A phenomenological pion-nucleon interaction is used to obtain pionic mass modification in presence of constant homogeneous magnetic field background at finite temperature and chemical potential in the real time formalism of thermal field theory. The magnetically modified propagator in its complete form is used to obtain the one loop self-energy for pions. For charged pions we find that the effective mass increases with the magnetic field at given temperature and chemical potential. Since the transverse momentum of charged pion is quantized and its contribution to Dyson-Schwinger Equation is large compared to the loop correction, the charged pion mass remains constant with both temperature and chemical potential for a given landau level. In order to unveil the role of the real part of the self-energy, we also calculate the effective mass neglecting the trivial shift. The effective mass for charged pions shows an oscillatory behaviour which is attributed to the thermal contribution of the self-energy. It is argued that the magnetic field dependent vacuum contribution to the self-energy influences the behaviour of the effective mass both qualitatively and quantitatively. We also find that very large field is necessary for neutral pions to condense.

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gives markedly different behaviour in comparison with the usual soft-cutoff approach.

It has been argued in [43] that the study of pion-nucleon interaction plays an important role in the behavioural description of the deconfinement critical temperature in terms of pion mass and isospin. Pion mass modification in presence of magnetic background has been calculated employing chiral perturbation theory in Ref. [44]. It has been shown that in presence of magnetic field, the charged pions are no longer the Goldstone modes and the critical temperature of chiral phase transition shows magnetic catalysis. In NJL approach [45], it has been found that there exists a sudden leap in the effective masses of charged pions near the same critical temperature from where the $\sigma$ and $\pi^0$ meson become nearly degenerate. Here also, the pseudo-critical temperature is found to increase with the increasing magnetic field. In Ref. [46] pion effective mass variation with $eB$ has been observed for large magnetic fields where Lowest Landau Level (LLL) approximation is reasonable. Also in Ref. [47], the medium modification of pion effective mass has been obtained in a self-consistent way with LLL approximation. However, it is a common trend to ignore the magnetic field dependent vacuum contribution of the self-energy function in case of mass and dispersion calculations. It has been shown [48, 49] (also see the references therein) that it can have significant influence on mesonic properties like effective mass, dispersion relation, decay width and spectral function. Merely on grounds of simplicity, it appears unreasonable to neglect this $eB$ dependent vacuum contribution apriori unless one compares the dependencies on other external parameters with it. It is also interesting to observe the interplay between the medium and the vacuum effect of the external magnetic field to find the complete $eB$ dependence of the physical properties.

In this article we revisit the mass modification of pions in presence of finite temperature and chemical potential in a homogeneous magnetic background with a well known pion-nucleon interaction in isospin-symmetric nuclear matter. Unlike NJL model, here pionic fields are treated as elementary. However, the non-trivial mass correction in presence of magnetic field occurs due to the modification of the nucleon propagators. The influence of the magnetic field dependent vacuum contribution in case of the pseudo-vector pion-nucleon interaction has been studied in detail. In the Dyson-Schwinger formalism, instead of restricting ourselves to the strong/weak field region, the full propagator is used to obtain the pion self-energy. Since the charged pion transverse momentum is quantized in magnetic field, we obtain Landau Level (LL) dependent self-energy and the corresponding Dyson-Schwinger Equation (DSE) is modified because of the presence of the transverse momentum. Thus we obtain LL dependent effective mass for charged pions. To see the importance of $eB$ dependent vacuum contribution we neglect the trivial shift. In case of neutral pions, although the evaluation is restricted to the special case where the external pion momentum is parallel to the field direction, this restriction does not put any constraint in case of mass calculation. More specifically, pion effective mass has been obtained with full magnetic field dependence up to one loop order. Effective mass variation with external magnetic field due to $eB$ dependent vacuum is compared with the $eB$ dependent thermal contribution. It is argued that neglecting vacuum contribution may even influence the qualitative predictions of the effective mass dependences for different pion species.

The article is organized as follows. In Sec.II we discuss the formalism for calculating the one loop pion self-energy function for phenomenological pion-nucleon interaction in presence of constant external magnetic field in dense thermal medium. The section comprises two subsections, one for the charged pions and the other for the neutral pion where the magnetic field dependent vacuum contribution and thermal contribution of the self-energy of the corresponding species are calculated. The Dyson-Schwinger equation that relates the effective mass with the real part of the self-energy is also obtained. Pion mass variation with respect to the independent parameters are presented in Sec.III. The effect of incorporation of magnetic field dependent vacuum part is also discussed. Finally we summarize our work in Sec.IV.

II. FORMALISM

The Dyson-Schwinger Equation(DSE) for the effective propagator of pion is given by

$$D^{-1}(q) = D_0^{-1}(q) - \Pi(q)$$  \hspace{1cm} (1)

where $D_0^{-1}(q) = q^2 - m^2 + i\epsilon$ and $\Pi(q)$ is the pion self-energy. One can obtain the effective mass by finding the pole of $D(q)$. Here, we are interested in finding the thermal modification of pion mass in presence of constant external magnetic field along with finite baryon density due to the effective pion-nucleon interaction, given by [50]

$$\mathcal{L}^{int}_{\pi NN} = -\frac{g_{\pi NN}}{m_\pi}\bar{\psi}5\gamma^\mu(\vec{\tau} \cdot \partial_\mu \vec{\tau})\psi.$$  \hspace{1cm} (2)

where $\psi$ is the two component nucleon field and $\vec{\tau} = (\sigma_x, \sigma_y, \sigma_z)$, with $\sigma_a$ denoting the $a$th Pauli spin matrix. The pionic fields are represented by the isovector $\vec{\tau}$. Expanding the interaction Lagrangian, one finds the Feynman diagrams for the one loop self-energy of pions as given in Fig. [1]
The real time thermal propagators can be decomposed into two parts as [51]

\begin{align}
\Pi^{11}_0(q,\mu_N, T) &= -ig_{\pi NN}^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^5 q^\mu S_p^{11}(k) \gamma^5 q^\nu S_p^{11}(k+q) \right] - ig_{\pi NN}^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^5 q^\mu S_n^{11}(k) \gamma^5 q^\nu S_n^{11}(k+q) \right] \\
\Pi^{11}_+(q,\mu_N, T) &= -2i g_{\pi NN}^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^5 q^\mu S_p^{11}(k) \gamma^5 q^\nu S_p^{11}(k+q) \right] \\
\Pi^{11}_-(q,\mu_N, T) &= -2i g_{\pi NN}^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^5 q^\mu S_p^{11}(k) \gamma^5 q^\nu S_p^{11}(k+q) \right]
\end{align}

