A DISCONTINUITY IN THE LOW-MASS INITIAL MASS FUNCTION

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ABSTRACT

The origin of brown dwarfs (BDs) is still an unsolved mystery. While the standard model describes the formation of BDs and stars in a similar way recent data on the multiplicity properties of stars and BDs show them to have different binary distribution functions. Here we show that proper treatment of these uncovers a discontinuity of the multiplicity-corrected mass distribution in the very-low-mass star (VLMS) and BD mass regime. A continuous IMF can be discarded with extremely high confidence. This suggests that VLMSs and BDs on the one hand, and stars on the other, are two correlated but disjoint populations with different dynamical histories. The analysis presented here suggests that about one BD forms per five stars and that the BD-star binary fraction is about 2\%--3\% among stellar systems.

Subject headings: binaries: general — open clusters and associations: general — stars: low-mass, brown dwarfs — stars: luminosity function, mass function

1. INTRODUCTION

Traditionally, brown dwarfs (BDs) are defined as (sub)stellar bodies with masses below the hydrogen burning mass limit (HBL), $m_H = 0.075 M_\odot$ for solar composition, and consequently they cool indefinitely after formation (Burrows et al. 1993; Chabrier & Baraffe 2000). Several attempts have been made to explain the formation of BDs by the same mechanisms as for stars, i.e. via fragmentation of a gas cloud and subsequent accretion (Adams & Fatuzzo 1996; Padoan & Nordlund 2002). If a gas cloud in a star-forming region fragments there will be a certain number of gas clumps with masses below the HBL. Unless the mass of the fragment is below the local Jeans mass it will contract in essentially the same way as higher-mass clumps and finally produce a single or multiple BD. This scenario predicts similar multiplicities and also a substellar initial mass function (IMF) as a continuous extension of the stellar one (e.g. the standard model with BDs in Kroupa et al. 2003). However, recent observations have shown that there is a lack of BD companions to low-mass stars (McCarthy, Zuckerman & Becklin 2003; Grether & Lineweaver 2006). We found a star-BD binary fraction among solar-type primaries of less than 1\% for close companions, the brown dwarf desert. This implies two populations of stellar and substellar objects, and that binaries are formed in each population separately (except for pairing due to post-formation dynamical exchanges). Observations e.g. by Reid et al. (2006) also show that most BD binaries have a primary-to-companion mass-ratio of $q > 0.8$ in contrast to the mass-ratio distribution of stellar binaries which has typically $q < 0.4$ (Duquennoy & Mayor 1991).

There are indications, e.g. Metchev & Hillenbrand (2005), that the BD desert may not be as dry for larger separations ($> 30 \text{ AU}$) as it is for smaller ones. Since new surveys using adaptive optics or new instruments like the upcoming James Webb Space Telescope might reveal more substellar companions to stars, the fraction of star-BD systems may increase.

Apart from the BD desert there are more hints for a separate population. For example, BDs and VLMSs have a relatively low binary fraction of about 15\% (Bouy et al. 2003; Close et al. 2003; Martin et al. 2003; Kraus, White & Hillenbrand 2006 and Law et al. 2007). For comparison, the stellar binary fraction is close to 100\% for the very young Taurus-Auriga association (TA, about 1 Myr; Duchêne 1999; Luhman et al. 2003) and about 40--50\% for other clusters and field stars (Lada 2006). The BD and VLMS binary fraction can be increased to a star-like binary fraction but only by postulating a large fraction of $\lesssim 5 \text{ AU}$ binaries (Jeffries & Maxted 2005). But such a semi-major axis distribution would again imply a discontinuity of its form between low-mass stars and VLMSs/BDs and is not supported by the radial-velocity survey of Joergens (2006). We therefore do not consider the star-like formation as a major mechanism for BDs.

It has also been argued that the low binary fraction of BDs can be understood as a continuous extension of a trend that can already be recognised from G-dwarfs to M-dwarfs (Luhman 2004a; Sterzik & Durisen 2003). Therefore the binary fraction alone cannot be taken as a strong evidence to introduce a separate population.

The most striking evidence for two separate populations is the empirical fact that the distribution of the separations and therefore the binding energies of BD binaries differs significantly from that in the stellar regime (Bouy et al. 2003; Burgasser et al. 2003; Martin et al. 2003; Close et al. 2003). This is shown in Figs. 1 and 2. The solid line is a Gaussian fit to the central peak of the histogram while the dash-dotted one refers to Basri & Reiners (2000) (a compressed Fischer & Marcy 1992 fit). BDs and very low-mass stars (VLMS) have a semi-major axis distribution limited to $\lesssim 15 \text{ AU}$, whereas M-, K- and G- dwarfs have a very broad and similar distribution (long-dashed and short-dashed curves; Close et al. 2003; Law, Hodgkin & Mackay 2007; Goodwin et al. 2007). There is also a dearth of BDs below 1 AU. Recent findings, e.g. by Guenther & Wuchterl (2003) and Kenyon et al. (2005), suggest a low number of such very close BD/VLMS binaries. The semi-major axis distribution of BDs/VLMSs binaries based on the data from Close et al. (2003) can be modelled with a log $a$ Gaussian centred at 4.6 AU ($\log a = 0.66$) with a half-peak width of $\sigma = 0.4$. It corresponds to an overall BD/VLMS binary fraction of $f_{\text{BD-VLMS}} = 0.15$. If data from Luhman (2004b),

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Joergens (2006) and Konopacky et al. (2007) are taken as hints to incomplete data between about 0.02 and 1 AU, the compressed Fischer & Marcy (1992) Gaussian from figure 4 in Basri & Reiners (2006) may provide an appropriate envelope. However, for an assumed BD mass of 0.07 $M_\odot$ and $f_{BD-BD} = 0.26$ their period distribution corresponds to a semimajor axis distribution with $\sigma = 0.85$ and is therefore still inconsistent with that of M- and G-dwarfs.

Although Konopacky et al. (2007) have recently found five VLMS binaries in TA with four of them having separations much larger than 15 AU, the sudden change of the orbital properties remains. In particular, they found two binaries with projected separations slightly above 30 AU and two others with separations between 80 and 90 AU. Possible implications of these discoveries are discussed in Section 5.4. However, the truncation near 15 AU cannot be derived from the stellar distribution through downsizing according to Newton’s laws (Close et al. 2003; Bouy et al. 2003). Not even dynamical encounters in dense stellar environments can invoke such a truncation near 15 AU (Burgasser et al. 2003). Using N-body simulations Kroupa et al. (2003) tested the hypothesis that BDs and stars form alike, and mix in pairs, and found that, despite of close dynamical encounters, the distribution of the semimajor axes of BD-binaries remains star-like. That is, dynamical encounters even in dense clusters cannot truncate the BD binary distribution near 15 AU. They further found that star-SD binaries would be much more frequent than actually observed. This classical hypothesis is rejected with high confidence.

With this contribution we study the implications of the observed change of binary properties on the underlying single-object initial mass function (IMF). For this purpose, we analysed the observational mass functions of TA (Luhman 2004a), the IC 348 cluster (Luhman et al. 2003), the Trapezium cluster (Muench et al. 2002) and the Pleiades cluster based on data by Dobie et al. (2002), Moraux et al. (2003) and the Prosser and Stauffer Open Cluster Database. For all the systems we analyse, we refer to the MF as the IMF, although strictly speaking this is not correct for the 130 Myr-old Pleiades. In Section 2 we shortly review the definition of the IMF and the role of multiplicity on the shape of the individual body IMF compared to the observed system IMF (IMF$_{sys}$). We show that the IMF that can be derived from an observed IMF$_{sys}$ does not need to be continuous even if the IMF$_{sys}$ does not show any discontinuity. In Section 3 we describe the statistical method how to fit a model IMF$_{sys}$ by combining BD and star IMFs as an approximation to the observed IMF$_{sys}$. Section 4 presents the results that indicate a discontinuity close to the HBL. Also the BD to star ratio from the model is calculated there. In Section 5 the results are discussed in the context of four alternative BD formation scenarios, i.e. embryo ejection, disk fragmentation, photoevaporation and ejection by close stellar encounters.

