Shocklets in the Comet Halley Plasma

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Abstract

Dust acoustic (DA) waves evolving into shocklets are investigated in the Comet Halley plasma system relaxing to Maxwellian, Kappa and Cairns distributions. Here dynamics of dust is described by the fully nonlinear continuity and momentum equations. A set of two characteristic wave nonlinear equations is obtained and numerically solved to examine the DA solitary pulse which develops into oscillatory shocklets with the course of time such as at time $\tau = 0$, symmetric solitary pulses are formed, which develop into oscillatory shocklets. It has been observed that variation in superthermality strongly affects the profiles of nonlinear DA structures in terms of negative potential, dust velocity and density.

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I. INTRODUCTION

In our solar system, billions of comets can be found circumnavigating the Sun. These are snowballs of frozen gases, rocks and cosmic dust particles. It is widely believed that like nucleus in an atom, nucleus of the comet is discrete and cohesive made of volatile solids containing all the mass of the comet. While orbiting around the Sun, when a comet approaches closer to the Sun, it expels dust particles and ionized gases in huge quantity into a behemoth like brightening head bigger than most planets. This dust and gases form a dusty plasma and is in the form of a tail that expands outwardly away from the Sun for millions of miles. These dust grains are electrically charged by virtue of plasma which permeates the solar system. From the observation of dust impact analyzer installed on Vega spacecraft supported that much of the carbon is trapped within dust grains [1–3].

Due to low geometric albedo ($\sim 0.04$), comets are considered darkest in the solar system. Whereas Halley’s comet is the most luminous one which appears after an interval of 76 years. Surface of the comet is covered with a dust mantle and has temperature of $\sim 300 - 400$ K where the latter was determined from the infrared spectrometer (IKS) installed on the Vega 2 spacecraft [4].

Whipple revitalize the discrete nucleus concept and developed a seminal model [5] which could potentially explain the dynamic affects on the orbits of comets (such as nongravitational recoil effects) as well as other vital features of cometary observations. Despite large volume ($\sim 500$ km$^3$), the small bulk density ($0.1 \sim 0.2 g$ cm$^{-3}$) can be due to the small mass of Halley’s comet ($\sim 5 \times 10^{16} - 10^{17} g$) which has been estimated independent of gravitational effects of its orbit [6].

H$_2$O is the main constituent of the volatile nucleus, usually volatiles like CO would be caged within the water - ice lattice and form clathrate hydrate. It is also anticipated that when water-ice sublimation starts, significant coma activity initiates and this happens at about distance of $2 - 3$ AU [7]. In a new comet water ice is of amorphous nature and transforms into crystalline structure as the kinetic energy of the cometary nucleus increases and reaches to a critical value. At this point usually an unanticipated nucleus burst out and this happens between about 3 and 6 AU [8].

Mendis in 1986 calculated the flights of the dust grains of different sizes (micron-submicron) that are anticipated to be expelled out of the cometary nucleus. Observational
data from Vega 1 & 2 and Giotto spaceports shows that plasma of comae of comet Halley contains ions of both positive and negative polarity other than cosmic dust and electrons. These ions are such as hydroxides (OH$^+$, OH$^-$), hydrogen (H$^+$, H$^-$), oxygen (O$^+$, O$^-$), silicon (Si$^+$, Si$^-$) etc [9, 10].

This kind of plasma with additional negative ions known as electronegative dusty plasma is chemically very reactive. Electronegative plasma is also found in other astrophysical environments such as D-region of the ionosphere, themesosphere, photosphere of Sun etc. and plays a vital role in the semiconductor technology. In most of these astrophysical and laboratory plasmas, electrons can stick to the surface of dust and so their density reduces and such system can also be treated as electron depleted plasma. Therefore, in this setting, role of negative ions becomes more crucial [10].

Observations reveal excitation of foreshock and bow shock like nonlinear structures at Halley’s comet. Shock waves are quite common nonlinear structures in astrophysical settings for instance are triggered when interaction of solar wind with the comets or magnetic field of different planets takes place, these waves are also anticipated to appear in galactic dust clouds at the beginning stage of star creation. Waves of compressional nature are generated when dust particles impact with solid surface of a spacecraft, in both the dust grain and surface of the spacecraft.

