Primordial magnetic field and spectral distortion of cosmic background radiation

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Abstract

The role played by a primordial magnetic field during the pre-recombination epoch is analysed through the cyclotron radiation (due to the free electrons) it might produce in the primordial plasma. We discuss the constraint implied by the measurement or lack thereof COBE on this primordial field.
1 Introduction

The existence of a primordial magnetic field has been the subject of many discussions. From the theory of turbulence in the primordial plasma developed by Gamow (1946) and Ozernoy and Chernin (1968), Harrison (1980) proposed a model for the generation of primordial magnetic field through the cosmic turbulence theory. Brecher and Blumenthal (1970) have suggested that a possible alignment of the baryonic and leptonic magnetic moments at early stages of cosmological evolution could have created a large scale magnetic field. The parity non conservation in weak interactions could also provide a mechanism of magnetic field generation as proposed by Vilenkin and Leahy (1982). Witten (1985) discussed a mechanism by which an electromagnetic current could be induced on cosmic strings, a mechanism that was shown to occur in most theories in which strings are present (Davis and Peter 1995). More recently Tajima et al. (1992) proposed that during the quark-hadron phase transition, density fluctuations could have generated magnetic fields whose expected order of magnitude would be much greater than in the case of the cosmic turbulence theory of Ozernoy. It is therefore not completely unrealistic to consider the existence of a primordial magnetic fields and therefore to investigate their cosmological consequences.

The influence of a primordial magnetic field has been considered in many physical cases (primordial nucleosynthesis, electromagnetic radiation). Wassermann (1978), in particular, analyzed the effects of a large-scale primordial magnetic field on the formation of large scale structures and, more recently, Coles (1992) pointed out the possible role of a primordial magnetic field in the cold dark matter scenario.

A magnetic field can induce a cyclotron radiation due to the acceleration of the free electrons in the primordial plasma which would yield an energy release. This radiation can be responsible for the existence of early distortions on the microwave background spectrum. The aim of this paper is to study the influence of this radiation on the distribution of photons, i.e. Kompaneets equation (Kompaneets 1957), and the $\mu$ and $y$ spectral distortions covered by COBE (Mather et al 1993), where $\mu$ is the chemical potential and $y$ the Compton distortion parameter, and to place a constraint on a primordial magnetic field. We introduce the equations of evolution in Section 2 and, in Section 3, we discuss the constraint given by COBE on the primordial magnetic field. Then the limitations of this simple model and its cosmological implications are discussed as well as the origin of such a primordial magnetic field.

2 The equations of evolution

All the equations of evolution are calculated in the case of an Einstein-De Sitter Universe and we shall assume for numerical calculations a value for the Hubble constant of $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ h.
2.1 Density

The density of the matter is known to be proportional to $a^3$ where $a$ is the factor of scale, so the matter density evolves as

$$n = n_o(1 + z)^3,$$

with the present matter density (at $z = 0$) being given by

$$n_o = \frac{3H_o^2}{8\pi G} \frac{\Omega_b}{1.4m_H} \sim 2.04 \times 10^{-7} \text{cm}^{-3}$$

where $G$ is the gravitational constant, $\Omega_b = 0.1$ the relative baryonic density with respect to the critical density $\Omega_b = \rho_b/\rho_c$, with $\rho_c = 3H_o^2/8\pi G$, and $m_H$ is the hydrogen mass.

2.2 Temperature

In the Universe the temperature of the radiation evolves as

$$T_r = T_o(1 + z),$$

where $T_o = 2.726$ Kelvins is the temperature of the cosmic background radiation (CMBR) at $z = 0$, which is given by the measurement of the COBE FIRAS instrument (Far InfraRed Absolute Spectrometer, see Mather et al. 1993).

