On the chemical equilibration of strangeness-exchange reactions in heavy-ion collisions

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Abstract

The strangeness-exchange reaction $\pi + Y \leftrightarrow K^- + N$ is shown to be the dynamical origin of chemical saturation and equilibration for $K^-$ production in heavy-ion collisions up to beam energies of $10 \text{ A·GeV}$. The hyperons occurring in this process are produced associatively with $K^+$ in baryon-baryon and meson-baryon interactions. This connection is demonstrated by the ratio of $K^-/K^+$ which does not vary with centrality and shows a linear correlation with the yield of pions per participant. At incident energies above AGS this correlation no longer holds due to the change in the production mechanism of kaons.

A fairly complete set of experimental results on $K^-$ production in heavy-ion collisions for beam energies from $1.5 \text{ A·GeV}$ up to about $10 \text{ A·GeV}$ has now become available from SIS [1–5] and from AGS [6–8]. These results have attracted a lot of interest as the yield of $K^-$ compared to the yield of $K^+$ is significantly higher in heavy-ion collisions than in elementary $N-N$ collisions [1–3]. Transport model calculations [9] lead to the interpretation that this effect may be caused by an attractive $K^-N$ potential. Alternatively, thermal-statistical models using bare particle masses were also shown to be very successful in describing particle yields (including the $K^-$ one) down to low incident energies around $1.5 \text{ A·GeV}$ [10–13].
A basic difference between $K^-$ production in heavy ion and in $N-N$ collisions can be attributed to strangeness-exchange reactions between secondaries [14] which are absent in $N-N$ collisions. In heavy-ion collisions more channels are available for anti–kaon production, namely strangeness-exchange processes like \( \pi + Y \rightarrow K^- + N \). The hyperons \( Y \) (i.e. \( \Lambda \) and \( \Sigma \)) are produced together with a \( K^+ \) since this is energetically the most favorable way to produce strange hadrons at low energies. These processes are usually included in transport models [9,15,17] which show the dominance of the strangeness-exchange channel. Yet, different conclusions have been drawn with respect to the influence of the \( K^-N \) potential. Some authors claim that one needs to reduce the \( K^- \) mass in the dense medium due to the attractive \( K^-N \) potential [9], while others [17] argue that the influence of the \( K^-N \) potential is negligible as the antikaons are produced at the end of the interaction chain where the density is low.

In this Letter we compare experimental results on \( K^- \) production in the energy range from 1.5 \( A \)-GeV up to RHIC energies with a specific assumption about the production mechanism of \( K^- \). We discuss in detail the question whether the experimental results support the idea of chemical equilibration of \( K^- \) in the strangeness-exchange channel. For this purpose we study the \( K^-/K^+ \) ratio as function of centrality and incident energy.

The experimental data on kaon production in heavy-ion collisions show that in a very broad energy range from SIS up to RHIC the \( K^-/K^+ \) ratio is independent of the collision centrality. This is illustrated in Fig. 1 for SIS, AGS, SPS and RHIC energies. Although at high incident energies this result could be expected since \( K^- \) is dominantly produced together with \( K^+ \), it is rather surprising to be also valid at SIS. Here, \( K^+ \) production is close to threshold (the threshold in \( N-N \) collisions is 1.58 GeV for \( K^+ \) being produced together with \( \Lambda \)), while \( K^- \) production is far below threshold as \( K^+K^- \) pair production requires 2.5 GeV in the laboratory frame. In central heavy ion collisions much higher densities are reached than in peripheral processes. As the number of multiple collisions increases with density, one would expect the centrality dependence of these two particles to be very different. This expectation is in clear contrast to the data as shown in Fig. 1.

In thermal-statistical models, the fact that the \( K^-/K^+ \) ratio is independent of centrality is natural. In the grand-canonical description of strangeness conservation, valid in high-energy heavy-ion collisions, particle densities are independent of the volume, thus also of the centrality or the number of participating nucleons \( A_{part} \). In a canonical description, as required at SIS energies, strange particle densities are strongly changing with the number of participants [10–12]. However, this dependence enters both in the \( K^+ \) and in the \( K^- \) production
Fig. 1. The $K^-/K^+$ ratio as a function of the number of participating nucleons $A_{\text{part}}$ obtained in heavy-ion collisions at SIS, AGS, SPS and RHIC incident energies [7,5,19,20]. The dotted lines represent the predictions of a statistical model[10–12].

and cancels in the ratio. A non–linear increase of $K^+$ and $K^-$ yields as a function of the centrality has indeed been found experimentally [5] and can also be seen in the upper two panels of Fig. 2.

The observation of a $K^-/K^+$ ratio being independent of the centrality (see Fig. 1), could already be considered as an indication of chemical equilibration of the strangeness production in low-energy heavy-ion collisions. However, similar results are also obtained in dynamical transport models as a consequence of the interplay between in–medium effects and the nuclear absorption of kaons [9,15]. Thus, to establish chemical equilibration of kaons requires further discussion which we present below.

