Introducing Transformer Degradation in Distribution Locational Marginal Prices

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Abstract—In this paper, we consider the day-ahead hourly operational planning problem of a radial distribution network hosting Distributed Energy Resources (DERs). We propose a tractable AC OPF formulation that optimizes DER schedules to minimize network costs including transformer degradation. Resulting spatiotemporal marginal costs provide Distribution Locational Marginal Prices (DLMPs) that can serve as price signals achieving system-wide optimal DER schedules. We show that sensitivity analysis unbundles DLMPs into additive marginal costs reflecting real/reactive power losses, voltage and ampacity constraints, and transformer degradation. We rely on an actual distribution feeder pilot study to compare optimal EV/PV schedules to popular open-loop scheduling options. Internalizing short term marginal asset degradation — focused on transformers — optimizes system cost, increases EV/PV hosting capacity and harvests their reactive power compensation capabilities.

Index Terms—Distribution Locational Marginal Prices, Short Term Transformer Degradation, Distributed Energy Resources.

I. INTRODUCTION

RAPIDLY growing Distributed Energy Resources (DERs), including clean, albeit volatile, renewable generation, combined heat and power micro generation, storage and flexible loads with storage-like properties and Volt/VAR control capabilities, e.g., Electric Vehicles (EVs) and Photovoltaic (PV) inverters, present a major challenge together with a still unexploited opportunity. With DERs emerging as a major user of distribution grid infrastructure, the grid is becoming increasingly active, distributed, dynamic, and challenging to plan and operate [1]. As such, DERs will have a profound impact on the adequacy of T&D assets, efficient grid operation, reliability, and security of supply. The key determinant of optimal DER scheduling will be their spatiotemporal value [2]–[4] that is expected to bring about fundamental changes in distribution planning and operation, as well as power markets. The aging of transformers is dependent upon thermal effects of loading. These works are mostly quantitative simulation studies considering various charging schedules and assessing their impact on transformer Loss-of-Life (LoL).

The life of a transformer is strongly related to its winding hottest spot temperature (HST) that drives insulation aging. Distribution transformer normal life expectancy is estimated at 20.55 years (180,000 hours), assuming operation at a continuous reference HST of 110°C (for insulation systems rated for a 65°C winding temperature on average). IEEE Standard C.57.91-2011 [13] and IEC Standard 60076-7:2018 [14] provide guidelines for transformer loading, detailed temperature calculations, and an exponential representation of the aging acceleration factor, i.e., the rate at which the transformer insulation aging is accelerated compared with the aging rate at 110°C. For HSTs in excess of (lower than) 110°C, the aging acceleration factor is greater (less) than 1.

Indicative results show that simple open-loop EV charging, e.g., delay until after midnight, can actually increase, rather than decrease, transformer aging [7]. They also show that the daily LoL of a residential transformer may almost double under a 50% EV penetration charging upon arrival [8].

Rooftop PVs, on the other hand, can have a beneficial impact on the transformer LoL [10], [11]. Results show that high PV penetration can significantly extend the life of distribution transformers in a suburban area [10]. More, while previous research determined insignificant synergy between the substation transformer LoL and charging EVs in the presence of rooftop PV (due to non-coincidence between peak hours of PV generation and EV charging), [11] shows that the transformer thermal time constant allows PV generation to reduce the transformer temperature when EVs are charging. Although the above works consider the DER impact on transformer aging, they do not include the transformer degradation in the optimization model. This is first outlined in [2]–[4], employing an exponential representation without considering transformer time constants. Two recent works use a linear approximation in [15] and [16] co-optimizing transformer aging with home energy management systems and EVs, respectively, while introducing binary variables; however, they do not consider the impact on the distribution network.

Emerging literature on capturing the network impact involves extending the concept of Locational Marginal Prices (LMPs) to the distribution network, hence producing prices that are commonly referred to as Distribution LMPs (DLMPs). In fact, there is a variety of approaches in defining DLMPs. In [17], DLMPs are determined in a social welfare optimization problem, using DC OPF, considering EV aggregators as price...
takers. Quadratic programming is used in [18] (as opposed to linear programming in [17]), to derive DLMPs that are announced to aggregators of EVs and heat pumps who optimize their energy plans. A linearized AC OPF model is employed in [19] for obtaining DLMPs that consider a reactive power price and voltage constraints. DLMPs are derived using an AC OPF formulation for radial networks in [20], which assumes that various DERs, such as distributed energy storage, distributed generators, microgrids, and load aggregators, can bid into a day-ahead distribution-level electricity market. [4] proposes one of the first AC OPF DLMP formulations incorporating transformer degradation. In a similar modeling approach, [2], [3], obtain real, reactive power and reserve DLMPs for radial networks and propose an iterative distributed architecture that captures the full complexity of DER inter-temporal preferences and physical system dynamics. A comprehensive analysis of various DLMP approaches is provided in [21].

B. Objectives and Contribution

Unlike a considerable body of the literature focusing on optimal open-loop DER scheduling, we focus on the derivation of time and location specific marginal costs of real and reactive power that are consistent with the optimal DER schedule in an adaptive/closed-loop sense. As such, we aim to discover distribution network time and node specific marginal costs in the presence of optimally responding DERs, and to understand and evaluate the components/building blocks/sources that constitute the marginal costs.

The contribution of this paper is three-fold: (i) we present a tractable formulation of the AC OPF operational planning problem for radial networks with a detailed cost representation of real/reactive power and transformer degradation in the presence of DERs (EVs, PVs); (ii) we derive DLMPs based on the spatiotemporal marginal cost, and relate them to price signals for optimal self-schedule of DERs, and (iii) we explore a realistic test case of an adapted distribution feeder and illustrate that compared to alternative schemes our approach reduces system cost, increases EV/PV hosting capacity and provides DLMPs that support the system optimal solution.

