A macroscopical entangled coherent state generator in a V configuration atom system

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Abstract

In this paper, we propose a scheme to produce a pure and macroscopically entangled coherent state. When a three-level ‘V’ configuration atom interacts with a doubly resonant cavity, under a strong classical driven condition, an entangled coherent state can be generated from vacuum fields. An analytical solution for this system in the presence of cavity losses is also given.

1. Introduction

The generation of Schrödinger cat states and entangled coherent states serves as the first step to use coherent states in quantum information processes such as quantum sweeping and quantum teleportation [1]. Numerous schemes have been proposed to generate such an entangled coherent state [2–4, 6–10]. A scheme based on a double electromagnetically induced transparency system has been proposed in [2]. As for ion trap systems, the vibrational motion of ions [3] and the entanglement swapping [4] seem promising to generate an entangled coherent state. Using Kerr nonlinearity and a 50/50 beam splitter, multidimensional entangled coherent states can be generated on the condition that the initial state is a coherent state and the interaction times are within certain range [5].

Cavity quantum electrodynamics (QED) has been shown to be a convenient environment to generate both Schrödinger cat states [6, 7] and entangled coherent states [8]. Recently, Solano et al proposed a scheme to entangle two cavity modes through the interaction of the cavity modes with a system of N two-level atoms inside the cavity [9]. In their scheme, the two cavity modes interact with the same atomic transition and will put some restrictions on these two cavity modes.

On the other hand, atomic coherence, which results from the coherent superposition of different quantum states, can lead to many novel phenomena. These include a correlated spontaneous emission laser (CEL) [10], lasing without inversion [11] and electromagnetically induced transparency [12], etc. It is known that a two cavity modes can be entangled when they interact with three-level atoms [10, 13] and atomic coherence plays an essential role in this entanglement generation [14, 15]. It was shown in [14] that a two-mode macroscopically entangled continuous-variable state could be created in a CEL system where the atomic coherence was induced by a classical driving field. The interaction between a ‘V’-type three-level atom and two thermal modes of a doubly resonant cavity was studied in [15] and it was shown that given a small amount of atomic coherence, the entanglement could be generated between these two thermal modes even when they initially had arbitrarily high temperatures. However, the question whether an entangled coherent state of two cavity modes can be generated through the interaction with a single three-level atom has still not been answered. Most recently, our group has investigated that under large detuning conditions, a ‘λ’ three-level atom system interacting with a two-mode field can entangle the two-mode field [16]. The main drawback of the schemes [5, 16] is that the initial coherent state is required, and the interaction time should be controlled accurately otherwise we cannot obtain an entangled coherent state.

In this paper, we propose a scheme to generate a macroscopically entangled coherent state through the resonant interaction between a two-mode field and a three-level ‘V’ (or ‘λ’) configuration atom. Comparing this scheme with [5, 16], we do not need to prepare the initial coherent state of the field while the amplitude of the entangled coherent state can be amplified, which means that the system can work as an entanglement generator. Also, we do not need to control the...
It is convenient to solve this system in a dressed state picture corresponding classical driving fields. The following form under the rotating-wave approximation:

\[
|a\rangle = \frac{1}{\sqrt{2}} (|B\rangle - |C\rangle), |b\rangle = \frac{1}{\sqrt{2}} [-\sqrt{2}\Omega_1|A\rangle + \Omega_2(|B\rangle + |C\rangle)],
\]

where

\[
A = \frac{1}{u} (-\Omega_1|b\rangle + \Omega_2|c\rangle), \quad \lambda_1 = 0,
\]

\[
B = \frac{1}{\sqrt{2}u} [u|a\rangle + \Omega_2|b\rangle + \Omega_1|c\rangle], \quad \lambda_2 = u,
\]

\[
C = \frac{1}{\sqrt{2}u} [-u|a\rangle + \Omega_2|b\rangle + \Omega_1|c\rangle], \quad \lambda_3 = -u,
\]

where

\[
u = \sqrt{\Omega_1^2 + \Omega_2^2}.
\]

It is easy to prove that the states \(|A\rangle, |B\rangle\) and \(|C\rangle\) compose a new orthogonal and complete basis of the three-level system.

