We compare strategies for evaporative and sympathetic cooling of two-species Fermi-Bose mixtures in single-color and two-color optical dipole traps. We show that in the latter case a large heat capacity of the bosonic species can be maintained during the entire cooling process. This could allow to efficiently achieve a deep Fermi degeneracy regime having at the same time a significant thermal fraction for the Bose gas, crucial for a precise thermometry of the mixture. Two possible signatures of a superfluid phase transition for the Fermi species are discussed.

Recent studies of ultracold dilute matter are bridging a gap between the idealized descriptions of quantum degenerate Bose and Fermi gases and their actual counterpart in strongly-interacting condensed matter systems like liquid $^4$He and electrons in superconducting materials. While experimental studies of interacting dilute Bose gases in the degenerate regime are ongoing since 1995, Fermi gases have been explored only more recently. Non-interacting, purely quantum-mechanical features of dilute Fermi gases have been observed in the degenerate regime, namely Pauli blocking and Fermi pressure. Phenomena involving their interacting nature are expected when fermions are highly degenerate. Important studies of strongly interacting degenerate Fermi gases have been recently reported for Fermi-Bose mixtures, and for two-component Fermi gases. Moreover, BCS-based models are predicting a superfluid phase based on Cooper pairing already invoked for the understanding of low-temperature superconductivity and superfluidity of $^3$He.

Current efforts to cool fermions seem ultimately limited by intrinsic heating sources in the case of evaporative cooling of two hyperfine states of fermions, and by the decreasing cooling efficiency of bosons in experiments using sympathetic cooling. Recently, we proposed the use of a two-color optical dipole trap to enhance the Fermi degeneracy temperature $T_F$ with respect to the Bose-Einstein critical temperature $T_c$ whenever a bosonic species is used to sympathetically cool a different fermionic species. This can be obtained by engineering different trapping potentials for the two species with proper detuning and intensities of two laser beams, as discussed in with particular regard to the static confinement features. In this Letter we analyze the dynamics of cooling and heating of the Fermi-Bose mixture in two-color optical dipole traps. It turns out that a deep Fermi degeneracy regime can be achieved, allowing at the same time for both precision thermometry and relatively simple signatures of fermion superfluidity.

In order to understand the efficiency limits of sympathetic cooling of a fermion-boson mixture let us consider the heat capacities $C(N, T)$ of the two species at a fixed number $N$ of particles as a function of temperature $T$. In the case of non-interacting gases confined into a harmonic potential $V(x, y, z) = \frac{1}{2}m(\omega _x^2x^2 + \omega _y^2y^2 + \omega _z^2z^2)$, the heat capacities can be evaluated numerically. In Fig. 1 we show the behavior of $C_b(N_b, T)$ and $C_f(N_f, T)$ for a mixture composed by the same number of bosons and fermions trapped in a crossed-beam optical dipole trap. Below the critical, $T_c = \zeta(3)^{-1/3}\hbar \omega _hN_b^{1/3}k_B^{-1}$, and Fermi, $T_F = 6^{1/3}\hbar \omega _fN_f^{1/3}k_B^{-1}$, temperatures, defined in terms of the average angular trap frequencies $\omega _b = (\omega _{xb}\omega _{yb}\omega _{zb})^{1/3}$ and $\omega _f = (\omega _{xf}\omega _{yf}\omega _{zf})^{1/3}$, the boson and fermion heat capacities vanish as $T^3$ and $T$, respectively. If $\omega _f = \omega _b$, the boson heat capacity becomes smaller than the fermion one below $T/T_F \approx 0.3$, strongly affecting the efficiency of sympathetic cooling for smaller $T/T_F$. This explains qualitatively the difficulty in reaching temperatures lower than $T/T_F \approx 0.25$ in the experiments reported in where $^7$Li-$^9$Li mixtures were used, and more in general in magnetically or single-color optically trapped Fermi-Bose mixtures. Were we able to increase the ratio $\omega _f/\omega _b$, the cooling efficiency of a boson-fermion mixture could be extended to much lower temperatures. As an example, in Fig. 1 we show that for $\omega _f/\omega _b = 10$, obtainable with bichromatic optical dipole traps, the heat capacity inversion takes place at $T/T_F \approx 10^{-2}$. The discussion can be made more quantitative by considering the dynamics of the system during forced evaporation and comparing a bichromatic trap to the single-color case.

