Supervised learning algorithms resilient to discriminatory data perturbations

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The actions of individuals can be discriminatory with respect to certain protected attributes, such as race or sex. Recently, discrimination has become a focal concern in supervised learning algorithms augmenting human decision-making. These systems are trained using historical data, which may have been tainted by discrimination, and may learn biases against the protected groups. An important question is how to train models without propagating discrimination. Such discrimination can be either direct, when one or more of protected attributes are used in the decision-making directly, or indirect, when other attributes correlated with the protected attributes are used in an unjustified manner. In this work, we i) model discrimination as a perturbation of data-generating process; ii) introduce a measure of resilience of a supervised learning algorithm to potentially discriminatory data perturbations; and iii) propose a novel supervised learning method that is more resilient to such discriminatory perturbations than state-of-the-art learning algorithms addressing discrimination. The proposed method can be used with general supervised learning algorithms, prevents direct discrimination and avoids inducement of indirect discrimination, while maximizing model accuracy.

Discrimination consists on treating somebody unfavorably because of their membership to a particular group, characterized by a protected attribute, such as race or gender. Freedom from discrimination is outlined as a basic human right by the Universal Declaration of Human Rights. Legal systems often prohibit discrimination in a number of contexts (1–4), for example the Civil Rights Acts of 1866 of the United States outlaw discrimination based on race in employment. Nowadays there is a growing appetite for introducing algorithmic decision-making systems and these systems introduce new concerns regarding discrimination. In principle, algorithmic systems can remove the biases associated with human judgment, increasing accuracy and fairness as well as transparency. In practice, however, there is a concern that these systems can perpetuate existing biases or introduce new ones, in a far from transparent manner (5–7). Given the nature of machine learning methods currently in use, a re-examination and through formalization of discrimination notions is necessary, and a large amount of research on this topic has emerged in computer science (8–22).

In the legal (1, 2) and social science (23–25) contexts, a key consideration serving as the basis for identifying discrimination is whether there is a disparate treatment or unjustified disparate impact on the members of some protected group. To prevent disparate treatment, the law often forbids the use of certain protected attributes, such as race or gender, in the decision-making, e.g., decisions about hiring, y ∈ Y. In statistical language, such hiring decisions shall be based on a set of relevant attributes, x ∈ X, and should not depend on the protected attribute, P(y|x, z₁) = P(y|x, z₂) for any z₁, z₂ ∈ Z, ensuring that there is no disparate treatment.¹ We refer to this kind of discrimination as direct discrimination, because of the direct use of the protected attribute z.

Historically, the prohibition of disparate treatment was circumvented by the use of variables correlated with the protected attribute as proxies. For instance, some banks systematically denied loans and services, intentionally or unintentionally, to certain racial groups based on the areas they lived in (26, 27), what is known as the phenomenon of “redlining”. Indirect discrimination is a particularly acute problem for machine learning data-rich systems, since they often can find surprisingly accurate surrogates for protected attributes when a large enough set of legitimate-looking variables is available, resulting in discrimination via association (28). In order to prevent this indirect discrimination, legal systems sometimes establish that the impact of a decision-making process should be the same across groups differing in protected attributes (24, 25), that is P(y|z₁) = P(y|z₂), unless there is a “justified reason” or “business necessity clause” for this disparate impact (1, 2). If there exists a valid business necessity then disparate impact is deemed legal — this precedence happened in the seminal case of Ricci v. DeStefano (29).

While the equalities standing for parity of treatment and impact differ in the conditioning on the relevant attributes, X, disparate impact can arise due to justified reasons that could be captured with conditioning on these relevant variables. Unfortunately, such relevant attributes might be strongly correlated with the protected attributes and their addition to a decision-making model could result in indirect discrimination via association (28). How to justify their impact in a way that unites the legal and algorithmic perspectives? The main challenge in introducing non-discriminatory learning algorithms lies in defining and constraining the indirect discrimination, while preventing direct discrimination (10).

In this paper, we consider a prevalent scenario of supervised learning, where a model supporting human decisions is trained on available data, i.e., a set of samples D = {xᵢ, zᵢ, yᵢ}. In principle, this model could represent any decision-making process, for instance: i) assigning a credit score for a customer, given her financial record x and her race z, or ii) deciding whether a given individual shall be hired to police, given her skills x and her gender z. The goal of a supervised learning algorithm is to obtain a function ̂y : X → Y that optimizes a given objective, e.g., the empirical risk function, R(Y, ̂Y) = Eᵢ[L(yᵢ, ̂y(xᵢ))], where the expectation is over observed samples and L is a loss function, e.g., quadratic loss, L(u, y) = (u − y)².

¹Throught the manuscript we use a shorthand notation for probability: P(y|x, z) ≡ P(Y = y|X = x, Z = z), where X, Y, Z are random variables and x, y, z are their instances.

