DARK MATTER AND DARK ENERGY FROM A SINGLE SCALAR FIELD AND COSMIC MICROWAVE BACKGROUND DATA

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ABSTRACT

Axions are likely to be the dark matter (DM) that cosmological data require. They arise in the Peccei-Quinn solution of the strong-CP problem. In a previous work we showed that their model has a simple and natural generalization that also yields dark energy (DE), in fair proportions, without tuning any parameter: DM and DE arise from a single scalar field and are weakly coupled in the present era. In this paper we extend the analysis of this dual-axion cosmology and fit it to WMAP data, by using a Markov chain technique. We find that ΛCDM, dynamical DE with a SUGRA potential, DE with a SUGRA potential and a constant DE-DM coupling, and the dual-axion model with a SUGRA potential fit data with a similar accuracy. The best-fit parameters are, however, fairly different, although consistency is mostly recovered at the 2σ level. A peculiarity of the dual-axion model with a SUGRA potential is to cause more stringent constraints on most parameters and to favor high values of the Hubble parameter.

Subject headings: cosmology: theory — dark matter — elementary particles

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1. INTRODUCTION

Models with density parameters Ω_0,de ≃ 0.7, Ω_0,m ≃ 0.3Ω_0,b ≃ 0.04 (for dark energy, whole nonrelativistic matter, and baryons, respectively), Hubble parameter h ≃ 0.7 (in units of 100 km s^{-1} Mpc^{-1}), and primeval spectral index n_s ≃ 1 fit most cosmological data, including cosmic microwave background (CMB) anisotropies, large-scale structure, and data on Type Ia supernovae (SNe Ia; Tegmark et al. 2001; de Bernardis et al. 2000; Hanany et al. 2000; Halverson et al. 2002; Spergel et al. 2003; Percival et al. 2002; Efstathiou et al. 2002; Riess et al. 1998; Perlmutter et al. 1999). The success of such ΛCDM models, also dubbed cosmic concordance models, does not hide their uneasiness. The parameters of standard cold dark matter (CDM) still need to be increased by one in order to tune dark energy (DE). Furthermore, if DE is ascribed to vacuum, this turns out to be quite a fine-tuning.

This conceptual problem was eased by dynamical DE models (Wetterich 1988, 1995; Ratra & Peebles 1988). They postulate the existence of an ad hoc scalar field, self-interacting through a suitable effective potential, which depends on a further parameter. In Ratra & Peebles (1988) and SUGRA (see below) models, this is an energy scale Λ (or an exponent α).

Within the frame of dynamical DE models, Mainini & Bonometto (2004, hereafter MB04) tried to take a step forward. Instead of invoking an ad hoc interaction, they refer to the field introduced by Peccei & Quinn (1977, hereafter PQ77; see also Weinberg 1978; Wilczek 1978) to solve the strong CP problem. If suitably tuned, such a scheme had already been shown to yield DM (Preskill et al. 1983; Abbott & Sikivie 1983; Dine & Fischler 1983). MB04 slightly modify the PQ77 scheme, replacing the Nambu-Goldstone (NG) potential introduced ad hoc by a potential admitting a tracker solution. This scheme solves the strong CP problem even more efficiently than the original PQ77 model. The Λ parameter of the tracker potential takes the place of the PQ77 energy scale, F_{PQ}. Fixing it in the range solving the strong CP problem yields DM and DE in fair proportions. Here we call this cosmology the “dual-axion” model. This model has several advantages with respect to both ΛCDM and ordinary dynamical DE: (1) it requires no fine-tuning; (2) it adds no parameter to the standard PQ77 scheme, which yields just DM; and (3) it introduces no field or interaction besides those required by particle physics. This scheme, however, leads to predictions (slightly) different from ΛCDM, for a number of observables. In principle, therefore, it can be falsified by data.

The essential peculiarity of the model is that it predicts a coupling between DM and DE. Coupled DE models were introduced by a number of authors (see, e.g., Amendola 2000, 2004; Gasperini et al. 2002; Perrotta & Baccigalupi 2002), as no direct evidence exists that DM particles follow geodesics. Most such models, however, introduce a further coupling parameter β. Its tuning fixes DM-DE coupling within an acceptable range. For instance, Amendola & Quercellini (2003) give limits on β deduced from a fit to Wilkinson Microwave Anisotropy Probe (WMAP) data (Spergel et al. 2003); Maccìò et al. (2004) restricted β even more, by studying the halo profiles produced in N-body simulations with coupled DE.

At variance from these pictures, the dual-axion scheme has no extra coupling parameter. The strength of the coupling is set by theory and, if this conflicts with data, the whole scheme is falsified. The only degree of freedom still allowed is the choice of the tracker potential. This freedom exists for any dynamical DE model. PQ77 also exploited it by choosing an NG potential. For the sake of definiteness, up to now, the dual-axion scheme has been explored only in association with a SUGRA potential. MB04 showed that, in this case, the dual-axion scheme predicts a fair growth of density fluctuations, hence granting a viable picture for the large-scale structure.

The recent detailed WMAP data on CMB anisotropies allow us to submit the dual-axion model to further stringent tests, by comparing it with other cosmologies such as ΛCDM and standard and interacting dynamical DE. This is done here using a multi-parameter Markov chain technique (see, e.g., Kosowsky et al. 2002; Christensen et al. 2001; Knox et al. 2001; Lewis & Bridle 2002). The results of this numerical approach can then be discussed and understood on physical bases.
models, however, perform nearly better than the others. Apparently, the best fit is obtained by dynamical DE based on a SUGRA potential, but its success is strictly marginal. The analysis of these models against CMB data is the main aim of this paper.

The plan of the paper is as follows. In § 2 we summarize the particle physics background to the dual-axion model. In particular, starting from § 2.3, we restrict our analysis to a particular form of DE potential, the SUGRA potential; using a different potential might change some quantitative results. In § 3 we describe the technique used to compare different models with CMB data and present (§ 3.2) the results of this comparison. We devote § 4 to a final discussion of the results.

2. A SINGLE SCALAR FIELD TO ACCOUNT FOR DM AND DE

Let us first remind the reader that the strong CP problem arises from the existence in quantum chromo dynamics (QCD) of multiple vacuum states. The set of the gauge transformations $\Omega(x)$ that join vacuum configurations can be subdivided into classes $\Omega_n(x)$, characterized by an integer $n$ (Jackiw & Rebbi 1976), setting their different asymptotic behaviors. Within each class, transformations can be distorted into each other with continuity, while this is impossible if they pertain to different classes.

Accordingly, in classical field theory there is no communication between different $n$ gauge sectors. In quantum field theory, on the other hand, tunneling is possible thanks to instanton effects, so that any vacuum state is a superposition of the vacua $|0_n\rangle$ (of the $n$th sector), of the kind $|0_n\rangle = \sum |0_n\rangle \exp(i\theta_n)$.

