Lepton mass generation and family number violation mechanism in the SU(6)$_L \otimes U(1)_Y$ model

Umberto Cotti
SISSA-ISAS, via Beirut 2-4, 34010 Trieste, Italia

Ricardo Gaitán
Centro de Investigaciones Teóricas, Facultad de Estudios Superiores-Cuautitlán, UNAM, A.P. 142, 54700 Cuautitlán Izcalli, Estado de México, México.

A. Hernández-Galeana
Departamento de Física, Escuela Superior de Física y Matemáticas del Instituto Politécnico Nacional, 07738 México D.F., México.

William A. Ponce
Departamento de Física, Universidad de Antioquia A.A. 1226, Medellín, Colombia

Arnulfo Zepeda
Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN A.P. 14-740, 07000 México D.F., México.

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Lepton family number violation processes arise in the SU(6)$_L \otimes U(1)_Y$ model due to the presence of an extra neutral gauge boson, $Z'$, with family changing couplings, and due to the fact that this model demands the existence of heavy exotic leptons. The mixing of the standard $Z$ with $Z'$ and the mixing of ordinary leptons with exotic ones induce together family changing couplings on the $Z$ and therefore nonvanishing rates for lepton family number violation processes, such as $Z \to e\bar{\mu}$, $\mu \to eee$ and $\mu \to e\gamma$. Additional contributions to the processes $\mu \to e\gamma$ and $\mu \to eee$ are induced from the mass generation mechanism. This last type of contributions may compete with the above one, depending on the masses of the scalars which participate in the diagrams which generate radiatively the masses of the charged leptons. Using the experimental data we compute some bounds for the mixings parameters and for the masses of the scalars.

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I. INTRODUCTION

In this communication we show how the tree level family changing neutral current (FCNC) phenomenon arises in the context of the SU(6)$_L \otimes U(1)_Y$ model [1] whose simplicity allows to tests several mechanisms for physics beyond the Standard Model (SM). Within this model we then compute the branching ratios for lepton family number violating (LFV) processes in the $e-\mu$ sector with the aim of bounding the light-heavy lepton mixing parameters of the model in this sector. In section II A we present the main features of the SU(6)$_L \otimes U(1)_Y$ model and in section II B those of its symmetry breaking. In section II C and II D we display the mixing effects in the leptonic sector and the FC couplings of the $Z$ to ordinary leptons; in section II E we deal with the mechanism which gives masses radiatively to the charged leptons and in section II F we compute the branching ratio for $Z \to e\bar{\mu}$ and $\mu \to eee$, both at tree level and for $\mu \to e\gamma$ at one loop.

II. THE MODEL

*and Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN A.P. 14-740, 07000 México D.F., México.

\textsuperscript{1}e-mail: rgaitan@servidor.unam.mx
A. Group Structure

The gauge group of the model is SU(6)_L ⊗ U(1)_Y, where SU(6)_L unifies the weak isospin SU(2)_L of Glashow-Weinberg-Salam, with an horizontal gauge group G_H, whose maximum expression is SU(3)_H. SU(2)_L ⊗ SU(3)_H is a maximal special subgroup of SU(6)_L. The thirty-five SU(6)_L generators, S_a, in the SU(2)_L ⊗ SU(3)_H basis are

\[ S_a = \frac{1}{2\sqrt{3}} \sigma_i \otimes 1_3 \leftrightarrow W_i, \quad \frac{1}{2\sqrt{2}} 1_2 \otimes \lambda_\alpha \leftrightarrow G_\alpha, \quad \frac{1}{2} \sigma_i \otimes \lambda_\alpha \leftrightarrow H_{i\alpha}, \]

where \( \sigma_i, i = 1, 2, 3 \), are the Pauli matrices, \( T_i = \frac{1}{2} \sigma_i \) are the SU(2)_L generators, \( \lambda_\alpha, \alpha = 1, 2, 3 \), are the Gell-Mann matrices, \( \frac{1}{2} \lambda_\alpha \) are the SU(3)_H generators, and \( I_3 \) and \( I_2 \) are \( 3 \times 3 \) and \( 2 \times 2 \) unit matrices, respectively. The corresponding gauge bosons are indicated. Notice that all the generators are equally normalized:

\[ \text{Tr} S_a S_b = \frac{1}{2} \delta_{ab}, \quad \text{Tr} T_i T_j = \frac{1}{2} \delta_{ij}, \quad \text{Tr} \frac{1}{2} \lambda_\alpha \lambda_\beta = \frac{1}{2} \delta_{\alpha\beta} \]

with the consequence that if \( g_6, g_2 \) and \( g_{3H} \) are the coupling constants of SU(6)_L, SU(2)_L and SU(3)_H, respectively, then they satisfy, at the unification scale, the relationships

\[ g_2 = \frac{1}{\sqrt{2}} g_6, \quad g_{3H} = \frac{1}{\sqrt{2}} g_6. \]

The 36 gauge bosons: 35 associated with the generators of SU(6)_L and one associated to U(1)_Y. Besides the standard gauge bosons \( W^3 \), \( W^8 \) and \( B \), we have 32 extra gauge bosons, which can be divided into four groups:

1. **(++)**, 12 charged gauge bosons which perform transitions among families. They couple to family changing charged currents (FCCC) and are related to \( \sigma_i \otimes \lambda_\alpha, i = 1, 2, \alpha = 1, 2, 4, 5, 6, 7 \).

