Dynamic Walking of Bipedal Robots on Uneven Stepping Stones via Adaptive-Frequency MPC

Junheng Li* and Quan Nguyen†, Member, IEEE

Abstract—This letter presents a novel Adaptive-frequency MPC framework for bipedal locomotion over terrain with uneven stepping stones. In detail, we intend to achieve adaptive gait periods with variable MPC frequency for bipedal periodic walking gait to traverse terrain with discontinuities without slowing down. We pair this adaptive-frequency MPC with kino-dynamics trajectory optimization to obtain MPC adaptive frequencies (in terms of sampling times), center of mass (CoM) trajectory, and foot placements. We use whole-body control (WBC) along with adaptive-frequency MPC to track the optimal trajectories from offline optimization. In numerical validations, our adaptive-frequency optimization and MPC framework have shown advantages over fixed-frequency MPC. The proposed framework can control the bipedal robot to traverse through uneven stepping stone terrains with perturbed stone heights, widths, and surface shapes while maintaining an average speed of 1.5 m/s.

Index Terms—Optimization, predictive control for linear systems, robotics.

I. INTRODUCTION

Uneven terrain locomotion has always been one of the most important problems that researchers aim to solve on bipedal robots via motion planning and control. The value of such capability will allow bipedal robots to perform robust locomotion in many real-world tasks, such as rescue and exploration missions with unknown terrains. Recent advancement in control strategies has allowed many successful integrations of control frameworks with bipedal robots.

For instance, on the one hand, Hybrid Zero Dynamics (HZD) model [1] is an effective control scheme employed on bipedal robots such as MABEL [2]. HZD on ATRIAS robot [3] has allowed more complex motion planning strategies to be integrated, such as gait libraries for stepping stones [4]. The gait library collected from offline optimization has allowed ATRIAS (2-D) to precisely place its foot on the stepping stones by online motion planning and position control. This position-control-based approach requires accurate terrain information, including the following stone distance and height, and is not robust to uneven terrain perturbations.

On the other hand, force-based control schemes on quadruped robots became more popular. Such control frameworks can be used with linearized dynamics models and constraints. The Quadratic Programming (QP)-based force control and Model Predictive Control (MPC) on quadruped robots [5], [6] both employ simplified rigid-body dynamics and have demonstrated effectiveness in stable locomotion over uneven terrain. We believe bipedal robots can also benefit from the robustness on uneven terrain with force-based control.

Our recent work on force-and-moment-based MPC schemes on bipedal robots with 5-Degree-of-Freedom (DoF) legs [7] has allowed stable 3-D locomotion with fixed gait periods (i.e., fixed-frequency MPC). However, due to the unawareness of the terrain, the robot cannot adapt its footsteps based on the terrain. The heuristic swing foot placement policy [8] of bipedal locomotion depends on both linear velocity and gait period. Hence, when maintaining a constant velocity during walking while aiming to vary step length, it can be achieved by adjusting the gait period of each step. By allowing varied sampling time lengths in MPC, we can achieve adaptive frequency in MPC to allow the robot to walk with varied gait periods for each step and achieve varied step lengths with a constant walking speed.

Kino-dynamics-based trajectory optimization (TO) has been introduced and used in many works on mobile-legged robots (e.g., [9], [10]). The framework has the advantage of simplified system dynamics while applying robot joint constraints. To synchronize the motion control and TO, we use the same simplified dynamics model in both MPC and TO, the same foot placement policy in swing foot control and TO, and the same
adaptive frequencies in TO and MPC. Hence TO optimizes the sampling times for the MPC to achieve varied step lengths.

Many related works (e.g., [11], [12], [13], [14]) that use TO for the bipedal gait and trajectory generation share a similarity in that the foot placement adaptation is included in the frameworks to optimize best capture point locations. In our work, exact foot placement on the stone is required to allow bipedal robots to overcome very narrow stepping stones. Hence, we pre-define the desired step locations in TO.