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If the dataset \( \tilde{D} \) is not tainted by discrimination, in which case we refer to it as \( D = \{ x_i, z_i, u_i \} \), such that \( u_i \in Y \) and \( (x_i, z_i) \in \tilde{D} \), then standard supervised learning algorithms can be applied to learn a non-discriminatory \( \tilde{y} \). If the dataset is tainted by discrimination, then a data science practitioner may desire, and, in principle, be obliged by law, to apply an algorithm that does not perpetuate this discrimination. This practitioner, however, may have no information whether the training dataset was tainted by discrimination \( (D) \) or not \( (\tilde{D}) \), so supervised algorithms that aim to prevent discrimination operate in a blind setting. A number of such algorithms have been developed by adding a constraint or a regularization to the objective function \((8–10, 12–17)\). Most of these algorithms prevent direct discrimination, but they do not provide a statistical justification of the impact of the relevant attributes \( X \) on the output \( Y \), what is necessary to avoid perpetuating indirect discrimination. For instance, the algorithms that put constraints on the aforementioned disparities in treatment and impact \((8–10)\) induce a discriminatory bias in model parameters if they are provided a non-discriminatory dataset \( D \) for training \((30)\).

Even if the designer knew that the training dataset is discriminatory, e.g., that \( Y \) is causally and involuntarily affected by \( Z \), there still remains the question of how to drop \( Z \) from the model without inducing indirect discrimination, that is without increasing the impact of relevant attributes \( X \) correlated with \( Z \) in an unjustified and discriminatory way.

To address these issues, we model discrimination as a perturbation of data-generating process, also known as a “dataset shift” \((31)\), that transforms \( D \) into \( \tilde{D} \). Then, we propose a measure of resilience to such perturbations, and develop a supervised learning algorithm that is resilient to discrimination.

**Resilience to potentially discriminatory perturbations.** Let us first consider a model of the unperturbed non-discriminatory output variable \( U \), expressed in terms of relevant variables \( X \). Samples of the output variable \( U \) are drawn from a probability distribution, i.e., \( u_i \sim P(u_i|x_i) \), or a corresponding probability density function if the output variables are continuous. In the remainder, we drop the subscript \( i \) for brevity. If this model has a causal interpretation \((32, 33)\), then the decisions \( U \) are not causally influenced by the protected attribute \( Z \), which is why we call \( U \) non-discriminatory. By contrast, we refer to the perturbed decisions, \( \tilde{Y} \), as potentially discriminatory. Perturbations of this kind were proposed before as random swaps of the class labels in a binary classification, i.e., \( y \sim P(y|u) \) \((34)\). However, to capture direct and indirect discrimination, it is important to consider discriminatory perturbations that depend, potentially causally, on the protected attribute \( Z \). We distinguish between direct and indirect discriminatory perturbations:

1. Directly discriminatory perturbations, \( y \sim P(y|x,z) \), resulting in \( E[y|x,z] \neq E[u|x] \).

2. Indirectly discriminatory perturbations, resulting in \( E[y|x] \neq E[u|x] \) and \( E[y|z] \neq E[u|z] \), through:

   a. a spurious proxy variable: \( y \sim P(y|x,s) \), where \( S \) is a variable that depends on \( Z \), but has no impact on the unperturbed non-discriminatory decisions \( U \), given the relevant variables \( X \),

   b. a relevant variable: \( y \sim P(y|x) \), what implies that the impact of one of the relevant features associated with \( Z \) on \( Y \) has changed in comparison to their influence on \( U \).

Note that direct discrimination, defined as \( E[y|x,z] \neq E[u|x] \), is equivalent to disparate treatment, i.e., \( P(y|x,z_1) \neq P(y|x,z_2) \) for any \( z_1, z_2 \in Z \), what amounts to a direct impact of the protected attribute on the output variable. Interestingly, indirect discrimination requires that \( E[y|z] \neq E[u|z] \), which means that the perturbation modifies the dependence of the output variable on the protected attribute, because the impact of a mediating variable, i.e., a relevant or spurious attribute, on the output variable is modified. This formulation resembles the aforementioned notion of disparate impact, introduced as \( P(y|z_1) \neq P(y|z_2) \). The key insight enabling the definition of indirect discrimination is its relational nature requiring comparisons of data-generating processes before and after the perturbation. In contrast, the definition of direct discrimination does not necessitate such relational comparisons.

