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A Local Evaluation of Global Issues in SUSY breaking

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Abstract

It is well known that there are different global (i.e. $M_P \to \infty$) limits of $N = 1$ supergravity. We distinguish between these limits and their relevance to low energy phenomenology. We discuss a) fermion mass matrices and recently proved theorems in global SUSY b) stability issues and SUSY breaking d) R-symmetry and a recently derived bound on the superpotential and e) FI terms in global and local SUSY.

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1 Introduction

There are two categories of theories which attempt to describe supersymmetry breaking phenomena at the TeV scale\(^1\). One is the class of GMSB theories which typically have a relatively low scale SUSY breaking, with the effective F-term of the SUSY breaking field(s) at scales around \(10^5\) GeV. The other consists of various gravity mediated theories in which the corresponding scale lies in a range \(10^{10}\) GeV − \(10^{12}\) GeV. These correspond to a very low mass (\(< 1\) eV) gravitino in the first class and to very heavy > 1 TeV ones in the latter case.

The literature on SUSY breaking seems to have split accordingly into two cultures. The first, the GMSB and dynamical SUSY breaking literature, is almost exclusively confined to global SUSY. While the necessity of canceling the cosmological constant (CC) is of course recognized - it is generally viewed as something to done at the end of the day by adding a constant to the superpotential. This of course is meaningful only in the context of supergravity since in global SUSY a constant in the superpotential is not of any physical significance as it disappears from the action. On the other hand the literature on the second - gravity mediation - class of models naturally has to be concerned with the full supergravity action from the beginning.

The point of this note is to evaluate various arguments made in the global SUSY breaking literature (for a recent review see for example [3]) from a local SUSY point of view. In the next section we define various relevant gravity decoupling limits and discuss some general issues concerning arguments derived in the context of global SUSY. In section 3 we argue that the scalar partner of the Goldstino - which is effectively the lightest neutral scalar in the theory - cannot be raised arbitrarily far above the scale of the gravitino. In section 4 we discuss some issues related to R-symmetry and its breaking in global and local SUSY. The final topic is concerned with the relation between Fayet-Illiopoulos terms in global and local supersymmetry, and may serve to clarify some aspects of the discussion of these issues initiated in [4, 5] (see also [6]).

2 Local to global

The action for supergravity at the two derivative level\(^2\) is uniquely determined in terms of the Kaehler potential \(K = K(\varphi, \bar{\varphi})\) the (holomorphic) superpotential \(W = W(\varphi)\), and gauge coupling functions \(f_a(\varphi)\) for each simple (or U(1)) factor group, where \(\varphi \equiv \{\phi^i\}, \phi^i = (\phi^i, \chi^i, F^i)\) is the set of chiral scalar fields in the theory. It is often convenient to define also the combination \(G = K + \ln|W|^2\) in which case at a generic point in field space (i.e. away from \(W = 0\)) the action can be written in terms of \(G\) and \(f\). The scalar potential is then given by the formula

\[
V = e^G(G_iG^i - 3) + V_D
\]

\[
= e^{K/M_P^2}(D_iWK_{\bar{ij}}D_j\bar{W} - 3|W|^2/M_P^2) + V_D
\]

\(G_i = \partial_iG, G_{\bar{i}j} = \partial_{\bar{i}}\partial_jG = K_{\bar{i}j}, G^{i\bar{j}} = G^{\bar{j}}_iG_j,\) and \(V_D\) is the D-term potential. In the second line we have for future reference explicitly indicated the scaling with respect to the Planck mass. The

\(^1\)For reviews see for example [1, 2].

\(^2\)We choose \(M_P \equiv (8\pi G_{\text{Newton}})^{-1/2} = 1\) except where for clarity it is explicitly written out. Note the dimensions (mass) \(|\varphi| = 1, |K| = 2, |W| = 3\).
Kaehler covariant derivative (on scalars) is defined as \( D_i W = \partial_i W + \frac{K_i}{M_P^2} \frac{W}{M_P} \). The squared gravitino mass is given by \( m^2_{3/2} = e^G = e^{K/M_P^2} |W|^2/M_P^4 \) when evaluated at a minimum of the potential.

It should be emphasized that these formulae depend only on the existence of a two derivative supersymmetric action. It is also important to stress that quantum effects are not expected to violate SUSY and will only change the functional form of these functions. Also according to the SUSY non-renormalization theorems \( W \) will not get perturbative corrections while \( f \) will only get such corrections at one loop though both can have non-perturbative corrections.