The effects of varying the $\theta$ vacuum can be recast into variations of a nonperturbative term

$$L_\theta = \frac{\alpha_s}{2\pi} \theta G \tilde{G}$$

(1)

(where $\alpha_s$ is the strong coupling constant and $G$ and $\tilde{G}$ are the gluon field tensor and its dual) in the QCD Lagrangian density. However, chiral transformations also change the vacuum angle, so that the $\theta$ parameter receives another contribution, arising from the electroweak (EW) sector, when the quark mass matrix $\mathcal{M}$ is diagonalized, becoming

$$\theta_{\text{eff}} = \theta + \arg \det \mathcal{M}.$$ 

(2)

The Lagrangian term given by equation (1) can be reset in the form of a four-divergence and causes no change of the equations of motion. However, it violates CP and, among various effects, yields a neutron electric moment $d_n \simeq 5 \times 10^{-10} \theta_{\text{eff}} e$ cm, conflicting with the experimental limit $d_n \leq 10^{-25} e$ cm, unless $\theta_{\text{eff}} \leq 10^{-10}$.

The point is that the two contributions to $\theta_{\text{eff}}$ are uncorrelated, so there is no reason why their sum should be so small.

PQ77 succeed in suppressing this term by imposing an additional global chiral symmetry $U(1)_{\text{PQ}}$, spontaneously broken at a suitable scale $F_{\text{PQ}}$. The axion field is suitably coupled to the quark sector. The details of this coupling depend on the model and may require the introduction of an ad hoc heavy quark (Kim 1979; Shifman et al. 1980; see also Dine et al. 1981; Zhitnitsky 1980). The $U(1)_{\text{PQ}}$ symmetry suffers from a chiral anomaly, so the axion acquires a tiny mass because of nonperturbative effects, whose size increases rapidly around the quark-hadron transition scale $\Lambda_{\text{QCD}}$. The anomaly manifests itself when a chiral $U(1)_{\text{PQ}}$ transformation is performed on the axion field, giving rise to a term of the same form of equation (1), which provides a potential for the axion field.

As a result, $\theta$ is effectively replaced by the dynamical axion field. Its oscillations about the potential minimum yield axions. This mechanism works independently of the scale $F_{\text{PQ}}$. Limits on it arise from astrophysics and cosmology, requiring that $10^{10}$ GeV $\leq F_{\text{PQ}} \leq 10^{12}$ GeV; in turn, this yields an axion mass that today lies in the interval $10^{-6}$ eV $\leq m_a \leq 10^{-3}$ eV.

In more detail, in most axion models, the PQ77 symmetry breaking occurs when a complex scalar field $\Phi = \phi e^{i\theta}/\sqrt{2}$, falling into one of the degenerated minima of an NG potential

$$V(\Phi) = \lambda (|\phi|^2 - F_{\text{PQ}}^2)^2,$$

(3)

develops a vacuum expectation value $\langle \phi \rangle = F_{\text{PQ}}$.

The CP-violating term, arising around quark-hadron transition when $\bar{q}q$ condensates break the chiral symmetry, reads

$$V_1 = \sum_q \text{Tr}(|0(T)|\bar{q}q|0(T)) m_q (1 - \cos \theta).$$

(4)

($\sum_q$ extends over all quarks), so that $\theta$ is no longer arbitrary but ruled by a suitable equation of motion. The term in square brackets, at $T \simeq 0$, approaches $m^2_{\pi} f^2$ (where $m_{\pi}$ and $f$ are the pion mass and decay constant). In this limit, for $\theta \ll 1$ and using $A = \theta F_{\text{PQ}}$ as the axion field, equation (4) reads

$$V_1 \simeq \frac{1}{2} q^2 m^2_{\pi} f^2 a^2 \frac{A^2}{F_{\text{PQ}}}.$$ 

(5)

Here $q(m_q)$ is a function of the quark masses $m_q$: in the limit of two light quarks ($u$ and $d$), $q = (m_u/m_d)^{1/2}(1 + m_u/m_d)^{-1}$. The $A$ field bears the right dimensions but will no longer be used below, and the axion degrees of freedom will be described through $\theta$ itself. Equation (5), however, shows that when $\langle \bar{q}q \rangle$ is no longer zero (since $T \leq \Lambda_{\text{QCD}}$), the axion mass decreases with temperature, approaching the constant value $m_a = m_q f_{\pi} F_{\text{PQ}}$ for $T \ll \Lambda_{\text{QCD}}$.

Accordingly, the equation of motion, in the small $\theta$ limit, reads

$$\ddot{\theta} + 2 \frac{\dot{a}}{a} \dot{\theta} + a^2 m^2_a \theta = 0$$

(6)

(here $a$ is the scale factor and dots yield differentiation with respect to conformal time; see next section), so that the axion field undergoes (nearly) harmonic oscillations as soon as $m_a$ exceeds the expansion rate; then, its mean pressure vanishes (Dine & Fischler 1983), leaving the axion as a viable candidate for cold DM.

MB04 replace the NG potential in equation (3) by a potential $V(\Phi)$ admitting a tracker solution (Wetterich 1988, 1995; Ratra & Peebles 1988; Ferreira & Joyce 1998; Brax & Martin 1999, 2000; Brax et al. 2000). The field $\Phi$ is complex and $V(\Phi)$ is $U(1)$ invariant, but there is no transition to a constant-value $F_{\text{PQ}}$, which is replaced by the modulus $\phi$ itself, slowly evolving over cosmological times. At a suitable early time, quantum dynamics starts to be fairly accounted for by the potential $V$; soon after, $\phi$ settles on the tracker solution in almost any horizon, although breaking the $U(1)$ symmetry, by the values assumed by $\theta$, in different horizons. Later on, when chiral symmetry breaks, dynamics also becomes relevant for the $\theta$ degree of freedom, as in the PQ77 case. At variance from it, however, this happens while $\phi$ continues its slow evolution, down to the present epoch, when it accounts for DE. Owing to the $\phi$ evolution, however, the axion mass also evolves, over cosmological times, for $T \ll \Lambda_{\text{QCD}}$ (see below).
The \( \Phi \) field, therefore, besides providing DM through its phase \( \theta \), whose dynamics solves the strong CP problem, also accounts for DE through its modulus \( \phi \).

This scheme holds for any DE potential admitting tracker solutions. To be more specific, MB04 use a SUGRA potential (Brax et al. 2000; Brax & Martin 1999, 2000), finding that, at the quark-hadron transition, \( \phi \) can be naturally led to have values \( \sim f_\Phi \), increasing up to \( \sim m_p = G^{-1/2} \) (the Planck mass), when approaching today. The only free parameter is the energy scale in the SUGRA potential, which must be \( \sim 10^{10} \) GeV. With this choice, \( \theta \) is driven to values even smaller than in the PQ77 case, so that CP is apparently conserved in strong interactions, while \( \Omega_{0,m}\Omega_{0,\text{de}} \) (the DE density parameter) and \( \Omega_{0,\phi} \) take fair values.

