Towards Port-Hamiltonian Approach for Modeling and Control of Two-wheeled Wheelchair

A Aula, R Akmeliawati, S Ahmad, T M Altalmas, and S N Sidek

Intelligent Mechatronics System Research Unit, Dept. of Mechatronics Engineering
International Islamic University Malaysia, Jalan Gombak, 50728 Kuala Lumpur, Malaysia

Email: rakmelia@iium.edu.my

Abstract. This paper introduces the modeling and control design of a two-wheeled wheelchair (TWW) based on structure-preserving port-Hamiltonian concept. In this paper, a model of TWW with features, including space-saving, four to two-wheel transformation, and adjustable seat height is proposed to increased mobility and independence of the user. Then, the mathematical model of a TWW in its balanced mode is derived. The model is based on the total energy in the system. The system is divided into subsystems whereby the interconnections which exist are utilized. The nonlinearity of the model is preserved using port-controlled Hamiltonian (PCH) system and made to advantage. The proposed controlled is designed based on the idea of PCH such that the energy balance in the system can be achieved while stabilizing the system.

1. Introduction

Many researches were conducted to find a better solution for helping the disabled people with different types of disabilities to be independent in their daily activities. Most of these researches on electric powered wheelchair were aimed to increase the comfort, handling and safety of the wheelchair [2-5]. Control algorithms were also embedded to improve its usability. However, existing mechanical designs and control algorithms of electric powered wheelchair do not give the users the full ability to act independently in life.

Though there are few wheelchair models that can adjust the seat to a certain height, only fewer of them that are actually operable in two wheels configuration. This feature increases user’s mobility by allowing them to maneuver in narrow spaces. This advantage was already recognized in JOE [6], a two-wheeled mobile robot, which is regarded as a pioneer in this field. This two-wheeled mobile robot, along with other similar robots [7] triggered the research on self-balancing robot, which is inspired by an inverted pendulum concept. These researches were then improved further for wheelchair application. Hence, the dawn of research on two-wheeled wheelchair.

Nakamura and Murakami [8-10] proposed a wheelchair on two wheels. They implemented the so-called disturbance observer (PADO) to balance and steer the wheelchair. A PADO was developed to estimate the disturbance on pitch angle to achieve a robust stabilization. Later, they added a reaction torque observer (RTOB) to realize a power-assist control on yaw direction. In [11-12], a new mechanism that transforms the wheelchair from four wheels to two wheels configuration was

1 Corresponding author.
proposed. Furthermore, they added an extendable feature to the second link which lifts the user to an eye-level height with a standing person. This feature increases user’s confident while having a conversation.

In the last decade, Port-controlled Hamiltonian (PCH) systems have emerged as an interesting class of nonlinear models suitable for a large number of physical applications. This modeling approach originates from the network modeling of energy-conserving lumped-parameter physical systems with independent storage elements. This kind of models encompasses a very large class of physical systems, containing the class of Euler-Lagrange models. The Hamiltonian function, used in this approach, is a good candidate of Lyapunov functions for many physical systems [13]. The name PCH systems refers to two major components of a control paradigm [14]:

- **Port**: the modeling approach is port-based, which successfully composes complex systems by means of power-preserving interconnections.
- **Hamiltonian**: the mathematical framework extends the Hamiltonian mechanics, which emphasizes energy function Hamiltonian as basic concept for modeling multi-physics system.

Latest applications of PCH systems include various types of control problem, structure-preserved modeling of nonlinear system, and stabilization technique, e.g. [15-19].

In this paper, an electric-powered TWW is modeled using PCH systems. The proposed TWW has a transformable wheel configuration, which turns the four-wheeled wheelchair into a two-wheeled wheelchair. This two-wheeled configuration has similar dynamics to a double-inverted pendulum on two wheels, which is a nonlinear, multi-variable, higher order, and unstable system. The PCH systems approach preserves the nonlinear structure of the system, hence preserving also its nonlinear dynamics [20].

