Investment-Specific Technological Change and the Brazilian Macroeconomy*

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Abstract
This study discusses the importance of investment-specific technological change for the Brazilian macroeconomy. We document evidence that a model that takes this specific type of technical progress into account is better suited to explain the Brazilian economy over the long term. We then present a DSGE [Dynamic Stochastic General Equilibrium] model with two sectors that incorporates technical progress in the investment goods sector and estimate the model for Brazil. The results demonstrate that productivity shocks in the investment goods sector are more volatile and persistent than in the final goods sector and that the output gap has a greater variance in the two-sector model. In addition, these results recommend a more rigorous monetary policy prescription to improve the economy’s well-being.

Keywords: Monetary Policy, DSGE Models, Potential Output.

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1. Introduction

In the Brazilian economy, there is strong evidence that technical progress has different meanings for the consumer goods sector than it does for the investment goods sectors. The most important clue in this direction is the trajectory of the price of investment goods (Wholesale Price Index [Índice de Preços por Atacado]; IPA-machines and equipment) in relation to consumer goods (IPA-consumer goods) as shown in figure 1,\(^1\) which shows a persistent downward trend and reveals strong evidence that productivity in the investment goods production sector grew persistently more than productivity in the consumer goods production sector.

![Graph](image)

Figure 1
Machines and Equipment Price Index / Consumer Goods Prices Index

However, to the best of our knowledge, this fact has been overlooked in the short-term and long-term macroeconomic models developed for the Brazilian economy. In this study, we aim to contribute in the following three ways regarding this aspect of the Brazilian macroeconomy:

i) we show evidence that technical progress in the investment goods production sector has different characteristics from the technical progress in the final goods production sector in Brazil, which is important in explaining long-term output;

\(^1\)Data are from the Institute of Applied Economic Research [Instituto de Pesquisa Econômica Aplicada – IPEA] and show the before-and-after results with respect to the adoption of a change in the method of calculating the indices.
ii) we develop a small-scale DSGE model that incorporates the foregoing feature; and

iii) we estimate the model for Brazil and assess the implications for the monetary policy-making process.

The importance of distinguishing the technical progress between the final goods and investment goods sectors in the macroeconomy was first addressed in Greenwood et al. (1997). These authors presented evidence that growth accounting for the USA shows a much more exact fit when the investment-specific technological change is included in the neoclassical model and that approximately 60% of the total factor productivity growth in the post-war period resulted from an increase in this sector’s productivity. Other studies corroborate this view, such as Whelan (2003) and Basu and Fernald (2009). Thus, in the first part of this study, we assess how the investment-specific technological change is important to explaining the long-term growth of the Brazilian economy using a simple extension of the Solow model.

In the second part of this study, we develop a DSGE model in which the investment goods production sector is subject to productivity shocks that are independent of the final goods production sector, and we consider the impact of the results derived from this model on prescriptions for monetary policy. Basu and Fernald (2009) developed a DSGE model to examine this question, but the structure of their model was different than the structure of the model proposed here, both in terms of the modeling strategy and because we included an additional structure that follows other models developed for the Brazilian economy, such as those of Vereda and Cavalcanti (2010) and (Castro et al., 2011, i.e., the SAMBA model). The DSGE model of Basu and Fernald (2009) examines productivity shocks in the durable goods sector in general, and not in the investment goods sector specifically. In our case, we include a structure with financial frictions, capital production adjustment costs, endogenous utilization of the capital capacity, and rigid prices in the investment sector.

Additionally, whereas Basu and Fernald (2009) calibrate the model using identical parameters for the persistence and volatility of the productivity shocks in the two sectors, we perform the estimation of such parameters as we estimate the DSGE model with Bayesian econometrics. As a result, we show that shocks in the investment goods sector are more persistent and more volatile than those in the consumer goods sector. As a consequence, the variance of the output gap is greater in the two-sector model, which suggests the prescription of a tighter monetary policy by the monetary authority. Such a result is contrary to that reached by Basu and Fernald (2009).

Basu and Fernald (2009) emphasize that the potential output would be more volatile in the two-sector model, and hence the output gap would have a smaller variance, which would imply weaker reactions in monetary policy. However, by
estimating the model and obtaining more appropriate parameters, we observe that including the investment goods sector by aggregating more volatile and persistent shocks actually increases the volatility of the output gap and suggests a stronger reaction from the Central Bank as the optimal policy, i.e., the opposite of what was suggested by the calibrated model.

2. The Two-Sector Model: Long Term

Can the standard Solow Model reproduce the patterns of decomposition of Brazilian growth? To answer this question, we must assess what the contribution of the capital accumulation predicted by the Solow model would be and compare it with the observed contribution under a growth accounting analysis.

In the decomposition of growth, the TFP (total factor productivity) growth rate is given by

\[ \dot{Y} = \alpha \dot{K} - (1 - \alpha) \dot{L} \]

In the steady-state of the Solow model, \( Y \), output, and \( K \), capital, grow at rate \((g + n)\), whereas \( L \), labor, grows at rate \( n \). Therefore, substituting into the previous equation, we have

\[ \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \]

Thus, the contribution of capital to growth is given by

\[ \dot{Y} = (1 - \alpha)g \]

\[ \dot{K} = \alpha(1 - \alpha)\frac{\dot{Y}}{Y} \]

\[ \dot{L} = n - (1 - \alpha)g \]

Thus, using the data obtained for the growth of TFP in Cardoso and Teles (2010), we can calculate the contribution of capital accumulation to the growth of the Brazilian economy predicted by the Solow model. Figure 2 compares the trajectory of the trend of the contribution of capital accumulation that is predicted by the Solow model with the trend observed for the Brazilian economy.

