Residual Symmetries for Neutrino Mixing with a Large $\theta_{13}$ and Nearly Maximal $\delta_D$

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The residual $Z_2^s(k)$ and $Z_2^c(k)$ symmetries induce a direct and unique phenomenological relation with $\theta_{13}(=0_{13})$ expressed in terms of the other two mixing angles, $\theta_{12}(=\theta_{12})$ and $\theta_{13}(=\theta_{23})$, and the Dirac CP phase $\delta_D$. $Z_2^s(k)$ predicts a $\theta_{13}$ probability distribution centered around $3^\circ \sim 6^\circ$ with an uncertainty of $2^\circ$ to $4^\circ$ while those from $Z_2^c(k)$ are approximately a factor of two larger. Either result fits the T2K, MINOS and Double Chooz measurements. Alternatively a prediction for the Dirac CP phase $\delta_D$ results in a peak at $\pm 74^\circ$ ($\pm 108^\circ$) for $Z_2^s(k)$ or $\pm 123^\circ$ ($\pm 57^\circ$) for $Z_2^c(k)$ which is consistent with the latest global fit. We also give a distribution for the leptonic Jarlskog invariant $J_\nu$ which can provide further tests from measurements at T2K and NOvA.

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Introduction — The T2K [1] and MINOS [2] experiments indicate a relatively large reactor angle $\theta_x$ for neutrino mixing. At the 90% C.L., T2K gives $0.03 (0.04) < \sin^2 2\theta_x < 0.28 (0.34)$, with zero Dirac CP phase, $\delta_D$, for normal (inverted) hierarchy while MINOS gives $0.01 (0.026) < \sin^2 2\theta_x < 0.088 (0.150)$ and Double Chooz [3] with $\sin^2 2\theta_{13} = 0.085 \pm 0.051$ at 68% C.L.

Many varied theoretical efforts have been made to understand this large $\theta_x$. Discrete groups such as $S_3$ [4], $A_4$ [11, 12], $S_4$ [6, 8], and the binary tetrahedral group $T'$ [8] have been quite popular while new possibilities are explored in [10]. Other efforts concentrate on perturbations from some featured zeroth-order mixing such as democratic [11, 12], bimaximal [8, 12, 13], tribimaximal [6, 13, 14], and tetra-maximal [15] patterns. More discussions can be found in [14].

In these papers symmetries or other model assignments are employed. We will show that phenomenological consequences of residual symmetries $Z_2^s(k)$ and $Z_2^c(k)$ can be readily established predicting not only $\theta_{13}$ to be large, fitting the T2K, MINOS and Double Chooz data, but also $\delta_D$ nearly maximal in good agreement with the latest global fits. This provides the first strong and direct evidence for residual symmetries.

Residual Symmetries — The symmetry that directly determines the lepton mixing pattern need not be the same as the full symmetry of the fundamental lagrangian. As the left-handed charged lepton and neutrino reside in a same $SU(2)_L$ doublet, they are governed by a common symmetry which must be broken. Otherwise they would share a same diagonalization matrix [17, 18], leading to trivial leptonic mixing. It is the residual symmetry that determines the mixing matrices, if indeed the mixing is believed to be determined by symmetry.

It is convenient to work in the diagonal basis of charged leptons [19]. To completely determine the mixing matrix, a product of two $Z_2$ symmetries is enough [18, 20]. One is the well-known $\mu-\tau$ symmetry [21], the other is $Z_2^c$ [18] which can be extended to accommodate a general solar angle [22] generated by,

$$G_1(k) = \frac{1}{2 + k^2} \begin{pmatrix} 2 - k^2 & 2k & 2k \\ 2k & k^2 & -2 \\ 2k & -2 & k^2 \end{pmatrix}.$$  (1)

There is another residual $Z_2^c(k)$ represented by $G_2(k) = G_1G_3$, where $G_3$ is the matrix for $\mu-\tau$ symmetry [18],

$$G_2(k) = \frac{1}{2 + k^2} \begin{pmatrix} 2 - k^2 & 2k & 2k \\ 2k & k^2 & -2 \\ 2k & -2 & k^2 \end{pmatrix}.$$  (2)

Since $\mu-\tau$ symmetry is just a first order approximation, indicated by the experimental data [13] and other considerations [21], it has to be broken. The remaining symmetry would be $Z_2^s(k)$ or $Z_2^c(k)$ but not both as they are not independent. However, their phenomenological consequences need not be the same as we show below. Note that, since the diagonal mass matrix of charged leptons is not degenerate, $G_1$ and $G_2$ only apply to the neutrino sector after the full symmetry is broken down to residual symmetries.