2. **(+o)**, 4 charged gauge bosons which do not make transitions among families but their couplings are family dependent. They couple to non-universal family diagonal charged currents (NUFDC) and are related to \( \sigma_i \otimes \lambda_\alpha, i = 1, 2, \alpha = 3, 8 \).

3. **(o+)**, 12 neutral gauge bosons which induce transitions among families. They couple to FCNC and are related to \( 1_2 \otimes \lambda_\alpha, \alpha = 1, 2, 4, 5, 6, 7 \).

4. **(oo)**, 4 neutral gauge bosons which couple non-universally without changing flavor, that is they couple to non-universal family diagonal neutral currents (NUFDNC) and are related to \( 1_2 \otimes \lambda_\alpha, \alpha = 3, 8 \).

Notice that at scales where families are not defined, that is at scales above the breaking of SU(2)_L ⊗ U(1)_Y, there is no clear distinction between the members of the (+o) set and those of the (++) one. Similarly, the members of the set (o+) and those of the set (oo) should be put in a unique class. In other words, there is no way to distinguish “family changing” from “non-universal flavor diagonal” attributes. For example a gauge boson associated with a generator of the form

\[ T_{H1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \]

in family space, will be identified as a family changing one, but applying a transformation in family space one can diagonalize \( T_{H1} \) to the form

\[ T'_{H1} = T_{H3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \]

and therefore transform the corresponding gauge boson into one with non-universal flavor diagonal couplings.

The fermionic content of the model is given by the following set of irreducible representations (irreps) of SU(6)_L ⊗ U(1)_Y, one for each color in the case of quarks (right handed fields are charge-conjugated to left handed ones):

\[
\begin{align*}
\{6, \frac{1}{2}\}_L &= (u, d, c, s, t, b)_L \equiv \psi^u_L(\frac{1}{2}), \\
\{\overline{6}, -1\}_L &= (e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau)_L \equiv \psi^s_L(-1), \\
\{1, -\frac{1}{3}\}_L &= q^u_L(-\frac{1}{3}), \\
\{1, \frac{2}{3}\}_L &= q^d_L(\frac{2}{3}), \\
\{1, 2\}_L &= l^e_L(2), \\
\{\overline{10}, 0\}_L &= \psi_{[\alpha\beta]}(0),
\end{align*}
\]
where \( I \) is a family index, \( \alpha \) and \( \beta \) are SU(6)_L tensor indices and \( u, d, \ldots \) refer to the up quark, down quark, \ldots fields. The label \( L \) refers to left handed Weyl spinors and the upper c symbol indicates a charge–conjugated field. The number in parenthesis stands for the hypercharge and the symbol \([\alpha\beta]\) indicates antisymmetric ordering, \(\phi_{AB} = \frac{1}{\sqrt{2}}(\phi_{AB} - \phi_{BA})\).

\[
\begin{pmatrix}
0 & N_1 & E_1^- & N_4 & E_2^- & N_6 \\
0 & N_5 & E_1^+ & N_7 & E_2^+ & N_8 \\
0 & N_2 & E_3^- & N_8 \\
0 & N_9 & E_3^+ \\
0 & N_3 \\
0 & 0
\end{pmatrix}_{L(6)}
\]

is the multiplet of exotic leptons. They are classified according to SU(2)_L into 3 triplets,

\[
\begin{pmatrix}
\frac{1}{\sqrt{2}}(N_4 + N_5) \\
E_1^+
\end{pmatrix}, \quad \begin{pmatrix}
\frac{1}{\sqrt{2}}(N_6 + N_7) \\
E_2^+
\end{pmatrix}, \quad \begin{pmatrix}
\frac{1}{\sqrt{2}}(N_8 + N_9) \\
E_3^+
\end{pmatrix},
\]

and six neutral singlets,

\[
N_1, \ N_2, \ N_3, \ \frac{1}{\sqrt{2}}(N_4 - N_5), \ \frac{1}{\sqrt{2}}(N_6 - N_7), \ \frac{1}{\sqrt{2}}(N_8 - N_9).
\]

Notice, from the structure of the \(\{6 (\frac{1}{2})\}_L\), that the SU(6)_L indices are arranged according to the following scheme:

\[
\begin{array}{c}
\leftarrow \text{SU}(3)_H \\
\uparrow \quad 1 \quad 3 \quad 5 \\
\text{SU}(2)_L \\
\downarrow \quad 2 \quad 4 \quad 6
\end{array}
\]

