The Preferred Frame and Poincaré Symmetry

Jakub Rembieliński †, Paweł Caban‡

Department of Theoretical Physics, University of Łódź
Pomorska 149/153, 90–236 Łódź, Poland

December 1996

Abstract

In this paper we describe a covariant canonical formalism for a free
time-like (massive) as well as space-like (tachyonic) particle in the frame-
work of nonstandard synchronization scheme. In this scheme one is able
to introduce absolute causality without breaking the Poincaré invariance.

1 Introduction

As is well known, special relativity, irrespectively of its great success in local
description of the reality, has a number of disadvantages. They are related to
the Minkowski space-time notion rather than to the very well experimentaly
supported Poincaré symmetry. The main difficulties are connected with the
absence of covariant canonical formalism and the lack of dynamics for systems
of relativistic particles [1]. Even more serious problems arise on the quantum
ground where in fact no fully consistent relativistic quantum mechanics for sys-
tems with finite degrees of freedom exists. In particular there is no covariant
notion of localizability and relativistic position operator [1]. In this paper we
briefly describe another point of view, preserving Poincaré symmetry but
changing the space-time notion. We apply this formalism to the simplest phys-
ical system—free particle and show that it is possible to introduce covariant
canonical formalism (Poisson structure) for both time-like and space-like parti-
cles. In the papers [3, 4, 5, 6] these ideas are applied to elaborate the hypothesis
of the tachyonic neutrino as well as to introduce a covariant notion of localiz-
ability on the quantum level.

†Submitted to the Proceedings of the XXI International Colloquium on Group Theoretical
Methods in Physics, 15–20 July, Goslar, Germany
‡E-mail address: jaremb@mvii.uni.lodz.pl, jaremb@krysia.uni.lodz.pl
§E-mail address: caban@mvii.uni.lodz.pl
2 Preliminaries

The main idea is based on two well known facts: (i) definition of a coordinate time depends on the synchronisation scheme; (ii) synchronisation scheme is a convention, because no experimental procedure exists which makes it possible to determine the one-way velocity of light without use of superluminal signals [7, 8, 9]. Therefore there is a freedom in the definition of the coordinate time. The standard choice is the Einstein–Poincaré (EP) synchronisation with the one-way light velocity isotropic and constant. This choice leads to the extremely simple form of the Lorentz group transformations but the EP coordinate time allows a covariant causality for time-like and light-like trajectories only. We choose a different synchronisation, namely that of Chang–Tangherlini (CT), preserving invariance of the notion of the instant-time hyperplane [3, 10]. In this synchronisation scheme the notion of causality is universal and space-like trajectories are physically admissible too. The price is the more complicated form of the Lorentz transformations incorporating transformation rules for velocity of distinguished reference frame (preferred frame). The EP and CT descriptions are entirely equivalent if we restrict ourselves to time-like and light-like trajectories; however a consistent description of tachyons is possible only in the CT scheme. A very important consequence is that if tachyons exist then the relativity principle is broken, i.e. there exists a preferred frame of reference, however the Lorentz symmetry is preserved.

The proper framework to this construction is the bundle of Lorentzian frames; the base space is simply the space of velocities of these frames with respect to the preferred frame. For this reason the transformation law for coordinates incorporates the velocity of distinguished frame. The preferred frame can be locally identified with the comoving frame in the expanding universe (cosmic background radiation frame) i.e. the reference frame of the privileged observers to whom the universe appears isotropic [11].

To be concrete the Lorentz group transformations in the mentioned bundle of frames have the following form [3]:

\[
\begin{align*}
  x' &= D(\Lambda, u)x \\
  u' &= D(\Lambda, u)u
\end{align*}
\]  

(1)

where for rotations \(D(R, u)\) has the standard form while for boosts it reads

\[
D(W, u) = \left( \begin{array}{cc}
  \frac{1}{W^0} & 0 \\
  -W^0 & I + \frac{W \otimes W^T}{(1+\sqrt{1+(W^0)^2})} - W \otimes u^T u^0
\end{array} \right).
\]  

(2)

