Conjugate mixed convection heat transfer with internal heat generation in a lid-driven enclosure with spinning solid cylinder

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ABSTRACT

The current study investigates conjugate mixed convection heat transmission with internal heat generation in a square enclosure driven by a sliding lid and a solid cylinder with a heat-conducting surface at its center. The enclosure has a stationary bottom wall that is kept at a constant hot temperature and a cold upper wall that moves consistently. The solid cylinder rotates both clockwise and counterclockwise at different angular speeds. Two-dimensional steady continuity, momentum, thermal energy equations, and boundary and interface conditions are solved using a commercial CFD tool based on the finite element method. By choosing Reynolds, Grashof, and Richardson numbers, as well as varying the rotating cylinder’s speed and direction under three different scenarios incorporating volumetric heat generation, parametric modeling of the mixed convection regime is carried out. The streamline and isotherm plots are used to illustrate qualitative findings. In contrast, the average Nusselt number, normalized Nusselt number, average drag coefficient, and average fluid temperature are used to assess quantitative thermal performance measures. This study reveals that the system’s thermal performance is less dependent on the solid cylinder’s rotational speed and direction. It successfully depicts the heat transfer enhancement with increasing Reynolds and Grashof numbers. A thorough study of the current facts can lead to the best choice of regulating parameters.

1. Introduction

Mixed convection is an essential thermo-fluid phenomenon that combines natural and forced convection within a domain of interest with the simultaneous action of buoyancy-driven flow caused by thermal gradient and the shear-driven fluid flow generated by the mechanical influence. Many engineering applications, e.g., heat exchanger fabrication, petroleum industry, furnaces, drying technology, crystal growth, material processing, chemical processing equipment, lubrication system, and so on, demand momentous involvement of mixed convection [1]. In recent years, conjugate mixed convection heat transfer has drawn substantial attention to the researcher’s community as many practical situations handle combined conduction-convection mechanisms. Moreover, some literature dealt with internal heat generation or absorption due to the involvement of exothermic-endothermic chemical reactions inside the system boundary, electrically conducting fluids or any other heat-generating physical interaction. For instance, internal heat generation occurs when electric current flows through a salt-water solution.

Heat transmission via mixed convection within a square or rectangular enclosure has a significant engineering application. A moving boundary of the enclosure enhances the performance of the fluid flow and heat transfer parameters. Many literatures [2, 3, 4, 5, 6] investigated fluid flow and heat transfer characteristics for mixed convection problems with differently heated lid-driven enclosures. Two major types of research works were traced in this regime. One was concerned with the enclosures having the horizontal wall(s) (top or/bottom) sliding at a constant velocity. Another one was a side wall-driven differentially heated enclosure where any of the vertical walls or both walls were moving at a uniform speed. Cheng [2] thoroughly investigated mixed convection in a top lid-driven differentially heated square enclosure for an extensive range of governing parameters. As the fluid flow shifted towards the forced convection regime, a significant enhancement of heat transfer performance was observed due to an increase in lid velocity. Nada and Chamkha [3] extended this work by adding an inclination effect to the enclosure. It was found that the inclination had an insignificant impact on local heat transfer and flow patterns within the forced convection-dominated regime, but it was significant in the case of the...
natural convection-dominated regime. In another work by Ibrahim and Hirpho [4], adding a magnetic field to the top lid-driven enclosure attenuated the convective flow and thus diminished heat transfer performance. Besides, a higher Richardson number was observed to enhance the heat transfer inside the enclosure. Further enhancement in heat transfer characteristics was achieved by Aljabair et al. [5] due to the utilization of nanofluid. For the case of sliding side walls of the enclosure, Oztop and Dagtekin [6] observed that heat transfer enhanced in the natural convection domain for opposing shear and buoyancy forces.

The above investigations were performed for lid-driven enclosures having no object inside. However, outcomes of other research works [7, 8, 9, 10, 11, 12] revealed a noticeable improvement in heat transfer performance when a stationary cylindrical object was present within the differentially heated enclosure. One of the remarkable research works in this regard was performed by Khanafer and Aithal [8]. They investigated the effect of the size and location of the cylinder and observed an increase in the average Nusselt number imposing both adiabatic and isothermal boundary conditions on the cylinder surface, although isothermal boundary conditions exhibited a thermal shielding effect. Optimal heat transfer performance was also found when the stationary cylinder was placed near the heated bottom wall, which was consistent with the findings of other literature [9, 10]. It was also noticed from the literature survey that an increase in the size of the stationary cylinder enhanced heat transmission inside the enclosure [8, 12, 13]. By placing a solid conducting cylinder instead of an adiabatic or isothermal cylinder, the conjugate effect involving both conduction and convection plays a vital role in the system's thermal performance. Hence, several numerical investigations of such problems [13, 14] had been performed, where the solid-fluid conductivity ratio was traced as another critical factor that significantly influenced thermal field and heat transfer characteristics.

Mixed convection associated with a rotating circular cylinder inside a differentially heated closed enclosure provided better flow circulation and overall heat transfer performance than stationary cylinder [15, 16, 17, 18, 19, 20]. The geometry of the enclosure and the cylinder substantially affected the heat transfer performance of the enclosure [16, 17, 18]. Besides, the impact of the cylinder's angular velocity, thermal conductivity, and thermal capacity on fluid flow and heat transfer characteristics was significant [15, 16, 18]. Sadr et al. [19] investigated by keeping all four walls of the square enclosure at a lower temperature and a rotating cylinder at the center of the enclosure with a relatively higher temperature. They concluded that when fluid flow inside the enclosure was dominated by natural convection, the heat transmission rate declined as the rotating speed of the cylinder increased. However, the impact of the location of the cylinder was not present in those research works [15, 16, 17, 18, 19], which was later investigated and then revealed its significant effect [20, 21].

