The influence of baryon-photon ratio on 21 and 92 cm brightness temperature

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Abstract.

An epoch before first stars and galaxies formation, so-called cosmic Dark Ages, still remains unobserved. One of the possibilities for observation phenomena took place during Dark Ages is studying hyperfine transition of such elements as hydrogen and deuterium. In this article there were examined physical processes in the Universe after recombination and evolution of 21 and 92 cm brightness temperature and studied influence of baryon-photon ratio on this radiation.

1. Introduction

Cosmic Dark Ages is an epoch after recombination (380 000 years after Big Bang [1]) lasted to first stars and galaxies formation. Imaging Universe during this period is difficult because, firstly there are no bright galaxies or quasars. Secondly barionic matter in the Universe during this epoch consists of almost cold neutral hydrogen and helium atoms, therefore there are almost no recombination lines, compton scattering of CMB (cosmic microwave background) photons and so on.

Nevertheless the possibility of experimental study of Dark Ages is given by hyperfine transitions in hydrogen and deuterium (H and D respectively) (21.1 cm and 91.6 cm lines). During the observations radio telescope measures antenna temperature, which is associated with brightness temperature of the source (\(T_b\)), which depends on intensity of the source. In this paper the where built the evolution of hydrogen and deuterium brightness temperature.

In this work there where used following parameters describing the standard ΛCDM cosmology: \(\Omega_m = 0.31\), \(\Omega_{\Lambda} = 0.69\), \(\Omega_b = 0.049\) (matter, dark energy and baryon densities divided by the critical density respectively), \(H_0 = 67 \text{ km s}^{-1}\text{Mpc}^{-1}\) - the Hubble constant [1].

2. The spin temperature

Effective spin temperature \(T_s\) defines distribution of atoms with different spin states:
Collisional coefficient: $\mu_n$ neglected [3].

For deuterium gas was cooling adiabatically due to Universe expansion: between hydrogen atoms and protons [8] (here collisions with photons, absorption and emission of $Ly\alpha$ photons (Wouthuysen-Field effect) and atomic spin-change collisions, so from [2] ($x$ is H or D)

$$T_s^x = \frac{(1 + \chi^x)T_k T_{CMB}}{T_k^x + \chi^x T_{CMB}}$$

where $T_k$ is kinetic gas temperature, $\chi^x = \chi^x_c + \chi^x_\alpha$ is sum of parameters for spin-change collisions and scattering of $Ly_0$ photons:

$$\chi^x_c = \frac{C^x_{+} - T_{CMB}^x}{A^x_{+} - T_{CMB}^x}, \quad \chi^x_\alpha = \frac{P_{+} - T_s^x}{A^x_{+} - T_{CMB}^x}$$

$P_{+} \propto P_\alpha$, $P_\alpha$ is $Ly\alpha$ scattering rate, $C^x_{+}$ is the collision rate, $A^x_{+}$ - Einstein coefficient. At $z \gg 10$, before first galaxies formation, $P_\alpha$ is small so it can be neglected [3].

For neutral hydrogen: $n_B/n_H = 1.3302$ [4] (there were H, D, $^3$He and $^4$He taken into consideration), $n_B = n_B n_\gamma (1+z)^3 (\eta_B$ is baryon-photon ratio), $A^H_{+} = 2.85 \times 10^{-15} \text{ s}^{-1}$. Collisional coefficient:

$$C^H_{+} = k_{10}^{HH} (T_k)n_H + k_{10}^{eH} (T_k)n_e + k_{10}^{pH} (T_k)n_p$$

where $k_{10}^{HH}$ is the scattering rate between two hydrogen atoms [5], [6], $k_{10}^{eH}$ is the scattering rate between hydrogen atoms and electrons [7], $k_{10}^{pH}$ is the scattering rate between hydrogen atoms and protons [8] (here collisions with $^4$He were neglected). For deuterium: $n_D = 2.61 \times 10^{-5} n_H$ [1] (here $n_D = n_{DF} + n_{DII}, n_H = n_{HI} + n_{HII}$), $A^D_{+} = 4.7 \times 10^{-17} \text{ s}^{-1}$. Collisional coefficient:

$$C^D_{+} = \sqrt{\frac{8kBT_k}{\mu_{DH}\pi} \sigma_{+}^{DH} n_H}$$

$\mu_{DH} = m_D m_H / (m_D + m_H) \approx m_H / 3$ is reduced mass, $\sigma_{+}^{DH}$ is spin-change cross-section [9].

