Generating Adjacency-Constrained Subgoals in Hierarchical Reinforcement Learning

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Abstract

Goal-conditioned hierarchical reinforcement learning (HRL) is a promising approach for scaling up reinforcement learning (RL) techniques. However, it often suffers from training inefficiency as the action space of the high-level, i.e., the goal space, is often large. Searching in a large goal space poses difficulties for both high-level subgoal generation and low-level policy learning. In this paper, we show that this problem can be effectively alleviated by restricting the high-level action space from the whole goal space to a \(k\)-step adjacency region centered by the current state using an adjacency constraint. We theoretically prove that the proposed adjacency constraint preserves the optimal hierarchical policy, and show that this constraint can be practically implemented by training an adjacency network that can discriminate between adjacent and non-adjacent subgoals. Experimental results on discrete and continuous control tasks show that our method outperforms the state-of-the-art HRL approaches.

1 Introduction

Hierarchical reinforcement learning (HRL) has shown great potentials in scaling up reinforcement learning (RL) methods to tackle large, temporally extended problems with long-term credit assignment and sparse rewards [31, 24, 2]. As one of the prevailing HRL paradigms, goal-conditioned HRL
framework [3, 30, 14, 34, 20, 16], which comprises a high-level policy that breaks the original task into a series of subgoals and a low-level policy that aims to reach those subgoals, has recently achieved significant success. However, the effectiveness of goal-conditioned HRL relies on the acquisition of effective and semantically meaningful subgoals, which stills remains a key challenge.

As the subgoals can be interpreted as high-level actions, it is feasible to directly train the high-level policy to generate subgoals using the external rewards as supervision, which has been widely adopted by previous researches [20, 19, 16, 14, 34]. However, although these methods require little task-specific design, they often suffer from training inefficiency. This is because the action space of the high-level, i.e., the goal space, is often as large as the state space. The high-level exploration in such a large action space results in inefficient learning. As a consequence, the low-level training also suffers as the agent tries to reach every possible subgoal produced by the high-level policy.

One effective solution for handling large action spaces is action space reduction or action elimination. However, it is difficult to perform action space reduction in general scenarios without additional information, since there is no guarantee that a restricted action set can still be expressive enough to form the optimal policy. There have been limited literature [35, 33] studying action space reduction in RL, and to our knowledge, there is no prior work studying action space reduction in the setting of HRL, since the information loss in the goal space can lead to severe performance degradation [19].

In this paper, we present an optimality-preserving high-level action space reduction method for goal-conditioned HRL. Concretely, we show that the high-level action space can be restricted from the whole goal space to a $k$-step adjacent region centered by the current state. Our main intuition is depicted in Figure 1: distant subgoals can be substituted by closer subgoals, as long as they drive the low-level to move towards the same “direction”. Therefore, given the current state $s$ and the subgoal generation frequency $k$, the high-level only needs to explore in a subset of subgoals covering states that the low-level can possibly reach within $k$ steps, instead of exploring in the whole goal space. By reducing the action space of the higher hierarchy, the learning efficiency of both the high-level and the low-level can be improved: for the high-level, a considerably smaller action space relieves the burden of exploration and value function approximation; for the low-level, adjacent subgoals provide a stronger learning signal as the agent can be intrinsically rewarded with a higher frequency for reaching these subgoals. Formally, we introduce a $k$-step adjacency constraint for high-level action space reduction, and theoretically prove that the proposed adjacency constraint preserves the optimal hierarchical policy. Also, to practically implement the constraint, we propose to train an adjacency network so that the $k$-step adjacency between all states and subgoals can be succinctly derived.

We benchmark our method on various tasks, including discrete control and planning tasks on grid worlds and challenging continuous control tasks based on the MuJoCo simulator [32], which have
While the high-level controller is motivated by the environmental reward, the low-level controller has
been widely used in HRL literature \cite{20, 16, 19, 7}. Experimental results exhibit the superiority of our
method on both sample efficiency and asymptotic performance compared with the state-of-the-art
HRL approach HIRO \cite{20}, demonstrating the effectiveness of the proposed adjacency constraint.

2 Preliminaries

We consider a finite-horizon, goal-conditioned Markov Decision Process (MDP) defined as a tuple
\((\mathcal{S}, \mathcal{G}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma)\), where \(\mathcal{S}\) is a state set, \(\mathcal{G}\) is a goal set, \(\mathcal{A}\) is an action set, \(\mathcal{P} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \mapsto \mathbb{R}\) is a state transition function, \(\mathcal{R} : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}\) is a reward function, and \(\gamma \in [0, 1]\) is a discount factor.

Following prior work \cite{14, 34, 20}, we consider a framework comprising two hierarchies: a high-level
counter with policy \(\pi^h_l(g|s)\) and a low-level controller with policy \(\pi^l_h(a|s, g)\) parametrized by
two function approximators, e.g. neural networks with parameters \(\theta_h\) and \(\theta_l\) respectively, as shown
in Figure 3. The high-level controller aims to maximize the external reward and generates a high-
level action, i.e. a subgoal \(g_t \sim \pi^h_l(g|s_t) \in \mathcal{G}\) every \(k\) time steps when \(t \equiv 0\) (mod \(k\)), where
\(k > 1\) is a pre-determined hyperparameter. It modulates the behavior of the low-level policy by
intrinsically rewarding the low-level for reaching these subgoals. The low-level aims to maximize the
intrinsic reward provided by the high-level, and performs a primary action \(a_t \sim \pi^l_h(a|s_t, g_t) \in \mathcal{A}\)
at every time step. Following \cite{20, 1}, we consider a goal space \(\mathcal{G}\) which is a sub-space of \(\mathcal{S}\) with a
known mapping function \(\varphi : \mathcal{S} \mapsto \mathcal{G}\). When \(t \not\equiv 0\) (mod \(k\)), a pre-defined goal transition process
\(g_t = h(g_{t-1}, s_{t-1}, s_t)\) is utilized.

