Estimation of transient boundary heat flux by using hybrid algorithm in a two dimensional laminar duct flow

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Abstract. Conjugate gradient method (CGM) with adjoint problem is a popular optimization algorithm in solving the inverse heat transfer problems. It starts with the initial guess solution of unknown parameters to be estimated which would be updated in an iterative procedure. The initial guess solution is one of the significant factors for the accuracy of estimation. In the current study, the Jaya algorithm has been developed to provide the initial guess solution to CGM. The resultant algorithm is called hybrid algorithm. The test problem considered here for the study is the estimation of transient boundary heat flux for two-dimensional hydrodynamically developed and thermally developing forced convective laminar duct flow. The hybrid algorithm is found to be robust and accurate than CGM.

1. Introduction
Inverse heat transfer problems (IHTP) aim to estimate the boundary conditions/properties which are difficult to be measured practically [1]. There are two types of optimisation algorithms used in IHTP: stochastic and gradient-based approach. Conjugate gradient method (CGM) with the adjoint problem is the most popular gradient-based method, while the Genetic algorithm (GE), Cuckoo Search algorithm, Jaya algorithm, etc. are the example of stochastic methods. There has been a lot of work available for boundary heat flux estimation for different cases of convection-diffusion flow [2,3,4] by using CGM. The main limitation of CGM is that it may converge to local minima and its accuracy is highly dependent on measurement errors. Many researchers have also used stochastic methods for IHTP [5,6,7] and are more accurate than CGM. A more economical and accurate algorithm can be developed by coupling CGM with any stochastic method. These types of algorithms are called hybrid algorithm [8,9].

Jaya is a newly developed parameter-less search-based algorithm by Rao [10]. In this paper, first, the Jaya algorithm is formulated for the estimation of transient boundary heat flux for two-dimensional hydrodynamically developed and thermally developing forced convective laminar duct flow. Then the hybrid algorithm is developed where the solution of the Jaya algorithm works as an input to the CGM. The comparison of the solution of the hybrid algorithm with CGM is done by considering different cases like triangular ramp heat flux profile and errors in temperature measurement.

2. Methodology
The methodology includes: (1) Solving the direct problem by the known value of heat flux and obtaining the temperature field (2) Formulate the IHTP and estimate the boundary heat flux by inverse algorithm from temperature data of direct problem.
2.1. Direct problem

Transient heat flux is applied at the bottom plate of a parallel plate channel in a two-dimensional hydrodynamically developed and thermally developing laminar flow as shown in Figure 1(a). The upper boundary is insulated. The channel has length $L = 1.0$ m and height $H = 0.1$ m. The air flowing inside with mean velocity $u_m = 0.033$ m/s and corresponding Reynolds number $Re = 429$.

Assuming the flow has a parabolic velocity profile given by the following equation:

$$u(y) = 6u_m \frac{y}{H} \left(1 - \frac{y}{H}\right)$$  \hspace{1cm} (1)

The governing equation for energy for the present case with boundary conditions are given by

$$\rho C_p \frac{\partial T}{\partial x} + \rho C_p \frac{\partial T}{\partial t} \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y}\right) = 0$$  \hspace{1cm} (2a)

at $y = H; t > 0$; $\frac{\partial T}{\partial y} = 0$  \hspace{1cm} (2b)

at $y = 0; t > 0$; $-k \frac{\partial T}{\partial y} = q(t)$

where $C_p$ is specific heat, $\rho$ is fluid density and $k$ is thermal conductivity of the fluid. The air is taken as working fluid with $\rho = 1.2$ kg/m$^3$, $C_p = 1005$ J/Kg K, and $k = 0.026$ W/mK. A code for solving Eq. (2) is developed using the finite volume method in the commercial software MATLAB R2016b for obtaining temperature data. This temperature data is used for the inverse problem

2.2. Inverse algorithms

The unknown boundary heat flux $q(t)$ is estimated by minimizing the least square-based objective function containing measured temperatures and calculated temperatures given by:

$$J(q(t)) = \int_{t_f}^{t_0} \sum_{m=1}^{M} [Y(X_m, Y_m; q(t)) - T_m(X_m, Y_m)]^2 dt$$  \hspace{1cm} (3)

where $M$ is the number of sensors, $t_f$ is the final time and $q(t)$ is the estimated heat flux. $Y$ is the calculated temperature obtained by solving the direct problem (Eq. (2)) while $T_m$ is the measured temperature at the location defined by $X_m$ and $Y_m$. 

