Optimal sector modelling of airspace based on optimization algorithms

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Abstract. Reasonable assessment and effective management of air space resources is the base for ensuring the safety and efficiency of air traffic management systems. Our research is focussed on optimal airspace organization that is an index to measure the ability of the airspace system to deliver services to meet air traffic demand. Optimal airspace organization is a key factor in reducing the workload for air traffic controllers. The purpose of the study is to present a method for optimal sector modeling with direct routes applicable to free route airspace with the help of mathematical modelling of airspace. A computational model is implemented in a numerical computing environment. As a result, the following was achieved: defining the horizontal boundaries of sectors, subjected to the following constraints; minimum time spent in a sector and balancing workload distribution of air traffic controllers in the sectors. After final optimization a minimum value of the optimality criterion is obtained which is 8.5 times lower than the initial value.

1. Introduction
Diseases such as COVID-19 have enormous impact on the European air traffic network. In the context of the COVID-19 pandemic, EUROCONTROL publishes a regular comprehensive assessment of the latest traffic situation in Europe and provides a comparison with the same period in 2019. The scenarios are for a gradual increase in traffic by -15% by February 2021 from current levels. A prerequisite for this is that air transport is one of the most popular means of transport and plays a major role in the economy of the country. This allows for the liberalization of air transport and the emergence of low-cost operators, combined with overall economic growth in European countries. The significant increase in the number of flights has resulted in an increased air traffic congestion and manifest contradiction between limited airspace resources and traffic demands. Forecasts indicate that global air traffic over Europe will increase by 2030. For this purpose, the effective management of air traffic resources is a key aspect of optimization Air traffic resources include two parts, namely, air traffic controllers and system resources, such as airspace [1]. Reasonable assessment and effective management of air traffic controllers and airspace resources are the base for ensuring the safety and efficiency of air traffic management systems.

Airspace capacity estimation is a major technique in the implementation of effective management and rational allocation of airspace resources. This assessment is directly linked to the delimitation of sectors and the planning of airspace resources. Airspace capacity assessment is carried out at an acceptable level of air traffic controller workload [2]. Each air traffic management system assesses the
level of airspace workload it can absorb and from here comes a variety of developed capacity assessment models to meet the needs of any air navigation service provider.

Studies on airspace capacity estimation date back to the theoretical research on runway capacity estimation conducted by Bowen and Pearcey in the late 1940s [3]. Since then, scientists and air traffic management experts have been proposing various theories and models for airspace modelling and capacity assessment. The proposed 2D algorithms for optimal airspace organization utilize real aircraft route data [4] and achieve an even distribution of air traffic controllers’ workload. Some authors further develop these algorithms using metaheuristic and genetic algorithms [5] and others statistical data analysis [1]. International Civil Aviation Organization (ICAO) recommends using the Directorate of Operation Research and Analysis Task (DORATASK) method for aerospace capacity evaluation through normative documents for different countries. The DORATASK method is an analytical method used for evaluation and workload analysis [6]. This method is based on Fast-Time Simulation (FTS). The ATC Capacity Analyzer (CAPAN) method is Fast Time Simulation (FTS) developed by Eurocontrol [7]. For the purposes of this study, a macroscopic workload model for capacity assessment is used [8]. This model quantifies the workload depending on the geometric characteristics of the sector, the direction of the air traffic flows and the frequency of conflicts between aircrafts [9].

Nowadays, air transport is built around strictly structured airspace and the use of an automated air traffic management system with centralized control (human - operator). Therefore, as traffic increases, airspace is divided into small sectors in order to avoid saturation. Boundary determination requires those constraints to be taken into account which limits the minimum size of the sectors. Necessary communication when aircraft enters and exits too many sectors also leads to an increase in the overall workload of the air traffic controller.

Mathematical simulation consists of evaluating the sectorization using simulation software and simulated traffic. This approach is essentially local and produces global sectorization, which is a juxtaposition of individually optimized regions of control [5].

The purpose of this study is to analyze and determine the optimal division of sectors of the free route airspace through uniform workload distribution of air traffic controllers based on mathematical modelling. The analysis identifies problems arising from the design of the route network and the determination of the boundaries of the sectors where the criterion of workload is minimized. Finding an optimal solution to these problems can be achieved through a combination of mathematical optimization methods.

2. Optimization process
Optimization in an air traffic management system is a purposeful activity in the sense of obtaining the best result of the state of the system under certain conditions. The object of optimization in the study is the process of airspace organization. The optimization process consists of creating mathematical modelling of a transportation network with optimal routes. A selection of mathematical indicators for quantitative assessment of the workload of air traffic controllers has been made and a criterion for optimality for balanced workload has been determined.