At low incident energies the strangeness-exchange reaction

$$\pi + Y \rightleftharpoons K^- + N.$$  

plays a key role in $K^-$ production. This process has a low threshold, approximately 180 MeV, and a large cross section [14].

If the rates for $K^-$ production are equal to those for $K^-$ absorption, the reaction in Eq. (1) is in chemical equilibrium. In this case the law-of-mass
action is applicable and leads to the following relation between particle yields

\[ \frac{[\pi] \cdot [Y]}{[K^-] \cdot [N]} = \kappa \]  

(2)

with \([x]\) being the multiplicity of particle \(x\) and \(\kappa\) the equilibration constant [16].

The above relation between kaon, hyperon and pion multiplicities required by detailed balance is a straightforward direct test of chemical equilibration in the final state. Before the above relation can be compared to measured yields, one needs to take into account that the pion density \([\pi]\) in Eq. (2) contains unequal contributions from \([\pi^+]\), \([\pi^0]\) and \([\pi^-]\) due to the isospin asymmetry of the colliding system. All available strangeness-exchange reactions (Eq. 1) can be subdivided into two groups: two channels involving \(\Lambda\) scattering of pions and five channels involving \(\Sigma\)'s. Listed below are the production channels with their corresponding isospin weight.
\[\begin{align*}
\pi^0 + \Lambda^0 &\Rightarrow K^- + p, \bar{K}^0 + n \quad 0.5 \\
\pi^- + \Lambda^0 &\Rightarrow K^- + n \quad 1.0 \\
\pi^- + \Sigma^+ &\Rightarrow K^- + p, \bar{K}^0 + p \quad 0.5 \\
\pi^- + \Sigma^0 &\Rightarrow K^- + n \quad 1.0 \\
\pi^0 + \Sigma^0 &\Rightarrow K^- + p, \bar{K}^0 + n \quad 0.5 \\
\pi^+ + \Sigma^- &\Rightarrow K^- + p, \bar{K}^0 + n \quad 0.5 \\
\pi^0 + \Sigma^- &\Rightarrow K^- + n \quad 1.0
\end{align*}\]

The relevant \(\pi\) multiplicities \(M^*(\pi)\) are obtained as follows: the contribution via the \(\Lambda\) channel is \(M^\Lambda(\pi) = 1/3[\pi^0] + 2/3[\pi^-]\). Correspondingly, the channels via \(\Sigma\) yield \(M^\Sigma(\pi) = 1/7[\pi^+] + 3/7[\pi^0] + 3/7[\pi^-]\). The multiplicity of \([\pi^0]\) is taken as \(0.5 \cdot ([\pi^+] + [\pi^-])\). The hyperons are produced together with \(K^+\) and \(K^0\) in equal rates, hence, \([Y] = [K^+] + [K^0] \approx 2 \cdot [K^+]\). The separation of the hyperons into \(\Lambda\) and \(\Sigma\) baryons is not possible experimentally. However, the relative yields of these particles can be estimated roughly using thermal occupancies. Neglecting the isospin asymmetry and within the limit of \(m_Y/T \gg 1\) which is valid at low incident energies, one gets

\[\frac{N_\Sigma}{N_\Lambda} \simeq \frac{g_\Sigma}{g_\Lambda} \left( \frac{m_\Sigma}{m_\Lambda} \right)^{3/2} \exp \left( -\frac{m_\Sigma - m_\Lambda}{T} \right).\]  

(3)

The mass difference \(m_\Sigma - m_\Lambda \approx 75\) MeV is approximately equal to the freeze-out temperature at beam energies around \((1 - 2)\) AGeV \([10,13]\). This together with the ratio of the spin–isospin degeneracy factors \(g_\Sigma/g_\Lambda = 3\) implies that \(N_\Lambda \approx N_\Sigma\) is a reasonable approximation. Hence, the probability to form a \(K^-\) with a \(\Lambda\) or with a \(\Sigma\) is the same. The multiplicity of pions in Eq. (2), due to the various isospin combinations, is then obtained as

\[M^*(\pi) = \frac{31}{42}[\pi^-] + \frac{11}{42}[\pi^+].\]  

(4)

The multiplicity of nucleons \([N]\) in Eq. (2) can be related to the average number of participants \(A_{\text{part}}\). Thus the law of mass action can be formulated directly through experimentally accessible observables leading to

\[\frac{2 \cdot [K^+]}{[K^-]} \cdot \frac{M^*(\pi)}{A_{\text{part}}} = \kappa.\]  

(5)

This relation also holds for reactions involving baryon exchange as in \(\Delta N \leftrightarrow KNN\) as \(\Delta\) corresponds to \(\pi N\).
In the two upper panels of Fig. 2 the dependencies of the $K^+$ and the $K^-$ multiplicities on $A_{\text{part}}$ are shown for Ni+Ni and Au+Au collisions at 1.5 AGeV [5]. The lines are functions $M \propto A_{\text{part}}^\alpha$ fitted to the data (solid lines for Au+Au, dashed lines for Ni+Ni) yielding similar values for $\alpha$ between 1.3 and 1.4 for both systems and both particle types. Consequently, the $K^-/K^+$ ratio is centrality independent, as seen in Fig. 1. The third panel of figure 2 shows that also the pion multiplicity per $A_{\text{part}}$ is approximately constant with centrality.

