Perturbative QCD description of mean jet and particle multiplicities in $e^+e^-$ annihilation

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Abstract

A complete numerical solution of the evolution equation for parton multiplicities in quark and gluon jets with initial conditions at threshold is presented. Data on both hadron and jet multiplicities in $e^+e^-$ annihilation are well described with a common normalization, giving further support to the picture of Local Parton Hadron Duality. Predictions for LEP-II energies are presented. Furthermore we study the sensitivity to the cutoff parameter $Q_0$ and the scale of $\alpha_s$.

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1. INTRODUCTION

Perturbative QCD calculations give a good description of data on jet observables as far as a scale of few GeV’s is involved in the process. For instance, in $e^+e^-$ annihilation the dependence of the mean jet multiplicity on the resolution parameter $y_c$ defined in the Durham algorithm is well reproduced by QCD calculations in absolute normalizations down to $y_c \sim 10^{-3}$, i.e. to a scale of a few GeV’s, when leading and next-to-leading terms in $\ln y_c$ are resummed and a matching with the full two-loop result is performed. In the softer region with scales of the order of 1 GeV or less, deviations from purely perturbative predictions are visible and this is usually explained in terms of purely non perturbative hadronization effects. However, following the idea of a soft hadronization mechanism, a simple picture of hadronization, known as local parton hadron duality, has been indeed formulated, where the same perturbative description of QCD parton cascade is stretched down to small scales of a few hundred MeV for the transverse momentum cutoff $Q_0$, and then parton predictions are directly compared to hadron distribution up to an overall normalization factor. The interesting feature of this picture is that it is surprisingly successful in describing the phenomenology of hadron production, as far as sufficiently inclusive observables, like particle multiplicity or inclusive energy spectra, are concerned.

Even though both parton multiplicities and jet multiplicities are described with the same type of evolution equation, two different sets of parameters, and in particular, two different normalization factors, were needed so far to describe phenomenologically the experimental results on the two observables. Here we show that both observables can be described in an unified way, if the complete QCD master equation which describes the evolution of a parton cascade is exactly solved. We refer to for further details on this calculation.

2. THEORETICAL FRAMEWORK

The perturbative QCD picture gives a probabilistic description of the production process of a multiparton final state in terms of a parton cascade. Formally, the physical information about this process is entirely contained in the evolution equation for the generating functional of the multiparton final state; in its more general formulation, this equation correctly takes into account angular ordering, energy conservation and the running coupling at the one-loop order; the choice of the transverse momentum as the variable which enters in the coupling incorporate at least part of higher order effects. The infrared cutoff $k_{\perp} > Q_0$ also provides one with a natural regularization of the infrared singularity in the running coupling. By appropriate differentiation of the equation for the generating functional, a
coupled system of two evolution equations for the mean parton multiplicities $N_q$ and $N_g$ in quark and gluon jets can be derived:

$$\frac{dN_q(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(k_{\perp})}{2\pi} \frac{1}{2} \Phi_{gg}(z) \{ N_q(\eta + ln z) + N_q(\eta + ln(1-z)) - N_q(\eta) \}$$

$$\frac{dN_g(\eta)}{d\eta} = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(k_{\perp})}{2\pi} \Phi_{gg}(z) \{ N_g(\eta + ln z) + N_g(\eta + ln(1-z)) - N_g(\eta) \}$$

The evolution of the parton densities is coupled through the heavy quark thresholds.

The argument of $\Phi_{AB}$ is the AP splitting functions for the process $A \to B$, $N_c$ and $n_f$ are the numbers of colours and flavours respectively, $\eta = ln \frac{Q^2}{\sqrt{s}}$, where $Q = Q_c \sin \frac{\pi}{2}$, with $Q$ the hard scale of the process ($\sqrt{s}$ in $e^+e^-$ annihilation) and $\Theta$ the maximum angle between the outgoing partons. $\alpha_s(k_{\perp})$ is the one-loop running coupling, given by $\alpha_s(k_{\perp}) = 2\pi / (b ln(k_{\perp}/\Lambda))$ with $b = (11 N_c - 2 n_f)/3$ and a smooth matching of the running through the heavy quark thresholds.

