Conceptual method for the detailed analytical calculation of capacitance matrix elements in variable capacitance synchronous machines

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Abstract
This article describes a detailed analytical method for obtaining the capacitance matrix elements in variable capacitance synchronous machines (VCMSs). The analytical modelling approach is performed by introducing a decentralized sinusoidal distribution of the stator/rotor electrodes and involves obtaining the capacitance matrix elements based on the electro motive force distribution in the air gap and calculating the induced electric charge in the stator/rotor electrode surfaces. The proposed analytical formulas are valid for both radial-field and axial-field VCMS structures. The analytical model is evaluated and well verified through analysing a multi-stack axial-field VCMS using 3D finite element simulation. Furthermore, a prototype of the VCMS is manufactured and tested to validate the results obtained from the proposed analytical model and finite element method simulations. Based on the results, the proposed model and analytical formulas are able to accurately estimate the capacitance matrix elements of the VCMSs.

1 | INTRODUCTION

Variable capacitance machines are a bunch of electric machines, based on Coulomb force. The Lorentz force consists of two parts; one part represents the force created between the static electric charges and is called the electrostatic field force (Coulomb force) and the other part represents the force created by the movement of electric charges in a magnetic field and named magnetic force. Coulomb force that can be produced is much smaller than the magnetic force. In this regard, the magnetic force is the dominant one in almost all the conventional electromagnets. Therefore, the small net force that can be produced in variable capacitance machines is the biggest defect of them. On the other hand, variable capacitance machines have a much lower cost, weight and volume due to the lack of expensive, heavy and bulky materials like magnets, iron and steel in their structure. Regarding the lack of complications, such as windings, they have a simpler structure, which makes them suitable for some robotic and medical applications [1–4]; for example a high-speed synchronous micro motor is introduced for intravascular imaging applications in [5]. On a larger scale, variable capacitance machines may be suitable for space applications because of low losses, high electrical efficiency, low weight, high transient response rate and the availability of suitable operating conditions such as vacuum. Moreover, the use of this type of machine in the magnetic field sensitive areas, which may cause unwanted magnetic fields, such as MRI/fMRI devices can be a good choice [6]. The use of electrostatic force for other interesting industrial applications such as micro-electromechanical systems [7], micro-switches [8], micro-pumps [9], micro-valves [10], electrostatic bearings [11,12] and electrostatic gyroscopes [13] is also observed.

The produced electrostatic force depends on the electric flux density and an increase in the electric flux density at the air

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gap can increase the produced torque. The electric flux density can be increased by increasing the permittivity of the dielectric material and/or increasing the electric field intensity between the electrodes, but it should be noticed that the created field should not exceed the tolerable field of dielectric material. Using liquid dielectric materials is one of the options convenient to increase torque. For example, Silicone Nitrile dielectric fluid for an electrostatically actuated hysteresis motor [14] is used as practical case and in spite of the creation of friction and complexity in construction, it obtains desirable results and caused a significant increase in the net torque. Another way to increase torque is to make the structural changes. The use of new geometry for electrodes in the linear motor introduced in [15], has increased the produced force. Owing to the significant friction produced by the slip rings, its application to the low-power machines causes problems. Thus, a structure is proposed in which the rotor electrodes are fed through the induction electrodes in stator and without direct contact [16]. Another proposed structure is a linear motor with skewing electrodes, which causes the motor to move more smoothly, increases the propulsion force and reduces the force ripple and noise [17]. Some excitation methods such as single-frequency, two-frequency and AC-DC methods are presented in [18] and the modulation drive method is proposed in [19] for feeding a small linear actuator.

Some of the previous studies have been devoted to modelling of variable capacitance machines. In the dynamic-transient modelling presented by [20] for a specific structure of the variable capacitance induction machines, the capacitance matrix coefficients are obtained by applying a numerical method. In references [16,21], modelling of a linear variable capacitance machine (VCM) machine is described considering the effects of capacitance imbalance, while the capacitance matrix is obtained both through numerical methods and the experimental measurements (EXP). Also, several simple and useful methods are introduced in [22–24] for measuring the self and mutual capacitances of the VCM machines experimentally. In this regard, finite element (FE)-based simulations have been well recognized as the other powerful conventional methodologies for evaluating all elements of the capacitance matrix [25–27]. Obviously, in comparison with 2D simulations, a 3D FE modelling can present the results more accurately as employed by [28] for estimating the capacitance and torque profiles of a switched VCM. Also occasionally, some other particular numerical approaches can be seen in the literatures such as ‘Comparison Method’, ‘Least Square Method’ and ‘Large Multiplier Method’ which are well accompanied by finite element method (FEM) to calculate the parameters of a VCM machine [29]. By the way, just knowing the values of all constants and coefficients of the capacitance matrix, the conventional classic methods or simplifying $dq$ and $ab$ transforms can be employed for modelling and performance prediction of a variable capacitance machine [30]. Also it can be applied for some performance improvement purposes if required by means of, for example, a modification in the supply voltage or etc. In this regard, some attractive designs can be found in references [21,22,31] where the VCM machines are connected to the external resonance circuits. In fact, commonly the external coils are connected to the rotor electrodes that cause lastly some increases in the output power via electrical LC resonance.

