The Analysis of the Negative Feedback Circuit in the Induction Sensors

Pengfei Lin¹, Chunsheng Lin¹*, Xingya Wu² and Ning Zhang¹

¹College of Weaponry Engineering, Naval University of Engineering, Wuhan 430033, China
²91278 unit, Lvshunkou in DaLian 116000, China

*Email: lcs_and_zh@163.com

Abstract. In this paper, the analysis method of the step response of the second-order system under different damping is applied to the feedback circuit analysis in the sensors. It analyzes the influence of different feedback resistors and mutual inductances on the system stability and pass band, which provides a theoretical basis for the matching of feedback resistors and mutual inductances in the circuit. It is of great significance to improve the performance of the sensor. When the parameters of the induction sensor are determined, the system performance can be more stable by designing appropriate mutual inductance and feedback resistance to make the damping ratio close to 0.8. Finally, we designed the feedback resistor $R_f=400\ \Omega$, and the mutual inductance $M=1$ to make sure damping ratio $\zeta$ be around 0.8 in the selected sensors.

1. Introduction

Magnetic induction sensor is mainly used to measure the earth's magnetic field, its main application is the exploration of underground resources [1-2] and the weak very low frequency signal detection [3-4]. The feedback circuit of the sensor is very important to improve the system performance.

In technology research, Y. Zhao etc. present a system-theoretic approach to quantitatively model a single Aux/IAA negative feedback loop, which can be implemented deterministically and stochastically [5]. Guiyuan Wang etc. propose the analytical calculation of the stability of a core fundamental gene regulation circuit involving positive and negative interlinked regulation and time-delays [6]. David C. Marciano etc. present an alternative mechanism that is simple and uses a frequently encountered network motif. Computational and experimental evidence shows that the self-correcting, negative-feedback gene regulation motif increases repressor expression in response to deleterious mutations and thereby precisely restores repression of a target gene [7].

In this paper, it analyzes the influence of mutual inductance and feedback resistance in the negative feedback circuit on the system performance in order to design the sensors with better performance.
2. The feedback circuit
In subsequent amplifier circuits, negative feedback circuit is usually used to suppress the phase
mutation and amplitude change.

2.1 Analysis of Negative Feedback Circuit

Figure 1. The negative feedback circuit diagram of the sensor.

When a negative feedback circuit is used for the sensor, the circuit is shown in figure1. The
frequency characteristic function of the system is expressed as follows [8-9].

\[
H(j\omega) = \frac{G}{1 - j\omega \omega_0 L_s C_s + j\omega \omega_0 R_s C_s + GM / R_f},
\]

(1)

\(G\) is the gain multiple of the amplifier, \(M\) is the mutual inductance between the feedback coil and
the sensor coil, and \(R_f\) is the feedback resistance. \(L_s, C_s, R_s\) is the distributed inductance, capacitance
and the resistance of the sensors respectively. Assuming that \(L_s=100, C_s=10^{-7}\) F, \(R_s=24880\) Ω.

When \(G=1\), mutual inductance is fixed and \(R_f\) takes different resistance values, the Bode diagram
of the characteristic function of the system is shown as follows:

Figure 2. Bode diagram of the sensor with different feedback resistors.

The Figure 2 shows that the value of feedback resistance is very important to the performance of
the sensor system. It shows that when the feedback resistance is very small to 10Ω, the pass band of
the sensor decreases, and the phase-frequency changes slowly. When the feedback resistance is 400Ω,
the characteristic curve of the system is similar to the system without feedback resistance, but the
stability is improved. When the resistance is very large, the influence of the feedback circuit on the
system becomes smaller, and it is the same as the system without feedback circuit.

When the feedback resistance is fixed, \(R_f=400\) Ω, and the mutual inductance is different, the Bode
diagram of the characteristic function of the system is shown in the Figure 3.
The Figure 3 shows that the proper mutual inductance is also important to the performance of the sensor system. When $M = 0.001$, the mutual inductance of the feedback circuit is too small, and it has little influence on the sensor system. When $M = 0.5$, the sudden change of the amplitude-frequency characteristics can be properly improved. When $M$ is too large, the pass band of the sensor system will be smaller, and it will make the performance of the system worse.

When the parameters of the sensor are determined, the proper matching of mutual inductance and feedback resistance of the feedback circuit of the system can improve the performance of the system. In this design, the feedback resistance $R_f = 400\Omega$, the mutual inductance $M = 1$. Thus, the performance of the system can be better.

