Fermion localization in a backreacted warped spacetime

Tanmoy Paul and Soumitra SenGupta

Department of Theoretical Physics, Indian Association for the Cultivation of Science, 2A & 2B Raja S.C. Mallick Road, Kolkata - 700 032, India.

We consider a five dimensional AdS warped spacetime in presence of a massive scalar field in the bulk. The scalar field potential fulfills the requirement of modulus stabilization even when the effect of backreaction of the stabilizing field is taken into account. In such a scenario, we explore the role of backreaction on the localization of bulk fermions which in turn determines the effective radion-fermion coupling on the brane. Our result reveals that both the chiral modes of the zeroth Kaluza-Klein (KK) fermions get localized near TeV brane as the backreaction of the scalar field increases. We also show that the profile of massive KK fermions shifts towards the Planck brane with increasing backreaction parameter.

I. INTRODUCTION

Eversince the original proposal of Kaluza-Klein (KK) regarding the existence of extra spatial dimension(s) it is often believed that our universe is a 3-brane embedded in a higher dimensional spacetime and is described through a low energy effective theory on the brane carrying the signatures of extra dimensions [1, 2].

Till date, Standard Model (SM) of particle Physics is the best suited model for describing the possible interactions between fundamental particles up to TeV scale. However, the Standard Model carries a quadratic divergence in the radiatively corrected Higgs mass which can be set to the desired value 126 GeV only through an unnatural fine tuning of the parameters of the theory. Among various models proposed to solve this fine tuning problem [3, 4], the Randall-Sundrum (RS) warped geometry model [5] earns a special attention since it resolves the gauge hierarchy problem without introducing any inherent potential [6, 7]. It has been demonstrated in [8, 9] that the modulus of RS scenario can be stabilized using GW prescription even by incorporating the backreaction of the stabilizing field. In such a braneworld scenario, several models were proposed by placing the standard model fields inside the bulk. Special care is taken that the localization property of a bulk fermion field is ensured inside the bulk, as it has been always neglected in GW proposal, its implications are subsequently studied in [10, 11]. It has been demonstrated in [12] that the modulus of RS scenario can be stabilized using GW prescription even by incorporating the backreaction of the stabilizing field. We aim to address this in the present work.

Our paper is organized as follows: The backreacted RS scenario and its modulus stabilization is described in section II. Section III addresses the localization property of bulk fermion field and its consequences. The paper ends with some concluding remarks.

II. BACKREACTED RS MODEL AND ITS MODULUS STABILIZATION

The action for the RS geometry with a stabilizing scalar field \( \Phi \) is given by

\[
S = \int d^4x \sqrt{|G|}(-M^4 R + \Lambda) + \int d^5x \sqrt{|G|}[(1/2)G^{MN} \partial_M \Phi \partial_N \Phi - V(\Phi)] - \int d^4x \sqrt{-g_{\text{vis}}\lambda_{\text{hid}}(\Phi)} - \int d^4x \sqrt{-g_{\text{vis}}\lambda_{\text{vis}}(\Phi)}
\]

where \( M \) is the five dimensional Planck scale, \( \Lambda \) is the bulk cosmological constant, \( G_{MN} \) is the five dimensional
metric, \(g_{hid}\) and \(g_{vis}\) are the induced metric on hidden and visible brane respectively. \(\lambda_{hid}\), \(\lambda_{vis}\) are the self interactions of scalar field (including brane tensions) on Planck and TeV branes. The background metric ansatz is given by,

\[
ds^2 = \exp [-2A(y)] \eta_{\mu\nu} dx^\mu dx^\nu - dy^2
\]

where \(A(y)\) is the warp factor. The bulk scalar field is assumed to depend only on the extra dimensional coordinate \(y\) \([12]\).

In order to get an analytic solution of the backreacted geometry, the form of the scalar field potential is chosen as \([12]\),

\[
V(\Phi) = (1/2)\Phi^2(u^2 + 4\kappa k) - (\kappa^2/6)u^2\Phi^4
\]

where \(k = \sqrt{-\kappa^2\Lambda/6}\). The potential contains quadratic as well as quartic self interaction of the scalar field and the two terms are connected by a common free parameter \(u\). Using this form of the potential, one obtains a solution of coupled gravitational-scalar field equations as:

\[
A(y) = k|y| + (\kappa^2/12)\Phi^2 \exp (-2uy) \tag{4}
\]

\[
\Phi(\phi) = \Phi P \exp (-uy) \tag{5}
\]