However, obtaining the comprehensive and simple analytical methods can be useful in accurately estimating the behaviours of the variable capacitance synchronous machines (VCSMs) and lead to the industrial development of this type of electrical machines. On the other hand, obtaining the capacitance matrix elements is the first step in modelling of variable capacitance machines. In this article, a new analytical method based on decentralized sinusoidal distribution of the VCSM electrodes is presented for obtaining the capacitance matrix elements by calculating the induced electric charges in the electrodes of each phase. The detailed analytical model is performed for axial field structure of VCSMs and with some minor changes, the proposed model is also validated for radial field machines. Using the proposed analytical method, the behaviour of various VCSM structures can be easily estimated without needing any complex simulations like FEM and so on.

2 | THE VCSM STRUCTURES

Figure 1 indicates the simplified structure of the tow-pole radial field (Figure 1a) and axial field (Figure 1b) VCSM machine. Stator electrodes are placed on the stator surface with three-phase arrangement named $a$, $b$ and $c$, and the DC.
excitation electrodes (group 2) are placed on the rotor surface. Distance between the electrodes and distance between the stator and rotor is determined according to the dielectric material between stator and rotor and the voltage of electrodes. The dielectric material can be air, vacuum [32,33], high-pressure gas [34] or a dielectric fluid [14], and each of them has some advantages and disadvantages. The use of dielectric gases and fluids, in addition to increasing the relative electrical conductivity and consequently increasing capacitance, can significantly increase the breakdown field, which results in the possibility of increasing the terminal voltage and hence increasing the capacitive power of the machine. On the other hand, the use of dielectric fluids and gases causes problems such as difficulty in machine construction and a significant increase in friction. In vacuum conditions, although the breakdown voltage extremely increases and the friction significantly reduces due to the removal of dielectric material, creating vacuum conditions and operating the machine in these conditions seems to be difficult because creating high vacuum conditions at very low pressures and maintaining this pressure requires relatively complex and expensive peripheral equipment. In addition, operating the machine under vacuum and transmitting the mechanical power to the environment outside the vacuum box is another important challenge that is not easy to create [32,33]. The use of air as a dielectric material in the machine is a simple and accessible option, but in this case, the electric field cannot be increased to more than 3kV/mm, which limits the applied voltages and consequently reduces the electrostatic power.

3 | CAPACITANCE MATRIX CALCULATION

It is conventional in electromagnetic machines that the magneto motive force (MMF) distribution is considered sinusoidal in the air gap that requires the implementation of the sinusoidal distribution of the windings in the stator/rotor space [35]. For variable capacitance machines, the distribution of the electro motive force (EMF) in the air gap can be considered sinusoidal. For this purpose, it can be assumed that the plates of each phase electrodes are not integrated and distributed sinusoidally at a distance of 180 electrical degrees. In order to implement this propose, instead of the electrodes of each phase being centrally located in the centre of each pole, the electrodes can be divided into several sections and distributed in a space equal to 180 electrical degrees, so that as the distance between the electrodes from the centre of the pole increases, the coverage surface of the electrodes decreases gradually. As the number of electrode sections in each phase increases, the distribution of the electrodes becomes closer to the sinusoidal distribution, and as a result, the corresponding EMF waveform will be closer to the sinusoidal waveform. It is clear that mechanical constraints prevent the achievement of an ideal sinusoidal distribution, and an approximate sinusoidal distribution is always available. Figure 2 shows a simplified schematic representation of the decentralized distribution of stator electrodes in the radial (Figure 2a) and axial field (Figure 2b) VCSMs. In this figure, the electrodes of each phase are divided into five sections and distributed in the space of each pole. Figure 3 shows the distribution of the electrodes and the resulting EMF, in which the distribution of the electrodes corresponds to the structure as shown in Figure 2. According to the Figure 3, the EMF waveform becomes closer to the sinusoidal waveform if the number of
electrode sections is large. Similar to the assumptions in the magnetic machines for the winding distribution, the analytical equations for the equivalent capacitances of each phase are obtained by assuming an ideal sinusoidal distribution of the electrodes.

