Saturation momentum scale extracted from semi-inclusive transverse spectra in high-energy pp collisions

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We extract an average saturation momentum scale from semi-inclusive transverse momentum spectra in high-energy pp collisions. An effective energy \( W^* \) is introduced into the average saturation momentum \( Q_{\text{sat}} \) to relate to the charged hadron multiplicity in the final state. By a scaling variable using \( Q_{\text{sat}} \) the semi-inclusive transverse momentum spectra exhibit the behavior of geometrical scaling; i.e., all semi-inclusive spectra observed at \( \sqrt{s} = 0.90, 2.76 \) and 7.00 TeV overlap one universal function. The particle density dependences of mean transverse momentum \( \langle p_T \rangle \) for three LHC energies scales in terms of \( Q_{\text{sat}}(W^*) \), and it is consistent with the expected geometrical scaling behavior. Furthermore, our model explains a scaling property of event-by-event \( p_T \) fluctuation measure \( \sqrt{C_m/(p_T)} \) at LHC energies for pp collisions, where \( C_m \) is two-particle transverse momentum correlator. Our analysis from the data evaluates a non-perturbative coefficient of the gluon correlation function.

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I. INTRODUCTION

The study of small collision systems in high multiplicity events is attracting considerable interest \([1]\) because of the collective phenomena which attribute to the formation of a strongly-interacting collectively-expanding quark-gluon medium \([2, 3]\). A remarkable similarity has been observed between strange particle production in pp collisions and that in Pb-Pb collisions, suggesting the possibility of deconfined QCD phase formation in small systems \([4]\). In such pp collisions, the charged particle pseudo-rapidity density rises as a power of energy \([7, 8]\), which can be explained by the theory of gluon saturation \([9, 10]\). Recombination of gluons \([11]\) in high particle number density state causes the saturation, and the gluon distribution function ceases growing from some intrinsic scale of the transverse momentum \( Q_s \) \([12]\). The Color Glass Condensate (CGC) \([13, 14]\) is an effective theory to describe saturated gluons with small \( x \) as classical color fields radiated by color sources at higher rapidity. The existence of \( Q_s \) which separates the degree of freedom into fast frozen color sources and slow dynamical color fields \([15]\) is the underlying assumption of the effective theory. The scaling of the limiting fragmentation curves \([16]\) is one of the crucial pieces of evidence for the picture of the CGC \([18, 19]\).

Another experimental evidence of CGC hypothesis is a geometrical scaling \([20, 21]\), (GS) confirmed originally in results on total \( \gamma^* p \) cross section \([22]\). If the saturation momentum \( Q_s \) is the only scale that controls \( p_T \) distribution, the distributions should exhibit GS behavior. In particular, the scaling property has been vigorously studied for pp collision data obtained at the Large Hadron Collider (LHC) energy \([23, 24]\) and GS is observed in single inclusive distributions of charged hadrons \([25]\). A term of the ‘geometrical’ of this GS comes from that survival probability of a color dipole is determined by the geometric relationship between the dipole size and the saturation radius given by \( Q_s^{-1} \) \([16, 27]\). Under an assumption that a local parton hadron duality \([28]\) as a hadronization model is appropriate, the hadron multiplicity density and the saturation momentum scale can be related as follows:

\[
\frac{dN_{ch}}{dy}_{y \geq 0; \text{incl}} \propto S_T(W) \frac{Q_{\text{sat}}^2(W)}{W}, (1)
\]

where \( S_T \) is an effective overlap transverse area of the colliding protons and \( Q_{\text{sat}} \) is a so-called average saturation momentum \([25]\) which should be distinguished from \( Q_s \). (See Section II for the exact definitions of \( Q_s \) and \( Q_{\text{sat}} \).) Let us suppose that GS holds not only for inclusive distributions but also for the semi-inclusive distributions \([29]\) i.e., inclusive distribution with fixed multiplicity or limited multiplicity class. In this case, since the l.h.s. of Eq. (1) turns to depend on the value of the multiplicity fixed, the r.h.s of it should depend on the multiplicity as well. Hence, the above assumption leads us to the following expression:

\[
\frac{dN_{ch}}{dy}_{y \geq 0; \text{semi-incl}} \propto S_T(W; X) \frac{Q_{\text{sat}}^2(W; X)}{W}, (2)
\]

