The FFX Correlator

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Abstract

We established a new algorithm for a correlation process in radio astronomy. This scheme consists of a 1st-stage Fourier transform as a filter and a 2nd-stage Fourier transform for spectroscopy. The “FFX” correlator stands for Filter and FX architecture, since the 1st-stage Fourier transform is performed as a digital filter, and the 2nd-stage Fourier transform is performed as a conventional FX scheme. We developed FFX correlator hardware not only for verifying the FFX scheme algorithm, but also for applying to the Atacama Submillimeter Telescope Experiment (ASTE) telescope toward high-dispersion and wideband radio observations at submillimeter wavelengths. In this paper, we present the principle of the FFX correlator and its properties, as well as evaluation results with the production version.

Key words: instrumentation: interferometers — instrumentation: spectrographs — radio continuum: general — radio lines: general — techniques: spectroscopic

1. Introduction

The signals received by the antennas obey the stationary stochastic process and then the ergodic process. The ergodic theory can be applied to the auto-correlation function for a spectrometer and the cross-correlation function for a radio interferometer. Under such conditions, S. Weinreb (1963) developed the first digital spectrometer. This digital spectrometer is called the XF correlator in which the correlation is calculated before a Fourier transform. Meanwhile, Chikada et al. (1987) developed the first FX correlator of another design, in which the Fourier transform is performed before a cross multiplication. Although there is a difference in the property between two basic designs, the obtained astronomical spectra of them were confirmed to be identical.

Determining the number of correlation lags in the XF scheme or of Fourier transform points in the FX scheme is essential for realizing a high-dispersion and wideband observation, because the frequency resolution is derived as

$$\Delta f = 1/(\Delta t_s N) = 2B/N,$$  \hspace{1cm} (1)

where $\Delta t_s$ is the sampling period, $N$ is the number of correlation lags or Fourier transform points, and the bandwidth of $B$ is equal to $1/(2\Delta t_s)$. The material size and cost of the correlator strongly depend on the sampling period, $\Delta t_s$, and the number of correlation lags or Fourier transform points, $N$.

The new XF architecture with a digital Tunable Filter Bank that is designed with the Finite Impulse Response (FIR) has been proposed and developed for the next-generation radio interferometers, the Expanded Very Large Array (EVLA) and the Atacama Large Millimeter/submillimeter Array (ALMA) (B. Carlson 2001; Escoffier et al. 2007). This is called the “FXF correlator”. The architecture of the FXF scheme can make the material size smaller compared with that of the conventional XF scheme. Since the digital filter allows a variety of observation modes [scientific and observational availability were shown in Iguchi et al. (2005)], the FXF scheme will provide the most appropriate specifications that meet the scientific requirements. This will lower the risk of over-engineering of the correlator.

The improved FX architecture with DFT filterbank was developed by J. Bunton (2000). The use of polyphase filter banks allows arbitrary filter responses to be implemented in the FX scheme (J. Bunton 2003). This is called the “Polyphase FX Correlator”. This scheme has a possibility to achieve spectral leakage of about $-120\text{dB}$. In particular, this performance is significant to suppress leakage from spurious lines mixed in receiving, down-converting or digitizing.

The FFX Correlator is a new algorithm for the correlation process in radio astronomy. The FFX scheme consists of 2-stage Fourier transform blocks, which perform the 1st-stage Fourier transform as a digital filter, and the 2nd-stage Fourier transform to achieve a higher dispersion. The first “F” of the FFX is the initial letter of the word “Filter”. In this paper, we present a new FFX architecture. The principle of the FFX scheme is presented in section 2, the properties of the FFX scheme in section 3, the algorithm verification and performance evaluation with the developed FFX correlator in sections 4 and 5, and a summary of this paper in section 6.

2. Principle of the FFX Correlator

This section shows the algorithm and a data-flow diagram of the signal processing in the Fourier transform of the FFX
Suppose that $x_n$ are digital waveforms at the correlator input from the astronomical radio signals that are received by the telescope. The inputs, $x_n$, are real digital signals at a sampling period of $\Delta t$, and obey the zero-mean Gaussian random variable. The suffix $n$ is an integer for time.

[Step 1] The correlator receives time-domain digital sampling signals from the Analog-to-Digital Converter (ADC), and accumulates them up to $N_n$ points.

[Step 2] The time-domain $N_n$-point data are transferred to the frequency domain by using the $N_n$-point discrete complex Fourier transform as follows:

$$X_p = \Delta t \sum_{n=0}^{N_n-1} x_n \exp \left( -j \frac{2 \pi p n}{N_n} \right).$$

where $X$ is the spectrum after the 1st Fourier transform.
the suffix \( p \) is an integer for frequency, and \( \Delta t_0 \) is equal to \( 1/(2B_{1st}) \) at a bandwidth of \( B_{1st} \); \( \Delta f_{1st} \) is the minimum frequency resolution of the 1st Fourier transform, which is equal to \( 1/(\Delta t_0N_n) \).