We can repeat the exercises by using three different measures of capital accumulation (1: Total Net Stock; 2: Net Machines and Equipment Stock; 3: Private Net Machines and Equipment Stock). Concurrently, we can compare these results separately for the three growth periods in accordance with an analysis of structural breaks conducted by Cardoso and Teles (2010). Thus, we observe that we would obtain the results presented in table 1 for the three growth periods of the Brazilian economy.

The results demonstrate that a simple Solow model predicts the variations of the contribution of capital accumulation for output growth surprisingly well, but it is not able to predict the contribution rate of capital accumulation quite so well and underestimates the contribution of capital accumulation to output growth per worker in all periods. The results are robust to the different Brazilian economy.
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Figure 2
Contribution of capital accumulation to growth

Table 1
Contribution of Capital Accumulation Predicted by the Solow Model

| Period   | Observed K1 |  | K2 |  | K3 | Predicted K1 |  | K2 |  | K3 |
|---------|-------------|---|----|---|----|-------------|---|----|---|----|
| 1950-1966 | 2.23% |  | 2.11% |  | 2.26% |  | 0.64% |  | 0.72% |  | 0.62% |
| 1968-1979 | 2.59% |  | 3.08% |  | 3.22% |  | 2.05% |  | 1.73% |  | 1.64% |
| 1980-2008 | 0.48% |  | -0.02% |  | 0.02% |  | -0.75% |  | -0.42% |  | -0.45% |
| The entire period | 1.44% |  | 1.24% |  | 1.33% |  | 0.26% |  | 0.39% |  | 0.33% |
growth stages and to different measures of capital stock; moreover, in low-growth periods, the divergence is generally greater.

Greenwood et al. (1997), Whelan (2003), and Basu and Fernald (2009) argue that an extension of the neoclassical model that includes the investment goods production sector considerably increases the model’s explanatory ability. The basic idea behind this result is that TFP in the capital goods sector grew more rapidly than in the consumer goods sector and that this component is the most important part of determining the contribution of capital accumulation to growth.

To see this idea more clearly, consider that the capital and consumption sectors are produced in accordance with the following production functions:

\[
I = K_I^\alpha (A_I L_I)^{1-\alpha}
\]

\[
C = K_C^\alpha (A_C L_C)^{1-\alpha} \equiv K_C^\alpha (Q^{1-\alpha} A_I L_C)^{1-\alpha} \equiv Q K_C^\alpha (A_I L_C)^{1-\alpha}
\]

At equilibrium, the prices are equal across sectors, i.e.,

\[
P_{KC} = Q K_C^\alpha (A_I L_C)^{1-\alpha} = \alpha K_I^\alpha - 1 (A_I L_I)^{1-\alpha} = P_{Ki}
\]

\[
P_{LC} = (1 - \alpha) Q K_C^\alpha A_I^{1-\alpha} L_C^{-\alpha} = (1 - \alpha) K_I^\alpha A_I^{1-\alpha} L_I^{-\alpha} = P_{Li}
\]

This result implies that the capital-labor ratio is balanced between sectors:

\[
\frac{L_C}{K_C} = \frac{L_I}{K_I} = \frac{L}{K}
\]

and also that the evolution of relative prices will depend on the difference in the pace of TFP growth between the two sectors:

\[
\frac{P_I}{P_C} = Q
\]

Finally, output per effective labor – as expressed in terms of the technology of the investment sector – will be

\[
y = \frac{Y}{A_I L} = \frac{I + C/Q}{A_I L} = A_I (L_I + L_C) \left( \frac{K/A_I L}{A_I L} \right)^\alpha = k^\alpha
\]

where \( k \equiv K/A_I L \).

As a result, in the steady state, capital per worker increases at a rate of \( g_I \), and the contribution of capital accumulation to output growth per worker will be

\[
\alpha g_I = \frac{\alpha}{1-\alpha} \frac{PTF_I}{PTF_I}
\]
Thus, there are two ways to assess whether this model is more suitable than the simple Solow model: i) Observe the evolution of relative prices; ii) Verify whether the contribution of the predicted capital accumulation better fits the observed contribution than it does in the simple model.

As argued in the introduction, the evolution of relative prices indicates that technical progress in the investment goods sector was greater than in the final goods sector. By verifying the contribution of capital accumulation predicted by the two-sector model, we arrive at the results shown in figure 3 and in table 2.

The two-sector model follows the Solow model in its ability to predict the variations in the contribution of capital accumulation to growth, and it exhibits a better fit to explain the level of accumulation rates, on average. In the period of high growth, the two-sector model exhibits a much better robust fit than the simple Solow model, regardless of the capital measure. In the low-growth period, the two-sector model exhibits a better fit when using a measure of total capital but an overestimated fit when other measures of capital are considered.

