Gaussian process regression for forest attribute estimation from airborne laser scanning data

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Abstract

While the analysis of airborne laser scanning (ALS) data often provides reliable estimates for certain forest stand attributes – such as total volume or basal area – there is still room for improvement, especially in estimating species-specific attributes. Moreover, while information on the estimate uncertainty would be useful in various economic and environmental analyses on forests, a computationally feasible framework for uncertainty quantifying in ALS is still missing. In this article, the species-specific stand attribute estimation and uncertainty quantification (UQ) is approached using Gaussian process regression (GPR), which is a nonlinear and nonparametric machine learning method. Multiple species-specific stand attributes are estimated simultaneously: tree height, stem diameter, stem number, basal area, and stem volume. The cross-validation results show that GPR yields on average an improvement of 6.2\% in estimate RMSE over a state-of-the-art k-nearest neighbors (kNN) implementation, negligible bias and well performing UQ (credible intervals), while being computationally fast. The performance advantage over kNN and the feasibility of credible intervals persists even when smaller training sets are used.

Keywords: forest inventory, LiDAR, area based approach, machine learning, Gaussian process

1 Introduction

Forest inventories based on airborne laser scanning (ALS) are becoming increasingly popular. Therefore, it is more and more important to have well performing methods for the estimation/prediction of stand attributes, such as basal area and tree height. Coupled with the prediction procedures, efficient methods for the quantification of prediction uncertainty are also urgently needed for forestry planning and assessment purposes [Kangas et al. 2018].

Operational forest inventories employing ALS data are most often implemented with the area based approach (ABA) [Næsset 2002]. In ABA, metrics used as predictor variables are calculated from the ALS returns within a plot or grid cell. Using training plots with field-measured stand attributes, a model is formulated between the stand attributes and
ABA metrics. This statistical model is then used to predict the stand attributes for each grid cell (e.g., Maltamo et al. 2014, Reutebuch et al. 2005) and the predictions are finally aggregated to the desired area, e.g., to a stand. Although tree species is among the most important attributes of forest inventory, the ALS research does not particularly reflect this. One reason for this is that in many biomes the number of tree species is so high that it is practically impossible to separate them by remote sensing. In the Nordic countries, however, the majority of the growing stock comes from three economically valuable tree species. The species-specific prediction is approached two ways in Nordic countries: in Norway, stands are stratified according to tree species by visual interpretation of aerial images before the actual ALS inventory (Næsset 2004), whereas in Finland, stand attributes are predicted by tree species using a combined set of metrics from ALS data and aerial images (Packalen and Maltamo 2007). In both approaches, aerial images are used to improve the discrimination of tree species.

Uncertainty estimation is a key component in strategic inventories that cover large areas (Mandallaz 2007). ALS can be used in that context too. For example, ALS metrics can be used as auxiliary variables in model-based (e.g., Ståhl et al. 2010) or model assisted (e.g., Gregoire et al. 2010) estimation of some forest parameter. Typically, sample mean and sample variance are estimated to the area of interest (e.g., 1000000 ha) using a certain number (e.g., 500) of sample plots and auxiliary variables covering all population elements. In the stand level forest management inventories, the situation is different: the point estimate and its confidence intervals are needed for each stand and there may not be any sample plots in most stands. Today, most ALS inventories can be considered as stand level management inventories.

Commonly used prediction methods in ABA, such as linear regression or kNN, produce only point estimates without accompanying uncertainty metrics. Plot or cell level prediction uncertainty has garnered some research interest in recent years and several methods of predicting plot/cell level variance have been proposed (Finley et al. 2013, Junttila et al. 2008, Magnussen et al. 2016). Recently, a Bayesian inference approach to quantify uncertainty within the framework of the ABA was proposed by Varvia et al. (2017). The main shortcoming in the method proposed in Varvia et al. (2017) is that it is computationally costly: wall-to-wall uncertainty quantification of a large forest area would require considerable computer resources.

Gaussian process regression (GPR) (e.g., Rasmussen and Williams 2006) is a machine learning method that provides an attractive alternative; compared with the more widely used machine learning methods, such as artificial neural networks (e.g., García-Gutiérrez et al. 2016, Niska et al. 2010), GPR also produces an uncertainty estimate for the prediction. Univariate GPR was tested for estimation of several total stand attributes by Zhao et al. (2011), where it was found to significantly outperform (log)linear regression.
In this paper, we propose a multivariate GPR for simultaneous estimation of species-specific stand attributes within ABA. The estimation accuracy of GPR is compared with kNN and the uncertainty quantification performance with the Bayesian inference method of Varvia et al. (2017). Furthermore, the effect of training set size on its performance is evaluated.

