Reliability estimation for the randomly censored pareto distribution

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ABSTRACT

Widespread applications of random censoring in life testing experiments to estimate reliability of engineering products or systems are available. Different parametric statistical models such as exponential, Rayleigh, Weibull and Maxwell distributions are used under random censoring scheme. In this paper, random censoring under Pareto distribution is considered. The maximum likelihood estimators (MLE’s) of the model parameters and survival function were derived along with Fisher information matrix and asymptotic confidence intervals. A simulation study was performed to observe the behavior of the MLE’s. The simulation results showed that the bias and MSE were reasonably small in all cases.

Keywords: Maximum likelihood estimation, Pareto distribution, Random censoring, Reliability, Survival analysis

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1. INTRODUCTION

In survival analysis and reliability theory, it is difficult to collect life time data for all the subjects under study because of time and cost constraints. The practitioners based on the model and available information use various types of censoring schemes. Conventional Type I and Type II censoring are common censoring schemes which has been is use in many situations in survival analysis and reliability engineering. Different censoring schemes are proposed and studied in the literature. Random censoring is used in which the time of censoring is not fixed but taken as random [1]. Another censoring scheme is the progressive and hybrid censoring which has also considered in the literature [2-3].

There is a widespread application of random censoring in life testing experiments and clinical trials where both the survival and censoring times are random. Abu-Taleb et al [4] studied random censoring using exponential for survival and censoring times. Saleem and Aslam [5] studied Bayesian analysis of the Rayleigh survival time assuming random censor time. Danish and Aslam [6] studied Bayesian inference for randomly censored Weibull distribution. Krishna et al. [7] studied estimation in Maxwell distribution with randomly censored data.

The Pareto distribution has been widely used in analysis of life time data from reliability survival and engineering point of view [8]. Sindhu et al. [9] considered Bayesian estimation of the left censored data from the Pareto type II distribution. Agharzadeh et al. [10] performed estimation and reconstruction based on left censored data from Pareto model. Usta and Gezer [11] studied reliability estimation in Pareto-I distribution based on progressively type II censored sample with binomial removals. Shou [12] studied estimation for the two-parameter Pareto distribution under progressive censoring with uniform removals.
Wu and Chang [13] studied inference in the Pareto distribution based on progressive type II censoring with random removals. Parsi et al. [14] studied simultaneous confidence interval for the parameters of Pareto distribution under progressive censoring.

In this paper the estimation problem for random censoring under Pareto distribution is considered, the survival times $X_i$’s and the censoring times $T_i$’s follow Pareto distributions. The maximum likelihood estimators for the model parameters and the survival functions are derived along with Fisher information matrix and asymptotic confidence intervals. The estimates of the survival function and asymptotic properties are given along with Fisher information matrix and asymptotic confidence intervals. The results of the simulation study are presented and discussed.

2. ESTIMATION OF THE PARAMETERS AND ASYMPTOTIC PROPERTIES

Let $X_1, X_2, ..., X_n$ be i.i.d. positive random variables (survival times) with the unknown survival function $S_1(x) = P(X_1 > x)$. Also, let $T_1, T_2, ..., T_n$ be i.i.d. positive random variables (censoring times) with the unknown survival function $S_2(x) = P(T_1 > t)$; $T$ is called a censoring variable. Assume that all $X_i$’s and $T_i$’s are independent variables. A randomly censored data set consists of $n$ i.i.d. pairs $(Y_i, D_i)$, where $Y_i = \text{min}(X_i, T_i)$ and $D_i$ is a binary random variable $D_i = I(X_i, T_i)$; $i = 1, 2, ..., n$; $D_i = 1$ if it is observed that $X_i \leq T_i$ and $D_i = 0$ if it is observed that $X_i > T_i$; $Y$ is called a censored survival or failure time random variable. Assuming $X_i$, the survival time, $i=1,...,n$, and $T_i$, censoring time, are independent Pareto distributions with probability density functions given by:

$$f_{X_i}(x_i) = \frac{\alpha}{x_i^{\alpha+1}} \quad 1 \leq x_i < \infty$$

and

$$f_{T_i}(t_i) = \frac{\beta}{t_i^{\beta+1}} \quad 1 \leq t_i < \infty$$

The survival function is

$$S(x_i) = P(X > x_i) = \frac{1}{x_i^\alpha}$$

Compared to the exponential distribution, the Pareto tail retain much more probability; and the survival function will decay slowly to zero; and it could be a good model to describe such data with heavy tailed behavior. If $(Y_i, D_i), i=1,...,n$ is observed, then the distribution of the Bernoulli random variable $D_i$ is

