Most production technologies require a uniform flow path of liquid from pressure distribution pipelines. To achieve this goal, it is proposed to introduce polymer additives into the liquid flow or to use converging distribution pipelines with a continuous longitudinal slot in the wall. To reduce the uneven operation of the distribution pipeline during discrete liquid dispensing, it is proposed to use cylindrical output rotary nozzles with a lateral orthogonal entry of the jet into the nozzle. The problem is the lack of methods for accurate hydraulic calculation of the operation of distribution pipelines. Adequate calculation methods are based on differential equations.

Finding the exact solution of the differential equation of fluid motion with variable path flow rate for perforated distribution pipelines is urgent, because it still does not exist. The available calculation methods take into account only the right angles of separation of the jets from the flow in the distribution pipeline. These methods are based on the assumption that the coefficient of hydraulic friction and the coefficient of resistance of the outlets are constant along the flow. A calculation method is proposed that takes into account the change in the values of these resistance coefficients along the distribution pipeline. The kinematic and physical characteristics of the flow outside the distribution pipeline are also taken into account. The accuracy of calculating the value of the flow rate of water distributed from the distribution pipeline has been experimentally verified. The error in calculating the water consumption by the method assuming that the values of the resistance coefficients are unchanged along the distribution pipeline reaches 18.75 %. According to the proposed calculation method, this error does not exceed 6.25 %. However, both methods are suitable for the design of pressure distribution pipelines, provided that the jet separation angles are straight.

Taking into account the change from 90° to 360° of the angle of separation of the jets from the flow in the distribution pipeline will expand the scope and accuracy of calculation methods.

Keywords: pressure distribution pipeline, pressure flow of liquid with variable travel flow

1. Introduction

Pressure distribution pipelines (DP) is common in many processes. In irrigation, they form the basis of sprinkler, drip (Fig. 1) and subsurface irrigation systems. DP is used for aeration of water in water treatment facilities in order to remove iron hydroxide, free carbon dioxide and hydrogen sulfide from water. DPs are part of aerators in biological wastewater treatment plants (aeration tanks (Fig. 2), air filters, biofilters) to ensure the vital activity of microorganisms (aerobic bacteria), mineralizing organic matter and other contaminants dissolved in wastewater. Pressure DPs are important components in fire-fighting water supply, in particular, in sprinkler (automatic) and deluge (semi-automatic) fire extinguishing systems. Pressure DP are the main functional elements of spray pools and cooling towers when cooling circulating water in nuclear and thermal power plants (NPP and TPP) [1, 2]. Turbines of TPPs and NPPs are also equipped with distribution pressure pipelines with directing nozzles that direct the jet of working steam to the blades of the turbine wheel [1]. Supply ventilation of the premises is a network of pressurized DP. In agricultural aviation, when spraying plants, pressure distribution pipelines are fixed under the wings of a propeller-driven aircraft [1]. Pressure DPs are widely represented in the technological processes of the chemical, petrochemical and oil industries, as well as in other production processes and technologies.

In practice, the uneven operation of pressure distribution pipelines is realized [3]. In a long DP, the highest working head $H$ is formed at its beginning. Therefore, the maxi-
The average velocity of the main flow is \( V \) in the DP section containing \( n \) outlets, according to the formula

\[

d\left( \frac{\alpha \cdot V^2}{2g} \right) + \frac{dp}{\rho g} + dz + i dx + \frac{\alpha_s (v\cos\beta - V) \cdot V}{g} \frac{dQ}{Q} = 0,
\]

(1)

where \( \alpha_s \) – the coefficient of momentum of the flow, \( \alpha_s = 1.03...1.05 \); \( V \) – the average velocity of the main flow inside the perforated pipeline; \( v \) – the velocity of the jet, disconnected from the main stream; \( p \) – the fluid pressure inside the DP; \( dz = \sin \alpha dx \) – geometric difference in the axis of the pipeline in its section of length \( dx \); \( \psi \) – the angle of inclination of the pipeline axis to the horizon; \( \Delta dx = dh_0 \) – friction head loss along the length of the pipeline; \( \beta \) – the angle between the vectors of the velocities \( V \) and \( v \); \( Q \) – the flow rate of the main fluid flow inside the DP.

For a cylindrical distribution pipeline DEMFVR (1) is simplified, since its cross-sectional area is \( \Omega = \text{const} \), and \( dQ = \Omega \cdot dV \), where \( Q = \Omega \cdot V \) [1]:

\[
\frac{\alpha_s (v\cos\beta - V) \cdot dV}{g} + \frac{dp}{\rho g} + \sin \psi \cdot dx + dh_0 = 0.
\]

Equations (1) and (2) do not have an exact solution to date.