The ratio $\kappa$ from Eq. (5) is shown in the lowest part of Fig. 1. It has been calculated separately for Au+Au and Ni+Ni from the fitted functions in the panels above. The experimental data obtained in Ni+Ni and Au+Au collisions at 1.5 AGeV confirm the validity of the law of mass action as $\kappa$ is independent of centrality within the experimental uncertainties. In addition, the value of the $K^-/K^+$ ratio is in very good agreement with the statistical model [12,11] as shown in Fig. 1. These results provide evidence that at SIS energies the $K^-$ yields follow the conditions for chemical equilibrium in the final state.

Recently, new experimental results for particle yields in central Au+Au collisions at AGS energies have become available [6–8] and allow for further tests of chemical equilibration of $K^-$ via the law of mass action. Figure 3 shows the $K^-/K^+$ ratios and the pion multiplicities both increasing with incident energy. Nevertheless, the equilibration constant $\kappa$, shown in the lower part of Fig. 3, appears to depend rather weakly on the incident energy indicating that the $K^-$ are produced via strangeness exchange in chemical equilibrium. Consequently, in the energy range between SIS and AGS one expects from Eq. (5) that there should be a linear correlation between the $K^-/K^+$ and $M^*(\pi)/A_{\text{part}}$ ratios.

As the results presented above strongly point to chemical equilibration in the strangeness exchange channel and the $K^-/K^+$ ratio is constant as a function of the centrality, one might be tempted to interpret the constancy also at high incident energies by equilibration in the strangeness-exchange channel. This is, however, not the case. Figure 4 shows the relation between the $K^-/K^+$ ratio and the pion multiplicity. As expected from the discussion above, up to top AGS energies a linear relation between these two quantities holds. At SPS and RHIC the $K^-/K^+$ ratio approaches unity while the pion multiplicity keeps increasing. At these high incident energies, the kaon production is dominated by $K^-K^+$ pair production.

Figure 4 shows the relation between the $K^-/K^+$ ratio and the pion yield per participant in AuAu/PbPb collisions obtained in the energy range from SIS up to RHIC. As seen in Fig. 4 a linear relation between these two quantities indeed holds up to AGS energies. This is a direct evidence for chemical equi-
liberation of $K^-$ production via the strangeness exchange reaction. For higher collision energies at SPS and RHIC the strangeness exchange processes are not any more the leading reactions for the $K^-$ production. At these incident energies, the kaon yield is dominated by direct $K^-K^+$ pair production and the linear relation between the $K^-/K^+$ and the pion/participant ratios no longer holds. At SPS and RHIC the $K^-/K^+$ ratio approaches unity while the pion multiplicity keeps increasing. The statistical model results obtained along a unified freeze-out curve [22] are seen in Fig. 4 to be consistent with data in the whole energy range. The change in slope seen in Fig. 4 appears for incident energies between 20–40 $A$-GeV where the composition of the collision fireball changes from baryonic to mesonic [23].

In summary, we have shown that in low energy heavy-ion collisions at SIS and AGS energies the data confirm the chemical equilibration of $K^-$ production. In this energy range the dominant process for $K^-$ production is due to the strangeness exchange channels $\pi + Y \leftrightarrow K^- + N$. These channels link the $K^-$ production to the yield of the $K^+$ production in $NN \rightarrow K^+\Lambda N$ reactions. If
Fig. 4. The relation between the $K^-/K^+$ ratio and the pion multiplicity $M^*(\pi)$ per participant $A_{part}$. Data are for central Au+Au/Pb+Pb collisions at SIS, AGS, SPS and RHIC energies [5,7,8,20,21]. The line is the statistical model result obtained along a unified freezeout curve of fixed energy/particle $\simeq 1$ GeV [22,23].

Strangeness exchange appears in chemical equilibrium then according to the law of mass action the $K^-/K^+$ ratio scales with the number of pions per participant. This scaling could be observed in two different ways:

- the $K^-/K^+$ ratio should be independent of centrality since $<\pi>/A_{part} = const.$ and
- with increasing incident energy an increase of the $K^-/K^+$ ratio should be proportional to $<\pi>/A_{part}$.

Both of these chemical equilibrium conditions are well consistent with the data up to AGS energies. At SPS and RHIC energies the condition (ii) is violated since the dominance of the strangeness-exchange reactions is replaced by direct $K^+K^-$ pair production which, however still preserves the centrality independence of the $K^-/K^+$ ratio.

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