THE HORIZON RUN N-BODY SIMULATION: BARYON ACOUSTIC OSCILLATIONS AND TOPOLOGY OF LARGE-SCALE STRUCTURE OF THE UNIVERSE

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ABSTRACT

In support of the new III survey, which will measure the baryon oscillation scale using the luminous red galaxies (LRGs), we have run the largest N-body simulation to date using 41203 = 69.9 billion particles, and covering a volume of (6.592 h−1 Mpc)3. This is over 2000 times the volume of the Millennium Run, and corner-to-corner stretches the whole way to the horizon of the visible universe. LRGs are selected by finding the most massive gravitationally bound, cold dark matter subhalos, not subject to tidal disruption, a technique that correctly reproduces the three-dimensional topology of the LRGs in the Sloan Survey. We have measured the covariance function, power spectrum, and the three-dimensional topology of the LRG distribution in our simulation and made 32 mock surveys along the past light cone to simulate the Sloan III survey. Our large N-body simulation is used to accurately measure the nonlinear systematic effects such as gravitational evolution, redshift space distortion, past light cone space gradient, and galaxy biasing, and to calibrate the baryon oscillation scale and the genus topology. For example, we predict from our mock surveys that the baryon acoustic oscillation peak scale can be measured with the cosmic variance-dominated uncertainty of about 5% when the SDSS-III sample is divided into three equal volume shells, or about 2.6% when a thicker shell with 0.4 < z < 0.6 is used. We find that one needs to correct the scale for the systematic effects amounting up to 5.2% to use it to constrain the linear theories. The uncertainty in the amplitude of the genus curve is expected to be about 1% at 15 h−1 Mpc scale. We are making the simulation and mock surveys publicly available.

Key words: cosmological parameters – cosmology: theory – large-scale structure of universe – galaxies: formation – methods: N-body simulations

Online-only material: color figures

1. INTRODUCTION

A leading method for characterizing dark energy is measuring the baryon oscillation scale using luminous red galaxies (LRGs). Baryon oscillations are the methods “least affected by systematic uncertainties” according to the Dark Energy task Force report (Albrecht et al. 2006). The baryon oscillation scale provides a “standard ruler” (tightly constrained by the Wilkinson Microwave Anisotropy Probe (WMAP) observations), which has been detected as a bump in the covariance function of the LRGs in the Sloan Survey and also in the 2dFGRS survey; see Percival et al. 2007 for a review.

This observation has prompted a team to propose the Baryon Oscillation Spectroscopic Survey (BOSS) for an extension of the Sloan Survey (Sloan III), which will obtain the redshifts of 1,300,000 LRGs over 10,000 deg2 selected from the Sloan digital image survey. This will make it possible to measure the baryon bump in the covariance function of the LRGs to a high accuracy as a function of redshift out to a redshift of z = 0.6. It is claimed that this allows, in principal, measurement of the Hubble parameter to an accuracy of 1.7% at z = 0.6, and allows us to place narrower constraints on w(z) (the ratio of dark energy pressure to energy density as a function of redshift). Systematic uncertainties are claimed to be of the order of 0.5% (Eisenstein et al. 2007a; Crocce & Scoccimarro 2008). These systematics are only as well controlled as knowledge of all nonlinear effects associated with LRGs and thus one needs large N-body simulations. Small N-body simulations—(256)3 particles, (512 h−1 Mpc)3 (Seo & Eisenstein 2005), show that nonlinear effects do change the covariance function of the LRGs relative to that in the initial conditions. The baryon bump in the covariance function is somewhat washed out, but can be recovered by reconstruction techniques moving the galaxies backward to their initial conditions using the Zel’dovich approximation (Eisenstein et al. 2007b). However, these simulations are inadequate because of their small box size which is only a factor of 4.7 larger than the baryon oscillation scale which is of order of 108 h−1 Mpc. It is very important to model the power spectrum accurately at large scales and to have a large box size so that the statistical errors in the power spectrum are small. After all, we need to measure the acoustic peak scale to an accuracy better than 1%.

Concerning the Baryon Acoustic Oscillation method applied to dark energy, Crotts et al. (2005) note that “systematic effects will inevitably be present in real data; they can only be addressed by means of studying mock catalogues constructed from realistic cosmological volume simulations” just as we are setting about to do here. According to Crotts et al. (2005), such modeling would typically improve the accuracy of the estimated dark energy parameters by 30%–50%. In support of this goal, we have previously completed a 20483 particle (4915 h−1 Mpc)3 Cold Dark Matter (CDM) Simulation (Park et al. 2005a). We have developed a new technique to identify LRGs by finding the most massive bound subhalos not subject to tidal disruption (Kim & Park 2006). This 20483 simulation successfully models the three-dimensional topology of the LRGs in the Sloan Survey.
correctly modeling within approximately 1σ level the genus curve found in the observations (Gott et al. 2009). By contrast the semianalytic model of galaxy assignment scheme applied to the Millennium Run by Springel et al. (2005) differed from the observational data on topology by 2.5σ indicating either a need for an improvement in its initial conditions (it used a bias factor that is too low and an \(n_s = 1\) power spectrum that have relatively low power at large scales according to WMAP-3 (Spergel et al. 2007)) or its galaxy-formation algorithm (compare Gott et al. 2008). The fact that our 2048\(^3\) N-body simulation modeled well the observational data on LRG topology suggests that the formation of LRGs is a relatively clean problem that can be well modeled by large N-body simulations using WMAP 5 year initial conditions.

Building on this success in modeling the formation and clustering of LRGs, in this paper, we will show first results from our very large Horizon Run N-body simulation in support of the BOSS in the Sloan III survey.

