SPIN EFFECTS IN LEPTON-NUCLEON SCATTERING: A THEORETICAL OVERVIEW

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I review recent theoretical results on polarized deep inelastic lepton nucleon scattering. Some specific issues, like $Q^2$ evolution of structure functions, the small $x$ behaviour, and the determination of polarized parton densities, are discussed in some detail.

1 Introduction

The physics of polarized nucleons has received increasing interest in the past few years, as we had the opportunity to see during the Workshop. Experimental progress in this field has been impressive, and prospect for the next future are even more interesting. A description of the present status of the field on the experimental side is given in another chapter of these Proceedings; here I will concentrate on the theoretical aspects.

The most important tool for studying the structure of polarized nucleons is polarized deep-inelastic scattering, and most of the work presented in our Workshop was devoted to the study of polarized structure functions; I conclude this introduction recalling a few basics concepts in this field. In Section 2, I will discuss the problem of scaling violation in polarized deep inelastic scattering, in the context of perturbation theory. In Section 3 I will review the theoretical status of the small $x$ behaviour of polarized structure functions, and in Section 4 I will discuss the extraction of polarized parton densities. Finally, in Section 5 I present my conclusions.

The antisymmetric part of the hadronic tensor, relevant for polarized deep inelastic lepton-nucleon scattering, is usually decomposed as

$$W^{A}_{\mu \nu}(x,Q^2) = \frac{m}{p \cdot q} \epsilon_{\mu \nu \rho \sigma} q^\rho \left[ s^\rho g_1(x,Q^2) + \left( s^\rho - \frac{s \cdot q}{p \cdot q} q^\rho \right) g_2(x,Q^2) \right], \quad (1)$$

where $m, s, p$ are the nucleon mass, spin and four-momentum respectively, $q$ is the virtual photon momentum, $Q^2 = -q^2$ and $x = Q^2 / (2p \cdot q)$. This parametrization applies to the case of neutral lepton currents; for charged current scattering three more structure functions must be introduced. It is easy to see that, for longitudinal polarizations $s$, $W^{A}_{\mu \nu}$ is dominated by the $g_1$ term in the Bjorken limit of large $Q^2$ at fixed $x$. 

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The first moment of $g_1$, 

$$
\Gamma_1(Q^2) = \int_0^1 dx g_1(x, Q^2)
$$

is a quantity of great interest, because it is related to the nucleon matrix element of the quark axial current $j_5^\mu = \sum_{i=1}^{n_f} \bar{\psi}_i \gamma^\mu \gamma_5 \psi_i$ whose experimental determination allows important tests of theoretical predictions of perturbative QCD, and possibly provides hints about non-perturbative effects. There are two main difficulties in extracting $\Gamma_1$ from measurements of $g_1$ at different values of $x$ and $Q^2$. First, all data points must be evolved to the same $Q^2$ before estimating the integral. Second, one needs an extrapolation criterion to estimate the contribution to $\Gamma_1$ from unmeasured regions of the $x$ range.

During the Workshop we have learned how both difficulties can be overcome.

### 2 Scale dependence

In the parton model, improved by QCD radiative corrections, the structure function $g_1$ has the following expression:

$$
g_1(x, Q^2) = \left(\frac{e^2}{2}\right) \int_x^1 dy \frac{dy}{y} \left[ C_s^g \left( \frac{x}{y}, \alpha_s(Q^2) \right) \Delta \Sigma(y, Q^2) 
\right.
\right. 
\left.
\left. + 2n_f C_g \left( \frac{x}{y}, \alpha_s(Q^2) \right) \Delta g(y, Q^2) + C_{qNS}^g \left( \frac{x}{y}, \alpha_s(Q^2) \right) \Delta q_{NS}(y, Q^2) \right],
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two different techniques (the operator product expansion in ref. 5 and the parton model in the light-like axial gauge for ref. 6) puts this important result on a firm ground.

The problem of evolving $g_1$ data points to the same $Q^2$, which has also been treated by methods based on the assumption that the evolution of $g_1$ is approximately equal to that of unpolarized structure functions, can now be consistently approached within next-to-leading order QCD. This involves assigning parton distributions at an initial values of $Q^2$ in terms of a set of free parameters, using eq. (3) and the evolution equations to compute $g_1$ for values of $x$ and $Q^2$ corresponding to data points, and performing a fit of the initial condition parameters. Such an analysis has been carried out by different groups; details on the results have been presented during the Workshop, and can be found in refs. 8, 9, 10. In section 4 I will discuss some specific aspects of these results.