The distinction between the two types of indirect discrimination is meaningful. Indirect discrimination via \( S \) is well-established in legal systems and social science \((24)\). For example, the Supreme Court of the United States ruled that usage of broad aptitude tests in hiring practices that disparately impact ethnic minorities must be relevant to the job \((35)\). Indirect discrimination via \( X \) is less known, but the aforementioned case of redlining \((26, 27)\) can be seen as its example. If a bank decides whether to give a loan to a customer, the zip code can contain some scarce information about the wealth of this customer, so to a small extent it could be used as a relevant attribute. However, the zip code also contains information about the ethnicity of the customer, if neighbourhoods are racially segregated, so denying loan based on the zip code alone causes unjustified disparate impact on ethnic minorities. In supervised learning, the indirect discrimination via \( X \) is commonly induced when the protected attribute \( Z \) is dropped before training and its proxy variables in \( X \) replace its predictive power. Furthermore, as we will see, it is more difficult to deal with indirect discrimination via \( X \) than via \( S \), because \( X \) cannot be dropped from a decision-making model without reducing model accuracy, whereas \( S \) can.

Once discrimination is seen as a perturbation of non-discriminatory data, then one can define a measure of resilience of a supervised learning algorithm to such perturbations, by measuring how close to the unperturbed decisions are the predictors generated by this algorithm, even if no knowledge is available about the type of perturbations present in the training data. We refer to the algorithm \( a \)’s solution, i.e., a predictor, obtained by training on the dataset \( D \) as \( a(x|D) \). Then, we define the resilience of a supervised learning algorithm \( a \) to perturbation \( \tilde{D} \) of data \( D \) as

\[
\Omega(D, \tilde{D}) = \frac{E_i [L (u_i, \hat{a}(x_i|D))] / E_i [L (u_i, a(x_i|D))]}{1},
\]

which is confined between 0 and 1, i.e., \( \Omega = 1 \) means that the algorithm is perfectly resilient to the data perturbation, whereas \( \Omega = 0 \) means it is not resilient at all. The \( \hat{a}(x_i|D) \) is a predictor of the non-discriminatory ground truth, trained on the unperturbed dataset \( D \), so the enumerator takes into account that \( U \) may be intrinsically random and unpredictable. The property that \( 0 \leq \Omega \leq 1 \) is ensured, if the learning algorithms yielding
\( \hat{u}(x|D) \) and \( a(x|\tilde{D}) \) optimise the same baseline objective function, e.g., both optimize empirical risk, but \( a \) ads to it some fairness criterion, regularization, or procedure.

The proposed measure of resilience describes how an algorithm trained on potentially discriminatory \( D \) performs when it is evaluated on non-discriminatory \( \tilde{D} \). In other words, we measure how well the algorithm deals with a discriminatory dataset shift. In general, dataset shifts happen when the training dataset is sampled from a different distribution than the test dataset used for evaluation (31). The introduced discriminatory perturbations are sub-types of "concept shift" that depend on the protected attribute (31).

In supervised learning, typically we constrain the set of models that could explain \( D \) to a certain family, e.g., generalized linear models. For simplicity, we assume that \( D \) is generated by the same family as \( \tilde{D} \). Next, we develop a supervised algorithm that prevents direct discrimination and prevents induction of indirect discrimination under this assumption. Later, we propose an evaluation framework for supervised algorithms preventing discrimination that is based on the introduced measure of resilience to perturbations. The framework makes the same assumption, although it is straightforward to extend it to the cases where this assumption is not met.

Proposed method for discrimination prevention. We develop a novel supervised learning procedure that yields predictors resilient to direct discrimination and the inducement of indirect discrimination against the groups defined by the protected attribute. Note that direct discrimination could be prevented by simply removing the protected attribute from the training data. By doing so, however, we could unwillingly induce indirect discrimination, because the relevant attributes that are correlated with the protected attribute would be used in place of the protected attribute, if we applied standard supervised learning algorithms. We conclude that the dependence of the predictor on the relevant attributes shall not come from the protected attribute, i.e., the inducement of indirect discrimination is not allowed.

Overall, the proposed resilient learning algorithm has two steps. In the first step, we train the model using all features, both protected \( Z \) and relevant \( X \), without any consideration of fairness. Most importantly, the protected attribute is available during the training, so the model does not use third variables as surrogates of the protected attribute, thus avoiding inducing indirect discrimination via \( X \). In this way, we estimate the true values of the parameters unaffected by perturbations that regulate the impact of the relevant variables \( X \) on \( Y \). Our estimates of these parameters are unbiased under the assumption that \( \hat{u}(x) \) and \( \hat{y}(x, s, z) \) belong to the same parametric family of functions and there is no model misspecification. In the second step of our method, we eliminate the influence of the protected attribute. This is done by using the model trained with all features but imputing the value of the protected attribute from a weighting distribution \( \pi \) that does not depend on any of the relevant features nor the dependent variable.