There are several important gravity decoupling limits:

- **a)** \( M_P \to \infty \), \( \frac{\phi}{M_P} \to 0 \), \( \frac{K}{M_P}, \frac{W}{M_P} \to 0 \) \( (3) \)
- **a’)** \( M_P \to \infty \), \( \frac{\phi}{M_P} \to 0 \), \( \frac{K}{M_P} \to 0 \), \( \frac{W}{M_P} = m_{3/2}M_P \neq 0 \) and \( < \infty \) \( (4) \)
- **b)** \( M_P \to \infty \), \( \frac{\phi}{M_P} \neq 0 \), \( \frac{Q}{M_P} \to 0 \), \( \frac{W}{M_P} \neq 0 \) and \( < \infty \) \( (5) \)

The first limit a) is the naive global limit. In this limit we recover the global SUSY expressions \( D_i W = \partial_i W \) and \( V = \partial_i W K^{ij} \partial_j \bar{W} \). However in the presence of SUSY breaking there would be a cosmological constant at the same scale as that of supersymmetry breaking. The second and third however are the phenomenologically relevant limits if we wish to incorporate the effects of SUSY breaking (i.e. generate soft SUSY breaking terms in the low energy effective theory) and cancel the cosmological constant that is generated when SUSY is broken. The second is the limit that should be taken if one wishes to have a phenomenologically relevant scenario like GMSB. However the gravitino mass goes to zero in this limit. The third is relevant to all gravity/moduli mediated scenarios and the gravitino mass is non-zero in the limit. Note that in b) we’ve separated the chiral fields \( \phi \) into “moduli” \( \Phi \) which typically take Planck scale vev’s and matter fields \( Q \) which have either zero or small vev’s (relative to the Planck scale).

The first and second derivatives of the potential are given by the following expressions (see for example [7]):

\[
\partial_i V = e^G(G_i + G^j \nabla_i G_j) + G_i V \tag{6}
\]

\[
V_{ij} = e^G(G_{ij} + \nabla_i G_k \nabla_j G^k - R_{ijkl} G^l G^k) + (G_{ij} - G_i G_j)V \tag{7}
\]

\[
V_{ij} = e^G(2\nabla_i G_j + G_k \nabla_i \nabla_j G_k) + (\nabla_i G_j - G_i G_j)V \tag{8}
\]

\( \nabla_i \) is the covariant derivative \( \nabla_i X_j = \partial_i X_j - \Gamma^k_{ij} X_k \), etc. The fermion mass matrix (ignoring mixing with the gravitino) is

\[
m_{ij} = e^{G/2}(G_{ij} + G_i G_j) = e^{K/2M_P^2} D_i D_j W, \tag{9}
\]

where \( D_i X_j = \nabla_i X_j + K_{ij} X_j/M_P^2 \).

In the limits a) and a’) above the fermion mass matrix goes to \( m_{ij} \to \partial_i \partial_j W \) which is the usual expression in global SUSY. However as is well known the fermions in a supergravity theory mix
with the gravitino and after unmixing the spin half mass matrix is actually \( \tilde{m}_{ij} \)

\[ \tilde{m}_{ij} = e^{G/2} \left( \nabla_i G_j + \frac{1}{3} G_i G_j \right) \]

\[ = e^{K/2} e^{-i\phi W} \left( D_i D_j W - \frac{2}{3} D_i W D_j W \right) \]

\[ \rightarrow \left( W_{ij} - \frac{2}{3} \partial_i W \partial_j W \right). \]

The last line is valid for the global limits \( (3)(4) \). Obviously if we had ignored gravity from the beginning this term would not have been there - but what the above illustrates is that these two theories i.e. the global theory and the limit of a local theory (in the presence of SUSY breaking - are not necessarily the same. Note that the second term goes to zero in the limit \( W \rightarrow \infty \) (keeping \( \partial W \) fixed) as it should. However in a limit where the CC is zero, even if there are no large (i.e. Planck scale) moduli fields, the second term is certainly not zero even in the naive decoupling limit. For example if \( W = c + \sigma_i \phi^i + \mu_{ij} \phi^i \phi^j + y_{ijk} \phi^i \phi^j \phi^k, K = \sum_i |\phi|^2 \), the global theory would give a fermionic mass matrix \( m_{ij} = \mu_{ij} + O(\phi_0) \) whereas even the naive limit \( (3) \) gives \( \tilde{m}_{ij} = \mu_{ij} - 2\sigma_i \sigma_j / 3c + O(\phi_0) \). Note that if we wish also to tune the CC to zero we would need to put \( \sigma_i = 3|\phi|^2 / M_P^2 \) which can only be satisfied in the decoupling limit if we take the limits \( (4)(5) \). In this case we would have the second term in the fermion mass matrix vanishing in the limit if we hold \( \sigma \) fixed. On the other hand if \( \sigma = O(M_P) \) then again we would pick up a non-zero contribution from the mixing with the gravitino. Of course the former limit corresponds to \( (4) \) where the gravitino mass is zero, while in the latter it is finite and non-zero and is in fact the limit \( (5) \).