**B. Symmetry Breaking**

The symmetry breaking is realized in three stages: at the scale \(M_1\)

\[
\text{SU}(6)_L \otimes \text{U}(1)_Y \xrightarrow{M_1} \text{SU}(2)_L \otimes \text{SU}(2)_H \otimes \text{U}(1)_Y
\]

and the six SU(2)_L exotic singlets get mass of order \(M_1\). This breaking is achieved \[9\] with a Higgs scalar in the irrep \(\phi_1 = \{105(0)\}\) and the horizontal symmetry is reduced to the special SU(2)_H subgroup generated by the set

\[
T_{H1} = \frac{1}{\sqrt{2}}(\lambda_1 + \lambda_6) \leftrightarrow \Sigma_1, \quad T_{H2} = \frac{1}{\sqrt{2}}(\lambda_2 + \lambda_7) \leftrightarrow \Sigma_2, \quad T_{H3} = \frac{1}{2}(\lambda_6 + \sqrt{3}\lambda_8) \leftrightarrow \Sigma_3,
\]

where the corresponding gauge fields have been indicated. Here the generators are conveniently normalized to \(\text{Tr}T_{H1}\)\(T_{H2} = 2\) so that \(T_{H3}\) has eigenvalues \(0, \pm 1\). It then follows that the coupling constants of SU(2)_H and SU(3)_H are related, at the unification scale, by

\[
g_{2H} = \frac{1}{2}g_{3H} = \frac{1}{2\sqrt{2}}g_5.
\]

In the second stage of the symmetry breaking chain

\[
\text{SU}(2)_L \otimes \text{SU}(2)_H \otimes \text{U}(1)_Y \xrightarrow{M_2} \text{SU}(2)_L \otimes \text{U}(1)_Y,
\]

at the scale \(M_2\), the exotic leptons which transform as triplets of SU(2)_L get a mass of order \(M_2\), and \(E_i^{+c}\) becomes the right–handed counterpart of \(E_i^-\). This step is implemented \[10\] with a Higgs \(\phi_2 = \{15(0)\}\).

The final stage of the symmetry breaking chain,
SU(2)_L \otimes U(1)_Y \xrightarrow{M_3} U(1)_{EM},

is achieved using a Higgs field \( \phi_3 = \mathcal{B}(1) \) with VEV’s in the neutral components \( \langle \phi_{3i} \rangle = v_i \) for \( i = 1,3,5 \). With \( \phi_3 \) the following mass term for quarks may be written

\[
\sum_{i=1}^{3} \gamma_i q_{iL}^c (-\frac{d}{2}) C \phi_{3a} \psi_{3L}^c (-\frac{d}{2}) + \text{h.c.} = (\gamma_u u^c + \gamma_c c^c + \gamma_t t^c)_L C (v_1 u + v_2 c + v_3 t)_L + \text{h.c.},
\]

where \( \gamma_i \) are Yukawa couplings of order 1 and \( c \) is the charge conjugation matrix. This mass term predicts tree–level zero mass for all the quarks, except for the top: \( m_t = \gamma v \) where \( \gamma = \sqrt{\gamma_u^2 + \gamma_c^2 + \gamma_t^2} \) and \( v = \sqrt{v_1^2 + v_2^2 + v_3^2} \). A similar mass term for the \( \tau \) is avoided postulating a \( Z_5 \) discrete symmetry that distinguishes quarks from leptons. In this way the Yukawa coupling of \( \phi_3 \) in the leptonic sector is completely absent.

### C. Mixing of \( Z^0 \) with heavy bosons

Among the 12 neutral gauge bosons related to FCNC, three are relatively light (those of SU(2)_H), with mass of the order of \( M_2 \). The other 9 are heavy, with mass of the order of \( M_1 \). If \( Z^0 \) mixes with this type of gauge bosons, the mixing with the first three will dominate, unless it vanishes as a consequence of some symmetry.

We are therefore interested in constructing the \( Z^0, \Sigma_1, \Sigma_2, \Sigma_3 \) mass matrix. The VEV’s of \( \phi_2 \),

\[
\langle \phi_2^{[3456]} \rangle = \langle \phi_2^{[1245]} \rangle = -\langle \phi_2^{[1236]} \rangle = V,
\]

produce the following \( \Sigma_1-\Sigma_2, \Sigma_3 \) mass terms:

\[
\mathcal{L}_{\Sigma^+\Sigma^-}^{m} \simeq \frac{3}{2} g_6^2 V^2 \left( \Sigma^+ \Sigma^- + \frac{1}{2} \Sigma^2 \right),
\]

where we do not consider the mixings with the heaviest gauge bosons. \( \phi_3 \) gives

\[
\mathcal{L}_{Z^0, \Sigma^0}^{m} \simeq \frac{1}{2} \Delta M_{\Sigma^0}^2 \Sigma_3^2 + \frac{1}{2} M_{Z^0}^2 Z^0 + \left( \frac{1}{2} M_{Z^0, \Sigma^0}^2 \Sigma^0 \Sigma^0 + \frac{1}{2} \Delta M_{\Sigma^2}^2, \Sigma^0 \Sigma^0 + \text{h.c.} \right),
\]

where

\[
Z^0 = \frac{g W^3 - g'B}{\sqrt{g^2 + g'^2}},
\]

