Magnetic moments of doubly heavy baryons in light-cone QCD

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The magnetic dipole moments of the spin-\(\frac{1}{2}\) doubly heavy baryons are extracted in the framework of light-cone QCD sum rule. The electromagnetic properties of the doubly heavy baryons encodes important information of their internal structure. The results for the magnetic dipole moments of doubly heavy baryons acquired in this work are compared with the predictions of the other theoretical approaches.

Keywords: Electromagnetic form factors, Magnetic moment, Doubly heavy baryon, Light-cone QCD sum rules

I. INTRODUCTION

A doubly charmed baryon was first reported by the SELEX Collaboration in the decay mode \(\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+\) with the mass \(M_{\Xi_{cc}^+} = 3519 \pm 1 \text{ MeV}\) \([1]\), however, other experimental groups, namely Belle \([2]\), FOCUS \([3]\), and BABAR \([4]\) could not find any evidence of the doubly heavy baryons in \(e^- e^+\) annihilations later. However, since the production mechanisms at these experiments were different from that of SELEX Collaboration, which studied collisions of a hyperon beam on fixed nuclear targets, these results had not ruled out the results of the SELEX Collaboration. In 2017, LHCb Collaboration discovered spin-\(\frac{1}{2}\) doubly heavy baryon \(\Xi_{cc}^{++}\) in the mass spectrum of \(\Lambda_c^+ K^- \pi^+ \pi^+\) with the mass \(M_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14 \text{ MeV}\) \([5]\). The examination for the doubly heavy baryons (DHB) may provide us with important information our understanding the nonperturbative QCD effects. One of the several aspects which makes the physics of doubly heavy baryons attractive is that the binding of two heavy quarks and a light quark ensures a unique point of view for dynamics of confinement. Therefore, the masses, decays and other properties of the (DHB) have been studied extensively in literature \([6–72]\).

In order to find out the inner structure of the baryons in the nonperturbative regime of QCD, the main challenges are the determination of the statical and dynamical features of baryons such as their coupling constants, magnetic moments, masses and so on, both experimentally and theoretically. As the electromagnetic features characterize essential aspects of the inner structure of hadrons, it is very important to investigate the baryon electromagnetic form factors, especially the magnetic dipole moments. The magnetic dipole moment is straight-forwardly regarding the charge and current distributions in the hadrons and these variables are directly connected to the spatial distributions of quarks and gluons inside the hadrons. The magnitude and sign of the dipole magnetic moment provide important information on structure, size and shape of hadrons. The magnetic dipole moments of the DHB have been studied in different theoretical models and approaches \([55–72]\).

In this study, the magnetic dipole moments of spin-\(\frac{1}{2}\) DHB (hereafter we will denote these states as \(B_{QQ}\)) are extracted in the framework of the light cone QCD sum rule (LCSR). The LCSR has already been successfully applied to extract dynamical and statical properties of hadrons for decades such as, form factors, masses, the electromagnetic multipole moments and so on. In this approach, the hadronic features are expressed in terms of the features of the vacuum and the light cone distribution amplitudes (DAs) of the hadrons in the process [for details, see for instance \([73–75]\)]. Since the magnetic dipole moment is expressed in terms of the properties of the DAs and the QCD vacuum, any uncertainty in these parameters reflects the uncertainty of the estimations of the magnetic dipole moments.

The rest of the paper is organized as follows: In Sections II, the details of the magnetic dipole moments calculations for the DHB with spin-\(\frac{1}{2}\) are presented. In the last section, we numerically analyze the sum rules obtained for the magnetic dipole moments and discuss the obtained results.

