Mathematical models of sunlight for dynamic lighting systems of agricultural objects

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Abstract. A promising method for illuminating agricultural objects is dynamic lighting with a change in the luminous flux and correlated color temperature of artificial optical radiation sources adapted to the natural dynamics of sunlight. The purpose of this study is to develop mathematical models of changes in illumination on the control surface and correlated color temperature of natural solar radiation during the day. The experimental method established a linear relationship between the correlated color temperature of sunlight and the surface illumination in the direction of the Sun. The regularity of time changes in the height of the Sun above the horizon was determined. The functional dependence of illumination on the height of the Sun above the horizon was revealed. The equations can be used to control artificial light sources in the systems of dynamic illumination of agricultural objects in order to increase the productivity of animals.

1. Introduction

It is known that optical radiation has a significant effect on the circadian rhythms of animals and humans [1]. Under artificial lighting conditions, special attention should be paid to the intensity, spectral composition and correlated color temperature of light sources [2]. The main hormones-regulators of the circadian rhythm "sleep-wake" are melatonin and cortisol. The higher the value of the correlated color temperature of the light source, the greater the effect of suppressing the production of melatonin in humans during illumination.

Light has a similar effect on farm animals, which significantly affects their productivity. For example, in cows, milk production is associated with the concentration of autogenous growth factor IGF-1, whose blood concentration is higher with a stronger suppression of melatonin production during the light period. In poultry, the secretion of gonadotropic hormones is associated with the concentration of melatonin which increase the productive qualities of hens.

Circadian rhythms are associated with a natural cyclical change in the level of illumination, which occurs due to the rotation of the Earth around its axis. They are observed in all living organisms. At the biological level, this is manifested in the daily cycle of secretion of melatonin and cortisol [3].

Like many other researchers [1, 4, 5], the authors adhere to the hypothesis, according to which the effectiveness of influence of artificial light is higher if the lighting system implements the dynamics of natural solar radiation during daily hours. An important feature of such lighting systems is a more directed impact on the system of circadian rhythms of biological objects.

The use of a dynamic lighting system for agro-industrial complex objects with a change in the level of the luminous flux of an artificial light source and the correlated color temperature of its optical radiation makes it possible to create an artificial light environment that is closer to natural conditions.
In recent years, this has become especially important, since under intensification of production, there is an increasing isolation of farm animals from natural conditions, and the artificial environment does not always meet the physiological needs of the animal body.

In such lighting systems, changes in the luminous flux and correlated color temperature of the optical radiation of light sources should be as close as possible to the dynamics of natural sunlight. In this case, to maintain a level of illumination, which changes in time, it is necessary to use a tracking automatic control system with an illumination sensor [6].

A literature analysis [7–10] allowed us to build a curve of illumination changes during daily hours, since the data on illumination changes are scattered and make it difficult to identify the exact functional dependence. Literature data did not allow us to describe the curve of changes in correlated color temperature of natural sunlight.

The purpose of this study was to build mathematical models of changes in illumination on the control surface and correlated color temperature of natural solar radiation during the day. These models can be used to control artificial lighting in systems of dynamic lighting of agricultural objects.

2. Materials and methods

The study of solar radiation was carried out using the experimental unit shown in Figure 1. The dynamics of illumination and correlated color temperature was investigated during the day. The values were measured with a digital colorimeter TKA-ITsT. To eliminate the angular error of the instrument, measurements were taken in a tracking mode in the direction of the Sun. The position of the photometric head was changed by rotating the knobs of fine movements in azimuth and altitude above the horizon. The measurement interval was 10 min. The research date was chosen close to the day of the autumnal equinox (September 23), taking into account weather conditions (from September 22 to October 4). The place of experiments was Krasnodar Territory, Russia (46° 19'44.5"N, 38° 46'37.4"E).

![Figure 1. Photos of the field experiment: a - location of the experiments; b - experimental setup, 1 - tripod; 2 - colorimeter TKA-ITsT; 3 - photometric head; 4 - handle of fine movements; 5 - counterweight](image)

The systems analysis, the theory of experimental research planning, the correlation analysis and the regression analysis were used. The experimental data were processed in Microsoft Office Excel. The regression coefficients of mathematical models were obtained using the "Search for solutions" software.
3. Results and discussion
An analysis of experimental data showed that in cloudy weather conditions there is a significant scatter of data due to random weather events. Therefore, a further study of the dynamics of illumination and correlated color temperature of natural light was carried out for solar radiation in cloudless weather conditions.

