Twin Higgs WIMP Dark Matter

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Dark matter (DM) without a matter asymmetry is studied in the context of Twin Higgs (TH) theories in which the LHC naturalness problem is addressed. These possess a twin sector related to the Standard Model (SM) by a (broken) $Z_2$ symmetry, and interacting with the SM via a specific Higgs portal. We focus on the minimal realisation of the TH mechanism, the Fraternal Twin Higgs, with only a single generation of twin quarks and leptons, and $SU(3)' \times SU(2)'$ gauge group. We show that a variety of natural twin-WIMP DM candidates are present (directly linked to the weak scale by naturalness), the simplest and most attractive being the $\tau'$ lepton with a mass $m_{\tau'} > m_{\text{Higgs}}/2$, although spin-1 $W'^\pm$ DM and multicomponent DM are also possible (twin baryons are strongly disfavoured by tuning). We consider in detail the dynamics of the possibly (meta)stable glueballs in the twin sector, the nature of the twin QCD phase transition, and possible new contributions to the number of relativistic degrees of freedom $\Delta N_{\text{eff}}$. Direct detection signals are below current bounds but accessible in near future experiments. Indirect detection phenomenology is rich and requires detailed studies of twin hadronization and fragmentation to twin glueballs and quarkonia and their subsequent decay to SM, and possible light twin sector states.

I. INTRODUCTION

Models based on the Twin Higgs (TH) mechanism [2–5] address the LHC fine-tuning problem and solve the little hierarchy problem by introducing a twin sector, that is, in its simplest realisation, a copy of the Standard Model (SM), regarding both field content and interactions. At tree level, the Higgs sector of the theory respects a global $SU(4)$ symmetry (in fact an $O(8)$ symmetry in the most attractive cases [2]) acting on the components of the pair of Higgs doublets $(H, H')$, where $H'$ is the twin Higgs doublet (throughout primes denote objects in the twin sector). A discrete $Z_2$ between the two sectors ensures equality of their couplings, which results in $SU(4)$-symmetric radiative corrections to the Higgs mass squared. The $SU(4)$ symmetry is broken at one loop order by radiative corrections to the Higgs quartic coupling and the SM Higgs is realised as a naturally light pseudo-Nambu-Goldstone boson of the approximate $SU(4)$. The $Z_2$ symmetry needs to be broken, explicitly or otherwise, for the SM Higgs to acquire a phenomenologically viable vacuum expectation value (vev), for an exact $Z_2$ would imply that the vev’s in the two sectors are equal, a possibility that is excluded by Higgs-coupling measurements. Denoting the SM and TH vev’s as $v \approx 246$ GeV and $f$ respectively, the fine-tuning arising from this difference of vev’s is $\sim 2v^2/f^2$, i.e. a mild $\sim 20\%$ tuning for the minimum experimentally allowed ratio $f/v \approx 3$. The physical light Higgs state, $h$, is shared between the SM and twin sectors with couplings to SM and twin sector states modified by $\cos(v/f) \approx 1 - v^2/2f^2$ and $\sin(v/f) \approx v/f$ respectively. It is important to also bear in mind that the TH theory needs to be UV completed at some cutoff scale $M_{\text{UV}} \leq 4\pi f$ (for definiteness we take $M_{\text{UV}} \approx 5$ TeV).

For the TH mechanism to operate, the twin sector does not need to be an exact copy of the SM, a reduced field content sufficing. This simplified version – the Fraternal Twin Higgs (FTH) [6] – has as its minimal ingredients twin weak $SU(2)'$ and colour $SU(3)'$ interactions, and a twin third generation consisting of top and bottom quarks $Q'$, $t'$ and $b'$, a lepton doublet $L'$ required by $SU(2)'$ anomaly cancellation, and a twin Higgs doublet $H'$. Right handed twin leptons may be added to the theory rendering the leptons massive, although they are not required by the TH mechanism. For the TH mechanism to still be effective without introducing significant extra tuning, the twin top Yukawa $y_t'$ can only differ by at most 1% from $y_t$ for $M_{\text{UV}} \approx 5$ TeV. The need for gauged twin $SU(3)'$ then becomes apparent: radiative corrections to $y_t$ and $y'_t$ could make them differ significantly at the weak scale even if they coincided at $M_{\text{UV}}$. A gauge coupling $g_t'$ differing by less than $\sim 15\%$ from $g_t$ at the cutoff [3] ensures that the running of $y_t$ and $y'_t$ is close enough. However, the running of the $g'_t$ and $g_t$ couplings, and thus the dynamical scales, differ because of the different field content and masses of the two sectors. With only one quark generation in the twin sector, and for the allowed range of values of $g_t'$, we have a twin confinement scale $\Lambda_{\text{QCD}}' \sim 0.5 - 20$ GeV [6]. (Unless otherwise stated we will take our default value as $\Lambda_{\text{QCD}}' \approx 3$ GeV; the preference for this choice, or larger, is justified in Section VIII.) The presence of an $SU(2)'$ gauge group in the twin sector also follows from fine-tuning considerations, with the twin $g_t'$ coupling allowed to differ by at most $\sim 10\%$ from the SM value. A gauged $U(1)'$ is not necessary from a naturalness perspective, although it remains an accidental global symmetry of the twin sector.

In this paper, we explore the possibilities for dark matter (DM) in TH scenarios, focusing on the minimal FTH
models in the case where a matter-antimatter asymmetry in the twin sector is not present and where $U(1)'$ is a global symmetry. (We reserve the study of both twin asymmetric DM, and the effects of a gauged $U(1)'$, for a companion paper [7].) Crucially for our later discussion, intrinsic to the success of the TH mechanism is the fact that the twin sector interacts with the SM via the Higgs portal with a strength determined by $f/v$, leading to specific predictions for DM signals. In addition, we will show that the most attractive twin DM candidates in the absence of an asymmetry are the twin leptons, which naturally have a ‘twin-WIMP miracle’ as they freeze out via twin weak interactions whose strength is set by $g'_2 \simeq g_2$ and $G'_F = (v/f)^2 G_F$ both of which are directly tied to SM weak interaction values by naturalness. Since the Higgs portal forces the SM and twin sector to be in equilibrium until temperatures well below the EFT cutoff, we have a purely thermal freeze-out scenario, at least for states that are not UV relics.

As asymmetric DM is not our concern here, we will primarily focus on the heavy twin quark sector, i.e. $m_{\nu'} \gg \Lambda_{QCD}$, which arises naturally for $y_t = y_\ell$ and $y_{\nu'} \simeq y_\tau$ and for the values of $f/v \geq 3$ that are allowed. Collider signals of the TH model in this regime were treated in detail in [6], and also considered in [8].

Finally we remark that the idea of a mirror world, either exact or partial, has a long and rich history [9-11], see e.g. [12]. For a recent review of aspects of mirror world physics we refer the reader to [13]. Often such theories lead to a variety of interesting DM candidates with overlap with those studied here. Here we are considering a particular, approximate mirror world interacting with the SM via the Higgs portal, as directly motivated by the LHC naturalness problem. Previous investigations of symmetric dark matter candidates in models with a similar philosophy include [14] and [15].

II. STABLE & METASTABLE TWINS

At temperatures well below the $SU(2)'$ phase transition, where anomaly effects are exponentially small, the FTH model has an accidental $U(1)$ global symmetry associated to twin baryon number $B'$. If Majorana mass terms for the right-handed twin leptons are forbidden (for example, due to a discrete symmetry), then there are also accidental $U(1)$ twin lepton number and twin ‘charge’ symmetries, with associated conserved numbers $L'$ and $Q'$. Ultimately, we might expect these global symmetries to be explicitly broken by higher dimensional operators, possibly connected with Planck-scale physics, or by terms from the UV completion of the TH models that connect the SM and twin sectors in ways beyond the TH-mandated Higgs portal interaction itself. For the purposes of this work we assume that these new interactions are sufficiently weak that the lightest states carrying $B'$, $L'$ and $Q'$ are stable on timescales $\gtrsim 10^8 H_0^{-1}$, although decaying DM is a natural possibility in TH models.