In general, the two-sector model exhibits a better fit than the simple model. Such a result naturally implies that understanding the technical progress in the investment goods sector is crucial to understanding Brazil’s long-term growth process and provides additional evidence that any model that seeks to observe potential output for economic policy objectives should consider that the technical progress of the investment goods sector is different from that of the consumer goods production sector.
Table 2
Contribution of Accumulation Predicted by the Solow Models

| Period     | K1     | K2     | K3     |
|------------|--------|--------|--------|
| Observed   |        |        |        |
| 1970-1979  | 2.30%  | 2.76%  | 2.97%  |
| 1980-2008  | 0.48%  | -0.02% | 0.02%  |
| Predicted (Solow) |        |        |        |
| Period     | K1     | K2     | K3     |
| 1970-1979  | 1.27%  | 1.06%  | 0.97%  |
| 1980-2008  | -0.75% | -0.42% | -0.45% |
| Predicted (2 sectors) |        |        |        |
| Period     | K1     | K2     | K3     |
| 1970-1979  | 2.60%  | 2.89%  | 2.77%  |
| 1980-2008  | 0.40%  | 0.66%  | 0.63%  |

3. Two-Sector Model: Short Term

In this section, we present the standard model that we will use to evaluate monetary policy in periods of crisis. The model is an extension of the standard New Keynesian model with price rigidity, in which the intermediate goods and capital goods production sectors, in addition to financial institutions (FIs), are included.

The model is described in simplified form in figure 4. Households save through loans to FIs, which in turn pass these loans on to firms that need capital to employ workers. Concurrently, the monetary authority can inject money into the financial system through purchases or sales of government bonds in open market transactions. Consequently, the interest rate charged to firms will be different from the rate that is paid to households because households cannot review their portfolio after the Central Bank’s intervention. Thus, the financial intermediary functions as a channel of monetary policy transmission.

The output of the economy is concentrated in the intermediate goods-producing firms, which employ workers and capital and are divided into two types: those that produce intermediate goods for use in the assembly of consumer goods and those that produce investment goods. These two types of intermediate goods-producing firms are subject to specific productivity shocks.

The firms producing consumer goods or investment goods only aggregate the production of intermediate goods-producing firms, so that they may be viewed as assemblers, and their function in the model is to allow the incorporation of price rigidity in a treatable form. Therefore, assemblers only buy intermediate goods, differentiate them and sell them to consumers and to firms producing capital goods.
3.1 Households

Households receive income from labor, from earnings, and from interest on deposits in the financial system in the previous period. Thus, at the beginning of the period, households decide whether they will use their resources on consumption or on deposits in the financial system to receive interest in the following period.

Households live infinitely, and each household $i$ seeks to maximize the expected utility function given by

$$\max E \sum_{t=0}^{\infty} \beta^t U_t(C_t, L_t)$$

such that

$$U_t(C_t, L_t) = S^C_t \left[ \frac{C_t^{1-\eta}}{1-\eta} - S^L_t \frac{L_t^{1+\omega}}{1+\omega} \right]$$

where $C$ symbolizes consumption; $L$ represents labor; and $S^C$ and $S^L$ are stochastic variables that incorporate intertemporal preference shocks and labor supply shocks, respectively. Additionally, $\eta$ is the coefficient of relative risk aversion, and $\omega$ is the marginal disutility of labor.

Temporal preference and labor supply are subject to shocks given, respectively, by the following:

$$\log S^C_t = (1 - \rho^C) \log S^C_{t-1} + \rho^C \log S^C_{t} + \epsilon_t^C$$
\[ \log S^L_t = (1 - \rho^L) \log S^L_{s,s} + \rho^L \log S^L_{t-1} + \epsilon^L_t \]

where \( \epsilon^C_t \rightarrow N(0, \sigma^C) \) and \( \epsilon^L_t \rightarrow N(0, \sigma^L) \).

Households are subject to the following budget constraint:

\[ C_t P_{C,t} + N_t = W_t L_t + R^n_t N_{t-1} + \Theta \]

where \( P_{C,t} \) is the price level of consumer goods, \( W \) is the level of wages, \( N \) represents deposits paid at the end-of-period, \( R^n \) is the gross nominal interest rate paid by the FIs, and \( \Theta \) symbolizes the earnings received.

When solving the households problem, the labor supply equation and the Euler equation for consumption are found in the following:

\[ S^L_t C^\eta_t L^\omega_t = \frac{W_t}{P_{C,t}} \]

\[ S^C_t C^{-\eta}_t P^\omega_t = \beta E_t \left[ S^C_{t+1} C^{-\eta}_{t+1} P^\omega_{t+1} \right] R^n_{t+1} \]

3.2 Firms

As previously explained, the production of the economy is divided among three types of firms: i) producers of consumer goods and investment goods; ii) firms producing intermediate goods; and iii) and firms producing capital goods.

3.2.1 Producers of consumer goods and investment goods

The two types of goods are produced in accordance with technology:

\[ Y_{x,t} = \left( \int_0^1 Y_{x,t}(j) \frac{\varphi_x^{-1}}{\varphi_x} dj \right) \frac{\varphi_x}{\varphi_x - 1} \]

where \( x \) indicates the sector to which the firm belongs, \( x \in I, C \), \( Y_{x,t} \) is the sector’s aggregate output, \( Y_{x,t}(j) \) is the firm’s intermediate output \( j \), and \( \frac{\varphi_x^{-1}}{\varphi_x} \) is the elasticity of substitution between intermediate goods.

