Magnetotransport in 2D lateral superlattices with smooth disorder: Quasiclassical theory of commensurability oscillations

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Commensurability oscillations in the magnetoresistivity of a two-dimensional electron gas in a two-dimensional lateral superlattice are studied in the framework of quasiclassical transport theory. It is assumed that the impurity scattering is of small-angle nature characteristic for currently fabricated high-mobility heterostructures. The shape of the modulation-induced magnetoresistivity $\Delta \rho_{xx}$ depends on the value of the parameter $\gamma \equiv \eta^2 q l / 4$, where $\eta$ and $q$ are the strength and the wave vector of the modulation, and $l$ is the transport mean free path. For $\gamma \ll 1$, the oscillations are described, in the regime of not too strong magnetic fields $B$, by perturbation theory in $\eta$ as applied to the case of one-dimensional modulation. At stronger fields, where $\Delta \rho_{xx}$ becomes much larger than the Drude resistivity, the transport takes the advection-diffusion form (Rayleigh-Bénard convection cell) with a large Péclet number, implying a much slower ($\propto B^{3/2}$) increase of the oscillation amplitude with $B$. If $\gamma \gg 1$, the transport at low $B$ is dominated by the modulation-induced chaos (rather than by disorder). The magnetoresistivity drops exponentially and the commensurability oscillations start to develop at the magnetic fields where the motion takes the form of the adiabatic drift. Conditions of applicability, the role of the type of disorder, and the feasibility of experimental observation are discussed.

I. INTRODUCTION

Electronic transport in semiconductor nanostructures is one of central issues of research in modern condensed matter physics, see e.g. \cite{B92} for reviews. In particular, transport properties of a two-dimensional electron gas (2DEG) subject to a periodic potential (lateral superlattice) with a period much shorter than the electron transport mean free path (but much larger than the Fermi wave length) have been intensively studied during the last decade. In a pioneering experiment [1], Weiss et al. discovered that a weak one-dimensional (1D) modulation with wave vector $q \parallel e_x$ induces strong commensurability oscillations of the magnetoresistivity $\rho_{xx}(B)$ (while showing almost no effect on $\rho_{yy}(B)$ and $\rho_{xy}(B)$), with the minima satisfying the condition $2 R_c / a = n - 1 / 4$, $n = 1, 2, \ldots$, where $R_c$ is the cyclotron radius and $a = 2 \pi / q$ the modulation wave length. The quasiclassical nature of these commensurability oscillations was demonstrated by Beenakker\cite{B93}, who showed that the interplay of the cyclotron motion and the superlattice potential induces a drift of the guiding center along $y$ axis, with an amplitude squared oscillating as $\cos^2(q R_c - \pi / 4)$ (this is also reproduced by a quantum-mechanical calculation, see\cite{C95}). While describing nicely the period and the phase of the experimentally observed oscillations, the result of\cite{B93} however, failed to explain the observed rapid decay of the oscillation amplitude with decreasing magnetic field. The cause for this discrepancy was in the treatment of disorder: while Ref.\cite{B93} assumed isotropic impurity scattering, in experimentally relevant high-mobility semiconductor heterostructures the random potential is very smooth and induces predominantly small-angle scattering, with the total relaxation rate $\tau^{-1}$ much exceeding the momentum relaxation rate $\tau^{-1}$. The theory of commensurability oscillations in the situation of smooth disorder was worked out by two of us in\cite{MW95}. It was found that the small-angle scattering drastically modifies the dependence of the oscillation amplitude on the magnetic field $B$, leading to a much stronger damping of the oscillations with decreasing $B$, in full agreement with experimental data (see also Monte-Carlo simulations\cite{M95} and numerical solution of the Boltzmann equation in\cite{B98}). The small-angle nature of the scattering plays also an important role in the theoretical description of the low-field magnetoresistance dominated by channeled orbits\cite{B98}. Summarizing, one can say that the theory of magnetoresistivity in 1D lateral superlattices is by now fairly well understood and provides a quantitative description of the experiment.