C. Paper Organization

The remainder of this paper is organized as follows. Section II provides the detailed model formulation of the operational planning optimization problem. Section III discusses the notion of DLMPs as price signals, and unbounds their components. Section IV presents the pilot study and lists numerical results and comparisons with alternative schemes. Section V concludes and provides directions for further research.

II. DETAILED MODEL FORMULATION

In this section, we present the formulation of an operational planning problem solved by a Distribution System Operator (DSO). For ease of exposition, we present the network model in Subsection II-A, the transformer model in Subsection II-B, the DER (PV/EV) constraints in Subsection II-C. We summarize the optimization problem in Subsection II-D.

A. Network Model

We consider a radial network with $N + 1$ nodes and $N$ lines. Let $\mathcal{N} = \{0, 1, \ldots, N\}$ be the set of nodes, with node 0 representing the root node, and $\mathcal{N}^+ \equiv \mathcal{N} \setminus \{0\}$. Let $\mathcal{L}$ be the set of lines, with each line denoted by the pair of nodes $(i, j)$ it connects —henceforth $ij$ for short, where node $i$ refers to the (unique due to the radial structure) preceding node of $j \in \mathcal{N}^+$. Transformers are represented as a subset of lines, denoted by $y \in \mathcal{Y} \subset \mathcal{L}$. For node $i \in \mathcal{N}$, $v_i$ denotes the magnitude squared voltage, $p_i$ and $q_i$ the net injection of real and reactive power, respectively. A positive (negative) value of $p_i$ refers to generation (consumption); similarly for $q_i$. Net injections at the root node refer to real and reactive power flowing from/to the transmission system. Net injections at node $j \in \mathcal{N}^+$ refer to the aggregate effect of DERs and conventional demand. For each line $ij$, with resistance $r_{ij}$ and reactance $x_{ij}$, $i_{ij}$ denotes the magnitude squared current, $P_{ij}$ and $Q_{ij}$ the sending-end real and reactive power flow, respectively.

We employ the branch flow model, introduced in [22], and recently revisited by [23]. The branch flow (a.k.a. DistFlow) equations are a substitute for the conventional AC power flow equations for a radial network. They are listed below, where we introduce the time index $t$ omitted previously for brevity; unless otherwise mentioned, $j \in \mathcal{N}^+$, and $t \in \mathcal{T}^+$, with $\mathcal{T} = \{0, 1, \ldots, T\}$, $\mathcal{T}^+ \equiv \mathcal{T} \setminus \{0\}$, and $T$ the optimization horizon.

\begin{align*}
    P_{01,t} &= p_{0,t} \rightarrow (\lambda^p_{01,t}), \\
    Q_{01,t} &= q_{0,t} \rightarrow (\lambda^q_{01,t}), \quad \forall t, \\
    P_{ij,t} - r_{ij}i_{ij,t} + p_{j,t} &= \sum_{k \in \mathcal{N}^+} P_{jk,t} \rightarrow (\lambda^p_{ij,t}), \quad \forall j, t, \\
    Q_{ij,t} - x_{ij}i_{ij,t} + q_{j,t} &= \sum_{k \in \mathcal{N}^+} Q_{jk,t} \rightarrow (\lambda^q_{ij,t}), \quad \forall j, t, \\
    v_{j,t} - 2r_{ij}P_{ij,t} - 2x_{ij}Q_{ij,t} &= (v_{i,t} + x_{ij}^2)i_{ij,t}, \quad \forall j, t, \\
    v_{i,t}, i_{ij,t} &= P_{ij,t}^2 + Q_{ij,t}^2, \quad \forall j, t.
\end{align*}

Briefly, (1)–(3) define the real and reactive power balance, $v_{i,t}$ the voltage drop, and $i_{ij,t}$ the apparent power but can be also viewed as the definition of the current. We supplement the model with voltage and current limits, as follows:

\begin{align*}
    v_{i,t} &\leq v_{i,t} \leq \bar{v}_i \rightarrow (\bar{p}_i, \bar{q}_i), \quad \forall i, t, \\
    i_{ij,t} &\leq i_{ij,t} \leq \bar{i}_{ij} \rightarrow (\bar{p}_{ij,t}, \bar{q}_{ij,t}), \quad \forall i, j, t,
\end{align*}

where $\underline{v}_i$, $\bar{v}_i$, and $\bar{i}_{ij}$ are the lower voltage, upper voltage, and line ampacity limits (squared), respectively. Dual variables of constraints (1)–(3), (4) and (7) are shown in parentheses.

The real (reactive) power net injections $p_{j,t}$, ($q_{j,t}$) include the aggregate effect of: (i) generation $p_{s,t}$, ($q_{s,t}$) of PV (rooftop solar) $s \in \mathcal{S}_j$, where $\mathcal{S}_j \subset \mathcal{S}$ is the subset of PVs (set $\mathcal{S}$) connected at node $j$; (ii) consumption $p_{e,t}$, ($q_{e,t}$) of EV $e \in \mathcal{E}_j$, where $\mathcal{E}_j \subset \mathcal{E}$ is the subset of EVs (set $\mathcal{E}$) that are connected at node $j$, during time period $t$, and (iii) consumption $p_{d,t}$, ($q_{d,t}$) of load $d \in \mathcal{D}_j$, where $\mathcal{D}_j \subset \mathcal{D}$ is the subset of loads (set $\mathcal{D}$) connected at node $j$. For clarity, the related definition constraints are listed below:

$$p_{j,t} = \sum_{s \in \mathcal{S}_j} p_{s,t} - \sum_{e \in \mathcal{E}_j} p_{e,t} - \sum_{d \in \mathcal{D}_j} p_{d,t}, \quad \forall j, t,$$
\[ q_{j,t} = \sum_{s \in S_j} q_{s,t} - \sum_{c \in E_j} q_{c,t} - \sum_{d \in D_j} q_{d,t}, \quad \forall j, t. \] (9)

