Strong BBP couplings for the charmed baryons

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Abstract

According to the Coleman-Glashow null theorem if all the symmetry breaking effects belong to the same regular representation (octet in the case of SU(3) and 15 in the case of SU(4)) and are generated in a tadpole type mechanism, the strangeness changing (charm changing) weak transitions generated through the $S_6$ ($S_9$) tadpole must vanish. Exploiting this null result, we find relations between the BBP coupling constants which allow us to write the coupling constants in terms of two parameters and baryon masses. Fixing these two parameters ($g_{NN\pi}$ and $g_{ΛΣ\pi}$) from experiments, we estimate the remaining coupling constants.

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1 Introduction

From the strong coupling constants extracted from data, it seems very well established that SU(3) invariance does not work for the BBP strong couplings, $g_{BjB_kP_i}$, the Yukawa coupling of the pseudoscalar mesons with baryons. Some form of symmetry breaking must be invoked in order to account for the observed couplings.

In a simple method [1,2] the SU(3) symmetry breaking at the BBP vertex could be accounted for by exploiting the Coleman-Glashow null theorem [3] for the tadpole type symmetry breaking. For the symmetry breaking effects, the medium strong (responsible for the mass differences in the internal symmetry multiplets), the electromagnetic and the weak effects, transforming as the members of the same octet, the strangeness changing scalar tadpole $S_6$ can be rotated away by a unitary transformation. Therefore, the strangeness changing transitions generated through $S_6$ must vanish. This is the Coleman-Glashow theorem. The current algebra and the Partially Conserved Axial Vector Current (PCAC) hypothesis have been used to write expression for the amplitudes of non-leptonic decays of hyperons in terms of the strong BBP coupling constants and baryon masses by Marshak et al. [4] and others [5] in the quark density model. The Coleman-Glashow theorem requires that these expressions for the parity conserving modes must all be equal to zero. The decays can, however, arise through current $\otimes$ current form of the weak interactions, which can be fairly satisfactorily explained on the basis of the standard model. Marshak et al. [4] have explicitly shown that these amplitude expressions due to tadpole indeed go to zero if all couplings and masses are SU(3) invariant. The null result has also been shown [6] to hold when the SU(3) symmetry for the couplings and for the masses is assumed broken via the tadpole mechanism. The requirement that each of the weak non-leptonic decay amplitude generated through the $S_6$ tadpole must be zero, then gives relations among the symmetry broken coupling constants and baryon masses. For the eight BBP coupling constants
involving pions and kaons, six relations are obtained [1,2]. Thus the eight coupling constants are determined in terms of only two parameters, no more than the SU(3) symmetric case. The SU(3) symmetry broken BBP couplings so obtained are in good agreement with the available experimental numbers [7-10].

The null result proved initially for the SU(3) case is valid for the SU(4) case also if the symmetry breaking effects belong to the regular representation 15. The current \( \otimes \) current weak Hamiltonian responsible for the hadronic weak decays of charmed baryons belongs to the 20'' and 84 representations of the SU(4). The tadpole part of the weak Hamiltonian will belong to the 15 representation. And it is this piece of the Hamiltonian which will give zero amplitude for the decays. The considerations can thus be generalised to the charm sector and relations for the strong couplings obtained. This was done [2] for some of the charm baryon coupling constants.

In this work, we do a detailed analysis and calculate all the strong coupling constants of charm baryons involving \( \pi \), K, D and \( D_s \) mesons by considering \( \pi \) and K emitting parity conserving decays whether kinematically allowed or not. Now that the hadronic properties of heavier baryons have become a real possibility, the strong coupling constants involving charmed baryons are of immediate interest for studying them, e.g., for studying weak hadronic decays of charm baryons [11,12]. We develop our formalism in section (2) and list our results in section (3) giving relations among various strong couplings. Numerical estimates are given in section (4).

2 Non-leptonic Baryon Decays

The matrix element for the baryon decay process

\[ B_j \rightarrow B_k + P_i \] (1)

can be expressed as

\[ M = -\langle B_k P_i | H_W | B_j \rangle = \bar{u}_{B_k}[iA + B \gamma_5]u_{B_j} \] (2)
where \( j, k \) represent internal symmetry (SU(3), SU(4)) indices for the initial and final baryons and \( i \) is the index for the emitted pseudoscalar meson. \( A \) and \( B \) are the pv (parity-violating) and the pc (parity-conserving) decay amplitudes respectively. This three hadron matrix element can be reduced [4,5] to the baryon-baryon transition matrix element of \( H_W \) by the application of current algebra along with the PCAC hypothesis as

\[
M = \frac{i}{f_P} \langle B_k | [Q^5_i, H_W] | B_j \rangle + M_P
\]  

where \( Q^5_i \) is the axial generator associated with the meson \( P_i \) and \( f_P \) is the corresponding pseudoscalar meson decay constant. The contribution from the pole diagram is contained in \( M_P \).

