Collective motion in quantum diffusive environment

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Abstract

The general problem of dissipation in macroscopic large-amplitude collective motion and its relation to energy diffusion of intrinsic degrees of freedom of a nucleus is studied. By applying the cranking approach to the nuclear many-body system, a set of coupled dynamical equations for the collective classical variable and the quantum mechanical occupancies of the intrinsic nuclear states is derived. Different dynamical regimes of the intrinsic nuclear motion and its consequences on time properties of collective dissipation are discussed.

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The appearance of dissipation for large-amplitude collective motion in nuclei is still an unsolved problem. The transport models of the nuclear collective motion like the linear response theory \cite{1} or the wall-formula approach \cite{2} assume a priori that the collective dynamics is adiabatically slow, such that the fast intrinsic nucleonic subsystem has always sufficient time to adjust to the large changes of collective deformation parameters. In that case one can say that statistical equilibrium for the fast intrinsic subsystem is established instantaneously providing the essentially Markovian equations of motion for the collective variables.

In the general case the adiabaticity of the collective motion must not be implied a priori, and one should consider selfconsistently dynamics of the collective and intrinsic nucleonic degrees of freedom. This is quite important when we are dealing with nuclear fission at high excitation energies or the initial stage of heavy ion collisions, i.e., when the typical times for the macroscopic collective and intrinsic nucleonic motions are of the comparable size. Here one would rather expect a non-Markovian collective dynamics caused by the complex energy flow between the macroscopic collective and intrinsic nucleonic modes.

Memory (non-Markovian) effects in a time evolution of the collective parameters have been studied within the linear response theory \cite{3}, the time-dependent shell-model theory \cite{4}, Fermi-liquid model \cite{5,6} and etc. If in all these approaches the main focus is made on the non-Markovian collective motion, we shall concentrate on the selfconsistent description of the dynamics of the collective and nucleonic degrees of freedom. The complex intrinsic nuclear motion at the high excitation energies can be described within random matrix theory. This provides a measure how different dynamic regimes of the nucleonic excitations show up in the corresponding dissipative properties of the collective motion.

The plan of the paper is as follows. In Sect. \textbf{II} we start from the cranking approach to nuclear many-body problem. Sect. \textbf{III} is devoted to the quantum-mechanical description of the intrinsic nuclear excitations. In Sect. \textbf{IV} we derive a system of coupled equations for the slow collective and fast intrinsic modes of the nuclear many-body motion and measure how the energy diffusion of the quantum-mechanical occupancies of the nuclear states defines the time properties of the collective friction. We apply our model to the description of nuclear fission dynamics on the part of descent from fission barrier to scission point in

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Finally, conclusions and discussion of the main results of the paper are given in the Summary.

II. NUCLEAR MANY-BODY SYSTEM

The total energy of the nucleus under collective excitation $\Xi_{\text{tot}}$ may be written as

$$
\Xi_{\text{tot}} = E_{\text{pot}}(q) + \frac{1}{2}B(q)\dot{q}^2 + E^*(t),
$$

(1)

where $q(t)$ is a single classical collective variable (a "nuclear deformation"), $E_{\text{pot}}$ is the collective potential energy, $B$ is the collective mass coefficient and $E^*$ is the excitation energy of the intrinsic nucleonic degrees of freedom. Writing the energy of the nucleus in the form of Eq. (1), we pick out explicitly the contribution from the virtual transition between the nuclear states, which gives rise to a collective kinetic energy term $(1/2)B\dot{q}^2$, and the contribution from the real nuclear transitions leading to the intrinsic excitation energy $E^*$.

Since the total energy of the nucleus is conserved, we can derive an equation of motion for the classical collective variable $q$ by differentiating with respect to time the both sides of Eq. (1),

$$
B(q)\ddot{q} = \frac{1}{2}\frac{\partial B(q)}{\partial q} \dot{q}^2 - \frac{\partial E_{\text{pot}}(q)}{\partial q} - \frac{1}{\dot{q}} \frac{dE^*(t)}{dt}.
$$

(2)

To study how the dissipation in the collective motion may arise, we shall derive an expression for the intrinsic excitation energy $E^*$.