### B. Transformer Degradation Model

For a given HST of the winding, \( \theta_H \), IEEE and IEC Standards [13], [14] provide the following exponential representation for the aging acceleration factor, \( F_{AA} \):

\[
F_{AA} = \exp \left( \frac{15000}{383} - \frac{15000}{\theta_H + 273} \right). \tag{10}
\]

In this subsection, we derive expressions for \( \theta_H \) that fit nicely with the branch flow model (linear recursive equations). We show that a piecewise linear approximation of (10), \( \tilde{F}_{AA} \), as illustrated in Fig. 1, allows us to embed the transformer degradation model in the branch flow formulation, by appending linear constraints and introducing the degradation cost in the objective function of the optimization problem.

1) **HST Calculations:** The winding HST at time period \( t \), \( \theta_H \), consists of the following components:

\[
\theta_H = \theta_A + \Delta \theta_T + \Delta \theta_R = \theta_A + \theta_T + \Delta \theta_H, \tag{11}
\]

where \( \theta_A \) is the ambient temperature, \( \theta_T \) is the top-oil temperature, \( \Delta \theta_T \) is the top-oil temperature rise over \( \theta_A \), and \( \Delta \theta_R \) is the winding HST rise over \( \theta_A \).

In [14], the HST is described as a function of time, for varying load and \( \theta_A \), using: (i) exponential equations, and (ii) difference equations. Both methods represent solution variation to the heat transfer differential equations. The differential equation for \( \theta_A \) is given by

\[
\Delta \theta_T^{TO} = \frac{1 + K^2}{1 + R} \theta_T^{TO} = k_{11} T \theta_T^{TO} dt + \theta_T^{TO} - \theta_A, \tag{12}
\]

where \( \Delta \theta_T^{TO} \) is the rise of top-oil temperature over ambient temperature at rated load, \( K_i \) is the ratio of the (current) load to the rated load, \( R \) is the ratio of load losses at rated load to no-load losses, \( \tau_T^{TO} \) is the time constant with recommended value 3 hours, \( k_{11} \) and \( n \) are constants with recommended values 1 and 0.8, respectively. Since the granularity for a day-ahead problem is \( \Delta t = 1 \) hour (less than half the recommended value of \( \tau_T^{TO} \)), employing the difference equations, we get the following recursive formula for \( \theta_A^{TO} \)

\[
\theta_A^{TO} = \frac{k_{11} \theta_T^{TO}}{k_{11} \theta_T^{TO} + \Delta t \theta_T^{TO - 1} + \Delta t} \left( \frac{1 + K^2}{1 + R} \right)^n + \theta_A + \Delta \theta_T^{TO} \]. \tag{13}

The initial value at \( t = 0 \), \( \theta_A^{TO} \), in case it is not known, can be calculated assuming a steady state, i.e., setting the derivative in the differential equation (13) to zero, yielding:

\[
\theta_A^{TO} = \theta_A + \Delta \theta_T^{TO} \left( \frac{1 + K^2}{1 + R} \right)^n. \tag{14}
\]

With respect to \( \Delta \theta_R^{H} \), it can be shown that both [13] and [14] yield the same results for the case of distribution (small) transformers. Because the winding time constant \( \tau_w \) has an indicative value of about 4 min (much less than the hourly granularity), the transient behavior, which is given by \( 1 - \exp \left( -\frac{t}{\tau_w} \right) \approx 1 \), vanishes. Hence, using (13), we get

\[
\Delta \theta_H = \Delta \theta_R \left( \frac{K_i^2}{m} \right)^n \tag{15}
\]

where \( \Delta \theta_R \) is the rise of HST over top-oil temperature at rated load, and \( m \) is a constant with recommended value 0.8.

The load ratio \( K_i \) can be defined with respect to the transformer nominal current (at rated load), denoted by \( I_N \). Using variables \( l_i \) (omitting the transformer index \( y \)), we have \( K_i^2 = l_i/l_N \), where \( l_N = \left( I_N^2 \right)^2 \). Hence, approximating the terms \( (1 + K_i^2)^n \) and \( (K_i^2)^n \) in (14) and (16), respectively, using the 1st order Taylor expansion of \( K_i^2 \) around 1, (equivalently of \( l_i \) around \( l_N \)), and replacing \( K_i^2 \) by \( l_i/l_N \), we get

\[
\frac{1 + K_i^2}{1 + R} \approx \frac{n R}{1 + (n - 1) R}, \quad \left( K_i^2 \right)^n \approx 1 + m(l_i/l_N - 1). \tag{16}
\]

Using (17) and (18), [14] and (14) and (16), respectively, yield

\[
\theta_i^{TO} \approx \frac{k_{11} T}{k_{11} T + \Delta t \theta_T^{TO - 1}} + \frac{\Delta t}{k_{11} T + \Delta t} \left( \frac{n R}{1 + (n - 1) R} \theta_A + \Delta \theta_T^{TO} \right). \tag{19}
\]

\[
\theta_i^{H} \approx \Delta \theta_R \left( \frac{m}{l_N} \right) \left( l_i/l_N - 1 \right). \tag{20}
\]

2) **Transformer Degradation Formulation:** The transformer degradation cost in the optimization horizon is represented by \( \sum_{y \in Y} \gamma_y f_y, t \), where \( \gamma_y \) is the hourly cost of transformer \( y \), and \( f_y, t \) its aging acceleration factor at time period \( t \).

