Effects of Compton Scattering on the Gamma-Ray Spectra of Solar Flares

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Abstract

Using the fully relativistic GEANT4 simulation toolkit, Monte Carlo simulation was applied to the transport of energetic electrons generated in solar flares, and resultant bremsstrahlung gamma-ray spectra were calculated. The solar atmosphere was approximated by 10 vertically stacked zones. The simulation took into account two important physical processes: that the bremsstrahlung photons emitted by precipitating relativistic electrons are strongly forward beamed toward the photosphere, and that the majority of these gamma-rays must be Compton backscattered by the solar atmosphere in order to reach the observer. Then, the Compton degradation was found to make the observable gamma-ray spectra much softer than predicted by simple analytic calculations. The gamma-ray signals were found to be enhanced by several conditions, including a broad pitch-angle distribution of the electrons, a near-limb flare longitude, and a significant tilt in the magnetic field lines if the flare longitude is rather small. These results successfully explain several important flare properties observed in the range of hard X-ray to gamma-ray, in particular including those obtained with Yohkoh. A comparison of the Yohkoh spectrum, from a GOES X3.7 class limb flare on 1998 November 22, with a simulation assuming a broad electron pitch-angle distribution suggests that gamma-rays from this particular solar flare were a mixture of direct bremsstrahlung photons and their Comptonization.

Key words: scattering: Compton scattering — Sun: flares — Sun: gamma rays

1. Introduction

Solar flares are sudden, brief, and powerful outbursts, releasing an energy of \(10^{28–34}\) ergs on a time scale of one second to several tens of minutes. In solar flares, both protons and electrons are accelerated to nonthermal energies. Although these energetically charged particles are strongly affected by the solar magnetic fields, and cannot usually be detected, we can utilize neutral secondary particles, such as neutrons and photons, to probe them. Among them, continua in the range of hard X-ray to gamma-ray, produced via the bremsstrahlung process, provide the best diagnostics of the flare-accelerated electrons.

Although high and spatial resolution images of flares in the hard X-ray and gamma-ray energies have become available by the Yohkoh HXT (Kosugi et al. 1991) and RHESSI (Lin et al. 2004), respectively, we still depend heavily on the spectral data when attempting to study nonthermal electrons. Actually, it has been an important issue in solar physics to calculate back the electron spectrum from the observed gamma-ray spectra. It is understood that flare-accelerated electrons having a power-law distribution penetrate the solar chromosphere and produce bremsstrahlung photons via “thick-target” emission. The thick-target emission is a process wherein nonthermal electrons impinge on a thick matter, and lose almost all of their energies through repeated Coulomb collisions while emitting X-ray/gamma-ray photons via bremsstrahlung.

In the thick-target condition, the spectrum of X-ray emitting electrons is formed by an equilibrium between the injection of newly accelerated electrons and the loss of their energies, predominantly through collisions. If the injected electron number spectrum is given by a power-law, as \(F(E) \sim E^{-\delta}\) (electrons keV\(^{-1}\) s\(^{-1}\)), with \(E\) being the electron kinetic energy and \(\delta\) a constant, called the electron index, then we expect the X-ray photon-number spectrum also to take a power-law form, as

\[
\frac{dJ(h\nu)}{d(h\nu)} \sim (h\nu)^{-\gamma},
\]

where \(\gamma\) is a quantity called the photon index. In 1974, Brown found an approximate relation,

\[
\gamma = \delta - 1,
\]

in the nonrelativistic regime, assuming that all of the emitted photons are collected. The photon spectrum thus becomes relatively flat, because of a “loss-flattening” effect working on the electrons; as they penetrate deeper into the target material, the spectrum gradually hardens, because lower-energy electrons have shorter time scales of energy loss through Coulomb collisions.

Besides the thick-target condition, another extreme case, called the thin-target condition (Brown 1971), was studied extensively. This represents a situation wherein a bunch of energetic electrons pass through a thin medium while emitting...
bremsstrahlung photons, but they leave the emission region with their almost unchanged energies. In the thin-target condition, the energy distribution of the incident electron is not significantly affected by collisions with the target. The emergent photon spectrum is steeper, because the electrons are free from the loss-flattening effect, and is characterized as (Brown 1971)

\[ \gamma = \delta + 1/2. \]  

Since then, there have been many attempts to improve equations (2) and (3), particularly toward the relativistic regime. Some utilized purely analytic calculations of various probability distributions, while others employed numerical solutions to the Fokker–Planck equation, or Monte Carlo techniques. For electron energies of up to 10 MeV, Bai (1982) investigated the angular dependence of the bremsstrahlung in the hard X-ray range, and discussed the associated electron transport. Murphy, Dermer, and Ramaty (1987) dealt with particle transport by employing a simple thick-target emission model, assuming an isotropic electron distribution. Dermer and Ramaty (1986) conducted a detailed study of the directional bremsstrahlung emission by anisotropically accelerated electrons. Miller and Ramaty (1989) made a calculation including the pitch-angle scattering of electrons by plasma waves. These previous works have indicated that the gamma-ray continua of solar flares can be generally explained in terms of chromospheric thick-target emission, produced by electrons having a power-law energy distribution with \( 2 < \delta < 4 \).

In spite of these extensive studies, an estimation of the electron spectra from flare gamma-ray data would not be self-contained, unless we take into account another important physical process; namely, Compton scattering. The relativistic bremsstrahlung from precipitating electrons must be significantly forward beamed, so that a majority of gamma-ray photons must be Compton backscattered by the solar materials in order to reach us. This process would strongly degrade the gamma-ray energies, and affect the photon spectrum to be observed. To treat this kind of physical condition, which involves multiple Compton scatterings, we clearly need to use Monte Carlo techniques.

The effect of Compton scattering has been studied by several authors, beginning, e.g., with Bai and Ramaty (1978), who used Monte Carlo simulations in such early days. Kontar et al. (2006) considered this issue using a semianalytic Green’s function by Magdziarz and Zdzierski (1995), and calculated the “primary” photon spectra from the observed ones. These works have clarified that Compton backscattering can significantly modify the bremsstrahlung spectra. However, these pioneering studies have treated electrons with sub-MeV kinetic energies, leaving higher energies unstudied.

