Observing the emergence of a quantum phase transition shell by shell

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Many-body physics describes phenomena that cannot be understood by looking only at the constituents of a system1. Striking examples are broken symmetry, phase transitions and collective excitations2. To understand how such collective behaviour emerges as a system is gradually assembled from individual particles has been a goal in atomic, nuclear and solid-state physics for decades3–6. Here we observe the few-body precursor of a quantum phase transition from a normal to a superfluid phase. The transition is signalled by the softening of the mode associated with amplitude vibrations of the order parameter, usually referred to as a Higgs mode7. We achieve fine control over ultracold fermions confined to two-dimensional harmonic potentials and prepare closed-shell configurations of 2, 6 and 12 fermionic atoms in the ground state with high fidelity. Spectroscopy is then performed on our mesoscopic system while tuning the pair energy from zero to a value larger than the shell spacing. Using full atom counting statistics, we find the lowest resonance to consist of coherently excited pairs only. The distinct non-monotonic interaction dependence of this many-body excitation, combined with comparison with numerical calculations allows us to identify it as the precursor of the Higgs mode. Our atomic simulator provides a way to study the emergence of collective phenomena and the thermodynamic limit, particle by particle.

A key element of our understanding of nature is that macroscopic systems are characterized by the presence of phase transitions and collective modes, which cannot be extrapolated from the two-body solution1. Although these effects in principle exist only in the thermodynamic limit, they are sometimes observed in surprisingly small systems. For instance, atomic nuclei consisting of only around 50 particles exhibit a collective mode spectrum consistent with a superfluid8. In liquid helium droplets, superfluidity has been found to set in for similar particle numbers4. Ultracold atoms offer the exciting possibility of studying the onset of many-body physics in systems with full tunability of interactions, particle number and single-particle spectra8. The emergence of a Fermi sea was observed in a one-dimensional trap in ref. 1. Two- and three-dimensional systems promise even richer physics such as quantum phase transitions and symmetry breaking, as well as degenerate energy levels and single-particle spectra akin to the shell structure of atoms and nuclei.

In this work, we observe the few-body precursor of a quantum phase transition. The measurement relies on our experimental breakthrough in the preparation of a tunable number of Fermionic atoms in the ground state of a two-dimensional (2D) harmonic potential. We study the interplay of the shell structure and Pauli blocking with the attractive interactions for closed-shell configurations. The competition of the gapped single-particle spectrum—given by the confinement—with the interactions gives rise to particular excitations exhibiting a non-trivial dependence on the attraction. We study this dependence and demonstrate that the modes consist of coherent excitations of particle pairs. Combined with a careful comparison with numerical calculations, this allows us to identify these excitations as few-body precursors of a Higgs mode associated with a quantum phase transition to a superfluid of Cooper pairs8 (see Methods). Comparing measured spectra for different atom numbers allows us to observe the approach towards the thermodynamic limit.

In the many-body limit, a Higgs mode has been observed in cold-atom, superconducting and ferromagnetic systems10–17. Our results pioneer the study of emergent quantum phase transitions and the associated Higgs mode starting from a few-body system.

Experimental setup

We perform our experiments with a balanced mixture of two hyperfine states of 6Li confined in a trap created by the superposition of an optical tweezer (OT) and a single layer of an optical lattice (see Fig. 1a). The radial trapping frequencies of $f_r = 1,000\,\text{Hz}$ are much smaller than the axial frequency $f_z = 6,800\,\text{Hz}$. Hence, for low temperatures and only a few occupied shells, the sample is in the quasi-2D regime and the dynamics along the third direction is frozen out. We prepare the ground state of up to 12 atoms in this trap by applying a novel spilling technique for quasi-2D systems based on the method in ref. 18. A magnetic field...
indicating the deterministic preparation of closed shells of atoms. Depths where the atom number reaches a plateau fluctuations are suppressed. Experimentally, we tune and control the interaction range at which to introduce interactions: whereas a gapless Fermi gas at zero temperature undergoes a phase transition from normal to superfluid at any attraction, a gap in the single-particle spectrum gives rise to a quantum phase transition from a normal to a superfluid phase at a certain critical interaction strength. For the 2D harmonic oscillator, this means that for partly filled (open) shells the particles in this system will pair for arbitrarily weak attraction, whereas for completely filled (closed) shells and weak attraction the system is dominated by the energy gap to empty shells and pairing is suppressed. Rich physics thus arises from the competition of interactions with the single-particle shell structure. Experimentally, we tune and control the interactions using a Feshbach resonance. Because the experiment is performed in a quasi-2D geometry there exists a two-body bound state for any attractive contact interaction. The binding energy $E_b$ of the pair uniquely characterizes the interaction strength. We explore an interaction range where the binding energy $E_b$ is much smaller than the axial confinement and the effective interactions can be treated as quasi-2D (see Methods).

