Localizing an Unknown Number of mmW Transmitters Under Path Loss Model Uncertainties

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Abstract—This work estimates the position and the transmit power of multiple co-channel wireless transmitters under model uncertainties. The model uncertainties include the number of the targets and the parameters of the path-loss model which enable the system to cope with changes in the weather conditions and in mmW ranges. The problem is solved by an unbiased estimator. The underlying complicated optimization problem has a combinatorial nature that selects the best grid points as the location of the targets. The combinatorial problem is converted to a convex form by means of \( l_1 \)-regularization, which enables locating off-grid targets. Simulations show that the proposed algorithm solves the problem with very high accuracy in the absence of noise and shadowing.

Index Terms—multi-source localization, \( l_1 \)-localization, mixed-integer programming, \( k \)-NN, internet of things

I. INTRODUCTION

It is envisaged that the majority of applications in the context of the internet of things and 5G mobile networks depend on the location awareness to deliver better services. This work studies a received signal strength (RSS)-based technique since it is simpler to implement and has lower costs, compared to the time difference of arrival (TDoA) or angle of arrival (AoA). Despite its lower positioning accuracy, RSS localization is beneficial in case precision can be somewhat compromised for price. The main challenge in RSS-based localization is the uncertainties about the path-loss model parameters [1]. This issue is addressed in this work. The RSS-based localization for a single target with unknown transmit power is studied in many publications, such as [2], where the transmit power cancels out upon dividing the RSS of two different receivers. The remaining of the problem is a standard multilateration problem. This technique is known as differential or ratio of RSS and is not applicable to multi co-channel targets.

A. Path Loss Model Uncertainties

In the free space communication, \( \alpha = 2 \) is a good approximate for the path-loss exponent (PLE). However, in other types of environments \( \alpha \) is different from 2. Indeed, its value not only depends on the propagation environment but also changes due to seasonal or weather conditions [3]. Furthermore, the free space model does not accurately describe the path loss in indoor environments. Numerous path loss models have been proposed based on measurements in different cities and locations [3, Sec. 3.11]. Furthermore, the estimation of the PLE has been the research topic of several works in the literature.

For instance, [4] exploits the probabilistic information about the distances between nodes and tries to estimate \( \alpha \). It also applies the technique of distance geometry problem if such probabilistic information is not available. However, it works only for one target with known transmit power. The advantage of such an idea is that it works with only RSS measurements and avoids distance calibrating. Calibrating based on distance measurements is costly, difficult, and even in some environments impossible since the distances between transmitter and receivers should be physically measured. Such a calibration is also prone to the changes in the values of \( \alpha \), which necessitates a new round of calibration. The existing works in the literature of localization can be classified to the following cases:

1) Both transmit power and PLE are known, e.g., [5].
2) Transmit power is unknown, PLE is known, e.g., [6].
3) PLE is known, transmit power is unknown, e.g., [7].
4) Neither transmit power nor PLE is known, e.g., [8].

Nevertheless, none of the existing works deals with the problem of multi-target localization in case of unknown PLE. In general, the severity of signal strength attenuation depends on a variety of factors, such as wavelength, gain and directivity of antennae, obstacles and big objects in the propagation environment, the height of antennae, and the existence of the line of sight (LoS) [3]. This also includes the shape and type of buildings and walls. Therefore, the Friis formula is not accurate in practice. Conventionally, a common way for network planning is using empirical models, which adds some correction terms to the basic path loss formula. One famous example of such a model is the Hata model [9], where the effect of antennae and other correction factors are included:

\[
P_t[\text{dB}] = P_s[\text{dB}] - (44.9 - 6.55 \log h_t) \log d + P,
\]

(1)

where \( P \) depends on the carrier frequency, the height \( h_t \) of transmit antenna as well as the height of receive antenna [3, Eq. 3.82]. This model is analogous to the free space path loss equation, by letting PLE be equal to \( \alpha = 4.49 - 0.655 \log h_t \). Another conventional method of predicting the path loss is using an average (effective) value of \( \alpha \) acquired by extensive radio filed measurements, which leads to an empirical path loss model. This approach is well-known in indoor scenarios, where the effect of walls and their types, number of floors or windows et cetera on path loss cannot be neglected. For instance, the model in [3, Eq. 3.94-3.95]:

\[
\frac{PL(d')}{\text{dB}} = \frac{PL(d_0)}{\text{dB}} + 10\alpha \log \left( \frac{d}{d_0} \right),
\]