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II. FORMALISM

In this section we derive the LCSR for the magnetic moments of spin-$\frac{1}{2}$ DHB. The starting point is to consider the following correlation function:

$$\Pi(p, q) = i \int d^4x e^{ipx} \langle 0 | \{ J_{BQQ}(x) \tilde{J}_{BQQ}(0) \} | 0 \rangle_{\gamma},$$

(1)

where $\gamma$ is the external electromagnetic field and $J_{BQQ}$ is the interpolating current having quantum numbers $J^P = \frac{1}{2}^+$. It is given as \cite{45}

$$J_{BQQ}(x) = \varepsilon^{abc} \left\{ (Q^T(x)Cq_b(x))\gamma_5Q_c(x) + \beta \left[ (Q^T(x)C\gamma_5q_b(x))Q_c(x) \right] \right\}$$

(2)

where $Q$ is the c or b-quark, $q$ is u, d or s-quark, $C$ is the charge conjugation matrix; and $a$, $b$, and $c$ are color indices; and $\beta$ is an arbitrary parameter that fixes the mixing of two local operators.

In order to obtain the sum rules for magnetic moments of the DHB the correlation function is calculated in two different ways: 1) In terms of the hadronic degrees of freedom, so called the hadronic side; 2) in terms of the Operator product expansion (OPE) over the twists of operators, and using the photon DAs which encode all nonperturbative effects, so called the QCD side. Then equating these two different representations of the correlation function to each other using the quark-hadron duality assumption. In order to suppress the contributions of the higher states and continuum we carry out Borel transformation, besides continuum subtraction to both sides of the acquired QCD sum rules.

We start to calculate the correlation function in terms of hadronic degrees of freedom including the physical properties of the baryons under consideration. To this end we insert intermediate states of $B_{QQ}$ into the correlation function. As a result, we obtain

$$\Pi^{Had}(p, q) = \frac{\langle 0 | J_{BQQ} | B_{QQ}(p) \rangle}{|p^2 - m_{BQQ}^2|} \langle B_{QQ}(p) | B_{QQ}(p + q) \rangle_{\gamma} \frac{\langle B_{QQ}(p + q) | \tilde{J}_{BQQ} | 0 \rangle}{(|p + q|^2 - m_{BQQ}^2)} + \ldots,$$

(3)

where $q$ is the momentum of the photon and dots refers to contribution of the higher states and continuum. The matrix elements in Eq.(3) are determined as

$$\langle 0 | J_{BQQ}(0) | B_{QQ}(p, s) \rangle = \lambda_{BQQ} u(p, s),$$

(4)

$$\langle B_{QQ}(p) | B_{QQ}(p + q) \rangle_{\gamma} = \varepsilon^\mu \bar{u}(p) \left[ (f_1(q^2) + f_2(q^2))\gamma_\mu + f_2(q^2) \frac{(2p + q)_\mu}{2m_{BQQ}} \right] u(p),$$

(5)

where $\lambda_{BQQ}$ is the residue and $u(p)$ is the Dirac spinor. Summation over spins of $B_{QQ}$ baryon is applied as:

$$\sum_s u(p, s)\bar{u}(u, s) = \not{p} + m_{BQQ},$$

(6)

Substituting Eqs. (3)-(6) in Eq. (1) for hadronic side we get

$$\Pi^{Had}(p, q) = \frac{\varepsilon^\mu \lambda_{BQQ}^2}{|(p + q)^2 - m_{BQQ}^2|^2} \left[ \frac{\not{p} + m_{BQQ}}{(2\not{p} + \not{q}) + m_{BQQ}} \right] \left[ (f_1(q^2) + f_2(q^2))\gamma_\mu + f_2(q^2) \frac{(2p + q)_\mu}{2m_{BQQ}} \right].$$

(7)

At $q^2 = 0$, the magnetic dipole moment is defined in terms of $f_1(q^2 = 0)$ and $f_2(q^2 = 0)$ form factors in the following way:

$$\mu_{BQQ} = f_1(q^2 = 0) + f_2(q^2 = 0).$$

(8)

As a result, the hadronic side of the correlation can be written in terms of magnetic dipole moment of the spin-$\frac{1}{2}$ doubly heavy baryon as,

$$\Pi^{Had}(p, q) = \frac{\lambda_{BQQ}^2}{|(p + q)^2 - m_{BQQ}^2|^2} \left[ \mu_{BQQ} m_{BQQ} \not{p} + \text{other structures} \right].$$

