THE LOSS-AVERSE NEWSVENDOR PROBLEM WITH QUANTITY-ORIENTED REFERENCE POINT UNDER CVAR CRITERION

WEI LIU
Department of Basic Science, Wuhan Donghu University
Wuhan 430212, China

SHIJI SONG
Department of Automation, Tsinghua University
Beijing 100084, China

YING QIAO*
Financial Department, Wuhan Business University
Wuhan 430056, China

HAN ZHAO
Department of Finance and Audit, Army Logistics University
Chongqing 401311, China

HUACHANG WANG
Department of Basic Science, Army Logistics University
Chongqing 401311, China

(Communicated by Hoang Xuan Phu)

Abstract. This paper studies a single-period inventory problem with quantity-oriented reference point, where the newsvendor has loss-averse preferences and conditional value-at-risk (CVaR) measure is introduced to hedge against his risk. It is shown there exists a unique optimal order quantity maximizing the CVaR of utility. Moreover, it is decreasing in loss aversion level, confidence level and target unit profit, respectively. Then we establish the sufficient conditions under which the newsvendor’s optimal order quantity may be larger than, equal to or less than the classical newsvendor solution. In particular, when the target unit profit is a convex combination of the maximum and minimum, the optimal order quantity is independent of price and cost parameters. Numerical experiments are conducted to illustrate our results and present some managerial insights.

2020 Mathematics Subject Classification. Primary: 90B05; Secondary: 91B42.

Key words and phrases. Newsvendor problem, conditional value-at-risk, loss aversion, reference point.

The paper is supported by National Key Research and Development Project of China (No. 2018YFB1702903), Research Project of Hubei Provincial Department of Education (No. B2020240), and Youth Foundation of Wuhan Donghu University (No. 2020dhzk005).

*Corresponding author: Ying Qiao.
1. **Introduction.** The single-period inventory problem, also known as the newsvendor problem, is a classical model in inventory management and has been extensively studied during the past decades. The review of the literature on this problem and its extensions can be found in Khouja [13] and Qin et al. [23].

Traditional newsvendor models typically assume the decision makers are risk-neutral and will select the expected profit-maximizing order quantity. However, some evidence (e.g., [2, 5, 6]) indicates that many human decisions defy this assumption and the actual order quantities are deviating from that maximizing the expected profit. Such deviation is referred to as decision bias in the literature ([31]). One major reason why bias exists is that the decision makers may have preferences instead of risk neutrality. Through numerical experiments, Kahneman and Tversky [12] propose prospect theory and suggest that people frame a problem around a reference point and have loss-averse preferences. More specifically, people will choose a reference point as an anchor in advance and then define incomes in terms of gains (i.e., above this point) and losses (i.e., below this point) with respect to it rather than final wealth. People are more averse to losses than they are attracted to same-sized gains, which is called loss aversion and intuitively appealing as well as supported in many fields.

Ever since the loss-averse newsvendor problem was proposed by Schweitzer and Cachon [26], loss-averse preferences have attracted considerable attention and a number of studies incorporate it into inventory management in the last two decades (see e.g., [16, 20, 31, 35]). Nevertheless, to our best knowledge, the existing papers pay little attention to the following two important issues. First, most of them are based on a zero reference point, and thus ignore the reference point effect. However, many works (e.g., [19, 40]) have shown that the reference point effect is prevalent in people’s decision-making behavior and as a cognitive bias may significantly affect people’s decisions. The interested readers are referred to Furnham and Boo [9] for detailed review of reference point effect. Second, the studies on portfolio management show that the loss-averse investors usually take on more risks (e.g., [7, 8, 10]). Thus, risk management should be considered in the inventory problem based on loss aversion and how to hedge against the potential risks for the loss-averse decision makers is becoming more important. However, there are now only a few researchers that notice this problem and investigate the loss-averse newsvendor model under conditional value-at-risk (CVaR) measure where reference point is not considered, see for example, Xu et al. [36] and Xu et al. [38]. The newsvendor’s ordering policy is still unclear when including reference point. This motivates the endeavour of this paper.

To fill this research gap, loss aversion, reference point effect and risk management are jointly considered in our paper. CVaR measure, which is initially proposed by Rockafellar and Uryasev [24, 25], has been widely used in risk and operation management as a coherent and easily-computed risk criterion (see, e.g., [4, 14, 34, 36, 38]). In view of this, it is introduced into our loss-averse system with reference point to hedge against the risks for the loss-averse newsvendor. We aim to shed light on the following problems: (1) When reference point is considered, how to determine the loss-averse newsvendor’s optimal order quantity maximizing the CVaR objective of utility? (2) How would the order quantity change as loss aversion level, confidence level or reference point increases? (3) Does the decision bias exist in this system? In other words, may the order quantity be larger than or less than the classical newsvendor solution?
In order to address the above problems, this paper investigates a loss-averse newsvendor model with quantity-oriented reference point under CVaR criterion. Here, we introduce a target unit profit as an anchor, and then the reference point is a linear function of the order quantity (33). The loss-averse newsvendor’s objective is to maximize the CVaR of utility under the quantity-oriented reference point by selecting the order quantity. To the best of our knowledge, this model has not been considered in the literature. Our paper differs from the existing studies on loss-averse newsvendor problem in three aspects. First, we obtain the newsvendor’s optimal order quantity maximizing CVaR of utility when considering reference point. Second, we provide insights into the effects of loss aversion, confidence level and target unit profit on the optimal order quantity. Finally, we demonstrate that decision bias exists in this system, and establish the sufficient conditions under which the optimal order quantity may be larger than (i.e., positive bias), equal to (i.e., no bias) or less than (i.e., negative bias) the classical newsvendor solution.

The remainder of this paper is organized as follows. In Section 2, we provide a review of the related literature. In Section 3, we introduce the proposed model and obtain the newsvendor’s optimal ordering policy, then make a complete sensitivity analysis of parameters. The numerical results are represented in Section 4. In Section 5, we conclude our paper.

2. Literature review. Based on the prospect theory established by Kahneman and Tversky [12], loss-averse preferences have drawn a great deal of attention and there is a growing body of research on the inventory problem based on loss aversion over the past decades. We here only review the papers on the loss-averse newsvendor model that is most related to ours. Note that some of them consider either reference point effect or CVaR measure, whereas we jointly consider these two factors in our loss-averse model.