### 2.1 Quark density model

The tadpole piece of the weak Hamiltonian in SU(3), for example, is written as

\[
H^\Delta s=1_W = G_{pv} P_7 + G_{pc} S_6
\]  

where \( S_6 \) and \( P_7 \) are the scalar and pseudoscalar quark densities transforming as \( \lambda_6 \) and \( \lambda_7 \) components of an SU(3) octet respectively.

The symmetry breaking in mass is described by

\[
M = M_0 + \delta M_s S_8 + \delta M_{em} S_3
\]  

The assumption that \( S_8 \) the medium strong, \( S_3 \) the electromagnetic and \( S_6 \) the weak symmetry breaking tadpoles belong to the same octet is called the universality hypothesis for the spurions. However, one can always find an SU(3) transformation such that the strangeness changing scalar tadpole \( S_6 \) can be rotated away leaving behind the medium strong and electromagnetic pieces \( S_8 \) and \( S_3 \) respectively as the diagonal eighth and third components of the SU(3) octet. The strangeness changing effects generated through \( S_6 \) must, therefore, vanish. Within the framework of current algebra this can be seen easily [4,5].
For the pc-weak Hamiltonian (4), the general expression (3) gives for the p-wave amplitudes

\[
\langle B'_k P_i | H_{PW}^C | B_j \rangle = -\frac{\sqrt{2}}{f_P} d_{ism} \langle B_k | P_m | B_j \rangle + \frac{a_{jm} g_{mki}}{M_j - M_m} \frac{M_k + M_j}{M_m} + \frac{g_{jmi} a_{nk}}{M_k - M_n} \frac{M_j + M_k}{M_n + M_j}
\]

(6)

where \( m, n \) are indices for the intermediate baryons in the s-channel and u-channel respectively. \( a_{jm} \) is the PC weak transition amplitude of \( B_j \to B_m \). \( M_j \) is the mass of \( B_j \), \( s \) is unitary index for the weak spurion with fixed value 6 for the strangeness changing weak baryon decays. Using the universality of the spurions and the SU(3) symmetry for the strong couplings and the baryon masses we see [3] that the decay amplitudes in equation (6) are all zero if we take \( \delta M_s = -\frac{2}{\sqrt{3}} f_\pi \).

It is straightforward to generalize this formalism to the charm sector. Quark-density weak Hamiltonian for charm changing (\( \Delta C = -1, \Delta S = 0 \)) mode is given by

\[
H_{W}^{\Delta C = -1, \Delta S = 0} = G_{pu} P_{10} + G_{pc} S_{9}
\]

(7)

and the mass breaking is given by

\[
M = M_0 + \delta M_0 S_{15} + \delta M_s S_8 + \delta M_{cm} S_3
\]

(8)

Here, mass breaking terms \( S_{15}, S_8 \) and \( S_3 \) and the weak symmetry breaking tadpole \( S_9 \) are assumed to belong to the same 15 representation of the SU(4). Now the index \( s \) in eq.(6) would become 9 for the charm changing and strangeness conserving decays.

3 Relations among strong couplings

By considering various p-wave decay modes in the quark density model and setting them equal to zero, we obtain relations containing strong coupling constants, meson decay constants and baryon masses. We list the results according to the decay modes involved.
3.0.1 \( \pi \) Couplings

There are ten SU(2) invariant couplings involving pions. The four couplings \( NN\pi, \Lambda\Sigma\pi, \Sigma\Sigma\pi, \Xi\Xi\pi \) exist for the SU(3) octet \((C=0)\) baryons \((N, \Lambda, \Sigma, \Xi)\) whereas the remaining \( \Sigma_c\Lambda_c\pi, \Sigma_c\Sigma_c\pi, \Xi_c\Xi_c\pi, \Xi_c\Xi_c'\pi, \Xi_c\Xi_c'\pi, \Xi_c\Xi_c'\pi \) for the \(3^*-(\Lambda_c, \Xi_c')\) and \(6-(\Sigma_c, \Xi_c, \Omega_c)\) of charm baryons \((C=1)\) and \(\Xi_{cc}\Xi_{cc}\pi\) for \(3-(\Xi_{cc}, \Omega_{cc})\) of \((C=2)\) charm baryons.

