Using Locality-sensitive Hashing for Rendezvous Search

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Abstract—The multichannel rendezvous problem is a fundamental problem for neighbor discovery in many IoT applications. The existing works in the literature focus mostly on improving the worst-case performance, and the average-case performance is often not as good as that of the random algorithm. As IoT devices (users) are close to each other, their available channel sets, though they might be different, are similar. Using the locality-sensitive hashing (LSH) technique in data mining, we propose channel hopping algorithms that exploit the similarity between the two available channel sets to increase the rendezvous probability. For the synchronous setting, our algorithms have the expected time-to-rendezvous (ETTR) inversely proportional to a well-known similarity measure called the Jaccard index. For the asynchronous setting, we use dimensionality reduction to speed up the rendezvous process. Our numerical results show that our algorithms can outperform the random algorithm in terms of ETTR.

Index Terms—multichannel rendezvous, locality-sensitive hashing.

I. INTRODUCTION

The multichannel rendezvous problem (MRP) that asks two users to meet each other by hopping over their available channels is a fundamental problem for neighbor discovery in many IoT applications. This problem has received much attention lately (see, e.g., the excellent book [1] and references therein). As classified in the paper [2], there are three key elements in the multichannel rendezvous problem: (i) users, (ii) time, and (iii) channels. Based on the assumptions on users, time, and channels, channel hopping (CH) algorithms can be classified into various settings. One of the most difficult settings, known as the obvious setting in the book [1], is the sym/async/hetero/local MRP. In such a setting, (i) sym: users are indistinguishable, (ii) async: users’ clocks may not be synchronized to the global clock, (iii) hetero: users may not have the same available channel sets, and (iv) local: the labels of the channels of a user are locally labeled, and they may not be the same as the global labels of the channels. In such a setting, nothing can be learned from a failed rendezvous attempt. It was shown in [3] that the expected time-to-rendezvous (ETTR) in such a setting is lower bounded \( \frac{n_1 + n_2 - n_{1,2}}{n_1 n_2} \), where \( n_1 \) and \( n_2 \) are the number of available channels of user 1 (resp. user 2), and \( n_{1,2} \) is the number of commonly available channels between users 1 and 2. The random algorithm in which each user randomly selects a channel from its available channel set performs amazingly well as the ETTR of the random algorithm is \( \frac{n_1 n_2}{n_{1,2}} \). Such an ETTR is very close to the lower bound when \( n_{1,2} \) is not too small. In other words, the random algorithm is nearly optimal in ETTR for the sym/async/hetero/local MRP.

The obvious setting, however, may not be realistic for practical IoT applications due to the following two reasons: (i) the channels are usually globally labeled (according to some international standards), and (ii) the available channel sets of two users are similar as they are in the same proximity. With the first feature (assumption), the rendezvous search problem falls into the category of the sym/async/hetero/global MRP in [3], where the labels of the channels of a user are globally labeled. For that setting, there are many existing algorithms that focus on the maximum time-to-rendezvous (MMTR). In particular, it has been shown that the MTTR of the CH sequences in [4]–[6] is \( O(\log \log N) n_1 n_2 \) for an MRP with \( N \) globally labeled channels. However, the ETTRs of these CH sequences are not as good as that of the random algorithm. Thus, one might pose the question of whether the ETTR can be further reduced by using the second feature.

To address such a question, we apply the locality-sensitive hashing (LSH) technique that is widely used in data mining. To the best of our knowledge, our work appears to be the first one to address the multichannel rendezvous problem by using the LSH technique for mining similar objects. As described in the book [7], LSH is a technique that hashes similar items into the same bucket with high probability. Since similar items are hashed into the same buckets, the technique maximizes hash collisions. If the available channel sets of two users are similar, then the multichannel rendezvous problem can be viewed as a data mining problem that maximizes hash collisions. In this paper, we first consider the synchronous setting, where the clocks of the two users are synchronized, i.e., the sym/sync/hetero/global MRP in [3]. For that setting, we propose the LSH algorithm (see Algorithm 1) that leads to an ETTR close to the inverse of the Jaccard index between the two available channel sets, i.e., \( \frac{n_1 + n_2 - n_{1,2}}{n_1 n_2} \). Such an ETTR is substantially smaller than the ETTR of the random algorithm. Moreover, we propose an improved version, the LSH2 algorithm (see Algorithm 2), with an MTTR bounded by \( \frac{N}{n_1 n_2} \), where \( N \) is the total number of globally labeled channels.
By conducting extensive simulations, we show that LSH2 outperforms the best algorithm in ETTR and MTTR for the sym/sync/hetero/global MRP.

