On the Concept of Variable Roles and its Use in Software Analysis

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Abstract—Human written source code in imperative programming languages exhibits typical patterns for variable use such as flags, loop iterators, counters, indices, bitvectors etc. Although it is widely understood by practitioners that these variable roles are important for automated software analysis tools, they are not systematically studied by the formal methods community, and not well documented in the research literature. In this paper, we study the notion of variable roles on the example of basic types (int, float, char) in C. We propose a classification of the variables in a program by variable roles, and demonstrate that classical data flow analysis lends itself naturally both as a specification formalism and an analysis paradigm for this classification problem. We demonstrate the practical applicability of our method by predicting membership of source files to the different categories of the software verification competition SVCOMP 2013.

I. INTRODUCTION

Programs written in imperative programming languages, (for example C, Java, Perl, Python etc.) share typical patterns of variable use, e.g. flags, loop iterators, counters, indices, bitvectors, temporary variables and so on. When recognising these patterns, a programmer has some expectations on how a variable can be used in the program, therefore we will call them variable roles. For example, from a piece of C code

```c
while(i<n) {a[i++]=0;}
```

one can deduce that `i` is a loop iterator and an array index, and from

```c
x=x&y
```

or by looking for code patterns (e.g. matching array indices in array subscripts), it is useful to know which variables are used as indexes for the array. Unfortunately, most state-of-the-art verification papers treat a program as a logical formula and either ignore such implicit information, or treat it as undocumented heuristics. In this paper, we argue that these heuristics deserve a more systematic study.

In this paper we define 14 variable roles with a standard data-flow analysis. Our definition serves at the same time as an algorithm to compute the roles. In order to choose the roles, we have manually investigated 5.2 KLOC of C code from cBench benchmark [11] and assigned roles to basic type variables (int, float and char). When choosing the roles, we were inspired by typical programming patterns for variable use in real life programs. We have chosen the roles in such a way that a small number of roles is able to classify each occurring program variable in the programs we considered.

As this short paper is reporting work in progress, we are currently exploring applications for variable roles. As mentioned earlier, variable roles can be used to create abstractions for software verification and to understand programs. We also conjecture that the frequency of variables of different roles in a program depends on the kind of the program, e.g. computationally intensive programs, device drivers or programs which extensively use dynamic data structures. We suggest that our method can be used to classify files (for example from benchmarks for different verification competitions) based on the number of variables of different roles.

We have implemented a prototype tool, which maps basic-type (int, float, char) variables in C programs to sets of roles. We then made the following experiment with the benchmarks from the software competition SVCOMP 2013 [2]. The competition distinguishes several categories of source files, e.g. device drivers, embedded systems, concurrent programs etc. This classification by human experts (who manually analysed and comprehended the source code) provides us with an excellent opportunity to benchmark our variable roles as follows: With our tool we computed the frequency of different roles in each category and used this data to train a machine learning tool to
predict the competition categories for new files. In a number of experiments we randomly selected a subset of the competition source files for training and used the remaining source files to check our prediction against human classification. The results of the experiments are encouraging - the prediction is successful in more than 80% of the cases. These results are particularly impressive because our choice of the roles was based on examples from cBench rather than SVCOMP.

A. Motivating examples

Consider the C programs from Figure 1. We will use these examples to informally introduce variable roles, formal definitions of which will be given in the next section.

The program in Figure 1(a) calculates the number of non-zero bits in variable \( x \). In every loop iteration a non-zero bit in \( x \) is set to zero and counter \( n \) is incremented. The loop continues until all bits are set to zero. Although variables \( x \) and \( n \) are declared of the same type int, however they are treated differently. For a human reading the program, statements \( n=0 \) and \( n=n+1 \) in the loop body signal that \( n \) is a counter (indeed, it is used to count the number of loop iterations). On the other hand, the value of variable \( x \) as an integer is not important for calculations, but rather individual bits in its binary representation matter.

We define the roles by putting constraints on the operations in which a variable occurs. We require that a bitvector must occur in at least one bitwise operation (bitwise AND, OR or XOR). For a counter variable we require that it can only change its value in increment or decrement statements. Alternatively, it can be reset to zero. By analysing all assignments to the variable \( n \) (lines 1 and 6) we make sure that it satisfies these constraints.

