Effects of Ringed Structures and Dust Size Growth on Millimeter Observations of Protoplanetary Disks

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Abstract

The growth of solids from submicron to millimeter and centimeter sizes is the early step toward the formation of planets inside protoplanetary disks (PPDs). However, such processes and their potential impact on the later stages of solid growth are still poorly understood. In this work, we test the hypothesis that most disks contain at least one ringed structure with a relatively small radius. We have carried out a large family of 1D two-fluid (gas+dust) hydrodynamical simulations by evolving the gas and dust motion self-consistently while allowing dust size to evolve via coagulation and fragmentation. We investigate the joint effects of ringed structures and dust size growth on the overall submillimeter and millimeter flux and spectral index of PPDs. Ringed structures slow down the dust radial drift and speed up the dust growth. In particular, we find that those unresolved disks with a high fragmentation velocity (~10 m s⁻¹) and a high dust surface density (~10 g cm⁻² in the ring) can have millimeter spectral indices as low as ~2.0, consistent with millimeter observations of faint disks in nearby star-forming regions. Furthermore, disks with more than one ringed structure can potentially reproduce brighter disks with spectral indices lower than ~2.5. Future multiwavelength high-resolution observations of these low-spectral-index sources can be used to test the existence of the ringed structures in the unresolved disks and differentiate the effects of dust size growth from optical depth.

Key words: accretion, accretion disks – planets and satellites: rings – protoplanetary disks – submillimeter: planetary systems

1. Introduction

The discovery of thousands of exoplanets over the last couple of decades has clearly shown that the birth of planets is a very efficient process in nature (e.g., Burke et al. 2015). The commonly accepted theory is that planets form in young disks orbiting pre-main-sequence (PMS) stars through the agglomeration of small micrometer-sized dust particles into kilometer-sized “planetesimals,” which are massive enough to gravitationally attract other solids in the disk (see review by Chiang & Youdin 2010). Grain growth from submicrometer sizes, which are the typical sizes of dust in the interstellar medium (ISM), to millimeter and centimeter sizes is thus the first step toward the formation of planetesimals inside young circumstellar disks. However, the formation of planetesimals is still poorly understood, due to (1) the low sticking efficiency of grains larger than ~1 mm to cm, and (2) the mutual dynamical interaction between dust grains and the gas in the disk. In the case of a gas-rich disk with density and temperature both decreasing with distance from the central star, small solids radially drift inward because of the aerodynamic drag of the gas orbiting at sub-Keplerian speeds (Weidenschilling 1977; Brauer et al. 2007). Models of the evolution of disk solids have predicted radial drift timescales that are too short (100 to 1000 orbits) to form planetesimals (see review by Johansen et al. 2014). Understanding dust coagulation might be of fundamental importance to understanding the formation of planetesimals.

Once dust particles grow from ISM-like sizes of micrometers to millimeter or centimeter sizes, they can be mainly traced by their continuum emission at submillimeter and longer wavelengths. Therefore, submillimeter observations have the potential of setting strong constraints on the initial stages of the planet formation process. In the last two decades, several observations of protoplanetary disks (PPDs) at submillimeter wavelengths have reported relatively shallow slopes with values of the millimeter spectral index 2.0 ≤ αₘₘₘ ≤ 3.0 (Fₙ ∝ ν⁻αₘₘₘ). These values are significantly lower than those measured for dust in the ISM and are taken as observational evidence for dust growth to millimeter-sized grains in disks under the assumption that the dust emission is mostly optically thin (e.g., Testi et al. 2003; Natta et al. 2004; Rodmann et al. 2006; Ricci et al. 2010a; Pinilla et al. 2014; Andrews 2015; Ansdell et al. 2018). Ricci et al. (2012) explored the effect of local optically thick emission regions in the millimeter emission without millimeter to centimeter grains. They found that optically thick regions with relatively small filling factors can reproduce the millimeter spectral indices of disks without grain growth, although the dust overdensities required by the optically thick effect are generally larger than those predicted by the typical known physical processes proposed in the literature. These processes include vortex formation, streaming instabilities, and disk viscosity transitions (e.g., Lovelace et al. 1999; Li et al. 2000, 2001; Klahr & Bodenheimer 2003; Rice et al. 2004; Fromang & Nelson 2005; Johansen & Youdin 2007; Johansen et al. 2009; Boss 2010; Regály et al. 2012).
Birnstiel et al. (2010b) have tested the grain growth model with millimeter observations of the dust continuum, and they found that it can naturally reproduce the observed millimeter slope if the radial drift of the dust is halted by an unknown mechanism. It is generally believed that some inhomogeneities in the gas density, either azimuthally symmetric, such as rings, or with azimuthal asymmetries, such as vortices, have to be invoked to slow down the dust radial drift because of the gas pressure structure (e.g., Pinilla et al. 2012, in the case of dust trapping by azimuthally symmetric rings). The dust can, therefore, be trapped effectively. Grains can collide frequently and stick together by van der Waals forces, which leads to the formation of large-size dust particles (e.g., Dominik & Tielens 1997; Poppe et al. 2000; Blum & Wurm 2008; Brauer et al. 2008; Birnstiel et al. 2010a). Such dust accumulation regions can appear as bright rings in PPDs observed from the dust continuum emission at millimeter wavelengths. There is now strong observational evidence for the existence of multiple rings (or rings sandwiched by gaps) in disks from high-resolution observations with the Atacama Large Millimeter Array (ALMA), for example, HL Tau (ALMA Partnership et al. 2015), TW Hya (Andrews et al. 2016; Tsukagoshi et al. 2016), HD 163296 (Isella et al. 2016), AA Tau (Loomis et al. 2017), Elias 24 (Cieza et al. 2017; Cox et al. 2017; Dipierro et al. 2018), AS 209 (Fedele et al. 2018), GY 91 (Sheehan & Eisner 2018), V1094 Sco (Andsell et al. 2018; van Terwisga et al. 2018), MWC 758 (Boehler et al. 2018), more disks in a recent survey of young disks in Taurus (Long et al. 2018), 20 disks from the Disk Substructures at High Angular Resolution Project (DSHARP) observed very recently (Andrews et al. 2018; Dullemond et al. 2018; Huang et al. 2018b), and 16 disks from ALMA archival data (van der Marel et al. 2019). These high-resolution observations have also revealed that these narrow rings have relatively high optical depth at the millimeter band (e.g., ALMA Partnership et al. 2015; Tsukagoshi et al. 2016; Andrews et al. 2018; Dullemond et al. 2018; Isella et al. 2016, 2018; Huang et al. 2018a, 2018b). Dullemond et al. (2018) find that the narrow radial size of the rings strongly supports the dust-trapping scenario. Such dust traps can also facilitate grain growth.

