Electrohydrodynamic lubrication model in point contacts with consideration of surface coating

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Abstract. An electrohydrodynamic lubrication (EHL) model in point contacts with consideration of surface coating is presented. The disturbance displacement caused by the coating on the surfaces is calculated using the finite element method (FEM). The effects of coating’s hardness and thickness on oil film pressure, thickness, and subsurface von Mises stresses are investigated. The results show that hard coating causes a significant increase in the oil film pressure, while soft coating has the opposite effect. A significant stress concentration occurs at the subsurface when there is a hard coating. And when the thickness of the coating reaches a certain value, the subsurface stress reaches its maximum. While when there is a soft coating, the maximum stress on the subsurface is slightly reduced.

Keywords: EHL, FEM, coating, deformation.

1. Introduction (Heading 1)
Surface coatings has been widely used in mechanical parts to protect the contact surface from wear and fatigue, such as gears, bearings [1]. The use of the coating on the surface of these parts can play a significant role in protecting and improving fatigue life. These components are often processed under electrohydrodynamic lubrication, so the influence of coatings on electrohydrodynamic lubrication has been increasingly studied.

Early in 1994, Elsharkawy et al introduced a solution for Newtonian and non-Newtonian lubrication EHL contact, with a rigid foundation was coated by an elastic layer [2, 3]. A central film thickness formula for coated line contacts is given when the coating’s thickness is small relative to the Hertzian contact radius [4]. Jaffer [5] and Elsharkawy [6] also investigated the effect of roughness on the coating. Jin provided a full numerical analysis of the EHL problem for point contact, but his cases are reduced to rigid substrates coated with an elastic layer [7, 8]. Liu et al developed an EHL model for coated surfaces in point contacts and the elastic deformation is calculated using the DC-FFT method (discrete convolution and fast Fourier transform) [9, 10].

All works mentioned above have simplified the model or require a complex mathematical algorithm, hence the application is limited. While the FEM method has great advantages in calculating the elastic deformation of the coated surfaces. In this paper, the FEM method is used to calculate the disturbance displacement caused by the coating on the surfaces. The effects of the surface coating’s hardness and thickness on oil film pressure, thickness, and subsurface stresses are investigated.
2. Model description
In rolling bearings, the contact between balls and raceway can be reduced to a contact between a plane and a ball, as illustrated in Fig. 1. Unlike a conventional EHL issue, the surface of the plane has a coating layer, the mechanical properties of which differ from the substrate. The previous formula for surface elastic deformation of linear elastic materials is no longer applicable. The influence of the coating on oil film thickness needs to be considered. In general, the EHL problem needs to solve three equations simultaneously: Reynolds, film thickness, and load balance equations.

![Figure 1. EHL model with coated surfaces in a point contact](image)

2.1. EHL model
The dimensionless form of the Reynolds equation is written as:

\[
\frac{\partial}{\partial x} \left( \varepsilon \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \varepsilon \frac{\partial P}{\partial y} \right) = \frac{\partial}{\partial x} (\bar{\rho} H)
\]  

Were

\[
\varepsilon = \frac{\bar{\rho} H^3}{\eta \lambda}, \lambda = \frac{6 \eta_0 R^2 u_s}{a^3 P_H}
\]

The following boundary condition shall be fulfilled.

\[
P(X, Y) = P(X, Y) \quad \text{if} \quad P(X, Y) > 0
\]

\[
P(X, Y) = 0 \quad \text{if} \quad P(X, Y) \leq 0
\]

The oil film thickness is calculated by

\[
H(X, Y) = H_0 + \frac{x^2}{2} + \frac{y^2}{2} + D + D_f
\]

In which D refers to the elastic deformation of the uncoated surface, and \(D_f\) is defined as the disturbed surface deformation caused by the coating. In this paper, the disturbed surface deformation \(D_f\) was calculated by using the FEM method. The elastic deformation of the uncoated surface is given by

\[
D = \frac{2}{\pi^2} \iint \frac{P(x', y') dX' dY'}{\sqrt{(X-x')^2 + (Y-y')^2}}
\]

The load balance equation must be satisfied.

\[
\iint P dX dY = \frac{2}{3} \pi
\]

The Roelands viscosity-pressure equation [11] and the Dowson and Higginson density-pressure equation [12] is given by
\[
\eta(p) = \eta_0 \exp \left\{ \frac{\alpha p a}{z} \left[ -1 + \left( 1 + \frac{p}{p_0} \right)^2 \right] \right\}
\] (4)

\[
\rho(p) = \rho_0 \left( 1 + \frac{5.8 \times 10^{-5} p}{1 + 1.7 \times 10^{-5} p} \right)
\] (5)