(2)
where $d_0$ is the reference distance and $\alpha$ varies between 1.81 to 5.22 in different locations. Depending on the number of measurement points and the size of the area, over which the measurements are averaged, predicting path loss using this model leads to different amounts of deviations from the actual value of path loss. In order to account for such uncertainty, an additive Gaussian zero-mean random variable (rv) $X_\sigma$, with the variance of $\sigma^2$, can be added to the formula:

$$PL(d)[\text{dB}] = PL(d_0)[\text{dB}] + 10\alpha \log\left(\frac{d}{d_0}\right) + X_\sigma,$$  \hspace{1cm} (3)

where $\alpha$ is an average value (slope of a linear curve fitting over measurement points) in a particular environment. Its value is usually higher than 2 in urban areas. Moreover, the actual value of path loss and thus $\alpha$ is subject to change. For instance, in an indoor scenario closing or opening windows may change the path loss level.

**B. Rain Fade in mmW**

The water molecules in any form of precipitation, e.g., rain, fog, humidity, or snow, intensify the attenuation of the electromagnetic waves [10, Ch. 7]. Precipitations are the main source of changes in the amount of path loss. The *International telecommunication union radiocommunication sector (ITU-R)* has provided methods in the *recommendation P.530* to calculate the excessive path loss imposed by precipitations. According to P.530 in addition to the conventional propagation effects such as fading and diffraction, the following reasons attenuate the signal, excessively:

- Absorption by the precipitations such as rain or snow.
- Atmospheric gases that absorb signal, i.e., *dielectric*.

In the frequencies over 10GHz, the rain absorption (rain fade) is severe as the size of the raindrops is comparable with the wavelength of signals. This effect becomes more severe in the case of mmW, over 30GHz, where the absorption by atmospheric gases and humidity as well as rain fade become more challenging. Based on ITU-R recommendation PN.837 such an excessive path loss in frequencies up to 40GHz and in LoS situation is given by

$$\gamma_r(\text{dB}_m) = a_0R^{b_0},$$  \hspace{1cm} (4)

where $R$ is the rate of rainfall in mm/hr exceeded for 0.01% of an average year (annual statistic), also $a_0$ and $b_0$ depend on local climate conditions. The recommended values of $R$ are provided by PN.837 for different climate zones. They can also be acquired from weather monitoring centers. In the literature there exist several publications that characterize the rain fade attenuation in different areas and climate zones, e.g., [11].

In summary, there are a few points to highlight:

1) RSS (in dB) decreases not only with a constant rate of $10\alpha$ (per decade) but also with an additional linear term $\gamma_r$ (per m), as the distance between the transmitter and receiver increases.

2) $\alpha$ is not always 2, but it lies into a range between $\alpha = 1$ to $\alpha = 6$. The values below 2 are relevant to the kinds of propagation channel which behave like a waveguide, such as tunnels. Contrarily, $\alpha$ represents environments with very harsh shadowing effects.

3) $\gamma_r \geq 0$ depends on many factors including the rainfall rate. Thus, it changes over time. More importantly, the values recommended by ITU-R or the estimated values thereupon can be inaccurate, which degrades the localization accuracy.

**C. Unknown Number of Targets**

Regarding uncertainties, the most challenging parameter to estimate is the number of targets since they are non-cooperative.

**D. The Contribution**

This work assumes a log-normal shadowing path-loss model, where multiple co-channel transmitters cause interference on one another. Thus, multilateration technique, unlike the single target case, is impossible. To the best of knowledge of the authors, there is no work with similar assumptions, except for [12–14], where, unlike this work, the number of targets and the PLE are assumed to be known. Furthermore, in this work, the path loss model is suitable for mmW ranges in different weather conditions. Since no work with similar assumptions and system model has been found, the results of this paper could not be compared with any other work, unfortunately.

The organization of this paper is as follows: the system model is described in Sec. II, while Sec. III presents the statistical properties of the RSS and the proposed $\ell_1$-localization technique. The performance of the presented technique is evaluated by computer simulations in Sec. IV.

**Notations**: Tab. I shows all mathematical notations of this paper.

**II. System Model**

The system of consideration consists of $N \in \mathbb{N}$ active targets and $K \in \mathbb{N}$ passive *sensor nodes* (SNs) with known positions. Each target transmits a signal with the unknown power $P \leq p_n \leq \bar{P}$, where $P$, $\bar{P} \geq 0$ are, the lowest and highest possible values for the transmit power.

