On gravitational waves generated during inflation

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Abstract. We study cosmological gravitational waves generated during inflation under the influence of a decaying cosmological “constant”. A non-perturbative contribution of the dynamical cosmological “constant” to the tensor modes is considered. As applications of the model we study the well-known cases \( \Lambda(t) = \sigma H^2 \) and \( \Lambda(t) = \vartheta H \). The spectrum of gravitational waves results scale invariant only for the first case, leaving this to new proposals of the form: \( \Lambda(t) = f(H, H^2) \), in order to include inflation in some \( \Lambda(t) \)-CDM models. We also found that the non-perturbative contributions of \( \Lambda(t) \), accelerate the decreasing of the amplitude of gravitational waves during a power-law inflationary stage, by an exponential factor.

1. Introduction
The relic background of gravitational waves is a prediction of inflationary cosmology [1, 2]. Their amplitude is related with the energy scale of inflation and the are observationally detectable by studies of B-mode polarization in the cosmic microwave background radiation (CMBR), if the energy of inflation is larger than ~ \( 3 \times 10^{15} \) GeV [3, 4, 5]. The technical mechanism for generating primordial gravitational waves has mainly two branches: the coordinate approach of Bardeen and the covariant formalism [6, 7, 8].

On the other hand, the present acceleration in the expansion of the universe is now accepted as an observational fact [9, 10]. The \( \Lambda \)-CDM model [11, 12, 13] is a concordance model that fits observations satisfactorily but unfortunately gives no more profound explanations as for instead on the nature or origin of the cosmological constant. Moreover, it still suffers from the cosmological constant and the cosmic coincidence problems [14].

A different proposal to alleviate these problems are the \( \Lambda(t) \)-CDM models [15]. In these models the dynamical cosmological constant represents a decaying vacuum energy capable to generate the observed acceleration in the cosmic expansion, without fall in contradictions with nucleosynthesis and predictions of structure formation. According to some models of QCD chiral phase transitions during the early universe, it is generated a vacuum energy density that scales as \( \Lambda \simeq m^3 H \), with \( m \simeq 150 \) MeV being the energy scale of QCD vacuum transitions and \( H \) denoting the Hubble parameter [15, 16, 17, 18, 19, 20, 21, 22]. Taking these results into account, two of the most employed models for \( \Lambda(t) \) are \( \Lambda(t) \simeq H^2 \) and \( \Lambda(t) \simeq m^3 H \) [23]. A cosmological scenario derived from \( \Lambda(t) \simeq m^3 H \) has been suted by S. Carneiro, J. S. Alcaniz and collaborators, establishing its observational viability starting from a radiation dominated epoch and
finishing in the present epoch [23]. However, their results do not consider the inflationary epoch.

The present lecture notes are based on our paper [24]. In this proceeding notes we study the generation of the relic background of gravitational waves during inflation under the presence of a dynamical cosmological constant. We consider a new approach in which a non-perturbative dependence of $\Lambda(t)$ in the tensor modes is taken. We work in the transverse-traceless (TT) gauge and consider two applications of the model.

2. Basics on $\Lambda(t)$-CDM Model
The Einstein field equations with cosmological constant read

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

(1)

where $R_{\mu\nu}$ is the Ricci tensor, $R$ is the Ricci scalar and $\Lambda$ is the cosmological constant. As it is well-known due to Bianchi identities $\nabla_{\mu} G_{\mu\nu} = 0$, and the Riemannian metricity condition $\nabla_{\mu} g_{\alpha\beta} = 0$, the Eq. (1) implies the conservation of the energy momentum tensor $\nabla_{\mu} T_{\mu\nu} = 0$.