III. INTRINSIC QUANTUM DIFFUSIVE DYNAMICS

We treat intrinsic nucleonic motion of the nucleus quantum–mechanically and start from the Liouville equation for the density matrix operator $\hat{\rho}$,

$$
i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}(q[t]), \hat{\rho}],
$$

(3)

where $\hat{H}$ is the nuclear many–body Hamiltonian. A moving basis is introduced as eigenstates of the time–independent Hamiltonian,

$$
\hat{H}(q)\Psi_n(q) = E_n(q)\Psi_n(q).
$$

(4)
That is determined by a set of the static many–body wave functions $\Psi_n$ and energies $E_n$ found at each fixed value of the collective variable $q$.

Now one can rewrite Eq. (3) as

$$i\hbar \frac{\partial \rho_{nm}}{\partial t} = \sum_n \{ W_{in} \rho_{nl} - W_{nl} \rho_{im} \}$$

with

$$\rho_{nl} = \exp \left( -i \int_0^t \omega_{nl}(q[t']) dt' \right) \langle \Psi_n | \hat{\rho} | \Psi_l \rangle,$$  

$$W_{nl} = \exp \left( -i \int_0^t \omega_{nl}(q[t']) dt' \right) \langle \Psi_n | i \frac{\partial}{\partial t} | \Psi_l \rangle,$$

and $\omega_{nl} = (E_n - E_l)/\hbar$.

By applying the Zwanzig’s projection method on the system of coupled equations (5), we obtain an equation of motion for the diagonal part of the density matrix $\rho_{nn}$

$$\frac{\partial \rho_{nn}(t)}{\partial t} = \int_0^t ds \sum_m \{ W_{nm}(t) W_{mn}(s) [\rho_{mm}(s) - \rho_{nn}(s)] + c.c. \},$$

which determines the intrinsic excitation of the nucleus $E^*$ at time $t$ and which should be supplemented by the normalization condition,

$$\sum_n \rho_{nn} = 1.$$  

c.c. in Eq. (8) stands for the complex conjugation.

Using Eqs. (7) and (4), functions $W_{nm}$ in Eq. (8) may be written as

$$W_{nm}(t) = i q(t) \exp \left( -i \int_0^t \omega_{nm} dt' \right) \frac{\langle \Psi_n | \hat{\rho} / \partial q | \Psi_m \rangle}{E_n - E_m} (q[t]).$$

Substituting these expressions into into Eq. (8) and assuming that the matrix elements $\langle \Psi_n | \hat{H} / \partial q | \Psi_m \rangle$ and energy distances $E_n - E_m$ rapidly fluctuate with time, one has

$$\frac{\partial \rho_{nn}(t)}{\partial t} = 2 \dot{q}(t) \sum_m \left[ \left| \langle \Psi_n | \hat{H} / \partial q | \Psi_m \rangle \right|^2 \right] \int_0^t ds \dot{q}(s) \cos \left( \frac{(E_n - E_m)}{\hbar(t-s)} \right) [\rho_{mm}(s) - \rho_{nn}(s)].$$

At high excitation energies, the nuclear spectrum is very complex and can be described by a random matrix theory. In the random matrix theory approach we ensemble average randomly distributed energy distances $E_n - E_m$ and $E_m$ and squared off-diagonal matrix elements $|\hbar|^2 \equiv \langle \Psi_n | \hat{H} / \partial q | \Psi_m \rangle|^2$,

$$\frac{\partial \bar{\rho}(E_n, t)}{\partial t} = 2 \dot{q}(t) \int_0^t ds \dot{q}(s) \int_{E_n}^{+\infty} dE_m R(|E_n - E_m|) \Omega(E_m) \cos \left( \frac{(E_n - E_m)}{\hbar(t-s)} \right) \cos \left( \frac{(E_n - E_m)}{\hbar(t-s)} \right) \frac{|\hbar|^2 \Omega(E_n, E_m, q[t]) [\bar{\rho}(E_m, s) - \bar{\rho}(E_n, s)]}{(E_n - E_m)^2},$$

(12)
where "bar" above a quantity means the corresponding ensemble averaged value and $\Omega$ is the average nuclear level–density. Here, it was assumed that the ensemble averaging over the energy distances and squared off-diagonal matrix elements can be performed independently. In Eq. (12), $R(|E_n - E_m|)$ is the two–level correlation function which is the probability density to find the state $m$ with energy $E_m$ in interval $[E_m, E + dE_m]$ at the average distance $|E_n - E_m|$ from the given state $n$ with energy $E_n$.

