Role of Chameleon Field in presence of Variable Modified Chaplygin gas in Brans-Dicke Theory

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In this work, we have considered FRW model of the universe for Brans-Dicke (BD) theory with BD scalar field as a Chameleon field. First we have transformed the field equations and conservation equation from Jordan’s frame to Einstein’s frame. We have shown in presence of variable modified Chaplygin gas, the potential function $V$ and another analytic function $f$ always increase with respect to BD-Chameleon scalar field $\phi$.

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I. INTRODUCTION

The present accelerated expansion of the universe are strongly believed under some observations of the luminosity-redshift relation for type-Ia supernovae [1, 2], measurements of CMBR, power spectrum of mass perturbation etc. The responsible candidate for the violation of strong energy condition is known as dark energy. Another simplest alternative candidate which includes the scalar field in addition to the tensor field in general relativity is Brans-Dicke (BD) theory. Recently, enormous interest in BD theory [3] can be found to the discovery by La et al [4] that the use of BD theory in place of general relativity can upgrade the problem of inflationary cosmology. This is possible as it arises naturally as the low energy limit due to the interaction of the BD scalar field. The field equations in this case are modified due to the coupling between matter Lagrangian and scalar field. In BD theory, there exists one dimensionless parameter $\omega$ and the effective gravitational constant $G$ which is inversely proportional to the scalar field $\phi$. Also, BD theory or its modifications become an important theory to explain the present cosmic acceleration [5] with some correction of Newtonian weak-field limit [6]. Einstein gravity can be reconstructed from it considering the condition $\omega \to \infty$ [7]. Several authors have done extensive amount of work on BD theory and generalized BD theory to solve different cosmological problems of inflation, quintessence etc [8 - 12].

There are different dark energy models which are basically considered as a scalar field, suggest a large correction to the Newton’s law. These are nearly massless scalar field and if exists on earth, should have been detected in local test of equivalence principle and as a fifth force. Recently, a different approach is introduced in general relativity by Khoury and Weltman [13], where a non-minimal interaction occurs between the quintessence scalar field and matter sector rather than the geometry and this interaction is introduced through an interference term in the action known as Chameleon scalar field. The important feature of this field is that due to the coupling to matter it obtains an effective potential i.e., an effective mass depends quite extremely on the background energy density. Many authors have been studied on Chameleon field in different aspects of accelerating universe [14 - 21].

A well known candidate for Q-matter is the exotic type of fluid named Chaplygin gas obeying the EOS $p = -\frac{B}{\rho}$ ($B > 0$) [22], where $p$ and $\rho$ are respectively the pressure and
energy density. A generalized version of this Q-matter having the form of EOS given by $p = -\frac{B}{\rho^\alpha}$ ($0 \leq \alpha \leq 1$) [23,24] and recently it was modified to the form $p = A\rho - \frac{B}{\rho^\alpha}$ ($A > 0$) [25-27], which is known as modified Chaplygin Gas. This model can represent the evaluation of the Universe starting from the radiation era to the ΛCDM model. There is one stage, when the pressure vanishes and the matter content is equivalent to a pure dust. Recently another modification have been made by Guo and Jhang [28]. They proposed variable Chaplygin gas model (VCG) where $B$ is a +ve function of the cosmological scale factor $a$. This $B(a)$ is related to the scalar potential if we take the Chaplygin gas Born-Infeld scalar field [29].

From these three models (BD theory, Chameleon field and VCG) we have construct our paper as follows: BD theory or its modifications have recently described as a driver of the present cosmic acceleration. It had been shown in [5] that BD theory can potentially generate sufficient acceleration in the matter dominated era even without including the exotic Q-field [5]. But this has problems to establish the transition from a decelerated to an accelerated phase. On the other hand, this transition is possible when a Chameleon scalar field have a non-minimal coupling with dark matter [20,21]. Here we consider the BD scalar field as a Chameleon type scalar field and the matter Lagrangian is coupled with an arbitrary analytical function of the field. Initially, we write the field equation in most general scalar tensor theory of gravity but the field equations become complicated for the gravitational and scalar fields. So we transfer these equation in Einstein frame using conformal transformation [30,31], in which the scalar and tensor degrees of freedom do not mix. Now we consider the universe filled with variable modified Chaplygin gas (VMCG) [32] and in general scalar tensor gravity, we graphically analyze the potential of the BD-Chameleon scalar field with respect to the field. We also analyze the analytical function of the BD-Chameleon scalar field.