In the case of $\Lambda(t)$-CDM models the dynamical cosmological constant $\Lambda(t)$ is added on the right hand side of Einstein equations in the form [15]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \Lambda(t) g_{\mu\nu} + 8\pi G T_{\mu\nu},$$

(2)

and this agrees with the idea that $\Lambda(t)$ represents a dynamical (decaying) vacuum energy. Thus on a Friedmann-Robertson-Walker (FRW) metrical background the Friedmann equation and the conservation equation derived from (2) as a consequence of Bianchi identities are

$$H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3},$$

(3)

$$\dot{\rho} + 3H (\rho + p) = -\frac{\dot{\Lambda}}{8\pi G},$$

(4)

where $H = \dot{a}/a$ is the Hubble parameter, the dot means time derivative. The equation (4) is like the standard continuity equation for a perfect fluid on cosmological settings, but with a source term $-\frac{\dot{\Lambda}}{8\pi G}$. However, this equation can be also written in the standard form if we define an effective energy-momentum tensor for a perfect fluid with an effective energy density $\rho_{\text{eff}} = \rho + (\Lambda/8\pi G)$ and an effective pressure $p_{\text{eff}} = p - (\Lambda/8\pi G)$.

3. Relic background of gravitational waves from $\Lambda(t)$-CDM model: the formalism
In the TT-gauge, the perturbed line element for a spatially flat FRW spacetime can be written in the form

$$ds^2_{\text{pert}} = dt^2 - a(t)^2 (\delta_{ij} + Q_{ij}) dx^i dx^j,$$

(5)

where the tensor modes $Q_{ij}(t, \vec{r})$ satisfy: $tr(Q_{ij}) = Q^i_i = 0$ and $Q^{i}_{ji} = 0$. The 3D spatial components of the metric can be written as $g_{ij} = -a^2(t)(\delta_{ij} + Q_{ij})$ whereas its 3D contravariant components can be linearly approximated by $g^{ij} = -a^{-2}(t)(\delta^{ij} - Q^{ij})$.

Due to the well-known fact that in the TT-gauge only space-space components contribute for tensor fluctuations, the dynamics of the tensor modes is in this case given by linearized Einstein equations free of sources

$$\delta R_{ij} - \frac{1}{2} \delta R g^{(b)}_{ij} - \left(\frac{1}{2} R^{(b)} - \Lambda(t)\right) Q_{ij} = 0,$$

(6)
where the label \((b)\) denotes quantities computed with the background metric derived from (5), \(\delta R_{ij}\) is the linearized Ricci tensor and \(\delta R\) is the Ricci scalar up to first order in \(Q_{ij}\).

By using (5), after some algebra the equations (6) can be put in the form

\[
\dot{Q}^i_j + 3HQ^i_j - \frac{1}{a^2} \nabla^2 Q^i_j + \Lambda Q^i_j = 0, \tag{7}
\]

where the dot is denoting derivative with respect the cosmic time. In order to study the impact of a dynamical cosmological constant on the background of gravitational waves during inflation, we must obtain the exact dependence of the tensor of metric fluctuations \(Q_{ij}\), with \(\Lambda(t)\). Thus, it is natural from (7) to expect the dependence:

\[
Q_{ij} = Q_{ij}(\bar{x}, \Lambda(t)), \tag{10}
\]

However, a better approximation can be made if we consider the case \(\Lambda = 0\) in (8), the series (8) can be written in the form

\[
Q_{ij} = h_{ij} \left[ 1 + \alpha_1 \Lambda + \frac{1}{2!} \alpha_2 \Lambda^2 + \mathcal{O}(\Lambda^3) \right]. \tag{9}
\]

In this moment, we may content ourselves with the expansion (9) up to linear order in \(\Lambda(t)\), as it is done for example in [3]. However, a better approximation can be made if we consider the case \(\alpha_1 = \alpha_0, \alpha_2 = \alpha_0^2, \ldots\). On this particular case, the expression (9) yields

\[
Q_{ij}(\bar{x}, \Lambda) = h_{ij}(\bar{x})e^{\alpha_0 \Lambda(t)}, \tag{10}
\]

where \(\alpha_0\) has units of \(\Lambda^{-1}\). The approximation (10) is non-perturbative in \(\Lambda(t)\) and this may be considered as an advantage. Moreover, this equation expresses the contribution of both the usual tensor metric fluctuations \(h_{ij}(\bar{x})\) (i.e. without any \(\Lambda\)) and \(\Lambda(t)\), to the general tensor metric fluctuations \(Q_{ij}\), in a separated way. In this letter we will work with the approximation (10).

