Hypercomplex Systems and Non-Gaussian Stochastic Solutions with Some Numerical Simulation of \(\chi\)-Wick-Type (2 + 1)-D C-KdV Equations

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1. Introduction

This article focuses on stochastic (2 + 1)-D C-KdV equation of the Wick-type with an NG parameter.

\[
\begin{cases}
U_t + \Phi_1(t) \circ \chi U \circ \chi V_{x_1} + \Phi_2(t) \circ \chi V \circ \chi U_{x_1} + \Phi_3(t) \circ \chi U_{x_1 x_1 x_1} = 0, \\
U_{x_1} + V_{y_1} = 0,
\end{cases}
\]

where \(\Phi_1, \Phi_2\) and \(\Phi_3\) are NG-\(\mathbb{H}^{1,q}\)-valued functions, and \(\circ \chi\) denotes the \(\chi\)-Wick product on distributions \(\mathbb{H}^{1,q}\) \(^{[1,2]}\). By means of the usual product in Equation (1), we introduce variable coefficients (2 + 1)-D C-KdV equations:

\[
\begin{cases}
u u_t + \phi_1(t) \nu v_{x_1} + \phi_2(t) v u_{x_1} + \phi_3(t) u_{x_1 x_1 x_1} = 0, \\
\nu u_{x_1} + v_{y_1} = 0,
\end{cases}
\]

where \(\phi_1(t), \phi_2(t)\) and \(\phi_3(t)\), functions which are integrable on \(\mathbb{R}_+, \) and \((t, x_1, y_1) \in \mathbb{R}_+ \times \mathbb{R}^2\).

The (2 + 1)-D C-KdV system is an important type of nonlinear equations in mathematics and physics. So, it is important to find solutions to this equation. The (2 + 1)-D C-KdV system has various applications in numerous fields of nonlinear science. One of the applications of the (2 + 1)-D C-KdV system is that it can be used to describe generic string...
dynamic properties in constant curvature spaces for strings and multistrings. Another application can be found in [3–9]. The (2 + 1)-D C-KdV Equation (2) describes nonlinear wave propagation in polarity–coordination systems. Moreover, if the problem is taken into account in a NG-stochastic framework, we can gain the NG-stochastic (2 + 1)-D C-KdV equation. In order to obtain exact stochastic solutions of the NGS (2 + 1)-D C-KdV equation, we consider this issue in an NGWN framework. Consequently, we dispute variable coefficients of stochastic (2 + 1)-D C-KdV Equation (1). The existence of a solution of non-self-governing fractional differential equations with integral impulse conditions is investigated and studied via the measurement of noncompactness. A novel analytical technique was studied to obtain solitary solutions for the fractional-order equation of nonlinear evolution; hence, the periodic solutions of a certain one-D differential equation were investigated. For more details, see [10–16]. Furthermore, Dumitru, et al. investigated and studied the fractional differential equation in mathematical physics, which is closely related with our work. For more details, see [17–21]. The functional analysis of stochastic white noise (SWN) in [6,22–29]: Stochastic WNFS was extensively studied for some nonlinear PDEs [2,30–32]. Okb El Bab, Zakarya, et al. [1,2,33], using the theory of HCS and NG-Wick calculus based on HCS with some SPDE and applications, studied some significant subjects related to the construction of NGWN analysis [34,35]. Recently, Zakarya, using HCS and NG-Wick calculus, sought to obtain stochastic Wick-type (3 + 1)-D-modified equations of the Benjamin–Bona–Mahony (BBM) type [36]. Very recently, N-soliton solutions to integrable equations were systematically studied with the Hirota direct method for both (1+1)-dimensional and (2 + 1)-dimensional integrable equations [37], and some important classes of novel equations in (2 + 1)-dimensions [38], and for nonlocal integrable equations [39].

The purpose of this article is giving the exact and stochastic (2 + 1)-D traveling wave solutions for (TWS) C-KdV Equation (2) and NG-stochastic (2 + 1)-D C-KdV Equation (1) in order to obtain NG-stochastic solutions of the (2 + 1)-D C-KdV equation. This problem was only considered in a NGWN framework, which means that the variable coefficients were considered to be stochastic (2 + 1)-D C-KdV Equation (1). We developed an NG-Wick calculus based on the theory of HCS for this purpose (L1(Q, dm(x))). We used the specific relation between WNA and theory of HCS [2]. We used this NG-parameter construction and F-expansion method to provide a number of families of (2 + 1)-D C-KdV equation solutions for solitary TWS, namely, (2) and NG-stochastic solutions of \( \chi \)-Wick-type (2 + 1)-D C-KdV Equation (1).

This article consists of five sections. In Section 1, we present the introduction. In Section 2, we used the main results obtained in [1,2] and the method of F-expansion to obtain the exact TWS for the stochastic style (2 + 1)-D C-KdV Equation (2). In Section 3, for the Wick-type stochastic (2 + 1)-D C-KdV equations, we obtain exact NGWN functional solutions (1). In Section 4, we support our findings with detailed examples. A summary and discussion are in Section 5.