These issues are of relevance to arguments where the discussion of supersymmetry breaking is primarily done within the global SUSY context. For instance from the point of view of SUGRA, the arguments made in \( [8] \) are actually dependent on the global limit \( (4) \) and also on having a flat Kaehler metric. To understand the issues involved from a local perspective, let us consider the full mass matrix

\[ M = \begin{pmatrix} M^2_{kl} & M^2_{kn} \\ M^2_{ln} & M^2_{mn} \end{pmatrix} \]

The sub-matrices are given by (evaluating \( (1)(8) \) at an extremum \( dV = 0 \))

\[ M^2_{lk} = m_l^j m_{jk} + \frac{1}{M_P^2} (K_{lk} |F|^2 - F_l F_k) - 2|m_{3/2}|^2 - R_{ikln} F^m F^n, \]

\[ M^2_{kn} = e^{K/2M_P^2} D_n D_k W F_i - D_k D_l W \tilde{m}_{3/2}, \]

where we’ve defined the gravitino mass function \( m_{3/2} \equiv e^{K/2M_P^2} W / M_P^2 \). Now even in the global limit \( (4) \)

\[ M^2_{lk} = m_l^j m_{jk} - R_{ikln} F^m F^n, \]

\[ M^2_{kn} = \nabla_n \nabla_k \nabla_i W F^i. \]

Thus the statement that at tree level a zero eigenvector \( v \) of the mass matrix \( m = [m_{ij}] \) becomes a zero eigenvector of the bosonic mass matrix \( M \) is not true in general even in the limits \( (3)(4) \). It
should be stressed that non-trivial terms in the Kaehler potential (and hence non-zero curvature) can be present even at tree level - for instance from classically integrating out heavy states that couple to light states. Thus the theorems proved in [8] are strictly valid only in the global limit (4) with a flat Kaehler metric. For instance the existence of a flat direction in O’R models (a so-called pseudo moduli space) depends in an essential way on the assumption of no higher dimension operators (i.e. those which are scaled by some mass which does not go to infinity with $M_P$) in $W$ or $K$.

In the limit (5) one needs to retain the second and third terms of (13) as well as the last term of (14). Needless to say the relationship between the fermionic zero mode and the bosonic one is completely lost - even for a flat Kaehler metric except in the limit (4) - since there are terms proportional to the gravitino mass which are absent in the fermion mass matrix. In fact the unit vector $v = F_i/|F|$ which defines the Goldstino direction is not a zero mode of $[m_{ij}]$: $v^i m_{ij} = 2\tilde{m}_3/2v_j - \text{after using } \partial_i V = 0$. On the other hand $F_i\tilde{m}_{ij} = -2D_iW/3V_0$. So the correct fermion mass matrix ($\tilde{m}$ the one which is relevant in the presence of a gravitino (13)) has a zero mode corresponding to $F_i/|F|^2$ provided that the CC is tuned to zero. Again this has no simple relation to the bosonic mass matrix.

The results proved in [8] have important consequence in dynamical SUSY breaking in the context of GMSB theories. These results are however valid only in the limit (4) and that too only provided that the Kaehler metric is flat.

## 3 sGoldstino mass

From (6) we see that at a extremum $\partial_i V = 0$ with zero CC i.e. $V_0 = 0$, we have $m_{ij}G^i = 0$ so that the Goldstino is the spinor $\chi_0^i = G^i\eta/3$, $\eta = G_i\chi_0^i$. The sGoldstino is its complex scalar partner which therefore has averaged squared mass (half the trace of the squared mass matrix, [9]) (when $V_0 = 0$)

$$M_{sg}^2 = \frac{1}{3}V_{ij}G^iG^j = \frac{1}{3}e^G(2G_{ij}G^iG^j - R_{ijkl}G^iG^jG^kG^l) = \frac{2}{3}K_{ij}F_iF_j - \frac{1}{|F|^2}R_{ijkl}F^iF^jF^kF^l. \quad (15)$$

$$= 2m_3^2 - \frac{1}{3M_P^2m_3^2}R_{ijkl}F^iF^jF^kF^l \quad (16)$$

In the last equality above we’ve imposed $V_0 = 0$. This formula immediately tells us that (since $F \sim M_P m_3/2$ when the cosmological constant is cancelled), unless the curvature on moduli space is enhanced way above the Planck scale or there is cancellation between the two terms, the sGoldstino mass is of order of the gravitino mass.

Several special cases are of some interest:
1. In any model in which the metric on moduli space is flat (as is the case with most dynamical SUSY breaking models) the (mean) sGoldstino mass is \( \sqrt{2} m^{3/2} \).

2. Einstein Kaehler space \( R_{ijkl} = \sigma K_{ij} K_{kl} \). In this case \( m_{sg}^2 = (2 - 3\sigma) m^{3/2} \). The no-scale model for instance corresponds to \( \sigma = 2/3 \).

