The notion of measurements is central for many debates in quantum mechanics. One critical point is whether a measurement can be regarded as an absolute event, giving the same result for any observer in an irreversible manner. Using ideas from the gedankenexperiment of Wigner’s friend it has been argued that, when combined with the assumptions of locality and no-superdeterminism, regarding a measurement as an absolute event is incompatible with the universal validity of quantum mechanics. We consider a weaker assumption: is the measurement event realised relatively to the observer when he only partially observed the outcome. We proposed a protocol to show that this assumption putting in conjunction with the natural assumptions of no-superdeterminism and locality is also not compatible with the universal validity of quantum mechanics.

I. INTRODUCTION

At the center of many orthodox interpretations of quantum mechanics is the assumption that the action of measuring a quantity on a physical system creates its actual value [1–3]. This prompts to suggest that the action of creating a value of a measured quantity is an ‘absolute event’ of this world, meaning that it is same for any observer and a process that cannot be reversed [4–7]. This perception of measurements in quantum mechanics has been highly debated. While it is supported by collapse models [8, 9] of the measurement process, it is not in line with other viewpoints which assume the universal validity of quantum mechanics [10–17]. Indeed, assuming universality of quantum mechanics suggests to model the measurement process by a unitary dynamics, which in principle can always be reversed [16]. As a result, the measurement may be undone and the value of the measured quantity can be erased, as if it never existed.

The subtleties of these different views are often discussed using a gedankenexperiment known as Wigner’s friend [18, 19]. In this scenario, the physicist Wigner has a friend in a box and this friend performs a measurement on a quantum system. For Wigner, the box undergoes a unitary evolution, but at a later point, Wigner may open the box and read out what his friend has measured. This, however, leads to a disagreement on when the absolute measurement event has taken place – when the friend measured or when Wigner opened the box? Variations of this scenario have sparked interesting discussions recently [4, 5, 20–24], where sometimes even the consistency of quantum mechanics was questioned.

Still, the question whether the event of a measurement can be reversed or happens in an absolute sense, is differently answered by different physicists. Consequently, this question should be answered with the help of experiments, and not only theoretical considerations. Recently, Bong et al. [6] provided a significant step in this direction. They analyzed a so-called local friendliness (LF) scenario, where the assumption of absolute measurement events (AOE) is combined with other natural assumptions. In their protocol, a measurement is carried out by Charlie, who plays the role of Wigner’s friend in a closed laboratory, carries out a measurement on his particle and output the outcome $b$. Alice, playing the role of Wigner, receives a signal $x \in \{0,1,2\}$. She asks Charlie if his outcome is $x$ or not. If it is $x$, Alice uses it as her output $a$. Otherwise, Alice continues to carry out a binary outcome measurement on the system and the whole laboratory to obtain an outcome $a \in \{0,1,2\}$ but $a \neq x$. Bob receives a signal $y \in \{0,1\}$ and performs correspondingly one of two possible binary outcome measurements on his particle and output the outcome $b$. 
Quantum mechanics is universally valid [5, 6]. A proof-of-principle experiment has been also carried out showing the disfavour of the assumption of absoluteness of a measurement event [6].

One may argue that assuming the absoluteness of a measurement with respect to every observer is a too strong assumption [25]. This is particularly clear under the viewpoint of universally valid quantum mechanics, in which the measurement can eventually be reversed, erasing any trace of existence of an outcome. This is the case notwithstanding Deutsch’s proposal [19] that Wigner can receive a message of the form ‘I see a definite outcome’ from his friend without destroying the superposition of his friend. Indeed, if universal quantum mechanics is assumed, the writing action of Wigner’s friend is also a unitary process. By the linearity of the unitary evolution, it is easy to show that such a unitary evolution would be uncorrelated with any state of the measurement device, be it in a definite state of a preferred basis or superpositions thereof.