In the nuclear case, the many–body Hamiltonian $\hat{H}$ obeys time–reversal symmetry implying the usage of Gaussian Orthogonal Ensemble (GOE) to model the nuclear spectrum. For a general mesoscopic system [10], $\hat{H}$ may not have a time–reversal symmetry and one has to use Gaussian Unitary Ensemble (GUE) of many–body levels. Correspondingly,

(i) For the GOE statistics [11]

$$R_{\text{GOE}}(x) = 1 - \left(\frac{\sin(\pi x)}{\pi x}\right)^2 + \left(\int_0^1 dy \frac{\sin(\pi xy)}{y} - \frac{\pi}{2}\right) \left(\frac{\cos(\pi x)}{\pi x} - \frac{\sin(\pi x)}{(\pi x)^2}\right),$$

(ii) while in the GUE case

$$R_{\text{GUE}}(x) = 1 - \left(\frac{\sin(\pi x)}{\pi x}\right)^2,$$

where $x \equiv |E_n - E_m| \Omega(E_n)$. The behaviour of the two–level correlation function $R(x)$ with the normalized level spacing $x$ for the different statistical ensembles (13) and (14) is shown in Fig. 1. The main difference between the GOE and GUE cases, seen in Fig. 1, is the behaviour of $R(x)$ at small energy spacings $x$. For the GOE statistics one has linear repulsion between levels, $R_{\text{GOE}} \sim x$, while the GUE statistics implies quadratic level repulsion, $R_{\text{GUE}} \sim x^2$.

On the other hand, $R_{\text{GOE}}$ and $R_{\text{GUE}}$ are similar at moderate spacings $x$, when the spectral correlations between levels consistently disappear.

The next ingredient of the statistical averaging procedure is the ensemble averaged squared matrix elements (EASME) $\langle|\hat{h}|^2\rangle$. It is rather clear that at high excitation energies the transition matrix elements between the complex many–body states should drop out with the energy distance between them. In order to characterize the $\langle|\hat{h}|^2(|E_n - E_m|)\rangle$–distribution, we introduce the strength of the distribution $\sigma^2$ and its width $\Gamma$. To clarify the physical meaning of the quantities $\sigma^2$ and $\Gamma$, one may use the random matrix approach of Ref. [12], where the nuclear many–body states are constructed on unperturbed basis states which are linearly coupled to external time–dependent classical variable $q(t)$, and complexity
is achieved by adding two–body interaction between them. In this approach $\sigma^2$ is the variance of the slopes, $\partial E_n/\partial q$, of the many–body energy levels. The strength of the two–body interaction, introduced to model the effect of residual interaction between nucleons, defines the spreading width $\Gamma_\mu$ of the squared off–diagonal matrix elements $|\langle n|\partial \hat{H}/\partial q|m \rangle|^2$, for example, via Fermi’s Golden Rule.

Thus, we present the ensemble averaged squared matrix elements (EASME) $|\hat{h}|^2$ in the following form

$$|\hat{h}|^2 = \frac{\sigma^2}{\sqrt{\Omega(E_n)\Omega(E_m)}} f(|E_n - E_m|/\Gamma),$$

(15)

where $f$ is a shape of the EASME’s distribution with the natural boundary conditions, $f(0) = const$ and $f(\infty) = 0$.

Going from the discrete energy levels $E_n$ and $E_m$ to continuous energy variables,

$$E \equiv E_n, \quad e \equiv E_m - E_n,$$

(16)

and substituting the expression (15) into Eq. (12), we obtain

$$\frac{\partial \bar{\rho}(E,t)}{\partial t} = \frac{\sigma^2}{\sqrt{\Omega(E)}} \bar{q}(t) \int_0^t ds \ \bar{q}(s) \times \int_{-\infty}^{+\infty} \frac{f(|e|/\Gamma)}{e^2} R(\Omega(E)|e|)cos(e/h(t - s))\sqrt{\Omega(E - e)}[\bar{\rho}(E - e, s) - \bar{\rho}(E, s)]de.$$  

(17)

