Derivation of RCM-driven potential evapotranspiration for hydrological climate change impact analysis in Great Britain: a comparison of methods and associated uncertainty in future projections

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Supplementary material

Sect. 1: PET methods and associated equations (for daily estimates)

| PET method                  | Equation                                                                 |
|-----------------------------|--------------------------------------------------------------------------|
| FAO56 (Allen et al., 1998)  | \[
PE_{\text{mm day}^{-1}} = \frac{\lambda^{-1}\Delta(R_n - G) + \gamma \frac{900}{T + 273} U_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34U_2)}
\] |
| Penman-Monteith (modified) (Kay et al., 2003) | \[
PE_{\text{mm day}^{-1}} = \frac{1}{\lambda} \frac{\Delta R_n}{\Delta + \gamma(1 + r_v/r_a)}
\] |
| Priestley-Taylor (Priestley and Taylor, 1972) | \[
PE_{\text{mm day}^{-1}} = \frac{1}{\lambda} \frac{\Delta}{\Delta + \gamma} (R_n - G)
\] |
| Turc (Turc, 1961)           | \[
PE_{\text{mm day}^{-1}} = 0.31 \frac{T}{T + 15} (R_m + 2.09) \left(1 + \frac{50 - RH}{70}\right)
\] for RH < 50%
\[
PE_{\text{mm day}^{-1}} = 0.31 \frac{T}{T + 15} (R_m + 2.09)
\] for RH > 50%
| Jensen-Haise (Jensen et al., 1990) | \[
PE_{\text{mm day}^{-1}} = \frac{1}{\lambda} 0.025(T + 3)R_{e}
\] |
| Makkink (Jacobs et al., 2009) | \[
PE_{\text{mm day}^{-1}} = \frac{1}{\lambda} \frac{R_n}{R_{e}} \frac{\Delta}{\Delta + \gamma} R_{e}
\] |
| Priestley-Taylor Idso-Jackson (Shuttleworth, 1993) | \[
PE_{\text{mm day}^{-1}} = \frac{1}{\lambda} \frac{\Delta}{\Delta + \gamma} \left(1 - \alpha\right) \left(0.25 + 0.5 \frac{n}{N} S_0 - \left(0.9 \frac{n}{N} + 0.1\right)(-0.02 + 0.26 \exp(-7.7 \times 10^{-4}T^2)) \sigma T^4\right)
\] |
Hamon

\( \text{PE}[\text{mm day}^{-1}] = \left( \frac{N}{12} \right)^2 \exp \left( \frac{T}{16} \right) \)

McGuinness-Bordne

\( \text{PE}[\text{mm day}^{-1}] = \frac{1}{\lambda} S_0 \left( \frac{T + 5}{68} \right) \)

Oudin

\[
\begin{align*}
\text{PE}[\text{mm day}^{-1}] &= \frac{1}{\lambda} S_0 \left( \frac{T + 5}{100} \right) \quad \text{if } T > -5^\circ \text{C} \\
\text{PE}[\text{mm day}^{-1}] &= 0 \quad \text{if } T \leq -5^\circ \text{C}
\end{align*}
\]

Blaney-Criddle

\( \text{PE}[\text{mm day}^{-1}] = k T p_d \) with \( p_d = 100 \frac{N_d}{\sum_{i=1}^{N} n_i} \)

Thornthwaite

\( \text{PE}' = 16 \left( \frac{10T}{1} \right)^2 \)

\[
a = 0.49239 + 0.01792 I - 7.71 \times 10^{-5} I^2 + 6.75 \times 10^{-7} I^3
\]

\( \text{PE}[\text{mm month}^{-1}] = \text{PE}' \frac{N_m D_m}{12 \times 30} \)
Sect. 2: Notations and used values of meteorological variables