Introducing this cost in the objective function allows us to replace (11) with the following set of inequalities:

\[
f_y, t \geq a_k \theta_H, \quad \forall y, t, k = 1, ..., M \tag{21}
\]

where we substituted \( \tilde{F}_{AA} \) with \( f_y, t \), and added indices \( y \) and \( t \) as required for completeness. Since \( \gamma_y > 0 \), at least one of the above inequalities will be binding (at equality).

Introducing indices \( y \) and \( t \) in (12) yields \( \theta_H^{y, t} = \theta_H^{y, t} + \Delta \theta_H^{y, t} \), where \( \theta_H^{y, t} \) and \( \Delta \theta_H^{y, t} \) are given by (19) and (20), respectively. Replacing \( \theta_H^{y, t} \) in (21), using (19) and (20), and substituting \( \theta_T^{y, t} \) with \( h_y, t \) (to simplify the notation), we get (\( \forall y \in Y, t \in T \))

\[
f_y, t \geq \alpha_1(a_k, y, t) + \alpha_2(a_k, y, t) + \beta_{y, k}, \quad \forall y, t, k = 1, ..., M \tag{22}
\]

with \( f_y, t \geq 0 \), and coefficients \( \alpha_1, \alpha_2, \) and \( \beta_{y, k} \) given by

\[
\alpha_1(a_k, y, t) = \alpha_k \sum_{y, t} \gamma_y f_y, t, \quad \alpha_2(a_k, y, t) = \sum_{y, t} \gamma_y f_y, t, \quad \beta_{y, k} = \sum_{y, t} \gamma_y f_y, t \tag{23}
\]
Also, substituting $\theta^{TO}_{y,z}$ with $h_{y,z}$ in (19), we get
\begin{equation}
  h_{y,z} = \gamma^{(1)} h_{y,z-1} + \gamma^{(2)} h_{y,z} + \delta_{y,z}, \quad \forall y, z, \tag{24}
\end{equation}
with coefficients $\gamma^{(1)}$, $\gamma^{(2)}$, and $\delta_{y,z}$ given by
\begin{equation}
  \gamma^{(1)} = \frac{k_{11} \tau^{TO}}{k_{11} \tau^{TO} + D_{y,z}}, \quad \gamma^{(2)} = \frac{k_{11} \tau^{TO}(1 + R_{y,z})}{k_{11} \tau^{TO}(1 + R_{y,z})}, \quad \delta_{y,z} = \frac{k_{11} \tau^{TO}}{k_{11} \tau^{TO} + D_{y,z}}, \quad \forall y, z, t. \tag{25}
\end{equation}

The initial condition for $h_{y,0}$, denoted by $h^{init}_{y,z}$, can be obtained from (13), replacing $R_{0}^{y}$ with $l_{0}^{y}/l^{y}$ (and adding the $y$ index). The initial condition constraint is as follows:
\begin{equation}
  h_{y,0} = h^{init}_{y,z} = \theta^{A}_{y,0} + \Delta \theta^{TO}_{y,z} \left( \frac{1 + R_{y,0} l_{0}^{y}}{1 + R_{y}} \right)^{n}, \quad \forall y. \tag{26}
\end{equation}

Summarizing, apart from the cost in the objective function, the transformer degradation formulation includes constraints (22), (24), and (26), with $f_{y,z} \geq 0$, and related coefficients defined by (23), and (25).

C. DER Constraints

1) PV Constraints: Due to the irradiation level $\rho_{t}$, the PV nameplate capacity $C_{s}$ is adjusted to $C_{s,t} = \rho_{t} C_{s}$, where $\rho_{t} \in [0, 1]$. PV constraints ($\forall s \in S$) are as follows:
\begin{align*}
  0 \leq p_{s,t} &\leq \tilde{C}_{s}, \quad p^{2}_{s,t} + q^{2}_{s,t} \leq C^{2}_{s}, \quad \forall s, t \in T_{I}, \tag{27} \\
  p_{s,t} &\equiv q_{s,t} = 0, \quad \forall s, t \notin T_{I}, \tag{28}
\end{align*}
with $p_{s,t} \geq 0$, and $T_{I} \subset T^{+}$ the subset of time periods for which $\rho_{t} > 0$. Constraints (27) impose limits on real and apparent power (implicitly assuming an appropriately sized inverter), whereas (28) imposes zero generation when $\rho_{t} = 0$.

2) EV Constraints: We consider an EV that is connected for $Z$ intervals, at nodes $j_{1}, \ldots, j_{Z}$. In the general case, the first and last intervals may not entirely fit within the time horizon. Hence, we define $T^{beg} = \{ z^{beg} \}$ and $T^{end} = \{ z^{end} \}$, for $z = 1, \ldots, Z$, the sets of time periods denoting an adjusted beginning and end, respectively, which considers only the part of the interval within the time horizon (it affects intervals $z = 1, Z$). We also define the set of time periods, $T_{z} = \{ z^{beg} + 1, \ldots, z^{end} \}$ of interval $z$, for $z = 1, \ldots, Z$, during which the EV is plugged in at node $j_{z}$. Subscript $e$ in the aforementioned set was omitted for simplicity; it is included next.