**Excitation spectra**

We utilize many-body spectroscopy to probe the effect of interactions on closed-shell configurations. The system is excited by modulating the axial confinement at frequencies far below the bandgap of the lattice, which modulates only the effective two-dimensional interaction strength between the different hyperfine states (see Methods). This interaction perturbation couples strongly to collective excitations driven by pairing correlations. After modulation, all atoms excited to higher states are removed by a second spilling procedure and we count the number of atoms remaining in the lowest two shells. The probability of detecting $N$ remaining atoms as a function of the modulation frequency is shown in Fig. 1c. The experiment is repeated 45 times for each drive frequency. We observe the lowest resonance at 1,890 Hz below twice the trap frequency, indicating mode softening. Remarkably, only the probability of detecting 4 atoms is increased, showing that this resonance consists of a coherent superposition of pair excitations. The second resonance at 2,060 Hz lies above twice the trap frequency consistent with a mean-field estimate. It is composed of single-particle excitations, as the probabilities for detecting both 4 and 5 atoms are enhanced.
energy is proportional to the atom density, thus predicting a larger interaction shift for the ground state than for the more dilute excited state. This higher resonance consists of single-particle excitations two shells up in energy. For the chosen drive strength and time there is a substantial probability of exciting the system more than once. This explains the enhanced probabilities of detecting 4 and 5 atoms.

Mean-field theory, however, completely fails to explain the lower resonance at 1,890 Hz, below twice the noninteracting trap frequency. Furthermore, the observed atom number distribution is strikingly different. Here only the probability of detecting 4 atoms is enhanced, whereas all other probabilities are flat. Thus, at this frequency it is only possible to excite a single pair of atoms and not individual atoms or two pairs. Both the energy and the atom number distribution are clear signatures of the collective nature of this excitation arising from the competition between the single-particle gap and the attractive interactions. The spectrum is obtained for a binding energy of $E_B = 0.33hf_r$, which is smaller than the single-particle gap and pairing is suppressed for closed shells owing to Pauli blocking. However, by exciting a coherent superposition of particle pairs from the completely filled shell the remaining atoms can enhance their overlap by occupying the now empty states and thereby gain pairing energy. The excited particles form a pair in the otherwise empty shell and have a lower energy compared to two noninteracting particles in the same shell. Thus, the pair excitation lies below twice the trap frequency.

Next, we investigate the competition between pairing and the shell structure in more detail by tuning their relative strength using a Feshbach resonance. The spectrum for different binding energies shown in Fig. 3a allows us to track the evolution of the different excitations discussed above. The branch highest in energy shows a monotonous increase of frequency with interaction, as expected, from the increasing mean-field shift. Remarkably, the lower two branches show a non-monotonic behaviour. As we shall discuss in detail below, they correspond to coherent excitations of pairs with angular momentum 0 and $\pm 2$ in small interactions the energy of these excitations decreases with increasing attraction. This is due to the increasing gain in binding energy and the larger pair correlations in the excited state. This picture breaks down above an interaction strength of $E_B = 1.1hf_r$, where the lower mode energies start to increase with the attraction. In this regime the binding energy is comparable to the radial trap frequency and pairing becomes important also for the closed-shell ground state. Here, it is energetically favourable to have an admixture of higher harmonic oscillator levels to form a pair. Consequently, the ground state has notable pairing correlations and its energy decreases faster than that of the excited states. We identify the position of the minimal excitation gap with the critical interaction strength.

