The Harari–Shupe preon model and nonrelativistic quantum phase space

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Abstract

We propose that the whole algebraic structure of the Harari–Shupe rishon model originates via a Dirac-like linearization of quadratic form $x^2 + p^2$, with position and momentum satisfying standard commutation relations. The scheme does not invoke the concept of preons as spin-$1/2$ subparticles, thus evading the problem of preon confinement, while fully explaining all symmetries embedded in the Harari–Shupe model. Furthermore, the concept of quark colour is naturally linked to the ordering of rishons. Our scheme leads to group $U(1) \otimes SU(3) \otimes SU(2)_L$ with two of the $SU(2)$ generators not commuting with reflections. An interpretation of intra-generation quark–lepton transformations in terms of genuine rotations and reflections in phase space is proposed.

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1. Introduction

The Standard Model (SM) is very successful in its description of the interactions of elementary particles. Yet, putting its successes aside, it contains many seemingly arbitrary features which indicate the need for a deeper explanation. In particular, while it was very natural to assume that the gauge principle known from electromagnetism should be extended to other interactions, the choice of $U(1) \otimes SU(3) \otimes SU(2)_L$ as the gauge group is dictated solely by experiment and remains unexplained at the level of theoretical principles. In other words, we do not know a simple theoretical reason that presumably underlies the emergence of internal symmetries and could explain the structure of SM generation.

Following the success of composite models throughout the history of physics, the proliferation of fundamental particles naturally led people to consider quarks and leptons as built of some constituents, dubbed “preons” by Pati and Salam [1]. The most interesting of such models is the Harari–Shupe model [2], which describes the structure of a single SM generation with the help of only two spin-$1/2$ “rishons” $V$ and $T$, of charges 0 and $+1/3$ respectively. This is shown in Table 1, where total charges and hypercharges of particles are also listed (for other preon models, see, e.g., [3]). However, though algebraically very economical, the rishon model has several drawbacks. These include: the issue of preon confinement at extremely small distance scales (when confronted with the uncertainty principle), the apparent absence of spin-$3/2$ fundamental particles, and the lack of explanation as to why the ordering of three rishons is important (this ordering gives rise to the “threeness” of the colour degree of freedom). These problems were addressed, e.g., in [4].

On the other hand, one has to be aware that explaining the existence of a multiplet of some symmetry in terms of particle

| $Q$ | $\nu_e$ | $\nu_R$ | $\nu_G$ | $\nu_B$ | $e^+$ | $d_R$ | $d_G$ | $d_B$ |
|-----|--------|--------|--------|--------|------|-------|-------|-------|
| 0   | $+\frac{2}{3}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+1$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ |
| $-1$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+1$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ |
constituents, though so successful in the old days, may be going too far. For example, Heisenberg was against such unwarranted “explanations” of symmetries [5]. His point of view was that at some point in the process of dividing matter again and again, the very concept of “dividing” loses its meaning.

Recently, a proposal along such general lines has been made by Bilson–Thompson, who suggests correspondence between the algebraic structure of the Harari–Shupe model and the topological properties of braids composed of three “helons” [6]. In his model, the “binding” of preons is topological in nature, and thus preons are not to be considered as confined point-like particles.

The approach presented below belongs to this very general line of reasoning with Harari–Shupe rishons considered to be purely algebraic “components”, and not ordinary confined point-like particles (the meaning of the term “algebraic component” will be fully explained as we proceed with the presentation of our proposal).

2. Spatial and internal symmetries

When searching for a principle underlying the appearance of quantum numbers corresponding to internal symmetries, one should note that some quantum attributes of elementary particles are clearly associated with the properties of classical macroscopic continuous space in which these particles move (e.g., spin). This suggests that internal quantum numbers could perhaps be also connected with the properties of some properly understood “classical space”. Such a point of view is held by several physicists, e.g., Penrose, who writes in [7]: “I do not believe that a real understanding of the nature of elementary particles can ever be achieved without a simultaneous deeper understanding of the nature of spacetime itself”.

Now, it should be noted that all the quantum numbers for which a connection with macroscopic classical space is known can be established using strictly nonrelativistic reasoning. This refers not only to spin and parity, but also to the existence of particles and antiparticles (and thus C-parity). Indeed, with antiparticles interpreted as particles moving backwards in time, it should be obvious that the existence of these two groups of objects is closely related to the existence of the operation of time reflection, and not to truly relativistic transformations. It may be formally shown that antiparticles also emerge when the strictly nonrelativistic Schrödinger equation is linearized à la Dirac [8].