Here \(W^\mu\) is the four-velocity of \((x')\) frame as seen by an observer in the frame \((x)\) while \(u^\mu\) is the four-velocity of the privileged frame as seen from the frame \((x)\). Notice that the time coordinate is rescaled by a positive factor only. The transformations (1) leaves invariant the metric form

\[
ds^2 = g_{\mu\nu}(u)\, dx^\mu \, dx^\nu
\]  

(3)
with
\[ [g_{\mu\nu}(u)] = \begin{pmatrix} 1 & u^0 u^T \\ u^T & -I + u \otimes u^T(u^0)^2 \end{pmatrix}. \] (4)

Interrelation with coordinates in the EP synchronization \((x^\mu_E)\) is given by
\[ x^0_E = x^0 + u^0 \vec{u} \vec{x}, \quad \vec{x}_E = \vec{x}. \] (5)

However the corresponding interrelations between velocities \(\vec{v}_E\) and \(\vec{v}\) obtained from (5) are singular for superluminal velocities.

### 3 Covariant canonical formalism for the free time-like (massive) particle

Let us consider in detail the case of a free particle associated with a time-like geodesics. The corresponding action \(S\) is of the form
\[ S_{12} = -m \int_{\lambda_1}^{\lambda_2} \sqrt{ds^2} \] (6)
where the square of the time-like line element
\[ ds^2 = g_{\mu\nu}(u) \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda^2 > 0 \] (7)
and the continuous affine parameter \(\lambda\) is defined along the trajectory as monotonically increasing as one proceeds along the curve in a fixed direction.

The equations of motion are obtained by means of the variational principle and reads
\[ \frac{d}{d\lambda} \left( \frac{\dot{x}^\mu}{\sqrt{g_{\mu\nu}(u)\dot{x}^\mu \dot{x}^\nu}} \right) = 0 \] (8)
with \(\dot{x}^\mu = \frac{dx^\mu}{d\lambda} \equiv w^\mu\). Now, we are free to take the path parameter as \(d\lambda = \sqrt{ds^2}\), so the four-velocity \(w^\mu\) satisfies
\[ w^2 = g_{\mu\nu}(u)w^\mu(u)w^\nu(u) = 1 \] (9)
and consequently
\[ \dot{w}^\mu = \ddot{x}^\mu = 0. \] (10)

Defining velocity in a standard way as \(\vec{v} = \frac{d\vec{x}}{d\lambda} = \frac{\vec{u}}{w}\), we can identify the Lagrangian of a free particle related to the action (6); by means of the formulas (3), (4) we have
\[ L = m \sqrt{(1 + u^0 \vec{u} \vec{v})^2 - (\vec{v})^2} \] (11)
Thus the canonical momenta read
\[ \pi_i = \frac{\partial L}{\partial v^i} = m \frac{v^i - u^0 u^i (1 + u^0 \vec{u} \vec{v})}{\sqrt{(1 + u^0 \vec{u} \vec{v})^2 - (\vec{v})^2}} = -m \omega_i \]  
where we have used eq. (9). The Hamiltonian is
\[ H = \pi_k v^k - L = \frac{m(1 + u^0 \vec{u} \vec{v})}{\sqrt{(1 + u^0 \vec{u} \vec{v})^2 - (\vec{v})^2}} = +m \omega_0 \]  
Therefore the covariant four-momentum \( k_\mu \) is
\[ k_0 = H = m \omega_0, \quad k_\mu = -\pi_\mu = m \omega \]  
i.e. \( k_\mu = m \omega_\mu \).

