Monitoring slope stability based on factor of safety estimated by back analysis of measured displacements

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Abstract. Field monitoring is essential for assessing the stability of natural as well as well-designed slopes to confirm the validity of the design. Many instruments and systems are available for monitoring the slope behavior. Interpretation of the monitoring results for assessing the stability of slopes is an important task. Displacements are usually plotted versus time, and their transition and rate of increase are observed and compared with the criteria. This is a common and useful practice, but it is based on an empirical method. Therefore, a method for assessing the stability of slopes on the basis of rock mechanics is required. This paper outlines a back analysis method originally proposed by Sakurai in 1987 to estimate the factor of safety from the measured displacements. Two case studies are demonstrated to confirm the validity of the method. By applying the back analysis method to natural and well-designed slopes, the time transition of the factor of safety can be estimated from the measured displacements. The applicability and limitations of the method are also discussed.

1. Introduction
In recent years, slopes in various areas of Japan have collapsed because of heavy rain and large-scale earthquakes. Therefore, it is important to properly assess the stability of slopes to reduce the risk of public disasters. In general, the stability of slopes is assessed by calculating the force balance using the mechanical constants obtained from geological surveys and laboratory and in situ tests. Field monitoring is essential for assessing the stability and confirming the validity of the design.

Many instruments and systems are available for monitoring the slope behavior. Extensometers and inclinometers are often used. GPS/GNSS is now a well-known method for monitoring three-dimensional displacements [1]. DInSAR has the potential to observe displacement distribution over extensive areas [2].

Interpretation of the monitoring results for assessing the stability of slopes is an important task. Displacements are usually plotted versus time, and their transition and rate of increase are observed. The results are compared with the threshold or the critical value for assessing the stability.

This is a common and useful practice, but it is based on an empirical method. Therefore, a method for assessing the stability of slopes on the basis of mechanics is required for field measurements. Back analysis of the displacement-monitoring data is one of the possible methods.

The back analysis method to estimate the factor of safety from measured displacements is outlined in Section 2. In Section 3, two case studies are demonstrated to confirm the validity of the method. The applicability and limitations of the method are discussed in Section 4.
2. Back Analysis Method

The back analysis method is presented as follows: First, the slip zone of the slope is geometrically estimated from the displacement vectors, and a numerical model is assumed. Second, the strength parameters along the slip zone are estimated by conducting a back analysis of the measured displacements. Finally, the factor of safety is computed using the strength parameters estimated via back analysis.

The back analysis method was originally proposed by Sakurai in 1987 and has been improved over the years [3]. In general, the stability of slopes at the design stage is assessed using the force balance on the basis of the theory of mechanics. On the contrary, the stability of slopes on site is assessed using the measured displacements (e.g., transition of measured displacements and its rate of increase). Sakurai pointed out that the method for assessing the slope's stability on site is different from that at the design stage. This implies an inconsistent assessment of the slope stability, which can be solved using the back analysis method.

The procedure of back analysis for estimating the strength parameters and the factor of safety is as follows [3]. Sakurai assumed that the deformational behavior of the sliding of a slope can be expressed by an anisotropic model with one of the main axes in the direction parallel to the slip layer. Introducing the anisotropic parameter $m$ proposed by Sakurai, the stress–strain relationship in the $x' − y'$ local coordinate system (figure 1) is

$$\{\Delta \sigma'\} = [D']\{\Delta e'\}$$ (1)

where

$$[D'] = \frac{E}{(1 - \nu - \nu^2)} \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & m(1 - \nu - 2\nu^2) \end{bmatrix}$$ (2)

$E$ and $\nu$ are the Young's modulus and Poisson's ratio, respectively. The anisotropic parameter $m$ is expressed by the ratio of the shear modulus $G$ and the Young’s modulus $E$.

