Mathematical model and numerical algorithm for studying suspension filtration in a porous medium considering the processes of colmatation and suffusion

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Abstract. Solution of an urgent problem related to technological process of filtering and dehydration of liquid solutions from fine particles and undesirable ionic compounds is discussed in the paper. There was developed a mathematical model and numerical algorithm taking into account the key factors and parameters that significantly affect considered processes. The problems related to the determination of the key parameters and their variation ranges are solved in the paper. Based on the analysis of conducted numerical experiments, the conclusions are formulated that serve as the basis for making appropriate management decisions.

1. Introduction

The process of purification and separating highly concentrated liquid solutions, and fresh water filtering from undesirable ions and fine particles using ion exchange filters, taking into account the pore colmatation of filter barriers, the pressure drop inside the filter chamber, the process of colmatation and suffusion, etc., is a complex non-stationary technological processes (CNTP).

As shows the analysis of experimental laboratory studies, the process of purification and separation of liquid solutions is affected by many external and internal factors of different specific weights. Therefore, the synthesis of the key parameters of CNTP, determination of their variation ranges and selection of an acceptable operating mode of a filtering unit are the most important tasks for the effective management and functioning of the process.

Many scientists dealt with the problem of mathematical modeling of CNTP and obtained significant theoretical and applied results.

In particular, to improve the removing efficiency of suspended solids from a solution by high-rate filters, a new design of the filtering unit was proposed in [1]. The filtration equation was obtained to describe the parallel filtration of a weak suspension through a granular and fibrous media; a mathematical model (MM) includes the equations of mass transfer, mass exchange and the dependences of colmatation effects and filtering medium parameters.
The problem of reservoirs permeability variation in the bottom-hole zone, and the problem of reducing the water intake capacity of injection wells due to the pore and channels colmatation of the reservoir by suspended solids were considered in [2]. The author has performed monitoring of corrosion, the observation of injected water composition and analysis of mechanical impurities in the injected water.

Water treatment in oil fields development using flooding was discussed in [3]. The authors of the paper showed that water impurities can enhance the rock mass colmatation up to the complete clogging of pores and to fractures. Therefore, the issues of improving the quality of injected water, aimed at improving the properties of the reservoir, have been studied from different points of view.

The problem of colmatation–suffusion filtration of disperse systems in a two-dimensional porous medium consisting of two zones with flowing and stationary fluids was studied in [4] taking into account diffusion phenomena. The influence of lateral and longitudinal diffusion and the phenomena of colmatation–suffusion on the reservoir properties of a porous medium were determined.

A four-component continuum finite-difference model for suffusion was considered in [5] and it was expanded by the process of self-filtration. A probabilistic study using the Monte Carlo method was carried out to analyze the effect of dispersion, spatial correlation length and cross-correlation of randomly distributed initial porosity and the content of small particles in the eroded mass.

A mathematical model was developed for the process of cleaning liquids from multicomponent harmful substances [6]. The model takes into account the interaction of hydrodynamic parameters, changes in hydraulic properties of the medium, and initial and boundary conditions. The influence of the pollutants content on the formation of sediment layer and the rate of fluid filtration, as well as the inverse effect on the filtration components, in particular, on the porosity and coefficient of particles separation, were studied.

Author of work [7] proposed a mathematical model of fluid filtration through a porous medium taking into account the process of colmatation and suffusion in a complex heterogeneous “filling” system. The influence of system parameters was taken into account through: 1) a dynamic change in the porosity of the backfill during the filtration process; 2) the coefficient of non-linear dependence of the filter, the porosity of the backfill; 3) the dependence of the process rate on the filtration coefficient and porosity. These relationships allowed an increase in adequacy of mathematical models for studying physical processes.

Analysis of the above-mentioned papers and CNTP studies showed that some authors are solving the problem of fluid filtering taking into account the colmatation and suffusion process in porous media at a constant filtration rate without considering the rate of particles deposition in filter pores; other authors solve this problem without considering the ion-exchange process of fluid filtration in porous media and the barodiffusion coefficient.

In our previous work [8], we have considered: firstly, the process of suspension filtering with a variation in filtration rate; secondly, with account for sediment layer formation on the surface of the filter and its compressibility; thirdly, with account for rate of gel particles deposition in the filter pores, where the filter pores are filled with gel particles and the pores of the filter are clogged. Carries studies shown that it is necessary to take into account the influence of these parameters on the change in porosity and permeability of the ion-exchange filter with time during the processes of colmatation and suffusion.

