Development of a mathematical model of wind waves in the area of the proposed construction of a hydraulic structure

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Abstract. The article presents a non-stationary two-dimensional hydrostatic model of wave propagation in the water area of the port of the Bay of Five Hunters, protected by a coastal protection structure in the form of a breakwater. The work uses the methods of mathematical modeling in the Delphi development environment. The results obtained are presented in graphical form, where different wave heights and maximum wave amplitudes are displayed using a color palette. The consistency of the obtained calculations with physical laws is shown, the effectiveness of the construction of a coastal protection structure is analyzed.

1. Introduction
The difficulty of obtaining reliable data on the characteristics of wind waves makes it difficult to predict wind waves and, consequently, to assess its impact on hydraulic structures [1-4]. Mathematical modeling is an urgent solution to this problem, especially for ensuring port activities and activities in the coastal zone, because this method allows, with a minimum of data, to simulate various characteristics of waves, their development and impact on economic activities [5].

In connection with the construction of the port in the Bay of Five Hunters, where the frequency of storm surges is rather high, a need arose for the construction of a coastal protection structure in the form of a breakwater and an assessment of its effectiveness. Previously, no studies were conducted for this study area [6-7]. The purpose of the study was to create a hydrodynamic model of the propagation of wind waves in the water area of the port protected by a breakwater. To achieve this goal, the following tasks were solved: determining the spatio-temporal variability of wind waves in the port water area, calculating and analyzing the spatial distribution of the main characteristics of wind waves - height and maximum amplitude, analyzing the effectiveness of constructing a coastal protection structure [8].

2. Materials and methods
To solve this problem, a non-stationary two-dimensional hydrostatic model of the motion of surface waves was developed in the Delphi development environment. We consider the water area of finite depth, where the sea depth is comparable to the wavelength [1; 6]. The level perturbation is given by the ratio:

\[ \xi = \xi_0 \cos(kx - \sigma t) \]  \hspace{1cm} (1)
Where \( \xi \) – is the level disturbance, \( \xi_0 \) – wave amplitude, \( k \) – wave number, \( \sigma \) – angular frequency.

The calculation of the propagation of wind waves is carried out according to a nonstationary two-dimensional model, in which a system of equations in the Boussinesq approximation (equations of motion and equations of continuity) is used to calculate the surface level disturbances

\[
\frac{du}{dt} = -g \frac{d\xi}{dx}
\]

\[
\frac{dv}{dt} = -g \frac{d\xi}{dy}
\]

\[
\frac{du}{dx} + \frac{dv}{dy} = -k \frac{d\xi}{th(kh)} \frac{dt}{dt}
\]

Where \( u,v \) – the components of the speed in the directions of the \( x \) and \( y \) axes; \( h \) – sea depth, \( \xi \) – surface level disturbance, \( g \) – gravity acceleration, \( k \) – wave number.

The system of equations is solved by the finite difference method on a displaced rectangular grid. When the derivatives along the axis are approximated by directly directed differences, we obtain the systems of equations

\[
u_{i,j} = u_{i,j} - g \frac{dt}{dx} (\xi_{i,j} - \xi_{i,j-1})
\]

\[
v_{i,j} = v_{i,j} - g \frac{dt}{dy} (\xi_{i,j} - \xi_{i-1,j})
\]

\[
\xi_{i,j} = \xi_{i,j} - \frac{th(kh)}{k} \frac{dt}{dt} ((u_{i+1,j+1} - u_{i,j}) + (v_{i+1,j} - v_{i,j})) + q_0 \sin(\frac{2\pi}{Tt})
\]

Where \( dI = d(x,y), T \) – period, \( q_0 \) – fictitious source.

On solid lateral boundaries, a boundary condition is established in the form of a non-leakage condition

\[
(U \bullet n) = 0
\]

Where \( U \) – is the velocity vector, \( n \) – normal to the lateral boundary.

At the liquid lateral boundary, the transmission condition is set, given by the impedance relation

\[
u = g \frac{\xi}{c}
\]

Where \( c \) – phase velocity.

The absence of shear stress of wind friction was set on the surface

\[
\tau_{0x} = \rho u K_u \frac{du}{dz} \bigg|_{z=0} = 0
\]

Where \( \tau_0 \) – surface shear stress component.

At the bottom, a sticking condition is set for the horizontal velocity components
\[ \rho_0 K_u \frac{dH}{dz} = \tau_{hx} \]  \hspace{1cm} (11)

Where \( \tau_{hx} \) – component of the shear stress of friction at the bottom [2].

