Versatile Design Protocol of Architectural Integration of Photovoltaics

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Abstract
By simply taking a fish-eye lens photograph once at the location, or performing the equivalent operation by computer graphics using geographical data of the location, one can see how much photovoltaic energy will be obtained in each season there. No restriction is required regarding the layout design of photovoltaic cells or installation location. As an eventual discovery, a theory of design rules pertaining to PV integration, which is characterized by “inner symmetry” was developed, as the integrated PV energy obtained remain unchanged towards any arbitrary rotation around an axis.

Keywords: photovoltaic; solar; versatile; design; architecture

Introduction
Photovoltaic (PV) is a typical renewable energy directly convertible from solar irradiation and its integration in the built environment is a new world wide trend in architecture.

Table 1 provides typical meteorological data of the Tokyo region. It suggest that more than half of the year, during the summer season, indirect irradiation energy from the sky surpasses direct irradiation energy from the sun.

Therefore, it is very important to establish a method of photovoltaic engineering which is able to provide reliable account of photovoltaic energy by indirect sky-irradiation as well as that by direct sunshine.

It is recognized that architectural PV integration can not be performed by separating design work from PV engineering. However, conventional method of PV engineering is unable to provide satisfactory answers for versatile architectural PV designs and location conditions, especially related to indirect irradiation from the sky.

The Versatile Design Protocol here provides an innovative method which makes possible such PV engineering of versatile roof design based upon insitu location conditions clearing architectural PV integration of its conventional uncertainties and design constraints.

As an eventual discovery, design rules pertaining to PV integration, which is characterized by “inner symmetry” were conducted on a theoretical basis, as the integrated PV energy obtained remains unchanged towards any arbitrary rotation around an axis.

One can install such a “PV system of inner symmetry” without having to give consideration to the azimuthal direction to which the whole of the PV system faces, which frees configuration design and installation works from the conventional azimuthal constraints.

Method
1) Sky-irradiation condition
A new methodology was developed to obtain the loc-
calized value of sunshine irradiation both from direct sunshine and indirect sky-irradiation at a specific location point where the PV system will be installed from regional data prepared by the authority.

In Fig. 1, a photovoltaic panel is placed at the point of origin “O” with its unit normal vector pointing to “n” in three dimensional space. The mesh-pattern is drawn on the surface of a concentric sphere so that sky-irradiation arrives at the surface of the photovoltaic cell through each elemental mesh becomes equal so far as the irradiation is spatially uniform over the sky.

Fig. 2a represents the virtual location of photovoltaic installation point surrounded by trees and a building. Fig. 2b is the supposed fish-eye view to be taken at the installation point to the zenith direction which is drawn by CG.

Figs. 3a, 3b, and 3c are superimposed images of fish-eye transforms of the same different directions over the same fish-eye view of a skyscape taken at the point of origin “O” in the zenith direction “Z” axis. The center of each web-like mesh locate the direction of the normal vector “n” respectively as inclined 15° from the zenith direction “Z”, which are rotationally symmetrical with each other.

The shadowed parts in the field of the fish-eye view represent images of surrounding objects such as building and trees which intercept the irradiation from the shadowed parts of the sky window to arrive at original point “O”.

One can measure the portion of the sky-irradiation energy arriving at the surface of the inclined solar cell as located by dividing the number of elemental meshes open to the sky “Nopn” in the field of the superimposed fish-eye view by the total number “N” of spatial elements in the meshed pattern as drawn over half of the sphere.

The value of sky-irradiation energy \( E_{sky} \) that arrives at the surface of the solar cell is calculated like as:

\[
E_{sky} = \left( \frac{N_{opn}}{N} \right) E_{HSKY}
\]

where \( E_{HSKY} \) is the value of the sky-irradiation energy that arrives at the horizontal plane located in plain field in the same region which is usually provided among the meteorological data issued from the authority.

The reflex flux from the surroundings can be well defined in the same manner as sky-irradiation based on the following assumption:

1) The reflex may be supposed to be completely diffusive.
2) The reflectance of the surrounding surface (albedo value) is given and the input energy on the surface is provided or well calculated.

2) Drawing the mesh-pattern

We introduce a new concept of spatial property around the axis “n” over the concentric circular area on half of the sphere called “coaxial solid angle” to be defined as:

\[
W = \frac{\pi (1 - \cos 2 \theta')}{2}
\]

We then draw concentric circles around the axis “n” on the sphere so that each difference in the value of W between the next enclosure circles become equal.