In real-life programs the use of variables can be ambiguous, when a variable which is used in one role for some time, is then used in an operation not typical for the role. For example consider a piece of C code \( x &= y; \ x <<= 1; \) where \( x \) is used as bitvector. Now for a slightly modified code \( x &= y; \ x *= 2; \) which is semantically equivalent to the first one it is arguable whether \( x \) is a bitvector, because using a bitvector in an arithmetic operation in general does not make sense. We intentionally do not require that a bitvector does not occur in non-bitwise arithmetic operations (e.g. multiplication), and therefore with our definition \( x \) is assigned the role BITVECTOR in both cases. We consider such a definition of BITVECTOR reasonable, because during the manual investigation of a large code base we observed that in most cases variables used in bitwise operations are in fact treated as a collection of bits. An exception to this rule is bitwise shift, which is often used to replace multiplication by a power of two and therefore not used in our definition. Another argument is that since a variable can be assigned more than one role at the same time, we can consider the roles assigned to a variable altogether and choose the appropriate ones.

The program in Figure 1(b) reads a decimal number from a text file and transforms it to numeric form. The result of computations is stored in variable \( \text{val} \). In contrast, variables \( \text{fd} \) and \( c \) both take as their values output of functions, which are not part of the program: \( \text{open()} \) and \( \text{read()} \) respectively. The difference between the variables is that \( c \) is later used in calculations, while \( \text{fd} \) is only passed to function \( \text{read()} \) as a black box - its value does not directly affect the result of computations. One can make an assumption that \( c \) is a character, because it is passed as input to \( \text{isdigit()} \) function, which checks whether its parameter is a decimal digit character. Even though \( \text{isdigit()} \) is declared to take parameter of type int, the documentation says that \( c \) is “character to be checked, casted to int”.

We define character, file descriptor and linear roles as follows. We require that a character variable must at least once be assigned either a character literal (e.g. \( c = \text{'a'} \)) or another character variable, or used in one of the functions from the standard C library (e.g. \( c = \text{getchar()} \) or \( \text{isdigit(c)} \)). Again we do not restrict the operations in which a character can occur, for example we allow that it is used in arithmetic operations, like a variable \( c \) in line 7. For a file descriptor we require that it is used at least once in a standard library function (e.g. \( \text{fd = open(path, flags)} \) or \( \text{read(fd, &c, 1)} \)). Finally, we require that a linear variable can be assigned only linear combinations of linear variables, and both \( c \) and \( \text{val} \) satisfy this constraint.

Contributions:

- We identify 14 variable roles that occur in practical programs.
- We implement a prototype tool for C which assigns one or more roles to basic-type variables.
- With our tool and machine learning techniques we predict the membership of a C programs in a competition category of the SW verification competition SVCOMP 2013. We get encouraging results in a number of experiments.
of space. For a program in C we translate every function to a program in C_{simpt}, and use this representation for intraprocedural analysis.

In the definitions below we use the following notation: \textbf{Var} denotes the set of program variables, \textbf{Num} - all scalar constant literals (e.g. 0, 0.5, ’a’) and \textbf{S}, \textbf{E} and \textbf{B} - the set of program statements, arithmetic and boolean expressions respectively. For the elements of these sets we use the same names in uncapsitalised version (e.g. \textit{var} for a program variable etc.).

For a program \textit{s} \in \textbf{S} the result of analysis \textit{R} is computed using the function \textit{Res}^R, which is defined as follows:

\[
\textit{Res}^R = \textit{Init}^R \bigcup \textit{gen}^R(s),
\]

where \textit{Init}^R \in \mathcal{P}(\text{Var}) is the initial set of variables, function \textit{gen}^R : \textbf{S} \cup \textbf{E} \cup \textbf{B} \rightarrow \mathcal{P}(\text{Var}) maps every statement and expression to a set of \textit{generated} variables, and the sign \bigcup is used as a placeholder for a set operation and must be instantiated for each analysis.

Analysis \textit{R} is therefore defined with a tuple \((\textit{Init}^R, \bigcup, \textit{gen}^R, c)\), where \(c \in \{f, o\}\) indicates how to evaluate \textit{Res}^R. When \(c\) is set to \textit{f}, a fixed point of \textit{Res}^R is computed, i.e. \textit{Res}^R is iteratively recalculated until it does not change. When \(c\) is set to \textit{o}, \textit{Res}^R is calculated in one iteration.