Notice that in the global limit (1) the mean sGoldstino mass is actually zero in the flat Kaehler metric case. Writing \( m^{3/2} m_P = \mu^2 < \infty \), as \( M_P \to \infty \), we have (16)

\[
M_{sg}^2 = -\frac{1}{\mu^4} R_{ijkl} F^i F^j F^k F^l.
\]

This means that unless the RHS is positive and non-zero one or other of the sGoldstinos is tachyonic. In the case that the metric is flat, if we also impose the condition of no tachyons then clearly there is locally a (complex) flat direction and both sGoldstinos have zero mass. Thus any sensible theory in the limit (1) must necessarily have a non-zero curvature in field space.

An example of a case where the sGoldstino mass is enhanced above the gravitino mass is the Kitano model \([10]\) in which

\[
K = S\bar{S} - \frac{(S\bar{S})^2}{\Lambda^2} + q\bar{q} + \bar{q}\bar{q} + \frac{\lambda^2}{(4\pi)^2} S\bar{S} \ln \left( \frac{(S\bar{S})^2}{\Lambda^2} \right), W = c + \mu^2 S + \lambda S q\bar{q}.
\]

Here \((q, \bar{q})\) are messengers, \( S \) is some gauge neutral state (for example a modulus of string theory and \( \Lambda \) is the scale at which for instance Kaluza-Klein states or string states have been integrated out. The last term in the Kaehler potential is a messenger one loop effect. This gives \( R_{SSS\bar{S}} = -4/\Lambda^2 \). \( S \) defines the direction of SUSY breaking . In this case

\[
M_{sg}^2 = \langle M_S^2 \rangle = 2m_{3/2}^2 (1 + 6\frac{M_P^2}{\Lambda^2} (1 + 6\frac{\lambda^2}{(4\pi)^4} \frac{M_P^2}{\Lambda^2})) \approx 12 \left( \frac{M_P m_{3/2}}{\Lambda} \right)^2.
\]

The last approximation follows from \( \Lambda < M_P \) and the stability condition for the \( S \) mass squared matrix which implies that \( \lambda/4\pi < \Lambda/M_P \), (see [10]) which is also the condition that the expansion in messenger loops makes sense. It would appear that by choosing the cutoff sufficiently low one can get a sGoldstino mass well above the gravitino mass scale. However this not correct. There are two stability conditions to satisfy at the minimum \( (S = \Lambda^2/\sqrt{12} M_P, q = \bar{q} = 0) \) - one coming from the mass matrix for \( S \) (which should include the contribution coming from messenger loops), and the other from that for the messengers. After tuning the CC to zero (giving \( \mu^2 \approx \sqrt{3} m_{3/2} M_P \)) one gets the following window for the messenger coupling:

\[
\frac{12\sqrt{3} m_{3/2} M_P^3}{(4\pi) \Lambda^4} < \frac{\lambda}{4\pi} < \frac{\Lambda}{M_P}.
\]

Furthermore the gaugino mass is given by [11]

\[
M \approx \frac{\alpha}{4\pi} \frac{F_S}{S} \approx \frac{\alpha}{4\pi} \frac{\sqrt{3} m_{3/2} M_P}{\lambda \Lambda^2/(2\sqrt{3} M_P)}
\]
In order to have an open window (20) the gravitino mass is bounded below and in order to have GMSB dominance it needs to be much smaller than the gaugino mass. For gaugino masses (and other soft parameters) at the weak scale (i.e. \( M \sim 100 GeV \)) this implies,
\[
10^{-5} GeV \leq m_{3/2} \ll 100 GeV.
\]
(22)

From (19) and (21) we also have
\[
M_S = 2 \sqrt{3} m_{3/2} \left( \frac{4 \pi \lambda M}{6 \alpha m_{3/2}} \right)^{1/2} \sim 10 \sqrt{\lambda \sqrt{m_{3/2} M}},
\]
(23)

where in the last relation we have used \( \alpha/4 \pi \sim 10^{-2} \) for the gauge coupling. Thus the modulus mass is constrained by the scale of soft masses - it is in fact well below that scale. Note that the above constraints (22) for the gravitino mass as well as the corresponding value for the sGoldstino mass, are incompatible with standard cosmology (for a recent discussion see [12]).

Let us see what happens in general O’Raifeartaigh (O’R) type models for SUSY breaking embedded in SUGRA. We take the following potentials :
\[
K = SS - \frac{(S\bar{S})^2}{\Lambda^2} + \sum_i q_i \bar{q}_i + \frac{1}{(4\pi)^2} tr|M|^2 \ln \frac{|M|^2}{\Lambda^2},
\]
(24)
\[
W = c + \mu^2 S + \mathcal{M}_{ij}(\bar{S})q_i \bar{q}_j
\]
(25)
where
\[
\mathcal{M}_{ij} = \lambda_{ij} S + m_{ij}.
\]
(26)
The superpotential here is written in the so-called canonical form - every O’R model can be rewritten in this form (see for example [8]). Then (21) will be replaced by
\[
M \approx \frac{\alpha}{4 \pi} F^S \partial_S \ln \det \mathcal{M}.
\]
(27)