where \( S_p^{11}(k) \) and \( S_n^{11}(k) \) are the 11-components of the thermal propagators for proton and neutron respectively. The real time thermal propagators can be decomposed into two parts as [51]

\[ S_p^{11}(k) = S_p(k) - 2i\eta(k) \text{Im} S_p(k) \]
and \[ S_n^{11}(k) = S_n(k) - 2i\eta(k) \text{Im} S_n(k) \]
with \[ \eta(k) = \theta(k^0) n^+_k + \theta(-k^0) n^-_k \]
and \[ n^+_k = \frac{1}{e^{\beta(\omega_k + \mu_N)} + 1}. \]

Here, \( S_p(k) \) is the momentum space representation of the fermionic propagator in presence of magnetic field, \( \theta \) denoting the unit step function and \( \beta = \frac{1}{T} \) is the inverse temperature in natural unit. Fermionic propagators in presence of magnetic field possess a phase factor which can not be taken as translationally invariant in general. However, in the current context, the phase factor can be removed by suitable gauge transformation [52] and we can work with the momentum space representation of the translationally invariant part which is given by [53]

\[ S_p(k) = -\sum_{n=-\infty}^{n=\infty} (-1)^n e^{-\alpha} \frac{\left[ (\vec{k})^2 + m^2 \right] \left( (1 - i\gamma^1 \gamma^2) L_n(2\alpha) - (1 + i\gamma^1 \gamma^2) L_{n-1}(2\alpha) \right) - 4\vec{k}_\perp L_{n-1}^1(2\alpha) \}}{\left( \vec{k}^2 - m^2 - 2\epsilon B + i\epsilon \right) \}} \]

where \( \alpha = -k^2_\perp / eB \) and \( L_n \equiv L_n^\alpha \) with \( L_n^\alpha \) representing the generalized Laguerre polynomials. The \( \epsilon \) in the denominator is an infinitesimal positive parameter. It should be mentioned here that in this article we use \( g^{\mu\nu}_|| = \text{diag}(1, 0, 0, -1) \) and \( g^{\mu\nu}_\perp = \text{diag}(0, -1, -1, 0) \) with metric defined as \( g^{\mu\nu} = g^{\mu\nu}_|| + g^{\mu\nu}_\perp \). A general four vector can be decomposed as \( a^\mu = a^\mu_|| + a^\mu_\perp \) with \( a^2_|| = a_0^2 - a_3^2 \) and \( a^2_\perp = -a_1^2 - a_2^2 \). The imaginary part of the propagator is

\[ \text{Im} S_p(k) = \pi \sum_{n=-\infty}^{n=\infty} (-1)^n e^{-\alpha} \frac{\left[ (\vec{k})^2 + m^2 \right] \left( (1 - i\gamma^1 \gamma^2) L_n(2\alpha) - (1 + i\gamma^1 \gamma^2) L_{n-1}(2\alpha) \right) - 4\vec{k}_\perp L_{n-1}^1(2\alpha) \}}{\left( \vec{k}^2 - m^2 - 2\epsilon B \right) \}} \times \delta(\vec{k}^2 - m^2 - 2\epsilon B). \]
However, the neutron propagator $S_n(k)$ is not influenced by the presence of the magnetic field and is given by

$$S_n(k) = -\frac{k + m}{k^2 - m^2 + i\epsilon} \quad \text{and} \quad \text{Im} S_n(k) = \pi (k + m) \delta(k^2 - m^2).$$

(7)

Now, the propagators have two distinct parts, one with the thermal distribution function and another without it. On this basis the self-energy function can be expressed as a sum of three different portions given by

$$\Pi_{0,\pm} = (\Pi_{0,\pm})_{\text{vac}} + (\Pi_{0,\pm})_\eta + (\Pi_{0,\pm})_{\eta^2}.$$  

(8)

The term with quadratic dependence on the distribution function is purely imaginary. As we are only interested in the real part of the self-energy, we have

$$\text{Re}(\Pi_{0,\pm}) \equiv \text{Re}(\Pi_{0,\pm}) = \text{Re}(\Pi_{0,\pm})_{\text{vac}} + \text{Re}(\Pi_{0,\pm})_\eta$$

where $\Pi$ represents the 11-component of the diagonal self-energy matrix (see e.g. [54]). Let us now consider the explicit forms of the real part of the self-energy for charged and neutral mesons separately.

### A. Charged pions

The medium independent vacuum self-energy will be same for the charged pions $\pi^+$ and $\pi^-$ and can be obtained as follows.

$$\Pi_{+,\text{vac}} = -2ig \pi_{NN} \int \frac{d^3k}{(2\pi)^3} \text{Tr} \left[ \gamma^5 \slashed{k} S_n(k) \gamma^5 \slashed{p} S_p(p = q + k) \right]$$

$$\quad = -2ig \pi_{NN} \int \frac{d^3k}{(2\pi)^3} \sum_{n=0}^\infty \frac{(-1)^n e^{-\alpha_p} \text{Tr} \left[ \gamma^5 \slashed{k} \gamma^5 \slashed{p} D_n(q + k) \right]}{(k^2 - m^2 + i\epsilon)(\slashed{p} - \slashed{q} - 2MeB + i\epsilon)}$$

$$\quad = \sum_{n=0}^\infty \int \frac{d^2k_\|}{(2\pi)^2} \frac{\mathcal{N}_n}{(k^2 - m^2 + i\epsilon)(\slashed{p} - \slashed{q} - 2MeB + i\epsilon)}$$

with

$$\mathcal{N}_n = -2g^2 \pi_{NN} (-1)^n e^{-\alpha_p} \text{Tr} \left[ \gamma^5 \slashed{k} \gamma^5 \slashed{p} D_n(q + k) \right]$$

$$\quad = -8g^2 \pi_{NN} (-1)^n e^{-\alpha_p} \left[ 4L_{n-1}(2\alpha_p) \{ (q^2 - q^2_\perp - 2q_\perp \cdot k_\perp)(q_\perp \cdot k_\perp) + q^2 k^2_\perp - 2(q^2_\perp + q_\perp \cdot k_\perp)(q_\perp \cdot k_\perp) \} \right.$$

$$\quad \left. + \{ (L_n(2\alpha_p) - L_{n-1}(2\alpha_p)) \{ -m^2 q^2 + 2q^2_\perp(q_\perp \cdot k_\perp) - q^2 k^2_\perp + 2(q_\perp \cdot k_\perp) \} \right.$$  

$$\quad \left. + (q^2 - q^2_\perp + 2q_\perp \cdot k_\perp)(q_\perp \cdot k_\perp) \right].$$