\section*{III. IMPLEMENTATION AND EXPERIMENTS}

We implemented a prototype tool using clang compiler. We handle pointers and function calls as follows: when a variable is assigned a pointer dereference (e.g. \texttt{n=*ptr, n=arr[i]}), or passed to a function as a parameter by reference, the result set does not change. To find a trade-off between safe and accurate analysis, we signal about such situations by assigning the variable the role "unresolved assignment". For some roles,
\[
\begin{align*}
n &= 0; \ y = ^1x; \\
\text{if} (x \neq 0) \{} & \\
\quad n &= n + 1; \\
\quad x &= x \land (x - 1); \\
\quad \text{while}^4 (x \neq 0) & \{} \\
\quad n &= n + 1; \\
\quad x &= x \land (x - 1); \\
\}\end{align*}
\]
(a) code from fig. 1(a) in C

(b) Result of the analysis

Fig. 3. Computation of roles

(a) Relative numbers of roles for 2 categories
- Control Flow and Integer Variables
- Linux Device Drivers 64-bit

| Relative number of roles, % | Ctrl Flow and Integer Variables | Linux Device Drivers 64-bit |
|-----------------------------|---------------------------------|-----------------------------|
| 50%                         | *                               | *                           |
| 40%                         | *                               | *                           |
| 30%                         | *                               | *                           |
| 20%                         | *                               | *                           |
| 10%                         | *                               | *                           |
| 0%                          | *                               | *                           |

(b) Prediction error in different settings

| Training set | % of all files | Error of choice 1 | Error of choice 2 |
|--------------|---------------|-------------------|-------------------|
| 90%          | 15.94%        | 2.90%             |                   |
| 80%          | 14.81%        | 5.93%             |                   |
| 70%          | 16.20%        | 7.98%             |                   |
| 60%          | 19.77%        | 7.98%             |                   |
| 50%          | 18.60%        | 8.54%             |                   |

Fig. 4. Comparison of categories and automatic classification of files

namely char and file descriptor, we use the information about function calls, and search for the calls of the functions from the standard library (e.g. `open`, `getchar` etc).

We compare the relative numbers of roles as shown in Figure 4a for categories "Control Flow and Integer Variables" and "Linux Device Drivers". As expected, we observe that boolean flags and branching operations as well as counters, arithmetic operations and constant assignments are typical for the first category, while Linux drivers extensively use bitvectors and pointers.

In our experiments we used a multiclass vector support machine \[4\] to predict the categories for files with some probability, e.g. the probability that the file is a driver is 60%, and that it is a concurrent program - 35% and so on. We translated the relative numbers of roles into the input format of a machine learning tool (http://www.cs.waikato.ac.nz/ml/weka) as follows: each source file represented one training example with the category corresponding to the class, and relative numbers of roles representing the vector of float attributes.

We ran the experiments for the sizes of the training sets from 90% to 50% of all files and obtained the prediction error of approximately 15% as shown in Figure 4b. When we analysed the second most probable choices, we observed the error of appr. 7%.

IV. Related work

The term variable roles was inspired by \[5\], which informally defines roles as patterns of how variables are initialised and updated. The authors have defined nine roles, implemented a tool for assigning roles to variables using static analysis and evaluated it on Pascal programs from textbooks. The work leaves open the question of formalising the notion of variable roles as well as of the possibility of applying the method to real-word programs.

\[6\] uses implicit knowledge in the form of programmer’s beliefs, i.e. propositional statements about program variables and functions, for bug finding. The authors use static analysis to extract must (e.g. “a pointer is not null”) and may (e.g. “calls to functions f() and g() should be paired”) statements. Since the project has grown into a commercial tool (Covcover), publicly available research results have been limited.

In \[7\] predicate abstraction over a fixed set of predicates is used to infer so called liquid types, i.e. refinement of types with a conjunction of propositional predicates (e.g. \(x > 0 \land x < 5\)). We consider this approach to be complementary to ours, because it does not use any information from the source code other than the transition relation, and concentrates on arithmetic properties of variables.

