Shapiro steps in charge-density-wave states driven by ultrasound

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We show that ultrasound can induce the Shapiro steps (SS) in the charge-density-wave (CDW) state. When ultrasound with frequency $\omega$ and a dc voltage are applied, the SS occur at the current $I \propto n \omega$ with integer $n$. Even and odd multiples of SS are represented by two couplings between the CDW and ultrasound. Although an ac voltage with frequency $\omega$ induces the SS at $I \propto n \omega$, the ultrasound bias enhances the odd multiples more strongly than the even ones. This is the difference between the ultrasound and the ac voltage. Since the SS cause abrupt peaks in the $dV/dI$, the extreme changes in the $I$-$V$ curve will be applied to a very sensitive ultrasound detector.

The charge-density wave (CDW) is a macroscopic quantum state with twice the Fermi wave vector ($2k_F$) induced by pairing of electrons and holes. The CDW is created by the electron-phonon interaction below the critical temperature of the Peierls transition $T_c$, which opens a semiconducting gap at a Fermi surface (FS) and increases resistivity [1]. The CDW is pinned by impurities or in the case that a spatial period of the CDW matches with a lattice constant, i.e., commensurability pinning. When an applied electric field surpasses a certain threshold, the CDW is depinned and begins to slide through the lattice, increasing conductivity [2–7]. Remarkable transport properties were observed in transition metal trichalcogenides such as NbSe$_3$ and TaS$_3$ [8–12]. A current generated by the sliding of CDW above the threshold dc field contains not only a dc but also oscillating components known as narrow-band noise [13–18]. These steps are induced by phase locking of an external frequency with an internal one. This mechanism is same to the Shapiro steps (SS) in a Josephson junction of superconductors. The steps of an I-V curve in the CDW state shall be referred to as “Shapiro steps” (SS).

Ultrasound has rich variety of applications. Ultrasonography is used in hospitals for biomedical diagnostics to view our bodies or to eradicate cancers. A touch sensor using ultrasound is installed in some monitors. The ultrasound at-tenuation technique was used to investigate the symmetry of superconducting order parameters [19, 20]. On a surface of a ferromagnet, a surface acoustic wave is used to control a spin current [21, 23]. Recently, electrical control of a surface acoustic wave toward quantum information technology has been realized [24]. In contrast to the electromagnetic waves, ultrasound does not radiate into free-space, allowing for coherent information processing and manipulation with a minimum cross-talk between devices and their environments [25]. The ultrasound is generated and detected by a piezoelectric material such as lithium niobate. Improved ultrasonic sensitivity will broaden its use in quantum information technologies. Recently, the SS by mechanical vibration have been reported [26].

In this paper, we show that ultrasound can induce SS in the CDW state. When ultrasound with frequency $\omega$ and a dc voltage are applied, the SS occur at the current $I \propto n \omega$ with integer $n$. An ac voltage with frequency $\omega$ also induces the SS at $I \propto n \omega$. However, depending on the settings, the odd multiples of SS will be enhanced rather than the even ones due to two couplings of CDW to ultrasound; one contributes to even multiples, and the other to odd multiples. This is the difference between the ultrasound and the ac voltage. Since the $dV/dI$ shows sharp peaks due to the SS (See Fig. 1), the drastic changes in the $I$-$V$ curve will be applied to a highly sensitive ultrasound detector.

The charge density in a CDW state $\rho(x, t)$ is written by

$$\rho(x, t) = \rho_0 + \rho_1 \cos[2k_F x + \theta(x, t)],$$

where averaged charge density $\rho_0$ and magnitude of $2k_F$-oscillation $\rho_1$ are constant. The dynamical behavior at low energies is given by

$$L_0 = \frac{1}{4\pi v'} \int dx \left[ \partial_\theta \theta(x, t)^2 - v^2 [\partial_\theta \theta(x, t)]^2 \right],$$