This paper is organized as follows. Section 2 summarizes the concept of Port-controlled Hamiltonian Systems. After developing the simplified model of TWW using MapleSim, the mathematical model is derived in section 3. Section 4 proposes the steps to design a controller based on PCH systems.

2. **Port-controlled Hamiltonian Systems**

2.1. **The Concept of Energy**

A dynamical system can be viewed as a set of simpler subsystems that exchange energy, (from theory of passivity, which is compatible with ‘interconnection theory’ [21]). Energy can serve as a lingua franca to facilitate communication among scientists and engineers from different fields. Most engineering applications are mixtures of electrical, mechanical and other domains. In PCH systems-based modeling; an energy-based perspective and the role of interconnection between subsystems provide the basis in modeling physical systems.

Port-controlled Hamiltonian system—sometimes referred also as port-Hamiltonian systems or generalized port-controlled Hamiltonian systems—is first introduced by A.J. van der Schaft [20]. This technique extends the theory of Hamiltonian mechanics. In this technique, the nonlinear dynamics of a system is modeled as the Hamiltonian equations of motion, which are derived from the total energy of the system $H$, also known as the Hamiltonian.

$$H(q, p) = \frac{1}{2} p^T M^{-1}(q) p + P(q) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + P(q)$$  

(2.1)

where $q$ is the generalized coordinates, $p$ is the generalized momenta, $p = M(q) \dot{q}$, $M$ is the $k \times k$ inertia (or generalized mass) matrix which is symmetric and positive definite for all $q$.

In PCH systems, any engineering domain (electrical, mechanical or hydraulic, with exception to thermodynamical) is introduced two variables, called power conjugate variables, whose product equals power. These variables are labeled as effort $e$ and flow $f$. For any given system (from any
domain), there exists a storage and dissipative element. Within each element, a power port, with \( P = e \cdot f \), is then defined. The subdomain classification in PCH framework is based on the Generalized Bond Graph (GBG) framework introduced in [22-23].

When modeling a physical system using this approach, it is viewed as an interconnection between energy storage elements, dissipative (resistive) elements, and the environment. The structure of any storage element is the following:

\[
\dot{z}(t) = f_z(t), \quad e_z(t) = \frac{dE}{dz}(z(t)),
\]

where \( f_z(t) \) is the flow variable of storage element, \( e_z(t) \) is the effort variable of storage element, \( z \) is the generalized state variable and \( E(z) \) is the stored energy. Figure 1 illustrates this relation.

![Figure 1](image)

**Figure 1.** The relationship between effort and flow variables in a storage element.

The change in energy of a storage element is always its external power flow, i.e.

\[
\dot{E}(z) = \frac{dE}{dz}(z(t))\dot{z} = e_z f_z.
\]

Thus, by construction, a product of effort and flow implies the integral of power with regards to time yields energy.

For a dissipative element, the energy dissipation \( D(x) \) is expressed by direct relation between effort and flow variables. In the linear case,

\[
e_R = Rf_R \Rightarrow P_{\text{diss}} = e_R f_R = Rf_R^2 \geq 0, \quad R \geq 0
\]

where \( f_R(t) \) and \( e_R(t) \) is the flow and effort variables of dissipative (resistive) element, respectively. In general, the dissipative power \( P_{\text{diss}} \geq 0 \), otherwise the element would generate energy instead of dissipating it. For the general nonlinear case, the relation can be expressed in the flow-controlled form (equation 2.5) or effort-controlled form (equation 2.6)

\[
e_R = \hat{f}_R(e_R),
\]

\[
f_R = \hat{e}_R(f_R).
\]

The power and energy balance of PCH system with storage and dissipative element are, respectively:

\[
e_S^T f_S + e_R^T f_R = 0,
\]

\[
\frac{d}{dt}H = e_S^T f_S + e_R^T f_R.
\]

The Hamiltonian equations of motion satisfies the general energy balance equation,
from which the controller will be designed. The designed controller will ensure the stability (with preserving structure of the Hamiltonian system and conservation of energy) of the system in the Lyapunov sense [23].

