Automatic Identification Method for Natural Frequency of Bridge Piers by Microtremor Measurement at Both Sides on Top of Pier

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The natural frequency of a bridge pier is used as an index to evaluate the soundness of pier foundations. However, using conventional approaches, it is sometimes difficult to obtain the natural frequency derived from microtremor, because the peak of the Fourier spectrum of the bridge piers derived from the microtremor does not appear clearly in many cases compared with that derived from the impact vibration test. In this paper, we propose a method that can automatically identify the natural frequencies using only derived from microtremor measurement results at both sides on the top of piers, without renewing the latest result of impact vibration test.

Keywords: microtremor, natural frequency, spread foundation, scour

1. Introduction

The river bed around pier foundations on railway bridges that cross rivers is scoured due to increases in the volume of water, resulting in instability, sinking and settling of the bridge piers, which may in turn cause serious accidents and transport disruption. Cases involving this type of chain of events have been reported on bridges with shallow and spread foundations [1].

The main method to evaluate the integrity of bridge piers is an impact vibration test [2] that identifies the natural frequency of a bridge pier from the responsive wave shape obtained by striking the bridge with a weight. This method determines whether the ground under the foundations has become unstable due to scouring by analyzing changes in the natural frequency of bridge piers before and after of the rising of the river. Performing an evaluation using this method however, requires a 30 kg weight, making it unsuitable for continuous measurement.

The authors propose a method to identify the natural frequency using a transfer function, which is obtained from the estimated ground vibration using microtremor measurement data measured at both ends of the pier top [3]. Furthermore, assuming the vibration of the bridge pier is a single-degree-of-freedom system, a new method for identifying the natural frequency was developed based on the premise that the transfer function is theoretically represented by a resonance curve [4], [5]. This paper describes the outline of the new identification method for analyzing disaster hazards, and its application to actual bridges to verify the validity of the method.

2. Outline of method for identifying natural frequency

2.1 Data processing flow

Figure 1 shows the flow of measurement data processing in the automatic identification method. Processes II, III, and IV in Fig.1, i.e. the method for identifying the natural frequency based on vibration theory is described in Section 2.2. Process V, or, the process for automatically identifying the natural frequency is described in Section 2.3.

Fig. 1 The flow of measurement data processing in the automatic identification method

2.2 Outline of the identification method of natural frequency based on vibration theory

2.2.1 Microtremor measurement on both sides of the top of the pier

Microtremor measurement data must be obtained by measuring the horizontal axis perpendicular to the bridge direction (hereinafter simply called horizontal axis) and the vertical axis at both ends (at upstream and at down-
stream) of the pier top. In this paper, all waveforms are one of velocity.

In the following explanation, sensor A is the sensor installed on the upstream side of the bridge pier top, and sensor B is the sensor installed on the downstream side.

When the measurement data is saved at a sample interval of $t_s$ seconds, velocity waveform from time $t - t_s$ to time $t$ (hereinafter referred to as velocity at time $t$) of horizontal axis and vertical axis are defined as $x_s(t)$ and $z_s(t)$ respectively. Similarly, sensor B is defined as $x_b(t)$ and $z_b(t)$ respectively. The direction of velocity is defined as positive, from upstream to downstream in the horizontal axis, and from bottom to top in the vertical axis.

### 2.2.2 Calculation of bridge pier oscillation center

In estimating the ground vibration, we obtain the position of the oscillation center provided that the bridge pier is oscillating around the rotational center (hereinafter referred to as primary vibration). Previous studies show that the position varies depending on the depth of penetration and does not exist necessarily at the bottom of the bridge pier [5]. The oscillation center position is calculated using the following method.

