Nuclear $\beta$-decay, Atomic Parity Violation, and New Physics

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Abstract

Determinations of $|V_{ud}|^2$ with super-allowed Fermi $\beta$-decay in nuclei and of the weak charge of the cesium in atomic parity-violation deviate from the Standard Model predictions by $2\sigma$ or more. In both cases, the Standard Model over-predicts the magnitudes of the relevant observables. I discuss the implications of these results for R-parity violating (RPV) extensions of the minimal supersymmetric Standard Model. I also explore the possible consequences for RPV supersymmetry of prospective future low-energy electroweak measurements.

24.80.+y, 11.30.Er, 12.15.Ji, 25.30.Bf, 23.40.Bw
The search for physics beyond the Standard Model (SM) is one of the primary objectives of present and future high-energy collider experiments. At the same time, there exist a variety of low- and medium-energy atomic and nuclear studies making important contributions in the search for new physics. For example, measurements of superallowed nuclear Fermi $\beta$-decay provide the most precise determination of the $ud$ element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $V_{ud}$. When combined with determinations of $|V_{us}|$ and $|V_{ub}|$ from $K$ and $B$ meson decays [1], nuclear Fermi $\beta$-decay provides a stringent test of CKM matrix unitarity. In the neutral current sector, the Boulder group has obtained a precise determination of the weak charge of the cesium atom, $Q_W$, using atomic parity-violation (APV). In both cases, the results deviate from the SM predictions by 2$\sigma$ or more. Denoting $|V_{ud}|^2_{SM}$ the value implied by CKM unitarity and $Q_{W}^{SM}$ the weak charge computed in the SM, one has [2–5]:

$$
\frac{(|V_{ud}|^2_{EX} - |V_{ud}|^2_{SM})}{|V_{ud}|^2_{SM}} = -0.0029 \pm 0.0014
$$

$$
\frac{(Q_{W}^{EX} - Q_{W}^{SM})}{Q_{W}^{SM}} = -0.016 \pm 0.006,
$$

where “EX” denotes the experimental value for the corresponding observable and where the experimental and systematic errors (including theoretical) have been combined in quadrature. The APV results correspond to a single experiment, whereas the $\beta$-decay results have been obtained by averaging over nine different decays. Interestingly, the relative deviations from the SM in both cases are negative.

Assuming the deviations in Eqs. (1-2) cannot be explained by conventional hadronic, nuclear, or atomic effects, they may hint at the presence of new physics. In this respect, the cesium APV result has sparked considerable recent attention. Among the more interesting possibilities is that an additional neutral weak gauge boson is the culprit behind the observed deviation. The sign of the observed deviation has a natural explanation in the context of $E_6$ theories [6–9]. The presence of an additional U(1) symmetry alone, however, would not help account for the longer-standing $\beta$-decay result.

In what follows, I investigate whether new physics scenarios exist which might account for both the common sign of the results in Eqs. (1-2) as well as the observed magnitudes. After making some general observations about the impact of new interactions on these observables, I illustrate using extensions of the minimal supersymmetric SM having R-parity violating (RPV) interactions. I show that low-energy electroweak data place severe constraints on this scenario. Nevertheless, at the 2$\sigma$ level, there exists a small but non-vanishing region in the parameter space of RPV couplings and sfermion masses which may account for the $\beta$-decay and APV results. I also show that, within this framework, consistency of the low-energy results with rare decay limits does not appear to require significant mass hierarchies in the sfermion spectrum. Finally, if RPV supersymmetry is responsible for the results in Eqs. (1-2), observable consequences may also follow for other prospective low-energy precision measurements. I discuss three such cases: (i) a measurement of the PV Möller scattering asymmetry, (ii) a determination of the weak charge of the proton using parity-violating electron scattering (PVES), and (ii) a measurement of ratios of APV observables for different atoms along an isotope chain. The sensitivity of all three measurements to new RPV interactions differs substantially from that of $\beta$-decay and APV. I discuss the conditions under which these new measurements may impose further constraints on the RPV parameter space.
In general, the presence of new physics may modify low-energy semileptonic electroweak observables in two ways: (i) directly, via a new semileptonic interaction or modification of the SM semileptonic interaction, and (ii) indirectly, through a modification of the relative normalizations of leptonic and semileptonic amplitudes. Indirect effects may arise because semileptonic SM amplitudes are expressed in terms of $G_\mu$, the Fermi constant measured in $\mu$-decay. In the SM, it is related to the semiweak couplings as

