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Measurements of electron density and temperature profiles in plasma produced by Nike KrF laser for laser plasma instability research

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A grid image refractometer (GIR) has been implemented at the Nike krypton fluoride laser facility of the Naval Research Laboratory. This instrument simultaneously measures propagation angles and transmissions of UV probe rays (λ = 263 nm, Δt = 10 ps) refracted through plasma. We report results of the first Nike-GIR measurement on a CH plasma produced by the Nike laser pulse (~1 ns FWHM) with the intensity of 1.1 × 10^{15} W/cm^2. The measured angles and transmissions were processed to construct spatial profiles of electron density (n_e) and temperature (T_e) in the underdense coronal region of the plasma. Using an inversion algorithm developed for the strongly refracted rays, the deployed GIR system probed electron densities up to 4 × 10^{21} cm^{-3} with the density scale length of 120 μm along the plasma symmetry axis. The resulting n_e and T_e profiles are verified to be self-consistent with the measured quantities of the refracted probe light. © 2015 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License. [http://dx.doi.org/10.1063/1.4927452]

I. INTRODUCTION

Knowing plasma conditions in the underdense coronal region is crucial to understanding physical processes of laser plasma instabilities (LPIs)1 in laser fusion research. The LPIs have been a major obstacle to achieving fusion ignition with high energy gain due to their destructive capabilities such as decrease of the laser energy deposition and preheat of the DT fuel before arrival of the imploding shock. In large scale plasmas relevant to laser fusion, two plasmon decay (TPD) and DT fuel preheat the target. These instabilities exist in the underdense coronal region near or lower than the quarter critical density region since the critical density increases strongly as the laser wavelength decreases (n_c ≈ λ^{-2}). For example, the quarter critical density is 2.3 × 10^{21} cm^{-3} for λ_{laser} = 351 nm and 4.5 × 10^{22} cm^{-3} for λ_{laser} = 248 nm. Experimental data are present in this density region. Hence, numerical simulation has been the primary way to assess n_e and T_e profiles in plasmas produced by modern laser drivers.

Grid image refractometry (GIR)10 is an optical probing technique for determining electron density profiles especially in long scale-length plasmas. It was proposed to access higher plasma densities by measuring propagation angles of probe rays refracted by plasma (Fig. 1). When probe light passes through a grid, it is broken up into a well-defined two dimensional array of small beamlets. The beamlets are then sent from the side and refracted through the plasma plume. Apparent cross-sectional profiles of the emerging beamlets can be imaged at two or more object planes for measuring individual deflection angles. One contrasting difference of GIR from plasma interferometry is its ability to trace each probe beamlet. This provides unique advantages of using GIR for diagnosing plasma parameters. First, the impact parameters of all the probe beamlets are known with GIR, allowing more

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FIG. 1. Concept of the GIR technique. Probe rays are refracted by the plasma and diagnosed with optics that image individual ray positions. The cross-sectional ray profiles can be imaged at two or more object planes to measure propagation angles of the emerging rays from the plasma.

rigorous comparisons with theoretical predictions. Second, the angle measurement requires not absolute locations of the object planes but the relative distance between them. Hence, GIR is not sensitive to the focusing error (between the object plane and the plasma symmetry axis) originating from shot-to-shot variation of the irradiating laser pointing on the target.\(^\text{11,28,29}\) And specifically in this article, the capability of measuring individual beamlet angles with known impact parameters was necessary to develop an iterative algorithm for constructing density profiles self-consistent with the measurement as described in Section III B.

One other difference of GIR from interferometry is that GIR does not depend on probe-beam coherence. The deflection angle measurement is thus associated with geometrical (rather than wave) optics, making GIR more flexible to large light deflections than interferometry.\(^\text{3}\) In addition, as pointed out by Craxton et al., the probe pulse duration for GIR can be much greater than the fringe-blurring time, and hence GIR does not require inconveniently short probe times.\(^\text{10}\) While the micron-size fringe resolution of interferometry is lost, the spatial resolution of GIR, determined by the grid spacing, is still adequate for long scale-length plasmas which are typical in laser fusion research. Therefore, one can expect to diagnose a higher density region with GIR for the plasmas in laser fusion research. Using 0.26 \(\mu\)m ultraviolet probe light and 50 \(\mu\)m grid spacing, the earliest GIR experiment measured a density profile up to \(n_e \sim 1 \times 10^{21} \text{ cm}^{-3}\).\(^\text{10}\)

The GIR method naturally provides a way to determine the electron temperature profile by simultaneously measuring beamlet transmissions (or absorptions) through the plasma. A shorter wavelength probe such as soft X-ray lasers,\(^\text{6,12}\) if practical for the GIR implementation,\(^\text{13}\) may be preferable to reach high electron density regions. However, the absorption of the X-ray probe is too small to measure in the coronal density region of interest suggesting that a longer wavelength probe should be used for the transmission measurement. We calculated absorption of 263 nm light through a plasma profile created by the FASTRAD3D code\(^\text{14,15}\) with Nike laser parameters for the LPI studies. The resulting absorption values were around 50% along the ray trajectories near the quarter critical density region. Hence, the GIR technique with ultraviolet probing was expected to be feasible for the simultaneous measurements of both temperature and electron profiles.