[Step 3] The extraction of the \( N_k \) points from the frequency domain \( N_n/2 \)-point data after the 1st Fourier transform is conducted as if filter and frequency conversions are performed simultaneously:

\[
X'_k = X_p \quad (p = k + k_0, \ k = 0, \ldots, N_k - 1), \tag{3}
\]

where \( k_0 \) is the minimum frequency channel in the extraction, and the suffix \( k \) is an integer for frequency.

[Step 4] The \( N_k \)-point data after an inverse Fourier transform is written as

\[
x'_m = \frac{1}{\Delta t_0N_k} \sum_{k=0}^{N_k-1} X_k \exp \left[ \frac{j2\pi(k - N_k/2)t}{N_k} \right], \tag{4}
\]

where \( x' \) is the time-domain signal after an inverse Fourier transform, the suffix \( l \) is an integer for time, and \( \Delta t_l \) is the sampling period after filtering at the bandwidth of \( B_{2nd}/\Delta t_l = 1/B_{2nd} = 1/(\Delta f_{1st}N_k) \).

[Step 5] By repeating the procedure from Step 1 to Step 4, the data are gathered up to \( N_m \) points as follows:

\[
x'_m = X'_{l+dN_k}, \tag{5}
\]

where \( m \) is \( l + dN_k \), and \( d \) is the number of repeating times of the procedure from Step 1 to Step 4.

**Table 1.** Definition of functions.

| Mark          | Explanation                        |
|---------------|------------------------------------|
| \( B_{1st} \) | Bandwidth of the input signals     |
| \( \Delta t_s \) | Sampling period of the input signals |
| \( N_n \)     | Number of the points of 1st FT     |
| \( \Delta f_{1st} \) | Minimum frequency resolution of 1st FT |
| \( N_k \)     | Number of the extraction times as a filter |
| \( B_{2nd} \) | Bandwidth after extraction         |
| \( \Delta t_f \) | Sampling period after filtering     |
| \( N_m \)     | Number of the points of 2nd FT     |
| \( \Delta f_{2nd} \) | Minimum frequency resolution of 2nd FT |

Note that FT is Fourier transform.

**Table 2.** Relationship among the functions.*

| Equation | \( \Delta t_s \) | \( \Delta f_{1st} \) | \( \Delta t_f \) | \( \Delta f_{2nd} \) |
|----------|------------------|---------------------|------------------|---------------------|
| (a)      | \( 1/(2B_{1st}) \) | \( 2B_{1st}/N_n \) | \( 1/(\Delta f_{1st}N_k) \) | \( 2B_{2nd}/N_m \) |
| (b)      | \( 2B_{1st}/N_n \) | \( 1/(\Delta f_{1st}N_k) \) | \( \Delta t_sN_k/N_k \) | \( B_{2nd}/N_m \) |
| (c)      | \( 1/(\Delta f_{1st}N_k) \) | \( \Delta t_sN_k/N_k \) | \( \Delta f_{1st}N_k/N_m \) | \( 1/(\Delta f_{1st}N_k) \) |
| (d)      | \( \Delta f_{2nd} \) | \( B_{2nd}/N_m \) | \( 1/(\Delta f_{1st}N_k) \) | \( 2B_{2nd}/N_n \times N_k/N_m \) |

* See table 1.

[Step 6] The time-domain \( N_m \)-point data after gathering are transferred to the frequency domain by using the \( N_m \)-point discrete complex Fourier transform as follows:

\[
x'_q = \Delta t_q \sum_{m=0}^{N_m-1} x'_m \exp \left( -j\frac{2\piqm}{N_m} \right), \tag{6}
\]

where \( X' \) is the spectrum after the 2nd Fourier transform, and the suffix \( q \) is an integer for frequency. The \( \Delta f_{2nd} \) is the minimum frequency resolution after the 2nd Fourier transform, which is equal to \( 1/(\Delta t_qN_m) = \Delta f_{2nd}N_k/N_m \).

The definitions of all functions used in this section are summarized in table 1. Also, a summary of the relationships among the functions (see table 1) are listed in table 2. The following relations were derived:

\[
\Delta f_{1st} = 2B_{1st}/N_n, \tag{7}
\]

\[
\Delta f_{2nd} = 2B_{2nd}/N_m \times N_k/N_m. \tag{8}
\]

The frequency resolution of the FFX scheme is determined by the number of the 1st Fourier-transform points \( (N_n) \), the number of the extractions as a filter \( (N_k) \), and the number of 2nd Fourier-transform points \( (N_m) \).