Thereafter, iso-gonic longitudinal division should be drawn around the same axis “n” on the sphere so that we can obtain “iso-projective mesh pattern” around the axis “n” over the sphere as shown in Fig. 1.

The fish-eye transformed image of the mesh pattern on the sphere over the fish-eye view taken at the point of origin “O” can be drawn by computer graphics as shown in Figs. 3a, 3b, and 3c.

3) Direct sunshine conditions

The location of direct sunshine which arrives at the point of origin “O” could be seen by superimposing fish-eye transforms of the solar orbits over the same fish-eye view as shown in Fig. 3d.

The direct sunshine irradiation conditions are given by insulation duration by solar time on each orbit which can be read by the hourly solar time scale from the meridian point as drawn in Fig. 3d. The location conditions and eventual PV is summarized in Table 2.

4) Inner symmetry of irradiation

A certain solar roof consisting of several oblique planes with their unit normal vectors \( n_1, n_2, \ldots, n_n \) (hereinafter referred to as “roof vectors”) is to be considered, where the following formula (1) should be realized by choosing the appropriate positive values;
Table 2. Location conditions and estimated PV energy at the location of Fig. 2a

| Location                        | Virtual place in Tokyo N35°45" as in Fig. 2a |
|---------------------------------|---------------------------------------------|
| **Seasonal Date**               | Winter solstice                             | Summer solstice |
| Regional data                   |                                             |                |
| Atmosphere transparency         | 0.8                                         | 0.63           |
| Direct irradiation on horizontal plane (MJ/m2.D) | 4.7                                         | 5.4            |
| Direct irradiation on normal plane (MJ/m2.D) | 11.8                                        | 6.7            |
| Sky irradiation on horizontal plane (MJ/m2.D) | 3.3                                         | 9.4            |
| Sunshine ratio (%)              | 53.5744                                     | 22.4193        |
| **Location conditions**         |                                             |                |
| Plane                           | ROOF-1                                      | ROOF-2         | ROOF-3         |
| PV capacity (W)                 | 10                                          | 10             | 10             |
| Inclination angle (°)           | 15°                                         | 15°            | 15°            |
| Azimuthal direction             | N                                           | E60°           | W60°           |
| Direct sunshine condition       | -2:30 ~ 2:30                                | -6:00 ~ 6:00   |
| Sky-irradiation condition       | 0.8111                                      | 0.85           | 0.7861         |
| **PV powers**                   |                                             |                |
| Direct sunshine PV power (W * Hrs/D) | 5.332                                        | 12.591         | 12.608         | 11.661 | 12.807 | 11.684 |
| Sky-irradiation PV power (W * Hrs/D) | 6.798                                        | 7.124          | 6.588          | 20.419 | 21.398 | 19.790 |
| Total irradiation PV power (W * Hrs/D) | 12.130                                       | 19.715         | 19.196         | 32.08  | 34.205 | 31.474 |

Fig. 3. Sky irradiation conditions and direct sunshine conditions
\[ \mu_1, \mu_2, \ldots, \mu_\nu, \text{ and } \mu^* \]
\[ \mu_1 n_1 + \mu_2 n_2 + \ldots + \mu_\nu n_\nu = \mu^* \varepsilon_z \quad \text{---(1)} \]
\( \varepsilon_z \) is the unit vector of “Z” axis which, in other words should be obtained from positive synthesis of the roof vectors.

\[ e_z \text{ is the unit vector of “Z” axis which, in other words should be obtained from positive synthesis of the roof vectors.} \]

As indicated in Fig. 4, \( \Delta s \) is a small elemental sky-area with its normal vector \( \Delta \sigma \) having its norm \( \| \Delta \sigma \| = \Delta s \), and \( \Delta \omega_1, \Delta \omega_2, \ldots, \Delta \omega_\nu \) are its orthogonal projections on the roof planes, and \( \Delta w \), upon the horizontal plane.

Then the following linear algebraic product:
\[ V(n_1, n_2, \ldots, n_\nu) \Delta s = \mu_1 \Delta \omega_1 + \mu_2 \Delta \omega_2 + \ldots + \mu_\nu \Delta \omega_\nu \quad \text{-----(2)} \]
will be introduced.

