Supersymmetry and $R_b$ in the light of LEP 1.5

John Ellis,¹ Jorge L. Lopez,² and D.V. Nanopoulos³,⁴

1CERN Theory Division, 1211 Geneva 23, Switzerland
2Department of Physics, Bonner Nuclear Lab, Rice University
6100 Main Street, Houston, TX 77005, USA
3Center for Theoretical Physics, Department of Physics, Texas A&M University
College Station, TX 77843–4242, USA
4Astroparticle Physics Group, Houston Advanced Research Center (HARC)
The Mitchell Campus, The Woodlands, TX 77381, USA

Abstract

We re-examine the possible magnitude of the supersymmetric contribution to $R_b \equiv \Gamma(Z^0 \rightarrow \bar{b}b)/\Gamma(Z^0 \rightarrow \text{all})$ in the light of the constraints imposed by the absence of light charginos at LEP 1.5, implementing also other available phenomenological constraints. We find the supersymmetric contribution to be $R_{\text{susy}}^b < 0.0017$, and discuss the extent to which this upper bound could be strengthened by future constraints on the chargino and top-squark masses. Such values of $R_{\text{susy}}^b$ tend to disfavor a supersymmetry explanation of the apparent $R_b$ discrepancy.
LEP 1 has, unfortunately, provided a showcase for the Standard Model, which has been tested successfully down to the per mille level. The measurements have proved to be sensitive to quantum corrections within the Standard Model, which have enabled the mass of the top quark to be predicted accurately, and may now be sensitive to the mass of the Higgs boson. The only possible blots on the Standard Model’s copybook have been suggested by the LEP measurements of $Z^0$ decays into $\bar{b}b$ and $\bar{c}c$. The preliminary measurements of $R_{b,c} \equiv \Gamma(Z^0 \to \bar{b}b, \bar{c}c)/\Gamma(Z^0 \to \text{all})$ reported at the Brussels and Beijing conferences disagree prima facie with the Standard Model at the levels of 3.7 and 2.5 standard deviations, respectively. Even if $R_c$ is fixed to its Standard Model value, at a considerable cost in $\chi^2$, the LEP 1 measurement of $R_b$ still disagrees with the Standard Model at the level of 3 standard deviations. It may well be that the apparent discrepancy is in fact due to a misestimation of the uncertainties associated with the simulation of the $\bar{b}b$ and $\bar{c}c$ final states, but it has been seductive to speculate that some new physics beyond the Standard Model may be coming into play.

One such speculation has been supersymmetry, and two specific scenarios to explain the $R_b$ discrepancy (but not the $R_c$ one) have been proposed. One has invoked a light chargino $\chi^+_1$ and a light top-squark $\tilde{t}_1$ close to the kinematic limits already excluded by new particle searches at LEP 1, and the other a light pseudoscalar Higgs boson $A$. These have inspired the hope in some quarters that one or more of these supersymmetric particles might be produced at LEP 2, and conceivably already in the intermediate-energy LEP 1.5 run recently completed. It should be pointed out, though, that it is has proved difficult in specific models to obtain a supersymmetric contribution to $R_b$ large enough to remove the apparent discrepancy, once one applies plausible phenomenological or theoretical constraints.

Preliminary results of the first part of the LEP 1.5 run have now been announced by the four LEP collaborations, and, to paraphrase Sherlock Holmes, “the curious incident was that the dog did nothing”. Specifically, all the four LEP collaborations have reported preliminary lower limits on the mass of the lighter chargino $\chi^+_1$: $m_{\chi^+_1} \gtrsim 65$ GeV if $m_{\chi^+_1} - m_{\chi^0_1} \gtrsim 10$ GeV (with some dependence on the sneutrino mass), where the $\chi^0_1$ is the lightest neutralino, which is assumed to be the lightest supersymmetric particle (LSP). Many people are aware that this news is particularly disappointing for advocates of the light ($\chi^+_1, \tilde{t}_1$) interpretation of the $R_b$ anomaly. The purpose of this note is to quantify the upper limit on the possible supersymmetric contribution to $R_b$ in the light of this preliminary LEP 1.5 result, as well as recent D0 constraints on the $\tilde{t}_1$ mass and updates of other experimental constraints on possible sparticle masses, limits on possible new physics effects in $Z^0, t$ and $b$ decay, and the absence of the lightest supersymmetric Higgs boson.

