resonant absorption in complicated plasma configurations: 
applications to multistranded coronal loop oscillations

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Abstract

We study the excitation and damping of transverse oscillations in a multistranded model of a straight line-tied coronal loop. The transverse geometry of our equilibrium configuration is quite irregular and more realistic than the usual cylindrical loop model. By numerically solving the time-dependent ideal magnetohydrodynamic equations in two dimensions, we show how the global motion of the whole bundle of strands, excited by an external disturbance, is converted into localized Alfvénic motions due to the process of resonant absorption. This process produces the attenuation of the transverse oscillations. At any location in the structure, two dominant frequencies are found: the frequency of the global mode or quasi-mode, and the local Alfvén frequency. We find that the mechanism of mode conversion, due to the coupling between fast and Alfvén waves, is not compromised by the complicated geometry of the model. We also show that it is possible to have energy conversion not only at the external edge of the composite loop, but also inside the structure. The implications of these results and their relationship with the observations are discussed.

Subject headings: MHD — Sun: corona — Sun: magnetic fields — waves
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1. Introduction

The resonant coupling of fast magnetohydrodynamic (MHD) waves and Alfvén waves has been an active research topic for many years. The most important applications are to laboratory plasmas, the solar corona, and pulsations in the Earth’s magnetosphere. In the present paper we concentrate on the application of resonant absorption as a possible candidate to explain the damping of transverse coronal loop oscillations (see Hollweg & Yang 1988; Goossens et al. 1992; Ruderman & Roberts 2002; Terradas et al. 2006a). The attenuation of the loop oscillations, interpreted as the kink modes, is due to the energy conversion from the global large-scale motions to localized Alfvénic motions.

The time-dependent resonant absorption mechanism has been traditionally analyzed for both driven and initial value problems using different equilibrium models. For the driven problem, this mechanism has been studied in smooth transition layers (Hollweg 1987; Davila 1987; Hollweg & Yang 1988), in slab models (Steinolfson & Davila 1993; Ofman et al. 1994; Ofman & Davila 1995; Ofman et al. 1995; De Groof & Goossens 2000, 2002), and in cylindrical tubes (Grossmann & Smith 1988; Poedts et al. 1989, 1990; Poedts & Kerner 1991; Sakurai et al. 1991; Goossens et al. 1995). For the initial value problem, similar studies have also focused on smooth interfaces (Ionson 1978; Lee & Roberts 1986), slabs (Steinolfson & Davila 1993), and cylinders (Ruderman & Roberts 2002; Terradas et al. 2006a). The effects of background flows have been investigated by Hollweg et al. (1990), Erdélyi & Goossens (1996), Tirry et al. (1998), Andries et al. (2000), and Andries & Goossens (2001).

The time-dependent results have been complemented by eigenmode calculations (see, for example, Tirry & Goossens 1996). The coupling of the fast kink mode with the Alfvén continuum produces the damping of the oscillations. In this case the global mode is usually called the quasi-mode and has a complex frequency. The theoretical models that have been used to calculate the quasi-modes are so far simple, but they are necessary in order to understand the main properties of resonant absorption. Recently, new effects have been investigated. The curvature effect has been studied by Van Doorsselaere et al. (2004b) and Terradas et al. (2006b), and the influence of stratification along the loop has been analyzed by Andries et al. (2005) and by Arregui et al. (2005). The effect of noncircularity of the tube cross section has been analyzed by Ruderman (2003), whereas the effect of the internal structure in loops, using a slab model, has been studied by Arregui et al. (2007).

The overall conclusion of these recent investigations is that these new ingredients do not significantly change the damping per period of the oscillations. This suggests that resonant absorption is a robust mechanism and that its efficiency is not easily affected by the considered effects.

Nevertheless, even the most recent models are still too simple compared with the real conditions in coronal loops. For example, there are no physical reasons to think that the cross section of coronal loops is perfectly cylindrical. Moreover, there is observational information that suggests that loops are not monolithic (as they are usually modeled), but that they are formed by bundles of individual strands considered as miniloops for which the heating and plasma properties are approximately uniform in the transverse direction (Schmelz et al. 2005). This view is supported by some authors (Martens et al. 2002; Klimchuk 2006), but not by others (Aschwanden 2005; Aschwanden & Nightingale 2005). Thus, one of the questions that arises from these observations is how the internal structure of coronal loops can modify the mechanism of resonant absorption.