2. THE IMF FOR INDIVIDUAL STARS AND SYSTEMS

2.1. Definition

The IMF is among the most important properties of a stellar population since it gives hints to the processes that form stars and BDs. Although we can only observe stellar populations at their given age there are data of several very young populations where the mass function is probably still very close to the initial one.

\[ \xi(m) = \frac{dn}{dm} = km^{-\alpha}, \]  

for $0.4 \lesssim m/M_\odot \lesssim 10$ and with $\alpha = 2.35$ and a normalisation constant $k$. The IMF is often expressed on the logarithmic mass scale and then becomes

\[ \xi_L(\log_{10}m) = \frac{dn}{d\log_{10}m} = (\ln 10) m \xi(m) = k_L m^{1-\alpha}, \]
An IMF is said to be flat if $\xi(m_{\log m})$ is constant, i.e. $\alpha = 1$. More recent work has shown there to be a flattening in the lower-mass regime of the observed mass function and, in the $\xi(m)$ representation, even a turn-over near the BD-star transition (Kroupa 2001; Reid, Gizis & Hawley 2002; Chabrier 2003).

Here, all objects from BDs to the most massive stars in a cluster are described by a continuous IMF, i.e. with a single population containing BDs as well as stars. The universal “canonical” IMF has $\alpha_{BD} \equiv \alpha_0 = 0.3 (0.01 \leq m/M_\odot \leq 0.075)$, $\alpha_1 = 1.3 (0.075 \leq m/M_\odot \leq 0.5)$, $\alpha_2 = 2.3 (0.5 \leq m/M_\odot \leq m_{\max})$, where $m_{\max}$ is the mass of the host cluster (Weidner & Kroupa 2006). The generally accepted wisdom has been that the IMF is continuous from above $0.01 M_\odot$ to $m_{\max}$ (e.g. Chabrier 2002).

### 2.2. Unresolved binaries

Star cluster surveys are usually performed with wide-field telescopes with limited resolution that do not resolve most of the binaries. Hence they yield, as an approximation, system MFs in which unresolved binary systems are counted as one object. The fraction of unresolved multiples can be taken as the total binary fraction of a cluster,

$$f_{\text{tot}} = \frac{N_{\text{bin, total}}}{N_{\text{sing}} + N_{\text{bin, total}}}, \quad (3)$$

where $N_{\text{sing}}$ is the number of singles (or resolved individual bodies) and $N_{\text{bin, total}}$ the number of (unresolved) binaries. Unresolved binaries increase the number of individual bodies, $N_{\text{body}}$, in a cluster such that

$$N_{\text{body}} = (1 + f_{\text{tot}})N_{\text{sys}}, \quad (4)$$

where $N_{\text{sys}} = N_{\text{sing}} + N_{\text{bin, total}}$ is the total number of systems. Note that a “system” is either a single body or a binary or a higher-order multiple. Here we ignore higher-order multiples, because they are rare (Goodwin & Kroupa 2005). The binary fraction in dependence of the mass of the primary star, $m_{\text{prim}}$, is

$$f(m_{\text{prim}}) = \frac{N_{\text{bin, total}}(m_{\text{prim}})}{N_{\text{sing}}(m_{\text{prim}}) + N_{\text{bin, total}}(m_{\text{prim}})}, \quad (5)$$

where $N_{\text{bin, total}}(m_{\text{prim}})$, $N_{\text{sing}}(m_{\text{prim}})$ are, respectively, the number of binaries with primary-star mass $m_{\text{prim}}$ and single-stars of mass $m_{\text{prim}}$.

### 2.3. The system mass function

The effect of this binary error on the appearance of the IMF can be described as follows: assume a stellar population with $f_{\text{tot}} < 1$ and with stellar masses with a minimum mass $m_{\min}$ and a maximum mass limit of $m_{\max}$. The minimum mass of a binary is $2m_{\min}$ while the mass-dependent binary fraction $f(m) = 0$ for $m_{\min} \leq m_{\sys} < 2m_{\min}$. A binary closely above $2m_{\min}$ can only consist of two stars near $m_{\min}$ making such binaries rare. For higher system masses, where a system can be a multiple or a single star, there are more possible combinations of primary and companion mass, so that the binary fraction increases with the system mass and approaches an upper limit for the most massive objects.

Fig. 3 shows the general shape of a system IMF for a flat (logarithmic scale) IMF with $m_{\min} = 0.1 M_\odot$, $m_{\max} = 1 M_\odot$ and $f_{\text{tot}} = 0.5$. The IMF$_{\text{sys}}$ is flat between $m_{\min}$ and $2m_{\min}$ ($\log_{10} m/M_\odot \approx -0.7$) and rises above $2m_{\min}$ to a maximum at $m_{\max}$. Systems with $m_{\sys} < 2m_{\min}$ can only be singles and the IMF$_{\text{sys}}$ in this region is just the IMF minus the mass function of objects that are bound to a multiple system. For masses $m_{\sys} > m_{\max}$, on the other hand, only binaries exist, and the IMF$_{\text{sys}}$ declines towards zero at $m_{\sys} = 2m_{\max}$, the highest mass possible for binaries. The sharp truncation of the IMF at $m = m_{\max}$ causes the sudden drop at $m_{\sys} = m_{\max}$, while the minimum peak at $m_{\sys} = m_{\max} + m_{\min}$ corresponds to the maximum of the binary mass function (IMF$_{\text{bin}}$, the IMF of binary system masses).

For a binary of this mass the primary and companion mass can be drawn from the whole IMF, and thus the number of possible combinations becomes maximal. It should be noted that natural distributions with smoother boundaries probably do not show such a double peak.

Mathematically and in the case of random-pairing, which is a reasonable approximation in the stellar regime (Malkov & Zinnecker 2001; Goodwin, Kroupa, Goodman & Burkert 2007), the binary mass function is just the integral of the product of the normalised IMF, $\xi = \xi/N_{\text{body}}$, of each component times the total number of binaries $N_{\text{bin, total}}$. Given the masses of the binary components A and B, $m_A$ and $m_B$, the binary mass $m_{\sys} = m_A + m_B$. Thus, $m_B = m_{\max} - m_A$. IMF$_{\text{bin}}$ can now be written as

$$\xi_{\text{bin}}(m_{\sys}) = \int_{m_{\min}}^{m_{\max}} \frac{dm}{m_{\sys}} \frac{dm - m}{m_{\max} - m} \hat{\xi}(m), \quad (6)$$

where $m_{\min}$ is the lower mass limit of all individual objects in the population. The upper limit of the integral, $m_{\max} - m_{\min}$, is the maximum mass of the primary component corresponding to a secondary component with $m_B = m_{\min}$.

The other extreme case of assigning the component masses is equal-mass pairing. In that case, eq. (6) simplifies to the IMF of one of the components and IMF$_{\text{bin}}$ is just the IMF shifted by a factor of 2 in mass and corrected for binarity using eqs. (3) and (4):

$$\xi_{\text{bin, equal}}(m_{\sys}) = 2m \xi(m), \quad (7)$$

This case is more applicable for BD binaries since their mass-ratio distribution peaks at a ratio $q = 1$ (Reid et al. 2006). However, due to the low overall binary fraction of BDs the effect of the mass-ratio distribution is quite small.

The IMF$_{\text{sys}}$ is just the sum of IMF$_{\text{bin}}$ and the IMF of the remaining single objects, $(1 - f_{\text{tot}})\xi$:

$$\xi_{\text{sys}}(m) = \xi_{\text{bin}}(m) + (1 - f_{\text{tot}})\xi(m). \quad (8)$$

Thus, to obtain the true individual star IMF, $\xi(m)$ has to be extracted from the observed $\xi_{\text{sys}}(m)$ for which a model for $\xi_{\text{bin}}(m)$ is required. That is IMF$_{\text{sys}}$ has to be corrected for unresolved binaries in the cluster or population under study. This leads to a significant increase of the numbers of objects at the low-mass end because low-mass objects contribute to both low-mass singles and intermediate-mass binaries and thus more individual objects are required to reproduce the observed IMF$_{\text{sys}}$. (Kroupa et al. 1991; Malkov & Zinnecker 2001). In the mass range $m_{\min}$ to $2m_{\min}$ systems can only be single because the system mass is the sum of the masses of the system members. Increasing the system mass beyond $2m_{\min}$ the binary contribution rises quickly and then asymptotically approximates a maximum value. Thus, the fraction of singles among M-dwarfs is higher than for G-dwarfs (Lada 2006), as one would expect for a stellar population ($> 0.08 M_\odot$), i.e. essentially without BDs.
Fig. 3.— The system IMF (IMF$_{sys}$, solid line) corresponding to a flat logarithmic IMF (i.e. $\alpha = 1$, dashed line) for $m_{\text{min}} = 0.1 \leq m/M_\odot \leq 1 = m_{\text{max}}$ with an overall binary fraction of 50%. The corresponding binary IMF (IMF$_{bin}$, eq. [6]) is shown by the thin dotted line. The system MF peaks at $1 M_\odot$ and is truncated for higher masses while there is a minor peak at $1.1 M_\odot$ corresponding to the peak of the binary IMF at $m_{\text{min}} + m_{\text{max}}$.