Under this basis, the atomic states can be written as

\[
|a\rangle = \frac{1}{\sqrt{2}} (|B\rangle - |C\rangle), |b\rangle = \frac{1}{\sqrt{2}} [-\sqrt{2}\Omega_1|A\rangle + \Omega_2(|B\rangle + |C\rangle)].
\]

We can then rewrite Hamiltonian, equation (1), under this new basis set and perform the following unitary transformation

\[
H_1 = e^{i\theta_1} H_1 e^{-i\theta_1}.
\]

where

\[
H_{\text{eff}} = \frac{1}{2u} [\Omega_1 g_1 (a_1 + a_1^*) |B\rangle - |C\rangle (|C\rangle - |B\rangle) e^{-2u\imath}],
\]

\[
+ |C\rangle (|B\rangle e^{2u\imath} + \sqrt{2}\Omega_2 (e^{2u\imath}|A\rangle - |A\rangle) e^{-2u\imath} + |C\rangle + g_2 \Omega_2|B\rangle (|B\rangle - |C\rangle (|C\rangle - |B\rangle) e^{-2u\imath} + |C\rangle + g_2 \Omega_2|B\rangle (|B\rangle - |C\rangle (|C\rangle - |B\rangle) e^{-2u\imath} + |C\rangle + g_2 \Omega_2|B\rangle (|B\rangle - |C\rangle (|C\rangle - |B\rangle) e^{-2u\imath}).
\]

In a strong driving regime, that is \(\Omega_2 \gg g_1, g_2\), we can realize a secular approximation (i.e. rotating-wave approximation) and eliminate the high frequency terms in equation (6) [9]. The effective Hamiltonian under this approximation is

\[
H_{\text{eff}} = \frac{1}{2u} [\Omega_1 g_1 (a_1 + a_1^*) |B\rangle - |C\rangle (|C\rangle - |B\rangle) e^{-2u\imath}].
\]

(7)

If our initial state of the atom–field combined system is \(|\Psi(0)\rangle = |a\rangle, 0, 0\rangle\), by using Hamiltonian, equation (7), we can have the state of the system as

\[
|\Psi(t)\rangle = \frac{1}{\sqrt{2}} (|B\rangle, \alpha, \beta - |C, -\alpha, -\beta\rangle).
\]

(8)

where \(\alpha = -\frac{g_1}{2} e^{-ut}\) and \(\beta = -\frac{g_2}{2} e^{-ut}\). We now apply the inverse unitary transformation on state equation (8) and change the basis set back to the original atomic states, and we have

\[
|\Psi(t)\rangle = \frac{1}{\sqrt{2}} (|\sqrt{2}|\langle e^{-iut}|\alpha, \beta\rangle + e^{iut}|\alpha, -\beta\rangle + |\langle e^{-iut}|\alpha, -\beta\rangle\rangle + |\langle e^{iut}|\alpha, \beta\rangle\rangle - e^{iut}|\alpha, -\beta\rangle\rangle).
\]

(9)

When the atom comes out from the two-mode cavity, we can use level-selective ionizing counters to detect the atomic state. If the internal state of the atom is detected to be \(|b\rangle, |c\rangle\) or \(|a\rangle\), the two-mode field will be projected into

\[
|\Psi(t)\rangle_{f_\pm} = \frac{1}{\sqrt{M_\pm}} [e^{-iut}|\alpha, \beta\rangle \pm e^{iut}|\alpha, -\beta\rangle],
\]

(10)

where

\[
M_\pm = 2 [1 \pm \cos 2ut \exp(-2|\alpha|^2 - 2|\beta|^2)].
\]

(11)

The state, equation (10), is a normalized one. The average photon number of the two modes of the cavity can be easily obtained as

\[
\langle N_1 \rangle = \frac{2|\alpha|^2}{M} \left(1 + e^{-2|\alpha|^2 - 2|\beta|^2} \cos 2ut\right),
\]

(12)

\[
\langle N_2 \rangle = \frac{2|\beta|^2}{M} \left(1 + e^{-2|\alpha|^2 - 2|\beta|^2} \cos 2ut\right).
\]

(13)