To set the scene for our study, we first recall that the Standard Model contribution to $R_b$ (for $m_t = 175$ GeV) is $R_{b}^{\text{SM}} = 0.2157$, whereas the reported experimental value (with $R_c$ constrained to the Standard Model value) is $R_{b}^{\text{exp}} = 0.2205 \pm 0.0016$. This means that a value of $R_{b}^{\text{susy}} \geq 0.0020$ would bring the supersymmetric $R_b$ prediction within the 95% C.L. interval, whilst a contribution $R_{b}^{\text{susy}} \geq 0.0030$ would
bring the prediction within one sigma of the experimental value.

In this note we consider the supersymmetric contributions to $R_b$ in the regime of light chargino and top-squark masses and small values of $\tan \beta$, where they may be enhanced \cite{4}. Enhancements to $R_b^{\text{susy}}$ may also occur for small values of the pseudoscalar Higgs mass ($m_A$) and large values of $\tan \beta$ \cite{5}, but this scenario now appears to be disfavored \cite{10}, and we do not consider it in what follows. The dominant contribution to $R_b^{\text{susy}}$ then depends on six parameters: those that parametrize the chargino sector ($M_2, \mu, \tan \beta$), the top-squark masses ($m_{\tilde{t}_1} < m_{\tilde{t}_2}$), and their mixing angle ($\theta_{\tilde{t}}$). We work in the context of the general Minimal Supersymmetric Standard Model (MSSM), without assuming a priori any relationship among these parameters that might result from unification conditions or dynamical models.

Following Ref. \cite{6}, we first sample a large number of six-plet choices of parameters, with those parameters that have the dimension of mass allowed to take random values in the interval (0 $\rightarrow$ 250) GeV, and with $\tan \beta$ restricted to the range 1 $\rightarrow$ 5. The total sample of approximately 365K six-plets is restricted in such a way that the most elementary LEP 1 lower bounds ($m_{\chi^\pm_1}, m_{\tilde{t}_1} > 45$ GeV) are satisfied. We then find a total of 1000 six-plets that yield $R_b^{\text{susy}} \geq 0.0020$. To examine in more detail the region of low values of $\tan \beta$, we have also generated and studied a “low-$\tan \beta$” sample (91K six-plets), for which $\tan \beta$ is restricted to the range 1 $\rightarrow$ 1.5.

In order to determine the upper bound on $R_b^{\text{susy}}$, we apply a series of experimental constraints to our large six-plet sample, as follows:

1. The invisible $\Gamma(Z \rightarrow \chi^0_1\chi^0_1)$ width should be less than 3.9 MeV, as can be inferred from the most recent LEP result $\delta \Gamma_{\text{inv}} = (-1.5 \pm 2.7)$ MeV \cite{2}.

2. The branching ratio $B(Z \rightarrow \chi^0_1\chi^0_2)$ should not exceed $10^{-4}$ \cite{11}.

3. The more restrictive LEP 1 lower limit on the chargino mass: $m_{\chi^\pm_1} > 47$ GeV, valid for $m_{\chi^0_1} < 42.5$ GeV and for the higgsino-like chargino \cite{12} required for an enhancement in $R_b^{\text{susy}}$.