Peebles (1968) has shown that the Compton scattering plays a role in the thermal balance between matter and radiation. At the beginning of the recombination period, matter and radiation were very close to thermal equilibrium. At the time of recombination, the species $H^+$ and $D^+$ (hydrogen and deuterium ions) recombine with the free electrons and so considerably reduce the ionisation fraction. Jones and Wyse (1985) have rederived the equations for the ionisation of the cosmic plasma during and before the recombination period. They find that before the recombination period, the medium is practically fully ionized. For this reason, in what follows, we consider only the pre-recombination period, and we take the approximation for the redshift $1 + z \sim z$.

Thus matter and radiation are coupled ($T_r = T_m$), and we have, during the pre-recombination period,

$$T_m = T_r \Rightarrow T_m = T_o z.$$

2.3 Magnetic field

Once the field is imprinted on the charged plasma it will remain there because the early Universe is a very efficient conductor. Its conductivity is inversely proportional to the collision cross section as was shown Turner and Widrow (1988).
A magnetic field frozen in a plasma accelerates all charged particles. In what follows, we shall only consider the motion of the free electrons, neglecting all heavy charged particles because of the large mass ratios that enter the equations. In the dipolar approximation, the Poynting flux gives the radiative emission $\Psi$ per unit volume (in erg cm$^{-3}$ s$^{-1}$)

$$\Psi = \frac{2}{3} \frac{e^4}{m_e^2 c^4} \frac{\langle v_e^2 \rangle}{c^2} B^2 n_e$$

at the frequency

$$\nu_{\text{cycl}} = \frac{eB}{2\pi m_e c},$$

where $e$ is the electric charge of an electron, $m_e$ his mass, $B$ the magnetic field, $n_e$ the density of electrons close to the matter density. The electron velocity $v_e$ is supposed to be of purely thermal origin, i.e.,

$$\langle v_e^2 \rangle = \frac{3kT_m}{m_e},$$

where $k$ is the Boltzmann constant. Moreover the medium is practically fully ionized, so we will assume $n_e \sim n_{\text{protons}} \sim n$. Therefore we can write

$$\Psi = \frac{2e^4 kT_m n_e B^2}{m_e^3 c^5} (1 + z)^4$$

for emissions occurring at redshift $z$.

The evolution of the magnetic field is due to the Cyclotron effect and the expansion. We now explore both effects to show that the most important reason why a primordial magnetic field should decrease with time even though the Universe is roughly a perfect conductor in the expansion.

### 2.4 Cyclotron effect

Let us introduce the magnetical energy density

$$\epsilon_{\text{mag}} = \frac{B^2}{8\pi \mu_0}$$

and the Cyclotron energy density $\epsilon_{\text{cycl}}$ which the magnetic field is responsible for. Energy conservation requires

$$\frac{d\epsilon_{\text{mag}}}{dt} = \frac{d\epsilon_{\text{cycl}}}{dt},$$

which leads to

$$\frac{d}{dt} \left( \frac{B^2}{8\pi \mu_0} \right) = -\Psi$$
where $\Psi$ is the radiative emission calculated above. We therefore obtain

$$\frac{dB}{dt} = -\frac{8\pi e^4 \mu_o k T_o n_o B (1 + z)^4}{m_e^2 c^3}$$

(12)

In the case of the radiation dominated Universe, we have

$$\frac{dz}{dt} = -(1 + z)^3 H^r_o$$

(13)

where $H^r_o$ is a constant (in the radiation dominated Universe) given by $H^r_o = 8\pi G \rho_{o,r}/3$ with the density of radiation $\rho_{o,r}$ at the redshift $z = 0$; thus

$$\frac{dB}{dz} = 2\omega (1 + z) B$$

(14)

where

$$\omega = \frac{4\pi e^4 k T_o n_o \mu_o}{m_e^2 c^3 H^r_o} \sim 2.1 \times 10^{-9}$$

(15)