Values used in the PET calculations calculated from meteorological inputs and the equations used to calculate them

| Symbol | Variable name                  | Units          | Description                                                                 | Formula                                                                 |
|--------|--------------------------------|----------------|----------------------------------------------------------------------------|------------------------------------------------------------------------|
| $\delta$ | Solar declination            | radians        | Angle between rays of the sun and the plane of the earth’s equator.       | $\delta = 0.4093 \sin \left( \frac{2\pi}{365} (J - 1.405) \right)$    |
|         |                                |                |                                                                            | With J Julian day number                                               |
|         |                                |                |                                                                            | Note that MORECS uses a different equation: $\delta = 0.41 \cos \left( \frac{2\pi(J-172)}{365} \right)$ |
| $C_p$  | Specific heat at constant      | MJ kg$^{-1}$°C$^{-1}$ | Amount of heat required to change a unit mass of a substance by one degree in temperature. |
|        | pressure (for water)           |                |                                                                            | $C_p = \gamma \epsilon \lambda$                                      |
|        |                                |                |                                                                            | With $\gamma$ in KPa$^{-1}$ $\lambda$ in MJ kg$^{-1}$ $P$ in KPa        |
|        |                                |                |                                                                            | Note this is a re-arrangement of the equation of the Psychrometric constant |
| $d_r$  | Relative earth-sun distance   |                | Distance between earth and sun varies through the year due to the ellipse orbit of the earth around the sun. |
|        |                                |                |                                                                            | $d_r = 1 + 0.033 \cos \left( \frac{2\pi}{365} J \right)$             |
| $\omega_s$ | Sunset hour angle            | radians        | Angle by which the ray of the sun reaches the earth’s surface.            | $\omega_s = \arccos \left( -\tan \Phi \tan \delta \right)$           |
|        |                                |                |                                                                            | With $\Phi$ latitude (+ is north, - is south)                         |
| $N$    | Maximum possible daylight      | hours          | Length of the period when the rays of the sun reach the earth’s surface.  | $N = \frac{24 \omega_s}{\pi}$                                        |
|        | length                        |                |                                                                            | Note that MORECS uses a different equation: $N = 24 - 2 \frac{12}{\pi} \arccos \left( \tan \delta \tan \phi + 0.0145 \cos \phi \sin \phi \right)$ |
| $e_s$  | Saturated water vapour        | kPa            | Equilibrium of rates of vapourisation and condensation for a given temperature that occurs at particular vapour pressure, the saturated vapour pressure. |
|        | pressure                      |                |                                                                            | $e_s = 0.6108 e \left( \frac{17.27}{T} \right)$                       |
|        |                                |                |                                                                            | With $T$ temperature in °C                                             |
| $e_s$  | Actual water vapour pressure  | kPa            | Actual water vapour pressure at dew point.                                | $e_s = 0.6108 e \left( \frac{17.27}{T_d} \right)$                     |
|        |                                |                |                                                                            | With $T_d$ temperature at dew point, °C                                |
| $\Delta$ | Gradient of vapour            | kPa°C$^{-1}$   | Gradient of vapour pressure curve is the slope of the non linear relationship between pressure and temperature. |
|        | pressure curve                |                |                                                                            | $\Delta = \frac{4098 e_s}{\left( 237.3 + T \right)^2}$               |
| $\lambda$ | Latent heat of                | MJ kg$^{-1}$   | Amount of energy needed for water to be                                   | $\lambda = 2.501 - 0.002361 T$                                       |
| Symbol | Variable name          | Units          | Description                                                                 | Formula                                                                 |
