CARIÑENA ORTHOGONAL POLYNOMIALS ARE JACOBI POLYNOMIALS

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1. Introduction

The relativistic Hermite polynomials (RHP) were introduced in 1991 by Aldaya et al. \[3\] in a generalization of the theory of the quantum harmonic oscillator to the relativistic context. These polynomials were later related to the more classical Gegenbauer (or more generally Jacobi) polynomials in a study by Nagel \[4\]. For this reason, they do not deserve any special study since their properties can be deduced from the properties of the well-known Jacobi polynomials - a more general class of polynomials that includes Gegenbauer polynomials - as underlined by Ismail in \[6\]. Recently, Cariñena et al. \[2\] studied an extension of the quantum harmonic oscillator on the sphere $S^2$ and on the hyperbolic plane and they showed that the Schrödinger equation can be analytically solved; the solutions are an extension of the classical wavefunctions of the quantum harmonic oscillator where the usual Hermite polynomials are replaced by some new polynomials - that we will call here Cariñena polynomials. In the next section, we show that these Cariñena polynomials are in fact Jacobi polynomials. The last section is devoted to the application of these results to the nonextensive context.

2. Relativistic Hermite and Cariñena polynomials: definitions and notations

2.1. Relativistic Hermite polynomials.

The relativistic Hermite polynomial $H_n^N$ of degree $n$ and parameter $N$ is defined by the Rodrigues formula

\[
H_n^N (X) = (-1)^n \left(1 + \frac{X^2}{N}\right)^{N+n} \frac{d^n}{dX^n} \left(1 + \frac{X^2}{N}\right)^{-N};
\]

examples of RHP polynomials are

\[
H_0^N (X) = 1; \quad H_1^N (X) = 2X; \quad H_2^N (X) = 2 \left(-1 + X^2 \left(2 + \frac{1}{N}\right)\right);
\]

\[
H_3^N (X) = 4 \left(1 + \frac{1}{N}\right) \left(X^3 \left(2 + \frac{1}{N}\right) - 3X\right)
\]

These polynomials are extensions of the classical Hermite polynomials $H_n (X)$ that are defined as

\[
H_n (X) = (-1)^n \exp (X^2) \frac{d^n}{dX^n} \exp (-X^2)
\]

and thus can be obtained as the limit case

\[
\lim_{N \to +\infty} H_n^N (X) = H_n (X).
\]
The RHP are orthogonal on the real line in the following sense

\[ \int_{-\infty}^{+\infty} H_n^N(X) H_m^N(X) \left( 1 + \frac{X^2}{N} \right)^{-N-\frac{m+n}{2}} \, dX = \frac{\sqrt{N} \pi n! \Gamma (2N + n) \Gamma (N + \frac{1}{2})}{(n + N) N^n \Gamma (2N) \Gamma (N)} \delta_{m,n}. \]

We note that this is an unconventional orthogonality since \( H_n^N(X) \) and \( H_m^N(X) \) are orthogonal with respect to a measure which depends on the degrees of these polynomials.

### 2.2. Gegenbauer polynomials.

The Gegenbauer polynomial \( C_n^\nu(X) \) of degree \( n \) and parameter \( \nu \neq 0 \) is defined by the Rodrigues formula

\[ C_n^\nu(X) = \alpha_{n,\nu} (-1)^n (1 - X^2)^{\frac{\nu}{2} - \frac{1}{2}} \frac{d^n}{dX^n} (1 - X^2)^{n+\nu-\frac{1}{2}} \]

with

\[ \alpha_{n,\nu} = \frac{(2\nu)_n}{2^n n! \left( \nu + \frac{1}{2} \right)_n}; \]

examples of Gegenbauer polynomials are

\[ C_0^\nu(X) = 1; \quad C_1^\nu(X) = 2\nu X; \quad C_2^\nu(X) = 2\nu (\nu + 1) X^2 - \nu; \]

\[ C_3^\nu(X) = 2\nu (\nu + 1) \left( \frac{2(\nu+2)}{3} X^3 - X \right). \]