2. Solitary TWS in Equation (2)

In this part of the report, we minimize Equation (1) into deterministic partial differential equations (PDEs) by using \( \chi \)-HT. In addition, the obtained PDEs could be translated into ordinary nonlinear differential equations (ODEs) by applying proper transformation. So, by using the proposed F-expansion method, we obtained many accurate solutions of Equation (2). Influenced by the \( \chi \)-HT of Equation (1), we obtained the following deterministic equations.

\[
\begin{align*}
\hat{U}_t(t, x_1, y_1, z) + \Phi_1(t, z)\hat{U}(t, x_1, y_1, z)\hat{V}_{x_1}(t, x_1, y_1, z) \\
+ \Phi_2(t, z)\hat{V}(t, x_1, y_1, z)\hat{U}_{x_1}(t, x_1, y_1, z) \\
+ \Phi_3(t, z)\hat{U}_{x_1}x_1(t, x_1, y_1, z) = 0, \\
\hat{V}_{x_1}(t, x_1, y_1, z) + \hat{V}_{y_1}(t, x_1, y_1, z) = 0,
\end{align*}
\]

(3)
where \( z = (z_1, z_2, \ldots) \in (\mathbb{C}^n)_c \) is a vector parameter. To seek solitary TWS of Equation (3), we took the transformations as follows: \( \Phi_1(t, z) := \phi_1(t, z), \Phi_2(t, z) := \phi_2(t, z), \Phi_3(t, z) := \phi_3(t, z) \), \( U(t, x_1, y_1, z) := u(t, x_1, y_1, z) = u(\hat{q}(t, x_1, y_1, z)) \) and \( V(t, x_1, y_1, z) := v(t, x_1, y_1, z) = v(\hat{q}(t, x_1, y_1, z)) \) with

\[
\hat{q}(t, x_1, y_1, z) = kx_1 + ky_1 + \mu \int_0^t \omega(\tau, z) d\tau + c,
\]

where \( k, l, \mu \) and \( c \) are free constants, and \( kl \mu \neq 0 \), \( \omega(t, z) \) is a nonzero variable function that is subsequently determined. As a result, Equation (3) can be converted into ODE as follows:

\[
\begin{align*}
\mu \omega u' + k\phi_1 u' + k\phi_2 v' + k^3\phi_3 u''' &= 0, \\
k' + lv' &= 0,
\end{align*}
\]

where \( ^{\prime} \) indicates the derivative for \( q \). According to the F-expansion method [31], we could express the solutions of Equation (3) as follows.

\[
\begin{align*}
u(t, x_1, y_1, z) &= u(q) = \sum_{i=0}^{N} a_i F^i(q), \\
v(t, x_1, y_1, z) &= v(q) = \sum_{i=0}^{M} b_i F^i(q),
\end{align*}
\]

where \( a_i \) and \( b_i \) are constants that are established later. Using the principle of homogeneous balance in Equation (4), we could obtain \( N = M = 2 \). So, Equation (5) becomes

\[
\begin{align*}
u(t, x_1, y_1, z) &= a_0 + a_1 F(q) + a_2 F^2(q), \\
v(t, x_1, y_1, z) &= b_0 + b_1 F(q) + b_2 F^2(q),
\end{align*}
\]

where \( b_0, b_1, b_2, a_0, a_1 \) and \( a_2 \), are constants that are found later. Using the F-expansion method and substituting (6) into (4) and the collection of all expressions that had the same power of \( F'(q) | F'(q) |^j \), \( (i = 0 \pm 1, 2, \ldots, j = 0, 1) \) as follows:

\[
\begin{align*}
\left\{ \begin{array}{l}
\mu \omega a_1 + k a_0 b_1 \phi_1 + k a_1 b_0 \phi_2 + k^3 a_1 \phi_3 P_2 \mid F' \\
+ [2 \mu \omega a_2 + 2 k a_0 b_2 \phi_1 + k a_1 b_1 \phi_1 + 2 k a_1 b_0 \phi_2 + k a_1 b_1 \phi_2 + 8 k^3 a_2 \phi_3 P_2] | F' \\
+ k [2 a_1 b_2 \phi_1 + a_2 b_1 \phi_1 + 2 a_2 b_1 \phi_2 + a_1 b_1 \phi_2 + 6 k^2 a_1 \phi_3 P_1] | F^2 F' \\
+ 2 k a_2 [b_2 \phi_1 + b_2 \phi_2 + 12 k^2 \phi_3 P_1] | F^2 F' = 0, \\
(k a_1 + l b_1) F' + 2 | k a_2 + l b_2 | | F F' = 0.
\end{array} \right.
\end{align*}
\]

