Evaluation of Equivalent Single-degree-of-freedom Method for Nonlinear Dynamic Response Analysis of Embankments

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A static push-over analysis method was used to assess nonlinear behavior of embankments. Then, based on the results of the analysis, a simple method for carrying out nonlinear seismic response analysis of an embankment using an equivalent-single-degree-of-freedom model was proposed. Seismic responses of embankments calculated using the proposed method agreed well with those obtained using the two-dimensional finite element method. This confirmed the validity of the proposed simple method for evaluating seismic behaviors.

The proposed method enables the dynamic responses of embankments to be evaluated easily compared to the conventional two-dimensional finite element method. The method is thus considered an efficient and expedient procedure for evaluating the seismic responses of lengthy railway embankments.

Keywords: railway embankment, nonlinear dynamic response, static push-over analysis, equivalent single-degree-of-freedom method

1. Introduction

Newmark’s sliding block analysis [1, 2] is normally used for evaluating the seismic response (sliding displacement) of railway embankments [3]. As the procedure is based on the results of a slope stability analysis, the Newmark’s sliding block analysis offers continuity with the existing seismic design for Level 1 earthquakes. In addition, as it offers a number of benefits such as a simpler sequence of calculations and a certain degree of reproducibility of the past events of damage [e.g.4-5], the analytical method has been used for seismic design not only of railway embankments but also of road embankments as well.

On the other hand, the Newmark’s sliding block analysis oversimplifies the actual phenomena, and some related defects have been pointed out [e.g.6-8]. As a result, this analytical method for seismic design has a limited scope of application. For example, seismic design of high embankments must take into account the amplification of seismic motion [3]. In order to help overcome these issues and associated limitations of the analytical method, a number of proposals have been made to be adopted in practical seismic design. In some of the modifications, for example, the finite element method is used to determine seismic amplifications of the embankments which are in turn used to obtain seismic motions that are then used as input to reduce soil strength in the Newmark’s sliding block analysis [e.g.9]. In practical applications, however, these modifications are rather complicated to be applied to all embankments with varying configurations. Furthermore, applying these modifications only to high embankments creates discontinuity in design.

Various other methods have been proposed to simplify the evaluation of the seismic behavior of embankments [e.g.10-16] as well as calculation of sliding displacements [e.g.7]. These methods, while considering all embankments configurations, either calculate seismic amplification theoretically or evaluate seismic behaviors using a simple single-degree-of-freedom model, assuming that the dynamic response characteristics of the embankments are elastic. For these methods to properly evaluate the behavior of embankments subjected to large earthquakes such as Level 2 seismic motion, however, the effect of nonlinearity of embankments must be taken into account.

This paper proposes a simple method for evaluating the nonlinear dynamic response of railway embankments subjected to large earthquakes. Specifically, the proposed method converts a continuous railway embankment supporting rail tracks into a single-degree-of-freedom model to analyze the seismic response at the crest of the embankment. The proposed method extends the concept of static push-over analysis used for level ground, representing it with a single-degree-of-freedom model [17], to simulate the seismic behavior of an embankment. This paper also attempts to verify the validity of the proposed method by comparing the seismic response of embankments obtained through the method with the response similarly obtained by finite element method.

2. Procedures for developing an equivalent-single-degree-of-freedom model of an embankment

Figure 1 shows a flow chart for developing an equivalent-single-degree-of-freedom model of an embankment. Firstly, the cross section of an embankment is modeled using two-dimensional finite elements (2DFEM). These finite
elements are given engineering properties obtained from the results of the field and laboratory tests, which enhances the accuracy in evaluation of dynamic response of the embankments. In this paper, the model is referred to as a 2D FEM model and is considered to give a correct answer.

Secondly, a simplified model is constructed. As the horizontal component is considered dominant in the deformation of an embankment during an earthquake, its behavior in the vertical direction can be ignored. Accordingly, elements of the same height can be replaced with a mass and a spring. Each mass is given a value corresponding to the product of its area specific to the height and its unit weight. Each spring is given a shearing rigidity that is specific to the height and the corresponding width. The procedure has been used in the past studies [14, 15], enabling the natural period of an embankment to be evaluated accurately. In this paper, the simplified model is referred to as a shear-soil-column model.

In the next step, the shear-soil-column model is subjected to a static push-over analysis [17] to establish the relationship between the degrading rigidity and increasing hysteretic damping with displacements of the embankment. Then, a one-mass model that meets both relationships is constructed. In this paper, the model is defined as an equivalent single-degree-of-freedom model. A static push-over analysis can be conducted on a 2D FEM model instead of a shear-soil-column model. Meanwhile, a finite element analysis of the embankments subjected to an earthquake demonstrated that the deformation of embankments was dominant in the shearing direction, indicating that the shear-soil-column model could be used to simulate the behavior of the 2D FEM model. In addition, the calculation cost is lower with the shear-soil-column model, which is an advantage for the actual design process. Based on the above, a static push-over analysis was conducted in this study on the simpler shear-soil-column model.

