Coupled Oscillator Dynamics in Brain EEG signals: Manifestation of synchronization and Across Frequency Energy Exchange by Neutral Turbulence

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ABSTRACT Phenomenological models, involving coupled oscillators of both deterministic and stochastic nature, have been invoked to describe the behavior of the EEG signals, with experimental validation. These models derive justification from the fact that the basic electronic unit for potential generation, incorporating capacitive, inductive, and resistive components, as well as the source, is modelled as damped driven oscillator, which can behave independently and may also exhibit synchronized dynamics with others, that can manifest in spatially correlated behavior observed in brain dynamics. Here, using a phase-space approach, we show clear evidence of oscillator dynamics to wave packet collapse and revival in the brain EEG signals, with distinct differences in healthy individuals and epileptic patients. The low frequency components exhibit single oscillator type behavior in a phase-space description, with the potentials as coordinates and their instantaneous changes as the corresponding velocities. The closed paths reveal periodic motion, well described by one linear oscillator or possibly coupled ones with limit cycle dynamics and synchronization at the macroscopic level. The epileptic patients reveal dynamical features of bistability, originating from non-linearity, a prominent feature in the signals from the epileptogenic zone and much enhanced during the periods of seizure. Analogy from the phase-space of oscillator dynamics reveals dominance of potential energy in the signals from the epileptogenic zone, as also in the patients during occurrence of seizure. The acceleration, arising from the change of velocity, is found to be particularly strong in epileptic patients, when the phase space shows bi-stability and bursty behavior. Wave behavior with characteristics of superposition emerges in the frequency range corresponding to the observed unstable periodic orbits that appear at 8-14Hz, centered at 10Hz. A modulated carrier wave is observed for all subjects at 18Hz, higher in width for patients. Coherent wave dynamics, with interference playing a key role in the wave packet collapse and revival, is observed starting from 18Hz, with the coherence getting significantly enhanced around 40-45Hz. Mechanism of intra frequency energy transfer is shown to be neutral turbulence.

INDEX TERMS Brain EEG signal and Epilepsy; Damp driven oscillator and Limit cycle; Wave packets and Coherence; Fourier power spectra and Unstable periodic orbits; Heisenberg Model and Neutral Turbulence

Significance Observation of oscillator behavior and wave phenomena characterized by superposition, wave packet collapse and revival are unequivocally established through a phase space approach, involving potential and its instantaneous changes, traditionally employed in studying electrical circuits. Clear observation of bistability, bursty dynamics, unstable periodic orbits and strong intra-band energy transfer through neutral turbulence are shown. Wave packet collapse and revival in brain EEG signals, with carrier wave modulation
is demonstrated, unveiling the domain of mono frequency oscillations, interference, and emergent coherent wave packet dynamics. The instantaneous approach reveals finer structures in phase space, as compared to global nonlinear time series analysis through embedding dimension, often hampered by insufficient data length and boundary effects.

I. INTRODUCTION

Human brain is central to all cognitive and sensory stimuli that control the body functions, producing complex biopotential signals. Dynamical behavior of the brain is reflected in the recording of the electroencephalogram (EEG) signals, a much-used approach to measure the neurophysiological activity of the brain in the form of electrical signals [1-5]. Seizure is a brain disease which occurs spontaneously, due to abnormal electrical activity occurring temporarily in nerve cells. Nearly 0.6–0.8% of the world’s population is reported to be suffering from epilepsy, with symptoms showing seizures [6]. Usually, EEG signals contain multiple frequency bands: the frequencies of the gamma waves are greater than 30Hz, those of the beta waves range from 13-30Hz, with alpha lying in 8-12 Hz, theta in 4-8 Hz, and delta being less than 4 Hz. During resting state, meditation or quiet periods of the brain, alpha waves are the dominant modes. It is reported that different brain states can be characterized by the occurrence of distinct oscillatory patterns from the EEG recordings in different frequency bands [7].