In the last equation, the integration limits over the energy spacing $e$ were extended to infinities since the time changes of the occupancy $\bar{\rho}(E,t)$ of the given nuclear state with the energy $E$ are mainly due to the direct interlevel transitions from the close–lying states located at the distances $|e| \ll E$. The same assumptions enable us to truncate expansion to $e^3$–order terms,

$$\sqrt{\Omega(E - e)}[\bar{\rho}(E - e, s) - \bar{\rho}(E, s)] = -\sqrt{\Omega(E)}\frac{\partial \bar{\rho}(E, s)}{\partial E}e$$

$$+ \frac{1}{2\sqrt{\Omega(E)}} \int dE \frac{\partial \bar{\rho}(E, s)}{\partial E} e^2 + \sqrt{\Omega(E)} \frac{\partial^2 \bar{\rho}(E, s)}{\partial E^2} e^2 + (\ldots)e^3 + O(e^4)$$

(18)

Substituting the expansion (18) into Eq. (17), the odd-\(e\) terms drop out and dynamical equation for the occupancy $\bar{\rho}(E,t)$ of the nuclear state with the energy $E$ becomes

$$\Omega(E) \frac{\partial \bar{\rho}(E,t)}{\partial t} \approx \sigma^2 \bar{q}(t) \int_0^t ds K(t - s)\bar{q}(s) \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \bar{\rho}(E, s)}{\partial E} \right],$$

(19)
where retardation of the $\bar{\rho}(E,t)$–dynamics is defined by a memory kernel $K(t-s)$ which is
defined by the Fourier transform of the product of the EASME’s energy distribution $f$ and
the two–level correlation function $R$,

$$K(t-s) = \frac{1}{\Gamma} Re \left( \int_{-\infty}^{+\infty} f(|e|/\Gamma) R(|e|\Omega(E)) exp\left(\frac{ie[t-s]}{\hbar}\right) de \right).$$  \hspace{1cm} (20)

Non–Markovian equation (19) describes the process of energy diffusion in the space of
highly excited many–body states. The memory effects in the intrinsic energy diffusion is
defined by the counterplay between a time spread of the memory kernel (20), $\tau \sim \hbar/\Gamma$, and
a typical collective time $\tau_{coll}$ (a duration of the physical process). Depending on the width
$\Gamma$ of the EASME’s energy distribution (15), we distinguish different dynamical regimes of
the intrinsic energy diffusion (19):

(i) $\hbar/\Gamma << \tau_{coll}$. In this case, $K(t-s)$ is sharply peaked around $t = s$, and one can
integrate by parts the right–hand side of Eq. (19) and keep only leading order term in a
small parameter $\hbar/\Gamma$. Thus, we obtain a Markovian limit of the intrinsic dynamics (19):

$$\Omega(E) \frac{\partial \bar{\rho}(E,t)}{\partial t} \approx \frac{\hbar \sigma^2 f(0)}{\Gamma} \dot{q}(t) \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \bar{\rho}(E,t)}{\partial E} \right].$$  \hspace{1cm} (21)

Here, the intrinsic energy diffusion is determined by the diffusion coefficient $D_E = \hbar \sigma^2 f(0) \dot{q}^2/\Gamma$ which grows with the square of the collective velocity $\dot{q}$ and drops out with
the increase of the width $\Gamma$. The latter feature of the quantum mechanical energy diffusion
can be understood as follows. The width $\Gamma$ of the EASME’s energy distribution defines an
effective number of states $N \sim \Gamma \Omega(E)$ coupled by the transition operator $\partial \hat{H}/\partial q$ at the
given excitation $E$. The initially occupied many–body state with energy $E$ will spread out
over $N$ neighboring states, resulting in a gradual equilibration of the quantum mechanical
intrinsic subsystem. The larger $\Gamma$, the closer the intrinsic subsystem to the equilibrium and
therefore, the weaker the energy diffusion.

(ii) $\hbar/\Gamma >> \tau_{coll}$. Now we can put approximately $K(t-s) \approx K(0)$ for the memory kernel
in Eq. (19) and get

$$\Omega(E) \frac{\partial \bar{\rho}(E,t)}{\partial t} \approx \sigma^2 K(0) \dot{q}(t) \Delta q(t) \frac{\partial}{\partial E} \left[ \Omega(E) \frac{\partial \bar{\rho}(E,t)}{\partial E} \right],$$  \hspace{1cm} (22)

where $\Delta q(t) = q(t) - q(t = 0)$ is the change of collective deformation of the nucleus. Here
the diffusion coefficient $D_E = \sigma^2 K(0) \dot{q} \Delta q$ is linearly proportional to the collective velocity
$\dot{q}$ and does not depend on the width $\Gamma$. 

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(iii) $\hbar/\Gamma \sim \tau_{\text{coll}}$. In the intermediate case, the memory effects in the intrinsic energy diffusion \(^{(19)}\) will be of maximal size.

