Temperature in warm inflation in non minimal kinetic coupling model

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Abstract
Warm inflation in the non minimal derivative coupling model with a general dissipation coefficient is considered. We investigate conditions for the existence of the slow roll approximation and study cosmological perturbations. The spectral index, and the power spectrum are calculated and the temperature of the universe at the end of the slow roll warm inflation is obtained.

1 Introduction
To describe the inflationary phase in the early universe [1, 2], many theories have been proposed which most of them are categorized into two classes: modified gravity models [3], and models with exotic fields dubbed as inflaton [4]. These groups may related to each other through some conformal transformations [5].

In a well known model, the responsible of the early accelerated expansion of the Universe is a canonical scalar field \( \phi \), slowly rolling down a nearly flat potential. Inflation lasts as long as the slow roll conditions hold. In this paradigm we encounter a cold universe at the end of inflation. After the cease of the slow roll conditions, the scalar field begins a rapid coherent oscillation and decays to ultra relativistic particles (radiation) reheating the Universe [6]. A natural candidate for this scalar field, as is proposed in [7], is the Higgs boson. In this context, adding a non-minimal coupling between the scalar field and scalar curvature is required for the renormalizability, and also consistency with the amplitude of density perturbations obtained via observations. Another model in which the inflaton is considered as the Higgs field is introduced in [8], where the scalar field has a non minimal
kinetic coupling term. This theory does not suffer from unitary violation and is safe of quantum corrections. In this framework, the inflation and the reheating of the Universe are discussed in the literature [9]. The same model, with a non canonical scalar field dark energy, is also employed to describe the present acceleration of the Universe [10]. In the aforementioned model, inflation and reheating happen in two distinct eras, but one can unify them by assuming an appropriate dissipative coefficient which permits the decay of inflaton to radiation during inflation: Warm inflation was first introduced for minimal coupling model [11]. Afterwards, numerous articles has been published in this subject [12, 13, 14]. Friction term for inflaton equation of motion is computed in [14]. Tachyon warm inflationary universe models are considered in [15].

In this work we consider warm inflation in non minimal derivative coupling model. We investigate slow roll conditions and also the temperature of the universe during the warm inflation for general dissipative coefficient. We study the cosmological perturbations and, based on observational parameters from PLANK2013 data, determine the temperature of the universe at the end of slow roll warm inflation.

We use units $\hbar = c = 1$ though the paper.

2 Preliminaries

The action of Gravitational Enhanced Friction (GEF) theory is given by [8]

$$S = \int \left( \frac{M^2}{2} R - \frac{1}{2} \Delta^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right) \sqrt{-g} d^4 x + S_{int} + S_r, \quad (1)$$

where $\Delta^{\mu \nu} = g^{\mu \nu} + \frac{1}{M^2} G^{\mu \nu}$, $G^{\mu \nu} = R^{\mu \nu} - \frac{1}{2} R g^{\mu \nu}$ is Einstein tensor, $M$ is a coupling constant with the dimension of mass, $M_P = 2.4 \times 10^{18} \text{GeV}$ is the reduced Planck mass, $S_r$ is the radiation action and $S_{int}$ describes the interaction of the scalar field with the other ingredient. In the absence of terms containing more than two time derivatives, we have not additional degrees of freedom in this theory. We calculate the energy momentum tensor,

$$T_{\mu \nu} = T_{\mu \nu}^{(c)} + T_{\mu \nu}^{(r)}, \quad (2)$$

by variation of the action with respect to the metric [16]. $T_{\mu \nu}^{(r)}$ is the radiation energy momentum tensor and $T_{\mu \nu}^{(\phi)}$ is the scalar field energy momentum tensor, consisting of parts coming from the minimal part: $T_{\mu \nu}^{(c)}$,

$$T_{\mu \nu}^{(\phi)} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu \nu} (\nabla \phi)^2 - g_{\mu \nu} V(\phi), \quad (3)$$

and parts coming from the non minimal derivative coupling section, $\Theta_{\mu \nu}$,

$$\Theta_{\mu \nu} = - \frac{1}{2} G_{\mu \nu} (\nabla \phi)^2 - \frac{1}{2} R \nabla_\mu \phi \nabla_\nu \phi + R^a_{\mu \alpha} \nabla_\alpha \phi \nabla_\nu \phi \quad (4)$$
\[\begin{align*}
+ R^\alpha_\mu \nabla_\alpha \varphi \nabla_\mu \varphi + R_{\mu \nu \alpha \beta} \nabla^\alpha \varphi \nabla^\beta \varphi + \nabla_\mu \nabla^\alpha \varphi \nabla_\alpha \nabla_\nu \varphi \\
- \nabla_\mu \nabla^\nu \varphi \Box \varphi - \frac{1}{2} g_{\mu \nu} \nabla^\alpha \varphi \nabla_\alpha \nabla_\beta \varphi + \frac{1}{2} g_{\mu \nu} (\Box \varphi)^2 \\
- g_{\mu \nu} \nabla^\alpha \varphi \nabla^\beta \varphi R_{\alpha \beta}.
\end{align*}\]