3.0.2 K couplings

The K couplings are \( \Lambda NK, \Sigma NK, \Xi\Lambda K, \Xi\Sigma K \) for the octet baryons corresponding to \(C=0\), \( \Xi_c'\Lambda_c K, \Xi_c'\Sigma_c K, \Xi_c\Lambda_c K, \Xi_c\Sigma_c K, \Omega_c\Xi_c K \) and \(\Omega_c\Xi_c'K\) from representations \(3^*\) and \(6\) corresponding to \(C=1\) and \(\Omega_{cc}\Xi_{cc}K\) from \(3\) corresponding to \(C=2\) baryons. The total number of coupling constants here is eleven.

3.1 Octet baryon strong couplings

From all the baryon decays in \(\Delta C = 0, \Delta S = 1\) mode, whether kinematically allowed or not, we get six independent relations for the coupling constants. These relations allow us to write all the eight pion and kaon couplings for the octet baryons in terms of two parameters for which we choose \(g_{NN\pi}\) and \(g_{\Lambda\Sigma\pi}\).

We consider the six decays \(\Sigma^+ \to n\pi^+, \Xi^0 \to \Sigma^-\pi^+, \Lambda \to p\pi^-, \Sigma^- \to n\pi^-, \Xi^0 \to pK^-\) and \(\Xi^- \to nK^-\) and compute their amplitudes by current algebra and soft pion technique [4] taking the quark density model for the weak Hamiltonian. Setting each of these amplitudes to zero, we get:

3.1.1 \(\Delta C = 0, \Delta S = 1\) : \(\pi\)-meson emitting decays of the hyperons

The decays \(\Sigma^+ \to n\pi^+\) and \(\Xi^0 \to \Sigma^-\pi^+\) yield two relations:

\[
G_{\Sigma\Sigma\pi} = -2G_{NN\pi} + \sqrt{3}G_{\Lambda\Sigma\pi} \tag{9}
\]

\[
G_{\Xi\Xi\pi} = G_{NN\pi} - \sqrt{3}G_{\Lambda\Sigma\pi} \tag{10}
\]
Here, we have used the following definition:

$$G_{B_jB_k\pi} = g_{B_jB_k\pi} \frac{2M_N}{M_{B_j} + M_{B_k}}$$

The decays $\Lambda \to p\pi^-$ and $\Sigma^- \to n\pi^-$ give

$$G_{\Lambda NK} = \frac{\sqrt{2}f_\pi}{\delta M_s}(\sqrt{2}G_{NN\pi} - \frac{2}{\sqrt{6}}G_{\Lambda\Sigma\pi})$$ (11)

$$G_{\Sigma NK} = \frac{\sqrt{2}f_\pi}{\delta M_s}(\frac{2}{\sqrt{6}}G_{NN\pi} - \sqrt{2}G_{\Lambda\Sigma\pi})$$ (12)

3.1.2 $\Delta C = 0$, $\Delta S = 1$: K-meson emitting decays of the hyperons

The decays $\Xi^0 \to pK^-$ and $\Xi^- \to nK^-$ give rise to the following relations:

$$G_{\Xi\Sigma K} = \frac{1}{2}(-\sqrt{3}G_{\Lambda NK} + G_{\Sigma NK})$$ (13)

$$G_{\Xi\Lambda K} = -\frac{1}{2}(G_{\Lambda NK} + \sqrt{3}G_{\Sigma NK})$$ (14)

Substitution of Eqs. (11) and (12) into the equations (13) and (14) allows us to express $g_{\Xi\Sigma K}$ and $g_{\Xi\Lambda K}$ in terms of $g_{NN\pi}$ and $g_{\Lambda\Sigma\pi}$,

$$G_{\Xi\Lambda K} = \frac{\sqrt{2}f_\pi}{\delta M_s}(-\sqrt{2}G_{NN\pi} + \frac{4}{\sqrt{6}}G_{\Lambda\Sigma\pi})$$ (15)

$$G_{\Xi\Sigma K} = \frac{\sqrt{2}f_\pi}{\delta M_s}(-\frac{2}{\sqrt{6}}G_{NN\pi})$$ (16)

Equations (9), (10), (11) and (12) already express the coupling constants $g_{\Sigma\Sigma\pi}$, $g_{\Xi\Xi\pi}$, $g_{\Lambda NK}$ and $g_{\Sigma NK}$ in terms of $g_{NN\pi}$ and $g_{\Lambda\Sigma\pi}$.