2. Literature review and problem statement

The authors of [4], who applied a number of simplifying assumptions and introduced the momentum exchange coefficient, made up a differential equation (3) for the impulse of forces acting on a local branch of the jet at an angle \( \beta = 90^\circ \) from the main flow flowing in the pressure distribution pipeline [4]:

\[
\frac{1}{\rho} \frac{dp}{dx} + \frac{\lambda}{2D} V_0^2 \left(1 - \frac{x}{L}\right)^{2z} + \frac{\alpha}{2x} \ln \left(1 - \frac{x}{L}\right) \frac{d \left(1 - \frac{x}{L}\right)^{2z}}{dx} = g l,
\]

(3)

where \( p \) – the hydrodynamic pressure; \( \rho \) – the density of the liquid; \( \lambda \) – the hydraulic coefficient of friction; \( D \) – DP diameter; \( V_0 \) – the flow rate at the entrance to the DP; \( L \) – the length of the perforated part of the DP; \( x \) – the current coordinate of the length of the DP; \( \alpha \) – the coefficient of change in the force impulse for the first outlet; \( \xi \) – (in the original \( \beta \) modified coefficient of change in the impulse of the force of jet detachment from the main stream flowing in the DP, \( \xi = \Delta V^2 / V^2 \); \( g \) – acceleration of gravity; \( I \) – longitudinal slope of the DP; \( z \) – the exponent, three variants of its value are considered \( (z<1, ~\text{when the force of the jet detachment momentum is greater than the friction force, and the velocity gradually increases along the flow, } z=1, \text{ when the friction force is equal to} \)
the jet detachment momentum force, and the velocity varies linearly along the stream, \( z > 1 \), in this case the friction force is greater than the impulse force, and the velocity gradually decreases along the flow.

Equation (3) does not fully reflect the work of a pressure head DP, since it is based on a number of simplifications. Applying a number of additional assumptions, the authors of [4] made an analysis of the probability of equation (3). However, no solution to equation (3) was provided. The reason for this is the laboriousness and lengthy process of solving the differential equation.

However, a number of approaches to the integration of DEMFVR (1) and (2) have been proposed [3]. The authors of [5], disregarding the loss of pressure for separating the mass in a long DP, and taking \( \beta = 90^\circ \) and \( \lambda = \text{const} \), compiled the DEMFVR for a pressure DP. However, the hydraulic coefficient of friction cannot be constant along the length of the DP, since the flow rate of the liquid along the flow decreases. According to the message of the author of work [6], the calculation according to the equation given in [5], at \( \lambda = \text{const} \), gives a deviation from the experimental data by 38 %. However, provided that \( \lambda \neq \text{const} \), the deviation decreased to 20 % [6].

In [7], when solving equations (1) and (2), a linear law of change in the flow rates of disconnected jets was adopted, which also does not correspond to the actual picture of the phenomenon under consideration. In addition, the authors of [7] neglected the change in the position of the axis of the main flow (the angle \( \psi (x) \) of its inclination to the horizon), the loss of energy of the flow for friction in the sections between the branches of the jets. However, a nonlinear law of variation of the main flow rate in the DP was adopted. However, according to the formula obtained with the listed assumptions, it is possible to calculate only a separate branch of the jet from the DP [7].

The method for calculating pressure distribution pipelines, given in [8], is based on the approach proposed by the author [9] when solving DEMFVR for pressure collecting pipelines, that is, for flows with a track mass connection. When solving the DEMFVR in work [8], the values of the following quantities were taken constant along the length of the DP:

- a) DP diameter \( D = \text{const} \);
- b) the distance between the outlet holes;
- c) coefficient of liquid flow through the outlet (\( \mu = \text{const} \));
- d) hydraulic coefficient of friction (\( \lambda = \text{const} \));
- e) the angle of branching of the jets (\( \beta = \text{const} = 90^\circ \));
- f) the angle of inclination of the DP axis to the horizon (\( \psi = \text{const} = 0^\circ \)).

As a result of mathematical transformations, the differential equation (4) with one unknown variable \( \bar{Q} \) was obtained in dimensionless form [8]:

\[
-\frac{d\bar{Q}}{dy} + \frac{d^2\bar{Q}}{dy^2} - (2 - m)\bar{Q} - \frac{\bar{Q}}{2f} = 0,
\]

where \( \bar{Q} = Q/Q_{\text{beg}} \);
- \( Q \) – the main flow rate in the section \( x \) DP;
- \( Q_{\text{beg}} \) – the same, at the beginning of the DP;
- \( \bar{h} = \sqrt{2gh}/V_{\text{beg}} \) – piezometric fluid head in the DP;
- \( f = \mu al/\Omega \);
- \( a = \Gamma_{01}/l \);
- \( m = \cos \beta/V \) – the coefficient of the detachable mass;
- \( v \) is the velocity of the jet outflow from the hole in the wall of the DP;
- \( V \) – the velocity of the main flow in the DP;
- \( \beta \) – the angle between the directions of movement of the detachable jet and the main flow;
- \( y = 1 - x \) – the relative distance measured upstream from the end of the DP;
- \( x = x/l \) – the same, following the flow from the beginning of the DP.

