Weak magnetic fields in early-type stars: failed fossils

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\begin{abstract}
Weak magnetic fields have recently been detected in Vega and Sirius. Here, we explore the possibility that these fields are the remnants of some field inherited or created during or shortly after star formation and are still evolving dynamically as we observe them. The time-scale of this evolution is given in terms of the Alfvén time-scale and the rotation frequency by $\tau_{\text{evol}} \sim \tau_{A} \Omega$, which is then comparable to the age of the star. According to this theory, all intermediate- and high-mass stars should contain fields of at least the strength found so far in Vega and Sirius. Faster rotators are expected to have stronger magnetic fields. Stars may experience an increase in surface field strength during their early main sequence, but for most of their lives field strength will decrease slowly. The length scale of the magnetic structure on the surface may be small in very young stars but should quickly increase to at least very approximately a fifth of the stellar radius. The field strength may be higher at the poles than at the equator.

Key words: magnetic fields – MHD – stars: magnetic fields – stars: early type – stars: chemically peculiar – stars: interiors.
\end{abstract}

1 INTRODUCTION

A bimodality in the magnetic properties of A stars is now clear (Aurière et al. 2007). The chemically peculiar Ap stars, accounting for $\approx 10\%$ per cent of the population, have non-time-variable large-scale fields of $\sim 200$ G to $\sim 30$ kG in strength. Detection limits (at least for large-scale fields) have now fallen to the gauss level, and from the lack of detections we infer a gap in the field strength distribution. Recent Zeeman polarimetric observations of two main-sequence A stars have finally revealed weak magnetic fields: in Vega a field of $0.6 \pm 0.3$ G was found (Lignièrè et al. 2009; Petit et al. 2010) and in Sirius $0.2 \pm 0.1$ G (Petit et al. 2011). The field geometry is poorly constrained, as is any time variability – although Petit et al. note that in Vega ‘no significant variability in the field structure is observed over a time span of one year’. This bimodality probably also exists in OB stars, kilogauss fields having been measured in a subset. See Donati & Landstreet (2009) for a review.

There are various possible origins or natures of these magnetic fields. In the stars with strong, large-scale fields, a ‘fossil’ equilibrium is likely (Cowling 1945; Braithwaite & Spruit 2004). The convective core probably contains a dynamo, but it is problematic getting this field up to the surface in a sensibly short time unless very thin flux tubes can be generated (Moss 1989; MacGregor & Cassinelli 2003), which is probably not consistent with observations. Moreover, when compositional gradients are considered, only magnetic fields much stronger than the equipartition value are able to reach the surface, which poses a further challenge to the core-dynamo hypothesis (MacDonald & Mullan 2004). In massive stars (above about $8 M_\odot$), a subsurface iron-ionization-driven convective layer can also host a dynamo, from where there is no difficulty for the resulting magnetic field to reach the surface (Cantiello et al. 2009; Cantiello & Braithwaite 2011); for solar metallicity field strengths of (very approximately) 5 to 300 G are predicted, depending on the mass and age of the star (higher fields in more massive stars and towards the end of the main sequence). These fields could give rise to various observational effects such as line profile variability and discrete absorption components. In intermediate-mass stars such as Vega and Sirius, there is a helium-ionization-driven convective layer beneath the surface, from where a dynamo-generated field can float to the surface in the same way – we explore this in a companion paper (Cantiello & Braithwaite, in preparation). This could produce fields of approximately the magnitudes observed. However, with current observations, it is difficult or impossible to distinguish between this and the hypothesis explored in this Letter, namely that these are dynamically evolving fields, with no need for any ongoing dynamo or other regenerative process. In the next section, we describe the generalities of the evolution of a magnetic field in a radiative star in the absence of regenerative processes, before discussing realistic scenarios in Section 3. In Section 4, the effects of convection, meridional circulation and differential rotation are looked at; differences in behaviour between the interior and near the surface are looked at in Section 5, and we summarize in Section 6.

