 10^{-9}$ | $4.88 \times 10^{-25}$ | $1.68 \times 10^{-18}$ | $4.67 \times 10^{-23}$ | $7.16 \times 10^{-15}$ |
| IB 2.58 $\times 10^{-9}$ | $1.68 \times 10^{-18}$ | |

| Type (m,n) | $m_{aa}(eV)$ | $\tilde{m}_1(eV)$ | $\epsilon_{aa}$ | $\epsilon$ | $Y_{B1}$ | $Y_{B3}$ |
|-----------|--------------|-----------------|----------------|--------|--------|--------|
| IC (4,2)  | $3.96 \times 10^{-1}$ | $0.3968$ | $1.21 \times 10^{-16}$ | $1.85 \times 10^{-13}$ | $5.12 \times 10^{-18}$ | $7.16 \times 10^{-10}$ |
| IC 2.78 $\times 10^{-9}$ | $1.37 \times 10^{-14}$ | $1.69 \times 10^{-13}$ | |
| IC 2.83 $\times 10^{-9}$ | $8.53 \times 10^{-21}$ | $1.47 \times 10^{-15}$ | $3.77 \times 10^{-20}$ | $5.80 \times 10^{-12}$ |
| IC (6,2)  | $3.97 \times 10^{-1}$ | $0.3968$ | $1.10 \times 10^{-16}$ | $1.25 \times 10^{-16}$ | $2.82 \times 10^{-21}$ | $4.34 \times 10^{-13}$ |
| IC 2.79 $\times 10^{-9}$ | $1.02 \times 10^{-16}$ | $1.01 \times 10^{-16}$ | $2.82 \times 10^{-21}$ | $4.34 \times 10^{-13}$ |
| IC 2.58 $\times 10^{-9}$ | $1.01 \times 10^{-16}$ | |

Table 6: Values of CP asymmetry $\epsilon$ and $\epsilon_{aa}$ and the baryon asymmetry $Y_{B1}$ and $Y_{B3}$ for degenerate models (IA, IB, IC) with $\tan^2\theta_{12} = 0.45$ without and with flavour effects respectively, using light neutrino mass matrices given in Appendix A. The entry (m, n) indicates the type of Dirac mass matrix as explained in the text.
| Type  | (m,n)  | $m_{\alpha\alpha}(eV)$ | $\bar{m}_1(eV)$ | $\epsilon_{\alpha\alpha}$ | $\epsilon$ | $Y_{B1}$   | $Y_{B3}$   |
|-------|--------|------------------------|----------------|--------------------------|----------|-----------|-----------|
| IIA   |        | $4.95 \times 10^{-2}$  |               | $5.86 \times 10^{-19}$  |          |           |           |
|       | (4,2)  | $1.22 \times 10^{-6}$  | $4.95 \times 10^{-1}$ | $7.46 \times 10^{-15}$  | $9.37 \times 10^{-13}$ | $2.08 \times 10^{-17}$ | $8.14 \times 10^{-13}$ |
|       | (6,2)  | $1.21 \times 10^{-6}$  |               | $7.47 \times 10^{-15}$  |          |           |           |
|       | (8,4)  | $1.21 \times 10^{-6}$  |               | $6.19 \times 10^{-17}$  |          |           |           |
| IIB   |        | $1.61 \times 10^{-2}$  |               | $1.36 \times 10^{-10}$  |          |           |           |
|       | (4,2)  | $6.22 \times 10^{-2}$  | $1.42 \times 10^{-1}$ | $2.94 \times 10^{-8}$  | $5.73 \times 10^{-6}$ | $4.42 \times 10^{-11}$ | $9.82 \times 10^{-11}$ |
|       | (6,2)  | $6.78 \times 10^{-2}$  |               | $7.98 \times 10^{-15}$  |          |           |           |
|       | (8,4)  | $6.78 \times 10^{-2}$  |               | $4.26 \times 10^{-8}$   |          |           |           |
|       |        | $1.61 \times 10^{-2}$  |               | $5.94 \times 10^{-19}$  |          |           |           |
|       | (4,2)  | $6.78 \times 10^{-2}$  | $1.52 \times 10^{-1}$ | $3.88 \times 10^{-10}$  | $2.81 \times 10^{-13}$ | $6.29 \times 10^{-13}$ |
|       | (6,2)  | $6.78 \times 10^{-2}$  |               | $3.88 \times 10^{-10}$  |          |           |           |
|       | (8,4)  | $6.78 \times 10^{-2}$  |               | $3.88 \times 10^{-10}$  |          |           |           |