Now by means of (10), writing the equation (7) in terms of the usual tensor fluctuations \(h_{ij}^t\) we arrive to

\[
\ddot{h}_{ij}^t + (3H + 2\alpha_0 \dot{\Lambda}) \dot{h}_{ij}^t - \frac{1}{a^2} \nabla^2 h_{ij}^t + \left( \alpha_0 \ddot{\Lambda} + \alpha_0^2 \dot{\Lambda}^2 + 3\alpha_0 H \dot{\Lambda} + \Lambda \right) h_{ij}^t = 0. \tag{11}
\]

Introducing the auxiliary field \(\chi_{ij}^t(t, \vec{r})\) defined by the transformation

\[
h_{ij}^t(t, \vec{r}) = e^{-\frac{1}{2} \int (3H+2\alpha_0 \dot{\Lambda}) dt} \chi_{ij}^t(t, \vec{r}), \tag{12}
\]

the equation (11) can be written as

\[
\ddot{\chi}_{ij}^t - \frac{1}{a^2} \nabla^2 \chi_{ij}^t - \left( \frac{9}{4} H^2 + \frac{3}{2} \dot{H} - \Lambda \right) \chi_{ij}^t = 0. \tag{13}
\]
Following the usual quantization process, we implement the Fourier expansion for $\chi_{ij}(t, \vec{r})$

$$\chi_{ij}^\dagger(t, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \sum_{\alpha=\pm, \times} (a_k^\alpha e_{ij}^\dagger + a_k^{\alpha\dagger} e_{ij}) \left[ a_k^{(\alpha)} e^{i\vec{k} \cdot \vec{r}} \xi_k(t) + a_k^\alpha e^{-i\vec{k} \cdot \vec{r}} \xi_k^\ast(t) \right],$$

with the creation and annihilation operators $a_k^\alpha$ and $a_k^{(\alpha)}$ respectively, satisfying

$$\begin{bmatrix} a_k^\alpha \, a_{k'}^{\alpha'} \end{bmatrix} = g^{\alpha\alpha'} \delta^{(3)}(\vec{k} - \vec{k'}),$$

$$\begin{bmatrix} a_k^\alpha \, a_{k'}^{\alpha'} \end{bmatrix} = \begin{bmatrix} a_k^{(\alpha)} \, a_{k'}^{(\alpha')} \end{bmatrix} = 0,$$

and where the following properties for the polarization tensor $e_{ij}$ hold

$$(a) e_{ij} = (a) e_{ji}, \quad k^i (a) e_{ij} = 0,$$

$$(a) e_{ii} = 0, \quad (a) e_{ij} [-\vec{k}] = (a) e_{ij}^\ast [\vec{k}].$$

We also require the modes $\xi_k(t)$ satisfy the commutation relation

$$\left[ \xi_k(t, \vec{r}), \xi_k^\ast(t, \vec{r}') \right] = i\delta^{(3)}(\vec{r} - \vec{r}'),$$

which for (14) reduces to

$$\xi_k \dot{\xi}_k^\ast - \xi_k^\ast \dot{\xi}_k = i,$$

with the asterisk denoting the complex conjugate operation. When this condition holds true the modes are normalizable on the UV-sector.

In view of (13) and (14), the modes $\xi_k$ obey the dynamical equation

$$\ddot{\xi}_k + \left( \frac{k^2}{a^2} - \frac{9}{4} H^2 - \frac{3}{2} \dot{H} + \Lambda \right) \xi_k = 0.$$

Thus, in principle, for a given $\Lambda(t)$ we can obtain a normalized solution for the $k$-modes $\xi_k$, just by solving (19) and imposing the normalization condition (18) to that solution.