Tracking optimal trajectory with only MPC is not optimal due to its inherent low sampling frequency, which is even lower with a long gait period. We pair the MPC with a higher-frequency Whole-body Control (WBC) for more accurate trajectory tracking. MIT Mini Cheetah quadruped robot [15], [16] and MIT Humanoid robot [17] both have demonstrated outstanding balancing performance during dynamical motion with the force-based MPC and WBC combination. We develop the WBC strategy to work with our bipedal force-and-moment-based MPC. WBC in [18] employed on bipedal robots [19], [20] validated the feasibility of a WBC-type control strategy in dynamic locomotion with periodic gaits. In our approach, We combine Kino-dynamics TO with adaptive-frequency MPC for bipedal robot traversing stepping stones and use WBC as low-level force-to-torque mapping and trajectory tracking control.

The main contributions of this letter are as follows:

- We allow bipedal robots to have adaptive foot placement and gait periods for each step by varying MPC frequencies and realizing it in control with adaptive-frequency MPC as our main locomotion controller.
- We enhance the adaptive-frequency MPC by kino-dynamics TO for optimal trajectory generation and use WBC as tracking control.
- We use the proposed framework in bipedal locomotion over uneven stepping stones. The proposed method allows the bipedal robot to maintain high speed at around 1.5 m/s when traversing uneven stepping stone terrains with height, width, and stone surface shape perturbations while only requiring minimal terrain knowledge.

The rest of this letter is organized as follows. Section II introduces the physical design parameters of the bipedal robot and the overview of the system architecture, including optimization and control. Section III presents the adaptive-frequency TO framework with the bipedal kine-dynamics model. Section IV presents the adaptive-frequency MPC framework. Some simulation result highlights and comparisons are presented in Section V.

II. Bipedal Robot Model and System Overview

A. Bipedal Robot Model

In this section, we present the bipedal robot model used for this letter. Our bipedal robot model is enhanced from our previous design in [7], a small-scale bipedal robot with 5-DoF legs. Presented in Figure 2, each of the robot legs consists of ab/ad, hip, thigh, calf, and ankle joints which are all actuated by Unitree A1 torque-controlled motor. A1 motor is a powerful joint motor with a 33.5 Nm maximum torque output and 21.0 rad/s maximum joint speed output while weighing only 0.6 kg.

In this bipedal leg design, we strategically placed all joint actuators on the upper thigh links, close to the hips, for mass concentration to minimize the leg dynamics during locomotion. Negligible leg mass is an important assumption in our force-and-moment-based simplified dynamics model in MPC [7]. More details about the physical design parameters are listed in Table I.

B. System Overview

The TO and control system block diagram is shown in Figure 3. The proposed framework is built around the adaptive-frequency MPC. In order to achieve optimized MPC frequencies, we pair the MPC framework with offline TO to generate adaptive MPC frequencies and desired trajectories based on the terrain. The TO framework uses terrain map to generate discrete optimization data including desired trajectory of robot state $x_{\text{des}} \in \mathbb{R}^2$, desired foot position $p_{n,\text{des}} \in \mathbb{R}^3$ for n-th foot, and discrete sampling time $d_t$ at time step i for MPC. The desired robot state $x_{\text{des}}$ include desired body Euler angles (roll, pitch, yaw).
and yaw) $\Theta_{des} = [\phi, \theta, \psi]^T$, desired position $p_{c,des}$, desired velocity of body CoM $\dot{p}_{c,des}$ and desired angular velocity $\dot{\omega}_{des}$. Reaction forces from MPC and swing leg control are input into WBC to be mapped to joint torques $\tau \in \mathbb{R}^{10}$. Joint feedback $q \in \mathbb{R}^{10}$ includes the joint positions of the bipedal robot.

### III. KINO-DYNAMICS-BASED ADAPTIVE-FREQUENCY TRAJECTORY OPTIMIZATION

Humans can walk with different stride lengths to adapt to the terrain and can allow swing foot to remain in the air for different periods of time. We intend to use adaptive-frequency TO to allow bipedal robots to walk with such characteristics.

We choose the kino-dynamics model in our TO for computation efficiency compared to using a full-dynamics model. The average solving time of offline TO in our approach is shown in Table II.

### A. Simplified Dynamics Model

We first present the force-and-moment-based simplified dynamics model we use in both the kino-dynamics TO and adaptive-frequency MPC framework introduced in the author’s previous work [7]. The simplified force-based dynamics model with ground reaction force and moment control inputs is shown in Figure 2. The control input consists of $u = [F_1; F_2; M_1; M_2] \in \mathbb{R}^{10}$, where $F_n = [F_{nx}; F_{ny}; F_{nz}]$, $M_n = [M_{nx}; M_{ny}; M_{nz}]$, leg $n = 1, 2$.

