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Optimized foam filling configuration in bi-tubular crush boxes; a comprehensive experimental and numerical analysis

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Abstract

This study focuses on introducing optimized foam filled double-walled energy absorbers. To this end, different geometries and foam densities is proposed with different foam configuration including foam inside the inner wall or between the tubes. A comprehensive experimental study has been done via construction and testing different unique structures. By extracting experimental force-displacement plots of all the structures, crashworthiness behavior of the structures is studied. Based on the decision making algorithm of COPRAS the best configuration is obtained using $F_{\text{max}}$, $\text{SEA}$ and $\text{CFE}$ as the decision functions. Finite element model of some selected structures are developed in LS-DYNA explicit dynamic software and validated using test results. Based on the best found configuration a new set of FE models are analyzed to create a meta-model by employing Response Surface Methodology (RSM) and the thicknesses of inner and outer walls are optimized via a multi-objective optimization algorithm.

1. Introduction

With the extensive growth of transportation industry, the number of vehicles as well as their speed is increasing rapidly. In this regard, road traffic injuries are one of the main causes of injuries [1, 2]. Improving the safety of vehicles by retaining energy absorbers has been the subject of the study recently. Alexander [3] was the first to investigate the crushing of circular tubes under axial loading. Wierzbicki and Abramowicz [4] theoretically investigated axial crushing of square tubes. By increasing use of aluminum membranes in automotive industry, researchers such as Langseth and Hopperstad [5] studied static and dynamic crushing response of aluminium extrusions, finding that dynamic mean force was considerably higher than static mean force. Energy absorption of braced elliptical tubes under lateral compression was first introduced by Wu and Carney [6] and further developed as multi-tube systems by Olabi et al [7, 8]. Since then, a wide-range of studies has been done on this subject. Study of foam filled sections by Santosoa et al [9] found that foam filler compressive strength has a linear relationship with mean crushing force. Sun et al [10] concluded that cellular material strain-rate sensitivity is associated with its base material properties as well as relating initial stress shock to mesoscopic structure. Zhang et al [11] experimented on dynamic response of graded metallic foams and showed that layer arrangement can affect energy absorption. Liu et al [12] investigated the effects of foam’s crushing speed on foam response and observed changes in foam’s deformation pattern. Zhang et al [13] developed mesoscopic numerical models of multi-layered metallic foams and showed that under impact loading the specimen are compacted layer by layer. Sun et al [14] analysed topological configuration of foam filled multi-cell tubes and found out that both cross-section and foam filler distributions have considerable effects on crashworthiness. Li et al [15] proposed a new configuration for foam filler based on biomimicry of ‘lotus root’ and found out that lotus foam fillers show lower peak crushing force. Dirgantara et al [16] experimentally investigated foam filled square columns and concluded that strain rate effects of foam core as well as the interaction between wall and foam play important roles in crushing behavior. Altin et al [17] utilized response surface method to optimize foal-filled multi-tubular circular tubes and found out that lateral
foam filling is superior to axial foam filling when considering specific energy absorption and crush force efficiency. Saeidi Googarchin et al [18] investigated the effects of taper angle and cell division on foam filled tubes. Results revealed an increase in SEA up to 5 times compared to base structure. A study of multi-cell square tubes [19] showed up to 90% improve in specific absorbed energy. Najafi and Rais-Rohani [20] investigated different multi-corner and multi-cell structures realizing that structural behaviour was highly reliant on cross-sectional geometry. Employing small and miniature conical frusta with different semi epical angle [21, 22], examining different cross-sectional geometries [23]. Introducing corrugations on lateral surface of the tube and laterally crushing it, as well as mixing the merits of corrugation with composite increases crush force efficiency while highlighting the importance of corrugation’s geometrical properties on crashworthiness [24–26]. Ahmadi and Asgari [27] considered corrugation on conical tubes and determined that unlike simple structures, adding corrugations increases SEA in oblique loading. Asgari et al [28] investigated conical tube and effects of introducing indentation and optimized them via different surrogate models. Results showed 17.6% increase in SEA and 24.63% decrease in Peak force for different types of indentations. Qiu et al [29] studied hexagonal tubes with different cross-sections and optimized them under different loading cases by using multi-objective particle optimization. Azimi and Asgari [30] mixed the conventional conical and circular tubes and found that peak force reduces without affecting other characteristics; an improvement under oblique loading was also observed. Recent studies also consider the derivation of the optimum design for the structure as a part of problem. Firouzi et al [31] proposed an H-shaped profile and optimized the control factors with respect to specific energy absorbed validating the results using a new theoretical solution. Qiu et al [32] attempted to optimize the numerical uncertainty and surrogate uncertainty using prediction variance of polynomial response surface to get more reliable response. Asanjarani et al [33] investigated tapered thin-walled square tubes and performed multi-objective optimization on them by considering indentations’ geometrical characteristics as variables.

