Scalar meson mediated nuclear $\mu^− - e^−$ conversion

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Abstract

We study the nuclear $\mu^− - e^−$ conversion in the general framework of the effective Lagrangian approach without referring to any specific realization of the physics beyond the standard model (SM) responsible for lepton flavor violation (LFV). We analyze the role of scalar meson exchange between the lepton and nucleon currents and show its relevance for the coherent channel of $\mu^− - e^−$ conversion. We show that this mechanism introduces modifications in the predicted $\mu^− - e^−$ conversion rates in comparison with the conventional direct nucleon mechanism, based on the contact type interactions of the nucleon currents with the LFV leptonic current. We derive from the experimental data lower limits on the mass scales of the generic LFV lepton-quark contact terms and demonstrate that they are more stringent than the similar limits existing in the literature.

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The study of lepton flavor violating (LFV) processes offers a good opportunity for shedding light on the possible physics beyond the Standard Model (SM). Muon-to-electron ($\mu^- \rightarrow e^-$) conversion in nuclei

$$\mu^- + (A, Z) \rightarrow e^- + (A, Z)^*$$  

is commonly recognized as one of the most sensitive probes of lepton flavor violation and of the related physics behind it (for reviews, see [1,2]).

At present, on the experimental side there is one running $\mu^- \rightarrow e^-$ conversion experiment, SINDRUM II [3], and two planned ones, MECO [4,5] and PRIME [6]. So far this LFV process has not been observed and experimental results correspond to the upper limits on the $\mu^- \rightarrow e^-$ conversion branching ratio

$$R_{\mu e}^A = \frac{\Gamma(\mu^- + (A, Z) \rightarrow e^- + (A, Z))}{\Gamma(\mu^- + (A, Z) \rightarrow \nu + (A, Z - 1))}.$$  

The current and expected limits from the above mentioned experiments are presented in Table I. As is known from previous studies (see, for instance, Ref. [2,7] and references therein) and will be discussed later in the present paper, these experimental bounds allow to set stringent limits on the mechanisms of $\mu^- \rightarrow e^-$ conversion and the underlying theories of LFV.

The theoretical studies of $\mu^- \rightarrow e^-$ conversion, presented in the literature, cover various aspects of this LFV process, elaborating adequate treatment of the structure effects [1,8,9] of the nucleus participating in the reaction and, considering underlying mechanisms of LFV at the level of quarks within different scenarios of physics beyond the SM (see [2] and references therein).

In general the $\mu^- \rightarrow e^-$ conversion mechanisms can be classified as photonic and non-photonic. In the former case the $\mu^- \rightarrow e^-$ conversion is mediated by the photon exchange between the LFV leptonic vertex and the ordinary electromagnetic nuclear vertex. The non-photonic mechanisms are based on the 4-fermion contact lepton-quark LFV interactions. These two categories of mechanisms differ significantly from each other since they receive different contributions from the new physics and require different treatment of the effects of the nucleon and the nuclear structure.

In the present paper we continue studying the non-photonic meson exchange mechanism of $\mu^- \rightarrow e^-$ conversion. Previously [7,10] we analyzed the vector-meson mediation of the $\mu^- \rightarrow e^-$ conversion. The contribution of this mechanism to the coherent $\mu^- \rightarrow e^-$ conversion results in some important issues for the physics beyond the SM absent in the case of the conventional direct lepton-nucleon interaction. Here, we extend our analysis to the scalar-meson exchange mechanism, which completes the study of meson exchange contributions to the coherent mode of the $\mu^- \rightarrow e^-$ conversion 1.