In the present work, we calculated not only the electron transport and bremsstrahlung emission, but also the photon propagation via the Compton process. For this purpose, we employed a Monte Carlo simulation toolkit, named GEANT, which is widely used in experimental high-energy physics. Our results imply that Compton scattering has significant effects on the observed gamma-ray spectra. Together with full-relativistic effects, this is expected to require revision in such relations as equations (2) and (3). We show the relevance of these results to actual flares, by comparing the simulated gamma-ray spectra with those observed with Yohkoh from a sample of solar flares.

2. Method of Simulation

2.1. The GEANT4 Toolkit

The GEANT4 Monte Carlo simulation toolkit (Agostinelli et al. 2003), developed by experimental high-energy physicists, has the following important features: Firstly, it allows a very flexible construction of the “geometry” in which particles interact. Secondly, it can “track” individual particles while monitoring their physical quantities, such as the energy, momentum, and position; the name GEANT in fact comes from “GEometry ANd Tracking”. Thirdly, it allows us to incorporate the desired interactions; in the present case, Coulomb scattering and bremsstrahlung for electrons, and Compton scattering for photons. Finally, the toolkit has been tested and calibrated extensively by many investigators under different conditions.

2.2. Particle Injection, Interaction, and Tracking

In our Monte Carlo simulation using GEANT4, we basically shoot a parallel beam of a large number of electrons, either vertically or slantly, into our “simple solar atmosphere” to be explained later. The electrons are assumed either to be monoenergetic, or to have a power-law energy distribution with an index \( \delta \). In the latter case, the initial electron energies are assumed to be in the range of 1–100 MeV, so as to simulate gamma-ray flares.

We implemented the following physical processes that we require our particles to obey in our simulation: ionization, multiple scattering, and bremsstrahlung for electrons, while photo-absorption, Compton scattering, and electron-positron pair creation for photons. We suppressed the production of secondary electrons for the sake of simplicity. Because those secondary electrons (typically less than 100 keV) have much lower energies than the primary ones, they can be considered to be negligible.

One of the essential features of GEANT4 is the step-by-step tracking of each individual particle, considering all of the implemented elementary processes. The particle-tracking algorithm roughly consists of the following steps: 1) The initial particle velocity is calculated. 2) Each physical process calculates the mean free path of the specified particle under the relevant interaction, and generates a random number around the mean. Then it proposes this random number as a step length. 3) Among various competing processes, the interaction that proposes the shortest physical length is adopted. 4) Track properties, such as the kinetic energy and momentum, of the current particle are updated.

Among the implemented processes, the Coulomb scattering of electrons needs special consideration, because its mean free path is much smaller than those of the other processes (e.g., \(~ 10^{-2}\) of that of bremsstrahlung for a 10 MeV electron). As a result, its full Monte Carlo treatment would demand too much computing times. Therefore, the expected total energy loss of an electron due to multiple Coulomb scatterings, in each step, which is determined by processes other than the Coulomb
scattering, itself, is computed based on semiempirical formulae by Messel and Crawford (1970). Fluctuations around the mean energy loss are expressed by generating random numbers after Lassila-Perini and Urban (1995). Similarly, the total spatial displacement caused by multiple scattering in each step is calculated based on the Lewis method (Lewis 1950), which solves a diffusion equation of electrons in the matter. Fluctuations around the expected mean are again represented which solves a diffusion equation of electrons in the matter.

In this way, the injected electrons and the bremsstrahlung-generated photons are tracked, until they escape out of the model boundary, or they become less energetic than a certain threshold energy, which we set at 1 keV. We collect these photons under a specified condition, and make their spectra.

The simulation of flare-accelerated electrons, precipitating onto the chromosphere, requires yet another physical process to be implemented: electron gyration around the solar magnetic fields. Its exact treatment, however, would make the simulation extremely time-consuming, because the gyration radius of a ~100 MeV electron in a ~100 G field is as small as ~10 m. Therefore, we decided not to take into account the gyration, but to consider slant injections instead; as illustrated in figure 1, injecting electrons at a polar angle $\alpha$, and collecting the emergent photons at another polar angle $\beta$, but all over the azimuth angle $\varphi$, are equivalent to simulating electrons gyrating with a constant pitch angle $\alpha$, as long as the field lines are close to normal to the photosphere. The case of tilted magnetic fields can be reproduced by the superposition of different injection angles, $\alpha$.

2.3. Electron Transport under a Simple Geometry

Before actually simulating the electron transport and gamma-ray emission employing a realistic model for the solar atmosphere, we carried out simpler simulations with two purposes in mind: to validate our simulation code by examining whether each of the relevant physical processes is correctly reproduced, and to grasp the essence of the physics to be investigated. We accordingly approximated the sun as a hydrogen gas box of 10000 km $\times$ 20000 km $\times$ 20000 km, having a uniform density of $3.2 \times 10^{-7}$ g cm$^{-2}$, which is the density at the solar photosphere. Hereafter, we call this model the “simple solar atmosphere model”; it is one of the three uniform target models employed in the present work.

We vertically injected $10^5$ monoenergetic electrons with an initial kinetic energy of 50 MeV into our “simple solar atmosphere”. In order to monitor how the Coulomb interaction affects the spectrum and angular distribution of the electrons, we prepared imaginary boundaries at different depths (100, 200, and 300 km from the injection surface), and collected electrons at each boundary. Figure 2a shows the electron spectra obtained in this way, while figure 2b gives the associated angular distributions. From these, we can see that the maximum energy of the spectrum decreases as the electrons penetrate deeper. Since a 100 km thick slab in our “solar atmosphere” has a column density of $3.2$ g cm$^{-2}$, an electron of energy $\sim 50$ MeV passing through it is expected to lose $\sim 15$ MeV, as we can easily calculate from the Bethe–Bloch formula. The results in figure 2a indeed meet this expectation. The geometrical thickness (10000 km) of the model is thus large enough to make it physically thick to Coulomb loss, for electrons with initial energies of up to $\sim 1.5$ GeV. Furthermore, the electron energy distribution is seen to broaden as the energy loss proceeds.

Figure 2b reveals that the injected electrons, initially forming a parallel beam, are gradually deflected due to Coulomb interactions with ambient atoms. The simulated angular distributions approximately consist of two components: a Gaussian component caused by small-angle multiple scatterings, and a power-law tail due to large-angle Rutherford scatterings.