Shocks are outcome of discontinuous disturbances traveling in a medium where an abrupt jump in physical quantities like density, pressure and temperature across some narrow region leads to such structures. Dissipation structures like kinematic viscosity (also known as momentum diffusivity) feature of collisions between dust and ions and the fluctuating charge of dust particles lead to the formation of dust acoustic (dust ion) DA (DIA) shock waves in dusty plasmas with $\Gamma_c << 1$ (coupling parameter). Whereas in opposite case when plasma is strongly coupled ($\Gamma_c >> 1$), shear and bulk viscosities are considered crucial in the excitation of DA or DIA shock waves of monotone and oscillatory nature [11]. Trigger of shock waves is an irreversible phenomena and contrary to nonlinear solitary waves, there is a sharp dissipation of energy and speed with the distance of the shock wave alone.

For the better understating of this important class of nonlinear waves in dusty plasma which has enormous significance in the astrophysical context, many experimental investigations are worth mentioning. For example first observation of shock waves in radio frequency 3D dusty (complex) plasma discharge under minute gravity condition was carried in the
PKE-Nefedov device by Samsonov et al. \cite{12}. Latter a compressional DA shock wave triggered with the aid of impulsive axial magnetic field was examined \cite{13}. In another interesting study, excitation of a bow shock in a 2D dusty plasma was examined where a super-sonic flow of micro sized negatively dust particles around a stationary object was used. this exactly mimics the aforementioned situation when shock waves are generated due to the impact of dust particles with the insitu spacecraft. Recently, Jaswal et al experimentally studied the evolution of DA shock waves compared their propagation characteristics numerical results based on Korteweg-de-Vries-Burgers (KdV-Burger) equation \cite{11}.

There has been extensive theoretical investigation of DA and DIA shock waves \cite{14, 19, 22}, in one of earlier works by Ghosh, Ehsan, and Murtaza \cite{11}, authors investigated shock waves deriving KdV-Burger equation for the Comet Halley plasma where nonsteady dust charge variation produces anomalous dissipation that leads to the shock wave. There authors transformed KdV burger equation to another one analogous to damped anharmonic oscillator and using fifth order Runge-Kutta technique, oscillatory nature shock amplitudes were studied. It was reported that shock wave is more dispersive in nature for a the higher strength of magnetic field. whereas for the parallel propagation a monotonic shock was observed.

In the present manuscript we aim to present a fully nonlinear theory to study large-amplitude DA waves developing into the DA shocklet dust acoustic in the Cometary plasma using diagonalization method \cite{23}. For the readers it is worth mentioning the term shocklet was introduced to represent sporadic steep fronts observed in high-speed turbulent compressible fluid and in space as flank formations associated with planetary-scale shocks \cite{20, 21}.

When solar wind passes through a comet makes it a complex plasma system which is far away from the state of equilibrium. In this scenario, cometary electrons and ions will asymptotically approach to a nonequilibrium steady state with power-law distributions, e.g., often fitted with the Cairns, Kappa or r, q distribution \cite{24, 26}.

To the best of authors’ knowledge this has not been studied earlier and results of the present investigation are useful for understanding the physics of formation of shocklets at Halley’s comet.

The paper is organized in the following manner: In Sec. II, the basic set of equations for the dust acoustic shock wave of comet Halley plasma are given. Section III deals with the derivation nonlinear equation. In Sec. IV and V provide quantitative analysis and
II. BASIC SET OF EQUATIONS

Consider a one dimensional, collisionless unmagnetized plasma, whose components are the four species consisting of electrons, singly ionized negatively and positively charged ions with dust grains that are negatively charged. We assumed that spherically shaped dust grains are carrying a constant charge because the time for the dust acoustic shocklets to be discussed is much smaller than that required for further significant change in dust charge. The condition of charge neutrality at equilibrium gives

\[ n_{i_+0} + n_{i_-0} = n_{e0} - \varepsilon Z_{d0} n_{d0} + n_{i_0}, \]

