CP Asymmetries in $B_d \to D^{*+}D^{*-}$ and $B_s \to D_s^{*+}D_s^{*-}$ Decays: 
$P$-wave Dilution, Penguin and Rescattering Effects

Xuan-Yem Pham
Laboratoire de Physique Théorique et Hautes Energies, Universités Pierre et Marie Curie (Paris 6) et Denis Diderot (Paris 7), Unité associée au CNRS, UMR 7589, France

Zhi-zhong Xing
Sektion Physik, Universität München, Theresienstrasse 37A, D-80333 München, Germany

Abstract

Determination of the $CP$-violating parameters $\sin 2\beta$ and $\sin 2\beta'$ is shown to be possible from $B_d^0 \to B_d^{*0} \to D^{*+}D^{*-}$ and $B_s^0 \to B_s^{*0} \to D_s^{*+}D_s^{*-}$ decays without doing the angular analysis. The $P$-wave dilution factors of these two asymmetries are found to be 0.89 and 0.90, respectively, using the factorization approximation and heavy quark symmetry. The penguin-induced corrections amount to about 2% and 3% in the corresponding $B_d$ and $B_s$ channels. Final-state rescattering effects could be handled by detecting the neutral modes $B_d^{0}$ and $B_d^{*0} \to D^{*0}D^{*0}$.

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1 Postal address: LPTHE tour 16/1er étage, Université Pierre et Marie Curie, BP 126, 4 Place Jussieu, F-75252 Paris CEDEX 05, France; Electronic address: pham@lpthe.jussieu.fr
2 Electronic address: xing@hep.physik.uni-muenchen.de
The study of B-meson decays appears to offer a unique opportunity to measure the quark mixing parameters, to investigate the nonperturbative confinement forces, and in particular to probe the origin of CP violation. Recently the CDF Collaboration \cite{1} has reported an updated direct measurement of the CP-violating parameter $\sin 2\beta$ in $B_d^0 \rightarrow J/\psi K_S$ decays, where

$$\beta = \arg \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{cd}} \right)$$

is known as an inner angle of the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle. The preliminary result $\sin 2\beta = 0.79_{-0.44}^{+0.41}$ (stat + syst) is consistent with the standard-model prediction. If the CDF measurement is confirmed, CP violation of the magnitude $\sin 2\beta$ should also be seen in $B_d^0 \rightarrow D^+ D^-$, $D^{*+} D^-$, $D^{*+} D^-$, and $D^{*+} D^-$ decays \cite{2, 3}, whose branching ratios are all expected to be of order $10^{-4}$. Indeed the decay channel $B_d^0 \rightarrow D^{*+} D^-$ has just been observed by the CLEO Collaboration \cite{4}. The measured branching ratio $B(D^{*+} D^-) = [6.2_{-2.9}^{+4.0} \text{(stat) } \pm 1.0 \text{ (syst)}] \times 10^{-4}$ is in agreement with the standard-model expectation. Further measurements of such decay modes will soon be available in the first-round experiments of KEK and SLAC B-meson factories as well as at other high-luminosity hadron machines.

A comparison between the value of $\sin 2\beta$ to be determined from $B_d \rightarrow D^{*+} D^-$ and that already measured in $B_d \rightarrow J/\psi K_S$ is no doubt important, as it may cross-check the consistency of the standard-model predictions. Towards this goal, a special attention has to be paid to possible uncertainties associated with the CP asymmetry in $B_d \rightarrow D^{*+} D^-$. One kind of uncertainty comes from the penguin contamination, as the weak phase of the penguin amplitude is quite different from that of the tree-level amplitude. In contrast, the CP asymmetry in the “gold-plated” modes $B_d^0 \rightarrow \bar{B_d}^0 \rightarrow J/\psi K_S$ is essentially free from the penguin-induced uncertainty, since the relevant tree-level and penguin amplitudes almost have the same CKM phases. Another kind of uncertainty arises from the $P$-wave dilution, because the final state $D^{*+} D^-$ is composed of both the CP-even ($S$- and $D$-wave) and the CP-odd ($P$-wave) configurations. Of course an analysis of the angular distributions of $B_d^0$ vs $\bar{B_d}^0 \rightarrow D^{*+} D^-$ transitions allows us to distinguish between the CP-even and CP-odd contributions \cite{5}. In this work we point out that the direct measurement of $\beta$ can be made in $B_d \rightarrow D^{*+} D^-$ decays without doing the angular analysis. Taking the P-wave dilution and the penguin contamination into account, one may write the characteristic measurable of indirect CP violation in $B_d \rightarrow D^{*+} D^-$ as follows \cite{5}:

