The non-linear redshift-space power spectrum: $\Omega$ from redshift surveys

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Accepted 1995 December 1. Received 1995 November 24; in original form 1995 October 6

ABSTRACT

We examine the anisotropies in the power spectrum by the mapping of real space to redshift space. Using the Zel’dovich approximation, we obtain an analytic expression for the non-linear redshift-space power spectrum in the distant observer limit. For a given unbiased galaxy distribution in redshift space, the anisotropies in the power spectrum depend on the parameter $f(\Omega) \approx \Omega^{0.6}$, where $\Omega$ is the density parameter. We quantify these anisotropies by the ratio, $R$, of the quadrupole and monopole angular moments of the power spectrum. In contrast to linear theory, the Zel’dovich approximation predicts a decline in $R$ with decreasing scale. This departure from linear theory is due to non-linear dynamics and is not a result of incoherent random velocities. The rate of decline depends strongly on $\Omega$ and the initial power spectrum. However, we find a scaling relation between the quantity $R/R_{\text{lin}}$ (where $R_{\text{lin}}$ is the linear theory value of $R$) and the dimensionless variable $k/k_{nl}$, where $k_{nl}$ is a wavenumber determined by the scale of non-linear structures. The scaling is weakly dependent on the initial power spectrum and is in good agreement with a large N-body simulation. This universal scaling relation greatly extends the scales over which redshift distortions can be used as a probe of $\Omega$. The scaling relation is in agreement with the observed quadrupole-to-monopole ratio from the 1.2-Jy IRAS survey, with a best estimate of $\Omega^{0.6}/b_{\text{lin}} = 0.6 \pm 0.2$ where $b_{\text{lin}}$ is the linear bias parameter.

Key words: cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

Statistical studies of the large-scale structure are mainly based on galaxy redshift surveys. Redshifts of galaxies differ from their distances as a result of peculiar velocities along the line of sight. This difference produces anisotropies in the observed galaxy distribution in redshift space ($s$-space). The anisotropies of clustering in $s$-space are manifested in the dependence of the correlation function on the direction of the pair separation or, equivalently, in the dependence of the power spectrum on the direction of the wavevector. These anisotropies offer the promise of measuring the cosmological density parameter, $\Omega$, on large scales. This is because gravitational instability predicts that the peculiar velocity causing the large-scale distortions in $s$-space depends on $f(\Omega) = d \ln D/d \ln a \approx \Omega^{0.6}$, where $D(t)$ and $a(t)$ are the linear growth and scale factors respectively. Therefore, given an approximation for gravity, one may obtain an estimate of $\Omega$ by quantifying the anisotropies, say, in the observed $s$-space power spectrum (Kaiser 1987; Hamilton 1992).

A useful way to characterize the anisotropies in $s$-space is through the angular moments of the $s$-space power spectrum, $P^s$; the lowest order moments, the monopole and quadrupole, are given by

$$M = \int_0^{+1} d\mu P^s(k, \mu)$$

and

$$Q = \frac{5}{2} \int_0^{+1} d\mu P^s(k, \mu)(3\mu^2 - 1), \quad (1)$$

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where $\beta = f(\Omega)/b$, and $b$ is the linear bias factor. Values of $\beta$ determined from s-space anisotropies span the range $\sim 0.4-1.0$ (Cole, Fisher & Weinberg 1995, hereafter CFW; Hamilton 1993, 1995; Fisher, Scharf & Lahav 1994a; Heavens & Taylor 1995; Fisher et al. 1994b; Loveday et al. 1995). One source of systematic error in these analyses is in modelling the non-linear gravitational evolution (Cole, Fisher & Weinberg 1994). In this Letter, we study the non-linear evolution of the s-space power spectrum. To describe non-linear dynamics we adopt the Zel'dovich approximation (Zel'dovich 1970). This approximation is an excellent tool for describing non-linear dynamics on large scales (e.g. Nusser et al. 1991). The Zel'dovich approximation has previously been used by Taylor (1993) and Schneider & Bartelmann (1995) to study the evolution of the real-space power spectrum.

In Section 2 we derive an analytic expression for the s-space power spectrum and find a strong dependence of the ratio $R$ on $\Omega$ and the initial conditions. However, in Section 3 we show that the quantity $R/R_m$ when plotted against the dimensionless variable $k/k_{nl}$, where $k_{nl}$ is a wavenumber determined by the scale of non-linear structures, acquires a universal shape which is only weakly dependent on $\Omega$ and the initial power spectrum. This relation can be very useful for estimating $\Omega$ from redshift surveys over a wide range of scales. As an example, we present a preliminary application to the 1.2-Jy IRAS survey. We conclude in Section 4.

