Hall Coefficient and electron-electron interaction of 2D electrons in Si-MOSFET’s.

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Recent experiments in silicon MOSFETs indicate that the Hall coefficient is independent of magnetic field applied at a small angle with respect to the plane. Below a scattering between spin-up and spin-down carriers is considered to be the main reason for the experimental observation. Comparison of two band models with experiment provides an upper limit for the electron-electron scattering time $\tau_{ee}$ in the dilute 2D electron system as a function of electron density $n_s$. The time $\tau_{ee}$ increases gradually with $n_s$, becoming much greater than the transport scattering time $\tau_p$ for densities $n_s > 4 \times 10^{11} \text{ cm}^{-2}$. Strong electron-electron scattering is found for $1.22 \times 10^{11} < n_s < 3 \times 10^{11} \text{ cm}^{-2}$, the region which is near to the apparent metal insulator transition.

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I. INTRODUCTION

Strongly interacting two-dimensional systems of electrons and holes have drawn intensive recent attention due to their anomalous behavior as a function of temperature and magnetic field [1]: the resistance exhibits metallic temperature-dependence above a critical density, $n_c$, raising the possibility of a metallic phase in two dimensions [2]. An additional intriguing characteristic of these systems is their enormous response to magnetic fields applied in the 2D plane of the carriers [3,4]. The resistivity increases substantially with in-plane magnetic field and saturates to a new value above a density-dependent characteristic magnetic field $H_{sat}$. Several experiments [3,4] have shown that the field $H_{sat}$ corresponds to the onset of full spin polarization of the 2D electron system.

With increasing in-plane magnetic field the system thus evolves from zero net spin polarization, with equal numbers of spin-up and spin-down electrons, to a completely spin-polarized state above the field $H_{sat}$.

Recent measurements of the Hall resistance in parallel magnetic field [10] have revealed another unexpected physical property: the Hall coefficient does not vary with parallel magnetic field for fields ranging from 0 to well above $H_{sat}$. This is in apparent contradiction with expectations based on straightforward arguments [11] that predict different mobilities of the spin-up and spin-down electrons and, therefore, a substantial variation of the Hall coefficient with in-plane magnetic field [10]. The purpose of the present paper is to provide a possible explanation of the behavior of the Hall coefficient.

The electron-electron ($e-e$) scattering is considered as the main reason for the invariance of the Hall coefficient with in-plane magnetic field. A comparison with experiment demonstrates that the frequency of the electron-electron scattering events $\nu_{ee} = 1/\tau_{ee}$ increases with decreasing electron density $n_s$. At density $1.22 \times 10^{11} < n_s < 2 \times 10^{11} \text{ cm}^{-2}$, the $e-e$ scattering rate $1/\tau_{ee}$ is found to be higher than the transport scattering rate $1/\tau_p$. This indicates that the $e-e$ interaction can be a dominant reason of decay of the electron states in the dilute 2D system near the apparent metal-insulator transition in the silicon MOSFET’s at temperature above 0.1K.

II. EXPERIMENT

The data used in the paper were obtained on two silicon MOSFETs of comparable mobility $\mu \approx 20,000 \text{ V/(cm²/s)}$ at $T = 4.2$ K. The samples were mounted on a rotating platform at the end of a low temperature probe. Measurements were taken at temperature...
$T = 0.235$ K and magnetic fields $H$ up to 12 T in a $^3$He Oxford Heliox system at City College of NY. The Hall coefficient was measured at higher magnetic fields up to 18T at the National High Magnetic Field Laboratory in Tallahassee, Florida. The details of the experiment are presented in the paper [12]. The Hall coefficient $R_H$, corresponding to different electron densities from $n_s = 1.22 \times 10^{11} \text{ cm}^{-2}$ to $n_s = 4.42 \times 10^{11} \text{ cm}^{-2}$, is shown in Fig.1. $R_H$ is a weak function of the magnetic field, varying by less than 3-7% over a wide range of magnetic field for all electron densities. For several electron densities, the arrows mark the magnetic field $H_s$ above which the longitudinal conductivity saturates [13] and 2D electrons are spin polarized completely [14]. A considerable change in $R_H$ is observed for the lowest electron density $n_s = 1.22 \times 10^{11} \text{ cm}^{-2}$ at $H > 4T$, where the system is in the insulating regime [16]. This behavior requires further investigation.

The longitudinal conductivity $\sigma$ depends strongly on in-plane magnetic field $H$ [13]. This indicates that the average mobility of the electrons changes substantially with magnetic field. In the absence of interactions between carriers, the mobility of spin-up and spin-down electrons should vary independently of each other. Taking into account the very different, variable Fermi velocities of spin-up and spin-down electrons in an in-plane field, we expect that the mobilities of the two subbands are most likely different in a magnetic field. The Hall coefficient measurements indicate, however, that the mobility of spin-up and spin-down electrons in the field $H$ are close to each other. Electron-electron interaction are a promising candidate to explain this result. Indeed, it is known [18], that due to the strong $e-e$ interband scattering the nonequilibrium averaged velocities of two different conducting bands in an external electric field are, in fact, the same. Below I will employ this approach for the 2D electron system, which is spin polarized by an external magnetic field.