2 Materials

The same test data as in Varvia et al. (2017) is used in this study. In this section, the data set is briefly summarized, for detailed description, see (e.g. Packalen et al., 2009, 2012). The test area is a managed boreal forest located in Juuka, Finland. The area is dominated by Scots pine (Pinus sylvestris L.) and Norway spruce (Picea abies (L.) Karst.), with a minority of deciduous trees, mostly downy birch (Betula pubescens Ehrh.) and silver birch (Betula pendula Roth.). The deciduous trees are considered as a single group.

The field measurements were done during the summers of 2005 and 2006. Total of 493 circular sample plots of radius 9 m are used in this study. The diameter at breast height (DBH), tree and storey class, and tree species were recorded for each tree with DBH larger than 5 cm and the height of one sample tree of each species in each storey class was measured. The heights of other trees on the plot were predicted using a fitted Näslund’s height model (Näslund, 1937). The species-specific stand attributes were then calculated using the measured DBH and the predicted heights. The stand attributes considered in this study are tree height ($H_{gm}$), diameter at breast height ($D_{gm}$), stem number ($N$), basal area ($BA$), and stem volume ($V$).

The ALS data and aerial images were captured in 13 July 2005 and 1 September 2005, respectively. The ALS data has a nominal sampling density of 0.6 returns per square meter, with a footprint of about 60 cm at ground level. The orthorectified aerial images contain four channels (red, green, blue, and near infrared). A total of $n_x = 77$ metrics were computed from the ALS point cloud and aerial images and used in ABA. The metrics include canopy height percentiles, the corresponding proportional canopy densities, the mean and standard deviation of the ALS height distribution, the fraction of above ground returns (i.e. returns with $z > 2$ m), and metrics computed from the LiDAR intensity. From the aerial images, the mean values of each channel were used along with two spectral vegetation indices (Packalen et al., 2009).
3 Methods

Let us denote a vector consisting of the stand attributes by $y \in \mathbb{R}^{15}$; the vector $y$ contains the species-specific (pine, spruce, deciduous) $H_{gm}$, $D_{gm}$, $N$, $BA$, and $V$, resulting in a total of $n_y = 15$ variables. The vector of predictors (ALS and aerial image metrics) is denoted by $x \in \mathbb{R}^{n_x}$.

The general objective is to learn a nonlinear regression model

$$y = f(x) + e,$$

where $e$ is an error term, from a set of $n_t$ training data $(Y_t, X_t)$.

Let $Y$ be a finite collection of points $y^{(i)} = f(x^{(i)}) + e^{(i)}$ concatenated in a long vector. In Gaussian process regression (e.g. Rasmussen and Williams, 2006), the joint probability distribution of these points $Y$ is modeled as a multivariate normal distribution, with mean $\mu_y$ and covariance $\Gamma_y$ written as functions of $x$:

$$\mu_y = m(x) = \mathbb{E}\{f(x)\},$$

$$\Gamma_y = K(x, x') = \mathbb{E}\{(f(x) - m(x))(f(x') - m(x'))^T\}.$$  

Let now $Y = [Y_t \ y^*]^T$, that is, $Y$ a vector consisting of the training data $Y_t$, and a new point $y^*$ which we want to estimate, using the corresponding measurement $x^*$. For simplification, we set $m(x) = 0$. The mean term mostly affects the behavior when extrapolating far away from the space covered by the training data. The joint distribution of $Y$ is then

$$[Y_t \ y^*] \sim \mathcal{N}\left(0, \begin{bmatrix} K(X_t, X_t) + \mathbf{E} & K(x^*, X_t)^T \\ K(x^*, X_t) & K(x^*, x^*) \end{bmatrix}\right),$$

where $\mathbf{E}$ is describes the covariance of the error $e$, i.e. uncertainty of the training data. In this work, we use $\mathbf{E} = 0.1 \mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix that contains the sample variances of the training data $Y_t$ on the main diagonal. For brevity, following shorthand notations are introduced:

$$K = K(X_t, X_t) \in \mathbb{R}^{n_y n_t \times n_y n_t} \quad (5)$$

$$K_* = K(x^*, X_t) \in \mathbb{R}^{n_y \times n_y n_t} \quad (6)$$

In GPR, the kernel matrices $K$ and $K_*$ are constructed based on a covariance function. In this study we use stationary Matérn covariance function with $\nu = 3/2$, fixed length scale $l = 10$, and $\sigma = 1$:

$$k(x, x') = \left(1 + \frac{\sqrt{3}d(x, x')}{10}\right) \exp\left(-\frac{\sqrt{3}d(x, x')}{10}\right),$$

where $d(x, x') = \sqrt{d_x^2 + d_y^2}$.


where the distance metric \( d(x, x') \) is the Euclidean distance. The covariance function \( k(x, x') \) describes the covariance between the vectors \( x \) and \( x' \) based on the distance between the vectors. The covariance function is the core component of GPR that specifies properties such as smoothness of the regressor.