The discrete symmetries $P$ and $C$ in the twin sector are maximally violated by $SU(2)'$ interactions but, in principle, $CP$ can remain conserved. Although we focus on the $CP$-preserving scenario, breaking of $CP$ in the twin sector, due, say, to an un-cancelled $\partial_{QCD}^\prime$ term, is allowed, and can have important consequences. We mention the changes this makes where appropriate.

We first consider the case where $m_{\nu'} \leq m_\tau$, $m_{\nu'} < m_{W'}$, meaning that both $\nu'$ and $\tau'$ are stable states and therefore automatic DM candidates. We will focus on the regime of heavy $\tau'$ ($m_{\tau'} \gtrsim m_h/2$), whereas we will allow $\nu'$ to be heavy or effectively massless. When $\nu'$ is effectively massless it will behave as dark radiation (DR), contributing to the number of effective neutrinos, $N_{\text{eff}}$. (We discuss the issue of $N_{\text{eff}}$ in detail in Section VIII.) If $m_{\nu'} + m_{\tau'} > m_{W'}$, the twin $W^\pm$ gauge bosons will also be stable, as automatically $m_{\nu'} + m_{\nu'} > m_{W'}$ in TH models, and therefore $W^\pm$ could contribute significantly to the DM density.

In the strong sector, things are more involved. One stable state is the lightest twin baryon, $\Delta'$, made out of three $b'$ quarks in a spin 3/2 state. In the absence of an asymmetry in the twin sector, $b'b'$ pairs annihilate efficiently into gluons rendering $\Delta'$ irrelevant as a DM candidate unless $m_{\nu'} \gtrsim 1$ TeV, a case we discuss in Section V (if the freeze-out temperature of the $b'$ quark-antiquark annihilations is below the phase transition temperature then $\Delta'$ (anti)baryons annihilate efficiently into glueballs and quarkonia). However, twin QCD glueball states are themselves of potential interest depending upon the UV completion of the TH theory. In the heavy quark regime the spectra of glueball and quarkonia are well separated (and relatively well known). The lightest glueball, whose mass is $m_0 \approx 6.8 \Lambda_{QCD}$ [16-17], has $J^{PC}$ quantum numbers $0^{-+}$ and therefore mixes with the Higgs, with mixing angle [6]

$$\theta = \frac{\alpha_3 v F_0}{6 \pi f^2 (m_h^2 - m_0^2)} \approx \frac{vm_0^3}{8 \pi^2 f^2 m_h^2} \quad (1)$$

with $F_0$ the $0^{++}$ glueball decay constant given by $F_0 = 3.06 m_0^3/(4 \pi \alpha_3)$ [17] and we have assumed $m_h^2 \gg m_0^2$ in the final step. It therefore promptly decays to light SM states ($\tau_0^{++} \sim 4 \times 10^{-10} \text{s}$ if we take $\Lambda_{QCD} = 3$ GeV and $f/v = 3$). In the case where the twin neutrino is light compared to $\Lambda_{QCD}$, all other glueballs decay, via $SU(2)'$ and heavy quark-loop induced interactions, to (eventually) combinations of the $0^{++}$ glueballs and $\nu'\nu'$ in appropriate angular momentum states, so leaving no stable twin-QCD states apart from $\Delta'$.

In the case where both $\tau'$ and $\nu'$ are heavy, such that twin-lepton pairs are not kinematically accessible in glueball decays, another two glueballs become potentially relevant: a $0^{+-}$ glueball, with mass $m_0^{-+} \approx 1.5 m_0$ [16,17], and a $1^{--}$ glueball, with mass $m_1^{--} \approx 1.7m_0$ [16,17]. These glueballs can be potentially long-lived metastable states, possibly with cosmologically long lifetimes, though this sensitively depends on the issues...
of additional SM–twin-sector interactions arising as from the UV completion and/or twin CP-violation. As we discuss in Section VI the freeze-out abundance of these glueballs is very small, ∼10^{-6}ρ_{DM}, so they are not significant gravitationally, though they may be constrained by CMB or cosmic ray observations if their lifetimes are long. Also, as we discuss in Section VII, both 0^{-} and 1^{-} glueballs have the potential to produce novel indirect detection signals if they decay to lifetimes in the ranges τ ∼ 10^{-10} s or τ ∼ 10^{11}–10^{12} s. Whether such lifetimes are achieved depends on the value of Λ_{QCD}^\prime, and on the UV completion, a subject we reserve for Section VI.

III. TWIN SU(3) PHASE TRANSITION

Before we can calculate the relic densities of the stable twin states, we first must consider the nature of the twin QCD phase transition, and whether it leads to significant dilution of relics by entropy production.

If m_q ≲ 8Λ_{QCD}, then the one-dynamical-quark-flavour QCD' phase transition is a smooth cross-over [18–20], with no significant non-equilibrium dynamics. As m_q is increased, the transition becomes second order at a critical value ∼8Λ_{QCD} (it should be noted that this upper limit is potentially uncertain by a few times Λ_{QCD}^\prime), and above that is (weakly) first order, as demonstrated by analytical arguments [18, 21] and lattice studies [22].

Investigating the dynamics of the first-order phase transition in the heavy-quark case, lattice studies give a critical temperature of T_{c} ≃ 1.26Λ_{QCD}^\prime [23], bubble wall surface tension σ ≃ 0.0155T_{c}^3, and latent heat \rho_{L} ≃ 1.4T_{c}^4 [24]. Since the pressure is continuous across the phase transition, the pressures of the confined and unconfined phases are equal at T_{c}. If the unconfined phase manages to supercool to a temperature T = (1 − δ)T_{c}, then a bubble of confined phase can grow due to the pressure difference between the phases, if it is large enough to overcome the surface tension, i.e. if it has radius R ≥ R_{c} = 2σ/\Delta P. The free energy cost of a ‘critical bubble’ of radius R_{c} is

$$\Delta F_{c} = 4\pi R_{c}^2 \sigma - \frac{4}{3} \pi R_{c}^3 \Delta P = \frac{16\pi}{3} \frac{\sigma^3}{(\Delta P)^2}. \tag{2}$$

Assuming that the supercooling is small, δ ≪ 1, so that

$$\Delta P = P_{G}(T) - P_{\gamma}(T) \approx -\delta T_{c}(s_{G}(T_{c}) - s_{\gamma}(T_{c})) = \delta \rho_{L}, \tag{3}$$

where P_{G} is the pressure in the confined phase, P_{\gamma} is the pressure in the unconfined phase, and s_{G}, s_{\gamma} are the corresponding entropy densities, we have

$$\frac{\Delta F_{c}}{T} \approx \frac{16\pi}{3} \frac{\sigma^3}{\rho_{L} T_{c}} \delta^{-2} \approx 3 \times 10^{-5} \delta^{-2}. \tag{4}$$

The rate per unit volume of bubble nucleation due to thermal fluctuations is set by \Gamma/V \sim T^4 e^{-\Delta F_{c}/T} [25]. This becomes comparable to H^4, i.e. one event per Hubble volume per Hubble time, when \Delta F_{c}/T \lesssim \log (T^4/H^4). Assuming that δ ≲ 1, this is equivalent to 3 × 10^{-5}δ^{-2} \lesssim 4\log(T_{c}/H(T_{c})) ≃ 160, so δ ≳ δ_{n} = 4 \times 10^{-4}. Since e^{-\Delta F_{c}/T} grows by an e-fold with a ≃ 10^{-6} drop in log T, the nucleation rate quickly becomes large as T drops.