(9)
The next step is to compute the correlation function in terms of quark-gluon degrees of freedom in the deep Euclidean region. Using the expression for interpolating current and Wick’s theorem, the QCD side of the correlation function can be written as,

$$
\Pi^{QCD}(p, q) = i e^{abc} e^{a'b'c'} \int d^4 x e^{ip \cdot x} \left\{ \gamma_5 S_Q^{c'}(x) \gamma_5 T_r[S_Q^{a'}(x)S_q^{b'}(x)] - \gamma_5 S_Q^{c'}(x) \tilde{S}_q^{b'}(x)S_Q^{a'}(x) \gamma_5 \\
+ \beta \left( \gamma_5 S_Q^{c'}(x) T_r[S_Q^{a'}(x)S_q^{b'}(x)] - \gamma_5 S_Q^{c'}(x) \gamma_5 S_q^{b'}(x)S_Q^{a'}(x) + S_Q^{a'}(x) \gamma_5 T_r[S_Q^{c'}(x)S_q^{b'}(x)] \right) \\
- S_Q^{c'}(x) \tilde{S}_q^{b'}(x) \gamma_5 S_Q^{a'}(x) \gamma_5 \right\} \left| 0 \right> \gamma
$$

where $\tilde{S}_q^{ij}(x) = C S_Q^{ijT}(x)$ and $S_q^{ij}(x)$ and $S_Q^{ij}(x)$ are the light and heavy quark propagators, respectively. The quark propagators are given as [76],

$$
S_q(x) = S_{free} - \frac{\bar{q}q}{12} \left( 1 - i \frac{m_q}{4} \right) - \frac{\bar{q}G_q}{192} x^2 \left( 1 - i \frac{m_q}{6} \frac{\sigma}{x^2} \right) - \frac{ig_s}{32\pi^2 x^2} G^{\mu\nu}(x) \left[ \frac{\sigma_{\mu\nu} + \sigma_{\mu\nu}^\prime}{x^2} \right], \quad (11)
$$

and

$$
S_Q(x) = S_{free} - \frac{g_s m_Q}{16\pi^2} \int_0^1 dv G^{\mu\nu}(vx) \left[ \sigma_{\mu\nu}^\prime + \frac{1}{2} \sigma_{\mu\nu} \right] \left[ \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + 2m_{\mu\nu} K_0(m_Q \sqrt{-x^2}) \right], \quad (12)
$$

where

$$
S_{free} = i \frac{x}{2\pi^2 x^2} - \frac{m_q}{4\pi^2 x^2}, \quad (13)
$$

$$
S_{free} = m_Q \left[ \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + i \frac{K_2(m_Q \sqrt{-x^2})}{(\sqrt{-x^2})^2} \right]. \quad (14)
$$

Here $K_{1,2}$ are Bessel functions of the second kind.

The correlation function in Eq. (10) includes different contributions: perturbative and nonperturbative. In case of perturbative contributions, when a photon is radiated from distant distance, one of the free quark propagators in Eq. (10) is replaced by

$$
S_{free} \rightarrow \int d^4 y S_{free}(x - y) A(y) S_{free}(y) \ , \quad (15)
$$

and the remaining two propagators are taken as the full quark propagators.