1Pseudoscalar and axialvector mesons do not contribute to the coherent $\mu^- \rightarrow e^-$ conversion.
II. GENERAL FRAMEWORK

The effective Lagrangian $L_{\text{eff}}$ describing the coherent $\mu^- - e^-$ conversion at the quark level can be written in the form \cite{8,10}

$$L_{\text{eff}}^q = \frac{1}{\Lambda_{\text{LFV}}^2} \left[ (\eta_{qV}^q j_{qV}^V + \eta_{AV}^q j_{qA}^V) J_{qV}^V + (\eta_{S\bar{S}S}^q j_{qS}^S + \eta_{P\bar{P}S}^q j_{qP}^P ) J_{qS}^S \right] , \quad (3)$$

where the lepton and quark currents are defined as:

$$j_{qV}^V = \bar{e} \gamma_\mu \gamma_5 q, \quad j_{qA}^V = \bar{e} \gamma_\mu \gamma_5 q, \quad j_{qS}^S = \bar{e} \gamma_\mu q, \quad j_{qP}^P = \bar{e} \gamma_\mu q . \quad (4)$$

In Eqs. (3) and (4) the summation runs over all the quark species $q = \{u, d, s, b, c, t\}$. The parameter $\Lambda_{\text{LFV}}$ with the dimension of mass is the characteristic high energy scale of lepton flavor violation attributed to new physics. The dimensionless LFV parameters $\eta^q$ in Eq. (3) depend on a concrete LFV model. In the present study we treat these parameters as phenomenological parameters to be constrained from the experiment.

The quark level Lagrangian (3) generates the effective lepton-nucleon interactions that can be specified in terms of an effective Lagrangian on the nucleon level

$$L_{\text{eff}}^{\text{n}} = \frac{1}{\Lambda_{\text{LFV}}^2} \left[ j_{\mu}^a (a_{V} (0) J_{\mu}^V (0) + a_{V} (3) J_{\mu}^V (3)) + j_{\mu}^b (a_{S} (0) J_{\mu}^S (0) + a_{S} (3) J_{\mu}^S (3)) \right] . \quad (5)$$

Here, the isoscalar $J^{(0)}$ and isovector $J^{(3)}$ nucleon currents are $J_{\mu}^V (k) = \bar{N} \gamma^\mu \tau^k N$ and $J_{\mu}^S (k) = \bar{N} \tau^k N$, where $N$ is the nucleon isospin doublet, $k = 0, 3$ and $\tau_0 \equiv \hat{I}$. The summation over the double indices $a = V, A$ and $b = S, P$ is implied in (5). The nucleon Lagrangian (5) is the basis for the derivation of the nuclear transition operators.

Naturally, the Lagrangian in terms of effective nucleon fields (5) is equivalent to the quark level Lagrangian (3). The former Lagrangian is supposed to appear after the hadronization from the quark level Lagrangian (3) and, therefore, must correspond to the same order $1/\Lambda_{\text{LFV}}^2$ in inverse powers of the LFV scale.

To make a bridge between the underlying LFV physics and $\mu^- - e^-$ observables one needs to relate the lepton-nucleon LFV parameters $\alpha$ in Eq. (5) to the lepton-quark LFV parameters $\eta$ in Eq. (3). This implies a certain hadronization prescription which specifies the way in which the effect of quarks is simulated by hadrons. In the absence of a true theory of hadronization we rely on some reasonable assumptions and models. In Refs. \cite{7,10} we considered the two mechanisms of nuclear $\mu^- - e^-$ conversion: direct nucleon mechanism and vector-meson exchange between nucleon and lepton currents. We found that the vector-meson exchange plays an important role in the coherent muon-electron conversion. Now, we extend this analysis to the scalar sector of the LFV Lagrangians in Eqs. (3) and (5).

As in the case of the vector currents, considered in Refs. \cite{7,10}, here we distinguish the following two hadronization mechanisms. The first one is the direct embedding of the quark currents into the nucleon (Fig.1a), which we call direct nucleon mechanism (DNM). The second mechanism consists of two stages (Fig.1b). First, the quark currents are embedded into the interpolating scalar meson fields which then interact with the nucleon currents. We call this possibility meson-exchange mechanism (MEM).
In general one expects all the mechanisms to contribute to the coupling constants $\alpha$ in Eq. (5). However, at present the relative amplitudes of different mechanisms are unknown. In view of this problem we assume for the first approximation, that only one mechanism is operative and estimate its contribution to the process in question. This allows us to evaluate the importance of a specific mechanism.