The Nike krypton fluoride (KrF) laser of the Naval Research Laboratory (NRL) is the highest energy krypton-fluoride laser in operation and provides unique environments for laser fusion research with its short wavelength (248 nm), large bandwidth (1–3 THz), and echelon-free induced spatial incoherence (ISI) beam smoothing.\(^\text{16–19}\) In the present paper, we report a new implementation of the GIR system on the Nike KrF laser facility and its application for measuring profiles of density and temperature in a plasma created for LPI research in the NRL KrF laser fusion program. In Sec. II, the arrangement and operation of the Nike-GIR system are described as well as the experimental parameters of the Nike laser for creating the plasma. In an attempt to probe a high density plasma region near the quarter critical density, the GIR optical system was designed using a steep (1/1.8) aspherical lens for the wide-angle collection of the refracted probe rays. In Sec. III, we present measured data, data reduction, and the profiles of density and temperature in the coronal region. The density profile was first constructed under the assumption of weak refraction. This, however, revealed significant discrepancies in the beamlet angles in the high density region close to the target surface. A feedback algorithm was developed to remove this inconsistency and the resulting self-consistent density profile solution is reported. The derived density profile was then used to estimate the temperature profile from the measured transmission data. A summary with future expectations is given in Sec. IV.

II. EXPERIMENT
A. Plasma Production

The plasma was created from a flat polystyrene (CH) target (190 \(\mu\)m thick and 1 mm wide) illuminated by the Nike main beams (\(\lambda = 248 \mu\)m and 0.8 ns FWHM). The individual laser beams were focused on the target surface by the final focusing lenses (f/40) within the 21° \(\times\) 21° square array as shown in Fig. 2(a). The target was mounted to be normal to the center axis of the incident beam lines. We measured the focal intensity profile of the overlapped laser at low laser energy prior to the full energy shot. The measured focal profile was well-fitted with a circular Gaussian distribution with 210 \(\mu\)m FWHM and used to represent the full energy laser profile. The quality of the actual full energy beam overlap was monitored by imaging the plasma with an X-ray pinhole camera (\(h\nu > 1 \text{ keV}\)). The standard Nike pulse shape diagnostics were operated to measure laser pulse shapes from 5 main beams.
We simultaneously operated time-resolved spectrometers for light detection at 300–550 nm and confirmed spectral signatures of TPD and the hybrid mode of TPD and SRS\cite{20,21} radiating from the plasma presented in this paper.

B. System layout

1. Refractometer

We implemented the GIR technique on the Nike target facility during the recent LPI experiment as shown in Fig. 2(a). The main design goal was to reach deep into the corona near the quarter critical density (of $\lambda = 248$ nm) region by collecting the probe light in a wide angle from the plasma. The probe laser of GIR was a short (~10 ps) 4th harmonic ($\lambda = 263$ nm) pulse of a Nd:YLF laser beam (energy ~100 $\mu$J). It was sliced into small beamlets passing through a copper-mesh grid of $51 \times 51$ $\mu$m periodicity with $28 \times 28$ $\mu$m square openings. The beamlet array was relayed by fairly slow ($f/12$) telescope optics onto the target center from the side with 1:1 magnification. The depth of focus of the telescope was measured to be 4 mm at the target. In order to let the beamlets pass through the more inner (higher density) region of the plasma, the beamlet illumination was slightly tilted toward the target surface by 8.7° from the side-on direction. Cartesian coordinates were defined as described in Fig. 2(b) and used throughout this paper.

The probe rays were collected within a cone angle of 32° using a steep ($f/1.8$) aspherical lens and sent to two imaging modules with CCD cameras.\cite{22} The cross-sectional beamlet profiles were imaged from two different object planes near ($x = 0.12$ mm) and off ($x = 1.52$ mm) the target center onto CCD#1 and CCD#2, respectively. The image resolutions were 2.1 and 4.3 $\mu$m/pixel for CCD#1 and #2, respectively. Narrow bandpass filters ($\lambda = 263$ nm, 3 nm FWHM) were positioned in front of the CCD cameras to block self-emission light from the plasma. The obtained CCD images provided apparent positions of the individual beamlets at the two selected object planes. The shift in the apparent positions of each beamlet was divided by the relative distance $\Delta x_{\text{obj}} = 1.40$ mm between the object planes to calculate the deflection angle. As mentioned in Sec. I, GIR requires the relative distance between the object planes for the angle measurement, and thus avoids a potentially significant measurement error when the absolute location of the plasma is not known accurately. In this experiment, physical positions of the object planes were accurately measured but the absolute plasma location might slightly change due to the actual pointing of the Nike main beams (overlapped) varying from shot to shot. The GIR angle measurement does not suffer from this uncertainty in the absolute location of the laser illumination. The accuracy of the relative distance between the object planes was within $\pm 30$ $\mu$m, i.e., smaller than 2% of $\Delta x_{\text{obj}}$. Sample GIR images taken with no target (Fig. 3) demonstrate that the image qualities were high enough for locating individual beamlets at both the object planes. It is notable that the focal depth (4 mm) of the $f/12$ relay telescope optics conveniently encompassed the object planes.

