Back-to-back correlations of boson-antiboson pairs for anisotropic expanding sources

Yong Zhang\textsuperscript{1}, Jing Yang\textsuperscript{1}, and Wei-Ning Zhang\textsuperscript{1,2}\textsuperscript{*}

\textsuperscript{1}School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian, Liaoning 116024, China
\textsuperscript{2}Department of Physics, Harbin Institute of Technology, Harbin, Heilongjiang 150006, China

Abstract

In the hot and dense hadronic sources formed in high energy heavy ion collisions, the particle interactions in medium might lead to a measurable back-to-back correlation (BBC) of boson-antiboson pairs. We calculate the BBC functions of $\phi \phi$ and $K^+K^-$ for anisotropic expanding sources. The dependences of the BBC on the particle momentum and source expanding velocity are investigated. The results indicate that the BBC functions increase with the magnitude of particle momentum and exhibit an obvious dependence on the direction of the momentum for the anisotropic sources. As the source expanding velocity decreases, the BBC function decreases when the particle momentum is approximately perpendicular to the source velocity, and the BBC function increases when the particle momentum is approximately parallel to the source velocity.

Keywords: back-to-back correlation, boson-antiboson pair, anisotropic expanding source, mass modification.

PACS numbers: 25.75.Gz, 25.75.Ld, 21.65.jk

\textsuperscript{*}wnzhang@dlut.edu.cn
I. INTRODUCTION

In the mid to late 1990s, it was shown [1, 2] that the mass modification of the particles in the hot and dense hadronic sources can lead to a squeezed back-to-back correlation (BBC) of boson-antiboson pairs in high energy heavy ion collisions. This BBC is the result of a quantum mechanical transformation relating in-medium quasiparticles to the two-mode squeezed states of their free observable counterparts, through a Bogoliubov transformation between the creation (annihilation) operators of the quasiparticles and the free observable particles [1–3]. The investigations of the BBC may provide another way for people to understand the thermal and dynamical properties of the hadronic sources in high energy heavy ion collisions, in addition to particle yields and spectra.

In Ref. [3], S. Padula et al. put forward the formulism of the BBC for the local-equilibrium expanding sources, and studied the BBC functions of $\phi\phi$. Recently, the BBC functions of $K^+K^-$ are also investigated [4] based on the formulism [3], and a method is suggested [5] to search for the squeezed BBC in the heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). In Ref. [6], we calculated the BBC functions of $\phi\phi$ and $K^+K^-$ using a Monte Carlo method and investigated the relativistic effect on the BBC functions. However, all these previous works are performed for spherical sources [3–6]. Motivated by the anisotropy of the hadronic sources formed in high energy heavy ion collisions, we will study here the BBC functions for anisotropic hadronic sources, using the relativistic Monte Carlo algorithm as in Ref. [6]. We will investigate the dependences of the BBC functions on the anisotropic source velocity and the angle between particle momentum and source velocity. As compared to the early spherical source models, the momentum-direction dependence of the BBC function for anisotropic sources provides additional signals for experimental detection.

In the next section we will give a brief description of the BBC formulas used in the calculations. In Sec. III, we will present the results of the BBC functions of $\phi\phi$ and $K^+K^-$ varying with particle momentum and source velocity. We will further examine the effect of source velocity on the BBC functions in Sec. IV, and finally provide the summary and conclusions in Sec. V.
II. BBC FUNCTIONS FOR ANISOTROPIC EXPANDING SOURCES

Denote $a_k$ ($a_k^\dagger$) as the annihilation (creation) operators of the freeze-out boson with momentum $k$ and mass $m$, and $b_k$ ($b_k^\dagger$) as the annihilation (creation) operators of the corresponding quasiparticle with momentum $k$ and modified mass $m_\nu$ in the homogeneous medium, they are related by the Bogoliubov transformation \cite{1,2}

$$a_k = c_k b_k + s_{-k}^* b_{-k}^\dagger,$$  \hspace{1cm} (1)

where

$$c_k = \cosh f_k, \quad s_k = \sinh f_k, \quad f_k = \frac{1}{2} \log (\omega_k/\Omega_k),$$ \hspace{1cm} (2)

$$\omega_k = \sqrt{k^2 + m^2}, \quad \Omega_k = \sqrt{k^2 + m_\nu^2}. \hspace{1cm} (3)$$