We then consider the asynchronous setting, i.e., the sym/async/hetero/global MRP in [3]. In the asynchronous setting, it is much more difficult for the two users to rendezvous as their clocks are not synchronized. To our surprise, a direct extension of the LSH2 algorithm to the asynchronous setting, called the LSH3 algorithm in Algorithm 3, can still achieve a 50% reduction of the ETTR of the random algorithm when the Jaccard index is almost 1. To further reduce the ETTR, we propose the LSH4 algorithm (see Algorithm 4) that uses LSH for dimensionality reduction. The LSH4 algorithm maps the available channel set of a user to a much smaller multiset with size $T_0$. We show that the ETTR of the LSH4 algorithm is smaller than that of the random algorithm if

$$T_0 \leq \frac{n_1 n_2}{n_1 + n_2 - n_1,2}. \quad (1)$$

The rest of the paper is organized as follows. In Section II, we briefly review the multichannel rendezvous problem and related works. In Section III, we consider the synchronous setting, in which we propose the LSH algorithm and the LSH2 algorithm and discuss the pros and cons of these two algorithms. In Section IV, we consider the asynchronous setting and propose the LSH3 algorithm and the LSH4 algorithm. In Section V, we provide simulation results for comparing our algorithms with the random algorithm and the best algorithm in the literature. We conclude the paper by discussing possible future works in Section VI.

II. THE MULTICHANNEL RENDEZVOUS PROBLEM

In this paper, we consider the multichannel rendezvous problem in a cognitive network with $N$ channels (with $N \geq 2$), indexed from 0 to $N - 1$. For neighborhood discovery, users first sense their available channels and rendezvous with other users on a commonly available channel to establish a communication link. We assume that the rendezvous protocol is done in a distributed manner, and each user uses a CH algorithm to hop over its available channels with respect to time. We also assume that time is slotted (the discrete-time setting) and indexed from $t = 0, 1, 2, \ldots$.

We consider the rendezvous problem with two users. The case with multiple users can be easily extended by using the rendezvous protocol with two users [3]. The available channel set for user $i$, $i = 1, 2$,

$$c_i = \{c_i(0), c_i(1), \ldots, c_i(n_i - 1)\},$$

is a subset of the $N$ channels, where $n_i = |c_i|$ is the number of available channels to user $i$. We assume that there is at least one channel that is commonly available to the two users (otherwise, it is impossible for the two users to rendezvous), i.e.,

$$c_1 \cap c_2 \neq \emptyset. \quad (2)$$

Let $n_{1,2} = |c_1 \cap c_2|$ be the number of common channels between these two users.

Define the time-to-rendezvous (TTR) as the number of time slots (steps) needed for these two users to hop to a common available channel. In this paper, we are interested in the expected time-to-rendezvous (ETTR) and the maximum time-to-rendezvous (MTTR). While ETTR is a measure of the average-case performance, MTTR is a measure of the worst-case performance.

There are several existing channel hopping algorithms in the literature for the sym/sync/hetero/global MRP [8]–[13]. Among them, SynMAC [8] performs best if the channel load is not a concern. As we only consider the rendezvous problem with two users, we do not consider the load constraints [11], [12] in this paper. In SynMAC, each user simply hops to channel $c$ at time $t$ in a period of $N$ time slots. As long as there exists a common channel between the two users, SynMAC guarantees the rendezvous of the two users within $N$ time slots. Thus, its MTTR is $N$, and it achieves the optimal MTTR for the sym/sync/hetero/global MRP with $N$ channels. However, the ETTR of SynMAC is $O(N)$ which is much larger than that of the random algorithm. We also note that both the SYNC-ETCH algorithm [10] and the criss-cross construction [13] are based on a tournament design [14], and they only work in the homogeneous setting (where all the $N$ channels are available channels to the two users). To apply these algorithms in the heterogeneous setting, one could use the channel rotation trick in [9], [11] to extend the CH sequence. However, the MTTR for such an extension is $O(N^2)$, which is much worse than $N$ in SynMAC. As such, we will use SynMAC and the random algorithm as our benchmarks for the synchronous setting.

