Flexural Analysis Combined with Freeze-thaw Depth for Reinforced Concrete Beams and Columns

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Abstract

This study presents a kinematic model for flexural analysis of RC beams and columns subjected to freeze-thaw action based on upper bound theorem. The developed model enables analytical derivation of the contribution of damaged concrete when actual deterioration profile is idealized as an assemblage of undamaged and damaged zones based on freeze-thaw depth obtained from concrete core specimens. The accuracy of the analysis is verified by comparing its predictions with available 21 RC columns and beams failing in flexure after freeze-thaw exposure. The predicted results show good agreement with the test results within error of 6% on average. Thereafter, the developed analysis predicts the ultimate moment capacity of a RC beam, which was taken from an existing bridge slab replaced because of the combined effect of frost damage and fatigue. Results demonstrate that the present analysis could support a rational decision-making regarding the need for repair or rehabilitation. This paper is the English translation from the authors’ previous work [Kanazawa, T., Nakamura, T., Sakaguchi, J. and Kawaguchi, K., (2021). “Flexural analysis combined with freeze–thaw depth for RC linear members.” Journal of Japan Society of Civil Engineers, Ser. E2 (Materials and Concrete Structures), 77(4), 177-186. (in Japanese)].

1. Introduction

Freeze-thaw action results in changes in material properties, affecting adversely and significantly the structural performance of existing reinforced concrete (RC) structures. For such deteriorated structures, regular visual inspections by experienced engineers have difficulty in making a rational decision on the future course of action. International code requirements have attempted to rate the existing capacity of frost-damage affected structural members (ACI 2019; Coronelli et al. 2020). However, no indicator has existed to correlate the residual structural capacity and information obtained from field investigation. The various approaches of structural assessment are, therefore, unable to support decision-making in regular maintenance practice. For example, three-dimensional finite element analysis (Gong and Maekawa 2018) is capable of predicting load-displacement relationships reflecting actual distribution of frost damage, requiring difficult judgments to use constitutive laws and verification of obtained solutions.

Existing works on freeze-thaw damage have been conducted mainly on plain concrete (Hasan et al. 2004; Li et al. 2017; Liu and Wang 2012). Limited experimental and analytical findings are still available on the mechanical behavior of RC structural members (Duan et al. 2017; Petersen et al. 2007; Xu et al. 2016), despite the recent progress in China (Liu et al. 2018; Rong et al. 2020). Hence, strength assessment is often replaced by durability assessment based on the relative dynamic modulus of elasticity (RDME) (Duan et al. 2014; Qin et al. 2016), which corresponds well with the mechanical properties of plain concrete. It is remarkable that frost damage changes failure modes of RC beams and columns from flexure to shear-dominance (Hayashida et al. 2014; Qin et al. 2017). One of the authors has developed a mechanical model that directly correlates the freeze-thaw depth with shear strengths. This model overcomes limitation of previous works (Hanjari et al. 2013; Su et al. 2019) that reflect qualitative relation between strength reduction of concrete and shear strengths, enabling shear strength derivation without using regression functions and empirically determined parameters. If both flexural and shear strengths could be associated with freeze-thaw depth, a simple and quantitative assessment would be possible to determine the requirement of repair or replacement based on the information obtained from core-sampling.

This study presents a rigorous analysis to assess residual flexural capacity of RC beams and columns, based on the upper bound theorem of limit analysis (Nielsen and Hoang 2011). It requires a concrete core taken from existing deteriorated structures as shown in...
Fig. 1, and allows direct correlation between freeze-thaw depth and flexural strengths. An actual cross-sectional distribution of frost damage is idealized as an assemblage of damaged and undamaged zones. This approximation enables the quantification of detrimental effect of damaged concrete by integration of strain velocity distribution. It also allows the computation of interaction diagrams between flexural capacity and axial load. Subsequently, the developed analysis is validated by comparison of its predictions with experimental data of 21 RC beams and columns currently available in the literature. Finally, the validated model predicts the residual flexural capacity of beam elements taken from an existing RC slab that was replaced because of serious frost damage. Information obtained from a concrete core sampled from this slab is employed for flexural assessment. Results highlight the possibility of rational flexural assessment according to the obtainable information from on-site inspections. All the symbols used in this paper are described in the appendix to avoid complications.

