CP-Violation with Bosons

G. Cvetić, M. Nowakowski
Inst. für Physik, Universität Dortmund, 4600 Dortmund 50, Germany

A. Pilaftsis
Inst. für Physik, Johannes-Gutenberg Universität, 6500 Mainz, Germany

ABSTRACT

We formulate the conditions under which a purely bosonic theory (without fermions) containing neutral spin-0 particles and vector (gauge) bosons violates the CP-symmetry through the presence of CP-even and CP-odd operators in the Lagrangian. This is done without reference to explicitly CP-violating scalar sector of extended standard models (SM) with two or more Higgs doublets. The Lagrangian expressed in the mass basis of the spin-0 fields can, however, in certain circumstances be identified with a part of the CP-violating SM with two Higgs doublets. It is instructive to consider the manifestation of CP-violation in the mass basis since this leads directly to suggestions of genuine CP-signals.
CP-violation has always been a subject of numerous investigations in particle physics [1]. One of the reasons is that CP-violation is an important ingredient in explaining the baryon asymmetry of the universe [2]. It is known that the strength of the CP-violation coming from the Kobayashi-Maskawa sector of the standard model (SM) is not sufficient to explain this asymmetry [3]. It is therefore important to consider also other sources of CP-violation. Recently, the CP-violating SM with two Higgs doublets has attracted a lot of attention [4] since it could give a natural explanation of the baryogenesis [3, 5].

Usually, such a theory is formulated in the isoweak basis, i.e. with $2SU(2)_L$-doublets. However, since the physical interaction terms are given in the mass basis of the particles, it is very instructive (and closer to the experimental situation) to view the manifestation of CP-violation from this end (mass basis). Later on, we can choose to identify the bosonic part of the Lagrangian with that of the extended SM. Our considerations of CP-violation are, in principle, independent of any specific choice of electroweak theory. From this more pragmatic point of view, CP-violation in the mass basis of the theory is manifested through the simultaneous presence of CP-even and CP-odd terms in the Lagrangian. More precisely, we cannot assign CP-eigenvalues to the mass eigenstates in such a way that the Lagrangian is CP-invariant. However, the coupling parameters are real. In our considerations, we will ignore fermions.

First, let us consider two well-known examples of Lagrangians in which at least two terms with different CP-transformation properties simultaneously appear to give a CP-violating theory.

One such example is the following interaction of a neutral spin-0 boson $\varphi^o$ with fermions ($f \bar{f}$)

$$\mathcal{L}_{\varphi^o f \bar{f}} = \varphi^o \{ \alpha \bar{\Psi} \Psi + i \beta \bar{\Psi} \gamma_5 \Psi \} \quad (\alpha, \beta \in \mathbb{R}).$$

(1)

It is known that the Lagrangian (1) is part of a spontaneously broken theory [4] where the vacuum is not an eigenstate of CP. The Lagrangian of this theory, as given in terms of the non-shifted field $\phi^o = v + \varphi^o$ ($v$ is the vacuum expectation value), is CP-invariant if we assign $CP(\phi^o) = -1$. It is then clear that $\varphi^o$ is not a CP-eigenstate. However, the fact that $\mathcal{L}_{\varphi^o f \bar{f}}$ is CP-nonconserving is independent of these involved considerations, i.e. independent of the assumption of the spontaneous symmetry breaking. Namely,
we can simply look at (1) as a given interaction Lagrangian of physical fields. It is then legitimate to perform a CP-transformation of the (neutral) physical field \( \phi^o \) and to try to assign \( \pm 1 \) CP quantum numbers to it. Since the Lagrangian (1) contains a linear combination of CP-even \( (CP(\bar{\Psi}\Psi) = 1) \) and CP-odd \( (CP(i\bar{\Psi}\gamma_5\Psi) = -1) \) fermionic operators, it is impossible to assign any CP-eigenvalue to \( \phi^o \) such that \( \mathcal{L}_{\nu f\bar{f}} \) is CP-invariant.