(11)

Here $\alpha_p = -p^2/\epsilon B$ and $p = q + k$ with $q$ being the external momentum of pions. The 2-dimensional $k_\|$ integration can be performed using standard Feynman parametrization and dimensional regularization technique to obtain

$$\mathcal{I}(n, q_\perp^2, q_\perp^2) = -8g^2 \pi_{NN} (-1)^n e^{-\alpha_p} \frac{i}{4\pi} \int_0^1 dx \left[ 4L_{n-1}(2\alpha_p) \frac{A}{\Delta_n} + \{ (L_n(2\alpha_p) - L_{n-1}(2\alpha_p)) \left( \frac{B_n}{\Delta_n} - q^2 x \ln \frac{\Delta_n}{\mu_0} \right) \right.$$  

with

$$A = q^2 k^2_\perp + 2q_\perp^2 q_\perp^2 + \{ (1 + 2x)q^2_\perp - 2(q_\perp \cdot k_\perp) \} (q_\perp \cdot k_\perp)$$

$$B_n = -2m^2 q^2 - x(1 - x) (q_\perp^2 - q^2_\perp - q^2_\perp) + 2(1 - x)(q^2_\perp(q_\perp \cdot k_\perp) + (1 - x)q^2 k^2_\perp - 2nx B q^2$$

$$\quad = B_0 - 2nx B q^2$$

$$\Delta_n = m^2 - x(1 - x)q^2_\perp - i\epsilon - (1 - x)k^2_\perp + 2nx B.$$

(12)
where \( \mu_0 \) is the scale which appears in the process of dimensional regularization. Now, with this \( I(n, k_\perp, q_\parallel^2, q_\perp^2) \), the summation in Eq. (10) can be taken inside the \( k_\perp \) integral which gives

\[
(\Pi_+)_{\text{vac}} = \frac{2}{\pi} \frac{g_{\pi NN}^2}{\mu_0} \int_0^1 dx \int \frac{d^2k_\parallel}{(2\pi)^2} e^{-\alpha_p} \left[ 4A \mathcal{S}_1 + \mathcal{S}_2 - q_\parallel^2 \mathcal{S}_3 \right]
\]

where

\[
\mathcal{S}_1 = \sum_n (-1)^n L_n^1 (2\alpha_p) \frac{1}{\Delta_n}
\]

\[
\mathcal{S}_2 = \sum_n (-1)^n \left( L_n (2\alpha_p) - L_{n-1} (2\alpha_p) \right) \frac{B_n}{\Delta_n}
\]

\[
\mathcal{S}_3 = \sum_n (-1)^n \left( L_n (2\alpha_p) - L_{n-1} (2\alpha_p) \right) \ln \frac{\Delta_n}{\mu_0}
\]

(13)

It is possible to find compact expressions for these summations by casting them into known series of Laguerre polynomials as discussed in detail in the appendix. The final expression of the vacuum contribution of the charged pion self-energy is given by

\[
(\Pi_+)_{\text{vac}} = \frac{g_{\pi NN}^2}{2\pi^2} \int_0^1 dx \int_0^1 \frac{dz}{z} \frac{z}{\mu} \left[ \text{sech}^2 \left( x \frac{eB}{\mu} \ln z \right) - \frac{1}{\eta} (q_\parallel^2 - q_\perp^2) + q_\parallel^2 q_\perp^2 (y^2 + x - y - xy^2)
\]

\[
- (1-x) \left( \frac{q_\parallel^2}{\eta} + \frac{\mu}{\ln z} q_\perp^2 \right)
\]

where

\[
\eta = (1-x) \frac{\ln z}{\mu} + \frac{1}{eB} \ln \left( x \frac{eB}{\mu} \ln z \right)
\]

\[
y = \frac{1}{\eta eB} \ln \left( x \frac{eB}{\mu} \ln z \right)
\]

(14)

One can observe here that instead of a 2-dimensional \( k_\perp \) integral and an infinite series summation, now we have one integration over the parameter \( z \) which is more convenient for numerical evaluation. Another important feature is that the expression is not in the form of any polynomial of \( eB \) which signifies its non-perturbative character. It should be pointed out that the scale \( \mu \) present here appears in the process of parametrization with \( z \) and is different from the scale \( \mu_0 \) that appeared from dimensional regularization of \( k_\parallel \). It can be shown from Eq. (14) that at \( eB = 0 \) the self-energy becomes

\[
(\Pi_+)_{\text{vac}}(eB = 0) = \frac{g_{\pi NN}^2 m_\pi^2 q_\perp^2}{\pi^2} \int_0^1 dx \ln \left( \frac{\Delta}{\mu} \right)
\]

which is exactly twice the contribution of \( nn \) loop in case of neutral pions as will be seen later. Thus the \( (\Pi_+)_{\text{vac}} \) can be decomposed as

\[
(\Pi_+)_{\text{vac}} = (\Pi_+)_{\text{vac}}(eB \neq 0) + (\Pi_+)_{\text{vac}}(eB = 0)
\]

(16)

where \( (\Pi_+)_{\text{vac}}(eB \neq 0) \) represents the external \( eB \) dependent part of the self-energy and will be used in the DSE to obtain the effective mass.

