Inner Market as a “Black Box”

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Abstract.

Each market has its singular characteristic. Its inner structure is directly responsible for the observed distributions of returns though this fact is widely overlooked. Big orders lead to doubling the tails. The behavior of a market maker with many or few “friends” who can reliably loan money or stock to him is quite different from the one without. After representing the inner market “case” we suggest how to analyze its structure.

Introduction

Recently several market models claimed to find self-organization in market behavior. Bak et al (1997) made a surprising claim that one singular feature - mimicking majority - can fix the Hurst exponent of the distribution of price returns. An obvious fact was overlooked - namely that an ordinary market player cannot know all the prices offered and asked at the market. (He knows only the prices of his market-maker.) The only people who possess such knowledge are market-makers.

Moreover, only on the level of market-makers one can answer the criticism of this model voiced by Levy and Solomon (1997) that the mimicking leads to “correlations between investments of large sets of individual investors of equal wealth”. Levy and Solomon claimed that such correlations were not observed at the level of the “big market”. However, this is perfectly true at the level of the “inner market”. It is clear that maker-makers try to hold their spread as majority do - not to loose money or clients. It would be interesting to repeat an experiment of ref. [1] for smaller number of agents.

Zhang (1999), on the basis of some empirical evidence, claimed that the price grows as a square root of market pressure (difference between number of buyers and sellers). However, these buyers and sellers, who were taken into account, bought and sold their stock at the Bid-Ask prices, or what is the same, bought and sold from

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market-makers. So we can reverse the picture claiming that market-makers somewhat “soften” the pressure by allowing price to grow slower than - as it might be thought - linearly. We will argue that this is because their response is somehow delayed.

In application of Minority Game to portraying the market dynamics, Giardina et al (2001) argue that random signals (orders) convert a competition between two (active and passive) strategies in something similar to the returns of random walk - this leads to the growth of the correlation in activity as square root of time. Still various features of such a market remained unexplained: Minority Game assumes two outcomes and it is unclear how to distinguish between buying and selling. As a result, only so-called market activity was investigated. Some important things, like truncation of density function for survival time of both strategies, remained unexplained. The major fact, however, which was overlooked is that “strategies” are the property of the only group in the market - market-makers. And two strategies discussed in ref (4) - passive and active (buying and selling) - are to hold a fixed Bid-Ask spread as other market-makers or to change it (lower Ask or raise Bid).

**Inner Market influence on the general market behavior**

Solomon and Richmond (2001) provided an excellent example of inner market influence on the final market phenomena. Introduction of a specialist (Osborne’s: the man handling mainly big orders) can double the tail of the distribution of the returns. Indeed, from matching a pair of two big orders one comes to joint distribution:

\[ P_1(v > x) = P(v > x)P(v > x). \]

This leads from a Pareto tail \( P(v > x) \propto x^{-\alpha} \) with \( \alpha = 3/2 \) to a Pareto tail \( P_1(v > x) \propto x^{-\beta} \) with \( \beta = 3 \), which is observed across the markets.

This phenomenon might be also accounted for by an observed “truncation” of the tails of Levy distribution across many markets and is worth further investigated. It is interesting that at many markets the specialist is removed at times of crises, so a natural suggestion would be to check whether “truncation” still exists at the time of crisis.

Another example of possible inner market influence on overall price can be seen from Langevin equation in the form developed by Bouchaud and Cont (1998). This purely phenomenological equation produced some global market features like bubbles, behavior at the time of crisis etc. Surely, their phenomenological description of the whole market is also relevant for “inner market” - market-makers are accustomed to fear (a) and greediness (b) no less than the rest of the population. Then price returns (u) satisfy the equation:

\[ u \]

1 Remarks, like “the best strategies tend to deteriorate”, are in contradiction to results of ref. [5].
\[ \frac{Du}{dt} = -cu + au - bu^2 + K\kappa(t). \]

To reproduce the empirical laws, especially long-ranged correlations, one must be careful about coefficient \( K \) before the noise term. A natural choice is to put \( K = u^2 \). However, this choice alone does not lead to a power-like tail for distribution of \( u \), as it is easy to see applying Focker-Planck equation (ref. [8], p. 230). To reproduce a power-like tail of distribution of returns one has to change the term \((-cu)\), representing market, to \((-cu^3)\). The smaller influence of this term on \( Du/dt \) for small \( u \)'s will reflect the fact that small trades are completely absorbed by market-makers without changes in prices.

Polynomial tail consists of two parts. Stratanovich’s term in the Focker-Planck equation brings \( u^{-1} \) to the density function and the “market” term \(-cu^3\) will bring \( u^{-c} \) and together they give \( u^{-1-c} \). As we saw before, a presence of a specialist may double the constant \( c \).

Market Basics

Many postulates and nuances of the market “mechanics” were explained by Osborne (1977), whose book was recently brought to light and popularized by McCauley (2000). Some of the basic market principles are:

1. The market should be continuous (market should suggest a spread at any time).
2. Holding inventories and money, a market-maker should not exceed certain limits.
3. Profit monies do not enlarge inventory but are put aside.

Nuances are: each market-maker could have friends who can allow him to relax his own bounds on inventories. This can be conveniently portrayed by a “friendship” matrix, whose entries \( G_{i,j} \) show ability by market-maker \( j \) to help market-maker \( i \). (One can chose another approach and assume further that each market-maker has his buying and selling curve for buying and selling blocks of different size - the exponent already may include the “friendship” matrix.) These two curves - for buying and selling - might be slightly different: this asymmetry was emphasized by Zhang (1999).

