Scaling laws for near barrier Coulomb and Nuclear Breakup

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We investigate the nuclear and the Coulomb contributions to the breakup cross sections of $^6$Li in collisions with targets in different mass ranges. Comparing cross sections for different targets at collision energies corresponding to the same $E/V_0$, we obtain interesting scaling laws. First, we derive an approximate linear expression for the nuclear breakup cross section as a function of $A^{1/3}_T$. We then confirm the validity of this expression performing CDCC calculations. Scaling laws for the Coulomb breakup cross section are also investigated. In this case, our CDCC calculations indicate that this cross section has a linear dependence on the atomic number of the target. This behavior is explained by qualitative arguments. Our findings, which are consistent with previously obtained results for higher energies, are important when planning for experiments involving exotic weakly bound nuclei.

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I. INTRODUCTION

The effect of the coupling of the breakup channel on the complete fusion of weakly bound, and especially, halo nuclei, is a subject that has been extensively investigated in the last years, both experimentally and theoretically. There are strong signatures that this coupling hinders the fusion cross section at energies above the barrier, and enhances the tunneling-dominated fusion below the barrier. Owing to the importance of the breakup channel at low energies, close to the Coulomb barrier, it is very interesting to investigate the dependence of the relative importance of its Coulomb and nuclear components, $\sigma_{\text{bu}}^{\text{C}}$ and $\sigma_{\text{bu}}^{\text{N}}$, respectively, on the mass number or charge of the target nucleus. There are results showing that the above mentioned effects of breakup coupling on fusion cross sections are less important for light systems than for heavy ones.

However, one cannot calculate the total breakup cross section $\sigma_{\text{bu}}$ as $\sigma_{\text{bu}} = \sigma_{\text{bu}}^{\text{C}} + \sigma_{\text{bu}}^{\text{N}}$ since the interference of these two components may be very strong, as demonstrated in [7,9]. From $\sigma_{\text{bu}}^{\text{C}}$ one can extract information about the collective response of halo and other weakly bound nuclei, such as the dipole response. By considering a fixed weakly bound projectile, one might expect that $\sigma_{\text{bu}}^{\text{C}}$ should increase with the charge of the target. For heavy targets, the Coulomb breakup should dominate over the nuclear breakup. On the other hand, the dependence of $\sigma_{\text{bu}}^{\text{N}}$ with the target nucleus characteristics is more difficult to predict. Two reported works [7,10] deal with the study of the nuclear breakup as a function of the target mass at high energies, around bombarding energies of some tens of MeV/n. According to those works, the nuclear breakup cross section behaves at a given value of the bombarding energy, $E_{\text{lab}}$, as,

$$\sigma_{\text{bu}}^{\text{N}} = P_1 + P_2 A_T^{1/3}. \tag{1}$$

Above, $P_1$ and $P_2$ are functions of the bombarding energy and the structure of the projectile. This formula, well substantiated by extensive continuum discretized coupled channel calculations (CDCC) performed in Ref. [7], was used to estimate the nuclear breakup cross section for a heavy target, using the experimental value of the cross section for collisions of the same projectile with a light target. In this case, the Coulomb breakup is much smaller and to a first approximation it can be neglected. Frequently [11,14], the nuclear contribution to the breakup cross section is estimated in this way and the Coulomb contribution is obtained subtracting this contribution from the experimental total breakup cross section. The Coulomb breakup cross section obtained in this way can be compared with the one given by expression [11,15,16],

$$\sigma_{\text{bu}}^{\text{C}} = \frac{16\pi}{9} \alpha \int dE_x n_{E_1}(E_x) \frac{dB(E_1)}{e^2 dE_x}, \tag{2}$$

where $\alpha$ is the fine structure constant, $n_{E_1}$ is the number of virtual photons, and $dB(E_1)/e^2 dE_x$ is the dipole response. Since $n_{E_1}$ is known, the above expression for $\sigma_{\text{bu}}^{\text{C}}$ is used to test models of the dipole response. However, assuming that the Coulomb breakup cross section...
can be given by the difference between the total and the nuclear breakup cross section may be very wrong. This procedure neglects the interference between the Coulomb and the nuclear breakup amplitudes, which may be quite important [7].

In Ref. [7], Hussein et al. performed CDCC calculations for the nuclear breakup of $^8$B, $^{11}$Be, and $^7$Be projectiles in collisions with several targets. The cross sections $\sigma_{\text{nu}}$ for different collision energies were plotted against the target mass. They concluded that the non-halo projectile $^7$Be seemed to obey the scaling law quite well. However, the halo nuclei required values and signs of $P_7$ of the target mass. They concluded that the non-halo projectile $^7$Be seemed to obey the scaling law quite well. However, the halo nuclei required values and signs of $P_1$ and $P_2$ which were not consistent with the simple scaling law obtained at higher energies.