At fig.1 there were plotted H and D spin temperatures ($T_s^H$ and $T_s^D$ respectively), gas kinetic temperature ($T_k$) and CMB temperature ($T_{CMB}$). Before $z \sim 300$: $T_k = T_{CMB}$; due to collisions (because $\chi_c \gg 1$) $T_s = T_k$. At $30 \lesssim z \lesssim 300$ (for hydrogen) 5 $\lesssim z \lesssim 300$ (for deuterium) gas was cooling adiabatically due to Universe expansion: $T_k \propto (1 + z)^2$. So $T_k < T_{CMB}$ ($T_{CMB} \propto (1 + z)$) and $T_s < T_{CMB}$ (due to collisions). At $z \lesssim 30$ (for
hydrogen) \( z \lesssim 5 \) (for deuterium) because of Universe expansion gas density decreases so collisions become ineffective and \( T_s \) drives to \( T_{CMB} \) (\( T_s^D \) is coupled to \( T_{CMB} \) at lower \( z \) because lifetime of the excited level is longer (\( A_H^+/A_D^+ = 61.35 \)) and \( \sigma_{DH}^+ \gg \sigma_{HH}^+ \) at low temperatures [8]).

3. Gas kinetic temperature
It is necessary to calculate gas kinetic temperature \( T_k \) to obtain \( T_s \) (following equation (2)). Gas kinetic temperature is defined by the following equation [10]:

\[
\frac{dT_k}{dz} = \frac{2T_k}{1 + z} - \frac{x_e}{1 + f_{He} + x_e} \frac{8\sigma_T u_{CMB} T_{CMB} - T_k}{3m_e c (1 + z) H(z)}
\]

where \( f_{He} = 0.08 \) is helium fraction, \( \sigma_T = 6.65 \times 10^{-25} \text{ cm}^2 \) is Thomson cross-section, \( u_{CMB} \) is the energy density of the CMB, \( x_e \) is free electron fraction.

Here the first term describes adiabatic cooling due to Universe expansion, the second term describes heating due to Compton scattering of CMB photons from the residual free electrons.

4. Ionization of H, D, \(^3\text{He}, \(^4\text{He}\)

The second term in the equation (3) depends on free electron fraction \( x_e \); it is necessary to calculate ionization fraction to find \( x_e \).

To calculate ionization fraction of H, D, \(^3\text{He}\) and \(^4\text{He}\), there were solved system of balance equations:

\[
\frac{d}{dt} \left[ \frac{n_{HI}}{n_H} \right] = \Gamma_{HI} n_e \frac{n_{HI}}{n_H} + \beta_{HI} n_e \frac{n_{HI}}{n_H} - \alpha_{HI} n_e \frac{n_{HI}}{n_H} \]

\[
\frac{d}{dt} \left[ \frac{n_{D}}{n_H} \right] = \Gamma_{D} n_e \frac{n_{D}}{n_H} + \beta_{D} n_e \frac{n_{D}}{n_H} - \alpha_{D} n_e \frac{n_{D}}{n_H} \]

\[
\frac{d}{dt} \left[ \frac{n_{^3\text{He}}}{n_H} \right] = \Gamma_{^3\text{He}} n_e \frac{n_{^3\text{He}}}{n_H} + \beta_{^3\text{He}} n_e \frac{n_{^3\text{He}}}{n_H} - \alpha_{^3\text{He}} n_e \frac{n_{^3\text{He}}}{n_H} \]

