Flavor Violation in Theories with TeV Scale Quantum Gravity

Zurab Berezhiani\textsuperscript{a} and Gia Dvali\textsuperscript{b}\textsuperscript{*}

\textsuperscript{a}Universit\`{a} dell’Aquila, I-67010 Coppito, L’Aquila, Italy, and
Institute of Physics, Georgian Academy of Sciences, Tbilisi, Georgia

\textsuperscript{b}Physics Department, New York University, 4 Washington Place, New York, NY 10003
and ICTP, Trieste, Italy

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Abstract

We study the effects of possible flavor-violating operators in theories with the TeV scale quantum gravity, in which the ordinary matter is localized on a 3-brane embedded in the space with $N$ extra dimensions, whereas gravity propagates in the bulk. These operators are scaled by the fundamental Planck mass $M_{Pl} \sim \text{TeV}$ and must be suppressed by the gauge family symmetries. We study suppression of the most dangerous and model-independent operators. Several points emerge. First, we show that the Abelian symmetries can not do the job and one has to invoke non-Abelian $U(2)_F$ (or $U(3)_F$) symmetries. However, even in this case there emerge severe restrictions on the fermion mixing pattern and the whole structure of the theory. In order not to be immediately excluded by the well-known bounds, the horizontal gauge fields must be the bulk modes, like gravitons. For the generic hierarchical breaking pattern the four-fermion operators induced by the tree-level exchange of the bulk gauge fields are unsuppressed for $N = 2$. For $N > 3$ the suppression factor goes as a square of the largest $U(2)_F$-non-invariant Yukawa coupling, which implies the lower bound $M_{Pl} > 10 \text{ TeV}$ or so from the $K^0 - \bar{K}^0$ system. Situation is different in the scenarios when flavor Higgs fields (and thus familons) live on a $(3 + N')$-brane of lower dimensionality than the gauge fields. The further suppression of gauge-mediated operators can be achieved by an explicit construction: for instance, if $U(2)_F$ is broken by a vacuum expectation value of the doublet, the troublesome operators are suppressed in the leading order, due to custodial $SO(4)$ symmetry of the Higgs-gauge quartic coupling.

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\textsuperscript{*}E-mail: gd23@is9.nyu.edu
A. Introduction

One of the fundamental mysteries of the Nature is the enormous hierarchy between the observable values of the weak interaction scale $M_W$ and the Planck scale $M_P$. A possible solution to this mystery may have to do with the fact that the fundamental scale of gravitational interaction $M_{Pf}$ is as low as TeV, whereas the observed weakness of the Newtonian coupling constant $G_N \sim M_P^{-2}$ is due to the existence of $N$ large ($\gg$ TeV$^{-1}$) extra dimensions into which the gravitational flux can spread out. At the distances larger than the typical size of these extra dimensions ($R$) gravity goes to its standard Einstein form. For instance, for two test masses separated by the distance $r \gg R$, the usual $1/r^2$ Newtonian low is recovered, and the relation between the fundamental and observed Planck scales is given by:

$$M_P^2 = M_{Pf}^{N+2} R^N$$

In such a theory, quantum gravity becomes strong at energies $M_{Pf}$, where presumably all the interactions must unify. For all the reasonable choices of $N$, the size of extra radii is within the experimental range in which the strong and electroweak interactions have been probed. Thus, unlike gravity the other observed particles should not ”see” the extra dimension (at least up to energies $\sim$ TeV). In ref. this was accomplished by postulating that all the standard model particles are confined to a 3 + 1-dimensional hyper-surface (3-brane), whereas gravity (as it should) penetrates the extra dimensional bulk. Thus, on a very general grounds, the particle spectrum of the theory is divided in two categories: (1) the standard model particles living on the 3-brane (brane modes); (2) gravity and other possible hypothetical particles propagating in the bulk (bulk modes). Since extra dimensions are compact, any 4 + $N$-dimensional bulk field represents an infinite tower of the four-dimensional Kaluza-Klein (KK) states with masses quantized in units of inverse radii $R^{-1}$. An important fact is that each of these states (viewed as a four dimensional mode) has extremely weak, suppressed at least as $M_P^{-1}$ couplings to the brane modes. The ordinary four-dimensional graviton, which is nothing but a lowest KK mode of the bulk graviton, is a simplest example. In what follows, this fact will play a crucial role as far as the other possible bulk particles are concerned.

