HEATING AND ACCELERATION OF INTRACLUSTER MEDIUM ELECTRONS BY TURBULENCE

VAHÉ PETROSIAN1,2 AND WILLIAM E. EAST3

Center for Space Science and Astrophysics, Department of Physics, Stanford University, Stanford, CA 94305;
vahep@stanford.edu, weast@stanford.edu

Received 2007 October 16; accepted 2008 March 17

ABSTRACT

We investigate the feasibility of bremsstrahlung radiation from “nonthermal” electrons as a source of hard X-rays from the intracluster medium of clusters of galaxies. With an exact treatment of the Coulomb collisions in a Fokker-Planck analysis of the electron distribution, we find that the severe difficulties with lifetimes of nonthermal particles found earlier by Petrosian in 2001 using a cold-target model remain problematic. We then address the possible acceleration of background electrons into a nonthermal tail. We assume a simplified but generic acceleration rate and determine the expected evolution of an initially Maxwellian distribution of electrons. We find that strong nonthermal components arise only for a rapid rate of acceleration, which also heats up the entire plasma. These results confirm the conclusion that if the observed nonthermal excesses are due to some process accelerating the background thermal electrons, then this process must be short lived.

Subject headings: acceleration of particles — galaxies: clusters: general — X-rays: general

Online material: color figures

1. INTRODUCTION

The classical picture of the intracluster medium (ICM) consisting of nearly excited isothermal hot gas, predominantly emitting the well-studied thermal bremsstrahlung (TB) radiation in the soft X-ray (SXR, ∼2–10 keV) region, has undergone considerable revisions in recent years. A considerable fraction of the observed clusters appear to be in the middle of merger processes with complex distributions of galaxies. There is also evidence for considerable deviations from isothermality: hot regions and cold fronts delineated perhaps by shocks resulting from the merger activity. An excellent example of this is the cluster RX J0658, also known as the Bullet Cluster, which has achieved considerable notoriety in recent years (see, e.g., Markevitch 2005; Bradac et al. 2006). In such clusters, there is also growing evidence for nonthermal activity, first observed as diffuse radio radiation from Coma. Recent systematic searches (see Giovannini et al. 1999; Giovannini & Feretti 2000) have detected similar radiation in more than 40 clusters that are classified either as relic or halo sources. There is little doubt that this radiation is due to synchrotron emission by a population of relativistic electrons. In the case of Coma, the radio spectrum may be represented by a broken power law (Rephaeli 1979), or a power law with a rapid steepening (Thierbach et al. 2003) or an exponential cutoff (Schlickeiser et al. 1987), implying the presence of electrons with similar spectra. Unfortunately, one cannot determine the energy of the electrons or the strength of the magnetic field from radio observations alone. Additional observations or assumptions are required. Equipartition or minimum total (particles-plus-field) energy arguments imply a population of relativistic electrons with a Lorentz factor γ ∼ 10^4 and a magnetic field strength B ∼ μG, in rough agreement with the Faraday rotation measurements (e.g., Kim et al. 1990). Rephaeli (1979) and Schlickeiser et al. (1987) also pointed out that these electrons, via inverse Compton (IC) scattering of cosmic microwave background (CMB) photons, should produce a broad spectrum of nonthermal hard X-ray (HXR) photons (similar to that observed in the radio band) around 50 keV. Detection of HXR radiation could break the degeneracy and allow determination of the magnetic field and the energy of the radiating electrons. In fact, because the energy density of the CMB radiation (with temperature T_0) u_{CMB} = 4 \times 10^{-13} (T_0/2.8 \text{ K})^4 \text{ erg cm}^{-3} is larger than the magnetic energy density u_B = 3 \times 10^{-14} (B/\mu G)^2 \text{ erg cm}^{-3}, one expects a higher flux of HXR than radio radiation.

HXR emissions (in the 20–80 keV range) at levels significantly above that expected from the thermal gas were detected by instruments on board the BeppoSAX and RXTE satellites from Coma (Rephaeli et al. 1999; Fusco-Femiano et al. 1999, 2004; Rephaeli & Gruber 2002), Abell 2319 (Gruber & Rephaeli 2002), and Abell 2256 (Fusco-Femiano et al. 2000, 2005; Rephaeli & Gruber 2003); a marginal (~3 σ) detection was also made from Abell 754, and an upper limit was placed on Abell 119 (Fusco-Femiano et al. 2003). We also note that a possible recent detection of nonthermal X-rays, albeit at lower energies, has been reported from a poor cluster, IC 1262, by Hudson et al. (2003). All these are nearby clusters in the redshift range 0.023 < z < 0.056. Notable recent exceptions at higher redshifts are RXTE observations of RX J0568 (z = 0.296; Petrostian et al. 2006, hereafter PML06) and Abell 2163 (z = 0.208; Rephaeli et al. 2006), where the HXR flux is consistent with the upper limit set by BeppoSAX (Feretti et al. 2001).5

It should also be noted that excess radiation was detected in the EUV region (0.07–0.14 keV) by the Extreme Ultraviolet Explorer from Coma (Lieu et al. 1996) and possibly from some other clusters, and in the 0.1–0.4 keV band by ROSAT, BeppoSAX, and XMM-Newton. A cooler (kT ∼ 2 keV) component or IC scattering

1 Department of Applied Physics, Stanford University, Stanford, CA 94305.
2 Kavli Institute of Particle Astrophysics and Cosmology, Stanford University, Stanford, CA 94305.
3 Department of Physics, Stanford University, Stanford, CA 94305.

4 The results of this paper have been challenged and rebutted by an analysis performed with different software by Rossetti & Molendi (2004) and Fusco-Femiano et al. (2007).
5 Note that the high-redshift observations are made relatively easier because of the (known) rapid increase with redshift of the CMB density (see PML06), so that, in principle, the cosmological evolution of these quantities can be investigated with simultaneous radio and HXR observations.
of CMB photons by lower energy ($\gamma \sim 10^3$) electrons are two possible ways that this excess radiation might be produced. However, some of the observations and the emission process are still controversial (see Bowyer 2003).