The individual beamlet probes were identified with two dimensional indices $(i, j)$ where $i = 0–14$ and $j = 0–15$ as
sensitivity of the laser cavity, the incident probe laser intensity profile was monitored for every shot as illustrated in Fig. 2(a). The probe laser intensity profile was sampled on the grid right after the probe light slicing and recorded by CCD#3 with 1:2 optical magnification. The recorded profile was then used to calibrate intensities of the emerging probe beamlets out of the plasma. To account for contamination of vacuum optics from previous shots, we obtained a set of reference GIR images with no target shortly before the Nike target shot. The transmission of the \((i,j)\)-th beamlet was then calculated by the following formula:

\[
Tr(i,j) = \frac{I_1(i,j) / I_0(i,j)}{I_1^{\text{preshot}}(i,j) / I_0^{\text{preshot}}(i,j)},
\]

where \(I_0^{\text{preshot}}\) and \(I_1^{\text{preshot}}\) are intensities in the preshot images recorded by CCD#3 and CCD#1, respectively, and \(I_0\) and \(I_1\) intensities in the target shot images taken by CCD#3 and CCD#1, respectively. It should be noted that the self emission contribution to \(I_1\) was pre-excluded for the transmission calculation, see Section III A.

3. Timing fiducial

The relative timing of the GIR probe snapshot and the Nike laser pulse was measured by the optical fiducial diagnostic as shown in Fig. 2(a). Partial intensity of one Nike main beam was picked off the beam line in the Nike propagation bay after the main amplifiers and then optically combined with the probe light sampled before entering the target chamber. The obtained light signal was resolved in time by a fast phototube (60 ps rise time) providing a fiducial timing for the GIR measurement. The measured timing between the two laser pulses (the Nike main beam and the probe light) was used to determine the actual snapshot time with respect to the drive laser pulse arrival on target.

The fiducial timing was pre-calibrated by firing both the Nike main beam at low energy and the GIR probe laser onto a thin UV beam splitter placed at the target center. The two beams were collinearly overlapped by the beam splitter and relayed into the fiducial phototube to measure the relative timing of the two laser pulses on target. The fiducial calibration was then completed by comparing timings of all of the four laser pulse events at the phototube: two delivered from the beam splitter at the target center and the other two from the fiducial diagnostic as described in the previous paragraph.

III. DATA AND RESULTS

A. Background emission subtraction

Our GIR optical system could pick up significant background intensities from thermal radiation of the hot plasma. The thermal light, due to its naturally broad angle radiation, can fill the collection angle more effectively than the highly directional GIR probe light. Moreover, the plasma emission was accumulated on the CCD sensors during the entire plasma evolution (>10 ns) while the probe laser was a snapshot flashing only in 10 ps. The narrow bandpass filters shown in the plasma-free images (Fig. 3). The impact parameters \((y_i, z_j)\) were defined by these unperturbed beamlet locations — the coordinates of the \((i,j)\)-th beamlet at \(x = 0\), i.e., in the \(yz\)-plane containing the center axis of the Nike beams. The target surface at \(x = 0\) was located at the mid-plane between \(z_{11}\) and \(z_{12}\) within \(\pm 15\ \mu m\) uncertainty. The impact parameters are, therefore, given by

\[
y_i, \mu m = -(51 \ \mu m) \times (i - i_0),
\]

\[
z_j, \mu m = \frac{51 \ \mu m}{\cos(8.7^\circ)} \times (j - j_0) \approx -(51 \ \mu m) \times (j - j_0),
\]

where \(i_0\) (=8) and \(j_0\) (=11.5) are the indices of locations of the center axis of the Nike beams and the target surface at \(x = 0\), respectively. It should be noted that when the target was loaded at the shooting position, the probe rays in the columns of \(j = 11–15\) were blocked by the solid volume of the target.

2. Transmission diagnostic

The Nike-GIR system was also equipped with the capability of measuring transmissions of the probe beamlets for the determination of plasma temperature profiles. Since probe laser performance varied from shot to shot due to the thermal
were placed in front of CCD#1 and #2 to block most of the background radiation, but the signal-to-background ratios of the GIR images were still measured to be as low as 1.5 with the available energy (∼100 µJ) of the GIR probe laser pulse. Hence, data reduction started with digital removal of the background. This step was necessary for making meaningful measurements of the beamlet transmissions. It is noted that the deflection angle measurement was insensitive to the detected background.

In contrast to the localized appearance of the beamlets, the background intensity varied smoothly over a fairly large area as shown in Fig. 4(a). Thus, the whole background profile was still well represented by intensities in the beamlet-free region where the localized beamlet areas were excluded. In other words, the missing background intensities in the excluded local areas could be recovered from the slowly changing trend of the background. We numerically separated the background profile via the following steps. First, the beamlet areas were manually excluded. Horizontal lineouts were then taken at selected vertical positions between the beamlet rows and fitted to polynomial functions. By performing spline interpolation along every vertical line across the obtained horizontal polynomials, the complete 2-dimensional background profile was recovered as displayed in Fig. 4(b). Last, the background distribution was subtracted from the original raw GIR image for the background-free image of the beamlets. The background-subtracted GIR images are displayed in Fig. 5.

As shown in Fig. 5(c), the timing fiducial diagnostic detected that the GIR probe laser flashed across the plasma 80 ps earlier than the peak time of the Nike laser pulse. The accompanying time-resolved LPI spectrometers observed signatures of TPD and the hybrid mode from the plasma near the laser peak time including the GIR snapshot time. This confirms that the GIR images presented in this paper were captured through a LPI-hosting plasma as intended to support the LPI research at Nike.

**FIG. 4.** GIR images taken by CCD#1 at the object plane at x = 0.12 mm. The plasma was produced by the laser intensity of 1.1 × 10^{15} W/cm² with the spot size of 210 µm FWHM. The target was mounted along the vertical axis. Dotted lines denote typical locations of the target center line at x = 0. Nike main beams were incident in the horizontal direction from right to left. (a) Raw image conveying both the thermal background and the beamlets refracted through the plasma. (b) Thermal background image digitally separated.

