Propagation of laser beams through curved interfaces of transparent media

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Abstract. Mathematical models of propagation of Gaussian, collimated and structured laser beams in transparent optically homogeneous media in the presence of curved interfaces are presented. Experimental techniques which are used for reconstruction of the interfaces profiles are described.

1. Introduction

The problems of studying the shape of a surface of interphase boundaries and the problems of propagation of optical beams through the interfaces with a given profile were considered in works [1–4]. Determining the shape of the interface between two media is an important task in many applications where it is necessary to take into account the refraction of light passing through this interface, i.e. in almost all optical research methods. In addition, determining the shape of a surface, for example, of a liquid droplet on a horizontal substrate, is an independent urgent task in such areas as the production of nanostructures [5], the creation of structured surfaces [6], medical diagnostics [7], and many others.

In [8] the technique was considered for reconstructing the surface of transparent solid and liquid media under assumption that one of the limiting surfaces is flat through which a probing laser beam passes, and the material the body is made of is optically homogeneous. In this case, the phase screen method can be used, which allows the construction of an nalytical algorithm for determining the surface profile based on the registration of the displacement of the elements of structured beams of a special type. In [8], the refraction of a plane laser beam on surfaces defined by polynomials of different orders was studied in the ray approximation. In the following research the class of the considered beams and surfaces will be expanded; in addition, wave models will be used to analyze the propagation of beams. The task of modeling the amplitude and phase distortions of a laser beam passing through a curved surface is formally similar to the task of modeling of the refraction of a beam in an optically inhomogeneous transparent medium; therefore, the approaches and methods described in [9–13] will be used to solve this problem.

2. The model of propagation of structured beams through curved interfaces in ray approximation

Let one of the surfaces of the considered transparent medium coincide with the plane $\zeta O\eta$, in which the initial structure of the probing laser beam is specified, and the other surface is curved and its relief is determined by the level function $h(\zeta, \eta)$, and the refractive index of the curved object differs from...
the refractive index of the surrounding medium by the value $\Delta n$. If the direction of probing is perpendicular to the plane $\xi O \eta$, then in the phase screen approximation the refractive displacement of geometrical optical rays in the plane $xOy$ located at a distance $z$ ($z \gg h(\xi, \eta)$ from the plane $\xi O \eta$ is determined by the relations
\begin{equation}
\begin{cases}
    x - \xi = z\Delta n \frac{\partial h}{\partial \xi}, \\
y - \eta = z\Delta n \frac{\partial h}{\partial \eta}.
\end{cases}
\end{equation}

It follows from (1) that refractive displacement of geometrical optical rays along $x$ and $y$ is determined by corresponding lateral gradient of the surface level.

Figure 1(a) shows a refractive image for the case of a point-structured beam, observed in the $xOy$ plane, for an inhomogeneity with a Gaussian surface profile shown in figure 1(b).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The model of propagation of a point-structured laser beam the curved interface: (a) - cross-section of the beam passing through the interface; (b) - shape of the interface with Gaussian profile.}
\end{figure}

The refractive displacement of the structural elements of the beam (points in this case) relative to their initial position can be recorded experimentally at a given distance $z$, and the components of the surface level gradient are defined as
\begin{equation}
\begin{cases}
    \frac{\partial h}{\partial \xi} = \frac{(x - \xi)}{z \cdot \Delta n}, \\
    \frac{\partial h}{\partial \eta} = \frac{(y - \eta)}{z \cdot \Delta n}.
\end{cases}
\end{equation}

From the components of the level gradient found from the system of equations (2), the surface level function $h(\xi, \eta)$ is determined. Since the components of the gradient can only be determined at a finite set of points, the intermediate values are based on quadratic interpolation.

As it is shown in [8], the solution of an inverse problem of beam refraction with the aim of reconstructing the surface profile can be considered as the problem of synthesizing such a surface relief that transforms a beam with the original discrete structure into a beam with some other predetermined structure. For example, a beam, which in the section at the entrance to the medium represents a family of segments, can be transformed in the observation plane into a beam representing in section a family of parabolas, circular arcs or other more complex curves.

Let the element of the beam section at the entrance to the medium be a plane curve defined parametrically as $\xi(t)$ and $\eta(t)$. At the output, it is required to obtain another curve described by a given parametric dependence $x(t)$ and $y(t)$. The parameter $t$ is a parameter of the geometrical optical ray, i.e. the ray corresponding to some value of $t$ passes through the points with coordinates $\xi(t)$, $\eta(t)$ at the entrance and $x(t)$, $y(t)$ at the observation plane. In some cases, for clarity, we can put $t = \xi$. Then, at the
entrance to the medium, the beam element will be described by the dependence \( \eta(\xi) \), which can be plotted in rectangular coordinates.