2.2. PCH-based Modeling

Consider the following nonlinear system in the form

\[
\dot{z} = f(z) + g(z)u, \quad y = h(z).
\]  

(2.10)

where \( f(z) \), \( g(z) \), and \( h(z) \) are nonlinear functions of the state \( z \), \( u \) is the input of the system and \( y \) is the output. Equation (2.10) can be rewritten as a general Port-controlled Hamiltonian systems form:

\[
\dot{z} = \mathbf{J}(z) \frac{\partial H(z)}{\partial z} + g(z)u, \quad y = g^T(z) \frac{\partial H(z)}{\partial z},
\]  

(2.11)  

(2.12)

where \( z = [q \ p]^T \) and \( \mathbf{J}(z) \frac{\partial H(z)}{\partial z} \) is the internal Hamiltonian dynamics which can be derived based on physical law(s). Equation (2.11) can be rewritten as

\[
\dot{q} = \frac{\partial H}{\partial p}(q, p) \quad (2.13)
\]

\[
\dot{p} = -\frac{\partial H}{\partial q}(q, p) \quad (2.14)
\]

where the Hamiltonian \( H \) is the total (kinetic and potential) energy as described by equation (2.1). \( \mathbf{J}(z) \) is an \( n \times n \) skew-symmetric matrix.

For a system with dissipation, equation (2.11) and (2.12) are rewritten as:

\[
\dot{z} = [\mathbf{J}(z) - \mathbf{R}(z)] \frac{\partial H(z)}{\partial z} + g(z)u, \quad y = g^T(z) \frac{\partial H(z)}{\partial z}.
\]  

(2.15)  

(2.16)

where \( \mathbf{R}(z) \) is a positive semi-definite symmetric matrix, which represents the dissipation term corresponds to the internal energy dissipation in the system.

Matrix \( \mathbf{J}(z) \) corresponds to the internal power-conserving interconnection structure of the physical systems due to:

1. Basic conservation laws, such as Kirchhoff’s laws;
2. Powerless constraints; kinematic constraints;
3. Transformers, gyrators, exchange between different types of energy.
   In many examples, the structure matrix \( J(z) \) will additionally satisfy an integrability condition.

   The main message of this approach is that Port-controlled Hamiltonian system is closer to physical modeling, and capturing more information than just energy-balance of passivity. Three major features of PCH systems are [14]: its scalability to very large interconnected multi-physics systems, its ability for incorporating nonlinearities while retaining underlying conservation laws, and its integration of the treatment of both finite-dimensional and infinite-dimensional components.

3. Modeling of Two-wheeled Wheelchair

3.1. 3D Model

Before deriving the mathematical model, a 3D frame-based model of the wheelchair is designed in MapleSim. This step helps in analyzing the interconnection relationship between elements for PCH systems derivation. From [24], the TWW system involves the following mechanism:

1. Transformation from four wheels to two wheels involves lifting the casters (front wheels) using certain mechanism;
2. The lower part of the system, which includes casters, main wheels, motor and motor housing, caster lifting mechanism; is assumed as first link;
3. The second part of the system, located atop the first one has a chair with extendable height; is assumed as second link;
4. While at two wheels, i.e. balancing mode, the system resembles a double-inverted pendulum on two wheels;

Figure 2 shows the 3D model of a TWW in balance mode, i.e. on two wheels. Its similar dynamics with a double-inverted pendulum is shown in figure 3. The definition of each symbol is described in table 1.