Although the IC interpretation seems natural, there is some difficulty with it. Soon after the discovery of HXR, it was realized that the relatively high observed fluxes require large numbers of relativistic electrons, and consequently a relatively low magnetic field for a given observed radio flux. For Coma, this requires a (volume-averaged) magnetic field of $B \sim 0.1 \mu G$, while equipartition gives $B \sim 0.4 \mu G$, and Faraday rotation measurements give a (average line-of-sight) field of $B_1 \sim 3 \mu G$ (Giovannini et al. 1993; Kim et al. 1990; Clarke et al. 2001; Clarke 2003). (In general the Faraday rotation measurements of most clusters give $B > \mu G$; see, e.g., Govoni et al. 2003.) Consequently, various authors (see, e.g., Enßlin et al. 1999; Blasi 2000) suggested that the HXR radiation is due to nonthermal bremsstrahlung (NTB) by a second population of nonthermal electrons with a power-law distribution in the 10–100 keV range. However, as shown by Petrosian (2001, hereafter P01), this process faces a more serious difficulty, which is hard to circumvent: the fact that bremsstrahlung is a very inefficient process. Compared to Coulomb losses, the bremsstrahlung yield is very small. For a particle with an energy $E$ much larger than that of background particles, $\gamma_{\text{brems}} \sim 3 \times 10^{-6}(E/25 \text{ keV})^{3/2}$ (see Petrosian 1973). Thus, for continuing production of a HXR luminosity of $4 \times 10^{43}$ erg s$^{-1}$ (observed for Coma), a power of $L_{\text{HXR}}/\gamma_{\text{brems}} \sim 10^{46}$ erg s$^{-1}$ must be continuously fed into the ICM, increasing its temperature to $T \sim 10^8$ K after $3 \times 10^7$ yr, or to $10^{10}$ K in a Hubble time, which indicates that the NTB emission phase must be very short lived. As pointed out in P01, a corollary of this is that it would be difficult to accelerate thermal particles to produce a nonthermal tail without excessive heating of the background plasma.

The above arguments, however, are not definitive.

1. The argument against the IC model is not as severe as stated above. There are several factors that may resolve the apparent discrepancy among different estimates of the magnetic field. First, the $B$-field value based on the Faraday rotation measure assumes a chaotic magnetic field with a scale of a few kpc, which is not a directly measured quantity (see, e.g., Carilli & Taylor 2002). Second, the accuracy of the quoted measurements has been questioned by Rudnick & Blundell (2003) and defended by Govoni & Feretti (2004) and others. Third, as pointed out by Brunetti et al. (2001), a strong gradient in the magnetic field can reconcile the difference between the volume and line-of-sight–averaged measurements. Finally, as pointed out in P01, this discrepancy can be alleviated by a more realistic electron spectral distribution (e.g., a spectrum with an exponential cutoff, as suggested by Schlickeiser et al. 1987) and/or a nonisotropic pitch-angle distribution. In addition, for a population of clusters, observational selection effects come into play and may favor Faraday rotation detection in high-$B$ clusters, which will have a weaker IC flux relative to synchrotron.

2. The spectral shape of the HXR emission is not very well constrained, so that a two-temperature model fits the observation as well as a single-temperature plus a power-law model (see, e.g., PML06). However, the second thermal component of electrons must have a much higher temperature than the gas responsible for the SXR emission. For production of HXR flux up to 50 keV, this requires a gas with $kT > 30$ keV and (for Coma) an emission measure about 10% of that of the SXR-producing plasma. Heating and maintaining the plasma to such high temperatures, in view of the rapid equilibration expected by classical Spitzer conduction, suffers from the same shortcoming as in the NTB case. In fact, as we shall see below, the thermal and nonthermal scenarios cannot be easily distinguished from each other. The acceleration mechanism energizes the plasma and modifies its distribution in such a way that both heating and acceleration take place.

3. The short timescale estimated above is based on energy losses of electrons in a cold plasma, which is a good approximation for electron energies $E \gg kT$. As $E$ nears $kT$, the rate of Coulomb loss (mainly due to electron-electron collision) decreases, but the bremsstrahlung rate (due to the electron-proton collision at these nonrelativistic energies) remains constant. There have been several attempts to address this issue. Blasi (2000), using a more realistic treatment of the Coulomb collision in a Fokker-Planck treatment based on coefficients derived by Nayakshin & Melia (1998, hereafter NM98), produced a nonthermal tail in the electron distribution that might explain the HXR observations from the Coma Cluster. Wolfe & Melia (2006), on the other hand, expanding on the results from NM98, use a covariant treatment of the kinetic equation and find that the result of energizing the plasma by turbulence is primarily to heat the plasma to higher temperatures on a short timescale, in agreement with P01. Finally, in a recent paper, Dögel et al. (2007) claim that in spite of the short lifetime of the test particles due to their Coulomb losses, the “particle distribution” lifetime is longer, and a power-law tail can be maintained without requiring the energy input estimated above.

In this paper, we address this problem not with the test-particle and cold-plasma assumption, but by carrying out a realistic acceleration and energization calculation of the ICM plasma by turbulence or any similar mechanism. In § 2, we describe our method of evaluating the influence of turbulence (or any other acceleration process) and Coulomb collisions on the spectral distribution of electrons in a hot plasma appropriate for the ICM. In § 3, we first present a test of our algorithm and then address the question of the lifetime of nonthermal tails. In § 4, we apply the method to the acceleration of thermal background particles by a generic acceleration model, present some results on the evolution of the distribution of electrons, and estimate the fraction of electrons that can be considered nonthermal. In § 5, we summarize our results, compare them with those from previous works, and present our conclusions. In the Appendices, we describe some technical details of our procedure.