Schweitzer and Cachon [26] are the first to investigate the loss-averse newsvendor problem. They show that compared with the risk-neutral newsvendor, the loss-averse one will select a smaller order quantity. Wang and Webster [31] extend their model by considering shortage cost and demonstrate that the loss-averse newsvendor may order larger than the risk-neutral one. Wang [30] studies the loss-averse newsvendor game under the proportional demand allocation rule. There exists a unique Nash equilibrium in this game. Liu et al. [17] study the newsvendor game with product substitution. Under a symmetry assumption, they point out that the supply chain inventory understocking will occur if loss aversion level is large enough. Ma et al. [21] assume the retailer has two ordering opportunities before the demand is realized, and derive the optimal order decisions at two different stages. Yu et al. [39] use robust optimization approach to investigate the loss-averse newsvendor problem and obtain the closed-form robust optimal order quantity. Vipin and Amit [29] study the newsvendor model with emergency orders and then extend it to a supply chain setting. Xu et al. [35] also investigate the model with emergency orders and show the retailer only needs to replenish the stock while does not purchase options under certain conditions. Moreover, different from above works that only focus on random demand, Liu et al. [15], Liu et al. [16], Liu et al. [18], and Ma et al. [20] all simultaneously consider supply uncertainty (random supply capacity or yield) in the inventory systems. They obtain the loss-averse newsvendor’s ordering policies and analyze the effects of loss aversion and supply uncertainty on them, respectively.
Although the loss-averse newsvendor problem has been studied in various contexts, most existing papers ignore the reference point effect and only a few pay attention to it. Herweg [11] investigates the expectation-based loss-averse newsvendor problem where both no shortage cost and shortage cost are considered, and obtains the first-order optimality conditions. Wu et al. [33] introduce a target unit profit as an anchor and investigate the loss-averse competitive newsvendor problem, in which proportional demand allocation and demand reallocation rules are considered, respectively. They find that a positive anchor always leads to inventory understocking, whereas a negative anchor may result in inventory overstocking. Bai et al. [1] also use this anchor to study the jointly pricing and ordering problem in the case of additive demand and multiplicative demand. Long and Nasiry [19], Wang and Wang [32] and Mandal et al. [22] all adopt a convex combination of the newsvendor’s maximum possible payoff and minimum possible payoff as the reference point. Long and Nasiry [19] show that prospect theory can explain the newsvendor’s ordering behavior without relying on risk attitudes if a proper reference point is chosen. Wang and Wang [32] point out that the loss-averse newsvendor’s optimal order quantity is always less than the loss-neutral newsvendor’s, whereas may be larger than, equal to or less than the classical newsvendor solution. Mandal et al. [22] study the newsvendor’s pricing and stocking decisions, and analyze the impacts of reference point on them.

Recently, to hedge against the loss-averse newsvendor’s risk, some papers study the loss-averse newsvendor problem under CVaR criterion. Xu et al. [36] investigate the newsvendor model and analyze the relation between loss aversion and fill rate target in the case with and without shortage cost. Xu et al. [38] also study this problem in which all or partial excess demands can be backlogged. They analyze the loss-averse newsvendor’s optimal order decisions to maximize the expected utility and CVaR of utility, respectively. Xu et al. [37] propose the legacy loss that is either the loss for excess order or the shortage penalty for lost sales. Then they study the newsvendor problem under three different objectives. Sun and Xu [27] think the loss from excess order has a more important effect on the retailer than revenue and then propose a loss aversion utility function. They demonstrate that the optimal order quantity maximizing CVaR of utility is less than that maximizing expected utility. Chan and Xu [3] further extend their model by considering shortage cost. Nevertheless, the above literature all ignores the reference point effect.

3. Model analysis. Consider a single-period inventory problem with stochastic customer demand and the loss-averse newsvendor. Before the selling period, the newsvendor will choose a target profit he expects to obtain as the reference point, and then make an order decision. Suppose the order will be completely fulfilled and delivered immediately. All unsold products will be salvaged at the end of the period and shortage cost is not considered. Notions concerned in this paper are listed in Table 1, and some is introduced when needed.

When the newsvendor’s order quantity is $Q$, his profit is

$$\pi(Q,D) = \begin{cases} (p - s)D - (c - s)Q, & 0 \leq D \leq Q, \\ (p - c)Q, & D > Q. \end{cases}$$

For any given $Q$, from above it follows that $\pi(Q,D)$ is increasing in $D$ when $0 \leq D \leq Q$ and then constant when $D > Q$. Therefore, the newsvendor’s minimum possible payoff is $(s - c)Q < 0$ while the maximum possible payoff is $(p - c)Q > 0$. 

\[\]
We assume the newsvendor has the following piecewise-linear loss aversion utility function:

$$U(\pi(Q, D)) = \begin{cases} 
\pi(Q, D) - \pi_0, & \pi(Q, D) \geq \pi_0, \\
\lambda(\pi(Q, D) - \pi_0), & \pi(Q, D) < \pi_0,
\end{cases}$$

(2)

where $\lambda \geq 1$ is the newsvendor’s loss aversion level and he is loss-neutral if $\lambda = 1$. The larger $\lambda$ is, the more loss-averse the newsvendor is. $\pi_0$ is the newsvendor’s reference point. Although there are multiple methods for setting reference point such as social preferences, mean demand and target profit ([1]), target profit is widely adopted in the newsvendor models with reference dependence (see e.g., [1, 19, 22, 32, 33]). Since the manager usually has a clear and definite target in practice while the target gross profit depends on the order quantity, it may be more realistic for him to choose a target unit profit instead of target gross profit as an anchor. For example, for new entrants to enter a competitive market, the average profit per unit sets the standard for the rest of the decision making ([33]). Therefore, similar to Bai et al. [1] and Wu et al. [33], we introduce a target unit profit $w_0 \in [s - c, p - c]$ as the anchor, where the minimum $s - c < 0$ is the loss per unit unsold product while the maximum $p - c > 0$ is the revenue per unit selling product. Then the target gross profit $\pi_0 = w_0Q$ is a linear function of the order quantity and referred to as the quantity-oriented reference point ([32]). The newsvendor will perceive losses if his profit at the end of the period is less than this point. Otherwise, he will perceive gains. Note that in most existing studies on the loss-averse newsvendor models (see e.g., [15, 31, 36]), the reference point $\pi_0$ is zero in (2), and thus, the reference point effect is ignored. Especially, Xu et al. [36] investigate the loss-averse newsvendor problem with zero reference point under CVaR measure, and we generalize their model to the nonzero reference point situation. Plugging (1) into (2) then
\[ U(\pi(Q, D)) = \begin{cases} (p - c - w_0)Q, & D \geq Q, \\ (p - s)D - (c - s + w_0)Q, & \frac{c - s + w_0}{p - s}Q \leq D < Q, \\ \lambda [(p - s)D - (c - s + w_0)Q], & 0 \leq D < \frac{c - s + w_0}{p - s}Q. \end{cases} \] (3)

The VaR and CVaR of utility \( U(\pi(Q, D)) \) are defined as (36)

\[ \text{VaR}_\alpha[U(\pi(Q, D))] = \sup \{y \in \mathbb{R} : P[U(\pi(Q, D)) \geq y] \geq \alpha\}, \] (4)

and

\[ CVaR_\alpha[U(\pi(Q, D))] = E[U(\pi(Q, D))|U(\pi(Q, D)) \leq \text{VaR}_\alpha[U(\pi(Q, D))]]. \] (5)

respectively, where \( \alpha \in [0, 1) \) is the newsvendor’s confidence level and reflects his degree of risk aversion. The larger \( \alpha \) is, the more risk-averse the newsvendor is. Specially, the CVaR of utility reduces to the loss-averse newsvendor’s expected utility if \( \alpha = 0 \). The newsvendor’s objective is to choose an order quantity \( Q \) to maximize \( CVaR_\alpha[U(\pi(Q, D))] \).

The following theorem characterizes the newsvendor’s optimal ordering policy.

