Topologically Protected Wormholes in a Type-III Weyl Phase

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Abstract

The observation of wormholes has proven to be difficult in the field of astrophysics. However, with the discovery of novel topological quantum materials it is possible to observe astrophysics and particle physics effects in condensed matter physics. In this work, we propose that wormholes can exist in a type-III Weyl phase, in addition, these wormholes are topologically protected, making them feasible to create and measure in condensed matter systems. Finally, several systems are proposed to host this phase and several experiments are put forward to confirm the existence of a Type-III Weyl phase.
INTRODUCTION

The discovery of topological quantum materials has opened a large path in experimental condensed matter physics. Initially, the first material discovered was the topological insulator which has an insulating bulk and a topologically non trivial surface state emerging from the Dirac cone. Interestingly these Dirac Fermions do not obey expected physics and behave relativistically[1–8]. These Dirac fermions have later been seen to violate Lorentz symmetry and form tilted type-II and critically tilted type-III Dirac cones[9–11]. These tilted Dirac cones have been predicted to have the same physics as black holes in certain cases.

In the presence of additional perturbations, the formation of Weyl Fermions[12] and their counterpart Weyl cone can be formed. These condensed matter excitation were first observed in condensed matter physics and have yet to be observed in high energy particle physics. Similarly to the Dirac cone, the edge states of the Weyl cone can be tilted to form type-II and type-III Weyl cones[13–16]. A plethora of type-I and type-II Weyl cones[17] have been discovered yet the discovery of a type-III Weyl Fermion phase has yet to be discovered conclusively[18, 19].

Wormholes are a yet undiscovered but physically plausible object that can exist within the framework of general relativity. Much work has been done in order to define wormhole topology, energy of formation, and experimental signatures. However, wormholes are largely considered improbable due to a need for a large amount of energy to form and maintain one within current models. The generation of a quasi-wormholes will prove valuable in understanding wormhole physics.

The type-III Dirac cone is predicted to host a direct analogue to a black hole where similar physics can be observed and measured with respect to the Dirac quasiparticles that experience the effects of the critically tilted Dirac cone. Limited experiments have been conducted in order to confirm the effects of the type-III Dirac phase and little to no materials have been discovered[20–25]. The counterpart to the Dirac black hole is the Weyl wormhole phase, in this work, it is predicted that wormholes can be formed and are topologically robust in the type-III Weyl phase. In addition, we predict the materials RB₆ (R = La, Sm, Ce), In₂Co₃S₂, and In₂Ni₃S₂ can host a Type-III Weyl fermion phase.
REALIZATION OF A TYPE-III WEYL PHASE

First we must discuss whether a type-III weyl phase can truly exist. The simple Hamiltonian that describes a 3D Weyl point can be described as

\[ H(k) = \pm vk \cdot \sigma \]  

From this formula, a Dirac point can be constructed with two 3D Weyl points of opposite chirality, thus forming the Dirac cone.

\[ H = \begin{bmatrix} H_K & 0 \\ 0 & H_{K'} \end{bmatrix} \]  

From this formulation, prototypical Type-I Weyl semimetals can be easily constructed and formed. However, we must introduce magnetism in order to tilt the Hamiltonian (and therefore the Weyl cone) so that be may generate a critically tilted weyl this which leads to a type-III Weyl phase. (where we add magnetism in the \( \hat{z} \) direction in order to allow for the Weyl cone to tilt). We add magnetism to the simple Weyl Hamiltonian model.
\[ H(k) = Ck_z \pm k \cdot \sigma \]

This realizes a Weyl point with +1 or -1 Chern number. The type of Weyl cone depends on the value of the parameter \( C \) where \( C > |1| \) is a Type-II Weyl semimetal [Fig 1(C)], \( C < |1| \) is a Type-I Weyl semimetal [Fig 1(A)], and \( C = -1 \) is a Type-III Weyl Semimetal [Fig 1(B)]. When the magnetic field is applied in the \( \hat{z} \) the Hamiltonian can be rewritten as:

\[ E_n = Ck_z \pm \sqrt{k_z^2 + \frac{2n}{l^2}} \]

