Pricing analysis of stock index linked structured financial products of Commercial Banks

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Abstract. Financial products of commercial banks are an investment tool designed and issued independently by commercial banks. Nowadays, commercial banks have issued a variety of financial products, and the theoretical research on this aspect mainly focuses on marketing, the analysis of influencing factors of return rate and etc, but involving less pricing. Most of the research on pricing is limited to products with a simple structure. The case of this study is a structured financial product related to stock index. And besides an in-depth analysis of the structure characteristics and risk characteristics of the product, this study also calculates out the theoretical pricing of the product on the basis of using the stationary time series for conditional heteroscedastic model and the Monte Carlo Simulation Method (with the aid of Eviews software), and compares it with the actual pricing set by the Agricultural Bank of China, and then determines the rationality of the actual pricing. And suggestions are given accordingly.

1. Introduction
Since China Everbright Bank launched the country's first bank financial products in 2001, structured financial products have gradually attracted Chinese's interests. With the rapid development of economy, residents' awareness of financial management has been strengthened. Their income has been greatly increased, and their demand for financial products has also been increasing. By the end of 2017, 555 commercial banks had issued a total of 85,800 financial products, an increase of 11,600 compared with that of the beginning of the year.

With the emergence of wealth management products, their disadvantages are more and more clearly exposed to the world. In particular, China lags far behind western countries in the core technology of structured financial products. Domestic banks are relatively passive due to their lack of experience and weak technology in pricing derivatives. In this context, experts in either theoretical or practical field need to further deepen their research on the risk pricing of financial products.

This paper presents an in-depth analysis of specific financial products ("Golden Key and Ruyi Portfolio", a structured financial product issued by Agricultural Bank of China). This product is a non-guaranteed floating income RMB structured financial products; the product income is linked to the CSI 300 index. Hopefully this study would present a deeper understanding of the risk pricing of this product. At the same time, it can also provide ideas for commercial banks to design financial products, and help investors understand financial products and make wiser decisions as well.
2. "Golden key Ruyi" financial products of Agricultural Bank of China

The financial products of Agricultural Bank of China are divided into six series, including "Ben li Feng", "Anxin-deli", "Anxin-express", "Anxin-lingdong", "Enterprising" and "Ruyi", which can meet customers of different levels, needs and risk types.

2.1. Product elements

Ruyi series is a structured wealth management product. Its capital are mainly invested in treasury bonds and other high credit rating products such as credit debt, private placement note, low-risk interbank fund business, or fixed-income investment instrument, non-standardised creditor rights issued by commercial banks and other qualified institutes, and trust plans which meet the regulatory requirements, and some other investments [1]. In the inter-bank bond market, the investment proportion of treasury bonds, central bank bills, financial bonds, buybacks, high-grade credit bonds, and non-public directional debt financing instruments, cash and deposits is about 20-40%. The investment proportion of money market instruments, investment trust plans, non-standardized debt and other fixed-income investment instruments is about 60-90%, and the above investment proportion is within the range of [-10%, 10%]. The bank adjusts the asset portfolio allocation dynamically according to the related investment market trend.

This paper takes Agricultural Bank of China's "Golden Key Ruyi", the 25th issue of 2018, as the object to study the bullish RMB financial products at the end of the CSI 300 index [2]. According to the product risk and customer equity statement, in this case, the product is a RMB financial product with non-breakeven floating income within 90 days (subject to the bank's early termination clause), and the product income is linked to the CSI 300 index. The details are as follows

| Table 1. "Golden key • Ruyi" 2018 25th CSI 300 index ending bullish RMB financial product specification |
|---------------------------------------------------------------|
| **The product name** | "Golden keys ruyi" the 25th issue of 2018 CSI-300 index final bullish continuous RMB financial products |
| **The investment currency** | Yuan |
| **Product risk rating** | Low and medium risk |
| **Financial period** | 90 days (subject to bank early termination clause) |
| **Product type** | Hook mark |
| **Income type** | The CSI 300 index |
| **Subscription opening date** | 2018-02-13 |
| **Closing date** | 2018-02-22 (11:00 am) |
| **Product commencement date** | 2018-02-23 |
| **Product expiration date** | 2018-05-24 |
| **Subscription amount** | Personal: the starting amount is RMB 50,000 Yuan, increasing by integer multiples of RMB 1,000 Yuan. For the company: the starting amount is RMB 100,000 Yuan, increasing by integer multiples of RMB 1,000 Yuan. |
| **Sales organization** | The national branch of Agricultural bank of China and its authorized branches. |
| **Sales channels** | Agricultural bank of China authorized outlets, online banking, palm banking, etc |
| **Actual days** | The number of natural days starting from the interest date (inclusive) to the expiry date (exclusive) of the product. |
| **The principal guarantee** | No principal guarantee is provided for this financial product |
2.2. Product income statement

Part of the income of the product is linked to the CSI 300 index to obtain floating income. The initial price refers to the closing price of the CSI 300 index on the value date of the product, and the final price refers to the closing price of the CSI 300 index two trading days before the product expires [3]. The price performance of the CSI 300 index is \((\text{final price/initial price - 1}) \times 100\%\).