Motivated by the fact that dust rings appear to be ubiquitous at least in the disk sample observed so far, we explore their consequences for the overall (sub)millimeter disk emission properties. For the sake of simplicity, we use a nonmonotonic disk viscosity profile (e.g., Figure 1) to produce a local gas surface density bump. We perform detailed two-fluid (gas + dust) hydrodynamics simulations, in which the dust coagulation and disk hydrodynamics evolution are treated together, and we calculate the dust continuum emission at several millimeter bands with detailed radiative transfer. Our main goals are to interpret the low spectral indices observed for many young disks and understand the link between the spectral slope and the dust dynamics in the disks. Based on the comparison between our simulations and the observed dust emission, we can constrain some fundamental parameters for the dust coagulation model and provide new insight into the mechanism of dust trapping and growth.

The paper is organized as follows. The numerical simulation method adopted in this work is presented in Section 2. We present the results in Section 3, and we summarize the main results and discuss the physical implications in Section 4.

2. Model

We envision the following scenario: the disk gas surface density profile contains at least one ring in the inner disk region (which is unresolved for most disks currently). By performing 1D hydrodynamical simulations using LA-COMPASS (Li et al. 2005, 2009; Fu et al. 2014), we study the gas plus dust dynamics spanning several million years, including dust coagulation and fragmentation self-consistently (Birnstiel et al. 2010a), and then we produce the dust emission at different millimeter wavelengths by making use of the RADMC-3D package (Dullemond et al. 2012) to compare with observations.

2.1. Hydrodynamics and Model Assumption

Without loss of generality, we consider a disk around a PMS star with a mass of 0.6 $M_\odot$ at a distance of 140 pc. The disk extends from 5 to 240 au. Disk self-gravity in gas and dust is not included. To create dust rings in such disks, we use varying disk viscosity radial profiles to achieve such configurations. The gas viscosity is adopted from the $\alpha$-prescription $v_g = \alpha_{vis}c_s h_g$ with $h_g$ being the gas disk height and $\alpha_{vis}$ the dimensionless viscosity parameter (Shakura & Sunyaev 1973). For simplicity, in our fiducial model, $\alpha_{vis}(r) = \alpha_0 = 10^{-2}$ for most of the disk. We also assume a rectangular gap with a low viscosity $\alpha_{vis}$ in its radial profile. The viscosity gap is centered at 15 au with a gap width $w_{ring} = 5$ au and a value for the viscosity in the dip $\alpha_{min} = \alpha_0/(1 + d_{ring}) = 1.7 \times 10^{-3}$, which is shown in Figure 1. As we will see, the viscosity gap can produce a gas bump, which can then slow down the radial drift of dust particles in the disk, creating a dust ring.

The existence of a dust ring in the disk, though a hypothesis in this paper, is well motivated by current high-resolution observations mentioned above. Actually, any mechanism that can produce a similar gas bump, even without invoking a viscosity jump, could produce results similar to the ones discussed here. Both a viscosity transition (e.g., Gammie 1996; Varnière & Tagger 2006) and the presence of a planet (e.g., Bryden et al. 1999; Papaloizou et al. 2007; D'Angelo et al. 2015; Jin et al. 2016; Dong et al. 2017; Liu et al. 2018; Zhang et al. 2018) could lead to such a dust ring configuration. The viscosity transition can be characterized by a gap in the viscosity around a few astronomical units (e.g., see review by Armitage 2011), which is slightly smaller than the radius of our

![Figure 1. Assumed disk viscosity radial profile $\alpha_{vis}$ for most of our models.](image)
viscosity gap. However, the inner edge and the radial width of the dead zone are actually not well constrained and will depend in general on the ionization sources and the physical structure of the disk, as well as on the intensity of the magnetic field in the disk itself. Given these uncertainties, it makes sense to explore a broad range of values for the central radius and width of the viscosity jump, as will be done in this work. For the planet–disk interaction scenario, some kinematic evidence of protoplanets forming within ringed-structure disks have been found recently based on the detection of a localized deviation from Keplerian velocity in HD 163296 (Pinte et al. 2018; Teague et al. 2018), although other possibilities (e.g., zonal flow) for the inner ringed structures cannot be fully ruled out yet.

The initial gas surface density profile $\Sigma_g(r)$ is set as follows:

$$\Sigma_g(r) = \Sigma_0 \left( \frac{r}{r_c} \right)^{\gamma - 1} \exp \left[ - \left( \frac{r}{r_c} \right)^2 \right],$$

(1)

where $r_c = 60$ au and $\gamma = 1$ unless otherwise stated. These parameters represent a typical disk as seen from observations (Andrews et al. 2010). The normalization of gas surface density $\Sigma_0$ is treated as a parameter listed in Table 1. The locally isothermal sound speed $c_s$ is chosen as

$$\frac{c_s}{v_K} = h_0 \left( \frac{r}{r_0} \right)^{0.25},$$

(2)

where $v_K(r)$ is the local Keplerian velocity, and $r_0 = 5$ au unless otherwise stated. The parameter $h_0$ is listed in Table 1 with a typical value of 0.04. This corresponds to a disk temperature profile of $T \propto r^{-0.5}$. Equation (2) also expresses the radial profile of the gas scale height $h_g/r$. The gas and dust fluids are evolved following the conservation of mass, radial, and angular momentum equations.

The continuity equation of gas is

$$\frac{\partial \Sigma_g}{\partial t} + \nabla \cdot (\Sigma_g v_g) = 0,$$

(3)

where $v_g$ is the gas fluid velocity. The momentum equation for the gas is

$$\frac{\partial (\Sigma_g v_g)}{\partial t} + \nabla (v_g \cdot \Sigma_g v_g) + \nabla P = -\Sigma_g \nabla \Phi + \Sigma_g f_g - \Sigma_d f_d,$$

(4)

where $f_d$ indicates the viscous force from the Shakura–Sunyaev disk (Shakura & Sunyaev 1973). We adopt an equation of state $P = \Sigma_g \gamma g$ for the gas component, where $P$ is the vertically integrated gas pressure. Here, $\Phi$, is the gravitational potential of the central star, $f_d$ is the summation of $f_d^i$ over $i$, and $f_d^i$ is the drag force between the gas and dust species $i$, which is defined as

$$f_d^i = \frac{\Omega_k}{St'}(v_g - v_d^i),$$

(5)

where $\Omega_k$ is the Keplerian angular velocity. Here, $St'$ is the Stokes number of the dust species $i$. We can estimate this feedback term based on Takeuchi & Lin (2002). In the Epstein regime for the disk and dust parameters of interest here, the Stokes number of the particles with dust radius $a$ in the midplane of the disk is defined as

$$St' = \frac{\pi \rho_a a^2}{2 \Sigma_g},$$

(6)

where $\rho_a$ is the solid density of the dust particles. The dust size corresponding to $St = 1$ is thus

$$a_{St=1} = \frac{2 \Sigma_g}{\pi \rho_a}.$$

(7)

The Stokes number in the Epstein regime can then be rewritten as $St' = a'/a_{St=1}$.