2.5 Expansion effect

The expansion contribution is calculated within the flux conservation framework (Turner and Widrow 1988) which gives here

$$B \cdot a^2 = \text{constant}$$

(16)

where $a$ is the scale parameter, this leads to

$$\frac{dB}{dz} = \frac{2B}{1 + z}$$

(17)

Eqs. (14) and (17) combined yield the actual evolution of the magnetic field, taking into account both effects, namely

$$\frac{dB}{dz} = 2\omega (1 + z) B + \frac{2B}{1 + z}$$

(18)

whose solution is easily found as

$$B = B_o (1 + z)^2 e^{2 \omega z + \omega z^2}$$

(19)

where $B_o$ is the magnetic field at $z = 0$. In our case where the distribution of photons is closed to a Bose-Einstein distribution we have $z < 10^6.4$ (see Danese & de Zotti 1977) thus $\exp(2\omega z + \omega z^2) \sim 1$ which lead to neglect the relaxation process and the evolution $B \sim B_o z^2$. Finally for the magnetic flux, we have

$$\Psi = A B_o^2 z^8$$

(20)

with $A = \frac{\omega H^r_o}{2\pi \mu_o} \sim 4.5 \times 10^{-30}$ c.g.s.
3 Evolution of the photons distribution

The Kompaneets equation characterizes the evolution of the photon distribution \( \eta \), or the dimensionless *occupation number* for the photon gas in equilibrium. This equation is, in fact, a simplified Boltzmann equation (Gould 1972), and assumes a uniform and isotropic photon gas, which can be written as

\[
\frac{\partial \eta}{\partial t} = \Sigma \Lambda_i + \Sigma \Gamma_i
\]  

(21)

where the terms \( \Lambda_i \) characterize the interactions (collisions) involving photons, and \( \Gamma_i \) represent sources and sinks of photons. These terms depend, in general, of the background.

In our case, namely the pre-recombination period, the dominant terms are expected to be Compton scattering \( \Lambda_{\text{compt}} \) and Bremsstrahlung \( \Gamma_{\text{brem}} \) (Danese and De Zotti 1977). However, with a magnetic field present, Cyclotron processes can participate, so in the evolution equation (21), we must add a cyclotron production term \( \Xi_{\text{cycl}} \)

\[
\frac{\partial \eta}{\partial t} = \Lambda_{\text{compt}} + \Gamma_{\text{brem}} + \Xi_{\text{cycl}}
\]  

(22)

We shall now recall and evaluate the three terms in turn

3.1 Compton scattering \( \Lambda_{\text{compt}} \)

The details of the calculation are given in Danese and De Zotti (1977), and it is found that

\[
\Lambda_{\text{compt}} = a_c \frac{1}{x_e} \frac{\partial}{\partial x_e} \left[ x_e^4 \left( \frac{\partial \eta}{\partial x_e} + \eta (1 + \eta) \right) \right]
\]  

(23)

where

\[
x_e = \frac{\hbar \nu}{kT_e}
\]  

(24)

and

\[
a_c^{-1} = \left( n_e \sigma_T \frac{kT_m}{m_e c^2} \right)^{-1} \sim 3.7 \times 10^{28} (1 + z)^{-4}
\]  

(25)

where \( \nu \) the frequency of photon, and \( \sigma_T \) is the Thomson cross section.

3.2 Bremsstrahlung processes \( \Gamma_{\text{brem}} \)

The expression of the bremsstrahlung effect is given by (Danese and De Zotti 1977)

\[
\Gamma_{\text{brem}} = K_o g(x_e) \frac{e^{-x_e}}{x_e^3} \left[ 1 + \eta (1 - e^{x_e}) \right]
\]  

(26)

where

\[
K_o = 2.46 \times 10^{-25} (1 + z)^{5/2}
\]  

(27)
and $g(x_e)$ is the Gaunt factor. Let us note that in our case where $x_e < 1$, the Gaunt factor may be estimated through the Born approximation

$$g(x_e) = \frac{\sqrt{3}}{\pi} \ln(2.25/x_e)$$  \hspace{1cm} (28)

