Directed diffraction without negative refraction

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Using the FDTD method, we investigate the electromagnetic propagation in two-dimensional photonic crystals, formed by parallel air cylinders in a dielectric medium. The corresponding frequency band structure is computed using the standard plane-wave expansion method. It is shown that within partial bandgaps, waves tend to bend away from the forbidden directions. This phenomenon has been explained in terms of negative refraction or ‘superlensing’ behavior, contrast to what has been conjectured.

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Ever since the suggestion that a perfect lens can be realized by so called Left Handed Materials (LHMs) or Negative Refraction Index Materials (NRIMs), a conceptual material first introduced by Veselago many years ago\textsuperscript{4}, the research on superlenses and LHMs has been skyrocketing in the mist of much debate. A great body of literature has been and continues to be generated. The research ranges from finding characteristics of LHMs, and negative refraction behavior (NRB), to fabricating composite materials which may reveal as LHMs. All this can be referred to in Refs.\textsuperscript{5}. In spite of some serious debates\textsuperscript{6}, the common consensus seems to agree that the negative refraction, perhaps equivalently the LHMs, has been confirmed\textsuperscript{7}.

Up to date, there mainly three aspects regarding negative refraction; this has caused some confusion to a certain degree. First, the concept of negative refraction was invented and specified with respect to LHMs, as originally proposed by Veselago\textsuperscript{2}. For composite materials, the individual components may be positively refractive. But when put together, the materials as a whole may behave effectively as a negative refracting medium, and a negative refraction index can be defined\textsuperscript{8}. The second is to regard the refraction into the same side as the incident wave as negative refraction\textsuperscript{9}. The second aspect can be observed for waves across the boundary between an isotropic and an anisotropic medium, and this is in fact irrelevant to the negative refraction in its original sense\textsuperscript{9}. The third aspect is to show negative refraction without employing or having to resort to an effective index of negative refraction.

In a recent communication, Luo et al\textsuperscript{10} described an all-angle negative refraction by photonic crystals, without negative effective index. By simulation, the authors demonstrated a frequency range so that for all incident angles one obtains a single negative-refracted beam. It was shown that a slab of photonic crystal can focus a point source on one side of the slab into a real point image on the other side even for the case a parallel sided slab of materials, that is, the slab slab imaging. This phenomenon has been connected to the superlensing phenomenon discussed by Veselago\textsuperscript{2} and Pendry\textsuperscript{3} for a slab of LHMs. The simulation in Ref\textsuperscript{10} has stimulated a number of further experimental and theoretical attempts in verifying the negative refraction behavior of photonic crystals\textsuperscript{11}.

Two problems, however, may concern with the superlensing phenomenon and the negative refraction discussed in Ref\textsuperscript{10}. The first question is whether the superlensing or the flat slab imaging can be explained by other mechanisms. The second question is whether there is indeed negative refraction as suggested. The first question has been answered in Ref\textsuperscript{11}. As shown in Ref\textsuperscript{11}, the imaging by a photonic crystal slab can be caused by partial bandgap effects or anisotropic scattering. The answer to the second question, in our view, is not conclusive. In this communication, we wish to shed some light to the second question, in the hope that more discussions can be stimulated.

We will consider exactly the same two dimensional photonic crystal systems as in Ref\textsuperscript{11}. Specifically, we consider a square lattice of identical air cylinders (holes) in a dielectric medium with dielectric constant $\epsilon = 12$. The lattice constant is denoted by a and the hole radius $r = 0.35a$. In accordance with Ref\textsuperscript{11}, we consider TE modes, that is, the magnetic field is kept parallel to the axis of the air cylinders.

In order to compute the electromagnetic propagation through the arrays of air cylinders, we have performed finite-difference time-domain (FDTD) simulations with perfectly matched layer (PML) boundary conditions. In the simulation, we scale all lengths by the lattice constant and the angular frequency by $2\pi c/a$ to make the system dimensionless.

First, we would like to duplicate the flat slab imaging observed in Ref\textsuperscript{11}. The band structure of the system is plotted in Fig. 1(a), in full agreement with the result in Fig. 3 of Ref\textsuperscript{11}.

