Numerical Solutions of Hybrid Nanofluids Flow Via Free Convection Over a Solid Sphere

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ABSTRACT

The purpose of the existing study is to examine how heat transfer enables consolidated by variations in the basic advantages of fluids in the existence of free convection with the assistance of suspended hybrid nanofluids. Iron-graphene oxide suspended in water as a hybrid nanofluid flow on a solid sphere is also considered in this work. The partial differential equations are gotten, for this problem, by transforming the mathematical governing equations using similarity equations (stream function). These partial differential equations are solved numerically by Keller-Box method and programmed by MATLAB program. The acquired numerical results are in excellent agreement with the preceding literature results. Graphical results of the influence of the hybrid nanofluid parameters on some physical quantities regarded to examine the behavior of hybrid nanofluid flow were attained, and they proved that hybrid nanofluid flow represents a more essential role in the operation of heat transfer than a regular nanofluid flow.

1. Introduction

At present time, the nano technology is continued to heat transfer enhancement, and it stays a matter of senior attention in the studies and sciences. When nanofluids expression was introduced by Choi and Eastman [1], considerable many searches have been widely achieved related to the characteristics of nanofluids on heat transfer and fluid flow. After that, two important kinds of simulations models permanently applied to discuss the demeanors of nanofluid, such as single-phase model and two-phase model, which are conducted by Tiwari and Das [2] and Buongiorno [3], respectively. Swalmeh et al., [4-8] and Alwawi et al., [9-12] used the one phase model to study the convection boundary layer flow in nanofluid over solid sphere and horizontal circular cylinder. And many papers investigated the two phase model like Sheikholeslami et al., [13], Garoosi et al., [14], Rea et al., [15]. Furthermore, many researchers reported distinct articles concerning the in boundary layer flow in a nanofluid. Noor et al., [16] studied the problem of convection heat transfer on stagnation point in micropolar nanofluid. The convection boundary-layer flow over a horizontal

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circular cylinder with a porous medium in presence of nanofluid were investigated by Rashad et al., [17]. Hussanan et al., [18] looked in convection heat transfer in micropolar nanofluids with oxide nanoparticles. Also, Hussain et al., [19] studied the analysis of micropolar nanofluid flow past a stretching surface [20-24].

A few years ago, experimental study of nanofluids called “hybrid nanofluids” are considered by Suresh et al., [25], which is suggested to present better heat transfer advantages and rheological conduct along with improved thermo-physical features. Hybrid nanofluid is an stretching of nanofluid which collected of two various nanoparticles suspended in the base fluid. Hybrid nanofluids are vastly utilized in numerous fields of enhancement heat transfer, such as electronic cooling, acoustics, coolant in machining, supercomputers, transportation, military, pharmaceutical, biomedical, nuclear safety. Devi and Devi [26] numerically studied the problem of hydro-magnetic hybrid Cu-Al₂O₃/water nanofluid flow. On stagnation-point flow of an aqueous TiO₂-Cu hybrid nanofluid on a wavy cylinder was considered by Yousefi et al., [27]. Hayat et al., [28] studied the rotating hybrid flow of Ag-CuO/H₂O nanofluid under radiation and partial slip boundary effects. Rehman et al., [29] conducted the three-dimensional in existence of micropolar hybrid nanofluid flow past an exponentially stretched surface. And in the survey articles, such as Hussien et al., [30], Tlili et al., [31], Murray [32], Hussain et al., [33], Babu et al., [34], Ahmadi et al., [35], Ali et al., [36].

Depending on the aforementioned above publications for this special hybrid nanofluid flow, the efforts were gone to inspect heat transfer elaboration in free convection flow of iron-graphene oxide suspended in water as a hybrid nanofluid over a solid sphere, with two boundary conditions, namely constant wall temperature (CWT) and constant heat flux (CHF). In the engineering field, the amount of enhancement of composition of nanofluid as hybrid nanofluid brings a lot of wide prospects especially in the development of modern industries. Besides, this study problem can be extended to another studies, like influences of magneto-hydrodynamics, micropolar, or Casson fluid, in presence of convection boundary layer flow with hybrid nanofluid. Hence, the numerical results for physical quantities can be gained for these influences parameters. Moreover, this research is an expansion and stretching of some previous research, check Manjunatha et al., [37], Waini et al., [38], Nadeem et al., [39], and Hamarsheh et al., [40].