Let us update the contribution of the direct nucleon mechanism derived in Ref. [8]. The relation between the quark-lepton and nucleon-lepton LFV parameters in Eqs. (3) and (5) takes in this case the form

$$
\sigma_{bS[DNM]}^{(3)} = \frac{1}{2} \eta_{bS}^{(3)} (G_S^u - G_S^d),
$$

$$
\sigma_{bS[DNM]}^{(0)} = \frac{1}{2} \eta_{bS}^{(0)} (G_S^u + G_S^d) + \eta_{bS}^{s} G_S^s,
$$

where $b = S, P$ and $\eta^{(0,3)} = \eta^u \pm \eta^d$ are the isoscalar and isovector quark couplings. The form factors $G_S^a$ are related to the scalar condensates in the nucleon [11]

$$
\langle p|\bar{u} u|p\rangle = G_S^u \bar{p} p, \quad \langle p|\bar{d} d|p\rangle = G_S^d \bar{p} p, \quad \langle p|\bar{s} s|p\rangle = G_S^s \bar{p} p, \\
\langle n|\bar{u} u|n\rangle = G_S^d \bar{n} n, \quad \langle n|\bar{d} d|n\rangle = G_S^s \bar{n} n, \quad \langle n|\bar{s} s|n\rangle = G_S^s \bar{n} n.
$$

Since the maximal momentum transfer $q$ in $\mu^- - e^-$ conversion is much smaller than the typical scale of the nucleon structure we can safely neglect the $q^2$-dependence of the nucleon form factors $G_S^a$. At $q^2 = 0$ these form factors are related to the corresponding meson-nucleon sigma-terms:

$$
\sigma_{\pi N} = \bar{m} [G_S^u + G_S^d], \quad \sigma_{K^* N}^{I=1} = \frac{\bar{m} + m_s}{4} [G_S^u - G_S^d], \quad y_N = \frac{2G_S^s}{G_S^u + G_S^d},
$$

where $\bar{m} = (m_u + m_d)/2$ and $m_s$ are the masses of current quarks; $y_N$ is the strangeness of the nucleon. For these parameters we use the following values [12,13] in our analysis

$$
\bar{m} = 7 \text{ MeV}, \quad m_s/\bar{m} = 25, \quad y_N = 0.2.
$$

The canonical value of the $\pi N$ sigma term $\sigma_{\pi N} = 45 \pm 8 \text{ MeV}$ [13] was originally extracted from the dispersive analysis of $\pi N$ scattering data taking into account chiral symmetry constraints. In particular, the value of the sigma-term, $\sigma_{\pi N} = 45 \pm 8 \text{ MeV}$, has been deduced from the analysis of two quantities: $\sigma_{\pi N}(t = 2M_\pi^2) = 60 \pm 8 \text{ MeV}$, the scalar nucleon form factor at the Cheng-Dashen point $t = 2M_\pi^2$, and the difference $\Delta_\sigma = \sigma_{\pi N}(2M_\pi^2) - \sigma_{\pi N}(0) = 15.2 \pm 0.4 \text{ MeV}$ [13]

as induced by explicit chiral symmetry breaking. The value of the isovector kaon-nucleon sigma-term $\sigma_{K^* N}^{I=1}$ was estimated in Ref. [14] using the baryon mass formulas:

$$
\sigma_{K^* N}^{I=1} \sim \frac{m_s + \bar{m}}{m_s - \bar{m}} \frac{m^2_\pi - m^2_S}{8m_P} = 48 \text{ MeV} \sim 50 \text{ MeV}.
$$

Substituting the above values of the hadronic parameters to Eq. (8) we obtain for the scalar nucleon form factors:

$$
G_S^u = 3.74, \quad G_S^d = 2.69, \quad G_S^s = 0.64.
$$
We have to point out that these values contain appreciable theoretical and experimental uncertainties, as seen from their derivation (for the possible error bars see, for instance, Ref. [15]). In our analysis of the DNM contribution to $\mu^- - e^-$ conversion we take the numbers from Eq. (6) as central values of the scalar nucleon form factors. Note, in Ref. [8] the different set of the values for the nucleon scalar form factors was derived on the basis of the QCD sum rules input parameters, which overestimates the pion-nucleon sigma-term and the strangeness of the nucleon.

