CVaR-based generation expansion planning of cascaded hydro-photovoltaic-pumped storage system with uncertain solar power considering flexibility constraints

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Abstract
The development of a high solar energy penetrated power system requires considerable flexibility to hedge the risk of solar power curtailment and power shortage. This paper explores how the generation portfolio of cascaded hydro-photovoltaic-pumped storage (CH-PV-PS) generation system will be appropriately designed to balance the overall planning costs and operational flexibility constraints. The proposed study relies on the generation expansion planning (GEP) model of the CH-PV-PS system, considering a full set of flexibility constraints. An index designated ramp-capability reserve shortage (RCRS) based on the conditional value at risk (CVaR) method is introduced to quantify the risk of solar power and load uncertainty. The piecewise linearization method and triangle method are developed to accommodate the non-linear terms in the proposed model. Finally, the case studies are conducted to demonstrate the applicability and effectiveness of the proposed model.

INTRODUCTION

Increasing the penetration of variable renewable energy is an inevitable way for the transition to a decarbonized power system, which is dominated by solar energy, wind energy, and hydroelectricity. The cascaded hydro-photovoltaic-pumped storage (CH-PV-PS) generation system concept originated in China aims to facilitate this transition [1–3]. However, the CH-PV-PS system always operates in an off-grid condition that escalates the challenges of meeting the need for flexibility to mitigate the mismatch between generation and consumption [4]. The long-term generation expansion planning (GEP) of the CH-PV-PS system considering flexibility constraints faces many difficulties and needs to be further studied.

The flexibility constraints are introduced to ensure the system’s adaptation to uncertainty and are of great significance to accommodate more renewable energies and improve the reliability of the power system connected with variable renewable energies. Ignoring the flexibility constraints might result in a significant underestimation of the curtailment of renewables and overall planning costs, leading to a sub-optimal planning result [5]. Belderbos and Delarue proposed a new system planning model on a power plant resolution using mixed-integer linear programming (MILP), taking into account technical flexibility constraints [6]. The results indicated that flexibility constraints have a critical impact on the optimal generation portfolio and cannot be ignored. Palmintier and Webster presented a generation expansion formulation for an 8760-hour chronological profile of load, demand, and wind [7]. It was solved as a single MILP optimization considering the operational flexibility constraints, pointing out that ignoring the flexibility constraints would create 35%–60% errors in the estimated carbon emissions with 20% renewables, resulting in an inability to meet tighter carbon emission regulations. The flexible resources planning to accommodate large-scale renewable energy has been considered in the current paper, including a battery energy storage system (BESS) [8], demand response (DR) [9], and integrated energy system (IES) [10]. The full set of flexibility constraints was introduced to the planning model mentioned above, and the results all illustrated effectively that the flexibility constraints cannot be ignored in the planning optimization. However, to the best of the author’s knowledge, there has been...
no description in the literature introducing the flexibility constraints into the CH-PV-PS generation system expansion planning and providing a detailed analysis. There is a lack of consideration for flexibility constraints in the GEP model of the CH-PV-PS system.

To deal with the uncertainty in the solar power output of a PV array, researchers have deployed a wide range of techniques. Stochastic optimization (SO) is an effective approach, where the power output of a PV array is assumed to follow a predetermined probability distribution [11]. The long-term planning model for the joint optimization of conventional thermal units, concentrating solar power (CSP), and storage devices have been proposed in [12], and the SO approach was employed to deal with the uncertainty of solar power. The robust optimization (RO) approach has the advantage that the uncertainties are represented by a robust set. The uncertain problem can thus be transformed into a deterministic problem under the worst-case scenarios [13]. The adaptive RO model was explored to reduce the uncertainty of power generation by integrating CSP plants with wind farms in [14]. However, the SO approach needs several scenarios predetermined by probability information, which may cause computational intractability. Besides, the RO approach can be immune to the worst cases of uncertainty realizations, so the planning results are conservative. Thus, the methods mentioned above are difficult to solve for the complex, large-scale CH-PV-PS planning problem.

The conditional value-at-risk (CVaR) method as an effective method to cope with the uncertainty of renewable energy has been applied to the power system optimization problem [15–18]. In [15], the flexible look-ahead dispatch model based on the CVaR method was proposed, and an index CVaR-WP was introduced to evaluate the risk of wind power accommodation. The results indicated that the CVaR method could avoid the over-conservativeness of traditional RO. In [16], the CVaR method was used to express the uncertainties of wind power in an IES planning model and transformed the stochastic probability model of wind power to a deterministic expression. Compared with the SO approach, the CVaR method did not require an accurate forecast power value and reduced the computational burden. However, the CVaR method is used to describe the uncertainty of single renewable energy (i.e. wind energy) in [15] and [16]. Consequently, the form of the CVaR method needs to alter if the uncertainty of both generations and loads is taken into account.

In this paper, we introduce the index designated the ramp-capability reserve shortage (RCRS) based on the CVaR method to measure the risk of solar power curtailment and power shortage. Here the CVaR method is applied to model uncertainties of solar power and power load and has the advantage of reducing the conservativeness of the conventional RO approach and the computational intractability of the SO approach. The GEP model of the CH-PV-PS system incorporating a full set of chronological flexibility constraints in operation is proposed to attempt a trade-off between planning cost and RCRS. The capacity and geographical allocation of the CH-PV-PS system can thus be optimized to mitigate the excess or shortage of solar power by utilizing our proposed model. The piecewise linearization method and triangle method are used in this paper to transform the non-linear model into a MILP model, which is easier to be solved.

According to the above literature survey, the major contributions of this paper are as follows:

(i) Developing a GEP model of CH-PV-PS system considering flexibility constraints to determine the optimal newly built capacity and geographical allocation to achieve higher solar energy penetration.
(ii) Modelling the uncertainties of solar power and power load based on the CVaR method avoids over-conservativeness and improves computing efficiency.
(iii) Analysing how the optimal generation mix will change towards a higher penetration of solar power and the necessity of considering flexibility constraints.