The other example is a purely bosonic Lagrangian which involves only spin-1 physical fields. Let \( V_\mu \) and \( W_\mu \) be neutral and charged vector fields, respectively. If we restrict ourselves to dimension-4 operators, we can construct the following interaction terms:

\[
\mathcal{L}^{(1)}_{\text{spin-1}} = i\kappa^{(1)} [W^-\mu W^+\nu - W^+\mu W^-\nu] + i\kappa^{(2)} W^-\mu W^+\nu V^{\mu\nu} + i\kappa^{(3)} W^+\mu W^-\nu V^{\mu\nu},
\]

(2)

\[
\mathcal{L}^{(2)}_{\text{spin-1}} = \kappa^{(3)} W^+\mu W^-\nu [\partial^\mu V^{\nu} + \partial^\nu V^{\mu}],
\]

(3)

where

\[
W^\mu_\nu = \partial^\mu W^\nu - \partial^\nu W^\mu,
\]

\[
V^\mu_\nu = \partial^\mu V^\nu - \partial^\nu V^\mu.
\]

(4)

If we set \( \kappa^{(1)} = \kappa^{(2)} = g \cos \theta_W \), then \( \mathcal{L}^{(1)}_{\text{spin-1}} \) describes the gauge interaction of \( Z \) with charged W-bosons in SM. The requirement of CP-invariance would now dictate \( J^{PC}(V) = 1^{--} \) for \( \mathcal{L}^{(1)}_{\text{spin-1}} \) and \( J^{PC}(V) = 1^{++} \) for \( \mathcal{L}^{(2)}_{\text{spin-1}} \). Therefore, as before, the linear combination \( \mathcal{L} = \mathcal{L}^{(1)}_{\text{spin-1}} + \mathcal{L}^{(2)}_{\text{spin-1}} \) violates CP. Note that, if we insist on having \( \mathcal{L}^{(3)}_{\text{spin-1}} \) as a part of an \( SU(2) \otimes U(1) \) gauge theory, then \( \kappa^{(3)} = 0 \) and the neutral vector boson is (in the absence of other, possibly CP-violating interactions) a 1^{--} boson.

We now raise the question: under what circumstances would the spin-0 neutral sector lead to CP-violation? The latter seems impossible for Lagrangians which contain only spin-0 neutral fields. Therefore, the situation here is more involved than in the case of the Lagrangians (1), or (2) and (3). Additionally, we have to include at least spin-1 bosons of SM in the Lagrangian to get such effects.

\[\text{We mention here that the quantum numbers } 1^{--} \text{ are exotic in a sense that such a particle cannot couple to fermion-antifermion pair if parity (P) and C are conserved.}\]
Let us start with the Lagrangian \((V = Z)\)

\[
\mathcal{L}_{H^o_i} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} + g_{ijk} H^o_i H^o_j H^o_k + \\
+ g_{ij} \left( H^o_i \overset{\leftrightarrow}{\partial}_\mu H^o_j \right) Z^\mu + \\
+ g_i^Z H^o_i Z_\mu Z^\mu + g_i^W H^o_i W^\mu W^{-\mu},
\]

(5)

where \(H^o_i\) are \(N\) spin-0 neutral fields which correspond to mass eigenstates, the indices \(i, j, k\) run over \(1, \ldots, N\) such that \(i \leq j \leq k\), and \(\overset{\leftrightarrow}{\partial}\) is the antisymmetrized derivative:

\[
H^o_i \overset{\leftrightarrow}{\partial}_\mu H^o_j = H^o_i \vec{\partial}_\mu H^o_j - H^o_j \vec{\partial}_\mu H^o_i.
\]

Note that in SM with one or two Higgs doublets, the terms containing physical neutral spin-0 particles (to up to 3rd power) represent just special cases of the Lagrangian (5). If we assume first that all coupling parameters \(g_{ijk}\) and at least one parameter \(g_{ij}\) in (5) are nonzero, then \(\mathcal{L}_{H^o_i}\) is invariant under CP only if \(J_{PC}(H^o_i) = 0^{++}\) and \(J_{PC}(Z) = 1^{-+}\). More precisely, \(\mathcal{L}_{H^o_i}\) is invariant under the transformations

\[
PH^o_i(t, \vec{x}) P^{-1} = \eta^P(H^o_i) H^o_i(t, -\vec{x}), \quad \eta^P(H^o_i) = 1,
\]

\[
CH^o_i(x) C^{-1} = \eta^C(H^o_i) H^o_i(x), \quad \eta^C(H^o_i) = 1,
\]

\[
PZ^\mu(t, \vec{x}) P^{-1} = \eta^P(Z) Z_\mu(t, -\vec{x}), \quad \eta^P(Z) = 1,
\]

\[
CZ^\mu(x) C^{-1} = \eta^C(Z) Z_\mu(x), \quad \eta^C(Z) = 1.
\]

(6)