2.1. Lagrangian Theory

In the dual-axion model we start from the Lagrangian

\[
\mathcal{L} = \sqrt{-g} \left[ g_{\mu
u} \partial_\mu \Phi \partial_\nu \Phi - V(\phi) \right],
\]

which can be rewritten in terms of \( \phi \) and \( \theta \), adding also the term breaking the \( U(1) \) symmetry. Then it reads

\[
\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} g_{\mu\nu} \left( \partial_\mu \phi \partial_\nu \phi + \phi^2 \partial_\mu \theta \partial_\nu \theta \right) - V(\phi) \right.
\]
\[
\left. - m^2(T, \phi) \phi^2 (1 - \cos \theta) \right].
\]

Here \( g_{\mu\nu} \) is the metric tensor. We assume that \( ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(d\tau^2 - \eta_{ij} dx_i dx_j) \), where \( a \) is the scale factor and \( \tau \) is the conformal time; Greek (Latin) indices run from 0 to 3 (1 to 3); and dots indicate differentiation with respect to \( \tau \). The mass behavior for \( T \sim \Lambda_{\text{QCD}} \) is detailed in § 2.2. The equations of motion, for the \( \phi \) and \( \theta \) degrees of freedom, read

\[
\ddot{\vartheta} + 2 \left( \frac{\dot{a}}{a} + \frac{\dot{\phi}}{\phi} \right) \dot{\theta} + m^2 a^2 \sin \theta = 0,
\]
\[
\ddot{\phi} + 2 \frac{\dot{a}}{a} \dot{\phi} + a^2 V'(\phi) = \phi \ddot{\theta}^2.
\]

[Notice that \( m^2(T, \phi) \phi^2 \) is \( \phi \) independent; see below.] In what follows, the former equation will always be considered when \( \sin \theta \approx \theta \). In particular, taking into account the condition \( \theta \ll 1 \), the expressions for the energy densities \( \rho_{\theta,\phi} = \rho_{\theta,\phi,\text{kin}} + \rho_{\theta,\phi,\text{pot}} \) and the pressures \( p_{\theta,\phi} = \rho_{\theta,\phi,\text{kin}} - \rho_{\theta,\phi,\text{pot}} \) are obtainable by combining the terms

\[
\rho_{\theta,\phi,\text{kin}} = \frac{\dot{\phi}^2}{2a^2} \dot{\theta}^2,
\]
\[
\rho_{\theta,\phi,\text{pot}} = m^2(T, \phi) \phi^2 (1 - \cos \theta) \simeq \frac{m^2(T, \phi)}{2} \phi^2 \dot{\theta}^2,
\]
\[
\rho_{\phi,\text{kin}} = \frac{\dot{\phi}^2}{2a^2}, \quad \rho_{\phi,\text{pot}} = V(\phi).
\]

When \( \theta \) undergoes many (nearly) harmonic oscillations within a Hubble time, \( \rho_{\theta,\phi,\text{pot}} \simeq \langle \rho_{\theta,\text{pot}} \rangle \) and \( \langle \rho_{\phi} \rangle \) vanishes (Dine & Fischler 1983). Under such conditions, using equations (9) and (10), it is easy to see that

\[
\dot{\rho}_\theta + \frac{3}{a} \dot{\rho}_\theta = \frac{m}{m} \rho_\theta, \quad \dot{\rho}_\phi + 3 \frac{\dot{a}}{a} (\rho_\phi + p_\phi) = - \frac{m}{m} \rho_\phi.
\]

When \( m \) is given by equations (15) and (16) below, \( \dot{m}/m = -\phi/\phi - 3.8\dot{T}/T \). At \( T \approx 0 \), instead, it is just \( \dot{m}/m \simeq -\phi/\phi \).

The \( \theta \) and \( \phi \) components account for DM and DE, respectively. Accordingly, below the indices \( \theta \) and \( \phi \) will be replaced by \( \text{dm} \) and \( \text{de} \).

Equation (12) clearly shows that an exchange of energy occurs between DM and DE. From this point of view, the MB04 model belongs to the set of coupled models treated by Amendola (2000, 2004). It is, however, characterized by a time-dependent coupling. In fact, in the small \( \theta \) limit, after averaging over cosmological times, the right-hand sides of equations (10) and (12) read \( C(\phi) \rho_\theta/a^2 \) and \( \pm C(\phi) \phi (\rho_\phi) \), if we set \( C(\phi) = 1/\phi \). Here \( C \) is the DE-DM coupling introduced by Amendola (2000, 2004), who, however, extensively studies only the case \( C = \beta (16\pi/3m_p^2)^{1/2} \), with constant \( \beta \). In the latter DE models, a \( \phi \)-MDE phase takes place after matter-radiation equivalence. It differs from a matter-dominated expansion because of the contribution of the kinetic part of the DE field to the expansion source. A regime of this kind is also present in the dual-axion model and is shown in Figure 5. Because of the \( \phi \) dependence, however, the DM-DE coupling, in the dual-axion model, weakens as we approach the present cosmological epoch.

The most stringent limits on \( \beta \) are set by nonlinear predictions (Maccì et al. 2004) and restrict \( \beta \) to values \( \lesssim 0.1 - 0.2 \), in order to avoid a too high concentration in DM halos. In turn, this is due to the behavior of the effective mass of DM particles, in the presence of the \( \beta \) coupling. A preliminary inspection indicates that the behavior expected here is opposite and that halo concentrations, on average, should be smaller than in \( \Lambda\text{CDM} \). This point, however, must be examined in much more detail.

Let us also note that the former equation (12) can be integrated soon, yielding \( \rho_{\text{dm}} \propto m a^3 \). In particular, this law holds at \( T \ll \Lambda_{\text{QCD}} \). Accordingly, at late times

\[
\rho_{\text{dm}} a^3 \phi \approx \text{const},
\]

so that the usual behavior \( \rho_{\text{dm}} \propto a^{-3} \) is modified by the energy flow from DM to DE. This modification is stronger when \( \phi \) varies rapidly and is damped when \( \phi \) attains a nearly constant behavior.

2.2. Axion Mass

According to equation (9), the axion field begins to oscillate when

\[
m(T, \phi) a \simeq 2 \left( \frac{\dot{a}}{a} + \frac{\dot{\phi}}{\phi} \right).
\]

In the dual-axion model, just as for PQ77, the axion mass rapidly increases when the chiral symmetry is broken by the formation of the \( \alpha \) condensate at \( T \sim \Lambda_{\text{QCD}} \). In the dual-axion model, however, the axion mass also varies later on because of the evolution of \( \phi \), when \( m(T, \phi) \) is

\[
m_0(\phi) = \frac{q(m_p) m \pi f_\alpha}{\phi} \simeq \frac{\Lambda_{\text{QCD}}}{\phi},
\]

with \( \mu_{\text{QCD}} \simeq 80 \) MeV. Since \( \phi \sim m_p \) today, the present axion mass \( m \sim 5 \times 10^{-13} \text{ eV} \). At high temperature, according to Gross et al. (1981),

\[
m(T, \phi) \simeq 0.1 m_0(\phi) \left( \frac{\Lambda_{\text{QCD}}}{T} \right)^{3.8}.
\]
This expression must be interpolated with equation (15) to study the fluctuation onset for \( T \sim \Lambda_{\text{QCD}} \). We report the results of the solution of the equations in § 2.1, obtained by assuming

\[
m(T, \phi) = \begin{cases} 
  m_0(\phi) \left( \frac{0.1^{1/3.8} \Lambda_{\text{QCD}}}{T} \right)^{3.8(1-a/\alpha)}, & a < a_c, \\
  m_0(\phi), & a > a_c,
\end{cases}
\]

with \( a_c = T_0/\Lambda_{\text{QCD}} 0.1^{1/3.8} = 2.16 \times 10^{-12} \); here \( T_0 = 2.35 \times 10^{-4} \) eV and \( \Lambda_{\text{QCD}} = 200 \) MeV \( \simeq 2.5 \mu_{\text{QCD}} \). Equation (17) ensures that \( m(T, \phi) \) and its first derivative are continuous and that the axion mass meets its low-\( T \) behavior \( m_0(T) \) at \( T \simeq 0.55 \Lambda_{\text{QCD}} \). The effects of selecting a different value for \( \Lambda_{\text{QCD}} \) were studied. The impact of the assumed \( T \) dependence was also considered. A change of \( \Lambda_{\text{QCD}} \), by a factor of 2, yields a variation of \( \Lambda \) in the SUGRA potential (see § 2.3) by not more than a few percent. Taking a power \( \nu \neq 1 \) of \( (1 - a/\alpha) \) in the exponent, instead, does not affect continuity but changes the rapidity with which the high-\( T \) regime given by equation (16) is met.