2. Statement of the problem
To carry out a comprehensive study, forecast, synthesis of the key parameters and their variation ranges, as well as making managerial decisions on CNTP based on the laws of hydrodynamics and kinetics of the process, the following MMs were developed:
\[
\frac{\partial mW}{\partial t} + \frac{\partial Wn}{\partial x} + \frac{\partial N}{\partial t} = \frac{\partial}{\partial x}\left(\chi \frac{\partial n}{\partial x}\right) + \frac{X_b}{\rho} \frac{\partial P}{\partial x};
\]
\[
\frac{\partial N}{\partial t} = \beta(n - n'), \quad N = \frac{n'}{a + bn'}
\]
\[
\frac{\partial W}{\partial t} + W \frac{\partial W}{\partial x} = -1 \frac{\partial P}{\partial x} + \mu \frac{\partial^2 W}{\partial x^2} - \frac{\mu H_0 W}{\rho^{H_0} (1 - \delta)^2};
\]
\[
\frac{\partial m\theta}{\partial t} + W \frac{\partial \theta}{\partial x} + \frac{\partial m\alpha}{\partial t} + (1 - m_0) \frac{\partial m\delta}{\partial t} = 0;
\]
\[
\frac{\partial \delta}{\partial t} = \lambda(\theta - \gamma\delta); \quad \theta = \frac{\alpha}{1 - \delta};
\]
\[
\frac{\partial m}{\partial t} = \omega_1 (m_0 - m) \mid \nabla p \mid - \omega_2 m\theta;
\]

under initial and boundary conditions:
\[
W(1, \theta = e \frac{W_0}{W}) = \varphi(x), \quad \delta = 0, \quad m = m_0 \quad \text{at} \quad t = 0
\]
\[
\frac{\partial W}{\partial x} = \frac{H_0^3}{Hk_0} \left[ \frac{P_0}{W (1 - \delta)^2} \right]; \quad \theta = 1 \quad \text{at} \quad x = 0
\]
\[
\frac{\partial W}{\partial x} = 0, \quad \frac{\partial \theta}{\partial x} = \frac{mH_0 \lambda (1 - m_0)}{W_0} (\gamma_0\delta - \theta), \quad \text{at} \quad x = 1
\]

where \( W \) - is the filtration rate; \( \theta \) - volume concentration of suspension in flowing mixture; \( \delta \) - the concentration of suspension of settled mass in filter pores; \( \alpha \) - the concentration of particles in suspension state; \( F \) - the filter area; \( \rho \) and \( \mu \) - the density and viscosity of suspensions; \( P \) - pressure in the unit column; \( H_0 \) - the filter thickness; \( k_0 \) - the coefficient of filter permeability before its operation; \( n \) and \( N \) - non-equilibrium concentrations of exchanging ions in solution per unit length of the sorption column; \( \beta \) - the effective constant of exchanging ions; \( \chi \) - the coefficient of longitudinal diffusion; \( \chi_b \) - the barodiffusion coefficient; \( a \) and \( b \) - the constant isotherms; \( \lambda \) - kinetic coefficient; \( n' \) - the concentration of ions in solution in equilibrium with the concentration of \( N \); \( \gamma \) - the dispersion coefficient; \( m_0 \), \( m \) - the initial porosity and porosity of the settled mass, \( \omega_1 \), \( \omega_2 \) - the coefficients characterizing the intensity of suffusion and pore colmatation; \( \mid \nabla p \mid \) - the pressure gradient modulus, \( t \) - time.

To solve the problem (1) - (3), assume that each suspended particle entering the filter unit pores can settle at any time, and the settled particle, on the contrary, can break off, and then: \( \alpha = \theta(1 - \delta) \).