3.2 K-meson couplings of charm baryons

We now consider the K-meson emitting weak decays of charmed baryons in the similar manner. $\Delta C = -1$, $\Delta S = 0$ decays $\Xi_{c^0} \to pK^-$, $\Xi_{c}^0 \to pK^-$, $\Lambda_c \to \Sigma^{0}K^+$, $\Sigma^{+}_c \to \Lambda K^+$ give rise to the following relations:

$$G_{\Xi_c\Lambda_c K} = \frac{1}{2\sqrt{6}}G_{\Lambda NK} + \frac{3}{2\sqrt{2}}G_{\Sigma NK}$$ (17)
\[ G_{\Xi_c\Sigma_c K} = \frac{1}{2\sqrt{2}} G_{\Lambda NK} - \frac{3}{2\sqrt{6}} G_{\Sigma NK} \]  
(18)

\[ G_{\Xi_c\Lambda_c K} = \frac{1}{2\sqrt{2}} G_{\Lambda NK} - \frac{3}{2\sqrt{6}} G_{\Sigma NK} = G_{\Xi'_c\Sigma_c K} \]  
(19)

\[ G_{\Xi_c\Sigma_c K} = \frac{3}{2\sqrt{6}} G_{\Lambda NK} + \frac{1}{2\sqrt{2}} G_{\Sigma NK} \]  
(20)

Similarly the consideration of the \( \Xi_c/\Xi'_c \rightarrow \Xi + K \) modes gives

\[ G_{\Omega_c\Xi_c K} = \frac{1}{2}(G_{\Lambda \Xi K} - \sqrt{3} G_{\Sigma \Xi K}) \]  
(21)

\[ G_{\Omega_c\Xi'_c K} = \frac{1}{2}(\sqrt{3} G_{\Lambda \Xi K} + G_{\Sigma \Xi K}) \]  
(22)

Further the decay \( \Omega_{cc} \rightarrow \Sigma_c^{++}K^- \) yields

\[ G_{\Omega_{cc}\Xi_{cc} K} = (\sqrt{3} G_{\Xi_{cc}\Sigma_c K} - G_{\Xi_{cc}\Sigma_c K}) \]  
(23)

Note that all the charm baryon K-couplings are related to the octet baryon K-couplings. Finally, these are expressed in terms of \( g_{\Lambda \Sigma \pi} \) and \( g_{NN\pi} \) as follows:

\[ G_{\Xi_c\Lambda_c \pi} = \frac{\sqrt{2}f_\pi}{\delta M_s} \left( \frac{2}{\sqrt{3}} G_{NN\pi} - \frac{5}{3} G_{\Lambda \Sigma \pi} \right) \]  
(24)

\[ -\frac{1}{\sqrt{2}} G_{\Omega_c\Xi_c K} = G_{\Xi_c\Lambda_c K} = G_{\Xi'_c\Sigma_c K} = \frac{\sqrt{2}f_\pi}{\delta M_s} \left( \frac{1}{\sqrt{3}} G_{\Lambda \Sigma \pi} \right) \]  
(25)

\[ -\frac{1}{\sqrt{2}} G_{\Omega_c\Xi_c K} = G_{\Xi_c\Sigma_c K} = \frac{\sqrt{2}f_\pi}{\delta M_s} \left( \frac{2}{\sqrt{3}} G_{NN\pi} - G_{\Lambda \Sigma \pi} \right) \]  
(26)

\[ G_{\Omega_{cc}\Xi_{cc} K} = \frac{\sqrt{2}f_\pi}{\delta M_s} \left( -\frac{2}{\sqrt{3}} G_{NN\pi} + 2G_{\Lambda \Sigma \pi} \right) = -\sqrt{2} G_{\Sigma NK} \]  
(27)

### 3.3 \( \pi \) meson couplings of charmed baryons

#### 3.3.1 \( \pi \)-meson emitting \( \Delta C = -1, \Delta S = 0 \) decays

These decays lead to the following relations:

\[ \sqrt{3} G_{\Sigma_c\Lambda_c \pi} - G_{\Sigma_c\Sigma_c \pi} = 2G_{NN\pi} \]  
(28)
from the $\Sigma^0_c \rightarrow p \pi^-$,
\[-2\sqrt{3}G_{\Xi_c{\Xi'}_c \pi} + 2G_{\Xi_c{\Xi'}_c \pi} = G_{\Lambda \Sigma \pi} + \sqrt{3}G_{\Sigma \Sigma \pi}\] (29)