An intriguing concern with mixed convection in lid-driven enclosures with a rotating cylinder is the double augmentation of flow mixing and recirculation due to the shear stress produced by the moving and the rotating surfaces and its overall impact on the imposed thermal gradient. Lid-driven closed enclosure with no cylinder, lid-driven enclosure with stationary circular cylinder, and lid-driven enclosure with rotating circular cylinder were extensively studied and compared by Khanafer and Aithal [22], which was found consistent with the findings of Chaterjee et al. [23]. Researchers identified a significant effect of the angular velocity of the rotating cylinder on the overall heat transfer enhancement [22, 23, 24]. As suggested by the nature of the heat transfer variation with rotational speed, a higher angular velocity improved the heat transfer but also caused Taylor instability [23]. Sometimes, the magnetic field interacts with the enclosure, as seen in micro-electronic devices, molten metal purification systems, etc. Heat transfer performance significantly depended upon the magnetic field applied to the lid-driven cavity with a rotating cylinder [25]. The effect of multiple rotating solid heat conducting cylinders was analyzed in recent years for a differentially heated lid-driven cavity, where remarkable enhancement of overall heat transfer was obtained due to the change of the lid velocity and the angular velocity of the cylinders [26, 27]. Table 1 includes related information on mixed convection published in early literature concerning a square enclosure with spinning circular cylinder(s). Here, the symbols \( Pr \), \( Ri \), \( Gr \), \( Re \), \( \Omega \), and \( d \) are referred to as Prandtl number, Richardson number, Grashof number, Reynolds number, cylinder speed ratio, the diameter of the cylinder, and the length of the cavity respectively.

Mustafa et al. [28] and Saieed et al. [29] extensively reviewed the mixed convective heat transfer in a lid-driven chamber, including the impact of geometry and dimensions, medium porosity, and angle of inclination, nanofluid properties, and magnetic field. However, their reviews did not reveal the effects of internal heat generation/absorption on a lid-driven chamber's heat transfer and flow properties. Volumetric heat generation or absorption inside an enclosure involves numerous engineering applications and has gotten ample attention in the research community. The volumetric heat generation was found to reduce the heat transfer for aiding flow in a side lid-driven differentially heated enclosure [30]. However, heat transfer enhancement was found in a top lid-driven enclosure with a corner heater when volumetric heat generation was applied [31]. No matter what the configuration of the enclosure, several researchers [32, 33, 34, 35, 36, 37] agreed that heat generation had a remarkable impact on the heat transfer phenomenon.

To the best of the authors' knowledge, no research has been conducted to date that focuses on the fluid flow field and conjugate heat transfer properties of a top lid-driven square enclosure with a revolving solid circular cylinder involving volumetric heat generation. The prime objective of this research is to numerically investigate the impact of volumetric heat generation on conjugate mixed convection characteristics in a differentially heated square cavity with an isothermally heated bottom wall, a cold sliding top wall, and adiabatic side walls. A solid circular cylinder is placed at the center of the enclosure, rotating clockwise or counterclockwise in its axis. In the presence of volumetric heat generation, the variation of the direction of cylinder rotation could open a new dimension to the heat transfer research arena. Besides, three different combinations of the governing parameters (\( Re \), \( Gr \), and \( Ri \)) are considered in the present study to investigate the systematic variation of thermal performance within the mixed convection regime.

2. Physical model

The physical domain is displayed in Figure 1. It contains an enclosure with an equal length and height of \( L \) and the requisite boundary and
interface settings in a Cartesian coordinate system. The bottom lid of the enclosure is motionless and is kept at a higher temperature of $T_h (> T_c)$, while the top lid of the enclosure is sliding at a constant speed of $u_0$ in the positive $x$-direction. There is insulation and no movement on either side wall. A stainless steel (AISI 304) solid cylinder with a diameter of $d$ and a thermal conductivity of $k_s = 14.9 \text{ W/mK}$ is rotating either clockwise (CW) or counterclockwise (CCW) at an angle of $\omega$ while keeping its center in the middle of the enclosure ($L/2$, $L/2$). The corresponding circumferential velocity of the cylinder is $u_s = \omega d/2$. Water as a heat-generating fluid, Prandtl number, $Pr = 5.85$, and thermal conductivity, $k_f = 0.61 \text{ W/mK}$ is poured into, fulfilling the remaining space of the enclosure. Gravity is acted vertically in the downward direction. The volumetric heat generation rate $Q (\text{W/m}^3)$ within the enclosure is considered.

