Temporal Self-Restoration of Compressed Optical Filaments

L. Bergé,1 S. Skupin,2 and G. Steinmeyer3
1CEA-DIF, DPTA, Bruyères-le-Châtel, 91297 Arpajon cedex, France
2Max-Planck-Institut für Physik Komplexer Systeme, Nöthnitzer Strasse 38, 01187 Dresden, Germany
3Max-Born-Institut für Nichtlineare Optik und Kurzzeitspektroskopie, Max-Born-Strasse 2a, 12489 Berlin, Germany
(Received 7 July 2008; published 17 November 2008)

We numerically investigate the propagation of a self-compressed optical filament through a gas-glass-gas interface. Few-cycle light pulses survive a sudden and short order-of-magnitude increase of nonlinearity and dispersion, even when all conservative estimates predict temporal spreading or spatial breakup. Spatiotemporal distortions are shown to self-heal upon further propagation when the pulse refocuses in the second gas. This self-healing mechanism has important implications for pulse compression techniques handled by filamentation and explains the robustness of such sources.

DOI: 10.1103/PhysRevLett.101.213901 PACS numbers: 42.65.Tg, 42.68.Ay, 52.38.Hb

The physics of ultrashort optical filaments has recently received increased attention as it enables self-compression of intense laser pulses into the few-cycle regime [1–7]. Femtosecond pulses with input powers close to the self-focusing threshold $P_{cr}$ indeed produce a single filament in which the spectrum broadens and the pulse duration decreases upon propagation. Experimental [2,3] as well as theoretical work [6,7] indicated that self-compression occurs in a limited parameter range, and small deviations from these ideal conditions may cause either spatial breakup into multiple filaments or temporal splitting of the pulse. For evidencing self-compression, experiments currently employ Brewster-windowed cells filled with noble gas at pressures controlling the ratio of input power over critical. Diagnostics are furthermore placed several tens of centimeters beyond the sample, making the pulse undergo a last stage of atmospheric propagation. A crucial point is then to preserve the main characteristics of the compressed pulse when it exits the output window. Several points concerning this issue, however, remain unclear. First, the pulse entering the glass sample undergoes a nonlinear index being three decades higher than in the gas. This renders the robustness of the exiting optical structure questionable as a light beam with several thousands of critical powers is expected to immediately break up by modulational instability [8–11]. Second, the much stronger dispersion of glass should severely alter the temporal propagation. For instance, a 1 mm thick sample with group-velocity dispersion (GVD) coefficient $k^{(2)} \approx 370\text{ fs}^2\text{ /cm}$ at 800 nm can double the duration of an 8 fs transform-limited Gaussian pulse [12].

In the following, we will show that, despite these apparent sources of instability, filamentary propagation holds a hidden self-healing mechanism for ultrashort pulses. Comparable phenomena have already been observed in several physical areas, e.g., in solid-state dynamics on atomic scales [13] and in plasma physics under magnetic confinement [14], explaining stabilization mechanisms in apparently unstable systems. In optics, spatial self-restoration of Bessel beam profiles [15] and of filament cores [16,17] has also been reported. In contrast to all of these previous observations, we report on simultaneous spatiotemporal self-restoration of light bullets having experienced a perturbation that should have immediately caused spatial and temporal decay of the filament. In this Letter, we identify the changes in a self-compressed filament when it leaves a cell filled with argon, propagates in a millimeter-long silica window, and finally travels through the atmosphere. We show that glass dispersion acts jointly with the Kerr response to widely stretch the pulse in time. Unexpectedly, the filament eventually recompresses in air to restore temporal profiles with few-cycle durations. This self-healing mechanism has not an anecdotal character, as it can be responsible for the remarkably short pulse durations seen in many self-compression experiments.

Propagation equations model the forward component of the pulse envelope $\mathcal{E}(x,y,z,t)$ coupled with the free electron density $\rho(x,y,z,t)$ [18]:

$$\begin{align*}
\partial_t \mathcal{E} &= \frac{i}{2k_0} T^{-1} \nabla_\perp^2 \mathcal{E} + iD \mathcal{E} - \frac{k_0}{2n_0^2 \rho_c} T^{-1} \rho \mathcal{E} - \frac{\sigma}{2} \rho \mathcal{E} \\
&\quad - \frac{U_j W(I) (\rho_{nl} - \rho)}{2I} \mathcal{E} \\
&\quad + i \frac{\omega_0}{c} n_2 T \int \mathcal{R}(t-t') |\mathcal{E}(t')|^2 dt' \mathcal{E}, \quad (1)
\end{align*}$$

