EXCLUSIVE ELECTROPRODUCTION OF PENTAQUARKS

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Exclusive electroproduction of a Κ or Κ∗ meson on the nucleon can give a Θ⁺ pentaquark in the final state. This reaction offers an opportunity to investigate the structure of pentaquark baryons at parton level. We discuss the generalized parton distributions for the \( N \to \Theta^+ \) transition and give the leading order amplitude for electroproduction in the Bjorken regime.

1 Introduction

There is increasing experimental evidence [1,2] for the existence of a narrow baryon resonance \( \Theta^+ \) with strangeness \( S = +1 \), whose minimal quark content is \textit{uudd} \( \bar{s} \). Triggered by the prediction of its mass and width in [3,4], the observation of this hadron promises to shed new light on our picture of baryons in QCD. A fundamental question is how the structure of baryons manifests itself in terms of the basic degrees of freedom in QCD, at the level of partons. This structure at short distances can be probed in hard exclusive scattering processes [5], where it is encoded in generalized parton distributions [6] (see [7] for a recent review). In Ref. [8] we introduced the generalized parton distributions (GPDs) for the transition from the nucleon to the \( \Theta^+ \) (denoted as \( \Theta \) below) and investigated electroproduction processes where these GPDs could be measured, hopefully already in existing experiments at DESY and Jefferson Lab.

2 Processes

We consider the electroproduction processes

\[ ep \to eK^0\Theta, \quad ep \to eK^{*0}\Theta, \]

where the \( \Theta \) subsequently decays into \( K^0p \) or \( K^+n \). Note that the decay \( K^{*0} \to K^-\pi^+ \) of the \( K^*(892) \) tags the strangeness of the produced baryon. In contrast, the observation of a \( K^0 \) as \( K_S \) or \( K_L \) includes a background from final states with a \( K^0 \) and an excited \( \Sigma^+ \) state in the mass region of the \( \Theta \), unless the strangeness of the baryon is tagged by the kaon in the decay mode \( \Theta \to K^+n \). Apart from their different experimental aspects the channels with \( K \) or \( K^* \) production are quite distinct in their dynamics. The crossed process \( K^+n \to e^+e^-\Theta \) could be analyzed along the lines of [9] at an intense kaon beam facility.

The kinematics of the \( \gamma^mp \) subprocess is specified by the invariants \( Q^2 = -q^2, W^2 = (p + q)^2, t = (p - p')^2 \), with four-momenta as given in Fig. [10]. We are interested in the Bjorken limit of large \( Q^2 \) at fixed \( t \) and \( x_B = Q^2/(2pq) \).
According to the factorization theorem for meson production \cite{10}, the Bjorken limit implies factorization of the $\gamma^* p$ amplitude into a perturbatively calculable subprocess at quark level, the distribution amplitude (DA) of the produced meson, and a generalized parton distribution (GPD) describing the transition from $p$ to $\Theta$ (see Fig. 1). The dominant polarization of the photon and (if applicable) the produced meson is then longitudinal, and the corresponding $\gamma^* p$ cross section scales like $d\sigma_L/(dt) \sim Q^{-6}$ at fixed $x_B$ and $t$, up to logarithmic corrections in $Q^2$ due to perturbative evolution.

3 The transition GPDs and their physics

To define the transition GPDs we introduce light-cone coordinates $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and transverse components $v_T = (v^1, v^2)$ for any four-vector $v$. The skewedness variable $\xi = (p - p')^+/(p + p')^+$ describes the loss of plus-momentum of the incident nucleon and is connected with $x_B$ by $\xi \approx x_B/(2 - x_B)$ in the Bjorken limit.