Very recently we reported [8] results of a study on this subject at lower energies, around the Coulomb barrier. We investigated the breakup process evaluating separate contributions from the Coulomb and from the nuclear fields, as well as the Coulomb-nuclear interference, through CDCC calculations. We performed calculations for collisions of the $^6$Li projectile with $^{59}$Co, $^{144}$Sm and $^{208}$Pb, at three energies very close to the Coulomb barrier ($E/V_b = 0.84$, 1.00 and 1.07, where $V_b$ is the Coulomb barrier). The choice of these systems was based on the availability of elastic scattering data in the literature. In this way, we were able to check the reliability of our calculations, which do not contain any adjustable parameter, through comparisons with the scattering data. The results showed a linear dependence of the Coulomb breakup cross section, $\sigma_{\text{nu}}$, with the charge of the target, for the same $E/V_b$ values. A linear dependence of the nuclear breakup cross section, $\sigma_{\text{nu}}$, was found as a function of $A_t^{1/3}$, similar to what was found at high energies [7], but for the same values of $E/V_b$, instead of $E_{\text{lab}}$. Furthermore, we have shown a strong interference between the two breakup components, in such way that $\sigma_{\text{nu}}$ is smaller than $\sigma_{\text{nu}} + \sigma_{\text{nu}}$ for all systems and all energies investigated and, for sub-barrier energies $\sigma_{\text{nu}}$ is even larger than $\sigma_{\text{nu}}$.

However, in ref. [8] we did not derive theoretically the dependence of $\sigma_{\text{nu}}$ with the target mass for the same $E/V_b$ values, and the calculations were restricted to energies very close to $V_b$. So, in this Brief Report, as a complement to ref. [8], we show the theoretical derivation of $\sigma_{\text{nu}}$ as a function of the target mass and we compare those results with similar CDCC calculations. Here, we consider collision energies in a wider range, from $E/V_b = 0.84$ to 3.0, and we also fill the gap existing between $^{59}$Co and $^{144}$Sm target nuclei, including in our investigation the $^6$Li + $^{120}$Sn system.

In the following we derive the low energy version of Eq. (1) for the nuclear breakup. We start with the Wong formula for the fusion cross section taken to be the total nuclear reaction cross section, without breakup [17],

$$\sigma_f = \frac{\Gamma}{E} \pi R^2 \ln \left[ 1 + \exp \left( \frac{h^2 (\Lambda_c + \Delta)^2}{2\mu R^2 \Gamma} \right) \right]. \quad (3)$$

where $\Gamma = h\omega/2\pi$ is an energy width related to the curvature of the Coulomb barrier ($h\omega$) and $R$ barrier radius. The variable $\Lambda_c$ is the critical angular momentum associated with fusion.

The reaction cross section including nuclear breakup but not Coulomb breakup (this contribution is of a long range nature and can not be described using the Wong formula), can be written as,

$$\sigma = \frac{\Gamma}{E} \pi R^2 \ln \left[ 1 + \exp \left( \frac{h^2 (\Lambda_c + \Delta)^2}{2\mu R^2 \Gamma} \right) \right]. \quad (4)$$

The nuclear breakup cross section is taken to be the difference [18],

$$\sigma_{\text{nu}} = \sigma_f - \sigma_f = \frac{\Gamma}{E} \pi R^2 \ln \left[ 1 + \exp \left( \frac{h^2 (\Lambda_c + \Delta)^2}{2\mu R^2 \Gamma} \right) \right]. \quad (5)$$

This expression can be simplified by expanding to lowest order in $\Delta/\Lambda_c$, to give,

$$\sigma_{\text{nu}} = 2 \frac{\pi}{k^2} \Lambda_c \Delta = 2\pi a \left[ 1 - \frac{V_b}{E} \right] (R_f + R_T) = P_1 + P_2 A_t^{1/3}, \quad (6)$$

where we have used $\Delta_c = (2\mu/h^2)(R_f + R_T)^2[E - V_b] = k^2[1 - V_b/E][R_f + R_T]^2$, and $\Delta = k^2[1 - V_b/E] a^2$, with $a$ being the diffuseness of the nuclear surface. Clearly $P_1 = 2\pi a (1 - V_b/E)R_f$ and $P_2 = 2\pi a r_0(1 - V_b/E)$.

