Research Article

Novel Robust Control of Stochastic Nonlinear Switched Fuzzy Systems

Hong Yang, Yu Zhang, and Le Zhang
School of Information Engineering, Shenyang University, Shenyang 110044, China

Correspondence should be addressed to Hong Yang; cherryyh@126.com

Received 18 June 2021; Accepted 16 August 2021; Published 6 September 2021

Abstract

This paper addresses the problem of designing novel switching control for a class of stochastic nonlinear switched fuzzy systems with time delay. Firstly, a stochastic nonlinear switched fuzzy system can precisely describe continuous and discrete dynamics as well as their interactions in the complex real-world systems. Next, novel control algorithm and switching law design of the state-dependent form are developed such that the stability is guaranteed. Since convex combination techniques are used to derive the delay independent criteria, some subsystems are allowed to be unstable. Finally, various comparisons of the elaborated examples are conducted to demonstrate the effectiveness of the proposed control design approach. All results illustrate good control performances as desired.

1. Introduction

As a typical hybrid system, switched systems have attracted much attention in the past decades because many physical systems can use this multimode system for mathematical modeling. Considering the wide application of switching power supply, the research on the switching system has never stopped, and some good results have been achieved [1–5].

One of the bridge of communication between the system and the fuzzy linear system is the T-S fuzzy dynamic model; the system makes the fuzzy system which has been greatly enriched; the research methods of nonlinear systems can be used to study the stability of linear system theory. The research literature [6–14] based on this kind of system has a representative, and in real-life problems, a lot has been applied to obtain considerable social and economic benefits.

From the middle of the 1980s, a number of analysis problems about T-S fuzzy control have appeared. And recently, switched systems have been extended further to encompass fuzzy control too. A novel fuzzy adaptive design is constructed for HFVs in spite of asymmetric time-varying constraints and actuator constraints. It can be assured finite-time convergence with the aid of a smooth switch between a fractional and a linear control [15]. Then, a Nussbaum function-based adaptive control method is proposed for high-order nonlinear systems with mixed control directions and dead-zone input [16].

Very notably, the stabilizability conditions and smoothness conditions for fuzzy switching control systems were reported. For the continuous-time case, a combination of hybrid systems and fuzzy multiple model systems was described and an idea of the fuzzy switched hybrid control was put forward [17]. Based on the T-S fuzzy systems, Tanaka et al. [18–20] introduced new switching fuzzy systems for more complicated real systems such as multiple nonlinear systems, switched nonlinear hybrid systems, and second-order nonholonomic systems. Such a switching fuzzy system model has two levels of structure, in which the first level is the region rule level and the second level is the local fuzzy rule level. In fact, it is switching according to the same premise variable. Stability conditions are given. In [21–23], some extension based on [19, 20] are given.

Because stochastic systems have many applications in the real world, including nuclear, biological, socio-economic, and chemical processes, the stability of stochastic systems has been widely concerned in the past decades. Moreover, the study of stochastic systems has been of great interest since stochastic modeling has come to play an
important role in many branches of engineering applications [24]. Many results on stochastic systems can be found in the literature. To mention a few, the control problem is studied in [25–27]. The filtering problem is investigated in [28], and the model reduction problem is considered in [29].

The switched stochastic system, as the product of the combination of the switched system and the stochastic system, has important theoretical research significance and practical application value. In recent years, switched stochastic control theory has become an important research tool in large class of practical engineering systems, such as switching transmission control, state estimation of distributed network control protocol, robust control of solar devices, and forced linear dynamic systems with communication network sequence [30, 31].

A new model of class of stochastic switched fuzzy time-delay systems is proposed in this paper. A system of this class is a switched system whose subsystems are all stochastic fuzzy time-delay systems. A switching law is designed to give robust stability. In contrast with the existing results, we study the switched stochastic fuzzy system without levels of the structure. The method provides a kind of different premise variable switching directly. In [18–20], the same premise variable switching with two levels of the structure is considered.