It should be also kept in mind that the connection between space and time is more subtle than the standard mathematical form of special relativity would suggest. Indeed, the latter form emerges only when the Einstein radiolocation prescription for the synchronization of distant clocks is adopted. However, distant clocks may be synchronized in various ways, reflecting the presence of a kind of gauge freedom related to the impossibility of measuring the one-way speed of light. With a suitable gauge even absolute simultaneity may be achieved, obviously without spoiling the successes of the theory of special relativity [9].

In view of the nonrelativistic origin of all quantum numbers for which their connection with the macroscopic classical arena has been established, the simplest expectation is that other observed quantum numbers of elementary particles may be also inferred through nonrelativistic reasoning [10].

3. Phase space as arena of physical processes

After restricting our considerations to the nonrelativistic approach, we observe that the description of the time evolution of a single particle may be provided either on the background of the three-dimensional position space, or on that of the six-dimensional phase space. Indeed, the Hamiltonian formalism—where position and momentum are independent variables—suggests that we may treat phase space as an arena of physical events. We now recall that our goal is to understand the origin of the quantum numbers of elementary particles, and that quantum mechanics works in phase space. Consequently, the choice of phase space as an arena for physical processes seems to be a proper choice for our purposes.

Consideration of nonrelativistic phase space as the arena of events permits a generalization of ordinary transformations of space to those of phase space. Obviously, if such generalized transformations are to be feasible, one has to add another physical constant, of dimension [momentum/position], which permits the expression of all six independent phase–space coordinates in terms of the same dimensional units. The actual value of this constant is completely irrelevant at this moment. It suffices to say here that—together with the Planck constant (and the velocity of light c)—a natural mass scale is then set. The introduction of such a constant was considered by many, in particular by Born, who observed that various physical quantities are invariant under the so-called “reciprocity” transformations $x \rightarrow p$, $p \rightarrow -x$ [11].

The choice of phase space as an arena is possible because physics does not deal with reality “directly”, providing only its descriptions instead. Consequently, different descriptions may be used to deal with the same physics, leading to the same (or similar) predictions. A well-known example of this general truth is provided by gauge theories, whose physical predictions are independent of the gauge.

4. Basic invariant and its linearization

The basic invariant in the standard description (3D arena of positions) is $x^2$. In the phase-space-based description we have to consider $p^2$ as well, which constitutes another fully independent invariant of this kind. If we want to maintain maximal symmetry between position and momentum, then only the combination $x^2 + p^2$ is admitted as a possible invariant in phase space (as considered also by Born [11]), with the relevant invariance group being $O(6)$.

We now consider $x$ and $p$ to be operators satisfying standard position–momentum commutation relations. When one requires restriction to the subgroup of $O(6)$ under which these commutation relations stay invariant, the resulting symmetry group is $U(1) \otimes SU(3)$, as is well known from the case of the standard 3D harmonic oscillator. The $U(1)$ factor takes care of the Born reciprocity transformations ($x' = -p$, $p' = +x$) and their squares, i.e., ordinary reflections ($x'' = -p'$, $p'' = -x$,
\( p'' = +x' = -p \), while \( SU(3) \) constitutes a generalization of the ordinary rotation group \( SO(3) \) [10]. The generator of \( U(1) \) in phase space is

\[
R^z = x^2 + p^2, \tag{1}
\]

where superscript \( z \) collectively denotes \((p, x)\) and indicates that we are dealing with the representation in phase space.

