Students’ thinking: arithmetic in solving a mathematical problem

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Abstract. The students in secondary school have difficulties making the transition from arithmetic to algebra. In order to obtain the students approach while solving the problem, the students are given a mathematical problem that can be solved by using arithmetic or algebraic. This study is implemented in class grade 7, and they have learnt the introduction of algebra. This research was using a qualitative method with description design. Six students who are a participant in this study have excellent communication and give different strategies while solving the problems. The students are given a mathematical test problem, and then the students are interviewed based on their answer sheet in the test in order to get in-depth information from their strategies. From the result of students are given, we acquire that the arithmetical approach is dominated in their strategy to solve the problem. Many of the students do not realize the using of the quantities of the unknown in that problem. Arithmetic is the approach that accomplished to coverage the students to solve the problem and the students just do numbers and operation, and focus in numeric answer. Most of the student in a secondary school in seventh grade is direct think arithmetically than algebraically.

1. Introduction

Several countries, including Indonesia, students are introduced to algebra after six years of learning arithmetic, geometry, and possibly some data topics for elementary school. In the following phase, they obtain the movement of the role of numbers, properties and arithmetic operations to the introduction of symbols, properties and conversions in general. Fong stated algebra curriculum, the formal introduction of algebra, in Singapore introduce at primary six. The emphasize of this level is on the developing of algebraic concepts and algebraic manipulation skills. Algebraic thinking is not mentioned explicitly, but the activities promote the development of algebraic thinking [1]. The students in elementary school are expected simultaneously develop their thinking arithmetically and algebraically, although there are different emphases in their idea, the arithmetic and algebra would come to be view as being inextricably interconnected [2].

Algebra is the gateways to higher mathematics, and also become the complex system in the world. In the other hand, algebra has a proposition for the students as a difficult subject and hard to learn [3-5]. Traditionally, most school mathematics separate the study of arithmetic and algebra. This separation makes algebra it more difficult for students to learn in later grade [6,7]. Instruction design needs reformation and new challenging, that nearly to algebra it can contribute maximal opportunities of the students learn algebra [8]. Otherwise, algebra becomes a building that blocks their path for the further.
Algebra is also known as the generalize arithmetic [9–13], and algebraic thinking involves the development of ways thinking within activity with or without symbolic [6]. Algebraic thinking is founded on numeracy and computational proficiency [14], but the process of development identified of algebraic thinking as a habit of mind enables of the students to express the mathematical structure and relationships, such as the structure in arithmetical and symbolic expression [3,15,16]. Thinking requires a problem or concreate situation that will be solved through a variation of the method. The solution of the student can be shown from their thinking arithmetic or algebraic way. Solving a problem by using either arithmetic or algebraic help students build arithmetic and algebraic ways of thinking about the problem.

Algebraic thinking is organized into two general categories (1) problem solving; and (2) representation and reasoning. Problem-solving knows what the student to do, have a variety method and strategies. Representation and reasoning include how the student solves the problem by using a model or tool to solve the problem.

The construction thinking from arithmetic to the algebra of the students rely on generalized arithmetic—the sign of mathematics and the mathematical thinking help construction of algebraic thinking. The process of the sign in the student thinking gives the importance process. The problem sign in arithmetic and algebra is still needed consideration. The students are merely aware of the sign. The role of transitivity and symmetry in the meaning of the equal sign is essential to process to construct the students mathematical thinking. Most students, even in the higher school, see the equal sign as the operationally view. Rarely of them see the equal sign as the relational view—this meaning of the equal sign making the transition of arithmetic to algebraic hard to learn.

In this time, our article gives an example that illustrates students answer while solving problem-solving by using the arithmetic and the awareness of the student about equal sign. Historically, algebra grows up from arithmetic. Nevertheless, there is a significant transition that students need to solve the problem with a variety approach. This approach begins in calculation then the new method that strictly to algebra.

2. Method
This study is qualitative methods with a descriptive design. The participants consisted of seventh grade in a private secondary school in Pekanbaru, Riau, and they have an average 12-13 years old. Six students took part in this study, the determination of these students was the suggestion of the teacher and the result of the answer sheet. The students have heterogenous academic and live around their school. The students have learnt the introduction of algebra. The teacher in this school is a cooperating and very kindness while doing this study. The student was given test, mathematical problems, and then the students were given opportunities 80 minutes to solve the problems. The purpose of the test was to obtain the student mathematical thinking, particularly in their thinking in arithmetically and algebraically. As the guidance of the problem, the researcher adopted the problem that Kieran suggest in their article in cognitive processes involved in learning algebra [17]. The data was collected by using a problem-solving task and interview. Furthermore, the analysis data in this study was using qualitative analyses data.