### 3. Properties of the FFX Correlator

The frequency responses for spectroscopic observations were finally derived by Fourier transforms in all schemes. For a finite length of the Fourier transform, the responses are multiplied by a rectangular window function, which corresponds to a convolving sinc function in the frequency domain. The frequency profile of the XF scheme becomes the shape of sinc function profile, while that of the FX scheme is a sinc squared function profile. This indicates that the FX scheme (including the FFX and polyphase FX schemes) is better than the XF scheme (including the FFX scheme) from the view points of the frequency profile and sharpness of individual frequency channels, and the spectral leakage.

For the realization of the high frequency resolution in the conventional FX scheme, the \( N \)-point complex FFT (Fast Fourier Transform) may be divided into \( N/2 \)-point FFTs, \( N \)-point twiddle factor multiplications, and \( N/2 \)-point second FFTs, because the circuit size of the LSI (Large-Scale Integration) is limited (Iguchi et al. 2002). The memory and circuit for the twiddle factor multiplications are critical. However, a high-frequency resolution can be realized in the FFX scheme without the twiddle factor multiplications.

The FFX scheme has advantages of selectivity for the frequency resolution and the bandwidth compared with other schemes. Comparable functions are also realized by implementing a digital LO circuits (Escoffier et al. 2007). The digital LO circuits need to be delicately designed so as to avoid spurious in mixing the digital LO signals due to rounding errors in the calculation process.

For investigating aliasing or foldover from frequencies over the bandedge in the FFX scheme, it is necessary and important to estimate the frequency response. As shown in [Step 3] in figure 1, the desired frequency response, equation (3), can be rewritten as

\[
X'_k = X'_p \quad (p = k + k_0, \ k = 0, \ldots, N_k - 1), \tag{9}
\]
Fig. 2. Frequency response of the 1st FFT stage in the FFX scheme, which works as a digital filter. (a) is the frequency response at the full range to investigate the stopband, and (b) is the closeup frequency response in the passband to investigate a ripple. The frequency response is multiplied by the window of the Bessel function, \( w(n) = J_0(4.24n/N_k) \).

\[
X_p = H_p \times X_p, \quad (10)
\]

\[
H_p = \begin{cases} 1 & (p = k_0, \ldots, k_0 + N_k - 1) \\ 0 & (p < k_0, \quad p > k_0 + N_k - 1). \end{cases} \quad (11)
\]

However, “\( H_{k_0} \)” should be replaced by zero to reduce the aliasing or foldover of noise from frequencies over the band-edge. Thus, in the FFX scheme, equation (11) needs to be replaced as

\[
H_p = \begin{cases} 1 & (p = k_0 + 1, \ldots, k_0 + N_k - 1) \\ 0 & (p < k_0 + 1, \quad p > k_0 + N_k - 1). \end{cases} \quad (12)
\]

According to the two (1st and inverse) Fourier transforms at the same resolution, the actual designed transfer function of the impulse response, which is derived with the square of sinc function, is represented as follows:

\[
|H(f)| = \sum_{p=0}^{N_n-1} H_p^2 \left\{ \frac{\sin[\pi (N_n \Delta t_s f - p)]}{\pi (N_n \Delta t_s f - p)} \right\}^2, \quad (13)
\]

\[
P(f) = \left\{ \sum_{p=0}^{N_n-1} H_p^2 \left\{ \frac{\sin[\pi (N_n \Delta t_s f - p)]}{\pi (N_n \Delta t_s f - p)} \right\} \right\}^2, \quad (14)
\]

where \( f \) is an arbitrary frequency, and also the response of a sinc function is caused by one Fourier transform. It can be confirmed that the designed transfer function approaches the desired frequency response by increasing \( N_n \), while keeping \( N_n/N_k \) constant. The filtering process in the FFX scheme is similar to that used in the design method of a Frequency Sampling Filter, FSF (Rabiner & Schafer 1971).

Also, to improve the frequency response, the window function before the 1st Fourier transform can be multiplied. In that case, equation (14) should be written as

\[
|H(f)| = \sum_{p=0}^{N_n-1} H_p^2 W(N_n \Delta t_s f - p) \left\{ \frac{\sin[\pi (N_n \Delta t_s f - p)]}{\pi (N_n \Delta t_s f - p)} \right\}^2, \quad (15)
\]
\[ P(f) = \left\{ \begin{array}{c} w(n), \\
N_0 - 1 \sum_{p=0}^{N_0-1} H_p^2 W(N_0 \Delta t_s f - p)^2 \frac{\sin[\pi (N_0 \Delta t_s f - p)]}{\pi (N_0 \Delta t_s f - p)} \end{array} \right\}, \]

