Designing constraint-based False Data Injection attacks for unbalanced distribution smart grids

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Abstract—Smart grid is equipped with various kinds of smart devices such as meters, sensors, and actuators, to name a few, and it can be characterized as the Internet of Things. The cyber domain of this cyber-physical system also has to deal with many cyber-threats, including the stealthy False Data Injection (FDI) attack. This man-in-the-middle data-driven attack is extremely dangerous due to its ability to disrupt the operation without being detected. The unbalanced distribution network is a highly probable target for an FDI attack due to its easy physical access. This paper will investigate an attack design scheme based on a nonlinear physical-constraint model that is able to produce an FDI attack with the same theoretical stealthy characteristic. To demonstrate the effectiveness of the proposed design scheme, simulations with the IEEE 13-node Test Feeder are conducted. The experimental results indicate that the false positive rate of the bad data detection mechanism is 100%. This figurative number opens a serious challenge for operators in maintaining the integrity of measurement data.

Index Terms—Cyber-physical system, cyber-security threat, distribution network, False Data Injection attack, Internet of Things, smart devices, smart grid, State Estimation, unbalanced.

I. INTRODUCTION

From the early stage of development, the task of supervising and controlling the interconnected power network was challenged by the geographical distances. This issue has been resolved by deploying a system of remote terminal units (RTUs) at critical nodes. The measured values from RTUs are sent to the control center as an input to the Energy Management System (EMS), for state estimation (ES). The estimated states, in turn, are the inputs for various control algorithms, that directly affect the operation of physical systems such as optimal power flows, contingency analysis, and economic dispatch. Previously, only the balanced transmission system is supervised using the linear DC-based low-accuracy SE module. With the growth of the power sector the state estimation requires a higher level of precision. The current information and computer technologies (ICTs) enables the SE module to be implemented with a sophisticated nonlinear AC-based large-scale high-accuracy model. The increasing penetration of distributed energy resources (DERs) such as solar or wind farm has transformed the unbalanced distribution network into a hybrid energy-Internet-of-Things system. Therefore, an AC-based SE module will be deployed for the task of monitoring and management of distribution network. In this paper, SE module is understood as an AC-based one.

Since the first released prototype in 1970 [1], several research groups have already proposed various paradigms for the SE module in the distribution network. Roytelman et al. [2], Baran et al. [3], Meliopoulos et al. [4] and Lu et al. [5] obtained extensive full-phase power based models. After that, Baran et al. tried to improve the computing efficiency of the module by introducing a current-based model [6]. Using the same ideas about the current-based model, Wang et al. [7] enhanced the working rate of the SE module. Along with the breakthrough of advanced technologies, the contemporary cyber-physical system is exposed to various cyber-security threats that normally exist in the communication system. Since Stuxnet [8] in 2010, the man-in-the-middle type False Data Injection attack has been getting more and more attention due to its devastating consequences.

Although there has been research about FDI attack against balanced transmission system, most of works did not target the state estimation for the unbalanced distribution network (DNSE). Deng et al. in [9] did an extensive job to enhance the insight of this topic, however, their scope of study is still about balanced system. This limitation is a major concern as the fundamental difference between the transmission system and the distribution network is about phase balancing. Apart from this work, almost no others address an FDI attack against the DNSE. This gap is expected to be filled soon as the evolution of distribution network with the integration of distributed generation necessitates the crucial role of DNSE. Given the fact that the distribution network is relatively more approachable (in terms of location, i.e., closer to residential areas), the system of meters and sensors are vulnerable to cyber attacks. An FDI attack in distribution network is not only feasible but also seems to be less difficult to establish than the one in transmission system.

The adversaries have various motivations to organize an FDI attack in distribution network. This type of attack is at utmost level of dangerous since it is completely stealthy under all the current examination measures equipped with SE module. The attackers can maximize the damage to the physical environment by a malevolent attack. A typical example for this situation is the 2015 Ukraine blackout [10]. That incident consists of a series of destructive actions, from phishing email to seize the control of SCADA (cyber domain) to remotely issuing the commands to switch off the power substations (physical domain). Consequently, a wide-area blackout (which was originated from the cyber domain) happened, dealing significant damage to the physical environment. In such sit-
cation, the operators at the control center have very little time to detect and respond, mostly too late to react. On the other hands, the attacker may employ the FDI attack scheme to gain economical advantage and benefits. For instance, he possesses a rooftop solar structure that is able to sell energy back to the distribution network and he wants to falsify the selling amount to earn more money from the utility companies. In the same manner, an electricity thief happens when the attackers may initialize an instability event that could make the system collapse, by triggering the relay protective system to wrongfully trip down certain lines due to the counterfeit over-threshold values. In the wake of the distributed generations era, the distribution network which is now having bi-directional power flows is more fragile than ever before.

With such awareness about the potential threats of FDI attack in distribution network, an in-depth investigation is necessary. Only when we can gather comprehensive insight about this FDI attack, we can expect to plan various strategies to mitigate or prevent it. The major contribution of this paper is to provide a comprehensive analysis of some possible ways to conduct FDI attack in the smart distribution network. There are five sections in this paper with the current one introducing the background and relevant works. Section II discusses some preliminary considerations that must be taken into account while working with distribution network, as well as steps in planning the design schemes for specific cases, and steps in devising a dedicated SE’s results assessment method based on the unbalanced load flow program. Section III provides a case study with the unbalanced IEEE 13-node Test Feeder to illustrate for the proposed attack design schemes. The obtained experimental results are presented in Section IV, demonstrate the theoretical stealth capability of the attack. Finally, concluding remarks and future research trend are highlighted in Section V.

