Multiuser Space–Time Line Code With Transmit Antenna Selection

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ABSTRACT In this paper, a multiuser space-time line code (MU-STLC) scheme is newly designed that concurrently delivers multiple STLC signals to multiple users, and a preprocessing matrix for the MU-STLC is derived based on the minimum mean square error criterion. The novel MU-STLC method retains the conventional STLC receiver structure so that each user linearly combines the received signals without using the full channel state information to decode the STLC signals. With more transmit antennas than the number of users having two receive antennas, a transmit antenna selection (TAS) scheme is investigated in combination with the proposed MU-STLC method, and the detection signal-to-interference-plus-noise ratio (SINR) is derived depending on a specific TAS pattern. The performance improvement obtained from the TAS is significant, yet finding the optimal TAS pattern is a combinatorial problem that requires prohibitively high computational complexity. To resolve this issue, a greedy TAS algorithm is also proposed that iteratively selects the transmit antenna maximizing the detection SINR in each greedy step. The numerical results verify the efficacy of the proposed MU-STLC system with the SINR-based greedy TAS algorithm in terms of the bit error rate performance and computational complexity. For example, comparing with a scheme that selects four antennas from eight antennas randomly to support four users, the proposed TAS scheme can reduce the required signal-to-noise ratio for achieving $10^{-3}$ bit-error-rate by approximately 6 dB when quadrature phase-shift keying is employed. Furthermore, the proposed method can achieve comparable performance to the optimal antenna selection scheme with the reduced computational complexity by $O(M^2)$ from $O(M^U+3)$, where $M$ and $U$ are the numbers of transmit antennas and selected antennas (or users), respectively.

INDEX TERMS Space-time line code, multiuser, transmit antenna selection, greedy algorithm.

I. INTRODUCTION Recently, a new full-rate full-spatial diversity achieving scheme, called space–time line code (STLC), was proposed in [1], [2]. The STLC scheme is fully symmetric with the space–time block code (STBC) [3]–[5], also known as Alamouti code [6], based on the reciprocity between a transmitter and a receiver. Precisely, an $M \times N$ STLC system with $M$-transmit and $N$-receive antennas is symmetric with an $N \times M$ STBC system that has $N$-transmit and $M$-receive antennas, where $M \geq 1$ and $N \in \{2, 3, 4\}$. Under the symmetric channel state information (CSI) conditions, i.e., full CSI is required at a transmitter for STLC whereas, at a receiver for STBC, the full spatial-diversity gain is achieved at both systems. The STLC can be interpreted as a precoded STBC using CSI at the transmitter (i.e., CSIT) so that CSI is not necessarily required at the STLC receiver (see Section II). Contrary to the STBC, the STLC can be readily scaled up to an arbitrarily large number of transmit antennas. Moreover, whereas an optimal beamforming scheme with CSIT requires a complexity order of $O(M^3)$ for finding the dominant singular vector, the complexity of the STLC linearly increases with the number of transmit antennas, $O(M)$, and the STLC scheme is more robust against the CSI uncertainty [2]. By the low-complexity linear processing for STLC encoding and decoding, scalability of the number of transmit antennas, and robustness against the CSI uncertainty, the STLC scheme has been applied to various communication systems requiring CSIT, such as multiuser systems [2], [7], two-way relay systems [8], [9], antenna shuffling systems [10], and machine learning-based blind decoding systems [11]. The spatial-domain coding in STLC was applied to a frequency
domain in an orthogonal-frequency division multiplexing system, and the efficacy of STLC was investigated [12]. Since the STLC allows a semi-blind detection at the receiver without full CSI, the STLC transmitter does not need to broadcast a long training or pilot sequence for channel estimation at the receiver, which prevents an unauthorized intruder, i.e., an eavesdropper, from estimating the channels, resulting in an improvement of the physical-layer security [13].

The conventional single-stream STLC was extended to a double STLC that could transmit two STLC streams for point-to-point communications in [14]. The two STLC streams can be separately transferred to two users, i.e., it works as a two-user STLC system. The double STLC scheme minimizes the inter-stream interference based on the minimum mean square error (MMSE) criterion; however, its design is not scalable to multiple users more than two. On the other hand, a multiuser-STLC scheme in [2] and [7] can support an arbitrary number of users. However, these schemes simultaneously transmit multiple data streams without considering multiuser (inter-stream) interferences, thus a large number of transmit antennas are required to suppress the multiuser interferences for reliable communications.