III. MODEL

Boltzmann transport theory provides a description of the classical magnetoconductivity and Hall resistance of carriers occupying several different subbands in $k$-space [17,18]. For example, the carriers in different subbands can have different electrical charge (electrons and holes) or different kinematic properties such as mass (light and heavy holes in GaAs [20]). The 2D electrons in a Si-MOSFET in a strong in-plane magnetic field occupy two subbands, one containing spin-up and the other spin-down electron states. Below I present a simple description of a two band model, based on analysis of the total averaged nonequilibrium momentum of each band in the mean free time approximation [15,18]. A more general approach can be found in reference [19].

In the absence of electron-electron scattering the spin-up and spin-down electrons can be considered independently. In this case we describe the relaxation of the total momentum $\vec{P}_{\uparrow \perp}$ of the nonequilibrium electron distribution in each spin subband by a scattering time $\tau_{\uparrow \perp}$. The electron-electron interaction (or scattering) induces a redistribution of the non-equilibrium momentum of carriers between two spin subbands and, therefore, changes the total momenta $\vec{P}_\uparrow$ and $\vec{P}_\downarrow$. To account for the intersubband redistribution of the nonequilibrium momentum an effective force $\vec{F}_{\text{int}}$ acting between spin-up and spin-down electron subbands is used [14]. The force $\vec{F}_{\text{int}}$ changes the total momentum of each spin sub-band. Newton's equations for total momentum $\vec{P}_\uparrow$, ($\vec{P}_\downarrow$) of the spin-up (spin-down) electrons in an external electric field $\vec{E}$ and a normal magnetic field $\vec{H}_\perp$ yield:

$$
\frac{d\vec{P}_\uparrow}{dt} = en_\uparrow \vec{v}_\uparrow \times \vec{H}_\perp - \frac{\vec{P}_\uparrow}{\tau_\uparrow} + \vec{F}_{\text{int}} \quad (1a)
$$

$$
\frac{d\vec{P}_\downarrow}{dt} = en_\downarrow \vec{v}_\downarrow \times \vec{H}_\perp - \frac{\vec{P}_\downarrow}{\tau_\downarrow} + \vec{F}_{\text{int}} >, \quad (1b)
$$

where $n_\uparrow$, $\vec{v}_\uparrow$, ($n_\downarrow$, $\vec{v}_\downarrow$) are the density and average velocity of spin-up (spin-down) electrons. The functional dependence of the force $\vec{F}_{\text{int}}$ was determined from the following observations [21]. We assumed that each interaction (or collision) between two particles does not depend on other particles. In this case the total effective force $\vec{F}_{\text{int}}$ should be proportional to the product of the densities of spin-up and spin-down electrons: $\vec{F}_{\text{int}} \sim n_\uparrow \times n_\downarrow$. In the absence of a relative drift of the spin-up electrons with respect to spin-down electrons ($\vec{v}_\uparrow = \vec{v}_\downarrow$) the force $\vec{F}_{\text{int}}$ is expected to be equal to 0. This result is, in fact, a consequence of invariance of the system under Galilean’s transformations, if we neglect variations of the electron distribution with drift velocity (in other words, if we consider the linear response of the system). In this approximation the force $\vec{F}_{\text{int}}$ is proportional to the difference between average velocities of spin-up and spin-down electrons: $\vec{F}_{\text{int}} \sim (\vec{v}_\uparrow - \vec{v}_\downarrow)$. Thus we use the following expression for the force $\vec{F}_{\text{int}}$:

$$
\langle \vec{F}_{\text{int}} \rangle = \alpha n_\uparrow n_\downarrow (\vec{v}_\uparrow - \vec{v}_\downarrow), \quad (2)
$$

where $\alpha$ is a constant. In stationary states the momenta $\vec{P}_\uparrow$, $\vec{P}_\downarrow$ of the spin subbands are constant. Therefore the left sides of Eq.(1) are equal to 0. Combining Eqs. (1) and (2) we obtain a linear relation between the electric field $\vec{E}$ and the momenta $\vec{P}_\uparrow$, $\vec{P}_\downarrow$ of the spin subbands:

$$
(i\omega_c + \epsilon_\uparrow)\vec{P}_\uparrow - \gamma_\uparrow \vec{P}_\downarrow = en_\uparrow \vec{E} \quad (3a)
$$
\[(i\omega_c + \epsilon)P_{\downarrow} - \gamma P_{\uparrow} = e n_1 E, \quad (36)\]

where \(P = P_x + i P_y\), \(E = E_x + i E_y\), \(\gamma_{\uparrow, \downarrow} = an_{\uparrow, \downarrow}/m\), \(\epsilon_{\uparrow, \downarrow} = 1/\tau_{\uparrow, \downarrow} + \gamma_{\uparrow, \downarrow}\), \(\omega_c = eH_c/me\) is the cyclotron frequency and \(m\) is the band mass of the electrons. The current density is given by

\[J = en_{\uparrow}v_{\uparrow} + en_{\downarrow}v_{\downarrow} = \frac{e}{m}(P_{\uparrow} + P_{\downarrow}) \quad (4)\]

