Phonon excitation for $^{4}\text{He}$ confined in nanometer-size uniform channel under pressure

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Abstract. We have studied the heat capacity of pressurized $^{4}\text{He}$ confined in the honeycomb structure of a nanometer-size uniform straight channel at low temperature. The temperature ($T$) dependence is almost in proportion to $T^3$ between 0.4 and 0.7 K, and a slight deviation from the $T^3$-dependence appears below this temperature region. The deviation is possibly explained by the beginning of the crossover to the one-dimensional (1D) phonon state which is continuous in the axial direction, from the 3D-like state with the excitation to discrete energy levels in the cross section.

1. Introduction
Properties of liquid $^{4}\text{He}$ in reduced dimensions are one of the interesting topics of quantum many-body systems. Recently, we have studied liquid $^{4}\text{He}$ confined in a one-dimensional (1D) nano-meter size channel by heat capacity and torsional oscillator measurements.[1] The heat capacity of liquid $^{4}\text{He}$ in the channel has a bend at a certain temperature $T_B$, and below this temperature a small amount of the superfluid fraction appears. This means that $^{4}\text{He}$ atoms enter a BEC-like low entropy state below $T_B$. In addition, it was found that the superfluid fraction shows the second growth far below $T_B$ under low pressure, accompanied by a dissipation. This two-stage growth is possibly a feature of the quasi-1D system.

To understand the temperature dependence of the superfluid fraction, it is of importance to study the elementary excitation at low temperature. In the 1D nano-meter channel, the excitation at low temperature is expected to be the 1D phonon. As the temperature is increased, the excitation is modified into the 3D-like phonon because of the thermal excitation to discrete energy levels in the cross section. In this proceeding, we first report a slight deviation from the $T^3$-dependence of the heat capacity at low temperature. Then, we discuss this behavior from the viewpoint of the crossover in the phonon excitation.

2. Experimental results
We have measured the heat capacity of liquid $^{4}\text{He}$ confined in a porous material, FSM [2], which forms a honeycomb structure of 1D uniform nanometer-size straight channel without interconnection. In the present experiments, we adopted FSM whose nominal radius of the channel is 1.4 nm. The measurements were performed in the pressure region below 2.5 MPa. The experimental details are explained elsewhere.[1]
Figure 1 shows the log-log plot of the heat capacity of $^4$He in the channel below 1 K. Under 0.03 MPa, the temperature dependence is almost in proportion to $T^3$ below 0.7 K. As the pressure is increased, the magnitude of the heat capacity is increased, and a slight deviation from the $T^3$-dependence appears below about 0.4 K. In the case of bulk liquid $^4$He, it is well known that the 3D phonon is dominant in the temperature region below about 0.7 K. The observed $T^3$-dependence suggests that the 3D-like phonon state is dominant, and the slight deviation below about 0.4 K is possibly due to the crossover to the 1D phonon state.

3. Discussion
To understand the temperature dependence, we calculated the phonon heat capacity of $^4$He confined in a cylindrical pore with the radius of $R$, in the same manner as Cole et al.[3] In this geometry, the eigenvalue problem of the Helmholtz equation for density fluctuations $\delta \rho (r)$ can be analytically solved as

$$\delta \rho (r) \propto e^{i(kz + m\varphi)} J_m(kr),$$

which describes the free propagation parallel to the $z$ direction, i.e., the pore’s axis. Here, $k$ is the linear wave number in the $z$ direction, $\varphi$ is the azimuthal variation, and $J_m(kr)$ is a Bessel function. The boundary condition that the normal component of the velocity vanishes at the wall of radius $r = R$ causes a discrete spectrum,

$$\omega^2 = c^2(k^2 + k_{mn}^2).$$

Here, $c$ is an isotropic phonon velocity, and the discretized wave number $k_{mn}$ is given by the $n$-th zero of the derivative of Bessel function $J_m'(\alpha)$, where $\alpha_{mn} = k_{mn}R$. Using the above spectrum, the total phonon heat capacity can be written as

$$\frac{C}{Nk_B} = \frac{k_BT}{\pi n_1 \hbar c} \sum_{mn} \int_0^{x_M} dx \frac{x^3}{\sqrt{x^2 - x_{mn}^2}} n(x)[1 + n(x)],$$

where $x = \hbar \omega/k_BT$, $x_{mn}$ is $x$ at $k = 0$ nm$^{-1}$, $x_M$ is a cut-off, and $n(x) = 1/(e^x - 1)$ is the Bose distribution function. Here, the 1D density $n_1 = \rho \pi R^2$ is obtained from the number density of bulk liquid $\rho$. The cut-off is chosen from a maximum wave vector $k_M = 10$ nm$^{-1}$, corresponding to the maxon excitation in the dispersion relation of liquid $^4$He.