\( g \) and \( g' \) are the coupling constants of SU(2)_L and U(1)_Y respectively,

\[
\Sigma^\pm = \frac{1}{\sqrt{2}} (\Sigma_1 \mp i \Sigma_2),
\]

and

\[
\Delta M_{\Sigma^3}^2 = \frac{1}{4} v^2 g^2, \quad M_{Z^0}^2 = \frac{1}{4} v^2 (g^2 + g'^2), \quad M_{Z^0, \Sigma^0}^2 = \frac{1}{\sqrt{3}} v^2 g \sqrt{g^2 + g'^2}, \quad \Delta M_{\Sigma^2}^2 = \frac{3}{4} v^2 g^2.
\]

Notice that \( \Sigma_2 \) decouples from \( Z^0 \) due to the h.c. term and that \( \Sigma_3 \) does not mix with any other. Thus only the \( Z^0-\Sigma_1 \) sector must be diagonalized and the mass matrix leads to

\[
\begin{pmatrix}
Z^0 \\
\Sigma_1
\end{pmatrix}
= \begin{pmatrix}
\cos \Theta & -\sin \Theta \\
\sin \Theta & \cos \Theta
\end{pmatrix}
\begin{pmatrix}
Z \\
Z'
\end{pmatrix}
\equiv R
\begin{pmatrix}
Z \\
Z'
\end{pmatrix},
\]

where

\[
\cos \Theta \simeq \frac{1}{K_3}, \quad \sin \Theta \simeq \frac{1}{K_3} \delta
\]

and

\[
K_3 = \sqrt{1 + \delta^2}, \quad \delta = 2 \sqrt{2} \left( \frac{M_Z}{M_{Z'}} \right)^2.
\]
The mass of the Z gauge boson is given by

\[ M_Z^2 \simeq \frac{M_W^2}{\cos^2 \theta_w} \left[ 1 - \left( \frac{v}{V} \right)^2 \right], \]  

(25)

where \( M_W \) is the mass of the charged boson \( W^\pm \) and \( \theta_w \) is the weak mixing angle given by

\[ g \sin \theta_w = g' \cos \theta_w = e. \]  

(26)

The mass for the \( Z' \) is given by

\[ M_{Z'}^2 \simeq \frac{9g^2}{2} V^2. \]  

(27)

D. Mixing of ordinary charged leptons with exotic ones and family changing couplings

Let \( \psi_0^a = (e, \mu, \tau, E_1, E_2, E_3)_{a}^\dagger \), \( a = L, R \), be a vector of gauge eigenstates in the electric charge \( q = -1 \) space and \( \psi_a \) the one corresponding to the mass eigenstates. Let

\[ \psi_0^a = U_a \psi_a, \]  

(28)

with

\[ U_a = \begin{pmatrix} A_a & E_a \\ F_a & G_a \end{pmatrix}. \]  

(29)

Let \( D_a \) and \( H_a \) be the 6 × 6 matrices that express the couplings of the \( Z^0 \) and \( \Sigma_1 \) to \( \psi_0^a \),

\[ -L^{\text{nc}} = \frac{e}{8 \theta_w \cos \theta_w} \sum_{a=L,R} \bar{\psi}_0^a \gamma^\mu (D_a, H_a) \psi_0^a \left( \frac{Z^0}{\Sigma_1} \right)_\mu, \]  

(30)

where

\[ D_L = \begin{pmatrix} -\frac{1}{2} + \sin^2 \theta_w & 0 \\ 0 & -1 + \sin^2 \theta_w \end{pmatrix} \otimes 1_3, \]  

(31a)

\[ D_R = \begin{pmatrix} \sin^2 \theta_w & 0 \\ 0 & -1 + \sin^2 \theta_w \end{pmatrix} \otimes 1_3, \]  

(31b)

\[ H_L = \begin{pmatrix} h & 0 \\ 0 & -h \end{pmatrix}, \quad H_R = \begin{pmatrix} 0 & 0 \\ 0 & h \end{pmatrix}, \quad h = -\frac{\sqrt{3}}{4} g \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \]  

(32)

then the coupling of the lowest mass neutral gauge boson mass eigenstate, \( Z \), to the light (ordinary) leptons, \( \psi_{\text{la}} = (e, \mu, \tau)^\dagger_{\text{la}} \), is given by

\[ -L^{\text{nc}}_Z = \frac{e}{\sin \theta_w \cos \theta_w} \sum_{a=L,R} \bar{\psi}_{\text{la}} \gamma^\mu K_a \psi_{\text{la}} Z_\mu, \]  

(33)

where

\[ K_L = \left[ -\frac{1}{2} (F_L^\dagger F_L) - \left( \frac{1}{2} - \sin^2 \theta_w \right) \right] \cos \Theta + (H_L)_{\text{ll}} \sin \Theta, \]  

(34a)

\[ K_R = \left[ - (F_R^\dagger F_R) + \sin^2 \theta_w \right] \cos \Theta + (H_R)_{\text{ll}} \sin \Theta, \]  

