Analysis of random vibration fatigue of aluminum alloy beam with a hole based on frequency domain method

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Abstract. Based on the frequency-domain method, the influence factors of vibration fatigue life of aluminum alloy beam with a hole was analyzed. Firstly, the finite element model of aluminum alloy beam with a hole was established, and modal analysis was performed to obtain the natural frequency and mode shape of each order. And then, harmonic response analysis was employed to determine the direction and natural frequency of vibration that has the greatest impact on the structure. After that, fatigue life of the structure was studied. The influence of model method, power spectral density, natural frequency order, frequency band, bandwidth and other factors on fatigue life was analyzed by changing excitation load conditions. Through the simulation, the results indicate that: Fatigue life calculated by Steinberg and Lalanne method is more conservative than that calculated by Dirlik method under the same excitation load. The accelerated fatigue life test can be performed by changing the power spectral density value. The power spectral density value of the excitation load at the first natural frequency plays a major role in the fatigue life. The quantitative relationship between central band fatigue and full frequency fatigue is established.

1. Introduction

Due to the harsh operating environment, aviation equipment have to bear complex random load, and fatigue failure is part of the main failure forms of aircraft structural parts [1-6]. Once the fatigue failure of aircraft structures occurs, it will result in a catastrophic accident. In recent years, as aircraft structures become increasingly lightweight, fly faster and more complex, the vibration environment becomes even more complex, and the random fatigue caused by the vibration environment attracts more and more attention from the engineering community [7]. Therefore, fatigue characteristics analysis of structures under random vibration load has important research value.
Random vibration fatigue refers to that, due to the action of random vibration load, the structure generates a dynamic response, which leads to fatigue failure[8]. A fatigue life estimation in the frequency domain is proven to have advantages in comparison to the time-domain estimation[9]. Under random load, the fatigue-life of the structure is tied to the loading frequency and the characteristics of the structure, such as inertial force and damping force. Especially when the natural frequency of the structure is on the brink of or overlaps with the frequency spectrum of the external force, resonance phenomenon is likely to occur and cause greater dynamic response, resulting in very serious damage[10, 11].

In recent years, many scholars have considered the random vibration fatigue of structures[12-17]. Relationships between fatigue damage assessment and counting methods are investigated by establishing when in frequency domain analysis, analytical solutions of expected damage are connected with a counting procedure assumption and how this is related to the rain-flow counting procedure[18]. In broadband load conditions, the definition of a period is not obvious, and a standard must be used to establish the period by pairing peaks and valleys [19, 20]. Several methods for damage evolution in the frequency domain have been developed in the last thirty years[17, 21-25]. Despite the number of contributions, due to the complexity of the matter, the mechanism of vibration-induced fatigue damage remains unclear, there is no clear distinction between structural vibration fatigue and static fatigue, and some areas require further study: 1. The differences of three fatigue life estimation models; 2. The influence of power spectral density value of excitation load on fatigue life; 3. The influence law of model method, power spectral density value, natural frequency order, frequency band, bandwidth and other factors on the fatigue life.

2. Fatigue analysis method based on frequency domain

In the study of vibration fatigue in frequency domain, the structure is simplified as a linear system. The power spectral density (PSD) function of stress response can be achieved by frequency-domain analysis, and the cumulative fatigue damage and fatigue life at the dangerous point of the structure can be obtained by using the PSD of stress response.

![Flow chart of random vibration fatigue analysis in frequency domain.](image)

**Figure 1.** Flow chart of random vibration fatigue analysis in frequency domain.
The key of frequency-domain fatigue life estimation is to select the appropriate fatigue life estimation model, and obtain the probability density function (PDF) of stress amplitude through the PSD of stress response. First, the frequency response analysis of the finite element model of the structure is made to obtain the PSD of stress response. The PDF of the stress amplitude is calculated by using the cumulative fatigue damage theory and certain failure criteria in combination with the fatigue life S-N curve of the materials used in the structure. The analysis process is illustrated in Figure 1. The following describes three frequency domain models of fatigue life estimation.

2.1. Steinberg method
For the random process with zero mean value, the power spectrum of stress response is equal to the power spectrum of stress amplitude. Steinberg[26] proposed a three interval methods based on Gaussian distribution and Miner linear cumulative damage theory. Steinberg believed that the distribution of stress amplitude has the following characteristics:

In 68.3% time, the stress amplitude is between $-1\sigma \sim 1\sigma$, in 95.4% time, the stress amplitude is between $-2\sigma \sim 2\sigma$ in 99.73% time, the stress amplitude is between $-3\sigma \sim 3\sigma$. It is assumed that in the remaining 0.27% time, the stress amplitude is greater than $6\sigma$ and will not cause any damage to the structure. According to Miner damage accumulation rule, the total damage of the structure is as follows:

$$D^ST = \frac{n_{1\sigma}}{N_{1\sigma}} + \frac{n_{2\sigma}}{N_{2\sigma}} + \frac{n_{3\sigma}}{N_{3\sigma}} = 0.6831f_0 + 0.2710f_0 + 0.0433f_0$$

