Stress state estimation in multilayer support of vertical shafts, considering off-design cross-sectional deformation

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Abstract. To determine the stress-strain state of multilayer support of vertical shafts, including cross-sectional deformation of the tubing rings as against the design, the authors propose an analytical method based on the provision of the mechanics of underground structures and surrounding rock mass as the elements of an integrated deformable system. The method involves a rigorous solution of the corresponding problem of elasticity, obtained using the mathematical apparatus of the theory of analytic functions of a complex variable. The design method is implemented as a software program allowing multivariate applied computation. Examples of the calculation are given.

Monitoring of shaft support sometimes reveals that the shape of tubing rings deviates from the designed circle. Deviation from the circular cross-section may result from low quality assembling of the tubing ring, use of different model tubings in the same ring, nonuniform deformation of of rocks during ice wall creation behind the installed lining, excessive local pressure of grouting mortar injection in the process of water imperviousness recovery of shaft support, tectonics, etc. Naturally, departure from the circular cross-section can, under certain conditions, reduce load-bearing capacity of shaft support and make the shaft unserviceable.

Developed at the Tula State University, the analytical method of combination (multi-layered) shaft support design based on the underground structure mechanics principle on interaction of rock mass and support as a uniform deformable system makes it possible to analyze load-bearing capacity of mine shaft support with regard to deviation of the actual geometry of tubing rings from the design crops-section.

The proposed approach extends the methodology of accounting for the influence exerted by factual cross-section of underground structures proposed by Dr Eng, Prof Fotieva and the theory of underground structure mechanics by Dr Eng, Prof Bulychev in the context of the multi-layered shaft support design: the support loads are not set 'a priory' but determined as radial and shear stresses during calculation of the whole rock mass–support system depending on geometrical and deformation characteristics of the underground structure constituents. The method rests on the earlier obtained and adapted analytical solution to the plane problem of elasticity on stress state of a noncircular multi-layered ring which simulates shaft support composed of cast iron tubes and concrete fill in a hole in an infinitely linearly deformed medium simulating rock mass [2, 3].

The mathematical model includes such factors that essentially influence stress state and load-bearing capacity of the compound support as: support design; dimension and actual shape of cross-section; deformation, rheological (within the linear hereditary creep theory) and permeability characteristics of rock mass and concrete layer; initial stress field conditioned by weight of overburden
(rock pressure), concrete support and tubing, as well as by groundwater pressure (with regard to seepage through the concrete fill).

Cast iron tubing is composed of elements with flat joints with bolted connections that are tension braces and is for this reason is modeled as a two-layer tube. The introduction of the additional inner layer is conditioned by the ring stiffeners installed at regular intervals; the influence of the radial stiffeners is disregarded as they only prevent liner plates between the ring stiffeners (Figure 1).

![Figure 1](image1.png)

**Figure 1.** Schematics of multilayer shaft support with periodically nonuniform layers: (a) nonuniform layer \( i \); (b) nonuniform layer 1 and uniform layer 2 simulating stiffeners and plates, respectively.

The analytical model (Figure 2) presents the shaft support as a three-layer ring where the outward layer with a thickness \( \Delta \) models concrete fill and the other two layers model the tubing cross-section. The boundaries of the layers differ from circle in the general case (the inside design circle boundary is shown by the dashed line).

The ring support the circular hole in the infinite ponderable linearly deformable isotropic (or viscoelastic) medium \( S_0 \) modeling surrounding rock mass with the deformation characteristics represented by the deformation modulus \( E_0 \), Poisson’s ratio \( \nu_0 \) and linear hereditary creep parameters \( \alpha_0 \), \( \delta_0 \). The layers in \( S_i \) have different deformation characteristics—deformation moduli \( E_i \) and Poisson’s ratios \( \nu_i \) (\( i = 1, 2, 3 \)).

![Figure 2](image2.png)

**Figure 2.** Analytical model.
So, the model assumes that the inside nonuniform layer formed by the ring stiffeners works as a quasi-uniform layer with the reduced deformation characteristics, and the displacements of the interface between plates and stiffeners are the same. The validity of such assumptions is proved by the research [12], including the shaft support designs.

The deformation characteristics of the quasi-uniform inside layer $S_3$ are governed by the reduced deformation modulus given by [2]:

$$E_3 = \mu E_a,$$

(1)

where $E_a$—deformation modulus of cast iron, MPa; $\mu$—reduction factor: $\mu = \frac{4a}{h}$; $a$—width of stiffeners (tubing design includes 4 stiffeners), m; $h = 1.5$ m—tubing height.