$$\Delta_d = \Im \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{cd}} \cdot \frac{(D^{*+} D^-|H_{\text{eff}}|B_d^0)}{(D^{*+} D^-|H_{\text{eff}}|B_d^0)} \right)$$

$$= \zeta_d (1 - \xi_d) \sin 2\beta,$$

where $\zeta_d$ and $\xi_d$ represent the $P$-wave dilution factor and the penguin-induced correction, respectively. We shall calculate both effects with the help of the effective weak Hamiltonian

\footnote{The CP violation induced solely by $B_d^0 \rightarrow \bar{B_d}^0$ mixing, like $\epsilon_k$ in the $K^0 - \bar{K}^0$ mixing system, is expected to be negligibly small (of order $10^{-3}$ or smaller \cite{5}) in the standard model.}
and the factorization approximation. Possible final-state rescattering effects in \( B_d \to D^{*+}D^{*-} \) decays will also be discussed.

A similar analysis will be made for \( CP \) violation in the decay modes \( B^0_s \) vs \( \bar{B}^0_s \to D^{*+}D^{*-} \). The preliminary signals of these transitions have been observed \([7]\). They are useful to extract the \( CP \)-violating phase

\[
\beta' = \arg \left( -\frac{V^*_{tb}V_{ts}}{V^*_{cb}V_{cs}} \right),
\]

whose magnitude is negligibly small (of order \( 1^\circ \) or smaller \([8]\) in the standard model. The associated \( CP \) asymmetry \( \Delta_s \), analogous to \( \Delta_d \) defined in Eq. (2), reads as

\[
\Delta_s = \text{Im} \left( \frac{V^*_{tb}V_{ts}}{V^*_{cb}V_{cs}} \cdot \frac{\langle D^+sD^*- | H_{\text{eff}} | B^0_s \rangle}{\langle D^+sD^*- | H_{\text{eff}} | B^0_s \rangle} \right) = \zeta_s (1 - \xi_s) \sin 2\beta',
\]

where \( \zeta_s \) and \( \xi_s \) denote the \( P \)-wave dilution factor and the penguin-induced correction, respectively. Under \( SU(3) \) invariance \( \zeta_s = \xi_d \) holds. We shall see later on that \( \zeta_d \approx \zeta_s \approx 0.9 \), while the magnitudes of \( \xi_d \) and \( \xi_s \) are only at the percent level.

2 Without loss of generality the amplitude of \( B^0_d \) or \( \bar{B}^0_d \) decay into \( D^{*+}D^{*-} \) can be written as a sum of three terms, i.e., the \( S \)-, \( D \)-, and \( P \)-wave components \([9]\):

\[
\langle D^+D^-|H_{\text{eff}}|B^0_d\rangle = a (\epsilon_+ \cdot \epsilon_-) + \frac{b}{m_{D^*}^2} (p_0 \cdot \epsilon_+) (p_0 \cdot \epsilon_-) + \frac{i}{m_{D^*}^2} c (\epsilon^a\epsilon^\gamma\delta \epsilon^+a\epsilon^-\beta p_+\gamma p_0\delta),
\]

\[
\langle D^+D^-|H_{\text{eff}}|\bar{B}^0_d\rangle = \bar{a} (\epsilon_+ \cdot \epsilon_-) + \frac{\bar{b}}{m_{D^*}^2} (p_0 \cdot \epsilon_+) (p_0 \cdot \epsilon_-) - \frac{i}{m_{D^*}^2} \bar{c} (\epsilon^a\epsilon^\gamma\delta \epsilon^+a\epsilon^-\beta p_+\gamma p_0\delta),
\]

where \( \epsilon_\pm \) denotes the polarization of \( D^{*\pm} \); \( p_0 \) and \( p_\pm \) stand respectively for the momenta of \( B_d \) and \( D^{*\pm} \) mesons; \( a, b, c \) and \( \bar{a}, \bar{b}, \bar{c} \) are complex scalars. To calculate these scalars we neglect effects from the annihilation-type quark diagrams which are anticipated to have significant form-factor suppression. In this case the effective weak Hamiltonian responsible for \( B_d \to D^{*+}D^{*-} \) decays can simply be written as \([10]\) \(^4\)