2. N-BODY SIMULATIONS

The first N-body computer simulation used 300 particles and was done by Peebles (1970). In 1975, Groth & Peebles made “cosmological” N-body simulations using 1500 particles with \(\Omega_m = 1\) and Poisson initial conditions (Groth & Peebles 1975). Aarseth et al. (1979) used 4000 particles. They found covariance functions and multiplicity functions quite like those observed for models with \(\Omega_m < 1\) and more power on large scales than Poisson (Gott et al. 1979; Bhavsar et al. 1981) as originally proposed theoretically (Gott & Rees 1975; Gott & Turner 1977). Indeed, the inflationary flat-lambda models popular today have \(\Omega_m < 1\) and more power on large scales than Poisson, just as these early simulations suggested. The results were reasonable from theoretical considerations of nonlinear clustering, concerning cluster (Gunn & Gott 1972), and void (Bertschinger 1985; Fillmore & Goldreich 1984) formation from small fluctuations via gravitational instability. The inflationary scenario and CDM brought realistic initial conditions for N-body models.

Geller & Huchra (1989)’s discovery of the CfA Great Wall of galaxies was unexpected. But no N-body simulations large enough to properly model structures as large as the Great Wall had yet been done. Then Park (1990), using 4 million particles, a peak biasing scheme, and a CDM, \(\Omega_m h = 0.2\) model, was able to simulate for the first time a volume large enough to properly encompass the CfA Great Wall. A slice through this simulation showed a Great Wall just like the CfA Great Wall.

Large cosmological N-body simulations have been very useful in understanding the distribution of galaxies in space and time, and provide us a powerful tool to test galaxy formation mechanisms and cosmological models. The current and future surveys of large-scale structure in the universe require even larger simulations. In this paper, we present one of such large simulations to study the distribution of galaxies in space and provide us a powerful tool to test galaxy formation.

Figure 1 shows how our simulation compares with other recent large simulations in terms of number of particles: the Millennium Run with 10 billion particles, and the recent French collaboration N-body simulation (Teyssier et al. 2009) using 68.7 billion particles. Our Horizon Run contains 69.9 billion particles. The French simulation size was 2000 \(h^{-1}\) Mpc while that of the Millennium Run was 500 \(h^{-1}\) Mpc. Figure 2 shows the sizes of these notable simulations to scale for comparison.

3. THE HORIZON RUN

Here we describe our Horizon Run N-body Simulation. We adopt the \( \Lambda \)CDM model of the universe with the WMAP 5 year parameters listed in Table 1. The initial conditions are generated on a 4120\(^3\) mesh with pixel size of 1.6\(h^{-1}\) Mpc. The size of the simulation cube is 6595 \(h^{-1}\) Mpc on a side, and 11,418 \(h^{-1}\) Mpc diagonally. Note that the horizon distance is 10,500 \(h^{-1}\) Mpc. We use 4120\(^3\) = 69.9 billion CDM particles whose initial positions are perturbed from their uniform distribution to represent the initial density field at the redshift of
corresponds to the age of the universe, 13.6 billion years. Linearly evolving the initial density field backward in time, the opening angle is 45° showing the matter density field in the past light cone all the way to the Earth (the observation point) at the past light cone. The subhalos are then found in the past light cone data. The effects of our choices of the starting redshift, time step, and force resolution are discussed in Appendix A, where we use the Zel’dovich redshifts and FoF halo multiplicity function to estimate the effects.

During the simulation, we located eight equally spaced (maximally separated) observers in the simulation cube, and saved the positions and velocities of particles at $z < 0.6$ as they cross the past light cone. The subhalos are then found in the past light cone data, and used to simulate the SDSS-III LRG survey. We assume that the SDSS-III survey will produce a volume-limited LRG sample with constant number density of $3 \times 10^{-4} (h^{-1} \text{ Mpc})^{-3}$. In our simulation, we vary the minimum mass limit of subhalos to match the number density of selected subhalos (the mock LRGs) with this number at each redshift. For example, the mass limit yielding LRG number density of $3 \times 10^{-4} (h^{-1} \text{ Mpc})^{-3}$ was found to be $1.33 \times 10^{12} h^{-1} M_\odot$ and $9.75 \times 10^{12} h^{-1} M_\odot$ at $z = 0$ and 0.6, respectively.

Our large volume allows us to much better model the true power at large scales, particularly in the case of the LRG distribution, which is critical to the baryon oscillation test. Figure 3 is a $64 h^{-1}$ Mpc thick slice through this simulation showing the matter density field in the past light cone all the way to the horizon. The thickness of the wedge is constant and the opening angle is 45°. The Earth (the observation point) is at the vertex and the upper boundary is the big bang surface at $z = \infty$. The distribution of the CDM particles is converted to a density field using the variable-size spline kernel containing 5 CDM particles, and the density field before $z = 23$ was obtained by linearly evolving the initial density field backward in time. The radial scale of the slice is the look-back time, so the upper edge corresponds to the age of the universe, 13.6 billion years located at the comoving distance of $10,500 h^{-1}$ Mpc.

4. MODELLING OF THE LUMINOUS RED GALAXIES

We use a subhalo finding technique developed by Kim & Park (2006; see also Kim et al. 2008) to identify physically self-bound (PSB) Dark Matter subhalos (not tidally disrupted by larger structures) at the desired epoch and identify the most massive ones with LRGs. This does not throw away information on the subhalos which is actually in the N-body simulations and would be thrown away in a Halo Occupation Distribution analysis using just simple friend-of-friend (FoF) halos, and importantly allows us to identify the LRGs without free fitting parameters. We have a mass resolution high enough to model the formation of LRGs whose dark halos are more massive than about $M_{30} \approx 8.87 \times 10^{12} h^{-1} M_\odot$ (total mass of 30 particles). Within the whole simulation cube we detect 127,890,474 and 92,043,641 subhalos at $z = 0$ and 0.5, respectively. The mean separation of these massive subhalos is $13.1 h^{-1}$ Mpc at $z = 0$.