Both firms solve the problem:

\[ \max_{Y_{x,t}(j)} P_{x,t} Y_{x,t} - \int_0^1 P_{x,t}(j) Y_{x,t}(j) dj \]

the result of which gives us the firms’ demands for intermediate goods, \( j \), given by

\[ Y_{x,t}(j) = Y_{x,t} \left( \frac{P_{x,t}}{P_{x,t}(j)} \right)^{\varphi} \]
3.2.2 Firms Producing Intermediate Goods

Firms producing intermediate goods are distributed in a continuum \( j \in (0, 1) \) that combines capital and labor for the production of goods that are offered under monopolistic competition. The production function of intermediate goods-producing firms is given by

\[
Y_{x,t}(j) = A_{x,t} K_{x,t}(j)^{\alpha_x} L_{x,t}(j)^{1-\alpha_x}
\]

where \( A_x \) represents the TFP in sector \( x \). Yields are subject to independently distributed shocks, whose stochastic processes are given by

\[
\log A_{C,t} = (1 - \rho^{A,C}) \log A_{C,ss} + \rho^{A,C} \log A_{C,t-1} + \epsilon^{A,C}_t
\]

\[
\log A_{I,t} = (1 - \rho^{A,I}) \log A_{I,ss} + \rho^{A,I} \log A_{I,t-1} + \epsilon^{A,I}_t
\]

where \( \epsilon^{A,C}_t \sim N(0, \sigma^{A,C}_t) \) and \( \epsilon^{A,I}_t \sim N(0, \sigma^{A,I}_t) \). Each firm determines the choices of inputs to be used to minimize costs, i.e.:

\[
\min_{L_{x,t}(j), K_{x,t}(j)} R_{t} W_{t} L_{x,t}(j) + R_{t} K_{x,t}(j)
\]

subject to the production function, where \( K_{x,t}(j) \) is capital stock, \( R_t \) is capital return, and \( R_{t}^{I} \) is the gross nominal interest rate paid by intermediary firms to the FIs. Note that the cost is affected by the interest charged by the FIs, such that liquidity shocks affect the firms’ allocation decisions.

From the CPOs, we understand the conditions that remunerations are paid in accordance with the marginal returns of the factors:

\[
R_{t}^{I} W_{t} = (1 - \alpha_x) A_{x,t} K_{x,t}^{\alpha_x} L_{x,t}^{-\alpha_x}
\]

\[
R_t = \alpha_x A_{x,t} K_{x,t}^{\alpha_x - 1} L_{x,t}^{1-\alpha_x}
\]

where the index \( j \) is omitted, assuming a symmetric equilibrium.

As previously discussed, the intermediate goods sector is subject to price rigidity. Thus, we assume that nominal prices follow a pattern of price readjustment that follows Calvo (1983): each firm can choose its price with a probability \((1 - \theta)\), regardless of the time elapsed since the last adjustment. Therefore, at each period \((1 - \theta)\), the producers can choose new prices, whereas only a fraction \( \theta \) maintain their prices unaltered.
3.2.3 Firms Producing Capital Goods

We assume that the producers of capital goods are subject to perfect competition and buy investment goods in \( t \) to augment the capital stock \( K_t \) and to restore the depreciated amount \((1 - \delta)K_{t-1}\), following exactly the same structure of Vereda and Cavalcanti (2010).

The production of new capital goods involves adjustment costs in accordance with the following law of motion:

\[
K_t = (1 - \delta) K_{t-1} + I_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right]
\]

where

\[
S \left( \frac{I_t}{I_{t-1}} \right) = \frac{\chi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2
\]

such that \( \chi \) is the parameter of sensitivity in the investments.

Subject to this technology, the representative capital producer chooses its investment \((I_t)\) and the repositioning of capital to maximize its profit by solving the following problem:

\[
\max_{u_t, K_t, I_t} E_t \sum_{t=0}^{\infty} \Xi_{0,t}(R_t U_t K_{t-1} - P_{t,t} \psi(U_t) K_{t-1} - P_{t,t} I_t)
\]

where \( \Xi_{0,t} \) is the stochastic discount factor, \( U \) represents the rate of capital utilization, and \( \psi(U_t) \) is the firm’s cost when it decides to provide capital below full capacity, assuming that

\[
\psi(U_t) = \psi_1(U_t - 1) + \frac{\psi_2}{2} (U_t - 1)^2
\]

where \( \psi_1, \psi_2 > 0 \) are sensitivity parameters of the installed capacity utilization.

To solve such a problem, we arrive at the three conditions below:

\[
R_t = \frac{P_{t,t}}{\Xi_{t,t+1}} = \psi_1 + \psi_2 (U_t - 1)
\]

\[
Q_t = E_t \sum_{t+1}^{\Xi_{t,t+1}} \left\{ \psi_1(U_{t+1} - 1) + \frac{\psi_2}{2} (U_{t+1} - 1)^2 \right\} + \psi_2 (U_t - 1)^2
\]

\[
P_{t,t} + Q_t \left[ 1 - \frac{\chi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 + \chi \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} - 1 \right) \right]
\]

\[
= E_t \Xi_{t,t+1} Q_{t+1} I_{t+1}^2 \chi \left( \frac{I_{t+1}}{I_t} - 1 \right)
\]
where \( Q \), also known as Tobin’s Q, represents the Lagrange multiplier for capital evolution.

The following implications of these equations have previously been discussed exhaustively in the literature (e.g., Vereda and Cavalcanti, 2010):

i) the rate of capital utilization will be equal to the cost of exploring one more unit of capital upon return of this use;

ii) the shadow price of capital decreases with a period’s ex ante real interest rate; and

iii) the current investment depends positively on its value in the previous period, on expectations about future investment returns and on the expected value of \( Q \) in the following period.