2. BASIC SCENARIO OF ACCELERATION

In this section, we consider a hot gas that is subject to some acceleration process. In a cluster, the hot gas is confined by the gravitational field of the total distribution of (dark and “visible”) matter. Relativistic particles, on the other hand, can cross a cluster of radius $R$ on a timescale of $T_{\text{cross}} = 3 \times 10^6(R/\text{Mpc})$ yr and can escape the cluster unless confined by a chaotic magnetic field or a scattering agent, such as turbulence, with a mean free path $\lambda_{\text{scatt}} \ll R$. For confinement on a Hubble timescale of $10^{10}$ yr, we need $\lambda_{\text{scatt}} < 10$ kpc. As stated above, the magnetic field is expected to be chaotic, and there are good arguments for the presence of turbulence, especially in clusters with recent merging episodes. For example, XMM-Newton observations indicate that in the Coma cluster, more than 10% of the ICM pressure is in turbulent form (Schuecker et al. 2004). In addition, theoretical

---

6 One possible way to circumvent the rapid cooling of the hotter plasma by conduction or rapid energy loss of the nonthermal particles is to physically separate them from the cooler ICM gas. Exactly how this can be done is difficult to determine, but strong magnetic fields or turbulence may be able to produce such a situation.
arguments (see, e.g., Brunetti & Lazarian 2007) and modeling (see, e.g., Kim 2007) of the motion of the galaxies within clusters indicate that turbulence will be present in clusters of galaxies.

As a result of the scattering from this turbulence, the particle pitch angle changes stochastically with the diffusion rate $D_{\mu \nu}$ (where $\mu$ is the cosine of the pitch angle). When the scattering time $\tau_{\text{scat}} = \lambda_{\text{scat}}/v \sim (1/D_{\mu \nu})$ is much less than the dynamic and other timescales of the particles (with velocity $v = c/\beta$), the pitch-angle distribution of the particles will be isotropic. Also as a result of this scattering, particles will be accelerated stochastically on a timescale of $\tau_{\text{diff}} = \beta^2/D_{\mu \nu}$, where $D_{\mu \nu}$ is the momentum diffusion coefficient. Particles may also undergo direct acceleration at a rate of $A(E)$ or timescale $\tau_{\text{ac}} \sim E/A(E)$, and will lose energy at a rate of $\dot{E}_L$ due to their interactions with background particles and fields.\(^7\)

The evolution of the energy and pitch-angle distribution function of plasma particles subjected to a stochastic acceleration process integrated over the volume of the turbulent region can then be described by the Fokker-Planck transport equation. Under the conditions specified above, this equation is considerably simplified. The transport equation describing the gyrophase and pitch-angle-averaged spectrum, $N(E, t)$, of the particles can be written as (see, e.g., Petrosian & Liu 2004)

\[
\frac{\partial N}{\partial t} = \frac{\partial^2}{\partial E^2} \left[(D(E) + D_{\text{ Coul}}(E))N\right] - \frac{\partial}{\partial E} \left[(A(E) - \dot{E}_L)N\right].
\]

For stochastic acceleration by turbulence,

\[
D(E) = \beta^2 D_{\text{pp}} \quad \text{and} \quad A(E) = D(E)\zeta(E)/E + dD(E)/dE
\]

describe the diffusive and systematic acceleration coefficients. Their value and evolution is determined by the energy density and spectrum of turbulence. Here, $\zeta(E) = (2 - \gamma^{-2})/(1 + \gamma^{-1})$ is a slowly varying function changing from 1\!/\!2 to 2 for $0 < E < \infty$. The term $dD(E)/dE$ would be absent if the diffusion term in equation (1) were written as $(\partial/\partial E)[D(E)(\partial/\partial E)N(E)]$, which is another commonly used form of the transport equation. The numerical results presented below are based on the code developed by Park & Petrosian (1995, 1996), which uses this form of the equation.

In what follows, we will not be concerned with the exact forms of these coefficients and will assume some very simple energy dependence. We will assume that they are constant in time, which is equivalent to having a constant density and spectrum of turbulence. Specifically, we will assume

\[
D(E) = \frac{E^2}{\zeta(E)\tau_0(1 + E_c/E)^q};
\]

so that for the alternate form of the transport equation used in our numerical code, we have a simple acceleration time,

\[
\tau_{\text{ac}} = \tau_0(1 + E_c/E)^q.
\]

The right panel of Figure 1 shows the acceleration time with $E_c = 0.2 \text{ (}\sim 25 \text{ keV)}$ for several values of the parameters $\tau_0$ and $q$ appropriate for ICM conditions, along with the Coulomb loss rate discussed in the next section. The rate at which energy would be added to the particles due to such turbulence is given by $\int_0^\infty A(E)N(E, t) dE$ (see Appendix A), which, assuming a particle

---

\(\text{Fig. 1.—Left: Various timescales for Coulomb collisions for a hot plasma with typical ICM parameters from eqs. (5), (11), (12), and (13); figure is from Liu (2006). Right: Acceleration timescale based on the model described by eq. (4) for } E_c = 0.2 \text{ and the specified parameters, and the total-loss timescale for ICM conditions. We use the effective Coulomb loss rate given in eq. (13). For completeness, the IC-plus-synchrotron losses for a CMB temperature of } T_{\text{CMB}} = 3 \text{ K and an ICM magnetic field of } B = 1 \mu \text{G are also included, but their influence appears at } E > 10^4 \text{ keV (see, e.g., P01). [See the electronic edition of the Journal for a color version of this figure.]}\)
distribution normalized to 1, is approximately \( \langle E \rangle / r_0 \), where \( \langle E \rangle \) is the average energy.

The remaining coefficient, \( E_i \), is the sum of the loss rates (defined here to be positive) due to Coulomb collisions (primarily with background electrons), synchrotron, IC scattering (of CMB photons), and bremsstrahlung. We will include all these terms in our analysis, but for nonrelativistic energies in the ICM, the dominant term is due to Coulomb collisions, and at low (mainly nonrelativistic) energies, which will be our main focus here, the Coulomb term will be the most important (see Fig. 1). As mentioned above, the previous analysis was based on the energy-loss rate due to Coulomb collisions with a “cold” ambient plasma (in which the target electrons have zero velocity),

\[
\dot{E}_{\text{Coul}}^\text{cold} = \frac{1}{(\tau_{\text{Coul}})^\beta}, \quad \text{where} \quad \tau_{\text{Coul}} \equiv \frac{4\pi r_0^3}{\ln \Lambda n}, \tag{5}
\]

and \( r_0 = e^2/(m_e^2) = 2.8 \times 10^{-13} \text{ cm} \) is the classical electron radius. For ICM conditions, the Coulomb logarithm \( \Lambda \sim 40 \) and density \( n \sim 10^{-3} \text{ cm}^{-3} \); therefore, \( \tau_{\text{Coul}} \sim 2.7 \times 10^7 \text{ yr} \). The cold-target loss rate is a good approximation when the nonthermal electron velocity \( v \gg v_{\text{th}} \), where \( v_{\text{th}} = (2kT/m_e)^{1/2} \) is the thermal velocity of the background electrons. This approximation becomes worse as \( v \rightarrow v_{\text{th}} \) and breaks down completely for \( v < v_{\text{th}} \), in which case the electron may gain rather than lose energy, as is always the case in the cold-target scenario. More general treatment of Coulomb loss is therefore desired. For a hot plasma, the above loss rate must be modified, and there will also be a nonzero Coulomb diffusion term \( D_{\text{Coul}}(E) \).