Before we proceed with analyzing the mock LRG sample, it will be interesting to see how well it reproduces the physical properties of the existing LRG sample. Kim et al. (2008) and Gott et al. (2008) has shown that the spatial distribution of the subhalos identified in the ACIDM simulation is consistent with the galaxies in the SDSS Main Galaxy DR4plus sample (Choi et al. 2007). The observed genus, the distribution of local density, and morphology-dependent luminosity function were successfully reproduced by subhalos that are ranked according to their mass to match the number density of galaxies. Moreover, the subhalos identified as LRGs are also shown to reproduce the genus topology of SDSS LRGs remarkably well (Gott et al. 2009). In Appendix B, we present a comparison between the correlation functions (CF) of the SDSS LRGs and simulated mock LRGs to examine how accurately our mock LRGs model the observed ones in terms of the two-point statistic. We find a reasonable match between them over the scales from $z \approx 1$ to $140 h^{-1}$ Mpc, particularly in the case of the shape of the CF. Based on our previous and present studies, we conclude that the observed BAO (Baryon Acoustic Oscillations) scale and large-scale topology, which this work is focusing on, can be very accurately calibrated using the mock galaxies identified from our gravity-only simulation. One of the reasons is that both are determined by the shape and not by the amplitude of the power spectrum.

It is interesting to know the fraction of mock LRGs that are central or satellite subhalos within FoF halos. This can be a test of consistency with other prescriptions for modeling LRGs (see Zheng et al. 2008 for the Halo Occupation Distribution modeling). For comparison with the SDSS LRG sample ($−23.2 \leq M_{0.3g} \leq −21.2$; Zehavi et al. 2005), we build 24 mock LRG samples from eight all-sky past light cone data with the same observation mask of the SDSS survey. Because the LRGs brighter than $M_{0.3g} = −23.2$ are extremely rare (Zheng et al. 2008), we do not apply the upper-mass limit to the mock samples (refer to Appendix B for detailed descriptions of generating SDSS LRG surveys). In these mock samples, we count the member galaxies belonging to each FoF halo utilizing the fact that our PSB method identifies the subhalos or mock LRGs

Table 1

| $N_p$ | $N_m$ | L$_{max}$ | N$_{sub}$ | $z_i$ | h | n | $\Omega_m$ | $\Omega_b$ | $\Omega_k$ | b | $m_p$ | $f_h$ |
|------|------|----------|----------|------|---|---|----------|----------|----------|---|------|------|
| 4120$^3$ | 4120$^3$ | 6592 | 400 | 23 | 0.72 | 0.96 | 0.26 | 0.044 | 0.74 | 1.26 | 2.96 x 10$^{11}$ | 160 $h^{-1}$ kpc |

Notes. Columns: (1) number of particles, (2) Size of mesh, Number of initial conditions, (3) Simulation box size in $h^{-1}$ Mpc, (4) Number of steps, (5) Initial redshift, (6) Hubble parameter in 100 km s$^{-1}$ Mpc$^{-1}$, (7) Primordial spectral index of $P(k)$, (8) Matter density parameter at $z = 0$, (9) Baryon density parameter at $z = 0$, (10) Dark energy density parameter at $z = 0$, (11) Bias factor, (12) Particle mass in $h^{-1} M_\odot$, and (13) Gravitational force softening length.

5 http://www.ksc.re.kr/eng/resources/resources1.htm
within the FoF halos. Among the mock LRGs in a FoF halo, we regard the most massive LRG as the central galaxy and the rest of them as satellites. The mass of a host system is obtained by summing up the masses of all member LRGs. We find that the fraction of the mock LRGs that are satellites in FoF halos is $5.4 \pm 0.1\%$. This satellite fraction is within the observed range of $5.2\%$--$6.2\%$ (Zheng et al. 2008).

5. BAO BUMP IN $\xi(r)$

Figure 4 shows the evolution of the CF of the matter density field from $z = 23$ to $z = 0$. The dashed curves are the linear CFs, and colored ones are the matter CFs measured at eight redshifts using the whole simulation data. The inset box demonstrates the change in the shape of CF near the baryon oscillation bump due to the nonlinear gravitational evolution in the matter field. In the inset box, the amplitude of the CF at $z = 23$ is scaled so that the peak of the baryon bump has unit amplitude, and then the amplitudes of the CFs at other epochs are scaled to match the CF at $z = 23$ at $r = 48 \, h^{-1} \, \text{Mpc}$. The amplitude of the bump decreases by $16\%$, and the peak location decreases by $4\%$ from the initial epoch to the present.

The top two curves of Figure 5 are the CFs calculated from all mock LRGs in the whole cube at $z = 0$ and 0.5. They are also shown in the inset box at the lower left corner. The curve with $1\sigma$ error bars is the mock LRG CF at $z = 0$. Errors are estimated from subcube results. The baryon oscillation bump is clearly visible. Notice the tiny amount of noise, because we are sampling such a large volume. In Figure 5, we also present the three-dimensional covariance function of the matter density.
Figure 4. Evolution of the correlation function of the matter density field at the epoches from $z = 0$ to 23. The dashed curves are the linearly evolved correlation functions, and the colored ones are the matter correlation functions measured from the horizon simulation. The inset box magnifies the matter correlation functions near the baryon oscillation bump with amplitudes to match at $r = 48\, h^{-1}\, \text{Mpc}$ after scaling the peak of the baryonic bump of matter correlation at $z = 23$ to unity.

(A color version of this figure is available in the online journal.)

Figure 5. Top curves: the real space correlation functions of the mock LRGs in the whole cube at $z = 0$ and 0.5. 3σ error bars are attached to the correlation function at $z = 0$. The matter density correlation functions and the linear theory correlation functions at $z = 0$ and 0.5 (bottom curves), are also shown.

(A color version of this figure is available in the online journal.)

field, and linear regime CF at $z = 0$ and 0.5 for comparison. The linear theory gives $107.6\, h^{-1}\, \text{Mpc}$ for the position of the BAO peak while in the mock LRG CFs they are $103.6\, h^{-1}\, \text{Mpc}$ and $104.7\, h^{-1}\, \text{Mpc}$ at $z = 0$ and 0.5, a difference of up to 3.7% and 2.7%, respectively. On the other hand, the matter density field has peak at $103.2\, h^{-1}\, \text{Mpc}$ and $105.0\, h^{-1}\, \text{Mpc}$ at these epoches, respectively.