In this study, we derive a multiuser-STLC (MU-STLC) scheme, which can deliver multiple STLC streams to multiple users. Owing to the STLC schemes can provide energy efficiency improvement at the receiver [8], the proposed multiuser-STLC (MU-STLC) scheme can be applicable to a system supporting multiple power/energy-limited devices, simultaneously. For the proposed MU-STLC transmission, the transmitter uses a preprocessing matrix designed in the minimum mean square error (MMSE) sense based on the CSI, and each user having two receive antennas decodes its data stream with the STLC method that can suppress the inter-stream interferences in the MMSE sense, when \( M \geq U \). The proposed MU-STLC can support multi-stream transmission for multiple users through preprocessing at the transmitter without changing the receivers, i.e., the conventional low-complexity STLC decoder is used for user receivers.

Furthermore, a transmit antenna selection (TAS) scheme is proposed to improve the signal-to-interference-plus-noise ratio (SINR) at each user. A TAS scheme alleviates the implementation cost of RF chains for multiple-input multiple-output (MIMO) systems retaining the spatial multiplexing and diversity gain. For example, a multimode TAS method allows any number of data streams to be dynamically selected providing array gain [15], and the feedback rate can be reduced for TAS with Alamouti coding [16]. Also, TAS methods can be combined with a wire-tap channel without CSIT, a massive MIMO channel, and a non-orthogonal multiple access system for secure communications [17]–[19], and the use of TAS has been investigated in spatial modulation and millimeter wave MIMO systems for performance enhancement [20], [21]. In an MU-STLC system, TAS is performed by finding the antenna subset minimizing the mean square error (MSE) (or equivalently maximizing the detection SINR), and the optimal TAS design requires \( O(M^{U+3}) \) computational complexity which is prohibitively large especially when \( M \) and/or \( U \) is large. In an attempt to reduce the complexity, an SINR-based greedy TAS algorithm is proposed. Through complexity analysis, it is shown that the complexity order of the proposed TAS method is \( O(M^5) \). Through numerical simulations, the complexity analysis is confirmed, and it is verified that the proposed MU-STLC with SINR-based greedy TAS can achieve near-optimal bit-error-rate (BER) performance with a noticeably reduced computational load. For example, comparing with a scheme that selects four antennas from eight antennas randomly to support four users, namely, \( M = 8 \) and \( U = 4 \), the proposed TAS scheme can reduce the required signal-to-noise ratio (SNR) for achieving \( 10^{-3} \) BER by approximately 6 \( \text{dB} \) when quadrature phase-shift keying (QPSK) is employed.

Notations: Superscripts \( T \), \( H \), \( * \), and \( −1 \) denote transposition, Hermitian transposition, complex conjugate, and inversion, respectively, for any scalar, vector, or matrix. The notations \( |x| \) and \( \|X\|_F \) denote the absolute value of \( x \) and the Frobenius-norm of matrix \( X \), respectively; \( I_m \) and \( 0_m \) represent an \( m \)-by-\( m \) identity matrix and a zero matrix, respectively; \( \text{tr}(A) \) is the trace operation of matrix \( A \); \( \text{blkdiag}(X_1 \cdots X_m) \) returns a block diagonal matrix whose main diagonal matrices are \( X_1 \cdots X_m \); and \( x \sim \mathcal{CN}(0, \sigma^2) \) means that a complex random variable \( x \) conforms to a complex normal distribution with a zero mean and variance \( \sigma^2 \). \( \mathbb{E}[x] \) stands for the expectation of random variable \( x \).

II. REVIEW AND COMPARISON OF SINGLE-USER STBC AND STLC SYSTEMS

We briefly introduce a \( 2 \times 1 \) STBC system in [6] and a \( 1 \times 2 \) STLC system in [1] to clarify the difference between them and to describe the encoding and decoding procedure of a \( 1 \times 2 \) STLC method which is extended to the new \( M \times 2U \) MU-STLC system.

A. \( 2 \times 1 \) STBC SYSTEM

As shown in Fig. 1, two information symbols \( s_{1,1} \) and \( s_{1,2} \) with \( \mathbb{E}[|x|^2] = 1 \) are encoded as [6]

\[
\begin{bmatrix}
  s_{1,1} \\
  s_{1,2}
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix},
\]

(1)

where \( s_{m,t} \) is the STBC symbols transmitted at the \( m \)th antenna and time \( t \). The receiver rearranges the received
signals $r_1$ and $r_2$ and then decodes the transmit symbols $x_1$ and $x_2$ using the orthogonality of the STBC encoded symbols in (1). The decoding SNR of STBC is given by

$$\rho_{\text{STBC}} = \frac{\gamma}{2\sigma_z^2},$$  \hspace{1cm} (2)

where $\gamma = |h_1|^2 + |h_2|^2$ is the effective channel gain and $\sigma_z^2$ is the per-antenna noise variance. Here, note that full diversity gain is achieved by using full CSI at the STBC receiver.