$$P(D_i = 1) = P(X_i \leq T_i) = \int_{x_i}^{\infty} \frac{\alpha}{x_i^{\alpha+1}} t_i^{\beta+1} dt_i dx_i = \frac{\alpha}{\alpha + \beta}$$

hence

$$P(D_i = 0) = \frac{\beta}{\alpha + \beta}$$

It can be shown that the joint probability density function of $(Y_i, D_i)$ is given by

$$f_{Y_i, D_i}(y_i, d_i, \alpha, \beta) = (\alpha + \beta)y_i^{-(\alpha + \beta)} d_i^\alpha \left( \frac{\beta}{\alpha + \beta} \right)^d_i, y_i > 0, \alpha, \beta > 0$$
Having in mind that, \( Y_i \) have a Pareto distribution with parameter \((\alpha + \beta)\), to derive the maximum likelihood estimators for \( \alpha \) and \( \beta \), the likelihood function should be used and is given by

\[
L(\alpha, \beta \mid y_1, \ldots, y_n, d_1, \ldots, d_n) = (\alpha + \beta)^n \left( \prod_{i=1}^{n} y_i^{-(\alpha + \beta) - 1} \right) \left( \frac{\alpha}{\alpha + \beta} \right)^{\sum y_i} \left( \frac{\beta}{\alpha + \beta} \right)^{\sum d_i}
\]

(6)

\[
L(\alpha, \beta \mid y_1, \ldots, y_n, d_1, \ldots, d_n) = (\alpha + \beta)^n \left( \prod_{i=1}^{n} y_i^{-(\alpha + \beta) - 1} \right) \left( \frac{\alpha}{\beta} \right)^{\sum y_i} \left( \frac{\beta}{\alpha + \beta} \right)^{\sum d_i}
\]

(7)

Taking logarithm of both sides

\[
L'(\alpha, \beta \mid y_1, \ldots, y_n, d_1, \ldots, d_n) = \ln(\alpha + \beta)^n + \ln \left( \prod_{i=1}^{n} y_i^{-(\alpha + \beta) - 1} \right) + \ln \left( \frac{\alpha}{\beta} \right)^{\sum y_i} + \ln \left( \frac{\beta}{\alpha + \beta} \right)^{\sum d_i}
\]

(8)

\[
= \ln(\alpha + \beta)^n + \sum_{i=1}^{n} \ln(y_i^{-(\alpha + \beta) - 1}) + \left( \sum_{i=1}^{n} d_i \right) \ln(\alpha) - \left( \sum_{i=1}^{n} d_i \right) \ln(\beta) + n \ln(\beta) - n \ln(\alpha + \beta)
\]

Taking partial derivatives of log likelihood function with respect to \( \alpha \) and \( \beta \), equating to zero, and checking the second partial derivatives to be less than zero, the following maximum likelihood estimators can be obtained:

\[
\hat{\alpha} = \frac{\sum_{i=1}^{n} d_i}{\sum_{i=1}^{n} \ln y_i}
\]

\[
\hat{\beta} = \frac{n - \sum_{i=1}^{n} d_i}{\sum_{i=1}^{n} \ln y_i}
\]

The above estimators are ratio estimators. The means and variances of these estimators are difficult to obtain; thus, and the performance of the estimators will be investigated via simulation. The information matrix is given by:

\[
I(\Theta) = \begin{bmatrix}
\frac{n}{\alpha(\alpha + \beta)} & 0 \\
0 & \frac{n}{\beta(\alpha + \beta)}
\end{bmatrix}
\]

Therefore, the asymptotic variance-covariance matrix, i.e. the inverse of the information can be expressed as:

\[
\sum = I^{-1}(\Theta) = \begin{bmatrix}
\frac{\alpha(\alpha + \beta)}{n} & 0 \\
0 & \frac{\beta(\alpha + \beta)}{n}
\end{bmatrix}
\]

The asymptotic distribution of MLE’s of the parameters \( \alpha \) and \( \beta \) can be written as

\[
(\hat{\alpha} - \alpha, \hat{\beta} - \beta) \approx N_2(0, I^{-1}(\Theta))
\]
Since $I^1(\Theta)$ involve the unknown parameters, they can be replaced by the corresponding ML estimates. Thus, the approximate $(1-\alpha)$ 100% confidence intervals for $\alpha$ and $\beta$, respectively, are

$$\alpha \pm \frac{Z_{\alpha/2}}{\sqrt{\hat{V}_{11}}} \quad \text{and} \quad \beta \pm \frac{Z_{\alpha/2}}{\sqrt{\hat{V}_{22}}}$$

where $\hat{V}_{11}$ and $\hat{V}_{22}$ are the estimated variances.

