Lorentz Invariance Violation and the Spectrum and Source Power of Ultrahigh Energy Cosmic Rays

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Abstract

Owing to their isotropy, it is generally believed that ultrahigh energy cosmic rays (UHECRs) are extragalactic in origin. It is then expected that interactions of these cosmic rays with photons of the cosmic background radiation (CBR) should produce a drastic reduction in their flux above an energy of about $5 \times 10^{19}$ eV (50 EeV), the so-called “GZK effect”. At present, the existence of this effect is uncertain owing to conflicting observational data and small number statistics. We show that a small amount of Lorentz invariance violation (LIV), which could turn off photomeson interactions of UHECRs with the CBR, could explain the UHECR spectrum as measured by AGASA which shows an excess of UHECRs at energies above 100 EeV. If new results from the Auger array agree with the AGASA spectrum, this may be interpreted as evidence for a small amount of LIV. If, on the other hand, the new results are consistent with the HiRes results favoring a GZK effect, this would place severe constraints on LIV and, by implication, on some Planck scale quantum gravity models. We also discuss the power requirements needed to explain the UHECR spectrum for a range of assumptions, including source evolution and LIV and show that in all cases our results disfavor a $\gamma$-ray burst origin for the UHECRs.

Key words: cosmic rays; Lorentz invariance; quantum gravity; gamma-ray bursts

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1 Introduction

Because of their extreme energy and isotropic distribution, it is believed that UHECRs are extragalactic in origin. Observations of UHECRs having energies in excess of $10^{20}$ eV (100 EeV) present difficult puzzles with regard to their origin, their nature, and their propagation [1]. After the discovery of the cosmic background radiation (CBR), Greisen [2] and Zatsepin and Kuzmin [3] pointed out that photomeson interactions should deplete the flux of cosmic rays with energies above $\sim 50$ EeV. One of us [4], using data on the energy dependence of the photomeson production cross section, then made a quantitative calculation of this “GZK effect” deriving the mean photomeson energy loss attenuation length for protons as a function of proton energy. These results indicated that the attenuation length of a proton with an energy greater than 100 EeV is less than 100 Mpc which is much less than the visible radius of the universe (see Figure 1 [4]). Thus, what is sometimes referred to as the GZK “cutoff” is not a true cutoff, but a suppression of the ultrahigh energy cosmic ray flux arising from a limitation of proton propagation through the cosmic background radiation.

More recently, ground based detectors have detected a double digit number of events with energies above 100 EeV, placing the existence of the GZK effect somewhat in doubt [5]-[10]. This is also because of a prima facie conflict between the observational results [6], [8] and is probably due, at least in part, to poor statistics [11]. The need to increase the number of observed events in order to improve the determination of the UHECR spectrum, particularly above 100 EeV, has resulted both in the development of the more sensitive Auger detector array [12] and the proposal of even more sensitive space-based detector missions such as EUSO [13] and OWL [14].

2 Violating Lorentz Invariance

With the idea of spontaneous symmetry breaking in particle physics came the suggestion that Lorentz invariance (LI) might be weakly broken at high energies [15]. Although no true quantum theory of gravity exists, it was suggested that LI might be violated in such a theory with astrophysical consequences [16]. It was also suggested that a possible natural abrogation of the GZK effect, resulting in the existence of significant fluxes of UHECRs at trans-GZK energies, could be caused by a small amount of Lorentz invariance violation (LIV) [15], [17], [18]. Recent work has placed strong constraints on such LIV, constraints which may have important implications for some quantum gravity and large extra dimension models [19]-[23]. However, only a very small amount of LIV could substantially curtail the photomeson production and $e^+e^-$ pair...
production that result from the high energy protons interacting with the CBR.

In this paper we investigate the observational implications of this possible effect of a very small amount of LIV, viz., that cosmic rays could indeed reach us after originating at distances greater than 100 Mpc without undergoing large energy losses from photomeson and pair production interactions. Such a small violation of Lorentz invariance might be produced by Planck scale effects [24], [25].