Let us now introduce the crucial step of our approach, i.e., the linearization of \( x^2 + p^2 \) à la Dirac. We achieve this by considering the square of

\[
A \cdot p + B \cdot x, \tag{2}
\]

with matrices \( A, B \) satisfying standard anticommutation relations:

\[
\{A_k, A_l\} = \{B_k, B_l\} = 2\delta_{kl}, \quad \{A_k, B_l\} = 0. \tag{3}
\]

We shall use the following representation:

\[
A_k = \sigma_0 \otimes \sigma_0 \otimes \sigma_1, \quad B_k = \sigma_0 \otimes \sigma_0 \otimes \sigma_2. \tag{4}
\]

Then the seventh anticommuting matrix of the Clifford algebra generated by \( A \) and \( B \) is

\[
B = iA_1A_2A_3B_1B_2B_3 = \sigma_0 \otimes \sigma_0 \otimes \sigma_3. \tag{5}
\]

One finds:

\[
(A \cdot p + B \cdot x)(A \cdot p + B \cdot x) = R^z + R^\sigma \equiv R, \tag{6}
\]

with \((p, x)\) referring to matrix space

\[
R^\sigma = \sum_k \sigma_k \otimes \sigma_0 \otimes \sigma_0 \equiv \sum_k R^\sigma_k. \tag{7}
\]

appearing here because \( x \) and \( p \) do not commute. Thus, operator \( R \) constitutes the total \( U(1) \) generator, a sum of contributions \( R^z \) from phase space and \( R^\sigma \) from matrix space. To proceed further, we find the eigenvalues of \( R^z \) and \( R^\sigma \).

The eigenvalues of \( R^z \) are obviously \( 3, 5, 7, \ldots \), while for \( R^\sigma \) there are eight eigenvalues: \(-3, +1, +1, +1, -1, -1, -1, +3 \). The lowest eigenvalue of \( R^z \) is \( +3 \) and we shall adopt it in the following as corresponding to some “vacuum”. The lowest absolute value of \( R^\sigma \) is \( +1 \), i.e., it is smaller than the minimal value of \( +3 \) allowed in the standard 3D harmonic oscillator. We shall discuss the meaning of this low value further on.

5. Recovering the Harari–Shupe model

We now adopt the lowest “no-excitation” value of \( +3 \) for \( R^z \), and propose to identify

\[
Q = \frac{1}{6} R = + \frac{1}{2} + \frac{1}{6} R^\sigma \tag{8}
\]

with the charge operator for the set of \( v_e, u_R, u_G, u_B, e^+ \), \( d_R, d_G, d_B \) shown in Table 1. The second term on the r.h.s. in Eq. (8) obviously corresponds to the hypercharge \( Y \) in the Gell–Mann–Nishijima–Glashow formula [13] \( Q = I_3 + Y/2 \) (with \( I_3 = +1/2 \)) if we identify:

\[
Y = \frac{1}{3} R^\sigma. \tag{9}
\]

In order to see strict correspondence with the rishon model, we introduce

\[
Y_k = \frac{1}{3} R^\sigma_k = \frac{1}{3} \sigma_k \otimes \sigma_k \otimes \sigma_3. \tag{10}
\]

Since all commutators of \( Y_k \) with themselves vanish:

\[
[Y_k, Y_l] = 0, \tag{11}
\]

it follows that \( Y_1, Y_2, Y_3 \) and \( Y \) may be simultaneously diagonalized. The eigenvalues of \( Y_k \) are \( \pm 1/3 \). Thus, we have \( 2^3 = 8 \) possibilities for \( Y = Y_1 + Y_2 + Y_3 \), as shown in Table 2. Strict correspondence with the rishon model is obvious. The value \( Y_k = -1/3 (+1/3) \) corresponds to rishon \( V(T) \), while the position of the rishon corresponds to the value of \( k \). Thus \( VTT \), corresponding to \( (Y_1, Y_2, Y_3) = (-1/3, +1/3, +1/3) \) is clearly different from \( VTV \) corresponding to \( (Y_1, Y_2, Y_3) = (+1/3, -1/3, +1/3) \), etc. In addition, there is no need for any “dynamical” preon confinement, as in our scheme the structure identified by Harari and Shupe corresponds to a mere group-theoretical procedure of adding the three components of \( Y \). The antiparticles of \( v_e, u, e^+ \), \( d \), i.e., \( \bar{v}_e, \bar{u}, e^- \), \( d \) (all of them with \( I_3 = -1/2 \)) are described by the complex-conjugate representation. One then finds [10] that \( Q = -1/2 + R^\sigma/6 \) (the sets of eigenvalues of \( R^\sigma \) and \( R^\sigma \) being identical), with \( Y, Y_k \) effectively changing their signs, exactly as in the Harari–Shupe model.