The State of Charge (SoC) of EV $e$ is described by variable $u_{e,t}$ for time periods $t \in T^{beg} \cup T^{end}$. SoC is reduced by $\Delta u_{e,z}$ after departure $z$ and until arrival $z+1$, for $z = 1, \ldots, Z-1$. EV constraints ($\forall e \in E$) are as follows:
\begin{align*}
  u_{e,z}^{beg} &\equiv u_{e,z}^{init}, \quad \forall e, \tag{29} \\
  u_{e,z}^{end} &\equiv u_{e,z}^{end} + \sum_{t \in T_{z}} p_{e,t}, \quad \forall e, z = 1, \ldots, Z-1, \tag{30} \\
  u_{e,z}^{beg} &\equiv u_{e,z}^{beg} - \Delta u_{e,z}, \quad \forall e, z = 1, \ldots, Z-1, \tag{31} \\
  u_{e,z}^{end} &\equiv u_{e,z}^{end} - \Delta u_{e,z}, \quad \forall e, t \in T^{end}, \tag{32} \\
  p_{e,t}^{2} + q_{e,t}^{2} &\leq C^{2}_{e}, \quad 0 \leq p_{e,t} \leq C_{r}, \quad \forall e, t \in \bigcup_{z=1}^{Z} T_{e,z}, \tag{33}
\end{align*}
with $p_{e,t}, q_{e,t} \geq 0$. Eq. (29) initializes the SoC ($u_{e,z}^{init}$) at $T_{z}^{beg}$, (30) and (31) define the SoC at the end/beginning of an interval, after charging/traveling, respectively. Constraints (32) impose a minimum SoC $u_{e,z}^{min}$ at the end of an interval as well as the limit of the EV battery capacity $C^{B}_{e}$, whereas (33) impose the limits of the charger $C_{r}$ (related to the size of the inverter) and the charging rate $C_{r}$. Lastly, (34) imposes zero consumption when the EV is not plugged in.

D. Optimization Problem Summary

The objective function of the operational-planning optimization problem—referred to as **Full-opt**—aims at minimizing the aggregate real and reactive power cost, with $c^{P}_{e}$ ($c^{Q}_{e}$) denoting the cost for real (reactive) power at the substation, as well as the transformer degradation cost. The real power cost $c^{P}_{e}$ is typically the LMP at the T&D interface, whereas $c^{Q}_{e}$ can be viewed as the opportunity cost for the provision of reactive power.

**Full-opt:**
\begin{equation}
  \begin{aligned}
    & \min_{v_{i,t}, q_{i,t}, p_{i,t}, q_{i,t}, u_{e,t}, q_{e,t}} \sum_{t \in T^{+}} \left( c^{P}_{t} p_{i,t}^{2} + c^{Q}_{t} q_{i,t}^{2} + \sum_{y \in Y} c_{y, t} f_{y,t} \right), \\
  & \text{subject to: } \text{Network constraints (1)-(9), transformer constraints (22), (24), (26), PV constraints (27), (28), and EV constraints (29)-(34), with } v_{i,t}, q_{i,t}, f_{y,t}, p_{i,t}, u_{e,t}, q_{e,t} \geq 0.
  \end{aligned} \tag{35}
\end{equation}

We note that the transformer related constraints, which are integrated into the branch flow model, are linear constraints. However, constraint (5) is a non-convex equality constraint. Following [23], we relax the equality to an inequality constraint, and substitute (5) with
\begin{equation}
  v_{i,t}, q_{i,t} \geq P^{2}_{i,t} + Q^{2}_{i,t}, \quad \forall j, t. \tag{36}
\end{equation}

The resulting relaxed AC OPF problem is a convex Second Order Cone Programming (SOCP) problem, which can be solved efficiently using commercially available solvers. We refer the interested reader to recent works that propose remedies for cases when the relaxation is not exact.

III. DISTRIBUTION LOCALIZATIONAL MARGINAL PRICES

In this section, we discuss the notion of DLMPs, which is based on the spatiotemporal marginal cost for real and reactive power. DLMPs are obtained by the dual variables of constraints (1)-(5). They represent the dynamic marginal cost for delivering (or consuming) real/reactive power (P-DLMP/Q-DLMP) at a specific location and time period.

In Subsection III-A, we elaborate on the optimality conditions of the operational planning problem, illustrating DLMPs as price signals for DERs. In Subsection III-B, we unbundle DLMPs into cost components using sensitivity analysis.

A. Optimality Conditions

A rather trivial remark reviewing the optimality condition is that $\lambda_{0,t}^{P} = c^{P}_{t}$, and $\lambda_{0,t}^{Q} = c^{Q}_{t}$, which essentially says that the DLMP at the root node equals the LMP for real power.
and the opportunity cost for reactive power. More importantly, optimality conditions illustrate that DLMPs provide the correct price signals for optimal DER self-scheduling.

Indeed, a PV can self-schedule to maximize its benefits over the daily cycle’s 24 hours by solving the following optimization problem, referred to as **PV-opt**:

$$\max_{p_e, t, t'} \sum_t \left( \lambda_{j_t, t}^P p_{e, t} + \lambda_{j_t, t}^Q q_{e, t} \right),$$  
subject to: PV constraints (27) and (28), where $\lambda_{j_t, t}^P$ and $\lambda_{j_t, t}^Q$ are parameters, representing the marginal cost, in fact the DLMP, at node $j_t$ where PV $s$ is installed, and at time period $t$. PV-opt has a straightforward interpretation: it maximizes the revenues of a PV that is charged/rewarded at $\lambda_{j_t, t}^P$ and $\lambda_{j_t, t}^Q$ for the provision of real and reactive power, respectively. It is trivial to show that the optimality conditions of PV-opt are included in the Full-opt if $\lambda_{j_t, t}^P = \lambda_{j_t, t}^P$, and $\lambda_{j_t, t}^Q = \lambda_{j_t, t}^Q$ (the asterisk denotes the optimal solution of Full-opt).

Similarly, an EV can self-schedule by solving **EV-opt**:

$$\min_{p_e, t, t'} \sum_t \left( \hat{\lambda}_{j_t, t}^P p_{e, t} + \hat{\lambda}_{j_t, t}^Q q_{e, t} \right),$$  
subject to: EV constraints (29)–(33), where node $j_t$ represents the node to which EV $e$ is connected at time period $t$. EV-opt is a cost minimization problem, whose optimality conditions, similarly to those of PV-opt, are also encountered in Full-opt.

