UNVEILING THE ETHICS BEHIND INEQUALITY MEASUREMENT: DALTON’S CONTRIBUTION TO ECONOMICS*

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This article assesses the importance of Dalton’s 1920 paper in the ECONOMIC JOURNAL for subsequent developments in income distribution analysis.

The signal achievement of Hugh Dalton, and Arthur Cecil Pigou, with whom his name is often coupled, was to provide a welfare economic basis for the measurement of income inequality. In 1920 Dalton published the volume Some Aspects of the Inequality of Incomes in Modern Communities (Dalton, 1920b), which extensively surveyed theories explaining income inequality and examined policies aimed at reducing inequality. In the article that appeared in the same year in the ECONOMIC JOURNAL, Dalton (1920a) investigated the analytical aspects of the measurement of income inequality. He wrote in his memoirs:

I had planned, as the final part of my Inequality of Incomes, a pretty full discussion of the measurement of the inequality of incomes and of the application of various rival measures to the available statistics. But this wide project, which no one even yet has carried out, shrank under the pressure of my timetable to a short article ... Rather an ingenious piece of writing, I still think, with some algebraical and differential decorations and drawing attention to some elegant theorems of little-known Italian economists, in whom, owing to my knowledge of the language, I made a temporary corner, reviewing a series of them in the ECONOMIC JOURNAL. But it was based on hypotheses which were a bit unreal

(Dalton, 1953, p. 107).

Dalton considered income, not as such, but as the determinant of individual welfare and, therefore, focused on the issues applicable in this case. He was not discussing the measurement of inequality in general – a major break with the tradition of preceding writers, particularly in the extensive Italian literature. Adopting this specific

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The article was subsequently included as an Appendix to the 2nd edition of his book published in 1925. Between 1921 and 1935, Dalton reviewed about a dozen books in Italian for the ECONOMIC JOURNAL. He had learnt Italian while serving in the artillery at the Italian front in the First World War (Dalton, 1953, p. 93).
‘instrumental’ perspective, he was able to penetrate much further and to relate the choice of inequality measures to the underlying concern with social welfare.

Dalton saw social welfare through resolutely utilitarian eyes, as in Pigou’s Wealth and Welfare (1912), which had introduced a key concept that underlay much of Dalton’s article: the principle of transfers. A mean-preserving transfer of income from a richer person to an (otherwise identical) poorer person, or what has come to be known as a Pigou–Dalton transfer, ‘must increase the sum of satisfaction’ (Pigou, 1912, p. 24). Importantly, it is assumed here that utility is a non-decreasing, concave function of income and identical for all persons. Although not stated in these terms, there is an equivalence between the statement that distribution $A$ can be reached from distribution $B$ by a sequence of equalising mean-preserving transfers and the statement that distribution $A$ has a higher level than $B$ of the sum of individual utilities.

Dalton used this perspective to evaluate a number of the summary measures of dispersion used by statisticians. He noted, for example, that the mean deviation is insensitive to equalising transfers on one side of the mean: ‘additional comforts for millionaires’ financed by ‘a tax levied on those whose incomes were just above the mean’ would be shown as leaving inequality unaffected (Dalton, 1920a, p. 352). In contrast, the standard deviation and the relative mean difference (Gini coefficient) are sensitive to such mean-preserving transfers. He then went on to enunciate three further principles to be applied where the total income or the total population is varied. None of the measures satisfy all principles but he concluded that, data allowing, the standard deviation and the Gini coefficient are to be preferred.

Dalton has in mind the distribution of income as a whole, drawing for example the Lorenz curve, showing the proportion of income received by different cumulative proportions of the population from the bottom upwards to the top. He refers to ‘income’, not to components of income such as wages or capital income. This is natural, given that he is ‘interested, not in the distribution of income as such, but in the effects of the distribution of income upon the distribution and total amount of economic welfare, which may be derived from income’ (Dalton, 1920a, p. 348). The article contains no empirical applications and he refers at the outset to the ‘inadequacy of the available statistics of the distribution of income in all modern communities’ (1920a, p. 348), a point to which we return at the end.2

1. The Initial Impact of the Paper

Dalton’s article did not go unnoticed. In the March 1921 issue of the Economic Journal, Gini, a leading figure in income distribution analysis at that time, expressed his admiration for ‘the simplicity and ease of the method which he suggests for measuring the inequality of economic welfare, on the hypothesis that the economic

2 In the preface to his book, Dalton writes that he had planned ‘to compare statistically the inequality of incomes in different communities’ (1920b, p. viii). Before the outbreak of the war, he had apparently collected a considerable amount of statistical material that struck him for its ‘inadequacy’. When he returned to his work after more than four years of military service, he however decided to put aside this material with a view to an early publication. The only data discussed at some length by Dalton (1920b, pp. 207–9) concern the division of the national income between workers and owners, and are drawn from Bowley (1919, 1920) and Stamp (1919).
welfare of different persons is additive’ (Gini, 1921; p. 124). Gini was however keen to underline the difference of such a welfare-based approach from that adopted by the Italian writers cited by Dalton, whose methods were ‘applicable not only to incomes and wealth, but to all other quantitative characteristics (economic, demographic, anatomical or physiological)’ (1921, p. 124). He evidently regarded the latter as a merit. It is a contrast that epitomises the two fold nature of income inequality measurement: the descriptive aspect – the account of factual diversity – and the normative aspect – the ethical judgement about this diversity. A few years later, in the Journal of the American Statistical Association, Yntema was also dismissive of Dalton’s welfare-based approach but on a somewhat different basis: Dalton’s procedure ‘encounters the difficulty of finding the function which relates the individual’s welfare to his income as well as the necessity of assuming identity between different individuals’ (Yntema, 1933, p. 423). It is a rejection of Dalton’s ethical approach on practical rather than conceptual grounds. Indeed, much of Yntema’s paper may be seen as a development of that by Dalton.

Possibly because of these practical and analytical difficulties, Dalton’s article does not appear, however, to have had much impact for many years. A search on standard databases such as Google Scholar, JSTOR and those of main international publishers identifies fewer than a dozen citations in Dalton’s lifetime (he died in February 1962). Passing references can be found in Castellano (1935), de Vergottini (1940), Rosenbluth (1951) and Cartter (1955), whereas Wedgwood (1929) and Mansfield (1954) stress the distinction between absolute and relative inequality measures. In a contribution to a volume of the Conference on Income and Wealth in 1952, Garvy argued that inequality should be measured not ‘against a manifestly unrealistic standard of either perfect equality or perfect inequality’ but rather against a ‘standard of a socially desirable or justifiable degree of inequality … reflecting given sets of economic, political, or ethical principles’ (Garvy, 1952; pp. 27, 30). He observed that ‘little seems to have changed with respect to the ‘theory of income distribution among persons’ since Dalton wrote his pioneering study thirty years ago’ (1952, p. 31) but did not pick up his hint of embodying the normative judgement within the inequality measure (rather than in the benchmark distribution). This happened fifteen years later, when Aigner and Heins (1967) straightforwardly generalised Dalton’s approach by considering a broader range of social welfare functions to measure equality. Their more general formulation allows also for a non-utilitarian interpretation, whereby the social welfare function can be seen as specified by an ‘egalitarian observer’. It does, however, share the same problem as Dalton’s original proposal: the dependence of the (in)equality measure on the unknown relationship that links income and welfare.