### B. ALTERNATIVE FORM OF 1 × 2 STLC SYSTEM

When we use a type-five STLC structure in [1, Table 4], the alternative model of $1 \times 2$ STLC shown in Fig. 2 is given by [14]

$$[s_1 \hspace{1cm} s_2] = \frac{1}{\sqrt{2}} \mathbf{h}^H \begin{bmatrix} x_1 \\ x_2 \\ -x_1^2 \\ x_2^2 \end{bmatrix},$$  \hspace{1cm} (3)

where $s_t$ is the STLC symbol transmitted at time $t$ and $\mathbf{h} = [h_1 \hspace{0.2cm} h_2]^T \in \mathbb{C}^{2\times 1}$ is the channel vector. Denoting $r_{n,t}$ is the received signal at antenna $n$ at time $t$, the four received signals are then written in a matrix form as follows:

$$\begin{bmatrix} r_{1,1} \\ r_{2,1} \\ r_{1,2} \\ r_{2,2} \end{bmatrix} = \mathbf{h} [s_1 \hspace{0.2cm} s_2] + \mathbf{z} \in \mathbb{C}^{2 \times 2},$$  \hspace{1cm} (4)

where $r_t \in \mathbb{C}^{2\times 1}$ is the received signal vector at time $t$; $\mathbf{z} = [z_1 \hspace{0.2cm} z_2] \in \mathbb{C}^{2\times 2}$ is a noise matrix whose $(n, t)$th element $z_{n,t}$ is the AWGN at $r_{n,t}$ distributed with $CN(0, \sigma^2_z)$.

By stacking $r_1 = [r_{1,1} \hspace{0.2cm} r_{1,2}]^T$ on the conjugate of $r_2 = [r_{2,1} \hspace{0.2cm} r_{2,2}]^T$, we construct the alternative form of the received signal vector as follows:

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1^H h_1^* + h_2^H h_2^* \\ h_1^H h_2^* - h_2^H h_1^* \\ (h_1 h_2^*)^* - (h_2 h_1^*)^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix},$$  \hspace{1cm} (5)

where $\mathbf{Q}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $x = [x_1 \hspace{0.2cm} x_2]$, and $z = [z_1 \hspace{0.2cm} z_2]$.

Using (5), the STLC combining procedure can be represented as below [14]:

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}^T \begin{bmatrix} I_2 \\ \mathbf{Q}_2^T \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = r_1 + Q_2^T r_2$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} h h^H + Q_2^T (h h^H)^* Q_2 \end{bmatrix} x + z'$$

$$= \sqrt{\gamma} x + z' \in \mathbb{C}^{2\times 1},$$  \hspace{1cm} (6)

where $z' = z_1 + Q_2^T z_2^* \epsilon$ and $E[z'(z')^H] = 2\sigma_z^2 I_2$.

The resulting decoding SNR after the STLC combining is readily derived from (6) as

$$\rho_{\text{STLC}} = \frac{\gamma}{2\sigma_z^2},$$  \hspace{1cm} (7)

which is identical to the decoding SNR of STBC in (2). From (7), it is verified that $1 \times 2$ STLC achieves the same performance as $2 \times 1$ STBC in terms of the diversity gain and array gain. Note that only partial CSI $\gamma$ is required at the STLC receiver. We extend the alternative model of $1 \times 2$ STLC to the proposed MU-STLC supporting multiple users in the next section.

### III. PROPOSED MU-STLC SYSTEM

We consider an $M$-by-$2U$ MU-STLC system, in which a transmitter with $M$-transmit antennas support $U$ users having two receive antennas for STLC as shown in Fig. 3. Here, we assume that the transmitter has a sufficient number of antennas as $M \geq U$. Let $x_{n,t}$ be the $t$th modulated information symbol that is transmitted to user $u \in U = \{1, 2, \ldots, U\}$, with $E[|x_{n,t}|^2] = \sigma^2_x$. Then, the proposed MU-STLC signals are defined as

$$S = \begin{bmatrix} s_{U,1} \\ s_{U,2} \end{bmatrix} = VX \in \mathbb{C}^{U \times 2},$$  \hspace{1cm} (8)

where $X = [X_1 \cdots X_u \cdots X_U]^T \in \mathbb{C}^{2U \times 2}$ and

$$X_u = \begin{bmatrix} x_{u,1} \\ x_{u,2} \end{bmatrix} \in \mathbb{C}^{2 \times 2}.$$  \hspace{1cm} (9)

Here, $V \in \mathbb{C}^{U \times 2U}$ is the MU-STLC preprocessing matrix, such that $\|V\|_F^2 = 1$. Herein, our objective is to design an optimal preprocessing matrix, $V$, to support multiple STLC users, who independently perform a single-user STLC decoding process in (6) to decode their information signals.