For the sym/async/hetero/global MRP, most existing algorithms focus on the MTTR. The best-known MTTR is $O(\log \log N)n_1n_2$ (see, e.g., [4]–[6]). However, their ETTRs are not as good as that of the random algorithm. The quasi-random (QR) algorithm [15] that uses the 4BSB encoding and the two-prime clock algorithm in [3] has a comparable ETTR to the random algorithm and an $O(\log N)n_1n_2$ MTTR. Another CH algorithm that has an $O(\log N)n_1n_2$ MTTR is QECH [16]. There are also periodic CH sequences with $O(N^2)$ periods that can achieve full rendezvous diversity, see, e.g., CRSEQ [17], DRDS [18], T-CH [19], DSCR [20], IDEAL-CH [2], RDSML-CH [21]. These CH sequences, along with random patching, can be used for the sym/async/hetero/global MRP to achieve $O(N^2)$ MTTR. However, according to their simulation results, their ETTRs are not as good as that of the random algorithm. One open question in the literature is whether there exist CH algorithms that have smaller ETTR than that of the random algorithm for the sym/async/hetero/global MRP.

III. THE SYNCHRONOUS SETTING

In this section, we consider the sym/sync/hetero/global MRP. In such a setting, (i) sym: users are indistinguishable, (ii) sync: users’ clocks are synchronized to the global clock,
(iii) hetero: users might not have the same available channel sets, and (iv) global: the labels of the channels of a user are the same as the global labels of the channels. We will use the locality-sensitive hashing (LSH) technique to generate channel hopping sequences that can outperform the random algorithm.

A. Locality-sensitive hashing

In this section, we briefly introduce the locality-sensitive hashing technique for mining large data sets. For a more detailed introduction, we refer to the book [7]. In data mining, objects are often represented by a binary vector of \( N \gg 1 \) features, indexed from 0 to \( N - 1 \). Given the binary vector of a specific object, we generate a set of \( K \) hash functions that are uniformly distributed in \([0, N - 1]\). For each hash function, it returns the index of the first nonzero element of a binary vector (in the modulo manner). If the \( K \) hash values of another vector are the same as that of the given vector, then that vector is declared to be similar to the given vector. By doing so, searching similar objects in a large dataset can be efficiently done by only checking the hash values.

The key rationale behind the LSH technique is that the probability that two binary vectors have the same hash value is approximately equal to the Jaccard index between these two binary vectors [7]. The Jaccard index is a widely used similarity measure between two sets \( S_1 \) and \( S_2 \), and it is defined as

\[
J = \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|}.
\]

Thus, the probability that the \( K \) hash values of two vectors are the same is roughly \( J^K \), which is very small unless the Jaccard index \( J \) is close to 1.

B. Channel hopping algorithms

To map the multichannel rendezvous problem to a data mining problem, we represent the available channel set of each user by one-hot encoding the available channel set into an \( N \)-dimensional binary vector. Since these two users are usually in the same proximity, it is reasonable to assume that the Jaccard index between the two available channel sets,

\[
J = \frac{|c_1 \cap c_2|}{|c_1 \cup c_2|} = \frac{n_{1,2}}{n_1 + n_2 - n_{1,2}},
\]

is not too small. As the LSH technique for data mining, we generate a sequence of uniformly distributed hash functions \( \{U(t), t \geq 0\} \). At time \( t \), each user selects the channel that is the first nonzero element from \( U(t) \) in its one-hot encoded vector. This is described in Algorithm 1.

In Figure 1, we illustrate how the LSH CH algorithm works. In this figure, the \( N \) channels are ordered in a ring. The outer ring represents the available channels of user 1 (colored in blue), and the inner ring represents the available channels of user 2 (colored in pink). At time \( t_1 \), a pseudo-random variable \( U(t_1) \) is selected in the ring (colored in yellow), and each user hops to the nearest available channel clockwise. In Figure 1, the nearest available channel of user 1 is the same as that of user 2, and thus both users rendezvous at time \( t_1 \). However, at time \( t_2 \), another pseudo-random variable \( U(t_2) \) is selected in the ring (colored in green). For \( U(t_2) \), the nearest available channel of user 1 is different from that of user 2, and thus both users fail to rendezvous at time \( t_2 \).