1) If the price performance of CSI 300 index is greater than or equal to 5%, investors will get a 6.10% annualized yield at maturity.

2) If the price performance of CSI 300 index is less than 5% and is greater than or equal to -5%, the annualized yield obtained by investors at maturity is: \(2.60\% + 0.35 \times (\text{price performance of CSI 300 index} + 5\%)\).

3) If the price performance of CSI 300 index is less than -5% and the ending price is no less than 1000, the investor will get 2.60% annualized yield at maturity.

4) If the price performance of CSI 300 index is less than -5% and the ending price is less than 1000, the investor will get 2.50% annualized yield at maturity. (The annualized yield to maturity is accurate to two decimal places, that is, 0.01%) Investors' financial return = financial capital \(*\) annualized yield to maturity \(*\) the number of actual financial days \(/365\) (accurate to two decimal places).

In the case of expiration (or early termination) of the product, the final income of the customer shall be calculated based on the actual investment annualized return rate of the financial product (after deducting relevant fees) and the actual survival days of the product. If the actual investment annualized yield of the wealth management product exceeds the expected maximum annualized yield, when the product expires (or terminates in advance or is redeemed by the customer), the excess will be regarded as floating management expense of the asset manager [4, 5].

3. Pricing analysis

The case studied in this paper, "Golden Key Ruyi portfolio", is a structured financial product with non-guaranteed floating income. The design principle of structured financial products is to use the decomposition technology of financial engineering to divide the principal of financial products into two parts, including a fixed part and floating part. Fixed part can be regarded as a kind of creditor right with stable income. The income of the floating part is related to the price fluctuation of the underlying asset, which is actually an option. The income of this part is not fixed, and the income depends on the accuracy of the future estimation of the underlying derivative. In the research, we assume that the financial market is mature [6, 7]. According to the principle of risk-free arbitrage, the total pricing of structured wealth management products in this market is the sum of fixed part pricing and floating part pricing. Therefore, in this chapter, we calculate the pricing of the two parts and then add them together, and the final result is the theoretical pricing of "Golden Key Ruyi portfolio". Then, we compare the theoretical pricing with the actual pricing of the products and observe whether the pricing of the products is reasonable.

3.1. Fixed part theoretical model

According to the product agreement, the "Golden Key Ruyi portfolio" financial products will repay the principal and interest in a lump sum on the maturity date of the products, so the fixed part of the pricing needs to be calculated by using the bond pricing formula [8], that is, the daily interest will be discounted and the principal will be added. The formula can be expressed as:

\[
P = \sum_{i=1}^{n} \frac{M + C}{(1 + r)^i}
\]

In the formula, \(P\) is the fixed income part of the pricing, \(M\) is the principal, \(C\) is the interest, \(n\) is the term, and \(r\) is the market discount rate. As it is necessary to take the actual situation in China and the characteristics of cases into consideration, SHIBOR is then taken as the market discount rate in the research.
3.2. Floating part of the theoretical model

In this paper, the Monte Carlo Simulation Method is used for the floating part of the pricing [9]. Monte Carlo Method is to firstly build a path model generated by other pricing methods. It then calculates repetitively to obtain multiple possible solutions. And then the average value of these solutions is the approximate value of the option pricing result in this study. In this paper, we first use the b-s model to find the path required by the Monte Carlo Method and then carry out the simulation 10,000 times along the path.

In order to obtain the path of asset price through Monte Carlo Simulation, the fluctuation law of the movement of the underlying asset must be found first. The sequence of financial assets, such as interest rate, exchange rate, stock return rate, etc., has the following three characteristics: first, the fluctuation of the sequence changes with time, sometimes changes can be explosive, other times very intense; second, compared with the normal distribution, the distribution of the sequence has a sharp peak and a thick tail. Probability values near the mean and the tail are larger than the normal distribution, while probability values in other regions are smaller than the normal distribution. Thirdly, the series has obvious Volatility Clustering, which is means a larger fluctuation tends to be followed by a larger fluctuation, while a smaller fluctuation by a smaller fluctuation. In order to eliminate the influence of volatility aggregation, Engel put forward the ARCH model in 1982. In this paper, the generalized autoregressive conditional heteroscedasticity model (GARCH model) is used to describe the fluctuation law of the movement of the underlying asset (CSI 300 index).