All this was to change in the 1970s, a change in fortunes that is illustrated in Figure 1. By 2014 Dalton’s article had received some 1,100 citations, according to Google Scholar, and the number has been growing: in the ten years from 2000 to 2009, it received 484 citations. To allow for the expansion in papers published, we have in Figure 1 expressed the citations relative to those for another paper from the Economic Journal in the 1920s that has also been cited some 1,100 times: Sraffa’s article on

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3 The pioneering study referred to in the citation is actually Dalton’s book but Garvy cites his article too.
‘The laws of returns under competitive conditions’ (Sraffa, 1926). Compared to Sraffa, Dalton’s work was completely ignored until the 1970s. This need not reflect a lack of interest for income distribution analysis, as revealed for instance by the intense debate around Pareto’s law of incomes (Bresciani-Turroni, 1939; Tommissen, 1969).

Dalton’s paper was ahead of time. To be fully understood it had to wait the post-war developments in social choice theory and the analysis of choice under uncertainty.

2. Fifty Years On: Revival of Interest

After some half century of neglect, the measurement of income inequality suddenly became again the subject of considerable interest in economics, and Dalton’s research moved centre stage. The turning point came with the research on social justice by Kolm (1969) and the article published by Atkinson in the *Journal of Economic Theory* in 1970. Kolm’s and Atkinson’s first contribution was to turn the Pigou–Dalton transfer principle (called ‘rectifiance’ by Kolm, 1969; p. 188) into ‘the criterion that is overwhelmingly used in the literature to introduce a concern for inequality into judgements about income distribution’ (Cowell, 2003, p. xv). But they advanced the subject beyond Dalton in two important respects.

First, a link was made to the Lorenz curve, which shows the proportion of total income received by successive proportionate groups cumulated from the bottom.

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*Note by A. B. Atkinson: Serge Kolm clearly has priority, in that his paper was originally presented at a conference of the International Economic Association in 1966; I came to the subject from a different direction and only became aware of his work after my own article had been accepted for publication.*

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Dalton (1920a) referred to the Lorenz curve but did not see the relationship with the principle of transfers: that where one distribution \( A \) can be reached from a distribution \( B \) by a sequence of equalising mean-preserving transfers, then the Lorenz curve for \( A \) lies inside that for \( B \). In this way, there is a direct link with a widely used statistical device. Where the Lorenz curves do not intersect, then two distributions are ranked in the same way by all social welfare functions satisfying the general concavity properties and by all inequality indices satisfying the transfer principle (and few other regularity conditions). This is a fundamental advancement, as it means that we may be able to rank one distribution as less, or more, unequal than another by only agreeing on the concavity properties and the principle of transfers. There may be no need to adopt a single, and hence potentially controversial, specification of the social welfare function. Of course, the ensuing ordering is only partial: wherever the Lorenz curves intersect, further restrictions on the welfare function may be necessary to rank the two distributions. The extensions of the Lorenz dominance conditions and the additional restrictions that may yield higher order dominance have been much studied. Dasgupta et al. (1973) and Rothschild and Stiglitz (1973) showed how additive separability and concavity of the social welfare function could be generalised to Schur-concavity. Kolm (1973, published as 1976) introduced the ‘principle of diminishing transfers’ whereby inequality is assumed to be more sensitive to a rich-to-poor income transfer if the richer person is located lower down in the income distribution, a question later re-examined by Atkinson (1973, published as 2008) and Shorrocks and Foster (1987). Shorrocks (1983) considered cases where the welfare assessment involves distributions with different mean incomes.

The only way to achieve complete ordering is to specify a single social welfare function. Here comes the second fundamental innovation, which is the recasting of Dalton’s approach in the income space, from the utility space, by means of the notion of ‘equally distributed equivalent income’, that is the level of income \( y_e \) which would give the same level of social welfare as the given distribution, when equally assigned to all individuals. Because of the concavity of the social welfare function, \( y_e \) is lower than mean income \( \mu \), and its proportional shortfall from \( \mu \) measures the (relative) welfare loss due to inequality. An inequality index can hence be defined as \( (1 - y_e)/\mu \). If the further assumption is made that the individual function of income is isoelastic, this formula yields the index

\[
I = \begin{cases} 
1 - \left[ \frac{1}{n} \sum_i \left( \frac{y_i}{\mu} \right)^{(1-\varepsilon)} \right]^{1/\varepsilon} & \varepsilon > 0, \varepsilon > 1 \\
1 - \left[ \Pi_i \left( \frac{y_i}{\mu} \right) \right]^{1/\varepsilon} & \varepsilon = 1 
\end{cases},
\]

where \( y_i \) denotes the income of person \( i \) and persons are assumed to be ranked by increasing income, so that \( i \) indicates their position in the income distribution.\(^5\)

Income is assumed to be non-negative. The parameter \( \varepsilon \) captures the aversion to inequality: the higher \( \varepsilon \), the more weight is attached to income transfers at the bottom

\(^5\) The more general class of generalised entropy (GE) indices (Toyoda, 1975; Cowell and Kuga, 1981) are given by \( [(1 - I)^{\varepsilon} - 1]/[\varepsilon(\varepsilon - 1)] \). The indices \( GE \) and \( I \) are ordinally equivalent where \( \varepsilon = 1 - x > 0 \).

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of the distribution relative to those at the top; as $\varepsilon$ tends to infinity, only $y_i/\mu$ matters. The index with $\varepsilon = 1$ corresponds to the first of the cases considered by Dalton (1920a), where he takes the individual function as $\log(y) + c$ and proposes as a measure of inequality

$$D = \frac{\log(\mu) + c}{\log(\gamma) + c},$$

(2)

where $\gamma$ is the geometric mean. In contrast to the measure (1), this depends on the value of $c$. (The second of the cases considered by Dalton corresponds to $\varepsilon = 2$ and leads to an inequality measure involving the harmonic mean). The advantage of the equally distributed income approach is that it is invariant with respect to linear transformations. Moreover, it has a straightforward intuitive interpretation as the ‘cost’ of inequality in terms of loss of total income. Note that there is nothing inherently utilitarian in this formulation. The social welfare function underlying (1) is the sum of identical concave transformations of individual incomes but social welfare is expressed in terms of incomes not utilities. Thus, the concavity of the social welfare function may represent the aversion to inequality of the evaluator rather than the degree of relative risk aversion of a utility function identical across all individuals.