The labelling of the algebraic components gives rise to colour and \( SU(3) \). Indeed, the nine generators of \( U(1) \otimes SU(3) \) are represented in our Clifford algebra by the \( U(1) \) generators \( A_k \) and \( B_k \), whose explicit form is given in [10]. Using this explicit form, it is straightforward to calculate that

\[
\sum_{a=1}^{8} (F^\sigma_a)^2 = 4 \left( 1 + \frac{1}{3} \sum_k \sigma_k \otimes \sigma_k \otimes \sigma_0 \right) = 4(1 + YB). \tag{12}
\]

Since \( [Y, B] = [Y, YB] = [B, YB] = 0 \), the matrix \( YB \) may be diagonalized simultaneously with \( Y \) and \( B \). The eigenvalues of \( YB \) are \(-1, +1/3, +1/3, +1/3 \), corresponding to the \( Y \) eigenvalues of \(-1, +1/3, +1/3, +1/3 \) (for \( B = +1 \)) and \( Y \) eigenvalues of \(+1, -1/3, -1/3, -1/3 \) (for \( B = -1 \)). Thus, for leptons \((YB = -1) \) one has \( \sum_{a=1}^{8} (F^\sigma_a)^2 = 0 \), while for quarks \((YB = +1/3) \) one has \( \sum_{a=1}^{8} (F^\sigma_a)^2 = 16/3 \). In our normaliza-
6. Weak isospin

In order to treat isospin in a standard way, one needs to put together \( v_e, u_R, u_G, u_B \) and \( e^-, d_R, d_G, d_B \) instead of \( v_e, u_R, u_G, u_B, \) and \( e^+, d_R, d_G, d_B \). Since the eigenvalues of \( YB \) are just \(-1, +1/3, +1/3, +1/3\), it follows that we may use matrix \( B \) to this end. In fact, within our Clifford algebra there are only four matrices which commute with the \( U(1) \otimes SU(3) \) generators. These are: the unit matrix, \( Y, B \), and \( YB \). Thus, in our (minimal) scheme we have

\[
I_3 = \frac{1}{2} B = \frac{1}{2} \sigma_0 \otimes \sigma_0 \otimes \sigma_3 \tag{13}
\]

The \( SU(2) \) counterparts of \( I_3 \), i.e., \( I_k = \frac{1}{2} \sigma_0 \otimes \sigma_0 \otimes \sigma_k \) \((k = 1, 2)\), commute with \( YB \). They do not commute with the generators \( F^\sigma_a \) of the original \( SU(3) \) for \( a = 1, 3, 4, 6, 8 \) [10]. However, one can modify \( F^\sigma_a \)'s by setting \( \tilde{F}^\sigma_a = F^\sigma_a \) for \( a = 2, 5, 7 \) (ordinary rotations) and \( \tilde{F}^\sigma_8 = F^\sigma_8 B \) for the remaining values of \( a \). Then, the \( \tilde{F}^\sigma_a \)'s still satisfy the \( SU(3) \) commutation relations, while commuting with all \( SU(2) \) generators [10]. In our scheme, the reflection operator \( P^\sigma \) is obtained as a particular rotation generated by \( R^\sigma \) [10]:

\[
P^\sigma = \exp\left(-i \frac{\pi}{2} R^\sigma \right) \tag{14}
\]

and it turns out to be proportional to \( I_3 \). Thus, two of the \( SU(2) \) generators do not commute with reflections. While the situation in the real world is certainly much more complex, this lack of commutativity seems to be an interesting byproduct of our approach. The generators \( F^\sigma_a \) or \( \tilde{F}^\sigma_a \) obviously commute with the reflection operator \( P^\sigma \). In conclusion, our scheme leads to \( U(1) \otimes SU(3) \) combined with \( SU(2) \). While for \( \tilde{F}^\sigma_8 \) the two groups: \( SU(3) \) and \( SU(2) \) also form a direct product, \( U(1) \) and \( SU(2) \) do not.