4. Empirical analysis

4.1. Fixed part

It can be seen from the product agreement of "Golden Key Ruyi portfolio" that the individual minimum sales amount of this product is RMB 50,000 Yuan, increasing by integer multiples of RMB 1,000 Yuan. Therefore, we set the unit as RMB 1000. According to the formula mentioned in 3.1, the six-month interest rate value of SHIBOR on May 14, 2018, which was 2.8875, was selected as the risk-free interest rate to discount. The principal was RMB 1000, it can be calculated that the theoretical pricing of the fixed part is \( P = 971.9354 \) Yuan.

4.2. Floating part

The simulation path required by Monte Carlo Simulation Method must be obtained before calculating the pricing of floating part. In this section, we first calculate the volatility of the underlying asset, then get the simulation path according to the b-s model, and finally use the software for multiple simulations. In the calculation of volatility, the daily closing prices of the solstice on January 25, 2017, and CSI 300 on December 25, 2017, were selected as the original data in this paper to establish a time series, which was named as series \( \{y\} \).Evievs9.0 was used to analyse the sample statistics.

According to the statistical results, the standard deviation is 238.01, which indicates that the price of the underlying asset is highly volatile. Looking at other statistics, the mean was 3690.77, the maximum 4227.57, and the minimum 3,337.70. The skewness is 0.33, greater than 0 of the standard normal distribution, and the kurtosis is 1.81. The Jarque-Bera value is 17.29, and the P value is 0. The null hypothesis is rejected.

In general, the price of financial products is subject to the Wiener process and do not subject to the stability test. Since the statistical characteristics of the non-stationary time series are not fixed and change with the change of time, it is difficult to infer the randomness of the subsequent sequence walk through the known information of the sequence. If the regression analysis of non-stationary sequence is forced, it will lead to "pseudo-regression". Therefore, the stationarity test should be carried out on the sequence before the regression analysis. Under the ADF unit root test for the underlying asset, it can be seen that the t statistic of the ADF is -0.6250, and the P value is 0.8612, which is greater than the various significance levels of 0.1~0.01 in the ADF test, and the null hypothesis that the sequence is a non-
stationary sequence cannot be rejected. Therefore, there is a unit root in the sequence \{y\}, showing a non-stationary trend.

![ADF test of CSI 300 index](image)

**Figure 1.** ADF test of CSI 300 index

Although financial sequences such as interest rate are non-stationary, their logarithmic sequences are stable in most cases. At this time, we carried out a differential stationary process on the sequence \{y\}. In order to reduce the rounding error, the natural logarithm of \{y\} is processed in the estimation, and the sequence \{x= DLN (y)\} is used as the dependent variable for the estimation. Therefore, the logarithm of \{y\} data of CSI 300 price series was taken and then the difference was made. The clustering phenomenon of fluctuations can be seen from the following time series diagram: the data fluctuates around 0, and the fluctuations are small in some time periods and large in some other time periods.

![Time series diagram of logarithmic rate of return](image)

**Figure 2.** Time series diagram of logarithmic rate of return

Statistical analysis was performed on \{x\}. According to the figure, the mean value, standard deviation and skewness of logarithmic price series of CSI 300 index were 0.0008, 0.0064 and -0.3316 respectively, indicating that the sequence distribution had a long left trailing tail. The kurtosis value was 5.19, higher than the kurtosis value of the normal distribution of 3, indicating that the price sequence has the characteristics of peak and thick tail. The J-B statistic is 48.84 and the P value is 0, rejecting the assumption that the logarithmic price sequence follows a normal distribution.
Stationarity test on the sequence \( \{x\} \) is performed. The test results are shown in the figure below. The value of t statistic is -11.7811, and the corresponding P value is 0, indicating that the sequence \( \{x\} \) is stable. Therefore, the logarithmic price sequence \( \{x\} \) of CSI 300 index was selected in the subsequent modelling process.