Figure 2 shows the flow chart for a static push-over analysis of an embankment. The flow is roughly the same as for a static push-over analysis [17] carried out for a level ground above the engineering bedrock where earthquake motion is input for seismic response analyses. The flow chart for the embankments was modified for this study to account for the tapering shape of the embankment and the interaction between the embankment and the supporting ground. The resulting flow is detailed below.

Step 1: The embankment configurations (height, gradient, and crest width) and physical properties (unit weight, initial shearing rigidity, damping characteristics and nonlinearity) are assigned.

Step 2: The embankment in Step 1 is then divided horizontally into \( k \) layers to construct a shear-soil-column model, having \( k \) degrees of freedom. The weight of the

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**Fig. 1** Flow chart for developing an equivalent-single-degree-of-freedom model of an embankment

**Fig. 2** Flow chart for static push-over analysis of an embankment
layer at each height is the integrated weight of the layer at the corresponding height of the embankment in Step 1. Each spring is given an averaged shearing rigidity of the soil at the relevant height with its height and width taken into account [14, 15]. Other physical properties are the same as those in Step 1. At the first step of the calculation, each element is assumed to be elastic and the shearing rigidity of each layer is defined as \( G_i^{(1)} = G_f(i) \), where \( G_i^{(1)} \) is the shearing rigidity of the spring in the \( i \)th layer in the first step of the calculation and \( G_f(i) \) is the initial shearing rigidity of the soil in the \( i \)th layer.

Steps 3 and 4: An eigenvalue analysis is conducted on the shear-soil-column model with thickness, weight and shearing rigidity \( G_i^{(N)} \) given for each of its layers to obtain primary-mode natural angular frequency \( \omega_i^{(N)} \), eigenvector \( \{u_i^{(N)}\} \), participation function \( PF_i^{(N)} \) and effective mass ratio \( EMR_i^{(N)} \). The eigenvector \( \{u_i^{(N)}\} \) is normalized so that its value at the crest of the embankment is one. Here, the superscript "\( (N) \)" means the results at the \( N \)th step.

Step 5: A small displacement is statically applied to each layer such that the incremental displacement at the crest of the embankment is \( \delta \). The vertical distribution of the displacement is calculated according to the primary-mode configuration obtained in Steps 3 and 4 using:

\[
\{\delta_i^{(N)}\} = \{\delta_i^{(N-1)}\} + \Delta \delta \cdot \{u_i^{(N)}\}
\]

where \( \{\delta_i^{(N)}\} \) is the displacement of the layer at each height at the \( N \)th step.

Step 6: From the strain in each spring determined in Step 5, tangential stiffness \( G_i^{(N-1)} \) and damping \( h_i^{(N-1)} \) are evaluated. Generally, shearing rigidity "\( G \)" of the \( G-\gamma \) relationship obtained from the laboratory test for dynamic soil properties represents secant stiffness. In this study, however, the aim is to determine instantaneous natural frequencies at the advanced stages of deformation, tangential stiffness \( G \) is chosen.

Step 7: Steps 3 to 6 are repeated until the displacement at the crest of the embankment reaches a preset value.

The results of the above analysis are evaluated in a manner similar to that of static push-over analysis for a level supporting ground [17]. From the above, the following relationships are obtained:
- Rigidity degradation rate \( G/G_0 \) - displacement \( \delta \)
- Hysteretic damping \( h \) - displacement \( \delta \)
- Participation function \( PF \) - displacement \( \delta \)
- Effective mass ratio \( EMR \) - displacement \( \delta \)

Based on these results, an equivalent single-degree-of-freedom model of an embankment is constructed. First, initial rigidities of springs are determined using the natural period \( T_i \) of an embankment at the first step of calculation. Then, a nonlinear constitutive model with parameters that meet the \( G/G_0 - \delta \) and \( h - \delta \) relationships described above, is selected.

When the response obtained by the equivalent single-degree-of-freedom model is converted to that of the crest of the embankment, the participation function \( PF \) is used. In the proposed method, the value of the participation function \( PF \) at the maximum displacement \( \delta_{\text{max}} \) is multiplied with the relative accelerations, relative velocities and relative displacements.