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2 RELAXATION TO EQUILIBRIUM AND TIME-SCALES

An arbitrary magnetic field in a star will, in the absence of any driving from differential rotation, convection or meridional circulation, evolve towards an equilibrium. We can estimate the time-scale on which this happens simply by examining the sizes of the various terms in the momentum equation:

\[ \frac{du}{dt} = -\frac{1}{\rho} \nabla P + g + \frac{1}{4\pi\rho} \nabla \times B \times B - 2\Omega \times u \]

\[ U \frac{\delta P}{\rho L} \quad g \quad \frac{B^2}{\rho L} \Omega U \]

\[ \omega_{\text{low}} \frac{\delta \rho}{\rho} \quad \omega_{\text{sound}} \quad \omega_{\text{L}} \quad \omega_{\Lambda} \quad \Omega \omega_{\text{flow}}, \]

where \( U, T \) and \( L \) are the characteristic velocity, time-scale and length scale of the flow – whereby the general flow relation \( U \sim L/T \) is assumed – and \( \omega_{\text{low}} \sim 1/T \) is the inverse time-scale of the evolution, \( \omega_{\text{sound}} = c_s/L \) is the inverse sound-crossing time-scale, \( \omega_{\text{L}} = \sqrt{g/L} \) is the inverse free-fall time-scale, \( \omega_{\Lambda} \sim \nu_A/L \) is the Alfvén frequency and \( \Omega \) is the angular velocity of the star’s rotation.

Note that the gravity term can be made to include the centrifugal force, which we therefore need not consider separately.

A star without a magnetic field will adjust on the free-fall time-scale into a spheroidal equilibrium where the pressure and gravity/centrifugal terms balance (although of course any oscillations have to be damped). If an arbitrary magnetic field is then added, the Lorentz force will give rise to motion on spheroidal shells, i.e. perpendicular to effective gravity. If the rotation of the star is slow – in the sense that \( \Omega \ll \omega_{\Lambda} \) the Lorentz force will be balanced by inertia and we have \( \omega_{\text{low}} \sim \omega_{\Lambda} \), i.e. we evolve towards equilibrium on an Alfvén time-scale. This has been confirmed in numerical simulations (Brathwaite & Spruit 2004; Brathwaite & Nordlund 2006). Once equilibrium is reached, the Lorentz force is balanced by non-spheroidal deviations of \( P \) and \( \rho \). Now, if the star is rotating fast – so that \( \Omega \gg \omega_{\Lambda} \) – the inertia term is small and during relaxation the Lorentz force is balanced instead by the Coriolis force, which gives \( \omega_{\text{low}} \sim \omega_{\Lambda}/\Omega \). Note that precisely this time-scale is found for the growth of various magnetohydrodynamic (MHD) instabilities, by e.g. Pitts & Tayler (1985) as well as in simulations of equilibrium formation (in a forthcoming publication). In the slow and fast rotating regimes, we have the following evolution time-scales

\[ \tau_{\text{evol}} = \omega_{\text{low}}^{-1} \]

\[ \omega_{\Lambda} \gg \Omega ; \]

\[ \tau_{\text{evol}} \sim 10^4 \text{yr} \frac{M/M_\odot}{(R/R_\odot)}^{1/2} \left( \frac{L}{R} \right) \left( \frac{B}{\mu G} \right)^{-1} , \]

\[ \omega_{\Lambda} \ll \Omega ; \]

\[ \tau_{\text{evol}} \sim 2 \times 10^{11} \text{yr} \frac{M/M_\odot}{(R/R_\odot)}^2 \left( \frac{L}{R} \right)^2 \left( \frac{B}{\mu G} \right)^{-2} \left( \frac{P}{\text{yr}} \right)^{-1} \]

where \( R \) and \( P \) are the stellar radius and rotation period.