### 3.3 Financial Institutions (FIs)

The financial intermediary operates in a competitive market with risk-free credit, taking loans from households, \( N_t \), and injections of money received from the monetary authority and re-loaned to firms. In this way, the activities of the financial system follow a simple working capital model suggested by Carlstrom and Fuerst (1995) in which the credit channel is explicitly included in the economy.

Thus, assuming that the banks act under perfect competition, they are faced with the following budget constraint:

\[
R^f_t (N_t + (g_t - 1)M_{t-1}) = R^n_t N_t
\]

i.e., an increase in the money supply implies a liquidity effect, reducing the interest paid by firms for working capital.

This equation is only a condition of zero profit. Therefore, the value received from loans to households and from the monetary authority translates into payment of wages by firms,

\[
(N_t + (g_t - 1)M_{t-1}) = W_t L_t
\]

### 3.4 Monetary Policy

The Central Bank chooses the rate of monetary expansion with the objective of adjusting the value of the interest rates of loans to firms. To do so, using the conditions (19) and (20), the Central Bank obtains the interest rate \( R^f_t \) when determining the period’s rate of monetary expansion according to the condition given by

\[
g_t = \left( \frac{R^n_t - R^f_t}{R^f_t} \right) N_t + \left( \frac{R^*_t - R^f_t}{R^f_t} \right) \frac{1}{M_{t-1}} + 1
\]

In turn, the Central Bank determines the desired interest rate of loans to firms \( R^f_t \) based on the Taylor rule, which reacts to deviations from the steady-state.
where \( \Pi_t = \frac{P_t}{P_{t-1}} \) is the rate of inflation, and the variables with subscript \( ss \) are steady-state values, \( a, b \) are the parameters of reaction of the Taylor rule and \( S_t^m \) is the monetary policy shock, where

\[
\log S_t^m = (1 - \rho^m) \log S_{ss}^m + \rho^m \log S_{t-1}^m + \epsilon_t^m
\]

where \( \epsilon_t^m \to N(0, \sigma^m) \).

As opposed to the usual interest rate, the interest rate to be determined by the Taylor rule is a real – not a nominal – interest rate because the loans are taken out at the beginning of the period and paid at the end of the period. This difference yields the parameter \( b = \hat{b} - 1 \), in which \( \hat{b} \) would be the parameter if the Taylor rule regulated the nominal interest rate.

### 3.5 Equilibrium

The model consists of the dynamics of the endogenous variables \( Y, C, I, L, L_C, L_I, K, K_C, K_I, W, R, R^n, R^f, P_C, P_I, U, Q, \Xi, N, A_C, A_I, S_C, S_L, S_m \), such that the conditions of optimization and restrictions presented in the previous sections are respected and the markets are in equilibrium, i.e.,

**Labor Market:**

\[
L_t = L_{C,t} + L_{I,t}
\]

**Capital Market:**

\[
K_t = K_{C,t} + K_{I,t}
\]

**Goods Market:**

\[
Y_{C,t} = C_t
\]

\[
Y_{I,t} = I_t
\]

\[
Y_t = C_t + I_t
\]

The appendix presents all the equations that determine the steady-state values of the variables, as well as the log-linearized equations that will serve as a basis for subsequent estimations and simulations.

### 4. Quantitative Analysis

#### 4.1 Data

The model was estimated using quarterly data from 2003Q1 until 2013Q4 (44 data points). For this estimation, five observable variables (\( C, I, L, P \) and \( U \)) were used, which are described in table 3, along with their respective series. This
set of observable variables was chosen after taking into account the availability of data to obtain the best possible identification of productivity shocks, in particular. DSGE models are designed to characterize a stationary economy. Thus, the first step was to deflate the nominal variables using the National Consumer Price Index [Índice Nacional de Preços ao Consumidor Amplo – IPCA]. Next, we removed seasonality and trends. For this purpose, we applied the program X-12-ARIMA and the difference of the logarithms. Figure 5 shows these transformed series in graphical form.

Table 3
Observable Variables of the Model

| Variable | Series | Source |
|----------|--------|--------|
| P        | IPCA (%a.m.) | IBGE /SMPC |
| I        | Capital – gross – R$ (millions) | IBGE/SCN |
| C        | Final consumption – households – R$ (millions) | IBGE/SCN |
| L        | Paid hours – industry – index (mean 2006 = 100)-SP | Fiesp |
| U        | Installed Capacity | CNI |

4.2 Calibration

The parameters whose values are relatively consensual and/or possibly observable were calibrated, and the relevant parameters in the analysis of shock propagation were estimated using Bayesian econometrics.
Vereda and Cavalcanti (2010) analyzed the dynamic properties of a DSGE model for Brazil, testing alternative parameterizations. These authors identified the range of values of some key parameters in the DSGE literature. Thus, the values of the parameters matching this study were used: the intertemporal discount factor ($\beta$), the rate of capital depreciation ($\delta$), the coefficient of relative risk aversion ($\eta$), and the marginal disutility of labor ($\omega$).