The measured brain potentials have been physically modelled earlier to be originating from a single oscillator at low frequency or multiple coupled ones, possibly with non-linearity at higher frequency range, leading both to synchronization and chaotic dynamics in different physiological conditions [8-12]. These oscillators can, on occasions, behave independently, with weak mutual coupling and experience non-linearity as has been observed experimentally through the phenomenon of bifurcation [13]. Non-linear time series analysis has been employed to infer about the fractal characteristics and to identify the effective degrees of freedom through the phenomenon of phase space recurrence and the largest Kolmogorov exponent, showing the diverging trajectories underlying chaotic behavior. The fact that the basic electronic unit for potential generation, incorporating capacitive, inductive, resistive components and a source, is a driven oscillator, has led to the use of the brain potential and its changes in time as coordinate and velocity, modelled, as mentioned above, by single or coupled oscillators.

As is well known, in nature, the dynamical behaviors are separated into two categories: particle and wave dynamics. Classical particle dynamics is described by Newtonian laws in terms of trajectory, well described in phase-space using coordinates and their instantaneous changes, the momenta. The simplest example of particle motion is that of a spring described by Hooke's law, resulting in mono frequency sinusoidal motion. Its generalization, the damped driven oscillator, finds much application in describing diverse physical phenomena, including those of potential generating electrical circuits, the key elements in brain dynamics. Coupled oscillators can show synchronized mono frequency behavior, resembling a single oscillator as also that of limit cycle.

Synchronization of localized oscillators can lead to collective spatial dynamics resembling wave motion. The wave dynamics is characterized by superposition, showing interference effects. This leads to an increase in amplitude, when waves interfere constructively and its decrease, arising from destructive interference, collapse and revival of wave packets being one such physical manifestation and the modulation of a carrier wave with higher frequency waves being another. All of these dynamical phenomena can be inferred through a phase-space approach by studying the brain potential values and its changes as the corresponding velocities (momenta for unit mass). Fourier space power spectra can faithfully reflect both periodic dynamics and self-similar behaviors, as also the unstable periodic orbits [14-16]. There are several studies devoted to the analysis of multiscale neuronal behavior [17-19]. Often these analyses use sophisticated nonlinear models and methods of study, whose accuracy may be curtailed due to finite data length and the intrinsic limitation of the model itself. Therefore, it is pertinent to employ reliable methods, well tested in diverse systems. It has been pointed out earlier that the brain dynamics at a global level may be simple enough to be tractable [20-26]. Neuronal oscillations in brain results into complex motion when studied using the EEG signals in a Phase space structure hence using a local approach here would bring out more nonlinear information from particle like motion or from wave behavior with use of nonlinear time frequency analysis. This is the primary objective of this research work.

This motivates us to take recourse to a local approach, where in the voltage and the change in voltage is considered as position and corresponding velocity, akin to the description of LCR circuit generating electrical voltages. The well-studied nonlinear harmonic oscillator model represented by the ordinary differential equations of the electrical LCR circuits is used in our analysis for equivalent representation of the complex neuronal oscillations through its EEG recordings. Hence the instantaneous phase-space approach used in this work reveals this behavior quite
transparently, as well as the subtle differences between the patients and healthy individuals. In the Fourier domain, one observes unstable periodic orbits (UPOs), earlier evoked for explaining observed features in brain dynamics. Significantly, the UPO is found to mark the emergence of wave dynamics, characterized by carrier wave modulation and wave packet collapse and revival. The time frequency analysis using Morlet wavelets precisely points out the local bursty behavior and the neutral turbulence being the mode of intra frequency energy transfer. The dynamics of the brain activity can evolve to either a fixed-point attractor, corresponding to a normal brain or to a limit cycle attractor corresponding to the pathological spike wave discharges observed in epileptic brains [27-29]. In the following section 2, the data description is presented along with our analysis methods. Section 3 discusses the results and highlights the observed particle and wave behavior characteristics for healthy and patients during seizure. Section 4 concludes the study with our inferences.