This model is built with these assumptions:
1. The system is divided into three subsystems: base, first link, and second link;
2. The model is constrained to move in \( x \)-direction only, i.e. the base only move straight forward and backward without turning and the two links rotate in \( x \)-direction only;
3. On the base, there are two identical motors to rotate the two main wheels;
4. On the first link, only the mass of the link itself is considered;
5. On the second link, there are two masses considered, the link’s mass itself and the load;
6. The extendable second link is assumed to be fully stretched;
7. The connections between each element in the system are assumed to be rigid and ideal, i.e. no spring-damper characteristics;

### Table 1. Nomenclature.

| Symbol | Definition |
|--------|------------|
| $m_0$  | Mass of the base, including the two wheels |
| $m_1$  | Mass of the first link |
| $m_2$  | Total mass of the second link, including the load |
| $L_1$  | Length of first link |
| $L_2$  | Total stretched length of second link |
| $\theta_1, \theta_2$ | Angular displacement of first and second link, respectively |
| $J_0, J_1, J_2$ | Moment of inertia of base, first and second link, respectively |
| $g$    | Gravity constant |

#### 3.2. Mathematical Model

Consider the TWW is in balancing mode, i.e. in two wheels as shown in figure 3. The system has two inputs, one torque $F$ applied to the base consisting of synchronous torques from identical left and right motors, and one torque $\tau$ from the motor between first and second link, which drives the second link. Table 2 completes the notation described in table 1.

### Table 2. Notation used in deriving the mathematical model.

| Symbol | Definition |
|--------|------------|
| $r$    | Radius of main wheel |
| $x$    | Linear displacement on x-axis |
| $F$    | Force acting on the base of the system |
| $\tau$ | Torque acting between first and second link |
| $i$    | Subsystem; $0 =$ base, $1 =$ first link, $2 =$ second link |
| $p_i$  | Linear momentum of $i$th subsystem |
| $K_i$  | Kinetic energy of $i$th subsystem |
| $U_i$  | Potential energy of $i$th subsystem |

As addressed in subsection 3.1, the model is divided into three subsystems, namely base, first link and second link.

On the base, the kinetic energy is considered from translational dynamic of the two wheels, which is the following:
\[ K_0 = 2 \left( \frac{1}{2} m_0 \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}_0^2 \right), \] (3.1)

\[ \theta_0 \] is the rotation angle of each wheel and \( J_0 \) is the inertia of the wheel. Since \( \dot{\theta} = \frac{\dot{x}}{r} \),

\[ K_0 = \left( m_0 + \frac{J_0}{r^2} \right) \dot{x}^2. \] (3.2)

Let \( a_0 = \left( m_0 + \frac{J_0}{r^2} \right) \). The potential energy is zero, i.e. \( U_0 = 0 \). The storage energy on the base, \( H_0 \), is the sum of the kinetic and potential energy \( K_0 + U_0 \). Rewriting \( H_0 \) in terms of momentum \( p_0 \),

\[ H_0(x, p_0) = \frac{1}{4 a_0} \frac{p_0^2}{a_0}. \] (3.3)

On the second subsystem, i.e. the first link, the kinetic energy is given as:

\[ K_1 = \frac{1}{2} m_1 L_1^2 \dot{\theta}_1^2. \] (3.4)

The potential energy acting on the first link due to gravitational acceleration is:

\[ U_1 = -m_1 g L_1 \cos \theta_1. \] (3.5)

Let \( a_1 = (m_1 g L_1) \) and \( J_1 = m_1 L_1^2 \) are constant. The total energy stored in the first link is

\[ H_1(\theta_1, p_1) = \frac{1}{2} \frac{p_1^2}{J_1} - a_1 \cos \theta_1. \] (3.6)

On the third subsystem, i.e. the second link, the kinetic energy is given as:

\[ K_2 = \frac{1}{2} m_2 L_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_2 L_2^2 \dot{\theta}_2^2 + m_2 L_2 L_2 \cos (\theta_1 - \theta_2). \] (3.7)

The potential energy by gravitational force is given by

\[ U_2 = -m_2 g \left( L_2 \cos \theta_1 + L_2 \cos \theta_2 \right). \] (3.8)