Let us first consider the energy-loss rate. This is obtained from the rate of exchange of energy between two electrons with energies \( E \) and \( E' \), which we write as

\[
\Delta E/\Delta t = G(E, E')/\tau_{\text{Coul}}. \tag{6}
\]

Here, \( G \) is an antisymmetric function of the two variables, such that the higher energy electron loses energy, and the lower energy electron gains energy. From equations (24)–(26) of NM98, we can write

\[
G(E, E') = \begin{cases} 
-\beta^{-1} & \text{for } E' > E, E \ll 1, \\
\beta^{-1} & \text{for } E' < E, E' \ll 1, \\
E'^{-1} - E^{-1} & \text{for } E, E' \gg 1.
\end{cases} \tag{7}
\]

The general Coulomb loss term is obtained by integrating over the particle distribution:

\[
\dot{E}_{\text{Coul}}^\text{gen}(E, t) = \frac{1}{\tau_{\text{Coul}}} \int_0^\infty G(E, E')N(E', t) dE'. \tag{8}
\]

Similarly, we can express the Coulomb diffusion coefficient as

\[
D_{\text{Coul}}^\text{gen}(E, t) = \frac{1}{\tau_{\text{Coul}}} \int_0^\infty H(E, E')N(E', t) dE'. \tag{9}
\]

From equations (35) and (36) of NM98, we get

\[
H(E, E') = \begin{cases} 
\beta^2/(3\beta') & \text{for } E' > E, E \ll 1, \\
\beta^2/(3\beta') & \text{for } E' < E, E' \ll 1, \\
1/2 & \text{for } E, E' \gg 1.
\end{cases} \tag{10}
\]

Thus, the determination of the distribution \( N(E, t) \) involves the solution of the combined integro-differential equations (1), (8), and (9), which can be solved iteratively. However, in many cases these equations can be simplified considerably. For example, if we are interested only in the “suprathermal” tail of the distribution, where the energy \( E \) is larger than that of the bulk of the population, and if these are mainly nonrelativistic (\( \beta' \ll 1 \)), then we can use the approximation \( G(E, E') = \beta^{-1} \) and \( H(E, E') = 2E'/\beta \approx G(E, E') \) (the second lines in eqs. [7] and [10]). In this case, the Coulomb loss term is absent, and we have a simple differential equation to solve. Another simplification arises when the bulk of the particles have a Maxwellian distribution, \( N(E') = n/\sqrt{2\pi}(kT/m_e^2)^{-3/2}e^{-E/m_e^2/kT} \), where \( kT \ll m_e^2 \).

Carrying out the integration of equations (8) and (9) over this energy distribution, and after some algebra, the net energy-loss (gain) and diffusion coefficient can be written as (see also Spitzer 1962; pp. 128–129; Benz 2002, eq. [2.6.28]; Miller et al. 1996)

\[
\dot{E}_{\text{Coul}} = \dot{E}_{\text{Coul}}^\text{cold} \left[ \text{erf}(\sqrt{x}) - 4\sqrt{x}e^{-x} \right], \tag{11}
\]

and

\[
D_{\text{Coul}}(E) = E_{\text{Coul}}^\text{cold} \left[ \frac{kT}{m_e^2} \text{erf}(\sqrt{x}) - 2\sqrt{x}e^{-x} \right], \tag{12}
\]

where \( \text{erf}(y) \) is an antisymmetric function of the two variables, such that the higher energy electron loses energy, and the lower energy electron gains energy. From equations (24)–(26) of NM98, we can write

\[
\dot{E}_{\text{Coul}} = \dot{E}_{\text{Coul}}^\text{cold} + \frac{dD_{\text{Coul}}}{dE} \tag{13}
\]

where we have used equations (11) and (12). Note that for relativistic test particles, we have the same expressions, as long as \( kT \ll m_e^2 \). The various Coulomb rates above are shown in the left panel of Figure 1 for typical ICM conditions; solid lines show energy loss, and dotted lines show energy gain. For the purpose of direct comparison of the loss and acceleration timescales, the effective Coulomb loss timescale (the sharply peaked curve) is shown in the right panel of Figure 1.

3. RELAXATION AND THERMALIZATION TIMESCALES

As a test of our algorithm, we first address the relaxation into a thermal distribution of particles with an initial Gaussian distribution (mean energy \( E_0 \) and width \( \Delta E \approx E_0 \)) subject only to elastic Coulomb collisions. The distribution should approach a Maxwellian with \( kT/m_e^2 = 2E_0/3 \), and total number and energy equal to that of the initial particles after several thermalization times (Spitzer 1962; Benz 2002)

\[
\tau_{\text{therm}} = 3.5\tau_{\text{Coul}}(kT/m_e^2)^{1.5}. \tag{14}
\]

Using only the Coulomb loss and diffusion terms for this value of the temperature, we show in Figure 2 the evolution of an initial narrow Gaussian electron spectrum toward the expected
Maxwellian distribution. As is evident, most of the particles settle down into a thermal distribution within several thermalization times. This is similar to the result found by Miller et al. (1996).