Poisson’s ratio of the cast iron layer $S_3$ is assumed $\nu_3 = 0.3$.

Gravity conditioned by the weight of overlying rocks and support, or by groundwater level is modeled by setting relevant initial compressive stresses in the medium $S_0$ and in the layers $S_i$ ($i = 1, 2, 3$).

To calculate the load due to overlying rock weight, the initial stresses in the medium $S_0$ are given by:

$$\sigma_x^{(0)(0)} = -\gamma H \lambda \alpha^*, \sigma_y^{(0)(0)} = -\xi \sigma_x^{(0)(0)},$$

(2)

where $\gamma$—averaged weight of overlying rocks; $H$—depth of the analyzed cross-section of the shaft; $\lambda$—lateral earth pressure coefficient; $\xi$—factor of nonuniformity of initial stress field; $\alpha^*$—correction factor to account for the process influences (lag between the support and the bottomhole advance, sequence of cast iron tubing and concrete fill installation, etc.) [2, 3].

The weight of the support is included in the model by setting initial stresses in the domains $S_i$ ($i = 1, 2, 3$) simulating the corresponding layer of the support structure:

$$\sigma_x^{(i)(0)} = \sigma_y^{(i)(0)} = -\gamma_i H \lambda_i,$$

(3)

where $\gamma_i$—specific weight of material of the support layer $S_i$; $\lambda_i = \frac{\nu_i}{1-\nu_i}$—later earth pressure coefficient in the given medium.

In the layer $S_3$ formed by the ring stiffeners of the tubing, the specific weight is calculated as $\gamma_3 = \gamma_{ci} \cdot \mu$, where $\gamma_{ci}$—specific weight of cast iron.

In the calculation of influence of the groundwater pressure, it is assumed that as a consequence of water seepage through the outward concrete layer, total water head recovers on the water-impermeable tubing column. Therefore, the initial stresses acting in the domains $S_i$ ($i = 0, 1$) are given by:

$$\sigma_x^{(i)(0)} = \sigma_y^{(i)(0)} = -\gamma_w H_w \ (i = 0, 1),$$

(4)

where $\gamma_w = 9.8$ kN/m$^3$—specific weight of groundwater; $H_w$—groundwater head, m.

Total stresses in the analyzed domains are presented as the sums of initial and secondary stresses conditioned by the formation of the supported hole. The initial stresses in rock mass before the shaft sinking are neglected.

The tubing layers $S_i$ ($i = 1, 2, 3$) and the medium $S_0$ deform jointly, i.e. the conditions of continuous vectors of displacements and total stresses are satisfied in in the lines $L_i$ ($i = 0, 1, 2$).

The boundary conditions in the formulated problem are:

— at the boundaries $L_i$ ($i = 0, 1, 2$)
\[
\begin{align*}
\tilde{\sigma}_n(i,i+1) &= \tilde{\sigma}_n(i,i), \quad \tilde{\tau}_n(i,i+1) = \tilde{\tau}_n(i,i), \quad u_n(i,i+1) = u_n(i,i), \quad u_\varphi(i,i+1) = u_\varphi(i,i),
\end{align*}
\]

—at the inner boundary \(L_3\)

\[
\begin{align*}
\tilde{\sigma}_n^{(3,3)}(i,i) &= 0, \\
\tilde{\tau}_n^{(3,3)}(i,i) &= 0,
\end{align*}
\]

where \(\tilde{\sigma}_n(i,i), \tilde{\tau}_n(i,i)\) \((i = 0, 1, 2; j = i, i + 1)\)—components of the total normal and shear stresses at the points in the domain \(S_j\) at the boundary \(L_j\); \(u_n(i,j), u_\varphi(i,j)\)—components of displacement vectors at the points of the contacting domains \(S_j\), along the normal \(n\) and tangent \(\varphi\) drawn to the boundary \(L_j\).

The formulations are linear, which allows using principle of superposition. Each problem is solved in terms of the fractions of the initial stress \(\sigma_x^{(0,0)}\). After the calculated stress is multiplied by the value of \(\sigma_x^{(0,0)}\), the results are summed up with regard to the most unfavorable combinations of loads.