### 3. ESTIMATION OF THE SURVIVAL FUNCTION AND ASYMPTOTIC PROPERTIES

To obtain the maximum likelihood estimators of the survival functions, $S_1(x)$ and $S_2(y)$. The following analysis can be performed. To fix $x$ and $y$ let

$$H(\Theta) = [S_1(x), S_2(y)] = [P(X_1 > x), P(Y_1 > y)]$$

Using invariance property of the MLE, the MLE of $S_1(x)$ and $S_2(y)$ can be expressed as:

$$\hat{P}_o(X_1 > x) = \int_x^{\infty} \frac{\alpha}{z^{\alpha+1}} \, dz = x^{-\alpha}$$

and

$$\hat{P}_o(Y_1 > y) = \int_y^{\infty} (\alpha + \beta)z^{-(\alpha+\beta)-1} \, dz = y^{-(\alpha+\beta)}$$

Thus the MLE of $H(\Theta) = [x^{-\alpha}, y^{-(\alpha+\beta)}]$ can be expressed as:

$$\hat{H}(\Theta) = [x^{-\hat{a}}, y^{-(\hat{a}+\hat{b})}]$$

The above estimators are ratio estimators; means and variances of these estimators are difficult to obtain, thus, the performance of the estimators will be investigated via simulation. Under certain regularity conditions, the asymptotic distribution can be given by [14]

$$\sqrt{n}[H(\Theta) - H(\Theta_0)] \rightarrow N(0, \frac{\partial H(\Theta)}{\partial \Theta} \sum \left(\frac{\partial H(\Theta)}{\partial \Theta}\right)^T)$$

The matrix of partial derivatives

$$\frac{\partial}{\partial \Theta} H(\Theta) = \begin{bmatrix} Ax^{-\alpha} & By^{-(\alpha+\beta)} \\ 0 & By^{-(\alpha+\beta)} \end{bmatrix}$$

where $A = -\frac{1}{\log x}$ and $B = -\frac{1}{\log y}$

Thus the asymptotic variance-covariance matrix of $\hat{H}(\Theta)$ is given by

$$\begin{bmatrix} \alpha + \beta \underline{\underline{\text{(}}A^2 \alpha x^{-2\alpha} + B^2 \beta y^{-2(\alpha+\beta)}\underline{\underline{\text{)}}}} & \underline{\underline{\text{B}}} \beta (\alpha + \beta) y^{-(\alpha+\beta)} \\ \underline{\underline{\text{B}}} \beta (\alpha + \beta) y^{-(\alpha+\beta)} & \underline{\underline{\text{B}}} \beta (\alpha + \beta) y^{-(\alpha+\beta)} \end{bmatrix}$$
4. SIMULATION

A Monte Carlo simulation was used to investigate the performance of the maximum likelihood estimators of \( \alpha \), \( \beta \), and the survival function \( P(X_1 > x_i) \) developed in the previous sections. For the survival function the value of \( x_i \), was considered as the mean of the survival time. The simulation study was conducted using different combinations of parameter values and sample sizes of 50, 150, and 300. The simulation results were based on 1000 replicates. The means and root mean square errors (RMSE) of the maximum likelihood estimators were calculated.

With combination of different settings of \( \alpha \), \( \beta \) and sample size \( n \), in every case we the following algorithm was used to obtain the simulation results:

*Step 1:* Generate \( x_1, x_2, \ldots, x_n \) from Pareto(\( \alpha \))

*Step 2:* Generate \( T_1, T_2, \ldots, T_n \) from Pareto(\( \beta \))

*Step 3:* Set \( Y_i = \min(X_i, T_i) \), \( i = 1, 2, \ldots, n \)

*Step 4:* For \( i = 1, 2, \ldots, n \), set

\[
D_i = \begin{cases} 
1 & \text{if } X_i \leq T_i \\
0 & \text{if } X_i > T_i 
\end{cases}
\]

