Little Higgs Models: New Approaches to the Hierarchy Problem

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In this note we present a review of the little Higgs models that stabilize the electroweak by realizing the Standard Model Higgs as a pseudo-Goldstone boson.

1. Introduction

The experimental results of the past decade indicate that the chiral Lagrangian describing EWSB works to one loop accuracy and irrelevant operators (such as $|D\omega|^4$, where $\omega$ are the Goldstone bosons eaten by $W^\pm$ and $Z^0$) are suppressed by a scale $\Lambda \gg 1$ TeV. This strongly indicates perturbative physics at 1 TeV and a physical Higgs boson $H = (h^0 + v)\omega$. However, this linear sigma model is unstable to quantum corrections leading to the hierarchy problem, meaning that the Standard Model is in an incomplete description of physics parametrically above the weak scale. To date, only the MSSM provides a solution to the hierarchy problem and weakly coupled physics at 1 TeV.

In this note we review models where the Higgs is a pseudo-Goldstone boson, $\Sigma = e^{i\sigma}/f$, for early attempts see [2]. Approximate global symmetries, $\Sigma \rightarrow L\Sigma R^T$, keep the Higgs vev small relative to the cut-off, $\Lambda \sim 4f$. The global symmetries are broken by couplings, however, the structure of the symmetry breaking prevents large corrections to the Higgs vev. At 100 GeV the model is a two Higgs doublet model with a Higgs potential identical to the MSSM. There is a mini-desert up to 1-3 TeV where new particles emerge that cancel the quadratic divergences of the Standard Model. The novelty of this set of models is that particles of the same spin cancel the respective quadratic divergences. Above the TeV scale there is another mini-desert until the 10-30 TeV scale where the description of the model becomes strongly coupled and new physics emerges. However, the Standard Model is insensitive to this physics.

2. Little Higgs Models

Little Higgs models are gauged non-linear sigma model with Lagrangians that can be divided into four pieces:

$$\mathcal{L} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{nlsm Kin}} + \mathcal{L}_{\text{Plaq}} + \mathcal{L}_{\text{Yuk}} \quad (1)$$

The first term is the standard kinetic term for gauge bosons. The second term is the nlsm kinetic terms that give mass to some of the gauge bosons. There is the possibility of adding a scalar potential, often called the plaquette potential, that gives mass to some of the nlsm fields. Finally there are the Yukawa couplings between the nlsm fields and fermions. To study the low energy dynamics of the theory we want to integrate...
out all the states that are classically massive and have an effective Lagrangian in terms of the classically massless fields. Some nlσm can be represented by theory spaces where points denote gauge groups, lines are nlσm fields, and faces are plaquette potentials. This was studied in depth in [4,6]. The classically massless scalars are the little Higgs and are associated with non-contractible loops in theory space. The little Higgs has a mass generated radiatively, but the size of the radiative corrections are parametrically smaller than the TeV scale.

In the limit of vanishing couplings the nlσm fields become exact Goldstone bosons under the symmetry Σ → LΣR†. This leaves the little Higgs exactly massless, but also without non-derivative interactions. Coupling constants break these chiral symmetries and allow masses for the little Higgs to be generated radiatively. Little Higgs models always require at least two couplings to communicate sufficient chiral symmetries and allow masses for the little Higgs to be generated radiatively. Little Higgs models always require at least two couplings to communicate sufficient chiral symmetry breaking to generate a mass radiatively, and therefore quadratic divergences first arise at two loop order. To ensure that there are no one loop quadratic divergences to the the little Higgs mass there are simple rules for theory space models [4,6]. The first rule is that if all nlσm fields are bi-fundamentals, then there are no one loop gauge quadratic divergences. If no plaquette contains any nlσm field more than once, then there one loop quadratic divergences to the little Higgs mass from the scalar potential.

Having removed the one loop quadratic divergences, there are always finite contributions to the little Higgs mass. We can estimate the size of these finite contributions by studying the theory beneath the scale of the classically massive modes. In this low energy theory there are a quadratic divergences and they are cut-off by the mass of the first heavy states and become finite corrections to the mass squared of the little Higgs. The mass of the heavy modes is roughly

\[ M_H \sim g f. \]

This means that the correction to the little Higgs mass from the low energy quadratic divergence is:

\[ \delta m_{\text{LH}}^2 \sim \frac{g^2}{16\pi^2} M_H^2 = \frac{g^2}{16\pi^2} g^2 f^2 = \left( \frac{g^2}{16\pi^2} \right)^2 \Lambda^2. \]

This is the same size as the two loop quadratic divergence in the full theory. Thus, even if we remove higher order quadratic divergences, we can not push the cut-off higher than two loop factors above the EW scale, setting \( \Lambda \sim 10 - 30 \text{ TeV} \). This still allows for a separation of scales \( m_{\text{LH}} \ll M_H \ll \Lambda \). Furthermore, this means that the states that stabilizes the electroweak scale is invisible to roughly 1 TeV.