Obviously, such a scenario requires various compatibility checks many of which were performed in [1], [2], [10], [12], [13], [15], [16]. It was shown that this scenario passes a variety of the laboratory and astrophysical tests. Most of the analysis was mainly concerned to check the ”calculable” consequences of the theory, ones that obviously arise and are possible to estimate in the field (or string) theory picture. On the other hand, there are constraints based on the effects whose existence is impossible to proof or rule out at the given stage of understanding the quantum gravity, but which are usually believed to be there. An expected violation of global quantum numbers by gravity is an example. There are no rigorous proofs

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1Witten suggested that string scale may be around the scale of supersymmetric unification $\sim 10^{16}$GeV. The possibility of having an extremely low string scale was also discussed by Lykken [4].
of such effect, nor any knowledge of what their actual strength should be. Yet, if an effect is there with a most naive dimensional analysis we expect it to manifest itself in terms of all possible gauge-symmetric operators suppressed by the Planck scale. Below we adopt this philosophy, which then imposes the severe constraints on the proposal of ref. [1], since now the fundamental gravity scale $M_{Pf}$ is as low as TeV! Issues regarding baryon and lepton number violation were discussed in [2], [11], [10] and some ways out were suggested. In the present paper we will discuss the flavor problem in TeV scale quantum gravity theories induced by higher order effective operators cutoff by the scale $M_{Pf}$ that can contribute to various flavor-changing neutral processes (FCNP), like $\bar{K}^0 - K^0$ or $D^0 - \bar{D}^0$ transitions, $\mu \rightarrow e \gamma$ decay etc.

It is normal that the flavor-violating interactions provide severe constraints to any new physics beyond the standard model. As we will see below the flavor problem provides severe constraints both on the symmetry structure of the theory and on the structure of fermion mass matrixes.

### B. The Problematic Operators and Gauge Family Symmetries

As said above, in the effective low energy theory below TeV, we expect all possible flavor violating four-fermion operators scaled by $M_{Pf}^2$. Some of these give unacceptably large contributions to the flavor changing processes and must be adequately suppressed. Let us consider what are the symmetries that can do the job. Usually one of the most sensitive processes to a new flavor-violating physics is the $K^0 - \bar{K}^0$ transition. Corresponding effective operator in the present context would have a form

$$\frac{(\bar{s}d)^2}{M_{Pf}^2}$$

This can only be suppressed by the symmetry that acts differently on $s$ and $d$ and therefore is a family symmetry. Thus, as a first requirement we have to invoke a gauge family symmetry. In an ordinary (four-dimensional) field theory there would be an immediate problem with this proposal. In order to adequately suppress the operator (2), the symmetry in question should be broken (well) below TeV. But there are well known lower bounds $\gg T e V$ on the scale of gauge flavor symmetry breaking. This bound comes from a tree-level exchange

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2 The unification of gauge couplings is another issue not to be addressed in this paper.

3 Unlike the baryon number non-conservation, gravity-mediated FCNP are only important for a very low scale quantum gravity theories: the lowest dimensional baryon-number-violating operators scaled as $M_{Pf}^{-4}$ are problematic even for theories with $M_{Pf} = M_P$, whereas the flavor problem disappears already for $M_{Pf} > 10^{7-8}$ GeV or so.

4 $N = 1$ supersymmetry can not be of much help due to the following reasons: first in any case it must be broken around TeV scale, and secondly the four-fermion interaction can arise from the Kähler metric, which can not be controlled by holomorphy.
of the horizontal gauge boson, that will mediate the same FCNP for which the symmetry was invoked! However, one has to remember that this is true as far as the four-dimensional field theory is concerned. Recall that in our case there are two type of particles, ordinary particles living on a brane and the bulk modes. If the horizontal gauge field is the bulk mode the situation is different. Now the coupling of each KK excitation to the ordinary particles will be enormously suppressed. This saves the scenario from being a priori excluded. The large multiplicity of the exchanged KK states however works against us and at the end puts severe constraint on the dimensionality of extra space, flavor breaking scale and the pattern of quark masses. We will discuss this in detail below.