**FIG. 5.** GIR images (background subtracted) of the data shot described in Fig. 4. (a) CCD#1 image taken at the object plane at 0.12 mm from the target center towards the collection lens. (b) CCD#2 image taken at the object plane at 1.52 mm from the target center towards the collection lens. (c) Drive laser pulse shape with the GIR probe laser timing (the vertical line) measured by the fiducial timing diagnostic. The GIR snapshot timing was 80 ps before the laser peak time (t_{peak} = 0.40 ns).
B. Electron density profile

The spatial distribution of the electron density was constructed in three steps: deflection angles of the probe light were extracted from the beamlet locations recorded in the GIR images, optical path differences (OPDs) of the individual beamlets were calculated from the angle data, and then the density profile was built by Abel-inverting the OPDs for the cylindrically symmetric plasma structure. We first performed the inversion process under the weak refraction approximation that has been commonly adopted in optical probing techniques. The obtained density profile, however, was inconsistent with the GIR measurements in the high density region, indicating that the effect of strong refraction had to be taken into account. A new inversion algorithm was then developed to find a self-consistent density profile, as will be described in this section.

The apparent beamlet displacements, \((y', z')\), were defined as the difference between the refracted probe ray positions (see Fig. 5) and the corresponding unperturbed preshot positions (see Fig. 3). Comparing these apparent locations in the two object planes at \(x = 0.12\) mm and 1.52 mm, the deflection angle components \(\theta_y\) and \(\theta_z\) in the \(y\) and \(z\) directions, respectively, were computed by the following formulas:

\[
\theta_y(i, j) = \tan^{-1}\left\{ \frac{y_{1,22}'(i, j) - y_{0,12}'(i, j)}{\Delta x_{\text{obj}}} \right\},
\]

\[
\theta_z(i, j) = \tan^{-1}\left\{ \frac{z_{1,22}'(i, j) - z_{0,12}'(i, j)}{\Delta x_{\text{obj}}} - \tan(8.7^\circ) \right\} + 8.7^\circ,
\]

where 1.52 and 0.12 represent the object planes at \(x = 1.52\) mm and 0.12 mm, respectively, \(i\) and \(j\) are indices of the incident beamlet indicating the \(i\)-th row and the \(j\)-th column (Fig. 3), and \(\Delta x_{\text{obj}}\) the distance between the two object planes (1.40 mm). It is noted that the impact parameters in Fig. 6 showing that the GIR system fulfills the optical path difference (OPDs) of the individual beamlet indicating the expected beamlet reference (OPDs) between the refracted probe ray positions \((9,8,6,3,0)\) and \((130, 180, 280, 440, 590)\) mm, respectively. (a) Deflection angles in \(z\), (b) Deflection angles in \(y\).

Assuming weak refraction \((\theta_y, \theta_z \ll 1\) radian) through the plasma, the deflection angles can be directly related to the optical path difference \((P)\) of the probe ray,

\[
(\theta_y, \theta_z) \approx \left( -\frac{\partial P}{\partial y}, \frac{\partial P}{\partial z} \right),
\]

where \(P = \int [1 - \mu(x, y, z)] ds\) is accumulated along the ray trajectory with the refractive index \(\mu = (1 - n_e/n_{e, 263nm})^{1/2}\). Note that when no plasma exists, the ray angles become \(\theta_y = \theta_z = 0^\circ\). Based on the formulas in Eq. (6), we obtained optical path differences, \(P_{\text{meas}}\) by integrating \(\theta_y\) and \(\theta_z\) in the \(y\) and \(z\) direction along selected pathways starting from a reference position. Beamlets in the outermost region \((z_0 = 590\) \(\mu m)) experienced the weakest refraction \((\theta_z \approx 1^\circ\) and \(\theta_y \approx 0^\circ\), and thus, we selected the \((i_0, 0)\)-th beamlet as the reference for the \(P_{\text{meas}}\) evaluation setting \(P_{\text{meas}}(i_0, 0)\) to zero. Note that \(i_0 = 8\) as defined in Eq. (1). The angle integration was then performed starting from \((y_8, z_0)\) to every beamlet location \((y, z)\) via two pathways: integrating \(\theta_z\) along \(z\) from \((y_8, z_0)\) to \((y, z)\) then \(\theta_y\) along \(y\) towards \((y, z)\) to yield \(P_1(i, j)\), and, if the angle data are available at every point along the pathway, integrating \(\theta_y\) along \(y\) from \((y_8, z_0)\) to \((y, z_0)\) then \(\theta_z\) along \(z\) towards \((y, z)\) to yield \(P_2(i, j)\). Since fewer beamlet points were available for the outer columns due to the circular shape of the viewing area as shown in Fig. 3, the actual pathway for the \(P_2(i, j)\) calculation was established with additional steps: when the circular viewing boundary was encountered during the \(y\)-integration, the \(z\)-integration was performed toward the next inner column then the \(y\)-integration was resumed in the new column. The \(P_{\text{meas}}\) components were evaluated as mean values of \(P_1\) and \(P_2\) and used for the density extraction. Ideally under the weak refraction approximation, the angle integration should be independent of the pathways, i.e., both \(P_1\) and \(P_2\) should be equal. We observed acceptable agreements in the optical path difference calculations: the average value of \(|P_1 - P_{\text{meas}}|/P_{\text{meas}}\) was 0.03 in the high density region where \(z_j \leq 180\) \(\mu m\).