3. Mathematical model

The continuity of mass, conservation of momentum, and conservation of energy equations comprise the governing equations of the current problem. Under the assumptions of two-dimensional, steady, laminar flow within the enclosure filled with incompressible, Newtonian fluid with constant thermo-physical properties, neglecting viscous dissipation and radiation and applying Boussinesq approximation for density change with temperature, the present mathematical model can be expressed as [25, 36]:

**Fluid domain:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

$$\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right). \tag{2}$$

$$\rho \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g(T-T_c). \tag{3}$$

**Solid domain:**

$$k_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)_s = 0. \tag{5}$$

where, the Cartesian coordinates systems are defined by $x$ and $y$, and $u$ and $v$ state the corresponding velocity components. Moreover, pressure, temperature, and gravitational acceleration are $P$, $T$, and $g$, respectively. Density ($\rho$), specific heat at constant pressure ($C_p$), dynamic viscosity ($\mu$), and thermal expansion coefficient ($\beta$) are the properties of the working fluid (water). For dimensional analysis of the above Eqs. (1), (2), (3), (4), and (5), the following scales (6) are used:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad P = \frac{P}{\rho u_0^2}, \quad \theta = \frac{T-T_c}{T_h-T_c}, \quad \Delta = \frac{QL^2}{k_f(T_h-T_c)}. \tag{6}$$

Table 2 shows the non-dimensional boundary and interface conditions of this problem, where $N$ designates dimensionless wall normal distance, $Re$ stands for rotational Reynolds number defined based on the circular cylinder’s circumferential velocity and diameter as follows (13),

| Parameters | Top | Bottom | Left | Right | Cylinder Surface |
|------------|-----|--------|------|-------|------------------|
| $U$        | 1   | 0      | 0    | 0     | 2(Y–0.5) (Re/Re)/(d/L) |
| $V$        | 0   | 0      | 0    | 0     | 2 (X–0.5) (Re/Re)/(d/L) |
| $\theta$   | 0   | 1      | $\partial/\partial X = 0$ | $\partial/\partial X = 0$ | $K (\partial\theta/\partial N)_s = (\partial\theta/\partial N)_w$ |

$$\rho C_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q. \tag{4}$$

Solid domain:

$$k_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0. \tag{5}$$

$Re = \frac{\rho u_0 L}{\mu} Gr = \frac{\rho u_0^2 L^3}{\mu^2} Ri = \frac{Gr^2}{Re_k}$.
Rec = \frac{\mu d}{\rho} = \frac{\rho wd^2}{2\mu}, \quad (13)

The fluid and solid domains, respectively, are denoted by the subscripts ‘f’ and ‘s’. The average Nusselt number (\(Nu\)) of the hot bottom surface, the normalized Nusselt number (\(Nunor\)), the average drag coefficient (\(Cd\)) on the sliding top lid, and the average fluid temperature (\(\Theta_{av}\)) inside the enclosure are assessed and expressed as performance parameters of the current system in the following ways [23]:

\[
Nu = - \frac{1}{\Theta} \int_0^1 \frac{\partial \Theta}{\partial Y} dX, \quad Nunor = \frac{Nu(Rec \neq 0)}{Nu(Rec = 0)},
\]

\[
Cd = \frac{2}{Rec} \int_0^1 \frac{\partial U}{\partial Y} dX, \quad \Theta_{av} = \frac{1}{A} \int_\Theta d\Theta.
\]

where, \(A\) symbolizes the non-dimensional area of the water domain in the square enclosure.

4. Numerical procedure

The non-dimensional Eqs. (7), (8), (9), (10), and (11) and the non-dimensional boundary conditions specified in Table 2 are solved with the commercial simulation software COMSOL Multiphysics 6.0, adopting Galerkin finite element technique. In this method, the computational domain is discretized into a finite number of elements made up of irregular rectangular or triangular cells. Triangular elements are used to create finite element equations. Figure 2 represents the mesh structures of the entire domain furnished with triangular mesh elements. Then, using Galerkin weighted residual method, a system of integral equations is developed by converting the governing nonlinear partial differential equations. The integration needed for each part of these equations is performed using the Gauss Quadrature Method. Following that, boundary conditions are applied to those nonlinear algebraic equations. Finally, by employing Newton Raphson Iteration Method, the system of those nonlinear algebraic equations is resolved, represented in matrices. The numerical solution error estimation and convergence criterion are set to a specific upper constraint. Later, the solution procedure is continued until the convergence conditions are fulfilled.

4.1. Grid refinement check

Before the model validation, a grid refinement check is performed to reduce processing time and increase the numerical accuracy of the simulated results. For improved computational efficiency, the mesh density and quality are carefully changed, and its respective performance is evaluated. Structured and non-uniform cells are created using physics-controlled mesh while accounting for all boundary conditions, heat transfer, and fluid flow. The meshes along the enclosure walls are more refined, especially near the circular cylinder surface. The generated grids are finally precise enough to represent the intricacies of the enclosure’s fluid systems and heat distribution. Table 3 shows grid refinement results.
in terms of the average Nusselt number. The optimum mesh type is investigated by taking several arrangements of elements ranging from extremely coarse to extremely fine. Finally, an extra fine mesh with 20730 elements is chosen, which has proved acceptable due to negligible change in the average Nusselt number. Detailed information on the selected 'extra fine mesh' type is provided in Table 4.

### Table 5. Ranges of non-dimensional governing parameters used for the present problem.

| Case | $Re$ | $Gr$ | $Ri$ | $Re_c$ |
|------|------|------|------|--------|
| 1    | 100  | $10^5$–$10^7$ | 0.1–10 | 2, 0, $-2$ |
| 2    | 31.623–316.23 | $10^8$ | 0.1–10 | 2, 0, $-2$ |
| 3    | 31.623–316.23 | $10^7$–$10^9$ | 1 | 2, 0, $-2$ |

#### 4.2. Model validation

The current model is justified numerically against the previously published research work of Khanafer and Aithal [22] before performing the parametric numerical simulation. Khanafer and Aithal [22] analyzed how the average Nusselt number varied with the variation of Richardson number for rotating isothermal circular cylinder inside a lid-driven square enclosure filled with air. Figure 3 shows quantitative verification of the present model for two distinct rotating directions. The numerical results obtained by the present model are pretty close to the published data. The maximum error in the current model's data is 1.11% compared with Khanafer and Aithal [22]. Hence, the current model can reasonably forecast the thermo-fluid characteristics of a heat-conducting spinning cylinder enclosed in a square enclosure.