### 3.1 Electric flux density

To calculate the air gap EMF and capacitance matrix, it is assumed that phase $a$ electrodes are sinusoidally distributed at a distance of 180 electrical degrees according to Figure 3 (curve $C_r$), where $\beta_p$ is the maximum angle covered by the electrodes or in other words the angle density of the electrodes and $\beta_h$ is the equivalent sinusoidally distributed angle of each phase. In addition, the EMF derived from the phase $a$ electrodes can be shown as Figure 3, where the EMF corresponding to the complete sinusoidal distribution of the electrodes is compared with the EMF corresponding to the distribution described in Figure 2. Therefore, the EMF of each stator phase can be written as (1) where $\gamma_a$, $\gamma_b$ and $\gamma_c$ represent the distribution of the stator electrodes and $v_a$, $v_b$ and $v_c$ are the voltages of the phases $a$, $b$ and $c$ electrodes, respectively.

$$\text{EMF}_a = \gamma_a v_a, \gamma_a = \left(\frac{\beta_p}{2}\right) \cos(q_a)$$
$$\text{EMF}_b = \gamma_b v_b, \gamma_b = \left(\frac{\beta_p}{2}\right) \cos\left(q_b - \frac{2\pi}{3}\right)$$
$$\text{EMF}_c = \gamma_c v_c, \gamma_c = \left(\frac{\beta_p}{2}\right) \cos\left(q_c + \frac{2\pi}{3}\right)$$

In the same way, the distribution of excitation electrodes and the excitation EMF can be written as (2), where $\beta_f$ is the maximum angle covered by the rotor electrodes and $v_f$ is the voltage applied by the DC power supply to the rotor electrodes.

$$\text{EMF}_f = \gamma_f v_f, \gamma_f = -\left(\frac{\beta_f}{2}\right) \sin(q_f)$$

To calculate the equivalent capacitances, it is assumed that the air gap length changes sinusoidally according to (3), where $\alpha_1$ and $\alpha_2$ are positive constants and consequently $(\alpha_1+\alpha_2)^{-1}$ and $(\alpha_1-\alpha_2)^{-1}$ are the minimum and maximum air gap length, respectively. In this way, given that the EMF is defined as the linear integral of the electric field intensity ($E$), the electric flux density ($D$) can be calculated as (4).

$$g(q_r) = \frac{1}{\alpha_1 - \alpha_2 \cos 2(q_r)}$$
$$D(q_r) = e\left(\frac{\text{EMF}}{g(q_r)}\right)$$

According to Equation (4), assuming that all voltage sources except $v_{a0}$ are off, the electric flux density caused by $v_a$ can be written as Equation (5).

$$D_a(q_r, \theta_r) = ev_a\left(\frac{\beta_p}{2}\right) \cos(q_r)\left[\alpha_1 - \alpha_2 \cos 2(q_r - \theta_r)\right]$$

Similarly, the electric flux density caused by $v_b$, $v_c$ and $v_f$ can be written as Equations (6), (7) and (8), respectively.

$$D_b(q_r, \theta_r) = ev_b\left(\frac{\beta_p}{2}\right) \cos\left(q_r - \frac{2\pi}{3}\right)\left[\alpha_1 - \alpha_2 \cos 2(q_r - \theta_r)\right]$$
$$D_c(q_r, \theta_r) = ev_c\left(\frac{\beta_p}{2}\right) \cos\left(q_r + \frac{2\pi}{3}\right)\left[\alpha_1 - \alpha_2 \cos 2(q_r - \theta_r)\right]$$
$$D_f(q_r) = -ev_f\left(\frac{\beta_f}{2}\right) \sin(q_r)\left[\alpha_1 - \alpha_2 \cos 2(q_r)\right]$$

### 3.2 Capacitance calculation

In order to achieve the equivalent capacitances, initially, the amount of induced electric charge must be calculated using Equation (9) on the entire surface of the electrodes of one phase of the VCSM machine.