By vanishing the coefficients of \( F'(q) | F'(q) |^j \), we obtain an algebraic system as follows:

\[
\begin{align*}
\mu \omega a_1 + k a_0 b_1 \phi_1 + k a_1 b_0 \phi_2 + k^3 a_1 \phi_3 P_2 &= 0, \\
2 \mu \omega a_2 + 2 k a_0 b_2 \phi_1 + a_1 b_1 \phi_1 + 2 k a_1 b_0 \phi_2 + k a_1 b_1 \phi_2 + 8 k^3 a_2 \phi_3 P_2 &= 0, \\
k [2 a_1 b_2 \phi_1 + a_2 b_1 \phi_1 + 2 a_2 b_1 \phi_2 + a_1 b_1 \phi_2 + 6 k^2 a_1 \phi_3 P_1] &= 0, \\
2 k a_2 [b_2 \phi_1 + b_2 \phi_2 + 12 k^2 \phi_3 P_1] &= 0, \\
k a_1 + l b_1 &= 0, \\
2 | k a_2 + l b_2 | &= 0.
\end{align*}
\]
By solving system 8, we obtain the following coefficients:

\[
\begin{align*}
    a_2 &= b_2 = 0, \quad a_0, b_0 = \text{arbitrary constant}, \\
    a_1 &= \frac{6k\phi_3(t,z)P_1}{\phi_2(t,z)}, \\
    b_1 &= -\frac{6k^2\phi_3(t,z)P_1}{\phi_2(t,z)}, \\
    \omega &= \frac{k^2a_0\phi_1(t,z) - l[kb_0\phi_2(t,z) + k^2\phi_3(t,z)P_2]}{\phi_2(t,z)}.
\end{align*}
\]  

(9)

By substituting from (9) into (6), we obtain the following general forms of the solutions of Equation (2):

\[
\begin{align*}
    u(t, x_1, y_1, z) &= a_0 + \frac{6k\phi_3(t,z)P_1}{\phi_2(t,z)} F(q), \\
    v(t, x_1, y_1, z) &= b_0 - \frac{6k^2\phi_3(t,z)P_1}{\phi_2(t,z)} F(q),
\end{align*}
\]

(10) (11)

with

\[
q(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \frac{k^2a_0\phi_1(\tau, z) - l[kb_0\phi_2(\tau, z) + k^2\phi_3(\tau, z)P_2]}{\phi_2(\tau, z)} d\tau.
\]

From Appendix A, we obtain special cases of solutions as follows:

**Case I.**

At \(P_1 = \frac{1}{4}, P_2 = \frac{1}{2}, P_3 = \frac{3}{4}\), we have \(F(q) \rightarrow ns(q) \pm ds(q)\). So,

\[
\begin{align*}
    u_1(t, x_1, y_1, z) &= a_0 + \frac{3lk\phi_3(t,z)}{2\phi_2(t,z)} \left[ ns \left( q_1(t, x_1, y_1, z) \right) \pm ds \left( q_1(t, x_1, y_1, z) \right) \right], \\
    v_1(t, x_1, y_1, z) &= b_0 - \frac{3k^2\phi_3(t,z)}{2\phi_2(t,z)} \left[ ns \left( q_1(t, x_1, y_1, z) \right) \pm ds \left( q_1(t, x_1, y_1, z) \right) \right],
\end{align*}
\]

(12) (13)

with

\[
q_1(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{2k^2a_0\phi_1(\tau, z) - l[kb_0\phi_2(\tau, z) + k^2\phi_3(\tau, z)(\lambda^2 - 2)]}{2l} \right\} d\tau.
\]

When, \(\lambda \rightarrow 0\), we find \(ns(q) \pm ds(q) \rightarrow 2 \csc(q)\). So, the solutions in Equations (12) and (13) become as follows:

\[
\begin{align*}
    u_2(t, x_1, y_1, z) &= a_0 + \frac{3lk\phi_3(t,z)}{\phi_2(t,z)} \csc \left( q_2(t, x_1, y_1, z) \right), \\
    v_2(t, x_1, y_1, z) &= b_0 - \frac{3k^2\phi_3(t,z)}{\phi_2(t,z)} \csc \left( q_2(t, x_1, y_1, z) \right),
\end{align*}
\]

(14) (15)

with

\[
q_2(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k^2a_0\phi_1(\tau, z) - l[kb_0\phi_2(\tau, z) - k^2\phi_3(\tau, z)]}{\phi_2(\tau, z)} \right\} d\tau,
\]
When, \( \lambda \to 1 \), we find \( \text{ns}(\varrho) \pm ds(\varrho) \to \coth(\varrho) \pm \text{csch}(\varrho) \). So, the solutions in Equations (12) and (13) become:

\[
u_3(t, x_1, y_1, z) = a_0 + \frac{3k\phi_3(t, z)}{2\phi_2(t, z)} \left[ \coth \phi_3(t, x_1, y_1, z) \pm \text{csch}(\phi_3(t, x_1, y_1, z)) \right],
\]

(16)

\[
v_3(t, x_1, y_1, z) = b_0 - \frac{3k^2\phi_3(t, z)}{2\phi_2(t, z)} \left[ \coth \phi_3(t, x_1, y_1, z) \pm \text{csch}(\phi_3(t, x_1, y_1, z)) \right],
\]

(17)

with

\[
\phi_3(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{2k^2a_0\phi_1(\tau, z) - lk[2b_0\phi_2(\tau, z) - k^2\phi_3(\tau, z)]}{2l} \right\} \, d\tau.
\]