The rest of this paper is organized as follows. Section 2 introduces the RCRS based on the CVaR method, which signifies the uncertainties of solar power and power load. Section 3 presents the mathematical formulation of the GEP model of the CH-PV-PS system. Section 4 details the solution methodology of the proposed model. Section 5 illustrates some numerical examples, and optimization results are analysed. Finally, concluding remarks are drawn in Section 6.

2 | CVAR-BASED RAMP-CAPABILITY RESERVE SHORTAGE CHARACTERIZATION

2.1 | RCRS of power system

The flexibility of the power system is defined here as the ability of a power system to deploy its flexible ramping resources to respond to changes in net load at a reasonable cost at a specific time scale [19]. The net load mainly consists of the power load and variable renewable generation (i.e. solar energy) in this paper. The time horizon is defined as the duration of net load change, equal to 1 hour in the following analysis. The RCRS of the power system corresponds to the deviation that the flexible system resources cannot meet the net load, as shown in Figure 1.

In Figure 1, I represent the upward net load at time t, which is caused by solar power output, drastic reduction, and drastic
load increase in a short time interval $t \rightarrow t + 1$. The upward ramp-capability reserve provided by flexible resources is limited at time $t$. Therefore, when the upward net load exceeds the upward ramp-capability reserve, there will be an upward RCRS. On the contrary, when solar power output increases significantly or load reduces drastically in a short time interval $t + 1 \rightarrow t + 2$, the net load will increase rapidly, as shown by II in Figure 1. If the downward ramp-capability reserve provided by flexible resources is insufficient at that time, the downward RCRS will arise.

2.2 Mathematical formulation of RCRS

Value at risk (VaR) refers to the maximum possible loss of an investment portfolio under a given confidence level in the market. However, numerous researches have shown that there are some defects with VaR. For instance, VaR is not a consistent risk measurement method and cannot provide an adequate picture of risks reflected in the extreme tail, which results in the underestimate of the investment risk. Therefore, based on VaR, Rockafellar and Uryasev [20] proposed CVaR, which refers to the conditional expectation of loss exceeding VaR for all conditions. As a common risk measurement method, CVaR calculated the probability of system loss to measure the operational risk of the system accurately. This paper adopts the CVaR method to deal with the uncertainty of solar power and load in planning optimization.

The uncertainty caused by solar power and power load can be statistically described using the normal distribution [11, 21]. However, the normal probability density function (PDF) generally uses $-\infty$ and $+\infty$ as the lower limits and upper limits, respectively, which is not suitable for the statistical description of solar power and power load due to specific lower and upper limits. Thus, the uncertainty related to solar generation and power load is modelled by normal PDF with specific upper and lower limits in this paper. The diagram of the PDF of solar power and power load is shown in Figure 2.
in an upward RCRS. Furthermore, if the actual value of solar power is less than $P_{\text{d,}i}^{\text{LL}}$, the total generation outputs will be insufficient, and the system will face the risk of power shortage, resulting in the downward RCRS. A similar analysis is applied to Figure 2(b).

Consider the case where there is upward RCRS in the system. In this case, the VaR values of solar power and load are $P_{\text{UL}}$, respectively. The CVaR is the conditional expected value that the risk loss exceeds the VaR. Thus, for a given node, the upper CVaR $\phi_{\text{up}}(P_{\text{L},i}, P_{\text{d},i})$ corresponding to the upward net load consisting of solar power and load demand can be calculated by (1):

$$
\phi_{\text{up}}(P_{\text{L},i}, P_{\text{d},i}) = \sum_{s \in \Omega} \phi_{\text{up}}(P_{\text{L},s}) + \sum_{s \in \Omega} \phi_{\text{up}}(P_{\text{d},s})
$$

Similarly, for the given node, the lower CVaR $\phi_{\text{down}}(P_{\text{L},i}, P_{\text{d},i})$ can be calculated by (2):

$$
\phi_{\text{down}}(P_{\text{L},i}, P_{\text{d},i}) = \sum_{s \in \Omega} \phi_{\text{down}}(P_{\text{L},s}) + \sum_{s \in \Omega} \phi_{\text{down}}(P_{\text{d},s})
$$

Therefore, in this case, the upward/downward RCRS for the given node can be calculated by (3a) and (3b), respectively. $RCR_{\text{up}}$ and $RCR_{\text{down}}$ denote the up/down ramp-capability reserves scheduled, respectively, as detailed in Section 3.5:

$$
RCR_{\text{up}} = \max \left\{ \phi_{\text{up}}(P_{\text{L},i}, P_{\text{d},i}) - RCR_{\text{up}}, 0 \right\} (3a)
$$

$$
RCR_{\text{down}} = \max \left\{ -\phi_{\text{down}}(P_{\text{L},i}, P_{\text{d},i}) - RCR_{\text{down}}, 0 \right\} (3b)
$$

3 | MODELLING FORMULATION

The structure of the CH-PV-PS generation system [1], which comprises cascaded hydro units (CHUs), PV unit, and PHES unit is shown in Figure 3. It is a typical structure that CHUs, PV and PHES are linked to the AC buses and send their electricity to the utility grids. The CHUs consist of hydroelectric units within a river basin. The PV unit is composed of a PV array and a DC/AC converter. The PHES unit consists of a pump turbine, a synchronous motor, and a full-size converter (FSC), which is used to transmit the energy between the storage and the utility grids. In addition, the control centre samples PV output power processes the operation data of the CH-PV-PS system and sends control instructions to CHUs and PHES.

