Research on the Multi-Scroll Chaos Generation Based on Jerk Mode

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Abstract

This paper presents the theory of the Multi-scroll chaotic attractor based on the Jerk model, designs the hardware circuit, and respectively gives the circuit simulation of the EWB and the numerical simulation of the MATLAB. By comparison, the results of two methods are consistent, which shows that the Jerk system has the physical realization. Finally, it gives a method to produce n×m scroll and the results of simulations.

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1. Introduction

Recently, attention has been shifted towards exploiting complicated dynamical properties such as chaos for technological applications [1-4]. Creating a chaotic system with a more complicated topological structure such as a multi-scroll or multi-wing attractor, therefore, becomes a desirable task and sometimes a key issue for many engineering applications.

In 2000, Sprott proposed the third-order autonomous chaotic system basing the Exhaustive Attack method of the computer to study the third-order Jerk system, which caused the concern of the researchers at home and abroad. The general form of Jerk system is $x'''' = f(x, x', x'')$, where, $x'$ is the first derivative of the location, namely speed; the second derivative is the acceleration; the third-order derivative is known as Jerk. Jerk

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system is simple and easy for circuit, besides, can be obtained the required frequency spectrum by changing the time scale. A potential application to transmit encrypted audio and image information is also given.

In this paper, it is analyzed the Jerk system by the numerical simulation of MATLAB and the circuit simulation of the EWB, respectively are analyzed, is given the results and a general method to produce n×m aligned array scroll.

2. Jerk basic dynamics of chaotic system

Files studying the following Jerk system [5, 6]:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= z \\
\dot{z} &= -x - y - az + f(x)
\end{align*}
\]  

(1)

When \( f(x) = \text{sgn}(x) + \text{sgn}(x + 2) + \text{sgn}(x - 2) \). If \( a = 0.6 \), the system can be produced four-scroll chaotic attractor, the simulation result is shown in Figure 1.

![Fig. 1. (a) The sequence of X; (b) The sequence of Y; (c) The sequence of Z; (d) Phase of the XY ; (e) Phase of the XZ; (f) Phase of the YZ](image)

If the right end of the system (1) is zero, can be found the equilibrium point of variable \( x \) of the system (1), namely, the zero of the nonlinear function \( f(x) \). If \( f(x) - x = 0 \), can be sought out four equilibrium points of the system (1): \((1,0,0)\), \((-1,0,0)\), \((3,0,0)\), \((-3,0,0)\).

By comparison, we can know that all of the slopes of the nonlinear function at those equilibrium points is equal to \(-1\), in addition, three turning points divide into four regions, which correspond to the positions where is generated the scroll, as the Figure 2.

![Fig. 2 The nonlinear function curve](image)
3. Multi-scroll generation and numerical simulation

According to the mechanism of the four-scroll attractor, we can further deduce the Jerk functional form of the n-scroll. Just adjusting the nonlinear function, such as increasing the number of zero, namely adding the equilibrium point of the system (1), the slope is always kept to be 1, the characteristic value of any equilibrium point will be satisfied Shil'nikov theorem. The scroll will be produced in the equilibrium point, and its position will be corresponding to the equilibrium point. In accordance with this law, we can produce the odd, even scroll, their function forms and the simulation results are shown as follows[7].

3.1. Even scrolls

If adjust the equilibrium point of the non-linear equations, can have a 6-scroll, 8-scroll, 10-scroll, even number-scroll, and the corresponding nonlinear equations are: The XY phase diagrams of the numerical simulation of 6-scroll, 8-scroll, and 10-scroll are shown in figure 3 as follows.

\[
\begin{align*}
\dot{x} &= \text{sgn}(x) + \text{sgn}(x+2) + \text{sgn}(x-2) + \text{sgn}(x+4) + \text{sgn}(x-4) \\
\dot{x} &= \text{sgn}(x) + \text{sgn}(x+2) + \text{sgn}(x-2) + \text{sgn}(x+4) + \text{sgn}(x-4) + \text{sgn}(x+6) + \text{sgn}(x-6) \\
\dot{x} &= \text{sgn}(x) + \text{sgn}(x+2) + \text{sgn}(x-2) + \text{sgn}(x+4) + \text{sgn}(x-4) + \text{sgn}(x+6) + \text{sgn}(x-6) + \text{sgn}(x+8) + \text{sgn}(x-8)
\end{align*}
\]

The XY phase diagrams of the numerical simulation of 6-scroll, 8-scroll, and 10-scroll are shown in figure 3 as follows.