The propagation channel is based on the log-normal shadowing attenuation model presented in [3]. In a multi-source scenario, the RSS $r_k$ at sensor $k$ is the sum of different terms corresponding to different target signals [15, 16]:

$$r_k = \sum_{n \in I_N} p_n d_{kn}^{-\alpha} \beta^{d_{kn}} 10^{\zeta_{kn}},$$  \hspace{1cm} (5)

where $d_{kn}$ is the distance between sensor $k$ and $n^{th}$ target, $\alpha$ is the path-loss exponent. $\zeta_{kn} \sim \mathcal{N}(0, \sigma_{kn}^2)$ is a zero-mean random variable.

**TABLE I**: Summary of general mathematical notations

| Notation | Description |
|----------|-------------|
| $N$      | set of all integer positive and non-zero numbers |
| $\mathbb{R}$ | set of all real numbers |
| $\mathbf{x}$ | column vector $\mathbf{x}$ with entries $x_i$ |
| $\mathbf{x}'$ | transpose of vector $\mathbf{x}$ |
| $\mathbf{X}$ | matrix $\mathbf{X}$ with entries $x_{ij}$ or $[\mathbf{X}]_{ij}$ |
| $\|\cdot\|_0$ | $\ell_0$-norm, i.e., the number of non-zero entries of $\mathbf{x}$ |
| $(\cdot)^*$ | optimal solution of an optimization problem |
| $\mathbf{x}$ | estimation of the unknown variable $x$ |
Gaussian random variable with the power of $\sigma^2_{k_n}$ that models the log-normal shadowing and is assumed to be identically and independently distributed (iid). The parameter $\beta = 10^{\frac{0}{10}}$ represents the rain fade.

In this work, $\beta$ is termed path-loss factor (PLF) analogous to the path-loss exponent (PLE). To be precise, it points out the fact that $\beta$ appears as a multiplicative term, and not as an exponent, in the path loss formula (5).

The coefficient $c_0$ depends on many factors such as the gains of antennae and the wavelength. Without loss of generality and for the sake of simplicity, it is assumed that $c_0 = 1$.

The thermal additive noise is neglected since shadowing has a much stronger effect on RSS compared to the thermal noise [5, 17]. Besides, the effect of additive noise can be somewhat compensated using methods of blind estimation of the noise power, e.g., [18].

Remark 1: The position estimation becomes very challenging if the number of targets is unknown. In what follows, the variable $\nu$ is introduced to estimate the actual number of targets. It is assumed $\nu$ is bounded between a minimum possible number $\bar{N} \in \mathbb{N}$ and a maximum number $\bar{N} \in \mathbb{N}$, $\bar{N} \geq N$, that is

$$\nu \in \{ \bar{N}, \cdots, \bar{N} \}. \quad (6)$$

Note in case the number of targets is known $N = \bar{N} = \bar{N}$. Otherwise, the choice of $\bar{N} = 1$ is reasonable since there should exist at least one active target.

Remark 2: In this work, the actual values of $\beta$, $\alpha$, and $\nu$ are unknown. It is further assumed that $\bar{\alpha} \leq \alpha \leq \bar{\alpha}$ and $\bar{\beta} < \beta \leq \bar{\beta}$. While $\bar{\beta} = 1$ corresponds to no rain fade conditions or frequencies below 3GHz, $\bar{\beta}$ is the minimum possible value of PLF, corresponding to the highest possible rain attenuation. According to ITU-R recommendations, the rain fade amounts to 0.035-0.04 dB/m for frequencies over 100 GHz in very heavy rain, i.e., over 120 mm/hr, and typhoon situations. Therefore, the value $\beta = 0.96 \leq 10^{-0.004}$ is considered, in this work, to be a lower bound of $\beta$. This is a loose lower bound since it corresponds to extreme weather conditions in mmW ranges, i.e., over 30GHz.