Now let us discuss what are the symmetries that one can use. In the limit of zero Yukawa couplings the standard model exhibits an unbroken flavor symmetry group

\[ G_F = U(3)_Q^L \otimes U(3)_u^R \otimes U(3)_d^L \otimes U(3)_e^R \]  

(3)

If one is going to gauge some subgroup of \( G_F \), the Yukawa coupling constants are to be understood as the vacuum expectation values of the fields that break this symmetry \[20\]. That is the fermion masses must be generated by the higher dimensional operators of the form:

\[ \left( \frac{\chi}{M} \right)_a^N H \bar{Q}_L Q_{La} \]  

(4)

where \( \chi \) are flavor-breaking Higgses. To take advantage of the problem, it is natural and most economical to assume that the above desired operators are generated by the same physics which induces the problematic ones. Thus we adopt that \( M \sim M_{Pf} \). An observed fermion mass hierarchy then is accounted by hierarchical breaking of \( G_F \). In the present paper we will not be interested how precisely such a hierarchy of VEVs is generated, but rather will look for its consequences as far as FCNP are concerned.

Now the large Yukawa coupling of the top quark indicates that at least \( U(3)_Q^L \otimes U(3)_{u_R} \rightarrow U(2)_Q^L \otimes U(2)_{u_R} \) breaking should occur at the scale \( \sim M_{Pf} \) and thus it can not provide any significant suppression. Therefore, the selection rules for the operators that involve purely \( Q_L^L \) and \( u_R \) states can be based essentially on \( U(2)_Q^L \otimes U(2)_{u_R} \) symmetry or its subgroups. The most problematic dimension six operators in this respect is (below we will not specify explicitly a Lorenz structure, since in each case it will be clear from the context):

\[ (\bar{Q}_L^a Q_{La})(\bar{Q}_L^b Q_{Lb}) + (\bar{Q}_L^a Q_{Lc})(\bar{Q}_L^b Q_{Ld})\epsilon_{ab} \epsilon_{cd} \]  

(5)

They both give a crudely similar effect. So let us for definiteness concentrate on the second one. Written in terms of initial \( s \) and \( d \) states (call it ‘flavor basis’) it has a form:

\[ (\bar{s}_L s_L)(\bar{d}_L d_L) - (\bar{s}_L d_L)(\bar{d}_L s_L) \]  

(6)

In general initial \( s \) and \( d \) states are not physical states and are related to them by \( 2 \times 2 \) rotation \( D_L \), which diagonalizes 1–2 block of the down quark mass matrix \( M^d \). The problem is that \( D_L \) is not in general unitary due to non-zero 1–3 and/or 2–3 mixing in \( M^d \). Note that this elements can be of the order of one, without conflicting with small 2–3 and 1–3 mixings in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, since CKM measures a
mismatch between rotations of $u_L$ and $d_L$ and not each of them separately. If so, then in the physical basis the disastrous operator

$$(\bar{s}_L d_L)(\bar{s}_L d_L)$$

(7)

will be induced with an unacceptable strength. This puts a severe constraint on the structure of $M^d$. In particular all the anzatses with both large $1-2$ and $2-3$ (or $1-3$) elements are ruled out. Note that smallness of $1-3$ and $2-3$ mixing in $M^d$ also works in favor of suppression of $B^0 - \bar{B}^0$ transitions from the same operator. Analogously, unitarity of $1-2$ diagonalization in $M^u$ suppresses the $D^0 - \bar{D}^0$ transition. In this respect the safest scenario would be the one in which, $1-2$ mixing in CKM comes mostly from down type masses, whereas $2-3$ mixing from ups.

Much in the same way the operator

$$(\bar{d}_R d^a_R \epsilon_{a\alpha\beta} \epsilon_{b\alpha\beta})(\bar{d}_R d^b_R)$$

(8)

gives unitarity constraint on a $1-2$ block of $D_R$, which can be somewhat milder since the operator (8) can in principle be suppressed by $U(3)_{d_R}$-symmetry, by a factor $\sim m_b/m_t$.

For the operators which involve both left and right-handed quark states, the suppression factors are more sensitive to what subgroup of the $G_F$ is gauged.

C. $L \times R$-Type Symmetries

If the left and the right-handed quark are transforming under different $U(2)_{FL} \otimes U(2)_{FR}$ flavor symmetries. The only possible unsuppressed operator is

$$(\bar{Q}_L^a Q_{La})(\bar{Q}_R^b Q_{Rb})$$

(9)

(plus its Fierz-equivalent combinations). Again, this is harmless only if $D_L$ and $D_R$ are nearly unitary, which brings us back to the constraint discussed above.