In case of nonperturbative contributions, a photon interacts with light quarks at large distance, the light quark propagator in Eq. (10) is replaced by

$$
S_{q}^{a'b'} \rightarrow \frac{1}{4} \left\{ \bar{q}^a \Gamma_i(q^a) \Gamma_i(q^a) \right\}, \quad (16)
$$

and remaining two propagators are replaced with the full quark propagators, and also including perturbative and nonperturbative contributions. Once Eq. (16) is inserted into Eq. (10), there seem matrix elements such as $\langle \gamma(q) | \bar{q}(x) \Gamma_i q(0) \rangle$ and $\langle \gamma(q) | \bar{q}(x) \Gamma_i G_{\alpha\beta}(0) \rangle$, representing the nonperturbative contributions. To compute the nonperturbative contributions, we need the matrix elements of the nonlocal operators between the vacuum and the photon states and these matrix elements are defined in terms of the photon DAs with definite twists, whose expressions are given in Ref. [77]. The QCD side of the correlation function can be acquired in terms of quark and gluons degrees of freedom by substituting photon DAs and expressions for heavy and light quark propagators in to Eq. (10).

The QCD sum rules for the magnetic dipole moments of the spin-$\frac{1}{2}$ DHB are obtained by equating the coefficients of the structure $\not{q}q$ from hadronic and QCD sides of the correlation function. The last step in deriving the sum rules for the magnetic dipole moments of the spin-$\frac{1}{2}$ DHB are applying double Borel transformations over the $p^4$ and $(p + q)^2$ on the both sides of the correlation function in order to suppress the contributions of higher states and continuum. Finally, we obtain:

$$
\mu_{BQQ} = \frac{m_{BQQ}^3}{\lambda_{BQQ}^2 m_{BQQ}} \Pi^{QCD}, \quad (17)
$$
The explicit forms of the $\Pi^{QCD}$ is given as follows:

$$
\Pi^{QCD} = -\frac{e_q}{7864320\pi^3} \left[ 3 m_q (-1 + \beta) \left\{ 3(-1 + \beta) \left( I[0, 5, 2] - 3 I[0, 5, 3] + 3 I[0, 5, 4] - 3 I[0, 5, 5] \right) + 80 m_e (1 + \beta) \pi^2 \right. \\
\times \langle \bar{q}q \rangle \left( 6 I[0, 3, 1] - 19 I[0, 3, 2] + 20 I[0, 3, 3] - 7 I[0, 3, 4] + 9 I[1, 2, 1] - 28 I[1, 2, 2] + 29 I[1, 2, 3] \\
- 10 I[1, 2, 4] \right) \right) + 20 \pi^2 \langle \bar{q}q \rangle \left\{ 16 m_c^2 (3 + 2 \beta + 3 \beta^2) \left( I[0, 3, 1] - 2 I[0, 3, 2] + I[0, 3, 3] \right) + 3(-1 + \beta)^2 \\
\times \left( 3 I[0, 4, 2] - 9 I[0, 4, 3] + 9 I[0, 4, 4] - 3 I[0, 4, 5] + 4 \left( I[1, 3, 2] - 3 I[1, 3, 3] + 3 I[1, 3, 4] - I[1, 3, 5] \right) \right) \right\} \\
+ 20 m_q^2 \pi^2 \langle \bar{q}q \rangle \left\{ 4 m_q m_c (-1 + \beta^2) \left( 27 I[0, 2, 1] - 84 I[0, 2, 2] + 87 I[0, 2, 3] - 30 I[0, 2, 4] + 54 I[1, 1, 1] \\
- 169 I[1, 1, 2] + 176 I[1, 1, 3] - 61 I[1, 1, 4] + 9 I[2, 0, 1] - 28 I[2, 0, 2] + 29 I[2, 0, 3] - 10 I[2, 0, 4] \right) + 24 m_c^2 \\
\times (3 + 2 \beta + 3 \beta^2) \left( 2 I[0, 2, 1] - 2 I[0, 2, 2] + I[0, 2, 3] + I[1, 1, 1] - 2 I[1, 1, 2] + I[1, 1, 3] \right) + 9 (-1 + \beta)^2 \\
\times \left( 2 I[0, 3, 2] - 6 I[0, 3, 3] + 6 I[0, 3, 4] - 2 I[0, 3, 5] + 7 I[1, 2, 2] - 21 I[1, 2, 3] + 21 I[1, 2, 4] - 7 I[1, 2, 5] \\
+ 2 I[2, 1, 2] - 6 I[2, 1, 3] + 6 I[2, 1, 4] - 2 I[2, 1, 5] \right) \right\} \right] \\
+ \frac{e_q}{15728640 \pi^3} \left[ 18 m_e (-1 + \beta^2) \left( I[0, 5, 2] - 3 I[0, 5, 3] + 3 I[0, 5, 4] - I[0, 5, 5] \right) + 5 \pi^2 \langle \bar{q}q \rangle \left\{ 8 m_e^2 (1 + \beta)^2 \\
\times \left( 2 I[0, 3, 2] - 4 I[0, 3, 3] + 2 I[0, 3, 4] + 3 I[1, 2, 2] - 6 I[1, 2, 3] + 3 I[1, 2, 4] \right) + 27 (-1 + \beta)^2 \left( - I[0, 4, 3] \\
+ 3 I[0, 4, 4] - 3 I[0, 4, 5] + I[0, 4, 6] - 4 I[1, 3, 3] + 12 I[1, 3, 4] - 12 I[1, 3, 5] + 4 I[1, 3, 6] - 2 I[2, 2, 3] \\
+ 6 I[2, 2, 4] - 6 I[2, 2, 5] + 2 I[2, 2, 6] \right) \right\} A[u_0] \right] \\
- \frac{e_q}{3145728 \pi} \left[ 6 m_e f_{3\gamma} (-1 - 4 \beta + 2 \beta^2) \left( I[0, 4, 1] - 3 I[0, 4, 2] + 3 I[0, 4, 3] - I[0, 4, 4] \right) I_1[A] - 3 \langle \bar{q}q \rangle (1 - \beta)^2 \\
\times \left( I[0, 4, 2] - I[0, 4, 3] + 3 I[0, 4, 4] - I[0, 4, 5] \right) I_1[V] - m_e f_{3\gamma} (1 - \beta^2) \left( 12 I[0, 4, 1] - 37 I[0, 4, 2] \\
+ 38 I[0, 4, 3] - 13 I[0, 4, 4] \right) I_1[Y] \right] \\
+ \frac{e_q \langle \bar{q}q \rangle}{1572864 \pi} \left[ - \left\{ 16 m_e^2 (1 + \beta)^2 \left( I[0, 3, 1] - 2 I[0, 3, 2] + I[0, 3, 3] \right) + 3(1 - \beta)^2 \left( I[0, 4, 2] - 3 I[0, 4, 3] \\
+ 3 I[0, 4, 4] - I[0, 4, 5] \right) \right\} I_2[T_1] - \left\{ 16 m_e^2 (1 + \beta^2) \left( I[0, 3, 1] - 2 I[0, 3, 2] + I[0, 3, 3] \right) - 6(1 - \beta)^2 \left( I[0, 4, 2] \\
- 3 I[0, 4, 3] + 3 I[0, 4, 4] - I[0, 4, 5] + 2 I[1, 3, 2] - 6 I[1, 3, 3] + 6 I[1, 3, 4] - 2 I[1, 3, 5] \right) \right\} I_2[T_2] \\
+ \left\{ 8 m_e^2 (1 + \beta^2) \left( I[0, 3, 1] - 2 I[0, 3, 2] + I[0, 3, 3] \right) + 3(1 - \beta)^2 \left( I[0, 4, 2] - 3 I[0, 4, 3] + 3 I[0, 4, 4] \\
- I[0, 4, 5] + 2 I[1, 3, 2] - 6 I[1, 3, 3] + 6 I[1, 3, 4] - 2 I[1, 3, 5] \right) \right\} I_2[T_3] + \left\{ 8 m_e^2 (1 + \beta^2) \left( I[0, 3, 1] - 2 I[0, 3, 2] + I[0, 3, 3] \right) + 3 \left( -1 + \beta^2 \left( I[0, 4, 2] - 3 I[0, 4, 3] + 3 I[0, 4, 4] - I[0, 4, 5] + 2 I[1, 3, 2] - 6 I[1, 3, 3] + 6 I[1, 3, 4] \\
- 2 I[1, 3, 5] \right) \right\} I_2[T_4] \right]\right]
\[-\frac{e_g \langle \bar{q}q \rangle}{1572864 \pi} \left( 8 m_Q^2 (1 + \beta)^2 \left( I[0,3,2] - 2 I[0,3,3] + I[0,3,4] \right) - 9(1 - \beta)^2 \left( - I[0,4,3] + 3 I[0,4,4] - 3 I[0,4,5] + I[0,4,6] - 2 I[1,3,3] + 6 I[1,3,4] - 6 I[1,3,5] + 2 I[1,3,6] \right) \right) \right]
\[+ \frac{e_g}{2621440 \pi} \left( - 3 \chi \langle \bar{q}q \rangle (1 - \beta)^2 \left( 2 I[0,5,3] - 6 I[0,5,4] + 6 I[0,5,5] - 2 I[0,5,6] + 5 I[1,4,3] - 3 I[1,4,4] + 3 I[1,4,5] - I[1,4,6] \right) \phi_\gamma[u_0] + 10 m_c f_{3\gamma}(1 - \beta^2) \left( I[0,4,2] - 3 I[0,4,3] + 3 I[0,4,4] - I[0,4,5] \right) \psi^\nu[u_0] \right) \tag{18}\]