We treat the dust component as a pressureless fluid. The dust feedback, that is, the drag forces between the gas and dust, are incorporated into the momentum equation for both the gas and dust (Fu et al. 2014). The continuity equation of dust species $i$ is

$$\frac{\partial \Sigma_d^i}{\partial t} + \nabla \cdot (\Sigma_d^i v_d^i) = \nabla \cdot \left( \Sigma_g D_d^i \nabla \frac{\Sigma_d^i}{\Sigma_g} \right).$$

(8)

Here, $D_d^i$, which describes the dust diffusivity, is related to the Stokes number $St$ as $D_d = \nu_g/(1 + St^2)$ (Youdin & Lithwick 2007), $\nu_g$ is the gas viscosity, $\Sigma_d^i$ and $v_d^i$ are the dust surface density and velocity for species $i$, respectively. The momentum equation for dust species $i$ after including the drag force from the gas is

$$\frac{\partial \Sigma_d^i v_d^i}{\partial t} + \nabla (v_d^i \cdot \Sigma_d^i v_d^i) = -\Sigma_d \nabla \Phi + \Sigma_d f_d^i.$$  

(9)

For illustrative purposes, the radial velocity of the dust can be estimated as

$$v_{r,d} = \frac{St^1 v_{r,g} - \eta v_K}{St^{-1} + St},$$

(10)

where $\eta = -\frac{c_s^2}{v_K} \frac{\partial \ln P}{\partial r}$, and

$$v_{r,g} \approx -\frac{3}{\Sigma_g \sqrt{f}} \frac{\partial}{\partial r} \left( \frac{\Sigma_g v_g \sqrt{f}}{\Sigma_g} \right).$$

(11)

is the radial velocity of the gas. Here we omit the superscript of $i$ for different dust species. We can see that the dust radial velocity is composed of two terms, namely gas drag (first term) and radial drift (second term) in Equation (10). Then the drag force between dust and gas for each dust species can be obtained by combining Equations (5), (10), and (11).

To model the dust size growth during disk evolution, we calculate the evolution of the dust size distribution, which is discretized logarithmically between 1 $\mu$m and 100 cm. We use 25 bins per size decade, which leads to 151 dust species with their size $a_d$. Such a dust size resolution can avoid an artificial dust size growth, due to some numerical diffusion in the Smoluchowski algorithm (Smoluchowski 1916), and ensure the convergence of dust growth rate (Ohtsuki et al. 1990).

The dust size evolution is computed within each spatial cell and is handled via an operator splitting approach between hydrodynamics and dust coagulation or fragmentation. Due to the high computational cost of solving dust coagulation, we implement a substepping routine, where the dust coagulation
Note. For all models, we adopt typical stellar parameters with $M_∗ = 0.6 M_\odot$, $R_∗ = 2 R_\odot$, and $T_{\text{eff}} = 3850$ K.

Module is executed every 50 time steps of the hydrodynamical simulations, which still gives a good calculation accuracy for the coagulation algorithm. The dust coagulation and fragmentation model follows Birnstiel et al. (2010a) with an explicit integration scheme by solving the Smoluchowski equation, which includes the radial drift and turbulent mixing as the sources of collision velocities. Turbulence is the major source of collision velocity. Since this velocity increases with Stokes number, we can derive the maximum size of grains that can grow before the impact velocity exceeds the fragmentation threshold velocity $v_f$, as (Birnstiel et al. 2012; Pinilla et al. 2012)

$$a_{\text{max}} = \frac{4 \Sigma_g v_f^2}{3 \pi \alpha_{\text{visc}} \rho_g c_s^2}. \quad (12)$$

Note that this $a_{\text{max}}$ is only valid for grains with $St \lesssim 1$. This size limit for dust coagulation is also referred to as the fragmentation barrier. The inverse dependence on viscosity parameter exists because the turbulent velocity depends on $\alpha_{\text{visc}}$.

We adopt the following procedure in simulations. Initially, only $1 \mu$m sized dust particles are included in the disk, and the surface density distribution follows the radial profile of the gas with an initial radial-independent dust-to-gas mass ratio of $0.01$. The evolution for each dust species $a_i$ and the gas component is determined by the mass and momentum equations described above. We can self-consistently consider the hydrodynamics of both the gas and multiple dust species with the full dust coagulation and fragmentation. Previous studies by Pinilla et al. (2016); see also Birnstiel et al. 2010a; Pinilla et al. 2012, 2014) have produced very interesting results on the effects of dust size growth during the disk evolution. In our current study, we treat the gas–dust dynamical interaction self-consistently, which goes beyond some of the previous work where the gas evolution is not considered (Pinilla et al. 2012, 2014). Birnstiel et al. (2010a; see also Pinilla et al. 2016)
has considered the viscous evolution of the gas, but without taking into account the full coupling (dust feedback) between the gas and dust.

Unless otherwise stated, we solve the 1D hydrodynamics equations with a logarithmically radial grid of \( n_r = 1024 \). An outflow inner boundary condition and outer boundary condition are imposed on the dust and gas, which allow the gas and dust to flow out and flow in from the boundary depending on their radial velocity.

2.2. Radiative Transfer

After having obtained the dust surface density distribution in disks, the dust continuum emission at different millimeter wavelengths was computed using the RADMC-3D package (Dullemond et al. 2012). A two-stage procedure is adopted for modeling the dust continuum radiative process. We first convert the 1D dust surface density produced from hydrodynamical simulation into a 3D distribution by assuming a dust scale height \( h_d(r) = 0.1 h_g(r) \) with azimuthal symmetry as adopted in previous works (e.g., Isella et al. 2016; Liu et al. 2018). The vertical density structure is simply scaled as \( \Sigma_d \propto e^{-z^2/2h_d^2} \). We use 300 grids along the radial direction with a grid refinement around the bump and 40 uniform grids in the \( \theta \) direction between 70° and 90° with a mirror symmetry in the equatorial plane. This grid can recover all of the dust mass from 1D hydrodynamics within a 5% uncertainty. A larger grid number does not change the results.

We then ran RADMC-3D simulations to compute the dust temperature \( T_d(r, z) \) contributed by the stellar radiation. For simplicity, the dust temperature for all dust species is assumed to be the same and controlled by the microsized dust particles. The dust opacity coefficient, which depends on grain sizes, chemical compositions, and shapes, is another important quantity for the modeling. We adopted the dust opacity as a function of wavelength and dust size from Isella et al. (2009) and Ricci et al. (2010b). Figure 2 presents the details of this model. We will also discuss how a different dust opacity can affect our results in the following sections (Semenov et al. 2003). The dust opacity is dominated by the diagonal region with \( \lambda < 2\pi a \) and \( \kappa \) decreases with \( a \).