Where \( l = \frac{eH}{c} \). With this we can visualize the landau levels for different parameters \( C \) in order to visualize the chiral dispersions of the Weyl cones. For the condition of a Type-I Weyl where \( C < |1| \) we select \( C = 0.5 \) [Fig 2(A)]. Here we see that the weyl cone is slightly tilted but is still preserves lorentz invariance, \( C = 1 \) [Fig 2(B)] shows a similar dispersion. When \( C = 5 \) [Fig 2(C)] The Weyl cone breaks lorentz invariance and forms a type-II over tilted Weyl dispersion. When \( C = -1 \) The Weyl cone becomes critically tilted [Fig 2(D)] and forms a Type-III weyl cone. In the Type-III Weyl phase is can be seen that the chiral edge mode has a linear dispersion in \( k \)-space and transitions from the hole-band to the electron-band.

**WORMHOLE EXPERIMENTAL SIGNATURES**

**ARPES**

Angle resolved photoemission spectroscopy (ARPES) is a valuable method in order to probe the Weyl states in order to discover a type-III weyl phase in predicted materials. The
issue is that we must first identify ideal material candidates to measure and the measurement can be subject to noise, etc. Ignoring experimental limitations, the ideal ARPES signature of a type-III Weyl phase is a Weyl line node that connects a hole-like conduction band to an electron-like valance band. In addition, this line must not be parabolic for a certain dispersion in $k$ (crystal symmetry can preserve this condition). In order for the Weyl line node to be confirmed to be chiral it would be ideal to conduct spin-resolved measurements in order to confirm that only one spin occupies the band between Weyl points. This measurement can take place at a synchrotron source with a Mott spin detector or with a pump-probe laser based setup utilizing a circularly polarized pump in order to preferentially select spins to detect with the probe laser. The Fermi surface of a type-III Dirac cone and a type-III Weyl cone will look the same without spin-resolved ARPES (an enclosed loop between two points). With spin-resolved ARPES, a $4\pi$ phase shift of spin across the entire loop will indicate a type-III Weyl phase while a type-III Dirac cone will have a $2\pi$ spin phase shift.

**Butterfly Magnetoresistance**

![Butterfly Magnetoresistance](image)

**FIG. 3:** **Butterfly magnetoresistance** (a) magnetoresistance near the Weyl line energy level (E) (b) $E + \delta$ a small distance from the Weyl line (c) $E + \delta$ a far way from the Weyl line level

Another way to confirm the existence of topological wormholes is to perform magnetoresistance measurements. When measuring the longitudinal resistance of a material as the magnetic field is rotated it is possible to measure the anisotropy in the sample in order to gain insight to the magnetoresponse in relation to different crystal axis. This response is typically called butterfly magnetoresistance because of how the anisotropy typically looks when plotted on a polar plot[27–29]. The magnetoresistance is a function of the electron
(hole) mobility in the sample. The mobility is also correlated with the Fermi velocity. We know from previous work that electrons (holes) that exist in the flat band will have zero Fermi velocity, this will lead to no magnetoresponse at the angle where the DOS of the Type-III Weyl cone lines up with the magnetic field [Fig 3(A)]. In order to simulate this response we construct a toy model of the variable Fermi velocity as a function of angle for different chemical potentials by using trigonometric functions. The actual magnetoresistance can be calculated by solving the conductivity tensor \( \sigma \)

\[
(\sigma)_{ik} = e \sum_{k,v} (v_{k,v})_i \frac{\delta g_{k,v}}{\delta E_j}
\]

by inverting the matrix longitudinal \( (p_{xx}) \) and traverse \( (p_{xy}) \) resistivity can be calculated.

\[
p_{xx} = \frac{\sigma_{yy}}{\sigma_{xx} \sigma_{yy} - \sigma_{xy}^2}, p_{xy} = \frac{\sigma_{xy}}{\sigma_{xx} \sigma_{yy} - \sigma_{xy}^2}
\]

In a type-III Weyl semimetal which is composed of two pairs of Weyl point we expect to see typical butterfly magnetoresponse at the Weyl line, but as the chemical potential moves away from the Fermi level we expect to see that response to decrease [Fig. 3(B,C)] and show less of an intense magnetoresponse.