Whereas the value of (2) becomes \( \mu^* \Delta w \) as:
\[ V(n_1, n_2, \ldots, n_\nu) \Delta s = \mu^* \varepsilon_z \Delta \sigma = \mu^* \Delta w \]

Also, it remains unchanged towards any continuous rotation around “Z” axis standing for:
\[ V(\Psi n_1, \Psi n_2, \ldots, \Psi n_\nu) \Delta s = V(n_1, n_2, \ldots, n_\nu) \Delta s \quad \text{-----(3)} \]
where \( \Psi \) is a rotation matrix around “Z” axis by rotation angle \( \phi \) as expressed:
\[ \Psi = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ V(\Psi n_1, \Psi n_2, \ldots, \Psi n_\nu) \Delta s = \]
\[ (\mu_1 \Psi n_1 + \mu_2 \Psi n_2 + \ldots + \mu_\nu \Psi n_\nu) \Delta \sigma = \]
\[ \Psi (\mu_1 n_1 + \mu_2 n_2 + \ldots + \mu_\nu n_\nu) \Delta \sigma = \]
\[ \Psi \mu^* \varepsilon_z \Delta \sigma = \mu^* \varepsilon_z \Delta \sigma \]

Obviously, \( \Psi \varepsilon_z = \varepsilon_z \), therefore,
\[ = \mu^* \varepsilon_z \Delta \sigma = \mu^* \Delta w \]

Namely, the sum of the value of the projected elemental area of sky on the roof planes remains unchanged towards any continuous rotation \( \Psi \) around the “Z” axis.

This result holds true even though negative values of the inner products incidental to the rotation are allowed.

In geometry, the negative value of the inner product means the elemental irradiation arrives on the reverse side of the plane, however, in PV integration, negative value of the inner product should partake a negative value of PV energy when irradiated on the reverse side of the PV cell.

In other words, while this principle should be applied to the PV system so that all values of the inner products remain positive in rotation, the value of the inner products all remain positive when every surface of the roof planes can be seen from \( \Delta s \) irrespective of any azimuthal rotation of the roof around the “Z” axis.

The nature of this “inner symmetry” regarding the elemental irradiant holds true when integrated over the domain where every surface of the roof planes can be seen placed in any azimuthal direction, and hereinafter, we call such a domain on the sky-sphere the “common visible domain”.

To materialize such a nature in PV properties on the roof, PV cells with their capacity balance corresponding to each coefficient value \( \mu_1, \mu_2, \ldots, \mu_\nu \) are placed on each plane.

Obviously, the solar roof being of an “inner symmetry” in nature does not necessarily have to be geometrically symmetrical as shown in Fig. 5 around the “Z” axis.

If the roof is geometrically symmetrical as any of Fig. 5, the value of coefficients becomes equal and PV cells of equal capacity should then be placed on each planes of the roof.

The context heretofore holds true in the case of direct sunshine irradiation by replacing \( \Delta \sigma \) by “sun vector” which locates the spatial direction of the sun.

5) An acquired nature of “inner symmetry”

The integrated PV energies both of direct and indirect irradiation remain unchanged towards any continuous irradiation.

\[ a \quad b \quad c \]

Fig. 5. Geometrically symmetrical solar roofs

Common visible domain

Fig. 6. Common visible domain and irradiation conditions inside it
Fig. 7. Photovoltaic sign

Fig. 8. Conceptual plan of Fishery Harbour, its bird’s-eye view and several view angles
rotation around the “Z” axis although the geometric symmetry of the roof is lesser, for example in Fig. 5, two times for Fig. 5a and Fig. 5b, and three times for Fig. 5c.

The integrated PV energy as inner property of the system thereby obtains a nature of “continuous symmetry” around the “Z” axis, as an acquired hidden nature.

6) Irradiation conditions replaced by horizontal plane

The solar roof of our concern having a nature of “inner symmetry” should consists of several geometric oblique planes in the form of a polyhedron rather than the obvious single horizontal plane.

However, physically, it is possible to treat such a polyhedral solar roof having a nature of “inner symmetry” as a single horizontal plane so far as irradiation within the “common visible domain” is concerned, which holds true both for direct sunshine irradiation and indirect sky-irradiation.