Although only EVs and PVs are modeled in this paper as dominant DER examples, other DERs can be treated similarly. The individual optimization problems suggest the following interpretation: nodal marginal costs are in fact the DLMPs at each node and can be construed as prices that elicit a price-taking DER to adapt fully and self-schedule to its socially optimal real/reactive power profile. In other words, had the DSO been able to determine and announce these DLMPs, DERs would have self-scheduled in a manner that is optimal for the system as a whole.

### B. Sensitivity Analysis

In this subsection we employ sensitivity analysis to derive the DLMP components. Assuming a system operating point that corresponds to the optimal solution, at time period $t'$, i.e., $P_{j_t, t'}^*, Q_{j_t, t'}^*, v_{j_t, t'}^*$, and $l_{j_t, t'}^*$, $\forall j \in N^+$, at the branch flow equations (2), we take partial derivatives w.r.t. $p_{j', t'}$ and $q_{j', t'}$, namely $\frac{\partial P_{j_t, t'}^*}{\partial p_{j', t'}}$, $\frac{\partial Q_{j_t, t'}^*}{\partial p_{j', t'}}$, $\frac{\partial v_{j_t, t'}^*}{\partial p_{j', t'}}$, and $\frac{\partial l_{j_t, t'}^*}{\partial p_{j', t'}}$, $\frac{\partial P_{j_t, t'}^*}{\partial q_{j', t'}}$, $\frac{\partial Q_{j_t, t'}^*}{\partial q_{j', t'}}$, $\frac{\partial v_{j_t, t'}^*}{\partial q_{j', t'}}$, and $\frac{\partial l_{j_t, t'}^*}{\partial q_{j', t'}}$. This yields $2NT$ systems with $4N$ linear equations each. The P-DLMP, $\lambda_{j_t, t'}^P$, is given by

$$\lambda_{j_t, t'}^P = c_{P, t'} \frac{\partial P_{j_t, t'}^*}{\partial p_{j', t'}} + c_{Q, t'} \frac{\partial Q_{j_t, t'}^*}{\partial p_{j', t'}} + \sum_{y, t} c_{y, t} \frac{\partial f_{y, t}}{\partial p_{j', t'}}.$$  

$$+ \frac{\partial P_{j_t, t'}^*}{\partial f_{j_t, t'}}.$$  

$$\frac{\partial Q_{j_t, t'}^*}{\partial f_{j_t, t'}} \frac{\partial v_{j_t, t'}^*}{\partial f_{j_t, t'}} \frac{\partial l_{j_t, t'}^*}{\partial f_{j_t, t'}},$$

where the real and reactive power components are given by

$$\frac{\partial p_{j_t, t'}}{\partial p_{j', t'}} = 1 + \sum_{i \in E} r_{ij} \frac{\partial l_{i, j', t'}}{\partial p_{j', t'}}.$$  

where the sums in (40) and (41) represent the aggregate real/reactive power marginal losses. The transformer cost component is given by

$$\sum_{y, t} c_{y, t} \frac{\partial f_{y, t}}{\partial p_{j', t'}}.$$

IV. PILOT STUDY

In this section, we illustrate the application of the proposed model on an actual 13.8 KV feeder of Holyoke Gas and Electric (HGE), a municipal distribution utility in MA, US. In Subsection IV-A we present input data and introduce several reasonable, though sub-optimal, EV and PV scheduling options we compare to the optimal schedule. In Subsection IV-B we discuss numerical results and findings.

#### A. Input Data

The feeder topology is shown in Fig. 2. Aggregate line and transformer data are listed in Table I. Voltage limits are 0.95 and 1.05 p.u. Fig. 2 shows load profiles (obtained from from [24], assuming a 0.95 and 0.85 power factor, for the residential and commercial node respectively, PV adjustment factor ($\rho_i$), and the ambient temperature. The opportunity cost for reactive power is assumed at 10% the value of the LMP.

#### TABLE I

| Distribution Lines | Transformers |
|--------------------|--------------|
| Type               | # Length (ft) | R (Ω/mile) | X (Ω/mile) | Amp. (A) | Size (KVA) | Type | # Res. | # Conm. | R (%) | X (%) | c_e ($) |
| ORU 35            | 12200        | 1.52       | 0.75       | 190      | 15        | 5.00 | 29     | 1             | 1.3   | 1.2   | 0.01    |
| ORU 9             | 9000         | 0.97       | 0.75       | 265      | 30        | 5.00 | 39     | 2             | 1.3   | 1.2   | 0.02    |
| ORU 56            | 10000        | 0.31       | 0.65       | 450      | 45        | 5.00 | 50     | 1             | 1.3   | 1.3   | 0.027   |
| ORU 66            | 10500        | 1.66       | 1.54       | 120      | 75        | 5.00 | 50     | 1             | 1.3   | 1.4   | 0.032   |
| ORU 7             | 5000         | 1.40       | 0.26       | 110      | 45        | 5.00 | 50     | 1             | 1.3   | 1.3   | 0.025   |
| US 7              | 4700         | 0.6        | 0.25       | 210      | 75        | 5.00 | 50     | 2             | 1.2   | 1.5   | 0.03    |
| US 8              | 8000         | 0.17       | 0.21       | 344      | 150       | 5.00 | 50     | 2             | 1.7   | 1.7   | 0.04    |
| US 11             | 12000        | 0.1        | 0.29       | 208      | 225       | 5.00 | 50     | 2             | 1.1   | 1.6   | 0.045   |
| US 9              | 4900         | 1.54       | 0.57       | 110      | 300       | 5.00 | 50     | 3             | 0.75  | 2.9   | 0.05    |
| US 6              | 2000         | 1.04       | 0.51       | 175      | 500       | 5.00 | 50     | 2             | 0.75  | 2.9   | 0.06    |

We focus our numerical experimentation on the two selected nodes shown in Fig. 2. They represent two 30-KVA transformers (with $R = 5$, $\Delta \theta_R = 5$, $\Delta \theta_H = 25$) that serve commercial and residential loads. In order to facilitate the illustration, but also emphasize the local effect that EVs and PVs have on a distribution feeder, we build our scenarios by allocating EVs and PVs only in these two nodes. This provides a more clear view of the results. At the commercial node, EVs are connected 9am–5pm (Z = 1) and should charge 12 KWh; at the residential node, they are connected 7pm–7am (Z = 2).
and should charge 18 KWh. At the time of departure, EVs should be fully charged. The battery capacity is 24 KWh, the maximum charging rate 3.3 KW/h, and the charger capacity 6.6 KVA. PVs are assumed to be 10 KVA rooftop solar.

