Trust from the past: Bayesian Personalized Ranking based Link Prediction in Knowledge Graphs

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Abstract
Link prediction, or predicting the likelihood of a link in a knowledge graph based on its existing state is a key research task. It differs from a traditional link prediction task in that the links in a knowledge graph are categorized into different predicates and the link prediction performance of different predicates in a knowledge graph generally varies widely. In this work, we propose a latent feature embedding based link prediction model which considers the prediction task for each predicate disjointly. To learn the model parameters it utilizes a Bayesian personalized ranking based optimization technique. Experimental results on large-scale knowledge bases such as YAGO2 show that our link prediction approach achieves substantially higher performance than several state-of-art approaches. We also show that for a given predicate the topological properties of the knowledge graph induced by the given predicate edges are key indicators of the link prediction performance of that predicate in the knowledge graph.

1 Introduction
A knowledge graph is a repository of information about entities, where entities can be any thing of interest such as people, location, organization or even scientific topics, concepts, etc. An entity is frequently characterized by its association with other entities. As an example, capturing the knowledge about a company involves listing its products, location and key individuals. Similarly, knowledge about a person involves her name, date and place of birth, affiliation with organizations, etc.

Resource Description Framework (RDF) is a frequent choice for capturing the interactions between two entities. A RDF dataset is equivalent to a heterogeneous graph, where each vertex and edge can belong to different classes. The class information captures taxonomic hierarchies between the type of various entities and relations. As an example, a knowledge graph may identify Kobe Bryant as a basketball player, while its ontology will indicate that a basketball player is a particular type of athlete. Thus, one will be able to query for famous athletes in the United States and find Kobe Bryant.

The past few years have seen a surge in research on knowledge representations and algorithms for building knowledge graphs. For example, Google Knowledge Vault [6], and IBM Watson [9] are comprehensive knowledge bases which are built in order to answer questions from the general population. As evident from these works, it requires multitude of efforts to build a domain specific knowledge graph, which are, triple extraction from natural language text, entity and relationship mapping [25], and link prediction [21]. Specifically, triples extracted from the text data sources using state of the art techniques such as OpenIE [8] and semantic role labeling [5] are extremely noisy, and simply adding noisy triple facts into knowledge graph destroys its purpose. So computational methods must be devised for deciding which of the extracted triples are worthy of insertion into a knowledge graph. There are several considerations for this decision making: (1) trustworthiness of the data sources; (2) a belief value reported by a natural language processing engine expressing its confidence in the correctness of parsing; and (3) prior knowledge of subjects and objects. This particular work is motivated by the third factor.

Link prediction in knowledge graph is simply a machine learning approach for utilizing prior knowledge of subjects and objects as available in the knowledge graph for estimating the confidence of a candidate triple. Consider the following example: given a social media post “I wish Tom Cruise was the president of United States”, a natural language processing engine will extract a triple
of the links differs widely. So, existing link prediction methods are not very suitable for this task. We build our link prediction method by borrowing solutions from recommender system research which accept a user-item matrix and for a given user-item pair, they return a score indicating the likelihood of the user purchasing the item. Likewise, for a given predicate, we consider the set of subjects and objects as a user-item matrix and produce a real-valued score to measure the confidence of the given triple. For training the model we use Bayesian personalized ranking (BPR) based embedding model [25], which has been a major work in the recommendation system. In addition, we also study the performance of our proposed link prediction algorithm in terms of topological properties of knowledge graph and present a linear regression model to reason about its expected level of accuracy for each predicate.

Our contributions in this work are outlined below:

1. We implement a Link Prediction approach for estimating confidence for triples in a Knowledge Graph. Specifically, we borrow from successful approaches in the recommender systems domain, adopt the algorithms for knowledge graphs and perform a thorough evaluation on a prominent benchmark dataset.

2. We propose a Latent Feature Embedding based link recommendation model for prediction task and utilize Bayesian Personalized Ranking based optimization technique for learning models for each predicate (Section 4). Our experiments on the well known YAGO2 knowledge graph (Section 5) show that the BPR approach outperforms other competing approaches for a significant set of predicates (Figure 1).