The following considerations may help to clarify the situation. In the slowly rotating case, the Lorentz force on each fluid element is directed in some sense towards equilibrium locations, and although they are forced to move on spherical shells (because of gravity and a positive radial entropy gradient) and such that the divergence of their velocities vanishes (because the field is too weak to cause significant gas compression), there is nothing stopping fluid elements from collectively moving directly towards the nearest available equilibrium. To simplify the situation, let us first consider the case of a single particle at a distance \( r \) from the origin, which experiences an acceleration \( r\omega_{\Lambda}^2 \) towards its equilibrium position at the origin. The inverse time taken to reach the origin is simply \( \omega_{\Lambda} \). The lone particle then performs oscillations. In a fluid, however, each fluid element will meet and interact with others along the way, and via reconnection events and so on the motion is damped. The equilibrium is therefore reached on an inverse time-scale comparable to \( \omega_{\Lambda} \). With fast rotation, a lone particle performs small epicycles of frequency \( 2\Omega \) superposed on a slower orbit about the origin of frequency \( \omega_{\Lambda}/2\Omega \). Again, in a fluid we expect this motion to be damped by reconnection events such that an equilibrium is reached on an inverse time-scale \( \omega_{\Lambda}^2/2\Omega \), albeit a different equilibrium from that in the slowly rotating case. And since the motion is on spherical shells, only the component of the Coriolis force perpendicular to gravity is important, so that the effect is greatest on the rotation axis and vanishes in the equatorial plane. Therefore, one would expect the field on the equatorial plane to reach some kind of equilibrium while the rest of the volume is still evolving. This may mean that the field strength is higher near the poles.

3 EVOLVING AND EQUILIBRIUM SCENARIOS

After several of these dynamic evolution time-scales have passed, an equilibrium is reached. A useful concept here is magnetic helicity, defined as \( H \equiv \int B \cdot A \, dV \), where the vector potential is given by \( B = \nabla \times A \). Helicity is perfectly conserved in the case of infinite conductivity (Woltjer 1958), and approximately conserved in fluids with finite (but high) conductivity. This can be explained in terms of helicity having units of energy times length: while magnetic energy is being dissipated on small length scales in reconnection zones, helicity is little affected. This has been confirmed in various laboratory and numerical experiments, e.g., by Chui & Moffatt (1995), Zhang & Low (2003) and Brathwaite (2010). Also, note that helicity is only conserved in a volume bounded by a magnetic surface (i.e. a surface with zero normal component of magnetic field), which clearly a star does not have, since the magnetic field does penetrate the stellar surface. However, there are good reasons to expect the bulk of the flux to self-connect within the star so that only a small fraction passes through the surface – because the gas in the star’s interior has been compressed more – in which case helicity should still be approximately conserved, as is confirmed in simulations (Brathwaite 2008). Therefore, in any case, relaxing towards a stable equilibrium entails evolving towards an energy minimum for the given value of helicity present. We can write \( H = L_B E \), where \( E \) is the magnetic energy (\( \sim B^2 R^2 \)) and \( L_B \) is some length scale, and clearly a lower energy state for a given helicity will have a higher \( L_B \). This explains why Ap stars often display simple dipole equilibria with \( L_B \sim R \), as this represents the lowest energy state. Some have more complex equilibria, where a local energy minimum has presumably been reached with smaller \( L_B \). The theory of equilibrium geometries was reviewed and explored by Brathwaite (2008). In any case, a relaxation towards equilibrium will necessarily increase \( L_B \). If the initial conditions have small \( L_B \) then most of the energy will be lost during the path to equilibrium, as magnetic energy is dissipated at the smaller scales through reconnection. As the field weakens, the evolution time-scale correspondingly increases, so that the total time required to reach an equilibrium depends essentially on the

\[ ^1 \text{Although the whole calculation is already very approximate, this is a justification for using } \Omega \text{ rather than } 2\Omega \text{ above, since the magnitude of the Coriolis parameter } f = 2\Omega \cos \theta \text{ averages over a spherical shell to } \Omega. \]
eventual strength of the equilibrium field rather than the initial field strength.

Note that $L_0$ is not the same as the characteristic length scale $L$ introduced in the last section. An analogy with a collection of left- and right-handed screws in a box helps us to explain the difference (A. Brandenburg, private communication). The screws have a size $L$ (shaped ‘fat’ so that they have the same size $L$ in all directions), ‘energy’ $E$, and helicity $H = \pm LE$, where the sign depends on the handedness. In total we have $N$ screws, so that the size $R$ of the box is given by $R^3 = NL^3$, and the total energy is $E = NE$. The total helicity $H$ depends on the relative numbers of right- and left-handed screws; if all the screws are of one kind, then the total helicity is $H = \pm NE, = \pm LE$. If, however, both handednesses are present, there is cancellation and the helicity is of smaller magnitude – if we write $H = L_0E$, then $L_0$ is smaller in magnitude than $L$ and may be positive or negative. In the case of a simple dipole-like torus MHD equilibrium with comparable toroidal and poloidal components, we have just one screw and $L = L_0 = R^2$.

