A Robust Gaussian Filter Corresponding to the Transmission Characteristic of the Gaussian Filter

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Abstract. A surface roughness profile of an object can be measured by extracting a mean line of the long wavelength component from the primary profile, and by subtracting it from the primary profile. This mean line is usually computed by convolving the traditional Gaussian filter (GF) and the primary profile. However, if an outlier exists in the primary profile, the output of a Gaussian filter will be greatly affected by the outlier. To solve the outlier problem, several schemes of robust Gaussian filter have been proposed. However there are several fatal problems that a mean line determined with respect to the measurement data containing no outliers does not agree with the mean line of the Gaussian filter output. To solve these problems, this paper proposes a new robust Gaussian filter based on a fast M-estimation method (FMGF) and the performance of the new robust Gaussian filter was experimentally clarified. As a result, if an outlier exists, the proposed method behaves a robust performance. If no outlier exists, the output wave pattern, RMSE and transmission characteristic accorded mutually with Gaussian filter.

1. Introduction
A surface roughness profile of an object to be measured is obtained by extracting a mean line for the long wavelength component from a primary profile by means of a low-pass filter of cut-off value \(\lambda_c\), and subtracting it from the primary profile. This mean line is usually determined by convolving a Gaussian filter with the primary profile \([1,2]\). However, if an outlier exists in the primary profile, the output of a Gaussian filter will be fatally affected from it \([3]\).

To solve the outlier problem, S.Brinkmann et al. proposed a robust Gaussian filter (RGF) \([4]\). However this RGF has a serious problem that its output becomes quite different from the GF output if no outliers exists in the measurement data. Therefore the filter is still not popularized in actual sites.

To solve these problems, the authors proposed a new robust Gaussian filter based on a fast M-estimation method (FMGF). If no outlier exists in measurement data, the FMGF output wave form accorded mutually with GF output \([5]\). The deviation between GF output and FMGF output is minimized when the data without outlier is applied. However a distribution characteristic of transmission ratio is not yet known completely. If this small deviation is caused by the difference in transmission rate of a specific wavelength, then this transmission characteristic of FMGF has also the deflection. If FMGF has such a deflection, FMGF is not used in the actual sites. And, it is not yet took into consideration that FMGF cannot realize 50% of transmission ratio at cut-off value \(\lambda_c\). Therefore, the transmission characteristic of the FMGF must be verified.

Generally, the transmission characteristic of the low-path filter is calculated by implementing Fourier transform of the weighting function of the filter. However, in such a case of RGF that it is difficult to
calculate the transmission characteristic because, in addition to a weighting function of RGF which cannot be directly expressed in a mathematical formula, the coordinate axis of the estimation function is the unique clue because this coordinate axis is perpendicular to the coordinate axis of the weighting function. Because FMGF weighting function is not directly given as a mathematical formula either, the above circumstances are the same. This problem may be solved by so-called Fast Convolution that Fourier transform of the convolution output of these two functions is equal to the product of the Fourier transform of each function. A mean line is obtained as the output of the convolution between the low-pass filter ad the h measurement data. However, a mean line is equal to the product between the Fourier transform of measurement data and the Fourier transform of the low-pass filter. From this fact, the Fourier transform of the low-pass filter can be provided by dividing the Fourier transform of measurement data from the Fourier transform of the mean line. This will be the transmission characteristic of the low-pass filter. However, as DFT (Discrete Fourier Transform) must be used as the substitute of Fourier Transform, the related problem called ringing or aliasing must be coped with.

Therefore, this paper established a computation method and verify its transmission characteristic of FMGF equal to transmission characteristic of GF.

2. Computation for transmission characteristic using discrete Fourier transform

2.1 Basic computation model

Here, N pieces of the measurement data is defined as $z = f(x_i)$ just in the same way of the weighting function of discretized low-pass filter as $l(x_i)$, and of a mean line of output by convolution of weighting function with $f(x_i)$, as $h(x_i)$, respectively.

$$h(x_i) = \sum f(x_i) l(x_i - x_i) = (f \otimes l)(x_i)$$

(1)

$\tilde{F}(u_k), \tilde{L}(u_k)$, and $\tilde{H}(u_k)$ are defined as the discrete Fourier transform of $f(x_i)$, $l(x_i)$, and $h(x_i)$, respectively. Assuming that $u_k = k/N$ leads to the following equation:

$$\tilde{H}(u_k) = \tilde{F}(u_k) \tilde{L}(u_k)$$

(2)

Using this equation let us go to the following equation for the low-pass filter the transmission function ($\tilde{L}(u_k)$).

$$\tilde{L}(u_k) = \frac{\tilde{H}(u_k)}{\tilde{F}(u_k)}$$

(3)