**Theorem 3.1.** There exists a unique optimal order quantity \( Q^* \) that satisfies:

\[ (\lambda - 1)(c - s + w_0)F\left(\frac{c - s + w_0}{p - s}Q^*\right) + (p - s)F(Q^*) = (1 - \alpha)(p - c - w_0). \] (6)

**Proof.** See Appendix A.

Then the effects of loss aversion, confidence level and target unit profit on the optimal order quantity will be analyzed.

**Corollary 3.2.** The newsvendor’s optimal order quantity \( Q^* \) is decreasing in \( \lambda, \alpha \) and \( w_0 \), respectively.

**Proof.** See Appendix A.

This corollary indicates that a higher loss aversion or confidence level will result in a lower inventory level. When loss aversion or confidence level increases, the newsvendor becomes more conservative. Since shortage cost is not considered and the potential loss or risk comes only from the possible excess order, the newsvendor will select a smaller order quantity to hedge against them. In addition, when the target unit profit increases, an outcome is more likely to be perceived as a loss. Thus the newsvendor will order less products based on a similar explanation. Comparing with the zero reference point case, a positive (negative) reference point will make the newsvendor order less (more) products. Note that the effects of loss aversion and target unit profit on the order quantity are in accordance with that found by Wu et al. [33], who investigate a loss-averse competitive newsvendor problem and do not consider the risk management.

The effects of price and cost parameters on the optimal order quantity are investigated by the following corollary.

**Corollary 3.3.** The newsvendor’s optimal order quantity \( Q^* \) is decreasing in \( c \) while increasing in \( s \) and \( p \).

**Proof.** See Appendix A.
This corollary shows that the newsvendor will order less products as purchasing cost increases, while order more products as salvage value or selling price increases, which are consistent with classical newsvendor model and intuitive.

Next we will compare the optimal order quantity maximizing CVaR of utility, maximizing expected utility, maximizing expected profit (with reference point) and maximizing expected profit (without reference point). Note that the optimal order quantity maximizing CVaR of utility is denoted by \( Q^* \) and satisfies (6). If \( \alpha = 0 \), then \( Q^* \) reduces to the optimal order quantity maximizing expected utility, which is denoted by \( Q^u \) and satisfies

\[
(\lambda - 1)(c - s + w_0)F \left( \frac{c - s + w_0}{p - s}Q^*_u \right) + (p - s)F(Q^*_u) = p - c - w_0.
\]

If \( \alpha = 0 \) and \( \lambda = 1 \), then \( Q^* \) reduces to the optimal order quantity maximizing expected profit (with reference point), which is denoted by \( Q^*_p = F^{-1} \left( \frac{p - c - w_0}{p - s} \right) \).

Further, if \( \alpha = 0 \), \( \lambda = 1 \) and \( w_0 = 0 \), then \( Q^* \) reduces to the optimal order quantity maximizing expected profit (without reference point), i.e., the classical newsvendor solution, which is denoted by \( Q^0 = F^{-1} \left( \frac{p - c}{p - s} \right) \). Then it follows from Corollary 3.2 that \( Q^* \leq Q^u \leq Q^*_p \). On the other hand, since \( F^{-1}(\cdot) \) is increasing, then \( Q^p_\alpha \leq Q^*_0 \) if \( 0 \leq w_0 \leq p - c \), while \( Q^p_\alpha > Q^*_0 \) if \( s - c \leq w_0 < 0 \). In light of the above analysis, we can directly obtain the following results.

**Corollary 3.4.** If \( 0 \leq w_0 \leq p - c \), then \( Q^* \leq Q^u \leq Q^*_p \leq Q^*_0 \). If \( s - c \leq w_0 < 0 \), then \( Q^* \leq Q^u \leq Q^*_p \) and \( Q^*_p > Q^*_0 \).

When reference point is considered, it follows from above corollary that the optimal order quantity maximizing CVaR of utility is always less than that maximizing expected utility, and further less than that maximizing expected profit (without reference point). Moreover, when \( 0 \leq w_0 \leq p - c \), that is, the target unit profit is nonnegative, the loss-averse newsvendor’s optimal order quantity \( Q^* \) is always less than the classical newsvendor solution \( Q^*_0 \). Thus, there exists negative decision bias in the loss-averse system. However, the relation between his optimal order quantity and the classical newsvendor solution is still unclear when \( s - c \leq w_0 < 0 \). To gain more managerial insights, we will discuss it.

Note that according to the proof of Theorem 3.1, \( Q^* \) satisfies

\[
\left. \frac{\partial H(Q,v^*)}{\partial Q} \right|_{Q=Q^*} = 0,
\]

where \( \frac{\partial H(Q,v^*)}{\partial Q} \) is (24) and \( H(Q,v^*) \) is concave. Let

\[
L(w_0) = \left. \frac{\partial H(Q,v^*)}{\partial Q} \right|_{Q=Q^*_0} = -w_0 - \frac{\alpha(p - c)}{1 - \alpha} - \frac{(\lambda - 1)(c - s + w_0)}{1 - \alpha} F \left( \frac{c - s + w_0}{p - s}Q^*_0 \right).
\]

**Theorem 3.5.** The relation between \( Q^* \) and \( Q^*_0 \) is as follows:

(i) When \( 0 \leq w_0 \leq p - c \), then \( Q^* \leq Q^*_0 \);

(ii) When \( s - c \leq w_0 < 0 \) and \( \frac{c - s}{p - s} < \alpha < 1 \), then \( Q^* < Q^*_0 \);

(iii) When \( s - c \leq w_0 < 0 \) and \( 0 \leq \alpha \leq \frac{c - s}{p - s} \), there exists a unique \( w^*_0 \in [s - c, 0) \) that satisfies \( L(w^*_0) = 0 \). If \( w_0 < w^*_0 \), then \( Q^* > Q^*_0 \); if \( w_0 = w^*_0 \), then \( Q^* = Q^*_0 \); otherwise, \( Q^* < Q^*_0 \).

**Proof.** See Appendix A. \(\square\)
Theorem 3.5 demonstrates that the optimal order quantity is always less than the classical newsvendor solution when the target unit profit is nonnegative. However, when it is negative, this is not always true. This theorem establishes the sufficient conditions under which the order quantity may be larger than, equal to or less than the classical newsvendor solution. More specifically, when \( s - c \leq w_0 < 0 \) and \( 0 \leq \alpha \leq \frac{c-s}{p} \), there exists a threshold of target unit profit \( w_0^* \in [s-c,0) \): (1) the loss-averse newsvendor will order larger than the classical newsvendor if \( w_0 < w_0^* \), that is, there exists positive bias; (2) the loss-averse newsvendor will order equal to the classical newsvendor if \( w_0 = w_0^* \), that is, there exists no bias; (3) the loss-averse newsvendor will order less than the classical newsvendor if \( w_0 > w_0^* \), that is, there exists negative bias. Note that Wang and Wang [32] indicate that the loss-averse newsvendor’s order quantity is larger than the classical newsvendor’s if the reference point is negative and less than a threshold. However, this theorem shows their result does not always hold when considering risk management simultaneously. When the anchor is negative (thus the reference point is negative) and the confidence level is large, then the effect of risk aversion that decreases the newsvendor’s order quantity will dominate the effect of negative anchor that increases the order quantity. Therefore, the loss-averse newsvendor will always order less than the classical one.