3. The Minimal Moose

The minimal theory space model [4] that has the requisite properties for EWSB has two gauge group and four nlσm fields and is illustrated in Fig. 1. The two gauge groups are taken to be \( SU(3) \) and \( SU(2) \times U(1) \) with the \( U(1) \) embedded as the \( T_8 \) generator of \( SU(3) \). There are two plaquettes added:

\[ L_{\text{Plaq.}} = \lambda_1 f^4 \text{Tr} X_1 X_2 X_3 X_4 + \text{h.c.} + \lambda_2 f^4 \text{Tr} X_2 X_3 X_4 X_1 + \text{h.c.} \]

This gauged nlσm breaks the \( SU(3) \times [SU(2) \times U(1)] \) gauge symmetry down to the diagonal \( [SU(2) \times U(1)] \) that is identified as the electroweak gauge group. The plaquettes give mass to one linear combination of nlσm fields and leave three massless. One is eaten by the massive \( SU(3) \) gauge bosons while two are physical and massless and are the little Higgs of the theory.

![Figure 1. Minimal theory space model.](image)

Since all the link fields are bi-fundamentals under the two gauge groups and the plaquettes contain each link field only once, there are no one loop quadratic divergences to the little Higgs masses.

The scalar potential for little Higgs, \( \Sigma_{1,2} \), is of the form \( \text{Tr} \Sigma_i \Sigma_j \Sigma_i^\dagger \Sigma_j^\dagger \) or in terms of the linearized modes:

\[ V(\sigma_1, \sigma_2) = \lambda_{\text{eff}} \text{Tr} [\sigma_1, \sigma_2]^2 + \cdots \]

\( \sigma_i \) see [4] for other classes of models.
where $\lambda_{\text{eff}}^{-1} = \lambda_1^{-1} + \lambda_2^{-1}$. Written in terms of the doublets, the potential is:

$$V(h_1, h_2) = \lambda_{\text{eff}} (|h_1|^2 - |h_2|^2)^2$$

(5)

This is identical to the Higgs potential inside the MSSM except that the coefficient is undetermined. Hence, the mass of the Higgs is undetermined, although we do expect $\lambda_{\text{eff}} \sim O(1)$, leading to a physical Higgs mass in the $100 - 300$ GeV range.

The Standard Model fermions are charged under the $SU(2) \times U(1)$ gauge group. It is convenient to write the fermions as triplets under the global $SU(3)$, i.e. $Q = (q, 0)$ and $U^c = (0, 0, u^c)$. They are coupled to the little Higgs, $\Sigma = \exp(i(\sigma_1 \pm \sigma_2))$, through operators such as:

$$y_f Q \Sigma U^c \sim y_q (h_1 \pm i h_2) u^c + \cdots$$

(6)

This operator induces a one-loop quadratically divergent contribution to the little Higgs mass. For everything but the top quark, the quadratic divergence is negligible. For the top quark, the one-loop quadratic divergence must be dealt with. By adding an additional massive vector-like fermion, $\chi$ and $\chi^c$, we can promote the quark doublet, $q$, to quark triplet under the global $SU(3)$, $Q = (q, \chi)$. By preserving the global symmetry of the coupling, we remove the one-loop quadratic divergence to the top quark. The quadratically divergent part of the one-loop Coleman-Weinberg effective potential is:

$$V_{\text{eff}} = -\frac{y^2 \Lambda^2 f^2}{16\pi^2} \text{Tr} P_R \Sigma P_L \Sigma^\dagger$$

(7)

where $P_R = (0, 0, 1)$ from the $U^c$ coupling and $P_L = (1, 1, 0)$ if without the vector-like fermion $\chi$ and becomes the identity matrix with $\chi$. When $P_L$ is the identity matrix, we preserve the $SU(3)$ symmetry and the effective potential is independent of $\Sigma$.

One loop log divergences from the top sector drives one linear combination of the Higgs to acquire a mass and the orthogonal combination are stabilized by one loop log divergences from the gauge and scalar sectors. This leads to phenomenologically acceptable EWSB driven radiatively much as in the MSSM. We refer the reader to [6] for more details.

4. Conclusions and Outlook

We have presented new models of EWSB that stabilize the weak scale to radiative corrections and have weakly coupled physics at 1 TeV. At the 100 GeV scale it is a 2HDM model with possibly additional scalar singlets and $SU(2)$ triplets. At 1 TeV there are additional vector bosons that cancel the SMs gauge $\Lambda^2$ divergences, additional scalars that cancel the Higgs quartic interaction $\Lambda^2$ divergence and additional coloured fermions that cancel the top’s $\Lambda^2$ divergence. This description is valid up to $10 - 30$ TeV where the brane becomes strongly coupled and new physics emerges.

There is a tremendous amount of physics that is still to be done: precision studies [9], experimental signature [9], exploring other little Higgs models [5,7], and UV completing the theory above 10 – 30 TeV.

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