2. Stochastic Nonlinear Switched Fuzzy System Model with Time Delay

A stochastic switched fuzzy model with time delay is considered:

\[ R^i_\sigma(t): \text{if } z_{\sigma(t)}(t) = M^i_{\sigma(t)} \ldots \text{and } z_{\sigma(t)}(t) = M^p_{\sigma(t)}, \text{then} \]
\[ dx(t) = \left[ A^i_{\sigma(t)}x(t) + A^i_{\sigma(t)}x(t - \tau) + B^i_{\sigma(t)}u^i_{\sigma(t)}(t) + F^i_{\sigma(t)}f^i_{\sigma(t)}x(t) \right] dt \]
\[ + D^i_{\sigma(t)}x(t) d\omega(t), \quad x_{\sigma}(\theta) = \phi(\theta), \theta \in [-\tau, 0], i = 1, 2, \ldots, N_{\sigma(t)}. \]  

Then, the \( i \)th substochastic fuzzy control system can be expressed as follows:

\[ R^i_l: \text{if } z_{il}(t) = M^i_{1l} \ldots \text{and } z_{ip}(t) = M^i_{pl}, \text{then} \]
\[ dx(t) = \left[ A^i_lx(t) + A^i_{il}x(t - \tau) + B^i_{il}u^i_{il}(t) + F^i_{il}f^i_{il}x(t) \right] dt \]
\[ + D^i_{il}x(t) d\omega(t), \quad l = 1, 2, \ldots, N_{l}, i = 1, 2, \ldots, m. \]

Assumption 1 (see [12]). There is an appropriate dimension real matrix \( \Phi \) and \( \Delta \) such that

\[ \| f(\delta(t), \delta(t - \tau) + f(\kappa, \kappa(t - \tau)) \| \]
\[ \leq \| \Phi(\delta(t) - \kappa(t)) \| + \| \Delta(\delta(t - \tau) - \kappa(t - \tau)) \|. \]

\[ dx(t) = \sum_{l=1}^{N_{l}} \eta_{il}(z(t)) \left[ A^i_lx(t) + B^i_{il}u^i_{il}(t) + F^i_{il}f^i_{il}x(t) + A^i_{il}x(t - \tau) \right] dt + D^i_{il}x(t) d\omega(t), \]
along with
\[ 0 \leq \eta_{il}(z(t)) \leq 1, \]
\[ \sum_{l=1}^{N_i} \eta_{il}(z(t)) = 1, \quad (6) \]
where
\[ \epsilon_{il}(z(t)) = \prod_{\rho=1}^{P} \Omega_{il}^\rho (z_{il}(t)), \quad \eta_{il}(z(t)) = \frac{\epsilon_{il}(z(t))}{\sum_{l=1}^{N_i} \epsilon_{il}(z(t))}, \quad (7) \]
where \( \Omega_{il}^\rho (z_{il}(t)) \) represents the membership function that \( z_{il}(t) \) belongs to the fuzzy set \( \Omega_{il}^\rho \).

Now, our purpose is to design a controller \( u_i = u_i(t) \) and a switching law such that system (5) is asymptotically stable.

A switching controller is employed to control the stochastic switched fuzzy time-delay model of (5). The switching controller consists of some simple subcontrollers. These subcontrollers will switch among each other to control the system of (5) according to an appropriate switching scheme. The switching controller is described by
\[ u_i(t) = -\sum_{a=1}^{N_i} \rho_{ia}(x(t))R_{ia}^TPx(t), \quad (8) \]
where \( \rho_{ia}(x(t)) \) takes the value of 0 or 1 according to a switching scheme discussed later, \( R \in \mathbb{R}^{q \times q} \) and \( P \in \mathbb{R}^{n \times n} \) are symmetric positive definite matrices to be designed, and \((\cdot)^T\) denotes the transpose of a matrix or a vector.

Combining (5) with (8), we get the augmented system:
\[ dx(t) = \left[ \sum_{l=1}^{N_i} \eta_{il} \tilde{A}_{il}x(t) + \sum_{l=1}^{N_i} \eta_{il} A_{il}x(t - \tau) + F_{il}f_j(t) \right] dt + \sum_{l=1}^{N_i} \eta_{il} D_{il}x \omega(t), \quad (9) \]
where \( \tilde{A}_d = A_d + B_d(-\sum_{l=1}^{N_i} \rho_{ia} R_{ia}^TP) \).

We have the following result.