It is important to note that the systematic effects in the baryonic bump in the LRG CF depend on the type of tracer. Because of the high statistical biasing in the distribution of LRGs the evolution of the LRG CF is different from that of the matter CF. In Figure 6, we show the ratio of the matter and mock LRG CFs relative to the linear CF. The bottom three curves are the matter CFs at $z = 0, 0.5$, and 2.6 divided by the linear CF evolved to the corresponding redshifts. The upper two solid curves are the mock LRG CFs at $z = 0$ and 0.5 also divided by the linear CF evolved to $z = 0$ and 0.5. It can be noted that the deviation from the shape of the linear CF is different in the cases of the matter and mock LRG CFs even though the difference is not large. For example, at the separations of $r = 48$ and $70\, h^{-1}\, \text{Mpc}$, the ratio between the LRGs and matter at $z = 0.5$ is about 4.65, but $r = 105\, h^{-1}\, \text{Mpc}$ the ratio is 4.52, a 2.7% difference. The long dashed lines in Figure 6 are the ratios of the mock LRG CF to the evolved matter CF. We can observe small enhancements of the ratio around the baryonic acoustic signature both at $z = 0$ and 0.5, and this enhancement is consistent with the results of Desjacques (2008) who used density maxima instead of halos. Therefore, it will be erroneous to use the nonlinear matter CF to correct for the observed LRG CF. It is very important to correctly model the LRGs in the N-body simulation that have the accurate degree of biasing, and to use them for the correction.

For direct comparison with the Sloan III data, we have made all-sky LRG surveys using the past light cone data generated at eight locations in our Horizon Run. The surveys are made so as to maintain the mean number density of LRGs as $3 \times 10^{-4}(h^{-1}\, \text{Mpc})^{-3}$. Figure 7 shows a fan shaped diagram of a random slice through a mock Sloan III survey.
In Figure 8, we present the CFs of the LRGs observed in the mock SDSS-III surveys. At each observer location, we make four SDSS-III LRG survey catalogs covering π stradian of the sky and extending out to z = 0.6. Each mock LRG sample is then divided into three redshift shells that contain roughly equal number of galaxies. The redshift boundaries between these shells are at z₁ = 0.396 and z₂ = 0.513. Three solid lines of Figure 8 are the mean CFs averaged over 32 mock surveys measured in these three shells. We add peculiar velocities along the line of sight to simulate the effects of redshift distortion, and measure the covariance function in the past light cone. The 1σ error bars for the CF of each redshift shell, obtained from 32 mock surveys, are attached to the CF of the shallow sample R₁(0 < z < z₁). In Figure 19 of Cabre & Gaztanaga (2009) who used SDSS LRG samples for their analysis. They tried to explain the BAO-amplitude shift by introducing possible systematic effects in the SDSS observation but they did not give any conclusion. However, it is interesting to observe a similar shift in the mock surveys which does not suffer from the systematic effect.

The local peak in the CF measured from individual mock survey data shows a large variation, and so we developed a method that can measure the BAO bump position more accurately. The position of the BAO bump was measured by fitting the observed CF to a set of template CFs. The template CFs are those selected from the simulated matter CFs from z = 23 to 0. As shown in Figure 4, the matter CF is accurately known, and shows a wide range of variation in the amplitude and shift of the BAO bump. To fit an observed CF to the templates, we calculate

$$\chi^2_m = \sum_{r_1 \leq r \leq r_2} [\xi(r) - A_m \xi_m(r - \delta_m)]^2,$$

where A_m and δ_m are the amplitude and shift parameters. Here, we set r₁ = 90 and r₂ = 115 h⁻¹ Mpc to safely include the bump. We numerically find the free parameters A_m and δ_m that minimize χ² to determine the best-fit template, and to estimate the position of the BAO bump.

A close inspection reveals that the peak location and amplitude of the BAO bump in the LRG CF slowly decrease as redshift decreases; the location of the BAO bump is at 102.0 ± 5.0, 102.8 ± 3.9, and 103.8 ± 4.1 h⁻¹ Mpc for R₁(0 < z < z₁), R₂(z₁ < z < z₂), and R₃(z₂ < z < 0.6) shells, respectively. Due to the sample variance the uncertainty
Figure 9. Colored lines: correlation functions of 16 mock SDSS-III surveys and their mean (black lines with error bars) in a 0.4 < z < 0.6 redshift shell. For clarity, we show only half the mock survey results. The position of the baryon oscillation bump is measured by fitting the correlation function of the mock LRGs to a set of template correlation functions.

(A color version of this figure is available in the online journal.)

Figure 10. Power spectra of matter fields at z = 0.5 (two bottom lines) and at z = 0 (two middle lines). Among matter power spectra, the solid lines are those from the simulation and the dotted lines are the linearly evolved ones. Top curves are those for the LRGs in redshift (topmost curve) and real spaces at z = 0.

(A color version of this figure is available in the online journal.)

in the BAO scale amounts to as much as 5.0%. For a comparison, the real space and redshift space CFs of all LRGs in the simulation box at z = 0.5 are also plotted in dotted and dashed curves, respectively. Their BAO bump peaks are located at 105.2 and 102.5 h⁻¹ Mpc, respectively.

The uncertainty in the BAO bump position can be reduced if larger samples are used. We measure the CF of mock LRGs located in a redshift shell with inner and outer boundaries at z = 0.4 and 0.6, respectively. This region contains more data than previous settings leading to smaller scatter around the average (see Figure 9). The mean value of the BAO peak position over 32 mock surveys is 103.2 h⁻¹ Mpc and the difference from the linear regime initial conditions is 4.4(±2.7)h⁻¹ Mpc. Thus if the BAO peak is measured as in our analysis, the peak location should be corrected upward by above value. The uncertainty in this systematic correction is also estimated from our 32 mock SDSS-III surveys. This gives us a benchmark idea of the extent of the systematic effects (and their uncertainty) encountered in measuring the position of the BAO bump from LRGs and comparing it with the baryon oscillation scale from the linear regime theory. As hoped for, the systematic effects due to nonlinear effects and biasing are small, and can be corrected for with high accuracy. The corrected value will only be in error by the uncertainty in the correction factor which is 2.5% when the data in this thick shell is used. Comparison of the individual 32 mock Sloan III surveys after such systematic corrections have been made will give us an accurate measure of the statistical accuracy of the results on w(z) that we should expect in the real survey.