*Figure 1. (a) Computational domain (b) CGM algorithm (c) Jaya Algorithm*
2.2.1 CGM algorithm

The formulation of CGM algorithm is well known in literature [1] and is not repeated here. The computational algorithm is shown in Figure 1(b).

2.2.2 Jaya algorithm

The Jaya algorithm starts with random selection of ‘P’ number of candidate solutions with ‘tn’ number of design variables (j =1,2…tn) [11]. Let best candidate obtains the best value of objective function and worst candidate obtains the worst value of objective function. If \( q_{j,p,G} \) is the value of \( j \)th variable for \( p \)th candidate during \( G \)th iteration, then this value is modified according to following equation:

\[
q(t)'_{j,p,G} = q(t)_{j,p,G} + r_{1j,G} (q(t)_{j,best,G} - |q(t)_{j,p,G}|) - r_{2j,G} (q(t)_{j,worst,G} - |q(t)_{j,p,G}|)
\]  

where, \( q(t)'_{j,p,G} \) is updated value of variable \( q(t)_{j,p,G} \), \( q(t)_{j,best,G} \) and \( q(t)_{j,worst,G} \) are best and worst solutions of \( j \)th candidate. The \( r_{1j,G} \) and \( r_{2j,G} \) are random numbers for \( j \)th variable during \( G \)th iteration and their range is [0, 1]. The computational algorithm of Jaya is shown in Figure 1(c).

2.2.3 Hybrid algorithm

Jaya algorithm is coupled with CGM for the development of hybrid algorithm wherein the Jaya algorithm provides the initial guess solution to CGM. As CGM gets better initial guess, the final estimation is more accurate. The stopping criterion for CGM is:

\[
J(q(x)) < \epsilon
\]  

where \( \epsilon \) is small number. For this work \( \epsilon \) is taken as \( 10^{-6} \). When measurement errors are present in measured data, the discrepancy principle [1] is used for stopping criteria which is given by,

\[
J(q(x)) < M\sigma^2 t_f
\]  

where \( \sigma \) is the standard deviation of measurement errors and \( t_f \) is the final time.

3. Results

The spatial grid size of 51×101 elements and temporal grid size of 31 (tn) are found to be optimum. The population size of hybrid algorithm is taken as \( P = 20 \) with only one sensor (\( M = 1 \)) located at \( X_m = 46dX \) and \( Y_m = 96dY \). Random population generated between \( q_{\text{min}} = 0 \) and \( q_{\text{max}} = 3000 \text{ W/m}^2 \). The accuracy of estimation is measured by root mean square (RMS) error given by

\[
\text{RMS} = \sqrt{\frac{1}{t_n} \sum_{i=1}^{t_n} [q_a(t) - q_e(t)]^2}
\]  

where, \( q_a(t) \) is actual heat flux and \( q_e(t) \) is estimated heat flux.

![Figure 2. Estimation of (a) triangular ramp heat flux profile by CGM and hybrid algorithm. (b) Effect of measurement error on accuracy of hybrid algorithm](image-url)
Figure 2(a) shows the comparison of the estimation of triangular heat flux profile by CGM and hybrid algorithm. The hybrid algorithm is estimating better (RMS = 10.17 W/m²) than CGM (RMS = 24.68 W/m²). However, the computational time (t_c) required for the hybrid algorithm is higher (t_c = 1352.43 s) than CGM (t_c = 1123.51 s). The effect of measurement error on the accuracy of estimation by the hybrid algorithm is shown in Fig. 2(b). It is shown that with increasing the value of σ, the accuracy of the algorithm decreases. For σ = 0.1 and σ = 0.2, the accuracy of the algorithm is almost same (RMS = 10.87 W/m² for σ =0.1; RMS = 10.91 W/m² for σ = 0.2) but for σ = 0.5, the accuracy of algorithm is minimum (RMS = 12.78 W/m²). However, the prediction of the shape of heat flux profile by the hybrid algorithm even with the measurement errors is quite remarkable.

4. Conclusions
The current work tested the ability of the Jaya algorithm coupled with CGM on the estimation of transient boundary heat flux in a two-dimensional convection-diffusion problem inside a parallel plate channel. The analysis is carried out for triangular heat flux profile and a comparison is made with CGM to test the accuracy of the hybrid algorithm. To examine the robustness of the hybrid algorithm different measurement errors have also been considered. The hybrid algorithm is found to be accurate than CGM.

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