The covariance function \( k(x, x') \) is used to construct univariate kernel matrices

\[
K(i, j) = k(x_t^{(i)}, x_t^{(j)}) \in \mathbb{R}^{n_t \times n_t} \tag{8}
\]

\[
K_s(1, j) = k(x^*, x_t^{(j)}) \in \mathbb{R}^{1 \times n_t}. \tag{9}
\]

To get from the univariate kernels to multivariate kernels used in \( \mathbf{4} \), the so-called separable kernel \( \text{[Bonilla et al.] 2008} \) is used:

\[
\mathbf{K} = \Gamma_y \otimes \mathbf{K} \tag{10}
\]

\[
\mathbf{K}_s = \Gamma_y \otimes \mathbf{K}_s, \tag{11}
\]

where \( \otimes \) is the Kronecker product, and \( \Gamma_y \in \mathbb{R}^{n_y \times n_y} \) is a (prior) covariance for \( y \). In this work, \( \Gamma_y \) is approximated by the sample covariance of \( Y_t \). It should be noted, that \( \sigma = 1 \) is chosen in the kernel function \( \text{[7]} \), because the (prior) variances of \( y \) are added to the covariance kernel in this step through \( \Gamma_y \).

From the joint density \( \text{[4]} \), the conditional density of \( y^* \) given the training data and the measurement \( x^* \) is

\[
y^* \mid Y_t, x_t, x^* \sim \mathcal{N}(\mathbf{m}(y^*), \Gamma_{y^*}), \tag{12}
\]

where

\[
\mu_{y^*} = \mathbf{K}_s(\mathbf{K} + \mathbf{E})^{-1}y_t \tag{13}
\]

\[
\Gamma_{y^*} = \Gamma_y - \mathbf{K}_s(\mathbf{K} + \mathbf{E})^{-1}\mathbf{K}_s^T. \tag{14}
\]

The predictive mean \( \mu_{y^*} \) is now the point estimate for the unknown vector of stand attributes \( y^* \) and \( \Gamma_{y^*} \) provides the estimate covariance.

Unlike kNN and certain other machine learning methods, GPR extrapolates outside the training data. As an unwanted side effect of this extrapolation behavior, GPR can produce unrealistic negative predictions for the stand attributes. Several statistically rigorous methods for constraining the GPR predictions have been proposed (e.g. \cite{Da Veiga and Marrel 2012} \cite{Jidling et al. 2017}), but these methods increase the computational cost significantly. Here, negative predictions are here simply set to zero.
3.1 Reference methods

The GPR point estimates are compared with a state-of-the-art kNN algorithm. We select ten predictors from the (transformed) data using a simulated annealing-based optimization approach of Packalen et al. (2012) and use the most similar neighbor (MSN) method for selecting the neighbors. The number of neighbors is chosen to be $k = 5$, as in Packalen et al. (2009, 2012). The predictor selection is done using the whole data set and leave-one-out cross-validation.

The prediction credible intervals provided by GPR are compared with the Bayesian inference approach (Varvia et al., 2017). In the Bayesian approach the posterior density:

$$
\pi(y|x) \propto \begin{cases} 
N(x|\hat{A}\phi(y) + \hat{\mu}_e|y, \hat{\Gamma}_e|y)N(y|\hat{\mu}_\theta, \hat{\Gamma}_y), & y \geq 0 \\
0 & y < 0,
\end{cases}
$$

is constructed based on the training data and the new measurement. The model matrix $\hat{A}$, conditional (residual) error statistics $\hat{\mu}_e|y$ and $\hat{\Gamma}_e|y$, and the prior statistics $\hat{\mu}_\theta$ and $\hat{\Gamma}_y$ are learned from the training data. The density (15) is then sampled using a Markov chain Monte Carlo method. The point estimate and 95% credible intervals are then calculated from the samples.

3.2 Performance assessment

The proposed GPR method is first evaluated using leave-one-out cross-validation (i.e. $n_t = 492$). From the results, relative root mean square error (RMSE%), relative bias (bias%), and credible interval coverage (CI%) are calculated. Credible interval coverage is the percentage of the test plots where the field measured value of a stand attribute lies inside the computed 95% prediction interval; CI% thus has the ideal value of 95%.

