Electromagnetic Interactions in Heavy Hadron Chiral Theory

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Electromagnetic interactions are incorporated into Heavy Hadron Chiral Perturbation Theory. Short and long distance magnetic moment contributions to the chiral Lagrangian are identified, and $M1$ radiative decays of heavy vector mesons and sextet baryons are studied. Using recent CLEO $D^*$ branching fraction ratio data, we fit the meson coupling to the axial vector Goldstone current and find $g_1^2 = 0.34 \pm 0.48$ for $m_c = 1700$ MeV. Finally, we obtain model independent predictions for total and partial widths of charm and bottom vector mesons.
A synthesis of Chiral Perturbation Theory and the Heavy Quark Effective Theory (HQET) has recently been developed \[1–5\]. This hybrid effective theory describes low energy strong interactions between light Goldstone bosons and hadrons containing a single heavy quark. Weak $b \to c$ transitions among heavy meson or baryon states can also be incorporated into this framework. In this letter, we extend the theory's formalism to include electromagnetism and then study the radiative decays of heavy vector mesons and sextet baryons.

To begin, we briefly review the basic elements of Heavy Hadron Chiral Perturbation Theory (HHCPT) \[1\]. The Goldstone bosons resulting from the chiral symmetry breakdown $SU(3)_L \times SU(3)_R \to SU(3)_{L+R}$ appear in the pion octet

$$\pi = \sum_{a=1}^{8} \pi^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta \\
\pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^+ \\
K^- & K^0 & -\sqrt{\frac{2}{3}} \eta
\end{pmatrix}$$

and are associated with the pion decay constant $f \approx 93$ MeV. These fields are arranged into the exponentiated matrix functions $\Sigma = e^{2i\pi\pi^0/f}$ and $\xi = \sqrt{\Sigma}$ that transform under the chiral symmetry group as

$$\Sigma \to L \Sigma R^\dagger$$

$$\xi \to L \xi U^\dagger = U \xi R^\dagger.$$  (2)

Here $L$ and $R$ represent global elements of $SU(3)_L$ and $SU(3)_R$, while $U$ acts like a local $SU(3)_{L+R}$ transformation. Chiral invariant terms that describe Goldstone boson self interactions are constructed from the fields in (2) and their derivatives.

Hadrons containing a heavy quark emit and absorb light Goldstone bosons with no appreciable change in their four velocities. They are consequently described by velocity dependent fields. In the meson sector, we introduce the operators $P_i(v)$ and $P_{i\mu}^*(v)$ that annihilate pseudoscalar and vector mesons with quark content $Q\pi$. When the suppressed heavy quark label carried by these fields corresponds to charm, their individual components are given by

$$(P_1, P_2, P_3) = (D^0, D^+, D_s^+)$$

$$(P_1^*, P_2^*, P_3^*) = (D^{*0}, D^{*+}, D_{s}^{*+}).$$  (3)

\[1\] This introductory discussion closely follows that presented in refs. \[4,5\] to which we refer interested readers for further details.
In the infinite quark mass limit, it is useful to combine the degenerate meson spin states into the $4 \times 4$ matrix field 

\[ H_i(v) = \frac{1 + \gamma_0}{2} \left[ -P_i(v)\gamma^5 + P^*_\mu(v)\gamma^\mu \right] \]  

(4a)

and its conjugate

\[ \overline{H}^i(v) = \gamma_0 H^i \gamma_0 = \left[ P^t_i(v)\gamma^5 + P^*_t\gamma^\mu \right] \frac{1 + \gamma_0}{2}. \]  

(4b)

$H$ carries a heavy quark spinor index and a separate light antiquark spinor index and transforms as an antitriplet under $SU(3)_L + R$ and doublet under $SU(2)_v$.

Baryons with quark content $Qqq$ enter into the theory in two types depending upon the angular momentum of their light degrees of freedom. In the first case, the light spectators are arranged in a symmetric spin-1 configuration that couples with the heavy spin-$\frac{1}{2}$ quark to form $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ states. When the heavy partner is taken to be charm, the spin-$\frac{1}{2}$ states are destroyed by the Dirac operators appearing in the symmetric sextet representation

\[ S = \sum_{i=1}^{6} S_i T_i^{(6)} = \begin{pmatrix} \Sigma_c^{++} & \sqrt{\frac{1}{2}}\Sigma_c^+ & \sqrt{\frac{1}{2}}\Xi_c^{0'} \\ \sqrt{\frac{1}{2}}\Sigma_c^+ & \Sigma_c^0 & \sqrt{\frac{1}{2}}\Xi_c^{0'} \\ \sqrt{\frac{1}{2}}\Xi_c^{0'} & \sqrt{\frac{1}{2}}\Xi_c^{0'} & \Omega_c^0 \end{pmatrix}. \]  