### 3. Validity of response evaluation using the equivalent single-degree-of-freedom model of an embankment

**1) Analytical conditions**

In this section, the behavior of embankments during earthquakes obtained using an equivalent single-degree-of-freedom model of an embankment is examined for its validity. Seismic response analyses were conducted using a 2D FEM, a shear-soil-column, and equivalent single-degree-of-freedom models. The results were then compared to verify the proposed method. Figure 3 shows the configurations of the embankment used in the analyses. It was for a double-track railway embankment of height \( H = 9 \) m, slope \( \alpha = 1:1.5 \) and crest width \( D = 10 \) m. The embankment was built with sandy soil of \( V_s = 200 \) m/s and \( \gamma = 20 \) kN/m\(^3\). The supporting ground was sandy soil of \( V_s = 400 \) m/s and \( \gamma = 20 \) kN/m\(^3\). Each analytical model was constructed on the basis of the conditions.

The 2D FEM model was divided into the elements shown in Fig. 4. The supporting ground was modeled as one covering a large area (100 m in width and 21 m in depth) to appropriately account for the interaction between the supporting ground and embankment. The bottom and sides of the analytical model had viscous boundaries (equivalent to \( V_s = 400 \) m/s and \( \gamma = 20 \) kN/m\(^3\)). The sides of the model were joined with one-dimensional ground with large masses.

The GHE-S model [18] was used as the nonlinear constitutive model of the ground. Parameters for the GHE-S model should preferably be determined from the results of dynamic soil properties tests of the embankment material. However, dynamic soil property tests have rarely been performed on embankments. Therefore, in this study the
results of static push-over analyses conducted on various types of standardized ground were used as parameters [17]. Table 1 shows the parameters assigned to the GHE-S model for each element. The reference strain $\gamma_{0.5}$ was calculated using a recommended formula [19]. Multi-spring elements [20] were used for the 2D FEM model to handle the complexity of the strain direction.

The shear-soil-column and equivalent single-degree-of-freedom models were constructed in accordance with the flow chart proposed in the previous section. For both the eigenvalue and static push-over analyses on these models, the supporting ground was treated as fixed or rigid. In the static push-over analysis, the final displacement at the crest of the embankment was set equal to 50 cm, and it was divided into 5000 steps ($\Delta \delta = 0.01$ cm). Results of the eigenvalue analysis on each model are shown in Table 2. Model shapes obtained from the 2D FEM model are shown in Fig. 5. The 2D FEM and shear-soil-column models give nearly the same natural frequencies, participation function values and effective mass ratios in the primary mode. With the 2D FEM model, the embankment rotates in the third mode as indicated in Fig. 5 (b), which is difficult to reproduce using the shear-soil-column model. Since the values of the participation function and effective mass ratio in that mode were relatively small, the impact of rotational mode on the embankment’s behavior during earthquake motion was considered small. The natural frequency in the 5th mode of the 2D FEM model was close to that in the secondary mode of the shear-soil-column model. This shows that the key vibration modes of embankments can be adequately reproduced using the shear-soil-column model. Needless to say, the natural frequency in the primary mode of the shear-soil-column model is equal to that of the equivalent-single-degree-of-freedom model.

Table 1  GHE-S model parameters (for 2D FEM model)

| C(0) | C(4(0)) | C(4(∞)) | C(4(1)) | C(4(1)) | $a$ | $h_{max}$ |
|------|---------|---------|---------|---------|-----|----------|
| 1.00 | 1.00    | 0.15    | 2.5     | 0.87    | 1.15| 1.30     | 0.21   |

Table 2  Results of eigenvalue analyses

(a) 2D FEM model

| Mode | Frequency (Hz) | Participation function Horizontal | Participation function Vertical | Effective mass ratio Horizontal | Effective mass ratio Vertical |
|------|----------------|----------------------------------|--------------------------------|-------------------------------|-------------------------------|
| 1    | 6.77           | 1.43                             | 0.00                           | 0.75                          | 0.00                          |
| 2    | 10.92          | 0.00                             | 0.68                           | 0.00                          | 0.15                          |
| 3    | 12.78          | 0.13                             | 0.00                           | 0.01                          | 0.00                          |
| 4    | 15.13          | 0.00                             | 0.24                           | 0.00                          | 0.02                          |
| 5    | 16.01          | -0.61                            | 0.00                           | 0.10                          | 0.00                          |

(b) Shear-soil-column model

| Mode | Frequency (Hz) | Participation function Horizontal | Effective mass ratio Horizontal |
|------|----------------|----------------------------------|-------------------------------|
| 1    | 7.00           | 1.45                             | 0.73                          |
| 2    | 17.38          | -0.72                            | 0.13                          |

(c) Equivalent single-degree-of-freedom model

| Mode | Frequency (Hz) | Participation function Horizontal | Effective mass ratio Horizontal |
|------|----------------|----------------------------------|-------------------------------|
| 1    | 7.00           | -                                | 1.00                          |

Figure 6 shows the results of a static push-over analysis carried out on the shear-soil-column model. The figure shows that as the displacement increases, the rigidity of embankment decreases while its hysteretic damping increases. Furthermore, as displacement increases, the participation function $PF_{G}$ gradually decreases. The effective mass ratio $EMR$ tends to increase slightly with displacement.