II. Methodology
For our analysis the EEG time series data of different subjects are obtained from the Department of Clinical Epileptology, University Hospital of Bonn, Germany [30]. The time series data consists of five sets, where first two datasets are extra-crannial recording of healthy volunteers during eyes open and eyes closed, whereas the intracranial recordings of patients during seizure free interval are obtained both from the hippocampal formation of the opposite hemisphere of the brain and recordings from within the epileptogenic zone. The fifth dataset is the recordings from patients obtained during the seizure activity. Each set contains 100 single channel EEG segments of 23.6 sec duration with each having 4097 recordings of electrical signal measured in μV. These segments were selected and cut out from continuous multichannel EEG recordings after visual inspection for artifacts by a subject expert. The data acquisition system sampling rate is 173.61 Hz. Since the selected patients belonged to different socioeconomic and age groups, we carried out the mean subtraction for each single channel signal to remove the average part of the signal and then normalized it by dividing the standard deviation for further analysis. Various methods have been developed and used on this dataset to characterize and identify seizure [31-32]. Evidently, for higher accuracy and use of AI tools for seizure prediction, more feature identification will be useful. The features identified and discussed in this paper are based on a local analysis of potentials and their changes, which finds useful while studying the behavior of LCR circuits, and hence can augment other markers.

For phase space analysis, we used the normalized EEG data as displacement and its rate of change as velocity that helps understand the system dynamics from a basic constituent component of physical motion, familiar from the study of LCR circuits, through damped driven oscillators. The phase space plots were generated from the EEG time series recordings with electrode potential values from the recording in the x-axis and the corresponding derivatives in the y-axis. These plotted trajectories reveal oscillatory patterns, periodic motion, UPO structures akin to well-studied electrical potential generating circuits characterized by the ordinary differential equation:

\[ x'' = -\omega^2 x, \text{ where } x'' = \frac{d^2 x}{dt^2} \text{ and } \omega^2 = k/m \]
\[ \ldots (1) \]

Diverse physical systems were described using equation (1) augmented with different damping, driving, and non-linear anharmonic terms. The observed oscillatory behavior is analogous to that of harmonic oscillator patterns that is known from Hooke's law.

\[ F = -kx \]  
\[ \ldots (2) \]

Here k is the spring constant and F is the acting force on the oscillator where x is expressed in the form, 
\[ x(t) = A\cos(\omega t + \varphi) \ldots (3) \]
Amplitude ‘A’ is controlled by the energy of the oscillations, 
\[ E = 1/2 \ kA^2. \]

Further to ascertain the unstable periodic orbit, the cumulative sum of the normalized data is used in the Fourier power spectral analysis. Nonlinear time series representing complex system behavior is characterized by unstable periodic orbits:

\[ P \subset \mathbb{R}^N, N \in \mathbb{N} \text{ and } x = x^1, x^2, \ldots x^N e^P, n \in \mathbb{Z} \ldots (4) \]

Where P is any number while x is the measurement set of a system.

\[ M: P \rightarrow P, x_{n+1} = M(x_n) \ldots (5) \]
Here generally equation (5) is considered as the equation of motion of a dynamically evolved system with x being the measurement set of the system, and M is the manifold of the dynamical system [33]. Further to understand the unstable periodic orbits and different frequency ranges like 18Hz, 40Hz, 40-45Hz, we performed an inverse Fourier transform, retaining the power for that region of interest, while removing other ranges. The results revealed many interesting coherence phenomena at various frequency ranges, showing distinct differences for the disease and healthy subjects. The reconstructed signal excluding the UPO region was used in our analysis to study the energy transfer phenomena for disease and healthy subjects. Various qualitative characteristics observed during the analysis are discussed in the subsequent sections.
III. Results and Discussion