In a similar way, we simulated the transport of energetic electrons, which has a power-law distribution over the 1–100 MeV range with $\delta = 1.2$. The spectra and angular distributions, measured at depths of 100, 200, 300, 400, and 500 km, are presented in figure 3. The maximum electron energy (initially at 100 MeV) thus decreases according to the Bethe–Bloch formula. Furthermore, the low-energy part of the spectrum gradually flattens, because the energy loss per unit length is roughly independent of the electron energy, and hence lower-energy electrons suffer a larger fractional energy loss than higher-energy ones. This is essentially the same as the “loss flattening” effect mentioned in section 1.

The electron index that we employed here, $\delta = 1.2$, is
Fig. 2. GEANT4 simulation of electrons with an initial kinetic energy of 50 MeV, injected vertically into a hydrogen target with a uniform density of $3.2 \times 10^{-7}$ g cm$^{-3}$ (the simple solar atmosphere). Data were collected at depths of 100 km (red), 200 km (blue), and 300 km (green). (a) Energy spectrum. (b) Distribution (per unit projected angle) of angular deflection, measured from the initial direction of injection.

Fig. 3. Same as figure 2, but when the initial electrons are distributed between 1 and 100 MeV with a power-law spectrum of photon index $\delta = 1.2$. Red, blue, green, cyan, and magenta indicate depths of 100, 200, 300, 400, and 500 km, respectively.

significantly smaller (flatter) than would be achieved in the standard diffusive shock acceleration process (Blandford & Ostriker 1978; Bell 1978). There are the following motivations. One is that the effects of Compton scattering are more clearly observed as $\delta$ becomes smaller. The other is that such a flat electron distribution could be realized in actual solar flares, via, e.g., direct electric acceleration. Hereafter, we hence consider both flat ($\delta < 2$) and steep ($\delta > 2$) electron distributions.

2.4. Bremsstrahlung Spectra

We investigated photon spectra created by the bremsstrahlung process under the two representative conditions: namely the thick-target and thin-target conditions explained in section 1. The GEANT4-simulated gamma-ray spectra are compared with analytic formulae, which are fully relativistic, unlike the nonrelativistic calculations of equations (2) and (3).
2.4.1. Thin-target emission

To validate the bremsstrahlung process under the thin-target condition, which is the simpler of the two, we let \(10^8\) monoenergetic electrons vertically penetrate our second simplified model atmosphere; namely a uniform hydrogen gas with a size of \((10^3\text{ km})^3\), having a density of \(1.0 \times 10^{-11} \text{ g cm}^{-2}\); this is henceforth called the “thin atmosphere model”. This model has a column density of \(1.0 \times 10^{-3} \text{ g cm}^{-2}\), so that the injected electrons lose their energies by no larger than 1%, in agreement with the thin-target condition. We collected information about each bremsstrahlung photon when it was created, and obtained the spectra shown in figure 4.

Also shown in figure 4 are analytically calculated spectra, using a fully relativistic formula (Schiff 1951; Koch & Motz 1959), as

\[
\frac{d\sigma}{dk} = \frac{2Z^2r_e^2}{137}\frac{dk}{k} \left[ \ln \left( \frac{\epsilon}{\epsilon_0} \right)^2 - \frac{2\epsilon}{3\epsilon_0} \ln M(0) + 1 - \frac{2}{b}\tan^{-1}b \right]
\]

\[
+ \frac{\epsilon_0}{2b^2} \ln(1+b^2) + \frac{4(2-b^2)}{3b^3}\tan^{-1}b - \frac{8}{3b^2} + \frac{2}{9} \right] \right],
\]

(4)

with

\[
b = \frac{2\epsilon_0Z^{1/3}}{11k}, \quad M(0) = \left( \frac{k}{2\epsilon_0} \right)^2 + \left( \frac{Z^{1/3}}{11} \right)^2.
\]

Here, \(d\sigma_i/dk\) is a cross section for an electron of total energy \(\epsilon_0 m_e c^2\) (including the rest mass energy) to interact with a target atom of charge Z, and emit a bremsstrahlung photon of energy \(km_e c^2\), thus achieving a final energy of \(\epsilon m_e c^2 \equiv (\epsilon_0 - k)m_e c^2\).

The classical electron radius is denoted by \(r_e\).

As can be seen in figure 4, the difference between our simulation and the analytical calculation is less than 10%, indicating that the bremsstrahlung process has been correctly implemented. In the fully relativistic regime (\(\epsilon_0 \gg 1, \epsilon \gg 1\)), in particular, the emitted gamma-ray spectrum has a power-law shape with a photon index of \(\gamma \simeq 1.0\), or an energy index of \(\simeq 0\), up to the maximum photon energy that is identical to the initial photon energy. This is reasonable, because equation (4) predicts \(d\sigma_i/dk \propto k^{-1}\) when \(\epsilon_0 \sim \epsilon\), and hence \(b \gg 1\).

Figure 5 shows the bremsstrahlung photon spectra calculated under the same thin-target condition, but when the electrons are assumed to have a power-law spectrum distributed from 1 to 100 MeV, with \(\delta = 0.8\) or 1.5. The simulated spectra again exhibit approximately power-law like shapes, with a steeper slope than that in the case of monoenergetic electrons, because softer electrons can obviously emit only lower-energy photons.

In figure 5, we also show photon spectra calculated analytically using equation (4), as

\[
N(k) = n_i \int_{k+1}^{E_{\text{max}}} \psi(\epsilon_0) J_\epsilon(\epsilon_0) \frac{d\sigma}{dk} d\epsilon_0,
\]

(5)

where \(N(k)\) is the photon number spectrum, \(J_\epsilon(\epsilon_0)\) the injected electron spectrum (\(\propto \epsilon_0^{-\delta}\) in the present case), \(n_i\) the target ion density, and \(\psi(\epsilon_0)\) the velocity of an electron having an energy \(\epsilon_0\). The simulation agrees with the analytic results within \(\sim 20\%\). Thus, the GEANT4 simulation and the fully relativistic analytic formalism consistently indicate \(\gamma \simeq 1.3\) for \(\delta = 0.8\) and \(\gamma \simeq 1.6\) for \(\delta = 1.5\), both measured in a typical gamma-ray energy range of 1 to 10 MeV. These photon indices are significantly smaller (flatter), particularly for \(\delta = 1.5\), when compared to the conventional nonrelativistic
Fig. 6. (left) Relation between the index $\delta$ of power-law distributed (1–100 MeV) electrons and the resulting thin-target bremsstrahlung photon index $\gamma$. The red lines is the calculation from equation (5). The blue crosses show the results of GEANT4 simulations, and the black line refers to the nonrelativistic formula represented by equation (3). The photon index refers to the 1–10 MeV range. (right) The same as left panel, but for thick target emission. The analytical results (red) are calculated from equation (8), while the blue line represents the nonrelativistic relation in equation (2).