As mentioned above, \( Q^* \) reduces to \( Q_u^* \) when \( \alpha = 0 \). Then according to Theorem 3.5, we have

**Corollary 3.6.** The relation between \( Q_u^* \) and \( Q_0^* \) is as follows:

(i) When \( 0 \leq w_0 \leq p - c \), then \( Q_u^* \leq Q_0^* \);

(ii) When \( s - c \leq w_0 < 0 \), there exists a unique \( w_0^* \in [s-c,0) \) that satisfies \( L(w_0^*) = 0 \). If \( w_0 < w_0^* \), then \( Q_u^* > Q_0^* \); if \( w_0 = w_0^* \), then \( Q_u^* = Q_0^* \); otherwise, \( Q_u^* < Q_0^* \).

This corollary shows that when the objective is to maximize expected utility, the loss-averse newsvendor with quantity-oriented reference point may order larger than, equal to or less than the classical newsvendor under certain conditions. This result is different from that in Schweitzer and Cachon [26] where reference point is not considered. They show that the loss-averse newsvendor will always order less than the classical newsvendor. The interpretations of this corollary are as follows. Note that Corollary 3.2 shows the higher the loss aversion level is, the less the order quantity is. Moreover, the lower (higher) the target unit profit is, the more (less) the order quantity is. When \( 0 \leq w_0 \leq p - c \), the loss-averse newsvendor with reference point \( (\lambda > 1) \) always orders less than the classical one \( (\lambda = 1 \text{ and } w_0 = 0) \) due to the effects of loss aversion and positive anchor. When \( s - c \leq w_0 < 0 \) and for any given \( \lambda \), there exists a threshold of target unit profit, above which the effect of loss aversion that decreases the newsvendor’s order quantity will dominate the effect of negative anchor that increases the order quantity. Therefore, the loss-averse newsvendor still orders less than the classical one. However, if \( w_0 \) is less than this threshold, then the effect of negative anchor will dominate the effect of loss aversion and thus the loss-averse newsvendor will order more than the classical one. In practice, the manager’s loss aversion level can be determined through experiments in advance, as shown in Tversky and Kahneman [28]. Then the threshold of target unit profit \( w_0^* \) can be calculated from (8). When the target unit profit \( w_0 \) is given subjectively, the relation between the manager’s optimal order quantity and the classical newsvendor solution can be easily obtained by comparing \( w_0 \) with \( w_0^* \).
Corollary 3.2 has demonstrated that when the confidence level increases, the newsvendor becomes more risk-averse and then orders less products. The following theorem investigates the impact of confidence level on the newsvendor’s expected utility.

**Theorem 3.7.** The newsvendor’s expected utility $E[U(\pi(Q^*, D))]$ under the optimal order quantity $Q^*$ is decreasing in $\alpha$.

*Proof.* See Appendix A.

This theorem indicates that as the confidence level increases, the newsvendor will obtain a lower expected utility if he orders less products to hedge against the potential risk. The higher the confidence level is, the lower his expected utility is. On the contrary, in order to obtain a higher utility, the newsvendor must select a larger order quantity, which enhances the risk of excess order and requires him to have greater risk tolerance. This confirms the fact that high risk implies high return while low risk comes with low return.

Then we will consider a special case in which target unit profit $w_0$ is a convex combination of the maximum $p-c$ and minimum $s-c$, i.e., $w_0 = \beta(p-c)+(1-\beta)(s-c)$, where $\beta \in [0, 1]$ is given subjectively in advance. In this case, the reference point $\pi_0 = w_0Q = \beta(p-c)Q+(1-\beta)(s-c)Q$ is a convex combination of the newsvendor’s maximum possible payoff $(p-c)Q$ and minimum possible payoff $(s-c)Q$, which is the same as that used by Long and Nasiry [19], Wang and Wang [32] and Mandal et al. [22]. Plugging above $w_0$ into (6) and combining with Corollary 3.2, we can directly obtain the following theorem.

**Theorem 3.8.** If $w_0 = \beta(p-c)+(1-\beta)(s-c)$ and $\beta$ is given subjectively in advance, then there exists a unique optimal order quantity $Q^*$ that satisfies

$$(\lambda - 1)\beta F(\beta Q^*) + F(Q^*) = (1 - \alpha)(1 - \beta),$$

and it is decreasing in $\lambda$, $\alpha$ and $\beta$, respectively.

This theorem indicates that when the target unit profit is a convex combination of the maximum and minimum, in other words, the reference point is a convex combination of the newsvendor’s maximum possible payoff and minimum possible payoff, then the optimal order quantity only depends on $\alpha$, $\beta$ and $\lambda$, while is independent of $p$, $c$ and $s$. Therefore, after chosen the coefficient of convex combination subjectively, the newsvendor does not consider the price and cost when he makes order decisions. More especially, if $\alpha = 0$ and $\lambda = 1$, then $Q^*$ reduces to the optimal order quantity maximizing expected profit (with reference point) and is $F^{-1}(1 - \beta)$, which only depends on $\beta$. In addition, the target unit profit $w_0$ will increase as $\beta$ increases, and thus the optimal order quantity will decrease.

4. **Numerical experiments.** In this section, we carry out numerical experiments to demonstrate our theoretical results and present some managerial insights. First, two experiments are conducted to illustrate the effects of loss aversion and confidence level on the optimal order quantity, respectively. Let $p = 50$, $c = 25$ and $s = 10$. The demand is a truncated normal random variable whose cumulative distribution function is defined as $F(x) = (G(x) - G(0))/(1 - G(0))$, where $G(x) = 1/\sqrt{2\pi\sigma} \int_{-\infty}^{x} e^{-(t-\mu)^2/2\sigma^2} dt$ and mean $\mu = 100$ as well as standard deviation $\sigma = 50$. In the first experiment, we fix $\alpha = 0.5$ and $w_0 = 10$, and vary the loss...
aversion level from 1 to 3 in steps of 0.1. In the second experiment, we fix $\lambda = 2$ and $w_0 = 10$, and vary the confidence level from 0 to 1 in steps of 0.05.

Figure 1 illustrates the optimal order quantities with respect to different loss aversion levels for the first experiment. As loss aversion level increases, the newsvendor’s optimal order quantity decreases. The higher the loss aversion level is, the less the order quantity is. This figure also shows that the loss-averse newsvendor ($\lambda > 1$) will order less than the loss-neutral one ($\lambda = 1$). Similarly, Figure 2 illustrates the optimal order quantities with respect to different confidence levels for the second experiment. When the loss-averse newsvendor becomes more risk-averse, he will order fewer products. Moreover, the order quantity maximizing CVaR of utility ($\alpha > 0$) is less than that maximizing expected utility ($\alpha = 0$). These results are intuitive and in accordance with Corollary 3.2. When loss aversion or confidence level increases, the newsvendor becomes more conservative and will order less products to hedge against the potential loss or risk.