Considering the rain fade in localization scenarios, in single target case, has recently received a significant attention. One important application of such a scenario is the find and rescue operation in emergency-related situations in extreme weather conditions. For instance, [19] estimates the rainfall intensity based on the received signal at a 4G mobile node using neural networks. Works [20, 21] consider the rain fade in GSM-1.8GHz for distance estimation (not the position) of a single target. In their scenario, only one receiver is needed since the PLE, PLF and transmit power are assumed to be known. The authors solve the problem using Newton-Raphson method and failed to see that the solution has the form of the Lambert $W$ function, which exists in closed-form. The closed-form solution is however shown in [22] in the context of underwater communication. Nevertheless, without assuming the transmit power is known, it becomes impossible to find a closed-form solution to this problem. The problem becomes even harder if the values for $\alpha$ and $\beta$ are to be estimated. The current work intends to do position estimation for the multi-target scenario, given the assumption that $\alpha$, $\beta$, transmit power of targets (different from one another), and most importantly the number of targets are unknown.

Remark 3: The path loss model (5) does not apply only to the rain fade and the frequencies over 10GHz. A similar path loss model has been also proposed by Devasirvatham in [23] for indoor multi-floor buildings, where the path loss follows the free space model, i.e., $\alpha = 2$ plus an additional linear term (dB/m), please see [3, Eq. 3.96].

The area of observation is assumed to be a square in the range of $[-w, w]$, $w \geq 0$ in both x- and y- axes, in the Cartesian coordinate system. The targets and sensors are randomly distributed within the area. The ordered pair $(\tilde{x}_k, \tilde{y}_k)$ stands for the coordinate of the $k^{th}$ sensor node, while target $n$ is located at the unknown position $(x_n, y_n)$. Assuming that the fusion center acquires the values of RSS $r_k$ of the $k^{th}$ sensor error-free upon successful communication from SN, it has to solve the following system of nonlinear equations

$$r_k = \sum_{\bar{n} \in \mathbb{N}} p_n \beta^{\sqrt{(x_n - \tilde{x}_k)^2 + (y_n - \tilde{y}_k)^2}} \alpha, \quad (7)$$

to estimate $N$, $\alpha$, $\beta$, the position $(x_n, y_n)$, and the transmit power $p_n$ of each of the $N$ targets.

Such a system of equations is extremely hard to solve. It is, nevertheless, solved in this work by a low-complexity heuristic. First, the area needs to be discretized into a grid of granularity of $G \in \mathbb{N}$ which means $G^2$ grid points. Let $G_C^w(x, y)$ be the grid set centered at the point $(x, y)$ of width $2w \geq 0$ and the granularity $G$, then $G_C^w(x, y)$ is defined by

$$\{ (x - w + (i - 1) \Delta_y, y - w + (j - 1) \Delta_y) \mid i, j \in \mathbb{N}_0 \}, \quad (8)$$

where $\Delta_y = \frac{2w}{G - 1}$ is the width of one grid square. Then, the defined grid consists of the grid points $(\tilde{x}_m, \tilde{y}_m) \in G_C^w(0, 0), m \in \mathbb{N}_0$. Fig. 1 depicts the example grid $G_C^w(0, 0)$.
III. $\ell_1$-Localization

Before solving the problem at hand, the statistical properties of the RSS at sensors need to be studied.

A. Sum of log normal random variables

The sum of log-normal ($\mathcal{LN}$) random variables has an unknown probability distribution function (pdf) [24], even for the sum of two random variables. By using the Fenton-Wilkinson [24] method, $r_k$ can be approximated by an $\mathcal{LN}$ random variable which has the same mean and variance as $r_k$. Let $R_k$ be a random variable from which the values of $r_k$ are drawn, with $r_k$ being the RSS at sensor $k$ due to $n^\text{th}$ target, i.e., $r_k = p_n d_{kn}^\alpha \beta d_{kn} 10^{\frac{\mu_k}{10}}$. Then, the mean and variance of $R_k$ are given by [13]:

$$E(R_k) = p_n d_{kn}^\alpha \beta d_{kn} b_{kn},$$

$$\text{Var}(R_k) = (p_n d_{kn}^\alpha \beta d_{kn})^2 (b_{kn}^2 - 1)b_{kn}^2,$$

where $b_{kn} = e^{\frac{(\ln 10)^2}{2} \sigma^2}$. It is assumed that all $\sigma_{kn}$ are equal and $b = b_{kn}$ is known. Since all the random variables $R_k$ are pairwise independent the mean $M_k$ and variance $V_k$ of the random variable $R_k = \sum_{n \in I} R_k$ reads

$$M_k = b g_k, \quad g_k := \sum_{n \in I} p_n d_{kn}^\alpha \beta d_{kn},$$

$$V_k = (b^2 - 1)b^2 h_k, \quad h_k := \sum_{n \in I} (p_n d_{kn}^\alpha \beta d_{kn})^2.$$