The purpose of this paper is to study the mechanism of resonant absorption and its efficiency in a rather arbitrary two-dimensional
distribution of plasma and to analyze the implications on the damping of transversal coronal loop oscillations in a multistranded loop model. Instead of calculating the eigenmodes of the structure, which would be difficult because of the complicated geometry, we investigate the dynamical response of the bundle of individual strands to an initial perturbation. Thus, we study the initial value problem of the excitation of the composite loop by solving the time-dependent two-dimensional ideal MHD equations. The equations are solved numerically, using an appropriate code. From the simulations, we study the motions in the loop structure and analyze how the energy from the global mode is converted into localized Alfvénic motions in the inhomogeneous regions of the loop.

This paper is organized as follows. In § 2 the multistranded loop model is described and the basic MHD equations are given. The numerical method used to solve the time-dependent equations is explained in § 3. In § 4 the results of the time-dependent problem for a single cylindrical inhomogeneous loop are compared with the eigenmode calculations. In § 5 the time-dependent problem for the multistranded loop is considered and several features of the resonant absorption mechanism are discussed, in particular the behavior in the inhomogeneous layer, and the energetics of the problem is investigated. Finally, in § 6 the main conclusions are drawn.

2. LOOP MODEL AND GOVERNING MHD EQUATIONS

The equilibrium magnetic field is straight, uniform, and pointing in the $z$-direction: $\mathbf{B} = B_0\hat{z}$. For applications to the solar corona, it is a good approximation to consider that the magnetic pressure is dominant over the gas pressure. This zero-$\beta$ approximation allows us to choose an arbitrary density profile. As a model of a bundle of loops, we use a superposition of $N$ tiny parallel cylinders with different radii and densities. In Cartesian coordinates, the cross section of the density of each individual strand is assumed to have the following form:

$$\rho_i(x, y) = \rho_0 \exp \left[ -\frac{(x-x_i)^2 + (y-y_i)^2}{a_i^2} \right],$$

where $\rho_0$ is the maximum density of the strand, $x_i$ and $y_i$ give the position of the strand axis, and $a_i$ is the strand radius. The density of the multistranded model is defined as

$$\rho(x, y) = \sum_{i=1}^{N} \rho_i(x, y) + \rho_{ex},$$

where $\rho_{ex}$ is the density of the external medium (which is assumed to be constant). Note that the density, $\rho_{in}$, of a single uniform cylindrical loop that has a radius $R$ and the same mass as the multistranded loop is simply

$$\rho_{in} = \frac{1}{R^2} \sum_{i=1}^{N} \rho_0 a_i^2 + \rho_{ex}.$$  

In Figure 1 the two-dimensional distribution of the density (the cross section of the composite loop) is plotted for a particular configuration based on equations (1) and (2). For this particular configuration the loop is composed of 10 individual strands with axes located at the following $x_i$ and $y_i$ coordinates: $(0, 0, 0.65, -0.65, -0.5, 0.5, 0.5, -0.5, -0.2, 0.2)$ and $(0.45, -0.65, 0, 0, -0.5, -0.5, 0.5, 0, 0.2)$. The radii of the strands ($a_i/R$) are $(0.2, 0.275, 0.25, 0.3, 0.2, 0.25, 0.2, 0.25, 0.25, 0.25)$, and the densities $(\rho_i/\rho_0)$ are $(0.8, 0.6, 0.4, 0.3, 0.5, 0.6, 0.4, 0.6, 0.4, 0.3)$. The external density is $\rho_{ex} = 1/3 \rho_{in}$, where $\rho_{in}$ is a reference density inside the loop.