It is natural to address a question of the effect of level statistics \(^{(13)-(14)}\) on the intrinsic energy diffusion. We believe that the energy diffusion will differ significantly for the statistical ensembles of levels \(^{(13)-(14)}\) only at quite small values of the width $\Gamma$, $\Gamma \leq 1/\Omega$, i.e., when the features of the nuclear spectrum at small spacings between levels show up; see Fig. 1. On the other hand, at quite large widths $\Gamma >> 1/\Omega$ the spectral statistics effect is of a minor role as far as the statistical ensembles of levels \(^{(13)-(14)}\) show the universal behavior at large level spacings. The latter regime is realized for the highly excited nuclei provided that the width $\Gamma$ of the EASME’s distribution \(^{(15)}\) may lie in a quite wide energy interval $\Gamma \sim (10^0 \div 10^6) \text{ eV}$.

We may illustrate quantitatively our general discussion of the intrinsic energy diffusion by calculating the memory kernel \(^{(20)}\) for a Lorentzian shape $f$ of the EASME’s energy distribution,

\[
f(|e|/\Gamma) = \frac{1/\pi}{(e/\Gamma)^2 + 1}.
\] \(^{(23)}\)

To estimate the spectral statistics effect, we evaluated the memory kernel $K(t - s)$ at $s = t$ for the different levels ensembles \(^{(13)-(14)}\). The corresponding results for $K(0)$ as a function of the reduced width $\Gamma \Omega(E)$ are shown in Fig. 2. As was discussed above, the level statistics play a role only for quite small parameters $\Gamma \Omega(E)$ and the effect from the spectral statistics on the intrinsic energy diffusion \(^{(19)}\) disappears at large widths $\Gamma \Omega(E)$.

For $\Gamma \Omega >> 1$, one can find analytically the memory kernel \(^{(20)}\)

\[
K(t - s) = \exp\left(-\frac{|t - s|}{\hbar/\Gamma}\right),
\] \(^{(24)}\)

leading to the following non–Markovian equation of motion for the occupancy $\bar{\rho}(E, t)$ of the
given nuclear state $E$,

\[
\Omega(E) \frac{\partial \bar{\rho}(E, t)}{\partial t} = \frac{\sigma^2}{\Gamma} \dot{q}(t) \int_0^t \exp\left(-\frac{|t - s|}{\hbar/\Gamma}\right) \dot{q}(s) \frac{\partial}{\partial E} \left[\Omega(E) \frac{\partial \bar{\rho}(E, s)}{\partial E}\right] ds.
\] \(^{(25)}\)
IV. INTRINSIC DIFFUSION AND COLLECTIVE DISSIPATION

Now we are able to obtain a dynamical equation for the intrinsic excitation energy of the nucleus $E^*(t)$,

$$E^*(t) = \sum_n E_n \bar{\rho}_{nn}(t) = \int_0^{+\infty} dE \, \Omega(E) \bar{\rho}(E, t),$$

which enters the equation of motion (2) for the classical collective variable $q(t)$. By using Eq. (19), one gets after partial integration

$$\frac{dE^*}{dt} = \frac{\sigma^2}{\Gamma} \bar{\dot{q}}(t) \int_0^t \exp\left(-\frac{|t-s|}{\hbar/\Gamma}\right) \dot{q}(s) \int_0^{+\infty} \frac{d\Omega(E)}{dE} \bar{\rho}(E, s) dE ds$$

We stress immediately that the collective motion is undamped for the constant nuclear level–density, $\Omega(E) = \text{const}$. In that case the intrinsic subsystem is not excited during the collective motion, $E^*(t) = E^*(t = 0)$ and therefore, due to the energy conservation condition (1), the collective energy is constant in time. This means that the growth of the average nuclear level–density $\bar{\Omega}$ with energy is the necessary condition for the collective dissipation. In the sequel, we will use the constant–temperature level–density,

$$\Omega(E) = c \cdot e^{E/T},$$

where $T$ is the temperature of the nucleus, and which leads us to non–Markovian collective dynamics,

$$B(q)\ddot{q}(t) = -\frac{1}{2} \frac{\partial B(q)}{\partial q} \dot{q}^2(t) - \frac{\partial E_{\text{pot}}(q)}{\partial q} - \frac{\sigma^2}{T} \int_0^t \exp\left(-\frac{|t-s|}{\hbar/\Gamma}\right) \dot{q}(s) ds.$$  