Following \cite{34, 20}, we adopt directional subgoals that represent the differences between desired states
and current states, where the goal transition function is set to \(h(g_{t-1}, s_{t-1}, s_t) = g_{t-1} + s_{t-1} - s_t\).
The reward function of the high-level policy is defined as:

\[
r^h_{kt} = \sum_{i=kt}^{kt+k-1} R(r|s_i, a_i), \quad t = 0, 1, 2, \ldots,
\]

which is the accumulation of the external reward in the time interval \([kt, kt + k - 1]\).

While the high-level controller is motivated by the environmental reward, the low-level controller has
no direct access to this external reward. Instead, the low-level is supervised by the intrinsic reward
that describes subgoal-reaching performance, defined as \(r^l_t = -D(g_t, \varphi(s_{t+1}))\), where \(D\) is a binary
or continuous distance function. In practice, we employ Euclidean distance as \(D\).

The goal-conditioned HRL framework above enables us to train high-level and low-level policies
concurrently in an end-to-end fashion. However, it often suffers from training inefficiency due to the
unconstrained subgoal generation process, as we have mentioned in Section 1. In the following
section, we will introduce the \(k\)-step adjacency constraint to remedy this issue.

3 Problem Formulation

In this section, we provide our theoretical results. Complete proofs of theorems are in the supple-
mentary material. We begin by introducing a distance measure that is used to decide whether a state
is “close” to another state. In this regard, common distance functions such as the Euclidean distance
are not suitable, as they often cannot reveal the real structure of the MDP. Therefore, we introduce
shortest transition distance, which equals to the minimum number of steps required to reach a target
state from a start state, as shown in Figure 2. In stochastic MDPs, the number of steps required is not
a fixed number, but a distribution conditioned on a specific policy. In this case, we resort to the notion
of hitting time (or first hit time) from stochastic processes. More formally, we define the shortest
transition distance by minimizing the expected hitting time over all possible policies.

**Definition 1.** Let \(s_1, s_2 \in \mathcal{S}\) and assume that \(s_2\) is reachable from \(s_1\). Then, the shortest transition
distance between \(s_1\) and \(s_2\), denoted as \(d_{xt}(s_1, s_2)\), is defined as:

\[
d_{xt}(s_1, s_2) \triangleq \min_{\pi \in \Pi} E[\mathcal{T}_{s_1, s_2} | \pi] = \min_{\pi \in \Pi} \sum_{t=0}^{\infty} tP(\mathcal{T}_{s_1, s_2} = t | \pi),
\]

where \(\Pi\) is the complete policy set and \(\mathcal{T}_{s_1, s_2}\) denotes the first hit time from \(s_1\) to \(s_2\).
The shortest transition distance is determined by a policy that connects states $s_1$ and $s_2$ in the most efficient way, which has also been studied in [6, 5]. This policy is optimal in the sense that it requires the minimum number of steps to reach state $s_2$ from state $s_1$. Compared with the dynamical distance [11], our definition here does not rely on a specific non-optimal policy. Also, we do not assume that the environment is reversible, i.e. $d_{st}(s_1, s_2) = d_{st}(s_2, s_1)$ does not hold for all pairs of states. Therefore, the shortest transition distance is a quasi-metric as it does not satisfy the symmetry condition. However, this limitation does not affect the following analysis as we only need to consider the transition from the start state to the goal state without the reversed transition.

Given the definition of the shortest transition distance, we now formulate the property of an optimal goal-conditioned policy $\pi^* : S \times G \mapsto A$ [29]. We have:

$$\pi^*(s, g) \in \arg \min_{a \in A} \sum_{s' \in S} P(s'|s, a) d_{st}(s', \varphi^{-1}(g)), \forall s \in S, g \in G. \tag{3}$$

We then consider the goal-conditioned HRL framework with high-level action frequency $k$. Compared to a flat goal-conditioned policy, in this setting the low-level policy is required to reach the subgoals within $k$ steps. The main difference is that within limited steps, only a subset of the original states can be reliably reached even with an optimal goal-conditioned policy. We introduce the notion of $k$-step adjacent region to describe this reachable subset of states.

**Definition 2.** Let $s \in S$. Then, its $k$-step adjacent region, denoted as $G_A(s, k)$, is defined as:

$$G_A(s, k) \triangleq \{g \in G | d_{st}(s, \varphi^{-1}(g)) \leq k\}. \tag{4}$$

Harnessing the property of $\pi^*$, we can prove that in deterministic MDPs, given an optimal low-level policy $\pi^l = \pi^*$, subgoals that fall in the $k$-step adjacency region of the current state can represent all optimal subgoals in the whole goal space in terms of the induced $k$-step low-level action sequence.

**Theorem 1.** Let $s \in S$, $g \in G$ and let $\pi^*$ be an optimal goal-conditioned policy. Under the assumptions that the MDP is deterministic and that the MDP states are strongly connected, for all $k \in \mathbb{N}_+$ satisfying $k \leq d_{st}(s, \varphi^{-1}(g))$, there exists a surrogate goal $\tilde{g}$ such that:

$$\tilde{g} \in G_A(s, k), \quad \pi^*(s|s_i, \tilde{g}) = \pi^*(s|s_i, g), \forall s_i \in \{\tau, i \neq k\}, \tag{5}$$

where $\tau := (s_0, s_1, \cdots, s_k)$ is the $k$-step state trajectory starting from state $s_0 = s$ under policy $\pi^*$.

Theorem 1 suggests that the $k$-step low-level action sequence output by an optimal low-level policy conditioned on a distant subgoal can be induced using a subgoal that is closer. Naturally, we can scale this result to a two-level goal-conditioned HRL framework, where the low-level is actuated not by a single subgoal, but by a subgoal sequence produced by the high-level policy.