where $v' \equiv v_F/v$ with Fermi velocity $v_F$ and $v$ is the velocity of Goldstone mode in the CDW state [5]. When dealing with one-dimensional electron systems, $\theta(x, t)$ is related...
to fermionic operators $\psi_{s\pm}$ of electrons using the bosonization

given by, $\psi_{s\pm} \sim \exp(\pm ik_F x \pm \theta_{s\pm})$, so as to $\theta(x, t) = \sum_{\theta_{s\pm} + \theta_{s\pm}}$ with spin $s$ [27–30]. According to these equations, $\theta$ denotes the 
fluctuation of electron density $\delta n_e$ around a FS. Considering
$2k_F = 2(\pi/L)(N/2)$ with system size $L$ and number of 
electrons $N$, $k_F$ corresponds to the electron density $N/L \equiv n_e$. 
Then, $(1/\pi)[\partial(\theta(x, t))/\partial x]$ is associated with $\delta n_e$, because of
the spatial derivative of the phase factors in $\psi_{s\pm}$ leads to the 
electron density, i.e., $\partial(2k_F x + \theta(x, t))/\partial x = 2k_F + \partial(\theta(x, t))/\partial x = \pi(n_e + \delta n_e)$ [27–30]. The continuity equation $\partial n + \partial_j j = 0$
defines the current $j$.

The key to transport characteristics is CDW pinning. We shall 
now look at a single impurity model. An impurity potential
at $x = 0$ is given by,

$$U_{\text{imp}} = -u_0 [\int dx \cos \{Q x + \theta(x, t)\} \delta (x)] (3)$$

where $Q \equiv 2k_F$, $\theta \equiv \theta(x = 0, t)$, $u_0 > 0$ and $\rho_1 > 0$. When 
ultrasound or surface acoustic wave is applied, the impurity
is moved from its initial position and the energy provided by
Eq. (6) is added,

$$U_{\text{snd}} = u_0 [\int dx \cos \{Q x + \theta(x, t)\} \delta (x - \xi(t)) + (\delta (x))] (5)$$

$$\sim -u_0 [\sin \theta \cdot \{Q + \partial \theta/\partial x\} \cdot \xi(t)] (6)$$

where $\partial \theta/\partial x \equiv \partial(\theta(x, t))/\partial x|_{x \rightarrow 0}$. At the impurity site, $x = 0$, the spatial variation of $\theta(x, t)$ is singular and its
derivative is not continuous. The right (left) derivative of $\theta(x, t)$ at 
the impurity site will be negative (positive) so as to fix the sign
of $\partial \theta/\partial x \cdot \xi(t)$. For $Q \cdot \xi(t) \equiv \eta \cos(\omega t)$, the pinning potential
and the coupling to ultrasound are given by,

$$U = U_{\text{imp}} + U_{\text{snd}}$$

$$\equiv -u_0 [\cos \theta + \sin \theta \cdot d(t)]$$

$$\equiv [\cos \theta + \sin \theta \cdot d(t)]$$

$$\eta \chi \cos \omega t + \lambda \cos \omega t$$

where $\chi = 1$, and $\lambda \equiv [\partial \theta/\partial x]/|Q| > 0$. In Eq. (7), the first 
term will be dominant in most circumstances because $\lambda$ relates to
$\delta n_e$ as $\lambda = |[\partial \theta/\partial x]/Q| = \delta n_e/n_e$. It is usually less than 1, i.e.,
$\lambda < 1$. Below, $\eta = 1$ is imposed for brevity and the following
results do not change qualitatively. The magnitude of the oscillation
at the impurity site is proportional to $\eta$, which can be regulated
by the ultrasonic input power.

