Remote state preparation and measurement of single photon

Arun K. Pati(1)

Institute of Physics, Sainik School Post, Bhubaneswar-751005, India
(1) School of Informatics, University of Wales, Bangor LL 57 1UT, UK

Abstract

Quantum information theory has revolutionized the way in which information is processed using quantum resources such as entangled states, local operations and classical communications. Two important protocols in quantum communications are quantum teleportation and remote state preparation. In quantum teleportation neither the sender nor the receiver know the identity of a state. In remote state preparation the sender knows the state which is to be remotely prepared without ever physically sending the object or the complete classical description of it. Using one unit of entanglement and one classical bit Alice can remotely prepare a photon (from special ensemble) of her choice at Bob’s laboratory. In remote state measurement Alice asks Bob to simulate any single particle measurement statistics on an arbitrary photon. In this talk we will present these ideas and discuss the latest developments and future open problems.

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email:akpati@iopb.res.in

I. INTRODUCTION

Quantum information theory is a vast area of interdisciplinary research which includes quantum computing, quantum complexity, quantum communication protocols, quantum cryptography, quantum entanglement, and so on [1]. In quantum communication protocols how to process the inaccessible quantum information contained in an unknown state using quantum entanglement and classical channels is an ongoing important area of research. The most cited example is quantum teleportation of an unknown state [2] from one place to another without ever physically sending the particle. Any quantum state can be teleported from Alice to Bob provided they share a maximally entangled pair and allow to communicate classically. Alice carries out a Bell-state measurement on the input state and one half of the entangled pair and sends the measurement outcomes via a classical channel to Bob. Then he performs a unitary operation on his particle to get the original state. In the process the original state is destroyed and a replica appears in accordance with no-cloning principle [3,4]. In a similar manner, one may interpret that since the original is being destroyed at sender’s
location it must appear somewhere else (i.e., at receiver’s location) in accordance with non-deletion principle [3]. Thus quantum information is ‘robust’ in some sense (and ‘fragile’ too)! If Alice and Bob do not share maximally entangled state rather non-maximally entangled state, then via non-maximally entangled measurement and classical communication Alice can teleport a single photon in a probabilistic manner [6].

However, if Alice has the complete classical information about a state she need not do teleportation. Instead, she can help Bob to prepare a quantum state at a remote location using prior entanglement and classical communication. It was shown that indeed Alice can prepare special class of qubits (either from polar or equatorial great circle) using one maximally entangled pair and one classical bit of communication [7]. In section I, I discuss mainly the exact remote state preparation of single photon. In section III, I discuss remote state measurement (RSM) of arbitrary photon via projection and generalised POVM measurements. In section IV, I present why Alice cannot ask Bob to simulate two particle measurement on an unknown photon with one unit of entanglement and one classical bit. In section V, I will mention recent developments in the context of remote state preparation and its generalisation to higher dimensional quantum system.

II. REMOTE STATE PREPARATION OF A SPECIAL PHOTON

In this section I discuss the remote state preparation protocol of a single photon chosen from a particular ensemble. The particular ensemble is known both to Alice and Bob, but the parameters describing the state in the ensemble is known only to Alice but unknown to Bob. This protocol requires only one classical bit to be communicated from Alice to Bob provided they share an Einstein-Podolsky-Rosen (EPR) pair. Here, we do not require a Bell-state measurement; only a local unitary operation and single particle von Neumann measurement is necessary. Moreover we do not need the physical presence of a qubit at Alice’s location. The possibility that a photon chosen from equatorial or polar great circles on a Poincare sphere can be remotely prepared arises from the isotropic nature of EPR state and impossibility of complementing an unknown state.

Consider a photon in a pure state $|\Psi\rangle$. An arbitrary photon can be represented as a linear superposition of two distinct polarisation states such as

$$|\Psi\rangle = \alpha|H\rangle + \beta|V\rangle,$$

where we can choose $\alpha$ to be real and $\beta$ to be a complex number, in general (up to $U(1)$ equivalence classes of states). $|H\rangle$ and $|V\rangle$ represents horizontal and vertical polarisation states of a photon. They are also called computational basis states, because they can represent classical information such as ‘0’ and ‘1’. The single photon state in (1) is a quantum bit (qubit) and can be represented by a point on the Poincare sphere $S^2$. Such a qubit is realized only in quantum world but not in the classical world. The qubit $|\Psi\rangle$ is known to Alice and unknown to Bob.