**Theorem 2.** Given the high-level action frequency $k$ and high-level planning horizon $T$, for $s \in S$, let $\rho^* = (g_0, g_k, \cdots, g_{(T-1)k})$ be the high-level subgoal trajectory starting from state $s_0 = s$ under an optimal high-level policy $\pi^{h*}$. Also, let $\tau^* = (s_0, s_k, s_{2k}, \cdots, s_{Tk})$ be the high-level state trajectory under $\rho^*$ and an optimal low-level policy $\pi^l$. Then, there exists a surrogate subgoal trajectory $\tilde{\rho}^* = (\tilde{g}_0, \tilde{g}_k, \cdots, \tilde{g}_{(T-1)k})$ such that:

$$\tilde{g}_{kt} \in G_A(s_{kt}, k), \quad Q^*(s_{kt}, \tilde{g}_{kt}) = Q^*(s_{kt}, g_{kt}), \quad t = 0, 1, \cdots, T - 1, \tag{6}$$

where $Q^*$ is the optimal high-level Q-function under policy $\pi^{h*}$.

Theorem 1 and 2 show that we can constrain the high-level action space to $k$-step adjacent regions without the loss of optimality. We formulate the high-level objective with this $k$-step adjacency constraint as:

$$\max_{\theta_h} \mathbb{E}_{\pi^h_{\theta_h}} \sum_{t=0}^{T-1} \gamma^t r^h_{kt}, \tag{7}$$

subject to $d_{st}(s_{kt}, \varphi^{-1}(g_{kt})) \leq k, \quad t = 0, 1, \cdots, T - 1$

where $r^h$ is the high-level reward defined by Equation (1), $\varphi^{-1} : G \mapsto S$ is the known inverse mapping of $\varphi$, and $g_{kt} \sim \pi^h_{\theta_h}(g|s_{kt})$. 


In practice, Equation (7) is hard to optimize due to the strict constraint. Therefore, we employ relaxation methods and derive the following unconstrained optimizing objective:

$$\max_{\theta_h} \mathbb{E}_{\pi_h} \sum_{t=0}^{T-1} \left[ r^{k_t}_h - \eta \cdot H \left( d_{st} \left( s^{k_t}_h, \varphi^{-1}(s^{t}_h) \right) \right) \right],$$

(8)

where $H(x) = \max(x/k - 1, 0)$ is a hinge loss function and $\eta$ is a balancing coefficient.

One limitation of the above formulation is the assumption that the MDP is deterministic. However, we note that many real-world applications can be approximated as environments with deterministic dynamics, and the stochasticity is mainly induced by the noise. It is thus natural to infer that the adjacency constraint preserves a near-optimal policy when the magnitude of the noise is small. We empirically show that our method still benefits learning when the stochasticity of the environment is limited in this condition, and leave rigorous theoretical analysis for future work.

4 HRL with Adjacency Constraint

Although we have formulated the adjacency constraint in Section 3, the exact calculation of the shortest transition distance $d_{st}(s_1, s_2)$ between two arbitrary states $s_1, s_2 \in \mathcal{S}$ remains complex and non-differentiable. In this section, we introduce a simple method to collect and aggregate the adjacency information from the interactions gathered by different policies. We then train an adjacency network using the aggregated information to approximate the shortest transition distance $d_{st}(s_1, s_2)$ in a parametrized form and enable practical optimization of Equation (8).

4.1 Parametrized Approximation of Shortest Transition Distances

As shown by prior research [23, 6, 5, 11], accurately computing the shortest transition distance is hard and often has the same complexity as learning an optimal low-level goal-conditioned policy. However, from the perspective of goal-conditioned HRL, we do not need a perfect shortest transition distance measure or a low-level policy that can reach any distant subgoals. Instead, only a discriminator of $k$-step adjacency is needed, and it is enough to learn a low-level policy that can reliably reach nearby subgoals (more accurately, subgoals that fall into the $k$-step adjacent region of the current state) rather than all potential subgoals in the goal space, which is generally much harder.

Given the demand above, here we introduce a simple approach to decide whether a subgoal satisfies the $k$-step adjacency constraint. We first note that Equation (2) can be approximated as follows:

$$d_{st}(s_1, s_2) \approx \min_{\pi \in \{\pi_1, \pi_2, \ldots, \pi_n\}} \sum_{t=0}^{\infty} t P(T_{s_1, s_2} = t | \pi),$$

(9)

where $\{\pi_1, \pi_2, \ldots, \pi_n\}$ is a finite policy set containing $n$ different deterministic policies. Obviously, if these policies are diverse enough, we can effectively approximate the shortest transition distance with a sufficiently large $n$. However, training a set of diverse policies separately is costly, while using one single policy to approximate the policy set ($n = 1$) [27, 28] often leads to non-optimality. To handle this difficulty, we exploit the fact that the low-level policy itself changes over time during the training procedure. We can thus build a policy set by sampling policies that emerges in different training stages. To aggregate the adjacency information gathered by multiple policies, we propose to explicitly memorize the adjacency information by constructing a binary $k$-step adjacent matrix of the explored states. The adjacency matrix has a same size as the number of explored states, and each element represents whether two states corresponding to the specific row and column are $k$-step adjacent. In practice, we use the agent’s trajectories, where the temporal distances between states can indicate their adjacency, to construct and update the adjacency matrix in a fixed frequency. More details are in the supplementary material.

In practice, using an adjacent matrix is not enough as this procedure is non-differentiable and cannot generalize to newly-visited states. To
We have presented our method of Hierarchical Reinforcement Learning with a learned adjacency network $\psi$. We consider possible extensions in this direction as our future work. Although the construction of an adjacent matrix limits our method to tasks with finite states, we can also handle continuous state spaces using goal space discretization (see our continuous control experiments in Section 5). For applications with vast state spaces, constructing a complete adjacent matrix will be problematic, but it is still possible to scale our method to these scenarios using specific feature construction or dimension reduction methods [21, 22, 4], or substituting the distance learning procedure with more accurate distance learning algorithms [6, 5] at the cost of learning complexity.