Fig. 4.— A population of stars and BDs with different binary fractions can result in a discontinuous IMF, even if the observed (system) IMF appears to be continuous. The binary fraction of the BDs is lower than that of stars and therefore a lower number, $N_{\text{bd}}$, of individual objects is required for the frequency of BD and stellar systems of a given mass being equal (eq. [13]). Note the overlap region indicated by the horizontal dotted lines: some star-like bodies may actually be physical BDs (upper line) while some BD-likes are indeed very-low-mass stars.

The method

3. THE METHOD

3.1. The parameter space

The most straightforward way to calculate the influence of binaries on the stellar/substellar statistics would be the Monte-Carlo method (Kroupa et al. 1991, 1993). However, for better (smoother) results and to reduce the computational efforts we do not use a Monte-Carlo approach here but a semi-analytical approach in which the binary-mass function is calculated via numerical integration of eq. [6] for each population. This can be done with a standard quadrature algorithm. This algorithm has been verified with a Monte-Carlo simulation of a few million random experiments, each being a random draw from the IMF. The Monte-Carlo method is used later to determine the BD-to-star ratio and the total binary fractions within defined mass-ranges (Sections 4.2 and 4.3). It makes use of the Mersenne Twister random number generator developed by Matsumoto & Nishimura (1998). Since there are only a few runs to be done (in contrast to hundreds of thousands of runs during an iterated parameter scan, see below) the simple Monte-Carlo approach with an appropriately large random sample is fully sufficient for this purpose.

The lack of star-BD binaries and the truncation of the BD binary separation distribution suggest two disjoint populations where binary components are taken only from the same regime rather than to a combined population. Random pairing over the whole mass regime is not considered further as it leads to too many star-BD binaries (Kroupa et al. 2003).

The approach here requires separate application of normalisations on both populations. For this purpose we define the population ratio

$$R_{\text{pop}} = N_1/N_2,$$

(9)

where $N_1$ is the number of individual bodies of the BD-like population ($m_{\text{BD}} \leq m \leq m_{\text{max, BD}}$) and $N_2$ ($m_{\text{0, star}} \leq m \leq m_{\text{max}}$) that of the star-like population. This must not be mixed-up with the BD-to-star ratio, $R$, (eq. [14]) that refers to physical BDs and stars separated by the HBL. The partial IMF for each population can be described by parameters $\alpha_{\text{BD}}$, $m_1$, and a normalisation constant. In this work a single power-law for BDs ($i = \text{"BD"}$) and a two-part power-law ($i = 1$ or $2$) for stars is applied. Thus, there is a mass border, $m_{\text{12}}$, separating the two power-law regimes. The parameters of both populations form a three-dimensional parameter space of the IMF model for each cluster. It has been found that the lower mass limits of the BD-like population, $m_{0, \text{BD}} = 0.01 M_\odot$,
and that of the star-like population, $m_{\text{0, stars}} = 0.07 M_\odot$, are suitable for all studied clusters. Furthermore, we focus on the canonical stellar IMF (Kroupa 2001) with $\alpha_1 = 1.3$ for $m \leq 0.5 M_\odot \equiv m_{12}$ and $\alpha_2 = 2.3$ for $m > 0.5 M_\odot$. The reason is that this canonical IMF has been verified with high confidence by other observations as well as theoretically. Thus, only the BD-like power-law, $\alpha_{\text{BD}}$, the normalisation of the BD-like population against the stellar one, $R_{\text{pop}}$, and the upper mass border, $m_{\text{max, BD}}$, of the BD-like regime are the parameters to be varied. Because of the sparse and probably incomplete datasets for IC 348 and the Pleiades, $\alpha_{\text{BD}}$ has also been set to the canonical value (i.e. $\alpha_{\text{BD}} = \alpha_0 = 0.3$) for these clusters, varying only $R_{\text{pop}}$ and $m_{\text{max, BD}}$.

Tab. 1 lists the variables and their range of variation for the Trapezium, IC 348 and the Pleiades. As the upper mass limits of stars in the clusters one can either take the maximum observed mass or a theoretical mass limit, as given by Weidner & Kroupa (2006). Because in our model the IMF is cut sharply rather than declining softly as shown in that work, we here set the mass limit somewhat below the standard opacity limit for fragmentation, $0.01 M_\odot$. After generating the model IMFs for both populations a binary mass function (eq. 6) is derived separately from the incomplete Gamma function (Press et al. 1992),

$$Q(\chi^2_\nu | \nu) = P(\chi^2_\nu \leq \chi^2_0). \tag{11}$$

The logarithmic mass error is not mentioned in the sources but is given by the photometric measurements and to a larger degree by uncertain theoretical models of stars and BDs. We assumed a value of $\Delta \log_{10} m = 0.05$ for the Trapezium and $\Delta \log_{10} m = 0.1$ for the others, corresponding to a relative error in the mass estimates of 12 and 26%, respectively. This error has been estimated from the width of the logarithmic bins in the observational data for the Trapezium, TA and IC 348. For the Pleiades, for which non-equally spaced data from different sources are given, we assume the same log mass error as for TA and IC 348. To take this into account the log$_{10} m$ values have been smoothed by a Gaussian convolution corresponding to a log-normal smearing of the masses.

After generating the model IMFs for both populations a binary mass function (eq. 6) is derived separately from the stellar and substellar IMF, i.e. there are formally no BD-like companions to star-like primaries, in accordance with the observations. This leads to consistency with the observed binary fraction (Section 4.3) and the BD desert. Note that as a result of the required overlap both the BD-like and the star-like population contain stars as well as BDs, thus we do have star-BD pairs in our description (more on this in Section 5).

Addition of the two resulting system IMFs leads to an overall IMF$_{\text{sys}}$ for the whole mass range. By adjusting the power-law coefficients and the population ratio $R_{\text{pop}}$, the IMF$_{\text{sys}}$ is fitted against the observational data such that $\chi^2$ is minimised. The prominent substellar peak in the Trapezium cluster below $0.03 M_\odot$ (Muench et al. 2002) is ignored here because it is well below the BD–star mass limit and therefore does not interfere with any feature there. Furthermore it is possibly an artefact of the BD mass-luminosity relation (Lada & Lada 2003).

3.3. Error estimation

The errors of the parameters are estimated from the marginal distribution of each parameter within the parameter space (Lupton 1993). The marginal probability density distribution, $p$, is

$$p(A_i) = \int \int L(A_1, A_2, A_3) dA_2 dA_3,$$

$$= \int \int L(A_1, A_2, A_3) dA_2 dA_3.$$

$$= \int \int L(A_1, A_2, A_3) dA_2 dA_3.$$
where $A_1, A_2, A_3$ correspond to $\alpha_{BD}, R_{\text{pop}}, m_{\text{max,BD}}$ and $i = 1 \ldots 3; j, k \neq i$. The likelihood $L$ is defined as

$$L(A_1, A_2, A_3) = N e^{-\chi^2/2}, \quad (13)$$

where the normalisation constant $N$ is such that the integral over the whole range is one. Note that $\chi^2$ instead of $\chi_j^2$ is used here. The integral is approximated by summation with equidistant stepping of each parameter. We conservatively rejected all parameter sets with $P < 0.27\%$ (corresponding to $\chi^2 > 3\sigma$; Section 3.2).