II. FALSE DATA INJECTION ATTACK DESIGN SCHEME FOR DISTRIBUTION NETWORK

A. Design considerations

In order to actively formulate strategies to deal with the potential threats of the FDI attack, the worst-case scenario must be taken into consideration. We assume that the hackers have enough resources to organize an FDI attack, i.e., there is a capability to collect all necessary information about the attack region (a sub-grid of the network) including topology, status of switches and breakers, steady-state measured values, and also to compromise whichever critical meters that is needed to override the measurement values. In short, the main work of the adversaries is to concentrate on constructing a completed set of false data that look like normal measured data from the viewpoint of the SE module. In this paper, the focus is on technical aspects of computing FDI attack vectors with the ability to completely bypass the bad data detector (BDD) of the DNSE module.

1) Models of components: Distribution network possesses several characteristics that distinguish itself from transmission system. It has no phase-transposing, is mostly radial in structure with the number of phases on each lateral varies from one to three; and most importantly, it has naturally unbalanced load. Both the representative models and analytic algorithms applied for distribution network are much different compared to transmission system. For instance, as the phase transposing is not applied, the combination of “self” and “mutual induction” into “phase induction” is no longer valid as is done for the transmission system (which results in the line impedance $Z = R + jX$ ($\Omega$)). Instead, the computation on each quantity must be conducted independently using Carson’s equations [11] that result in individual self and mutual impedances (of simplified model) presented in the matrix format as below:

$$Z_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}$$

\(1\)

2) Models of measurements: Let’s assume that digital meters to collect measurement quantities (active and reactive powers) are equipped at both the terminals of each phase on every branch and at positions where power is injected. Given the unbalanced characteristics of the distribution network, employing per-phase representative model in designing FDI attack is inadequate. Instead, a full 3-phase power-flow measurement model as in Eq. (2) and (3) that includes all the unique features of the distribution network, is employed:

$$P_{ij}^{ph} = \sum_{l=a,b,c} V_{ij}^{ph} \{V_{ij}^{ph} [G_{ij}^{ph,1} \cos(\theta_{ij}^{ph} - \theta_i^{ph}) + B_{ij}^{ph,1} \sin(\theta_{ij}^{ph} - \theta_i^{ph})]\}$$



$$Q_{ij}^{ph} = \sum_{l=a,b,c} V_{ij}^{ph} \{V_{ij}^{ph} [G_{ij}^{ph,1} \sin(\theta_{ij}^{ph} - \theta_i^{ph}) - B_{ij}^{ph,1} \cos(\theta_{ij}^{ph} - \theta_i^{ph})]\}$$

where $i, j = 1, 2, \ldots, N$ with $N$ is the total number of nodes, and $ph$ denotes phase $a, b, \text{ or } c$ in a distribution smart network. $G$ and $B$ are real and imaginary parts of the elements of $Z_{abc}$. Observing the above formulas, all the state variables (phase quantities: voltage magnitude $V_i^{ph}$ and voltage angle $\theta_i^{ph}$) are included in each individual one-phase measurement ($P_{ij}^{ph}$ or $Q_{ij}^{ph}$) computation. All the three phases in the distribution network are coupled, which means that any change happens in one phase will result in alterations in the quantities of the other phases. This feature makes the attack design scheme for distribution network more challenging than the counterpart in transmission system since the latter one does not have to consider that interaction.
3) Models for the State Estimation: When it comes to FDI attack, regardless of type of system (either transmission system or distribution network), the target is the same SE module of the EMS package. The relationship between measurements and state variables is given as:

\[ z = h(x) + \epsilon \]  

(4)

where

- \( z \): a vector of measurements (power flows, power injections, voltage magnitudes and angles). In distribution network, all the measurements are phase quantities.
- \( x \): a vector of state variables (voltage magnitudes and angles at every phase of each node).
- \( h \): a vector of nonlinear functions represent the relationship between measurement values and state variables.
- \( \epsilon \): a vector of measurement errors which is assumed to have Gaussian distribution with zero mean.

Given a set of acquired measurements, an SE algorithm will iteratively solves, for instance, the most common weighted least square optimization problem as in (5), to obtain the set of state variables \( \hat{x} \):

\[
\min F(\hat{x}) = (z - h(\hat{x}))^T \cdot W \cdot (z - h(\hat{x}))
\]

(5)

In the above equation, \( W \) is the weighting matrix whose elements correspond to the inverse of the individual measurements’ accuracy. The existence of bad measurement data due to various reasons is detected and eliminated by comparing the normalized residual with a threshold \( \tau \) as if:

\[ ||z - h(x)|| > \tau \]

(6)

B. Design scheme

Corresponding to linear DC-based and nonlinear AC-based SE models, the attack models are also divided into DC-based and AC-based attacks. The DC-based FDI attack model can easily bypass the criterion (6) just by adding an attack vector \( a \) that is the product of the linearized matrix \( H \) and an arbitrary contaminated vector \( c \) to the current measurement vector \( z \). However, FDI attack aimed at an AC-based SE module is far more complicated. This kind of attack targets a specific region, called attack area. Let the whole grid be divided into two regions: region 1 is the area under attack and the rest of the network is region 2. The corresponding sets of measurements, state variables and nonlinear representative functions are \( \{z_1, z_2\} \), \( \{x_1, x_2\} \), and \( \{h_1, h_2\} \), respectively. The relationship between measurements and state variables becomes:

\[
\begin{bmatrix}
  z_1 \\
  z_2 
\end{bmatrix} =
\begin{bmatrix}
  h_1(x_1, x_2) \\
  h_2(x_2) 
\end{bmatrix} +
\begin{bmatrix}
  \epsilon_1 \\
  \epsilon_2 
\end{bmatrix}
\]