### 4.2 Combining HRL and Adjacency Constraint

With a learned adjacency network $\psi$, we can now incorporate the adjacency constraint into the goal-conditioned HRL framework. According to Equation (8), we introduce an adjacency loss $L_{\text{adj}}$ to replace the original strict adjacency constraint and minimize the following high-level objective:

$$L_{\text{high}}(\theta_h) = -\mathbb{E}_{\pi_h^0} \sum_{t=0}^{T-1} [\gamma^t r_{kt}^h - \eta \cdot L_{\text{adj}}], \quad (12)$$

where $L_{\text{adj}}$ is derived by replacing $d_{st}$ with $\tilde{d}_{st}$ defined by Equation (10) in the second term of (8):

$$L_{\text{adj}}(\theta_h) = H(\tilde{d}_{st}(s_{kt}, \varphi^{-1}(g_{kt})|\phi)) = \max (||\psi_{\theta}(\varphi(s_{kt})) - \psi_{\theta}(g_{kt})||_2 - \epsilon_k, 0), \quad (13)$$

where $g_{kt} \sim \pi_h^0(g|s_{kt})$. Equation (13) will output a non-zero value when the generated subgoal and the current state have an Euclidean distance larger than $\epsilon_k$ in the adjacency space, indicating non-adjacency. It is thus consistent with the $k$-step adjacency constraint.

### 5 Experimental Evaluation

We have presented our method of Hierarchical Reinforcement learning with $k$-step Adjacency Constraint (HRAC). Our experiments are designed to answer the following questions: (1) Can $k$-step adjacency constraint promote the generation of adjacent subgoals? (2) Can HRAC improve the sample efficiency and overall performance of goal-conditioned HRL? (3) Can HRAC outperform other strategies that also aim to improve learning efficiency of HRL, e.g., the hindsight technique?
5.1 Environment Setup

We employed two types of tasks with discrete and continuous state and action spaces to evaluate the effectiveness of our method, as shown in Figure 5. Discrete tasks include Key-Chest and Maze, where the agents are spawned in grid worlds with injected stochasticity and need to accomplish tasks that require both low-level control and high-level planning. Continuous tasks include Point Gather, Ant Maze and Point Maze Sparse, where the first two tasks are widely-used benchmarks in HRL community [7, 20, 19, 16], and the third task is a more challenging navigation task with sparse reward. In all tasks, we used a pre-defined 2-dimensional goal space that represents the $x, y$ position of the agent. More details of the environments are in the supplementary material.

5.2 Comparative Experiments

To comprehensively evaluate the performance of HRAC with different HRL implementations, we employed two different HRL instances for different tasks. On discrete tasks, we used off-policy TD3 [9] for high-level training and on-policy A2C, the synchronous version of A3C [18], for the low-level. On continuous tasks, we used TD3 for both the high-level and the low-level training, following prior work [20], and discretized the goal space to $1 \times 1$ grids for adjacency learning.

We compared HRAC with the following baselines. (1) HIRO [20]: one of the state-of-the-art goal-conditioned HRL approaches. (2) HIRO-B: A baseline analogous to HIRO, using binary intrinsic reward for subgoal reaching instead of shaped reward. (3) HRL-HER: A baseline that employs hindsight experience replay (HER) [11] to produce alternative successful subgoal-reaching experiences as complementary low-level learning signals [16]. (4) Vanilla: Kulkarni et al. [14] use absolute subgoals instead of directional subgoals and adopt a binary intrinsic reward setting. For fair comparison, all baselines used the same hierarchical architecture as HRAC. More details of the baselines are in the supplementary material.

The learning curves of HRAC and baselines across all tasks are plotted in Figure 6. In the Maze task with dense reward, HRAC achieves comparable performance with HIRO and outperforms other baselines, while in other tasks HRAC consistently surpasses all baselines both in sample efficiency and asymptotic performance. We note that the performance of the baseline HRL-HER matches...
5.3 Ablation Study and Visualizations

We also compared HRAC with several variants to investigate the effectiveness of each component. (1) **HRAC-O**: An oracle variant that uses a perfect adjacent matrix directly obtained from the environment. We note that compared to other methods, this variant uses the additional information that is not available in many applications. (2) **NoAdj**: A variant that uses an adjacency training method analogous to [27, 28], where no adjacent matrix is maintained. The adjacency network is trained using states directly sampled from stored trajectories, under the same training budget as HRAC. (3) **NegReward**: This variant implements the $k$-step adjacency constraint by penalizing the high-level with a negative reward when it generates non-adjacent subgoals, which is used by HAC [16].

We provide learning curves of HRAC and these variants in Figure 7. In all tasks, HRAC yields similar performance with the oracle variant HRAC-O while surpassing the NoAdj variant by a large margin, exhibiting the effectiveness of our adjacency learning method. Meanwhile, HRAC achieves better performance than the NegReward baseline, suggesting the superiority of implementing the adjacency constraint using a differentiable adjacency loss, which provides stronger supervision than a penalty. We also empirically studied the effect of different balancing coefficient $\eta$. Results are shown in Figure 8 which suggest that generally a large $\eta$ can lead to better and more stable performance.

Finally, we visualize the subgoals generated by the high-level policy and the adjacency heatmap in Figure 9. Visualizations indicate that the agent does learn to generate adjacent and interpretable subgoals. We provide additional visualizations in the supplementary material.