![Figure 4](image-url)  
**Figure 4.** (a) Growth of average Nusselt number of the hot surface and (b) the variation of normalized Nusselt number as a function of $Ri$ and $Gr$ for different $Re_c$ at $Re = 100$ (color online).

![Figure 5](image-url)  
**Figure 5.** (a) Distribution of average drag coefficient on sliding lid, and (b) progress of average fluid temperature of the enclosure as a function of $Ri$ and $Gr$ for multiple values of $Re_c$ at $Re = 100$ (color online).
5. Results and discussion

After completing a successful grid refinement check and model validation, the parametric simulation is carried out for the present problem keeping the following physical and governing parameters constant: \( Pr = 5.85, K = 24.43, \Delta = 10, \) and \( d/L = 0.2. \) The ranges of governing parameters selected for the parametric simulation are summarized in Table 5 under three sets of cases of investigation. For each case, the growth of the average Nusselt number of the hot wall, the variation of the normalized Nusselt number for the case of rotating cylinder with respect to the stationary cylinder, the change of average drag coefficient on the sliding lid of the enclosure and the observation of the average bulk water temperature inside the enclosure calculated using (14) are quantitatively presented. Besides, the qualitative observations of flow and thermal fields in contour plots of stream function and temperature are carried out systematically for each set of parametric variations of the mixed convective governing parameters.

5.1. Effect of \( Re_c \) and \( Gr \) at a fixed value of \( Re \)

Figure 4 depicts the impact of rotational speed and directions of the solid cylinder in a lid-driven enclosure on the system's thermal performance for various Richardson and Grashof numbers. The results also provide data for comparison for a stationary cylinder inside the enclosure. Figure 4(a) indicates that the rotational direction of the solid cylinder has no significant influence on the heat transfer enhancement compared to a stationary cylinder at high \( Gr \) and \( Ri; \) instead, the enhancement primarily relies on the temperature difference between the top and bottom lids. The figure depicts that when the free convection dominates, the Nusselt number increases. It is because the bottom surface of the enclosure is kept stationary at a higher temperature. Due to the increasing Grashof number, the thermal gradient across the enclosure increases significantly. Hence, heat transfer becomes higher when free convection dominates, which is consistent with the findings obtained by

![Streamline Visualization](image-url)
Khanafer and Aithal [22] and Selimefendigil and Oztop [25]. For a constant lid velocity \((Re = 100)\), Figure 4(b) depicts the change of the normalized Nusselt number with the variation of Richardson and Grashof numbers. Though the effect of rotational direction for the specified rotational speed on heat transfer enhancement is minor, the clockwise rotation has somewhat improved heat transfer compared to a stationary cylinder. On the other hand, the counterclockwise rotation has a marring effect on heat transfer due to the negative impact of the flow velocity caused by the revolving cylinder on the induced fluid flow by the sliding top lid. The variation of the average Nusselt number along the hot wall as a function of Richardson number \((0.1 < Ri < 10)\) and rotational Reynolds number \((-2 \leq Re_c < 2)\) when Reynolds number is fixed at 100 can be expressed by the correlation (15) within the limiting range:

\[
Nu = 4.563Re^{0.1577} - 0.0255Re_c,
\]

where the value of the square of the correlation coefficient becomes \(R^2 = 0.9908\).

Figures 5(a) and (b) illustrate the variations of the average drag coefficient of the moving lid and average fluid temperature of the enclosure, respectively, with the variation of Richardson and Grashof numbers for different rotational Reynolds numbers of the cylinder at constant Reynolds number based on lid velocity of the enclosure. The average drag coefficient reduces with increasing Richardson and Grashof numbers since the speed of the moving fluid inside the enclosure increases due to a greater temperature gradient, as seen in Figure 5(a). Moreover, the influences of the rotating direction on average drag coefficient and average fluid temperature are insignificant except for slightly enhancing and opposing effects for clockwise and counterclockwise directions, respectively. The average fluid temperature exhibits similar behavior to the average Nusselt number for various values of \(Re_c\) as shown in Figure 4(a),

![Figure 7. Visualization of isotherms for several values of Ri and Gr at (a)–(c) \(Re_c = -2\) (CW), (d)–(f) \(Re_c = 2\) (CCW) and (g)–(i) \(Re_c = 0\) inside the enclosure when Re is constant at 100 (color online).](image)
since the fluid temperature attains higher thermal stratification and stabilization with increasing temperature gradient at higher $Gr$ and $Ri$. The variation of the average drag coefficient for the cold top moving lid as a function of Richardson number ($0.1 \leq Ri \leq 10$) and rotational Reynolds number ($-2 \leq Re_c \leq 2$) when Reynolds number is fixed at 100 can be expressed by the correlation (16) within the limiting range:

$$C_d = \left(0.0042Ri^{-0.0598} - 0.00012Ri - 1.1668 \times 10^{-5}Re_c\right)^{0.159},$$

(16)

where the value of the square of the correlation coefficient becomes $R^2 = 0.998$.