The goal is to find $\mu_k$ and $\sigma_k$ such that the mean and the variance of the random variable $e^{\mu_k + \sigma_k X}$, where $X$ is a standard normal random variable, equate with the ones of $R_k$:

$$M_k = e^{\mu_k + \frac{\sigma^2}{2}},$$

$$V_k = e^{2\mu_k + 2\sigma^2}.$$  

By so doing the approximation $\ln r_k \approx \mu_k + \sigma_k X$ is achieved. This results in

$$\mu_k = 2 \ln (M_k) - \frac{1}{2} \ln (M_k^2 + V_k),$$

$$\sigma_k^2 = \ln (M_k^2 + V_k) - 2 \ln M_k.$$  

B. The General Idea of the Solution

In this work, the equation (5) is solved by a heuristic to estimate the number of targets, their positions, and values of transmit power as well as PLE and PLF. The heuristic is iterative and performs a sequence of actions at iteration $i$, given a grid and the estimation values from the previous iteration. The concept of these actions is explained below:

(i) Error minimization: Minimizing an error function w.r.t to the variables $\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\alpha}, \hat{\beta}, \hat{\nu}$.

(ii) $\ell_1$-relaxation: Since an optimization that involves the $\ell_0$-norm is $NP$-hard, here this norm is relaxed to $\ell_1$-norm, i.e., $s_m \in [0, 1]$, in order to convexify the underlying optimization. On the other hand, due to the fact that $\Phi$ does not hold incoherence properties, the optimal vector $\hat{s}$ is not $N$-sparse. To over come this shortcoming, a cluster-and-average scheme is devised.

(iii) Clustering: Let the set $\Pi M_0$ be the index set of $M_0$ largest entries of the optimal vector $\hat{s}$, given $M_0 < M$. Then, the set of all positions $(\hat{x}_m + \hat{d}_x, \hat{y}_m + \hat{d}_y)$, $\forall m \in \Pi M_0$ is represented by $\Pi M_0$. Using the k-means clustering, $\Pi M_0$ can be partitioned into $\Pi_1, \cdots, \Pi_N$, where $\Pi N \subset \Pi M_0 \subset G(N, G), \forall n \in I_N$.

(iv) Averaging: Then, the averaging rules

$$\hat{x}_n = \frac{\sum_{m \in \Pi_n} s^*_m (\hat{x}_m^{-1} + \hat{d}_x)}{\sum_{m \in \Pi_n} s^*_m},$$

$$\hat{y}_n = \frac{\sum_{m \in \Pi_n} s^*_m (\hat{y}_m^{-1} + \hat{d}_y)}{\sum_{m \in \Pi_n} s^*_m},$$

$$\hat{p}_n = \frac{\sum_{m \in \Pi_n} s^*_m (\hat{p}_m^{-1} + \hat{d})}{\sum_{m \in \Pi_n} s^*_m},$$

are employed to update the position and power estimation. Fig. 2 shows how the cluster-and-average improves positioning performance. Note that $M_0 = N$ means that only the $N$ largest entries of $s^*$ and their corresponding grid points are selected as the position estimation. This exclude averaging since each $\Pi_n$ has only one member.

(v) Grid update: Then, a sub-grid of $G^2$ points is formed around each estimate point and the power $\hat{p}_n$ will be associated with each point. This results in a set $G$ of $NG^2$ grid points and the set $P$ of their power values.

$$G(N, G) = \bigcup_{n \in I_N} G^2(\hat{x}_n, \hat{y}_n),$$

$$\tilde{h}_k = \sum_{m \in \Pi_n} (\hat{p}_m d_{km}^{-\alpha} \beta d_{km})^2.$$  

It needs to be mentioned that the accent tilde appearing over a variable means the area is discretized to $M$ grid points, while the index $m$ refers to the $m^\text{th}$ grid point, for which the variable stands. The (Taylor) linearization introduces the variables $\hat{d}_x, \hat{d}_y, \hat{d}, \hat{d}_\alpha, \hat{d}_\beta$, and $\hat{d}_\nu$ which are used for iterative update of the variables.
At each iteration, the variables are updated using:
\[
\alpha^i = \alpha^{i-1} + \nu \partial \alpha \theta^i, \quad \beta^i = \beta^{i-1} + \nu \partial \beta \theta^i.
\]