The BBC function is defined as \cite{1,2}

$$C(k, -k) = 1 + \frac{|G_s(k, -k)|^2}{G_c(k, k)G_c(-k, -k)},$$ \hspace{1cm} (4)

where $G_c(k_1, k_2)$ and $G_s(k_1, k_2)$ are the chaotic and squeezed amplitudes, respectively, as

$$G_c(k_1, k_2) = \sqrt{\omega_{k_1}\omega_{k_2}} \langle a_{k_1}^\dagger a_{k_2} \rangle,$$ \hspace{1cm} (5)

$$G_s(k_1, k_2) = \sqrt{\omega_{k_1}\omega_{k_2}} \langle a_{k_1} a_{k_2} \rangle,$$ \hspace{1cm} (6)

where $\langle \cdots \rangle$ means ensemble average.

For local-equilibrium expanding sources with the sudden freeze-out assumption, $G_c(k_1, k_2)$ and $G_s(k_1, k_2)$ can be expressed as \cite{2,7}

$$G_c(k_1, k_2) = \frac{K_{1,2}^0}{(2\pi)^3} \frac{\bar{F}(\omega_{k_1} - \omega_{k_2})}{\int d^3 r e^{i(k_1 - k_2) \cdot r} \rho(r)} \times \left\{|c_{k_1,k_2}(r)|^2 n_{k_1,k_2}(r) + |s_{k_1,k_2}(r)|^2 [n_{k_1,k_2}(r) + 1] \right\}, \hspace{1cm} (7)$$

$$G_s(k_1, k_2) = \frac{K_{1,2}^0}{(2\pi)^3} \frac{\bar{F}(\omega_{k_1} + \omega_{k_2})}{\int d^3 r e^{i(k_1 + k_2) \cdot r} \rho(r)} \times \left\{s_{k_1,k_2}^*(r) c_{k_2,k_1}(r) n_{k_1,k_2}(r) + c_{k_1,k_2}(r) s_{k_2,k_1}^*(r) [n_{k_1,k_2}(r) + 1] \right\}, \hspace{1cm} (8)$$

where

$$c_{k_1,k_2}(r) = \cosh[f_{k_1,k_2}(r)], \quad s_{k_1,k_2}(r) = \sinh[f_{k_1,k_2}(r)],$$ \hspace{1cm} (9)

$$f_{k_1,k_2}(r) = \frac{1}{2} \log \left[ K_{1,2}^\nu u_{\mu}(r)/K_{1,2}^{*\nu} u_{\nu}(r) \right],$$ \hspace{1cm} (10)
\[ n_{k_1, k_2}(\mathbf{r}) = \exp \left\{ -\left[ K_{1,2}^{\mu} u_\mu(\mathbf{r}) - \mu_{1,2}(\mathbf{r}) / T(\mathbf{r}) \right] \right\}, \]  

where, \( K_{1,2}^{\mu} = (k_1^{\mu} + k_2^{\mu}) / 2 \) and \( K_{1,2}^{*\mu} = (k_1^{*\mu} + k_2^{*\mu}) / 2 \) are the pair four-momenta of the particles and the quasiparticles in medium, \( u^\mu(\mathbf{r}) = \gamma_v [1, \mathbf{v}(\mathbf{r})], \mu_{1,2}(\mathbf{r}), \) and \( T(\mathbf{r}) \) are the source four-velocity, the pair chemical potential, and the source temperature at particle freeze-out, respectively. In Eqs. (7) and (8), \( \tilde{F}(\omega_{k_1} + \omega_{k_2}) \) is a factor of the Fourier transform of source emission time distribution, \( \rho(\mathbf{r}) \) is source density distribution.