In the following we assume that the $\Theta$, which we treat as a stable hadron, has spin $J = \frac{1}{2}$ and isospin $I = 0$. Different theoretical approaches predict either $\eta_\Theta = 1$ or $\eta_\Theta = -1$ for the intrinsic parity of the $\Theta$, and we will give our discussion for the two cases in parallel \cite{11}. The hadronic matrix elements that occur in the electroproduction processes \cite{11} at leading-twist accuracy are

\begin{align}
F_V &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \bar{\Theta} (\gamma^+ s(\frac{1}{2}z) | p) \bigg|_{z^+ = 0, z_T = 0} , \\
F_A &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \bar{\Theta} (\gamma^+ \gamma_5 s(\frac{1}{2}z) | p) \bigg|_{z^+ = 0, z_T = 0} ,
\end{align}

with $P = \frac{1}{2}(p + p')$, where here and in the following we do not explicitly label the hadron spin degrees of freedom. We define the corresponding $p \to \Theta$ transition GPDs by

\begin{align}
F_V &= \frac{1}{2P^+} \left[ H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)\alpha}{m_\Theta + m_N} u(p) \right] , \\
F_A &= \frac{1}{2P^+} \left[ \tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{m_\Theta + m_N} u(p) \right]
\end{align}
Figure 2. Wave function representation of the $p \rightarrow \Theta$ GPDs in the different regions of $x$. The blobs denote light-cone wave functions, and all possible configurations of spectator partons have to be summed over. The overall transverse position of the $\Theta$ is shifted relative to the proton.

for $\eta_\Theta = 1$ and by

$$F_V = \frac{1}{2P^+} \left[ \bar{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \bar{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{m_\Theta + m_N} u(p) \right],$$

$$F_A = \frac{1}{2P^+} \left[ H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i \sigma^{+\alpha} (p' - p)_{\alpha}}{m_\Theta + m_N} u(p) \right]$$ (4)

for $\eta_\Theta = -1$. The scale dependence of the matrix elements is governed by the non-singlet evolution equations for GPDs [6,12], with the unpolarized evolution kernels for $F_V$ and the polarized ones for $F_A$.

The value of $x$ determines the partonic interpretation of the GPDs. For $\xi < x < 1$ the proton emits an $s$ quark and the $\Theta$ absorbs a $d$ quark, whereas for $-1 < x < -\xi$ the proton emits a $\bar{d}$ and the $\Theta$ absorbs an $\bar{s}$. The region $-\xi < x < \xi$ describes emission of an $sd$ pair by the proton. In all three cases sea quark degrees of freedom in the proton are involved. The interpretation of GPDs becomes yet more explicit when the GPDs are expressed as the overlap of light-cone wave functions for the proton and the $\Theta$. As shown in Fig. 2 the proton must be in at least a five-quark configuration for $\xi < |x| < 1$ and at least a seven-quark configuration for $-\xi < x < \xi$.

As shown in [13], GPDs contain information about the spatial structure of hadrons. A Fourier transform converts their dependence on $t$ into the distribution of quarks or antiquarks in the plane transverse to their direction of motion in the infinite momentum frame. This tells us about the transverse size of the hadrons in question. The wave function overlap can also be formulated in this impact parameter representation, with wave functions specifying transverse position and plus-momentum fraction of each parton as shown on Fig. 2. We see in particular that for $\xi < |x| < 1$ the transverse positions of all partons must match in the proton and the $\Theta$, including the quark or antiquark taking part in the hard scattering. For $-\xi < x < \xi$ the transverse positions of the spectator partons in the proton must match those in the $\Theta$, whereas the $s$ and $\bar{d}$ are extracted from the proton at the same transverse position (within an accuracy of order $1/Q$ set by the factorization scale of the hard scattering process). Note that small-size quark-antiquark pairs with net strangeness are not necessarily rare in the proton, as is shown by the rather large kaon pole contribution to the $p \rightarrow \Lambda$ GPDs [14]. The upshot of our discussion is that the $p \rightarrow \Theta$ transition GPDs probe the partonic structure of the $\Theta$, requiring the plus-momenta and transverse positions of its partons to match
with appropriate configurations in the nucleon. The helicity and color structure of the parton configurations must match as well.