where \( n_{p0} \) denotes the equilibrium number density of the \( p \)-th species (viz., \( p = i_+ \) for positively charged ions, \( p = i_- \) for negatively charged ions, \( p = e \) for electrons, \( p = d \) for dust particles and \( Z_{d0} \) is the equilibrium dust charge state. The following set of one dimensional nonlinear fluid equations are used, which describe the dynamics of DA solitary and shock waves:

\[ \frac{\partial n_d}{\partial t} + v_d \frac{\partial n_d}{\partial x} + n_d \frac{\partial v_d}{\partial x} = 0, \]  
\[ \frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} - \frac{Z_{d0} e}{m_d} \frac{\partial \phi}{\partial x} + \frac{3n_d v_d^2}{n_{d0}} \frac{\partial n_d}{\partial x} = 0 \]

and

\[ \frac{\varepsilon_0}{e} \frac{\partial^2 \phi}{\partial x^2} = Z_{d0} n_d + n_e + n_{i_-} - n_{i_+}, \]

where \( v_{Td} = (k_B T_d / m_d)^{1/2} \), \( v_d \) and \( \phi \) represents the dust thermal speed, dust fluid velocity and the electrostatic potential. As the singly charged negative, positive ions and electrons are assumed to be in thermal equilibrium, so we can described their densities by the following expressions:

\[ n_s (\phi) = n_{s0} \exp \left( \frac{e\phi}{k_B T_s} \right), \]  
\[ n_{i_+} (\phi) = n_{i_+0} \exp \left( - \frac{e\phi}{k_B T_{i_+}} \right) \]

for the case when electrons, singly charged negative and positive ions are taken as non-thermal, we can expressed the number densities for Kappa distributed electrons and singly charged negative and positive ions by the following expressions:

\[ n_s (\phi) = n_{s0} \left[ 1 - \left( \frac{\kappa - 3}{2} \right)^{-1} \frac{e\phi}{k_B T_s} \right]^{-\kappa+1/2}, \]

conclusions, respectively.
and for the case of Cairns distributed electrons and singly charged negative and positive ions densities are given by \cite{23, 26}

\[ n_s(\phi) = n_{s0} \left\{ 1 + \Gamma \left( -\frac{e\phi}{k_B T_s} \right) + \Gamma \left( -\frac{e\phi}{k_B T_s} \right)^2 \right\} \exp \left( \frac{e\phi}{k_B T_s} \right), \]  

\[ n_{i+}(\phi) = n_{i+0} \left\{ 1 + \Gamma \left( \frac{e\phi}{k_B T_{i+}} \right) + \Gamma \left( \frac{e\phi}{k_B T_{i+}} \right)^2 \right\} \exp \left( -\frac{e\phi}{k_B T_{i+}} \right), \]

where the subscript \( s \) equals \( e \) for electrons and \( i- \) for negatively charged ions. Here, \( \Gamma = \frac{4\alpha}{1 + 3\alpha} \) comprise of Cairns parameter \( \alpha \) that determines the population of nonthermal particles. It is important to mention here that if we consider \( \kappa \to \infty \) and \( \Gamma \to 0 \) in Eqs. (6−9), respectively, then we can directly replicate the Maxwell-Boltzmann density distribution for electrons, positively and negatively charged ions.

**III. NONLINEAR EVOLUTION EQUATION**

In order to normalized the above set of Eqs. (1−9) we apply the scaled parameters \cite{23} as \( N_d = n_d/n_{do}, \ V_d = v_d/c_d, U = e\phi/k_B T_e, \xi = x/\lambda_0 \) and \( \tau = t\omega_{pd} \) where \( C_{da} = \omega_{pd} \lambda_0 = (Z_{d0} k_B T_e/m_d)^{1/2} \) represents the DA speed with \( \omega_{pd} = (Z_{d0}^2 e^2 n_{do}/\varepsilon_0 m_d)^{1/2} \) the dust plasma frequency and \( \lambda_0 = (\varepsilon_0 k_B T_e/Z_d e^2 n_{do})^{1/2} \) is the scale length. Hence, the normalized dust momentum and continuity equations can be written as

\[ \frac{\partial V_d}{\partial \tau} + V_d \frac{\partial V_d}{\partial \xi} - \frac{\partial U}{\partial \xi} + 3 \frac{T_d N_d}{Z_{d0} T_e} \frac{\partial N_d}{\partial \xi} = 0, \]  

and

\[ \frac{\partial N_d}{\partial \tau} + V_d \frac{\partial N_d}{\partial \xi} + N_d \frac{\partial V_d}{\partial \xi} = 0 \]  