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ (V_{cb}V^*_{cd}) \sum_{i=1}^2 (c_i Q_i^c) - (V_{tb}V^*_{td}) \sum_{i=3}^{10} (c_i Q_i) \right] + \text{h.c.},
\]

where \( c_i \) (for \( i = 1, \ldots, 10 \)) are the Wilson coefficients, and

\[
Q_i^c = (\bar{d}_a c_\beta)_{V-A} (\bar{c}_\beta b_a)_{V-A},
\]

\(^4\)Here we assume the top-quark dominance in the penguin loops and neglect the small strong phases induced by absorptive parts of the up and charm penguin loop-integral functions \([11]\).
\[ Q_2^c = (\bar{d}c)_{V-A}(\bar{c}b)_{V-A}, \]
\[ Q_3 = (\bar{d}b)_{V-A}(\bar{c}c)_{V-A}, \]
\[ Q_4 = (\bar{d}_\alpha b_\beta)_{V-A}(\bar{c}_\beta c_\alpha)_{V-A}, \]
\[ Q_5 = (\bar{d}b)_{V-A}(\bar{c}c)_{V+A}, \]
\[ Q_6 = (\bar{d}_\alpha b_\beta)_{V-A}(\bar{c}_\beta c_\alpha)_{V+A}, \] (7)

as well as \( Q_7 = Q_5, Q_8 = Q_6, Q_9 = Q_3 \) and \( Q_{10} = Q_4 \). Here \( Q_3, \cdots, Q_6 \) denote the QCD-induced penguin operators, and \( Q_7, \cdots, Q_{10} \) stand for the electroweak penguin operators. The factorization approximation allows us to single out a common hadronic matrix element from \( \langle D^* D^* | H_{\text{eff}} | B_0^0 \rangle \) and another one from \( \langle D^* D^* | H_{\text{eff}} | \bar{B}_0 d \rangle \):

\[ M = \frac{G_F}{\sqrt{2}} \langle D^* | (\bar{c}d)_{V-A} | 0 \rangle \langle D^* | (\bar{b}c)_{V-A} | B_0^0 \rangle, \]
\[ \bar{M} = \frac{G_F}{\sqrt{2}} \langle D^* | (\bar{d}c)_{V-A} | 0 \rangle \langle D^* | (\bar{c}b)_{V-A} | \bar{B}_0^0 \rangle. \] (8)

In this approach it should be noted that the relevant Wilson coefficients and the hadronic matrix elements need be evaluated in the same renormalization scheme and at the same energy scale \([10]\). Furthermore, we follow the standard procedure of Ref. \([12]\) to decompose \( M \) and \( \bar{M} \) in terms of three form factors \( A_{1}^{BD^*}(q^2), A_{2}^{BD^*}(q^2) \) and \( V^{BD^*}(q^2) \). Then the scalars \( (a, b, c) \) and \( (\bar{a}, \bar{b}, \bar{c}) \) are found to be

\[ (a, b, c) = G_F \frac{\sqrt{2}}{\chi (\bar{a}, \bar{b}, \bar{c})}, \]
\[ (\bar{a}, \bar{b}, \bar{c}) = G_F \frac{1}{\sqrt{2}} \chi^*(\bar{a}, \bar{b}, \bar{c}) \] (9)

where

\[ \chi = (V_{cb}^* V_{cd}) c_x - (V_{td}^* V_{td}) (c_y + c_z) \] (10)

with

\[ c_x = \bar{c}_2 + \frac{\bar{c}_1}{3}, \]
\[ c_y = \bar{c}_4 + \frac{\bar{c}_3}{3}, \]
\[ c_z = \bar{c}_{10} + \frac{\bar{c}_9}{3}; \] (11)