**Case II.**

At \( P_1 = 1, P_2 = -(1 + \lambda^2) \) and \( P_3 = \lambda^2 \), then \( F(\varrho) \to \text{ns}(\varrho) \). So, we find

\[
u_4(t, x_1, y_1, z) = a_0 + \frac{6lk\phi_3(t, z)}{\phi_2(t, z)} \text{ ns } (\phi_4(t, x_1, y_1, z)),
\]

(18)

\[
v_4(t, x_1, y_1, z) = b_0 - \frac{6k^2\phi_3(t, z)}{\phi_2(t, z)} \text{ ns } (\phi_4(t, x_1, y_1, z)),
\]

(19)

with

\[
\phi_4(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{2k^2a_0\phi_1(\tau, z) - lk[2b_0\phi_2(\tau, z) + k^2\phi_3(\tau, z)(\lambda^2 - 2)]}{l} \right\} \, d\tau.
\]

When \( \lambda \to 0 \), we find \( \text{ns}(\varrho) \to \csc(\varrho) \). So, the solutions in Equations (18) and (19) become as follows:

\[
u_5(t, x_1, y_1, z) = a_0 + \frac{6lk\phi_3(t, z)}{\phi_2(t, z)} \csc (\phi_5(t, x_1, y_1, z)),
\]

(20)

\[
v_5(t, x_1, y_1, z) = b_0 - \frac{6k^2\phi_3(t, z)}{\phi_2(t, z)} \csc (\phi_5(t, x_1, y_1, z)),
\]

(21)

while when \( \lambda \to 1 \) we find \( \text{ns}(\varrho) \to \coth(\varrho) \). So, the solutions in Equations (18) and (19) become:

\[
u_6(t, x_1, y_1, z) = a_0 + \frac{6lk\phi_3(t, z)}{2\phi_2(t, z)} \coth (\phi_5(t, x_1, y_1, z)),
\]

(22)

\[
v_6(t, x_1, y_1, z) = b_0 - \frac{6k^2\phi_3(t, z)}{2\phi_2(t, z)} \coth (\phi_5(t, x_1, y_1, z)),
\]

(23)

with

\[
\phi_5(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k^2a_0\phi_1(\tau, z) - lk[b_0\phi_2(\tau, z) - 2k^2\phi_3(\tau, z)]}{l} \right\} \, d\tau.
\]

**Case III.** At \( P_1 = 1, P_2 = (2 - \lambda^2) \) and \( P_3 = 1 - \lambda^2 \), then \( F(\varrho) \to \csc(\varrho) \). So, we find

\[
u_7(t, x_1, y_1, z) = a_0 + \frac{6lk\phi_3(t, z)}{\phi_2(t, z)} \csc (\phi_6(t, x_1, y_1, z)),
\]

(24)
\[
v_7(t, x_1, y_1, z) = b_0 - \frac{6k^2\phi_3(t, z)}{\phi_2(t, z)} \cos(\varphi_6(t, x_1, y_1, z)), \tag{25}
\]
with
\[
\varphi_6(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k^2a_0\phi_1(t, z) - lk[2b_0\phi_2(t, z) + k^2\phi_3(t, z)(2 - \lambda^2)]}{l} \right\} dt.
\]

When \( \lambda \to 0 \) we find \( \cos(\varphi) \to \cot(\varphi) \). So, the solutions in Equations (24) and (25) become as follows:
\[
u_8(t, x_1, y_1, z) = a_0 + \frac{6lk\phi_3(t, z)}{\phi_2(t, z)} \cot(\varphi_7(t, x_1, y_1, z)), \tag{26}
\]
\[
\varphi_7(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k^2a_0\phi_1(t, z) - lk[2b_0\phi_2(t, z) + 2k^2\phi_3(t, z)]}{l} \right\} dt.
\]

while when \( \lambda \to 1 \) we find \( \cos(\varphi) \to \csc(\varphi) \). So, the solutions in Equations (24) and (25) become:
\[
u_9(t, x_1, y_1, z) = a_0 + \frac{6lk\phi_3(t, z)}{\phi_2(t, z)} \csc(\varphi_8(t, x_1, y_1, z)), \tag{28}
\]
\[
\varphi_9(t, x_1, y_1, z) = b_0 - \frac{6k^2\phi_3(t, z)}{\phi_2(t, z)} \csc(\varphi_8(t, x_1, y_1, z)), \tag{29}
\]
with
\[
\varphi_8(t, x_1, y_1, z) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k^2a_0\phi_1(t, z) - lk[2b_0\phi_2(t, z) + k^2\phi_3(t, z)]}{l} \right\} dt.
\]

Remark 1. There are many results for Equation (2) that are based on checking various coefficients values \( P_1, P_2 \) and \( P_3 \) (see Appendices A–C). The above cases clearly illustrate the degree to which our approach is valid.