Although investigation of thin-walled tubes in crashworthiness and effects of different parameters including cross-section and foam filling have been significantly considered in previous studies but also in most cases these parameters have been studied individually. In the other words, the interactions between foam filling configuration, type of cross-section and multi-wall tubes have not been considered, to the best of author’s knowledge. Based on this finding, configuration of foam filling and its effects on crushable tubes with different cross sections as well as crashworthiness of foam filled multi-walled tubes could be still a matter of importance. On the other hand, the wide-ranging experiments which validated via numerical simulations could provide a more comprehensive means of comparison for readers as well as a better understanding of the phenomena. It is also worth mentioning that finding the proper arrangement of foam filling beside the cross-section type of multi-walled tube could help designers to make a better decision while such type of optimization is not as straightforward as common problems having explicit objective functions. This framework has been provided via combination of efficient decision making and optimization algorithms.

Based on these facts, in the present study, conventional foam filled circular and square tubes are firstly studied and developed to double-walled. The effects of aluminium foam filler placement as well as different foam densities are examined experimentally for double-walled tubes of different cross sections. Based on a comprehensive experimental and numerical study, the best crush-box is chosen using by using a multi criteria decision making approach called Complex Proportional Assessment of Alternatives (COPRAS). The finite element model of the candidate model is developed in LS-DYNA explicit dynamic finite element software package and validated via experimental tests. After validating numerical model of the best crush box, using Response Surface methodology (RSM) a meta-model definition for energy absorption characteristics are derived by taking thickness of inner and outer tube as design variables. Finally by setting weighing factors for specific energy absorbed (SEA), Initial peak force (Fmax) and Crush force efficiency (CFE) best possible structure is identified. Results indicate the superiority of double walled circular tubes to square ones, as well as limitation caused by using denser foam fillers.

2. Experimental study

2.1. Configuration of specimens

As shown in figure 1 six different geometries and foam filling configurations were proposed in this study. An outer tube with a diameter of 83.25 mm was chosen for all circular structures while outer tubes with an edge of 58.5 mm are used for all square structures. These dimensions were chosen in a way that the cross-sectional area of the circular tubes is the peripheral circle of the square ones. Figure 2 depicts the dimensions of the tube as well as the relationship between circular and square tubes. Lengths of all tubes are 120 mm. All of specimens are made of Aluminium 4043AA.
The thicknesses of the tubes are between 1 mm and 2 mm. Aluminium foams with different densities of $\rho_1$, $\rho_2$ and $\rho_3$, which are 400, 500 and 600 kg m$^{-3}$ respectively, were embedded in tubes as shown in figure 1. Classifying the structures is as the following:

- **NF**: structures with no foam
- **FI**: structures with their inner wall filled with foam
- **MF**: structures filled with foam between the two walls

The diameter ratio of the circular tubes to peripheral circles varies as 0.4, 0.5 and 0.6. C and S stand for cylindrical and square cross sections of the tubes. For instance, S0.4NF is a double-walled square tube with no foam between its walls and a ratio of 0.4 between its peripheral circles. To keep the tubes in centre, a couple of thin aluminium pins in form of ‘+’ are stationed at the bottom of structures.

Table 1 summarizes all the crush-boxes used in the experimental tests with their expanded description and their acronyms which are used in the context.

Aluminium foam was developed using liquid metallurgy procedure. The process involves steps including:

1. Melting alloy in the furnace: Aluminum alloy powder is first mixed and compacted using uniaxial compression
| Schematics | Description                  | Abbreviation | Schematics | Description                  | Abbreviation |
|------------|------------------------------|--------------|------------|------------------------------|--------------|
| Square 0.5 Foam-Filled $\rho$3 Inside | S0.5FI$\rho$3 |             | Cylinder 0.4 No Foam | C0.4NF |
| Square 0.5 Foam-Filled $\rho$1 Middle | S0.5FM$\rho$1 |             | Cylinder 0.5 No Foam | C0.5NF |
| Square 0.5 Foam-Filled $\rho$2 Middle | S0.5FM$\rho$2 |             | Cylinder 0.6 No Foam | C0.6NF |
| Square 0.5 Foam-Filled $\rho$3 Middle | S0.5FM$\rho$3 |             | Square 0.4 No Foam | S0.4NF |
| Schematics Description | Abbreviation | Schematics Description | Abbreviation |
|-----------------------|--------------|-----------------------|--------------|
| Cylinder 0.4 Foam-Filled $\rho_1$ Inside | C0.4FI$\rho_1$ | Square 0.5 No Foam | S0.5NF |
| Cylinder 0.4 Foam-Filled $\rho_2$ Inside | C0.4FI$\rho_2$ | Square 0.6 No Foam | S0.6NF |
| Cylinder 0.4 Foam-Filled $\rho_3$ Inside | C0.4FI$\rho_3$ | Square 0.3 Foam-Filled $\rho_1$ Inside | S0.3FI$\rho_1$ |
| Cylinder 0.4 Foam-Filled $\rho_3$ Middle | C0.4FM$\rho_3$ | Square 0.3 Foam-Filled $\rho_2$ Inside | S0.3FI$\rho_2$ |
in steel die; next the mix is sintered for 3.5 h at 550 °C, 2) Dispersion of TiH₂ particles in the melt: the melt temperature is raised to a constant value of 700 °C and TiH₂ foaming agent is added to the melt, 3) Mixing the blend steadily and fast (an average rotational speed of 1100 rpm for the duration of 60–180 s), 4) Moving the mixture to crucible and allow the foam to form, 5) Allowing the foam to cool down (for 10 to 12 min) and ejecting it. Mixing speed, homogeneity of mixture, TiH₂ particles dispersion time as well as temperature of the process could affect the quality of the foam. Figure 3 shows the development procedure.