As an example, consider the construction of a surface profile that maps a straight line segment to a parabola segment. We take \( \xi \) as a parameter, then at the entrance of the medium \( \eta = a\xi \) is the equation of some straight line passing through the origin of coordinates. In the observation plane, we put \( x = \xi, \ y = b\xi^2 \) - parametric specification of a parabola with apex at the origin. Then from (2) it follows

\[
\frac{\partial h}{\partial \xi} = 0,
\]

\[
\frac{\partial h}{\partial \eta} = \frac{b}{x^2} (\eta^2 - \eta^2/2).
\]

Hence, we can find the surface profile

\[
h(\xi, \eta) = \frac{b}{x^2} \eta^2 - \eta^2/2.
\]

Corresponding transformation of a beam at \( a = b = 1 \) is depicted in figure 2(a). In this case the straight line is mapped to one branch of the parabola. Figures 2(b) and 2(c) also show the transformation of a plane beam into a parabolic one, but when it passes through a surface of the form

\[
h(\xi, \eta) = \frac{b}{x^2} \frac{\xi^2 \eta^2}{2} - \frac{\eta^2}{2}.
\]

Figures 2(a), 2(b), and 2(c) show the transformation of a plane beam into a parabolic one.

3. The experimental techniques for studying propagation of laser beams through a liquid droplet on a transparent substrate

The shape of a wide collimated laser beam or laser sheet was recorded on the screen after passing from bottom to top through a transparent liquid droplet lying on a horizontal glass substrate. The experimental setup described in [14–16] was created for this purpose. It consists of a helium-neon or semiconductor laser with power less than 10 mW, an optical system for formation of probing beam, a rotating prism, a glass substrate with the investigated droplet, a paper screen and a digital camera. Distilled water was usually used as the liquid to create the droplet, but salt and sugar aqueous solutions and glycerol were also used. The substrates made of different materials (glass of different brands, coated glass, glass with different cleaning methods) were used to create different wetting conditions. The examples of images obtained in the experiments with a wide collimated laser beam with diameter of 10 mm are shown in figure 3.
In the center of the presented images, a direct shadow image of a droplet (dark circle) can be seen against the background of a bright image of unbent part of the collimated laser beam. The largest area in the presented images is occupied by the refracted part of the beam. Since the droplet acts as a plano-convex liquid collecting lens, the part of the beam that passed through the droplet is first focused at a certain distance from the droplet (this distance depends on the refractive index of the liquid and the radius of curvature of the upper surface of the droplet and changes during evaporation), and then diverges. In these experiments, the paper screen was located at distances from the substrate exceeding twice the focal length of the droplet. The rays corresponding to the perimeter of this refractive image pass through the edge of the droplet and the angle of their deflection depends on the wetting contact angle of the droplet on the substrate. This is the basis of the method for measuring the wetting contact angle, described in work [15]. A characteristic feature of the edge of the refractive images is the presence of curvilinear concave sections (the so-called "arcs"), the number and size of which, as shown in [14], depend on the roughness and degree of cleaning of the substrate surface.

Figure 4 shows the examples of images obtained by probing droplets of distilled water and glycerol with a volume of 10 μl with a plane-structured beam (laser sheet) at different positions of the screen relative to the substrate.

As can be seen from the above images, at small distances from the substrate to the screen (less than the focal length of the droplet), the part of the laser plane passing through the droplet is twisted in the form of a loop, and at a large distance (beyond the focus) it turns into a kind of hyperbolic line. In this case, obviously, the higher the refractive index, the closer the droplet focus to the substrate and the greater the deflection of refracted rays at the same distance from the substrate to the screen.

In [16] it was shown that with layer-by-layer probing of a droplet on a glass substrate by a laser plane, it is possible to restore the shape of the droplet outer surface by fitting the model parameters in order to obtain the smallest deviation from the experimental results.

4. Models of wave beam propagation in the presence of curved interfaces of optically homogeneous media

4.1. Calculated results for an astigmatic laser beam propagating across the boundary defined by harmonic function

Figure 5 illustrates the geometric parameters of the problem when an astigmatic (plane) laser beam passes through a "wavy" surface defined by harmonic function. With the introduction of time as an additional parameter, it is possible to consider the dynamics of the intensity in the cross section of the beam. In addition, distortions of the beam envelope are considered in the case of its oblique incidence on the interface at small angles of deviation of the direction of incidence from the normal to the unperturbed surface.
Figure 4. The examples of experimental images of laser sheet passing through a droplet of distilled water (a), (c) and glycerol (b), (d) at different distances from the substrate to the screen: (a), (b) - 8 mm; (c), (d) - 19 mm.

