Internal Model Based Active Disturbance Rejection Control

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Abstract

The basic active disturbance rejection control (BADRC) algorithm with only one order higher external state observer (ESO) proves to be robust to both internal and external disturbances. An advantage of BADRC is that in many applications it can achieve high disturbance attenuation level without requiring a detailed model of the plant or disturbance. However, this can be regarded as a disadvantage when the disturbance characteristic is known since the BADRC algorithm cannot exploit such information. This paper proposes an internal model based ADRC (IADRC) method, which can take advantage of knowing disturbance characteristic to achieve perfect estimation of the disturbance under some mild assumptions. The effectiveness of the proposed method is validated by comprehensive simulations and comparisons with the BADRC algorithm.

Index terms— Active disturbance rejection control, Internal model principle, Disturbance estimation, Sinusoidal disturbance, Extended state observer

1 Introduction

Rejecting unknown disturbances in dynamical systems is a fundamental control problem with various applications such as friction compensation during stick-slip motion [1], disturbance reduction in gyroscopes [2,3], active noise control [4], sinusoidal disturbances rejection of vibrating structures [5,6], control of robot manipulators [7], rotating mechanisms control [8], and nano-positioning [9,10]. This problem is usually solved by applying the internal model principle (IMP) for which a general solution is given in [11] in the case of linear systems. The IMP states that if the disturbance model can be accurately obtained and embedded in the controller, the disturbance can be entirely canceled. On the other hand, when there is no information available about the disturbance, IMP is no longer effective.

Active disturbance rejection control (ADRC) was proposed by Han [12] as an alternative paradigm for control system design [13,14], and since it is a model-free approach, it has the inherent advantages of rejecting nonlinearities, uncertainties and disturbances. Fruitful simulation results as well as the experimental results have been reported in various applications [15-19]. In these applications, the unknown parts (unknown nonlinearities, uncertainties and external disturbances) are treated as a total disturbance and estimated by an extended state observer (ESO). It has been proven that if the total disturbance or its first derivative is bounded, the estimate error is bounded and can be arbitrarily reduced [20]. However, the ESO has some limitations: (1) if the total disturbance is not a constant, the estimate error can only be bounded but not zero; (2) the disturbance information cannot be used. An example given in [21] shows that perfect estimation of even a simple sinusoidal disturbance cannot be achieved by basic ESO.

In this paper, we will propose an internal model based active disturbance rejection control (IADRC) in consideration of the disturbance information. The disturbance is separated into two parts, which are the part that can be modeled and the part that cannot be modeled. The former part is estimated by a disturbance observer with estimate error exponentially converging to zero. The unmodeled part with unknown nonlinearities and uncertainties are together treated as an extended state (total disturbance) of the system and estimated using ESO with a bounded error. It is shown that the modeled part is captured perfectly and the unmodeled part is regarded as a constant.
during the estimation and compensation from the IMP point of view. It is also illustrated that the performance of BADRC is improved significantly by IADRC when the disturbance information is used, and the more we know about the disturbance, the better IADRC performs.

The remainder is organized as follows. The problem statement is described in section 2. In section 3 a special class of disturbance that can be modeled as an output of a fully excited linear system is considered. Two adaptive estimation algorithms for the disturbance are proposed based on the known disturbance information. The IADRC is designed and analyzed in section 4. Simulation examples are given in section 5, and conclusions are drawn in section 6.