![Fig. 2 Conceptual diagram of primary vibration of bridge pier [3]](image)

As shown in Fig. 2, it is assumed that the position of the oscillation center is at a depth of $h_0$ from the top. Here, $a$ and $b$ are determined so that the distance between the two sensors becomes $(a + b) \times h_0$. Next, the velocity of horizontal axis during ground vibration is defined as $x_s(t)$, and the velocity of vertical axis during ground vibration is defined as $z_s(t)$. The velocity of the horizontal axis components of sensors A and B, the vibration components due to the primary vibration of the structure are defined as $x_m(t)$ and $x_m(t)$, respectively. The vibration components due to the primary vibration of the structure of the velocity of the vertical axis components of sensors A and B are defined as $z_m(t)$ and $z_m(t)$, respectively. Assuming that the vibration waveform measured at the top of the bridge pier is the sum of the primary vibration and ground vibration of the structure, and ignoring the effects of structures other than the bridge pier such as high-order vibration and girder, we can get approximations (1) and (2) as follows:

$$x_s(t) = x_m(t) + x_s(t)$$  \hspace{1cm} (1)

$$z_s(t) = z_m(t) + z_s(t)$$  \hspace{1cm} (2)

Also, equation (3) is obtained from the geometric relationship shown in Fig. 2, and (4) is obtained from (1) to (3).

$$x_s(t) = z_s(t)/\tan \alpha = 1/a \cdot z_s(t)$$  \hspace{1cm} (3)

$$x_s(t) - x_s(t) = 1/a \cdot [z_s(t) - z_s(t)]$$  \hspace{1cm} (4)

As the primary vibration is dominant in the frequency band near the natural frequency of the bridge pier, the Fourier amplitude on the bridge pier is generally several times greater than the ground vibration, though it depends on the magnitude of the damping constant. If the natural frequency at time $t$ is $f_c$, the velocities $f_{bp}x_s(t)$ and $f_{bp}z_s(t)$ where the primary vibration is dominant can be obtained by applying a bandpass filter to the velocity data $x_s(t)$ and $z_s(t)$ at time $t$ if they include the frequency $f_c$. The relationship between $f_{bp}x_s(t)$ and $f_{bp}z_s(t)$ can be approximately expressed as follows (5) from (4).

$$f_{bp}x_s(t) = \frac{1}{a} \cdot f_{bp}z_s(t)$$  \hspace{1cm} (5)

Similarly, the velocities $f_{bp}y_s(t)$, and $f_{bp}y_s(t)$ where the primary vibration is dominant can be obtained by applying a bandpass filter that includes the frequency $f_c$ to the velocity data $y_s(t)$ and $y_s(t)$ at time $t$. The relationship between $f_{bp}x_s(t)$ and $f_{bp}z_s(t)$ can be approximately expressed as follows (6):

$$f_{bp}x_s(t) = -\frac{1}{b} \cdot f_{bp}y_s(t)$$  \hspace{1cm} (6)

When the relationships between $f_{bp}x_s(t)$ and $f_{bp}y_s(t)$, and between $f_{bp}x_s(t)$ and $f_{bp}x_s(t)$ are plotted on a plane consisting of horizontal axis and vertical axis, (5) and (6) draw a trajectory perpendicular to the center of vibration. In actual vibration, the trajectory varies, but since it can be approximated by a straight line passing through the origin, following (7) and (8) is obtained by the least-squares method.

$$a = \frac{n\Sigma_{i=1}^n [f_{bp}x_s(t) \cdot f_{bp}y_s(t)] - \Sigma_{i=1}^n f_{bp}x_s(t) \Sigma_{i=1}^n f_{bp}y_s(t)}{n\Sigma_{i=1}^n [f_{bp}x_s(t)]^2 - \left(\Sigma_{i=1}^n f_{bp}x_s(t)\right)^2}$$  \hspace{1cm} (7)

$$b = \frac{n\Sigma_{i=1}^n [f_{bp}x_s(t) \cdot f_{bp}y_s(t)] - \Sigma_{i=1}^n f_{bp}x_s(t) \Sigma_{i=1}^n f_{bp}y_s(t)}{n\Sigma_{i=1}^n [f_{bp}y_s(t)]^2 - \left(\Sigma_{i=1}^n f_{bp}y_s(t)\right)^2}$$  \hspace{1cm} (8)

Here, $a + b$ is obtained from (7) and (8). By dividing the distance between the sensors A and B by this value, the height $h_0$ of the oscillation center is obtained. In calculating $a$ and $b$, we have to be careful in considering the frequency band of the bandpass filter. This technique will be discussed in the Section 2.3.