In any case, we can imagine three different scenarios corresponding to three different sets of initial conditions at the birth of the star.

(i) High $E$, high $H$. The star is born with a strong magnetic field so that initially the evolution time as given by (2) or (3) $\tau_{\text{evol}} \ll \tau_{\text{MS}}$, the main-sequence lifetime, so that the field begins evolving relatively quickly. The star also has a large helicity (which necessarily means that the initial length scale $L_0$ is not too small). This means that at the predetermined equilibrium we still have a sufficiently strong field so that $\tau_{\text{evol}} \ll \tau_{\text{MS}}$, and since this $\tau_{\text{evol}}$ at equilibrium is the time taken to reach equilibrium, we will observe a ‘fossil’ equilibrium field except in extremely (and probably unobservably) young stars.

(ii) High $E$, low $H$. As in (i), initially $\tau_{\text{evol}} \ll \tau_{\text{MS}}$. The small helicity comes from $L_0$ being very small, because $L$ is also small (remember that $L_0 \lesssim L$), or $L$ is large but the field has a high degree of symmetry (e.g. two equal and opposite screws), or a combination of the two. The predestined equilibrium field strength is very low and the evolution time-scale at equilibrium is greater than the star’s lifetime, so an equilibrium is never reached. We observe at time $t$ a non-equilibrium field with $\tau_{\text{evol}} \sim t$.

(iii) Low $E$, low $H$. So little flux survived star formation that right from the beginning $\tau_{\text{evol}} \gg \tau_{\text{MS}}$ and we observe a non-equilibrium field which has evolved little since the star’s birth.

We can see from (2) and (3) that the kilogauss fields in some stars (the Ap/Bp stars and more massive stars with large-scale fields) correspond to scenario (i), since they have $\tau_{\text{evol}} \ll \tau_{\text{MS}}$. Putting the measured Vega and Sirius field strengths into (3) and setting $L = R$ gives an evolution time-scale longer than the stellar ages, which are of the order of $2-5 \times 10^8$ yr. That points prima facie towards scenario (iii), but there are several reasons that scenario (ii) seems likelier. All we need is to reduce the time-scale in (3) by two or three orders of magnitude. A smaller length scale $L$ seems likely, say $L \sim R/5$ would shorten the time-scale by one and a half orders of magnitude. A shorter length scale could also mean that the observed field strengths are an underestimate, since some of the detail in the small scales is lost to averaging. Also, it seems very likely that the field strength in the middle of the star is greater than that at the surface (see below), which would also reduce the dynamical evolution time-scale.

An additional reason to favour scenario (ii) over (iii) is that it puts weaker demands on any solution to the well-known flux problem in star formation, the problem of reconciling the enormous magnetic flux threading the interstellar medium with the much smaller flux in the stars which are born out of it (see e.g. Braithwaite 2012). To make even the most strongly magnetized Ap star, all but $10^{-3}$ of the flux must be lost; to make Sirius’ field, less than $10^{-8}$ can be retained. And finally, scenario (iii) can be ruled out if, as expected, the young star contained a dynamo field powered by either differential rotation or convection (see the next section) which died away, in which case $E$ would initially be reasonably high (although $H$ could be high or low).

The question remains, though, why these stars should have so little magnetic helicity. Since helicity is a measure of asymmetry (more of one kind of screw than the other), it implies a lack of asymmetry in star formation or in a protostellar convective dynamo. Also puzzling is the huge difference in magnetic helicity present in Ap stars and other A stars; Ap stars perhaps suffered some symmetry-breaking event during their formation.

In this sense, it is interesting to note that Ap stars show a complete lack of binary companions with periods shorter than about 3 d (Carrier et al. 2002). It could be that Ap stars are created through the merger of short-period binaries (Tutukov & Fedorova 2010), with their magnetic field resulting from some dynamo process active during the merging phase (see e.g. Soker & Tylenda 2007; Ferrario et al. 2009). The symmetry breaking involved in the violent process of merging two stars could lead to a much higher helicity. The likely large amount of mass lost during the merger process, together with the presence of a magnetic field, could then lead to a rapid spin-down of the merger product, explaining why Ap stars are generally slow rotators. This slow rotation of Ap stars has itself long been a major puzzle (see D’Angelo & Spruit 2011, for recent progress).