(1)

where $n_{1\sigma}, n_{2\sigma}$ and $n_{3\sigma}$ represents the actual number of cycles corresponding to the stress level less than or equal to $1\sigma, 2\sigma, 3\sigma$. Also, $N_{1\sigma}, N_{2\sigma}, N_{3\sigma}$ is the fatigue life on the S-N curve of the material at the stress level of $1\sigma, 2\sigma, 3\sigma$. $f_0$ represents the root mean square value of stress.

2.2. Dirlik method
Based on a large number of Monte Carlo simulation in the time domain, the empirical model of probability density function of the amplitude of a simple cycle and rain flow cycle is obtained. The Dirlik[22] method considers that the empirical formula for the probability density function of the amplitude of the broadband stochastic process is a combination of an exponential distribution and two Rayleigh distribution.

$$p_{DK}(s) = \frac{D_1}{Q} \exp\left(-\frac{Z}{Q}\right) + \frac{D_2Z}{Q} \exp\left(-\frac{Z^2}{2R^2}\right) + D_3Z\exp\left(-\frac{Z^2}{2}\right)$$

(2)

where:
\[ Z = \frac{s}{2\sqrt{m_0}} \quad \chi_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}} \quad \gamma = \frac{m_2}{\sqrt{m_0m_4}} \]  

\[ D_1 = \frac{2(\chi_m - \gamma^2)}{1 + \gamma^2} \quad D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R} \quad D_3 = 1 - D_1 - D_2 \]  

\[ Q = \frac{1.25(\gamma - D_3 - D_2R)}{D_1} \quad R = \frac{\gamma - \chi_m - D_1^2}{1 - \gamma - D_1 - D_1^2} \]  

and where the following equation:

\[ m_n = \int_0^\infty \omega^n S(\omega) d\omega \]  

defines the \( n \)-th spectral moment of the PSD signal.

According to Dirlik’s probability density function distribution model of rain circulation amplitude and miner’s linear fatigue cumulative damage theory, the structural fatigue damage is calculated as follows:

\[ D^{DK} = \frac{E(p)}{C} \int_0^\infty S^k p^{DK}(S) dS \]  

where \( k \) and \( C \) depends on the fatigue strength curve of the materials (through the \( S^K N = C \) relation).

2.3 Lalanne method

This method is suitable for wideband random signals. The empirical formula of the amplitude probability density function of wideband random process is the combination of Gaussian distribution and Rayleigh distribution according to certain weight proportion[27].

\[ p_{LE}(S) = \frac{B_1 \exp\left(-\frac{S^2}{2\sqrt{m_0}(1-\gamma^2)}\right) + B_2 \exp\left(-\frac{QS^2}{2m_0}\right)}{\sqrt{m_0}} \]  

where:

\[ B_1 = \frac{(1-\gamma^2)^{1/2}}{2\pi} \quad B_2 = \frac{S\gamma}{2\sqrt{m_0}} \]  

\[ Q = 1 + \text{erf}\left[\frac{S\gamma}{\sqrt{2m_0(1+\gamma^2)}}\right] \]
Here, \( \text{erf}(x) \) represents the error function:

\[
\text{erf}(x) = \int_0^x (2\pi)^{-1/2} \exp(-\frac{y^2}{2}) \, dy
\]

According to the probability density function distribution model of rain-flow circulation amplitude of Lalanne method and miner's linear fatigue cumulative damage theory, the structural fatigue damage is calculated as follows:

\[
D_{LE} = \frac{E(p)}{C} \int_0^\infty S^k p_{LE}(S) \, dS
\]

3. Structural dynamics simulation analysis

3.1. Test object

In order to study the influence of dynamic characteristics on the fatigue life of structural members, the beam with a hole is selected as the research object, and the geometric model is shown in Figure 2. Thickness of the beam is 3 mm, with the left side at the clamped fixed end. In the simulation test, the root position of the specimen is restrained by fixed support, and random vibration load is applied in the form of acceleration PSD flat spectrum.

Aluminum alloy 6061-T6 is selected as the test piece material, and the basic mechanical properties at room temperature are shown in Table 1. The stress S-N curve of the material is shown in Figure 3.

**Table 1.** Mechanical Properties of aluminum alloy 6061-T6[14].

| Features | Young modulus | Poisson ratio | Tensile ultimate strength | Density |
|----------|---------------|---------------|---------------------------|---------|
| Unit     | GPa           | –             | MPa                        | kg·m\(^{-3}\) |
| Value    | 68.948        | 0.33          | 310.26                     | 2849    |
Figure 3. S-N curve of aluminum alloy 6061-T6[14].