and

\[ \bar{a} = m_{D^*} f_{D^*} (m_B + m_{D^*}) A_1^{BD^*}(m_{D^*}^2), \]
\[ \bar{b} = -2m_{D^*}^3 f_{D^*} \frac{A_2^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}}, \]
\[ \bar{c} = -2m_{D^*}^3 f_{D^*} \frac{V^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}}. \] (12)
Note that in Eq. (11) the effective Wilson coefficients $\bar{c}_i$ are independent of the energy scale and the renormalization scheme [14]. It is clear that $c_x$, $c_y$ and $c_z$ stand for the tree-level, gluonic penguin and electroweak penguin contributions, respectively. Taking the top-quark mass $m_t = 174$ GeV and the strong coupling constant $\alpha_s(m_b) = 0.21$, one finds $c_x = 1.045$, $c_y = -0.031$ and $c_z = -0.0014$ [13]. Note also that the form factors in Eq. (12) are related to one another in the heavy quark symmetry [13]. The $q^2$ dependence of the form factors, given by the common slope of the universal Isgur–Wise function, allows us to extrapolate from $q^2_{\text{max}} = (m_B - m_{D^*})^2$ to $q^2 = m_{D^*}^2$. Then we get

$$V^{BD^*}(m_{D^*}^2) = A_{2}^{BD^*}(m_{D^*}^2) = \frac{(m_B + m_{D^*})^2}{m_B(m_B + 2m_{D^*})} A_{1}^{BD^*}(m_{D^*}^2).$$

(13)

Therefore the ratios $\tilde{b}/\tilde{a} = \tilde{c}/\tilde{a}$ depend only upon the meson masses $m_B$ and $m_{D^*}$.

Now let us determine the $\zeta_d$ and $\xi_d$ parameters in the $CP$-violating quantity $\Delta_d$ defined in Eq. (2). For this purpose we sum over the polarizations of $D^{*+}$ and $D^{*-}$ mesons [14]. After a lengthy but straightforward calculation, we arrive at

$$\sum_{(\text{pol})} \left( \frac{\langle D^{*+}D^{*-}\cdots \tilde{D}^0_d \rangle}{\langle D^{*+}D^{*-}\cdots \tilde{H}_0 \rangle} \right) = \frac{(2 + \kappa^2)(\bar{a}a^*) + (\kappa^2 - 1)^2(\bar{b}b^*) + \kappa(\kappa^2 - 1)2(\bar{a}b^* + \bar{b}a^*) - 2(\kappa^2 - 1)(\bar{c}c^*)}{(2 + \kappa^2)|a|^2 + (\kappa^2 - 1)^2|b|^2 + 2\kappa(\kappa^2 - 1)\Re(ab^*) + 2(\kappa^2 - 1)|c|^2}$$

$$= \frac{\chi^*}{\chi} \frac{(2 + \kappa^2)a^2 + (\kappa^2 - 1)^2b^2 + 2\kappa(\kappa^2 - 1)\bar{a}b - 2(\kappa^2 - 1)c^2}{(2 + \kappa^2)a^2 + (\kappa^2 - 1)^2b^2 + 2\kappa(\kappa^2 - 1)\bar{a}b + 2(\kappa^2 - 1)c^2},$$

(14)

where $\kappa = (p_+ \cdot p_-)/m_{D^*}^2 = (m_B^2 - 2m_{D^*}^2)/(2m_{D^*}^2)$. As a result, we obtain

$$\zeta_d = \frac{m_B^3 - 3m_Bm_{D^*}^2 + 10m_{D^*}^3}{m_B^3 + m_Bm_{D^*}^2 + 2m_{D^*}^3},$$

(15)

and

$$\xi_d = \frac{c_y + c_z}{c_x} \cdot \frac{\cos 2\beta}{\cos \beta} \cdot \left| \frac{V_{tb}V_{td}}{V_{cb}V_{cd}} \right|.$$ 

(16)

It is remarkable that the result obtained in Eq. (15) for the $P$-wave dilution factor $\zeta_d$ relies only on the heavy quark symmetry. Therefore the value of $\zeta_d$ is independent of specific models for the form factors. To estimate the penguin-induced correction (i.e., the $\xi_d$ parameter), $|(V_{tb}V_{td})/(V_{cb}V_{cd})| = 1$ and $\beta = 26^\circ$ are typically taken. The former is favored by current data on quark mixing [14], and the latter is consistent with $\sin 2\beta = 0.79$ observed by the CDF Collaboration [14]. Then we find

$$\zeta_d = 0.89,$$

$$\xi_d = -0.021.$$ 

(17)