We see that the loop density has an inhomogeneous distribution with quite an irregular cross section and an irregular boundary. This model has a complex geometry compared with the usual cylindrical or elliptical tubes that are usually used to study loop oscillations. In this so-called spaghetti model (see Bogdan & Fox 1991; Keppens et al. 1994), the distance between the strands is quite small, and a strong dynamical interaction between them is expected. This kind of model has been previously considered in the context of the scattering and absorption of sound waves in composite sunspot models.

To study small-amplitude perturbations in this equilibrium, we use the linearized ideal MHD equations. The equilibrium depends on $x$ and $y$, but it is independent of the longitudinal $z$-coordinate. For this reason, we perform a Fourier analysis in this direction, assuming a dependence of the form $e^{-ik_z z}$, where $k_z$ is the vertical wavenumber. We concentrate on the fundamental mode, with $k_z = \pi/L$, where $L$ is the length of the loop (here we use $L = 20R$). Under these assumptions, the MHD equations are

$$\rho \frac{\partial v_x}{\partial t} + \frac{B_0}{\mu} \left( ik_x b_x + \frac{\partial b_z}{\partial x} \right) = 0,$$

$$\rho \frac{\partial v_y}{\partial t} + \frac{B_0}{\mu} \left( ik_y b_y + \frac{\partial b_z}{\partial y} \right) = 0,$$

$$\frac{\partial b_z}{\partial t} + B_0 ik_x v_x = 0,$$

$$\frac{\partial b_z}{\partial t} + B_0 ik_y v_y = 0,$$

$$\frac{\partial b_z}{\partial t} + \frac{\partial b_z}{\partial x} + \frac{\partial b_z}{\partial y} = 0.$$
where \( \mathbf{r} = (r_x, r_y, 0) \) is the velocity and \( \mathbf{b} = (b_x, b_y, b_z) \) is the perturbed magnetic field. These equations can be rewritten as a system of five equations with five real variables using the transformations \( b_x \rightarrow ib_y \) and \( b_y \rightarrow -ib_x \) (these variables are shifted by \( \pi/2 \) in the \( z \)-direction with respect to \( r_x, r_y, \) and \( b_z \)). The equations are complemented with an initial perturbation located in the external medium. For simplicity, we assume that it has the following form:

\[
v_y(x, y) = v_{y0} \exp \left[ -\frac{(y - y_0)^2}{w^2} \right],
\]

where \( v_{y0} \) is the amplitude of the perturbation (this parameter is arbitrary, since we are in the linear regime), \( y_0 \) is the position of the center of the disturbance, and \( w \) is its width (here we use \( y_0 = 3R \) and \( w = R \), which means that the perturbation is located at a distance of \( 3R \) from the center of the loop). All other MHD variables are initially set to zero. This perturbation is a planar pulse that produces the excitation of fast MHD waves that propagate and interact with the loop structure.

In the excitation of fast MHD waves and especially Alfvén waves, an important quantity in our model is the local Alfvén frequency,

\[
\omega_A(x, y) = k_z v_A(x, y) = k_z \frac{B_0}{\sqrt{\mu \rho_0(x, y)}},
\]

where \( v_A \) is the Alfvén speed. The Alfvén frequency distribution in our equilibrium depends on \( x \) and \( y \) in a quite complicated manner, as we can see in Figure 1 (see the contours of constant \( \omega_A \), represented with solid lines). The fact that \( \omega_A \) varies with position plays a key role in the resonant conversion of wave energy from global large-scale motions to localized Alfvén modes.

### 3. Method

From the numerical point of view, the problem that we are studying has basically two difficulties. The first one is related to the small spatial scales that are generated in the inhomogeneous layers, so we must make sure that we are properly resolving these layers. This point is crucial in order for us to have the correct energy conversion rates and consequently the right damping times for the transversal oscillations. Due to the phase mixing process (see Heyvaerts & Priest 1983), which is especially important at the inhomogeneous layers, the maximum time for which the simulations can be run is determined by the phase mixing length, which is defined as the length over which the phase of neighboring Alfvén waves differs by \( 2\pi \) (Mann et al. 1995; Wright & Rickard 1995):

\[
L_{ph} = \frac{2\pi}{t(d\omega_A/dx)}.
\]

Therefore, the typical spatial lengths decrease quickly with time. The shortest wavelength that can be resolved numerically with a uniform grid spacing of \( \Delta x \) is \( \lambda = 2\Delta x \). We can run the simulations until the phase mixing length is equal to the shortest wavelength (\( L_{ph} = \lambda \)). Thus, the maximum simulation time is simply

\[
t_{\text{max}} = \frac{\pi}{\Delta x (d\omega_A/dx)}.
\]

We have checked that this condition is not violated in the simulations (note that it also has to be satisfied in the \( y \)-coordinate).