This extremely large nucleation rate means that, once δ is somewhat larger than δ_{n}, the latent heat released from bubble nucleation and expansion must heat us back above the nucleation temperature, or the transition must complete entirely in a very small fraction of a Hubble time (in the most extreme limit, for δ ∼ 10\delta_{n} we have e^{-\Delta F_{c}/T} \sim O(1), and nucleation effectively happens everywhere at once). The rapid-completion case would arise if e.g. the heat capacity of the supercooled unconfined phase was very high, so that the expansion necessary to cool it from T_{c} to (1 − δ)T_{c} removed almost all of the latent heat of the transition. In the former case, the confined phase bubbles grow, releasing latent heat, until the temperature has been brought back up to T_{c}, where the pressures are equal. Once there, the confined phase bubbles can only grow as Hubble expansion removes energy from the mixture. In this way they grow until they occupy the entire volume, with the temperature remaining constant at T_{c}.

In either case, the out-of-equilibrium part of the phase transition, during which there is a pressure difference driving confined phase bubble expansion, occurs at temperatures only very slightly below T_{c}, so can only produce a small amount of entropy. The maximum possible increase in the overall entropy density occurs in the rapid-transition case, where the pressure-driven growth transforms all of the volume from the unconfined to the confined phase, in which case the overall increase in entropy density is

$$\Delta s \approx \frac{\Delta P}{T_{c}} \approx \text{few} \times \delta_{n} \frac{\rho_{L}}{T_{c}} \sim 10^{-3} T_{c}^3. \tag{5}$$

In the (naively more likely) case where most of the transition occurs in quasi-equilibrium, the maximum entropy production is even smaller, Δs ≲ \rho_{L}T_{c}/T \sim 2 \times 10^{-7} T_{c}^3, since only a fraction ∝ δ_{n} of the volume is converted by out of equilibrium growth. Either way, there is no significant effect on relic densities. If the transition is simply a cross-over, as occurs for light enough b', then there is again no significant entropy production. For a crossover or a quasi-equilibrium transition, there will not be significant gravitational radiation production, rendering the FTH model less encouraging for gravity wave signals than the other scenarios discussed in [26].

We also remark that if twin-CP is violated by \theta_{QCD} ≠ 0 in the pure SU(3) case with no light quarks, the transition remains first order but with parameters such as the critical temperature, T_{c}(\theta), and the latent heat, now depending on \theta_{QCD} in a periodic fashion but otherwise poorly constrained, either analytically [27] or from lattice studies [28]. These studies indicate that for \theta_{QCD} ≪ \pi the transition temperature decreases T_{c}(\theta) < T_{c}(0), while
the strength of the transition slowly increases. However the properties of the phase transition at \( \theta_{\text{QCD}} \neq 0 \) are not firmly established, especially once \( \theta_{\text{QCD}} \) is not small. Thus to be conservative, in the cases with twin-\( CP \) violation we take \( \theta_{\text{QCD}} \ll \pi \), a limit that is sufficient for our purposes.

IV. TWIN TAU DARK MATTER

Having argued in Section III that the twin-QCD phase transition leads to no significant dilution of relics by entropy production we now proceed to calculate the freeze-out density of the stable twin-sector states. We start with the simplest case, that of the twin \( \tau \) lepton (since \( U(1)' \) is not gauged, the situation for \( \nu' \) is identical to that of \( \tau' \) assuming a suitable Yukawa coupling giving a Dirac mass). In most of parameter space the annihilation of \( \tau' \)'s dominantly proceeds via twin-\( SU(2) \) weak interactions into the (assumed lighter) \( b'\) quarks/quarkonia and \( \nu'\nu' \) pairs. Annihilation via the Higgs, with couplings that are given by \( \frac{v}{\sqrt{2}} \frac{h}{\Lambda} \tau' \), is subdominant apart from a narrow resonance region around \( m_{\tau'} \approx m_b/2 \).

Figure 1 shows the contribution to the present energy density of the Universe from \( \tau' \) species, normalized to the observed DM density for different values of \( f/v \). This calculation, along with the other relic density calculations throughout this paper, was performed using the MicrOMEGAs software package [30]. For \( f/v \approx 3 \), the least tuned case, the observed DM density is obtained for \( m_{\tau'} \approx 63 \text{ GeV} \). Larger values of the ratio \( f/v \), which imply worse tuning, result in larger DM masses[1].

We emphasise that, for (symmetric) twin DM candidates with \( O(0.1) \) Yukawa couplings, we can obtain the correct relic density since the couplings and mass of the \( W' \) bosons are set by the TH mechanism to be \( g_3' \approx g_2 \) and \( M_{W'} \approx (f/v) M_{W^\pm} \). These are constrained by naturalness to be close to SM weak interaction values, giving rise to a natural ‘twin-WIMP-miracle’.

Turning now to direct detection, scattering of \( \tau' \) with SM nuclei occurs, at tree level, via Higgs exchange, and this process sets the scattering cross section in direct detection experiments. In Figure 2 we show the spin independent scattering cross section per nucleon on SM nuclei for \( \tau' \) DM as a function of \( m_{\tau'} \) and for values of \( f/v \) such that the correct DM abundance is obtained. For \( f/v \approx 3 - 5 \) (tuning 20 – 8%), the predicted direct detection signatures fall below current experimental bounds[31] but above the region of parameter space that will be probed in the very near future by LUX [31]. Larger (more tuned) values of \( f/v \) will be probed by next-generation experiments such as LUX-ZEPLIN (LZ) [32].

V. MULTICOMPONENT, \( W' \), & \( \Delta' \) DARK MATTER

In the case where the sum of \( \tau' \) and \( \nu' \) masses is larger than the \( W' \) mass, the latter is not able to decay. In the regime where \( m_{\tau'} \approx m_{\nu'} \) and \( m_{\nu'}, m_{\tau'} < m_{W'} \), this implies that all three states are stable and may significantly contribute to the DM energy density, opening a possibility for a 3-component DM scenario.

Figure 3 shows the contribution to the DM energy density of these three particle species (normalized to the observed value) for different values of the twin weak coupling, that we allow to vary by 10% from its central value \( g_2' = g_2 \approx 0.64 \). For concreteness, we have taken \( m_{\nu'} = m_{\tau'} \approx 0.55 m_{W'} \), with \( m_{W'} = g_2' f/v/2 \). As one can see, the observed DM energy density is only achieved for relatively large values of the ratio \( f/v \), where the fine-tuning is in the range 5% to 1%. This occurs since \( \tau' \) and \( \nu' \) are forced to be heavier than considered in the \( f/v \approx 3 \) case, so their annihilation cross sections set by \( m^2/f^4 \) are larger — to compensate, \( f \) must be increased.

As can be read off from Figure 3, the correct DM abundance is obtained for \( f/v \approx 9.7 \) for \( g_2' = g_2 \), which implies a tuning of approximately 2%. In this case, \( \tau' \) and \( \nu' \)

\[ \text{FIG. 1. Contribution to the energy density of the Universe from } \tau' \text{ species normalized to the observed DM energy density as a function of } m_{\tau'}. \text{ Light (dark) pink area indicates the 2-sigma bounds from invisible Higgs width and modified couplings to visible sector particles in the case } f/v = 3 (3.5), \text{ whereas } f/v = 4, 5 \text{ remain unconstrained in the region of parameter space shown. Note that, if } \Delta_{\text{QCD}} \text{ is large enough so that } m_\nu \gtrsim 2m_{\nu'}, \text{ then annihilations of low-mass } \tau' \text{ will have significant non-perturbative corrections. However, this regime generically leads gives too high a } \tau' \text{ density, so is not of primary concern here.} \]
species would contribute roughly 75% to the DM energy density, with $W'$ species making for the remaining 25%.

Regarding direct detection experiments, the predicted spin independent scattering cross section per nucleon for all three particle species is of order $\sim 10^{-46}\text{cm}^2$ for values of the masses that result in the correct DM abundance. This lies around an order of magnitude below LUX current projected sensitivity for the range of masses considered, which means that next-generation direct detection experiments such as LZ \cite{32} will be able to cover the relevant region of parameter space.