The Lagrangian

\[
\mathcal{L}^{(2)} = \mathcal{L}_{H^o_i} + \mathcal{L}_{\text{spin}^{-1}}^{(2)}
\]

is then also CP-invariant, but the Lagrangian

\[
\mathcal{L}^{(1)} = \mathcal{L}_{H^o_i} + \mathcal{L}_{\text{spin}^{-1}}^{(1)}
\]

(8)

violates CP (it is P-invariant, but violates C). The reason is that we cannot assign any CP quantum numbers to the fields to make \(\mathcal{L}^{(1)}\) CP-invariant. At least two neutral spin-0 particles must be present in order to violate CP in \(\mathcal{L}^{(1)}\) in such a case (note that SM with

\[\text{Note that the assigned eigenvalues of P yield P-invariant } \mathcal{L}_{H^o_i} \text{ and } \mathcal{L}_{\text{spin}^{-1}}^{(j)} (j=1,2), \text{ which implies that the question of CP-violation reduces in our cases to the question of C-violation.}\]
two Higgs doublets contains three neutral Higgses. We can then indeed interpret the $H_i^o$ fields as linear combinations of CP-even and CP-odd (non-physical) fields. Note that it is irrelevant whether this mixing takes place on a more fundamental level, e. g. in a two Higgs SU(2)-doublet model, or if (8) is an effective Lagrangian of composite fields $H_i^o$.

The Lagrangian (8) is not the only CP-violating combination. Typical charged scalar field interaction of the form

$$L_{ZH+H^-} = ig_{ZH+H^-}(H^+ \frac{i\sigma_3}{2} \partial_\mu H^-)Z^\mu$$

is CP-invariant only if again $J^{PC}(Z) = 1^{--}$. Therefore, the same arguments as before lead us to the statement that the Lagrangians

$$L_{H^oH^\pm} = L_{H^o} + L_{ZH+H^-},$$

$$L^{(3)} = L^{(1)}_{\text{spin-1}} + L_{H^o} + L_{ZH+H^-}$$

also violate the CP-symmetry.

These conclusions were arrived at when assuming that all coupling parameters $g_{ijk}$ and at least one $g_{ij}$ in $L_{H_i^o}$ (eq.(5)) are nonzero. On the other hand, it is possible to restore CP-invariance in $L^{(3)}$ (or $L^{(1)}$) by putting some coupling parameters to zero. For instance, if there are just two neutral $H_i^o$ fields (i=1,2), then we restore it if, for example

$$g_{211} = g_{222} = g_2^Z = g_2^W = 0$$

- by assigning $J^{PC}(H_1^o) = O^{++}$, $J^{PC}(H_2^o) = O^{+-}$ (exotic quantum numbers).

Similarly, for three physical neutral $H_i^o$-fields, CP in $L^{(3)}$ (or $L^{(1)}$) is restored if:

1. There is at least one specific $H_i^o$ (say $H_1^o$) which appears in all nonzero $(H_j^oH_k^oH_i^o)$-terms as an odd power (as $(H_1^o)^1$ and/or $(H_1^o)^3$) and the corresponding other couplings are zero:

$$g_{222} = g_{333} = g_{112} = g_{113} = g_{23} = g_2^Z = g_3^Z = g_2^W = g_3^W = 0.$$ 

(Possible assignments are: $J^{PC}(H_1^o) = O^{++}$, $J^{PC}(H_2^o) = J^{PC}(H_3^o) = O^{+-}$.)
2. There is at least one specific $H^o_i$ (say $H^o_1$) which appears in all nonzero $(H^o_jH^o_kH^o_l)$-terms as an even power (as $(H^o_1)^2$ and/or zeroth power) and the corresponding other couplings are zero:

$$g_{122} = g_{133} = g_{123} = g_{23} = g_1^Z = g_1^W = 0.$$  (13)

(Possible assignments are: $J^{PC}(H^o_1) = O^{-\pm}$, $J^{PC}(H^o_2) = J^{PC}(H^o_3) = O^{++}$.) One such specific case is SM with two Higgs doublets and no CP-violation ($\xi = 0$, where $<0 | \Phi^o_2 | 0 > = v_2 e^{i\xi}$, in the notation of ref. [8]).

We necessarily obtain CP-violation in $L^{(3)}$ (or $L^{(1)}$), if at least one of the terms $(H^o_i \partial^\mu H^o_j)Z^\mu$ is nonzero and the terms $(H^o_jH^o_kH^o_l)$ satisfy neither 1. nor 2. (i.e., each of the three $H^o_i$’s appears in $H^o_jH^o_kH^o_l$-terms at least once as an odd power and at least once as an even power) - for example, if all the nonzero $g_{jkl}$ are: $g_{123}, g_{112}, g_{223}$ and $g_{233}$. 