By variation of the action (1) with respect to the scalar field \(\varphi\), the equation of motion for the homogeneous and isotropic scalar field in the presence of a dissipative term can be expressed as

\[
(1 + 3 H^2 M^2) \ddot{\varphi} + 3 H (1 + 3 H^2 M^2) \dot{\varphi} + V'(\varphi) + \Gamma \dot{\varphi} = 0,
\]

where \(H = \dot{a} / a\) is the Hubble parameter, a “dot” is the differentiation with respect to the cosmic time \(t\), ”prime” is differentiation with respect to the scalar field \(\varphi\), and \(\Gamma \dot{\varphi}\) is the friction term adopted phenomenologically to describe decay of the \(\varphi\) field and its energy transfer into the radiation bath. \(\Gamma\) in general is a function of \(\varphi\) and temperature [17, 18]. The Friedman equation for this model is given by

\[
H^2 = \frac{1}{3 M_p^2}((1 + 9 H^2 M^2) \dot{\varphi}^2 / 2 + V(\varphi) + \rho_r),
\]

where \(\rho_r\) is the energy density of the radiation, which can be written as [17]

\[
\rho_r = \frac{3}{4} T S.
\]

\(S\) is the entropy density and \(T\) is the temperature. The energy density and pressure of homogenous and isotropic scalar field are given by

\[
\rho_\varphi = \left(1 + \frac{9 H^2 M^2}{M^2}\right) \dot{\varphi}^2 / 2 + V(\varphi),
\]

and

\[
P_\varphi = (1 - 3 H^2 M^2) \dot{\varphi}^2 / 2 - V(\varphi) - 2 H \dot{\varphi} \ddot{\varphi},
\]

respectively. By continuity equation for the total system \(\dot{\rho} + 3 H (\rho + P) = 0\), and also the equation of motion (5), we obtain

\[
\dot{\rho}_r + 4 H \rho_r = \Gamma \dot{\varphi}^2,
\]

which gives the rate of entropy production as

\[
T(\dot{S} + 3 HS) = \Gamma \dot{\varphi}^2.
\]
3 Slow roll approximation

In the previous section we pointed out to the equations needed to describe the scalar field and radiation evolutions in an interacting nonminimal coupling model. Hereafter we consider the slow roll approximation:

\[ \ddot{\phi} \ll 3H\dot{\phi} \quad \dot{H} \ll H^2 \quad (1 + \frac{9H^2}{M^2})\dot{\phi}^2 \ll V(\varphi). \]  

(12)

The entropy density satisfies

\[ TS \ll V(\varphi) \quad \dot{S} \ll 3HS. \]  

(13)

For a positive potential, the slow roll conditions give rise to the inflation. Neglecting the second order derivative, we can write the equation of motion of the scalar field as

\[ \dot{\phi} \approx -\frac{V'(\varphi)}{3HU(1 + r)}. \]  

(14)

where

\[ U = 1 + \frac{3H^2}{M^2} \quad r = \frac{\Gamma}{3HU}. \]  

(15)

\( r \) is the ratio of thermal damping component to the expansion damping. During the slow roll warm inflation, the potential energy of the scalar field is dominant, and therefore the Friedman equation becomes

\[ H^2 \approx \frac{1}{3M_p^2} V(\varphi). \]  

(16)

We have also

\[ ST \approx U_r \dot{\phi}^2. \]  

(17)

By the equation (16) we can write \( U \) as the function of potential

\[ U = 1 + \frac{V(\varphi)}{M^2M_p^2}. \]  

(18)

We employ the following set of parameters to characterize the slow roll:

\[ \delta = \frac{M_p^2}{2} \left( \frac{V'(\varphi)}{V(\varphi)} \right)^2 \frac{1}{U(\varphi)}, \]  

(19)

\[ \eta = M_p^2 \frac{V''(\varphi)}{V(\varphi)} \frac{1}{U(\varphi)}, \]  

(20)

\[ \beta = M_p^2 \frac{\Gamma'(\varphi) V'(\varphi)}{\Gamma(\varphi) V(\varphi)} \frac{1}{U(\varphi)}, \]  

(21)

\[ \epsilon = -\frac{\dot{H}}{H^2}. \]  

(22)
To express slow roll conditions in terms of these parameters, we need to calculate $\dot{U}$ and $\dot{r}$. We have

$$\dot{U} = \frac{6HH}{M^2},$$

(23)

therefore

$$\frac{\dot{U}}{H} = -2\epsilon(U - 1),$$

(24)

and

$$\frac{\dot{r}}{H} = -\beta \frac{r}{r + 1} + \epsilon r \left(3 - \frac{2}{U}\right).$$

(25)

Using the relation (16), one can obtain $\epsilon$ as a function of $\delta$ and $r$

$$\epsilon = \frac{\delta}{1 + r}.$$  

(26)

From (14) we can derive

$$\frac{\ddot{\varphi}}{H^2\dot{\varphi}} = -\eta \frac{1}{r + 1} + \delta(3 + \frac{2}{U}) \frac{1}{(1 + r)^2} + \beta \frac{r}{(1 + r)^2}.$$  

(27)

The slow roll conditions can be expressed as

$$\epsilon \ll 1, \quad \delta \ll 1, \quad \eta \ll 1, \quad \beta \ll 1.$$  

(28)