Next, we consider thermalization or an energy-loss timescale of nonthermal populations of electrons (with isotropic pitch-angle distribution) added to a background thermal plasma. We first consider what one may call the test particle case, where the nonthermal tails contain a much smaller amount of energy than the background particles. Alternatively, we assume that the energy lost by the injected nonthermal tail is radiated or conducted away, such that the background temperature stays constant. In this case, we can use the thermal form of the coefficients given by equations (12) and (13) calculated for constant temperature. We consider two different forms of nonthermal tails: one a Gaussian spectrum of electrons with mean energy $E_0$ and width $\Delta E$, and another a power-law tail starting at some energy $E_0 > kT$. The top two panels of Figure 3 show the evolutions of these nonthermal tails. As is evident, the nonthermal distribution is depleted to a tenth of its original size within several cold-target Coulomb loss times, $t_{\text{Coul}}^{\text{cold}} = E/E_{\text{Coul}}^{\text{cold}} = \sqrt{2} t_{\text{therm}} n_0^{1/3}$, appropriate for an initial energy $E_0$.

Finally, we consider cases similar to those above, but without the “test particle” assumption. In these cases, the nonthermal tails contain a significant amount of energy, with the result that the energy lost by them heats the plasma and changes its temperature. This in turn changes the loss and diffusion coefficients. We evaluate the total particle evolution using two different methods. In the first method, we obtain an estimated temperature at each time step and use the new temperature to update the coefficients according to equations (12) and (13). Our procedure for advancing the temperature to its new value is described in Appendix A. The second and more accurate but also more time-consuming method is to use equations (8) and (9) to calculate the values of the coefficients at each time step. The results obtained from these two methods are essentially indistinguishable (see Appendix A for comparison). The bottom panels of Figure 3 show the evolutions of the total (thermal-plus-tail) spectrum of electrons with initial forms similar to those shown in Figure 3. As the tail is dissipated, the temperature increases to its final value within a time of $100 t_{\text{therm}}$ of the original temperature of 1 keV, or 2–3 times the cold-target loss time for $E_0 = 20$ keV particles.

The above results show that the conclusions based on the cold-plasma approximation are good order-of-magnitude estimates, and that using the more realistic hot-plasma relations changes these estimates by factors of 2 or 3. Consequently, the estimates made in P01 based on a cold-target assumption are modified by similar factors; the required input energy will be lower, and the timescale for heating will be longer by the same factor. This agrees qualitatively with Figure 3 of Dogiel et al. (2007) but does not support their other claims about long lifetimes of power-law tails, which are based on a less realistic treatment of the problem (see below).

4. HEATING AND ACCELERATION OF ELECTRONS

In this section, we investigate the evolution of the spectra of ICM electrons subject to diffusion and acceleration by turbulence, and to diffusion and energy losses due to Coulomb collisions using the equations described in §2. We also include synchrotron, IC, and bremsstrahlung losses, which have an insignificant effect on the final results for ICM conditions. We start with an ICM of $kT = 8$ keV and $n = 10^{-3}$ cm$^{-3}$, and assume a continuous injection of turbulence at a rate that allows its energy density to remain constant, resulting in time-independent diffusion and acceleration rates (i.e., parameters $q$, $E_c$, and $\tau_0$ in eq. [4] are constants). However, the Coulomb coefficients must be updated. Again, we use two different approaches. In the first, we estimate a new temperature at each time step, using the fitting prescription described in Appendix B, and calculate the coefficients based on equations (12) and (13). This is accurate at low acceleration rates, where the deviation from a Maxwellian distribution is small, but at higher rates these deviations become large, and we use the more accurate method described above. At each time step, we use the new distribution of the electrons and equations (8) and (9) to calculate the values of the Coulomb coefficients.

Figures 4 and 5 show the evolution of initially Maxwellian distributions subject to various acceleration models. Figure 4 shows the evolution for the three acceleration models $(q = -1, 0, 1; E_c = 0.2)$ shown in Figure 1 (right) with the smallest values of $\tau_0$, while Figure 5 shows the evolution for the two other values of $\tau_0$, but with $q = 1$ only. For each model, we show the spectrum at several evenly spaced time steps, beginning with the initial distribution. In addition, we plot a best thermal fit (see Appendix B) to the final distribution and a nonthermal residual to this distribution. The general feature of these results is that the turbulence causes both acceleration and heating in the sense that the spectra at low energies resemble a thermal distribution but have a substantial deviation from this quasi-thermal distribution at high energies, which can be fitted by a power law over a finite energy range. Alternatively, one can fit the broad distribution by a multitemperature model. In most cases, there is no distinct nonthermal tail. In general, the distributions are broad and continuous, and as time progresses they become broader and shift to higher...
energies; the temperature increases and the nonthermal tail becomes more prominent. Comparing different values of the parameter \( q \), which determines the low-energy behavior of the acceleration model, we can see that for higher (lower) values of \( q \), the fraction of nonthermal particles is greater (smaller). For the highest rate of turbulence \( \nu / C^2_{28} = 0.013 \nu / C^2_{28} \text{Coul} / C^4_{24} 10^5 \text{yr} \), as shown in Figure 4, the \( q = 1 \) model (with a reduced acceleration rate at low energies) mostly produces heating. For \( \tau_0 > \tau_{\text{Cold}} \), there is very little of a nonthermal tail, and most of the turbulent energy goes into heating. We fit all spectra to a best thermal distribution, and the remainder is called the nonthermal part. The initial and final temperatures, the fraction of particles in the quasi-thermal component \( N_{\text{th}} \), and the ratio of nonthermal to thermal energies \( R_{\text{nonth}} \) are shown in Figures 4 and 5.

We calculate the above parameters, as well as the index \( \delta = -d \ln N(E)/d \ln E \), for several time steps up to the time \( t = \tau_0 \). In Figure 6, we show the evolution with time of these parameters for all nine of the acceleration models shown in the right panel of

![Graphs showing the evolution of nonthermal and thermal distributions.](image-url)
Figure 1, grouped by the value of the parameter $q$. From these figures, we can see that in all cases, except for the one with $q = 1$ and $\tau_0/\tau_{\text{cold}} = 0.013$, the temperature increases by more than a factor of 2 by $t = \tau_0$. As expected, we can also see that faster acceleration rates lead to more pronounced nonthermal components with flatter tails (corresponding to a smaller $\delta$), more particles (corresponding to smaller $N_{th}$), and more energy (corresponding to higher $R_{nonth}$). In addition, it is evident that increasing the acceleration rate at low energies (by increasing $q$) leads to larger temperature increases.