(5)

Their spin-$\frac{3}{2}$ counterparts are annihilated by the corresponding Rarita-Schwinger field $S^*_\mu$. We again combine the Dirac and Rarita-Schwinger operators into the “super” fields

\[ S^{ij}_\mu(v) = \frac{1}{3} (\gamma_\mu + v_\mu)\gamma^5 S^{ij}(v) + S^{*ij}_\mu(v) \]  

\[ \overline{S}^{ij}_\mu(v) = -\frac{1}{3} \overline{S}^{ij}(v)\gamma^5 (\gamma_\mu + v_\mu) + \overline{S}^{*ij}_\mu(v). \]  

(6)

Then $S_\mu$ transforms as a sextet under $SU(3)_L + R$, doublet under $SU(2)_v$, and is an axial vector.

The spectators in the second case are bound together into an antisymmetric spin-0 state. The resulting $J^P = \frac{1}{2}^+$ baryons are assigned to the field $T_i(v)$, which is an $SU(3)_L + R$ antitriplet and $SU(2)_v$ doublet. When $Q = c$, the components of $T_i$ are the singly charmed baryons

\[ (T_1, T_2, T_3) = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+). \]  

(7)
These antitriplet baryons can alternatively be arranged into the antisymmetric matrix

\[
T = \sum_{i=1}^{3} T_i T^3_i = \begin{pmatrix}
0 & \sqrt{\frac{1}{2}} \Lambda^+_{c} & \sqrt{\frac{1}{2}} \Xi^0_{c} \\
-\sqrt{\frac{1}{2}} \Lambda^+_{c} & 0 & \sqrt{\frac{1}{2}} \Xi^0_{c} \\
-\sqrt{\frac{1}{2}} \Xi^+_{c} & -\sqrt{\frac{1}{2}} \Xi^0_{c} & 0
\end{pmatrix}
\]  

(8)

where \((T^3_i)_{jk} = \epsilon_{ijk}/\sqrt{2}\).

We can now construct the zeroth order effective chiral Lagrangian that describes the low energy interactions between light Goldstone bosons and heavy hadrons in the infinite heavy quark mass limit. The leading order terms must be hermitian, Lorentz invariant, light flavor and heavy quark spin symmetric, and parity even. We can also readily incorporate electromagnetism into the hybrid chiral theory by gauging a \(U(1)_{EM}\) subgroup of the global \(SU(3)_L \times SU(3)_R\) symmetry group. Only long wavelength photons with energies less than the chiral symmetry breaking scale explicitly remain in the low energy theory while short wavelength modes are integrated out. In \(d = 4 - \epsilon\) dimensions, the effective Lagrangian looks like

\[
L^{(0)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\mu - \epsilon f^2}{4} \text{Tr}(D^\mu \Sigma^\dagger D_\mu \Sigma) \\
L^{(0)}_v = \sum_{Q=c,b} \left\{ -i \text{Tr}(H^{i'}_i v \cdot D H^{i'}_i) - i S_{ij}^{\mu} v \cdot D S_{ij}^{\mu} + (M_S - M_T) S_{ij}^{\mu} S_{ij}^{\mu} + iT^i v \cdot D T_i \\
+ g_1 \text{Tr}(H^{i'}_i (A)_{ij}^{\sigma} \gamma^5 H^{j'}_j) + ig_2 \varepsilon^{\mu\nu\sigma\lambda} S_{ik}^{\mu} v^{\nu}(A^\sigma)_j^{i} (S^\lambda)_j^{jk} \\
+ g_3 \left[ \epsilon_{ijk} T^i (A^{\mu})_j^{i} S_{kl}^{\mu} + \epsilon_{ijk} S_{kl}^{\mu} (A^{\mu})_j^{i} T_i \right] \right\}.
\]  

(9a)

(9b)

The Goldstone bosons explicitly couple to the matter fields through the axial vector combination

\[
A^\mu = \frac{i}{2} (\xi^\dagger D^\mu \xi - \xi D^\mu \xi^\dagger).
\]  