Imagine that Alice and Bob are far away from each other. Alice wants a photon in the state $|\Psi\rangle$ at Bob’s place. One way would be to prepare a photon in her lab and send through an optical fiber over to Bob’s lab. Alternately, she can send the classical description of the photon over an ordinary communication channel (telephone line) and Bob prepares himself
accordingly. In the first case, some one else on the way can take the photon that Alice has sent and in the second case, to transmit two real numbers one needs to send infinite number of bits, in principle. From communication point of view this would be very expensive. However, both the problems can be over come if Alice and Bob have shared previously one half of the photons from an EPR source, which is given by

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B).$$

(2)

This type of entangled state of photons can be created by parametric down-conversion process where an input photon of given frequency (called pump photon) can decay inside a non-linear crystal into two photons with maximal quantum correlation.

Now the protocol goes like this. Alice is having first particle and Bob is having the second. Since Alice knows the state she can chose to measure her photon in any basis she wants. Furthermore, she knows how to relate the computational basis states \{\(|H\rangle, |V\rangle\}\} and the arbitrary “qubit basis” \{\(|\Psi\rangle, |\Psi_\perp\rangle\}\} in a unitary manner. The unitary operator is the standard \(SU(2)\) operator. This is given by

$$|\Psi\rangle = U(\alpha, \beta)|H\rangle = \alpha|H\rangle + \beta|V\rangle$$

$$|\Psi_\perp\rangle = U(\alpha, \beta)|V\rangle = \alpha|V\rangle - \beta^*|H\rangle.$$  

(3)

An important property of the EPR state \(|\Psi^-\rangle_{AB}\) is that it is invariant under local unitary operator \(U_A \otimes U_B\) operation where the same \(U\) acts on both the subsystems, i.e.,

$$U_A(\alpha, \beta) \otimes U_B(\alpha, \beta)|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}[|\Psi\rangle_A|\Psi_\perp\rangle_B - |\Psi_\perp\rangle_A|\Psi\rangle_B] = |\Psi^-\rangle_{AB}.$$  

(4)

Now another remarkable property follows from the above invariance nature of EPR singlet is that if a subsystem undergoes an evolution, then the other subsystem undergoes a de-evolution or vice versa. It is something which is really counter intuitive and has no classical analog! It is expressed by the following equation:

$$U_A(\alpha, \beta) \otimes I_B|\Psi^-\rangle_{AB} = I_A \otimes U_B(\alpha, \beta)^\dagger|\Psi^-\rangle_{AB}. $$  

(5)

This says that when both the photons are in an EPR state, then if one subsystem goes forward in time and the other one is silent (done nothing), it is equivalent to first subsystem being silent and the second goes back reverse in time. However, this evolution cannot be seen at individual level, because the state of either photon is completely unpolarized (i.e. a random density matrix \(I/2\)). The evolution leading to state preparation is possible only after local measurement and sending the classical information. To me this lies at the heart of remote state preparation of special class of photons.

Now Alice applies \(U_A(\alpha, \beta)^\dagger\) to her particle and as a result the state becomes

$$U_A(\alpha, \beta)^\dagger \otimes I_B|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}[|H\rangle_A|\Psi_\perp\rangle_B - |V\rangle_A|\Psi\rangle_B].$$  

(6)

She performs a von Neumann projection onto horizontal and vertical basis and sends one classical bit to Bob. If her outcome is \(|H\rangle\) Bob’s photon would be in \(|\Psi_\perp\rangle\) and if she gets
|V⟩ then Bob’s photon would be in the desired state |Ψ⟩. For example, if Alice chooses to prepare a photon from polar great circle, i.e., |Ψ⟩ = α|H⟩ + β|V⟩ (with α and β both are real), Bob will apply \(i\sigma_y\) to |Ψ⊥⟩ = β|H⟩ - α|V⟩ or do nothing after receiving the classical information from Alice. Alternatively, if Alice chose to prepare a photon from equatorial circle on Poincare sphere such as |Ψ⟩ = \(\frac{1}{\sqrt{2}}(|H⟩ + e^{iφ}|V⟩)\) then in this case Bob can get |Ψ⟩ from |Ψ⊥⟩ = \(\frac{1}{\sqrt{2}}(|H⟩ - e^{iφ}|V⟩)\) by applying \(σ_z\) or do nothing after receiving one classical bit. Thus Alice can prepare any photon either from polar or equatorial circle using this protocol at a remote location.