In contrast, the situation with two-dimensional (2D) superlattices is by far less clear. Experimental studies of transport in samples with weak 2D modulation (which can be obtained by superimposing two 1D modulations with equal amplitudes and orthogonal wave vectors)\cite{B95} have demonstrated that the commensurability oscillations can be observed in this case as well, but their amplitude is to some extent suppressed as compared to the case of 1D modulation. However, no clear answer concerning the conditions for this suppression, as well as its magnitude, can be drawn from the literature.

The present state of the theory is equally controversial. It was stated\cite{M95} that within the quasiclassical approach a contribution to the resistivity tensor induced by a 2D modulation,

$$V(x, y) = \eta E_F (\cos qx + \cos qy), \quad \eta \ll 1,$$

(1)
where $E_F$ is the Fermi energy, is given by a trivial generalization of the result of (for 1D modulation), yielding
\[ \Delta \rho_{xx}^{2D} = \Delta \rho_{yy}^{2D} = \Delta \rho_{1D}(q \parallel e_x). \] (2)

Experimental deviations from the results of this perturbative-in-$V$ approach were attributed to inadequacy of the quasiclassical treatment. In complete contrast, the authors of Ref. \textsuperscript{22} claimed that the perturbation theory of modulation fails in 2D. They argued, in particular, that since in a symmetric ($V_x = V_y$) 2D modulation equipotential contours (along which the guiding center drifts) are closed, the modulation-induced correction to resistivity should vanish.

The aim of this paper is to present a systematic theoretical analysis of the quasiclassical transport in a weak 2D superlattice in the presence of a smooth random potential. We will show that the overall picture is much richer than any of the two above-mentioned extremes (proposals of Refs. \textsuperscript{20} and of Ref. \textsuperscript{22}). We will also demonstrate that, as in the case of a 1D modulation, the nature of the disorder crucially affects the magnetoresistivity.

It is worth mentioning that in Ref. \textsuperscript{21} a purely quantum-mechanical interpretation of experimentally observed commensurability oscillations was proposed, related to the miniband structure of Landau levels induced by modulation. The authors of Ref. \textsuperscript{19} argued that this structure suppresses the quasiclassical effect of the modulation. We note, however, that for typical experimental parameters the miniband splitting is small compared to the total scattering rate $1/\tau_s$, implying that the disorder broadening washes out the miniband structure. In this situation, the quasiclassical theory should provide an adequate description of the effect. Indeed, the large amplitude of commensurability oscillations (of the same order as in samples with 1D modulation) observed in many experiments strongly suggests their classical origin. Only recently, first experimental observations of the miniband structures (smeared considerably by disorder) in the regime of strong Shubnikov-de Haas oscillations have been reported.\textsuperscript{23} In any case, the quasiclassical theory of the commensurability oscillations should serve as a reference point also for analysis of the quantum effects.

\section*{II. ADVECTION-DIFFUSION TRANSPORT IN A 2D PERIODIC POTENTIAL}

In the absence of disorder and in a sufficiently strong magnetic field $B$ (the condition will be discussed in Sec. \textsuperscript{IV} below) the motion of a particle in the periodic potential \textsuperscript{17} is a drift along the equipotential contours $V_{\text{eff}}(x, y) = \text{const}$. Here $x$ and $y$ are the coordinates of the guiding center, and $V_{\text{eff}}$ is obtained by averaging \textsuperscript{17} along the cyclotron orbit,
\[ V_{\text{eff}}(x, y) = \eta E_F J_0(qR_c)(\cos qx + \cos qy). \] (3)

where $R_c = v_F/\omega_c$ is the cyclotron radius, $v_F$ the Fermi velocity, and $\omega_c = eB/mc$ the cyclotron frequency. The drift velocity $v_d(x, y)$ is given by
\[ v_d(x, y) = \frac{1}{eB} \nabla V_{\text{eff}}(x, y) \times B. \] (4)