### 2.2.3 Estimating ground vibration

Similar to (2), $z_s(t)$ can be approximated by (9). Also, (10) is obtained geometrically.

$$z_s(t) = z_m(t) + z_s(t)$$  \hspace{1cm} (9)

$$z_s(t) = -a/b \cdot z_m(t)$$  \hspace{1cm} (10)

From (2), (9), and (10),

$$z_m(t) = a / (a + b) \cdot [z_s(t) - z_s(t)]$$  \hspace{1cm} (11)
Equation (12) is obtained from (3), and (11).

\[ x_n(t) = \frac{1}{(a + b)} \cdot \{z_n(t) - z_{n-1}(t)\} \]  

(12)

And, equation (13) is obtained from (1), and (12).

\[ x_g(t) = x_n(t) - \frac{1}{(a + b)} \cdot \{z_n(t) - z_{n-1}(t)\} \]  

(13)

Finally, by substituting the values of \(a\) and \(b\) obtained by (7) and (8) into (13), we can estimate \(x_g(t)\), the velocity of horizontal axis during ground vibration. In addition, from the measurement using the model bridge pier and measurements from an actual bridge, we confirmed that the ground vibration can be properly estimated by (13).

2.2.4 Fitting to the theoretical formula

Let us assume the simplification, where the vibration of the bridge pier is a one-degree-of-freedom system with viscous damping that is forcibly excited from the ground.

In this assumption, the transfer function expressed by the Fourier amplitude ratio between the microtremors at the top of the bridge pier and at the bottom of the foundation can be expressed as a resonance curve with the natural frequency and damping constant as parameters. Here, the Fourier transform of the horizontal axis waveform at time \(t\), is defined as \(x^h(\omega)\), and the Fourier transform of the horizontal axis waveform during ground vibration obtained by (13) is \(x_g(\omega)\). The resonance curve of those values is expressed by (14) [6].

\[ \frac{x^h(\omega)}{x^g(\omega)} = \frac{1 + \left(\frac{2hf}{\omega_f} \right)^2}{\left(1 - \left(\frac{f}{f_0}\right)^2\right)^2 + \left(\frac{2hf}{\omega_f} \right)^2} \]  

(14)

Where \(f\) is the frequency (Hz), \(f_0\) is the natural frequency of the bridge pier (Hz), and \(h\) is the damping constant. We can obtain \(f_0\) and \(h\) at time \(t\) (hereinafter referred to as \(f(t)\) and \(h(t)\) respectively) by fitting the theoretical expression of (14) to the transfer function obtained by data processing as mentioned above.

2.3 Automatic calculation algorithm for identifying natural frequency of bridge pier by dividing process [4]

In the method for identifying the natural frequency described in the previous section, it is important to obtain accurate values of \(a\) and \(b\). For this purpose, bandpass filter processing in a frequency band including the natural frequency \(f(t)\) is necessary. However, since the natural frequency value is unknown in the calculation process, it is difficult to set the frequency band of the bandpass filter in advance. Furthermore, on actual bridge piers, as scouring progresses, the depth of root penetration and ground conditions at the bottom of the bridge pier change. In addition, the oscillation center position may change with the fluctuation of the water pressure applied to the bridge pier.

For these reasons, the values of \(a\) and \(b\) cannot be fixed to a constant for each bridge pier, and must be calculated for each set of fresh measurements. Therefore, the procedure in the previous section has to be divided and calculated for each frequency band as follows: after excluding some of the obtained natural frequencies with some condition settings, we should identify the most probable natural frequencies among them.

2.3.1 Dividing the measured waveform for each frequency band

The obtained measurement data is divided using a band pass filter as shown below.

1) The natural frequency of the pier is usually 1 Hz or more and less than 20 Hz, though it depends on the structure and the condition of the foundation ground. The frequency band in this range is divided into \(n\) bands. As shown in Fig.3, each frequency band has a frequency width \(b\) which overlaps by \(m\), such as 1 to \(f_{k-1}\) to \(f_{k}\) Hz set in 1), coefficients \(a\) and \(b\) are found using (7) and (8), to obtain the height of the oscillation center.

2) For each \(k\)th frequency band \((f_{k-m} \leq f \leq f_k)\) Hz set in 1), coefficients \(a\) and \(b\) are found using (7) and (8), to obtain the height of the oscillation center.