from the $\Xi^0_c \rightarrow \Sigma^+ \pi^-$,
\[-2\sqrt{3}G_{\Xi_c{\Xi'}_c \pi} + 2G_{\Xi_c{\Xi'}_c \pi} = \sqrt{3}G_{\Lambda \Sigma \pi} - G_{\Sigma \Sigma \pi}\] (30)

from the $\Xi^0_c \rightarrow \Sigma^+ \pi^-$,
\[\sqrt{2}G_{\Xi_c{\Xi'}_c \pi} = \frac{1}{\sqrt{2}}G_{\Sigma_c{\Sigma'}_c \pi} - \frac{3}{\sqrt{6}}G_{\Lambda_c \Sigma_c \pi}\] (31)

from the $\Xi^{++}_{cc} \rightarrow \Sigma^+_c \pi^+$. 

3.3.2 $\pi$-meson emitting $\Delta C = 0$, $\Delta S = 1$ decays of charm baryons

However, the $\Delta C = 0$, $\Delta S = 1$ decay $\Omega^{+}_{cc} \rightarrow \Xi^{++}_{cc} \pi^-$ leads to
\[G_{\Omega^{+}_{cc} \Xi^{++}_{cc} K} = -2\sqrt{2}\frac{f_{\pi}}{\delta M_s}G_{\Xi^{++}_{cc} \pi}\] (32)

From the K-couplings $G_{\Omega^{+}_{cc} \Xi^{++}_{cc} K}$ turns out to be equal to $-\sqrt{2}G_{\Sigma N K}$. Substituting the expression of $G_{\Sigma N K}$ in terms of $G_{\Sigma N \pi}$ and $G_{\Lambda \Sigma \pi}$, we obtain
\[G_{\Xi^{++}_{cc} \pi} = G_{\Sigma N \pi} - \sqrt{3}G_{\Lambda \Sigma \pi} = G_{\Xi \Xi \pi}\] (37)

This relation combined with Eqns. (28) and (31) leads to
\[G_{\Lambda_c \Sigma_c \pi} = G_{\Lambda \Sigma \pi}\] (34)
\[G_{\Sigma_c \Sigma_c \pi} = \sqrt{3}G_{\Lambda \Sigma \pi} - 2G_{\Sigma N \pi} = G_{\Sigma \Sigma \pi}\] (35)

We have thus been able to write the three charmed baryon couplings $\Sigma_c \Lambda_c \pi$, $\Sigma_c \Sigma_c \pi$, and $\Xi_{cc} \Xi_{cc} \pi$ in terms of the two basic parameters $g_{NN \pi}$ and $g_{\Lambda \Sigma \pi}$. To express the remaining $\pi$-mesons couplings in terms of these parameters, we consider now the $\Delta C = 0$, $\Delta S = 1$ decays in $\Xi^0_c \rightarrow \Lambda_c + \pi^-$ modes. This decay involves the mass breakings both due to $\delta M_c$ and $\delta M_s$ unlike the decays considered so far which involve either of the two mass breakings. Ignoring the term involving $\delta M_s / \delta M_c$, we get
\[G_{\Xi_c{\Xi'}_c \pi} = -\frac{G_{\Lambda \Sigma \pi}}{2}\] (36)
Using this result, the relations (29) and (30) yield the following sumrules:

\[ G_{\Xi_c \Xi_c \pi} = -\frac{\sqrt{3}}{2}G_{\Lambda \Sigma \pi} + G_{NN\pi} \] (37)

\[ G_{\Xi'_c \Xi'_c \pi} = -\frac{5}{2\sqrt{3}}G_{\Lambda \Sigma \pi} + G_{NN\pi} \] (38)

### 3.4 D-meson couplings of charm baryons

We have eleven \( g_{BB'D} \) SU(2) invariant coupling constants. The seven

\[ g_{\Lambda_c ND}, \ g_{\Sigma_c ND}, \ g_{\Xi'_c \Lambda D}, \ g_{\Xi'_c \Sigma D}, \ g_{\Xi_c \Lambda D}, \ g_{\Xi_c \Sigma D}, \ g_{\Omega_c \Xi D} \]

correspond to \( C = 1 \rightarrow C = 0 + D \) mode and the four

\[ g_{\Xi_c \Sigma c D}, \ g_{\Xi_c \Lambda_c D}, \ g_{\Omega_{cc} \Xi_c D}, \ g_{\Omega_{cc} \Xi'_c D} \]

correspond to \( C = 2 \rightarrow C + 1 + D \) mode.