The argument of $\alpha_s$ is chosen to be the “Durham” transverse momentum

$$\vec{k}_{\perp} = min(z, 1-z) \kappa$$

The boundaries of the integral over the parton momentum fractions $z$ are determined by the lower cutoff $k_{\perp} > Q_c$ and given by $z_c = (Q_c \sqrt{2})/Q = \sqrt{2 y_c} = e^{-\eta}$. Since $z_c \leq \frac{1}{2}$, one finds $y_c \leq \frac{1}{2}$ and then $\eta \geq ln 2$.

The coupled system of evolution equations must of course be supplemented by a suitable set of initial conditions; they are fixed at threshold to be:

$$N_q(\eta) = N_g(\eta) = 1 \quad \text{for} \quad 0 \leq \eta \leq ln 2.$$  

This master equation contains the correct leading and next-to-leading terms in an $\sqrt{\alpha_s}$ expansion, but it has been obtained by using approximations valid in the limit of soft emission (for instance, the definition of $k_{\perp}$ given in (2) comes from the Durham definition of $k_{\perp}$ in the limit of small $z_c$). In other words, non-logarithmic terms have been neglected in the derivation of the master equation, and these terms give a non-negligible contribution in the region of small $\eta$ (i.e., poor resolution (large $y_c$) or low cms energy (small $Q$)). To improve the description in this region, the contribution of $O(\alpha_s)$ in (1) has been replaced by the explicit result for $e^+e^- \to 3$ partons in $O(\alpha_s)$:

$$N_{corr}^{e^+e^-}(y_c) = 2 N_q(y_c) - 2 N_q^{(1)}(y_c) + N^{3\to jet}(y_c).$$

where $N_q^{(1)}$ is obtained by taking the first iteration of (1) with the initial conditions (2) and the full $O(\alpha_s)$ contribution is given by:

$$N^{3\to jet}(y_c) = 2 \int_{1/2}^{1} d z_1 \int_{0}^{z_1} d z_2 \Theta(d_{23} - y_c)$$

$$C_F \alpha_s(k_{\perp}) \frac{z_1^2 + z_2^2}{(1 - z_1)(1 - z_2)}$$

$$d_{23} = min \left( \frac{z_2}{z_3}, \frac{z_3}{z_2} \right) (1 - z_1) > y_c$$

where $z_1$ ($z_2$) denote the quark (antiquark) and $z_3 = 2 - z_1 - z_2$ the gluon momentum fractions ($C_F = 4/3$).

3. PHENOMENOLOGICAL ANALYSIS

3.1. Multiplicities in $e^+e^-$ annihilation

The average jet multiplicity at $Q = 91$ GeV as a function of the resolution parameter $y_c = Q^2/Q^2$, defined via the Durham algorithm, is shown in Figure 1 (lower data points). Also shown in the lower part of the Figure is the theoretical prediction in absolute normalization of the jet multiplicity at the same cms energies of 91 GeV, as obtained from (3). The predictions depend on the single parameter $\Lambda$ only, so we can use the LEP-1 data on jet multiplicity to fix the best value of the QCD scale $\Lambda$. The best value of $\Lambda$ turns out to be $500 \pm 50$ MeV, in qualitative agreement with previous results within the MLLA framework.