More specifically, in the case of frequentist decision theory, our method for discrimination prevention is as follows. In the first step, we obtain the full predictor \( \hat{y}(x, z) \) by minimizing the corresponding expected value of our loss function, e.g., the empirical risk \( \hat{R}(Y, \hat{Y}) \). In the second step, we eliminate the dependence on the protected variable \( z \) by replacing it with a counterfactual random variable \( z' \) with a mixing distribution \( \pi(z') \) independent from other variables, yielding \( \hat{y}_z(x) = \sum z' \hat{y}(x, z') \pi(z')dz' \), which we refer to as an average predictor imputing the protected attribute. Methods preventing discrimination trade accuracy to fulfill fairness objectives (12).

Here, we search for the optimal mixing distribution, \( \pi^*(z') \), that minimizes the empirical risk, \( \hat{R}(\hat{Y}, \bar{Y}_x) \), while all parameters of the full predictor \( \hat{y}(x, z) \) are fixed, i.e.,

\[
\pi^* = \arg \min_{\pi} \mathbb{E}[L(y_i, \hat{y}_z(x))] \tag{2}
\]

This optimization problem is convex for quadratic loss function. Thus, the optimal weighting distribution can be found by applying disciplined convex programming with constraints ensuring that \( \pi(z') \) is a distribution, i.e., \( \sum z', \pi(z') = 1 \) and \( \pi(z') \geq 0 \) for all \( z' \) (36). Once the optimal mixing distribution is known, the optimal imputing predictor can be computed,

\[
\hat{y}^*(x) = \sum z' \hat{y}(x, z') \pi^*(z')dz' \tag{3}
\]

which is the solution of the proposed learning algorithm.

An evaluation framework estimating resilience to perturbations. In real datasets, we typically have access only to the potentially perturbed decisions \( Y \) and we do not know \( U \). The definitions of discriminatory perturbations allow us to generate synthetic datasets perturbed with direct or indirect discrimination for which we know non-discriminatory ground truths. In this setting, we reason that learning algorithms that prevent discrimination should be resilient to such synthetic perturbations and should retrieve predictors that are close to the non-discriminatory ground truth. This is a challenging task, because the training method does not have access to that ground truth, but only to its perturbed version, \( \tilde{D} \). In our evaluation framework (Figure 1), we generate random datasets \( D \) and \( \tilde{D} \) from the same family of functions and measure the resilience to discriminatory perturbations of various learning algorithms preventing discrimination, including our proposed algorithm.

More specifically, we apply our theory and evaluation framework to the case of generalized linear models, which govern the expectation value of the output variable to be \( \hat{u}(x) = \sigma(\alpha x) \), where \( \sigma \) is inverse of the link function. For instance, in the case of binary dependent variables, as in logistic regression, the function \( \sigma \) is a sigmoid function. Next, we define discrimination as a perturbation of \( \hat{u} \) that in general can be represented as \( \bar{y}(x, z, s) = \bar{u}(x) + f(x, z, s) \). Under the assumption that \( \bar{u} \) and \( \bar{y} \) belong to the same family of models, we can represent the perturbations as \( \bar{y}(x, z, s) = \sigma(\Delta x + \beta z + \gamma s) \).

Using this framework, we measure the resilience of several state-of-the-art learning algorithms for discrimination prevention, which we introduce next.
State-of-the-art learning algorithms addressing discrimination. Several methods have been proposed to train machine learning models that prevent a combination of disparate treatment and impact (8–10). These methods, however, induce indirect reverse discrimination, by negatively affecting the members of advantaged group (30). Other studies propose novel mathematical notions of fairness, such as equalized opportunity, \( P(\hat{y}|y = 1, z = 0) = P(\hat{y}|y = 1, z = 1) \), and equalized odds, \( P(\hat{y}|y, z = 0) = P(\hat{y}|y, z = 1) \) (13, 15–17), or parity mistreatment, i.e., \( P(\hat{y} \neq y|z = 0) = P(\hat{y} \neq y|z = 1) \) (12). These methods at first look promising, but they too induce indirect discrimination (see Appendix A). Overall, about twenty such fairness objectives and their implementations have been proposed (37), and recent works expose the impossibility of simultaneously satisfying multiple non-discriminatory objectives, such as equalized opportunity and parity mistreatment (38–40). In other words, there exist multiple supervised learning methods for preventing discrimination, but they are often mutually exclusive. There is a need to find ways to compare these methods with objective measures.

We evaluate several of these learning algorithms in the following section. For this evaluation, we select a diverse set of methods that aim to prevent discrimination through different objectives: disparate impact (41), disparate mistreatment (10, 12), preferential fairness (14), equalized odds (13), a convex surrogate of equalized odds (17), and a causal database repair (22). In most cases, we use implementations of these algorithms as provided by authors. All of these methods were implemented for the case of discrete decisions \( Y \). We re-implemented one of these methods so that it works for the case of continuous \( Y \) (10, 12).