We choose the state variables as $[\Theta; p_c; \omega; p_c]$ and control inputs as $u$, then the simplified dynamics equation, expressed in the world frame, can be represented as

$$
\frac{d}{dt} \begin{bmatrix} \Theta \\ p_c \\ \omega \\ \dot{p}_c \end{bmatrix} = A \begin{bmatrix} \Theta \\ p_c \\ \omega \\ \dot{p}_c \end{bmatrix} + Bu + \begin{bmatrix} 0_{3 \times 1} \\ 0_{3 \times 1} \\ 0_{3 \times 1} \\ 0_{3 \times 1} \end{bmatrix}$$

(1)

$$A = \begin{bmatrix} 0 & 0 & R_c & 0 \\ 0 & 0 & 0 & I_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_c = \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2)

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (p_1 - p_c) \times & (p_1 - p_c) \times & I_G & I_L & I_L & I_G & I_G & I_L & I_G & I_L \end{bmatrix}$$

(3)

where $s$ denotes sine operator, and $c$ denotes cosine operator. Note that $R_c$ is simplified by the assumption of small roll and pitch angles $\phi \approx 0, \theta \approx 0$ [7].

In equation (3), $I_G \in \mathbb{R}^{3 \times 3}$ represent the rotational inertia of the rigid body in the world frame, which is approximated by $I_G = I_L^T R_c I_L$. $p_c$ represents the Cartesian coordinate of the contact point on nth foot. $L$ is the selection matrix to enforce the 5-D control input, $L = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$, $\times$ denotes the cross-product operation. (i.e., $(p_1 - p_c) \times F_n$)

To form a linear state-space dynamics equation for TO and MPC, we choose to include gravity $g$ as a dummy state variable $x = [\Theta; p_c; \omega; p_c; g] \in \mathbb{R}^{15}$ in equation (1) to form $x(t) = A_c x + B_c u$. Where continuous-time matrices $A_c \in \mathbb{R}^{15 \times 15}$ and $B_c \in \mathbb{R}^{15 \times 10}$ are modified from $A$ and $B$.

To use this linearized dynamics equation in TO and MPC, we translate it to discrete-time form at $t$th step:

$$x[i + 1] = \hat{A}_d x[i] + \hat{B}_d u[i]$$

(4)

$$\hat{A}_d = I_{15} + A_c \cdot dt_i, \quad \hat{B}_d = B_c \cdot dt_i$$

(5)

### B. Trajectory Optimization Problem Formulation

The adaptive-frequency TO is an offline multiple-shooting discretization method [21] to optimize the robot’s CoM trajectory, foot placements, and MPC frequencies of each stride based on the terrain map.

The optimization variable $X \in \mathbb{R}^{10(N+1)}$ includes

$$X = [x_N; \ p_{N,1}; \ p_{N,2}; \ q_N; \ u_N; \ dt_0, \ldots, dt_N]$$

(6)

where $dt_0 \ldots dt_N$ are discrete sampling times between each two time steps with $N$ total time steps. Subscript N indicates the variable is vertically concatenated from initial time step to time step $N + 1$. We define one complete bipedal walking gait to have 5 time steps of stance phase and 5 time steps of swing phase. (i.e., one gait cycle contains 10 optimization time steps) To achieve adaptive frequency in MPC and walking gait, we allow the TO to optimize $dt_i$ and ensure every $5 dt_i$ are unchanged, and thereby the gait period of each step $l$ is the summation of 5 time steps.