Let \( a_{21} = (m_2 L_2) \), \( a_{22} = (m_2 L_4 L_2) \), \( a_{23} = (m_2 g L_1) \), \( a_{24} = (m_2 g L_2) \), and \( J_2 = m_2 L_2^2 \). The total energy stored in the second link is:

\[ H_2(\theta_1, \theta_2, p_1, p_2) = \frac{1}{2} \frac{p_1^2}{a_{21}} + \frac{1}{2} \frac{p_2^2}{a_{22}} - a_{22} \cos (\theta_1 - \theta_2) - a_{23} \cos \theta_1 + a_{24} \cos \theta_2. \] (3.9)

The total energy of the system is the sum of Hamiltonian of base, first link, and second link, i.e.
\[ H_{\text{Total}}(q, p) = \frac{1}{2} p^T M^{-1}(q)p + U(q), \]  
(3.10)

where \( q \), \( p \), and \( M \) are the vector of generalized coordinate, vector of generalized momenta, and matrix of inertia, respectively. For our system, 
\[ q = [x, \theta_1, \theta_2]^T, \]  
(3.11)
\[ p = [p_0, p_1, p_2]^T. \]  
(3.12)

Recall the general Hamilton’s equations of motion from equation (2.13) and equation (2.14):
\[ \frac{dq}{dt} = \frac{\partial H}{\partial p}, \]  
(3.13)
\[ \frac{dp}{dt} = -\frac{\partial H}{\partial q}. \]  
(3.14)

Substituting equation (3.11) and (3.12) into equation (3.13) and (3.14) yields
\[ \frac{dx}{dt} = \frac{\partial H_{\text{Total}}}{\partial p_0} = \frac{1}{2} p_0, \]  
(3.15)
\[ \frac{d\theta_1}{dt} = \frac{\partial H_{\text{Total}}}{\partial p_1} = \left( \frac{1}{J_1} + \frac{1}{a_{21}} \right) p_1, \]  
(3.16)
\[ \frac{d\theta_2}{dt} = \frac{\partial H_{\text{Total}}}{\partial p_2} = \frac{p_2}{J_2}, \]  
(3.17)
\[ \frac{dp_0}{dt} = -\frac{\partial H_{\text{Total}}}{\partial \theta_1} = -\left( a_1 \sin \theta_1 + a_{21} \sin(\theta_1 - \theta_2) + a_{22} \sin \theta_1 \right), \]  
(3.19)
\[ \frac{dp_2}{dt} = -\frac{\partial H_{\text{Total}}}{\partial \theta_2} = -\left( -a_{22} \sin(\theta_1 - \theta_2) - a_{24} \sin \theta_2 \right). \]  
(3.20)

Equation (3.15) implies that the momentum on base, i.e. \( p_0 = 2a_0 \dot{x} \), is comprised of two momenta from two main wheels. Equation (3.16) implies that the angular momentum of first link, i.e. \( p_1 = \dot{\theta}_1 \left( \frac{J_1 a_{21}}{J_1 + a_{21}} \right) \), is influenced by the mass of second link. On the second link, however, the angular momentum \( p_2 \) is affected only by its own inertia \( \dot{\theta}_2 J_2 \), as shown by equation (3.17). Equation (3.18) confirms equation (3.2) that the potential energy is zero. Equation (3.19) and equation (3.20) suggest that the two links are affected by downward potential energy due to gravitational acceleration.

From d’Alembert’s principles, the interconnections are obtained as follows:
\[ F = m_0 \ddot{x}, \]  
(3.21)
\[ \tau = J_2 \ddot{\theta}_2 + \left( a_{22} \sin(\theta_1 - \theta_2) + a_{24} \sin \theta_2 \right) \]  
(3.22)

Therefore, the complete PCH-based model of a two-wheeled system is as follows.
where $J = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$ and $g = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$.

### 4. Controller Design

In this section a proposition for controller design using Port-controlled Hamiltonian systems is introduced.

It is well-known that the standard feedback interconnection of two PCH systems is again a PCH system [20]; which can be exploited for stability analysis and control purposes.