The calculation was performed for the bulk density of liquid $^4$He $\rho = 21.9$ nm$^{-3}$. We used the phonon velocity of 140 m/s, which was evaluated from fitting the experimental data to the $T^3$-dependence of 3D phonon.[4] And as the radii, we adopted typical values for $^4$He confined in the
honeycomb structure of FSM, and the structure itself. The smallest radius of 0.8 nm corresponds to the radius of the inert layer tube. The middle one of 1.6 nm is from the circumradius of the hexagonal cross section of the pore, and the largest one of 2.2 nm, from the half of the lattice constant of the honeycomb structure.

The calculated heat capacity is summarized in Fig. 2. At around the lowest temperature, all heat capacities show the $T$-linear dependence, corresponding to the 1D phonon state, $(mn)=(00)$. As the temperature is increased, it deviates upward from the $T$-linear dependence. This deviation occurs at the highest temperature for the smallest radius, due to a large gap of the discrete energy levels. The difference from the 1D phonon heat capacity exceeds 10% at about 0.34, 0.17 and 0.13 K for 0.8, 1.6, and 2.2 nm, respectively. At these temperatures, the ratio in population of the second excited state at $k = 0$ nm$^{-1}$ to the ground state is about 0.1%.

In comparison with the present experiments, the 1D phonon state for 0.8 nm exists at high temperature where the $T^3$-dependence was observed. On the other hand, the temperature dependence above around 0.3 K for the radius above 1.6 nm is very close to the experimental results. Thus, assuming that the phonon propagates in the inert layer similarly to the liquid, we derived the heat capacity per atom by dividing the observed heat capacity by the total amount of $^4$He in the pore. Here, the amount of $^4$He was obtained using the bulk density under the same pressure. Under the fixed radius of 1.6 nm, we fitted the data below 0.5 K with only one fitting parameter of the phonon velocity, $c$. As shown in Fig. 3 (a), the temperature dependence is well explained. The obtained phonon velocity is shown as a function of pressure in Fig. 3 (b). It varies from 160 to 110 m/s, as the pressure is increased. Although the value is small compared with the sound velocity of bulk $^4$He, it is similar to that of the phonon velocity obtained for the film region in the previous work.[5] The pressure dependence is, however, opposite to that of the bulk. Its origin remains an open question.

Finally, we comment on the case of 2.2 nm. The experimental data are also well fitted under the radius of 2.2 nm, using the product of $n_1$ and $c$ as the fitting parameter. However, since the elastic constant of single-layer silicate is not known, it is difficult to determine whether the phonon can propagate in the complex system of $^4$He and silicate layer.

The heat capacity measurements down to the lower temperature will reveal whether the 1D phonon state appears. And the absolute value of the specific heat in the region of 1D phonon state may give us the information of the effective radius.

4. Summary
We have studied the heat capacity of pressurized $^4$He confined in the honeycomb structure of a nanometer-size uniform straight channel. At low temperature, a slight deviation from the $T^3$-
Figure 3. (a) Temperature dependence of the specific heat. Solid curves represent the fitted results to Eq. (3). Symbols are the experimental data. (b) Phonon velocity obtained by the fitting.

dependence was observed. This temperature dependence is possibly explained by the beginning of crossover to the 1D phonon state.

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References
[1] Taniguchi J, Fujii R, and Suzuki M 2011 Phys. Rev. B 84 134511
[2] Inagaki S, Koiwai A, Suzuki N, Fukushima Y, and Kuroda K 1996 Bull. Chem. Soc. Jpn. 69 1449
[3] Cole M W and Hernández E S 2002 Phys. Rev. B 65 092501
[4] Taniguchi J and Suzuki M 2011 Phys. Rev. B 84 054511
[5] Matsushia Y, Taniguchi J, Toda R, Begami H, Matsushita T, Hieda M, and Wada N 2008 J. Low Temp. Phys. 150 342