2. Development of flexural analysis combined with frost damage depth

2.1 Analytical framework

To compute ultimate flexural capacity based on the upper bound theorem, the authors extend the flexural analysis for RC columns without deterioration (Doi 1980) to those with frost damage. The following assumptions are introduced.

(a) Concrete and steel reinforcement are in a plane stress state.
(b) Cross-sectional distribution of strain velocity follows the assumption that plane sections remain plane.
(c) An actual strain distribution under the effect of frost damage is idealized as an assemblage of undamaged and damaged zones with no discontinuity.
(d) Perfect bonding exists between steel reinforcement and concrete.
(e) Concrete and steel reinforcement respectively behaves as rigid-perfectly plastic material and elastic-perfectly plastic material as shown in Fig. 2 (Nielsen and Hoang 2011).

Fig. 1 Schematic representation of the model application in practical use.

Fig. 2 Constitutive model of concrete (left) and steel reinforcement (right).
Here, dimensionless quantities $\alpha$ and $\beta$, representing the damage depth and the strength reduction because of freeze-thaw action, respectively, are defined as follows:

$$\alpha = \frac{2D}{h} \quad (1)$$

$$\beta = \frac{f'_{c,dam}}{f'_c} \quad (2)$$

Although the peak strength of a rigid-plastic body is modified to ensure the equivalence of strain energy with real concrete (Exner 1979), no such modification is considered here because few reports have focused on the appropriate modification factor for frost-damage affected concrete.

### 2.2 Derivation of ultimate moment

Figure 3 illustrates the strain velocity distribution of RC structural members under the combined effect of symmetric moment and axial loading as shown in Fig. 1. The damaged zone is introduced by its depth of $\alpha$ [Eq. (1)]. The continuous curvature rate $\phi$ is assumed over the whole section, following the assumption (c) made in Section 1. The symbol $\epsilon$ denotes the strain velocity at the mid-height of cross-section. One can calculate the velocity distribution along the $y$-coordinate of Fig. 3.

Integration of Eq. (4) yields the contribution of concrete and longitudinal reinforcement in the damaged and undamaged zones.

$$W_i = \sigma_y \int f(y) dA_y + \sigma_y \int f(y) dA_y$$

In this equation, $f(y) = \epsilon + \phi \gamma$ represents the strain velocity distribution along the $y$-coordinate of Fig. 3.

Integration of Eq. (4) yields the contribution of concrete and longitudinal reinforcement in the damaged and undamaged zones.

$$W_i = \int f(y) dy + \beta \int f(y) dy + \int \left( \epsilon + \frac{x}{2} \phi \right) dA_y$$

One obtains the energy dissipation rate of internal forces from Eq. (5) as shown below.

$$W_i = N_e + M \phi \quad (6)$$

Under the principle of virtual work, equilibrium between Eqs. (5) and (6) yields the ultimate moment capacity $M$ as seen in Eq. (7), which is an upper bound solution because it stems from the compatibility condition (strain velocity distribution of Fig. 3) and the yield criteria of Fig. 2; the force equilibrium is disregarded. One minimizes Eq. (7) with respect to $\epsilon/\phi$, obtaining the upper bound solution closest to the exact solution.

$$M = N_e \left( \frac{\epsilon}{\phi} \right)^2 + \frac{N_e}{2} \left( \rho - \frac{2(N + 2S)}{N_{00}} \right) \left( \frac{\epsilon}{\phi} \right) + \rho(1 - \alpha + \alpha \beta) N_0 \quad (7)$$

Minimizing Eq. (7) to find $M$ corresponds to draw the $M-N$ interaction diagram (Wight and MacGregor 2016) because $\epsilon/\phi$ varies with the magnitude of axial force $N$. The five states (I to V) shown in Table 1 and Fig. 4 constitute the $M-N$ interaction diagram, depending on the yielding state of concrete and longitudinal reinforcement.