Small variations of the values of $\tau'$ and $\nu'$ masses around the case we have considered do not make a significant difference to our conclusions, except when $m_{\nu'} \sim m_{\nu}$, but $m_{\nu'} < m_{W'}$. In this case, the $W'$ is no longer stable and then only $\tau'$ and $\nu'$ species would contribute to the DM density. In this 2-component DM scenario, sufficient annihilation requires $m_{W'}$ and $m_{\nu'}$ to be in the mass range above $m_b/2$, automatically evading invisible Higgs width constraints. The different contribution to the DM density from the two particle species would depend solely on the ratio of their masses: for equal masses, both components would contribute 50%, whereas if they differ by approximately 10 GeV the right DM abundance requires $m_{\nu'} \approx 70\text{ GeV}$ (therefore $m_{W'} \approx 80\text{ GeV}$) and $\nu'$ and $\tau'$ species would make for 65% and 35% of DM respectively. Regarding direct detection signals, this 2-component scenario is analogous to the single-component case discussed in Section IV, for interactions between the DM species and the visible sector proceed only via Higgs exchange.

Finally we turn to the most complicated of the possible twin DM candidates, the $\Delta'$ baryon. Although for light $b'$ quarks, and in the absence of a matter-antimatter asymmetry (a subject of a companion paper \cite{7}), the spin-3/2 $\Delta'$ baryons efficiently annihilate to glueballs and quarkonia, leaving an interestingly small freeze-out density, this is no longer the case if the $b'$-quarks, and thus the $\Delta'$ baryons, are sufficiently heavy, $m_{b'} \gtrsim 1\text{ TeV} \gg \Lambda_{QCD}$. To estimate the freeze-out density of such states let us consider the case where the freeze-out temperature is well above $\Lambda_{QCD}$, a situation that applies if $m_{b'}$ is sufficiently large. In this case we may self-consistently work with $b'$ quarks and twin-gluons, and first calculate the freeze-out density of $b'$ quarks, via a leading annihilation cross section to two gluons that parametrically goes as $\sigma v \sim (\alpha_s/m_{b'})^2$. We find that a numerical evaluation of the annihilation rate leads to a freeze-out temperature $T_f \sim m_{b'}/30$ and gives a substantial freeze-out density of $b'$ quarks and anti-quarks only once $m_{b'} \gtrsim 1\text{ TeV}$, which implies very large $f/v \gtrsim 30$ and thus a very badly tuned TH model (here we have taken $y_{b'} \lesssim 0.2$ so as not to have yet further tuning at 1-loop). We therefore come to the conclusion that the $\Delta'$ baryon in the absence of a matter-antimatter asymmetry is a poor DM candidate in TH models. (We remark that the freeze-out density esti-
mated this way provides the most optimistic estimate of the minimum $b'$ mass. The reason is that the $b'$ quark and anti-quark densities do not simply translate, via a factor of $1/3$, into the final freeze-out density of $\Delta'$ baryons and anti-baryons. Post-twin-confinement only a proportion of $b'$-(anti-)quarks end up in $\Delta'$ (anti-)baryons, and thus are stable relics, compared to the number that form $b\overline{b}$ quarkonia and therefore quickly decay. We expect this proportion to be an $O(1)$ number but we are not aware of any reliable calculation of the ratio.)

VI. GLUEBALL METASTABILITY AND RELIC DENSITY

If the twin QCD sector had no couplings to other states, then after the phase transition, the glueball bath would behave roughly as non-relativistic strongly-self-interacting DM (since all of the glueballs have mass $\gtrsim 5.6 T^2$). That is, while number-changing (e.g. $3 \rightarrow 2$) interactions were still fast enough to maintain number equilibrium, its temperature would decrease only logarithmically with the scale factor \[^{33}\], with its energy density decreasing as $\rho \propto 1/(a^3 \log(a/a_0))$, where $a_0$ is a constant.

However, even in the absence of light twin sector states, the Higgs mixing portal with the SM necessarily provides a coupling to lighter states, and in particular, means that most of the glueballs decay rapidly to the SM. The question then becomes whether the relic population of (meta)stable glueballs is small enough not to be cosmologically dangerous.

To calculate the relic population of (meta)stable glueballs, we need to know the point at which their number-changing interactions freeze out. The last such interactions to freeze out will be processes with two particles in the initial state. Inspecting the glueball spectrum, if states have their equilibrium abundances then we expect the fastest such processes to be annihilation to lighter glueballs, e.g. $0^{-+}0^{-+} \rightarrow 0^{++}0^{++}$. We can therefore perform the usual calculation for the relic density of annihilating DM. If we parameterise the annihilation cross section as $\langle \sigma v \rangle \sim C/(\Lambda_{\text{QCD}}')^2$, then the present-day relic density is (assuming no significant entropy injection)

$$\Omega h^2 \simeq 3.8 \times 10^{-10}(\Lambda_{\text{QCD}}'/\text{GeV})^2 C^{-1},$$

in comparison to the present-day DM relic density $\Omega_{\text{DM}} h^2 \simeq 0.12$ \[^{34}\]. This assumed that the SM and glueball temperatures were equal at the freeze-out time. This will be the case when the decay rates of some of the glueballs are sufficiently fast—roughly, faster than the Hubble rate at freeze-out. If it is not the case, then as described above, the glueballs will be at a higher temperature than the SM, as their temperature will have fallen only logarithmically with the scale factor. Comparing the $0^{++} \rightarrow \text{SM}$ decay rate (from the mixing of Eq(1)) to the Hubble rate at freeze-out, the former is larger for $\Lambda_{\text{QCD}}' \gtrsim 0.6 \text{ GeV}$, so we are in the fast-decay regime for most of the $\Lambda_{\text{QCD}}'$ range of interest.

As $\Omega/\Omega_{\text{DM}} \simeq \text{few} \times 10^{-9}(\Lambda_{\text{QCD}}'/\text{GeV})^2 C^{-1}$, the relic density of stable glueballs will have no significant gravitational effects, and if the metastable glueballs decay well before recombination time ($\sim 10^{13}\text{s}$), they will not inject enough energy to observably disrupt BBN or the CMB spectrum \[^{35,37}\]. However, an energy injection of $\gtrsim 10^{-10}$ of the DM energy density can, depending on the injection time and channels, have observational consequences if it occurs around recombination time or later, either through CMB effects, or via cosmic ray observations \[^{37,38}\]. Thus, it may be a requirement that the meta-stable glueballs have lifetimes shorter than $\sim 10^{13}\text{s}$.

As an aside, note that the situation is different if $m_{\nu}$ is light enough that the lightest twin QCD states are mesons rather than glueballs. The lightest meson state is a $0^{-+}$, so in the absence of lighter twin sector states, will decay through higher dimensional operators, as per the $0^{-+}$ glueball discussed below. The difference is that its annihilations must now produce SM final states, rather than purely twin-QCD states, so will be much too suppressed to give it a sub-DM abundance. Number-changing interactions (e.g. $3 \rightarrow 2$ processes) will reduce its density, since other twin QCD states can decay to the SM, but since we are working in the regime of meson masses significantly larger than 1 GeV, the number-changing interactions generally freeze out before the abundance can be reduced to sub-DM levels \[^{39}\]. Thus, it appears that, in the absence of twin sector CP violation and lighter twin sector states, the lightest $b'$ meson must decay before BBN time in order to avoid dangerous energy injection. Since there are dimension-6 operators that could lead to this decay, this condition is in principle easily satisfied.