$$q_{\text{NET}} = \int_{\varphi_1}^{\varphi_2} D(q)dS, \quad dS = \left(\frac{r_o^2 - r_i^2}{2}\right) r d\varphi$$

where, $dS$ is the area element of the electrode plates, $r_o$ and $r_i$ represent the outer radius and inner radius of the axial field variable capacitance machine as shown in Figure 1(a), respectively. Therefore, by replacing Equation (5) in Equation (9) with respect to the sinusoidal distribution of the electrodes, the amount of induced charge on the electrodes of the phase $a$ caused by $v_a$ can be written as Equation (10). Finally, as a result, the self-capacitance of the phase $a$ is obtained as Equation (11).

$$q_{aa}(\theta_r) = c_{a0}v_a + \int_{-\pi/2}^{\pi/2} D_a(q_r, \theta_r) \left(\frac{r_o^2 - r_i^2}{2}\right) \gamma_a d\varphi$$
$$= c_{a0}v_a + \int D_a(q_r, \theta_r) \left(\frac{r_o^2 - r_i^2}{2}\right) \gamma_a d\varphi$$
$$+ \frac{ev_a}{\alpha_1 - \alpha_2 \cos 2(\theta_r)}\left(\frac{\beta_p}{2}\right)^2 \frac{\pi}{2}\left[\alpha_1 - \alpha_2 \cos 2(\theta_r)\right]$$
$$c_{aa}(\theta_r) = c_{a0} + \frac{ev_a}{\alpha_1 - \alpha_2 \cos 2(\theta_r)}\left(\frac{\beta_p}{2}\right)^2 \frac{\pi}{2}\left[\alpha_1 - \alpha_2 \cos 2(\theta_r)\right]$$
where, $c_l$ is the leakage capacitance of the stator. Similarly, other stator self-capacitances can be calculated through the following equations:

$$c_{bk}(\theta_r) = c_b + \varepsilon \left( \frac{r_o^2 - r_i^2}{2} \right) \left( \frac{\beta_i^2}{2} \right) \left[ \alpha_1 - \frac{\alpha_2}{2} \cos 2 \left( \theta_r - \frac{2\pi}{3} \right) \right]$$

(12)

$$c_{ab}(\theta_r) = c_b + \varepsilon \left( \frac{r_o^2 - r_i^2}{2} \right) \left( \frac{\beta_i^2}{2} \right) \left[ \alpha_1 - \frac{\alpha_2}{2} \cos 2 \left( \theta_r + \frac{2\pi}{3} \right) \right]$$

(13)

The induced electric charge and self-capacitance of the rotor electrodes are achieved as Equations (14) and (15), respectively.

$$q_{ff} = \int \frac{2\pi}{\tau} D_f(\varphi_r) \left( \frac{r_o^2 - r_i^2}{2} \right) (r_f d\varphi_r)$$

(14)

$$q_{ff} = \varepsilon \varphi_f + \varepsilon \varphi_f \left( \frac{r_o^2 - r_i^2}{2} \right) \left( \frac{\beta_i^2}{2} \right) \left[ \alpha_1 + \frac{\alpha_2}{2} \right]$$

$$c_{ff} = \varepsilon \varphi_f + \varepsilon \varphi_f \left( \frac{r_o^2 - r_i^2}{2} \right) \left( \frac{\beta_i^2}{2} \right) \left[ \alpha_1 + \frac{\alpha_2}{2} \right]$$

(15)

where, $c_f$ is the leakage capacitance of the rotor. The rotor-side self-capacitance can be moved to the stator-side as follows:

$$c_{ff} = \left( \frac{\beta_f}{\beta_f} \right) c_{ff}$$

(16)

To calculate the mutual capacitance between phases $a$ and $b$, the electric charge induced by $\varphi_b$ on the phase $a$ electrodes must be calculated and then $c_{ab}$ is obtained as Equation (18).

$$q_{ab}(\theta_r) = \int_{-\pi/2}^{\pi/2} D_b(\varphi_a, \theta_r) \left( \frac{r_o^2 - r_i^2}{2} \right) (r_f d\varphi_r)$$

(17)

$$q_{ab}(\theta_r) = -\varepsilon \varphi_b \left( \frac{r_o^2 - r_i^2}{2} \right) \left( \frac{\beta_i^2}{2} \right) \left[ \alpha_1 + \frac{\alpha_2}{2} \cos 2 \left( \theta_r - \frac{\pi}{3} \right) \right]$$

$$c_{ab}(\theta_r) = -\varepsilon \left( \frac{r_o^2 - r_i^2}{2} \right) \left( \frac{\beta_i^2}{2} \right) \left[ \alpha_1 + \frac{\alpha_2}{2} \cos 2 \left( \theta_r - \frac{\pi}{3} \right) \right]$$