(7)

The vector \( h(x) \) is based on physical laws such as the Kirchhoff’s Current Law (KCL) and the Kirchhoff’s Voltage law (both originate from the law of Conservation of Energy), and Ohm’s law. Any set of values \( \hat{z} \) which strictly complies with the physical laws that constituted the measurement models, is able to provide a converged solution for the optimization problem (5). If another set of this kind exists in the proximity of a steady state operating point, that set of measurements is also able to bypass the examining process of the BDD.

In this paper, the above FDI attack design principle is applied, in which the set of manipulated state variables \( \hat{x} = \{\hat{x}_1, x_2\} \) and the set of malicious measurements \( \hat{z} = \{\hat{z}_1, z_2\} \) will be acquired directly. A new pseudo steady state near the genuine one is created to deceive the SE module. Because it is a pseudo steady state, it meets the criterion (6):

\[
\begin{bmatrix}
  \hat{z}_1 \\
  \hat{z}_2 
\end{bmatrix} =
\begin{bmatrix}
  h_1(\hat{x}_1, x_2) \\
  h_2(x_2) 
\end{bmatrix} < \tau
\]

(8)

The above proposed method is fundamentally different from all the previous approaches (for balanced transmission system) in the literature. Other algorithms try to find the contaminated vector \( c \), the set of manipulated state variables \( \hat{x} = \{x_1 + c, x_2\} \), and the set of mixture of malicious and normal measurements \( \hat{z} = \{z_1 + a, z_2\} \), respectively in that order. The normalized residual is qualified if:

\[
\begin{bmatrix}
  z_1 + a \\
  z_2 
\end{bmatrix} -
\begin{bmatrix}
  h_1(x_1 + c, x_2) \\
  h_2(x_2) 
\end{bmatrix} < \tau
\]

(9)

Hence, the condition for the attack being hidden is:

\[
a - h_1(x_1 + c, x_2) + h_1(x_1, x_2) = 0
\]

\[\leftrightarrow a = h_1(x_1 + c, x_2) - h_1(x_1, x_2)\]

(10)

Attack vector \( a \) and contaminated vector \( c \) are mutually dependent for a successful attack. Therefore, all elements of the contaminated vector \( c \) cannot be arbitrarily selected as Anwar et al. claimed in [12]. In addition, this approach could possibly produce cumulative errors due to enduring through various computation stages.

The proposed attack design scheme in this paper has two main stages which are illustrated in Fig. 1. The first stage is to identify all possible attack areas. In an area of attack the measurements are compromised. This concept was first defined in [13], where the first condition to guarantee a successful attack is: the attack region must be enclosed by nodes with power injection only. This is important as the type of attack area helps to allocate resources for launching attack. In the next step the constraint-based attack model is acquired for the attack areas from the previous stage. The output of this step is a set of manipulated state variables of a false steady state. This set is fed into the measurement model to compute the set of measurements with false data injection. The relevant technical details are presented below.

1) Attack Area: The attack area should be as small as possible because of two reasons: (i) a smaller attack area would require less resource to launch the attack, e.g. the number of meters that the adversary have to compromise; (ii) the changes in a smaller area probably take less attraction of the operators, thus increase the chance of being undetected.

As discussed above, the necessary condition for an attack area to be invisible is having a boundary of nodes with power injection only. Since an FDI attack will modify the state variables at various nodes, the involved measurements will be altered accordingly. All these alterations must be "assigned" to some nodes around in order to avoid any unreasonable
inconsistency. Nodes with no power injection can only adjust by the changes in the power flow along the connected branch, thus making the attack area larger. Meanwhile, nodes with power injection (either incoming or outgoing) can justify any change without further expanding the attack area by adjusting the metered value of the local power injection. Algorithm 1 presents the entire process of identifying the attack area in a distribution network.

There are two types of attack areas that exist within a distribution network: (a) attack area does not contain any no-injection node, and (b) attack area contains no-injection node. Fig. 2 illustrates the general prototypes of these two attack areas. Although the only feature that separates two types of attack area is the existence of a node with power injection, the attack design schemes are quite different. The appearance of node with no power injection aggravates the complexity of the design problem as various physical-law based constraint equations must be formulated and solved. On the other hand, the design process for an attack area without no power injection node is more straightforward as it does not require the solution of the set of nonlinear equations (11), hence significantly reduce the amount of computations.

Algorithm 1: Finding FDI Attack Area for Distribution Network

**Input:** All $Z_{abc}$ of branch $ij$, Initial node $k$

**Output:** Area of Attack $\Omega_A$

1. Scanning all $Z_{abc}$
   if $(Z_{kia} \neq 0)$ or $(Z_{kib} \neq 0)$ or $(Z_{kic} \neq 0)$ then
      Add node $k$ to $\Omega_A$
   else
      $k++$
   end if
2. Scanning $\Omega_A$:
   if (typeofnode($i$) = injection) then
      Move to the next node in $\Omega_A$
   else
      Back to 1
   end if

Figure 1: The FDI attack design scheme for distribution network.