In higher mass stars, the lifetimes are shorter and a dynamically evolving field should be correspondingly stronger. For instance, in a star of age 3 Myr, the field strength should be 10 times greater than in Vega and Sirius. However, as mentioned above, in stars above about 10 $M_\odot$ (at solar metallicity) it is likely that a contemporary dynamo in subsurface convection layers can produce a greater field strength than that expected of a dynamically evolving field (Cantiello & Braithwaite 2011).

4 DRIVING

Now we examine the assumption made above that the magnetic field is not affected by convection, meridional circulation and differential rotation. As mentioned above, a dynamo in the convective core is unlikely to produce an observable effect at the surface, but there are also convective layers nearer the surface. In A stars, the most promising to produce fields is a thin helium-ionization layer lying just 1 per cent in radius below the photosphere – this is the subject of the companion paper: Cantiello & Braithwaite (in preparation). There are one or two more convective zones even closer to and at the photosphere but the energetics are unfavourable (Petit et al. 2011). If not actually producing the observed field, these convective layers could in principle affect a failed-fossil field. Whilst the interaction of convection with an imposed large-scale magnetic field is poorly understood, it seems plausible that small-scale diffusive processes
allow the two to co-exist (see e.g. Moss & Tayler 1969), although then the convection may only have a small-scale effect on a failed-fossil field and not alter its observational appearance.

Meridional circulation and differential rotation of some kind could be present in A stars, as a field of the magnitude observed in Sirius and Vega will be unable to inhibit it (see below). The picture of a field evolving dynamically undisturbed by outside influence is therefore probably too simple. However, the following statements can be made. First of all, differential rotation on its own can only change the azimuthal component of the field, rather than the poloidal component which is seen at the surface. Let us assume the star begins its life with a fair degree of differential rotation and a magnetic field of arbitrary geometry. If the magnetic field is above some certain strength, fulfilling the condition $\alpha_{\ast}/\Omega^2 > q^2/3n\pi\Omega^2$, where $\eta$ is the magnetic diffusivity and $q = r/\Omega/\Omega$ is the dimensionless rotation rate, then the magnetic field should damp the differential rotation without losing its original geometry and strength (Spruit 1999, and references therein). In an A star with rotation period 0.5 d, the critical field strength is about 15 G. The differential rotation is damped on the phase-mixing time-scale $\tau_p \sim \tau_{Ohmic}a^3$, where $\tau_{Ohmic}$ (see below) is of the order of $10^{12}$ yr, so that for a 15 G field we have $\tau_p \sim 10^5$ yr, very short compared to the stellar lifetime.

This time-scale is also shorter than the time-scale $\tau_{evol}$ on which an equilibrium is approached, unless the field is much stronger, about 600(L/R) G in a star with $P = 0.5$ d, stronger than a failed-fossil field. The field therefore evolves towards its equilibrium after the differential rotation has been damped.

If, on the other hand, the field strength is below this threshold, the field will suffer ‘rotational smoothing’ whereby its non-axisymmetric component is removed (Raeder 1986). This takes place over a time-scale $\tau_{smooth} \sim \tau_{Ohmic}(1/\Omega)^{2/3}$ (Spruit 1999).

Given that the rotation period is normally a lot shorter than the Alfven time-scale, the rotational smoothing time-scale is clearly much shorter than the phase-mixing time-scale, shorter than any other evolution time-scale. After this smoothing, the star is still differentially rotating but the angular velocity is constant on field lines. The field then evolves towards its equilibrium, which will obviously affect the rotation law.

However, before either of these two states is reached, an instability in the toroidal field may set in (Tayler 1973). This could lead to a self-sustaining dynamo mechanism (Spruit 2002; Braithwaite 2006) whereby the instability converts energy from the toroidal to poloidal parts of the field, and the new poloidal field is wound up by differential rotation to create new toroidal field. Eventually, this will destroy the original differential rotation; once the dynamo has reached saturation, the damping occurs on a time-scale $\tau_{DS} \sim \Omega^{-3/2}a^{1/2} \Omega_{Bu} r_{Bu}$, where $\tau_{KH}$ and $\Omega_{Bu}$ are the Kelvin–Helmholtz time-scale and the breakup spin, which works out at $\sim$300 yr.