1. The Diagonal $U(2)_F$

From the point of view of an anomaly cancellation, the most economic possibility would be to gauge a diagonal subgroup of $G_F$ under which all fermions are in the fundamental representation. Then at scales below $M_{PF}$ we are left with an effective $U(2)_F$ symmetry. In such a case one encounters an option whether $U(2)_F$ is a chiral or vector-like symmetry. It turns out that the chiral $U(2)_F$ is inefficient to suppress a large flavor violation, whereas the vector-like one can do the job, provided a stronger restriction on the fermion mass pattern is met.

Unfortunately, however, the vector-like flavor symmetry $U(2)_F$ allows the fermion mass degeneracy, and unnatural conspiracies would be needed for explaining their observed splitting. In this view, it would be most natural if the vector-like $U(2)_F$ is supplemented by some (discrete of continuous) gauge chiral piece of the full chiral ($L \times R$) symmetry.
To support the first statement it is enough to consider an operator:

\[(\bar{Q}_L^a d_L^b)(\bar{Q}_{Ra} d_{Ra})\]  

(10)

This is invariant under the chiral-$U(2)_F$ times an arbitrary combination of extra $U(1)$-factors. Obviously this operator is a disaster, since it directly contains an unsuppressed four-Fermi interaction (2). Thus chiral $U(2)_F$ cannot protect us. On the other hand the vector-like $U(2)$ suppresses (10). Analogous (self-conjugate) operators in this case would have the form:

\[(\bar{Q}_L^a d_R)(\bar{d}^b_R Q_{Lb}), \quad (\bar{Q}_L^a d_{Ra})(\bar{d}^b_R Q_{L,b})\epsilon_{ab}\epsilon^{\alpha\beta}\]  

(11)

which contains no (2) term in the flavor basis. The requirement that it will not be induced in the physical basis simply translates as a requirement that

\[D_L D_L^+ = D_L D_L^+ = D_R D_R^+ = 1\]  

(12)

with a great accuracy. In other words, this means that the fermion mass matrices should be nearly Hermitian.\(^6\) If this is satisfied other operators do not cause an additional constraints, since they are further suppressed. For instance, non-self-conjugate operators:

\[(\bar{Q}_L^a d_{Ra})(\bar{Q}_L^b d_{Rb}), \quad (\bar{Q}_L^a d_{Ra})(\bar{Q}_L^b d_{Rb})\epsilon_{ab}\epsilon^{\alpha\beta}\]  

(13)

carry two units of weak isospin and must be suppressed by extra factor $\sim (\frac{M_W}{M_{P1}})^2$. In conclusion, we see that all working versions converge to the requirement (12).

2. Why Abelian Symmetries Cannot work?

Although our analysis was quite general, one may wonder whether by considering non-Abelian symmetries, one is not restricting possible set of solutions: for example requirement of $SU(2)$ symmetry restricts the possible charge assignment under the additional $U(1)$ factors which otherwise could be used for the same purpose. In other words, one may ask, whether instead of non-Abelian symmetries one could have invoked a variety of $U(1)$ factors and by properly adjusting charges of the different fermions get the same (or even stronger) suppression of FCNP. We will argue now that this is not the case, and even (neglecting esthetics and various technical complications, like anomalies) if one allows completely arbitrary charge assignment under an arbitrary number of $U(1)$-factors, the problem cannot be solved. The reason for this is that no Abelian symmetry can forbid the operators of the form:

\[C_{ab}(\bar{Q}_L^a Q_{La})(\bar{Q}_L^b Q_{Lb})\]  

(14)

and similarly for right-handed fermions. Since no non-Abelian symmetry is invoked the coefficients $C_{ab}$ are completely arbitrary. Due to this fact there is no choice of fermion mass

\(^6\)This might be rather natural in the context of the left-right symmetric model $SU(2)_L \times SU(2)_R \times U(1)$
matrixes which would avoid appearance of either \((\bar{s}d)^2\) or \((\bar{uc})^2\) unsuppressed vertexes. Since \(C_a\) are arbitrary, non-appearance of any of these operators would mean that the flavor and the physical quark states are equal (that is the masses are diagonal in a flavor basis). But this is impossible to be the case in both up and down sectors simultaneously due to non-zero Cabibbo mixing \(\sin \theta_C = 0.22\). Thus at least one of these operators should be induced and the suppression factor cannot be smaller than

\[
\sin \theta_C^2/M_{Pf}^2
\]

This gives rise to an unacceptably large contribution to either \(\bar{K}^0 - K^0\) or \(\bar{D}^0 - D^0\) transitions. The only way to avoid the problem would be a conspiracy between the \(C_{ab}\) coefficients, which can be guaranteed by non-Abelian \(U(2)\) symmetry, (subject to unitarity of \(U_L, U_R, D_L\) and \(D_R\) transformations).