One obtains Eqs. (8) to (12) by minimizing Eq. (7) with respect to $\epsilon/\phi$ in each state. $\lambda$ represents the lower-to-upper longitudinal reinforcement ratio; $\lambda = 1$ means the symmetrically reinforced section as observed in Fig. 4. An example of the $M-N$ interaction diagram obtained from Eqs. (8) to (12) under $\lambda = 1$ is demonstrated in Fig. 5. Each state is identified by the solid and dashed lines. The black and red lines in Fig. 5 show, respectively, the undamaged and damaged cases, the latter of which is calculated using $\alpha = 0.3$ and $\beta = 0.7$. The significant influence of damaged zone is apparent in the States I, II and III because the concrete in compression zone has considerable contribution. On the other hand, almost no difference between the undamaged and damaged cases is observable in States IV and V, where the limited zone of concrete is under compression. The derivation of Eqs. (8) to (12) is supplemented in the appendix.
Let us note that the distinction of each state is made not by $\phi$ but by the axial force $N$ in Eqs. (8) to (12). This replacement is understandable from the force equilibrium. Eq. (13) is the equilibrium equation along horizontal direction ($\lambda = 1$) in State III. The external axial force is equilibrated only by concrete because the upper and lower longitudinal reinforcement is yielding in compression and tension, respectively.

$N = h \left( \frac{h}{2} (1 - \alpha) + \frac{\epsilon}{\phi} + \frac{\alpha \beta h}{2} \right)$  (13)

One finds Eq. (14) by arranging Eq. (13) with respect to $\dot{\epsilon}/\dot{\phi}$.

$\frac{\dot{\epsilon}}{\dot{\phi}} = \frac{h}{\frac{h}{2} (N_0 - \rho)}$  (14)

**Figure 4** represents that the States II–III and III–IV are distinguished by $\dot{\epsilon}/\dot{\phi} = \pm \delta/2$. Substituting $\dot{\epsilon}/\dot{\phi} = \pm \delta/2$ into Eq. (14) and arranging with respect to $\dot{\epsilon}/\dot{\phi}$.

| Table 1 Description of each state. |
|-------------------------------------|
| Concrete in compression $\sigma_r = f_y$ |
| Upper longitudinal reinforcement $\sigma_y = f_y$ | $\sigma_y = f_y$ | $\sigma_y = f_y$ | No yielding | $\sigma_y = -f_y$ |
| Lower longitudinal reinforcement $\sigma_y = f_y$ | No yielding | $\sigma_y = -f_y$ | $\sigma_y = -f_y$ | $\sigma_y = -f_y$ |
| Neutral axis depth Under lower reinforcement | Between upper and lower reinforcement | Above upper reinforcement |

Note that $\sigma_y = f_y$ and $\sigma_y = -f_y$ signify the yielding in compression and tension, respectively.

**State I**: $N = \frac{h}{2} (\rho + \mu) + (1 + \lambda) S_y \leq N < \rho N_0 + (1 + \lambda) S_y$

$M = \left( \rho (1 - \alpha) + \alpha \beta - \frac{2 (N - (1 + \lambda) S_y)}{N_0} \right) M_0 + \frac{1 - \lambda}{2} M_1$  (8)

**State II**: $N = \frac{h}{2} (\rho + \mu) + (1 - \lambda) S_y \leq N < \frac{h}{2} (\rho + \mu) + (1 + \lambda) S_y$

$M = \left( (1 - \alpha + \mu) + \alpha \beta - 2 (1 + \mu) - \alpha \right) M_0 + M_1 - \frac{\delta}{2} N$  (9)

**State III**: $N = \frac{h}{2} (\rho - \mu) - (1 + \lambda) S_y \leq N < \frac{h}{2} (\rho - \mu) + (1 - \lambda) S_y$

$M = \alpha \beta (1 - \beta) + 4 \rho N_0 - 4 \frac{N}{N_0} M_0 + (1 + \lambda) M_1$  (10)