C. Linearizing the Error Function \( f_k \)

The first order Taylor series expansion is now deployed to linearize the error function \( f_k = \log r_k - \tilde{\mu}_k \). The first derivative of \( f_k \) w.r.t the variable \( \theta \) is given by
\[
\frac{\partial f_k}{\partial \theta} = -\frac{2}{\nu} \frac{\partial \tilde{\mu}_k}{\partial \theta} + \frac{\partial \tilde{\mu}_k}{\partial \theta} g_k + \frac{1}{2} \frac{\partial \tilde{\mu}_k^2}{\partial \theta} + (b^2 - 1) \tilde{\mu}_k^2,
\]
where \( \theta \) stands for \( \bar{x}_m, \bar{y}_m, \tilde{\mu}_m, \alpha, \) and \( \beta \). The derivatives of the functions \( g_k \) and \( h_k \) read
\[
\frac{\partial g_k}{\partial \bar{x}_m} = \tilde{p}_m \beta_{km} (\bar{x}_m - \bar{x}_m)(\alpha - \tilde{d}_{km} \ln \beta),
\]
\[
\frac{\partial h_k}{\partial \bar{x}_m} = 2 \tilde{p}_m \tilde{d}_{km} \beta_{km} (\bar{x}_m - \bar{x}_m),
\]
\[
\frac{\partial g_k}{\partial \bar{y}_m} = \tilde{p}_m \beta_{km} (\bar{y}_m - \bar{x}_m)(\alpha - \tilde{d}_{km} \ln \beta),
\]
\[
\frac{\partial h_k}{\partial \bar{y}_m} = 2 \tilde{p}_m \tilde{d}_{km} \beta_{km} (\bar{y}_m - \bar{y}_m),
\]
\[
\frac{\partial g_k}{\partial \alpha} = -\sum_{m \in M} (\tilde{p}_m \tilde{d}_{km} \beta_{km} \ln \tilde{d}_{km}),
\]
\[
\frac{\partial h_k}{\partial \alpha} = -2 \sum_{m \in M} (\tilde{p}_m \tilde{d}_{km} \beta_{km})^2 \ln \tilde{d}_{km},
\]
\[
\frac{\partial g_k}{\partial \beta} = \sum_{m \in M} (\tilde{p}_m \tilde{d}_{km} \beta_{km} \ln \tilde{d}_{km}),
\]
\[
\frac{\partial h_k}{\partial \beta} = 2 \sum_{m \in M} (\tilde{p}_m \tilde{d}_{km} \beta_{km})^2 \ln \tilde{d}_{km},
\]
\[
\frac{\partial \tilde{g}_k}{\partial \beta} = \tilde{p}_m \tilde{d}_{km} \beta_{km},
\]
\[
\frac{\partial \tilde{h}_k}{\partial \beta} = 2 \tilde{p}_m \tilde{d}_{km} \beta_{km},
\]
where \( \psi_{k'k} \) and \( q_{km} \) are the entries of the pre-processing matrix \( \Psi \) and the matrix \( Q = \Psi \Phi \) that is defined by
\[
\Phi = \text{orth}(\Phi^r)', \quad \Psi = \text{orth}(\Phi^r)'.
\]

The symbol \( \dagger \) stands for the Moore–Penrose inverse and orth(X) is an orthogonal basis for the range of matrix \( X \). The authors in [25] apply such a pre-processing by multiplying both sides of the equation \( r = \Phi s \) with \( \Psi \) since the sensing matrix \( \Phi \) does not possess the incoherence property. The entry \( km \) of the sensing matrix \( \Phi \) at \( i \)th iteration is given by
\[
\phi_{km} = \frac{P + \tilde{P}}{2} (d_{km}^{\alpha-1} (\beta^{i-1}) d_{km}^{\beta^i-1}).
\]
The constraint equations (24e) to (24h) guarantee that the estimates \( \hat{\rho}_n, \hat{\alpha}, \hat{\beta}, \) and \( \hat{N} \) are in their admissible ranges, given the fact that the number of targets must be an integer. Moreover, (24i) together with (24j) relaxes \( \|s\|_0 \) to its \( \ell_1 \)-norm. Having incorporated the path loss model uncertainties as well as the number of targets, Alg. 1 summarizes the \( \ell_1 \)-localization technique to jointly estimate the number, position, and transmit power of the targets as well as the values of PLE and PLF. Note that the estimation of \( N \) can change from iteration to iteration. After the iteration \( I_1 \) the variables \( s_m \) become ineffective, since \( \mu \) is set to zero. Furthermore, the number of grid points becomes \( N \) since \( G \) becomes one.