![Fig. 3. (a)The XY phase of 6-scroll (b) The XY phase of 8-scroll (c) The XY phase of 10-scroll](image)

3.2. Odd scrolls

If adjust the equilibrium point of the non-linear equations, can have a 3-scroll, 5-scroll, 7-scroll, odd number-scroll, and the corresponding nonlinear equations are:

\[
\begin{align*}
\dot{x} &= \text{sgn}(x+1) + \text{sgn}(x-1) \\
\dot{x} &= \text{sgn}(x+1) + \text{sgn}(x-1) + \text{sgn}(x+3) + \text{sgn}(x-3) \\
\dot{x} &= \text{sgn}(x+1) + \text{sgn}(x-1) + \text{sgn}(x+3) + \text{sgn}(x-3) + \text{sgn}(x+5) + \text{sgn}(x-5)
\end{align*}
\]

The XY phase diagrams of the odd number-scroll are shown in figure 4 as follows.

![Fig. 4.(a) The XY phase of 3-scroll (b) The XY phase of 5-scroll (c) The XY phase of 7-scroll](image)
4. The EWB circuit implementation of multi-scroll

4.1. EWB hardware circuit design

System (1) is the differential equation, integrating on both sides of system (1) will be obtained:

\[
\begin{align*}
\dot{x} &= \int y \, dt \\
\dot{y} &= \int z \, dt \\
\dot{z} &= \int (-x - y - az + f(x)) \, dt 
\end{align*}
\]  

(2)

From the formula (2), we can find that the core of the circuit includes three parts: the integrator, the nonlinear function generator and the adders. For example, the circuit for producing 4-scroll is shown in figure 5.

![Fig. 5 the Jerk circuit diagram of the 4-scroll](image)

4.2. Results

If take the circuit as shown in figure 5 working in software work environment of Multisim, the four-scroll chaotic attractor will be obtained as shown in Figure 6(a). In the generation part of the symbolic function of the function generator, if we increase the number of equilibrium point, the number of scroll will be increased. For example, in order to increase the scroll of the equilibrium point, we just need to change the resistance to the resistance, and increase a resistance between and the ground which ensures that the sum of the resistances between and the ground is. Then, connecting two more group operational amplifier in circuit as shown in the figure 5 will be used for producing 6-scroll chaos. Therefore, just slightly modifying the non-linear generator will be used for controlling the number of scroll, using this method, we can gain the XY phase diagram of the 6-scroll, 7-scroll as shown in figure 6 (b) (c).

![Fig. 6. (a) The XY phase of 4-scroll; (b)The XY phase of 6-scroll; (c)The XY phase of 7-scroll](image)

5. The numerical simulation of the n * m array scroll

From the results of part 2, recursion of the dimension of the scroll can be used for getting the Jerk equation of the n×m array scroll. A case study of 6×6 scrolls is for explaining.
System equation becomes:

\[
\begin{align*}
\dot{x} &= y - f(y) \\
\dot{y} &= z \\
\dot{z} &= -x - y - az + f(x) + f(y)
\end{align*}
\]

(3)

Where:

\[
\begin{align*}
f(x) &= \text{sgn}(x) + \text{sgn}(x+2) + \text{sgn}(x-2) + \text{sgn}(x+4) + \text{sgn}(x-4) \\
f(y) &= \text{sgn}(y) + \text{sgn}(y+2) + \text{sgn}(y-2) + \text{sgn}(y+4) + \text{sgn}(y-4)
\end{align*}
\]

The numerical simulation results are shown in figure 7.

![Image](image.png)

Fig.7. (a) The XY phase of 6×6-scroll; (b) The XYZ phase of 6×6-scroll

6. Conclusion

In this article, the most simple nonlinear function is used for obtaining Jerk chaos, comparing the equilibrium point of the non-linear function with the location of the scroll can be used for finding the method of producing the multi-scroll, otherwise, the numerical simulation and the circuit simulation can be used for verifying the generation of the multi-scroll. The results show that changing the number of zeros of nonlinear functions can be quickly and efficiently carried out the control of the number of scroll. The one-dimensional scroll can be gained the n× m-dimensional scroll signal generation method by changing the system equation, the numerical simulation on this show that the scheme is feasible. This paper is important for the chaos secure communication, the image encryption and the M-ary transmission for chaotic communications.

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