Data Unfolding in $W$ Mass Measurements at LEP2

V.Kartvelishvili $a^)$
Department of Physics and Astronomy,
Schuster Laboratory, University of Manchester,
Manchester M13 9PL, U.K.

R.Kvatadze $a^)$
Joint Institute for Nuclear Research,
Dubna, Moscow Region, RU-141980, Russia

Abstract

The use of an unfolding procedure is proposed as an alternative method of extracting the $W$ boson mass from the data measured at LEP2, which may improve the accuracy of this measurement. The benefits of the direct unfolding method based on the Singular Value Decomposition of the response matrix are demonstrated on the example of $W$ mass determination from the charged lepton energy spectra.

$^a)$ On leave from High Energy Physics Institute, Tbilisi State University, Tbilisi, GE-380086, Republic of Georgia.
1 Introduction

One of the main physics motivations for the LEP energy upgrade is the precise measurement of the $W$-boson mass. This is necessary to test the Standard Model of the electroweak interactions, and together with the top quark mass would allow to set more significant limits for the Higgs boson mass. The existing information on the $W$-boson mass comes from the measurements in $\bar{p}p$ interactions at the Tevatron [1] and CERN [2]. In these experiments, transverse momentum distributions of the charged leptons, missing transverse momentum spectra, and transverse mass distributions of the charged lepton-neutrino system were used to obtain the $W$ mass, resulting in the world average $80.26 \pm 0.16$ GeV [3].

Various methods have been proposed for the precision determination of the $W$ mass in $e^+e^-$ annihilation at LEP2 [4, 5]. Extensive tests and studies resulted in the following conclusions:

- The direct reconstruction of the $W$ mass from the final state particles is considered to be the most promising method, if one uses the four constraints from the energy-momentum conservation and the assumption that the $W$'s in the intermediate state have equal masses. Two decay channels — $q\bar{q}q\bar{q}$ (four jets) and $q\bar{q}l\nu$ (two jets plus leptons) — can be used in this analysis. If colour reconnection [6] and Bose-Einstein correlations [7] do not lead to additional significant errors, then the $W$ mass can be measured with the precision of about 60 MeV (statistical) and 40 MeV (systematic), for a single LEP2 experiment [5]. Otherwise, one should concentrate on the decay channel of two jets plus leptons, which is free of these complications, but results in a larger statistical error of $\approx 80$ MeV per experiment.

- $W$ mass measurement from the rise of the $W^+W^-$ cross section near the threshold gives the error competitive, but not better than the direct reconstruction method.

- Measurement of the charged lepton energy spectra does not allow one to achieve comparable precision for $W$ mass at any energy of LEP2.

In this paper we consider the semileptonic decay channel $jet + jet + l + \nu$. The actual measurement gives the following information:

- The distribution of the hadronic and electromagnetic energy deposited into the calorimeters.

- Momenta of charged hadrons.

- The energy-momentum of the charged lepton.

Obviously, the complete event is lost if the charged lepton escapes detection, hence the geometric coverage is vital. The major error comes from the limited acceptance of hadron detection and finite resolution of energy-momentum measurements, so that the invariant mass of the measured hadronic system is significantly lower than the mass of the decayed $W$.
This also prevents the overall energy-momentum conservation constraints to be used directly to determine the 4-momentum of the neutrino, unless the rescaling of the hadronic jet energies is performed. The overall kinematical fit of the whole event allows one to achieve the required accuracy only if the masses of the two W-bosons in the event are assumed to be equal to each other. So, each event in this approach gives just a single entry into the W mass distribution instead of two, thus effectively reducing the available statistics. It is argued, however, that the gain in the precision of the resulting W-boson mass should justify the use of this procedure \cite{4,5}.

