Quantum bit commitment and the reality of the quantum state

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Quantum bit commitment (QBC) is insecure in the standard non-relativistic quantum cryptographic framework, essentially because Alice can exploit quantum steering to defer making her commitment. Two assumptions in this framework are that: (a) Alice knows the ensembles of evidence \( E \) corresponding to either commitment; and (b) system \( E \) is quantum rather than classical.

Here, we show how relaxing assumption (a) or (b) can render her malicious steering operation indeterminable or inexistent, respectively. Finally, we present a secure protocol that relaxes both assumptions in a quantum teleportation setting. Without appeal to an ontological framework, we argue that the protocol’s security entails the reality of the quantum state, provided retrocausality is excluded.

I. INTRODUCTION

Bit commitment (BC) is a cryptographic mistrustful task between two adversarial parties Alice and Bob, wherein Alice commits a bit \( a \) by submitting as evidence a quantum system, possibly after multiple rounds of communication between them, and later she unveils \( a \). The security requirement is that the evidence must be binding on her, while hiding \( a \) from Bob until her unveiling. BC is important because it can serve as a primitive for important crypto-tasks, among them coin flipping, oblivious transfer, secure multi-party computation, signature schemes and zero-knowledge proofs. Except by invoking computational assumptions, a trusted third party, or relativistic constraints on signaling [1–6], secure BC is conventionally not believed to be possible.

We briefly recapitulate a version of the standard insecurity argument against (nonrelativistic) quantum BC (QBC) [9,10]. Suppose \( \mathcal{E}^\alpha = \{ |\tilde{\chi}_0^\alpha \rangle = \sqrt{\rho_0^\alpha} |\chi_0^\alpha \rangle \} \) denotes the ensemble of un-normalized states of the evidence, corresponding to Alice’s commitment to bit \( \alpha \), which Alice submits to Bob. For her commitment to be concealed from Bob, it is required that

\[
\rho_B^0 = \rho_B^1, \tag{1}
\]

where \( \rho_B^\alpha = \sum_j |\tilde{\chi}_j^\alpha \rangle \langle \tilde{\chi}_j^\alpha | \) and \( \alpha \in \{0, 1\} \). To cheat, Alice submits the second register of the purification

\[
|\Psi\rangle = \sum_j |\phi_j^0 \rangle |\tilde{\chi}_j^0 \rangle \tag{2}
\]

as her evidence system, where \( |\phi_j^0 \rangle \) are elements of an orthonormal basis. To unveil \( \alpha = 0 \), she measures the first register in the basis \( \{ |\phi_j^0 \rangle \} \) at unveiling time and announces the outcome \( j \). To unveil \( \alpha = 1 \), she steers \( |\Psi\rangle \) in \( E \) into the ensemble \( \mathcal{E}^1 \) before measurement, i.e., she measures the first register in the basis \( \{ |\phi_j^1 \rangle = \mathcal{U} |\phi_j^0 \rangle \} \), where matrix \( \mathcal{U} \equiv \{ U_{jk} \} \) is defined by

\[
|\tilde{\chi}_k^0 \rangle = \sum_j U_{jk} |\tilde{\chi}_j^0 \rangle, \tag{3}
\]

i.e., the unitary linking the two ensembles [11]. In summary, perfect concealment against Bob allows Alice to cheat by remotely steering Bob’s state to either ensemble. More generally, relaxing Eq. (1), we may let \( \rho_B^0 \approx \rho_B^1 \), which would correspondingly diminish Alice’s ability to cheat.

Interestingly, an analogous attack on a toy bit commitment protocol using steering correlations can be demonstrated [12] in the toy theory due to Spekkens [13], which features steering but not nonlocality.

In response to the steering attack, various works have studied cheat sensitive QBC, where Bob’s security requirement is relaxed by letting \( \rho_B^0 \neq \rho_B^1 \) (cf. [14], and references therein). But others have questioned whether this framework of mistrustful quantum cryptography is broad enough to truly rule out secure QBC (see [15–21] and references therein), and our work lends credence to this point of view. A key observation in these works is that, unlike (say) the no-cloning or no-signaling theorems, the no-go argument for (non-relativistic) QBC does not seem to invoke a simple and broad physical principle, and thus appears to be tied to a specific framework.

Our point of departure is to draw attention to two assumptions implicit in the no-go theorems in the standard framework:

**Assumption (a):** That Alice knows the two final ensembles.

**Assumption (b):** Quantumness of the evidence \( E \).

Our approach is to explore the possibility of securing QBC protocols by relaxing these two assumptions.

Regarding Assumption (a), consider a QBC protocol in which Alice lacks full knowledge of \( \{ |\tilde{\chi}_j^\alpha \rangle \} \), say because the two ensembles are determined through secret choices of Bob. Her ignorance may prevent her from computing the cheat unitary \( \mathcal{U} \), in view of Eq. (3). But, it may well end up over-empowering Bob. Thus, we want a protocol that relaxes Assumption (a), but with safeguards to guarantee the concealment of evidence \( E \), essentially via secrecy injected by Alice. Thus, we expect such a possible protocol to be “double-blind”. These ideas are discussed in, and form the basis of our protocol P1 in, Section [11]
Regarding Assumption (b), a QBC protocol in which evidence \( E \) is fully classical is trivially protected against the steering attack of [24]. At first sight, the classicality of \( E \) would appear to reduce such a protocol to a classical bit commitment scheme [22], for which information-theoretic arguments exist preventing unconditional security. However, irrespective of the security status of QBC protocol obtained by relaxing Assumption (b), it is clear that such a reduction is not the case, given that there will be intermediate stages involving quantum communication and quantum operations. It may be helpful to think of the QBC protocol with a classical evidence \( E \), as one that implements a “classical-valued quantum one-way function”, i.e., an operation with a classical output, whose difficulty to invert comes from quantum nonclassicality, rather than from computational complexity.