We now turn to the question of the lifetimes of the metastable $0^{-+}$ and $1^{+-}$ glueballs mentioned in Section II. Let us first consider the case where there exist new SM–twin-sector interactions in the UV completion of the TH theory that allow the $0^{-+}$ and $1^{+-}$ glueballs to decay. For the $0^{-+}$ glueball the lowest dimension effective operators conserving total CP that allow the glueball to directly decay to the SM sector are of dimension 7, e.g. operators of the schematic form $\bar{\psi} \gamma_5 q \times \text{tr}(G' G')/M^3$ (here $G'$ are the $SU(3)'$ field strengths, while $q$ stands in for SM fermions). These lead to a lifetime, $\tau_{0^{-+}} \sim 10^{-12}\text{s} (M/5\text{TeV})^6(3\text{GeV}/\Lambda_{\text{QCD}}')^7$, irrelevantly short for astrophysical purposes (though interesting for displaced vertices at the LHC) unless the scale suppressing the operator is raised to $M \gtrsim 500\text{TeV}$. On the other hand, the leading effective operators for the $1^{+-}$ glueball are of dimension 10, e.g. operators of the schematic form $\bar{\psi} \gamma_\mu \gamma_5 q \times \partial^\mu G' G' /M^6$. These lead to a lifetime $\tau_{1^{+-}} \sim 10\text{s} (M/5\text{TeV})^2(3\text{GeV}/\Lambda_{\text{QCD}}')^{13}$, which is of interest for indirect detection signals as discussed in Section VII.

On the other hand, if only those interactions present...
in the IR effective theory are considered then decays of the $1^{++}$ glueball compatible with conservation of both angular momentum and $CP$ involve combinations of on- and off-shell glueballs, which then decay to the SM via the Higgs portal. For example, $1^{++}$ can decay to $0^{++}$ and an off-shell $0^{++} / h$, in a $l = 1$ state. Since both $C$ and $P$ are violated in this process, and the only interactions in the twin sector that violate both $C$ and $P$ are $SU(2)'$, the decay of the $1^{++}$ glueball needs to proceed through twin weak interactions involving both axial and vector currents. We can estimate its decay rate taking into account the leading heavy quark corrections to the vector and axial currents, and assuming that the decay to SM final states is via off-shell $0^{++} / h$ mixing. Using the (dominant) one-loop $b'$ quark correction to the vector and axial $SU(2)'$ currents \cite{40,41}:

$$
\delta J_{\mu}^V = \frac{g_{\tau}}{16\pi^2 m_{b'}^4} \partial_\alpha \left[ \frac{1}{4} G_{\sigma\tau} \left\{ G^{\sigma\tau}, G_{\alpha\mu} \right\} - \frac{14}{49} G_{\mu\sigma} \left\{ G^{\sigma\tau}, G_{\tau\alpha} \right\} \right]
$$

$$
\delta J_{\mu}^A = \frac{g_{\tau}}{48\pi^2 m_{b'}^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr}(G^{\rho\sigma} \partial_\alpha G^{\sigma\tau} + 2 G^{\tau\alpha} \partial_\sigma G^{\rho\sigma}),
$$

the decay rate of the $1^{++}$ glueball is estimated to be (ignoring dimensionless numbers and assuming $m_{b'} \gg m_{0}^*$),

$$
\Gamma_{1^{++}} \propto \frac{\nu^2 \Gamma_h(m_{0}^*)}{m_{b'}^2 m_{b}^4 m_{h}^4} m_{0}^{22} \tag{9}
$$

and since $\Gamma_h(m_{0}^*) \sim m_{0}^* \sim m_{0}$ we see that $\Gamma_{1^{++}} \sim m_{0}^{23}$. Thus the decay rate of the $1^{++}$ glueball depends very strongly on the mass of the glueballs, or equivalently on the twin confinement scale. For example, for $f/v = 3$,

$$
\tau_{1^{++}} \sim \left( \frac{m_{b'}}{15 \text{GeV}} \right)^{12} \times \left\{ \begin{array}{l}
10^9 \text{ s for } \Lambda_{QCD}' \approx 1.5 \text{ GeV} \\
10^2 \text{ s for } \Lambda_{QCD}' \approx 3 \text{ GeV}
\end{array} \right. \tag{10}
$$

i.e. changing $\Lambda_{QCD}'$ by a factor of 2 leads to a 7-order of magnitude difference in the lifetime of the $1^{++}$ glueball! All that can be said is that, for reasonable values of the parameters of the model, it appears that the $1^{++}$ glueball may be a long lived state. Since it ultimately decays to SM states, even a tiny freeze-out density of $1^{++}$ glueballs may be dangerous, specifically if the $1^{++}$ lifetime is close to or after recombination time. We therefore from now on assume that, in the case where effects from higher dimensional operators can be neglected, $\Lambda_{QCD}'$ and $m_{0}$ are such that $\tau_{1^{++}} \ll t_{\text{CMB}} \simeq 10^{13}$ s, preferring larger values of $\Lambda_{QCD}'$ or smaller values of $m_{0}$.

We finish by emphasising that the stability or duration of metastability of the $0^{+}$ and $1^{++}$ glueballs depends sensitively on whether the twin neutrinos are heavy or not, the mass scales $m_{\nu'}$ and $\Lambda_{QCD}'$, whether twin $CP$ is conserved, and finally the nature of the interactions in the UV completion between the twin and SM sectors in addition to the Higgs portal that might allow the glueballs to more quickly decay.

\section{VII. INDIRECT DETECTION}

As well as leading to DM-SM scatterings in direct detection experiments, the Higgs mixing portal between the twin and SM sectors may result in SM energy injections from astrophysical DM-DM annihilations, leading to potentially observable indirect detection signals. For the twin DM candidates considered in this paper, annihilations proceed to lighter twin sector states. For $\tau'$ and $\nu'$ DM, the dominant annihilation channel is to a $b'b'$ pair via $s$-channel $Z'$ exchange, due to the colour factor enhancement of the final states available. Since $SU(3)'$ is confining, the $b'b'$ state will fragment into some number of twin glueballs and mesons — in the heavy-quark limit, we would expect dominantly glueballs.

As previously discussed, if the $\nu'$ are light, then most of the glueballs will decay down to the lightest $0^{++}$ state by $b'b'$ emission, with the $0^{++}$ state then decaying via mixing with the SM Higgs. If there are no light hidden sector states, then some of the glueballs may be metastable, while others will decay via the SM Higgs. In either case, we expect some combination of invisible decay products — $\nu'$ or stable glueballs — and of off-shell Higgs particles, $h^*$. The latter will have some distribution of mass squared values determined by the glueball masses and mass splittings.

For $\Lambda_{QCD}'$ greater than a few GeV, most of these $h^*$ should be above the $b\bar{b}$ threshold. In that case, the SM injection products will, to a good approximation, be a spectrum of $b\bar{b}$ with energies determined by the fragmentation process. For DM of mass $m_{\text{DM}} \gtrsim 63 \text{ GeV}$ as considered here, the most sensitive probes of such astrophysical energy injection are cosmic ray antiprotons and gamma rays. The most up-to-date constraints on antiproton injection from dark matter annihilations come from the AMS-02 experiment \cite{42,43}. This sees an antiproton spectrum which is higher, at energies $\gtrsim 20 \text{ GeV}$, than expected in some models of astrophysical secondary production and propagation. If one takes these observations as disfavouring those models, and places constraints on the DM antiproton contribution assuming the secondary production models that better fit the AMS-02 data, one obtains constraints such as those of \cite{43}, which finds that DM annihilating to $b\bar{b}$ with a thermal freeze-out cross section is disfavoured below $m_{\text{DM}} \approx 100 \text{ GeV}$. If instead one includes the lower-secondary-production models in one’s uncertainty, the upper bound on the DM cross-section increases by almost an order of magnitude, and DM annihilating to $b\bar{b}$ is still viable in the entire mass range (though at the lower end, necessarily contributing significantly to the observed AMS-02 spectrum).

The best gamma-ray constraints on DM annihilation to hadrons, in this mass range, come from the FERMI
LAT instrument. In particular, a recent FERMI analysis of dwarf spheroidal galaxies [44] places strong constraints on DM annihilating to $\bar{b}b$ with a thermal freeze-out cross section, disfavouring NFW-profile DM in these galaxies with $m_{\text{DM}} < 100\, \text{GeV}$. Taking into account uncertainties as to the DM profile in these galaxies, and/or using the limits from the better-constrained diffuse Milky Way halo [45], relaxes these constraints, disfavouring $m_{\text{DM}} \lesssim 20\, \text{GeV}$.