3) For each \(k\)th frequency band \((f_{k} \leq f \leq f_{k+1})\) Hz set in 1), the ground vibration \(x_g(t)\) is estimated with (13).

4) For each frequency band set in 1), the Fourier amplitude ratio is obtained from the ground vibration \(x_g(t)\) estimated in 3) and the vibration \(x_g(t)\) on the bridge pier. These are fitted with the theoretical formula (14) to find the optimum \(f(t)\), \(h(t)\) and the coefficient of determination at that time, where the possible value of \(f(t)\) is set to the frequency band set in 1). The range of possible values for the damping constant \(h(t)\) is set to a slightly expanded (approximately 0.01 to 0.40) value from the range of possible general attenuation constants in consideration of the occurrence of calculation errors.

5) For all the divided frequency bands, the calculation process from 2) to 4) is performed.
2.3.2 Exclusion by position of oscillation center

The oscillation center position is usually located near the bottom surface and is slightly above when the penetration depth is larger [5], [7]. It is determined that the natural frequency of the bridge pier is not included in this frequency band when the oscillation center is significantly deeper than the bottom surface or near the top of the pier.

2.3.3 Exclusion of results for identifying natural frequencies on boundary values

When \( f(t) \) fitted by the processing in paragraph 2.3.1 4) has the best coefficient of determination on the boundary value set as a possible value of \( f(t) \), it is determined that the natural frequency is not included in this frequency band, for the following reason: even if the value of \( a_n + b_n \) is obtained by chance despite the natural frequency not being included in the frequency band \( f_{k-1} \) to \( f_k \) Hz, the coefficient of determination is considered to be the best on the boundary value that is closer to the natural frequency as shown in Fig.4. Similarly, in the case of \( h(t) \) results with the best determination coefficient on the boundary value are excluded.

2.3.4 Exclusion of results with a relatively small coefficient of determination

If the natural frequency is not included in the frequency band \( (1+ (n-1)m) \) to \( (1+ (n-1)m+k) \) Hz, the correct value of \( a_n + b_n \) cannot be obtained and the ground vibration \( x_g(t) \) cannot be estimated correctly. Therefore, since a correct transfer function cannot be obtained, even if fitting can be performed within the range of the relevant frequency band in the processing of 2.3.1 4), it is out of the original theoretical formula and the coefficient of determination becomes small. Consequently, \( f(t) \) with the highest coefficient of determination among the fitted results is the natural frequency as shown in Fig.4.

3. Example of proposed method applied to actual pier

3.1 Target bridge piers

This Chapter introduces a case where the method described in Chapter 2 was applied to actual bridge piers. The results were compared and verified with the natural frequency obtained in the impact vibration test. Table 1 shows a total of 12 piers with 7 bridges that were evaluated using the new method. The foundations were either spread foundations (some with wood piles) or caisson foundation. Of these, examples applied to the bridge A are excerpted in Section 3.2, and the overall results are described in Section 3.3.

3.2 Application example of Bridge A

Bridge A was constructed in 1935 and is a single-line bridge consisting of a total of 12 girders with a span length of 152 m and a simple through plate girder of length of 13.5 m. The substructure is a concrete pier with an oval cross section with a wooden pile foundation. Among these piers, P4 and P5 were measured. They are adjacent bridge piers with the same structure, where the height from the bottom to the top is about 6.7 m as shown in Fig.5.

The two bridge piers had different depths of penetration, about 1.1 m for P4 and about 5.1 m for P5. At the time of microtremor measurement, P4 was submerged in the river, but P5 was not. From the impact vibration test conducted before the microtremor measurement, the natural frequency of P4 was established to be 6 Hz, and for P5 to be 11 Hz. However, since P5 had a deeper pen-