Classical bit commitment based on a one-way function \( f \) works as follows: Alice computes \( f(K_{\text{priv}}, K_{\text{pub}}, a) \), where \( K_{\text{priv}}, K_{\text{pub}} \) and \( a \) are Alice’s private key, public key and commitment. She submits \( E = \{ f(K_{\text{priv}}, K_{\text{pub}}, a), K_{\text{pub}} \} \) as her evidence. Later, at the unveiling stage, she submits the message \( \{ K_{\text{priv}}, a \} \). Bob accepts her commitment after checking that it reproduces \( E \). This is only computationally secure, since with sufficient computational resources, Bob could invert \( E \) to derive \( a \).

In the proposed quantum realization of the classical-valued one-way function \( f \), the basic idea is that this function will have the form \( g = f(K_{\text{pub}}, K_{\text{priv}}, L_{\text{pub}}, L_{\text{priv}}, a) \), where \( L_{\text{pub}} \) and \( L_{\text{priv}} \) are the public and private inputs of Bob, and the output of \( f \) is generally probabilistic. Unlike in classical bit commitment, here all inputs of Alice and Bob consist in general of quantum state preparations, rotations and measurements. The hope is that the privacy of Bob’s input will bind Alice, while the privacy of Alice’s input will provide the requisite one-wayness to conceal the commitment. Thus, as with relaxing Assumption (a), we require the double-blindness feature. These ideas are discussed in, and form the basis of our protocol P2 in Section III.

In summary, with both protocols P1 and P2, Alice is unable to launch the steering-based attack, though for different reasons. In protocol P1, this happens because Alice can’t determine the malicious operation \( U \) required to steer Bob’s ensemble, whereas in protocol P2, this happens because no such \( U \) exists. As it turns out, P1 is still vulnerable to a less devastating, probabilistic attack by Alice, based on her superposing her commit actions. As a result, protocol P1 lacks certification of classicality (CC) [23]. On the other hand, P2 is secure even in this stronger (CC) scenario, and is our main proposal.

With a foundational objective in mind, we propose protocol P3 in Section IV which is an extension of P2, together with the use of entanglement. The security of protocol P3 is shown to demonstrate the reality of the quantum state, under the exclusion of retrocausality.

II. PROTOCOL P1, AND THE STEERING-BASED ATTACK

A BC protocol has three phases: a commit phase, at the end of which Alice submits an evidence of her having committed to a specific value \( a \); a holding phase, during which her commitment remains valid; finally, an unveil phase, where she opens her commitment, and reveals supporting information.

The commit and unveil phases of protocol P1 are as follows. A holding phase of arbitrary duration between these two phases is, in principle, allowed. Although protocol P1 requires quantum memory for the holding phase, yet with a slight modification, this can be avoided. The protocol makes use of quantum encryption, which is the task whereby \( n \) qubits can be maximally mixed using \( 2n \) bits of a random private key [24]. This relies on the fact that given any qubit state \( \rho \), \( \frac{1}{2}(\rho + X\rho X + Y\rho Y + Z\rho Z) = \frac{1}{2}I \), where \( X, Y \) and \( Z \) are the Pauli operators. Here, we use the notation wherein \( |0\rangle \) and \( |1\rangle \) are the eigenstates of the Pauli Z operator, while \( |\pm\rangle \) are eigenstates of Pauli X.

**Commit phase**: (C1)\(_{P1}\) Bob transmits to Alice \( 2n \) “single-blind” (i.e., unknown to Alice) random qubit states \( |\phi^{(a)}_{j}\rangle \in \{|0\rangle, |1\rangle, |\pm\rangle\} \), where \( a \in \{0,1\} \), \( 0 \leq j < n \), indicating the two sets to her. (Alternatively, he could submit halves of Bell states, deferring measurement in X or Z basis on the “home” qubits until later.) Additionally, he supplies \( Q \) extra qubits prepared in pure states unknown to her (or as halves of singlets), where \( Q \gg n \). (C2)\(_{P1}\) Alice prepares \( Q \) “decoy” qubits by quantum encrypting the states of the extra qubits. We denote the \( 2Q \) bits of encryption information \( R_Q \). To commit to bit \( a \), she inserts the \( n \) states \( |\phi^{(a)}_{j}\rangle \) at positions \( W \) among the \( Q \) decoys, and then rearranges all \( n + Q \) qubits using permutation \( P \). (C3)\(_{P1}\) She transmits back to Bob these \( n + Q \) qubits as evidence \( E \) of her commitment.

**Unveil phase**: (U1)\(_{P1}\) Alice announces \( a \), \( P \), \( R_Q \) and positions \( W \). She returns the \( n \) qubits of her non-commit state \( |\phi^{(a)}_{j}\rangle \). (U2)\(_{P1}\) Bob extracts the \( n \) commit qubits from evidence \( E \) using information \( P \) and \( W \). He verifies that they are the states \( |\phi^{(a)}_{j}\rangle \). Further, he verifies that the \( n \) non-commit qubits returned in step (U1)\(_{P1}\) are the states \( |\phi^{(a_i)}_{j}\rangle \). Finally, using information \( P \) and \( R_Q \), he checks that the \( Q \) decoys are indeed the extra qubits sent by him.

The protocol assumes the availability of quantum memory. However, this is not essential. As in the BB84 protocol [25], Bob can measure, just after the commit phase ends, each evidence qubit \( j \) in a BB84 basis (X or Z). When Alice opens her commitment, he verifies that there is outcome agreement on all qubits where her measurement basis \( j \) matches that of the preparation of \( |\phi^{(a)}_{j}\rangle \). Overall, this doesn’t affect the following security arguments, except to roughly halve the security parameter from \( n \) to \( \frac{n}{2} \).