(18)

Other stator mutual capacitances can be obtained as:

$$c_{ac}(\theta_r) = -\varepsilon \left( \frac{r_o^2 - r_i^2}{2} \right) \left( \frac{\beta_i^2}{2} \right) \left[ \alpha_1 + \frac{\alpha_2}{2} \cos 2 \left( \theta_r + \frac{\pi}{3} \right) \right]$$

(19)

In order to calculate the mutual capacitance between the phase $a$ and $f$ (excitation electrodes), the electric charge
TABLE 1  Design parameters of the designed multi-stack VCFSM machine

| Symbol | Quantity | Value |
|--------|----------|-------|
| εr    | Dielectric constant | 1 (air) |
| Nstk  | Number of stacks | 3 |
| Ne    | Number of stator layers | 6 |
| Nr    | Number of rotor layers | 3 |
| p     | Number of poles | 128 |
| ψ     | Pole pitch | 2.8125° |
| βs    | Equivalent stator electrode angle | 1.571° |
| βf    | Equivalent rotor electrode angle | 1.571° |
| Rss   | Inner radius | 40 mm |
| Rus   | Outer radius | 80 mm |
| g     | Air gap length | 1 mm |
| Nph   | Number of stator phases | 3 |
| Vm    | Stator voltage amplitude | 320 V |
| f     | Stator voltage frequency | 1 kHz |
| Vf    | Excitation voltage | 2.6 kV |
| ns    | Synchronous speed | 937.5 rpm |

\[
c_{af}(\theta_r) = -\epsilon \left( \frac{r_a^2 - r_i^2}{2} \right) \left( \frac{\beta_s}{2} \right) \left( \frac{\beta_f}{2} \right) \left( \frac{\pi}{2} \right) \left[ \alpha_1 + \frac{\alpha_2}{2} \right] \sin(\theta_r) (22) \]

The rotor-side capacitances can be moved to the stator-side using Equation (25).

\[
\begin{bmatrix}
  c_{sf}' \\
  c_{bf}' \\
  c_f'
\end{bmatrix} =
\begin{bmatrix}
  \frac{\beta_s}{2} \\
  \frac{\beta_f}{2} \\
  \frac{\pi}{2}
\end{bmatrix}
\begin{bmatrix}
  c_{af} \\
  c_{bf} \\
  c_f
\end{bmatrix} (25)
\]

The obtained values are calculated for the axial field machine (Figure 1b). In order to obtain the capacitance matrix elements for radial field VCFSM as shown in Figure 1a, it is sufficient to replace only \( \frac{r_a^2 - r_i^2}{2} \) with any r, l, where r is the average radius and l is the axial length of the air gap in the radial field structure of the variable capacitance machine.

3.3 | Dynamic equations of the VCSM

In order to describe the dynamic behaviour of the motor, it is necessary to obtain the equations describing the relations between different parameters of the motor.
These parameters include variables such as source current, voltage and electric charge of the electrodes and dynamic variables of motion including angular velocity and displacement. Hence, assuming that the positive direction of the stator currents is out of the terminals, the differential equations of the machine can be written as follows:

\[
[I(t)] = [R]^{-1}[V(t)] + \frac{d[Q(t)]}{dt} \\
=[R]^{-1}[V(t)] + \left( [C(\theta_r)] \frac{d[V(t)]}{dt} + \omega_m \frac{d[C(\theta_r)]}{d\theta_m} [V(t)] \right)
\]

(26)
where:

\[
[R] = \text{diag}\left[ r_s, r_s, r_s, \frac{1}{2}r_f \right], [f(t)] = \left[ -i_a, -i_b, -i_c, i_f \right]^T
\]

\[
[V(t)] = \left[ v_a, v_b, v_c, v_f \right]^T, [Q(t)] = \left[ q_a, q_b, q_c, q_f \right]^T
\]

\[
[C(\theta_r)] = \begin{bmatrix}
        c_{aa} & c_{ab} & c_{ac} & c_{af} \\
        c_{ba} & c_{bb} & c_{bc} & c_{bf} \\
        c_{ca} & c_{cb} & c_{cc} & c_{cf} \\
        c_{da} & c_{db} & c_{dc} & c_{df}
\end{bmatrix}
\]

\[
\theta_m = \frac{(2\theta_r)}{P}
\]

where \( r_s = 1/(\omega c \tan\theta) \), where \( \omega \) is the electrical angular speed of rotor, \( c \) is the corresponding parallel capacitance and \( \tan\theta \) is the loss tangent of the dielectric material [36]. In addition, the dynamic equations of motion are given in Equation (28). In this way, the equations describing the dynamic behaviour of machine are obtained as a system of differential equations.