After determining the stationarity of the \( \{x\} \) sequence, we also need to determine whether the lag term of the sequence will affect the closing price. At this point, we performed autocorrelation test and partial autocorrelation test on the \( \{y\} \) sequence (FIG.6) and \( \{x\} \) sequence (FIG. 7). They can be used to identify the stationarity of sequences and the order of the ARMA model of logarithmic sequences.
In financial markets, the variance of time series residuals usually changes with time, and the resulting volatility also changes. This can be shown in the fluctuation chart of the Shanghai and Shenzhen 300 index price series \( \{ x \} \). Therefore, before regression analysis, heteroscedasticity test should be carried out on the sequence. First, the logarithmic rate of return sequence was de-averaged to obtain the sequence \( w \), and let \( z = w^2 \), the 18-order lag autocorrelation function test is carried out on the \( z \) sequence, and the results are shown in the figure.

![Figure 6. Autocorrelation function of Ln{x}](image)

![Figure 7. Autocorrelation function of Ln return residual](image)

As shown in the figure above, it has high significance and high order autocorrelation at all significance levels. Therefore, the null hypothesis that there is no ARCH effect is rejected, that is, the price sequence of CSI 300 index has the ARCH effect. This paper uses this model to describe the track of CSI 300 return sequence. The GARCH (2, 1) test was performed on \( w \) sequences, assuming that the residues follow normal distribution, \( t \) distribution and GED generalized error distribution, respectively.

![Figure 8. GARCH (2, 1) test results](image)

The sum of the coefficients of ARCH and GARCH is 0.97866, less than 1, which satisfies the constraint condition of the parameter. But it's already close to 1, which suggests that the random error term still has a strong lasting impact. At this time, the GARCH model equation is:
This is the basic equation of Monte Carlo simulation. The CSI 300 index closed at 3,218.450 at the beginning of the period. According to the formula for calculating product income, the value of the option part is 25.5435, which is discounted without risk, and the discount rate is still 2.8875%, and the value of the floating part is 24.8266. Plus the fixed part, the total value is 996.76 Yuan, which is less than 1,000 Yuan of theoretical value, so this product is issued at a premium.

5. Conclusion

Through the above empirical analysis, we get that the theoretical price of the "Golden Key • Ruyi portfolio" product is 996.76 Yuan, while the actual price of the product of Agricultural Bank of China is 1000 Yuan, slightly issued at a premium of 3.24 Yuan. While it looks within a reasonable range, the premium is smaller. However, in combination with the minimum sales scale of the product, individual investors will start from 50,000 Yuan and institutional investors will start from 100,000 Yuan. Therefore, for individual investors who purchase the product with the minimum amount, the product will be sold at a premium of 162 Yuan, and for institutional investors who purchase the product with the minimum amount; the product will be sold at a premium of 324 Yuan.

For consumers, the premium part of the product is within a reasonable range, and there is no excessive premium issue, and it has no adverse conditions for ordinary investors. It is an optional structural wealth management product, with moderate returns and risks, which is suitable for conservative and prudent investors. For commercial banks, this product is one of the main financial products promoted by Agricultural Bank of China, which represents the top level of low-risk financial products designed by Agricultural Bank of China. The fixed part and floating part are designed reasonably.

But we also need to know that the essence of structured wealth management products is to invest part of the principal in the future market, and the yield of the products is closely related to the price trend of the underlying asset. Therefore, the judgment of the future price trend of the underlying asset and the design of the bank's yield structure are crucial to the income of financial products.

In the long run, structured financial products, with their unique functions and advantages, will still play an important role in the international capital market and financial derivatives market, and China's structured financial products will also enter a more rapid development trend. Throughout the development of the financial market, the relationship between structured financial products and macro economy is inseparable. At present, the financial derivative market in China is still not perfect, which to some extent has caused a certain negative impact on the development of structured financial products. The follow-up research needs to be further combined with the macro environment to meet the different needs of various investors, which is a realistic problem to be considered in the pricing of such products. Combined with the development status of China's structured financial products, their circulation is relatively large, with certain representativeness. The research results of this paper show that there is a certain difference between the actual price and the theoretical price of this kind of products. The essence of this phenomenon reflects that China's commercial banks do not adopt accurate econometric models for the pricing of this kind of products at present, which makes the market exist in a bad circular environment. Unreasonable pricing will not only prevent commercial Banks from making profits, but also discourage investors from investing, thus hinders the whole wealth management product market.

At present, the research on the pricing of structured financial products is based on the condition of a complete market, and the underlying asset conforms to the geometric Brownian motion. Future research can try to further improve the pricing of such products under the assumption of an incomplete market. In the later research, it can be considered to introduce the model of random volatility and further modify the b-s model to get the research results more consistent with the market.

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