These polynomials are orthogonal with respect to the measure \( (1 - X^2)^{\nu-\frac{1}{2}} \, dX \) on \([-1, +1]\) for \( \nu \neq 0 \)

\[ \int_{-1}^{+1} C_n^\nu(X) C_m^\nu(X) (1 - X^2)^{\nu-\frac{1}{2}} \, dX = \frac{\pi 2^{1-2\nu} \Gamma (n + 2\nu)}{n! (n + \nu) \Gamma (\nu)} \delta_{m,n}. \]

### 2.3. Cariniéna polynomials.

The Cariniéna polynomial of degree \( n \) and parameter \( N \in \mathbb{R} \) is defined by the Rodrigues formula

\[ H_n^N(X) = (-1)^n \left( 1 + \frac{X^2}{N} \right)^{N+\frac{1}{2}} \frac{d^n}{dX^n} \left( 1 + \frac{X^2}{N} \right)^{n-N-\frac{1}{2}} \]

where \( X \in \mathbb{R} \) for \( N > 0 \) and \( X \in [-\sqrt{-N}, +\sqrt{-N}] \) when \( N < 0 \). Since the cases \( N > 0 \) and \( N < 0 \) differ greatly, we’ll denote - for reasons that will appear clearly in the following - the Cariniéna polynomials with negative parameter as

\[ H_n^N(X) = C_n^\nu(X), \nu = -N. \]

The Cariniéna polynomials are orthogonal with respect to the measure \( (1 + \frac{X^2}{N})^{-N-\frac{1}{2}} \, dX \) on the real line for \( N > 0 \), and on the interval \([-\sqrt{\nu}, +\sqrt{\nu}]\) with respect to the measure \( (1 - \frac{X^2}{\nu})^{-\nu-\frac{1}{2}} \, dX \) when \( N < 0 \):

\[ \int_{\mathbb{R}} H_n^N(X) H_m^N(X) \left( 1 + \frac{X^2}{N} \right)^{-N-\frac{1}{2}} \, dX = a_n \delta_{m,n}. \]
and
\[
\int_{-\sqrt{\nu}}^{+\sqrt{\nu}} C_n^\nu(X) C_m^\nu(X) \left(1 - \frac{X^2}{\nu}\right)^{-\frac{\nu}{2}} dX = b_n \delta_{m,n}
\]
for some constants \(a_n\) and \(b_n\).

3. Links between Relativistic Hermite and Cari\'nena polynomials

The following theorems show that the family of Cari\'nena polynomials is related to the set of RHP and Gegenbauer polynomials in a simple way.

**Theorem 1.** The Cari\'nena polynomial \(H_N^N(X)\) of degree \(n\) and parameter \(N > 0\) is related to the RHP polynomial \(H_N^N(X)\) of same degree \(n\) and parameter \(N\) as

\[
H_N^N(X) = \left(\frac{N}{N'}\right)^{\frac{N}{2}} H_n^N\left(X \sqrt{\frac{N}{N'}}\right)
\]

with

\[N = N' + 1/2 - n.\]

**Proof.** Denote \(N = N' + 1/2 - n\); then

\[H_N^N(X) = (-1)^n \left(1 + \frac{X^2}{N}\right)^{N+n} \frac{d^n}{dX^n} \left(1 + \frac{X^2}{N'}\right)^{-N}.\]

But by the Rodrigues formula (2.1)

\[(-1)^n \left(1 + \frac{X^2}{N}\right)^{N+n} \frac{d^n}{dX^n} \left(1 + \frac{X^2}{N'}\right)^{-N} = \left(\frac{N}{N'}\right) H_n^N\left(X \sqrt{\frac{N}{N'}}\right),\]

so that the result holds. \(\square\)

The same kind of result is now obtained for Cari\'nena polynomials with negative parameter, where the Gegenbauer polynomials now play the role of the RHP polynomials.