Figure 6(a)-(i) show the influence of Richardson and Grashof numbers on the streamlines for three different values of the rotational Reynolds number of the cylinder at $Re = 100$. These figures show that the rotational direction of the solid cylinder has a noticeable effect on the streamline pattern, especially in the case of CW direction of rotation. However, the variation in the stream function contour profiles is relatively low, which is to be expected given that the flow velocity induced by the rotary cylinder is very low in comparison to the speed of the moving lid. Thus the flow pattern is mainly dominated by the moving lid. The figures also show the effect of changing Richardson and Grashof numbers on the streamline patterns. At higher Richardson/Grashof numbers, inertia force is negligible, and buoyancy force dominates heat transfer. Hence, the streamlines and eddies are more scattered than those of lower Richardson numbers.

Figure 7(a)-(i) depict the influence of the solid cylinder’s rotating direction on isotherms for different values of $Ri$ and $Gr$ at a fixed value of $Re (= 100)$. A higher $Ri$, as predicted, indicates greater natural convection activity within the enclosure. These figures show that at lower $Gr$, lower thermal gradient is observed within the enclosure compared to the
case of higher $Gr$. Thus, $Ri$ and $Gr$ should be kept higher to achieve a more uniform temperature inside the enclosure. These phenomena have significant design implications in practical applications.

5.2. Effect of $Re$ and $Re$ at a fixed value of $Gr$

Figure 8(a) depicts the average Nusselt number of the heat source as a function of $Ri$ and $Re$, showing the effect of the rotational Reynolds number defined based on the rotational speed of the solid cylinder. The values of $Nu$ drop with the increase of Richardson number at constant Grashof number since the Reynolds number based on the moving lid speed decreases simultaneously. With the decrease in lid velocity, the effective movement of fluid becomes a bit slower, which eventually reduces heat transfer from the bottom wall. Besides, the cylinder's direction and speed play a vital role in heat transfer due to the constant thermal gradient between the cold top and hot bottom walls. These can be identified in Figure 8(b) as similarly observed in the previous case (see Figure 4(b)). The variation of the average Nusselt number along the hot wall as a function of Richardson number ($0.1 \leq Ri \leq 10$) and rotational Reynolds number ($-2 \leq Re_{c} \leq 2$) when Grashof number is fixed at $10^{4}$ can be expressed by the correlation (17) within the limiting range:

$$Nu = 4.3352Ri^{-0.0284} - 0.0284Re_{c},$$

(17)

where the value of the square of the correlation coefficient becomes $R^2 = 0.991$.

Changes in average drag coefficient and average fluid temperature with different rotational directions are presented in Figure 9(a) and (b), respectively, as a function of $Ri$ and $Re$. With the decrease of $Re$, the effect of natural convection increases. Therefore, the fluid being heated at the lower wall moves up at a high velocity at lower $Re$. Thus, the average

\[ \begin{array}{ccc}
Ri = 0.1 \text{ and } Re = 316.23 & Ri = 1\text{ and } Re = 100 & Ri = 10\text{ and } Re = 316.23 \\
(a) & (b) & (c) \\
Ri = -2 \text{ (CW)} & Ri = 2 \text{ (CCW)} & Ri = 0 \\
(d) & (e) & (f) \\
Ri = 0 \\
(g) & (h) & (i) \\
\end{array} \]
The average fluid temperature inside the enclosure also increases with increasing $Ri$ and decreasing $Re$. The variation of the average drag coefficient for the cold top moving lid as a function of Richardson number ($0.1 \leq Ri \leq 10$) and rotational Reynolds number ($-2 \leq Re \leq 2$) when Grashof number is fixed at $10^4$ can be expressed by the correlation (18) within the limiting range:

$$C_d = 0.4235Ri^{0.421} - 3.057 \times 10^{-4}Re,$$  

where the value of the square of the correlation coefficient becomes $R^2 = 0.9998$. Besides, the variation of average fluid temperature due to the rotational direction of the cylinder is also visible in this case, which further confirms the uniform distribution of fluid temperature throughout the whole domain for the case of CW rotation.

**Figure 10**(a)-(i) visualize the streamline plots for different rotational directions and circumferential speeds of the cylinder and several values of $Ri$ and $Re$ at constant $Gr$. When the heat transmission moves from forced convection to natural convection mechanism with increasing $Ri$ irrespective of rotational direction and speed of the cylinder, one can notice that the upward vortex generation suppresses significantly, which shifted towards the cylinder. It is because the lower velocity of the top lid at low $Re$ allows the fluid to move more freely and to follow the path of the circulating zone created around the cylinder based on the direction of its rotation. Some disconnected eddies are clearly visible in the case of pure mixed convection heat transfer ($Ri = 1$) due to the low-pressure zone downstream of the cylinder. Therefore, changing the rotational direction of the cylinder from CW to CCW, significant formation of eddies is observed at the upper-right side of the cylinder. The rotational speed of the cylinder also affects the appearance of eddies, which is also depicted in the same plot.

**Figure 11.** Visualization of isotherms for several values of $Ri$ and $Re$ at (a)-(c) $Re = -2$ (CW), (d)-(f) $Re = 2$ (CCW) and (g)-(i) $Re = 0$ inside the enclosure when $Gr$ is constant at $10^4$ (color online).
Figure 11(a)–(i) depict the isotherm plots at constant $Gr$ with varying $Ri$ and $Re$ and the rotational direction and speed of the cylinder. Isotherms are significantly changed with the change of $Re$, i.e., the change of moving lid speed. At lower $Re$ which implies dominating natural convection phenomena, hot circulating air encompasses a more extensive area inside the enclosure. The low-speed top cold lid cannot force the moving air from the heated bottom wall for a faster cool-down process. Conversely, the isotherms show a uniform distribution of cool circulating air around the cylinder at a higher value of $Re$ for the same reason stated above. However, the rotational velocity and direction of the cylinder do not significantly affect the isotherm plots, as seen in Figure 11.