Switching to the rotated coordinates $X = (x + y)/\sqrt{2}$, $Y = (y - x)/\sqrt{2}$, we find the components of the drift velocity
\[ v_{dX} = -v \sin(\pi Y/d) \cos(\pi X/d), \] (5)
\[ v_{dY} = v \sin(\pi X/d) \cos(\pi Y/d), \] (6)
where $d = a/\sqrt{2}$ and
\[ v = \frac{\eta v_F qR_c J_0(qR_c)}{\sqrt{2}}, \]
\[ \simeq \frac{\eta v_F (qR_c)^{1/2} \cos(qR_c - \pi/4)}{\sqrt{2}} \] (7)

(in the last line we assumed $qR_c \gg 1$). Eqs. (5), (6) determine an incompressible flow $v_{dX} = \partial \Psi/\partial Y$, $v_{dY} = -\partial \Psi/\partial X$, with a stream function
\[ \Psi(X, Y) = \frac{vd}{\pi} \cos(\pi X/d) \cos(\pi Y/d). \] (8)

The streamlines of this flow are closed and belong to one of the cells $n - 1/2 < X/d < n + 1/2$, $m - 1/2 < Y/d < m + 1/2$, with integer $m$ and $n$, which form a square lattice. The streamlines $X = d(n+1/2)$ and $Y = d(m+1/2)$ separating the cells are called separatrices.

We turn now to the effect of disorder. The small-angle scattering leads to diffusion of the guiding center with the diffusion constant
\[ D = \frac{v_F^2 \tau}{2(\omega_c \tau)^2} \] (9)

(we assume $\omega_c \tau \gg 1$). The limits of applicability of this diffusion approximation will be discussed below, see Sec. \textsuperscript{IV}. We are thus left with a transport problem of the advection-diffusion type. This kind of problem has been studied in the context of hydrodynamics since long ago, see \textsuperscript{24} for review. More specifically, with the periodic stream function \textsuperscript{17} we are facing the problem of the diffusive transport in a Rayleigh-Bénard convection cell \textsuperscript{17}. The nature of the transport depends crucially on the value of the Péclet number
\[ P = \frac{vd}{D} \sim \frac{\eta q l}{(qR_c)^3/2}, \] (10)

where $l = v_F \tau$ is the transport mean free path. If $P \ll 1$, the impurity scattering dominates over the advection. In this case the particle “does not realize” that the equipotential contours are closed, and the correction to the conductivity tensor due to the periodic potential can be calculated using perturbative expansion in $P$. Therefore, in
this regime, the perturbative expression in the modulation strength \( \eta \) is valid, yielding

\[
\frac{\Delta \rho_{xx}}{\rho_0} = \frac{\eta^2 q l}{4} Q \sinh \pi \mu J_{\nu}(Q) J_{-\nu}(Q),
\]

(11)

where \( \rho_0 = \hbar e^2 k_F l \) is the Drude resistivity (\( k_F \) is the Fermi wave vector),

\[
\mu = \frac{Q}{q v_F \tau_s} \left[ 1 - \left( 1 + \frac{\tau_s Q^2}{\pi} \right)^{-1/2} \right],
\]

(12)

and we introduced the dimensionless parameter \( Q = q R_c \) convenient to characterize the strength of the magnetic field. At low magnetic fields, \( Q \gg Q_{\text{dis}} \), with \( Q_{\text{dis}} = (2q l / \pi)^{1/3} \), the oscillations are exponentially damped by disorder, and the magnetoresistivity saturates at the value

\[
\frac{\Delta \rho_{xx}}{\rho_0} = \frac{\eta^2 q l}{4} \equiv \gamma
\]

(13)

