Abstract—By combining the results from a self-consistent 2D dusty plasma fluid model and a 3D N-body code, the equilibrium position and crystal structure were determined for dust particles levitated in the sheath in a modified Gaseous Electronics Conference (GEC) reference cell, in which the lower electrode was heated or cooled. The Debye length, charge and electric field were reconstructed on a sub-millimeter scale by applying a previously developed, independent method. However, this method seems to overestimate the charge, and hence underestimate the electric field. Even corrected for this fact, the results show that the dust is levitated on the plasma side of the Bohm point. The ion drag, which is not fully taken into account, probably plays an important role in the force balance.

Index Terms—Dusty plasma, fluid model, N-body model, sheath electric field, thermophoresis.

I. INTRODUCTION

The sheath in a confined plasma is the volume where the nearly constant potential in the bulk is connected to the potential on the walls. Strong electric fields repel electrons from and attract ions to the walls. Hence, the form of the sheath electric field is of interest for plasma applications, like surface coating, deposition and etching, and other applications in the automotive, microchip, and solar cell industries [1]–[3].

In dusty plasma experiments, performed for instance in a modified GEC reference cell, like the one discussed in this paper, these electric fields levitate dust against gravity. The vertical force balance and the radial balance between a confining potential and the inter-particle forces typically result in thin two-dimensional dust crystals. These crystals allow the study of many solid state phenomena, such as waves and phonons, melting and solidification, super-diffusion, Mach cones, and turbulence on spatial and temporal scales accessible with ordinary optical techniques [4]–[7].

The properties of these systems depend on the dust charge and the Debye length, which depend on the properties of the sheath. Measurement of the electric field using probes is too disruptive to obtain reliable results, while optical measurements close to the electrode (either by passively observing plasma emission [8], or by using some form of induced fluorescence) are technically difficult [9]. Fortunately, dust particles themselves can act as probes in the sheath, basically being tiny floating Langmuir probes.

Many dust-as-probe techniques have been employed in the past: oscillating particles by applying an additional low-frequency potential, manipulating particles with lasers [10], or adding perturbations to the plasma (for instance a sudden change in DC bias) have all been examined. Other studies used the dust levitation height to determine the sheath edge, observing multiple particles of different sizes simultaneously to determine the sheath electric field profile [11].

There are many difficulties with these techniques. First, measuring the vertical force balance requires knowledge of both the dust charge and the electric field and a combination of techniques is required to obtain the electric field profile. Secondly, in the case of sheath-edge determinations, additional techniques such as plasma emission observation or probe measurements are required to justify the results, since it is not clear a priori why the dust should float at the sheath edge. Finally, many techniques depend on perturbing the plasma. Since the plasma and the dust are necessarily coupled (especially for many dust particles), this adds uncertainty to the measurements.

In this paper, we discuss a method to obtain the dust charge and Debye length for a dust crystal levitated in the sheath of a modified GEC cell, confined radially by a parabolic potential created by a shallow circular cutout in a plate placed on top of the lower electrode, by using top-view images of the crystal and measuring the radius and inter-particle distances, as explained in [12]. Once the charge is obtained, we proceed to trace the sheath electric field using thermophoresis to adjust the equilibrium height of the dust, a method which does not depend on perturbing the plasma. Results obtained with a combination of a self-consistent dusty plasma fluid model and a N-body code are employed for this discussion.

II. DUST CHARGE DETERMINATION

Following [12], we assume that the potential above the cutout in a plate placed on top of the powered electrode has a parabolic radial dependence, and write the potential as a function of the height with a radial shift,

$$ \phi = \phi(z - h(r)), $$

$$ h(r) = cr^2, $$

where the constant $c$ will be defined later.
Next, we consider the vertical force balance on dust particles with mass $m_D$ and charge $q_D$ located in the sheath. The forces acting on the particles include the upward electrostatic force due to the vertical sheath electric field, the force of gravity, with acceleration $g$, the thermophoretic force, $F_{th}$, due to the applied heating or cooling of the lower electrode (thermophoretic forces without electrode heating or cooling are negligible), and finally the ion drag force, $F_{id}$, due to ions accelerated downwards in the sheath:

$$m_D \ddot{z} = -m_D g - q_D \frac{d\phi(z - h(r))}{dz} + F_{th} - F_{id} = 0. \tag{3}$$