**State IV**: $N = \frac{h}{2} (\rho - \mu) - (1 - \lambda) S_y \leq N < \frac{h}{2} (\rho - \mu) + (1 - \lambda) S_y$

$M = \left( (1 - \alpha - \mu) + \alpha \beta - 2 (1 - \mu) + \alpha \right) M_0 + \lambda M_1 + \frac{\delta}{2} N$  (11)

**State V**: $N = - (1 + \lambda) S_y \leq N < \frac{h}{2} (\rho - \mu) - (1 + \lambda) S_y$

$M = \rho (1 - \alpha) + \alpha \beta - \frac{2 (N + (1 + \lambda) S_y)}{N_0} \right) M_0 + \frac{1 - \lambda}{2} M_1$  (12)

Let us note that the distinction of each state is made not by $\phi$ but by the axial force $N$ in Eqs. (8) to (12). This replacement is understandable from the force equilibrium. Eq. (13) is the equilibrium equation along horizontal direction ($\lambda = 1$) in State III. The external axial force is equilibrated only by concrete because the upper and lower longitudinal reinforcement is yielding in compression and tension, respectively.

**Figure 5** An example of the M–N interaction diagram obtained from Eqs. (8) to (12).
to \( N \), one obtains the range of axial forces \( N \) in Eq. (10).

\[
N = \left\{ \begin{array}{ll}
\frac{N_s}{2} (\rho + \mu) & \text{when } \frac{\dot{\varepsilon}}{\phi} = \frac{\delta}{2} \\
\frac{N_s}{2} (\rho - \mu) & \text{when } \frac{\dot{\varepsilon}}{\phi} = -\frac{\delta}{2}
\end{array} \right.
\]

is substituted into Eq. (14) \( (15) \).

\[ \alpha = 0.40 \frac{n}{n_{\text{max}}} \]  

(16)

Simultaneous heat and moisture transfer analysis (Matsumoto et al. 2001) offers an alternative way to estimate the damage depth, but it requires many parameters to fix appropriately according to the various conditions of existing structures. Using Eq. (16) is considerable simplification because the minimum temperature during freeze-thaw tests and concrete mixture have a non-negligible effect on degradation under the same number of freeze-thaw cycles. However, the authors assume that such simplification is essential for practical implementation if analytical predictions show satisfactory agreement with test results of Table 2. For the dimensionless quantity of \( \beta \), the results of compression test summarized in Table 3 were used. Each tested specimen was prepared from the same concrete mixtures as corresponding beams and columns.

3.2 Discussion of model validity

Figures 7 to 12 portray comparisons between the analytically obtained flexural capacity \( (M_{\text{ana}}) \) and the experimentally obtained one \( (M_{\text{exp}}) \), drawn as the parabo-
las and plots. Table 2 lists the values of $M_{\text{ana}}$ and $M_{\text{exp}}$ for each case. Table 4 demonstrates that the close agreement for both undamaged and damaged specimens is readily apparent within error of 6% on average, despite the significant simplification about damage depth.

The present analysis overestimated a few results of column specimens: $N1(300)$ and $N2(300)$ in Fig. 7, and $R100F1$ and $R100F2$ in Fig. 11. This discrepancy could be attributed to exclude the strength reduction factor (Exner 1979) for a rigid-perfectly plastic material.

It is noteworthy that the model validation does not include the test results belonging to the States I and II, which have significant neutral axis depth. All the collected data belonged to the States III and IV as seen in Table 2 because massive sections lead to smaller influence of axial force than narrower sections do.