Note that if $\frac{H^2}{M^2} \to 0$ our model reduces to warm inflation in minimal coupling model [11], and if $r \to 0$ and $\frac{H^2}{M^2} \to 0$ we recover the standard slow roll inflation [19]. By using the relations (19), (20), and (21), we get:

$$\frac{1}{H} \frac{dt}{d\ln(TS)} = \epsilon(1 + 2(3 - \frac{1}{2U}) + \beta \frac{-1 + r}{(1 + r)^2} - 2\eta \frac{1}{1 + r}).$$  

(29)

In our study, we take $r \gg 1$ and consider the high friction limit $U \approx \frac{3H^2}{M^2} \gg 1$, hence $\frac{1}{H} \frac{d\ln(TS)}{dt} = \frac{1}{H} \left(\frac{\dot{T}}{T} + \frac{\dot{S}}{S}\right) \ll 1$.

The number of efolds during slow roll warm inflation is

$$N = \int_{t_s}^{t_{end}} H dt = \int_{\varphi_s}^{\varphi_{end}} \frac{H}{\dot{\varphi}} d\varphi = -\int_{\varphi_s}^{\varphi_{end}} \frac{3H^2U(1 + r)}{V'(\varphi)} d\varphi,$$

(30)

where $\varphi_*$ and $\varphi_{end} = \varphi(t_{end})$ are the values of the scalar field at the horizon crossing ($t_*$), and at the end of inflation, ($t_{end}$). By horizon crossing (or horizon exit) we mean the time at which a pivot scale exited the Hubble radius during inflation. Using the Friedman equation the above relation becomes

$$N = \frac{1}{M_p^2} \int_{\varphi_*}^{\varphi_{end}} \frac{V(\varphi)}{V'(\varphi)} U(1 + r) d\varphi.$$  

(31)
At the end of this section, by choosing the form of $\Gamma$ and the potential, we derive more specific results. We adopt the (general) damping term proposed in [17]

$$
\Gamma = \Gamma_0 \left( \frac{\varphi}{\varphi_0} \right)^a \left( \frac{T}{T_0} \right)^b,
$$

where $a$ and $b$ are two arbitrary integers and $\varphi_0, \Gamma_0, T_0$ are constant, and consider the power law potential

$$
V(\varphi) = \lambda \varphi^n,
$$

where $n$ and $\lambda$ are two constants. By using relation (18), and in high friction limit for $r \gg 1$, after some computations we obtain

$$
\rho_r = \frac{\Gamma \dot{\varphi}^2}{4H}.
$$

By inserting $\dot{\varphi}$ from (14), into the above equation we obtain

$$
\rho_r = \frac{V'(\varphi)^2}{4\Gamma V(\varphi)} = \frac{\sqrt{3} M_p V'(\varphi)^2}{4\Gamma \sqrt{V(\varphi)}}.
$$

Using (32) and (33), $\rho_r$ is obtained as

$$
\rho_r = \sqrt{3} M_p^2 \varphi_{a0}^n \frac{T_b^b}{4\Gamma_0} \times \frac{\varphi^{(\frac{3n}{2} - 2 - a)}}{T_b^b}.
$$

We can write radiation energy density as a function of temperature,

$$
\rho_r = \frac{g \pi^2}{30} T^4,
$$

where $g$ is the number of degree of freedom for ultra relativistic particles. By relations (36,37) temperature of the universe may derived as a function of $\varphi$

$$
T = A \varphi^{\left(\frac{3n-4-2a}{2n+4}\right)},
$$

where in this relation $A$ is given by

$$
A = \left[ \frac{15\sqrt{3} M_p n^2 \lambda^4 \varphi_{a0}^n \varphi_0^b}{2\Gamma_0 g \pi^2} \right]^{1+\frac{b}{b}}.
$$

The slow roll parameters may be now expressed as

$$
\delta = \frac{M^2 M_p^4 n^2}{2\lambda} \frac{1}{\varphi^{n+2}},
$$

$$
\eta = \frac{M^2 M_p^4 n^2 (n - 1)^2}{\lambda} \frac{1}{\varphi^{n+4}}.
$$
and the number of efolds is given by
\[ N = \frac{1}{3M_p^2} \int_{\varphi_{\text{end}}}^{\varphi_*} V(\varphi) \Gamma \frac{d\varphi}{V(\varphi)H}, \] (42)
where \( \varphi_* = \varphi(t_*) \) and \( t_* \) is the time at the horizon crossing. By using (38) and assuming \( \varphi_* \ll \varphi_{\text{end}} \), the number of e fold becomes
\[ N = \frac{\Gamma_{\text{0}} A_b (4 + b)}{\sqrt{3M_p n \sqrt{\lambda \phi_a_0}} \times \frac{4a + nb - 2n + 8}{4a - 2n + 8}}. \] (43)
For \( b = 0 \), the relation (43) reduces to
\[ N = \frac{4\Gamma_{\text{0}} A_0 (4 + 0)}{\sqrt{3M_p n \sqrt{\lambda \phi_a_0}} \times \frac{4a + 0 + 8}{4a - 2 + 8}}. \] (44)
where \( \frac{d^2}{dt^2} = H^2 (1 - \epsilon) \) implies that the inflation ends when \( \epsilon \approx 1 \). Putting \( \epsilon \approx 1 \) back into (26) gives \( \delta \approx 1 + r \) and if \( r \gg 1 \), at the end of warm inflation we have \( \delta \approx r \).