The receive signals with TAS are then modeled as

$$\begin{bmatrix} r_{U,1} \\ r_{U,2} \end{bmatrix} = \mathbf{HPS} + \mathbf{z} \in \mathbb{C}^{2U \times 2},$$  \hspace{1cm} (10)

where $r_{n,t} \in \mathbb{C}^{2\times 1}$ is the received signal vector, whose $n$th element $r_{n,u,t}$ is the received signal at the $n$th receive antenna of user $u$ at time $t$; $H = [h_1 \cdots h_m \cdots h_U]^T$ and $h_m \in \mathbb{C}^{2\times 1}$ is a channel vector between the $m$th transmit antenna and users, which is static for $t = 1$ and 2 and $E[h_m h_m^H] = I_M$; $P = [p_1 \cdots p_U]$ and $p_u$ is an $M$-by-1 antenna selection vector for the $U$th STLC symbols, namely $s_{U,1}$ and $s_{U,2}$, whose $i$th element $p_{k,i} = 1$ if the $i$th transmit antenna is selected, and $p_{k,i} = 0$ otherwise; and $Z \in \mathbb{C}^{2U \times 2}$ is a noise matrix whose elements are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean.

Note that the STLC preprocessing matrix $V$ in (8) enables the user to decode the STLC signals through the simple linear combining without full CSI, and the STBC structure in (9) will provide the full spatial diversity gain of the STLC users and symmetric properties to the STLC.
and variance $\sigma_z^2$. Here, $\sum_k p_{k,i} = 1$, $\forall i$ for the orthogonal antenna selection. If the $k$th antenna is selected, $\sum_i p_{k,i} = 1$, and otherwise, $\sum_i p_{k,i} = 0$.

Let $\mathcal{L} = \{l_1, \ldots, l_u, \ldots, l_M\}$ be the set of the selected antenna indices, where $l_i \in \{1, \ldots, M\}$ is the antenna index selected for user $u$, and $l_i \neq l_{i'}$ when $u \neq u' \in \mathcal{U}$. Then the received signals in (10) can be rewritten as

$$H_{PS} + Z \triangleq H_{L} VX + Z \in \mathbb{C}^{2U \times 2},$$

where $H_{L}$ is the selected channel matrix, which is given as

$$H_{L} = [h_{l_1}, \ldots, h_{l_u}, \ldots, h_{l_M}] \in \mathbb{C}^{2U \times U} \quad (12)$$

For the MU-STLC decoding, (11) is alternatively reconstructed in a linear form as

$$r \triangleq \begin{bmatrix} r_{1,1}^H & \cdots & r_{U,1}^H & r_{1,2}^H & \ldots & r_{U,2}^H \end{bmatrix}^T$$

$$= \begin{bmatrix} H_{L} \sqrt{V} \left( (H_{L} V)^* Q_{2U} \right) \end{bmatrix} x + z \in \mathbb{C}^{2U \times 1},$$

where

$$x = \begin{bmatrix} x_{1,1} x_{1,2} \ldots x_{u,1} x_{u,2} \ldots x_{U,1} x_{U,2} \end{bmatrix}^T; \quad (14a)$$

$$Q_{2U} = \text{blkdiag} \left( Q_{2}, \ldots, Q_{2} \right) \in \mathbb{C}^{2U \times 2U}; \quad (14b)$$

$$Q_{2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$  

(14c)

and $z \in \mathbb{C}^{2U \times 1}$ is the corresponding AWGN vector with $\mathbb{E}[zz^H] = \sigma_z^2 I_{2U}$.

User $u$ performs STLC combining in (6) as $ru_u + Q_{2}^T r_{u,2}^*$; therefore, the multiuser combined-STLC received signals are represented as follows:

$$y = \begin{bmatrix} I_{2U} & Q_{2U} \end{bmatrix} r$$

$$= \begin{bmatrix} I_{2U} & Q_{2U} \end{bmatrix} \left( H_{L} \sqrt{V} \left( (H_{L} V)^* Q_{2U} \right) \right) x + \begin{bmatrix} I_{2U} & Q_{2U} \end{bmatrix} z$$

$$= \left( H_{L} \sqrt{V} + Q_{2U}^T (H_{L} V)^* Q_{2U} \right) x + z'$$

$$= \left( H_{L} Q_{2U}^* \right) \left( V^* Q_{2U} \right) x + z'$$

$$\triangleq H_{L} V x + z',$$

(15)

where

$$H_{L} \triangleq \begin{bmatrix} H_{L} \quad Q_{2U}^* \end{bmatrix} \in \mathbb{C}^{2U \times 2U},$$

$$V \triangleq \begin{bmatrix} V^T \quad Q_{2U}^T V^H \end{bmatrix} \in \mathbb{C}^{2U \times 2U},$$

and the combined AWGN vector $z' \in \mathbb{C}^{2U \times 1}$ conforms to the distribution $CN(0_{2U}, 2\sigma_z^2 I_{2U})$.