**Theorem 2.** The Cari\'nena polynomial \(C_\nu^n(X)\) of degree \(n\) and parameter \(N = -\nu < 0\) is related to the Gegenbauer polynomial \(C_\nu^n\) of same degree \(n\) and parameter \(\nu\) as

\[
C_\nu^n(X) = \frac{1}{\alpha_{\nu,n}} \nu^{-\frac{\nu}{2}} C_n^{\nu} \left(\frac{X}{\sqrt{\nu'}}\right),
\]

**Proof.** With \(\nu = -N\), we deduce

\[C_\nu^n(X) = (-1)^n \left(1 - \frac{X^2}{\nu}\right)^{-\frac{\nu}{2}} \frac{d^n}{dX^n} \left(1 - \frac{X^2}{\nu}\right)^{n+\nu-\frac{\nu}{2}}.\]

It can be easily checked that

\[\left(1 - \frac{X^2}{\nu}\right)^{-\frac{\nu}{2}} \frac{d^n}{dX^n} \left(1 - \frac{X^2}{\nu}\right)^{n+\nu-\frac{\nu}{2}} = \frac{1}{\alpha_{\nu,n}} \left(\frac{1}{\nu}\right)^{\frac{\nu}{2}} C_n^{\nu} \left(\frac{X}{\sqrt{\nu'}}\right),\]

so that the result holds. \(\square\)
We now use Nagel’s identity \[ (3.3) \]

\[
H_N^n(X) = \frac{n!}{N^n} \left( 1 + \frac{X^2}{N} \right)^{\frac{N}{2}} C_N^n \left( \frac{X/\sqrt{N}}{\sqrt{1 + X^2/N}} \right)
\]

that connects the RHP polynomials \( H_N^n(X) \) with the Gegenbauer polynomials \( C_N^n(X) \); we show that the same kind of connection can be derived between Cariñena polynomials with positive parameter \( \mathcal{H}_N^n(X) \) and Cariñena polynomials with negative parameter \( \mathcal{C}_n^\nu(X) \) as follows.

**Theorem 3.** The Cariñena polynomial \( \mathcal{H}_N^n(X) \) of degree \( n \) and parameter \( N > 0 \) is related to the Cariñena polynomial \( \mathcal{C}_n^\nu(X) \) of same degree \( n \) and parameter \( \nu \) by the following formula

\[
\mathcal{H}_n^N(X) = \alpha_{n,\nu} n! \left( \frac{\nu}{\sqrt{N}} \right)^{\frac{N}{2}} (1 + X^2)^{\frac{N}{2}} C_n^\nu \left( \frac{X/\sqrt{\nu}}{\sqrt{1 + X^2/\nu}} \right)
\]

where

\[
\nu = N + 1/2 - n.
\]

**Proof.** This is a direct consequence of Nagel’s identity \[ (3.3) \] and equalities \[ (3.1) \] and \[ (3.2) \].

These results are summarized in Table 1.

**Table 1. Summary of the results**

| Relativistic Hermite \( H_N^n(X) \) | Cariñena \( \mathcal{H}_N^n(X) \) with \( N > 0 \) |
| Nagel’s identity \[ (3.3) \] | \[ (3.3) \] |
| Gegenbauer \( C_n^\nu(X) \) | Cariñena \( \mathcal{C}_n^\nu(X) \) with \( N = -\nu < 0 \) |

4. **The nonextensive setup**

In the nonextensive theory, the classical Shannon entropy of a probability density \( f_X \)

\[
H = -\int f_X \log f_X
\]

is replaced by the so-called Tsallis entropy

\[
H_q = \frac{1}{1-q} \int (f_X - f_X^q)
\]

where \( q \) is a positive real number called the nonextensivity parameter. It can be checked by L’Hospital rule that

\[
H = \lim_{q \to 1} H_q.
\]

The canonical distribution in the classical \( q = 1 \) case - that is the distribution with maximum entropy and given variance \( \sigma^2 \) - is known to be Gaussian distribution

\[
f_X(X) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{X^2}{2\sigma^2} \right).
\]
The polynomials orthogonal on the real line with respect to the Gaussian measure are the Hermite polynomials \((2.2)\); the Hermite functions are defined as

\[
h_n(X) = \exp \left( -\frac{X^2}{2} \right) H_n(X) ; \quad n \geq 0
\]

and verify the simple orthogonality property

\[
\int_{\mathbb{R}} h_n(X) h_m(X) \, dX = \sqrt{\pi} 2^n n! \delta_{m,n}.
\]