2.2. Test results

Energy absorption capacity and structure’s capability is measured using force-displacement curve of the crush-box. These characteristic are required to make a practical assessment between crush-boxes with different geometrical and material properties. The foremost prevalent parameters are as the followings:

Initial peak load: To begin progressive folding the crushing force needs to surpass the first maxima load, which is usually the highest peak load overall, otherwise crush-box exhibits rigid behavior and will not provide any absorption capacity.

Specific absorbed energy (SEA): The efficiency of absorbed energy EA. with respect to its mass, where EA. is equal to the area under force displacement curve as below:

\[
SEA = \int_0^d f_a \, dx \quad \frac{m}{m}
\]

where \( f_a \) is the resultant impact force and \( d \) stands for total deflection of the crush-box

Crush force efficiency (CFE): is defined as the ratio of the mean force, in total deflection distance, to initial peak force. The closer CFE is to unity results in more desirable crush-box. In other words, CFE is a measure of undesired difference between mean crush force and initial peak force.

\[
CFE = \frac{F_{\text{mean}}}{F_{\text{peak}}} \times 100
\]

To assess the crashworthiness of the aforementioned structures, an INSTRON8502 universal Testing machine with the maximum capacity of 25 Tons was used as shown in figure 4. The machine crushed the structures axially with the movement speed of 25 mm min⁻¹ up to 2/3 of the total length (8 cm).

In this part of study, force-displacement graphs of structures are compared to each other with respect to their foam density, foam placement and diameter ratio.
It is necessary to axially crush the foams separately, to understand their energy absorption capacity. As displayed in figure 5, foams which are less dense provide a long plateau region which is desirable. On the other hand foam \( p_3 \) starts to densify right after elastic region and does not offer a plateau region, meaning employing too much of this density could result in early rigidification of structure which should be avoided.

2.2.1. Effects of diameter ratio
As seen in figure 6 square crush-boxes with no foam, S’X’NF, depict similar behaviour to circular ones. Peak load \( F_{\text{max}} \) of S0.4NF structure is higher than the other two, which could be due to its higher thickness and lower diameter ratio. Figure 6(b) represents circular structures with no foam, CXNF; similar to figure 6(a) structure with larger diameter ratio has led to lower initial peak load.

2.2.2. Effects of foam placement
In this part of the study foams with similar density are placed in different possible geometrical configurations to consider the impact of foam placement on Energy absorption characteristics.

Figure 7(a) investigates the effect of foam \( p_1 \) placement in square structures with diameter ratio of 0.5. While S0.5p1FM starts with greater initial peak load, due to larger foam cross sectional area, which necessitates higher crushing force. Based on the deformation shapes of Square FM crush-boxes, it could be inferred that placing
foams in between the two tubes would lead to separation of the edges in corners; the structure’s edges started to fall apart and parts of foam fall out of structure at about 4 cm; because of this the load force decreases gradually but smoothly. As shown in figure 7(a) densification of S0.5p1FI occurs sooner than the other two cases. Figure 7(b) and (c) represent the effects of foam \( p_2 \) and \( p_3 \) on mentioned structure. The reason for force decline in S0.5p2FM and S0.5p3FM are similar to S0.5p1FM case in figure 7. Higher peak loads \( F_{\text{max}} \) are due to using denser foam \( p_2 \) and \( p_3 \).

Placement effects of foam \( p_3 \) in C0.4 tubes, could be seen in figure 8 as foam \( p_3 \) is dense, it leads the structure to densify faster in the crush-box with middle foam. Crush-box with inner foam has similar \( F_{\text{max}} \) to NF, but absorption capacity of them are evidently different, meaning that foams with lower densities could be used without aggregative results on \( F_{\text{max}} \). It is also notable that circular crush-box with no foam shows diamond folding pattern while with inner foam the pattern changes to concertina mode. Similar to square crush-boxes, placement of dense foam in between the tubes impedes progressive folding mode. As seen in figure 8 C0.4p3FM crush-box has formed fewer number of plastic hinges compared to the other two cases.

2.2.3. Effects of foam density

This part tries to study the effects of using different foam density in similar placement. Figure 9 investigates S0.5FM (p1, p2 and p3) structures with different foam densities. Unlike to square structures with inner foam placement which all have identical \( F_{\text{max}} \) in crush-boxes with middle foam, denser foam leads to higher initial peak force. This could be reasoned due to larger cross sectional area of the foam in FM set. As mentioned in section 2.2.2, placing foam in between the two tubes causes rupture in corners of the outer tube, hindering the complete use of crashworthiness capacity.
Figure 10 represents circular crush-boxes with different foam densities inside the inner tubes, C0.4p (p1, p2 and p3) FI. Initial peak load of these structures are at the same level. This could be interpreted as the marginal effect of inner foam density on initial peak load which is negligible. Early desification of C0.4p2FI and C0.4p3FI p3 . FI is related to their higher foam densities.
Table 2 provides the EA characteristics for all aforementioned structures. After extracting the data from the force-displacement curves of all the structures, using the equations in section 22 these characteristics namely, SEA, \( F_{\text{max}} \) and CFE which are the bedrock of crashworthiness and hold important ground in analysing and comparing are extracted.