where \( S_T \) and \( Q_{\text{sat}} \) generally depend on both the collision energy \( W \) and a set of unknown initial conditions \( X \). As the variable \( X \) related to the initial conditions...
that affect the charged particle multiplicity $N_{ch}$ of the final states, for example, the impact parameter of pp collisions, the internal structure or shape of the proton can be considered. Therefore, to consider Eq. (2) may be comparable to introduce an impact parameter for a proton-color dipole collision. Furthermore, incorporating changes in multiplicity due to $Q_{sat}$ may also be relevant to take into account intrinsic fluctuations in saturation momentum. Such fluctuation has been introduced by McLerran and Praszalowicz to explain the experimental results on $dN_{ch}/dy$ in pA collisions. They assumed that the saturation momentum fluctuates around the mean value according to a Gaussian probability distribution and the fluctuations contribute to the increase of particle density. Since the fluctuation of $Q_{sat}$ is thought to be an intrinsic physical property existing not only in pA but also in pp collisions, one may extend Eq. (23) in Ref. 35 to the following for pp collisions in the central rapidity region:

$$\frac{dN_{ch}}{dy} \big|_{y \approx 0, \text{fluc}} = S_{T} Q_{sat}^{2}(W) I_{\text{fluc}}(W). \tag{3}$$

We interpret the multiplicity increase by the factor $I_{\text{fluc}}$ as that $Q_{sat}$ depends on some initial condition $X$. Hence, one may write $Q_{sat}(W; X) = Q_{sat}(W) I_{\text{fluc}}(W)$ because $I_{\text{fluc}}$ is considered to be due to the fluctuation in the initial condition $X$. Inspired by these works on the fluctuating initial conditions or fluctuating $Q_{sat}$, we shall phenomenologically investigate those effects on particle densities and transverse momentum distributions. To this end, we introduce an effective energy $W^*$ instead of $W$ in Eq. (1):

$$Q_{sat}(W; X) \equiv Q_{sat}(W^*), \tag{4}$$

where $W^*$ is a function of $dN_{ch}/dy$. The idea of the effective energy in high energy hadron multi-particle production has a rather long history (see, for example, 39) and concerning to leading particle effects or inelasticity, see also 41. We take effective energy $W^*$ into the saturation momentum scale, and study GS for semi-inclusive distribution. Therefore, the primary purpose of this paper is to extract $Q_{sat}(W^*)$ from semi-inclusive transverse momentum distributions observed. Then we try to explain the characteristics of the experimental results of event-by-event mean transverse momentum fluctuations.

This paper is organized as follows. In the following Section II we briefly review GS hypothesis and we confirm that it holds well for inclusive transverse spectra observed in pp collisions at LHC energies. Then, we determine the universal function of GS used throughout this paper. In section III, the effective energy $W^*$ is determined from the semi-inclusive transverse spectra with classified by the charged multiplicity. By redefining the scaling variables with $Q_{sat}(W^*)$, we show that the transverse momentum spectra observed in the different multiplicity classes at the different collision energies scale to the universal function found in Section II. We also show that the multiplicity dependence of the mean transverse momentum scales with $Q_{sat}(W^*)$. Furthermore, we analyze the scaling behavior of a normalized fluctuation measure of transverse momentum and consider it as a result of the correlation between particles generated from color flux tubes. We close with Section IV containing the summary and some concluding remarks.

II. GS FOR INCLUSIVE $p_T$ DISTRIBUTION

The transverse momentum spectra of various energies for pp collisions never scale with variable $p_T$ because their absolute values and slopes depend on the colliding energy $W$. However, for high energy collisions in which the number of soft gluons inside the proton saturates, the transverse momentum spectrum depends only on a scaling variable defined by $\tau^{1/2} = p_T/Q_{sat}(x)$. In such high energy pp collisions, the local parton hadron duality expects that charged hadron spectra in the final state is proportional to the gluon spectra in the initial state. In the case, one can obtain the following GS relation with $\tau$ 41

$$1 \frac{1}{S_{T} 2\pi p_T d\rho d\eta} \frac{d^2N_{ch}}{p_T d\eta} = F(\tau), \tag{5}$$

where $F(\tau)$ is a so-called universal function. Here, the squared saturation momentum $Q_{sat}^2$ is given by $-\lambda$ power of Bjorken $x$, and especially in the central rapidity region ($y \approx 0$), we obtain 42

$$Q_{sat}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda} = Q_0^2 \left( \frac{p_T}{x_0 W} \right)^{-\lambda}, \tag{6}$$

where $x_0$ and $Q_0$ are constants. By rearranging $p_T$ contained in the scaling variable $\tau$, one can write

$$\tau = \left( \frac{p_T}{Q_{sat}(W)} \right)^{2+\lambda}, \tag{7}$$

and define average saturation momentum as

$$Q_{sat}(W) \equiv Q_0 \left( \frac{x_0 W}{Q_0} \right)^{\lambda/(2+\lambda)}. \tag{8}$$