The insight behind the LSH CH algorithm is that the \( n_1 + n_2 - n_{1,2} \) common channels partition the ring into \( n_1 + n_2 - n_{1,2} \) line graphs. If \( U(t) \) falls in a line graph associated with a common channel, then the two users rendezvous. Otherwise, they do not. If the numbers of nodes of these line graphs are evenly distributed, then the probability that \( U(t) \) falls in a line graph associated with a common channel is \( \frac{n_{1,2}}{n_1 + n_2 - n_{1,2}} = J \). Since \( \{U(t), t \geq 0\} \) are assumed to be independent, one would expect the ETTR of Algorithm 1 to be close to \( 1/J \).

However, in some of our simulations, we find that the ETTR of Algorithm 1 could sometimes be much larger than \( 1/J \) if the numbers of nodes of the \( n_1 + n_2 - n_{1,2} \) line graphs are not evenly distributed. This happens when the available channels of a user are contiguous (see Figure 2 for an illustration). The leading channel in a contiguous block of available channels has a much larger probability of being selected than the other available channels in the same block. This is because a non-leading available channel in the block can only be selected when \( U(t) \) is exactly the same as that channel. To remedy this, we add a pseudo-random permutation \( \pi_t \) of the \( N \) channels that reorders the \( N \) channels randomly so that each available channel of a user can be hashed roughly with an equal probability.

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**Algorithm 1** The LSH CH algorithm

**Input:** A set of available channels \( c = \{c_0, c_1, \ldots, c_{n-1}\} \) that is a subset of the \( N \) channels \( \{0, 1, \ldots, N-1\} \), and a sequence of uniformly distributed pseudo-random variables \( \{U(t), t \geq 0\} \) in \([0, N-1]\).

**Output:** A CH sequence \( \{c(t), t = 0, 1, \ldots\} \) with \( c(t) \in c \).

1. Generate CH sequence \( \{c(t), t = 0, 1, \ldots\} \) with \( c(t) = c_i^*, \) where \( i^* = \arg\min_{0 \leq i \leq n-1} (c_i - U(t)) \mod N \).
Fig. 2: Normally, available channels are evenly distributed (as shown in the drawing on the left). However, sometimes available channels are contiguous (as shown in the drawing on the right), and the leading channel in a contiguous block has a much larger probability of being selected.

Another drawback of the LSH CH algorithm in Algorithm 1 is that its maximum time-to-rendezvous (MTTR) is not bounded. This is because nothing is learned from a failed rendezvous attempt by using independent uniformly distributed random variables \( \{U(t), t \geq 0\} \) in Algorithm 1. In fact, when a user uses \( U(t) = u \) to select a channel at time \( t \), and it results in a failed rendezvous, then the user knows that the value \( u \) should not be used again. To learn from a failed rendezvous attempt, one should use sampling without replacement. For this, we replace the sequence of uniform hash functions \( \{U(t), t \geq 0\} \) by a pseudo-random permutation \( \pi_2 \) of the \( N \) channels. This leads to the following improved algorithm, called the LSH2 CH algorithm in Algorithm 2.

**Algorithm 2 The LSH2 CH algorithm**

**Input:** A set of available channels \( c = \{c_0, c_1, \ldots, c_{n-1}\} \) that is a subset of the \( N \) channels \( \{0, 1, \ldots, N-1\} \), and two pseudo-random permutations \( \pi_1 \) and \( \pi_2 \) of the \( N \) channels.

**Output:** A CH sequence \( \{c(t), t = 0, 1, \ldots, N-1\} \) with \( c(t) \in c \).

1. Let \( c(t) = \pi_2^i \), where \( i^* = \text{argmin}_{0 \leq i < n-1}((\pi_1(c_i) - \pi_2(t)) \mod N) \).

**Theorem 1:** Consider the sym/sync/hetero/global MRP described in Section III. Assume that the two users use Algorithm 2 to generate their CH sequences and that the two random permutations \( \pi_1 \) and \( \pi_2 \) are independent. Then the MTTR is bounded by \( N \), and the ETTR approaches \( 1/J \) when \( N \to \infty \).

**Proof:** We first show that the MTTR is bounded above by \( N \). Let \( \pi_2^{-1} \) be the inverse permutation of \( \pi_2 \). Suppose that channel \( c \) is a common channel. Then we have from Algorithm 2 that both users hop on channel \( c \) at time \( \pi_2^{-1}(\pi_1(c)) \). As such, the two users rendezvous within \( N \) time slots.