(34b)
In this section we describe a mechanism to generate radiatively the masses for the ordinary charged leptons. With the additional Higgs scalars $\phi_4 = \{70(1)\}$ and $\phi_5 = \{15(-2)\}$ introduced in Ref. [1], and using as a seed the mass of the exotic charged lepton $E_3^-$, we have a contribution coming from the diagrams of Fig. 1, where the couplings are given by

$$\eta \psi^T_{\alpha \beta} C \psi^\alpha_{\gamma L}(0) C \eta^\gamma_{\beta L}(-1) \phi^\alpha_{\phi}(1) + h.c.$$  \hspace{1cm} (38)$$

$$+ \psi^T_{\alpha \beta} C \sum_i M^2 \left( \eta^\gamma_{\beta L}(2) \phi^\alpha_{\phi}(2) \right) + h.c.$$  \hspace{1cm} (39)$$

$$+ \lambda \phi^\beta_{\phi} \phi^\alpha_{\phi} + h.c.$$  \hspace{1cm} (40)$$

where $\eta$ and $\eta_L$ are the Yukawa couplings of order 1. The mixing of the double charged scalar fields needed in the diagrams of Fig. 1 are generated when $\phi_2$ and $\phi_3$ take VEV’s. The $\Sigma_{ij}(0)$ mass term generated from this diagram is

$$\Sigma_{ij}(0) = \frac{\lambda_i \lambda_j}{16 \pi^2} M_2 \sum_i O_{ij} f(M_2, M_i)$$  \hspace{1cm} (41)$$

where $\lambda_i$ and $\lambda_j$ are the couplings in the vertices, $O$ is the unitary matrix which diagonalizes the mass matrix of the scalars, $M_i$ are the eigenvalues, and

$$f(M_2, M_i) = \frac{1}{M_i^2 - M_2^2} \left[ M_i^2 \ln \frac{M_i^2}{M_2^2} - M_i^2 + M_2^2 \right]$$  \hspace{1cm} (42)$$

This one loop contribution may be written as $h_{ij} v_{ij} = \Sigma_{ij}(0)$, and therefore it leads to an increase in one unit of the rank of the charged leptons mass matrix. In this way the mass of the $\tau$ is generated. To generate the masses for $\mu$ and $e$ we introduce two new color singlet scalars $\chi_1 = \{21(2)\}$ and $\chi_2 = \{1(4)\}$, and a new Higgs $\phi_6 = \{21(-2)\}$, which allow the couplings

$$H^5 \psi^T_{\alpha \beta}(1) C \psi^\alpha_{\gamma L}(1) \chi^\alpha_{\gamma L} + h.c.$$  \hspace{1cm} (43)$$

$$+ H^5_{ij}(e^+)^T_{\alpha \beta} C (e^+)^\alpha_{\gamma L} \chi^\alpha_{\gamma L} + h.c.$$  \hspace{1cm} (44)$$

$$+ \lambda_i \chi^\alpha_{\gamma L} \chi^\alpha_{\gamma L} + h.c.$$  \hspace{1cm} (45)$$

These couplings generate the mass terms in the $e - \mu$ sector coming from the diagrams of Fig. 2 and from those diagrams starting with the right handed chirality from the same Fig. 2. The appropriate mixing among the $\chi$ scalars is generated when $\phi_6$ takes VEV’s. The contribution from these diagrams can be written again in the form $h_{ij} v_{ij}$ and therefore the rank of the charged lepton mass matrix is again increased in one unit. In this way the muon obtain mass. Finally the mass of the electron is generated through a diagram similar to those of Fig. 2, but with the muon in the internal fermion line.
III. CONSTRAINTS

A. Family diagonal process $Z \rightarrow e\bar{e}$

Mixing effects also modify slightly the rate for family diagonal processes $[3,4]$. Consider for example $Z \rightarrow e\bar{e}$, whose branching ratio is given by

$$B(Z \rightarrow e\bar{e}) \simeq \frac{1}{\Gamma_{\text{tot}}} \frac{G_F M_Z^3}{8\sqrt{2}\pi} \left( |g_V|^2 + |g_A|^2 \right)$$

(46a)

$$= \frac{1}{\Gamma_{\text{tot}}} \frac{G_F M_Z^3}{3\sqrt{2}\pi} \left( |\Lambda_{e\bar{e}}^e + \Xi_{e\bar{e}}^{\epsilon} \Theta - \frac{1}{2} + \sin^2 \theta_w |^2 + |\Lambda_{e\bar{e}}^{e\epsilon} + \Xi_{e\bar{e}}^{e\epsilon} \Theta + \sin^2 \theta_w |^2 \right) + O(\theta^2).$$

(46b)

Since the agreement of the SM predictions with the experimental data for these processes is better than 0.1\% (the experimental value of $\Gamma(Z \rightarrow l\bar{l})$ is $83.83 \pm 0.27$ against the theoretical one equal to $83.97 \pm 0.07$), the quantities $\Lambda_{e\bar{e}}^e + \Xi_{e\bar{e}}^{e\epsilon} \Theta$ are bounded practically by the experimental uncertainty in the data $[6]$, 

$$B_{e\bar{e}} \equiv B(Z \rightarrow e\bar{e}) = (3.366 \pm 0.008) \times 10^{-2}.$$ 