The procedure to obtain the thermal part of the self-energy is relatively simpler and only the final expressions are presented. The real part of the thermal contribution of the self-energy for \( \pi^+ \) is given by

\[
\text{Re}(\Pi_+)_{\eta} = \sum_{l=0}^{\infty} \int \frac{d^3k}{(2\pi)^3} \left[ \mathcal{N}_l(k^0 = -q^0 + \omega_p^+ n_{p,l}^1) + \mathcal{N}_l(k^0 = -q^0 - \omega_p^- n_{p,l}^-)
\]

\[
+ \mathcal{N}_l(k^0 = \omega_k) n_k^+ + \mathcal{N}_l(k^0 = -\omega_k) n_k^-
\]

\[
\right] \frac{1}{2 \omega_k \left( (q^0 - \omega_k)^2 - (\omega_p)^2 \right)}
\]

(17)

where the expression for \( \mathcal{N}_l \) is given in Eq. (11) for the dummy index \( l = n \). One can observe from Eq. (3) that

\[
\Pi_+^{11}(-q, \mu_N, T) = \Pi_+^{11}(q, \mu_N, T).
\]

(18)
Thus the expression of \( \text{Re}(\Pi_\pm) \) can be easily obtained from \( \text{Re}(\Pi_+)_\eta \) by successively changing \( k \to k - q \) and \( q \to -q \).

It should be mentioned here that unlike the vacuum case, the infinite sum could not be performed analytically and as a result the Laguerre polynomials remain in the numerator within the single sum structure of the thermal contribution.

As in the case of the external particles are charged, the external transverse momentum suffers Landau quantization in presence of \( eB \). Thus to obtain the effective mass of \( \pi^\pm \) as a function of temperature, chemical potential and external magnetic field, we now solve the DSE of charged pions given by

\[
m^*_{\pi^\pm} - m^2_{\pi^\pm} + \text{Re}\Pi_\pm(m^*_{\pi^\pm}, q^2_\perp) = (2n + 1)eB(eB) - (2n + 1)eB = 0
\]

where \( \text{Re}\Pi_\pm \) contains the sum of explicit \( eB \) dependent vacuum and thermal contributions. Here, \( m^*_{\pi^\pm} \) and \( m_{\pi^\pm} \) denotes the renormalized mass and the effective mass of the charged pions respectively.

### B. Neutral pions

At first we consider the magnetic field dependent vacuum contributions from \( pp \) and \( nn \) loops. Here, the vacuum contribution refers to the part of the self-energy which is independent of the thermal distribution functions and can be written as

\[
(\Pi_0)_{\text{vac}} = (\Pi_0)^{pp}_{\text{vac}} + (\Pi_0)^{nn}_{\text{vac}}.
\]

For the \( pp \) loop

\[
(\Pi_0)^{pp}_{\text{vac}} = -ie^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \gamma^5 \gamma^\mu S_\mu(k) \gamma^5 \gamma^\nu S_\nu(p = q + k) \right]
\]

\[
= -ie^2 \int \frac{d^2k_\parallel}{(2\pi)^2} \int d^2k_\perp \sum_{n,l=0}^{\infty} \frac{(-1)^{n+l}e^{-i(\alpha_\parallel + \alpha_\perp)}}{k^2_\parallel - m^2 - 2neB + ic} \text{Tr} \left[ \gamma^5 \gamma^\mu D_\mu(n)(k) \gamma^5 \gamma^\nu D_\nu(p) \right]
\]

where \( D_n(k) = [(k_\parallel + m)] \{ (1 - i\gamma^1\gamma^2)L_n(2\alpha) - (1 + i\gamma^1\gamma^2)L_{n-1}(2\alpha) \} - 4k_\perp^4 L_{n-1}(2\alpha) \}. \]

At this point, one can observe that with \( q_x = q_y = 0 \) the \( k_\perp \) integration can be done analytically and standard Feynman parametrization technique can be applied to obtain

\[
(\Pi_0)^{pp}_{\text{vac}} = -i \sum_{n,l=0}^{\infty} \int \frac{d^2k_\parallel}{(2\pi)^2} \int_0^1 dx \frac{N^{n,l}_{\parallel}(k_\parallel)}{[k_\parallel + xq_\parallel]^2 - \Delta_{nl}^2}.
\]

Dropping the odd terms after the momentum shift \( k_\parallel \to k_\parallel - xq_\parallel \), we get

\[
N^{n,l}_{\parallel}(k_\parallel) = 4\pi e^2 \int_{\text{vac}} eB \left[ 4eB q_\parallel^2 n\delta_{n-1,l-1} + (\delta_{n,l} + \delta_{n-1,l-1}) \right. \\
\left. \times [q_\parallel^2 (-m^2 + x^2 q_\parallel^2 - xq_\parallel^2) + 2k_\parallel \cdot q_\parallel^2) - q_\parallel^2 k_\parallel^2] \right]
\]

\[
\Delta_{nl} = m^2_n - x(1-x)q_\parallel^2 + x(m^2_l - m^2_n)
\]

where

\[
m^2_n = m^2 + 2neB - ic.
\]
After the momentum integration we obtain
\[
(\Pi_0)_{\text{pp}}^{\text{vac}} = \frac{eB}{4\pi^2 g_{\pi NN}} \int_0^1 dx \sum_{n,l=0}^{\infty} \frac{[4eBn\delta_{n-1,l-1} + (\delta_{n,l} + \delta_{n-1,l-1})\{-m^2 - m_n^2 - x(m_l^2 - m_n^2)\}]}{\Delta_{nl}}
\]
\[
= \sum_{n,l=0}^{\infty} \frac{[4eBn\delta_{n-1,l-1} + (\delta_{n,l} + \delta_{n-1,l-1})\{-m^2 - m_n^2 - x(m_l^2 - m_n^2)\}]}{\Delta_{nl}}
\]
\[
= \sum_{n,l=1}^{\infty} \frac{[4eBn\delta_{n,l} + 2\delta_{n,l}\{-m^2 - m_n^2 - x(m_l^2 - m_n^2)\}]}{\Delta_{nl}} - \frac{2m^2}{\Delta}
\]
\[
\text{with } \Delta_{00} = \Delta = m^2 - x(1-x)q_{||}^2 - i\epsilon
\]
\[
= -\frac{2m^2}{eB} \sum_{n=1}^{\infty} \frac{1}{n + \frac{\Delta}{2eB}}. \frac{2m^2}{\Delta}
\]
\[
\text{where } \psi(\alpha) = \frac{d\ln(\Gamma(\alpha))}{d\alpha}
\]