II. BASIC EQUATIONS AND SOLUTIONS

We consider here the Brans-Dicke scalar field $\phi$ act as a Chameleon field and we consider the action for the chameleon scalar field $\phi$ in self-interacting Brans-Dicke (BD) theory [31] as: (choosing $8\pi G_0 = c = 1$)
\[ S = \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega}{\phi} \phi^\alpha \phi_{\alpha} - V(\phi) + f(\phi)\mathcal{L}_m \right] \]  

(1)

where \( V(\phi) \) is the self-interacting potential for the BD-Chameleon scalar field \( \phi \) and \( \omega \) is the BD parameter. Unlike the usual Einstein-Hilbert action, the matter Lagrangian \( \mathcal{L}_m \) is considered as \( f(\phi)\mathcal{L}_m \), where \( f(\phi) \) is some arbitrary analytical function of \( \phi \). This term makes possible the non-minimal interaction between the cold dark matter and BD-Chameleon scalar field.

The metric of a homogeneous and isotropic universe in the FRW model is

\[ ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \]  

(2)

where \( a(t) \) is the scale factor and \( k (= 0, +1, -1) \) is the curvature scalar. Now, in BD theory, the Einstein’s field equations for the above space-time symmetry are

\[ H^2 + \frac{k}{a^2} - \frac{\omega}{6} \frac{\dot{\phi}^2}{\phi^2} + H \frac{\dot{\phi}}{\phi} = \frac{f(\phi)}{3\phi} \rho + \frac{1}{6} \frac{V(\phi)}{\phi} \]  

(3)

\[ \frac{2}{a} \ddot{a} + H^2 + \frac{k}{a^2} + \frac{\omega}{2} \frac{\dot{\phi}^2}{\phi^2} + 2H \frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{f(\phi)}{\phi} p + \frac{1}{2} \frac{V(\phi)}{\phi} \]  

(4)

and the wave equation for the BD-Chameleon scalar field \( \phi \) is

\[ \frac{\ddot{\phi}}{\phi} + 3H \frac{\dot{\phi}}{\phi} = \frac{\rho - 3p}{2(2\omega + 3)\phi} \left( f(\phi) - \frac{1}{2} \phi \frac{df}{d\phi} \right) + \frac{2}{2\omega + 3} \left( V(\phi) - \frac{1}{2} \phi \frac{dV}{d\phi} \right) \]  

(5)

The energy conservation equation is given by

\[ \dot{\rho} + \frac{\dot{a}}{a}(\rho + p) = -\frac{3}{4}(\rho + p) \frac{f(\phi)}{f(\phi)} \]  

(6)

Here we consider the variable modified Chaplygin gas with equation of state [32]

\[ p = A\rho - \frac{B}{\rho^\alpha}, A > 0, B > 0, 0 \leq \alpha \leq 1 \]  

(7)

where \( B \) is not constant. In the BD theory, the effective gravitational constant is given by \( G = \frac{G_0}{\phi} \), which is indeed not a constant. Now considering the the transformation

\[ \bar{g}_{\mu\nu} = \phi g_{\mu\nu} , \]  

(8)
so that the equations in the Jordan’s frame reduce to the equations in the Einstein’s frame. So let us define the new variables in Einstein’s frame as in the following:

\[ \psi = \ln \phi, \quad \alpha^2 = \phi a^2, \quad \rho = \phi \bar{\rho}, \quad p = \phi \bar{p}, \quad f = \phi \bar{f}, \quad V = \phi^2 \bar{V}, \quad d\tau = \sqrt{\phi} \, dt \]  