4. The lightest Higgs boson should be heavier than the LEP 1 limit ($m_h \sim 40$ GeV). The mass of this Higgs boson acquires a large quantum correction at the one-loop level, which is dominated by the top–top-squark loop \cite{13}. Casting the one-loop correction in terms of the observable top-squark parameters ($m_{\tilde{t}_1,2}, \theta_{\tilde{t}}$) alone, one obtains \cite{14}

$$M_Z^2 \left\{ \cos^2 2\beta + \gamma \left( \frac{m_t}{M_Z} \right)^4 \left[ \ln \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} + (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) \sin^2 \theta_{\tilde{t}} \ln \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) \left( \frac{\sin^2 \theta_{\tilde{t}}}{4m_t^2} \right)^2 g \left( \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right) \right] \right\}$$

\footnote{We take the mass of the charged Higgs boson ($H^\pm$) to be large, so as to minimize the contribution to $R_b^{\text{susy}}$ from the $H^\pm - t$ loop, which is always negative. This means that our results are conservative upper bounds.}
with \( \gamma = \frac{3\alpha}{4\pi \sin^2 \theta_W \cos \theta_W} \) and \( g(r) = 2 - \frac{r^4+1}{r^4} \ln r \) \([g(1) = 0, g(r) \leq 0]\). Other one-loop corrections and the largest of the two-loop corrections are not expected to be large \([13]\), and are probably no larger than uncertainties in the approximations used, so we do not incorporate them.

5. The branching ratio \( B(b \rightarrow s\gamma) \) should fall in the range \((1 - 4) \times 10^{-4}\). This interval is a conservative interpretation of the latest CLEO result \( B(b \rightarrow s\gamma)^{\text{exp}} = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \) \([16]\), which should cover the theoretical uncertainties in the calculation of \( B(b \rightarrow s\gamma) \), principally due to higher-order perturbative QCD corrections in the Standard Model contribution.

6. The branching ratio \( B(t \rightarrow bW) \) has been determined by CDF to be \(0.87^{+0.13}_{-0.32}\) \([17]\). We therefore require \( B(t \rightarrow \text{new}) < 0.45 \), where “new” includes in our case the \( t \rightarrow \tilde{t}_1, 2\chi_0^{\pm 1}, 2\tilde{\ell} \) decay channels, when kinematically allowed. More restrictive upper limits on \( B(t \rightarrow \text{new}) \) have been considered elsewhere \([6, 18]\).

7. The D0 Collaboration has included a region in the \((m_{\chi_0^{\pm 1}}, m_{\tilde{t}_1})\) space, assuming that \( m_{\tilde{t}_1} < \{m_{\chi_0^{\pm 1}}, m_{\tilde{t}_1}, m_{\tilde{\nu}}\} \) \([19]\). These restrictions insure that the dominant \( \tilde{t}_1 \) decay mode is via the one-loop process \( \tilde{t}_1 \rightarrow c\chi_1^0 \).

8. The new LEP 1.5 lower limit on the chargino mass \( m_{\chi_1^{\pm}} \gtrsim 65 \text{ GeV} \), valid as long as \( m_{\chi_1^{\pm}} - m_{\chi_0} \geq 10 \text{ GeV} \) \([8]\). A more precise formulation of the limit must await the publication of their results by the LEP collaborations: it depends on the sneutrino mass and on the wino/higgsino content of the chargino. It seems to us that the above limit is conservative, applying when the sneutrino is heavy, or when the chargino is higgsino-like, which is the case of relevance for obtaining a large value of \( R_{\text{ SUSY}} \). We also discuss later the effect of decreasing the restriction on the chargino-neutralino mass difference to about 5 GeV, as might be achieved in the final analysis.

Motivated by the requirement that any stable supersymmetric relic particle should be electromagnetically neutral and have no strong interactions \([21]\), we also require that neither the lightest top-squark nor the lightest chargino should be the lightest supersymmetric particle, \( i.e., \{m_{\chi_1^{\pm}}, m_{\tilde{t}_1}\} > m_{\chi_0} \).