2.3. Complex proportional assessment method

Using a multi-criteria decision making method is essential to make a comparison between tested structures with different geometrical characteristics and choose the structure with best crashworthiness characteristics. Kaklauskas et al \[34\] developed Complex proportional assessment method COPRAS which assumes direct and proportional dependences of the significance and utility degree of the available alternatives under the presence of mutually conflicting criteria. By means of selected criteria and their corresponding weights, performance of structures become comparable, making it possible to rank them and come up with the best decision. COPRAS steps are described as below \[35\].

1. Generating a decision matrix A that describes structures and design factors

Where \( a_{ij} \) is the performance value of \( j \)th design factor for \( i \)th structure. \( m \) is the number of compared structures and \( n \) refers to criterion number.

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]  

(1)

2. Generating dimensionless decision matrix \( R \) to make criteria comparable to each other using the following formula

\[
R = \begin{bmatrix}
\alpha_{1j} \\
\alpha_{2j} \\
\vdots \\
\alpha_{mj}
\end{bmatrix} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}}
\]  

(2)

3. Finding weights of each design factor as below \[35\]

- Comparing \( N \) sets of \( 2 \) design factors where \( N = n(n - 1)/2 \) and \( n \) refers number of design factors
- The significant factor has a score of 3 and insignificant factor has a score of 1, if both factors are equally significant then both have a score of 2.
- Summing up the scores for each design factor: \( W_j \)
- Calculating weight coefficients \( r_j \) by dividing \( W_j \) to global total score \( N^*4 \).
4. Generating weighted normalized decision matrix $D$ as below:

$$D = [y_{ij}] = r_{ij} \times w_j.$$  

(3)

5. Summing of beneficial and non-beneficial attributes

A beneficial attribute is one that should be maximized and a non-beneficial attribute is one that should be minimized. As an example it is desirable to minimize $F_{\text{max}}$ so it is a non-beneficial attribute. The following represents the sum of beneficial and non-beneficial attributes respectively.

$$S_{+ij} = \sum_{i=1}^{n} y_{+ij}$$

(4)

$$S_{-ij} = \sum_{i=1}^{n} y_{-ij}.$$  

(5)

where $y_{+ij}$ and $y_{-ij}$ denotes to normalized weighted values of beneficial and non-beneficial attributes respectively.

6. Determining the smallest $S_{-j}$ as $S_{\text{min}}$.

Figure 9. Force versus displacement of rectangular structures with $\rho_1$, $\rho_2$ and $\rho_3$ foam densities and different foam placements (a) middle (b) inside.
7. Determining relative priority and significance \( Q \).
Relative significance \( Q \) defines the satisfaction degree of a design concept. In other words, maximizing relative significance \( Q \) leads to selecting the best design concept. \( Q \) is defined as the following:

\[
Q_i = S_{+i} + \frac{S_{-\min} \sum_{j=1}^{m} S_{-j}}{S_i \sum_{j=1}^{m} (1/S_{-j})},
\]

(6)

8. Determining the quantitative utility (\( U \))

\( U_i \) could be used to rank the structures consequently

\[
U_i = \frac{Q_i}{Q_{\max}} \times 100\%.
\]

(7)

where the structure with \( U = 100 \) is our best design.

Table A1 depicts energy absorption characteristics of all tested structures. Using equations (2) and (3) weighted normalized matrix is shown in table A2.
Table 3. COPRAS method characteristics for all the structures.

|      | S+      | S−      | 1/S−    | Q       | U (%)   |
|------|---------|---------|---------|---------|---------|
| C0.4NF | 0.03779 | 0.02286 | 43.7442 | 0.06546 | 91.8892 |
| C0.5NF | 0.02858 | 0.02121 | 47.158  | 0.05841 | 81.9951 |
| C0.6NF | 0.03073 | 0.02077 | 48.1362 | 0.06118 | 85.882  |
| S0.4NF | 0.04094 | 0.02228 | 44.8879 | 0.06933 | 93.277  |
| S0.5NF | 0.03714 | 0.02005 | 49.8747 | 0.06869 | 96.4181 |
| S0.6NF | 0.03041 | 0.02037 | 49.1036 | 0.06146 | 86.2806 |
| S0.5FIρ1 | 0.03701 | 0.02161 | 46.2777 | 0.06628 | 93.04402|
| S0.5FIρ2 | 0.03806 | 0.02418 | 41.3556 | 0.06422 | 90.1416 |
| S0.5FIρ3 | 0.03935 | 0.02348 | 42.593  | 0.06629 | 93.054  |
| S0.5FMρ1 | 0.03196 | 0.02538 | 39.0397 | 0.05669 | 79.5802 |
| S0.5FMρ2 | 0.03277 | 0.02813 | 35.5445 | 0.05526 | 77.5642 |
| S0.5FMρ3 | 0.028  | 0.03392 | 29.4821 | 0.04665 | 65.4879 |
| C0.4FIρ1 | 0.04231 | 0.02335 | 42.8227 | 0.06939 | 97.4097 |
| C0.4FIρ2 | 0.04535 | 0.02443 | 40.9293 | 0.07124 | 100     |
| C0.4FIρ3 | 0.04301 | 0.0239  | 41.8494 | 0.06948 | 97.5268 |
| C0.4FMρ3 | 0.04039 | 0.06089 | 16.4236 | 0.05098 | 71.5576 |

Figure 11. The schematic of FE model.