Table 7: Values of CP asymmetry $\epsilon$ and $\epsilon_{\alpha\alpha}$ and the baryon asymmetry $Y_{B1}$ and $Y_{B3}$ for inverted hierarchical models (IIA, IIB) without and with flavour effects respectively for $\tan^2 \theta_{12} = 0.50$, using light neutrino mass matrices given in Appendix A. The entry $(m, n)$ indicates the type of Dirac mass matrix as explained in the text.
| Type  | (m,n) | $m_{\alpha\alpha}(eV)$ | $\bar{m}_{\alpha}(eV)$ | $\epsilon_{\alpha\alpha}$ | $\epsilon$ | $Y_{B1}$ | $Y_{B3}$ |
|-------|-------|------------------------|------------------------|------------------------|--------|--------|--------|
| IIA   |       | $4.94 \times 10^{-2}$  | $6.76 \times 10^{-19}$ |                       |        |        |        |
|       | (4,2) | $1.56 \times 10^{-6}$  | $4.95 \times 10^{-2}$  | $9.07 \times 10^{-15}$ | $1.12 \times 10^{-12}$ | $2.49 \times 10^{-16}$ | $7.90 \times 10^{-12}$ |
|       | (6,2) | $1.55 \times 10^{-6}$  | $4.95 \times 10^{-2}$  | $7.28 \times 10^{-17}$ | $9.00 \times 10^{-15}$ | $2.00 \times 10^{-18}$ | $6.34 \times 10^{-14}$ |
|       | (8,4) | $1.55 \times 10^{-6}$  | $4.95 \times 10^{-2}$  | $4.89 \times 10^{-21}$ | $7.53 \times 10^{-17}$ | $1.67 \times 10^{-20}$ | $5.35 \times 10^{-16}$ |
| IIIB  |       | $1.99 \times 10^{-2}$  | $9.04 \times 10^{-11}$ |                       |        |        |        |
|       | (4,2) | $5.74 \times 10^{-2}$  | $1.36 \times 10^{-1}$  | $2.13 \times 10^{-8}$  | $4.02 \times 10^{-6}$ | $3.25 \times 10^{-10}$ | $7.53 \times 10^{-10}$ |
|       | (6,2) | $6.14 \times 10^{-2}$  | $1.43 \times 10^{-1}$  | $1.78 \times 10^{-10}$ | $3.33 \times 10^{-8}$ | $2.57 \times 10^{-12}$ | $5.96 \times 10^{-12}$ |
|       | (8,4) | $6.14 \times 10^{-2}$  | $1.42 \times 10^{-1}$  | $1.78 \times 10^{-14}$ | $2.71 \times 10^{-10}$ | $2.09 \times 10^{-14}$ | $4.86 \times 10^{-14}$ |