The single impurity model can account for several important
aspects of CDW transport properties. This is the limit of
strong pinning. When pinning is weak enough that a single 
impurity cannot pin the phase of the CDW, the pinning
potential is determined by balancing the increase in kinetic energy
owing to charge density change i.e., the second term of Eq.
(2), and an energy gain from phase-dependent impurity pinning
energy [6–7]. These are characterized by the pinning frequency in the
optical conductivity [6–7]. The spatial distortions of the phase in the 
ground state caused by the random distribution of impurities are an important characteristic
of impurity pinning. If the quantity of interest is not sensitive
to such spatial fluctuation, the pinning will be characterized
by an averaged potential proportional to $\cos \theta(x, t)$, where
$\theta(x, t)$ is an averaged phase within an impurity distribution-determined domain [53]. Hence, the above discussion will be 
valid even in a weak pinning region.

Since electrons move together with lattice distortion, the effective
mass of CDW is large and the dynamics of phase motion
can be treated classically. Similar to the SS in a Josephson junction
of superconductors, the transport properties of CDW can be calculated by,

$$\alpha \frac{d^2 \theta}{dt^2} + \beta \frac{d \theta}{dt} + U'(\theta) = F(t), \quad (10)$$

with $U'(\theta) = dU/d\theta$. Equation (10) is interpreted as the 
acceleration $\alpha d^2 \theta/dt^2$ by the effective driving force, $F_{\text{eff}} = F(t) - U'(\theta)$, with the friction term $\beta d\theta/dt$. In an overdamped
region, $\alpha \sim 0$, the first term in Eq. (10) is neglected. When 
ultrasound is used to drive a CDW, the transport properties are
determined by,

$$\beta \frac{d \theta}{dt} + u_0 [\sin \theta + \cos \theta \cdot d(t)] = f_{\text{dc}}$$

where $f_{\text{dc}}$ corresponds to a dc voltage. Below, we will use the
following parametrization,

$$\frac{d \theta}{dt} + U'(\theta) = V$$

$$U'(\theta) = A [\sin \theta + \cos \theta \cdot d(t)].$$

The current $I \equiv \pi j$ is calculated by averaging of $\partial \theta(t)/\partial t$ in
time as,

$$I = \frac{1}{\pi} \int_0^T \frac{\partial \theta(t)/\partial t}{\lambda} \lambda$$

Solving Eqs. (12) and (13), the $I$-$V$ curves are obtained as shown
in Fig. 2 with $\omega = 0.25$, $\chi = 1.0$ and $\lambda = 1.0$ (orange), 0.5 (blue), and 0.0 (red). The SS appear at $I/\omega = n$ for $\lambda = 1.0$ and 0.5. For $\lambda = 0.0$, however, the SS appear at only odd multiples, i.e., $I/\omega = 2n+1$, and even multiples are
suppressed. This is a critical distinction between ultrasound and
an ac voltage. When the system is driven by the ac voltage, the
equation of motion is given by [14–15].

$$\frac{d \theta}{dt} = V - A \sin \theta - n_{\text{ac}} \cos(\omega t)$$

Solving Eq. (15), the $I$-$V$ curves are obtained as shown in
Fig. 3 with $\omega = 0.25$ and $n_{\text{ac}} = 0.5$ (red), 0.25 (blue), and 0.0 (black). The SS appear at $I/\omega = n$ similar to a Josephson junction
of superconductors. In the case of Eq. (15), assuming $\theta = at - b \sin(\omega t)$ with constants $a$ and $b$, and
using the relation, $e^{\pm \sin \theta} = \sum_n J_n(z)e^{i\mp \theta}$, with Bessel function
$J_n(z)$, the potential term is transformed as, $\sin \theta = \Im \left[ \sum_n J_n(b) e^{i(\theta - ma)2\pi} \right]$. When $a = \omega$, the time average of $\sin \theta$ is time-independent and leads to the SS. Since another
potential term, such as \( \sin \theta \cos \omega t \), is multiplied by the oscillation in the ultrasound bias Eq. (13), analytical estimation of the SS is problematic. However, using the guess, \( \theta = at - b \cos(\sin \omega t) \), the potential term is written as, \( \sin \theta = 1 \text{Im} \left[ \sum_{m=0}^{\infty} J_m(b) J_m(n)(1 + (-1)^m e^{i(m \omega t - \text{int} \omega t)}) \right] \). Then, the SS appear at \( I/\omega = n \) with integer \( n \) except for \( \lambda = 0.0 \), where only odd multiples appear.