Figure 12. Stress-strain curve of AA4043.
Applying equation (6) relative significance $Q_i$ is determined for each structure. $U_i$ denotes to ranking of structures. As shown below in table 3 COPRAS method characteristics for all the structures are summarized. The best structure determined by COPRAS is C0.4F$\rho_2$I which is the bi-circular tube with diameter ratio of 0.4 and foam-filled inner tube. This tube is next used for numerical study.
3. Finite element modelling and validation of numerical simulation

The selected structure chosen by COPRAS algorithm, bi-circular tube with diameter ratio of 0.4 and foam-filled inner tube, is first modelled to validate the numerical simulation. The schematic of FE model is shown in figure 11. Crush-box undergoes axial quasi-static load using ‘PRESCRIBED_MOTION_RIGID’ for the top plate. The fixed plate at the bottom is not tied to crush-box similar to experimental tests. ‘CONTACT_NODE_TO_SURFACE’ is used for interaction between rigid plates and crush-box. All contacts have a dynamic coefficient of 0.2 and static

| Experiment NO. | t_in | t_out | Mass (kg) | EA (N.m) | SEA (N.m/kg) | F_max (kN) | displacement (m) | F_avg (kN) | CFE |
|----------------|------|-------|-----------|----------|--------------|------------|-----------------|------------|-----|
| 1              | 1    | 1     | 0.11769   | 1238.09  | 10520.1      | 26287.2    | 0.07204         | 17186.94   | 0.6538 |
| 2              | 1    | 2     | 0.20211   | 2215.61  | 10962.4      | 53748.2    | 0.07204         | 30756.82   | 0.5722 |
| 3              | 1    | 3     | 0.2865    | 4106.4   | 14333        | 81271.7    | 0.07204         | 57004.41   | 0.7014 |
| 4              | 2    | 1     | 0.15094   | 2102.75  | 13931        | 37052.8    | 0.07204         | 29190      | 0.7878 |
| 5              | 2    | 2     | 0.2535    | 3082.24  | 13099.2      | 62906      | 0.07204         | 42787.11   | 0.6802 |
| 6              | 2    | 3     | 0.3197    | 4980.02  | 15577.2      | 94644.3    | 0.07204         | 69131.81   | 0.7304 |
| 7              | 3    | 1     | 0.1842    | 2778.17  | 15082.4      | 46342      | 0.07204         | 38566.11   | 0.8322 |
| 8              | 3    | 2     | 0.2686    | 3836.3   | 14282.6      | 72503.8    | 0.07204         | 53254.93   | 0.7345 |
| 9              | 3    | 3     | 0.353     | 5640.32  | 15978.3      | 105516     | 0.07204         | 78298.08   | 0.0     |

Table 5. Adequacy measures of three responses.

| Source      | RMSE   | R²       | Adjusted R² | Predicted R² |
|-------------|--------|----------|-------------|--------------|
| N.m kg⁻¹    | 272.366| 0.99242  | 0.979788    | 0.914025     |
| F_max (kN)  | 1768.46| 0.996648 | 0.99553     | 0.992377     |
| CFE         | 0.021303| 0.970344 | 0.920916    | 0.918448     |

Table 6. RSM confirmation experiments.

| t_in(mm) | t_out(mm) | SEA. (N.m/kg) | F_max. (kN) | CFE |
|----------|-----------|---------------|-------------|-----|
| predicted| 10635.27  | 25340.39      | 0.651025    |
| numerical| 10520.08  | 26287.16      | 0.653815    |
| Error (%)| −1.09499  | 3.601644      | 0.426714    |
| predicted| 10968.1   | 53965.38      | 589833      |
| numerical| 10962.42  | 53748.17      | 5239        |
| Error (%)| −0.0518   | −0.40413      | −3.07452    |
| predicted| 14212.08  | 82590.37      | 0.686593    |
| numerical| 14332.98  | 81271.66      | 0.701406    |
| Error (%)| 0.843519  | −1.6226       | 2.11187     |
| predicted| 13628.48  | 35849.39      | 0.775762    |
| numerical| 13531.01  | 37052.76      | 0.787796    |
| Error (%)| 2.171656  | 3.24772       | 1.527505    |
| predicted| 13232.05  | 64474.38      | 0.68013     |
| numerical| 13099.17  | 62905.95      | 0.680176    |
| Error (%)| −1.01438  | −2.49329      | 0.006748    |
| predicted| 15746.78  | 93099.37      | 0.74245     |
| numerical| 15577.15  | 94644.25      | 0.730439    |
| Error (%)| −1.08896  | 1.632302      | −1.64441    |
| predicted| 15269.68  | 46358.39      | 0.846959    |
| numerical| 15082.35  | 46342.04      | 0.832206    |
| Error (%)| −1.24202  | −0.03528      | −1.7278     |
| predicted| 14144     | 74983.38      | 0.716887    |
| numerical| 14282.58  | 72503.84      | 0.734512    |
| Error (%)| 0.970292  | −3.41987      | 2.399533    |
| predicted| 15929.48  | 103608.4      | 0.744767    |
| numerical| 15978.25  | 105516        | 0.742049    |
| Error (%)| 0.365229  | 1.807906      | −0.36623    |