(where we will use below the parameter \( \gamma \) introduced here in order to classify different transport regimes). Note that at still lower magnetic fields, \( Q > Q_{\text{ch}} \), with \( Q_{\text{ch}} = 2 / \eta \), and for a sufficiently strong modulation, \( \eta^3 / 2 q l \gg 1 \), an additional strong magnetoresistivity occurs, dominated by the channeled orbits. We will not consider this region of magnetic fields in the present paper. In strong fields, \( Q \ll Q_{\text{dis}} \), the amplitude of oscillations increases as \( B^3 \),

\[
\frac{\Delta \rho_{xx}}{\rho_0} = \frac{(\eta q l)^2}{\pi Q^3} \cos^2 \left( Q - \frac{\pi}{4} \right) .
\]

(14)

Note that the condition of validity of Eq. (14), \( P \ll 1 \), is equivalent to \( \Delta \rho_{xx} / \rho_0 \ll 1 \).

In the opposite limit of large Péclet number, \( P \gg 1 \), the transport is determined by a narrow boundary layer around the square network of separatrices (“stochastic web”). The width \( d_b \) of this layer can be estimated from the condition that the particle can diffuse through the layer and oscillate that the Péclet number \( \tau Q \ll 1 \). We find for the typical shift

\[
\frac{\Delta \rho_{xx}}{\rho_0} \gg 1, \text{ its } B\text{-dependence changes to } \Delta \rho_{xx} / \rho_0 \propto B^{3/4} .
\]

The two formulas (14) and (17) match at \( Q = Q_P \equiv [0.13 (\eta q l)^2]^{1/3} \), their conditions of validity being \( Q \gg Q_P = \rho_0 \), and \( Q \ll Q_P \), respectively. This behavior of the magnetoresistivity is illustrated in Fig. 1.

FIG. 1. Schematic representation of the magnetoresistivity \( \Delta \rho_{xx}(B) \) induced by the modulation in the case \( \gamma \ll 1 \). Characteristic points \( B_{\text{dis}} \) and \( B_P \) on the magnetic field axis (corresponding to \( Q = Q_{\text{dis}} \) and \( Q = Q_P \), see the text) are shown. Below \( B_{\text{dis}} \) the oscillations are exponentially damped and the magnetoresistivity saturates at \( \Delta \rho_{xx} = \gamma \), while at \( B = B_P \) the \( B^3 \)-behavior of \( \Delta \rho_{xx} \) changes to a much slower, \( B^{3/4} \)-increase.

In the following two sections we will analyze conditions of validity of the two approximation used in the above derivation: the drift approximation for the motion in the periodic potential and the diffusion approximation for the impurity scattering. We will also briefly discuss the transport regimes which take place when one of these approximations is violated.

III. ROLE OF ADIABATICITY OF THE MOTION IN THE PERIODIC POTENTIAL: DRIFT VS CHAOTIC DIFFUSION

The drift approximation reflects the adiabatic nature of the electron motion in the periodic potential and is applicable provided the shift \( \delta \) of the guiding center after one cyclotron revolution is small compared to the modulation period \( a \). Noting that \( \delta X = (2\pi / \omega_c) v_d x \) and that according to Eq. (17) the average squared drift velocity is equal to

\[
\langle v_{dX}^2 \rangle = \frac{\eta^2}{4 \pi} v_F^2 Q \cos^2 (Q - \pi/4) ,
\]

(18)

we find for the typical shift \( \delta = \langle \delta X^2 \rangle^{1/2} \),

\[
\frac{\delta}{a} = \frac{\eta}{(4 \pi)^{1/2} Q^{3/2} \cos (Q - \pi/4)} .
\]

(19)
Therefore, the adiabatic condition $\delta/a \ll 1$ can be rewritten as $Q \ll Q_{ad}$, where $Q_{ad} = (4\pi/\eta^2)^{1/3}$. Comparing $Q_{ad}$ with the value $Q_{dis}$ determining the point where the drift motion breaks down due to the impurity scattering (see above), we find

$$\frac{Q_{dis}}{Q_{ad}} = \left(\frac{\eta^2 q l}{2\pi^2}\right)^{1/3}.$$  \hspace{1cm} (20)

We see that this ratio is determined by the parameter $\gamma$ defined in Eq. (13), which governs also the relation between $Q_{dis}$ and $Q_P$,

$$\frac{Q_P}{Q_{dis}} \simeq (0.2\eta^2 q l)^{1/3}.$$  \hspace{1cm} (21)

Therefore, one should analyze separately two regimes, $\gamma \ll 1$ and $\gamma \gg 1$.