2.2. Calculation of the disturbed surface deformation

In this paper, the disturbed surface deformation was calculated by using the FEM method. The FEM model of the contact domain with a surface coating is illustrated in Fig.2. Since the oil film pressure distribution is symmetrical, it only needs to be solved in the half domain. The domain with sizes of -30a \leq x \leq 30a, 0 \leq y \leq 30a, 0 \leq z \leq 60a was selected, where 'a' stands for the radius of the Hertzian contact zone. To reduce the calculation time, the grid is locally refined in areas where the stress is concentrated or at the interface of the coating and substrate, and general grids are used in other areas.

Figure 2. The FEM model of the contact domain with a surface coating

In the FEM model, the elastic deformation of the surface shall be computed twice. For the first time, the elastic modulus of the coating is set to be identical to substrate material (i.e. Ec=Es, equivalent to uncoated contact), and the surface deformation \(d_1\) is obtained. For the second time, the elastic modulus of the coating is set to be different from the substrate material, and the surface deformation \(d_2\) is obtained. The difference between \(d_2\) and \(d_1\) is entirely due to the different properties of the coating. Thus, the disturbed surface deformation can be calculated as follows:

\[
d_f = d_2 - d_1
\] (6)

The dimensionless form of disturbed surface deformation can be written as

\[
D_f = \left( d_2 - d_1 \right) \frac{R}{a^2}
\] (7)

Where \(a^2/R\) is the dimensionless factor of oil film thickness.

The flow chart of the entire calculation process is shown in Fig 3. The oil film pressure, thickness, and load should converge at the same time, which requires an iterative approach. In this paper, the influence of temperature is ignored.
3. Parameter setup
In this paper, the effects of coating’s properties on oil film pressure, thickness, and subsurface stress are mainly investigated. The load is set as 25N and the effective radius of curvature is 0.02 m. The Elastic modulus of the substrate $E_s$ is fixed at 210 GPa and the load is 25N, which corresponding uncoated Hertzian pressure is 0.86 GPa and the Hertzian contact radius is 235 ums.

The elastic modulus of the coating $E_c$ are considered 105 GPa and 420 GPa. The thickness of the coating varies between 40 and 160um. The principal parameters used in this document are given in Table 1.

Table 1. principal parameters used in this paper

| Parameter                                              | Value          |
|--------------------------------------------------------|----------------|
| The effective radius of curvature $R$ (m)              | 0.02           |
| Inlet viscosity of the lubricant $\eta$ (pa.s)         | 0.04           |
| Inlet density of the lubricant (kg/m$^3$)              | 846            |
| Pressure-viscosity coefficient (GPa$^{-1}$)            | 15.8           |
| The entrancement speed $u_0$ (m/s)                     | 1              |
| Elastic modulus of substrates $E_s$ (GPa)              | 210            |
| Elastic modulus of the coating $E_c$ (GPa)             | 105;210;420    |
| Poisson’s ratio of substrates $\nu_s$                  | 0.3            |
| Poisson’s ratio of coating $\nu_c$                     | 0.3            |
| Hertzian contact radius ‘$a$’ (um)                     | 235            |
| The thickness of the coating $t_c$(um)                 | 40-160         |

4. Results and discussion
In this work, effects of coating’s hardness and thickness on oil film pressure, thickness, and subsurface stress are mainly studied. Fig. 4 shows a typical result of the impact of the coating on the thickness and
pressure of the oil film. The results indicate that the coating significantly affects the oil film pressure. The coating with a high elastic module causes a significant increase in the oil film pressure, whereas the coating with a low elastic module has the opposite effect. The coating makes little difference to the thickness of the oil film. The hard coating slightly narrows the contact area, while the soft coating does the opposite.

Figure 4. Effects of coating’s hardness on oil film pressure and thickness

Fig. 5 and Fig. 6 show the effects of coating’s thickness and hardness on oil film pressure. The results show that for the hard coating, the pressure of the oil film increases as the thickness of the coating increases, whereas the soft coating has a reverse effect. The maximum pressure of the oil film increases or decreases approximately linearly with the increase of the coating thickness. When the thickness of the coating reaches 160um, the maximum oil film pressure changes by about 15% compared to the uncoated surface.