Note that $Q_{sat}$ is the solution,

$$Q_{sat}^2 = Q_0^2 \left( \frac{Q_{sat}}{x_0 W} \right)^{-\lambda}, \tag{9}$$

which obtained by substituting $Q_{sat}$ into both $Q_s$ and $p_T$ of Eq. 6. We show $Q_s$ and $Q_{sat}$ as a function of $p_T$ in Fig. I for the case of $\lambda = 0.22$, $x_0 = 1.0 \times 10^{-3}$, $Q_0 = 1.0 \text{ GeV}/c$ 41, and we will fix the values from now on. Since $Q_s$ is less dependent on $p_T$ for $p_T \gtrsim 0.5 \text{ GeV}/c$, one may use $Q_{sat}$ instead of $Q_s$ to define the scaling variable $\tau$. The values of the average saturation momentum $Q_{sat}$ obtained from inclusive $p_T$ spectra at energy $W = (\sqrt{s} =) 0.90, 2.76, 7.00 \text{ TeV}$ are 0.99, 1.11.
The long and short dashed line represents $Q_s$ energy dependence to cancel the energy dependence of $Q_\text{angle}$ and circle symbols) and average saturation momentum $s_\text{sat}$ (horizontal solid lines) for $W = 0.90$, 2.76 and 7.00 TeV. The intersections of the line and the dotted curve give the average saturation momentum $Q_\text{sat}$ at each $W$. For $W = 0.90$, 2.76 and 7.00 TeV, $Q_\text{sat} = 0.99$, 1.11, 1.21 GeV/c, respectively.

and 1.21 GeV/c, respectively. As shown in Fig. 2 experimental data observed by ALICE [43] and CMS Collaboration [44] suggests the validity of GS especially in the region of $q = 1.134$, $\kappa = 0.1293$ and $\lambda = 0.22$. Gluon saturation is physics of the intermediate energy scale $Q_\text{sat}$, while GS observed in the final state is physics of energy scale $T_\text{eff}$ which is much lower than $Q_\text{sat}$. Therefore, the parameter $\kappa$ in Eq. (10) may have a physical meaning of a linkage between two energy scales of $Q_\text{sat}$ and $T_\text{eff}$. Substituting Eq. (12) into Eq. (11) yields the expression of the universal function of Eq. (10) in the case of

$$T_\text{eff} = \kappa Q_\text{sat}. \quad (13)$$

III. GS FOR SEMI-INCLUSIVE $p_T$ DISTRIBUTION

A. Extraction of saturation momentum scale

To phenomenologically incorporate the effects of fluctuations of saturation momentum $Q_\text{sat}$, we introduce the effective energy $W^*$ as mentioned in Section I. Our central assumption is that the semi-inclusive $p_T$ distribution scales to the same universal function $F(\tau)$ as the inclusive one (i.e., Eq. (11)) with $q = 1.134$, $\kappa = 0.1293$ and $\lambda = 0.22$, providing that the appropriate saturation momentum $Q_\text{sat}(W^*)$ is used. Since $S_T$ in Eq. (4) now depends on the multiplicity, we require GS for the semi-

FIG. 1. Saturation momentum $Q_s$ (dotted curve with triangle and circle symbols) and average saturation momentum $Q_\text{sat}$ (horizontal solid lines) for $W = 0.90$, 2.76 and 7.00 TeV. Experimental data (indicated by triangles or circles) are observed by ALICE Collaboration [43] and CMS Collaboration [44]. The solid curve is the universal function $F(\tau)$ with $q = 1.134$, $\kappa = 0.1293$ and $\lambda = 0.22$ (See Eq. (10)). The effective interaction cross-sectional area $S_T = 22.66 \text{ GeV}^{-2}$ is used.

FIG. 2. The transverse momentum distributions exhibit geometrical scaling behavior for pp collisions at $W = 0.90$, 2.76 and 7.00 TeV. Experimental data (indicated by triangles or circles) are observed by ALICE Collaboration [43] and CMS Collaboration [44]. The solid curve is the universal function $F(\tau)$ with $q = 1.134$, $\kappa = 0.1293$ and $\lambda = 0.22$ (See Eq. (10)). The effective interaction cross-sectional area $S_T = 22.66 \text{ GeV}^{-2}$ is used.