2 THE REDSHIFT-SPACE POWER SPECTRUM IN THE ZEL'DOVICH APPROXIMATION

Let $v$, $q$, $x$ and $s$ be, respectively, the comoving peculiar velocity, the initial coordinate (i.e. Lagrangian), the present real-space (r-space) coordinate (i.e. Eulerian), and the s-space coordinate of a particle in units of km s$^{-1}$. The Zel'dovich approximation states that the displacement vector, $d$, from $q$ to $x$ is related to the peculiar velocity by $f(\Omega)^{-1}v$; accordingly, the s-space coordinate, $s$, is given by

$$s = x + (v - \mathbf{i})t = q + [1 + f(\Omega)]d(q, t),$$

where $\mathbf{i}$ is the unity matrix and $\mathcal{P} = \mathbf{H}^T$ is a matrix which projects the velocity vector along the line of sight denoted by the unit vector $\mathbf{i}$. In order to simplify the calculations, we work in the 'distant observer' limit where the line of sight is well approximated by a fixed direction. Having specified the mapping (3), we can readily write down a formal equation for the matter density field in s-space as

$$\delta^i(s) = \int \delta^i \delta_{s} [s - q - (1 + f\mathcal{P})d - 1],$$

where the $s$ superscript denotes s-space and $\delta_{s}$ is the Dirac delta function.

By Fourier transforming (4) we find

$$\delta^i = \int \delta^i \delta_{s} [s - q - (1 + f\mathcal{P})d - 1].$$

The second term in brackets reflects the Fourier transform of the mean density and contributes only to the $k = 0$ mode. From (5) we see that, for $k \neq 0$,

$$\langle \delta^i \delta^i \rangle = \int \delta^i \delta_{s} \exp[ik(T + f\mathcal{P})d + ik^T(1 + f\mathcal{P})d'].$$

The great simplification of the Zel'dovich approximation is that the displacement is proportional to the initial peculiar velocity. If we assume that the initial fluctuations were Gaussian, then $\langle \delta^i \delta^i \rangle$ is specified by the correlation function of the initial displacement field, $C(r) = \langle d \mathcal{P} d' \rangle$, where $r = q' - q$. After performing the integral in (8), and comparing with (7), we find

$$P^*(k) = \int d^3r e^{-ikr} \times \exp\{k(T + f\mathcal{P})[C(r) - C(0)](1 + f\mathcal{P})k\}.$$
where \( P_{in}(k) \) is the initial \( r \)-space power spectrum.

To first order in \( C \), equation (9) reduces to

\[
P_{in}(k, \mu) = D(\mu)P_{in}(k)(1 + f(\mu))^2.
\]

This is the well-known result of linear theory (Kaiser 1987); the agreement is not surprising since to lowest order the Zel'dovich approximation is equivalent to linear theory.

So far we have assumed that galaxies trace the matter distribution. In reality, we should allow for some bias between the galaxy and matter distributions. Here we make the hypothesis that, when the fluctuations are small, the galaxy and matter density fluctuations are related by a linear biasing relation. However, with the growth of fluctuations into the non-linear regime at the present time, the bias factor becomes a non-linear function of the density. With this hypothesis, the parameter \( f(\Omega) \) in (9) is simply replaced by \( \beta_{in} = f(\Omega)/b_{lin} \), where \( b_{lin} \) is the initial linear bias factor scaled to the present using linear theory.

The integral in (9) can be numerically evaluated to yield the non-linear \( s \)-space power spectrum for a given initial power spectrum and a value of \( \Omega \). We introduce a sharp small-scale cut-off, \( k_c \), in the initial power spectrum in an attempt to remedy the failure of the Zel’dovich approximation in regions where orbit mixing has occurred. We consider a family of cold dark matter (CDM)-like models specified by the shape parameter \( \Gamma \) as defined by Efstathiou, Bond & White (1992). We quantify the anisotropies in \( P^s \) at each \( k \) by the ratio \( R = Q/M \) where \( Q \) and \( M \) are computed according to (1). Fig. 1 shows \( R \) for a variety of different \( \Gamma \), \( \sigma_8 \), and \( \Omega \) (see caption). All curves except the light solid one were generated using a small-scale cut-off of \( 2\pi k_c = 5 \, h^{-1} \, \text{Mpc} \). For comparison, the light solid curve was derived with a smaller truncation of \( 2\pi k_c = 1 \, h^{-1} \, \text{Mpc} \). We see that, fortunately, \( R \) is only weakly sensitive to the cut-off.