**Remark 1.** For \( P \) in equation (8), we consider the single Lyapunov function method. The single Lyapunov function method is to find the same positive definite function for all subsystems of the switched system, which decreases along the trajectory of each working subsystem, so as to ensure the stability of the switched system.

\[ \sum_{j=1}^{m} \lambda_{ij} \begin{bmatrix} -P B_{ia} R_{ia}^TP - P B_{ia} R_{ia}^TP + A_{ij}^TP + PA_{ij} & \frac{1}{4} \left( D_{ij}^T + D_{ij} \right) P \left( D_{ij} + D_{ij}^T \right) + S \\ A_{ij}^TP & -S \end{bmatrix} < 0, \quad j, \theta, a = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m, \quad (10) \]

**3. Main Results**

**Theorem 1.** Suppose there exist positive definite symmetric matrices \( P, R, \) and \( S \) and convex combination constants \( \lambda_{ij} > 0 \) such that the following matrix inequalities (10) are satisfied; then, system (9) is asymptotically stable via the state-feedback controller (13) under the switching law (11):

\[ \sigma(t) = \arg \min \left\{ \begin{array}{l} \sum_{j=1}^{m} \lambda_{ij} \begin{bmatrix} -P B_{ia} R_{ia}^TP - P B_{ia} R_{ia}^TP + A_{ij}^TP + PA_{ij} & \frac{1}{4} \left( D_{ij}^T + D_{ij} \right) P \left( D_{ij} + D_{ij}^T \right) + S \\ A_{ij}^TP & -S \end{bmatrix} < 0, \\
\quad j, \theta, a = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m \end{array} \right. \]

\[ \begin{bmatrix} \mathbf{z}(t) \end{bmatrix} \leq \max \begin{bmatrix} \mathbf{z}(t) \end{bmatrix} \\ \begin{bmatrix} \mathbf{z}(t) \end{bmatrix} < 0, \quad j, \theta, a = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m \]

\[ \begin{bmatrix} \mathbf{z}(t) \end{bmatrix} < 0, \quad j, \theta, a = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m \]
Proof. From (11), we know that, for any $\Xi(t) \neq 0$, it holds that

\[
\sum_{i=1}^{m} \lambda_{ij} \Xi(t) \left[ -PB_{ia}R_{ij}^T P - PB_{ij}R_{ia}^T P + PA_{ij} \right] = \Xi(t) < 0. \tag{12}
\]

Note that (12) holds for any $j, \theta \in \{1, 2, \ldots, N_i\}$ and $\lambda_{ij} > 0$; then, there exists at least an $i$ such that, for any $j_i$,

\[
\sum_{i=1}^{m} \lambda_{ij} \Xi(t) \left[ -PB_{ia}R_{ij}^T P - PB_{ij}R_{ia}^T P + PA_{ij} \right] \Xi(t) < 0. \tag{13}
\]

Thus, the switching law defined by (11) is well defined.

Now, define a quadratic Lyapunov–Krasovskii functional candidate:

\[
V(x(t)) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(\theta)Sx(\theta)d\theta, \tag{14}
\]

which is positive definite since $P$ and $S$ are positive definite matrices.

The partial differential operator of the system is as follows:

\[
LV(x(t)) = 2x^T(t)P \sum_{i=1}^{M} \eta_{ia} \left[ -A_{ia}x(t) + A_{ia}x(t-\tau) \right] + \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{N_i} \eta_{ij} \left( D_{ij}x(t) \right)^T 2P(D_{ij}x(t))
\]

From (16), we have

\[
x^T(t) \left( -\sum_{a=1}^{N_i} (\rho_{ia} - \eta_{ia}) (B_{ia}R_{ia}^T P) + A_{ia}^T P \right) x(t) = x^T(t) \left( -\sum_{a=1}^{N_i} (\rho_{ia} - \eta_{ia}) (B_{ia}R_{ia}^T P) + A_{ia}^T P \right) x(t).
\]

On the other case, we can obtain

\[
\tilde{A}_{ia}^T P + P \tilde{A}_{ia} = \left( -\sum_{a=1}^{N_i} \rho_{ia} R_{ia}^T P \right)^T B_{ia}^T P + A_{ia}^T P + PB_{ia} \left( -\sum_{a=1}^{N_i} \rho_{ia} R_{ia}^T P \right) + PA_{ia}.
\]