2.2 How to give measurement data

The measurement data to calculate transmission characteristic is enforced by the random data of the small amplitude to base line which is designed by considering following conditions carefully: An original function does not become discontinuous, the discrete Fourier transform has a periodicity, and the both ends of the data coincide and are connected smoothly. In order to do this, base curve was designed to be becomes point symmetric as shown in Eq.(4) below, and a 2nd-order B-spline function was utilized and 1st-order differential coefficient was introduced for coinciding the values at both ends of data. Here, $L$ is an evaluation length and $C$ is a constant of amplitude.

$$z = \begin{cases} 
- C \left( x - \frac{1}{4} L \right)^2 + \frac{CL^2}{16} & \text{for } 0 \leq x \leq \frac{L}{2} \\
C \left( x - \frac{3}{4} L \right)^2 - \frac{CL^2}{16} & \text{for } \frac{L}{2} \leq x \leq L 
\end{cases}$$

(4)
3. Experiments

Experiment is conducted to determine a transmission characteristic of FMGF. Figure 1 shows the primary profile \( f(x_i) \), of which evaluation length \( L=4\text{mm} \) in the x-direction. This primary profile has a base line, which is enforced by the random data where max. \( 1\mu\text{m} \) to a base curve becomes as a periodic function of \( 25\mu\text{m} \) amplitude as shown in Eq.(4). The number \( N \) of data is 200, and the GF width and the cut-off value are both \( \lambda_c=0.8\text{mm} \). The sampling interval in the z-direction of FMGF was set at \( \Delta z=4\mu\text{m} \), and the parameters for dividing the basic width were set at \( m=7,9 \) and \( 11 \), respectively.

![Primary profile](image)

**Figure 1** Test data (Primary profile)

3.1 Computation for determination of transmission characteristic

Figure 2 shows transmission characteristic of FMGF, which is computed by Eq.(3) from primary profile \( f(x_i) \) and FMGF output \( l(x_i) \) and the mean line \( h(x_i) \). The computation of the discrete Fourier transform of the data of \( N=200 \) was executed by Chirp-Z transform. The horizontal axis is a logarithm axis showing the ratio of space frequency to cut-off value \( \lambda_c \), and the vertical axis is the transmission ratio.

![Transmission characteristic](image)

**Figure 2** Transmission characteristic of the FMGF
In the case of m=7, Figure 2(a) shows that similar to transmission characteristic of theoretical GF generally. However characteristic of the short wavelength area is different from theoretical GF. However Table 1 shows a transmission ratio in cut-off value \( \lambda_c \) is 0.5044 and the error with transmission characteristic of filter GF of width \( \lambda_c \) is less than 1%, and almost agree in the band of the long wavelength than \( \lambda_c \).

In the case of m=9, Figure 2(b) shows that almost all characteristics becomes coincident except that some characteristics of the low wavelength area are different from GF. The transmission ratio at the cut-off value \( \lambda_c \) is 0.5090 where the error with the transmission ratio of filter GF of width \( \lambda_c \) is less than 0.1% (Table 1).

In the case of m=11, Figure 2(c) shows that a transmission ratio becomes almost perfectly coincident with theoretical GF in all wavelengths area. The transmission ratio at cut-off level \( \lambda_c \) is 0.5085 (Table 1) where the transmission ratio of filter GF of width \( \lambda_c \).

The transmission ratio differs slightly from the theoretical GF, however, it coincides with the theoretical GF at the cut-off wave length level \( \lambda_c \). Figure 3 shows the deviation of the transmission ratio among FMGF and theoretical GF and the theoretical GF at the cut-off wave length level \( \lambda_c \). The deviation with the transmission ratio of FMGF and theoretical GF becomes up to 1%. However, the deviation with the transmission ratio of FMGF and transmission ratio of the theoretical GF at the cut-off wave length level \( \lambda_c \) is very few less than \( 10^{-3}\% \). It was clarified that such an agreement occurs in the first calculation step of FMGF to apply the theoretical GF at the cut-off wave length level \( \lambda_c \). If the width of GF is wider than \( \lambda_c \) then the consistency between the transmission characteristic of FMGF and transmission characteristic of theoretical GF become highly precise.

| Filter Remarks | Fast M-estimation type | GF Filtersize \( \lambda_c \) |
|----------------|------------------------|-----------------------------|
| m=7            | m=9                    | m=11                        |
| Transmission ratio | 0.5044                | 0.5090                      | 0.5085                      | 0.5085                      |

![Figure 3](image.png)
4. Conclusion

The method for determining the transmission characteristic of FMGF by the computation using the discrete Fourier transform has established in this paper. Therefore, FMGF became practical to the actual site, and it is verified that the filter actually behaves in a robust way against outliers [5], and that its output value became in good agreement with that of a GF in terms of measurement data other than outliers.

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