5.3. Effect of $Re_c$, $Re$, and $Gr$ at a fixed value of $Ri$

Finally, the effects of all governing parameters keeping $Ri$ constant at 1, the pure mixed convection case, are assessed in quantitative and qualitative examinations of the system performance. From Figure 12(a), it can be concluded that increasing rotational velocity in the clockwise direction of the cylinder and increasing $Re$ and $Gr$ simultaneously increases the average Nusselt number. Higher lid velocity forces the fluid to circulate inside the enclosure and eventually takes much greater heat from the heated wall. However, heat transfer does not enhance significantly by changing the direction of rotation of the cylinder (see Figure 12(b)) when $Ri = 1$. With increasing both $Re$ and $Gr$, heat transfer increases sharply due to the increasing velocity of the top lid and higher temperature gradient by which fluid inside the enclosure is driven faster towards the bottom wall. Eventually, the circulating fluid takes more heat from the heated bottom surface. The normalized values of the average Nusselt number, as shown in Figure 12(b), indicate that the Nusselt number’s alteration due to different rotational directions of the cylinder is significantly less at higher $Re$ and $Gr$. The variation of the average Nusselt number along the hot wall as a function of Grashof
number \((10^3 \leq Gr \leq 10^5)\) and rotational Reynolds number \((-2 \leq Re_c \leq 2)\) when Richardson number is fixed at unity can be expressed by the correlation (19) within the limiting range:

\[
Nu = 0.1742Gr^{0.3484} - 0.0483Re_c,
\]  

where the value of the square of the correlation coefficient becomes \(R^2 = 0.993\).

The fact that the average drag coefficient on the moving lid and the average temperature of the fluid decrease with increasing the same parameters \((Re\) and \(Gr)\) by keeping \(Ri\) constant is depicted in Figure 13(a) and (b), respectively. With the increase of \(Gr\), either buoyancy force will increase or viscous force will decrease. As a result, fluid moves faster inside the enclosure for rising values of \(Gr\). On the other hand, the moving lid speed will increase at higher \(Re\), making the circulation of cold mixing fluid more prominent inside the enclosure. Besides, the cooling process becomes uniform and faster around the circular cylinder due to the quicker fluid movement. The direction of rotation in this case \((Ri = 1)\) does not influence the drag coefficient. In contrast, the average fluid temperature shows a noticeable change in the direction of rotation of the cylinder, especially at low \(Re\) and \(Gr\). Average fluid temperature decreases gradually with increasing \(Re\) and \(Gr\). The variation of the average drag coefficient for the cold top moving lid as a function of Grashof number \((10^3 \leq Gr \leq 10^5)\) and rotational Reynolds number \((-2 \leq Re_c \leq 2)\) when Richardson number is fixed at unity can be expressed by the correlation (20) within the limiting range:

\[
C_d = 30.952Gr^{-0.4647} - 7.6 \times 10^{-5}Re_c,
\]  

where the value of the square of the correlation coefficient becomes \(R^2 = 0.998\).

Like the previous two cases, Figures 14(a)–(i) and 15(a)–(i) depict the effects of the cylinder's rotational direction and the rotational velocity

Figure 14. Visualization of streamline for several values of \(Gr\) and \(Re\) at (a)–(c) \(Re_c = -2\) (CW), (d)–(f) \(Re_c = 2\) (CCW) and (g)–(i) \(Re_c = 0\) inside the enclosure when \(Ri\) is constant at 1.
and the combined effect of $Gr$ and $Re$ on streamline and isotherm plots, respectively. For $Ri = 1$, from Figure 14, it can be concluded that eddies created on the upper side of the enclosure are shifted towards the right wall with the increase of $Re$ and $Gr$. The higher velocity of the top lid and the higher temperature gradient drive the fluid toward the right wall. Compared with the stationary cylinder ($Re = 0$), the rotating cylinder increases/decreases the formation of eddies significantly based on the direction of rotation which plays a vital role in changing the streamline pattern. With the rise of lid velocity and thermal gradient across the enclosure's upper and lower walls, the average fluid temperature is decreased significantly for the same reason as described before (see Figure 13(b)). This obvious fact is illustrated in Figure 15. Thermal fields slightly affected by the rotational velocity and the direction of rotation of the cylinder are also shown in the same plot.

6. Conclusion

The present study numerically investigates two-dimensional steady laminar mixed convection heat transfer characteristics with internal heat generation in a lid-driven enclosure consisting of a rotating heat conducting cylinder. The primary outcomes of this study are as follows:

- Heat transfer enhancement is strongly dependent on the sliding velocity of the moving cold lid and the thermal gradient across the differentially heated top and bottom walls.
- The thermal performance of the system has a weak dependence on the rotational speed and the direction of the rotating solid cylinder.
- To minimize the drag force on the moving lid and to achieve a more uniform temperature inside the enclosure, the Reynolds number

\[ Gr = 10^3 \text{ and } Re = 31.623 \]
\[ Gr = 10^4 \text{ and } Re = 100 \]
\[ Gr = 10^5 \text{ and } Re = 316.23 \]

Figure 15. Visualization of isotherms for several values of $Gr$ and $Re$ at (a)-(c) $Re = -2$ (CW), (d)-(f) $Re = 2$ (CCW) and (g)-(i) $Re = 0$ inside the enclosure when $Ri$ is constant at 1 (color online).
should be kept higher, resulting in a faster sliding velocity of the top moving lid.