(24)

It should be noted that \(\delta_{n-1} = 0\) here as Laguerre polynomial with negative index is taken to be zero. Finally, after regularization in the \(\overline{MS}\) scheme, the vacuum part of self-energy for the \(pp\) loop with \(\vec{q}_\perp\) becomes
\[
(\Pi_0)_{\text{pp}}^{\text{vac}} = \frac{g_{\pi NN}^2 m_n^2 q_{||}^2}{32\pi^2} \int_0^1 dx \left[\psi\left(\frac{\Delta}{2eB}\right) + \frac{eB}{\Delta}\right].
\]

(25)

Note that it contains the pure vacuum part i.e one without explicit \(eB\) dependence as well as the explicit magnetic field dependent vacuum contribution. The vacuum contribution from \(nn\) loop is given by
\[
(\Pi_0)_{\text{vac}}^{\text{nn}} = -ig_{\pi NN}^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[\gamma^5 q S_n(k) \gamma^5 q S_n(p = q + k)\right]
\]
\[
= -i \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{(k^2 - m^2 + i\epsilon)(p^2 - m^2 + i\epsilon)}
\]
\[
\text{where } N(k) = 4g_{\pi NN}^2 \left[\left(2(k \cdot q)^2 + q^2(k \cdot q - k^2 - m^2)\right)\right].
\]

(26)

This is a divergent integral and the momentum integration can be performed after standard Feynman parametrization. After regularization in \(\overline{MS}\) scheme the finite part of the vacuum self-energy for the \(nn\) loop can be obtained as
\[
(\Pi_0)_{\text{vac}}^{\text{nn}} = g_{\pi NN}^2 \frac{m_n^2 q_{||}^2}{32\pi^2} \int_0^1 dx \ln \left(\frac{\Delta}{\mu}\right) \text{ where } \vec{q}_\perp = 0
\]

(27)

and \(\mu\) is the scale of the theory having dimension of square mass. At zero magnetic field, the self-energy contribution of the \(pp\) loop must coincide with the contribution from the \(nn\) loop in isospin symmetric matter. Now since the complete form of the propagators is used in order to derive Eq.(26), we obtain a non-perturbative result as long as expansion in terms of \(eB\) is concerned. For that reason, we can not simply put \(eB = 0\) there to obtain the zero field contribution. Instead, we obtain a perturbative expansion of the \(pp\) result around \(eB = 0\). The \(eB \to 0\) expansion of Eq.(25) neglecting \(O(eB)^2\) term is given by
\[
(\Pi_0)_{\text{vac}}^{\text{pp}} = g_{\pi NN}^2 \frac{m_n^2 q_{||}^2}{32\pi^2} \int_0^1 dx \ln \left(\frac{\Delta}{2eB}\right)
\]
\[
= g_{\pi NN}^2 \frac{m_n^2 q_{||}^2}{32\pi^2} \int_0^1 dx \left[\ln \left(\frac{\Delta}{\mu}\right) - \ln \left(\frac{2eB}{\mu}\right)\right].
\]

(28)

The first term in the square bracket exactly matches with Eq.(27) whereas the second term diverges at \(eB = 0\). Hence, to match the two expressions identically i.e irrespective of the value of external momentum we must modify Eq.(26) as,
\[
(\Pi_0)_{\text{vac}}^{\text{pp}} = g_{\pi NN}^2 \frac{m_n^2 q_{||}^2}{32\pi^2} \int_0^1 dx \left[\psi\left(\frac{\Delta}{2eB}\right) + \frac{eB}{\Delta} + \ln \left(\frac{2eB}{\mu}\right)\right].
\]

(29)
By demanding identical contributions from pp and nn loop at vanishing magnetic field, one in fact imposes here a physical condition to extract out the finite part of the self energy.

We now turn to the thermal contribution. The real part of the thermal contribution for proton-proton(pp) loop can be obtained following a similar procedure employed in case of charged pions and is given by

$$\text{Re}(\Pi_0^{pp})_\eta = \sum_{n,l=0}^{\infty} \int \frac{dk_z}{(2\pi)^3} \mathcal{P} \frac{N^{n,l}_\| (k^0 = -q^0 + \omega_p n_p^+)}{2\omega_p \{(q^0 - \omega_p)^2 - (\omega_k^0)^2\}} + \frac{N^{n,l}_\| (k^0 = -q^0 - \omega_p n_p^-)}{2\omega_p \{(q^0 + \omega_p)^2 - (\omega_k^0)^2\}}$$

$$+ \frac{N^{n,l}_\| (k^0 = \omega_k^0 n_k^+)}{2\omega_k \{(q^0 + \omega_k)^2 - (\omega_k^0)^2\}} + \frac{N^{n,l}_\| (k^0 = -\omega_k^0 n_k^-)}{2\omega_k \{(q^0 - \omega_k)^2 - (\omega_k^0)^2\}}$$

where

$$N^{n,l}_\| (k) = 4\pi g_{\pi NN}^2 eB \left[ 4eB q^2 \{n_{\delta n-1,l-1} + (2 (q \cdot k) + q^2 (-k^2 - m^2 + q \cdot k)) (\delta_{n,l} + \delta_{n-1,l-1}) \} \right]$$

with $\omega_k^0 = \sqrt{k^2 + m^2 + 2neB}$ and $\omega_p^0 = \sqrt{q^2 + m^2 + 2leB}$. Here $\mathcal{P}$ represents the principle value of the argument. In case of neutron-neutron(nn) loop

$$\text{Re}(\Pi_0^{nn})_\eta = \int \frac{dk_z}{(2\pi)^3} \mathcal{P} \frac{N(k^0 = -q^0 + \omega_k n_k^+)}{2\omega_k \{(q^0 - \omega_k)^2 - (\omega_k^0)^2\}} + \frac{N(k^0 = -q^0 - \omega_k n_k^-)}{2\omega_k \{(q^0 + \omega_k)^2 - (\omega_k^0)^2\}}$$

$$+ \frac{N(k^0 = \omega_k n_k^+)}{2\omega_k \{(q^0 + \omega_k)^2 - (\omega_k^0)^2\}} + \frac{N(k^0 = -\omega_k n_k^-)}{2\omega_k \{(q^0 - \omega_k)^2 - (\omega_k^0)^2\}}$$