### 3.3 Cyclotron production $\Xi_{cycl}$

The emitted power due to cyclotron effect is given by (in erg s$^{-1}$)

$$P_{cycl} = \frac{\Psi}{n} = X B_o^2 (1 + z)^5$$  \hspace{1cm} (29)

with

$$X = \frac{\omega H_o}{2\pi \mu_o n_o} \sim 2.2 \times 10^{-24} \text{ergs}^{-1} \text{Gauss}^{-2}$$

Finally we deduce the cyclotron contribution to the photons distribution $\eta$

$$\Xi_{cycl} = P_{cycl} \delta(\epsilon - \epsilon_{cycl}) = X z^5 B_o^2 \delta(\epsilon - \epsilon_{cycl})$$  \hspace{1cm} (30)

where the energy $\epsilon = h\nu$ and the cyclotron energy

$$\epsilon_{cycl} = h\nu_{cycl} = \frac{he}{2\pi m_e c} B_o z^2$$  \hspace{1cm} (31)

### 3.4 Evolution of the photons distribution

Danese and De Zotti (1977) and Salati (1992) have discussed the influence of the Compton scattering and Bremsstrahlung on the distribution of photons. Thomson collisions (Compton scattering) redistribute the additional energy, and the spectrum approach a Bose-Einstein distribution with chemical potential $\mu$.

$$\eta(x_e) = \frac{1}{e^{x_e+\mu} - 1}$$  \hspace{1cm} (32)

The radiation spectrum is of Bose-Einstein type with a chemical potential $\mu$, the simultaneous action of the Bremsstrahlung leads to a Planck distribution (i.e. $\mu \rightarrow 0$). Thus a production process of photons work to diminish the potential $\mu$, which is characteristic to the energy released, Danese and de Zotti give

$$\mu \sim \begin{cases} 3 \ln[0.85(1 + \frac{\Delta \epsilon}{\epsilon_r})] & \text{if } \mu \gg 1 \\ 1.4 \frac{\Delta \epsilon}{\epsilon_r} & \text{if } \mu \ll 1 \end{cases}$$

Nevertheless the cyclotron emission is produced only at the cyclotron frequency. The Bose-Einstein distribution are not altered by the Cyclotron process, we have no effects on the Kompaneets equation. The Cyclotron process will induce only spectral distortions due to electromagnetic energy emission in the CMBR.
4 Spectral distortions due to cyclotron process

The distortions depend on the time at which the energy is released into the primordial plasma. Sunyaev and Zeldovich (1970) discussed the problem of the interaction of matter and radiation and heating of the primeval plasma. More recently Salati (1992) has presented the possibilities of scenario of the spectral distortions of the microwave background radiation, which depend on the redshift.

\[ z > z_P, \text{ redshift at which the double Compton reaction on the electron } e: \]

\[ e + h\nu \rightarrow e + h\nu_1 + h\nu_2 \]

which produces a double emission of photons \( h\nu_1 \) and \( h\nu_2 \), becomes inefficient. Because of this mechanism, any spectral distortion is smoothed and CMBR is not affected. The photon distribution is always a Planckian distribution. The value of \( z_P = 10^{6.4} \) was calculated by Danese and De Zotti (1977).

\[ z_{BE} < z < z_P, \]

In this range, the radiation spectrum relaxes towards a Bose-Einstein (BE) distribution faster than the expansion time whereas the double Compton emission is slower (see for instance Danese and De Zotti 1977, Bond 1988, Salati 1992). The average photon energy becomes larger than for a Planckian spectrum but in thermal equilibrium, which means a nonzero chemical potential \( \mu \). Danese and De Zotti (1977) have given the lower limit \( z_{BE} = 10^{4.7} \) and shown that the chemical potential can be related with the energy release.