3.2. Finite element model and modal analysis

The tetrahedron 3D mesh method is used to divide the specimen from the bottom to the top, and the average cell side length is defined as 3mm, with a total of 8055 cells and 11479 nodes. Using the modal analysis module, the natural frequency and mode shape of the first six modes of the test piece are calculated, as shown in Table 2 and Figure 4.

Table 2. Natural frequency of the first six modes.

| Mode no. | 1     | 2     | 3     | 4     | 5     | 6     |
|----------|-------|-------|-------|-------|-------|-------|
| Frequency (Hz) | 64.0  | 275.5 | 417.1 | 534.6 | 1578.6| 1616.9|

Figure 4. Mode shapes of the first six modes.
3.3. Stress response analysis

Continue to analyze the cloud chart of root mean square distribution of von-Mises stress response of each unit of the test piece (as shown in Figure 5). It can be seen that the stress maximum response of the structural piece is at the neck of the notch, were prone to fatigue failure.

Figure 5. Von-Mises stress nephogram of specimen under ANSYS simulation.

3.4. Harmonic response analysis

Then, select the nodes close to the dangerous parts, and calculate the frequency of dynamic response results based on the stress, as shown in Figure 6. By comparing the three curves in the figure, we can get:

1. The stress response value in the X axis is far greater than that in the Y and Z axis, so we mainly analyze the stress in X axis.
2. The response curve takes the maximum value at the 1-th, 4-th and 5-th natural frequency.
3. The response peak value 699.1 MPa²/Hz of the first order natural frequency is far greater than the other two.

Figure 6. Frequency response curve of dangerous parts based on stress.
4. Vibration fatigue simulation analysis

By calculating the fatigue life of the specimen, Influence factors of vibration fatigue life are analyzed from five aspects: calculation model of fatigue life, PSD value of excitation load, central frequency band range of natural frequency, order of natural frequency and same frequency band.

4.1. Influence analysis of fatigue life calculation model

Including the first six modal frequencies, the frequency band is 10~2000Hz, and the PSD value is 0.2 g²/Hz. Three different wide-band random load models are selected for fatigue calculation to explore the influence of them on the structural fatigue life. From Table 3, it can be concluded that fatigue life calculated by Steiberg and Lalanne method is more conservative than that calculated by Dirlik method under the same excitation load.

| Number of profile | Estimation Model | Frequency Band (Hz) | PSD Magnitude (g²/Hz) | Fatigue Life (s) |
|-------------------|-----------------|---------------------|----------------------|-----------------|
| A1                | Steiberg        | 10 ~ 2000           | 0.2                  | 7.778×10⁴      |
| A2                | Dirlik          | 10 ~ 2000           | 0.2                  | 2.217×10⁵      |
| A3                | Lalanne         | 10 ~ 2000           | 0.2                  | 1.914×10⁵      |

4.2. Influence analysis of PSD value of excitation load

The frequency band of the PSD spectrum of excitation load is 10 ~ 2000Hz, and six different values (Table 4) are selected: 0.05 g²/Hz, 0.10 g²/Hz, 0.15 g²/Hz, 0.20 g²/Hz, 0.25 g²/Hz and 0.30 g²/Hz. The model selects the Lalanne method (the same below), and the fatigue life calculation results are presented in Figure 7. It can be observed that there is a linear relationship between the PSD value of the excitation load and the value of the fatigue life. It is verified that the accelerated fatigue life test can be performed by changing the PSD value.

Figure 7. Test results of group B.
Table 4. Test group B.

| Number of profile | Estimation Model | Frequency Band(Hz) | PSD Magnitude(g²/Hz) |
|-------------------|------------------|--------------------|----------------------|
| B1                | Lalanne          | 10 ~ 2000          | 0.05                 |
| B2                | Lalanne          | 10 ~ 2000          | 0.10                 |
| B3                | Lalanne          | 10 ~ 2000          | 0.15                 |
| B4                | Lalanne          | 10 ~ 2000          | 0.20                 |
| B5                | Lalanne          | 10 ~ 2000          | 0.25                 |
| B6                | Lalanne          | 10 ~ 2000          | 0.30                 |

4.3. Influence analysis of natural frequency center frequency band range

It can be seen from the stress frequency response curve (Figure 8) of the fatigue dangerous part of the test piece that the first-order natural frequency of the structure has the greatest influence on the fatigue life. In this paper, the first-order natural frequency is taken as the center, and the frequency band range \((f_i(1-\alpha), f_i(1+\alpha))\) is selected, as shown in Table 5.

Figure 8. The central frequency range of the first natural frequency.