### 2.2 Analysis of Impulse Response and Step Response of Magnetic Induction Coil

According to the equivalent circuit diagram of the induction sensor, the following formula can be obtained from the Kirchhoff theorem.

\[
\begin{align*}
L_i \frac{di(t)}{dt} + \frac{1}{C_o} \int i(t) \, dt + R_i i(t) &= e(t) \\
U_o(t) = \frac{1}{C_o} \int i(t) \, dt
\end{align*}
\]

then the transfer function is written in standard form.

\[
H(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

In the formula (3), the $\omega_n = \frac{1}{\sqrt{L_i C_o}}$, and the $\zeta = \frac{R_i C_o}{2\sqrt{L_i C_o}}$.

When $R_i^2 C_o^2 - 4L_i C_o > 0$, it is $\zeta > 1$, the transfer function has two real poles, it is an over-damped system; when $0 < \zeta < 1$, The transfer function has two complex poles, and it is under-damped system; when $\zeta = 1$, the transfer function had two identical poles, and it is critical damper system.[10,11,12].

(1) $0 < \zeta < 1$, Under-damped Systems

Then the transfer function is written as:

\[
H(s) = \frac{\omega_n^2}{(s + \zeta \omega_n)^2 + \omega_n^2}
\]
In the formula (4), the \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \)

Order \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \), The step response and impulse response are obtained as follows.

\[

t(t) = 1 - e^{-\omega_d t} \left( \cos \omega_n t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \omega_n t \right)
\]

\[
h(t) = \frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\omega_d t} \sin \omega_n \sqrt{1 - \zeta^2} t
\]

(2) \( \zeta = 1 \), Critical Damping
\( \zeta = 1 \), the transfer function has two equal poles. Then the transfer function can be expressed as follows.

\[
H(s) = \frac{\omega_n^2}{(s + \omega_n)^2}
\]

Step response and impulse response are as follows.

\[
u(t) = 1 - e^{-\omega_t} (1 + \omega_n t)
\]

\[
h(t) = \omega_n^2 t e^{-\omega_t}
\]

(3) \( \zeta > 1 \), Over Damping
The transfer function has two different poles of negative real numbers, the transfer function is expressed as follow:

\[
H(s) = \frac{\omega_n^2}{(s + \zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1})(s + \zeta \omega_n - \omega_n \sqrt{\zeta^2 - 1})}
\]

The two poles of the transfer function are as follows:

\[
s_1 = -\zeta \omega_n - \omega_n \sqrt{\zeta^2 - 1}, s_2 = -\zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1}
\]

Step response and impulse response are as follows.

\[
u(t) = 1 + \frac{\omega_n}{2 \sqrt{\zeta^2 - 1}} \left( \frac{e^{s_1 t} - e^{s_2 t}}{s_1 - s_2} \right)
\]

\[
h(t) = \frac{\omega_n}{2 \sqrt{\zeta^2 - 1}} e^{-\zeta \omega_n t} - \frac{\omega_n}{2 \sqrt{\zeta^2 - 1}} e^{-\omega_n \sqrt{\zeta^2 - 1} t}
\]

(4) Comparison of three cases

\[\text{Figure 4. The impulse response and the step response of the second-order system.}\]

The Figure 4 shows that when \( \zeta = 0.4, 0.6, 0.8 \), the response time is shorter, which avoids the oscillation caused by too small damping ratio and shorten the response time caused by too small...
damping ratio. It takes about several seconds to reach stable. When the $\zeta \geq 1$, the step response curve rises slowly. The response of the system has no oscillation, but the adjustment time is longer, it takes more than 10 seconds to reach stable in Figure 4. It seriously affects the accuracy of system. Therefore, in the design of the system, the damping ratio should be reasonably determined and it should be controlled in 0.4–0.8.

3. Effect of Feedback Circuit on System Response
When the system adopts negative feedback circuit, the damping ratio of the sensor can be expressed as follows [13].

$$\zeta = \frac{R \cdot C \cdot G + M / R_f}{2 \sqrt{L \cdot C}} \quad (11)$$

Then the transfer function of the system is expressed as follows [14].

$$H(j\omega) = \frac{G}{1 - \omega^2 L \cdot C + j\omega (R \cdot C + G \cdot M / R_f)} \quad (12)$$

3.1 Effect of Feedback Resistance $R_f$ on System Response
The damping ratio is not only related to the parameters of the induction sensor, but also to the mutual inductance coefficient $M$ and the feedback resistance $R_f$ of the feedback circuit. When the parameters of the induction sensor is determined and the mutual inductance coefficient $M=1\text{H}$, the step response and impulse response of the system with different feedback resistors are drawn as follows.

![Figure 5](image)

(a) The impulse response and (b) the step response of the sensor with different feedback resistors.

The Figure 5 shows that the appropriate matching of feedback resistance directly affects the damping ratio. It shows that the system transits from under-damped state to over-damped state with the increase of resistance. When $R_f=10\Omega$, the over-damped coefficient reaches 16.2048, which makes the system response time longer and it is difficult to achieve stability; When $R_f=10000\Omega$, 1 000\Omega and 400\Omega, the system is under-damped and tends to be stable after a short period of oscillation. This is consistent with conclusion that the damping ratio should be controlled between 0.4 and 0.8 in the previous analysis. At the same time, when $R_f = 400\Omega$ and the damping ratio $\zeta = 0.7887$, the system has the best performance.