where \(\Phi P\) is the value of the scalar field on Planck brane. From eqn. (4), it can be argued that \(\kappa \Phi P\) controls the deviation of the warp factor from RS model and thus \(\kappa \Phi P\) is known as scalar field backreaction parameter. Moreover \(\lambda_{hid}\) and \(\lambda_{vis}\) can be obtained from the boundary conditions on branes as,

\[
\lambda_{hid} = 6k/\kappa^2 - u\Phi P^2 \tag{6}
\]

\[
\lambda_{vis} = -6k/\kappa^2 + u\Phi P^2 \exp (-2uy) \tag{7}
\]

where \(r_c\) is the compactification radius of the extra dimension. Once the solutions of \(A(y)\) and \(\Phi(y)\) are obtained (see eqn. (4) and eqn. (5)), the modulus can be stabilized using GW prescription. It has been demonstrated in \([17]\) that the interbrane separation in backreacted RS scenario is stabilized at a value given by,

\[
k \pi r_c = \frac{k}{u} \ln \left( \frac{\kappa \Phi P}{2\sqrt{1 + \frac{\kappa^2}{6}}} \right) \tag{8}
\]

After stabilizing the modulus of RS scenario with the modification due to backreaction of the stabilizing scalar field, we now show how the backreaction parameter \((\kappa \Phi P)\) affects the localization of fermion field within the five dimensional spacetime.

### III. Fermion Localization

Consider a bulk massive fermion field propagating on a background geometry model characterized by action in eqn. (1). The lagrangian for the Dirac fermions is given by

\[
L_{Dirac} = e^{-4A(y)}[\overline{\Psi} i\Gamma^a D_a \Psi - m_5 \overline{\Psi} \Psi]
\]

where \(\Psi = \overline{\Psi}(x^\mu, y)\) is the fermion field and \(m_5\) is its mass. \(\Gamma^a = (\epsilon^{A(y)}\gamma^\mu - i\gamma^5)\) denotes the five dimensional gamma matrices where \(\gamma^\mu\) and \(\gamma^5\) represent 4D gamma matrices in chiral representation. Curved gamma matrices obey the Clifford algebra i.e. \([\Gamma^a, \Gamma^b] = 2G^{ab}\).

The covariant derivative \(D_a\) can be calculated by using the metric in eqn. (2) and is given by

\[
D_\mu = \partial_\mu - \frac{1}{2} \Gamma_\mu^a \Gamma^4 A'(y)e^{-A(y)}
\]

Using this set up, the Dirac lagrangian \(L_{Dirac}\) turns out to be,

\[
L_{Dirac} = e^{-4A(y)}\overline{\Psi}[i\epsilon^{A(y)}\gamma^\mu \partial_\mu + \gamma^5(\partial_y - 2A'(y)) - m_5]\Psi \tag{9}
\]

We decompose the five dimensional spinor via Kaluza-Klein (KK) mode expansion as \(\Psi(x^\mu, y) = \sum \chi^n(x^\mu)\xi^n(y)\), where the superscript \(n\) denotes the nth KK mode. \(\chi^n(x^\mu)\) is the projection of \(\Psi(x^\mu, y)\) on the 3-brane and \(\xi^n(y)\) is the extra dimensional component of 5D spinor. Left \((\chi_L)\) and right \((\chi_R)\) states are constructed by \(\chi^n_L = \frac{1}{2}(1 + \gamma^5)\chi^n\). Thus the KK mode expansion can be written in the following way :

\[
\Psi(x^\mu, y) = \sum \chi^n_L(x^\mu)\xi^n_L(y) + \chi^n_R(x^\mu)\xi^n_R(y) \tag{10}
\]

Substituting the KK mode expansion of \(\Psi(x^\mu, y)\) in the Dirac field lagrangian given in eqn.(9), we obtain the following equations of motion for \(\xi^n_L, R\) as follows :

\[
e^{-A(y)}[\pm(\partial_y - 2A'(y)) + m_5]\xi^n_{L,R}(y) = 0
\]

where \(m_n\) is the mass of nth KK mode. The 4D fermions obey the canonical equation of motion \(i\gamma^\mu \partial_\mu \chi^n_{L,R} = m_n \chi^n_{L,R}\). Moreover eqn.(11) is obtained provided the following normalization conditions hold :

\[
\int_0^\pi dy e^{-3A(y)} \xi^n_L \xi^n_{L,R} = \delta_{m,n} \tag{12}
\]

\[
\int_0^\pi dy e^{-3A(y)} \xi^n_L \xi^n_R = 0 \tag{13}
\]

In the next two subsections, we discuss the localization scenario for massless and massive KK modes respectively.