As an expansion of the spherical sources \[3–6\], we take here the source density distribution and velocity as

\[ \rho(\mathbf{r}) = C e^{-(x^2 + y^2)/(2R_T^2)} e^{-z^2/(2R_L^2)} \theta(\sqrt{x^2 + y^2} - 2R_T) \theta(|z| - 2R_L), \]  

where \( C \) is a normalization constant, \( R_T \) and \( R_L \) are the Gaussian radii of the source in the transverse and longitudinal directions, \( a_x, a_y, \) and \( a_z \) are three velocity parameters. The emission time distribution is taken to be the typical exponential decay \[2–6\],

\[ F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t}, \]  

and its Fourier transform square is \( |\tilde{F}(\omega_k + \omega_{-k})|^2 = [1 + (\omega_k + \omega_{-k})^2 \Delta t^2]^{-1} \). In the calculations, we take the parameter \( \Delta t \) to be 2 fm/c as in \[3–6\], and take \( R_T = 5 \) fm, \( R_L = 4 \) fm. The chemical potential of boson-antiboson pairs, \( \mu_{1,2}(\mathbf{r}) \), is taken to be zero, and \( T(\mathbf{r}) \) is taken as fixed freeze-out temperatures, approximately, \( T_f = 140 \) MeV for \( \phi \) meson and \( T_f = 170 \) MeV for \( K \) meson \[3–6\], respectively.

We show in Fig. 1 the BBC functions of \( \phi \phi \) in \( m_\star \)-k plane for the anisotropic sources with different velocity parameters. For a fixed mass shift, the BBC functions increase with momentum magnitude of particle. The peaks of the BBC functions for the sources with smaller longitudinal velocity are higher than those with larger longitudinal velocity. The pattern of BBC function for \( a_x = a_y = 0.4 \) is almost the same as that for \( a_x = 0.5 \) and \( a_y = 0.3 \), because the two sources have almost the same transverse velocity.

In Fig. 2 we plot the BBC functions of \( K^+K^- \) for the sources with different velocity parameters. As compared with the BBC functions of \( \phi \phi \), the BBC functions of \( K^+K^- \) are wider in \( m_\star \) distribution, and have lower peak values.
FIG. 1: The BBC functions of $\phi \phi$ in $m_*-k$ plane for the anisotropic sources.

FIG. 2: The BBC functions of $K^+K^-$ in $m_*-k$ plane for the anisotropic sources.
III. THE DEPENDENCE OF BBC FUNCTION ON PARTICLE MOMENTUM DIRECTION

For anisotropic sources, the BBC functions depend not only on the magnitude of particle momentum, but also on its direction. We introduce

$$\cos \alpha = \frac{k_z}{|k|}, \quad \cos \beta = \frac{k_x}{|k_T|}, \quad \left( |k_T| = \sqrt{k_x^2 + k_y^2} \right).$$

(15)

to describe the direction of particle momentum. The variation of the BBC functions of $\phi \phi$ with $\cos \beta$ for the anisotropic sources are shown in Fig. 3, where $m_*$ is taken as 1050 MeV corresponding approximately to the peaks of the BBC functions in Fig. 1. For the sources with isotropic transverse velocity ($a_x = a_y$), the BBC functions are independent of $\cos \beta$, and the values of the BBC functions at large $k$ decrease slightly with increasing longitudinal velocity. The BBC functions for the sources with anisotropic transverse velocity ($a_x > a_y$) show an obvious dependence of transverse momentum direction. For the sources with $a_x > a_y$, the average velocities $\langle v_x \rangle > \langle v_y \rangle$, and the values of the BBC functions at large $k$ reach maximums/minimums when the transverse momentum is parallel with the smaller/larger transverse velocity ($v_y/v_x$) direction.

FIG. 3: The BBC functions of $\phi \phi$ in $\cos \beta$-$k$ plane for the anisotropic sources. $m_* = 1050$ MeV.
FIG. 4: The BBC functions of $\phi\phi$ in $\cos\alpha$-$k$ plane for the anisotropic sources. $m_* = 1050$ MeV.

We plot in Fig. 4 the BBC functions of $\phi\phi$ in $\cos\alpha$-$k$ plane for the anisotropic sources with the same parameters as Fig. 3. For the sources with $a_z = 0.3$, the average transverse velocity is larger than the average longitudinal velocity, $\langle v_T \rangle > \langle v_z \rangle$. The values of the BBC functions at large $k$ for the sources with $a_z = 0.3$ have maximums/minimums when particle momentum is parallel with the smaller/larger velocity ($v_z/v_T$) direction. On the other hand, there is $\langle v_z \rangle > \langle v_T \rangle$ for the sources with $a_z = 0.6$. In this case the values of the BBC functions at large $k$ have maximums/minimums when particle momentum is parallel with the smaller/larger velocity ($v_T/v_z$) direction.