On large scales, \( R \) asymptotically approaches \( R_{lin} \) given in (2), but decreases rapidly with increasing wavenumber. This decline is because linear theory overpredicts the peculiar velocity associated with a given density field (Nusser et al. 1991) and consequently overestimates the amplitude of anisotropies in the power spectrum on scales where non-linear effects are important. Our formalism accounts for multi-valued zones in which the flow in \( r \)-space is laminar even though particle orbits have crossed in \( s \)-space. On scales where this effect is significant, the quadrupole moment reverses sign and \( R \) becomes negative. It is important to distinguish this effect from that of incoherent velocities in virialized regions (Fingers of God). The scale at which \( R \) crosses zero defines a non-linear scale corresponding to a characteristic length \( k_n \). The value of \( k_n \) is determined by the initial power spectrum as well as \( \Omega \). The \( \Omega \) dependence is due to the fact that, for a given \( s \)-space density field, the predicted velocity is proportional to \( f(\Omega) \). Hence the non-linear scale in \( s \)-space is a function of \( \Omega \).

3 UNIVERSAL SCALING RELATION

In the Zel’dovich approximation the peculiar velocity is specified by the \( r \)-space density field, independently of the initial conditions (Nusser et al. 1991). Since the anisotropies in \( s \)-space are a reflection of this density/velocity relation, we expect the ratio \( R \) to be a weak function of the initial power spectrum when expressed in terms of the dimensionless variable \( \tau = k/k_{nl} \). In order to verify this ansatz, we plot, in Fig. 2, the ratio \( R/R_{lin} \) against the variable \( \tau \) for each of the curves in Fig. 1. In addition, we show results from the large \( \Omega = 0.3 \) \( N \)-body simulation of CFW. This figure shows that, indeed, the ratio \( R/R_{lin} \) acquires a universal shape over a large range of scales (over two orders of magnitude). The agreement of the Zel’dovich predictions with the simulation is remarkable. The small differences between the different curves may be attributed to two effects. First, the non-linear density/velocity relation in Fourier space involves coupling between different modes; this coupling may depend on the initial power spectrum. Secondly, the failure of the Zel’dovich approximation in regions where orbit mixing has occurred is especially pronounced by models with large power on small scales; in reality, however, caustics are preserved and therefore one might expect the scaling relation to hold more tightly.

A simple empirical fit to the scaling relation (valid for \( \tau \leq 1 \)), shown by the solid dots in Fig. 2, is

\[
\frac{R}{R_{lin}} = \frac{1 - \tau^2}{1 + \frac{1}{2} \tau^{3/2}}.
\]

This relation permits an estimation of the parameter \( \Omega \) or, more precisely, \( \beta_{lin} \) on scales with \( \tau \leq 1 \) where non-linear effects are important. Here we use the relation (14) to estimate \( \beta_{lin} \) from the 1.2-Jy IRAS survey (Fisher et al. 1995). The ratio \( R \) for this survey versus scale was taken from CFW (fig. 7). The wavenumber at which \( R \) crosses zero was estimated to be \( k_{nl} = 0.22 \, h \, \text{Mpc}^{-1} \). The three solid curves in Fig. 1 are
ratios $R$ (CFW). Since the scaling solution is not exact, we add the scatter in the different curves in Fig. 2 in quadrature with the quoted IRAS error to derive a total error in the ratio, $R$. The $\chi^2$ fit gives $\beta_\text{lin}=0.6 \pm 0.2$.

4 DISCUSSION

We have presented a calculation of the anisotropies in the s-space power spectrum based on the Zel'dovich approximation. The Zel'dovich approximation shows clearly that the decline of the quadrupole-to-monopole ratio on large scales is due to non-linear, yet coherent, peculiar velocities. Despite a strong dependence on $\Omega$ and the initial power spectra, the s-space anisotropies obey a universal scaling relation when expressed as a function of the variable $k/k_n$.

This scaling relation extends analyses of redshift distortions into the non-linear regime with the only free parameter being $\beta_\text{lin}$. This is a marked improvement over phenomenological models for the distortion, which require an unrealistically high random velocity component to explain the decline of $R$ with scale (e.g. Peacock & Dodds 1994; CFW).

We have not incorporated the effects of random velocities. The main manifestation of random velocities is the so-called Finger of God effect, arising from the redshift stretching of virialized regions of rich clusters. The velocity dispersion in these regions can be of the order of $\sim 1000$ km s$^{-1}$, yet rich clusters are rare in the Universe and can, in principle, be excised from the analysis. Field galaxies have a random velocity component in addition to their coherent motions. However, the amplitude of this component is small. The one-dimensional velocity dispersion of all galaxies, including those in clusters, is estimated to be $\sim 140-200$ km s$^{-1}$ (Miller, Davis & White 1995).

Measurements of the redshift distortions in the planned Sloan Digital Sky Survey (Gunn & Knapp 1993) and Anglo-Australian 2dF galaxy survey should provide accurate determinations of $R$ over a wide range of scales. Our formalism is especially suitable for the estimation of $\Omega$ from these surveys.

ACKNOWLEDGMENTS

KBF gratefully acknowledges the financial support of the National Science Foundation. AN acknowledges the support of a PPARC research fellowship. We thank Michael Strauss and Avishai Dekel for helpful comments. We thank Shaun Cole and David Weinberg for providing results from the N-body simulation.

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