According to [8], for relatively short DPs, the piezometric line constantly grows along the main flow (Fig. 3, curve a). In distribution pipelines of intermediate length, it first decreases, has an inflection point curve \( (x_{\text{inf}}) \), and then increases (Fig. 3, curve b). For long DPs, the piezometric line decreases along the flow and reaches its lowest position at the end of DP (Fig. 3, curve c).

For DP with different relative lengths (short, intermediate, long), different solutions of eq. (4) are obtained under the condition of a turbulent regime of fluid motion. For example, for a long DP, the roots of equation (4) are as follows [8]:

\[
\bar{Q} = \frac{sh(\bar{ky})}{shk}, \quad \bar{h} = k - ch(\bar{ky})
\]

where the coefficient \( k \) is found from expression (6):

\[
k = 2Bk + \tan(\bar{ky}) = 0.
\]

or according to the graph shown in Fig. 4 [8].

Equations (4)–(6) [8] give good results for calculating DP, provided that the diameter of the DP and the distance between the outlet holes are constant along the length of the DP, its longitudinal axis is horizontal \( (\psi = 0^\circ) \), the jets are disconnected from the DP at right angles \( (\beta = 90^\circ) \), and the hydraulic coefficient of friction \( \lambda(x) \) and the coefficient \( \mu(\xi) \) of the fluid flow rate through the outlet nozzles are constant along the DP. However, there are no DPs with such characteristics in nature.

![Fig. 3. Scheme of operation and the type of piezometric lines for pressure head DP.](image)

![Fig. 4. Graph for determining the coefficient k to formulas (5) at different values of: 1 – (1); 0.8 – (2); 0.6 – (3); 0.4 – (4); 0.2 – (5); 0 – (6) [8]](image)
In the literature, many results of an experimental study of the operation of DP have been published. It has been found that a decrease in the uneven distribution of liquid along the pressure head DP is achieved by introducing polymer applications into the flow [10]. The latter reduce turbulent friction in cylindrical pipelines and change the hydraulic resistance of pipes of variable diameter [11].

With the help of numerical and physical experiments [12, 13], a decrease in the unevenness of the track distribution of liquid with DP equipped with:
- a) rectangular outlet openings;
- b) the same, round;
- c) a longitudinal continuous rectangular slit.

At the beginning of the DP, a branching of the jets from the flow in the DP at an angle β equal to 68° was formed. Along the DP, with a decrease in the flow energy, the angle β increased and at the end of the DP was 90° [12]. Numerous experiments have found that the slightest irregularity of the path distribution of liquid with DP is provided by a continuous slot [13].

The uneven distribution of water along the conical DP is less than that along the cylindrical DP [14]. The smallest unevenness is achieved when the inlet diameter $D_{\text{beg}}$ of the tapered DP is twice the final diameter $D_{\text{out}}$. The subsequent increase in the ratio of diameters $D_{\text{beg}}/D_{\text{out}}$ caused an increase in the unevenness of the track distribution of water [14]. On the basis of an analytical analysis of literature sources, the authors of [15] came to the conclusion that the slightest uneven distribution of liquid from a pressure head DP can be achieved in a diffuser DP with a continuous slot in its wall.

A decrease in the uneven distribution of liquid along the DP is also achieved by reducing the distance between the outlets and increasing their area along the flow. The kinematic characteristics of the outlet jets are influenced by the shape of the outlet holes arranged in the wall of a rectangular DP and the thickness of the DP wall [16]. With an increase in the wall thickness of the DP from 2 to 10 mm, the coefficient of narrowing of the flow $\alpha$ at the outlet from the hole increased, and the uneven distribution of water along the flow decreased. The uneven operation of the DP was weakened with a decrease in its duty cycle (duty cycle). Less irregularity of the track distribution of water is observed when it flows through the holes arranged in the DP with thicker walls. The results obtained in [16] suggest that this tendency will increase with the outflow of water through the packing.

Investigation [17] of water leakage through holes arranged in the wall of the irrigation pipeline, it was found that the value of the flow coefficient $\mu$ depended on the angle β. The angle β, in turn, was a function of the flow rate and pressure in the pipeline, in the alignment of the outlets [17].

An effective method [18] is to regulate the unevenness of the track distribution of liquid with DP using cylindrical rotary nozzles with a lateral orthogonal entry of the jet into the nozzle. By returning the nozzles with respect to their longitudinal axes, the value of the angle β changes in the range from 0 to 360° and, accordingly, the unevenness of the DP operation is significantly regulated [18].

The influence of the geometric parameters and the shape of the branching of the jets from the flow in the DP on the uneven distribution of the working fluid with the DP in the heat exchanger with parallel flows were experimentally investigated [19]. For a two-phase medium (liquid together with gas), the slightest irregularity of the DP operation was obtained when ascending flows entered vertical pipes [20]. Reducing the uneven distribution of methanol from DP into the catalyst microchannels during steam reforming can be a key factor for increasing the productivity of hydrogen production [21].