Let \( E_\ell \) be the event that the two users hop to a common channel at time \( \ell \). We will show that

\[
P(E_\ell) = J.
\]

To show this, let us consider a directed ring with \( N \) nodes, indexed from 0, 1, \ldots, \( N-1 \) (see Figure 1). In the ring, there is a directed edge from node \( i \) to node \( (i + 1) \mod N \). Let \( \hat{c}_0, \ldots, \hat{c}_{n_1+n_2-n_1,2-1} \) be the \( n_1+n_2-n_{1,2} \) common channels. Since \( \pi_1 \) is a random permutation, \( \pi_1(\hat{c}_i), i = 0, 1, \ldots, n_1 + n_2 - n_{1,2} - 1 \), are randomly placed one by one (without replacement) into the ring. Let

\[
B = \{\pi_1(\hat{c}_i), i = 0, 1, \ldots, n_1 + n_2 - n_{1,2} - 1\}.
\]

Denote by \( \sigma_1 \) be the \((i+1)\)th smallest number in \( B \), i.e.,

\[
\sigma_0 < \sigma_1 < \ldots < \sigma_n_{1,2} - n_{1,2} - 1.
\]

Now associate each node in the ring with the nearest neighbor (including itself) in \( B \), and this partitions the ring into \( n_1 + n_2 - n_{1,2} \) line graphs, where the \( \ell \)th line graph contains nodes between node \( \sigma_{i} \mod n_1+n_2-n_{1,2} + 1 \) and node \( \sigma_\ell \), \( \ell = 0, \ldots, n_1 + n_2 - n_{1,2} - 1 \). Node \( \sigma_\ell \) is called the representing node of the \( \ell \)th line graph, and a line graph is called a rendezvous line graph if its representing node, say node \( \sigma_\ell \), is mapped from a common channel, i.e., \( \pi_2^{-1}(\sigma_\ell) \) is a common channel of the two users. As there are \( n_{1,2} \) common channels, there are \( n_{1,2} \) rendezvous line graphs (among the \( n_1+n_2-n_{1,2} \) line graphs). Let \( B_2 \) be the set of nodes in the \( n_{1,2} \) rendezvous line graphs. According to Algorithm 2, the two users hop to a common channel at time \( t \) if and only if \( \pi_2(t) \) is a node in \( B_2 \). Since \( \pi_2(\hat{c}_i), i = 0, 1, \ldots, n_1 + n_2 - n_{1,2} - 1 \), are randomly placed one by one into the ring, it follows from the symmetry of a random permutation that the numbers of nodes in the \( n_1 + n_2 - n_{1,2} \) line graphs are identically distributed. Thus,

\[
P(E_\ell) = \frac{E[|B_2|]}{N} = \frac{n_{1,2}}{n_1 + n_2 - n_{1,2}} = J.
\]

Note that the events \( \{E_\ell, t = 0, 1, \ldots, \tau\} \) for a finite time \( \tau \) are not independent. However, since \( \pi_2(t) \) is a random permutation, the events \( \{E_\ell, t = 0, 1, \ldots, \tau\} \) converges to independent Bernoulli trials when \( N \to \infty \). This implies that the time-to-rendezvous converges to a geometric distribution with mean \( 1/J \) when \( N \to \infty \).

**IV. THE ASYNCHRONOUS SETTING**

Now, we extend the LSH algorithms to the asynchronous setting. Specifically, we consider the sym/sync/hetero/global MRP. In such a setting, (i) sym: users are indistinguishable, (ii) async: users’ clocks might not be synchronized to the global clock, (iii) hetero: users might not have the same available channel sets, and (iv) global: the labels of the channels of a user are the same as the global labels of the channels.

**A. Direct extension**

One simple extension is to use the LSH2 algorithm. As the clocks of the two users may not be synchronized, the LSH2 algorithm cannot guarantee the rendezvous of the two users. For example, if all the channels are available to the two
users, i.e., \( n_1 = n_2 = N \), and their clocks are shifted by one time slot, then the two users using the LSH2 algorithm will hop to two different channels in every time slot. To avoid the scenario that the two users never rendezvous, we need to add “randomness” into the rendezvous algorithm. This can be easily done by using sampling with replacement in the LSH algorithm. As a combination of the LSH algorithm and the LSH2 algorithm, we propose the LSH3 algorithm in Algorithm 3 for the sym/async/hetero/global MRP.