**Algorithm 1**

The \( \ell_1 \)-localization heuristic to jointly estimate the transmit power, positions of targets as well as the parameters of the path loss model in the presence of precipitation uncertainty.

**initialization:**
- set the grid granularity \( G \in \mathbb{N} \)
- set the area width \( 2w \geq 0 \)
- \( \delta \leftarrow \frac{w}{G} \)
- \( N \leftarrow \hat{N} \)
- set the area width \( w \leftarrow \frac{w}{G} \)
- let \( \hat{p}_n = \frac{1}{2}(P + \bar{P}), (\hat{x}_n, \hat{y}_n) = (0, 0), \forall n \in \hat{N} \)
- let \( \alpha^0 = 2, \beta^0 = 1 \)
- let \( M_0 = G^2 \)
- set the number of iterations by setting \( I_1, I_2 \in \mathbb{N} \)
- \( \mu \leftarrow 1 \)

for \( \ell \leftarrow 1 \) to \( I_1 + I_2 \) do

if \( \ell = I_1 + 1 \) then

\( \mu \leftarrow 0 \)

\( G \leftarrow 1 \)

end if

let \( M = NG^2 \) be the number of grid points

define \( G(\nu, G) \) and \( \mathcal{P}(\nu, G) \) using (19)

let \( (\hat{x}_n^{-1}, \hat{y}_n^{-1}) \in G(N, G) \) and \( \bar{p}_n^{-1} \in \mathcal{P}(N, G) \)

find \( s_m^*, d\hat{x}_m, d\hat{y}_m, d\bar{p}_m, \nu^*, \delta^* \) and \( \delta^* \) using (24)

find \( \mathbb{I}_M, \mathbb{I}_1, \cdots, \mathbb{I}_\nu \), using k-means clustering

update \( \hat{p}_n, \hat{x}_n, \hat{y}_n, \forall n \in \hat{N} \) using (18)

update \( \alpha^*, \beta^* \) using (20)

end for

\( \mathcal{X} \leftarrow \{(\hat{x}_n, \hat{y}_n, \hat{p}_n) \mid \forall n \in \hat{N} \} \)

\( A \leftarrow \{\hat{N}, \hat{\alpha}, \hat{\beta} \mid \hat{N} = \nu^*, \hat{\alpha} = \alpha^{I_1+I_2}, \hat{\beta} = \beta^{I_1+I_2}\} \)

return \( \mathcal{X} \) and \( A \)

**iv. Simulations**

Since there is no work with the same assumptions as this paper, the results cannot be compared with any other works, unfortunately. The only work, except for the previous papers [12, 13] of the authors, that has similar assumptions is [26]. It, nevertheless, deals with a fingerprinting problem. Therefore, a fair comparison with its results is not straightforward. In what follows the performance of the proposed \( \ell_1 \)-localization is evaluated by means of computer simulations.

In the simulation setup \( \bar{P} = 1, P = 0.5 \) and \( w = 1 \)Km are chosen. The results are the outcome of \( J = 5000 \) simulation realizations, in each of which the position of sensors and realization of \( \zeta_{kns} \)s are random, while the transmit power and position of targets are always the same. Let the estimated position of the \( n \)th target at \( j \)th realization be denoted by \( (\hat{x}_n^j, \hat{y}_n^j) \). Then, positioning root mean square error (RMSE) in meters is defined by

\[
\delta = \sqrt{\frac{1}{JN} \sum_{j=1}^{J} \sum_{n=1}^{N} (\hat{x}_n^j - x_n)^2 + (\hat{y}_n^j - y_n)^2}.
\]