Once a normalized solution for the $k$-modes is obtained, we are in position to calculate the spectrum for tensor fluctuations on Super-Hubble scales i.e. on the IR-sector. The amplitude of the tensor fluctuations $<h^2>_{IR} = <0| h^i_j h_i^j |0>$ on the IR sector is given in this case by

$$\langle h^2 \rangle_{IR} = \frac{1}{2\pi^2} e^{-\int (3H + 2a\Lambda) dt} \int_0^{\epsilon k H} \frac{dk}{k} k^3 |\xi_k \xi_k^\ast|_{IR},$$

where $\epsilon = k_{max}^{IR}/k_P \ll 1$ is a dimensionless parameter, being $k_{max}^{IR} = k_H(t_i)$ the wave-number related to the Hubble radius at the time $t_i$, which is the time when the modes re-enter to the horizon at the end of inflation. The $k_P$ is the Planckian wave-number. For a typical Hubble parameter at the end of inflation $H = 0.5 \cdot 10^{-9} M_P$, values of $\epsilon$ on the range of $10^{-5}$ to $10^{-8}$ correspond to the number of e-foldings $N_e = 63$. 

4. Applications

4.1. The case of a decaying $\Lambda(t)$

Let us considering a power-law inflationary stage, thereby the scale factor is taken to be $a(t) = a_0(t/t_0)^p$, with $a_0$ being the present value of the scale factor, $t_0$ the present time, and $p > 1$ a constant parameter compatible with an accelerated expansion. The equation for the modes (19) reduces in this case to

$$\ddot{\xi}_k + \left(\frac{k^2}{a_0^2} t^{-2p} - \frac{(9/4)p^2 - (3/2)p}{t^2} - \Lambda(t)\right) \xi_k = 0. \tag{21}$$

For a decaying $\Lambda(t)$ of the form $\Lambda(t) = \varpi/t^2$, with $\varpi$ being a constant parameter, the expression (21) reads

$$\ddot{\xi}_k + \left(\frac{k^2}{a_0^2} t^{-2p} - \frac{(9/4)p^2 - (3/2)p + \varpi}{t^2}\right) \xi_k = 0. \tag{22}$$

The general solution for (22) is given by

$$\xi_k(t) = C_1 \Gamma(1 + \nu) \frac{k^{\nu}(t/t_0)^{1-p}}{\sqrt{1 + 4\beta/(2(p-1))}} \frac{\varpi}{(9/4)p^2 - (3/2)p - \varpi}, \tag{23}$$

where $J_\nu$ is the Bessel Function, $z(t) = \frac{k}{a_0(p-1)}(t/t_0)^{1-p}$, and $\nu = \sqrt{1 + 4\beta/(2(p-1))}$, with $\beta = (9/4)p^2 - (3/2)p + \varpi$. Unfortunately, this solution is not normalizable. A normalizable solution of (22) can be obtained when we write (22) in terms of the conformal time $\tau$. Thus, for the conformal time during inflation $\tau = [a_0/(t_0^p(1-p))]t^{1-p}$ the equation (22) gives

$$\frac{d^2\xi}{d\tau^2} - \frac{p}{1-p} \left(\frac{1}{\tau}\right) \frac{d\xi}{d\tau} + \left(\frac{\kappa^2 - \kappa_0^2}{\tau^2}\right) \xi_k = 0, \tag{24}$$

where $\kappa = (t_0^p/a_0^2)k$ and $\kappa_0^2 = (1-p)^{-2}[(9/4)p^2 - (3/2)p - \varpi]$. The general solution of this equation is given in terms of the first and second kind Hankel functions $H^{(1)}_\mu$ and $H^{(2)}_\mu$ in the form

$$\xi_k(\tau) = A_1 \tau^{-\frac{1}{2(\nu-1)}H^{(1)}_\mu}[w(\tau)] + A_2 \tau^{-\frac{1}{2(\nu-1)}H^{(2)}_\mu}[w(\tau)] \tag{25}$$

being $w(\tau) = k\tau$ and $\mu = [1/(2(p-1))]\sqrt{1 + 4\kappa_0^2(p-1)^2}$. The normalized solution to (24) is then

$$\xi_k(\tau) = i \sqrt{\frac{\pi}{4}} \left[\frac{(p-1)t_0}{a_0^{1/p}}\right]^{-\frac{1}{2(\nu-1)}} \tau^{-\frac{1}{2(\nu-1)}H^{(1)}_\mu}[w(\tau)]. \tag{26}$$