$$F(\tau) = \left[1 + (q - 1) \frac{\tau^{1/(\lambda + 2)}}{\kappa}\right]^{1/(q-1)}, \quad (10)$$

where the non-extensive parameter $q = 1.134$ and $\kappa = 0.1293$ are used. In this way, the transverse momentum distribution indeed exhibits GS behavior for pp collisions in the LHC energy range. It seems to be appropriate to shortly comment on the average saturation momentum $Q_\text{sat}$ and an effective temperature $T_\text{eff}$ (or a slope parameter) $T_\text{eff}$ here. In case of Tsallis-type distribution function, $T_\text{eff}$ is defined as

$$T_\text{eff} = \frac{1}{2\pi p_T d^2 N_{\text{ch}}/dy}_{y=0; \text{incl}} = C \left[1 + (q - 1) \frac{p_T}{T_\text{eff}}\right]^{-1/(q-1)}. \quad (11)$$

Here, one may interpret the constant $C$ as $S_T$. Since the transverse spectra experimentally observed exhibits good GS behavior, the effective temperature $T_\text{eff}$ must have energy dependence to cancel the energy dependence of

$$p_T = Q_0 \left(\frac{x_0 W}{Q_0}\right)^{\frac{1}{\tau}} \frac{1}{\tau}, \quad (12)$$

which is obtained from Eq. (11). Hence, the property of the GS determined the energy dependence of $T_\text{eff}$ and that $T_\text{eff}$ should be proportional to $Q_\text{sat}$. Gluon saturation is physics of the intermediate energy scale $Q_\text{sat}$, while GS observed in the final state is physics of energy scale $T_\text{eff}$ which is much lower than $Q_\text{sat}$. Therefore, the parameter $\kappa$ in Eq. (10) may have a physical meaning of a linkage between two energy scales of $Q_\text{sat}$ and $T_\text{eff}$. Substituting Eq. (12) into Eq. (11) yields the expression of the universal function of Eq. (10) in the case of

$$T_\text{eff} = \kappa Q_\text{sat}. \quad (13)$$
inclusive spectra as the following:

\[ \frac{1}{S_T(N_{ch})} \frac{1}{2\pi p_T} \frac{d^2N_{ch}}{dp_T dy} \bigg|_{y \approx 0; \text{semi-incl}} = F(\tau), \quad (14a) \]

where the scaling variable \( \tau \) is to be modified by

\[ \tau^{1/(2+\lambda)} = \frac{p_T}{Q_{sat}(W^*)}. \quad (14b) \]

In order to determine the multiplicity dependence of \( W^* \) in Eq. (14), we fit Eq. (14a) to data on \( p_T \) spectra at energy 0.90 TeV for the accepted number of charged particles \( n_{acc} = 3, 7 \) and 17 observed ALICE Collaboration [48] and at energy 0.90, 2.76 and 7.00 TeV for the average track multiplicity \( n_{tracks} = 40, 63, 75, 98, 120 \) and 131 observed CMS Collaboration [44]. Figure 3 and 4 show the results of fitting with \( S_T F \) to ALICE and CMS data, respectively. Besides, Table II shows the values of \( W^* \) (multiplied by \( x_0 \)) and effective radius \( R_T \) (instead of area \( S_T \)) obtained by the fit. Table II also shows the value of \( Q_{sat} \) obtained from \( W^* \) and the minimum value of \( \chi^2 \) (denoting by \( \chi^2_{\min} \)) in each fitting. As shown in Fig. 5, we confirm that the semi-inclusive transverse momentum spectra depicted in Figs 3 and 4 scale in terms of the scaling variable \( \tau \) of Eq. (14b). Note that the solid curve (the universal function \( F \)) in Fig. 4 is exactly the same as that given for the inclusive distribution in Fig. 1. We show \( Q_{sat} \) and \( R_T \) as function of \( dN_{ch}/dy \) in Fig. 6. Both \( Q_{sat} \) and \( R_T \) increases by a third power of \( dN_{ch}/dy \). The curves depicted by broken lines in the left panel (for \( Q_{sat} \)) and the right panel (for \( R_T \)) of FIG.

6 are given as a function of \( dN_{ch}/dy \):

\[
\begin{align*}
Q_{sat} \left[ \text{GeV}/c \right] &= \begin{cases} 
0.677 + 0.150 \left( \frac{dN_{ch}}{dy} \right)^{1/3} & (W = 0.90 \text{ TeV}), \\
0.725 + 0.150 \left( \frac{dN_{ch}}{dy} \right)^{1/3} & (W = 2.76 \text{ TeV}), \\
0.749 + 0.150 \left( \frac{dN_{ch}}{dy} \right)^{1/3} & (W = 7.00 \text{ TeV}),
\end{cases} \\
R_T \left[ \text{fm} \right] &= \begin{cases} 
0.039 + 0.400 \left( \frac{dN_{ch}}{dy} \right)^{1/3} & (W = 0.90 \text{ TeV}), \\
0.006 + 0.396 \left( \frac{dN_{ch}}{dy} \right)^{1/3} & (W = 2.76 \text{ TeV}), \\
0.006 + 0.392 \left( \frac{dN_{ch}}{dy} \right)^{1/3} & (W = 7.00 \text{ TeV}),
\end{cases}
\end{align*}
\]