In this way, we repeated the GEANT4 simulation with difference values of the electron index, and calculated the bremsstrahlung photon index over an energy of 1–10 MeV. The obtained relation between the electron index and the photon index is shown in figure 6a, in comparison with the prediction by equation (3). We can see that the formula of equation (3) in the nonrelativistic regime, namely $\gamma = \delta + 1/2$, is no longer valid in the present relativistic regime, with the discrepancy that increases with electron indices.

The cross section of bremsstrahlung as a function of the emitted photon energy and the angle is given by Schiff (1951) and Koch and Motz (1959) as

$$
dsigma_{\theta, \gamma} = \frac{4Z^2\epsilon_{\gamma}^2}{137} \frac{d\kappa}{k} y^2 \left\{ \frac{16\gamma^2\epsilon_{\gamma}}{y^2 + 1} + \frac{(\epsilon_{\gamma} + \epsilon)^2}{y^2 + 1} \right\} \ln M(y) + \left[ \frac{\epsilon_{\gamma}^2 + \epsilon^2}{y^2 + 1} - \frac{4\gamma^2\epsilon}{y^2 + 1} \right] \ln M(y).
$$

where the “reduced photon angle”, $y$, together with its function, $M(y)$, is given by

$$
y = \epsilon_0\theta_0, \quad \frac{1}{M(y)} = \left( \frac{k}{2\epsilon_0\epsilon} \right)^2 + \left[ \frac{Z^{1/3}}{111(y^2 + 1)} \right]^2.
$$

and $\theta_0$ is the initial angle of an emitted photon. Figure 7 shows equation (6) as a function of $\epsilon_0\theta_0$, where the results of the simulation using the thin atmosphere model are superposed. Thus, the whole emission becomes concentrated roughly within $\epsilon_0\theta_0 \leq 1$; the emission is strongly forward peaked to within $\theta_0 \leq \epsilon_{10}^{-1}$.

2.4.2. Thick-target emission

The thick-target emission is significantly more complex than the thin-target case, because it results from electrons, while their energies change continuously due to Coulomb scattering. Moreover, it has a higher practical importance, because the flare hard X-rays and gamma-rays are thought to be produced mainly through this process in the solar chromospheric and denser regions.

Under the thick-target condition, we first investigated the emission from monoenergetic electrons, as we did for the thin-target case. We shot $10^7$ electrons with an initial kinetic energy of 10 MeV or 50 MeV into a hydrogen target of which the size was $(10^4 \text{ km})^3$; this is our third simplified model atmosphere, called the “thick atmosphere model”. Figure 8 shows the spectra produced by collecting all photons emitted by each electron while its energy gradually decreased down to $\lesssim 1$ keV. Compared with the thin-target emission from monoenergetic electrons, the photon spectra are somewhat steeper and roll more prominently toward the maximum...
energy. Obviously, this is because the electrons keep their energies losing while radiating.

In figure 8, we also show the results of analytic calculations, which are formulated as

$$N_{\text{ini}}(\epsilon, k) = n_i \int_{\epsilon_{\text{ini}}}^{\epsilon_{\text{max}}} \frac{v \, d\sigma}{d\epsilon} \, d\epsilon,$$

where $\epsilon$ is the time, $\epsilon_{\text{ini}}$ is the initial total energy of an electron, $d\epsilon/dt$ its energy loss rate, $v$ the instantaneous electron velocity, and $n_i$ the ion density of the target plasma. And $d\sigma/dk$ refers to equation (4). Again, the simulation and analytic calculation agree within $\pm 20\%$.

We investigated the thick-target emission from power-law distributed electrons, with $\delta = 0.8$ or 1.5, over the 1–100 MeV range. Employing $10^6$ electrons, we collected information about the photons in the same way as in the preceding simulations, and obtained figure 9. Designating the electron injection rate (electrons cm$^{-2}$s$^{-1}$) by $f(\epsilon_{\text{ini}})$ and the target area by $A$, and employing equation (7), the photon production rate is analytically given by

$$N(k) = A \int_{\epsilon_{\text{min}}}^{\epsilon_{\text{max}}} \frac{N_{\text{ini}}(\epsilon_{\text{ini}}, k) f(\epsilon_{\text{ini}}) \, d\epsilon_{\text{ini}}}{\int_{\epsilon_{\text{ini}}}^{\epsilon_{\text{max}}} \frac{v \, d\sigma}{d\epsilon} \, d\epsilon}.$$

This prediction is again shown in the figure. The simulation agrees within $\sim 20\%$ with the analytic calculation via equation (8).

As can be seen from figure 9, the photon spectra emitted by power-law distributed electrons under the thick-target condition again exhibit the power-law shape up to $\sim 10$ MeV, similarly in the case of thin-target emission (figure 5). The obtained photon index is $\gamma \approx 1.4$ for $\delta \approx 0.8$ and $\gamma \approx 1.5$ for $\delta \approx 1.5$. In contrast, the hardest portion of the spectra exhibits a rather opposite trend, and cannot become as hard as implied by equation (2), since the maximum energy of electrons decreases as they propagate deeper through the thick target.

Figure 6b is the same comparison as figure 6a, but for the thick-target condition. The photon indices in this case are generally smaller compared with thin-target emission, because of the loss-flattening effect (section 1). In addition, $\gamma$ depends more weakly on $\delta$ than in the thin-target case.

3. Solar Simulation

3.1. A More Realistic Model Atmosphere

In the previous section, we treated the solar atmosphere as a uniform gas slab. This is basically reasonable, because the energy losses of electrons and photons are primarily determined by the column density of matter along its path. The actual solar atmosphere, with a strong vertical density gradient, could be transformed into such a uniform zone, as long as all of the particles are traveling vertically. However, particle trajectories with significant transverse components can no longer be conserved by such a transform, and hence we need to construct a more realistic solar model, while taking into account its vertical density gradients. We here refer to “Harvard-Smithsonian reference atmosphere” (Gingerich et al. 1971) for the solar mass-density profile, and approximate this numerical table analytically by an exponential function,

$$\rho(z) = 3.19 \times 10^{-7} \exp\left(-\frac{z}{h}\right).$$
photosphere, \( \rho(z) \) is the mass density in units of \( \text{g cm}^{-3} \), and \( h \) is the scale height, which is \( \sim 400 \text{km} \) for \( z < 0 \) (the solar interior) and \( \sim 110 \text{km} \) for \( z > 0 \) (the coronal region).