(9)

Now the BD field equations (3), (4) and the conservation equation (6) can be reduced to the Einstein frame as

\[ \frac{\ddot{\alpha}^2}{\alpha^2} + \frac{k}{\alpha^2} = \left( \frac{2\omega + 3}{12} \right) \psi^2 + \frac{\bar{f}(\psi)\bar{\rho}}{3} + \frac{\bar{V}(\psi)}{6} \]  

(10)

\[ 2\frac{\dddot{\alpha}}{\dot{\alpha}} + \frac{\ddot{\alpha}^2}{\alpha^2} + \frac{k}{\alpha^2} = -\left( \frac{2\omega + 3}{4} \right) \psi^2 - \bar{p}\bar{f}(\psi) + \frac{\bar{V}(\psi)}{2} \]  

(11)

and

\[ \dot{\rho}' + 3\frac{\ddot{\alpha}^2}{\dot{\alpha}}(\bar{\rho} + \bar{p}) = -\frac{1}{4} (\bar{p} - 3\bar{p})\psi' - \frac{3}{4} (\bar{\rho} + \bar{p})\bar{f}' \]  

(12)

where, \( \dot{} \) represents the derivative w.r.t. new time variable \( \tau \). Also using the above transformation and considering the variable term \( B = B_0\phi^{\alpha+1} \), the EOS of the variable modified Chaplygin gas (7) becomes

\[ \bar{p} = A\bar{\rho} - \frac{B_0}{\bar{\rho}^\alpha} \]

(13)

where \( B_0(>0) \) is a constant. This EOS represents only the modified Chaplygin gas in the Einstein’s frame. From the equations (12) and (13) we can construct a linear differential equation in \( z \) (where, \( \bar{\rho} = z^{\alpha+1} \)) as

\[ \frac{dz}{d\tau} + z(\alpha + 1) \left( 3(1 + A)\frac{\ddot{\alpha}}{\dot{\alpha}} + \frac{3}{4}(1 + A)\frac{\ddot{f}}{f} + \frac{1 - 3A}{4}\psi' \right) = (\alpha + 1) \left( 3B_0\frac{\ddot{\alpha}}{\dot{\alpha}} + \frac{3}{4}B_0\frac{\ddot{f}}{f} - \frac{3}{4}B_0\psi' \right) \]

(14)

For simplicity we take only the radiation case \( A = 1/3 \) and in this case the integrating factor (I.F.) becomes \((\alpha^4\bar{f})^{\alpha+1}\). We also assume \( \bar{\alpha}^4\bar{f} = \phi^n \), where \( n \) is a positive constant. After solving the above the differential equation, we get the solution of \( \bar{\rho} \) as

\[ \bar{\rho} = \left[ \frac{3}{4}B_0 \left( \frac{\alpha + 1}{n(\alpha + 1) + 1} \psi + C\psi^{-n(\alpha+1)} \right) \right]^{\alpha+1} \]

(15)

where \( C \) is an integrating constant. Now let the transformed Hubble parameter is defined by \( \frac{\ddot{\alpha}}{\dot{\alpha}} = \bar{H} \) then \( \bar{H}' = \bar{a}\bar{H} \frac{dt}{d\alpha} \). Now we take a new assumption \( \psi = \psi_0\bar{a}^m \) where \( m \) is a positive
constant. Using this new assumption the solution of $\bar{\rho}$ becomes

$$\bar{\rho} = \left[ \frac{3}{4} B_0 \left( 1 - \frac{\alpha + 1}{n(\alpha + 1)} \psi_0 a^m + C(\psi_0 a^m)^{-n(\alpha + 1)} \right) \right]^{1 \over \alpha + 1}$$

(16)

Now putting $\bar{H}^2 = \bar{Y}$ and considering the previous all assumptions, we have from the equations (10) and (11) a differential equation in $\bar{Y}$ as

$$\frac{d\bar{Y}}{d\bar{a}} + \frac{2\omega + 3}{2} \psi_0^2 m^2 \bar{a}^{2m-1} \bar{Y} = -(\bar{\rho} + \bar{p}) \psi_0^n \bar{a}^{mn-5} + \frac{2k}{\bar{a}^3}$$