respectively. Here, it is interesting to find a particle number density \( dN^*/dy \) to give \( W^*(dN^*/dy) = W \). From Eq. (15b), we find \( dN^*/dy = 9.1, 17, \) and 29 for inclusive distribution at energy \( W =0.90, 2.76 \) and 7.00 TeV, respectively.

B. Mean transverse momentum

Next, we turn our attention to the average transverse momentum \( \langle p_T \rangle \) observed in the semi-inclusive distributions. The average saturation momentum \( Q_{sat}(W^*) \) should be proportional to \( \langle p_T \rangle \) in GS framework [49]. As seen in ALICE [50] and CMS [44] experimental results in the left panel of Fig. 7, the multiplicity density dependences of \( \langle p_T \rangle \) observed at 0.90, 2.76 and 7.00 TeV do not scale in terms of \( dN_{ch}/dy \). However, since GS holds
TABLE I. The values of $x_0W^*, Q_{sat}, R_T$, and minimum chi-squared $\chi^2_{\text{min}}$ obtained from the fitting to the semi-inclusive transverse momentum distribution observed by ALICE [48] with multiplicity class $n_{acc} = 3, 7, 17$ (accepted number of charged particles per inelastic event in the range $|\eta| < 0.8$) and by CMS [44] with multiplicity class $n_{\text{tracks}} = 40, 63, 75, 98, 120$ and 131 (average number of true tracks multiplicity in the range $|\eta| < 2.4$). For assignment from $n_{acc}$ to $\langle n_{ch} \rangle$ in ALICE data, we use the results presented in Table 2 of Ref. [48]. The particle densities $dN_{ch}/dy$ at central rapidity region, are estimated by $\langle n_{ch} \rangle/\Delta \eta$ for simplicity. In this table, effective interaction radius $R_T \equiv \sqrt{S_T/\pi}$ is shown instead of area $S_T$. $X_+^{a+b}$ denotes $X(=x_0W^*, Q_{sat}, R_T)$ giving $\chi^2_{\text{min}}$ and $a, b$ mean a boundary $X - b \leq X \leq X + a$ giving $\chi^2 = 1.5\chi^2_{\text{min}}$.

| $\sqrt{s}$ (TeV) | $n_{acc}$ | $\langle n_{ch} \rangle/\Delta \eta$ | $x_0W^*$ [GeV] | $Q_{sat}$ [GeV/c] | $R_T$ [fm] | $\chi^2_{\text{min}}$/dof |
|-------------------|-----------|-------------------------------|----------------|-----------------|----------|------------------|
| 0.90              | 3         | 4.8/1.6                       | 0.18$^{+0.16}_{-0.09}$ | 0.84$^{+0.06}_{-0.06}$ | 0.63$^{+0.09}_{-0.07}$ | 97.5/33  |
| 0.90              | 7         | 10.0/1.6                      | 1.39$^{-0.3}_{+0.6}$  | 1.03$^{+0.04}_{-0.04}$ | 0.69$^{+0.04}_{-0.05}$ | 20.5/33  |
| 0.90              | 17        | 22.5/1.6                      | 9.6$^{+7.0}_{-4.3}$   | 1.24$^{+0.08}_{-0.08}$ | 0.82$^{+0.08}_{-0.07}$ | 23.2/33  |