Figure 2: Two types of attack area exist in distribution network

2) FDI Attack Model: From the set of nodes inside an attack area $\Omega_A$, we can calculate the number of changeable state variables. For every node in each phase, there are two accompanied state variables, voltage magnitude $V_{i}^{ph}$ and voltage angle $\theta_{i}^{ph}$. However, all the nodes on the boundary of the attack area must be removed from the set. If these nodes are also modified, the measurements on the connected branches, outside the attack area, will be altered as well. This might introduce inconsistencies that could be detected. Furthermore, there is a kind of state variable with predefined value that any imposed adjustment will attract attention, for instance, voltage magnitude and voltage angle of a slack bus. For that reason, the attack should always avoid that type of nodes. The complete...
set of all changeable state variable can be obtained through Algorithm 2.

**Algorithm 2: Identifying changeable state variables**

**Input:** Area of Attack $\Omega_A$

**Output:** The set of changeable state variable $SV$

No. of $SV$ $n \leftarrow 2 \times Size\omega f(\Omega_A)$

$SV = \{ |V|, \theta_j | j \in \Omega_A \}$

while $(j \in \Omega_A)$ do

if (type of node $(j) = \text{Slack}$) then

$n = n - 2$

Remove $|V|$ and $\theta_j$

else if (type of node $(j) = \text{PV}$) then

$n = n - 1$

Remove $|V|$

else

$j++$

end if

end while

If the attack area is $(\alpha_1)$, the set of changeable state variable are solely located at the initial node. Given the omission of no power injection node, the attack design here goes straight to the stage of computing the set of false data. A new pseudo steady state is created, then all the new measurements are calculated accordingly. Any change on the branches are adjusted by means of changing the measured value of power injections at the local as well as the adjacent nodes.

Dealing with $(\alpha_2)$-type attack area is a bit more complicated as it requires more effort to apply the nonlinear constraint-based attack model to find the set of manipulated state variables. The model is represented by a set of constraint equations that has the basis in the law of Conservation of Energy. In order to launch an ideal stealthy FDI attack on a predefined attack area $\Omega_A$, all possible alterations must happen locally. It means that all power exchanges with the outside regions must be kept unchanged in order to maintain the seamlessly transition between the attack area and the rest of the grid. In addition, the characteristic of no power injection node must also be guaranteed. These requirements are satisfied by following the two rules below:

- Algebraic sum of all power flows at a no power injection node must be equal to zero. By complying with this condition, the relationships that constitutes the $h$ function in (1) are preserved.
- Regarding the attack area, the sum of all changes in every branches plus the sum of all injection variations must be equal to zero. By complying with this condition, the consistencies between powers inside and outside the attack area are preserved.

Given the set of no power injection node is $\Omega_{A0}$, the constraint-based FDI attack model is constructed as:

$$\sum_{j \in \Omega_{A0}} z_{jk}^p (\hat{x}_1, x_2) = 0$$

$$\sum_{j \in \Omega_{A0}} \Delta z_{jk}^p (\hat{x}_1, x_2) + \sum_{i,j \in \Omega_A} \Delta z_{ij}^p (\hat{x}_1, x_2) = 0 \quad (11)$$

As per-phase analysis is conducted, each rule must be composed for every phase in the attack coverage. In a same manner, a new pseudo steady state is created by an initialization, then this attack model outputs the manipulated state variables to compute the malicious measurements. By keeping the state variables at boundary unchanged, all the alterations are held inside the attack area. The changes of measurements on branches in the attack region ($P_{ij}, Q_{ij}$) will be justified by changing the measured values of the power injections at all power injection nodes. The active and reactive power flows are calculated by Eqs. (2) and (3), for all the phases in order to update the new meters’ values.

The explanations of related quantities, superscripts and subscripts using in the above constraint-based attack model are provided below:

- $P_{ph}, Q_{ph}$ - Real and reactive power measured on phase $ph$ on branch from node $i$ to node $j$ that we obtained from load flow result (denoted by “o”).
- $P_{ijon}, Q_{ijon}$ - Real and reactive power measured on phase $ph$ on branch from node $i$ to node $j$ that has been changed due to new state variables applied at one or both terminals of the branch (denoted by “n”).
- $P_{ph}, Q_{ph}$ - Real and reactive power injection measured on phase $ph$ at node $i$ that we obtained from load flow result (denoted by “o”).
- $P_{ph}, Q_{ph}$ - Real and reactive power injection measured on phase $ph$ at node $i$ that has been changed due to new state variables applied at terminal (denoted by “n”).
- $P_{ph}, Q_{ph}$ - Power flow losses of phase $ph$ on branch $ij$ that we obtained from load flow result, ($P_{ph}^{Lijo} = P_{ijon}^{ph} + P_{ph}^{Lijo}, Q_{Lijo}^{ph} = Q_{ijon}^{ph} + Q_{ph}^{Lijo}$).
- $P_{ph}, Q_{ph}$ - Power flow losses of phase $ph$ on branch $ij$ that has been changed due to new state variables applied at one or both terminals of the branch, ($P_{ph}^{Lijo} = P_{ijon}^{ph} + P_{ph}^{Lijon}, Q_{Lijo}^{ph} = Q_{ijon}^{ph} + Q_{ph}^{Lijon}$).