3 Kinematical Considerations

A simple formulation for breaking LI by a small first order perturbation in the free particle Lagrangian which leads to a renormalizable treatment has been given by Coleman and Glashow [18]. Using this formalism, these authors point out that different particles can have different maximum attainable velocities (MAVs) which can be different from c. If we denote the MAV of a particle of type $i$ by $c_i$ and the difference $c_i - c_j \equiv \delta_{ij}$ then Coleman and Glashow have shown that for interactions of protons with CBR photons of energy $\epsilon$ and temperature $T_{CBR} = 2.73$ K, pion production ($p + \gamma \rightarrow N + \pi$’s) is kinematically forbidden to take place and thus CBR photomeson production interactions are turned off if

$$\delta_{p\pi} > 5 \times 10^{-24} (\epsilon/T_{CBR})^2. \tag{1}$$

The condition for significantly increasing the threshold for electron-positron pair production interactions ($p + \gamma \rightarrow p + e^+ + e^-$) is given by [25]

$$\delta_{e\gamma} > \frac{(m_p + m_e)m_p}{E_f^2}. \tag{2}$$

which is $O(10^{-22})$ for a fiducial energy $E_f = 100$ EeV. The relation given by eq. (2) is similar to that derived for $\gamma\gamma$ pair production in Ref. [19]. These values are much smaller than the constraint $\delta_{e\gamma} < \sim 10^{-15}$ found in Ref. [19] and also smaller than the conditional constraint of $\delta_{e\gamma} < O(10^{-20})$ derived in Ref. [26]. Thus, given even a very small amount of LI violation, both photomeson and pair-production interactions of UHECR with the CBR can be turned off.
4 Calculations

We consider a power-law source spectrum that will be capable of explaining the UHECR data after propagation effects are considered. We first describe our model for propagating the high energy protons. We then calculate the effects of violating Lorentz invariance on the photomeson and pair production energy loss mechanisms resulting from the high energy protons interacting with the CBR. We derive and show the predicted spectra to be observed at the Earth that result from including LIV effects in our models. We end by giving the power density requirements of the source spectrum needed to produce the observed UHECR flux.

We first reconsider the standard picture for the propagation of high energy protons, including energy losses resulting from cosmological redshifting, pair production and pion production through interactions with cosmic background radiation (CBR) photons (see above). We shall assume for this calculation a flat ΛCDM universe with a Hubble constant of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, taking $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. [27]

The energy loss owing to redshifting for a ΛCDM universe is then given by

$$-(\partial \log E/\partial t)_{\text{redshift}} = H_0[\Omega_m(1 + z)^3 + \Omega_\Lambda]^{1/2}. \quad (3)$$

The combined energy loss rate for both pion and pair production for protons in collisions with photons of the 2.7 K background at the present epoch ($z = 0$) is defined as

$$-(\partial \log E/\partial t)_{\gamma p} \equiv r_{\gamma p} = r_\pi(E) + r_{e^+e^-}(E), \quad (4)$$

where the energy loss rates $r(E)$ are defined at $z = 0$. The photon CBR number density increases as $(1 + z)^3$ and the CBR photon energies increase linearly with $(1 + z)$. The corresponding energy loss for protons at any redshift $z$ is thus given by

$$r_{\gamma p}(E, z) = (1 + z)^3r[(1 + z)E]. \quad (5)$$

Taking a flat Einstein-de Sitter universe with $\Lambda = 0$ model does not significantly affect the results.
We take the photomeson loss rate, \( r_{\pi}(E) \), from Ref. [4] and the pair-production loss rate, \( r_{e^+e^-}(E) \) from Ref. [28]. Figure 1 shows the proton energy loss rates at \( z = 0 \) as a function of energy.

Let \( q(E_i, z) \) be the volume emissivity of particles produced by UHECR sources at redshift \( z \) at an initial energy \( E_i \). We assume that \( q(E_i, z) \) has a power-law energy dependence of the form

\[
q(E_i, z) = K(z) E_i^{-\Gamma} \tag{6}
\]
Fig. 2. The “maximum” energy a proton should be observed with at Earth (i.e., “the GZK cutoff energy”) if it is produced at redshift $z$.