III. SCALAR MESON CONTRIBUTION

In the following we turn to the scalar meson-exchange mechanism of $\mu^- - e^-$ conversion. Although the status of scalar mesons is still unclear [16] we think it is reasonable to study their effect in the $\mu^- - e^-$ conversion since their contribution is associated with the experimentally most interesting coherent mode of this exotic process.

The lightest unflavored scalar mesons are the isoscalar $f_0(600)$ and the isotriplet $a_0(980)$ states. The former in the context of the nonlinear realization of chiral symmetry can be treated as a resonance in the $\pi\pi$ system (see detailed discussion, e.g. in Refs. [17,18]). For simplicity we neglect a possible small strange content of the isoscalar meson and treat this state as $\bar{u}u + \bar{d}d$.

We derive the LFV lepton-meson effective Lagrangian in terms of the interpolating $f_0$ and $a_0$ fields. Retaining all the interactions consistent with Lorentz invariance, we obtain the general form of this Lagrangian:

$$L_{\text{eff}}^{\text{ls}} = \frac{\Lambda_H^2}{\Lambda_{\text{LFV}}^2} \left[ (\xi_S f_0^S + \xi_P f_0^P) f_0 + (\xi_S^{a_0} f_0^S + \xi_P^{a_0} f_0^P) a_0^0 \right]$$

with the unknown dimensionless coefficients $\xi$ to be determined from the hadronization prescription. We assume that the Lagrangian to be generated by the quark-lepton Lagrangian (3), and, therefore, all its terms have the same suppression $\Lambda_{\text{LFV}}^2$ with respect to the large LFV scale $\Lambda_{\text{LFV}}$. Another scale in the problem is the hadronic scale $\Lambda_H \sim 1 \text{ GeV}$ which adjusts the physical dimensions of the terms in Eq. (12). In the Lagrangian in Eq. (12) we neglect derivative terms since their contribution to $\mu^- - e^-$ conversion is suppressed by a factor $(m_{\mu}/\Lambda_H)^2 \sim 10^{-2}$.

In order to relate the parameters $\xi$ of the Lagrangian (12) to the “fundamental” parameters $\eta$ of the quark-lepton Lagrangian (3) we use an approximate method based on the standard on-mass-shell matching condition [19]

$$\langle \mu^+ e^- | L_{\text{eff}}^{\text{iq}} | S \rangle \approx \langle \mu^+ e^- | L_{\text{eff}}^{\text{ls}} | S \rangle,$$

where $|S = f_0, a_0\rangle$ are the on mass-shell scalar meson states. We solve equation (13) using the quark current matrix elements

$$\langle 0 | \bar{u} u | f_0(p) \rangle = \langle 0 | \bar{d} d | f_0(p) \rangle = m_{f_0}^2 f_{f_0},$$

$$\langle 0 | \bar{u} u | a_0^0(p) \rangle = - \langle 0 | \bar{d} d | a_0^0(p) \rangle = m_{a_0}^2 f_{a_0}.$$