$$\begin{align*}
\partial_t \rho &= W(I) (\rho_{nl} - \rho) + \sigma \rho I / U_j - \rho / \tau_{rec}, \quad (2)
\end{align*}$$

where $I = |\mathcal{E}|^2$, $z$ is the propagation variable, and $t$ is the retarded time in a reference frame moving with group velocity $1/k^{(1)}$ at laser midfrequency $\omega_0$; $k(\omega) = n(\omega) \omega / c$, $k_0 = k(\omega_0)$, $n_0 = n(\omega_0)$, and $T = 1 + (i / \omega_0) \partial_\lambda$ [19]. $D = \sum_{n=2} \sum_{k(\alpha)} (k^{(n)} / n!) (i \partial_\lambda)^n$ is the dispersion operator formally involving the derivatives $k^{(n)} \equiv \partial^n k / \partial \omega^n |_{\omega_0}$. Linear dispersion curves $n(\omega)$ are taken from Refs. [20,12,21] for argon, silica, and air, respec-
tively. At 800 nm, \(n_0\) takes the value 1 for gases and 1.45 for glass. The leading GVD term in \(D [K^{(2)}]\) is about 3 orders of magnitude larger in glass than in the two gases; 
\[ \rho_c = 1.73 \times 10^{21} \text{ cm}^{-3} \]
is the critical plasma density. The nonlinear Kerr response contains only an instantaneous contribution \(R(t) = \delta(t)\) when the pulse self-focuses in argon. It includes a Raman-delayed contribution \(R(t) = (1 - f)\delta(t) + f \theta(t) \frac{1}{\omega_0^2} e^{-t/\tau_0} \sin(\omega_0 t)\), with ratio \(f\) in silica [22,23] and in air [24]. Plasma sources are driven by the Perelomov, Popov, and Terent’ev ionization rate \(W(t)\) [25] in argon and air and by the Keldysh rate for crystals in glass [26]. In the argon cell, we assume a uniform pressure of 0.5 bar, leading to a nonlinear Kerr index \(n_2 = 5 \times 10^{-20} \text{ cm}^2/\text{W} \) at 800 nm. The Kerr index for glass is 
\[ n_2 = 3.2 \times 10^{-16} \text{ cm}^2/\text{W} \]
and \(n_2 = 2.5 \times 10^{-19} \text{ cm}^2/\text{W} \) in air at atmospheric pressure [24]. All other parameter values used in this Letter, including ionization potentials \(U_j\), avalanche cross sections \(\sigma\), neutral densities \(n_{\text{ne}}\), and electron recombination time for silica \(\tau_{\text{rec}}\), can be found in Refs. [3] (argon) and [18] (glass, air). Simulations have been performed in full 3D \( (\nabla^2 = \partial_x^2 + \partial_y^2)\) and radially symmetric \( (\nabla^2 = r^{-1} \partial_r \partial_r, r = \sqrt{x^2 + y^2})\)
geometries, yielding analogous results. The starting pulse, being Gaussian, is focused by a lens of focal length \(f = 50 \text{ cm}\) into an argon cell with a 1.5 m maximum length. The input waist at \(1/e^2\) and full width at half maximum (FWHM) duration are \(w_0 = 500 \mu\text{m}\) and \(\tau_0 \approx 30 \text{ fs}\), respectively. The input power \(P_{\text{in}}\) is equal to one critical power in argon, i.e., 
\[ P_{\text{cr}} = \frac{\lambda_0^2}{2\pi n_{\text{ne}} n_2} = 20.4 \text{ GW} \]
The following analysis is divided into three steps devoted to pulse propagation in argon, inside the silica window, and in air.

1. Filamentation in argon.—With one critical power, the pulse develops a light bullet dynamics [7], characterized by a primary focusing sequence, followed by a second, limited refocusing [Fig. 1(a)]. Minimum FWHM duration is attained when the plasma relaxes, and the filament is compressed to about 6 fs at \(z \approx 0.7 \text{ m}\) [Fig. 1(b)]. The three subplots of Fig. 1(c) show intensity distributions at distances \(z = 0.5, 1\), and 1.5 m, where the on-axis fluence \(\langle F = \int |E|^2 dt \rangle\) reaches 1.7, 0.2, and 0.05 J/cm², respectively. These distances define the position of the exit glass window.