Mostly, the dust temperature effect is neglected because of low values of dust temperature \( T_d \) and also because of the large dust charge state so we can neglect the last term in Eq. (10) which gives the following form

\[ \frac{\partial V_d}{\partial \tau} + V_d \frac{\partial V_d}{\partial \xi} - \frac{\partial U}{\partial \xi} = 0 \]
In the current work, we are intended to study nonstationary, one dimensional nondispersive fully nonlinear DAW’s. Thus, by neglecting the dispersive term viz., $\partial^2_x \phi = 0$ in Eq. (3) and by utilizing Eqs. (4) and (5) for thermal distributed particles and Eqs. (6–9) for the case of non-thermal distributed particles, yields the following results

$$N_{d,b}(U) = \left\{ \begin{array}{ll}
[-\delta_+ \exp U + \exp \left(-\frac{U}{\sigma_+}\right) - \delta_- \exp \left(\frac{U}{\sigma_-}\right)]/\alpha_d \\
\text{for Maxwellian distributed ions and electrons}
\end{array} \right.$$

$$\left[-\delta_+ \left(1 - \frac{U}{(\kappa-\frac{3}{2})}\right)^{-\kappa+1/2} + \left(1 + \frac{U}{\sigma_+ (\kappa-\frac{3}{2})}\right)^{-\kappa+1/2} - \delta_- \left(1 - \frac{U}{\sigma_- (\kappa-\frac{3}{2})}\right)^{-\kappa+1/2} \right]/\alpha_d$$

for Kappa distributed distributed ions and electrons

$$\left[-\delta_+ \eta_1(U) \exp U + \eta_2(U) \exp(-\frac{U}{\sigma_+}) - \delta_- \eta_3(U) \exp(\frac{U}{\sigma_-})\right]/\alpha_d$$

for Cairns distributed ions and electrons

Here $\eta_1(U) = (1 - \Gamma U + \Gamma U^2)$, $\eta_2(U) = \left(1 + \frac{U}{\sigma_+} + \frac{U^2}{\sigma_+^2}\right)$ and $\eta_3(U) = \left(1 - \frac{U}{\sigma_-} + \frac{U^2}{\sigma_-^2}\right).$ The subscript $b = m$ for Maxwellian distributed ions and electrons, $b = \kappa$ for Kappa distributed non-Maxwellian negatively and positively charged ions and electrons and $b = c$ for Cairns-distributed non-Maxwellian negatively and positively ions and electrons. Where $\sigma_+ (= T_{i+}/T_e)$ represents the positive ion to electron temperature ratio, $\sigma_- (= T_{i-}/T_e)$ represents the negative ion to electron temperature ratio, $\delta_+ (= n_{e0}/n_{i+0})$ is the electron to positive ion density ratio and $\delta_- (= n_{i-0}/n_{i+0})$ is taken as negative to positive ion density ratio which can be written in terms of dust concentration as $\alpha_d = 1 - \delta_+ - \delta_-$. It is worth mentioning here that the inequality $\delta_+ + \delta_- < 1$ always holds in order to have positive dust concentration.

Substituting Eq. (13) into Eqs. (11) and (12) and carrying out differentiations w.r.t space-time coordinates ($\xi , \tau$) leads to the following form of equation

$$\frac{\partial U}{\partial \tau} + V_d \frac{\partial U}{\partial \xi} - \chi_b (U) \frac{\partial V_d}{\partial \xi} = 0 \quad (14)$$
where

