Two-dimensional Magnetic Field Dependence of Current-Voltage Characteristics of Rectangular, Hexagonal and Triangle Shape Superconducting Junctions and DC-SQUIDs

Akiyoshi NAKAYAMA1,2, Norimichi WATANABE1,2, Susumu ABE1, Yohei NISHI1, and Yoichi OKABE3

1 Department of Engineering, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama, 221-8686, Japan
2 High-tech Research Centre, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama, 221-8686, Japan
3 The University of the Air, 2-11 Wakaba, Mihama-ku, Chiba-shi, 261-8586, Japan

E-mail: nakayama@ee.kanagawa-u.ac.jp

Abstract. Modulation characteristics of Josephson current through rectangular, hexagonal and triangle shape superconducting junctions and dc-SQUIDs by two-dimensional scanning of external magnetic field have been studied. Two pairs of Helmholtz coils were controlled by two current sources with GPIB (General Purpose Interface Bus) system to obtain the magnetic field dependence automatically. In the case of rectangular shape junction, the dependence of Josephson current on $H_x$-$H_y$ plane has become the product of the two Fraunhofer patterns in the direction $H_x$, $H_y$ parallel to each edge of this rectangular shape. The periods of these modulations were inversely proportional of each length of the rectangular shape perpendicular to the magnetic field. In the case of hexagonal shape junction, we have found hexagonal symmetry in the dependence of Josephson current on the $H_x$-$H_y$ plane. Six ridges of the modulation surface were clearly observed in the directions of six edges of the hexagonal shape (0, 60, 120, 180, 240, 300 degrees). In triangle shape junctions, six ridges of the modulation surface were also clearly observed in the directions of three edges of the triangle shape. In the case of dc-SQUIDs, the modulation pattern was the product of the sensitive modulation of the period of the magnetic flux quantization of the hole of the vertical dc-SQUID and the insensitive modulation pattern of each square junction itself.

1. Introduction
Josephson effect is important both in analog and digital application of superconductive electronics and in fundamental physics. In Josephson tunnel junctions, a gauge-invariant-phase-difference across the tunnel barrier can be modulated by an external magnetic field [1, 2]. At each point of the junction, superconducting current density is a sine function of this gauge-invariant-phase-difference. The total current that can flow without voltage drop is the sum of the superconducting current over the junction area. The modulation of the total Josephson current $I_c$ of the superconducting junctions has usually been observed by one-dimensional scanning of the applied magnetic field [1, 3]. However, by this one-dimensional scanning method, only course information of the current distribution would be...
obtained. Recently, the external magnetic field has been scanned in two dimensions and the two-dimensional surfaces of the \( I_{c}(H_x, H_y) \) dependence has been measured [2,4-6].

The two-dimensional surfaces of the \( I_{c}(H_x, H_y) \) dependence change according to the shape of the junction area. In the previous Eucas report [6], the modulation of Josephson currents through a triangle shape junction, has first be obtained. Along the pass across the barrier of the junction, the gauge-invariant-phase-difference between the two superconducting electrodes can be considered from the sum of the phase difference and the line integral of vector potential of electro-magnetic field. This gauge-invariant-phase-difference is modulated in the perpendicular direction to the external magnetic field inside the barrier region, where the modulation wavelength is inversely proportional to the magnitude of the applied magnetic field. From this modulation pattern of the Josephson current by the magnetic field, uniformity of the tunnel barrier can be confirmed [2]. Specially shaped junctions defined by quartic polynomial [3,7] and normal-distribution-function [8] have been fabricated for X-ray spectroscopy.

In this paper, the external magnetic field dependences of the Josephson current \( I_{c}(H_x, H_y) \) of various shape junctions are fabricated. In sec. 2, measurement method for the \( I_{c}(H_x, H_y) \) dependence is described. The measured \( I_{c}(H_x, H_y) \) dependence is presented in sec. 3. A cylinder pulley model for explaining the measured result is presented also in sec. 3. Conclusions are presented in sec. 4.

2. Experiment

For junction fabrication, niobium thin films are deposited by magnetron sputtering in a vacuum system with a load-lock chamber. Tunnel barriers are formed by a natural oxidation of Al layer in pure oxygen in an oxidation chamber. Two-dimensional magnetic field dependence of the Josephson current has been measured by using two pairs of Helmholtz coils. Two current sources for driving these Helmholtz coils have been controlled by GPIB system in order to obtain magnetic field dependences automatically [5].

3. Results and Discussion

We have fabricated different shaped junctions such as rectangular, hexagonal and triangle shape superconducting junctions and also have fabricated dc-SQUIDs.