Considering the \( \pi \)-meson emitting \( \Delta C = -1, \ \Delta S = 0 \) decays of the charm baryons, we obtain the following relations:

\[ G_{\Xi_c \Sigma_D} = \frac{\sqrt{2}f_\pi}{4\delta M_c}(-\frac{3}{\sqrt{6}}G_{\Lambda \Sigma \pi} + \frac{3}{\sqrt{2}}G_{\Sigma \Sigma \pi}) \] (39)

from the decay \( \Xi^0_c \rightarrow \Sigma^- \pi^+ \),

\[ G_{\Xi_c \Sigma_D} = -\frac{\sqrt{2}f_\pi}{4\delta M_c}(-\frac{3}{\sqrt{2}}G_{\Lambda \Sigma \pi} + \frac{3}{\sqrt{6}}G_{\Sigma \Sigma \pi}) \] (40)

from the decay \( \Xi^0_c \rightarrow \Sigma^- \pi^+ \),

\[ G_{\Lambda_c ND} = \frac{\sqrt{2}f_\pi}{4\delta M_c}(-6G_{NN\pi} + 2\sqrt{3}G_{\Lambda_c \Sigma \pi}) \] (41)

from the decay \( \Lambda_c \rightarrow n \pi^+ \),

\[ G_{\Sigma_c ND} = \frac{\sqrt{2}f_\pi}{4\delta M_c}2\sqrt{3}(G_{\Sigma_c \Sigma \pi} + G_{NN\pi}) \] (42)

from the decay \( \Sigma^+_c \rightarrow n \pi^+ \),
\[ G_{\Omega, \Xi D} = \frac{\sqrt{2}f_\pi}{4\delta M_c} 2\sqrt{6}G_{\Xi \pi} \] (43)

from the decay \( \Omega^0 \rightarrow \Xi^0\pi^+ \),

\[ G_{\Xi c\Lambda D} = -\frac{\sqrt{2}f_\pi}{4\delta M_c} 3\sqrt{2}(G\Lambda\Sigma\pi + G\Xi_c\Xi_c\pi + \frac{1}{\sqrt{3}}G\Xi_c\Xi_c\pi) \] (44)

from the decay \( \Xi^+_c \rightarrow \Lambda\pi^+ \),

\[ G_{\Xi cc\Sigma D} = \frac{\sqrt{2}f_\pi}{4\delta M_c} 2\sqrt{6}(-G\Lambda\Sigma\pi + G\Xi_c\Xi_c\pi + \sqrt{3}G\Xi_c\Xi_c\pi) \] (45)

from the decay \( \Xi^{++}_c \rightarrow \Lambda^+\pi^+ \),

\[ G_{\Xi_{cc}\Lambda c D} = -\frac{\sqrt{2}f_\pi}{4\delta M_c}(2\sqrt{3}G\Lambda\Sigma\pi + \sqrt{3}G\Xi_c\Xi_c\pi) \] (46)

from the decay \( \Xi^{++}_c \rightarrow \Sigma^+_c\pi^+ \),

\[ G_{\Omega_{cc}\Xi c D} = \frac{\sqrt{2}f_\pi}{4\delta M_c}(6G\Xi_c\Xi_c\pi - 2\sqrt{3}G\Xi_c\Xi_c\pi) \] (48)

from the decay \( \Omega^+_c \rightarrow \Xi^{0}_c\pi^+ \),

\[ G_{\Omega_{cc}\Xi c' D} = \frac{\sqrt{2}f_\pi}{4\delta M_c}(6G\Xi_c\Xi_c\pi - 2\sqrt{3}G\Xi_c\Xi_c\pi) \] (49)

from the decay \( \Omega^+_c \rightarrow \Xi^{0}_c\pi^+ \).

Thus, all the D-meson strong couplings are related to the \( \pi^-\) meson couplings which in turn have been expressed in terms of the two basic parameters \( g_{\Lambda\Sigma\pi} \) and \( g_{NN\pi} \).