But why cannot Alice prepare any arbitrary photon state even though she has complete knowledge. Here comes Bob’s knowledge in manipulating photon states! Similar to no-cloning and no-deleting principles, we know that an arbitrary unknown state cannot be complemented. Here complementing operation means creating a state that is orthogonal to the given state. Though we can design a device (called NOT gate) which can take |H⟩ → |V⟩ and |V⟩ → |H⟩ there is no universal NOT gate which can take an unknown qubit |Ψ⟩ → |Ψ⊥⟩ as it involves an anti-unitary operation. Anti-unitary operations correspond to improper rotations and cannot be implemented by physical operations (called Completely Positive Maps). Thus, Bob cannot convert a photon in an orthogonal state (which he gets half of the time) since it is unknown to him. Thus, a doubly infinity of bits of information (corresponding to two real numbers) cannot be passed all the time with the use of entanglement by sending just one classical bit.

III. REMOTE STATE MEASUREMENT OF PHOTON

In an interesting protocol called “classical teleportation” of a qubit it is aimed to simulate any possible measurement on a qubit known to Alice but unknown to Bob using hidden variables and classical communications. Remote state measurement (RSM) protocol is an outcome of quantum version of the above protocol. At a first glance, it appears that in our RSP scheme one can remotely prepare an arbitrary known photon state one half of the time, so Bob might not be able to simulate the measurement statistics all the time as Bob cannot get a unknown photon from the orthogonal photon state. However, a little thought shows that there is no problem with Bob for simulating the measurement statistics on the complement photon. This is because the quantum mechanical probabilities and transition probabilities are invariant under unitary and anti-unitary operations which is the famous Wigner’s theorem. This says that for any two non-orthogonal states if \(|⟨Ψ|Φ⟩|^2 = |⟨Ψ'|Φ'|⟩|^2\) then |Ψ⟩, |Φ⟩ are related to |Ψ⟩, |Φ⟩ either by unitary or anti-unitary transformations. For example, if Bob wants to measure an observable \(⟨b,σ⟩\), with the projection operator \(P_±(b) = \frac{1}{2}(I ± b,σ)\), then the probability of measurement outcome in the state \(ρ_Ψ = |Ψ⟩⟨Ψ| = \frac{1}{2}(I + n,σ)\) is given by

\[P_±(ρ_Ψ) = \text{tr}(P_±(b)ρ_Ψ) = \frac{1}{2}(1 ± b.n).\]  

But suppose Bob gets \(ρ_Ψ⊥ = |Ψ⊥⟩⟨Ψ⊥| = \frac{1}{2}(I - n,σ)\). In this case the measurement gives a result

\[P_±(ρ_Ψ⊥) = \text{tr}(P_±(b)ρ_Ψ⊥) = \frac{1}{2}(1 ± b.n).\]
The probabilistic outcomes in (7) and (8) are different. However, Bob can always choose his apparatus (by reversing the direction of $b$) such that he can make $P_{\pm}(\rho) = P_{\pm}(\rho_{\perp})$. Note that Bob cannot reverse the direction of $n$ but can in principle reverse the direction of $b$. So even if Bob cannot get a qubit from a complement qubit (half of the time) still he can get the same measurement outcomes from it. Therefore, Bob can always simulate efficiently the statistics of his measurements on a qubit known to Alice but unknown to him, provided they share an EPR pair and communicate one classical bit. In the classical case, if Alice and Bob do not share entanglement they need to send 2.19 bits on the average [12].

Bob can also simulate probabilistic outcomes via generalized measurements such as POVMs $F_\mu$ with $\sum_\mu F_\mu = 1$. They can be described by the elements $F_\mu = \frac{1}{2}(|f_\mu|I + f_\mu \sigma)$, where $f_\mu$’s are vectors on Poincare sphere [12]. The probability of measurement outcome in the state $\rho$ and $\rho_{\perp}$ are given by $P_\mu(\rho) = \text{tr}(F_\mu \rho) = \frac{1}{2}(|f_\mu| + f_\mu \cdot n)$ and $P_\mu(\rho_{\perp}) = \text{tr}(F_\mu \rho_{\perp}) = \frac{1}{2}(|f_\mu| - f_\mu \cdot n)$, respectively. These outcome are different, but Bob can flip the vectors $f_\mu$ to make these simulations identical as if he has a qubit at his disposal. This is possible again with one unit of entanglement and one classical bit, whereas classically, if Alice and Bob share hidden variables to simulate POVM outcomes Alice and Bob have to communicate on the average 6.38 bits (counting forward and backward communications) [12]. Thus use of quantum entanglement saves 5.38 bits and backward communication in remote simulation of POVMs.