We further assume a redshift evolution of the volume emissivity of the UHECR sources to be of a form similar to that of the star formation rate, viz.,

$$K(z) = K(0)(1 + z)^{(3 + \zeta)}.$$  \hspace{1cm} (7)
We first calculate the initial energy \( E_i(z) \) at which a proton is created at a redshift \( z \) whose observed energy today is \( E \) following the methods detailed in Refs. [29] and [30]. We neglect the effect of possible small intergalactic magnetic fields on the paths of these ultrahigh energy protons and assume that they will propagate along straight lines from their source. We then compute the “GZK energy” for UHECRs versus redshift of origin using the ΛCDM model with the cosmological parameters given above. This is defined as the energy at which the spectrum drops to \( 1/e \) of its original value owing to photomeson production. We have updated our result from Ref. [30] to reflect the ΛCDM cosmology used in this paper and the new result is given in Figure 2. It can be seen from Figure 2 that the protons of energy above 10 EeV can not have originated further than a redshift of \( z \sim 0.4 \).

The observed ultrahigh energy cosmic ray spectrum flattens above 3 EeV [31], [5]. This flattening is usually interpreted as being caused by the emergence of a harder extragalactic component which dominates the spectrum at energies above 3 EeV, whereas below this energy the observed spectrum is dominated by a more steeply falling galactic component.

In our calculations we assume a power law extragalactic UHECR source spectrum with a value for \( \Gamma \) of 2.6 which, even after modification by energy losses, dominates above \( \sim 3 \) EeV.⁵ For this reason, and because the UHECR data have smaller error bars at lower energies, we have chosen to normalize our propagation-modified extragalactic component at 3 EeV. Also, if LIV turns off the GZK effect, the maximum observable proton energy is no longer limited by it. Because the only significant energy loss for protons with energy above 500 EeV then comes from redshifting (see Figure 1), a power-law source spectrum will produce a power-law UHECR spectrum with the same spectral index. Therefore, for the purposes of our calculations, we assumed an arbitrary maximum UHECR energy of 500 EeV in the source spectrum. However, in the case of LIV with a positive value of \( \delta_{\gamma\gamma} \sim O(10^{-23}) \), the maximum proton energy can be limited by the vacuum Čerenkov effect [18],[19].

As in ref. [30], we have considered two scenarios for the UHECR emissivity corresponding to different values for the redshift evolution index, \( \zeta \), viz., (1) a constant comoving source density distribution \( (\zeta = 0) \), i.e. no source evolution, [3]

³ We have tried using a flatter source spectrum with \( \Gamma = 2.35 \), but that this does not produce a spectrum that fits the observational data.
Fig. 3. Predicted spectra for an $E^{-2.6}$ source spectrum with source evolution (see text) shown with pair-production losses included and photomeson losses included (solid curve) and turned off (dashed curve). The curves are shown with ultrahigh energy cosmic ray spectral data from Fly's Eye (triangles), AGASA (circles), and HiRes monocular data (squares). They are normalized to the data at 3 EeV (see text).

out to a maximum redshift, $z_{\text{max}} = 6$, and (2) an emissivity evolution with a redshift dependence proportional to the star formation rate, taken to be a source luminosity evolution corresponding to $\zeta = 3.6$ for $0 < z < 2$ with no further evolution out to $z_{\text{max}} = 6$ [32].
Fig. 4. Predicted spectra for an $E^{-2.6}$ source spectrum with source evolution (see text) shown with pair-production losses turned off and with photomeson production losses included (dashed curve) and turned off (dot-dashed curve). The curves are shown with ultrahigh energy cosmic ray spectral data from *Fly’s Eye* (triangles), *AGASA* (circles), and *HiRes* monocular data (squares). They are normalized to the data at 3 EeV (see text.)