The work [22] presents the results of studies of DP mounted in chemical devices, in work [23] – in solar collectors, in article [24] – in nuclear reactors. The results of experimental studies serve as the basis for constructing mathematical models of fluid flows, in particular, differential equations.

In theoretical and experimental studies [4–24], the issues of creating a method for calculating DP at indirect angles of jet separation (β=90°) and taking into account the variable along the flow value of the hydraulic friction coefficient ($\lambda(x)\neq\text{const}$) and the coefficient of fluid flow through nozzles ($\mu_{(x)}\neq\text{const}$). The reason for this is the difficulty of taking into account all the characteristics of the flows in the pressure head DP when solving the DEMFVR. This problem, in addition to taking into account the value of the angle β≠90°, was eliminated in [25].

Based on the design scheme (Fig. 5) of the pressure flow with jet separation, the author of [25] supplemented the differential equation of fluid motion with variable flow rate (2) with the term $p_{\text{out}} (dQ/\psi)$. This term takes into account the force $S$, which replaces the action of the part of the detachable jet thrown back in section 3-3. Thus, let’s obtain equation (7) [25]:

$$\alpha_{s}(v \cos \beta - 2V) dV + \frac{d}{\rho g} + \frac{p_{\text{out}}}{\rho g} \cos \theta + \sin \psi \cdot dx + dh_{i} = 0. \tag{7}$$

The variables $V, dV, v, dp, p_{\text{out}}, dh_{i}, dx$ in equation (7) [25] are expressed not because of the flow rate $Q_{i}$, as in the works analyzed above [5, 7–9], but because of the total head $H(t)$ at the distribution pipe, under the influence of which the jets leak, and through an independent variable distance $x$. The angles $\beta, \theta, \psi$ and $\xi$ (Fig. 5–7) vary from 0° to 360°, and $m_{(t)} = (v_{(t)} \cos \beta)/V_{(t)} = \text{const}$, where $\xi$ is the angle between the velocity vectors $v_{(t)}$ of the detachable jet (in the outlet section of the outlet-hole) and $V_{(t)}$ of the external flow (Fig. 6).
As a result of solving the supplemented DEMFVR (7), the author [25] obtained dependence (8) for calculating the flow rate in pressure distribution pipelines laid in an external fluid flow. Since it is advisable to calculate the DP with a stroke from the end to the head, the x-axis is directed against the flow in DP, which is reflected in equation (8) and in Fig. 7.

\[
\frac{b}{\xi} \left( \frac{1}{H_{\text{in}}^{\text{out}}} + \frac{b_x}{4g \Omega} \frac{\alpha_x \cos \beta_x}{2g} \right) - \frac{b_x}{4H_{\text{in}}^{\text{out}}} \left[ \frac{\lambda_{\text{in}} x l_x}{D_{\text{in}}^2} + \left( \frac{b_x}{\rho g} \frac{P_{\text{out}}}{\rho g} \sin \psi + \frac{f_c}{2g} \right) \right] = 0 \quad (8)
\]

The total liquid head in the pressure distribution pipeline (Fig. 7) is calculated according to the dependence (9). Formula (9) was obtained by differentiating equation (8) [25]:

\[
H_{\text{in}}^{\text{out}} = \left\{ \frac{H_{\text{in}}^{\text{out}}}{2g \Omega} G \left[ \frac{\lambda_{\text{in}} x l_x}{D_{\text{in}}^2} + \left( \frac{b_x}{\rho g} \frac{P_{\text{out}}}{\rho g} \sin \psi + \frac{f_c}{2g} \right) \right] \right\}^{1/2} \quad (9)
\]

where in equations (8), (9) [25]:

- \( \beta_{i,k} \) - the angle between the vectors of the average velocity \( \bar{v} \) of the main fluid flow in the DP and the cutoff jet \( \bar{v} \) (Fig. 5–7);
- \( \omega_{i,k} \) - the angle between the vectors of the average velocity \( \bar{v} \) of the main fluid flow in the DP and the force \( \bar{S} \), replacing the action of the rejected part of the detachable jet \( \bar{S} = \bar{S}_{\text{det}} = 180° \) (Fig. 5);
- \( \xi_{i,k} \) - the angle between the vectors of velocities \( \dot{v}_{i,k} \) of the detachable jet and the external flow \( \bar{v}_{\text{ext}} \) (Fig. 6);
- \( D_{i,k} \) - the angle between the horizontal 0-0 and the l-axis of the flow (Fig. 7);
- \( \lambda_{i,k} \) - inner DP diameter;
- \( b_{x,k} \) - coefficient constant in the calculated section \( i-k \);
- \( n_{i-k} \) - the number of outlets per unit of length DP, m \(^{-1}\); \( \mu_i \) - coefficient of discharge of the outlet-hole; \( \omega_{\text{out}} \) - area of one outlet-hole;
- \( x_i \) - the index at the symbols of the parameters of the equations indicates their belonging to the beginning of the calculated section \( i-k \), that is, to the \( i \)-th cross-section of the DP;
- \( \kappa \) - empirical coefficient, \( \kappa = 0.6–0.9 \);
- \( V_{i,k} = Q_{i,k}/\omega_{i,k} \) - the average water flow rate inside the DP;
- \( H_{i,k} = \omega_{i,k} \) - the cross-sectional area of DP;
- \( v_{i,k} = \sqrt{gH} = ah_{i,k} \) - the velocity of the jet outflow through the outlet in the \( i \)-th section of the DP, \( a = \sqrt{g} \); the multiplier \( \cos \phi_{i,k} \) in the expression

