Effect of characteristics of unsaturated soils on the stability of slopes subject to rainfall

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ABSTRACT

This research aims to investigate the influence of characteristics of unsaturated soils on the stability of slopes subject to rainfall by using a coupled hydro-mechanical approach. In practice, distribution of the soil suction and moisture content at the shallow depth of a slope is considerably affected by the near-ground environment and climate change, e.g., vegetation, precipitation, evapotranspiration, and exposure to sun light, etc. and shallow slope failures are mainly governed by the unsaturated soil behavior at the shallow depth. The relationship between the soil suction and soil moisture content is important in characterizing the mechanical behavior of unsaturated soils. Parameters quantifying the soil-water characteristic curve (SWCC) are used to represent the characteristics of unsaturated soils. In addition, the degree of saturation at the residual state in the soil also affects SWCC, and the subsurface flow in a slope is governed by the hydraulic conductivity of the soil. Thus, influence of parameters ($\alpha$, $\gamma$, $n$, and $S_i$) in SWCC, the hydraulic conductivity of the soil ($K_{sat}$) in saturated conditions, and the degree of saturation ($S_i$) at the residual state on the stability of slopes subject to rainfall is investigated. Major findings summarized from this research are: (1) $\alpha$, $n$, and $S_i$ values of the soil considerably affect the variation of the factor of safety (FS) of the slope with time during rainfall, specifically at the later stage of the rainfall activity. (2) Influence of $m$ values on the stability of a slope during rainfall is minor. (3) The hydraulic conductivity at saturated conditions ($K_{sat}$) plays an important role in the stability of a slope during rainfall. Variation of the factor of safety of the slope with time during rainfall is not noticeable if the $K_{sat}$ value of the soil is low.

Keywords: unsaturated soil slopes, soil-water characteristic curve, seepage, factor of safety, soil suction

1. INTRODUCTION

Intensive rainfall is considered the most devastating factor to trigger the failure of a slope, specifically the shallow landslip, in most parts of the world. Infiltration in the slope induced by rainfall results in an increase in the soil moisture content and soil unit weight and a decrease in the soil shear strength, which in turn the stability of the slope drops to some extent. In conventional slope stability analysis, rainfall induced slope failures were analyzed in terms of variation of water level and probable change in the soil shear strength in the slope. The effect of variation of soil suction in the slope subjected to rainfall was not taken into account in the slope stability analysis. Fredlund et al. (1978) proposed the mathematical relationship between unsaturated soil shear strength and the soil suction. The relationship between the unsaturated soil shear strength and soil moisture content was further proposed by Vanapalli et al. (1996) and Vanapalli & Fredlund (2000). In a rainfall activity, once the soil moisture content can be determined in an unsaturated soil slope, the shear strength of the soil in the slope can be estimated. Researches on the coupled hydro-mechanical analysis for the slope subjected to rainfall has been carried out extensively during the past decades (Crosta and Frattini 2003; Lan et al. 2005; Tofani et al., 2006; Tsai, 2011). Seepage in the unsaturated soil slope during rainfall can be taken into account in the stability analysis of the slope, and soil suction and degree of saturation of the soil in the slope can be calculated.

The distribution of soil moisture content and soil suction in the unsaturated soil slope during rainfall plays an important role in the stability of the slope. Soil suction governs the mechanical behavior of unsaturated soils, which in turn affects the stability of a slope. The soil suction has a strong relationship with the soil moisture content, and their relationship is normally characterized using soil-water characteristic curve (SWCC). Brooks & Corey (1964), van Genuchten (1980) and Fredlund & Xing (1994) proposed mathematical equations in association with parameters to model SWCC of the soil. The behavior of the subsurface flow in unsaturated soil slope may be noticeably affected by the unsaturated soil characteristics. Rahimi, et al. (2010) indicated that fitting parameters of soil-water characteristic curve noticeably influence the
stability of soil slopes with poor drainage ($K_{sat} < 0.0001 \text{m/s}$) more than that with good drainage ($K_{sat} > 0.0001 \text{m/s}$). The hydraulic conductivity of the soil at saturated conditions has a unique effect on the stability of both good and poor drainage soil slopes.