\[
d\omega_m/dt = \left( \frac{1}{J} \right) (T_e - T_f - B\omega_m), \omega_m = d\theta_m/dt
\]

where \( \theta_m = (2\theta_r)/P \) defined as the angular position of rotor, \( P \) is the number of poles, \( \omega_m \) is the mechanical angular speed, \( J \) is the moment of inertia, \( B \) is the coefficient of friction, \( T_i \) is the input torque and \( T_e \) is the electrostatic torque given by Equation (29).

\[
T_e = \frac{1}{2} |V(t)|^2 d[C(\theta_r)] \begin{bmatrix} V(t) \end{bmatrix}
\]

4 | CASE STUDY

To evaluate the analytical modelling, a multi-stack axial field machine is designed and its performance is investigated through the proposed analytical formulas and 3D FEM simulations. Also, an experimental prototype is manufactured, and the results of practical measurements are compared with the FEM simulation and the analytical model. Theoretical calculations are performed by calculating the capacitance matrix using the proposed analytical formulas. Then, the dynamic
differential equations are implemented in MATLAB-SIMULINK software and the results are presented. The design parameters of the machine are shown in Table 3. Figure 6 shows the simplified tow-pole structure of the designed machine. As shown in Figure 4, the electrodes of each stator phase are arranged on a separate stack. Each of the layers of the rotor and stator contains tow conductive metal electrodes on each side mounted on the plates made of insulating material and create a tow-pole machine. Each stack of the machine consists of one layer of rotor in the middle and two layers of one of the stator phases that are located on both sides of the rotor layer. The rotor electrodes are arranged in a similar manner on both surfaces of all rotor layers, and the stator layers of stack 2 and stack 3 relative to the stack 1, are shifted by 120 and 240 electrical degrees, respectively. Given that the air gap is considered uniform, therefore \( \alpha_2 = 0 \) and \( X_q = X_s \) (\( X_s \) is defined as synchronous reactance). Moreover, because the electrodes of each phase are designed in separate stacks, there is no coupling between the different phases of stator, and as a result some elements of the capacitance matrix are equal to zero. In other words \( c_{ab} = c_{ac} = c_{bc} = 0 \). The only difference between the designed machine and the basic structure shown in Figure 2 is that in the designed machine, the electrodes of each phase are arranged in a separate stack. Also, for each layer of the rotor, two stator layers are placed on both sides of the rotor. These

| Symbol | Quantity                  | Calculation method | Value (pF) | Error (%) |
|--------|--------------------------|--------------------|------------|-----------|
| \( C_{rs} \) | Stator capacitance       | THE                | 417.6      | 2.3       |
|        |                          | FEM                | 420.8      | 1.6       |
|        |                          | EXP                | 427.6      | -         |
| \( C_{rf} \) | Rotor capacitance       | THE                | 1259       | 1.0       |
|        |                          | FEM                | 1262       | 0.7       |
|        |                          | EXP                | 1271       | -         |

Abbreviations: EXP, experimental measurement; FEM, finite element method; THE, theoretical calculations.
changes have been made to increase the power of the designed machine, which is not the main scope of this study. Therefore, there is no difference in terms of modelling and calculating the capacitances between the designed machine and the basic structure of the VCMS machines and the proposed method is valid for all VCMS structures with flat electrode plates.

Figure 5 shows the VCMS model simulated in 3D FEM software and Figure 6 describes the images of different parts of the prototype constructed based on the simulated model with the parameters given in Table 1. In order to make stator and rotor layers, the printed circuit board (PCB) manufacturing technology is used. The thickness of the rotor and stator plates is 0.5 and 1.6 mm, respectively, and their material is made of fiberglass. Also, the electrodes placed on the stator and rotor plates are of 18 microns in thickness and made of copper. The very thin plastic layers are placed on the stator plates to prevent the direct contact of the electrodes.

5 | DISCUSSION AND RESULTS

In this section, in order to evaluate the proposed model, the results of analytical calculations are compared with the results of 3D FE simulations and EXP.