At this point, the field will continue to evolve dynamically, the field strength decaying. There may be some symmetry about the rotation axis.

Differential rotation is enforced by the presence of a meridional circulation (Zahn 1992). This circulation is driven by the temperature imbalance arising in rotating stars, with a time-scale given by the Eddington–Sweet time-scale $\tau_{ES} \sim \tau_{KH}(\Omega/\Omega_{Bu})^{-3/2}$, For Vega, rotating at about 90 per cent of breakup (Yoon et al. 2010), this time-scale is about 1 Myr, much shorter than its main sequence and comparable to the phase-mixing time-scale. Therefore, in rapidly rotating intermediate-mass stars, meridional circulation can not only play an important role in keeping the star differentially rotating, but also might have a direct impact on the observable field, for example advecting magnetic flux to or from the surface of the star. The complex interaction of such circulation with a fossil or failed-fossil field is therefore an interesting subject, but is beyond what we can do here.

5 EFFECTS NEAR THE SURFACE

In the interior of the star, the Ohmic time-scale is given by $\tau_{Ohmic} \sim L^2/\eta$, where $L$ is a characteristic length scale and $\eta$ is the magnetic diffusivity. Numerically, $\tau_{Ohmic} \sim (L/R)^{10}12$ yr. Therefore, the field in the bulk of the star evolves purely dynamically unless $L$ is rather small, which seems unlikely given that very small values of $L$ at the surface make the fields practically unobservable. However, lower conductivity near the surface of the star will cause the field to relax to a potential state there. For instance, in a 2 $M_\odot$ of age 3 $\times 10^{12}$ yr, the field should be potential down to a depth $=0.1R_\ast$, calculated by equating the age to $\tau_{Ohmic}$, where $D_{Ohmic}$ is depth and $\eta$ the magnetic diffusivity at that depth. The length scale $L_{surf}$ and field strength $B_{surf}$ at the surface now depend on those at depth $D_{Ohmic}$: $L_{Ohmic}$ and $B_{Ohmic}$. If $D_{Ohmic} > D_{surf}$, then we expect both $L_{surf} \approx L_{Ohmic}$ and $B_{surf} \approx B_{Ohmic}$. If, on the other hand, $D_{Ohmic} < D_{surf}$, then we expect instead $L_{surf} \approx D_{Ohmic}$ and $B_{surf}$ to be rather less than $B_{Ohmic}$, perhaps by a factor of $L/D_{Ohmic}$ (by assuming a statistical $\sqrt{N}$ effect, where $N$ is the number of random elements, i.e. the number of patches of positive or negative radial field component at depth $D_{Ohmic}$ within a horizontal distance $D_{Ohmic}$ of the point in question).

Therefore, if the star is born with a very much weaker field at the surface than lower down, which is not implausible if flux is conserved to some extent during the formation of the star, or if the field is created during formation by the Tayler–Spruit mechanism (see Section 4), then the field strength on the surface may actually increase with time. This increase would be strongest at the beginning of the main sequence, since it is then that the Ohmic depth sinks fastest. In Fig. 1, $D_{Ohmic}$ is plotted against age, and Fig. 2 illustrates

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The fractional depth (i.e. depth/R_\ast), as a function of age, above which the field is Ohmically relaxed (potential) and dynamically relaxed (in equilibrium). A 2 $M_\odot$ model is used. The dynamic penetration depth assumes a field strength of 0.6 G at the photosphere and a rotation period of 0.5 d (appropriate for Vega), and is shown for three different profiles in the magnetic field of the form $V_B = N B/\partial \ln P$.}
\end{figure}

$^3$ Magnetic fields created by the Tayler–Spruit mechanism might also reach the stellar surface (Mullan & MacDonald 2005).
Figure 2. The effect of the increasing Ohmic penetration depth (dashed lines), above which the field is approximately potential. Upper panel: initially, the field is ‘shielded’ from the surface (solid field lines) but as Ohmic relaxation proceeds downwards (to the dashed line) the field relaxes to a potential state (dotted lines), the field strength on the surface increasing. Its amplitude and geometry are comparable to those present at the Ohmic penetration depth. Lower panel: if, however, the magnetic field in the inner layers is structured on a smaller scale, the potential field emerging at the surface will be rather weak.

how the surface field strength can increase or not, depending on the length scale $L_{\text{Ohm}}$.