We suggest a different way of extracting the mass distributions of W-bosons from the same data. Our method is based on the direct unfolding of the measured data straight into the W mass. This analysis requires a high statistics Monte Carlo simulation of the response matrix describing the whole measurement process. The method does not use the assumption of equal W masses and does not require any rescaling of measured hadronic jet energies. All necessary unfolding (i.e. corrections for efficiencies, acceptances and resolutions of various detectors used for the measurement) is performed in a consistent way in one go, in the framework of the regularized unfolding procedure based on the Singular Value Decomposition of the response matrix \cite{8}. The unfolded W-boson mass distribution will have its natural decay width, not broadened by the detector resolution effects, and can be fitted to obtain $M_W$.

The suggested procedure is presented in some detail in the following section. The general description of the unfolding method used in this paper is given elsewhere \cite{8}. The latter has been tested on various examples and is stable and reliable, provided the response matrix is known precisely enough. In order to apply the suggested procedure to the channel jet + jet + l + ν one has to have a detailed Monte Carlo simulation of this process, including full detector response for hadrons and the charged lepton, with the statistics of at least an order of magnitude larger than the expected number of measured events. In the absence of this information, in Section \cite{8} we have restricted ourselves to the case when only the single charged lepton energy spectrum is used to unfold the W mass distribution. This example is easier to simulate but is more difficult to unfold, as the W mass distribution in this case is hidden behind a non-trivial functional dependence in addition to the usual detector effects. Nevertheless, it incorporates many features of the full problem, and demonstrates the ability of our method to deal with multi-dimensional distributions. Some conclusions are drawn out in Section \cite{8}.

2 Description of the procedure

Consider the Monte Carlo simulation of the process

$$e^+ + e^- \rightarrow W^+ + W^- \rightarrow jet + jet + l + \nu.$$ (1)

Let $M_1$ be the mass of the W which decayed leptonically, while $M_2$ is the mass of the second W, which decayed hadronically. The distributions of both $M_1$ and $M_2$ essentially follow the Breit-Wigner curve, apart from phase space factors. Each generated event corresponds to an entry into a two-dimensional histogram $M_1$ vs. $M_2$, and there is no reason why the two projections of the latter onto the axes $M_1$ and $M_2$ should be different from each other, apart from obvious statistical fluctuations.
After the measurement of each event one gets the distribution of the measured energy over the calorimeter cells, and the momenta of charged hadrons and the charged lepton. Define the initial four-vector \( P \equiv (\sqrt{s}, \vec{0}) \), the measured four-vector of the lepton \( p_l \equiv (E_l, \vec{p}_l) \) and the summary four-vector of the whole measured hadronic system \( p_h \equiv (E_h, \vec{p}_h) \). The initial energy \( \sqrt{s} \) and the lepton mass \( \sqrt{E_l^2 - \vec{p}_l^2} \) are known, so one has no more than 7 independent measured components, some of which do not contain any significant information on the masses of the decaying \( W \)s. Four non-trivial invariant combinations of the 3 vectors defined above can be combined to calculate the following quantities (lepton masses have been neglected):

- the invariant mass of the measured hadronic system \( M_h \),
  \[
  M_h^2 = p_h^2;
  \]

- the invariant mass of the system recoiling from the measured hadrons \( M_A \),
  \[
  M_A^2 \equiv (P - p_h)^2 = M_h^2 + \sqrt{s}(\sqrt{s} - 2E_h);
  \]

- the invariant mass of missing particles \( M_{\text{miss}} \),
  \[
  M_{\text{miss}}^2 \equiv (P - p_h - p_l)^2 = M_A^2 - 2E_l(\sqrt{s} - E_h + |\vec{p}_h| \cos \theta),
  \]
  where \( \theta \) is the angle between \( \vec{p}_l \) and \( \vec{p}_h \).

As far as a non-negligible portion of hadrons escapes detection, the actual value of \( M_h \) is significantly smaller and \( M_A \) correspondingly larger than \( M_W \), while \( M_{\text{miss}} \) is far from zero.