The radial electrostatic force is considered to be a harmonic restoring force that stabilizes the dust crystal against the repulsive inter-particle interactions, which gives

$$m_r \ddot{r} = -q_D \frac{d\phi(z - h(r))}{dr} \equiv -kr. \tag{4}$$

Using partial differentiation and defining the prime to denote the partial derivative with respect to the argument of a function, we can use equation [3] to rewrite the radial electric field. This gives

$$-q_D h'(r) \phi'(z - h(r)) = h'(r) [m_D g - F_{th} + F_{id}]. \tag{5}$$

Inserting the definition for $h(r)$, we find for $k$

$$k = 2c [m_D g - F_{th} + F_{id}], \tag{6}$$

which means that we have expressed the coefficient for the radial restoring force in terms of the vertical equilibrium forces and $c$.

The dust particles suspended in the sheath will form a crystal lattice. The equation of state for a crystal consisting of $N$ particles, with inter-particle spacing $\Delta$, interacting through an interaction potential $V(\Delta)$ is given by [12]

$$P = -\frac{1}{N} \frac{d}{d(V(\Delta)/\Delta)} \frac{\sqrt{3} dV(\Delta)}{d(\sqrt{3} \Delta^2/2)}. \tag{7}$$

Using appropriate boundary conditions, two equations can be derived relating the radius where the inter-particle distance goes to infinity, $R_\infty$, and the central inter-particle spacing $\Delta_0$ to the Debye length, $\lambda_D$, and the dust charge:

$$R_\infty^2 = \frac{3}{k} \left(3 + \frac{\Delta_0}{\lambda_d}\right) V(\Delta_0), \tag{8}$$

and

$$N = \frac{2\pi \sqrt{3}}{k \Delta_0} \left(1 + \frac{1}{\lambda_d}\right) V(\Delta_0). \tag{9}$$

In the above, $V$ is assumed to be a screened Coulomb potential $V(r) = q_D^2 \exp(-r/\lambda_D)/4\pi\varepsilon_0 r$. $R_\infty$ is related to the crystal radius, $R_M$, and the outer inter-particle spacing, $s_M$, through $R_\infty \approx R_M + s_M \sqrt{3/2}$. Solving for $V(\Delta_0)$ and substituting in equation [8] gives an equation for the Debye length:

$$\lambda_D = \Delta_0 \left[\frac{A - S}{3S - A}\right], \tag{10}$$

where we have defined the total crystal surface area $A = \pi R_\infty^2$, and the total surface area covered by $N$ Wigner-Seitz cells measured at the center: $S = \sqrt{3} \Delta_0^2 N/2$. Using this equation for $\lambda_D$, we find for the dust charge:

$$q_D = \frac{4\pi\varepsilon_0 \Delta_0 k R_\infty^2}{\sqrt{3} \left(3 + \frac{3S - A}{A - S}\right)} \exp\left(\frac{A - 3S}{A - S}\right). \tag{11}$$

Measuring the central inter-particle spacing, the maximum radius of the dust crystal, the outer inter-particle spacing, and the total number of particles in the crystal allows us to determine the Debye length. We can then derive the dust charge provided we have established the value for $k$.

In order to determine $k$, we need to know the vertical equilibrium forces, as well as $c$. In our model the melamine-formaldehyde (MF) particles are monodisperse spheres. This means we know the gravitational force. The only significant thermophoretic force is the one we apply by heating/cooling the lower electrode. For particles with a diameter below 4 microns, the ion drag can be significant in the sheath [12]. In our simulations we use 2 micron diameter particles. The fluid model self-consistently solves for the ion drag, although the N-body code does not include the effect of the ion drag. For the current analysis, we therefore neglect the effect of the ion drag, although we will discuss its possible importance later.

As mentioned, the radial confinement potential is provided by a cylindrical depression in the lower electrode. $c$ is determined by the geometry of this cutout: the narrower and deeper the cutout, the steeper the potential well. In [12] spherical cutouts were used, so that the value of $c$ could be directly related to the radius of curvature of the cutout $R_c$ through $c = 0.5/R_c$. Following this approach, we approximate the cylindrical cutout by the sphere that exactly touches the lowest point of the cutout as well as the edge. The radius of the modeled cutout is 12.5 mm, and it is 0.5 mm deep. From $R_c \cdot \sin(\theta) = 12.5$ mm and $R_c \cdot \cos(\theta) = R_c - 0.5$ mm, we find an effective radius of curvature for our cutout of 160 mm. We thus use $k = (m_D g - F_{th})/0.16 [N/m]$.