The social welfare function approach leads to a class of inequality measures that are alternatives to the standard statistical measures, such as the standard deviation and Gini coefficient that Dalton ended up championing. This way of approaching the problem of measuring inequality has two significant consequences. First, it brings home to researchers that there exists a mapping from (in)equality indices to social welfare functions and vice versa. Any summary inequality measure reflects a certain set of value judgements which can be unveiled by looking at the characteristics of the underlying social welfare function. Thus, Newbery (1970) suggested rejecting the Gini coefficient on the ground that its ranking of income distributions cannot be supported by any additively separable social welfare function, to which Sheshinski (1972) replied that there is no reason to assign additive separability a particular significance and proposed a non-additive function generating the same ordering as the Gini coefficient. Moving beyond the Gini coefficient, Blackorby and Donaldson (1978) showed more generally how to uncover the social judgement hidden in any equality index.

The second merit of the parameterisation in (1) is that the different views concerning distributional justice are not only made explicit through the degree of inequality aversion $\varepsilon$ but also easily accommodated, in statistical applications, by simply considering a range of values for such a parameter. Atkinson (1970) took $\varepsilon$ varying from 0 to 2.5, although in most applications the range taken is typically narrower. The study prepared for the OECD by Sawyer (1976) used values of 0.5 and 1.5. LIS, a cross-national data centre in Luxembourg, publishes key figures on income inequality in about 40 countries using the values of 0.5 and 1 (http://www.lisdatacenter.org/data-access/key-figures/download-key-figures/). The US Census Bureau uses the values of 0.25, 0.5 and 0.75 (DeNavas-Walt et al., 2013). Consideration of different values is aided by the fact that $-\varepsilon$ is the elasticity of the social marginal value of income. This way of presenting the distributional trade-offs has been popularised by Okun (1975) in terms of the ‘leaky bucket experiment’. He asked: how much loss can be justified in making a progressive transfer of a marginal unit of income from a donor to a recipient?

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The relative valuation of marginal changes in incomes at different locations in the income distribution then determines the acceptable loss. If the donor has income four times that of the recipient, then the three values of $e$ used by the US Census Bureau correspond to the donor’s income being valued at 0.71, 0.5 and 0.35 times that of the recipient. Alternatively, they imply assuming that the acceptable loss in social welfare of a progressive transfer can increase up to 29%, 50%, and 65% respectively, an illustration of the fact that the higher $e$, the higher the evaluator’s dislike for inequality.\footnote{Formally, a loss $l$ is socially acceptable up to the point at which $z(1-l) = 1$, where $z$ is the ratio of the income of the donor to that of the recipient.}

3. The Pigou–Dalton Principle of Transfers

Since the rediscovery of Dalton’s contribution around 1970, the literature on the measurement of inequality has been growing rapidly. Subsequent research took many directions, including the important extension of the welfare-based approach to poverty measurement (Sen, 1976a; Blackorby and Donaldson, 1980a; Clark et al., 1981; Foster et al., 1984; Atkinson, 1987; Foster and Shorrocks, 1988) and the development of multidimensional measures of inequality (Kolm, 1977; Atkinson and Bourguignon, 1982). It is beyond our scope to examine these various strands, and we simply refer to the many excellent surveys (Sen, 1973, 1997; Lambert, 1989; Cowell, 2000, 2011; Jenkins and Van Kerm, 2009). Here, we only focus on two issues that follow very directly from Dalton’s contribution: the transfer principle and, in the next Section, the distinction between relative and absolute indices.

Dalton considered a range of existing measures of inequality in the light of their welfare implications; the natural alternative is to specify the desired set of properties and to derive the implied measures. The Pigou–Dalton transfer principle is a central tenet of such an axiomatic approach. The idea that a mean-preserving income transfer from a richer person to someone poorer should decrease inequality is intuitively convincing, and several conceptual arguments can motivate its adoption (Cowell, 2003, p. xv). Yet, people differ in their views on inequality, and the transfer principle is not immune from disagreement. A striking finding of experiments conducted in Europe, Oceania, Israel and the US to elicit university students’ attitudes to inequality is that ‘there is a substantial body of opinion which rejects the principle in its pure form, although of these many were prepared to go along with the “borderline” view that a rich-to-poor transfer might leave inequality unchanged’ (Amiel and Cowell, 1999, p. 45).

Also statistical practice need not abide by the principle. The most popular measure of dispersion in labour economics is the variance of logarithms. Typically the ‘log-variance’ of earnings follows naturally from models of wage determination and lends itself to be nicely decomposed by population subgroups. However, this measure violates the transfer principle. These violations might not be too important in empirical applications, but Foster and Ok (1999, pp. 901, 907) argue that the variance of logarithms is ‘capable of making very serious errors’ and recommend completing its evidence with that derived from ‘one or more of the standard, well-behaved inequality measures.’ Another example is provided by the core indicator of income inequality

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used to monitor social cohesion in the European Union: the ‘income quintile share ratio (S80/S20)’, that is the ratio of total (equivalised) disposable income received by the top 20% of the population to that received by the bottom 20%. This measure of inequality is insensitive to any income transfer occurring within the bottom, middle or top income group – no different in this respect from the mean deviation criticised by Dalton.

Experimental evidence and statistical practice weaken the unanimous acceptance of the transfer principle and thus implicitly challenge the axiomatic approach, particularly because ‘there is in the literature no obvious alternative assumption to be invoked if the transfer principle were to be abandoned’ (Amiel and Cowell, 1999, p. 47). These considerations point to the need to explore a broader set of alternative axioms and to expand the range of measures that inequality scholars tend to regard as appropriate. There is room to allow for a greater plurality of ethical views, in theory as well as in practice.

4. Absolute and Relative Inequality Indices

While there is virtually unanimity about the transfer principle, theoretical research is more open as regards the choice between ‘scale invariance’ and ‘translation invariance’. With the former property, an inequality index is unaffected by an equal proportional change of all incomes; with the latter, it is instead left unchanged by equal additions to (or subtractions from) all incomes. This absolute criterion was imaginatively advocated by Kolm (1973, published as 1976, p. 419) as follows: ‘in May 1968 in France, radical students triggered a student upheaval which induced a workers’ general strike. All this was ended by the Grenelle agreements which decreed a 13% increase in all payrolls. Thus, labourers earning 80 pounds a month received 10 pounds more, whereas executives who already earned 800 pounds a month received 100 pounds more. The Radicals felt bitter and cheated; in their view, this widely increased income inequality’. Kolm’s example looks persuasive. Yet, much of its appeal fades away when we consider income reductions rather than increases. Atkinson cites the case of the sailors of the British Navy, Atlantic Fleet, at Invergordon, who in 1931 opposed a shilling a day reduction in their pay since ‘they did not regard it as fair that they should bear a bigger proportionate cut than the officers’ (Atkinson, 1983, p. 6).