The formulation of the nonlinear programming (NLP) problem is as follows. The optimization objective is to drive the linear velocity close to a constant command $\dot{\bar{p}}_{cmd}^l$ and minimize the ground reaction force to maximize efficiency.

$$\min_{X} \sum_{i=0}^{N} \| p_{c,i} - \dot{\bar{p}}_{cmd}^l \|^2 + u[i]^T \beta u[i]$$

(7)

s.t.  
Initial Condition: $x_0 = x[0]$  
Final Condition: $x_f = x[N]$  
Dynamics: $x[i + 1] = \hat{A}_d x[i] + \hat{B}_d u[i]$  
$$q_{min} \leq q_n[i] \leq q_{max} \quad 0.02 \leq dt_i \leq 0.05$$

(8)

Stance phase Constraints:

$$P_{stance}[i] = P_{stance}[i - 1]$$

(8f)

Friction Cone: $\sqrt{F_{nx}^2 + F_{ny}^2} \leq \mu |F_{nz}|$  
$$\tau_{min} \leq \tau_n[i] \leq \tau_{max}$$

(8g)

Swing phase Constraints:

$$u_n[i] = 0$$

(8i)

$$p_{swing}[i - 1] \leq p_{swing}[i]$$

(8j)

$$p_{l}(\text{terrain}) = p_{swing,l} + \frac{t_{stance}}{2} p_{c,l}$$

(8k)

where $x_f$ in equation (8b) is the target final condition acquired from the terrain map. Equation (8d) constrains the joint limits. Function $\mathbb{IK}$ denotes the Inverse Kinematics calculation based on CoM and foot positions. To enforce the periodic

| Cases: | 4 stones | 5 stones | 6 stones | 7 stones |
|-------|----------|----------|----------|----------|
| Solving time: | 6.73s | 7.93s | 10.15s | 12.23s |
walking gait, the stance and swing phases are alternated every 5 \(dt\) for each leg in TO. During stance phase, equation (8f) ensures the stance foot is fixed. Here the NLP formulation allows the use of nonlinear friction cone constraint (8g), from which we have observed better locomotion performance in optimization results compared to using linearized friction pyramid constraints (i.e., equation (10b)), without introducing a significant extra computation cost. Equation (8h) enforces joint torque limits. \(J^T_n\) is the contact Jacobian of the \(n^{th}\) leg. During swing phase of the \(n^{th}\) leg at time step \(i\), equation (8i) enforces all contact forces and moments \(u_n[i] = [F_n[i]; M_n[i]]\) to be zero, where the forces and moments control input \(F_n\) and \(M_n\) are defined in Section III-A. Equation (8j) ensures the foot does not move backward during the swing. Lastly, the foot placement policy [5], [7], [8] is applied in equation (8k). This equation is the core bridging the TO and adaptive frequency MPC. The TO framework can adapt to the most optimal \(dt\) for each stride based on how far one stride needs to be to overcome the terrain while keeping the robot’s linear velocity constant. \(t_{\text{stance}}\) represents the total time the stance foot spends on the ground, which is the summation of 5 time steps at step \(l\). The placement at touch-down for each step \(l\) is acclimated to the terrain (i.e., each step is on a stepping stone).

IV. ADAPTIVE-FREQUENCY CONTROL WITH VARIED GAIT PERIODS

In this section, we present a force-and-moment-based MPC with adaptive frequency in bipedal walking gait with varied stride lengths to overcome discontinued terrains without slowing down or coming to a complete stop. The TO introduced in Section III-A promises optimal sampling times \(dt\) for each stride to achieve adaptive frequency, which can also be interpreted as the gait period for each stride.

A. Adaptive-Frequency MPC for Bipedal Locomotion

First, we present the adaptive-frequency MPC that works with varied frequencies from the TO results. Both MPC and TO use the same simplified dynamics shown in Figure 2.

A formulation of the MPC problem with finite horizon \(k\) can be written in the following form,

\[
\min_{x, u} \sum_{i=0}^{k-1} ||x[i+1] - x^{\text{ref}}[i + 1]||^2 + ||u||^2_R (9)
\]

s.t. \(\dot{x} = \hat{A}_d[i]x[i] + \hat{B}_d[i]u[i]\) \hspace{1cm} (10a)

\(-\mu F_nz \leq F_nx \leq \mu F_nz\) \hspace{1cm} (10b)

\(-\mu F_nz \leq F_ny \leq \mu F_nz\) \hspace{1cm} (10c)

\(0 < F_{\text{min}} \leq F_nz \leq F_{\text{max}}\) \hspace{1cm} (10d)

The objective of the problem is to drive state \(x\) close to the specified \(x^{\text{ref}}\) and minimize \(u\). These objectives are weighted by diagonal matrices \(R_i \in \mathbb{R}^{15 \times 15}\) and \(R_i \in \mathbb{R}^{10 \times 10}\).