To study the scaling of the spectrum towards the many-body limit, we fill one more shell in our trap, working with 12 particles. The corresponding excitation spectrum is shown in Fig. 3b. Qualitatively, the spectra for $N = 12$ and 6 show the same features. For the larger system the number of states that are shifted upwards in energy above $2hf_r$ increases, rendering it impossible to resolve a single well defined excitation peak. Importantly, the minima of the pair excitation branches below $2hf_r$ deepen and move to smaller interaction strengths for larger particle numbers, as is evident from the resonance positions shown in Fig. 3c.

**Many-body picture**

Crucially, the qualitative behaviour of this spectrum and its evolution from the few- to the many-body limit can be understood from theory. In the thermodynamic limit, a closed-shell system undergoes a quantum phase transition from a normal to a superfluid phase with increasing attraction. As a generic feature of quantum phase transitions, this gives rise to a collective mode that goes soft, that is, the excitation gap closes at the transition point. In the case at hand, the lowest collective mode corresponds to the coherent excitations of time-reversed pairs across the gap. The energy cost of these excitations vanishes at the transition point, indicating that the system is spontaneously forming Cooper pairs. From a broken-symmetry perspective, the mode corresponds to amplitude vibrations in the order parameter (pairing strength) around its average value, which is zero in the normal phase.
The interactions are modulated at the resonance frequency of the lower pair excitation mode for variable times. The probabilities for 4 and 6 particles coherently oscillate out of phase, revealing the pair character of the excitation. Coupling to other states is negligible, as seen from the almost constant value of $P_{N=4} + P_{N=6}$ and the fact that $P_{N=4}$ and $P_{N=6}$ converge to the same value. Thus, the ground state plus the lower Higgs mode precursor can be described as a coherently driven two-state system. We fit $P_{N=4}$ with an exponentially damped Rabi oscillation. The data are taken for $E_x = 0.57\hbar f$. The error bars represent the standard error of the mean. For each modulation time the measurement is repeated between 177 and 181 times.

and non-zero in the superfluid phase. In the superfluid phase, this mode is referred to as the Higgs mode.

The pair excitation modes we observe in the experiment are the few-body precursors of the Higgs mode. Owing to the finite particle number, the phase transition is broadened to a crossover and the gap does not close completely. However, the lowest excitation corresponding to zero angular momentum retains the non-monotonic dependence on interactions and the pair correlation character. Adding more particles to the system decreases the minimal gap, consistent with an eventual complete gap closure in the many-body limit. When the Fermi energy increases, the relative importance of the single-particle gap decreases, and the minimal gap moves towards smaller binding energies. Both the softening of the mode and the shift to smaller critical binding energies when approaching the many-body limit are clearly visible when going from two to three closed shells. This interpretation is explicitly confirmed by comparing to a numerical diagonalization of the microscopic Hamiltonian (see Methods). The higher non-monotonic branch in fact corresponds to two nearly degenerate modes consisting of coherent excitations of pairs with angular momentum $\pm 2\hbar$. They are precursors of Higgs modes of a superfluid with higher-angular-momentum Cooper pairs. Although modulating the interaction strength does not add angular momentum, these modes are visible owing to a slight breaking of the circular symmetry of the trap (see Methods).

Coherent drive
At all interaction strengths shown in Fig. 3, the Higgs mode precursor is a well defined excitation: The linewidth is of the order of 10 Hz and is thus much smaller than the excitation energy. To probe the stability of the excited state we drive the 6-particle system for variable times at the frequency of the lower pair excitation mode. We observe oscillations between the probabilities of detecting 4 and 6 particles in the lowest two shells, indicating the coherent formation and destruction of a pair in the excited shell at a Rabi rate of $8.0 \pm 0.1$ Hz (see Fig. 4). The $1/e$ decay rate (where $e$ is Euler’s number) of the oscillation of $4.5 \pm 0.5$ Hz gives a quantitative upper limit on the lifetime of the pair excitation mode, exceeding the transition frequency of $1.480 \text{ Hz}$ by a factor of more than 300. The long lifetime of the excited state can be attributed to the discrete level spectrum of our trap. No decay channels to single-particle excitations are available that conserve the energy of the isolated system.