We see from Eq. (29) that the non–Markovian character of the intrinsic nuclear dynamics (19) gives rise to the presence of memory effects in the macroscopic collective motion. Correspondingly, the Markovian limits of the intrinsic energy diffusion (21) and (22) would correspond to the Markovian collective motion. Indeed, for the quite broad energy distributions of the EASME (15), $\hbar/\Gamma << \tau_{\text{coll}}$, we obtain

$$B(q)\ddot{q}(t) = -\frac{1}{2} \frac{\partial B(q)}{\partial q} \dot{q}^2(t) - \frac{\partial E_{\text{pot}}(q)}{\partial q} - \frac{\hbar \sigma^2}{\Gamma T} \dot{q}(t).$$

Here an ordinary friction force with the friction coefficient $\hbar \sigma^2/(\Gamma T)$ appears as a result of the Markovian intrinsic energy diffusion (21).

In the opposite case of the intrinsic dynamics (22), when the EASME’s distribution is strongly peaked, $\hbar/\Gamma >> \tau_{\text{coll}}$, we obtain a friction–less limit of the collective motion,

$$B(q)\ddot{q}(t) = -\frac{1}{2} \frac{\partial B(q)}{\partial q} \dot{q}^2(t) - \frac{\partial E_{\text{pot}}(q)}{\partial q} - \frac{\sigma^2}{T} (q(t) - q_0),$$

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when the retarded force in the right-hand side of Eq. (29) is reduced to a pure conservative force $\sigma^2(q - q_0)/T$.

V. NUCLEAR FISSION CALCULATIONS

Even within a very simple one-dimension model for the collective dynamics (29), we may calculate quantities which can be estimated from experimental observables. Let us consider a symmetric fission of the highly excited $^{236}U$. The classical collective variable $q(t)$ can be chosen as the elongation of axial symmetric nuclear shape measured in units of the radius $R_0 = r_0 A^{1/3}$ of the nucleus. The collective potential energy from saddle point to scission $E_{pot}(q)$, shown in Fig. 3, is approximated by an inverted parabolic potential [13, 14],

$$E_{pot}(q) = 8 \text{ MeV} - \frac{1}{2} \hbar \omega_f B(q_0)(q - q_0)^2,$$

(32)

where $\hbar \omega_f = 1.16 \text{ MeV}$, $q_0$ is the initial (saddle-point) deformation of the nucleus, $q_0 = q(t = 0) = 1.6$, and $B(q)$ is the collective mass coefficient derived for the incompressible and irrotational nuclear fluid,

$$B(q) = \frac{1}{5} AmR_0^2 (1 + \frac{1}{2q^2}),$$

(33)

with the nucleonic mass $m$. The scission point $q_{sc}$ can be obtained from the following condition [14]

$$E_{pot}(q_0) - E_{pot}(q_{sc}) = 20 \text{ MeV}.$$

(34)

The initial collective kinetic energy is taking to be equal to $1 \text{ MeV}$.

Characterizing the intrinsic nuclear motion, we adopt the initial temperature of the nucleus $T = 2 \text{ MeV}$ and estimate the strength $\sigma^2$ of the EASME’s distribution within the Nilsson model for single-particle nuclear states in an anisotropic harmonic oscillator potential, see Ref. [15]:

$$\sigma^2 = \frac{3m^2 \omega_0^3 Ar_0^4}{560 \pi \hbar},$$

(35)

with $\hbar \omega_0 = 41/ A^{1/3} \text{ MeV}$.

Using Eq. (29), we calculated numerically from Eq. (29) the time, $t_{sc}$, of the nuclear descent from the top of fission barrier $q_0$ to the scission point $q_{sc}$ (34). The corresponding results for the saddle-to-scission time $t_{sc}$ are plotted in Fig. 4 as a function of the width $\Gamma$ of the Lorentzian distribution of the EASME (15). As can be seen from Fig. 4, the
time for the nuclear descent $t_{sc}$ decreases with the increase of the width $\Gamma$ of the EASME’s distribution \cite{15}. In order to explain such kind of behavior of $t_{sc}$, we represent the retarded force in the right–hand side of Eq. \eqref{29} as a sum,