\[
\frac{d}{dt} \left[ \frac{n_{^4\text{He}}}{n_H} \right] = \Gamma_{^4\text{He}} n_e \frac{n_{^4\text{He}}}{n_H} + \beta_{^4\text{He}} n_e \frac{n_{^4\text{He}}}{n_H} - \alpha_{^4\text{He}} n_e \frac{n_{^4\text{He}}}{n_H} \]

\[
\frac{d}{dt} \left[ \frac{n_{^3\text{He}}^+}{n_{^3\text{He}}} \right] = \Gamma_{^3\text{He}^+} n_e \frac{n_{^3\text{He}^+}}{n_{^3\text{He}}} + \beta_{^3\text{He}^+} n_e \frac{n_{^3\text{He}^+}}{n_{^3\text{He}}} - \alpha_{^3\text{He}^+} n_e \frac{n_{^3\text{He}^+}}{n_{^3\text{He}}} \]

\[
\frac{d}{dt} \left[ \frac{n_{^4\text{He}^+}}{n_{^4\text{He}}} \right] = \Gamma_{^4\text{He}^+} n_e \frac{n_{^4\text{He}^+}}{n_{^4\text{He}}} + \beta_{^4\text{He}^+} n_e \frac{n_{^4\text{He}^+}}{n_{^4\text{He}}} - \alpha_{^4\text{He}^+} n_e \frac{n_{^4\text{He}^+}}{n_{^4\text{He}}} \]

\[
\frac{d}{dt} \left[ \frac{n_{^3\text{He}^{++}}}{n_{^3\text{He}^+}} \right] = \Gamma_{^3\text{He}^{++}} n_e \frac{n_{^3\text{He}^{++}}}{n_{^3\text{He}^+}} + \beta_{^3\text{He}^{++}} n_e \frac{n_{^3\text{He}^{++}}}{n_{^3\text{He}^+}} - \alpha_{^3\text{He}^{++}} n_e \frac{n_{^3\text{He}^{++}}}{n_{^3\text{He}^+}} \]

\[
\frac{d}{dt} \left[ \frac{n_{^4\text{He}^{++}}}{n_{^4\text{He}^+}} \right] = \Gamma_{^4\text{He}^{++}} n_e \frac{n_{^4\text{He}^{++}}}{n_{^4\text{He}^+}} + \beta_{^4\text{He}^{++}} n_e \frac{n_{^4\text{He}^{++}}}{n_{^4\text{He}^+}} - \alpha_{^4\text{He}^{++}} n_e \frac{n_{^4\text{He}^{++}}}{n_{^4\text{He}^+}} \]
$\frac{d}{dt} \left[ \frac{n_{\text{He}II}}{n_{\text{He}}} \right] = \Gamma_{\text{He}II} n_e \frac{n_{\text{He}II}}{n_{\text{He}}} + \beta_{\text{He}II} n_e \frac{n_{\text{He}II}}{n_{\text{He}}} - \beta_{\text{He}II} n_e \frac{n_{\text{He}II}}{n_{\text{He}}} - (\alpha_{\text{He}II} + \xi_{\text{He}II}) n_e \frac{H_{\text{e}II}}{n_{\text{He}}} + \alpha_{\text{He}III} n_e \frac{n_{\text{He}III}}{n_{\text{He}}} \tag{5}$

$\frac{d}{dt} \left[ \frac{n_{\text{He}III}}{n_{\text{He}}} \right] = \Gamma_{\text{He}III} n_e \frac{n_{\text{He}II}}{n_{\text{He}}} + \beta_{\text{He}III} n_e \frac{n_{\text{He}II}}{n_{\text{He}}} - \alpha_{\text{He}III} n_e \frac{n_{\text{He}III}}{n_{\text{He}}} \tag{6}$

$n_e = n_{\text{HI}} + n_{\text{HII}} + 2n_{\text{He}III} \tag{7}$

Here $n_e$ is free electron fraction, $n_{\text{HI}}, n_{\text{HII}}$ are number densities of neutral and ionized hydrogen, $n_{\text{He}I} = n_{\text{He}I} + n_{\text{He}II}$ is the total number density of hydrogen; $n_{\text{He}II}, n_{\text{He}III}, n_{\text{He}III}$ are number densities of neutral, singly-ionized and doubly-ionized helium, $n_{\text{He}e} = n_{\text{He}I} + n_{\text{He}II} + n_{\text{He}III}$ is total number density of helium.