Then the other two experiments are presented to illustrate the effect of target unit profit on the optimal order quantity, and make comparisons between the optimal order quantity $Q^*$ and the classical newsvendor solution $Q^*_0$. In the first experiment, we vary $w_0$ from $s - c = -15$ to 0 in steps of 1. In the second experiment, we vary $w_0$ from 0 to $p - c = 25$ in steps of 1. For each experiment, let $\lambda = 2$ and three

(a) $s - c \leq w_0 \leq 0$

(b) $0 \leq w_0 \leq p - c$
different confidence levels are considered: \( \alpha = 0.2, \alpha = 0.5 \) and \( \alpha = 0.8 \). Other parameters are the same as above.

Figure 3 illustrates the optimal order quantities with respect to target unit profits under different confidence levels. The optimal order quantity always decreases as target unit profit increases, and changes faster for lower confidence level, while slower for higher confidence level. For an example of comparing the lines of \( \alpha = 0.2 \) and \( \alpha = 0.5 \) in Figure 3(a), the optimal order quantity when \( \alpha = 0.2 \) decreases more rapidly than that when \( \alpha = 0.5 \). This shows that if the loss-averse newsvendor is more risk-averse, then the change to target unit profit has a smaller effect on his order quantity. Moreover, we can calculate the classical newsvendor solution \( Q_0^* = 117.06 \). In the case of \( w_0 \leq 0 \), Figure 3(a) shows that the optimal order quantity \( Q^* \) is always less than \( Q_0^* \) when \( \alpha = 0.5 \) and \( \alpha = 0.8 \). However, when \( \alpha = 0.2 \), there exists a threshold of target unit profit \( w_0^* = -6.70 \), below which \( Q^* \) is greater than \( Q_0^* \), and above which \( Q^* \) is less than \( Q_0^* \). On the other hand, in the case of \( w_0 \geq 0 \), Figure 3(b) illustrates that \( Q^* \) is always less than \( Q_0^* \) for three different confidence levels. These imply that the loss-averse newsvendor may order larger than, equal to or less than the classical one, which is influenced by the target unit profit and confidence level of the newsvendor. These results are consistent with Theorem 3.5 and provide managerial insights. The newsvendor will order more products for smaller target unit profit. If the target unit profit is negative and small enough, and meanwhile the newsvendor has a greater risk tolerance (i.e., a lower confidence level), then his order quantity can be more than the classical newsvendor. Note that although the truncated normal demand distribution and some parameter values are used, according to Corollary 3.2 and Theorem 3.5, the monotonicity of optimal order quantity and the existence of threshold \( w_0^* \) are independent of them.

5. Conclusion. In this paper, we investigate a single-period inventory problem where the newsvendor is loss-averse and reference point is quantity-oriented. Here, we introduce a target unit profit as an anchor, and then the reference point is a liner function of the order quantity. CVaR measure is introduced to hedge against the loss-averse newsvendor’s risk and his objective is to maximize the CVaR of utility. By considering loss aversion, reference point effect and risk management jointly, we characterize the structural properties of the newsvendor’s optimal procurement strategies. Specifically, there exists a unique optimal order quantity and it is decreasing in loss aversion level, confidence level and target unit profit. It is also decreasing in purchasing cost while increasing in selling price and salvage value, which are in accordance with classical newsvendor model. Moreover, the optimal order quantity is always less than that maximizing expected utility, and further less than that maximizing expected profit (with reference point). When comes to the loss-averse system with reference point and classical system, we establish the sufficient conditions under which the optimal order quantity maximizing CVaR of utility or maximizing expected utility may be larger than, equal to or less than the classical newsvendor solution. More precisely, when the target unit profit \( w_0 \in [s - c, 0) \) and confidence level \( \alpha \in [0, \frac{c - s - p - s}{p}] \), the loss-averse newsvendor with reference point will order larger than the classical newsvendor if the target unit profit below a threshold. This result is significantly different from that in the case without considering reference point. We also find that the newsvendor’s expected utility decreases as confidence level increases. In particular, when the target unit profit is a convex combination of the maximum and minimum where the coefficient is given
subjectively in advance, the newsvendor's optimal order quantity is independent of price and cost parameters. The numerical experiments are conducted to illustrate the impacts of loss aversion, confidence level and target unit profit on the optimal order quantity, then make comparisons between it and the classical newsvendor solution. These results are consistent with our theoretical conclusions.

If the shortage cost is considered, the newsvendor’s minimum possible payoff, i.e., the lower bound of reference point, will become more complicated and then the difficulty for study is increased largely. Thus we ignore it in this paper. However, some works (e.g., [31, 36]) have shown that the shortage penalty has an important effect on the order decisions. The newsvendor’s ordering policy when the shortage cost is also included deserves further study. Moreover, considering the CVaR of the profit as a constraint will be a direction for future research.

Appendix A.

Proof of Theorem 3.1. Define an auxiliary function

\[ H(Q, v) = v - \frac{1}{1 - \alpha} E[v - U(\pi(Q, D))]^+ = v - \frac{1}{1 - \alpha} \left\{ \int_0^{v + \lambda(c - s + w_0)Q} [v - \lambda(p - s)x + \lambda(c - s + w_0)Q]^+ dF(x) \right. \\
+ \int_{v + \lambda(c - s + w_0)Q}^{Q} [v - (p - s)x + (c - s + w_0)Q]^+ dF(x) \\
\left. + \int_{Q}^{+\infty} [v - (p - c - w_0)Q]^+ dF(x) \right\}. \]  

(10)

Rockafellar and Uryasev [24] show that the optimal order quantity maximizing CVaR_\alpha[U(\pi(Q, D))] is equal to that to problem max_{Q \geq 0} \left\{ \max_{v \in \mathbb{R}} H(Q, v) \right\}. Thus, for any given Q, we first solve the inner problem max_{v \in \mathbb{R}} H(Q, v) and consider the following four cases:

**Case 1.** \( v \leq -\lambda(c - s + w_0)Q \).

Here,

\[ H(Q, v) = v. \]  

(11)

Then \( \frac{\partial H(Q, v)}{\partial v} = 1 > 0 \) and \( H(Q, v) \) is increasing in \( v \).

**Case 2.** \( -\lambda(c - s + w_0)Q < v \leq 0 \).

In such a case,

\[ H(Q, v) = v - \frac{1}{1 - \alpha} \int_0^{v + \lambda(c - s + w_0)Q} [v - \lambda(p - s)x + \lambda(c - s + w_0)Q] dF(x). \]  

(12)

Then

\[ \frac{\partial H(Q, v)}{\partial v} = 1 - \frac{1}{1 - \alpha} F \left[ \frac{v + \lambda(c - s + w_0)Q}{\lambda(p - s)} \right], \]  

(13)

and it is easy to calculate that \( \frac{\partial^2 H(Q, v)}{\partial v^2} < 0 \), thus \( H(Q, v) \) is concave. We have

\[ \frac{\partial H(Q, v)}{\partial v} \bigg|_{v = -\lambda(c - s + w_0)Q} = 1 > 0. \]

If \( \frac{\partial H(Q, v)}{\partial v} \bigg|_{v = 0} = 1 - \frac{1}{1 - \alpha} F \left( \frac{c - s + w_0}{p - s}Q \right) \leq 0 \), that
is $Q \geq \frac{p-s}{c-s+w_0} F^{-1}(1-\alpha)$, there exists a unique optimal solution $v^*$ that satisfies
\[
\frac{\partial H(Q,v)}{\partial v} \bigg|_{v=v^*} = 0
\]
and then $v^* = \lambda(p-s)F^{-1}(1-\alpha) - \lambda(c-s+w_0)Q$.