3 Small-$x$ behaviour

The problem of small (and large) $x$ extrapolation of $g_1$ is closely related to that of $Q^2$ evolution. In the approach of refs. 8, 9, 10, after the fit parameters have been determined, the $x$ behaviour of $g_1$ outside the measured range is completely fixed, and the integral can be taken over the whole range from 0 to 1. The associated uncertainty is controlled by the errors on the fit parameters and implicitly by the functional form chosen for the parametrization. This procedure allows to estimate the contribution to $\Gamma_1$ from the unmeasured region taking into proper account the effect of $Q^2$ evolution. Such effects are not taken into account if, for example, the small-$x$ contribution to the first moment is estimated by assuming that in the small $x$ region $g_1$ is equal to the average of the last two data points; in fact $Q^2$ evolution at a given value of $x = \bar{x}$ is determined by convolution integrals, and therefore it is affected by the value of the structure function at all values $x > \bar{x}$.

Evolution effects at small $x$ are particularly sensitive to higher order perturbative corrections, which are enhanced by powers of $\alpha_s \log^2(1/x)$. It has been checked in ref. 9 that this is not yet a problem in the $x$ range probed by present experiments. However, the resummation of these contributions, although unsupported by appropriate factorization theorems, leads to a power-like rise of both the singlet and the non-singlet components of $g_1$ as $x \to 0$. In the singlet case this rise is so strong as to produce a divergent first moment of $g_1$. The impact of these corrections on the evolution of the next-to-leading order best-fit non-singlet parton distributions, which already display a strong power-like rise at small $x$, is rather small 14. However it is important
to bear in mind that in the unpolarized case, where factorization theorems do exist, and the double logs are known to cancel down to single logs, there still seems to be some disagreement between the results expected theoretically from resummations and the HERA data on $F_2$.

4 Polarized parton distributions

As mentioned in Section 2, in refs. 8, 9, a next-to-leading order fit to deep inelastic scattering data has been performed; this procedure requires the determination of polarized parton densities in the nucleon. Measurements of $g_1$ are not sensitive to the individual quark distributions, but only to their sum, weighted by their squared charges; for this reason, presently available data do not allow a satisfactory determination of individual quark densities, but only to the their sum, weighted by their squared charges; for this reason, presently available data do not allow a satisfactory determination of individual quark densities, and some ad hoc assumption on the flavor structure has to be made; for example, in ref. 9 all non-singlet quark distributions are assumed to have the same $x$ dependence, while in ref. 8 an SU(3)-symmetric structure of antiquark distributions is assumed. Future data on semi-inclusive deep inelastic scattering 15 may help to disentangle the distributions of individual flavours.

The solution of the coupled Altarelli-Parisi equations in the singlet sector involves the determination of the polarized gluon density $\Delta g$, which is therefore measured indirectly through scaling violation, just as in the unpolarized case. On the other hand, the polarized gluon distribution, and in particular the value of its first moment, is particularly important here, because it affects the determination of the singlet axial charge. It is therefore interesting to know to what extent present data allow a determination of $\Delta g$. In fig. 1 I present six different parametrizations of $\Delta g$, obtained with the procedure outlined above using available data on polarized deep inelastic scattering with proton and deuteron targets.

The three parametrizations of ref. 8, labelled by AB, OS and AR, differ because they were obtained in different subtraction (factorization) schemes for collinear divergences; the curves labelled by GS-A, GS-B and GS-C, instead, were all obtained in the $\overline{\text{MS}}$ subtraction scheme, and they differ because a different large $-x$ behaviour was chosen for each of them.

The subtraction scheme dependence is particularly subtle in the polarized case because of the extra ambiguity related to the way chiral symmetry is broken by the regularization procedure (see ref. 13 for a complete discussion of this point). This is reflected in an ambiguity in the size of the first moment of the gluon coefficient function $C_g$ (which starts at order $\alpha_s$). In particular, the usual $\overline{\text{MS}}$ prescription in dimensional regularization must be supplemented with suitable finite counterterms in order to recover non-singlet axial current
conservation. In this scheme, adopted in refs. 8, 10, the first moment of the gluon coefficient function \( C^1_g \) vanishes. In a different, widely-adopted class of schemes, one fixes the finite counterterms in such a way that the first moment of the polarized quark distribution is scale independent, which implies \( C^1_g = -\frac{\alpha_s}{4\pi} \). In this class of schemes, the quark singlet distribution, being conserved by evolution, can be interpreted in a natural way as the total helicity carried by quarks. The three factorization schemes used in ref. 9 belong to this class. The quark distributions change by an amount proportional to \( \Delta g \) when going from one class of schemes to the other; this change is compensated by a corresponding modification of coefficient functions and two-loop Altarelli-Parisi kernels, so that physical quantities like \( g_1 \) remain unaffected, up to next-to-next-to-leading order terms.