Outlook
In conclusion, we have shown that systems consisting of only a few particles exhibit precursors of a quantum phase transition to a superfluid phase with an associated Higgs mode present in the thermodynamic limit. In addition to the emergence of pairing, the degree of control achieved over this mesoscopic system will allow us to study thermalization in isolated quantum systems and fermionic superfluidity at its fundamental level. As a next step, we will go beyond the excitation spectrum studied here and investigate the emergence of pair correlations across the precursor of the normal-to-superfluid transition directly in momentum space. Another interesting question to be explored concerns the emergence of Cooper pairs and Goldstone modes with increasing system size. Since Goldstone modes are driven by phase fluctuations of the order parameter, they exist when the phase can be defined on a length scale much smaller than the system size, or equivalently, when the superfluid gap is much larger than the trap level spacing.

Online content
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Experimental sequence
The experimental sequence starts by transferring a gas of $^4$Li atoms from a magneto-optical trap into a red-detuned crossed-beam optical dipole trap. Here, we make use of radio-frequency pulse sequences to prepare a balanced mixture of the two hyperfine states $|1\rangle$ and $|3\rangle$ of the ground state of $^4$Li. We label the hyperfine states according to their energy from lowest to highest, $|1\rangle$ to $|6\rangle$.

After a first evaporative cooling stage in the crossed-beam optical dipole trap we transfer approximately 1,000 atoms into a tightly focused OT. In the OT, quantum degeneracy is reached by spilling around 20 atoms close to the ground state of the approximately harmonic confinement by the procedure described in ref. 18. Subsequently, we begin the crossover to a quasi-2D system (see Extended Data Fig. 1). This is achieved with an adiabatic transfer of the atoms from the effective confinement of the OT alone with $f_r/f_z = 5:1$ to $f_r/f_z = 1:7$ in a combined potential of OT and a single layer of a one-dimensional optical lattice. To this end, we lower the radial trap frequency $f_r$ of the OT from approximately 20 kHz to 1 kHz. This is done by ramping the pattern displayed on a spatial light modulator in 20 ms, which changes the aperture of the optical setup creating the OT. This changes the waist of the OT from about 1 μm to about 5 μm. The transfer is performed at a magnetic field of 750 G in order to have sizeable coupling between the different states. The axial confinement is solely defined by the optical lattice with $f_z = 6.8$ kHz. The measurements with 6 atoms were performed at a final radial trap frequency of $f_r = 1,001$ Hz and the 12-atom data were taken at $f_r = 992$ Hz.

In the combined trap we create closed-shell configurations of the quasi-2D harmonic oscillator, by applying a magnetic gradient of approximately 70 G cm$^{-1}$ in the axial direction and reducing the power of the OT such that only the lowest one to three energy shells remain bound. The spilling procedure is performed at 750 G, where the interaction energy is sufficiently small that one recovers the noninteracting shell structure. After preparation, we increase the OT power back up until we recover the trap frequencies and aspect ratio discussed in the main text.

The measurements of the excitation spectrum are performed by a sinusoidal modulation of the power of either the OT or of the optical lattice with frequency $f_{seq}$ for $t = 400$ ms. To detect excitations, we implement a final spilling stage after which we transfer all the remaining trapped atoms from the OT back into the magneto-optical trap. In the magneto-optical trap we are able to determine the total atom number in both spin states with a fidelity exceeding 99%. The latter is achieved by integrating the total fluorescence signal of the magneto-optical trap we are able to determine the total atom number from the OT back into the magneto-optical trap. In the quasi-2D system the lowest monopole excitation is at $f_z = 6.8$ kHz. The measurements with 6 atoms were performed at a final radial trap frequency of $f_r = 1,001$ Hz and the 12-atom data were taken at $f_r = 992$ Hz.

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The trap frequencies are determined using the same sequence as explained above. The only difference is that the system is excited by modulating the confinement at a magnetic field of $B = 568$ G, that is, at the zero crossing of the scattering length. In the noninteracting system the lowest monopole excitation is at $f_z = 6.8$ kHz. This allows us to extract the frequencies by modulating the harmonic confinement in the respective directions. The frequencies for $f_r$ and $f_z$ were measured at $E_B = 0$ in Figs. a and b. For the axial direction we find $f_z = 6,803$ ± 21 Hz.

