Active Disturbance Rejection Adaptive Command Filtered Control of Electrohydraulic Actuator

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Abstract. This paper proposed an active disturbance rejection adaptive command filtered control (CFC) scheme for tracking control of electrohydraulic actuator subjected to both parametric uncertainties and uncertain nonlinearities. The proposed controller is developed by combining extended observer technology and adaptive CFC backstepping method. The adaptive law is used to deal with parameter uncertainty; matching uncertainties and mismatching uncertainties are estimated by two extended state observers and compensated in a feedforward way. Parameter self-adaptation can greatly reduce parameter uncertainty and ease the estimation burden of the extended state observer. CFC technology is employed to deal with the inherently complex explosion problem of classic backstepping technology. Therefore, the design of the controller avoids high-gain feedback, reduces tracking errors, and decreases online calculation burden. The proposed controller can guarantee asymptotic stability and expected control performance. Comparative simulation results are acquired to verify the tracking performance of the proposed approach.

Keywords: hydraulic actuator, command filtered control, extended state observer, uncertain nonlinear.

1. Introduction

Hydraulic servo system is widely used in various industrial fields due to its irreplaceable role, such as in [1], small size-to-power ratios, high response, high stiffness, and high load capability. High-performance control is a challenge due to the inherent modeling uncertainties and uncertainty behaviors in hydraulic servo systems. To overcome this problem, many advanced nonlinear control algorithms have been introduced, for example, adaptive control [2], RISE-based control [3], robust adaptive control[4], and adaptive robust control[5], the high tracking performance of all of above nonlinear controllers must utilize high feedback gain, which is not suitable for industrial applications due to high measurement noise and neglect of high-frequency dynamics.

Is there a way to deal with both large modeling uncertainties and not requiring high feedback gain? Disturbance observer-based control may be an effective method. Especially in [6], an active disturbance rejection control (ADRC) was introduced to solve the modeling uncertainties and external interference. The core design of ADRC is to treat the unknown dynamics as an extended state, and then estimate and compensate for generalized disturbances using extended state observers in a feedforward way. To
simplify the ADRC controller, a linear ESO is proposed, and a stability analysis is performed [7]. Recently, ADRC research has made some progress [8]. These studies demonstrate the effectiveness of the ADRC method. However, the parameter uncertainties are not considered by the above literature, which will increase the calculation and learning burden of the extended state observer. In [9], Yao proposed a new control scheme that integrates adaptive backstepping control and dual extended state observer. This method effectively solves the problem of burden. However, classic backstepping control has an inherent complexity explosion problem. An adaptive command filtered control (CFC) [10] is proposed to solve the multiple derivative problem through filter technology, and the control accuracy is better than dynamic surface control (DSC) due to the consideration of filter error compensation [11].

In this paper, a novel control design method is proposed. CFC technology combined with the parameters for adaptive law and the dual extended state observers, ensured the tracking error and estimation error of the system, reducing the calculation load, and is more suitable for industrial applications. In designing an appropriate Lyapunov function, the asymptotic stability of the control system is guaranteed. Simulation results prove the effectiveness of the method.

2. Nonlinear model of hydraulic system
The system under consideration is pump-controlled servo hydraulic system. It is composed of a fixed-displacement hydraulic pump, a servo motor, and a double-rod cylinder. According to the second Newton’s law, the dynamics of the piston can be described by

\[ \ddot{x}_p = \frac{(P_a A - P_b A - F_f)}{m}. \] (1)

where \( F_f \) represents the friction; \( A \) is the annulus area; \( P_a \) and \( P_b \) are pressure in two chambers; \( \ddot{x}_p \) is the acceleration of the piston; \( m \) refers to the equivalent mass. Neglecting the external leakage, the continuity equations at the actuator sides a and b are

\[
\begin{align*}
\dot{P}_a &= \beta_c (q_b \omega_m - Q_i + Q_2 - A \ddot{x}_p) / (V_{01} + A x_p) \\
\dot{P}_b &= \beta_c (q_b \omega_m + Q_i - Q_2 + A \ddot{x}_p) / (V_{02} - A x_p)
\end{align*}
\] (2)