As is evident from these results, in most cases there is a large rise in temperature before a significant nonthermal tail is produced. Noteworthy among these results is the case with a high acceleration rate and $q = 1$ (which means that the acceleration rate increases with energy), where a promising spectrum to explain the HXR observation is obtained. Unfortunately, this spectrum appears before about $3 \times 10^5$ yr, and at its rate of energization, the electrons will achieve relativistic temperatures and energies on timescales $>10^8$ yr.

5. SUMMARY AND CONCLUSION

The aim of this paper is to explore the possibility that the HXR excesses observed in several clusters of galaxies could be produced by bremsstrahlung emission by a population of “nonthermal” electrons. Since the observed HXR tails can be fitted to a multitemperature or one-temperature+power-law model, the same will be true for the electron distribution. Based on cold-target Coulomb loss rates in P01, we showed that these scenarios must be short lived, since otherwise there would be extensive heating of the ICM gas. Here, we carry out a more detailed analysis of the problem that includes the fact that for some low-energy electrons, the cold-target assumption is not accurate. We
derive exact forms for the energy-loss and diffusion coefficients for an arbitrary distribution of electrons to be used in the particle kinetic equation for determination of the spectral evolution of the electrons. We also devise approximate recipes that can be utilized more readily. We test our algorithm by evaluating the rate of relaxation of an arbitrary distribution of particles into a Maxwellian one under the influence of Coulomb collisions alone. The thermalization occurs within the expected time.

We first evaluate the survival time of nonthermal tails, such as a distinct high-energy bump (Gaussian shape) or a power law, in a background plasma with parameters appropriate for ICM conditions. We assume two conditions. In one scenario, which we refer to as the “test particle case,” it is assumed that the energy of the tail is insignificant, or that its input is radiated or conducted away, such that the background temperature remains constant. In a second scenario, this assumption is relaxed, and the evolution of the temperature is explicitly determined. We compare the survival times of such tails with what one would get in a cold-target scenario in P01. We find that the lifetimes of the tails are increased by a factor of 2 or 3. This reduces the severity of the difficulty of producing the HXR tails by the nonthermal tails are increased by a factor of 2 or 3. This reduces the severity of the difficulty of producing the HXR tails by the nonthermal process discussed in P01 but does not alleviate it completely.

In P01, it was also claimed that because of the large Coulomb losses, the acceleration of background thermal particles into a distinct nonthermal tail would be accompanied by a catastrophic heating, and that long-lived (∼10^5 yr) nonthermal electrons would need energies greater than a few hundred MeV to avoid this difficulty. Here, we have carried out a detailed analysis of the evolution of nonthermal electron spectra under the influence of a generic acceleration (or more correctly energizing) process, while suffering Coulomb (and all radiative) losses. The results confirm the earlier claim; in general, an initial Maxwellian spectrum evolves into a fairly broad spectrum without a distinct nonthermal tail. The resulting spectra can be decomposed in many ways. In most cases, the spectra are dominated by a single-temperature Maxwellian. Consequently, we decompose the spectra into a Maxwellian core (and determine its density and temperature), and the remainder is lumped into a nonthermal tail. We refer to these components as heated and accelerated electrons, respectively. Our results can be summarized as follows:

1. At energizing rates smaller than the thermalization rate of the background plasma, there is very little acceleration. The primary effect is heating of the plasma at a rate equal to the energizing rate. Therefore, in order to avoid excessive heating, the timescale of the energizing process must be comparable to the Hubble time, or be short lived and last less than the thermalization time, which is <10^5 yr.
2. A corollary of this is that in the steady state situation, where the energizing rate is equal to the radiative loss rate, there will not be a nonthermal tail. At temperatures of a few keV, the radiative loss is dominated by the bremsstrahlung process, which is very slow compared to the Coulomb scatterings.
3. At higher energizing rates, a distinguishable nonthermal tail is developed, but this is again accompanied by an unacceptably high rate of heating. For example, for τ₀ ∼ 5 × 10^6 yr and q = 1, about 10% of electrons end up in a nonthermal tail, but the background temperature is increased to 20 keV from an initial value of 8 keV within 5 million years.
4. A well-developed tail occurs only for the relatively fast energizing case with τ₀ ∼ 10^−2 τ_Coul. The heating rate for such cases is relatively slow. However, this phenomenon is short lived, and the vast majority of the particles will be accelerated to high energies if this level of turbulence is maintained for times of order τ_Coul ∼ 10^5 yr. The only way to avoid a catastrophic heating would be for the energy process to last ∼10^5 yr.
5. However, to explain the HX observations by NTB would require episodic energizing once every τ_therm(E ∼ 50 keV) ∼ 10^6 yr before the nonthermal tail is dissipated through heating the plasma, so that at the end, on average, the result would be a hotter plasma and less of a nonthermal tail, similar to a case with slower acceleration.

These results disagree with those presented by Dogiel et al. (2007), where it is claimed that nonthermal tails can be maintained for long periods, and that acceleration by turbulence can produce power-law tails without excessive heating. However, their results seem to be based on cold-target Coulomb loss rates. They
also introduce an energy diffusion term that depends on a constant temperature and that would be absent for a cold-target case. Because this temperature is fixed, heating is not allowed. The crucial fact that these coefficients depend on the exact shape of the particle spectra is not included in their calculations. Our results also disagree with those of Blasi (2000). This is somewhat puzzling, because unlike Dogiel et al. (2007), Blasi uses an algorithm similar to ours, except that he calculates turbulence coefficients based on an assumed spectrum of turbulence. In fact, we tried to reproduce his results using the given turbulence spectrum and the exact Coulomb coefficients from NM98. To begin with, as noted in Wolfe & Melia (2006), we found that the entire spectrum became accelerated to higher energies on timescales much shorter than $10^8$ yr unless a low-energy cutoff in the turbulence was introduced. Following Wolfe & Melia’s (2006) prescription and setting the turbulence to zero for $\beta < 0.5$, we still found much more heating than in Blasi (2000). We found that the temperature of the nonthermal component rose from 7.5 to 9.7, 12.7, and 44 keV after 3, 4, and 5 $\times 10^8$ yr, respectively, compared to a temperature of 8.2 keV, which Blasi’s results show after $5 \times 10^8$ yr. This and our general results on the acceleration agree qualitatively with the results of Wolfe & Melia (2006), which also show that with such a turbulence model, the electrons will be heated before $10^8$ yr to a temperature too high to match observations. However, we should also note that these authors state that the Fokker-Planck and Coulomb coefficient formulation based on NM98 (which we use here) may suffer from some numerical problems. For example, using NM98 coefficients, they claim that an initially Maxwellian distribution with $kT = 0.1m_e c^2$ and subjected only to Coulomb interactions changes by 15% after 4 Spitzer times. We found, however, that we were able to achieve less than 1% deviation [defined here as $100 |N_{th}(E) - N(E)|/N(E)$] after 10 Spitzer times.