**Case 3.** $0 < v \leq (p-c-w_0)Q$.

Here,
\[
H(Q,v) = v - \frac{1}{1-\alpha} \left\{ \int_0^{c-s+w_0} \left[ v - \lambda(p-s)x + \lambda(c-s+w_0)Q \right] dF(x) \right. \\
+ \left. \int_{c-s+w_0}^{\frac{v+(c-s+w_0)Q}{p-s}} \left[ v - (p-s)x + (c-s+w_0)Q \right] dF(x) \right\}.
\]

Then
\[
\frac{\partial H(Q,v)}{\partial v} = 1 - \frac{1}{1-\alpha} F \left( \frac{c-s+w_0}{p-s} Q \right)
\]
and further $\frac{\partial^2 H(Q,v)}{\partial v^2} < 0$, which shows that $H(Q,v)$ is concave. If $\frac{\partial H(Q,v)}{\partial v} \bigg|_{v=0} = 1 - \frac{1}{1-\alpha} F \left( \frac{c-s+w_0}{p-s} Q \right) > 0$ and $\frac{\partial H(Q,0)}{\partial v} \bigg|_{v=(p-c-w_0)Q} = 1 - \frac{1}{1-\alpha} F(Q) \leq 0$, that is $F^{-1}(1-\alpha) \leq Q < \frac{p-s}{c-s+w_0} F^{-1}(1-\alpha)$, then the optimal solution is $v^* = (p-s)F^{-1}(1-\alpha) - (c-s+w_0)Q$.

**Case 4.** $v \geq (p-c-w_0)Q$.

In such a case,
\[
H(Q,v) = v - \frac{1}{1-\alpha} \left\{ \int_0^{c-s+w_0} \left[ v - \lambda(p-s)x + \lambda(c-s+w_0)Q \right] dF(x) \right. \\
+ \int_{c-s+w_0}^{\frac{v+(c-s+w_0)Q}{p-s}} \left[ v - (p-s)x + (c-s+w_0)Q \right] dF(x) \\
+ \int_{p-s}^{\infty} \left[ v - (p-c-w_0)Q \right] dF(x) \right\}.
\]

Then
\[
\frac{\partial H(Q,v)}{\partial v} = 1 - \frac{1}{1-\alpha} \leq 0,
\]
which implies that $H(Q,v)$ is decreasing in $v$ and thus the optimal solution is $v^* = (p-c-w_0)Q$.

In light of the above analysis, for any given $Q$, the optimal solution $v^*$ to problem $\max_{v \in \mathbb{R}} H(Q,v)$ can be expressed as
\[
v^* = \begin{cases} 
\lambda(p-s)F^{-1}(1-\alpha) - \lambda(c-s+w_0)Q, & Q \geq \frac{p-s}{c-s+w_0} F^{-1}(1-\alpha), \\
(p-s)F^{-1}(1-\alpha) - (c-s+w_0)Q, & F^{-1}(1-\alpha) \leq Q < \frac{p-s}{c-s+w_0} F^{-1}(1-\alpha), \\
(p-c-w_0)Q, & 0 \leq Q < F^{-1}(1-\alpha).
\end{cases}
\]
Next we will solve the problem \( \max_{Q \geq 0} \left[ \max_{v \in \mathcal{H}} H(Q, v) \right] = \max_{Q \geq 0} H(Q, v^*) \) and distinguish the following three cases:

**Case 1.** \( Q \geq \frac{p-s}{c-s+w_0} F^{-1}(1-\alpha) \).

Here, plugging \( v^* \) into (12), then

\[
H(Q, v^*) = \lambda(p-s)F^{-1}(1-\alpha) - \lambda(c-s+w_0)Q - \frac{1}{1-\alpha} \int_0^{F^{-1}(1-\alpha)} \lambda(p-s)[F^{-1}(1-\alpha) - x]dF(x).
\]

Since

\[
\frac{\partial H(Q, v^*)}{\partial Q} = -\lambda(c-s+w_0) < 0,
\]

then \( H(Q, v^*) \) is decreasing in \( Q \).

**Case 2.** \( F^{-1}(1-\alpha) \leq Q < \frac{p-s}{c-s+w_0} F^{-1}(1-\alpha) \).

In such a case, plugging \( v^* \) into (14), then

\[
H(Q, v^*) = (p-s)F^{-1}(1-\alpha) - (c-s+w_0)Q - \frac{1}{1-\alpha} \times \\
\left\{ \int_{\frac{c-s+w_0}{p-s}}^{c-s+w_0} [(p-s)(F^{-1}(1-\alpha) - \lambda x) + (\lambda-1)(c-s+w_0)Q]dF(x) \\
+ \int_0^{\frac{c-s+w_0}{p-s}} (p-s)[F^{-1}(1-\alpha) - x]dF(x) \right\}.
\]

Since

\[
\frac{\partial H(Q, v^*)}{\partial Q} = -(c-s+w_0) - \frac{1}{1-\alpha}(\lambda-1)(c-s+w_0)F\left(\frac{c-s+w_0}{p-s}Q\right) < 0,
\]

then \( H(Q, v^*) \) is decreasing in \( Q \).

**Case 3.** \( 0 \leq Q < F^{-1}(1-\alpha) \).

Here, plugging \( v^* \) into (16), then

\[
H(Q, v^*) = (p-c-w_0)Q - \frac{1}{1-\alpha} \times \\
\left\{ \int_{\frac{c-s+w_0}{p-s}}^{c-s+w_0} [(p-c-w_0)Q - \lambda(p-s)x) \\
+ \lambda(c-s+w_0)Q]dF(x) + \int_0^{\frac{Q}{p-s}} (p-s)(Q-x)dF(x) \right\}.
\]

We have

\[
\frac{\partial H(Q, v^*)}{\partial Q} = (p-c-w_0) - \frac{1}{1-\alpha} \times \\
\left[ (\lambda-1)(c-s+w_0)F\left(\frac{c-s+w_0}{p-s}Q\right) + (p-s)F(Q) \right].
\]
and further \( \frac{\partial^2 H(Q, v^*)}{\partial Q^2} < 0 \), which shows \( H(Q, v^*) \) is concave. Since \( \left. \frac{\partial H(Q, v^*)}{\partial Q} \right|_{Q=0} = (p - c - w_0) > 0 \) and \( \left. \frac{\partial H(Q, v^*)}{\partial Q} \right|_{Q=F^{-1}(1-\alpha)} < 0 \), there exists a unique \( Q^* \) satisfies

\[
\frac{\partial H(Q, v^*)}{\partial Q} \bigg|_{Q=Q^*} = 0, \text{ i.e., (6)}.
\]

Combining above three cases, \( Q^* \) is the optimal solution to problem \( \max_{Q \geq 0} H(Q, v^*) \).