Figure 5. Effects of coating’s thickness and hardness on oil film pressure
Figure 6. Effects of coating’s thickness and hardness on the maximum pressure of the oil film

The coating not only affects the pressure distribution of the oil film but also affects the subsurface stress in the contact area. Fig 7(a) shows the subsurface stress distribution when there is no coating, and Fig 7(b) and Fig 7(c) show the subsurface stress distribution when there is an 80um thick hard coating and a soft coating, respectively. A significant stress concentration can be seen to occur at the subsurface when there is a hard coating, and the maximum stress increases by 31%. While when there is a soft coating, the maximum stress on the subsurface is slightly reduced.

Figure 7. Effects of coating’s thickness and hardness on subsurface von Mises stress (N/m2)

For hard coatings, the maximum stress on the subsurface is located at the junction of the coating and the substrate. And as the thickness of the coating increases within a certain range, a greater stress
concentration occurs, as shown in the Fig. 7(b) and (d). If the coating and the substrate are not tightly bonded, it is more likely to cause the coating to fatigue spall.

Fig. 8 shows the effects of coating’s hardness and thickness on subsurface von Mises stress. In the case of hard coatings, when the thickness of the coating reaches a certain value, the subsurface stress reaches its maximum. Then, as the thickness of the coating increases, the subsurface stress decreases slightly but still far greater than the value of the subsurface stress of the uncoated surface. Similar results can be obtained with soft coatings.

**Figure 8.** Effects of coating’s hardness and thickness on subsurface von Mises stress

5. Conclusion
An EHL model in point contacts with consideration of surface coating is presented. The effects of the surface coating’s hardness and thickness on oil film pressure, thickness, and subsurface stresses are investigated. The main conclusions were obtained as follows:

- A coating with a high Elastic modulus will cause a significant increase in the oil film pressure in the contact area, while a coating with a low Elastic modulus has the opposite effect. The coating makes little difference to the thickness of the oil film.
- A significant stress concentration occurs at the subsurface when there is a hard coating. And when the thickness of the coating reaches a certain value, the subsurface stress reaches its maximum. While when there is a soft coating, the maximum stress on the subsurface is slightly reduced.

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References
[1] W. Habchi, "A numerical model for the solution of thermal elastohydrodynamic lubrication in coated circular contacts," Tribology International, vol. 73, pp. 57 - 68, 2014.
[2] A. A. Elsharkawy and B. J. Hamrock, "EHL of Coated Surfaces: Part I—Newtonian Results," Journal of Tribology, vol. 116, no. 1, pp. 29-36, 1994.
[3] A. A. Elsharkawy and B. J. Hamrock, "EHL of Coated Surfaces: Part II—Non-Newtonian Results," Journal of Tribology, vol. 116, no. 4, pp. 786 - 793, 1994.
[4] M. Hlavacek, "A central film thickness formula for elastohydrodynamic lubrication of cylinders with soft incompressible coatings and a non-Newtonian piecewise power-law lubricant in steady rolling motion," (in English), Wear, vol. 205, no. 1-2, pp. 20-27, Apr 1997.
[5] Jaffar MJ. On the microelastohydrodynamic lubrication of an elastomeric bonded layer. Tribology International. 2002; 35 (3):193. doi:10.1016/S0301-679X(01)00115-3.

[6] Elsharkawy AA, Holmes MJA, Evans HP, Snidle RW. Micro-elastohydrodynamic lubrication of coated cylinders using coupled differential deflection method. Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology. 2006; 220 (1): 29 - 41.

[7] Jin, Z. M. (2000). "Elastohydrodynamic lubrication of a circular point contact for a compliant layered surface bonded to a rigid substrate Part 1: Theoretical formulation and numerical method." Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology 214 (3): 267 - 279.

[8] Jin, Z. M. (2000). "Elastohydrodynamic lubrication of a circular point contact for a compliant layered surface bonded to a rigid substrate Part 2: Numerical results." Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology 214 (3): 281 - 289.

[9] Liu, Y., et al. (2007). "An Elastohydrodynamic Lubrication Model for Coated Surfaces in Point Contacts." Journal of Tribology 129 (3): 509 - 516.

[10] Liu, Y., et al. (2008). "Effect of Stiff Coatings on EHL Film Thickness in Point Contacts." Journal of Tribology 130 (3).

[11] Roelands, C. J. A. Correlation Aspects of Viscosity-Temperature-Pressure Relationship of Lubricating Oils. The Netherlands: Delft University of Technology; 1966.

[12] Higginson, D. D. G. R.: Elasto-hydrodynamic lubrication: The fundamentals of roller and gear lubrication. Pergamon, Oxford (1966).