3.1 | Objective function

The objective function minimizes the overall system cost by minimizing annualized investment cost $C_{\text{inv,CH-PV-PS}}$ of CH-PV-PS system, annual system operation cost $C_{\text{op}}$ and CVaR related to RCRS $CR_{\text{RCR}}$:

$$
\text{min Cost} = \min \left( C_{\text{inv,CH-PV-PS}} + C_{\text{op}} + C_{\text{RCR}} \right) (4)
$$

such that,

$$
C_{\text{inv,CH-PV-PS}} = \gamma_{\text{CRF}} \sum_{i \in \Omega_{\text{CHU}}} j_{\text{inv}}^{\text{CHU},i} C_{\text{cap,CHU},j}^{j_{\text{inv}}} + \sum_{j \in \Omega_{\text{PHES}}} j_{\text{inv}}^{\text{PHES},j} C_{\text{cap,PHES},j}^{j_{\text{inv}}} (5)
$$
The investment cost of the CH-PV-PS system, formulated in (5), includes annualized investments of newly built cascaded hydro units and PHES units. The total annual operation cost includes start-up costs, fuel costs, and upward/downward ramp-capability reserve costs of existing thermal units in the first line, start-up/shut-down costs and upward/downward ramp-capability reserve costs of cascaded hydro units in the second line, and start-up costs and upward/downward ramp-capability reserve costs of PHES units in the third line, as shown in (6). The CVaR related to RCRS, formulated in (7), is calculated by multiplying the upward/downward RCRS in (3) with the price for RCRS.

### 3.2 Cascaded hydro units operation constraints

\[
P_k(t) = c_{d,k}(V_k(t))^2 + c_{a,k}Q_k^{up}(t)^2 + c_{a,k}Q_k^{down}(t)Q_k^{up}(t)
+ c_{a,k}V_k(t) + c_{d,k}Q_k^{down}(t) + c_{6,k}v_k \in \Omega_{CHU}, \forall t \in T
\]

\[
P_k \leq P_k(t) \leq \bar{P}_k \leq Cap_{CHU}, v_k \in \Omega_{CHU}, \forall t \in T
\]

\[
V_k(t + 1) = V_k(t) + \left(Q_k^{up}(t) + Q_k^{down}(t) - Q_k^{up}(t)\right) \cdot \Delta t
\]

\[
Q_k^{up}(t) = Q_{\text{up}}(t) + \sum_{n \in \Omega_{CHU}} \left(Q_{\text{up}}(t) + Q_{\text{app}}(t)\right)
\]

\[
\forall k \in \Omega_{CHU}, \forall t \in T
\]

\[
\bar{V}_k \leq V_k(t) \leq \underline{V}_k, \forall k \in \Omega_{CHU}, \forall t \in T
\]

\[
\bar{Q}_k^{up} \leq Q_k^{up}(t) \leq \underline{Q}_k^{up}, \forall k \in \Omega_{CHU}, \forall t \in T
\]

\[
\sum_{k \in \Omega_{CHU}} \left(P_k(t) + P_k(t) + \sum_{n \in \Omega_{CHU}} \left(P_k(t) - P_k(t)\right) + \sum_{l \in \Omega_{CHU}} P_l(t)
+ \sum_{n \in \Omega_{CHU}} P_{\text{up}}(t) \geq \sum_{n \in \Omega_{CHU}} P_{\text{up}}(t), \forall t \in T, \forall k \in N_b
\]

The power output [22] of the \( k \)th cascaded hydro unit and its limit is presented in (8) and (9), respectively. The water balance constraint, formulated in constraint (10), ensures the water balance in the time dimension of a single cascaded hydro unit and the space dimension of upstream and downstream cascaded hydro units. Reservoir volume, water discharge, and water outflow for the \( k \)th cascaded hydro unit are limited by constraints (11)–(13), respectively. Finally, the ramping constraint, formulated in (14) and (15), ensures that upward/downward flexibility provided by cascaded hydro units should not interrupt the ramp-capability of units.

### 3.3 PHES units operation constraints

\[
0 \leq x_{\text{up}}(t) + x_{\text{down}}(t) \leq 1, \forall k \in \Omega_{PHES}, \forall t \in T
\]

\[
0 \leq P_{\text{up}}(t) \leq x_{\text{up}}(t), R_{\text{up}} \leq x_{\text{up}}(t)Cap_{PHES}, \forall k \in \Omega_{PHES}, \forall t \in T
\]

\[
0 \leq P_{\text{down}}(t) \leq x_{\text{down}}(t), R_{\text{down}} \leq x_{\text{down}}(t)Cap_{PHES}, \forall k \in \Omega_{PHES}, \forall t \in T
\]

\[
\sum_{k \in \Omega_{PHES}} \left(P_{\text{up}}(t) \cdot \sqrt{\eta_{\text{up}}} - P_{\text{down}}(t) / \sqrt{\eta_{\text{down}}} \right) = 0, \forall k \in \Omega_{PHES}
\]

The PHES units can provide sufficient ramp-capability reserves with the advantage of short start-up/shut-down times and no minimum up/down times. As formulated in (16), the generating-pumping mutual exclusion constraint ensures that PHES units cannot be in the generating and pumping status simultaneously. The power output of PHES units is limited by constraints (17) and (18). Constraint (19) refers to the energy balance of a single PHES unit.

### 3.4 Energy balance constraints

\[
\sum_{k \in \Omega_{CHU}} \left(P_k(t) + P_k(t) + \sum_{n \in \Omega_{CHU}} \left(P_k(t) - P_k(t)\right) + \sum_{l \in \Omega_{CHU}} P_l(t)
+ \sum_{n \in \Omega_{CHU}} P_{\text{up}}(t) \geq \sum_{n \in \Omega_{CHU}} P_{\text{up}}(t), \forall t \in T, \forall k \in N_b
\]
The power loads should be matched with total generators net output from cascaded hydro units, PHES units, PV units, and the energy balance is guaranteed in (20). The DC power flow equation and the power flow constraint are guaranteed in (21) and (22).