3.2 Effect of Mutual Inductance on System Response
When the parameters of the magnetic induction sensor have been determined, the feedback resistance $R_f = 400\Omega$, and the mutual inductance is different, the impulse response and step response of the system are drawn as follows.
Figure 6. The impulse response and the step response of the sensor with different mutual inductances.

The Figure 6 shows that the mutual inductance of the feedback circuit also directly affects the damping ratio. With the increase of mutual inductance, the system transfer from under-damped state to over-damped state. When $M=5H$ and $M=10H$, the system is in over-damped state, and the response time of the system is longer, so it is difficult to achieve stability. When $M=0.1H$ and $M=1H$, the system is under-damped state and tends to be stable after short-time oscillation, this is consistent with conclusion that the damping ratio should be controlled between 0.4 and 0.8 in the previous analysis. At the same time, when $M=1H$ and damping ratio $\zeta=0.7887$, the system has the best performance.

In conclusion, when the parameters of the induction sensor are determined, the system performance can be more stable by designing appropriate mutual inductance and feedback resistance to make the damping ratio close to 0.8. In this design, the feedback resistance $R_f=400\Omega$ and the mutual inductance $M=1$, and the damping ratio $\zeta=0.7887$.

4. Conclusion
In this article, it analyzes the influence of mutual inductance and feedback resistance in the negative feedback circuit on the system performance in order to design the sensors with better performance. It concludes that if the resistor of the sensor $R=2.488\times10^4\Omega$, the feedback resistor $R_f=400\Omega$, and the mutual inductance $M=1$ to make sure damping ratio $\zeta$ be around 0.8, and the sensitivity of the sensor can reach the optimal performance. It can conclude that when the parameters of the induction sensor are determined, the system can achieve the optimal stability performance by matching mutual inductance and feedback resistance reasonably. This is very essential to the stability of the sensor in the design.

References
[1] Cao B, Qin Q M and Li B Sh. Day and night alter-nation of natural ultra-low frequency electromagnetic field and its stability analysis. Journal of Beijing University: Science and Technology, 2008, 44(6):897-901.
[2] Wang Y Zh, Cheng D F, Wang J, et al. Research of broad frequency difference magnetic field sensor based on nanocrystalline alloy. Chinese Journal of Sensors and Actuators, 2007, 20(9):1967-1970.
[3] He J Sh. Wide field electromagnetic sounding method. Journal of Central South University: Science and Technology, 2010,41 (3):1065-1072.
[4] Xu X X. The application of magnetotelluric sounding to the investigation of ore resources at depth. Geophysical & Geo chemical exploitation, 2011, 35(1):17-19,23.
[5] Y.Zhao etc. Theoretical modeling of the Aux/IAA negative feedback circuit in plants. South African Journal of Botany. 2015, 09:16-19
[6] Guiyuan Wang etc. Stability and hopf bifurcation analysis in a delayed three-node circuit involving interlinked positive and negative feedback loops. Mathematical Biosciences. 2019, 04:50-64

[7] David C. Marciano etc. Negative feedback in genetic circuits confers evolutionary resilience and capacitance. Cell Reports. 2014, 07:1789-1795.

[8] Shao Y Q, Wang Y Zh and Cheng D F, et al. Development of broad frequency magnetic field sensor based on flux feedback. Chinese Journal of Scientific Instrument, 2010, 31(11):2461-2466.

[9] Shao Y Q, Cheng D F, Wang Y Zh, et al. Research of high sensitivity inductive magnetic sensor. Chinese Journal of Scientific Instrument, 2012, 33(2):349-355.

[10] Wang H J. Characteristics of damping coefficient effect on transient electromagnetic signal. Chinese Journal of Geophysics, 2010, 53(2):428-434.

[11] Claude C. Closed loop applied to magnetic measurements in the range of 0.1-50 MHz Review of Scientific Instruments, 2006, 77:1-7.

[12] Ji Y J, Lin J, Yu S B, et al. A study on solution of transient Electromagnetic response during transmitting current turn-off in the ATTEM system. Chinese J. Geophys (in Chinese). 2006, 49(6):1884-1890.

[13] Zang Y L, Xin M W. Quantitative Analysis of Eddy Current Loss in Ferromagnetic Materials. Journal of Shandong University of Technology (Natural Science Edition), 2003 (3): 82-85.

[14] Duan Z, Li T B, Li Y S, et al. Application of TEM for geologic prediction at Tongluoshan Tunnel, Modern Tunneling Technology (in Chinese), 2008, 45(2):58-62.