2 Problem formulation

Consider the nonlinear single-input-single-output system

\[
\begin{align*}
\dot{x} &= \bar{A}x + \bar{f}(\bar{x}, \omega_1) + \bar{b}(u + \omega_2) \\
\bar{y} &= \bar{c}x,
\end{align*}
\]  

(1)

with the state vector \( \bar{x} = [x_1, x_2, \cdots, x_n]^T \), the control input \( u \in R \), shift matrix \( \bar{A} \in R^{n \times n} \), \( \bar{f}(\bar{x}, \omega_1) \) an entirely unknown nonlinear smooth function, and \( \bar{b}(\bar{x}, \omega_1) = [0, \cdots, 0, f_n(\bar{x}, \omega_1)]^T \). \( \bar{b} = [0, \cdots, 0, b_n]^T \) with \( b_n \) a known constant, \( \omega_1, \omega_2 \in R \) are bounded unknown time-varying disturbances and \( \bar{c} = [1, 0, \cdots, 0] \). System (1) can be rewritten as

\[
\begin{align*}
\dot{x} &= Ax + b(u + d_2) + f \\
y &= cx
\end{align*}
\]  

(2)

where \( A \in R^{(n+1) \times (n+1)} \), \( b = \bar{b}^T, 0 \)^T, \( c = \bar{c}, 0 \), \( x = [\bar{x}^T, x_{n+1}]^T \) and \( f = [0, \cdots, 0, x_{n+1}]^T \). 

\( d_2 \) is part of the matched disturbance \( \omega_2 \) that has some known information. The total matched disturbance \( d \) is defined as \( d = d_1 + d_2 \), where \( d_1 := x_{n+1}/b_n \), and \( d_2 \) is the output of following system

\[
\begin{align*}
\dot{w} &= Sw, \\
d_2 &= h^Tw \\
w(0) &= w_0
\end{align*}
\]  

(3)

with \( w \in R^s \) and \( w_0 \) is selected that \( w \) are fully excited.

Remark 1 The extended state \( x_{n+1} \) that is entirely unknown can be viewed as the lumped unknown disturbance consisting of unknown nonlinearities, uncertainties of the plant and unknown part of external disturbances. \( d_1 \) can be considered as part of the total matched disturbance that is entirely unknown.

For the system and the disturbances, we have the following assumptions,

A. 1 \( f_n(\bar{x}, \omega_1) \) is unknown, but \( f_n(\bar{x}, \omega_1) \) or \( f_n(\bar{x}, \omega_1) \) is bounded with \( f_n(0, 0) = C_1 \) where \( C_1 \) is an unknown constant.

A. 2 The matrix \( S \) has no zero eigenvalues.

A. 3 \( h \) is an unknown constant vector.

A. 4 The matrix \( S \) is entirely known.

A. 5 The matrix \( S \) is unknown, but \( s \), the dimension of \( S \) is known.

Our problem is to design an output feedback controller to stabilize the origin with the ability to reject the disturbance \( d_2 \) exponentially (thus perfectly) when \( \omega_1 = 0 \) by making full use of the known information of the external disturbance and simultaneously to reject \( d_1 \) in the frame of ADRC.
3 Disturbance observer design and analysis

The idea is as follows. we use an ESO to estimate the internal uncertainty and a disturbance observer to estimate the external disturbance exponentially, and then compensate the total disturbance.

If the external disturbance does not exist in system (1), that is, \( d_2 = 0 \) in system (2). In this case, the extended state observer can be designed as

\[
\dot{p} = Ap + bu + l (\bar{y} - cp),
\]

where \( p, l \in \mathbb{R}^{n+1} \) and \( l \) is chosen such that \( A - lc \) is Hurwitz. It is difficult to estimate the real states because of the unknown disturbance \( d \). \( A - lc \) and \( S \) have exclusive eigenvalues for that we have assumption A.1 and the selection of \( l \), so unique solution \( Q \in \mathbb{R}^{(n+1)\times s} \) of the following Sylvester equation

\[
QS = (A - lc) Q + bh^T,
\]

for a given \( S \) exists [22]. By defining \( q := Qw \), (5) implies

\[
\dot{q} = (A - lc) q + bd_2.
\]

Remark 2 Since \( h \) is unknown, no matter whether \( S \) is known or not, the solution \( Q \) cannot be obtained from (5), and the observer (6) is unimplementable for that \( d_2 \) is unknown.

In order to obtain \( p \) and \( q \), we have the following lemma [23].