$$G^{SM}_\mu = \frac{g^2}{8M_W^2} + \text{rad. corr.} \equiv \frac{g^2}{8M_W^2}[1 + \Delta r_\mu]$$

where “rad. corr.” and $\Delta r_\mu$ denote the appropriate radiative corrections to the tree-level $\mu$-decay amplitude. The presence of new leptonic physics modifies the relation (3) as

$$G_\mu = \frac{g^2}{8M_W^2}(1 + \Delta r_\mu + \Delta_\mu) = \frac{G^{SM}_\mu}{\sqrt{2}}(1 + \Delta_\mu)$$

where $\Delta_\mu$ denotes the new physics correction to the tree-level SM $\mu$-decay amplitude. When the SM is used to compute $\beta$-decay or APV amplitudes, one requires $g^2/M_W^2$ as input. Since $G_\mu$ is one of the three most precisely measured electroweak input parameters, it is standard to rewrite $g^2/M_W^2$ in terms of $G_\mu$ using Eq. (3). Thus, the presence of $\Delta_\mu$ would modify the normalization of the $\beta$-decay and APV amplitudes via Eq. (4).

In the case of PV neutral current amplitudes, an additional $\Delta_\mu$-dependence arises from the determination of the weak mixing angle. At tree-level in the SM, the weak charge is given by

$$Q_W^0 = Z(1 - 4x) - N$$

where $x \equiv \sin^2 \theta_W$ is computed in terms of $\alpha$, $G_\mu$, and $M_Z$ from the relation

$$x(1 - x) = \frac{\pi \alpha}{\sqrt{2G_\mu M_Z^2}(1 - \Delta r - \Delta_\mu)}$$

and where the precise values of $x$ and the radiative corrections $\Delta r$ depend on the choice of renormalization scheme. The $\Delta_\mu$-dependence of $\sin^2 \theta_W$ in Eq. (5) translates into a corresponding dependence of $\Delta_\mu$ in $Q_W$.

In order to delineate the effects of new leptonic and semileptonic physics in the semileptonic observables of interest here, it is useful to define effective Fermi constants for the latter:

$$G_F^\beta = G_\mu |V_{ud}| (1 - \Delta r_\mu + \Delta r_\beta - \Delta_\mu + \Delta_\beta)$$

$$G_F^{PV} = G_\mu Q_W^0(\Delta_\mu)(1 - \Delta r_\mu + \Delta r_{PV} - \Delta_\mu + \Delta_{PV})$$

where $\Delta r_\beta$ and $\Delta r_{PV}$ denote the appropriate SM radiative corrections to the charged current $\beta$-decay and neutral current PV amplitudes, respectively, and where $\Delta_\beta$ and $\Delta_{PV}$ denote the corresponding semileptonic new physics corrections. The $\Delta_\mu$-dependence of $Q_W^0$ arises for the reasons discussed above. The experimental results imply that

\[1\text{It is conventional to define the SM weak charge as } Q_W^{SM} = Q_W^0(1 - \Delta r_\mu + \Delta r_{PV}).\]
\[
\frac{G_F^{\beta,\text{EX}}}{G_F^{\beta,\text{SM}}} < 1 \quad \text{(9)}
\]
\[
\frac{G_F^{PV,\text{EX}}}{G_F^{PV,\text{SM}}} < 1 \quad \text{(10)}
\]

where the SM values are computed using \(\Delta_\mu = \Delta_\beta = \Delta_{PV} = 0\). The conventional interpretation of the reduction in effective Fermi constants is given in Eqs. (9-10).