In (2), \( \beta_c \) is the effective bulk modulus; \( V_{01} \) and \( V_{02} \) are the total volume of the chambers. \( Q_i \) and \( Q_2 \) are flow rates from the oil pump source. Define the state variables as \( x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9] \), where \( x_3, x_4 \) are the velocity and position of the piston; \( P_1 = P_a - P_b \) is the load pressure of the hydraulic actuator. \( Q_2 \) is internal leakage \( (Q_2 = K_{leak} P_1) \); \( K_{leak} \) is the internal leakage coefficient; \( Q_4 \) and \( Q_5 \) are flows into and out of the pump \( (Q_4 = Q_5 = q_b \omega_m) \). \( q_b \) is the displacement of the hydraulic pump (flow per revolution). \( \omega_m \) is the motor speed. The system can be expressed as

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= A z_3 - B z_4 - C r_s + f(t) \\
\dot{z}_3 &= (\beta/(V_{01} + A z_4) + \beta/(V_{02} - A z_8))q_b \omega_m + \beta/(V_{01} + A z_4) (Q_i - K_{leak} z_5 - A z_2) - \beta/(V_{02} - A z_8) (Q_2 + K_{leak} z_5 + A z_2) - q(t)
\end{align*}
\] (3)

where \( B \) is the coefficient of the modeled damping and viscous friction; \( C \) is the Coulomb friction and \( s_f \) is a shape function; \( \mathbf{r}(t) \) is other disturbance; \( q(t) \) is the modeling errors.

3. Active disturbance rejection adaptive command filtered control design

3.1. Design model
In order to simplify the state-space equation, define \( \theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7]^\top \), where \( \theta_1 = B, \theta_2 = C \), \( \theta_3 = \beta Q_1, \theta_4 = \beta Q_2, \theta_5 = \beta K_{leak}, \theta_6 = \beta, \theta_7 = \beta A \beta \); The hydraulic system can be described in a state space form as
where \( f_5 = 1/(V_{01} + Ax_1) \), \( f_6 = x_3/(V_{01} + Ax_1) \), \( f_4 = Ax_2/(V_{01} + Ax_1) \), \( f_5 = 1/(V_{02} - Ax_1) \), \( f_6 = x_3/(V_{02} - Ax_1) \).

**Assumption 1:** The desire position \( x_d \) and velocity \( \dot{x}_d \) are smooth and bounded. In the hydraulic system, supply pressure \( P_s \), return pressure \( P_r \), \( P_a \) and \( P_t \) are bounded.

**Assumption 2:** The parameters \( \beta, K_{tub}, Q_1, Q_2 \) are defined range; the defined parameters set \( \theta \) are bounded; \( \theta_{\text{max}} \) and \( \theta_{\text{min}} \) are adaptive parameter bounds, \((\theta_{\text{min}} \leq \theta \leq \theta_{\text{max}})\).

### 3.2. Projection mapping

In the following section, discontinuous projection mapping is introduced to solve the problem of parametric uncertainty. Set \( \hat{\theta} \) denote the estimate of \( \theta \), \( \hat{\theta} \) is the estimation error \((\hat{\theta} = \theta - \theta)\). A discontinuous projection can be defined as

\[
\text{Proj}_h(Y) = \begin{cases} 
0 \text{ if } \hat{\theta} = \theta_{\text{max}} \text{ and } Y > 0 \\
0 \text{ if } \hat{\theta} = \theta_{\text{min}} \text{ and } Y < 0 \\
Y, \text{ otherwise}
\end{cases}
\]  

(5)

The adaptation law can be given by

\[
\dot{\hat{\theta}} = \text{Proj}_h(I\tau)
\]  

(6)

where \( I \) is positive diagonal matrix; \( \tau \) is an adaptation function. The projection mapping can guarantee

\[
\hat{\theta}^T (I^{-1} \text{Proj}_h (I\tau) - \tau) \leq 0
\]  

(7)

### 3.3. Extended state observer design

In this paper, two state observers were established to estimate the uncertainty in each channel. All the system states are available; \( x_1 \) and \( x_2 \) are additional states, respectively. Two linear ESOs [9] can be given as

\[
\begin{align*}
\dot{\hat{x}}_1 &= \dot{x}_2 + 2\omega_1(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_2 &= \frac{1}{m}(Ax_2 + \hat{\theta}^T \varphi_2) + \hat{x}_3 + 3\omega_1^2(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_3 &= \omega_1^3(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_4 &= \hat{\theta}^T \varphi_3 + \dot{\hat{x}}_2 + 2\omega_1(x_3 - \hat{x}_2) \\
\dot{\hat{x}}_5 &= \omega_2(x_3 - \hat{x}_2)
\end{align*}
\]  