The PCH systems use the system’s interconnection structure and its Hamiltonian (i.e. its total energy) as the primary means for modeling and control. Theoretically [14], the Hamiltonian of the system assumes its minimum at its stable equilibrium state. So, by introducing dissipation (or damping) in the controller, the energy in the system decreases until the minimum of energy or, equivalently, the desired equilibrium configuration is reached, i.e. the power balance of the (closed-loop) system is then reduced to

$$\frac{d}{dt}H_{cl} = \overline{\sigma}_R^T \overline{f}_R \leq 0$$  \hspace{1cm} (4.1)

where $\overline{\sigma}_R^T \overline{f}_R$ refers to the product of dissipative effort and flow of a dissipative element, which is the dissipated power. By standard Lyapunov theory, equation (4.1) implies that the Hamiltonian can be used as Lyapunov function candidate, which is already known in classical Hamiltonian mechanics. However, in some cases, Hamiltonian $H$ cannot be used as a Lyapunov function, i.e. there is no minimum in $H$. To solve this issue, a Lyapunov function of combination of the total energy $H$ and another conserved quantity is proposed. In PCH systems, this methodology is called the Energy-Casimir method [25].

In general, for any PCH systems with dissipation (see equation (2.11)) or without dissipation (see equation 2.15), there exist a conserved quantity $C$, called Casimir function, as being the solutions of

$$\frac{\partial^T C}{\partial z}(z) J(z) = 0$$  \hspace{1cm} (4.2)

The time-derivative of $C$ along the PCH system (see equation (4.3)) shows that it remains constant along the trajectory of the PCH system, regardless of the form of the PCH system, thus proving that it is invariant of the Hamiltonian $H$.

$$\frac{dC(z)}{dt} = \frac{\partial^T C}{\partial z}(z) \dot{z}$$  \hspace{1cm} (4.3)

Substituting equation (2.11) to equation (4.3) yields:
\[
\frac{\text{d}C(z)}{\text{d}t} = \frac{\partial^T C(z)\partial H(z)}{\partial z} + \frac{\partial^T C(z)}{\partial z}g(z)u
\]

(4.4)

According to equation (4.2), for autonomous system or if \(\frac{\partial^T C(z)}{\partial z}g(z)u = 0\), the function \(C(z)\) is constant. For a system with dissipation (equation (2.15)), the Casimir functions are obtained from both \(J(z)\) and \(R(z)\), i.e.:
\[
\frac{\partial^T C(z)}{\partial z}J(z) = 0,
\]
\[
\frac{\partial^T C(z)}{\partial z}R(z) = 0.
\]

(4.5)

The existence of Casimir function \(C(z)\) also hinted that it can be used for stability analysis of the PCH system by using it as a Lyapunov functions [25-27].

Recall a nonlinear PCH plant system with dissipation from equation (2.15) and (2.16):
\[
\dot{z} = [J(z) - R(z)]\frac{\partial H(z)}{\partial z} + g(z)u,
\]
\[
y = g^T(z)\frac{\partial H(z)}{\partial z}.
\]

(4.7)
(4.8)

The proposed controller is of the PCH form. Thus, both the controller and the plant are treated as subsystems with the interconnection as shown in figure 4.

**Figure 4.** A standard feedback interconnection used by most plant with controller system

with \(e\) and \(e_c\) are external error signals.

**Proposition 4.1.** Consider the proposed PCH controller system as follows, with the interconnection as shown in figure 4.
\[
\dot{\xi} = [J_c(\xi) - R_c(\xi)]\frac{\partial H_c(\xi)}{\partial \xi} + g_c(\xi)u_c,
\]
\[
y_c = g_c^T(\xi)\frac{\partial H_c(\xi)}{\partial \xi}
\]

(4.5)
(4.6)
Remark 4.1: The interconnection is defined as follows:

\[ u = -y_c + e, \]
\[ u_c = y + e_c. \]  \hfill (4.7)