Lemma 1 The state variable \( x \) can be expressed as

\[
x = p + q + \varepsilon,
\]

where \( p \) is from (4) with \( q \) from (6) and \( \varepsilon \) satisfying

\[
\dot{\varepsilon} = (A - lc) \varepsilon.
\]

The state estimation is solved if an estimate of \( q \) is obtained. Considering (6), the problem to be solved is to estimate the states and unknown input to a minimum phase linear dynamic system.

In order to design the disturbance observer, a reformulation of the system (3) is first introduced. A controllable pair \((F,g)\) with \( F \in \mathbb{R}^{s \times s} \) Hurwitz and \( g \in \mathbb{R}^s \) are selected. For a matrix \( S \) satisfying A.2 which also implies the pair \((S,Q(1))\) observable, there exists a non-singular \( M \in \mathbb{R}^{s \times s} \) satisfying the following Sylvester equation [22]

\[
MS - FM = gQ(1),
\]

where \( Q(i) \) denotes the \( i \)-th row of \( Q \). Let \( \eta := Mw \) which implies

\[
\dot{\eta} = F_0 \eta,
\]

where

\[
F_0 = MSM^{-1} = F + g\psi_1^T
\]

and \( \psi_1^T = Q(1)M^{-1} \). In the new coordinate \( \eta \), \( q \) and \( d_2 \) can be expressed as

\[
q = \Psi^T \eta,
\]

where \( \Psi^T = QM^{-1} = [\psi_1, \psi_2, \psi_3, \cdots, \psi_{n+1}]^T \), and

\[
d_2 = \psi_u^T \eta,
\]

with \( \psi_u^T = h^T M^{-1} \).

From (10) and (11), we know that if an estimate of \( \eta \) is provided and \( \psi, \psi_u \) are obtained, the estimate of \( q \) and \( d_2 \) are obtained, thus the state estimation is solved. From (8) and Lemma 1, we have

\[
\dot{\eta} = F\eta + g(\bar{y} - p_1 - \varepsilon_1),
\]
where \( p_1 = p(1) \) and \( \bar{y} = x_1 \), indicating that the observer for \( \eta \) should be designed as

\[
\dot{\xi} = F\xi + g(\bar{y} - p_1).
\]  

(12)

Define \( e_\eta := \eta - \xi \). We have \( \dot{e}_\eta = Fe_\eta - gc\varepsilon \), which together with (7) imply

\[
\begin{bmatrix}
\dot{\varepsilon} \\
\dot{\xi}
\end{bmatrix} =
\begin{bmatrix}
F & -gc \\
0 & A - lc
\end{bmatrix}
\begin{bmatrix}
e_\eta \\
\varepsilon
\end{bmatrix}.
\]

Since \( F \) and \( A - lc \) are both Hurwitz, \( e_\eta \) converges to zero exponentially.

How to get \( \psi \) and \( \psi_u \) depends on whether \( S \) is known or not. From \( q \) and \( d_2 \) in (10) and (11) to the observer (6), we have

\[
\begin{align*}
\psi^T_i F_o &= \psi^T_{i+1} - l_i \psi^T_1, i = 1, \ldots, n - 1, \\
\psi^T_n F_o &= \psi^T_{n+1} - l_n \psi^T_1 + b_n \psi_u^T, \\
\psi^T_{n+1} F_o &= -l_{n+1} \psi^T_1.
\end{align*}
\]

S is invertible under assumption A.2, relating (9) we have \( F_o \) invertible. Then we get

\[
\begin{align*}
\psi^T_i F_o &= \psi^T_{i+1} - l_i \psi^T_1, i = 1, \ldots, n - 1, \\
\psi^T_{n+1} &= -l_{n+1} \psi^T_1, \\
\psi^T_u &= (\psi^T_n F_o + l_n \psi^T_1 - \psi^T_{n+1}) / b_n.
\end{align*}
\]

From (13), we know that if \( \psi_1 \) is obtained, then \( \Psi \) and \( \psi_u \) are obtained. We will show how to get \( \psi_1 \) based on \( S \).