The solution of the linear equations (3) gives the sub-band momenta \(P_{\uparrow}\) and \(P_{\downarrow}\) as a function of external electric and magnetic fields. The longitudinal and Hall conductivities of the system were obtained using the real and imaginary parts of Eq.(4), respectively:

\[
\sigma_{xx} = \frac{(\epsilon_{\uparrow} - \gamma_{\uparrow}) \left[n_{\uparrow}(\epsilon_{\uparrow} + \gamma_{\uparrow}) + n_{\downarrow}(\epsilon_{\downarrow} + \gamma_{\downarrow})\right] + (\epsilon_{\downarrow} - \gamma_{\downarrow}) \left[n_{\downarrow}(\epsilon_{\downarrow} + \gamma_{\downarrow}) + n_{\uparrow}(\epsilon_{\uparrow} + \gamma_{\uparrow})\right]}{(\epsilon_{\uparrow} - \gamma_{\uparrow}) \left[n_{\uparrow}(\epsilon_{\uparrow} + \gamma_{\uparrow}) + n_{\downarrow}(\epsilon_{\downarrow} + \gamma_{\downarrow})\right] + (\epsilon_{\uparrow} - \gamma_{\uparrow}) \left[n_{\downarrow}(\epsilon_{\downarrow} + \gamma_{\downarrow}) + n_{\uparrow}(\epsilon_{\uparrow} + \gamma_{\uparrow})\right] + \omega_c^2(\epsilon_{\uparrow} + \epsilon_{\downarrow})(n_{\uparrow} + n_{\downarrow})}.
\]

\[
\sigma_{xy} = \frac{\omega_c \left[n_{\uparrow}(\epsilon_{\downarrow} + \gamma_{\downarrow}) + n_{\downarrow}(\epsilon_{\uparrow} + \gamma_{\uparrow})\right](\epsilon_{\uparrow} + \epsilon_{\downarrow}) - (\epsilon_{\uparrow} - \gamma_{\uparrow}) \left[n_{\downarrow}(\epsilon_{\downarrow} + \gamma_{\downarrow}) + n_{\uparrow}(\epsilon_{\uparrow} + \gamma_{\uparrow})\right]}{(\epsilon_{\uparrow} - \gamma_{\uparrow}) \left[n_{\uparrow}(\epsilon_{\uparrow} + \gamma_{\uparrow}) + n_{\downarrow}(\epsilon_{\downarrow} + \gamma_{\downarrow})\right] + (\epsilon_{\uparrow} - \gamma_{\uparrow}) \left[n_{\downarrow}(\epsilon_{\downarrow} + \gamma_{\downarrow}) + n_{\uparrow}(\epsilon_{\uparrow} + \gamma_{\uparrow})\right] + \omega_c^2(\epsilon_{\uparrow} + \epsilon_{\downarrow})(n_{\uparrow} + n_{\downarrow})}.
\]

The Hall resistivity was obtained by inverting the conductivity tensor \(\sigma\).

In order to evaluate the strength of the electron-electron scattering quantitatively we consider a model for the in-plane field dependence of the longitudinal conductivity \(\sigma(H)\). One possible explanation of the field dependence of \(\sigma(H)\) was proposed recently by Dolgopolov and Gold (DG) [11]. They have argued that the screening of electrically charged impurities by the 2D electrons depends substantially on the populations of the spin-up and spin-down subbands and therefore varies with in-plane magnetic field. This causes the resistance to increase with field \(H\), saturating when the electrons reach full spin polarization at \(H > H_s\). In the DG approach [11] the average scattering probabilities, \(1/\tau_{\uparrow, \downarrow}\), are calculated corresponding to spin-up and spin-down electrons in a parallel magnetic field. The longitudinal conductivity is the sum of contributions of spin-up and spin-down bands: \(\sigma = \sigma^\uparrow + \sigma^\downarrow = e^2n_{\uparrow}\tau_{\uparrow}/m + e^2n_{\downarrow}\tau_{\downarrow}/m\), where \(n_{\uparrow, \downarrow} = n_0(1 \pm \xi)\) is the density of the spin-up (spin-down) electrons and \(\xi = H/H_s, (H < H_s)\) is the degree of spin polarization of the 2D system. We use their expression for the conductivity \(\sigma(H)\) [11] to find the scattering times \(\tau_{\uparrow}(H)\) as a function of parallel magnetic field \(H\). In accordance with the eq.(5,6) the variations of the Hall coefficient \(R_H(H)\) are result of different mobilities of spin-up and spin-down electrons in a magnetic field \(H\). Therefore we expect a similar dependence of the \(R_H(H)\) (and the \(e-e\) scattering rate) for any other models of magnetoconductivity corresponding to the experiment, in which the bare mobilities of spin-up and spin-down electrons are different considerably.