Equation (10b) describes inequality constraints on contact friction pyramid. Equation (10c) describes the bounds of reaction forces. Equation (10d) enforces the swing leg exerts zero contact forces or moments.

The translation of the proposed MPC problem into Quadratic Programming (QP) form to be efficiently solved can be found in many related and previous works [5], [7].

B. Whole-Body Control

With adaptive-frequency MPC, in a step with a long gait period, the sampling frequency can be as low as only 20 Hz. We choose to combine MPC with WBC to ensure more accurate tracking performance compared to MPC alone. The WBC is an established level-low control method to map reaction forces to joint torques on legged robots [16], [17].

We adapt the WBC to work with force-and-moment-based MPC control input and allow bipedal walking gait with varied gait periods. The WBCs used in [16] and [18] are paired with a high-frequency joint PD controller to track desired joint position and velocity in addition to computing joint torques based on prioritized tasks. Both CoM and swing foot position control are parts of the WBC tasks. Our WBC framework only uses torque output from QP optimization and does not require joint tracking. Instead, we choose to continue using Cartesian space PD swing foot control [7] to track optimal foot placement from TO. With this approach, the WBC tasks are reduced to only driving CoM position and rotation for each stride based on how far one stride needs to be to overcome the terrain while keeping the robot’s linear velocity constant. \(t_{\text{stance}}\) represents the total time the stance foot spends on the ground, which is the summation of 5 time steps at step \(l\). The placement at touch-down for each step \(l\) is acclimated to the terrain (i.e., each step is on a stepping stone).

The full joint space equation of motion is,

\[
M\ddot{q} + C + g = \begin{bmatrix} 0 \\ \tau \end{bmatrix} + \tau_b
\]

(11)

\(q\) is a linear vector space containing both entries of body state (i.e., CoM position vector and Euler angles) and joint states components, \(\hat{q} = [\dot{q}_b; \dot{\hat{q}}]\), where \(\dot{q}_b \in \mathbb{R}^6\), \(\ddot{\hat{q}} \in \mathbb{R}^{10}\), and \(\tau_b = J^T qu\).

The desired acceleration of the CoM tracking task uses the optimal CoM trajectory from the TO as reference \(x^{\text{des}}\), and is computed based on a PD control law,

\[
\ddot{x}^{\text{des}} = K_p^{\text{WBC}}(x^{\text{des}}_c - x_c) + K_p^{\text{WBC}}(\ddot{x}^{\text{des}}_c - \ddot{x}_c)
\]

(12)

And the acceleration command \(\ddot{x}_c^{\text{cmd}}\) is calculated by a similar task-space projection algorithm in [16].

Now the WBC-QP problem to compute the minimized relaxation components of MPC ground reaction force \(\Delta u\) and joint acceleration command \(\Delta \ddot{q}\) is as follows,

\[
\min_{\Delta \ddot{q}, \Delta u} ||\Delta \ddot{q}|| + ||\Delta u||_K
\]

(13)

s.t. \(S_h[M(\Delta \ddot{q} + \ddot{\hat{q}}^{\text{cmd}}) + C + g - J^T f(\Delta u + u)] = 0\) \hspace{1cm} (14a)

\(u_{\text{min}} \leq \Delta u + u \leq u_{\text{max}}\) \hspace{1cm} (14b)

\(\tau_{\text{min}} \leq \tau \leq \tau_{\text{max}}\) \hspace{1cm} (14c)

In equation (13), \(H \in \mathbb{R}^{16 \times 16}\) and \(K \in \mathbb{R}^{10 \times 10}\) are diagonal weighting matrices for each objective. Equation (14a) is a dynamics constraint to control the floating base dynamics. Selection matrix \(S_h \in \mathbb{R}^{6 \times 16}\) consists of 1s and 0s to identify the float base joints.