In addition to conducting a leave-one-out cross-validation, the effect of the number of training plots is evaluated. Training set sizes from $n_t = 20$ to $n_t = 400$ are tested with a stepping of 20. The cross-validation is performed by first randomly sampling $n_t$ plots to be used as a training set and then randomly selecting a single test plot from the remaining 493$−n_t$ plots. This procedure is repeated 2000 times for each $n_t$ value. This way the number samples for each tested $n_t$ stays constant. The effect of training set size is only evaluated for GPR and kNN, due to the high computational cost of the reference Bayesian inference approach.
4 Results and discussion

4.1 Species-specific attributes

The RMSE\% comparison between the GPR predictions, kNN and Bayesian inference is shown in the Figure 1 for all estimated stand attributes. The numerical RMSE\% value is shown above each bar. For pine, which is the dominant species in the study area, the GPR and kNN estimates have fairly equivalent performance: GPR is slightly better for all the stand attributes except height and basal area. The Bayesian linear inference estimates are notably worse. In the minority species (spruce and deciduous), the GPR estimates have consistently better RMSE\% than kNN or Bayesian linear. On average, the relative improvement over kNN is 6.2%.

Figure 2 shows a similar comparison of relative bias between the evaluated methods for all
the estimated stand attributes. The numerical bias% value is printed for each bar. GPR estimates show smaller than 2% absolute bias for all the stand attributes, except the spruce basal area and volume. kNN shows small bias in the spruce attributes, but has a large bias in deciduous basal area and volume. The Bayesian linear results show notable bias in $N$, $BA$, and $V$.

Figure 2: Relative bias of Bayesian linear, GPR, and kNN estimates for pine (top), spruce (middle), and deciduous (bottom). Numerical values are shown on top of the bars.

The CI coverages of GPR and the reference Bayesian inference method are compared in Figure 3. The numerical CI% value is shown above each bar; the ideal value is here 95%. Both the Bayesian linear and GPR estimates fall short of the 95% target, that is, both produce prediction intervals that are too narrow. In general, the performance is roughly similar, but varies between stand attributes. GPR intervals seem to be the most overconfident on height and diameter; this is most evident in the deciduous variables.
Figure 3: CI% of Bayesian linear and GPR estimates for pine (top), spruce (middle), and deciduous (bottom). Numerical values are shown on top of the bars.

4.2 Total attributes

Point estimates and credible intervals for the total stem number, basal area, and stem volume were calculated from the species-specific results. The point estimates were computed by summing up the corresponding species-specific estimates. The GPR prediction interval for the total attributes is acquired from the prediction covariance $\mathbf{\Gamma}_{y_\ast}$, because summation is a linear transformation.

The results for the total attributes are shown in Figure 4. In RMSE%, GPR estimates show the best performance. Bayesian linear estimates have lower RMSE% than kNN in the total basal area and volume, while kNN is better in the stem number. In the relative bias, GPR has fairly low bias and performs worst in the total stem volume. kNN has consistent slight bias, while the Bayesian linear estimates show large bias in the stem number. In credible interval coverage, GPR performs well in the total basal area and volume, producing nearly ideal interval width. The performance in the stem number is worse and roughly equal to
the Bayesian linear method.

4.3 Effect of training set size

RMSE% versus training set size is shown in Figure 5. The dashed minimum line corresponds to the species-specific stand attribute with the lowest RMSE%, maximum to the stand attribute with the highest RMSE%, and mean is the average over the stand attributes. As expected, the RMSE% increases for both methods when the training set size decreases. GPR keeps the performance advantage over kNN even when using smaller training sets. The improvement in performance for training sets larger than c. 200 plots is fairly small.

Figure 6 shows the relative bias as a function of the training set size. When the training set size decreases, the bias increases in both positive and negative directions, but with a general negative tendency. Smaller training sets are less likely to cover the full range of variation of the stand attributes in the population, which results in underestimation of large values: this would explain the observed tendency in bias. The largest negative biases produced by
Figure 5: Lowest, average, and the highest relative RMSE as a function of training set size for GPR and kNN estimates.

kNN are consistently larger than in GPR.