From (16), we have
Let

\[ \rho_{ia} = \frac{1 + \text{sgn}(x^T(t)PB_{ia}P)x(t))}{2}, \]

\[ \text{sgn}(z) = \begin{cases} 
1, & \text{if } z > 0, \\
-1, & \text{otherwise.} 
\end{cases} \quad (18) \]

From (17), we have

\[ x^T(t)(\tilde{A}_{il}^T P + PA_{il} + S)x(t) \]

\[ = x^T(t) \left( \sum_{a=1}^{N_i} \eta_{ia} \left( \begin{array}{c} -PB_{ia}P - PB_{ia}R_{ia}T^TP \\ A_{il}^T P + PA_{il} + S \end{array} \right) \right) \]

\[ = x^T(t) \left( \begin{array}{c} 1 + \text{sgn}(x^T(t)PB_{ia}P)x(t)) \\ 2 \end{array} \right) \frac{1}{2} \left( \begin{array}{c} (B_{ia}R_{ia}T^TP)^T P \\ \text{sgn}(x^T(t)PB_{ia}P)x(t)) \end{array} \right) \]

\[ \leq \sum_{a=1}^{N_i} \eta_{ia} x^T(t) \left( \begin{array}{c} -PB_{ia}R_{ia}P - PB_{ia}R_{ia}T^TP \\ A_{il}^T P + PA_{il} + S \end{array} \right) x(t) \]

\[ -2 \sum_{a=1}^{N_i} \left( \frac{1}{2} \left( \eta_{ia} - \frac{1}{2} \right) \right) \left| x^T(t)PB_{ia}P(x(t)) \right| \]

As \( \eta_{ia} - (1/2) \in \left[-(1/2), (1/2)\right] \), due to the property of the switched fuzzy model with time-delay, it can be shown that (19) satisfies the following inequality:

\[ x^T(t)(\tilde{A}_{il}^T P + PA_{il} + S)x(t) \leq \sum_{a=1}^{N_i} \eta_{ia} x^T(t) \left( \begin{array}{c} -PB_{ia}R_{ia}P - PB_{ia}R_{ia}T^TP \\ A_{il}^T P + PA_{il} + S \end{array} \right) x(t). \]

Thus, it is easy to see that

\[ x^T(t)(\tilde{A}_{il}^T P + PA_{il} + S)x(t) \leq \sum_{a=1}^{N_i} \eta_{ia} x^T(t) \left( \begin{array}{c} -PB_{ia}R_{ia}P - PB_{ia}R_{ia}T^TP \\ A_{il}^T P + PA_{il} + S \end{array} \right) x(t). \]

\[ \text{From (17), we have} \]

\[ \text{LV}(x(t)) \leq \sum_{i=1}^{N_l} \eta_{il} \sum_{j=1}^{N_j} \eta_{ij} \sum_{a=1}^{N_i} \eta_{ia} x^T(t) \]

\[ \left( \begin{array}{c} -PB_{ia}R_{ia}P - PB_{ia}R_{ia}T^TP \\ A_{il}^T P + PA_{il} + S \end{array} \right) x(t) \]

\[ + \sum_{i=1}^{N_l} \eta_{il} \sum_{j=1}^{N_j} \eta_{ij} x^T(t)P_iA_{il}x(t - \tau) \]
Considering (21), we have

\[
LV(x(t)) \leq \sum_{i=1}^{N_i} \sum_{j=1}^{N_i} \sum_{a=1}^{N_i} \eta_{il} \sum_{j=1}^{N_i} \eta_{il} x(t) x(t - \tau) \leq \sum_{i=1}^{N_i} \eta_{il} \sum_{j=1}^{N_i} \eta_{il} x(t) x(t - \tau)
\]

\[
= \begin{bmatrix}
-PA_{il} & 2\epsilon_i x(t) \Phi_i \\
PA_{il} & -S + 2\epsilon_i \Delta_i \Delta_i
\end{bmatrix}
\]

In order words, we formulate the finding of \( R \) and \( P \) of (22) into the LMI problem. Considering (22) and multiplying both sides of (22) by the matrix diag\([P^{-1}, I]\), we restate as follows.