6 Related Work

How to effectively learn policies with multiple hierarchies has been a long-standing topic in RL. Goal-conditioned HRL [3, 30, 14, 34, 20, 16] aims to answer this question with a framework that separates high-level planning and low-level control using subgoals. Recent advances in goal-conditioned HRL mainly focus on improving the learning efficiency of this framework. Nachum et al. [20, 19] propose an off-policy correction technique to stabilize training, and address the problem of goal space representation learning using a mutual-information-based objective. However, the subgoal generation process in their approaches is unconstrained and supervised only by the external reward, and thus they may still suffer from training inefficiency. Levy et al. [16] use hindsight techniques [1] to train multi-level policies in parallel and also punish the high-level for generating
subgoals that the low-level fails to reach. However, they directly obtain the reachability measure from the environment, using the environmental information that is not available in many scenarios. There are also researches [17, 13, 15, 27, 25, 12] focusing on unsupervised acquisition of subgoals based on potentially pivotal states. However, these subgoals are not guaranteed to be well-aligned with downstream tasks and thus are often sub-optimal.

Several prior work [27, 5, 12] construct an environmental graph for high-level planning and search nearby graph nodes to be reachable subgoals for the low-level. However, these approaches hard-code the planning process based on domain-specific knowledge, and thus are limited in scalability. Nasiriany et al. [22] use goal-conditioned value functions to measure the feasibility of subgoals, but a pre-trained goal-conditioned policy is required. There are also researches that study the more general topic of goal generation in RL [8, 21, 26]. Compared with our method, these methods only have a flat architecture and therefore cannot successfully solve tasks that require complex high-level planning.

Finally, our method relates to previous researches that study transition distance or reachability [23, 27, 28, 6, 11]. Most of these work learn the transition distance based on RL [23, 6, 11], which tend to have a high learning cost. Savinov et al. [27, 28] propose a supervised learning approach for reachability learning. However, the metric they learned depends on a certain policy used for interaction and thus could be sub-optimal compared to our learning method.

7 Conclusions

We presented a novel k-step adjacency constraint for goal-conditioned HRL framework to address the issue of training inefficiency, with the theoretical guarantee of preserving the optimal policy in deterministic MDPs. We show that the proposed adjacency constraint can be practically implemented with an adjacency network. Experiments on several testbeds with discrete and continuous state and action spaces demonstrate the effectiveness of our method. Future work include extending the proposed framework to tasks with high-dimensional state spaces and leveraging the learned adjacency network to improve learning efficiency in more general scenarios.

Broader Impact

This work may promote the researches in the field of HRL and RL, and has potential real-world applications such as the robotics. The main uncertainty of the proposed method might be the fact that the RL training process itself is somewhat brittle, and may break in counterintuitive ways when the reward function is misspecified. Also, since the training data of RL heavily depend on the training environments, designing unbiased simulators or real-world training environments is important for eliminating the biases in the data collected by the agent.

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A Proofs of Theorems

A.1 Proof of Theorem 1

Proof. Under the assumption that the MDP is deterministic and all states are strongly connected, there exists at least one shortest state trajectory from $s$ to $g$. Without loss of generality, we consider one shortest state trajectory $\tau^* = (s_0, s_1, s_2, \ldots, s_n)$, where $s_0 = s$, $s_n = \varphi^{-1}(g)$ and $d_{st}(s, \varphi^{-1}(g)) = n$. For all $k \in \mathbb{N}_+$ and $k \leq d_{st}(s, \varphi^{-1}(g)) = n$, let $\tilde{g} = \varphi(s_k)$, and let $\tau = (s_0, s_1, s_2, \ldots, s_k)$ be the $k$-step sub-trajectory of $\tau^*$ from $s_0$ to $s_k$. Since $s_0$ and $s_k$ is connected by $\tau$ in $k$ steps, we have that $d_{st}(s_0, \varphi^{-1}(\tilde{g})) = d_{st}(s_0, s_k) \leq k$, i.e., $\tilde{g} \in G_A(s, k)$. In the following, we will prove that $\pi^*(|s_i, \tilde{g}|) = \pi^*(|s_i, g|)$, $\forall s_i \in \{\tau, i \neq k\}$.

We first prove that the shortest transition distance $d_{st}$ satisfies the triangle inequality, i.e., consider three arbitrary states $s_1, s_2, s_3 \in \mathcal{S}$, then $d_{st}(s_1, s_3) \leq d_{st}(s_1, s_2) + d_{st}(s_2, s_3)$. Let $\tau^*_{12}$ be one shortest state trajectory between $s_1$ and $s_2$ and let $\tau^*_{23}$ be one shortest state trajectory between $s_2$ and $s_3$. We can concatenate $\tau^*_{12}$ and $\tau^*_{23}$ to form a trajectory $\tau^*_{13} = (\tau^*_{12}, \tau^*_{23})$ that connects $s_1$ and $s_3$. Then, by Definition 1, we have $d_{st}(s_1, s_3) \leq d_{st}(s_1, s_2) + d_{st}(s_2, s_3)$.

Using the triangle inequality, we can prove that the sub-trajectory $\tau = (s_0, s_1, s_2, \ldots, s_k)$ is also a shortest trajectory from $s_0 = s$ to $s_k$. Assume that this is not true and there exists a shorter trajectory from $s_0$ to $s_k$. Then, by Definition 1, we have $d_{st}(s_0, s_k) < k$. Since $(s_k, s_{k+1}, \ldots, s_n)$ is a valid trajectory from $s_k$ to $s_n$, we have $d_{st}(s_0, s_k) \leq n - k$. Applying the triangle inequality, we have $d_{st}(s_0, s_n) \leq d_{st}(s_0, s_k) + d_{st}(s_k, s_n) < k + n - k = n$, which is in contradiction with $d_{st}(s, \varphi^{-1}(g)) = d_{st}(s_0, s_n) = n$. Thus, our original assumption must be false, and the trajectory $\tau = (s_0, s_1, s_2, \ldots, s_k)$ is a shortest trajectory from $s_0$ to $s_k$.