These preliminary studies can be used to improve the survey strategy and reconstruction methods before the survey starts. In the simulations we are omniscient since we know where each of the LRGs actually originated, so this should help us to design and improve the reconstruction techniques. As the data is being collected, and the selection function of the LRGs being detected becomes accurately known, it will be possible to improve the accuracy of these studies as the survey continues. Comparisons with the N-body simulations should be of critical value, allowing one to calibrate the dark energy experiment with high accuracy.

6. BAO WIGGLES IN P(k)

A BAO bump in the correlation has a wiggle pattern in the power spectrum (hereafter PS). Also similar changes, the shift and degrading of shapes, happen to the baryonic wiggles of PS. Recently, Seo et al. (2008) reported that they have detected sub percentage level of shift of the BAO wiggle after applying a fitting to the matter PS. Therefore, it is valuable to check whether LRGs, as a biased peak, have a similar level of shift or they have different scale of shift compared to the matter field. Figure 10 shows the mock LRG power spectra in real (the second top line) and redshift (topmost line) spaces at z = 0 and matter PS at z = 0.5 (bottom line) and 0. Figure 11 shows the evolution of the simulated matter PS normalized by the smoothed PS (Psm) around baryonic scales (Eisenstein & Hu 1998). As can be seen, the nonlinear gravitational evolution affects the wiggle pattern from small scales making them degraded in shape and buried in nonlinear background with time. Oscillatory features at k ≥ 0.1h / Mpc experience a severe distortion in shape while the features on the larger scales show less deviations from the linear shapes.

To quantify the nonlinear distortion of the BAO feature in the PS of mock LRGs, we followed the analysis of Seo et al. (2008) who introduced the shift (α) and degrading (Σm) parameters in
the polynomial fitting functions. The contrast of the baryonic feature is obtained by subtracting the smoothed background PS ($P_{lm}$) of no wiggle form from the linear PS ($P_{lin}$):

$$P_b(k) = P_{lin}(k) - P_{sm}(k).$$

The fitting function is organized as

$$P_{lin}(k) = B(k)P_m(k/\alpha) + A(k),$$

where $B(k)$ reflects the scale-dependent biasing and mode coupling, and $A(k)$ is added to account for shot noise. For the $\chi^2$-fit, we adopt the polynomial forms:

$$A(k) = \sum_{i=0}^{7} a_i k^i,$$

$$B(k) = \sum_{i=0}^{2} b_i k^i.$$  \hspace{1cm} (4)

This functional form provides a reasonable fit to the data and, therefore, we will not consider other alternatives listed in Seo et al. (2008). And $P_{m}(k/\alpha)$ is the shifted model PS with a degrading factor:

$$P_{m}(k) = P_{m}(k/\alpha) \exp\left(-k^2\sum_m^2/2\right) + P_{sm}(k).$$  \hspace{1cm} (5)

The linear PS subtracted by the smoothed power is suppressed by a Gaussian damping of length, $\Sigma_m$. Seo et al. (2008) fixed it as $\Sigma_m = 7.6 \, h^{-1}$ Mpc for the matter field at $z = 0.3$. However, we treat it as a free parameter here because we are dealing with biased subhalos while Seo et al. used the matter particles. The shift parameter, $\alpha$, is added to account for the shift of the baryonic features due to the nonlinear evolution and biasing.

In Figure 12 shown are the ratio of the LRG PS to the smoothed nonlinear PS ($P_{lin}^n(k)$). It shows the contrast of the BAO wiggles over the smooth background PS. The smoothed PS in the nonlinear regime is obtained by applying

$$P_{lin}^n = B(k)P_m(k/\alpha) + A(k),$$

where we adopt the same values of $A(k)$ and $B(k)$ as previously obtained when performing the fitting to the simulated power spectrum of the LRGs.

The fitting results are $(1 - \alpha) = -0.020$, $\Sigma_m = 14.9 \, h^{-1}$ Mpc, and $\chi^2$/DOF = 0.43 at $z = 0$ and, $(1 - \alpha) = -0.023$, $\Sigma_m = 9.0 \, h^{-1}$ Mpc, and $\chi^2$/DOF = 0.47 at $z = 0.5$ for LRGs in redshift space. For the real-space data, $\alpha$ is unchanged while $\Sigma_m$ decreases slightly. The negative value of $1 - \alpha$ means that the baryonic feature of the LRGs is moving toward small scales relative to the linear PS. We detected a 2% wiggle shift for the LRGs while Seo et al. (2008) found 0.45% at $z = 0.5$. Also we obtain $\Sigma_m = 9.0 \, h^{-1}$ Mpc at $z = 0.3$ which was adopted for matter field by Seo et al. (2008). The more negative shift and larger degrading parameter for the mock LRGs relative to the background matter density field, again demonstrate the tracer dependence of the nonlinear effects. We have also applied above analysis to the matter field in real space. The value of $1 - \alpha$ is $-0.0025$, $-0.0025$, $-0.001$, and 0 at $z = 0$, 0.5, 2, and 23, respectively. Also we obtain $\Sigma_m = 8.25$, 6.60, 3.75, and $0 \, h^{-1}$ Mpc for each redshift. These results are consistent with Seo et al. (2008).
7. GENUS TOPOLOGY OF LARGE SCALE DISTRIBUTION OF LRGs

Through the genus topology analysis one can check whether or not the mock LRGs in our simulation correctly reproduce the spatial distribution of the observed LRGs. At scales larger than the correlation length the LRG genus curve is well-approximated by the Gaussian random phase curve with expected small shifts due to nonlinear effects and biasing.