In Fig. 7, the resulting \(P_{\text{meas}}\)’s are plotted against the impact parameters \(y\) and \(z\), showing good cylindrical symmetry with respect to \(y = 0\). The \(P_{\text{meas}}\) values, however, did not fall to a sufficiently low level at the lateral edges \((|y|\)
Results under small angle approximation. (a) Abel-inverted electron densities at impact parameters of $z_{0,8,6,3} = 130, 180, 280,$ and $440 \, \mu m$, respectively. Solid lines are the least-square Gaussian fit curves. The deviations between $P_i(i, j)$ and $P_\text{meas}(i, j)$ are smaller than the symbol sizes.

~400 $\mu m$ of the GIR viewing area, so the lack of measurements in the large $y$ region could cause uncertainty in the inversion process. We circumvented this problem by extrapolating the $P_\text{meas}$ data into the large $y$ region, i.e., fitting the data to a reasonable formula. Hinted by the Gaussian distribution of the Nile laser intensity at the target surface, we adopted a Gaussian form for the fitting. As shown in Fig. 7, the $P_\text{meas}$ data were excellently fitted to the least-squares Gaussian curves with standard deviations smaller than 2% of the peak values. Hence, we assumed that the OPD profiles were Gaussian over a sufficiently large area. The obtained Gaussian formulas of $P_\text{meas}$'s were then used in the inversion process for the density profile extraction.

In order to construct the cross-sectional density profile at a fixed distance $z = z_j$, Abel inversion requires OPD values, $P_i(i, j)$, accumulated along straight line trajectories parallel to the $x$-axis passing the given impact parameter points $(0, y_j, z_j)$. Under the weak refraction approximation, the measured OPDs ($P_\text{meas}$) were considered to be accumulated on the unaltered straight lines (8.7°-tilted) of the incident probe beamlets through the plasma. Assuming the plasma profile was cylindrically symmetric and locally linear in $z$ near the $z = z_j$ plane, one can approximate $P_\text{meas}(i, j)$ to the OPD along the straight line parallel to the $x$-axis, $P_0(i, j) \equiv \int [1 - \mu(x, y, z_j)] dx$, as follows:

$$P_\text{meas}(i, j) \approx \int_{-\infty}^{\infty} \{1 - \mu(x, y, z_j + x \tan(-8.7^\circ))\} ds$$

$$\approx \int_{-\infty}^{\infty} \left[1 - \mu(x, y, z_j)\right] \frac{dx}{\cos(8.7^\circ)} \approx P_0(i, j).$$

(7)

The Abel inversion then reveals the plasma refractive index $\mu(x, y, z)$ from the OPD profile as follows:

$$1 - \mu(r, z = z_j) = -\frac{1}{\pi} \int_0^\infty \frac{dP_0(y, z = z_j)}{dy} \frac{dy}{\sqrt{y^2 - r^2}},$$

(8)

where $r = \sqrt{x^2 + y^2}$ and $P_0(y, z_j)$ is the Gaussian curve from the $P_\text{meas}$ data fitting at the impact parameter $z_j$. Another advantage of inverting the analytical fit curve was the avoided numerical error amplification associated with the differentiation of experimental data. Repeating the inversion process at every $z_j$ parameter, the 3-dimensional density profile $n_e(r, z_j) = n_{\text{ref, 263 nm}}(1 - [\mu(r, z_j)]^2)$ was constructed for the $z$ distances of 130–590 $\mu m$ from the target surface (Fig. 8(a)). The resulting densities ranged from $5 \times 10^{19}$ cm$^{-3}$ up to 3.3 $\times 10^{21}$ cm$^{-3}$ at the symmetry axis, $r = 0$.

Due to the large deflection angles (up to 25° as shown in Fig. 6) observed in this experiment, the validity of the weak refraction approximation was in question. We investigated the self consistency of the obtained density profile with the measured angles to address this question. The ray-tracing method was used to calculate the beamlet angles refracted by the density profile shown in (a).
profile was first established from the Abel-inverted \( n_e \) distribution by bicubic spline interpolation \(^{(21)}\) and then ray trajectories were numerically traced through the density profile for the individual incident probe beamlets. The agreement between the measured deflection angles and the ray-traced ones of the emerging rays from the plasma profile is displayed in Figs. 8(b) and 8(c). Good agreement was observed in the relatively low density region \( (n_e \leq 2 \times 10^{21} \text{ cm}^{-3}) \) as expected from its small deflection angles. Discrepancies were, however, clearly appreciable in the higher density region, suggesting that the weak refraction approximation results in underestimated electron densities.

An iterative numerical algorithm was designed to find a self-consistent density profile that minimizes the deflection angle discrepancies (Fig. 9). Starting with the density profile constructed under the weak refraction approximation, we computed numerical quantities during each iteration stage for every probe beamlet: \( P_0(i,j) \), the OPD integrated along the parallel line to the \( x \)-axis with the impact parameters \( y_i \) and \( z_j \), and \( \theta_y, \theta_z \) for\( (i,j) \), the deflections angles in \( y \) and \( z \), respectively, determined by ray-tracing through the given \( n_e \) profile. \( P_{\text{ray}}(i,j) \) was then calculated by integrating \( \theta_y, \theta_z \) for\( (i,j) \) along the pathways as earlier for the \( P_{\text{meas}}(i,j) \) calculation from the measured deflection angles, see Eq. (6). Differences between \( P_{\text{ray}} \) and \( P_{\text{meas}} \) were then negatively fed back into \( P_0 \) as follows:

\[
P'_0(i,j) = P_0(i,j) - \{ P_{\text{ray}}(i,j) - P_{\text{meas}}(i,j) \}.
\]  

The computed OPD profile \( P'_0(i,j) \) was then Abel-inverted to update the \( n_e \) profile for the next iteration stage. The performance of the algorithm converged to a stable solution fairly rapidly — no significant changes were observed after four iterations.