In the case $\gamma \ll 1$ we have $Q_{ad} \gg Q_{dis} \gg Q_P$. The first inequality means that the effect of the randomization of the guiding center position by the periodic potential is weaker than the analogous effect of the random potential. Therefore, the former effect can be neglected, and the point $Q = Q_{ad}$ is irrelevant for the behavior of magnetoresistivity. We also note that $Q_{ch} \gg Q_{ad}$, so that the region of the low-field magnetoresistivity (channeled orbits), $Q \gtrsim Q_{ch}$ is parametrically separated from the region of commensurability oscillations, $Q \lesssim \min\{Q_{dis}, Q_{ad}\}$.

The assumed condition $\gamma \ll 1$ is typically fulfilled in an experiment if the modulation strength $\eta$ is of order of a few percent. For example, taking typical experimental values of the parameters, $q l = 1000$, $\eta = 0.03$, we get $\gamma \simeq 0.22$ and $Q_P \simeq 4.9$, $Q_{dis} \simeq 8.6$, $Q_{ad} \simeq 24$, $Q_{ch} \simeq 67$, so that the above inequalities $Q_P \ll Q_{dis} \ll Q_{ad} \ll Q_{ch}$ are indeed reasonably satisfied.

We turn now to the consideration of the opposite limit $\gamma \gg 1$, which is realized for sufficiently strong modulation. Now we have $Q_{ad} \ll Q_{dis} \ll Q_P$, implying that $Q_{dis}$ and $Q_P$ become irrelevant. In the whole region $Q \gtrsim Q_{ad}$ the non-adiabatic electron dynamics leads to chaotic diffusion. The transport in this regime is similar to that in the region $Q \gg Q_{dis}$ for $\gamma \ll 1$, with disorder replaced by the modulation-induced chaos. Therefore, the magnetoresistivity is given in this regime by Eq. (13) (note that $\Delta\rho_{xx}$ in this formula does not depend explicitly on disorder), with the oscillations being exponentially damped.

With magnetic field increasing beyond the point $Q = Q_{ad}$ the system enters the adiabatic regime, with the advection-diffusion type of transport, as considered in Sec. I. One can thus expect Eq. (17) to be applicable. It is clear, however, that Eq. (17) does not match the formula (13) [valid for $Q \gg Q_{ad}$] at $Q \sim Q_{ad}$. Let us demonstrate that there will be an additional, logarithmically narrow, intermediate regime between the regions of validity of these two formulas. The reason is that even at $Q \ll Q_{ad}$ the picture of adiabatic drift in a periodic potential $\square$ is only approximate, though its violation is exponentially weak. This leads, in the absence of any disorder, to the formation of a stochastic boundary layer of a width $\tilde{d}_b$ falling off exponentially with magnetic field $\square$.

$$\frac{\tilde{d}_b}{d} \propto \exp\left(-\frac{\omega d}{2v}\right)\exp\left[-\frac{\pi}{2\sqrt{2}}\cos(Q - \pi/4)\right]\left(\frac{Q_{ad}}{Q}\right)^{3/2}.$$  \hspace{1cm} (22)