(8)

where the estimation error \( \hat{x} = x - \hat{x} \), \( \hat{x}_a = x_a - \hat{x}_a \), \( \hat{x} \) is the estimation of \( x \); \( \hat{x}_a \) are estimation of the extended states; \( \omega_i \) are the turning parameters; \( x_{a1} = f(t)/m, x_{a2} = g(t), \varphi_2 = [-\dot{x}_2, -s_2, 0, 0, 0, 0] \), 

\[
\varphi_3 = [0, 0, f_1 u_1, f_2, -(f_3 + f_6), -(f_4 + f_5), f_5]^T
\]

There are the following way of the definitions of the extended states [9]. The system model (4) can be rewritten as
\[
\begin{align*}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= 1/m (Ax_3 + \dot{\theta}^T \varphi_2) - 1/m (\ddot{\theta}^T \varphi_2) + x_{e1} \\
\dot{x}_{e1} &= h_1(t) \\
\dot{x}_3 &= \dot{\theta}^T \varphi_3 - \dot{\theta}^T \varphi_3 + x_{e2} \\
\dot{x}_{e2} &= h_2(t)
\end{align*}
\]  
(9)

where \( h_1(t) \) and \( h_2(t) \) are the rate of change of the uncertainty. Then, the state estimation errors is

\[
\begin{align*}
\dot{\eta} &= \omega_{\text{r}1} A_1 \eta - B_1 \frac{1}{m} \dot{\theta}_e \varphi_2 + B_2 \frac{h_1(t)}{\omega_{\text{r}1}}, \\
\dot{\chi} &= \omega_{\text{r}2} A_2 \chi - D_1 \dot{\theta}^T \varphi_3 + D_2 \frac{h_2(t)}{\omega_{\text{r}2}}
\end{align*}
\]  
(10)

where \( \eta = [\tilde{x}_1, \tilde{x}_2, \omega_{\text{r}1}, \tilde{x}_3, \omega_{\text{r}2}]^T \), \( \chi = [\tilde{x}_3, \tilde{x}_e, \omega_{\text{r}3}]^T \), \( B_1 = [0, 1, 0]^T \), \( B_2 = [0, 0, 1]^T \), \( D_1 = [1, 0]^T \), \( D_2 = [0, 1]^T \).

The matrixes A are Hurwitz, so there exist definite matrixes P satisfying the equation \( A^T P + PA = -I \); where I is identity matrix.

### 3.4. CFC controller design

In order to avoid the complex derivation of the traditional backstepping control law, a command filter backstepping method is proposed. Commander filter is

\[
\begin{align*}
\dot{\varphi}_{i,1} &= \omega_i \varphi_{i,2} \\
\dot{\varphi}_{i,2} &= -2\zeta \omega_i \varphi_{i,2} - \omega_i (\varphi_{i,1} - \alpha_i)
\end{align*}
\]  
(11)

where \( x_{i+1,e}(t) = \varphi_{i,1} \) and \( \dot{x}_{i+1,e}(t) = \omega_i \varphi_{i,2} \) are the outputs of each filter; \( \omega_i \) is a natural frequency; \( \zeta \) is a damping parameter. The filter initial conditions are \( \varphi_{i,1}(0) = \alpha_i(0), \varphi_{i,2}(0) = 0 \), \( i=1, 2 \).

The tracking errors for the command filtered backstepping approach are defined as

\[
\bar{x}_i = x_i - x_{w_i}, \; i = 1, 2, 3
\]  
(12)

where \( x_{w_i} \) is the command filtering output and \( x_i \) is the desired reference signal, and \( x_{w_i} = x_d \). Choose the virtue control functions input \( \alpha_1, \alpha_2, \alpha_3 \) as follows

\[
\begin{align*}
\alpha_1 &= -k_1 \bar{x}_1 + \bar{x}_d \\
\alpha_2 &= -k_2 \bar{x}_2 + mx_{2e} - \bar{x}_1/\omega_2 + \hat{\theta}_1 x_2 + \hat{\theta}_2 s_f - \hat{f}(t) \\
\alpha_3 &= \bar{u}_s + \bar{u}_s
\end{align*}
\]  
(13)

The error compensating signals \( \gamma_1, \gamma_2, \gamma_3 \) are defined as the derivative equation

\[
\begin{align*}
\dot{\gamma}_1 &= -k_1 \gamma_1 + \gamma_2 + (x_{2e} - \alpha_1) \\
\dot{\gamma}_2 &= -k_2 \gamma_2 - \gamma_1/\omega_2 + A \gamma_3 + (Ax_{3e} - \alpha_2) \\
\dot{\gamma}_3 &= -k_3 - A \gamma_2 \omega_3/\omega_3
\end{align*}
\]  
(14)