3. Finite element modelling and validation of numerical simulation

The selected structure chosen by COPRAS algorithm, bi-circular tube with diameter ratio of 0.4 and foam-filled inner tube, is first modelled to validate the numerical simulation. The schematic of FE model is shown in figure 11. Crush-box undergoes axial quasi-static load using 'PRESCRIBED_MOTION_RIGID' for the top plate. The fixed plate at the bottom is not tied to crush-box similar to experimental tests. 'CONTACT_NODE_TO_SURFACE' is used for interaction between rigid plates and crush-box. All contacts have a dynamic coefficient of 0.2 and static
coefficient of 0.3. ‘Automatic surface to surface’ algorithm is employed to model the interactions between ‘inner foam filler and inner tube’ and ‘circular tubes’. To account for self-interactions of shell components ‘Automatic single surface’ algorithm is considered. ‘BELYSCHKO-LIN-TSAY’ element formulation with 5 integration points through thickness is considered to model the thin components. 1.5 mm element size was selected after mesh sensitivity analysis for shell components.

A flat shoulder specimen of AA4043 aluminium was used according to UNS 94043 standard in uniaxial tension test to get stress-strain curve. Mat123-modified-piecewise-linear-elasticity [36] with density of 2690 kg/m³, young modulus of 70 GPa, poisson’s ration of 0.34 and 70 Mpa yield stress was used to model the aluminium. Stress-strain curve is shown in figure 12. Mat63_Crushable_foam [36] with density of 500 kg/m³, young modulus of 317.5 MPa and stress-strain curve properties in figure 12 are used to model the foam filler.

Deformation patterns of C0.4Fρ2I in numerical model and experimental test are shown in figure 13. Force-displacement response of C0.4Fρ2I crush-box in FE simulation and experimental test is provided in figure 14. As seen in this figure, numerical model is able to capture the overall behaviour of the crush-box and deformability pattern of it. Based on the data in the said figure, it is evident that numerical model has predicted initial peak force $F_{\text{max}}$ and mean load $F_{\text{avg}}$ with acceptable accuracy.

4. Optimization of selected structure

4.1. Response surface model
In order to do optimizations study for finding the best configuration and dimensions of the crush box energy absorber having a theoretical model could of great importance. On the other hand, reaching an analytical solution for energy absorption characteristics of crush-boxes based on their geometrical and material
characteristics could be very challenging sometimes. Response surface methodology (RSM) is one of the available methods which provide an empirical solution to express the energy absorption characteristics of crush-box in terms of desired variables.

In this study, using this method, 9 design points were proposed and numerically simulated. Thicknesses of internal and external shown as \( t_{in} \) and \( t_{out} \) tubes were selected as input design factors. Range of inner and outer thicknesses are from 1 mm to 3 mm with step-se of 1 mm in design points.

Using the statistical software, Design Expert, via the numerical data in table 4 which has been validated earlier in figure 14, the RSM is applied. The adequacy measures including f-value, \( R^2 \), Adjusted \( R^2 \), Predicted \( R^2 \) has been used in selecting the best models. A step-wise regression method was used to fit the polynomial equation to the experimental data and to identify the relevant model terms [37, 38].

Table 4 summarizes the number of experiments and the recommended design of experiment distribution by the design expert software. Consequently, the table shows energy absorption characteristics of these design points extracted from the finite element simulation after running all 9 experiments. In addition, force-displacement curves of 3 selected design samples including experiments number 2, 6 and 8 are depicted in figure 15.

Implementing the step-wise regression method using the design expert software the resulting data about analysis of variance for each response has been gathered and shown tables A3, A4, and A5.

Table 5 shows the relative parameters of adequacy measures for validating the adequacy of the models which good agreement between adjusted \( R^2 \) and predicted \( R^2 \) for all three models was noticed indicating the RSM models holds legitimacy. The obtained statistical parameters are then employed to find the best polynomial fit for \( SEA, CFE \) and \( F_{\text{max}} \). Linear, quadratic and cubic polynomial functions of \( SEA, CFE \) and \( F_{\text{max}} \) in terms of \( t_{in} \) and \( t_{out} \) has been examined (as illustrated in 3 A, 4 A, and 5 A). Linear function for \( F_{\text{max}} \) and Quadratic functions for \( SEA \) and \( CFE \) are chosen as they presented better fits for the corresponding output. Root mean square error
RMSE, R squared, Adjusted R squared and predicted R squared are applied to measure the accuracy of these models. Smaller RMSE values, or R^2 values closer to 1 show better significance of the meta-model.

Obtained results show that the models and the regression method which was quadratic, linear and quadratic for SEA, F_{max} and CFE, respectively, are adequate and the RSM holds validity.

Equations (13)–(15) summarize the mathematical models in terms of actual factors which were fitted via the design expert software to each response using the said regression method. Response surfaces plots of SEA, F_{max} and CFE are shown in Figure 16.

\[
\text{SEA} = 13232.07 + 1587.97t_{\text{in}} - 1059.17t_{\text{out}} - 729.25t_{\text{in}}^2t_{\text{out}} - 676.00t_{\text{in}}^2 + 1455.60t_{\text{out}}^2
\]

Figure 18. Contour plot of (a) SEA, (b) F_{max}, (c) CFE.