We have developed tools for analyzing the genus topology of large scale structure in the universe (Gott et al. 1986; Hamilton et al. 1986; Gott et al. 1987; Gott et al. 1989; Vogele et al. 1994; Park et al. 2005a; Park et al. 2005b). We smooth the spatial distribution of galaxies to construct density contour surfaces. We measure the genus as a function of density, allowing comparison with the topology expected for Gaussian random phase initial conditions, as predicted in a standard big bang inflationary model (Guth 1981; Linde 1983) where structure originates from random quantum fluctuations in the early universe. We smooth the galaxy distribution with a Gaussian smoothing ball of radius $R_G$, where $R_G$ is chosen to be greater than or equal to the correlation length to reduce the effects of nonlinearity, and greater than or equal to the mean particle separation to reduce the shot noise effects.

Density contour surfaces are labeled by $\nu$, where the volume fraction on the high density side of the density contour surface is $f$:

$$f = \frac{1}{\sqrt{2\pi}}\int_{\nu}^{\infty} \exp(-x^2/2)dx.$$  

The genus curve is given by the topology of the density contour surface:

$$g(\nu) = \# \text{ of donut holes} - \# \text{ of isolated regions}$$  

(Gott et al. 1986). An isolated cluster has a genus of $-1$ by this definition. Since $g(\nu)$ is equal to minus the integral of the Gaussian curvature over the area of the contour surface divided by $4\pi$, we can measure the genus with a computer program (CONTOUR3D; see Gott et al. 1986, 1987). For Gaussian random phase initial conditions:

$$g(\nu) = A(1 - \nu^2)\exp(-\nu^2/2),$$  

where $A = \langle k^2 \rangle/(2\pi^2)$ and $\langle k^2 \rangle$ is the average value of $k^2$ integrated over the smoothed power spectrum (Hamilton et al. 1986; Adler 1981; Doroshkevich 1970; Gott et al. 1987). The $f = 50\%$ median density contour ($\nu = 0$) shows a predicted sponge-like topology (holes), the $f = 7\%$ high density contour ($\nu = 1.29$) shows isolated clusters while the $f = 93\%$ density contour ($\nu = -1.29$) shows isolated voids.

When the smoothing length is much greater than the correlation length, fluctuations are still in the linear regime and since fluctuations in the linear regime grow in place without changing topology, the topology we measure now should reflect that of the initial conditions, which should be Gaussian random phase according to the theory of inflation (Gott et al. 1987; Melott 1987). Small deviations from the random phase curve give important information about biased galaxy formation and nonlinear gravitational effects as shown by perturbation theories and large $N$-body simulations (cf. Matsubara 1994; Park et al. 2005a). All previous studies have shown a sponge-like median density contour as expected from inflation (Gott et al. 1986, 1989, 2009; Moore et al. 1992; Vogele et al. 1994; Canaveses et al. 1998; Hikage et al. 2002, 2003; Park et al. 2005b)

Small deviations from the Gaussian random phase distribution are expected because of nonlinear gravitational evolution and biased galaxy formation and these can now be observed with sufficient accuracy to do model testing of galaxy formation scenarios.

The genus curve can be parameterized by several variables. The amplitude $A$ of the genus curve, proportional to $\langle k^2 \rangle^{3/2}$ of the smoothed power spectrum, gives information about the power spectrum and on the phase correlation of the density distribution. Shifts and deviations in the genus curve from the overall theoretical random phase case can be quantified by the following variables:

$$\Delta \nu = \int_{-1}^{1} g(\nu)d\nu / \int_{-1}^{1} g_{RF}(\nu)d\nu, \quad (10)$$

$$A_V = \int_{-2}^{-1.2} g(\nu)d\nu / \int_{-2}^{-1.2} g_{RF}(\nu)d\nu, \quad (11)$$

$$A_C = \int_{1.2}^{2} g(\nu)d\nu / \int_{1.2}^{2} g_{RF}(\nu)d\nu, \quad (12)$$

where $g_{RF}(\nu)$ is the genus of the random phase curve (Equation (8)) best fit to the data. Thus, $\Delta \nu$ measures any shift in the central portion of the genus curve. The theoretical curve (Equation (8)) has $\Delta \nu = 0$. A negative value of $\Delta \nu$ is called a "meatball shift" caused by a greater prominence of isolated connected high-density structures which push the genus curve to the left. This can be due to nonlinear galaxy clustering and bias associated with galaxy formation. $A_V$ and $A_C$ measure the observed number of voids (and clusters), respectively, relative to those expected from the best fitting theoretical curve.

Park et al. (2005a) show that $A_V < 1$ can result from biasing in galaxy formation because voids are very empty and can coalesce into a few larger voids. A value of $A_C < 1$ can occur because of nonlinear clustering, when clusters collide and merge, and if there is a single large connected structure like the Sloan Great Wall. As Park et al. (2005a) have shown, with Matsubara’s (1994) formula for second-order gravitational nonlinear effects alone, one has the result that $A_V + A_C = 2$ at all scales, so if we observe both $A_V$ and $A_C$ to be less than 1, for example, biased galaxy formation must be involved.

Gott et al. (2009) compared observational genus curves from the SDSS survey with a few recent $N$-body simulations. The semi-analytic model of galaxy assignment scheme applied to the Millennium run (Springel et al. 2005) gives reasonable values for $A_V$ and $A_C$, but its value of $\Delta \nu$ differs from the observational data by 2.5 $\sigma$, showing less of a “meatball” shift than the data. This indicates a need for an improvement in either its initial conditions (it used a bias factor that is too low according to WMAP 5 year results) or its galaxy formation algorithm (Gott et al. 2008).