where, $m_Q$ is the mass of the c or b-quark, $m_q$ is the mass of the u, d or s-quark, $e_Q$ is the electric charge of the c or b-quark, $e_q$ is the electric charge of the the u, d or s-quark, $\chi$ is the magnetic susceptibility of the quark condensate, $m_0^2 = \langle \bar{q}g_\gamma \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle / \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle$ is the quark condensate. The reader can find out some details about the computations such as Borel transformations and continuum subtraction in Refs. [78, 79].

The functions $I[n, m, l] = f_{3\gamma}(u_0), I_3[A]$, and $I_3[I\lambda]$, $I_3[I\lambda\xi]$ are defined as:
\[
I[n, m, l] = \int_{4m_Q^2}^{s_0} ds \int_0^1 dt e^{-s/M^2} s^n (s - 4 m_Q^2)^m \nu^l,
\]
\[
I_1[A] = \int D_{\alpha_i} \int_0^1 dv A(\alpha_q, \alpha_q, \alpha_q) \delta(\alpha_q + (1 - v)\alpha_g - u_0),
\]
\[
I_2[A] = \int D_{\alpha_i} \int_0^1 dv A(\alpha_q, \alpha_q, \alpha_g) \delta(\alpha_q + (1 - v)\alpha_g - u_0),
\]
\[
I_3[A] = \int_0^1 du A(u).
\]

### III. NUMERICAL ANALYSIS AND CONCLUSION

In this section, we perform numerical analysis for the spin-$\frac{1}{2}$ DHB. We use $m_u = m_d = 0$, $m_s = 96_{-4}^{+8}$ MeV, $m_c = 1.28 \pm 0.03$ GeV, $m_b = 4.18_{-0.03}^{+0.04}$ GeV, $[80]$, $M_{\Xi_{bb}^{++}} = 3519 \pm 1$ MeV [1], $M_{\Xi_{bb}^{++}} = 3621.40 \pm 1.13$ MeV [5], $f_{3\gamma} = -0.0039$ GeV [77], $\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3$ GeV $^3$ [81], $m_0^2 = 0.8 \pm 0.1$ GeV $^2$ and $\chi = -2.85 \pm 0.5$ GeV$^{-2}$ [82].