In this case the cutoff \( \Lambda \) can in principle be decoupled from the soft mass scale since one could take (roughly speaking) \( \lambda^{-1} m \gg S_0 \sim \Lambda^2/M_P \). In this case one can in fact have a standard cosmological scenario. Nevertheless the stability constraints impose restrictions which are rather unnatural. Schematically these constraints now take the form,
\[
\frac{m_{3/2} M_P}{|\lambda^{-1} \mathcal{M}|} < \lambda < 4 \pi \frac{|\lambda^{-1} \mathcal{M}|}{\Lambda}
\]
(28)

where \( |\mathcal{M}| \) stands for the scale of the messenger mass matrix. This (together with (27) and \( M \sim 100 GeV \)) gives only the lower bound \( m_{3/2} > 0.1 eV \) and so is compatible with the standard cosmological scenario. However in this case we also have the bounds
\[
\Lambda < 10^9 GeV, |m| < 10^4 GeV
\]
(29)
The bound on \( m \) is of course quite unnatural for a SUSY preserving term. Secondly the cutoff \( \Lambda \) is expected to be some physical scale such as the Kaluza-Klein scale of string theory. Thus this bound on \( \Lambda \) would imply a KK scale which is unnaturally low. In fact it is often the case in the GMSB literature, that the cut off is assumed to be at the GUT scale! Thus it appears that the only way we can get a low enough gravitino mass (as well as a high sGoldstino mass) in these theories is by making two unnatural choices of mass parameters.
4 R-axion and SUSY breaking

4.1 Global SUSY bound

Let us first recapitulate the argument of [13] within the global supersymmetric context. Let the theory have a R-symmetry generated by the Killing vector \( k^i(\Phi) \). Here \( i \) labels the chiral superfields of the theory. For a linearly realized R-symmetry \( k^i = iq^i\Phi^i \) (no sum) where \( q^i \) is the R-charge of \( \Phi^i \). Since under an R-symmetry the superpotential transforms, we have the Killing equation (using the notation \( \partial_i \equiv \partial/\partial \Phi^i \))

\[
k^i W_i = i2W,
\]

where as usual we’ve taken the charge of \( W \) to be two. Let the Kaehler potential of the theory be \( K(\Phi, \bar{\Phi}) \) so that the Kaehler metric is \( K_{ij} \). For any pair of vectors \( U = \{U^i\}, V = \{V^i\} \), we define the inner product \( < U, V >= \underbrace{V, U}_\equiv K_{ij}V^i\bar{U}^j \). Then putting \( \bar{U}^j = K^{ji}\partial_j W \) and \( V^i = k^i \) the Killing equation (30) becomes

\[
< U, V >= i2W \quad (31)
\]

Let the scalar component of \( \Phi^i = \phi^i \). If one or more of the charged fields acquire a non-zero vacuum expectation value \( \phi^i_0 \), the R-symmetry is spontaneously broken and we may isolate the axion field by writing \( \phi^i = \phi^i_0 e^{iq^i a(x)} \). The kinetic term for the R-axion then becomes

\[
K_{ij}\partial_i\Phi^j\partial_j\bar{\Phi} \rightarrow K_{ij}k^i_0k^j_0(\partial a)^2
\]

so that the axion decay constant \( f_a \) is given by (here and in what follows the subscript 0 indicates evaluation at the minimum of the potential),

\[
f_a^2 = K_{ij}k^i_0k^j_0 = < V_0, V_0 > .
\]

Then using the Cauchy-Schwarz inequality and equation (31) we have,

\[
4|W_0|^2 = | < U_0, V_0 > |^2 \leq < U_0, U_0 > < V_0, V_0 > = |F|^2 f_a^2, \quad (32)
\]

where \( |F|^2 = K^{ij}\partial_i W \partial_j \bar{W} \).

4.2 SUGRA bound

The above inequality is valid only in global SUSY. As we pointed out earlier the superpotential has no meaning in and of itself in global supersymmetry. Only the derivatives of the superpotential have physical significance. Thus the formula (32) makes sense only in the context of supergravity. However supergravity is invariant under the Kaehler transformations \( K \rightarrow K+\Lambda+\bar{\Lambda}, W \rightarrow e^{-\Lambda}W \). The above formula is not invariant under these transformations and hence is not valid as it stands in supergravity. Let us therefore work out the Kaehler invariant form of the above discussion.

Since the Kaehler potential is invariant under the R-symmetry we have

\[
k^i \partial_i K + \bar{k}^j \partial_j \bar{K} = iq^i \phi^i \partial_i K - iq^i \bar{\phi}^i \partial_i \bar{K} = 0 \Rightarrow q^i \phi^i \partial_i K = q^i \bar{\phi}^i \partial_i \bar{K}. \quad (33)
\]
In supergravity the order parameter for SUSY breaking, i.e. the F-term (for the moment we ignore gauge interactions) is given by the expression

$$F_i = e^{K/2} D_i W \rightarrow e^{(\tilde{A}-\Lambda)/2} F_i \Rightarrow |F_i| \rightarrow |F_i|$$ (34)

where the second pair of relation indicate the Kaehler transformation properties and we’ve set $M_P = 1$.