|--------|------------------------|----------------|------------------------------------------------------------------------------|-------------------------------------------------------------------------|
| v      | vaporisation          |                | transformed from a liquid to a gas, approximated as $\lambda=2.45 \text{ MJkg}^{-1}$. | With $T$ as 20°C                                                         |
| $P$    | Atmospheric pressure   | kPa            | The change of pressure due to altitude                                       | $P = 101.3 \left(\frac{293 - 0.0065z}{293}\right)^{5.26}$ With $z$ elevation above sea level |
| $\rho_a$ | Mean air density      | Kgm$^{-3}$     | The mass of air per unit volume. It depends on the atmospheric pressure $P$ and temperature $T$ | $\rho_a = \frac{P}{T_v R}$ With $T_v$ the virtual temperature: $T_v = 1.01(T + 273) \text{ °K (T in °C)}$ and $R$ specific gas constant for dry air ($=0.287 \text{ kJkg}^{-1}\text{ °K}^{-1}$) |
| RH     | Relative humidity      | %             | Amount of water the air can hold at a certain temperature. In other words the percentage ratio of actual vapour pressure to saturated vapour pressure. | $\text{RH} = 100 \frac{e_a}{e_s}$ |
| $f$    | Cloudiness factor      | [-]           | Amount of cloud cover in the atmosphere, related to number of bright sunshine hours in a day. Different coefficients can be used for humid and arid areas. Using the longwave coefficients for arid areas a simplified version of the formula can be derived. A second expression is given by Jensen 1990. In this formula the cloudiness factor is expressed as the effect of clouds on short-wave global radiation. The simplified version is the one used in this paper. | Shuttleworth, 1993 $f = \left(\frac{a_c b_s}{a_s + b_s}\right) \frac{n}{N} + \left(\frac{b_c}{a_s + b_s}\right) a_s$ With: $n$ as bright sunshine hours (h), $a_s$ is fraction of extraterrestrial radiation ($S_0$) for $n=0$, $a_s + b_s$ is fraction of extraterrestrial radiation for $n>0$, $a_s$ and $b_s$ are long wave coefficients for clear skies. $N$ is the maximum possible daylight hours Simplified version (Allen et al., 1998) $f = 0.9 \frac{n}{N} + 0.1$ Jensen, 1990 $f = a_s \frac{R_s}{S_0} + b_c$ With $R_s$ solar (short-wave) radiation (MJ m$^{-2}$day$^{-1}$) |
| $G$    | Soil heat flux         | MJm$^{-2}$month$^{-1}$ | Energy that moves from the surface to subsurface soil by conduction, depends on | Monthly formulation (Shuttleworth, 1993) |
| Symbol | Variable name | Units | Description | Formula |
|--------|---------------|-------|-------------|---------|
| G     | soil temperature fluctuations |       |             | $G = 0.14(T_{\text{month2}} - T_{\text{month1}})$ |
| γ     | Psychrometric constant (for water) | KPa°C⁻¹ | Describes the thermodynamic properties of moist air at a constant pressure. Relates the partial pressure of water in the air to the air temperature | $γ = \frac{c_p P}{ελ}$  
With $c_p$, specific heat of moist air  
$c_p = 1.013 \times 10^{-3}$ MJ/kg°C⁻¹  
$P$, atmospheric pressure  
$ε$, ratio of molecular weight of water vapour to that of dry air:  
$ε = 0.622$ |
| S₀    | Extraterrestrial radiation | MJmm⁻²day⁻¹ | The amount of solar energy that reaches the top of the atmosphere. Depends on angle of sun radiation and length of day. | $S₀ = 37.62d_ω(ω_σ\sin\phi\sinδ + \cosφ\cosδ\sinω_x)$  
Shuttleworth, 1993 |
| Rₛ    | Solar radiation | MJm²day⁻¹ | Amount of energy measured at the earth’s surface including direct and diffuse short-wave radiation | Generalised form (Jensen et al., 1990)  
$R_s = S_0(a_σ + b_σ\frac{n}{N})$  
Here $a_σ = 0.25$ and $b_σ = 0.50$ |
| Rₙₛ   | Net solar radiation | MJm²day⁻¹ | That part of the incident short wave radiation that is captured at the ground (reflection losses are taken into account), in other words, the absorbed incoming solar radiation. | Shuttleworth, 1993  
$R_{ns} = (1 − α)R_s$  
With $α$, albedo |
| Rₙₙ   | Net long-wave radiation | MJm²day⁻¹ | Incoming (atmosphere to ground) minus outgoing (ground to atmosphere) long-wave radiation | $R_{nl} = ε'αT^4$  
With $ε'$, net emissivity between atmosphere and ground (given for average conditions):  
$ε' = 0.34 − 0.139\sqrt{ε_x}$  
$σ$, Stefan-Boltzmann constant:  
$σ = 4.903 \times 10^{-8}$ MJK⁻¹m⁻²day⁻¹  
$T$, mean air temperature in °K |
| Rₙ   | Net radiation | MJm²/day | Difference between the net solar radiation and the long-wave radiation | $R_n = R_{ns} − R_{nl}$ |
Maps of mean MORECS PET and PET derived from HadRM3-Q0 for the 1961-1990 period.
August

Baseline

FAO56
Panman-Monteith (mod)
Priestley-Taylor
Turc

Jensen-Haise
Makkink
Priestley-Taylor / HttpSession
Hann

McGuinness-Bodine
Cudini
Blaney-Criddle
Thornthwaite

MORECS

Potential evapotranspiration (mm/month)
Potential evapotranspiration (mm/month)
PET percentage change between averages values calculated for the 1961-1990 and 2040-2069 time slices from HadRM3-Q0
Potential evapotranspiration changes (%)