In optical dipole traps forced evaporative cooling is obtained by continuously decreasing the depth of the confining potential energy via proper control of the laser.
power. The detailed dynamics for a single-color optical dipole trap has been discussed in [10], where scaling laws for all the relevant parameters of evaporative cooling were obtained. A fundamental quantity in forced evaporation is the ratio between the potential energy depth experienced by the trapped atoms and their temperature, \( \Delta U/k_b T \equiv \eta \). It has been shown that thermodynamic equilibrium is assured if \( \eta \) is kept constant, even if \( \Delta U \) and \( T \) are time dependent [17]. This implies that during cooling the temperature of the atomic cloud is always well-defined. The condition of constant \( \eta \) determines the time dependence of the potential energy depth [10]

\[
\frac{\Delta U(t)}{\Delta U_i} = \left(1 + \frac{t}{\tau}\right)^{\varepsilon_U},
\]

where \( \Delta U_i \) is the initial potential depth, \( \varepsilon_U = -2(\eta' - 3)/\eta' \), and \( \tau^{-1} = (2/3)\eta'(\eta - 4)\exp(\varepsilon_U) \) for \( \eta' = \eta + (\eta - 5)/(\eta - 4) \) and \( \gamma_i \) being the initial elastic collision rate. Once the time dependence of \( \Delta U \) is known, all other relevant quantities, e.g. number of particles, temperature, phase space density, and elastic scattering rate, are obtained by scaling laws similar to [10] with possibly different exponents, \( \varepsilon_N, \varepsilon_T, \varepsilon_P, \) and \( \varepsilon_\gamma \). In Table 1 we report the values of these exponents for three different values of \( \eta \) realistically achievable in experimental situations, and the corresponding time constant \( \tau \).

In the case of a mixture of bosonic and fermionic species, Eq. [11] describes the potential energy depth of bosons \( \Delta U_b \) [13]. This quantity, in turns, fixes the laser power \( P \) necessary to create the confining potential well. The potential energy depth of fermions \( \Delta U_f \) is then determined as a function of \( P \). By using the scaling law [11], we have studied evaporative cooling strategies for single-color and two-color optical dipole traps with \(^6\)Li-\(^{23}\)Na mixtures, recently brought to quantum degeneracy for both species in a magnetic trap [11]. Analogous considerations hold for all the combinations of the only two available stable Fermi alkali isotopes, \(^6\)Li and \(^{40}\)K, refrigerated through the largely available Bose coolers, \(^{23}\)Na and \(^{87}\)Rb. In the single-color case only a red-detuned laser beam is present and its power \( P_2 \) is decreased continuously as shown by the dashed line in the upper panel of Fig. 2. The ratio of the fermionic and bosonic average trapping frequencies has the constant value \( \omega_f/\omega_b = 1.96 \) determined by the ratios of the two species and the different detunings of the atomic transition wavelengths with respect to the red-detuned laser wavelength. In the two-color situation a coaxial blue-detuned beam focused on the center of the existing optical dipole trap is turned on at time \( t_0 \) and for \( t \geq t_0 \) its power \( P_2 \) is maintained at a constant ratio with the red-detuned laser power, \( P_2/P_1 = \) constant. By demanding a smooth time-dependence for \( \Delta U_b(t) \) as described by Eq. [11] - see central panel of Fig. 2 - together with the fact that \( \Delta U_b = \Delta U_b(P_1, P_2) \), a discontinuity at \( t = t_0 \) also for the red-detuned laser power is required. For \( t \geq t_0 \) the ratio \( \omega_f/\omega_b \) can be ideally increased to an arbitrary high value by choosing a proper ratio \( P_2/P_1 \). The abrupt increase (decrease) of the fermionic (bosonic) average trapping frequency due to the turning-on of the blue-detuned laser determines a corresponding decrease (increase) of the Fermi (critical) temperature. As shown in the lower panel of Fig. 3, this produces an increase of \( T/T_c \) and a decrease of \( T/T_F \) with respect to their corresponding smooth evolutions in the case of a single-color trap. Therefore, the presence of the blue-detuned laser helps to both maintain the Bose gas in a non-condensed state and allow for a deeper degeneracy condition of the Fermi gas. At the same time, evaporative cooling is less efficient as a consequence of the weakening of the confinement caused by the blue-detuned beam. However, this is not an issue since in the latest stage of evaporation we estimate elastic scattering rates of order \( \Gamma_{el} \approx 10^2 \text{ s}^{-1} \) even taking into account the suppression of scattering induced by Pauli blocking [15]. Also, the blue-detuned beam gives