5.1 | Finite element analysis

The FE simulation is performed using Ansys Maxwell 3D software. Air, copper and fiberglass are selected as the materials of the dielectric, electrodes and plates, respectively. The simulation is performed when the rotor position gradually changes from the non-alignment position to the full alignment position. As a result, the capacitance matrix and the field distribution are achieved. Figure 7 shows the distribution of electric field intensity in the air gap and around the electrodes. The results are recorded in the non-alignment, half alignment and full alignment position of the rotor and stator electrodes when their potential difference is 3 kV. The electric field distribution on the hypothetical plane located at the centre of the air gap indicates that the field is nearly uniform in the electrode alignment area and the highest field intensity is observed in this area. As the rotor electrodes move from the non-alignment position (Figure 7(a)) to the full alignment position (Figure 7(c)), the electric field intensity increases gradually.

5.2 | Capacitance characteristics

Regarding the limitation in the minimum area of the electrodes and the high number of poles to generate more power, each
phase of stator is designed in a separate stack. Therefore, the mutual capacitance between the phases of stator is zero. Also, due to the uniform air gap, the self-capacitances of each phase are constant. On the other hand, there is a large leakage capacitance because of the radiational nature of the electric field and similarity of the dielectric material (air) with the surrounding environment.

Figure 8 shows the mutual capacitance characteristic between the phase a of stator and the rotor electrodes achieved by theoretical calculations using the proposed analytical model (THE), FEM and EXP. In order to obtain the capacitance matrix through FEM simulation, first the designed machine model is implemented in the software environment. Then, by defining the voltage terminals for the electrodes of each stator phase and the rotor excitation electrodes, the self-capacitances and mutual-capacitances for each position of the rotor are obtained. By moving the rotor from the full-alignment position to the non-alignment position, the capacitance characteristic is obtained and the 4x4 matrix is formed. An LCZ metre is used to measure the capacitance characteristic [23,24]. The measurement procedure involves changing the position of the rotor electrodes from the non-alignment position with the stator electrodes to the full alignment position and all the capacitance values are accurately measured. As shown in Figure 8, the results of the proposed analytical model are in good agreement with the results of FEM simulations and EXP. In addition, self-capacitances of the stator and the rotor achieved by theoretical calculations, FEM simulations and EXP are reported in Table 2. As shown in Table 2, the error amounts of the theoretical calculations and FEM simulations for the stator self-capacitance are 2.34% and 1.60%, respectively. Also, the error amounts for the rotor self-capacitance are 1% and 0.7%, respectively.

5.3 | Open-circuit test

Figure 9a illustrates the voltage induced to each phase of the stator electrodes when the rotor electrodes are connected to 2600 V DC excitation voltage and the stator phases are open-circuited. In this condition, the phase-to-ground voltage of the stator is about 300 V when the rotor rotates at a speed of 937.5 rpm (in this case, the frequency of the stator voltage is equal to 1 kHz). In addition the amplitude of the open-circuit voltage remains constant as the rotor speed changes. The fast fourier transform (FFT) analysis and harmonic spectrum of the OC voltage are shown in Figure 9b. The value of the THD index of OC voltage is 3.02% in FEM simulation and 2.01% in EXP.
Figure 10 shows the open-circuit (OC) curve and the error curve obtained through the proposed analytical model, FEM simulations and EXP. The results of all three methods are well matched. The maximum error value for the analytical method that occurs at an excitation voltage of 3 kV is 32.6 V for the open-circuit test, which is equivalent to 8.83%.

In view of the fact in electromagnetic machines that the induced voltage in the stator windings depends on the rotor current, the no-load characteristic is defined as open circuit voltage of stator in terms of the rotor current variations. But in variable capacitance machines, the induced voltage in the stator circuit depends on the voltage of the rotor electrodes. Therefore, the no-load characteristic is defined as OC voltage of stator in terms of excitation voltage. Since the permittivity of known dielectric materials is low, despite the high excitation voltage in the rotor, it is observed that the induced voltage in the stator electrodes is not noticeable. Therefore, the produced power in VCIM machines is very small and this is the biggest defect of this type of electrical machines.

5.4 | Short-circuit test

Figure 11a shows the current passing through the stator circuit when the excitation voltage is equal to 2.6 kV and stator electrodes are short-circuited. The amplitude of the SC current is about 0.9 mA and is obtained when the rotor speed is 937.5 rpm. Therefore, frequency of the current signal is equal to 1 kHz. From the FFT analysis presented in Figure 11b, the value of total harmonic distortion (THD) index of the SC current is 9.30% in FEM simulation and 11.41% in EXP. Figure 12b illustrates that unlike the open-circuit voltage, the short-circuit current is directly related to the change in rotational speed. Accordingly, the amplitude of the SC current increased by increasing the rotational speed of the rotor. Based on the error diagram shown in Figure 12b, the maximum error value for the analytical method that occurs at a stator frequency of 3 kHz is 0.30 mA for the back MMF current, which is equivalent to 10.52%.