At first glance, it appears that a positive value for \(\Delta_\mu\) would reduce the effective Fermi constants from their SM values and explain the sign of the observed deviations without requiring the interpretation of Eqs. (1-2). In the case of APV, however, the \(\Delta_\mu\)-dependence of \(Q_W^0\) cancels against the \(\Delta_\mu\)-induced modification of the overall normalization, yielding a negligible net effect from \(\Delta_\mu\) on \(G_{PV}^F\).

To see this cancellation explicitly, one may expand \(Q_W^0(\Delta_\mu)\) to first order in \(\Delta_\mu\) using Eq. (6), yielding
\[
G_{PV}^F \approx G_\mu Q_W^0 (1 - \Delta r_\mu + \Delta r_{PV} + \xi \Delta_\mu + \Delta_{PV}), \quad \text{(11)}
\]
where
\[
\xi = -1 - (4Z/Q_W^0)\lambda_x
\]
\[
\lambda_x \approx \frac{x(1-x)}{1-2x} \frac{1}{1-\Delta r}. \quad \text{(12)}
\]

For cesium, \(\xi \approx 0.05\) when the weak mixing angle is defined in the \(MS\) scheme. Thus, while a non-zero value for \(\Delta_\mu\) might account for the reduction in \(G_\beta^F\) from its SM value, it is an unlikely source of the 1.6% reduction in \(G_{PV}^F\). Instead, one must look to new semileptonic neutral current interactions to generate the observed APV effect.

Extensions of the minimal supersymmetric standard model (MSSM) containing RPV interactions can generate tree-level contributions to \(\Delta_\mu\), \(\Delta r_\beta\), and \(\Delta r_{PV}\). The MSSM is a popular candidate for SM extensions. Although no direct evidence for supersymmetry (SUSY) has yet been obtained, there exist compelling theoretical arguments as to why it should be correct (for a review, see Ref. [10]). The MSSM can be extended to include terms in the superpotential which do not conserve the quantum number \(P_R = (-1)^3(B-L)+2S\), where \(B\) and \(L\) denote baryon and lepton number, respectively, and \(S\) is the spin of a given particle. Such RPV interactions result in the Lagrangians [12]
\[
\mathcal{L}_{RPV} = \lambda_{ijk}[\tilde{\nu}_i^L \tilde{e}_k^R \tilde{e}_j^L + \tilde{e}_j^L \tilde{e}_k^R \tilde{\nu}_i^L + (\tilde{e}_j^L)^*(\tilde{\nu}_i^L)\tilde{e}_k^L - (i \leftrightarrow j)] + \text{h.c.}
\]
\[
+ \lambda'_{ijk}[\tilde{\nu}_i^L \tilde{d}_k^R \tilde{d}_j^L + \tilde{d}_j^L \tilde{d}_k^R \tilde{\nu}_i^L + (\tilde{d}_k^R)^*(\tilde{\nu}_i^L)\tilde{d}_j^L - (i \leftrightarrow j)] + \text{h.c.}
\]
\[
- \tilde{\nu}_i^L \tilde{d}_k^R \tilde{e}_j^L - \tilde{d}_j^L \tilde{d}_k^R \tilde{\nu}_i^L - (\tilde{d}_k^R)^*(\tilde{e}_j^L)\tilde{\nu}_i^L] + \text{h.c.}, \quad \text{(14)}
\]

where the \(i,j,k\) indices denote generation and where the \(\tilde{f}\) denotes the supersymmetric partner of the corresponding fermion \(f\). Both the \(\lambda\) and \(\lambda'\) terms in Eq. (14) violate lepton number conservation.