Correlation Between Mixing Angles — With a single $Z_2^s(k)$, a correlation between the three mixing angles and

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the Dirac CP phase can be derived. In particular,
\[ \cos \delta_D = \frac{(s_a^2 - c_a^2 s_x^2)(c_a^2 - s_x^2)}{4 c_a s_a c_x s_x}, \]
\[ \cos \delta_D = \frac{(s_a^2 s_x^2 - c_x^2)(c_a^2 - s_x^2)}{4 c_a s_a c_x s_x}, \]
for \( \Sigma_2(k) \) and \( \bar{\Sigma}_2(k) \) respectively. Note that only physical quantities are involved which gives the possibility of robust physical predictions. By implementing the measured values of the three mixing angles, a prediction of \( \delta_D \) can be made. Or, (3) can be solved for \( \theta_z \),
\[ \sin \theta_z = p \left[ \pm \sqrt{c_D^2 + \cot^2 2\theta_D - c_D} \right] \tan 2\theta_D (\tan \theta_x)^p, \]
with \( c_D \equiv \cos \delta_D \) while \( p = \pm 1 \) for \( \Sigma_2(k) \) or \( \bar{\Sigma}_2(k) \). Solutions for the \( \pm \) sign within the parenthesis are equivalent through a redefinition \( (\theta_x, \delta_D) \rightarrow (-\theta_x, \delta_D + \pi) \) leaving no effect on the measured physical quantity \( \sin^2 \theta_z \). This is also true for the overall \( p \). The difference comes from the exponent \( p \) leading to a \( (\tan \theta_x)^p \approx 1/2 \) factor between the \( \Sigma_2(k) \) and \( \bar{\Sigma}_2(k) \) predictions.

The main feature of (4) can be seen by expanding it to the leading order. As the reactor angle \( \theta_x \equiv \delta_x \) is small and the atmospheric angle \( \theta_a = 45^\circ \), \( \delta_a \) is nearly maximal, (4) reduces to,
\[ \frac{\delta_x}{\delta_a} = -p \frac{(\tan \theta_x)^p}{\cos \delta_D}. \]

Eqs.(3)–(5) are general and direct. To demonstrate this, three examples are provided. Eq.(3) was first obtained in a minimal seesaw model [22] with \( \mu-\tau \) and CP softly broken and \( \Sigma_2(k) \) retained exactly, befitting the situation discussed here. A special case with \( k = 2 \), which constrains the mixing matrix to be trimaximal, is studied in [23]. Even an "unphysical" bimaximal solution [24] can be covered as a marginal example. Note that the first two examples are obtained in model-dependent and perturbative ways while the last one comes from a pure symmetry analysis.

The ratio (5) of the deviation of the reactor angle from zero and that of the atmospheric angle from maximal is given by the solar angle and the CP phase. Its absolute value is a minimum when \( \delta_D \) equals 0 or \( \pi \), \( |\delta_x| \geq (\tan \theta_x)^p |\delta_a| \). Alternately (3) can be solved exactly with \( \cos \delta_D = \pm 1 \) to give an absolute lower bound \( \sin \theta_x \geq (\tan \theta_x)^p |c_a - s_a|/|c_a + s_a| \). An upper bound can also be obtained but since \( c_a \approx s_a \) it is larger than 1.

**Numerical Predictions** – A nonzero \( \theta_x \) has been consistent with global fits for several years. The first hint appears in [22] at only 0.9σ C.L.. It persists in all subsequent global fits [26,30] and increases steadily to about 3σ [31,32] as summarized in Table I.

| \( \sin^2 \theta_1(\theta_x) \) | \( \sin^2 \theta_2(\theta_x) \) | \( \sin^2 \theta_3(\theta_x) \) |
|------------------------|------------------------|------------------------|
| Best Fit               | (33.6°)                | (40.4°)                | (8.3°)                |
| 1σ Range              | 0.291-0.324            | 0.39-0.50              | 0.013-0.028           |
|                       | (32.7-34.7°)           | (38.7-45.0°)           | (6.6-9.6°)            |

TABLE I: The global fit for the neutrino mixing angles.

The fits can be classified into two categories depending on the result for the atmospheric angle \( \theta_a \) which is persistently maximal in [27,28,30,31] while an apparent deviation from 45° is claimed possible in [26,29,32] due to a subleading effect [33].

![FIG. 1: (Color Online) Predicted distributions of \( \theta_x \).](image)

From (4) the distribution of \( \theta_x \) can be derived by using asymmetric Gaussian distributions \( P \) as,
\[ \frac{dP(\theta_x)}{d\theta_x} = \int f^p(\theta_x) P(s_x^2) ds_x^2 ds_x^2 d\delta_D 2^{-1}, \]
where \( f^p \equiv \frac{1}{\lambda} \delta(\theta_x - \arcsin s_x) \) are \( \delta \)-functions that pick out the predicted value \( s_x \equiv \text{RHS of (4)} \) for \( \theta_x \), given a concrete input of \( p, s_x^2, s_{\bar{x}}^2 \), and \( \delta_D \). There is also a mirror contribution for negative \( \theta_x \) that is not shown – hence the \( \frac{1}{2} \) prefactor. We take \( \delta_D \) to be evenly distributed in \( [0,2\pi] \), as in (4), or replace it with a specific value. The integration (6) can be simulated with scattering points or the delta function can be converted to one for \( \theta_x \) and the other integrals done numerically. The results are shown in Fig. 1 for both \( \Sigma_2(k) \) and \( \bar{\Sigma}_2(k) \). We will first discuss the results from \( \Sigma_2(k) \).

