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The arithmetic basilica: a quadratic PCF arboreal Galois group. (English) [Zbl 07569757]

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Summary: The arboreal Galois group of a polynomial \( f \) over a field \( K \) encodes the action of Galois on the iterated preimages of a root point \( x_0 \in K \), analogous to the action of Galois on the \( \mathbb{Q}/\mathbb{Z} \)-power torsion of an abelian variety. We compute the arboreal Galois group of the postcritically finite polynomial \( f(z) = z^2 - 1 \) when the field \( K \) and root point \( x_0 \) satisfy a simple condition. We call the resulting group the \textit{arithmetic basilica group} because of its relation to the basilica group associated with the complex dynamics of \( f \). For \( K = \mathbb{Q} \), our condition holds for infinitely many choices of \( x_0 \).

MSC:
- 37P05 Arithmetic and non-Archimedean dynamical systems involving polynomial and rational maps
- 11R32 Galois theory
- 14G25 Global ground fields in algebraic geometry

Keywords:
arithmetic dynamics; arboreal Galois representations

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References:
[1] Aitken, Wayne; Hajir, Farshid; Maire, Christian, Finitely ramified iterated extensions, Int. Math. Res. Not., 855-880 (2005)
[2] Anderson, Jacqueline; Bouw, Irene I.; Ejder, Ozlem; Girgin, Neslihan; Karemaker, Valentijn; Manes, Michelle, Dynamical Belyi maps, (Women in Numbers Europe II (2018), Springer: Springer Cham), 57-82
[3] Bartholdi, Laurent; Grigorchuk, Rostislav; Nekrashevych, Volodymyr, From fractal groups to fractal sets, (Fractals in Graz 2001 (2003), Birkhäuser: Birkhäuser Basel), 25-118
[4] Benedetto, Robert L.; Faber, Xander; Hutz, Benjamin; Juul, Jamie; Yasufuku, Yu, A large arboreal Galois representation for a cubic postcritically finite polynomial, Res. Number Theory, 3, Article 29 pp. (2017)
[5] Benedetto, Robert L.; Juul, Jamie, Odoni’s conjecture for number fields, Bull. Lond. Math. Soc., 51, 237-350 (2019)
[6] Boston, Nigel; Jones, Rafe, Arboreal Galois representations, Geom. Dedic., 124, 27-35 (2007)
[7] Bridy, Andrew; Tucker, Thomas J., Finite index theorems for iterated Galois groups of cubic polynomials, Math. Ann., 373, 37-72 (2019)
[8] Bush, Michael R.; Hindes, Wade; Looper, Nicole R., Galois groups of iterates of some unicritical polynomials, Acta Arith., 181, 57-73 (2017)
[9] Ferraguti, Andrea; Micheli, Giacomo, An equivariant isomorphism theorem for mod \( \mathfrak{p} \) reductions of arboreal Galois representations (2019), preprint, Available at
[10] Ferraguti, Andrea; Pagano, Carlo; Casazza, Daniele, The inverse problem for arboreal Galois representations of index two (2019), preprint, Available at
[11] Gottesman, Richard; Tang, Kwokfung, Quadratic recurrences with a positive density of prime divisors, Int. J. Number Theory, 6, 1027-1045 (2010)
[12] Gratton, Chad; Nguyen, Khoa; Tucker, Thomas J., ABC implies primitive prime divisors in arithmetic dynamics, Bull. Lond. Math. Soc., 45, 1194-1208 (2013)
[13] Hindes, Wade, Average Zsigmondy sets, dynamical Galois groups, and the Kodaira-Spencer map, Trans. Am. Math. Soc., 370, 6391-6420 (2018)
[14] Hindes, Wade, Classifying Galois groups of small iterates via rational points, Int. J. Number Theory, 14, 1403-1426 (2018)
[15] Ingram, Patrick, Arboreal Galois representations and uniformization of polynomial dynamics, Bull. Lond. Math. Soc., 45, 301-308 (2013)
[16] Jones, Rafe, Galois representations from pre-image trees: an arboreal survey, (Actes de la Conférence “Théorie des Nombres et Applications” (2013), Pub. Math. Besançon), 107-136
[17] Jones, Rafe; Manes, Michelle, Galois theory of quadratic rational functions, Comment. Math. Helv., 89, 173-213 (2014)
[18] Juul, Jamie, Iterates of generic polynomials and generic rational functions, Trans. Am. Math. Soc., 371, 809-831 (2019)
[19] Juul, Jamie; Krieger, Holly; Looper, Nicole; Manes, Michelle; Thompson, Bianca; Walton, Laura, Arboreal representations for rational maps with few critical points (2018), preprint, Available at

[20] Juul, Jamie; Kurlberg, Pär; Madhu, Kalyani; Tucker, Tom J., Wreath products and proportions of periodic points, Int. Math. Res. Not., 3944-3969 (2016)

[21] Kadets, Borys, Large arboreal Galois representations (2018), preprint, Available at

[22] Looper, Nicole, Dynamical Galois groups of trinomials and Odoni’s conjecture, Bull. Lond. Math. Soc., 51, 278-292 (2019)

[23] Nekrashevych, Volodymyr, Self-Similar Groups (2005), American Mathematical Society: American Mathematical Society Providence

[24] Odoni, R. W.K., The Galois theory of iterates and composites of polynomials, Proc. Lond. Math. Soc. (3), 51, 3, 385-414 (1985)

[25] Pink, Richard, Profinite iterated monodromy groups arising from quadratic polynomials (2013), preprint, Available at

[26] Serre, Jean-Pierre, Propriétés galoisiennes des points d’ordre fini des courbes elliptiques, Invent. Math., 15, 259-331 (1972)

[27] Specter, Joel, Polynomials with surjective arboreal Galois representations exist in every degree (2018), preprint, Available at

[28] Stoll, Michael, Galois groups over Q of some iterated polynomials, Arch. Math. (Basel), 59, 239-244 (1992)

[29] Swaminathan, Ashvin A., On arboreal Galois representations of rational functions, J. Algebra, 448, 104-126 (2016)

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