We first demonstrate the oscillator behavior in the low frequency components of the EEG signals. The phase space description reveals mono-frequency modulations, representable by one linear oscillator at the macroscopic level. The epileptic patients show dynamical features of coupled oscillators, with non-linearity manifesting in the observed bifurcation and limit cycle behavior in phase-space. In particular, the bistability is found to be stronger in the epileptogenic zone and during the periods of seizure. The phase space plots for the normalized first channel recordings shown in Fig 1 as a characteristic example, clearly shows the existence of single particle oscillator-like behavior at macroscopic level with closed orbits of varying size. The phase space plots in Figs 1c, and 1d possibly reveal limit cycle and bi-stability behavior, stronger in the epileptogenic zone and during the periods of seizure. It is worth emphasizing that secondary limit cycle behavior during the seizure is the characteristic feature earlier reported for rats [26]. The multiple converging loops in the phase space shows the multi-periodic nature of the dynamics, with the random phase change of the periodic orbits indicating the intermittency behavior of the subject under study. The limit cycle behavior at each of the brain frequency bands appears to provide a more accurate representation of the EEG signal than the one based on chaotic phenomena.

It is worth emphasizing, our introduced local approach of phase space analysis used to describe the oscillating particle behavior of the EEG signal represented in Fig 1 corroborates with the earlier reported various global approaches [8-9,11,34-35]. From Fig 1, it is observed that the phase space plot for the epileptogenic zone is prolonged with compact width, indicating dominance of potential energy. The release of the same may be an indication of bursty behavior as observed during the period of seizure. The energy stored here is possibly getting transferred to intra frequency bands of the brain, as observed and reported in this work. From Fig 1g, the observed acceleration in phase space reveals the large amplitude bursty behavior for the epileptic case, with higher potential and lower kinetic energy as also the phenomena of inter band energy transfer. It is worth emphasizing that our phase space analysis is an instantaneous approach, akin to the one traditionally used in mechanics, unlike the phase space study in quantifying recurrence through embedding time delay and dimension [12]. The observed bounded smaller variations in velocity, the bursty behavior arising from its sudden changes resulting in strong acceleration for the epileptic case may have physical significance in modelling finer aspects of seizure dynamics. As is well known, the transition from periodic motion to chaotic behavior can occur through several routes, like bistability in dynamical systems or due to the presence of short unstable periodic orbits separating these two motions. The unstable periodic orbits manifest in the Fourier power spectrum as a local enhancement of power at certain frequencies. For this purpose, we analyzed the Fourier power spectrum of the data sets, as shown in Fig 2. The analyzed time series is the accumulated fluctuations obtained from the cumulative sum of the series after subtracting the mean and normalizing the raw data. The Fourier power at higher frequencies showed power law behavior: \( P(f) \sim f^{-\alpha} \), as has been noted earlier.
(d) Phase space plot of patient's signal during the period of seizure shows behavior like that of a coupled oscillator with emergence of limit point. The inset figure shows the same with only 200 data points, the arrow showing its evolution.

(e) From the phase space analysis of other representative channel (40th) for patients during the seizure, bimodal structures are observed for a few orbits which are spread out, while most of them are tightly packed, showing significant increase in potential energy.

(f) For patients during seizure from the 40th channel recordings of coordinate, velocity and acceleration in phase space 3D reveal much less spread in velocity, rapid irregular variations in a much bigger fraction of closed orbits and much more potential energy as compared to Fig 1c.

(g) For patients during seizure from the 1st channel recordings between the coordinate and acceleration depicting the increase in the potential energy and corresponding decreased kinetic energy, possibly indicative of a limit cycle behavior during the period of seizure.

FIGURE 1. Phase Space diagram for different subjects.