**Proof of Corollary 3.2.** Let

\[
M(Q^*) = (\lambda - 1)(c - s + w_0) \left( \frac{c - s + w_0}{p - s} Q^* \right) + (p - s)F(Q^*) - (1 - \alpha)(p - c - w_0).
\]

Then

\[
\frac{\partial M(Q^*)}{\partial Q^*} = \frac{(\lambda - 1)(c - s + w_0)^2}{p - s} f \left( \frac{c - s + w_0}{p - s} Q^* \right) + (p - s)f(Q^*) > 0,
\]

\[
\frac{\partial M(Q^*)}{\partial \lambda} = (c - s + w_0) \left( \frac{c - s + w_0}{p - s} Q^* \right) > 0,
\]

\[
\frac{\partial M(Q^*)}{\partial \alpha} = p - c - w_0 > 0,
\]

and

\[
\frac{\partial M(Q^*)}{\partial w_0} = (\lambda - 1)F \left( \frac{c - s + w_0}{p - s} Q^* \right) + \frac{(\lambda - 1)(c - s + w_0)Q^*}{p - s} f \left( \frac{c - s + w_0}{p - s} Q^* \right) + (1 - \alpha) > 0.
\]

Since \( M(Q^*) = 0 \), applying the implicit function theorem to it then we have

\[
\frac{dQ^*}{d\lambda} = - \frac{\frac{\partial M(Q^*)}{\partial Q^*}}{\frac{\partial M(Q^*)}{\partial \lambda}} < 0,
\]

\[
\frac{dQ^*}{d\alpha} = - \frac{\frac{\partial M(Q^*)}{\partial Q^*}}{\frac{\partial M(Q^*)}{\partial \alpha}} < 0
\]

and

\[
\frac{dQ^*}{dw_0} = - \frac{\frac{\partial M(Q^*)}{\partial Q^*}}{\frac{\partial M(Q^*)}{\partial w_0}} < 0,
\]

respectively, which imply the results.

**Proof of Corollary 3.3.** It follows from (25) that

\[
\frac{\partial M(Q^*)}{\partial c} = (\lambda - 1)F \left( \frac{c - s + w_0}{p - s} Q^* \right) + \frac{(\lambda - 1)(c - s + w_0)Q^*}{p - s} f \left( \frac{c - s + w_0}{p - s} Q^* \right) + 1 - \alpha > 0,
\]

\[
\frac{\partial M(Q^*)}{\partial s} = - (\lambda - 1)F \left( \frac{c - s + w_0}{p - s} Q^* \right) - \frac{(\lambda - 1)(c - s + w_0)(p - c - w_0)Q^*}{(p - s)^2} f \left( \frac{c - s + w_0}{p - s} Q^* \right) - F(Q^*) < 0,
\]

\[
\frac{\partial M(Q^*)}{\partial w_0} = \frac{(\lambda - 1)(c - s + w_0)Q^*}{p - s} f \left( \frac{c - s + w_0}{p - s} Q^* \right) + (1 - \alpha) > 0.
\]
Proof of Theorem 3.7.

\[
\frac{\partial M(Q^*)}{\partial p} = -\frac{(\lambda - 1)(c - s + w_0)^2}{(p - s)^2} Q^* f \left( \frac{c - s + w_0}{p - s} Q^* \right) + \alpha - \bar{F}(Q^*). \tag{35}
\]

Since from (6) we have \((p - s)F(Q^*) < (1 - \alpha)(p - c - w_0) < (1 - \alpha)(p - s)\), that is \(\alpha < \bar{F}(Q^*)\), then \(\frac{\partial M(Q^*)}{\partial p} < 0\).

Applying the implicit function theorem to \(M(Q^*) = 0\) and combining with (26), then we have

\[
\frac{dQ^*}{dc} = -\frac{\partial M(Q^*)}{\partial c} \bigg/ \frac{\partial M(Q^*)}{\partial Q^*} < 0, \tag{36}
\]

\[
\frac{dQ^*}{ds} = -\frac{\partial M(Q^*)}{\partial s} \bigg/ \frac{\partial M(Q^*)}{\partial Q^*} > 0, \tag{37}
\]

and

\[
\frac{dQ^*}{dp} = -\frac{\partial M(Q^*)}{\partial p} \bigg/ \frac{\partial M(Q^*)}{\partial Q^*} > 0, \tag{38}
\]

respectively, which imply the results. \(\square\)

**Proof of Theorem 3.5.** When \(0 \leq w_0 \leq p - c\), from Corollary 3.4 we have \(Q^* \leq Q^*_0\).

When \(s - c \leq w_0 < 0\), it follows from (8) that \(L(w_0)\) is decreasing in \(w_0\) and

\[
L(0) = -\frac{\alpha(p - c)}{1 - \alpha} - \frac{(\lambda - 1)(c - s)}{1 - \alpha} F \left( \frac{c - s}{p - s} Q^*_0 \right) < 0, \tag{39}
\]

and

\[
L(s - c) = \frac{1}{1 - \alpha} \left[ c - s - \alpha(p - s) \right]. \tag{40}
\]

If \(\frac{c - s}{p - s} < \alpha < 1\), then \(L(s - c) < 0\). Thus we always have \(L(w_0) < 0\) when \(w_0 \in [s - c, 0)\), which implies \(\frac{\partial H(Q^*_0)}{\partial Q^*_0} \bigg|_{Q^*_0 = Q^*_0} < 0\) and then \(Q^* \leq Q^*_0\).

If \(0 \leq \alpha \leq \frac{c - s}{p - s}\), then \(L(s - c) \geq 0\). There exists a unique \(w^*_0 \in [s - c, 0)\) that satisfies \(L(w^*_0) = 0\). If \(w_0 < w^*_0\), then \(L(w_0) > 0\), which shows \(Q^* > Q^*_0\); if \(w_0 = w^*_0\), then \(L(w_0) = 0\), which shows \(Q^* = Q^*_0\); otherwise, \(L(w_0) < 0\), which shows \(Q^* < Q^*_0\). \(\square\)

**Proof of Theorem 3.7.** When the newsvender’s order quantity is \(Q^*\), from (3) it follows that his expected utility is

\[
E[U(\pi(Q^*, D))] = \int_{0}^{\frac{c - s + w_0}{p - s} Q^*} \lambda[(p - s)x - (c - s + w_0)Q^*] dF(x)
+ \int_{\frac{c - s + w_0}{p - s} Q^*}^{Q^*} [(p - s)x - (c - s + w_0)Q^*] dF(x)
+ \int_{Q^*}^{+\infty} (p - c - w_0)Q^* dF(x), \tag{41}
\]

and then

\[
\frac{\partial E[U(\pi(Q^*, D))] }{\partial Q^*} = -\frac{(\lambda - 1)(c - s + w_0)F \left( \frac{c - s + w_0}{p - s} Q^* \right)}{(p - s)F(Q^*) + (p - c - w_0)}. \tag{42}
\]
Combining with (7) and note that \( Q^* \leq Q^*_u \), we have

\[
\frac{\partial E[U(\pi(Q^*, D))]}{\partial Q^*} = (\lambda - 1)(c - s + w_0) \left[ F\left( \frac{c - s + w_0}{p - s} Q^*_u \right) - F\left( \frac{c - s + w_0 Q^*}{p - s} \right) \right] \\
+ (p - s)[F(Q^*_u) - F(Q^*)] \geq 0.
\]

Since \( Q^* \) is decreasing in \( \alpha \), then

\[
\frac{\partial E[U(\pi(Q^*, D))]}{\partial \alpha} = \frac{\partial E[U(\pi(Q^*, D))]}{\partial Q^*} \cdot \frac{\partial Q^*}{\partial \alpha} \leq 0,
\]

which implies the result.

\[\square\]

REFERENCES

[1] T. Bai, M. Wu and S. X. Zhu, Pricing and ordering by a loss averse newsvendor with reference dependence, Transportation Research Part E: Logistics and Transportation Review, 131 (2019), 343–365.