In Figure 4 we plot both the interpolated mass behavior and the first \( \phi \) oscillations. Changing \( \nu \) just slightly displaces the interpolating curve between \( \sim a_0 \text{QCD} \) and \( \sim 1.5 a_0 \text{QCD} \), causing a minor phase shift. Let us finally summarize that equations (15) and (16), as well as the interpolation given by equation (17), ensure that \( m(T, \phi) \phi \) is \( \phi \) independent, as is required to give the equation of motion the form of equation (10).

2.3. Using the SUGRA Potential

Most of the above features are general and do not depend on the choice of the potential \( V \). We now assume that

\[
V(\Phi) = \frac{\Lambda^{\alpha+4}}{\phi^{\alpha}} \exp \left( \frac{4\pi\phi^2}{m_p^2} \right) \]

(SUGRA potential; see Brax & Martin 1999, 2000; Brax et al. 2000). Apparently \( V(\Phi) \) depends on the two parameters \( \Lambda \) and \( \alpha \). When they are independently assigned, \( \Omega_{\text{de}} \) is also fixed. Here we prefer to use \( \Omega_{\text{de}} \) and \( \Lambda \) as independent parameters. The latter scale is related to \( \alpha \) as shown in Figure 1. This relation is almost independent of the presence of DE-DM coupling. The slight shifts due to the couplings, evaluated through the expressions of this paper, are just slightly above the numerical noise and are also shown in Figure 1.

The potential given by equation (18) does not depend on \( \theta \) and, in the radiation-dominated era, admits the tracker solution

\[
\phi^{\alpha+2} = g_0 \Lambda^{\alpha+4} a^2 \tau^2,
\]

with \( g_0 = \alpha (\alpha + 2)^2 \). This tracker solution, characterizing SUGRA models at very high \( z \), is abandoned because of the DE-DM coupling, when the term \( \phi \phi^2 \) exceeds \( a^2 V' \), and the field enters a different tracking regime:

\[
\phi^2 = \frac{1}{2} \rho_{\text{dm}} a^2 \tau^2.
\]

Figure 2 is a landscape of the behavior of densities, starting from the high-\( z \) tracking regime given by equation (19), passing then to the new intermediate tracking regime, and reaching the regimes when DE density eventually exceeds first radiation (at \( z \sim 100 \)), then baryons (at \( z \sim 10 \)) and DM (at \( z \sim 3 \)). In Figure 3 we show the evolution of the DE field \( \phi \). In Figure 4 we focus on the Q-H transition and magnify the scale dependence of the mass and the onset of \( \phi \) oscillations. Finally, in Figure 5 we show the behaviors of the density parameters \( \Omega_i \) (\( i = r, b, \theta, \phi \), i.e., radiation, baryons, DM, DE) versus the scale factor \( a \). More detailed pictures of the single transitions are shown in MB04.

2.4. Parameter Fixing

In general, once \( \Omega_{0,\text{de}} \) and \( h \) are given, a model with dynamical (coupled or uncoupled) DE is not yet determined, as \( \alpha \) or \( \Lambda \) are still to be fixed. When fitting WMAP data here we mostly refer to the energy scale \( \Lambda \) and remind the reader that, when \( \Omega_{0,\text{de}} \) and \( \Lambda \) are also given, \( \alpha \) is fixed.

Other potentials show similar features, which implies that, in general, dynamical DE models depend on one more parameter than, e.g., \( \Lambda_{\text{CDM}} \); accordingly, one can expect that they will more easily accommodate observational data. In principle, data fitting is even more facilitated by DM-DE coupling, which adds a further parameter (the strength of coupling) to the theoretical plot.
In the dual-axion model with a SUGRA potential, no such arbitrariness exists. If this model, where a suitable DM-DE coupling is present, succeeds in fitting observational data, this will not be favored by extra parameters. The coupling, in particular, depends on no parameter, and the very scale \( \Lambda \) is set by theoretical consistency arguments, similarly to what happens for the \( F_{\text{PQ}} \) scale in the PQ77 approach.

Let us follow the behavior of \( \rho_{\text{dm}} \) backward in time, until the very beginning of the oscillatory regime, when the approximation \( \theta \ll 1 \) begins to hold. Using the scaling law given by equation (13) until then and, at earlier times, the more general laws given by equation (11), together with equation (14), we build a system yielding the scale factor \( a_h \) when the fluctuations start and the scale \( \Lambda \) in the SUGRA potential given by equation (18).

It turns out that \( a_h \) and \( \Lambda \) are almost independent of \( \Omega_{0, \text{dm}} \) and \( h \), and we now outline the qualitative reasons for this feature. According to equations (11) and (15), at the \( a_h \) scale it must be

\[
\rho_{\text{h, dm}} \simeq m_h^2 \phi^2 (a_h) \epsilon^2 (a_h) = \left( \frac{m_h}{m_0} \right)^2 \mu_{\text{QCD}}^2 (a_h),
\]

(21)

where \( m_h = m(a_h) \), while \( \epsilon^2 \equiv 2(1 - \cos \theta) \) is \( \sim \theta^2 \) at the onset of the oscillatory regime. Owing to equation (14), however, the oscillation onset occurs when the axion mass is approximately the inverse of \( a_h \phi (a_h) \), and the latter quantity can be related to the temperature \( T_h \), via the Friedmann equation. This yields that \( T_h^2 \simeq m_p m_\phi / 8 \) and, again using equation (15), that

\[
T_h \simeq 5 \times 10^{-2} \mu_{\text{QCD}} \left( \frac{m_p}{m_\phi} \frac{m_h}{m_0} \right)^{1/2}.
\]

(22)

Let us assume that now \( \phi \sim m_p \) and use the law given by equation (13) from now to \( a_h \). It is then

\[
T_h \simeq \mu_{\text{QCD}} \left[ 5 \times 10^3 \left( \frac{m_h}{m_0} \right)^3 \epsilon^2 (a_h) \right]^{1/5},
\]

(23)

provided that \( \mu_{\text{QCD}} \simeq 4 \times 10^{11} T_0 \) and \( \rho_{0, \text{dm}} \simeq 3 \times 10^4 T_0^4 \Omega_{0, \text{dm}} h^2 \).

The temperature \( T_h \) and the scale \( a_h \) are then essentially model independent because of the power \( \frac{1}{2} \) on the right-hand side of equation (23). If we consider the plots in Figure 4, obtained through a numerical integration for \( \Omega_{0, \text{dm}} = 0.27 \) and \( h = 0.7 \), we see that \( (m_h / m_0)^2 \epsilon^2 \sim 10^{-2} \) to \( 10^{-3} \), so that \( T_h \) falls around \( \Lambda_{\text{QCD}} \). However, no appreciable displacement can be expected just varying \( \Omega_{0, \text{dm}} \) and \( h \), even though we suppose that this induces substantial variations of the factor \( (m_h / m_0)^2 \epsilon^2 (a_h) \), which, however, cannot exceed unity or lie below \( 10^{-3} \).

