THEOREM ABOUT THE CHANGE OF RESONANCE FREQUENCIES OF VIBRATIONS OF MECHANICAL SYSTEMS WITH FRICTION

There are the insufficiently known phenomena, resulting in the operating damages of knots and details of machines, among that most dangerous are a friction, wear, and dynamic tiredness. They behaves to the mechanical systems, the elements of that are bound by inter se forces of dry friction, that is widely widespread in a technique, especially in the knots of contact of elements with a friction. It results in the origin of new effects the account of that is needed for providing of increase of reliability of work of machines and mechanisms. At the decision of practical tasks usually use the simplified charts that are characterized the eventual number of degrees of freedom, although the real mechanical system has an endless large number of degrees of freedom. The study of vibrations of the mechanical systems on such models allows to get basic conformities to law of influence of interesting factors on dynamic descriptions of the system. In the theory of vibrations are of interest research of questions of cooperation of forces of friction, operating in the system, and force vibrations. However paid attention to the question of influence of forces of friction on the size of frequency of resonant vibrations of the mechanical systems. Under the inlaid constructions we understand such, when one elements of construction are located into other and here constrained inter se on opposite surfaces by forces of dry friction. The selection of class of the inlaid constructions, among the nonlinear mechanical systems, requires the row of clarifications. Consists the feature of design of one mass models of the inlaid constructions in that they have two degrees of freedom. It results in appearance at the inlaid constructions of new properties, what and dedicated hired.

Keywords: mechanical system; degrees the freedoms; inlaid constructions; operating damages; friction; wear; dynamic systems; calculation charts; oscillating mechanical systems; nonlinear mechanical systems.

Formulation of the problem

One of the problems facing modern science is to ensure the reliability of machines [1, 4]. Among many phenomena that lead to operational damage to machine components and components, friction, wear and dynamic fatigue are the most dangerous. First of all, this refers to mechanical systems, the elements of which are interconnected by the forces of dry friction. Such systems are widely used in engineering, especially in the nodes of contacting elements with friction [2]. If the contact of the body with the counter body occurs on opposite surfaces (we call such constructions nested), then this leads to the appearance of new effects, which are necessary to ensure the increased reliability of machines.

It is known that a real mechanical system has an infinite number of degrees of freedom. However, when solving practical problems, simplified schemes are usually used, which are characterized by a finite number of freedom degrees. In such calculation schemes, some (the lightest) parts of the system are considered completely devoid of mass and are represented in the form of deformable inertia-free bonds, while the bodies behind which the inertia property is retained in the calculation scheme are considered material points [1]. The study of the oscillations of mechanical systems on such models makes it possible to obtain the basic regularities of the influence of the factors of interest on the dynamic characteristics of the system.

A special place in the theory of oscillations is given to investigations of the interaction of frictional forces acting in the system and forced oscillations. At the same time, this issue is mainly discussed in two aspects: friction, as a source of self-oscillations and oscillations, is a mechanism for controlling friction. As a result, attention is not paid to the effect of friction forces on the frequency of the resonant oscillations of mechanical systems. This is connected, perhaps, with the fact that this issue is considered resolved. However, the selection of a class of embedded constructions (among nonlinear mechanical systems) requires some refinement.

Nested we will call the construction, some elements of which are located inside the others and are connected to each other on opposite surfaces by the forces of dry friction, and the ratio of the maximum tangential force spent on overcoming the bonds caused by touching the elements when removing them from the state of rest to the same load, which compresses the touch elements, provided there is no connection between them on opposite surfaces, greater than the true coefficient of friction of rest.

Nested constructions correspond to a mechanical model of a certain type. The peculiarity of the model lies in the fact that the single-mass model of the embedded construction has two degrees of freedom [2]. This leads to the appearance of new properties in nested constructions.

The aim of the article is to determine the influence of the frictional force on the magnitude of the oscillation frequency of a mechanical system.