3.5 \( D_s \)-meson couplings of charm baryons

There are seven \( g_{BB'D_s} \) couplings;

\[ g_{\Lambda c\Lambda D_s}, \ g_{\Sigma_c\Sigma D_s}, \ g_{\Xi_c\Xi D_s}, \ g_{\Xi' c\Xi D_s} \]
corresponding to $C = 1 \rightarrow C = 0 + D_s$ modes, and
\[ g\Xi_c\Xi_c D_s, \quad g\Xi'_c\Xi'_c D_s, \quad g\Omega_{cc}\Omega_c D_s \]
corresponding to $C = 2 \rightarrow C = 1 + D_s$ modes. Applying our formalism to $K^+$ meson emitting $\Delta C = -1$, $\Delta S = 0$ decays, we obtain the following relations:

\[ G_{\Lambda_c\Lambda D_s} = \frac{\sqrt{2} f_K}{4\delta M_c} 2\sqrt{2} G_{\Lambda N K} \]  
(50)

from the decay $\Lambda_c^+ \rightarrow \Lambda K^+$,

\[ G_{\Sigma_c\Sigma D_s} = -\frac{\sqrt{2} f_K}{4\delta M_c} 2\sqrt{6} G_{\Sigma N K} \]  
(51)

from the decay $\Sigma_c^0 \rightarrow \Sigma^- K^+$,

\[ G_{\Xi_c\Xi D_s} = \frac{\sqrt{2} f_K}{4\delta M_c} 2\sqrt{3} G_{\Xi N K} \]  
(52)

from the decay $\Xi_c^0 \rightarrow \Xi^- K^+$,

\[ G_{\Xi'_c\Xi D_s} = \frac{\sqrt{2} f_K}{4\delta M_c} 2\sqrt{3} G_{\Lambda N K} \]  
(53)

from the decay $\Xi'_c^0 \rightarrow \Xi^- K^+$,

\[ G_{\Xi_{cc}\Xi'_{cc} D_s} = \frac{\sqrt{2} f_K}{4\delta M_c} (-3G_{\Lambda N K} - \sqrt{3} G_{\Sigma N K}) \]  
(54)

from the decay $\Xi_{cc}^+ \rightarrow \Xi_{cc}^0 K^+$,

\[ G_{\Xi_{cc}\Xi'_{cc} D_s} = \frac{\sqrt{2} f_K}{4\delta M_c} (-\sqrt{3}G_{\Lambda N K} - 3G_{\Sigma N K}) \]  
(55)

from the decay $\Xi_{cc}^+ \rightarrow \Xi_{cc}^0 K^+$,

\[ G_{\Omega_{cc}\Omega_c D_s} = \frac{\sqrt{2} f_K}{4\delta M_c} \sqrt{6} (\sqrt{3}G_{\Lambda N K} - G_{\Sigma N K}) \]  
(56)

from the decay $\Omega_{cc}^+ \rightarrow \Omega_{cc}^0 K^+$. Note that all $g_{BB'D_s}$ couplings are related to the two octet baryon K-couplings $g_{\Sigma N K}$ and $g_{\Lambda N K}$ which have already been written in terms of $g_{NN\pi}$ and $g_{\Lambda\Sigma\pi}$.  

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4 Numerical Work

To obtain the numerical values of the symmetry broken coupling constants, we need the values of $g_{NN\pi}$, $g_{\Lambda \Sigma \pi}$, $f_\pi$, $f_K$, $\delta M_s$ and $\delta M_c$. We take

$$\frac{g_{NN\pi}^2}{4\pi} = 14.6, \frac{g_{\Lambda \Sigma \pi}^2}{4\pi} = 11.1$$

from Pilkuhn et al. [7],

$$f_\pi = 132 \text{MeV}, \quad f_K = 166 \text{MeV},$$

and the mass breakings

$$\delta M_s = -160 \text{MeV}, \quad \delta M_c = -1150 \text{MeV}.$$ 

are determined from the baryon mass differences. Values of the coupling constants obtained are tabulated in the third column of table 1 and they are compared with the SU(4) symmetric values shown in the second column.

5 Discussion

With the help of Coleman-Glashow null result, we are able to determine symmetry breaking at the pseudoscalar BBP vertices without introducing any new parameters. The estimated coupling constants are way off from the symmetric values and are in good agreement with experimental values available at present.

The experimental values for $g_{\Sigma \Sigma \pi}^2/4\pi$ from the compilation of Nagels et al. [9] and for $g_{\Lambda \Sigma K}^2/4\pi$ and $g_{\Sigma \Sigma K}^2/4\pi$ quoted from Baillon et al. [10] are respectively 13.4 $\pm$ 2.1, 20.4 $\pm$ 3.7 and 1.9 $\pm$ 3.2. The estimated value of $g_{\Lambda c \Sigma c \pi}^2/4\pi$ is in good agreement with the current algebra calculation [13] and estimated value [14] using Melosh transformation and PCAC. We find that the strong couplings to charm mesons are reduced in the presence of the SU(4) flavour symmetry breaking. This provides a significant test of the formalism considered here.
With more studies on charmed baryons, the coupling constants are likely to find use in calculations, for example, of charmed baryon-antibaryon bound states [15], baryon-exchange forces [16], charmed baryon weak decays [17,18], where because of the lack of knowledge on broken coupling constants, invariant couplings have been used. For weak hadronic decays [11,12], use of the symmetry broken coupling constants affects the pole contributions significantly.