- Although the rotational direction of the cylinder affects the formation and the movement of eddies inside the enclosure, it exhibits negligible influence on heat transfer enhancement.

- For pure mixed convection, simultaneous changes of Re and Gr affect the overall heat transfer and average drag coefficient of the moving lid.

- For all three cases of the variation of mixed convection parameters, the clockwise rotation of the cylinder retains almost the highest enhancement of convective heat transfer within the enclosure.

The following investigations can be performed in future works:

- Further studies can be conducted by changing the heated or moving lid walls (e.g., double sliding lids-aiding/opposing flows).

- Higher values of Reynolds number beyond the laminar range can be considered for investigating the transitional change of thermal performance.

- The range of rotational speed considered in the simulation is marginal, which can be extended to higher values to observe heat transfer enhancement.

Declarations

**Author contribution statement**

Md. Jisan Mahmud, Ahmed Imtiaz Rais, Md. Rakib Hossain: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Sumon Saha: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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**Data availability statement**

Data will be made available on request.

**Declaration of interest’s statement**

The authors declare no competing interests.

**Additional information**

No additional information is available for this paper.

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**References**

[1] M.M. Gulzar, A. Aslam, M. Waqas, M.A. Javed, K. Hosseinizadeh, A nonlinear mathematical analysis for magneto-hydropyclic-tangent liquid featuring simultaneous aspects of magnetic field, heat source and thermal stratification, Appl. Nanosci. 10 (2020) 4513–4518.

[2] T.S. Cheng, Characteristics of mixed convection heat transfer in a lid-driven square cavity with various Richardson and Prandtl numbers, Int. J. Therm. Sci. 50 (2011) 197–205.

[3] E.A. Nada, A.J. Chamkha, Mixed convection flow in a lid-driven inclined square enclosure filled with a nanofluid, Eur. J. Mech. B Fluid 29 (2010) 472–482.

[4] W. Ibrahim, M. Hirpo, Finite element analysis of mixed convection flow in a trapezoidal cavity with non-uniform temperature, Heliyon 6 (2020) e06933.

[5] S. Aljahar, A.L. Ekaid, S.H. Ibrahim, J. Alsheba, Mixed convection in sinusoidal lid driven cavity with non-uniform temperature distribution on the wall utilizing nanofluid, Heliyon 7 (2021), e06907.

[6] H.P. Otop, I. Dagtekin, Mixed convection in two-sided lid-driven differentially heated square cavity, Int. J. Heat Mass Transfer 47 (2004) 1761–1769.

[7] S. Saha, A.K. Hussein, S. Saha, A. Hussain, Mixed convection in a tilted lid-driven square enclosure with adiabatic cylinder at the center, Heat Technol. 29 (1) (2011) 143–156.

[8] K. Kanaar, S.M. Athial, Laminar mixed convection flow and heat transfer characteristics in a lid-driven cavity with a circular cylinder, Int. J. Heat Mass Transfer 66 (2013) 201–209.

[9] H. Zheng, M.Y. Ha, H.S. Yoon, Y.G. Park, A numerical study on mixed convection in a lid-driven cavity with a circular cylinder, J. Mech. Sci. Technol. 27 (1) (2013) 273–286.

[10] M.K. Triveni, R. Panua, Natural and mixed convection study of isothermally heated cylinder in a lid-driven square enclosure filled with nanofluid, Aram J. Sci. Eng. 46 (2021) 2505–2525.

[11] M.S.H. Thakur, M. Islam, A.U. Karim, S. Saha, M.N. Hasan, Numerical study of laminar mixed convection in a Cu-water nanofluid filled lid-driven square cavity with an isothermally heated cylinder, AIP Conf. Proc. 2211 (1) (2019), 070017. AIP Publishing LLC.

[12] Z.A.S. Raizah, Mixed convection in a lid-driven cavity filled by a nanofluid with an inside circular cylinder, J. Nanofluids 6 (2017) 927–939.

[13] M.M. Billah, M.M. Rahman, U.M. Sharif, N.A. Rahim, R. Saidur, M. Hasansuzzaman, Numerical analysis of fluid flow due to mixed convection in a lid-driven cavity having a heated circular hollow cylinder, Int. Commun. Heat Mass Transfer 38 (2011) 1093–1103.

[14] M.M. Rahman, M.A. Alam, M.A.H. Mamun, Finite element analysis of mixed convection in a rectangular cavity with a heat-conducting horizontal circular cylinder, Nonlinear Anal. Model Control 14 (2) (2009) 217–247.

[15] W.S. Fu, C.S. Cheng, W.J. Shieh, Enhancement of natural convection heat transfer of an enclosure by rotating a circular cylinder, Int. J. Heat Mass Transfer 37 (1994) 1885–1897.

[16] V.A.F. Costa, A.M. Raimundo, Steady mixed convection in a differentially heated square enclosure with an active rotating circular cylinder, Int. J. Heat Mass Transfer 53 (5-6) (2010) 1208–1219.

[17] C.C. Liao, C.A. Lin, Mixed convection of a heated rotating cylinder in a square enclosure, Int. J. Heat Mass Transfer 72 (2014) 9–22.