### Table 2 List of specimens used for model validation.

| Spec.  | Dimensions | Concrete-related | Steel reinforcement related | Axial force | Ultimate moment |
|--------|------------|------------------|----------------------------|-------------|-----------------|
|        | $b$ mm | $h$ mm | $\delta$ mm | $\mu$ | $f'$ N/mm$^2$ | $\alpha$ | $\beta$ | $S_0$ kN | $M_0$ kNm | $N$ kN | $M_{\text{exp}}$ kNm | $M_{\text{ana}}$ kNm | State |
| N1(0)  | 200 | 200 | 160 | 0.800 | 34.2 | 0.00 | 1.00 | 157 | 25.0 | 0.20$\beta N_0$ | 48.9 | 46.7 | III |
| N2(0)  | 200 | 200 | 160 | 0.800 | 34.2 | 0.00 | 1.00 | 157 | 25.0 | 0.32$\beta N_0$ | 55.8 | 49.4 | III |
| N1(300) | 200 | 200 | 160 | 0.800 | 27.6 | 0.400 | 0.807 | 157 | 25.0 | 0.20$\beta N_0$ | 36.7 | 47.0 | III |
| N2(300) | 200 | 200 | 160 | 0.800 | 27.6 | 0.400 | 0.807 | 157 | 25.0 | 0.32$\beta N_0$ | 39.0 | 49.1 | III |
| Z-C1   | 200 | 200 | 160 | 0.800 | 53.7 | 0.00 | 1.00 | 225 | 36.0 | 301 | 66.9 | 61.9 | III |
| Z-C2   | 200 | 200 | 160 | 0.800 | 49.6 | 0.130 | 0.924 | 225 | 36.0 | 301 | 60.6 | 61.8 | III |
| Z-C3   | 200 | 200 | 160 | 0.800 | 44.1 | 0.270 | 0.823 | 225 | 36.0 | 301 | 56.5 | 61.0 | III |
| NF0    | 240 | 240 | 190 | 0.790 | 30.2 | 0.00 | 1.00 | 248 | 47.2 | 301 | 92.2 | 91.1 | III |
| NF1    | 240 | 240 | 190 | 0.790 | 29.4 | 0.200 | 0.974 | 248 | 47.2 | 301 | 91.1 | 90.0 | III |
| NF2    | 240 | 240 | 190 | 0.790 | 28.5 | 0.400 | 0.944 | 248 | 47.2 | 301 | 88.2 | 88.2 | III |
| R50F0  | 240 | 240 | 190 | 0.790 | 26.9 | 0.00 | 1.00 | 248 | 47.2 | 301 | 86.6 | 86.5 | III |
| R50F1  | 240 | 240 | 190 | 0.790 | 25.2 | 0.200 | 0.937 | 248 | 47.2 | 301 | 83.1 | 83.9 | III |
| R50F2  | 240 | 240 | 190 | 0.790 | 22.3 | 0.400 | 0.828 | 248 | 47.2 | 301 | 79.0 | 79.8 | II |
| R100F0 | 240 | 240 | 190 | 0.790 | 26.1 | 0.00 | 1.00 | 248 | 47.2 | 301 | 81.5 | 84.3 | III |
| R100F1 | 240 | 240 | 190 | 0.790 | 23.1 | 0.200 | 0.884 | 248 | 47.2 | 301 | 71.5 | 81.5 | III |
| R100F2 | 240 | 240 | 190 | 0.790 | 17.3 | 0.400 | 0.661 | 248 | 47.2 | 301 | 65.4 | 73.6 | III |
| B-C    | 150 | 250 | 210 | 0.840 | 37.3 | 0.00 | 1.00 | 169 | 35.5 | 0.00 | 42.3 | 36.6 | IV |
| B0-20  | 150 | 250 | 210 | 0.840 | 34.0 | 0.100 | 0.912 | 169 | 35.5 | 0.00 | 41.5 | 36.5 | IV |
| B0-40  | 150 | 250 | 210 | 0.840 | 32.6 | 0.200 | 0.874 | 169 | 35.5 | 0.00 | 38.8 | 36.5 | IV |
| B0-60  | 150 | 250 | 210 | 0.840 | 32.2 | 0.300 | 0.863 | 169 | 35.5 | 0.00 | 38.1 | 36.6 | IV |
| B0-80  | 150 | 250 | 210 | 0.840 | 31.9 | 0.400 | 0.855 | 169 | 35.5 | 0.00 | 36.7 | 36.7 | IV |

Note that the definition of each notation is given in the appendix.