Variable names and comments as an additional source of knowledge about a program have been systematically studied in program comprehension. The Latent Semantic Indexing technique \[8\] allows to query on the program source code using words in natural language, and to obtain a list of functions ranked with a similarity characteristic. The latter is calculated from the number of occurrences of the words from the query in variable names and comments of a function. The rules for naming variables in real-word programs are studied in \[9\], and \[10\] suggests a method for expanding abbreviated identifiers to full words. We regard using these techniques in our approach as future work.

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APPENDIX

Syntax of the language C_{simpl}

E ::= var | num | E aop E | E bitop E | bitnot E | avar[E]
aop ::= + | - | * | /
bitop ::= bitor | bitand | bitxor
B ::= E compop E | not B | B logop B
compop ::= = | != | > | < | ≥ | ≤
logop ::= ∧ | ∨
S ::= var := E | avar[E] := E | if B then S else S | S ; S | skip | while B do S | call p(Par)
Par ::= E | Par, E
P ::= proc p (VD) begin VD C end | Proc;Proc | ε
VD ::= var x | VD ; VD | ε
Prog ::= begin VD Proc end

Note: 1) the first parameter of a function is the returned value
2) avar is an array variable

Fig. 5. Syntax of C_{simpl}

Definition of variable roles

IsVar(var) = \{var\}
IsVar(num) = ∅
IsVar(e1 aop e2) = IsVar(e1 bitop e2) = IsVar(bitnot e) = IsVar(b1 logop b2) = IsVar(not b) = IsVar(avar[e]) = ∅
IsVar(s) = ∅

BITVECTOR

gen(var := e) = gen(e) \cup \{\{var\}\} if e := e1 bitop e2
\{∅\} otherwise
gen(avar[e1] := e2) = gen(ax[e1]) \cup gen(e2)
gen(if b then s1 else s2) = gen(b) \cup gen(s1) \cup gen(s2)
gen(s1; s2) = gen(s1) \cup gen(s2)
gen(skip) = ∅
gen(while b do s) = gen(b) \cup gen(s)
gen(call p(e1, ..., en)) = \bigcup_{1 \leq i \leq n} gen(ei)
gen(var) = gen(num) = ∅
gen(e1 aop e2) = gen(e1) \cup gen(e2)
gen(bitnot e) = IsVar(e) \cup gen(e)
gen(b1 logop b2) = gen(b1) \cup gen(b2)
gen(b) = gen(b)
gen(avar[e]) = gen(e)

FILE_DESCR

gen(var := e) = gen(ax[e1] := e2) = ∅
gen(if b then s1 else s2) = gen(s1) \cup gen(s2)
gen(skip) = ∅
gen(while b do s) = gen(s)
gen(call p(e1, ..., en)) = \begin{cases} 
IsVar(e1) & \text{if } p := \text{open and and } 3 \leq n \leq 4 
IsVar(e2) & \text{if } (p := \text{read or } p := \text{write}) \text{ and } \text{en} = 4 
∅ & \text{otherwise} 
\end{cases}
gen(e) = gen(b) = ∅

Fig. 6. One run-positive roles
One-run positive roles (cont.)

**ARRAY_INDEX**

| Expression                | Genation |
|---------------------------|----------|
| gen(var := e)             | gen(e)   |
| gen(avar[e1] := e2)       | IsVar(e1) ∪ gen(e2) |
| gen(if b then s1 else s2) | gen(b) ∪ gen(s1) ∪ gen(s2) |
| gen(s1; s2)               | gen(s1) ∪ gen(s2) |
| gen(skip)                 | ∅        |
| gen(while b do s)         | gen(b) ∪ gen(s) |
| gen(call p(e1, ..., en))  | \bigcup_{1 \leq i \leq n} gen(ei) |

**ARRAY_SIZE**

| Expression                | Genation |
|---------------------------|----------|
| gen(var := e)             | 0        |
| gen(avar[e1] := e2)       | 0        |
| gen(if b then s1 else s2) | gen(s1) ∪ gen(s2) |
| gen(s1; s2)               | gen(s1) ∪ gen(s2) |
| gen(skip)                 | ∅        |
| gen(while b do s)         | gen(s)   |
| gen(call p(e1, ..., en))  | \begin{cases} IsVar(e2) & \text{if } p := \text{malloc and } n=2 \\ ∅ & \text{otherwise} \end{cases} |
| gen(e) = gen(b)           | 0        |