**Case 1: The matrix \( S \) is entirely known.**

Since \( M \) is non-singular, from (9), we know that the matrix \( F_o \) has the same eigenvalues with the matrix \( S \) and then \( \psi_1 \) can be obtained. Without losing generality, \( F \) and \( g \) are selected as

\[
F = A - b\alpha F, g = [0, 0, \cdots, 0, 1]^T.
\]

(14)

The characteristic polynomial coefficients of \( S \) and \( F \) are

\[
\begin{align*}
\alpha_S &= [\alpha_0, \alpha_1, \cdots, \alpha_{s-2}, \alpha_{s-1}]^T, \\
\alpha_F &= [f_0, f_1, \cdots, f_{s-2}, f_{s-1}]^T,
\end{align*}
\]

thus

\[
\psi_1 = \alpha_F - \alpha_S.
\]

(15)

Then \( \psi_i, i = 2, \cdots, n + 1 \) and \( \psi_u \) are computed from (13). Therefore, the external disturbance is estimated as

\[
\hat{d}_2 = \psi_u^T \xi.
\]

(16)

To have a summarization, when \( S \) is known, we can get \( \hat{d}_2 \) with following steps:

**Procedure 1:**

S1. Select \( F \) and \( g \) with the form (14) and determine \( \alpha_S \) and \( \alpha_F \);

S2. Compute \( \psi_1 \) using (15) and \( \psi_i, i = 2, \cdots, n + 1 \) and \( \psi_u \) using (13);

S3. Obtain the \( \xi \) using (12);

S4. Get \( \hat{d}_2 \) using (16).

**Case 2: The matrix \( S \) is unknown, but \( s \), the dimension of \( S \) is known**

In this case, we know that \( \psi_1 \in R^s \). Since \( S \) is unknown, we cannot obtain \( \psi_1 \) through **Procedure 1**. Suppose that \( \hat{\psi}_1 \) is the estimate of \( \psi_1 \), then \( \hat{\xi} \), the estimate of \( \xi \) is updated by

\[
\hat{\xi} = F \hat{\xi} + g \hat{\psi}_1^T \xi.
\]

(17)
Define $e_\xi = \zeta - \zeta$, we have
\[ \dot{e}_\xi = F e_\xi + g c + g \psi_1^T e_\eta + g \xi^T \tilde{\psi}_1. \]

Define $e = \begin{bmatrix} e_\xi^T & e_\eta^T & e^T \end{bmatrix}^T$, we have
\[ \dot{e} = A_e e + \phi(t) \tilde{\psi}_1, \]
where $A_e = \begin{bmatrix} F & g \psi_1^T & gc \\ 0 & F & -gc \\ 0 & 0 & A - lc \end{bmatrix}$ and $\phi(t) = \begin{bmatrix} g \xi^T \\ 0 \\ 0 \end{bmatrix}$. $A_e$ is Hurwitz for that both of $F$ and $A - lc$ are Hurwitz, therefore, for a given positive definite symmetric matrix $Q_c$, there exists a positive definite symmetric matrix $P_c$ satisfying the Lyapunov equation
\[ A_c^T P_c + P_c A_c = -2Q_c. \]

Selecting $\Gamma \in \mathbb{R}^{s \times s}$ as a positive definite matrix, the Lyapunov candidate function is selected as
\[ V \left( e, \tilde{\psi}_1 \right) = \frac{1}{2} \left( e^T P_c e + \tilde{\psi}_1^T \Gamma^{-1} \tilde{\psi}_1 \right), \]
whose first derivative is
\[ \dot{V} \left( e, \tilde{\psi}_1 \right) = -e^T Q_c e + \tilde{\psi}_1^T \phi(t) P_c e + \tilde{\psi}_1^T \Gamma^{-1} \dot{\tilde{\psi}}_1, \]
by setting
\[ \dot{\tilde{\psi}}_1^T \phi(t) P_c e + \tilde{\psi}_1^T \Gamma^{-1} \dot{\tilde{\psi}}_1 = 0, \]
which indicates that
\[ \dot{\tilde{\psi}}_1 = -\Gamma^{-1} \phi^T(t) P_c e, \] (18)
we have
\[ \dot{V} \left( e, \tilde{\psi}_1 \right) = -e^T Q_c e, Q_c > 0, \]
so $e$ and $\tilde{\psi}_1$ is bounded and from the well known Barbalat Lemma we know that $\lim_{t \to \infty} e(t) = 0$.