Here $p$, $m_S$ and $f_S$ are the scalar-meson four-momentum, mass and decay constant, respectively. The quark operators in Eq. (14) are taken at $x = 0$. In the numerical calculations we use the following values of scalar meson masses [16]:
The coupling constants $f_S$ in Eqs. (14) can be estimated using the linear $\sigma$-model in the case of the $f_0$ meson [20] and by QCD sum rules in the case of the $a_0$ [21]. In the linear $\sigma$-model one has the following relationship [20]:

$$\langle 0|\bar{u}u|f_0(p)\rangle = m_{f_0}^2 \frac{\sqrt{N_c}}{2\pi},$$

where $N_c = 3$ is the number of quark colors. Comparing Eqs. (14) and (16) we get

$$f_{f_0} = \frac{\sqrt{N_c}}{2\pi} = 0.28.$$  

The coupling constant $f_{a_0}^0$ of the neutral $a_0^0$ meson is related to the coupling constant $f_{a_0}^\pm$ of the charged $a_0^\pm$ state due to isospin invariance:

$$f_{a_0}^\pm = f_{a_0}^0 \sqrt{2},$$

with the definition $\langle 0|\bar{d}u|a_0^-(p)\rangle = m_{a_0}^2 f_{a_0}^\pm$. The value of $f_{a_0}^\pm$ was estimated in Ref. [21]:

$$f_{a_0}^\pm = \frac{0.0447 \text{ GeV}^3}{m_{a_0}^2 (m_s - \hat{m})}.$$  

Combining Eqs. (19) and (18) we have

$$f_{a_0}^0 = 0.19.$$  

Solving Eq. (13) with the help of Eqs. (14), we obtain the expressions for the coefficients $\xi$ of the lepton-meson Lagrangian (12) in terms of the generic LFV parameters $\eta$ of the initial (3) lepton-quark effective Lagrangian:

$$\xi_{b}^{a_0} = \left(\frac{m_{a_0}}{\Lambda_H}\right)^2 f_{a_0}^0 \eta_{bS}^{(3)}, \quad \xi_{b}^{f_0} = \left(\frac{m_{f_0}}{\Lambda_H}\right)^2 f_{f_0} \eta_{bS}^{(0)},$$

where $b = S, P$ and $\eta^{(0,3)} = \eta^u \pm \eta^d$.

For our analysis we also need the effective Lagrangian describing the interactions of the scalar mesons with nucleons. We take it in the following form:

$$\mathcal{L}_{SN} = \bar{N} \left[ g_{a_0 N N} \bar{a}_0 \gamma^\mu \gamma_5 f_0 N \right].$$

For the meson-nucleon couplings $g_{SN}$ we adopt the central values

$$g_{a_0 N N} \simeq g_{f_0 N N} \simeq 5$$

used in phenomenological description of nucleon-nucleon interactions and recently also calculated in the chiral unitary approach [18]. Since our analysis does not pretend to high accuracy we do not supply the error bars for the meson-nucleon couplings $g_{SN}$. 

\[ m_{f_0} = 500 \text{ MeV}, \quad m_{a_0} = 984.7 \text{ MeV}. \]
Now, having specified the interactions of the scalar mesons $f_0, a_0$ with leptons and with nucleons we can derive the scalar meson-exchange contributions to the $\mu^- e^-$ conversion. This contribution can be expressed in the form of the nucleon-lepton effective Lagrangian (5) which arises in second order in the Lagrangian $L_{eff} + L_{SN}$ and corresponds to the diagram in Fig.1b. We estimate this contribution only for the coherent $\mu^- e^-$ conversion process. In this case we disregard all the derivative terms of nucleon and lepton fields. Neglecting the kinetic energy of the final nucleus, the muon binding energy and the electron mass, the square of the momentum transfer $q^2$ to the nucleus has a constant value $q^2 \approx -m^2_\mu$. In this approximation the meson propagators convert to $\delta$-functions leading to effective lepton-nucleon contact type operators. Comparing them with the corresponding terms in the Lagrangian (5), we obtain the scalar meson exchange contribution to the coupling constants of this Lagrangian:

$$
\alpha_{bS[MEM]}^{(3)} = \beta_{a_S(0)} \eta_{bS}^{(0)} \\
\alpha_{bS[MEM]}^{(0)} = \beta_{f_0} \eta_{bS}^{(0)}
$$

with $b = S, P$ and the coefficients

$$
\beta_{a_0} = \frac{g_{a_0 NN} f_{a_0} m^2_{a_0}}{m^2_{a_0} + m^2_\mu}, \quad \beta_{f_0} = \frac{g_{f_0 NN} f_{f_0} m^2_{f_0}}{m^2_{f_0} + m^2_\mu}.
$$