**UNRES_ASSIGN**

| Expression                | Genation |
|---------------------------|----------|
| gen(var := e)             | \begin{cases} \{\text{var}\} & \text{if } e := \text{ax[e1]} \\ ∅ & \text{otherwise} \end{cases} |
| gen(avar[e1] := e2)       | 0        |
| gen(if b then s1 else s2) | gen(s1) ∪ gen(s2) |
| gen(s1; s2)               | gen(s1) ∪ gen(s2) |
| gen(skip)                 | ∅        |
| gen(while b do s)         | gen(s)   |
| gen(call p(e1, ..., en))  | ∅        |
| gen(e) = gen(b)           | 0        |

**OUTPUT**

| Expression                | Genation |
|---------------------------|----------|
| gen(var := e)             | gen(e)   |
| gen(avar[e1] := e2)       | IsVar(e1) ∪ gen(e2) |
| gen(if b then s1 else s2) | gen(b) ∪ gen(s1) ∪ gen(s2) |
| gen(s1; s2)               | gen(s1) ∪ gen(s2) |
| gen(skip)                 | ∅        |
| gen(while b do s)         | gen(b)   |
| gen(call p(e1, ..., en))  | \bigcup_{1 \leq i \leq n} gen(ei) |

**INPUT**

| Expression                | Genation |
|---------------------------|----------|
| gen(var := e)             | gen(e)   |
| gen(avar[e1] := e2)       | IsVar(e1) ∪ gen(e2) |
| gen(if b then s1 else s2) | gen(b) ∪ gen(s1) ∪ gen(s2) |
| gen(s1; s2)               | gen(s1) ∪ gen(s2) |
| gen(skip)                 | ∅        |
| gen(while b do s)         | gen(s)   |
| gen(call p(e1, ..., en))  | \bigcup_{2 \leq i \leq n} IsVar(ei) if } p := \text{printf} \bigcup_{1 \leq i \leq n} IsVar(ei) if } (p := \text{sprintf or } p := \text{fprintf}) |
| gen(e) = gen(b)           | ∅        |

Fig. 7. One run-positive roles (cont.)
One-run positive roles (cont.)

**BRANCH_COND**

\[
\begin{align*}
gen(\text{var} := e) &= \emptyset \\
\text{gen}(&\text{if } b \text{ then } s_1 \text{ else } s_2) &= \text{vars}(b) \\
\text{gen}(s_1; s_2) &= \text{gen}(s_1) \cup \text{gen}(s_2) \\
\text{gen}(&\text{skip}) &= \emptyset \\
\text{gen}(\text{while } b \text{ do } s) &= \text{gen}(s) \\
\text{gen}(&\text{call } p(e_1, \ldots, e_n)) &= \emptyset \\
\text{gen}(e) &= \emptyset \\
\text{vars}(\text{num}) &= \emptyset \\
\text{vars}(\text{var}) &= \{\text{var}\} \\
\text{vars}(b_1 \text{ logop } b_2) &= \text{vars}(b_1) \cup \text{vars}(b_2) \\
\text{vars}(\text{not } b) &= \text{vars}(b) \\
\text{vars}(e_1 \text{ compop } e_2) &= \text{vars}(e_1 \text{ aop } e_2) = \text{vars}(e_1 \text{ bitop } e_2) = \text{vars}(e_1) \cup \text{vars}(e_2) \\
\text{vars}(\text{binot } e) &= \text{vars}(e)
\end{align*}
\]

**USED_IN_ARITH**

\[
\begin{align*}
gen(\text{var} := e) &= \text{gen}(e) \\
\text{gen}(\text{avar}[e_1] := e_2) &= \text{gen}(e_1) \cup \text{gen}(e_2) \\
\text{gen}(\text{if } b \text{ then } s_1 \text{ else } s_2) &= \text{gen}(b) \cup \text{gen}(s_1) \cup \text{gen}(s_2) \\
\text{gen}(s_1; s_2) &= \text{gen}(s_1) \cup \text{gen}(s_2) \\
\text{gen}(\text{skip}) &= \emptyset \\
\text{gen}(\text{while } b \text{ do } s) &= \text{gen}(b) \cup \text{gen}(s) \\
\text{gen}(\text{call } p(e_1, \ldots, e_n)) &= \bigcup_{1 \leq i \leq n} \text{gen}(e_i) \\
\text{gen}(e_1 \text{ aop } e_2) &= \text{IsVar}(e_1) \cup \text{IsVar}(e_2) \cup \text{gen}(e_1) \cup \text{gen}(e_2) \\
\text{gen}(e_1 \text{ bitop } e_2) &= \text{gen}(e_1) \cup \text{gen}(e_2) \\
\text{gen}(\text{binot } e) &= \text{gen}(e) \\
\text{gen}(b_1 \text{ logop } b_2) &= \text{gen}(b_1) \cup \text{gen}(b_2) \\
\text{gen}(\text{not } b) &= \text{gen}(b) \\
\end{align*}
\]