| $\sqrt{s}$ (TeV) | $n_{\text{tracks}}$ | $\langle n_{ch} \rangle/\Delta \eta$ | $x_0W^*$ [GeV] | $Q_{sat}$ [GeV/c] | $R_T$ [fm] | $\chi^2_{\text{min}}$/dof |
|-------------------|----------------------|-------------------------------|----------------|-----------------|----------|------------------|
| 0.90              | 40                   | 40/4.8                        | 0.77$^{+0.16}_{-0.14}$ | 0.97$^{+0.02}_{-0.02}$ | 0.84$^{+0.04}_{-0.02}$ | 10.5/18  |
| 0.90              | 63                   | 63/4.8                        | 1.45$^{+0.33}_{-0.39}$ | 1.04$^{+0.03}_{-0.03}$ | 0.98$^{+0.05}_{-0.04}$ | 21.8/18  |
| 0.90              | 75                   | 75/4.8                        | 1.72$^{+0.22}_{-0.45}$ | 1.06$^{+0.04}_{-0.03}$ | 1.05$^{+0.05}_{-0.05}$ | 17.6/18  |
| 2.76              | 40                   | 40/4.8                        | 1.05$^{+0.33}_{-0.24}$ | 1.00$^{+0.03}_{-0.03}$ | 0.81$^{+0.04}_{-0.03}$ | 19.8/18  |
| 2.76              | 63                   | 63/4.8                        | 2.23$^{+1.15}_{-0.80}$ | 1.08$^{+0.05}_{-0.05}$ | 0.93$^{+0.07}_{-0.05}$ | 41.0/18  |
| 2.76              | 75                   | 75/4.8                        | 2.92$^{+1.79}_{-1.17}$ | 1.11$^{+0.05}_{-0.06}$ | 0.99$^{+0.07}_{-0.06}$ | 46.4/18  |
| 2.76              | 98                   | 98/4.8                        | 3.94$^{+2.24}_{-1.54}$ | 1.15$^{+0.05}_{-0.05}$ | 1.10$^{+0.07}_{-0.06}$ | 32.4/18  |
| 7.00              | 40                   | 40/4.8                        | 1.06$^{+0.35}_{-0.19}$ | 1.01$^{+0.03}_{-0.02}$ | 0.81$^{+0.03}_{-0.03}$ | 19.1/18  |
| 7.00              | 63                   | 63/4.8                        | 2.47$^{+1.25}_{-0.88}$ | 1.09$^{+0.05}_{-0.05}$ | 0.92$^{+0.07}_{-0.05}$ | 41.1/18  |
| 7.00              | 75                   | 75/4.8                        | 3.24$^{+1.71}_{-1.17}$ | 1.12$^{+0.05}_{-0.05}$ | 0.98$^{+0.07}_{-0.05}$ | 40.9/18  |
| 7.00              | 98                   | 98/4.8                        | 4.95$^{+3.20}_{-2.00}$ | 1.17$^{+0.06}_{-0.06}$ | 1.07$^{+0.07}_{-0.06}$ | 47.1/18  |
| 7.00              | 120                  | 120/4.8                       | 6.25$^{+3.92}_{-2.17}$ | 1.20$^{+0.06}_{-0.05}$ | 1.17$^{+0.07}_{-0.06}$ | 30.1/18  |
| 7.00              | 131                  | 131/4.8                       | 8.25$^{+5.67}_{-3.25}$ | 1.23$^{+0.07}_{-0.06}$ | 1.18$^{+0.07}_{-0.07}$ | 35.6/18  |

FIG. 5. Geometrical scaling of the semi-inclusive transverse momentum spectra in terms of the scaling variable $\tau$ defined by Eq. (10). The experimental data are observed by ALICE [48] (open symbols) with multiplicity class $n_{acc} = 3, 7, 17$ and by CMS [44] (closed symbols) with multiplicity class $n_{\text{tracks}} = 40, 63, 75, 98, 120$ and 131.
even in the semi-inclusive distributions, one expects that \( dN \) lie on a straight line regardless of colliding energy \( W \). In our previous work, \cite{49} we studied it in high energy pp collisions.

The right panel of Fig.\ 7 shows a result of the conversion of the \( dN/d\eta \) dependence of \( \langle p_T \rangle \) on the left side into the dependence of \( Q_{\text{sat}}(W^*) \). The difference in scaling curves between ALICE and CMS seems to be due to that in acceptance employed in each observation. Thus, the behavior of GS is observed not only in the inclusive distributions but also in the semi-inclusive distributions in high energy pp collisions.

C. Normalized fluctuation measure of transverse momentum

A prominent scaling behavior emerges in an event-by-event mean \( p_T \) fluctuations in pp collisions at LHC energies \cite{51,52}. In our previous work, \cite{10} we studied it focusing only on the energy \( W = \sqrt{s} = 0.90 \) TeV, and we did not discuss the geometrical scaling behavior by extending the analysis to other energies. In this paper, we analyze transverse momentum fluctuation data obtained at \( \sqrt{s} = 0.90, 2.76, 7.00 \) TeV using \( Q_{\text{sat}}(W^*) \) and \( S_T(N_{\text{ch}}) \) without changing the basic idea of the model proposed in Ref.\cite{10}. The fluctuation measure is essentially a two-particle distribution as defined below,