**C. Assessment method**

As in the discussion above, the DNSE will be implemented to satisfy the rising demand for real-time system monitoring of the distributed smart grid. The adoption of DNSE is currently at a much lower level compared with the corresponding package for transmission system. Various prestigious power-oriented software vendors, such as DgSILENT, only provide customer with a balanced-only SE module. In order to overcome this limitation, a dedicated assessment process is devised in this project. The foundation of this assessment method is the consistency between the SE results and the load flow results. The FDI attack model generates a set of measurements that, in turn, will create a pseudo steady state that the state estimation routine considers as a genuine operating point. Thus, an FDI attack will completely bypass the bad data detection process if its loading values can generate the load flow results that match perfectly with the falsified values. A simulation tool from DgSILENT, PowerFactory, which is well-known for its capability of providing a comprehensive unbalanced load flow result, is selected to detect the designed attack. Fig. 3 illustrates the general idea of the
detection process. The details are sequentially presented below. Subscript 1 is used for manipulated values (quantities within the attack area) and subscript 2 for unchanged values (quantities outside the attack area) as in Section II-A.

1) Acquiring steady-state values from load flow, denoted by script 0: These include steady-state measurements \( z_0 = \{ \text{PI}_0, \text{PF}_0 \} \), and steady-state variables \( x_0 = \{ \text{SV}_0 \} \), which will be used as the input to the attack design process. \text{PI} is the power injection at nodes, either injected into (generated) or drawn from (consumed) a node. \text{PF} is the power flow measurement \( (P_{ij} \text{ or } Q_{ij}) \), indicating the algebraic power flows at the two terminals of each branch in the network (lines, transformers etc.). \text{SV} is the state variable (voltage magnitude or voltage angle).

2) False data generation: Feeding the steady-state sets \( \{z_0, x_0\} \) into the attack design scheme, we obtained attack design results:

\[
\hat{z} = z_1 + z_2 = (\text{PF}_1 + \text{PF}_2) + (\text{PI}_1 + \text{PI}_2) \\
\hat{x} = x_1 + x_2 = (\text{SV}_1 + \text{SV}_2)
\]

(12)

3) Extracting the set of power injection \( \text{PI} \) and then inputting as the set of demands for the unbalanced load flow program: The set of power injection includes loading profile of the pseudo steady-state. Running the unbalanced load flow with such input will produce the results of pseudo steady-state measurements \( z_n = \{ \text{PI}_n, \text{PF}_n \} \), and pseudo steady-state state variables \( x_n = \{ \text{SV}_n \} \).

4) Conducting element-by-element comparisons then drawing conclusion: Each corresponding element of the two couple sets \( \{\hat{z}, z_n\}, \{\hat{x}, x_n\} \) will be compared. Based on our observations from the experimental result for the case of 1-phase equivalent transmission system, the maximum mismatches are always smaller than 1%. If ALL the comparisons produce the results that are less than this threshold, it is reasonable to conclude that the \( \hat{z} \) set will definitely bypass the BDD of SE module.

III. CASE STUDY

The IEEE 13-node Test Feeder (Fig. 4) is chosen as the test system following the recommendation from the Test Feeder Working Group of the IEEE’s Distribution System Analysis Subcommittee [15] for a task related to state estimation. It has 11 overhead lines and underground cables with seven different configurations for various 1-, 2-, or 3-phase laterals. This system load is diverse and unbalanced. In this investigation, for the sake of simplicity at early stage of work, the distributed load along the line from node 632 to node 671 is disabled. The nominal voltage of this distribution network is 4.16 kV line-to-line or 2400 V line-to-neutral. All the meter readings and pre-attack state variables are collected from the unbalanced load flow results with the default load profile.

In this section, a case study of the IEEE 13-node Test Feeder is investigated and presented following the order of the proposed attack design scheme. For convenience in representing various quantities, all the nodes will be assigned to new aliases as in Table I (these aliases are also indicated in Fig. 4).

| Node | New Name | Node | New Name |
|------|----------|------|----------|
| 650H | 01       | 650L | 02       |
| 632  | 03       | 645  | 04       |
| 646  | 05       | 633  | 06       |
| 634  | 07       | 671  | 08       |
| 692  | 09       | 675  | 10       |
| 684  | 11       | 652  | 12       |
| 611  | 13       | 680  | 14       |

Table I: Aliases of nodes in the IEEE 13-node Test Feeder.
After imposing one or several initial adjustments onto the selected changeable state variables, the design process goes straight towards the stage of computing malicious measurements. One thing to bear in mind is that no matter how many state variables at node 10 we change, the following power flows must be calculated and then updated to the corresponding meters: $P_{1009}^{a,b,c} \cdot Q_{1009}^{a,b,c} \cdot P_{0910}^{a,b,c} \cdot Q_{0910}^{a,b,c}$. After obtaining those malicious values, the power injections at nodes can be calculated. As the load at node 09 has two phases $a$ and $c$ only, the changes on the line of phase $b$ must be justified at node 08 instead.

2) $(a_2)$-type Attack Area $\Omega_{A1}$: The full 3-phase diagram of an $(a_2)$-type attack area $\Omega_{A1}$ is illustrated in Fig. 6. In order to keep all the changes happened only inside the attack area, all the state variables at node 08 must be kept unchanged. Consequently, there are 8 changeable state variables within the attack area $\Omega_{A1}$:

$$\text{SV} = \{a_{11}^a, a_{11}^b, a_{11}^c, b_{11}^a, b_{11}^b, b_{11}^c, c_{11}^a, c_{11}^b, c_{11}^c\}$$ (13)

Since the attack area $\Omega_{A1}$ possesses only one no-injection node, 11, and it has only two phase (a and c), the number of no-injection constraint equation related to this node is 4.