In the nonextensive case, the canonical distributions are called \(q\)-Gaussian distributions and read, for \(q < 1\)

\[
f_X(X; q) = \frac{\Gamma \left( \frac{2-q}{1-q} + \frac{1}{2} \right)}{\Gamma \left( \frac{2-q}{1-q} \right) \sigma \sqrt{\pi d}} \left( 1 - \frac{X^2}{d \sigma^2} \right)^{\frac{1}{1-q}} ; \quad d = 2 \frac{2-q}{1-q} + 1
\]

and for \(1 < q < \frac{5}{3}\)

\[
f_X(X; q) = \frac{\Gamma \left( \frac{1}{q-1} \right)}{\Gamma \left( \frac{1}{q-1} - \frac{1}{2} \right) \sigma \sqrt{\pi (m-2)}} \left( 1 + \frac{X^2}{(m-2) \sigma^2} \right)^{\frac{1}{1-q}} ; \quad m = \frac{2}{q-1} - 1
\]

It can be easily checked that the limit case

\[
\lim_{q \to 1} f_X(X; q)
\]

coincides with the Gaussian distribution \((4.1)\).

To our best knowledge, the polynomials orthogonal with respect to the \(q\)-Gaussian distributions - the extensions of the Hermite polynomials - have not been studied in the non-extensive theory. They can be deduced from the results of Section 2, and are indicated in Table 2\(^1\). In the case \(q < 1\), the Gegenbauer polynomials are the polynomials orthogonal with respect to the \(q\)-Gaussian distribution. In the case \(q > 1\), either the Cariñena or the Relativistic Hermite polynomials are orthogonal with respect to the \(q\)-Gaussian measure; in the relativistic case, this measure should also depend on indices \(m\) and \(n\), what is not the case in the Cariñena case.

In all cases, the corresponding orthogonal function and the domain of definition is given; each of the corresponding orthogonal functions - let us call it generically \(w_n(X)\) - verifies the orthogonality property

\[
\int_I w_n(X) w_m(X) \, dX = K_n \delta_{m,n}
\]

for some constant \(K_n\).

It turns out that the orthogonal functions above cited describe the behaviours of physically significant systems:

- as shown in \([2]\), the probability density that describes the harmonic oscillator on a 2-dimensional surface of constant negative curvature \(\kappa\) (typically the hyperbolic plane) are

\[
f_{m,n,N}(y, z) = |\hat{y}_n^{N-m-\frac{1}{2}}(y)|^2 |\hat{y}_m^{N}(z)|^2
\]

\(^1\)Note that we consider here the \(q\)-Gaussian distributions with scaling constants normalized to 1
CARIÑENA ORTHOGONAL POLYNOMIALS ARE JACOBI POLYNOMIALS

|                         | $q$                     | orthogonal function                                      | domain         |
|-------------------------|-------------------------|----------------------------------------------------------|----------------|
| Gegenbauer polynomials  | $\frac{2\nu-3}{2\nu-1} < 1$ | $c_n^{\nu}(X) = (1 - X^2)^{\frac{1}{2} - \frac{1}{4\nu}} C_n^{\nu}(X)$ | $[-1;1]$       |
| Cariñena polynomials    | $\frac{2N+3}{2N+1} > 1$ | $h_n^{N}(X) = \left(1 + \frac{X^2}{N}\right)^{-\frac{N}{2} + \frac{1}{4}} H_m^{N}(X)$ | $\mathbb{R}$   |
| Relativistic Hermite    | $\frac{2+N+m+n}{1+N+m+n} > 1$ | $h_n^{N}(X) = \left(1 + \frac{X^2}{N}\right)^{-\frac{1}{2} + \frac{1}{4}} H_m^{N}(X)$ | $\mathbb{R}$   |

**Table 2.** polynomials orthogonal with respect to the $q-$Gaussian measure and the corresponding orthogonal functions

with $z = \frac{x}{\sqrt{1+y^2}}$ (note that $y$ and $z$ are not independent variables). The parameter $\mathcal{N}$ here is defined as

$$\mathcal{N} = -\frac{ma}{\hbar \kappa} > 0,$$

• as shown in the same reference, the probability density that describes the harmonic oscillator on a 2-dimensional surface of constant positive curvature $\kappa$ (typically the sphere) are

$$g_{m,n,\nu}(y,z) = |c_{n+m+\frac{1}{2}}^{\nu}(y)|^2 |c_{m}^{n}(z)|^2$$

with $z = \frac{x}{\sqrt{1+y^2}}$ and $\nu = \frac{ma}{\hbar \kappa} > 0$.