As an example, the parameter distribution for the Trapezium is illustrated in Fig. 5. The interval around the median which contains 68% of the scanned parameter sets is taken as the $\sigma$ measure of the errors. The interval limits and the medians are shown among the best-fit results in Tab. 3. Apparently, the median of the probability density distribution does not always coincide with the best-fit value. The reason for this is that the distribution is slightly asymmetric for most parameters and clusters with the Pleiades being the by far worst case. This results in asymmetric errorbars. Note that there are sets of $\alpha_{BD}, R_{\text{pop}}$ and $m_{\text{max,BD}}$ that are within these error limits but with a $\chi^2 > 3\sigma$, and thus the error limits may be slightly over-estimated.

Another possibility to illustrate the statistical significance of a fit is by its confidence contours within a two-dimensional subspace of the parameter space, as is shown later in Section 4.2.

Fig. 5.— The marginal probability density distributions of the parameters $\alpha_{BD}, \log_{10} R_{\text{pop}}$ and $\log_{10} (m_{\text{max,BD}}/M_\odot)$ as an estimate of the errors for the Trapezium cluster fit (see Section 3.3). The peak-width engulfing 68% of the whole parameter sample (thus referring to $\approx 1\sigma$, dashed vertical lines) is taken as the error for each parameter. Because the median (solid vertical line) of the sample does not match the best-fit value (dash-dotted line) in all cases, both are listed in Tab. 3.

Fig. 6.— Four different IMF models applied to the Trapezium cluster. Top panel: A classical continuous-IMF fit (dashed line) for the Trapezium cluster with random-pairing over the whole BD and stellar mass range. The method of converting the IMF to the IMF$_{sys}$ (solid curve) is essentially the same as for the two-component IMFs discussed in this paper but with no allowance for an overlap or a discontinuity in the BD-star transition region. Although the general shape of the observational histogram (thin steps) is represented by this fit this model leads to an unrealistic high fraction of star-BD binaries and is therefore discarded. Second panel: The Trapezium IMF fitted with two separate IMFs, $\xi_{BD}$ (dotted line) and $\xi_{\text{star}}$ (dashed line), as in Fig. 5 (the two-component IMF). Third panel: The same but with the sum IMF of both component IMFs shown here as the possible appearance of the IMF if all binaries could be resolved. Apparently, there is a “hump” between about 0.07 $M_\odot$ and 0.2 $M_\odot$, bracketing the probable overlap region of the BD-like and star-like populations. Bottom panel: Trapezium fitted with two separate IMFs but with no overlap. The binary correction leads to the dip near the hydrogen-burning limit. However, the fit is only slightly worse than that one with an overlap.

4. RESULTS
4.1. The IMF for BDs and stars

Fig. 6 demonstrates the results for different models (continuous IMF, two-component IMF with and without overlap region) for the Trapezium cluster. The continuous IMF is shown for illustration only. Its underlying model assumes a single population containing BDs as well as stars (Kroupa et al. 2003). Instead of an overlap as for the two-component model the mass-border between the regime of the BD slope $\alpha_{BD}$ and the (canonical) stellar regime is varied. The total binary fraction, $f_{\text{tot}}$, is set to 0.4 with random-pairing among the entire population. Our calculations give a best fit with $\alpha_{BD} = -0.4 \pm 0.2, m_{\text{max,BD}} = m_{\text{stellar}} = 0.093 \pm 0.01$ (and canonical stellar IMF) which is comparable to the canonical IMF for BDs and stars (Kroupa 2001). As already mentioned in Section 4 it leads to a high number of star-BD binaries, $f_{\text{star-BD}} = \ldots$
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The best-fit BD-like power law coefficients, $\alpha_{BD}$, population ratios, $R_{pop}$, and BD-like upper mass limits, $m_{\text{max,BD}}$. The uncertainties are derived in Section 3. Note that $\alpha_{BD}$ is set to the canonical value for IC 348 and the Pleiades.

| Cluster   | Median $\pm 1\sigma$ | Best fit | $\chi^2$ | $P$  |
|-----------|----------------------|----------|----------|------|
|           | $\alpha_{BD}$ | $\log_{10} R_{pop}$ | $m_{\text{max,BD}}$ | $\alpha_{BD}$ | $\log_{10} R_{pop}$ | $m_{\text{max,BD}}$ | $\chi^2$ | $P$  |
| Trapezium | $-0.57^{+0.10}_{-0.09}$ | $0.22^{+0.04}_{-0.05}$ | 0.45 | -0.66 | 0.22 | 0.23 | 0.99998 |
| TA        | $-0.68^{+0.13}_{-0.15}$ | $0.11^{+0.04}_{-0.03}$ | 0.00 | -0.62 | 0.12 | 2.69 | 0.020 |
| IC 348    | $-0.57^{+0.23}_{-0.25}$ | $0.22^{+0.06}_{-0.05}$ | 0.3(c) | -0.51 | 0.22 | 1.01 | 0.416 |
| Pleiades  | $-0.82^{+0.44}_{-0.27}$ | $0.07^{+0.02}_{-0.00}$ | 0.3(c) | -0.80 | 0.06 | 1.20 | 0.257 |

Fig. 8.— The two-component model with canonical stellar IMF for a slightly modified TA observational histogram (black solid steps). The peak at 1 $M_\odot$ and the dent around 0.3 $M_\odot$ have been slightly flattened without leaving the errorbars of the original data (grey steps and bars). This fit has a reasonable confidence of about $P = 0.25$, about 12 times larger than for the original histogram ($P = 0.02$, see Tab. 3).

15% ± 4% altogether (see Section 4.3), while Kroupa et al. (2003) found 7%–15% star-BD binaries with $a \gtrsim 30$ AU in their standard model with stars and BDs. Furthermore, the different orbital properties of BD-BD and star-star binaries (Fig. 1) cannot be explained by a single-population model without further assumptions. Therefore, it is not considered further here.

The two-component IMF model (second panel) accounts for the empirical binary properties of BDs and stars. Apart from this it also fits the observational data (histogram) slightly better than the continuous model, especially in the BD/VLMS region and the “plateau” between 0.1 and 0.5 $M_\odot$.

Objects with equal mass and composition appear as equal in observations, even if they have been formed in different populations. Thus it is hard to determine their formation history. A high-resolution survey that resolves most of the multiples would yield an overall mass function from the lowest-mass BDs to the highest-mass stars. However, if such a resolved individual mass function is composed of two overlapping populations, it may be possible to detect its imprints as an excess of objects within the overlap region. To illustrate this (third panel), we construct such an overall IMF by simply adding the two IMF components from the second panel. BDs and stars are paired separately from their respective mass range. It is clear that the addition of the two IMFs leads to discontinuities, i.e. in the overlap region the continuous IMF increases steeply at the minimum stellar mass and drops again at the upper mass limit of the BD-like regime. Even with a smoother drop at each end of the BD-like and star-like IMF it is likely to be detectable once most of the multiples have been resolved. Thus, a discovery of such a “hump” within the resolved IMF would strongly support the two-component model.
The stellar IMF is canonical, i.e. $\alpha_{BD} = 0$ (see Section \ref{section:stellar_imf} for details) of $\alpha_{BD}$ and the BD-like to star-like ratio $R_{HBL}$ at $m = 0.075 M_\odot$ (eq. \ref{eq:bd_ratio}) for the Trapezium, TA, IC 348 and the Pleiades. The stellar IMF is canonical, i.e. $\alpha_1 = 1.3$ and $\alpha_2 = 2.3$ while the upper BD mass limits, $m_{\text{lim, BD}}$, are the best-fit values from Tab. \ref{table:best_fit}. All fitting parameters outside the solid line are rejected with 95%, and those outside the dashed line are rejected with 99% confidence. The optimum of the Trapezium and TA is marked by the filled circle while the cross marks the standard/canonical configuration with a continuous IMF (i.e. $\log_{10} R_{HBL} = 0$) and $\alpha_{BD} = 0.3$. As can be seen, it is well outside both levels (even for arbitrary $\alpha_{BD}$). For the Pleiades it is still well outside at least for reasonable choices of $\alpha_{BD}$. Note that the optimum for $\alpha_{BD}$ for IC 348 and the Pleiades does exist in this 2D cross-section but not in the full 3D parameter space.