3.5 Flexibility constraints

\[
\begin{align*}
\sum_{s \in b} P_s(t) + \sum_{k \in b} P_k(t) + \sum_{\xi \in b} \left( P_{\xi \uparrow}(t) - P_{\xi \downarrow}(t) \right) + \sum_{i \in I_a} P_i(t) \\
+ \sum_{m \in b} P_{s, m}^L(t) \leq \sum_{m \in b} P_{s, m}^U(t), \forall t, \forall b \in N_b
\end{align*}
\]

(20b)

\[
P_i(t) = (\theta_i(t) - \theta_i^{\prime}(t))/X_i, \forall t, \forall i \in I_a
\]

(21)

\[-P_i^{\text{max}} \leq P_i(t) \leq P_i^{\text{max}}, \forall t, \forall i \in I_a
\]

(22)

The flexible up/down ramp-capability reserves, as formulated in (23) and (24), are provided by existing thermal units, newly built cascaded hydro units, and newly built PHES units. The ramp-capability reserve of thermal units is constrained by ramping limit, maximum power output, and minimum power output of thermal units, as shown in (25). The ramp-capability reserve of cascaded hydro units is formulated in (26) and is limited by ramping limit, maximum power output, and minimum power output of cascaded hydro units. In this paper, we assume that the upward/downward ramp ratio of PHES is very high, and no start-up and shut down times [23] Therefore, the ramp-capability reserve of PHES units is only constrained by the upper/lower power output limit, as formulated in (27).

4 SOLUTION METHODS

4.1 Piecewise linearization method for linearizing the objective function

The non-linear integral terms in the penalty cost \( C_{\text{RCS}} \) related to RCSR for the curtailment of solar power and power load, as shown in (7), are challenging to find an effective solution. The piecewise linearization method [24] deals with the non-linear integral terms in this paper. The piecewise linearization method can transform the non-linear problem into a MILP problem. Figure 4 shows the piecewise linearization of the non-linear terms \( \Phi_{s}^{\text{up}}(P_{s, m}^U) \) and \( \Phi_{s}^{\text{up}}(P_{s, m}^L) \) corresponding to the upward RCSRs.

The piecewise linearization process of \( \Phi_{s}^{\text{up}}(P_{s, m}^U) \) is illustrated in Figure 4(a). First, we introduce a number \( O + 1 \) of sampling coordinates \( \{M_s, 1, \ldots, M_s, O \} \), where the sampling coordinates are defined as breakpoints on the \( P_{s, m}^U \) axis. Thus, the \( P_{s, m}^U \) axis is divided into \( O \) intervals by these break- points. Then, let us introduce a continuous variable \( \alpha_{i, m, o} \) associated with the interval \( [M_{s, o}, M_{s, o+1}] \) \( (o = 1, 2, \ldots, O) \). To use the above techniques in a MILP solver, it is necessary to include in the binary variables \( \beta_{i, m, o} \) associated with the \( o \)th interval \( [M_{s, o}, M_{s, o+1}] \) \( (o = 1, 2, \ldots, O) \). The approximate value of \( \Phi_{s}^{\text{up}}(P_{s, m}^U) \) can then be obtained by imposing the constraints (28) and (29):

\[
\Phi_{s}^{\text{up}}(P_{s, m}^U) = \sum_{o \in O} \left( \alpha_{i, m, o} + \beta_{i, m, o} \right) \forall m \in \Omega_s
\]

(28)

\[
\begin{align*}
\sum_{o \in O} \alpha_{i, m, o} &= P_{s, m}^U, \forall m \in \Omega_s \\
\sum_{o \in O} \beta_{i, m, o} &= 1, \forall m \in \Omega_s
\end{align*}
\]

(29)

\[
\beta_{i, m, o} \cdot P_{s, m, o} \leq \alpha_{i, m, o} \leq \beta_{i, m, o} \cdot P_{s, m, o+1} \forall m \in \Omega_s, \forall o \in O
\]

(27)
Similarly, the approximate value of \( \phi^{\text{up}}(P^{UL}) \) in Figure 4(b) can be transformed to (30) and (31):

\[
\phi^{\text{up}}(P^{UL}) = \sum_{d \in O} \left( \alpha_{d,n,o} \cdot \beta_{d,n,o} + b_{d,n,o} \right), \forall n \in \Omega_d \tag{30}
\]

\[
\begin{align*}
\sum_{d \in O} \alpha_{d,n,o} &= \phi^{\text{IL}}(P^{UL}), \forall n \in \Omega_d \\
\sum_{d \in O} \beta_{d,n,o} &= \phi^{\text{IL}}(P^{UL}), \forall n \in \Omega_d \\
\beta_{d,n,o} \cdot \rho_{d,n,o} &\leq \alpha_{d,n,o} \cdot \rho_{d,n,o}, \forall n \in \Omega_d, \forall o \in O
\end{align*}
\tag{31}
\]

Similarly, the non-linear terms \( \phi^{\text{down}}(P^{UL}) \) and \( \phi^{\text{down}}(P^{UL}) \) corresponding to the downward RCRS can be piecewise linearized using the same above techniques.

### 4.2 Triangle method for linearizing the two variables constraints

In the proposed model, two continuous variables are included in the non-linear constraint (8). To cope with this non-linear constraint, the triangle method [25] is proposed to accommodate the two variables in (8). The triangle method is still a piecewise linearization approximation of functions of two variables. Constraint (8) indicates that the power output of the cascaded hydro units is related to the volume reservoir and water discharge and can be transformed to (32):

\[
P_k(t) = f(V_k(t), Q_k(t)), \forall k \in \Omega_{CHU}, \forall t \in T \tag{32}
\]

Consider \( X + 1 \) sampling coordinates \( r_1, \ldots, r_{X+1} \) on the \( V \) axis and \( Y + 1 \) sampling coordinates \( q_1, \ldots, q_{Y+1} \) on the \( Q \) axis. Let consider a surface of vertices \( (r_1, q_1), (r_1, q_{Y+1}), (r_{X+1}, q_1), (r_{X+1}, q_{Y+1}) \) limited by (32). The surface is partitioned into \( X \times Y \) small rectangles using the triangle method of two variables, as shown in Figure 5.