In our case, the uncertainty in the spectrum of $h^*$ energies, and so $\bar{b}b$ energies produced, as well as the uncertainty regarding the invisible decay fraction, mean that we cannot make precise predictions for comparison with published limits. However, both the softening of the $\bar{b}b$ injection spectrum, and the invisible decay fraction, will act to decrease the strength of these constraints relative to straightforward $\bar{b}b$ injection, and in combination with the astrophysical uncertainties, this means that these models are not ruled out by current data. However, especially at low tuning, and so low $f/\nu$, they predict indirect detection signals potentially within the range of future observations.

Particularly interesting from the point of view of future measurements are the tentative hints of a GeV-scale gamma-ray excess from the Galactic Centre (GC) [46, 47] in FERMI data, which have now been corroborated by the FERMI collaboration itself [48]. This excess has a spectral and spatial profile compatible with $\sim 40\, \text{GeV}$ DM annihilating to $\bar{b}b$, with approximately thermal freeze-out cross section (or with $\sim 10\, \text{GeV}$ DM annihilating to leptons). While our DM masses are somewhat higher, the softer spectrum expected may plausibly give a good spectral fit. The higher mass and invisible decay fraction of our model would have some difficulty replicating the approximately-thermal annihilation cross section derived in current analyses, but systematic astrophysical uncertainties are such that this may decrease with future observations. Alternatively, if $\Lambda^2_{\text{QCD}}$ is low enough that a substantial fraction of the $h^*$ produced have mass-squared below the $\bar{b}b$ threshold, then injection to $\tau^+\tau^-$ and to lighter quarks may be important, likely lowering the required annihilation cross section (e.g. [49]). The potential compatibility with this existing signal provides additional motivation for better understanding the twin QCD fragmentation process.

Another interesting possibility is that the fragmentation process produces an appreciable number of metastable glueballs, which may have a long enough lifetime to travel for astrophysically-relevant distances before decaying. For this effect to be significant in galactic DM observations, the lifetime of the glueballs would need to be on kiloparsec ($\sim 10^{11}\, \text{s}$) scales (just shorter than the lifetimes constrained by the CMB power spectrum).

A different and more striking signal may occur for smaller lifetimes — if the lifetime is $\gtrsim$ the radius of the Sun, $R_\odot \simeq 2\, \text{s}$, but not much greater than the Earth-Sun distance, then metastable glueballs produced by annihilations of captured DM in the Sun can escape before decaying, and their decay products could be detected in charged cosmic rays or gamma rays [50, 53]. The solar capture rate of DM scattering spin-independently (and momentum-independently) from SM nuclei is, taking a fiducial mass of $m_{\text{DM}} = 100\, \text{GeV}$, $\Gamma_\odot \simeq 10^{20}\, \text{s}^{-1} \frac{m_{\text{DM}}^2}{10^{-8}\, \text{cm}^2}$ [54] — once enough DM has accumulated in the Sun, an equilibrium will be reached in which the annihilation rate is half of the capture rate, $\Gamma_{\text{ann}} = \frac{1}{2}\Gamma_\odot$. Since we have a precise prediction for the scattering rate off SM nuclei, for given DM mass, this determines the annihilation rate in the Sun (for annihilation cross sections of the magnitude we are considering, the equilibrium rate is reached in much shorter than the lifetime of the Sun). Ref. [51] considers the decay of metastable DM annihilation products into leptonic final states, finding that FERMI gamma-ray observations place an upper bound of $\Gamma_{\text{ann}} f_{\text{dec}} \lesssim 5 \times 10^{19}\, \text{s}^{-1}$, where $f_{\text{dec}} = e^{-\frac{R_\odot}{\gamma\tau}} - e^{-\frac{AU}{\gamma\tau}}$ is the fraction of events, for a mediator of proper lifetime $\tau$ with boost factor $\gamma$, in which the decay occurs between the Sun and the Earth. This is in the scenario where DM of mass $100\, \text{GeV}$ annihilates inside the Sun to two mediator particles, each of which then decay to two leptons. In our case, only some fraction of the DM annihilation energy will go to metastable glueballs, and the gamma-ray spectrum produced by each of these will generally be softer, so we would expect the constraints to be less severe. However, for lifetimes $\sim 1\, \text{s}$, but less than a few times $10^3\, \text{s}$, there is some possibility that future observations could place limits on this scenario, or see a signal. Additionally, in the same way that constraints on an excess of high-energy electron/positrons from the Sun have been used to set limits on scenarios with leptonic decays of the mediators [53, 55, 56], a similar analysis applied to antiprotons may provide stronger constraints on the Higgsportal models considered here.

VIII. EQUILIBRATION OF SECTORS

If the $\nu'$ are light, then we do not expect any of the glueballs to have cosmologically relevant lifetimes. Also, for most glueballs the decays involving $\nu'$ will occur at a faster rate than decays involving the SM. Since $\nu'$ interactions with SM states are very suppressed, this raises the possibility that most of the entropy in the twin QCD sector may be transferred to the $\nu'$, and remain there decoupled from the SM. As discussed below, if the $\nu'$ are light, then they must be effectively massless as regards early-universe cosmology. Still, they could potentially give a large DR contribution, which would be in conflict with cosmological constraints on the extra number $\Delta N_{\text{eff}}$ of effective neutrino species.

The minimum $\nu'$ energy density we might get would arise from the $\nu'$ being in equilibrium with the SM until after the twin QCD phase transition. In this case, since there were $g_\nu \simeq 75$ effective relativistic degrees of freedom in the SM bath, and SM neutrinos decou-
ple from the SM bath when $g_{SM} = 10.75$, we have $T_{\nu} \simeq \left( \frac{75}{100} \right)^{1/3} T_{\nu}^* = 1.9 T_{\nu}^*$ at late times, assuming that the $\nu'$ are light enough to behave as DR. Writing the energy density in DR at around CMB times as $\rho_{DR} \equiv (3 + \Delta N_{eff})\rho_{\nu,SM}$ (where $\rho_{\nu,SM} \equiv \frac{1}{2} \left( \frac{11}{4} \right)^{4/3} \rho_N$ is the naive SM neutrino energy density), the $\nu'$ give a contribution to the effective number of neutrino species of $\Delta N_{eff} \simeq (1.9)^{-4} = 0.075$. If $\nu'$ are also light, the contribution is doubled. Non-instantaneous decoupling of the SM neutrinos gives a contribution of $\Delta N_{eff} = 0.046$ from the SM alone. Planck has measured $\Delta N_{eff} = 0.15 \pm 0.2$ [34] (the $\pm 0.2$ being a simplified summary of a rather complex set of constraints: see [34] for details), so such a twin sector contribution is entirely compatible with present-day constraints. However, future observations, including improved astrophysical determinations of the Hubble constant [34], large-scale structure surveys [57], and a future cosmic-variance CMB polarisation mission [58], may be able to measure $N_{eff}$ to an accuracy of $\sigma(N_{eff}) \sim 0.05$, potentially testing the presence of a twin $\nu'$ bath.

Lowering the temperature at which the phase transition occurs will lower the number of effective degrees of freedom in the SM bath, so will raise the contribution to $\Delta N_{eff}$, up to $\sim 0.1$ for temperatures just above the SM QCD phase transition. Below the SM QCD phase transition, the minimal contribution becomes $\sim 0.5$, which is observationally marginal. However, we will see below that we need to take $\Lambda_{QCD} \sim 2.5$ GeV for sufficiently fast equilibration in any case, which is well above $\Lambda_{QCD}$.

If the $\nu'$ are too heavy to behave as DR until CMB times, they will constitute an extra (possibly hot) DM abundance. For $m_{\nu'}$ greater than a few keV, and less than weak scale values, the $\nu'$ will constitute a warm/cold DM abundance with far too high a relic density. If $m_{\nu'}$ is smaller than that, but still greater than $\mathcal{O}(10\,\text{eV})$, then they will constitute too large a warm/hot DM abundance (e.g. [59]). So, if $m_{\nu'} < m_h/2$, it is required to be less than a few eV, the same applying to $\nu''$.