*Step 5:* Set the maximum likelihood estimates as follows

\[
\hat{\alpha} = \frac{\sum_{i=1}^{n} d_i}{\sum_{i=1}^{n} \ln y_i} \quad \text{and} \quad \hat{\beta} = \frac{n - \sum_{i=1}^{n} d_i}{\sum_{i=1}^{n} \ln y_i}
\]

S6: Repeat steps S1-S5 for 1000 iterations, compute the mean and mean square error of the maximum likelihood estimates for \( \alpha \) and \( \beta \). The results are displayed in Tables 1-3. The following remarks can be drawn based on these results:

- The results indicate that the bias and MSE’s were reasonably good in all cases.
- As the shape of the parameters \( \alpha \) and \( \beta \) increases (curves decays faster), the RMSE of the maximum likelihood estimators of the parameters \( \alpha \) and \( \beta \) increases.
- As the survival times curve decreases faster (\( \alpha \) increases) and censoring times curve decays slower (\( \beta \) decreases), the RMSE decreases. As survival times curve slowly decreases (\( \alpha \) decreases) and censoring times curve decays faster (\( \beta \) increases), the RMSE increases.
- As expected the bias and RMSE decrease as the sample sizes increases.

Means and RMSE of MLE’S of \( \alpha \) and \( \beta \), \( P(X_1 > x_i) \), \( \hat{P}(X_1 > x_i) \), \( n = 50 \), \( n = 150 \), \( n = 300 \) as shown in Tables 1, 2 and 3.

| \( \alpha \) | \( \hat{\alpha} \) | RMSE | \( \beta \) | \( \hat{\beta} \) | RMSE | \( P(X_1 > x_i) \) | \( \hat{P}(X_1 > x_i) \) | RMSE |
|---|---|---|---|---|---|---|---|---|
| 2 | 2.02535 | 0.41439 | 2 | 2.04536 | 0.42040 | 0.25000 | 0.25532 | 0.06860 |
| 2 | 2.02742 | 0.49390 | 4 | 4.08861 | 0.73840 | 0.25000 | 0.25906 | 0.08334 |
| 4 | 4.06462 | 0.71462 | 2 | 2.04346 | 0.31641 | 0.31641 | 0.31690 | 0.06187 |
| 4 | 4.05071 | 0.82878 | 4 | 4.09072 | 0.84080 | 0.31641 | 0.32032 | 0.07190 |
| 4 | 4.03601 | 0.91022 | 6 | 6.14759 | 1.66677 | 0.31641 | 0.32350 | 0.08030 |
| 4 | 0.05484 | 0.98780 | 8 | 8.17723 | 1.47679 | 0.31641 | 0.32353 | 0.08665 |
| 6 | 6.10108 | 1.33143 | 4 | 4.07434 | 0.93154 | 0.33490 | 0.33552 | 0.06586 |
| 8 | 8.12923 | 1.42924 | 4 | 4.08692 | 1.02753 | 0.34361 | 0.34366 | 0.06245 |
| 6 | 6.07607 | 1.24312 | 6 | 6.12608 | 1.26120 | 0.33490 | 0.33843 | 0.07235 |
| 6 | 6.05401 | 1.36534 | 9 | 9.22139 | 1.75015 | 0.33490 | 0.34154 | 0.08071 |
| 6 | 6.08226 | 1.48170 | 12 | 12.2683 | 2.21519 | 0.33490 | 0.34148 | 0.08706 |
| 9 | 9.15162 | 1.69715 | 6 | 6.11149 | 1.39731 | 0.34644 | 0.34686 | 0.06604 |
| 12 | 12.1938 | 2.14386 | 6 | 6.17038 | 1.54130 | 0.35200 | 0.35192 | 0.06256 |
Table 2. Means and RMSE of MLE’s of $\alpha$ and $\beta$, $P(X_1 > x_j)$. $\hat{P}(X_1 > x_j)$, $n=150$