After running our large sample of six-plets through the above set of experimental and theoretical constraints, we find that no points with \( R_{\text{ SUSY}} > 0.0020 \) survive. The main reason for this result is the new LEP 1.5 constraint on the chargino mass. This could have been anticipated, as Refs. \([1, 8, 10]\), which did not have access to the new data, found regions of parameter space with \( R_{\text{ SUSY}} > 0.0020 \), even after enforcing most of the constraints enumerated above. We conclude that a supersymmetric solution to the \( R_b \) anomaly is less likely in the light of LEP 1.5. This conclusion holds for both our “regular” sample and our “low-tan\( \beta \)” sample. Moreover, these results rely only on the present LEP 1.5 result, with the chargino-neutralino mass difference required to be more than 10 GeV, and are in fact independent of the constraint on the Higgs-boson mass (item 4 above). We should add that our full sample contains
a small fraction of points with very low values of the neutralino masses (few GeV), which manage to pass all LEP 1 constraints (see also [3, 21]) and are not subjected to the known limits on the gluino mass as we do not impose the GUT relation among gaugino masses. These points are, however, all excluded by either the $B(b \to s\gamma)$ constraint (item 5) or the LEP 1.5 constraint (item 8).

Next we look for the largest achievable values of $R_{b}^{\text{susy}}$. In Fig. 1, we show $(R_{b}^{\text{susy}})_{\text{max}}$ as a function of the lightest chargino mass ($m_{\chi^{\pm}_{1}}$), for both signs of $\mu$. The top curves (“None”) give the raw results obtained from the full sample of parameter six-plets, whereas the (solid) bottom curves (“All”) give the limiting values when all the above constraints are applied, in which case we find the absolute upper limit

$$R_{b}^{\text{susy}} < 0.0017.$$  

(2)

Of particular importance in excluding values of $\tan \beta \approx 1$ is the Higgs mass constraint (item 4 above). As has already been mentioned, this constraint is worthy of further theoretical refinement, and may soon be strengthened by LEP itself. The effect of not enforcing this constraint is represented by the dashed lines in Fig. 1. Note that this constraint is superseded by the LEP 1.5 constraint for $m_{\chi^{\pm}_{1}} \lesssim 65$ GeV. We also display as dotted lines the further restriction that may be obtained should the LEP 1.5 be strengthened to exclude chargino-neutralino mass differences down to about 5 GeV, assuming that the lower bound on the chargino mass remains at 65 GeV. We note that if it were possible to obtain an absolute lower bound of 65 GeV on the chargino mass, then only values of $R_{b}^{\text{susy}} < 0.0010$ would be possible. Future runs at LEP 2 energies should be able to probe chargino masses as large as 90 GeV, which would imply $R_{b}^{\text{susy}} < 0.0005$, should no chargino signal be observed.

The Tevatron should also be able to constrain $(R_{b}^{\text{susy}})_{\text{max}}$ by setting lower limits on the chargino mass. Indeed, D0 has just released its first limits on chargino-neutralino production and decay into trilepton final states [23]. The limits are on the trilepton rates, i.e., $\sigma(pp \to \chi^{\pm_{1}}\chi^{0_{2}}X) \cdot B(\chi^{\pm_{1}} \to \ell) \cdot B(\chi^{0_{2}} \to 2\ell)$, which can be translated into limits on the chargino mass once one calculates the trilepton branching ratio. The latter depends on the detailed spectrum of sleptons and squarks (which we do not consider), and may be enhanced if there are light sleptons [24], in which case the D0 limits imply $m_{\chi^{\pm}_{1}} \gtrsim 55$ GeV [23]. The possibility of light sleptons will soon be explored at LEP, and the D0 sensitivity to trileptons is expected to increase significantly once the full data set is analyzed.

With a view to present and future top-squark searches at LEP and the Tevatron, we have also studied the dependence of $(R_{b}^{\text{susy}})_{\text{max}}$ on the lightest top-squark mass. This is shown in Fig. 2 for the “None” and “All” cases (with the Higgs mass constraint included and allowing a chargino-neutralino mass difference of up to 10 GeV). Direct top-squark searches at the Tevatron are underway, but so far have

---

2Note that just as LEP 1.5 has not been able to set an absolute lower limit on the chargino mass because of the experimental limitation of a minimal chargino-neutralino mass difference, the same could happen at LEP 2 energies. This limitation may be overcome by resorting to a hard photon tag, as recently discussed in Ref. 22.
concentrated on top-squark decays via $t \rightarrow c\chi^0_1$. This decay is dominant as long as $m_{\tilde{t}_1} < \{m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^-_1}, m_{\tilde{\nu}}\}$. With this restriction, D0 has excluded a region in the $(m_{\tilde{\chi}^0_1}, m_{\tilde{t}_1})$ plane \[19\]. This region is not very constraining for our present purposes, but it is expected that top-squark masses as large as 130 GeV could be explored with the data ($\sim 100 \text{pb}^{-1}$) already accumulated. As Fig. 2 shows, a lower bound of this magnitude would impose new severe restrictions on the allowed values of $R^{\text{susy}}_b$.