(47)

From eq. (46b) it follows that

$$|\Lambda_{e\bar{e}}^e + \Xi_{e\bar{e}}^{e\epsilon} \Theta - \frac{1}{2} + \sin^2 \theta_w |^2 + |\Lambda_{e\bar{e}}^{e\epsilon} + \Xi_{e\bar{e}}^{e\epsilon} \Theta + \sin^2 \theta_w |^2 = c B_{e\bar{e}},$$

(48)

where $c^{-1} = \frac{1}{\Gamma_{\text{tot}}} \frac{G_F M_Z^3}{3\sqrt{2}\pi} = 0.2675 \pm 0.0005$. Therefore we obtain, in a neighborhood of $|\Lambda_{e\bar{e}}^e + \Xi_{e\bar{e}}^{e\epsilon} \Theta| = 0$ and with $\sin^2 \theta_w = 0.2237 \pm 0.0010$, the bounds

$$|\Lambda_{e\bar{e}}^e + \Xi_{e\bar{e}}^{e\epsilon} \Theta| < \text{few} \times 10^{-3}.$$ 

(49)

B. Constraints from $Z \rightarrow e\bar{\mu}$

With the approximation $M_Z \gg m_\mu, m_e$ and taking into account that experimental limits for $Z \rightarrow e\bar{\mu}$, Fig. 3 exist only for the sum of the charge states of particles and antiparticles states, the branching ratio is

$$B(Z \rightarrow e\bar{\mu} + \mu\bar{e}) \simeq \frac{2}{|g_V|^2 + |g_A|^2} \left( |g_V^\mu|^2 + |g_A^\mu|^2 \right)$$

(50a)

$$\simeq \frac{4}{|g_V|^2 + |g_A|^2} \left( |\Lambda_{L}^{e\mu} + \Xi_{L}^{e\mu} \Theta|^2 + |\Lambda_{R}^{e\mu} + \Xi_{R}^{e\mu} \Theta|^2 \right) + O(\theta^2).$$

(50b)

which leads to

$$|\Lambda_{L}^{e\mu} + \Xi_{L}^{e\mu} \Theta|^2 + |\Lambda_{R}^{e\mu} + \Xi_{R}^{e\mu} \Theta|^2 < c B_{e\bar{\mu}},$$

(51)

where $c^{-1} = \frac{4}{|g_V|^2 + |g_A|^2} = 0.536$ (using the conventional SM branching ratio $0.0337$ for $B_{l\bar{l}}$ and the standard values for $g_V$ and $g_A$) and where $B_{e\bar{\mu}}$ is defined by the numerical value of the experimental upper bound $[6]$

$$B_{e\bar{\mu}} \equiv B(Z \rightarrow e\bar{\mu} + \mu\bar{e}) < 1.7 \times 10^{-6} \equiv \bar{B}_{e\bar{\mu}}.$$ 

(52)

This means that the ordinary-ordinary off diagonal mixing parameters $\Lambda_{e\bar{\mu}}^{e\mu}$ are bounded to lie in a circular region centered at $(-\Xi_{L}^{e\mu} \Theta, -\Xi_{R}^{e\mu} \Theta)$ and of radius $9.5 \times 10^{-4}$. 
C. Constraints from $\mu \to e\bar{e}e$

Since $m_\mu \gg m_e$ and ignoring possible contributions from scalars to the process, the branching ratio $B(\mu \to e\bar{e}e)$, described by Fig. 3 is given by

$$\frac{B(\mu \to e\bar{e}e)}{B(\mu \to e\bar{\nu}_e\nu_\mu)} = \frac{1}{2} \left[ 3 \left( |g_V^{ee}|^2 + |g_A^{ee}|^2 \right) \left( |g_V^{e\mu}|^2 + |g_A^{e\mu}|^2 \right) + 2\Re \left( g_V^{ee} g_A^{e\mu*} \right) 2\Re \left( g_V^{e\mu} g_A^{ee*} \right) + \frac{M^2}{M_Z^2} \Re \left[ 3 \left( g_V^{ee} g_V^{ee*} + g_A^{ee} g_A^{ee*} \right) \left( g_V^{e\mu} g_V^{e\mu*} + g_A^{e\mu} g_A^{e\mu*} \right) \right] + \left( g_V^{ee} g_V^{ee*} + g_A^{ee} g_A^{ee*} \right) \left( g_V^{e\mu} g_V^{e\mu*} + g_A^{e\mu} g_A^{e\mu*} \right) \right] + \frac{1}{2} M_Z^2 \left[ 3 \left( |g_V^{ee}|^2 + |g_A^{ee}|^2 \right) \left( |g_V^{e\mu}|^2 + |g_A^{e\mu}|^2 \right) + 2\Re \left( g_V^{ee} g_A^{e\mu*} \right) 2\Re \left( g_V^{e\mu} g_A^{ee*} \right) \right] \right] \sim 4 \left[ \left( 2 - \frac{1}{2} + s^2_{\theta_w} \right)^2 + \left( -\frac{1}{2} + s^2_{\theta_w} \right)^2 \right] |\Lambda_\mu^e + \Xi_\mu^e e\bar{e}e|^2 + \left( \left( -\frac{1}{2} + s^2_{\theta_w} \right)^2 + 2 \left| s^2_{\theta_w} \right|^2 \right) |\Lambda_\mu^R + \Xi_\mu^e e\bar{e}e|^2 + O(\Theta^2), \quad (53a)$$