If the theory has an R-symmetry then the superpotential satisfies equation (30). So we get

$$k^i F_i = (2i + k^i K_i) e^{K/2} W = i(2 + \sum_i q^i \phi^i K_i) e^{K/2} W$$

Evaluating this at the minimum of the potential and identifying the gravitino mass as $m_{3/2} = e^{K/2} |W|_0$, we have

$$|<k, F>_0| = 2 + \sum_i q^i \phi^i \partial_i |K_0| m_{3/2}.$$ 

Using the Cauchy-Schwarz inequality we get (again putting $f_a^2 = <k_0, k_0>$) instead of (32), the following;

$$f_a^2 < F, F >|_0 \geq |2 + \sum_i q^i \phi^i \partial_i |K_0| m_{3/2}.$$ (35)

Note however that this is still not Kaehler invariant! The reason is that the R-symmetry equation is not Kaehler covariant. However if we assume that the theory is such that the sum on the right hand side is positive then we can write, (restoring $M_P$)

$$f_a^2 < F, F >|_0 \geq 4 m_{3/2}^2 M_P^2.$$ (36)

However as we’ve observed this is model dependent. Nevertheless it is the obvious Kaehler invariant generalization of (32). It should be observed however that the requirement of setting the cosmological constant (CC) to zero implies that this is actually an inequality for the axion decay constant. Since we must fine tune the parameters of the theory such that

$$<F, F>_0 \equiv F_i K^{ij} \bar{F}_j|_0 = 3 m_{3/2}^2 M_P^2,$$ (37)

we get from (36),

$$f_a^2 \geq \frac{4}{3} M_P^2.$$ (38)

Actually it is clear from (35) and (37) that, since for a generic theory the first factor on the RHS of that inequality i.e. the expression $4M_P^2 |1 + \frac{1}{2M_P^2} \sum_i q^i \phi^i \partial_i |K_0|^2$ is Planck scale, we will have a Planck scale axion decay constant,

$$f_a^2 \gtrsim M_P^2,$$

and this is of course both Kaehler invariant and valid for generic theories with a spontaneously broken R-symmetry.
5 Global limits of a local SUSY model with FI terms

5.1 Classical issues

Let us now discuss the issue of Fayet-Illiopoulos terms from the point of view of the different global limits. In [4] certain problems with extending global theories with FI terms to SUGRA were discussed (see also [5][6]). In [14] a different approach is taken. Here we discuss some aspects of [14] that are relevant to the issues that we've addressed in this note. There is of course no conflict between the statements made there (or in this note) and the arguments of [4] since the class of SUGRA theories that we consider do not have a well defined global limit.

We begin by introducing a SUGRA theory with an FI term that is manifestly consistent, at least at the classical level. The model is given by

\[ G \equiv K + \hat{\xi} V + M_P^2 \ln \frac{|W|^2}{M_P^8} \]

with

\[ K = \bar{S} e^V S + \sum \bar{\Phi} \Phi, \quad W = \left( \frac{S}{M_P} \right) \frac{\hat{\xi}/M_P^2}{W_I(\Phi)} \]

In the simplest version discussed \( W_I \) is taken to be a constant. However in general we can take it to be a gauge invariant holomorphic function involving the other chiral fields \( \Phi \) as well as possibly \( S \) itself. The superfield \( G \) is then invariant under the gauge transformations

\[ V \rightarrow V + i(\Lambda - \bar{\Lambda}) \]
\[ S \rightarrow e^{-i\Lambda} S \]

with the other fields transforming appropriately. Since by well-known arguments off-shell supergravity can be expressed entirely in terms of the function \( G \) (and the holomorphic gauge coupling function \( f(\Phi) \)) this model gives a gauge invariant supergravity with an FI term.

Now the global theory is well defined even with the addition of an FI term \( \xi V \) to the invariant Kaehler potential \( K \); under the gauge transformation \( V \rightarrow V + i\Lambda - i\bar{\Lambda} \), this term is invariant because of the \( \int d^4\theta \) integral. In SUGRA however the Kaehler potential becomes an exponential and will not give an invariant theory unless one adds a harmonic (i.e. chiral plus anti-chiral) piece that transforms in such a fashion as to cancel the gauge variance of the \( \xi V \) term. So after a Kähler transformation the full superspace integral is taken to be (with \( E \) being the full superspace supervielbein determinant)

\[-3M_P^2 \int d^4\theta E e^{-[K + \hat{\xi} V + \hat{\xi}(\ln(S/M_P) + h.c.)]/3M_P^2} .\]