Thus, if the $\nu'$ are light, then they are certainly much lighter than the glueball mass splittings. In this case, as mentioned above, all of the glueballs are unstable, either against decays involving $\nu'$ or $h^*$. The question of interest then becomes how much energy density from the twin QCD bath ends up in the eventually decoupled $\nu'$ and SM baths, and whether the $\nu'$ represent an observationally significant DR abundance.

In the event that all of the entropy density of the twin QCD bath ends up in the $\nu'$, we can estimate the resulting contribution to $\Delta N_{eff}$ as above. If $m_{\nu'} \gtrsim 5$ GeV, then during the first stages of its Boltzmann suppression, the $b'$ coupling to the SM is large enough to equilibrate the SM and twin gluon sectors. Thus, the scenario resulting in maximum $\nu'$ energy density is that in which the SM and twin QCD sectors decouple shortly after the $b'$ go non-relativistic. In this case, the entropy density of the gluon bath, corresponding to $\sim 16$ effective relativistic dof, stays within the twin sector. Since all of this gets transferred eventually to the $\nu'$, then if (as expected) they have a quasi-thermal spectrum, they will have a temperature $\sim \left( (16 + 2 \times \frac{3}{4})/(2 \times \frac{3}{4}) \right)^{1/3} \simeq 2.3$ times the $a^{-3}$ scaled SM temperature before the phase transition. So, they contribute $\Delta N_{eff} \simeq (2.3/1.9)^4 \simeq 2.0$.

A $\Delta N_{eff}$ contribution this large is incompatible with the existing cosmological observations. In fact, even if only the entropy of the twin QCD bath just about the critical temperature is transferred to the $\nu'$, then the resulting $\Delta N_{eff} \simeq 0.4$ is in tension with observations. It is therefore a requirement that most of the entropy in the unconfined phase of the QCD bath is transferred to the SM, rather than to the $\nu'$. Logically, this could occur in a number of ways:

- If twin QCD interactions with the $\nu'$ and SM are too slow to transfer significant entropy to either of them in the first Hubble time, then the only process that can become important is gluon/meson decays. Hence, the energy transferred to the $\nu'$ and SM is set by the overall branching ratio of these decays. However, in our scenarios, $SU(2)'$ mediated processes are generally faster than Higgs portal ones, and we expect the branching ratio to favour $\nu'$ over SM, so this does not solve the problem.

- If twin QCD interactions with both the $\nu'$ and SM are fast enough, then $\nu' \rightarrow QCD' \leftrightarrow S M$ interactions may be fast enough to bring the $\nu'$ and SM sectors (almost) to equilibrium, and so to (almost) restore the $\Delta N_{eff} = 0.075$ situation derived at the beginning of this section. (Note that direct interactions between $\nu'$ and SM states are extremely suppressed, since diagrams must involve $SU(2)'$ gauge bosons and multiple Higgs's. The $\nu'$ may have a small interaction with the twin Higgs, giving them a $\lesssim \text{eV}$ mass, but this coupling is again far too small to bring about equilibration.)

It is not trivial for interactions during the confined phase to be fast enough, since the glueball bath is non-relativistic, and its equilibrium number density falls very fast as the temperature drops. Energy transfer from the glueball bath to the SM needs to be fast enough that it is still operating until the SM and $\nu'$ have reached almost the same temperature.

In the confined phase, the fastest processes transferring energy $QCD' \rightarrow S M$ are glueball and meson decays, for example, the $0^{++} \rightarrow h^* \rightarrow S M$ decay discussed previously, and possibly SM-QCD' scatterings. Parameterising the effective SM decay rate of the glueball bath as $\rho_G = -\Gamma_{G ightarrow SM \rho_G} + \ldots$, and solving the Boltzmann equations numerically shows that $\Gamma_{G ightarrow SM} \gtrsim 10H_c$ is required to significantly reduce the $\nu'$ abundance, where $H_c$ is the Hubble rate at the start of the phase transition – see Figure 4. Since $H \propto T^2$, and the decay rates go as higher powers of $\Lambda_{QCD}$ (e.g. $\Gamma_{G ightarrow \nu'} \propto \Lambda_{QCD}^7$), increasing $\Lambda_{QCD}$, and so $T_c$, makes it easier to fulfil this condition, which
is satisfied for $\Lambda_{\text{QCD}}' \gtrsim 2.5\text{ GeV}$ for our fiducial parameters. Since the decay and scattering rates of most of the glueball and meson states have not been calculated, the precise bound is uncertain. However, because these rates increase as much higher powers of the mass scale than the Hubble rate does, such uncertainty in the rates at a given energy has a small effect on the $\Lambda_{\text{QCD}}'$ bound.

The most natural DM candidate is the twin tau lepton $\tau'$, which requires masses $m_{\tau'} > m_{\nu}/2$ in order to provide the observed DM abundance. In particular, we find $m_{\nu'} \approx 63 - 130\text{ GeV}$ for ratios $f/v = 3 - 5$, which implies a very mild $20 - 8\%$ tuning. DM scattering off SM nuclei happens via Higgs exchange, and direct detection signatures lie below LUX current bounds \[30\] but within LUX future sensitivity \[31\]. A very natural possibility within the Fraternal Twin Higgs scenario is that of multicomponent DM. In particular, DM made of both $\tau'$ and $\nu'$ species arises naturally as soon as the masses of both states are of similar size, though the phenomenology of this scenario is very similar to that of single-$\tau'$ DM, modulo the lack of any $\Delta N_{\text{eff}}$ contribution. In the case where $m_{\nu'} + m_{\nu'} > m_{W'}$, $W'^\pm$ gauge bosons become stable and may significantly contribute to the DM density, leading to a scenario where DM is made of three different species of twin particles ($\tau'$, $\nu'$ and $W'^\pm$). We find, however, that this three-component scenario requires large values of $f/v$, that result in fine-tuning between $5 - 1\%$, substantially worse than the single- or two-component cases. We briefly mention the possibility of twin baryon DM, where the DM particle would be a $\Delta'$ baryon made of three $b'$ quarks. The efficient annihilation of $\bar{b}b'$ pairs via twin strong interactions requires extremely large quark masses if the observed DM abundance is to be reproduced. In particular, we find that masses $m_{\nu'} \gtrsim 1\text{ TeV}$ are required, which translates into $f/v \gtrsim 30$ for $y_\tau' \approx 0.2$, and therefore a fine-tuning worse than $0.5\%$.

Regarding possible indirect detection signatures, annihilation of DM particles happens mainly to $\bar{b}b'$ pairs, which mostly results in glueball final states as a result of the fragmentation process in the twin QCD sector. Glueballs will decay both into SM final states and into invisible twin sector states (either to $\bar{\nu}'\nu'$ in the case of light $\nu'$, or to (meta)stable twin glueballs). The former will consist mostly of $\bar{b}b$ pairs, giving broadly WIMP-like indirect detection phenomenology, though with a presently non-calculable injection spectrum. The masses of our DM candidates are large enough that there is no inconsistency with current data, though future observations should probe relevant regions of parameter space. Decays of metastable glueballs may also have cosmological / astrophysical consequences, though the strong dependence of these lifetimes on $\Lambda_{\text{QCD}}'$, and on the UV physics, means that precise predictions are not possible.

In the case where $m_{\nu'}$ is small, a non-zero contribution to the number of effective neutrinos, $\Delta N_{\text{eff}}$, is a prediction of the theory. This contribution depends on the value of the twin confinement scale and we find that for $\Lambda_{\text{QCD}}' \gtrsim 2.5\text{ GeV}$, the $\nu'$ bath remains in equilibrium with the SM bath after the twin QCD phase transition and $\Delta N_{\text{eff}} - \Delta N_{\text{eff,SM}} \approx 0.075$, a value compatible with current bounds but within reach of future measurements. If the $\nu'$ and SM baths fell out of equilibrium before the twin QCD transition, all the energy in twin glueballs is damped into the $\nu'$ bath, in which case we would find $\Delta N_{\text{eff}} - \Delta N_{\text{eff,SM}} \approx 2$, strongly disfavoured.
by current bounds. In the case of heavy twin neutrinos \((m_{\nu'} > m_h/2)\), there are no light twin sector states, so no contribution to \(\Delta N_{\text{eff}}\).