The error vector after the detection of $x$ from (15) is then defined as

$$e = cy - x = (cH_{L}V_{t} - I_{2U}) x + cz',$$

(17)

where $1/c$ is the real-value effective channel gain from STLC. Let us define $W_{t}$ as

$$W_{t} \triangleq c V_{t}$$

$$= \begin{bmatrix} V \\ V^* Q_{2U} \end{bmatrix}.$$  

(18)

From the definition in (18), we can derive that

$$c = \frac{\|W_{t}\|_F}{\sqrt{2\|V\|_F}} = \frac{\|W_{t}\|_F}{\sqrt{2}},$$

(19)

and rewrite the error vector in (17) as

$$e = (H_{L} W_{t} - I_{2U}) x + \frac{1}{\sqrt{2}} \|W_{t}\|_F z'.$$

(20)

The MSE, $J(W_{t})$, is then derived as (21) at the bottom of the next page. The optimal $W_{t}$ that minimizes MSE can be obtained by solving the first-order optimality condition, i.e.,

$$\frac{\partial J(W_{t})}{\partial W_{t}^*} = \sigma_z^2 H_{L}^H (H_{L} W_{t} - I_{2U}) + 2U \sigma_z^2 W_{t}$$

$$= 0_{2U}.$$  

(22)

From (22), the optimal $W_{t}$ is obtained as follows:

$$W_{t,o} = H_{L}^H \left( H_{L} H_{L}^H + \frac{2U \sigma_z^2}{\sigma_z^2} I_{2U} \right)^{-1}$$

$$= \left[ \begin{array}{c} H_{L}^H \\ H_{L}^H Q_{2U} \end{array} \right] \Delta_{L}.$$  

(23)
From (18) and (23), we obtain the following equality
\[ \Delta_{\mathcal{L}} = \left( H_{\mathcal{L}} H_{\mathcal{L}}^H + \frac{2U \sigma_z^2}{\sigma_x^2} I_{2U} \right)^{-1} \]
\[ = \left( [H_{\mathcal{L}} Q_{2U}^H H_{\mathcal{L}}^H] \frac{H_{\mathcal{L}}^H}{H_{\mathcal{L}}^H Q_{2U}^H} \right) + \frac{2U \sigma_z^2}{\sigma_x^2} I_{2U} \right)^{-1} \]
\[ = \left( H_{\mathcal{L}} H_{\mathcal{L}}^H + Q_{2U}^H H_{\mathcal{L}}^H Q_{2U} + \frac{2U \sigma_z^2}{\sigma_x^2} I_{2U} \right)^{-1}. \] (24)

From (18) and (23), we obtain the following equality
\[ W_{t,o} = \left[ H_{\mathcal{L}}^H \Delta_{\mathcal{L}} \right] \frac{H_{\mathcal{L}}^H}{H_{\mathcal{L}}^H Q_{2U}^H \Delta_{\mathcal{L}}} \]
\[ = c \left[ V^* Q_{2U} \right], \] (25)
which shows the optimal \( V_o \), i.e., \( V_o = \frac{1}{2} H_{\mathcal{L}}^H \Delta_{\mathcal{L}} \). Considering the definition that \( \|V\|_F^2 = 1 \), \( c = \|H_{\mathcal{L}}^H \Delta_{\mathcal{L}}\|_F \) and the MMSE-based optimal MU-STLC preprocessing matrix \( V_o \) is obtained from (25) as
\[ V_o = \frac{H_{\mathcal{L}}^H \Delta_{\mathcal{L}}}{\|H_{\mathcal{L}}^H \Delta_{\mathcal{L}}\|_F} \in \mathbb{C}^{U \times 2U}. \] (26)

In summary, the MU-STLC transmitter transmits \( S \) in (8) using the MU-STLC preprocessing matrix \( V_o \) in (26), and user \( u \) obtain the estimates of \( x_{u,1} \) and \( x_{u,2} \), denoted by \( \hat{x}_{u,1} \) and \( \hat{x}_{u,2} \), respectively, as (from (15) and (17))
\[ \begin{bmatrix} \hat{x}_{u,1} \\ \hat{x}_{u,2} \end{bmatrix} = c \begin{bmatrix} J_2 \\ Q_2^* \end{bmatrix} \begin{bmatrix} r_{u,1} \\ r_{u,2} \end{bmatrix}, \] (27)
which is the same as the conventional single-user STLC decoding process [1], [14]. Here, note that the effective channel gain, i.e., \( 1/c \), is not required to detect phase-shift keying (PSK) modulated symbols, whereas it needs to be estimated for the non-PSK modulated symbol detection. In other words, only partial CSI is required at the STLC users (receivers). The effective channel gain can be estimated through the blind estimation schemes [22]–[24] and/or a training-based machine learning algorithm [11].