The model independence of \( a_h \) implies that the scale \( \Lambda \) is also almost model independent. The numerical result \( a_h \simeq 10^{-13} \) is consistent with \( \Lambda \simeq 1.5 \times 10^{10} \) GeV, but neither value will change much just by varying \( \Omega_{0, \text{dm}} \). In practice, when \( \Omega_{0, \text{dm}} \) goes from 0.2 to 0.4, log\( _{10} (\Lambda / \text{GeV}) \) (almost) linearly runs from 10.05 to 10.39 and \( a_h \) steadily lies at the eve of the quark-hadron transition.

The residual dependence of \( \alpha \) and \( \Lambda \) on \( \Omega_{0, \text{dm}} \) is plotted in Figure 6, together with the corresponding values for \( w \) at \( z = 0 \). Significantly smaller values for \( \Lambda \) are obtainable only for unphysically small DM densities.

The only way to modify the result is to consider values of \( \phi \) significantly different from \( m_p \) today. In fact, the rest of the plot can be understood as an effect of the position reached by the \( \phi \) field, in the SUGRA potential, at the present time. The peak of \( w \) corresponds to a maximum of kinetic energy and occurs if the minimum of the SUGRA potential is attained today. Today lies in the proximity of the minimum for a fairly wide \( \Lambda \) interval. Accordingly, a significant \( \Lambda \) variation, theretofore, yields just a modest shift of \( \Omega_{0, \text{dm}} \), as is shown in the bottom panel. For still greater \( \Lambda \)-values, \( \phi \) would still be in its preminimum descent. For even smaller amounts of DE, the present \( \phi \) configuration would still lie in the \( \phi \)-MD era, when kinetic energy dominates for DE, although the very DE contribution to the overall energy density becomes negligible.

A model with DE and DM given by a single complex field, based on a SUGRA potential, therefore bears a precise prediction on the scale \( \Lambda \), for the observational \( \Omega_{0, \text{dm}} \) range. Moreover, only the observational \( \Omega_{0, \text{dm}} \) range (0.2–0.4) corresponds to \( \phi \sim m_p \) today.

The rest of this paper is devoted to a comparison of this and other models against WMAP data. In such a comparison, however,
were potential, when varying the present fraction of DM. The faster shape of the coupling, where the scale dominates, as the strength of the coupling is gauged by an extra parameter as in standard coupled DE models. In the latter models, however, suitably coupled, about whose nature no assumption is made, just their two independent dark components, DM and DE, /C30. The precision estimates of the anisotropy power spectrum WMAP parameter values compatible with cosmological observables and 

$\Omega_{\text{dm}} \equiv \Omega_{0,\text{dm}} \Omega_{0,h} + \Omega_{0,\text{de}}$. As possible extensions of ΛCDM cosmologies, several works considered models with a fixed state parameter $w \equiv p_{\text{de}}/\rho_{\text{de}}$ (e.g., Spergel et al. 2003; Pogosian et al. 2003; Bean & Doré 2004; Tegmark et al. 2004; Melchiorri 2004) or adopted $\omega$-dependent parameterizations of $w(z)$ interpolating between early-time and late-time values (e.g., Corasaniti et al. 2004; Jassal et al. 2005; Rapetti et al. 2005). A general conclusion was that current data mostly allow one to constrain only the present state parameter, $w(z = 0) \lesssim -0.80$.

In this work we consider, instead, three classes of dynamical DE, requiring the introduction of additional parameters specifying the physical properties of the scalar field: (1) ΛCDM dynamical DE requires the introduction of $\lambda = \log_{10} (\Lambda/\text{GeV})$, yielding the energy scale in the potential given by equation (12). (2) In constant-coupling DE, the coupling parameter $\beta = C(3m_p^2/16\pi)^{1/2}$ is also needed. (3) In the case of the dual-axion model, the last parameter is excluded, for it is simply $C = \phi^{-1}$. The scale $\Lambda$ is also constrained by the requirement that $\Omega_{0,\text{de}}$ lie in a fair range (also solving the strong CP problem). Hence, in the dual-axion model, $\Lambda$ and $\Omega_{0,\text{de}}$ are no longer independent parameters. However, we consider a wider class of coupled DE models, which we call $\phi^{-1}$ models, leaving $\lambda$ as a free parameter. Our aim is to test whether WMAP data constrain it into the region turning a $\phi^{-1}$ model into a dual-axion model.

In the use of MCMC, as well as in any attempt to fit CMB data to models, a linear code providing $C_l$ values is needed. Here we use our optimized extension of CMBFAST (Seljak & Zaldarriaga 1996) to inspect cosmologies 1, 2, and 3. Then, the likelihood of each model is evaluated through the publicly available code by the WMAP team (Verde et al. 2003) and accompanying data (Hinshaw et al. 2003; Kogut et al. 2003).

3.1. Implementing an MCMC Algorithm

An MCMC algorithm samples a known distribution $\mathcal{L}(x)$ by means of an arbitrary trial distribution $p(x)$. Here $\mathcal{L}$ is a likelihood and $x$ is a point in the parameter space. The chain is started from a random position $x$ and moves to a new position $x'$, according to the trial distribution. The probability of accepting the new point is given by $\mathcal{L}(x')/\mathcal{L}(x)$; if the new point is accepted, it is added to the chain and used as the starting position for a new step. If $x'$ is rejected, a replica of $x$ is added to the chain and a new $x'$ is tested.

In the limit of infinitely long chains, the distribution of points sampled by an MCMC describes the underlying statistical process. Real chains, however, are finite and convergence criteria are critical. Moreover, a chain must be required to fully explore the high-probability region in the parameter space. Statistical properties estimated using a chain that has yet to achieve good convergence or mixing may be misleading. Several methods exist to diagnose mixing and convergence, involving either single long chains or multiple chains starting from well-separated points in the parameter space, as the one used here. Once a chain passes convergence tests, it is an accurate representation of the underlying distribution.

In order to ensure mixing, we run six chains of $\sim 30,000$ points each, for each model category. We diagnose convergence by requiring that, for each parameter, the variances both of the single chains and of the whole set of chains ($W$ and $B$, respectively) satisfy the Gelman & Rubin test (Verde et al. 2003; Gelman & Rubin 1992), $R < 1.1$ with

$$R = \frac{(N - 1)/N W + (1 + 1/M)B}{W}.$$ (24)
Here each chain has $2N$ points, but only the last $N$ points are used to estimate variances, and $N$ is the total number of chains. In most model categories considered, we find that the slowest parameter to converge is $\lambda$.

3.2. Results

The results of this paper mostly concern the fit of the double-axion model with *WMAP* data. Let us, however, also briefly outline the features of the model.

Quite generally, cosmological models fitting observations require a triple coincidence between DM, baryon, and DE densities, the last one occurring just at the present epoch. Any such coincidence requires the tuning of a suitable parameter. It is then natural to look for an underlying physics, able to predict the parameters values that cosmology requires.

If DM is made of axions, we must tune the $F_{PQ}$ parameter in the NG potential involving a $\Phi$ field, whose phase $\theta$ is then the axion field. Here we saw that the PQ77 model does not strictly require an NG potential. A potential causing no immediate settling at an energy minimum, but inducing a slow rolling along a tracking solution, achieves the same aims. If such potential is SUGRA, it contains an energy scale $\Lambda$ to be tuned, instead of $F_{PQ}$. If this is done, to yield the required amount of DM, this scheme provides, as an extra bonus, a fair amount of DE, which is prescribed to interact with DM in a peculiar way, never studied in the literature.