Considering each rectangle corresponding to intervals \([v_X, r_{X+1})\) and \([q_1, q_{Y+1})\) \((x = 1, 2, \ldots, X; y = 1, 2, \ldots, Y)\), we introduce continuous variables \( y_{xy} \in [0, 1] \) and associate binary variables \( \mu_{xy} \) and \( \omega_{xy} \), one for each upper and lower triangle of the rectangle, with boundary values \( \mu_{xy}^* = \mu_{xy}^{\text{up}} = \mu_{xy}^{\text{down}} = \mu_{xy}^{\text{mid}} = 0 \). Then, the equality constraints in (32) can be approximated by using the triangle method, and the non-linear programming problem of two variables can thus be transformed into a MILP problem by imposing the following constraints:

\[
f(V_k, Q_k^{\text{up}}) = \sum_{x=1}^{X} \sum_{y=1}^{Y} y_{xy} f(V_k, Q_k^{\text{up}}), \forall k \in \Omega_{CHU} \tag{33}
\]

\[
\begin{align*}
V_k &= \sum_{x=1}^{X} \sum_{y=1}^{Y} y_{xy} V_k, \forall k \in \Omega_{CHU} \\
Q_k^{\text{up}} &= \sum_{x=1}^{X} \sum_{y=1}^{Y} y_{xy} Q_k^{\text{up}}, \forall k \in \Omega_{CHU} \\
y_{xy} &\leq \mu_{xy}^* + \mu_{xy}^{\text{down}} + \mu_{xy}^{\text{mid}} + \mu_{xy}^{\text{up}} + \mu_{xy}^{\text{up}} \chi_{x=1, y=1} \chi_{x=1, y=1} + \mu_{xy}^{\text{up}} \chi_{x=1, y=1} + \mu_{xy}^{\text{up}} \chi_{x=1, y=1}
\end{align*}
\tag{34}
\]

### 5 CASE STUDY

In this section, the proposed CH-PV-PS optimal planning model is implemented on the IEEE 14-bus and IEEE
118-bus test systems to illustrate the effectiveness of the proposed approach. The MILP problem is formulated with YALMIP and solved by GUROBI 9.1.0 using MATLAB R2016b on a 2.5-GHz Intel Core i5-10300H processor with 16GB of RAM.

5.1 IEEE 14-bus test system

Tests are carried on the IEEE-14 bus test system. The structure of the 14-bus test system with the river basin is given in Figure 6. The PV units, PV1 and PV2, are connected to busbar 12 and 13, respectively. The solar power and power load are assumed to follow normal distributions for simplicity. The expected values of total load and PV output are scaled down from the actual data. The standard deviations of solar power and power load are assumed as 30% and 20%, respectively. The capital recovery factor is presumed to 8%, $\xi_{up}$ and $\xi_{down}$ are assumed as $300$/MW and $120$/MW, respectively. Parameters of candidate generators and existing thermal generators are shown in Tables 1 and 2, respectively. Characteristic parameters of candidate cascaded hydro units’ location is depicted in Table 3.

### TABLE 1 Parameters of candidate generators

|                  | Cascade hydro unit | PHES unit |
|------------------|--------------------|-----------|
|                  | #1     | #2     | #3     | #1 | #2 |
| Capacity (MW)    | 50     | 150    | 250    | 40 | 80 |
| Capital cost (× 10^6 $/MW) | 4.60   | 4.55   | 4.51   | 5.36 | 5.84 |
| Start-up cost ($) | 22     | 26     | 30     | 35 | 40 |
| Shut-down cost ($) | 22    | 26     | 30     | /  |
| Upward ramp-capability reserve cost ($/MW) | 20     | 20     | 20     | 15 | 15 |
| Downward ramp-capability reserve cost ($/MW) | 15     | 15     | 15     | 10 | 10 |
| Ramping limits (MW/min) | 0.42 | 1.25   | 2.08   | /  |
TABLE 2 Parameters of existing thermal generators

|       | G1  | G2  |
|-------|-----|-----|
| Capacity (MW) | 150 | 80  |
| Start-up cost ($) | 270 | 300 |
| Price to provide energy ($/MW·h) | 49  | 47  |
| Upward ramp-capability reserve cost ($/MW) | 25  | 25  |
| Downward ramp-capability reserve cost ($/MW) | 20  | 20  |
| Ramping limits (MW/min) | 0.25 | 0.12 |

TABLE 3 Characteristic parameters of candidate cascaded hydro units' location

| Location | Historical inflows (m³/s) | Max. water level (m) | Min. water level (m) |
|----------|---------------------------|----------------------|----------------------|
| 1        | 1112                      | 1800                 | 1680                 |
| 2        | 1119                      | 1640                 | 1520                 |
| 3        | 1175                      | 1495                 | 1385                 |
| 4        | 1184                      | 1321                 | 1300                 |
| 5        | 1054                      | 1155                 | 1100                 |
| 6        | 1265                      | 1011                 | 963                  |
| 7        | 2119                      | 790                  | 700                  |
| 8        | 2402                      | 540                  | 500                  |
| 9        | 3108                      | 370                  | 360                  |

TABLE 5 Comparison of the costs with or without considering the flexibility constraints

| Cost (×10^6$) | Without flexibility constraints | With flexibility constraints |
|---------------|--------------------------------|----------------------------|
| Investment cost | Cascaded hydro unit | 187.41 | 210.23 |
| PHES unit     | –                          | 46.98 |
| Total         | 187.41                      | 257.21 |
| Operation cost | Thermal unit              | 56.26 | 27.54 |
| Cascaded hydro unit | –                    | 0.04 | 1.16 |
| PHES unit     | –                          | 15.34 |
| Total         | 56.30                       | 44.04 |
| RCRS cost     | Upward RCRS cost           | 68.79 | 0.08 |
| Downward RCRS cost | –                | 27.56 | 0.00 |
| Total cost    | 340.06                      | 301.33 |

5.2 Comparison of optimal planning results with or without considering the flexibility constraints

In this section, the CH-PV-PS planning results with flexibility constraints in the proposed model are compared with conventional planning without flexibility constraints. Without considering flexibility constraints, the conventional planning model is optimized based on merit order curves, and the flexibility constraints formulated by (23)–(27) are disregarded. The detailed planning results and costs with or without considering the flexibility constraints at 50% solar power penetration are shown in Tables 4 and 5.