Fig. 9.— Contour plots of 5% and 1% significance levels from $\chi^2$ fitting (see Section \ref{section:sig_level} for details) of $\alpha_{BD}$ and the BD-like to star-like ratio $R_{HBL}$ at $m = 0.075 M_\odot$ (eq. \ref{eq:bd_ratio}) for the Trapezium, TA, IC 348 and the Pleiades. The stellar IMF is canonical, i.e. $\alpha_1 = 1.3$ and $\alpha_2 = 2.3$ while the upper BD mass limits, $m_{\text{lim, BD}}$, are the best-fit values from Tab. \ref{table:best_fit}. All fitting parameters outside the solid line are rejected with 95%, and those outside the dashed line are rejected with 99% confidence. The optimum of the Trapezium and TA is marked by the filled circle while the cross marks the standard/canonical configuration with a continuous IMF (i.e. $\log_{10} R_{HBL} = 0$) and $\alpha_{BD} = 0.3$. As can be seen, it is well outside both levels (even for arbitrary $\alpha_{BD}$). For the Pleiades it is still well outside at least for reasonable choices of $\alpha_{BD}$. Note that the optimum for $\alpha_{BD}$ for IC 348 and the Pleiades does exist in this 2D cross-section but not in the full 3D parameter space.

To illustrate the effect of the overlap region, a two-component IMF without an overlap is shown in the bottom panel. Like the continuous IMF this approach gives a slightly worse fit, especially between 0.1 and 0.2 $M_\odot$ ($-1 \leq \log_{10} m/M_\odot \leq -0.7$). However, the “dip” in this region is only weak.

The results of the two-component IMF model for all clusters are shown in Fig. \ref{fig:binary_fraction} the power-law coefficients being listed in Table \ref{table:best_fit}. For the Pleiades and IC 348 the BD IMF slope has been kept standard, i.e. $\alpha_{BD} = 0.3$ (Kroupa 2001), because the sparse, and in the case of the Pleiades most probably incomplete data, do not allow useful confidence limits to be placed on $\alpha_{BD}$. For the same reason the highly uncertain observational data for $\log_{10} (m/M_\odot) \geq 0.65$ have not been used for fitting (but are still plotted for completeness).

As can be seen the BD/VLMS and stellar IMFs do not meet at the BD-star boundary. The number density of individual BDs near the BD-star border is about one third of that of individual low-mass M-dwarfs. This discontinuity cannot be seen directly in the observational data because it is masked by the different binary fractions for different masses (Section \ref{section:binary_fraction}). Furthermore, there are large uncertainties in the mass-determination of stars in stellar groups such as TA which may probably lead to even larger errorbars of the bins than shown in the observational data. Even within the given errorbars certain variations of these data provide a much better fit to the canonical stellar IMF with $P = 0.25$ (original: $P = 0.02$, Tab. \ref{table:best_fit}). This has been done in Fig. \ref{fig:sig_level} by low-
ering the peak near 1 \( M_\odot \) and somewhat rising the “valley” around 0.3 \( M_\odot \) corresponding to the re-shuffling of 9 stars out of 127 (Tab. 1) by 0.1 to 0.4 \( M_\odot \) (one bin-width), e.g. through measurement errors. This suggests that, in agreement with Kroupa et al. (2003), the TA IMF might not necessarily be inconsistent with the canonical IMF.

Except for the Pleiades all clusters show features near the peak in the low-mass star region that are slightly better fitted with a separate BD population and an overlap region. Although these features alone do not reject the continuous IMF model, they might be taken as a further support of the argumentation towards a two-populations model.

It should be mentioned that the continuous IMF model (first panel) still fits the observed Trapezium and Pleiades MF with high confidence (while failing for the other clusters) but leads to VLMS binary properties that are inconsistent with the observed properties as mentioned in Section 1. A possible extension of our modelling would thus be to fit both the observed IMF and the observed binary statistics.

4.2. BD to star ratio

We analysed the BD-to-star ratio \( R \) of TA, Trapezium and IC 348, defined as

\[
R = \frac{N(0.02–0.075 M_\odot)}{N(0.15–1.0 M_\odot)},
\]

where \( N \) is the number of bodies in the respective mass range. The mass ranges are chosen to match those used in Kroupa & Bouvier (2003) in their definition of \( R \). Since BDs below 0.02 \( M_\odot \) are very difficult to observe and since TA and IC 348 do not host many stars above 1 \( M_\odot \) (in contrast to Trapezium and the Pleiades), we restricted the mass ranges to these limits.

The Pleiades cluster is difficult to handle due to a lack of BD data. Moreover, at an age of about 130 Myr Barrado y Navascués, Stauffer & Jayawardhana (2004) it has undergone dynamical evolution, and massive stars have already evolved from the main-sequence which affects the higher-mass end of the IMF (Moraux et al. 2004).

Another relevant quantity is the ratio \( R_{\text{HBL}} \) of BD-like to star-like objects at the HBL, i.e. the classical BD-star border,

\[
R_{\text{HBL}} = \frac{\xi_{\text{BD}}(0.075 M_\odot)}{\xi_{\text{star}}(0.075 M_\odot)}. \tag{15}
\]

In the classical continuous IMF approach it is one by definition, because otherwise the IMF would not be continuous. In a two-component IMF its value depends on the shapes of the BD-like and the star-like IMF as well as on the BD-to-star ratio. The evaluation of eq. (15) yields \( R_{\text{HBL}} = 0.17 \) for the Trapezium, \( R_{\text{HBL}} = 0.30 \) for TA, \( R_{\text{HBL}} = 0.22 \) for IC 348 and \( R_{\text{HBL}} = 0.3 \) for the Pleiades. Although \( R_{\text{HBL}} \) is not an input parameter here but is calculated from the two IMFs and their relative normalisation, \( R_{\text{pmp}} \), it could easily be used as one instead of \( R_{\text{pmp}} \) while calculating the latter from \( R_{\text{HBL}} \).

Fig. 9 shows the contour plots of the 5% and 1% significance ranges in the \( \sigma_{\text{BD}} - R_{\text{HBL}} \) space for Trapezium, TA and IC 348 and the Pleiades. The significance values are calculated from \( \chi^2 \), via the incomplete Gamma function as described in Section 3. The contours mark the regions outside which the hypothesis of a two-component IMF with a single power-law for BDs and a double-power law for stars has to be rejected with 95% or 99% confidence. Also shown (by a cross) is the standard configuration with \( \sigma_{\text{BD}} = 0.3 \) and \( \log R_{\text{HBL}} = 0 \) for the continuous standard IMF. This point is outside both levels for all three clusters, and at least for Trapezium and TA it is well outside even for arbitrary \( \sigma_{\text{BD}} \). In other words, the corresponding hypothesis of a continuous IMF has to be rejected with at least 99% confidence.

The size of the non-rejection areas can be used for an estimate of the errors of \( \sigma_{\text{BD}} \) and \( R_{\text{HBL}} \). However, one has to keep in mind that these are only maximum possible deviations with all the other parameters kept at the optimum. The non-rejection areas in the full 3 dimensional parameter space are therefore expected to be somewhat narrower.

4.3. Binary fraction

Additionally, the binary fraction for each cluster as well as the total binary fraction is calculated for the best-fit models. The fraction of binaries as a function of the primary mass, \( f(m_{\text{prim}}) \), among all systems is shown in Fig. 10. For stars the binary fraction is a monotonic function of the primary mass in agreement with the data (Lada 2006), at least for the Trapezium, IC 348 and the Pleiades. For BDs, however, it is flat due to the equal-mass pairing. In the case of random pairing it would approach zero for very-low mass BDs (Fig. 11). The true mass-ratio function grows monotonically with the mass ratio and becomes very steep near \( q = 1 \), as shown by Reid et al. (2006). Thus, for BDs the true binary fraction is probably closer to the equal-mass case than the random pairing case.

For comparison, Fig. 11 also shows the binary fraction, \( f_c(m_{\text{prim}}) \), for the Trapezium that would result from a continuous IMF. Although the overall shape is very similar to that of the two-component model the binary fraction near the BD-star transition is significantly higher for low-mass stars than the observed values while being approximately equal for stars above 1 \( M_\odot \). Thus, a continuous IMF cannot fit the obser-
vational data as good as a two-population IMF even if \( f_{\text{tot}} \) is reduced.

The total fractions of BD-BD binaries, \( f_{\text{BD-BD}} \), of star-star-binaries, \( f_{\text{star-star}} \), and the fraction of (very-low-mass) star-BD binaries, \( f_{\text{star-BD}} \), are of further interest. Pairs of the latter type consist of two objects of the BD-like or star-like population (see Section 5 for the motivation and definition of the populations) but where the primary object is a star (\( m_{\text{prim}} \geq 0.075 M_\odot \)) within the BD-VLMS overlap region between 0.07 and 0.15 \( M_\odot \), while the companion is a true, physical BD with a mass below 0.075 \( M_\odot \).