[2] A. O. Brown and C. S. Tang, The impact of alternative performance measures on single-period inventory policy, J. Ind. Manag. Optim., 2 (2006), 297–318.

[3] F. T. Chan and X. Xu, The loss-averse retailer’s order decisions under risk management, Mathematics, 7 (2019), 595.

[4] Y. Chen, M. Xu and Z. G. Zhang, A risk-averse newsvendor model under the CVaR criterion, Operations Research, 57 (2009), 1040–1044.

[5] T. Feng, L. R. Keller and X. Zheng, Decision making in the newsvendor problem: A cross-national laboratory study, Omega, 39 (2011), 41–50.

[6] M. Fisher and A. Raman, Reducing the cost of demand uncertainty through accurate response to early sales, Operations Research, 44 (1996), 87–99.

[7] C. Fulga, Portfolio optimization under loss aversion, European J. Oper. Res., 251 (2016), 310–322.

[8] C. Fulga, Portfolio optimization with disutility-based risk measure, European J. Oper. Res., 251 (2016), 541–553.

[9] A. Furnham and H. C. Boo, A literature review of the anchoring effect, The Journal of Socio-Economics, 40 (2011), 35–42.

[10] J. Guo and X. D. He, Equilibrium asset pricing with Epstein-Zin and loss-averse investors, J. Econom. Dynam. Control, 76 (2017), 86–108.

[11] F. Herweg, The expectation-based loss-averse newsvendor, Econom. Lett., 120 (2013), 429–432.

[12] D. Kahneman and A. Tversky, Prospect theory: An analysis of decision under risk, Econometrica, 47 (1979), 263–291.

[13] M. Khouja, The single-period (news-vendor) problem: Literature review and suggestions for future research, Omega, 27 (1999), 537–553.

[14] B. Li, P. Hou, P. Chen and Q. Li, Pricing strategy and coordination in a dual channel supply chain with a risk-averse retailer, International Journal of Production Economics, 178 (2016), 154–168.

[15] W. Liu, S. Song, B. Li and C. Wu, A periodic review inventory model with loss-averse retailer, random supply capacity and demand, International Journal of Production Research, 53 (2015), 3623–3634.

[16] W. Liu, S. Song, Y. Qiao and H. Zhao, The loss-averse newsvendor problem with random supply capacity, J. Ind. Manag. Optim., 13 (2017), 1417–1429.

[17] W. Liu, S. Song and C. Wu, Impact of loss aversion on the newsvendor game with product substitution, International Journal of Production Economics, 141 (2013), 352–359.

[18] W. Liu, S. Song and C. Wu, The loss-averse newsvendor problem with random yield, Transactions of the Institute of Measurement and Control, 36 (2014), 312–320.

[19] X. Long and J. Nasiry, Prospect theory explains newsvendor behavior: The role of reference points, Management Science, 61 (2015), 3009–3012.

[20] L. Ma, W. Xue, Y. Zhao and Q. Zeng, Loss-averse newsvendor problem with supply risk, Journal of the Operational Research Society, 67 (2016), 214–228.

[21] L. Ma, Y. Zhao, W. Xue, T. C. E. Cheng and H. Yan, Loss-averse newsvendor model with two ordering opportunities and market information updating, International Journal of Production Economics, 140 (2012), 912–921.
[22] P. Mandal, R. Kaul and T. Jain, Stocking and pricing decisions under endogenous demand and reference point effects, European J. Oper. Res., 264 (2018), 181–199.

[23] Y. Qin, R. Wang, A. J. Vakharia, Y. Chen and M. M. H. Seref, The newsvendor problem: Review and directions for future research, European J. Oper. Res., 213 (2011), 361–374.

[24] R. T. Rockafellar and S. Uryasev, Optimization of conditional value-at-risk, Journal of Risk, 2 (2000), 21–41.

[25] R. T. Rockafellar and S. Uryasev, Conditional value-at-risk for general loss distributions, Journal of banking & finance, 26 (2002), 1443–1471.

[26] M. E. Schweitzer and G. P. Cachon, Decision bias in the newsvendor problem with a known demand distribution: experimental evidence, Management Science, 46 (2000), 404–420.

[27] J. Sun and X. Xu, Coping with loss aversion in the newsvendor model, Discrete Dyn. Nat. Soc., (2015), Art. ID 851586, 11 pp.

[28] A. Tversky and D. Kahneman, Advances in prospect theory: Cumulative representation of uncertain, Journal of Risk and Uncertainty, 5 (1992), 297–323.

[29] B. Vipin and R. K. Amit, Loss aversion and rationality in the newsvendor problem under recourse option, European J. Oper. Res., 261 (2017), 563–571.

[30] C. X. Wang, The loss-averse newsvendor game, International Journal of Production Economics, 124 (2010), 448–452.

[31] C. X. Wang and S. Webster, The loss-averse newsvendor problem, Omega, 37 (2009), 93–105.

[32] R. Wang and J. Wang, Procurement strategies with quantity-oriented reference point and loss aversion, Omega, 80 (2018), 1–11.

[33] M. Wu, T. Bai and S. X. Zhu, A loss averse competitive newsvendor problem with anchoring, Omega, 81 (2018), 99–111.

[34] M. Wu, S. X. Zhu and R. H. Teunter, A risk-averse competitive newsvendor problem under the CVaR criterion, International Journal of Production Economics, 156 (2014), 13–23.

[35] X. Xu, F. T. S. Chan and C. K. Chan, Optimal option purchase decision of a loss-averse retailer under emergent replenishment, International Journal of Production Research, 57 (2019), 4594–4620.

[36] X. Xu, C. K. Chan and A. Langevin, Coping with risk management and fill rate in the loss-averse newsvendor model, International Journal of Production Economics, 195 (2018), 296–310.

[37] X. Xu, Z. Meng, R. Shen, M. Jiang and P. Ji, Optimal decisions for the loss-averse newsvendor problem under CVaR, International Journal of Production Economics, 164 (2015), 146–159.

[38] X. Xu, H. Wang, C. Dang and P. Ji, The loss-averse newsvendor model with backordering, International Journal of Production Economics, 188 (2017), 1–10.

[39] H. Yu, J. Zhai and G.-Y. Chen, Robust optimization for the loss-averse newsvendor problem, J. Optim. Theory Appl., 171 (2016), 1008–1032.

[40] X.-B. Zhao and W. Geng, A note on “Prospect theory and the newsvendor problem”, J. Oper. Res. Soc. China, 3 (2015), 89–94.