#### A. Massless KK mode

For massless mode, equation of motion of \(\xi_L, R\) takes the following form (taking \(\frac{\kappa \Phi P}{\sqrt{2}} = l\))

\[
\exp (-ky - \frac{l^2}{6} e^{-2uy})[\pm(\partial_y - 2k + \frac{2l^2}{3} e^{-2uy}) + m_5]\xi_{L,R}(y) = 0
\]

\(\text{(14)}\)
where we use the form of warp factor i.e. \( A(y) = ky + \frac{7}{6} e^{-2uy} \). Solution of eqn. (14) is given by:

\[
\xi_{L,R}(y) = \sqrt{\frac{k}{e^{2k\pi r_c} - 1}} \left[ 1 + \exp \left( \frac{2l^2}{3} e^{-u\pi r_c} \right) \right]^{1/2} \exp \left( \frac{l^2}{6} e^{-2uy} e^{2ky} \right)
\]

for \( m_5 = 0 \); and

\[
\xi_{L,R}(y) = \sqrt{\frac{k}{e^{2k\pi r_c} - 1}} \left[ 1 + \exp \left( \frac{2l^2}{3} e^{-u\pi r_c} \pm \frac{2m_5}{k} \right) \right]^{1/2} \exp \left( \frac{l^2}{6} e^{-2uy} e^{2ky} \right)
\]

for \( m_5 \neq 0 \).

The overall normalization constants in eqn. (15) and eqn. (16) are determined by using the normalization condition presented earlier in eqn. (12). It may be noticed that left and right chiral modes have the same solution when \( m_5 = 0 \), but the degeneracy between the two chiral modes lifted in the presence of non-zero bulk fermionic mass term.

It is worthwhile to study how the localization scenario depends on the backreaction parameter \( l \) as well as the bulk mass parameter \( (m_5) \).

**Effect of backreaction parameter**

From eqn. (15), we obtain the Figure 1 between \( \xi_{L,R} \) and \( y \) for various values of backreaction parameter. The constant \( y \) hypersurfaces at \( y = 0 \) and \( y = 36 \) represent Planck and TeV branes respectively. We focus into the region near the TeV brane (see figure 1) to depict the localization properties of the left and right modes.

**FIG. 1.** \( \xi_{L,R} \) vs \( y \) for \( k = 1 \), \( \frac{u}{k} = 0.2 \) and \( m_5 = 0 \)

Figure 1 clearly demonstrates that for \( m_5 = 0 \), the two chiral modes get more and more localized on TeV brane as the backreaction parameter comes close to \( l = 25 \) (needed to solve the gauge hierarchy problem for \( \frac{u}{k} = 0.2 \) from a lower value. Thus the solution of hierarchy problem and the localization of fermion are linked with each other. The figure also reveals that larger the backreaction parameter \( l \), the localization of both the chiral modes of massless fermions becomes sharper near the visible brane.

On the other hand, for small values of \( l \), the fermions are clearly localized deep inside the bulk spacetime and without any backreaction (i.e. \( l = 0 \)) we retrieve the RS solution where the fermions are peaked away from the visible brane. Thus without any bulk mass term, the fermions can be localized at different regions inside the bulk by adjusting the value of backreaction parameter.

From eqn. (16), we obtain the plots of left and right chiral modes for various \( l \) in presence of non-zero bulk fermionic mass.

**FIG. 2.** \( \xi_L \) vs \( y \) for \( k = 1 \), \( \frac{u}{k} = 0.2 \) and \( m_5 = 0.5k \)

**FIG. 3.** \( \xi_R \) vs \( y \) for \( k = 1 \), \( \frac{u}{k} = 0.2 \) and \( m_5 = 0.5k \)

Figure 2 and figure 3 reveal that as the backreaction parameter increases, the peak of both left and right chiral wave function get shifted towards the visible brane.