Consider the security against Bob. Intuitively, the
large number of Alice’s decoy qubits swamp the relatively small number of coding qubits \(|\phi_j^{(a)}\), making it impossible for him to determine \(a\). More quantitatively, until Alice’s announcement of the information \(P\) and \(W\) in (U1)\(p_1\), the state of the evidence is

\[
\rho_B^0 = C_W \left( \bigotimes_j \left( |\phi_j^{(a)}\rangle \langle \phi_j^{(a)}| \right) \otimes \left( \frac{1}{2} \right)^\otimes Q \right),
\]

where \(C_W\) represents the uniform mixture over all \(|Q+n\rangle\) combinations of interpolating the \(n\) scrambled qubits \(|\phi_j^{(a)}\rangle\) among the \(Q\) decoy qubits inserted by Alice. We note that from Bob’s viewpoint, all decoys inserted by Alice are in the state \(\frac{1}{2}\). Clearly, for any given \(n\), the state \(\rho_B^0\) approaches \(\rho_B^1\) closer for larger \(Q\), thereby allowing satisfaction of Eq. (1) to any required degree.

We indicate this showing that \(\rho_B^1\) for either \(a\), can be made arbitrarily close to \(\left( \frac{1}{2} \right)^\otimes (n+Q)\) in \(Q\) terms in the fidelity \(F(Q,n)\). Then, in view of Eq. (1), for given \(n\) and sufficiently large \(Q \gg n\), it follows (Appendix A) that the fidelity \(F(Q,n) \equiv F \left( \rho_B^0, \left( \frac{1}{2} \right)^\otimes (Q+n) \right)\) satisfies

\[
F(Q,n) \geq 1 - 2^{-Q(1-H(n/Q))},
\]

where \(H(x) = -x \log(x)-(1-x) \log(1-x)\) is the Shannon binary entropy. For a given \(n\), fidelity \(F(Q,n)\) is thus seen to approach unity exponentially as \(Q\) increases, meaning that Bob can hardly find out \(a\). Even though Bob knows \(|\phi_j^{(0)}\rangle\) and \(|\phi_j^{(1)}\rangle\) for each \(j\), Alice’s evidence for either commit bit is close to 1/2. There is no advantage for Bob even if he transmits halves of Bell state \(|\Phi^+\rangle \equiv \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\) since under Alice’s method of concealing her commitment by inserting decoys is symmetric to any choice of states he makes.

As regards security against Alice, crucial to protection from the steering attack is the fact that both ensembles \(E^{\alpha}\) are unknown to Alice because of the single-blindness feature, i.e., the dropping of assumption (a). But, as we noted earlier, merely Alice’s ignorance of the ensembles \(E^{\alpha}\) may not guarantee that she can’t determine \(U\).

To see this, the situation can be formalized as follows: Bob prepares and sends to Alice an unknown-to-her (“single-blind”) state \(\xi\), Alice then returns to Bob an element of the ensemble \(T_\alpha(\xi) (\alpha \in \{0,1\})\) as evidence. As Bob varies the blind information \(\xi\), clearly the two ensembles \(T_\alpha(\xi)\) are also rotated in some way. In general, \(T_0(\xi)\) and \(T_1(\xi)\) need not be indistinguishable under arbitrary variation of \(\xi\).

However, with the indistinguishability condition Eq. (1), the ensembles corresponding to \(T_0(\xi)\) and \(T_1(\xi)\) indeed co-rotate in such a way that the steering unitary \(U\) linking them is invariant.

To prove this, we allow for the encoding ensembles to be randomly chosen by Bob, say through a secret parameter \(\mu\) he holds, i.e., \(E^{\mu(\alpha)} \equiv \{ V^{\alpha}_\mu \xi_j^{\alpha} \}\), for unitaries \(V^{\alpha}_\mu\). Alice has a quantum computer, which entangles the evidence system with an auxiliary system she holds, in such a way that it realizes the positive operator-valued measure (POVM) that generates the ensemble \(|\chi_\alpha^{\mu}\rangle\) for any commitment \(\alpha\) she chooses.

If Bob applies his secret transformation, then generated ensemble is correspondingly rotated. Now if \(V^{\alpha}_\mu \neq V^{\mu}_\mu\), then in general, this would imply that

\[
\sum_j V^{\alpha}_\mu \xi_j^{\alpha} \langle \chi^{\alpha}_j | V^{\mu}_\mu \xi_j^{\mu} \rangle \neq \sum_j V^{\mu}_\mu \xi_j^{\mu} \langle \chi^{\alpha}_j | V^{\mu}_\mu \xi_j^{\mu} \rangle,
\]

even if \(\rho_B^0 \equiv \sum_j |\xi_j^{\alpha}\rangle \langle \xi_j^{\alpha}| = \sum_j |\xi_j^{\mu}\rangle \langle \xi_j^{\mu}| \equiv \rho_B^1\), as per Eq. (1). Therefore, in general, to order to preserve indistinguishability Eq. (1) when \(\mu\) is varied, we require that the two ensembles be rotated identically, i.e.,

\[
V^{\mu}_\mu = V^{\alpha}_\mu \equiv V^{\mu}_\mu.
\]
can indeed achieve this asymptotically, she will fail (with probability exponentially close to 1 in $Q$) to convince Bob that the states of qubits in the other ports than $k$ were indeed prepared from the extra qubits he had supplied.

Lastly, suppose she performs the honest commitment operation for $a = 0$ and naively tries to unveil $a = 1$, or vice versa, then the probability that she can escape his check in step (U2)$_{P1}$ in general vanishes or is exponentially small in $n$, since the fidelity of the two possible encoding states is $\prod_{j=1}^{n}|\langle \phi_{j}^{(0)}|\phi_{j}^{(1)}\rangle|^2$.

Interestingly, despite being impervious to the steering attack, protocol P1 does not offer a “certificate of classicality” (CC) $[23]$; i.e., the probabilities $p_a$ to unveil $a$ only satisfy the weaker condition $p_0 + p_1 = 1$ $[23]$, rather than the stronger requirement that precisely one of $p_0$ and $p_1$ should be 1 and other 0.

The reason is that Alice would be able to launch an attack, wherein she creates a superposition of the two commit actions by entangling the quantum computer performing them, with a suitably prepared auxiliary. We shall refer to this as the “superposition attack”. Although less devastating than the steering attack, clearly it is more persistent.