Let the maximum positioning error at \( j \)th iteration, i.e.,

\[
\delta_j = \max_{n \in \hat{N}} \sqrt{ (\hat{x}_n^j - x_n)^2 + (\hat{y}_n^j - y_n)^2}.
\]

be a sample drawn from the distribution of a random variable, e.g., \( \Delta \), Then, the error function

\[
P_d := Pr(\Delta > d) = 1 - F_\Delta (d),
\]

stands for the probability that at least one of the targets is localized with an error of more than \( d \) meters. Note that \( F_\Delta \) is the empirical cumulative distribution function (cdf) of the error \( \Delta \). Similarly, the RMSE of the transmit power is defined by

\[
\bar{p} = \sqrt{\frac{1}{NJ} \sum_{j=1}^{J} \sum_{n \in \hat{N}} \rho_n^j},
\]

where \( \rho_n^j := (p_n - \hat{p}_n^j)^2 \) is the square error of the estimated power of target \( n \) at \( j \)th realization. The \( \ell_1 \)-localization technique in Alg. 1 is simulated for three different scenarios:
The error probability $P_d$ against positioning error $d$ for $N = 2$ targets achieved by Alg. 1. The transmit power of the targets are unknown. The PLE and PLF are unknown and their actual values are $\sigma = 3.4$ and $\beta = 0.99655$. The algorithm is deployed with $I_1 = 8$ and $I_2 = 12$ number of iterations. The values of $\delta$, $\rho$, $\sigma$ and $\beta$ are shown in the legend.

1) The number of targets is known, but PLE and PLF are unknown, see Fig. 3. The actual value of the PLE is chosen to be $\alpha = 3.4$, while the actual value of PLF is $\beta = 10^{-0.0015} \approx 0.99655$. This value means the rain fade attenuation $\gamma_r = 15$dB/km, e.g., the rainfall rate $R = 100$mm/hr at 24GHz [11, Fig. 7]. In the problem (24) $\alpha = 1$, $\beta = 6$, $\beta = 0.96$ and $N = N$.  

2) The number of targets is unknown and can be between $N = 1$ and $N = 4$, as shown in Fig. 4. The values of $\alpha = 2$ and $\beta = 1$ are considered as known, which indicates that $\sigma = \alpha = 2$ and $|\beta| = 1$.  

3) Neither the number of targets, nor PLE nor PLF is known, see Fig. 5. In this scenario, $N = 1$, $N = 4$ and the actual values of $\alpha$ and $\beta$ are chosen to be $3.4$ and $0.99655$, respectively. In the constraint (24f) $\alpha = 1$ and $\beta = 6$. In the constraint (24g) also $\beta$ is set to 0.96.

In all these three scenarios the transmit power of the targets are unknown. Under the aforementioned assumptions, all algorithms in [12–14] fail since uncertainties in values of $N$, $\alpha$, and $\beta$ are overlooked.

As it is evidenced by the figures 3 to 5, the parameters of the path loss model can be well estimated using the proposed algorithm. For instance for $N = 2$ and $\gamma \to \infty$, in case the number of targets is correctly estimated $Pr(\Delta > 1\text{mm}) \approx 0$. Figures 4a and 5a shows that for the case $N = 1, 2$, the probability of the correct estimation of $N$ is between 87%-100%. Obviously, for $N > 2$ targets a much bigger number of sensors is required for a successful localization.

Furthermore, for $\gamma = 40$ the positioning error of all the targets is very unlikely to be more than 10m, if the number of sensors is sufficient, i.e., $K > 10$. In the strong shadowing conditions, i.e., higher values of $\sigma$, more SNs can be deployed to make the estimation more reliable. This is hopefully viable since RSS-based localization requires inexpensive and unsophisticated sensors, on the one hand. On the other hand, the proposed $\ell_1$-localization method has a low-complexity and can solve the problem for higher values of $K$, efficiently.

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Fig. 5: The localization performance achieved by Alg. 1, for an unknown number of targets under path loss model uncertainties. In the algorithm $N$ can be between $\hat{N} = 1$ and $N = 4$. The PLF and PLF are assumed unknown and their actual values are $\alpha = 3.4$ and $\beta \approx 0.99655$. The algorithm is deployed with $G = 5$, $I_1 = 8$ and $I_2 = 12$ and the number of sensors is $K = 20$. 

(a) The probability $\Pr(\hat{N} = N)$ against number of targets $N$.

(b) The error probability $P_{e}$ against positioning error $d$. The probability shows only those cases that the number of targets are correctly estimated. The values of $\delta$, $\bar{\gamma}$, $\bar{\beta}$ and $\bar{\alpha}$ are shown in the legend.