2.1. Transportation network model
Initially, mathematical modelling of a transportation network with direct routes is performed, which is applicable to free route airspace. The design of the transport network allows for flight planning on the shortest route between entry and exit waypoints. The announced waypoints and possible direct routes of the aircraft are used for input data. The waypoints can be defined as entry and exit, as well as to determine the parity of the flight level for the flight of these waypoints – even and odd. This allows one input even waypoint to be connected to another output even waypoint as well as an odd input waypoint to an odd output waypoint. It is also assumed that these direct routes are one-way ones. On one direct route an aircraft will be able to fly either only on even or odd flight, depending on the route directions. In this way, the transport network can be optimized and its horizontal projection determined.
The horizontal projection of the transportation network is considered as a graph, whose nodes correspond to the intersection points of the possible routes and whose links represent these routes. Since each waypoint and each intersection point can be represented by two coordinates (x, y), the model is called a 2D model [5]. Aircraft flows passing through intersection points create conflicts between these aircraft, which must be detected and resolved by air traffic controllers.

Let \( V = \{v_1, v_2, ..., v_n\} \) be a finite set, the elements of which are the intersection points and \( E = \{e_1, e_2, ..., e_m\} \) be a finite set, the elements of which are the routes. The connecting function \( f_G: E \to V \times V \) compares on each couple ordered of intersection points. Then \( G(V, E, f_G) \) is a finite oriented multigraph, in which each waypoint \( v_i \in V \) is represented by a point in the plane, and each route \( e \in E \) is such that \( f_G(e) = \{v_i, v_j\} \) with a track starting at point \( v_i \) and ending at \( v_j \). For \( i \neq j \) it follows that \( f(e_i) \neq f(e_j) \) and \( G(V, E, f_G) \) is a finite oriented graph. Since function \( f_G \) is unique, it follows that each couple ordered pair of \( V \times V \) could be found at most only once in the graph and therefore the set \( E \) is defined as a subset of \( V \times V \), i.e. the final oriented graph is a relation over the Cartesian product \( V \times V \). In this way, each direct route is unambiguously determined by a single pair of two waypoints. Mathematical modelling of the transportation network allows us to determine and plan the optimal flight route.

2.2. Determining the number of sectors

The optimal organization of airspace requires an assessment of aircraft performance. When dividing the airspace into sectors, the minimum time spent in a sector and the maximum speed of aircraft flying through the sector shall be assessed.

Let airspace \( S \) be divided into \( n \) number of sectors \( s_1, s_2, ..., s_n \). Therefore,

\[
S = s_1 + s_2 + ... + s_n = \sum_{j=1}^{n} s_j, \tag{1}
\]

where \( S \) is the area of airspace, \( s_j \) is the area of sector \( j \), and \( n \) is the number of sectors.

Initially, it is assumed that the sectors have the shape of an elementary rectangle and \( s_1 = s_2 = ... = s_n \), then \( S = n \cdot s_j \). Thus \( s_j \) can be represented as:

\[
s_j = (V_m t)^2, \tag{2}
\]

where \( V_m \) is the maximum speed of the aircraft and \( t \) is the minimum time spent in a sector \( j \).

If \( V_m = \text{const} \) then \( s_j = f(t) \), and if \( S = \text{const} \) then the number of sectors depends on \( s_j \). Therefore \( n = f(t) \).

The flight time spent in a sector is considered in the interval \([t_{\text{min}}, t_{\text{max}}]\) at a given function \( n = f(t) \). Points \( t_0, t_1, ..., t_n \) are used to divide the interval \([t_{\text{min}}, t_{\text{max}}]\) into subintervals \([t_{j-1}, t_j]\), \( j = 1, 2, ..., n \). In each subinterval a random point \( \xi_j \) is chosen, as \( t_{j-1} \leq \xi_j \leq t_j \) and the product \( s_j \) is found from the values of the function at this point \( f(\xi_j) \) and the length of each sub-interval \( \Delta t_j = t_{j+1} - t_j \), such as:

\[
s_j = (V_m t)^2 = f(\xi_j) \times \Delta t_j, j = 1, 2, ... n. \tag{3}
\]

Therefore, the sum of these products can be represented as:

\[
S = \sum_{j=1}^{n} f(\xi_j) \times \Delta t_j, \tag{4}
\]
In the expression (3), \( s_j \) are the areas of the elementary rectangles at \( j = 1, 2, \ldots, n \). If \( n \to \infty \), then it follows that \( \Delta t_j \to 0 \) and \( n = f(t) \) is a linear function.

For each sector, a point called a sector centre is chosen at random. The number of randomly generated sector centres \( n \) is equal to the total number of calculated sectors \( K \).