This result indicates that the penguin contamination in $\Delta_d$ is negligibly small, while the $P$-wave dilution to $\Delta_d$ should be taken seriously.
For $B_s^0 \rightarrow D_s^{*+} D_s^{-}$ decays, the $\zeta_s$ and $\xi_s$ parameters appearing in the $CP$ asymmetry $\Delta_s$ can be evaluated in the same way. The relevant results are obtained, through the discrete transformation from $d$ to $s$ (the so-called U-spin reflection) in the above formulas, as follows:

$$\zeta_s = \frac{m_{B_s}^3 - 3m_{B_s}m_{D_s^+}^2 + 10m_{D_s^0}^3}{m_{B_s}^3 + m_{B_s}m_{D_s^0}^2 + 2m_{D_s^+}^3},$$

(18)

and

$$\xi_s = \frac{c_y + c_z}{c_x} \cdot \frac{\cos 2\beta'}{\cos \beta'} \cdot \frac{|V_{tb}V_{ts}|}{|V_{cb}V_{cs}|}.$$  

(19)

For illustration, we typically take $|(V_{tb}V_{ts})/(V_{cb}V_{cs})| = 1$ and $\beta' = 0.5^\circ$ to calculate the value of $\xi_s$. We obtain

$$\zeta_s = 0.90,$$

$$\xi_s = -0.031.$$  

(20)

Indeed $\zeta_s = \zeta_d$ is expected to hold exactly under SU(3) symmetry. The result in Eq. (20), similar to that in Eq. (17) for $B_d \rightarrow D^{*+} D^{*-}$ modes, implies that in $\Delta_s$ the penguin contamination is negligibly small but the $P$-wave dilution is significant.

3. Now we proceed to discuss possible final-state rescattering effects in $B_d \rightarrow D^{*+} D^{*-}$ decays, which were not taken into account in the above analysis. The $\Delta B = +1$ and $\Delta B = -1$ parts of $\mathcal{H}_{\text{eff}}$ in Eq. (6) have the isospin structures $|1/2, +1/2 \rangle$ and $|1/2, -1/2 \rangle$, respectively. They generally govern the transitions $B_u^+ \rightarrow D^{*+} \bar{D}^0$, $B_0^0 \rightarrow D^{*+} D^{*-}$, $B_0^0 \rightarrow D^{*0} \bar{D}^0$ and their charge-conjugate processes. The final state of each decay mode can be in either $I = 1$ or $I = 0$ isospin configuration, therefore rescattering effects are possibly present. We find that the isospin relations

$$\langle D^{*+}\bar{D}^0|\mathcal{H}_{\text{eff}}|B_u^+\rangle = A_1,$$

$$\langle D^{*+}D^{*-}|\mathcal{H}_{\text{eff}}|B_0^0\rangle = \frac{1}{2}(A_1 + A_0),$$

$$\langle D^{*0}\bar{D}^0|\mathcal{H}_{\text{eff}}|B_0^0\rangle = \frac{1}{2}(A_1 - A_0).$$  

(21)

where $A_1$ and $A_0$ are the $I = 1$ and $I = 0$ isospin amplitudes, hold separately for three transition amplitudes with the same helicity ($\lambda = -1$, 0 or +1). The same isospin relations can be obtained for $B_u^- \rightarrow D^{*-} D^0$, $B_0^+ \rightarrow D^{*+} D^{*-}$ and $B_0^0 \rightarrow D^{*0} \bar{D}^0$ decays in terms of the corresponding isospin amplitudes $\bar{A}_1$ and $\bar{A}_0$.