Furthermore, it is transformed into the $x − y$ global coordinates as follows:

$$\{\Delta \sigma\} = [D]\{\Delta e\}$$ (3)

where

$$[D] = [T][D'][T]^T$$ (4)

$[T]$ is a transformation matrix expressed as,

$$[T] = \begin{bmatrix} \cos^2 \alpha & \sin^2 \alpha & -2\sin \alpha \cos \alpha \\ \sin^2 \alpha & \cos^2 \alpha & 2\sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha & -\sin \alpha \cos \alpha & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix}$$ (5)

$\alpha$ is the angle between the $x'$ and $x$ axes. If $m = 1/2(1 + \nu)$, then Eq. (3) represents an isotropic elastic material.

The anisotropic parameters and Young’s modulus are back-analyzed to minimize the following residual error of displacements:

$$f = \sum_{i=1}^{N}(u_i^c - u_i^m)^2 \rightarrow \text{min.}$$ (6)

where $u_i^m$ and $u_i^c$ represent the measured and computed displacements, respectively, and $N$ is the number of measurements.

In the case of an existing slope, it is assumed that the displacement is caused by the change in the anisotropic parameter. This is because the external force (e.g., gravity and stress released due to excavations) has already been applied before starting the measurement.
Therefore, the computed displacement $u'_i$ in Eq. (6) that corresponds to the actual measured displacement can be obtained using the following equation.

$$u'_i = u'_i - u^0_i$$  \hspace{1cm} (7)

where $u'_i$ is the computed displacement after the anisotropic parameter is changed, and $u^0_i$ is the displacement caused by the external force before starting the measurement. $u^0_i$ can be calculated by the ordinary analysis (e.g., FEM, etc.).

After determining $E$ and $m$ via back analysis of the measured displacements, the procedure for estimating the strength parameters and the factor of safety is as follows.

1. Shear modulus $G$ is determined using the back-analyzed parameters $E$ and $m$ as,

$$G = mE$$  \hspace{1cm} (8)

2. The critical shear strain $\gamma_0$ is defined as the ratio of the shear strength $\tau_c$ against the shear modulus $G$, i.e., $\gamma_0 = \tau_c / G$ (figure 2(a)). The critical shear strain is determined via laboratory tests as shown in figure 3. From the figure, it can be seen that the relationship between both parameters does not depend on the types of soil and rock. The critical shear strain $\gamma_0$ can then be estimated from $G$ using the following relationship:

$$\gamma_0 = 4G^{-2/7}$$  \hspace{1cm} (9)

3. Hence the shear strength parameter $\tau_c$ is obtained as,

$$\tau_c = G\gamma_0$$  \hspace{1cm} (10)

4. Assuming the internal friction angle $\phi$, the cohesion $c$ can be determined as (figure 2(b))

$$c = \frac{(1 - \sin \phi)\tau_c}{\cos \phi}$$  \hspace{1cm} (11)

5. The factor of safety can then be calculated using the conventional limit equilibrium method with the strength parameters estimated as above.
3. Application of Back Analysis – Case Studies

In this section two case studies are demonstrated to confirm the validity of the back analysis described in Section 2. The time transition of the factor of safety is estimated from the measured displacements.

3.1. A large rock slope under long-term monitoring

3.1.1. Geographical features and monitoring data. The first case-study involves a large rock slope with a height of 370 m and a width of 645 m. Displacement monitoring via surveying methods has been performed at this site for over 40 years since 1979. Figure 4 shows the transition of the accumulated displacements measured using an electronic distance meter (EDM) and through level survey. The displacement increased at a rate of 9–27 mm/year in the first eight years of monitoring. Collecting and draining of groundwater by drainage boring and tunnel since 1979 reduced the rate of displacement to 3–12 mm/year after the late 1980s.
3.1.2. Conditions of analysis. Figure 5 shows the finite element mesh. The slip zone is assumed to be parallel to the direction of the measured displacements at K-1, K-2, …, and K-5. Back analysis of the measured displacements as described in Section 2 was performed four times when the displacement rate changed (CASE-A1, CASE-A2, CASE-A3, and CASE-A4 in figure 4). The Young’s modulus and the anisotropic parameter along the slip zone are then obtained. The unit weight and Poisson’s ratio are assumed to be 16 kN/m² and 0.3, respectively. To save computation time the Genetic Algorithm is applied to derive the optimum solution of Eq. (6) from a large number of combinations of Young’s modulus and anisotropic parameters [4].