3. We apply a linear regression model to quantitatively analyze the correlation between the prediction accuracy for each predicate and the topological structure of the induced subgraph of the original Knowledge Graph. Our studies show that metrics such as clustering coefficient or average degree can be used to reason about the expected level of prediction accuracy (Section 5.3, Figure 2).

2 Related Work

There is a large body of work on link prediction in knowledge graph. In terms of methodology, factorization based and related latent variable models [3, 13, 22, 23], graphical model [14], and graph feature based method [17, 18] are considered.

There exists large number of works which focus on factorization based models. The common thread
among the factorization methods is that they explain the triples via latent features of entities. \cite{[2]} presents a tensor based model that decomposes each entity and predicate in knowledge graphs as a low dimensional vector. However, such a method fails to consider the symmetry property of the tensor. In order to solve this issue, \cite{[22]} proposes a relational latent feature model, RESCAL, an efficient approach which uses a tensor factorization model that takes the inherent structure of relational data into account. By leveraging relational domain knowledge about entity type information, \cite{[3]} proposes a tensor decomposition approach for relation extraction in knowledge base which is highly efficient in terms of time complexity. In addition, various other latent variable models, such as neural network based methods \cite{[6][27]}, have been explored for link prediction task. However, the major drawback of neural network based models is their complexity and computational cost in model training and parameter tuning. Many of these models require tuning large number of parameters, thus finding the right combination of these parameters is often considered more of an art than science.

Recently graphical models, such as Probabilistic Relational Models \cite{[10]}, Relational Markov Network \cite{[21]}, Markov Logic Network \cite{[13][24]}, have also been used for link prediction in knowledge graph. For instance, \cite{[24]} proposes a Markov Logic Network (MLN) based approach, which is a template language for defining potential functions on knowledge graph by logical formula. Despite its utility for modeling knowledge graph, issues such as rule learning difficulty, tractability problem, and parameter estimation pose implementation challenge for MLNs.

Graph edge based approaches assume that the existence of an edge can be predicted by extracting features from the observed edges in the graph. Lao and Cohen \cite{[17][15]} propose Path Ranking Algorithm (PRA) to perform random walk on the graph and compute the probability of each entity pair, and use any favorable classification model, such as logistic regression and SVM, to predict the probability of missing edge between an entity pair in a knowledge graph.

It has been demonstrated \cite{[11]} that no single approach emerges as a clear winner. Instead, the merits of factorization models and graph feature models are often complementary with each other. Thus combining the advantages of different approaches for learning knowledge graph is a promising option. For instance, \cite{[20]} proposes to use additive model, which is a linear combination between RESCAL and PRA. The combination results in not only decrease the training time but also increase the accuracy. \cite{[15]} combines a latent feature model with an additive term to learn from latent and neighborhood-based information on multi-relational data. \cite{[6]} fuses the outputs of PRA and neural network model as features for training a binary classifier. Our work strongly aligns with this combination approach. In this work, we build matrix factorization based techniques that have been proved successful for recommender systems and plan to incorporate graph based features in future work.

3 Background and Problem Statement

**Definition 3.1.** We define the knowledge graph as a collection of triple facts $G = (S, P, O)$, where $s \in S$ and $o \in O$ are the set of subject and object entities and $p \in P$ is the set of predicates or relations between them. $G(s, p, o) = 1$ if there is a direct link of type $p$ from $s$ to $o$, and $G(s, p, o) = 0$ otherwise.

Each triple fact in knowledge graph is a statement interpreted as “A relationship $p$ holds between entities $s$ and $o$”. For instance, the statement “Kobe Bryant is a player of LA Lakers” can be expressed by the following triple fact (“Kobe Bryant”, “playsFor”, “LA Lakers”).

**Definition 3.2.** For each relation $p \in P$, we define $G_p(S_p, O_p)$ as a bipartite subgraph of $G$, where the corresponding set of entities $s_p \in S_p$, $o_p \in O_p$ are connected by relation $p$, namely $G_p(s, o) = 1$.