In this attack on protocol P1, Alice prepares an auxiliary $A'$ in the state $\frac{1}{\sqrt{2}}(|\gamma_0\rangle_{A'} + |\gamma_1\rangle_{A'})$, and through a joint interaction with all $2n$ qubits received from Bob plus $A'$, produces the state:

$$\sum_{a=0}^{1} \gamma_a |a\rangle_{A'} \otimes |\Phi^{(a)} \rangle_{\text{keep}} \otimes |\Phi^{(a)} \rangle_{\text{encrypt}},$$

where $\sum_{a} |\gamma_a|^2 = 1$ and the subscripts “keep” and “encrypt” refer to the action of retaining state $|\Phi^{(a)} \rangle$ and transmitting the system corresponding to $|\Phi^{(a)} \rangle$ after insertion of decoys, respectively. Alice measures register $A'$ just before unveiling, leading to either $a = 0$ or $a = 1$ being chosen randomly, with probability $|\gamma_a|^2$.

This lack of CC of the commit bit is a feature shared with some other proposed QBC protocols, such as the relativistic BC protocols $[1, 4]$ and also $[13, 14]$. In practice, it is questionable whether a dishonest Alice would opt for such a random cheat strategy in a stand-alone application of bit commitment, but the fact remains that the protocol doesn’t bind her to commit deterministically, which undermines the composability of this QBC protocol in a larger application, which may require the commitment to have a specific classical value.

We next present protocol P2, that closes the above security gap, essentially by relaxing Assumption (b) mentioned in Section II as being implicit in the standard framework. Obviously, neither the steering attack nor the superposition attack would be possible if the two system $E$ is classical, typically encoding classical information based on outcomes of a measurement determined by her commitment. We stress that the evidence $E$, although by itself classical, is generated by quantum operations (which are restricted by nonclassical features such as non-commutation, no-cloning, measurement disturbance, etc.), so that the present protocol won’t be reducible to a purely classical BC (which is known, by classical information theoretic arguments, to guarantee only computational, and not unconditional, security $[22]$.)

But this would seem to endanger the concealment against Bob. To fight this threat, Alice must initially prepare the states. However, this would introduce the new threat of Alice’s taking advantage of her preparation knowledge. Therefore, Bob must randomize these states in some way. Thus the encoding states should be “double-blind”, unknown to both Alice and Bob. These are the considerations behind the following protocol.

## III. PROTOCOL P2, AND THE SUPERPOSITION-BASED ATTACK

The scheme P2 can be considered (as noted in Section II) as a classical-valued quantum realization of a (probabilistic) one-way function. The commit and unveiling phases are as follows, with an intervening holding phase of arbitrary duration. Even from a practical standpoint, there is no time constraint on the storage of the evidence, since it is classical.

**Commit phase:** (C1)$_{P2}$ Alice transmits to Bob $2n$ qubits randomly prepared in states $|\psi_k\rangle \in \{|0\}, |1\rangle, |\pm\rangle\}$. (C2)$_{P2}$ Bob randomizes their bases (by randomly applying either identity $I$ or Hadamard $H$ to each of them), randomizes their bits by quantum encryption, and finally also randomly scrambles the qubits according to some permutation operation $P$. (C3)$_{P2}$ Bob transmits to Alice these double-blind states, denoted $|\phi_j\rangle$. (C4)$_{P2}$ Alice picks out $n$ of the transmitted states, and asks Bob to reveal his randomizing operations for these qubits. Upon receiving this information, she verifies that they are indeed qubits she had prepared. These $n$ check qubits are discarded. (C5)$_{P2}$ To commit to bit $a = 0$ (resp., $a = 1$), she measures the remaining $n$ states $|\phi_j\rangle$ in the basis $Z$ (resp., $X$). The $n$-bit random outcome string is denoted $M$. (C6)$_{P2}$ She announces $M$ as evidence of her commitment.

**Unveil phase:** (U1)$_{P2}$ Alice announces $a$ and her preparation information of the qubits $|\psi_k\rangle$. (U2)$_{P2}$ From the latter, Bob obtains complete classical knowledge of all $2n$ states $|\phi_j\rangle$. (U3)$_{P2}$ Bob verifies that the string $M$ is compatible with the measurement of states $|\phi_j\rangle$ in the basis $Z$ (resp., $X$) if $a = 0$ (resp., $a = 1$).

Consider the security against Bob. Prior to the unveil phase, he has no classical information about the preparation of $|\psi_j\rangle$, and hence of $|\phi_k\rangle$, which is derived from $|\psi_j\rangle$ via random rotations and rearrangements, but without measurements. Therefore, knowledge of the string $M$ reveals to him nothing about $a$. Let $R$ denote the classical information about Bob’s randomization operations in step (C2)$_{P2}$ and $H(\cdot)$ classical conditional entropy. Then P2 satisfies the condition:

$$H(a|M, R) = 1,$$ (9)
where $\alpha$ is the commitment random variable. Eq. $(9)$ replaces Eq. $(1)$ as the condition of security against Bob appropriate to this scenario. Bob can’t substitute his own states in step $(C3)_{P2}$, nor measure Alice’s qubits, since such actions would generate disturbance that would almost certainly be detected in check $(C4)_{P2}$, where Alice verifies that Bob has returned her own qubits after a unitary and rearrangement operation.

As to security against Alice, she can’t launch a steering attack or even a superposition attack for the trivial reason that her commit evidence is now classical information, and thus is in principle unsteerable and unsuperimposable. In other words, a malicious steering operation $U$ simply doesn’t exist. This stands in contrast to the situation with protocol P1, where the operation $U$ does exist, but Alice can’t ascertain it.

Of course, other forms of attacks must be considered. Intuitively, security against Alice comes from the fact that because of her ignorance of $R$, she is maximally ignorant of the states by measuring which she generates string $M$. Thus she can’t confidently unveil a fake measurement basis. Now, even without the scrambling action, the basis and bit randomizations ensure that Alice has no information about the returned state $|\phi_j\rangle$. However, given her initial preparation in step $(C1)_{P2}$ and her final measurement in step $(C5)_{P2}$, Alice can launch a local entanglement based attack of the following kind.