We remark that, independent of twin sector DM opportunities, our analysis highlights cosmological and astrophysical constraints on TH related models. The \(\Delta N_{\text{eff}}\) bounds are independent of whether there is a twin sector DM abundance, while the relic density calculations of Sections IV and V demonstrate that, for some mass parameters, the minimal TH model naively produces a super-DM abundance of stable states, which would need to be reduced by introducing new decay operators or hidden sector states.

Finally, we note that similar investigations of DM in Twin Higgs models have been carried out by other groups \([60\) and \([61]\).

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[1] Z. Chacko, H.-S. Goh, and R. Harnik, Phys.Rev.Lett. 96, 231802 (2006), hep-ph/0506256.
[2] Z. Chacko, H.-S. Goh, and R. Harnik, JHEP 0601, 108 (2006), hep-ph/0512088.
[3] Z. Chacko, Y. Nomura, M. Papucci, and G. Perez, JHEP 0601, 126 (2006), hep-ph/0510273.
[4] R. Barbieri, T. Gregoire, and L. J. Hall (2005), hep-ph/0509242.
[5] N. Craig and K. Howe, JHEP 1403, 140 (2014), 1312.1341.
[6] N. Craig, A. Katz, M. Strassler, and R. Sundrum (2015), 1501.05310.
[7] I. García García, R. Lasenby, and J. March-Russell, Phys. Rev. Lett. 115, 121801 (2015), 1505.07410.
[8] D. Curtin and C. B. Verhaaren (2015), 1506.06141.
[9] S. Blinnikov and M. Y. Khlopov, Sov.J.Nucl.Phys. 36, 472 (1982).
[10] E. D. Carlson and S. Glashow, Phys.Lett. B193, 168 (1987).
[11] R. Foot, H. Lew, and R. Volkas, Phys.Lett. B272, 67 (1991).
[12] L. Okun, Phys.Usp. 50, 380 (2007), hep-ph/0606202.
[13] R. Foot, Int.J.Mod.Phys. A29, 1430013 (2014), 1401.3965.
[14] D. Poland and J. Thaler, JHEP 0811, 083 (2008), 0808.1290.
[15] S. El Hedri and A. Hook, JHEP 1310, 105 (2013), 1305.6608.
[16] C. J. Morningstar and M. J. Peardon, Phys.Rev. D60, 034509 (1999), hep-lat/9901004.
[17] Y. Chen, A. Alexandru, S. Dong, T. Draper, I. Horvath, et al., Phys.Rev. D73, 014516 (2006), hep-lat/0510074.
[18] R. D. Pisarski and F. Wilczek, Phys.Rev. D29, 338 (1984).
[19] C. Alexandrou, A. Borici, A. Feo, P. de Forcrand, A. Galli, et al., Phys.Rev. D60, 034504 (1999), hep-lat/9811028.
[20] M. Fromm, J. Langelage, S. Lottini, and O. Philipsen, JHEP 1201, 042 (2012), 1111.4953.
[21] L. Yaffe and B. Svetitsky, Phys.Rev. D26, 963 (1982).
[22] M. Panero, Phys.Rev.Lett. 103, 232001 (2009), 0907.3719.
[23] S. Borsanyi, G. Endrodi, Z. Fodor, S. Katz, and K. Szabo, JHEP 1207, 056 (2012), 1204.6184.
[24] B. Beinlich, F. Karsch, and A. Peikert, Phys.Lett. B390, 268 (1997), hep-lat/9608141.
[25] L. P. Csernai and J. I. Kapusta, Phys.Rev. D46, 1379 (1992).
[26] P. Schwaller (2015), 1504.07263.
[27] M. M. Anber, Phys.Rev. D88, 085003 (2013), 1302.2641.
[28] M. D’Elia and F. Negro, Phys.Rev.Lett. 109, 072001 (2012), 1205.0538.
[29] G. Belanger, F. Boujdema, A. Pukhov, and A. Semenov, Comput.Phys.Commun. 185, 960 (2014), 1305.0237.
[30] D. Akerib et al. (LUX), Phys.Rev.Lett. 112, 091303 (2014), 1310.8214.
[31] M. Horn et al. (LUX), Nucl.Instrum.Meth. A784, 504 (2015).
[32] D. Malling, D. Akerib, H. Araujo, X. Bai, S. Bedikian, et al. (2011), 1110.0103.
[33] E. D. Carlson, M. E. Machacek, and L. J. Hall, Astrophys.J. 398, 43 (1992).
[34] P. Ade et al. (Planck) (2015), 1502.01589.
[35] K. Jedamzik, Phys.Rev. D74, 103509 (2006), hep-ph/0604251.
[36] W. Hu and J. Silk, Phys.Rev.Lett. 70, 2661 (1993).
[37] T. R. Slatyer, Phys.Rev. D87, 123513 (2013), 1211.0283.
[38] L. Dugger, T. E. Jeltema, and S. Profumo, JCAP 1012, 015 (2010), 1009.5988.
[39] Y. Hochberg, E. Kuflik, T. Volansky, and J. G. Wacker, Phys.Rev.Lett. 113, 171301 (2014), 1402.5143.
[40] D. B. Kaplan and A. Manohar, Nucl.Phys. B310, 527 (1988).
[41] J. E. Juknevich, JHEP 1008, 121 (2010), 0911.5616.
[42] AMS-02 Collaboration, Phys.Rev.Lett. 113, 171301 (2014), 1402.5143.
[43] K. N. Abazajian, N. Calm, S. Horiuchi, and M. Kapling-
hat, Phys.Rev. \textbf{D90}, 023526 (2014), 1402.4090.

[48] S. Murgia, \textit{presented at LLNL Lattice BSM workshop, 24 Apr. 2015}, URL \url{https://lattice.llnl.gov/meetings/2015/beyond-standard-model-physics/presentations/2015-04-24/1600_MURGIA.pdf}.

[49] J. M. Cline, G. Dupuis, Z. Liu, and W. Xue, Phys. Rev. \textbf{D91}, 115010 (2015), 1503.08213.

[50] B. Batell, M. Pospelov, A. Ritz, and Y. Shang, Phys.Rev. \textbf{D81}, 075004 (2010), 0910.1567.

[51] P. Schuster, N. Toro, and I. Yavin, Phys.Rev. \textbf{D81}, 016002 (2010), 0910.1602.

[52] P. Schuster, N. Toro, N. Weiner, and I. Yavin, Phys.Rev. \textbf{D82}, 115012 (2010), 0910.1839.

[53] M. Ajello, W. Atwood, L. Baldini, G. Barbiellini, D. Bastieri, et al., Phys.Rev. \textbf{D84}, 032007 (2011), 1107.4272.

[54] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke, et al., JCAP \textbf{0407}, 008 (2004), astro-ph/0406204.

[55] J. Casaus (AMS), J.Phys.Conf.Ser. \textbf{531}, 012007 (2014).

[56] V. Mikhailov, O. Adriani, G. Barbarino, G. Bazilevskaya, R. Bellotti, et al., Bull.Russ.Acad.Sci.Phys. \textbf{79}, 298 (2015).

[57] S. Hannestad, J. Hamann, and Y. Y. Wong, J.Phys.Conf.Ser. \textbf{485}, 012008 (2014).

[58] S. Bashinsky and U. Seljak, Phys.Rev. \textbf{D69}, 083002 (2004), astro-ph/0310198.

[59] M. Viel, G. D. Becker, J. S. Bolton, M. G. Haehnelt, M. Rauch, et al., Phys.Rev.Lett. \textbf{100}, 041304 (2008), 0709.0131.

[60] N. Craig and A. Katz (2015), 1505.07113.

[61] M. Farina (2015), 1506.03520.