\(^2\)This cancellation was first noted in Ref. [11] in the context of oblique corrections to electroweak observables.
At low-energies, the exchange of a sfermion between SM fermions yields four-fermion effective interactions. Upon Fierz reordering, these interactions take on the structure of the corresponding effective current-current interactions in the SM. Consequently, one expects $\mathcal{L}_{\text{RPV}}$ to induce corrections to low-energy electroweak observables. In the present context, one may express these corrections in terms of the quantities $\Delta_{12k}(\tilde{e}_R^k)$, $\Delta'_{11k}(\tilde{d}_R^k)$, $\Delta'_{1j1}(\tilde{q}_L^i)$, where

$$\Delta_{12k}(\tilde{e}_R^k) = \frac{\left|\lambda_{12k}\right|^2}{4\sqrt{2}G^\text{SM}_\mu M^2_{\tilde{e}_R^k}}. \tag{15}$$

with $\tilde{e}_R^k$ being the exchanged slepton, and where $\Delta'_{11k}(\tilde{d}_R^k)$ and $\Delta'_{1j1}(\tilde{q}_L^i)$ are defined as in Eq. (13) but with $\lambda_{12k} \rightarrow \lambda'_{11k}$, $M^2_{\tilde{e}_R^k} \rightarrow M^2_{\tilde{d}_R^k}$ and $\lambda_{12k} \rightarrow \lambda'_{1j1}$, $M^2_{\tilde{e}_R^k} \rightarrow M^2_{\tilde{q}_L^i}$, respectively. In terms of these quantities, which are non-negative, one has

$$\Delta - \Delta_0 \approx \Delta'_{11k}(\tilde{d}_R^k) - \Delta_{12k}(\tilde{e}_R^k) \tag{16}$$
$$\Delta_{\text{PV}} + \xi\Delta_\mu \approx 0.05\Delta_{12k}(\tilde{e}_R^k) - 2\left(\frac{2Z + N}{N}\right)\Delta'_{11k}(\tilde{d}_R^k) \tag{17}$$
$$+ 2\left(\frac{2N + Z}{N}\right)\Delta'_{1j1}(\tilde{q}_L^i).$$

In arriving at the expression in Eqs. (17) I have omitted small contributions to the tree-level amplitude involving $1 - 4\sin^2\theta_W$. Note that $\Delta'_{11k}$ and $\Delta_{12k}$ cancel against each other in the $\beta$-decay amplitude. In contrast, the impact of $\Delta_{12k}$ on the PV amplitude is suppressed while the effects of the $\lambda'$ terms are enhanced by the factors $2(2Z + N)/N \sim 2(2N + Z)/N \sim 5$.

Typically, limits on the RPV interactions of Eqs. (14) are obtained assuming all but one of the $\lambda_{ijk}$ and $\lambda'_{ijk}$ vanish. In the present case, however, a common explanation for the $\beta$-decay and APV results does not obtain if only one of the terms in Eq. (14) is non-vanishing. For example, taking $\Delta_{12k} > 0$ but $\Delta'_{11k} = 0 = \Delta'_{1j1}$ could not account for the common sign of both the $\beta$-decay and APV deviations. Similarly, taking $\Delta_{12k} = 0$ but either $\Delta'_{11k} \neq 0$ or $\Delta'_{1j1} \neq 0$ would not generate the observed phases. A potentially successful scenario may arise when the both a leptonic and a semi-leptonic RPV interaction occur.

To illustrate, consider the case in which $\Delta_{12k} > \Delta'_{11k} > \Delta'_{1j1} = 0$. In Fig. 1 I show the values of these corrections needed to account for the low-energy results at the 2$\sigma$ level. By themselves, these results allow $\Delta_{12k}$ and $\Delta'_{11k}$ to differ from zero over considerable ranges. A further restriction on the allowed region is obtained by studying the results of $\pi\ell_2$ decays. The ratio

$$R_{\ell/\mu} = \frac{\Gamma(\pi^+ \rightarrow e^+\nu_e + \pi^+ \rightarrow e^+\nu_e\gamma)}{\Gamma(\pi^+ \rightarrow \mu^+\nu_\mu + \pi^+ \rightarrow \mu^+\nu_\mu\gamma)} \tag{18}$$