**Problem Statement:** For every predicate $p \in P$ and given an entity pair $(s, o)$ in $G_p$, our goal is to learn a link recommendation model $M_p$ such that $x_{s,o} = M_p(s, o)$ is a real-valued score.

Due to the fact that the produced real-valued score is not normalized, we compute the probability $Pr(y_{s,o}^p = 1)$, where $y_{s,o}^p$ is a binary random variable that is true iff $G_p(s, o) = 1$. We estimate this probability $Pr$ using the logistic function as follows:

$$Pr(y_{s,o}^p = 1) = \frac{1}{1 + exp(-x_{s,o})}$$

Thus we interpret $Pr(y_{s,o}^p = 1)$ as the probability that a vertex (or subject) $s$ in the knowledge graph $G$ is in a relationship of given type $p$ with another vertex (or the object) $o$.

4 Methods

In this section, we describe our model, namely Latent Feature Embedding Model with Bayesian Personalized Ranking (BPR) based optimization technique that we propose for the task of link prediction in a knowledge graph. In our link prediction setting, for a given predicate $p$, we first construct its bipartite subgraph
Then we learn the optimal low dimensional embeddings for its corresponding subject and object entities \( s_p \in S_p, o_p \in O_p \) by maximizing a ranking based distance function. The learning process relies on Stochastic Gradient Descent (SGD). The SGD based optimization technique iteratively updates the low dimensional representation of \( s_p \) and \( o_p \) until convergence. Then the learned model is used for ranking the unobserved triple facts in descending order such that triple facts with higher score values have a higher probability of being correct.

### 4.1 Latent Feature Based Embedding Model

For each predicate \( p \), the model maps both its corresponding subject and object entities \( s_p \) and \( o_p \) into low-dimensional continuous vector spaces, say \( U_p^p \in \mathbb{R}^{1 \times K} \) and \( V_p^p \in \mathbb{R}^{1 \times K} \) respectively. We measure the compatibility between subject \( s_p \) and object \( o_p \) as dot product of its corresponding latent vectors which is given as below:

\[
(4.2) \quad x_{s_p, o_p} = (U_p^s)(V_p^p)^T + b_p^o
\]

where \( U_p \in \mathbb{R}^{S \times K} \), \( V_p \in \mathbb{R}^{O \times K} \), and \( b_p \in \mathbb{R}^{K \times K} \). \( |S| \) and \( |O| \) denote the size of subject and object associated with predicate \( p \) respectively. \( K \) is the number of latent dimensions and \( b_p^o \in \mathbb{R} \) is a bias term associated with object \( o \). Given predicate \( p \), the higher the score of \( x_{s_p, o_p} \), the more similar the entities \( s_p \) and \( o_p \) in the embedded low dimensional space, and the higher the confidence to include this triple fact into knowledge base.

### 4.2 Bayesian Personalized Ranking

In collaborative filtering, positive-only data is known as implicit feedback/binary feedback. For example, in the eCommerce platform, some users only buy but do not rate items. Motivated by [23], we employ Bayesian Personalized Ranking (BPR) based approach for model learning. Specifically, in recommender system domain, given user-item matrix, BPR based approach assigns the preference of user for purchased item with higher score than un-purchased item. Likewise, under this context, we assign observed triple facts higher score than un-purchased item. Likewise, under this context, we assign observed triple facts higher score than un-purchased item. The last term in Equation 4.3 is a \( l_2 \)-norm regularization term used for model parameters \( \Theta_p = \{U_p, V_p, b_p\} \) to avoid overfitting in the learning process. In addition, the logistic function \( \sigma(\cdot) \) in Equation 4.3 is defined as \( \sigma(x) = \frac{1}{1+e^{-x}} \).

Notice that the Equation 4.3 is differentiable, thus we employ the widely used SGD to maximize the objective. In particular, at each iteration, for given predicate \( p \), we sample one observed entity pair \((s_p, o_p^+)^+\) and one unobserved one \((s_p, o_p^-)^-\) using uniform sampling technique. Then we iteratively update the model parameters \( \Theta_p \) based on the sampled pairs. Specifically, for each training instance, we compute the derivative and update the corresponding parameters \( \Theta_p \) by walking along the ascending gradient direction.