### Table 3 Specimen details used to determine $\beta$.

| Specimen details | References |
|------------------|------------|
| Average strength of five cube specimens (100 mm $\times$ 100 mm $\times$ 100 mm) | Xu et al. 2016 |
| Cylindrical specimens (diameter and height of 100 mm) taken from each RC column | Qin et al. 2017 |
| Average strength of three prismatic specimens (100 mm $\times$ 100 mm $\times$ 400 mm) | Liu et al. 2018 |
| Average strength of three cube specimens (150 mm $\times$ 150 mm $\times$ 150 mm) | Dual et al. 2017 |

### Table 4 Accuracy of the developed analysis.

| Specimens | Average value of $M_{\text{exp}} / M_{\text{ana}}$ | Coefficient of variation (%) |
|-----------|----------------------------------|-----------------------------|
| 7 undamaged specimens | 1.06 | 5.93 |
| 14 damaged specimens | 0.95 | 10.6 |
| Total (21 specimens) | 1.01 | 10.6 |

### 3.3 Analysis of beam specimens with unbonded reinforcement

Assuming perfect bonding between steel reinforcement and concrete is unavoidable in limit analysis because derivation of an analytical solution becomes quite difficult by consideration of bond behavior (Nielsen and Hoang 2011). It is useful to know the discrepancy of the present analysis with concrete members having unbonded reinforcement because existing frost-damage...
affected structures suffer from bond deterioration. The three specimens with unbonded steel reinforcement (Takiguchi et al., 1976) will be analyzed as tabulated in Table 5. The bond was removed to fill the concave-convex surface of deformed bars with paraffin wax; there was no loss of concrete cross-section unlike previous studies (Cairns and Zhao 1993; Zhang and Raoof 1995). The results in Table 5 show that values of $N_{exp}/N_{ana}$ lie within the error of 20%.

4. Flexural assessment of an existing RC slab replaced because of severe frost damage

4.1 Description of the analyzed specimens

Figure 13 portrays schematic drawings of the analyzed RC beams. The specimen No. 0 was fabricated as the reference specimen for comparison; the specimen No. 1 was a portion of an existing RC slab (200 mm thickness) that had been in-service for 53 years. Regular inspection for this slab detected the anomaly: water leakage, exposed steel reinforcement, leaching of lime and several cracks on its underside. Then, the detailed survey revealed the spalling of concrete cover and resultant exposure of steel reinforcement on the topside as well. The partial replacement of the damaged slab was performed as an emergency repair. The cause of such serious damage is inferred to be a combined effect of frost damage and fatigue loading.

The specimen No. 1 has bent bars (black line in Fig. 13) to resist the negative bending moment occurring on the supports under service condition. To ensure the proper anchorage of existing reinforcing bars, the bar ends were cut off and then welded to the steel plate. The subsequent re-casting was carried out by ready-mixed concrete of nominal strength of 27 MPa.

A concrete core with the height of 200 mm was taken from the same slab as the specimen No. 1 to investigate the inside. The ultrasonic pulse velocity of the core was measured along the thickness direction of the slab as Table 5 shows.
shown in Fig. 14. The RDME profile (Fig. 14 underside) was also depicted using Eq. (17) (Ogata et al. 2002). The damage depth was determined as $D = 30 \text{ mm}$ ($\alpha = 0.30$) in accordance with the values of RDME.

$$E_d = 4.0387v^2 - 14.438v + 20.708$$ (17)

In Eq. (17), the units of ultrasonic pulse velocity $v$ and dynamic modulus of elasticity $E_d$ are km/s and GPa, respectively.

The inputs to the present analysis were inferred to be as tabulated in Table 6 based on these data. The values of $S_y$ were calculated by the cross-sectional area of upper longitudinal reinforcement and its yield strength (not nominal but measured value). The difference of $S_y$ between the specimen No. 0 and No. 1 results from the yield strength of each reinforcement. $\beta$ of specimen No. 1, representing the reduction of compressive strength, was determined by averaging the compressive strengths of three concrete cores taken from the original slab.