Algorithm 3 The LSH3 CH algorithm

**Input:** A set of available channels \( c = \{c_0, c_1, \ldots, c_{n-1}\} \) that is a subset of the \( N \) channels \( \{0, 1, \ldots, N-1\} \), a pseudo-random permutations \( \pi_i \) of the \( N \) channels, and a sequence of uniformly distributed pseudo-random variables \( \{U(t), t \geq 0\} \) in \([0, N-1]\).

**Output:** A CH sequence \( \{c(t), t = 0, 1, \ldots\} \) with \( c(t) \in c \).

1. Generate CH sequence \( \{c(t), t = 0, 1, \ldots\} \) with \( c(t) = c_i^* \), where \( i^* = \arg\min_{0 \leq i \leq n-1} ((\pi_i(c_i) - U(t)) \mod N) \).

Assume that the two users use Algorithm 3 to generate their CH sequences and that the random permutations \( \pi_i \) and the sequence of uniformly distributed random variables \( \{U(t), t \geq 0\} \) are independent. Under these assumptions, we derive an approximation for the rendezvous probability of the LSH3 algorithm in the asynchronous setting. Due to space limitation, the derivation is not shown here (and it can be found in the full report in ArXiv with the same title). Recall that \( E_t \) is the event that the two users hop to a common channel at time \( t \). Then

\[
P(E_t) \approx \frac{2n_1}{n_1 + n_2 - n_{1,2}} + \frac{n_1}{2n_1 - n_{1,2}} + \frac{n_1}{n_1 + n_2 - n_{1,2}} + \frac{n_1}{n_1 + n_2 - n_{1,2}} \tag{8}
\]

Since the events \( \{E_t, t = 0, 1, \ldots\} \) are independent Bernoulli trials when \( N \to \infty \), the time-to-rendezvous converges to a geometric distribution with mean \( 1/P(E_t) \) when \( N \to \infty \). Thus, we can approximate the ETTR by \( 1/P(E_t) \) as in Theorem 1. In particular, when \( n_{1,2} = n_1 = n_2 \), the approximation from (8) for the rendezvous probability of the LSH3 algorithm is \( 2/n_{1,2} \), which is twice of that the random algorithm. Thus, the LSH3 algorithm achieves a 50% reduction of the ETTR when the Jaccard index is almost 1.

B. Dimensionality reduction

Our second approach is to use LSH for dimensionality reduction. The idea is to map the available channel set of a user to a much smaller multiset (that might have duplicated elements). For this, we use the LSH2 algorithm to generate a CH sequence with the length \( T_0 \). Let \( \tilde{c}_i = [\tilde{c}_i(0), \tilde{c}_i(1), \ldots, \tilde{c}_i(T_0 - 1)] \) be the multiset from the CH sequence by user \( i \), \( i = 1 \) and 2. The size of the multiset \( T_0 \) is fixed, and it should be much smaller than \( n_1 \) and \( n_2 \) for dimensionality reduction. Since the multisets are generated by the LSH2 algorithm, we have from Theorem 1 that there are, on average, \( JT_0 \) times such that \( \tilde{c}_1(t) = \tilde{c}_2(t), \) \( t = 0, 1, 2, \ldots, T_0 - 1 \). Thus, the size of the intersection of the two multisets is roughly \( JT_0 \). In the asynchronous setting, suppose that at each time slot, each user randomly selects an element from its multiset and hops to that channel. Then at time slot \( t \), the two users will rendezvous with probability roughly \( JT_0 / (T_0^2) \), and the ETTR for such an algorithm is roughly \( T_0 / J \), which is only \( T_0 \) times larger than that in the synchronous setting. Moreover, it is smaller than the ETTR of the random algorithm if

\[
T_0 \leq \frac{n_1n_2}{n_1 + n_2 - n_{1,2}}. \tag{9}
\]

One problem with the above argument is that there is a nonzero probability that the intersection of the two multisets is empty. If this happens, the two users never rendezvous. One quick fix is to use a mixed strategy. With probability \( p \) (resp. \( 1 - p \)), a user selects a channel from its multiset (resp. available channel set). This leads to the LSH4 algorithm in Algorithm 4.