Results from the evaluation framework. We use the proposed evaluation framework to test whether different supervised learning algorithms are resilient to various dataset shifts. First, we generate a synthetic set of 10 000 samples \( \{x, z, s\} \) by sampling them from a standard multivariate normal distribution with a random correlation matrix (42). The variable \( z \) is converted to a binary value with the sign function. Second, we generate the non-discriminatory ground truth decisions, either as draws from normal distribution with unit variance, or by sampling from a standard multivariate normal distribution with a random correlation matrix (42). The variable \( z \) is converted to a binary value with the sign function.

Fig. 2. Average resilience of learning algorithms to non-discriminatory perturbations (the leftmost column) and discriminatory perturbation (the remaining three columns), for logistic regression (upper part) and linear regression models (lower part). On average, the proposed optimal imputing predictor (red bars) is more accurate w.r.t non-discriminatory ground truth than state-of-the-art methods addressing discrimination (orange bars). The error bars correspond to 95% confidence intervals of the expectation, obtained via bootstrapping.

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1. https://github.com/jmikko/fair_ERM
2. https://github.com/mbilalzafar/fair-classification
3. https://github.com/gpleiss/equalized_odds_and_calibration
4. https://github.com/mikko/fair_ERM
where coefficients \( \alpha_j \sim \text{Uniform}[0, 5] \) for \( j \in \{1, 2\} \). The resulting set of samples constitute the dataset \( D = \{x, z, u\} \). Third, we sample the perturbed decisions, \( y \sim P(y|x, z, s) \), which is the same family of distributions as \( P(u|x) \). These perturbed decisions constitute the dataset \( \tilde{D} = \{x, z, y\} \) that will be used by learning algorithms as a training dataset. These perturbations may or may not be discriminatory, depending on how they affect the expected perturbed outcomes:

1. no discrimination: \( \tilde{y}(x) = \tilde{u}(x) = \sigma(\alpha_1 x_1 + \alpha_2 x_2) \),
2. direct discr. via \( Z \): \( \tilde{y}(x, z) = \sigma(\alpha_1 x_1 + \alpha_2 x_2 + \beta z) \),
3. indirect discr. via \( S \): \( \tilde{y}(x, s) = \sigma(\alpha_1 x_1 + \alpha_2 x_2 + \gamma s) \),
4. indirect discr. via \( X \): \( \tilde{y}(x) = \sigma((\alpha_1 + \tilde{\alpha}) x_1 + \alpha_2 x_2) \),

where coefficients \( \tilde{\alpha}, \beta, \) and \( \gamma \) are drawn from \( \text{Uniform}[-5, 5] \). In the case of each perturbation, we receive a set of potentially discriminatory samples, \( \tilde{D} \).

These perturbed datasets are then used to train a model, using various state-of-the-art supervised methods for discrimination prevention (10, 12–14, 17) and the proposed learning algorithm (Equation 3). To compare the effectiveness of different methods preventing discrimination we measure the resilience, \( \Omega(D, \tilde{D}) \), computed for the squared loss function. For each learning algorithm, the procedure of data generation and training is repeated 1000 times, each time with a different correlation matrix \( \Sigma \) and model parameters \( \alpha, \tilde{\alpha}, \beta, \gamma \). Then, we report the resilience averaged over these trials, \( \mathbb{E}[\Omega] \), measured separately for each type of data perturbation (Figure 2).

When the learning algorithms preventing discrimination are applied to non-discriminatory data, they shall fall back to a traditional learning algorithm to avoid biases in inference. However, most of the algorithms tested here do not achieve this result (the leftmost column in Figure 2), except for two algorithms: the game-theoretic method based on envy-freeness (“Zafar EF” in Figure 2) (14) and our algorithm. The methods equalizing overall misclassification rate, false negative rate, or related measures (e.g., “Zafar OMR” in Figure 2) (11) introduce indirect discrimination (see Appendix A), same as the methods that leverage parity treatment and impact (8–10, 30).

As expected, the resilience of all methods decreases when they are trained on the datasets with discriminatory perturbations (the three right columns in Figure 2). However, the proposed learning algorithm (the red bars in Figure 2) is more resilient to direct and indirect discriminatory perturbations.
than other supervised methods aiming to prevent discrimination (10, 12–14, 17). The second best method is consistently the game-theoretic method based on envy-freeness, however this algorithm allows direct discrimination via $Z$.