The compensated tracking error signals are designed as

\[
\nu_i = \bar{x}_i - \gamma_i, \; i = 1, 2, 3
\]  
(15)

The actual control input is \( u = \bar{u}_s + \bar{u}_s \).
3.5. Main results

Setting the adaptation function \( \nu \) is

\[
\tau = \omega_2 \varphi \nu + \omega_3 \varphi_3 \nu_3 + \mu_2 \eta^T P_1 B_1 \frac{\varphi_2}{m \omega_1} + \mu_3 \chi^T P_2 D_j \varphi_3
\]  

(17)

**Theorem 1:**

For a nonlinear pump-controlled electrohydraulic actuator system (3), after a finite time \( t_0 \), the modeling errors can be ignored \( h_1(t) = h_2(t) = 0 \), the system exists parameter uncertainty, constant disturbances. Thus, under the assumptions 1-2, properties (7) is satisfied, the virtual controllers (13), the projection adaptation law (6), the adaptive function (17), the ESO (8), can guarantee that all closed-loop signals are bounded; The signal \( \nu(t) \rightarrow 0 \) as \( t \rightarrow \infty \) and \( \lim_{t \to \infty} |\tilde{x}| \leq \frac{5\omega}{2k_0} \).

**Proof:** Define the Lyapunov function as

\[
V_o = \frac{1}{2} \nu^T \nu + \frac{1}{2} \omega_2 m u_2^2 + \frac{1}{2} \omega_3 \nu_3^2 + \frac{1}{2} \mu_2 \eta^T P_1 \eta + \frac{1}{2} \mu_3 \chi^T P_2 \chi + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}
\]

As a consequence

\[
\dot{V}_o \leq -\lambda_{\text{min}}(\nu^T \nu + \eta^T \eta + \chi^T \chi) = -W
\]

All signals are bounded. Based on error dynamics, the derivative \( W \) is bounded, \( W \) is uniformly continuous. By using Barbalat’s lemma, \( W \rightarrow 0 \) as \( t \rightarrow \infty \), so theorem 1 is proven.

4. Simulation results

To prove the above design, simulation results for the hydraulic system were used. The following actual parameters are: \( m=11.5 \text{kg}, A = 6.4 \times 10^{-4}, \beta = 700 \times 10^5, \nu_0 = V_T = 0.000483, K_{\text{tank}} = 2.4 \times 10^{-11}, \)

\( q_b = 6 \times 10^{-6}, Q = 1.8 \times 10^{-6} \), friction using luGre model [5] parameters are \( l_1 = 416, l_2 = 14.9, l_3 = 458.9, f_r = 525, \eta_f = 360.5 \). A sampling period 0.001s is used in all simulations.

The control objective is to track a desired trajectory \( x_d = 30 \arctanh (\sin(\pi t)) [1 - e^{-t}] / 0.7854 \).

To verify the effectiveness of the proposed design, the following two controllers are compared.

1) ADRACFC: This is the active disturbance rejection adaptive command filtered controller proposed in this paper. The controller parameters are: \( k_1 = 6000, k_2 = 3500, k_3 = 100, \omega_1 = \omega_2 = 1000, \)
\( \omega_1 = 5, \omega_2 = 8, \mu_2 = 1 \times 10^{16}, \mu_3 = 0.5 \); the parameter adaptation rates are set \( \Gamma = \text{diag} \{ 6 \times 10^{-9}, 1.5 \times 10^{-11}, 1 \times 10^{-18}, 0.001, 1 \times 10^{-11}, 230, 0.001 \} \).

2) PI: This is the proportional integral controller. The controller gains are \( k_p = 8000, k_i = 400 \), which are the P-gain and I-gain, respectively.

Simulation results for the two controllers are in Fig.1-2. The position tracking performance of ADRACFC is shown in Fig.1. We can see that the controller has excellent tracking performance.

The corresponding tracking error of the two control algorithms is shown in Fig.2. The tracking error of the proposed controller is less than the PI controller, which verify the effectiveness of the ADRACFC algorithms.
5. Conclusion
In this paper, an active disturbance rejection adaptive command filtered control method is proposed for hydraulic systems of both parametric uncertainties and unknown modeled disturbances. The proposed controller integrates extended observer technology and parameter adaptation law to overcome parameter uncertainties and unmodeled disturbances problems. The utilization of command filtered control technology reduces the system's online calculation burden and is more suitable for industrial applications. Extensive comparative simulation results are acquired to illustrate the effectiveness of the proposed scheme.

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