The choice between relative and absolute criteria raises the more general question as to how the shape of the social welfare function (defined over incomes) changes as we move outwards, or inwards, in income space. One answer is given by the purely relative class (1). Another answer is given by the family of absolute measures proposed by Kolm (1976):

$$K = \frac{1}{k} \log \left[ \frac{1}{n} \sum e^{k(x - y_i)} \right],$$

(3)

where $k$ is a parameter capturing the degree of inequality aversion: the larger $k$, the more weight is attached to lower incomes; when $k$ tends to infinity, $K$ tends to the

7 The distinction between absolute and relative indices is clearly made by Dalton but he only notices that they differ in their measurement unit (currency unit versus real number).

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difference \((\mu - y_1)\). Unlike its relative counterpart \(I\), the index \(K\) expresses the cost of inequality in terms of the absolute amount of income that could be subtracted from the mean \(\mu\) without affecting the level of social welfare, that is, \(K = \mu - y_e\), where the value of \(y_e\) differs from the one for the index \(I\), as the underlying social welfare function is different (Blackorby and Donaldson, 1980b). When all incomes are increased by the same amount, \(K\) does not change; on the other hand, a proportional increase of all incomes leads to a rise of \(K\), while it would leave unaltered any relative measure. Kolm also proposed an ‘intermediate’ class of measures, which have the property of decreasing when all incomes are augmented by the same amount and of increasing when all incomes go up in the same proportion. Other intermediate measures of inequality have subsequently enriched the literature (Bossert and Pfingsten, 1990; Zoli, 1999; Zheng, 2004).

The contrast between relative and absolute measures is likely to be of particular significance in contexts where income differences are wide, such as in analyses of the world income distribution. Whether absolute, relative or intermediate, currently used measures of inequality impose tight constraints on how social marginal valuation varies with income. Yet, we may want to go beyond the standard pattern of declining sensitivity to transfers as incomes rise in order to allow for other patterns: for instance, one where sensitivity to transfers is first increasing and then decreasing, like with the Gini coefficient. Or we may want measures that blend the concern for poverty with that for inequality. As argued by Atkinson and Brandolini (2010), there is a case for considering more flexible measures.

5. Looking to the Future

Dalton (1920a, p. 361) ends his article with a plea for improvement in statistical information: ‘this paper may be compared to an essay in a few of the principles of brickmaking. But, until a greater abundance of straw is forthcoming, these principles cannot be put to the test of practice’. In this respect, the statistical agencies and the economics profession can be said to have responded magnificently since Dalton’s time. The quantity and quality of data on the distribution of income has improved out of all recognition. Indeed, after the revival of interest in measuring inequality at the end of the 1960s, there has been an explosion in the availability of unit record data. In many countries, measures of inequality can be derived, with a lag of a year or so, from data on individual incomes derived from household surveys or from administrative records. We even today know more about the UK at the time when Dalton was writing as a result of historical studies using income tax data: the share of the top 1% in total gross income in 1919 was 19.6% (Atkinson, 2005).

Yet there is still some distance to go in empirical implementation. In particular, the data on income distribution need to be integrated into the macroeconomic indicators of economic performance. The social welfare function approach that has built on Dalton (1920a) provides the basis for such integration via the calculation of equally distributed equivalent income, or distributionally adjusted national income (Jenkins, 1997). Or, if one wishes to employ the Gini coefficient, then we have the real national income of Sen (1976b), equal to national income times \(1 - G\),
where $G$ is the Gini coefficient. This in turn requires an investment in more timely distributional data.

In theoretical terms, we highlight the issues raised by the application of inequality measurement to variables other than income, as is inherent in multi-dimensional indicators of inequality. The title of Dalton’s article referred deliberately to the ‘inequality of incomes’. He did draw a parallel with the inequality of rainfall (1920a, p. 348) but he was very clear that he was considering measures applicable to income, not to other economic variables, and certainly not to the host of different variables evoked by Gini in his comment (see earlier quotation). Even the move to considering inequality of wealth means that we have to think seriously about negative levels. When inequality measures are applied to other dimensions, such as health status, then we have to return to the underlying assumptions and ask how far they are applicable to the new variable. It may, for example, be that absolute measures are more appropriate than relative in this case, while it is not clear how the transfer principle can be applied to health status.

Dalton invited social scientists to reflect on the welfare meaning of the measures of inequality used to study income distribution. He did so from the utilitarian perspective but the meaning of his intuition was broad and amenable to reinterpretation by people with different views about distributive justice. It took half a century and substantial advancements in neighbouring fields, such as social choice theory and the theory of decision under risk, before his seminal contribution could grow into a fertile research field. After roughly another half a century, the theory of inequality measurement is still attracting considerable attention. Indeed, the enduring legacy of Dalton is the admonition to think seriously about the implications of our measurement tools, especially in a field where the assessment of objective differences is inherently intertwined with our normative views.

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Appendix A. Dalton, H. (1920). ‘The measurement of the inequality of incomes’, Economic Journal, vol. 30(119), pp. 348–61.
THE MEASUREMENT OF THE INEQUALITY OF INCOMES

1. It is generally agreed that, other things being equal, a considerable reduction in the inequality of incomes found in most modern communities would be desirable. But it is not generally agreed how this inequality should be measured. The problem of the measurement of the inequality of incomes has not been much considered by English economists. It has attracted rather more attention in America, but it is in Italy that it has hitherto been most fully discussed. The importance of the problem has been obscured by the inadequacy of the available statistics of the distribution of income in all modern communities. To such statistics as we have, no very fine measures can be applied. The improvement of these statistics is the business of statisticians, but the problem of measuring and comparing the inequalities, which improved statistics would more precisely reveal, should be capable of theoretical solution now. No complete solution is presented in this paper, but only a discussion of certain points of principle and method.

2. First, as to the nature of the problem. An American writer has expressed the view that "the statistical problem before the economist in determining upon a measure of the inequality in the distribution of wealth is identical with that of the biologist in determining upon a measure of the inequality in the distribution of any physical characteristic." But this is clearly wrong. For the economist is primarily interested, not in the distribution of income as such, but in the effects of the distribution of income upon the distribution and total amount of economic welfare, which may be derived from income. We have to deal, therefore, not merely with one variable, but with two, or possibly more, between which certain functional relations may be presumed to exist.

A partial analogy would be found in the problem of measuring the inequality of rainfall in the various districts of a large agricultural area. From the point of view of the cultivator, what is

1 Persons, Quarterly Journal of Economics, 1908–9, p. 431.
important is not rainfall as such, but the effects of rainfall upon the crop which may be raised from the land. Between rainfall and crop there will be a certain relation, the discovery of which will be a matter of practical importance. The objection to great inequality of rainfall is the resulting loss of potential crop. The objection to great inequality of incomes is the resulting loss of potential economic welfare.