For each cluster we define

\[
\begin{align*}
    f_{\text{BD-BD}} &= \frac{N_{\text{BD-BD}}}{N_{\text{sys,BD}}} \\
    f_{\text{MD-MD}} &= \frac{N_{\text{MD-MD}}}{N_{\text{sys,MD}}} \\
    f_{\text{star-star}} &= \frac{N_{\text{star-star}}}{N_{\text{sys,star}}} \\
    f_{\text{star-BD}} &= \frac{N_{\text{star-BD}}}{N_{\text{sys,star}}},
\end{align*}
\]

where \( N_{\text{BD-BD}} \) is the number of all BD-BD binaries (i.e. all objects have masses \( \leq 0.075 M_\odot \)), \( N_{\text{MD-MD}} \) that of all M-dwarf-M-dwarf (MD-MD) binaries, \( N_{\text{star-star}} \) that of all star-star binaries and \( N_{\text{star-BD}} \) the number of mixed (star-BD) binaries (composed of a VLMS and a BD). Furthermore, \( N_{\text{sys,BD}} \) is the number of all BD systems (including single BDs), \( N_{\text{sys,MD}} \) that of all M-dwarf systems and \( N_{\text{sys,star}} \) the number of all systems with a star as primary and with a stellar, BD or no companion. As for the BD-to-star ratio we also applied the (primary) mass ranges from Kroupa & Bouvier (2003) on the BD and star sample from each cluster but with no gap between the BD and the stellar regime, i.e. 0.02 \( M_\odot \leq m_{\text{prim}} \leq 0.075 M_\odot \) for BDs and 0.075 \( M_\odot \leq m_{\text{prim}} \leq 1 M_\odot \) for stars. Additionally, the binary fraction of M-dwarfs is calculated in the same way as the stellar one but in the mass range 0.075 \( M_\odot \leq m_{\text{prim}} \leq 0.5 M_\odot \). The uncertainty limits of the binary fractions and BD-to-star ratios have been derived from those of the IMF slopes by applying the minimum and maximum slopes.

The results are shown in Tab. 4. The binary fractions for BDs vary only slightly between 12 and 15% while the stellar binary fraction, \( f_{\text{star-star}} \), is about 70% for TA and 30–40% for the others. They are apparently slightly lower than the binary fractions that are set for the star-like population because of the non-constant distribution binary as shown in Fig. 10. The binary mass function (eq. 6) is smaller in the mass region below 1 \( M_\odot \) and thus the binarity is below-average if the focus is set on this region.

Furthermore, in the overlap region the relatively low binary fraction of VLMSs from the BD-like population also contribute to \( f_{\text{star-star}} \), which results in an even lower value of \( f_{\text{star-star}} \). This trend is more emphasised for the M-dwarf binary fraction, \( f_{\text{MD-MD}} \), which is about 10% lower than \( f_{\text{star-star}} \) for each cluster but still much larger than \( f_{\text{BD-BD}} \).

The star-BD binary fraction \( f_{\text{star-BD}} \) is of special interest since it is the measure for the “dryness” of the BD desert. Note that due to the equal-mass pairing used here for BD-like binaries, the BD-like population formally does not contribute to \( f_{\text{star-BD}} \). All star-BD binaries are from the star-like regime which extends down to \( m_{\text{prim}} = 0.07 M_\odot \), i.e. into the BD mass regime. For the Trapezium, IC 348 and the Pleiades our two-component models yield values between 2% and 2.5% whereas TA shows \( f_{\text{star-BD}} \approx 5 \% \). For comparison, the continuous IMF from the top panel of Fig. 6 corresponds to \( f_{\text{star-BD}} = 15 \% \pm 4 \% \).

5. DISCUSSION: BROWN DWARFS AS A SEPARATE POPULATION?

5.1. An apparent discontinuity

By correcting the observed MFs for unresolved multiple systems a discontinuity in the IMF near the BD/VLMS region emerges. We have also tried to model continuous single-body IMFs but we find this hypothesis of continuity to be inconsistent with the observed MFs given the observational data on the binary properties of stars and BDs. We have shown that the empirically-determined difference in the binary properties between BDs/VLMSs on the one hand and stars on the other, and the empirical finding that stars and BDs rarely pair, implies a discontinuity in the IMF near the BD/VLMS mass.

Thus, the discontinuity in binary properties, which has already been interpreted to mean two separate populations Kroupa & Bouvier (2003), also implies a discontinuity in the IMF. This relates the probably different formation mechanism more clearly to the observational evidence.

We have performed a parameter survey allowing the IMF parameters \( \alpha_{\text{BD, pop}}, R_{\text{pop}} \) and \( m_{\text{max, BD}} \) to vary finding that the canonical stellar IMF (\( \alpha_1 = 1.3, \alpha_2 = 2.3 \)) cannot be discarded even for TA and that the BD/VLMS discontinuity is required for all solutions. The discontinuity uncovered in this way, if measured at the classical BD-star border, is of a similar magnitude for the stellar clusters studied (0.17 \( \leq R_{\text{HBL}} \leq 0.30 \)), supporting the concept of a universal IMF, which is, by itself, rather notable.

We recommend calling these populations BD-like and star-like with respect to their formation history. These populations have probably overlapping mass ranges since there is no physical reason for the upper mass limit of the BD-like population to match the lower mass limit of the star-like one. Furthermore, the best-fit models suggest such an overlap between 0.07 \( M_\odot \) and about 0.2 \( M_\odot \). According to this classification through the formation history, BD-like objects would include VLMSs, while star-like ones would include massive BDs.

The overlap region implies that BD-like pairs can consist of a VLMS-BD pair, and that a star-like binary can consist of a stellar primary with a massive BD as a companion. As can be seen from Tab. 4, the star-BD fractions are similar (except for the dynamically unevolved TA) and that the star-BD binary fraction, \( f_{\text{star-BD}} \), is about 2%–3%. This is somewhat higher than the value of \(< 1 \% \) BD companion fraction found for nearby stars but still far below the value expected for a single population model (at least 7%–15% in Kroupa et al. (2003) and 15% ± 4% in our calculations). Simulations by Bate et al. (2003), Bate & Bonnell (2005) and Batel (2005) predict a fraction of about 2% star-BD binaries (actually one M-dwarf with a BD companion out of 58 stars formed in three independent calculations). Note that, in our model, \( f_{\text{star-BD}} \) is a prediction of the required over-
lap region and is sensitive to the overlapping range. As our model for the Pleiades IMF suggests, the overlapping range might be considerably smaller than that we have found for the other clusters. Furthermore, we did not fit the lower mass border, $m_{\text{0,star}}$, of the star-like population but simply assumed a value of 0.07 $M_\odot$, which is well in the BD mass regime (and is the major source of BD binaries in our Pleiades model). A slight increase of $m_{\text{0,star}}$ by only 0.01 $M_\odot$ would cause the Pleiades star-BD binary fraction to drop to nearly zero.

Several authors doubt the existence of two separate populations. Most recently, Eisloffel & Steinacker (2007) summarise that the observational community in general prefers the model of star-like formation for BDs. They mention the detection of isolated proto-(sub)stellar “blobs” in the Ophiuchus B and D clouds, which may support the theory of star-like formation for BDs. It remains unclear though how many of these blobs will actually form BDs instead of dissolving from lack of gravitational binding energy. Goodwin & Whitworth (2007) refer to a private communication with Ake Nordlund stating that the pure turbulence theory predicts about 20 000 transient cores for every actual prestellar core of about 0.1 $M_\odot$.

We recall that one of the main reasons for the existence of a separate population is the semi-major axis distribution of BD binaries (Fig. 1). But Luhman et al. (2007) also mention a wide binary BD in Ophiuchus with a separation of about 300 AU. Indeed, a small number of wide BD binaries are known. However, it can be doubted that the occasional discovery of a wide BD binary may expand the narrow semi-major axis distribution (Fig. 1) to a star-like one. The striking evidence posed by the lack of BD companions to stars is a strong indication for two populations. It is usually ignored by the community though. We note that even if there actually may be some BDs that formed star-like they are most probably a minority.