In a very general case it would be preferable to include all directly measured quantities into the analysis, but this is clearly problematic because of the huge dimension of the resulting problem. In this analysis we will use just two measured variables: \( M_A \) and

\[
M_B \equiv \sqrt{M_h^2 - M_{\text{miss}}^2}.
\]

The better the detector is, the closer the measured values of \( M_A \) and \( M_B \) are to \( M_1 \) and \( M_2 \), correspondingly. Any event from the 2-dimensional plot of the generated masses of \( W \) bosons \( M_1 \) vs. \( M_2 \) is transformed by the measurement process to an entry into the measured two-dimensional histogram \( M_A \) vs. \( M_B \). To be more precise, some of the generated events will be washed away because of the limited acceptance.

The overall response matrix of the measurement \( \hat{A}_{ij,kl} \) can be defined as the probability for an event generated in bin \( kl \) of the initial \( M_1 \) vs. \( M_2 \) histogram to find itself in the bin \( ij \) of the measured \( M_A \) vs. \( M_B \) histogram. The measurement process is now simulated by the multiplication (convolution) of the probability response matrix \( \hat{A}_{ij,kl} \) by the generated 2-dimensional histogram \( H^{(12)}_{\text{sim}} \equiv M_1 \) vs. \( M_2 \), yielding the simulated “measured” histogram \( H^{AB,\text{sim}} \equiv M_A \) vs. \( M_B \):
We will assume that the same response matrix relates the true (yet unknown) two-dimensional mass distribution $H^{(12)}$ to a genuine measured histogram $H^{AB}$:

$$\sum_{kl} \hat{A}_{ij,kl} H^{(12)}_{kl} = H^{AB}_{ij}. \quad (7)$$

With the known response matrix $\hat{A}$, one can usually fold (convolute) different simulated distributions $H^{(12)}$ with various $M_W$ in order to determine which value of $W$ mass corresponds to the measured pattern in the r.h.s. Alternatively, one can use the eq.(7) to unfold $H^{(12)}$ from the measured histogram. It is well known that the latter procedure is unstable, and without proper stabilization (regularization) measures would yield totally useless rapidly oscillating solutions. However, using the rather simple and still very efficient unfolding algorithm developed in [8] it is possible to perform the unfolding quite successfully. The built-in regularization effectively suppresses the spurious oscillations, leaving only statistically significant components of the solution.

The unfolding procedure [8] takes the response matrix $\hat{A}$ and the measured r.h.s. $H^{AB}$ as input and outputs the unfolded 2-dimensional mass distribution of the two $W$ bosons $H^{(12)}$, together with its error matrix and the inverse of the latter. The unfolded histogram $H^{(12)}$ can then be projected onto its axes, to obtain separately mass distributions for leptonically and hadronically decayed $W$s. As the two distributions must be identical, their sum should be fitted to obtain the $W$ mass. Note that the unfolded mass spectrum contains strong bin-to-bin correlations, and the inverse of the full error matrix must be used in the fit.

The above analysis can be easily extended to include three or even four measured variables without increasing too much the computing power requirements: this extension would increase the number of equations in the system (7) but not the number of unknowns, so that the most time-consuming part of the unfolding procedure [8] would remain unchanged.

3 Example: $W$ mass from single lepton spectrum

In order to demonstrate the use of the above method without involving complicated simulations of the detection of hadronic decays, a study of a simpler case was performed where the $W$ mass was extracted from the shape of the energy spectrum of the single charged lepton. Both $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ decays have been used. The main sensitivity of the lepton energy spectrum towards the mass of the $W$-boson lies in the end-points. In the conventional analysis [4, 5] the statistical precision even for the most sensitive region of energies is not expected to be better than 300 MeV.