The results from the 6th iteration are plotted in Fig. 10, indicating that the solution is reasonably consistent with the measured quantities of the experiment. The \( \theta_y \)-plot (Fig. 10(b)) demonstrates that the self-consistency was remarkably improved for the inner beamlets at \( z_8 \) and \( z_9 \) and left relatively unchanged from the weak refraction solution for the outer ones. The \( \theta_y \)-plot (Fig. 10(c)) also shows enhanced agreement with the measured angles. As displayed in Fig. 10(a), the algorithm resulted in increased \( n_e \) values from the ones given by the weak refraction approximation. At the innermost distance \( (z=130 \mu \text{m}) \), the accessible density was as high as \( 4.1 \times 10^{21} \text{ cm}^{-3} \). This density level exhibits the diagnostic capability of the GIR, using the ultraviolet probe light, to examine electron densities up to \( 0.23n_{ec} \) for the Nike laser (\( \lambda \))

FIG. 9. Flow chart of algorithm to find a density profile with self-consistency between the ray-traced beamlet angles \( \theta_y \) and \( \theta_z \) and the measured ones \( \theta_y, \theta_z \). The OPD discrepancy \( P_{\text{ray}}(i,j) - P_{\text{meas}}(i,j) \) is used as the tracking error to converge towards the desired output \( \theta_y, \theta_z \) and \( \theta_{y,z} \).
Electron density profiles along the target normal at the plasma center ($r = 0$). The triangles were obtained from the weak refraction approximation and the circles from the self-consistent solution at the 6th iteration. The solid line is the exponential fit curve to the self-consistent solution (circles). The horizontal dashed line indicates the quarter critical density of the 248 nm wavelength light as shown in Fig. 11. The lateral densities near or less than the quarter critical density of the third harmonic light ($\lambda = 351$ nm).

FIG. 11. Electron density profiles of $n_e (x, 0, z)$ as shown in Fig. 11. The lateral densities near or less than the quarter critical density of the 248 nm wavelength light as shown in Fig. 11. The solid line is the exponential fit curve to the self-consistent solution (circles). The horizontal dashed line indicates the quarter critical density of the 248 nm wavelength light as shown in Fig. 11. The lateral densities near or less than the quarter critical density of the third harmonic light ($\lambda = 351$ nm).

By fitting the density values along the $z$-axis to an exponential form, $n_e(z) = n_{e0} \exp(-z/L_n)$, the density scale-length ($L_n$) was estimated to be 120 $\mu$m over the broad region of densities near or less than the quarter critical density of the 248 nm wavelength light as shown in Fig. 11. The lateral profiles of $n_e$ in the $xy$ planes at $z = z_j$ were virtually Gaussian in $r$ with the full width of $\sim 600$ $\mu$m at the half maximum.

C. Electron temperature measurement

In addition to the experimental determination of the electron density profile, the GIR method inherently provides a diagnostic way to evaluate plasma temperatures from its capability of the simultaneous measurement of the probe beamlet transmissions. Assuming that the probe light was collisionally absorbed with free electrons in the corona of the plasma, one can derive a formula to estimate the electron temperature out of the measured ray transmission (or absorption) data.

For non-magnetized plasma, the collisional absorption coefficient $K$ is given by:

$$K = \frac{\nu_e}{c} \frac{n_e}{n_{e0} 263 \text{ nm} \mu}, \quad (10)$$

where $\nu_e$ is the electron collision frequency, $c$ is the light speed, and $\mu$ is the local refractive index in the plasma as mentioned earlier. The collision frequency for the fully ionized plasma has the form:

$$\nu_e = \frac{4\sqrt{2\pi}}{3} \frac{n_e Z_i e^4 l n \Lambda_e}{m_e^{0.5} (k_B T_e)^{3/2}}, \quad (11)$$

where $Z_i$ is the average ion charge, $m_e$ the electron mass, $k_B$ the Boltzmann constant, $e$ the electron charge, $T_e$ the electron temperature, and $ln \Lambda_e$ the Coulomb logarithm of collisions between electrons. For $k_B T_e \geq 10$ eV, we used:

$$ln\Lambda_e = 24.4 - ln(n_{e0}^{0.5}/n_{e0}^{0.5}/T_e, eV), \quad (12)$$

with $n_e$ in cm$^{-3}$ and $T_e$ in eV. The ray transmission $Tr(y, z)$ can then be calculated by integrating Eq. (10) in $x$,

$$Tr(y, z) = e^{-\int_{-\infty}^{\infty} K(x, y, z) dx} = e^\frac{-2\int_{-\infty}^{\infty} \frac{K(r, z)}{\sqrt{r^2 - y^2}} dr}{2}. \quad (13)$$

Rewriting Eq. (13) as

$$ln[1/Tr(y, z_j)] = 2 \int_{|y|}^{\infty} \frac{K(r, z_j)}{\sqrt{r^2 - y^2}} dy, \quad (14)$$

we get the Abel-inverted formula for $K$,

$$K(r, z_j) = \frac{1}{\pi} \int_{r}^{\infty} \frac{dTr(y, z_j)}{Tr(y, z_j)} \frac{dy}{\sqrt{y^2 - r^2}}, \quad (15)$$

Since the $n_e$ profile was given by the angle measurement and $K$ is a function of $n_e$ and $T_e$, one may use Eq. (15) to extract the $T_e$ profile out of the transmission data via Abel inversion.