Table 4 shows that the only candidate cascaded hydro units #2 and #3 are invested without considering the flexibility constraints. The planning results are entirely different when considering flexibility constraints. The system prefers to invest more flexible PHES units, which upward/downward ramp ratio is very high, and no start-up and shut down times when accounting flexibility constraints. The coordinated cascaded hydro and pumped storage system is scheduled to provide a ramp-capability reserve to compensate for the upward/downward RCRS while considering flexibility constraints. Besides, the investment of cascaded hydro unit prefers to be allocated to downstream candidate location.

Table 5 shows that the overall economy is better while considering flexibility constraints. Considering flexibility constraints, the system can provide enough upward/downward ramp-capability reserves, and the mismatch between generation and consumption mitigates, which leads to a significant reduction of RCRS cost. Moreover, even though ramp-capability reserves and the cost of investment increase when accounting flexibility constraints, the total cost is still reduced by 11.39%. Figure 7 shows the hourly power balance with or without considering the flexibility constraints. The proposed model (with flexibility constraints) significantly reduces solar power curtailment compared with the conventional planning model (without flexibility constraints). This is because the variability of PV units and the flexibility limits of the CH-PV-PS generation system are considered in the proposed model.

5.3 CH-PV-PS system optimal planning results at different level of solar power shares

The CH-PV-PS capacity planning results under 5%, 10%, 15%, 30%, 50%, and 75% solar power penetration considering the flexibility constraints are compared in Figure 8.

No PHES unit is invested when the solar energy penetration is below 15%, as the investment cost of a cascaded hydro unit is lower and can offer enough ramp-capability to accommodate low penetration of solar energy. However, the more flexible PHES units are built from 15% to 75% solar energy penetration, as enough flexibility is required with a higher solar energy.
penetration. Those PHES units are higher in investment cost but more flexible in dealing with the variability of solar energy. The annual operation cost and annualized investment cost with different solar energy penetration are depicted in Figure 9.

The increase in investment cost of more flexible generation is associated with a higher penetration of solar energy. Moreover, the cost for RCRS reduces significantly with increasing investment of flexible PHES units. The candidate PHES units, PHES #1 and PHES #2 are built at 75% of solar energy penetration, and the cost for RCRS is only $6.3 \times 10^6$. In addition, the cost for ramp-capability reserves increases significantly with a higher solar energy penetration. At 5% solar energy penetration, the thermal units and cascaded hydro units are scheduled to provide a ramp-capability reserve, and the cost for ramp-capability reserves is only $3.3 \times 10^6$. Candidate PHES unit #1 is invested at 15% solar energy penetration, and the ramp-capability reserve is provided by the coordinated cascaded hydro and pumped storage system. When the solar energy penetration increases to 75%, the ramp-capability reserve is mainly provided by PHES units, and the cost for ramp-capability reserve reaches $3.4 \times 10^7$.

5.4 | Impact of system parameters

5.4.1 | Impact of price for RCRS

The prices for RCRS are related to the system operator’s attitude to risk. Lower prices for RCRS indicate that the investor of the CH-PV-PS system is a risk-taker who is willing to exchange a higher risk for a lower cost. On the contrary, higher prices for RCRS indicate that the CH-PV-PS investor is a risk averter, particularly for a higher investment cost and a lower risk. The results under different prices for RCRS are shown in Figure 10.

As illustrated by Figure 10, when the prices for RCRS are low ($\xi_{up}/\xi_{down}$ are $60/24$ MW), respectively), the PHES units are not installed when the solar energy penetration is below 30%. Moreover, the optimal planning of the system prefers to pay the costs for solar power curtailment and power shortage instead of investing newly capacity. Only PHES #1 is installed even if solar energy penetration increases to 75%. On the contrary, when the prices for RCRS are high ($\xi_{up}/\xi_{down}$ are $600/240$ MW), respectively), the optimal planning of the system prefers to install PHES units that are more flexible and efficient. The PHES unit #1 is installed at 10% solar energy penetration, and two candidate PHES units are both invested when solar penetration reaches 50%. It is indicated that the proposed approach can be used to guide the set of prices for wind curtailment and power shortage to enhance the system’s flexibility and economy.

5.4.2 | Impact of probability interval

The probability intervals $[p_{e_{min}}^{L}, p_{e_{max}}^{L}]$ are here presumed as 10%, 20%, 30%, 40%, 50%, 60%, and 70% of their maximum intervals $[p_{e_{min}}^{max}, p_{e_{max}}^{max}]$ of solar power and loads. Optimal costs under different probability intervals are shown in Figure 11.

With the increase of the probability intervals, ramp-capability reserves cost, RCRS cost, and the total costs decreased. With the increase of probability intervals, the lower and upper limits $p_{e_{min}}$, $p_{e_{max}}$ are approaching the minimum and maximum values.
with the increase of the interval, the flexibility, and economy of the CH-PV-PS system are improved.

5.5 Scalability of proposed model

To illustrate the scalability of the proposed model, we analyse a case study on the IEEE 118-bus test system [26]. A schematic of the modified network is shown in Figure 12. There are six integrated PV units at bus 2, 14, 39, 52, 75, 106. The expected values and standard deviations of load and PV output are consistent with the previous study on the 14-bus test system. Similar to the above-mentioned test system, candidate equipment comprises 9 cascaded hydro units and 6 PHES units. The following cases are conducted to analyse the proposed model.