Now in this expression, let us take the global limit

\[ M_P^2 \rightarrow \infty, \quad E \rightarrow 1, \]
\[ \xi = \frac{\hat{\xi}}{M_P^2} \rightarrow O(\frac{1}{M_P^2}) .\]

\[ ^3\text{This model along with a discussion of its global limits was circulated amongst a few workers in the field in October 2010. Since then a similar model has been published by other authors [15].} \]

\[ ^4\text{Quantum anomalies can be cancelled by adding additional fields and or Green-Schwarz terms as we shall see.} \]
The last relation implies keeping \( \hat{\xi} \) fixed as we take the limit. Then we get

\[
\int d^4 \theta [-3M_P^2 + (K + \hat{\xi}V + \hat{\xi}(\ln S/M_P + \text{h.c.}) + O(1/M_P^2))] \to \int d^4 \theta (K + \hat{\xi}V)
\]  

(45)

In the last expression we have used the chirality of \( \ln S \) (i.e. \( D_\alpha \ln \bar{S} = D_\alpha \bar{S}/\bar{S} = 0 \)). Note that the apparent singularity of the formalism is irrelevant since all that means is (see for example the discussion of the potential in the last section) that for \( \xi < 1 \) there is no gauge invariant \( S = 0 \) minimum of the potential.

On the other hand if \( \xi \neq 0 \) is fixed in the limit \( M_P^2 \to \infty \) (whether or not \( \hat{\xi} \) is quantized in Planck units) we get in the limit (43) instead of (45) the expression,

\[
\int d^4 \theta (K + \xi M_P^2 (V + \ldots))
\]

which means that the limit does not exist!

It is in fact instructive to study the case \( \xi < 1 \) in a little more detail in the simple model where \( W_I \) is independent of \( S \) and is just a function of a neutral field \( \Phi \). Note that in this model \( K = S e^V S + \Phi \Phi \). The potential in the global limit (3) (with \( \hat{\xi} \) fixed) is

\[
V_{\text{global}} = \left| \frac{\partial W_I}{\partial \Phi} \right|^2 + \frac{g^2}{8} (S \bar{S} + \hat{\xi})^2,
\]  

(46)

and there is a gauge invariant ground state \( S = 0 \) regardless of the value of \( \hat{\xi} \) (> 0). Note also that at this minimum supersymmetry is broken and (assuming there is a solution to \( \partial W_I/\partial \Phi = 0 \))

\[
V_{\text{global,0}} = \frac{g^2}{8} \hat{\xi}^2
\]

On the other hand before taking the limit the potential of the theory is

\[
V = e^{(S \bar{S} + \Phi \Phi)/M_P^2} \left( \frac{S \bar{S}}{M_P^2} \right)^{\hat{\xi}/M_P^2} \left[ \frac{|W_I|^2}{S \bar{S}} (\frac{\hat{\xi}}{M_P^2} + \frac{\bar{S} S}{M_P^2})^2 + |\partial_\theta W_I + \frac{\bar{\Phi}}{M_P^2} W_I|^2 - 3 \frac{|W_I|^2}{M_P^2} \right] + \frac{g^2}{8} (S \bar{S} + \hat{\xi})^2
\]  

(47)

Now the global limit in which \( M_P^2 \to \infty \) with \( W_I, \hat{\xi}, S, \Phi \) fixed gives us back (46). On the other hand in the limit a’) (see (11)) (with \( W_I/M_P \equiv \mu^2 \) fixed but without assuming that \( s_0 \to O(1/M_P^2) \)), we have

\[
V \to e^{SS/M_P^2} [\mu^4 S \bar{S}/M_P^2 + |\partial_\theta W_I|^2 - \mu^4] + \frac{g^2}{8} (S \bar{S} + \hat{\xi})^2,
\]  

(48)

which again has a minimum at \( S = 0 \). However as can be seen from (47), for \( \xi = \hat{\xi}/M_P^2 < 1 \), the minimum of the potential (47) is at (for finite \( M_P \))

\[
\frac{S_0 \bar{S}_0}{M_P^2} \neq 0.
\]
Thus in this case the existence of the gauge invariant minimum is an artifact of the decoupling limit.

The model is easily generalized to the case when the superpotential of the global theory \( W_I \) is not independent of the field \( S \). In this case we need at least one other field which is charged (with charge \(-1\)). The invariant superpotential is

\[
W_I = W_I(S, \Phi),
\]

where \( \Phi \) now stands for a set of fields which must have at least one charged field. The invariant Kaehler potential plus FI term is taken as before to be

\[
\hat{K} = K + \hat{\xi} V + \hat{\xi} \ln(S \bar{S}/M^2_P).
\]