3. **NG-Stochastic Solutions of Equation (2)**

In this section, by applying HT to obtain NG-stochastic solutions, we use the results of the above section to Wick-type \( (2 + 1) \)-D C-KdV Equation (2). The characterization of trigonometric and exponential functions ensures that there is a bounded set open \( G \subset \mathbb{R}_+ \times \mathbb{R}^2, q < \infty, M > 0 \), such that the solution \( u(t, x_1, y_1, z) \) of Equation (3) and all its partial derivatives in Equation (3) are uniformly bounded for \( (t, x_1, y_1, z) \in G \times \mathbb{O}_q(M) \), continuous with respect to \( (t, x_1, y_1) \in G \) for all \( z \in \mathbb{O}_q(M) \) and analytic with respect to \( z \in \mathbb{O}_q(M) \), for all \( (t, x_1, y_1) \in G \). From Theorem 2.1 in [34], there exists \( U(t, x_1, y_1) \in H^q \) such that \( u(t, x_1, y_1, z) = \tilde{U}(t, x_1, y_1)(z) \) for all \( (t, x_1, y_1, z) \in G \times \mathbb{O}_q(M) \) and \( U(t, x_1, y_1) \) solves Equation (2) in \( H^q \).

For this reason, by using the inverse \( \chi \)-HT to the above results of Section 2, we have NG-stochastic functional solutions of Equation (1) as follows.

**I) NG-Stochastic Solutions of JEF Type:**
\[
U_1(t, x_1, y_1) = a_0 + \frac{3lk\phi_3(t)}{2\phi_2(t)} \lambda \left[ \eta(t, x_1, y_1) \pm ds\lambda(t, x_1, y_1) \right], \tag{30}
\]
\begin{align}
V_1(t, x_1, y_1) &= b_0 - \frac{3k^2 \Phi_3(t)}{2 \Phi_2(t)} \diamondsuit \left[ n_\chi (q_1(t, x_1, y_1)) \pm ds_\chi (q_1(t, x_1, y_1)) \right], \quad (31) \\
U_2(t, x_1, y_1) &= a_0 + \frac{6lk \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi n_\chi (q_2(t, x_1, y_1)), \quad (32) \\
V_2(t, x_1, y_1) &= b_0 - \frac{6k^2 \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi n_\chi (q_2(t, x_1, y_1)), \quad (33) \\
U_3(t, x_1, y_1) &= a_0 + \frac{6lk \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \csc_\chi (q_3(t, x_1, y_1)), \quad (34) \\
V_3(t, x_1, y_1) &= b_0 - \frac{6k^2 \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \csc_\chi (q_3(t, x_1, y_1)), \quad (35)
\end{align}
with
\begin{align}
q_1(t, x_1, y_1) &= kx_1 + ly_1 + \int_0^t \left\{ \frac{2k^2 a_0 \Phi_1(\tau) - lk[2b_0 \Phi_2(\tau) + k^2 \Phi_3(\tau)(m^2 - 2)]}{2l} \right\} d\tau, \\
q_2(t, x_1, y_1) &= kx_1 + ly_1 + \int_0^t \left\{ \frac{2k^2 a_0 \Phi_1(\tau) - lk[2b_0 \Phi_2(\tau) + k^2 \Phi_3(\tau)(m^2 - 2)]}{2l} \right\} d\tau, \\
q_3(t, x_1, y_1) &= kx_1 + ly_1 + \int_0^t \left\{ \frac{k^2 a_0 \Phi_1(\tau) - lk[2b_0 \Phi_2(\tau) + k^2 \Phi_3(\tau)(2 - m^2)]}{l} \right\} d\tau.
\end{align}

(II) NG-Stochastic Solutions of the Trigonometric Type

\begin{align}
U_4(t, x_1, y_1) &= a_0 + \frac{3lk \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \csc_\chi (q_4(t, x_1, y_1)), \quad (36) \\
V_4(t, x_1, y_1) &= b_0 - \frac{3k^2 \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \csc_\chi (q_4(t, x_1, y_1)), \quad (37) \\
U_5(t, x_1, y_1) &= a_0 + \frac{6lk \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \csc_\chi (q_4(t, x_1, y_1)), \quad (38) \\
V_5(t, x_1, y_1) &= b_0 - \frac{6k^2 \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \csc_\chi (q_4(t, x_1, y_1)), \quad (39) \\
U_6(t, x_1, y_1) &= a_0 + \frac{6lk \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \cot_\chi (q_5(t, x_1, y_1)), \quad (40) \\
V_6(t, x_1, y_1) &= b_0 - \frac{6k^2 \Phi_3(t)}{\Phi_2(t)} \diamondsuit \chi \cot_\chi (q_5(t, x_1, y_1)), \quad (41)
\end{align}
with

\[ q_4(t, x_1, y_1) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k_a^2 \Phi_1(\tau) - lk[b_0 \Phi_2(\tau) - k^2 \Phi_3(\tau)]}{l} \right\} \, d\tau, \]

\[ q_5(t, x_1, y_1) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k_a^2 \Phi_1(\tau) - lk[b_0 \Phi_2(\tau) + 2k^2 \Phi_3(\tau)]}{l} \right\} \, d\tau. \]

