Optimization the first frequency modal shape of a tensural displacement amplifier employing flexure hinge by using Taguchi Method

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Abstract. The high amplification ratio, high frequency, and larger workspace always ask in compliant mechanisms. Thus, in this investigation optimized design dimension to select an optimal model for a tensural displacement amplifier employing flexure hinge by utilizing Taguchi method based on finite element analysis (FEA). The Solidworks software was applied to design the model, the first modal shape frequency was gained by FEA in ANSYS. The orthogonal L27 was applied to design 27 experiments. The data was collected from 27 cases which were analyzed by signal to noise, ANOVA and regression analysis (RA). The simulation results indicated that thickness of flexure hinge (TOFH) made significant increasing the 1st natural frequency when its rises from 0.3 mm to 0.7 mm and decreased as incline angle between two flexure hinges rises from 0° to 0.5°. The phenomena also confirmed by S/N analysis, ANOVA and RA. The optimal value of the frequency obtained 214.06 Hz is good agreement with the forecasted value of 198.1758 Hz with deviation error of 7.15%.

1. Introduction

The traditional mechanism used universal joint to connect parts with each other such as revolute joints and translation joints. In the joints always present clearance due to manufacture and assembly. This problem caused vibration lead to friction phenomena and wear joint caused failure for mechanism. In order to eliminate this problem, FH has many different shapes utilized changing to traditional joints. In this study design and optimize a tensural displacement amplifier employing flexure hinge with new dimensions which achieved high displacement.

Many FHs with different profile were designed and developed such as right circular, right angle, parabolic, elliptic, V-shape, corner filleted, cycloidal, hyperbolic and hybrid FH were designed to replace traditional joints. For example, Xu and Li [1] applied the Euler-Bernoulli beam theory to estimate the displacement amplification ratio (DAR) and compared FEA and experiment. In 2015, Keqi Qi et al. [2] investigated the DAR of bridge-type compliant mechanism flexure hinge (BTCMFH) by using elastic beam theory (EBT) and is verified by finite element method (FEM) and previous publication. Liu and Yan [3] stated a new analytical method based on EBT to investigate effects of external load on the DAR, and was confirmed by FEM in ANSYS. The power preservation law and EBT were applied by Ling et al. [4] to improve a method for bridge-type, rhombus-type compliant mechanisms, and the obtained results were compared with the FEM in ANSYS and experiment. A BTCMFH fully compliant was designed and investigated by Choi et al. [5] with concentration force and distribution force. The results were compared with experiment and previous publication. Ma et al.
[6] stated that the DAR rises as the thickness of a flexure hinge (TOFH) decreases, and the FEA and a mathematic model was applied to verify the problem. A modular and assembled statics modeling tool were applied by Ling et al. [7] to analyze and design of a wide variety of flexure hinges was applied in the precision positioning stage. The previous investigation and FEM also was applied to confirm this problem. The DAR was obtained by the stiffness matrix method and compare with FEM and experiment to ensure high rigidity, large magnification, high-precision tracking, and high-accuracy positioning. Dao and Huang [8] applied Grey-Taguchi to optimize two degree of freedom compliant mechanism. Chen et al. [9] investigated DAR of a tensural displacement amplifier mechanism using flexure hinge based on the kinetostatic model, amplification ratio obtained 40 and compared with FEM and experiment.

The target of the paper presents a design and optimizes the effects of design parameters on the first frequency modal of a tensural displacement amplifier employing flexure hinge by using the Taguchi method based on the FEM in ANSYS.

2. Modeling and methodology

2.1. Modeling of mechanism

The tensural displacement amplifier compliant mechanism employing FHs was shown in Figure 1. Figure 1(a) present the 3D model and Figure 1(b) is the projection of the 3D model. The high of the model is 100 mm, the di of the model changes from 110 mm to 120 mm. TOFH changes from 0.3 mm to 0.7 mm, incline angle between two FHs changes from 0° to 0.5°, thickness of mechanism (TOM) changes from 10 mm to 20 mm, width of FH (WOFH) changes from 2 mm to 6 mm, width of mechanism (WOM) changes from 110 mm to 120 mm and the other dimensions are constant. The mechanism moves to according to the y-axis direction at output position when input external force of displacement at input position as presented in Figure 1(b).

![Figure 1. The tensural displacement amplifier employing flexure hinge ((a) 3D, (b) 2D).](image)

2.2. Analysis finite element method

First, Solidworks 2018 software was used to create the mechanism and then was import into static structural tool in ANSYS Workbench. The material aluminum AL-7075 with 72 GPa Young’s modulus, 0.33 Poisson’s ratio and 503 MPa Yield strength was selected for the mechanism. Figure 2(a)
presented the model meshed. The meshed model was meshed by automatic in ANSYS software. The number of the finite elements and number of nodes achieved 26145 elements and 50571 nodes. The model was fixed at four points A, B, C, D and input displacement at point E as shown in Figure 2(b).