2.3. **Optimality criterion**

Air traffic controllers provide air traffic control, ensure that aircraft are on the right routes and resolve conflicts that arise in the transportation network. When aircraft moves from one sector to another, they coordinate with adjacent air traffic services unit. It is assumed that the air traffic controller workload is formed by the following main activities: monitoring, conflict detection and resolution and coordination [9]. In the case of a model, based on the transportation network, the monitoring of the workload is proportional to the flow, the air traffic controller workload in resolving conflicts arising from the passage of flows through vertexes and coordination depends on the flows crossing the boundaries of the sectors. The aim is to determine the optimal division of sectors of the free route airspace by uniform workload distribution of air traffic controllers for each sector and minimize air traffic flows crossing the boundaries of these sectors. This aim is expressed by the optimality criterion on uniform workload of air traffic controllers \( J \), with the following restrictions:

- convexity – the resulting sectors must be convex in relation to the routes of the aircraft;
- minimum determined time spent in a sector.

The following mathematical indicators were used to quantify the workload. The monitoring workload indicator \( w_1(k) \) in sector \( k \) is as follows:

\[
    w_1(k) = \sum_{l_{ij} \in S_k} \frac{l_{ij}^k}{V_{ij}} f_{ij},
\]

where \( l_{ij} \) is the length of the route from the entrance waypoint \( i \) to the exit waypoint \( j \), \( l_{ij}^k \) is the length of the route from \( i \) to \( j \) in sector \( k \), \( V_{ij} \) is the maximum speed of the aircraft on the route from \( i \) to \( j \) and \( f_{ij} \) is the air traffic flow intensity from \( i \) to \( j \).

The conflict workload indicator \( w_2(k) \) in sector \( k \) is as follows:

\[
    w_2(k) = \sum_{r \in S_k} p_r,
\]

where \( r \) is an intersection point, \( S_k \) is an area of sector \( k \) and \( p_r \) is traffic density,

\[
    p_r = \sum_{f_{ij} \in r} f_{ij}.
\]

The coordination workload indicator \( w_3(k) \) in sector \( k \) is as follows:

\[
    w_3 = \sum_{i \notin j \in S_k} f_{ij},
\]

where \( f_{ij} \) is the air traffic flow intensity from \( i \) to \( j \).

The total workload indicator \( w(k) \) in sector \( k \) is defined as the sum of these three indicators:

\[
    w(k) = w_1(k) + w_2(k) + w_3(k).
\]

As an objective function, determining the optimal division of sectors of the free route airspace by uniform total workload distribution of air traffic controllers for each sector:
\[ J = \sum_{k=1}^{K} |w(k) - W_{av}| \rightarrow \min, \tag{10} \]

where \( W_{av} \) is the average workload.

The total workload \( W \) in \( K \) sectors is defined as:

\[ W = \sum_{k=1}^{K} w(k), \tag{11} \]

where \( W \) is the total workload and \( K \) is the total number of sectors.

An initial model of the airspace is determined by randomly generating the location of \( n \) number of points, centers of Voronoi cells, for which the value of the optimality criterion is calculated. The optimization process consists of \( n \) number of iterations by randomly generating a new location of the centers of the Voronoi diagram at the same traffic intensity and with minimum time spent in a sector. For each iteration, the value of the optimality criterion is calculated. The minimum value of the optimality criterion determines the optimal solution of the divided airspace with even distribution of the workload.

A computer program is developed to incorporate the optimization algorithm in order to minimize calculation time by orders of magnitude and also to eliminate the possibility of human error. For that purpose, a proprietary programming language in a multi-paradigm numerical computing environment was used. MATLAB 2019 was chosen mainly because of the friendly user interface and the fact that it is common programming language used by engineers. Although full vectorization of the code was not achieved, loops were used only when necessary. This further reduces the required processing time. The use of computer systems also eases the data management and the visualization of the final results.

3. Results

A geometric algorithm was used to calculate the intersections between simulated orthodromes. The algorithm is applied to the transportation network in figure 1, for which 598 intersection points are determined and the distances of the possible routes along the network are calculated.

![Figure 1. Intersections of direct routes.](image)

The determination of the sum of products (4) is obtained using the value of the function at the midpoints \( \xi_j \) of the sub-intervals (3). The calculated dependence of the number of sectors on the total area of the airspace \( S = 97804 \text{ NM}^2 \) and the maximum speed of aircraft flying through the sector \( V_{max} = 460 \text{ kt} \) is presented graphically in figure 2.
Interval \([t_{\text{min}} = 12 \text{ min}, t_{\text{max}} = 20 \text{ min}]\) is divided into subintervals, as \(t_0 = 1\) and \(t_n = 60\). Then 12 sectors at \(t_{\text{min}} = 12 \text{ min}\), 8 sectors at \(t_{\text{min}} = 15 \text{ min}\) and 4 sectors at \(t_{\text{min}} = 20 \text{ min}\) were calculated.