(III) NG-Stochastic Solutions of the Hyperbolic Type

\[ U_7(t, x_1, y_1) = a_0 + \frac{3k^4 \Phi_3(t)}{2\Phi_2(t)} \sqrt{\chi} \left[ \coth^{\hat{\chi}} (q_6(t, x_1, y_1)) \pm \csc^{\hat{\chi}} (q_6(t, x_1, y_1)) \right], \]

\[ V_7(t, x_1, y_1) = b_0 - \frac{3k^4 \Phi_3(t)}{2\Phi_2(t)} \sqrt{\chi} \left[ \coth^{\hat{\chi}} (q_6(t, x_1, y_1)) \pm \csc^{\hat{\chi}} (q_6(t, x_1, y_1)) \right], \]

\[ U_8(t, x_1, y_1) = a_0 + \frac{6k^2 \Phi_3(t)}{2\Phi_2(t)} \sqrt{\chi} \coth^{\hat{\chi}} (q_7(t, x_1, y_1)), \]

\[ V_8(t, x_1, y_1) = b_0 - \frac{6k^2 \Phi_3(t)}{2\Phi_2(t)} \sqrt{\chi} \coth^{\hat{\chi}} (q_7(t, x_1, y_1)), \]

\[ U_9(t, x_1, y_1) = a_0 + \frac{6k^2 \Phi_3(t)}{2\Phi_2(t)} \sqrt{\chi} \csc^{\hat{\chi}} (q_8(t, x_1, y_1)), \]

\[ V_9(t, x_1, y_1) = b_0 - \frac{6k^2 \Phi_3(t)}{2\Phi_2(t)} \sqrt{\chi} \csc^{\hat{\chi}} (q_8(t, x_1, y_1)), \]

with

\[ q_6(t, x_1, y_1) = kx_1 + ly_1 + \int_0^t \left\{ \frac{2k_a^2 \Phi_1(\tau) - lk[2b_0 \Phi_2(\tau) - k^2 \Phi_3(\tau)]}{2l} \right\} \, d\tau, \]

\[ q_7(t, x_1, y_1) = kx_1 + ly_1 + \int_0^t \left\{ \frac{2k_a^2 \Phi_1(\tau) + lk[b_0 \Phi_2(\tau) - k^2 \Phi_3(\tau)]}{l} \right\} \, d\tau, \]

\[ q_8(t, x_1, y_1) = kx_1 + ly_1 + \int_0^t \left\{ \frac{k_a^2 \Phi_1(\tau) - lk[b_0 \Phi_2(\tau) + 2k^2 \Phi_3(\tau)]}{l} \right\} \, d\tau. \]

**Remark 2.** By various composition of the functions \( \Phi_1, \Phi_2 \) and \( \Phi_3 \), we obtain a different type of non-Gaussian stochastic solutions of Equation (2) from Equations (30)-(47).

4. Example

The Wick version of the function is ordinarily difficult to evaluate. So, in this section, we introduce the solutions of Equation (2) via the non-Wick version. Let \( W_t^X = B_t^X \) be NG white noise where \( B_t^X \) is the \( \chi \)-Brownian motion. We have \( \chi \)-HT \( W_t^X(z) = \sum_{i=1}^{\infty} z_i \int_0^t \eta_i(s) \, ds \) [34]. Since \( \exp^{\hat{\chi}}(B_t) = \exp(B_t - \frac{t}{2}) \), we have \( \cot^{\hat{\chi}}(B_t) = \cot(B_t - \frac{t}{2}) \), \( \csc^{\hat{\chi}}(B_t) = \csc(B_t - \frac{t}{2}) \), \( \coth^{\hat{\chi}}(B_t) = \coth(B_t - \frac{t}{2}) \) and \( \csc^{\hat{\chi}}(B_t) = \csc(B_t - \frac{t}{2}) \). Suppose that
\(\Phi_1(t) = \psi_1\Phi_3(t), \Phi_2(t) = \psi_2\Phi_3(t)\) and \(\Phi_3(t) = \Gamma(t) + \psi_3W_t\), where \(\psi_1, \psi_2\) and \(\psi_3\) are free constants, and \(\Gamma(t)\) is a function that is integrable on \(\mathbb{R}_+\). Therefore, \(\Phi_1(t)\Phi_2(t)\Phi_3(t) \neq 0\). So, we have the functional NGWN solutions of Equation (2) as follows:

\[
U_{10}(t, x_1, y_1) = a_0 + \frac{3lk}{\psi_2} \csc (\Omega_1(t, x_1, y_1)), \quad (48)
\]

\[
V_{10}(t, x_1, y_1) = b_0 - \frac{3k^2}{\psi_2} \csc (\Omega_1(t, x_1, y_1)), \quad (49)
\]

\[
U_{11}(t, x_1, y_1) = a_0 + \frac{6lk}{\psi_2} \csc (\Omega_1(t, x_1, y_1)), \quad (50)
\]