![Figure 2. (a) The meshing model, (b) Insert Fixed support and input displacement for model.](image)

3. **Optimization**

In order to obtain maximum value of the 1st natural frequency, the mathematical models need to determine first and then applied the optimal mathematical models. However, if the mathematics models have error very high, then the models are unacceptable. Thus, Taguchi method was proposed to optimize the 1st natural frequency [8, 10].

“The larger the better” approach,

\[
S/N = -10\log\left(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i}\right)
\]  

(1)
where, the variance of $y_i$ is the observed data of each characteristic at the ith experiment, $n$ is the quantity of experiments and $i$ is the ith experiment, respectively.

In this paper, the Minitab 18 program was used to create the Taguchi method, S/N analysis, analysis of variance (ANOVA) and regression analysis. The obtained results are stated in the Results and Discussion section of this paper.

4. Results and discussion

4.1. Influence of thickness

Figure 3. The 1st frequency modal shape with different TOFH ((a) TOFH = 0.3 mm, (b) TOFH = 0.5 mm, (c) TOFH = 0.7 mm).

The first frequency modal shape was presented in Figure 3 that pointed out that this frequency rises from 71.599 Hz to 195.16 Hz when TOFH rises from 0.3 mm to 0.7 mm with input displacement is equal to zero according to Y axis direction, with TOM of 10 mm, incline angle between two FHs of $0.3^\circ$, WOM of 115 mm, WOFH of 4 mm. The simulation results are higher references [9].

4.2. Influence of incline angle

The first frequency modal shape was depicted in Figure 4 that indicated its decreases from 60.536 Hz to 52.94 Hz when incline angle rises from $0^\circ$ to $0.5^\circ$ with input displacement is equal to zero according to Y axis direction, with TOM of 15 mm, TOFH of 0.3 mm, WOFH of 6 mm, WOM of 120 mm. The simulation results are higher references [9].
Figure 4. The 1st frequency modal shape with different Incline angle ((a) 0°, (b) 0.3°, (c) 0.5°).

4.3. Optimization of the first frequency modal shape

In Section 4.1 and 4.2 revealed that the first frequency modal shape is strongly impacted by TOFH and incline angle between two FHs. In order to obtain maximum value of the first frequency modal, the design parameters and their level need to optimize as listed in Table 1. Table 2 presented orthogonal arrays, simulation results and S/N ratio which the orthogonal arrays were designed by Minitab, the simulation results were obtained by ANSYS 18.0 and S/N ratios results were obtained by Minitab or by using Equation (1).

Table 1. Factors and their level.

| Factors                          | Unit | Levels |
|---------------------------------|------|--------|
| Thickness of mechanism          | x mm | 10     |
| Thickness of flexure hinge      | y mm | 0.3    |
| Incline angle between 2 FHs     | z degree | 0.3    |
| Width of mechanism              | t mm | 110    |
| Width of flexure hinge          | w mm | 2      |
Table 2. Orthogonal array and results.