Different modulation schemes
As discussed above, we use two different schemes to drive excitations above the closed-shell ground state. All the data shown in the main text, except the spectra taken at $E_B = 0$, are recorded by modulating the optical lattice. We modulate the depth of the axial confinement at frequencies well below its bandgap, in order not to create excitations along this tightly confined direction. The wavefunction adiabatically follows the potential change and is compressed periodically. The effective 2D interaction is obtained by integrating out the wavefunction along the third direction. Thus this effectively modulates only the two-dimensional binding energy. The strengths of the modulation correspond to a change of $E_B$ by approximately 2%. For reference, we compare the modulation of the radial trapping potential with the modulation of interactions at 300 G. We find that both schemes lead to different relative transition probabilities for the Higgs and the other excited modes (see Extended Data Fig. 3). The locations of the respective excitations in the spectrum remain unaffected. The qualitative result that a modulation of interaction strength leads to an increased transition matrix element of the Higgs mode precursor is consistent with the few-body calculation by ref. 9. Consequently, $f_r$ modulation was applied for all the data shown in the main text, except for the spectra taken at $E_B = 0$.

Anisotropy and anharmonicity
The small size of our OT results in a finite anharmonicity of the trapping potential. The transition frequency from the lowest shell two shells up is about 10% larger than the transition frequency from the second shell to the fourth shell. The anharmonicity extracted from the noninteracting spectra matches our expectations, owing to the finite size of the optical tweezer. Its waist of around 5 μm is of the same magnitude as the harmonic oscillator length $l_{ho} = \sqrt{\hbar/\omega_{ho}} = 1.3$ μm and the atoms probe the non-harmonic parts of the trap. In addition, the trap shows a slight anisotropy $(\omega_x - \omega_y)/(\omega_x + \omega_y)$ of approximately 2%. These corrections should not affect the qualitative behaviour of the measured spectra for the interacting system. However, they might quantitatively change the coupling strengths to the different modes and the exact shape of the Higgs mode precursor. We neglect the influence of the anharmonicity for calculating the binding energy. A comparison of the calculated and measured excitation energies for two particles, shown in Extended Data Fig. 2, confirms that this is only a small effect, as expected.

Numerical modelling
We model the experiment using a trapping potential of the form

$$V = V_{2D}(x, y) + \frac{1}{2} m\omega^2 z^2.$$ (1)
Here, \( V_3 \) describes the potential in the \( x-y \) plane, which is provided by the Gaussian profile of the OT so that
\[
V_3(x, y) = \frac{A}{2} \hbar \omega_x \times \left[ 1 - e^{-\frac{(x^2 + y^2)}{\gamma_x y^2}} \right],
\]
with the lowest order harmonic trapping frequency \( \omega_x \). The parameter \( \gamma_x \) controls the ratio of the trap frequencies in the \( x \)- and \( y \)-directions and hence the anisotropy. The parameter \( A \) is the depth of the trap (in units of \( \hbar \omega_x \)) and determines the anharmonicity.

Pure contact interactions in three dimensions are represented by a term \( g_{3D} (\mathbf{r}_i - \mathbf{r}_j) \). We assume \( g_{3D} \gg g_{2D} \) so that the fermions reside in the lowest harmonic oscillator state along the \( z \)-direction. Consequently, the \( z \)-direction can be integrated out, yielding a quasi-2D model with an effective coupling strength \( g_{2D} \) (ref. 8). Since the integral depends on the wavefunction, \( g_{2D} \) increases with the confinement in the \( z \)-direction. It then follows that modulating \( \omega_z \) will modulate the effective 2D coupling strength, which is an efficient way to excite the Higgs mode precursor.