4 Cosmological perturbations

In this section we consider the evolution equation for the first order cosmological perturbations of a system containing inflaton and radiation. In the Newtonian gauge, scalar perturbations of the metric can be written as [20]
\[ ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)\delta_{ij}dx^i dx^j. \] (45)
The energy momentum tensor splits into radiation \( T_r^{\mu\nu} \) and inflaton part \( T_{\phi}^{\mu\nu} \),
\[ T^{\mu\nu} = T_r^{\mu\nu} + T_{\phi}^{\mu\nu}. \] (46)
\( T_{\phi}^{\mu\nu} \) is the energy momentum tensor of the inflaton, introduced in the second section. We have modeled the radiation field as a perfect barotropic fluid. We have
\[ T_r^{\mu\nu} = (\rho_r + P_r)u_\mu u_\nu + P_r g^{\mu\nu}, \] (47)
where \( u_r \) is four velocity of radiation fluid and \( \overline{\rho}_r = 0 \) and \( \overline{\mu}_r = -1 \). "Bar" denotes unperturbed quantities. By considering the normalisation condition \( g^{\mu\nu} u_\mu u_\nu = -1 \), we obtain
\[ \delta u^0 = \delta u_0 = \frac{h_{00}}{2}. \] (48)
\( \delta u^i \) is an independent dynamical variable. We can define \( \delta u_i = \partial_i \delta u \) [20]. Energy transfer between the two components is described by a flux term [21]
\[ Q_\mu = -\Gamma u^\nu \partial_\mu \varphi \partial_\nu \varphi, \] (49)
associated to the field equations
\[ \nabla_\mu T^{\mu \nu} = Q^\nu, \quad (50) \]
and
\[ \nabla_\mu T^{\mu \nu}_\phi = -Q^\nu. \quad (51) \]

From relation (50) we deduce \( Q_0 = \dot{\rho}_r + 3H(\rho_r + P_r) \), which is the continuity equation for radiation field in the presence of interaction. Similarly, the equation (51) becomes \(-Q_0 = \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi)\). Perturbations to the energy momentum transfer are described by the energy transfer
\[ \delta Q_0 = -\delta \dot{\Gamma} \phi^2 + \Phi \dot{\phi}^2 - 2\Gamma \delta \phi \dot{\phi}, \quad (52) \]
and the momentum flux
\[ \delta Q_i = -\Gamma \dot{\phi} \delta \mu_i. \quad (53) \]

By variation of equation (50) as \( \delta (\nabla_\mu T^{\mu \nu}_r) = \delta Q^\nu \), for the zeroth (0-0) component we obtain
\[ \delta \rho_r + 4H \delta \rho_r + \frac{4}{3} \rho_r \nabla^2 \delta u - 4 \dot{\Psi} \rho_r = -\Phi \dot{\phi}^2 + \delta \Gamma \phi^2 + 2 \delta \phi \dot{\phi}, \quad (54) \]
and for the \( i-th \) component we derive
\[ 4 \rho_r \ddot{u} + 4 \dot{\rho}_r \delta u^i + 20H \rho_r \delta u^i = -a^2 [3 \Gamma \dot{\phi} \delta \phi + \partial_i \delta \rho_r + 4 \rho_r \partial_i \dot{\phi}]. \quad (55) \]

Equation of motion for perturbation of the scalar field can be calculated by variation of (51) as \( \delta (\nabla_\mu T^{\mu \nu}_\phi) = -\delta Q^\nu \) giving
\[ \left(1 + \frac{3H^2}{M^2}\right) \ddot{\phi} + \left(1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2}\right)3H + \Gamma \dot{\phi} + \delta V'(\phi) + \dot{\phi} \delta \Gamma \right) \]
\[ -(1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2}) \nabla^2 \delta \phi = \]
\[ -2V'(\phi) + 3\Gamma \dot{\phi} - \frac{6H \dot{\phi}}{M^2} (3H^2 + 2\dot{H}) - \frac{6H^2 \dot{\phi}}{M^2} \Phi \]
\[ + (1 + \frac{9H^2}{M^2}) \phi \dot{\phi} + \frac{2H \phi \nabla^2 \Phi}{M^2} + 3(1 + \frac{9H^2}{M^2} + \frac{2H}{M^2} + \frac{2H \dot{\phi}}{M^2}) \dot{\phi} \]
\[ + \frac{6H \dot{\phi}}{M^2} \frac{\nabla^2 \Psi}{a^2} - \frac{2(\dot{\phi} + H \dot{\phi})}{M^2} \nabla^2 \Psi a^2, \quad (56) \]
for the zeroth component. By using perturbation to the Einstein field equation \( G_{\mu \nu} = -8\pi G T_{\mu \nu} \), one can obtain the evolution equation for perturbation parameters, which for the 0 – 0 component is
\[ -3H \dot{\Psi} - 3H^2 \Phi + \frac{\nabla^2 \Psi}{a^2} = 4\pi G \right] -(1 + \frac{18H^2}{M^2}) \phi^2 \Phi - \frac{9H \phi^2}{M^2} \dot{\Psi} \quad (58) \]
\[ + \frac{\phi^2}{M^2} \frac{\nabla^2 \phi}{a^2} + V(\phi) \delta \phi + (1 + \frac{9H^2}{M^2}) \phi \delta \phi - \frac{2H \phi \nabla^2 (\delta \phi)}{M^2} - \frac{\delta \rho_r}{a^2} + \delta \rho_r \],
and the $i-i$ components are