The second difficulty is the effect of the boundaries on the loop dynamics. Even if we apply transparent boundary conditions (zero-order extrapolation) at the limits of the computational domain, it is important for the boundaries to be located far from the loop. Note that a trapped mode in the loop has always an evanescent part in the external medium, which might be affected by the conditions that we apply at the boundaries.

To solve these difficulties, we have used a resolution that is sufficiently high to resolve the different scales (a grid of \( 4000 \times 4000 \) points is sufficient), and to avoid significant reflections, we have located the domain limits far from the loop, at \( x_B = \pm 8R \) and \( y_B = \pm 8R \). Nevertheless, in order to better interpret and visualize the results, the plots are displayed in a smaller spatial domain \((-1.6R, 1.6R) \times (-1.6R, 1.6R)\).

To numerically solve the time-dependent MHD equations, equations (4)–(8), together with the initial condition, equation (9), we use the code CLAWPACK (LeVeque 2002). This code implements a wide class of high-resolution finite volume methods for solving linear or nonlinear hyperbolic problems. Due to resolution requirements, we have run the parallelized version of the code on a cluster of computers.

### 4. Test Case: Resonant Absorption in an Inhomogeneous Cylindrical Loop

Due to the different scales involved in the problem, the modeling of resonant absorption is challenging from the numerical point of view. For this reason, before the “spaghetti model” is studied, it is necessary to check that the numerical method we are using is appropriate to this problem. As a test, we consider a single inhomogeneous cylindrical loop in two dimensions. To facilitate comparison of the time-dependent results with those from previous eigenmode studies, we choose the same density profile as in Ruderman & Roberts (2002), Van Doorsselaere et al. (2004a), and Terradas et al. (2006a); i.e., a sinusoidal variation in density across the nonuniform layer. For such a configuration and for the \( m = 1 \) mode (the kink mode), the damping per period (the damping time, \( \tau_d \), over the period, \( P \)) in the limit of a thin tube and a thin boundary is

\[
\frac{\tau_d}{P} = \frac{2 R}{\pi} \frac{\rho_m + \rho_ex}{\rho_m - \rho_ex},
\]

where \( l \) is the thickness of the nonuniform layer and \( R \) is the loop radius. For thick layers, the eigenvalue problem has to be solved numerically (see Van Doorsselaere et al. 2004a; Terradas et al. 2006a). In our composed loop model there is a wide range of thicknesses, and for this reason we have calculated the eigenmodes of the cylindrical loop numerically (see Terradas et al. [2006a] for details about the method used to perform these calculations). In Figure 2 (top), the damping per period for a single cylindrical loop calculated from equation (13) and the eigenmode calculations are represented as a function of the thickness of the layer. The small differences between the two curves are simply due to the fact that equation (13) is inaccurate for thick layers (see also Van Doorsselaere et al. 2004a).

To compare the results of these eigenmode calculations with the time-dependent problem, we need to place an initial perturbation in the system that excites the kink mode (the eigenmode calculations that we want to compare with are for this mode, \( m = 1 \)). The planar pulse perturbation given by equation (9) is the kind of disturbance that has such a property, because an external perturbation hardly excites high-order harmonics such as the fluting modes, and the sausage modes are leaky in our configuration (see Terradas et al. 2007a). We have run the code with
this perturbation and have studied the subsequent evolution. Figure 2 (bottom) shows the plot of $v_y$ at the center of the loop as a function of time for a particular thickness of the layer (see Fig. 4 of Terradas et al. [2006a] for the analogous simulation in one dimension). After a short transitory phase, the loop oscillates with the quasi-mode period, and the amplitude of the mode is attenuated due to the energy conversion in the inhomogeneous layer. From the results of the simulations we calculate the period, $P$, by performing a periodogram. The damping time, $\tau_d$, is numerically estimated by fitting an exponential function of the form $\exp(-t/\tau_d)$ to the envelope of the curve. We have performed several simulations, varying the width of the layer, and the numerically obtained damping rates are represented with circles in Figure 2 (top) as a function of the thickness of the layer. We find the expected dependence with $l/R$, and we see that the deviations from the eigenmode calculations are quite small. It is important to remark that, although we model a cylindrical tube, in these simulations we use Cartesian coordinates, and we still obtain the right damping rates.