- The sum of active power flows of phase $a$:
  $$\sum_{11} P_{11}^a = 0 \Leftrightarrow P_{1108n}^a + P_{1112n}^a = 0$$ (14)

- The sum of reactive power flows of phase $a$:
  $$\sum_{11} Q_{11}^a = 0 \Leftrightarrow Q_{1108n}^a + Q_{1112n}^a = 0$$ (15)

- The sum of active power flows of phase $c$:
  $$\sum_{11} P_{11}^c = 0 \Leftrightarrow P_{1108n}^c + P_{1113n}^c = 0$$ (16)

- The sum of reactive power flows of phase $c$:
  $$\sum_{11} Q_{11}^c = 0 \Leftrightarrow Q_{1108n}^c + Q_{1113n}^c = 0$$ (17)

Next, we need to identify the constraint equations related to changes in power injection at nodes and changes in power loss on branches. For the current attack area $\Omega_{A1}$, four constraint equations of this type will be formed:

- The sum of all changes in active-related quantities for phase $a$ must equal to 0:
  $$(P_{12n}^a - P_{12o}^a) + (P_{08n}^a - P_{08o}^a) + (P_{L0811n}^a - P_{L0811o}^a) + (P_{L1112n}^a - P_{L1112o}^a) = 0$$ (18)

- The sum of changes in reactive-related quantities for phase $a$ must equal to 0:
  $$(Q_{12n}^a - Q_{12o}^a) + (Q_{08n}^a - Q_{08o}^a) + (Q_{L0811n}^a - Q_{L0811o}^a) + (Q_{L1112n}^a - Q_{L1112o}^a) = 0$$ (19)

- The sum of changes in active-related quantities for phase $c$ must equal to 0:
  $$(P_{13n}^c - P_{13o}^c) + (P_{08n}^c - P_{08o}^c) + (P_{L0811n}^c - P_{L0811o}^c) + (P_{L1113n}^c - P_{L1113o}^c) = 0$$ (20)

B. Constructing the constraint-based FDI attack model

1) $(a_1)$-type Attack Area $\Omega_{A3}$: In the traditional distribution network where radial structure dominates, the attack area type of $(a_1)$ is omnipresent. The FDI attack here is launched by arbitrarily changing one or several state variables at the initial node while keeping all the other nodes around to be unchanged. The number of changeable state variables at one node is up to six (three voltage magnitudes and three voltage angles for three phases). For the case of attack area $\Omega_{A3}$, we can choose from the set $\{V_{10}^a, V_{10}^b, V_{10}^c, \theta_{10}^a, \theta_{10}^b, \theta_{10}^c\}$. Meanwhile, the set of state variable at nodes 08 and 09 must be kept unchanged in order to avoid the expansion of attack region.
The sum of changes in reactive-related quantities for phase $c$ must equal to 0:

$$
(Q_{13n}^c - Q_{13o}^c) + (Q_{08n}^c - Q_{08o}^c) + (Q_{L0811n}^c - Q_{L0811o}^c) + (Q_{L1113n}^c - Q_{L1113o}^c) = 0
$$

Figure 6: The full 3-phase diagram of attack area $\Omega_{A1}$.

In total, the attack model has 8 constraint equations in companion with 8 changeable state variables. At the first sight, it seems that we would have an overdetermined problem to solve because the proposed design scheme requires to devote one state variable for initialization. However, the detailed analysis of the set of constraint equations has shown a different result. First, we have $P_{12a}^o = -P_{1211a}^o$.

Next, we will delve deeper into the last four constraint equations in order to reveal their true forms. As observed from the detailed 3-phase diagram in Fig. 6, the circuit of phase $c$ including $\{08, 11, 13\}$ and the circuit of phase $a$ including $\{08, 11, 12\}$ are identical. In each circuit of phase, the equations relevant to active and reactive power are corresponding. Therefore, we just only need to investigate one and then obtain the similar results for the rest.

Consider (18), we already have $P_{12a}^n = -P_{1211a}^n$ and $P_{12o}^n = -P_{1211o}^n$. Next, we focus on the region around node 08-09 (that is extracted and illustrated as in Fig. 7) and obtain the following relationship: $P_{08}^a + P_{09}^a + P_{0811}^a + P_{0803}^a + P_{0910}^a = 0$

Figure 7: Region around node 08-09.

As we determine to keep the state variables at 03, 08-09, and 10 unchanged, the two power flows $P_{0803}^a$ and $P_{0910}^a$ will be unchanged accordingly. Consequently, any change in power injection $P_{08a}^o$ will be reflected on $(P_{09}^a + P_{0811}^a)$. Let’s suppose to keep the metered value of load at node 09, $P_{09}^a$, to be unchanged then all the changes in $P_{08}^a$ will only reflected on $P_{0811}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$. Meanwhile, the branches’ power losses are calculated by adding together two opposite power flows from two terminals. We have $P_{L0811n}^a - P_{L0811o}^a = P_{0811n}^a + P_{0811o}^a$.
opposite sign, we have:

\[(P_{1108n}^a - P_{1108o}^a) + (P_{1122n}^a - P_{1122o}^a) = 0\]
\[\Rightarrow (P_{1108n}^a + P_{1122n}^a) - (P_{1108o}^a + P_{1122o}^a) = 0\]  \hspace{1cm} (22)

We already obtained the value of \(P_{1108n}^a + P_{1122n}^a\) from steady state and it exactly equals to 0. Hence, the constraint equation (18) becomes \(P_{1108n}^a + P_{1122n}^a = 0\), or the constraint equation (14). In conclusion, the requirement for unchanged power in the attack area has already been fulfilled by the requirement for energy conservation at zero-injection node. Also, applying the same procedure as above, the constraint equations (19), (20), and (21) all have reduced forms resemble to the constraint equations (15), (16), and (17). Consequently, there are only four constraint equations (14)-(17) for this attack area.