For each predicate \( p \), given a training triple \((s_p, o_p^+, o_p^-)^-\), the gradient of BPR objective in Equation 4.3 with respect to \( U_p^s, V_p^p, V_p^p, b_p^o, b_p^o^- \) can be computed as follows:

\[
\frac{\partial \text{BPR}}{\partial U_p^s} = \frac{\partial \ln \sigma(x_{s_p, o_p^+} - x_{s_p, o_p^-}) - 2\lambda U_p^s}{\partial U_p^s} = \frac{\partial \ln \sigma(x_{s_p, o_p^+} - x_{s_p, o_p^-})}{\partial x_{s_p, o_p^+}} \times \frac{\partial x_{s_p, o_p^+}}{\partial U_p^s} - \frac{\partial x_{s_p, o_p^-}}{\partial U_p^s}
\]

\[
= \frac{1}{\sigma(x_{s_p, o_p^+} - x_{s_p, o_p^-})} \times \sigma'(x_{s_p, o_p^+} - x_{s_p, o_p^-}) \times (V_{o_p^+} - V_{o_p^-}) - 2\lambda U_p^s
\]

\[
= (1 - \sigma(x_{s_p, o_p^+} - x_{s_p, o_p^-})) \times (V_{o_p^+} - V_{o_p^-}) - 2\lambda U_p^s
\]

(4.4)

We obtain the following using similar chain rule derivation.

\[
\frac{\partial \text{BPR}}{\partial V_p^p} = (1 - \sigma(x_{s_p, o_p^+} - x_{s_p, o_p^-})) \times (U_p^s - 2\lambda U_p^s)
\]

(4.5)

\[
\frac{\partial \text{BPR}}{\partial b_p^o} = (1 - \sigma(x_{s_p, o_p^+} - x_{s_p, o_p^-})) \times (-V_p^p - 2\lambda V_p^p)
\]

(4.6)
Algorithm 1 Bayesian Personalized Ranking Based Latent Feature Embedding Model

Input: latent dimension $K$, $G$, target predicate $p$

Output: $U^p$, $V^p$, $b^p$

1: Given target predicate $p$ and entire knowledge graph $G$, construct its bipartite subgraph $G_p$
2: $m = \text{number of subject entities in } G_p$
3: $n = \text{number of object entities in } G_p$
4: Generate a set of training samples $D_p = \{(s_p, o_p^+, o_p^-)\}$ using uniform sampling technique
5: Initialize $U^p$ as size $m \times K$ matrix with 0 mean and standard deviation 0.1
6: Initialize $V^p$ as size $n \times K$ matrix with 0 mean and standard deviation 0.1
7: Initialize $b^p$ as size $n \times 1$ column vector with 0 mean and standard deviation 0.1
8: for all $(s_p, o_p^+, o_p^-) \in D_p$ do
9: Update $U^p$ based on Equation (4.9)
10: Update $V^p$ based on Equation (4.10)
11: Update $b^p$ based on Equation (4.11)
12: Update $b_p^-$ based on Equation (4.12)
13: Update $b_p^+$ based on Equation (4.13)
14: end for
15: return $U^p$, $V^p$, $b^p$

4.3 Pseudo-code and Complexity Analysis

The pseudo-code of our proposed link prediction model is described in Algorithm 1. It takes the knowledge graph $G$ and a specific target predicate $p$ as input and generates the low dimensional latent matrices $U^p$, $V^p$, $b^p$ as output. Line 1 constructs the bipartite subgraph of predicate $p$, $G_p$ given entire knowledge graph $G$. Line 2-3 compute the number of subject and object entities as $m$ and $n$ in resultant bipartite subgraph $G_p$ respectively. Line 4 generates a collection of triple samples using uniform sampling technique. Line 5-7 initialize the matrices $U^p$, $V^p$, $b^p$ using Gaussian distribution with 0 mean and 0.1 standard deviation, assuming all the entries in $U^p$, $V^p$ and $b^p$ are independent. Line 8-14 update corresponding rows of matrices $U^p$, $V^p$, $b^p$ based on the sampled instance $(s_p, o_p^+, o_p^-)$ in each iteration. As the sample generation step in line 4 is prior to the model parameter learning, thus the convergence criteria of Algorithm 1 is to iterate over all the sampled triples in $D_p$.