III. NUMERICAL MODELS

The fluid model self-consistently solves for the coupled plasma and dust parameters, but is unable to resolve the inter-particle interactions on the microscopic level and hence provides no information on the crystal structure. The N-body code solves the inter-particle interactions, thus providing the crystal properties, but the plasma properties are not self-consistently solved. The fluid model is therefore used in this study to provide the external confinement, as well as the input parameters for the N-body code, for given discharge settings. We briefly discuss both models; a complete description can be found in the references provided in the sections below.

A. N-body code: box-tree

Box-tree [13] integrates the equations of motion for particles moving in a simulation box under prescribed external forces, while calculating the interaction between the particles using a tree-method employing multipole expansions for the
electrostatic forces. The box and the external forces can be set to represent for instance dust particles in Saturn’s F-Ring [14], or particles in a laboratory crystal [15]. In this study, the external forces considered include the electrostatic force of the sheath, gravity and thermophoresis. Since the plasma properties are not determined by box-tree, the electric field, dust charge, and Debye length were set using values obtained from the fluid model. Dust particles were then introduced with random positions (but zero initial velocity), and their kinetic energy was dissipated through a drag force representing the neutral drag until an equilibrium crystal was formed.

B. The fluid model

The fluid model employed [16] solves the continuity equations for the electrons and positive argon ions, as well as for the electron energy density, assuming a drift-diffusion equation for the fluxes. The sources and sinks for the electrons (and electron energy density) include excitation of argon atoms, ionization and recombination on the dust particles. The ions are assumed to locally dissipate their energy in collisions with the neutral atoms. Therefore, an explicit equation for the ion energy density is not solved. The electric field is found from Poisson’s equation, including the dust charge density. These equations are initially iterated on sub-RF timescales without the presence of dust, until the solution has become periodic over a RF-cycle. Then, the addition of the dust is simulated by adding source terms for the dust, below the upper electrode.

The transport of the dust fluid is solved by assuming a balance between the neutral drag and the other forces, which allows for a drift-diffusion type equation for the dust. Gravity is a constant force which only depends on the dust size. The electrostatic force is calculated from the time averaged electric field and the dust charge, which is calculated from the local plasma parameters using Orbital Motion Limited (OML) electron- and ion currents, including the effect of charge-exchange collisions on the ion current [17], [18].

The ion drag force is calculated from the local ion flux interacting with the dust particles. The ion collection cross-section is derived from OML theory. The ion scattering cross-section includes the effects of scattering beyond the Debye length [19], the anisotropic screening caused by significant ion drift [20], and the effect of charge-exchange collisions [21].

In order to compute the thermophoretic force, the neutral gas temperature profile is calculated by iterating the power balance. The sources include the dissipation of energy by the ions, as well as heating through atoms impinging on the hot dust particle surfaces. In order to find the dust particle surface temperature, a balance is solved between the recombination of ions and electrons on the surface on the one hand, and the thermal radiation of the dust particles and the conduction to the gas on the other hand. The temperature of the surrounding walls and electrodes sets the boundary conditions. These can be changed to include heating or cooling of surfaces.

C. Approach

The known variables are the particle mass, the effective radius of our cutout, the applied temperature gradient and hence the thermophoretic force, calculated through

$$F_{th} = -\frac{32}{15} \kappa T \frac{a^2}{v_T} \nabla T_{gas}.$$  \hspace{1cm} (12)

with $\kappa_T = 0.01772$ the heat conduction coefficient for argon, $a$ the particle radius, and $v_T$ the thermal velocity of the background gas. We therefore know $\kappa_T$, since we are not concerned with the ion drag force right now.

Given a set of discharge parameters, i.e. the pressure and input power, we obtain the Debye length, the dust charge, and the electric field from the fluid model. These are then used as input in the box-tree model. Once the dust crystal has reached equilibrium, the observables are obtained from the crystal (i.e. the number of particles in the crystal, the crystal radius, the central and outer inter-particle spacing) and the method of [12] discussed above is used to reconstruct the Debye length, charge and electric field. By varying the thermophoretic force, the charge and electric field profiles are obtained throughout the sheath region. This provides a method to determine whether or not this results in an acceptable reconstruction of the electric field, and also if using thermophoresis allows us to probe the electric field throughout the sheath.