In conclusion, with this simple numerical experiment we demonstrate that we are using a reliable tool with which to study resonant absorption. Note that, contrary to some preceding works, in which a large amount of resistivity is used mainly to avoid numerical problems, here we are in ideal MHD conditions. Hence, the system dynamics is not dominated by the resistive regime; in that case the behavior in Figure 2 (top) would be completely different, because the damping time would be independent of $l/R$ (see, for example, Fig. 2 in Terradas et al. 2006a).

5. RESULTS: MULTISTRANDED MODEL

We now use the multistranded model shown in Figure 1 and study how the system evolves due to a planar pulse perturbation (eq. [9]). The initial pulse produces a displacement of the whole ensemble of strands, mainly in the $y$-direction; i.e., the direction in which the initial perturbation propagates. The initial stage of the evolution of the bundle of loops is dominated by a complicated set of internal reflections of the wave front between the different strands. During this transitory phase, several wave fronts propagating from the bundles into the external medium are found. They correspond to the emission of the leaky modes, which represent fast-radiating MHD waves with short periods and fast attenuation (see Cally [1986, 2003] and Terradas et al. [2007b] for an analysis of such modes in a single cylindrical loop). After the transitory phase, the system periodically oscillates with certain frequencies. Hereafter, we concentrate on this stage of the dynamics.
5.1. Frequency Analysis

We first investigate the characteristic frequencies of the system. An analysis of the results of the simulations shows that inside the loop, there are essentially two dominant frequencies at each point. One is the collective frequency of the bundle of loops (different from the kink frequency of the individual strands), and the other is the local Alfvén frequency, $\omega_A$. The collective frequency is the result of the excitation of the global mode of the system, which represents the emergent behavior of the entire loop. On the other hand, the local Alfvén frequency is due to the excitation of the global mode of the system, which represents the emergent behavior of the entire loop. On the other hand, the local Alfvén frequency is due to the excitation of the Alfvén continuum modes. As an example, in Figure 3 the $v_y$ velocity component as a function of time (top) and its power spectrum (bottom) are represented at an interior point (see label 1 in Fig. 1). The two dominant frequencies, which are the local Alfvén frequency with the largest power, are clearly identified in the power spectrum. Note the strong decrease of the amplitude with time. This attenuation is found almost everywhere at the interior points and suggests a collective motion of the bundle of tubes. We have also represented in Figure 3 (top) the other component of the velocity, $v_x$. Its amplitude is smaller than that of the $v_y$-component, and it does not show attenuation with time.

There are particular locations in the structure at which the behavior of the signal is completely different. An example is shown in Figure 4, which shows the data that correspond to a position situated near the loop edge (see label 2 in Fig. 1). Now, instead of a damped oscillation, the amplitude of the $v_y$ velocity component grows with time while the $v_x$-component decreases. There is just one peak in the power spectrum because the local and the global frequencies coincide. As we will show in the following sections, this is the place where the energy conversion takes place.

Outside the structure, oscillations tending to the Alfvén frequency are found everywhere; see, for example, the velocity represented in Figure 5 (see label 3 in Fig. 1). These oscillations are basically produced by the wake of the initial fast MHD perturbation and have a large amplitude compared with the amplitude inside the loop (see Fig. 3). This wake oscillates (in the limit of large times) at the local Alfvén frequency (see Terradas et al. 2005), which is precisely where the dominant peak is found in the power spectrum. However, it is worth noting that near the loop edge, but in the external medium, the quasi-mode frequency is also present. This is not surprising, since the quasi-mode has an evanescent tail in the external medium.