Interestingly, our learning algorithm has also significantly larger resilience than the traditional learning algorithm (with or without protected attribute; see blue bars in Figure 2), for every type of discriminatory perturbation, except for the indirect discrimination via $X$. This result holds true both for logistic regression model (upper part of Figure 2) and linear regression (lower part of Figure 2). For instance, for the linear regression model, the proposed method achieves maximal resilience to directly discriminatory perturbations. In the case of indirect discrimination via $X$, the proposed algorithm has the same resilience as the traditional algorithm. It is impossible for the proposed algorithm to address indirect discrimination via $X$, because $X$ impacts the unavailable non-discriminatory $U$, in contrast to indirect discrimination via $S$, which can be partially tackled by not including the variable $S$ in the training dataset, what increases slightly the resilience of the proposed method over the traditional method.

**Perturbed and missing relevant attributes.** Apart from the perturbations of the output variable $U$, the perturbed dataset, $\tilde{D}$, could also include the perturbations of some of the relevant attributes $X_1$, in which case we refer to these relevant attributes as $\tilde{X}_1$. For instance, Jim Crow laws required literacy to decide whether an individual has a voting right, while ethnic minorities had systematically limited access to education (43). If some $X_1$ is suspected to be affected by discriminatory perturbations, then we shall construct a respective model for these variables, in which they are treated as output variables. Then, one can obtain an estimator of $X_1$ from $\tilde{X}_1$ by applying the proposed algorithm. The computed optimal imputing predictor, which serves as an estimator of $X_1$, can be applied to also obtain an estimator of $U$ from $Y$ that is formed using the perturbed $\tilde{X}_1$. We apply this procedure within our evaluation framework by modeling a perturbation of $X_1$ yielding $\tilde{X}_1$ (see Appendix B). We measure the resilience of the learning algorithms to this perturbation, finding that the proposed learning algorithm prevents direct discrimination in $X$ and as a consequence in $U$ (left side of Figure 3), under a linear model of $X$ and either a logistic or linear model of $Y$.

In real-world settings, relevant attributes are often unknown or their measurements are unavailable. We model this scenario by removing $X_1$ from the training dataset $D$, while keeping it unchanged in $\tilde{D}$. Then, we measure the resilience of learning algorithms to missing relevant attributes. We distinguish between the case where the missing relevant attribute is discriminatory, $X_1$, and the case where the missing attribute is not affected by discrimination, $X_1$, although there exists an association between that attribute and the protected variable. The proposed learning algorithm is more resilient to missing discriminatory attribute than the other methods (middle column in Figure 3). When the missing attribute is non-discriminatory, the proposed algorithm performs slightly worse than the traditional algorithm (right column in Figure 3), which uses the protected attribute to obtain a more accurate predictor.

This drop in performance does not happen when all relevant attributes are introduced into the model, what creates an incentive for a data science practitioner to identify relevant features. Note also that, under the proposed learning method, the effect of adding proxy variables of the protected attribute on the predictor is mitigated, in contrast to the traditional learning algorithm, where proxy variables are used in place of the protected attribute. These two points taken together mean that the proposed algorithm allows and encourages a search for relevant features while inhibiting propagation of discrimination.

**Discussion and limitations of the evaluation framework.** The proposed evaluation framework could have other specifications than the ones studied in this manuscript. First, the functional forms of the non-discriminatory ground truth model and its perturbations may influence the results of the proposed evaluation framework. In future work, these perturbations could be measured via experiments or observational studies to generate more realistic perturbations. Second, these results also depend on the distributions of all variables and the parameters of the used models, although our explorations show that the presented results are qualitatively robust. Third, it is important to understand how model misspecification influences these results: the two models used to generate the ground truth and to train on observations could differ, violating the assumption that they belong to the same parametric family. Future research shall develop this evaluation framework to make it more comprehensive and realistic, potentially enabling it as a benchmark for novel training methods that are resilient to discriminatory perturbations of data.

**Evaluation on real-world datasets.** While we have shown the resilient performance of our method in the evaluation framework, it remains to show whether the performance over synthetic dataset can translate to the empirical performance where we do not know the true data generating process. To evaluate this effect, we conduct the empirical analysis of our method over real-world datasets and compare its performance with the other algorithms addressing discrimination (10, 12–14, 17). We focus on binary classification task on two real datasets commonly used for the evaluation in fairness literature: the COMPAS recidivism dataset (5) and German Credit Dataset (44) (see Appendix C). For COMPAS, we use the binary labels for race as a protected attribute. Similarly, for German Credit dataset, we use the gender of individuals as a protected attribute.

Since in real-world scenarios we typically do not have access to non-discriminatory ground truth, $U$, as we did in the synthetic evaluation framework, we measure traditional accuracy and demographic disparity as a proxy of discrimination. Demographic disparity is defined as $DD = |P(\hat{y} = 1 | z = 0) − P(\hat{y} = 1 | z = 1)|$ (10, 22). Note that even a perfectly non-discriminatory model can produce non-zero demographic disparity if underly-
ing data is unfair, as we argued in the previous sections. While other measures have been proposed and used in the context of real-world applications (5), such as disparity in false positive rate or positive predictive value (see Appendix C), these measures and other measures derived from the confusion matrix are determined for any given dataset by accuracy and demographic disparity (or any other such two measures for that matter) (38–40, 45). In this experiment, we report the mean and standard deviation of these measures computed via 5-fold cross-validation (22).