\[
\chi_b(U) = \begin{cases} 
\exp\left(-\frac{U}{\sigma_+}\right) - \delta_- \exp\left(\frac{U}{\sigma_-}\right) - \delta_+ \exp U/\delta_+ \exp U + \frac{1}{\sigma_+} \exp\left(-\frac{U}{\sigma_+}\right) + \frac{\delta_-}{\sigma_-} \exp\left(\frac{U}{\sigma_-}\right) \\
\quad \text{for Maxwellian distributed pair ions and electrons} \\
-\delta_+\left(1 - \frac{U}{(\kappa - \frac{3}{2})}\right)^{-\kappa+1/2} + \left(1 + \frac{U}{\sigma_+(\kappa - \frac{3}{2})}\right)^{-\kappa+1/2} - \delta_-\left(1 - \frac{U}{\sigma_-(\kappa - \frac{3}{2})}\right)^{-\kappa+1/2} \\
\delta_+c_k\left(1 - \frac{U}{(\kappa - \frac{3}{2})}\right)^{-\kappa-1/2} + \frac{\sigma_-}{\sigma_+}\left(1 + \frac{U}{\sigma_+(\kappa - \frac{3}{2})}\right)^{-\kappa-1/2} + \frac{\delta_-}{\sigma_-}c_k\left(1 - \frac{U}{\sigma_-(\kappa - \frac{3}{2})}\right)^{-\kappa-1/2} \\
\quad \text{for Kappa distributed pair ions and electrons} \\
\delta_+\eta_1(U) \exp U - \eta_2(U) \exp\left(-\frac{U}{\sigma_+}\right) + \delta_-\eta_3(U) \exp\left(\frac{U}{\sigma_-}\right) \\
-\delta_+\left[\eta_1(U) + \zeta_1(U)\right] \exp U - \left[\frac{\eta_2(U)}{\sigma_+} - \zeta_2(U)\right] \exp\left(-\frac{U}{\sigma_+}\right) - \left[\frac{\delta_-\eta_3(U)}{\sigma_-} + \delta_-\zeta_3(U)\right] \exp\left(\frac{U}{\sigma_-}\right) \\
\quad \text{for Cairns distributed pair ions and electrons}
\end{cases}
\]

(15)

Here \(\zeta_1(U) = (-\Gamma + 2\Gamma U)\), \(\zeta_2(U) = \left(\frac{\Gamma}{\sigma_+} + \frac{2\Gamma U}{\sigma_+}\right)\), \(\zeta_3(U) = \left(-\frac{\Gamma}{\sigma_-} + \frac{2\Gamma U}{\sigma_-}\right)\) and \(c_k = (2\kappa - 1)/(2\kappa - 3)\) is the parameter that represent hot electron superthermality effects and for \(\kappa \to \infty, c_k \to 1\) displaying the reduction of Kappa distribution to standard Maxwellian distribution. Equations (12) and (14) are the normalized nonlinear coupled equations, which describes the large-amplitude nonstationary DA shock waves.

We can express Eqs.(12) and (14) in the matrix form:

\[
\frac{\partial}{\partial \tau} \begin{bmatrix} U \\ V_d \end{bmatrix} + \begin{bmatrix} V_d & -\chi_b(U) \\ -1 & V_d \end{bmatrix} \frac{\partial}{\partial \xi} \begin{bmatrix} U \\ V_d \end{bmatrix} = 0
\]

(16)

where the parameter \(\chi_b(U)\) for each distributions is expressed as given in Eq. (15).

The square matrix on the left hand side of Eq. (16) can be diagonalized by means of a diagonalizing matrix technique. The two eigen values of the square matrix can be resolved by \(\det(A - \lambda I) = 0\), given by

\[
\lambda_{\pm,b} = V_d \pm \sqrt{\chi_b(U)}
\]

(17)

where square matrix and unit matrix are given by

\[
A = \begin{bmatrix} V_d & -\chi_b(U) \\ -1 & V_d \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

(18)

The square matrix in Eq. (16) taken as \(A\), can be diagonalized by means of a diagonalizing...
matrix $C$ whose columns are the eigenvectors of $A$, so that

$$C^{-1}AC = \begin{bmatrix} \lambda_{+,b} & 0 \\ 0 & \lambda_{-,b} \end{bmatrix}$$  \hspace{1cm} (19)$$

where

$$C = \begin{bmatrix} 1 & 1 \\ -\sqrt{1/\chi_b(U)} & \sqrt{1/\chi_b(U)} \end{bmatrix}$$  \hspace{1cm} (20)$$

