Breaking Halo Occupation Degeneracies with Marked Statistics

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ABSTRACT

We show that a suitably defined marked correlation function can be used to break degeneracies in halo-occupation distribution modeling. The statistic can be computed on both 3D and 2D data sets, and should be applicable to all upcoming galaxy surveys. A proof of principle, using mock catalogs created from $N$-body simulations, is given.

Key words: cosmology: large-scale structure

1 INTRODUCTION

In recent years our ability to describe galaxy clustering has advanced dramatically. The halo model (Seljak 2000; Peacock & Smith 2000, see e.g. Cooray & Sheth 2002 for a review) has provided us with a physically informative and flexible means of describing galaxy bias - the relation between galaxies and the underlying dark matter halos. The key insight is that an accurate prediction of galaxy clustering requires knowledge of how galaxies are apportioned between and distributed within halos - the halo occupation distribution or HOD. Combined with the theoretically predicted spatial distribution of halos from e.g. $N$-body simulations, a specified HOD makes strong predictions about a wide array of galaxy clustering statistics. The formalism is now widely used in the interpretation of galaxy clustering and to infer cosmological parameters from large-scale galaxy surveys.

Much of the recent work on fitting HODs has used the two-point galaxy correlation function as the observation of choice. While the galaxy correlation function provides very strong constraints on the HOD, there exist degeneracies between the inferred HOD and the underlying cosmology. Much of this degeneracy arises because a change in the cosmology, and hence the halo population, can be compensated to a large extent by a change in the galaxy halo occupation. This modified halo occupation apportions galaxies differently amongst halos of different mass than the fiducial model. Not surprisingly, combining the galaxy correlation function with a second observable with different sensitivities to the HOD can lift such degeneracies (Zheng & Weinberg 2005) tightening the constraints and allowing one to simultaneously constrain the cosmological world model and HOD (see e.g. Abazajian et al. 2003). A number of such observables have been considered in the literature. Galaxy-galaxy lensing has the potential to directly measure the mass of the halos hosting a galaxy population. Cross-correlating with another galaxy sample selected to live in high (or low) mass halos can help to break degeneracies, as can redshift space distortions (which are very sensitive to the satellite fraction) or peculiar velocities. Another observation is the abundance of rich clusters of galaxies, which can constrain the number density of massive halos. While all of these approaches are certainly valid, and will continue to be used in the future, a disadvantage is that they generally require additional observations or measurements, with the associated modeling and additional systematics that must be calibrated. A natural question, therefore, is whether one can break these degeneracies using only the data going into the clustering statistics themselves?

Higher order clustering measurements provide one such approach (e.g. Kulkarni et al. 2007). A particularly convenient choice for our purposes is a marked correlation function (Beisbart & Kerscher 2000; Beisbart, Kerscher, & Mecke 2002; Gottlöber et al. 2002; Sheth & Tormen 2004; Sheth, Connolly, & Skibba 2005; Harker et al. 2006; Wechsler et al. 2006) can lift degeneracies in the HOD or between the HOD and the cosmology. Such marked correlation functions are straightforward to compute with the same set of observations, and require no additional data nor understanding of the survey or algorithms beyond those required for basic clustering statistics. This paper serves as a proof-of-principle, by demonstrating that the degeneracy between HOD and the amplitude of the primordial fluctuation spectrum is broken with a simple density mark in simulations.
2 A WORKED EXAMPLE

2.1 Degeneracies

To illustrate our point we consider a mock galaxy sample, with characteristics similar to an $L^*$ sample, at $z \sim 0.1$, and try to analyze this in two cosmologies that only differ in the normalization of the primordial power spectrum: $\sigma_8 = 0.8$ and 1.0. In both cosmologies a good fit to the 2-point function can be found (Fig. 1), but the HOD differs (Fig. 2) because the halo mass function is different in the two cosmologies.

The fiducial galaxy sample was generated, and the fits were done, by populating N-body simulations with galaxies using an HOD prescription. We use a halo model which distinguishes between central and satellite galaxies with a mean occupancy of halos: $N(M) = \langle N_{\text{cen}}(M_{\text{halo}}) \rangle$. Each halo either hosts a central galaxy or does not, while the number of satellites is Poisson distributed about a mean $N_{\text{sat}}$. We parameterize $N(M) = N_{\text{cen}} + N_{\text{sat}}$ with 5 parameters (e.g. Zheng et al. 2003)

$$N_{\text{cen}}(M) = \frac{1}{2} \frac{\ln(M_{\text{cut}}/M)}{\sqrt{2\sigma}} \erfc \left( \frac{\ln(M_{\text{cut}}/M)}{2\sigma} \right)$$

and

$$N_{\text{sat}}(M) = \left( \frac{M - \kappa M_{\text{cut}}}{M_1} \right)^\alpha$$

for $M > M_{\text{cut}}$ and zero otherwise. Different functional forms have been proposed in the literature, but the current form is flexible enough for our purposes here.