2. Mathematical Formulation

The problem of steady laminar free convection boundary layer flow in presence of an incompressible hybrid nanofluid, on a solid sphere, is investigated. The x-axis measured in the circumference of the solid sphere surface motion from the lower stagnation point \((x \approx 0)\), and the y-axis is measured perpendicular to it. Also, the constant wall temperature (CWT) \((T_w)\) and constant heat flux (CHF) \((q_w)\) boundary conditions, are studied in this problem, as shown in Figure 1. \(T_w\) is the wall temperature, \(q_w\) is the heat flux constant, \(T_\infty\) the ambient temperature of the fluid which does not change, \(g\) is the gravity vector which affects in the opposite direction.

![Fig. 1. Schematic physical model](image-url)
Subject to the above suppositions, the continuity, momentum, and thermal equations, of the boundary layer flow of the hybrid nanofluids, on a solid sphere, are obtained by Salleh et al., [48], Manjunatha et al., [37]

\[
\frac{\partial(\bar{\rho} \bar{u})}{\partial x} + \frac{\partial(\bar{\rho} \bar{v})}{\partial y} = 0,
\]

\[
\rho_{hf} \left( \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = \mu_{hf} \left( \frac{\partial^2 \bar{u}}{\partial x^2} \right) - (\beta)_{hf} g (T - T_\infty) \sin \frac{x}{a},
\]

\[
\bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} = \alpha_{hf} \left( \frac{\partial^2 \bar{T}}{\partial y^2} \right),
\]

subject the two boundary conditions defined by Swalmeh et al., [6] and Nazar et al., [41] as

\[
\bar{u} = 0, \bar{v} = 0, T = T_w \text{ (CWT)} \text{ or } \frac{\partial T}{\partial y} = \left( \frac{- \bar{q}_w}{k_f} \right) \text{ (CHF)}, \text{ as } \bar{y} = 0,
\]

\[
\bar{u} \to 0, T \to T_\infty, \text{ as } \bar{y} \to \infty.
\]

Here, \((\bar{u}, \bar{v})\)-velocity components along \((x, y)\) coordinate respectively, \(\bar{r}(\bar{x})\) is the radial distance from the symmetrical axis to the surface of the sphere, \(a\) is constant. \(\rho_{hf}, \mu_{hf}, g, (\beta)_{hf}, (\alpha)_{hf} = \frac{k_{hf}}{(\rho c_p)_{hf}}\) are the density of hybrid nanofluid, viscosity of hybrid nanofluid, gravity acceleration, coefficient of thermal expansion of hybrid nanofluid, and Thermal diffusivity coefficient of the hybrid nanofluid, respectively. \(T, T_w\) are the temperature of fluid and ambient temperature, \(k_f\) and \(k_{hf}\) are the thermal conductivity of based fluid and hybrid nanofluid, \(\bar{q}_w\) is constant heat flux. The thermo-physical properties for used nanoparticles and based fluid are presented in Table 1. Also, the hybrid nanofluid properties are displayed by Table 2.

**Table 1**

| Physical properties | Water | GO | Fe |
|---------------------|-------|----|----|
| \(k \text{ (W/mK)}\) | 0.613 | 5000 | 9.7 |
| \(\rho \text{ (kg/m}^3\) | 997.1 | 1800 | 5180 |
| \(\rho c_p \text{ (J/kgK)}\) | 4179 | 717 | 670 |
| Pr | 6.2 | ... | ... |

**Table 2**

Thermo-physical model [37]