Eq. (6) clearly shows that for a fixed $E/V_b$ and for a given projectile, the cross section scales linearly with the radius of the target, as it was verified in ref. [8]. The above formula for the nuclear breakup cross section represents the scaling at low energies and reduces to the one discussed in [7] at higher energies. Of course the factor $(1 - V_b/E)$ is only meaningful at above-barrier energies. As the energy is lowered below the barrier, tunneling takes over and one must rely on a different approximation. Once we have derived this scaling law, the next step is to confirm its validity by calculating the nuclear breakup cross sections through CDCC calculations. The CDCC method [19, 20] is the most suitable approach to deal with the breakup process, which feeds states in the continuum, whose wave functions are grouped in bins or wave packets that can be treated similarly to the usual bound inelastic states, since they are described by square-integrable wave functions. In the present work we extend the calculations already presented in ref. [8], and so we will not repeat the details here, since they can be found in that reference and in Refs. [19, 21]. Only the main aspects of the method and some specific details of the calculations will be mentioned in the following. It is assumed that $^6$Li projectiles breakup into a deuteron and an alpha particle, and so it is used the cluster model in...
which $^6$Li is described as a bound state of the $d+\alpha$ system and the breakup channel is represented by the continuum states of this system, as it was successfully done in previous works \[22\], \[23\]. The calculations were performed using the computer code FRESCO \[24\]. In the cluster model, the projectile-nucleus interaction is written as

$$V(R, r, \xi) = V_{\alpha-T}(R, r, \xi) + V_{d-T}(R, r, \xi),$$ (7)

where $R$ is the vector joining the projectile’s and target’s centers, $r$ is the relative vector between the two clusters ($d$ and $\alpha$), and $\xi$ stands for any other intrinsic coordinate describing the projectile-target system. The continuum states of $^6$Li are discretised as in Refs. \[21\] \[25\] \[26\]. The interaction between the $d$ and the $\alpha$ clusters within $^6$Li is given by a Woods-Saxon potential, with the same parameters as in Refs. \[21\] \[25\] \[26\]. The real parts of the $V_{\alpha-T}(R, r, \xi)$ and $V_{d-T}(R, r, \xi)$ interactions are given by the double-folding S˜ ao Paulo potential \[27\]. We have assumed that the mass densities of the $d$ and $\alpha$ clusters, required for the double-folding calculation, can be approximated by the charge densities multiplied by two, whereas the mass densities of the targets were taken from the systematic study of Ref. \[27\]. The imaginary parts of $V_{\alpha-T}(R, r, \xi)$ and $V_{d-T}(R, r, \xi)$ were chosen as to represent short-range fusion absorption, corresponding to assume ingoing wave boundary conditions. The CDCC calculations include also inelastic channels, corresponding to collective excitations of the targets. The channels selected for the three targets reported in ref. \[8\] were already described in that paper. For the $^{120}$Sn nucleus, the excitation included was the one-phonon quadrupole ($2^+$, $E^* = 1.1714$ MeV) first order vibration. The values of the deformation parameters were obtained from Ref. \[28\] and \[29\] for the quadrupole and octupole deformations, respectively.

Figure 1 shows the nuclear breakup cross sections obtained with our CDCC calculations, as functions of $A_{1/3}^2$. The systems are the ones mentioned above and we consider 5 different energies in panels a), b), c), d) and e). The notation is explained in the figure. The straight lines are linear fits to the points. We see that the results are very well fitted by the lines in all cases, in agreement with the expression of Eq. \[4\]. One observes that although this equation was derived for energies above the barrier, it seems to remain valid even slightly below the barrier.

Now we consider Coulomb breakup. Fig. 2 shows CDCC calculations of $\sigma_{cb}^{\text{nu}}$ for the same systems and collision energies of the previous figure. Now the results are plotted against the atomic number of the target. We observe that the points are very well described by the linear fits represented by solid lines. This scaling for $\sigma_{cb}^{\text{nu}}$ can be understood from the low energy behavior of the Coulomb dissociation cross section \[30\]. The electromagnetic coupling matrix-elements are proportional to $Z_T^2$, which should lead to a $Z_T^2$ dependence in the Coulomb breakup cross section, whereas the cross sections for reaction channels are proportional to a $1/E$ factor \[31\]. Since in each panel the collision energy corresponds to the same $E/V_n$ ratio, and $V_n$ is roughly proportional to $Z_T$, one gets a $1/Z_T$ factor. The combination of these two dependences leads to the linear dependence obtained in Fig. 2.

The above discussion about the scaling of the nuclear
and Coulomb breakup cross sections at near-barrier energies should be useful in the study of low energy fusion of weakly bound nuclei as it provides means to assess the feasibility of performing a given experiment which aims to discern the influence of breakup on fusion.

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