**LOOP_IT**

\[
\begin{align*}
gen(\text{var} := e) &= \text{gen}(\text{avar}[e_1] := e_2) = \emptyset \\
\text{gen}(\text{if } b \text{ then } s_1 \text{ else } s_2) &= \text{gen}(s_1) \cup \text{gen}(s_2) \\
\text{gen}(s_1; s_2) &= \text{gen}(s_1) \cup \text{gen}(s_2) \\
\text{gen}(\text{skip}) &= \emptyset \\
\text{gen}(\text{while } b \text{ do } s) &= \text{varsB}(b) \cap \text{varsS}(s) \\
\text{gen}(\text{call } p(e_1, \ldots, e_n)) &= \emptyset \\
\text{gen}(e) &= \text{gen}(b) \\
\text{varsB}(e_1 \text{ compop } e_2) &= \text{IsVar}(e_1) \cup \text{IsVar}(e_2) \\
\text{varsB}(b_1 \text{ logop } b_2) &= \text{varsB}(b_1) \cup \text{varsB}(b_2) \\
\text{varsB}(\text{not } b) &= \text{varsB}(b) \\
\text{varsS}(\text{var} := e) &= \{\text{var}\} \\
\text{varsS}(\text{if } b \text{ then } s_1 \text{ else } s_2) &= \text{varsS}(s_1) \cup \text{varsS}(s_2) \\
\text{varsS}(s_1; s_2) &= \text{varsS}(s_1) \cup \text{varsS}(s_2) \\
\text{varsS}(\text{skip}) &= \emptyset \\
\text{varsS}(\text{while } b \text{ do } s) &= \text{varsS}(s) \\
\text{varsS}(\text{call } p(e_1, \ldots, e_n)) &= \emptyset \\
\end{align*}
\]

Fig. 8. One run-positive roles (cont.)
**Fixed-point negative roles**:

\[ \text{Init}^R = \text{Vars} \cup \emptyset, \ c = F \]

**LINEAR**

\[
\begin{align*}
\text{gen}(\text{var} := e) &= \begin{cases} \{\text{var}\} & \text{if lin}(e) = \text{false} \\ \emptyset & \text{otherwise} \end{cases} \\
\text{gen}(\text{avar}[e1] := e2) &= \emptyset \\
\text{gen}(\text{if b then s1 else s2}) &= \text{gen}(s1) \cup \text{gen}(s2) \\
\text{gen}(s1; s2) &= \text{gen}(s1) \cup \text{gen}(s2) \\
\text{gen}(\text{skip}) &= \emptyset \\
\text{gen}(\text{while b do } s) &= \text{gen}(s) \\
\text{gen}(\text{call p(e1, ..., en)}) &= \emptyset \\
\text{gen}(e) &= \text{gen}(b) = \emptyset
\end{align*}
\]

\[
\begin{align*}
\text{lin}(\text{num}) &= \text{true} \\
\text{lin}(\text{var}) &= \begin{cases} \text{true} & \text{if var} \in \text{Res}^{\text{LINEAR}} \\ \text{false} & \text{otherwise} \end{cases} \\
\text{lin}(e1+e2) &= \text{lin}(e1) \land \text{lin}(e2) \\
\text{lin}(e1*2) &= \begin{cases} \text{lin}(e1) & \text{if } e1 \in \text{Num} \\ \text{true} & \text{otherwise} \end{cases} \\
\text{lin}(e1 \text{ bitop } e2) &= \text{lin}(\text{bitnot } e) = \text{lin}(e1/e2) = \text{false}
\end{align*}
\]