Figure 7 shows the CI% of the GPR estimates versus the training set size. The average CI% increases slightly as training set size is decreased, the lowest CI% increases considerably, while the highest CI% drops somewhat. The generally too narrow credible intervals signify overconfidence in the predictions, which implies that either the GP model is not optimal in its current formulation, or the stand attributes do not sufficiently explain the variation in the predictors. The latter explanation might cause that there are usually contradicting training data (i.e. training points that are close in the stand attribute space, but distant in the predictor space) in large training sets, which might partly explain the slight improvement of CI% when training set size decreases. With the Bayesian inference approach, on the contrary, a substantial drop in CI% in smaller training set sizes would be expected based on the results by Varvia et al. (2017).
4.4 Discussion

Conceptually, GPR is a non-parametric machine learning method that has similarities with kNN. Thus, many approaches proposed for improvement of kNN estimates within ABA could be also utilized to further improve GPR estimates. GPR seems to be insensitive to multicollinearity and quite large numbers of predictors can been used simultaneously (Zhao et al., 2011). In this paper, fairly traditional ABA metrics were used, adding additional predictors, such as $\alpha$-shape (Vauhkonen et al., 2008) or composite metrics (Zhao et al., 2009), could potentially improve prediction performance. Additionally, dimension reduction, for example by using principal component analysis (Junttila et al., 2015), would probably improve performance when using small training sets.

The prediction step of GPR is not computationally much more costly than using kNN. The most computationally expensive part is the GPR model training, which requires computing
Figure 7: Lowest, average, and the highest CI% as a function of training set size for GPR estimates.

the matrix inverse of a large matrix (see the equations (13) and (14)). However, the matrix inverse can be precomputed for a given set of training data. After this, computing the prediction and the prediction interval only requires calculating matrix products.

The present work used fixed length scale, covariance function, and error magnitude, because finding the optimal values for these (hyper)parameters automatically is generally a nonconvex and computationally difficult optimization problem. The values, $l = 10$ for the correlation length, and 10% variance for the error $e$, were found by manual testing. The unoptimal choice of these parameters might explain some of the tendency to produce too narrow prediction intervals. Additionally, several commonly used covariance functions were tested; Matérn 3/2 covariance function was found to be the best performing. Further research is still needed on finding the optimal model formulation.

Due to extrapolation, GPR can produce unrealistic negative predictions. In this study,
these negative values were simply set to zero. The truncation affected mostly the bias, which increased in the positive direction 0-5%. The positive bias% values of the GPR (Figure 2) predictions for basal area and volume of spruce resulted from the truncation. The effect on CI% was negligible.

In this study, GPR showed better reliability in all considered stand attribute predictions except the mean height of pine (the RMSE of basal area predictions of pine were practically the same for GPR and kNN) and the relative improvement of GPR predictions over the state-of-the-art method kNN were rather large being on average 6.2%. This is contrary to the earlier studies where the reliability of ALS based forest inventory system has been examined by comparing different estimation methods. For example, Maltamo et al. (2015) used visual pre-classification of aerial images to divide the study data into strata according to the main tree species and stand development stages. The aim was to improve species-specific estimates by applying more homogeneous reference data in kNN but the results were contradictory. The pre-classification did improve the accuracy of some species-specific stand attributes compared to kNN estimates, which applied whole study data as reference but in each attribute case some species-specific estimates were more accurate without pre-classification. It is also notable that usually the accuracy of minor tree species did not improve whereas in the present study the improvement was substantial especially for the minor species.

Similar contradictory results have been obtained when comparing different statistical methods, such as neural networks or Bayesian approach (e.g. Niska et al., 2010; Varvia et al., 2017). For example, Niska et al. (2010) obtained more accurate species-specific volume estimates using neural networks at plot level than kNN but on the other hand kNN was more accurate on the stand level. Räty et al. (2018) compared kNN estimates in which the species-specific estimates were obtained either by simultaneous imputation for all the species (as in this study) or by separate imputation for each species. The results concerning separate imputations were promising, but again, the results were contradictory.

5 Conclusions

In this article, the feasibility of Gaussian process regression for the estimation of species-specific stand attributes within the area based approach was evaluated. In addition to testing the prediction performance, the prediction credible intervals were also evaluated. GPR estimates were compared with a state-of-the-art kNN-based algorithm and a linear Bayesian inference based method. The effect of training set size on the performance was also examined.
The GPR estimates showed on average a 6.2% relative improvement in RMSE over the reference kNN method in the leave-one-out cross-validation, generally smaller bias, and credible interval performance on par with the linear Bayesian inference. The GPR estimates kept the advantage even when tested using smaller training set sizes. Especially the credible interval performance proved robust with respect to the training set size.

The excellent performance of GPR, the low computational cost that is in the same scale as kNN, and that it provides prediction intervals make GPR an attractive method to use in forestry applications. Especially the plot level prediction uncertainty information provides many potential improvements in forest planning.

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