| Properties of nanofluid | Properties of hybrid nanofluid |
|--------------------------|--------------------------------|
| 1. \((\beta)_{nf} = (\gamma_2 (\beta) + (1 - \gamma_2)(\beta)_{sf})\) | \((\beta)_{hf} = (1 - \gamma_2)[(1 - \gamma_1) (\beta)_{f} + \gamma_1 (\beta)_{sf}] + (\gamma_2 (\alpha)_{sf})\) |
| 2. \((\mu)_{nf} = \frac{k_{sf}}{(1 - \gamma_2)^{2.5}}\) | \((\mu)_{hf} = \frac{k_{hf}}{(1 - \gamma_2)^{2.5}}\) |
| 3. \((\rho c_p)_{nf} = (\gamma_2 (\rho c_p)_{sf} + (1 - \gamma_2)(\rho c_p)_{sf})\) | \((\rho c_p)_{hf} = (1 - \gamma_2)[(1 - \gamma_1)(\rho c_p)_{f} + \gamma_1 (\rho c_p)_{sf}] + \gamma_2 (\rho c_p)_{sf}\) |
| 4. \((\alpha)_{nf} = \frac{k_{hf}}{(\rho c_p)_{hf}}\) | \((\alpha)_{hf} = \frac{k_{hf}}{((\rho c_p)_{hf})}\) |
| 5. \(\frac{k_{hf}}{k_f} = \frac{(k_s + 2 k_f)}{(k_s + 2 k_f) + \gamma_2 (k_f - k_s)}\) | \(\frac{k_{hf}}{k_f} = \frac{k_{hf}}{(k_s + 2 k_f)} - 2 \gamma_2 (k_f - k_s) + \gamma_1 (k_f - k_s)\) |
| \(\frac{k_{hf}}{k_f} = \frac{k_{hf}}{(k_s + 2 k_f) + \gamma_2 (k_f - k_s)}\) | \(\frac{k_{hf}}{k_f} = \frac{k_{hf}}{(k_s + 2 k_f)} - 2 \gamma_1 (k_f - k_s)\) |
| \(\frac{k_{hf}}{k_f} = \frac{(k_{s1} + 2 k_f)}{(k_{s1} + 2 k_f) + \gamma_1 (k_f - k_{s1})}\) | \(\frac{k_{hf}}{k_f} = \frac{(k_{s1} + 2 k_f)}{(k_{s1} + 2 k_f) + \gamma_1 (k_f - k_{s1})}\) |
where \( \gamma_2 \) is the volume fraction for the Fe, \( \gamma_1 \) is the volume fraction for GO. \((\gamma_2 = \gamma_1 = 0)\) are represent to a regular Newtonian fluid. Besides, to simplify the above problem equations, we get the following non-dimensional variables, as follows [9,10]

\[
\begin{align*}
    r &= \left( \frac{x}{a} \right), \quad x = \left( \frac{x}{a} \right),
    \\
y &= (Gr)^{(1/4)} \left( \frac{\nu}{a} \right), \quad \theta = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \quad u = (Gr)^{(-1/2)} \left( \frac{\alpha \nu}{v_f} \right), \quad v = (Gr)^{(-1/4)} \left( \frac{\alpha \nu}{v_f} \right) \text{(CWT)},
    \\
    y &= (Gr)^{(1/5)} \left( \frac{\nu}{a} \right), \quad \theta = (Gr)^{(1/5)} \left( \frac{T - T_{\infty}}{\alpha \nu q_f} \right), \quad u = (Gr)^{(-2/5)} \left( \frac{\alpha \nu}{v_f} \right), \quad v = (Gr)^{(-1/5)} \left( \frac{\alpha \nu}{v_f} \right) \text{(CHF)}.
\end{align*}
\]

\(Gr = g(\beta_f)(T_{w} - T_{\infty}) \alpha^3 / \nu^2 (CWT), Gr = g(\beta)(\frac{a \alpha q_f}{k_f}) \alpha^3 / \nu_f^2 (\text{CHF}),\)

such that \(Gr\) is the Grashof number. \(\nu_f\) is Kinematic viscosity of the fluid. The above non-dimensional variables along (5) have utilized in Eq. (1) to Eq. (4) to obtain the next non-dimensional equations, subject to two boundary conditions, namely constant wall temperature and constant heat flux.

\[
\begin{align*}
    u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= 0, \quad \text{(6)}
    \\
    u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{\rho_f}{\rho_{hf}} \frac{\mu_f}{(1-\gamma_1)^{2/5}(1-\gamma_2)^{2/5}} \left( \frac{\partial^2 u}{\partial y^2} \right) + \frac{1}{\rho_{hf}} \left( 1 - \gamma_2 \right) \left( 1 - \gamma_1 \right) \rho_f + \gamma_1 \frac{\rho_{s1} \beta_{s1}}{\beta_f} + \gamma_2 \frac{\rho_{s2} \beta_{s2}}{\beta_f} \theta \sin x, \quad \text{(7)}
    \\
    u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} &= \frac{1}{Pr} \left( \frac{k_{hf} / k_f}{(1-\gamma_2)(1-\gamma_1) \rho_f + \gamma_1 \frac{\rho_{c1} \rho_{p1}}{\rho_f} + \gamma_2 \frac{\rho_{c2} \rho_{p2}}{\rho_f}} \right) \left( \frac{\partial^2 \theta}{\partial y^2} \right), \quad \text{(8)}
\end{align*}
\]

where, \(Pr = \frac{\nu_f}{\alpha_f}\) is the Prandtl number.