**IV. TRANSMIT ANTENNA SELECTION STRATEGIES**

In the previous section, we design the MU-STLC preprocessing matrix. We now propose a TAS strategy that maximizes the detection SINR. To this end, we derive the detection SINR of the MU-STLC system using the optimal preprocessing \( V_o \) in (26). By substituting \( W_{t,o} \) in (23) into \( W_t \) in (21), the minimum MSE, denoted by \( J_{\min} \), is derived as follows:
\[ J_{\min} = \text{tr} \left( \sigma_x^2 (H_t W_{t,o} - I_{2U}) (H_t W_{t,o} - I_{2U})^H \right) \]
\[ + 2U \sigma_z^2 W_{t,o} W_{t,o}^H \]
\[ = \text{tr} \left( \sigma_x^2 (W_{t,o} - I_{2U}) (H_t W_{t,o} - I_{2U}) \right) \]
\[ + 2U \sigma_z^2 W_{t,o} W_{t,o}^H \]
\[ = \text{tr} \left( \sigma_x^2 (W_{t,o} - I_{2U}) (H_t W_{t,o} - I_{2U}) + 2U \sigma_z^2 W_{t,o} W_{t,o}^H \right) \]
\[ \equiv \sigma_x^2 \left( -I_{2U} - H_t H_t^H \right) \Delta_{\mathcal{L}} \right) \]
\[ = \sigma_x^2 \text{tr} \left( (\Delta_{\mathcal{L}}^{-1} - H_t H_t^H) \Delta_{\mathcal{L}} \right) \]
\[ = \sigma_x^2 \text{tr} \left( \frac{2U \sigma_z^2}{\sigma_x^2} I_{2U} \Delta_{\mathcal{L}} \right) \]
\[ = 2U \sigma_z^2 \text{tr} \left( \Delta_{\mathcal{L}} \right), \] (28)
where \((a)\) comes from (22) and \((b)\) follows the definition in (24). Consequently, the detection SINR is derived as follows:

\[
\rho_L = \frac{\mathbb{E}[x^H x]}{\mathbb{E}[\mathcal{J}]_{\min}} - 1 = \frac{\sigma_s^2}{\sigma_r^2 \text{tr} (\mathbf{A}_L)} - 1. \tag{29}
\]

The TAS that maximizes the detection SINR in (29) is then designed by solving the following optimization problem:

\[
\max_{\mathcal{L} \subseteq \mathcal{A}_M} \rho_{\mathcal{L}} \equiv \min_{\mathcal{L} \subseteq \mathcal{A}_M} \text{tr} (\mathbf{A}_L), \tag{30}
\]

where \(\mathcal{A}_M\) is the set of all possible selected antenna combinations \(\mathcal{L}\) from \(M\) transmit antennas, for example, \(\mathcal{A}_4 = \{|1, 2|, \{1, 3|, \{1, 4|, \{2, 3|, \{2, 4|, \{3, 4|\}\} when \(M = 4\) and \(U = 2\).

In general, for \(M\) transmit antennas,

\[
\binom{M}{U} = \frac{M(M - 1) \times \cdots \times (M - U + 1)}{U!}
\]

combinations are possible for the TAS pairs. Since \(O(M^3)\)-complexity is typically required to compute (29), the overall computational complexity of the proposed TAS is \(O(M^U+3)\), which is prohibitive if \(M\) is massive.

To reduce the TAS complexity, the channel-norm (CN)-based greedy TAS algorithm shown in Algorithm 1 can be considered.