Algorithm 4 The LSH4 CH algorithm

**Input:** A set of available channels \( c = \{c_0, c_1, \ldots, c_{n-1}\} \) that is a subset of the \( N \) channels \( \{0, 1, \ldots, N-1\} \), two pseudo-random permutations \( \pi_1 \) and \( \pi_2 \) of the \( N \) channels, and two parameters \( T_0 \) and \( p \).

**Output:** A CH sequence \( \{c(t), t = 0, 1, \ldots, N-1\} \) with \( c(t) \in c \).

1. Generate the multiset \( \tilde{c} = [\tilde{c}(0), \tilde{c}(1), \ldots, \tilde{c}(T_0 - 1)] \) by letting \( \tilde{c}(t) = c_i^* \), where \( i^* = \arg\min_{0 \leq i \leq n-1} ((\pi_i(c_i) - \pi_2(t)) \mod N) \).
2. With probability \( p \) (resp. \( 1 - p \)), randomly select an element \( c \) from \( \tilde{c} \) (resp. \( c \)) and let \( c(t) = c \).

One can approximate the ETTR of the LSH4 algorithm by considering the four cases: (i) with probability \((1 - p)^2\), both users select their channels from the available channel sets, (ii) with probability \(p(1 - p)\), user 1 selects its channel from its multiset, and user 2 selects its channel from its available channel set, (iii) with probability \(p(1 - p)\), user 2 selects its channel from its multiset, and user 1 selects its channel from its available channel set, and (iv) with probability \(p^2\), both users select their channels from the multisets. As argued before, the probability for a successful rendezvous in the fourth case is roughly \( J/T_0 \). Also, the probability of a successful rendezvous in the first case is simply the rendezvous probability of the random algorithm, i.e., \( \frac{n_1^2}{n_1n_2} \). For the second case, we have to average over all the random permutations \( \pi_1 \) for user 1, and the probability that user 1 selects a channel \( c \) (after the averaging) is the same as that of the random algorithm. Hence the probability of a successful rendezvous in the second case is also the same as that of the random algorithm, i.e., \( \frac{n_1^2}{n_1n_2} \). The argument for the third case is the same as that for the second case. Averaging over these four cases, the probability of a successful rendezvous in a time...
slot is \((1 - p^2) \frac{n_{1,2}}{n_1 n_2} + p^2 \frac{J}{n_0}\). As such, the ETTR of the LSH4 algorithm is approximately

\[
\frac{1}{(1 - p^2) \frac{n_{1,2}}{n_1 n_2} + p^2 \frac{J}{n_0}}.
\]  

(10)

V. SIMULATIONS

A. Numerical results in the synchronous setting

In this section, we compare our LSH CH algorithms in Algorithm 1 and Algorithm 2 with the random algorithm and the SynMAC algorithm [8].

For our experiments, we first specify the number of channels \(N\), the number of common channels \(n_{1,2}\), the number of available channels \(n_1\) for user 1, and the number of available channels \(n_2\) for user 2. We then randomly select \(n_{1,2}\) channels from \(\{0, 1, \ldots, N - 1\}\) as the common channels of the two users, i.e., \(c_1 \cap c_2\). Remove these channels from \(\{0, 1, \ldots, N - 1\}\) to form a set of channels \(C\). Randomly select two disjoint channel sets \(A_1\) and \(A_2\) from \(C\), where \(A_1\) has \(n_1 - n_{1,2}\) channels and \(A_2\) has \(n_2 - n_{1,2}\) channels. Add the channels in \(A_1\) (resp. \(A_2\)) to \(c_1 \cap c_2\) to form the available channel set \(c_1\) for user 1 (resp. \(c_2\) for user 2). By doing so, the Jaccard index \(J\) between the two available channel sets is \(\frac{n_{1,2}}{n_1 + n_2 - n_{1,2}}\). For each experiment, we conduct the simulation for 10,000 time slots and record the rendezvous time slots. The ETTR of each experiment is the average of the TTRs in the 10,000 time slots, and the MTTR of each experiment is the maximum of the TTRs in the 10,000 time slots. We conduct 10,000 experiments and obtain our estimates for ETTR and MTTR by averaging over the ETTRs and MTTRs of these 10,000 experiments.