We omit preexponential factors here. The underlying physics is analogous to that discussed in Sec. I in the case of a large Péclet number, with the only difference that the diffusion is now not due to disorder but rather due to violation of adiabaticity. The width $\tilde{d}_b$ exceeds the disorder-induced one, $d_b$, in a narrow range $\gamma > Q \gg Q_{ad}$, where $Q_{ad} \sim O_{ad}\langle\ln\gamma\rangle^{-2/3}$. Within this logarithmically narrow interval of magnetic fields, the transport is still fully determined by the modulation, with the macroscopic diffusion coefficient $D_{eff} \sim \nu d_b$, and the magnetoresistivity drops off exponentially (Fig. 3),

$$\Delta\rho_{xx}/\rho_0 \propto \exp\left[-\frac{\pi}{2\sqrt{2}}\cos(Q - \pi/4)\right]\left(\frac{Q_{ad}}{Q}\right)^{3/2},$$  \hspace{1cm} (23)

from $\Delta\rho_{xx}/\rho_0 \sim \gamma$ at $Q \sim Q_{ad}$ to $\Delta\rho_{xx}/\rho_0 \sim \gamma^{1/2}$ at $Q \sim Q_{ad}'$, with a simultaneous rapid development of commensurability oscillations. At $Q < Q_{ad}'$ the disorder-induced diffusion becomes more efficient than that due to violation of adiabaticity of the motion in the periodic potential, so that Eq. (14) becomes applicable, and the magnetoresistivity starts to increase with magnetic field as $B^{3/4}$, as illustrated in Fig. 2.

FIG. 2. Schematic representation of the magnetoresistivity $\Delta\rho_{xx}(B)$ in the case $\gamma \gg 1$. The magnetoresistivity starts to drop exponentially and the commensurability oscillation appear at the value $B_{ad}$ of the magnetic field where the motion in the periodic potential takes the form of an adiabatic drift. At $B \sim B_{ad}$ the disorder starts to dominate over the non-adiabatic effects, leading to a $B^{3/4}$-increase of the oscillation amplitude.
IV. ROLE OF THE RANGE OF DISORDER

In this section we discuss the condition of validity of the diffusion approximation used in Sec. II. This approximation corresponds formally to the limit of zero time interval between two successive scattering events. In a real situation this interval is, however, finite, its average value being equal to \( \tau_s \). Correspondingly, the r.m.s. shift of the guiding center in a scattering event is given by

\[
\delta_s \equiv \sqrt{\langle (\delta x)^2 \rangle} = \sqrt{\langle (\delta y)^2 \rangle} = (2D\tau_s)^{1/2} = R_c(\tau_s/\tau)^{1/2} . \tag{24}
\]

The ratio \( \tau_s/\tau \ll 1 \) is of the order of \( 1/(k_F\xi)^2 \), where \( \xi \) is the correlation length of the random potential set by the width of the undoped spacer between the 2DEG and the impurity layer, since an electron is typically scattered by a small angle \( \delta \phi \sim 1/k_F\xi \). In order for the diffusion-advection description of the transport presented in Sec. II to be justified, the following two conditions should be fulfilled: (i) \( \delta_s \ll a \) and (ii) \( \tau_s \ll a/v \), which can be equivalently rewritten as

\[
Q \ll 2\pi(\tau/\tau_s)^{1/2} \equiv Q_{\text{diff}}^{(1)} \tag{25}
\]

and

\[
Q \cos^2(Q - \pi/4) \ll 8\pi^3 \left( \frac{\tau}{\tau_s} \right)^2 \left( \frac{1}{(2\eta l)^2} \right) \equiv Q_{\text{diff}}^{(2)} . \tag{26}
\]

respectively. Let us estimate typical values of \( Q_{\text{diff}}^{(1)} \) and \( Q_{\text{diff}}^{(2)} \). Using the same parameters as above (\( ql = 1000 \), \( \eta = 0.03 \)) and the ratio \( \tau/\tau_s = 50 \) (characteristic for the best of the undoped spacer between the 2DEG and the impurity layer, we find \( Q_{\text{diff}}^{(1)} \approx 45 \) and \( Q_{\text{diff}}^{(2)} \approx 690 \), so that the diffusion approximation is justified in the whole range of interest, \( Q \ll Q_{\text{diff}} \), where the oscillations are observed. We see, however, that this conclusion relies heavily on the small-angle nature of the scattering, \( \tau/\tau_s \gg 1 \). If we would assume in the above example isotropic scattering, \( \tau/\tau_s = 1 \), we would get \( Q_{\text{diff}}^{(1)} \approx 6.3 \) and \( Q_{\text{diff}}^{(2)} \approx 0.28 \), so that the diffusion approximation would not be valid in the whole region of oscillations. We analyze now what happens if at least one of the conditions (25), (26) is violated. The following three situations can be distinguished:

(A) Eq. (25) violated, Eq. (26) fulfilled. In this case the picture of the transport is as follows: the guiding center drifts during a time \( \sim \tau_s \) along an equipotential contour to a small distance \( \sim v\tau_s \ll a \), then makes a large jump of a range \( \sim \delta_s \gg a \), etc. The disorder-induced correction to resistivity is thus equal to

\[
\frac{\Delta \rho_{xx}}{\rho_0} = \frac{2\langle v_{\parallel}^2 \rangle \tau_s}{\delta_s^2} = \frac{v^2 \tau_s}{2R_c^2} \ll 1 , \tag{27}
\]

with \( v \) given by Eq. (6). This result can be obtained by a perturbative expansion of the type used in \( \rho_{xx}/\rho_0 \propto \eta^2 \), which is determined by the fact that a drifting particle does not close a contour before it is scattered far away. The correction (27) shows according to (7) the conventional commensurability oscillations \( \propto \cos(Q - \pi/4) \), but it is small, \( \Delta \rho_{xx}/\rho_0 \ll 1 \), since it can be presented in the form

\[
\frac{\Delta \rho_{xx}}{\rho_0} \sim \left( \frac{v\tau_s}{a} \right)^2 \left( \frac{a}{\delta_s} \right)^2 ,
\]

both factors being much less than unity.

(B) Both Eqs. (25) and (26) violated. In this case the particle typically completes the drift contour before it is scattered to a large distance \( \delta_s \gg a \). The resistivity correction can thus be estimated as

\[
\frac{\Delta \rho_{xx}}{\rho_0} \sim \frac{\rho_{\text{diff}}}{\rho_0} = \frac{a^2 \tau}{R_c^2 \tau_s} \ll 1 . \tag{28}
\]

Since (28) does not depend on the drift velocity, the magnetoresistivity does not show oscillations as a function of \( Q \). More precisely, there will be a remnant of oscillations in the form of narrow dips in the vicinity of zeros of the drift velocity, \( Q = \pi(n - 1/4) \), where Eq. (26) will be fulfilled and the resistivity will be given by Eq. (27).

(C) Eqs. (25) fulfilled, Eq. (26) violated. The transport in this regime is determined by a boundary layer of the width \( \delta_s \) near separatrices, so that

\[
D_{\text{eff}} \sim \frac{a\delta_s}{\tau_s} ,
\]

and, correspondingly,

\[
\frac{\rho_{xx}}{\rho_0} = \frac{D_{\text{eff}}}{D} \sim \frac{a(\tau/\tau_s)^{1/2}}{R_c} \gg 1 . \tag{29}
\]

As in the case (B), Eq. (29) does not depend on \( v \), so that the oscillations are washed out, except for narrow vicinities of the points \( Q = \pi(n - 1/4) \), where the condition (29) will be restored and thus the result (17) [or, still closer to a zero of the drift velocity, Eq. (14)] will be applicable.