Table 8: Values of CP asymmetry $\epsilon$ and $\epsilon_{\alpha\alpha}$ and the baryon asymmetry $Y_{B1}$ and $Y_{B3}$ for inverted hierarchical models (IIA, IIIB) without and with flavour effects respectively for $\tan^2\theta_{12} = 0.45$, using light neutrino mass matrices given in Appendix A. The entry $(m, n)$ indicates the type of Dirac mass matrix as explained in the text.
| Type (m,n) | $m_{\alpha\alpha}(eV)$ | $\tilde{m}_1(eV)$ | $\epsilon_{\alpha\alpha}$ | $\epsilon$ | $Y_{B1}$ | $Y_{B3}$ |
|-----------|------------------|------------------|------------------|--------|------|------|
| IIIA      | $2.17 \times 10^{-4}$ | $7.44 \times 10^{-9}$ | $1.19 \times 10^{-6}$ | $3.43 \times 10^{-5}$ | $8.65 \times 10^{-10}$ | $1.79 \times 10^{-8}$ |
| IIIA (4,2)| $4.13 \times 10^{-2}$ | $4.38 \times 10^{-2}$ | $3.31 \times 10^{-5}$ | $1.79 \times 10^{-12}$ | $7.01 \times 10^{-11}$ | $1.53 \times 10^{-10}$ |
| IIIA      | $2.05 \times 10^{-3}$ | $1.79 \times 10^{-12}$ | $3.70 \times 10^{-7}$ | $3.43 \times 10^{-5}$ | $1.79 \times 10^{-12}$ | $1.27 \times 10^{-12}$ |
| IIIA (8,4)| $3.20 \times 10^{-3}$ | $1.79 \times 10^{-12}$ | $3.70 \times 10^{-7}$ | $3.43 \times 10^{-5}$ | $1.79 \times 10^{-12}$ | $1.27 \times 10^{-12}$ |

| Type (m,n) | $m_{\alpha\alpha}(eV)$ | $\tilde{m}_1(eV)$ | $\epsilon_{\alpha\alpha}$ | $\epsilon$ | $Y_{B1}$ | $Y_{B3}$ |
|-----------|------------------|------------------|------------------|--------|------|------|
| IIIB      | $2.29 \times 10^{-4}$ | $7.90 \times 10^{-9}$ | $1.35 \times 10^{-6}$ | $4.82 \times 10^{-5}$ | $1.17 \times 10^{-9}$ | $1.62 \times 10^{-8}$ |
| IIIB (4,2)| $4.18 \times 10^{-2}$ | $4.52 \times 10^{-2}$ | $4.69 \times 10^{-5}$ | $1.50 \times 10^{-12}$ | $3.40 \times 10^{-5}$ | $1.33 \times 10^{-10}$ |
| IIIB      | $3.20 \times 10^{-3}$ | $1.50 \times 10^{-12}$ | $3.37 \times 10^{-5}$ | $1.02 \times 10^{-16}$ | $2.81 \times 10^{-7}$ | $1.10 \times 10^{-12}$ |
| IIIB (8,4)| $3.31 \times 10^{-1}$ | $6.13 \times 10^{-1}$ | $2.00 \times 10^{-11}$ | $2.81 \times 10^{-7}$ | $5.04 \times 10^{-13}$ | $1.10 \times 10^{-12}$ |
| IIIB      | $2.82 \times 10^{-1}$ | $2.80 \times 10^{-7}$ | $2.80 \times 10^{-7}$ | $2.80 \times 10^{-7}$ | $2.80 \times 10^{-7}$ | $2.80 \times 10^{-7}$ |