Substituting the values of the meson coupling constants and masses we obtain for these coefficients

$$
\beta_{a_0} = 0.93, \quad \beta_{f_0} = 1.32,
$$

These values should be considered as rough estimates in view of the uncertainties in the scalar meson masses and couplings.

**IV. CONSTRAINTS ON LFV PARAMETERS FROM $\mu^- e^-$ CONVERSION**

From the Lagrangian (5), following the standard procedure, one can derive the formula for the branching ratio of the coherent $\mu^- e^-$ conversion. To leading order in the nonrelativistic reduction the branching ratio takes the form [1]

$$
P_{\mu e^-}^{coh} = \frac{Q}{2\pi \Lambda^4_{LFV}} \frac{p_e E_e (M_p + M_n)^2}{\Gamma_{\mu c}},
$$

where $p_e, E_e$ are 3-momentum and energy of the outgoing electron, $M_{p,n}$ are the nuclear $\mu^- e^-$ transition matrix elements and $\Gamma_{\mu c}$ is the total rate of the ordinary muon capture. The factor $Q$ takes the form

$$
Q = |\alpha_{VV}^{(0)} + \alpha_{VV}^{(3)} \phi|^2 + |\alpha_{AV}^{(0)} + \alpha_{AV}^{(3)} \phi|^2 + |\alpha_{SS}^{(0)} + \alpha_{SS}^{(3)} \phi|^2 + |\alpha_{PS}^{(0)} + \alpha_{PS}^{(3)} \phi|^2 + 2\text{Re}\{ (\alpha_{VV}^{(0)} + \alpha_{VV}^{(3)} \phi)(\alpha_{SS}^{(0)} + \alpha_{SS}^{(3)} \phi)^* + (\alpha_{AV}^{(0)} + \alpha_{AV}^{(3)} \phi)(\alpha_{PS}^{(0)} + \alpha_{PS}^{(3)} \phi)^* \}
$$

in terms of the parameters of the lepton-nucleon effective Lagrangian (5) and the nuclear structure factor.
\[ \phi = (M_p - M_n)/(M_p + M_n), \] (29)

that is typically small for the experimentally interesting nuclei.

The nuclear matrix elements \( M_{p,n} \) have been calculated in Refs. [8,9] for the nuclear targets \( ^{27}\text{Al}, ^{48}\text{Ti} \) and \( ^{197}\text{Au} \). We show their values in Table II together with the experimental values of the total rates \( \Gamma_{\mu e} \) of the ordinary muon capture [22] and the 3-momentum \( p_e \) of the outgoing electron. Using the quantities from Table II we find for the dimensionless scalar lepton-nucleon couplings of the Lagrangian (5) the following limits

\[ \alpha^{(k)}_{bS} \left( \frac{1\text{GeV}}{\Lambda_{LFV}} \right)^2 \leq 1.2 \times 10^{-12} \left[ B^{(k)}(A) \right]^{-2}, \] (30)

with \( k = 0, 3 \). Here the scaling factors \( B^{(k)}(A) \) depend on the target nucleus \( A \) used in an experiment setting the upper limit \( R_{\mu e}^A(\text{Exp}) \) on the branching ratio of \( \mu^- - e^- \) conversion: \( R_{\mu e}^A \leq R_{\mu e}^A(\text{Exp}) \). The numerical values of these factors for the experiments discussed in the introduction have been calculated with the help of Table II and are given in Table I.