Since $\tilde{\psi}_1$ is unknown, we cannot get $P_c$, and the updating law (18) is not implementable. Suppose that $P_c$ is of the form
\[ P_c = \text{diag} \{ P_1, \gamma_1 P_1, \gamma_2 P_2 \}, \]
where $P_1$ and $P_2$ are positive definite matrices satisfying
\[ F^T P_1 + P_1 F = -2Q_1, \] (19)
and
\[ (A - lc)^T P_2 + P_2 (A - lc) = -2Q_2, \]
with $Q_1$ and $Q_2$ selecting as positive definite matrices and $\gamma_1$ and $\gamma_2$ are positive constant. Thus $Q_c$ can be selected as
\[ Q_c = \begin{bmatrix} Q_1 & \frac{P_1 \gamma_1}{\gamma_2} & \frac{P_1 \gamma_1}{\gamma_2} \\ \frac{P_1 \gamma_1}{\gamma_2} & \gamma_1 Q_1 & \gamma_1 \gamma_2 Q_1 \\ \frac{P_1 \gamma_1}{\gamma_2} & \gamma_1 \gamma_2 Q_1 & \gamma_1 \gamma_2 \gamma_2 Q_2 \end{bmatrix}. \]

Obviously, $Q_c$ is symmetric and by selecting $\gamma_1$ and $\gamma_2$ sufficiently large, $Q_c$ will be positive definite. Then the updating law (18) can be rewritten as
\[ \dot{\tilde{\psi}}_1 = -\Gamma^{-1} \xi g^T P_1 e_\xi, \] (20)
yielding
\[ \dot{\tilde{\psi}}_1 = \Gamma^{-1} \xi g^T P_1 e_\xi. \] (21)
(21) is implementable for that $\Gamma$ and $g$ are selected, $P_1$ is computed by (19), $\xi$ is updated by (12), $e_\xi = \xi - \zeta$ where $\zeta$ is updated by (17).
The updating law (21) ensures that \( \lim_{t \to \infty} \dot{\hat{\psi}}_1 = 0 \), indicating that \( \hat{\psi}_1 \) will converge to a constant vector, but no guarantee that \( \tilde{\psi}_1 \) converges to zero. It can be proven that \( \tilde{\psi}_1 \) converges to zero iff \( \xi g^T \) is persistently excited. Computing

\[
\int_{t_0}^{t_0+T_0} \xi g^T (\xi g^T)^T d\tau = \|g\|^2 \int_{t_0}^{t_0+T_0} \xi^T d\tau,
\]

where \( \xi = \xi(\tau) = \eta(\tau) - e_\eta(\tau) \), we have \( \xi g^T \) is persistently excited iff \( \eta \) is persistently excited for that \( e_\eta \) converges to zero exponentially. Since

\[
\int_{t_0}^{t_0+T_0} \eta \eta^T d\tau = \|M\|^2 \int_{t_0}^{t_0+T_0} w w^T d\tau,
\]

we have \( \eta \) is persistently excited iff \( w \) is persistently excited, which can be realized by selecting a proper \( w_0 \).