| \( \eta \) | \( \varepsilon_U \) | \( \varepsilon_T \) | \( \varepsilon_N \) | \( \varepsilon_P \) | \( \varepsilon_\gamma \) | \( (\gamma_i \tau)^{-1} \) |
|---|---|---|---|---|---|---|
| 5 | -0.80 | -0.60 | 0.60 | -1 | 2.2 \times 10^{-2} |
| 10 | -1.45 | -0.28 | 1.89 | -1 | 2.0 \times 10^{-3} |
| 15 | -1.62 | -0.19 | 2.25 | -1 | 3.6 \times 10^{-5} |

FIG. 1: Heat exchange for harmonically trapped Bose-Fermi mixtures. The single particle heat capacity of non-interacting fermions (dot-dashed) and bosons (solid) is shown versus temperature for two different values of the trap frequency ratio \( \omega_f/\omega_b \). Arrows evidence the different \( T/T_F \) values below which the boson heat capacity becomes smaller than the fermion one. We consider a case with \( N_b = N_f = 10^6 \), \( m_b = m_f \), and \( \omega_b = \omega_f = \omega_z/\sqrt{2} \).

**TABLE I**: Evaporative cooling scaling exponents for the potential energy depth \( \varepsilon_U \), the temperature \( \varepsilon_T \), the number of particles \( \varepsilon_N \), the phase-space density \( \varepsilon_P \), and the elastic collision rate \( \varepsilon_\gamma \), for three values of the evaporation parameter \( \eta \). We also report the time constant \( \tau \) in terms of the initial elastic scattering rate \( \gamma_i = N_b m_b \sigma \omega_z^2/(2\pi^2 k_b T) \), where \( N_b \), \( \omega_b \), and \( T \) are the initial values of the number of bosons, their average angular trap frequency and temperature, respectively, and \( \sigma \) the elastic cross-section.
FIG. 2: Evaporative cooling strategies for an optically trapped $^6$Li-$^{23}$Na mixture. Time evolution of red-detuned and blue-detuned laser powers (upper panel), fermion and boson trap depths (central panel) and fermion and boson trap frequencies (lower panel) for a single-color optical dipole trap (dashed lines) and a two-color optical dipole trap (solid lines). The laser powers are fixed by the condition that $\Delta U/b_\omega$.

The laser powers are chosen at $\lambda_1=1064$ nm and $\lambda_2=532$ nm.

the dominant contribution to the heating of the mixture due to Rayleigh scattering of the sodium atoms, but this is largely compensated by the cooling power of the Bose gas even in the latest stage of the evaporation. Other sources of heating, like hole heating or technical laser noise, can be made negligible with respect to the heating induced by Rayleigh scattering. In particular, fluctuations in the laser power ratio $P_2/P_1$ could induce instabilities and parametric heating especially in the interesting regime where the ratio $\omega_i/\omega_b$ is made large. For $\omega_i/\omega_b = 10$, a power ratio stability of 0.1-1% is required, which is within the capability of current laser stabilization techniques.