Now, getting back to the cosmic time, the normalized solution (26) reads

$$\xi_k(t) = i \sqrt{\frac{\pi}{4(p-1)}} t^{1/2} H^{(1)}_\mu[w(t)], \tag{27}$$

with $w(t) = [kt_0^p/(a_0(p-1))]t^{1-p}$. In view of (27), the amplitude for gravitational waves on Super-Hubble scales ($k \gg k_H$), according to the expression (20) gives

$$\langle h^2 \rangle_{IR} = \frac{t_0^{(3-2\mu)p}}{8\pi^3} \frac{(2a_0)^{2\mu}}{(p-1)^{1-2\mu}} \Gamma^2(\mu) \tau^{\gamma-\frac{2\mu}{p\tau^2}} (ck_H)^{3-2\mu}, \tag{28}$$

where $\gamma = 1 - 3\mu - 2\mu(1-p)$. Therefore for the spectrum of gravitational waves in this case we obtain

$$P_g(k) = \frac{t_0^{(3-2\mu)p}}{8\pi^3} \frac{(2a_0)^{2\mu}}{(p-1)^{1-2\mu}} \Gamma^2(\mu) \tau^{\gamma-\frac{2\mu}{p\tau^2}} k^{3-2\mu} \bigg|_{k=ck_H}. \tag{29}$$
Clearly for $\mu \approx 3/2$ the spectrum is nearly scale invariant. The scale invariance is achieved when the condition
\[ \frac{1 + 3p(3p - 2) - 4\varpi}{9(p - 1)^2} = 1, \]  
(30)
is valid. The solution of (30) for $\varpi$ is then $\varpi = 3p - 2$. The accelerated expansion during inflation $p > 1$ leaves to the restriction: $\varpi > 1$. Hence, a nearly scale invariant spectrum for gravitational waves under the presence of a $\Lambda(t) = \varpi/t^2$ is perfectly possible if we restrict ourselves to $\varpi > 1$.

In the particular case of $\Lambda(t) = \sigma H^2$ we would have $\varpi = \sigma p^2$. Thus the condition (30) reduces to
\[ \frac{1 + (9 - 4\sigma)p^2 - 6p}{9(p - 1)^2} = 1. \]  
(31)
Then for $\sigma$ we have the restriction $\sigma = (3p - 2)/p^2$. Hence, for $p > 1$ we obtain $0 < \sigma \leq 9/8 \simeq 1.125$.

4.2. The case of a Cosmological Constant

In the case of a cosmological constant $\Lambda_0$ with a de-Sitter expansion $a(t) = a_0 e^{H_0 t}$, with $H_0$ being the constant value of the Hubble parameter during inflation, the equation for the modes (19) reads
\[ \ddot{\xi}_k + \left( \frac{k^2}{a_0^2} e^{-2H_0 t} - \frac{9}{4} H_0^2 - \Lambda_0 \right) \xi_k = 0. \]  
(32)
This equation has for solution
\[ \xi_k(t) = D_1 \mathcal{H}^{(1)}_\lambda[x(t)] + D_2 \mathcal{H}^{(2)}_\lambda[x(t)], \]  
(33)
being $\mathcal{H}^{(1,2)}$ the first and second type Hankel functions, $\lambda = [1/(2H_0)]\sqrt{9H_0^2 + 4\Lambda_0}$ and $x(t) = [k/(a_0 H_0)] e^{-H_0 t}$. Invoking (18) and taking a Bunch-Davies vacuum condition, the normalized solution for (32) is given by
\[ \xi_k(t) = \frac{i\sqrt{3\pi}}{2a_0^{1/4}} \mathcal{H}^{(1)}_\lambda[x(t)]. \]  
(34)

By using (34) the amplitude for gravitational waves on cosmological scales (20), acquires the form
\[ \langle h^2 \rangle_{IR} = \frac{3}{\pi^3} \frac{2^{-3+2\lambda}}{\sqrt{\Lambda_0}} \frac{\Gamma^2(\lambda)}{(a_0 H_0)^{-2\lambda}} e^{-(3-2\lambda)H_0 t} e^{3-2\lambda}, \]  
(35)
and the spectrum for gravitational waves results in this case
\[ \mathcal{P}_g(k) = \frac{3}{\pi^3} \frac{2^{-3+2\lambda}}{\sqrt{\Lambda_0}} \frac{\Gamma^2(\lambda)}{(a_0 H_0^2)^{-2\lambda}} e^{-(3-2\lambda)H_0 t} k^{3-2\lambda} \bigg|_{k = \epsilon k_H}. \]  
(36)
It can be easily check from (36) that the spectral index for gravitational waves under the presence of a cosmological constant is $n_{gw} = 4 - 2\lambda$. Thus, in principle, for values of $\Lambda_0$ obeying $4\Lambda_0/(9H_0^2) \ll 1$, we may have a nearly scale invariant spectrum $n_{gw} \simeq 1$.