Table 5. Test group C.

| Number of profile | Center band range | Frequency band (Hz) | PSD Magnitude (g²/Hz) | Number of profile | Center band range | Frequency band (Hz) | PSD Magnitude (g²/Hz) |
|-------------------|-------------------|---------------------|-----------------------|-------------------|-------------------|---------------------|-----------------------|
| C1                | 5%                | 60.8~67.2           | 0.2                   | C9                | 45%               | 35.2~92.8           | 0.2                   |
| C2                | 10%               | 57.6~70.4           | 0.2                   | C10               | 50%               | 32.0~96.0           | 0.2                   |
| C3                | 15%               | 54.4~73.6           | 0.2                   | C11               | 55%               | 28.8~99.2           | 0.2                   |
| C4                | 20%               | 51.2~76.8           | 0.2                   | C12               | 60%               | 25.6~102.4          | 0.2                   |
| C5                | 25%               | 48.0~80.0           | 0.2                   | C13               | 65%               | 22.4~105.6          | 0.2                   |
The fatigue life calculation results are shown in Figure 9. It can be seen from the curve in the figure that:

1. With the increase of central frequency band range, the fatigue life shows a downward trend.
2. Compared with the second stage (\( \alpha \geq 0.2 \)), the decline trend of the first stage fatigue life value is more significant.

Figure 9. Test results of group C.

4.4. Influence analysis of natural frequency order

The former fourth-order modal frequency is taken as the center, and the same bandwidth \( \Delta f \) is took as 100 Hz. PSD magnitude is 0.2 g²/Hz (Table 6), and the fatigue life of the test piece is calculated respectively. The results show that the fatigue damage of the structure is mainly caused by the center frequency band of the first natural frequency, while the contribution ratio of the rest order of natural frequency is very small.

| Number of profile | Mode no. | Frequency band (Hz) | Band width (Hz) | PSD Magnitude (g²/Hz) | Fatigue life (s) | Damage contribution ratio |
|-------------------|---------|---------------------|-----------------|------------------------|-----------------|--------------------------|
| D0                | 1–6     | 10–2000             | 2000            | 0.2                    | 1.914×10⁵       | 100%                     |
| D1                | 1       | 14–114              | 100             | 0.2                    | 1.946×10⁵       | 98.35%                   |
| D2                | 2       | 225–325             | 100             | 0.2                    | 5.109×10⁴       | 3.746×10⁻⁴              |
| D3                | 3       | 367–467             | 100             | 0.2                    | 5.322×10⁴       | 3.596×10⁻⁴              |
| D4                | 4       | 484–584             | 100             | 0.2                    | 9.027×10⁴       | 2.120×10⁻⁴              |
4.5. Influence analysis of frequency band

Including the first-order and second-order natural frequencies, the frequency interval of 14–284 Hz is divided into equal intervals. The bandwidth of each segment $\Delta f = 10$ Hz, and the PSD magnitude is 0.20 g$^2$/Hz. There are 28 groups of excitation loads (as shown in Table 7).

Table 7. Test group E.

| Number of profile | Bandwidth (Hz) | Number of profile | Bandwidth (Hz) | Number of profile | Bandwidth (Hz) | Number of profile | Bandwidth (Hz) |
|-------------------|----------------|-------------------|----------------|-------------------|----------------|-------------------|----------------|
| E1                | 14–24          | E8                | 84–94          | E15               | 154–164        | E22               | 224–234         |
| E2                | 24–34          | E8                | 94–104         | E16               | 164–174        | E23               | 234–244         |
| E3                | 34–44          | E10               | 104–114        | E17               | 174–184        | E24               | 244–254         |
| E4                | 44–54          | E11               | 114–124        | E18               | 184–194        | E25               | 254–264         |
| E5                | 54–64          | E12               | 124–134        | E19               | 194–204        | E26               | 264–274         |
| E6                | 64–74          | E13               | 134–144        | E20               | 204–214        | E27               | 274–284         |
| E7                | 74–84          | E14               | 144–154        | E21               | 214–224        | E28               | 284–294         |

![Figure 10](image.png)

**Figure 10.** Test results of group E.

The simulation results of fatigue life (Figure 10) show that:

1. The lower the excitation frequency, the greater the fatigue damage caused by the same interval bandwidth.
2. The slope of the fatigue life value at the first natural frequency is the largest, however the slope of the second order is close to 0.

5. Conclusions

Under random vibration, fatigue life of typical beam with a hole in the design of an aircraft has been analyzed in this paper. The simulation results demonstrate that:

1. Fatigue life calculated by Steiberg and Lalanne method is more conservative than that calculated by Dirlik method under the same excitation load.
2. The accelerated fatigue life test can be performed by changing the power spectral density value.
3. The PSD value of the excitation load at the first natural frequency plays a major role in the fatigue life.
4. The quantitative relationship between central band fatigue and full frequency fatigue is established.

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