Let us assume, as is reasonable in a preliminary discussion, that the economic welfare of different persons is additive, that the relation of income to economic welfare is the same for all members of the community, and that, for each individual, marginal economic welfare diminishes as income increases. Then, if a given income is to be distributed among a given number of persons, it is evident that economic welfare will be a maximum, when all incomes are equal. It follows that the inequality of any given distribution may conveniently be defined as the ratio of the total economic welfare attainable under an equal distribution to the total economic welfare attained under the given distribution. This ratio is equal to unity for an equal distribution, and is greater than unity for all unequal distributions. It may, therefore, be preferred to define inequality as this ratio minus unity, but for comparative purposes this modification of the definition is unnecessary. Inequality, however, though it may be defined in terms of economic welfare, must be measured in terms of income.

3. Starting from the above definition, it is clear that, if we assume any precise functional relation between income and economic welfare, we can deduce a corresponding measure of inequality. It is also clear that, under this procedure, no one measure of inequality will emerge, whose appropriateness will be independent of the particular functional relation assumed.

The procedure suggested may be illustrated by two examples. Take, first, the hypothesis that proportionate additions to income, in excess of that required for “bare subsistence,” make equal additions to economic welfare. This is Bernoulli’s hypothesis, except that economic welfare is substituted for satisfaction.¹

Then, if \( w = \) economic welfare and \( x = \) income, we have—

\[
dw = \frac{dx}{x}
\]

or

\[
w = \log x + c.
\]

If \( x_1, x_2, \ldots x_n \) are individual incomes, whose arithmetic mean

¹ A discussion of the distinction, if any, between economic welfare and satisfaction lies outside the scope of this paper.
is $x_a$ and geometric mean $x_g$, the corresponding measure of inequality is, by our definition—

$$\frac{n \log x_a + nc}{n \log x_g + nc} = \frac{\log x_a + c}{\log x_g + c}.$$

If we assume that, when $x=1$, $w=0$, then $c=0$, and our measure of inequality becomes $\frac{\log x_a}{\log x_g}$. It may, at first sight, be thought that a still simpler, and practically equivalent, measure will be $\frac{x_a}{x_g}$, but this simplification raises a question to which further reference will be made below.

The above hypothesis, however, is not satisfactory. Apart from the difficulty that only income in excess of that required for “bare subsistence” is taken into account, it is clear that too rapid a rate of increase of economic welfare is assumed, when income becomes large. After a certain point it is pretty obvious that more than proportionate additions to income will generally be required, in order to make equal additions to economic welfare. To be even tolerably realistic, a formula connecting income with economic welfare should satisfy the following conditions.

(1) Equal increases in economic welfare, at any rate after income is greater than a certain amount, should correspond to more than proportionate increases in income; (2) economic welfare should tend to a finite limit, as income increases indefinitely; (3) economic welfare should be zero for a certain amount of income, and negative for smaller amounts. These conditions are satisfied, if we assume that the relation of economic welfare to income is of the form $dw = \frac{dx}{x^2}$, so that $w = c - \frac{1}{x}$, where $c$ is a constant. For then, however large $x$ becomes, $w$ can never become larger than $c$, and, when $x$ is less than $\frac{1}{c}$, $w$ is negative. If we adopt this formula, which appears to be a good compromise of its kind between plausibility and simplicity, the corresponding measure of inequality is—

$$\frac{nc - \frac{n}{x_a}}{nc - \frac{n}{x_h}} = \frac{\frac{c}{x_a}}{\frac{c}{x_h}},$$

where $x_h$ is the harmonic mean of the individual incomes, and

1 If it were practicable to fix a unit of economic welfare, it would have to be fixed, in relation to the unit of income, so that both these attributes of $c$ would hold good. There is no theoretical objection to this.
c, as already stated, the reciprocal of the minimum income, which yields positive economic welfare.

Both the measures of inequality obtained above are simple in form and have a certain theoretical elegance. But neither is readily applicable to statistics. The arithmetic mean is, indeed, easily calculated from perfect statistics, and fairly easily approximated to from imperfect statistics, but the corresponding calculations for the geometric and harmonic means are very laborious, when the number of individual incomes is large, and the corresponding approximations, especially for the harmonic mean, are practically impossible, where the statistics show more than a small degree of imperfection. The first of the two measures, moreover, involves an estimate of the income necessary for "bare subsistence," and the second an estimate of the minimum income which yields positive economic welfare. And neither of these estimates are easily made. Nor, of course, have we really any precise knowledge of the functional relation between income and economic welfare.

4. Failing such precise knowledge, we may still lay down certain general principles, which shall serve as tests, to which various plausible measures of inequality may be submitted. We have, first, what may be called the principle of transfers. Maintaining the assumptions laid down in Section 2 above, we may safely say that, if there are only two income-receivers, and a transfer of income takes place from the richer to the poorer, inequality is diminished.¹ There is, indeed, an obvious limiting condition. For the transfer must not be so large, as more than to reverse the relative positions of the two income-receivers, and it will produce its maximum result, that is to say, create equality, when it is equal to half the difference between the two incomes. And we may safely go further and say that, however great the number of income-receivers and whatever the amount of their incomes, any transfer between any two of them, or, in general, any series of such transfers, subject to the above condition, will diminish inequality.² It is possible that, in comparing two distributions, in which both the total income and the number of income-receivers are the same, we may see that one might be able to be evolved from the other by means of a series of transfers

¹ Compare Pigou, Wealth and Welfare, p. 24.
² Inequality is certain to be diminished by a series of transfers such that all transfers from A, the richer, to B, the poorer, still leave A richer than, or just as rich as, B. But if some of the transfers make B richer than A, it is possible that the effects of the series of transfers might cancel out and leave the inequality the same as before.
of this kind. In such a case we could say with certainty that the inequality of the one was less than that of the other.

5. Let us now apply the principle of transfers to various measures of dispersion used by statisticians for measuring inequality in general. A distinction may be drawn between measures of relative dispersion and measures of absolute dispersion. Measures of relative dispersion will be simply numbers, while measures of absolute dispersion will be, in the present case, numbers of units of income. Most of the general measures of dispersion proposed by statisticians are measures of absolute dispersion, but are easily transformed into measures of relative dispersion, when divided by an appropriate divisor.

Consider first the mean deviation from the arithmetic mean. This measure is the sum of two parts, one of which comprises the deviations above, the other the deviations below, the mean.\(^1\) It is a bad measure, judged by the principle of transfers, for it is unaffected by transfers within either part, provided that no income previously above the mean is reduced below it, and conversely. The transfer of a given sum from incomes above the mean to incomes below it, as, for example, by the provision of old age pensions for persons of small incomes from the proceeds of a tax on large incomes, would obviously reduce the mean deviation. But it would be unaffected, if such pensions were provided by a tax levied on those whose incomes were just below the mean, or if additional comforts for millionaires were provided from a tax on those whose incomes were just above the mean, provided that none of the latter were reduced below the mean by the tax.

The mean deviation is a measure of absolute dispersion. If we divide it by the arithmetic mean, we obtain what we may call the relative mean deviation, which is equally insensitive to transfers wholly above or wholly below the mean.