Therefore, we propose to investigate a weaker assumption than AOE: the persistence of the measurement event relative to the observer who has gained partial information about the measurement outcome. That is to say, an observer holding a coarse description of the outcome, is convinced of the existence of its fine description. We can refer to this assumption as ‘relative event by incomplete information (REII)’. We show that this assumption can also be rejected by the universal validity of quantum mechanics under the assumptions of no-superdeterminism and locality. The idea is to allow the measurement by Wigner’s friend to have at least three outcomes, for example $a = 0, 1, 2$; see Fig. 1. In this scenario, it is possible for Wigner to ask his friend whether his measurement has yielded outcome in a certain coarse set, say $\{0, 2\}$, without accessing its fine, exact value. It is interesting to see that, unlike the original Wigner friend scenario discussed in Ref. [4, 6, 19], even when quantum mechanics is universally valid, in general, Wigner is not able to reverse the whole experiment of his friend; the coarse description of the outcome remains valid to him and his friend. REII assumes that this coarse record implies the existence of a fine-grain description of the outcomes, although it could have been forgotten by the friend. That quantum mechanics violates this assumption can be interpreted as: this coarse record is all to be there; a fine-grain description is still ‘unspeakable’. Our results show that not only the existence of measurement events is relative [21], but the nature of the events themselves is defined by how the observer perceives the outcomes.

\footnote{Indeed, the evolution is expected to map $|i\rangle$\lspan{\text{"blank"}}\rangle$ to $|i\rangle$\lspan{\text{\textit{I see a definite outcome}}}\rangle$, where $|i\rangle$ is the state of the measurement device pointing to outcome $i$, and the second factor describes the state of the paper carrying the message from Wigner’s friend. By linearity, this also implies that the unitary evolution maps $|\psi\rangle$\lspan{\text{\textit{I see a definite outcome}}}\rangle$ to $|\psi\rangle$\lspan{\text{\textit{I see a definite outcome}}}\rangle$ for any state $|\psi\rangle$ of the measurement device.}

\section{THE MAIN PROTOCOL}

To illustrate the idea, we consider here a minimal protocol. Subtleties and further elaborations are to be discussed later in Sec. III. Consider Alice and Bob sharing two particles at two different locations. Alice stores her particle in a laboratory, in which she has Charlie playing the role of Wigner’s friend. In each run of the protocol, Charlie performs a measurement with three outcomes $c \in \{0, 1, 2\}$ on the particle. Then Alice receives a signal $x \in \{0, 1, 2\}$. Given the signal $x$, she asks Charlie if his measurement outcome is $c = x$. If this is the case, Alice outputs $a = x$, else she makes a binary outcome measurement on the particle and use the outcome to decide the output among $\{0, 1, 2\}$ with the constraint that $a \neq x$. On the other hand, Bob receives a signal $y \in \{0, 1\}$ in each run of the protocol, based on which he chooses one of two measurements to perform on his particle to get a binary outcome $b \in \{0, 1\}$. Over many runs of the protocol, the collected data allows one to compute the statistics $p(a, b|x, y)$. In the following, we then derive explicit inequalities based on the assumption of the realisation of measurement event to the observer who partly gains the information about the outcomes (Alice in this case) combined with other natural assumptions. Still, the inequalities are violated in quantum mechanics, if its universal validity holds.

\subsection{Relative event by incomplete information (REII)}

Because Alice has obtained part of the information about the outcomes of Charlie’s measurement in each of the runs, REII directly implies that the value of $c$ is realized in every run of the protocol. So, for every given $x$ and $y$ in each of the runs, Alice can assume a joint distribution $P(a, b, c|x, y)$ such that the observed data $p(a, b|x, y)$ is given by marginalizing the irrelevant outcome $c$,

$$p(a, b|x, y) = \sum_c P(a, b, c|x, y).$$ (1)

Here and in the following we use the lower case letter $p$ to indicate that $p(a, b|x, y)$ is experimentally accessible, rather than being hypothetical like $P(a, b, c|x, y)$ with a capital $P$. Since if $c = x$ then $a = x$, we have $P(a = x|c, x, y) = \delta_{xc}$. This means that the existence of $c$ can be consistently revealed when it is read. Noticing $P(a = x, b|c, x, y) = P(b|a = x, c, x, y)P(a = x|c, x, y)$, this then implies that

$$P(a = x, b|c, x, y) = \delta_{xc} P(b|a = x, c, x, y) = \delta_{xc} P(b|c, x, y).$$ (2)

\footnote{Notice that, if we are to apply our assumption of relative event by incomplete information to the scenario in Ref. [6], the existence of $c$ is not implied when Alice does not inquire Charlie at all.}
It is unknown how to reject this thesis purely by its own. However, when combined with two other seemingly natural assumptions: freedom of choice (or no-superdeterminism) and locality, it has a stringent constraint on the observable statistics $p(a, b|x, y)$.