From the limits in Eq. (30) one can deduce individual limits on the terms contributing to the coefficients \( \alpha^{(0,3)}_{bS} \), assuming the absence of significant cancellations (unnatural fine-tuning) between the different terms. In this way, using Eqs. (6), (24), we derive constraints on the 4-fermion quark-lepton LFV couplings of the Lagrangian (3) for the two considered mechanisms of hadronizations: the direct nucleon mechanism (DNM) and the meson-exchange mechanism (MEM). The corresponding limits are shown in Table 3. Following the common practice, we presented these limits in terms of the individual mass scales, \( \Lambda_{ij} \), of the scalar quark-lepton contact operators in Eq. (3). In the conventional definition [23] these scales determine the couplings \( z_{ij} = g^2/\Lambda_{ij}^2 \) of the 4-fermion contact terms of the form \( z_{ij}(\bar{l}_i l_j)(\bar{q}q) \) with a fixed \( g^2 = 4\pi \). Historically [23] this definition of \( \Lambda_{ij} \) originates from substructure models and corresponds to the compositeness scale in the strong coupling regime \( g^2/4\pi = 1 \). Thus, the individual mass scales \( \Lambda_{ij} \), introduced in this way, are related to our notations according to the formula:

\[ \frac{\eta^{(k)}_{bS}}{\Lambda_{LFV}^2} = \frac{4\pi}{\left( \Lambda_{\mu e}^{(k,b)} \right)^2}, \] (31)

with \( k = 0, 3, s \) and \( b = P, S \).

Let us compare our limits for these mass scales with the corresponding limits existing in the literature. The limits on \( \Lambda_{\mu e} \) can be derived from the experimental bounds on the rates of the \( \pi^+ \rightarrow \mu^+\nu_e, \pi^0 \rightarrow \mu^+ e^- \) decays [24]. Despite the scalar quark current does not directly contribute to these processes the limits come from the gauge invariance with respect to the SM group which relates the couplings of the scalar and pseudoscalar lepton-quark contact operators. Typical limits from these processes are of \( \Lambda_{\mu e} \geq \text{few TeV} \). In the future experiments at the LHC it is also planned to set limits on the mass scales of various contact quark-lepton interactions from the measurement of Drell-Yan cross sections in the high dilepton mass region [25,26]. In this case typical expected limits for the scales of the lepton flavor diagonal contact terms are \( \Lambda_{ll} \geq 35 \text{ TeV} \). We are not aware of the corresponding analysis for the limits on the LFV scales, like \( \Lambda_{\mu e} \), expected from the experiments planned at the LHC. However, one may expect these limits to be significantly stronger (typically by...
an order of magnitude) than the above cited limits for the lepton flavor diagonal mass scales $\Lambda_{ll}$. This is motivated by the fact that, usually a flavor diagonal process have much more SM background than the LFV processes. A comparison of the above limits with the limits in Table III, extracted from $\mu^- - e^-$-conversion, shows that the latter are more stringent.

The following comment is in order. As follows from Eq. (28) and Table II, the contribution of the isovector couplings $\alpha^{(3)}$ to the $\mu^- - e^-$-conversion rate, Eqs. (27) and (28), is suppressed by a small nuclear factor $\phi$. On the other hand, the information on the isovector couplings may be important for the phenomenology of the physics beyond the SM allowing one to distinguish the contribution of d and u quarks. In this respect, the contribution of the scalar mesons is important, since it is comparable on magnitude with the DNM mechanism. Taking this contribution into account we extracted reasonable constraints on mass scales of the isovector quark-lepton contact terms $\Lambda_{\mu e}^{(3)}$ presented in Table 3. The strange quark contributions are not affected by the meson exchange because we neglected the possible strange quark component of the scalar mesons.