We report here the presence of short unstable periodic orbits (UPOs) at 8-14Hz brainwaves from the Fourier power spectra in EEG time series. In the power spectra, one observes the presence of small peaks in the mid frequency domain for the healthy subjects, not present in other samples, due to dynamical instability from the short periodic orbits [36,37] or the so-called unstable orbits. It is worth emphasizing that these periodic orbits via the Gutzwiller formalism play an important role in determining the spectrum in the semi-classical approach [38]. Further, it has been reported that these short time periodic orbits determine the long-range spectral properties, and these are non-universal and system specific in nature [39]. Observed unstable periodic orbits of EEG recordings from the Fourier domain in Figs 2a, 2b reveal the presence of instability in normal brains and interestingly the instability appears predominantly at the low frequency alpha wave centered at 10Hz with the band spreading from 8Hz to 14Hz. The absence of the same in patients during seizure is discernible in the EEG signals, reported in this work. These deviations are observed from the power enhancement in the mid frequency domain in cases of healthy subjects and suppression of the same in the epilepsy case. The presence of short time periodic orbits highlights prominent features of the signals, as these are specific to certain functionalities in the brain [40]. Further, we analyzed the nature of unstable periodic orbits by reconstructing the signal from the Fourier transform of the UPO frequency region. The subsequent wavelet analysis, using the Morlet wavelet, having a Gaussian window with commensurate sinusoidal sampling functions, identified the wave front modulation as well as collapse and revival. Complex wavelet functions extract information about both amplitude and phase and are better adapted for capturing oscillatory behavior whereas a real wavelet function yields only one type of information and are used to isolate peaks or discontinuities. Therefore, to capture the structured variations, oscillatory nature, and time varying information, Morlet wavelet with a gaussian window and sinusoidal sampling function compliments for our analysis of EEG signal and is a natural choice. The rapid convergence in the scalograms i.e., the CWT coefficients further justified our choice [48]. From Figs 2c, 2d, the presence of wave packet collapse and revival is seen in the frequency range 8-14Hz from our analysis, lying in the domain of alpha and beta brain waves. Reduced dominance of alpha and beta brain waves for epilepsy patients, as compared to healthy subjects and the oscillatory region dominance for these frequencies possibly indicates a transition from single particle to limit cycle behavior. Changes in this frequency range of the brain also reveals intra frequency transfer of energy becoming prominent during periods of seizure.
(b) From patient’s (during seizure) Fourier power spectra the reduction in UPO is discernible and a characteristic feature for subject state classification.

(c) Reconstructed signal of the UPO range with time frequency localization scalogram for a healthy subject during eye closed showing strong oscillatory regions.

(d) Reconstructed signal of the UPO range with time frequency localization for a patient during seizure shows reduced oscillatory regions.

FIGURE 2. Fourier power spectra of all datasets, showing local power enhancement for healthy subjects in the mid-frequency domain, not present in the patient’s dataset, revealing a much broader frequency distribution with significant power. Also, from Figs 2c and 2d the reconstructed signal for the UPO region from Fourier power spectra shows oscillatory regions and bursty behavior.

(b) Reconstructed signal at 18Hz for healthy subject (eye closed) showing wave packet dynamics with reduced carrier wave modulation.

(c) Reconstructed signal at 18Hz showing wave packet dynamics with higher power observed from time frequency localization for patients (no seizure) signal from hippocampal zone.

(d) Reconstructed signal at 18Hz showing wave packet dynamics, wider carrier wave with higher power observed from time frequency localization for patients (no seizure) signal from epileptogenic zone.

(e) Reconstructed signal at 18Hz depicts wave packet dynamics with much higher power observed from time frequency localization for patients during the period of seizure. The carrier wave modulation gets wider for patients suffering modulations.
from seizure in comparison to healthy subjects.

(f) Reconstructed signal at 40Hz depicting stronger interference and wave packet dynamics, compared to that of 18Hz for healthy subjects (eye closed).

(g) Reconstructed signal at 40-45Hz showing stronger interference as compared to 18Hz for healthy subjects (eye closed) with manifestation of wave superposition.

(h) Reconstructed signal at 40Hz showing stronger wave packet dynamics with higher power observed from time frequency localization for patients during seizure compared to healthy subjects.