Furthermore, since the density is lower near the surface, the dynamic time-scale is also shorter, and the field should be dynamically relaxed there. Equating an age $3 \times 10^8$ yr to the dynamic evolution time-scale $\tau_{\text{dyn}}$ using depth $D_{\text{dyn}}$ as the length scale $L$, a field strength (constant with depth) of 0.6 G and a rotation period of 0.5 d give a depth $\approx 0.2 R_\ast$ – note that unlike the Ohmic depth, the dynamic penetration depth does depend on the field strength and rotation speed. The depth of this surface against time is plotted in Fig. 1 (dotted line), together with the depths assuming a field strength–depth relation of the form $V_B \equiv \partial \ln B / \partial \ln P$, with values 1/4 and 1/2. The latter is unrealistic but corresponds to $B \propto \rho^{7/3}$ in a radiative star, which one might naively expect from flux conservation during formation, and is shown for completeness. As with Ohmic relaxation, the length scale and field strength at the surface now depend on those at the dynamic penetration depth $D_{\text{dyn}}$. As far as length scales are concerned, we probably have the same effect, namely that $L_{\text{surf}}$ is comparable to the larger of $D_{\text{dyn}}$ and $L_{\text{dyn}}$. However, the relations between field strengths are different, since dynamic evolution can only reduce the field strength seen at the surface, as new flux cannot be brought upwards since the gas is constrained to move on spherical shells.

Therefore, in Vega and Sirius we would expect the field to be approximately potential down to a depth $\approx 0.1 R_\ast$, and in dynamic equilibrium down to roughly a depth $\approx 0.2 R_\ast$, while the field is continuously evolving below. Given the above considerations, we can infer that the length scale of the field at the surface should be at least as great as about $0.2 R_\ast$. Deep inside the star, we can still have the dynamic evolution time-scale comparable to the star’s age – we simply need (see Section 2) a length scale of less than about $0.2 R_\ast$ and/or a stronger field than that seen at the surface. In young stars, it is not obvious whether the field strength at the surface should increase or decrease with time, since we have two competing effects.\(^\dag\) Certainly, though, its characteristic length scale can only increase. Finally, note that in more massive stars the two depths to which the field should be potential and in equilibrium will be somewhat lower, mainly because the stars are younger.

6 CONCLUSIONS

We have presented the hypothesis that the weak magnetic fields found recently on the main-sequence A stars Vega and Sirius do not arise from any ongoing regenerative or dynamo mechanism, but that they are dynamically evolving. This means that the part of the Lorentz force perpendicular to gravity is balanced not by the pressure-gradient force (as in the case of equilibria in Ap/Bp stars) but by the Coriolis force. The time-scale of this evolution is given in terms of the Alfvén time-scale and rotation angular velocity by $r_A^2 \Omega$, which is then equal to the age of the star.

By inverting (3), we get an upper limit to the field strengths expected; a more quantitative prediction is prevented mainly by a large uncertainty in the quantity and radial distribution of magnetic flux in the protostar, which could be improved by a better understanding of protostellar dynamos. However, the theory does provide some predictions for some observable properties and correlations. The predictions are that (i) the length scale of magnetic structure at the surface of A stars should be no less than approximately a fifth of the stellar radius, but that smaller length scales may be present in more massive or young stars, (ii) essentially all A stars should have fields of strength at least comparable to the Vega and Sirius fields, (iii) younger stars (more massive or otherwise) should tend to have stronger fields, except that very young stars might experience a field strength increase, and (iv) faster rotators should have stronger fields (in fact the factor-of-3 difference in field strengths between Vega and Sirius is exactly what one expects from age and spin correlations).

There should not be any detectable time variability over observable time-scales, and finally, it would be natural to expect some pattern connecting the rotation axis to the magnetic field, i.e. some undetermined kind of symmetry about the rotation axis. These predictions contrast with those of the subsurface dynamo hypothesis, where one would expect time variability, probably structure on smaller length scales and an increase of the magnetic field strength with the stellar age (Cantiello & Braithwaite, in preparation).

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\(^\dag\) Note that in contrast to some other situations near stellar surfaces, magnetic buoyancy is insignificant here since the buoyant velocity scales as $B^2$. 


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