The fiducial galaxy sample is generated from the $\sigma_8 = 0.8$ simulation. It has a number density of $1.5 \times 10^{-3} h^3 \text{Mpc}^{-3}$ and a correlation length of about $7 h^{-1}\text{Mpc}$. All errors are computed by Monte-Carlo methods, dividing the simulation into disjoint regions. For definiteness we consider a survey of volume $(250 h^{-1}\text{Mpc})^3 \simeq 1.6 \times 10^7 h^{-3}\text{Mpc}^3$, similar to the corresponding Sloan Digital Sky Survey sample, and scale the covariance matrices to that volume. This yields diagonal errors on $\xi(r)$ of around 5 – 10% and bin-to-bin correlation of 15-80%. When fitting HOD models to these data the best fits are “good” fits, and the parameter values are well within the range of HOD parameters seen for similar galaxy samples, and so both cosmologies are acceptable a priori.

It is clear (Fig. 1) that the two-point correlation function by itself cannot distinguish between the two models $\Delta \chi^2 < 1$ for 8 data points. The next sections demonstrate that a simple density mark, measurable from the spatial distribution of galaxies strongly discriminates between these models.

2.2 Marked correlation functions

The marked correlation function generalizes the standard correlation function by weighting galaxies by a numerical “mark”. If the mark of the $i^{th}$ object is $m_i$, then the marked correlation function is defined as (e.g. Sheth, Connolly, & Skibba 2003, Eq. 3)

$$M(r) = \frac{1}{n(r)\bar{m}^2} \sum_{i,j} m_im_j,$$

where the sum is over all pairs of objects $(i,j)$ with separation $r_{ij} = r$, $n(r)$ is the number of pairs, and the mean mark, $\bar{m}$, is calculated over all objects in the sample. Note that, unlike $\xi$, $w_p$ or $w$, no random catalog is needed in the computation of $M(r)$. It is convenient to divide out the clustering of the average sample since $M(r) \neq 1$ then implies a difference in clustering by objects with different marks. The above expression can be applied in 2D or 3D, with angular or linear bins.

1 We use the full covariance matrix when quoting significance levels.
The choice of mark depends on the application. In the example above, we would like a mark that encodes information about which halos host which galaxies. One set of such marks would be functions of the local density, which can be computed in a number of ways, e.g. the distance to the nth nearest neighbor, the number of neighbors within a fixed metric aperture, spline kernel interpolation (e.g. Dehnen 2001) or kernel deprojection (e.g. Eisenstein 2003). Massive halos tend to host more galaxies with higher density than lower mass halos, so if the HOD is changed we expect the density-marked correlation function to differ. The relation of local density to the number density of groups or clusters (which are known to break degeneracies in model fitting, e.g. Zheng & Weinberg 2007) can be complex, but is easily calculable from a mock catalog.

What function of $\rho$ should we choose as our mark? The choice is arbitrary, but $\rho^n / (\rho_0^n + \rho^n)$ has the nice property that it tends to zero for $\rho \ll \rho_0$ and unity for $\rho \gg \rho_0$, the rapidity of the transition being controlled by $n$. This means the dynamic range in the mark is limited, which leads to more stable results. If we are concerned that our density estimator may be noisy, which is often true in practice, we should choose a low value of $n$. Hereafter we choose $n = 1$.

2.3 Different HODs, different marks

We now measure the local-density marked correlation function for our two examples HODs. To begin we imagine that we can use spectroscopy or multi-band photometry to select a sample of galaxies in a slice $\pm 50 h^{-1} \text{Mpc}$. At our fiducial $z \simeq 0.1$, or $\chi_z \simeq 300 h^{-1} \text{Mpc}$, this corresponds to $\Delta z / (1 + z) \sim 15\%$. In this 2D slice we estimate the density using spline kernel interpolation with 4 nearest (in projection) neighbors. Not surprisingly, we find that this density is much higher for objects which live in massive halos than for those which live in smaller halos. As the width of the slice is increased the contrast in density between high and low mass halos is reduced, but the trend remains the same. Note that our goal is not to optimize the density estimator, but to demonstrate that a useful estimator may be computed even for samples with limited redshift information. Of course, the exact choice of estimator will depend on the data set being considered.

