Nonlinear two-dimensional temperature-dependent impedance and the ac power absorption by vortices in a tilted washboard pinning potential

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Abstract. The influence of an ac current of arbitrary amplitude and frequency on the dc and ac magnetoresistivity tensors at arbitrary value of the Hall effect is considered. These results are based on the exact solution (in terms of a matrix continued fraction) of the Langevin equation for a two-dimensional nonlinear guided vortex motion in a tilted cosine pinning potential. The well known results of Coffey and Clem regarding the temperature-dependent vortex motion in the linear impedance problem are substantially generalized and new effects in the microwave power absorption by vortices are discussed.

1. Introduction

One of the most popular experimental methods for the investigation of the vortex dynamics in type-II superconductors is the measurement of the complex ac response in the radiofrequency and microwave ranges. When the Lorentz force acting on the vortices is alternating, then due to the pinning the ac resistive response acquires imaginary (out-of-phase) component. Due to this reason measurements of the complex ac response versus frequency ω can give important information on the pinning forces.

The very early model of Gittleman and Rosenblum (GR) [1] considered oscillations of damped vortex in a garmonic pinning potential. The complex vortex resistivity ρv in the GR’s model is

\[(\rho_v/\rho_f) = i(\omega/\omega_p)/[1 + i(\omega/\omega_p)],\]  

(1)

where ρf is the flux-flow resistivity and ωp is the depinning frequency. As follows from Eq. (1), pinning forces dominate at low frequencies (ω ≪ ωp) where ρv is nondissipative, whereas at high frequencies (ω ≫ ωp) frictional forces dominate and the vortex resistivity is dissipative. As the GR’s model was developed for zero temperature and could not account for the thermally activated flux flow and creep, which are very pronounced in HTCS, there was a need for a more general model for the ac vortex dissipation at different temperatures and frequencies.

For this purpose the vortex equation of motion was supplemented with Langevin force which was assumed to be Gaussian white noise with zero mean. In the limit of small ac current (i.e. for a nontilted cosine pinning potential) this new equation of motion gives the complex resistivity ρv which generalizes the GR’s Eq. (1) has the form (see Eq. (8) in [2])

\[(\rho_v/\rho_f) = [i(\omega/\omega_0) + \nu_0]/[1 + i(\omega/\omega_0)],\]  

(2)
where \( \nu_{00} \) is a creep factor that grows monotonically with temperature increasing from \( \nu_{00} = 0 \) (no flux creep) to \( \nu_{00} = 1 \) (flux flow regime) and \( \omega_0 \) is a characteristic frequency (nonmonotonic in temperature) which, in absence of creep, corresponds to the depinning frequency \( \omega_p \). In this way the temperature-dependent ac-driven vortex motion problem has been exactly solved so far only for the one-dimensional (1D) nontilted cosine pinning potential at a small oscillation amplitude of the vortices by Coffey and Clem (CC) in [2]. However, the examination of a strong ac-driving (that is interesting both for theory and for different high-frequency or microwave applications) requires to consider strongly tilted pinning potential.

The aim of this work is to use a new theoretical approach which substantially generalizes the CC's results because the two-dimensional (2D) Langevin equation for the nonlinear guided motion in a tilted cosine PPP in the presence of a strong ac current at arbitrary value of the Hall effect have been exactly solved [5]. New analytical formulas for 2D temperature-dependent linear impedance tensor \( \hat{Z}_L \) in the presence of a dc current which depend on the angle \( \alpha \) between the current density vector and the guiding direction of the washboard PPP are presented. An influence of a subcritical or overcritical dc current on the time-dependent stationary ac longitudinal and transverse resistive vortex response (on the dimensionless frequency of an ac-driving \( \Omega \)) in terms of the nonlinear impedance tensor \( \hat{Z} \) is studied.