The possible shapes of DE-DM interaction considered up to now involved an ad hoc coupling parameter. No such parameter exists here. For instance, a standard model of coupled SUGRA involves a density parameter $\Omega_{0,\text{dm}}$, an energy scale $\Lambda$, and a coupling parameter $\beta$. Here, not only does $\beta$ no longer exist, but also $\Lambda$ is fixed once $\Omega_{0,\text{dm}}$ is assigned. The model is therefore highly constrained and data can easily falsify it. The only available degree of freedom, to modify it, amounts to replacing the SUGRA potential with another potential shape. Another reservation to be borne in mind is that axion models have a contribution to DM coming from the decay of topological structures, which were not considered here.

These arguments outline the significance of the fit of the model with *WMAP* data. The basic results of this work concern this fit and are summarized in Tables 1–3. For each model category we list the expectation values of each parameter and the associated variance; we also list the values of the parameters of the best-fitting models. The corresponding marginalized distributions are plotted in Figures 10–12, while joint two-dimensional confidence regions are shown in Figures 7–9.

The values of $\chi^2$ for each category of models can be compared, taking into account the number of degrees of freedom. This comparison is shown in Table 4. The smallest $\chi^2$ is obtained for the uncoupled SUGRA model, which performs slightly better than $\Lambda$CDM. Differences, however, are really small and yield no support to any model category.

It must be reiterated, however, that the $\phi^{-1}$ models, whose fitting results are reported in Table 3 and Figures 9 and 12, include the dual-axion model, but many other cases as well. Our approach was meant to test whether CMB data carry information on $\lambda$ and how this information fits the $\lambda$ range turning a $\phi^{-1}$ model into the dual-axion model.

Let us also sum up Tables 1 and 2 and the corresponding figures, concerning uncoupled or constant-coupling SUGRA models, and emphasize that *WMAP* data provide no real constraint on $\Lambda = \log (\Lambda/\text{GeV})$, when allowed to vary from roughly $-12$ to $16$. No limitation exists even on its sign. On the contrary, when a $\phi^{-1}$ coupling is set, loose but precise limitations on $\lambda$ arise, as is shown in Table 3. In the presence of this coupling, the 2 $\sigma$ $\Lambda$ interval ranges from $-10$ to $3 \times 10^{10}$ GeV, including the range required by the dual-axion model.

In fact, in this case the location of peaks in spectra is more strictly related to the $\phi$ evolution. In turn, only a restricted set of $\Lambda$-values allows an $l$ dependence of multipoles consistent with data. Figure 13 shows this fact for anisotropy, while Figures 14 and 15 are predictions for the TE and E polarization spectra, respectively.

Figure 13 also shows why no model category neatly prevails. At large $l$ all best-fit models yield similar behaviors. In turn this shows that discrimination could be achieved by improving large angular scale observations, especially for polarization, so as to reduce errors on small-$l$ harmonics.

4. DISCUSSION

Our analysis does not concern the double-axion model only, but the whole spectrum of models built using a SUGRA potential.

**TABLE 1**

| $x$ | $\langle x \rangle$ | $\sigma_x$ | $x_{\text{max}}$ |
|---|---|---|---|
| $\Omega_{0,\text{dm}} h^2$ | 0.025 | 0.001 | 0.026 |
| $\Omega_{0,\text{tot}} h^2$ | 0.12 | 0.02 | 0.11 |
| $h$ | 0.63 | 0.06 | 0.58 |
| $\tau$ | 0.21 | 0.07 | 0.28 |
| $n_s$ | 1.04 | 0.04 | 1.08 |
| $A$ | 0.97 | 0.13 | 1.11 |
| $\lambda$ | 3.0 | 7.7 | 13.7 |

**TABLE 2**

| $x$ | $\langle x \rangle$ | $\sigma_x$ | $x_{\text{max}}$ |
|---|---|---|---|
| $\Omega_{0,\text{dm}} h^2$ | 0.024 | 0.001 | 0.024 |
| $\Omega_{0,\text{tot}} h^2$ | 0.11 | 0.02 | 0.12 |
| $h$ | 0.74 | 0.11 | 0.57 |
| $\tau$ | 0.18 | 0.07 | 0.17 |
| $n_s$ | 1.03 | 0.04 | 1.02 |
| $A$ | 0.92 | 0.14 | 0.93 |
| $\beta$ | $-0.5$ | 7.6 | 8.3 |

**TABLE 3**

| $x$ | $\langle x \rangle$ | $\sigma_x$ | $x_{\text{max}}$ |
|---|---|---|---|
| $\Omega_{0,\text{dm}} h^2$ | 0.025 | 0.001 | 0.026 |
| $\Omega_{0,\text{tot}} h^2$ | 0.11 | 0.02 | 0.09 |
| $h$ | 0.93 | 0.05 | 0.98 |
| $\tau$ | 0.26 | 0.04 | 0.29 |
| $n_s$ | 1.23 | 0.04 | 1.23 |
| $A$ | 1.17 | 0.10 | 1.20 |
| $\lambda$ | 4.8 | 2.4 | 5.7 |

**Note.**—For each parameter $x$, the expectation value $\langle x \rangle$, variance $\sigma_x$, and maximum likelihood values $x_{\text{max}}$, in the seven-dimensional parameter space, are shown.

**Note.**—The parameter space is seven-dimensional, and parameter values are shown as in the Table 1.
Fig. 7.—Marginalized distributions for the seven-parameter SUGRA model with no priors (solid line), BBNS prior (long-dashed line), or HST prior (dot-dashed line). Short-dashed (dotted) vertical lines show the boundaries of the 68.3\% c.l. (95.4\% c.l.) interval; for $\lambda$ only upper limits are shown. [See the electronic edition of the Journal for a color version of this figure.]
Fig. 8.—Same as Fig. 7, but for the eight-parameter constant-coupling model. For $\lambda$ and $\beta$ only the upper c.l. boundaries are shown. [See the electronic edition of the Journal for a color version of this figure.]
Fig. 9.—Same as Fig. 7, but for the seven-parameter $\phi^{-1}$ model. [See the electronic edition of the Journal for a color version of this figure.]
Fig. 10.—Joint two-dimensional constraints for SUGRA models. Light (dark) shaded areas delimit the region enclosing 68.3% (95.4%) of the total points. [See the electronic edition of the Journal for a color version of this figure.]
This is useful also to evaluate the significance of the fit of the dual-axion model.

4.1. Uncoupled and Coupled SUGRA Models

A first point we must therefore outline is that SUGRA uncoupled models are consistent with WMAP data. The ratio $w = p/\rho$ for most of these models is less than or approximately $-0.80$ at $z = 0$. However, they exhibit a fast variation of $w$, which already attains values of approximately $-0.6$ at $z \sim 1-2$. This sharp decrease does not conflict with data, and these models perform even better than $\Lambda$CDM. For uncoupled or constant-coupling SUGRA models, the analysis in the presence of priors leads to analogous conclusions.