Table 9: Values of CP asymmetry $\epsilon$ and $\epsilon_{\alpha\alpha}$ and the baryon asymmetry $Y_{B1}$ and $Y_{B3}$ for normal hierarchical models (IIIA, IIIB) without and with flavour effects respectively for $\tan^2 \theta_{12} = 0.50$, using mass matrices given in Appendix A. The entry $(m, n)$ indicates the type of Dirac mass matrix as explained in the text.
| Type  | (m,n) | $m_{\alpha\alpha}(eV)$ | $\tilde{m}_1(eV)$ | $\epsilon_{\alpha\alpha}$ | $\epsilon$ | $Y_{B1}$ | $Y_{B3}$ |
|-------|-------|------------------------|-------------------|--------------------------|----------|----------|----------|
| IIIA  |       | 8.94×10^{-4}          | 8.64×10^{-8}     |                          |          |          |          |
| IIIA  | (4,2) | 3.01×10^{-2}          | 4.28×10^{-2}     | 1.24×10^{-5}            | 2.22×10^{-4} | 5.71×10^{-8} | 2.00×10^{-7} |
| IIIA  |       | 1.19×10^{-2}          | 2.10×10^{-4}     |                          |          |          |          |
| IIIA  | (6,2) | 8.78×10^{-4}          | 6.95×10^{-2}     | 1.61×10^{-7}            | 4.34×10^{-6} | 6.86×10^{-10} | 1.39×10^{-9} |
| IIIA  |       | 3.42×10^{-2}          | 4.18×10^{-6}     |                          |          |          |          |
| IIIA  | (8,4) | 3.44×10^{-2}          | 6.96×10^{-2}     | 1.91×10^{-11}           | 5.06×10^{-8} | 7.99×10^{-12} | 1.62×10^{-11} |
| IIIA  |       | 3.44×10^{-2}          | 5.05×10^{-8}     |                          |          |          |          |
| IIIB  |       | 2.09×10^{-4}          | 7.18×10^{-9}     |                          |          |          |          |
| IIIB  | (4,2) | 3.99×10^{-2}          | 4.19×10^{-2}     | 1.13×10^{-6}            | 3.09×10^{-5} | 8.13×10^{-9} | 1.85×10^{-7} |
| IIIB  |       | 1.77×10^{-3}          | 2.98×10^{-5}     |                          |          |          |          |
| IIIB  | (6,2) | 1.93×10^{-5}          | 1.85×10^{-12}    |                          |          |          |          |
| IIIB  |       | 3.04×10^{-1}          | 3.29×10^{-7}     | 3.74×10^{-5}            | 7.37×10^{-10} | 1.62×10^{-9} |
| IIIB  | (8,4) | 2.54×10^{-1}          | 3.71×10^{-5}     |                          |          |          |          |
| IIIB  |       | 1.93×10^{-5}          | 1.26×10^{-10}    |                          |          |          |          |
| IIIB  | (8,4) | 3.06×10^{-1}          | 2.22×10^{-11}    | 3.09×10^{-7}            | 6.06×10^{-12} | 1.13×10^{-11} |          |
| IIIB  |       | 2.55×10^{-1}          | 3.09×10^{-7}     |                          |          |          |          |

Table 10: Values of CP asymmetry $\epsilon$ and $\epsilon_{\alpha\alpha}$ and the baryon asymmetry $Y_{B1}$ and $Y_{B3}$ for normal hierarchical models (IIIA, IIIB) without and with flavour effects respectively for $\tan^2\theta_{12} = 0.45$, using mass matrices given in Appendix A. The entry $(m, n)$ indicates the type of Dirac mass matrix as explained in the text.
and Table 2 for $\tan^2 \theta_{12} = 0.5$ and 0.45 respectively. They obey $\mu - \tau$ symmetry and predict tribimaximal mixings as expected. In Table 3 and Table 4 the three heavy right-handed neutrino masses are extracted from the right-handed Majorana mass matrices so constructed through the inverse seesaw formula, for three choices of diagonal Dirac neutrino mass matrices. The corresponding baryon asymmetry $Y_B$ are estimated following sections 2 and 3, for degenerate model(IA,IB,IC), inverted hierarchical models(IIA,IIB) and normal hierarchical models(IIIA, IIB) respectively as indicated in the Tables (5-10).

In these calculations we have focussed on two issues :(i) dependence of $Y_B$ on lepton flavours, (ii) dependence of $Y_B$ on type of Dirac neutrino mass matrix. We have found strong dependence on the type of Dirac neutrino mass matrix, where down-quark type mass matrix $(\lambda^4, \lambda^2, 1)\nu$ leading to highest contribution and charged lepton mass matrix $(\lambda^6, \lambda^2, 1)\nu$ and up-quark type mass matrix $(\lambda^8, \lambda^4, 1)\nu$ in decreasing order with factor of 100, in all cases. The enhancement in flavoured leptogenesis is also a common feature for all cases, and such enhancement is also dependent on the type of the Dirac neutrino mass matrix.