We consider 4 scheduling options for EVs and PVs:

1) “BaU” (Business as Usual): EVs “dumb” charge at full rate upon arrival with p.f. = 1. PVs operate with p.f. = 1.

2) “ToU” (Time-of-Use): EVs charge responding to the LMP with p.f. = 1. PVs operate with p.f. = 1.

3) “PQ-opt”: EVs and PVs are scheduled in order to minimize real and reactive power cost, subject to voltage and ampacity constraints. The objective function does not include the transformer degradation cost. As such, PQ-opt is a special case of Full-opt where transformer degradation costs and related transformer constraints are excluded.

4) “Full-opt”: EVs and PVs are scheduled by solving Full-opt, and thus taking into account all cost components and constraints including transformer degradation costs.

In each of the above scheduling options, we consider several DER penetration scenario instances at the nodes of interest: EVs equal 0, 3, and 6, and rooftop PV installations equal 0, 30, and 60 KVA (i.e., 0, 3, and 6 units of 10 KVA rooftop solar, respectively). The scenario with 0 EVs and PVs is used as the base case for comparison purposes.

B. Numerical Results

To handle properly initial conditions (transformer temperature and EV SoC) in the daily cycle, we require them to coincide at the beginning and at the end of the cycle, t = 0, and t = 24. Table II shows the system cost differences (in $) for real/reactive power (P/Q), transformer degradation, and total cost compared to the base case (no EVs and no PVs). It also shows the aggregate LoL (in hours) of the two 30-KVA transformers (commercial and residential) under the different scheduling options and EV/PV penetrations. As expected, Full-opt achieves the lowest total cost for the system, since it co-optimizes real/reactive power and transformer cost, while maintaining low aggregate LoL. PQ-opt performs better if we consider the combined effect of real and reactive power (P and Q columns in Table II), since it co-optimizes real and reactive power. However, it exhibits some very high values of aggregate LoL and transformer degradation cost, as a result of high reactive power provision. Unlike PQ-opt and Full-opt, ToU and BaU do not take advantage of DER reactive power provision capabilities and may occasionally achieve even lower aggregate LoL.

At the commercial node, BaU and ToU produce essentially identical EV schedules (LMPs are increasing during the day). At the residential node, ToU shifts the BaU EV profile by 3 hours (EVs start charging at 10pm). The specific LMPs and EV/PV penetrations render the ToU option sustainable transformer LoL-wise; different LMPs and/or higher penetration levels may result in worse performance of the ToU option.

We next investigate and discuss EV-only (PV: 0), PV-only (0 EVs), and EV-PV synergy scenarios. Note that real power P-DLMPs and reactive power Q-DLMPs reported for scheduling options other than Full-opt, are not the optimal clearing prices. Instead, they represent the location and time specific marginal cost of real and reactive power for DER operation associated with each scheduling option. As such, when compared to the Full-opt DLMPs, they represent system-wide cost reduction incentives suggesting higher or lower real and reactive DER power injections relative to those obtained in each of the suboptimal scheduling options.

1) EV Only: In Fig. 5 we show P-DLMPs and Q-DLMPs for a relatively low (3 EV) penetration. Small differences are observed, caused primarily by the provision of reactive power. The commercial node experiences higher Q-DLMPs for BaU/ToU compared to PQ-opt/Full-opt, suggesting that a higher provision of reactive power would be desirable. At the residential node, P-DLMPs under BaU are higher for hours 20-22, suggesting a preference for a 3-hour shift in EV consumption; this coincides with the result of other scheduling options where EVs also start charging at 10pm. Q-DLMPs indicate the preference for more reactive power in the BaU/ToU scheduling options. Indeed, Full-opt DLMPs schedule higher reactive power resulting in lower system-wide cost. Sensitivity analysis results are shown in Fig. 5. They illustrate the cost components of P-DLMPs for Full-opt, and Q-DLMPs for BaU and Full-opt, at the commercial node. P-DLMPs are similar across all scheduling options; their components range from 1.7% to 7.5% for the real power losses, from 0.2% to 0.9% for the reactive power losses and from 0.2% to 1.7% for the transformer degradation components. For Q-DLMPs, we observe that during hours 10-17 (when EVs are plugged in), the marginal cost components for real and reactive power are lower under Full-opt, while the transformer degradation component becomes negative. The reason is that there is a reverse reactive power flow at that node and those time periods, hence the sensitivity of the current w.r.t. to reactive power injection becomes negative. For Full-opt Q-DLMPs, the cost components for real power losses range from 8% to 30.7%, for
reactive power losses from 1% to 3.7%, and for transformer degradation from -2% to 5%.