CP-invariance of the $(H^o_jH^o_kH^o_l)$-terms alone would imply that $J^{CP}(H^o_1) = O^{++}$, and similarly for $L^{(1)}_{spin-1}$. It is worth noticing that in general $L_{H^o_i} + L^{(1)}_{spin-1}$ or $L_{H^o_i} + L_{ZH+H^+}$, with all couplings $g_{ijk}$ and at least one $g_{ij}$ being nonzero and the number of spin-0 particles $N \geq 2$, suffice to establish CP-violation in these models even though we may have additional interaction terms like quartic couplings, etc.

The mass basis in which we have written the Lagrangians has the advantage that we can now look directly for processes that would reveal CP-violation in the bosonic sector of the Lagrangian. In general, this could be realized directly, by predicting and observing asymmetries of the form

$$\Delta_{AB} = dN(\mid A \rangle \rightarrow \mid B \rangle) - dN(\mid \bar{A} \rangle \rightarrow \mid \bar{B} \rangle),$$  (14)

where $\mid A \rangle$ and $\mid B \rangle$ are CP-conjugated states and $dN$ is the number of events. On the other hand, CP-violation could be observed (and predicted) also indirectly, by two
nonzero amplitudes of the form

\[ T(\mid A \rangle \to \mid B \rangle_{CP=+1}) \]

\[ T(\mid A \rangle \to \mid B \rangle_{CP=-1}). \quad (15) \]

The possibility (15) implies that one should construct CP-even and CP-odd states (components). Feynman rules are usually formulated in the basis of spin or helicity eigenstates which are not eigenstates of CP. This, of course, does not mean that one cannot perform discrete symmetry tests in the helicity basis; but we prefer to do it in the basis of orbital angular momentum eigenstates which have the advantage of being also CP-eigenstates (see below).

Restricting ourselves to tree level processes (i.e., we do not take into account higher derivative couplings like \( H_i^o W_{\mu \nu}^{-} W^{+\mu \nu} \), \( H_i^o Z^{\mu \nu} Z_{\mu \nu} \), which can be effectively generated at one-loop level), we observe that the two-particle final states of decay processes, as predicted by (CP-violating) Lagrangian \( \mathcal{L}^{(3)} \) (eq. (10)), are either S-wave, or P-wave (derivative coupling). Taking in (15) \( \mid A \rangle \) as the spin-0 particle state (\( \vec{J} = \vec{L} + \vec{S} = \vec{0} \)) and \( \mid B \rangle \) as a particle-antiparticle state, then the latter state when produced at tree level is an S-wave (\( \vec{L}_{\text{final}} = \vec{0} = \vec{S}_{\text{final}} \)) and cannot contain both components of CP (since \( CP(B) = (-1)^{S_{\text{final}}} = +1 \) for boson-antiboson final state). If we want to have a mixture of both CP-components, we have to consider \( 1 \to 3 \) processes like

\[ H_i^o \to W^+ W^- Z, \]

\[ H_i^o \to H^+ H^- Z, \quad (16) \]

These processes are the simplest examples which would give a genuine signal of CP-violation in the neutral spin-0 sector (without the inclusion of fermions). Of course, we assume that the mass spectrum of the theory allows kinematically these decays. A correct procedure to show that the final states of these reactions are mixtures of \( CP = 1 \) and \( CP = -1 \) components would be to perform a partial wave analysis of the amplitudes in terms of orbital and spin angular momenta for 3-particle final state. This basis has the properties

\[ P \mid l\sigma; L\Sigma \rangle = \eta^P_3 (-1)^{l+L} \mid l\sigma; L\Sigma \rangle, \]

\[ C \mid l\sigma; L\Sigma \rangle = \eta^C_3 (-1)^{l+\sigma} \mid l\sigma; L\Sigma \rangle, \quad (17) \]
where $l$ and $\sigma$ are the relative orbital angular momentum and the resultant spin of the particle-antiparticle pair (1-2), $L$ and $\Sigma$ are the corresponding quantum numbers of the neutral particle 3 and the subsystem (1-2) as a whole \([11]\). The phases $\eta_3$’s are the intrinsic quantum numbers of the neutral particle 3 (which we will take to be $Z$, i.e. the final state is $W^+W^-Z$). In a CP-violating theory, $\eta_3$’s are, in principle, conventional. Through the detected angular distribution $dN/d\Omega$, it should be possible to show whether or not both CP components $(W^+W^-Z)_{CP=1}$ and $(W^+W^-Z)_{CP=-1}$ are produced in the decay process. We plan to do such an analysis in the near future, in the case of SM with two Higgs doublets ($\xi \neq 0$ \([8]\)). Here, we will give another argument for the presence of CP-even and CP-odd final states in (16). It suffices to show this for the first of these two processes \([11]\). The line of arguments for the other is similar.