**COUNTER**

\[
\begin{align*}
\text{gen}(\text{var} := e) &= \begin{cases} \emptyset & \text{if } e \in \text{Num} \text{ and } e::=0 \\ \{\text{var}\} \setminus \text{sumd}(e) & \text{otherwise} \end{cases} \\
\text{gen}(\text{avar}[e1] := e2) &= \emptyset \\
\text{gen}(\text{if b then s1 else s2}) &= \text{gen}(s1) \cup \text{gen}(s2) \\
\text{gen}(s1; s2) &= \text{gen}(s1) \cup \text{gen}(s2) \\
\text{gen}(\text{skip}) &= \emptyset \\
\text{gen}(\text{while b do } s) &= \text{gen}(s) \\
\text{gen}(\text{call p(..., modi ei, ...)}) &= \emptyset \\
\text{gen}(e) &= \text{gen}(b) = \emptyset
\end{align*}
\]

\[
\begin{align*}
\text{sumd}(\text{num}) &= \emptyset \\
\text{sumd}(\text{var}) &= \{\text{var}\} \\
\text{sumd}(e1+e2) &= \begin{cases} \text{IsVar}(e1) & \text{if } e2 \in \text{Num} \\ \text{IsVar}(e2) & \text{if } e1 \in \text{Num} \\ \emptyset & \text{otherwise} \end{cases} \\
\text{sumd}(e1-e2) &= \begin{cases} \text{IsVar}(e1) & \text{if } e2 \in \text{Num} \\ \emptyset & \text{otherwise} \end{cases} \\
\text{sumd}(e1*e2) &= \text{sumd}(e1/e2) = \text{sumd}(\text{bitop } e2) = \text{sumd}(\text{bitnot } e) = \emptyset
\end{align*}
\]

**CONST_ASSIGN**

\[
\begin{align*}
\text{gen}(\text{var} := e) &= \begin{cases} \emptyset & \text{if isConst}(e) = \text{true} \\ \{\text{var}\} & \text{otherwise} \end{cases} \\
\text{gen}(\text{avar}[e1] := e2) &= \emptyset \\
\text{gen}(\text{if b then s1 else s2}) &= \text{gen}(s1) \cup \text{gen}(s2) \\
\text{gen}(s1; s2) &= \text{gen}(s1) \cup \text{gen}(s2) \\
\text{gen}(\text{skip}) &= \emptyset \\
\text{gen}(\text{while b do } s) &= \text{gen}(s) \\
\text{gen}(\text{call p(..., modi ei, ...)}) &= \emptyset \\
\text{gen}(e) &= \text{gen}(b) = \emptyset
\end{align*}
\]

\[
\begin{align*}
\text{isConst}(\text{num}) &= \emptyset \\
\text{isConst}(\text{var}) &= \begin{cases} \text{true} & \text{if } \text{var} \in \text{Res}^{\text{CONST_ASSIGN}} \\ \emptyset & \text{otherwise} \end{cases} \\
\text{isConst}(e1 \text{ aop } e2) &= \text{isConst}(e1) \land \text{isConst}(e2) \\
\text{isConst}(e1 \text{ bitop } e2) &= \text{isConst}(e1) \land \text{isConst}(e2) \\
\text{isConst}(\text{bitnot } e) &= \text{isConst}(e)
\end{align*}
\]

Fig. 9. Fixed-point negative roles
Fixed-point negative roles (cont.)

**BOOL**

\[
\begin{align*}
gen(var:=e) &= \begin{cases}
\emptyset & \text{if } isBool(e) = \text{true} \\
\{\text{var}\} & \text{otherwise}
\end{cases} \\
\text{gen(avar[e]:=e2)} &= \text{gen(e1)} \cup \text{gen(e2)} \\
\text{gen(if b then s1 else s2)} &= \text{gen(b)} \cup \text{gen(s1)} \cup \text{gen(s2)} \\
\text{gen(s1; s2)} &= \text{gen(s1)} \cup \text{gen(s2)} \\
\text{gen(skip)} &= \emptyset \\
\text{gen(while b do s)} &= \text{gen(b)} \cup \text{gen(s)} \\
\text{gen(call p(e1, ..., en))} &= \bigcup_{1 \leq i \leq n} \text{gen(ei)}
\end{align*}
\]