4 Scattering amplitude and cross section

The scattering amplitude for longitudinal polarization of photon and meson at leading order in $1/Q$ and in $\alpha_s$ readily follows from the general expressions for meson production given in [7]. One has

\[
A_{\gamma^* p \to K^0 \Theta} = i e \frac{8\pi \alpha_s}{27} \frac{f_K}{Q} \left[ I_K \int_{-1}^{1} \frac{dx}{\xi - x - i\epsilon} \left( F_A(x, \xi, t) - F_A(-x, \xi, t) \right) \right. \\
+ J_K \int_{-1}^{1} \frac{dx}{\xi - x - i\epsilon} \left( F_A(x, \xi, t) + F_A(-x, \xi, t) \right), \\
A_{\gamma^* p \to K^{*0} \Theta} = i e \frac{8\pi \alpha_s}{27} \frac{f_{K^{*0}}}{Q} \left[ I_{K^{*0}} \int_{-1}^{1} \frac{dx}{\xi - x - i\epsilon} \left( F_V(x, \xi, t) - F_V(-x, \xi, t) \right) \right.
\]

\[
+ J_{K^{*0}} \int_{-1}^{1} \frac{dx}{\xi - x - i\epsilon} \left( F_V(x, \xi, t) + F_V(-x, \xi, t) \right),
\]

independently of the parity of the $\Theta$. In (5) we have integrals

\[
I = \int_{0}^{1} \frac{dz}{z(1-z)} \phi(z), \\
J = \int_{0}^{1} \frac{2z - 1}{z(1-z)} \phi(z), \tag{6}
\]

over the twist-two distribution amplitudes of either $K^0$ or $K^{*0}$. Note that we cannot easily guess the relative sign of the transition GPDs at $x$ and $-x$, since they do not become densities in any kinematical limit. As a consequence we cannot say whether the terms with $I$ or with $J$ tend to dominate in the amplitudes (5).

To leading accuracy in $1/Q^2$ and in $\alpha_s$ the cross section for $\gamma^* p$ for a longitudinal photon on a transversely polarized target is

\[
\frac{d\sigma}{dt} = \frac{64\pi^2 \alpha_W^2 \alpha_s^2}{729} \frac{f_{K^{*0}}^2}{Q^2} \frac{\xi^2}{1 - \xi^2} \left( S_U + S_T \sin \beta \right), \tag{7}
\]

where $\beta$ is the azimuthal angle between the hadronic plane and the transverse target spin. The cross section for an unpolarized target is simply obtained by omitting the $\beta$-dependent term. To have concise expressions for $S_U$ and $S_T$ we define

\[
H(x, \xi, t) = I_{K^{*0}} \int_{-1}^{1} \frac{dx}{\xi - x - i\epsilon} \left( H(x, \xi, t) - H(-x, \xi, t) \right) \\
+ J_{K^{*0}} \int_{-1}^{1} \frac{dx}{\xi - x - i\epsilon} \left( H(x, \xi, t) + H(-x, \xi, t) \right), \tag{8}
\]

and analogous expressions $E$, $\tilde{H}$, $\tilde{E}$ for the other GPDs. For $\eta_0 = 1$ we have

\[
S_U = (1 - \xi^2)\tilde{H} |^2 + \left( \frac{m_\Theta - m_N}{m_\Theta + m_N} \right)^2 t \xi^2 \tilde{E}^2 - \left( \xi + \frac{m_\Theta - m_N}{m_\Theta + m_N} \right) 2\xi \text{Re}(\tilde{E}^* \tilde{H}), \\
S_T = -\sqrt{1 - \xi^2} \frac{\sqrt{4t - 1}}{m_\Theta + m_N} 2\xi \text{Im}(\tilde{E}^* \tilde{H}) \tag{9}
\]
Table 1. Combinations of transition GPDs multiplying $I$ and $J$ in the hard scattering formula \( \Theta \) and its analogs for the listed channels.