where \( W(f) \) is the response after the window function, \( w(n) \), is transferred to the frequency domain by a Fourier transform. If the window is a rectangular window function, \( W(f) \) becomes a sinc function. In that case, equation (16) is consistent with equation (14). There are the following famous window functions: Hanning, Hamming, Blackman, and Kaiser. By making a good choice of the window function, the first sidelobe of the stopband in the frequency response will be improved. It is well known that the first sidelobe levels with Backman and Kaiser as the window are better than those with Hanning and Hamming. For the FFX scheme, it is found that the Bessel function of the zeroth-order \( J_0 \) is better than others. This frequency response is shown in figure 2. The first and second sidelobe levels are about −34 dB, the fifth and sixth sidelobe levels are about −50 dB, and the higher-order sidelobe levels will be better than −60 dB (see figure 2a). The stopband response of the FFX scheme is not better than that of the polyphase FX scheme (J. Bunton 2000) and that of the FFX scheme for EVLA (B. Carlson 2001). On the other hand, the ripple response in the passband is less than 0.4 dB peak-to-peak (see figure 2b). This performance is better than that of the polyphase FX scheme.

Note that the stopband performance to suppress the spurious lines can be improved by installing “the detection and cancellation techniques of high and low-frequency spurious lines” into the FFX scheme (Y. Chikada found this algorithm in the development of ALMA/ACA correlator).

### 4. Requirements and Specifications for the Development of an FFX Correlator

The development of FFX correlator hardware is significant for verifying the FFX scheme algorithm. It is necessary to define the requirements and specifications of the hardware of the FFX correlator. The hardware size can be optimized for verifying the algorithm. On the other hand, there were scientific requests for an application to the Atacama Submillimeter Telescope Experiment (ASTE), which is a new project to install and operate a 10-m submillimeter telescope at a high-latitude site (4800 m) in the Atacama desert of northern Chile (Ezawa et al. 2004). Under these situations, the FFX correlator hardware for the algorithm verification was specified by also considering the scientific requirements for submillimeter astronomy, including an application to ASTE.

In the case of spectroscopic observations of atomic/molecular line emissions, their line widths were extended by Doppler shifts along a line of sight with movement of the interstellar matter (ISM). In nuclear regions of external galaxies and Ultra Luminous Infrared Galaxies (ULIRGs), the molecular clouds that have various velocity components can be observed simultaneously, and the line widths of the observed atomic/molecular emission lines are extended. For example, the line width of the CO line emission of an external galaxy is sometimes extended to more than 800 km s\(^{-1}\) (e.g., Narayanan et al. 2005), which corresponds to about 920 MHz in \(^{12}\)CO \((J = 3–2)\) \((v_{\text{rest}} \sim 345.796 \text{GHz})\) and about 2.2 GHz in \(^{12}\)CO \((J = 7–6)\) \((v_{\text{rest}} \sim 806.652 \text{GHz})\), by rotating around its nuclear region. On the other hand, in order to evaluate the kinematics of protoplanetary disks and the internal structure of molecular clouds in the Milky Way, it is also necessary to resolve their thermal line widths using a spectrometer with high-frequency dispersion; for instance, a frequency resolution of 32 kHz corresponds to a velocity resolution of 0.032 km s\(^{-1}\) at 1 mm wavelength.

Because a full bandwidth of more than 3 GHz is required, the FFX correlator needs to achieve a processing speed of 8192 Mega sample per second (Msps). In this case, for realizing two-type spectral resolutions of about 5 MHz and less than 32 kHz, the FFX correlator must meet at least the specifications given in table 3.

### 5. Development and Evaluation of the FFX Correlator

A correlation processing block diagram of the FFX correlator is shown in figure 3. The FFX correlator consists of a DTS-R (Data Transmission System Receiver) module, a Correlation Module, and a Monitor & Control Computer. In the DTS-R module, the DTS-R Board or the ADC (Analog-to-Digital Converter) Board is implemented as an EIB (Electrical Input Interface Board). The input data rate of the FFX correlator is about 48 Giga bit per second (Gbps) with 3-bit quantization at a sampling frequency of 8192 or 4096 Msps, which is 8192 Msps \(x\) 3 bits \(x\) 2 IF or 4096 Msps \(x\) 3 bits \(x\) 4 IFs. The DCDCB (Delay Correction and Data Configuration Board) effectively distributes the input signals to the next boards for parallel correlation processing. For data processing at a throughput of 8192 Msps, the correlation is performed with 16 parallels, and both of the real and imaginary parts are used in FFT. The data are sent to each 16-parallel CORB (Correlation Board) per one-segment length. In the correlation mode of the FFX scheme, the data are sent per total segment length, which is determined considering signal process, including the second stage of FFT. The final correlation output is obtained by adding 16-parallel correlation results. The correlation output is sent to the Monitor & Control Computer via a LAN cable. Switching between the FX processing and the FFX processing is normally operated by setting the command into the Monitor & Control Computer.

The correlation processing flow of the FFX correlator is shown in figure 4. All main logics are implemented in FPGAs (Field Programmable Gate Array).