with $\omega_k = \sqrt{k^2 + m^2}$ and $\omega_p = \sqrt{(q + k)^2 + m^2}$. It should be mentioned here that in case of pp loop, the expression is obtained with the simplifying assumption that $q \perp = 0$ so that the $q \perp$ integral can be performed exactly using the orthogonality condition of generalized Laguerre polynomials which renders the products of Laguerre polynomials into simple Kronecker Deltas and in turn makes it trivial to convert the double summation structure of the self-energy into a single sum over Landau Levels. Obviously, the same assumption does not provide any such simplification for nn loop. In this case also $q \perp = 0$ is taken for consistency. Thus, the one loop vacuum self-energy of neutral pion becomes

$$\Pi_0^{(\Pi_0)^{pp}}_{\text{vac}} = (\Pi_0)^{pp}_{\text{vac}} + (\Pi_0)^{nn}_{\text{vac}}$$

$$= 2g_{\pi NN}^2 m_\pi^2 q_0^2 \int_0^1 dx \left[ \psi \left( \frac{\Delta}{2eB} \right) + eB + \ln \left( \frac{2eB}{\mu} \right) + \ln \left( \frac{\Delta}{\mu} \right) \right]$$

$$= 2g_{\pi NN}^2 m_\pi^2 q_0^2 \int_0^1 dx \left[ \psi \left( \frac{\Delta}{2eB} \right) + eB + \ln \left( \frac{2eB}{\Delta} \right) \right] + 2g_{\pi NN}^2 m_\pi^2 q_0^2 \int_0^1 \frac{dx}{2\pi^2} \ln \left( \frac{\Delta}{\mu} \right)$$

$$= (\Pi_0)_{\text{vac}}(eB \neq 0) + (\Pi_0)_{\text{vac}}(eB = 0).$$

Unlike the charged pion case, the external transverse momentum of the neutral pions is continuous. To obtain the effective mass of the neutral pions as a function of $T$, $\mu_N$ and $eB$, we solve the DSE given by

$$m_\pi^2 - m_\pi^2 = \text{Re} \Pi_0 (m_\pi^2, q = 0, eB) = 0$$

where $\text{Re} \Pi_0$ contains the sum of explicit $eB$ dependent vacuum and thermal contribution and the renormalized pion mass in vacuum $m_{\pi^0}$ is taken to be same as $m_\pi^\pm$ and will be denoted as $m_\pi$ in subsequent sections.

### III. RESULTS AND DISCUSSIONS

In this section we present the numerical results obtained by solving the DSE given in Eq.(33). We have taken $f_{\pi NN}^2/4\pi = 0.0778$ and $m_\pi = 0.14$ GeV. The nucleon mass is taken as 0.938 GeV. In case of evaluating the effective masses, we have summed up to 300 Landau levels of the loop particles. For stronger magnetic fields i.e $eB > 0.1$ GeV$^2$, the results are found to be much lower than the maximum Landau Level used. However, for $eB \rightarrow 0$, $m_\pi \rightarrow m_\pi^\pm$. 


more than 200 landau levels are needed to produce a convergent numerical result. In case of charged pions we have taken the scale $\mu$ as the square of the nucleon mass. At first we solve the DSE for the charged pions given by

$$m^* - m^2 + Re\Sigma(m^*, q^2) = (2n + 1)eB - (2n + 1)eB = 0$$

where $m^* = m^*_{\pi^+}$ when $Re\Sigma_{\pi^+}$ is used. The variation of $m^*$ with $eB$, $T$ and $\mu_N$ for $\pi^+$ is shown in Fig.2. It can be seen from Fig.2(a) that the plots are completely dominated by the trivial Landau quantization of the external pion. In other words, for each value of Landau Level, the linear $(2n+1)eB$ term present in Eq.(19) affects the effective mass much more compared to the one loop self-energy correction. Moreover, as the temperature and $\mu_N$ dependence of the effective mass comes only through the self-energy, one can expect that in the $eB$ dominated scale, they will be insignificantly small. Accordingly $m^*$ seems to be independent of the variation of $T$ and $\mu_N$ as shown in (b) and (c) part of Fig.2. It should be mentioned here that as the DSE of $\pi^-$ is different from that of $\pi^+$ only in the expression of $Re\Sigma$, the $m^*$ plots for $\pi^-$ will be exactly superimposed on those of $\pi^+$. However, differences between the two charged species can be observed in Fig.3 where the variation of the real part of the self-energy with the invariant mass is shown for two different values of $eB$ with first three Landau Levels. Although both of the self-energies decrease with invariant mass and remain negative throughout, the small oscillatory behaviour present in case of $Re\Sigma_{\pi^+}$ can not be observed for $Re\Sigma_{\pi^-}$. To unveil the contribution of the real part of the self-energy correction,
the effective mass due to the magnetic field dependent vacuum part decreases monotonically with

now we neglect the trivial shift and solve

$$m_{\pi^\pm}^2 - m^2 + \text{Re}\Pi_k(q^2, \vec{q}=0, eB) = 0.$$  \hspace{1cm} (35)

In other words, the effective mass is measured with respect to the trivial Landau shift. This kind of situation may occur in principle when different species of charged particles are present in the system and all of which undergo equivalent trivial Landau shifts. In that case the real part of the self-energies will play the deciding role in the characterization of the effective masses. Moreover, comparing Eq. (33) and Eq. (35), one can observe that Eq. (35) brings down the charged pions in equal footing with the neutral pions only in the difference of the self-energy.

In Fig. 4, the effective mass variation as a function of external magnetic field has been shown at a given temperature 160 MeV and chemical potential of 200 MeV. Considering magnetic field dependent vacuum contribution and the thermal contribution of the self-energy separately, one can compare with the total contribution as shown. When only the thermal contribution is taken into account, it can be noticed that with the increase of external magnetic field, the effective mass of the charged pions develops smooth oscillations. However, no such oscillations have been observed for neutral pion which instead, shows marginal increase in the thermal effective mass with $eB$. On the other hand, the effective mass due to the magnetic field dependent vacuum part decreases monotonically with $eB$ for $\pi_0$ as well as for $\pi_{\pm}$. However, the decreasing nature is more pronounced in case of neutral pion. It is clear from the figure that the field dependent vacuum part of the self-energy can influence the $eB$ dependence of the effective mass significantly. One can notice that, for neutral pions, even the qualitative nature of the $eB$ dependence of the effective mass changes i.e from a slowly increasing nature it becomes a decreasing function of $eB$ due to the incorporation of the vacuum part.