The closed-loop system is represented by the following

\[
\begin{bmatrix}
\dot{x} \\
\dot{\xi}
\end{bmatrix} = \left[ J_{CL}(x, \xi) - R_{CL}(x, \xi) \right] \begin{bmatrix}
\frac{\partial H_{CL}(x)}{\partial x} \\
\frac{\partial H_{CL}(\xi)}{\partial \xi}
\end{bmatrix} + \begin{bmatrix}
g(x) & 0 \\
0 & g_c(\xi)
\end{bmatrix} \begin{bmatrix}
e \\
e_c
\end{bmatrix},
\tag{4.8}
\]

\[
\begin{bmatrix}
y \\
y_c
\end{bmatrix} = \begin{bmatrix}
g(x) & 0 \\
0 & g_c(\xi)
\end{bmatrix} \begin{bmatrix}
\frac{\partial H_{CL}(x)}{\partial x} \\
\frac{\partial H_{CL}(\xi)}{\partial \xi}
\end{bmatrix},
\tag{4.9}
\]

which is again a Port-controlled Hamiltonian System.

In the control system, it is the input \( u \) that we want to know, as it is the output from the controller, as illustrated by figure 4. To determine \( H_c \), the following steps is suggested:

1. Define state variables of the system, i.e. \( x \);
2. Using the property \( \frac{\partial^T C}{\partial z}(z)J(z) = 0 \), obtain Casimir function \( C \);
3. Define Lyapunov function \( V \) as

\[
V = H_p + H_c + C,
\tag{4.10}
\]

with \( H_p, H_c \) are the Hamiltonian of the plant and the controller, respectively.
4. \( H_c \) is determined such that \( \dot{V} \leq 0 \), to guarantee stability;
5. From [25], taking the time-derivative of \( H_c \), we can design \( u \).

\[
\frac{dH_c}{dt} = -u^T y. \tag{4.11}
\]

5. Conclusion

In this paper, a model of two-wheeled wheelchair based on Port-controlled Hamiltonian systems was presented. The nonlinearity of the model is preserved using PCH system. Furthermore, a PCH-based controller was also proposed. Due to this, the closed-loop system was also a PCH system, hence preserving the nonlinear structure.

In the future, the derived model and the proposed controller will be implemented into hardware experiment.

Acknowledgments

This work is supported by The Ministry of Science, Technology, and Innovation (MOSTI) Malaysia under project grant SF12-012-0041.

References

[1] Karmarkar A M, Dicianno B E, Cooper R, Collins D M, Matthews J T, Koontz A, Teodorski E
[2] Ding D and Cooper R A 2005 Electric powered wheelchairs *IEEE Cont. Syst.* **25**(2) pp 22-34

[3] Zhan J 2009 A pressure cushion for control of intelligent wheelchair movements *IEEE 10th Int. Conf. on Computer-Aided Industrial Design and Conceptual Design* (26-29 November 2009) pp 155-157

[4] Kuramatsu T and Murakami T 2010 Force sensorless power-assist control of yaw motion direction for two wheels driven wheelchair *World Automation Congress (Kobe, Japan, 19-23 September, 2010)* pp 1-6

[5] Fan J 2011 Motion control of intelligent wheelchair based on sitting postures *Int. Conf. on Mechatronics and Automation (Beijing, 7-10 August 2011)* pp 301-306

[6] Grasser F, D’Arrigo A, Colombi S and Rufer A C 2002 JOE: a mobile, inverted pendulum *IEEE Trans. Industr. Electr.* **49**(1) pp 107-114

[7] Lee H J, Kim H W and Jung S 2010 Development of a mobile inverted pendulum robot system as a personal transportation vehicle with two driving modes: TransBot *World Automation Congress (Kobe, Japan, 19-23 September 2010)* pp 1-5

[8] Nakamura A and Murakami T 2008 Trajectory Tracking Control of a Two Wheels Driven Wheelchair *Asia Int. Symp. on Mechatronics (Japan, 2008)* pp 255-259

[9] Nakamura A and Murakami T 2009 A stabilization control of two wheels driven wheelchair *IEEE/RSJ Int. Conf. on Intelligent Robots and System (St. Louis, USA, 11-15 October 2009)* p 4863