• the harmonic oscillator in the relativistic context as described by \[3\] has probability density

$$f_{n,N}(X) = |h_{n}^{N}(X)|^2$$

where the parameter $N > 0$ is defined as

$$N = \frac{mc^2}{\hbar \omega}$$

so that the non-relativistic limit $c \to +\infty$ corresponds to the classical $N \to +\infty$ Hermite polynomials.

Thus the behaviour of the harmonic oscillator - in either the relativistic case or the case of constant curvature geometries - can be related to the nonextensive framework, giving in each case an explicit physical interpretation of the nonextensivity parameter $q$ in terms of the physical constants of the harmonic oscillator, as shown in Table 3. We note that in the case of the harmonic oscillator on the sphere or on the hyperbolic plane, the densities (4.2) and (4.3) are separable functions in the variables $z$ and $y$, each term inducing a different value of $q$.

| positive curve $\kappa$ | $\frac{2\nu-3}{2\nu-1}$ | $\frac{\nu+m-1}{\nu+m}$ |
|-------------------------|-------------------------|-------------------------|
| negative curve $\kappa$ | $\frac{2N+3}{2N+1}$ | $\frac{N+m+1}{N-m}$ |
| RHP                     | $1 + \frac{m+m+n}{1+N+m+n}$ |

**Table 3.** values of the nonextensivity parameter $q$ associated with the three harmonic oscillators
5. A natural bijection

5.1. the relativistic harmonic oscillator. Let us denote
\[ f_{n,N}(X) = |h_n^N(X)|^2, g_{n,\nu}(Y) = |c_n^\nu(Y)|^2 \]
the probability densities associated to the orthogonal functions studied above. There exists a geometric interpretation of Nagel’s formula in the framework of non-extensivity: the ground state distribution \( f_{0,N}(X) = |h_N^0(X)|^2 \) of a \( q \)-Gaussian system \( X \) with \( q > 1 \) coincides with the ground state \( |c_0^\nu(Y)|^2 \) of a \( q \)-Gaussian system \( Y \) provided that
\[ Y = \frac{X/\sqrt{N}}{\sqrt{1 + X^2/N}} \text{ and } \nu = N. \]

Our main result is that this geometric interpretation holds not only for the ground state, but for all states of the relativistic harmonic oscillator as follows

Theorem 4. If \( X \sim f_{n,N} \) then the random variable
\[ (5.1) \quad Y = \frac{X/\sqrt{N}}{\sqrt{1 + X^2/N}} \]
is distributed according to \( g_{n,\nu} \) with \( \nu = N. \)

Proof. The distribution of \( Y \) defined by (5.1) is
\[ f_Y(Y) = \left(1 + \frac{X^2}{N}\right)^\frac{N}{2} f_{n,N}\left(\frac{\sqrt{N}Y}{\sqrt{1-Y^2}}\right) \]
or equivalently
\[ f_Y(Y) = (1 - Y^2)^{-\frac{N}{2}} (1 - Y^2)^{n+N+1} |H_n^N\left(\frac{\sqrt{N}Y}{\sqrt{1-Y^2}}\right)|^2 \]
\[ = (1 - Y^2)^{n+N-\frac{N}{2}} \left(1 + \frac{Y^2}{1-Y^2}\right)^n |C_n^N(Y)|^2 \]
\[ = (1 - Y^2)^{N-\frac{N}{2}} |C_n^N(Y)|^2 = g_{n,\nu}(Y). \]

5.2. The harmonic oscillator on the sphere and on the hyperbolic plane.
We now extend the preceding result to the case of the harmonic oscillator on spaces of constant curvature. We set to one all scaling constants for simplicity.