![Figure 5. Numerical model.](image)

3.1.3. Results of back analysis. To confirm the validity of the back-analyzed parameters, the displacements calculated using these parameters are compared with the measured displacements as illustrated in figure 6 for CASE-A4. Figure 6 illustrates both displacements for CASE-A4. From this comparison, it can be observed that the calculated displacements generally agree with the measured one, although the direction of displacement at K-1 and K-5 and the amount of the displacement at K-4 and K-5 are slightly different. In this study, these discrepancies can be accepted and will be identified as a future issue in improving the results to be close to the measured displacement.

The back-analyzed mechanical constants for the four cases are shown in table 1. It is found that the shear modulus $G$ tended to decrease as the measured displacement increases. The shear strength $\tau_c$ and the cohesion $c$ tended to decrease. The back-analyzed strength parameters are compared with the result of the block shear test conducted near the slip zone in the drainage tunnel of this slope in 2006, where the internal friction angle $\phi$ was 21.8° and the cohesion $c$ was 0.13 MPa. The cohesion estimated using Eqs. (8)–(11) for CASE-A3 (12/2001) and CASE-A4 (04/2016) are 0.14 MPa and 0.11 MPa, respectively (see table 1), close to the cohesion obtained by the in situ test (0.13 MPa).

Finally, the factor of safety is calculated using a limit equilibrium method with the back-analyzed cohesion from the measured displacements. The time transition of the factor of safety in figure 7 shows a gradual decreased to 1.2. However, the slope is stable at present due to the effect of additional countermeasures of groundwater drainage.

![Figure 6. Comparison between calculated and measured displacements for CASE-A4.](image)

![Figure 7. Time transition of the factor of safety.](image)
3.2. A cut slope along a highway

3.2.1. Geographical features and monitoring data. The second case study was performed on a well-designed cut slope along a highway with a height of 25.4 m and a width of 45.7 m [5]. The slope is composed of sedimentary rocks significantly affected by weathering. After detecting several cracks on the surface of the slope, the continuous displacement monitoring using GPS was started. Figure 8 shows the monitoring results of the three-dimensional displacement at G-1 (see figure 9). In early July 2009, the displacement increased up to about 20 mm due to rainfall and the slope was stable. However, all components of the three-dimensional displacement significantly increased, exceeding 200 mm in July 2010 during continuous heavy rainfall for a few days. As a result, the slope collapsed.

3.2.2. Conditions of analysis. Figure 9 shows the finite element mesh. The slip zone was assumed to be parallel to the direction of the displacements measured by GPS at G-1. Back analysis of the measured displacements as described in Section 2 was performed three times when the displacement

| Table 1. Results of back analysis. |
|-----------------------------------|
|                                 |
| Young’s modulus: $E$ (MPa)       |
| Anisotropic parameter: $m$       |
| (sliding zone)                   |
| Shear modulus: $G$ (MPa)         |
| Critical shear strain: $\gamma_0$ (°) |
| Shear strength: $\tau_c$ (MPa)   |
| Internal friction angle: $\phi$ (assumed) |
| Cohesion: $c$ (MPa)              |
| Factor of safety                 |
| CASE-A1  1,037                  |
| CASE-A2  612                    |
| CASE-A3  518                    |
| CASE-A4  270                    |
| 0.033                             |
| 0.023                             |
| 0.019                             |
| 0.024                             |
| 34.5                             |
| 14.6                             |
| 9.7                              |
| 6.4                              |
| 1.5                              |
| 1.9                              |
| 2.1                              |
| 2.4                              |
| 0.50                             |
| 0.27                             |
| 0.20                             |
| 0.15                             |
| 20                               |
| 20                               |
| 20                               |
| 20                               |
| 0.35                             |
| 0.19                             |
| 0.14                             |
| 0.11                             |
| 2.3                              |
| 1.5                              |
| 1.3                              |
| 1.2                              |

Figure 8. Measured displacement at G-1 by GPS.
rate changed (CASE-B1, CASE-B2, and CASE-B3). The displacement increased by 1, 20, and more than 200 mm in CASE-B1, CASE-B2, and CASE-B3, respectively. The Young’s modulus and anisotropic parameter at the slip zone were obtained as shown in Table 2. The unit weight and the Poisson’s ratio were assumed to be 16 kN/m³ and 0.3, respectively.