Given the constructed $G_p$ as input, the time complexity of the update rules shown in Equations (4.9) (4.10) (4.11) (4.12) (4.13) is $O(cK)$, where $K$ is the number of latent features. The total computational complexity of Algorithm 1 is then $O(|D_p| \cdot cK)$, where $|D_p|$ is the total size of pre-sampled triples shown in line 4 of Algorithm 1.

5 Experiments and Results

This section presents our experimental analysis of the Algorithm 1 for thirteen unique predicates in the well known YAGO2 knowledge graph [12]. We construct a model for each predicate and describe our evaluation strategies, including performance metrics and selection of state-of-the-art methods for benchmarking in section 5.1. We aim to answer two questions through our experiments:

1. How does our approach compare with related work for link recommendation in knowledge graph?

2. For a predicate $p$, can we reason about the link prediction model performance $M_p$ in terms of the structural metrics of the bipartite graph $G_p$?

Table 1 shows the statistic of various YAGO2 relations used in our experiments. # Subjects and # Objects represent the number of subject and object entities associated with its corresponding predicate. The last column shown in Table 1 shows the number of facts for each relation in YAGO2. We run all the experiments on a 2.1 GHz Machine with 4GB memory running Linux operating system. The algorithms are implemented in Python language along with NumPy and SciPy libraries for linear algebra operations. The
Figure 1: Link Recommendation Comparison on YAGO2 Relations

(a) HR Comparison among different link recommendation methods
(b) ARHR Comparison among different link recommendation methods
(c) AUC Comparison among different link recommendation methods

Figure 2: Quantitative Analysis Between Graph Topology and Link Recommendation Model Performance

(a) Graph Density and HR
(b) Graph Density and ARHR
(c) Graph Density and AUC
(d) Graph Average Degree and HR
(e) Graph Average Degree and ARHR
(f) Graph Average Degree and AUC
(g) Clustering Coefficient and HR
(h) Clustering Coefficient and ARHR
(i) Clustering Coefficient and AUC
other model parameters, we fix learning rate $\alpha$ and number of latent factors $K$ for comparison.

class recommendation methods as baseline approaches in recommender system domain, we consider the following state-of-the-art one-class item recommendation [23] in recommender system.

the problem we solve in this paper is similar to the one-class item recommendation problem.

During the model evaluation stage, we use three popular metrics, namely Hit Rate (HR), Average Reciprocal Hit-Rank (ARHR), and Area Under Curve (AUC), to measure the link recommendation quality of our proposed approach in comparison to baseline methods. HR is defined as follows:

$$HR = \frac{\#hits}{\#subjects}$$

where $\#subjects$ is the total number of subject entities in test set, and $\#hits$ is the number of subjects whose object entity in the test set is recommended in the size-N recommendation list. The second evaluation metric, ARHR, considering the ranking of the recommended object for each subject entity in knowledge graph, is defined as below:

$$ARHR = \frac{1}{\#subjects} \sum_{i=1}^{\#subjects} \frac{1}{p_i}$$

where if an object of a subject is recommended for connection in knowledge graph which we name as hit under this scenario, $p_i$ is the position of the object in the ranked recommendation list. As we can see, ARHR is a weighted version of HR and it captures the importance of recommended object in the recommendation list.

The last metric, AUC is defined as follows:

$$AUC = \frac{1}{\#subjects} \sum_{s \in \text{subjects}} \frac{1}{|E(s)|} \sum_{(o^+, o^-) \in E(s)} \delta(x_{s, o^+} > x_{s, o^-})$$

Where $E(s) = \{(o^+, o^-) | (s, o^+) \in S_{test} \cap (s, o^-) \notin (S_{test} \cup S_{train})\}$, and $\delta()$ is the indicator function.

For all of three metrics, higher values indicate better model performance. Specifically, the trivial AUC of a random predictor is 0.5 and the best value of AUC is 1.