where we have assumed $\left( \frac{M^2}{M_Z^2} \right)^2 \sim \Theta$, $\Lambda_\mu^{e\mu} \lesssim \Theta$ (remember that $\Lambda_\mu^{e\mu}$ is second order in the ordinary-exotic mixing) and we have taken into account the stringent limits obtained in eq. (49) from which

$$|\Lambda_\mu^{ee} + \Xi_\mu^{ee} e\bar{e}e - \frac{1}{2} + s^2_{\theta_w}| \simeq \left| -\frac{1}{2} + s^2_{\theta_w} \right|, \quad (54a)$$

$$|\Lambda_\mu^{ee} + \Xi_\mu^{ee} e\bar{e}e + s^2_{\theta_w}| \simeq \left| s^2_{\theta_w} \right|. \quad (54b)$$

Using the experimental limits [8]

$$B_{\mu e\bar{e}e} \equiv B(\mu \to e\bar{e}e) < 1.0 \times 10^{-12} \equiv \tilde{B}_{\mu e\bar{e}e} \quad (55)$$

and $s^2_{\theta_w} = 0.2237$, the constraints on the mixing parameters are

$$0.203 |\Lambda_\mu^{e\mu} + \Xi_\mu^{e\mu} e\bar{e}e|^2 + 0.176 |\Lambda_\mu^{e\mu} + \Xi_\mu^{e\mu} e\bar{e}e|^2 < c_\mu \tilde{B}_{\mu e\bar{e}e} \quad (56)$$

where $c_\mu = (4B(\mu \to e\bar{\nu}_e\nu_\mu))^{-1} \simeq 0.25$. Eq. (56) is more stringent than eq. (52).

A possible contribution from scalars to this process come from the fermionic mass-generation mechanism through a complete penguin diagram as in the Fig. 3.

D. Constraints from $\mu \to e\gamma$

In this section we analyze the lepton flavor violation process $\mu \to e\gamma$ arising in the model mainly from the $Z^0 - \Sigma_1$ mixing and from the effect induced of the fermionic mass-generation mechanism at one loop level.

The contribution to this process coming from the mixing of ordinary with exotic leptons is negligible compare with the above mention sources. This is a consequence of the approximations obtained in eqs.(54a) and (54b), because these results means that the modifications to the couplings of the $Z$ and $Z'$ to the charged leptons are not sensitive to the mixing of ordinary with exotic leptons. In the other hand when we write the gauge eigenstates in terms of the mass eigenstates, eq.(28), the dominant contribution come from the diagonal elements of the $U$ matrix, that is we are working in the limit of very small off-diagonal elements of the matrix $U$.

Contributions from the would be Goldstone bosons are forbidden in the model. The reason is that the amplitude for $\mu \to e\gamma$ is of the form $u_2(p_2)\sigma^{\mu\nu}q_\mu\epsilon_\mu\bar{u}_1(p_1)$, where $q = p_1 - p_2$ and $\epsilon_\mu$ is the polarization of the photon. The
process involves therefore a flip of helicity and the lowest order contribution would arise from diagrams of the type of Fig. 7b. However, as was mentioned in Section II B, \( \phi_3 \) is forbidden, by a \( Z_5 \) discrete symmetry, to couple to leptons.

Contributions from \( Z^0 - \Sigma_1 \) mixing are given through the diagrams of Fig. 7a, and from the mechanism of mass generation by inserting a photon in the internal lines of the diagram of Fig. 7b, and the conjugate diagram of Fig. 7c, which produce off-diagonal mass terms in the \( e - \mu \) sector. Note that contributions from the diagrams of Fig. 7b and Fig. 7d are proportional to the mass of the muon, while the contribution from diagrams of Fig. 7c and Fig. 7e are proportional to the mass of the electron. That is, the dominant contribution from the \( Z^0 - \Sigma_1 \) mixing is to \( \mu \tau \leftrightarrow e\gamma \). By simplicity and economy we align the VEVs of \( \phi_6 \) such that \( \chi_2 \) mixes only with \( \chi_{\mu\tau} \). In this case, the contribution from the mass-mechanism is of the form \( \mu_L \leftrightarrow e_R \gamma \) through the diagram of Fig. 7b. The calculation of the diagram of Fig. 7b, without the photon yields the result

\[
m_{12} = \frac{\lambda_1 \lambda_2}{16\pi^2} m_\tau \ln \frac{M^2}{M^2_\mu} \cos \alpha \sin \alpha \sim m_\mu, \tag{57}\]

where \( \lambda_1, \lambda_2 \) are the couplings of the vertices,

\[
\begin{pmatrix}
\chi_2 \\
\chi_{\mu\tau}
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
\chi_2' \\
\chi_{\mu\tau}'
\end{pmatrix}, \tag{58}
\]

\( M_1 \) and \( M_2 \) are the mass eigenvalues, \( \chi_2 \) and \( \chi_{\mu\tau} \) are the mass eigenstates, with the approximation \( M_1, M_2 \gg m_\tau \).