We construct our new solar atmosphere model as a series of boxes (or “zones”) stacked in the vertical direction. Individual zones are defined to have different densities, but to share approximately the same column density; the density within each of them is assumed to be uniform. Each zone is assumed to have a lateral extent of \( 2 \text{ km} \). Each zone is assumed to have approximately the same column density of \( 3.5 \text{ g cm}^{-2} \).

According to the Harvard-Smithonian model, the total column density above \( z = 0 \) is \( 3.5 \text{ g cm}^{-2} \), which corresponds to a depth of \( \sim 100 \text{ km} \) in our simple atmosphere model. Also, the sum of the 5 coronal zones in our new model gives an overall column density of \( 3.5 \text{ g cm}^{-2} \), thus faithfully representing the actual solar atmosphere for \( z > 0 \). While traversing each zone, a 100 MeV electron loses \( \sim 15 \text{MeV} \) in total, of which only \( \sim 20\% \) is in radiation, while the rest is in Coulombic loss.

In simulating the solar interior \((z < 0)\), we use 10 thicker zones having \( 3.5 \text{ g cm}^{-2} \) each; again, this corresponds to a depth of \( \sim 100 \text{ km} \) in our simple atmosphere model. With the overall column density of \( 3.5 \text{ g cm}^{-2} \) below \( z = 0 \), our new model goes down to a depth of \( \sim 500 \text{km} \), below the solar photosphere, in comparison with equation (9). This depth is considered to be sufficient, because a 100 MeV electron, injected at the top of our model, would lose \( \sim 100\% \) of its initial energy by the time it reaches the bottom of the deepest zone. Furthermore, the 10 thicker zones, when summed up, have a Compton optical depth of \( \sim 10 \), for a vertically precipitating photons; few photons would penetrate deeper.

Although our “solar atmosphere” is comprised of pure hydrogen, the actual solar atmosphere contains helium to \( \sim 10\% \) by number, or \( \sim 25\% \) by mass. Since the ionization loss of electrons is proportional to the electron number density, the helium would decrease the ionization loss per unit mass column density by \( \sim 10\% \). Bremsstrahlung per unit mass should remain the same, because a helium atom has a 4-times higher mass than a proton, but a 4-times higher bremsstrahlung cross section at the same time. Compton scattering depends only on the number of electrons, and hence would decrease by \( \sim 10\% \) if helium was included.

### 3.2. Vertical Injection

To examine the flare photon spectra, we vertically injected \( 10^6 \) electrons into our solar atmospheric model from a height of \( z = 10000 \text{ km} \), and tracked them from that height to \( \sim 500 \text{km} \), as they lose energy via Coulomb scattering and emit bremsstrahlung photons. The electrons are assumed to be distributed initially in the \( 1-100 \text{MeV} \) range with a power-law index of \( \delta = 1.2 \). The electrons, initially moving in the vertical direction, gradually become deflected, as indicated in figure 3; in the actual configuration, this corresponds to pitch-angle scattering. Nevertheless, as represented by figure 3 in red, still \( 98\% \) of the electrons are contained within \( \sim 5^\circ \) of the vertical direction when they have reached the photosphere. Therefore, the vertical injection employed here is valid as long as the magnetic fields are sufficiently normal to the solar photosphere, the electrons’ pitch angle is initially concentrated near \( \sim 0^\circ \), and non-Coulombic (e.g., magnetohydrodynamic, or due to magnetic mirroring) pitch-angle scatterings can be neglected.

The bremsstrahlung photons emitted by the almost vertically moving electrons are strongly forward collimated, as indicated by figure 7. Therefore, most of them are expected to enter the atmosphere and undergo Compton scattering, or photoabsorption. Although some of these photons will die, others will be scattered back to escape out of the photosphere. We collected these outcoming photons at different viewing angles, \( \beta \) (figure 1), and averaged the results over \( 0 \leq \varphi < 2\pi \).

Figure 11 shows the photon spectra obtained in this way for equal intervals in \( \cos \beta \) (hence, over an equal solid angle). In figure 11, spectra with a small \( \beta \), simulating disk-center flares, exhibit a clear “knee”, which represents the effect of single Compton scatterings. In the case of \( \beta \sim 0 \) (blue in figure 11), the knee energy becomes \( \sim (1/2) m_e c^2 \), because of the strong energy degradation in the single Compton backscatterings. Photons with energies above the knee result...
Fig. 11. Monte Carlo simulated spectra of gamma-rays emergent from our “solar atmosphere”, when 1–100 MeV electrons with $\delta = 1.2$ are vertically injected into it. The photons, mostly Compton backscattered, are averaged over the observer’s azimuth $\varphi$ (figure 1), and collected at different viewing angle $\beta$; red, green, and blue crosses represent the photons satisfying $0.1 < \cos \beta < 0.2$, $0.5 < \cos \beta < 0.6$, and $0.9 < \cos \beta < 1.0$, respectively.

As $\beta$ gets larger in figure 11, the knee energy increases, because the angle needed in a single Compton scattering decreases. Nevertheless, the spectrum remains much softer (e.g., $\gamma \sim 3$ in the 0.2–0.6 MeV range) than is expected when the Compton effects are not taken into account ($\gamma \sim 1.3$; figure 6b). The observed photon flux below the knee energy decreases as $\beta$ increases and approaches $90^\circ$, due to the same limb-darkening effect as in visible light.

3.3. Slant Injection

As we confirm in figure 11, bremsstrahlung photons back-scattered from the solar photosphere exhibit very steep spectra as the result of Compton degradation. In order to explain the origin of the observed flare spectra, often extending beyond 1 MeV, we must then consider photons that reach us after Compton scatterings with much smaller angles. For this purpose, we had to consider electrons penetrating into the atmosphere with shallow angles (i.e., large $\alpha$), namely “slant injection”.