(10a)

They also communicate via the vector field

\[
V^\mu = \frac{1}{2} (\xi^\dagger D^\mu \xi + \xi D^\mu \xi^\dagger)
\]  

(10b)

\(2\) Meson contributions are written in terms of the dimension-3\(\frac{3}{2}\) field \(H' = \sqrt{M_H} H\) so that all heavy mass dependence is removed from the leading order Lagrangian.
that appears inside the heavy hadron covariant derivatives $\mathcal{D}^\mu H'_i = \partial^\mu H'_i - H'_i (\mathcal{V}^\mu)^i_j - i\mu^\nu e A^\mu \left[ Q Q H'_i - Q Q Q H'_i - Q Q Q H'_i \right]$

\[
\mathcal{D}^\mu S_{ij} = \partial^\mu S_{ij} + (V^\mu)^i_k S_{kj}^i + (V^\mu)^j_k S_{ik}^j - i\mu^\nu e A^\mu \left[ Q S_{ij} + Q S_{ij} + Q S_{ij} \right]
\]

(11)

The remaining Goldstone covariant derivatives in (9) and (10) are given by

\[
\mathcal{D}^\mu \Sigma^i_j = \partial^\mu \Sigma^i_j - i\mu^\nu e A^\mu \left[ Q \Sigma^i_j - Q \Sigma^i_j \right]
\]

\[
\mathcal{D}^\mu \xi^i_j = \partial^\mu \xi^i_j - i\mu^\nu e A^\mu \left[ Q \xi^i_j - Q \xi^i_j \right]
\]

(12)

where

\[
\mathcal{Q} = \begin{pmatrix} Q_1 & Q_2 & Q_3 \\ Q_2 & Q_3 & Q_1 \\ Q_3 & Q_1 & Q_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}
\]

(13)

denotes the light quark electric charge matrix.

Spin symmetry violating contributions to the chiral Lagrangian enter at $O(1/m_Q)$. Among these are heavy quark magnetic moment terms which mediate $M1$ radiative transitions. As we will see, these terms are completely fixed by heavy quark number conservation. This simple but crucial observation allows one to use experimental meson decay information to determine the parameter $g_1$.

Recall that the photon gauge field couples to the conserved current that counts heavy quark number in the underlying QCD theory as well as in the low energy HQET and HHCPT. The original QCD current appears in its well-known Gordon decomposed form as

\[
J^{QCD}_\mu = \bar{Q}(p')\gamma_\mu Q(p) = \frac{1}{2m_Q} \bar{Q}(p') \left[ (p' + p)_\mu + i\sigma_{\mu\nu}(p' - p)_\nu \right] Q(p).
\]

(14)

Running down in energy to the heavy quark thresholds and invoking the velocity superselection rule to set $p^{(r)} = m_Q v + k^{(r)}$, one can match this tree level current onto the corresponding HQET expression

\[
J^{HQET}_\mu = \bar{h}^{(q)}(v) + \frac{i}{2m_Q} \left( v_\mu + \frac{\hat{v}_\mu}{2m_Q} \right) + \frac{1}{2m_Q} \sigma_{\mu\nu}(\hat{v}_\nu + \hat{v}_\nu) \bar{h}^{(q)}(v).
\]

(15)

3 We distinguish the photon field $A^\mu$ from the axial vector Goldstone current $A^\mu$ by writing the former in calligraphy type. Similarly, we let $Q$ represent the electric charge operator, which is different from the heavy quark symbol $Q$. 

4
The $A^{\mu}$ gauge field couples to this current in the Lagrangian

$$L^{(HQET)}_v = \sum_{Q=c,b} \left\{ \bar{h}_v^{(Q)}(i\gamma^\mu)h_v^{(Q)} + a_1O_1 + a_2O_2 + a_3O_3 \right\}$$

where the $O(1/m_Q)$ $O_i$ operators are constructed from either symmetric or antisymmetric combinations of two HQET covariant derivatives $D^{\mu} = \partial^{\mu} - i\mu^{\psi/2}gG^a\sigma^{\mu}_a - i\mu^{\psi/2}eA^{\mu}\cdot Q$:

$$O_1 = \frac{1}{2m_Q}\bar{h}_v^{(Q)}(iD)^2h_v^{(Q)}$$

$$O_2 = \frac{\mu^{\psi/2}g}{4m_Q}\sigma^{\mu\nu}T_ah_v^{(Q)}G^{\mu\nu}_a$$

$$O_3 = \frac{\mu^{\psi/2}eQ}{4m_Q}\bar{h}_v^{(Q)}\sigma^{\mu\nu}h_v^{(Q)}F^{\mu\nu}.$$ (17c)

The $a_i$ coefficients of these dimension-five operators are thus fixed by (15) and equal unity at lowest order [9].