2. Linear ac response - new analytical results

Here we assume that \( (dc+ac) \) \( \alpha \)-dependent [5] current density \( j = j^{dc} + j^{ac} e^{i\omega t} \) and the alternating current is small \( (j^{ac} \ll 1) \). Below we consider an approximate analytical expression for the linear impedance \( Z_{1l}(\omega) \) at arbitrary value of \( j^{dc} \) within the frames of the method of effective eigenvalue (see details in subsections 5.6 and 5.7 of Ref. [4]). Following this approach we can express the dimensionless linear impedance in terms of the modified Bessel functions \( I_\nu(z) \) as

\[
Z_{1l}(\omega, g, j^{dc}) = 1 - \frac{1}{2} \frac{I_{1+\mu}(g)}{I_\mu(g) (\lambda + i\omega \tau)} + \frac{I_{1-\mu}(g)}{I_{-\mu}(g)(\lambda^* + i\omega \tau)},
\]

where

\[
\lambda = I_\mu(g) I_{1+\mu}(g) / [2 \int_0^g I_\mu(t) I_{1+\mu}(t) dt]
\]

is an effective eigenvalue [4], \( \mu \equiv igj^{dc}, \tau \) is the relaxation time, and \( g \) is the dimensionless inverse temperature [5]. It follows from Eqs. (3) and (4) that at \( \omega = 0 \)

\[
Z_{1l}(\omega = 0, g, j^{dc}) = d[j^{dc} \nu_0(j^{dc})]/dj^{dc} = 1 - \text{Re}[(2/I^*_\mu(g)) \int_0^g I_\mu(t) I_{1+\mu}(t) dt],
\]

where \( \nu_0(j^{dc}) \) is the static probability of vortex overcoming the potential barrier [5]. Note also that the right-hand side of Eq.(5) is the exact expression for the dimensionless static differential resistivity in an analytical form. In the limit \( j^{dc} = 0 \) from Eq.(5) follows the well-known result of Coffey and Clem [2]. Actually, in this limit \( \mu = 0 \) and \( \lambda = \lambda^* = I_0(g) I_1(g)/|I_0^2(g) - 1| \). As a result \( Z_{1l}(\omega, g, j^{dc} = 0) = Z_{1l}^0 = (\rho_c/\rho_f) \) (see Eq.(2)) where \( \nu_{00} \equiv \nu_0(j^{dc} = 0) = 1/I_0^2(g) \) is the flux creep factor [2], \( \omega_0 = \lambda \omega_p \) and \( \omega_p = 1/\tau \). In the limit of zero temperature \( g \to \infty \) we have that \( \nu_0(j^{dc}) \to 0, \omega_0 \to \omega_p \) and the results of Gittlemann and Rosenblum [1] are valid.

3. Nonlinear impedance

Below we present the graphical analysis of the frequency-dependent ac and dc nonlinear responses calculated in the transverse geometry [5] (\( \alpha = 0 \)) at low temperatures \( g > 10 \). Let us consider strong nonlinear effects in the ac impedance of a sample subjected to a pure ac driven dimensionless current density \( \xi^a \cos \omega t \), where \( \xi^a \equiv |j^{ac}|/j_c \). Figures 1(a,b) show the
Figure 1. The ac resistivity $\rho_1(\xi^a)$ (a) and reactivity $\zeta_1(\xi^a)$ (b) for various $\Omega = 0.01(1), 0.1(2), 0.2(3), 0.4(4), 0.7(5), 1(6), \xi^d = 0.01, g = 100$. The resistivity $\rho_{1,T}^+(\xi^d)$ (c) for various $\xi^a$. The frequency dependence of $\rho_{1,T}^+(\Omega)$ (d) for various $g$, (I) $\xi^d = 1$ (solid lines), (II) $\xi^d = 0$ (dotted lines).

dimensionless ac resistivity $\rho_1$ and reactivity $\zeta_1$ versus ac current density $\xi^a$ for different dimensionless frequencies $\Omega \equiv \omega\tau$ at very low temperature ($g=100$). As can be seen from the Fig. 1(a), when $\Omega$ is very small, the $\rho_1(\xi^a)$ shows several characteristic features: a threshold $\xi^c$ value and a subsequent parabolic rise, above the threshold, with associated steplike structures. The threshold current density where a sudden increase in $\rho_1(\xi^a)$ starts may be defined as critical current density $\xi^a_c$. The step height decreases with $\xi^a$ increasing. The reactivity $\zeta_1(\xi^a)$ shows nearly periodic dynamic $2\pi$-jumps of the vortex coordinate $x$ occurring as the drive current density $\xi^a$ is increased (see Fig. 1(b)). When $\Omega$ becomes large, both the threshold and steps in $\rho_1(\xi^a)$ disappear and the amplitude of the $x$-jump in $\zeta_1(\xi^a)$ becomes larger. Also, the $x$-jump moves to large values of $\xi^a$ and the spacing in $\xi^a$ between $x$-jumps becomes large which results in $\rho_1$ and $\zeta_1$ approaching unity. Because the abrupt $2\pi$-jumps of the dimensionless vortex coordinate $x$ correspond to the overcoming by vortex of the potential barrier between two neighboring potential wells at nonzero temperature, our curves $\rho_1(\xi^a)$ and $\zeta_1(\xi^a)$ are smoothed due to the influence of a finite temperature. It is worth noticing that the magnitude of $\rho_1(\xi^a)\Omega)$ in Fig. 1(a) at $\xi^a < 1$ is approximately equal to a constant which progressively increases with $\Omega$ increasing. From a physical viewpoint it corresponds to the enhancement of power absorption with the growth of $\Omega$ due to the increasing of the viscous losses accordingly to GR’s mechanism.