In Figs. 5 and 6, we plot the BBC functions of $K^+K^-$ for the anisotropic sources, in $\cos\beta$-$k$ and $\cos\alpha$-$k$ planes, respectively. Here $m_*$ is taken as 650 MeV corresponding approximately to the peaks of the BBC functions in Fig. 2. One can see that the BBC functions have the similar dependence of particle momentum direction as that of $\phi\phi$ BBC functions.
FIG. 5: The BBC functions of $K^+K^-$ in $\cos \beta$-$k$ plane for the anisotropic sources. $m_\ast = 650$ MeV.

FIG. 6: The BBC functions of $K^+K^-$ in $\cos \alpha$-$k$ plane for the anisotropic sources. $m_\ast = 650$ MeV.
IV. THE EFFECT OF SOURCE VELOCITY ON BBC FUNCTIONS

We have seen that the BBC functions are sensitive to the magnitude and direction of particle momentum for the anisotropic source. Next, we further examine the effect of source velocity on the BBC functions. From Eqs. (7) — (11), one sees that the BBC functions will increase when \( n_{k_{-}k} \) decreases [2]. Because \( n_{k_{-}k} \) is related to \( k^{\mu}u_{\mu} \), the effect of source velocity on the BBC function will vary not only with the magnitude of the velocity, but also with the angle, \( \theta \), between the velocity and particle momentum.

We plot in Fig. 7 the BBC functions of \( \phi \phi \) in \( m_{*}-\cos \theta \) plane for the sources with the anisotropic velocity and almost zero velocity. (a) and (b) \( k = 500 \text{ MeV/c} \); (c) and (d) \( k = 1000 \text{ MeV/c} \).

We plot in Fig. 7 the BBC functions of \( \phi \phi \) in \( m_{*}-\cos \theta \) plane for the source with the anisotropic velocity [panels (a) and (c)] and the source with almost zero velocity [panels (b) and (d)], for the fixed \( k = 500 \) and \( 1000 \text{ MeV/c} \). The effect of velocity leads to the smaller/larger values of the BBC functions at large/smaller \( \cos \theta \), as compared to the approximately static source. In Fig. 8 we plot the BBC functions of \( \phi \phi \) in \( \cos \theta - k \) plane for the sources with the anisotropic velocity and almost zero velocity. For the anisotropic expanding source, The BBC function at a large \( k \) and small \( \cos \theta \) is larger than that for the almost static source, and the BBC functions at larger \( k \) decrease with \( \cos \theta \) rapidly.
FIG. 8: The BBC functions of $\phi \phi$ in $\cos \alpha$-$k$ plane for the sources with the anisotropic velocity and almost zero velocity. $m_* = 1050$ MeV.

FIG. 9: The BBC functions of $K^+K^-$ in $m_*$-$\cos \theta$ plane for the sources with the anisotropic velocity and almost zero velocity. (a) and (b) $k = 500$ MeV/c; (c) and (d) $k = 1000$ MeV/c.

In Figs. 9 and 10 we plot the BBC functions of $K^+K^-$ in $m_*$-$\cos \theta$ and $\cos \theta$-$k$ planes for the sources with the anisotropic velocity and almost zero velocity. The effect of the source velocity on the BBC functions are similar to that on the BBC functions of $\phi \phi$, but a little smaller.
FIG. 10: The BBC functions of $K^+K^-$ in $\cos \theta$-$k$ plane for the sources with the anisotropic velocity and almost zero velocity. $m_\pi = 650$ MeV.

FIG. 11: The $\cos \theta$-dependence of the average BBC functions of $\phi \phi$ over particle momentum regions (a) $0 < k < 500$ MeV/c and (b) $500 < k < 1000$ MeV/c, for the sources with the different values of the velocity parameters, $a_x$, $a_y$, $a_z$, and the corresponding average source velocity, $\bar{v}$. Here, $m_\pi = 1050$ MeV.