$$
(3H^2 + 2\dot{H})\Phi + H(3\dot{\Psi} + \Phi) + \frac{\nabla^2(\Phi - \Psi)}{3a^2} + \ddot{\Psi} = (59)
$$

$$
4\pi G\left[\left(\frac{3H^2 + 2\dot{H}}{M^2}\right)^2 - \frac{\dot{\varphi}^2}{M^2} + \frac{8H\ddot{\varphi}\ddot{\varphi}}{M^2}\right]\Phi + \frac{3H\dot{\varphi}^2}{M^2}\Phi
$$

$$
+ \frac{\dot{\varphi}^2}{M^2}\nabla^2\Phi + \left[\left(\frac{3H^2 + 2\dot{H}}{M^2}\right)^2 + \frac{2\dot{\varphi}\ddot{\varphi}}{M^2}\right]\Phi + \frac{\dot{\varphi}^2}{M^2}\nabla^2\Psi
$$

$$
- V(\varphi)\delta\varphi - \left[(-1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2})\dot{\varphi} + \frac{2H\ddot{\varphi}}{M^2}\right]\delta\varphi
$$

$$
- \frac{2H\dot{\varphi}}{M^2}\delta\varphi + \frac{2(\ddot{\varphi} + H\dot{\varphi})}{M^2}\nabla^2(\delta\varphi) - \frac{3a^2}{3a^2} + \delta P_r].
$$

By relation $-H\partial_i\Phi - \partial_i\dot{\Psi} = 4\pi G(\rho + P)\partial_i\delta u$, from 0 $- i$ component of field equation we have

$$
H\Phi + \dot{\Psi} = 4\pi G[\frac{3H^2}{M^2}\Phi + \frac{\dot{\varphi}^2}{M^2}\dot{\Psi} + (1 + \frac{3H^2}{M^2})\dot{\varphi}\delta\varphi - \frac{2H\dot{\varphi}}{M^2}\delta\varphi] (60)
$$

$$
+ (\rho_r + P_r)\delta u].
$$

These six equations (55-60), generally describe the evolution of perturbations. To obtain physical mode, we impose the below condition on the perturbation [20]

$$
\Phi = \Psi.
$$

(61)

We consider the quantities in momentum space via Fourier transform, therefore the spatial parts of these quantities are $e^{ikx}$ where $k$ is the wave number of the corresponding mode. So by replacing $\partial_j \rightarrow ik_j$ and $\nabla^2 \rightarrow -k^2$, and defining

$$
\delta u = -\frac{a}{k}ve^{ikx},
$$

(62)

we can write the equation (55) as

$$
\rho_r\dot{v} + \dot{\rho}_rv + 4H\rho_rv = \frac{k}{a}[\rho_r\Phi + \frac{\delta\rho_r}{4} + \frac{3}{4}G\delta\varphi].
$$

(63)

During warm inflation the background and perturbation satisfy the slow roll approximation. In other words the background and perturbations vary slowly in time (e.g. $\dot{\Phi} \ll H\Phi$). In the continue, we consider non minimal derivative coupling at high friction limit. Also we consider modes with wavenumbers satisfying $\frac{k}{a} \ll H$. By applying these conditions to the equation (64), we obtain

$$
\frac{\delta\rho_r}{\rho_r} = -\Phi + \frac{\delta\Gamma}{\Gamma}.
$$

(64)

Similarly, (63) reduces to

$$
v = \frac{k}{4aH}[\Phi + \frac{\delta\rho_r}{4\rho_r} + \frac{3\Gamma\delta\varphi}{4\rho_r}],
$$

(65)
and the equation (56) takes the form
\[
\left[(1 + \frac{3H^2}{M^2})3H + \Gamma\right] \delta \dot{\phi} + \delta V'(\phi) + \phi \delta \Gamma = \]
\[-[2V'(\phi) + 3\Gamma \dot{\phi} - \frac{3H^2}{M^2}(6H\dot{\phi})]\Phi. \tag{66}
\]

We derive also
\[
H \Phi = 4\pi G\left[\frac{3H^2\dot{\phi}^2}{M^2} \Phi + (1 + \frac{3H^2}{M^2})\dot{\phi} \delta \phi - \frac{4a}{3k} \rho_r v\right]. \tag{67}
\]

From relations (64-67) we can calculate \( \delta \phi \) as a function of \( H, \Gamma, \) and \( V(\phi) \),
\[
\delta \phi \approx CV' \exp (-\Im(\phi)) \tag{68}
\]
where \( \Im(\phi) \) is defined as
\[
\Im(\phi) \equiv -\int \left( \frac{V'}{V} \frac{1}{1+r} + \frac{2 + 5r}{V^2} \frac{1}{2(r+1)^2} \left[ 1 + \frac{3r}{4} - \frac{\beta r}{16(1+r)} \right] \right) d\phi. \tag{69}
\]

For cold inflation \( r = 0 \) we have \( \delta \phi \approx CV' \), in agreement with ref [17].