Running down further in energy from the heavy quark thresholds to the chiral symmetry breaking scale $\Lambda_{\chi}$, we match the electromagnetic pieces of operators $O_1$ and $O_3$ onto the following short distance contributions to the HHCPT Lagrangian:

$$L^{(short)}_v = \sum_{Q=c,b} \left\{ -\frac{1}{2m_Q}\text{Tr}(\bar{H}^{(Q)}(iD)^2H^{(Q)}) - \frac{\mu^{\psi/2}Qe(m_Q)}{4m_Q}\text{Tr}(\bar{H}'\sigma^{\mu\nu}H'F^{\mu\nu}) 
- \frac{1}{2m_Q}\bar{S}^{ij}_{\lambda}(iD)^2S^{ij}_{\lambda} - \frac{\mu^{\psi/2}Qe(m_Q)}{4m_Q}\bar{S}^{ij}_{\lambda}\sigma_{\mu\nu}S^{ij}_{\lambda}F^{\mu\nu} 
+ \frac{1}{2m_Q}\bar{T}^{i}(iD)^2T_{i} + \frac{\mu^{\psi/2}Qe(m_Q)}{4m_Q}\bar{T}^{i}\sigma_{\mu\nu}T_{i}F^{\mu\nu} \right\}.$$ (18)

These $O(1/m_Q)$ terms describe the interaction of photons with the heavy quark constituent inside a $H'$, $S$ or $T$ hadron. Consequently, the Lorentz and flavor indices for the hadrons’ light degrees of freedom are trivially contracted. Heavy quark number conservation determines the ratio of the operator coefficients in (18) to the kinetic terms in the zeroth order Lagrangian [9]. The conserved HQET current therefore matches onto

$$J^{HHCPT}_\mu = -\text{Tr}\bar{H}'^{\mu}\left[v_{\mu} + \frac{i}{2m_Q}(\bar{\partial}_{\mu} - \bar{\partial}_{\mu}) + \frac{1}{2m_Q}\sigma_{\mu\nu}(\bar{\partial}^{\nu} + \bar{\partial}^{\nu})\right]H'_{i}^{\mu} 
- \bar{S}^{ij}_{\lambda}\left[v_{\mu} + \frac{i}{2m_Q}(\bar{\partial}_{\mu} - \bar{\partial}_{\mu}) + \frac{1}{2m_Q}\sigma_{\mu\nu}(\bar{\partial}^{\nu} + \bar{\partial}^{\nu})\right]S^{ij}_{\lambda}^{\mu} 
+ \bar{T}^{i}\left[v_{\mu} + \frac{i}{2m_Q}(\bar{\partial}_{\mu} - \bar{\partial}_{\mu}) + \frac{1}{2m_Q}\sigma_{\mu\nu}(\bar{\partial}^{\nu} + \bar{\partial}^{\nu})\right]T_{i}^{\mu}. $$ (19)
in the low energy chiral theory.

Photons also couple to the light brown muck inside heavy hadrons leaving the spins of their heavy quark constituents unaltered. Such long distance interactions generate additional electromagnetic contributions to the effective Lagrangian at the $\Lambda_\chi$ scale. We focus upon just the induced magnetic moment terms:

$$L^{(\text{long})}_\nu = \frac{\mu e^2}{\Lambda_\chi} \left\{ c_H \text{Tr}(\overline{H}^i \sigma_{\mu\nu} F^{\mu\nu}) + i c_S S_{\mu,i,j} (Q^i_k S_{\nu}^{kj} + Q^i_{k'} S_{\nu}^{ik'}) F^{\mu\nu} \right\} \right.$$  \hspace{1cm} (20)

A few points about these long distance operators should be noted. Firstly, the suppressed heavy quark spinor indices in these spin symmetry preserving terms are simply contracted. Their light Lorentz and flavor indices on the other hand are nontrivially arranged. Secondly, the coefficients $c_H$, $c_S$ and $c_{ST}$ are a priori unknown. But naive dimensional analysis suggests that they are of order one \([10]\). Finally, there is no long distance magnetic moment interaction for just the antitriplet baryon since the photon field cannot couple to its spinless light degree of freedom.