The beamlet transmissions were measured at $x = 0.12$ nm by Eq. (3) as plotted in Fig. 12(a). In the outer region ($z_j \geq 390 \mu$m) where the peak densities were lower than $n_e = 248$ nm or $0.46n_{e0}$ for the third harmonic laser ($\lambda = 351$ nm) of other laser facilities in the inertial confinement fusion research.

FIG. 12. Probe light transmissions measured by the GIR image taken at $x = 0.12$ nm. (a) Beamlet transmission data plotted with respect to $y_j$ impact parameters. The triangles represent beamlets collected at $z_j \geq 390 \mu$m, the circles at $z_8 = 280 \mu$m, the crosses at $z_j = 230 \mu$m, and the diamonds at $z_8 = 180 \mu$m. No transmission data are available at $z_8 = 130 \mu$m due to the strong refraction effect. (b) Best fits to the transmission data at $z_8 = 280 \mu$m (circles). The dashed line is the transmission fit with a constant temperature profile ($T = 500$ eV), the dotted line the best transmission fit with a Gaussian temperature profile ($T_{peak} = 1380$ eV with $400 \mu$m FWHM), and the solid line the corresponding Gaussian temperature.
4 × 10^{20} \text{ cm}^{-3}, the plasma was observed to be quite transparent (attenuated less than 10\%) to the UV probe light and the temperature extraction was thus not feasible for the poor signal-to-noise ratio of the absorption. In the inner region \((z_j \leq 340 \mu m)\) where electron densities were higher, the transmission values (25\%–80\%) became more favorable for the temperature estimate, but available data were limited by the beamlet overlaps caused by the strong refraction. The number of the overlapped beamlets might be reduced if the object plane was taken at a slightly negative \(x\) position away from the collection lens.\(^{26}\) Ray-tracing the incident beamlets through the density profile obtained in Sec. III B, we found that most beamlet overlaps were avoided if the object plane was chosen at \(x = -(0.1–0.3) \text{ mm}\) instead of \(x = 0.12 \text{ mm}\). This will be tested in future experiments at Nike. In this paper, we analyzed the transmission data collected from non-overlapped beamlets at \(z_5 = 280 \mu m\) for the temperature investigation.

The obtained transmission data appeared to be insufficient to meet the requirement for the Abel inversion process. The absorption — or \(ln(1/Tr)\) in Eq. (14) — did not decrease enough within the detection area and, different from the refraction angle case, it was difficult to find an acceptable data trend into the far region (\(|y| > 400 \mu m\)). Hence, instead of seeking a temperature profile via Abel inversion, we took a reverse approach: we assumed a \(T_e\) profile, calculated transmissions of the probe rays through the measured density profile using Eq. (10)–(13), and then compared them with the measured transmission values. The first trial was made with a constant temperature profile. We repeated the above steps for various \(T_e\) values ranging from 300 eV to 3500 eV in search of a reasonable match to the measured data. The resulting transmission profiles, however, were too narrow to represent the measured transmission data as plotted in Fig. 12(b). This implies that the temperature should generally decrease as the \(|y|\)-value increases, since transmission decreases with temperature as seen in Eq. (10). One plausible candidate for the temperature shape was the Gaussian form given by

\[
T_e(r, z_j) = T_{e0}e^{-r^2/(0.5\Delta_T)^2}ln(2),
\]

where \(T_{e0}\) and \(\Delta_T\) are the peak temperature at the center and the full width at the half maximum, respectively. We again repeated the steps for various \(T_{e0}\)'s (300–3000 eV with a 10 eV step) and \(\Delta_T\)'s (200–1000 \(\mu m\) with a 10 \(\mu m\) step). The best fit occurred with the temperature profile of \(T_{e0} = 1380\) eV and \(\Delta_T = 400\) \(\mu m\) showing reasonable agreement with the measured transmission data as plotted in Fig. 12(b).

### IV. SUMMARY AND FUTURE PROSPECTS

We added a GIR diagnostic suite to the Nike target facility and measured deflection angles and transmissions of ultraviolet (\(\lambda = 263\) nm) probe rays traveling through a plasma produced for laser fusion research. The GIR system demonstrated the capability of diagnosing the probe rays refracted over a wide range of angles (0°–25°) as designed to measure plasma parameters in the underdense coronal region of interest.

The measured deflection angle profiles were processed to extract electron densities. The density profile obtained under the typical assumption of weak refraction, however, showed significant discrepancies between the computed ray trajectories and the measured ray angles in the inner region where strong refraction should take place. We thus developed a numerical algorithm to construct density profiles in which the probe ray trajectories were iteratively modified with the measured deflection angles. This approach was realizable with the GIR capability of measuring individual probe ray angles with known impact parameters. The resulting 3-dimensional density profile exhibited strong self-consistency with the GIR observations. We report that electron densities were accessible up to \(4.1 \times 10^{21} \text{ cm}^{-3}\) which is equivalent to 0.23\(n_e\) and 0.46\(n_e\) of \(\lambda_{laser} = 248\) nm and 351 nm, respectively, and the density scale length was 120 \(\mu m\) along the center axis of the Nike laser in the coronal plasma.