At this point, there are four constraint equations and eight changeable state variables. Thus, we can launch an attack by initializing an adjustment on a state variable arbitrarily, keeping three others to be unchanged, and then formulating the four constraint equations with the rest four state variables. The problem of designing an FDI attack now requires solving a set of four nonlinear constraint equations for four unknowns. To illustrate the process, we will choose voltage angle of phase \(a\) at node 12, \(\theta_{12}^a\), as the initial point to launch an attack. In addition, three state variables will be kept unchanged are \(V_{12}^c\), \(V_{11}^c\), and \(\theta_{11}^c\). Finally, we have to solve a set of constraint equations \((14)-(17)\) for four unknowns \(\{V_{12}^a, V_{12}^c, \theta_{11}^a, \theta_{13}^c\}\). Before generating the set of manipulated state variables, a care must be taken to guarantee the correctness of results. Therefore, we must examine the result of steady state first. It means that we will launch an attack with an initial adjustment amount of zero to the state variable \(\theta_{12}^a\). As the solving process for the set of constraint equations yielded a set of state variables that matches exactly with the steady state values gathered from running load flow program, it is certain that the set of constraint equations is formed appropriately.

IV. EXPERIMENTAL RESULTS

The outcomes of the attack model in the previous section are the sets of manipulated state variables which are used to compute malicious measurements. These sets of malicious measurements are then fed into the dedicated Assessment Method (discussed in Section II-C), yielding various experimental results as presented below.

A. Attack area \(\Omega_{A3}\)

Several experiments are conducted with various small alterations to state variables at node 10. Using the proposed assessment procedure, very small differences between the calculated values and the simulation results are achieved. Fig. 8 provides some insight about the average differences (in percentage) between values acquired from attack design scheme and their corresponding results obtained from the assessment procedure. In general, the gaps normally fall in the region from 0.035% to 0.06%, which is small enough to negate any dissimilarity. As the loading values from the attack design process can recreate the steady state with a consistent load flow results, the set of malicious measurements is able to gain a “good” reputation from the viewpoint of the SE’s BDD module. The detection rate is 0% since the BDD fails to uncover all the injected malicious measurements. In addition, because 100% of these measurements are recognized as normal, the false positive rate is firmly 100%.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
State Var & Steady State & Attack State \\
\hline
\(\theta_{12}^a\) & -5.066 deg & -4.966 deg \\
\(V_{12}^a\) & 2.3537 kV/0.9800 pu & 2.3549 kV/0.9805 pu \\
\(V_{12}^c\) & 2.3657 kV/0.9850 pu & 2.3654 kV/0.9848 pu \\
\(\theta_{13}^a\) & -5.154 deg & -5.122 deg \\
\(\theta_{13}^c\) & 116.517 deg & 116.518 deg \\
\hline
\end{tabular}
\caption{Comparing state variables from steady state and attack state.}
\end{table}

Due to a slight adjustment of +0.1 degree applied for \(\theta_{12}^a\), the values of various state variables are altered accordingly. Although most of changes are minuscule, the gaps between steady state values and attack values are significant, as shown in the second and the third columns of Table III. However, the most important concern is the mismatch between the attack design results and the experimental values obtained from the simulation. Based on the empirical data of SE results for the balanced 1-phase equivalent model under attack, the differences between two individual corresponding member never surpass 1%. In this case, as indicated by the last column of Table III, the absolute values of difference in percentage are

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8}
\caption{Differences (in percentage) between the state variables obtained from attack design process and from the assessment procedure using PowerFactory 2017.}
\end{figure}
Table III: Comparisons of the steady state measurements, the malicious measurements due to an FDI attack, and the simulation results.

| Measurement | Steady State | Attack | PF2017 | % Diff |
|-------------|--------------|--------|--------|--------|
| P_{1108}^k (kW) | -123.81 | -103.03 | -103.03 | ~0 |
| P_{1112}^k (kW) | 123.81 | 103.03 | 103.03 | ~0 |
| Q_{1106}^k (kVar) | -82.84 | -105.30 | -105.23 | 0.067 |
| Q_{1112}^k (kVar) | 82.84 | 105.30 | 105.23 | 0.067 |
| P_{1108}^c (kW) | -167.83 | -172.62 | -172.61 | 0.006 |
| P_{1112}^c (kW) | 167.83 | 172.62 | 172.61 | 0.006 |
| Q_{1106}^c (kVar) | 17.83 | 11.26 | 11.29 | -0.266 |
| Q_{1112}^c (kVar) | -17.83 | -11.26 | -11.29 | -0.266 |
| P_{1211} (kW) | -122.93 | -102.17 | -102.17 | ~0 |
| P_{0811}^c (kW) | 124.02 | 103.25 | 103.25 | ~0 |
| Q_{1211}^c (kVar) | -82.59 | -104.99 | -104.90 | ~0 |
| Q_{0811}^c (kVar) | 83.05 | 105.49 | 105.41 | 0.076 |
| P_{1311} (kW) | -167.45 | -172.21 | -172.21 | ~0 |
| P_{0811}^c (kW) | 168.21 | 172.99 | 172.98 | 0.006 |
| Q_{1311}^c (kVar) | 18.22 | 11.67 | 11.70 | -0.257 |
| Q_{0811}^c (kVar) | -17.58 | -10.98 | -11.01 | -0.273 |
| P_{12}^p (kW) | 122.93 | 102.17 | 102.17 | ~0 |
| Q_{12}^p (kVar) | 82.59 | 104.99 | 104.99 | ~0 |
| P_{11}^p (kW) | 167.45 | 172.21 | 172.21 | ~0 |
| Q_{11}^p (kVar) | 78.80 | 85.32 | 85.32 | ~0 |
| Q_{11}^{cap} (kVar) | -97.00 | -96.99 | -97.00 | -0.01 |
| P_{06}^c (kW) | 382.00 | 374.00 | 374.00 | ~0 |
| Q_{06}^c (kVar) | 204.00 | 173.02 | 173.00 | 0.012 |
| P_{08}^c (kW) | 375.00 | 366.00 | 366.00 | ~0 |
| Q_{08}^c (kVar) | 217.00 | 229.00 | 229.00 | ~0 |