(i) Reconstructed signal at 40-45Hz showing stronger interference and wave packet dynamics with higher power, faster superposition of waves observed from time frequency localization for patients during seizure.

FIGURE 3. Reconstructed signal at 18Hz showing wave packet dynamics with carrier wave modulation showing energy transfer. Figs. from 3f to 3i show reconstructed signals at 40Hz and 40-45Hz showing stronger wave packet dynamics.

Coherence wave packet dynamics, characterized by collapse and revival, is observed starting from around 18Hz, clearly indicated from the Fourier domain in Figs 3a to 3e. The unambiguous wave packet dynamics is observed, with bursty power patterns visualized from the time frequency localization through wavelets, with higher power for patients during seizure as compared to healthy subjects. One also observes carrier wave modulation, visible for all subjects at 18Hz with higher width for patients showing wave energy transfer in the intra frequency domain. The observed wave phenomena with varying amplitude are being ascribed as modulation of the carrier wave. The observed wave packet dynamics from the Fig 3 explains strength of the carrier wave modulation at different brain frequencies reconstructed from the Fourier domain. It has been earlier reported that fast oscillations with frequencies >40 Hz are a promising biomarker of the epileptogenic zone [25,27]. Oscillations in the gamma frequency band at 40 Hz of the EEG also play a critical role during behavioral wakefulness and cognition. Several experimental observations have been reported showing that neocortical oscillations at the gamma frequency (30–100 Hz) band, mainly around 40 Hz, are involved in cognitive functions [24]. Increase in gamma power typically appears during behaviors that are characterized by the cognitive processing of external precepts or internally generated thoughts and images. Gamma activity has also been observed during alert or attentive wakefulness, not only in humans, but also in animals. Well known coherence behavior is observed at the gamma wave region that connects the conscious to unconscious state of the brain. From our analysis in human EEG, it is observed from Figs. 3f to 3i that the coherence gets significantly enhanced around 40-45Hz.

(a) Heisenberg model’s energy transfer phenomena of neutral turbulence (K=-5/3) from the reconstruction signal without the UPO range 8-14Hz frequency for healthy (eye closed).

(b) Heisenberg model’s energy transfer phenomena of neutral turbulence (K=-5/3) from the reconstruction signal without the UPO range 8-14Hz frequency for healthy (eye closed).
Intra frequency energy transfer is observed, both at the junction of alpha and beta and beta and gamma waves, which is particularly prominent during the onset of seizure. The mechanism is established using the Heisenberg model, revealing that neutral turbulence is more prominent vis-a-vis viscous dissipation [41-44]. For quantification, we have analyzed the normalized global wavelet power with frequency in logarithmic scale and applied Heisenberg model fitting for both $K=5/3$ and $K=7$. The plots are shown in Fig. 4. The presence of neutral turbulence can be inferred through the Heisenberg model with $K=5/3$. It is prominent in comparison to the viscous dissipation regime $K=7$, as is observed from Fig. 4 of the supplementary material of this manuscript. The system is observed to make a transition at 10Hz. This 10Hz transition regime in the neutral turbulence is the domain of observed unstable periodic orbits, as is evident from the Fourier space study. This is the frequency where the transfer of power occurs that becomes prominent for patients during seizure. We also analyzed the energy transfer phenomenon [45-47], by reconstructing the time series from the Fourier space, excluding the UPO range 8-14Hz for use in the Heisenberg model. Once these characteristic frequencies are removed at 8-14Hz in UPO range, the universal scale free behavior is manifest in the Heisenberg model. The observed transition at 10Hz in the UPO range is identified as the energy transfer region, reported here. The fit of neutral turbulence is particularly good in the low frequency range up to 8Hz for almost all subjects. It is evident that neutral turbulence is the mechanism of intra frequency energy transfer, both at the junction of alpha and beta, beta, and gamma waves, that manifest during onset of seizure, when turbulence is observed to be significantly enhanced.