#### 5.1. Delay Correction and Data Configuration Board (DCDCB)

##### 5.1.1. Delay correction

The FFX correlator has a delay correction circuit per bit for every 3-bit sampling signal. Delay tracking for every single bit...
is realized by extracting a 64-sample length from 128 time-sequential samples that are produced by splitting the output from FIFO (First In, First Out) memory in two, and shifting one side or the other in one-clock phase (see figure 5). The delay correction circuit has a FIFO memory of 1.024 Mega samples to each bit for delay tracking. In the case of 8192 Msp/s, the delay correction range is \( \pm 512 \) kilo samples, which is \( \pm 62.5 \mu s \) \((= \pm 18.75 \text{ km})\).

5.1.2. Data distribution

Given the operation speed of the device (FPGA etc.), 16-parallel correlation processing is essential to achieve a throughput of 8192 Msp/s. To do this, the input signals are divided by one segment length of FFT and sent to the Correlation Board (CORB) so that each parallel processing is performed separately.

In FFX processing, the data are divided by a single segment length of the second-stage FFT. From table 3, the input signals are divided into

\[1024 \times \frac{4096}{8} = 512 \kappa,\]  

(17)

where \( \kappa \) is 1024 \((2^{10})\). This value determines the one segment length of this FFT processing; 64-parallel signals are converted to 4-parallel after being output to each distribution buffer (see figure 6).

5.2. Correlation Board (CORB)

5.2.1. Operation format

The operations of the correlation processing were performed using a 16-bit floating point format. The FFT calculation is expressed as
and the range is from $\pm 1/32768$ to $131008$. When the index ($e$) is 0, it is regarded as 0 instead of $2^{-16}$, irrespective of the mantissa.

This not only simplifies the processing, but also improves the compatibility with IEEE single-precision floating-point data. The IEEE single-precision floating-point has a 32-bit representing sign ($S$): 1 bit, exponential ($E$): 8 bits, and mantissa ($Ma$): 23 bits, and is expressed as

$$-1^S \times 2^{127E} \times (1 + Ma/2^{23}),$$

and they have following relations:

$$S = s,$$  

$$E = e + 111,$$  

$$Ma = ma \times 8192(2^{13}).$$

The conversion between 16-bit floating point and IEEE single-precision floating point is also shown in figure 7.

### 5.2.2. Parallel processing of two datasets

Since FFT is the linear response, the real and imaginary parts of the input can be used separately. Since the signals received by an antenna are the real part only, two datasets are combined into one complex dataset for FFT processing (see figure 8). One dataset is inserted into a real part, and then the next dataset is inserted into an imaginary part. These two datasets are shifted with one-FFT segment length ($M$), and are then processed as one complex data. The combination of two datasets is performed by Read/Write sequence control of the received buffer (= input buffer in FPGA). This method can reduce the material size of the correlator.

#### 5.2.3. F-block (FX mode)

FX processing is performed with $16\hat{k}$-point FFT. In terms of the throughput, pipeline processing is applied to each FFT stage processing. To reduce the memory size in the pipeline processing, $16\hat{k}$-point FFT is realized by being divided into three parts: 32-point FFT, 32-point FFT, and 16-point FFT (see figure 9). The input signals are multiplied by the window function between the input buffer and FFT, so that the input buffer size is based on the 3-bit signals. FFT is realized by changing the twiddle factor according to the processing stages. This helps to minimize the ROM of the twiddle factors.

#### 5.2.4. F-block (FFX mode)

The FFX mode is shown in figure 10. The first-stage FFT ($1\hat{k}$-point FFT) is performed in the same manner as 32 points of the FX mode. The second-stage FFT ($4\hat{k}$-point FFT) is serially processed with one butterfly computing unit. Since $2 \times 8$-point data are obtained during every single process of $1\hat{k}$-point first-stage FFT, 4096-point data is obtained by repeating the process of the first-stage FFT 256 times. The required time for this process is:

$$512 \times 256 = 128\hat{k} [CLK].$$

Compared with these processing times, the processing time

$$4096 \times \log_2(4096)/2 = 24\hat{k} [CLK].$$

(23)
of 4096-point FFT with one butterfly computing unit is much shorter.

5.2.5. Window function processing

After re-allocating to two datasets, the signals are multiplied by a window function. Any of the following window functions are selected: None (rectangular window), Hanning, Hamming, and Blackman. Window functions are generated in the CPU according to the selected command (“WINDOW”) from the Monitor & Control Computer before the correlation process starts.

5.2.6. Data conversion

The input signals go through 3-bit processing. By setting a command (“SMPLBIT”) from the Monitor & Control Computer, the input signals are processed as 1-bit or 2-bit data. Also, conversion to 1-bit data with the (M) middle bit only is available. The normalized threshold value for each bit of sampling data is given as shown in figure 11. Based on the above threshold value, the converted value of the input data is calculated as shown in figure 12. The input signals are converted into a 16-bit floating-point operation format before FFT.