A recent analysis of APV and other semi-leptonic data in terms of leptoquark interactions has been reported in Ref. [1]. In that analysis, no new purely leptonic interactions were included. These authors find – as noted here – that the APV and charged current decay results are not consistent with $\Delta'_{11k} > 0$ in the absence of new leptonic physics.
has been measured precisely at PSI \[16\] and TRIUMF \[17\]. Comparing the Particle Data Group average \[1\] with the SM value as calculated in Ref. \[18\] one has

$$\frac{R_{e/\mu}^{EX}}{R_{e/\mu}^{SM}} = 0.9958 \pm 0.0033 \pm 0.0004$$  \hspace{1cm} (19)

where the first error is experimental and the second is theoretical. In terms of RPV interactions, one has

$$\frac{R_{e/\mu}^{SM}}{R_{e/\mu}^{SM}} = 1 + 2 \left[ \Delta_{11k}^{'}(\bar{d}_k) - \Delta_{21k}^{'}(\bar{d}_k) \right]$$  \hspace{1cm} (20)

Note that the leptonic correction $\Delta_{12k}$ to the overall normalization cancels from the ratio of these charged current decays, leaving only the new semileptonic contributions. Assuming $\Delta_{11k}(\bar{d}_k) > \Delta_{21k}(\bar{d}_k) = 0$ one obtains strong upper bounds on $\Delta_{11k}(\bar{d}_k)$ from the results in Eq. (20). The corresponding $2\sigma$ bounds are also shown in Fig. 1.

In principle, an additional restriction on the allowed region arises from the the self-consistency of electroweak parameters. For example, one may relate $G^SM_F$ to other parameters in the SM \[13,14\]

$$G^SM_F = \frac{\pi \alpha}{\sqrt{2} M_w^2 \sin^2 \theta_W (M_Z)^4_r MS (1 - \Delta r (M_Z)^4_r MS)} ,$$  \hspace{1cm} (21)

where $\Delta r (M_Z)_{MS}$ denotes a radiative correction to this relation in the $\overline{MS}$-scheme. From a comparison of $G_\mu$ with the value of the Fermi constant computed according to Eq. \[21\], one obtains the $2\sigma$ limits\[4\]\[5\]

$$-0.0035 < \Delta_{12k} < 0.0040 \hspace{1cm} (22)$$

This constraint is also shown in Fig. 1 (similar constraints can be obtained in other renormalization schemes.) At this level, the bounds from Eq. \[22\] do not significantly impact the allowed region. The approximate centroid of the allowed is given by ($\Delta_{12k} = 0.0025$, $\Delta_{11k} = 0.0010$). This point corresponds to a -0.15% shift in $G^SM_F$ and a -0.5% change in $G^{PV}_F$ from the SM values.

In general, experimental limits on flavor-changing neutral currents and other rare processes impose stringent limits on products of the $\lambda_{ijk}$ and $\lambda_{ijk}^{'}$ couplings when two or more are simultaneously non-vanishing. The case considered above is no exception. However, when the purely leptonic correction $\Delta_{12k}(\bar{e}_k)$ involves the exchange of a $\tau$ slepton ($k = 3$), the limits from rare processes do not appear to rule out the simultaneous occurrence of a leptonic and semi-leptonic RPV interaction. For example, if the $\lambda_{123}$ and $\lambda_{11k}^{'}$ ($k = 2$ or 3 but not both) interactions are both non-zero, then the decays $B^0 \rightarrow \tau^{\pm} \mu^{\pm}$ (k=3) or

\[4\] Note that $\Delta_{12k} \geq 0$ according to Eq. \[13\].