The boundary conditions (4) become

\[
\begin{align*}
    u &= 0, \quad v = 0, \quad \theta = 1 \text{ (CWT) or } \frac{\partial \theta}{\partial y} = -1 \text{ (CHF), as } y = 0, \quad \text{(9)}
    \\
    u &\rightarrow 0, \quad \theta \rightarrow 0, \quad \text{as } y \rightarrow \infty.
\end{align*}
\]

To solve the Eq. (6) to Eq. (8), subject to the boundary conditions (9), we assume the following variables [9]

\[
\psi = xr(x) f(x, y), \quad \theta = \theta(x, y) \quad \text{(10)}
\]

such that \(\psi\) is the stream function defined as

\[
u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad \text{and } v = \frac{1}{r} \frac{\partial \psi}{\partial x} \quad \text{(11)}
\]
which satisfies the continuity Eq. (6), then the Eq. (6) to Eq. (8) become

\[
\frac{\rho_f}{\rho_{hnf}} \left( \frac{\mu_f}{(1 - \gamma_1)^2 \gamma_1 - \gamma_2} \right) \frac{\partial \theta}{\partial x} = \frac{1}{\rho_{nf}} \left[ (1 - \gamma_1) \rho_f + \gamma_1 \frac{\rho_{sw}}{\beta_f} \right] + \frac{1}{\rho_{nf}} \left[ (1 - \gamma_2) \rho_f + \gamma_2 \frac{\rho_{sw}}{\beta_f} \right] \theta \sin \theta \frac{x}{x} = x \left( \frac{\partial f}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial \theta}{\partial y} \right),
\]

subject to the boundary conditions

\[
f = \frac{\partial f}{\partial y} = 0, \theta = 1 \ (\text{CWT}) \text{ or } \frac{\partial \theta}{\partial y} = -1 \ (\text{CHF}), \text{ as } y = 0,
\]

\[
\frac{\partial f}{\partial y} \to 0, \theta \to 0, \text{ as } y \to \infty.
\]

It can be seen that at the lower stagnation point of the sphere, \( x \approx 0 \), the above equations reduce to the following ordinary differential equations:

\[
\frac{\rho_f}{\rho_{hnf}} \left( \frac{\mu_f}{(1 - \gamma_1)^2 \gamma_1 - \gamma_2} \right) \frac{\partial \theta}{\partial x} = \frac{1}{\rho_{nf}} \left[ (1 - \gamma_1) \rho_f + \gamma_1 \frac{\rho_{sw}}{\beta_f} \right] + \frac{1}{\rho_{nf}} \left[ (1 - \gamma_2) \rho_f + \gamma_2 \frac{\rho_{sw}}{\beta_f} \right] \theta \sin \theta \frac{x}{x} = 0,
\]

The boundary conditions become

\[
f(0) = f'(0) = 0, \theta = 1 \ (\text{CWT}) \text{ or } \theta' = -1 \ (\text{CHF}), \text{ as } y = 0,
\]

\[
f' \to 0, \theta \to 0, \text{ as } y \to \infty,
\]

where primes denote differentiation with respect to \( y \).