5.2.7. X-block (FX mode)

The composition of an X-block in normal FX processing is shown in figure 13. Correlation processing is performed in the 8k-channel either of USB or LSB. For improving the sensitivity loss (see table 6 of S. Okumura et al. 2000), 5 2-channel frequency binning is performed to generate 4k-channel data (Frq. bin). Integration of 100 ms is conducted in time integration (S.T. Intg). If the integration takes longer than 100 ms, it is performed in the Long-Term Accumulation and Output Board (LTAOB) at the time of 16-parallel data synthesis.

In the OTF mode, the data are compressed from 8k frequency channels to 32 frequency channels by frequency binning. With a time component of 1 ms, the correlation buffer area is changed every 1 ms. The output to the Long-Term Accumulation and Output Board (LTAOB) is performed every 100 ms.

Prior to correlation processing, the data from the F-block needs some pre-processing, such as splitting two datasets, USB/LSB (upper and lower sideband) selection, and 90-degree and 180-degree phase switching demodulations. In the splitting process of two datasets, the input data are added and subtracted, and then the two-data components combined in the F-block are split. When LSB is selected, the sign of the split imaginary component is inversed. On the other hand, when USB is selected, the split data are sent to the correlation process without any additional processing. In 90-degree and 180-degree phase-switching demodulations, switching between real and imaginary, and sign inversion are performed according to the two phase-switching signals.

5 ALMA Memo No. 350 (http://www.alma.nrao.edu/memos/html-memos/abstracts/abs350.html).
5.2.8. X-block (FFX mode)

X-block in the FFX processing is shown in figure 14. In the FFX processing, X-block has a dual structure considering the processing speed of the device. The first-stage FFT is the same as that of FX mode; however, frequency binning is not performed. Split of two datasets in the second-stage FFT is not performed in X-block, since the process is performed in F-block. The results of the second-stage FFT are output together with USB and LSB. Two-channel frequency binning is performed for improving the sensitivity loss (see table 6 of S. Okumura et al. 2001).^3

5.2.9. \( \Delta W \) correction

\( \Delta W \) correction is performed based on the baseline. A \( \Delta W \) correction coefficient is generated within FPGA. Firstly, \( \Delta W_0(t) \), the gradient of \( \Delta W \) to time change, is evaluated, and then the value of each frequency channel is calculated (see figure 15). A circuit diagram of the line graphs given above is shown in figure 16.

For every 1 ms, the initial value (Init) is provided by the Monitor & Control Computer, and read as an initial gradient \( [\Delta W_0(0)] \). In the border between segments, the previous value is multiplied by the gradient (Grad) data to generate a gradient of a new segment \( [\Delta W_0(t)] \). In an arbitrary time \( t \), the initial value (the value of DC) is set to 0. \( \Delta W \) is set for every 128 channels in the full bandwidth of the 8\( k \) channel, which means that the full bandwidth (8\( k \) channel) is corrected with 64 steps. Although the initial value of \( \Delta W_0(t) \) can be set for every 1 ms, a given value is set within 128 channels. The gradient of \( \Delta W_0(t) \) is the gradient variation of \( \Delta W \) per one segment length for FFT. The Monitor & Control Computer specifies a set of the initial value and gradient approximately every 1 s.

5.3. Long-Term Accumulation/Output Board (LTAOB)

The final correlation value is calculated by adding the output of 16 correlators. The composition of the Long-Term Accumulation/Output Board (LTAOB) is shown in figure 17. When the correlation result is output, the data is converted into the format of IEEE single-precision floating point.

In FX processing, the number of the output frequency channels is normally 4\( \hat{k} \) (= 4096). Thus, the output data size per one correlation is

\[
4\hat{k} \times 32 \times 2 \times 3 = 768\hat{k} \text{ bits}, \tag{24}
\]

where 32 is the number of single-precision floating bits, 2 is the complex, and 3 is the number of correlations; auto-correlations of \( x \) and \( y \), and a cross-correlation between them. Assuming that the minimum integration time is 0.1 s, the estimated output speed is approximately 7.7 Mbps. In FFX processing, since the number of the output frequency channels is 512 + 2\( \hat{k} \), the data size per one correlation is 480\( \hat{k} \) bits. Consequently, the estimated output speed with 0.1-s integration time is approximately 4.8 Mbps. Correlation results are sent to the Monitor & Control Computer using the TCP/IP protocol of 100 BaseT-Ether.