There are several amplitudes contributing to $H_i^o \to W^+W^-Z$ if the underlying dynamics is described by $\mathcal{L}^{(1)}$ of eq. (8). The following three are possible also in the minimal SM or in a CP-conserving version of SM with two Higgs doublets:

\[
\begin{align*}
T(H_i^o \to W^+W^* \to W^+W^-Z), \\
T(H_i^o \to W^-W^{++} \to W^-W^+Z), \\
T(H_i^o \to Z^*Z \to W^+W^-Z). 
\end{align*}
\]

(18)

The star in (18) denotes an off-shell intermediate particle state. It follows that only states with $CP(W^+W^-Z) = +1$ contribute to the diagrams of (18). The crucial point is now that the CP-properties of the final state (as given in (17)) are determined only by the Lorentz structure of the interaction terms (when adopting the convention that the internal $CP = \eta^C_3 \eta^P_3$ of the final particles is +1) and are independent of the magnitude of the (real) coupling parameters. The conclusion that $CP(W^+W^-Z) = +1$ in (18) is then the same for a CP-conserving and CP-violating theory (although the initial state is a linear combination of both CP-eigenstates with $CP = +1$ and $CP = -1$ in a CP-violating theory). However, a fourth amplitude contributing to the process in a CP-violating theory does not exist in the minimal SM, namely

\[
\sum_j T(H_k^o \to H_j^oZ \to W^+W^-Z). 
\]

(19)
This amplitude contains a momentum-dependent vertex \( (H_{k}^{0} \leftrightarrow \partial_{\mu} H_{j}^{0})Z^{\mu} \) and yields final states with only the \( CP = -1 \) component (both types of amplitudes, (18) and (19), yield nonzero contributions if CP is violated). The reason is that the diagram corresponding to (19) exists also in the CP-conserving SM with two Higgs doublets (\( \xi = 0 \)), with \( H_{k}^{0} \) being the "pseudoscalar" \( (J^{PC}(H_{k}^{0}) = 0^{+-}) \). Consistently, we have in (19) only \( CP = -1 \) component in the initial and in the final state in \( \xi = 0 \) case. In \( \xi \neq 0 \) case, the initial state in (19) is a linear combination of two CP-eigenstates with different eigenvalues, and the final state has only \( CP = -1 \) (by the same arguments as above for (18)).

From the experimental data for the angular distribution

\[
d\Gamma(H_{k}^{0} \to W^{+}W^{-}Z)/d\Omega_{1}d\Omega_{2},
\]

it should in principle be possible to disentangle, through a partial wave analysis in the orbital angular momentum basis, the contributions to \( \Gamma(H_{k}^{0} \to W^{+}W^{-}Z) \) of the components of the final states with \( CP = +1 \) from those with \( CP = -1 \). It is important to note that such a signal would be, if obtained from experiments, a genuine signal of CP-violation, i.e. a signal independent of any specific theoretical assumptions.

If we already knew that the underlying theory is SM with two Higgs doublets, then the evidence of the decays \( H_{i}^{0} \to ZZ \) (or \( H_{i}^{0} \to W^{+}W^{-} \)) for all \( i = 1, 2, 3 \) at tree level would be a signal for CP-violation. However, this would amount to first "proving" the theory in order to prove CP-violation. We regard as more realistic to deal directly with the processes of eq. (16).

In conclusion, we state that we have considered here CP-violation originating from the bosonic sector with neutral spin-0 particles. Not every such CP-violating theory can be identified with SM containing two Higgs doublets, since, in principle, even the existence of just two neutral spin-0 particles can lead to CP-violation. We emphasized how CP-violation can manifest itself in the mass basis of the neutral spin-0 particles, the considerations being in principle independent of any specific assumptions (SSB, etc.) on how the physical, CP-violating interactions came about.

\[3^{\text{Note that states with opposite CP are orthogonal to each other and hence the interference terms of amplitudes (18) and (19) yield zero in the decay width } \Gamma(H_{k}^{0} \to W^{+}W^{-}Z).} \]
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