**Algorithm 1** Channel-Norm-Greedy-Based TAS Algorithm

1. Initialization: \(\mathcal{L} = \emptyset\) and \(\mathcal{M} = \{1, \ldots, M\}\)
2. for \(u = 1 : U\) do
3. Find the antenna that has the strongest norm of the channel vector \(\mathbf{h}_m\), such that \(m = \arg \max_{m \in \mathcal{M}} \|\mathbf{h}_m\|\)
4. Update the selected antenna set: \(\mathcal{L} = \mathcal{L} \cup \{m\}\)
5. Update the antenna set: \(\mathcal{M} = \mathcal{M} \setminus \{m\}\)
6. end for

The norm of the \(2U\)-by-1 channel vector for all \(m \in \mathcal{M}\), i.e., \(\|\mathbf{h}_m\|\), \(\forall m \in \mathcal{M}\), is required for Algorithm 1, and thus the computational complexity is \(O(UM)\). Although the computational complexity can be significantly reduced by the CN-based greedy TAS algorithm compared to the optimal TAS strategy in (30), the performance improvement of the TAS is marginal as shown in Section V. To further improve the TAS gain, we propose a novel greedy algorithm that initially selects the transmit antenna with the largest \(\|\mathbf{h}_m\|\) and then iteratively finds the antenna minimizing the trace of MSE (or equivalently maximizing the detection SINR) from remaining transmit antennas in each greedy step. The proposed SINR-based greedy TAS algorithm is summarized in Algorithm 2.

Next, we consider the computational complexity of Algorithm 2. For the \(i\)th greedy step, (24) in line 8 needs to be computed \(|\mathcal{M}|\) times, and each calculation has a complexity of \(O(U^3)\) since \(\mathbf{H}_L\) is a \(2U\)-by-1 matrix. Noting that \(|\mathcal{M}| = M - i - 1\) in the \(i\)th greedy step, the upper bound of the overall computational complexity is given by

\[
O \left( \sum_{i=1}^{U-1} (M - i - 1)(2U)^3 \right) = O \left( U^3(2MU - M - U - U^2 + 2) \right) \leq O \left( MU^4 \right) \leq \frac{a}{a} O \left( M^5 \right), \tag{31}
\]

where \((a)\) comes from the fact that the maximum supportable number of users is limited by \(M/2\), i.e., \(U \leq M/2\).

**V. PERFORMANCE EVALUATION AND DISCUSSION**

In this section, we compare the performance of various schemes that can support multiple users. In the first part, the received SINRs of multuser STLC schemes are compared. In the second part, the computational complexity of the SINR-greedy TAS and optimal TAS methods are compared for the proposed MU-STLC scheme. In the last part, BERs of the proposed MU-STLC and the conventional space-division multiple access (SDMA) schemes in [25]-[27] are compared.

**A. SINR COMPARISON**

The transmitter has \(M\) transmit antennas to support \(U\) users \((M \geq U)\). For the TAS schemes, the proposed SINR-greedy TAS scheme is compared to a random TAS scheme, CN-greedy scheme, and optimal TAS scheme. Furthermore, we compare the received SINRs of the proposed MU-STLC scheme and the existing STLC scheme in [2]. The compared systems are summarized as follows:

- \((M \rightarrow U) \times 2U\) MU-STLC w/ random TAS: This is an MU-STLC system, where \(U\) transmit antennas are randomly selected from \(M\) antennas for MU-STLC.

**Algorithm 2** SINR-Greedy-Based TAS Algorithm

1. Initialization: \(\mathcal{L} = \emptyset\) and \(\mathcal{M} = \{1, \ldots, M\}\)
2. The same as procedure 3 of Algorithm 1
3. Update the selected antenna set: \(\mathcal{L} = \mathcal{L} \cup \{m\}\)
4. Update the antenna set: \(\mathcal{M} = \mathcal{M} \setminus \{m\}\)
5. for \(i = 1 : U - 1\) do
6. for \(m = 1 : |\mathcal{M}|\) do
7. Construct a partial channel matrix: \(\mathbf{H}_L \triangleq [\mathbf{H}_L \mathbf{h}_{M(m)}] \in \mathbb{C}^{2U \times (i+1)}\), where \(\mathcal{M}(m)\) is the \(m\)th element of \(\mathcal{M}\)
8. Compute \(\mathbf{A}_L\) using \(\mathbf{H}_L\) in (24).
9. \(\text{cost}(m) = \text{tr}(\mathbf{A}_L)\)
10. end for
11. Select an antenna index: \(m^* = \arg \min_m \text{cost}(m)\)
12. Update the selected antenna set: \(\mathcal{L} = \mathcal{L} \cup \{\mathcal{M}(m^*)\}\)
13. Update the antenna set: \(\mathcal{M} = \mathcal{M} \setminus \{\mathcal{M}(m^*)\}\)
14. end for
**FIGURE 4.** SINR of multiuser STLC schemes when $M = 80$ and $U = 2$. '(M → U)' represents that $U$ transmit antennas are selected from $M$ antennas.

**FIGURE 5.** SINR of multiuser STLC schemes when $M = 8$ and $U = 2$. '(M → U)' represents that $U$ transmit antennas are selected from $M$ antennas.