There is also the interpretation of the BD desert being a “low $q$ desert” rather than an absolute mass-dependent drop in the companion mass function. Grether & Lineweaver (2006) find a low-mass-companion desert of solar-type primary stars between approximately 0.01 and 0.06 $M_\odot$. They find this interval to be dependent on the primary star mass and therefore predict M-dwarfs to have BD companions, and that M-dwarfs ought to have a companion desert between a few Jupiter masses and the low-mass BD regime. However, this interpretation does not address the different orbital properties of BDs and stars as well as the different $q$ distribution.

5.2. Implications for the formation history

Can the existence of such a discontinuity, i.e. the formation of two separate populations, be understood theoretically? Although BDs and stars appear to be distinct populations the formation of BDs is likely to be connected to star formation. Bate, Bonnell & Bromm (2002) and Bate et al. (2003) show that BDs significantly below 0.07 $M_\odot$ cannot form in a classical way since the minimum mass they need for stellar-type formation would also lead to progressive accretion and growth to stellar mass unless they are in regions with very low mass in-fall rates. But such regions are very rare, because the prevailing densities and temperatures cannot achieve the required Jeans masses, as also stressed by Goodwin & Whitworth (2007). For this reason, Adams & Fatuzzo (1996) expected BDs to be rare. Only a small fraction of the BDs, especially those at the high-mass end of the BD regime, may form this way if the surrounding gas has been consumed by star-formation processes just after the proto-BD has reached the Jeans mass. To explain the actually higher BD frequency (per star) in recent surveys the accretion process has to be terminated or impeded somehow (Bonnell, Larson & Zinnecker 2007).

Also the above-mentioned differences in the distribution of the semimajor axes between BDs and stars cannot be explained by a scaled-down star formation process, because that would imply a continuous variation and a much broader semimajor axis distribution for BDs and VLMSs that has not been observed (Kroupa et al. 2003). While binary stars show a very broad distribution of their semimajor axes peaking at about 30 AU the semimajor axes of BDs are distributed around about 5 AU with a sharp truncation at 10–15 AU (Fig. 1). No smooth transition region between both regimes can be recognised (Close et al. 2003). In a high-angular-resolution survey Law et al. (2007) found that the orbital-radius distribution of binaries with V-K < 6.5 appears to differ significantly from that of cooler (and thus lower-mass) objects, suggesting a sudden change of the number of binaries wider than 10 AU about the MS spectral type. This is in agreement with our finding of a possible BD-like population that extends beyond the hydrogen-burning mass limit into the VLMS regime.

In a radial-velocity survey of Chamaeleon I, Joergens (2006) has found evidence for a rather low binary fraction below 0.1 AU while most companions found in that survey orbit their primaries within a few AU. For this reason an extreme excess of close BD binaries that cannot be resolved by imaging surveys appears to be unlikely. A larger binary fraction than about 15% would thus not be plausible. Basri & Reiners (2006) suggest an upper limit of 26% ± 10% for the BD binary fraction based on their own results (11% ± 0.07%) for separations below 6 AU and the survey by Close et al. (2003) (15% ± 7%)

| Cluster     | $f_{\text{BD-BD}}$ | $f_{\text{MD-MD}}$ | $f_{\text{star-star}}$ | $f_{\text{star-BD}}$ | $R$     | $R_{\text{HBL}}$ |
|-------------|-------------------|--------------------|------------------------|-----------------------|---------|------------------|
| Trapezium   | 0.13 ± 0.01       | 0.30 ± 0.01        | 0.34 ± 0.01            | 0.023 ± 0.002         | 0.18 ± 0.03 | 0.17 ± 0.04     |
| TA          | 0.15 ± 0.01       | 0.64 ± 0.06        | 0.69 ± 0.05            | 0.046 ± 0.005         | 0.27 ± 0.09 | 0.30 ± 0.11     |
| IC 348      | 0.13 ± 0.02       | 0.29 ± 0.02        | 0.33 ± 0.02            | 0.021 ± 0.004         | 0.20 ± 0.10 | 0.22 ± 0.11     |
| Pleiades    | 0.13 ± 0.02       | 0.33 ± 0.03        | 0.37 ± 0.03            | 0.025 ± 0.002         | 0.28 ± 0.18 | 0.28 ± 0.18     |

| Cluster     | $f_{\text{BD-BD}}$ | $f_{\text{MD-MD}}$ | $f_{\text{star-star}}$ | $f_{\text{star-BD}}$ | $R$     | $R_{\text{HBL}}$ |
|-------------|-------------------|--------------------|------------------------|-----------------------|---------|------------------|
| Trapezium   | 0.13 ± 0.01       | 0.30 ± 0.01        | 0.34 ± 0.01            | 0.023 ± 0.002         | 0.18 ± 0.03 | 0.17 ± 0.04     |
| TA          | 0.15 ± 0.01       | 0.64 ± 0.06        | 0.69 ± 0.05            | 0.046 ± 0.005         | 0.27 ± 0.09 | 0.30 ± 0.11     |
| IC 348      | 0.13 ± 0.02       | 0.29 ± 0.02        | 0.33 ± 0.02            | 0.021 ± 0.004         | 0.20 ± 0.10 | 0.22 ± 0.11     |
| Pleiades    | 0.13 ± 0.02       | 0.33 ± 0.03        | 0.37 ± 0.03            | 0.025 ± 0.002         | 0.28 ± 0.18 | 0.28 ± 0.18     |
for separations greater than 2.6 AU) by simple addition of the results. This is nearly consistent with a BD binary fraction of 15%, since the survey is neither magnitude- nor volume-limited. However, they admit that their value may be overestimated since the objects with separations between 2.6 and 6 AU are counted twice. We note further that even a total BD binary fraction of 25% \( (f_{\text{BD-BD}} = 0.25) \), although outside the error limits of our best-fit models, would only lead to a minor change in the fitted IMFs.

It has been argued \( \text{Basri} \& \text{Reiners} \) \( 2006 \) that the lower binary fraction of BDs is just the extension of a natural trend from G-dwarfs to M-dwarfs \( (\text{Figs. 10, 11}) \). Our contribution has shown that this trend can be understood by the simple fact that there are many fewer possibilities to form a binary near the lower-mass end than for higher component masses. The observational data are in better agreement with a minimum mass, \( m_{\text{min}} \), near the hydrogen-burning mass limit and a low overall binary fraction of BD-like objects than with an “all-in-one” IMF from the lowest-mass BDs to the upper stellar mass limit, as shown in Fig \( 11 \). This observed trend thus appears as an additional enforcement of the two-populations model of BDs and stars.

Given that the conditions for a star-like formation of BDs are very rare \( \text{(Bate et al.} \text{)} \text{2003)} \), four alternative formation scenarios for BDs apart from star-like formation can be identified, namely

1. formation of wide star-BD binaries via fragmentation of a proto- or circumstellar disk and subsequent disruption by moderately close encounters,
2. formation of BDs as unfinished stellar embryos ejected from their birth system,
3. removal of the accretion envelopes from low-mass protostars via photo-evaporation and
4. removal of the accretion envelopes due to extremely close stellar encounters \( \text{Price} \& \text{Podsiadlowski} \text{1995)} \).

Scenario 4. can be ruled out as the major BD formation mechanism because the probability of such close encounters, with required fly-by distances typically below 10 AU \( \text{(Kroupa} \& \text{Bouvier} \text{2003)} \) for efficient disruption of accretion envelopes (less than a tenth of those proposed by \text{Thies} \& \text{Thies} \text{2005} \) for triggered planet formation), is far too low for such a scenario being a significant contribution to BD formation. The photo-evaporation model, as studied by \text{Whitworth} \& \text{Zinnecker} \text{2004) \), also cannot be the major mechanism of BD formation \( \text{Kroupa} \& \text{Bouvier} \text{2003)} \). It predicts a variation of the IMF with the population number and density of the host cluster. In dense star-burst clusters (young globular clusters) with a larger number of O/B stars or even modest clusters such as the ONC with a dozen O/B stars compared to TA, a larger fraction of low-mass stars would have halted in growth. This would result in a bias towards M-dwarfs, since many of them would be failed K or G dwarfs. In contrast to this prediction, \text{Briceno et al.} \text{2002) \) and \text{Kroupa et al.} \text{2003) \) show that the IMFs of TA and ONC are very similar in the mass range 0.1–1 \( M_{\odot} \), while globular clusters likewise have a low-mass MF similar to the standard form \( \text{Kroupa} \text{2001)} \).