\[
C_m = \frac{\int d^2p_{T_1} \int d^2p_{T_2} m(m-1)}{d^4n_{\text{ch}}/d^2p_{T_1} d^2p_{T_2}} (p_{T_1} - \langle p_T \rangle)(p_{T_2} - \langle p_T \rangle),
\]

where \( m = dN_{\text{ch}}/d\eta \times |\Delta \eta| \) is the multiplicity in the pseudorapidity window \( |\Delta \eta| \). Since the universal function characterizing the behavior of GS is essentially one particle inclusive distribution, two-particle correlation function \cite{54} as shown below is required to obtain the two-particle distribution as used in Eq.\cite{10}:

\[
C(p_{T_1}, p_{T_2}) = \frac{d^4n_{\text{ch}}}{d^2p_{T_1} d^2p_{T_2}} / \frac{d^2n_{\text{ch}}}{d^2p_{T_1} d^2p_{T_2}}.
\]

It is known that a gluon two-particle correlation function takes the following simple geometrical form in the CGC / Glasma framework \cite{55,57}:

\[
C_{\text{GFT}}(p_{T_1}, p_{T_2}) = 1 + \frac{\kappa^2}{S_T^2 Q_{\text{sat}}^2},
\]

where \( \kappa \) is a non-perturbative constant, and the evaluation of this constant is a challenging problem in theoretical physics.

On the other hand, we consider an extreme model in which the correlation in momentum space between gluons is inherited to that between hadrons in the final state. Since the transverse size of color flux tubes stretching between the receding protons is expected to be of order in \( 1/Q_{\text{sat}} \), one may write the following correlation function commonly found in Bose-Einstein correlation (BEC) analysis:

\[
C(p_{T_1}, p_{T_2}) = 1 + \left( S_T \ [\kappa Q_{\text{sat}}] \right)^n \exp \left( -\frac{(p_{T_1} - p_{T_2})^2}{\sigma^2 |K_{\text{sat}}|^2} \right),
\]

where \( n \) and \( \sigma \) are model parameters. Here, \( \kappa \) is the parameter that appears in the universal function Eq.\cite{10} which connects the saturation \( Q_{\text{sat}} \) (intermediate energy)}
TABLE II. The best fit values of the parameters $\sigma$ and $n$ to the experimental data on the event-by-event fluctuation of mean $p_T$ observed by ALICE Collaboration [51] and results of evaluation of $\kappa_2$ by Eq. (20).

| $\sqrt{s}$ [TeV] | $\sigma$ | $n$ | $\chi^2_{\text{min}}$/dof | $\kappa_2$ |
|------------------|----------|-----|----------------------------|------------|
| 0.90             | 1.14     | -0.74 | 4.26/23                 | 1.79       |
| 2.76             | 1.07     | -0.82 | 8.77/45                 | 1.42       |
| 7.00             | 1.10     | -0.90 | 11.3/64                 | 1.58       |

scale and hadronization energy scale. Since $\kappa$ and $Q_{\text{sat}}$ always appear together in the inclusive distribution, there must be such property also in the two particle distribution in Eq. (19). Note also that the term $S_T Q_{\text{sat}}^2$ in Eq. (19) is proportional to the number of flux tubes [32], especially when $n = -1$, it can be interpreted as chaoticity of the BEC effect [33]. Another parameter $\sigma$ is for adjusting the size of the flux tube, and if the model is valid, its value is expected to be approximately 1. ALICE observed normalized fluctuation measure $\sqrt{C_m}/(p_T^{\text{sat}})$ over the three energies of LHC, i.e., $\sqrt{s} = 0.90, 2.76$ and 7.00 TeV, and they found almost no energy dependence in them (See Fig. 8). Our model based on GS easily explain the reason why the measure $\sqrt{C_m}/(p_T^{\text{sat}})$ hardly depends on the collision energy: i.e., By noting that $p_T = Q_{\text{sat}} \tau^{1/(2+\lambda)}$, $(p_T^{\text{sat}}) \propto Q_{\text{sat}}$, and $m \propto S_T Q_{\text{sat}}^2$, one can represent the measure as a function of the scaling variable $\tau$ except for the term $S_T Q_{\text{sat}}^2$ in the correlation function Eq. (19). However, as shown by the Eqs. (15a) and (15b), the energy dependence of both $Q_{\text{sat}}$ and $S_T$ are considerably small. Therefore, it is explained that $\sqrt{C_m}/(p_T^{\text{sat}})$ is almost independently of the colliding energy $W$ in our model.