Having identified the short and long distance magnetic moment terms in the low energy chiral theory, we can now study $M1$ radiative transitions between meson and baryon states. Since the hyperfine splitting between charmed pseudoscalar and vector meson partners is only slightly greater than a pion mass, the electromagnetic decay $D^* \rightarrow D\gamma$ competes with the strong process $D^* \rightarrow D\pi$. The greater phase space for the electromagnetic transition offsets its inherently smaller amplitude. Bottom vector mesons must radiatively decay because pion emission is kinematically forbidden. So these $M1$ meson processes are of genuine phenomenological interest. Similar considerations apply to the baryon transitions.

The vector meson and sextet baryon radiative decay rates are readily determined from the magnetic moment terms in \((18)\) and \((20)\):

$$\Gamma(P_i^* \rightarrow P_i \gamma) = \frac{2}{3} \left( \frac{M_{P}}{M_{P^*}} \right) \left( \frac{M_{P^*}^2 - M_P^2}{M_{P^*}^2} \right)^3 \left[ \frac{Q_Q}{4m_Q} \alpha_{EM}(m_Q)^{1/2} + \frac{c_H}{\Lambda_\chi} Q_4 \alpha_{EM}(\Lambda_\chi)^{1/2} \right]^2$$ \hspace{1cm} (21a)

$$\Gamma(S_{\ast}^i \rightarrow S_i \gamma) = \frac{1}{18} \left( \frac{M_{S}}{M_{S^*}} \right) \left( \frac{M_{S^*}^2 - M_S^2}{M_{S^*}^2} \right)^3 \left[ \frac{Q_Q}{m_Q} \alpha_{EM}(m_Q)^{1/2} + 2 \frac{c_S}{\Lambda_\chi} \text{Tr}(T_{(6)}^i \dagger QT_{(6)}^i) \alpha_{EM}(\Lambda_\chi)^{1/2} \right]^2$$ \hspace{1cm} (21b)

$$\Gamma(S_{\ast}^i \rightarrow T_j \gamma) = \frac{1}{6} \left( \frac{M_{T}}{M_{S_{\ast}}} \right) \left( \frac{M_{S_{\ast}}^2 - M_T^2}{M_{S_{\ast}}^2} \right)^3 \left[ \frac{c_{ST}}{\Lambda_\chi} \text{Tr}(T_{(3)}^j \dagger QT_{(6)}^j) \alpha_{EM}(\Lambda_\chi)^{1/2} \right]^2.$$ \hspace{1cm} (21c)
One can clearly identify the short and long distance contributions to these partial widths from their electric charges and associated inverse mass scales. The corresponding strong interaction decay rates are derived from the Goldstone axial vector couplings in the leading order Lagrangian (22b):

\[
\Gamma(P^* \to P_j \pi^a) = \frac{g_1^2}{48\pi f^2} \left( \frac{M_P}{M_{P^*}} \right)^3 \left[ \frac{M_{P^*}^2 - (M_P + m_\pi)^2}{(M_{P^*} - (M_P - m_\pi))^2} \right]^{3/2} |(T^a)_{ji}|^2
\]

\[
\Gamma(S^* \to S_j \pi^a) = \frac{g_2^2}{144\pi f^2} \left( \frac{M_S}{M_{S^*}} \right)^3 \left[ \frac{M_{S^*}^2 - (M_S + m_\pi)^2}{(M_{S^*} - (M_S - m_\pi))^2} \right]^{3/2} |\text{Tr}(T^a_{(6)} \gamma^a T^a_{(6)})|^2
\]

\[
\Gamma(S^* \to T_j \pi^a) = \frac{g_3^2}{24\pi f^2} \left( \frac{M_T}{M_{S^*}} \right)^3 \left[ \frac{M_{S^*}^2 - (M_T + m_\pi)^2}{(M_{S^*} - (M_T - m_\pi))^2} \right]^{3/2} |\text{Tr}(T^a_{(3)} \gamma^a T^a_{(6)})|^2
\]