For the parameters related to the monetary side, the sensitivities of the interest rate, such as the output ($a$) and the rate of inflation ($b$), were obtained from Castro et al. (2011). Meanwhile, those related to the structure of the firms were calibrated from two studies. The participation of capital in the output was obtained from Kanczuk (2002), whereas the price rigidity index ($\theta$) and the elasticity of substitution between intermediate goods ($\varphi$) were obtained from Lim and McNelis (2008). Table 4 summarizes the information concerning the calibration of the parameters.

| Parameters | Value | Source                        |
|------------|-------|-------------------------------|
| $\beta$    | 0.985 | Vereda and Cavalcanti (2010)  |
| $\delta$   | 0.025 | Vereda and Cavalcanti (2010)  |
| $\eta$     | 2     | Vereda and Cavalcanti (2010)  |
| $\psi$     | 1.5   | Vereda and Cavalcanti (2010)  |
| $a$        | 0.16  | Castro et al. (2011)         |
| $b$        | 2.43  | Castro et al. (2011)         |
| $\alpha$   | 0.39  | Kanczuk (2002)               |
| $\theta$   | 0.85  | Lim and McNelis (2008)       |
| $\varphi$  | 6     | Lim and McNelis (2008)       |

4.3 Priors and Posteriors

The priors used follow the calibration of Basu and Fernald (2009) for the parameters governing the stochastic processes of shocks and for the weight of the investment sector in the economy ($\omega_I$). The parameters governing the capital adjustment costs, $\chi$ and $\psi$, follow Vereda and Cavalcanti (2010). The posterior distributions of the parameters were calculated using the Metropolis-Hasting algorithm, which uses the Markov chain Monte Carlo (MCMC) simulation procedure. This section shows the multivariate result of the convergence of MCMC and the posterior values of the parameters.

4.3.1 Testing the Convergence of the MCMC

To determine whether the result of the estimation was adequate, we performed the MCMC diagnostic to test the convergence of the posterior distribution. The
multivariate MCMC diagnostic of this estimation (figure 6) indicates that the Markov chains converged in both the ‘interval’ statistic and in the second and third moments (m2 and m3). The univariate diagnostic analysis also exhibited satisfactory results (statistical convergence), with only some fluctuation in the statistics of the autoregressive parameters ($\rho$s) (see figures in the appendix).

4.3.2 Estimated Values

Given the prior distributions of the parameters, the posterior distributions were estimated using an MCMC process with 200,000 iterations, a 0.4 scale value, and five parallel chains for the Metropolis-Hastings algorithm. The results of the Bayesian estimation are shown in table 5 and in figure 7. These graphs are particularly important because they serve to detect problems and to analyze the reliability of the results. The posterior distributions of relevant parameters in this study visibly follow a normal distribution. There are problems that we note in relation to the prior value for the exogenous shocks and for the autoregressive parameters, but the strategy was to keep the same prior mean and standard deviation value for each parameter within its group (autoregressive parameter and exogenous shocks), which enhances the gap between the prior and the posterior values.

The most important result of the values obtained is the difference in values between $\rho^{AC}$ and $\rho^{AI}$, and between $\epsilon^{AC}$ and $\epsilon^{AI}$, which indicates that productivity shocks in the investment goods sector are more volatile and persistent than productivity shocks in the consumer goods sector. Such values imply that the use of a model that does not include the investment goods sector for purposes
of policy prescription can identify shocks erroneously and therefore reach values for the potential output and the output gap that are different from real values. Because the output gap is an important variable for monetary policy-making in and of itself and for obtaining inflation expectations by agents, monetary policy is always biased when it does not incorporate this source of volatility.

5. Model Properties

Using estimated values to conduct model simulations and to compare these versions with a model with only one sector that sells its products to firms that assemble consumer goods and investment goods equally, we find that the mean variance of the output gap of the two-sector model is 60.44% greater than the mean variance of the output gap of the one-sector model. Thus, the predicted output gap in the model without specific technical progress for the investment goods sector is considerably less volatile. Consequently, when using this type of model to prescribe monetary policy action, the suggested reaction would generally be less rigorous than necessary to stabilize the output. Thus, we can assert that this result is the main implication of this model for purposes of monetary policy. At this point, it is notable that the same parameters of monetary policy reaction were used in both simulations. Therefore, we can posit that monetary policy must be stricter to have the same output gap variance predicted by the one-sector model.
Figure 7
Model priors and posteriors
There are two possible explanations for the difference in the volatility of the gap in the two cases: the difference in the estimated values, and the asymmetry of the model, as shocks to the productivity of the investment goods sector have an effect on the production of both final goods and investment goods at a later time, creating a feedback component effect as if waves repeating in the sea. To analyze which of the effects would be the most important for the output gap, we calculated what the variance of the gap would be using the same values for the shocks parameters in the two sectors. The result was of a variance only 0.35% greater than in the one-sector model. Therefore, the difference in the parameters values constitutes the main reason for the difference in the variance values obtained for the gap.

In addition, we present the dynamics of the linearized model using impulse responses with a focus on productivity shocks and monetary policy. Figures 11 to 16 in the Appendix show that the results go in the correct direction – in accordance with theoretical expectations – and are statistically significant. The most interesting results are the opposite responses of the capital series in the face of productivity shocks in the consumer goods and investment goods sectors. The basic reason for this result is that, although productivity shocks in the consumer goods production sectors affect only the demand for capital, productivity shocks in the investment goods sector primarily affect the supply of capital. It is also notable that variations in capital in the short term are strongly determined by changes in capacity utilization ($U$) rather than by changes in the rate of investment.

Regarding monetary policy shocks, it is possible to note the importance of financial frictions in the model as shocks in interest rates alter the prices of factors paid by firms by increasing $R^f$ and reducing capital procurement, labor, and, as a result, economic output. A crucial aspect in this case is that a shock of this nature concomitantly affects the consumer goods and investments goods production sectors with more significant impacts because these shocks affect investment (and not just capital capacity utilization) and are thus more persistent.