2. Crossing the silica window.—At high enough fluences \(>0.1 \text{ J/cm}^2\), only a fraction of the forward pulse can be transmitted through a silica surface. According to Ref. [27], only 20% of the forward pulse is expected to transmit the glass surface at \(z = 0.5 \text{ m}\), whereas this fraction increases to 90% and 100% for \(z = 1\) and 1.5 m, respectively. Conditions beyond \(z = 1 \text{ m}\) \((I < 2 \times 10^{13} \text{ W/cm}^2)\) therefore avoid damage of silica [28]. Figure 2(a) summarizes our results. The output profiles of Fig. 1(c) multiplied by the previous transmission rates are used as initial conditions of Eqs. (1) and (2) solved for silica. The 3D and radially symmetric simulation results almost superimpose; hence, no severe azimuthal distortions affect the intensity profiles in the \((x, y)\) plane. The pulse captured at \(z = 0.5 \text{ m}\) (pulse I) begins to diffract over 1 mm, but it refocuses and collapses just afterwards. Although experimentally irrelevant, this extreme configuration already points out the importance of the first few millimeters of propagation dominated by pulse dispersion. Triggered by beam fluences \(>1.5 \text{ J/cm}^2\) in argon, the surface plasma created by pulse I should, however, act as a filter transmitting only the leading edge of the pulse [29], thereby rendering the underlying assumption of a fixed temporal profile invalid. For pulses II and III having much lower fluences, the assumption of a fixed profile still holds. In the following, we thus concentrate on these last two configurations only.

When the window is positioned at \(z = 1 \text{ m}\) (pulse II), again dispersion first prevails but does not prevent beam collapse over 5 mm-long paths. The input beam has a mean waist of 240 \(\mu\text{m}\) and conveys time slices with a maximum power of \(\sim 7100 P_{\text{cr}}\), where \(P_{\text{cr}} = 2.19 \text{ MW}\) here denotes the critical power in glass. In contrast, the low-intensity pulse III disperses rather slowly over the same distance and beyond. The maximum input power, however, is still about \(8000 P_{\text{cr}}\), as the beam waist entering the sample has been enlarged to \(\sim 0.6 \text{ mm}\). Rows (II) and (III) of Fig. 2 depict the evolution of pulses II and III, respectively, in the \((x, t)\) plane \((y = 0)\). Pulse II enters the dielectric with a time extent of \(\sim 8 \text{ fs}\). After 0.5 mm in silica, this pulse decays.
and temporal radii of Gaussian pulses by
\[ R = \frac{p}{2\pi c T^2} \]
and \( R \) in the nonlinear index, as no plasma density plays a negligible role

periodic spatial modulations leading to MF is practically

\[ \frac{1}{4} R^2 R_{zz} = 1 - \frac{\rho}{2T}; \quad \frac{1}{4} T^3 T_{zz} = \delta \left( \delta + \frac{T p}{2R^2} \right). \] 

In Eq. (3), \( \rho = P_m/P_c \) and \( \delta = 2\pi n_0 R_{fil}^2 k^2 / \lambda_0 T^2 \) is

the normalized GVD coefficient for the input filament with

size \( R_{fil} \) and temporal radius \( T_{fil} \). \( \delta \) takes very large values

\( \gg 200 \) when using durations and waists of pulses II and III. Since the growth of \( T(z) \) is driven by \( \delta^2 \) (dispersion) and \( \delta \times \rho \) (Kerr response),

the beam collapse characterized by the vanishing \( R(z) \rightarrow 0 \) at finite \( z \) can efficiently be delayed

in the limits \( \delta, \rho \gg 1 \). Both GVD and cubic nonlinearity

contribute to enlarge the pulse duration. Typically, FWHM
durations of \( \sim 25 \) fs are attained over \( 0.5 \) mm by

the initially 8 fs-long pulse II, whereas linear dispersion alone

allows a temporal broadening limited to \( \sim 10 \) fs.

3. Air propagation.—Because diagnostics are usually

positioned tens of centimeters away from the cell,

the beam finally propagates in atmosphere. Figures 3(a)

and 3(b) show the temporal evolution of pulses II and III,

respectively, as they exit a \( 0.5 \) mm-thick window and are

fully transmitted in air. These two configurations are char-

acteristic of filaments diagnosed either at moderately high

intensities (II) or close to the end of the diffraction stage

(III). These pulses contain time slices with maximum

powers of \( \sim 3.5 P_c \) in air. This fact contributes to make

them refocus and thus recompress. Besides the local Kerr

response, self-healing also proceeds from the spatial phase

curvature imparted by the silica window. This phase

modification is a key process, which causes a focusing

lens effect in air. Collapse is not arrested by plasma gen-

into a broader structure of \( \sim 33 \) fs, forming a pedestal that

extends deep into the trailing region. After \( 1 \) mm, its
duration increases to \( 50 \) fs. Pulse III first exhibits a duration

of \( 13 \) fs, which then broadens to \( 28 \) fs over \( 0.5 \) mm in glass

and more afterwards.