The important aspect of the problem which can be tested in this example is the simultaneous dependence of the measured quantity — the charged lepton energy $E_l$ — upon both $W$ masses in the same event:

$$\frac{M_1^2}{2(E_1 - P_1)} \geq E_l \geq \frac{M_1^2}{2(E_1 + P_1)}, \quad (8)$$

$$E_1 = \frac{s + M_1^2 - M_2^2}{2\sqrt{s}}, \quad P_1 = \sqrt{E_l^2 - M_1^2}, \quad (9)$$
where $M_i, P_i$ and $E_i$ denote the masses, momenta and energies of the $W$-bosons, with the index $i = 1$ referring to the parent $W$ and $i = 2$ to the recoil one. So, an event in the two-dimensional histogram $M_1$ vs. $M_2$ corresponds to an entry into the distribution of the charged lepton energy $E_l$. The convolution equation takes the form:

$$
\sum_{mn} \hat{A}^{(l)}_{i,mn} H^{(12)}_{mn} = H^{(l)}_i,
$$

with $\hat{A}^{(l)}$ being the response matrix for each lepton type $l = e, \mu$, while $H^{(l)}_i$ is the measured lepton energy distribution.

In order to obtain the matrices $\hat{A}^{(e)}$ and $\hat{A}^{(\mu)}$, 2 · $10^5$ events of $W$ pair production with an electron or a muon in the final state have been generated at the energy 183 GeV, using PYTHIA 5.7 [9] Monte Carlo event generator. The detector response was simulated using the following assumptions:

- Only leptons within the polar angle $\Theta$ in the range $20^0 < \Theta < 160^0$ are detected.
- The energy resolution for electrons is $\Delta E_e/E_e = 0.2/\sqrt{E_e}\,\text{(GeV)}$.
- The momentum resolution for muons is $\Delta P_\mu/P_\mu = 10^{-3}P_\mu\,\text{(GeV/c)}$.

The unfolding method of ref. [8] requires the introduction of the regularization parameter $\tau$, whose optimal value depends on both the properties of the response matrix and the statistical accuracy of the measured data. The most reliable way of estimating the correct $\tau$ in this case is to apply the procedure to a simulated spectrum where the true distribution is known, and to minimize the deviation of the unfolded distribution from the true one.

Over 40 sets of data at different $W$ mass values from the interval $80.25 \pm 0.25$ GeV have been generated, each consisting of 1700 electrons or muons in the final state. This number corresponds to the integrated luminosity of 500 pb$^{-1}$ at 183 GeV. The generated and the unfolded mass distributions were then compared to each other, and the optimal values of the regularization parameter $\tau$ were obtained, different for $e$ and $\mu$ samples. After that, the procedure was applied to independent test sets of 1700 events with various $W$ mass values from the same interval $80.25 \pm 0.25$ GeV. For each sample, the unfolded two-dimensional $W$ mass distribution was projected onto the axes and the sum of the projections was fitted by a Breit-Wigner parameterization, using the full inverse error matrix. An example with $M_W = 80.50$ GeV using the muon sample only is presented in Fig.1.

There are three contributions to the overall error of the $W$ mass determined by the above procedure. First, the purely statistical error comes from the fit of the unfolded distribution: $\Delta M_W^{\text{stat}} \approx 250$ MeV for the combined $e/\mu$ sample, slightly better than the 300 MeV expected from the lepton end-point energy measurement. Second, the systematic error of the regularized unfolding method connected to the choice of the parameter $\tau$: $\Delta M_W^{\text{sys}} \approx 100 \div 120$ MeV. The last set of uncertainties may come from the inadequate event generator program, the inadequate description of the detector and the statistical errors in the response matrix itself. The first can be estimated by using several Monte Carlo programs, while the second requires the detailed understanding of the detector.
response. In this particular example, however, there is no ambiguity in the description of the leptonic $W$ decay, and the detector resolution of the lepton energy-momentum measurement is expected to be known well enough. As for the statistical errors in the response matrix, their effects can be eventually made negligible compared to the statistical accuracy of the measured data. In our example they do not exceed 10 MeV.

4 Discussion

The example with $W$ mass determination using the single charged lepton spectra shows that direct unfolding of the immediately measured data into the required distribution is a powerful method of data analysis. Errors obtained by our procedure are slightly smaller compared to those from the end-point analysis [5] despite the higher c.m.s. energy and accurate accounting for the $W$ width in our case — the two effects which would inevitably make the error of the conventional analysis larger than the quoted 300 MeV. In the case when the full measured information on both the lepton and the hadrons is used, our approach can also result in an important improvement of accuracy, because of the following reasons:

- Each measured event $e^+e^- \rightarrow W^+W^- \rightarrow jet + jet + l + \nu$ gives two entries into the final $W$ mass distribution, thus effectively increasing the statistics compared to the constrained fit method [4, 5], which gives one entry with the “average mass” $(M_1 + M_2)/2$.