Extended Data Fig. 6a shows the calculated as well as experimentally observed spectrum for the \( N=6 \) particle system, with \( V=20 \) and \( y=0.99 \). The numerical calculation includes states up to \( E_{\text{cut}}=10 \hbar \omega_0 \), and up to a many-body energy of \( 25 \hbar \omega_0 \). We see that there is reasonable agreement between theory and experiment and that all qualitative features in the spectrum are recovered by the calculations. In particular, the existence of two non-monotonic Higgs branches is confirmed by the calculations.

The lowest branch connects smoothly to the \( L_z = 0 \) Higgs mode for the isotropic case, whereas the two higher modes connect smoothly to the \( L_z = \pm 2h \) Higgs modes when the anisotropy goes to zero (\( y=1 \)). The latter modes have a higher energy because they describe Cooper pairing with finite angular momentum, and are almost degenerate owing to the small anisotropy. Since our numerics include only positive angular momentum states, only one of the \( L_z = \pm 2h \) Higgs modes is visible in the calculated spectrum. In the experimental data they appear as a single resonance owing to the small energy splitting for small anisotropy.

Extended Data Fig. 6b shows the spectrum weighted with the matrix element
\[
|G| \sum_{L_i J} (\delta(r_k - r_j) |E) |^2,
\]
which gives the coupling between the ground state \(|G\rangle \) and the excited state \(|E\rangle \) when the interaction strength is modulated in the experiment. The slight breaking of the circular symmetry leads to the coupling of the ground state to all three Higgs modes. This plot also highlights why the manifold of states around the energy \( \hbar \omega_0 \) is not observed experimentally: These states correspond to exciting one fermion one shell up, which changes the angular momentum by \( \pm h \). The trap anisotropy leads solely to quadrupole excitations, where the angular momentum changes by \( \Delta L_z \approx \pm 2h \). As a result, these modes are not visible in the experimental spectrum.

Limitations of the model

We note that the agreement between the theoretical and experimental spectra becomes worse in the region where the binding energy is substantially larger than the critical binding energy. There are two main reasons that explain this behaviour. First, this region corresponds to the few-body precursor of the BEC regime of dimers and the large binding energy requires a cut-off beyond what is numerically feasible. Second, the modelled potential only approximates the actual experimental confinement. The fitting of the potential parameters is performed only for a small set of experimental values, all within the lowest few harmonic shells. This allows for a qualitative simulation of the low-lying excitation spectra observed in the few-body experiments but we did not perform a systematic fit including all observed modes. This is because the experimental trap is not precisely described by equation (2): there are deviations away from the Gaussian profile especially close to the continuum. More importantly, the experimental aspect ratio \( \omega_x/\omega_y \approx 6.8 \) implies that three-dimensional effects become important at higher energies. In this regime, effects such as excitations to higher axial states or the confinement-induced effective range \( \rho \) have to be considered. Including such effects would enable us to achieve a more quantitative agreement...
but is at the same time computationally very challenging. In addition, this would require very precise control and knowledge of the experimental potential at high energies beyond what is achieved here. We estimated that, in the regime relevant for this work, these effects do not change the results qualitatively and therefore do not include them in our model.

In conclusion, the overall agreement between the numerical calculations and the experiment confirms the physical interpretation of the data. In particular, we are indeed observing a few-body precursor of a quantum phase transition with the associated emergence of a Higgs mode. We note that obtaining an even better quantitative agreement requires a more accurate determination of the shape of the trap, inclusion of three-dimensional physics for higher energies, and the use of substantially larger computational resources. All qualitative features of the low energy spectrum, however, were found to be insensitive towards these effects.

We finally note that simpler theoretical approaches are not able to capture the non-monotonic behaviour of the pair excitation mode. Mean-field theory would predict only modes above \(2\omega_c\). Second-order perturbation theory would capture the initial decrease of the excitation frequency of the pair mode. It would however fail to describe the non-monotonic behaviour at larger interaction strengths, as this is caused by non-perturbative pairing correlations.

Data availability

The data that support the findings of this study are available from the corresponding authors upon reasonable request. Source data are provided with this paper.