6. Conclusions

In this study, we investigated the role of technical progress that is specific to the investment goods production sector in the Brazilian macroeconomy. The results suggest that this specific type of technical progress is crucial to understanding the process of long-term growth and to develop appropriate policies for the short term.

In the first part of the study, it was demonstrated that the growth accounting in Brazil can be better understood when one model with different trajectories of technical progress for both the consumer goods and investment goods sectors is adopted, which mirrors the results reported by Greenwood et al. (1997), Whelan (2003), as well as Basu and Fernald (2009) for the U.S. economy.

In the second part of the study, we developed a DSGE model with independent investment goods and consumer goods production sectors and included com-
ponents in the model that are common to models widely used for the Brazilian economy, such as the model developed by Vereda and Cavalcanti (2010) and Castro et al. (2011) – the SAMBA model. The main contributions are empirical in nature, namely, that this model is the first to estimate the structural model with productivity shocks specific to the investment goods sector and that the results found move in the opposite direction than results based on calibrations performed with identical values for the parameters of the stochastic process that governs productivity shocks.

In this respect, the main conclusion of the study is that the output gap is much more volatile when considering the two-sector model because the productivity shocks in the investment goods sector are more volatile and persistent, which implies that a monetary policy that disregards these specific shocks may have an insufficient reaction to stabilize output around potential.
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Appendix

A. Steady-State Equations

Assuming \( \alpha = \alpha_C = \alpha_I \) and \( \varphi = \varphi_C = \varphi_I \) and considering the Steady-State values: \( A_{ss} = 1 \); \( U_{ss} = 1 \); and \( \Pi_{x,ss} = 0 \), we have:

\[
C_{ss}P_{C,ss} + N_{ss} = W_{ss}L_{ss} + R_{ss}^\alpha N_{ss} \quad (A.1)
\]

\[
C_{ss}^\alpha L_{ss}^\omega = \frac{W_{ss}}{P_{C,ss}} \quad (A.2)
\]

\[
\frac{1}{\beta} = R_{ss}^n \quad (A.3)
\]

\[
Y_{x,ss} = K_{x,ss}^{\alpha}L_{x,ss}^{1-\alpha} \quad (A.4)
\]

\[
R_{ss}^f \frac{W_{ss}}{P_{x,ss}} = \left( \frac{\varphi - 1}{\varphi} \right) (1 - \alpha) \frac{Y_{x,ss}}{L_{x,ss}} \quad (A.5)
\]

\[
R_{ss} = \left( \frac{\varphi - 1}{\varphi} \right) \alpha \frac{Y_{x,ss}}{K_{x,ss}} \quad (A.6)
\]

\[
1 = \left( \frac{\varphi - 1}{\varphi} \right) \left( 1 - \alpha \right) \frac{R_{ss}^f W_{ss}}{1 - \alpha} \left[ \frac{1 - \alpha}{\alpha} \right] \frac{R_{ss}}{R_{ss}^f W_{ss}} \quad (A.7)
\]

\[
\Xi_{ss} = \beta \quad (A.8)
\]

\[
I_{ss} = \delta K_{ss} \quad (A.9)
\]

\[
\frac{R_{ss}}{P_{f,ss}} = \psi_1 \quad (A.10)
\]

\[
\frac{Q_{ss}}{\Xi_{ss}} = Q_{ss}(1 - \delta) - R_{ss} \quad (A.11)
\]

\[
\Xi_{ss} = \beta \quad (A.12)
\]

\[
P_{f,ss} = -Q_{ss} \quad (A.13)
\]

\[
N_{ss} = W_{ss}L_{ss} \quad (A.14)
\]
\[ R^n_{ss} = R^I_{ss} \quad (A.15) \]

\[ Y_{ss} = (1 - \omega_I)C_{ss} + \omega_I I_{ss} \quad (A.16) \]

\[ K_{ss} = (1 - \omega_I)K_{C,ss} + \omega_I K_{I,ss} \quad (A.17) \]

\[ L_{ss} = (1 - \omega_I)L_{C,ss} + \omega_I L_{I,ss} \quad (A.18) \]

where \( \omega_I \in (0, 1) \) denotes the participation of the investment goods sector in the economy.

B. Log-Linearized Model

B.1 Households

Budget Restriction

\[ C_{ss}P_{C,ss}(\tilde{C}_t + \tilde{P}_{C,t}) + N_{ss}\tilde{N}_t = W_{ss}L_{ss}(\tilde{W}_t + \tilde{L}_t) + R^n_{ss}N_{ss}(\tilde{R}^n_t + \tilde{N}_{t-1}) \quad (B.1) \]

Intratemporal Substitution

\[ 0 = \tilde{S}^L_t + \eta\tilde{C}_t + \omega\tilde{L}_t - \tilde{W}_t + \tilde{P}_{C,t} \quad (B.2) \]

Euler Equation

\[ 0 = \tilde{S}^C_t - \tilde{S}^C_{t+1} + \eta(\tilde{C}_{t+1} - \tilde{C}_t) + \tilde{N}_{C,t+1} - \tilde{R}^n_{t+1} \quad (B.3) \]

B.2 Firms

Production Functions

\[ 0 = \tilde{C}_t - \tilde{A}_{C,t} - \alpha\tilde{K}_{C,t} - (1 - \alpha)\tilde{L}_{C,t} \quad (B.4) \]

\[ 0 = \tilde{I}_t - \tilde{A}_{I,t} - \alpha\tilde{K}_{I,t} - (1 - \alpha)\tilde{L}_{I,t} \quad (B.5) \]