5.3. Embryo ejection

\text{Reipurth} \text{2000) and Reipurth} \& \text{Clarke} \text{2001) introduced the formation of BDs as ejected stellar embryos as the alternative scenario 2. If a forming protostar in a newborn multiple system is ejected due to dynamical instability its accretion process is terminated and the object remains in a protostellar state with only a fraction of the mass compared to a fully developed star. Since the final mass is physically independent of the hydrogen fusion mass limit one would not expect the mass range of ejected embryos to be truncated at the HBL and thus expect an overlap region between these populations. This fully agrees with the requirement of having to introduce such an overlap region in order to fit the observed IMF_{sys} in Section 3.\)

This model gives some hints to understand the low BD binary fraction as well as the truncation of the semi-major-axis distribution of BDs. The decay of a young multiple system of three or more stellar embryos typically leads to the ejection of single objects but also to the ejection of a small fraction of close binaries. In order to survive the ejection, the semi-major axis of such a binary must be significantly smaller (by a factor of about three) than the typical orbital separation within the original multiple system. A similar explanation is that the orbital velocity of the BD binary components has to be higher than the typical ejection velocity in order to keep the interaction cross-section of the binary with other system members small. Indeed, the velocity dispersion of BDs in the embryo-ejection model shown in \text{Kroupa} \& \text{Bouvier} \text{2003) is \( \lesssim 2 \text{km s}^{-1} \) for the majority of the BDs. This is in good agreement with the Keplerian orbital velocity of each BD-binary member of about 1.5–2 \( \text{km s}^{-1} \) for an equal-mass binary of 0.05–0.08 \( M_{\odot} \) and \( a \approx 10 \text{AU} \). The majority of BD binaries have smaller separations and, consequently, higher orbital velocities and are bound tighter. This would set the low-binding energy cut near \( E_{\text{bind}} = 0.2 \text{pc}^2 \text{Myr}^{-1} \) in Fig. 2.\)

There have been numerical simulations of star formation and dynamics, e.g. \text{by Bate et al.} \text{2003) and Umbreit et al.} \text{2005) \), in which binaries are produced via ejection that show remarkably similar properties to the actually observed ones. \text{Umbreit et al.} \text{2005) describe the formation of BDs from decaying triple systems. Their simulations predict a semi-major axis distribution between about 0.2 and 8 AU \( \text{(see Fig. 8 in their paper)} \), peaking at 3 AU. This is slightly shifted towards closer separations compared with the results by \text{Close et al.} \text{2003) \) but still in agreement with the observational data. In contrast to this, \text{Goodwin} \& \text{Whitworth} \text{2007) doubt the frequent formation of close BD binaries via ejection, arguing that hydrogen-burning stars which formed via ejection were almost always single.\)

In further qualitative support of the embryo ejection model, \text{Guieu et al.} \text{2006) describe a deficit by a factor of two of BDs near the highest-density regions of TA relative to the BD abundance in the less dense regions that can possibly be explained by dynamical ejection and consequently larger velocity dispersion of stellar embryos, i.e. BDs. A star-like fragmentation scenario would result in an opposite trend since the Jeans mass is smaller for higher densities, thus allowing gas clumps of lower mass to form (sub)stellar bodies. Contrary to this, \text{Luhman} \text{2006) did not find any evidence for a different spatial distribution of BDs and stars in TA. Recently, Kumar \& Schmeja \text{2007) have found that substellar objects in both the Trapezium and IC 348 are distributed homogeneously within twice the cluster core radii while the stellar populations display a clustered distribution. They conclude that these distributions are best explained with
A discontinuity in the low-mass IMF

5.4. Disk fragmentation and binary disruption

The fragmentation of proto-binary disks with subsequent disruption of a star-BD binary is another promising alternative scenario. A disk can fragment during the accretion process if it reaches a critical mass above which the disk becomes gravitationally unstable against small perturbations. Fragmentation may be triggered by an external perturbation, i.e. in-falling gas clumps or a passing neighbour star. The latter mechanism may also be capable of triggering fragmentation in relatively low-mass circumstellar disks within about 15 AU while larger disks or widely-separated binaries would be disrupted.

Furthermore, Bate et al. (2003) suggest from their simulations that the binary fraction via ejection might by as small as about 5% (see also Whitworth et al. 2007).

5.5. Summary

Both the embryo-ejection model and disk fragmentation with subsequent wide-binary disruption explain the above-mentioned connection between stars and BDs since BDs start to form like stars before their growth is terminated due to their separation from their host system or from lack of surrounding material in the outer parts of a circumstellar disk. It is obvious that the formation rate of these embryos is proportional to the total star formation rate.

For these reasons these formation mechanisms appear to be the most likely ones for BDs and some VLMSs and BD/VLMS binaries. It cannot be decided yet which scenario is the dominant mechanism. This may depend on the size and the density of the star-forming region. We expect, on the other hand, the classical star-like formation scenario to be of some importance only for the most massive BDs.

The possibility of two different alternative BD formation mechanisms (disk fragmentation and embryo ejection) may lead to another discontinuity in the intermediate mass BD IMF since both scenarios correspond to different binary fractions as mentioned above. The currently available data, however, are far from being sufficient for a verification of this prediction.

6. CONCLUSIONS

The different empirical binary properties of BDs and stars strongly imply the existence of two separate but mutually related populations. We have shown that if the IMF of BDs and stars is analysed under consideration of their binary properties then there is a discontinuity in the transition region between the substellar and stellar regime that is quite independent of the host cluster. The discontinuity in the IMF near the HBL is a strong logical implication of the disjunct binary properties and suggests splitting-up the IMF into two components, the BD-like and the star-like regime. An alternative but equivalent description would be to view the stellar IMF as a continuous distribution function ranging from about 0.07 to 150 $M_{\odot}$ (Weidner & Kroupa 2004) and a causally connected but disjoint distribution of (probably mostly) separated ultra-low-mass companions and ejected embryos with masses ranging from 0.01 $M_{\odot}$ to 0.1–0.2 $M_{\odot}$. While the canonical stellar IMF is consistent with the observed stellar MFs at least for

$$r_{\text{disk}} \gtrsim 150 \text{AU} \left( \frac{m}{M_{\odot}} \right),$$

where the disk is cool enough to allow a substellar clump to undergo gravitational collapse. For a primary star below 0.2 $M_{\odot}$, this minimum radius therefore becomes less than 90 AU which is in remarkable agreement with the two wider VLMS binaries found by Konopacky et al. (2007). Such large distances to the primary star allow the formation of BD-BD binaries as well as the survival of circum-substellar disks up to about 10–30 AU, depending on the total mass of the pre-substellar core and the mass of the primary star. Furthermore this scenario explains the existence of wide star-BD binaries. In addition, such a wide star-BD binary can be disrupted by moderately close encounters of about 100–200 AU (i.e. a distance similar to the star-BD orbital radius), the disruption of such systems appears to be likely in contrast to the disruption of accretion envelopes as required in the already rejected scenario 4.
the Trapezium, IC 348 and the Pleiades, the sub-stellar IMF of at least the Trapezium and TA has a power-law index that is consistent with the canonical value $\alpha = 3.0$. Within the error limits, our analysis does not reject the canonical power-law indices for BDs and stars for any cluster.

The discontinuity is often masked in the observational data due to a mass-overlap of both populations in the BD-VLMS region as well as the higher apparent masses of unresolved binaries compared to single objects in the observed IMF$_{sys}$. The discontinuity in the number density near the HBL is a step of approximately a factor of three to five (Tab. 2). This implies a general dependency between both populations and is, as far as we can tell, consistent with the scenario of disrupted wide binaries (Goodwin & Whitworth 2007) as well as with the truncated-star scenario (e.g. as an ejected stellar embryo, Reipurth & Clarke 2001) since the number of unready stars is directly correlated with the total amount of star formation in the host cluster. Both embryo-ejection and wide-binary disruption are also consistent with the properties of close binary BDs.

Our results (Tab. 3) suggest that about one BD is produced per 4–6 formed stars. This suggests the necessity of a distinct description of BDs and stars as well as the connection between these two populations through their formation process.

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