Best-fit cosmological parameters exhibit some dependence on the model. First, the opacity $\tau$ is pushed to values exceeding the $\Lambda$CDM estimates (see also Corasaniti et al. 2004). This can be understood in two complementary ways. (1) Dynamical DE models, in general, exhibit a stronger ISW effect, as the field $\phi$ itself varies during the expansion, and DE effects extend to greater $z$. This increases $C_l^T$ in the low-$l$ plateau (e.g., Weller & Lewis 2003). To compensate for this effect, the fit tends to shift the primeval spectral index $n_s$ to a greater value. Owing to the $\tau$-$n_s$ degeneracy, this is then compensated for by increasing $\tau$. (2) In dynamical DE models, the expected TE correlation, at low $l$, is smaller than in $\Lambda$CDM (Colombo et al. 2003). A given observed correlation level, therefore, requires a greater $\tau$. In any case, values of $\tau \sim 0.07$ remain consistent with data within less than $2\sigma$.

Greater $\tau$-values push upward $\Omega_bh^2$ estimates, although best-fit values are consistent with $\Lambda$CDM within $1\sigma$. Adding a prior on $\Omega_bh^2 = 0.0214 \pm 0.0020$ (BBNS estimates; see, e.g., Kirkman et al. 2003) lowers $h$, within $1\sigma$ from Hubble Space Telescope (HST) findings. We therefore also consider the effect of a prior on $h$. In Figures 7 and 8 the effects of priors are shown by the dashed line (prior on $\Omega_bh^2$) and the dot-dashed line (prior on $h$).

The former prior affects mainly reionization and $n_s$; $\tau$ and $n_s$ are lowered to match WMAP’s findings, and the high tail of the $\tau$ distribution is partly suppressed. The physical analysis of primeval objects causing reionization (e.g., Ciardi et al. 2003; Ricotti & Ostriker 2004) could, however, hardly account for values of $\tau \sim 0.3$, which are still allowed but certainly not required.

The latter prior favors greater $h$-values. In the absence of coupling, this favors low-$\lambda$ models, closer to $\Lambda$CDM. In fact, the sound horizon at decoupling is unaffected by the scale $\Lambda$, while the comoving distance to last scattering band is smaller for greater $\lambda$-values. Then, as $\lambda$ increases, lower $h$-values are favored to match the angular position of the first peak. In the presence of coupling, there is a simultaneous effect on $\Omega_\Lambda h^2$, as greater $\Omega_\Lambda h^2$-values yield a smaller sound horizon at recombination, so that the distribution on $h$ is smoother.

A previous analysis of WMAP limits on constant-coupling models had been carried out by Amendola & Quercellini (2003). Their analysis concerned potentials $V$ fulfilling the relation $dV/d\phi = BV^N$, with suitable $B$ and $N$. Furthermore, they assume that $\tau \equiv 0.17$. Our analysis deals with a different potential and allows more general parameter variations. The constraints on $\beta$...
we find are less severe. However, it must be emphasized that $\beta \gtrsim 0.1-0.2$ seems forbidden by a nonlinear analysis of structure formation (Macciò et al. 2004).

4.2. Dual-Axion Models

Let us consider then $\phi^{-1}$ models, which generalize dual-axion models to an arbitrary $\Lambda$ scale, used as a parameter to fit data. Parameters are better constrained in this case, although the overall model likelihood is similar. This is made evident by Figure 12. In particular, at variance from the former case, the energy scale $\Lambda$ is significantly constrained and, within 2 $\sigma$, constraints are consistent with the double-axion model.

Several other parameters are constrained, similarly to coupled or uncoupled SUGRA models. What is peculiar to $\phi^{-1}$ models is the range of favored $h$-values: the best-fit 2 $\sigma$ interval does not extend much below 0.85.

This problem is slightly more severe for the dual-axion model. The point is that this model naturally tends to displace the first $C_l^T$ peak to greater $l$ (smaller angular scales) than coupling does, in any case. But, in the absence of a specific coupling parameter, the very intensity of coupling, in these models, depends just on the scale $\Lambda$. Increasing $\Lambda$ requires a more effective compensation, favoring greater values of $h$.

Apart from the possibility that $h$ is currently underestimated, let us review other substantial options that are still to be investigated. (1) The contributions of topological singularities to DM were not considered here, and taking them into account could naturally increase the amount of DM, yielding smaller $\Lambda$-values for the double-axion model; Figure 6, however, shows that this is hardly an efficient solution. (2) The choice of a SUGRA potential is, however, arbitrary; the dual-axion model does not require SUGRA. Other potentials could possibly yield the same coupling intensity in agreement with a smaller $h$.

5. CONCLUSIONS

The first evidence of DM arose some 70 years ago, although only in the late 1970s did limits on CMB anisotropies make it evident that a nonbaryonic component had to be dominant. DE can also be dated back to Einstein’s cosmological constant, although only SN Ia data revived it, soon followed by data on CMB and deep galaxy samples.

Axions have been a good candidate for DM since the early 1980s, although various studies, as well as the occurrence of SN1987a, strongly constrained the PQ77 scale around values $10^{11}$ GeV. Contributions to DM from topological singularities (cosmic string and walls) narrowed the constraints to $F_{\text{PQ}}$. Full agreement on the relevance of these contributions has not yet been attained and, in this paper, they are still disregarded.

The fact that DM and DE can both arise from scalar fields, just by changing the power of the field in effective potentials, has already stimulated the work of various authors. A potential like that of equation (18) was considered in the so-called spintessence model (Boyle et al. 2002; Gu & Hwang 2001). According to the choice of parameters, $\Phi$ was shown to behave as either DM or DE. In the frame of a model of tachyon DE, Padmanabhan & Choudhury (2002) also built a model in which DM and DE arise

Fig. 12.—Same as Fig. 10, but for $\phi^{-1}$ models. Here cosmological parameters are more stringently constrained than in other models. [See the electronic edition of the Journal for a color version of this figure.]
from a single field. The possibility that both DM and DE arise from the solution of the strong CP problem was also suggested by Barr & Seckel (2001). Their model, however, does not deal with dynamical DE and aims to explain why the vacuum energy is so finely tuned, while DM is simultaneously provided.

Here we deal with the possibility that the field, which solves the strong CP problem, simultaneously accounts for both DE and DM. The angle $\theta$ in equation (1), as in the PQ77 model, is turned into a dynamical variable, i.e., the phase of a scalar field $\phi$. While $\theta$ is gradually driven to approach zero, by our cosmic epoch, in our model $\phi$ gradually increases and approaches $m_\phi$, yielding DE. Residual $\theta$ oscillations, yielding axions, account for DM.

The main topic of this paper is, however, the fit of this and other models with WMAP data. We compared $\Lambda$CDM, SUGRA dynamical, and coupled DE models, as well as a scheme we dub the $\Lambda$CDM dual-axion model. In principle, this strong impact of $\phi$ variations on the detailed ISW effect, as they affect both DE pressure and energy density, as well as DE-DM coupling. In this way, the stronger effects of $\phi$ variations on the detailed ISW effect, as they affect both DE pressure and energy density, as well as DE-DM coupling. In this way, the strongly varying $\phi$ could badly disrupt the fit and make $\phi$ models significantly farther from the data. This does not occur, and the observational $\Lambda$ range agrees with the dual-axion model at a 2 $\sigma$ level.

