Recent progress towards a chiral effective field theory for the NN system

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Since Weinberg’s proposal two decades ago, chiral effective field theory in the NN sector has been developed and applied up to order \(O((Q/M_{\pi})^4)\). In principle it could provide a model-independent description of nuclear force from QCD. However, in spite of its huge success, some open issues such as the renormalization group invariance and power counting, still remain to be solved. In this talk we refine the chiral effective field theory approach to the NN system based on a renormalization group analysis. Our results show that a truly model-independent description of NN system can be obtained by a new power counting which treats the subleading order corrections perturbatively.

Keywords: Chiral effective field theory, nucleon-nucleon interaction.

1. Introduction

Based on the symmetries of Quantum chromodynamics (QCD) in the low energy region (\(\leq 1 \text{ GeV}\)), chiral effective field theory (\(\chi\)EFT) enables calculations of strong interaction in the non-perturbative region. However, unlike the pion-pion and pion-nucleon section, where the power counting—the key ingredient which guarantees the intrinsic consistency of an EFT—is given clearly from the vertices generated by the chiral Lagrangian, the power counting in nucleon-nucleon (NN) case is hindered by the infrared enhancement and cannot be obtained straightforwardly. The first step out of the NN problem, as suggested by Weinberg,\(^1\) is to apply the power counting to the NN potential level first, and then sum the amplitude by iterating the potential in Schrodinger or Lippmann-Schwinger (LS) equation with an ultraviolet cutoff \(\Lambda\). Currently, this prescription (the
so-call Weinberg power counting (WPC)) has been carried out to next-to-
next-to-next-to-leading order (N^{3}LO)^{a}, and has became the standard of
many conventional calculations.\(^{2–4}\) However, since Weinberg prescription
only applies power counting to the potential level, the systematic control
of the theory could be lost in the final amplitude.

2. Renormalization group analysis

One way to check whether a proposed scheme is under control is to per-
form the RG-analysis. RG-analysis carried out at leading order (LO), up to
next-to-next-to-leading order (NNLO) and N^{3}LO based on WPC indicate
that the conventional implementation of WPC fails to fulfill the RG re-
quirement once the ultraviolet cutoff of the iteration $\Lambda > 1$ GeV.\(^{5–8}\) Since
the chiral expansion is established in powers of $Q/M_{hi}^{b}$, some authors\(^{9}\)
have questioned whether the theory is valid once intermediate states have
$p \sim \Lambda > M_{hi}$. Thus, it makes little sense to perform RG-analysis for $\Lambda > 1$
GeV, even the final on-shell $Q << M_{hi}$.

A second point of view\(^{10–16}\) takes the final amplitude as a partial sum of
the (infinitely many) diagrams, then under the assumption that a reason-
able separation of scales exists\(^{c}\), in any EFT one should be able to organize
those diagrams in a systematic way to absorb the unimportant physics into
contact term(s) order by order after a proper renormalization. Thus, as long
as $Q << M_{hi}$, the impact of high-energy physics (which is well-represented
by the contact term(s)) in the final amplitude should reduce as the increase
of $\Lambda$, since the contribution from physics haven’t been integrated out (i.e.,
from $\Lambda$ to $\infty$) becomes smaller and smaller.

The answer of the above in-debating issue actually depends on how the
diagrams are organized. It was shown that due to the fine-tuning of low
energy constants and a Wigner bond-like effect,\(^{17}\) once a cutoff $\Lambda > 1$ GeV
is adopted the renormalization is effectively dominated by one contact term
under the WPC scheme.\(^{8}\) Moreover, a full-iteration of some type of irre-
ducible two-pion-exchange diagrams could result in a pole-like structure.\(^{18}\)
Therefore, if one insists to build a NN potential based on $\chi$EFT and utilizes
it later in a conventional way (e.g., inserts it as a potential in Schrodinger

\(^{a}\)Note that the order here is defined based on the pion-exchange (long-range) part of
the potential, which does not necessary equal to the order at the final NN amplitude.
\(^{b}\)Here $Q \equiv (p, m_{r})$, $p$ the NN c.m. momentum, $m_{r}$ the pion mass, and the breakdown
scale $M_{hi}$ is nominally $m_{r} \sim 4\pi f_{\pi}$.
\(^{c}\)In the case where there is no reasonable separation of scales, EFT is impossible.
or LS equation), he or she needs to stay in $500 < \Lambda < 1000$ MeV. The consequence is that a full RG-based analysis becomes inapplicable.

To allow a full RG-analysis, one must give up treating the whole chiral potential non-perturbatively. In other words, for the NN case there exists no “ideal potential” (in the traditional sense) to be extracted or derived. Some parts of the diagrams have to be included perturbatively. Recent works\textsuperscript{10–13,15,16} which treat the subleading chiral potentials in the framework of Distorted-Wave-Born-Approximation enable a full RG- and power counting analysis. Once the $\Lambda$–dependence is under well-control, the estimation of the theoretical error becomes much easier, i.e., the error is given by $O(Q^{n+1}/M^{n+1}_{hi})$ up to order-$n$ in the new power counting scheme.

We must point out that the lack of a RG-analysis cannot rule out the possibility that WPC under the specified range of $\Lambda$ could generate final amplitudes which has the correct power counting, but there is no way to check this so far. On the other hand, a RG-correct scheme could converge too slowly to be useful. Thus, before the full implementation to few- and many-body calculations, one cannot determine the superiority of either scheme. Nevertheless, it is of importance to start with a scheme which allows a full RG-analysis first, then check the power counting step by step to build the theory on a more solid ground.

### 3. New Power counting and Future task

The new power counting developed so far\textsuperscript{11–13} can be summarized as:

1. The LO potential needs to be iterated to all order\textsuperscript{4}, and all subleading chiral potentials are included perturbatively as represented diagrammatically in Fig. 1.

2. The contact terms are determined by RG-analysis to guarantee the correct RG-behavior. As a general rule, for potentials which are singular and attractive at LO (i.e., $V_{LO}(r \to 0) \approx -\frac{1}{r^n}$ with $n \geq 3$), all contact terms need to be promoted one order earlier with respect to WPC.

Phase shifts from the above new scheme are evaluated up to NNLO, and the agreement with the Nijmegen phase-shift analysis\textsuperscript{19} is comparable to those from WPC at the same order.

Further tasks such as refining power counting with Lepage plot\textsuperscript{20} or similar techniques, including Delta (1232) contribution, and deciding the power counting for high partial-waves ($l \geq 2$) are under investigation.

\textsuperscript{4}At least for those spin-triplet and $l \leq 1$ partial-waves.
Fig. 1. Diagrammatic representation of the new power counting in the case where the $O(Q)$ contribution is absent. Here $T$, $G$ and $V$ denote the $T$-matrix, propagator and chiral potential. The order of $T$ and $V$ is indicated in the superscript.

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