We have also performed the same analysis for the $b_z$ component, which is proportional to the total pressure perturbation ($P_T = b_z B_0/v\mu$). We have found a large amount of power at the global mode frequency and very weak power around the local Alfvén frequency. This is because pure Alfvén modes do not perturb the total pressure and, although in our configuration Alfvén modes are coupled with fast modes, they still keep a strong incompressible character.

![Fig. 4.— Top: Time evolution of the $v_x$ (dashed line) and $v_y$ (solid line) velocity components at a position near the loop edge: $x = 0.9R, y = 0$ (see label 2 in Fig. 1). Bottom: Power spectrum of the $v_y$ velocity component. The local Alfvén frequency is represented with a dot-dashed line.](image1)

![Fig. 5.— Top: Time evolution of the $v_x$ (dashed line) and $v_y$ (solid line) velocity components at a position near the loop edge: $x = 1.4R, y = 0$ (see label 3 in Fig. 1). Bottom: Power spectrum of the $v_y$ velocity component. The local Alfvén frequency (i.e., for this particular point, the external Alfvén frequency) is represented with a dot-dashed line.](image2)
Fig. 6.—Spatial distribution of the $v_x$ velocity component (top) and the $v_y$ velocity component (bottom) at $t = 40\tau_A$. The corresponding velocity field is plotted in the top panel of Fig. 7. Contours of constant Alfven frequency are represented with solid lines.

5.2. Velocity Field

To explain the different behavior of the velocity at different locations in the structure, we need to understand the evolution of the whole system instead of looking at individual points. In Figure 6 the velocity components $v_x$ and $v_y$ are plotted at $t = 40\tau_A$. Inside the loop, the $v_x$-component is quite uniformly distributed, while the $v_y$-component has a complicated spatial structure that basically coincides with the spatial distribution of the strands (see the contours). At this particular instant, the higher the density of the strand, the smaller the amplitude of the $v_y$-component. On the other hand, the velocity components outside the loop have a very smooth spatial distribution. The motion of the whole structure is clearer if we represent the velocity field constructed using the $v_x$- and $v_y$-components. The result is shown in the top panel of Figure 7. We clearly see that all the strands are moving in the negative $y$-direction and that the external medium reacts in a very organized way to the displacement of the bundle of loops. The plasma at the bottom of the loop ($y \approx -R$) is pushed sideways, whereas the plasma at the top of the structure ($y \approx 0.8R$) tends to fill the region that has been displaced downward. This motion is the equivalent of the kink mode in a cylindrical tube.

At later times (see the middle and bottom panels in Fig. 7), large-amplitude velocities develop, especially near the external edge of the composite loop. This is the consequence of the energy conversion between the global mode and the Alfven modes. Due to this process, the global oscillation gradually loses its energy, and its amplitude is attenuated in time (see, for example, the length of the arrows at the centers of the individual strands in Fig. 7 at $t = 160\tau_A$). This explains the attenuation of $v_y$ that can be seen in Figure 3. On the other hand, the amplitude of the Alfven modes increases at the resonant layers where the energy conversion takes place (see the large arrows at the loop boundaries in Fig. 7, bottom). We already found this behavior in Figure 4.

Since the Alfven frequency changes with position, the Alfvenic motions get out of phase very quickly due to phase mixing. The consequence of this process is visible at the external boundaries where strong shear motions develop. To see these motions in detail, we have concentrated on the small region marked with a square box in Figure 7 (bottom). The result is plotted in Figure 8. We see that the field is organized and that near the edge of the loop, the velocity vectors are aligned with the contours of constant Alfven frequency (solid lines). It is also clear that on neighboring magnetic surfaces, the motions are essentially in opposite directions. This is even clearer in Figure 9, where we have plotted a cut of $v_y$ at $y = 0$ (solid line). This figure also shows how this component of the velocity field (which is mainly parallel to the magnetic surfaces around $x = R$) evolves with time. As is expected from the phase mixing process, the typical spatial scales of the shear motions decrease with time. The amplitude of the velocity decreases in the internal part of the structure, whereas it develops short wavelengths around the resonant positions. We find that the typical wavelengths agree with the phase mixing lengths, $L_{ph}$, predicted by equation (11) (compare the lengths of the lines in the legend of the plot with the wavelengths of $v_y$ at $x = R$).