3.1. Rectangular Junctions

We have fabricated rectangular junctions and measured the modulation of the Josephson current. Numerical simulation of magnetic field dependence of the Josephson current through superconducting junction is also obtained. No magnetic field by the junction current across the barrier is considered in this calculation. A distribution of the gauge-invariant-phase-difference (hereafter, phase difference) between the two electrodes has to obey following conditions. First, at each point in the junction area, superconducting current is a sin function of the phase difference. Secondly, the phase difference should change perpendicular direction to the external magnetic field as shown in Fig. 1, where the wavelength of this modulation is inversely proportional to the absolute value of this magnetic field. Finally, in the phase difference there is an arbitral phase factor. In Fig. 1(a), the simulated modulation pattern of Josephson current by two-dimensional scanning of the applied magnetic field is shown. In the case of square junctions, dependence of Josephson current obtained upon the external field \( H_x, H_y \) parallel to aluminium-oxide layer, have become the product of the two Fraunhofer patterns in the direction \( H_x, H_y \) parallel to each edge of this square junction area [5,6]. In a rectangular junction of this study, the dependence of Josephson current upon the external field \( H_x, H_y \) have also become the product of the two Fraunhofer patterns, however the modulation period is inversely proportional to the length of the rectangular shape, because the catching area of the magnetic flux is proportional to the edge length perpendicular to the flux. Figures 1(b)-(f) show the current distribution of the maximum Josephson current at each point of the \( (H_x, H_y) \) plane shown in Fig. (a), where an arbitral phase factor has to be chosen to make the total current as maximum value. In Figs. (b) and (d) the phase is \( \pi/2 \) at
the centre point of the junction as maximum current flows. However, in Figs. (c), (e) and (d), the phase is \(-\pi/2\) at the centre point of the junction. Simulated pattern shows good agreement to the experimental data as shown in Figs. 1 and 2. Around the main peak in the center of the \((H_x, H_y)\) plane of the \(I_c-H\) \((H_x, H_y)\) dependence, sub peaks have also been observed in four directions at 0, 90, 180 and 270 degree.

Fig. 1. (a) \(I_c-H\) \((H_x, H_y)\) dependence of rectangular junction (simulation) (b-f) current distribution inside the junction area of the maximum current \(I_c\) at each point on \((H_x-H_y)\) plane

Fig. 2. \(I_c-H\) \((H_x, H_y)\) dependence of rectangular junction (measured)

3.2. Hexagonal Junctions

We have also made hexagonal shape junctions. In the dependence of Josephson current \(I_c\) upon the magnetic field \(H\) \((H_x, H_y)\), hexagonal symmetry has been found as shown in Fig. 3. Six ridges of the modulation surface were clearly observed in the directions of six edges of the hexagonal shape symmetry (0, 60, 120, 180, 240, 300 degrees).
3.3. Triangle Junctions
In triangle shape junctions, six ridges of the modulation surface were also clearly observed in the directions of three edges of the triangle shape as shown in Fig. 4. In order to explain these $I_c$-$H$ ($H_x$, $H_y$) dependences intuitively, a cylinder pulley model can be used. Figure 5 shows this cylinder model explaining the $I_c$-$H$ ($H_x$, $H_y$) dependence of triangle junctions. In this figure, the weight of the triangle shape can be stretched on the side surface of a cylinder pulley, in which the angle to the horizontal plane should equal to the phase difference at each point of the junction. Magnification of stretching equals the absolute value of the magnetic field. Total weight is kept constant in this stretching. The maximum current $I_c$ equals the maximum force sustaining a cylinder pulley in static balance. Figure 5(a) shows the case the external magnetic field is applied perpendicular to the edge. In this case (a) just at the time the triangle weight surrounds a cylinder pulley even times, the perfect balance can be obtained and the maximum current $I_c$ equals 0. Figure 5(b) shows the case the external magnetic field is applied parallel to the edge. In this case (b), perfect balance cannot be obtained even if we change the absolute value of the magnetic field. So, six ridges appear in the directions of the hexagonal shape symmetry (0, 60, 120, 180, 240, 300 degrees) in the $I_c$-$H$ ($H_x$, $H_y$) dependence.
3.4. DC-SQUIDs

We have also fabricated vertical type dc-SQUIDs and measured the $I_c-H(H_x, H_y)$ dependence as shown in Fig. 6. The magnetic field component $H_x$ penetrates the hole of the SQUID. In the $I_c-H(H_x, H_y)$ dependence there are two modulation patterns: the sensitive modulation period (80A/m) of the magnetic flux quantization of the hole of the vertical dc-SQUID and the insensitive modulation pattern $(\sin X/X)$ of each square junction itself.

4. Conclusions

We have fabricated various shaped Nb/AlO$_x$/Nb junctions by using magnetron sputtering. Two-dimensional dependences of the Josephson current through rectangular, hexagonal and triangle shape...
Nb/AlOx/Nb junctions and dc-SQUIDs upon the external magnetic field $\mathbf{H} = (H_x, H_y)$ have been studied using two pairs of the Helmholtz coils.

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