Fig. 3 shows the DLMPs for the 6 EV penetration scenario. The PQ-opt scheduling option results in a 5 to 9 fold higher aggregate transformer LoL relative to Full-opt. The DLMP spikes are caused by a higher transformer cost component. Low transformer costs across all hours are associated with generally smoother DLMP profiles. At the commercial node, P-DLMP spikes for BaU/ToU. High P-DLMPs for PQ-opt indicate that this scheduling option’s charging rates are too high. Q-DLMPs exhibit positive spikes for BaU/ToU, implying that EVs should provide more reactive power. Negative spikes for PQ-opt suggest EVs provide excessive reactive power under the PQ-opt option. We note that Full-opt scheduling results in practically zero Q-DLMPs (→ 0°) during the hours that EVs are plugged in. Indeed, reactive power DLMPs “tank” during these hours which are associated with high reactive power production under Full-opt. Nevertheless, despite the fact that reactive power income decreases with tanking Q-DLMPs, Full-opt results in lower P-DLMPs as well whose net impact on overall system marginal cost from EV charging is lower! Full-opt Q-DLMPs support the system-optimal solution by incentivizing EVs to provide reactive power at a rate that is smaller than their charger capability, while the PQ-opt scheduling option “brute-forces” EVs to fully utilize their reactive power production capability resulting in negative Q-DLMPs which significantly increase the imputed system-wide marginal cost of the EVs. Fig. 4 reports the sensitivity analysis of Q-DLMP components. It is interesting to note how the marginal transformer degradation cost varies across scenarios and scheduling options illustrating the incentives implied by the respective DLMPs. Similar remarks can be made for the residential node.

Lastly, we also tested 9 and 12 EV scenarios. Full-opt scheduling can still accommodate such high penetrations, whereas other scheduling options fail.

2) PV Only: PV penetration of 30 kVA is rather mild revealing small differences. However, for the 60 kVA penetration, PQ-opt yields very high values of aggregate LoL. Similar to the EV-only scenarios, DLMPs under PQ-opt scheduling suggest that both reactive and real power provision is excessive during several hour periods. Interestingly, DLMPs reflecting binding voltage constraints are also observed (the upper limit

### Table II

**Aggregate System Cost Difference (in $) and LoL (in hours)**

| EVs | Options | P | Q | Tr. | Total | P | Q | Tr. | Total |
|-----|---------|---|---|-----|-------|---|---|-----|-------|
| 6 EVs | BaU | 6.85 | 0.06 | 2.87 | 9.78 | 10.25 | 0.15 | 4.17 | 14.77 |
|      | Total | 6.25 | 0.04 | 0.86 | 7.15 | 10.85 | 0.09 | 8.02 | 19.98 |
|      | PQ-opt | 5.99 | 1.44 | 5.62 | 8.77 | 11.17 | 4.64 | 12.95 | 161.75 |
|      | Full-opt | 6.07 | 2.24 | 0.31 | 4.48 | 11.45 | 5.26 | 16.85 | 28.21 |
| 3 EVs | BaU | 3.41 | 0.03 | 0.09 | 3.53 | 13.66 | 0.40 | 13.04 | 26.70 |
|      | Total | 3.11 | 0.02 | 0.04 | 3.17 | 13.96 | 0.11 | 14.13 | 27.19 |
|      | PQ-opt | 2.84 | 1.43 | 0.04 | 1.40 | 14.55 | 5.25 | 24.90 | 45.70 |
|      | Full-opt | 2.84 | 1.43 | 0.04 | 1.40 | 14.55 | 5.25 | 24.90 | 45.70 |
| 0 EVs | BaU | 17.03 | 0.13 | 0.09 | 17.26 | 33.74 | 0.23 | 33.96 | 67.60 |
|      | Total | 17.03 | 0.13 | 0.09 | 17.26 | 33.74 | 0.23 | 33.96 | 67.60 |
|      | PQ-opt | 17.03 | 0.13 | 0.09 | 17.26 | 33.74 | 0.23 | 33.96 | 67.60 |
|      | Full-opt | 17.03 | 0.13 | 0.09 | 17.26 | 33.74 | 0.23 | 33.96 | 67.60 |

**Fig. 4. DLMPs at commercial node, 6 EVs. Q-DLMP components.**
is reached at hours 6 and 7).

Experimentation with 90 and 120 KVA PV penetration scenarios showed that Full-opt can significantly increase PV hosting capacity “curtailment” real and reactive power as needed, and, thus, avoiding excessive transformer temperatures and LoL. Notably, if we consider self-scheduling PVs responding to optimal DLMP-based signals, Full-opt DLMPs elicits economically efficient “curtailment”.

3) EV - PV Synergy: With the exception of the PQ-opt scheduling option at the residential node, the 3 EV and 30 KVA PV scenario is easily sustainable with low aggregate LoL. The synergy of EVs and PVs during hours 6, 7, 20, and 21 results in high reverse reactive power flows that overload the transformer. P-DLMPs exhibit positive spikes and Q-DLMPs negative spikes in those hours. Increasing EVs to 6 and/or PVs to 60 KVA results in very high LoL for PQ-opt. On the other hand, as expected, Full-opt achieves sustainability with low LoL under all scenarios.

V. CONCLUSIONS AND FURTHER RESEARCH

We have shown the significance of distribution network asset degradation costs exemplified by service transformer marginal LoL. More significantly, we have shown that optimal DLMP driven scheduling of DERs can achieve significantly higher capacity of distribution networks in hosting sizable PV and EV adoption. We proposed appropriate AC OPF models for granular marginal costing on distribution networks, and a real distribution network carried out a feeder pilot study to obtain supporting numerical results. Comparison with popular open-loop DER scheduling options provided solid evidence that optimal DLMP-based clearing markets can bring about significant economic efficiencies and support the sustainability/adequacy of current distribution network infrastructure in the presence of high DER (PV, EV, and the like) adoption.

Our ongoing/future research aims at extending the proposed approach to 3-phase network representations, and employing decomposition approaches and distributed algorithms for dealing with systems consisting of multiple feeders and large numbers of diverse DERs transcending EVs and PVs to include microgenerators, smart buildings with pre-cooling/heating capable HVAC, smart appliances, storage, and new technologies.

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