In the considered airspace, 8 points were chosen at random, which are the centers of 8 Voronoi cells. Since the formation of the Voronoi diagram is preceded by Delaunay's triangulation, the initial distribution is relatively good and covers the whole space in a balanced way. The Voronoi diagram shown in figure 3 is a 2D projection applied to the 598 intersection points taken from figure 1 in 8-sector airspace.

For the initial airspace model (before optimization), the mathematical expectation of the distribution of intersection points 74.75, the standard deviation 29.80 and the optimality criterion \(J=5430.38\) are calculated.

The optimization process consists of 100 iterations with air traffic intensity of 114 aircraft per hour and \(t_{\text{min}} = 15 \text{ min}\). In figure 4, optimal solution for aerospace divided into 8 sectors, containing 598 intersection points, 114 aircraft per hour and interval of \(t_{\text{min}} = 15 \text{ min}\) is graphically presented.
The value of the optimality criterion for this iteration is $J = 636.58$, which is reduced 8.5 times compared to the initial airspace model. The value of the standard deviation from the mean distribution of the intersection points is 23.56 compared to the initial iteration of 29.80. The optimal solution in figure 4 has changed the boundaries of the sectors, improving the distribution of the intersections by 7.9 %.

The estimate of the distribution of workload indicators for sector 1 and sector 8 is presented in figure 5 and figure 6. For every iteration after the optimization process is done, the algorithm sums the values for $\omega_1(k)$, $\omega_2(k)$ and $\omega_3(k)$. The lowest summation value must be found. This is achieved by first taking the sum of all values for $\omega_1(k)$, $\omega_2(k)$ and $\omega_3(k)$ for the first iteration, than the algorithm continues to probe values as follows; if the current summation value is a number, smaller than the lowest summation value of all previous iterations this new value is set as the new “check for lower” value in the algorithm. This allows the algorithm to find the optimality criterion on uniform workload of air traffic controllers for the summation values of $\omega_1(k)$, $\omega_2(k)$ and $\omega_3(k)$ for the whole iteration cycles.

The value of the standard deviation of the monitoring workload is 714.81 and decreases 2.20 times compared to the initial iteration of 1571.02. The value of the standard deviation of the conflict workload indicator is 66.12, but increases by 2.36% compared to the initial iteration of 64.57. This increase is due to the redistribution of air traffic by sector as a result of the redistribution of intersections. The value of the standard deviation of the coordination workload is 129.74 and decreases 1.64 times compared to the initial iteration of 212.79. When applying this method, a decrease in the total workload indicator $w(k)$ is observed. The value of the standard deviation is 826.80 and decreases 2.20 times compared to the initial iteration of 1820.92.

The sectors modelled with this method are convex with respect to the routes of the aircraft, which is obtained by means of the Voronoi diagram. The better value of the standard deviation from the average workload shows a more uniform distribution of intersections and air traffic by sectors.
Therefore, the obtained minimum value of the optimality criterion shows that the airspace model of figure 4 is optimal in terms of balanced workload distribution.

4. Analysis of the obtained results
The results of the allocation of airspace to sectors show that as the flight time spent in the sector decreases, the number of sectors increases and the size of sectors $s_j$ decreases. The application of this algorithm has its limitations as it must be taken into account that air traffic controllers need sufficient volume of airspace and sufficient time to service its traffic. Therefore, the size of the sectors cannot be reduced indefinitely. The sectors obtained using the herein presented method are convex geometric figures. The mathematical method using Voronoi diagram satisfies the practical requirements for the sectors.

The results of solving the presented optimization problem show that the set goal for determining sectors with uniform workload has been solved successfully. The presented indicators for the types of workload of the air traffic controllers adequately reflect the real practical problems in the definition of the sectors. The repeated reduction of the criterion for optimality and the better value of the standard deviation from the mean workload shows an even distribution of the workload by sectors. Therefore, the obtained minimum value of the optimality criterion shows that the airspace model of figure 4 is optimal in terms of balanced workload distribution.

5. Conclusion
This study proposes a method for determination of sector boundaries through mathematical modelling of a transport network with direct routes. The method is relevant and applicable to free route airspace. The analysis of the results of the conducted experiments shows that with the used mathematical optimization algorithms an optimal sector modelling is obtained according to the balanced distribution of the workload of the air traffic controllers. The applied optimization methods allow for even distribution of the total workload by reducing the monitoring and coordination workload in each sector. The minimization of the value of the optimality criterion 8.5 times compared to the initial value is obtained by optimal distribution of the intersections and aircraft flows. This is achieved by using the random search method to change the location of the generated points on the centres of the Voronoi diagram.

The presented method allows for repeated resolution of the problem of sector modeling in real time depending on the changes in traffic.

The presented algorithm for sector modelling can be improved by adding weights to the indicators of the types of workload in the general criterion of optimality. The results obtained are encouraging and other numerical optimization methods can be tested, such as correlation and regression data analysis, genetic methods, and others.

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