Consider next the standard, or mean square, deviation from the arithmetic mean, i.e., the square root of the arithmetic average of the squares of deviations from the arithmetic mean. The standard deviation is perfectly sensitive to transfers,\(^2\) and thus passes our first test with distinction. Dividing the standard deviation by the arithmetic mean yields the coefficient of variation, which we may call the relative standard deviation.

\(^1\) Thus if \(S_1\) is the sum of the deviations of incomes greater than the mean and \(S_2\) the sum of the deviations of incomes less than the mean, the mean deviation \(= \frac{1}{n}(S_1 + S_2)\), where \(n\) is the total number of incomes.

\(^2\) For, if \(\delta\) be the initial standard deviation of any distribution of \(n\) incomes, and \(\delta'\) the standard deviation after an amount \(h\) has been transferred from an income \(x_1\) to an income \(x_2\), all other incomes remaining the same, we have \(n(\delta'^2 - \delta^2) = 2h(x_1 - x_2) - 2h^2\). Therefore \(\delta = \delta'\), only if \(h = 0\) or if \(h = x_1 - x_2\).
deviation by the arithmetic mean, we obtain what may be called the relative standard deviation. This, too, is perfectly sensitive to transfers.

Consider next Professor Bowley's quartile measure of dispersion, \( \frac{Q_3 - Q_1}{Q_3 + Q_1} \), where \( Q_1 \) and \( Q_3 \) are quartiles.\(^1\) This is a measure of relative dispersion. It is sensitive to transfers, in so far as these involve movements of the quartiles, but not otherwise. In this respect it is somewhat more sensitive than the mean deviation, but much less sensitive than the standard deviation.

An interesting measure of dispersion, which has not, I think, hitherto attracted the attention of English writers, is Professor Gini's mean difference, which, as applied to incomes, is the arithmetic average of the differences, taken positively, between all possible pairs of incomes.\(^2\) It may be shown that this mean difference is equal to the weighted arithmetic mean of deviations from the median, the weights being proportionate to the number of incomes, increased by one, which are intermediate in size between the median and the income whose deviation is being considered.\(^3\) The mean difference, thus defined, is a measure of absolute dispersion. Dividing it by the arithmetic mean, we obtain a measure of relative dispersion, which may be called the relative mean difference. The mean difference, whether absolute or relative, is perfectly sensitive to transfers.

Another interesting measure of inequality is based upon what some writers have called a Lorenz curve.\(^4\) (See next page.) This is a simple and convenient graphical method of exhibiting any distribution of income, provided that our interest is confined to proportions, rather than absolute amounts, both of total income and of the number of income-receivers.

Along the axis \( Ox \) are measured percentages of the total income, and along the axis \( Oy \) the minimum percentages of the total number of income-receivers, who receive various percentages of the total income. For example, if the richest 20 per cent. of the population received 75 per cent. of the total income, this fact would determine one point \((x=75, y=20)\), upon the Lorenz curve. A perfectly equal distribution

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\(^1\) Compare Bowley, Elements of Statistics, p. 136.
\(^2\) See, for a discussion of this measure, Gini, Variabilità e Mutabilità.
\(^3\) Gini, ibid., pp. 32–33.
\(^4\) Originally proposed by Mr. M. O. Lorenz, Publications of the American Statistical Association (1907), Vol. ix, pp. 209 ff. M. Stéailles also recommended it, apparently independently, in 1910 in his Répartition des Fortunes en France, pp. 56–7. Sir Leo Chiozza Money had already given hints of this measure in his Riches and Poverty, first published in 1905.
would be represented by the straight line $OP$ inclined at an angle of $45^\circ$ to either axis. An unequal distribution would be represented by a curve, such as $OQP$, lying below the line $OP$. If $MP$ is perpendicular to $OM$, $OM = MP = 100$, and an obvious measure of inequality is the area enclosed between the Lorenz curve and the line of equal distribution $OP$. The larger this area, the larger the inequality.

A remarkable relation has been established between this measure of inequality and the relative mean difference, the former measure being always equal to half the latter.\(^1\)

Something will be said below concerning Professor Pareto's well-known measure of the inequality of incomes. But this measure cannot be tested, with reference to the principle of transfers, since it is based upon a supposed law, according to which, if the total income and the number of income-receivers are known, the distribution is uniquely determined.

6. So far, then, as tested by the principle of transfers, the standard deviation, whether absolute or relative, and the mean difference, whether absolute or relative, are good measures; Professor Bowley's quartile measure is a very indifferent measure; the mean deviation, whether absolute or relative, is a bad measure; and Professor Pareto's measure evades judgment. But the scope of the principle of transfers, as a test of measures of inequality, is narrowly limited. It can only be applied to some cases—and by no means to all—in which both the total income and the number of income-receivers are constant, and distribution varies.\(^2\)

\(^1\) For a mathematical proof of this see Ricci, *L'Indice di Variabilità*, pp. 22–24. The proof was first given, apparently, by Professor Gini. Another most elegant proposition, due to Professor Ricci (*ibid.*, pp. 32–33), is that, if any straight line be drawn parallel to the line of equal distribution, then all the Lorenz curves, to which this straight line is a tangent, represent distributions having the same relative mean deviation.

\(^2\) Professor Pigou (*Wealth and Welfare*, p. 25 n.) uses the following argument to prove that, in these circumstances, a reduction in the standard deviation will probably increase aggregate satisfaction. "If $A$ be the mean income and
It cannot be applied when either the total income or the number of income-receivers varies, or when both vary simultaneously. For these more general cases further tests are required, and three general principles suggest themselves as serviceable for this purpose.

7. We have, first, what may be called the principle of proportionate additions to incomes. It is sometimes suggested that proportionate additions to, or subtractions from, all incomes will leave inequality unaffected. But, if the definition of inequality given above be accepted, this is not so. It appears, rather, that proportionate additions to all incomes diminish inequality, and that proportionate subtractions increase it. This is the principle of proportionate additions to incomes just referred to. A general proof of this principle presents difficulties, and is not attempted here, but the proof in two important special cases is easy. For, first, assume, using the same notation as in Section 3 above, that the relation of income to economic welfare is \( w = \log x \). Then, if \( \delta \) be the inequality of any given distribution, we have

\[
\delta = \frac{\log x_a}{\log x_g}
\]

Let all incomes be multiplied by \( \theta \) and let \( \delta' \) be the inequality of the new distribution.

\[
a, a, \ldots \text{ deviations from the mean, aggregate satisfaction, on our assumption}
\]

\[
= n f(A) + (a_1 + a_2 + \ldots) f' + \frac{1}{2!} (a_1^2 + a_2^2 + \ldots) f'' + \frac{1}{3!} (a_1^3 + a_2^3 + \ldots) f''' + \ldots
\]