3. Result and Discussion
3.1. Solving problem in arithmetic or algebraic approach
The transition from arithmetic to algebra is difficult for the students, even those for who are sufficiently proficient in arithmetic. The student needs many adjustments [6], [18]. Kieran said that the students who have a successful transition from arithmetic to algebra require the following adjustments in their developing an algebraic way of thinking. There are (1) a focus on relations and not merely on the calculation of numeric answer; (2) a focus on operations as well as their inverses, and on the related ideas of doing/undoing; (3) a focus on both representing and solving a problem rather than on merely solving it; (4) a focus on both numbers and letters, rather than on numbers alone, and (5) refocusing of the meaning of the equal sign. These adjustments certainly fall within the domain in arithmetic, but they represent an alteration the direction of developing ideas of algebra.
Our analysis of Indonesia curricula showed that there are lacking interconnection between arithmetic and algebra as a common goal in elementary and secondary school [19]. In this section, we present the problem that may help the students adjust developing their algebraic ways of thinking in the seventh grade.

The idea of this problem is to use both arithmetic and algebraic approaches to solve the mathematical problem. It is a following problem given to the students: Dani went to visit his grandmother who gave him Rp, 5,000.00. Then he bought a book costing Rp 4,500.00. If he has Rp 9,000.00 left, how much money did he have before visiting his grandmother?. It is common initially solving by the arithmetic approach for the students. However, the algebraic approaches take part in that problem.

When the problem gives to the students, most the students solve by using the arithmetic approach. The students do not realize the algebraic way in that problem. Here the example answer the student give from a problem.

![Student solve the problem by an arithmetic approach](image)

Figure 1. Student solve the problem by an arithmetic approach

Figure 1 shows that the students answer using the arithmetic way to solve the problem. The students can identify the problem from the numerical given in that problem. The student gives the answering by arithmetic than algebraic way. The arithmetic way is dominant for the students, although the student has proficiency in arithmetic. The students give calculating and try to find the solution—the student focus on the calculation of the numerical answer. the student [2,18,20].

The result signals that the students can handle the numerical task, but it is much more difficult for the student to solve by the algebraic way. The step from arithmetic is sufficiently for the students to solve the problem, and the student is hard to find the algebra solution, however, the students in grade seven. To know the student thinking, the researcher confirms about the students’ answer. The following excerpts of the student and researcher do get in-depth information of the students.

**Researcher**: why you begin solving the problem by subtracted 5,000 and 4,500?
**Student**: because 5,000 is enough to pay a book. So I subtract 5,000 and 4,500 then I get 500. if money is 9,000 left, I subtract 9,000 and 500 finally I got 8,500.

The student identifies the problem from the known number then tried to find a logical reason to find their solution. It is a compelling reason how the student tries to connect their daily life using the money, whether the money enough to pay a book or not. It indicates that the student solves the problem by arithmetically than algebraically, but their development of his thinking about the unknown closely appearance. Every stage that student is a conjecture in our expected. The student needs experience how to use the equation that will be useful and understand how the problem to solve it. Sometimes the student needs more explanation about their strategy, either arithmetically or algebraically. It is necessary to see the step by step that their following in the next strategy [21]. It might common to appear as the transition of the student not to see the equation as the solving problem. The students should encourage algebraic ways to come to the see of the equation as the solver the problem.

3.2. The meaning of the symbols and signs in arithmetic and algebra

Arithmetic and algebra have the same many symbols and signs such as the equal sign and the addition and subtraction signs. However, algebra leads arithmetic, the interpretation meaning of the symbols in
algebra difference from the arithmetic. In arithmetic, the equal sign is used more to signal as the result than to express the symmetric and transitive relative. In algebra, the equal sign as a relational symbol. The previous research has shown that many students have an operational view than relational view [22, 23]. In this case, Indonesian students also think the equal sign as the operational view, however the students in the higher schools. Here the example of student answer and their view of the equal sign as a signal "do something".

Figure 2. Student see the equal sign as an operational view

Figure 1 shows that seventh grade often write $9,000 - 4,500 = 13,500 - 5,000 = 8,500$. The balance and the transitivity of the equal sign are often violated. The student has a conception view that the equal sign as an operational than a relational symbol. The equal signs read as "it gives" as a left-to-right directional signal. The previous research was also found in this case. Vermeulen in South Africa said that students in grade 5 dan 6 have misconceptions about equal sign [22] and the other researcher, Knuth et al., also found this case, the students response the equal sign as operational means "add the numbers" or "the answer" [24]. At this initial level, students interpret the equal sign as operationally [3]. In algebra, the equal sign is a signal in relationally. The different meaning of these symbols should be student awareness to solve the problem. In algebra function of the equal sign as relationally will be used in solving the equation $ax + b = c + dx$. The unability of the student solve the equation problem will claim that gap from arithmetic to algebra [17, 25]. The importance of the equal sign is a fundamental concept while students solve the equations. The transition from arithmetic to algebraic is arguably most salient in algebra school. The different meaning of the equal sign need transformation for the student while facing in arithmetic and algebraic.

4. Conclusion

By looking at the students' mathematical thinking, we can see the students in seventh-grade lead solve the problem by using arithmetic way. The students are not yet moving to algebraic way. As the teacher, we should be aware of this transition and make the student realize their mathematical thinking. Algebra thinking takes the role as their transition. How they give reason from their answer, it will give our attention to prepare algebra learning

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