To calculate the magnitudes of $A_1$, $A_0$ and $\bar{A}_1$ and $\bar{A}_0$, we assume again that transition amplitudes from the annihilation-type quark diagrams are negligible due to their strong form-factor suppression. This implies that $B_0^0 \rightarrow D^{*0} \bar{D}^0$ and its charge-conjugate process, which occur only through the quark diagrams illustrated in Fig. 1, would be forbidden if there

5For example, hadronic matrix elements of the type $\langle D^{*+}D^{*-}|(\bar{c}c)_{V-A}|0\rangle(0|(\bar{b}d)_{V-A}|B_0^0)$ depend on the annihilation form factor $F_{\text{ann}}(m_B^2) \sim 1/m_B^2$ [17] and the mass difference of two final-state mesons in a constituent U(2,2) quark model [18].
were no final-state rescattering. In other words, \( A_1 = A_0 \) and \( \bar{A}_1 = \bar{A}_0 \) would hold, if the rescattering effect were absent. Following this argument we make use of the factorization approximation to calculate \( A_1 \) (or \( \bar{A}_1 \)) and \( A_0 \) (or \( \bar{A}_0 \)), and account for final-state interactions at the hadron level by incorporating the elastic rescattering phases \( \delta_1 \) and \( \delta_0 \) \([19]\). We then arrive at the factorized isospin amplitudes as follows:

\[
A_1 = \chi M e^{i\delta_1}, \quad A_0 = \chi M e^{i\delta_0};
\]

and

\[
\bar{A}_1 = \chi^* \bar{M} e^{i\delta_1}, \quad \bar{A}_0 = \chi^* \bar{M} e^{i\delta_0},
\]

where \( M \) and \( \bar{M} \) are given in Eq. (8). One can see that \( |A_0| = |A_1| \) and \( |\bar{A}_0| = |\bar{A}_1| \) hold in this factorization approach. It becomes clear that the transitions \( B_d^0 \to D^{*-0} \bar{D}^{*0} \) would be forbidden, if there were no final-state rescattering effects (i.e., if \( \delta_0 = \delta_1 \)). As a consequence, one obtains

\[
\langle D^{*-0} | \mathcal{H}_{\text{eff}} | B_d^0 \rangle = \chi M \cos \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2},
\]

\[
\langle D^{*-0} | \mathcal{H}_{\text{eff}} | \bar{B}_d^0 \rangle = \chi^* \bar{M} \cos \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2};
\]

and

\[
\langle D^{*0} \bar{D}^{*0} | \mathcal{H}_{\text{eff}} | B_d^0 \rangle = i\chi M \sin \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2},
\]

\[
\langle D^{*0} \bar{D}^{*0} | \mathcal{H}_{\text{eff}} | \bar{B}_d^0 \rangle = i\chi^* \bar{M} \sin \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2}.
\]

We see that the isospin phases can be cancelled out in the ratio of two charge-conjugate decay amplitudes, thus the previous results of \( \zeta_d, \xi_d \) and \( \Delta_d \) are unchanged even in the presence
of final-state rescattering effects. Of course this conclusion depends on the factorization hypothesis used above. Whether it is valid or not can be checked, once the relevant branching ratios are measured. For example, Eqs. (24) and (25) lead to two experimentally testable relations among the branching ratios of three correlative decay modes:

\[
|\langle D^{++}D^{*-}\rangle_{\text{eff}}|B_u^0 \rangle|^2 + |\langle D^{*0}\bar{D}^{*0}\rangle_{\text{eff}}|B_u^0 \rangle|^2 = |\langle D^{**}\bar{D}^{**}\rangle_{\text{eff}}|B_u^0 \rangle|^2 ,
\]

\[
|\langle D^{++}D^{*-}\rangle_{\text{eff}}|\bar{B}_d^0 \rangle|^2 + |\langle D^{*0}\bar{D}^{*0}\rangle_{\text{eff}}|\bar{B}_d^0 \rangle|^2 = |\langle D^{*+}\bar{D}^{*0}\rangle_{\text{eff}}|\bar{B}_d^0 \rangle|^2 .
\]

(26)

It can easily be shown that $$|M|^2 = |\bar{M}|^2$$ will hold, if one sums over the polarizations of two final-state vector mesons. In this case, the two (rectangular) triangle relations in Eq. (26) are congruent with each other.

It is worth remarking that the detection of $$B_d \rightarrow D^{*0}\bar{D}^{*0}$$ transitions will be crucial: (a) if their branching ratios in comparison with those of $$B_d \rightarrow D^{*+}D^{*-}$$ are too small to be observed, then the final-state rescattering effects should be negligible ($$\delta_1 - \delta_0 \approx 0$$) and the naive factorization approach might work well; (b) if their branching ratios are more or less comparable with those of $$B_d \rightarrow D^{*+}D^{*-}$$, then a quantitative isospin analysis should be available, allowing us to extract the isospin phase differences through Eq. (21). In case (b), a measurement of CP violation of the magnitude sin 2$$\beta$$ in $$B_d^0$$ vs $$\bar{B}_d^0$$ $$\rightarrow D^{*0}\bar{D}^{*0}$$ decays should be quite likely.