In Fig. 11 we show the dependences of the averaged BBC functions of $\phi \phi$ on the angle between the particle momentum and source velocity, for the sources with the different values of the velocity parameters, $a_x$, $a_y$, $a_z$, and the corresponding average source velocity,
Here, \( m_* \) is taken as 1050 MeV, the momentum regions are \( 0 < k < 500 \text{ MeV}/c \) and \( 500 < k < 1000 \text{ MeV}/c \) for Figs. 11(a) and 11(b), respectively. By comparing the results of the BBC functions for the expanding sources and the static source, one sees that as the source velocity increases, the average BBC function increases when the particle momentum is perpendicular to the velocity, and the average BBC function decreases when the particle momentum is nearly parallel to the velocity. The BBC function is greatest when the particle momentum is perpendicular to the source velocity at cos \( \theta = 0 \), and the BBC is smallest when the particle momentum is parallel to the source velocity at cos \( \theta = 1 \).

\[
\langle C(k,k) \rangle_k
\]

**FIG. 12:** The cos \( \theta \)-dependence of the average BBC functions of \( K^+K^- \) over particle momentum regions (a) \( 0 < k < 500 \text{ MeV}/c \) and (b) \( 500 < k < 1000 \text{ MeV}/c \), for the sources with the different values of the velocity parameters, \( a_x, a_y, a_z \), and the corresponding average source velocity, \( \bar{v} \). Here, \( m_* = 650 \text{ MeV} \).

In Fig. 12 we show the dependences of the averaged BBC functions of \( K^+K^- \) on the angle between the particle momentum and source velocity, for the sources with the different values of the velocity parameters, \( a_x, a_y, a_z \), and the corresponding average source velocity, \( \bar{v} \). Here, \( m_* \) is taken as 1050 MeV, the momentum regions are \( 0 < k < 500 \text{ MeV}/c \) and \( 500 < k < 1000 \text{ MeV}/c \) for Figs. 12(a) and 12(b), respectively. At cos \( \theta \sim 0 \), the average BBC function increases with increasing source expansion velocity. While at cos \( \theta \sim 1 \), the average BBC function decreases with the increasing source expansion velocity.
V. SUMMARY AND CONCLUSION

As an extension of the previous works of the BBC functions for spherical hadronic expanding sources [3–6], we calculate the BBC functions of $\phi \phi$ and $K^+K^-$ for the anisotropic expanding sources. As compared to the spherical source models, the anisotropic sources are a more realistic case for the hadronic sources formed in high energy heavy ion collisions, and the investigations of the BBC functions for the anisotropic sources may provide additional signals for experimental detection. The values of the BBC functions for the expanding sources depend not only on the magnitude of particle momentum, but also on its direction. For the sources with the average transverse velocities, $\langle v_x \rangle > \langle v_y \rangle$, the values of the BBC functions at large particle momentum reach maximums (or minimums) when the transverse momentum is parallel with the smaller (or larger) velocity direction. For the sources with $\langle v_z \rangle \leq \langle v_T \rangle$ [or $\langle v_z \rangle > \langle v_T \rangle$], the values of the BBC functions at large particle momentum have maximums (or minimums) when the momentum is parallel with $v_z$ (or $v_T$). We further investigate the effect of the anisotropic source velocity on the BBC functions. As the source expansion velocity increases, the BBC function increases when the particle momentum is perpendicular to the source velocity, and the BBC function decreases when the particle momentum is parallel to the source velocity.

Acknowledgments

This research was supported by the National Natural Science Foundation of China under Grant No. 11275037.

[1] M. Asakawa and T. Csörgő, Heavy Ion Physics 4 (1996) 233; arXiv:hep-ph/9612331
[2] M. Asakawa, T. Csörgő and M. Gyulassy, Phys. Rev. Lett. 83, (1999) 4013.
[3] S. S. Padula, G. Krein, T. Csörgő, Y. Hama, P. K. Panda, Phys. Rev. C 73 (2006) 044906.
[4] D. M. Dudek, S. S. Padula, Phys. Rev. C 82 (2010) 034905.
[5] S. S. Padula, O. Socolowski, Jr., Phys. Rev. C 82 (2010) 034908.
[6] Yong Zhang, Jing Yang, Wei-Ning Zhang, Chin. Phys. C 39 (2014) 034103; arXiv:1406.6446.
[7] A. Makhlin and Yu. M. Sinyukov, Sov. J. Nucl. Phys. 46 (1987) 354; Yad. Phys. 46 (1987) 637; Yu. M. Sinyukov, Nucl. Phys. A566 (1994) 589c.