\[
(2\alpha_n + \alpha \cos \phi_{i,k}) V_{i,k}
\]

of those participating in the first square brackets, which takes into account the effect of the dynamic head of the flow flowing inside the DP on the surface of the free section of the detachable jet:

- \( \rho \) - density of the liquid inside the DP and outside it;
- \( n_{i-k} \) - coefficient of compression of the jet in the outlet (nozzle);
- \( P_{\text{out}} \) - fluid (or air) pressure outside the DP;
- \( \lambda_{i,k} \) - hydraulic coefficient of friction (Darcy coefficient);

The value of the Reynolds criterion for the flow in the distribution pipeline:

\[
\frac{Q_{i,k}D_{i,k}}{\Omega_{i,k}}<10, \quad \frac{Q_{i,k}D_{i,k}}{\Omega_{i,k}}<500
\]

The zone of the turbulent regime is assumed to be constant \( \lambda_{i,k} \). In the calculations, it is taken into account that the laminar flow regime is realized when \( \Re_{i,k} \leq 2320 \). The zone of the turbulent regime of fluid movement includes three sections of hydraulic resistance, respectively: a section of hydraulically smooth pipes at \( \Re_{i,k} \left( \Lambda_{i,k}/D_{i,k} \right) < 10 \), a section of pre-square resistance of hydraulically rough pipes at \( 10 \leq \Re_{i,k} \left( \Lambda_{i,k}/D_{i,k} \right) \leq 500 \) and a section of quadratic resistance of hydraulic pipes at \( \Re_{i,k} \left( \Lambda_{i,k}/D_{i,k} \right) > 500 \).

The value of the Reynolds criterion for the flow in the distribution pipeline:

\[
\Re_{i,k} = \frac{Q_{i,k}D_{i,k}}{\Omega_{i,k}v_{i,k}}
\]

where \( v_{i,k} \) - the kinematic viscosity.
The discharge coefficient of the outlet holes (nozzles) \( \mu_{(i)} \), as well as the Darcy coefficient \( \lambda(x) \), is variable along the DP. For a cylindrical outlet nozzle at \( \lambda_{(i)} > 10 \), \( W_{(i)} > 200 \) and full compression and sharp leading edges, the value \( \mu_{(i)} \) can be calculated using empirical formulas obtained from the data [26]. For example, for the ratio \( l/d=1...1.5 \), \( \lambda_{(i)} = 10^1...10^2 \), or \( l/d=2...5 \), \( \lambda_{(i)} = 50...15 \times 10^4 \), or \( l/d=10...50 \), \( \lambda_{(i)} = 80...15 \times 10^4 \), \( \lambda_{(i)} = 80...15 \times 10^4 \); the values \( \mu_{(i)} \) are found by the formula (11) [26]:

\[
\mu_{(i)} = \frac{1}{1.23 + \frac{58.1}{\lambda_{(i)} l/d}}. \tag{11}
\]

The accuracy of calculating the distribution of pressure head DP was verified by experiments on experimental DP.

5. Experimental verification of methods of hydraulic calculation of pressure head DP

An experimental stand with gravitational water supply to the distribution pipeline was used (Fig. 8) [27]. The head at the entrance to the DP was equal to 3740 mm. The inner diameter of the investigated DP is \( D = 8.21 \) mm, and its length is 2644 mm (Fig. 9). The experimental DP is equipped with eleven outlet nozzles with inner diameters \( d = 3.2 \) mm and lengths equal to 25 mm (Fig. 10). The distances between the nozzles were not the same along the DP, but multiples of 20D. The experimental distribution pipeline had a duty cycle of 1.67. The outlet nozzles and fittings for connecting the impulse lines from the piezometers were welded to the experimental DP at an angle \( \beta = 90^\circ \) (Fig. 10). DP material – stainless steel [1]. DP with a horizontal axis (\( \psi = 0^\circ \)) and DP with angles \( \psi \) of inclination of their axes to the horizon equal to 5.3° and 354.7° (Fig. 11) were investigated.