3. Results
The initial data of the model is an array of depths for the computational grid (Fig. 1), the parameters of the incoming waves are set using a fictitious source. The output of the model is the obtained wave characteristics (wave height and amplitude).

When solving this problem, a rectangular grid with dimensions of 420x385 nodes was used with a vertical step of 2 m and 1 m horizontally. Figure 1 shows the location of the coastal protection structure (breakwater), which in this model is used as a solid wall; the direction of propagation of incoming waves is also shown; at the port entrance, the liquid boundary condition is used.

The calculation was carried out until the sea waves were fully established. Based on the results of the calculations, maps of the propagation of wave heights were built at the initial moment of time, after 6.12 and 30 minutes (figure 2-5).

**Figure 1.** Map of the depths of the Pyati Okhotnikov Bay (m) indicating the location of the coastal protection structure (breakwater) in the port water area and indicating the direction of propagation of incoming waves.

**Figure 2.** Wave height distribution map (cm) at the initial moment of time.

**Figure 3.** Wave height distribution map (cm) after 6 minutes.
4. Discussion

According to the results of calculations, at the initial moment of time, the incoming wave front undergoes a transformation (figure 2), the front unfolds, in connection with the distribution of depths in the water area. Reaching the coast, the waves approach it along the normal (figure 3 and 4), are reflected and propagate further along the water area, thus, a classical refraction pattern is observed, which repeats when the wave front reaches a solid boundary. The results obtained also include the phenomenon of diffraction [2; 3]. When waves enter the water area, the zones of light and shadow, which are located behind the breakwater, are well separated (figure 3). The shade zone is considered the best place for anchorage of ships because of the least influence of waves on ships standing in the port [4]. It was determined that it took less than 30 minutes to fully establish the waves, the picture of the steady-state waves has a complex cellular structure, with a decrease in the wave height towards the periphery of the cell (figure 5). As a result of calculations, the maximum wave height was 7.5m. The highest wave heights are observed at solid boundaries and in the center of the port water area, due to the convergence of waves reflected from solid boundaries.

The maximum amplitude of wind waves in the water area was considered with averaging every 1000 seconds. To identify the port areas with the highest concentration of wind waves with the maximum amplitude, maps of the spatial distribution of the maximum amplitudes of wind waves were built with averaging over 1000, 20000, 50000 seconds (figure 6-8).
Figure 6 shows that the maximum amplitude is observed at solid boundaries and reaches, near the coast, near the port entrance, 1 meter. On most of the rough surface, the amplitude is about 70-80 cm, further from the passage to the port water area, the amplitude decreases and changes from 40 to 10 cm at the extreme waves.

In Fig. 7, the waves are distributed over almost the entire water area. The maximum amplitudes are observed near the coast at the entrance to the port and are 1 meter, there is a large accumulation of wave energy, which also leads to the presence of a large number of waves with an amplitude of about 80 cm. In a large part of the investigated water area, especially along the coastal protection structure, waves with an amplitude of about 50 cm are observed. The smallest amplitudes are noted in the central part of the port water area and are 10 cm.

When averaged over 50,000 seconds (figure 8), the picture has a very complex structure. The maximum amplitudes are also observed when entering the port near the coast and exceed 1 meter. The number of waves with the maximum amplitude has increased in comparison with the previously considered case, and the zone of occurrence of waves with the maximum amplitude increases and moves deeper into the water area. Wind waves with an amplitude of about 80 cm throughout the entire water area are especially concentrated in the areas at the port entrance and along the coastline. The minimum amplitude of 10 cm is observed locally and practically does not occur. Waves with an amplitude of 40-60cm prevail.

After the establishment of waves, the picture is complex, most of the water area is covered by waves with amplitudes of about 60 cm, the maximum value of the wave height is about 1.5 m. The rest of the values are 40 cm on average. The concentration of wind waves with a maximum height of about 1 m occurs at the entrance to the port water area and in the central part at the last stages of the calculation, where the greatest accumulation of wave energy is observed.

5. Conclusion
Based on the results of calculations, it can be concluded that the influence of coastal protection structures on waves within the port water area takes place, reducing the wave height and reducing the impact on port facilities [2; 4]. The presented model for calculating wave propagation in the port water area shows the picture of wave propagation in the water area and calculates the main wave parameters. The obtained results of the model do not contradict the physical laws [9-10].

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