In our context, the energy release is a magnetic contribution due to the cyclotron radiation, so we have

\[ \mu \sim 1.4 \int_{t_{BE}}^{t_P} \frac{\Psi dt}{a_{bb} T_r^4} = 1.4 \int_{z_{BE}}^{z_P} \frac{\Psi}{a_{bb} T_r^4} \frac{dz}{H_0 (1 + z)^3} \]

where \( t_{BE} \) and \( t_P \) represent the age of the Universe respectively at \( z_{BE} \) and \( z_P \), so

\[ \mu \sim \frac{1.4 \omega B_o^2}{2 \pi a_{bb} T_r^4 \mu_o} \int_{z_{BE}}^{z_P} (1 + z) \frac{dz}{(1 + z)^3} \]

\[ z_{rec} < z < z_{BE}, \text{ where } z_{rec} \text{ corresponds to the redshift of the recombination of hydrogen.} \]

The last measurement of the cosmic microwave background spectrum derived from the FIRAS instrument on the COBE satellite give stringent limits on energy release, the dimensionless cosmological distortion parameter being limited to \( \mu < \mu_{FIRAS} = 3.3 \times 10^{-4} \).

Thus we obtain a constraint on the primordial magnetic field

\[ B_o < \sqrt{\frac{4 \pi a_{bb} T_r^4 \mu_{FIRAS} \mu_o}{1.4 \omega z_P^2}} \]

\[ (34) \]

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i.e. numerically $B_o < 3.44 \times 10^{-10}$ Gauss
In this range the chemical potential relaxes towards zero ($\mu \rightarrow 0$), while the photons and electrons still exchange energy through the Compton diffusion. The energy release yields a distortion on the Compton distortion parameter $y$. Danese and De Zotti (1977) have shown that the energy release is related to $y$, so we can write

$$y = \frac{1}{4} \int^{t_{rec}}_{t_{BE}} \frac{\Psi dt}{a_0 b_0 T^4_r} = \frac{\omega B^2_o}{8\pi a_0 b_0 T^4_o \mu_o} \int^{z_{BE}}_{z_{rec}} (1 + z) dz. \quad (35)$$

The measure done by FIRAS gives $y < y_{FIRAS} = 2.5 \times 10^{-5}$, which again can be expressed as an upper limit for the magnetic field at $z = 0$, namely:

$$B_o < \sqrt{\frac{16\pi a_0 T^4_o y_{FIRAS} \mu_o}{\omega z^2_{BE}}} \quad (36)$$

which gives $B_o < 1.12 \times 10^{-8}$ Gauss. So the best upper limit is given by the Bose Einstein distortion giving $B_o < 3.44 \times 10^{-10}$ Gauss
This value is more restrictive than the constraint on a relic magnetic field (cluster-sized and unidirectional) such as was recently measured in the Coma cluster halo by Kim et al. (1990) who obtained an upper limit of $B < 2 \times 10^{-8}$ Gauss.

The origin of a magnetic field having these orders of magnitude at $z = 0$ is far from obvious. One interesting possibility, as pointed out by Thompson (1990), relies on the existence of superconducting currents trapped in cosmic strings. Such currents, spacelike or timelike, would be carried by the strings at velocities approaching that of light at the time they are formed. These strings, whose precise motion in the primordial cosmological plasma has not yet been investigated in detail, would carry at least (in the low energy case), roughly $10^6$ A and $1$ C m$^{-1}$ (Peter, 1992). Signore and Sánchez (1991) discussed the millimiter and radioastronomical constraints on the cosmological evolution of superconducting strings, but they did not consider the possible magnetic field produced. It is clear that the resulting electromagnetic effects deserves further investigation, including in particular the precise determination of the remnant magnetic field as well as its coherence length.

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\footnote{in most models at the Grand Unified scale, but also, as proposed by Peter (1992-a), following a suggestion by Carter (1990), possibly at much lower energy scale, in practice close to that of the electroweak symmetry breaking.}
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