Basics of doubly heavy tetraquarks

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Abstract. We outline the most important results regarding the stability of doubly heavy tetraquarks $QQ\bar{q}\bar{q}$ with an adequate treatment of the four-body dynamics. We consider both color-mixing and spin-dependent effects. Our results are straightforwardly applied to the case of all-heavy tetraquarks $QQ\bar{Q}\bar{Q}$. We conclude that the stability is favored in the limit $M_Q/m_q \gg 1$ pointing to the stability of the $bb\bar{u}\bar{d}$ state and the instability of all-heavy tetraquarks.

1. Introduction

Despite the impression given by the recent flurry of studies dealing with multiquark states, flavor-exotic multiquarks have already a long history [1] and have motivated an abundant literature (see Ref. [2] for a recent compendium). In the pioneering work of Ader, Richard and Taxil [1] it was shown that $QQ\bar{q}\bar{q}$ four-quark configurations become more and more bound when the mass ratio $M_Q/m_q$ increases. The critical value of $M_Q/m_q$ for binding is somewhat model dependent.

Currently, a broad theoretical consensus about the existence of a stable axial vector doubly bottom tetraquark has been reached. Lattice QCD calculations find unambiguous signals for a stable $J^P = 1^+$ bottom-light tetraquark [3]. Based on a diquark hypothesis, Ref. [4] uses the discovery of the $\Xi^{++}_{cc}$ baryon to calibrate the binding energy in a $QQ$ diquark. Assuming that the same relation is true for the $bb$ binding energy in a tetraquark, it concludes that the axial vector $bb\bar{u}\bar{d}$ state is stable. The Heavy-Quark Symmetry analysis of Ref. [5] predicts the existence of narrow doubly heavy tetraquarks. Using as input for the doubly bottom baryons, not yet experimentally measured, the diquark-model calculations of Ref. [4] also lead to a bound axial vector $bb\bar{u}\bar{d}$ tetraquark. Other approaches, using Wilson twisted mass lattice QCD [6], also find a bound state. Few-body calculations using quark-quark Cornell-like interactions [7], simple color magnetic models [8], QCD sum rule analysis [9], or phenomenological studies [10] come to similar conclusions. More doubtful has become the prediction about the stability of all-heavy tetraquarks [11].

In the present note, we stress that a careful treatment of the few-body problem is required before drawing any conclusion about the existence of stable states in a particular model. There is, indeed, a dramatic spread of strategies: some authors use the full machinery of a variational method based on correlated Gaussians or hyperspherical expansion, and others use a crude trial wave function or a cluster approximation.
2. General results based on symmetry breaking

The analogy between the stability of few-charge systems and multiquarks in additive chromoelectric potentials offers a good guidance for identifying the favorable configurations. There are, however, some differences mainly due to the color algebra replacing the simpler algebra of electric charges. Unlike in the case of the positronium molecule, the equal-mass tetraquarks are unstable in the chromoelectric model with frozen color wave functions [12, 13]. In both the atom and quark cases, the four-body system and its threshold, after simple rescaling, are governed by a generic Hamiltonian

\[ H = \sum_{i} \frac{\vec{p}_i^2}{2m_i} - \text{c.o.m.} + \sum_{i<j} g_{ij} v(r_{ij}), \quad \sum_{i<j} g_{ij} = 2, \tag{1} \]

with \( v(r) = -1/r \) in the atomic case, and \( v(r) = -a/r + b r \) in the quark case [14].

In quantum mechanics, the minimum of a Hamiltonian containing a symmetric and an antisymmetric term is always lower than the minimum of the symmetric part. From this result, one can analyze the effect of symmetry breaking in systems of four-charged particles. Let us first consider the hydrogen molecule, \( M^+ M^- m^- m^- \). The Hamiltonian for this system reads,

\[ H = \frac{\vec{p}_1^2}{2 M} + \frac{\vec{p}_2^2}{2 M} + \frac{\vec{p}_3^2}{2 m} + \frac{\vec{p}_4^2}{2 m} + V = H_0 + H_1 \]

\[ = \left[ \sum_{i} \frac{\vec{p}_i^2}{2 \mu} + V \right] + \left( \frac{1}{4 M} - \frac{1}{4 m} \right) (\vec{p}_1^2 + \vec{p}_2^2 - \vec{p}_3^2 - \vec{p}_4^2), \tag{2} \]

where \( 2 \mu^{-1} = M^{-1} + m^{-1} \). The C-parity breaking term, \( H_1 \), lowers the ground state energy of \( H \) with respect to the \( C \)-parity even part, \( H_0 \), which is simply a rescaled version of the Hamiltonian of the positronium molecule. Since \( H_0 \) and \( H \) have the same threshold, and since the positronium molecule is stable, the hydrogen molecule is even more stable, and stability improves when \( M/m \) increases. Clearly, the Coulomb character of \( V \) hardly matters in this reasoning. The key property is that the potential does not change when the masses are modified.