**B. Freedom of choice and locality**

The assumption of the freedom of choice demands that the random inputs $x$ and $y$, are uncorrelated with any relevant variable in the experiment [26]. In this context, this means that $x$ and $y$ are independent from the variable $c$, $P(c|x, y) = P(c)$. One therefore can write $P(a, b, c|x, y) = P(a, b|c, x, y)P(c)$, and hence

$$p(a, b|x, y) = \sum_c P(a, b|c, x, y)P(c). \quad (3)$$

The freedom of choice is justified if Charlie makes the measurement before Alice and Bob make the choices $x$ and $y$. This relied on the honesty of Charlie and the functionality of his device [6]. Interestingly, part of this problem can also be addressed within our protocol; see Sec. III. The locality assumption implies that Bob's measurement result $b$ does not depend on Alice's input $x$ and there is an analogue independence between $a$ and $y$. Then

$$P(a, c, x, y) = P(a|c, x) \quad \text{and} \quad P(b|c, x, y) = P(b|c, y). \quad (4)$$

It should be emphasized that this notion of signal locality is weaker than the so-called local causality [26] and the probabilities $P(a|b, c, x, y)$ and $P(b|a, c, x, y)$ in general cannot be further simplified.

We have deliberately used the locality notion from Ref. [6], resembling that of the localfriendliness. Had we used the stronger assumption such as local causality, Bell-like correlations were to be obtained [4–6]. We emphasize that, in either case the assumption of REH is used instead of that of AOE.

**C. Combining the assumptions**

The combination of these assumptions may, in accordance with the terminology developed in Ref. [6], be called local friendliness under incomplete information (LFIC). Using this combination, we find that the probability distribution $p(a, b|x, y)$ must take the form

$$p(a, b|x, y) = \sum_c P_{NS}(a, b|c, x, y)P(c), \quad (5)$$

$$p(a = x, b|x, y) = \sum_c \delta_{xc}P(b|c)P(c), \quad (6)$$

where $P_{NS}(a, b|c, x, y)$ is a probability distribution constrained only by the assumption of locality and freedom of choice. On the one hand, Eq. (5) can be understood as a direct consequence of Eqs. (3, 4). On the other hand, Eq. (6) requires the consistency of the measurement outcomes when they are read in Eqs. (2, 5).

It is interesting to see that Eq. (5) and Eq. (6) are a combination of the no-signaling model [27, 28] and the local hidden variable model [29]. To mimic the scenario described in Ref. [6], we can adjust the protocol to reproduce the original LF model as follows: to respond to Alice's query, her friend just outputs the outcome of his measurement, which is also Alice’s final output in the next step of the protocol. Then Eqs. (5, 6) reduce to $p(a, b|x, y) = \sum_c \delta_{xc}P(b|c)P(c)$ for all $x$, which is the original LF model. In this case, the LF model is also a local hidden variable model since there is effectively only a single measurement on Alice's side. However, the LFIC correlation polytope is strictly larger than LF polytope as mentioned in Footnote 2. In fact, here one can choose a cross-section in correlation space, such that both the LHV and LF correlations are empty, while the LFIC correlations are not.

**D. The correlation polytope**

The correlations that are constrained by Eqs. (5, 6) form a polytope, to which there are 60 facets [30]. Among those, many coincide with the facets of the no-signaling polytope. The remaining 32 facets can be grouped into 4 inequivalent classes by considering the symmetry between measurements and outcomes. Among these facets, two classes only involve at most one measurement of Bob. Therefore, this makes no difference between the no-signaling model and the local hidden variable model, thus they are omitted here. The other two representative inequalities of relevant classes are

$$Z_1 := p(A_0 = 0, B_0 = 0) + p(A_1 = 1, B_1 = 0) - p(A_2 = 1, B_0 = 0) + p(A_2 = 1, B_1 = 1) \geq 0, \quad (7)$$

$$Z_2 := p(A_0 = 1, B_0 = 0) + p(A_0 = 2, B_1 = 1) + p(A_1 = 1, B_0 = 1) - p(A_1 = 1, B_1 = 1) \geq 0. \quad (8)$$

Note that the inequality (7) includes 3 measurements on Alice’s side, but inequality (8) includes only 2 measurements on Alice’s side. The inequality (7) resembles the well-known Clauser-Horne-Shimony-Holt inequality, if the event $A_2 = 2$ is never realized, that is, whenever the query is $A_2$, Charlie always reply with a negative answer. A similar discussion applies for inequality (8) if the event $A_0 = 0$ is never realized.