Summarizing the results for the regimes (B) and (C), one can conclude that if the condition \( Q \ll Q_{\text{diff}} \) is violated, the modulation-induced resistivity is strongly suppressed (as compared to what the diffusion approximation would predict) and the commensurability oscillations are washed out. This explains, in particular, why essentially no oscillations were observed in the numerical simulations of Ref. 20 for a symmetric square superlattice [4]. These authors assumed a model of isotropic scattering, \( \tau_s = \tau \), which typically leads to the violation of both inequalities (25) and (26), as shown above. The resulting magnetoresistivity is weak, Eq. (28), and shows only remnants of oscillations, in agreement with the simulations in [4].
V. SUMMARY AND DISCUSSION

In this article we have presented a systematic theoretical analysis of the quasiclassical magnetotransport of 2D electrons in a 2D lateral superlattice. The amplitude of the modulation-induced magnetoresistivity, and in particular of the commensurability oscillations, depends crucially on the nature of disorder (see Sec. IV). We assumed small-angle impurity scattering induced by a smooth random potential characteristic for high-mobility heterostructures. In the region of existence of commensurability oscillations the transport is determined by the interplay of the advection in the periodic potential and the diffusion due to impurity scattering. The shape of the magnetoresistivity depends on the dimensionless parameter $\gamma$, Eq. (3).

For small $\gamma$ (corresponding typically to a modulation strength not exceeding a few percent) the magnetoresistivity is given by the perturbative formulas (11) - (14) up to the point $Q \sim Q_{P}$, where the correction $\Delta \rho_{xx}$ becomes of the order of the Drude resistivity $\rho_{0}$. For higher magnetic fields the Péclet number $P$ characterizing the advection-diffusion problem becomes large and the transport is determined by a narrow boundary layer around a square network of separatrices. As a result, the $B^{3}$-dependence of the oscillation amplitude characteristic for the perturbative ($Q > Q_{P}$) regime crosses over to a much slower $B^{3/4}$-increase at $Q < Q_{P}$, see Eq. (17).

For $\gamma \gg 1$ (which is typically valid for the modulation strength $\eta$ larger than $10 \div 15\%$) the oscillations are damped at low magnetic fields not by disorder (as in the perturbative regime) but by the modulation-induced chaotic diffusion. The oscillations become observable at $Q \sim Q_{ad}$ where the motion of electrons in the superlattice potential acquires the form of adiabatic drift. Since the violation of adiabaticity is exponentially small, the magnetoresistivity drops exponentially in a logarithmically narrow interval of magnetic fields, $Q_{ad} \ln^{-2/3} \gamma < Q < Q_{ad}$, Eq. (23). At higher magnetic fields the impurity scattering starts to dominate over the non-adiabatic processes and thus to determine the diffusion constant of the advection-diffusion problem, so that the commensurability oscillations take the same form (23) as in the large-$P$ limit of the $\gamma \ll 1$ regime.

We have demonstrated that the transport regimes discussed in Sec. I and II, i.e. the perturbative regime (advection-diffusion with small $P$), the stochastic web regime (advection-diffusion with $P \gg 1$), and the chaos-dominated regime with an exponential fall-off of the magnetoresistivity at the adiabaticity threshold, are within the range of typical experimental parameters. Experimental identification of these regimes and a quantitative analysis of the experimental data on the basis of the theory presented here is thus not only highly desirable but also practically feasible.

Finally, let us note that in this paper, as well as in the earlier study of 1D modulation, we treated disorder as a collision term in the Boltzmann equation, thus neglecting memory effects. On the other hand, it has been shown recently that for smooth disorder these effects may become important with increasing magnetic field, eventually leading to an adiabatic character of the motion in a random potential or a random magnetic field, and consequently to an exponential drop of magnetoresistivity (in the absence of any modulation). Though, to our knowledge, this behavior has not yet been observed experimentally in the low-field magnetotransport of a 2D electron gas, the progress in fabrication of better samples with larger spacers may make it experimentally relevant in the near future. Also, a positive magnetoresistance due to memory effects, followed by the adiabatic fall-off of the resistivity is observed in magnetotransport of composite fermions near half-filling of the lowest Landau level, where the effective disorder has the form of a smooth random magnetic field. Understanding of the effect of a 1D or 2D periodic modulation in the situation of such a non-Boltzmann transport in the unmodulated system would require a separate theoretical analysis.

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