The fit results to the experimental data by Eq. (19) are shown by solid lines in Fig. 8. Besides, we show values of the parameter both $\sigma$ and $n$ giving $\chi^2_{\text{min}}$ in Table II. The values of $n$ obtained from the fits are $-0.74$ to $-0.90$, which are larger than $-1$, and Eq. (19) can be compared with the Eq. (15) in the Glasma framework. Evaluating the typical momentum scale of BEC as $|p_{T1} - p_{T2}|^2 \sim [2\kappa Q_{\text{sat}}]^2 \approx [200 \text{MeV}]^2$, the comparison leads us to a rough estimation of $\kappa_2$ as the following:

$$\kappa_2 \sim \frac{1}{\kappa^2} \exp \left( -\frac{4}{\sigma} \right). \quad (20)$$

Table II also shows the values of $\kappa_2$ evaluated by Eq. (20). Since there are considerable variations in the extracted values of $\kappa_2$ from experimental data based on the Glasma framework, its value is not known to be as accurate as an order of 1 [52]. It is interesting to note that the values of $\kappa_2$ extracted from our model are comparable to the estimation by the Glasma framework, although the picture for particle correlation of each other is different.

IV. SUMMARY AND CONCLUDING REMARKS

In this paper, we have phenomenologically investigated a possibility that the gluon saturation momentum scale may fluctuate from event to event in high energy pp collisions. The average saturation momentum $Q_{\text{sat}}(W^*)$ should depend on some initial condition, such as impact parameter, the internal structure, and shape of colliding protons. If the local parton-hadron duality hypothesis is strictly correct, $Q_{\text{sat}}(W^*)$ must link to observables in the final state of the charged hadrons. In order to extract $Q_{\text{sat}}(W^*)$ that governs the multiplicity of the final states, we assumed that the semi-inclusive transverse momentum spectra exhibit geometrical scaling behavior independently of its fixed multiplicity and its colliding energy. Furthermore, the universal function is assumed to be the same as that of the inclusive distribution. Through the effective energy $W^*$ defined by Eq. (4), we determined average saturation momentum $Q_{\text{sat}}(W^*)$ for the semi-inclusive distributions. We have shown that the transverse momentum distribution of various multiplicity class at $\sqrt{s} = 0.90, 2.76$ and 7.00 TeV do scale in terms of the scaling variable $\tau^{1/(2+\lambda)} = p_T/Q_{\text{sat}}(W^*)$. We also confirm that $Q_{\text{sat}}(W^*)$ dependence on the average transverse momentum also scales to a straight line, which is consistent with the behavior expected from GS.

It may be essential to note on works by Korus and Mrówczyński [58, 59]. They have introduced a multiplicity-dependent temperature and related the non-trivial behavior of fluctuations in the transverse momentum to that in the multiplicity distribution. On the other hand, recall that the energy dependence of $T_{\text{eff}}$ cancels with that of $p_T$ in GS because $T_{\text{eff}}$ and $p_T$ have the same energy dependence except for the constant $\kappa$ (see Eq. (13)). Since $W^*$ results in multiplicity dependence on $Q_{\text{sat}}$, and hence on the effective temperature $T_{\text{eff}}$, we find that our model based on GS and the model proposed in Refs. [58, 59] certainly have similar theoretical structure. The reason why is almost no dependence on collision energy in the fluctuation measure $\sqrt{C_m}/(p_T^{\text{sat}})$ is that the energy dependences on $Q_{\text{sat}}(W^*)$ and $S_T$ are considerably small in addition to the fact that the semi-inclusive transverse momentum spectrum shows the behavior of geometrical scaling. Korus and Mrówczyński [58, 59], on the other hand, argue that the reason for it is that the fluctuation in the multiplicity distribution is almost independent of energy. In fact, the normalized $q$-moment values of $C_2, C_3, C_4$ for the multiplicity distribution in the central rapidity region $|\eta| < 0.5$ are almost independent of the collision energy [60, 61].

In this paper, we thought that the two-particle Bose-Einstein correlation between identical gluons produced from flux tubes could explain the experimental results of the fluctuation measure. The measure $\sqrt{C_m}/(p_T^{\text{sat}})$ can be fitted by Eq. (19) nicely, in which the correlation between gluons is considered to remain between charged particles after hadronization. Comparing Eq. (13) with
FIG. 8. Experimental data on event-by-event mean transverse momentum fluctuation \[51\] and fit results of our model (Eq. (19)) to the data. The pseudo-rapidity window \[|\Delta \eta| = 0.8\] is employed. Table II shows the values of the parameters that give the least chi-square for the fitting of \[\sqrt{C_{m}/\langle p_{T} \rangle}.\] We have excluded two small \[dN_{ch}/dy\] (=1.8 and 2.4) data points with from the fits.

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