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Table Ia

**Strong \( \pi/K \)-Coupling Constants**

| Coupling \( BB'P \) | Symmetric Value of \( g_{BB'P}^2/4\pi \) | Symmetry Broken Value of \( g_{BB'P}^2/4\pi \) |
|---------------------|---------------------------------|---------------------------------|
| \( NN\pi \)        | 14.60*                          | 14.60*                          |
| \( \Lambda\Sigma\pi \) | 11.10*                         | 11.10 *                         |
| \( \Sigma\Sigma\pi \) | 3.50                           | 14.03                           |
| \( \Xi\Xi\pi \)     | 3.80                           | 1.50                            |
| \( \Lambda_c\Sigma_c\pi \) | 11.10                          | 46.79                           |
| \( \Sigma_c\Sigma_c\pi \) | 3.50                           | 59.39                           |
| \( \Xi_c\Xi_c\pi \)  | 0.88                           | 17.17                           |
| \( \Xi_c\Xi'_c\pi \) | 2.77                           | 13.59                           |
| \( \Xi'_c\Xi'_c\pi \) | 0.98                           | 0.08                            |
| \( \Xi_{cc}\Xi_{cc}\pi \) | 3.80                           | 15.00                           |
| \( \Lambda\Sigma\pi \)        | 10.80                          | 18.20                           |
| \( \Sigma\Sigma\pi \)        | 3.80                           | 0.90                            |
| \( \Xi\Lambda\pi \)         | 0.002                          | 2.20                            |
| \( \Xi\Sigma\pi \)         | 14.60                          | 24.69                           |
| \( \Xi_c\Sigma_c\pi \)      | 1.75                           | 29.02                           |
| \( \Xi'_c\Sigma_c\pi \)     | 5.55                           | 22.91                           |
| \( \Xi_c\Lambda_c\pi \)     | 5.55                           | 29.92                           |
| \( \Xi'_c\Lambda_c\pi \)    | 1.95                           | 0.10                            |
| \( \Omega_c\Xi_c\pi \)      | 3.50                           | 66.77                           |
| \( \Omega_c\Xi'_c\pi \)     | 11.10                          | 52.97                           |
| \( \Omega_{cc}\Xi_{cc}\pi \) | 7.60                           | 28.46                           |
### Table Ib

**Strong $D/D_s$-Coupling Constants**

| Coupling     | Symmetric Value of $g_{BB'P}^2/4\pi$ | Symmetry Broken Value of $g_{BB'P}^2/4\pi$ |
|--------------|---------------------------------------|------------------------------------------|
| $\Lambda_cND$ | 10.8                                  | 0.90                                     |
| $\Sigma_cND$  | 3.80                                  | 0.05                                     |
| $\Xi_c\Lambda D$ | 5.70                                  | 0.09                                     |
| $\Xi_c\Sigma D$ | 1.90                                  | 0.03                                     |
| $\Xi'_c\Lambda D$ | 1.80                                  | 0.18                                     |
| $\Xi'_c\Sigma D$ | 5.40                                  | 0.58                                     |
| $\Omega_c\Xi_cD$ | 7.60                                  | 0.15                                     |
| $\Xi_{cc}\Sigma_cD$ | 29.20                                | 7.22                                     |
| $\Xi_{cc}\Lambda_cD$ | 0.002                                | 0.34                                     |
| $\Omega_{cc}\Xi'_cD$ | 0.005                                | 0.38                                     |
| $\Omega_{cc}\Xi_cD$ | 14.60                                | 4.03                                     |
| $\Lambda_c\Lambda D_s$ | 7.20                                  | 1.05                                     |
| $\Xi'_c\Xi D_s$  | 10.80                                 | 1.95                                     |
| $\Sigma_c\Sigma D_s$ | 7.60                                  | 0.18                                     |
| $\Xi_c\Xi D$    | 3.80                                  | 0.11                                     |
| $\Xi_{cc}\Xi_cD_s$ | 14.60                                | 6.03                                     |
| $\Xi_{cc}\Xi'_cD_s$ | 0.002                                | 0.56                                     |
| $\Omega_{cc}\Omega_cD_s$ | 29.20                                | 13.41                                    |