The main improvement of our new approach in comparison to the old results is that we can now describe experimental data down to very small values of the resolution parameter $y_c$; therefore,
Figure 1. Data on the average jet multiplicity at $Q = 91$ GeV and the average hadron multiplicity (assuming $N = \frac{3}{4} N_{ch}$) at different cms energies with $Q_c = 0.507$ GeV as a function of $y_c$. The solid lines show the predictions for the hadron multiplicity (upper curve) and for the jet multiplicity at different LEP cms energies (91, 133, 161 and 172 GeV) (lower curves), obtained by using eq. (4) with parameters $K_{all} = 1$, $\Lambda = 0.5$ GeV and $\lambda = 0.015$. The right most data point for hadrons refers to pions only.

after having properly taken into account the effects of energy conservation, the validity of the perturbative picture can be extended well below the scale of 2-3 GeV which was usually claimed to be the soft scale where hadronization effects beyond perturbative QCD take place. In the Figure the predictions on the jet multiplicity at LEP-2 cms energies are also shown. These curves could be compared with new data recently collected at CERN. After having fixed the first parameter $\Lambda$, we can now ask ourselves whether it is possible to describe also the cms energy dependence of hadron multiplicity within the same framework. In this case, we can still choose two free parameters, the hadronic scale related to the $k_\perp$ cutoff $Q_0$ and the normalization parameter $K_{all}$, which relates parton and hadron multiplicities as $N_{all} = K_{all}N_{corr}^{e^+e^-}$. It turns out that a common description is indeed possible, with parameters given by $K_{all} = 1$ and $\lambda = \ln Q_0/\Lambda = 0.015 \pm 0.005$ (i.e., $Q_0 \sim 507$ MeV). Figure 1 (upper data points) shows the data on hadron multiplicities from $\sqrt{s} = 1.6$ (only pions) up to 172 GeV taken as $N_{all} = \frac{3}{4} N_{ch}$ as a function of the resolution parameter $y_c = Q_0^2/Q^2$, together with the perturbative predictions of (4) with the aforementioned parameters. It is clear from the Figure that the perturbative approach can describe quantitatively the hadron multiplicity even at very small cms energies, thus extending again the range of validity of the perturbative picture. We refer to [7] for a more detailed comparison of our new numerical results with previous analytical approximate solutions. Let us stress that a common description of both hadron and jet multiplicities is possible with the common normalization $K_{all} = 1$, which is a natural value, since it provides the correct boundary conditions at threshold for both hadrons and jets.

It is interesting to study the dependence of our
results on the infrared cutoff $Q_0$; one could indeed stop the parton evolution at a larger scale of the order of 1-2 GeV and then fill the gap between partonic predictions and experimental data using purely nonperturbative hadronization models, as implemented in the most used Monte Carlo models. The perturbative predictions for the parton multiplicity with different values of the infrared cutoff $Q_0$ are shown in Fig. 2. Notice that in our picture most of the particles ($\sim 3/4$) are produced in the very last stage of the evolution for $0.5 Q_0 \leq 1$ GeV, where Monte Carlo models already stopped the perturbative phase. A different value of $Q_0$ also changes the energy dependence of the predicted parton multiplicity. It is indeed remarkable that the right energy dependence of particle multiplicity can be reproduced with a small value of the infrared cutoff $Q_0$ and a normalization factor $K_{all} = 1$. A residual correlation among the two parameters exists, so a variation of one of the two parameters within 30% can be accommodated by adjusting the other one.

It is important to stress that the evolution variable $\eta = \ln \frac{Q}{Q_c}$ in (1) is given by the ratio of the two dimensional scales which enter in the problem, i.e., the external scale $Q$ and the resolution scale $Q_c$. Therefore, the same evolution equation controls both the cms energy dependence of the cluster multiplicity, with clusters defined at a given resolution (where $Q$ varies and $Q_c$ is fixed), and the resolution dependence of the jet multiplicity at a fixed hard scale $Q$ (where $Q$ is fixed and $Q_c$ varies). There is indeed only one place where the two scales enter separately and not through their ratio, namely the expression of the running coupling $\alpha_s$, where only $Q$ (and $\Lambda$) but not $Q_c$ appears. Therefore, if one would switch off the running of the coupling, the equation would depend only on the dimensionless variable $\eta$ and there would be no difference at all be-
between jet and hadron multiplicities. It is then clear that the difference between the two multiplicities entirely comes from the running of the coupling. Notice that this difference goes up to a factor 10 in the region of large \( y_c \) (i.e., small \( cm\) energy for hadron multiplicity), and this factor is within this picture completely related to the increased coupling at small transverse momentum. This result further confirms previous results on energy spectra \[8,11\]. The effect of the running of the coupling is visible not only in the difference between the two curves, in particular at large values of \( y_c \), but it is also directly visible in the strong rise of the jet multiplicity in the high resolution region (low \( y_c \)) (see Fig. 2). Indeed, the jet multiplicity increases roughly by a factor 3 if one lowers the resolution cutoff from 0.9 down to 0.5 GeV, as expected by the quick growing of the running coupling close to the Landau singularity (which is however screened by the \( Q_c \) cutoff). Such results are qualitatively consistent with expectations from the Double Log Approximation \[8,11\].