In the upper panel of Fig. 3 we show the evolution of the ratio $C_b/C_F$ between the heat capacities of the bosonic and fermionic species - a figure of merit of the sympathetic cooling efficiency. The heat capacities have been evaluated numerically taking into account the time evolution of the relevant quantities, in particular the diminishing number of bosons during forced evaporation which strongly affects the time-dependence of $C_b$ (while the effect of many-body interactions, evaluated in $^{[21]}$, gives a much weaker time-dependence). The boson heat capacity in the single-color trap becomes smaller than the fermion one at times $\gamma_t \geq 6200$ when most of the bosons are condensed ($T/T_c \simeq 0.25$, $T/T_F \simeq 0.1$). On the other hand, in the two-color trap $C_b/C_F$ maintains values much larger than unity and it is possible to reach $T/T_F \simeq 0.02$ while $T/T_c \gtrsim 0.3$. The presence of a larger bosonic thermal cloud for the two-color case even at the latest stage of fermion cooling allows for a more precise thermometry. The estimate of the temperature for the Fermi-Bose mixture is indeed obtained by fitting the tail of the normal Bose component superimposed to the condensate fraction. As the temperature is lowered the thermal component shrinks in amplitude and size therefore lowering the accuracy of the measurement. This effect is mitigated in the two-color trap.

Superfluidity of the Fermi gas is expected below the critical temperature for the onset of atomic Cooper pairs $^{[3]} T_{BCS} \simeq 5/3 \exp(-\pi/2k_F|a-1|)T_F$, where $k_F$ is the Fermi wavevector and $a$ the elastic scattering length of fermions. Besides leaving freedom to apply arbitrary homogeneous magnetic fields to enhance the scattering length through tuning to a Feshbach resonance $^{[22, 23, 24]}$, our bichromatic configuration allows also for
an independent increase of $k_F$ due to the higher achievable densities. The resulting $T_{\text{BCS}}/T_F$ are within the explorabile range which corresponds, as seen in the lower panel of Fig. 3, to $T/T_F \approx 2 \cdot 10^{-2}$. The presence of a superfluid state could be evidenced by using the same blue-detuned beam used to deconfine the bosons as a mechanical stirrer for the fermion cloud. Thus, in analogy to already performed experiments on Bose condensates, one could look at a finite threshold for the onset of a highly dissipative regime or of a drag force. The stirring of the Fermi gas occurs in the presence of both a bosonic thermal cloud and a Bose condensed component. These last give rise to heating at all stirring velocities and at a critical velocity lower than for the one expected for the Fermi superfluid, respectively. However, due to their low density in the latest cooling stage, the contributions to the heating induced by the stirring of the Bose components are much smaller than the Rayleigh heating. To discriminate against the bosonic cloud background one could take advantage of the recently proposed manipulations of an ultracold cloud with Raman beams by creating a directional critical velocity for the superfluid Fermi component. An alternative signature for superfluidity consists in looking at the bulge in the density profile predicted below $T_{\text{BCS}}$. Here, again, the presence of a thermal cloud for the bosons makes this background simpler to discriminate against any fermion superfluidity signature due to the well controllable Gaussian-shaped profile of the former, its weaker interactions with the Fermi gas, and the broader Thomas-Fermi profile of the Bose-condensed component caused by the shallower confinement.

In conclusion, our analysis of evaporative and sympathetic cooling in a two-color optical dipole trap shows that a deep Fermi degeneracy regime can be achieved by efficiently exploiting the cooling capability of a Bose gas with large heat capacity. The fact that the cooler need not be in the condensed phase gives also larger flexibility for choosing the Bose species. One could reconsider the use of $^{133}\text{Cs}$ which, due to its large mass and small recoil temperature, can be efficiently cooled to very low temperatures by purely optical means, therefore ensuring robust conditions in terms of temperature and heat capacity to start evaporative cooling. More generally, the presence of a larger thermal component for the Bose gas does not interfere significantly with the Fermi component, rather it allows for a more accurate thermometry and a more controllable background against possible signatures of the fermion superfluid phase.

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