In order to visualize the polytope, we compute a two-dimensional cross-section of the polytope with a plane defined by three points (see Fig. 2 and for details Appendix A). To have a comparison, we have presented the cross-section with the polytope of the statistics which is constrained only by the locality condition, which is referred to as the non-signalling polytope. As expected, this polytope contains the polytope of the LFIC statistics. Interestingly, the LF polytope, which coincides with
Alice performs an arbitrary measurement and outputs the outcome. This is because the events $A_0 \neq 0$ and $A_1 \neq 1$ have not appeared in Eq. (7), hence the choice of those two measurements does not affect the violation. If $x = 2$, she makes a unitary evolution to disentangle Charlie and his device from the qutrit-qubit system, bringing the latter back to $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ and effectively undoing Charlie’s measurement. She then performs the measurement of $\sigma_x$ on the qutrit in the subspace spanned by $|0\rangle$ and $|1\rangle$. Bob is making one of the two measurements $[-\sigma_x - \sigma_z]/\sqrt{2}$ and $[-\sigma_x + \sigma_z]/\sqrt{2}$. This setup in fact allows for a maximal violation of inequality (7) by $Z_1 = (1 - \sqrt{2})/2 \approx 0.2071$, corresponding to the point $Q_1$ in Fig. 2. This violation is robust with respect to mixing white noise to the state via $\rho(p) = p|\psi\rangle\langle\psi| + (1 - p)\mathbb{I}/6$ as long as $p \geq \frac{1}{2}\left(3\sqrt{2} + 1\right) \approx 0.616781$. A similar setup can be designed to attain the distribution $Q_2$ in Fig. 2, maximally violating inequality (8). More details are provided in Appendix A.

III. SUBTLETIES AND AN ALTERNATIVE PROTOCOL

In our main protocol, we have assumed that Charlie makes the measurement first and then Alice asks for the outcome. In principle, this assumption does not hold if Charlie is not honest or the device does not function properly. For example, Charlie can wait for the inquiry from Alice and then implement the measurement, or Charlie can answer without referring to the exact measurement outcome. In principle, Alice can always open the box to check whether Charlie’s answer is consistent with the outcome of the measurement device or not. Thus, we can simply assume that Charlie’s answer is consistent with the outcome of the measurement device. This however cannot guarantee the no-superdeterminism assumption if the outcome of the measurement (which Charlie supposedly implemented before) can be impacted by Alice’s inquiry. Here we propose another protocol to fix this loophole. That is, to make sure that Charlie’s answer reflects the outcome of the measurement, which does not depend on Alice’s input used in the statistics.

For each run,

- Charlie makes a measurement and obtains one of four outcomes $c \in \{0, 1, 2, 3\}$;
- Alice receives a random number $t \in \{0, 1, 2, 3\}$ and inquires Charlie whether $c$ equals $t$;
- Alice receives a random number $x \in \{0, 1, 2, 3\}$ and inquires Charlie whether $c$ equals $x$;
  - If $c = x$, Alice outputs $a = x$;
  - If $c \neq x$, Alice continues to make a measurement and obtains an outcome $a \in \{0, 1, 2, 3\}\setminus\{x\}$.
• Bob receives the input $y \in \{0,1\}$, make a measurement and obtains an outcome in $b \in \{0,1\}$;

The statistics after many runs is collected to estimate $p(a,b|x,y)$.

According to this protocol, although $c$ could depend on $t$, it is independent of $x$ under the assumptions of REII and no-superseding. In the special case that $c \neq t$ and $x \neq t$ for any fixed $t$, the LFIC model of the current protocol just reduces to our main protocol. As we have discussed already, this model cannot describe the statistics predicted by quantum mechanics.

IV. DISCUSSION AND CONCLUSION

The violation of the inequalities (7) and (8) by quantum theory is due to the fact that the qutrit at Alice’s side still maintains entanglement with Bob’s qubit after that Charlie responds negatively to the question “Is $x = 2$”. This points towards a relevant discussion about the nature of the measurement process. Originally, according to von Neumann [32], a measurement of a degenerate observable leads to full decoherence in the entire eigenbasis of the observable, such that the post-measurement state is diagonal in this basis. It was then recognised by Lüders [33] that this is not the appropriate formulation. If a degenerate measurement is made, then according to the Lüders’ rule the coherence in the degenerate subspaces is unaffected by that. In fact, the validity of Lüders’ rule has recently been observed experimentally [34].