None of the heavy hadron electromagnetic and strong partial widths have been directly measured. However, values for \(D^*\) branching fraction ratios are known [11]:

\[
R_0^0 = \frac{\Gamma(D^{*0} \to D^{0}\gamma)}{\Gamma(D^{*0} \to D^{0}\pi^0)} = 0.572 \pm 0.057 \pm 0.081
\]

\[
R_+^0 = \frac{\Gamma(D^{*+} \to D^{+}\gamma)}{\Gamma(D^{*+} \to D^{+}\pi^0)} = 0.035 \pm 0.047 \pm 0.052.
\]

(23)

Taken in conjunction with the isospin relation

\[
R_+^\pi = \frac{\Gamma(D^{*+} \to D^{0}\pi^+)}{\Gamma(D^{*+} \to D^{+}\pi^0)} = 2.21 \pm 0.07,
\]

(24)

these data yield the following branching fractions:

\[
D^{*+} \to D^{0}\pi^+ \quad 68.1 \pm 1.0 \pm 1.3% \quad (25a)
\]

\[
D^{*+} \to D^{+}\pi^0 \quad 30.8 \pm 0.4 \pm 0.8% \quad (25b)
\]

\[
D^{*+} \to D^{+}\gamma \quad 1.1 \pm 1.4 \pm 1.6% \quad (25c)
\]

\[
D^{*0} \to D^{0}\pi^0 \quad 63.6 \pm 2.3 \pm 3.3% \quad (25d)
\]

\[
D^{*0} \to D^{0}\gamma \quad 36.4 \pm 2.3 \pm 3.3% \quad (25e)
\]

\[4\] These very recent CLEO values differ significantly from Particle Data Group world averages [12].
Using the branching fraction ratios for the two independent $D^*$ charge modes in (23), we can deduce the parameters $c_H/\Lambda_\chi$ and $g_1^2$ that enter into the heavy meson electromagnetic and strong decay rates respectively. To extract these unknown couplings from the $D^*$ data and to predict the $B^*$ widths, we must specify numerical values for the charm and bottom mass parameters $m_c$ and $m_b$. Since these quark masses are sources of large theoretical uncertainty, we perform the fit twice. First we assume $(m_c, m_b) = (1500 \text{ MeV}, 4500 \text{ MeV})$, and then we take $(m_c, m_b) = (1700 \text{ MeV}, 5000 \text{ MeV})$. Reasonable estimates for the heavy quark masses are covered by the range between these two sets of input values.

From the charm vector meson ratios, we find two equations for the two unknowns:

$$g_1^{-2} \left[ \frac{1}{2m_c} \alpha_{EM}(m_c)^{1/2} - \frac{c_H}{\Lambda_\chi} \alpha_{EM}(\Lambda_\chi)^{1/2} \right]^2 = \frac{9}{128\pi f^2} \left\{ \frac{\left( M_{D^*+}^2 - (M_{D^0} + m_{\pi^0})^2 \right) \left( M_{D^*+}^2 - (M_{D^+} - m_{\pi^0})^2 \right)}{(M_{D^*+}^2 - M_{D^+}^2)^2} \right\}^{3/2} R_\gamma^+ \tag{26}$$

$$g_1^{-2} \left[ \frac{1}{4m_c} \alpha_{EM}(m_c)^{1/2} + \frac{c_H}{\Lambda_\chi} \alpha_{EM}(\Lambda_\chi)^{1/2} \right]^2 = \frac{9}{512\pi f^2} \left\{ \frac{\left( M_{D^*0}^2 - (M_{D^0} + m_{\pi^0})^2 \right) \left( M_{D^*0}^2 - (M_{D^0} - m_{\pi^0})^2 \right)}{(M_{D^*0}^2 - M_{D^0}^2)^2} \right\}^{3/2} R_\gamma^0.$$ 

Following the suggestion of naive dimensional analysis, we choose the roots of these quadratic equations that yield values for $g_1^2$ of order unity. The results of the parameter fit are then listed as functions of the charm quark mass in Table 1:

| Coupling          | $m_c = 1500 \text{ MeV}$          | $m_c = 1700 \text{ MeV}$          |
|-------------------|-----------------------------------|-----------------------------------|
| $c_H/\Lambda_\chi$| $(-0.68 \pm 0.50)/(1000 \text{ MeV})$ | $(-0.60 \pm 0.44)/(1000 \text{ MeV})$ |
| $g_1^2$           | $0.43 \pm 0.61$                   | $0.34 \pm 0.48$                   |