In step $(C1)_{P2}$ she sends to Bob half a singlet $|\Phi^+\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)$. Her singlet will now have been modified through Bob’s actions according to

| Bob’s action | Alice’s state |
|--------------|---------------|
| $II, IZ$     | $|00\rangle \pm |11\rangle$ |
| $IX, IY$     | $|01\rangle \pm |10\rangle$ |
| $HI, HX$     | $|0+\rangle \pm |1-\rangle$ |
| $HZ, HY$     | $|0-\rangle \pm |1+\rangle$ , |

where the normalization factor has been dropped out. In step $(C5)_{P2}$, Alice measures in the standard Bell basis $\{|00\rangle, |11\rangle, |01\rangle, |10\rangle\}$. Suppose she finds outcome $|00\rangle - |11\rangle$. From Eq. $(10)$ it follows that Bob could not have applied the three operations: $II, IX, IY$.

Now the four honest states that would result under Bob’s eight possible randomization actions are:

| Bob | $|0\rangle$ | $|1\rangle$ | $|+\rangle$ | $|-\rangle$ |
|-----|------------|------------|------------|------------|
| $II$ | $|1\rangle$ | $|+\rangle$ | $|-\rangle$ |
| $IX$ | $|0\rangle$ | $|+\rangle$ | $|-\rangle$ |
| $IY$ | $|1\rangle$ | $|0\rangle$ | $|-\rangle$ | $|+\rangle$ |
| $IZ$ | $|0\rangle$ | $|1\rangle$ | $|-\rangle$ | $|+\rangle$ |
| $HI$ | $|+\rangle$ | $|-\rangle$ | $|0\rangle$ | $|1\rangle$ |
| $HX$ | $|+\rangle$ | $|-\rangle$ | $|1\rangle$ | $|0\rangle$ |
| $HY$ | $|+\rangle$ | $|1\rangle$ | $|0\rangle$ |
| $HZ$ | $|+\rangle$ | $|0\rangle$ | $|1\rangle$ |

Note that only the last five rows in Eq. $(11)$ are applicable in this case. Alice can announce an arbitrary bit as her measurement outcome in step $(C6)_{P2}$, say $M_j = 0$. Then, to unveil $a = 0$ (resp., $a = 1$), she must claim $|\psi_j\rangle = |0\rangle$ (resp., $|\psi_j\rangle = |-\rangle$), since this would be consistent with her having applied any of these five operations, whereas if $M_j = 1$, then to unveil $a = 0$ (resp., $a = 1$), she must claim $|\psi_j\rangle = |1\rangle$ (resp., $|\psi_j\rangle = |+\rangle$), in view of Eq. $(11)$. Therefore, Bob’s scrambling action is necessary, in addition to his bit and basis randomizing actions.

Given the exponentially large number of ways to permute the qubits, with Bob’s bit, basis and position randomization in step $(C2)_{P2}$, Alice is fully uncertain about which of the four states $|0\rangle, |1\rangle, |+\rangle, |-\rangle$ is each qubit $|\phi_j\rangle$, even given her preparation information of $|\psi_j\rangle$. Therefore, Alice is constrained to execute the measurements in $(C5)_{P2}$ as per protocol and to announce a honest $M$ at step $(C6)_{P2}$. To cheat, at best she simply unveils the wrong basis. This would be detected with probability $3/4$ in view of Eq. $(11)$, implying that her probability to escape detection will be $(3/4)^n$.

That Alice is forced to measure the qubit states $|\phi_j\rangle$ in step $(C5)_{P2}$ guarantees that protocol P2 carries a CC. Therefore, it can be safely composed with other instances of protocol P2 or other tasks in a larger application. Protocol P2 is our principal proposal for an experimental implementation, though in practice the simplified (without quantum memory) protocol P1 may suffice.

IV. PROTOCOL P3, AND THE REALITY OF THE QUANTUM STATE

From a foundational (rather than cryptographic) perspective, we find it instructive to consider the following protocol P3, which is a simple extension of protocol P2. We will show that the security proof of P3 reduces to that of P2, in the sense that the security of P2 guarantees that of P3.

In the commit phase, the steps $(C1)_{P3}$ through $(C6)_{P3}$ are the same as the steps $(C1)_{P2}$ through $(C6)_{P2}$, respectively, except: in $(C3)_{P3}$, Bob additionally transmits $\frac{n}{2}$ qubits, which are halves of singlets, to Alice; in $(C5)_{P3}$, in addition to all actions of $(C5)_{P2}$ additionally Alice measures the singlet-halves sent by Bob in the basis $Z$ or $X$ basis, depending on whether each bit in the second half of $M$ is 0 or 1, respectively. The outcomes for these measurements constitutes $\frac{n}{2}$-bit string $M_2$; and in $(C6)_{P3}$, Alice transmits the first $\frac{n}{2}$ bits of $M$ as before (denote this string by $M_1$). Further, she transmits the $\frac{n}{2}$ bits $M_2$. Thus, Alice’s classical evidence $E$ in this is the set of two strings $\{M_1, M_2\}$.

Similar, the unveiling phase of protocol P3, the steps $(U1)_{P3}$ through $(U3)_{P3}$ are the same as the steps $(U1)_{P2}$ through $(U3)_{P2}$, respectively, except: in $(U1)_{P3}$, Alice additionally transmits the remaining $\frac{n}{2}$ bits of $M$; in $(U3)_{P3}$, Bob checks that the outcomes $M_2$ are compatible with measuring his halves of the singlets in the bases specified by these remaining bits of $M$.

Instead of a detailed proof of security of P3, it will
suffice to show that its security proof reduces to that of P2. In protocol P3, set $n \rightarrow 2n$, and suppose that Alice and Bob ignore the $M_2$ part. Then, it is clear that the resulting protocol is at least as secure as protocol P2 on $n$ bits. If P2 is secure (as we saw it is), then so is P3. We wish to draw attention to the fact that $M_2$ reaches Bob at the end of the commit phase, entailing that the singlet halves on his side are already “collapsed”.