**D. Electroweak Higgs-Mediated Flavour Violation.**

Existence of the \(M_{Pf}\)-suppressed operators brings another potential source of flavour violation, mediated by the electrically neutral component \((H^0)\) of the standard model Higgs. In the standard model this source is absent since the couplings of \(H^0\) are automatically diagonal in the physical basis. This is not any more true if higher dimensional operators with more Higgs vertices are involved [18]. For instance, add the lowest possible such operator

\[
\left( g_{ab} + h_{ab} \frac{H^+ H}{M_{Pf}^2 + \ldots} \right) H \bar{Q}^a_L q^b_R
\]

where \(g_{ab}\) and \(h_{ab}\) are constants. After \(H\) gets an expectation value the fermion masses become

\[
M_{ab} = \left( g_{ab} + h_{ab} \frac{|\langle H \rangle|^2}{M_{Pf}^2 + \ldots} \right) \langle H \rangle
\]

whereas the Yukawa couplings of the physical Higgs are

\[
Y_{ab} = g_{ab} + 3h_{ab} \frac{|\langle H \rangle|^2}{M_{Pf}^2 + \ldots}
\]

In the absence of flavour symmetries the matrixes \(g_{ab}\) and \(h_{ab}\) are arbitrary \(3 \times 3\) matrices and thus \(M_{ab}\) and \(Y_{ab}\) are not diagonal in the same basis. This induces an unacceptably large flavour violation. In the present context however according to Eq(4), \(g_{ab}\) and \(h_{ab}\) must be understood as the VEVs of the horizontal Higgs scalars

\[
g_{ab} \sim h_{ab} \sim \left( \frac{\chi}{M} \right)^N_{ab}
\]

and thus obey an *approximately same* hierarchy [21]. This can reduce the resulting flavor violation to an acceptable level. For instance, adopting anzats \(Y_{ab} \sim \sqrt{m_a m_b}/M_W\) [22], where \(m_a\) are masses of physical fermions, the resulting flavour violation can be below the experimental limits.
E. decay $\mu \rightarrow e\gamma$, etc.

The suppression of lepton-flavour violating processes through dimension-6 operators goes much in the same spirit as discussed above for quarks. The constrains on the $M^l$ mixing angles can be satisfied easier since not much is known about the lepton mixing angles. Dimension five operators can be more problematic. For instance the lowest operator inducing $\mu \rightarrow e\gamma$ transition is:

$$\frac{H}{M_{Pf}^2} \bar{e} \sigma^{\mu\nu} \mu F_{\mu\nu}$$

This has the same chirality structure as the $m_{\mu e}$ mass term, and thus we expect to be suppressed by the same flavour symmetry that guarantees its smallness. An exact strength of the suppression factor is very sensitive to the mixing in the charged lepton matrix and can be as large as $\frac{m_\tau}{M_{Pf}}$ for the maximal $e - \mu - \tau$ mixing angles. But can be zero if mixing is absent.

The experimental limit $\text{Br}(\mu \rightarrow e\gamma) < 5 \times 10^{-11}$ translates into

$$\lambda_{e\mu} < 3 \times 10^{-6} \cdot \left(\frac{M_{Pf}}{1 \text{ TeV}}\right)^2$$

which is satisfied for $\lambda_{e\mu} \sim \sqrt{\frac{m_e m_\mu}{\langle H \rangle}} = 4 \cdot 10^{-5}$ and $M_{Pf} \sim 3$ TeV. Analogously, for the $\tau \rightarrow \mu\gamma$ decay $\text{Br}(\tau \rightarrow \mu\gamma) < 3 \times 10^{-6}$ translates into

$$\lambda_{\mu\tau} < 3 \times 10^{-2} \cdot \left(\frac{M_{Pf}}{1 \text{ TeV}}\right)^2$$

which is well above the geometrical estimate $\lambda_{\mu\tau} = \sqrt{\frac{m_\mu m_\tau}{\langle H \rangle}} = 2.5 \cdot 10^{-3}$.