After averaging over \( \delta_D \), the probability distribution peaks around 3° with an asymmetric width from 2° to
4°. This is in sharp contrast with the distribution given by the previous global fit [23], which peaks at 0°. From [5] we can see that δ2 is proportional to δu. With θa significantly deviating from the maximal value, the predicted δ2 must increase accordingly. In the global fit adopted in [22], the central value of θa is about 43° and the maximal value is well within the 1σ range. Hence, there is no apparent nonzero peak in the predicted distribution of θa. For the latest global fit [32] of θa the central value is about 40.4°, while the maximal value is at the edge of the 1σ region. This significant change in θa leads to a clear nonzero prediction of θx. As θa only contributes as an overall factor in [5], its deviation will not change the conclusion for θx. For example, a different treatment of the reactor data leads to 3.7% difference in tan θs. The best fit values of sin^2 θx(θs) vary from 0.02 (8.1°) to 0.04 (11.5°) which are still covered by our predictions. This is also treated in [32] but with a much smaller variation, approximately 20% in sin^2 θx.

The measured θx [1,2] is not independent of the Dirac CP phase δD. But this does not affect the matching between the experimental result and the theoretical predictions. Figure 1 shows that averaging over possible δD values gives a best fit value and deviation which resemble those with vanishing δD. As the experimental fit of θx depends only slightly on δD while the theoretical prediction is sensitive to it, as shown in [3], varying δD can effectively improve the matching. For example, using δD = 60° moves the peak to the MINOS and Double Chooz central values while δD = 70° for that of T2K.

Since |δx| ≥ |δu| tan θs, the prediction for θx with vanishing δD is the most conservative in the sense that it gives the smallest prediction for θx. The prediction with δD uniformly distributed also peaks at 3° but has an extended tail to higher θx. This is because, while δD is uniformly distributed, cos δD is not. Its distribution varies as (sin δD)^{-1} which is relatively suppressed for small cos δD. Thus, the most conservative region is the most probable one. For example, the probability for |cos δD| ≤ 0.1 (0.2, 0.3, 0.4) is just 6% (13%, 19%, 26%) corresponding to δD = 84° (78°, 73°, 66°) respectively. Most of the significant region lies between δD = 0° and approximately δD = 60°. Within this region, the θx peak varies from approximately 3° to around 6° and the width changes from roughly 2° ~ 4° to almost 4° ~ 8°. This is the region covered by MINOS result 2.9°(4.6°) < θx < 8.6°(11.4°) and 5.3° < θx < 10.8° for Double Chooz at 1σ level while T2K has 5.0°(5.8°) < θx < 16.0°(17.8°) at 90% C.L.

The above discussion also applies to the case of Z(k). The only difference is the factor of about 2 coming from the exponent p in [1]. As θx < 10° can be treated as small perturbation, this will induce approximate factors of 2 in the peak location and 1/2 in its height relative to the predictions from Z(k). The result is still in good agreement with the data and the global fits.

This consistency between the data and our prediction of a large θx provides the first nontrivial indication of the viability of residual symmetries Z2 and Z2(k). The correlation between the mixing angles [3] is independent of the group parameter k, and is obtained in a direct way, making the result quite robust.

The change in the global fit also alters our prediction of the Dirac CP phase δD [23]. As shown in Fig. 2(a) the most probable value of δD is no longer maximal. This is also caused by the shifted central value of θa. As δu deviates further from θs, maximal δD becomes less probable as indicated by [3]. Instead it peaks around ±74° for Z2. Notice that a mirror solution in [11] can be obtained through (θx, δD) → (−θx, δD + π) generating another peak around ±106°. These are in perfect consistency with the indication of −74°(−110°) for inverted
(normal) hierarchy [31]. Although no concrete number is provided, a nonzero CP phase also appears in [32]. For $\overline{\theta}_R(k)$, the predicted $\cos \delta_D$ [31] is larger than [3a] by a factor of 2. Consequently, the peak moves to around $\pm 123^\circ (-57^\circ)$.

The distribution of the leptonic Jarlskog invariant $J_\nu$ is shown in Fig. 2(b). These predictions can be tested at T2K [34] and at NOvA [34].

Conclusions – Phenomenological consequences of the residual $\mathbb{Z}_2^1(k)$ and $\mathbb{Z}_2^0(k)$ symmetries are compared with data and global fits. Although not independent, their predictions are different. A large reactor angle $\theta_\nu$ peaking around 3° or 6° which is consistent with T2K, MINOS and Double Chooz can be obtained and the Dirac CP phase $\delta_D$ has peaks at $\pm 74^\circ (+195^\circ)$ or $\pm 123^\circ (57^\circ)$ in excellent agreement with the latest global fits. This provides the first strong and direct support for $\mathbb{Z}_2^1(k)$ and $\mathbb{Z}_2^0(k)$ as residual symmetries of neutrino mixing. Further confirmation may come from the measurement of the leptonic Jarlskog invariant $J_\nu$ at T2K or NOvA.

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