3.2. Ratio of hadron multiplicities in quark and gluon jets

The numerical solution of the coupled system of evolution equations (1) provides one with new results on the mean multiplicity \( N_g \) in gluon jets, and then also for the ratio of hadron multiplicities in quark and gluon jets, \( r = N_g / N_q \). This observable has received a lot of attention both theoretically and experimentally. Theoretically, different predictions were derived for the mean parton multiplicity in a full hemisphere of the gluon jet produced by a primary \( gg \) colour singlet state. The asymptotic value of \( r = 9/4 \) has been successively reduced to smaller values, by including next-to-leading order and even part of higher order correction (see \[6,7\] for a discussion of this point). Experimentally, a completely inclusive configuration similar to the theoretical predictions has recently been realized in the decay \( Y \rightarrow gg\gamma \) by the CLEO Collaboration \[8\] and in \( e^+e^- \rightarrow 3 \) jets with nearly parallel \( q \) and \( \bar{q} \) recoiling against the gluon by the OPAL Collaboration \[9\].

By numerically solving the evolution equation
(1) with initial condition \( r \) (without any finite order correction term yet), the prediction for the ratio \( r \) is further reduced with respect to the previous approximate calculations and can describe the OPAL data for jets of 40 GeV very well. One fails, however, to reproduce the CLEO data for jets of about 5 GeV. This failure could be due to an important contribution from the fixed order correction term, not yet included so far.

### 3.3. Another definition of transverse momentum

All results we have shown so far have been obtained by using the Durham-inspired definition of transverse momentum given in (3). With this method, a value of the parameter \( \lambda \) consistent with the previous finding from the energy moments \( \Delta \) has been found; however, the scale \( \Lambda \) (or, equivalently, \( Q_0 \)) has turned out to be larger than the value of \( \approx 250 \text{ MeV} \) used in the description of energy spectra \( \Delta \). In the latter case, the alternative definition of \( k_\perp = z(1-z)\kappa \) has been used. To check whether this difference plays an important role, an alternative calculation with this different definition of the transverse momentum has been performed. As shown in Fig. 3, a good description of experimental data is indeed possible also in this case, but with a lowered scale \( \Lambda \sim 0.35 \text{ GeV} \) and the same \( \lambda \) parameter. The remaining difference to the earlier \( \Lambda \) comes from having taken the new scale \( \kappa = \sqrt{2}E \) instead of \( \kappa = E \) in the new calculation. Therefore, the new results presented here are completely consistent with previous studies of the energy spectra.

### 4. CONCLUSIONS

The complete QCD evolution equations for quark and gluon jets have been numerically solved. By including also the full \( O(\alpha_s) \) correction for \( e^+e^- \) annihilation, a common description of the resolution dependence of jet multiplicity at LEP-1 and of the \( cms \) energy dependence of hadron multiplicity in the whole \( cms \) energy range has been achieved with two free parameters only and an overall normalization factor consistent with 1. The ratio of hadron multiplicity in gluon and quark jets at LEP-1 is described as well. For such inclusive observables the perturbative description turned out to be valid also in extreme kinematical domains, thus extending the region of validity of the perturbative approach supplemented by the Local Parton Hadron Duality picture. Further tests of this picture and, in particular, of its validity for soft particles and for less inclusive observables are certainly needed; some of them are discussed in \( \Delta \).

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