With the dust temperature \( T_d \) obtained from RADMC-3D and the dust surface density \( \Sigma_d \) for each dust species derived from hydrodynamical simulations, we can calculate the continuum emission for each dust species by ray-tracing with RADMC-3D. The surface brightness of the continuum emission can be approximated as

\[
I(r) = B_\nu(T_d(r))(1 - e^{-\tau_\nu(r)}),
\]

where the optical depth \( \tau_\nu(r) = \kappa_\nu \Sigma_d(r) / \cos \theta \), \( B_\nu \) is the Planck function, and \( \theta \) is the disk inclination. We assume the disk inclination angle of \( \theta = 45^\circ \) and a position angle of \( -30^\circ \) for all our models without losing generality. Using these quantities, we compute the corresponding spectral energy distribution and the radial profile of the surface brightness at several wavelengths between 0.89 mm and 7.0 cm using RADMC-3D.

In regions of the disk where the emission at a given wavelength is optically thick, the dust continuum flux and slope are fully determined by \( B_\nu(T_d(r)) \), which results in a spectral slope of 2.0 in the Rayleigh–Jeans limit. Conversely, in the optically thin regime, the dust emission is related to the opacity coefficient as \( F_\nu \propto \kappa_\nu B_\nu \). If the emission is in the Rayleigh–Jeans limit, then \( F_\nu \propto \nu^{2+\beta} \), where \( \beta \) is the slope of the dust opacity coefficient \( (\kappa_\nu \propto \nu^\beta) \). Note that \( \beta \) changes as dust grows bigger, and \( \beta \) can approach zero once a dust particle grows to a size larger than a few millimeters, as seen in the lower panel of Figure 2. This suggests that dust growth can be an important factor for setting the spectral shape of the dust continuum in the optically thin regions of the disk (e.g., Wilner et al. 2000, 2005; Testi et al. 2001, 2003; Draine 2006).

3. Results

In this section, we describe our results for modeling the gas and dust dynamics for various disks, and we discuss the comparison between the dust continuum observations of some young disks observed at millimeter wavelengths and the predictions of our models. Based on the comparisons with
global spectral index and disk brightness with current observations, we aim to set some constraints on the disk and coagulation parameters (e.g., disk viscosity, fragmentation velocity). The different behaviors of radial profiles for the spectral indices observed at different wavelengths in the future can also be used to break the model degeneracy in producing a low spectral index.

We first discuss the dust dynamics based on the coagulation and fragmentation model presented above. The main model parameters are listed in Table 1; otherwise their default values are adopted.

### 3.1. Fiducial Model

Here we start by discussing the model labeled as m1v2 in Table 1, which is characterized by \( \Sigma_0 = 4.3 \text{ g cm}^{-2} \) and fragmentation velocity \( v_f = 10^3 \text{ cm s}^{-1} \). The initial disk gas mass is \( 0.01 M_\odot \).

After incorporating the low-viscosity rectangular gap between 12.5 and 17.5 au as shown in Figure 1, we ran the numerical simulation for the two-fluid gas and dust evolution with dust coagulation and fragmentation for 2.1 Myr, which corresponds to 14,700 orbits at 5 au. The dust settles down into an equilibrium state with a balance of coagulation and fragmentation after \(~2\) Myr.

The radial profile of the gas surface density is shown at different times in the upper panel of Figure 3. We generically separate this profile into the bump region and the smooth region in the following discussions. The gas bump is built up quickly in the viscosity-gap region. In the case of mass conservation without disk wind, steady accretion implies that \( r \Sigma_v \nu_g = \) constant, and \( \nu_g \propto \Sigma_g/r = \alpha_{vis} \nu_v \Sigma_g/r \), so that \( \Sigma_g \propto \alpha_{vis} \). Therefore, a lower \( \alpha_{vis} \) in the gap region leads to a higher gas surface density. The minimum Toomre Q parameter (\(~10\)) is reached in the gas bump region at the initial stage, justifying that the disk is gravitationally stable during the whole evolution.

We also show the radial distribution of the total dust surface density \( \Sigma_d \), which sums up all of the dust surface densities of each species \( \sigma_d \), in the middle panel of Figure 3. The dust accumulation in the gas bump region is clearly shown in the radial distribution of the dust surface density plot. We can see that the dust-to-gas ratio can increase from the initial value of 0.01 to 0.5 at the final stage in the dust-trapping region, where the dust feedback could become important (Fu et al. 2014; Miranda et al. 2017).

A well-defined power-law distribution for the sizes of the dust particles in the dust-trapping region is shown in the lower panel of Figure 3. The red dotted line shows the fitted power law with an index of 0.4, which is equivalent to a power-law index of \( q = 3.6 \) for the number density distribution \( n(a) \propto a^{-q} \), close to the value of 3.5 for the dust size distribution in the ISM (Mathis et al. 1977).

The evolution of the total disk gas and dust masses is shown in Figure 4, where the masses within the “bump” region 12.5–17.5 au are plotted as well. As expected, the total disk mass has been decreased significantly by accretion during the 2.1 Myr evolution. The relative fraction of disk gas mass within the bump has increased. The total dust mass has been reduced by about 30%, and, by 2.1 Myr, most of the dust mass is now residing within the bump region. The ring is essential to retaining the dust mass from the rapid radial drift and loss through the disk boundary.

We further show the 2D contour plot of the dust surface density distribution for all dust species at all stellocentric radii throughout the disk in Figure 5. The fragmentation barrier, which was defined in Equation (12), is represented as the red dashed line. It can be seen that the bump in the fragmentation barrier that limits the dust size growth can be attributed to both the smaller viscosity and the larger gas surface density. The grain sizes corresponding to \( q_s = 1 \) (refer to Equation (7)) are shown as the white line, which has the same slope as the gas surface density, that is, a ringed structure around 15 au as in the
upper panel of Figure 3. The fact that all of the grain species in this model lie below the white line representing \( a_{\text{St}} = 1 \) indicates that \( \text{St} < 1 \) applies to all grains.

As expected, the dust spatial distribution shows significant trapping in the gas bump region, as shown in Figure 5 (see also the middle panel of Figure 3). This is because the radial inward motion of the dust is significantly slowed down as a result of the positive pressure \( (\Sigma_\text{d} c_s^2)^{-1} \) gradient in the inner edge of the gas bump. The large local concentration of dust grains retained in the gas bump can also facilitate the growth of dust particles from the initial 1 \( \mu \)m size to a few centimeters through frequent collisions and sticking of small grains. Some particles above the fragmentation barrier shown in Figure 5 are due to particle diffusion. The larger particles drift faster, which leads to a sharper trap in the inner edge of the bump for the larger grains. Both dust trapping and size growth are important in determining the dust continuum emission.