Somewhat stronger constraints come from the electron and neutron EDMs. For example, the experimental limit $d_e < 0.3 \cdot 10^{-26}$ e·cm implies

$$\text{Im} \lambda_e < 10^{-9} \cdot \left(\frac{M_{Pf}}{1 \text{ TeV}}\right)^2$$

therefore, for $\lambda_e$ taken of the order of the electron Yukawa coupling constant, $\lambda_e \sim \frac{m_e}{\langle H \rangle} \sim 10^{-6}$ with a phase order 1, one needs to take $M_{Pf} > 30$ TeV or so. Analogous constraint emerges from the light quarks (i.e. neutron) EDM.

F. Gauging Flavor Symmetry in the Bulk

Up to now we were discussing suppression of $M_{Pf}$-cutoff operators by the gauge flavor symmetries. What about flavor-violation mediated by the horizontal gauge fields? Naively, one encounters a puzzle here: in order to suppress quantum-gravity-induced operators, $G_F$ must survive at scales below $M_{Pf}$, but in this case the gauge bosons can themselves induce problematic operators. Situation is very different if the horizontal gauge bosons are the bulk fields.

Generic procedure of gauging an arbitrary gauge symmetry in the bulk was discussed in details in [10] and it was shown that: 1) an effective coupling of the bulk gauge-field
and its KK partners to the brane modes is automatically suppressed by $g_4 \sim M_{Pf}/M_P$ and, 2) whenever the symmetry is broken on the brane (only) the mass of the gauge field is suppressed by $\sim M_{Pf}/M_P$ independently of the number of extra dimensions. In the other words, as it should be, the symmetry broken on the brane is "felt" by the brane fields much stronger then by the bulk modes. This is not surprising, since the bulk modes "spent" much more time in the bulk where symmetry is unbroken.

Consider a gauge field of some symmetry group $G$ propagating in the bulk. We will assume the scale of the original 4+$N$-dimensional gauge coupling to be $g_{(4+N)} \sim M_{Pf}^{-2}$. From 4-dimensional point of view, this gauge field represents an infinite number of KK states out of which only the zero mode $A^0_{\mu}$ shifts under the 4-dimensional local gauge transformation, whereas its KK partners are massive states. All these states couple to the gauge-charged matter localized on the brane through an effective four-dimensional gauge coupling

$$g_4^2 \sim 1/(RM_{Pf})^N \sim M_{Pf}^2/M_P^2$$

Consequently if any of the Higgs scalars localized on the brane gets nonzero VEV $\langle \chi(x_A) \rangle = \delta(x_A - x^0_A)\langle \chi \rangle$ (where $x^0_A$ are coordinates of the brane) all the bulk states get a minuscule mass shift

$$\delta m^2 \sim \langle \chi \rangle^2 M_P^2/M_P^2$$

On the other hand if the Higgs scalar is a bulk mode and its VEV is not localized on the brane, but rather is constant in the bulk, the gauge fields get an unsuppressed mass shift

$$\delta m^2 \sim \langle \chi \rangle^2 M_{Pf}^2/M_P^2$$

Note that the bulk scalar $\chi$ has dimensionality of $(mass)^{1+N/2}$.

What is the implication of these facts for the flavor symmetry? Let $G_F = U(3)_F$ be a vector-like family symmetry under which all the fermions are triplets and let us estimate flavor violation induced by exchange of its gauge field. Consider for instance $M^0 - M^0$ transitions (where $M = K, B, D$). Obviously, if $U(3)_F$ was unbroken, all the four-fermion operators induced by their exchange would never contribute to any of these processes, since the only possible invariant is

$$q^a q_a q^b q_b$$

However, $U(3)_F$ must be broken in order to account for the hierarchy of fermion masses. Let $\chi$ be the Higgs that does this breaking. We can consider three options:

1. **Breaking occurs only on the brane.** Unfortunately this option is ruled out due to the following reason. If $U(3)_F$ is broken on the brane, then there must be massless pseudo-scalar modes localized on it. These are Goldstone bosons (familons [24]) of broken $U(3)_F$ which have both flavor-diagonal and flavor-non-diagonal couplings to the ordinary matter, suppressed by $\langle \chi \rangle$. This is excluded due to various astrophysical and laboratory reasons [24,23]. At a first glance, in the present context this statement may appear as a surprise, since by assumption $U(3)_F$ is a gauge symmetry and thus troublesome familons must be eaten up by the gauge fields. Recall however that the gauge coupling is abnormally small (and the scale of symmetry breaking is not large). In such a situation it is more useful to
argue in terms of the massless Goldstones, rather than massive gauge bosons. Thus we are left with the following option.