We then examined the case of slant injection, by shooting $10^6$ power-law electrons (in the 1–100 MeV energy range), into our realistic solar model. Specifically, we repeated the GEANT4 simulation under the same condition, but assuming this time an injection angle of $\alpha = 80^\circ$ (figure 1), instead of the vertical injection ($\alpha = 0^\circ$) considered in the previous subsection. We thus injected the electrons from a height of 100 km (i.e., neglecting the uppermost two zones in figure 10), so as to keep them from escaping sideways before arriving at thicker regions. Figure 12 shows the results of this simulation for the same three intervals of $\beta$ as figure 11, again averaged over the observer’s azimuth angle, $\varphi$. The spectrum becomes considerably harder than that in the case of vertical electron injection, and extends much beyond $\sim 1$ MeV when $\beta$ is rather large ($0.1 < \cos \beta < 0.2$). In this case, electrons make grazing incidence on the atmosphere, and some of their bremsstrahlung photons become nearly forward scattered to leave the zone.
with a small energy loss, thus producing the hard power-law continuum.

In this way, we Monte Carlo simulated slant injections for a family of electron spectra with various indices, $\delta$ (again distributed over 1–100 MeV), assuming two representative injection angles of $\alpha = 60^\circ$ and $\alpha = 80^\circ$. Figure 13 summarizes the photon index, $\gamma$, of 1–10 MeV gamma-rays, collected over the most favorable viewing angle of $0.1 < \cos \beta < 0.2$ (i.e., rather grazing to the solar photosphere) and integrated over the observer’s azimuth. Thus, even with the extreme assumption of $\alpha = 80^\circ$, the simulated gamma-ray spectra become significantly softer (typically by $\sim 1$ in $\gamma$) than the original bremsstrahlung spectra before the photons are Compton scattered (reproduced in figure 13 in blue). Moreover, $\gamma$ further increases by $\sim 0.5$ as $\alpha$ decreases from $80^\circ$ to $60^\circ$. These results imply that the Compton scattering significantly softens the emergent photon spectra, of which the effect depends sensitively on $\alpha$ and $\beta$.

An interesting prediction of figure 13 is that the observed gamma-ray photon index should appear in a relatively narrow range of $\gamma = 1.7$–2.6, regardless of the electron index, $\delta$. Of course, we would observe considerably larger values of $\gamma$ when either $\alpha$ or $\beta$ is rather small. However, such a spectral component would not be easily detected in the MeV energy region.

Although our simulations of the slant injection neglected the uppermost two coronal zones, their effects may not necessarily be negligible, because the large injection angle of $\alpha = 80^\circ$ (or $60^\circ$) effectively increases the electron path length in each zone by $(\cos 80^\circ)^{-1} = 5.8$ [or $(\cos 60^\circ)^{-1} = 2.0$] times. Since the electron energy losses in these two regions, if properly taken into account, would reduce the maximum electron energies while flattening the low-energy electron slope (figure 3), the emergent photon flux above $\sim 1$ MeV would be further suppressed if the electron injection in actual flares takes place at a height much exceeding the assumed 100 km. In other words, our simplification employed in simulating the slant injection makes the case more conservative.

4. Discussion

4.1. Summary of Simulations

In order to investigate the effects of Compton scatterings on the solar gamma-ray spectra, we conducted Monte Carlo simulations of energetic electrons, impinging on the solar atmosphere, using GEANT4 as our basic toolkit. We modeled the solar atmosphere as vertically stacked parallel hydrogen zones, each assumed to be uniform. The density gradient was expressed by assigning higher densities to lower zones. We injected electrons power-law distributed over the 1–100 MeV range, changing their spectral index $\delta$ and incident angle $\alpha$. The results of the present simulations can be summarized as follows:

1. The observed photons hardly reach MeV energies if the electrons are injected vertically, because the bremsstrahlung-produced gamma-rays must be Compton backscattered to reach the observer, and hence their energies are subject to strong Compton degradation.

2. The emergent gamma-ray spectrum, integrated over the observer’s azimuth, gradually hardens as the injection angle $\alpha$ increases (becoming more “slant”), and also as the observer’s polar angle $\beta$ becomes larger.

3. As long as $\alpha$ and $\beta$ are both relatively large, the azimuthally averaged 1–10 MeV gamma-ray spectra exhibit $\gamma = 1.7$–2.6 over a relatively wide range of $\delta$.

These results generally agree with, and further extend, previous studies by, e.g., Bai and Ramaty (1978) and Kontar et al. (2006).

4.2. Effects Ignored in the Simulation

There are some effects that were not considered in our simulation. Among them, those of the secondaries electrons and helium were already described in subsections 2.2 and 3.1, respectively. As the electron energy increases, its synchrotron energy loss becomes significant. However, in the chromospheric region in which we are interested, the magnetic field intensity is typically $\sim 100$ G, and hence the rate of synchrotron loss of 100 MeV electrons is only $\sim 1\%$ of their Coulomb loss, and only $\sim 5\%$ of their bremsstrahlung loss. This is because the Coulomb and bremsstrahlung losses are rather large due to the high matter density. Thus, the synchrotron process can be neglected.

We also ignored the inverse Compton process between flare-accelerated electrons and ambient photons, because electrons with the assumed maximum energy, 100 MeV, can produce at most hard X-rays up to $\sim 100$ keV by scattering off the solar visible photons. However, if the electrons have a very flat spectrum well extending to the energy above 350 MeV, the inverse Compton process may become a key process in creating gamma-ray photons.

The size of the world used in our simulation, $10000 \times 10000 \times 20000$ km, simulates the typical size of a foot point of the magnetic tube. The number of photons that escaped from the boundaries below the photosphere was less than $10^{-4}$ of that of produced photons, and energies of these escaped photons were less than 100 keV each. Hence, we ignored those photons in the simulation. Possible effects of the relatively low heights of our slant injection were already considered in subsection 3.3.

In the present simulation, we truncated the power-law electron spectrum at 100 MeV. In order to examine whether this abrupt cutoff would produce any artifact, we repeated some of the simulations by changing the maximum cutoff energy to 1 GeV. We thus have found that $\gamma$ decreases only slightly, typically 0.1, or less, if $\delta$ is larger than 2.

Finally, a separate gamma-ray component could be produced without the Compton effect, simply if the electron spectrum consists of two power-law components with different slopes. Even so, the good agreement between the predicted range of the power-law indices and the Yohkoh data suggests that Compton scattering plays an important role (see the next section).