V. SUMMARY

We analyzed the nuclear $\mu^- - e^-$ conversion in the general framework the effective Lagrangian approach without referring to any specific realization of the physics beyond the standard model responsible for lepton flavor violation. The two mechanisms of the hadronization of the underlying effective quark-lepton LFV Lagrangian have been studied: the direct nucleon (DNM) and the scalar meson-exchange (MEM) mechanisms. We showed that the scalar meson-exchange contribution is comparable on magnitude with the DNM mechanism and, therefore, can modify the limits on the LFV lepton-quark couplings derived on the basis of the conventional direct nucleon mechanism. These results lead us to the conclusion that the meson exchange mechanism may have an appreciable impact on the phenomenology of the LFV physics beyond the standard model and, therefore, should be taken into account in the analysis of the LFV effects in hadronic and nuclear semileptonic processes.

From the experimental upper bounds on the $\mu^- - e^-$-conversion rate we extracted the lower limits on the mass scales of the LFV lepton-quark contact terms involved in this process and showed that they are more stringent than the similar limits existing in the literature.

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### TABLES

**Table I.** The experimental upper limits on the $\mu^- - e^-$ conversion branching ratio $R_{\mu e}^A$ and the values of the scaling factors $B^{(0,3)}(A)$ from Eq. (30) for the running and forthcoming experiments.

| Target, Experiment | $R_{\mu e}^A$ $\times 10^{-13}$ (90% C.L.) | $B^{(0)}(A)$ | $B^{(3)}(A)$ |
|-------------------|----------------------------------------|-------------|-------------|
| $^{48}\text{Ti}$, SINDRUM II [3] | $6.1\times 10^{-13}$ | 1 | 0.3 |
| $^{27}\text{Al}$, MECO [5] | $\sim 2 \times 10^{-17}$ † | 11.5 | 1.7 |
| $^{197}\text{Au}$, SINDRUM II [3] | $\sim 6 \times 10^{-13}$ † | 1.27 | 0.46 |
| $^{48}\text{Ti}$, PRIME [6] | $\sim 10^{-18}$ † | 28 | 8.8 |

† expected upper limits.

**Table II.** Transition nuclear matrix elements $M_{p,n}$ from Eqs. (27), (29) and other useful quantities (see the text).

| Nucleus | $p_e (fm^{-1})$ | $\Gamma_{\mu e} (x10^6 s^{-1})$ | $M_p (fm^{-3/2})$ | $M_n (fm^{-3/2})$ |
|---------|-----------------|-----------------------------|-----------------------------|-----------------------------|
| $^{27}\text{Al}$ | 0.531 | 0.71 | 0.047 | 0.045 |
| $^{48}\text{Ti}$ | 0.529 | 2.60 | 0.104 | 0.127 |
| $^{197}\text{Au}$ | 0.485 | 13.07 | 0.395 | 0.516 |
Table III. Lower limits on the individual mass scales, $\Lambda_{\mu e}$, of the scalar quark-lepton contact operators in Eq. (3) inferred from the experimental upper bounds on the branching ratio of $\mu^- - e^-$ conversion for the direct nucleon mechanism (DNM) and the meson exchange mechanism (MEM). The superscript notation is $b = P, S$. The values of the scaling factors $B^{(0,3)}(A)$ for the running and some planned $\mu^- - e^-$ conversion experiments are given in Table 2.

| LFV Mass Scale | DNM               | MEM               |
|---------------|-------------------|-------------------|
| $\Lambda_{\mu e}^{(0,b)}$ | $5.8 \times 10^3 B^{(0)}(A)$ TeV | $3.7 \times 10^3 B^{(0)}(A)$ TeV |
| $\Lambda_{\mu e}^{(3,b)}$ | $2.3 \times 10^3 B^{(3)}(A)$ TeV | $3.1 \times 10^3 B^{(3)}(A)$ TeV |
| $\Lambda_{\mu e}^{(s,b)}$ | $2.6 \times 10^3 B^{(0)}(A)$ TeV | no limits |
FIGURES

Fig.1: Diagrams contributing to the nuclear $\mu^--e^-$ conversion in the scalar channel: direct nucleon mechanism (a) and meson-exchange mechanism (b).