The theory of the coupled hydro-mechanical analysis for analyzing the stability of unsaturated slopes has been well developed during the past decade. This framework can model the seepage and variation of soil suction in the unsaturated soil in a slope subjected to rainfall and is considered more convincing compared with the conventional methods, e.g. the conventional limit equilibrium method. The parameters quantifying the SWCC can be used to represent the characteristics of unsaturated soils. The van Genuchten (VG) model is used to describe the mathematical expression of the SWCC in this study. Three parameters ($\alpha$, $m$, and $n$) are used in the VG model, and they are relevant to the soil constituents. In addition, the degree of saturation at residual state in the soil also affects SWCC. The subsurface flow in unsaturated soil slopes is governed by the hydraulic conductivity of the soil at saturated conditions and unsaturated characteristics of the soil. Thus, this paper focuses mainly on studying the influence of parameters ($\alpha$, $m$, and $n$) in SWCC, the hydraulic conductivity ($K_{sat}$) in saturated conditions, and the degree of saturation ($S_r$) at residual state on the variation of the factor of safety of a slope with time subject to rainfall.

2 METHODS AND MATERIALS

2.1 Basic theory

In a rainfall activity, the transient groundwater flow takes place in the unsaturated soil slope. The hydraulic head in the soil changes with respect to time in the slope. The Richard’s equation was used to model the groundwater flow in the saturated and unsaturated soil, as shown in eq. (1).

$$c(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[ K_x(h) \frac{\partial H}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K_y(h) \frac{\partial H}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K_z(h) \frac{\partial H}{\partial z} + 1 \right]$$  

(1)

where $C(h)$ is specific water capacity ($=\partial \theta/\partial h$); $\theta$ is the volumetric water content of the soil; $h$ is the hydraulic head in the soil; $t$ is time; $K_v$, $K_s$ and $K_t$ is the hydraulic conductivity of the soil in $x$-, $y$- and $z$-direction, respectively. The specific water capacity is relevant to the soil water characteristic curve (SWCC). The van Genuchten (1980) model (VG model) is used to describe the mathematical expression of the SWCC in this study, as shown in eq. (2).

$$S(h) = S_e + (S_e - S_r)[1+(\alpha h l)^n]^g$$  

(2)

$$K(S) = K_{sat} K_{rel}(S)$$  

(3)

$$K_{rel}(S) = S_e^g \left[ 1 - \left(1 - S_e^{-1/g}\right)^{-g} \right]$$  

(4)

$$S_e = \frac{(S_{res} - S_{sat})}{(S_{sat} - S_{res})}$$  

(5)

Where $h$ is the hydraulic head; $S$ is the degree of saturation of the soil at a given hydraulic head; $S_{sat}$ is the degree of saturation of the soil at residual state; $S_{res}$ is the degree of saturation of the soil at saturated conditions; $S_e$ is the effective degree of saturation; $\alpha$, $n$, $g$ are empirical parameters, and they are relevant to the soil constituents; $K$ is the hydraulic conductivity of the unsaturated soil; $K_{rel}$ is the relative permeability of the soil; $K_{sat}$ is the hydraulic conductivity of the soil at saturated conditions. The hydraulic conductivity of the soil in $x$- and $y$-direction is presumed to be identical in this study. The soil shear strength at various stages during rainfall can be calculated based on the soil suction and soil moisture content obtained.

2.2 Numerical model

The coupled hydro-mechanical FE scheme was used in calculating the factor of safety of the slope subjected to rainfall by using the computer software PLAXIS 2D. The slope in the study has a uniform gradient of 30°, and the slope length is 50 m. The slope is covered with top soil with a thickness of 6 m. The finite element mesh for the study slope is shown in Fig. 1. The factor of safety of the slope in the FE analysis is calculated based on the strength reduction method. The water table is assumed to be at a deep depth. The soil suction at the soil-rock interface is assumed to be null, and it increases in a hydrostatic distribution from the interface to the top of the slope surface.

2.3 Soil parameters

The soil parameters used in the numerical analyses were obtained in the laboratory through soil samples extracted using soil core samplers at a slope site covered with residual soils. The tri-axial compression tests were conducted to obtain the strength parameters of the soil. The hydraulic conductivity at saturated conditions was also measured in the laboratory, and $K_{sat}$ values in $x$- and $y$-direction are assumed to be identical. Pressure plate extractors were used to obtain the soil-water-characteristic curve (SWCC), and curve fitting of the experimental data was carried out using the mathematical expression of SWCC proposed by van Genuchten (1980) (VG model). The parameters ($\alpha$, $m$, $n$) to describe the SWCC for the soil at the site are listed in Table 1, and the SWCC for the
soil is shown in Figure 2. The strength parameters of the soil at the site are illustrated in Table 2.