To analyze the markedly different behaviour of the thermal contribution in the effective masses of $\pi^+$ and $\pi^-$, the real part of the self-energy is shown in Fig. 5. Real part of the thermal contribution is plotted as a function of $q^0$ for two different values of chemical potential, $\mu_N=0$ and 200 MeV with $|q_z|=200$ MeV. External parameters $eB$ and $T$ are set at 0.1 GeV$^2$ and 160 MeV respectively.

FIG. 3: Real parts of the thermal contributions are plotted as a function of $q^0$ for two different values of chemical potential, $\mu_N=0$ and 200 MeV with $|q_z|=200$ MeV. External parameters $eB$ and $T$ are set at 0.1 GeV$^2$ and 160 MeV respectively.
small windows within the given range of the magnetic field values, in which the mass hierarchy of different pion species gets altered. Moreover, the possibility of a large window exists due to the fact that, for $eB > 0.1 \text{ GeV}^2$ the decreasing rate of the effective mass for $\pi^+$ is higher in comparison with that of $\pi^-$ i.e at even higher values of magnetic field, the negatively charged pions become more massive than positively charged pions. However, in extremely large magnetic background, the chiral power counting does not hold anymore [46] which imposes serious restrictions on the validity of the model calculation.

Effective mass as a function of chemical potential is presented in Fig.4(c) and 4(d) for two different values of magnetic field, $eB=0.1 \text{ GeV}^2$ and $0.2 \text{ GeV}^2$ keeping the temperature fixed at $T=160 \text{ MeV}$. In a similar fashion, the temperature dependence is plotted in Fig.4(e) and 4(f) with constant $\mu_N=200 \text{ MeV}$ and with $eB=0.1 \text{ GeV}^2$ and $0.2 \text{ GeV}^2$ as before. In case of $\pi^+$ and $\pi^0$, the effective mass slightly increases with $T$ whereas for $\pi^-$, it almost remain independent of temperature variation which is analogous to the $\mu_N$ dependence. However, unlike the temperature dependence, careful observation suggests that for $eB = 0.2 \text{ GeV}^2$, $m_{\pi^-}$ follows a decreasing trend with $\mu_N$. Moreover, in plots with higher $eB$ value, there exists a noticeable initial mass difference between neutral and charged pions. One can also observe that the rate of increase of effective mass as a function of $T$ as well as $\mu_N$ for all the pion species gets reduced for higher magnetic fields.
FIG. 4: $eB$ dependence of the effective mass due to the magnetic field dependent vacuum contribution and the thermal contribution of the self-energy are shown at $T=160$ MeV and $\mu_N = 200$ MeV. Effective mass variations for the charged pions are plotted in the upper panel whereas the lower panel describes the neutral pions. The total self-energy contribution is also shown in each case for comparison.

FIG. 5: Real parts of the thermal contributions are plotted as a function of $q^0$ for two different values of chemical potential, $\mu_N = 0$ and $200$ MeV with $|q_z|=200$ MeV. External parameters $eB$ and $T$ are set at $0.1$ GeV$^2$ and $160$ MeV respectively.
FIG. 6: Effective mass variations with $eB$, $T$ and $\mu_N$ are shown in the three horizontal panels respectively. First row contains two different plots corresponding to two different values of $\mu_N$ (50 and 200 MeV respectively) with $T=160$ MeV. At the same temperature, second row describes the $\mu_N$ dependence for $eB=0.10$ and 0.20 GeV$^2$. Likewise for temperature variation, same set of $eB$ values are chosen for two different plots in the third row with fixed $\mu_N$ (200 MeV).
IV. SUMMARY AND CONCLUSIONS

We have evaluated the one loop pion self-energy in presence of constant homogeneous magnetic field for finite temperature and chemical potential. As far as the strength of the external magnetic field is concerned, we have not made any approximation and used the complete form of the fermionic propagator represented in terms of a sum over infinite landau levels. We have used the real time formalism in the evaluation of the thermal part of pion self-energy.

We have solved LL dependent DSE to obtain the effective masses as a function of different external parameters. It is shown that by taking the trivial Landau shift term, the effective mass increases with $eB$ for the charged pions. Although the real part of the self-energy depends on $T$, $\mu$ and $eB$ it is sub-leading in comparison to the trivial Landau shift. Thus the effective mass of the charged pions remain constant as a function of both $T$ and $\mu$ for a given $eB$. To extract the contribution of the real part of the self-energy, we also solve the DSE by neglecting the trivial shift. It is shown that the effective masses of the charged pions possess oscillatory behaviour. However, the same oscillatory behaviour is not seen in case of the neutral pions. We have also shown that the oscillatory behaviour with finite chemical potential is not similar for $\pi^+$ and $\pi^-$. With increasing chemical potential, the oscillation in the effective mass of positive pion is found to be enhanced while that of $\pi^-$ gets reduced. Along with the thermal contribution, the magnetic field dependent vacuum contribution is also taken into account. Our results suggest that the external magnetic field dependent vacuum part of the self-energy significantly influences not only the quantitative behaviour but also the qualitative behaviour of the effective mass.