We now remark on the physical interpretation of the security of P3, which is the main motivation behind its proposal. In the standard Hilbert space formalism, when distant players Alice and Bob, mutually at rest, make measurements on an entangled quantum state, such that Bob measures after Alice in their common reference frame, both agree that there is a spacelike update to the description of Bob’s state as a result of her measurement. However, Bob’s reduced density operator remains unchanged. This situation lies at the heart of the dilemma regarding whether Bob’s state’s changed status as a result of Alice’s measurement is an objective transformation (i.e., a genuine ontological change in the state of Nature) or a subjective transformation (i.e., just a Bayesian update of her knowledge) of his state.

Intriguingly, this familiar dilemma becomes less ambiguous in the entanglement scenario of P3. Let Alice and Bob be separated by a finite distance, and at rest in each other’s reference frames. They have synchronized their clocks, which keep the same time $t_A = t_B = t$. In their common coordinate system, the absolute past is defined by the event set $T^- = \{(t,x) : t < 0\}$, the absolute future by $T^+ = \{(t,x) : t > 0\}$ and the absolute present by $T^0 = \{(0,x)\}$. Further, let $W_B$ denote the world line of Bob, and $W_B^- = T^- \cap W_B$, $W_B^+ = T^+ \cap W_B$, i.e., the past and future segments of Bob’s world line, and let event $W_B^0 = T^0 \cap W_B$.

In this frame, we denote by $e_A \equiv (0,0)$ the event of Alice’s entanglement breaking (EB) via measurements on singlet halves in step (C5) of P3. In this instant, she knows that she has “collapsed” – i.e., irreversibly prepared– Bob’s state in favor of one particular commitment. We denote this remote preparation event in $W_B$ by $e_B$.

Is this preparation of Bob’s state at event $e_B$ an objective or subjective transformation? To answer this, we shall employ an operational framework, i.e., one not based on an ontological model, but simply on the inputs and measured outputs of the communication involved. First, we define a feature $X$ associated with a system $S$ as operational if $X$ can be described in terms of probabilities for outcomes of quantum physical measurements. Features that must refer to an ontological model for QM aren’t operational. A transformation $T$ is said to be objective if there is an operational feature $X$ of system $S$ such that $X$ is altered under $T$. Ordinary classical transformations are manifestly objective, but this framework allows a larger set of transformations to be characterized as objective. More precisely, it enlarges the set of objective transformations to include not just detectable transformations, but also its superset of verifiable transformations. All classical transformations are detectable, and hence also verifiable. (For example, if the color of a classical objected turned from red to blue, this may be verified, and indeed detected.) But, we shall find that quantum theory allows verifiable transformations that aren’t detectable.

We define an “EPR-certificate” as the $2 \times 2$-bit outcome and basis specification (namely, whether the basis is $Z$ or $X$) for a set of $\frac{q}{2}$-qubit pure states, that can pass Bob’s check for outcome matching in step (U3)\textsubscript{P3} with complete certainty, in support of some commitment. In the terminology of [20], the EPR-certificate associates an “element of reality” to the commitment encoded in the $\frac{q}{2}$ home qubits of Bob. Let $p(a,C)$ denote the probability that a candidate EPR-certificate $C$ can (upon being checked) pass the test in (U3)\textsubscript{P3}, and further, that $p^*(a) \equiv \max_C p(a,C)$. Clearly, the probability $p^*(a)$ so defined is an operational feature associated with Bob’s state.

Now, the security of protocol P3 implies that after Alice’s EB event, there exists a specific EPR-certificate for commitment $a$ and none for $\overline{a}$. Note that this is so even though she doesn’t have the EPR-certificate’s complete classical description. Thus, $p^*(a|\emptyset) = 1$ whilst $p^*(\overline{a}|\emptyset) < 1$, which entails that:

$$p^*(0|\emptyset) \neq p^*(1|\emptyset) \tag{12}$$

where where the overline on $\emptyset$ indicates negation, and $\emptyset$ is the statement asserting that Alice executed an EB in step (C5)\textsubscript{P3}. (We shall ignore the effect of noise, since we are concerned with the situation in principle.)

When Alice hasn’t yet executed this step, she knows that her choice is uncorrelated with Bob’s preparation in the strong sense that there aren’t any EPR-certificates in the universe. Thus,

$$p^*(0|\emptyset) = p^*(1|\emptyset). \tag{13}$$

Bob’s state is symmetric with respect to both possible commitments. This symmetry is also consistent with the assumption of Alice’s free will, in that her choice is unrelated to Bob’s preparation.

In the present framework, Eqs. (12) and (13) together imply that that Alice’s EB measurement induces an objective transformation of Bob’s state, whereby the original symmetry in Bob’s state with respect to both commitments gets verifiably broken.

The objective nature of the symmetry breaking event $e_B$ signifies that Alice’s remote preparation must correspond to a definite spacetime event on Bob’s world line. Where to position $e_B$ on Bob’s world line? From Alice’s perspective, in view of the constraints imposed by Eqs. (12) and (13), $e_B$ must be identified precisely with $W_B^0$ (see Figure 1), which is spacelike separated from $e_A$.

Observers in relative motion with respect to Alice may make other claims, complicating the picture, to which we return in Section IV B. Now, consider an alternative to the above scenario, as seen in the Alice/Bob reference frame.
the set of tests of nonlocality in quantum mechanics or general non-signaling probability theories. Here, the correlations don’t admit a “clean” causal story in the sense that there is no unequivocal case for a definite time ordering behind the observed correlations (cf. [33]). At best, one can show that nonlocal correlations observed in these protocols, if “extended” to a more deterministic ontological model, would entail signaling at the ontic level (with no preference for Alice or Bob in regard to the direction of causation) [34]. These protocols constitute the class XS, or “extension-signaling”.