Our analysis of the assumption of relative event by incomplete information (REII) also highlights the difference between the viewpoints of von Neumann and Lüders. If we treat Alice’s query about $i$ and Charlie’s measurement as a single measurement $\tilde{c}_i$ implemented and read by Alice, it constitutes a dichotomic degenerate measurement on the qutrit. The post-measurement state following the Lüders’ rule still has quantum coherence in a 2-dimensional subspace, and thus entanglement with a remote party can still remain. In comparison, we see that von Neumann’s rule is similar to the assumption of REII.

The violation of our inequalities can also be seen as related to an interesting remark by Peres in connection to the concept of contextuality [35–37]. There, in order to justify the contextuality assumption, he argued that it is essential that for a three-outcome measurement, Alice can construct a device to measure whether the system gives some outcome, e.g., 2, and then, at a later stage or even in a different lab, decide how to complete the measurement.

While the main protocol presented in Sec. II was of a particularly simple form, we stress that our findings also hold for related scenarios that are more sophisticated; see the analysis in Sec. III. Another interesting extension would be to consider this quantum correlation sets with incomplete information beyond bipartite cases. The investigation of a multipartite all-versus-nothing-like proof such as the one discussed in Ref. [5] under incomplete information would also be very interesting.

Finally, asking for an experimental test would require the manipulation of an entire lab. This brings us back to Bell’s famous question [38]: What exactly qualifies some physical systems to play the role of ‘measurer’? Does Charlie’s lab need to contain a physicist with a PhD? Still, proof-of-principle demonstrations, in the spirit of Ref. [6] would be highly desirable.

Acknowledgments.—The authors would like to thank Eric G. Cavalcanti and Howard Wiseman for critical comments on the early version of this work. Discussions with Fabian Bernards, Matthias Kleinmann were fruitful. We acknowledge Adán Cabello for support of the discussions, and the University of Siegen for enabling our computations through the OMNI cluster. This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation, project numbers 447948357 and 440958198), the Sino-German Center for Research Promotion (Project M-0294), the ERC (Consolidator Grant 683107/TempoQ) and the German Ministry of Education and Research (Project QuKuK, BMBF Grant No. 16KIS1618K). Z.P.X. acknowledges support from the Humboldt foundation. J.S. acknowledges support from the House of Young Talents of the University of Siegen. H.C.N. acknowledges the Unitary fund for supporting his attendance at the Wigner’s friend workshop, and the participants for exciting discussions.

Appendix A: Details of the main protocols

1. Details of the polytopes of correlations

The set of correlations $p(a,b|x,y)$ defined by Eqs. (5) and (6) in the main text forms a polytope. The polytope can be fully characterized by its facets, which are described by inequalities. The inequalities are derived from the definition Eqs. (5) and (6) using a linear programming solver [39]. There are 60 such inequalities to the polytope, among which 32 do not coincide with the ones from the no-signalling (NS) polytope. These 32 inequalities can classified into 4 inequivalent classes, which are not related by the permutation of measurements and outcomes. The representatives
of these 4 inequivalent classes are
\[ p(A_0 = 0, B_0 = 0) + p(A_1 = 1, B_1 = 0) - p(A_2 = 1, B_0 = 0) + p(A_2 = 1, B_1 = 1) \geq 0, \] 
(A1)
\[ p(A_0 = 1, B_0 = 0) + p(A_0 = 2, B_1 = 1) + p(A_1 = 1, B_0 = 1) - p(A_1 = 1, B_1 = 1) \geq 0, \] 
(A2)
\[ p(A_0 = 0, B_0 = 1) + p(A_1 = 1, B_0 = 1) - p(A_2 = 1, B_0 = 1) \geq 0, \] 
(A3)
\[ p(A_0 = 1, B_0 = 1) + p(A_0 = 2, B_0 = 1) - p(A_1 = 1, B_0 = 1) \geq 0. \] 
(A4)

Additionally, the polytope is also constrained by three inequivalent hyperplanes which are not from the NS polytope
\[ -p(A_0 \neq 0) + p(A_1 = 1) + p(A_2 = 2) = 0, \] 
(A5)
\[ -p(A_0 \neq 0, B_1 = 1) + p(A_1 = 1, B_0 = 1) + p(A_2 = 2, B_0 = 1) = 0, \] 
(A6)
\[ -p(A_0 \neq 0, B_1 = 1) + p(A_1 = 1, B_1 = 1) + p(A_2 = 2, B_1 = 1) = 0. \] 
(A7)

It turns out that, the inequalities in from Eq. (A3) to Eq. (A7) also hold if quantum mechanics is assumed. Hence, only the inequalities in Eq. (A1) and Eq. (A2) are nontrivial.