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Appendix A: Evaluation of the summation

In Eq.(13) we find that the vacuum contribution for the charged pion self-energy possess the sum of three different infinite series $S_1$, $S_2$ and $S_3$. Here we discuss the procedure to obtain the compact expressions for these summations one by one. These compact expressions will be useful for evaluating the subsequent $k_{\perp}$ integral as will be seen below.

\[
S_1 = \sum_{n=0}^{\infty} (-1)^n L_{n-1}^1 (2\alpha_p) \frac{1}{z^n} \\
= \sum_{n=0}^{\infty} (-1)^n L_{n-1}^1 (2\alpha_p) \int_0^1 \frac{dz}{\mu} z^t z^{2nxeB/\mu} \text{ with } t = \frac{\Delta}{\mu} - (1 - x) \frac{k^2}{\mu} - 1 \\
= \int_0^1 \frac{dz}{\mu} z^t (-z^{2xeB/\mu})(1 + z^{2xeB/\mu})^{-2} \exp \left[ \frac{2\alpha_p(-z^{2xeB/\mu})}{-z^{2xeB/\mu} - 1} \right] \tag{A.1}
\]

where we have used the identity

\[
\sum_{n=0}^{\infty} L_n^a(x)z^n = (1 - z)^{-a-1} \exp \left[ \frac{xz}{z - 1} \right] \text{ for } |z| \leq 1. \tag{A.2}
\]

In a similar way with $\theta = z^{2xeB/\mu}$ we find

\[
S_2 = \int_0^1 \frac{dz}{\mu} z^t \exp \left[ \frac{2\alpha_p \theta}{1 + \theta} \right] \left[ B_0 - \frac{4xeBq^2 \alpha_p \theta}{1 + \theta} \right] \tag{A.3}
\]

\[
\sum_{n=0}^{\infty} \left[ L_n(x) - L_{n-1}(x) \right] z^n = \frac{-xz}{(1 - z)^2} \exp \left[ \frac{-xz}{1 - z} \right] \text{ for } |z| \leq 1 \tag{A.4}
\]
is used. $S_3$ is a divergent series and to extract the momentum dependent finite part we use derivative regularization as follows.

$$S_3 = \sum_{n=0}^{\infty} \left( -1 \right)^n (L_n(2\alpha_p) - L_{n-1}(2\alpha_p)) \ln \frac{\Delta_n}{\mu_0}$$

$$\frac{\partial S_3}{\partial q_{||}^2} = \sum_{n=0}^{\infty} \left( -1 \right)^n (L_n(2\alpha_p) - L_{n-1}(2\alpha_p)) \left( -x \right) \frac{(1-x)}{\Delta_n}$$

$$= \int_0^1 \frac{dz}{\mu} z^t x(1-x) \exp \left( \frac{2\alpha_p \theta}{1+\theta} \right)$$

$$S_3 = \int_0^1 \frac{dz}{\ln z} z^t \exp \left( \frac{2\alpha_p \theta}{1+\theta} \right)$$

where we use

$$\sum_{n=0}^{\infty} \left[ L_n(x) - L_{n-1}(x) \right] z^n = \exp \left( - \frac{xz}{1-z} \right) \text{ for } |z| \leq 1. \quad (A.5)$$

It might seem that the scale is absent here but in fact is hidden in $\theta = z^{2xeB/\mu}$. Moreover, one can observe that the $S_3$ is obtained after an indefinite integral over $q_{||}^2$ which must contain an integration constant independent of $q_{||}^2$. In fact this constant must be infinity as the series we started with is divergent in nature. However this procedure extracts out the finite momentum dependent part that we require and the infinite contribution can be taken care by redefining the scale $\mu$ in such a way that it renormalizes the bare mass to the physical one. The vacuum self-energy now becomes

$$(\Pi_+)^{\text{vac}} = g_{\pi NN}^2 \frac{2}{\pi} \int_0^1 dx \int \frac{d^2 k_{\perp}}{(2\pi)^2} e^{-\alpha_p} \int_0^1 \frac{dz}{\mu} e^{t \ln z} \exp \left( \frac{2\alpha_p \theta}{1+\theta} \right) \left[ - \frac{4A\theta}{(1+\theta)^2} + B_0 - \frac{4xeBq^2\alpha_p \theta}{(1+\theta)^2} - \frac{\mu}{\ln z} q_{\perp}^2 \right]$$

$$\times \left[ - \frac{4A\theta}{(1+\theta)^2} + B_0 - \frac{4xeBq^2\alpha_p \theta}{(1+\theta)^2} - \frac{\mu}{\ln z} q_{\perp}^2 \right]$$

where

$$\eta = \frac{(1-x) \ln z}{\mu} + \frac{1}{eB} \tanh \left( \frac{xeB}{\mu} \ln z \right)$$

$$y = \frac{1}{\eta eB} \tanh \left( \frac{xeB}{\mu} \ln z \right). \quad (A.6)$$

Now, we shift the $k_{\perp} \to k_{\perp} - yq_{\perp}$. Dropping the odd terms we get

$$(\Pi_+)^{\text{vac}} = g_{\pi NN}^2 \frac{2}{\pi} \int_0^1 dx \int \frac{d^2 k_{\perp}}{(2\pi)^2} e^{-\alpha_p} \int_0^1 \frac{dz}{\mu} e^{t \ln z} \exp \left( \frac{2\alpha_p \theta}{1+\theta} \right) \left[ - \frac{4\theta}{(1+\theta)^2} A + B_0 + \frac{4\theta}{(1+\theta)^2} xq_2 \left( k_{\perp}^2 + (1-y)^2 q_{\perp}^2 \right) - \frac{\mu}{\ln z} q_{\perp}^2 \right]$$

with

$$\bar{A} = q^2 (k_{\perp}^2 + y^2 q_{\perp}^2) + 2xq_2 q_{\perp} \left( 1 + 2x \right) q_{\perp} \left( - q_{\perp}^2 \right)$$

and

$$\bar{B}_0 = -2m^2 q^2 + 2 \left( x-y \right) (1-x) q_{\perp}^2 \left( 1 - x \right) q_{\perp}^2 \left( k_{\perp}^2 + y^2 q_{\perp}^2 \right). \quad (A.7)$$

Now, the 2 dimensional $k_{\perp}$ integral can be easily evaluated using the standard Gaussian integral identities given by

$$\int d^2 k_{\perp} e^{-\eta k_{\perp}^2} = \frac{\pi}{\eta}$$

$$\int d^2 k_{\perp} k_{\perp}^2 e^{-\eta k_{\perp}^2} = \frac{\pi}{\eta^2}$$

$$\int d^2 k_{\perp} (q_{\perp} \cdot k_{\perp})^2 e^{-\eta k_{\perp}^2} = -\frac{\pi}{2\eta^2} q_{\perp}^2 \text{ all with } \Re(\eta) < 0. \quad (A.8)$$
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