Multiplying Eq. (16) by $C^{-1}$ from the left gives the diagonalized system of equations

$$\frac{\partial \Psi_{\pm}}{\partial \tau} + \lambda_{\pm,b} \frac{\partial \Psi_{\pm}}{\partial \xi} = 0$$  \hspace{1cm} (21)$$

Here the new variables are $\Psi_{\pm} = V_d \mp F(U)$ with $F(U) = \int_0^U \left[ \frac{1}{\chi_b(U)} \right]^{\frac{1}{2}} dU$. A straightforward wave solution is found by taking either $\Psi_+$ or $\Psi_-$ to zero. Setting $\Psi_-$ to zero, we obtain $V_d = -F(U)$ and $\Psi_+ = 2U_b$. Since $\Psi_+$ can be written as function of $V_d$, so we can write Eq. (21) for $\Psi_+$ as

$$\frac{\partial V_d}{\partial \tau} + \lambda_{+,b}(U) \frac{\partial V_d}{\partial \xi} = 0$$  \hspace{1cm} (22)$$

where eigen value can be expressed in its new form as $\lambda_{+,b}(U) = -F(U) + \sqrt{\chi_b(U)}$ where $\chi_b(U)$ for Maxwellian, Cairns and Kappa distributed positively and negatively ions and electrons are given by Eq (15). Since we know that $V_d$ is a function of $U$ we can also write an equation similar to Eq. (22) as

$$\frac{\partial U}{\partial \tau} + \lambda_{+,b}(U) \frac{\partial U}{\partial \xi} = 0$$  \hspace{1cm} (23)$$

The above equation interprets the self steepening of the negative potential $U$, the general solution of Eq. (23) is $U = U_0 [\xi - \lambda_{+,b}(U) \tau]$, where $U_0$ is a function of one variable and is resolved by the initial condition for $U$ at $\tau = 0$, where $\lambda_{+,b}(U)$ is the effective phase speed which is a function of $U$, makes the general solution nonlinear and may therefore self-steepen which develops into oscillatory shocks with the course of time as depicted from the numerical plots shown in Fig. 1. The speeds with which these kind of shock fronts propagate is stated by Rankine Hugoniot condition $v_{\text{shock}} = \{\Lambda_L(U_L) - \Lambda_R(U_R)\} / (U_L - U_R)$, where $U_R(U_L)$ represents the value of $U$ on the right side (on the left side) of shock front, and $\Lambda_L(U_L)$ and $\Lambda_R(U_R)$ gives the left and right flux functions, where the flow function is defined as $\Lambda(U) = \int_0^U \lambda_+(U) dU$. 

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IV. QUANTITATIVE ANALYSIS

In this section, we solve numerically Eq. (23) for exploring the temporal evolution of large amplitude localized DA waves in Comet Halley plasma with Maxwellian and non Maxwellian like Kappa and Cairns distributed ions and electrons. For illustration we have chosen some typical numerical parameters of the Comet Halley taken at $\sim 10^4$ km from the nucleus \([10, 29]\) such as are $n_{d0} \sim 10^5$ m$^{-3}$, positive ion density $n_{i,0} \sim 2 \times 10^8$ m$^{-3}$, $m_{+i} = m_{-i} \sim 1.67 \times 10^{-27}$ kg, $T_e = T_{i+} = T_{i-} = 100$ eV, mass density for ice dust grains $\rho_d \sim 9 \times 10^2$ kg/m$^{-3}$, $Z_{d0} = 10^5$ dust grains with diameter $r_0 \approx 5 \mu$m and we used equilibrium charge neutrality condition to obtain the negative ions to positive ions density ratio $\delta_-(= n_{i-,0}/n_{i+,0})$. For these parameters, we have dust plasma frequency $\omega_{pd} = 1.79 \times 10^{-5}$ Hz, dust fluid velocity $C_d = 1.30 \times 10^{-5}$ m/s, characteristic lengths $C_d/\omega_{pd} = 0.743$ m and initial condition for electrostatic potential is taken as $U = U_0 \text{sech}[\xi/d]$ with amplitude $U_0 = -0.15$ and pulse width $d = 2.3$.