[10] Kawamura T and Murakami T 2011 Vibration suppression for uprizing control of two-wheel driven wheelchair *37th Ann. Conf. on IEEE Industrial Electronics Society (Melbourne, 7-10 November 2011)* p 3323

[11] Goher K, Ahmad S and Tokhi O M 2010 A new configuration of two wheeled vehicles: Towards a more workspace and motion flexibility *4th Ann. IEEE System Conference (San Diego, California, 5-8 April 2010)* pp 524-528

[12] Ahmad S and Tokhi O M 2008 Modelling and Control of a Wheelchair on Two Wheels *2nd Asia Int. Conf. on Modeling & Simulation (Kuala Lumpur, 13-15 May 2008)* pp 579-584

[13] Wei A, Wang Y and Hu X 2012 Adaptive robust parallel simultaneous stabilization of two uncertain port-controlled Hamiltonian systems subject to input saturation *Chinese Control Conf. (Hefei, 25-27 July 2012)* pp 727-732

[14] Duindam V, Macchelli A, Stramigioli S, and Bruyninckx H 2009 Modeling and Control of Complex Physical Systems: The Port-Hamiltonian Approach (New York: Springer-Verlag)

[15] Ravi B and Dey B 2010 Stabilizing a Flexible Beam on a Cart: A Distributed Port-Hamiltonian Approach *J. Nonlinear Sci.* **20** pp 131-151

[16] Peza-Solis J F,Silva-Navarro G and Castro-Linares R 2010 Control of a rigid-flexible two-link robot using Passivity-based and Strain-feedback approaches *7th Int. Conf. on Electrical Engineering, Computing Science and Automatic Control (Mexico City, 8-10 September 2010)* pp 476-481

[17] Yang B, Li H, Kang Z and Jiang H 2012 Hamiltonian-based binocular visual servoing of camera-in-hand robotic systems *Proc. Int. Conf. on Modeling, Identification & Control (Wuhan, China 24-26 June 2012)* pp 388 - 393

[18] Dirkz D A and Scherpen J M A 2012 Adaptive control of port-Hamiltonian systems *Proc. 19th Int. Symp. on Mathematical Theory of Networks and Systems (Budapest 5-7 July 2010)* pp 1503-1508

[19] Dirkz D A and Scherpen J M A 2012 Structure Preserving Adaptive Control of Port-Hamiltonian Systems *IEEE Trans. Auto. Cont.* **57** pp 2880-2885

[20] van der Schaft A J 2000 Port-controlled Hamiltonian Systems: towards a theory for control and design of nonlinear physical systems *J. Society of Instrument and Control Engineers of Japan* **39** pp 91-98
[21] Ortega R, Loria A, Nicklasson P J and Sira-Ramirez H 1998 Passivity-based Control of Euler-Lagrange Systems (London: Springer-Verlag)

[22] Breedveld P C 1982 Thermodynamic Bond Graphs and the Problem of Thermal Inertance J. Franklin Inst. 314(1) pp 15-40

[23] van der Schaft A J 1986 Stabilization of Hamiltonian systems Nonlinear Anal. Theory Methods Appl. 10(10)1021-35

[24] Aula A, Akmeliawati R, Ahmad S, Altalmas T and Sidek S N 2013 Integrated Design, Modeling and Analysis of Two-wheeled Wheelchair for Disabled. Proc. 16th Int. Conf. on Climbing and Walking Robots (Sidney, Australia) pp 141-152

[25] van der Schaft A J 2004 Port-Hamiltonian systems: network modeling and control of nonlinear physical systems Advanced dynamics and control of structures and machines ed H Irschik and K Schlacher (New York: Springer)

[26] Ortega R, van der Schaft A J, Mareels I and Maschke B 2001 Putting Energy Back in Control IEEE Cont. Sys. 21(2) pp 18-33

[27] OrtegaR, van der Schaft A J, Maschke B and Escobar G 2002 Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems Automatica 38 pp 585-596