Moreover, using the solution of \( \xi_{L,R}(y) \) (in eqn. (16)), we obtain the effective coupling [26] between radion and...
zeroth order fermionic KK mode as follows:

\[
\lambda_L = \sqrt{\frac{k}{24M^4}} e^{\frac{A(\pi r_c)}{2}} \exp \left[ \frac{i^2}{3} \exp (-2u \pi r_c) \right] \\
\left[ 1 + \exp \left( \frac{2i^2}{3} e^{-u \pi r_c} - \frac{2m_5}{k} \right) \right] \left( \frac{e^{(k-2m_5)\pi r_c}}{e^{(k-2m_5)\pi r_c} - 1} \right) \tag{17}
\]

for left handed chiral mode and,

\[
\lambda_R = \sqrt{\frac{k}{24M^4}} e^{\frac{A(\pi r_c)}{2}} \exp \left[ \frac{i^2}{3} \exp (-2u \pi r_c) \right] \\
\left[ 1 + \exp \left( \frac{2i^2}{3} e^{-u \pi r_c} + \frac{2m_5}{k} \right) \right] \left( \frac{e^{(k-2m_5)\pi r_c}}{e^{(k-2m_5)\pi r_c} - 1} \right) \tag{18}
\]

for right handed mode. It is evident that the effective radion-fermion coupling increases (for both left and right chiral mode) with the backreaction parameter. It is expected because the peak of both left and right chiral wave function get shifted towards the visible brane as the backreaction parameter increases.

**B. Massive KK mode**

In this section, we study the localization of higher Kaluza-Klein modes. For massive KK modes, equation of motion for fermionic wave function is given as,

\[
\exp (-ky - \frac{i^2}{6} e^{-2uy}) (\pm (\partial_y - 2k + \frac{2iu}{3} e^{-2uy}) + m_5) \xi_{L,R}^n (y) = -m_n \xi_{L,R}^n (y) \tag{19}
\]

Recall that \(m_n\) is the mass of \(n\)th KK mode. Using the rescaling \(\tilde{\xi}_{L,R} = e^{\frac{A(\pi)}{2}} \xi_{L,R}\), we find that the two helicity states, \(\xi_L\) and \(\xi_R\) satisfy the same equation of motion and is given by,

\[
\tilde{\xi}^n(y) [\frac{k^2}{4} (1 + \frac{i^2}{3} \frac{u}{k}) + (m_n^2 - \frac{k^2}{4} (1 + \frac{i^2}{3} \frac{u}{k})) - m_5^2 \exp (2ky + \frac{i^2}{3} e^{-2uy})] \tilde{\xi}_{L,R}^n (y) = 0 \tag{20}
\]

Solution of eqn. (20) is given by Bessel function as follows:

\[
\xi_{L,R}^n (y) = \sqrt{k} \exp \left( -\frac{5i^2}{12} e^{-2uy} \right) \Gamma \left( 1 + \frac{\sqrt{3k + i^2 u}}{4\sqrt{3k}} BesselI_{\frac{-\sqrt{3k + i^2 u}}{4\sqrt{3k}}} \right) \left( e^{-ky \frac{(3k^2 + 12m_5^2 + 12m_5^2 + k^2 u)}{4\sqrt{3k}}} \right) \tag{21}
\]

The mass spectrum can be obtained from the requirement that the wave function is well behaved on the brane. Demanding the continuity of \(\xi_{L,R}^n\) at \(y = 0\) and at \(y = \pi r_c\) gives the mass term as follows:

\[
m_n^2 = e^{-2A(\pi)} [\xi^2 (n^2 + 2n + 1) + m_5^2] \tag{22}
\]

where \(n = 1, 2, 3, ...\). Now from the requirement of solving the gauge hierarchy problem, the warp factor at TeV brane acquires the value as \(A(\pi) = 36\) which produces a large suppression in the right hand side of eqn. (22) through the exponential factor. Since \(k, m_5 \sim M\), the mass of KK modes \((n = 1, 2, 3...\) comes at TeV scale.

Using the solution of \(\xi_{L,R}^n (y)\) (in eqn. (21)), we determine the coupling between massive KK fermion modes and the radion field, given by

\[
\lambda^{(n)} = \sqrt{\frac{k}{24M^4}} e^{\frac{A(\pi)}{2}} \exp \left( -\frac{5i^2}{6} e^{-2uy} \right) \left( \Gamma \left( 1 + \frac{\sqrt{3k + i^2 u}}{4\sqrt{3k}} BesselI_{\frac{-\sqrt{3k + i^2 u}}{4\sqrt{3k}}} \right) \right) \left( e^{-ky \frac{(3k^2 + 12m_5^2 + 12m_5^2 + k^2 u)}{4\sqrt{3k}}} \right)^2 \tag{23}
\]

where \(\lambda^{(n)}\) is the coupling between \(n\)th KK fermion mode and the radion field. Eqn. (23) clearly indicates that \(\lambda^{(n)}\) decreases with increasing backreaction parameter.