The curvature perturbation \( \Re \) is defined as \( \Re = \Phi - k^{-1} aHv \), which in the slow roll large scale limit is a constant [17]. Using \( \Re \sim C \) and the equation (68), the density perturbation becomes [17]
\[
\delta \frac{\Delta}{H} \approx \left(2M_P^2 P^5\right) \exp(-2\Im(\phi)) \frac{2}{V'} \delta \phi. \tag{70}
\]
In this relation \( \delta \phi \) is the fluctuation of the scalar field during the warm inflation [11]
\[
\delta \phi^2 \approx \frac{k_F^2 T}{2\pi^2}, \tag{71}
\]
where \( k_F \) is the freeze out scale. To calculate \( k_F \), we must determine the time at which the damping rate of relation (56) falls below the expansion rate \( H \). At the freeze out time, \( t_F \), the freeze out wavenumber, \( k_F = \frac{k}{a(t_F)} \),

is given by
\[
k_F = \sqrt{\frac{\Gamma H + 3H^2(1 + \frac{3H^2}{M^2})}{3H^2 U(1 + r)}}, \tag{72}
\]
therefore the density perturbation becomes
\[
\delta^2_{\Delta} \approx \left(\frac{2M_P^2}{5} \right)^2 \left[ \frac{\exp(-2\Im(\phi))}{V'(\phi)^2} \right] \delta \phi^2. \tag{73}
\]
We can rewrite this relation in the following form
\[
\delta^2_{\Delta} \approx \left(\frac{4M_P^4}{25} \right)^2 \left[ \frac{\exp(-2\Im(\phi))}{V'(\phi)^2} \right] \sqrt{3H^2 U(1 + r) T}. \tag{74}
\]
The spectral index for the scalar perturbation is given by

\[ n_s - 1 = \frac{d \ln \delta_H^2}{d \ln k}, \quad (75) \]

where this derivative is computed at the horizon crossing \( k \approx aH \). Finally, we obtain

\[ n_s - 1 = \frac{2\eta}{(1 + r)} - \frac{\delta}{2(1 + r)} - \frac{\beta(1 + 5r)}{2(1 + r)^2} - \frac{\delta(2 + 5r)(4 + 3r)}{2(1 + r)^2} + \frac{\delta \beta r(2 + 5r)}{8(1 + r)^4} \quad (76) \]

Afterwards, we will consider the warm inflation at high friction limit for the power law potential \( V(\phi) = \lambda \phi^n \) and also the power law dissipation coefficient \( \Gamma = \Gamma_0 (\frac{\phi}{\phi_0})^\alpha \). Therefore, from relations (19,20,21), one can see that the slow roll parameters become

\[ \delta \approx \frac{n^2}{2} \alpha, \quad \eta \approx n(n - 1)\alpha, \quad \beta \approx a \alpha, \quad (77) \]

where

\[ \alpha = \frac{M^2 M_p^4}{\lambda} \phi^{-(n+2)}. \quad (78) \]

For \( r \gg 1 \), we have

\[ \Im \approx -\ln (\Gamma V(\frac{\phi}{\phi_0})), \quad (79) \]

therefore

\[ \delta_H^2 \approx \left( \frac{4M_p^4}{25 \times 3^{\frac{1}{2}}M_p^3} \right) \frac{\Gamma_0^2 V^3}{V'^2} T. \quad (80) \]

With our power law choices for the potential and dissipation coefficient, (80) reduces to

\[ \delta_H^2 \approx \left( \frac{4M_p^4}{25 \times 3^{\frac{1}{2}}M_p^3 \phi_0^n} \right)^\frac{2}{3} \lambda^2 \phi_0^{(2n+2+\frac{5a}{2})} T. \quad (81) \]

Subscript (*) denotes the value of a quantity at the horizon crossing. The spectral index becomes

\[ n_s - 1 = \frac{2\eta}{r} - \frac{8\delta}{r} - \frac{5\beta}{2r}. \quad (82) \]

We can rewrite this relation as

\[ n_s - 1 = -\frac{n\alpha}{r} \left[ 2n + 2 + \frac{5\alpha}{2} \right], \quad (83) \]

where \( r \) is given by

\[ r = \frac{\Gamma_0 M^2 M_p^3}{\sqrt{3} \phi_0^\alpha \lambda^2} \phi^{(a-\frac{3a}{2})}. \quad (84) \]
By inserting the value of $\varphi$ at the horizon crossing in (83) we get
\[
n_s - 1 = -\frac{n M_p \sqrt{3 \lambda \varphi_0^2}}{\Gamma_0} \left[ 2n + 2 + \frac{5a}{2} \right] \varphi_*^{-\frac{(a+2-\frac{n}{2})}{2}}
\] (85)

5 Evolution of the universe and temperature of the warm inflation

In this section, using our previous results, we intend to calculate the temperature of warm inflation as a function of observational parameters via the method introduced in [22]. By the temperature of warm inflation, we mean the temperature of the universe at the end of warm inflation. For this purpose, we divide the evolution of the universe into three parts as follows

I— from $t_\ast$ (horizon exit) until the end of slow roll warm inflation, denoted by $t_e$. In this era, the potential of the scalar field is the dominant term in the energy density.