We next incorporate the potential effects of LIV on the proton energy loss rate into our model. In the first case (A) considered, this is implemented by nullifying the energy loss from photomeson production as a possible consequence of LIV. In addition to this case, we present the results obtained for two other possible scenarios *viz.*, (B) the case where the pair production energy loss mechanism is nullified but not that from photomeson production
Fig. 5. Predicted spectra for an $E^{-2.6}$ source spectrum with no evolution (see text) shown with pair-production losses included and photomeson losses included (solid curve) and turned off (dashed curve). The curves are shown with ultrahigh energy cosmic ray spectral data from Fly’s Eye (triangles), AGASA (circles), and HiRes monocular data (squares). They are normalized to the data at 3 EeV (see text).

and, (C) the case where the energy losses from both photomeson production and pair production are nullified. In all of these cases, we have imposed an arbitrary cutoff energy of 500 EeV on the postulated source spectrum.
Fig. 6. Predicted spectra for an $E^{-2.6}$ source spectrum with no evolution (see text) shown with pair-production losses turned off and with photomeson production losses included (dashed curve) and turned off (dot-dashed curve). The curves are shown with ultrahigh energy cosmic ray spectral data from Fly’s Eye (triangles), AGASA (circles), and HiRes monocular data (squares). They are normalized to the data at 3 EeV (see text.)

5 Conclusions

The results of our calculations, along with the spectra obtained for the standard LI case, are given in Figures 3 and 4 for the source evolution model and Figures 5 and 6 for the no evolution ($\zeta = 0$) model. All of these figures also
show the HiRes [8], Fly’s Eye [7] and AGASA [6] data. These spectra are normalized at an energy of 3 EeV above which energy the extragalactic cosmic ray component is assumed to be dominant and below which we assume that the galactic cosmic ray component dominates (see, e.g., Ref. [1]). The results shown are calculated for “on” and “off” energy losses for both photomeson production and pair production for protons. As can be seen in Figures 3 and 5, the LI case fits the HiRes spectrum, while the LIV case fits the AGASA spectrum.

It can also be seen from Figs. 3 - 6 that, in principle, a very small amount of LI violation can eliminate the GZK “cutoff”. Also, when pair-production is turned off, the ~ 10 EeV “bite” in the predicted spectrum is eliminated. Of course, when both interactions are turned off, all of the features in the predicted spectrum disappear and only a power-law remains. Contrary to the discussion of Ref. [25], as can be seen from the figures, the present data cannot be used to put constraints on LI violation in pair-production interactions.

The UHECR source power density requirements for the source evolution and no evolution cases are different. The absolute minimum power density required is found by integrating the source spectrum times particle energy from 3 EeV to 500 EeV. Since we have normalized this spectrum at 3 EeV where photomeson production is unimportant, we only have four cases to consider:

(1) With source evolution and no pair production losses, we find that the minimum local \((z = 0)\) power density required is \(1.2 \times 10^{30} \) W Mpc\(^{-3}\).

(2) With source evolution and including pair production losses, we find that the local minimum power density required is \(1.5 \times 10^{31} \) W Mpc\(^{-3}\).

(3) With no evolution and no pair production losses, we find that the local minimum power density required is \(7.7 \times 10^{30} \) W Mpc\(^{-3}\).

(4) With no evolution and including pair production losses, we find that the local minimum power density required is \(2.2 \times 10^{31} \) W Mpc\(^{-3}\).

In all cases, we note that the local minimum power density required is orders of magnitude larger than that estimated if the sources are \(\gamma\)-ray bursts and if we equate their power output in UHECRs with their power output in \(\gamma\)-rays. \(\text{viz.}, \sim 2 \times 10^{28} \) to \(\sim 7 \times 10^{28} \) W Mpc\(^{-3}\) [33], [34]. Of course, the extragalactic component should naturally extend below 3 EeV. If it does so with \(\Gamma = 2.6\), this can further increase the source energy requirements.

We consider the source evolution case to be a more probable scenario as the frequency of astrophysical production sites, regardless of specific mechanism,
is likely to be closely tied to the star formation rate for various reasons.