4.3. Comparison with Observations

In order to evaluate the significance of the results of our simulation, let us compare them with actual observations of
hard X-ray and gamma-ray emission from solar flares. For the purpose, we must also relate the simulation parameters, $\alpha$ and $\beta$, to the observational parameters, including the pitch-angle distributions of electrons, the tilt of the magnetic field lines, and the flare longitude on the solar disk. As observational data, we utilize 40 flares that show significant gamma-ray emission, selected out of 2788 X-ray flares observed from 1991 October to 2001 December by Yohkoh (Matsumoto et al. 2005). Yohkoh, the Japanese solar satellite launched in 1991, was able to take four-color hard X-ray flare images with the Hard X-ray Telescope (HXT) (Kosugi et al. 1991, 1992) and perform spectro-photometry over the 0.2–30 MeV range with the Gamma-Ray Spectrometer (GRS) (Yoshimori et al. 1991).

4.3.1. Vertical Magnetic Fields

As a very simple case, we may consider a condition where the magnetic field lines are perpendicular to the photosphere. In this case, we can identify $\alpha$ with the pitch-angle of an electron, and the effect of electron gyration around the magnetic field lines is fully represented by our photon collection method, which takes an average over $\varphi$. If the electrons have a narrow pitch-angle distribution around 0, the strong Compton degradation, described in subsection 3.2 and shown in figure 11, will make the observed photon spectrum very steep, and will prevent it from reaching MeV energies. If, on the other hand, the pitch-angle distribution is rather broad, electrons with larger values of $\alpha$ will emit harder spectra, as represented in figure 12. Conversely, as long as the field lines are nearly vertical, flares with significant gamma-ray emission must involve broad pitch-angle distributions of electrons.

Analyzing the 40 gamma-ray flares detected by Yohkoh, Matsumoto et al. (2005) found that their gamma-ray (typically 1 MeV) to hard X-ray ($\sim$70 keV) flux ratios scatter largely (by an order of magnitude), with no correlations to their hard X-ray spectral slopes. Hence, they conclude that there is a hidden parameter that causes the gamma-ray flux to vary significantly from the hard X-ray extrapolation, in such a way that the gamma-ray component often appears as a separate spectral hard tail (see their figure 6). Given the present results, this parameter is very likely to be the electron pitch-angle distribution, with broader distributions yielding higher gamma-ray to hard X-ray flux ratios.

It has been confirmed that the viewing angle, $\beta$, as well as the electron injection angle, $\alpha$, strongly affected the flare gamma-ray spectra (section 3). Assuming again the vertical field geometry, we can identify $\beta$ with the angle $\theta_\nu$, which our line-of-sight to the flare makes relative to the normal to the photosphere. Hereafter, we simply call $\theta_\nu$ "the flare-viewing angle". Using the flare longitude, $l$, and latitude, $b$, it can be calculated as

$$\theta_\nu = \cos^{-1}(\cos b \cos l).$$  (10)

To examine the effect of $\beta$, we plot in figure 14 the gamma-ray to hard X-ray hardness ratio of the 40 Yohkoh flares, as a function of $\theta_\nu$, calculated via equation (10). Figure 14 is similar to figure 7 of Matsumoto et al. (2005), but differs from their figure in that $\theta_\nu$ is used instead of $|l|$. As predicted by our simulation, the hardness ratio clearly increases with $\theta_\nu$; this effect was already known as "gamma-ray limb brightening" in solar flares using SMM data (Vestrand et al. 1987). Vestrand et al. (1987) interpreted this effect in terms of the radiation anisotropy; on the supposition that the electrons have a pitch-angle distribution which increases with angle from the outward normal, then higher-energy continua are expected to be more strongly beamed parallel to the photosphere, because harder bremsstrahlung photons are more strongly forward-beamed than softer ones, even when emitted by electrons of the same energy. We presume that this mechanism can explain at least part of the observed limb-hardening effects, while the Compton backscattering process must be enhancing them. In particular, our scenario can explain limb hardening even when the electrons are mostly directed downward.

For a more quantitative comparison of our simulation with the Yohkoh observations, let us limit ourselves to limb flares, which have absolute longitudes larger than 60°, because gamma-ray data of disk flares are of poorer photon statistics than those of limb flares, as is clear from figure 14, and because these disk flares are more contaminated by nuclear gamma-ray lines (Matsumoto 2002), which are unrelated to the accelerated electrons. Figure 15 shows a scatter plot between two hardness ratios of the 40 sample flares, one calculated in the hard X-ray range (57–93 keV vs. 33–57 keV), while the other is in the gamma-ray range (1.43–6.21 MeV vs. 0.22–1.43 MeV). In this "color-color" plot, the dashed line represents the condition that the two colors imply the same photon index, $\gamma$, of which the values are given in the figure. The hard X-ray (or gamma-ray) photon index of a flare can be found by drawing a vertical (or horizontal) line from the data point, and by reading its intersection with the dashed line.

In figure 15, we can see that the gamma-ray spectral index of the 40 flares is distributed in a typical range of 1.7–2.5. This agrees very well with the prediction by our Monte Carlo simulations (figure 13; subsection 3.3), on condition that the electrons in these flares have broad pitch-angle distributions beyond $\alpha \sim 60°$. Further, in figure 15, a majority of flares are distributed to the left side of the single power-law locus. Such a concave-shaped wide-band spectrum can be naturally explained as a consequence of a broad pitch-angle distribution of electrons, because electrons with small pitch angles are
the photosphere and the flare-viewing angle long as the magnetic lines are assumed to be perpendicular to large pitch angles emit much harder spectra. A superposition

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hard X-ray intensity ratio for limb flares (figure 14, with us, the gamma-ray spectra are expected to soften, on the narrow pitch-angle distribution. If the lines are tilted toward energies, e.g., a few MeV, even if the electrons have a rather tilted away from us, the observed spectra tend to reach higher expected to have a prominent effect; if the field lines are case, a field-line tilt on the sky plane is considered to have

some aspects of that case in a qualitative way.

4.3.2. Slant magnetic fields

As a more general case, we may briefly consider tilted magnetic-field conditions. In this case, it is somewhat difficult to directly compare the results of our simulation with the observational data, because the electron gyration can no longer be faithfully emulated by our method of taking an average over the photosphere normal (i.e., our line-of-sight) without depending on the tilt direction. Then, an electron with a fixed pitch angle, \( \alpha_0 \), is expected to sweep a range of injection angle, from \( \alpha = |\alpha_0 - \theta| \) to \( \alpha = \alpha_0 + \theta \). Therefore, a larger value of \( \theta \) is favorable for delivering strong gamma-ray signals to the observer, even though \( \beta \) is still limited to the unfavorable value of \( \sim 90^\circ \). If \( \alpha_0 \) and \( \theta \) are both rather large, so that \( \alpha_+ > 90^\circ \), the electron is implied to be approaching us over a certain phase of its gyration, thus allowing a considerable fraction of the bremsstrahlung photons to directly reach us without being Compton scattered. Therefore, a disk flare may become a strong gamma-ray source with a limited condition, that the electrons have a broad pitch angle and that the magnetic field lines are significantly tilted.