Our analysis of this problem has been limited to consideration of energizing the electrons. Eventually, on a $\sim 2000 \times$ longer timescale, the electrons will come into thermal equilibrium with protons, and the estimated temperatures will be reduced by a factor of 2. We have not considered the possibility of directly energizing the proton by the same process that heats up the electrons. In that case, the situation becomes more complex, and would depend on the relative rate of energy input for electrons and protons. However, we note that HXRs in the $>20$ keV range can also be produced via interactions between low-energy thermal electrons and nonthermal protons, with energies greater than 40 MeV in a process one may call “inverse bremsstrahlung.” In the rest frame of the protons, the electrons will have the requisite velocity to produce HXR photons. The Coulomb loss rate of the nonthermal protons, being mainly due to their encounters with the thermal protons, will be 43 times longer. This may increase the bremsstrahlung yield by an undetermined factor that will depend on the details of the electron and proton acceleration rates and energy dependence. Treating this problem is beyond the scope of this paper, where our main goal has been to clarify the situation with the electrons, as summarized above. In future works, we will consider this more complex problem of electron and proton acceleration.

These results also indicate that HXRs are produced by relativistic electrons via inverse Compton processes with energies similar to those needed for the diffuse radio emission. As shown in P01 (see also Brunetti et al. 2001), these electrons must be injected and reaccelerated in the ICM. For a recent analysis of this reacceleration by turbulence, see Brunetti & Lazarov (2007).

We thank Siming Liu and Wei Liu for extensive discussions of the problem of acceleration. This work was supported by the NASA grants NNG046A66G and NN07AG65G and by a Stanford University VPUE grant.

**APPENDIX A**

**TIME EVOLUTION OF TEMPERATURE**

When the Coulomb energy-loss (gain) rate is given as a function of temperature as in equation (11), an expression for the time evolution of this temperature can be derived based on energy conservation. In the case where there is no turbulence, and the only energy exchange occurs through Coulomb interactions, the total energy of the system should remain constant. The rate of change in total energy of the system, $\partial E_{\text{tot}}/\partial t = (\partial/\partial t) \int_0^\infty N(E, t) E \, dE$, can be rewritten using equation (1) along with the no-flux boundary condition (see Park & Petrosian 1995, eq. [3]) and the assumption that $D_{\text{Coul}}(E)N(E)$ vanishes at the boundaries as

$$\frac{\partial E_{\text{tot}}}{\partial t} = - \int_0^\infty \hat{E}_{\text{hot}}(E, t) \, dE.$$  \hspace{1cm} (A1)

Now we can consider the Coulomb energy-loss term to be a function of temperature, which is in turn a function of time, $\hat{E}_{\text{Coul}}(E, T(t))$. At some initial time $t = t_0$, we assume that the temperature is set so that $\partial E_{\text{tot}}/\partial t(t_0) = 0$ (for example, the initial distribution is Maxwellian, and the energy-loss term temperature is set to the temperature of the distribution). Therefore, in order to conserve energy at all times, we require that $\partial^2 E_{\text{tot}}/\partial t^2 = 0$. Based on equation (A1), this means that

$$\frac{\partial T}{\partial t} = \frac{\int_0^\infty \hat{E}_{\text{Coul}}(E, T) \, (\partial N/\partial t)(E, t) \, dE}{\int_0^\infty (\partial \hat{E}_{\text{Coul}}/\partial T)(E, T) N(E, t) \, dE}.$$  \hspace{1cm} (A2)

A new value of $T$ can be calculated at each time step and used for the calculation of the Coulomb rates (eqs. [12] and [13]) to be used in the solution of equation (1) at the next time step.

As long as the particle distribution is nearly Maxwellian, this method gives results similar to the more computationally intensive method, which uses equations (8) and (9) to calculate the coefficients at each time step. Figure 7 shows a comparison of these two methods using the same conditions as those in the bottom left panel of Figure 3.
Fig. 7.—Comparison of the two different methods for calculating time-dependent Coulomb coefficients using the same conditions as in the bottom left panel of Fig. 3. In the left panel, the dashed line is the initial particle distribution, the solid lines [N(E)] were calculated according to the method described in the appendix, which uses a time-dependent temperature parameter, and the dotted lines [Nt(E)] were calculated according to eqs. (8) and (9). The right panel shows the relative difference in the distributions calculated using these two different methods. [See the electronic edition of the Journal for a color version of this figure.]

APPENDIX B
FITTING METHODS

The following provides details on the fitting methods used in §3. Given a particle distribution N(E), the thermal component was determined by a two-parameter fit. Consider a Maxwellian distribution Nth(E) = A√Ee^(-Emc^2/kT), where A and T are the two free parameters. For E \leq kT/m_ec^2, we have ln[Nth(E)] \approx 1/2 ln E + ln A. Therefore, A was determined by finding the intercept of the tangent line to the distribution in log-log space, approximately 3 orders of magnitude below the energy at which the distribution peaks. This ensures that the particle distribution and the fitted Maxwellian agree at low energies. The value of T was then determined by finding the largest such value such that Nth(E) < N(E) across the energy range. This gives the largest thermal component contained within the particle distribution. Note that for the acceleration models with q = 0, the distribution was too broad to fit well to a Maxwellian at lower energies, and A and T were instead determined by requiring the peaks of Nth(E) and N(E) to coincide. The total number of thermal particles is given by Nth = (A√π/2)(kT/m_ec^2)^3/2, and the total number of nonthermal particles is given by Nnonth = 1 - Nth. The ratio of nonthermal energy to total energy was calculated as Rnonth = 1 - [(3/2)kT]\^Etot, where \^Etot = \int_0^\infty N(E)E dE. The power-law index was calculated from the nonthermal component Nnonth(E) = N(E) - Nth(E) as δ = -[d ln Nnonth(E)]/d ln E at an energy 2 orders of magnitude above the energy at which the nonthermal component peaks.