**Fig. 13.** $C_l^T$ spectra for the best-fit SUGRA (solid line), constant-coupling (dotted line), $\phi^{-1}$ coupling (dashed line), and dual-axion (dot-dashed line) models. The dual-axion model results from considering only those $\phi^{-1}$ with $9.5 < \lambda < 10.5$. The binned first-year WMAP data are also plotted. [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 14.** Best-fit $C_l^T$ spectra. [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 15.** Best-fit $C_l^T$ spectra. [See the electronic edition of the Journal for a color version of this figure.]

**Table 4**

| Model          | $\chi^2_{\text{red}}$ | Probability (%) |
|----------------|------------------------|-----------------|
| No coupling    | 1.064                  | 5.0             |
| $\beta$ coupling | 1.066                  | 4.7             |
| $\phi^{-1}$ coupling | 1.074                  | 2.9             |
| Dual axion     | 1.081                  | 2.0             |
| $\Lambda$CDM   | 1.066                  | 4.7             |

Notes.—For each class of DE considered, the table lists the number of degrees of freedom (dof), the reduced $\chi^2_{\text{red}}$, and the corresponding probability of the best-fit model. Figures for $\Lambda$CDM and dual-axion models are also included. $\Lambda$CDM models have 1342 degrees of freedom, uncoupled and $\phi^{-1}$ have 1341, while fixed $\beta$ has 1340.
The success would be complete if the favored range of values of the Hubble parameter ($h \sim 0.8-1$) could be slightly lowered. This range is, however, obtained only for a SUGRA potential. This choice is not compulsory and, moreover, contributions to axion DM due to topological singularities were also disregarded. Furthermore, primeval fluctuations were assumed to be strictly adiabatic, while in axion models a contribution from isocurvature modes can be expected. This could legitimately affect the apparent position of the first peak in the anisotropy spectrum, thus completing the success of the model in a fully self-consistent way.

REFERENCES

Abbott, L., & Sikivie, P. 1983, Phys. Lett. B, 120, 133
Amendola, L. 2000, Phys. Rev. D, 62, 043511
———. 2004, Phys. Rev. D, 69, 103524
Amendola, L., & Quercellini, C. 2003, Phys. Rev. D, 68, 023514
Barr, S. M., & Seckel, D. 2001, Phys. Rev. D, 64, 123513
Bean, R., & Doré, O. 2004, Phys. Rev. D, 69, 083503
Boyle, L. A., Caldwell, R. R., & Kamionkowski, M. 2002, Phys. Lett. B, 545, 17
Brax, P., & Martin, J. 1999, Phys. Lett. B, 468, 40
———. 2000, Phys. Rev. D, 61, 103502
Brax, P., Martin, J., & Riazuelo, A. 2000, Phys. Rev. D, 62, 103505
Christensen, N., Meyer, R., Knox, L., & Lucy, B. 2001, Classical Quantum Gravity, 18, 2677
Ciardi, B., Ferrara, A., & White, S. D. M. 2003, MNRAS, 344, L7
Colombo, L. P. L., Mainini, R., & Bonometto, S. A. 2003, in When Cosmology and Fundamental Physics Meet, ed. V. Le Brun, S. Basa, & A. Mazure (Paris: Frontier Group), 223
Corasaniti, P. S., Kunz, M., Parkinson, D., Copeland, E. J., & Bassett, B. A. 2004, Phys. Rev. D, 70, 083006
de Bernardis, P., et al. 2000, Nature, 404, 955
Dine, M., & Fischler, W. 1983, Phys. Lett. B, 120, 137
Dine, M., Fischler, W., & Srednicki, M. 1981, Phys. Lett. B, 104, 199
Dunkley, J., Bacher, M., Ferreira, P. G., Moodley, K., & Skordis, C. 2005, MNRAS, 356, 925
Efstathiou, G., et al. 2002, MNRAS, 330, L29
Ferreira, G. P., & Joyce, M. 1998, Phys. Rev. D, 58, 023503
Gasperini, M., Piazza, F., & Veneziano, G. 2002, Phys. Rev. D, 65, 023508
Gelman, A., & Rubin, D. B. 1992, Stat. Sci., 7, 457
Gross, D., Pisarski, R., & Yaffe, L. 1981, Rev. Mod. Phys., 53, 43
Gu, J.-A., & Hwang, Y.-Y. P. 2001, Phys. Lett. B, 517, 1
Halverson, N. W., et al. 2002, ApJ, 568, 38
Hanany, S., et al. 2000, ApJ, 545, L5
Hinshaw, G., et al. 2003, ApJS, 148, 135
Jackiw, R., & Rebbi, C. 1976, Phys. Rev. Lett., 37, 172
Jassal, H. K., Bagla, J. S., & Padmanabhan, T. 2005, MNRAS, 356, L11
Kim, J. E. 1979, Phys. Rev. Lett., 43, 103
Kirkman, D., et al. 2003, ApJS, 149, 1
Knox, L., Christensen, N., & Skordis, C. 2001, ApJ, 563, L95
Kogut, A., et al. 2003, ApJS, 148, 161
Kosowsky, A., Miloslavjevic, M., & Jimenez, R. 2002, Phys. Rev. D, 66, 63007
Lewis, A., & Bridle, S. 2002, Phys. Rev. D, 66, 103511
Maccio, A., Quercellini, C., Mainini, R., Amendola, L., & Bonometto, S. A. 2004, Phys. Rev. D, 69, 123516
Mainini, R., & Bonometto, S. A. 2004, Phys. Rev. Lett., 93, 121301 (MB04)
Melchiorri, A. 2004, preprint (astro-ph/0406652)
Padmanabhan, T., & Choudhury, T. R. 2002, Phys. Rev. D, 66, 081301
Pecei, R. D., & Quinn, H. R. 1977, Phys. Rev. Lett., 38, 1440 (PQ77)
Percival, W. J., et al. 2002, MNRAS, 337, 1068
Perlmutter, S., et al. 1999, ApJ, 517, 565
Perrotta, F., & Baccigalupi, C. 2002, Phys. Rev. D, 65, 123505
Pogosian, D., Bond, J. R., & Contaldi, C. 2003, preprint (astro-ph/0301310)
Preskill, J., Wise, M., & Wilczek, F. 1983, Phys. Lett. B, 120, 127
Rapetti, D., Allen, S. W., & Weller, J. 2005, MNRAS, 360, 555
Ratra, B., & Peebles, P. J. E. 1988, Phys. Rev. D, 37, 3406
Ricotti, M., & Ostriker, J. P. 2004, MNRAS, 350, 539
Riess, A. G., et al. 1998, AJ, 116, 1009
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Shifman, M. A., Vainshtein, A. I., & Zakharov, V. I. 1980, Nucl. Phys. B, 166, 493
Spergel, D. N., et al. 2003, ApJS, 148, 175
Tegmark, M., Zaldarriaga, M., & Hamilton, A. J. 2001, Phys. Rev. D, 63, 43007
Tegmark, M., et al. 2004, Phys. Rev. D, 69, 103501
Verde, L., et al. 2003, ApJS, 148, 195
Weinberg, S. 1978, Phys. Rev. Lett., 40, 223
Weller, J., & Lewis, A. M. 2000, MNRAS, 346, 987
Wetterich, C. 1988, Nucl. Phys. B, 302, 668
———. 1995, A&A, 301, 321
Wilczek, F. 1978, Phys. Rev. Lett., 40, 279
Zhitnisky, A. P. 1980, Soviet J. Nucl. Phys., 31, 260