The physical quantities of interest in this problem are the local skin friction coefficient \( C_f \), the Nusselt number \( Nu \) and local wall temperature \( \theta_w \), and they can be written as

\[
(C_f = \frac{Gr^{-3/4}a^2}{\mu_f v_f} \tau_w, Nu = \frac{a}{k_f(T_f - T_w)} q_w) \ (\text{CWT}), (\theta_w = \theta(x, 0), C_f = \frac{Gr^{-0.5}a^2}{\mu_f v_f} \tau_w) \ (\text{CHF}),
\]

where \( \tau_w \) is Surface shear stress, and defined as

\[
\tau_w = \mu_{hnf} \left( \frac{\partial u}{\partial y} \right)_y, q_w = -k_{hnf} \left( \frac{\partial T}{\partial y} \right)_y \bigg|_{\tilde{y}=0},
\]

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By using the non-dimensional variables (5) and boundary conditions (9) the local skin friction coefficient \( C_f \) and Nusselt number \( Nu \) become

\[
C_f = Gr^{-1/4} \left( \frac{\mu_f}{(1 - \gamma_1)^{2.5}(1 - \gamma_2)^{2.5}} \right) x \frac{\partial^2 f}{\partial y^2} (x, 0),
\]

\[
Gr^{-1/4} Nu = - Gr^{-1/4} \frac{k_f}{\kappa_{nf}} \left( \frac{\partial \theta}{\partial y} \right) (x, 0), \quad (CWT) \tag{20}
\]

3. Results and Discussions

Numerical solutions of the nonlinear partial differential equations, Eq. (12) and Eq. (11), under the two boundary conditions, such that (CWT) and (CHF), Eq. (14), are solved employing the Keller-box method available in the Matlab software. For a long time, this method well-known solver that has been vastly utilized by many researchers to solve the convection boundary layer flow problems, see Keller [43], Nazar [44], Tham et al. [45], and Hamarsheh, et al. [40]. Thermo-physical properties of used based fluids and nanoparticles in this study are shown in Table 1. The numerical solutions are gained using an initial estimation provided at an initial profile, at the lower stagnation point of the sphere \( x = 0 \), and follow up around the sphere up to \( x = 100^\circ \). In the special case, viscous Newtonian fluid, we get a suitable step size \((x,y)\), and then comparing theses solutions with formerly published numerical results reported by Alwawi et al., [10] and Cheng [46]. We checked that the present results are in good agreement accuracy, as displayed in Table 3 and Table 4.

| \( x^\circ \) | Cheng [46] | Alwawi et al., [10] | Present |
|---|---|---|---|
| 0° | 0.4576 | 0.4576 | 0.4576 |
| 10° | 0.4565 | 0.4565 | 0.4565 |
| 20° | 0.4534 | 0.4534 | 0.4533 |
| 30° | 0.4481 | 0.4480 | 0.4480 |
| 40° | 0.4407 | 0.4406 | 0.4405 |
| 50° | 0.4310 | 0.4310 | 0.4311 |
| 60° | 0.4191 | 0.4194 | 0.4192 |
| 70° | 0.4049 | 0.4053 | 0.4050 |
| 80° | 0.3881 | 0.3886 | 0.3882 |
| 90° | 0.3686 | 0.3693 | 0.3685 |

| \( x^\circ \) | Huang and Chen [47] | Alwawi et al., [9] | Present |
|---|---|---|---|
| 0° | 0.0000 | 0.0000 | 0.0000 |
| 10° | 0.2138 | 0.2123 | 0.2133 |
| 20° | 0.4247 | 0.4157 | 0.4250 |
| 30° | 0.6299 | 0.6252 | 0.6288 |
| 40° | 0.8265 | 0.8201 | 0.8259 |
| 50° | 1.0118 | 1.0033 | 1.0105 |
| 60° | 1.1828 | 1.1676 | 1.1811 |
| 70° | 1.3376 | 1.3198 | 1.3366 |
| 80° | 1.4708 | 1.4519 | 1.4713 |
| 90° | 1.5818 | 1.5609 | 1.5809 |
In this section, understanding the impact of hybrid nanofluid parameters $\gamma_1$ and $\gamma_2$ on physical quantities for the problem of free convection boundary layer flow in a hybrid nanofluid is discussed. In this research, the nanoparticle of graphene oxide is added to the base fluid with nanoparticle volume fraction $\gamma_1$ equal to 0.1. Subsequently, Fe is added with a different value of nanoparticle volume fraction $\gamma_2$ to compose the hybrid nanofluid namely Fe-GO/water. The hybrid nanofluid parameter $\gamma_2$ values are studied from 0.007 to 0.06. Variations of the skin friction coefficient, local wall temperature, and the local Nusselt number, as well as the velocity and temperature profiles, are offered in plotted form through Figure 2 to Figure 8, with two boundary conditions (CWT) and (CHF). The influences of $Nu$ and $\theta_w$, are demonstrated in Figure 2 and Figure 3. It illustrates that an increase in $\gamma_2$ increases $Nu$ (CWT) and $\theta_w$ (CHF), with an increase in the values in $x$. On the other hand, it is observed that there is a drop in the fluid moment in Nusselt number $Nu$ along with the momentum angle $x$-direction. But the opposite case happens, there is a raise in the fluid moment in local wall temperature along with the momentum the angle $x$-direction. It is cleared that in the subsistence of a charismatic field, the change in the heat transfer $Nu$ and $\theta_w$ of hybrid nanofluid (Fe-GO/H$_2$O) is higher than that of nanofluid (Fe/H$_2$O). Therefore, we understand that the expected heat transfer rate can be got by a convenient complex of nanoparticle magnitude.

