Hollow nuclear matter

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It is generally considered that an atomic nucleus is always compact. Based on the isospin-dependent Boltzmann nuclear transport model, here I show that large block nuclear matter or excited nuclear matter may both be hollow. And the size of inner bubble in these matter is affected by the charge number of nuclear matter. Existence of hollow nuclear matter may have many implications in nuclear or atomic physics or astrophysics as well as some practical applications.

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I. INTRODUCTION

One generally considers that an atomic nucleus is always compact. However, it has been theoretically argued that for a nuclear system the total energy can be decreased if a bubble configuration is created \cite{4,5}. In fact, the possible exotic bubble or toroidal configuration of atomic nuclei has been theoretically discussed for many decades \cite{4–12}. In 1946, Wilson has already studied spherical bubble nuclei \cite{4}. In 1967, Siemens and Bethe studied spherical bubble nuclei using a liquid drop model \cite{5}. And later on, based on a liquid drop model (LDM) with shell correction energy, Wong studied spherical bubble nuclei \cite{5}. And later on, based on a liquid drop model (LDM) with shell correction energy, Wong studied known \textbeta-stable nuclei and found spherical bubbles \cite{10}. Furthermore, Moretto et al. showed that bubbles at finite temperature may be stabilized by the inner vapor pressure \cite{13}. Borunda and López, using the hydrodynamic equations coupled to a simple but realistic equation of state, argued that the hollow configuration is a direct consequence of the liquid-like behavior of compressed nuclear matter \cite{14}. And transport simulations of nuclear collisions studied by W. Bauer et al. indicate the possibility of bubble configuration in nuclear matter at finite temperature \cite{15}. Recently, J. Dechargé et al. performed a self-consistent microscopic Hartree–Fock–Bogoliubov (HFB) calculations using the effective Gogny interaction and found stable bubble solutions of some specific superheavy nuclei (250 \(\lesssim\) Z \(\lesssim\) 280, 780 \(\lesssim\) A \(\lesssim\) 920) \cite{10,13}. These results are qualitatively similar to previous studies based on the liquid drop model (LDM) using Strutinsky shell correction method \cite{0,13} and phenomenological shell model potentials \cite{17,18}.

Unlike the probe of bubble formation in heavy-ion collisions \cite{15}, the hollow configuration of an atomic nucleus is in fact hard to probe \cite{1}. Till now the existence of the bubble nucleus is still in debate. It is thus necessary to re-study whether the hollow atomic nucleus exists or not by completely different theoretical methods. Based on the isospin-dependent Boltzmann nuclear transport model (nuclear initialization, single nucleon potential, nucleon-nucleon cross section and Pauli-blocking are all isospin-dependent), here I show that atomic nuclei with super-large mass number or with excitation energy may both be hollow. Besides practical applications, the existence of hollow nuclear matter may have many implications in quantum many-body theory, nuclear physics, atomic physics and the configuration of neutron stars, etc.

II. THE BOLTZMANN NUCLEAR TRANSPORT MODEL

To simulate the formation of the hollow configuration of an atomic nucleus, we use the isospin-dependent Boltzmann nuclear transport model, in which nucleon co-ordinates in initial nuclei are randomly given in the sphere with radius R = 1.2A\(^{1/3}\) (A is the mass number of nucleus) and their momenta are randomly given in the Fermi sea \cite{19}. The isospin-dependence of nucleon Fermi momentum is considered. We use the Skyrme-type parametrization for the isoscalar mean field, which reads \cite{19}

\[ U(\rho) = A(\rho/\rho_0) + B(\rho/\rho_0)^\gamma. \]  

Where \(\sigma = 1.3, A = -232\) MeV accounts for attractive part and \(B = 179\) MeV accounts for repulsive part. These choices correspond to an incompressibility coefficient \(K = 230\) MeV \cite{20}. Considering the nucleon-nucleon Short-Range-Correlations (SRC), we let the kinetic symmetry energy be -6.71 MeV \cite{21}. The symmetry potential becomes \cite{22}

\[ U_{\text{sym}}(\rho, \delta) = 38.31(\rho/\rho_0)^\gamma \times (\pm \delta + (\gamma - 1)\delta^2), \]  

where \(\delta = (\rho_n - \rho_p)/\rho\) is the isospin asymmetry of nuclear medium and \(\gamma = 0.3\). In the above, we let the value of symmetry energy at saturation density be 31.6 MeV.
The corresponding symmetry energy becomes

$$E_{\text{sym}} = -6.71(\rho/\rho_0)^{2/3} + 38.31(\rho/\rho_0)\gamma.$$ (3)

In this model, the in-medium nucleon-nucleon ($NN$) elastic cross section is factorized by the product of a medium correction factor and the free baryon-baryon scattering cross section \[26\], i.e.,

$$\sigma_{\text{medium}}^{NN,\text{elastic}} = \left(\frac{1}{3} + \frac{2}{3} e^{-u/0.54568}\right) \times (1 + 0.85\delta) \times \sigma_{\text{free}}^{NN,\text{elastic}},$$ (4)

where $u = \rho/\rho_0$ is the nuclear reduced density and $\rho_0$ is nuclear saturation density. To find the stable state of atomic nuclei in the simulations, we use 1000 test-particles for each nucleon.