5.1 Experimental Setting

For our experiment, in order to demonstrate the performance of our proposed link prediction model, we use the YAGO2 dataset and several evaluation metrics for all compared algorithms. Particularly, for each relation, we split the data into a training part, used for model training, and a test part, used for model evaluation. We apply 5-time leave one out evaluation strategy, where for each subject, we randomly remove one fact (one subject-object pair) and place it into test set $S_{test}$ and remaining in the training set $S_{train}$. For every subject, the training model will generate a size-N ranked list of recommended objects for recommendation task. The evaluation is conducted by comparing the recommendation list of each subject and the object entity of that subject in the test set. Grid search is applied to find regularization parameters, and we set the values of parameters used in section 4.2 as $\lambda_s = \lambda_{o^+} = \lambda_{o^-} = 0.005$. For other model parameters, we fix learning rate $\alpha = 0.2$, and number of latent factors $K = 50$ respectively. For parameter in model evaluation, we set $N = 10$.

In order to illustrate the merit of our proposed approach, we compare our model with the following methods for link prediction in a knowledge graph. Since the problem we solve in this paper is similar to the one-class item recommendation in recommender system domain, we consider the following state-of-the-art one-class recommendation methods as baseline approaches for comparison.

1. **Random (Rand):** For each relation, this method randomly selects subject-object entity pair for link recommendation task.

2. **Most Popular (MP):** For each predicate in knowledge base, this method presents a non-personalized ranked object list based on how often object entities are connected among all subject entities.

3. **MF:** The matrix factorization method is proposed by LE, which uses a point-wise strategy for solving the one-class item recommendation problem.

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Table 1: Statistics of Various Relations in YAGO2 Dataset

| Relation                  | # Subjects | # Objects | # of Facts in YAGO2 |
|---------------------------|------------|-----------|---------------------|
| Import                    | 142        | 62        | 391                 |
| Export                    | 140        | 176       | 579                 |
| isInterestedIn            | 358        | 213       | 464                 |
| hasOfficialLanguage       | 583        | 214       | 964                 |
| dealWith                  | 131        | 124       | 945                 |
| happenedIn                | 7121       | 5526      | 12500               |
| participatedIn            | 2330       | 7943      | 16889               |
| isConnectedTo             | 2835       | 4391      | 33581               |
| hasChild                  | 10758      | 12800     | 17320               |
| influence                 | 8056       | 9153      | 25819               |
| wroteMusicFor             | 5109       | 21487     | 24271               |
| edited                    | 549        | 5673      | 5946                |
| owns                      | 8330       | 24422     | 26536               |

software is available online for download [3]

For our experiment, in order to demonstrate the performance of our proposed link prediction model, we use the YAGO2 dataset and several evaluation metrics for all compared algorithms. Particularly, for each relation, we split the data into a training part, used for model training, and a test part, used for model evaluation. We apply 5-time leave one out evaluation strategy, where for each subject, we randomly remove one fact (one subject-object pair) and place it into test set $S_{test}$ and remaining in the training set $S_{train}$. For every subject, the training model will generate a size-N ranked list of recommended objects for recommendation task. The evaluation is conducted by comparing the recommendation list of each subject and the object entity of that subject in the test set. Grid search is applied to find regularization parameters, and we set the values of parameters used in section 4.2 as $\lambda_s = \lambda_{o^+} = \lambda_{o^-} = 0.005$. For other model parameters, we fix learning rate $\alpha = 0.2$, and number of latent factors $K = 50$ respectively. For parameter in model evaluation, we set $N = 10$.

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1. **Random (Rand):** For each relation, this method randomly selects subject-object entity pair for link recommendation task.