In Fig. 1(c) the dimensionless ac resistivity $\rho_{1,T}^+(\xi^d)$ ( where $\xi^d \equiv |j^d|/j_c$) which is proportional to $\rho_{1,T}^{ac,+}$, and calculated at different values of $\xi^a$ is shown. These $\rho_{1,T}^+(\xi^d)$ dependences demonstrate several main features. First, the curves, calculated at $\xi^d \ll 1$ show the
progressive shrinking of the flux creep range (where the $\rho_{\text{dc}}^+(\xi^d) \ll 1$) with the $\xi^a$ increasing. If we define the $\xi_d^c(\xi^a)$ as the dependence of the dc critical current on the value of a small ac driving, then we can show that $\xi_c^d \simeq 1 - \xi^a$ at $\xi^a \ll 1$. Second, the appearance of a high peak in $\rho_{\text{dc}}^+(\xi^d)$ near $\xi^d = 1$ for $\xi^a \to 0$ follows from an examination of the dynamic dc resistivity (taken in the vicinity of the $\xi^d$) which equals to the derivative of the dc CVC with respect to the $\xi^d$ and is strongly enhanced at $T \to 0$. Third, as it was shown recently in [6], a strong enhancement of the effective diffusion coefficient $D$ of an overdamped Brownian particle in a tilted washboard potential near the critical tilt may occur; that, in our case, $D(\xi^d)$ may have a peak in the vicinity of $\xi^d = 1$.

The consequences of the $D$-enhancement we analyze with the aid of Fig. 1(d) where the frequency dependence of $\rho_{\text{dc}}^+(\Omega|\xi^d = 0)$ (monotonic curves) and $\rho_{\text{dc}}^+(\Omega|\xi^d = 1)$ (nonmonotonic curves) calculated at $\xi^d = 0$ for three different temperatures ($g = 10, 20, 50$) are shown. The monotonic curves $\xi^d = 0$ agree with the results of Coffey and Clem [2] who, in fact, calculated the temperature dependence of the depinning frequency in a nontilted cosine pinning potential. In contrast to this monotonic behaviour, the nonmonotonic curves ($\xi^d = 1$) demonstrate two characteristic features. First, an anomalous power absorption ($\rho_{\text{dc}}^+(\xi^d) \simeq 1.6 \div 2.8$) at very low frequencies. Second, a deep minimum for the power absorption ($\rho_{\text{dc}}^+(\xi^d) \simeq 0.3 \div 0.6$) at $T$-dependent $\Omega_{\text{min}}$. The appearance of this frequency- and temperature dependent minimum at $\xi^d = 1$ may be related to the resonance activated reduction of the mean escape time of the Brownian particle due to an oscillatory variation of the pinning barrier height [7].

4. Conclusion
In conclusion, it was shown how pronounced nonlinear effects appear in the \textit{ac} response and the linear response solutions are recovered from the nonlinear \textit{ac} response in the weak \textit{ac} current limit. Up to now we have considered only the vortex motion problem. For the future experimental verification of our theoretical findings we should keep in mind that they may be applied directly only for thin-film superconductors in the form of naturally grown (for example, in the untwinned $a$-axis oriented YBCO film [8]) and artificially prepared washboard pinning structures [9]. An application of our results for more general cases should take into account that they may be supplemented by consideration of the complex penetration length and the quasiparticle contribution in the way as it was made in the seminal CC’s paper [2].

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