Because the evolution equations imply that at leading order the first moment of \( \alpha_s \Delta g \) does not vanish at large \( Q^2 \), this scheme dependence persists asymptotically and is potentially large if the first moment of the gluon distribution is large. On the other hand the gluon distribution, and in particular its first moment, is not affected, at this order, by the scheme change, and therefore parametrizations of \( \Delta g \) obtained in different schemes, like those presented in fig. 1, can be directly compared. We see that five of the six parametrizations considered are quite close with each other in shape; the sixth one, namely the GS-C parametrization, was obtained by forcing a particular large-\( x \) behaviour of the initial conditions; the quality of the resulting fit is only slightly worse.
than for the others. This suggests that present data are not precise enough to determine the exact behaviour of $\Delta g$ in the whole $x$ range. However, I would like to stress that enormous progress has been made in the determination of $\Delta g$, and in particular that the situation has significantly improved since deuteron data have become available. Furthermore, it is interesting to observe that, using presently published data, the first moment of $\Delta g$ can be determined with an accuracy of about 50%; the result of ref.\(^8\), for example, is

$$\eta_g = \int_0^1 dx \Delta g(x, 1\text{ GeV}^2) = 1.52 \pm 0.74,$$  \hspace{1cm} \hspace{1cm} (5)$$

to be contrasted with the fact that proton data alone would even be consistent with $\eta_g = 0$. The fits of ref.\(^8\) are consistent with the result in eq. \(5\). The uncertainty quoted in eq. \(5\) will improve very much in the near future, with the inclusion of recent deuteron data from the SMC Collaboration, and particularly if polarized deep inelastic scattering data will be collected at HERA. This possibility has been discussed during our Workshop\(^1\), and it has been concluded that an accuracy of about 30\% for $\eta_g$ is achievable at HERA, assuming a luminosity of 200 pb\(^{-1}\) in the polarized configuration. The important point here is that HERA measurements will help in determining $\eta_g$ because they will allow measurements of $g_1$ in the same $x$-range as previous experiments, but at much larger values of $Q^2$.

It is apparent from the above discussion that a direct experimental determination of $\Delta g$ would be of great interest, because it would provide an independent check on the results obtained from scaling violation. There are several proposals in this direction. A direct measurements of the polarized gluon distribution can be performed by studying two-jet production with polarized beams at HERA, a process dominated by photon-gluon fusion\(^1\) for relatively small $x$. In ref.\(^1\) the conclusion is reached that a good measurement of $\Delta g$ for $x$ below 0.2 is achievable at HERA in the polarized configuration, if the designed conditions of energy and luminosity will be reached. This estimate is based on leading order calculations, since a next-to-leading order calculation of the cross section for polarized dijet production is lacking.

A second possibility is the production of heavy quark pairs in photon-proton collisions. This possibility is being consider by the COMPASS collaboration in the deep-inelastic scattering regime\(^1\). It has been recently pointed out\(^2\) that charm asymmetries can be measured at HERA in the polarized configuration, and that significant constraints on $\Delta g$ will be put by this kind of measurements. Also in this case, a next-to-leading order calculation is unfortunately not available.
5 Conclusion and outlook

Important progress has been made in the study of the structure of polarized nucleons in the recent past. The structure function $g_1$ is the quantity which has undergone the most detailed analyses, since it is relatively easily accessible from the experimental point of view. The perturbative study of $g_1$ has now reached a level of accuracy which is comparable to the one we are used to in the case of unpolarized structure functions, since both coefficient functions and Altarelli-Parisi kernels are known at next-to-leading order. The results of the different analyses performed are consistent with each other, and indicate that the density of polarized gluons in the nucleons is significantly different from zero. Future data will considerably improve our knowledge of polarized parton distributions; scaling violation will be studied at a higher level of accuracy, and direct measurements of polarized asymmetries are expected in the near future.

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