The pore volume of the distribution pipeline was calculated by the formula

\[
f = n \omega / \Omega, \tag{12}
\]

where \( \omega \) – the cross-sectional area of the outlet nozzle, \( \omega = \pi d^2 / 4 = 3.14 \times 0.0032^2 / 4 = 8.0425 \times 10^{-6} \) m²; \( d \) – its inner diameter, \( d = 3.2 \) mm (Fig. 9); \( n \) – the number of nozzles on the DP, \( n = 11 \); \( \Omega \) – the cross-sectional area of the experimental DP, \( \Omega = \pi D^2 / 4 = 3.14 \times 0.00821^2 / 4 = 52.939 \times 10^{-6} \) m²; \( D \) – its diameter, \( D = 8.21 \) mm. DP with \( f = 11 \times 8.0425 \times 10^{-6} / 52.939 \times 10^{-6} = 1.67 \) and DP with \( f = 8.0425 \times 10^{-6} / 52.939 \times 10^{-6} = 1.22 \) were investigated. To obtain DP with lower values of the duty cycle, some of the outlet nozzles on the DP were muffled.

The applied experimental DP is similar in size to DP, which are used in the overwhelming majority in practice in the technological processes of various industries.
The maximum average water velocity at the entrance to the experimental distribution pipeline with $f=1.67$ was $V_{\text{beg}}=Q_{\text{beg}}/\Omega=0.16\times10^{-3}/52.939\times10^{-6}=3.022$ m/s, where $\Omega=0.16\times10^{-3}$ m$^3$/s (Fig. 12, b). It met the value of the Reynolds criterion $Re=VD/\nu=3.022\times8.21\times10^{-3}/1.01\times10^{-6}=24568$, where $\nu=1.01\times10^{-6}$ m$^2$/s is the kinematic viscosity of water at a temperature of 20 °C.

The purpose of this experimental study is to check the correctness of the solutions of the DEMFVR. The theoretical calculations made by the methods [8] and [25] were...
compared with the experimental results. The experimental DP was made in a geometric scale of 1:1 with respect to the distribution pipelines of drip irrigation systems for orchards (Fig. 1) and berry fields and at a scale of 1:3–1:4 in relation to DP used in agricultural aviation.

Fig. 11. The angle $\psi$ reference scheme is the inclination of the DP axis to the horizon

Hydraulic resistance in pressure pipeline flows is mainly created by frictional forces. The similarity of the action of friction forces in pressure pipeline flows of liquids is achieved while ensuring the conditions of geometric, kinematic and dynamic similarity, if for each pair at the corresponding points of nature and model the Reynolds criterion will have the same values.

For comparison, let’s take DPs used in agricultural aviation when spraying plants, for example, with pesticides. With their mass concentration in water $10^{-3}$–$10^{-4}$, the viscosity of the pesticide solution is practically the same as the viscosity of the solvent, that is, water. The value of the Reynolds criterion is calculated for the alignment at the beginning of the DP before the first outlet nozzle. The distribution pipe of an aircraft sprayer with an outer diameter of 40 mm and a wall thickness of 5.5 mm has an inner diameter of 29 mm. With an average velocity of an aqueous solution of a pesticide at the entrance to the DP equal to 0.90 m/s, the Reynolds criterion for the flow section in front of the first outlet nozzle $Re_{nature}=0.90\cdot 29\cdot 10^{-3}/1.01\cdot 10^{-6}=25842$. The value of the Reynolds criterion for the model (experimental DP) is calculated above. It is equal to $Re_{model}=V_{2}D/\nu=3.0228\cdot 21.10^{-3}/1.01\cdot 10^{-6}=24568$. The values of the Reynolds criterion calculated for nature and model differ by $\{25842–24568\}/25842\cdot 100\%≈4.93\%$, which is an acceptable deviation.

5.2. Comparison of calculation results with experimental data

The values of the water flow rate in the horizontal distribution pipeline at $\psi=0^\circ$ and PT with $\psi=5.3^\circ$ and $\psi=354.7^\circ$ (Fig. 11), calculated from dependencies (8)–(11) [25] practically coincide with the data of the physical experiment (Fig. 12).

The results of calculating the piezometric heads and water flow rate in the DP, obtained by formulas (8)–(11) [25] at $\psi=0^\circ$ and $\beta=90^\circ$, are also compared with the results of calculations by formulas (5) and (6) [8] as well as with experimental data (Fig. 13). In order to start calculating the DP according to the method [8], it is necessary to set the value of the flow rate of the liquid at the beginning of the DP. Since its value is unknown, and its calculation is laborious, then $Q=160.2\cdot 10^{-6}$ m$^3$/s, obtained experimentally on an experimental DP with $D=8.21$ mm (Fig. 8–10). According to the method [8], only the piezometric head is calculated at the outlet nozzles (without taking into account the velocity head) and provided that the DP is horizontal ($\psi_{0}=0^\circ$), and the jet separation angles are straight ($\beta_{0}=90^\circ$). In addition, by the method of [8], the hydraulic coefficient of friction $\lambda_{0}$ and the coefficient of the detachable mass $m_{0}=\xi_{0}\cos\beta_{0})/V$ are taken to be constant along the length of the DP.