### Table 6 Inputs for the strength assessment of each specimen.

| Specimens | $b$ (mm) | $h$ (mm) | $\delta$ (mm) | $\mu$ (–) | $S_y$ (kN) | $f'_{c}$ (N/mm²) | $\alpha$ (–) | $\beta$ (–) | $N$ (kN) |
|-----------|-----------|-----------|---------------|------------|-------------|----------------|-------------|-------------|----------|
| No. 0 (undamaged) | 360 | 200 | 110 | 0.550 | 130 | 31.2 | 0.00 | 1.00 | 0.00 |
| No. 1 (damaged) | 340 | 200 | 110 | 0.550 | 99.9 | 31.2 | 0.300 | 0.670 | 0.00 |

| Table 7 Accuracy of the analysis for each specimen. |

| Specimens | $M_{exp}$ (kN m) | $M_{ana}$ (kN m) | $M_{exp}/M_{ana}$ |
|-----------|-----------------|-----------------|-------------------|
| No. 0 (undamaged) | 32.2 | 30.9 | 1.04 |
| No. 1 (damaged) | 20.3 | 23.6 | 0.86 |

### 4.2 Results and discussion

The $M$–$N$ interaction diagram of Fig. 15 portrays the comparison between the experimental results and analytical prediction, respectively shown as the plots and curves. The range of States III, IV and V is represented in Fig. 15 because these specimens were subjected to no axial force. The analysis accurately predicts the experimental results for specimen No. 0 with no deterioration as listed in Table 7. The accuracy slightly decreases for the specimen No. 1, but it retains the error of 14%. The assumption of perfect bonding might lead to this overestimation of test result because the unbonded specimens of B–O–0.0–NB and B–R–0.0–NB in Table 5 were overestimated as well.

The effect of bond deterioration on flexural capacity is often discussed in terms of the experimental findings that the concrete compressive strain at extreme fiber increases significantly as the neutral axis depth becomes narrower than that of ordinary RC beams (Cairns and Zhao 1993; Zhang and Raoof 1995). The flexural capacity in States IV and V reduces as the neutral axis
depth decreases. Such tendency was not reflected by assuming the perfect bonding.

It is a rare example that the plots of test results in Fig. 15 belong to the State V. All the specimens listed in Table 2 were classified as State III or IV. This is because the specimens No. 0 and No. 1 have relatively large cover depth of 45 mm and width $b$ as seen in Table 6. When the neutral axis lies above the upper longitudinal reinforcement (State V), the damaged concrete in compression zone can afford to equilibrate with the tensile force of lower longitudinal reinforcement. Readers can compare Fig. 5 with Fig. 15. The former and latter respectively demonstrates little and considerable difference between the undamaged and damaged response under the States IV and V.

5. Conclusions

This study presents a flexural analysis incorporating the adverse effect of freeze-thaw depth into the integration of strain velocity distribution. The formulation is based on the upper bound theorem of limit analysis, yielding the $M$–$N$ interaction diagram with no empirically determined parameters and regression approach. The developed analysis was validated by comparison of four experimental campaigns including 21 RC columns and beams. Subsequently, the validated model predicted the residual flexural capacity of RC beam taken from the existing slab replaced because of serious frost damage.

For ease of practical use, actual deterioration profile is idealized as an assemblage of undamaged and damaged zone. The damage depth is limited up to $\alpha = 0.40$ for the model validation. The developed analysis corresponds well with the available experimental results within the error of $\pm 6\%$ on average. Further validation is necessary to the data belonging the States I and II, but sufficient level of accuracy is obtained despite such simplification. In addition, the explicit formulation reveals its applicability of $\mu < \rho \leq 1$, which actual examples are rarely beyond. An interesting finding is that, the frost damage has little influence on flexural capacity in the States IV and V because the slight contribution of concrete in compression zone i.e., damaged zone is observed. Application of this analysis to existing structures requires a core sample to estimate the damage depth $\alpha$ as shown in Fig. 14. The information obtained from detailed inspection usually gives the value of $\beta$, representing the strength reduction because of freeze-thaw action. This analytical framework offers a practical approach to correlate directly the damage depth with residual flexural capacity.