$$-rac{\sigma^2}{T} \int_0^t \exp\left(-\frac{t-s}{\hbar/\Gamma}\right) \dot{q}(s) ds = -\gamma(t, \hbar/\Gamma) \dot{q}(t) - \tilde{C}(t, \hbar/\Gamma)(q(t) - q_0), \quad (36)$$

where $\gamma$ and $\tilde{C}$ are the time–dependent friction and stiffness coefficients, respectively. The separation \eqref{36} of the retarded force is general in the sense that it always contains the time–irreversible (the friction part) and time–reversible (the conservative part) contributions. In fact, the memory effects in the collective dynamics \eqref{29} give rise to the friction, $\gamma(t) \dot{q}(t)$, and lead to the renormalization of the stiffness of the nuclear many–body system,

$$C = -B(q_0)(\hbar \omega f)^2 + \tilde{C}(t, \hbar/\Gamma), \quad (37)$$

see Eqs. \eqref{29} and \eqref{36}. It is important that $\tilde{C}$ is always positive resulting in the additional hinders of the nuclear descent from the fission barrier, see Ref. \cite{16}. The relative sizes of the friction and the dynamic conservative forces in \eqref{36} are defined by the time–spread of the exponential kernel, $\hbar/\Gamma$. If the dynamic stiffness $\tilde{C}$ is expected to increase monotonically with $\hbar/\Gamma$, the friction coefficient $\gamma$ is a non–monotonic function of the memory time $\hbar/\Gamma$. At the limit of relatively small values of $\hbar/\Gamma$ (the large–widths limit which we consider here), both the friction and the dynamic conservative contributions drop out with the memory time explaining the decay of the saddle–to–scission time $t_{sc}$ with the width $\Gamma$ of the EASME’s distribution.

By using our previous estimations of the saddle–to–scission time done in Ref. \cite{16} for the same one–parametric nuclear shape parameterization \cite{32}–\cite{34}, $t_{sc} \sim (6 \div 12) \cdot 10^{-21}s$, we can conclude from Fig. 4 that the width $\Gamma$ of the Lorentzian distribution of the EASME lies in the interval $10 \text{ MeV} \leq \Gamma \leq 20 \text{ MeV}$.

We also calculated the dependence of collective kinetic energy at the scission point $E_{ps}$ on the width $\Gamma$, see Fig.5. As far as the nuclear descent gets faster with the width of the EASME’s distribution, the collective energy of the nucleus at the scission point will increase with $\Gamma$. The estimated interval for the width, $10 \text{ MeV} \leq \Gamma \leq 20 \text{ MeV}$, obtained from our saddle–to–scission calculations (see Fig. 4), gives realistic values of the pre–scission kinetic energy $1 \text{ MeV} \leq E_{ps} \leq 3 \text{ MeV}$ \cite{16}. 
VI. SUMMARY

In attempt to describe selfconsistently the nuclear many–body dynamics undergoing along macroscopic collective path and intrinsic excitations, we have applied the cranking approach \((1)–(2)\) to the nucleus. We have introduced a single time–dependent classical variable \(q(t)\) to characterize the slow collective nuclear motion, while the fast intrinsic modes of the motion have been treated quantum–mechanically within the Liouville equation \((3)\) for the nuclear density matrix. Applying the Zwanzig’s projection method \((7)\), the intrinsic nuclear dynamics has been reduced to the equation of motion \((12)\) for the occupancies of nuclear states. To obtain Eq. \((12)\), we have averaged the intrinsic dynamics over the randomly distributed level spacings \(\epsilon\) and squared matrix elements \(|h|^2\) of the transition operator \(\partial \hat{H}/\partial q\), where \(\hat{H}\) is the nuclear many–body Hamiltonian. The distribution of the ensemble averaged matrix elements (EASME) \(|h|^2\) has been taken in the general form \((15)\), where decay of \(|h|^2\) with the energy distance between states \(\epsilon\) has been characterized by the width \(\Gamma\) and we have assumed that \(|h|^2\) drops out strongly with excitation energy through the average nuclear level–density \(\Omega\).