According to Eq.(5,6), the behavior of the Hall coefficient as function of \(H\) depends on two parameters. One is the strength of
FIG. 3. The Hall coefficient as a function of the in-plane magnetic field at high electron density. The solid lines are numerical fit corresponding to the two band model (see eq. (5,6)). The parameter $B = (\omega_c \tau_0)_{H=H_s}$ was obtained as a ratio of the $R_{xy}(H = H_s)/\rho(H = 0)$ for each curve. The spin polarization field $H_s$ was found from the saturation of the magnetoconductivity [13]. The arrows indicate the field $H_s$ for each electron density. The ratio of the transport scattering time $\tau_p$ to $\tau_{ee}$, found from the fitting, are labeled.

the $e-e$ interaction, which we characterize by a factor $A = \alpha n_0 \tau_0/m = \tau_0/\tau_{ee}$, the ratio of the inter subband $e-e$ scattering rate to the transport scattering rate at $H = 0$T. The second parameter is $B = (\omega_c \tau_0)_{H=H_s}$ calculated at field $H = H_s$, where the time $\tau_0$ is the scattering time at $H = 0$ T without $e-e$ interaction ($A = 0$). In Fig. (2a) we show the Hall coefficient calculated for different scattering times $\tau_0$ (B) without $e-e$ scattering ($A = 0$). The Hall coefficient depends substantially on the parameter $B$. The behavior of the Hall coefficient for different $B$ and $A = 0$ is similar to results obtained earlier, neglecting the intersubband $e-e$ scattering [22,23]. In the experiment, the parameter $B$ was varied between 0.1 and 3.4 depending on electron density and the angle $\phi$ between the magnetic field and the plane of the electrons. In accordance with Fig.2a for all angles and electron densities measured excepting the highest, we should expect a substantial change of the Hall coefficient in the absence of $e-e$ scattering ($A = 0$). In Fig.2b we present the Hall coefficient as a function of in-plane magnetic field at $B = 0.3$ and different rate of $e-e$ scattering. The Hall coefficient behavior depends substantially on the $e-e$ interband scattering. For $A = 1$ the variation of the Hall coefficient is less than 4%.

FIG. 4. (a) The ratio of the electron-electron scattering rate $1/\tau_{ee}$ to the rate of the transport scattering $1/\tau_p$ at different electron densities $n_s$. The shadow area indicates the possible values of the $e-e$ scattering rate in the dilute 2D electron system. (b) the transport scattering rate $1/\tau_p$ vs $n_s$.

IV. COMPARISON WITH EXPERIMENT

We now compare the results of the two band model presented above with experiment. The parameter $B = (\omega_c \tau_0)_{H=H_s}$ was found from experiment as the ratio of the Hall resistance $\rho_{xy} = H_{\perp}/(nec)(H = H_s)$ at magnetic field $H = H_s$ to the longitudinal resistivity $\rho_{xx} = m/(n e^2 \tau_0)$ at $H = 0$T. The strength of the $e-e$ scattering (coefficient $A$) was varied to obtain agreement with experiment. A comparison is shown on Fig.3 for
four different electron densities. The accuracy of the experiment is not sufficient to extract the value of the $e - e$ scattering time $\tau_{ee}$ in the system at electron densities below $n_s = 3 \times 10^{11} \text{cm}^{-2}$. The main difficulty is that the time $\tau_{ee}$ appears to be too short and comparable with the mean free time of the electrons $\tau_p$ at $n_s < 3 \times 10^{11} \text{cm}^{-2}$. We determined an upper limit for $\tau_{ee}$ or lower limit for the spin scattering rate $\nu_{ee}$ for electron densities $n_s < 3 \times 10^{11} \text{cm}^{-2}$. The ratio of the electron-electron scattering rate to the transport scattering rate extracted from the experiment is presented in Fig.4. The transport scattering rate $1/\tau_p$, obtained from the Drude conductivity (neglecting the $e - e$ mass enhancement), is shown for comparison. The transport scattering rate $1/\tau_p$ is comparable or even higher than the spin scattering rate $1/\tau_{ee}$ at temperature $T > 0.1K$.

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