IV. RESULTS

Here we present the results for an argon plasma at 200 mTorr pressure and 2 Watts of absorbed power in the geometry of a modified GEC cell. We introduce 1000 two micron diameter MF particles, while varying the temperature on the lower, powered electrode.

In the calculation of the thermophoretic force in box-tree, it is assumed that the temperature gradient is simply given by the difference between the temperatures of the electrodes divided by the distance between the electrodes, i.e. no large gradients are expected. Figure 1 shows two gas temperature profiles along the central axis, obtained with the fluid model; one where the lower electrode was cooled to 20°C below room-temperature, and one where the lower electrode was cooled to 20°C below room-temperature. We see that in the first case, the vertical temperature gradient between the electrodes is negligible and certainly too small to create a significant thermophoretic force. The second case clearly shows that the temperature profile

![Height above lower electrode (mm)](image-url)

Fig. 1. The vertical temperature profile above the center of the electrode for the case where the lower electrode is at room-temperature, and for the case where it is cooled to 20°C below room-temperature.
varies linearly with height and shows no local gradients; in other word, the temperature gradient is indeed simply the temperature difference divided by the distance between the electrodes.

Figure 2 shows a fit of the electric field, obtained self-consistently with the fluid model. The electric field in the sheath decreases linearly with height, whereas in the bulk, the field can be well approximated by a third order polynomial. For the mentioned experimental settings and geometry there is no extended quasi-neutral bulk where the electric field vanishes, which has also been observed experimentally.

The equilibrium position of the dust fluid was also calculated while varying the electrode temperature. Figure 4 shows the obtained levitation height above the electrode. Clearly, the levitation height increases exponentially with applied temperature. This is consistent with the change in levitation height observed in experiments, as is presented elsewhere in this issue [22].

The above results from the fluid model were used to prescribe the external forces in box-tree, which was then run at each temperature setting, until one minute real-time was simulated. At this point the dust crystal had become stable. A top-view image of one quadrant of the dust crystals obtained for electrode temperatures equal to room-temperature and 20° below room-temperature are shown in figure 5. Clearly, the crystal expands with increasing temperature, as would be expected from the behavior of $k$ in equation 6.

The dust levitation height obtained with box-tree is also shown in figure 4. The height increases linearly with the temperature, rather than exponentially, as for the fluid model results. The primary difference between the models is the missing ion drag calculation in box-tree, implying that the ion drag is important for the levitation of 2 micron particles in the plasma.

The reconstructed Debye length, dust charge and electric field are shown in figure 6 as a function of the height above the lower electrode. The Debye length increases from 750 µm to almost 1.3 mm, while the negative dust charge ranges from -17,000 to -34,000 electron charges. The electric field changes...
from -75 V/m to -7 V/m and can be better fitted by a third-order polynomial than a straight line, indicating that the dust is not levitated in the region where the field varies linearly.

The values given for the Debye length and the dust charge are overestimated probably from an incorrect value of the radial confinement, through the calculation of $k$, since this confinement depends on the ion drag force. Compared to the fluid model, the reconstructed Debye length is too large by a factor of 7, whereas the dust charge is overestimated by a factor of 5. This leads to an underestimated electric field.

The ion drag clearly plays an important role, as evident from the linear increase in levitation height in box-tree where the ion drag is neglected, versus exponential increase in the fluid model, which self-consistently calculates the ion drag force.

Overall, the method shows promise. Adding the ion drag force in box-tree is a necessary extension, both for the vertical equilibrium height, as well as to obtain the proper radial confinement. Using thermophoresis in a dusty discharge with two distinct particle sizes also provides a promising prospect: since the thermophoretic force depends on the particle size, the vertical shift will be different for two crystals of different size. By applying thermophoresis, it is thus possible to squeeze two crystals together, study the interaction between the dust clouds and any possible change in crystal properties. Since this could be achieved without changing the plasma properties directly, it would remove some of the ambiguity involved in analyzing dusty plasma systems.

Fig. 6. The Debye length, dust charge, and vertical electric field obtained with box-tree, together with a linear fit and a third-order polynomial fit to the electric field.

V. DISCUSSION AND CONCLUSION

By combining a self-consistent fluid model and an N-body code, the charge, Debye length and electric field in the sheath can be reconstructed on sub-mm length scales. The ion drag force in box-tree together with a linear fit and a third-order polynomial fit to the electric field.

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