To pick a reasonable value of $\rho_0$, we note that halos of $10^{12} h^{-1} M_\odot$ host $\mathcal{O}(10)$ galaxies in our models and cover $\sim (5 h^{-1} \text{Mpc})^2$ in projection. Projected over $\pm 50 h^{-1} \text{Mpc}$ the background density is $\sim 0.1 (h^{-1} \text{Mpc})^{-2}$, so massive halos are $\sim 20$ more dense than the mean ($\bar{\rho}$). We pick $\rho_0 = 25 \bar{\rho}$ as a convenient round number, though our conclusions are not sensitive to the exact choice.

Figure 3 shows that this marked correlation function on sub-Mpc scales is different for our two samples reflecting the differences in the HOD (Fig. 2). How discriminatory is this measure? For our fiducial volume, $\Delta \chi^2 \simeq 33$ for the two marked correlation functions, compared with $\Delta \chi^2 < 1$ for the unweighted correlations. Since almost all of the difference comes from the lowest 4 data points, the two models can be strongly discriminated (> 99% assuming Gaussian errors). The distribution of the marks is almost the same in the two samples, and the difference in $M(r)$ remains even if we rescale the marks in one model to match the distribution in the other, showing that the difference is robust. We also note that the relevant measure of error for $M(r)$ comes from the Monte-Carlo estimation of the covariance matrix. Simply scrambling the marks allows us to test for a density dependence of the correlation function (Sheth, Connolly, & Skibba 2003), which is detected in all of our catalogs at extremely high significance, but does not tell us how to compare different $M(r)$ to each other.

Our initial choice of slice width, $\pm 50 h^{-1} \text{Mpc}$, was possibly optimistic for surveys at higher redshift. As we increase the width of the slice the density contrast decreases and the significance by which we can differentiate the models is also decreased. For a slice $\pm 125 h^{-1} \text{Mpc}$ in width, i.e. the full depth of our fiducial ($250 h^{-1} \text{Mpc}$) survey, using the same mark as above, the two models in Figure 3 differ by $\Delta \chi^2 = 19$. It is easily conceivable that a different choice of $\rho_0$, or a higher power of $\rho$ in the mark could increase the discriminatory power of $M(r)$, but this is already reasonably significant given that only the 4 points with $r < 1 h^{-1} \text{Mpc}$ contribute. If it proves impossible to select slices as thin as $\pm 125 h^{-1} \text{Mpc}$ for some particular sample, it is always possible to jointly analyze samples using cross-correlation marks where one of the samples can be well isolated in distance (e.g. red galaxies with strong breaks). Such statistics would need to be analyzed on a case-by-case basis.

3 CONCLUSIONS

Although the galaxy two-point correlation function has proved to be extremely useful in modeling the relationship between galaxies and dark matter, it does not exhaust the information in the data. One degeneracy that cannot be broken by the correlation function alone is between the HOD and cosmology - within the context of the correlation function, one is free to re-arrange galaxies to compensate for differences in the halo mass function. The number of such
degeneracies will only increase as we attempt more detailed mappings of galaxies to dark matter in the future. This note provides proof of principle that such degeneracies can be lifted by marked correlation functions.

An important advantages of marked correlations is that they do not involve multiple data sets. Indeed, for the example presented here, the mark required no additional information beyond the spatial distribution of galaxies (and survey mask), which one needed to compute the correlation function in the first place. Using the same data set considerably simplifies any modeling step. Furthermore the marked correlation function can be computed with the same code and at the same time as the standard correlation function, for which optimized algorithms exist. This is a non-negligible advantage when one considers the need to repeatedly compute it for mock samples while modeling, estimating errors, etc.

Rapid advances in computational power and algorithm development have made it reasonably straightforward to simulate the distribution of dark matter halos in large volumes for almost any cosmological model. Combined with a halo occupation approach this makes “forward modeling” of almost any galaxy statistic possible. This implies that statistics which use all of the galaxy information, in the manner of standard large-scale structure $n$-point statistics, can be just as useful as those which try to identify special subsets of galaxies (‘cluster’ vs. ‘field’). As our ability to model a wider array of observations (e.g. weak lensing, Sunyaev-Zel’dovich decrement or X-ray flux) matures, similar methods can be applied to these observations, bypassing the need to relate particular features in a map with individual 3D structures.

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