Outside the gas bump region, the dust surface density decreases significantly with time, due to its radial drift. In addition, the fragmentation barrier defined as in Equation (12) is significantly lower outside the bump region than inside the bump due to the radial viscosity profile we adopt, which leads to the average dust size (weighted by the dust surface density) being much smaller in the smooth region of the disk. This also explains an increase in dust size with a decreasing stellocentric radius \( r \) because the dust surface density becomes increasingly higher toward the inner region of the disk, which results in a higher fragmentation barrier.

We then calculate the dust temperature using RADMC-3D as described in Section 2.2. The radial profile of the temperature in the midplane of the disk is shown in Figure 6. The temperature profile is very consistent with the \( r^{-0.5} \) scaling expected for a disk heated by the central star, except for a bump around 15 au (Armitage 2010). The dust temperature profile is also roughly in agreement with the profile in our hydrodynamical simulation based on Equation (2), which is shown as a black dotted line in Figure 6. The temperature bump is associated with the dust bump feature in the same region, which results from the more efficient absorption (Bjorkman & Wood 2001). A higher dust temperature in the bump could decrease the dust size in that region. But this is a small modification in dust size, so it cannot significantly influence the dust emission, since the current maximum size is much bigger than \( \sim 1 \) mm. It is expected that the temperature in the midplane is lower than that at the disk surface in the outer disk region, while the temperature in the inner edge of the disk is close to the temperature at the disk surface determined by the stellar heating. This results in the steep slope for the midplane temperature profile in the inner disk. The total emission from the dust in this very inner region is, however, negligible, due to the significant dust depletion discussed above.

Due to the dust accumulation in the gas bump region, the local effective optical depth \( \tau_{\text{eff}} \) increases significantly. The radial profiles of the effective optical depth \( \tau_{\text{eff}} \) for all dust species, which are obtained by summing the optical depth over all dust species, are shown in the upper panel of Figure 7. We show the optical depth at two wavelengths, 1 and 7 mm, and at two epochs, close to the initial stage (0.04 Myr) and at the end.

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**Figure 4.** Time evolution of the disk gas and dust mass. Solid (dashed) lines correspond to gas (dust) mass. Black lines show the mass from the ring region, while red ones show the total disk mass. For the purpose of this presentation, the gas masses are multiplied by \( 10^{-2} \).

**Figure 5.** Dust surface density distribution for different dust species at time 2.1 Myr for model m1v2. The red dashed line indicates the fragmentation barrier, which represents the maximum size of the particles before they reach fragmentation velocities. The white solid line denotes the grain size corresponding to a Stokes number of unity, which shows the same shape as the gas surface density \( \Sigma_g \). The black and gray vertical dashed lines indicate the location of \( r = 13.5 \) au and \( r = 15.0 \) au, respectively, between which we extract the dust size distributions plotted in the lower panel of Figure 3.

**Figure 6.** Radial profile of the dust temperature at the equatorial plane of a disk for our fiducial model. For comparison, we also show the temperature profiles with different disk scale height \( h_0 \) and gas surface density profile index \( \gamma \). The dust temperature profile in the outer disk is close to the \( r^{-2.5} \) power law shown as the black dotted line, which is the temperature profile of our fiducial model adopted in hydrodynamical simulations based on Equation (2).
Two evolution stages (2.1 Myr). At the wavelength of 1 mm, the dust-trapping region is optically thick, while other regions are still optically thin at the end of the simulation (2.1 Myr). At the longer wavelength of 7 mm, the emission becomes marginally optically thick even in the dust-trapping region. Note that the optical depth in the ring is significantly higher than that inferred from DSHARP samples (Dullemond et al. 2018). A lower gas surface $\Sigma_0$ can make a better agreement with these observations. Furthermore, the inclusion of dust scattering can result in an optically thick disk appearing as optically thin (Zhu et al. 2019). The dust optical depth in the trapping region does not evolve significantly with time, but it decreases significantly outside the trapping region at millimeter wavelengths. This is mostly due to the dust radial drift effect.

After we obtained the dust size and radial distribution, its temperature, and optical depth for all dust species, we can obtain the radial profile of the dust continuum emission at different submillimeter and millimeter bands with a detailed radiative transfer calculation using RADMC–3D, which is shown in the lower panel of Figure 7. The flux at different bands can be obtained by integrating the dust continuum emission over the whole disk. We can then calculate the corresponding global spectral index and the 1 mm dust continuum flux. The results for model m1v2 are shown in Table 1 and Figure 8. In Figure 8, we show the time variation of the spectral index calculated between three different submillimeter to millimeter wavelengths as a function of the 1 mm flux density. The 1 mm flux decreases significantly at the early stage of the evolution, and then shows less temporal variations after $\sim 1.7$ Myr. Such a decreasing flux is a consequence of the dust radial evolution as shown in the middle panel of Figure 3. In the early stages of the disk evolution, the region outside the trapping region contributes a comparable fraction to the total flux of the disk. The outer disk contribution is significantly reduced as time evolves to $\sim 1.7$ Myr. This is also confirmed by the radial profile of the optical depth, as shown in the upper panel of Figure 7. The emission outside the bump region could be initially optically thick for a massive disk (as is the case for model m1v2; see the upper panel of Figure 7), and it then becomes optically thin due to the dust radial drift. The spectral slope from the optically thin emission in the smooth region can be as large as about 3.5 since its value is determined by a population of small-sized dust, as shown in the lower panel of Figure 7. Therefore, the global spectral index could stay at a low value initially due to a large fraction of optically thick emission. It then increases with time rapidly when the emission begins to transfer to the optically thin regime, as long as the flux contribution from the smooth region is still comparable to that from the bump. As the disk evolves further, the contribution from the smooth region becomes increasingly negligible, which means that the global spectral index is mainly determined by the emission from the bump, and then the global spectral index decreases with a decreasing flux, as shown in Figure 8.

In addition to the spectral index $\alpha_{1.0-3.0\, \text{mm}}$, we further calculate the index between 0.89 and 1.3 mm, or $\alpha_{0.89-1.3\, \text{mm}}$, which can be compared with the observations from the ALMA Lupus survey (Ansdell et al. 2018). It can be seen that the time evolution patterns of the spectral index in these two spectral windows are quite similar, due to the closeness of the two windows.