![Figure 9. Numerical model.](image)

**Table 2. Results of back analysis.**

|                     | CASE-B1 | CASE-B2 | CASE-B3 |
|---------------------|---------|---------|---------|
| Young’s modulus: $E$ (KPa) | 1,000   | 1,000   | 1,000   |
| Anisotropic parameter: $m$ (sliding zone) | 0.330   | 0.070   | 0.004   |
| Shear modulus: $G$ (KPa) | 330.0   | 70.0    | 3.7     |
| Critical shear strain: $\gamma_0$ (%) | 5.5     | 8.6     | 19.8    |
| Shear strength: $\tau_c$ (KPa) | 18.1    | 6.0     | 0.7     |
| Internal friction angle (assumed): $\phi$ (^) | 20      | 20      | 20      |
| Cohesion: $c$ (KPa) | 12.7    | 4.2     | 0.5     |
| Factor of safety | 1.3     | 1.0     | 0.8     |

![Figure 10. Comparison between calculated and measured displacements for CASE-B3.](image)

![Figure 11. Transition of the factor of safety.](image)

3.2.3. **Results of back analysis.** To confirm the validity of the back analysis, the displacements calculated using the back-analyzed parameters were compared with the measured displacements as illustrated in Figure 10 for CASE-B3. Both displacements show a good agreement.

The back-analyzed mechanical constants in CASE-B1, CASE-B2 and CASE-B3 are shown in Table 2. The shear modulus $G$ is smaller in CASE-B3 than in CASE-B1 and CASE-B2, and the shear strength $\tau_c$ and the cohesion $c$ are also rather small. Figure 11 shows the decrease in the factor of safety from 1.3 in CASE-B1 to 1.0 in CASE-B2 due to heavy rainfall. Finally, it became less than 1.0 in CASE-B3 during a long-term heavy rainfall, and the slope collapsed. Before the slope failure, the road was closed to traffic and therefore, no accident occurred. This proves that the back analysis method can be useful for safe road management.
4. Discussion

The back analysis method described in Section 2 can estimate the strength parameter “cohesion” from the measured displacements and can provide the transition of the factor of safety with time. The strength parameter obtained in the first case study agreed with that obtained in the in situ test. The factor of safety in the second case study corresponded to the actual collapse behavior of the slope. Therefore, the back analysis of the measured displacement can be useful for assessing the stability of slopes. However, the following points should be taken into consideration as future issues for improving the method.

(1) The numerical models of these case studies have been assumed based on the slip zone of the slope, which was estimated from the direction of the measured displacements. However, these are not unique and the back-analyzed mechanical constants may depend on the assumed models. Therefore, a few models are needed to be employed for back analysis to know the possible ranges of back-analyzed mechanical constants.

(2) The back analysis method requires the measured displacements. Usually, measurement instruments are installed after some deformation has occurred at the slopes. Thus, the initial value of the displacements cannot then be measured. The back-analyzed mechanical constants and factor of safety are affected by the values of unmeasured displacements. To obtain reliable results from back analysis, a suitable scenario and an appropriate method for estimating the unmeasured displacements are required. The Synthetic Aperture Radar could be used to achieve this purpose [2].

5. Conclusion

In this paper, a back analysis method of the measured displacements was described to estimate strength parameters and the factor of safety of slopes. Two case studies were demonstrated to confirm the validity of the method. It was concluded that the transition of the factor of safety can be provided with time for assessing the stability of slopes. Further study is required for assuming an appropriate model and for evaluating unmeasured displacements before the installation of measurement devices.

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