A special case that deserves our attention is when $\Lambda_0 = \vartheta H_0$. As we mentioned before, in this case $H_0 = \vartheta/3$ and $\Lambda_0 = \vartheta^2/3$, and therefore $\lambda = \sqrt{21}/2 \simeq 2.2913$. Unfortunately, for this value
of $\lambda$ the spectral index becomes $n_{gw} \simeq -0.5826$, which is far away from the scale invariance, thus entering in contradiction with observations, which from WMAP7 + BAO + $H_0$-Mean analysis give $n_{gw} \simeq 0.963 \pm 0.012$ [25].

We may interpret this result for $\Lambda(t) = \vartheta H(t)$, as during inflation, at least with respect to the spectrum of gravitational waves, it seems more convenient to use $\Lambda(t) = \sigma H^2(t)$ than $\Lambda(t) = \vartheta H(t)$. However, the fact that the former time dependence $\Lambda(t) = \vartheta H(t)$ works very well for the radiation-dominated, matter-dominated and the present epochs [23, 26], suggests that to include inflation we might consider $\Lambda(t) = f(H, H^2)$, in such a way that during inflation the dominant term is $H^2$ and after inflation it becomes sub-dominant in front of $H$. One example of this kind of time dependence for $\Lambda(t)$ would be $\Lambda(t) = (1 - \eta_{sr})H^2 + \eta_{sr} H$, where $\eta_{sr}$ is the slow-roll parameter during inflation defined by $\eta_{sr} = -\dot{H}/H^2$. As it is well-known during inflation $\eta_{sr} \ll 1$ while when inflation ends $\eta_{sr} = 1$. Thus, we obtain the desired dominance of $H^2$ during inflation and of $H$ for the resting epochs.

5. Conclusions
In this lecture notes we have shown the impact of a dynamical cosmological constant over the background of gravitational waves generated during inflation. Our results have some differences with respect to the ones we can find in the literature, for example, we incorporate a nonperturbative contribution of the dynamical cosmological constant $\Lambda(t)$ to the tensor modes (see eq.(10)). The fact that the contribution is non-perturbative in $\Lambda(t)$ allows us mainly, to calculate new contributions of $\Lambda(t)$ on the spectrum and the mean square amplitude $<h^2>$ on the IR-sector (cosmological scales), at all orders in $\Lambda(t)$. We found that the presence of $\Lambda(t)$ speeds up the decreasing of the gravitational waves amplitude during a power law inflation by an exponential factor.

Two applications were in order. In the case of $\Lambda(t) = \sigma H^2$, for a power law expansion, we obtain a scale invariant gravitational spectrum at the end of inflation for $0 < \sigma < 1.25$. In the case of $\Lambda = \vartheta H$ we obtain a spectral index for gravitational waves $n_{gw} \simeq -0.5826$, which is in contradiction with observational data. Therefore, due to the fact that $\Lambda(t) = \vartheta H$ seems to work very well from the radiation dominated epoch to the present time, one way to extend a $\Lambda(t)$-ΛCDM model to inflation, compatible with observations, would be of the form $\Lambda(t) = f(H, H^2)$. One proposal we can make is $\Lambda(t) = (1 - \eta_{sr})H^2 + \eta_{sr}H$, where $\eta_{sr}$ is the slow-roll parameter during inflation defined by $\eta_{sr} = -\dot{H}/H^2$. In this manner, $\eta_{sr} \ll 1$ during inflation and $\eta_{sr} = 1$ when it ends, reproducing thus the desired dominance of $H^2$ during inflation and of $H$ for the resting periods in the cosmic history.

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