**2. Breaking occurs in the bulk.** In this case familons are the bulk modes and are totally safe by the same reason as the bulk gravitons [10]. Thus the dominant contribution to the flavor-violation is provided by the gauge components. Let us estimate this contribution to the effective four-Fermi operators mediating $M^0 - \bar{M}^0$ processes. This comes from the tree-level exchange of infinite tower of $KK$ states. The mass of each individual $KK$ mode is

$$m_K^2 = \langle \chi \rangle^2/M_{Pf}^N + \frac{|n|^2}{R^2}$$

(28)

where $|n| = \sqrt{n_A^2}$ and $n_A$ are integers. The second contribution is flavor-universal. Thus for the heavy states the flavor violation will be suppressed by $\sim \langle \chi \rangle^2 R^2/M_{Pf}^N |n|^2$. Note that the same scalars are responsible for the fermion masses through

$$\int dx^{4+N} \delta(x_A) \frac{\chi^6}{M_{Pf}^{1+N/2}} HQ_La^b_R$$

(29)

where dimensionality of the denominator comes from the fact that $\chi$ is a bulk mode. Thus $U(3)_F$-violating mass can be parameterized as

$$\langle \chi \rangle^2/M_{Pf}^N = (\lambda M_{Pf})^2$$

(30)

where $\lambda$ is roughly the Yukawa coupling of the fermion (e.g. for $U(2)_H$ gauge bosons $\lambda$ can be taken to be $\sim m_c/\langle H \rangle \sim 10^{-2}$ or so). It is useful to evaluate contributions of the modes $\frac{|n|^2}{R^2} < (\lambda M_{Pf})^2$ and $\frac{|n|^2}{R^2} > (\lambda M_{Pf})^2$ separately. Each of the first states generates an operator scaled by $\frac{g^2}{(\lambda M_{Pf})^2}$ whereas their multiplicity is roughly $\sim (\lambda M_{Pf} R)^N$. Therefore their combined effect gives an effective four-fermion regulator

$$\sim \frac{\lambda^{N-2}}{M_{Pf}^2}$$

(31)

The flavor violating contribution of the modes with $\frac{|n|^2}{R^2} > (\lambda M_{Pf})^2$ is crudely given by the sum

$$\sum_{n_A} g^2_R \frac{R^4}{|n|^4} (\lambda M_{Pf})^2 \sim g^2_R \frac{R^4 (\lambda M_{Pf})^2 |n|^{N-4}}{|n|_{\min}^{N-4}} |n|_{\max}^{N-4}$$

(32)

and it is power-divergent for $N > 4$. Cutting off from above this sum at $|n|_{\max} \sim (M_{Pf} R)$ we get that the amplitude goes as

$$\sim \frac{\lambda^2}{M_{Pf}^2} \left(1 - \lambda^{N-4}\right)$$

(33)

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The situation is analogous to the case of the low energy supersymmetry, when the dominant coupling of gravitino is provided by its goldstino component!
Thus we see that the dominant contribution comes from the lowest modes. Combining everything we get that for \( N > 3 \) the operators scale as
\[
\sim \frac{\lambda^2}{M_{Pf}}
\] (34)
and are problematic even if \( \lambda \sim 10^{-2} - 10^{-3} \). The case \( N < 4 \) is even more problematic and, in particular, there is no suppression for \( N = 2 \). At a first glance this may appear as a surprise: since in the limit \( \lambda \to 0 \) the transition should be absent. Note however, that this does not contradict to above result, since for \( \lambda M_{Pf} \) becoming smaller than the meson mass \( m_M \) an additional power suppression \( \sim \frac{\lambda M_{Pf}}{m_M} \) must appear in the flavor violating transition amplitude. Thus, we are lead to the third option.