7. Genuine \( SU(4) \) transformations

The appearance of the eigenvalues of \( R^\sigma \) equal to \( \pm 1 \), i.e., smaller in the absolute value than the minimal value of \( +3 \) allowed by the 3D harmonic oscillator, requires explanation in the phase-space language. We will now show that such low eigenvalues correspond to quark position–momentum commutation relations having been modified when compared to those in the lepton case. In order to see this, we need to find transformations from the lepton sector to the quark sector. To this end, let us consider six “genuine” \( SU(4) \) generators \( F^\sigma_{+n} \) \((n = 1, 2, 3)\) that—together with the nine generators of \( U(1) \otimes SU(3) \)—form fifteen rotation generators in our Clifford algebra [12]:

\[
F^\sigma_{+n} = \frac{1}{2} \epsilon_{nkl} \sigma_k \otimes \sigma_l \otimes \sigma_3 \tag{15}
\]

\[
F^\sigma_{-n} = \frac{1}{2} (\sigma_0 \otimes \sigma_n - \sigma_n \otimes \sigma_0) \otimes \sigma_0 \tag{16}
\]

We shall study transformations of \( YK B = \frac{1}{2} Y_k \otimes \sigma_0 \) and \( YB = \frac{1}{2} Y \otimes \sigma_0 \) induced by finite rotations generated by \( F^\sigma_{+n} \).

Before the transformation, \( y_k \)'s \((y_k = \sigma_k \otimes \sigma_k)\) diagonalize (simultaneously) as follows:

\[
y_1 \rightarrow \begin{bmatrix} -1 \\ +1 \end{bmatrix}, \quad y_2 \rightarrow \begin{bmatrix} +1 \\ -1 \end{bmatrix}, \quad y_3 \rightarrow \begin{bmatrix} +1 \\ -1 \end{bmatrix} \tag{17}
\]

so that

\[
y \rightarrow \begin{bmatrix} +1 \\ +1 \\ -3 \end{bmatrix}, \quad \leftarrow \text{colour #}'s \begin{cases} 1, \\ 3, \\ 0 \text{ (lepton),} \\ 2. \end{cases} \tag{18}
\]

As an example, we focus here on \( F^\sigma_{-2} \)-generated rotations:

\[
\tilde{y}_k B = e^{i \phi F^\sigma_{-2}} Y_k B e^{-i \phi F^\sigma_{-2}} \tag{19}
\]

for \( \phi = \pm \pi/2 \) (for the general case and for rotations generated by \( F^\sigma_{+3} \) see [12]).

After the above transformation, the \( \tilde{y}_k \)'s diagonalize as:

\[
\tilde{y}_1 \rightarrow \begin{bmatrix} -1 \\ +1 \end{bmatrix}, \quad \tilde{y}_2 \rightarrow \begin{bmatrix} +1 \\ -1 \end{bmatrix}, \quad \tilde{y}_3 \rightarrow \begin{bmatrix} +1 \\ -1 \end{bmatrix} \tag{20}
\]

and

\[
\tilde{y} \rightarrow \begin{bmatrix} +1 \\ +1 \\ -3 \end{bmatrix}, \quad \leftarrow \text{colour #}'s \begin{cases} 1, \\ 3, \\ 2, \\ 0 \text{ (lepton).} \end{cases} \tag{21}
\]

Thus, for \( \phi = \pm \pi/2 \), transformation (19) interchanges the lepton with the quark of colour #2, while leaving the remaining two quark colours unchanged. Rotations by \( \pm \pi/2 \) generated by \( F^\sigma_{+2} \) lead to the same result (see [12]).
8. Phase-space interpretation of colour

The meaning of the quark–lepton interchange of the previous section may be understood in terms of phase-space concepts through analyzing the invariance of expression \( A \cdot p + B \cdot x \) under \( F_{xln}^{\pi/2}_z \)-generated transformations. The \( F_{xln}^{\pi/2}_z \)-generated transformation corresponds to a rotation in the position space relative to the momentum space. Thus, if one chooses to work in momentum representation of the standard 3D picture, in which the \( B \cdot x \) term is not present, the \( A \cdot p \) term does not change when going from the lepton sector to the quark sector. Hence, the same connection between the (algebraic) spin and momentum should exist for both lepton and quarks. However, the connection between position and momentum gets modified. In fact, the phase-space counterpart of Eq. (19) leads for general \( \phi \) to new momenta \( \tilde{p} \) and positions \( \tilde{x} \) satisfying the following commutation relations [12]:

\[
[\tilde{x}_k, \tilde{x}_l] = [\tilde{p}_k, \tilde{p}_l] = 0, \tag{22}
\]

\[
[\tilde{x}_k, \tilde{p}_l] = i \Delta_{kl} \tag{23}
\]

with

\[
\Delta = \begin{bmatrix}
\cos 2\phi & 0 & \sin 2\phi \\
0 & 1 & 0 \\
-\sin 2\phi & 0 & \cos 2\phi
\end{bmatrix}. \tag{24}
\]