Finally, let $\alpha: \mathcal{S} \times \mathcal{S} \mapsto \mathcal{A}$ be an inverse dynamics model, i.e., given state $s_t$ and the next state $s_{t+1}$, $\alpha(s_t, s_{t+1})$ outputs the action $a_t$ that is performed at $s_t$ to reach $s_{t+1}$. Then, employing Equation 3, for $i = 0, 1, \ldots, k - 1$ we have $\pi^*(|s_i, g|) = \alpha(s_i, s_{i+1})$ given that $\tau^*$ is a shortest trajectory from $s_0$ to $\varphi^{-1}(g)$, and $\pi^*(|s_i, \tilde{g}|) = \alpha(s_i, s_{i+1})$ given that $\tau$ is a shortest trajectory from $s_0$ to $\varphi^{-1}(\tilde{g})$. This indicates that $\pi^*(|s_i, \tilde{g}|) = \pi^*(|s_i, g|)$, $\forall s_i \in \{\tau, i \neq k\}$. 

A.2 Proof of Theorem 2

Proof. Using Theorem 1, we have that for each subgoal $g_{kt}$, $t = 0, 1, \ldots, T - 1$, there exists a subgoal $\bar{g}_{kt} \in G_A(s_{kt}, k)$ that can induce the same low-level $k$-step action sequence as $g_{kt}$. This indicates that the agent’s trajectory and the high-level reward $r_{kt}^{nh}$, defined by Equation 1, remain the same for all $t$ when replacing $g_{kt}$ with $\bar{g}_{kt}$. Then, using the high-level Bellman optimality equation for the optimal $Q$ function

$$
Q^*(s_{kt}, g_{kt}) = r_{kt}^{nh} + \gamma \max_{g \in \mathcal{G}} Q^*(s_{k(t+1)}, g) 
$$

(14)

and $Q^*(s_{kt}, g) = 0$, $\forall g \in \mathcal{G}$ as $s_{kt}$ is the final state of $\tau^*$, we have $Q^*(s_{kt}, \bar{g}_{kt}) = Q^*(s_{kt}, g_{kt})$, $t = 0, 1, \ldots, T - 1$. 

B Implementation Details

B.1 Adjacency Learning

Constructing and updating the adjacent matrix. We use the agent’s trajectories to construct and update the adjacent matrix. Concretely, the adjacent matrix is initialized to an empty matrix at the beginning of training. Each time when the agent explores a new state that it has never visited before, the adjacent matrix is augmented by a new row and a new column with zero elements, representing the $k$-step adjacent relation between the new state and explored states. When the transition distance between two states in one trajectory is not larger than $k$, then the corresponding element in the adjacent matrix will be labeled to 1, indicating the adjacency. (The diagonal of the adjacent matrix will always be labeled to 1.) Although the transition distance between two states based on a single trajectory is often larger than the real shortest transition distance, it can be easily shown that the
adjacent matrix with this labeling strategy can converge to the optimal adjacent matrix asymptotically with sufficient trajectories sampled by different policies. In practice, we employ a trajectory buffer to store newly-sampled trajectories, and update the adjacent matrix in a fixed frequency using the stored trajectories. The trajectory buffer is cleared after each update.

Training the adjacency network. The adjacency network is trained by minimizing the objective defined by Equation (11). We use states evenly-sampled from the adjacent matrix (i.e. from the set of all explored states) to approximate the expectation, and train the adjacency network each time after the adjacent matrix is updated with new trajectories. Note that by explicitly aggregating the adjacency information using an adjacent matrix, we are able to achieve the uniform sampling of all explored states and thus achieve a nearly unbiased estimation of the expectation, which cannot be realized when we directly sample state-pairs from the trajectories (see the following comparison with Savinov et al. [27, 28] for details).

Embedding all subgoals with a single adjacency network is enough to express adjacency when the environment is reversible. However, when this condition is not satisfied, it is insufficient to express directional adjacency using one adjacency network, as the parametrized approximation defined by Equation (10) is symmetric for \( s_1 \) and \( s_2 \). In this case, one can use two separate sub-networks to embed \( g_1 \) and \( g_2 \) in Equation (10) respectively using the structure in [29].

Comparison with Savinov et al. Savinov et al. [27, 28] also propose a supervised learning approach for learning the adjacency between states. The main differences between our method and theirs are: 1) We use trajectories sampled by multiple policies to construct training samples, while they only use trajectories sampled by one specific policy; 2) We use an adjacent matrix to explicitly aggregate the adjacency information and sample training pairs based on the adjacency matrix, while they directly sample training pairs from trajectories. These differences lead to two advantages of our method: 1) By using multiple policies, we achieve a more accurate adjacency approximation, as shown by Equation (9); 2) By maintaining an adjacent matrix, we can uniformly sample from the set of all explored states and realize a nearly unbiased estimation of the expectation in Equation (11), while the estimation by sampling state-pairs from trajectories is biased. As an example, consider a simple grid world environment in Figure 10(a) where states are represented by their \( x, y \) coordinates. In this environment, states \( s_1 \) and \( s_2 \) are non-adjacent since they are separated by a wall. However, it is hard for the method by Savinov et al. to handle this situation as these two states rarely emerge in the same trajectory due to the large transition distance, and thus the loss induced by this state-pair is
Algorithm 1 HRAC

**Input:** High-level policy $\pi^h_0$, parametrized by $\theta_h$, low-level policy $\pi^l_0$ parametrized by $\theta_l$, adjacency network $\psi_\phi$ parametrized by $\phi$, state-goal mapping function $\phi$, goal transition function $h$, high-level action frequency $k$, the number of training episodes $N$, adjacency learning frequency $C$, empty adjacent matrix $\mathcal{M}$, trajectory buffer $\mathcal{B}$.

Sample and store trajectories in the trajectory buffer $\mathcal{B}$ using a random policy.