But we know that \( a_1 + a_2 + \ldots = 0 \). We know nothing to suggest whether the sum of the terms beyond the third is positive or negative. If, therefore, the third and following terms are small relatively to the second term, it is certain, and, in general, it is probable that aggregate satisfaction is larger, the smaller is \( (a_1^2 + a_2^2 + \ldots) \). This latter sum, of course, varies in the same sense as the \( \ldots \) standard deviation." This argument would be strong, if all deviations were small, i.e. if inequality were already very small. But when, as is the case in all important modern communities, a number of the deviations are very large, it is quite likely that successive terms in the expansion will go on increasing (numerically) for some time, and this is specially likely as regards the series of alternate terms, which involve deviations raised to even powers. This likelihood will vary according to the form of the function \( f \), but it seems clear that the third and following terms cannot, in general, be neglected. It follows that, in general, there is no certainty and only a somewhat low and problematical degree of probability, that a reduction in the standard deviation will increase satisfaction. There is no reason to suppose that it is not at least equally probable that a reduction in certain other measures of dispersion will have the same effect. One good test of the relative appropriateness of various measures of the inequality of incomes would be the relative probability that a reduction in such measures would increase economic welfare (or satisfaction), on the assumption that both the total income and the number of income receivers were constants. But the evaluation of such relative probabilities presents difficulties.

1 See, e.g., Taussig, Principles of Economics, II, p. 485.
Then \( \delta' = \frac{\log \theta + \log x_a}{\log \theta + \log x_g} \), and, since \( x_a > x_g \), we have \( \delta > \delta' \), if \( \log \theta > 0 \), that is to say, if \( \theta > 1 \).

Similarly, \( \delta < \delta' \), if \( \theta < 1 \).

That is to say, proportionate additions to all incomes diminish inequality and proportionate subtractions increase it.\(^1\) This is true, if \( x \) is the total income of any individual. \( A \) fortiori, it is true, if \( x \) is surplus income in excess of "bare subsistence." For equal proportionate additions to surplus income involve larger proportionate additions to total income, when the latter is large, than when it is small. A series of transfers from richer to poorer will, therefore, transform proportionate additions to surplus incomes into proportionate additions to total incomes.

Next assume that the relation of income to economic welfare is \( w = c - \frac{1}{x} \). Then, if \( \delta \) be the inequality of any given distribution, we have \( \delta = \frac{c - \frac{1}{x_a}}{c - \frac{1}{x_h}} \).

Let all incomes be multiplied by \( \theta \) and let \( \delta' \) be the inequality of the new distribution.

Then \( \delta' = \frac{c - \frac{1}{\theta x_a}}{c - \frac{1}{\theta x_h}} \), and we have \( \delta > \delta' \), if \( (x_a - x_h)(\theta - 1) > 0 \).

But \( x_a > x_h \). \( \therefore \delta > \delta' \), if \( \theta > 1 \).

Similarly, \( \delta < \delta' \), if \( \theta < 1 \).

That is to say, proportionate additions to all incomes diminish inequality, and proportionate subtractions increase it.

8. If the principle of proportionate additions to incomes thus enunciated be provisionally accepted as true generally,\(^2\) and not merely for the particular hypotheses just examined, a second principle follows as a corollary. This may be called the principle of equal additions to incomes, and is to the effect that equal additions to all incomes diminish inequality and equal subtractions increase it. Here, again, a direct general proof presents

\(^1\) If we write \( \delta = x_a/x_p \), instead of \( \delta = \log x_a/\log x_p \), proportionate additions or subtractions will leave inequality unaffected. It follows that \( x_a/x_p \) is not a mere simplification of the measure \( \log x_a/\log x_p \) arrived at in section 3 above, but is a distinct, and inferior, measure.

\(^2\) The additions must, of course, be genuine. Inequality in this country would not be diminished by reckoning everyone's income in shillings, instead of in pounds. Units of money income in any two cases to be compared must have approximately equal purchasing power.
difficulties, though several writers have regarded the principle as so obvious that no proof is required. But as a corollary of the preceding principle the proof is easy. For, let the total additional income involved in proportionate additions to all incomes be redistributed among income-receivers in such a way as to make equal, instead of proportionate, additions to all incomes. Then the addition to maximum economic welfare attainable is the same in both cases. But the addition to economic welfare actually attained is obviously greater, when additions to incomes are equal, than when they are proportionate. Therefore, inequality is smaller after equal additions have been made than after proportionate additions have been made, the total additional income being the same in both cases. But proportionate additions reduce inequality. Therefore, a fortiori, equal additions reduce inequality. 

9. The third principle may be called the principle of proportionate additions to persons, and is to the effect that inequality is unaffected if proportionate additions are made to the number of persons receiving incomes of any given amount. This, again, is easily proved. For the maximum economic welfare attainable and the economic welfare actually attained will both have been increased in the same proportion, and hence their ratio will be unaltered.

10. We may now test, by means of these three principles, the measures of inequality which have already been tested by means of the principle of transfers. Simple mathematical operations yield the following results:

| Effect of | Proportionate Additions to Incomes | Equal Additions to Incomes | Proportionate Additions to Persons |
|-----------|-----------------------------------|---------------------------|----------------------------------|
| Absolute Mean Deviation | Increased | Unchanged | Unchanged |
| Relative Mean Deviation | Unchanged | Diminished | Unchanged |
| Absolute Standard Deviation | Increased | Unchanged | Unchanged |
| Relative Standard Deviation | Unchanged | Diminished | Unchanged |
| Bowley's Quartile Measure | Unchanged | Diminished | Unchanged |
| Absolute Mean Difference | Increased | Unchanged | Unchanged |
| Relative Mean Difference | Unchanged | Diminished | Unchanged |

Here the three absolute measures of dispersion give one set of identical results, and the four relative measures another. None

1 "An equal addition to everyone's income ... obviously makes incomes more equal than they would otherwise be." Cannan, *Elementary Political Economy*, p. 137. See also Loria, *La Sintesi Economica*, p. 369.

2 Or alternatively, the total additional income being given, a distribution involving equal additions to all incomes may be evolved from a distribution involving proportionate additions to all incomes by means of a series of transfers from richer to poorer.
of the seven measures pass the test of proportionate additions to incomes, but the relative measures come nearer to doing so than the absolute measures.¹ The relative measures pass the test of equal additions to incomes, but the absolute measures do not. All seven measures pass the test of proportionate additions to persons. We may therefore eliminate the three absolute measures from further consideration. As between the four relative measures, the order of merit established by reference to the principle of transfers may stand, so far, unchanged, viz.:

1 and 2. Relative standard deviation and relative mean difference (bracketed equal).
3. Bowley’s quartile measure.
4. Relative mean deviation.