1. Comparison with or without considering flexibility constraints: The detailed planning results and costs with or without considering flexibility constraints are illustrated in Tables 6 and 7, respectively.

Table 6 depicts the comparison of planning results with or without considering flexibility at 50% solar power penetration. More flexible PHES units are installed while considering the flexibility constraints because of its high energy supply efficiency. The coordinated cascaded hydro and pumped storage system is scheduled to provide enough ramp-capability.

| Location/capacity of cascaded hydro units (MW) | Without flexibility constraints | With flexibility constraints |
|-----------------------------------------------|--------------------------------|-----------------------------|
| 10*/50, 14*/150, 18*/250, 19*/50, 23*/150, 27*/250 | 9*/250, 10*/50, 14*/150, 18*/250, 23*/150, 27*/250 |
| Busbar/capacity of PHES units (MW) | Without flexibility constraints | With flexibility constraints |
| 6/40 | 6/40, 19/80, 46/80 |
FIGURE 11  Optimal costs under different probability intervals

FIGURE 12  Structure of 118-bus test system
TABLE 7  Comparison of the costs with or without considering the flexibility constraints

| Cost ($ \times 10^6$) | Without flexibility constraints | With flexibility constraints |
|----------------------|---------------------------------|-----------------------------|
| Investment cost ($ \times 10^6$) | 421.87 | 530.64 |
| Operation cost ($ \times 10^6$) | 98.09 | 90.86 |
| RCRS cost ($ \times 10^6$) | 187.87 | 1.01 |
| Total cost ($ \times 10^6$) | 707.83 | 622.51 |

Numerical cases have been conducted to verify the effectiveness of the proposed methods. Compared with the conventional planning model without considering the flexibility constraints, the proposed model can reduce solar power curtailment and improve the system economy. The lack of flexibility constraints leads to a failure to meet the ramp-capability reserve requirements, resulting in a sub-optimal generation planning. Besides, developing a higher solar energy penetrated power system involves higher operational flexibility. With a higher solar energy penetration, those units (i.e., PHES unit), which are more expensive but flexible, are preferred to be invested and will become one of the dominating technologies to provide ramp-capability reserves.

The proposed model may be used in generation planning of any other hybrid system that aims to balance the overall costs and its limited operational flexibility and immunize its uncertainty modelling against conservatism and computational intractability. However, as an interesting perspective, other flexible resources (e.g., DR and transmission expansion planning) will also essential and need to be considered in the future.

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NOMENCLATURE

- $T$: set of time periods, index $t$
- $N_b$: set of bus nodes, index $b$
- $L_b$: set of transmission lines connected to busbar $b$, index $l$
- $\Omega_s$: set of PV units, index $m$
- $\Omega_d$: set of load demands, index $n$
- $\Omega_{CHU}^{i}$: set of candidate cascaded hydro units, index $i$
- $\Omega_{PHES}^{j}$: set of candidate PHES units, index $j$
- $\Omega_{CHU}^{k}$: set of existing thermal units, index $u$
- $\Omega_{CHU}^{\Omega}$: set of newly built cascaded hydro units, index $k$
- $\Omega_{PHES}^{\Omega}$: set of newly built PHES units, index $z$
- $\Omega_{CHU}^{\Omega}$: set of upstream cascaded hydro units, index $\upsilon$
- $i_{\text{inv}}$: investment cost per capacity ($$/MW)
- $c_u$: price of existing thermal unit $u$ to provide energy ($$/MW-h)
- $\gamma_{CRF}$: capital recovery factor
- $Cap_{\text{inv}}^{i}$: capacity of the candidate cascaded hydro unit/PHES unit (MW)
- $\Delta s$: total seconds per hour (s)
- $\eta_{\text{round}}$: round trip efficiency of PHES unit $\zeta$
- $P_{\text{max}}$: capacity of transmission line $l$ (MW)
- $RCRS$: hourly upward/downward RCRS (MW)
- $RCSR_{\upsilon}^{(t)}$: hourly upward/downward ramp-capability reserves in period $t$ (MW)
- $P_{\upsilon}^{(t)}$: power output of existing thermal unit $u$ (MW)
\( P_k(t) \) power output of cascaded hydro unit \( k \) (MW)

\( V_k(t) \) reservoir volume of cascaded hydro unit \( k \) in period \( t \) (m³)

\( q^{\text{up}}_{\Omega}/q^{\text{down}}_{\Omega} \) start-up/shut-down cost ($)

\( \Omega_{\text{up}}/\Omega_{\text{down}} \) offered cost of upward/downward ramp capability reserve ($/MW)

\( \xi^{\text{up}}_{\Omega}/\xi^{\text{down}}_{\Omega} \) price for upward/downward RCRS ($/MW)

\( Q_k^{\text{up}}/Q_k^{\text{down}} \) lower/upper limits of power output of cascaded hydro unit \( k \) (MW)

\( P_{\text{up}}/P_{\text{down}} \) upper limits of power output of PHES unit \( \gamma \) in generating/pumping status (MW)

\( V_{\gamma}/V_{\gamma} \) lower/upper limits of reservoir volume of cascaded hydro unit \( k \) (m³)

\( Q_{\text{dis}}^{\text{up}} / Q_{\text{dis}}^{\text{down}} \) lower/upper limits of water discharge of cascaded hydro unit \( k \) (m³/h)

\( Q_{\text{out}}^{\text{up}} / Q_{\text{out}}^{\text{down}} \) lower/upper limits of water outflow of cascaded hydro unit \( k \) (m³/h)

\( RU_k/RD_k \) upward/downward ramping limits of cascaded hydro unit \( k \) (MW/h)

\( a_{b_{t+1}}/b_{y_{t+1}} \) coefficients of piecewise linearization function

\( \rho, \rho, \rho \) value of \( \theta \) break point

\( S_{\gamma}(t)/S_{\gamma}(t) \) hourly upper/lower CVaR corresponding to net loads (MW)