**MATERIALS CANDIDATES FOR TYPE-III WEYL PHASES**

In order to form a type-III Weyl phase in a crystal lattice it is necessary to satisfy several conditions. Firstly perfectly flat bands must exist with a large enough momentum dispersion to connect a two bands (or a band must be flat for a period between these two bands). The band must be chiral and connect a hole-like band to an electron-like band, this condition allows for inversion symmetry to be preserved (from a band inversion). Materials that satisfy this condition only satisfy a type-III Dirac semimetal phase, therefore in order to break time reversal symmetry and turn the Dirac cone into two Weyl nodes magnetism must also exist in these materials in order for there to be a type-III Weyl phase. The best materials that satisfy these conditions are Weyl-Kondo (WKSM) semimetals[30] in heavy fermion systems[31–33], WKSM are formed when a heavy fermion system, typically consisting of a rare-earth element, is cooled to a point where the f-bands begin to hybridize with the d-bands breaking inversion symmetry near the Fermi-level and forming a flat bands that are connected to the conduction and valance bands[34]. Typically, a Weyl cone is formed in the gap which could
have a flat Fermi arc which is a signature of a type-III Weyl semimetal. Ideal systems for the Type-III Weyl phase are materials with two d-bands that cross the Fermi-level which when hybridized with the flat f-bands form heavily tilted Weyl edge modes with a flat chiral edge mode. Excellent materials candidates for this are the rare-earth hexaborides RB$_6$ (R = La, Sm, Ce) [35–41] which form the ideal case to form type-III Weyl semimetals. These materials have already been studied both theoretically and experimentally with ARPES and mangetotransport measurements, however, the possibility of a type-III Weyl semimetal state has never been considered in the context of the flat bands near the Fermi surface [Fig. 4(A-C)]. These materials should be revisited in order to probe the possibility of this new phase existing in these materials. In addition to WKSM, magnetic Weyl semimetal offer a good selection of materials that can host crystalline symmetry protected flat bands. Trigonal systems such as R3m (No. 166) can protect the existence of flat bands. This work identifies In$_2$Co$_3$S$_2$ and In$_2$Ni$_3$S$_2$ as excellent candidates that host flat bands near the Fermi level in similar space groups.

![SmB$_6$ band structure](image)

**FIG. 4:** **SmB$_6$ band structure:** (A) Bulk band structure of SmB$_6$ (B) Surface band structure of the (001) plane (C) Constant energy contour near the flat band (100 meV) below the Fermi surface

**CONCLUSION**

In conclusion, this work has outlined the parameters that can create a topologically protected wormhole in a type-III Weyl semimetal. In addition we provide a theoretical argument for the probability of there existing type-III Weyl phases that can host these
phases. Finally, materials R\(_6\) (R = La, Sm, Ce), In\(_2\)Co\(_3\)S\(_2\), and In\(_2\)Ni\(_3\)S\(_2\) that host flat bands needed for critically tiled Weyl cones

**METHODS**

The band structure calculations were carried out using the density functional theory (DFT) program Quantum Espresso (QE)[42], with the generalized gradient approximation (GGA)[43] as the exchange correlation functional. Projector augmented wave (PAW) pseudo-potentials were generated utilizing PSlibrary[44]. The relaxed crystal structure was obtained from materials project[45, 46]. The energy cutoff was set to 60 Ry and the charge density cutoff was set to 270 Ry for the plane wave basis, with a k-mesh of 25 × 25 × 25. High symmetry point K-path was generated with SSSP-SEEK path generator[47, 48]. The Wannier tight binding Hamiltonian was generated from the non-self consistent calculation with Wannier90[49]. The surface spectrum[50] was calculated with Wannier Tools[51]. In order to model the \(f\) bands of Sm, DFT + U is implemented with \(U = 7\) and \(J = 0.83\).

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