Construct the adjacent matrix $\mathcal{M}$ using the trajectory buffer $\mathcal{B}$.

Pre-train $\psi_\phi$ using $\mathcal{M}$ by minimizing Equation (11).

Clear $\mathcal{B}$.

for $n = 1$ to $N$ do

  Reset the environment and sample the initial state $s_0$.
  $t = 0$.

  repeat

    if $t \equiv 0 \pmod{k}$ then

      Sample subgoal $g_t \sim \pi^h_0(g|s_t)$.

    else

      Perform subgoal transition $g_t = h(g_{t-1}, s_{t-1}, s_t)$.

    end if

    Sample low-level action transition $a_t \sim \pi^l_0(a|s_t, g_t)$.
    Sample next state $s_{t+1} \sim P(s|s_t, a_t)$.
    Sample reward $r_t \sim R(r|s_t, a_t)$.
    Sample episode end signal $\text{done}$.

    $t = t + 1$.

  until $\text{done}$ is true.

  Store the sampled trajectory in $\mathcal{B}$.

  Train high-level policy $\pi^h_0$ according to Equation (12) and (13).

  Train low-level policy $\pi^l_0$.

  if $n \equiv 0 \pmod{C}$ then

    Update the adjacent matrix $\mathcal{M}$ using the trajectory buffer $\mathcal{B}$.

    Fine-tune $\psi_\phi$ using $\mathcal{M}$ by minimizing Equation (11).

    Clear $\mathcal{B}$.

  end if

end for

very likely to be dominated by the loss of other nearer state-pairs. Meanwhile, our method treat the loss of all state-pairs equally, and can therefore alleviate this phenomenon. Empirically, we employed a random agent (since the random policy is stochastic, it can be viewed as multiple deterministic policies, and is enough for adjacency learning in this simple environment) to interact with the environment for 20,000 steps, and trained the adjacency network with collected samples using both methods. We visualize the LLE of state embeddings and two adjacency distance heatmaps by both methods respectively in Figure 10(b) and 10(c). Visualizations validate our analysis, showing that our method does learn a better adjacency measure in this scenario.

B.2 Algorithm Pseudocode

We provide Algorithm 1 to detail the training procedure of HRAC. Some training details are omitted for brevity, e.g. the detailed training process of the low-level policy.

B.3 Environment Details

**Maze.** This environment has a size of $13 \times 17$, with a discrete 2-dimensional state space representing the $x, y$ position of the agent and a discrete 4-dimensional action space corresponding to actions moving towards four directions. The agent is provided with a dense reward to facilitate exploration, i.e., $+0.1$ each step if the agent moves closer to the goal, and $-0.1$ each step if the agent moves farther. Each episode has a maximum length of 200. Environmental stochasticity is introduced by replacing the action of the agent by a random action each step with a probability of 0.25.
Key-Chest. This environment has a size of $13 \times 17$, with a discrete 3-dimensional state space in which the first two dimensions represent the $x$, $y$ position of the agent respectively, and the third dimension represents whether the agent has picked up the key (1 if the agent has the key and 0 otherwise). The agent has the same action space as the Maze task. The agent is provided with sparse reward of +1 and +5, respectively for picking up the key and opening the chest. Each episode ends if the agent opens the chest or runs out of the step limit of 500. The random action probability of the environment is also 0.25.

Point Gather. This environment has a size of $20 \times 20$, with a continuous state space including the current position and velocity of the robot, the current time step $t$, and the depth readings defined by the standard Gather environment. The point robot has a 2-dimensional continuous action space corresponding to the pivot rotation and the accelerator. We use the point robot pre-defined by Rllab, limiting its low-level action to $(\pm 0.2, \pm 0.1)$, corresponding to acceleration and rotation respectively. The point robot is spawned at the center of the map and needs to gather apples while avoiding bombs. Both apples and bombs are randomly placed in the environment at the beginning of each episode. The agent receives a positive reward of +1 for each apple and a negative reward of −1 for each bomb. Each episode terminates at 1000 time steps.

Point Maze Sparse. This environment has a size of $20 \times 20$, with the same state and action spaces as the Point Gather task, except that the agent does not observe depth readings. The target position (goal) is set at the position $(2.0, 9.0)$ in the center corridor. The agent is rewarded by +1 only if it reaches the goal, which is defined as having a Euclidean distance that is smaller than 1 from the goal. At the beginning of each episode, the agent is randomly placed in the maze except at the goal position. Each episode is terminated if the agent reaches the goal or after 500 steps.

B.4 HRAC and Baseline Details

HRAC. For discrete control tasks, we adopt a binary intrinsic reward setting: we set the intrinsic reward to 1 when $|s_x - g_x| \leq 0.5$ and $|s_y - g_y| \leq 0.5$, where $(s_x, s_y)$ is the position of the agent and $(g_x, g_y)$ is the position of the desired subgoal. For continuous control tasks, we adopt a dense intrinsic reward setting based on the negative Euclidean distances $-\|s - g\|_2$ between states and subgoals.

HIRO. Following [20], we restrict the output of high-level to $(\pm 10, \pm 10)$, representing the desired shift of the agent’s position. By limiting the range of directional subgoals generated by the high-level, HIRO can roughly control the Euclidean distance between the absolute subgoal and the current state in the raw goal space rather than the learned adjacency space.

HRL-HER. As HER [1] cannot be applied to the on-policy training scheme in a straightforward manner, in discrete control tasks where the low level policy is trained using A2C, we modify its implementation so that it can be incorporated into the on-policy setting. For this on-policy variant, during the training phase, we maintain an additional episodic state memory. This memory stores states that the agent has visited from the beginning of each episode. When the high-level generates a new subgoal, the agent randomly samples a subgoal mapped from a stored state with a fixed probability 0.2 to substitute the generated subgoal for the low-level to reach. This implementation resembles the “episode” strategy introduced in the original HER.