\( x_{\gamma_{\text{up}}}/x_{\gamma_{\text{down}}} \) binary variable of start-up/shut-down status

\( x_{\gamma_{\text{up}}}/x_{\gamma_{\text{down}}} \) binary variable of investment status of candidate cascaded hydro unit/PHES unit

\( x_{\gamma_{\text{up}}}/x_{\gamma_{\text{down}}} \) power output of PHES unit \( \gamma \) in generating/pumping status (MW)

\( x_{\gamma_{\text{up}}}/x_{\gamma_{\text{down}}} \) water inflow/ natural inflow of cascaded hydro unit \( k \) in period \( t \) (m³/h)

\( x_{\gamma_{\text{up}}}/x_{\gamma_{\text{down}}} \) water discharge/water spillage of cascaded hydro unit \( k \) in period \( t \) (m³/h)

\( x_{\gamma_{\text{up}}}/x_{\gamma_{\text{down}}} \) binary indicator of generating/pumping status of PHES unit \( \gamma \)

**REFERENCES**

1. Wu, F., et al.: An optimal wavelet packets basis method for cascade hydro- PV-pumped storage generation systems to smooth photovoltaic power fluctuations. Energies 12(24), 1642–1664 (2019)

2. Chen, L., et al.: Smoothing photovoltaic power fluctuations for cascade hydro-PV pumped storage generation system based on a fuzzy CEEDAN. IEEE Access 7, 172718–172727 (2019)

3. Lou, N., et al.: Two-stage congestion management considering virtual power plant with cascade hydro-photovoltaic-pumped storage hybrid generation. IEEE Access 8, 186335–186347 (2020)

4. Lei, M., et al.: An MPC-based ESS control method for PV power smoothing applications. IEEE Trans. Power Electron. 33(3), 2136–2144 (2018)

5. Chen, X., et al.: Power system capacity expansion under higher penetration of renewables considering flexibility constraints and low carbon policies. IEEE Trans. Power Syst. 33(6), 6240–6253 (2018)

6. Belderbos, A., Delarue, E.: Accounting for flexibility in power system planning with renewables. Int. J. Elect. Power Energy Syst. 71, 33–41 (2015)

7. Palmintier, B.S., Webster, M.D.: Impact of operational flexibility on electricity generation planning with renewable and carbon targets. IEEE Trans. Sustainable Energy 7(2), 672–684 (2016)

8. Luo, F., et al.: Coordinated operational planning for wind farm with battery energy storage system. IEEE Trans. Sustainable Energy 6(1), 253–262 (2015)

9. Li, C., et al.: Flexible transmission expansion planning associated with large-scale wind farms integration considering demand response. IET Gener. Transm. Distrib. 9(15), 2276–2283 (2015)

10. Clegg, S., Mancarella, P.: Integrated electrical and gas network flexibility assessment in low-carbon multi-energy systems. IEEE Trans. Sustainable Energy 7(2), 718–731 (2016)

11. Yu, D., et al.: Risk-constrained stochastic optimization of a concentrating solar power plant. IEEE Trans. Sustainable Energy 11(3), 1464–1472 (2020)

12. Du, E., et al.: The role of concentrating solar power toward high renewable energy penetrated power systems. IEEE Trans. Power Syst. 33(6), 6630–6641 (2018)

13. Verástegui, F., et al.: An adaptive robust optimization model for power systems planning with operational uncertainty. IEEE Trans. Power Syst. 34(6), 4606–4616 (2019)

14. Chen, R., et al.: Reducing generation uncertainty by integrating CSP with wind power: an adaptive robust optimization-based analysis. IEEE Trans. Sustainable Energy 6(2), 583–594 (2015)

15. Li, P., et al.: Flexible look-ahead dispatch realized by robust optimization considering CVaR of wind power. IEEE Trans. Power Syst. 33(5), 5330–5340 (2018)

16. Li, Z., et al.: Probability-interval-based optimal planning of integrated energy system with uncertain wind power. IEEE Trans. Ind. Appl. 56(1), 4–13 (2020)

17. Zhang, Y., et al.: Conditional value at risk-based stochastic unit commitment considering the uncertainty of wind power generation. IET Gener. Transm. Distrib. 12(2), 482–489 (2018)

18. Farhoumandi, M., et al.: Generation expansion planning considering the rehabilitation of aging generating units. IEEE Trans. Smart Grid 11(4), 3384–3393 (2020)

19. Lanno, E., et al.: Evaluation of power system flexibility. IEEE Trans. Power Syst. 27(2), 922–931 (2012)

20. Rockafellar, U., et al.: Stansilav: optimization of conditional value-at-risk. J. Risk 2(3), 21–41 (2000)

21. Zou, K., et al.: Distribution system planning with incorporating DG reactive capability and system uncertainties. IEEE Trans. Power Syst. 3(1), 112–123 (2012)

22. Hou, W., et al.: Data-driven multi-time scale robust scheduling framework of hydro-thermal power system considering cascade hydropower station and wind penetration. IET Gener., Transm. Distrib. 13(6), 896–904 (2018)

23. Apostolopoulou, D., McCallloch, M: Optimal short-term operation of a cascaded hydro-solar hybrid system: a case study in Kenya. IEEE Trans. Sustainable Energy 10(4), 1878–1889 (2019)

24. Ali, H.R., et al.: A trajectory piecewise-linear approach to nonlinear model order reduction of wind farms. IEEE Trans. Sustain. Energy 11(2), 894–905 (2020)

25. D’Ambrosio, C., et al.: Piecewise linear approximation of functions of two variables in MILP models. Oper. Res. Lett. 38(1), 39–46 (2010)

26. Ivonne, P., et al.: An extended IEEE 118-bus test system with high renewable penetration. IEEE Trans. Power Syst. 33(1), 281–289 (2018)

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