The plane laser beam considered earlier in the problems of laser refractography [9–12] is essentially an astigmatic Gaussian beam with an elliptical cross-section, which is specified by two characteristic dimensions $w_1$ (along x-axis) and $w_2$ (along y-axis), and $w_1 \ll w_2$ or $w_2 \ll w_1$.

Figure 5. Geometrical illustration of the mathematical model.
The beam axis makes an angle $\alpha$ with z-axis at $z = 0$. Further we will assume that $\sin \alpha << 1$. Let the complex amplitude of the laser beam at the entrance to the medium at $z = 0$ be equal to $E(x, y, 0)$

$$E(x, y, 0) = \exp \{i[kx \sin \alpha]\} A^0(x, y, 0),$$

where $A^0(x, y, z)$ is the beam envelope for an unperturbed surface.

$$A^0(x, y, z) = A^0_x(x, z)A^0_y(y, z) = A^0 \exp \left\{-\frac{x^2}{w^2_1} - \frac{y^2}{w^2_2}\right\},$$

which can be represented as product of $A^0_x(x, z)$, and $A^0_y(y, z)$ - factors, one of which depends on x and other on y.

In the case of one-dimensional problem, the profile of a wavy surface (in the presence of a standing wave) and the distance between antinodes $\Lambda/2$ can be given by the function

$$h(x, t) = h_0 \left(1 + 2\delta h \sin(\Omega t) \cos(Kx)\right),$$

$$\delta h = \frac{\Delta h}{h_0} << 1.$$

where $h_0$ is the path in an unperturbed medium with refractive index $\Delta n$, $\Delta h$ is the amplitude of surface disturbance (in the presence of a traveling wave of length $\Lambda$, $K = \frac{2\pi}{\Lambda}$ is the wave number), $\Omega$ is the corresponding frequency.

The corresponding displacement of the beam elements is given by relation (1), and the intensity envelope $|A|^2$ at a known displacement is calculated by the method described in [17].

The subsequent figures 6–7 demonstrate the change in the beam envelope depending on the distance $z$ from the plane of exit from the medium to the plane of observation and the time instant $t$ (the dimensionless parameter $T = \frac{\Omega t}{2\pi}$ is used) for different values of $w_1, w_2$ at $\alpha \neq 0$.

Figure 6. The beam envelope for different points in time $t$ ((a) $- T = 0.4$; (b) $- T = 0.7$) in the case of $w_1 << \Lambda (\Lambda = 1 \text{ mm}, w_1 = 0.05 \text{ mm}, w_2 = 1 \text{ mm})$ at $\sin \alpha = 0.05, z = 100 \text{ mm}, \delta h = 4 \times 10^{-5}$.

### 4.2. Modeling the effects of changing the intensity in the cross section of a Gaussian beam

Figures 8–9 show the processes of focusing and defocusing of the laser beam passing through the focusing and defocusing surfaces of the Gaussian types when parameter $z$ changes. The characteristic radius of the Gaussian surface is 1 mm, the beam radius is 2 mm. Similar patterns are observed when simulating a dynamic change in the gradient of the surface relief.
Figure 7. The beam envelope in the case of $w_1 >> A$ ($A = 0.15$ mm, $w_1 = 1$ mm, $w_2 = 0.1$ mm), $z = 10$ mm, $\sin \alpha = 0.05$, $\delta h = 3 \cdot 10^{-5}$ for different values of $T$: (a) $T = 0$, (b) $T = 0.05$, (c) $T = 0.1$, (d) $T = 0.25$, (e) $T = 0.4$, (f) $T = 0.5$. 
Modeling the effects of changing the intensity in the cross-section of the beams forms the basis for modeling the direct shadow images of optical inhomogeneities of the medium, obtained with help of wide collimated and defocused laser beams.

5. Conclusion
Direct problems of propagation of laser beams of different types in the presence of curved interfaces between media with different values of the refractive index are solved. A technique has been developed for reconstructing the surface profile from refractive images of sections of probing discretely structured laser beams. The technique can be used for experimental diagnostics of surface irregularities of transparent objects, study of the shape of liquid droplets and films on the surface of solids [14–16], dynamic surface changes, etc. Synthesis of surfaces for a given transformation of the shape and structure of the beam is of independent interest.

The mathematical models developed to describe the refraction of wave beams are generalized to the case of curved interfaces. Examples of transformation of Gaussian and astigmatic beams, respectively, when they pass through surfaces described by quadratic and periodic functions are considered.

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