In Figure 3, we show the ETTRs as a function of the Jaccard index when \(N = 64\), \(n_1 = n_2 = 15\). As \(J = \frac{n_{1,2}}{n_1 + n_2 - n_{1,2}}\), this figure can also be interpreted as a function of the number of common channels. As shown in Figure 3, increasing the Jaccard index decreases the ETTR for all these three algorithms. Moreover, we find that LSH2 has a great improvement compared with the random algorithm and SynMAC. Specifically, the LSH2 algorithm achieves 43% improvement compared with SynMAC. As expected, the experimental results of the random algorithm are very close to the theoretical values, i.e., \(n_1 n_2 / n_{1,2}\). While the ETTR of LSH2 is slightly higher than the derived theoretical value \(1/J\) in Theorem 1. This is because the number of channels \(N\) is only 64. We also conduct simulations for other choices of channel numbers, including \(N = 128, n_1 = n_2 = 30\) and \(N = 256, n_1 = n_2 = 60\). When the number of channels increases from 64 to 256, the simulation results confirm that the ETTR approaches the theoretical value \(1/J\).

In Figure 4, we compare the MTTRs of the LSH algorithm and the LSH2 algorithm with the random algorithm and SynMAC. As shown in Figure 4, the (measured) MTTR of the random algorithm is much larger than the other algorithms as it is not bounded in theory. We also find that the MTTR of LSH is slightly larger than SynMAC when the Jaccard index is small. It is because LSH uses sampling with replacement, and it is more likely to repeatedly select the same value of \(U(i)\) when the Jaccard index is small. By using sampling without replacement in LHS2, the MTTR of LSH2 is greatly improved, and it outperforms SynMAC for all the selections of the Jaccard index. To conclude, our numerical results show that there is a roughly a 41% to 49% reduction in the ETTR and MTTR for LSH2 when compared with SynMAC.

B. Numerical results in the asynchronous setting

In Figure 5 and Figure 6, we compare the ETTRs and the MTTRs in the asynchronous setting with \(N = 256, n_1 = n_2 = 60\). For the LSH4 algorithm, we consider two parameter settings: (i) \(T_0 = 20\) and \(p = 0.5\), and (ii) \(T_0 = 20\) and \(p = 0.75\). As shown in Figure 5, the LSH3 algorithm outperforms the random algorithm when the Jaccard index is larger than 0.3. In particular, when the Jaccard index is almost 1, the ETTR of the LSH3 algorithm is only one-half of that of the random algorithm. On the other hand, for the two choices of \(p\), the LSH4 algorithm is significantly better than the random algorithm for the whole range of the Jaccard index in this experiment. We note that there are some discrepancies between the approximations for the LSH3 algorithm (from (8))
and the LSH4 algorithm (from (10)) and the simulation results. This is because the events $E_i$’s are not independent. Also, as shown in Figure 6, the MTTRs of the LSH3 algorithm and the LSH4 algorithm are smaller than that of the random algorithm.

One possible extension of our work is to embed the multiset into a periodic CH sequence with full rendezvous diversity. By doing so, the MTTR can still be bounded with high probability.

VI. CONCLUSION

In this paper, we used the locality-sensitive hashing technique for the multichannel rendezvous problem. For the synchronous setting, we proposed two algorithms: LSH in Algorithm 1 and LSH2 in Algorithm 2. We proved that the MTTR of LSH2 is bounded by $N$, and the ETR of LSH2 converges to $1/\theta$ when $N \to \infty$. Our simulation results show that LSH2 outperforms not only the random algorithm but also the best algorithm in the literature. For the asynchronous setting, we also proposed two algorithms: LSH3 in Algorithm 3 and LSH4 in Algorithm 4. LSH3 is a direct extension of LSH2, and it still outperforms the random algorithm when the Jaccard index is large. On the other hand, LSH4 uses the idea of dimensionality reduction to speed up the rendezvous process, and it outperforms the random algorithm in the whole range of the Jaccard index in our experiment. However, like the random algorithm, the MTTR of the LSH4 is not bounded.

Fig. 5: Comparisons of the ETRs in the asynchronous setting with $N=256$, $n_1 = n_2 = 60$.

Fig. 6: Comparisons of the MTTRs in the asynchronous setting with $N=256$, $n_1 = n_2 = 60$.

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