**Theorem 2.** Suppose there exist positive definite matrices \( Q, S, \) and \( R \) and convex combination constants \( \lambda_{ij} > 0 \) such that the following matrix inequalities are satisfied; then, system (9) is asymptotically stable via the state-feedback controller (8) under the switching law (11).

\[
L_\text{V} (x(t)) \leq \sum_{i=1}^{N_i} \sum_{j=1}^{N_i} \sum_{a=1}^{N_i} \eta_{il} \sum_{j=1}^{N_i} \eta_{il} \left[ x(t) \right]^T
\]

\[
\begin{bmatrix}
-PA_{il} & 2\epsilon_i x(t) \Phi_i \\
PA_{il} & -S + 2\epsilon_i \Delta_i \Delta_i
\end{bmatrix}
\]

is asymptotically stable via the state-feedback controller (8) under the switching law (11).

Once we have \( Q = P^{-1}, \Psi = S^{-1}, \) and \( S \) and \( R \) from (23), the switching controllers (8) can be constructed.

Hence, we can conclude that the closed-loop system of (9) is asymptotically stable if the following stability condition of (23) is satisfied and the switching scheme of (8) is applied:

\[
\sum_{i=1}^{N_i} \sum_{j=1}^{N_i} \sum_{a=1}^{N_i} \eta_{il} \sum_{j=1}^{N_i} \eta_{il} \left[ x(t) \right]^T
\]

\[
\begin{bmatrix}
-PA_{il} & 2\epsilon_i x(t) \Phi_i \\
PA_{il} & -S + 2\epsilon_i \Delta_i \Delta_i
\end{bmatrix}
\]

\[
< 0, \quad j_i, 0, a = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m.
\]

\[
\Lambda_{ij_i} = -B_{ij_i} RB_{ij_i} + Q + \epsilon_i^2 F_{ij_i} T_{ij_i} + S,
\]

\[
(24)
\]

4. Illustrative Examples and Results

In recent years, the research results show that the chaotic system has a great applying value in practical engineering
system. In [32], the chaotic system model is designed. The system is two different chaotic systems, so the switch system is simple and easy to operate.

This kind of chaotic system equation is expressed as follows:

\[
\begin{align*}
\dot{x} &= a(z-x), \\
\dot{y} &= bx - dxz, \\
\dot{z} &= xy - cy - gz, \\
\dot{x} &= a((z-y), \\
\dot{y} &= bx - dx^2, \\
\dot{z} &= xy - cy - gz.
\end{align*}
\]

Security is a very important performance of the modern synchronous communication research. Chaotic synchronization communication system using multiple correlation system switching has a huge role for the safety of aging performance of chaotic synchronization and secure communication in the field of communications. The actual engineering control systems also have a large number of such switching system and good control effects.

The established stochastic nonlinear switched fuzzy system model is as follows:

\[
\begin{align*}
R^1_1: \text{if } z_2(t) \text{ is } M^1_{11}, \text{ then} \\
&dx(t) = [A_{11} x(t) + A_{111} x(t - \tau) + B_{111} u_1(t) + F_{11} f_1(t)] dt + D_{11} x(t) d\omega(t), \\
R^2_1: \text{if } z_2(t) \text{ is } M^1_{1}, \text{ then} \\
&dx(t) = [A_{12} x(t) + A_{112} x(t - \tau) + B_{112} u_1(t) + F_{12} f_1(t)] dt + D_{12} x(t) d\omega(t), \\
R^1_2: \text{if } z_2(t) \text{ is } M^2_{21}, \text{ then} \\
&dx(t) = [A_{21} x(t) + A_{211} x(t - \tau) + B_{211} u_2(t) + F_{21} f_2(t)] dt + D_{21} x(t) d\omega(t), \\
R^2_2: \text{if } z_2(t) \text{ is } M^2_{21}, \text{ then} \\
&dx(t) = [A_{22} x(t) + A_{212} x(t - \tau) + B_{212} u_2(t) + F_{22} f_2(t)] dt + D_{22} x(t) d\omega(t),
\end{align*}
\]