11. Can Professor Pareto’s measure be brought into this order of merit? This is a relative measure, which is only applicable when distribution is approximately of the form \( y = \frac{A}{x^a} \), where \( x \) is any income, \( y \) the number of incomes greater than \( x \), and \( A \) and \( a \) constants for any given distribution, but variables for different distributions.² Assuming this formula for distribution, which, as Professor Bowley has shown,³ is the same thing as assuming that the average of all incomes greater than \( x \) is proportional to \( x \), Professor Pareto treats \( a \) as the measure of inequality, in the sense that, the greater \( a \), the greater inequality. It follows mathematically that “neither an increase in the minimum income nor a diminution in the inequality of incomes can come about, except when the total income increases more rapidly than the population.”⁴ In other words, increased production per head is both a necessary condition and a sufficient guarantee of a diminution of inequality.

Professor Pareto’s law, about which much has been written both by way of criticism and of qualified appreciation, implies a uniformity in distribution, which makes it impossible to apply either the principle of transfers or the principle of equal additions to incomes. Like the four other measures just considered, it is

¹ It should be noticed that, if we are comparing the inequality of two distributions by means of a measure which is unchanged by proportionate additions to incomes, it is not necessary that the unit of money income in the two distributions should have approximately the same purchasing power.
² Compare Pareto, Cours d’Economie Politique, II, pp. 305 ff, and Manuel d’Economie Politique, pp. 391 ff.
³ Measurement of Social Phenomena, p. 106.
⁴ Cours, II, pp. 320–1.
unchanged both by proportionate additions to incomes and by proportionate additions to persons. It has been suggested that this measure, where it is applicable, will be in general accord with other plausible measures of dispersion. But, in view of the investigations of Italian economists, this is very doubtful. It seems on the whole more likely, though the question requires further study, that, in order to bring it into general accord with other measures, the Pareto measure should be inverted, so that, the greater \( a \), the smaller inequality. But such an inversion will explode Professor Pareto's alleged economic harmonies, and it will follow, according to his law, that increased production per head will always mean increased inequality!

According to Professor Gini, many actual distributions of income approximate to the formula

\[
n = \frac{1}{c} s^\delta, \text{ or } \log n = \delta \log s - \log c,
\]

where \( s \) is the total income of the \( n \) richest income-receivers and \( \delta \) and \( c \) are constants for any given distribution. He proposes \( \delta \) as a measure of inequality, or "index of concentration," as he prefers to call it, such that, the greater \( \delta \), the greater inequality. This formula is a more convenient variant of Professor Pareto's, such that \( \delta = \frac{a}{a - 1} \), and, as \( a \) diminishes from any quantity greater than one down to one, \( \delta \) increases up to infinity.

The equation \( \log n = \delta \log s - \log c \) is easily transformed into that of a Lorenz curve. For, if \( N \) is the total number of income-receivers and \( S \) the total income, we have

\[
\log N = \delta \log S - \log c.
\]

\[
\text{or } \log \frac{n}{N} = \delta \log \frac{s}{S}.
\]

Putting \( \frac{n}{N} = \frac{y}{100} \) and \( \frac{s}{S} = \frac{x}{100} \), we have the equation of a Lorenz curve,

\[
\log \frac{y}{100} = \delta \log \frac{x}{100}
\]

or

\[
\frac{y}{100} = \left( \frac{x}{100} \right)^{\delta}.
\]

1 See, e.g., Pigou, *Wealth and Welfare*, pp. 25 and 72.
2 See Bresciani, *Giornale degli Economisti*, August 1905, pp. 117-8, and January 1907, pp. 27-8. Ricci, *L'Indice di Variabilità*, pp. 43-6, Gini, *Variabilità* p. 65 and pp. 70-1. Compare also Persons, *Quarterly Journal of Economics*, 1908-9, pp. 420-1, and Benini, *Principii di Statistica Metodologica*, p. 187. Professor Benini inverts Professor Pareto's measure, but apparently without realising that he has done so.
3 *Ibid.*, pp. 72 ff.
The area enclosed between this Lorenz curve and the line of equal distribution is—

\[
\begin{align*}
    & \Delta = (100)^2 \int_0^{100} \frac{x}{(100)^3} \, dx \\
    & = (100)^2 \left( \frac{1}{\delta + 1} \right).
\end{align*}
\]

Thus, the greater \( \delta \), the larger is the above area, and the larger the relative mean difference.\(^1\) There is thus some ground for believing, though I do not here definitely commit myself to the belief, that the reciprocal of Professor Pareto's measure is a mere variant of the relative mean difference, in the particular case, when distribution is approximately according to Pareto's law. In this particular case, then, Professor Pareto's measure would have no independent significance, and, in the more general case, when distribution may depart widely from Pareto's law, the measure has, of course, no general significance at all. It will, therefore, be provisionally set aside in this discussion.

12. Returning to the four measures set out in order of merit at the end of Section 10, this order is based on theoretical advantages. But account must also be taken of practical applicability to statistics. Both the relative mean deviation and the quartile measure are more easily applicable than either of their two rivals to perfect statistics, and applicable, with less risk of serious error, to imperfect statistics. As regards perfect, or nearly perfect, statistics, the advantage of the former pair over the latter relates only to laboriousness and not to accuracy, and is not, therefore, a matter of great importance. But, as regards markedly imperfect statistics, such as are actually available, the advantage relates to accuracy as well as to laboriousness and is, therefore, vital.

The provisional conclusion which suggests itself, is as follows. When statistics are so imperfect, that neither the relative standard deviation nor the relative mean difference can be applied with any expectation of reasonable accuracy, we must make shift with the relative mean deviation and the quartile measure. It is some palliation of the comparative insensitiveness

\(^1\) This index \( \delta \) has been used by several Italian writers in enquiries into distributions of income. See, e.g., Savorgnan, La Distribuzione dei Redditi nelle Provincie e nelle Grandi Città dell' Austria, and Porru, La Concentrazione della Ricchezza nelle Diverse Regioni d'Italia.
to transfers, which is a defect of both the latter measures, that each is sensitive to many possible transfers, to which the other is insensitive. If, therefore, both give the same result in any particular comparison, their evidence is to some extent corroborative.

If statistics are so far improved that the relative standard deviation and the relative mean difference are applicable, these are to be preferred to the two measures just mentioned. If a single measure is to be used, the relative mean difference is, perhaps, slightly preferable, owing to the graphical convenience of the Lorenz curve. Probably, however, it will be desirable, at any rate for some time to come, not to rely upon the evidence of a single measure, but upon the corroboration of several. Given perfect, or nearly perfect, statistics, it is worth while considering whether corroboration may not also be sought from the measure \[ \log \frac{x_a}{x_y} \], applied, for the sake of simplicity, to total incomes, and not to surplus incomes in excess of the requirements of "bare subsistence." For this measure passes our test of proportionate additions to incomes, which none of the other four survivors do. In most practical cases, no doubt, these five measures will give results pointing in the same direction, but in some cases they may not do so.

Meanwhile, the chief practical necessity is the improvement of existing statistical information, especially as regards the smaller incomes. This paper may be compared to an essay in a few of the principles of brickmaking. But, until a greater abundance of straw is forthcoming, these principles cannot be put to the test of practice.

Hugh Dalton

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