At this point it is worth noting that, due to mode coupling, Alfven modes are excited everywhere in the structure, even at locations far from the resonance. The excitation of these modes is already known, especially in driven problems (see, for example, Mann et al. 1995; Tirry et al. 1997; Goossens & De Groof 2001), but there are still several questions about these modes that need to be addressed, such as, for example, the amount of energy that is deposited in the Alfven modes. In any case, far from the resonance, the amplitude of these modes is smaller than the amplitude of the global mode (on average). We see in Figure 3 (bottom) that for this internal position, the amplitude of the global oscillation in the $v_y$-component is around 1.5 times the amplitude of the Alfven modes (the power of the peaks, which is proportional to the square of the amplitude, is around 24 and 10). A more detailed analysis, which is beyond of the scope of this work, is required.
5.3. Energy Distribution

The process of mode conversion is even clearer if we study the energetics of the system. For this reason we now focus on the wave energy density. Since in our model we adopt the zero-$\beta$ approximation, there are only contributions from the kinetic and magnetic energy to the total wave energy:

$$E = \frac{1}{2} \left[ \rho_0 (v_x^2 + v_y^2) + \frac{1}{\mu} (b_x^2 + b_y^2 + b_z^2) \right].$$

This quantity is represented in Figure 10 at different times (same as in Fig. 7). The plots show that the system evolves from a situation in which the energy is more or less uniformly distributed to a state in which it is concentrated around the resonant layers. The wave energy is spatially distributed in such a way that it follows the irregular geometry of the loop edge (see the bright areas on the left and right loop boundaries). Out of the resonances the
energy decreases with time—see the interior points—except for the area around $x = 0.2R, y = -0.2R$, where we can find some energy for large times. It should be also noted that the system evolves such that the resonance energy width decreases with time. This behavior of the wave energy at the resonance has been described in detail by Mann et al. (1995); see also the equivalent results in a cylindrical inhomogeneous loop: Fig. 8 of Terradas et al. 2006a). To see these effects more clearly, we have concentrated on a slice at $y = 0$ (see Fig. 11) and have plotted the wave energy density at different times. Figure 11 shows that the energy width decreases with time at the external boundaries, basically at $x = R$ and $x = -R$. We also see a peak around $x = 0.3R$, and the energy decreases with time in the ranges $-0.8 < x/R < 0.1$ and $0.5 < x/R < 0.7$.

5.4. Resonant Magnetic Surfaces

It is quite evident from the velocity and energy analysis where the energy conversion takes place. However, we can be more precise in the determination of the resonant magnetic surface or surfaces. Once we know the frequency of the global mode, the location of the resonances is essentially where this frequency coincides with the local Alfvén frequency. In Figure 1, contours of the Alfvén frequency, $\omega_A$, are represented together with the global frequency (see the thick contours). In this plot we find that there are two resonant magnetic surfaces. One surface is located at the external edge of the composite loop, and the other resonant magnetic surface is located inside the structure (see the small hole around $x = 0.2R, y = -0.2R$). It is precisely at these locations where the energy maps show enhanced energy (see, for example, Fig. 10, bottom) and is also where the highest velocity amplitudes are found (see Fig. 7). Thus, for this particular multistranded configuration, resonant absorption not only takes place at the external boundaries, but some energy is also deposited in the internal part of the composite structure.

5.5. Damping Time

Due to the simultaneous excitation of the quasi-mode and the local Alfvén modes, the determination of the damping rates of the loop oscillation can be quite difficult (see, for example, the time series in Fig. 3). However, there is a simple way to estimate the damping time. We have taken uniformly distributed regions inside the loop and have added up the velocity field at a given
Fig. 12.—Time evolution of the spatially averaged velocity component as a function of time. The solid, dot-dashed, and dotted lines correspond to the (normalized) averages in the square regions marked with labels a, b, and c in Fig. 10 (bottom), respectively. The profile of the three signals is almost the same, since the loop is oscillating with the global mode. The damping per period is $\tau_d/P = 2.3$.