We now try to estimate the entanglement of state, equation (10). We note that for a general normalized and
number increase. The entanglement reaches its maximum
chosen for equations (12) and (13). As we have analysed
maximum state only when \( \beta \rightarrow \infty \). Therefore, the
parameters are \( g_1 = g_2 = 1, \Omega_1 = 100, \Omega_2 = 200. \)

nonorthogonal entangled coherent state,
\[ |\Psi\rangle = \mu |\alpha\rangle \tilde{\beta} + v |\gamma\rangle \delta, \tag{14} \]
we can define \( |0\rangle = |\alpha\rangle, |1\rangle = (|\gamma\rangle - |\tilde{\beta}\rangle)/\sqrt{1 - |\tilde{\beta}|^2} \) with
\( p_1 = (|\alpha\rangle |\tilde{\beta}\rangle) \) for the first subsystem and define \( |0\rangle = |\beta\rangle, |1\rangle =
\( (|\gamma\rangle - |\tilde{\beta}\rangle)/\sqrt{1 - |\tilde{\beta}|^2} \) with \( p_2 = (|\beta\rangle |\tilde{\beta}\rangle) \) for the second subsystem. The entanglement of the state, equation (14),
can be measured on the orthogonal basis \( |0,0\rangle, |0,1\rangle, |1,0\rangle \)
and \( |1,1\rangle \) [4, 17]. We recall that the concurrence of a
state can be used to estimate the entanglement for such a
state. The concurrence [18] of a state is defined as
\[ C = \max(0, 2 \lambda_{\max} - \sum_{i=1}^{4} \lambda_i), \]
where \( \lambda_i \) is the square roots of the eigenvalues of the matrix
\[ R = \rho (\sigma_1 \otimes \sigma_2) \rho^{\dagger} (\sigma_1 \otimes \sigma_2) \]
and \( \sigma_i \) are Pauli matrices. The concurrence of state, equation
(14), is [4, 17]
\[ C = 2 |\mu| v \sqrt{1 - |p_1|^2} (1 - |p_2|^2). \tag{15} \]
Whenever \( C > 0 \), the state, equation (14), is an entangled
state. The concurrence of the state generated from our system
(equation (10)) is
\[ C = \sqrt{[1 - \exp(-4|\alpha|^2)][1 - \exp(-4|\beta|^2)]}, \tag{16} \]
Figure 2 shows the time evolution of the concurrence
(equation (16)) and the average total photon number (equations
(12) and (13)) of our state. Here, the positive sign has been
chosen for equation (16) and the negative sign has been chosen
for equations (12) and (13). As we have analysed in
[19], due to the phase \( e^{i\omega t} \), the states \( |\Psi(t)\rangle_1 \) and
\( |\Psi(t)\rangle_2 \) almost have no difference. One can see it from
the expression of equation (15). The high frequency term
related with \( \cos 2\omega t \) actually can be ignored. The
property is different from the two state \( 1/\sqrt{2}(|\alpha, \alpha\rangle \pm |\alpha, -\alpha\rangle) \) is exact one ebit and its
entanglement is always 1 while \( 1/\sqrt{2}(|\alpha, \alpha\rangle + |\alpha, -\alpha\rangle) \) is maximum state only when \( \alpha \to \infty \).
From figure 2, we see that during short time evolution the concurrence curve shows high frequency oscillation which comes from a classical field. As time elapses, the entanglement and average photon number increase. The entanglement reaches its maximum
value 1 after a specific time and in the mean time we can obtain a large number of photons in the cavity. This can be clearly understood from equation (16). If \( \alpha \) and \( \beta \) are
large enough, \( \alpha \) and \(-\alpha, \beta \) and \(-\beta \) will be orthogonal, i.e.,
\[ |\alpha - \alpha\rangle = \exp(-2|\alpha|^2) = 0, |\beta - \beta\rangle = \exp(-2|\beta|^2) = 0, \]
so that the state equals \( 1/\sqrt{2}(|0,0\rangle + |1,1\rangle) \). Therefore, the
concurrence \( C = 1. \) One also sees that with the increase in \( \alpha \)
and \( \beta \) with respect to time, the average photon number is thus increased.