Note that the opacity we have adopted (Isella et al. 2009; Ricci et al. 2010b) is higher by a factor of a few ($\sim 5$) than that adopted in other works (e.g., Semenov et al. 2003; Birnstiel et al. 2018). To test the sensitivity of our spectral results on the
choice of dust opacity, we adopt a norm silicates (NRM) dust opacity model.\textsuperscript{11} It turns out that the global spectral index around 1 mm changes only slightly, and the final disk flux \(F_{1\text{ mm}}\) decreases by a factor of \(\sim 2\). The decrease in the total flux occurs mainly because the optical depth in the trapping region becomes lower. Such modifications do not significantly change our main conclusions. In the following discussion, we will only adopt the opacity from Isella et al. (2009; Ricci et al. 2010b) unless otherwise stated.

### 3.2. Dependence on Parameters

In this section, we study how our main results depend on the model parameters we have used. The main parameters we will explore include the fragmentation velocity \(v_f\), the initial gas surface density \(\Sigma_0\), disk global viscosity \(\alpha_0\), viscosity gap (ring) width \(w_{\text{ring}}\), gap (ring) depth \(d_{\text{ring}}\), gap (ring) location \(r_{\text{ring}}\), disk scale height \(h_0\), and disk gas profile index \(\gamma\), which are listed in Table 1. In the following, we evolve all of the disk models for 2.1 Myr when the dust/gas dynamics has reached the equilibrium state.

#### 3.2.1. Fragmentation Velocity

We first study the effect of fragmentation velocity \(v_f\) of the dust coagulation model. We ran several models with \(v_f\) in the range of \(1 \times 10^2\) to \(3 \times 10^3\) km s\(^{-1}\) while fixing all other parameters. The final global spectral index and 1 mm flux for models with varying fragmentation velocity are shown in the upper left panel of Figure 9.

In comparison with the case of \(v_f = 1 \times 10^3\) km s\(^{-1}\) in Figure 8, the spectral index around 1 mm \(\alpha_{1.0-3.0\text{ mm}}\) is insensitive to \(v_f\) until \(v_f\) decreases to \(1 \times 10^2\) km s\(^{-1}\). The spectral index \(\alpha_{1.0-3.0\text{ mm}}\) for a very small \(v_f = 1 \times 10^2\) km s\(^{-1}\) can reach a large value of \(\sim 3.5\). The two effects contribute to this large index. As \(v_f\) becomes very small (i.e., lowering the fragmentation barrier), it limits the maximum grain size obtained from coagulation. With the decrease of \(v_f\) by one order of magnitude, the maximum dust size decreases by two orders of magnitude, giving the largest grain sizes around only \(\sim 2\) mm. In addition, without the large dust grains, the dust emission contributed by the small dust becomes optically thin. Such a large population of small-sized grains with optically thin emission thus produces emission with a high spectral index.

When the fragmentation velocity becomes even larger (\(v_f = 3 \times 10^3\) km s\(^{-1}\)), most of the dust mass now resides in the largest dust grains, reducing the overall surface density in smaller grains. Because opacity is typically dominated by the small grains, the effective optical depth can get significantly reduced, becoming mostly optically thin. As a result, the total millimeter flux, which is dominated by the dust bump region, also decreases significantly when \(v_f\) becomes large enough, as shown in the upper left panel of Figure 9.

The spectral index \(\alpha_{0.8-1.3\text{ mm}}\) for different \(v_f\) is also shown in Figure 9, which is similar to \(\alpha_{1.0-3.0\text{ mm}}\). The submillimeter to millimeter fluxes for most of the faintest disks in the Lupus sample can be reproduced by models with a high fragmentation velocity \(\sim 30\) m s\(^{-1}\). However, it is difficult to use models with high fragmentation velocity to explain the disks with low spectral indices, as discussed below.

\textsuperscript{11} http://www2.mpia-hd.mpg.de/homes/henning/Dust_opacities/Opacities/RI/new ri.html

### 3.2.2. Disk Mass

Another important disk parameter is the disk mass, which, in our models, is determined by the gas surface density \(\Sigma_0\). In addition to the fiducial model, we chose three other values for \(\Sigma_0\), as listed in Table 1. The most massive disk corresponding to \(\Sigma_0 = 12.8\) g cm\(^{-2}\) (model labels including m0) has a total disk mass of \(0.03 M_\odot\). We also find that, for this model, the minimum Toomre \(Q\) parameter in the gas bump region can approach 3 during its evolution. The dependence of the total disk flux \(F_{1\text{ mm}}\) on \(\Sigma_0\) is rather straightforward. An increasing \(\Sigma_0\) will result in a higher \(F_{1\text{ mm}}\), as shown in the upper right panel of Figure 9, because more dust can be available initially. But the increasing flux is not linearly related to \(\Sigma_0\), especially when the emission is dominated by the optically thick regions. We find that the disk flux is very likely saturated at a flux level of \(\sim 100\) mJy, as shown in Table 1 (and also the upper right panel of Figure 9). In this optically thick regime, the global spectral index is insensitive to \(\Sigma_0\) because this is determined solely by the spectral slope of the Planck function and does not depend on the size of the emitting dust.

When the initial dust surface density is decreased by two orders of magnitude through the decrease in \(\Sigma_0\), the dust surface density becomes low globally, and most of the dust mass is retained in the small particles. As a result, the whole disk, even the dust-trapping region, becomes optically thin. At the same time, due to the lower fragmentation barrier \(a_{\text{max}}\), which linearly depends on the gas surface density \(\Sigma_0\), the maximum dust size also decreases with the decreasing \(\Sigma_0\). In this case, not only does the dust continuum flux decrease with \(\Sigma_0\), but also the global spectral index \(\alpha_{1.0-3.0\text{ mm}}\) increases to values reflecting the dust opacity index \(\beta\) of grains with sizes much smaller than \(1\) mm.

We have also made a comprehensive parameter study for the combined effects of different \(v_f\) and \(\Sigma_0\), and we list their results in Table 1 and Figure 10.

We find several trends that are described below. For a fixed \(v_f\), the dependence of \(\alpha_{\text{mm}}\) on \(\Sigma_0\) is as follows. When \(v_f\) is as high as \(3 \times 10^3\) km s\(^{-1}\), \(\alpha_{\text{mm}}\) is insensitive to \(\Sigma_0\) because the fragmentation barrier \(a_{\text{max}}\) is already larger than 1 cm even for the lowest values of \(\Sigma_0\) explored in this study. However, the global spectral index is usually larger than 2.5, except for extremely low mass disk (i.e., model m3v1). Such a low-mass disk is below the detection limit of current observations, while for the case of low \(v_f\), \(\alpha_{\text{mm}}\) (1 mm flux \(F_{1\text{ mm}}\)) is anticorrelated (correlated) with \(\Sigma_0\), due to the optical depth effect. Based on our parameter survey for different \(\Sigma_0\) and \(v_f\) listed in Table 1, the tentative anticorrelation between \(\alpha_{\text{mm}}\) and \(F_{1\text{ mm}}\) found by Ansdell et al. (2018) for some faint sources, as well as the low spectral index for some slightly bright disks from different surveys (Ricci et al. 2012; Ansdell et al. 2018), is consistent with a fragmentation velocity in the range of \(3 \times 10^2\) to \(1 \times 10^3\) km s\(^{-1}\). This conclusion still holds when we use the opacity from Semenov et al. (2003). As there are still some uncertainties for the dust opacity and disk parameters (the viscosity profile and the associated gas bump structures, as we will discuss later), we should point out that such a constraint on the fragmentation velocity could be not so stringent.