In order to calculate the internal current (Back MMF current), the short-circuit (SC) test is implemented. Pursuant to Figure 13, the internal current increased with increasing the excitation voltage while the excitation current is very small. According to the error curve illustrated in Figure 12c, the maximum error value for the analytical method that occurs at an excitation voltage of 3 kV is 0.10 mA for the short-circuit test, which is equivalent to 10.02%.

Because the current of variable capacitance machine is very small, the ohmic losses of these machines are negligible and their electrical efficiency is very high. The major losses in the variable capacitance machines are mechanical losses, such as losses caused by friction in bearings, slip ring and the friction
caused by viscosity of the dielectric material used in the distance between electrodes. Regarding the low power of variable capacitance machines, these mechanical losses are effective and tangible.

In VCSMs, exactly the opposite of magnetic machines, if the rotor electrodes be connected to a constant source when the stator electrodes are open-circuit, a voltage with constant amplitude is induced in the stator electrodes, and if the stator electrodes are short-circuited, a current corresponding to the rotational speed will be created in the stator circuit known as the back MMF current. Pursuant to the previous sentences, it is observed that the MMF in variable capacitance machines is a dual for the EMF in the electromagnetic machines. In addition, the open-circuit conditions in electromagnetic machines are

**Figure 13** Short-circuit test. (a): Internal current in terms of excitation parameters, (b): Short-circuit curve and (c): Error curve
similar to the short-circuit conditions in variable capacitance machines, and vice versa. This is another interesting duality of these machines.

As illustrated in the harmonic spectra shown in Figures 8b, 9b and 11b, the amplitude of the second to tenth harmonics is very small compared to the fundamental frequency. Since the distribution of the electrodes is assumed to be completely sinusoidal in theoretical calculations performed through the proposed formulas, no harmonics are seen in the spectra. In the FEM simulations and EXP, small amounts of capacitance harmonics, voltage harmonics and current harmonics are observed. Given the half-wave symmetry in the waveforms, it is obvious that even harmonics should not appear and the amplitude of the even harmonics is very small and negligible.

According to the error curves shown in Figure 10b, 12b and 13c, it is clear that the FEM simulation results are very close to the EXP. But there is more error in the results obtained through proposed theoretical calculations. The causes of this error can be attributed to the assumption of an ideal decentralized sinusoidal distribution of the stator and rotor electrodes and also the absence of field leakage consideration in the theoretical calculations. In order to reduce these errors, some methods in future can be provided for accurate analytical calculation of leakage capacitance so that the field leakage effect can also be modelled. Table 3 shows the amount of error calculated in the tests performed at the point where the maximum amount of error occurred and also at the main operating point of the designed multi-stack VCSM prototype machine. According to this table, the amount of error calculated through FEM simulation is obtained between 1.8% and 5.5%. This error is between 8% and 14% for the analytical calculations.

Presence of damper cages on the rotor face of the common magnetic synchronous machines, is very essential regarding their main duties on stability, shortening the rotor oscillations and regulating the positive and negative harmonic content rotating fields. Similar subjects would be much attractive while being questionable for the VCSMs due to unavailable previously reported literatures on configurations, structures, design procedures, assembly challenges, modellings, equivalent circuits and dynamic-transient performances related to damper electrodes which require further works. In addition, employing a VCSM machine in the applications such as rotational speed measurement seems very interesting thanking low weight and volume values of the device, low electrical/mechanical losses and fast transient response that can be considered as our current project and hopefully will be reported in the near future.

## 6 | Conclusion

The main results and achievements of this article can be itemized as follows:

- Obtaining the detailed analytical formulas for calculating the elements of the capacitance matrix based on the electric charges induced on the stator/rotor electrode surfaces
- Implementing the 3D FE simulation to evaluate the accuracy of the proposed analytical modelling of the capacitance matrix
- Evaluating the accuracy of the proposed analytical model and 3D FEM simulation by studying the behaviour of a multi-stack VCSM prototype using some tests/measurements on the manufactured prototype

The obtained results of this study show that the amounts of error of the proposed method are about 10% and the proposed analytical model and the formulas presented in this article are able to accurately calculate the elements of the capacitance matrix and estimate the behaviour of VCSMs.

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