| $\alpha$ | $\hat{\alpha}$ | RMSE $\beta$ | $\hat{\beta}$ | RMSE | $P(X_1 > x_j)$ | $\hat{P}(X_1 > x_j)$ | RMSE |
|---|---|---|---|---|---|---|---|
| 2 | 2.01499 | 0.23961 | 2 | 2.01413 | 0.23135 | 0.25000 | 0.25076 | 0.04059 |
| 2 | 2.01753 | 0.28949 | 4 | 4.02014 | 0.40678 | 0.25000 | 0.25186 | 0.04918 |
| 4 | 4.02558 | 0.41826 | 2 | 2.02200 | 0.28626 | 0.31641 | 0.31633 | 0.03733 |
| 4 | 4.02999 | 0.47922 | 4 | 4.02826 | 0.46271 | 0.31641 | 0.31662 | 0.04662 |
| 4 | 4.03125 | 0.53591 | 6 | 6.03428 | 0.64446 | 0.31641 | 0.31752 | 0.04807 |
| 4 | 4.03507 | 0.57899 | 8 | 8.04029 | 0.81757 | 0.31641 | 0.31750 | 0.05160 |
| 6 | 6.03479 | 0.65988 | 4 | 4.04445 | 0.51701 | 0.33490 | 0.33490 | 0.03951 |
| 8 | 8.05115 | 0.83652 | 4 | 4.04400 | 0.57353 | 0.34361 | 0.34337 | 0.03765 |
| 6 | 6.04498 | 0.71883 | 6 | 6.04239 | 0.69406 | 0.33490 | 0.33497 | 0.04294 |
| 6 | 6.04687 | 0.80387 | 9 | 9.05144 | 0.96668 | 0.33490 | 0.33557 | 0.04835 |
| 6 | 6.05261 | 0.86840 | 12 | 12.06942 | 1.22635 | 0.33490 | 0.33579 | 0.05192 |
| 9 | 6.05261 | 0.86840 | 6 | 6.06660 | 0.77551 | 0.34644 | 0.34662 | 0.03961 |
| 12 | 12.07673 | 1.25479 | 6 | 6.06600 | 0.86000 | 0.35200 | 0.35171 | 0.03771 |

Table 3. Means and RMSE of MLE’s of $\alpha$ and $\beta$, $P(X_1 > x_j)$. $\hat{P}(X_1 > x_j)$, $n=300$

| $\alpha$ | $\hat{\alpha}$ | RMSE $\beta$ | $\hat{\beta}$ | RMSE | $P(X_1 > x_j)$ | $\hat{P}(X_1 > x_j)$ | RMSE |
|---|---|---|---|---|---|---|---|
| 2 | 2.00122 | 0.16211 | 2 | 2.01420 | 0.17441 | 0.25000 | 0.25135 | 0.02780 |
| 2 | 2.00160 | 0.19295 | 4 | 4.02854 | 0.30462 | 0.25000 | 0.25192 | 0.03300 |
| 4 | 4.00474 | 0.28276 | 2 | 2.01401 | 0.20940 | 0.31641 | 0.31701 | 0.02553 |
| 4 | 4.00244 | 0.32421 | 4 | 4.02840 | 0.34881 | 0.31641 | 0.31755 | 0.02921 |
| 4 | 4.00807 | 0.35491 | 6 | 6.03764 | 0.47956 | 0.31641 | 0.31730 | 0.03189 |
| 4 | 4.00319 | 0.35859 | 8 | 8.05708 | 0.60924 | 0.31641 | 0.31804 | 0.03466 |
| 6 | 6.00618 | 0.44772 | 4 | 4.02682 | 0.38529 | 0.33490 | 0.33562 | 0.02703 |
| 8 | 8.00949 | 0.56551 | 4 | 4.02802 | 0.41801 | 0.34360 | 0.34414 | 0.02575 |
| 6 | 6.00365 | 0.48632 | 6 | 6.04260 | 0.52322 | 0.33490 | 0.33598 | 0.02940 |
| 6 | 6.01210 | 0.53237 | 9 | 9.05646 | 0.71934 | 0.33490 | 0.33572 | 0.03209 |
| 6 | 6.00479 | 0.57884 | 12 | 12.08562 | 0.91386 | 0.33490 | 0.33644 | 0.03488 |
| 9 | 9.00927 | 0.67150 | 6 | 6.04023 | 0.57794 | 0.34644 | 0.34644 | 0.34731 |
| 12 | 12.01424 | 0.84827 | 6 | 6.04203 | 0.62821 | 0.35200 | 0.35251 | 0.02570 |

5. CONCLUSION

The MLE’s for model parameters and survival functions were derived along with the Fisher information matrix and asymptotic confidence intervals. A simulation study verified the behavior of the MLE’s. The results showed that the bias and MSE were reasonably small in all cases. It is shown that as the shape parameters for Pareto distributions increases (i.e. curves decays faster), the RMSE of the MLE’s increases. As the survival times curve decreases faster and censoring times curve decays slower, the RMSE decreases and via versa.

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