where

\[
\begin{align*}
A_{11} &= \begin{bmatrix}
-11 & 3 \\
1 & -1
\end{bmatrix}, \\
A_{12} &= \begin{bmatrix}
-21 & 4 \\
3 & -2
\end{bmatrix}, \\
A_{21} &= \begin{bmatrix}
-9 & 4 \\
5 & -1
\end{bmatrix}, \\
A_{22} &= \begin{bmatrix}
-15 & 7 \\
8 & -2
\end{bmatrix}, \\
B_{11} &= \begin{bmatrix}
-1 \\
0
\end{bmatrix}, \\
B_{12} &= \begin{bmatrix}
-1 \\
1
\end{bmatrix}, \\
B_{21} &= \begin{bmatrix}
-1 \\
0
\end{bmatrix}, \\
B_{22} &= \begin{bmatrix}
3 \\
2
\end{bmatrix}, \\
D_{11} &= \begin{bmatrix}
2 & -1 \\
3 & 5
\end{bmatrix}, \\
D_{12} &= \begin{bmatrix}
-5 & 7 \\
-6 & 4
\end{bmatrix}, \\
D_{21} &= \begin{bmatrix}
5 & 3 \\
2 & 2
\end{bmatrix}, \\
D_{22} &= \begin{bmatrix}
-4 & 5 \\
2 & 3
\end{bmatrix}, \\
A_{111} &= \begin{bmatrix}
2 & 5 \\
2 & -2
\end{bmatrix}, \\
A_{112} &= \begin{bmatrix}
5 & 3 \\
-6 & -2
\end{bmatrix}, \\
A_{121} &= \begin{bmatrix}
2 & 5 \\
7 & -8
\end{bmatrix}, \\
A_{122} &= \begin{bmatrix}
6 & 8 \\
-2 & -3
\end{bmatrix}.
\end{align*}
\]
The membership functions are as follows:

\[
M_{11}^1 (x_2 (t)) = 1 - \frac{1}{1 + e^{-2x_2(t)}},
\]

\[
M_{11}^2 (x_2 (t)) = \frac{1}{1 + e^{-2x_2(t)}},
\]

\[
M_{21}^1 (x_2 (t)) = 1 - \frac{1}{1 + e^{-\frac{1}{2}(x_2(t) - 0.3)}},
\]

\[
M_{21}^2 (x_2 (t)) = \frac{1}{1 + e^{-\frac{1}{2}(x_2(t) - 0.3)}}.
\]

The nonlinearities \( f(t) \) in (1) are

\[
f_1 (t) = \begin{bmatrix} 0.4x_1 (t) + 0.5x_2 (t) \\ 0.7x_1 (t) \end{bmatrix} \sin t
\]

\[
+ \begin{bmatrix} 0.4x_1 (t - \tau) + 0.4x_2 (t - \tau) \\ 0.4x_1 (t - \tau) + 0.7x_2 (t - \tau) \end{bmatrix} \sin t,
\]

\[
f_2 (t) = \begin{bmatrix} 0.4x_1 (t) + 0.5x_2 (t) \\ 0.7x_1 (t) \end{bmatrix} \cos t
\]

\[
+ \begin{bmatrix} 0.4x_1 (t - \tau) + 0.4x_2 (t - \tau) \\ 0.4x_1 (t - \tau) + 0.7x_2 (t - \tau) \end{bmatrix} \cos t,
\]

which satisfy Assumption 1 with

\[
\Phi_1 = \Phi_2 = \begin{bmatrix} 0.4 & 0.5 \\ 0.7 & 0 \end{bmatrix},
\]

\[
\Delta_1 = \Delta_2 = \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.7 \end{bmatrix}.
\]

Using MATLAB to solve (23) with parameters \( j_t, a_t, t = 1, 2, \tau = 1 \), and \( \lambda_{ij} = 1 \), the following matrices are obtained:

\[
P = \begin{bmatrix} 0.5232 & 0.1451 \\ 0.1451 & 0.2512 \end{bmatrix},
\]

\[
Q = \begin{bmatrix} 0.2816 & 0.0261 \\ 0.0261 & 0.2134 \end{bmatrix},
\]

\[
S = \begin{bmatrix} 0.4133 & 0.3241 \\ 0.3241 & 0.1341 \end{bmatrix},
\]

\[
R = 5.2.
\]

The simulation result under initial condition \([-10 -15]^T\) is depicted in Figure 1.