By the FDTD method, we have simulated the wave propagation across a flat (11)-oriented slab of lattice of air cylinders. Hereafter, the slabs of photonic crystals are all placed in air, in line with the Ref\textsuperscript{11}. The geometric parameters are given in the figure caption. Fig. 1(b) shows a snapshot of the magnetic field for a point source transmitting the continuous-wave of frequency 0.192, which lies within the regime of so called all-angle negative re-
fraction (AANR). Note that we have also done with frequency 0.195 as in Ref., the results are qualitatively the same. Here we indeed observe the formation of a ‘point’ image on the right hand side of the slab, confirming the result depicted by Fig. 4 in Ref.

If such an imaging property can be attributed to the occurrence of the superlensing effect (referring to the bottom paragraph on the second page of Ref., two wave propagation schemes can be identified. These are illustrated by Fig. 1(c) and (d) respectively. Here, Fig. 1(c) shows the original sketch for the superlensing. The superlensing effect will not only focus a single image on the other side of the slab, but also another focused image within the slab. These two images must go hand in hand. The image inside the slab, however, is not obvious from Fig. 1(b). We note that the imaging behavior has been misinterpreted in Ref. The second possible scheme is described by Fig. 1(d). That is, a plane wave incident upon a slab of negative refraction structures will be negatively refracted inside the slab and recovers to its original travelling direction when returning to the outside medium which is positive refracting.

While the point source scheme has been discussed in Ref., here we will concentrate on the second scheme, i.e., Fig. 1(d). We wish to verify whether the negative refraction depicted in Fig. 1(d) or asserted in the right panel of Fig. 2 in Ref. actually occurs within the AANR regime, as suggested in Ref. For the purpose, we consider electromagnetic propagation through much larger slabs and plot the waves inside the slabs with high resolution, again, by the standard FDTD method.

In Fig. 2, we plot the images of the magnetic intensity fields when a collimated plane wave, mimicked by the Gaussian beam, is incident at various angles onto a slab of the above photonic crystals. Although many incident angles have been taken into account, here we only show the results for four incident angles. The slab measures 49.5x92 in terms of lattice constant. The frequency is 0.192, which is within the AANR range declared in Ref. The flat slab is (11) oriented, as in the simulation of Ref. The detailed information about the setup can be referred in the figure and the figure caption. Here we see that no matter what angle the wave is incident onto the slab, the wave always tends to travel along the [11] direction of the square lattice. This phenomenon has also been confirmed by plotting the snapshots of the magnetic field. It was shown in Ref. that the [11] direction, in which there is a passing band, serves as a main guiding channel for wave propagation, and an focused image can be formed across the slab. In the present case, the outgoing waves are more less parallel to the incident waves. Here, however, we fail to observe the negative refraction expected for the superlens as discussed in Ref. or that depicted in Fig. 1(d). We also verify with other frequencies within the partial bandgaps, yielding similar results.

We have also investigated electromagnetic propagation through a slab of the photonic crystals with different orientations. As example, we show the case with the [11] direction being tilted leftward at 22.5 degree. The results are shown in Fig. 3. Here the flat slab has the same dimension as that in Fig. 2. The normal direction at the incident interface lies exactly in the middle of the [11] and [10] directions of the square lattice of the air cylinders. All the physical parameters are the same as in the above and in Ref. Here we, again, observe that at any incident angle, the waves are more or less bent towards the [11] direction. In the case of Fig. 3(d), we also observed a secondary path along the direction perpendicular to the indicated [11] direction. This can be expected to be due to the symmetry of the square lattice.

To exclude the boundary of the slab as a possible cause for the bending, we have also performed simulations on various slab sizes. Two examples are shown in Fig. 4: in (a) the slab height is increased, while in (b) the slab horizontal length is enlarged. The slab is (11) oriented. The plane wave is incident at the angle of 22.5 degree. Here, the waves inside the slab travel along the [11] direction. Comparing the results in Figs. 2 and 4, one can conclude that the wave deflection behavior inside the slab is not caused by the boundaries.