Both normal hierarchical models(IIIA,IIIB) predict good results consistent with observations for the case with Dirac neutrino mass matrix in Table (9-10). Inverted hierarchical model(IIB) with $(m, n) = (4, 2)$ also leads to acceptable results and it is not yet ruled out. However inverted hierarchical model(IIA) is completely ruled out as seen in Table (7-8). The degenerate models (IA,IB,IC) with $(m, n) = (4, 2)$ in the case of flavoured leptogenesis still show reasonable prediction in Table (5-6).

The present analysis is extended for $\mu - \tau$ symmetric mass matrices $m_{LL}$ with $\tan^2 \theta_{12} = 0.45$. The analysis indicates an enhancement in the baryon asymmetry by a factor of one. In some left-right symmetric SO(10) GUT, Dirac neutrino mass matrix is considered as charged lepton type mass matrix. In such condition only normal hierarchical model leads to good prediction consistent with data.

The present analysis if considered as an additional criteria for the discrimination of neutrino mass models, may lead to normal hierarchical model as the most favourable choice of nature. This conclusion is consistent with other conditions such as stability criteria under quantum radiative corrections in MSSM. Moreover, the normal hierarchical model also leads to a good prediction with the Type II seesaw formula as well [32].
Appendix A: Possible patterns of neutrino mass models obeying $\mu - \tau$ symmetry with two parameters $\epsilon$ and $\eta$

Left-handed Majorana neutrino mass matrices which obey $\mu - \tau$ symmetry\cite{11,12} have the following form

$$m_{LL} = \begin{pmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{pmatrix} m_o$$

This predicts an arbitrary solar mixing angle $\tan 2\theta_{12} = \frac{2\sqrt{2}Y}{(X-Z-W)}$, while the predictions on atmospheric mixing angle is maximal ($\theta_{23} = \pi/4$) and Chooz angle is zero. We parametrise the mass matrices (with only two parameters $\epsilon$ and $\eta$) whereby the solar mixing is fixed at tribimaximal mixings for all possible patterns of neutrino mass models:

1. Deg Type A (IA) ($m_i = m_1, -m_2, m_3$)

$$m_{LL} = \begin{pmatrix} \epsilon - 2\eta & -\epsilon & -\epsilon \\ -\epsilon & \frac{1}{2} - \eta & -\frac{1}{2} - \eta \\ -\epsilon & -\frac{1}{2} - \eta & \frac{1}{2} - \eta \end{pmatrix} m_o$$

with input values: $\epsilon = 0.66115$, $\eta = 0.16535$, $m_o = 0.4eV$.

2. Deg Type B (IB) ($m_i = m_1, m_2, m_3$)

$$m_{LL} = \begin{pmatrix} 1 - \epsilon - 2\eta & \epsilon & \epsilon \\ \epsilon & 1 - \eta & -\eta \\ \epsilon & -\eta & 1 - \eta \end{pmatrix} m_o$$

with input values: $\epsilon = 8.314 \times 10^{-5}$, $\eta = 0.00395$, $m_o = 0.4eV$.

3. Deg Type C (IC) ($m_i = m_1, m_2, -m_3$)

$$m_{LL} = \begin{pmatrix} 1 - \epsilon - 2\eta & \epsilon & \epsilon \\ \epsilon & -\eta & 1 - \eta \\ \epsilon & 1 - \eta & -\eta \end{pmatrix} m_o$$

with input values: $\epsilon = 8.314 \times 10^{-5}$, $\eta = 0.00395$, $m_o = 0.4eV$. 