Commutation relations (23) become diagonal if

\[
\phi = 0, \pm \pi/2, \pm \pi, \pm 3\pi/2, \ldots. \tag{25}
\]

The cases with \( \phi = 0, \pm \pi \) are trivial (the latter being equivalent to ordinary rotation by \( \pm \pi \) around the second axis), and since \( 3\pi/2 = \pi/2 + \pi \), only \( \phi = \pm \pi/2 \) is of real interest. This is the case of the quark–lepton interchange from Eq. (18) to Eq. (21). A similar conclusion is reached when the \( F_{xln}^{\pi/2}_z \)-generated rotations are considered.

In our scheme, therefore, transformations between a lepton and three quarks (with the same \( I_3 \)) correspond to transformations between four forms of position–momentum commutation relations:

\[
[x_k, p_l] = i \Delta_{kl} \tag{26}
\]

with four different possibilities for diagonal \( \Delta_{kl} \):

\[
\begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \\ -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix} \tag{27}
\]

and the standard meaning of positions and momenta. Since going from a lepton to any of the three types of quarks requires a (particular) genuine rotation in phase space, this transformation cannot be effected in our ordinary 3D world.

The fact that none of the three additional sets of commutation relations above is rotationally invariant does not entitle us to dismiss the presented approach, as the argument below indicates. The point is that in the real world we never probe individual quarks. Instead, we always probe quark aggregates, i.e., hadrons.

This is reflected also in the description provided by the Standard Model, in which photons or weak bosons couple to \( SU(3) \)-singlet quark currents, i.e., to \( \bar{q} \ldots q \) bilinears summed over colour, or, in other words, to objects with meson-like (and not quark-like) properties. Thus, the SM description does not allow us to “see” a quark of a fixed colour.

We expect this general qualitative idea to work in our case as well. Our scheme certainly admits the formation of \( SU(3) \)-singlets (and \( SO(3) \) scalars) out of \( SU(3) \)-triplets. It has to be studied further whether in our description, which is richer than the standard 3D one, quark aggregates of the expected properties can be constructed (presumably in the form of appropriate combinations of tensor products). In other words, the question is whether one can make our quarks “conspire” in such a way that the resulting aggregate—as a whole—behaves in a proper way under rotations. Such a study obviously touches on the issue of confinement and is beyond the scope of the present Letter. However, since the scheme includes the rotation group (and, consequently, must involve all its representations), a positive answer seems quite possible here. Furthermore, it has to be stressed that the conceptual basis of the approach, i.e., the choice of phase space as the arena of physical processes, combined with the introduction of more symmetry between momentum and position, and linearization a la Dirac—looks so natural that it certainly justifies further studies.

9. Reflections in phase space—isospin

Transition between sectors of different \( I_3 = \pm \frac{1}{2} \) is achieved by

\[
\tilde{X} = I_3 X I_3^{-1}, \tag{28}
\]

with \( I_3 \) satisfying \( I_3 = -I_3 I_3 I_3^{-1} \). We may take \( I_3 = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \), whence \( \tilde{X} = X \) and \( \tilde{B} = -B \). The invariance of expression \( A \cdot p + B \cdot x \) requires then that the corresponding transformation in phase space be:

\[
\tilde{p} = p, \quad \tilde{x} = -x, \tag{29}
\]

i.e., we get reflection in six dimensions. Under the transformation of Eq. (28) the sign of the imaginary number \( i \) is unchanged. Consequently, the original commutation relations

\[
[x_k, p_l] = +i \Delta_{kl} \tag{30}
\]

are replaced with

\[
[x_k, p_l] = -i \Delta_{kl}. \tag{31}
\]

The operation leading from Eq. (30) to Eq. (31) is not the same as complex conjugation since the latter changes the sign of both \( p_k = -i \frac{i}{2x_k} \) and \( i \), while leaving (real) \( x_k \) untouched. The two possibilities of Eqs. (30), (31) exist because the imaginary unit which is to appear on the r.h.s. of position–momentum commutation relations may be arbitrarily chosen as \( +i \) or \( -i \). With \( \Delta \rightarrow -\Delta \) the four cases of Eq. (27) are now extended to eight, thus exhausting all possibilities.
10. Conclusions