| Trial No. | x   | y   | z   | t   | w   | The 1st frequency | S/N of the 1st frequency |
|----------|-----|-----|-----|-----|-----|------------------|-------------------------|
| 1        | 10  | 0.3 | 0   | 110 | 2   | 35.195           | 30.9296                 |
| 2        | 10  | 0.3 | 0.3 | 115 | 4   | 53.199           | 34.5181                 |
| 3        | 10  | 0.3 | 0.5 | 120 | 6   | 65.065           | 36.2670                 |
| 4        | 10  | 0.5 | 0   | 115 | 6   | 127.62           | 42.1184                 |
| 5        | 10  | 0.5 | 0.3 | 120 | 2   | 53.106           | 34.5029                 |
| 6        | 10  | 0.5 | 0.5 | 110 | 4   | 109.29           | 40.7716                 |
| 7        | 10  | 0.7 | 0   | 120 | 4   | 153.67           | 43.7318                 |
| 8        | 10  | 0.7 | 0.3 | 110 | 6   | 217.6            | 46.7532                 |
| 9        | 10  | 0.7 | 0.5 | 115 | 2   | 80.427           | 38.1080                 |
| 10       | 15  | 0.3 | 0   | 110 | 2   | 28.47            | 29.0877                 |
| 11       | 15  | 0.3 | 0.3 | 115 | 4   | 43.051           | 32.6797                 |
| 12       | 15  | 0.3 | 0.5 | 120 | 6   | 53.4             | 34.5508                 |
| 13       | 15  | 0.5 | 0   | 115 | 6   | 104.83           | 40.4097                 |
| 14       | 15  | 0.5 | 0.3 | 120 | 2   | 43.086           | 32.6867                 |
| 15       | 15  | 0.5 | 0.5 | 110 | 4   | 88.648           | 38.9534                 |
| 16       | 15  | 0.7 | 0   | 120 | 4   | 126.99           | 42.0754                 |
| 17       | 15  | 0.7 | 0.3 | 110 | 6   | 179.31           | 45.0721                 |
| 18       | 15  | 0.7 | 0.5 | 115 | 2   | 65.28            | 36.2956                 |
| 19       | 20  | 0.3 | 0   | 110 | 2   | 24.323           | 27.7203                 |
| 20       | 20  | 0.3 | 0.3 | 115 | 4   | 36.706           | 31.2947                 |
| 21       | 20  | 0.3 | 0.5 | 120 | 6   | 46.39            | 33.3229                 |
| 22       | 20  | 0.5 | 0   | 115 | 6   | 91.064           | 39.1869                 |
| 23       | 20  | 0.5 | 0.3 | 120 | 2   | 36.953           | 31.3513                 |
| 24       | 20  | 0.5 | 0.5 | 110 | 4   | 75.743           | 37.5869                 |
| 25       | 20  | 0.7 | 0   | 120 | 4   | 109.89           | 40.8192                 |
| 26       | 20  | 0.7 | 0.3 | 110 | 6   | 156.09           | 43.8675                 |
| 27       | 20  | 0.7 | 0.5 | 115 | 2   | 55.035           | 34.8128                 |

The mean values of S/N and natural frequency was presented in Table 3, Table 4, respectively. The graph for S/N and mean values were depicted in Figure 5(a) and Figure 5(b), respectively. Figure 5(a) and Table 3 indicated optimal combination design dimension namely thickness of mechanism of 10 mm, TOFH of 0.7 mm, incline angle of 0°, WOM of 110 mm, WOFH of 6 mm (x1y3z1t1w3), with corresponding mean values are 99.46 Hz, 127.14 Hz, 89.12 Hz, 101.63 Hz, 115.70 Hz, respectively. The ANOVA result was presented in Table 5. It’s identified that design dimensions have significant effects on the natural frequency with contributing percent 6.13% x, 52.42% y, 1.4% z, 3.82% t, 35.85% w and 0.38% deviation error. The F-value and P-value make sure condition larger than 2 and less than 0.05, respectively. The results are good agree with FEA. The R-square, R-square(adj), R-square(pred) gained 99.62%, 93.39%, 98.93%, respectively.

The regression equation (RE) was obtained by Minitab 18.0 as presented in Equation (2). The graph compared between the FEA value and forecasted regression model are good agree as drawn in Figure 6. Because two curves lie near each other.

\[ f = (13.75 - 0.1548x + 11.376y - 1.298z - 0.1016t + 0.925w)^2 \]
Figure 5. (a) S/N plot and (b) Mean plot for the first frequency modal.

Table 3. Response table for S/N ratios.

| Level | x   | y   | z   | t   | w   |
|-------|-----|-----|-----|-----|-----|
| 1     | 38.63 | 32.26 | 37.34 | 37.86 | 32.83 |
| 2     | 36.87 | 37.51 | 36.97 | 36.60 | 38.05 |
| 3     | 35.55 | 41.28 | 36.74 | 36.59 | 40.17 |
| Delta | 3.08 | 9.02 | 0.60 | 1.27 | 7.34 |
| Rank  | 3   | 1   | 5   | 4   | 2   |

Table 4. Response table for means.

| Level | x   | y   | z   | t   | w   |
|-------|-----|-----|-----|-----|-----|
| 1     | 99.46 | 42.86 | 89.12 | 101.63 | 46.87 |
| 2     | 81.45 | 81.15 | 91.01 | 73.02 | 88.58 |
| 3     | 70.24 | 127.14 | 71.03 | 76.50 | 115.70 |
| Delta | 29.22 | 84.28 | 19.98 | 28.61 | 68.83 |
| Rank  | 3   | 1   | 5   | 4   | 2   |
Table 5. Analysis of variance for transformed response.

| Source | DF | Seq SS  | Contribution | Adj SS | Adj MS | F-Value | P-Value |
|--------|----|---------|--------------|--------|--------|---------|---------|
| x      | 2  | 10.906  | 6.13%        | 10.906 | 5.4532 | 130.42  | 0.000   |
| y      | 2  | 93.226  | 52.42%       | 93.226 | 46.6131| 1114.83 | 0.000   |
| z      | 2  | 2.483   | 1.40%        | 2.4832 | 1.2416 | 29.70   | 0.000   |
| t      | 2  | 6.789   | 3.82%        | 6.7895 | 3.3947 | 81.19   | 0.000   |
| w      | 2  | 63.755  | 35.85%       | 63.7546| 31.8773| 762.40  | 0.000   |
| Error  | 16 | 0.669   | 0.38%        | 0.0418 |        |         |         |
| Total  | 26 | 177.829 | 100.00%      |        |        |         |         |

R-sq = 99.62%, R-sq(adj) = 99.39%, R-sq(pred) = 98.93%

Figure 6. Residual plot for the first frequency modal shape.