The used instantaneous phase space approach, which clearly shows the presence of a particle type single oscillator behavior at low frequencies, with closed trajectories in phase space, indicative of either oscillator dynamics or limit cycle behavior. Bi-stability and coupled oscillations manifest at still higher frequencies, both in the epileptic zones as well as during the periods of seizure. Prior to that, one observes a decrease in kinetic energy with simultaneous increase in the potential energy for both above cases. The wave behavior begins to emerge from 18 Hz and higher frequencies, showing interference and wave packet collapse and revival. The Fourier domain analysis clearly reveals these behaviors, as well as that of unstable periodic orbits in the boundary of regular and chaotic dynamics, the latter showing power law decay signifying self-similarity. The local time-frequency domain wavelet analysis reveals the nature of wave packet dynamics as carrier wave modulation and wave packet collapse and revival. The process of inter modal energy transfer is identified as Heisenberg’s neutral turbulence. The coupled synchronous behavior leads to spatial correlation indicative of energy exchange at different scales. This has been clearly seen in the Heisenberg energy transfer phenomenon. Further study of the rate of change of the velocity shows the presence of forces operative at time zones where rapid changes of velocity are observed. This process is found to be operative in a large frequency range for the normal brains and in the low frequency domain for the seizure patients.

IV. Conclusion

In conclusion, an instantaneous phase space approach introduced in this work for the study of EEG signals akin to the nonlinear model representation for motion of the electrical LCR circuit provides a unified picture of a diverse range of brain behavior. Manifestation of single particle oscillator like behavior at macroscopic level, progressively leading to limit cycle and bistability, as also wave packet collapse and revival, originating from the domain of unstable periodic orbits, opens up new avenues to explore and validate a number of features about brain dynamics surmised earlier. The observed particle behavior was well demonstrated through various distinct phase space structures for different subjects. The fact that some of these features are much stronger in the epileptogenic zone and during the periods of seizure for patients shows the potential use of this instantaneous method for prediction and medical intervention. Analogy from the phase-space of oscillator dynamics reveals significantly enhanced potential energy contribution in the signals from the epileptogenic zone, as also in the patients before the occurrence of seizure. Coherence wave packet dynamics, evident from collapse and revival, is observed here, that emerges around 18Hz inferred from the Fourier domain analysis of the phase space power-spectra. The Fourier analysis also reveals the unstable periodic orbits, observed around 10Hz for healthy subjects, indicative of the presence of instability. The observed coherence gets significantly strengthened around 40-45Hz as inferred from a local time frequency domain approach. Reported wave packet collapse and revival observed from reconstructed signals at 18 Hz, 40-45Hz from the nonlinear local approach is akin to...
semi-classical approach of UPO studied earlier via the Gutzwiller formalism as in the ref [38]. Our obtained results confirm the understanding. The energy transfer phenomena at higher frequency as observed in our study is critical to the understanding of the phase transition in the system from conscious to unconscious state of the brain. Neutral turbulence is established as the mechanism of intra frequency energy transfer through the Heisenberg model during the analysis. Use of local approach with nonlinear analysis and feature extraction in this work had resulted into the validation of the understanding of energy transfer phenomena and distinct evolved structures for healthy and patients during seizure.

APPENDIX

Supplementary material to this article can be found with this manuscript.

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The authors declare that they have no known competing financial interest or personal relationships that could have appeared to influence the work reported in this paper. The obtained data for our analysis is from free public database published by Department of Clinical Epileptology, University Hospital of Bonn, Germany [30]. As per the reference [30] the original data collection methods were carried out in accordance with relevant guidelines and regulations where all experimental protocols were approved by the University of Bonn’s ethics and licensing committee and written consents were obtained from all subjects by the author in ref [30] to make the data available in public for further research. The EEG data used in this analysis could be downloaded from http://www.epileptologie-bonn.de/cms/upload/workgroup/lehneretz/eegdata.html.

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