Theorem 5. Consider the harmonic oscillator on the hyperbolic plane described by its coordinates \((x, y)\) and with distribution
\[ f_{m,n,N}(z, y) = |h_m^{N-m-\frac{1}{2}}(y)|^2 |h_n^{N}(z)|^2 \]
(with \( z = \frac{x}{\sqrt{1+x^2+y^2}} \)). If this system is transformed as
\[ (5.2) \quad X = \frac{x}{\sqrt{1+x^2+y^2}}; Y = \frac{y}{\sqrt{1+x^2+y^2}} \iff x = \frac{X}{\sqrt{1-X^2-Y^2}}; y = \frac{Y}{\sqrt{1-X^2-Y^2}} \]
then the new system \((X,Y)\) follows the distribution

\[
g_{m,n,\nu}(Z,Y) = |c_m^{\nu+m+\frac{1}{2}}(Y)|^2 |c_n^n(Z)|^2
\]

where

\[
\nu = N - m - n.
\]

Proof. As a function of \((x,y)\), the density of the harmonic oscillator writes, in terms of Gegenbauer polynomials,

\[
f_{m,n,N}(x,y) = (1 + y^2)^n (1 + x^2 + y^2)^{-N-m-n} |C_n^{N-m-n} \left( \frac{y}{\sqrt{1 + y^2}} \right) |^2 |C_m^{N-m+\frac{1}{2}} \left( \frac{x}{\sqrt{1 + x^2 + y^2}} \right) |^2
\]

We now perform the change of variable \((5.2)\); the distribution of the new system is obtained as

\[
\tilde{f}_{m,n,N}(X,Y) d\mu(X,Y) = f_{m,n,N}(x,y) d\mu(x,y)
\]

with the measure

\[
d\mu(x,y) = \frac{dxdy}{\sqrt{1 + x^2 + y^2}}; d\mu(X,Y) = \frac{dXdY}{\sqrt{1 - X^2 - Y^2}}
\]

and since the Jacobian of the transformation \((x,y) \mapsto (X,Y)\) is

\[
J = \det \begin{bmatrix}
\frac{1+y^2}{(1+x^2+y^2)^{\frac{3}{2}}} & \frac{-xy}{(1+x^2+y^2)^{\frac{3}{2}}} \\
\frac{-xy}{(1+x^2+y^2)^{\frac{3}{2}}} & \frac{1+x^2}{(1+x^2+y^2)^{\frac{3}{2}}}
\end{bmatrix} = (1 + x^2 + y^2)^{-2} = (1 - X^2 - Y^2)^2
\]

we deduce

\[
\tilde{f}_{m,n,N}(X,Y) = (1 - X^2 - Y^2)^{-1} (1 - X^2 - Y^2)^{N-m+\frac{1}{2}} \left( 1 + \frac{Y^2}{1 - X^2 - Y^2} \right)^n
\]

\[
\times |C_n^{N-m-n} \left( \frac{Y}{\sqrt{1 - X^2}} \right) |^2 |C_m^{N-m+\frac{1}{2}} (X) |^2
\]

\[
= (1 - X^2 - Y^2)^{N-n-m-\frac{1}{2}} (1 - X^2)^n |C_n^{N-m-n} \left( \frac{Y}{\sqrt{1 - X^2}} \right) |^2 |C_m^{N-m+\frac{1}{2}} (X) |^2.
\]

But since the distribution of the harmonic oscillator on the sphere reads, in terms of Gegenbauer polynomials, \((5.3)\)

\[
g_{m,n,\nu}(X,Y) = (1 - Y^2)^{m-\frac{1}{2}} (1 - X^2 - Y^2)^{\nu} |C_n^{\nu+m+\frac{1}{2}} (Y) |^2 |C_m^{\nu} \left( \frac{X}{\sqrt{1 - Y^2}} \right) |^2,
\]

we deduce that

\[
\tilde{f}_{m,n,N}(X,Y) = g_{m,n,\nu}(Y,X)
\]

with

\[
\nu = N - m - n.
\]

6. Conclusion

We have shown that the Cariñena orthogonal polynomials are Jacobi polynomials; moreover, there exists a natural bijection between the negative and the positive curvature cases. These results hold only in the two dimensional case.
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