One can use the same reasoning to study the stability of four-charged particles when \( C \)-parity is preserved but particle symmetry is broken, in other words the \( M^+ M^- m^- m^- \) configuration. The Hamiltonian is that of Eq. (2) by exchanging \( 2 \leftrightarrow 3 \). The same arguments used above lead to the conclusion that this configuration gains binding with respect to the threshold \( (M^+ m^-) + (M^- m^+) \) that it shares with \( H_0 \). However, there is another threshold that lies lower, \( (M^+ M^-) + (m^+ m^-) \). This threshold gains more from the symmetry breaking than the four-body molecule, and, indeed, it is found that the molecule becomes unstable for \( M/m \geq 2.2 \).

The above arguments can be directly translated to four-quark systems: the \( QQ\bar{q}\bar{q} \) configuration becomes more and more bound when the mass ratio \( M_Q/m_q \) increases. This has been first established in Ref. [1], and discussed and confirmed in further studies. Arguments based on diquarks, as e.g. [4], might considerably overestimate the binding, as analyzed in [2]. There are many variants of the so-called diquark model. An extreme point of view is that diquarks are almost-elementary objects, with their specific interaction with quarks and between them. In the case of doubly heavy baryons \( QQq \) there is obviously a \( QQ \) clustering which makes it tempting to use a two-step approach: first a \( (QQ) \) diquark and then a \( (QQ)q \) quasi-meson, as the diquark has the same color 3 as an antiquark. The exercise can be repeated for the \( QQ\bar{q}\bar{q} \) states. For simplicity, we consider only the case of a frozen 33 color wave function, i.e., the Hamiltonian (1). In Fig. 2, we compare the exact solution of (1) with the approximation consisting of first computing the \( QQ \) diquark with \( r_{12}/2 \) alone and \( qq \) with \( r_{34} \) alone, and then \( (QQ)(\bar{q}\bar{q}) \) as a meson with a potential \( r_{12,34} \) and constituent masses \( 2 M \) and \( 2 m \).

A remaining problem is to understand why the positronium molecule lies slightly below its dissociation threshold, while a chromoelectric model associated with the color additive rule does
3. Color dynamics.

In the heavy-quark limit, the lowest lying tetraquark configuration resembles the helium atom [5], a factorized system with separate dynamics for the compact color $3 Q Q$ nucleus and for the light quarks bound to the stationary color 3 state, to construct a $Q Q \bar{q} \bar{q}$ color singlet. This argument has been mathematically proved and numerically checked time ago [18], see the probabilities for the axial vector $b b \bar{u} \bar{d}$ tetraquark shown in Table II (note that the 66 probability in a compact $Q Q \bar{q} \bar{q}$ tetraquark tends to zero for $M_Q \rightarrow \infty$).

The $\tilde{\lambda}_i, \tilde{\lambda}_j$ model of Eq. (1), with a pairwise potential due to color-octet exchange, induces mixing between $33$ and $66$ states in the $Q Q - \bar{q} \bar{q}$ basis. If one starts from a $33$ state with $Q Q$ in a spin triplet, and, for instance $\bar{q} \bar{q} = \bar{u} \bar{d}$ with spin and isospin $S = I = 0$, then its orbital
Figure 2. Left panel: Effect of color-mixing on the binding of $QQ\bar{u}\bar{d}$. The tetraquark energy calculated with only the color $\bar{3}3$ configurations (dark curve) and with the 66 components (light grey curve). Right panel: Effect of the spin-spin interaction of the binding of $QQ\bar{u}\bar{d}$. The tetraquark energy calculated with (light grey line) and without (dark line) the chromomagnetic term. The faint dotted solid lines stand for the threshold. Figures are colored online.

wave function is mainly made of an $s$-wave in all coordinates. It can mix with a color $\bar{6}6$ with orbital excitations in the $\vec{x}$ and $\vec{y}$ linking $QQ$ and $\bar{q}q$, respectively. A minimal wave function in this sector can be chosen as:

$$\Psi_6 \propto \vec{x}.\vec{y} \exp(-a \vec{x}^2 - b \vec{y}^2), \quad \text{or} \quad \Psi_6 \propto \exp[-a_{12} \vec{x}^2 - a_{34} \vec{y}^2 - \alpha(\vec{r}_{13}^2 + \vec{r}_{24}^2) - \beta(\vec{r}_{14}^2 + \vec{r}_{23}^2)] - \{\alpha \leftrightarrow \beta\}. \quad (8)$$

To illustrate the role of color-mixing we use the potential AL1 [19]. Its central part is a Coulomb-plus-linear potential. Its spin-spin part is a regularized Breit-Fermi interaction, with a smearing parameter that depends on the reduced mass.