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Author contributions

L.B. and M.H. contributed equally to this work. L.B., M.H. and K.S. performed the measurements and analysed the data. J.B., S.M.R. and G.M.B. developed the theoretical framework. J.B. performed the numerical calculations. P.M.P. and S.J. supervised the experimental part of the project. All authors contributed to the discussion of the results and the writing of the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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Extended Data Fig. 1 | Experimental protocol. The sequence can be separated into three parts. First, several evaporation and spilling stages are combined with a transfer from a quasi-1D to a quasi-2D trap geometry. This is needed to prepare closed-shell ground state configurations of up to 12 atoms. Second, we excite the system at some defined frequency $f_{\text{exc}}$ and magnetic offset field $B$ using a sinusoidal modulation of either the radial or axial confinement. Third, detection is implemented by spilling to the ground state a second time and a transfer of all remaining atoms to the magneto-optical trap, where we count them.
**Extended Data Fig. 2 | Excitation spectrum for two particles.** We define the two-body excitation energy $E_{\text{exc}}$ as the energy difference between the ground state and the lowest monopole excitation of the two atom system. It is measured using the same modulation scheme as for the Higgs mode. The system is initialized with one filled shell, that is, two particles. The analytical solution of the two-body problem (solid line) shows good agreement with the measurement (blue points). The systematic uncertainty of around 2% on the measured radial and axial trap frequencies that enters into the analytical solution is indicated by the grey error band. Residual systematic deviations can be explained by the trap anharmonicity. For the measurement (blue points) error bars are extracted from the fit to the spectrum and are smaller than the data points.
Extended Data Fig. 3 | Comparison of different modulation schemes.
A modulation of the radial trap frequency leads to similar transition probabilities for the pair excitation mode and the higher excited states (top). In contrast, a modulation of the axial confinement effectively only modulates the interaction strength and couples predominantly to the pair excitation mode. Excitations to higher states are suppressed by this modulation scheme (bottom). This qualitative observation agrees with the coupling elements that were predicted in ref. 9. The two modulation amplitudes have been chosen such that they lead to a similar response of the pair excitation mode. The data are taken for $E_A = 0.09 hf_r$. For this measurement the radial trap frequency was $2f_r = 1,660$ Hz. Error bars show the standard error of the mean. Each data point is the average of at least 24 measurements.
Extended Data Fig. 4 | Probabilities of different atom numbers remaining in the lowest two shells for the $N = 6$ initial state.

The probabilities for different remaining atom numbers after modulating the $6$-atom ground state with a defined frequency given on the $y$-axis and subsequent removal of excited atoms. All possible excitations manifest themselves by a reduced probability of remaining in the ground state of $6$ atoms ($a$–$f$). We find that the lowest excitation, or Higgs mode, mostly consists of excitations to four atoms ($c$), while the higher excited peaks are predominantly generated by the loss of a single atom ($b$). For each setting, the experiment is repeated between 42 and 47 times.
Extended Data Fig. 5 | Probabilities of different atom numbers remaining in the lowest three shells for the $N=12$ initial state. a–f. The probabilities for different remaining atom numbers after modulating the 12-atom ground state with a defined frequency given on the y axis and subsequent removal of excited atoms. All possible excitations manifest themselves by a reduced probability of remaining in the ground state of 12 atoms (a). We find that lowest excitation, or Higgs mode, mainly consists of excitations to ten atoms (c), while the higher excited peaks are predominantly generated by the loss of even more atoms (d–f). For each setting the experiment is repeated between 19 and 63 times.
Extended Data Fig. 6 | Numerically calculated excitation spectrum for 6 particles. a, The level spectrum obtained by exact diagonalization with parameters $A = 20$ and $\gamma = 0.99$ for the potential as well as the experimental results. The calculation includes states up to $E_{\text{cut}} = 10\hbar\omega_r$ and up to a many-body energy of $28\hbar\omega_r$. For comparison the experimental data are shown by green diamonds. Values and errors bars are obtained as in Fig. 3c. b, The numerically calculated excitation spectrum for a modulation of the interaction strength. As in the experiment we observe that this modulation couples to two non-monotonous modes. We note that the calculations performed for b employ a smaller cut-off ($E_{\text{cut}} = 6\hbar\omega_r$ and a maximal many-body energy of $24\hbar\omega_r$) than in a owing to the computational demand in calculating the matrix element (equation (4)).