To summarize, both a high fragmentation velocity \(v_f\) and high dust mass can produce a global spectral index \(\alpha_{\text{mm}}\) smaller than 2.5, which is consistent with the millimeter observations for some young PPDs (Ricci et al. 2012; Ansdell et al. 2018), as shown in Figure 9. Such a model degeneracy can be broken...
by checking the variation of the spectral index radial profile with different wavelengths, which will be discussed in Section 3.3. When both the fragmentation velocity and dust mass are below a critical value such that the fragmentation barrier \( a_{\text{max}} < 1 \) mm and the effective optical depth \( \tau_{\text{eff}} \ll 1 \), the global spectral index \( \alpha_{\text{mm}} \) > 3.0.

### 3.2.3. Viscosity

The viscosity is expected to influence the gas bump properties and modify the dust dynamics as well. We simulate several models with different viscosity parameters \( \alpha_0 \). The spectral index \( \alpha_{\text{mm}} \) at different bands and 1 mm flux are listed in Table 1 and are shown in Figure 9.

According to Equation (12), the maximum dust size increases as viscosity decreases. This affects the optical depth significantly. Indeed, as shown in Figure 11 where the run is similar to our fiducial run except that \( \alpha_0 = 5 \times 10^{-3} \) is a factor of 2 lower, the fragmentation barrier in the whole disk \( a_{\text{max}} \) is larger. This leads to two consequences. One is that \( F_{1\text{mm}} \) decreases with decreasing \( \alpha_0 \) because the dust in the whole disk can coagulate to a larger size thanks to the larger fragmentation barrier, which results in the emission shifting from the optically thick to thin regime for most of the disk region. The other is that \( \alpha_{\text{mm}} \) becomes increasingly larger. It turns out that the dust bump region is still optically thick and it gives a spectral index around 2.0, but the flux aside from the bump region can
growth. The value of global spectral index $\alpha_{0.8-1.3 \text{ mm}}$ increases slightly, but it is still smaller than 3.0. In order to reproduce a spectral index lower than 2.5, the gap depth for the gas viscosity, or equivalently the gas density contrast, should be larger than $\sim 3$.

In the situation where the gas bump is produced via a mechanism different from a viscosity gap, such a gap should not be used in the coagulation/fragmentation calculation. This is because the real viscosity should be smooth in the whole disk, and our viscosity gap is only used to generate the bump. This has been tested for our fiducial model. We find that the maximum dust size reached in the bump region decreases by a factor of 6, which is $\sim 2 \text{ mm}$. The millimeter flux and the spectral index are 42.2 mJy and 2.46, respectively, which are actually close to our fiducial model results.

We have also adjusted the gap width by using different $w_{\text{ring}}$, 2.5, 7.5, and 10.0 au, of the viscosity profile. The results are listed in Table 1 and shown in Figure 9. We find that the final spectral slopes and $F_{1 \text{ mm}}$ are insensitive to changes in $w_{\text{ring}}$ between 2.5 and 10 au.

The dependence of disk emissions on the ring location is investigated by placing the viscosity gap at $r_{\text{ring}} = 25$, 50, and 75 au. The results are shown in Figure 9. In general, the 1 mm flux increases slightly and the spectral slope increases when $r_{\text{ring}}$ moves outward. This is because the dust mass interior to the dust bump region, $r \lesssim r_{\text{ring}} - w_{\text{ring}}/2$, gets larger. This region has a relatively steep spectral slope because it is optically thin emission from small dust grains. The emission from the dust-trapping region can still be optically thick. Therefore, as $r_{\text{ring}}$ increases, both the global spectral index and the total disk flux increase. As shown in Figure 9, the global spectral indices from these models give somewhat higher values than the observations.

3.2.5. Gas Scale Height and Surface Density Profile

We further consider two other disk parameters, gas scale height $h_0$ and gas surface density profile $\gamma$. The gas scale height can influence the gas temperature (pressure) profile. We show one example for the variation of the midplane dust temperature with the disk scale height in Figure 6. The temperature is slightly higher because of a higher sound speed. The $\gamma$ parameter can change the total amount of dust. The models labeled as m1h1(2) and m1b1(2) with the corresponding results are shown in Table 1. For all these models, the spectral index and millimeter flux variations are only modest. This is because the dust-trapping region is mostly optically thick, and in the optically thick regime the emission depends on temperature, not on the grain size distribution. The small variation of the flux and spectral index is due to the small variation in the dust temperature in the dust-trapping region, as shown in Figure 6.

3.2.6. No Viscosity Gap and No Ring

In order to quantify the role of the viscosity gap/ring, we study the case with $\alpha_{\text{rms}} = \alpha_0 = 10^{-2}$ throughout the whole disk and all other disk parameters the same as in our fiducial model m1v2. At the end of the 2.1 Myr evolution, the total dust mass is only $1.3 \times 10^{-6} M_\odot$, or $\lesssim 1\%$ of the original dust is retained in the disk and is two orders of magnitude smaller than for model m1v2, as shown in the left panel of Figure 12. In addition, low dust surface density severely limits the ability of dust particles to grow to larger sizes, as shown in the right
panel of Figure 12. In fact, almost all of the bigger grains will drift radially inward due to their larger Stokes number.

We then repeat the same radiative transfer calculations to produce the dust continuum emission for this disk model. The obtained values for the spectral index for the emission within 100 au from the star are quite large, reaching values of ≈ 4.0, due to an inefficient size growth and an optically thin emission for the dust. As expected, the flux at 1 mm is only 5.0 mJy, due to the low total dust mass. It is difficult to decrease the global spectral index down to the observed values by changing the dust size growth and an optically thin emission for the dust. As expected, the flux at 1 mm is only 5.0 mJy, due to the low total dust mass. It is difficult to decrease the global spectral index down to the observed values by changing the spectral index between 0.8 and 1.3 mm is about 2.0, but at longer wavelengths, such as at 3.0–7.0 mm, it reaches much higher values in the same region. The reason is as follows. First, this is due to the optically thin emission at longer wavelength, as shown in the lower left panel of Figure 13. Second, in the optically thin regime, the grains cannot grow to a size larger than 1.0 mm because of the lower fragmentation velocity.