5.4. Operation Mode

The FFX correlator is divided into four F parts, in principle (not physically). Each F part in a FX mode and the first FFT stage of the FFX mode are operated at 2048 MHz, and each F part at the second FFT stage of the FFX mode are operated at 32 MHz. Also, the digital signals input from EIB are distributed to arbitrary F parts by using the command ("FCHSEL") from the Monitor & Control Computer. In the FX mode and the first FFT stage of the FFX mode, the digital signals of 8192 Msps are operated by combining two F parts (= 2 x 2048 MHz),
Table 4. Operation mode in a FX mode.

| Bandwidth | Spectral points | Spectral resolution | Velocity resolution at 1 mm | Correlation |
|-----------|-----------------|---------------------|-----------------------------|-------------|
| 4096MHz   | 4096            | 1MHz                | 1.0 km/s^{-1}               | 2AC–1CC     |
| 2048MHz   | 4096            | 0.5MHz              | 0.5 km/s^{-1}               | 4AC–1CC     |

Table 5. Operation mode in a FFX mode.

| Stage | Bandwidth | Spectral points | Spectral resolution | Velocity resolution at 1 mm | Correlation |
|-------|-----------|-----------------|---------------------|-----------------------------|-------------|
| 1st   | 4096MHz   | 512             | 8 MHz               | 8.0 km/s^{-1}               | 2AC–1CC     |
| 2nd   | 64 MHz    | 2048            | 31.25 kHz           | 0.031 km/s^{-1}             | 2AC–1CC     |
| 1st   | 128 MHz   | 4096            | 8 MHz               | 8.0 km/s^{-1}               | 2AC–1CC     |
| 2nd   | 32 MHz    | 2048            | 15.625 kHz          | 0.016 km/s^{-1}             | 4AC–2CC     |
| 1st   | 2048 MHz  | 512             | 4 MHz               | 4.0 km/s^{-1}               | 2AC–1CC     |
| 2nd   | 64 MHz    | 4096            | 15.625 kHz          | 0.016 km/s^{-1}             | 2AC–1CC     |
| 1st   | 2048 MHz  | 512             | 4 MHz               | 4.0 km/s^{-1}               | 2AC–1CC     |
| 2nd   | 128 MHz   | 8192            | 15.625 kHz          | 0.016 km/s^{-1}             | 1AC         |
| 1st   | 2048 MHz  | 512             | 4 MHz               | 4.0 km/s^{-1}               | 2AC–1CC     |
| 2nd   | 96 MHz    | 6144            | 15.625 kHz          | 0.016 km/s^{-1}             | 1AC         |
| 2nd   | 32 MHz    | 2048            | 15.625 kHz          | 0.016 km/s^{-1}             | 1AC         |

4AC: Auto-Correlation (H1 and V1, H2 and V2), 2CC: Cross-Correlation (H1V1, H2V2).
2AC: Auto-Correlation (HH and VV), 1CC: Cross-Correlation (HV).
1AC: Auto-Correlation (HH or VV).

5.6. Evaluation and Discussion

Figure 19 shows a block diagram of the measurement setup of the frequency response of the FFX Correlator. To investigate the frequency response, it is useful to use the CW signal, which can measure the folding effects by sweeping the frequency range of 0 to 4096 MHz. The white noise is important to measure the frequency response, because the astronomical signals obey the Gaussian random variable. To obtain input signals that are approximated to the zero-mean Gaussian probability, mixing of the CW signal with the white noise is necessary.

The frequency response of the FFX correlator when CW is included is written as

\[ P_{on}(f) = a_{on} \times [\lvert C(f) \rvert^2 + \lvert N(f) \rvert^2] \times H_1(f)H_2^*(f) \times |H_D(f)|^2, \]

(25)

where \( C(f) \) is the frequency response of the CW signal, \( N(f) \) is the frequency response of the white noise from the ASTE analog backend subsystem, \( H_1 \) and \( H_2 \) are the frequency response by different transmission paths, in which \( H_1 = H_{a1}H_{b1} \) and \( H_2 = H_{a2}H_{b2} \) (see figure 19), and \( H_D \) is the frequency response of the FFX correlator, including the effects of requantization and the folding noise after the second FFT stage can be changed.

All of the operation modes available in this FFX correlator are listed in tables 4 and 5.

5.5. Hardware of the FX Correlator

The Hardware of the FX Correlator is shown in figure 18. The DTS-R module consists of two Electrical input Interface Boards (EIBs), two Delay Correction and Data Configuration Boards (DCDCBs), and one DTS-R Monitor & Control Boards (DRMCBs). The Correlation module consists of eight Correlation Boards (CORBs), a Long-Term Accumulation/Output Board (LTAOB), and one Correlation Monitor & Control Board (CORMCB). Each module is connected to an independent Power module.