Figures 1 represents the time enhancement of normalized (a) electrostatic negative potential (b) dust velocity (c) dust number density as a function of position $\xi(= x/\lambda_0)$ for dotted, solid and dashed curves corresponding to different values of parameters $\kappa(= 2, 3, 10)$ for Kappa distribution in the right section [Figs. 1(a)-(c)] and for Cairns distributed ions and electrons by increasing the parameters $\alpha(= 0.1, 0.3, 0.5)$ for dotted, solid and dashed curves that determines the population of the non thermal ions and electrons are shown in the right panel [Figs. 1(d)-(f)] with fixed electron to positive ion number density ratio $\delta_+ = 0.984$, negative to positive ion number density ratio $\delta_- = 0.011$, positive ion to electron temperature ratio $\sigma_+ = 1$ and negative ion to electron temperature ratio $\sigma_- = 1$. It is displayed that the negative potential pulses overlap on one another at time $\tau = 0$, without showing any impact of superthermality. Although, the pulses symmetry break with the passage of time $\tau = (3, 4.5, 6)$ s and consequently, solitary pulses transform into an oscillatory shocks with enhanced self steepness and wave amplitude. The similar trend for potential profile is obtained for Cairns distributed ions.

Variation of ion and electron superthermality in Kappa distribution results in the reduction of amplitude of solitary pulses and shocklets this results for the reason that the effective phase speed $\lambda_{+,-}(U)$ become larger at a smaller value of $\kappa$ and vice versa. The effect of increasing pair ions and electrons superthermality with the time evolution of non
linear structures associated with the dust fluid velocity and density profiles reduces the wave amplitudes and pulse width which are better apparent as compare to the profiles of negative potential. On the contrary in comparison with Kappa distribution, the Cairns distributed non thermal pair ions and electrons with the time evolution of non linear structures related with the negative potential, dust fluid velocity and density profiles leads to the reduction of wave self steepness, amplitude and pulse width of solitary waves and shocklets with increasing the parameter $\alpha (= 0.1, 0.3, 0.5)$. These results may be explained by looking at the particle distribution functions curves of Kappa and Cairns distribution. Kappa distribution at high energies represent power law tails, however Cairns distribution displays strong variations in shoulders as compared with the tails so Kappa distribution is the most energetic distribution, whereas Cairns distribution is least energetic which outcomes in the reduction of linear and nonlinear effective phase speed with reduced self steepness in comparison with Kappa distribution.

Figure 2 displays temporal development of nonlinear DA waves related with the normalized (a) negative potential, (b) dust velocity and (c) dust density profiles as a function of $\xi$ position for Maxwellian distribution with fixed $\delta_+ = 0.984, \delta_- = 0.011$ and $\sigma_+ = \sigma_- = 1$. It is shown that as time progresses the solitary pulse stimulations propagate with non linear phase speed over the right hand side of the plot and evolve into shocks with increase wave amplitude and self steepness. It is important to mention here that DA excitations for the glimpses of potential, dust fluid velocity and dust density in case of Kappa distributed non thermal pair ions and electrons for high value of $\kappa = 50$ and for Cairns distributed non thermal pair ions and electrons at $\alpha = 0$ almost coincides with the case of Maxwellian as depicted for Kappa distribution in the left panel [Figs. 3 (a)-(c)] and for Cairns distribution in the right panel [Figs. 3 (d)-(f)].

Figure 4 and 5 manifests the variation of normalized dust number density with ratio of electron to negative ion number density $\beta = n_{e0}/n_{i-0}$ [Fig. 4] and with electron to negative ion temperature ratio $T = T_e/T_{i-}[\text{Fig. 5}]$ for Maxwellian (blue curve) Kappa (red curve) and Cairns (black curve) distributed ions and electrons respectively with fixed $\delta_- = 0.011, \kappa = 2, \alpha = 0.1$ and $\sigma_+ = \sigma_- = 1$. It is determined that increasing value of $\beta$ results in an enhancement of dust number density $N_d$. On the other hand, by increasing the electron to negative ion temperature ratio $\xi$ means an increase in the dust number density.

It is evident from the curves of Figures 4 and 5 that dust number density is higher
for Kappa distribution and least for Cairns distribution and intermediate for Maxwellian distribution.