It is important to note the complete absence of multi-

filamentation (MF) in our simulations. Estimates from

modulational instability theory [9], however, predict an

onset of MF and subsequent collapse upon short distances

\( \Delta z_{MF} \approx n_0 P_c / \lambda_0 I_{max} \sim 0.1 \) mm for peak intensities \( I_0 \geq

3 \) TW/cm\(^2\) impinging on the glass surface. In fact, as GVD

rapidly damps the peak intensity, the amplification of

periodic spatial modulations leading to MF is practically

prevented. Here the plasma density plays a negligible role

in the nonlinear index, as \( \Delta n_{NL} \approx n_2 I_{max} - \rho_{max} / 2n_0 \rho_c \sim

1-5 \times 10^{-3} \). We also numerically checked that high-order

dispersion, pulse steepening, and Raman scattering are of

minor influence.

Besides, it turns out that both GVD and the Kerr

response are responsible for the strong increase of the pulse duration that exceeds predictions by the

well-known linear formula for Gaussian pulses \( \tau_{lin}(z) \approx

\tau_0 \sqrt{1 + 16(\ln 2)^2 k^2 / \tau_0^2} \) [12]. Denoting the spatial

and temporal radii of Gaussian pulses by \( R(z) \) and \( T(z) =

\tau(z) / 2\sqrt{\ln 2} \), respectively, a two-scaled variational princi-

FIG. 2 (color online). (Top) Peak intensity of the filament exiting the argon cell at (I) \( z = 0.5 \) m, (II) \( z = 1 \) m, and (III) \( z = 1.5 \) m and propagating into the silica window over millimeter distances. Solid (dotted) curves refer to 3D (radial) computations. (Middle and bottom) Rows (II) and (III) detail the \( (x, t) \) evolution of the pulse intensity profiles II and III, expressed in TW/cm\(^2\), inside the exit window.

FIG. 3 (color online). On-axis temporal evolution of (a) pulse II and (b) pulse III in the atmosphere, after exiting a 0.5 mm-thick window. (c) Temporal profiles compressed in air at the distances \( z = 0.25 \) (solid line) and \( 0.5 \) m (dashed line) for pulse II. (d) Same for pulse III at \( z = 0.5 \) m (solid line) and \( z = 1 \) m (dashed line).
Despite this large stretching in time, durations in the 10 fs filamentation and significantly increases pulse durations. Between dispersion and Kerr self-focusing inhibits multi-gas-glass-gas interface. In the silica window, the interplay generality of the self-healing mechanism. The beam into the last gaseous medium. This underlines the accumulated inside the glass window, which then focuses phenomenon again follows from the nonlinear spatial phase observed that self-restoration still takes place. This phenomenon is caused by loss mechanisms, such as raindrops, apertures, or wire grids, the self-restoration is here induced by a discontinuity of the nonlinear properties inherent to the materials.

Figure 4 summarizes the history of on-axis spectra around the pump frequency ($\omega_0 = 2.35$ PHz) along the propagation steps 1–3. Figure 4(a) describes pulse II, passing through the glass window and being transmitted to air. The first focusing event in argon creates red- and blue-shifted spectral wings caused by self-phase modulation (red curve). Next, GVD drives the propagation in the dielectric, which lowers the saturation intensities and weakens self-phase modulation, such that the blueshifted wing drops out (blue curve). Figure 4(b) provides the same pieces of information for pulse III. At weaker intensities, the spectral evolution follows that of pulse II, although linear dispersion becomes more important and keeps the spectrum closer to that exiting argon (black curve).

It is worth emphasizing that other compressor schemes based, e.g., on an Ar-glass-Ar interface were also numerically simulated (not shown). By decreasing the pressure in the second gas cell until reaching subcritical powers, we observed that self-restoration still takes place. This phenomenon again follows from the nonlinear spatial phase accumulated inside the glass window, which then focuses the beam into the last gaseous medium. This underlines the generality of the self-healing mechanism.

In conclusion, numerical simulations cleared up the dynamics of self-compressed filaments when they cross a gas-glass-gas interface. In the silica window, the interplay between dispersion and Kerr self-focusing inhibits multi-filamentation and significantly increases pulse durations. Despite this large stretching in time, durations in the 10 fs range restore themselves upon further propagation in the second gas. This new self-healing mechanism acts in both space and time. Unlike previous scenarios where the perturbation is caused by loss mechanisms, such as raindrops, apertures, or wire grids, the self-restoration is here induced by a discontinuity of the nonlinear properties inherent to the materials.