| $\gamma^* p \to K^0 \Theta$ | $I$ | $J$ |
|-------------------------------|-----|-----|
| $F_{p\to\Theta} (x) - F_{p\to\Theta} (-x)$ | $F_{p\to\Theta} (x) + F_{p\to\Theta} (-x)$ |
| $\gamma^* p \to K^0 \Sigma^+$ | $F_{p\to\Sigma^+} (x) - F_{p\to\Sigma^+} (-x)$ | $-[F_{p\to\Sigma^+} (x) + F_{p\to\Sigma^+} (-x)]$ |
| $\gamma^* p \to K^+ \Sigma^0$ | $-[2F_{p\to\Sigma^0} (x) + F_{p\to\Sigma^0} (-x)]$ | $2F_{p\to\Sigma^0} (x) - F_{p\to\Sigma^0} (-x)$ |
| $\gamma^* p \to K^+ \Lambda$ | $-[2F_{p\to\Lambda} (x) + F_{p\to\Lambda} (-x)]$ | $2F_{p\to\Lambda} (x) - F_{p\to\Lambda} (-x)$ |

for $K$ production and

$$S_U = (1 - \xi^2) |\mathcal{H}|^2 - \left( 2 \xi (m_{\Theta}^2 - m_N^2) + t \right) |\mathcal{E}|^2 - \left( \xi + \frac{m_{\Theta} - m_N}{m_{\Theta} + m_N} \right) 2 \xi \text{Re} (\mathcal{E}^* \mathcal{H}) ,$$

$$S_T = \sqrt{1 - \xi^2} \sqrt{\frac{t_0 - t}{m_{\Theta} + m_N}} 2 \text{Im} (\mathcal{E}^* \mathcal{H})$$

(10)

for $K^*$ production. If $\eta_{\Theta} = -1$ then $\Theta$ describes $K^*$ production and $\bar{\Theta}$ describes $K$ production. We see that one cannot determine the parity of the $\Theta$ from the leading twist cross section (7) without knowledge about the dependence of $\mathcal{H}$, $\mathcal{E}$, $\bar{\mathcal{H}}$, $\bar{\mathcal{E}}$ on $t$ or $\xi$.

There are arguments that theoretical uncertainties cancel at least partially in suitable ratios of cross sections. Processes to compare with are given by $ep \to eK^0 \Sigma^+$, $ep \to eK^+ \Sigma^0$, $ep \to eK^+ \Lambda$ or their analogs for vector kaons, with the production of either ground state or excited hyperons. Their amplitudes are given as in (9) with an appropriate replacement of matrix elements $F_V$ or $F_A$ listed in Table 1. Isospin invariance further gives $F_{p\to\Sigma^+} = \sqrt{2} F_{p\to\Sigma^0}$.

5 Conclusions

We have investigated exclusive electroproduction of a $\Theta^+$ pentaquark on the nucleon at large $Q^2$, large $W^2$ and small $t$. Such a process provides a rather clean environment to study the structure of pentaquark at parton level, in the form of well defined hadronic matrix elements of quark vector or axial vector currents. In parton language, these matrix elements describe how well parton configurations in the $\Theta$ match with appropriate configurations in the nucleon (see Fig. 2). Their dependence on $t$ gives information about the size of the pentaquark. Channels with production of pseudoscalar or vector kaons and with a proton or neutron target carry complementary information. The transition to the $\Theta$ requires sea quark degrees of freedom in the nucleon, and we hope that theoretical approaches including such degrees of freedom will be able to evaluate the matrix elements given in (9).

At modest values of $Q^2$ the leading approximation in powers of $1/Q^2$ and of $\alpha_s$ on which we based our analysis may receive considerable corrections. The associated theoretical uncertainties should be alleviated by comparing $\Theta$ production to the
production of $\Sigma$ or $\Lambda$ hyperons as reference channels. In any case, even a qualitative picture of the overall magnitude and relative size of the different hadronic matrix elements accessible in the processes we propose would give information about the structure of exotic baryons (see also Ref. [15] for the case of hybrid mesons) well beyond the little we presently know.

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