Optimization of Amplitude and Frequency Modulated Magnetic Field Parameters in a Square Mold Wall

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Abstract
The possibility of optimization of parameters of amplitude and frequency modulated rotating magnetic field (AFM RMF) acting on the copper wall of a mold with a square cross-section aimed at an increase of the efficiency of electromagnetic impact on the mold in the process of continuous casting of a steel billet has been theoretically substantiated.

Key words: continuous casting of steel billet, mold, rotating magnetic field, amplitude and frequency modulation, resonance.

Introduction
To improve the continuous ingot quality, a variety of methods of action on liquid metal are used and constantly modernized [1]. In the presented study, we analyze the possibility of optimizing the non-stationary electromagnetic forces excited in the wall of the crystallizer by means of an amplitude-frequency modulated rotating magnetic field (AFM RMF) [2], which ensures a maximal amplitude of electro-magnetic fields (EMF) in it. The authors suggest a method of optimization of non-stationary electro-magnetic forces excited in the mold wall using AFM RMF. The conditions for the resonant energy transfer by a part of the Fourier-pack harmonics to the mold wall are determined.

Problem Formulation and Solution
A problem of excitation of oscillations in the wall of a square copper mold using an AFM RMF source is examined for the optimization (determining the maximal amplitude of the vectorial potential) of this influence. Electrodynamic processes in the mold wall arising under the action of a circularly polarized AFM RMF excited by an ideal inductor are described in rectangular coordinates \((x,y,z)\) by a dimensionless equation for the \(z\) – component of the vectorial potential \(a_z\) \((\mathbf{b} = \text{rot} \mathbf{a})\)

\[
\frac{1}{\pi^2} \left( \frac{\partial^2 a_z}{\partial x^2} + \frac{\partial^2 a_z}{\partial y^2} \right) - \omega_0 \frac{\partial a_z}{\partial t} = U(a_z) = 0,
\]

(1)

where \(\omega_0 = \frac{\mu_0 \sigma m k^2}{\pi^2}\) – dimensionless frequency of EMF in a mold wall with specific electrical conductivity \(\sigma_m\) and with the half of external mold side length \(x_0, \mu_0\) – magnetic permeability in the vacuum, \(\omega_0\) – carrier angular frequency of AFM RMF, \(t\) – dimensionless time.

Boundary conditions for Equation (1) are determined in the first approximation by the magnetic induction \(\mathbf{b}\) of the RMF excited by an ideal inductor in whose circular boring the mold is located (Fig.1).

In rectangular Cartesian coordinates, these conditions have the following forms:

\[
\begin{align*}
\frac{\partial a_z}{\partial x} \Bigg|_{x=1} &= \frac{(x - i y) \phi(t)}{\sqrt{1 + x^2}}, & \frac{\partial a_z}{\partial x} \Bigg|_{x=x_0} &= \frac{(x - i y) \phi(t)}{\sqrt{x_0^2 + y^2}} e^{-d} \\
\frac{\partial a_z}{\partial y} \Bigg|_{y=1} &= \frac{(x - i y) \phi(t)}{\sqrt{1 + x^2}}, & \frac{\partial a_z}{\partial y} \Bigg|_{y=-y_1} &= \frac{(x + i y_1) \phi(t)}{\sqrt{x^2 + y_1^2}} e^{-d}
\end{align*}
\]

(2)

(3)
where \( d = \frac{\langle \delta \rangle}{2 \rho} \), \( \langle \delta \rangle \) is a dimensional averaged mold wall thickness which is determined by the following formula:

\[
\langle \delta \rangle = \frac{\delta_0}{2} \int_0^1 \sqrt{1 + y^2} \, dy = 1.148 \delta_0
\]

\( \delta_0 \) – dimensional thickness of the mold wall, \( \Delta p = \sqrt{\frac{\mu_0 \sigma_m \omega_0}{\rho \rho_0}} \) – depth of magnetic field penetration in copper mold.

Fig.1: Scheme for calculation of electrodynamical processes in a square copper mold section; \( x \) and \( y \) are dimensionless Cartesian coordinates.

According to [3], the function of time \( \phi(t) \) can be presented in the following form:

\[
\phi(t) = (1 + xe^{ib_0 a t}) \sum_{n=-\infty}^{\infty} J_n(\xi) e^{i \pi n \Omega n t}
\]

where \( \chi \) is the depth of amplitude modulation, \( \omega_a \) – dimensionless angular velocity of amplitude modulation, \( J_n(\xi) \) – Bessel function of the \( n \) order, \( \Omega n \) - dimensionless angular velocity of the \( n \)-th Fourier harmonic, \( \xi = \frac{\Delta \omega}{\omega_f} \) – frequency modulation index, \( \Delta \omega \) – angular frequency deviation, \( \omega_f \) – angular frequency of frequency modulation.

To solve the problem (1) with boundary conditions (2), we represent vectorial potential \( \alpha_z \) in the form

\[
\alpha_z = \alpha_z - F(x, y) \phi(t)
\]

where

\[
F(x, y) = \iint_{x_1-y_1}^{x_1+y_1} \frac{(x-iy)}{\sqrt{x^2 + y^2}} (e^{-f_1(x)} + e^{-f_1(y)}) \, dx \, dy;
\]

\[
f_1(x) = \frac{1-x}{1-x_1} d; \quad f_1^*(y) = \frac{1-y}{1+y_1} d.
\]

The transformation (5) reforms a semi-uniform problem (1) into the following semi-uniform problem:

\[
U(\alpha_z) = U[F(x, y) \phi(t)], \quad \frac{\partial \alpha_z}{\partial x} \bigg|_{L_0} = \frac{\partial \alpha_z}{\partial y} \bigg|_{L_1} = 0
\]

where \( L_0, L_1 \) are external and internal contours of the mold cross-section (Fig. 1).
We look for the solution of the problem (5) by Galerkin’s method in the form

\[ \alpha_x = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} T_{kl}(t) u_k(x) v_l(y) \]  

where \( u_k(x) = \sin \left( k\pi \frac{1-x}{x_1 - \delta} \right) \), \( v_l(y) = \sin \left( l\pi \frac{1-y}{y_1 - \delta} \right) \), \( x_1 = 1 - \delta \), \( y_1 = 1 - \delta \), \( \delta = \frac{\delta}{x_0} \).