\[
V_{11}(t, x_1, y_1) = b_0 - \frac{6k^2}{\psi_2} \csc (\Omega_1(t, x_1, y_1)), \quad (51)
\]

\[
U_{12}(t, x_1, y_1) = a_0 + \frac{6lk}{\psi_2} \cot (\Omega_2(t, x_1, y_1)), \quad (52)
\]

\[
V_{12}(t, x_1, y_1) = b_0 - \frac{6k^2}{\psi_2} \cot (\Omega_2(t, x_1, y_1)), \quad (53)
\]

with

\[
\Omega_1(t, x_1, y_1) = kx_1 + ly_1 + \left(\frac{k^2a_0\psi_1 - k|b_0\psi_2 - k^2|}{l}\right) \left\{ \int_0^t \Gamma(\tau) d\tau + \psi_3 |B_1 - \frac{t^2}{2}| \right\},
\]

\[
\Omega_2(t, x_1, y_1) = kx_1 + ly_1 + \left(\frac{k^2a_0\psi_1 - k|b_0\psi_2 + 2k^2|}{l}\right) \left\{ \int_0^t \Gamma(\tau) d\tau + \psi_3 |B_1 - \frac{t^2}{2}| \right\},
\]

and

\[
U_{13}(t, x_1, y_1) = a_0 + \frac{3lk}{2\psi_2} \left[ \coth (\Omega_3(t, x_1, y_1)) \pm \csch (\Omega_3(t, x_1, y_1)) \right], \quad (54)
\]

\[
V_{13}(t, x_1, y_1) = b_0 - \frac{3k^2}{2\psi_2} \left[ \coth (\Omega_3(t, x_1, y_1)) \pm \csch (\Omega_3(t, x_1, y_1)) \right], \quad (55)
\]

\[
U_{14}(t, x_1, y_1) = a_0 + \frac{6lk}{2\psi_2} \coth (\Omega_4(t, x_1, y_1)), \quad (56)
\]

\[
V_{14}(t, x_1, y_1) = b_0 - \frac{6k^2}{2\psi_2} \coth (\Omega_4(t, x_1, y_1)), \quad (57)
\]

\[
U_{15}(t, x_1, y_1) = a_0 + \frac{6lk}{\psi_2} \csch (\Omega_5(t, x_1, y_1)), \quad (58)
\]
\[ V_{15}(t,x_1,y_1) = b_0 - \frac{6k^2}{\psi_2} \operatorname{csch} (\Omega_5(t,x_1,y_1)), \]

with

\[ \Omega_3(t,x_1,y_1) = kx_1 + ly_1 + \left( \frac{2k^2a_0\psi_1 - lk[b_0\psi_2 - k^2]}{2l} \right) \left\{ \int_0^t \Gamma(\tau)d\tau + \psi_3[B_1 - \frac{l^2}{2}] \right\}, \]

\[ \Omega_4(t,x_1,y_1) = kx_1 + ly_1 + \left( \frac{k^2a_0\psi_1 - lk[b_0\psi_2 - 2k^2]}{l} \right) \left\{ \int_0^t \Gamma(\tau)d\tau + \psi_3[B_1 - \frac{l^2}{2}] \right\}, \]

\[ \Omega_5(t,x_1,y_1) = kx_1 + ly_1 + \left( \frac{k^2a_0\psi_1 - lk[b_0\psi_2 + k^2]}{l} \right) \left\{ \int_0^t \Gamma(\tau)d\tau + \psi_3[B_1 - \frac{l^2}{2}] \right\}. \]

For suitable parametric choices, we plotted three-dimensional graphics of the solution \{\(U_{10}(t,x_1,y_1)\), \(V_{10}(t,x_1,y_1)\)\} of \(\chi\)-Wick-type \((2 + 1)\)-D C-KdV equations for Figures 1 and 2.

**Figure 1.** (a–c) Three-dimensional plots of \(U_{10}(t,x_1,y_1)\) without the noise effect, when \(t = 1, 1.5\) and 2; (d–f) 3D plots of \(V_{10}(t,x_1,y_1)\) without the noise effect, when \(t = 1, 1.5\) and 2.

**Figure 2.** (a–c) Three-dimensional plots of \(U_{10}(t,x_1,y_1)\) with the noise effect when \(t = 1, 1.5\) and 2; (d–f) 3D plots of \(V_{10}(t,x_1,y_1)\) with the noise effect, when \(t = 1, 1.5\) and 2.
5. Conclusions

The (2 + 1)-D coupled KdV Equation (2) can describe the diffusion of nonlinear waves in polarity symmetry schemes. When a problem is considered in a NG-stochastic system, we can obtain the NG-stochastic (2 + 1)-D C-KdV equation [1,2,33]. In order to obtain NG-stochastic solutions of the (2 + 1)-D C-KdV equation, we just considered this topic in a NG-white noise climate, that is, we considered variable coefficient stochastic (2 + 1)-D C-KdV Equation (1). For this reason, we developed an NG-Wick calculus-based theory of HCS $L_{1}(Q, dm(x))$. We used a direct relation to the study of white noise and HCS [2]. We employed this construction of NG-parameters and the F-expansion method to provide several families with solitary TWS of (2 + 1)-D C-KdV Equation (2) and NG-stochastic solutions of $\chi$-Wick-type (2 + 1)-D C-KdV Equation (1). The obtained solutions were JEF functional solutions, and trigonometric and hyperbolic forms. Through suitable parametric choices, we also plotted three-dimensional graphics of some NG-white-noise-induced solutions for SPDEs. In future work, we will apply this solution approach by using fractional differential equations.