V. CONCLUSION

Anticipating the widely deployment of the State Estimation package for the emerging smart distribution network with integrated energy resources, this paper focuses on one of the utmost cyber-security threat, the False Data Injection attack. A comprehensive investigation of constraint-based FDI attack scheme is presented to demonstrate its feasibility and stealthy characteristics. Firstly, a thorough review of all typical features of distribution-level grid is discussed in order to emphasize challenges one must take on to organize the attack. Next, a detailed attack design scheme is proposed with special consideration given to the naturally unbalanced characteristic of the objective. An empirical-based dedicated assessment procedure is also devised to validate the simulation results.

A case study with the IEEE-13 node Test Feeder is conducted and examined to prove the correspondence between expected and experimental results.

The success of this proposed attack design scheme has posed a huge challenge to the future operation of smart grid operators. It means that as long as an FDI attack is systematically elaborated (such that it can create a pseudo steady state), all the contemporary data filtering mechanisms will definitely fail to accomplish the assigned missions. Hence, it is a crucial matter to build up a new detection method that has capability to handle this kind of threat. Therefore, one of our future priority work focuses is about a new detection method.

In addition, the investigation of attack must be expanded to various alternative scenarios, e.g. when the adversaries have limited attack resources or have no privilege to approach the control center. Such case studies will enhance the knowledge about threats in cyber-physical system, thus greatly contribute to improve the system’s security.

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REFERENCES

[1] F. C. Schwepe and J. Wildes, “Power system static-state estimation, part i: Exact model,” IEEE Transactions on Power Apparatus and systems, no. 1, pp. 120–125, 1970.
[2] I. Roytelman and S. Shahidehpour, “State estimation for electric power distribution systems in quasi real-time conditions,” IEEE Transactions on Power Delivery, vol. 8, no. 4, pp. 2009–2015, 1993.
[3] M. E. Baran and A. W. Kelley, “State estimation for real-time monitoring of distribution systems,” IEEE Transactions on Power Systems, vol. 9, no. 3, pp. 1601–1609, 1994.
[4] A. S. Meliopoulos and F. Zhang, “Multiphase power flow and state estimation for power distribution systems,” IEEE Transactions on Power Systems, vol. 11, no. 2, pp. 939–946, 1996.
[5] C. Lu, J. Teng, and W.-H. Liu, “Distribution system state estimation,” IEEE Transactions on Power systems, vol. 10, no. 1, pp. 229–240, 1995.
[6] M. E. Baran and A. W. Kelley, “A branch-current-based state estimation method for distribution systems,” IEEE transactions on power systems, vol. 10, no. 1, pp. 483–491, 1995.
[7] H. Wang and N. N. Schulz, “A revised branch current-based distribution system state estimation algorithm and meter placement impact,” IEEE Transactions on Power Systems, vol. 19, no. 1, pp. 207–213, 2004.
[8] “The Real Story of Stuxnet.” [Online]. Available: https://spectrum.ieee.org/telecom/security/the-real-story-of-stuxnet.
[9] R. Deng, P. Zhuang, and H. Liang, “False data injection attacks against state estimation in power distribution systems,” IEEE Transactions on Smart Grid, vol. 10, no. 3, pp. 2871–2881, 2018.
[10] G. Liang, S. R. Weller, J. Zhao, F. Luo, and Z. Y. Dong, “The 2015 ukraine blackout: Implications for false data injection attacks,” IEEE Transactions on Power Systems, vol. 32, no. 4, pp. 3317–3318, 2016.
[11] W. H. Kersting, Distribution system modeling and analysis. CRC press, 2006.
[12] A. Anwar and A. N. Mahmood, “Vulnerabilities of smart grid state estimation against false data injection attack,” in Renewable energy integration. Springer, 2014, pp. 411–428.
[13] G. Hug and J. A. Giampapa, “Vulnerability assessment of ac state estimation against false data injection cyber-attacks,” IEEE Transactions on Smart Grid, vol. 3, no. 3, pp. 1362–1370, 2012.
[14] “DigSILENT’s PowerFactory 2017 SP4.” [Online]. Available: https://www.digsilent.de/en/newsreader/digsilent-releases-powerfactory-2017-sp4.html.
[15] K. Schneider, B. Mather, B. Pal, C.-W. Ten, G. Shirek, H. Zhu, J. Fuller, J. Pereira, L. Ochoa, L. De Araujo et al., “Analytic considerations and design basis for the ieee distribution test feeders,” *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3181–3188, 2017.