3. Gauge fields and familons on the branes of different dimensionality. Imagine that flavor Higgs fields are not 3-brane modes and live in space of larger dimensionality. But, unlike gauge fields, they can only propagate in the bulk of less \( N' < N \) dimensions. That is assume that they live on \( 3 + N' \)-brane which contains our 3-brane Universe as a subspace. When the breaking occurs on the \( 3 + N' \)-brane, both, masses of bulk gauge fields and couplings of familons to the standard model fermions will be suppressed by volume factors. The question is whether for certain values of \( N' < N \), both gauge and familons-mediated processes can be adequately suppressed. Consider first some gauge-mediated FCNP. Since \( \chi \)-s are \( 3 + N' \)-dimensional fields, according to (29) their VEV can be parameterized as
\[
\langle \chi \rangle^2 / M_{Pf}^{N'} = (\lambda M_{Pf})^2
\] (35)
The resulting flavor-non-universal mass for each gauge KK mode is
\[
m_{fv}^2 = \frac{(\lambda M_{Pf})^2}{(M_{Pf} R)^{N-N'}}
\] (36)
or if translated in terms of \( M_P, m_{fv}^2 = (\lambda M_{Pf})^2 (M_{Pf}/M_P)^{2(N-N')/8} \). For the gauge-mediated flavor-violation to be suppressed, this should be smaller than the typical momentum transfer in the process \( M^0 - \bar{M}^0 \). If this is the case, then the contribution to the process from the light (\( <<< m_M \)) and heavy (\( >>> m_M \)) modes go as \( m_{fv}^2 m_M^{N-4}/M_{Pf}^2 \) and \( m_{fv}^2/M_{Pf}^2 \) respectively and are suppressed. Now let us turn to the familon couplings. Since \( \chi \)-s are \( 4 + N' \) dimensional fields, so are the familons and their effective decay constant is \[10\]
\[
1/(\lambda M_{Pf}) (M_{Pf} R)^{N'/2}
\] (37)
Again because of the bulk-multiplicity factor their emission rate is amplified. For instance the star-cooling rate becomes \[10\]
\[
\sim (\lambda M_{Pf})^{-2} (T / M_{Pf})^{N'}
\] (38)
where \( T \) is the temperature in the star. This is safe for \( N' > 2 \) even for \( M_{Pf} \sim \)TeV. Contribution from light and heavy modes to familon-mediated flavor-violating amplitudes are suppressed as
\[
\frac{1}{(\lambda M_{Pf})^2 (m_M/M_{Pf})^{N'}}
\] (39)
Finally let us consider flavor-violating operators induced by horizontal Higgses. This can be analyzed much in the same way as was done above for the gauge fields. If the non-zero VEV occupies the whole $3 + N'$-brane volume where Higgs can freely propagate, then the resulting dangerous operators are scaled as the largest of (31) and (33) where now $N$ must be understood as the number of the dimensions where Higgs can propagate. This is very much like gauge-contribution in the case of bulk-breaking. The difference is that horizontal Higgses have an extra suppression factor $\sim (m_W/M_{Pl})^2$ and therefore are relatively safer.

1. Custodial $SO(4)_F$

We must stress that there may very well be the group-theoretical cancellations which can weaken the above constraints. For instance, imagine that $U(2)_F$ symmetry is broken by a doublet VEV $\chi_a$. Gauge boson masses generated in this way are automatically $SU(2)$ invariant due to the custodial global $SO(4)$ symmetry (just like in the standard model) of the coupling

$$g^2 (\chi^a \chi_a) A^a_\nu A^{\nu a}$$

As a result in the leading order $U(2)_F$ non-invariant operator structure must cancel out.

G. Conclusions

Adopting philosophy that the quantum gravity explicitly breaks global symmetries via all possible operators scaled by powers of $M_{Pl}^{-1}$, we studied some implications of this fact for the flavor-violation in theories with TeV scale quantum gravity [1]. In these theories the ordinary fermions are localized on a 3-brane embedded in space with $N$ new dimensions. We have discussed most dangerous and model independent operators and their suppression by gauged family symmetries. Non-Abelian symmetries (such as $U(2)_F$) broken below TeV seem to be necessity in this picture, but in no way they are sufficient for FCNP-suppression. Additional constraints come out for the structure of the fermion mass matrices and the high-dimensional bulk properties of the horizontal gauge fields. All the ”safe” versions seem to converge to the structures in which, at best, only two generations can have significant mixing per each mass matrix. In particular, this rules out all possible ”democratic” structures: when mixing is maximal among all three families.

To suppress gauge-mediated flavor violation and avoid standard bounds on the scale of flavor symmetry breaking, the horizontal symmetry should be gauged in the bulk. If breaking occurs in the bulk, the flavor violation is somewhat reduced only for large enough number of new dimensions ($N > 2$). On the other hand if breaking occurs in a subspace with $N' < N$ the gauge-mediated contribution can be strongly suppressed, but unless $N'$ is also large, the would be familons, that are localized on a $3 + N'$-brane, can mediate unacceptable flavor-violation.
Combining all the potential sources, it seems that unless implementing an extra source of suppression ”by construction” FCNP are pretty close to their experimental limits.

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