\[5\] For a similar analysis in terms of the oblique parameters, see Ref. \[15\].
\[ \tau \rightarrow \mu K^0 \text{ (k=2)} \] can occur via the exchange of a \( \tilde{\nu}_e \). The corresponding branching ratios are \( \frac{B(\tau \rightarrow \mu K^0)}{1 \times 10^{-3}} \) and \( \frac{B(B^0 \rightarrow \tau^\pm \mu^\pm)}{8.3 \times 10^{-4}} \). These results imply that

\[ \sqrt{|\lambda'_{12k}^1 \lambda_{123}|} < 0.11 \text{ (} M_{\tilde{\nu}_e}/100 \text{ GeV)} \] (25)

\[ \sqrt{|\lambda'_{113} \lambda_{123}|} < 0.036 \text{ (} M_{\tilde{\nu}_e}/100 \text{ GeV)} \] (26)

By comparison, taking \( \Delta_{12k}(\tilde{e}_R^k) = 0.0025 \) and \( \Delta'_{11k}(\tilde{d}_R^k) = 0.0010 \) as above would require

\[ |\lambda_{12k}| = 0.041 \text{ (} M_{\tilde{e}_e}/100 \text{ GeV)} \] (27)

\[ |\lambda'_{11k}| = 0.026 \text{ (} M_{\tilde{d}_d}/100 \text{ GeV)} \] .

If \( M_{\tilde{e}_e} \sim M_{\tilde{d}_d} \), then Eqs. (27) imply

\[ \sqrt{|\lambda'_{11k} \lambda_{12k}|} \sim 0.033 \text{ (} M_f/100 \text{ GeV)} \] ,

where \( M_f \) is a common sfermion mass scale. Comparing Eqs. (28) and (25,26), one sees that the \( \beta \)-decay and APV results and rare decay limits can be accommodated in the RPV MSSM without requiring mass hierarchies in the soft SUSY-breaking sector.

The viability of RPV supersymmetry in the present context would be further constrained by improved limits on rare \( B \) and \( \tau \) decays. In the light flavor sector, it may be tested by future low-energy electroweak measurements. New measurements of pion, neutron, and Fermi nuclear \( \beta \)-decay will further test the deviation of \( G_F^R \) from the SM value. A new determination of the \( ^{10}\text{C}(0^+, \text{g.s.}) \rightarrow ^{10}\text{B}(0^+, 1.74 \text{ MeV}) \) branching ratio \cite{20} yields a value for \( G_F^R \) consistent with the SM value, though the errors are considerably larger than those corresponding to Eq. (1). A 0.7\% determination of the neutron \( \beta \)-decay asymmetry parameter \( A \) has been obtained at ILL \cite{21}. When combined with the world average for the neutron lifetime, the new value for \( A \) implies an even smaller value for \( G_F^R \) than obtained from the average of superallowed decays, with a similar uncertainty. A future, precise determination of \( A \) is underway at Los Alamos.

Among neutral current studies, a PV Möller scattering experiment is planned for SLAC \cite{22}. The Möller asymmetry is sensitive to the leptonic correction \( \Delta_{12k} \). At tree level, one has

\[ \delta_e = A_{LR}(ee)/A_{LR}^{SM}(ee) \approx - \left[ 1 + \left( \frac{4}{1 - 4 \sin^2 \theta_W} \right) \lambda_x \right] \Delta_{12k}(c_R^k) \] .

Including the \( \mathcal{O}(\alpha) \) electroweak corrections in \( A_{LR}^{SM} \) \cite{23} leads to \( \delta_e \approx -31 \Delta_{12k}(c_R^k) \). The expected precision for this experiment is \( \pm 7\% \). Thus, a result implying \( \delta_e^{EX} > 0.11 \) would begin to impact the 2\( \sigma \) constraints in Fig. 1.