V. CONCLUSIONS

In this paper we have studied the formation of large-amplitude dust acoustic waves and their evolution into shocklets with time for a an unmagnetized Maxwellian and non-Maxwellian Kappa/Cairns distributed comet Halley plasma comprising of electrons, negative and positive ions in addition to negatively charged dynamical dust grains. We worked out the fully nonlinear hydrodynamic equations, momentum and continuity equations alongwith a quasineutrality equation for the dust fluid by employing diagonalization method technique, a set of two characteristic wave equations are found which are then solved numerically. Our numerical results reveals the existence of oscillatory shocks in a Maxwellian and non-Maxwellian dusty plasma with the course of time. The effects of ion superthermality $\kappa$ and $\alpha$ parameter density are investigated on the profiles of solitary and oscillatory shock waves resulting considerable variations in the wave amplitudes and widths with the temporal development. The effect of electron to negative ion temperature ratio $\xi$ and electron to negative ion density ratio $\beta$ on dust number density are also investigated and found that it is higher in case of Kappa distributed negative and positive ions and electrons lower for Cairns distribution and intermediate for Maxwellian distribution. It was found that the drift solitary and shock wave amplitude is lower for cairns distributed pair ions and electrons as compare to Kappa and Maxwellian distributions. To the best of author’s knowledge present study has not done before and is important for the comet Halley plasmsa.

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Figure Captions

Figure 1 (color online): Temporal change of the normalized (a) negative potential $U (= e\phi/k_BT_e)$, (b) dust fluid velocity $V_d = v_d/c_d$, and (c) dust density $N_d (= n_d/n_{d0})$ is plotted against the normalized position $\xi (= x/\lambda_0)$ by taking different values of $\kappa = 2$ (dotted curve), $\kappa = 3$ (solid curve), and $\kappa = 10$ (dashed curve) in the left section and $\alpha = 0.1$ (dotted curve), $\alpha = 0.3$ (solid curve), and $\alpha = 0.5$ (dotted curve) in the right section with fixed values of $\delta_+ = 0.984$, $\delta_- = 0.011$ and $\sigma_+ = \sigma_- = 1$.

Figure 2 (color online): Temporal change of the normalized (a) negative potential $U (= e\phi/k_BT_e)$, (b) dust velocity $V_d = v_d/c_d$, and (c) dust density $N_d (= n_d/n_{d0})$ is plotted against the normalized position $\xi (= x/\lambda_0)$ for Maxwellian distributed pair ions and electrons with fixed values of $\delta_+ = 0.984$, $\delta_- = 0.011$ and $\sigma_+ = \sigma_- = 1$.

Figure 3 (color online): Temporal change of the normalized (a) negative potential $U (= e\phi/k_BT_e)$, (b) dust velocity $V_d = v_d/c_d$, and (c) dust density $N_d (= n_d/n_{d0})$ is plotted against the normalized position $\xi (= x/\lambda_0)$ for Kappa distribution with $\kappa = 50$ is plotted in the left section and for Cairns distribution with $\alpha = 0$ is plotted in the right section with fixed values of $\delta_+ = 0.984$, $\delta_- = 0.011$ and $\sigma_+ = \sigma_- = 1$.

Figure 4 (color online): The normalized dust density $N_d (= n_d/n_{d0})$ is plotted against the electron to negative ion number density $\beta = n_{e0}/n_{i-0}$ for Maxwellian (blue curve) Kappa (red curve) and Cairns (black curve) distributed pair ions and electrons respectively with...
fixed $\kappa = 2$, $\alpha = 0.1$, $\delta_- = 0.011$ and $\sigma_+ = \sigma_- = 1$.

Figure 5 (color online): The normalized dust density $N_d (= n_d/n_{d0})$ is plotted against the electron to negative ion temperature ratio $T = T_e/T_{i-}$ for Maxwellian (blue curve), Kappa (red curve) and Cairns (black curve) distributed pair ions and electrons respectively with fixed $\kappa = 2$, $\alpha = 0.1$, $\delta_- = 0.011$ and $\sigma_+ = \sigma_- = 1$. 