REFERENCES

Benz, A. O. 2002, Plasma Astrophysics (2nd ed.; Dordrecht: Kluwer)
Blasi, P. 2000, ApJ, 532, L9
Bowyer, S. 2003, in ASP Conf. Ser. 301, Matter and Energy in Clusters of Galaxies, ed. S. Bowyer & C.-Y. Hwang (San Francisco: ASP), 125
Bradač, M., et al. 2006, ApJ, 652, 937
Brunetti, G., & Lazarian, A. 2007, MNRAS, 378, 245
Brunetti, G., Setti, G., Feretti, L., & Giovannini, G. 2001, MNRAS, 320, 365
Carilli, C., & Taylor, G. B. 2002, ARA&A, 40, 319
Clarke, T. E., 2003, in ASP Conf. Ser. 301, Matter and Energy in Clusters of Galaxies, ed. S. Bowyer & C.-Y. Hwang (San Francisco: ASP), 185
Clarke, T. E., et al. 2001, ApJ, 547, L111
Dogiel, V. A., Ko, C. M., Kuo, P. H., Hwang, C. Y., Ip, W. H., Birkinshaw, M., Colafrancesco, S., & Prokhorov, D. A. 2007, A&A, 461, 433
Enßlin, T. A., Lieu, R., & Biermann, P. 1999, A&A, 344, 409
Feretti, L., Fusco-Femiano, R., Giovannini, G., & Govoni, F. 2001, A&A, 373, 106
Fusco-Femiano, R., Landi, R., & Orlandini, M. 2005, ApJ, 624, L69
---. 2007, ApJ, 654, L9
Fusco-Femiano, R., et al. 1999, ApJ, 513, L21
---. 2000, ApJ, 534, L7
---. 2003, A&A, 398, 441
---. 2004, ApJ, 602, L73
Fusco-Femiano, R., & Orlandini, M. 2005, ApJ, 624, L69
Fusco-Femiano, R., & Orlandini, M. 2005, ApJ, 624, L69
Fusco-Femiano, R., et al. 2006, ApJ, 652, 948 (PML06)
Galaxies, ed. S. Bowyer & C.-Y. Hwang (San Francisco: ASP), 501
Gruber, D., & Rephaeli, Y. 2002, ApJ, 538, 877
Hudson, D. S., Henriksen, M. J., & Colafrancesco, S. 2003, ApJ, 583, 706
Kim, K. T., et al. 1990, ApJ, 355, 29
Kim, W.-T. 2007, ApJ, 667, L3
Lieu, R., et al. 1996, Science, 274, 1335
Liu, W. 2006, Ph.D. thesis, Stanford University
Markevitch, M. 2005, in The X-ray Universe 2005, ed. J. Koglin (Paris: ESA)
Miller, J. A., Larosa, T. N., & Moore, R. L. 1996, ApJ, 461, 445
Nayakshin, S., & Melia, F. 1998, ApJS, 114, 269 (NM98)
Park, B. T., & Petrosian, V. 1995, ApJ, 446, 699
---. 1996, ApJS, 103, 255
Petrosian, V. 1973, ApJ, 186, 291
---. 2001, ApJ, 557, 506 (P01)
Petrosian, V., & Liu, S. 2004, ApJ, 547, 125
Petrosian, V., & Liu, S. 2004, ApJ, 610, 550
Petrosian, V., Madejski, G., & Luli, K. 2006, ApJ, 562, 948 (PML06)
---. 2001, ApJ, 557, 506 (P01)
Gruber, D., & Rephaeli, Y. 2002, ApJ, 538, 877
Hudson, D. S., Henriksen, M. J., & Colafrancesco, S. 2003, ApJ, 583, 706
Kim, K. T., et al. 1990, ApJ, 355, 29
Kim, W.-T. 2007, ApJ, 667, L3
Lieu, R., et al. 1996, Science, 274, 1335
Liu, W. 2006, Ph.D. thesis, Stanford University
Markevitch, M. 2005, in The X-ray Universe 2005, ed. J. Koglin (Paris: ESA)
Miller, J. A., Larosa, T. N., & Moore, R. L. 1996, ApJ, 461, 445
Nayakshin, S., & Melia, F. 1998, ApJS, 114, 269 (NM98)
Park, B. T., & Petrosian, V. 1995, ApJ, 446, 699
---. 1996, ApJS, 103, 255
Petrosian, V. 1973, ApJ, 186, 291
---. 2001, ApJ, 557, 506 (P01)
Petrosian, V., & Liu, S. 2004, ApJ, 547, 125
Petrosian, V., Madejski, G., & Luli, K. 2006, ApJ, 562, 948 (PML06)
Rephaeli, Y. 1979, ApJ, 227, 364
Rephaeli, Y., & Gruber, D. 2002, ApJ, 579, 587
———. 2003, ApJ, 595, 137
Rephaeli, Y., Gruber, D., & Arieli, Y. 2006, ApJ, 649, 673
Rephaeli, Y., et al. 1999, ApJ, 511, L21
Rossetti, M., & Molendi, S. 2004, A&A, 414, L41
Rudnick, L., & Blundell, K. M. 2003, ApJ, 588, 143
Schlickeiser, R., Sievers, A., & Thieemann, H. 1987, A&A, 182, 21
Schuecker, P., Finoguenov, A., Miniati, F., Böhringer, B., & Briel, U. G. 2004, A&A, 426, 387
Spitzer, L. 1962, Physics of Fully Ionized Gases (2nd ed.; New York: Interscience)
Thierbach, M., Klein, U., & Wielebinski, R. 2003, A&A, 397, 53
Wolfe, B., & Melia, F. 2006, ApJ, 638, 125