3. The two-mode field in the leak cavities
In order to obtain an analytic solution of a density matrix, here we do not consider the atomic level decay. We now consider the
effects of cavity losses upon the entanglement generation of the two-mode field. The master equation is
\[ \dot{\rho} = -i[H_{\text{int}}, \rho] + \kappa (2a_1a_1^{\dagger} \rho - \rho a_1^{\dagger} a_1 - a_1^{\dagger} a_2 \rho - \rho a_1^{\dagger} a_2). \tag{17} \]
For simplicity, we assume that the two modes have the same cavity decay rates \( \kappa \). We still assume that the atom is initially injected into state \( |\alpha\rangle \). Thus, we only need to work on the
subspaces \( |B\rangle \) and \( |C\rangle \). Using the superoperator technique
[21] and Hausdorff similarity transformation [22], we deduce that
\[ \rho = |B, \alpha', \beta', \rangle \langle B, \alpha', \beta', | + |C, -\alpha', -\beta'\rangle \langle C, -\alpha', -\beta'| - q(\exp(-2|\alpha|^2)|B, \alpha', \beta', \rangle \langle B, \alpha', \beta' | + \exp(2|\alpha|^2)|C, -\alpha', -\beta'| - \beta'| \langle B, \alpha', \beta' |) \tag{18} \]
where
\[ \alpha' = -i\Omega_1 g_1 (1 - e^{-\kappa t}), \quad \beta' = -i\Omega_2 g_2 (1 - e^{-\kappa t}), \quad q = \exp \left[ \frac{\Omega_1^2 g_1^2 + \Omega_2^2 g_2^2}{2\kappa^2} (2\kappa t + 4 e^{-\kappa t} - e^{-2\kappa t} - 3) \right] \tag{19} \]
with \( P_1 = \exp(-2|\alpha|^2), P_2 = \exp(-2|\beta|^2). \) In the process of
calculation, equation (19), we need to successively use the
operator disentangled equation \( \exp(A + B) = \exp \left[ \frac{\exp(A) - 1}{\eta} \right] \) if \( [A, B] = \eta A \). After
the atom comes out from the cavity, we measure the atomic state
again. Suppose the internal state of the atom is detected to be
in \( |\alpha\rangle \), the two-mode field will be projected into
\[ \rho = \frac{1}{S} \left[ |\alpha', \beta'\rangle \langle \alpha', \beta' | + q \exp(2|\alpha|^2)|\alpha', \beta'\rangle \langle -\alpha', -\beta'| + q \exp(-2|\alpha|^2)|\alpha', \beta'\rangle \langle -\alpha', -\beta'| + |\alpha', \beta'\rangle \langle \alpha', \beta' | + |\alpha', -\beta'\rangle \langle -\alpha', -\beta'| + |\alpha', -\beta'\rangle \langle -\alpha', -\beta'| \right], \tag{20} \]
with
\[ S = 2 + 2P_1 P_2 \cos 2\omega t. \]
Now the field state is in a mixed entangled state. The difference
between the states, equations (10) and (20), mainly lies in the
factor of \( q \) except for the change in the coherent amplitude \( \alpha' \).
Here \( q \) is the key factor to destroy the entanglement. With the
time evolution, \( q \) will achieve its asymptotic value zero so that
the entanglement will be destroyed completely. If there are no
cavity losses, $q$ will be 1, therefore the state, equation (20), will be exactly the same as equation (10).

To measure the entanglement, we will still use the concurrence. Let $|0\rangle = (|\alpha\rangle', |1\rangle) = (\langle -\alpha| - P_1|\alpha\rangle')/M_1$ with $P_1 = (|\alpha\rangle'| - \alpha\rangle') = \exp(-2|\alpha|'^2)$, $M_1 = \sqrt{1 - |P_1|^2}$ for field 1, and $|0\rangle = (|\beta\rangle', |1\rangle) = (\langle -\beta| - P_2|\beta\rangle')/M_2$ with $P_2 = (|\beta\rangle'| - \beta\rangle') = \exp(-2|\beta|'^2)$, $M_2 = \sqrt{1 - |P_2|^2}$ for field 2. The density matrix of the fields will be

$$
\rho = \frac{1}{S}
\begin{pmatrix}
1 + P_1^2 + 2qP_1P_2 \cos 2ut & P_1M_2(P_1P_2 + q e^{-2uti})