4.3.3. Case of a broad pitch-angle distribution

Now that broad pitch-angle distributions of accelerated electrons are considered to be an important element for observing intense gamma-ray emission, it may be imperative to simulate such a case, and to compare the results with gamma-ray spectra from actual solar flares. Therefore, we repeated our solar simulation, assuming that the electrons have a continuous incident-angle distribution, as

\[
\alpha \propto \frac{3}{2} \left(1 - \cos^2 \alpha\right)
\]

which corresponds to \( M_2(\cos \phi) \) in Dermer and Ramaty (1986) except for covering only the range of \( 0 \leq \cos \alpha \leq 1 \). When the magnetic fields are vertical, equation (11) becomes identical to the pitch-angle distribution of the accelerated electrons. Under this angular distribution, we injected \( 10^7 \) electrons in the same way as in section 3, assuming the incident electron spectrum to have a canonical index of \( \delta = 2.0 \), and to extend again over 1–100 MeV. Collecting the produced gamma-rays over a viewing-angle range of \( 0.1 \leq \cos \beta \leq 0.2 \), and convolving the results with an approximate response of the Yohkoh GRS (Yoshimori et al. 1991), we obtained the spectrum shown in figure 16 in red.

To compare the solar flare with this simulation, we chose a GOES X3.7 class flare of 1998 November 22, 06:37 UT, which is one of the brightest gamma-ray events among the 40 Yohkoh flares (Matsumoto et al. 2005). Since this is an extreme limb flare located at S31W90, our viewing angle, \( \beta \), should be close to \( 90^\circ \), in agreement with the simulation condition. Actually, the HXT highest-band images of this flare reveal intense hard X-ray emission from a pair of magnetic loop foot points, both located right on the solar west limb (Matsumoto 2002). The GRS data, accumulated over a period of 06:37:22–06:40:22, are shown in figure 16 in blue, after subtracting background (accumulated for 06:56:14–06:59:02), but without removing the detector response. Thus, the spectrum is featureless, and the 2.23 MeV neutron capture line is undetectable with its equivalent width being \(< 10 \) keV (Matsumoto 2002). Therefore, this flare is considered to be a typical “electron-dominated” event.

In figure 16, the simulation and the actual data agree reasonably well, at least over the 1–10 MeV band. When the
thick-target emission. We should instead find \( \Gamma = \Gamma_{\text{GRS}} \) given figure 15. Incidentally, we would obtain \( \Gamma = \frac{1}{\alpha} \) for the GRS response is considered, these spectra can be approximated by \( \Gamma = 2.0 \pm 0.1 \) (Matsumoto 2002), which is typical in figure 15. Incidentally, we would obtain \( \Gamma \sim 1.7 \) for the assumed \( \delta = 2.0 \) in figure 13, if we were observing direct thick-target emission. We should instead find \( \Gamma = 2.2 \) (for \( \alpha = 80^\circ \)) to \( \Gamma = 2.5 \) (for \( \alpha = 60^\circ \)), if the Compton backscattering dominates. Therefore, the values of \( \Gamma \sim 2.0 \) found in figure 16 are in between these two cases. Indeed, an inspection of the Monte Carlo photons revealed that \( \sim 60\% \) of them are Compton-scattered gamma-rays, while the remaining \( \sim 40\% \) are direct bremsstrahlung events. Given the good agreement between the two data sets, it is possible that the actual gamma-rays from the 1998 November 22 flare are also a mixture of the direct and scattered photons.

5. Conclusion

In order to quantitatively estimate the spectra of energetic electrons generated in solar flares, based on the observed gamma-ray spectra, we numerically studied the electron transport and gamma-ray emission in the solar atmosphere. As elementary processes, we mainly considered Coulomb scattering, bremsstrahlung, and Compton scattering. We modeled the solar atmosphere with a vertical stack of parallel zones with different densities, and neglected the magnetic fields.

As a result of our simulation, we found that the thick-target gamma-ray spectra emitted from the magnetic loop footpoint can hardly reach MeV energies, because of heavy Compton degradation, unless the electrons make rather grazing angles to the photosphere, and we are observing forward-scattered gamma-rays. When this Compton effect is taken into account, the traditional \( \delta \) vs. \( \gamma \) relation is drastically modified. The pitch-angle distribution of flare-accelerated electrons is suggested to be broad and to extend to large angles for those flares of which the emitted photon spectra reach MeV energies. Actually, a gamma-ray spectrum simulated under electron injection with a broad pitch-angle distribution reproduced reasonably well the Yohkoh spectrum of the 1998 November 22 flare.

In terms of actual flares, the above results can be translated into the following conclusions:

1. When the magnetic field lines are perpendicular to the photosphere, and the electrons have a narrow pitch-angle distribution around 0, the gamma-ray spectra are very soft and cannot reach MeV energies.
2. Even in the same field configuration, the gamma-ray spectra become harder if the electrons have larger pitch angles. Therefore, the electrons producing gamma-rays are inferred to have relatively large pitch angles.
3. Limb flares are expected to have harder spectra than disk flares, because of their smaller angles of Compton scattering. This agrees with the observed fact that the gamma-ray to hard X-ray intensity ratio increases as the flare longitude becomes closer to the solar limb.
4. In the case of a broad pitch-angle distribution, electrons with small pitch angles emit photons with flatter spectra. This can explain another Yohkoh result, that the gamma-ray slope tends to be flatter than the contemporaneous hard X-ray slope.
5. Under the conditions of \( \delta = 1–3 \) and \( \alpha > 60^\circ \), the values of \( \gamma \) obtained in our simulations in the 1–10 MeV range are concentrated over 1.7–2.5. This agrees again with the Yohkoh observations.
6. A disk flare may become a strong gamma-ray source under a limited condition, that the electrons have a broad pitch angle and the magnetic field lines are significantly tilted.
7. The gamma-ray spectra of actual solar flares are expected to be an appropriate mixture of the direct and Compton-scattered photons.

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