We have also explored the dependence of $(R^{\text{susy}}_b)_{\text{max}}$ on $B(b \rightarrow s\gamma)$ and $B(t \rightarrow \text{new})$. We find that more stringent experimental limits will decrease further the size of the allowed region in parameter space, but will not necessarily impose important new restrictions on $(R^{\text{susy}}_b)_{\text{max}}$.

Requiring rather light top-squark masses may entail a degree of fine-tuning in the top-squark mass matrix, such as large values of $A_t$. In the limit $\tan \beta \approx 1$ this situation may lead to minima of the electroweak scalar potential that break electric or color charge \[25\]. We do not include these constraints in the present analysis, as these would only further constrain the allowed region of parameter space.

Before concluding, we note that imposing further theoretical constraints on the parameter space, such as those that follow from universal supersymmetry breaking masses at the GUT scale and radiative electroweak breaking, tend to reduce $(R^{\text{susy}}_b)_{\text{max}}$ very substantially \[4, 6\]. Consulting Fig. 1 in Ref. \[6\], one can see that $R^{\text{susy}}_b \lesssim 0.0002$, after the new LEP 1.5 limit is imposed.

Even without imposing such additional theoretical constraints, the central result (2) of our analysis suggests that the previously most plausible supersymmetric scenario for accommodating the apparent anomaly in $R_b$ is now so severely constrained that it no longer appears able to resolve this experimental discrepancy with the Standard Model. In the absence of any other promising explanation from beyond the Standard Model, it may be necessary to review carefully the calculation and simulation of the Standard Model contributions to $R_b$ and related measurements. LEP 1.5 has done much to clarify the prospects of a supersymmetric resolution of this LEP 1 anomaly, and further stages of LEP should be able to cement our conclusion.

**Acknowledgments**

We thank Carlos Wagner and James White for helpful discussions. The work of J. L. has been supported in part by DOE grant DE-FG05-93-ER-40717, and that of D.V.N. has been supported in part by DOE grant DE-FG05-91-ER-40633.
References

[1] J. Ellis, G.L. Fogli, and E. Lisi, CERN-TH/95-202 (hep-ph/9507424), and references therein.

[2] P. Renton, Rapporteur talk at the International Symposium on Lepton and Photon Interactions at High Energies, High Energy Physics, Beijing (August 1995), Oxford preprint OUNP-95-20 (1995).

[3] G. Altarelli and R. Barbieri, Phys. Lett. B 253 (1990) 161; M. Boulware, D. Finnel, Phys. Rev. D 44 (1991) 2054; A. Djouadi, G. Girardi, C. Verzegnassi, W. Hollik and F. Renard, Nucl. Phys. B 349 (1991) 48; G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. B 369 (1992) 3; G. Altarelli, R. Barbieri, and F. Caravaglios, Nucl. Phys. B 405 (1993) 3; G. Altarelli, R. Barbieri, and F. Caravaglios, Phys. Lett. B 314 (1993) 357.

[4] J. D. Wells, C. Kolda, and G. L. Kane, Phys. Lett. B 338 (1994) 219.

[5] D. Garcia, R. Jimenez, and J. Sola, Phys. Lett. B 347 (1995) 321; D. Garcia and J. Sola, Phys. Lett. B 357 (1995) 349.

[6] X. Wang, J. L. Lopez, and D. V. Nanopoulos, Phys. Rev. D 52 (1995) 4116.

[7] G. Kane, R. Stuart, and J. Wells, Phys. Lett. B 354 (1995) 350; E. Ma and D. Ng, hep-ph/9508338; Y. Yamada, K. Hagiwara, and S. Matsumoto, hep-ph/9512227.