The final joint torques can be calculated as

\[
\begin{bmatrix} 0 \\ \tau \end{bmatrix} = M(\Delta \ddot{q} + \ddot{\hat{q}}^{\text{cmd}}) + C + g - J^T f(\Delta u + u)
\]
As for swing leg, the joint torques $\tau_{\text{swing},n} \in \mathbb{R}^5$ are computed separately by inverse Jacobian $J_{\nu,n}^\top$ of leg $n$,

$$
\tau_{\text{swing},n} = J_{\nu,n}^\top F_{\text{swing},n}.
$$

(16)

Where swing foot force is determined by a simple PD control law,

$$
F_{\text{swing},n} = K_P(p_{n,\text{des}} - p_n) + K_D(\dot{p}_{n,\text{des}} - \dot{p}_n).
$$

V. RESULTS

In this section, we will present highlighted results for validation of our proposed adaptive-frequency control and TO framework. The readers are encouraged to watch the supplementary simulation videos.¹

We validate our proposed approach in a high-fidelity physical-realistic simulation in MATLAB Simulink with Simscape Multibody library. We also use Spatial v2 software package [22] to acquire coefficients of dynamics equations in WBC and CasADi [23] for offline TO.

Firstly, we present the comparison between MPC-only control vs. MPC+WBC in tracking sinusoidal height command with double-leg stance. Due to low sampling frequency, previous works usually only use MPC as locomotion control and use QP-based force control as balance/stance control for its higher frequency (e.g., [6], [7], [17]). Figure 5 shows the comparison of simulation snapshots between the two approaches, it can be observed that the WBC+MPC approach we proposed performed ideally in height tracking while the MPC-only approach failed over time.

Secondly, we compare the locomotion performance over stepping stones in simulation with the following approaches.

1) With fixed-frequency MPC (33.3Hz) + WBC,
2) With adaptive-frequency MPC + WBC
3) With adaptive-frequency MPC + WBC + TO

¹https://youtu.be/8hLihy96ICg

As can be seen in Figure 6(a), the approach with fixed-frequency control cannot adapt the foot placement based on the stepping stone gap distance. In Figure 6(b), the adaptive-frequency MPC+WBC framework with manually-input gait

![Fig. 4. Motion Snapshots of Uneven Stepping Stone Locomotion: a). TO results. b). Simulation results of various cases with terrain perturbations.](image)

![Fig. 5. Motion Snapshots of Height Command Tracking.](image)

![Fig. 6. Motion Snapshots of Uneven Stepping Stone Locomotion Comparison of 3 Approaches.](image)
periods based on the terrain shows improvement from the fixed gait period case. However, it cannot achieve precise foot placement on stepping stones, therefore, failed after only a few stones. Our proposed approach, shown in Figure 6(c), with both adaptive-frequency control and TO succeeding in this terrain. Figure 7 shows the velocity tracking performance with our proposed approach 3).

We also would like to present the solver computation times for several tasks in CasADi with IPOPT solver in MATLAB R2021b. As a benchmark, the PC platform we use for offline TO has an AMD Ryzen 5-5600X CPU clocked at 4.65GHz. In Table II, we measure the solving time of the proposed adaptive-frequency TO. The cases are categorized into the number of stepping stones. We run the TO with 30 randomized terrain setups for each case to find average performance.

Lastly, we present the uneven stepping stone terrain locomotion results with our proposed approach. In realistic scenarios, the stepping stone surface shapes, heights, and widths may vary. Hence the errors and disturbances in a vision-based terrain map acquisition system may hinder the accuracy of terrain information. In our approach, we can allow the terrain map in the optimization framework to be simplified to uniformly sized stepping stones with varied center-to-center distances, shown in Figure 4(a). We then use this optimization result to control the robot to traverse the terrains with various perturbations, shown in Figure 4(b). These terrain perturbations include varied stepping stone widths, heights, and surface shapes. In the above simulation results, the linear velocity the robot maintained during the task is 1.5 m/s. The stone center-to-center gap distance is 15 to 30 cm. The maximum stone height perturbation is 5 cm. The stone width is 4 to 10 cm.

VI. CONCLUSION

In conclusion, we introduced an effective adaptive-frequency MPC and optimization framework for bipedal locomotion over terrains with discontinuities such as stepping stones with varied gait periods and step lengths. In addition, we also introduced the adaptive-frequency trajectory optimization framework to generate optimal sampling times for each step, CoM trajectory, and foot positions based on the terrain. We paired MPC with WBC for more accurate tracking control performance. Through numerical validation in simulation, we successfully allowed the robot to walk over a series of uneven stepping stones with perturbations while maintaining the robot’s average linear velocity at 1.5 m/s.