The masses of the $\Omega_{cc}^{+}, \Xi_{bb}^{0}, \Xi_{bb}^-$ and $\Omega_{bb}^-$ baryons are borrowed from Ref. [31, 32], in which the mass sum rules have been used in computing them. These masses are computed to have the following values: $M_{\Omega_{cc}^+} = 3.73 \pm 0.2$ GeV, $M_{\Xi_{bb}^0} = 9.96 \pm 0.9$ GeV, $M_{\Xi_{bb}^-} = 9.96 \pm 0.9$ GeV, $M_{\Omega_{bb}^-} = 9.97 \pm 0.90$ GeV. In order to specify the magnetic dipole moments of DHB, the value of the residues are needed. The residues of the DHB are computed in Refs. [31, 32]. These residues are calculated to have the following values: $\lambda_{\Xi_{cc}} = 0.16 \pm 0.03$ GeV$^3$, $\lambda_{\Omega_{cc}} = 0.18 \pm 0.04$ GeV$^3$, $\lambda_{\Xi_{bb}} = 0.44 \pm 0.08$ GeV$^3$, $\lambda_{\Omega_{bb}} = 0.45 \pm 0.08$ GeV$^3$. The parameters used in the photon distribution amplitudes are given in [77].

The sum rules for the magnetic dipole moments of the DHB depend on three auxiliary parameters, namely the continuum threshold $s_0$, Borel mass parameter $M^2$ and mixing parameter $\beta$. We shall find their working region such that the magnetic dipole moments are practically independent of these parameters according to the standard prescriptions in QCD sum rules. The continuum threshold is not totally arbitrary, it is chosen as the point at which the excited states and continuum begin to contribute to the computations. To designate the working region of the $s_0$, we enforce the conditions of OPE convergence and pole dominance. In this respect, we choose the value of the continuum threshold within the interval $s_0 = (16 - 20) GeV^2$ for $\Xi_{cc}$, $s_0 = (18 - 22) GeV^2$ for $\Omega_{cc}$, $s_0 = (16 - 20) GeV^2$ for $\Xi_{bb}$ and $s_0 = (118 - 122) GeV^2$ for $\Omega_{bb}$ baryons. The working window for $M^2$ is acquired by requiring that the series of OPE in QCD side is convergent and the contribution of higher states and continuum is adequately suppressed. Our numerical analysis shows that these conditions are fulfilled when $M^2$ change in the regions: $4 GeV^2 \leq M^2 \leq 6 GeV^2$ for $\Xi_{cc}$, $5 GeV^2 \leq M^2 \leq 7 GeV^2$ for $\Omega_{cc}$, $10 GeV^2 \leq M^2 \leq 14 GeV^2$ for $\Xi_{bb}$ and $11 GeV^2 \leq M^2 \leq 15 GeV^2$ for $\Omega_{bb}$ baryons. In Fig. 1, we plot the dependencies of the magnetic dipole moments on $M^2$ at several fixed values of the continuum threshold $s_0$. We observe from the figures that the magnetic dipole moments show relatively weak dependence on the variations of the Borel mass parameter and continuum threshold in their working regions. The sum rules are expected to be independent of the mixing parameter $\beta$ and it is chosen $\beta = \pm 2$ [31].
FIG. 1: The dependence of the magnetic dipole moments for spin-$$\frac{1}{2}$$ DHB on the Borel parameter squared $$M^2$$ at different fixed values of the continuum threshold: (a), (c) and (e) for the doubly charmed baryons, (b), (d) and (f) for the doubly bottomed baryons.
Our final results for the magnetic dipole moments are given in Table I. For comparison, in the same Table we present the estimations of other approaches on the magnetic dipole moments of the spin-1/2 DHB. The errors in the given results originate because of the variations in the calculations of the working regions of \( M^2 \) and \( s_0 \) as well as the uncertainties in the values of the input parameters and the photon DAs. We should also stress that the primary source of uncertainties is the variations with respect to \( s_0 \) and the results weakly depend on the choices of the Borel mass parameter. We also would like to mention that in table and figure, the absolute values are presented since it is not possible to designate the sign of the residue from the mass sum rules. Therefore, we cannot estimate the signs of the magnetic dipole moments.