In the limit of high excitations of the nucleus, we have obtained the non–Markovian diffusion equation \((19)\) for the time evolution of the intrinsic nuclear occupancies. The time features of the intrinsic energy diffusion \((19)\) are defined by the relation between the time scale \(\hbar/\Gamma\) and the characteristic time scale of the collective motion \(\tau_{coll}\). We have found that at fairly broad energy distributions of the EASME \((15)\), i. e., when \(\hbar/\Gamma << \tau_{coll}\), Markovian regime \((21)\) of the intrinsic energy diffusion is observed with the diffusion coefficient quadratically depending on the collective velocity \(\dot{q}\) and inversely proportional to the width \(\Gamma\). In the opposite case of quite small widths \(\Gamma\), \(\hbar/\Gamma >> \tau_{coll}\), we also found the normal (Markovian) regime of the intrinsic energy diffusion but with the diffusion coefficient linearly proportional to the collective velocity \(\dot{q}\) and not depending on the width \(\Gamma\).

We have investigated how the level spacing statistics can influence the intrinsic energy diffusion \((19)\). Only in the case of quite small widths of the EASME’s distribution, \(\Gamma\Omega \leq 1\), the significant difference of the intrinsic dynamics for the Gaussian orthogonal (GOE) \((13)\) and Gaussian unitary (GUE) \((14)\) ensembles of levels is expected. Such a difference would disappear as far as the product \(\Gamma\Omega\) becomes larger and larger. We may explain that by the fact that the transitions between the nuclear states may be sensitive to the level statistics.
only when the coupling between states is of order of the average level spacing, \( \Gamma \sim 1/\Omega \), i.e. when the different small–spacing behavior of the GOE and GUE statistics may shows up. At high nuclear excitations, we have believed that the product \( \Gamma \Omega >> 1 \) and therefore, one can neglect the role of the spectral statistics on the intrinsic energy diffusion (19). We have illustrated quantitatively this feature of the intrinsic dynamics by applying the Lorentzian distribution (23) of the EASME (15), see Fig. 3.

Our next goal was to calculate the nuclear fission’s characteristics within our approach. By using the constant–temperature level–density (28), we have derived non–Markovian equation of motion (29) for the classical collective variable \( q(t) \), where the influence of the intrinsic quantum motion on the collective dynamics is determined by the retarded friction force. Then the non–Markovian collective dynamics (29) has been applied to describe descent of the nucleus \(^{236}U\) from the top of fission barrier to the scission point approximating the collective potential energy on this path by the inverted parabolic potential (32) [13, 14].

We have calculated the time of the nuclear descent, \( t_{sc} \) (Fig. 4), and the collective kinetic energy at the scission point, \( E_{ps} \) (Fig. 5), as a function of the width \( \Gamma \) of the EASME’s distribution. We have found that the nuclear descent is hindered with the decrease of \( \Gamma \) due to the ordinary friction force contribution and the additional conservative dynamic force caused by the presence of memory effects in Eq. (29) [16]. The relative size of the memory effects decreases with the width of the EASME’s distribution and, at \( \Gamma \to \infty \), we have friction–less limit of the collective motion, see Eq. (31). From the calculations of the saddle–to-scission time and the pre–scission kinetic energy we have estimated the value of the width \( \Gamma \), \( 10\ MeV \leq \Gamma \leq 20\ MeV \), which is consistent with the previous estimations of the analogous quantity done in Refs. [17, 18].

VII. ACKNOWLEDGMENTS

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[1] H. Hofmann, P. J. Siemens, Nucl. Phys. A257, 165 (1976).
VIII. FIGURE CAPTIONS

Fig. 1: The two–level correlation function \( R(x) \) vs the normalized level spacing \( x \) for the different Gaussian ensembles of Eqs. (13) and (14) of energy levels.

Fig. 2: Dependence of the non–Markovianity of the intrinsic nuclear dynamics (19) on the reduced width \( \Gamma \Omega(E) \) of the Lorentzian distribution (23) of the EASME. The dependence is shown for the different spectral statistics (13) and (14).

Fig. 3: Dependence of the collective potential energy \( E_{pot} \) on the nuclear shape parameter \( q \) during the descent from the top of fission barrier \( q_0 \) to the scission point \( q_{sc} \) (34).

Fig. 4: The saddle–to–scission time \( t_{sc} \) of the symmetric fission of the \( ^{236}U \), calculated
from Eq. (29), is shown as a function of the width $\Gamma$ of the Lorentzian distribution of the EASME (15).

Fig. 5: The collective kinetic energy at the scission point $E_{ps}$ vs the width $\Gamma$ of the Lorentzian distribution of the EASME (15).
The diagram shows a plot of $K(0)$ vs. $\Gamma \Omega$ for two different ensembles: GOE and GUE. The curves represent the distribution of eigenvalues for these ensembles.