The power consumption of the filter module is 400W, while that of the output module is 600W. The total AC power is 750 W at 1-phase 100–220 VAC 50/60 Hz (100 V ± 10% or 220 V ± 10%). The total weight is 71.3 kg.
downsampling. Bandpass calibration is essential for estimating the CW power accurately, because the bandpass response becomes a time-variable due to outdoor air temperature. The frequency response without the CW signal is written as

$$P_{\text{off}}(f) = a_{\text{off}} \times |N(f)|^2 \times |H_1(f)H_2^*(f)|^2 \times |H_D(f)|^2.$$  \hspace{1cm} (26)

The ADCs work as 1-bit performance (Okuda & Iguchi 2008). In that case, it is important to adjust the power as precisely as possible so as to avoid any high-order spurious effects. The frequency responses of $P_{\text{on}}(f)$ and $P_{\text{off}}(f)$ depend on the relative power of the CW signal to the white noise, and also the threshold levels in quantization. To correct these effects, we need to calibrate the bandpass by sensitively adjusting the continuum floor level of $P_{\text{off}}$ to that of $P_{\text{on}}$ in the data analysis.

These values are $a_{\text{on}}$ and $a_{\text{off}}$. From equations (25) and (26), the frequency response in a FFX mode is written as

$$P_{\text{on}}^F(f) = a_{\text{on}} \times \left[ |C(f)|^2 + |N(f)|^2 \right]$$
$$\times H_1(f)H_2^*(f) \times |H_D^E(f)|^2,$$  \hspace{1cm} (27)

while the frequency response without the CW signal can be written as

$$P_{\text{off}}^F(f) = a_{\text{off}} \times |N(f)|^2$$
$$\times H_1(f)H_2^*(f) \times |H_D^E(f)|^2,$$  \hspace{1cm} (28)

and then the CW frequency response including the response of the measurement system is derived as...
Similarly, the frequency response in a FX mode is written as
\[
P_{\text{on}}^\text{N}(f) = a_{\text{on}} \times |C(f)|^2 \times H_1(f)H_2(f) \times |H_D^\text{N}(f)|^2,
\]
while the frequency response without the CW signal can be written as
\[
P_{\text{off}}^\text{N}(f) = a_{\text{off}} \times |N(f)|^2 \times H_1(f)H_2(f) \times |H_D^\text{N}(f)|^2.
\]
From equations (29) and (32), we can derive the frequency response in a FFX mode from the correlated spectra obtained in the FFX and FX modes as
\[
|H_F^\text{E}(f)|^2 = \frac{P_{\text{off}}^\text{F}(f)}{P_{\text{off}}^\text{N}(f)} \times |H_D^\text{N}(f)|^2.
\]

The measurement results show that the effective bandwidth is about 59.28 MHz, which was obtained by passband responses of about 1 dB at 2046.40625 and 2106.28125 MHz, -3 dB at 2045.3125 and 2107.28125 MHz, and by a stopband response with the first sidelobe of about -20 dB. The measurement results are quite consistent with the theoretical curve in the passband, both
bandedges (sharpness), and the first sidelobe levels. In the stopband response, except for these responses, however, it is shown that there are differences between the theoretical curve and the measured results. This problem is probably due to the non-linear response of 1-bit ADC and the precision of the data-reduction process in this measurement method. The cross-modulation distortion is strongly generated in digitizing the CW signals at 1 bit. This character complicates the data-reduction method, and will reduce the measurement precision. If ADCs with 3 bits or more are feasible, this problem will be relaxed.

Finally, the measurement results show that the theory of the FFX scheme can be confirmed, and the development of the FFX Correlator was successfully realized.

6. Summary

There are two basic designs of a digital correlator: the XF-type, in which the cross-correlation is calculated before a Fourier transformation, and the XF-type, in which a Fourier transformation is performed before cross multiplication. To improve the XF-type correlator, we established a new algorithm for the correlation process, that is called the FFX scheme. The FFX scheme demonstrates that the realization of a stop-band response with first and second sidelobes of $-34 \text{ dB}$ and higher-order sidelobes of $-60 \text{ dB}$ is technically feasible. The FFX scheme consists of 2-stage Fourier transform blocks, which perform a 1st-stage Fourier transform as a digital filter, and a 2nd-stage Fourier transform to achieve higher dispersion. The FFX scheme provides flexibility in setting the bandwidth within the sampling frequency.

The input data rate of the developed FFX correlator is about 48 Giga bit per second (Gbps) with 3-bit quantization at a sampling frequency of 8192 or 4096 Msp, which is $8192 \text{ Msp} \times 3 \text{ bits} \times 2 \text{ IF} \times 4096 \text{ Msp} \times 3 \text{ bits} \times 4 \text{ IFs}$. We have successfully evaluated the feasibilities of the FFX correlator hardware. Also, this hardware will be installed and operated as a new spectrometer for ASTE.

We successfully developed the FFX correlator, measured its performances, and demonstrated the capability of a wide-frequency coverage and high-frequency resolution of the correlation systems. Our development and measurement results will also be useful and helpful in designing and developing the next-generation correlator.

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