Table 7. Criteria of numerical optimization.

| Name   | Goal   | Lower limit | Upper limit | Importance | Weight |
|--------|--------|-------------|-------------|------------|--------|
| t_{in}(mm) | Is in range | 1            | 3            | 3          | 1      |
| t_{out}(mm) | Is in range | 1            | 3            | 3          | 1      |
| SEA(N.m/kg) | Maximize | 10520.1      | 15978.3     | 3          | 1      |
| F_{max}(kN.) | Minimize | 26287.2      | 105516      | 3          | 1      |
| CFE      | Maximize | 0.5722       | 0.8322      | 1          | 1      |
Table 6 summarizes the numerical values of the predicted values and the error percentages which are within acceptable tolerances showing the models are valid. Regression lines of RSM values versus FEM models in figure 17 with the coefficient of determination above 0.97 also indicates that the model can firmly explain the variability of RSM responses around its mean.

4.2. Multi response optimization

Regarding the best inner thickness and outer thickness for the structures, the minimum range of 1 mm and maximum range of 3 mm were selected for design parameters which are summarized in table 7. For the responses, SEA and CFE should be maximum while F_max should be minimum to get the best thicknesses corresponding with the designated response parameters. Also the importance of 3, 3 and 1 were implied for SEA, F_max and CFE, as SEA and F_max hold a more important ground in comparison to CFE. Via these conditions, the best answer will be chosen which have the highest ‘desirability’ factor which in this case is t_in of 3 mm and t_out of 1 mm. All the possible responses with the highest desirability are summarized in table 8.

Using visual inspection the best thicknesses can be deduced based on the desired response conditions. As it was mentioned before SEA, F_max and CFE need to be maximum, minimum and maximum. Estimated fitted surfaces of equations (13)–(15) are shown in figure 16 in form of 3D and are shown in figure 18 in form of contours, Knowing that higher SEA values is desired, ‘top right’ and ‘bottom right’ regions of figure 18(a) have the thicknesses best suited for crashworthiness. In figure 18(b) initial peak force F_max is desired to be minimum. In figure 18(c) crush force efficiency CFE is preferred to be maximum, which make ‘right bottom’ region best suited.

5. Conclusion

To investigate the effects of cross-section and foam filling in energy absorption capacity of double-thin walled crush boxes, foam filled specimens with different cross-sections were tested and compared with no-foam samples. Experimental investigations were carried out on double walled square and double walled circular tubes by placing foam fillers with different densities inside or in-between them resulting in 16 unique arrangements. The following results could be concluded based on this investigation:

- Progressive folding of the structure would be prevented while using densities higher than 500 kg m⁻³, and early densification of the foam will happen which is not desirable.
- Lower diameter ratio between the inner and outer walls results in higher initial peak force.
- Denser foams increase the chances of outer wall separation having a negative effect on energy absorption.
- By employing foams with higher densities in-between walls initial peak force will be increased, while no such effect was seen for inner foams.

The best structure was determined by using Complex Proportional Assessment method in accordance with specific energy absorption, peak force and crush force efficiency responses. Applying COPRAS method showed circular double-walled tube with inside foam ‘C0.4FI/2’ provided best characteristics. The finite element model of best structure was next validated to build up a response surface model with respect to inner and outer wall thickness. A multi-objective optimization was next performed on response surface, with the goal of minimizing...
peak force, and maximizing specific energy absorption as well as crush force efficiency. Optimization results show that, with regards to selected importance factor in this study, thicker inner wall and thinner outer wall are desired for maximum efficiency. It is clear that different importance factors would indicate different values for desired thicknesses.

Appendix. COPRAS calculations and Analysis of variance for RSM.

These tables gather data about analysis of variance for each response and show the significant model terms. The F-value of the quadratic models indicates that the models are significant with only 0.01% chance such that the 'Model F-value' could have occurred due to noise. The same tables show the other adequacy measures R2, adjusted R2 and predicted R2. If a good agreement between the adjusted R2 and predicted R2 were noticed for models, it indicates models are adequate [33]. Related statistical parameters could be defined as follow.

\[ SSE = \sum_{i=1}^{k} (y_i - \bar{y})^2 \]  
\[ SST = \sum_{i=1}^{k} (y_i - \hat{y})^2 \]  
\[ RMSE = \sqrt{\frac{SSE}{n}} \]  
\[ R^2 = 1 - \frac{SSE}{SST} \]  
\[ R_{adj}^2 = 1 - \left( 1 - R^2 \right) \frac{n - 1}{n - f - 1} \]

where \( n \) are \( f \) the total number of design points and the number of design factors respectively. \( y_j \) is FEA value, \( \bar{y} \) is mean FEA value, and \( \hat{y} \) is corresponding predicted value of \( y_j \). SSE. is called sum of squared errors whereas SST is total sum of squares [23].

The Model F-value of 78.57 implies the model is significant. There is only a 0.22% chance that an F-value this large could occur due to noise. P-values less than 0.0500 indicate model terms are significant.