Similar to the earlier experiment with synthetic data, we compare our method with existing supervised-learning methods for comparison for fairness (10, 12–14, 17). We report the results in Figure 4. Our method achieves the lowest demographic disparity and the highest accuracy for German Credit data. For the COMPASS data it also achieves the top accuracy, while yielding medium demographic disparity. Methods that achieve lower disparity also have lower accuracy, e.g., "Zafar (2015)" for COMPAS. Results for other measures of disparity can be found in Figure 6 in Appendix C.

Conclusions. Our results shed a new light on the problem of discrimination prevention in supervised learning. First, we propose a formal definition of direct and indirect discrimination, inspired by research in humanist fields (24). This allows us to design a new evaluation framework for discrimination prevention in supervised learning by seeking methods that are resilient to various discriminatory perturbations. Second, we show that state-of-the-art methods addressing discrimination often return biased predictors when they are trained on datasets that are not affected by discrimination. Third, we propose a novel learning algorithm, whose solution is an average predictor imputing the protected attributes, which is resilient to direct and indirect discriminatory perturbations, thus performing better than the state-of-the-art methods in the proposed evaluation framework.

The proposed learning algorithm performs better in the evaluation framework than the traditional learning algorithm when there is direct discrimination via the protected attribute or indirect discrimination via a spurious variable, what justifies its use in the circumstances where discrimination could have affected the training dataset. In real-world scenarios, it is often unclear whether all relevant attributes are taken into account – the proposed learning algorithm performs better in these scenarios than traditional learning methods if discrimination is present. In the scenarios where discrimination does not affect the training data, the proposed learning algorithm returns unbiased predictors, unless relevant attributes are missing. By contrast, in the scenario where there is no discrimination and attributes are missing, the proposed method returns more biased models than the traditional learning algorithm. These results suggest that legal systems could enforce algorithmic learning methods inhibiting discrimination when evidences of discrimination are found in a society, but once discrimination is not present any more and the availability of relevant attributes is limited, then traditional learning methods return less biased estimators.
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Appendix A

Additional synthetic experiments Here, we present the results from a synthetic scenario proposed by (30), modified slightly as follows. Using this example, we show how state-of-the-art learning algorithms addressing discrimination induce it even when the training data is non-discriminatory.

To this end, we sample 1000 observations from the data-generating process below:

\[
\begin{align*}
    z_i &\sim \text{Bernoulli}[0.5] \\
    \text{hair_length}|z_i = 0 &\sim 35 + \beta[2, 2] \\
    \text{hair_length}|z_i = 1 &\sim 35 + \beta[2, 7] \\
    \text{work_exp}|z_i = 0 &\sim \text{Poisson}[25] - \text{Normal}[20, \sigma = 0.2] \\
    \text{work_exp}|z_i = 1 &\sim \begin{cases} \\
        \text{Normal}[10, \sigma = 2] &\text{with probability 0.2} \\
        \text{Normal}[15, \sigma = 2] &\text{with probability 0.8} 
    \end{cases} \\
    p_i &\sim f(-25.5 + 2.5 \times \text{work_exp}) \text{ where } f(x) = \frac{1}{1 + e^{-x}} \\
    y_i|\text{work_exp} &\sim \text{Bernoulli}[p_i] 
\end{align*}
\]

This synthetic data represents the historical hiring process where the protected attribute is a candidate’s gender, z. The data has the following properties: i) the hiring decision has been made based on the work experience only; thus, it is non-discriminatory data; ii) since women on average have less work experience than men, men have been hired at higher rate than women historically; and iii) women tend to have longer hair than men. Therefore, a model that uses hair length in its decision-making can induce indirect discrimination. Additionally, we introduced modification to this synthetic data with respect to the original scenario (30). The work experience of male candidates now follows a bi-modal distribution (i.e., a mixture of two normal distributions) with one peak at 10 and another at 15. We trained a method for discrimination prevention (12) under three different fairness constraints: equalized missclassification rate, false positive rate (FPR), false negative rate (FNR) \(\dagger\).

Figure 5 demonstrates the indirect discrimination induced by models under various fairness objectives. We observe the following. First, none of the models found the true data generating process (dashed line) even though the training data is non-discriminatory. Second, each triangle points represent the candidates affected by indirect discrimination: we observe that the model under the FPR objective (the center figure) rejects male candidates due to their shorter hair (male-characteristics). Finally, we present the relative utility of various models (10, 12–14, 17) under this synthetic data in Table 1.