To show the advantages of the proposed method, we now compare the method with the traditional PDC fuzzy controller. Obviously, the global control of the traditional PDC fuzzy controller is

\[
u_i (t) = \sum_{i=1}^{N} \eta_j K_j x(t).
\]

The state-feedback gains of subsystems are obtained as

\[
K_{11} = \begin{bmatrix} 0.4623 & 0.6826 \\ -2.1751 & 3.2715 \end{bmatrix},
\]

\[
K_{12} = \begin{bmatrix} 1.2815 & 0.8361 \\ -2.3813 & 1.2642 \end{bmatrix}.
\]

For the stochastic switched fuzzy system, the simulation result under the same initial condition \([-10 -15]^T\) is depicted in Figure 2. Figures 1 and 2 indicate that the proposed method gives better results.

5. Conclusion

In this paper, the stability of stochastic nonlinear switched fuzzy with input time-delay systems is studied. The switched fuzzy control system with stochastic factors has good control.
References

[1] X.-M. Sun, G.-P. Liu, W. Wang, and D. Rees, "L∞-Gain of systems with input delays and controller temporary failure: zero-order hold model," *IEEE Transactions on Control Systems Technology*, vol. 19, no. 3, pp. 699–706, 2011.

[2] L. G. Wu and W. X. Zheng, "Weighted H∞ model reduction for linear switched systems with time-varying delay," *Automatica*, vol. 45, no. 1, pp. 186–193, 2009.

[3] L. G. Wu, Z. G. Feng, and W. X. Zheng, "Exponential stability analysis for delayed neural networks with switching parameters: average dwell time approach," *IEEE Transactions on Neural Networks*, vol. 21, no. 9, pp. 1396–1407, 2010.

[4] M. S. Ali, "On exponential stability of neutral delay differential system with nonlinear uncertainties," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 6, pp. 2595–2601, 2012.

[5] M. Liu, D. W. C. Ho, and Y. Niu, "Stabilization of Markovian jump linear system over networks with random communication delay," *Automatica*, vol. 45, no. 2, pp. 416–421, 2009.

[6] B. Jiang, H. R. Karimi, Y. Kao, and C. Gao, "Notice of violation of ieee publication principles: a novel robust fuzzy integral sliding mode control for nonlinear semi-markovian jump T-S fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3594–3604, 2018.

[7] Y. Hong, H. Zhang, and Q. Zheng, "Asynchronous H∞ filtering for switched T-S fuzzy systems and its application to the continuous stirred tank reactor," *International Journal of Fuzzy Systems*, vol. 20, no. 5, pp. 1470–1482, 2018.

[8] W.-J. Wang, Y.-J. Chen, and C.-H. Sun, "Relaxed stabilization criteria for discrete-time T-S fuzzy control systems based on a switching fuzzy model and piecewise Lyapunov function," *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, vol. 37, no. 3, pp. 551–559, 2007.

[9] D. J. Choi and P. Park, "Guaranteed cost controller design for discrete-time switching fuzzy systems," *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 1, pp. 110–119, 2004.

[10] X. J. Su, P. Shi, L. G. Wu, and Y. D. Song, "A novel approach to filter design for T-S fuzzy discrete-time systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 6, pp. 1114–1129, 2012.

[11] L. G. Wu and D. W. C. Ho, "Fuzzy filter design for Itô stochastic systems with application to sensor fault detection," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 1, pp. 233–242, 2009.

[12] L. G. Wu and W. X. Zheng, "L2-L∞ control of nonlinear fuzzy Itô stochastic delay systems via dynamic output feedback," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 39, no. 5, pp. 1308–1315, 2009.

[13] T. C. Lin, S. W. Chang, and C. H. Hsu, "Robust adaptive fuzzy sliding mode control for a class of uncertain discrete-time nonlinear systems," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 1, pp. 347–359, 2012.

[14] W. H. Ho, S. H. Chen, I. T. Chen, J. H. Chou, and C. C. Shu, "Design of stable and quadratic-optimal static output feedback controllers for TS-fuzzy-model-based control systems: an integrative computational approach," *International Journal of Innovative Computing, Information and Control*, vol. 8, no. 1, pp. 403–418, 2012.