[8] L. Rolandi, H. Dijkstra, D. Strickland and G. Wilson, representing the ALEPH, DELPHI, L3 and OPAL collaborations, Joint Seminar on the First Results from LEP 1.5, CERN, Dec. 12th, 1995.

[9] A. Akhundov, D. Bardin, and T. Riemann, Nucl. Phys. B 276 (1986) 1; J. Bernabeu, A. Pich, and A. Santamaria, Phys. Lett. B 200 (1988) 569; W. Beenaker and W. Hollik, Z. Phys. C40, 141 (1988); F. Boudjema, A. Djouadi, and C. Verzegnassi, Phys. Lett. B 238 (1990) 423; A. Blondel and C. Verzegnassi, Phys. Lett. B 311 (1993) 346.

[10] J. Wells and G. Kane, hep-ph/9510372.

[11] M. Acciarri, et. al. (L3 Collaboration), Phys. Lett. B 350 (1995) 109.

[12] See, e.g., D. Decamp, et. al. (ALEPH Collaboration), Phys. Reports 216 (1992) 253.

[13] Y. Okada, M. Yamaguchi, and T. Yanagida, Prog. Theor. Phys. 85 (1991) 1 and Phys. Lett. B 262 (1991) 54; J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B 257 (1991) 83 and Phys. Lett. B 262 (1991) 477; H. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815.
[14] J. L. Lopez and D. V. Nanopoulos, Phys. Lett. B 266 (1991) 397.

[15] M. Diaz and H. Haber, Phys. Rev. D 46 (1992) 3086; R. Hempfling and A. Hoang, Phys. Lett. B 331 (1994) 99; M. Carena, J. Espinosa, M. Quiros, and C. Wagner, Phys. Lett. B 355 (1995) 209; M. Carena, M. Quiros, and C. Wagner, hep-ph/9508343.

[16] T.E. Browder and K. Henscheid, University of Hawaii and Ohio State University preprint, UH 511-816-95 and OHSTPY-HEP-E-95-010 (1995), to appear in Progress in Nuclear and Particle Physics, Vol. 35.

[17] J. Incandela (CDF Collaboration), FERMILAB-CONF-95-237-E (July 1995).

[18] S. Mrenna and C.-P. Yuan, hep-ph/9509424.

[19] S. Abachi, et. al. (D0 Collaboration), “Search for light top squarks in $p\bar{p}$ collisions at $\sqrt{s} = 1.8 \text{ TeV}”$, December 1995 (submitted to Phys. Rev. Lett.).

[20] J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, Nucl. Phys. B 238 (1984) 453.

[21] J. Feng, N. Polonsky, and S. Thomas, hep-ph/9511324.

[22] C.-H. Chen, M. Drees, and J. Gunion, hep-ph/9512230.

[23] S. Abachi, et. al. (D0 Collaboration), hep-ex/9512004.

[24] J. L. Lopez, D. V. Nanopoulos, X. Wang, and A. Zichichi, Phys. Rev. D 48 (1993) 2062 and Phys. Rev. D 52 (1995) 142.

[25] See e.g., P. Langacker and N. Polonsky, Phys. Rev. D 50 (1994) 2199.
Figure 1: The maximum attainable value of $R_{b}^{\text{susy}}$ versus the chargino mass for both signs of $\mu$, when no constraint has been applied (“None”) and when all the constraints described in the text have been applied (“All”). The dashed lines indicate the effect of not enforcing the Higgs-mass constraints, and the dotted lines indicate the possible further restriction should future LEP 1.5 searches exclude a chargino-neutralino mass down to about 5 GeV.
Figure 2: The maximum attainable value of $R_{b}^{\text{susy}}$ versus the top-squark mass for both signs of $\mu$, when no constraint has been applied (“None”) and when all constraints have been applied (“All”). It can be seen from this plot how the expected future direct limits on $m_{\tilde{t}_{1}}$ from the Tevatron will constrain $R_{b}^{\text{susy}}$ further.