(17)

From conformal transformation we have $\bar{a} = (\ln \phi)^m \psi_0$ which implies $d\bar{a} = \left( \frac{1}{m} \frac{1}{\phi} \right) \phi \psi_0^m$. Now using equation (13) and (16), we have a transformed form of the equation (17) given by

$$\frac{d\bar{Y}}{d\phi} + \frac{2\omega + 3}{2} \phi \ln \phi \bar{Y} = -\left[ \frac{4}{3} \left( \frac{3B_0}{4} \left( 1 - \frac{\alpha + 1}{n(\alpha + 1)} \ln \phi + C(\ln \phi)^{-n(\alpha + 1)} \right) \right) \right]^{1 \over \alpha + 1}$$

$$-B_0 \left\{ \frac{3B_0}{4} \left( 1 - \frac{\alpha + 1}{n(\alpha + 1)} \ln \phi + C(\ln \phi)^{-n(\alpha + 1)} \right) \right\}^\frac{1}{\alpha + 1} \psi_0^n \left( \ln \phi \right)^{n-\frac{1}{m}} + \frac{2k}{m \phi} \left( \ln \phi \right)^{-\frac{2}{m-1}} \psi_0^2 \left( \ln \phi \right)^{\frac{1}{n-1}}$$

(18)

Using all the substitutions and the equation (9),(10) we get the expression for the potential $V$ (after simplification) as

$$V = \phi^2 \left[ \left( 6 - \frac{2\omega + 3}{2} m^2 (\ln \phi)^2 \right) \bar{Y} + 6k \left( \frac{\ln \phi}{\phi_0} \right)^{-\frac{2}{m}} \right]$$

$$-2(\ln \phi)^{n-\frac{1}{m}} \psi_0^n \left\{ \frac{3B_0}{4} \left( 1 - \frac{\alpha + 1}{n(\alpha + 1)} \ln \phi + C(\ln \phi)^{-n(\alpha + 1)} \right) \right\}^{\frac{1}{\alpha + 1}}$$

(19)

where $\bar{Y}$ is the solution of the equation (18).

The analytical function $f$ in terms of $\phi$ is given by

$$f = \phi \psi_0^n \left( \frac{\ln \phi}{\phi_0} \right)^{\frac{an-1}{mn}}$$

(20)

We now graphically represent $V$ against $\phi$ and $f$ against $\phi$. From the figure 1 we see that considering BD scalar field as Chameleon, the potential $V(\phi)$ is sharply increasing.
Fig. 1 shows the variation of $V$ against $\phi$ for flat (red curve), closed (blue curve), open (yellow curve) universe respectively and fig. 2 shows the variation of $f$ against $\phi$ for

$$\alpha = 0.5, B_0 = 4, C = 1, m = 4, n = 2, \psi_0 = 1, \omega = -1/3.$$ 

with respect to the field and from figure 2 we see that the analytical function $f(\phi)$ is also increasing as field increases for some particular values of the parameters i.e., for $\alpha = 0.5, B_0 = 4, C = 1, m = 4, n = 2, \psi_0 = 1, \omega = -1/3$.

### III. DISCUSSIONS

Here we have considered initially general scalar tensor Brans-Dicke theory of gravity having BD scalar field as Chameleon and then using conformal transformation we rewrite the field equations in Einstein’s frame. Now considering the universe filled with variable modified Chaplygin gas and using some particular assumptions, we solve the density parameter for the radiation era. Now we graphically analyzed the potential of BD-Chameleon scalar field with respect to the field. We see from figure 1 that considering BD scalar field as a Chameleon and in presence of VMCG the potential is increasing with the increase of the field. We also see from the figure 2 that the analytical function of the field which is coupled with the matter Lagrangian in the action is also increasing with the increase of the BD-Chameleon scalar field.
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