In Table I, we compare our predictions with the results obtained using other approaches, such as quark model (QM) [68], relativistic three-quark model (RTQM) [58], nonrelativistic quark model in Faddeev approach (NRQM) [56], relativistic quark model (RQM) [57], skyrmion model [70], MIT bag model [60], nonrelativistic quark model (NRQM) [60], chiral constituent quark model \( \chi \)CQM [62], relativistic harmonic confinement model (RHM) [67], lattice QCD [64] and heavy baryon chiral perturbation theory (HBChBT) [65]. From a comparison of our results with the estimations of other approaches we see that for the \( \Xi^{++}_{ccu} \) baryon, all results more or less consistent with each other except the results of Ref. [62,70], which are quite different. For the \( \Xi^{+}_{ccd} \) and \( \Omega^{+}_{ccu} \) baryons, consistent with Refs. [64] and approximately two times smaller than other predictions. For the \( \Xi^{0}_{ccbd} \) baryon we see that all results more or less, are similar except the results of Ref. [65,67], which are large. For the \( \Xi^{0}_{bbd} \) baryon, all results more or less consistent with each other except the results of Ref. [66], which are small. For the \( \Omega^{++}_{bbu} \) baryon, our predictions are larger than other predictions. As can be seen from this Table, various models lead to quite different predictions for the magnetic dipole moments of DHB, which may be used to distinguish these models.

In conclusion, we have calculated the magnetic dipole moments of the spin-1/2 DHB in the framework of light-cone QCD sum rule. The electromagnetic properties of the DHB encodes important information of their internal structure and geometric shape. We performed a comparison of our results with the estimations of various theoretical approaches existing in literature. The agreement of the estimations with some (but not all) theoretical estimations is good. We hope our analysis may be helpful for future experimental measurements.

### IV. ACKNOWLEDGEMENTS

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### TABLE I: Magnetic dipole moments of the spin-\(\frac{1}{2}\) DHB (in units of \(\mu_N\)).

| Approaches                  | \(\Xi^{++}_{ccu}\) | \(\Xi^{+}_{ccd}\) | \(\Omega^{+}_{ccu}\) | \(\Xi^{0}_{ccu}\) | \(\Xi^{0}_{ccbd}\) | \(\Omega^{++}_{bbu}\) |
|-----------------------------|--------------------|--------------------|----------------------|-----------------|------------------|---------------------|
| QM [68]                     | -0.12              | 0.80               | 0.69                 | -               | -                | -                   |
| RQM [57]                    | -0.10              | 0.86               | 0.72                 | -               | -                | -                   |
| Skyrmion [70]               | -0.47              | 0.98               | 0.59                 | -               | -                | -                   |
| NQM [60]                    | -0.20              | 0.79               | 0.64                 | -               | -                | -                   |
| \(\chi\)CQM [62]            | 0.006              | 0.84               | 0.70                 | -               | -                | -                   |
| Lattice QCD [64]            | -                 | 0.425              | 0.413                | -               | -                | -                   |
| MIT Bag model-I [66]        | 0.11               | 0.72               | 0.66                 | -0.43           | 0.09             | 0.04                |
| MIT Bag model II[71]        | -0.11              | 0.72               | 0.64                 | -0.58           | 0.17             | 0.11                |
| RTQM [58]                   | 0.13               | 0.72               | 0.67                 | -0.53           | 0.18             | 0.04                |
| NRQM [56]                   | -0.20              | 0.78               | 0.63                 | -0.69           | 0.23             | 0.10                |
| RHM [67]                    | -0.17              | 0.85               | 0.74                 | -0.89           | 0.32             | 0.16                |
| HBChBT [65]                 | -0.25              | 0.85               | 0.78                 | -0.84           | 0.26             | 0.19                |
| This work                   | 0.23 ± 0.05        | 0.43 ± 0.09        | 0.39 ± 0.09          | 0.51 ± 0.09     | 0.28 ± 0.04      | 0.42 ± 0.05         |

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