NoAdj. We follow the training pipeline proposed by [27, 28], where no adjacent matrix is maintained. Training pairs are constructed by randomly sampling state pairs $(s_i, s_j)$ from the stored trajectories. The samples with $|i - j| \leq k$ are labeled as positive with $t = 1$, and the samples with
Table 1: Hyper-parameters used in discrete control tasks. “K-C” in the table refers to “Key-Chest”.

| Hyper-parameters          | Values                  | Ranges                  |
|---------------------------|-------------------------|-------------------------|
| **High-level TD3**        |                         |                         |
| Actor learning rate       | 0.0001                  |                         |
| Critic learning rate      | 0.001                   |                         |
| Replay buffer size        | 10000 / 20000 for Maze / K-C | {10000, 20000}          |
| Batch size                | 64                      |                         |
| Soft update rate          | 0.001                   |                         |
| Policy update frequency   | 2                       | {1, 2}                  |
| $\gamma$                 | 0.99                    |                         |
| High-level action frequency $k$ | 10                      |                         |
| Reward scaling            | 1.0                     |                         |
| Exploration strategy      | Gaussian ($\sigma = 3.0/5.0$ for Maze / K-C) | {3.0, 5.0}          |
| Adjacency loss coefficient $\eta$ | 10                      | {1, 5, 10, 20}          |
| **Low-level A2C**         |                         |                         |
| Actor learning rate       | 0.0001                  |                         |
| Critic learning rate      | 0.0001                  |                         |
| Entropy weight            | 0.01                    |                         |
| $\gamma$                 | 0.99                    |                         |
| Reward scaling            | 1.0                     |                         |

$|i - j| \geq Mk$ are negative ones with $l = 0$. The hyper-parameter $M$ is employed to create a gap between the two types of samples, where in practice we use $M = 4$.

**NegReward.** In this variant, every time the high-level generates a subgoal, we use the adjacency network to judge whether it is $k$-step adjacent. If the subgoal is non-adjacent, the high-level will be penalized with the reward $-1$.

### B.5 Network Architecture

For the hierarchical policy network, we employ the same architecture as HIRO [20] in continuous control tasks, where both the high-level and the low-level use TD3 [9] algorithm for training. In discrete control tasks, we use two networks consisting of 3 fully-connected layers with ReLU nonlinearities as the low-level actor and critic networks of A2C, and use the same high-level TD3 network architecture as the continuous control task. The size of the hidden layers of both low-level actor and critic is $(300, 300)$. The output of the high-level actor is activated using the $\tanh$ function and scaled to fit the size of the environments.

For the adjacency network, we use a network consisting of 4 fully-connected layers with ReLU nonlinearities in all tasks. Each hidden layer of the adjacency network has the size of $(128, 128)$. The dimension of the output embedding is 32.

We use Adam optimizer to train all networks.

### B.6 Hyper-parameters

We list all hyper-parameters used in the discrete and continuous control tasks respectively in Table 1 and Table 2 and list the hyper-parameters used for adjacency network training in Table 3. “Ranges” in the tables show the ranges of hyper-parameters considered, and the hyper-parameters without ranges are not tuned.

### C Additional Visualizations

We provide additional subgoal and adjacency heatmap visualizations of the Maze and Key-Chest tasks respectively in Figure 11 and Figure 12.
Table 2: hyper-parameters used in continuous control tasks.

| Hyper-parameters                  | Values       | Ranges          |
|-----------------------------------|--------------|-----------------|
| **High-level TD3**                |              |                 |
| Actor learning rate               | 0.0001       |                 |
| Critic learning rate              | 0.001        |                 |
| Replay buffer size                | 200000       |                 |
| Batch size                        | 128          |                 |
| Soft update rate                  | 0.005        |                 |
| Policy update frequency           | 1            |                 |
| \(\gamma\)                       | 0.99         |                 |
| High-level action frequency \(k\) | 10           |                 |
| Reward scaling                    | 0.1 / 1.0 for Ant Maze / others |          |
| Exploration strategy              | Gaussian (\(\sigma = 1.0\)) |          |
| Adjacency loss coefficient \(\eta\) | 10.0         | {1, 5, 10, 20} |
| **Low-level TD3**                 |              |                 |
| Actor learning rate               | 0.0001       |                 |
| Critic learning rate              | 0.001        |                 |
| Replay buffer size                | 200000       |                 |
| Batch size                        | 128          |                 |
| Soft update rate                  | 0.005        |                 |
| Policy update frequency           | 1            |                 |
| \(\gamma\)                       | 0.99         |                 |
| Reward scaling                    | 1.0          |                 |
| Exploration strategy              | Gaussian (\(\sigma = 1.0/0.2\) for Ant / Point) |   |

Table 3: Hyper-parameters used in adjacency network training.

| Hyper-parameters                  | Values       | Ranges          |
|-----------------------------------|--------------|-----------------|
| **Adjacency Network**             |              |                 |
| Learning rate                     | 0.0002       |                 |
| Batch size                        | 64           |                 |
| \(\epsilon_k\)                   | 1.0          |                 |
| \(\delta\)                       | 0.2          |                 |
| Steps for pre-training            | 50000        |                 |
| Pre-training epochs               | 50           |                 |
| Online training frequency (steps) | 50000        |                 |
| Online training epochs            | 25           |                 |
Figure 11: Additional subgoal and adjacency heatmap visualizations of the Maze task, based on a single evaluation run. The agent (A), goal (G) and subgoal (g) at different time steps in one episode are plotted. Colder colors in the adjacency heatmaps represent smaller shortest transition distances.
Figure 12: Additional subgoal and adjacency heatmap visualizations of the Key-Chest task, based on a single evaluation run. The agent (A), key (K), chest (C) and subgoal (g) at different time steps in one episode are plotted. Colder colors in the adjacency heatmaps represent smaller shortest transition distances.