II— from $t_e$ until recombination era, denoted by $t_{rec}$.

III— from $t_{rec}$ until the present time $t_0$.

Therefore the number of e-folds from horizon crossing until now becomes
\[
N = \ln \left( \frac{a_0}{a_*} \right) = \ln \left( \frac{a_0}{a_{rec}} \right) + \ln \left( \frac{a_{rec}}{a_e} \right) + \ln \left( \frac{a_e}{a_*} \right) = N_I + N_{II} + N_{III}
\] (86)

5.1 Slow roll

During the slow roll warm inflation, the scalar field rolls down to the minimum of the potential and ultra relativistic particles are generated. In this period the positive potential energy of the scalar field is dominant and therefore expansion of the universe is accelerated. By relations (43) and (85), for high damping term $r \gg 1$, the number of e-folds during warm inflation becomes
\[
N_I = \frac{(2n + 2 + \frac{5a}{2})}{(a + 2 - \frac{n}{2})(1 - n_s)}.
\] (87)

We need to calculate scalar field and temperature at the end of slow roll. Inflation ends at the time when $r(\varphi_{end}) \approx \delta(\varphi_{end})$. From equations (77) and (84), we can calculate the scalar field at the end of inflation as
\[
\varphi_{end}^{-\frac{(a+2-\frac{n}{2})}{2}} \approx \frac{2\Gamma_0}{n^2 \sqrt{3 \lambda M_p \varphi_0^2}}.
\] (88)

At the end of inflation the radiation energy density gains the same order as the energy density of the scalar field
\[
\rho_{end} \approx V(\varphi_{end}) = \lambda \left( \frac{n^2 \sqrt{3 \lambda M_p \varphi_0^2}}{2\Gamma_0} \right)^{-\frac{(a+2-\frac{n}{2})}{2}}.
\] (89)


From equation (38) we deduce that the temperature of the universe at the end of inflation is

\[ T_{\text{end}} \approx \left( \frac{\lambda}{2C_r} \right)^{\frac{1}{2}} \times \left( \frac{2\Gamma_0}{n^2\sqrt{3}\lambda_c^2 M_p} \right)^{\frac{1}{4(a-2-a^2)}}. \]  

(90)

5.2 Radiation dominated and recombination eras

At the end of the warm inflation, the universe enters a radiation dominated epoch. During this era the universe is filled of ultra-relativistic particles which are in thermal equilibrium, and experiences an adiabatic expansion during which the entropy per comoving volume is conserved: \( dS = 0 \) [23]. In this era the entropy density, \( s = S a^{-3} \), is derived as [23]

\[ s = \frac{2\pi^2}{45} g T^3, \]  

(91)

So we have

\[ \frac{a_{\text{rec}}}{a_{\text{end}}} = \frac{T_{\text{end}}}{T_{\text{rec}}} \left( \frac{g_{\text{end}}}{g_{\text{rec}}} \right)^{\frac{1}{2}}. \]  

(92)

In the recombination era, \( g_{\text{rec}} \) is related to photons degrees of freedom and as a consequence \( g_{\text{rec}} = 2 \). Hence

\[ N_{II} = \ln \left( \frac{T_{\text{end}}}{T_{\text{rec}}} \left( \frac{g_{\text{end}}}{2} \right)^{\frac{1}{2}} \right). \]  

(93)

By the expansion of the universe, the temperature diminishes: \( T(z) = T(z = 0)(1 + z) \), where \( z \) is the redshift parameter. So we can state \( T_{\text{rec}} \) in terms of \( T_{\text{CMB}} \) as

\[ T_{\text{rec}} = (1 + z_{\text{rec}})T_{\text{CMB}}. \]  

(94)

We have also

\[ \frac{a_0}{a_{\text{rec}}} = (1 + z_{\text{rec}}), \]  

(95)

hence

\[ N_{II} + N_{III} = \ln \left( \frac{T_{\text{end}}}{T_{\text{CMB}}} \left( \frac{g_{\text{end}}}{2} \right)^{\frac{1}{2}} \right). \]  

(96)

5.3 The temperature in the warm inflation

We have determined the number of e-folds appearing in the right hand side of (86). To determine the warm inflation temperature we require to determine \( \mathcal{N} \) in (86). By assuming \( a_0 = 1 \), the number of e-folds from the horizon crossing until the present time is obtained as \( \mathcal{N} = \exp(\Delta) \), where

\[ \Delta = \frac{1}{a_*} = \frac{H_*}{k_0} = \frac{V(\varphi_*)^{\frac{1}{2}}}{\sqrt{3k_0 M_p}}. \]  

(97)
By relations (97,96,86) we can obtain

$$T_{\text{end}} = T_{\text{CMB}} \left( \frac{2}{g_{\text{end}}} \right)^{\frac{1}{3}} \exp \left( \mathcal{N} - \frac{(2n + 2 + \frac{5a}{2})}{(a + 2 - \frac{n}{2})(1 - n_s)} \right). \quad (98)$$