P_1M_2(P_1P_2 + q e^{2uti}) & P_2M_1(P_2M_2 + q e^{-2uti})

M_1M_2(P_1P_2 + q e^{-2uti}) & M_2M_1(P_2M_2 + q e^{2uti})

M_1M_2P_2 & M_2M_1P_2

M_1M_2P_2 & M_2M_1P_2

M_1M_2P_2 & M_2M_1P_2
\end{pmatrix}.
$$

(21)

Although calculation of the density matrix $\rho$ is very tedious, the eigenvalues of $R$ and concurrence still can be evaluated. The concurrence is

$$
C = \frac{2M_1M_2}{S} - q.
$$

(22)

One can easily check that without the loss of the cavity, the factor $q$ will be 1, and the expression (22) will be the same as equation (16) for the positive sign. The average photon number for the two modes can be obtained as

$$
N_1 = \frac{2|\alpha|^2}{S}(1 - qP_1P_2 \cos 2ut),
$$

$$
N_2 = \frac{2|\beta|^2}{S}(1 - qP_1P_2 \cos 2ut).
$$

(23)

Figure 3 shows the time evolution of the concurrence and the total average photon number $N = N_1 + N_2$ of the two-mode cavity in the presence of cavity losses. Not surprisingly, the average photon number of the two-mode field will decrease with the increase in $\kappa$. The concurrence increases first and then drops down to zero. The reason for concurrence dropping is the presence of the factor $q$. As time passes, $q$ will be smaller and smaller so that the entanglement is destroyed. Therefore, in our scheme, the high-$Q$ doubly resonant cavity is preferred.

4. Discussion and conclusion

We now briefly address the experimental feasibility of the proposed scheme. The required atomic level configuration can be achieved in the Rydberg atom’s circular atomic level [23]. The radiative lifetime is about $3 \times 10^{-5}$ s, which is much longer than those for noncircular Rydberg states [24]. Even in free space, the atoms would propagate a few metres at thermal velocity before decaying. So, the atomic radiative decay is thus negligible along the 20 cm path inside the apparatus. (In our model, we do not consider atomic spontaneous decay, therefore a circular level Rydberg atom is a good choice.) For a circular Rydberg atom, the coupling constant is $g = 2\pi \times 24$ kHz, if the interacting time $t \approx 6.6 \times 10^{-4}$ s (much smaller than the radiative lifetime $3 \times 10^{-5}$ s), $g \tau \sim 100$ correspond to the maximum value in figure 2(b), the coherent state $\alpha = \frac{|\Omega_1|}{\sqrt{g}} \sim 35i$ (for $\Omega_1 = \Omega_2$). Considering the loss of the cavity, $\alpha' = \frac{|\Omega_1|}{\sqrt{g}}(1 - e^{-\kappa t}) \sim -15.6i$ (for $\Omega_1 = \Omega_2, \kappa = 0.05$). Therefore, based on a cavity QED technique presently, the proposed scheme might be realizable. As to the atomic projection detection, if the upper two atomic levels $|c\rangle$ and $|b\rangle$ are not degenerate, when the atom comes out from the cavity, the three field ionization detectors $D_a, D_b$ and $D_c$ can be used for the three levels, respectively. If the atomic lifetime is not long enough, one must include the decay of the excited atomic level. The analytic solution cannot be obtained. One can numerically solve it. Definitely, the decay of the atom will be bad for the coherence of the system. In order to obtain a strong entanglement, the high-$Q$ doubly resonant cavity and long lifetime atom should be a first choice.

In conclusion, by employing a three-level 'W'-configuration atom interacting with a two-mode cavity field, under a strong driven condition, we can produce an entangled coherent state from vacuum state, which means that the system can work as an entanglement generator. The average photon number of the two-mode cavity can be large. The scheme has its advantages. We do not need to control the interacting time accurately. It just affects the amplitude of the entanglement and has no effect on the entanglement. The two cavity modes interact with different atomic transitions so that it is easy to drive two classical fields separately. The produced entangled coherent state can be easily differentiated just by differentiable polarization.

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