2. **Most Popular (MP):** For each predicate in
for all the YAGO2 predicates used in the experiment, our proposed model consistently outperforms MF based method, which demonstrates the empirical experience that pairwise ranking based method achieves much better performance than pointwise regression based method given implicit feedback for link recommendation task. Compared with Popularity based recommendation method MP, our method obtains better performance for most predicates. For example, predicates such as “participate”, “connect”, “hasChild”, and “influence”, our proposed model achieves more than 10 times better performance in terms of both HR and ARHR. However, for several predicates such as “import”, “export”, and “language”, MP based method performs the best among all the competing methods. The good performance of MP is owing to the semantic meaning of specific predicate. For instance, “import” represents Country/Product relation in YAGO2, which indicates the types of its subject and object entities are geographic region and commodity respectively. For such a predicate, most popular object entities such as food, cloth, fuel are linked to most of the countries, which helps MP based method obtain good link recommendation performance.

5.3 Analysis and Discussion

Figure 1 shows that the link prediction model performance widely varies from predicate to predicate in the YAGO2 knowledge base. For example, the HR of predicate “dealsWith” is significantly better than “own”. Thus it is critical that we quantitatively understand the model performance across various relations in a knowledge graph. Recall from the Problem Statement that given a predicate $p$, our model $M_p$ only accounts for the bipartite subgraph $G_p$. Motivated by [19], we study the impact of resultant graph structure of $G_p$ on the performance of $M_p$.

For each predicate $p$, we compute several graph topology metrics on its bipartite subgraph $G_p$ such as graph density, graph average degree, and clustering coefficient. Figure 2 shows the quantitative analysis between graph structure and link prediction model performance of each predicate. In each subfigure, x-axis represents the computed graph topology metric value of each predicate and y-axis denotes our proposed link prediction model performance in terms of HR, ARHR, and AUC. Each cross point shown in blue represents one specific YAGO2 predicate used in our experiments. Then we developed a linear regression model to understand the correlation between link prediction model performance and each graph metric. For each linear regression curve shown in red color, we also report its slope, intercept, and correlation coefficient (rvalue) to capture the association trend.

From Figure 2, both graph density and graph average degree show strong positive correlation signal with proposed link prediction model as demonstrated by rvalue. As our approach is inspired by collaborative filtering for recommender systems that accept a user-item matrix as input, for resultant graph of each predicate, higher graph density indicates higher matrix density in user-item matrix, which naturally leads to better recommendation performance in recommender system domain. Similar explanation can be adapted to graph average degree. For the clustering coefficient, it shows strong negative correlation signal with link prediction model performance. For instance, in terms of AUC, the rvalue is around $-0.69$. As clustering coefficient (cc) is the number of closed triples over the total number of triples in graph, smaller value of cc indicates lower fraction of closed triples in the graph. Based on the transitivity property of a social graph, which states the friends of your friend have high likelihood to be friends themselves [28, 30], it is relatively easier for link prediction model to predict (i.e., hit) such link with open triple property in the graph, which leads to better link prediction performance.

6 Conclusion and Future Work

Inspired by the success of collaborative filtering algorithms for recommender systems, we propose a latent feature based embedding model for the task of link prediction in a knowledge graph. Our proposed method provides a measure of “confidence” for adding a triple into the knowledge graph. We evaluate our implementation on the well known YAGO2 knowledge graph. The experiments show that our Bayesian Personalized Ranking based latent feature embedding approach achieves better performance compared with two state-of-art recommender system models: Most Popular and Matrix Factorization. We also develop a linear regression model to quantitatively study the correlation between the performance of link prediction model itself and various topological metrics of the graph from which the models are constructed. The regression analysis shows strong correlation between the link prediction performance and graph topological features, such as graph density, average degree and clustering coefficient.

For a given predicate, we build link prediction models solely based on the bipartite subgraph of the original knowledge graph. However, as real-world experience suggests, the existence of a relation between two entities can also be predicted from the presence of other relations, either direct or through common neighbors. As an example, the knowledge of where someone studies
and who they are friends with is useful to predict possible workplaces. Incorporating such intuition as “social signals” into our current model will be the prime candidate for an immediate future work. Another future work would be to update the knowledge graph based on the newer facts that become available over time in streaming data sources.

Acknowledgement
This work was supported by the Analysis In Motion Initiative at Pacific Northwest National Laboratory, which is operated by Battelle Memorial Institute, and by Mohammad Al Hasan’s NSF CAREER Award (IIS-1149851).

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