III. RESULTS AND DISCUSSIONS

![FIG. 1: (Color online) Time evolution of the contour density distribution of the hypothetical superheavy atomic nucleus $^{900}$X in X - Y plane with the Boltzmann nuclear transport model. Density becomes larger as color changes from light to dark. The bubble configuration appears after 50 fm/c in the compact nucleus $^{900}$X.](image1)

![FIG. 2: (Color online) Time evolution of the contour density distribution of the hypothetical superheavy atomic nucleus $^{900}$X in X - Y plane with the Boltzmann nuclear transport model. Density becomes larger as color changes from light to dark. The bubble configuration appears after 40 fm/c in the compact nucleus $^{900}$X.](image2)

Based on the isospin-dependent Boltzmann nuclear transport model, I made simulations on the formation of the bubble configuration of some hypothetical superheavy atomic nuclei. Interestingly, I found the bubble configuration of hypothetical superheavy atomic nuclei but the charge number dependence is not very evident. Fig. 1 shows the time evolution of the contour density distribution of the hypothetical superheavy atomic nucleus $^{900}$X in X - Y plane with the Boltzmann nuclear transport model. At the initial time, I fix nucleon co-ordinates randomly in spheroidal atomic nuclei. It is seen that as time increases a bubble steadily appears in the compact hypothetical heavy atomic nucleus. This simulation of atomic nucleus tells us that such atomic nucleus with large mass number tends to internally hollow atomic nucleus. Unlike previous discussions in Ref. [1], in the simulation, we also see similar internally hollow atomic nucleus with the same mass number as $^{900}$X but different charge number, such as atomic nucleus $^{900}$X as shown in Fig. 2. While comparing Fig. 2 with Fig. 1 we find the size of inner bubble depends on the charge number of the nucleus. This point is somewhat consistent with the study in Ref. [1, 9].

However, for relatively light atomic nucleus such as the stable existed element $^{197}$Au, in the evolution of simulation, we did not see such internally hollow state (as shown in Fig. 3) unless giving an excitation energy. Shown in Fig. 4 is the time evolution of the contour density dis-
Density becomes larger as color changes from light to dark. The bubble configuration does not appear in the compact nucleus $^{197}$Au.

FIG. 3: (Color online) Time evolution of the contour density distribution of the relatively light atomic nucleus $^{197}$Au in X - Y plane with the Boltzmann nuclear transport model. The giving average excitation energy (an average energy per nucleon increased relative nuclear ground state) is 5 MeV per nucleon solute. It is seen that as time increases a bubble steadily appears in the compact light atomic nucleus $^{197}$Au. Because the surface tension is relatively strong for relatively light atomic nuclei, to form bubble configuration in compact nucleus, one has to give excitation energy for relatively light atomic nucleus.

Comparing Figs. 1 – 4, one may conclude that only large block ground-state nuclear matter or excited nuclear matter are internally hollow. The main reason is that the surface tension is relatively strong for small block nuclear matter but it becomes weak for large block nuclear matter.

The formation of bubble or toroidal nuclear matter was in fact predicted by nuclear transport model at lower incident beam energy $^{15, 27, 30}$. It is thus also interesting to see if such internally hollow nuclear matter is also formed in nucleus-nucleus collisions by our isospin-dependent transport model. Shown in Fig. $^{3}$ is the $^{197}$Au + $^{197}$Au head-on collision at lower incident beam energy 35 MeV/nucleon. One can see that internally hollow nuclear matter is formed in the nucleus-nucleus collisions as time increases. And inner halo in the bubble of nuclear matter is also seen $^{1, 2}$.

As for how to probe internally hollow nuclear matter formed in heavy-ion collisions, one can probe the bubble formation by using the radial velocity of emitting nucleons in nucleus-nucleus collisions, as shown in Fig. 6, to deduce whether internally hollow nuclear matter is formed or not. If the bubble configuration is formed in nucleus-nucleus collisions, the radial velocity of nucleons in the center of reaction system should be roughly null due to inverse pressure in the bubble as shown in Fig. 5 and Fig. 6.

The results presented here are obtained from the isospin-dependent Boltzmann nuclear transport model, one might question their dependence upon the particular inputs of the transport model. In this respect, I made simulations by varying the symmetry energy parameter $\gamma$ from 0.3 to 1.5 (corresponding very soft and stiff symmetry energies). I found the effects of symmetry energy on the results presented here are very small. While changing the parameters of the isoscalar mean field, i.e., letting the incompressibility coefficient $K = 230 \rightarrow 200$ MeV, one sees relatively large changes of the results presented here. Therefore the incompressibility coefficient used in the transport model needs to be carefully selected $^{20}$.

Just as the discovery of fullerene cluster in 1985 $^{31}$, internally hollow nuclear matter may have many implications in nuclear or atomic physics or astrophysics such as the physics of Neutron stars as well as some practical applications. Existence of internally hollow atomic nucleus
energies may both be internally hollow. And nuclear matter formed in heavy-ion collisions also trends to inner hollow. One can use the methods of heavy-ion collisions to probe the bubble configuration in nuclear matter. Because the existence of internally hollow atomic nuclei may promote the developments of quantum many-body theory and nuclear theory, etc., further studies on the formation as well as its observation signals from nuclear theories and nuclear experiments are urgently needed.

IV. CONCLUSIONS

Based on the nuclear transport model, it is shown that hyperheavy nuclei or general nuclei with excitation energy may both be internally hollow. And nuclear matter formed in heavy-ion collisions also trends to inner hollow. One can use the methods of heavy-ion collisions to probe the bubble configuration in nuclear matter. Because the existence of internally hollow atomic nuclei may promote the developments of quantum many-body theory and nuclear theory, etc., further studies on the formation as well as its observation signals from nuclear theories and nuclear experiments are urgently needed.

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