[18] M. Alam, Kamruzzaman, F. Ahsan, M.N. Hasan, Mixed convection heat transfer inside a differentially heated square enclosure in presence of a rotating heat conducting cylinder, AIP Conf. Proc. 1754 (1) (2016), 050035. AIP Publishing LLC.

[19] A.N. Sadr, M. Shekaramiz, M. Zarinfar, A. Esmaeily, H. Khoobtash, D. Tohghraie, Simulation of mixed-convection of water and nano-encapsulated phase change material inside a square cavity with a rotating hot cylinder, J. Energy Storage 47 (2022), 103606.

[20] S.H. Hussein, A.K. Hussein, Mixed convection heat transfer in a differentially heated square enclosure with a conductive rotating circular cylinder at different vertical locations, Int. Commun. Heat Mass Transfer 38 (2) (2011) 263–274.

[21] M. Alam, Kamruzzaman, F. Ahsan, S. Saha, M.N. Hasan, Effect of location of a rotating circular cylinder and heat source on mixed convection heat transfer characteristics inside a square enclosure with discrete heater at the bottom wall, AIP Conf. Proc. 1851 (1) (2017), 020101. AIP Publishing LLC.

[22] K. Kanaar, S.M. Athial, Mixed convection heat transfer in a lid-driven cavity with a rotating circular cylinder, Int. Commun. Heat Mass Transfer 86 (2017) 131–142.

[23] D. Chatterjee, S.K. Gupta, B. Mondal, Mixed convective transport in a lid-driven cavity containing a nanofluids and a rotating circular cylinder at the center, Int. Commun. Heat Mass Transfer 56 (2014) 71–78.

[24] S. Dalvi, S. Bali, D. Nayka, S.H. Kale, T.B. Gohil, Numerical study of mixed convective heat transfer in a lid-driven enclosure with rotating cylinders, Heat Transfer Asian Res. 48 (4) (2019) 1180–1203.

[25] F. Selimfendigil, H.F. Otop, Numerical study of MHD mixed convection in a nanofluid filled lid driven square enclosure with a rotating cylinder, Int. J. Heat Mass Transfer 78 (2014) 741–754.

[26] E.H. Chowdhury, A.A. Mehedi, D.K. Paul, S. Saha, M. Ali, Conjugate pure mixed convection in a differentially heated lid-driven square cavity with two rotating cylinders, AIP Conf. Proc. 2324 (1) (2021), 050023. AIP Publishing LLC.

[27] D.K. Paul, A.A. Mehedi, E.H. Chowdhury, S. Saha, M. Ali, M.R. Amin, Conjugate mixed convection in a differentially heated lid-driven square cavity with rotating cylinders, in: ASME International Mechanical Engineering Congress and Exposition 64/591, American Society of Mechanical Engineers (ASME), 2020, p. V011T11A017.

[28] M.A.S. Mustafa, H.M. Hussain, A.A. Ahtan, L.J. Habeeb, Review on mixed convective heat transfer in different geometries of cavity with lid driven, J. Mech. Eng. Res. Dev. 43 (7) (2020) 12–25.

[29] A.N. A Saied, M.A. S Mustafa, S.K. Ayed, L.J. Habeeb, Review on heat transfer enhancement in cavity with lid driven, J. Mech. Eng. Res. Dev. 43 (7) (2020) 356–373.

[30] Ali J. Chamkha, Hydromagnetic combined Convection flow in a vertical lid-driven cavity with internal heat Generation or absorption, Numer. Heat Transfer A: Appl. 41 (5) (2002) 529–546.

[31] S. Sivasankeran, V. Sivakumar, A.K. Hussein, P. Prakash, Mixed convection in a lid-driven two-dimensional square cavity with corner heating and internal heat generation, Numer. Heat Transfer A: Appl. 65 (3) (2019) 269–286.

[32] S. Saha, M.N. Hasan, G. Saha, M.Q. Islam, Effect of inclination angle on mixed convection in a lid-driven square enclosure with internal heat generation or
absorption, in: Proceedings of International Conference on Mechanical, Industrial and Energy Engineering, 2010. Paper No. ME10-149, Khulna, Bangladesh.

[33] A.K.M.S. Islam, G. Saha, S. Saha, M.Q. Islam, Effect of inclination angle on mixed convection in a lid driven enclosure with internal heat generation, in: Proceedings of the 12th Asian Congress of Fluid Mechanics, 2008. Paper No. 12acfm-128, Daejeon, Korea.

[34] L.K. Saha, K.M.S. Uddin, M.A. Taher, Effect of internal heat generation or absorption on MHD mixed convection flow in a lid driven cavity, Am. J. Appl. Math. 3 (1–1) (2015) 20–29.

[35] A. Mahmoudi, I. Mejri, M.A. Abbassi, A. Omri, Analysis of the entropy generation in a nanofluid-filled cavity in the presence of magnetic field and uniform heat generation/absorption, J. Mol. Liq. 198 (2014) 63–77.

[36] F. Selimefendigil, H.F. Öztöp, Analysis of MHD mixed convection in a flexible walled and nanofluids filled lid-driven cavity with volumetric heat generation, Int. J. Mech. Sci. 118 (2016) 113–124.

[37] N.A. Bakar, R. Roslan, Mixed convection in a lid-driven horizontal cavity in the presence of internal heat generation or absorption, J. Adv. Res. Numer. Heat Transfer 3 (1) (2020) 1–11.