The Model F-value of 891.93 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. P-values less than 0.0500 indicate model terms are significant.

The Model F-value of 19.60 implies the model is significant. There is only a 1.69% chance that an F-value this large could occur due to noise. P-values less than 0.0500 indicate model terms are significant.

Table A1. EA characteristics of all tested structures.

|        | \( E_{\text{max}} \) (kN) | \( \text{SEA} \) (N.m kg\(^{-1}\)) | \( \text{CFE} \) |
|--------|---------------------------|---------------------------------|------------|
| C0.4NF | 60546.7                   | 13542.9                         | 0.61504    |
| C0.5NF | 56163.6                   | 10154.7                         | 0.47619    |
| C0.6NF | 55022.3                   | 10835.8                         | 0.5223     |
| S0.4NF | 59004                     | 14701.6                         | 0.66264    |
| NF     | 53104.4                   | 12885.4                         | 0.65716    |
| S0.6NF | 53938.3                   | 10493                           | 0.54498    |
| S0.5Flp1 | 57232                   | 12465.8                         | 0.70152    |
| S0.5Flp2 | 64043.7                  | 13159.3                         | 0.67891    |
| S0.5Flp3 | 62183.1                  | 13402.0                         | 0.72507    |
| S0.5FMp1 | 67749.2                  | 10657.7                         | 0.6193     |
| S0.5FMp2 | 74514                    | 10850.1                         | 0.64458    |
| S0.5FMp3 | 89836.3                  | 9495.7                          | 0.32291    |
| C0.4Flp1 | 61849.6                  | 15012.9                         | 0.70695    |
| C0.4Flp2 | 64710.8                  | 16102.4                         | 0.75659    |
| C0.4Flp3 | 63288                    | 14976.6                         | 0.754      |
| C0.4FMp3 | 161266                   | 15263.6                         | 0.57123    |
| sum    | 1104451                   | 204018                          | 10.1594    |
Table A2. Weighted normalized matrix.

| C0.4NF | 0.02286 | 0.02768 | 0.01011 |
| C0.5NF | 0.02121 | 0.02076 | 0.00783 |
| C0.6NF | 0.02077 | 0.02213 | 0.00859 |
| S0.4NF | 0.02228 | 0.03005 | 0.01089 |
| S0.5NF | 0.02161 | 0.02548 | 0.01153 |
| S0.6NF | 0.02187 | 0.02215 | 0.00859 |
| C0.4FI | 0.02335 | 0.03069 | 0.01162 |
| C0.5FI | 0.02443 | 0.03291 | 0.01244 |
| C0.6FI | 0.0239  | 0.03061 | 0.01239 |
| C0.4FM | 0.06089 | 0.0312  | 0.00939 |

Table A3. ANOVA table for SEA reduced quadratic model.

| Source | Sum of Squares | df | Mean Square | F-value | p-value |
|--------|----------------|----|-------------|---------|---------|
| Model  | 2.914E + 07    | 5  | 5.828E + 06 | 78.57   | 0.0022  | significant |
| A-tin  | 1.513E + 07    | 1  | 1.513E + 07 | 203.99  | 0.0007  |
| B-tout | 6.731E + 06    | 1  | 6.731E + 06 | 90.75   | 0.0025  |
| AB     | 2.127E + 06    | 1  | 2.127E + 06 | 28.68   | 0.0127  |
| A²     | 9.140E + 05    | 1  | 9.140E + 05 | 12.32   | 0.0392  |
| B²     | 4.238E + 06    | 1  | 4.238E + 06 | 57.13   | 0.0048  |
| Residual | 2.225E + 05 | 3  | 74170.85    |         |         |
| Cor Total | 2.936E + 07 | 8  |            |         |         |

Table A4. ANOVA table for Fmax reduced linear model.

| Source | Sum of Squares | df | Mean Square | F-value | p-value |
|--------|----------------|----|-------------|---------|---------|
| Model  | 5.579E + 09    | 2  | 2.789E + 09 | 891.93  | < 0.0001 | significant |
| A-tin  | 6.626E + 08    | 1  | 6.626E + 08 | 211.88  | < 0.0001 |
| B-tout | 4.916E + 09    | 1  | 4.916E + 09 | 1571.98 | < 0.0001 |
| Residual | 1.876E + 05 | 6  | 3.127E + 06 |         |         |
| Cor Total | 5.598E + 09 | 8  |             |         |         |

Table A5. ANOVA table for CFE reduced quadratic model.

| Source | Sum of Squares | df | Mean Square | F-value | p-value |
|--------|----------------|----|-------------|---------|---------|
| Model  | 0.0446         | 5  | 0.0089      | 19.60   | 0.0169  | significant |
| A-tin  | 0.0242         | 1  | 0.0242      | 53.31   | 0.0053  |
| B-tout | 0.0017         | 1  | 0.0017      | 3.67    | 0.1514  |
| AB     | 0.0047         | 1  | 0.0047      | 10.44   | 0.0482  |
| A²     | 0.0014         | 1  | 0.0014      | 3.16    | 0.1737  |
| B²     | 0.0125         | 1  | 0.0125      | 27.44   | 0.0135  |
| Residual | 0.0014 | 3  | 0.0005      |         |         |
| Cor Total | 0.0459 | 8  |             |         |         |

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