2.4 Design rainfall
The precipitation input in the analysis is a 24-hour design precipitation hyetograph with a 25-year return period based on frequency analyses on the data collected from 17 rain gauge stations, where precipitation data has been recorded for more than 20 years, in southern Taiwan and Horner equations, as shown in Fig. 3.

3 RESULTS AND DISCUSSION
The factor of safety of the slope subjected to rainfall was analyzed for various soil parameters, i.e. strength parameters (c and φ), hydraulic conductivity at saturated conditions (Ksat), and unsaturated soil parameters (α, m, n). The sensitivity of the soil parameter on the factor of safety of the unsaturated slope subjected to rainfall is analyzed and discussed.

Fig. 2. SWCC for the soil used in the numerical analysis.

![SWCC for the soil used in the numerical analysis.](image)

3.1 Effect of soil parameter α
Slope stability analyses were carried out for three α values (α=0.5, 2, and 7 1/m) in association with the soil parameters listed in Tables 1 and 2. Variation of the factor of safety (FS) with time for different α values is presented in Fig. 4. The difference in FS values for different α values at the early stage (first 10 hours) of the rainfall is not significant, however, the FS value drops considerably for the scenario of low α value (=0.5 1/m). The FS value at a given time greater than 12 hours (later stage of the rainfall) decreases with decreasing α value. The α parameter is related to the air-entry pressure in SWCC. When the α value is low, the soil particle tends to be small. For the slope with low α values, rainfall induced infiltration in the slope may result in a noticeable decrease in the soil shear strength during rainfall, specifically at a high rainfall intensity (10 to 15 hours in the design rainfall used in this study), which in turn a greater decrease in the FS value is expected compared with that with higher α values. In addition, the FS value rebound to some extent as the rainfall intensity downgrades to a certain amount (e.g. 14-15 hours in the design rainfall).

Table 1 Flow parameters used in the numerical analysis.

| Flow parameters | Sres | Ssat | α (1/m) | n | g | Ksat (m/s) |
|-----------------|------|------|---------|---|---|------------|
| Soil            | 0.3  | 1    | 2       | 1.41 | -0.5| 1×10^-4    |
| Rock            | -    | 1    | -       | -  | -  | 1×10^-6    |

Table 2 Soil parameters used in the mechanical analysis of the numerical model.

| Items                        | Soils         | Rocks         |
|------------------------------|---------------|---------------|
| γsat (kN/m³)                 | 18.1          | 24.5          |
| γsat (kN/m³)                 | 20.6          | 24.5          |
| E (kPa)                      | 15E+3         | 80E+3         |
| v (Poisson’s ratio)          | 0.3           | 0.2           |
| c’ (kPa)                     | 25            | 98            |
| φ’ (°)                       | 25            | 30            |
| Material model               | Mohr-Coulomb  | Mohr-Coulomb  |
| Drainage type                | Drained       | Non-porous    |

3.2 Effect of soil parameter g
Various g values (g=-0.3, -0.5, and -0.9) in association with the soil parameters listed in Tables 1 and 2 were used in the slope stability analyses. Variation of the factor of safety (FS) with time for different g values is shown in Fig. 5. The results indicate that g values have minor influence on the FS value of the slope subjected to rainfall, while at the later stage of the rainfall, the FS value of the slope increases slightly with g values. The g parameter controls mainly the slope of the soil water characteristic curve, and
small g values result in a steeper slope gradient at high suction. The higher the g values are, the greater the soil particle tends to be.

3.3 Effect of soil parameter n

Three n values (n=1.1, 1.41, and 2) in association with the soil parameters listed in Tables 1 and 2 were used in the slope stability analyses. The n parameter governs mainly the curvature near the air-entry value, and large n values produce a sharp corner near the air-entry value (Fredlund and Xing, 1994). Variation of the factor of safety (FS) with time for different n values is shown in Fig. 6. The soil parameter n has a noticeable influence on the FS value of the unsaturated soil slope during rainfall. At the early stage of the rainfall, the FS value decreases with increasing n values. Nevertheless, the decrease in FS value with time during rainfall for large n values illustrates a slower rate compared that with low n values. The initial soil moisture content in the slope for soils with large n values is lower than that with low n values prior to the rainfall activity, which in turn the hydraulic conductivity of the soil is low for the soil with low n values.