[15] M. L. Lv, Y. M. Li, W. Pan, and S. Baldi, "Finite-time fuzzy adaptive constrained tracking control for hypersonic flight vehicles with singularity-free switching," *IEEE/ASME Transactions on Mechatronics*, 2021.

[16] M. L. Lv, W. W. Yu, and S. Baldi, "Fuzzy adaptive tracking control of high-order nonlinear dynamics with mixed control directions," in *Proceedings of the 39th Chinese Control Conference*, pp. 634–639, Shenyang, China, July 2020.

[17] R. Palm and D. Driankov, "Fuzzy switched hybrid systems - modeling and identification," in *Proceedings of the IEEE International Conference on Fuzzy Systems*, pp. 130–135, Anchorage, AL, USA, May 1998.

[18] K. Tanaka, M. Iwasaki, and H. O. Wang, "Stability and smoothness conditions for switching fuzzy systems," in *Proceedings of the American Control Conference*, pp. 2474–2478, Chicago, IL, USA, June 2000.

[19] K. Tanaka, M. Iwasaki, and H. O. Wang, "Stable switching fuzzy control and its application to a Hovercraft type vehicle," in *Proceedings of the IEEE International Conference on Fuzzy Systems*, pp. 804–809, San Antonio, TX, USA, May 2000.

[20] K. Tanaka, M. Iwasaki, and H. O. Wang, "Switching Control of an R/C Hovercraft: stabilization and smooth switching," *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, vol. 31, no. 6, pp. 853–863, 2001.

[21] H. Ohtake, K. Tanaka, and H. O. Wang, "Switching fuzzy control for nonlinear systems," in *Proceedings of the IEEE International Symposium on Intelligent Control*, pp. 281–286, Houston, TX, USA, October 2003.

[22] D. J. Choi and P. Park, "State-feedback controller design for discrete-time switching fuzzy systems," in *Proceedings of the IEEE Conference on Decision and Control*, pp. 191–196, Las Vegas, NV, USA, December 2002.

[23] D. J. Choi and P. Park, "Guaranteed cost controller design for discrete-time switching fuzzy systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 3, pp. 110–119, 2004.

[24] D. Higham, "An algorithmic introduction to numerical simulation of stochastic differential equations," *SIAM Review*, vol. 43, no. 3, pp. 525–546, 2001.
[25] Y. Niu, D. W C Ho, and J. Lam, "Robust integral sliding mode control for uncertain stochastic systems with time-varying delay," *Automatica*, vol. 41, no. 5, pp. 873–880, 2005.

[26] S. Xu and T. Chen, "H∞ output feedback control for uncertain stochastic systems with time-varying delays," *Automatica*, vol. 40, no. 12, pp. 2091–2098, 2004.

[27] J. Ren, G. He, and J. Fu, "Robust H∞ sliding mode control for nonlinear stochastic T-S fuzzy singular Markovian jump systems with time-varying delays," *Information Sciences*, vol. 535, pp. 42–63, 2020.

[28] Z. Wang, Y. Liu, and X. Liu, "H∞ filtering for uncertain stochastic time delay systems with sector-bounded nonlinearities," *Automatica*, vol. 44, no. 5, pp. 1268–1277, 2008.

[29] J. Sun, H. Zhang, Y. Wang, and S. Sun, "Dissipativity analysis on switched uncertain nonlinear T-S fuzzy Systems with stochastic perturbation and time delay," *Journal of the Franklin Institute*, vol. 357, no. 18, pp. 13410–13429, 2020.

[30] J. P. Hespanaha, "A model for stochastic hybrid systems with application to communication networks," *Nonlinear Analysis*, vol. 62, pp. 1353–1383, 2005.

[31] S. Oh and S. Sastry, "Approximate estimation of distributed networked control systems," in *Proceedings of the American Control Conference*, pp. 997–1002, New York, NY, USA, July 2007.

[32] Y. Z. Liu, C. S. Jiang, C. S. X. X. Lin, and L. Shi, "A class of switchable 3D chaotic systems," *Acta Physica Sinica*, vol. 56, no. 6, pp. 3107–3112, 2007.