With the estimate of \( \hat{\psi}_1 \), we obtain the estimate of \( F_o \) as \( \hat{F}_o = F + g \hat{\psi}_1^T \), and the estimate of \( \hat{\psi}_i, i = 2, \ldots, n + 1 \) and \( \hat{\psi}_u \) as

\[
\begin{align*}
\hat{\psi}_i^T_{i+1} &= \hat{\psi}_i^T \hat{F}_o + l_i \hat{\psi}_1^T, \quad i = 1, \ldots, n - 1, \\
\hat{\psi}_{n+1}^T &= -l_{n+1} \hat{\psi}_1^T \hat{F}_o^{-1}, \\
\hat{\psi}_u^T &= \left( \hat{\psi}_n^T \hat{F}_o + l_n \hat{\psi}_1^T - \hat{\psi}_{n+1}^T \right) / b_n.
\end{align*}
\]

(22)

Spontaneously we get the estimate of the external disturbance

\[
\hat{d}_2 = \hat{\psi}_u^T \xi.
\]

(23)

To have a summarization, when \( S \) is unknown, we can get \( \hat{d}_2 \) with following steps:

**Procedure 2:**

S1. Select \( F \) and \( g \) with the form (14) and \( Q_1 \), then compute \( P_1 \) from (19);

S2. Obtain \( \xi \) using (12);

S3. Obtain \( \zeta \) using (17);

S4. Update \( \hat{\psi}_1 \) using (21);

S5. Compute \( \hat{\psi}_i, i = 2, \ldots, n + 1 \) and \( \hat{\psi}_u \) using (22);

S6. Get \( \hat{d}_2 \) using (23).

4  The internal model based active disturbance rejection control design and analysis

**Theorem 1** Considering the dynamic system (3) satisfying assumption A(2) and the following output feedback observer

\[
\begin{align*}
\dot{v} &= Av + bu + l (\bar{y} - y) \\
y &= cv
\end{align*}
\]

and the control input

\[
u = -k^Tv,
\]

the closed-loop system described under the state \( z = [x^T, v^T]^T \) is asymptotically stable when \( l \) is selected that \( A - lc \) is Hurwitz and \( k \) is selected as \( k = [k^T, 1]^T \) where \( k \) is selected such that \( \bar{A} - \bar{b}k \) is Hurwitz.

**Remark 3** In fact, (24) and (25) are the BADRC for system (3).
With the estimate of the external disturbance, the controller is then designed as
\[ u = u_c + u_d, \]
where
\[ u_d = -\hat d_2, \]
and \( u_c \) is generated by
\[ \dot v = Av + bu_c + l (\bar y - cv), \]
\[ u_c = -k^T v. \] (26)

**Remark 4** In [26], the input to get \( v \) is \( u_c \) rather than \( u \), which is reasonable for that the external disturbance \( d_2 \) is compensated and \( u_c \) can be seen as the feedback control when there is no disturbance.

**Closed-loop system stability analysis:** We consider the stability of the original system (2) under the control [26]. Defining \( \hat v = x - v \), we have
\[ \dot v = (A - lc) \dot v + b\hat d_2 + f, \]
and
\[ \dot x = (A - bk) x + bk\dot v + b\hat \sigma d_2 + f, \]
which together imply
\[
\begin{bmatrix}
\dot x \\
\dot \nu
\end{bmatrix} =
\begin{bmatrix}
A - bk & bk \\
0 & A - lc
\end{bmatrix}
\begin{bmatrix}
x \\
\dot \nu
\end{bmatrix} +
\begin{bmatrix}
b & b_f \\
b & b_f
\end{bmatrix}
\begin{bmatrix}
\hat d_2 \\
\dot x_{n+1}
\end{bmatrix}.
\]
Since \( \hat d_2 \) and \( \dot x_{n+1} \) are bounded, the overall system is input-to-state-stable (ISS).