Table 1. Relative utility of various fairness models (10, 12–14, 17) trained with the synthetic data

| Method | \(\mathbb{E}[\text{Utility}]\) |
|---|---|
| Mixture (ours) | 1.000 |
| Zafar (2018) (14) | 0.997 |
| Zafar (2017) (12) with FNR | 0.838 |
| Zafar (2017) (12) with Missclass. | 0.777 |
| Donini (2018) (17) | 0.634 |
| Zafar (2017) (12) with FPR | 0.570 |
| Hardt (2016) (13) | 0.328 |
| Zafar (2016) (10) | 0.179 |

Appendix B

Experiment Setup. We report the performance of the model by Donini et al (17) with SVM with linear kernel. The regularization parameter C was tuned via grid search with \(C \in \{0.01, 0.1, 1\}\). We report the statistics of (12) when the model is optimized to equalize missclassification rates between two groups. The implementation of the models we used for the experiment (10, 12–14, 17) are readily available online:\*\*\*

Modeling discrimination in the relevant attributes. To account for the discrimination in a component of \(X_i\), we generate the dataset \(D\) in a slightly different way than in the main evaluation framework. Namely, after drawing the correlation matrix \(\Sigma\), we modify it to ensure that \(x_1 = \delta x_2\), where \(\delta\) is another coefficient and \(Z\) does not influence \(X_1\). From this non-discriminatory \(X_1\), we create its perturbed version, \(\tilde{x}_1 = x_1 + \epsilon/2\). Finally, the perturbed output variable is formed by using \(\tilde{x}_1\) in place of \(x_1\), that is \(\bar{y} = \sigma(x_1 \tilde{x}_1 + \alpha_2 x_2)\), whereas the non-discriminatory output variable is formed as usual, \(u = \sigma(x_1 \tilde{x}_1 + \alpha_2 x_2)\).

Appendix C

Experiment Setup. Similar to the synthetic experiment, we report the performance of the model by Donini et al (17) with SVM with linear kernel. The regularization parameter \(C\) was tuned via grid search with \(C \in \{0.01, 0.1, 1\}\). We report the

\*\*\* We also trained a model while simultaneously optimizing both FPR and FNR; however, the learned model returned trivial predictions where all candidates are rejected.

\*\*\*\* https://github.com/mlblizalafar/ fair-classification

\*\*\*\*\* https://github.com/gpleiss/ equalized_odds_and_calibration

\*\*\*\*\*\* https://github.com/jmikko/fairERM
Following the synthetic data proposed by (30), we show how machine learning models under different fairness constraints (12) can return biased predictors even when the training data is non-discriminatory. We observe the following: i) none of the ML models (solid lines) found the true data generating process (dashed line) and ii) each triangular region between the decision boundary (solid lines) and true model (dashed line) is where the indirect discrimination happens. In particular, we observe that a group of male candidates are adversely affected by the model under the FPR objective (the center figure). Those candidates are rejected due to their short hair, or male-like characteristics.

| Metric | Description and Definition |
|--------|-----------------------------|
| DD | Demographic Disparity: $|P(\hat{y} = 1 | z = 0) - P(\hat{y} = 1 | z = 1)|$ |
| PPD | Positive Predictive Disparity: $|P(y = 1 | \hat{y} = 1, z = 0) - P(y = 1 | \hat{y} = 1, z = 1)|$ |
| FPD | False Positive Disparity: $|P(\hat{y} = 1 | y = 0, z = 0) - P(\hat{y} = 1 | y = 0, z = 1)|$ |

Table 2: Summary of discrimination metrics used in our experiments

statistics of (12) when the model is optimized to equalize misclassification rates between two groups.

**COMPAS Dataset.** The ProPublica COMPAS dataset (5) contains the records of 7214 offenders in Broward County, Florida in 2013 and 2014. COMPAS also provides binary label for each data if the individual shows high sign of recidivism. We use the race (African American, Caucasian) as the sensitive features. This dataset also includes information about the age, job type, housing type of applicants, the total amount in saving accounts, checking accounts and the total amount in credit, the duration in month and the purpose of loan applications.

**German Credit Dataset.** German Credit Dataset (44) provides information about 1000 individuals and the corresponding binary labels describing them as creditworthy ($y_i = 1$) or not ($y_i = 0$). Each feature $x_i$ includes 20 attributes with both continuous and categorical data. We use the gender of individuals as the sensitive feature. This dataset also includes information about the age, job type, housing type of applicants, the total amount in saving accounts, checking accounts and the total amount in credit, the duration in month and the purpose of loan applications.

**Fig. 6.** Additional experiment with real-world dataset using Positive Predictive Disparity (PPD) and False Positive Disparity (FPD). The lower these values are, the more likely that models are fair.