Gott et al. (2009) recently measured the 3D topology of the LRGs from the Sloan Survey and measured small deviations from the Gaussian form using $\Delta \nu$, $A_V$, and $A_C$ from Equations (10) to (12). There is only a small amount of noise in the genus curve due to the large sample size. The results are compared with the genus curves averaged over twelve mock LRG surveys that are performed in a 2048$^3$ particle $\Lambda$CDM simulation (Gott et al. 2009). The mean of the mock surveys fits the observations extraordinarily well, including the fact that the number of voids, as measured by the depth of the valley at $\nu < -1$ or by $A_V$, is less than the number of clusters, as
measured by the depth of the valley at $\nu > 1$ or by $A_C$, which is also seen in the observations. The fact that the clusters outnumber the voids is due to nonlinear biasing and gravitational evolution effects, and they are modeled by the simulations very well. The observed amplitude of the genus curve $A$ at $R_G = 21$ and $34 h^{-1}$ Mpc scales agrees extremely well with the mean of the mock $N$-body catalog. This indicates that the shape of the power spectrum is also modeled extremely well. The observed values of $A, \Delta A, A_V$, and $A_C$ are within approximately $1 \sigma$ of those expected from the 12 mock catalogs, all without any free fitting parameters (see Gott et al. 2009 for details). This suggests that finding the LRGs is a clean problem, which can be calculated using CDM particles and gravity. The genus topology is an important check that the $N$-body simulations here are doing a good job identifying the LRGs.

We now present the genus topology analysis of the Horizon Run. Figure 13 compares the genus curves of the matter density field (dashed line) and mock LRG distribution (thick solid line) calculated from the whole simulation cube using the periodic boundary condition. The distribution of our mock LRGs in the simulation cube is smoothed with a smoothing length of $20 h^{-1}$ Mpc. The curves are compared with the random phase Gaussian curve (thin solid line) corresponding to the $\Lambda$CDM model we adopted is plotted for a comparison. The uncertainty in the genus at $\nu = 0$ is $0.19\%$ for the mock LRG distribution.

(A color version of this figure is available in the online journal.)

Figure 13. Genus curves of the matter density field (dashed line) and mock LRG distribution (thick solid line) in real space at redshift $z = 0$ calculated from the whole simulation cube. The random phase Gaussian curve (thin solid line) corresponding to the $\Lambda$CDM model we adopted is plotted for a comparison. The uncertainty in the genus at $\nu = 0$ is $0.19\%$ for the mock LRG distribution.

Figure 14 shows the mean genus curves averaged over 32 mock SDSS-III LRG surveys. Each survey is a cone-shaped volume extending out to redshift $z = 0.6$ along the past light cone of our Horizon Run, and the whole survey cone is used to calculated the genus curve. Peculiar velocities are added to radial distances of LRGs to mimic the redshift distortion effects. Since the mock LRGs have the mean density of $3 \times 10^{-4} (h$/Mpc$)^3$ or a mean separation of $15 h^{-1}$ Mpc, the smoothing length of $20 h^{-1}$ Mpc is large enough to beat down the shot noise effects. Also shown are the real space genus curves of the mock LRGs at redshift $z = 0.5$ and of the random phase Gaussian density field corresponding to the $\Lambda$CDM model adopted in our simulation. Note that there is no fitting involved. The $3 \sigma$ error bars are estimated based on the variance in these parameters seen in 64 quarter-sized subsamples. For mock SDSS-III LRG survey results the estimates are the mean values averaged over 32 mock SDSS-III surveys, and the error bars are for a single survey. The linear regime predictions are also given for comparison.

In Figure 14 we show the mock LRG genus curves for the entire cube in real (thin solid line) and redshift space (thick solid line) at redshift $z = 0$. The redshift space distortion is mimicked by adding the $x$-component of peculiar velocities of mock LRGs to their positions. The smoothing scale is again $20 h^{-1}$ Mpc. The dotted line is the matter genus curve in redshift space. It can be seen that the redshift space distortion effects make the amplitude of the genus curve decrease. But the genus curve of mock LRGs is much less affected by the redshift distortion when compared with that of matter field. This is again due to the strong statistical biasing of the LRGs. These curves are compared with the genus curve of mock LRGs in real space at redshift $z = 0.5$ (dashed line).

The genus curves of mock LRGs deviate from the linear regime curve because of nonlinear biasing and gravitational effects. Table 2 lists the genus-related parameters at the Gaussian smoothing scale of $20 h^{-1}$ Mpc for LRGs measured from the whole simulation cube and from the 32 mock SDSS-III surveys. For the whole simulation cube results (the second and third columns) the error bars are estimated based on the variance in these parameters seen in 64 quarter-sized subsamples. For mock SDSS-III LRG survey results the estimates are the mean values averaged over 32 mock SDSS-III surveys, and the error bars are for a single survey. The linear regime predictions are also given for comparison.
It can be seen that the amplitude of the observed genus curve of the mock SDSS-III LRGs is only 0.6% different from that of the linear theory expectation, and will be determined with 1.5% uncertainty. The non-Gaussian parameters $\Delta V$, $A_C$, and $A_V$ show statistically significant deviations from the Gaussian curve, which is characteristic of the LRGs identified in our simulation.

8. CONCLUSIONS

Baryon oscillations are believed to be the method of characterizing dark energy “least affected by systematic uncertainties” according to the Dark Energy Task Force Report. But the baryon oscillation is affected by all kinds of nonlinear effects, so comparison with large $N$-body simulations is essential to control the systematics. Gott et al. (2009)’s study of the topology of the LRG clustering in the current SDSS shows that large $N$-body simulations are accurately modeling these nonlinear gravitational and biased galaxy formation effects.

To aid in calibrating nonlinear gravitational and biasing effects in the future Sloan-III baryon oscillation scale survey (BOSS), we have made here a large $N$-body simulation: $4120^3 = 69.9$ billion particles in a $(6592 h^{-1} \text{ Mpc})^3$ box, which models the formation of LRGs in this region. Using improved values over those adopted by the Millennium Run, which was based on WMAP-1 initial conditions (Spergel et al. 2003), our simulation has more power at large scales ($n_s = 0.953$ versus $n_s = 1$, which should make for a higher frequency of large scale structures), and more accurate normalization at $8 h^{-1} \text{ Mpc}$ scale ($\sigma_8 = 0.796$ while Millennium Run used $\sigma_8 = 0.9$).