Labor demand

\[ 0 = \tilde{R}^I_t + \tilde{W}_t - \tilde{P}_{C,t} - \tilde{C}_t + \tilde{L}_{C,t} \quad (B.6) \]

\[ 0 = \tilde{R}^I_t + \tilde{W}_t - \tilde{P}_{I,t} - \tilde{I}_t + \tilde{L}_{I,t} \quad (B.7) \]
Capital demand
\[ 0 = \tilde{R}_t - \tilde{P}_{C,t} - \tilde{C}_t + \tilde{K}_{C,t-1} \] (B.8)
\[ 0 = \tilde{R}_t - \tilde{P}_{I,t} - \tilde{I}_t + \tilde{K}_{I,t-1} \] (B.9)

Phillips Curves
\[ \tilde{\Pi}_{C,t} = \beta \tilde{\Pi}_{C,t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \left[ (1 - \alpha)(\tilde{W}_t + \tilde{R}_t) - \tilde{A}_{C,t} + \alpha \tilde{R}_t \right] \] (B.10)
\[ \tilde{\Pi}_{I,t} = \beta \tilde{\Pi}_{I,t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \left[ (1 - \alpha)(\tilde{W}_t + \tilde{R}_t) - \tilde{A}_{I,t} + \alpha \tilde{R}_t \right] \] (B.11)

Law of Motion of Capital
\[ K_{ss}\tilde{K}_t = (1 - \delta)K_{ss}\tilde{K}_{t-1} + I_{ss}\tilde{I}_t \] (B.12)

Return on Capital Production
\[ 0 = \frac{R_{ss}}{P_{I,ss}}(\tilde{R}_t - \tilde{P}_{I,t}) - \psi_2\tilde{U}_t \] (B.13)

Tobin’s Q
\[ \frac{Q_{ss}(\tilde{Q}_t - \tilde{\Xi}_{t+1})}{\Xi_{ss}} = (1 - \delta)Q_{ss}\tilde{Q}_t - R_{ss}(\tilde{R}_{t+1} + \tilde{U}_{t+1}) + \psi_1 P_{I,ss}\tilde{U}_{t+1} \] (B.14)
\[ P_{I,ss}\tilde{P}_{I,t} + \left( 1 - \frac{\chi}{2} \right) Q_{ss}\tilde{Q}_t + \chi(\tilde{I}_t - \tilde{I}_{t-1}) = \chi\Xi_{ss}Q_{ss}P_{I,ss}^2(\tilde{I}_{t+1} - \tilde{I}_t) \] (B.15)

Stochastic discount factor
\[ \tilde{\Xi}_{t+1} = \eta(\tilde{C}_t - \tilde{C}_{t+1}) - \tilde{\pi}_{t+1} + \tilde{S}_C^{I+1} - \tilde{S}_C^{I} \] (B.16)

B.3 Financial Institutions/Central Bank

Zero Profit
\[ \tilde{R}_n + \tilde{N}_t = \tilde{R}_{I,t} + \tilde{W}_t + \tilde{L}_t \] (B.17)

Taylor Rule
\[ R_{ss}\tilde{R}_{I,t} = aY_{ss}\tilde{Y}_t + b\Pi_{ss}\tilde{N}_t \] (B.18)
B.4 Market Clearing

Labor Market

\[ \tilde{L}_t = (1 - \omega_t)\tilde{L}_{C,t} + \omega_t \tilde{L}_{I,t} \]  
(B.19)

Capital Market

\[ \tilde{K}_t = (1 - \omega_t)\tilde{K}_{C,t} + \omega_t \tilde{K}_{I,t} \]  
(B.20)

Goods Market

\[ Y_{ss} \tilde{Y}_t = C_{ss} \tilde{C}_t + I_{ss} \tilde{I}_t \]  
(B.21)

B.5 Shocks

Productivity

\[ \tilde{A}_{C,t} = \rho^{A,C} \tilde{A}_{C,t-1} + \epsilon_t^{A,C} \]  
(B.22)

\[ \tilde{A}_{I,t} = \rho^{A,I} \tilde{A}_{I,t-1} + \epsilon_t^{A,I} \]  
(B.23)

Intertemporal Preference

\[ \tilde{S}^C_t = \rho^C \tilde{S}^C_{t-1} + \epsilon_t^C \]  
(B.24)

Labor Supply

\[ \tilde{S}^L_t = \rho^L \tilde{S}^L_{t-1} + \epsilon_t^L \]  
(B.25)
C. Diagnostic Tests

![Graphs showing diagnostic tests for model parameters.](image)

Figure 8
Univariate diagnostic of the Main Model Parameters.
Figure 9
Univariate diagnostic of the Autoregressive Model Parameters.
Figure 10
Univariate diagnostic of the Exogenous Shocks of the Model.
D. Impulse Response Functions – IRFs

Figure 11
IRFs of Productivity in the Consumer Goods Sector

Figure 12
IRFs of Productivity in the Consumer Goods Sector (cont.)
Figure 13
IRFs of Productivity in the Investment Goods Sector

Figure 14
IRFs of Productivity in the Investment Goods Sector (cont.)
Figure 15
IRFs of the Interest Rate

Figure 16
IRFs of the Interest Rate (cont.)