3. Numerical method for solving the problem

The problem (1) - (3), it is described by a system of nonlinear partial differential equations and is difficult to obtain an analytical solution. Based on the above, the finite-difference method is used for numerical integration; to increase the approximation order, the vector-difference scheme with an accuracy of \( 0(h^2) \) is used and the system of differential equations in a dimensionless form [8] is obtained:
\[
\frac{\partial n_1}{\partial t} + \gamma \frac{\partial n_2}{\partial t} + \frac{\partial N_1}{\partial t} = \lambda \left( \frac{\partial n_1}{\partial x} \right) + \chi_s \partial \rho; \quad (4)
\]

\[
\frac{\partial n_2}{\partial t} + \gamma \frac{\partial n_1}{\partial t} + \frac{\partial N_2}{\partial t} = \lambda \left( \frac{\partial n_2}{\partial x} \right) + \chi_s \partial \rho; \quad (5)
\]

\[
\frac{\partial W_1}{\partial t} + W_1 \frac{\partial W_2}{\partial x} - W_1 \frac{\partial \theta_1}{\partial t} = -E_u \frac{\partial \rho}{\partial x} + \frac{1}{Re} \frac{\partial^2 W_1}{\partial \xi_1^2}; \quad (6)
\]

\[
\frac{\partial W_2}{\partial t} + W_2 \frac{\partial W_1}{\partial x} - W_2 \frac{\partial \theta_2}{\partial t} = -E_u \frac{\partial \rho}{\partial x} + \frac{1}{Re} \frac{\partial^2 W_2}{\partial \xi_2^2} - \frac{1}{Re_1} \left( 1 - \theta_3 \right)^2; \quad (7)
\]

\[
\frac{\partial \xi_1}{\partial t} + \frac{\partial W_1\theta_1}{\partial x} + \frac{\partial \xi_2\alpha}{\partial x} + (1 - m_0) \frac{\partial \xi_1\theta_1}{\partial t} = 0; \quad (8)
\]

\[
\frac{\partial \xi_2\alpha}{\partial x} + \frac{\partial W_2\theta_1}{\partial x} + (1 - m_0) \frac{\partial \xi_2\theta_2}{\partial t} = 0; \quad (9)
\]

where \( \theta_3 = \frac{1}{0} \int_{2 - \theta_1}^1 \theta_1 (1 - \theta_1) \, dx; \quad \theta_3 = \frac{1}{2} \int_{2 - \theta_2}^0 \theta_2 (1 - \theta_2) \, dx. \)

Boundary conditions of problem (4) - (9) have the following form:

\[
W_i = W_2 = 1, \quad \xi_1 = \xi_2 = 0, \quad \theta_1 = \theta_2 = e^{-\lambda mH_0 (1-m_0)x}, \quad \text{at } t=0; \quad (10)
\]

\[
W_i = W_2 = 0; \quad n_1 = n_2 = n(0), \quad N_1 = N_2 = 0, \quad \text{at } x=0; \quad (11)
\]

\( \theta_1 = \theta_2 = 1, n_1 = n_2 = 1. \)
\[
\frac{\partial W_1}{\partial x} = \frac{\partial W_2}{\partial x} = 0, \quad \frac{\partial \theta_1}{\partial x} = \frac{m_1 H_0 \lambda (1 - m_0)}{W_0} \left( \gamma_0 \delta_1 - \theta_1 \right), \\
\frac{\partial \theta_2}{\partial x} = \frac{m_2 H_0 \lambda (1 - m_0)}{W_0} \left( \gamma_0 \delta_2 - \theta_2 \right), n_1 = n_2 = 0.
\]

(12)

At approximating the system of equations (4) - (9) by an implicit scheme we get:

\[
\begin{align*}
    a_{1i} n_{i+1,j} - b_{1i} n_{i,j} + c_{1i} n_{i,j-1} - d_{1i} n_{2,j-1} + e_{1i} n_{2,j-1} &= -f_{1i}; \\
    a_{2i} n_{2,j+1} - b_{2i} n_{2,j} + c_{2i} n_{2,j-1} - d_{2i} n_{1,j-1} + e_{2i} n_{1,j-1} &= -f_{2i}.
\end{align*}
\]

(13)

where the solution to the system is sought in the form

\[
\begin{align*}
    n_{1i} &= A_i n_{1,i+1} + B_i n_{2,i+1} + C_{1i}; \\
    n_{2i} &= A'_i n_{2,i+1} + B'_i n_{1,i+1} + C'_{1i}.
\end{align*}
\]

The coefficients in system (13) are determined from the recurrence relations [9]. From the system of equations (6) we obtain

\[
\begin{align*}
    a_{1i} W_{1i+1} - b_{1i} W_{1i} + c_{1i} W_{1i-1} - d_{1i} W_{2i} + e_{1i} W_{2i-1} &= -f_{1i}; \\
    a_{2i} W_{2i+1} - b_{2i} W_{2i} + c_{2i} W_{2i-1} - d_{2i} W_{1i} + e_{2i} W_{1i-1} &= -f_{2i}.
\end{align*}
\]

(14)

and the solution to system (14) is sought in the form

\[
\begin{align*}
    W_{1i} &= A_i W_{1,i+1} + B_i W_{2,i+1} + C_{1i}; \\
    W_{2i} &= A'_i W_{2,i+1} + B'_i W_{1,i+1} + C'_{1i}.
\end{align*}
\]

The coefficients included in system (14) are determined from recursive relations [9].