We have made 32 mock surveys of the Sloan III survey, which should allow us to correct for small systematic effects in the measurement of the physical scale of the baryon oscillation peak as a function of redshift. We predict from our mock surveys that the BAO peak scale can be measured with the cosmic variance-dominated uncertainty of about 5% when the SDSS-III sample is divided into three equal volume shells, or about 2.6% when a thicker shell with $0.4 < z < 0.6$ is used. We find that one needs to correct the scale for the systematic effects amounting up to 5.2% to use it to constrain the linear theories.

The amplitude of the genus curve measured in redshift space in the mock SDSS III survey is, in the mean, expected to be equal to the linear regime prediction to an accuracy of about 2% at the smoothing scale of $20 h^{-1} \text{ Mpc}$. So the systematic evolution correction in the amplitude of the genus curve will be tiny. Also, the rms uncertainty in the amplitude in the genus curve due to the cosmic variance as measured in one SDSS-III survey is only 1.5% at $20 h^{-1} \text{ Mpc}$ scale. Since the uncertainty scales with inverse square root of the number of resolution elements (or equivalently with $R_{G,5}$), this implies the uncertainty of about 1.0% at $15 h^{-1} \text{ Mpc}$ scale, which will be the mean separation of the SDSS-III LRGs. In subsequent papers, we shall show how measuring the amplitude of the genus curve at different smoothing length makes possible an independent measure of scale complimenting the baryonic oscillation bump method for measuring $w(z)$.

As dark energy makes up three quarters of the universe today, making such improvements in characterizing dark energy are particularly exciting. Much ancillary science can be done with this large $N$-body simulation; detailed modeling of the integrated Sachs–Wolf effect and comparison with counts of LRGs in the sky, for example. We can search for Great Walls and voids to estimate their frequency and size (Gott et al. 2005). Here, our large volume size and WMAP-5 based initial conditions (Komatsu et al. 2009) should be of great benefit. We will make the simulation data available to the community (See http://astro.kias.re.kr/Horizon-Run/ for information). Please cite this paper when the simulation data is used.

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APPENDIX A

THE STARTING REDSHIFT AND THE NUMBER OF TIME STEPS

The TreePM code (Dubinski et al. 2004) used in this paper adopts the first-order Lagrangian perturbation scheme to generate the initial Zel’dovich displacement (for comparison with the second-order scheme, refer to Crocce et al. 2006, and references therein). The starting redshift of the simulation should be chosen so that the Lagrangian shifts of particles are not larger than the pixel spacing. This is because, if the initial displacement is larger than a pixel spacing, a particle may not fully experience the local differential gravitational potential (or Zel’dovich force) and, consequently, the shift calculated by using the first-order scheme could not fully reflect the fluctuations at the pixel scale. To quantitatively estimate the proper starting redshift, we introduce the Zel’dovich redshift, \( z_k \), of a particle defined as the epoch when its Zel’dovich displacement becomes equal to the pixel size either in \( x \)-, \( y \)-, or \( z \)-direction. In Figure 16, we show the distribution function of the Zel’dovich redshift, \( z_k \), for three simulations tagged by the mean particle separation (\( d \)), or equivalently the pixel size. To measure the distribution, we perform an initial setting moving 256\(^3\) particles from the initial conditions defined on a 256\(^3\) size mesh. One should acknowledge a lack of the large-scale power causing a possible underestimation of \( z_k \) especially for the small \( d \) simulation. Each distribution shows a power-law increase with \( z_k \) and a sharp drop after a peak. It can be seen that no particle experiences a shift larger than a pixel size at \( z = 23 \), when \( d = 1.6 h^{-1} \) Mpc, which justifies our choice of the initial redshift.

We adopted 400 time steps to evolve the CDM particles from \( z = 23 \) to 0. We set the force resolution to 0.1 \( d_{\text{mean}} \). Here, we present a test justifying our choice of the number of time steps.

Due to this relatively larger step size and lower force resolution, one may raise a question about the simulation accuracy whether it may have sufficient time and force resolutions to correctly model the formation of halos and subhalos.

One of the simplest ways to judge whether a simulation retains a sufficient power to resolve small structures is to compare the simulated FoF halo mass function with the well known functions obtained from very high resolution simulations with a special emphasis on the population of less massive halos. This is because coarse time step size or poor force resolution may destroy smaller structures more easily. In Figure 17, Sheth & Tormen (1999; dotted)’s fitting function is shown at \( z = 0 \) (upper) and 0.5 (lower). The corresponding FoF halo mass functions from our simulation are shown by filled squares (\( z = 0 \)) and open circles (\( z = 0.5 \)). This comparison demonstrates that the Horizon Run resolves structures with mass down to about \( M_{30} \approx 8.87 \times 10^{12} h^{-1} M_{\odot} \), which is the mass of 30 simulation particles. This mass limit is below the expected lower mass limit of the BOSS LRGs, and therefore formation of the dark halos associated with the BOSS LRGs are accurately simulated by the Horizon Run.
The density of the LRGs as a function of redshift. We apply the redshift-invariant multiplicity limit of the corresponding mock LRGs (dark halos) should be a nearly constant over redshift (Zehavi et al. 2005), the lower-mass parameters. As the number density of LRGs in the sample S1 is the spatially flat on the intermediate scales (3 h⁻¹ Mpc) seems slightly higher than that of the observation by 5%–15% WMAP 3 year limits (shaded region) of the projected CFs measured from the 24 simulated LRG samples. These CFs match the observed counterparts well except that it is slightly higher at 5 h⁻¹ Mpc ≲ r_p ≲ 20 h⁻¹ Mpc, which was also seen in the three-dimensional CF. This deserves further analysis and will be investigated in a separate paper.

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Figure 18. Left: the three-dimensional CFs of mock LRG and SDSS LRG. The shaded region shows the 1σ distribution of CFs of the 24 mock surveys while the filled circles are the measured CFs of the SDSS-II LRG sample. The solid line marks the median value of the mock surveys. For comparisons, we overplot those CFs of SDSS LRG subsamples under various magnitude limits (Zehavi et al. 2005). Right: Same as the left panel but for the projected CF. (A color version of this figure is available in the online journal.)