With regards to “direct sunshine”, one can calculate the minimum PV energy obtained from the polyhedral solar roof from that of a single horizontal plane based upon the direct irradiation condition within the “common visible domain”.

With regards to “sky-irradiation condition” of the polyhedral roof, one can provide its minimum value by replacing that of the horizontal plane within the “common visible domain” such as replacing those of Fig. 3a, 3b, and 3c with that of Fig. 6a.

In either case, the substitutive value of the PV capacity Ch assigned to the horizontal plane to replace those C1, C2, ..., Cn on the polyhedral roof is assumed as follows:

\[ Ch = C_1 \cos \lambda_1 + C_2 \cos \lambda_2 + ... + C_n \cos \lambda_n \]

where \( \cos \lambda_1, \cos \lambda_2, ..., \cos \lambda_n \) are the value of the inner products between \( \mathbf{e}_z \) and each unit roof vector.

**Table 3. Estimated PV Energy of Fishery Harbor Around “Seto Nai Kai Sea”**

| Season       | Summer solstice | Spring equinox | Autumn equinox | Winter solstice |
|--------------|-----------------|----------------|----------------|-----------------|
| Atmosphere transparency | 0.66 | 0.71 | 0.71 | 0.79 |
| Direct sunshine on horizontal plane (M3n=2.D) | 7.5 | 7.4 | 7.1 | 6.3 |
| Direct sunshine on normal plane (M3n=2.D) | 9.7 | 12.1 | 10.9 | 10.1 |
| Sky-irradiation on horizontal plane (M3n=2.D) | 9.6 | 6.7 | 7.3 | 3.6 |
| Sunshine ratio (PV) | 30.56 | 38.25 | 38.92 | 43.74 |
| Roof unit | 38 planes 6 planes 15 planes | 38 planes 6 planes 15 planes | 18 planes 4 planes 15 planes | 18 planes 4 planes 15 planes |
| Inclination angle (°) | 22 15 18 | 22 15 18 | 22 15 18 | 22 15 18 |
| Sky-irradiation condition | 0.8860 0.9330 0.9806 | 0.8860 0.9330 0.9806 | 0.8860 0.9330 0.9806 | 0.8860 0.9330 0.9806 |
| Direct sunshine condition | -5.20 -5.30 -6.30 | -4.10 -4.40 -5.20 | -4.10 -4.40 -5.20 | -2.15 -3.20 -4.00 |
| Solar cells mounted | 64 19.2 4.5 | 64 19.2 4.5 | 64 19.2 4.5 | 64 19.2 4.5 |
| Total PV (KW/hr/D) | 38,372 9,480 | 38,372 9,480 | 38,372 9,480 | 38,372 9,480 |
| Total energy (KW/hr/D) | 20.36 34.57 8.444 | 20.36 34.57 8.444 | 20.36 34.57 8.444 | 20.36 34.57 8.444 |
| Number of building units | 6 2 46 | 6 2 46 | 6 2 46 | 6 2 46 |
| PV in total building units (KW/hr/D) | 127,714 162,744 927.13 | 107,889 137,604 785.588 | 105,428 134,946 771.098 | 51,006 72,798 423.476 |
| PV all total (KW/hr/D) | 1217.788 | 1030.856 | 1011.464 | 547.28 |

**Installation example**

Fig. 7 shows one of the installation example of such PV sign systems having a nature of “inner symmetry” designed by the author and installed in Showa Kinen Park by the Japanese Government.

Fig. 8 shows a conceptual plan of “Fishery Harbor”, the roofs of its built facilities are composed of unit PV roofs each having a nature of “inner symmetry” and the total capacity of solar cells mounted is estimated to be about 277.8KW with its PV energies as calculated in Table 3 using meteorological data around “Seto Nai Kai Sea”.

**Conclusion**

Architectural integration of PV will be provided with powerful engineering tool by this Protocol enabling versatile design at any location and under all conditions, which will bring about both freedom and beauty in architectural configuration as well as performance reliability.

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The author is grateful for having been given the opportunity to perform design and engineering of the PV systems for Japanese Government implementation such as PV signs and lightning.

Also it is a great honor for the author to receive “NEW ENERGY AWARD” of the year of 2001 from the “NEW ENERGY FOUNDATION” for the development of the “Versatile Design Protocol”.

**References**

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