Table 1

These results only weakly depend upon $\Lambda_\chi$ through the logarithmic running of the fine structure constant. Therefore, a very precise numerical value for the chiral symmetry
breaking scale need not be specified. However, if one reasonably assumes $\Lambda_\chi \approx 1000$ MeV, then the value for $c_\mu$ turns out to be of order one and is consistent with our earlier expectations. We also note for comparison that the nonrelativistic quark model estimate for the squared Goldstone axial vector parameter is $0.7 \lesssim g_1^2 \lesssim 1.0$ \cite{3}. The HHCPT central value for this coupling is therefore of the same order of magnitude but smaller than the quark model number. The large error bars on $g_1^2$ reflect the 200\% uncertainty in the measurement \cite{25} of the $D^{*+} \rightarrow D^+\gamma$ branching fraction. Improvements in the experimental value will yield more precise estimates for this basic chiral Lagrangian parameter.

Having found $c_\mu/\Lambda_\chi$ and $g_1^2$, we can now obtain model independent predictions for the total and partial widths of all $D^*$ and $B^*$ vector mesons. Our predictions are summarized in Table 2:
Current upper bounds on $D^*$ widths are about an order of magnitude greater than the central values quoted here, while no $B^*$ decay information is yet available. Comparison of these theoretical results with experimental data must therefore be left for the future.

To conclude, we comment upon several possible extensions of this work. In the meson sector, a number of refinements of our leading order analysis should be pursued. Perturbative QCD corrections, subleading $O(1/m_Q)$ and $SU(3)_{L+R}$ breaking effects, and calculable

| Width (MeV) | $m_c = 1500$ MeV $m_b = 4500$ MeV | $m_c = 1700$ MeV $m_b = 5000$ MeV |
|------------|----------------------------------|----------------------------------|
| $\Gamma(D^{*+})$ | $(12.44 \pm 12.27) \times 10^{-2}$ | $(9.70 \pm 9.56) \times 10^{-2}$ |
| $\Gamma(D^{*+} \to D^+\pi^0)$ | $(3.56 \pm 5.06) \times 10^{-2}$ | $(2.77 \pm 3.94) \times 10^{-2}$ |
| $\Gamma(D^{*+} \to D^0\pi^+)$ | $(7.83 \pm 11.13) \times 10^{-2}$ | $(6.10 \pm 8.68) \times 10^{-2}$ |
| $\Gamma(D^{*+} \to D^+\gamma)$ | $(1.06 \pm 1.05) \times 10^{-2}$ | $(0.83 \pm 0.81) \times 10^{-2}$ |
| $\Gamma(D^{*0})$ | $(6.49 \pm 7.94) \times 10^{-2}$ | $(5.06 \pm 6.19) \times 10^{-2}$ |
| $\Gamma(D^{*0} \to D^0\pi^0)$ | $(5.36 \pm 7.63) \times 10^{-2}$ | $(4.18 \pm 5.94) \times 10^{-2}$ |
| $\Gamma(D^{*0} \to D^0\gamma)$ | $(1.13 \pm 2.20) \times 10^{-2}$ | $(0.88 \pm 1.71) \times 10^{-2}$ |
| $\Gamma(B^{*+}) = \Gamma(B^{*-} \to B^+\gamma)$ | $(8.46 \pm 11.94) \times 10^{-4}$ | $(6.60 \pm 9.31) \times 10^{-4}$ |
| $\Gamma(B^{*-}) = \Gamma(B^{*-} \to B^0\gamma)$ | $(1.63 \pm 2.61) \times 10^{-4}$ | $(1.27 \pm 2.03) \times 10^{-4}$ |

Table 2
nonanalytic terms from Goldstone boson loop diagrams may all be systematically incorporated into the HHCPT framework to yield improved values for the meson parameters and decay rates. $D_s^*$ and $B_s^*$ decays can also be worked out and studied in a straightforward fashion. For the sextet baryons, the present absence of branching ratio data precludes our determining the baryon couplings $(c_s/\Lambda_X, g_2^2)$ and $(c_{ST}/\Lambda_X, g_3^2)$ as well as the widths of the spin-$3/2$ states in precisely the same manner as their meson analogues. Nonetheless, such baryon data will eventually become available in the future. So the enhancements mentioned above for the mesons ought to be carried out for the baryons as well. Finally, the scope of HHCPT can be broadened to include higher resonances such as the $D_1$ and $D_2^*$ states [14]. Electromagnetic interactions for these meson and baryon excitations may be incorporated into the theory along the same lines as those for the heavy hadron $H'$, $S$ and $T$ ground states.

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