IV. REMOTE JOINT STATE MEASUREMENT

Next we ask the question: Can Alice ask Bob to simulate any joint measurement on two unknown photon states with the use of one ebit and one classical bit? The general answer is ‘no’. Suppose Bob at his disposal has another photon in an unknown quantum state $|\Phi\rangle = a|H\rangle + b|V\rangle$, with $\rho_\Phi = \frac{1}{2}(I + m_\sigma)$, in addition to a photon that Alice would be preparing. Now Bob wants to do a joint measurement on the state of the first photon (being in $|\Psi\rangle$ or in $|\Psi_{\perp}\rangle$) and the second photon in the state $|\Phi\rangle$. Suppose the joint measurement operation is an entangled operator $\Pi$ given by

$$\Pi = \frac{1}{4}(I \otimes I + (r_\sigma \otimes I + I \otimes s_\sigma) + \sum_{ij} t_{ij} \sigma_i \otimes \sigma_j) \quad (9)$$

where $r$ and $s$ are real vectors and $t_{ij}$ are real coefficients. In terms of density matrices, Bob’s two photons can be in a state (when Alice gets $|1\rangle$)

$$\rho \otimes \rho_\Phi = \frac{1}{4}[I \otimes I + (n_\sigma \otimes I + I \otimes m_\sigma) + (n_\sigma) \otimes (m_\sigma)] \quad (10)$$

or in a state (when Alice gets $|0\rangle$)

$$\rho_{\perp} \otimes \rho_\Phi = \frac{1}{4}[I \otimes I - (n_\sigma \otimes I + I \otimes m_\sigma) - (n_\sigma) \otimes (m_\sigma)] \quad (11)$$

The question is can Bob make the following two probabilities equal, i.e., if $\text{tr}[\Pi \rho \otimes \rho_\Phi] = \text{tr}[\Pi \rho_{\perp} \otimes \rho_\Phi]$? Explicitly, they are given by
\[
\text{tr}[\Pi \rho_\Psi \otimes \rho_\Phi] = \frac{1}{4}[1 + r.n + s.m + \sum_{ij} t_{ij} n_i m_j]
\]

\[
\text{tr}[\Pi \rho_\Psi \perp \otimes \rho_\Phi] = \frac{1}{4}[1 - r.n + s.m - \sum_{ij} t_{ij} n_i m_j] 
\]

(12)

In general these two probabilities are not equal and there is no way for Bob to make them equal either. This is because Bob can only manipulate with his measuring device described by parameters \( r \) and \( s \). By flipping the sign of \( r \) and \( s \) he cannot make these probabilities identical. However, examination of the above equation reveals that if the additional quantum state \( |\Phi\rangle \) is known to Bob, then he can flip \( m \) and \( r \) to make these probabilities equal. Thus remote joint state measurement is possible on an arbitrary photon that Alice wanted, together with a known photon state.

V. FURTHER GENERALISATION AND OPEN QUESTIONS

Remote state preparation and measurement protocols have brought out various features of quantum and classical resources used in quantum communication. Unlike in teleportation where resources are fixed for the task, here it is possible to have trade-offs. In last three years, important progresses have been made in understanding these trade-offs. Soon after the exact RSP protocol, Lo has [13] conjectured that if Alice wants to prepare remotely an arbitrary qubit it may still require two classical bits as in the case of quantum teleportation. Bennett et al have generalised RSP for arbitrary qubits, higher dimensional Hilbert spaces and also RSP of entangled systems. If one does not restrict the number of entangled pairs used, then asymptotically one can prepare an arbitrary qubit with one classical bit [14]. Subsequently, Devetak and Berger have proposed a low entanglement RSP protocol [15] for arbitrary quantum states. The exact and minimal resource consuming RSP protocol is generalised to higher dimension by Zeng and Zhang [16]. It was found that it is not possible to have RSP in arbitrary higher dimension with minimal resources. There are restrictions on the dimension of the Hilbert space for which RSP can be realized. Leung and Shor have given a stronger proof of Lo’s conjecture for RSP of arbitrary quantum state [17]. Remote preparation of ensemble of mixed states has been studied by Berry and Sanders [18]. In addition, as a first step the exact RSP and RSM protocol for qubit [8] have been implemented using NMR devices [19,20] over atomic distances. However, long distance implementation of RSP of special and arbitrary qubits and RSM of arbitrary qubits would be very welcome in future. Even though RSP could be generalised to higher dimensional Hilbert space, similar generalisation of RSM to higher dimensional quantum systems is still elusive. It may be even impossible! So in that sense only qubits (photons) enjoy the RSM protocol.

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