With the help of the relation $$\delta H(k_0) = \frac{5}{2M_p} \mathcal{P}_s(k_0)^{\frac{1}{2}}$$ we express the equation (81) as

$$\mathcal{P}_s(k_0) \approx \left( \frac{32}{3^{\frac{1}{3}} M_p^2 \eta^2} \right) \left( \frac{\Gamma_0}{\varphi_0} \right)^{\frac{5}{2}} \lambda^2 \varphi^2_{*} \left( \frac{19n + 12 + 18a}{2} \right)^{\frac{1}{2}} \exp \left( - \frac{(2n + 2 + \frac{5a}{2})}{(a + 2 - \frac{n}{2})(1 - n_s)} \right). \quad (99)$$

In the above equation $$T_*$$ is the temperature of the universe at the horizon crossing where the relation $$T_* \approx A \varphi \left( \frac{2n + 2}{a + 2 - \frac{n}{2}} \right)$$ holds, thus

$$\mathcal{P}_s(k_0) \approx \left( \frac{32\lambda^2}{\eta^2} \right) \left( \frac{\Gamma_0}{\varphi_0} \right)^{\frac{9}{2}} \left( \frac{15}{2\sqrt{3} \pi \eta g^{-2}} \right) \varphi^2_{*} \left( \frac{19n + 12 + 18a}{2} \right)^{\frac{1}{2}} \exp \left( - \frac{11}{3(1 - n_s)} \right). \quad (100)$$

By relations (97,96,86) one obtains

$$T_{\text{end}} = T_{\text{CMB}} \left( \frac{2}{g_{\text{end}}} \right)^{\frac{1}{3}} \exp \left( \mathcal{N} - \frac{(2n + 2 + \frac{5a}{2})}{(a + 2 - \frac{n}{2})(1 - n_s)} \right). \quad (101)$$

For $$a = 2$$, $$n = 2$$ and $$\varphi_0 = 1$$ we can calculate $$T_{\text{end}}$$ as a function of the parameters of the model

$$T_{\text{end}} = T_{\text{CMB}} \left( \frac{2}{g_{\text{end}}} \right)^{\frac{1}{3}} \exp \left( - \frac{(2n + 2 + \frac{5a}{2})}{(a + 2 - \frac{n}{2})(1 - n_s)} \right). \quad (102)$$

By sitting $$g_{\text{end}} = 106.75$$, which is the ultra relativistic degrees of freedom at the electroweak energy scale, and from PLANCK 2013 for pivot scale $$k_0 = 0.05 \text{Mpc}^{-1}$$ in one sigma level giving $$\mathcal{P}_s(k_0) = (2.20 \pm 0.056) \times 10^{-9}$$ and $$n_s = 0.9608 \pm 0.0054 \ [24]$$, the temperature of the universe at the end of warm inflation becomes $$T_{\text{end}} \approx 6.23 \times 10^{11} \text{GeV}$$. This is comparable with the reheating temperature in the context of cold inflation produced in the era of rapid oscillation, determined as $$T_{\text{reheating}} \approx 6.53 \times 10^{12} \text{GeV} \ [9]$$.

The lower and upper limits on $$T_{\text{end}}$$ depend on uncertainty of $$n_s$$ and $$\mathcal{P}_s(k_0)$$. At one sigma level we have

$$2.21 \times 10^{10} \text{GeV} \ll T_{\text{end}} \ll 5.32 \times 10^{12} \text{GeV}. \quad (103)$$

The range of the temperature, lies below the upper bound scale assumed in the literature which is about the GUT scale $$T_{\text{max}} \approx 10^{16} \text{GeV}$$.

By considering the big bang nucleosynthesis (BBN), and on the base of the data derived from large scale structure and also cosmic microwave background (CMB), a
lower bound $T_{\text{min.}} \simeq 4 \text{MeV}$, is obtained in [25] which is consistent with our result.

Up to first order Taylor expansion, the relative uncertainty is

$$\frac{\sigma(T)}{T} = \sqrt{\frac{\sigma^2(P_s)}{T^2} \left( \frac{\partial T}{\partial P_s} \right)^2 + \frac{\sigma^2(n_s)}{T^2} \left( \frac{\partial T}{\partial n_s} \right)^2} = 3.41.$$  \hspace{1cm} (104)

At the end let us note that the two conditions that we have used for calculation of the temperature, i.e. $r \gg 1$ and $\frac{H^2}{M^2} \gg 1$, lead to

$$\lambda \Gamma_0 \gg \frac{198 M_p^4 M^3}{1 - n_s},$$  \hspace{1cm} (105)

which by using the the Planck data [26], becomes

$$\lambda \Gamma_0 \gg 5051 M_p^4 M^3.$$  \hspace{1cm} (106)

6 Summary

We considered warm inflation in the framework of non minimal derivative coupling model in high friction regime. After an introduction to the model, we studied the slow roll conditions and e-folds number and then specified them in terms of the parameters of the model for a power law potential and a general power law dissipation factor. By studying The cosmological perturbations, we obtained the power spectrum and the spectral index. We used these quantities to determine the temperature of the universe in terms of $T_{CMB}$. It was shown that this temperature has the same order as the reheating temperature produced after the cold inflation in the nonminimal derivative coupling model.

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