The simultaneously measured ray transmission data were also examined to estimate electron temperatures. We found that a Gaussian temperature profile with \(T_{peak} = 1.4\) keV and \(400\) \(\mu m\) FWHM was acceptable to represent the transmission data measured at the distance of 280 \(\mu m\) from the target surface where the peak electron density was \(1.4 \times 10^{21} \text{ cm}^{-3}\). The Abel inversion, however, was not applicable to the temperature profile extraction due to the insufficient data limited by the size of the GIR detection area (diameter of 0.8 mm) on the target. The next experiment will be performed with a wider viewing area of the GIR (diameter of 1.6 mm) for more thorough experimental determination of the spatial profiles of the plasma parameters.

We are also planning an extension to a shorter wavelength ultraviolet probe for the GIR measurements adopting a fifth harmonic (\(\lambda = 213\) nm) generator for the Nd:YAG laser. The ray-trace calculation was performed to investigate propagation of the 213 nm probe light through the plasma density profile obtained in this paper. The computed ray trajectories suggested that the fifth harmonic light might provide insight into a deeper coronal region with densities ~20% higher than the ones reached by the 263 nm probe light. The new probing system will be implemented for the next GIR experiments at Nike.

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### APPENDIX: ERROR ANALYSIS

For the cylindrically symmetric plasmas varying smoothly over the grid periodicity, major uncertainties in extraction of the plasma profiles arise from the measurement errors of refraction angles and transmissions and their propagation through the inversion processes. To estimate the uncertainties with the given measurement errors, we computed the
propagation errors using the self-consistent solutions of density and temperature obtained in this paper.

The accuracy of the refraction angle measurement was limited by the uncertainty in the relative distance $\Delta x_{obj}$ between the GIR object planes and the image resolutions of CCD#1 and #2. Hence, the errors in the measured angles can be given by

$$\Delta \theta_{y,z} = X \pm 0.02 \theta_{y,z},$$  \hspace{1cm} (A1)

where $X$ is a random variable distributed between $\pm0.25^\circ$ representing the accuracy due to the CCD image resolutions and $0.02 \theta_{y,z}$, the error contribution of $\Delta x_{obj}$ to the angles. To analyze the error propagation in the density extraction, we simulated the experiment by ray-tracing the probe beamlets through the 3-dimensional self-consistent density profile to compute refracted beamlet angles (Fig. 10), putting the measurement uncertainties on the calculated angles as shown in Eq. (A1), and then applying the inversion algorithm to these refraction angle data (Fig. 9). The resulting density profiles illustrated that the measurement errors propagated through the inversion process yielding 3% errors in densities in the inner region near the quarter critical density.

The uncertainties in the obtained density profile and the transmission measurement are two sources of error in the temperature determination as exhibited in Eqs. (10)–(13). Due to the lack of the 3-dimensional solution for the temperature profile in the current work, the temperature uncertainty was investigated in a plane parallel to the target surface using the 2-dimensional profiles of density and temperature obtained at $z = z_0$, see Figs. 10(a) and 12(b). The initial transmission data were calculated for side-on probe rays through these profiles. Here, the probe rays were distributed between $y = -800$ and $800 \mu m$ with $50 \mu m$ separation for the next GIR experiment planned with the wider viewing area. Both the density profile and the computed transmission values were then equipped with errors: the 3% density uncertainty and an appropriate transmission measurement error, respectively. This 1-dimensional array of transmission data was processed by the classical Abel-inversion given in Eq. (15) and then temperatures were calculated from the reconstructed absorption coefficients and the density profile with 3% error. Last, comparison between the resulting temperature profiles and the input temperature data was made to estimate the possible accuracy of the temperature measurement. Assuming no error in the transmission measurement, the contribution of the density uncertainty to the temperature profile was computed to be 4%. Using the transmission data collected in the outer region at $z < z_0 \geq 390 \mu m$ where the plasma was fairly transparent as plotted in Fig. 12(a), we estimated the transmission measurement error to be about 6%. This measurement error was randomly distributed in the computed transmission data and then propagated through the inversion process for the temperature extraction. The reconstructed profiles showed that 10% uncertainty in the temperature measurement was caused by the errors of 6% in transmission and 3% in density.

One application of the GIR measurements is to study the thresholds of laser plasma instabilities. For example, the theoretical threshold of TPD instability in inhomogeneous plasma was predicted with $T_e$ and $L_n$ values near the critical density region as follows: \cite{86}

$$I_{th} \propto \frac{T_e}{L \lambda_{laser} (1 - \alpha)},$$  \hspace{1cm} (A2)

where $I_{th}$ is the threshold laser intensity in vacuum, $\lambda_{laser}$ the wavelength of the incident laser in vacuum, and $\alpha$ the absorption ratio of the laser intensity through the incident pathway to the quarter critical density region. With the current errors in measuring density and temperature profiles, the uncertainty in the predicted threshold intensity was estimated to be 12%. This is smaller than the measurement errors in the incident laser intensity of the previous LPI experiment at the Nike laser facility,\cite{11} indicating that the theoretical prediction can be examined within the uncertainty imposed by the LPI experiment.

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The optical quality of the GIR system was numerically investigated by tracing probe rays through the f/1.8 collection lens (spherical or aspherical) along the GIR imaging lines. The results suggested that use of the aspherical collection should be necessary for reducing optical aberration which could cause strong image distortion if an f/1.8 spherical optic was used instead.

The ray trajectories were computed by solving the eikonal equation in the density profile using the fourth-order Runge-Kutta technique.

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