The Main material

Let us consider the effect of friction on the frequency of free vibrations of a single-mass mechanical system, which is a load of mass $m$ suspended on a spring in rigidity $k$, in parallel to which a damping element with a damping coefficient $C$ (Fig. 1). The differential equation of cargo movement, as is known, has the form [3-6]:

$$m\ddot{x} + c\dot{x} + kx = 0.$$

The solution of this equation is known:
Fig. 1. The calculation scheme of a single-mass model of a mechanical system with one degree of freedom.

\[ x = e^{\frac{C_1}{2m}} \left[ C_1 \cos pt + C_2 \cos pt \right], \]

here \( C_1, C_2 \) are arbitrary constants; \( p \) is the frequency of the natural oscillations.

\[ p = \sqrt{\frac{k - C^2}{4m^2}}. \]

From the last equality it is seen that with increasing resistance the frequency of the oscillations of the load decreases, which is confirmed by experiments.

We apply the external force \( Q(t) = Q_0 \sin \omega t \) to the load. Let us see how the increase in the damping factor affects the value of the frequency of the resonance oscillations of the load. The resonance in the system will no longer occur when the ratio of the frequency of the forced oscillations to the frequency of the natural oscillations of the system without friction \( p \) is equal to 1, but at lower frequencies, that is, the maxima of the curves of the dynamical system dependence on the ratio \( \omega / p \) will be shifted to the left of the value \( \omega / p = 1 \) [6]. Thus, for the system under consideration, an increase in the damping coefficient \( C \) leads to a decrease in the value of the frequency of the resonance oscillations of the load. The calculation of a mechanical system consisting of \( n \) masses and \( n \) dampers is given in [7]. And in this case, an equal increase in the damping coefficients in \( n \) dampers leads to a decrease in the magnitude of the frequency of the resonance oscillations of the goods. A further increase in damping leads to an end to the oscillatory movements of the system.

From the definition of this nested construction, it follows that the nested structures correspond to mechanical models of a certain type. One of such models with two degrees of freedom (with one mass and one knot of friction) is shown in Fig. 2. A feature of nested constructions is the condition \( S = n \), where \( S \) is the number of damping elements, \( n \) is the number of degrees of freedom.

Let us examine, in the general case, the dependence of the natural frequencies of oscillations of a mechanical system on the magnitude of the frictional force. In this case, consider systems in which the number of damping elements is equal \( (S = n) \), and less number of degrees of freedom, which corresponds to nested structures.

Consider the oscillations of a mechanical system consisting of two masses \( m_1 \) and \( m_2 \) (we take \( m_1 = m_2 = m \)), connected with the base and each other by means of elastic elements with stiffness \( k \) and damping elements with a damping coefficient \( C \) (Fig. 3).

We apply to the masses an external force

\[ Q = Q_0 \sin t. \]

The equations of oscillations of the system have the form [6]:

\[ \ddot{x}_1 = \frac{k_3}{m_1} x_1 - \frac{(k_1 + k_3)}{m_1} x_1 - \frac{C_1}{m_1} \dot{x}_1 + \frac{Q_0}{m_1} \sin \omega t, \]

\[ \ddot{x}_2 = \frac{k_3}{m_2} x_2 - \frac{(k_1 + k_3)}{m_2} x_2 - \frac{C_2}{m_2} \dot{x}_2 + \frac{Q_0}{m_2} \sin \omega t. \]

Analysis of the results of calculation of the resonance frequencies of the oscillations of the system under consideration as a function of the value of the damping coefficient (with parameters \( m_1 = m_2 = m = 1 \text{ kg}, \ k_1 = k_2 = k_3 = 200 \text{ n/m} \) and calculations of many mass systems carried out using numerical methods allow us to formulate the theorem on the change in resonance frequencies of oscillations of mechanical systems in the presence of friction.

Fig. 2. The calculation scheme of a single-mass mechanical system with two degrees of freedom.