**FIGURE 6.** SINR of multiuser STLC schemes when $M = 8$ and $U = 4$. '(M → U)' represents that $U$ transmit antennas are selected from $M$ antennas.

- $(M → U) \times 2U$ MU-STLC w/ CN-greedy TAS: Channel-norm-based greedy (CN-greedy) algorithm, i.e., Algorithm 1, is used to select $U$ transmit antennas from $M$ for MU-STLC.

- $(M → U) \times 2U$ MU-STLC w/ SINR-greedy TAS: SINR-based greedy (SINR-greedy) algorithm, i.e., Algorithm 2, is used to select $U$ transmit antennas from $M$ antennas for MU-STLC.
Here, it is shown that the proposed MU-STLC w/ SINR-greedy scheme achieves near-optimal TAS performance. However, the performance of a CN-greedy TAS scheme is similar to that of a random TAS scheme. This implies that the CN-greedy does not capture the multiuser interference effectively. On the other hand, the conventional multiuser STLC scheme in [2] outperforms the proposed MU-STLC scheme when SNR is low, however, the SINR of the conventional multiuser STLC scheme is saturated as SNR increases. Furthermore, note that the conventional multiuser STLC scheme uses the whole transmit antennas, i.e., $M$, whereas the proposed MU-STLC uses $U$ selected transmit antennas.

In Fig. 5, the SINRs are compared when the number of transmit antennas is small, i.e., $M = 8$. In general, the SINRs of all schemes decrease as $M$ decreases. Here, the SINR achieved by the proposed MU-STLC with SINR-greedy TAS scheme is slightly lower than the MU-STLC with optimal TAS scheme. However, the SINR of the conventional multiuser STLC decreases significantly, approximately 10 dB. This is because the multiuser STLC schemes in [2] do not consider multiuser (inter-stream) interferences, thus the insufficient number of transmit antennas cannot effectively suppress the multiuser interferences. As the number of users increases to $U = 4$, the conventional multiuser STLC schemes do not achieve comparable performance to the proposed MU-STLC schemes as shown in Fig. 6.

B. COMPLEXITY COMPARISON OF TAS METHODS

As shown in Fig. 7, where the run time is shown in log scale, the complexity of the optimal TAS increases exponentially as $M$ increases when $U = M/2$. On the other hand, the complexity of the proposed SINR-based greedy TAS increases moderately. As observed here, the complexity increases as $M$ increases and the complexity analyses match well with the numerical results.
C. BER COMPARISON
BER is evaluated for verifying the proposed SINR-based greedy TAS, where the proposed MU-STLC transmitter has eight transmit antennas to support \( U \) users \((M = 8 \geq U)\), and QPSK is employed. For the BER comparison, we consider two SDMA schemes in [25]–[27]. One SDMA scheme uses \( U \) transmit antennas, for the fair comparison with the MU-STLC using \( U \) selected antennas. The other SDMA scheme uses \( M \) transmit antennas.

In Fig. 8, the BER performance is compared across the system SNR, i.e., \( \sigma_x^2/\sigma_z^2 \), when \( U = 4 \). From the results, we can observe that the performance improvement from the CN-based greedy TAS is marginal. On the other hand, the proposed SINR-based greedy TAS achieves near-optimal performance. The \( 4 \times 8 \) SDMA achieves poor performance due to the lack of the degree of freedom for beamforming after the spatial division as \( U > M/2 \). Note that \( 8 \times 8 \) SDMA achieves the best performance, yet it uses eight transmit antennas (not four) and full CSI at both transmitter and receiver. Even though the SINR-based TAS STLC uses four selected antennas, it outperforms the \( 8 \times 8 \) SDMA system in the high-SNR regime due to the antenna selection diversity.

In Fig. 9, the BERs are evaluated across \( U \) when \( M = 8 \) and the SNR is 10 dB. Due to the increase of the multiuser interference, the BER performance is degraded for all schemes as \( U \) increases. The proposed SINR-based TAS outperforms \( M \times 2U \) SDMA when \( U > M/2 \). It is verified that the TAS can improve BER performance regardless of the number of users. Furthermore, the proposed SINR-based greedy TAS achieves near-optimal performance with significantly reduced computational complexity. In Fig. 7, the complexity analyses in Section IV are verified by comparing with run-time simulation results.

VI. CONCLUSION
In this study, we designed an MU-STLC system supporting multiple STLC users, and then proposed a novel SINR-based greedy TAS scheme. The proposed method significantly improves the BER performance at the cost of a moderate increase in the computational load. The proposed MU-STLC can be applied to a multiuser multi-antenna system requiring low-complexity and low-cost user devices.

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