4. Discussion of results

To conduct a comprehensive study of CNTP, an algorithm and a software tool have been developed and computer-aided experiments (CE) have been conducted. As seen from the conducted CE, the rate of fluid passed through the filter barriers decreases with time due to gel particles colmatation in filter pores and a sediment layer formation on the surface of the filtering unit column. A noticeable decrease in filtration rate, seen from the analysis of numerical calculations, is observed in the interval from \( t = 8 \) h to \( t = 18.5 \) h. It was also observed that the concentration of suspended particles inside the filter began to increase with time. A noticeable increase in the concentration of gel particles inside the filter occurred first in the upper layers of porous media; when the filtration time was more than 18.5 hours the process of colmatation gradually occurred in the lower layers of the filter barriers.

Computational experiments were carried out at different values of fluid supply rate to the column of the filtering unit. From the analysis of CE, it is seen that in solutions filtering technology, the initial rate of fluid supply to the filter column of the unit and the barriers thickness and filter initial porosity play a significant role. CE have shown that with decreasing thickness of the filter barrier, the filtration rate increases exponentially. It should be noted that when the fluid is supplied at a high rate to the unit column at the initial stages, it increases the filter performance, and then, due to the process of colmatation, the switching time of the filtering unit is shortened and the hydraulic pressure in the filter increases. The maximum pressure value is reached at \( t = 9.5 \) hours.

It has been established by CE that during the process of solution filtering at a constant pressure, the bulk of the gel particles settle on the upper layer of the filter and a sediment layer forms; it subsequently plays the role of a filter, and the settling rate of gel particles at a change in the filtration
time from $t \geq 0.01$ to $t \geq 2.02$ h grows slowly and, starting from $t \geq 4.03$, it noticeably increases with time, a sharp increase being observed at $t \geq 6.04$ h.

CEs were also performed for various values of the barodiffusion coefficient. Analysis of the calculations showed that with an increase in the barodiffusion coefficient, the rate of exchanging ions in the solution and filter barriers increases. This in turn leads to a reduction in filter operating time and an increase in pressure inside the column of the filtering unit. It has been established by CE that the maximum saturation of the filter pores with ions and gel particles occurs in the upper layers of the filter barriers and the filter operating time increases with increasing sizes of gel particles in solution.

As follows from the results of CE, the time of filter pores colmatation with gel-particle depends on the rate of fluid filtration, initial concentration of the filtrate, the pore size of the filter, and the diameters of the gel particles in solution.

To validate the adequacy of the developed mathematical apparatus, the calculated data (model) are compared with the experimental data obtained by Baileys and it does not exceed 5% (figure 1).

![Figure 1. Comparison of the change in filtering rate over time (— - output curve calculated according to the full model; • - experimental data).](image)

Comparison of the obtained numerical calculations (Fig. 1) with experimental data shows that the developed TP model adequately describes the process as a whole and the difference between the calculated data obtained as a result of numerical integration of the problem and experimental data does not exceed 5-6%.

5. Conclusion

From the analysis of numerical calculations it was found that: the filtration rate of the suspension decreases over time due to gel particles colmatation in the filter pores and the formation of a sediment layer on the surface of the filtering column of the unit; in the process of suspension filtering, the initial rate of fluid supply to the column of the unit, the barrier thickness and initial porosity of the filter play a significant role; an increase in the rate of fluid supply to the filter column at the initial stages of the filtering process leads to an increase in filter productivity, and then due to the process of colmatation, the switching time of the filter unit is reduced and the hydraulic pressure in the filter increases; with an increase in the barodiffusion coefficient, the rate of change of the exchanging ions in solution and filter barriers increases, and the maximum saturation of filter pores with ions and gel particles occurs in the upper layers of the filter barriers, the filter operating time increases with increasing size of the gel particles in solution.

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