A strictly stronger class than VS is the set of two-party protocols in any of which Bob can unilaterally detect distant Alice’s input. These protocols are denoted S (for “detectable-signaling”, or simply, “signaling”). Obviously, protocols S are prohibited by both QM and special relativity when Alice’s and Bob’s measurements are spacelike separated.

The containments among these classes of two-party protocols are depicted in Figure 2. It may be helpful to think of S, VS and XS as signaling analogues of the computational complexity classes P, NP and PSPACE.

The verifiable signal in a VS protocol, since it causes an objective transformation of Bob’s state, requires a causal channel to mediate it. (If the transformation were only subjective, then clearly there is no such requirement.) However, relativistic causality forbids any dynamic mechanism being responsible for it. We are thus led to conclude that the quantum state vector itself, whose reduction forms the basis for predicting the remote preparation, must supply this causal channel. In this sense, the state vector is a real entity.

Some points are worth noting here. Firstly, observe

Figure 1. Committer Alice and recipient Bob are distant observers at rest in each other’s reference frame. The two vertical lines denote their respective worldlines. Alice’s free choice of bit a at the EB event e_A = (0, 0) prepares Bob’s system at event e_B on his world line. Possible locations of e_B include W^0_B (in Alice’s present), e^B (in the causal future) and e^T_B (in the causal past).

A. Forward-time influence

Suppose e_B is positioned later on his worldline (region \( W^+_B \)), say at e^T_B, which is lightlike or timelike separated from e_A. Then, there is a worldline segment \( \omega^T \) between \( W^0_B \) and \( e^T_B \) (Figure 1) for which neither Eq. (12) nor Eq. (13) would hold true. This would correspond to a breakdown in quantum correlations (cf. [31]), and in general lead to violation of conservation laws for spin angular momentum, etc. The possibility of such breakdown is experimentally ruled out by data from loophole-free Bell inequality violation tests [32].

Barring retrocausality (discussed below in Section IV B), Alice thus concludes that her remote preparation of Bob’s state is associated with a spacelike influence occurring across the interval \( d^- \) (Figure 1). There is no overt conflict with relativistic no-signaling, since Bob can’t unilaterally detect this influence at e_B, but can only verify it later on. Warded differently, this superluminal influence corresponds to a verifiable signal, but not a detectable signal.

We denote by VS (“verifiable-signaling”) the set of two-party protocols, such as P3, which permit a verifiable signal. For protocols in this class, one can construct a “causal story” at the operational level. In the story associated with P3, Alice is the sender and Bob the receiver.

A class of two-party protocols that is strictly weaker is
that we deduce the reality of the quantum state appealing only to operational criteria, and without recourse to an ontological framework \[12\]. Secondly, our argument for the reality of the quantum state didn’t require quantum nonlocality but only quantum teleportation, which is a weaker resource. Indeed, local entangled quantum states can still teleport above the classical fidelity threshold \[27\]. Thirdly, the essence of our argument could be couched in a teleportation setting without invoking bit commitment. The latter mainly serves to provide a situation that is amenable in this framework for defining the objectivity of Alice’s remote preparation of Bob’s state.

In regard to the second point above, verifiable signaling can be shown to occur even in a theory without nonlocality, but permitting teleportation, such as Spekkens’ toy theory \[13\], where the state of a particle can be modelled epistemically (i.e., as a probability distribution over certain ontic elements). It can be shown that both the steering-based attack on bit commitment in the standard framework \[12\], as well as our solution to this problem via protocols P1, P2 and P3, are possible in this toy theory. Our above argument for the reality of the state vector, adapted to a toy version of protocol P3, isn’t contradicted by the epistemicity of the toy state, but rather implies that Alice’s choice at \(e_A\) produces a remote ontic disturbance at \(e_B\), i.e., a remote disturbance in the ontic elements underlying Bob’s state.

\section{B. Retrocausal influence}

Backward-time influence has been considered as a viable alternative to superluminal influence because it offers attractive features like time-symmetry, Lorentz invariance and retaining local-realism \[28\] \[10\]. In the present situation, note that the forward-time and also the simultaneous (in the reference frame of Alice/Bob) spacelike influence, discussed in Section \[IV\] \[A\] would lead certain observers in relative motion to Alice to expect a breakdown in quantum correlations of the type discussed above or in \[31\]. These are observers in whose reference frame events in the segment \(\omega_1\) happen before \(e_A\). To prevent this, one can posit that these causal influences are transmitted through a wider future cone than the future light-cone, but the required “speed of information” would have to be several orders faster than light speed to account for current experimental verification of quantum nonlocal correlations (cf. \[31\], and references therein).

But, if one swallow the bitter pill of backward-time causation, then a covariant solution to avoid predicting a breakdown of nonlocal correlations in any reference frame, is to further broaden this “wider causal cone” until it coincides with the past light-cone rooted at \(e_A\), so that the verifiable signal is transmitted (or, state vector “collapses”) along the boundary of event \(e_A\)’s causal past \[41\]. In the context of protocol P3, \(e_B\) would be positioned backward on Bob’s worldline at \(e_B\) on \(W_B\) which is lightlike separated from \(e_A\) (Figure \[1\]). This has the effect of extending the domain of validity of Eq. \[12\] backwards to include the region \(\omega_1\). However, the protocol classes in Figure \[2\] carry over, though with the “signaling” in the context of \(VS\) and \(XS\) should be understood as retrocausal influences.

Such retrocausality would infringe on the definition of Alice’s free will, since it would imply that Bob’s ensemble in the causal past is correlated with Alice’s choice. Presumably, one can try to redefine free will in this situation to exclude such correlations from its definition.

If we accept this retrocausal model, then one can presumably imagine some dynamic effect propagating at light-speed backward in time from \(e_A\) and preparing Bob’s state at \(e_B\), and the above argument for attributing reality to the quantum state, no longer holds. Hence, we require the assumption of absence of retrocausality to arrive at our conclusion of the reality of the quantum state.