![Fig. 12. Relative change in the track distribution of water along the DP with a duty ratio $\phi=1.22$: 1–3 – experimental data; 4–6 – calculated curves calculated by formulas (8)–(11) [25]; 11, 4 – $\psi=0^\circ$; 2, 5 – $\psi=5.3^\circ$; 3, 6 – $\psi=354.7^\circ$; $L$ – the length of the perforated part of the DP; the x-axis is directed along the flow ($\beta=90^\circ$; $Q_{ref}=0$)](image)

![Fig. 13. Comparison of the calculation results: a – piezometric pressure inside the DP; b – the same, water consumption; 1 – experimental data (head at the last nozzle $H_{0}=0.104$ m, $Q_{0}=0$); 2 – curves obtained according to formulas (8)–(11) [25] at $\phi=0.6$ and the distance between the outlet nozzles as in the experimental DP (Fig. 9); 3 – the same, according to the formulas (5) [8] with a uniform arrangement of the outlet nozzles; 4 – curves obtained by formulas (8)–(11) [25] at $\phi=0.6$ under the condition of uniform arrangement of the outlet nozzles ($\psi=0^\circ$; $\beta=90^\circ$; the x axis is directed along the water flow in the DP; calculation curves 2 and 4 are made going counterflow from the end of the DP)](image)
According to the method [8], according to Fig. 4, the DP is divided into five parts equal in length, and the parameters $Q$ and $H$ are calculated for the sections at the border of these sections. When calculating according to the method [8], the hydraulic friction coefficient $\lambda = 0.03 = \text{const}$ was taken for the entire length of the DP, as for the section of hydraulically rough pipes in the zone of turbulent motion.

In order to correctly compare the method [8] with the method [25], curves 4 (Fig. 13) were additionally calculated using formulas (8)–(11), for the case when the initial nozzles are located with a constant step, since this can be calculated by the method [eight].

The method [25] takes into account the change along the DP of the values of the hydraulic friction coefficient $\lambda (x)$ and the flow rate coefficient of the nozzles $\mu (x)$. In order to start the calculation by the method [25], it is necessary to set the head value $H(o)$ at the last outlet nozzle located at the end of the DP, which is not difficult. The experimental value $H(o) = 0.104$ m was taken. The $\lambda (x)$ coefficient was calculated using the formulas known from hydraulics, and the packing flow rate $\mu (x)$ was calculated using the formula (11).

The flow rate of water in the DP at its beginning in the section of packing 1 (Fig. 9), calculated according to [25] (curve 2 in Fig. 13, b), is equal to 0.00017 m$^3$/s. The water flow rate obtained according to [8] (curve 3) is equal to 0.00013 m$^3$/s. The experimental value (array of points 1) of the water flow rate at the nozzle 1 is 0.00016 m$^3$/s. Consequently, the water flow rate at the beginning of the DP, calculated according to [25], is 6.25 % higher than the experimental value of $\alpha$, the flow rate obtained according to [8] is 18.75 % less than the experimental value. Thus, the hydraulic calculations of the water flow rate for the remaining length of the DP (nozzles 2–11) according to formulas (8), (9) [25] for pressure DP with jet diversion angles $\beta = 90^\circ$ practically coincide with the experimental data and differ by 25, 0–39.7 % of the calculation results according to the method [8]. It should be recalled that the methodology [8] does not provide the calculation of DP with an uneven arrangement of the outlet nozzles.

6. Discussion of the results of the analysis of methods for calculating pressure distribution pipelines

The data obtained as a result of physical and numerous experimental studies of fluid flows in pressure head DP are the basis for constructing mathematical models of these fluid flows.

Differential equations of motion for a fluid with variable flow rate (DEMFR) (1) and (2) [3] have not received a rigorous mathematical solution from 1937, that is, from the time of their derivation. The reason for the long search for a solution to the DEMFR is the objective complexity of the problem. Nevertheless, teams of authors of works [27–30] are working to eliminate this problem.

Equation (3) [4], compiled in 2017, is based on the assumption that the hydraulic coefficient of friction $\lambda (x)$ is constant along the distribution pipeline and includes a number of other simplifications. Consequently, it cannot correctly reflect the real hydrodynamic processes in the pressure head DP. In addition, equation (3) requires a solution for possible practical application.

The method [5] of the hydraulic calculation of the pressure distribution pipeline is also based on the differential equation (2) [3]. However, the authors of [5] laid down in the solution the condition that $\lambda (x) = \text{const}$ along the DP. Provided that $\lambda (x) = \text{const}$, the method [5], according to the author [6], gives the deviation of the calculated values from the experimental data by 38 %. However, equations suitable for practical calculations of pressure head DP at $\beta = 90^\circ$ were obtained in [8, 25].

Methodology for calculating pressure head DP [8], originally created for the design of DP with a longitudinal horizontal axis ($\varphi = 0^\circ$) and right angles of jet diversion ($\beta = 90^\circ$). Also in work [8], it is assumed that the hydraulic coefficient of friction $\lambda (x)$ and the coefficient $\mu (x)$ of the liquid flow rate through the outlet nozzles are constant along the DP. In addition, equations (1) and (2) [3], as well as expressions (4)–(6) [8] obtained on their basis, were drawn up under the condition that the liquid flows from the DP into a stationary atmosphere. Therefore, these equations do not take into account the hydrodynamic pressure of the external fluid flow.