The energy as a function of $M/m$ without and with color-mixing is shown in the left panel of Fig. 2. The ground state of the $QQ\bar{u}\bar{d}$, candidate for stability with $J^P = 1^+$, has its main component with color $\bar{3}3$, and spin $\{1,0\}$ in the $QQ - \bar{u}\bar{d}$ basis. The main admixture consists of 66 with spin $\{1,0\}$ and an antisymmetric orbital wave function of which (8) is a prototype, and of 66 with spin $\{0,1\}$ with a symmetric orbital wave function. Note how the diquark hypothesis and color mixing have opposite effects that tend to cancel in the charm sector.

4. Spin-dependent corrections

In Ref. [20] it was acknowledged that, within current models, a pure additive interaction such as (1) will not bind $cc\bar{q}q$, on the sole basis that this tetraquark configuration benefits from the strong $cc$ chromoelectric attraction that is absent in the $Q\bar{q} + Q\bar{q}$ threshold. When $qq = ud$, there is in addition a favorable chromomagnetic interaction in the tetraquark, while the threshold experiences only heavy-light spin-spin interaction, whose strength is suppressed by a factor $m/M$.

For illustration, we use the again the potential AL1 [19]. The results are shown in the right panel of Fig. 2 for $QQ\bar{u}\bar{d}$, as a function of the mass ratio $M/m$. The system $bb\bar{u}\bar{d}$ is barely bound without the spin-spin term, though the mass ratio $m_b/m_q$ is very large. It acquires its binding energy of the order of 150 MeV when the spin-spin is restored. The system $cc\bar{u}\bar{d}$ is clearly unbound when the spin-spin interaction is switched off. This is shown here for the AL1 model, but this is true for any realistic interaction, including an early model by Bhaduri et al. [21]. The case of $cc\bar{u}\bar{d}$ is actually remarkable. Here the binding requires both the color mixing of 33 with 66, and the spin-spin interaction. Moreover, the binding is so tiny that it cannot be obtained with a simple variational method. One needs either a fully converged expansion on a basis of
correlated Gaussians, or a hyperspherical expansion up to a grand orbital momentum $K_{\text{max}}$ of the order of 12. Semay and Silvestre-Brac [19], who used the AL1 potential, missed the binding, but their method of systematic expansion on the eigenstates of an harmonic oscillator is not very efficient to account for the short-range correlations. Janc and Rosina [22] were the first to obtain binding with such potentials, and their calculation was checked in Ref. [7]. The stability of $cc\bar{u}\bar{d}$ with $J^P = 1^+$ is with respect to the nominal $DD^*$ threshold. Depending on its binding energy, it decays into $DD\pi$ or $DD\gamma$. The $bb$ analog decays weakly.

5. Conclusions

The four-body problem of tetraquarks is rather delicate, especially for systems at the edge of stability. The analogy with atomic physics is a good guidance to indicate the most favorable configurations. However, unlike the positronium molecule, the all-heavy configuration $QQQ\bar{Q}$ is not stable if one adopts a standard quark model and solve the four-body problem correctly. The mixing of the 33 and 66 color configurations is important, especially for states very near the threshold. This mixing occurs by both the spin-independent and the spin-dependent parts of the potential.

Approximations are welcome, especially if they shed some light on the four-body dynamics. The diquark-antidiquark approximation is not supported by a rigorous solution of the 4-body problem, but benefits of a stroke of luck, as the erroneous extra attraction introduced in the color 33 channel is somewhat compensated by the neglect of the coupling to the color 66 channel.

Finally, $cc\bar{u}\bar{d}$ with $J^P = 1^+$ is at the edge of binding within current quark models. For this state, all contributions should be added, in particular the mixing of states with different color structure, and the four-body problem should be solved with extreme accuracy. In comparison, achieving the binding of $bb\bar{u}\bar{d}$ looks easier. Still, with a typical quark model, the stability of the ground state cannot be reached if spin-effects and color mixing are both neglected.

Acknowledgments

Work funded by Ministerio de Economía, Industria y Competitividad and EU FEDER under Contract No. FPA2016-77177 and by Generalitat Valenciana PrometeoII/2014/066.

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