\section{V. CONCLUSIONS AND DISCUSSIONS}

The no-go theorem for quantum bit commitment (QBC) in the standard non-relativistic framework is a consequence of the fact that Alice can exploit quantum steering to unveil either commit bit on Bob’s system, if he can’t distinguish the mixtures corresponding to the two commitments \[4\] \[9\]. However, various authors have questioned whether this framework is general enough to cover, and thus rule out, all possibilities for QBC \[15\] \[16\] \[18\] \[20\].

In line with this argument, here we identify two assumptions implicit in the standard framework: (a) that Alice’s submitted evidence exists in an ensemble known to her; and (b) that \(E\) is a quantum— and not a classical— system.

Relaxing the assumption (a), we construct a QBC protocol (named “P1”), whose security against the steering-based attack arises from the fact that Alice is unable to determine the malicious steering operator \(U\). However, protocol P1 still allows Alice to attack probabilistically, and as a result, P1 lacks “certification of classicality” (CC) \[23\], in common with various relativistic bit commitment protocols \[1\] \[3\] \[5\] \[8\].

Relaxing the second assumption above, we present a second protocol (named “P2”) that is secure in the stronger sense of also guaranteeing CC. Because the submitted evidence is classical, there trivially exists no malicious steering operator \(U\) and furthermore, the superposition-based probabilistic attack is impossible. In both protocols, security against simpler attacks makes use of the single-blindness or double-blindness feature through the use of randomization of the state by quantum encryption, particle rearrangement, etc. Protocol P2 is most suitable for experimental implementation, though the quantum-memory-less variant of protocol P1 may be practically sufficient.

Finally, we propose a third protocol (named “P3”), which relaxes both assumptions in a teleportation setting, and is motivated for a foundational purpose. Here,
the role of entanglement and teleportation in protocol P3 is vital for the argument of the reality of the quantum state. Alice’s free will and the protocol’s security are invoked to argue that her remote preparation of Bob’s system is an objective transformation in the weaker sense that she produces a verifiable preparation of Bob’s state (leading to cryptographic security), rather than in the stronger sense of being unilaterally detectable (which would have led to superluminal signaling). It is argued that, barring retrocausality, this remote objective transformation entails a superluminal influence which, by virtue of relativistic causality, can’t be pinned down on any dynamical mechanism. This is used as the basis to argue for the objectivity of Alice’s remote preparation, and hence for the reality of the quantum state.

All our protocols are conceptually and experimentally simple. They allow for a holding phase of an indefinite time period even with current technology, in contrast to the relativistic BC protocols $[1, 2, 3, 6]$, which require an increasingly complex, continued and carefully timed communication to extend the holding phase (the current experimental record being 24 hours).

Finally, it is hoped that our results open up new possibilities in mistrustful quantum cryptography, in particular, highlighting the difficulty in—and consequent care needed for—determining the full scope of the most general framework appropriate to mistrustful quantum cryptography. It also uncovers a basic relationship between secrecy and the nature of physical laws, which would be useful for devising cryptographic axioms to derive quantum mechanics.

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Appendix A: State of the evidence

We conservatively assume that all states $|\phi^{(a)}\rangle$ for a given $a$ are identical, say $|0\rangle$. Under the stated assumptions, Alice’s evidence is in the state

$$\rho_B^a = C_W \left[ |0\rangle \langle 0| \otimes \left[ \frac{1}{2} \right] \otimes^{Q} \right],$$

$$= \left( 2^{Q+n} - \sum_{j=1}^{n} \frac{(Q+n)}{(Q+j)} \right)^{-1} \mathbb{I}^*,$$

$$\Sigma_{j=0}^{n-1} \frac{(Q+n)}{(Q+j)} \right)^{-1} \mathbb{I}^* \tag{A1}$$

where $\mathbb{I}^*$ is the density matrix in the Hilbert space $\mathcal{H}_2^{\otimes Q+n}$ of $2^{Q+n}$ qubits, which is diagonal and equal-weighted in the computational basis, with precisely the components with Hamming weight greater than $Q$ vanishing.

For a fixed integer $t$, and integer $T \to \infty$, the truncated binomial series satisfies the bound $[42]$

$$\lim_{T \to \infty} \left( \frac{T}{t} \right)^{-1} \sum_{j=0}^{t} \left( \frac{T}{j} \right) = \frac{1}{T-t+1} + \frac{t(t-1)}{(T-t+1)(T-t+2)} + \cdots$$

$$= 1 + \frac{1}{T-t+1} + \left( \frac{t}{T-t+1} \right)^2 + \cdots$$

$$= \frac{T-t+1}{T-2t+1}. \tag{A2}$$

Setting $T \equiv n + Q$ and $t \equiv n-1$ here, one finds

$$\sum_{j=0}^{n-1} \frac{(n+Q)}{j} \leq \left( n+Q \right) \frac{Q+2}{Q-n+3}. \tag{A3}$$

Substituting this in Eq. (A1), we find that, for large $Q \gg n$, the number of non-vanishing entries in $\mathbb{I}^*$ is bounded below by $v(Q,n) = 2^{Q+n} - \frac{(n+Q)}{(n+1)} \frac{Q+2}{Q-n+3} \approx 2^{Q+n} - \frac{(Q+n)}{(n+1)} \approx 2^{Q+n} - 2^{Q+n}(1 - \frac{1}{Q+n})$, where the Stirling approximation $\binom{N}{n} \approx N^H(p)$, has been used.

The fidelity between states $\rho$ and $\sigma$ is given by

$$\text{Tr}(\sqrt{\sigma^* \rho \sqrt{\sigma}}).$$

$$\text{Tr}(\sqrt{\rho_B^a} \otimes \sqrt{\sigma}) \approx 2^{-(Q+n)/2} \text{Tr}(\sqrt{\rho_B^a} \otimes \sqrt{\sigma}) \geq 2^{-(Q+n)/2} \sqrt{\text{Tr}(\rho_B^a \otimes \sigma)}.$$
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