- The unfolding restores the natural width of the $W$, which is smaller than the expected resolution.

- The use of a well-balanced unfolding procedure allows us to retain all statistically significant data, simultaneously avoiding the amplification of systematic errors which usually takes place if separate correction routines are applied at various stages of the analysis.

The most important systematic error specific to the direct unfolding method itself comes from the uncertainty in the determination of the regularization parameter $\tau$. This error can be reliably estimated using various simulated “measured” distributions, and can be shown to decrease when the statistical significance of the measured data increases [8]. The effects due to the initial state radiation can be either included into the response matrix (as we have done in our example), or accommodated at a later stage of the analysis. A more subtle source of errors is the response matrix: any deviation of the Monte Carlo event generator or the detector model from reality may potentially result in a systematic effect in the unfolded distribution. However, we believe that the huge experimental data

\[\text{If the overall uncertainty in } M_W \text{ were dominated by the gaussian-distributed detector resolution effects the two approaches would lead to identical statistical errors, as the gaussian for the average mass would be narrower. However, the significant portion of the measured } W \text{ width in the constrained fit method comes from the natural width of the } W \text{ boson which obeys the Breit-Wigner (or indeed Cauchy) distribution. The latter remains invariant under the averaging: clearly, the } W \text{ lifetime should not change as long as the two } W \text{s decay independently.}\]
accumulated by the four LEP collaborations will eventually allow one to minimize those uncertainties. The hadronic jets in $W$ decays should be quite similar to the ones originating from the $Z$ peak, and the variation in the jet energy can be accounted for by the proper Lorentz boost. Finally, the effects of the statistical fluctuations in the response matrix can be made negligible compared to the influence of the statistical errors in the measured data by the proper increase of the generated statistics. In any case, by combining the results of various methods of analysis one can achieve better understanding of the overall uncertainty in $W$ mass determination.

5 Acknowlegdements

We are grateful to R. Barlow, N. Kjaer, A. Lebedev, A. Olshevski, T. Shears and D. Ward for helpful discussions. We would like to thank DELPHI collaboration and CERN for their kind hospitality during our stay at CERN, where this work has been finalized.

References

[1] F.Abe et al., Preprint FERMILAB-PUB-95/035, 1995.
[2] J.Alitti et al., Phys. Lett. B276 (1992) 354.
[3] See e.g. K.Hagiwara, Talk given at 17th Int. Symposium on Lepton-Photon Interactions, Beijing, 10-15 August 1995.
[4] A.Böhm and W.Hoogland (Eds.), ECFA Workshop on LEP 200, Preprint CERN 87-08, ECFA 87/108, June 1987.
[5] G.Altarelli et al., The Workshop on Physics at LEP2, Preprint CERN-TH/95-151, CERN-PPE/95-78, June 1995.
[6] G.Gustafson et al, Phys. Lett. B209 (1988) 90.
[7] L.Lönnblad and T.Sjöstrand, Phys. Lett. B351 (1995) 293.
[8] A.Höcker and V.Kartvelishvili, Preprint MC-TH-95/15, LAL-95/55, August 1995; hep-ph 9509307 (to be published in NIM A).
[9] T.Sjöstrand, PYTHIA 5.7 and JETSET 7.4, Physics and Manual, Preprint CERN-TH.7112/93, February 1994.
Figure 1: $W$ mass distribution unfolded from a sample of 1700 simulated events with final state muons. The errors shown correspond to the diagonal elements of the error matrix. The smooth curve describes the fit performed using the full error matrix including bin-to-bin correlations, resulting in $M_W = 80.50 \pm 0.33$ GeV.