Nonlinear parameters were given to the equivalent single-degree-of-freedom model so that it could reproduce as closely as possible the results of the static push-over analysis conducted on the shear-soil-column model (Fig. 6). The GHE-S model was used as the nonlinear constitutive model applying the parameters shown in Table 3. The reference displacement, $\delta_{0}$, was chosen as the displacement where $G/G_{0}$ was equal to 0.5 [17]. Figure 7 shows the $G/G_{0} = \delta/\delta_{0}$ and $h = \delta/\delta_{0}$ relationships calculated using these parameters. Figure 7 indicates that the equivalent single-degree-of-freedom model can adequately reproduce the static push-over analysis results from the shear-soil-column model.

For the 2D FEM model, L2 seismic motion spectra II [21], which is used for the seismic design of railway structures, was chosen as the input motion. The seismic wave simulates the shaking of an inland earthquake of $M_{w}$ 7.0, occurring in the upper crustal rock immediately below the engineering supporting where earthquake motion is input for numerical analyses. Table 4 shows the three cases analyzed in this study. In Case 1, the supporting ground was fixed or considered rigid, and the embankment was treated as linearly elastic body. In Case 2, the analysis considered the interaction between the supporting ground and embankment.

(2) Case 1: Elastic embankment with the supporting ground fixed or rigid

In Case 1, responses at the crest of the embankment were calculated ignoring nonlinearity of the embankment material and assuming a fixed supporting ground. A participation function $PF = 1.45$, which represents elasticity, was used for converting the responses of the shear-soil-column and equivalent single-degree-of-freedom models to those at the crest of the embankment. Acceleration time-history waveforms and response spectra at the crest of the embankment for each model are shown in Fig. 8.

Figure 8 shows that the responses are almost identical for all the models. This suggests that the seismic behavior of two-dimensional embankments is largely dominated by the primary-mode, and their behavior can be reproduced to a sufficient degree by a simple single-degree-of-freedom model that takes into account natural periods and participation functions.

(3) Case 2: Nonlinear behavior of embankment taken into account with elastic supporting ground

The analysis in Case 2 accounted for the interaction between the supporting ground and embankment. For both the shear-soil-column and equivalent single-degree-
of freedom models, the effect of the supporting ground was represented by a dashpot at the bottom of each model. The damping force was calculated while considering the initial effective mass ratio (0.73) of the primary mode in the total weight of the embankment. The participation functions $PF$ for the models considered the values at their maximum displacements throughout the analysis time. With these settings, dynamic analysis was conducted on each model. Response waveforms and spectra obtained at the crest of the embankment for the models are shown in Fig. 9. As shown in the figure, the response of embankment in both the shear-soil-column and equivalent single-degree-of-freedom models is largely the same as that in the 2D FEM model.

The above confirms that the simple single-degree-of-freedom model can adequately evaluate the seismic response of embankments even when they are nonlinear and there is interaction between the embankment and supporting ground.

4. Conclusion

In this study, a method was developed for analyzing nonlinear responses of embankments using an equivalent single-degree-of-freedom model. The following findings were obtained:
- A method for constructing an equivalent single-degree-of-freedom model of an embankment was proposed. Specifically, a static push-over analysis was conducted on the embankment to identify the trends in decreasing rigidity and increasing attenuation with displacement. Then, pa-

![Graphs and tables related to the text content]

| Case | Nonlinearity of embankment | Ground condition |
|------|-----------------------------|-----------------|
| 1    | Linear                      | Fixed           |
| 2    | Nonlinear                   | Elastic         |

![Graphs and tables related to the text content]
parameters that satisfied the results of the static push-over analysis were selected for constructing a single-degree-of-freedom model.

- Responses of a 9-meter height embankment were evaluated using 2D FEM and equivalent single-degree-of-freedom models. Responses of the equivalent single-degree-of-freedom model were roughly equal to those of the 2D FEM model. Sliding displacements calculated using the response waveforms of the embankment were roughly the same for both the 2D FEM model and the proposed method. In other words, seismic responses of embankments can be adequately evaluated by using an equivalent single-degree-of-freedom model.

By using the single-degree-of-freedom model, nonlinear responses of embankments can be readily analyzed. The method therefore makes it possible to more easily evaluate the responses of long railway structures. The results of dynamic soil properties tests of embankment material, if available, can further improve the accuracy of the response evaluations. By applying the sliding displacement of embankments method [e.g., 3, 8], with seismic response inputs obtained as above, it is possible to evaluate displacement while taking into consideration the dynamic responses of embankments, which has been ignored heretofore.

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