6. DISCUSSION AND CONCLUSIONS

We have studied the mechanism of resonant absorption by solving the time-dependent problem of the excitation of oscillations in a complicated multistranded coronal loop. We have shown that the mode conversion takes place in quite irregular geometries such as the one studied in this paper, and that regular magnetic surfaces (considered in previous works) are not necessary for this mechanism to work efficiently. This suggests that resonant absorption is a very robust damping mechanism. Although we have analyzed a particular system, the behavior found in the present equilibrium is expected to be quite generic of inhomogeneous plasma configurations. Since inhomogeneity is certainly present in coronal loops, the resonant coupling between fast and Alfvén modes can hardly be avoided in a real situation. For this reason, resonant absorption seems to be a natural damping mechanism.

The fact that the loop is not monolithic does not much affect the global oscillatory properties. Although the loop is composed of different strands, we find a dominant frequency almost everywhere in the structure, which indicates a global motion. It is interesting to compare the frequency of this global mode or quasi-mode with the frequency of the equivalent homogeneous loop with radius $R$ and the same mass. Using equation (3), we find that the equivalent cylindrical loop should have an internal density of $\rho_{in} = 0.67\rho_{00}$. The kink mode period for a cylindrical tube with this internal density is $P = 28.4\tau_A$. This value is in good agreement with the period of the quasi-mode estimated from the numerical simulations of the multistranded model, which is around $P = 28.0\tau_A$. This indicates that the internal structure does not change the global behavior of the loop much (see also Arregui et al. 2007). Nevertheless, it may have some effect on the location of the energy deposition. We have found that in our model there is also energy conversion inside the loop, although it is small compared to the energy at resonances in the external layers.

Although both the quasi-mode and the local Alfvén modes are excited at any time and at any position, we have been able to estimate the damping time of the quasi-mode by performing averages of the dominant velocity component in different regions of the loop. The average eliminates the local Alfvén modes and enhances the global mode and is essentially what the observations provide: a sort of integration along the line of sight (but in the displacement instead of the velocity).

On the other hand, it must be noted that a system of $N$ tubes, such as the one studied in this paper, is expected to have a large number of eigenmodes. For example, in a configuration with just two loops, Luna et al. (2008) have shown that there are four kink-like eigenmodes and that an external disturbance in general excites these four modes. However, these authors have shown that since their frequencies are very similar, it is very difficult to distinguish between the different eigenmodes in a time-dependent study. For this reason, the interpretation of the global mode found in our multistranded loop as a sum of different eigenmodes with similar frequencies and similar damping times needs to be considered. This could explain the small differences in the period and damping time of the velocity averages in different regions of the structure. A detailed analysis of this issue is out of the scope of this paper, but it is clear that the calculation of the eigenmodes of complicated configurations is very important. In this regard, analytical studies based on scattering theory will allow us to calculate the eigenmodes of a system of $N$ tubes (M. Luna et al. 2008, in preparation) and to make progress in this direction.

Finally, it must be noted that we have concentrated on the linear regime. If the amplitude of the oscillations becomes large in the inhomogeneous layer due to mode conversion, nonlinear terms might be important and the efficiency of resonant absorption could be altered. However, diffusion or viscosity may prevent the development of large amplitudes around the resonant layer. In addition, there are some results that indicate that in the nonlinear regime, the heating at the resonant layers may produce significant changes in the equilibrium configuration (see Ofman et al. 1998; Klimchuk 2006). Hence, the initial value problem needs to be studied using the full nonlinear equations, which basically means that, since the Fourier analysis would no longer be possible in the $z$-direction, the problem becomes three-dimensional. Due to the grid resolution requirements in the layers, the three-dimensional problem has a high computational cost. The numerical study presented here for the two-dimensional problem is a preliminary step toward investigating resonant absorption in more realistic three-dimensional models, including, for example, twisted or tangled magnetic fields.

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