Fig. 3. Design diagram of two-mass mechanical systems with dampers.
Theorem. Introduction to the mechanical system with n degrees of freedom s of dissipative elements leads to the fact that the resonant frequencies of the newly formed system with respect to the system without friction are located as follows:

where \( s = n \)

\[ p_k > p_n^s \ (k = 1, 2, \ldots n); \]

where \( s < n \) for n and \((n-s)\) degrees of freedom \( p_k \leq p_n^s < p_{s+1} \ (k = 1, \ldots, n-1). \)

Evidence:
1. Consider a mechanical system in which the number of damping elements is equal to the number of degrees of freedom \((n = s)\). The equation of motion in the system has the form [8]:

\[ MX + CX + SX = Q, \]

where \( M \) is the mass matrix; \( C \) is the damping matrix; \( S \) is the matrix of rigidity.

It was shown in [6, 8] that in this case the with equation of motion in normal coordinates will have the form:

\[ \ddot{x}_i + 2n_i \dot{x}_i + p_i \dot{x}_i = q_i, \quad i = 1, 2, 3 \ldots n, \]

where \( 2n \) is the damping constant.

Each of the \( n \) equations is unrelated to all the others. Therefore, the dynamic displacement is related to the \( i \)-th form of oscillations.

The dynamic displacements corresponding to the \( i \)-th form of the oscillations of the system, in the presence of damping, are determined by the expression [6]:

\[ x_i = e^{-n_i t} (x_{0i} \cos p_i t + \frac{\dot{x}_{0i} + n_i x_{0i}}{p_i} \sin p_i t). \]

The circular frequency with damped oscillations is determined from the expression:

\[ \nu_{bi} = \sqrt{p_i - n_i^2} = p_i \sqrt{1 - \gamma_i^2}, \]

where \( p_i \) is the circular frequency of undamped oscillations; \( C_i \) is the corresponding value of the damping coefficient.

Thus, \( p_i > p_n \).

2. Consider a mechanical system in which the number of damping elements is less than the number of degrees of freedom \((n > s)\). Suppose that the \( s_i \) damping element is converted into a rigid connection \((C \to \infty)\). The coupling equation can be represented by the expression [8]:

\[ A_{11} q_1 + A_{12} q_2 + \ldots + A_{1n} q_n = 0; \]

\[ (a_{11} p^2 - c_{11}, \ldots, a_{1n} p^2 - c_{1n}) ; \]

\[ a_{n1} p^2 - c_{n1}, a_{n2} p^2 - c_{n2}, \ldots, a_{nn} p^2 - c_{nn} \]

Conclusions
The properties of oscillatory systems formulated in the theorem are revealed in numerical investigation and can be generalized by the following provisions:

1) The steady-state oscillations occur with respect to the position of stable equilibrium with the frequency of the disturbing force.

2) If the frequency of the perturbing force is close to one of the natural frequencies of the system, then the amplitudes of the oscillations of all masses increase and reach local maxima, the values of which depend on the frictional forces.

3) Local extrema of the resonance curve for \( n = s \) are attained at frequencies less than the natural frequencies of the system.

4) Local extrema of the resonance curve \( n > s \) are attained at frequencies not less than the natural frequencies of the system.

5) In systems with small friction, the change in the coefficients of friction at any place causes a change in the resonance amplitudes of the oscillations: as the friction increases, the amplitudes decrease. In systems with high friction, it is possible to select a friction value.
относятся к механическим системам при наличии трения. Пределы введенных конструкций, в особенности, малопригодны для изучения звуковых колебаний механических систем, особенно в узлах контактирования элементов с трением.

При виреншенных практических задачах закономерными строятся дополнительные схемы, которые характеризуются квадратными числом степеней свободы, хотя реальная механическая система имеет конечное число степеней свободы. Выводы из этого представляют интерес для изучения колебаний механических систем в многочисленных приложениях в различных областях техники.

ВВЕДЕНИЕ

Сущность работы заключается в том, чтобы выделить класс вложенных конструкций, среди нелинейных механических систем, которые имеют две степени свободы. Это приводит к появлению нового класса вложенных конструкций новых свойств, чему и посвящается данная работа.

Ключевые слова: механическая система; степень свободы; вложенные конструкции; эксплуатационные повреждения; трение; износ; динамические системы; расчетные схемы; колебательные механические системы; нелинейные механические системы.