Notice that
\[ k^2 = g^{\mu \nu} (u) k_\mu k_\nu = m^2 \]  
and the condition \( H = m \omega_0 \geq m \) holds in each inertial frame. The Hamilton equations for massive particle in this synchronization have the form
\[ \frac{d\vec{x}}{dt} = \frac{\partial H}{\partial \pi}, \quad \frac{d\pi_k}{dt} = -\frac{\partial k_0}{\partial \pi} = \vec{v}, \quad \frac{dk}{dt} = -\frac{\partial H}{\partial \vec{x}} = 0. \]  
From the second equation it follows that \( \frac{d\vec{x}}{dt} = 0 \). It is important that in this synchronization we can define a Poincaré covariant Poisson structure contrary to the standard EP synchronization case. Namely, the unique definition of a Poisson bracket of two observables \( A \) and \( B \) is given by
\[ \{A, B\} = -\left( \delta^{\mu \nu} - \frac{uk^\mu}{uk} \right) \left( \frac{\partial A}{\partial \pi^\mu} \frac{\partial B}{\partial \pi^\nu} - \frac{\partial B}{\partial \pi^\mu} \frac{\partial A}{\partial \pi^\nu} \right) \]  
with \( uk = u_\mu k^\mu = u_0 k^0 \); this last equality follows from the fact that \( u_k = g_{k \mu} (u) u^\mu = 0 \).

It is easy to see that the Poisson bracket defined by the relation (16) satisfies all necessary conditions:
— It is linear with respect to the both factors, antisymmetric, satisfying the Leibniz rule and fulfill the Jacobi identity;
— It is manifestly Poincaré covariant in the CT synchronization;
— It is consistent with the Hamilton equations;
— It is easy to check that the dispersion relation (14), \( k^2 = m^2 \), is consistent with this bracket i.e. \( \{k^2, k_\mu\} = \{k^2, x^\mu\} = 0 \); therefore we do not need to introduce a Dirac bracket.

4 Covariant canonical formalism for the free space-like (tachyonic) particle

As is well known in the standard EP synchronization in special relativity space-like trajectories are ruled out because of causality breaking. On the other hand,
in the CT synchronization scheme of special relativity the time component is only rescaled by a positive factor under Lorentz group transformations. Therefore it is possible to introduce notion of absolute causality and consequently to overcome all difficulties of the standard approach [3]. Now, the canonical formalism for a space-like particle can be developed in the complete analogy with the time-like case. The corresponding action functional reads

$$S_{12} = -\kappa \int_{\lambda_1}^{\lambda_2} \sqrt{-ds^2}$$

and under the appropriate choice of the affine parameter \(\lambda\) we have

$$\dddot{x}^\mu = 0, \quad \omega^\mu = \dot{x}^\mu, \quad \omega^2 = -1$$ (18)

Let us focus our attention on the last constraint in the eq. (18). Obviously it defines an one-sheet hyperboloid; in particular in the preferred frame (for \(u = \tilde{u} = (1, \vec{0})\) \(g_{\mu\nu}(\tilde{u}) = \eta_{\mu\nu}\), so \(\eta_{\mu\nu} w^\mu(\tilde{u})w^\nu(\tilde{u}) = -1\), like in the EP synchronization. However, there is an important difference; namely under Lorentz boosts the zeroth component \(w^0(u)\) of \(w^\mu\) is rescaled by a positive factor only (see eq. (2)) i.e. \(w^0(u') = 1/W_0 w^0(u)\). Therefore, contrary to the EP synchronization, in this case points of the upper part of the above hyperboloid (satisfying \(w^0(u) > 0\)) transforms again into points of the upper part. This allows us to define consistently the velocity of a tachyon:

$$\vec{v} = \frac{d\vec{x}}{dx^0} = \frac{\vec{v}}{w^0}$$ (19)

because now, for each observer, the tachyon speed is finite (i.e. \(|\vec{v}| < \infty, w^0 > 0\).

We see that the infinite velocity is a limiting velocity, like in the non-relativistic case (it corresponds to \(w^0 = 0\) which is an invariant condition). Notice that the constraint relation in eq. (18) implies that velocity of a tachyon moving in a direction \(n\) is restricted by the inequality

$$|\vec{v}| = \frac{1}{1 - \vec{n}\vec{u}w^0} < |\vec{v}| < \infty.$$ (20)

Furthermore, the transformation law for velocities in the CT synchronization, derived from (1-2) reads

$$\vec{v}' = W^0 \left[ \vec{v} + W \left( \frac{(W\vec{v})}{1 + \sqrt{1 + (W^2)}} - W_0 (\vec{u}\vec{v}) - 1 \right) \right]$$ (21)

We see that the transformation law (21) is well defined for all velocities (sub- and superluminal). Recall that in the EP scheme tachyonic velocity space does not constitute a representation space for the Lorentz group. A technical point is that the space-like four-velocity cannot be related to a three-velocity in this
case by the relation \( \vec{v}_E = \frac{\vec{w}_E}{w_E} \), because \( w_0^2 \) can take the value zero for a finite Lorentz transformation.