Substituting (7) into (6) and accomplishing the procedures of Galerkin’s method, we obtain

\[ \frac{1}{\pi} \frac{\partial T_{kl}}{\partial t} + P_{kl} T_{kl} = \frac{P_{KL}(\text{Int}_1 - i\text{Int}_2)\phi(t) - (\text{Int}_3 - i\text{Int}_4) \frac{1}{\pi} \frac{\partial \phi(t)}{\partial t}}{\text{Int}_0} \]  

where

\[ P_{kl} = \frac{1}{\omega^2} \left( k^2 \delta^2 + \frac{1}{2} l^2 (2 - \delta) \right) ; \text{Int}_0 = \int_{x_1 - \delta}^{x_1} \int_{y_1 - \delta}^{y_1} u_k(x)v_l(y) dx dy \]

\[ \text{Int}_1 = \int_{x_1 - \delta}^{x_1} \int_{y_1 - \delta}^{y_1} \Delta F_k u_k(x)v_l(y) dx dy; \text{Int}_2 = \int_{x_1 - \delta}^{x_1} \int_{y_1 - \delta}^{y_1} \Delta F_l u_k(x)v_l(y) dx dy \]

\[ \text{Int}_3 = \int_{x_1 - \delta}^{x_1} \int_{y_1 - \delta}^{y_1} F_k u_k(x)v_l(y) dx dy; \text{Int}_4 = \int_{x_1 - \delta}^{x_1} \int_{y_1 - \delta}^{y_1} F_l u_k(x)v_l(y) dx dy \]

We look for the solution of Equation (8) in the form

\[ T_{kl} = (A_{Rkl} + iA_{Ikl}) e^{\Omega_{n} t} \]  

Substituting (9) into (8), we obtain

\[ A_{Rkl} = \frac{-1}{\text{Int}_0} \left[ \frac{P_{kl}}{\pi^2} \int_{x_1 - \delta}^{x_1} \int_{y_1 - \delta}^{y_1} (\Delta \phi_k u_k(x)v_l(y) + (\text{Int}_3 - \text{Int}_4) \frac{1}{\pi^2} \frac{\partial \phi(t)}{\partial t}) dx dy \right] \]

\[ A_{Ikl} = \frac{1}{\text{Int}_0} \left[ \frac{P_{kl}}{\pi^2} \int_{x_1 - \delta}^{x_1} \int_{y_1 - \delta}^{y_1} (\Delta \phi_k u_k(x)v_l(y) - (\text{Int}_3 - \text{Int}_4) \frac{1}{\pi^2} \frac{\partial \phi(t)}{\partial t}) dx dy \right] \]

Exploring (11) for the presence of extremes using the condition

\[ \frac{\partial A_{Rkl}}{\partial \Omega_{n}} = 0 \]

we obtain

\[ \Omega_{n}^2 + 2P_{kl}(P_{kl}\text{Int}_4 + \text{Int}_2) - P_{kl}^2 = 0 \]  

Owing to the symmetry of the problem with respect to variables \( x \) and \( y \),

\[ \text{Int}_1 = \text{Int}_2; \text{Int}_3 = \text{Int}_4 \]

The solution of Equation (12) looks as follows

\[ \Omega_{nr} = P_{kl}(\sqrt{2} - 1) = 0.414P_{kl} \]
Since

\[ \Omega_{nr} = 1 + n\omega_f \quad (14) \]

we obtain

\[ \omega_{fr} = \frac{(\Omega_{nr} - 1)}{n} \quad (15) \]

where \( \omega_{fr} \) is the resonance frequency of the RMF frequency modulation.

As an example, we present a plot of the dependence of the vectorial potential amplitude \( A_{kln} \) on the \( \Omega_n \) frequency harmonic in the copper mold wall with a square cross-section (Fig. 1). Parameters for calculation: carrier frequency of the current \( f_0 = 10 \) Hz; billet cross-section 150x150 mm\(^2\); \( \delta_0 = 12 \) mm; \( \Omega_0 = 0.015 \), dimensionless resonance frequency \( \Omega_{nr} = 14.1 \).

![Graph](image.png)

Fig. 2: Dependence of the vectorial potential amplitude \( A_{kln} \) on the frequency of \( \Omega_n \) Fourier-pack harmonics in the square cross-section mold.

In view of the fact that the AFM RMF is characterized by a superposition of two Fourier-packs (see Equation (4)), the total amplitude of resonance harmonics of the vector potential is more than twice greater than when using a harmonic field.

**Conclusion**

Results of the analysis show that the amplitudes of resonance values of the vectorial potential exceed more than twice the amplitudes of the vectorial potential corresponding to harmonic currents used at the existing continuous casting machine. Since the electromagnetic body forces are proportional to a squared vectorial potential, the effect of force impact grows more than four times. This point indicates to the possibility of development the highly-efficient continuous casting technology using the amplitude-and-frequency modulation of rotating magnetic field acting on the copper wall of the mold.

**References**

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