However, in practice, DPs are used, laid in an external flow of liquid or gas. For example, with a dispersed discharge of treated wastewater into the water intake, the jets that poured out of the DP holes are warmer than the water of a river or sea. Then they float to the surface of the reservoir. In summer, on the contrary, it is colder, so they sink to the bottom.

In agricultural aviation, jets v flow from the DP into an air stream, the velocity $V$ of which, for example, when using the AN-2 aircraft, is equal to 37.8–44.4 m/s. Also, the angle $\beta$ between vectors and is not always straight.

The considered methods [3, 7–9], as their analysis showed, do not take into account:

a) the value of the angle $\beta$ between the vectors $\vec{v}$ and $\vec{V}$, other than $90^\circ$;

b) the angle of inclination of this axis PT to the horizon;

c) the change along the DP of the velocities of the flow $\vec{V}$ and detachable jets $\vec{v}$;

d) change in the values of the coefficient $\mu (x)$ of the liquid flow rate through the outlet;

e) change in the hydraulic coefficient of friction $\lambda (x)$ along the DP;

f) kinematic and physical parameters of the flow outside the DP.

The reason for the neglect in the works [5, 7–9] of the named parameters of the DP and the kinematic characteristics of the flow in it and the detached jets is the simplified solution of equations (1) and (2) [3], when these quantities are neglected. The neglect of these values was eliminated in [25]. The originality of the calculated equations (8) and (9) [25] lies in the fact that when solving the initial equation (2) [3], the variables along the distribution pipeline, the values of the hydraulic friction coefficient $\lambda (x) = \text{const}$ and the coefficient of fluid flow through the outlet nozzles $\mu (x) = \text{const}$.

Thus, the accuracy of the hydraulic calculation of pressure head DP has increased. Water consumption at the beginning of DP, calculated by formulas (8) and (9) [25], exceeds the experimental value by 6.25 %, and the flow rate obtained from expressions (5) and (6) [8] is 18.75 % less from the experimental value (Fig. 13, b). The advantage of the technique [25] over the techniques [5, 7–9] is that equations (8) and (9) take into account the kinematic and physical parameters of the external fluid flow that washes the distribution pipeline.
In the practical application of equations (8) and (9) [25], it should be taken into account that they give a practical coincidence of the calculated results with the experimental data only at $\beta = 90^\circ$. So, the urgent task of hydromechanics is the creation of a mathematical model of fluid pressure flows with variable path flow rate, which provides accurate hydraulic calculation of distribution pipelines and collecting pipelines at angles of $0^\circ \leq \beta \leq 360^\circ$. The creation of such a model in the future will provide predictable regulation of the distribution pipeline operation, in particular, a controlled decrease in the unevenness of the path distribution of the liquid. Further development of research consists in improving equations (8), (9) [25] with respect to taking into account any values of the angles $\beta$ other than straight lines. To achieve this goal, the values of the flow coefficient $\mu(x)$ of the outlet cylindrical nozzles installed on the DP are experimentally established at different values of the jet detachment angle $\beta$ ($0^\circ \leq \beta \leq 360^\circ$) and at different values of the Reynolds criterion [28].

The authors of [31] continue to work on the creation of a reliable method for calculating pressure head DP. Simultaneously with the study of DP, work is being carried out to improve the design of pressure collecting pipelines [32, 33]. The latter require an approach similar to that used for calculating pressure distribution pipelines [29, 33], which is reflected in the works [1, 30].

**7. Conclusions**

1. Studies carried out on an experimental distribution pipeline with gravitational water supply made it possible to determine the accuracy of calculations of the distribution of water along the pressure head DP, obtained by comparative methods. The inner diameter of the distribution pipe is $8.21 \text{ mm}$, which is close to the DP used in practice. The experimental DP is equipped with eleven outlet nozzles with jets detaching from it at right angles. The value of the Reynolds criterion for the flow at the beginning of the DP was equal to $24568$. Experimental DP is geometrically and hydrodynamically similar to DP, which are common in technological processes of various industries, for example, in pressure irrigation systems.

2. The water flow rate calculated by the method [25] is $6.25\%$ higher than the experimental value, and the flow rate obtained by [8] is $18.75 \%$ lower than the experimental value. The method [25] for calculating pressure distribution pipelines provides more accurate calculation results in comparison with the method [8]. The increased accuracy of calculations by the method [25] is achieved by taking into account the variable values along the DP of the hydraulic friction coefficient $\lambda(x)$ and the coefficient $\mu(x)$ of the fluid flow rate through the original nozzles. The question of taking into account the angle $\beta_{\text{det}}$ between the vectors of the average velocity of the main fluid flow in the DP and the detachable jet $\vec{v}$, provided that it varies within $0^\circ \leq \beta_{\text{det}} \leq 360^\circ$ remains unresolved.

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