4.4. Predicted optimization

In this section, the forecasted value is calculated and verified by the optimization models was simulation in ANSYS R18.0, as based the optimal combination parameters. First, the mean value of the first frequency modal were listed in Table 4, Second, \( \mu_f \) was obtained by using Equation (3) and Table 4. The total mean value is equal to 83.72 Hz and \( \mu_f \) are obtained, as follows:

\[
\mu_f = f_m + \sum_{i=1}^{q} (f_0 - f_m) = x_1 + y_3 + z_1 + t_1 + w_3 - 4f_m
\]

\[
= 99.46 + 127.14 + 89.12 + 101.63 + 115.7 - 4 \times 83.72 = 198.1758 \text{ (Hz)}
\]

The predicted result is larger than the result presented in reference [10].

The 95% confidence interval to verify \( CI \) was obtained using Equation (4), at \( \alpha = 0.05 \), \( fe = 16 \), \( F_{0.05}(1,16) = 4.494 \) [10], \( Ve = 0.0418 \), \( R = 10 \), \( Re = 1 \), \( n = 27 \)

\[
CI_{CE} = \pm \sqrt{F_{\alpha}(1, fe)Ve\left(\frac{1}{n_{eff}} + \frac{1}{R e}\right)}
\]

\[
= \pm \sqrt{4.494 \times 0.0418 \times \left(\frac{1}{27/10}\right) + 1} = \pm 0.5657, \quad 197.6101 < f_{\text{confirmation}} < 198.7415
\]

The \( CI_{CE} \) values was obtained by Equation (4) verified that the regression values are closed to the simulation values with deviation ±0.5657 at 95% confidence interval.
Figure 7. The modal shape frequency with the optimal combination dimension x1y3z1t1w1.

The optimal value of model with optimal dimension was illustrated in Figure 7, the optimal nature frequency was achieved 214.06 Hz which are good agree with the predicted value of 198.1758 Hz with deviation error 7.15%.

5. Conclusions
The S/N analysis result was pointed out optimal combination dimension for the mechanism based on FEA. Besides, the S/N and ANOVA result also confirmed that FEA revealed design dimensions have significantly influenced on the 1st modal shape frequency. The first is TOFH, the second is WOFH, the third is TOM, the fourth is WOM and the final is incline angle between two FHs. The optimal value of the first modal shape frequency was obtained of 214.06 Hz by Taguchi method with the optimal combination parameter x1y3z1t1w3 compared with the predicted value of 198.1758 Hz are good agree with deviation error 7.15%. The RE values obtained near to the simulation values with $CI_{CE}$ of $\pm 0.5657$ at confidential level 95% and R-square achieved over 98%.

Acknowledgments
The authors acknowledge and thank the Ministry of Science and Technology of the Republic of China for their partial financial support of this study under Contract Number MOST 107-2622-E-992-013 - CC3.

References
[1] Xu Q and Li Y 2011 *Mechanism and Machine Theory* vol 46 pp 183-200
[2] Qi Keqi, Xiang Yang, Fang Chao, Zhang Yang and Yu Changsong 2015 *Mechanism and Machine Theory* vol 87 pp 45-56
[3] Liu P and Yan P 2016 *Mechanism and Machine Theory* vol 99 pp 176-188
[4] Ling M, Cao J, Zeng M, Lin J and D J Inman 2016 *Smart Materials and Structures* vol 25 pp 075022
[5] K B Choi, J J Lee, G H Kim, H J Lim and S G Kwon 2018 *Mechanism and Machine Theory* vol 121 pp 355-372
[6] Ma H W, Yao S M, Wang L Q and Zhong Z 2006 *Sensors and Actuators A: Physical* vol 132 pp 730-736
[7] Ling M, Cao J, Jiang Z and Lin J 2017 *Mechanism and Machine Theory* vol 107 pp 274-282
[8] Dao T P and Huang S C 2017 *Microsystem Technologies* vol 23 pp 4815-4830
[9] Chen G, Ma Y and Li J 2016 *Sensors and Actuators A: Physical* vol 247 pp 307-315
[10] Ranjit K Roy 2010 *Society of Manufacturing Engineers*