All the results shown above indicate that in certain ranges of frequencies, the photonic crystals can serve as a guiding medium that directs the wave diffraction or deflection. The fact that the waves can be bent into a certain direction may allow for novel applications in controlling optical flows as suggested in Ref.

One may ask for the reason for the discrepancies between the present results and the analysis of the superlensing phenomenon discussed in Ref. Here we would like to share our thoughts. The details will be published elsewhere. The problem with the conjectured superlensing may lie in the approach to the energy flow inside the slab. The usual approach mainly relies on the curvatures of frequency bands to infer the energy flow.

As documented in Ref., an energy velocity is defined as

$$\vec{v}_e = \frac{1}{\omega} \sum_{\omega} \frac{\bar{J}_R d^3r}{\bar{U}_K d^3r},$$

where $\bar{J}_K$ and $\bar{U}_K$ are the energy flux and energy density of the eigenmodes, and the integration is performed in a unit cell. It can be shown that thus defined energy velocity equals the group velocity obtained as $\vec{v}_g = \nabla_K \omega(K)$. Therefore it is common to calculate the group velocity to infer the energy velocity and subsequently the energy flows or refraction of waves. It has been discussed elsewhere, whether the net current flow through a unit cell really follows the direction of $\vec{v}_e$ remains unclear. We note here that the average flux through a surface may be defined as

$$\langle \bar{J} \rangle = \frac{1}{\bar{n}} \int S \bar{J}.\bar{n}.$$

where $\bar{n}$ is the unit normal vector of the surface $S$. Clearly, the volume averaged current within a unit cell does not necessarily correspond to actual energy flows.

In summary, we have used the standard FDTD method to simulate electromagnetic propagation across various flat slabs of photonic crystals. No negative refraction behavior has been discovered in the frequency regime.
thought to be the regime for all angle negative refraction. The present results, however, are consistent with the previous simulation on the flat slab imaging by the standard multiple scattering theory. Although the present analysis has been focused on TM modes, it can be extended to TE modes. Some results of TE modes have been reported in Ref. 11.

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Figure Captions

Fig. 1 (a) The band structure of a square lattice of air holes embedded in the dielectric medium \( \epsilon = 12 \). The lattice constant is \( a \) and the radius of the cylinders is 0.35\( a \). \( \Gamma M \) and \( \Gamma X \) denote the [11] and [10] directions respectively. (b) The magnetic field \( H_z \) of a point source and its image across a photonic crystal slab called superlens in Luo et al. The slab measures 11.31x31.11, and the frequency is 0.192. (c) The conceptual layout of a superlensing phenomenon, adapted from Veselago and Pendry. (d) The illustration of negative refraction of a plane wave across a flat slab, by analogy with the right panel of Fig. 2 in Ref. 11.

Fig. 2 The intensity image of the magnetic fields across a flat photonic crystal slab made of air holes in dielectric \( \epsilon = 12 \). The principle directions of the crystal are drown in the figure, as [11] and [10]. A plane wave is incident to the slab with various incident angles in degrees with respect to the [11] direction: (a) 0; (b) 22.5; (c) 45; and (d) 67.5. The slab measures as 49.5x92 in terms of lattice constant. For clarity, the air holes are not shown in the figure.

Fig. 3 The intensity image of the magnetic fields across a flat photonic crystal slab. The principle directions of the crystal are drown in the figure, as [11] and [10]. The [11] direction is tilted towards left by an angle of 22.5 degree with reference to the normal direction of the slab surface. A plane wave is incident to the slab with various incident angles in degrees with respect to the normal direction of the slab surface: (a) 0; (b) 22.5; (c) 45; and (d) 67.5. The slab measures as 49.5x92 in terms of lattice constant.

Fig. 4 The intensity image of the magnetic fields across two flat photonic crystal slabs. The principle directions of the crystal are drown in the figure, as [11] and [10]. The two slabs measure in terms of lattice constant as (a) 70x92;(b) 49.5x120. A plane wave is incident to the slab with the incident angle of 22.5 degree with respect to the normal direction of the slab surface.
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