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4: Inverted Hierarchical mass matrix with $m_3 \neq 0$:

$$m_{LL}(IH) = \begin{pmatrix} 1 - 2\epsilon & -\epsilon & -\epsilon \\ -\epsilon & 1/2 & 1/2 - \eta \\ -\epsilon & 1/2 - \eta & 1/2 \end{pmatrix} m_0.$$ 

Inverted Hierarchy with even CP parity in the first two mass eigenvalues (IIA) ($m_1 = m_1, m_2, m_3$): $\eta/\epsilon=1.0, \eta=0.0048, m_0 = 0.05eV$.

Inverted Hierarchy with odd CP parity in the first two mass eigenvalues (IIB) ($m_i = m_1, -m_2, m_3$): $\eta/\epsilon=1.0, \eta=0.6607, m_0 = 0.05eV$.

5: Normal Hierarchical mass matrix Case (i) with $m(1,1) = X \neq 0$ type-IIIA:

$$m_{LL}(NH) = \begin{pmatrix} -\eta & -\epsilon & -\epsilon \\ -\epsilon & 1 - \epsilon & -1 \\ -\epsilon & -1 & 1 - \epsilon \end{pmatrix} m_0$$

with input values: $\eta/\epsilon=0.0, \epsilon=0.175, m_0 = 0.029eV$.

6: Normal Hierarchical mass matrix Case (ii) with $m(1,1) = X = 0$; type-IIIB:

$$m_{LL}(NH) = \begin{pmatrix} 0 & -\epsilon & -\epsilon \\ -\epsilon & 1 - \epsilon & -1 + \eta \\ -\epsilon & -1 + \eta & 1 - \epsilon \end{pmatrix} m_0$$

with input values: $\eta/\epsilon=0.0, \epsilon=0.164, m_0 = 0.028eV$. The textures of mass matrices for degenerate (IA, IB, IC), inverted hierarchy (IIA, IIB) as well as normal hierarchy (IIIA, IIIB) have the potential to decrease the solar mixing angle from the tribimaximal value, without sacrificing $\mu-\tau$ symmetry. This is possible through the identification of ‘flavour twister’ $\eta/\epsilon \neq 0$ \[11, 12\]. The values of $\epsilon$ and $\eta$ for $\tan^2\theta_{12}=0.45$ are collected from \[11, 12\].

**Acknowledgement**

One of us HZD would like to thank CSIR for the Senior Research Fellowship as financial assistance for carrying out her research work.

**References**

[1] M. Gell-Mann, P. Ramond and R. Slansky in Supergravity, Proceeding of the Workshop, Stony Brook, New York, 1979, Edited by P. Van Nieu-
menhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, KEK Lectures, 1979 (unpublished); R. N. Mahapatra and G. Senjanovic, Phy. Rev. Lett. 44, 912 (1980).

[2] Sacha Davidson, Enrico Nardi, Yosef Nir, \texttt{arXiv:0802.2962} and further references therein.

[3] J. Dunkley et al, Astrophys. J. Suppl., 180, (2009) 306; 0803.0586[astro-ph].

[4] A. D. Sakharov, JETP Lett. 5 (1967) 24.

[5] V. A. Kuzmin, V. A. Rubakov, M. E. Shaposhnikov, Phy. Lett. 155B (1985) 36.

[6] M. Fukugita and T. Yanagida, Phy.Lett. 174B (1986) 45.

[7] M. A. Luty, Phy.Rev. D45 (1992) 455.

[8] E. W. Kolb, M. S. Turner, \textit{The Early Universe}, Addision - Wesely, New York (1990).

[9] U. Sarkar, \texttt{hep-ph/9810247}; M. Flanz, E. A. Paschos, U. Sarkar, Phys. Lett. B345 (1995) 248-252.

[10] D. Falcone, Phy. Rev D66, (2002) 053001.

[11] N. Nimai Singh, H. Zeen Devi, Mahadev Patgiri, \texttt{arXiv: 0707.2713}.

[12] N. Nimai Singh, H. Zeen Devi, Abhijit Borah, S. Somorendro Singh \texttt{arXiv: 0911.1488}

[13] Riccardo Barbieri, Paolo Creminelli, Alessandro Strumia, and Nikoloas Tetradsis \textit{Nucl. Phys.} B575:61-77, 2000.