The calculation of the shaft support cross-section in watered rock mass includes the ascending force of groundwater [5].

When stresses at the internal and external boundaries of the quasi-uniform inside layer \(S_3\) are found, the stresses on the stiffeners are calculated. The this effect, the stresses in \(S_3\) are multiplied by the factor \(k = \frac{1}{\mu}\).

The influence of viscoelastic deformation of rocks susceptible to creep can be taken into account based on the theory of linear hereditary creep using the method of variable moduli, according to which the deformation characteristics of the medium \(S_0\), included in the elasticity problems solution, are presented as functions of time [1].

After introduction of complex potentials \(\varphi_j(\zeta), \psi_j(\zeta)\), characterizing stress state in the domains \(S_j\) \((j = 0, 1, 2, 3)\) connected with the additional stresses and displacements by the Kolosov–Muskhelishvili formulas [4], the set problem reduces to the boundary value problem of the theory of functions of a complex variable and is solved using the apparatus of the conformal mapping and complex series.

The described solution is implemented in a computer program for multi-variant calculations in practical shaft support design.

Figure 3. Schematic shapes of the internal boundary of tubing rings: (a) first section (rings 110–115) and (b) second section (rings 148–158).
The resultant maximum deviation from the design circle is depicted in Figure 3 in terms of two sections of rubbing rings (magnification by 10 times): section 1 (Figure 3a) lies in chalky clay, tubing 7.0–50, concrete fill thickness 960 mm; section 2 (Figure 3b) lies in rock salt, tubing 7.0–60, concrete fill layer 1160 mm thick.

The calculated normal and normal tangential stresses in the shaft support in case of departure from the design circular cross-section are only compressive; shearing stresses in the normal (radial) cross-sections of the support are insignificant.

The criterion of the real ring shape was assumed the value $\varepsilon = \frac{\delta}{R} \times 100\%$, where $\delta$—decrease in the design radius $R$ at the point of the local displacement.

The resultant curves of increase in the normal tangential stresses $\Delta \sigma$ (%) in the tubing and concrete fill in sections 1 and 2 (as in Figure 1) versus $\varepsilon$ are plotted in Figure 4 as the solid and dashed lines, respectively. The lines marked by 1 are for the cast iron tubing, the lines marked by 2 are for the concrete fill.

The calculations show that with the thinner layer of the concrete fill, the departure of the tubing ring from the design circle has higher influence on the stress state of the support than in the case of the thicker concrete layer. It follows from the analysis of the plots in Figure 4 that the higher displacement of the support inward the shaft results in the increase $\Delta \sigma$ in the tubing ring and concrete fill almost by the linear law; the maximum $\Delta \sigma$ due to the deviation of the design shape of tubing ring is not higher than 20% (ring 153) and is within 7% in all other cases.

![Figure 4](image-url)  
Figure 4. Relative increase in the stress $\Delta \sigma$, %, in the support in sections 1 and 2 of the shaft as against the decrease in the radius $\varepsilon$.

The highest deviation of the tubing ring shape from the design in ring 153 conforms with the maximum increase in stresses.

When the support loads are set in accord with [5], the stresses in the support do not exceed limiting values and the required load-bearing capacity of the support is ensured.

The scientific provisions included in the described calculation method have many times been approved in practical support designs for cage and skip hosting shafts in potassium and potash-magnesium salt mines in the area Perm, Bashkiria, Volgograd and Kaliningrad.

Conclusions
For the purpose of calculating stress state in the multilayer support of mine shafts with regard to the deviation of the tubing ring cross-section from the design geometry, the analytical method has been
developed. The method is implemented as a computer program for the multi-variant practical calculations.

References
[1] Amusin BZ and Linkov AM 1974 On the use of the method of variable modules for solving a class of linear hereditary creep problems Izv. AN SSSR. Mekh. Tverd. Tela No 6 pp 162–166
[2] Bulychev NS 1994 Mechanics of Underground Structures: University Textbook Moscow: Nedra (in Russian)
[3] Bulychev NS, Fotieva NN and Streltsov EV 1986 Designing and Computation of Support for Permanent Roadways Moscow: Nedra (in Russian)
[4] Muskhelishvili NI 1966 Some Basic Problems of Mathematical Elastic Theory Moscow: Nauka (in Russian)
[5] Mine and Support Design Manual Moscow: Stroyizdat 1983 (in Russian)