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Appendix A. ODE and Jacobi Elliptic Functions

Relation between values of $(P_{1}, P_{2}, P_{3})$ and corresponding $F(\varphi)$ in ODE.

\[
(F'(\varphi))^{2} = P_{1}F^{4}(\varphi) + P_{2}F^{2}(\varphi) + P_{3},
\]

| $P_{1}$ | $P_{2}$ | $P_{3}$ | $F(\varphi)$ |
|--------|--------|--------|-------------|
| $-1$   | $2 - \lambda^{2}$ | $\lambda^{2} - 1$ | $\text{dn} \varphi$ |
| $1$    | $-1 - \lambda^{2}$ | $\lambda^{2}$ | $\text{ns} \varphi = \frac{1}{\text{sn} \varphi}$, $\text{dc} \varphi = \frac{\text{dn} \varphi}{\text{cn} \varphi}$ |
| $\lambda^{2} - 1$ | $2 - \lambda^{2}$ | $-1$ | $\text{nd} \varphi = \frac{1}{\text{dn} \varphi}$ |
| $1 - \lambda^{2}$ | $2 - \lambda^{2}$ | $1$ | $\text{sc} \varphi = \frac{\text{sn} \varphi}{\text{cn} \varphi}$ |
| $-\lambda^{2}(1 - \lambda^{2})$ | $2\lambda^{2} - 1$ | $1$ | $\text{sd} \varphi = \frac{\text{sn} \varphi}{\text{dn} \varphi}$ |
| $1$ | $2 - \lambda^{2}$ | $1 - \lambda^{2}$ | $\text{cs} \varphi = \frac{\text{cn} \varphi}{\text{sn} \varphi}$ |
| $1$ | $2\lambda^{2} - 1$ | $-\lambda^{2}(1 - \lambda^{2})$ | $\text{ds} \varphi = \frac{\text{dn} \varphi}{\text{sn} \varphi}$ |
| $\lambda^{4}$ | $\frac{2 - \lambda^{2}}{2}$ | $\frac{1}{4}$ | $\frac{\text{sn} \varphi + i\text{cn} \varphi}{\sqrt{1 - \lambda^{2} + \text{dn} \varphi}}$, $\frac{\text{cn} \varphi}{\sqrt{1 - \lambda^{2} + \text{dn} \varphi}}$ |
| $\lambda^{2}$ | $\frac{2 - \lambda^{2}}{2}$ | $\frac{\lambda^{2}}{4}$ | $\text{sn} \varphi \pm i\text{cn} \varphi$, $\frac{\text{dn} \varphi}{\sqrt{1 - \lambda^{2} + \text{sn} \varphi \pm i\text{cn} \varphi}}$, $\frac{\lambda \text{sn} \varphi}{1 + \text{dn} \varphi}$ |
| $\frac{1}{4}$ | $\frac{1 - 2\lambda^{2}}{2}$ | $\frac{1}{4}$ | $\text{ns} \varphi \pm \text{cs} \varphi$, $\frac{\text{cn} \varphi}{\sqrt{1 - \lambda^{2} + \text{sn} \varphi \pm \text{dn} \varphi}}$, $\frac{\lambda \text{sn} \varphi}{1 + \text{dn} \varphi}$ |
| $\frac{1}{4}$ | $\frac{2 - \lambda^{2}}{2}$ | $\frac{\lambda^{2}}{4}$ | $\text{ns} \varphi \pm \text{ds} \varphi$ |
Appendix B

When $\lambda \to 0$, JEF degenerates into trigonometric functions as follows:

$$
sn\varrho \to \sin \varrho, cn\varrho \to \cos \varrho, dn\varrho \to 1, sc\varrho \to \tan \varrho, sd\varrho \to \sin \varrho, cd\varrho \to \cos \varrho,
nsn\varrho \to \csc \varrho, ncn\varrho \to \sec \varrho, ndn\varrho \to 1, csc\varrho \to \cot \varrho, dsn\varrho \to \csc \varrho, dcn\varrho \to \sec \varrho.
$$

Appendix C

When $\lambda \to 1$, JEF degenerates into hyperbolic functions as follows:

$$
sn\varrho \to \tanh \varrho, cn\varrho \to \varrho, dn\varrho \to \varrho, sc\varrho \to \sinh \varrho, sd\varrho \to \sinh \varrho, cd\varrho \to 1,
nsn\varrho \to \coth \varrho, ncn\varrho \to \cosh \varrho, ndn\varrho \to \cosh, csc\varrho \to \varrho, dsn\varrho \to \varrho, dcn\varrho \to 1.
$$

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