\[
\begin{align*}
\text{gen(var)} &= \text{gen(num)} = \emptyset \\
\text{gen(e1 aop e2)} &= \text{IsVar(e1)} \cup \text{IsVar(e2)} \\
\text{gen(e1 bitop e2)} &= \text{IsVar(e1)} \cup \text{IsVar(e2)} \\
\text{gen(bitnot e)} &= \text{IsVar(e)}
\end{align*}
\]

\[
\begin{align*}
isBool(var) &= \begin{cases}
\text{true} & \text{if } var \in \text{Res}_BOOLEAN \\
\text{false} & \text{otherwise}
\end{cases} \\
isBool(num) &= \begin{cases}
\text{true} & \text{if } num::=0 \text{ or } num::=1 \\
\text{false} & \text{otherwise}
\end{cases} \\
isBool(b) &= \text{true} \\
isBool(e1 aop e2) = isBool(e1 bitop e2) = isBool(bitnot e) = isBool(b) = false \\
isBool(e1 bitop e2) &= false \\
isBool(bitnot e) &= false
\end{align*}
\]

Fig. 10. Fixed-point negative roles (cont.)

**CHAR**

\[
\begin{align*}
gen(var:=e) &= \begin{cases}
\{\text{var}\} & \text{if } isChar(e) = \text{true} \\
\emptyset & \text{otherwise}
\end{cases} \\
\text{gen(avar[e]:=e2)} &= \emptyset \\
\text{gen(if b then s1 else s2)} &= \text{gen(s1)} \cup \text{gen(s2)} \\
\text{gen(s1; s2)} &= \text{gen(s1)} \cup \text{gen(s2)} \\
\text{gen(skip)} &= \emptyset \\
\text{gen(while b do s)} &= \text{IsVar(e1)} \text{ if } (n=1 \text{ and } p ::= \text{getchar}) \\
&\quad \text{or } (n=2 \text{ and } (p ::= \text{getc or p ::= fgetc or p ::= tolower or p ::= toupper}) \\
&\quad \text{or } p ::= \text{toupper or p ::= isalnum or p ::= isblank or p ::= iscntrl or p ::= isdigit or p ::= isgraph or p ::= islower or p ::= isprint or p ::= isputnct or p ::= isspace or p ::= isupper or p ::= isxdigit)) \\
&\quad \text{otherwise} \\
\text{gen(call p(e1, ..., en))} &= \text{IsVar(e2)} \text{ if } (n=2 \text{ and } (p ::= \text{putchar or p ::= tolower}) \\
&\quad \text{or } p ::= \text{toupper or p ::= isalnum or p ::= isblank or p ::= iscntrl or p ::= isdigit or p ::= isgraph or p ::= islower or p ::= isprint or p ::= isputnct or p ::= isspace or p ::= isupper or p ::= isxdigit)) \\
&\quad \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\text{gen(e)} &= \text{gen(b)} = \emptyset \\
isChar(num) &= \begin{cases}
\text{true} & \text{if num is character literal} \\
\text{false} & \text{otherwise}
\end{cases} \\
isChar(var) &= \begin{cases}
\text{true} & \text{if } var \in \text{Res}_CHAR \\
\text{false} & \text{otherwise}
\end{cases} \\
isChar(e1 aop e2) &= isChar(e1 bitop e2) = isChar(bitnot e) = isChar(b) = \emptyset
\end{align*}
\]

Fig. 11. Fixed-point positive roles

**One-run negative roles:** \(Init^R = \emptyset, \bigcup = \setminus, c = F\)

**SYNT_CONST**

\[
\begin{align*}
gen(var:=e) &= \{\text{var}\} \\
\text{gen(avar[e]:=e2)} &= \emptyset \\
\text{gen(if b then s1 else s2)} &= \text{gen(s1)} \cup \text{gen(s2)} \\
\text{gen(s1; s2)} &= \text{gen(s1)} \cup \text{gen(s2)} \\
\text{gen(skip)} &= \emptyset \\
\text{gen(while b do s)} &= \text{gen(s)} \\
\text{gen(call p(e1, ..., en))} &= \emptyset \\
\text{gen(e)} &= \text{gen(b)} = \emptyset
\end{align*}
\]

Fig. 12. One-run negative roles