The use of design equations based on lower bound solutions is not always appropriate for strength assessment of existing structures (fib 2010). Nonlinear finite element analysis is a powerful tool to consider real but complex deterioration phenomena; it is challenging to implement regular maintenance practice. On the other hand, upper bound analysis does not require different idea to formulate failure mode of shear and flexure. It enables the derivation of analytical solutions under the minimum necessary parameter, namely damage depth. Combination of the flexural and existing shear analysis (Kanazawa and Ushiwatari 2021) gives a consistent solution for strength assessment of freeze-thaw affected RC structural members.

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Appendix

The derivation of Eqs. (8) to (12) from Eq. (3) is supplemented under \( \lambda = 1 \) for simplicity. It should be noted that the integration of Eq. (3) yields different results for each state. Specifically, one obtains Eq. (A.1) by substituting \( \sigma_0 = f_y \) and \( \sigma_0 = -f_y \), respectively corresponding to yielding in compression and tension, into the second term of Eq. (3).

Table A.1 Notations used in this study.

| Notation | Description |
|----------|-------------|
| \( A \) | Cross-sectional area of concrete |
| \( A_1 \) | Cross-sectional area of longitudinal reinforcement |
| \( A_{1s} \) | Cross-sectional area of upper longitudinal reinforcement |
| \( A_{2s} \) | Cross-sectional area of lower longitudinal reinforcement |
| \( b \) | Width of cross-section |
| \( D \) | Damage depth |
| \( E_d \) | Dynamic modulus of elasticity |
| \( f_c \) | Compressive strength of undamaged concrete |
| \( f_{c, dam} \) | Compressive strength of damaged concrete |
| \( f_y \) | Yield strength of longitudinal reinforcement |
| \( f(y) \) | Strain velocity distribution |
| \( h \) | Total depth of cross-section |
| \( M \) | Ultimate moment capacity |
| \( M_0 \) | Ultimate moment capacity obtained by analysis |
| \( M_{exp} \) | Ultimate moment capacity obtained by experiments |
| \( M_{y} \) | Yield moment of upper (or lower) longitudinal reinforcement defined by \( S_y, \delta \) |
| \( N \) | Axial force |
| \( N_0 \) | \( bh f_y \) |
| \( n \) | Number of freeze-thaw cycles |
| \( n_{max} \) | Maximum number of freeze-thaw cycles for each case of experiments |
| \( S_y \) | Yield capacity of upper (or lower) longitudinal reinforcement |
| \( v \) | Ultrasonic pulse velocity |
| \( W_i \) | Energy dissipation rate of internal forces |
| \( W_e \) | Energy dissipation rate of external forces |
| \( \alpha \) | Dimensionless quantity of damage depth, defined by Eq. (1) |
| \( \beta \) | Dimensionless quantity of strength reduction, defined by Eq. (2) |
| \( \delta \) | Distance between lower and upper longitudinal reinforcement |
| \( \varepsilon \) | Strain velocity at mid-height of cross-section |
| \( \lambda \) | \( A_{1s} / A_1 \) ratio |
| \( \mu \) | Dimensionless quantity defined as \( \delta / h \) |
| \( \rho \) | Dimensionless quantity defined as \( 1 - \alpha + \alpha \beta \) |
| \( \sigma_c \) | Concrete stress |
| \( \sigma_y \) | Stress of longitudinal reinforcement |
| \( \phi \) | Curvature rate |
The derivation process of Eqs. (8) to (12) is supplemented by minimizing $M$ with respect to $\frac{\varepsilon}{\phi}$. One substitutes $\frac{\varepsilon}{\phi}$ satisfying Eq. (A.3) into Eq. (7) and Eq. (A.2) to obtain Eqs. (8), (12) and (10), respectively. Equations (9) and (11) are obtainable by substituting $\frac{\varepsilon}{\phi} = \pm \delta / 2$ into Eq. (7).

$$\frac{\partial M}{\partial \left(\frac{\varepsilon}{\phi}\right)} = 0$$ (A.3)

Finally, all the notations used in this paper are summarized in Table A.1.