[14] A. Abada, Habib Aissaoui and M. Losada \textit{Nucl. Phys.} B728:55-66, 2005

[15] H B Nielsen, Y. Takanishi \textit{Phys. Lett.}, B507:241-251, 2001.

[16] O. Vives \textit{Phys. Rev.}, D73:073006, 2006.
[17] Apostolos Pilaftsis and Thomas E. J. Underwood, Phys. Rev., D72:113001, 2005.

[18] Asmaa Abada, Sacha Davidson, Francois-Xavier Josse-Michaux, Marta Losada, and Antonio Riotto, JCAP, 0604:004, 2006.

[19] Enrico Nardi, Yosef Nir, Esteban Roulet, and Juan Racker, JHEP, 0601:164, 2006.

[20] A. Abada et al, JHEP, 0609:010, 2006.

[21] M. C. Gonzalez-Garcia, Michele Maltoni, ArXiv: 0704.1800. Phys. Rept.460(2008)1-129.

[22] An incomplete list: P. F. Harrison, W. G. Scott, Phys. Lett. B547, 219(2002); C. S. Lam, Phys. Rev. D71, 093001(2005); hep-ph/0503157; W. Grimus, hep-ph/0610158; W. Grimus, L. Lavoura, hep-ph/0611149; A. S. Joshipura, hep-ph/0512252; T. Kitabayashi, M. Yasue, Phys. Lett. B490, 236(2000); E. Ma, Phys. Rev. D70, 031901(2004); K. S. Babu, R. N. Mohapatra, Phys. Lett. B532, 77(2002); T. Fukuyama, H. Nishiura, hep-ph/9702253; K. Fuki, M. Yasue, hep-ph/0608042; A. Gosal, Mod. Phys. Lett. A19, 2579(2004); hep-ph/0304090; T. Ohlsson, G. Seidl, Nucl. Phys. B643, 247(2002); Riazuddin, arXiv:0707.0912.

[23] Y. H. Ahn, Sin Kyu Kang, C. S. Kim, Jake Lee, hep-ph/0610007; hep-ph/0602160.

[24] Yoshio Koide, Phys. Rev. D69, 093001(2004); Yoshio Koide, H. Nishiura, K. Matsuda, T. Kikuchi, T. Fukuyama, Phys. Rev. D66, 093006(2002); Koichi Matsuda, H. Nishiura, Phys. Rev. D73, 013008(2006).

[25] Y. Koide, E. Takasugi, arXiv:0706.4373.

[26] R. N. Mohapatra, S. Nasri, Hai-Bo Yu, Phys. Lett. B636, 114(2006).

[27] P. F. Harrison, D. H. Perkins, W.G. Scott, Phys. Lett. B530, 167(2002); P. F. Harrison, W. G. Scott, Phys. Lett. B557, 76(2003).
[28] E. Ma, Phys. Rev. D70, 03191(2004); Mod. Phys. Lett. A21, 2931(2006); S. F. King, Michal Malinsky, Phys. Lett. B645, 351(2007); R. N. Mohapatra, S. Hoi, Hai-Bo Yu [hep-ph/0605020]; Xiao-Gang He, A. Zee, [hep-ph/0607163]; Florian Plentinger, Werner Rodejohann, [hep-ph/0507143].

[29] N. Nimai Singh, Monisa Rajkhowa, Abhijit Borah, J. Phys. G: Nucl. Part. Phys. 34, 345(2007); [hep-ph/0603154] [hep-ph/0603189].

[30] E.K.Akhmedov, M. Frigerio and A. Y. Smirnov, JHEP 0309, 021(2003), [hep-ph/0305322].

[31] P. M. Fishbane and P. Kaus, J. Phys.G:Nucl. Part. Phys.25(1999)1629-1640.

[32] A.K.Sarma, H.Z.Devi and N.Nimai Singh, Nucl.Phys.B765(2007)142-153.