The black line in Fig. 3 represents the threshold \( V = A \), at which the solution changes its behavior. At \( \eta_{ac} = 0 \), Eq. (15) is analytically solved as, \( V \tan(\theta(t)/2) = A + (V^2 - A^2)^{1/2} \tan(V^2 - A^2)^{1/2} t/2 \). For \( V < A \), the solution is monotonously merging to a constant, whereas, for \( V > A \), the solution oscillatory increases or decreases with time. Considering Eq. (14), the increase or decrease of \( \theta \) in time leads to a finite value of \( I \). Because of the phase-locking, the solution at the SS do not change its gradient.

In Fig. 3 without the \( \lambda \)-term in Eq. (9), the SS appear only at odd multiples. However, only with the \( \lambda \)-term, i.e., \( \chi = 0.0 \), the \( I-V \) curves are presented in Fig. 4 for \( \lambda = 1.0 \) (green) and 0.5 (purple). Notably, the SS appear at even multiples, i.e. \( I/\omega = 2n \), which is in sharp contrast to Fig. 2. The \( \lambda \)-term in Eq. (9), \( \cos \theta - \cos \omega t \), is not equivalent to \( \cos(\theta - \omega t) + \cos(\theta + \omega t) \) due to \( \chi = 0.0 \). Hence, \( \theta = at - b \cos(\sin \omega t) \) will be one of the possibilities of even multiples. Another reason is that the external force proportional to \( \chi \) is nearly equivalent to the \( 2\omega \)-oscillation.

We discovered that ultrasound can drive the CDW state in two ways, namely \( \chi \)- and \( \lambda \)-terms in Eq. (9), which cause odd and even multiples in the SS, respectively. By definition \( \chi = 1 \) and \( \lambda = \Delta \theta/\Delta t /Q = \delta n_e/n_e < 1 \), the odd multiples due to the \( \chi \)-term will be dominant in the \( I-V \) curves. This will help to distinguish between an ultrasound contribution and an electromagnetic one. For example, the red curve in Fig. 2 is driven by \( 0.5 \cdot \cos \theta \cdot \cos \omega t \) and another red curve in Fig. 3 is driven by \( 0.5 \cdot \cos \omega t \). Just a factor of \( \cos \theta \) leads to completely different results. When ultrasound changes the dielectric constant of a material, an electromagnetic field would be simultaneously applied together with the ultrasound. In this case, the SS appear all of the integer multiples as \( I/\omega = n \). The step-width at even multiples will be suppressed, if the produced electromagnetic field weakens. The electromagnetic bias is proportional to \( \cos \omega t \), while the ultrasound bias has an additional factor \( \cos \theta \) such as, \( \cos \theta \cdot \cos \omega t \) and \( \cos \theta \cdot \cos \omega t \). Even though the difference is just a factor of \( \cos \theta \), the step-widths are determined by their functional shapes. In the case of electromagnetic bias, all step-widths decrease with \( \eta_{ac} \) as shown in Fig. 3. There is not much of a distinction between even and odd multiples. In Fig. 5, on the other hand, the step-width at \( I/\omega = 1 \) and 2 are plotted as a function of \( \lambda \) for \( \chi = 1.0 \). The
The $\lambda$-dependence of the step-width at $I/\omega = 1$ (green) and 2 (red) with $\omega = 0.25$, $A = 0.5$, and $\chi = 1.0$. The inset shows the $I$-$V$ curve (blue), which is the same as that in Fig. 2 and its derivative $dV/dI$ (black). The peaks corresponding to the steps in the $I$-$V$ curve appear at $I/\omega = n$ with integer $n$.

The $I$-$V$ curve driven by ultrasound with $\omega = 0.25$, $A = 0.5$, $\chi = \lambda = 1.0$, and $\gamma = 0.5$ (red), 1.0 (blue), and 2.0 (green).
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