2. Details of the cross-section and quantum violation

We use three points \( N_0, Q_1, Q_2 \) to determine the plane for the cross-section in Fig. 2. The point \( N_0 \) is obtained in the NS model, the points \( Q_1, Q_2 \) can be achieved in quantum theory. In fact, \( Q_1 \) is one optimal solution for inequality (7) in the main text, \( Q_2 \) is an optimal solution for inequality (8) in the main text in quantum theory. The exact values of those three points are collected in Table I.

| \( N_0 \) | 1 | \( A_0 = 1 \) | \( A_0 = 2 \) | \( A_1 = 1 \) | \( A_1 = 2 \) | \( A_2 = 1 \) | \( A_2 = 2 \) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1/2 | 1/2 | 1/2 | 0 | 1/2 | 1/2 |
| \( B_0 \) | 1 | 0 | 1/2 | 0 | 0 | 0 | 1/2 |
| \( B_1 \) | 1 | 0 | 1/2 | 0 | 0 | 1/2 | 0 |

| \( Q_1 \) | 1 | \( A_0 = 1 \) | \( A_0 = 2 \) | \( A_1 = 1 \) | \( A_1 = 2 \) | \( A_2 = 1 \) | \( A_2 = 2 \) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1/2 | 0 | 1/2 | 0 | 1/2 | 0 |
| \( B_0 \) | 1 | \( \alpha \) | 0 | \( \alpha \) | 0 | \( \alpha \) | 0 |
| \( B_1 \) | 1 | \( \beta \) | 0 | \( \beta \) | 0 | \( \alpha \) | 0 |

| \( Q_2 \) | 1 | \( A_0 = 1 \) | \( A_0 = 2 \) | \( A_1 = 1 \) | \( A_1 = 2 \) | \( A_2 = 1 \) | \( A_2 = 2 \) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1/2 | 1/2 | 1/2 | 0 | 0 | 1/2 |
| \( B_0 \) | 1 | \( \beta \) | \( \alpha \) | \( \alpha \) | 0 | 0 | \( \alpha \) |
| \( B_1 \) | 1 | \( \beta \) | \( \alpha \) | \( \alpha \) | 0 | 0 | \( \alpha \) |

TABLE I. The probability distributions \( N_0, Q_1, Q_2 \), where \( \alpha = (\sqrt{2} - 1)/4\sqrt{2}, \beta = (\sqrt{2} + 1)/4\sqrt{2} \).

The point \( Q_1 \) can be obtained using the state \( \frac{|00\rangle + |11\rangle}{\sqrt{2}} \) and the measurement directions
\[ |A_0 = 2\rangle = |A_1 = 2\rangle = |A_2 = 2\rangle = |2\rangle, \] 
(A8)
\[ |A_0 = 0\rangle = |A_1 = 0\rangle = |0\rangle, \] 
(A9)
\[ |A_0 = 1\rangle = |A_1 = 1\rangle = |1\rangle, \] 
(A10)
\[ |A_2 = 0\rangle = |+\rangle, |A_2 = 1\rangle = |\rangle, \] 
(A11)
\[ B_0 = \frac{-\sigma_x - \sigma_z}{\sqrt{2}}, B_1 = \frac{-\sigma_x + \sigma_z}{\sqrt{2}}. \] 
(A12)

The point \( Q_2 \) can be obtained using the state \( \frac{|00\rangle + |11\rangle}{\sqrt{2}} \) and the measurement directions
\[ |A_0 = 0\rangle = |A_1 = 2\rangle = |A_2 = 1\rangle = |2\rangle, \] 
(A13)
\[ |A_0 = 1\rangle = |A_2 = 0\rangle = |0\rangle, \] 
(A14)
\[ |A_0 = 2\rangle = |A_2 = 2\rangle = |1\rangle, \] 
(A15)
\[ |A_1 = 0\rangle = |\rangle, |A_1 = 1\rangle = |+\rangle, \] 
(A16)
\[ B_0 = \frac{\sigma_x - \sigma_z}{\sqrt{2}}, B_1 = \frac{-\sigma_x - \sigma_z}{\sqrt{2}}. \] 
(A17)
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