The amplitudes obtained by computing the one loop diagrams are

\[
iA_B = (C_B \bar{e}i) \frac{\sigma_{\mu\nu}q_{\nu}e_\mu}{m_\mu + m_e} \frac{1 + \gamma_5}{2} \mu \tag{59a}\]

and

\[
iA_M = (C_M \bar{e}i) \frac{\sigma_{\mu\nu}q_{\nu}e_\mu}{m_\mu + m_e} \frac{1 - \gamma_5}{2} \mu, \tag{60a}\]

where \( A_B \) comes from the \( Z^0 - \Sigma_1 \) mixing, \( A_M \) from the mechanism of mass generation and the coefficients \( C_B \) and \( C_M \) are given by

\[
C_B = \frac{5 e}{12\sqrt{2}\pi} \alpha m_\mu (m_\mu + m_e) \frac{\sin \Theta \cos \Theta}{\cos \theta_w M_2^2} \tag{61a}\]

and

\[
C_M = \frac{e}{M_1^2} m_\mu (m_\mu + m_e) \frac{1}{\ln \frac{M^2}{m_\mu^2}} \left[ \ln \frac{M^2}{m_\mu^2} - \frac{M_1^2}{M_2^2} \left( \ln \frac{M_2^2}{m_\mu^2} \right) \right] \tag{61b}\]

Comparing with the general expressions

\[
\text{im} [\bar{f}_1(p_1) \rightarrow f_2(p_2) + \gamma(q)] = (\bar{e}_2(p_2)i) \frac{\sigma_{\mu\nu}q_{\nu}e_\mu}{m_1 + m_2} [F(0)^Y_{21} + F(0)^A_{21}]_5 u_1(p_1), \tag{62a}\]

\[
B(\mu \rightarrow e\gamma) = \frac{m_\mu}{8\pi} \left( 1 - \frac{m_e}{m_\mu} \right)^2 \frac{1 - m_e^2}{m_\mu^2} \left[ |F(0)^Y_{21}|^2 + |F(0)^A_{21}|^2 \right] \tag{62b}\]

for a process \( f_1 \rightarrow f_2 + \gamma \) for a real photon, from ref. 8. In our case

\[
F(0)^Y_{21} = \frac{1}{2} (C_M + C_B), \tag{63a}\]

\[
F(0)^A_{21} = \frac{1}{2} (C_B - C_M) \tag{63b}\]

and in the limit \( m_\mu \gg m_e \) we get

\[
B(\mu \rightarrow e\gamma) = \frac{m_\mu}{16\pi} \left( C_B^2 + C_M^2 \right) \tag{64a}\]

In the approximation \( M_1 \sim M_2 \) and using the experimental limit 10

\[
B_{\mu e\gamma} \equiv B(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11} \equiv B_{\mu e\gamma}, \tag{65}\]

the constraints on the mixing parameters are \( |\Theta| < 4.2 \times 10^{-5} \), \( M_{Z'} > 24 \text{ TeV} \) and \( M_1 > 200 \text{ TeV} \).
IV. CONCLUSIONS

In this article we have analyzed the consequences of simultaneous mixing of the Z gauge boson with an horizontal neutral gauge boson and the mixing of ordinary charged leptons with exotic ones in the context of the model SU(6)_L ⊗ U(1)_{Y}. We concentrated on the effects of the above mixing on lepton family violation processes in the e–μ sector. The charged leptons in this model obtain mass radiatively by introducing new exotic scalar particles. This new scalars also contribute to lepton violation processes. In order to be consistent with experiments we find that their masses must be heavier than 200 TeV, |Θ| < 4.2 × 10^{-5}, and M_{Z'} > 24 TeV. We also determine that: i) Since Ξ_{eμ} are order O(1) and Θ is very small, the Λ_{eμ} are bounded by each one of the processes to lie in a circular region centered in (−Ξ_{eμ} L, −Ξ_{eμ} R) and with radius depending on the considered branching ratio, ii) The radius fixed by eq. (56) is 2.2 × 10^{-6}.

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FIG. 1. Diagram generating the τ mass term

FIG. 2. Diagrams generating mass terms in the e – μ sector

FIG. 3. Diagrams contributing to the Z → eμ̄ decay.

FIG. 4. Contribution to the μ → eee process.

FIG. 5. Induce contribution to the μ → eee process coming from the mass generation mechanism.

FIG. 6. Diagrams forbidden by the Z_5 discrete symmetry.

FIG. 7. Contribution to the μ → eγ decay coming from the Z^0 – Σ_1 mixing.
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
Figure 7