Let us identify the Lagrangian of a free tachyon related to the action (17); we obtain

\[
L = \kappa \sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}
\]

Thus the canonical momenta read

\[
\pi_k = \frac{\partial L}{\partial \dot{v}^k} = \frac{\kappa \left[ v^k - u^k u^0 (1 + u^0 \vec{u} \vec{v}) \right]}{\sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}} = -\kappa \omega_k
\]

The Hamiltonian has the following form

\[
H = \pi_k v^k - L = \frac{\kappa (1 + u^0 \vec{u} \vec{v})}{\sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}} = +\kappa \omega_0
\]

Therefore the covariant four-momentum \( k_\mu \) of tachyon is \( k_\mu = \kappa \omega_\mu \).

Notice that

\[
k^2 = g^{\mu\nu} (u) k_\mu k_\nu = -\kappa^2
\]

and the energy \( H = \kappa \omega_0 \) has in each inertial frame a finite lower bound corresponding to \( |\vec{v}| \rightarrow \infty \), i.e.

\[
E > \frac{\kappa \sqrt{1 - (u^0)^2 \cos \phi}}{\sqrt{1 - \left( \frac{\kappa \sqrt{1 - (u^0)^2 \cos \phi}}{\sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}} \right)^2}} \equiv \mathcal{E}(u^0, \phi)
\]

where \( \cos \phi = \frac{\vec{u} \vec{v}}{|\vec{u}| |\vec{v}|} \).

Therefore, contrary to the standard case, the energy of tachyon is always restricted from below by \( \mathcal{E}(u^0, \phi) > -\infty \). Moreover, if we calculate the contravariant four-momentum \( k^\mu = g^{\mu\nu} (u) k_\nu = \kappa \omega^\mu \) we obtain that

\[
k^0 = \frac{\kappa}{\sqrt{(\vec{v})^2 - (1 + u^0 \vec{u} \vec{v})^2}} > 0
\]

which confirm our statement that the sign of \( k^0 \) is Lorentz invariant also for tachyons. Finally, the Poisson structure can be introduced in a full analogy with the time-like case.

**Acknowledgments**

One of us (JR) is grateful to Professor H.D. Doebner for his kind invitation to the XXI International Colloquium on Group Theoretical Methods in Physics.
References

[1] K. Sundermeyer, “Constrained Dynamics”, Springer–Verlag, Berlin, 1982.

[2] H. Bacry, “Laocalizability and Space in Quantum Physics”, Springer–Verlag, Berlin, 1988.

[3] J. Rembieliński, “Tachyons and the preferred frame” to appear in Int. J. Mod. Phys. A [hep-th/9607233].

[4] J. Ciborowski and J. Rembieliński, “Experimental results and the hypothesis of the tachyonic neutrinos” The talk presented at 28th International Conference on High Energy Physics, Warsaw, July 1996, [hep-ph/9607477].

[5] J. Ciborowski and J. Rembieliński, “Tritium decay and the hypothesis of the tachyonic neutrinos” submitted to Phys. Lett. B.

[6] P. Caban and J. Rembieliński, “Localization of quantum states and the preferred frame”, Preprint, Łódż University, 1996.

[7] M. Jammer, In “Problems in the Foundations of Physics”. North-Holland, Bologne, 1979.

[8] C. M. Will, Phys. Rev. D45, 403 (1992).

[9] R. Mansouri and R.U. Sexl, Gen. Relativ. Grav. 8, 479, 515, 809 (1977).

[10] J. Rembieliński, Phys. Lett. A78, 33 (1980).

[11] S. Weinberg, “Gravitation and Cosmology”, J. Wiley & Sons, New York, 1972.