## 5 Simulation examples

Consider the following system
\[
\begin{align*}
\dot x_1 &= x_2 \\
\dot x_2 &= f_2 (x, \omega_1) + b_2 (u + \omega_2), \\
y &= x_1
\end{align*}
\]
where \( b_2 = 3 \) is a known constant, \( f_2 (x, \omega_1) = 0 \) and \( \omega_2 = \sigma_0 + r \sin(\sigma T + \varphi) \). So \( d_1 = \sigma_0 / b_2 \), and \( d_2 \) is of the form
\[ d_2 = r \sin(\sigma T + \varphi), \]
where \( r = 0.8, \sigma = 2 \) and \( \varphi = \pi / 5 \). \( d_2 \) can be rewritten as (3) where \( h = [r \cos \varphi, r \sin \varphi]^T \), and \( w = [\sin \sigma T, \cos \sigma]^T \) with \( S = \begin{bmatrix} 0 & \sigma \\ -\sigma & 0 \end{bmatrix} \) and \( w_0 = [0, 1]^T \).

We first consider the case \( S \) is known. With **Procedure 1**, we select \( F = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \), \( g = [0, 1]^T \), the we get \( F_0 = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \), \( \psi_1 = [-2, 3]^T \), \( \psi_2 = [-102, 133]^T \), \( \psi_3 = [900, 1500]^T \), and \( \psi_u = [7168/3, 1116]^T \). The simulation results are shown in Figs.1 and 2. In Fig.1, \( x_{1a} \) and \( x_{2a} \) are states with respect to (w.r.t) the BADRC while \( x_{1m} \) and \( x_{2m} \) are states w.r.t the proposed IADRC. Fig.1 shows that the proposed algorithm can reject the unknown disturbance more effective than the BADRC. Fig.2 reveals the reason: by exploiting the known information of the disturbance, the perfect estimation of \( d_2 \) is achieved while there exists a phase lag when the disturbance is estimated only by using ESO.

We then consider the case \( S \) is unknown. With **Procedure 2**, we select \( F = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \), and \( g = [0, 1]^T \), \( \Gamma = 40000 I_2 \) and \( Q_1 = 150 I_2 \). By solving Lyapunov equation (19), we have \( P_1 = \begin{bmatrix} 300 & -150 \\ -150 & 150 \end{bmatrix} \). Results are shown in Figs.3, 4 and 5. As shown in Fig.4, perfect estimates for \( d_1 \) and \( d_2 \) are achieved for that the estimate of \( \psi_1 \)
Figure 1: States revolution w.r.t ADRC and IADRC (known $S$)

Figure 2: Disturbances and their estimates (known $S$)

Figure 3: States revolution w.r.t IADRC (unknown $S$)

Figure 4: Disturbances and their estimates (unknown $S$)

Figure 5: Estimates of $\psi_{11}$ and $\psi_{21}$

Figure 6: States revolution w.r.t IADRC (unknown $S$)
converges to its real value $[-2, 3]^T$ as shown in Fig. 5. The total matched disturbance $d$ is perfectly estimated thus being fully compensated, therefore the system states converges to zero at the steady state shown in Fig. 3.

A more complex case is considered here. Suppose that $f_2(x, \omega_1) = x_1^2 + x_2^2 + \sin(\pi t)$, therefore $d_1 = f_2/b_2 + \sigma_0$ and $d_2 = r \sin(\sigma t + \varphi)$. $S$ is unknown and parameters are chosen the same as above. Results are shown in Figs. 6, 7 and 8. Since $x_3 = f_2 + b_2\sigma_0$ is not a constant, as shown in Fig. 7, no perfect tracking for $d_1$ can be reached, which leads to the estimates of $\psi_1$ oscillating around its true value in a small region as shown in Fig. 8. Therefore, there exists oscillation in system states around 0 as shown in Fig. 6.

6 Conclusions

The principle of ADRC from the internal model principle point of view was presented in this paper. An improved ADRC that can properly exploit known information about the disturbance was proposed. Depending on whether the dynamics of $S$ is known or not two adaptation algorithms were provided. Moreover, it was shown that when $S$ is unknown, it is required to estimate it, whereas the system states are the only elements that need to be estimated when $S$ is known. Simulation results show that IADRC is of significant improvement compared to the BADRC.

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