Eqn. (21) indicates the relation between \(\xi_{L,R}\) and \(y\) for various values of \(l\) from which one can find the dependence of localization for massive KK fermion modes on the backreaction parameter. From this, the behaviour of the first KK mode \((n = 1)\) is described in figure 4.

![Figure 4. \(\xi_{L,R}^n\) vs y for k = 1, \(\pi = 0.2\) and \(\frac{m_5}{k} = 0.5\)](image-url)

Figure 4 clearly depicts that the wave function for first massive KK mode gets more and more localized near Planck brane with increasing value of backreaction parameter. As a result, the coupling parameter decreases near the visible brane as the backreaction parameter increases.

Moreover, it can also be shown (from eqn. (21)) that as the order of KK mode increases from \(n = 1\), the localization of fermions becomes sharper near Planck brane.
Before concluding, it may be mentioned that the bulk fermion mass term \( (m_f) \) also affects the localization of fermion field. Using the solution of \( \xi_{l,r}(y) \) presented in eqn. (16), it can be shown that for a fixed value of backreaction parameter, the left chiral mode of zeroth KK fermion has higher peak values on TeV brane as the bulk fermion mass increases where as the right chiral mode shows the reverse nature, which is in agreement with [24].

**IV. CONCLUSION**

We consider a five dimensional AdS compactified warped geometry model with two 3-branes embedded within the spacetime. For the purpose of modulus stabilization, a massive scalar field is invoked in the bulk and its backreaction on spacetime geometry is taken into account. In this scenario, we study how the backreaction parameter affects the localization of a bulk fermion field within the entire spacetime. Moreover, we also explore how the fermion localization depends on the bulk mass parameter. Our findings are as follows:

1. **For massless KK mode** -
   - In the absence of bulk fermion mass, left and right chiral modes can be localized at different regions in the spacetime by adjusting the value of backreaction parameter \( (l) \). However, the localization of both the chiral modes becomes sharper near TeV brane as the value of \( l \) increases.
   - In the presence of non-zero bulk fermion mass, the left as well as right mode get more and more localized as the backreaction parameter becomes larger. Correspondingly the overlap of fermion wave function with the visible brane increases with \( l \), which is depicted in figure (2) and figure (3).

2. **For massive KK mode** -
   - The effective coupling between radion and zeroth order fermionic KK mode is obtained (in eqn. (17) and eqn. (16)). It is found that the radion-fermion coupling (for both left and right chiral mode) increases with the increasing value of backreaction parameter. This is a direct consequence of the fact that the peak of the left and right chiral mode get shifted towards the visible brane as the backreaction parameter increases. This in turn enhances the radion to fermion decay amplitude.
   - For a fixed value of backreaction parameter, the left chiral mode has higher peak values on TeV brane as the bulk fermions become more and more massive where as the right chiral mode shows a reverse nature.

Before concluding, it may be mentioned that the bulk fermion mass term \( (m_f) \) also affects the localization of fermion field. Using the solution of \( \xi_{l,r}(y) \) presented in eqn. (16), it can be shown that for a fixed value of backreaction parameter, the left chiral mode of zeroth KK fermion has higher peak values on TeV brane as the bulk fermion mass increases where as the right chiral mode shows the reverse nature, which is in agreement with [24].
[16] J. Lesgourgues, S. Pastor, M. Peloso, L. Sorbo, Phys.Lett. B489 411 (2000)
[17] A. Das, T. Paul and S. SenGupta, Modulus stabilisation in a backreacted warped geometry model via Goldberger-Wise mechanism, arXiv:1609.07787 [hep-ph]
[18] B. Bajc, G. Gabadadze, Phys.Lett. B474, 282 (2000)
[19] P. Smyth, K.S. Stelle, Nucl.Phys. B790, 89 (2008)
[20] S. Chang, J. Hisano, H. Nkano, N. Okada, M. Yamaguchi, Phys.Rev. D62, 084025 (2000)
[21] R. Koley, S. Kar, Mod.Phys.Lett. A20, 363 (2005)
[22] R. Koley, J. Mitra, S. SenGupta, Phys.Rev.D78, 045005 (2008)
[23] Y. Grossman, M. Neubert, Phys.Lett. B474, 361 (2000)
[24] R. Koley, J. Mitra, S. SenGupta, Phys.Rev. D79, 041902(R) (2009)
[25] S. Das, D. Maity and S. Sengupta, JHEP, 0805 042 (2008)
[26] T.G. Rizzo, JHEP 06 056 (2002).