At the initial stage, experimental data were sampled; outliers were eliminated, and data were averaged. It was revealed an insignificant asymmetry of the shapes of curves of changes in illumination and correlated color temperature before and after noon caused by the planetary motion of the Earth. In order to simplify the type of models, it was assumed that the dynamics of natural light is symmetric relative to the noon. Insignificant asymmetry of measurements was not taken into account. Further processing of the experimental data was carried out for the averaged experimental curves.

Based on the results of the preliminary processing, graphs of time changes in illumination ($E$) and correlated color temperature of natural light ($CCT$) in the direction of the Sun were built (Figure 2, a and Figure 2, b).

![Figure 2. Changes in the direction of the Sun: a - illumination on the surface; b - correlated color temperature of sunlight](image)

A visual comparison of the graphs (Figure 2) shows that the curves of changes in illumination and correlated color temperature have similar shapes. A linear relationship has been established between the correlated color temperature of sunlight and the level of illumination:

$$CCT = aE + b, \text{[K]}.$$  \hspace{1cm} (1)

The model coefficients are: $a = 3027.1$, $b = 2545.8$.

In model (1), illumination is normalized relative to the average maximum value $E_{\text{max}} = 127.65$ klx of the experimental data. The graphic interpretation of model (1) (Figure 3, orange line) shows a high convergence with the experimental data (Figure 3, blue curve). The coefficient of determination of model (1) is $R^2 = 0.9972$. 

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Figure 3. Dependence of the correlated color temperature of natural light on illumination on the surface in the direction of the Sun

It is advisable to search for the functional dependence \( E = f(t) \) through the height of the Sun above the horizon, which can be described by the cosine function with a high accuracy \( (R^2 = 0.9987) \) (Fig. 4):

\[
h = h_m \cos \left( \frac{2\pi}{T}(t_m - t) \right) + h_0, \text{[deg]}. \tag{2}
\]

The harmonic function period \( T = 24 \text{ h} \), the amplitude is \( h_m = 44.48^\circ \), the constant is \( h_0 = -4.39^\circ \) and the time of a maximum is \( t_m = 5.5 \text{ h} \).

Figure 4. Time changes in the height of the sun above the horizon

We assume that the power of solar optical radiation is linearly dependent on its height above the horizon. Taking into account the Weber – Fechner law, the dependence \( E = f(h) \) can be described by a logarithmic function whose graph passes through the points \((0; 0)\) and \((h_m + h_0; 1)\):

\[
E = \frac{1}{c} \ln \left( \frac{h}{h_m + h_0}(e^c - 1) + 1 \right), \text{[p.u.]} \tag{3}
\]

The model coefficient \( c = 2.1507 \). The results of calculating the illumination level by model (3) are shown in Figure 5. The visual convergence of the calculated data with the experimental ones (Figure 5) is confirmed by the high value of the coefficient \( R^2 = 0.9926 \).
Figure 5. Dependence of illumination on the height of the Sun above the horizon

The models (1) - (3) allow high-precision calculations of the illumination level ($R^2 = 0.9926$) and correlated color temperature ($R^2 = 0.9972$) at any time (Figure 6).

Figure 6. Time variations in surface illumination (a) and correlated color temperature of sunlight (b)

4. Conclusion

A promising method for illuminating agricultural objects is dynamic lighting with a variation in the luminous flux and correlated color temperature of artificial optical radiation sources adapted to the natural dynamics of sunlight.

The experimental method established a linear relationship (1) between the correlated color temperature of sunlight and the surface illumination in the direction of the Sun. The regularity (2) of time changes in the height of the Sun above the horizon was determined. The functional dependence (3) of illumination on the height of the Sun above the horizon was revealed.

Equations (1) - (3) can be used to control artificial light sources in the systems of dynamic illumination of agricultural objects, in order to increase the productivity of animals. When implementing various lighting programs, it may be necessary to scale the illumination level, illumination and correlated color temperature over time. The developed models (1) - (3) are easy to implement. The change in the length of the day is made by adjusting the time of noon $t_m$, and changes in the maximum illumination level – by changing the value $E_m$.

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