Note that there is now an ambiguity in the choice of the last term. For instance if there is only one other charged field in the set \( \Phi \) (say \( \tilde{S} \) with charge \(-1\)) then an equally valid extension to SUGRA of the original global theory would have instead \( \hat{K} = K + \hat{\xi} V - \hat{\xi} \ln(\tilde{S} \bar{\tilde{S}})/M^2_P \). This means that there will be several possible SUGRA extensions of a given global theory. In any case the point is that (taking for comparison with the previous discussion the extension (50), we will essentially have the same potential as (47) with some simple modifications. Namely we have

\[
V = e^{(SS + \sum \Phi \Phi)/M^2_P} \left( \frac{SS}{M^2_P} \right)^{\hat{\xi}/M^2_P} \left[ \frac{|W_I|^2}{SS} | \frac{\hat{\xi}}{M^2_P} + \frac{S \bar{S}}{M^2_P} \frac{\partial \ln W_I}{\partial \ln S} \right]^2 + | \partial_\Phi W_I + \frac{\Phi}{M^2_P} W_I |^2
\]

\[
-3 \frac{|W_I|^2}{M^2_P} + \frac{g^2}{8} (SS + \sum q_\Phi |\Phi|^2 + \hat{\xi})^2.
\]

Finding the minima of this potential is of course much more complicated. However it is clear that as in the simpler case analyzed before, for \( \hat{\xi} < M^2_P \) the potential \( V \rightarrow +\infty \) for both limits \( S \bar{S} \rightarrow 0, +\infty \), and so the minimum will be at \( |S|_0 \neq 0 \). Thus quite generally it is the case that when \( 0 < \xi < 1 \) the minimum of the potential will not be \( U(1) \) symmetric except in the global limit. Generically supersymmetry will also be broken.

Finally we should stress that even if it were the case that the gauge group is compact so that \( \hat{\xi} \) is quantized in Planck units \([17][18]\), the theory can exist as a valid effective field theory if the scale of the superpotential \( W_I \) is chosen to be well below the Planck scale.

### 5.2 Quantum issues

So far the discussion of this model has been entirely classical. In fact at the quantum level there is an anomaly in the model, which can be dealt with either by adding extra fields (as in \([15]\)) or by including a Green-Schwarz anomaly canceling sector.

Let us consider extending the model so that the \( U(1) \) anomaly is cancelled by the Green-Schwarz mechanism. So we take the model of \([10]\) (for simplicity without the \( \Phi \) field) and add a field \( T \) with the non-linear \( U(1) \) transformation rule

\[
T \rightarrow T - iMA,
\]
and an additional (gauge invariant) Kaehler potential\footnote{Note this is to be contrasted with the case of a $\Delta K = (T + \bar{T} + MV)^2$ where by a redefinition $T = T' - \xi/2M$ the FI term can be removed as discussed in \cite{6, 15}.}

$$\Delta K = -3 \ln (T + \bar{T} + MV).$$ \hspace{1cm} (53)

Note that again we’ve set $M_P = 1$. To incorporate anomaly cancellation we now take the gauge coupling function to be

$$f = \frac{1}{g^2} + b_{UU}T,$$ \hspace{1cm} (54)

where $b_{UU}$ is the pure gauge anomaly coefficient. In addition we will need to add a curvature squared term with a coefficient $b_{RR}T$ (where $b_{RR}$ is the mixed gauge gravitational anomaly coefficient) to cancel the mixed anomaly.

The potential is positive definite because of the no-scale form of $\Delta K$;

$$V = V_F + V_D,$$

$$= \frac{|W_I|^2}{(T + \bar{T})^3} e^{S\bar{S}} (S\bar{S})^{\xi-1} (S\bar{S} + \xi)^2$$

$$+ \frac{g^2}{8(1 + \frac{g^2}{2} b_{UU}(T + \bar{T}))} (S\bar{S} + \xi - \frac{3M}{T + \bar{T}})^2.$$

For $\xi > 1$ it has a SUSY minimum at $S\bar{S} = 0$, $\Re T = 3M/2\xi$ with $V_0 = 0$. Note that a non-zero value of $\xi$ is crucial for avoiding runaway behavior in the $\Re T$ direction. On the other hand for $\xi \leq 1$ $\Re T$ runs away to infinity at the global minimum. There is however a local minimum where SUSY and gauge invariance are broken if the condition for a certain quartic in $\Re T$ to have real roots, is satisfied by the coefficients $M, \xi$.

6 Conclusions

In this work we have elaborated on the well known phenomenon that a generic supergravity theory can have different global limits. The usual action of global supersymmetry is simply one such limit. If one starts from that limit to build supergravity only a limited class will emerge. The point is when supersymmetry is broken the relevant limit which would lead to a zero CC (or at least one which is parametrically smaller than the SUSY breaking scale) is not the one which would have been obtained as a global supersymmetric theory where gravity was not taken into account. We’ve pointed out in this paper that issues related to the scalar partner of the goldstino (which has the potential to cause cosmological moduli problems) can be meaningfully addressed only within the SUGRA context. Also we show that a bound on the superpotential which may be derived in the global SUSY context disappears when it is rederived in the context of SUGRA and the cosmological constant is tuned to zero. Finally we addressed issues relating to Fayet-Illiopoulos terms in SUGRA and global SUSY from the perspective of these different limits.
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