Market Dynamics

The basic answer to the most empirical laws above is the distribution of time intervals for market-makers before changing their spread after trading a big amount of stock, or in the language of Minority Game, of changing his “passive strategy” to the “active” one. We assume that a market-maker does not like to leave his inventory

\footnote{This, however, should be further tuned to adjust it to the lucid “geometrical” interpretation of different market phenomena (like bubbles etc) delineated in the ref. [7]}
unbalanced overnight and certainly - not over weekend. Therefore he is ready to sell his extra stock for a lower price or to buy it back for a higher price. How much lower or higher? It depends on the closedness of the market-maker’s horizon (week-end). (The same feature can be observed at any “liquid” fruit-vegetable market - on Friday afternoon prices go down dramatically).

This scheme implies that the price drops when a market-maker is unable to recover symmetry in his inventories for some time depending on his “patience” or “fear”. We have to evaluate the average time he can hold his current spread before going to an “active” strategy - to lower his Ask or raise his Bid - which automatically will change the market price. We should assume that his “patience” is also restricted (“truncated”) by closedness of the end of the day and even more by the end of the week. This “truncation” alone can explain power-type tails of the “passive” time intervals distribution.

The “root square law” in activity correlations for small time intervals found in Giardina et al (2001) was based on stochastic mathematics suggested in Godreche and Luck (1999). The major conclusion was that the “root square law” comes from the very fact of randomness of incoming signals and a resulting $x^{-3/2}$ distribution of time switches between two strategies. Our goal is discover the same law in the framework described above.

Mathematical model

We assume that each market-maker $j$ has his maximal asymmetry limit, denoted $\text{Limit}(j)$ and number of friends which can be encoded in “friendship” matrix $G_{i,j}$. Randomly, at time $t$ a market-maker trades an amount of stock, denoted $\text{Order}(t)$, which is also taken from distribution $w^{-\alpha}$ (because orders are proportional to general wealth distribution as convincingly, albeit differently, argued in ref. [3], [4], [5]). Then, if the $\text{Order}$ exceeds his $\text{Limit}$, he waits for $\Delta t$ time which reflects his “fear” and his wealth (ability to accept the $\text{Order}$)

$$e^{-\Delta t(|\text{Order}(t_0) - \text{Limit}(j) - \Sigma_i G_{i,j}(t)|)} < F(j) \quad (\ast),$$

where $F(j)$ is his “patience” or “fear” and term $\Sigma_i G_{i,j}(t)$ shows a current ability of his friends to help him. Clearly, if the asymmetry exceeds his limits, he is unable to help his friends. At the point in time where the inequality (\ast) no longer holds, and meantime the reverse (trade back) order did not occur, a market-maker become “active” - he lowers or raises his spread. This time interval might be calculated as

$$\Delta t = \log(1/F(j))/|\text{Order}(t_0) - \text{Limit}(j) - \Sigma_i G_{i,j}(t)| + \eta(t_k),$$

These $\text{Limits}$ are distributed according to Pareto law $\sim w^{-3/2}$.

“Fears” can be thought as uniformly distributed in $(0,1)$ across $j$’s.
where positively distributed $\eta(t_k)$ represents randomly appearing opposite orders which can increase his readiness to hold his old spread or, if these orders will be comparable to the first one, even cancel his desire to change his spread.

Whether the distribution of time holdings of the spread according to this strategy has $x^{-3/2}$ tail should be further clarified. However, this scheme provides a missing ingredient of ref. [4], namely upper truncation of the tail of that distribution. The important factor has to be considered - the closedness of the end of the day or the end of the week $\delta t$. With unbalanced inventory a market-maker before the week-end likely changes his spread to recover the balance.

This “truncation” was an important part of the argument given in ref. [4] for the variogram of activity to behave as a square root of time for short times.

**Further Research**

Our next effort will be to uncover the hidden features of the market: number of market-makers, their inventory limits and their friendship matrix. The question, of course, is how. The simplest argument can be borrowed from Solomon and Richmond [6]: the lowest limit (wealth) among market-makers might define exponent of the distribution of their inventory limits. So from tail $x^{-\alpha}$ one can find the lower limit as proportional to $1/(1 - 1/\alpha)$ and thus - the wealth of the poorest market-maker. However, many “friends” of the poorest market-maker can change the tail of the exponent.

We give one more example. Short-ranged correlations in distributions of returns (period from 4 to 15 minutes) can be observed across all markets (see, e.g., [13, p. 54]). This period shows how quickly the price is “recovered” on average or, in our terminology, how quickly an average market-maker can trade back an average amount of stock to recover a previous spread. So a period from 4 to 15 minutes may uniquely characterize a particular market and exact “inverse” methods have to be developed to uncover hidden (from public) variables. And before answering about average strategy of an average player of a “big market” we can answer some questions about market inner circle.

**Conclusions**

We suggested a straightforward approach to attack numerous general laws observed in time series of price returns. The market, no matter how big, is governed by a small group of people called market-makers, whose Pareto-like limits for inventories and - more important - particular behavior (their fear of unbalanced inventories) redefine distribution laws of the entire market. Their “strategies” amount to keeping fixed or to changing their spreads by different amounts. In either case, their true strategy is to uphold fundamental principles of the market.
It means that the so-called “free” economy is in fact predefined by properties of a small group of people who are ready to do their utmost to support its spirit and first principles. In social terminology we call them bureaucracy or government. In the terminology of “market ecology”, together with “predates and prey” (speculators and producers - see [14]) they are true hosts of the forest - foresters.

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