For $$B_s^0$$ and $$\bar{B}_s^0$$ decays into the $$D_s^{**+}D_s^{*-}$$ state the similar isospin analysis does not exist. The SU(3) symmetry between $$B_d \rightarrow D^{*+}D^{*-}$$ and $$B_s \rightarrow D_s^{+}D_s^{-}$$ channels, however, allows one to conjecture possible final-state interactions in the latter. As one has seen in Eqs. (15) and (18), the SU(3) breaking effect is indeed rather small and even negligible.

4 We have calculated the indirect CP asymmetries ($$\Delta_d$$ and $$\Delta_s$$) in $$B_d \rightarrow D^{*+}D^{*-}$$ and $$B_s \rightarrow D_s^{+}D_s^{-}$$ decay modes. It has been shown that a quite clean determination of the CP-violating parameters sin 2$$\beta$$ and sin 2$$\beta'$$, with no help of the angular analysis, is in practice possible. The penguin contamination in either $$\Delta_d$$ or $$\Delta_s$$ is negligibly small. The P-wave dilution factors of $$\Delta_d$$ and $$\Delta_s$$ (i.e., $$\zeta_d$$ and $$\zeta_s$$) are found to be 0.89 and 0.90, respectively, in the factorization approximation and heavy quark symmetry.

As $$\zeta_d$$ does not deviate too much from unity, a large CP asymmetry in $$B_d^0$$ vs $$\bar{B}_d^0$$ $$\rightarrow D^{*+}D^{*-}$$ transitions is expected within the standard model. If the penguin-induced correction to the indirect CP violation in $$B_d^0$$ vs $$\bar{B}_d^0$$ $$\rightarrow D^+D^-$$ is also negligible, then a comparison between the CP asymmetries in $$D^+D^-$$ and $$D^{*+}D^{*-}$$ modes allows us to directly determine the P-wave dilution factor, i.e.,

$$\zeta_d = \frac{\Delta_d(D^{*+}D^{*-})}{\Delta_d(D^+D^-)} .$$

(27)

Of course such a measurement is very useful, in order to check the theoretical value of $$\zeta_d$$ obtained in Eq. (15).

Although the CP asymmetry $$\Delta_s$$ is expected to be vanishingly small in the standard model (because of the smallness of $$\beta'$$), its magnitude could significantly be enhanced if there were new physics in $$B_s^0$$-$$\bar{B}_s^0$$ mixing. For illustration let us consider a kind of new physics that does
not violate unitarity of the CKM matrix and has insignificant effects on the penguin channels of the decay modes under discussion [8]. It may introduce an additional CP-violating phase into $B^0_d \bar{B}^0_d$ or $B^0_s \bar{B}^0_s$ mixing. In this case the overall mixing phases of two systems become

$$\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \Rightarrow \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} e^{i\phi_{NP}},$$

$$\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*} \Rightarrow \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*} e^{i\phi'_{NP}},$$

(28)

in which $\phi_{NP}$ and $\phi'_{NP}$ denote the CP-violating phases induced by new physics. The weak phases that can be extracted from the CP asymmetries $\Delta_d$ and $\Delta_s$ turn out to be

$$\beta \Rightarrow \beta + \frac{\phi_{NP}}{2},$$

$$\beta' \Rightarrow \beta' + \frac{\phi'_{NP}}{2},$$

(29)

respectively. To distinguish $\beta$ and $\beta'$ from the phase combinations in Eq. (29), one has to study the CP asymmetries in some other neutral $B$-meson decays [8].

In conclusion, we point out that the CP-violating parameters in $B_d \rightarrow D^{*+}D^{*-}$ and $B_s \rightarrow D^{*+}D^{*-}$ decays can be determined without measuring their angular distributions. The approach advocated here may be complementary to the angular analysis considered in the literature. Hopefully both will soon be confronted with the data from $B$-meson factories.

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