Also, $\Gamma_x$ is photo-ionization rate, $\alpha_x$ is recombination rate, $\xi_x$ is dielectronic recombination rate, $\beta_x$ is collisional ionization rate; they where taken from [11]. These functions depend on gas kinetic temperature, so it is necessary to solve them with equation (3).

5. The brightness temperature

The brightness temperature is the temperature that black body would have to emit observed intensity. To find it let us consider the equation of radiative transfer for intensity $I_\nu$ in the absence of scattering along the path:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \tag{8}$$

where $ds = cdt$, $j_\nu$ is an emission coefficient, $\alpha_\nu$ is an absorption coefficient.

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

here $\tau_\nu = \int \alpha_\nu(s)ds$ is an optical depth, $S_\nu = j_\nu/\alpha_\nu$.

To simplify calculations there were used Rayleigh-Jeans limit, appropriates for frequencies $\nu$ that much smaller then the peak frequency of CMB spectrum: $I_\nu = 2k_B T^2 c^2/\nu^2$. So after rewriting (8) we can get equation of radiative transfer for light from radio source with brightness temperature $T_b$ through the cloud with spin temperature $T_s = \frac{c^2 j_\nu}{2k_B^2 \nu^2 \alpha_\nu}$. So the observed brightness temperature at frequency $\nu$:

$$T_b^{\text{obs}} = T_b e^{-\tau_\nu} + T_s (1 - e^{-\tau_\nu}) \tag{9}$$

Taking into consideration that optical depth is small at relevant redshifts ($10 \lesssim z \lesssim 1000$) ($x$ is H or D):
6. Dependence of 21 cm brightness on baryon-photon ratio

On the fig.3 there were plotted dependencies of 21 cm brightness on z with different values of baryon density $\Omega_b$; solid curve corresponds to the modern value of $\Omega_b = 0.049$ [1] - solid curve

On the fig.4 there was plotted dependence of 21 cm brightness on baryon-photon ratio $\eta = n_B/n_\gamma$; dashed line corresponds to the modern value of $\eta = 6.1 \times 10^{-10}$. From [12]:

$$
\eta = 273.4 \times 10^{-10} \Omega_b h^2
$$
\( h = 0.67 \) is dimensionless Hubble constant.

Without loss of generality let us consider that \( \eta \) increases, so \( n_H \) will increase too (\( n_H \propto \eta \)) and \( x_e \) will decrease therefore gas kinetic temperature \( T_k \) will decrease. Coefficient describes collisions \( \chi_c \propto C_{+}\) will increase; it drives spin temperature \( T_s \) to \( T_k \), thus \( T_s \) will decline as \( T_k \). So \( \tau \propto n_H/T_s \) will decrease and \( \delta T_b = \frac{T_s - T_{CMB}}{1 + z}\) will decrease too.

7. Conclusions
In this paper there were considered physical processes during cosmic Dark Ages and there were calculated H and D brightness temperature, gas kinetic temperature and CMB temperature. At fig.1 there were plotted dependence of H and D spin temperature (\( T^H_s \) and \( T^D_s \) respectively), gas kinetic temperature (\( T_k \)) and CMB temperature (\( T_{CMB} \)) on redshift \( z \).

In this work there were examined dependencies of 21 cm brightness temperature on baryon density (fig.3) and dependence on baryon-photon ratio \( \eta = n_B/n_\gamma \) (fig.4). \( \Omega_B \) and \( \eta \) are the key parameters in modern cosmology and hence we can see a strong dependence on these parameters.

These measures potentially could become a new instrument for independent measurement of barion-photon ratio during Dark Ages.

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