3.4 Effect of coefficient of permeability (K_{sat}) of the soil

Three K_{sat} values (K_{sat}=0.001, 0.0001, and 0.000001 m/s) in association with the soil parameters listed in Tables 1 and 2 were used in the slope stability analyses. The K_{sat} parameter affects the variation of the hydraulic conductivity of the unsaturated soil with time during rainfall. The K_{sat} value has a minor influence on the FS value at the early stage of the rainfall, as shown in Fig. 7. For the soil with low K_{sat} values, the infiltration induced by rainfall is low, and the variation of the FS values with time during rainfall is low. For the soil with higher K_{sat} values, rainfall induced infiltration in the slope is high compared with that with low K_{sat} values. The soil moisture content in the slope with high K_{sat} values increases greater than that with low K_{sat}, and the FS value for the slope with high K_{sat} values decreases greater than that with low K_{sat} values. Nevertheless, the FS value for the slope with high K_{sat} values slightly increases as the rainfall intensity starts to lower down after 13 hours since the onset of the rainfall.

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**Fig. 4.** Effect of α values on the FS value of the slope.

**Fig. 5.** Effect of g values on the FS value of the slope.

**Fig. 6.** Effect of n values on the FS value of the slope.

**Fig. 7.** Effect of K_{sat} values on the FS value of the slope.
3.5 Effect of degree of saturation at residual state ($S_r$)

Soil constituents affect the degree of saturation ($S_r$) at residual state during drying. Soils with higher $S_r$ value indicate that the soil has more fine materials. Figure 8 presents the influence of $S_r$ values ($S_r=1\%$, $10\%$, and $20\%$) on the FS values of the unsaturated soil slope during rainfall. The degree of saturation of the soil in the slope prior to the rainfall is high as the $S_r$ value is high. Rainfall induced infiltration in the slope with higher $S_r$ values is greater than that with lower $S_r$ values, and deformations in the slope is greater for the soil with higher $S_r$ values. The FS values for three $S_r$ values used in this study are close at the early stage of the rainfall. As the $S_r$ value of the soil is low, decrease in the FS value of the slope during rainfall tends to be low compared with that with higher $S_r$ values. The FS value for the $S_r$ value of $1\%$ is $8\%$ higher than that for the $S_r$ value of $10\%$ following the rainfall, and the FS value for the $S_r$ value of $20\%$ is $3\%$ lower than that for the $S_r$ value of $10\%$ following the rainfall.

Fig. 8. Effect of $S_r$ values on the FS value of the slope.

4 CONCLUSIONS

This paper investigates the influence of characteristics of unsaturated soils on the stability of unsaturated soil slopes subject to rainfall by using a coupled hydro-mechanical approach. The characteristics of unsaturated soils is illustrated through the fitting parameters in the soil-water characteristic curve proposed by van Genuchten (1980). In addition, the hydraulic conductivity in saturated conditions and the degree of saturation of the soil at residual state are also parameters influencing the unsaturated soil behavior during rainfall. Some of the findings concluded from this research are: (1) $\alpha$ values slightly affect the FS values at the early stage of the rainfall, whereas FS values for soils with lower $\alpha$ values drop more than that with higher $\alpha$ values, (2) $g$ values have minor influence on the FS value of the unsaturated soil slope subjected to rainfall, (3) FS values decrease with increasing $n$ values at the early stage of the rainfall, while rate of the decrease in FS value with time during rainfall for larger $n$ values is lower than that with low $n$ values, (4) the variation of the FS values with time during rainfall is low for soils with low $K_{sat}$, and FS values for soils with higher $K_{sat}$ values decrease greater than that with lower $K_{sat}$ values during rainfall, (5) as the $S_r$ value of the soil is low, decrease in the FS value of the slope during rainfall tends to be low compared with that with higher $S_r$ values at the later stage of the rainfall, while the difference in the FS value for different $S_r$ values at the early stage of the rainfall is low.

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