As a comparison to our fiducial model with \( v_f = 1 \times 10^3 \text{ cm s}^{-1} \) (model m1v2), the spectral index at the dust-trapping region can reach 2.0 both at the short waveband (0.8–1.3 mm) and long waveband (3.0–7.0 mm), which is a consequence of the large optical depth.

As shown in these cases, the different radial behavior of the spectral index at multiple millimeter wavelengths can be used to break the model degeneracy as well as set some constraints on the dust size. This can be tested by future high-resolution observations of ALMA and ngVLA at longer wavelengths.

4. Summary and Discussions

In this work, we present detailed 1D hydrodynamical simulations of PPDs to study the two-fluid (gas+dust) coevolution with a state-of-the-art dust evolution code (Li et al. 2005, 2009; Birnstiel et al. 2010a; Fu et al. 2014) with dust size growth via coagulation and fragmentation (Birnstiel et al. 2010a). This allows us to self-consistently obtain the quasi-steady-state gas and dust distributions including dust feedback, dust radial drift, and coagulation/fragmentation balance. The comparisons with observations are then made using the RADMC-3D code for dust continuum radiative transfer.

12 These constraints will be affected by the uncertainties in the dust opacity or the unknown porosity of the particles.
By assuming a viscosity gap in the inner region of the disk (∼15 au), a gas bump as well as a dust trap are produced. With a systematic parameter study for different disk gas and dust parameters and coagulation velocities, we find the following:

1. A gas and dust ring at the inner disk can produce a lower global spectral index.

2. High fragmentation velocities \( v_f \) (1000 cm s\(^{-1}\) ≤ \( v_f \) ≤ \( 3 \times 10^3 \) cm s\(^{-1}\)) can facilitate efficient dust size growth (Figure 5), which produces a dust millimeter spectral index \( \alpha_{\text{mm}} \) close to 2.0, even for optically thin emission. For lower values of \( v_f \) (300 to 1000 cm s\(^{-1}\)), a high dust surface density in the dust-trapping region leads to optically thick emission, which then results in a low \( \alpha_{\text{mm}} \) as well (Figures 8, 9).

3. The high flux and low spectral index sources from a few surveys (Ricci et al. 2012; Ansdell et al. 2018) can be explained by a fragmentation velocity \( v_f \) in the range 300–1000 cm s\(^{-1}\). The tentative negative relation between \( \alpha_{\text{mm}} \) and \( F_{1\text{mm}} \) shown by Ansdell et al. (2018) for the faint PPD sources can also be interpreted by a relatively low \( v_f \) (≤1000 cm s\(^{-1}\)) with a variation of dust mass surface density. This is simply because the optical depth becomes smaller as the dust mass and millimeter flux decrease (Figure 9).

4. The spectral index at millimeter wavelengths \( \alpha_{\text{mm}} \) is not very sensitive to the gas bump width \( w_{\text{bump}} \), gap depth \( d_{\text{gap}} \), disk scale height \( h_0 \), and disk profile index \( \gamma \). A very low global disk viscosity \( \alpha_0 \lesssim 10^{-3} \) cannot reproduce the observed low global spectral index.

5. The one-ringed disk models can reproduce the observed properties of the faint part of the observed disks (≤100 mJy) reasonably well, but they cannot reproduce the brightest disks in the observational surveys (Figure 9). We expect that multiple rings could be responsible for these bright disks, as discussed later.

6. While both large-size grains and optically thick emission contribute to a small spectral index \( \alpha_{\text{mm}} \lesssim 2.5 \), future high-resolution observations, such as by ALMA and ngVLA, can distinguish these two effects based on their spatial distribution properties (Figures 7, 13).

In this work, we adopt a viscosity parameter \( \alpha_0 = 10^{-2} \) for most of our disks. This value is a factor of a few higher than some observationally constrained values based on the vertical thinness of some disks with rings and gaps, for example, HL Tau (Pinte et al. 2016; \( \alpha_{\text{vis}} \sim 3 \times 10^{-3} \)) and HD 163296 (Flaherty et al. 2017; \( \alpha_{\text{vis}} \lesssim 3 \times 10^{-3} \)). On the one hand, such a global low \( \alpha_{\text{vis}} \) could reflect the low turbulence in the ringed/gap regions. Therefore, it indicates that some regions in the disk could be indeed magnetorotational instability (MRI).
inactive compared with the theoretical expectation of MRI (Balbus & Hawley 1991). On the other hand, for a lower viscosity $\alpha_{\text{vis}}$ in a very thin disk, we find that an even lower fragmentation velocity ($\lesssim 3 \text{ m s}^{-1}$) is required to produce a low spectral index ($\alpha_{\text{mm}} \lesssim 2.5$). One issue we should point out is that we keep the viscosity profile $\alpha_{\text{vis}}(r)$ independent of dust evolution. This is contrary to the expectation of the MRI dead-zone scenario. Dzyurkevich et al. (2013) have shown that $\alpha_{\text{vis}}$ depends on ionization fraction, which depends on dust properties.

We have ignored the dust scattering effect in the radiative transfer. Recently, Zhu et al. (2019; see also Liu 2019) found that dust scattering can reduce the emission from an optically thick region. It leads to the apparent interpretation that bright disks tend to be harder. It can also modify the spectral slope at the millimeter band depending on the variation of the dust albedo with wavelength. However, the modification of the spectral shape is not significant (close to the expectation of 2.0) if the dust size is much larger than $\lambda/2\pi r$, which is the case in the dust-trapping region in most of our models. This issue will be addressed with detailed radiative transfer calculations in the future.

We have also explored the effects of multiple rings. A very low global spectral index (smaller than 2.5) can be obtained with a reasonable combination of disk parameters and fragmentation velocity ($\sim 10 \text{ m s}^{-1}$), and, more importantly, the disk becomes systematically brighter. Multiple rings can increase the fraction of optically thick emission regions, thus significantly increasing the disk total flux while still keeping low values for the global spectral indices around millimeter wavelengths. These are consistent with recent ALMA surveys for 12 disks in the Taurus star-forming region (Long et al. 2018) and 18 single-disk systems in the DSHARP program (Andrews et al. 2018; Huang et al. 2018b), where disks with multiple substructures tend to be brighter, although an observational bias might exist.

The ringed structures we inferred could be the location to trigger further planetesimal formation. If the ringed structure is associated with the MRI dead zone, the less turbulence in the gas bump would result in lower collision speeds for particles, which would be more suitable for further grain growth. Such a gas bump can also slow down the rapid radial drift to allow more time for planetesimal formation. In addition, the concentration of particles in high densities can trigger planetesimal formation via streaming instability (Youdin & Goodman 2005) and gravitational collapse (e.g., see review by Pinilla & Youdin 2017). The planetesimal formation process can remove some millimeter-size dust particles, and hence reduce the millimeter flux of the disk, which makes our model even less able to explain the brighter disks.

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