![Fig. 2. The influence of $\gamma_2$ on $Nu$](image)

![Fig. 3. The influence of $\gamma_2$ on $\theta_w$](image)

The characteristics of $C_f$ are studied in Figure 4 and Figure 5. These figures signalize that with the increasing values of $\gamma_2$, the flow in local skin friction quantity is faster for both the nanofluid and the combination of nanofluids. Besides, the local skin friction values for hybrid nanofluid (Fe-GO/H$_2$O) are greater than nanofluid (Fe/H$_2$O), with two (CWT) and (CHF) boundary conditions. Also, It indicates that when the nanoparticle volume fraction values $\gamma_2$ are raising, the $C_f$ quantity values are reducing, in presence of the (Fe-GO/H$_2$O) and (Fe/H$_2$O).

Figure 6 to Figure 9 depicted the behaviours of $\gamma_1$ and $\gamma_2$ on temperature and velocity profiles. It presents that an increase in $\gamma_2$ increases the temperature but the velocity decreases, with an associated thickness of the boundary layer increases. Physically, the boost in temperature and decrease in velocity between the surface and the ambient hybrid nanofluid is due to the increase in the concentration of nanoparticle density. It deduces that we get the optimizations in convection currents. In addition, GO-Fe/H$_2$O has high temperature and velocity profiles compared to Fe/Water with an increase in the values of $\gamma_2$. 
Fig. 4. The influence of $\gamma_2$ on $C_f$

Fig. 5. The influence of $\gamma_2$ on $C_f$

Fig. 6. The influence of $\gamma_2$ on $\theta(0, y)$

Fig. 7. The influence of $\gamma_2$ on $\theta(0, y)$

Fig. 8. The influence of $\gamma_2$ on $(\partial f / \partial y)(0, y)$

Fig. 9. The influence of $\gamma_2$ on $(\partial f / \partial y)(0, y)$
4. Conclusions

Research on the convection heat transfer boundary layer flow in presence of hybrid nanofluid are considered significant value in engineering sciences. To address the affair, this article presents mathematical model and numerical results for the heat transfer impacts for free convection in hybrid nanofluid flow over a solid sphere. The appropriate similarity transformation is utilized to convert the governing equations into partial differential equations. These partial differential equations are numerically solved by Keller box method and programmed with MATLAB program. The obtained numerical results for the effects of hybrid nanofluid parameters on the engineering interesting physical quantities are investigated in an attempt to discuss them through several figures and tables. Also, (CWT) and (CHF) boundary conditions have been considered in this investigation. The following significant observations are concluded of this study, as follows

i. When the nanoparticle volume fraction parameter $\gamma_2$ increases, the values of local Nusselt number, local wall temperature, and temperature profile are increased, of both regular nanofluid and hybrid nanofluid flows

ii. The local skin friction coefficient and the velocity profile are increased by an increment of nanoparticle volume fraction $\gamma_2$ of both regular nanofluid and hybrid nanofluids flows.

iii. Fe-GO/water hybrid nanofluid has a higher temperature and velocity profile compared with Fe/water nanofluid with an increase in the nanofluid parameter $\gamma_2$.

iv. Also, Fe/ Water nanofluid has lower local skin friction, local wall temperature, and local Nusselt number than Fe-GO/ Water hybrid nanofluid, with (CWT) and (CHF) boundary conditions.

v. Hybrid nanofluid flow represents a more essential character in the operation of heat transfer than a regular nanofluid flow.

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