If the GZK effect is confirmed, eq. (1) places a very strong constraint on the LIV parameter $\delta_{p\pi}$. If the pair-production “bite” is also clearly exhibited in data with better statistics, eq. (2) then similarly constrains $\delta_{ep}$.

If, on the other hand, LIV is the explanation for a missing GZK effect, one can also look for the absence of a potential “pileup” spectral feature at $\sim 70$ EeV [35] and for the absence of ultrahigh energy photomeson neutrinos. Such additional future observational tests should decide the issue.

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References

[1] F.W. Stecker, J. Phys. G 29 (2003) R47, e-print astro-ph/0309027.
[2] K. Greisen, Phys. Rev. Letters 16 (1966) 748.
[3] G.T. Zatsepin and V.A. Kuz’min, Zh. Eks. Teor. Fiz., Pis’ma Red. 4 (1966) 144.
[4] F.W. Stecker, Phys. Rev. Letters 21 (1968) 1016.
[5] M. Nagano and A.A. Watson, Rev. Mod. Phys. 72 (2000) 689.
[6] M. Takeda, et al., Astropart. Phys 19 (2003) 447.
[7] D.J. Bird et al., Astrophys. J. 441 (1994) 144.
[8] R.U. Abbasi et al., Phys. Rev. Letters 92 (2004) 151101.
[9] L. Anchordoqui and H. Goldberg, Phys. Letters B583 (2004) 213.
[10] S.P. Knurenko et al., e-print astro-ph/0411484.
[11] D. De Marco, P. Blasi and A.V. Olinto, Astroparticle Phys. 20 (2003) 53.
[12] J. Swain, Proc. CIPANP, in press, e-print astro-ph/0309515.
[13] L. Scarsi, Nuovo Cimento 24C (2001) 471.
[14] F.W. Stecker et al. Nucl. Phys. B 136C (2004) 433, e-print astro-ph/0408162.
[15] H. Sato and T. Tati, Progr. Theor. Phys. 47 (1972) 1788.
[16] G. Amelino-Camilia et al., *Nature* **393** (1998) 763.
[17] D.A. Kirzhnits and V.A. Chechin, * Yad. Fiz.* **15** (1972) 1051.
[18] S. Coleman and S.L. Glashow, *Phys. Rev.* **D59**, (1999) 116008.
[19] F.W. Stecker and S.L. Glashow, *Astroparticle Phys.* **16** (2001) 97.
[20] F.W. Stecker, *Astroparticle Phys.* **20** (2003) 85.
[21] T. Jacobson, S. Liberati and D. Mattingly, *Nature* **424** (2003) 1019.
[22] T. Jacobson, S. Liberati, D. Mattingly and F.W. Stecker, *Phys. Rev. Letters* **93** (2004) 021101.
[23] G. Amelino-Camilia, e-print astro-ph/0410076.
[24] R. Aloisio, P. Blasi, P.L. Ghia and A.F. Grillo, *Phys. Rev. D* **62** (2000) 053010.
[25] J. Alfaro and G. Palma, *Phys. Rev. D* **67**, (2003) 083003.
[26] O. Gagnon and G.D. Moore, *Phys. Rev. D* **70** (2004) 065002.
[27] O. Lahav and A.R. Liddle, *Phys. Letters B592* (2004) 1.
[28] G.R. Blumenthal, *Phys. Rev. D* **1** (1970) 1596.
[29] V.I. Berezinsky and S.I. Grigor’eva, *Astron. Astrophys* **199** (1988) 1.
[30] S.T. Scully and F.W. Stecker, *Astroparticle Phys.* **16** (2002) 271.
[31] D.J. Bird et al., *Astrophys. J.* **424** (1994) 491.
[32] A.J. Bunker et al., *MNRAS* in press, e-print astro-ph/0403223.
[33] F.W. Stecker, *Astroparticle Phys.* **14** (2000) 207.
[34] M. Schmidt, *Astrophys. J.* **552** (2001) 36.
[35] F.W. Stecker, *Nature* **342** (1989) 401.