We have proposed that the Harari–Shupe model should be understood solely in terms of the built-in symmetry, without the need to introduce “confined preons”. This symmetry has been shown to follow in a natural way from a change in the concept of arena on which physical processes occur, i.e., from a shift from the ordinary 3D space to the 6D phase space. In our scheme, the two rishons \( V \) and \( T \) correspond precisely to two eigenvalues \((-1/3, +1/3)\) of the “partial hypercharge” \( Y_k = \sigma_k \otimes 1 \otimes \sigma_3 / 3 \) that emerges from the consideration of phase-space transformations. The value of \( k = 1, 2, 3 \) corresponds to the position of rishon in the Harari–Shupe model. Thus, for any \( k \), rishons \( V \) and \( T \) are just two different eigenvalues of a single algebraic entity. All this explains why the ordering of rishons is important, leads to the \( SU(3) \) colour degree of freedom, and removes the arbitrariness present in the original Harari–Shupe scheme. Since each rishon corresponds to just one direction in the ordinary 3D world, the concept of spin cannot be applied to it: individual rishons do not possess spin. The very idea of “dividing” loses its original meaning.

While getting rid of several drawbacks of the Harari–Shupe scheme, our approach clearly has its problems. In particular, we have not proposed any explicit link to the gauge principle. In that respect, therefore, we have not yet improved on the original ideas of Harari. He speculated that the gauge structure is absent at the rishon level, but emerges at the composite level, writing in [2] that the dynamics at the rishon level “should somehow re-produce currently accepted theories”. In fact, Harari suggested that gauge bosons are composed of rishons as well. In our scheme, however, quarks and leptons are not composite objects at all, i.e., they are definitely point-like when viewed in the standard 3D framework. Neither our quarks nor leptons have any internal structure in the ordinary sense, and the same is expected of gauge bosons. Thus, our model is in fact not a preon model at all. It just provides a possible explanation of the symmetry between quarks and leptons, as identified by Harari and Shupe, but without any subparticle structure. It shows that our tendency to explain such a symmetry in terms of “preons” may be misleading. Obviously, our gauge bosons have to possess symmetry properties corresponding to those of the phase space. However, in order to deal with the gauge bosons and be internally consistent, one needs to address the issue of gauge invariance in the phase-space language (see, e.g., [14]). In our opinion, the problem here is related to a general difficulty in joining different descriptions, often formulated at different levels, and possibly involving completely different formalisms (or “dynamics” as Harari put it). We believe, however, that symmetry survives the change of description formalism (as, e.g., rotation symmetry does in the transition from the classical to quantum description), and therefore we think that the origin of the SM symmetry group lies in the symmetries of phase space (or else is intimately related to them).

Another problem is that, although in our approach weak isospin is automatically connected with the lack of invariance under reflections, there does not seem to be a strict correspondence to the pattern of parity violation built into the Standard Model. Then there is the problem of mass (including the question of the existence of Higgs particle). With both the Planck constant and the new constant of dimension [momentum/position] needed in our approach, the natural mass scale is set when the velocity of light \( c \) is added. It seems therefore that one should expect the scheme to be able to say something about mass. In fact, the issue of mass constituted one of the questions from which our approach originally started, and some symmetry-based conjectures have already been made [10, 12]. While a more explicit proposal (presumably at the level of phase-space-induced algebra) is still missing, we hope that our approach has the potential to provide a different angle on the problem of mass.

The general idea behind our scheme is that (at least some of) the internal symmetries built into the Standard Model and the related quantum numbers represent an image of the symmetries of nonrelativistic quantum phase space (or underlie these symmetries). This idea is in strict analogy to the well-known connection between spin (parity) and the symmetry properties of ordinary 3D space. The presented proposal constitutes a kind of “minimal solution”, in which a simple mathematical structure realizes and reflects the basic physico-philosophical idea. A better description of the real world is expected to require a variation on the theme. If the origin of internal symmetries is indeed connected with phase-space properties, then a better understanding of our macroscopic world should follow from the studies of elementary particles.

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