In the semi-leptonic sector, additional experiments are planned in APV. These measurements will consider ratios of PV observables along an isotope chain in order to reduce the
effect of atomic theory uncertainties. For example, if $A_{PV}(N)$ denotes an APV observable for an isotope with $N$ neutrons, one may consider

$$\mathcal{R} = \frac{A_{PV}(N') - A_{PV}(N)}{A_{PV}(N') + A_{PV}(N)} \approx \frac{Q_w(N') - Q_w(N)}{Q_w(N') + Q_w(N)} .$$  \tag{30}$$

Letting $\mathcal{R} = \mathcal{R}^{SM}(1 + \delta_R)$, where $\mathcal{R}^{SM}$ denotes the value in the SM, one has \[7,24\]

$$\delta_R \approx 2 \left( \frac{2Z}{N' + N} \right) \left[ -2\lambda_x \Delta_{12k}(\tilde{e}_R^k) + 2\Delta'_{11k}(\tilde{d}_R^k) - \Delta'_{ij1}(\tilde{q}_L^j) \right] - \left( \frac{N'}{\Delta N} \right) (Z\alpha)^2 \left( \frac{3}{7} \right) \delta(\Delta X_N) .$$  \tag{31}

Here, I have followed Refs. \[25,26\] and approximated the nucleus as a sphere of constant neutron and proton densities out to radii $R_N$ and $R_P$, respectively. The parameter $\Delta X_N = (R_{N'} - R_N)/R_P$ and $\delta \Delta X_N$ denotes the uncertainty in this quantity. Note that unlike the correction to the PV amplitude for a single isotope, the dependence of $\delta_R$ on the purely leptonic new physics is not negligible. Given the allowed region in Fig. 1, the first term in Eq. (31) could range between -0.0006 and 0.0019. Although one anticipates an experimental uncertainty in $\delta_R$ of $\sim 0.001 - 0.003$, the uncertainty in the nuclear structure term is likely to be larger \[7\]. The sensitivity of isotope ratio measurements to possible RPV effects is thus complicated by nuclear structure uncertainty.

Alternatively, one may access the RPV corrections with a PV electron scattering (PVES) measurement of the proton’s weak charge. The relative shift induced in this case is

$$\delta_P = \Delta Q_w^p/Q_w^p \approx \left( \frac{2}{1 - 4\sin^2 \theta_W} \right) \left[ -2\lambda_x \Delta_{12k}(\tilde{e}_R^k) + 2\Delta'_{11k}(\tilde{d}_R^k) - \Delta'_{ij1}(\tilde{q}_L^j) \right] ,$$  \tag{32}$$

where a small contribution to the coefficient of $\Delta_{12k}$ proportional to $(1 - 4\sin^2 \theta_W)$ has been omitted for simplicity of illustration. Note that – apart from the latter – the dependence of $Q_w^p$ on new RPV physics is the same as that of $\mathcal{R}$, to first order in the new interactions. This feature is general and applies to situations other than the RPV SUSY scenario discussed here \[4\]. From Eq. (32), one would expect $-0.03 \leq \Delta Q_w^p/Q_w^p \leq 0.4$ for the allowed region in Fig. 1. Alternatively, a 3% determination of $Q_w^p$ would begin to tighten the $2\sigma$ allowed region if $\delta_P^{\text{ex}} \sim -0.02$. Recently, a letter of intent to measure $Q_w^p$ at the 3-5% level with PVES at the Jefferson Lab has appeared \[27\]. In contrast to the situation with the isotope ratios, the interpretation of a 3% PVES determination $Q_w^p$ does not appear to be limited by strong interaction uncertainties. Such a measurement could place new and interesting constraints on the possibility of low-energy RPV effects.

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FIGURES

FIG. 1. The 2σ constraints on RPV corrections $\Delta_{12k}(\tilde{e}_R^k)$ and $\Delta_{11k}'(\tilde{d}_R^k)$ from precision electroweak data. Dark solid lines give constraints from superallowed nuclear $\beta$-decay. Dashed lines indicate APV constraints, while light vertical solid line corresponds to bounds of Eq. (22). Dot-dashed line gives upper bound from $\pi\ell_2$ decays. The allowed region is indicated by shading.