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Table 1  Actual pier specifications and natural frequency identification results

| Bridge | Pier | Height of pier (m) | Pier structure/Foundation type | Natural frequency (Hz) | New method A-B (within ±5%) | Conventional method (by dominant frequency) |
|--------|------|-------------------|-------------------------------|-----------------------|-----------------------------|-----------------------------------------------|
| A      | P4   | 6.7               | Concrete/Spread foundation    | 6.0                   | 6.0                         | ○                                             |
|        | P5   | 6.7               | Concrete/Spread foundation    | 12.1                  | 11.0                        | —                                             |
| B      | P1   | 7.3               | Concrete/Spread foundation    | 12.6                  | 12.8                        | ○                                             |
|        | P4   | 7.5               | Concrete/Spread foundation    | 12.7                  | 12.6                        | ○                                             |
| C      | P19  | 25.9              | Concrete/Caisson foundation   | 5.6                   | 5.6                         | ○                                             |
| D      | P19  | 25.9              | Concrete/Caisson foundation   | 5.1                   | 5.1                         | ○                                             |
| E      | P2   | 10.3              | Brick and stone/Spread foundation | 7.6                   | 7.3                         | ○                                             |
| F      | P3   | 17.4              | Reinforced Concrete/Spread foundation | 9.1                   | 9.2                         | ○                                             |
|        | P4   | 17.4              | Reinforced Concrete/Spread foundation | 11.3                  | 11.3                        | ○                                             |
| G      | P112 | 10.3              | Reinforced Concrete/Spread foundation | 9.6                   | 9.7                         | ○                                             |
|        | P113 | 10.3              | Reinforced Concrete/Spread foundation | 2.7                   | 2.7                         | ○                                             |

Fig. 4  Conceptual diagram of exclusion of identified natural frequencies
etration depth and high soundness, the natural frequency was somewhat unclear compared to P4.

The sampling rate for microtremor measurement was 200 Hz, and time taken for each measurement was 5 minutes. Figure 6 shows the Fourier amplitude spectrum of the waveform of horizontal axis obtained in the microtremor measurement. The dominant frequency in this figure is significantly different from the result of the impact vibration test, especially for P5. This was therefore a case where it would be difficult to apply the conventional method for identifying the natural frequency from the dominant frequency.

For each pier, Fig. 7 shows the Fourier amplitude ratio obtained with the new method and the fitting result that is the final solution. From the figure, the automatically calculated natural frequency was 6.0 Hz for P4 and 12.1 Hz for P5. Compared with the impact vibration test results mentioned above, P4 was almost the same. On the other hand, although the calculated value of P5 is slightly larger, it is generally considered to be a reasonable result considering that the result of the shock vibration test was somewhat unclear.

3.3 Example of application to whole bridge

Figure 8 shows the natural frequency obtained by applying the new method to the 12 actual piers shown in Table 1, compared with the results of the impact vibration test. In the figure, the natural frequencies obtained with the new method are indicated by ○, and those with the conventional method with a +. The natural frequency of only 4 piers could be determined using the conventional method, but with the new method using microtremor data, the natural frequency of all 12 piers could be identified.

Next, the deviation between the natural frequencies obtained with the new method and those obtained with the impact vibration test was considered: in Fig.8 the differences in natural frequency obtained in the impact vibration tests, in relation to those obtained with the new method, are plotted in the range of ±5% and ±10%. In determining the integrity of the bridge pier, according to the maintenance management standard [8], the ratio of the current natural frequency to the initial value of the natural frequency is used as the soundness index value, and assessment bands are set every 15%. Therefore, if the deviation of the natural frequency from the new method with respect to the impact vibration test is within ±5%, the above assessment bands can be evaluated. As such, it is considered that when the deviation of the natural frequency was within ±5%, the assessment bands were suitable. The results of application of the method to each pier are shown in Table 1: results showed that assessment bands were applicable to 10 of the 12 piers whose natural frequencies were identified with the new method. This confirmed that the applicability of natural frequency evaluation based on microtremor measurements could be greatly improved by the new method.

For the remaining two piers (one is bridge A, P5), al-
though the deviation was within $\pm 10\%$, it was still possible to identify the natural frequency. These results demonstrate that the applicability of the new method is broader than the conventional method.

4. Conclusions

A new method was developed to automatically identify the natural frequency of bridge piers using microtremor measurements on both sides of the top of piers, which was applied to an actual bridge to verify its practical validity. Results confirmed that the applicability of natural frequency evaluation using microtremor measurements could be greatly improved by using the new method.

Future work will concentrate on obtaining measurements using this method over the long term, in order to clarify application conditions. It is also necessary to generalize the method to set the bandpass filter to match the structural conditions of the pier and exclusion conditions when identifying the natural frequency.

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