Ductile failure viewed as a final stage of the autowave process of plastic flow localization

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Abstract. The localized plasticity development was studied for a wide range of pure metals and alloys in single-crystal and polycrystalline state as well as nonmetallic materials, alkaline halide crystals, ceramics and rocks. Using photographic and digital versions of speckle photography, the localization of plastic flow in the test samples was investigated on specially designed units having high spatial and temporal resolution. According to the analysis given here, the different stages of plastic flow are related both qualitatively and quantitatively.

1. Introduction

The kinetics of plastic flow in solids was studied experimentally from the yield point to failure. The results obtained are generalized in the monograph by Zuev et al. (2011). In the course of plastic flow development there is a strong tendency towards space-time inhomogeneity, which is a most complex problem one comes up against when dealing with the plastic deformation. The findings presented in this monograph give an answer to this problem. This view finds support in recent studies of Asharia et al. (2008) and Fressengeas et al. (2009).

Thus one of the intriguing features of plastic flow is spontaneous layering of material to cause the emergence of mobile or immobile localization nuclei on the deforming sample. Of particular interest is the localized plasticity pattern observed for the stage of linear work hardening. The work hardening coefficient for the latter stage, \( \theta \equiv E^\prime \times d\sigma / d\varepsilon = \text{const} \) (here \( \sigma \) and \( \varepsilon \) are stress and strain, respectively, and \( E \) is Young’s modulus). The space-time periodic plastic flow behavior was examined experimentally. The values obtained are \( \lambda \approx 10^{-3} \text{ m} \) and \( \Gamma \approx 10^{3}...10^{4} \text{ s} \). Hence, the propagation rate of localized plasticity nuclei, \( V_{aw} = \lambda / \Gamma \approx 10^{3}...10^{4} \text{ m/s} \). The spontaneous formation of nuclei is illustrated for the stage of linear work hardening in Figure 1. Note that the latter pattern differs from those emergent at the other flow stages.

![Figure 1](image.png)

Figure 1. A typical example of spontaneous material layering in the deforming sample of polycrystalline Al. Dark fringes correspond to plastic deformation nuclei

The problem of intermittent plastic flow behavior was addressed using specially developed method of speckle photography described by Zuev et al. (2008, 2012, 2014). The processing of experimental
data enabled discovery of the following regularities defining the space-time periodicity of deformation localization.

(i) The propagation rate of localized plasticity nuclei, \( V_{aw} \), is inversely proportional to the work hardening coefficient:
\[
V_{aw}(\theta) = V_0 + \Xi/\theta - \theta^2
\]

(ii) The process of plastic flow localization is described by the quadratic dispersion relation
\[
\omega(k) = \omega_0 + \alpha(k - k_0)^2 \sim k^2
\]
where \( \omega = 2\pi/T \) and \( k = 2\pi/\lambda \) are frequency and wave number, respectively.

(iii) The grain size dependence of spatial period is described by the logistic function
\[
\lambda(\delta) = \lambda_0 + \frac{a/\lambda_0}{1+C \cdot \exp(-a/\delta)}
\]

Evidently, the coefficients in equations (1) through (3) require a non-empirical explanation. Nonetheless, these equations are found to be valid for any material, which has a stage of linear work hardening, no matter what its structure and composition. Hence, the localization phenomenon observed for the deforming solid is addressed herein on the level of universals.

2. Localized Plasticity Autowaves

The challenge now is to explain the nature of the localized plasticity phenomenon. It has been long recognized that the deformation involves both elastic and plastic wave processes, the latter waves named after Kolsky (1963). The processes of interest have apparently all the salient features of wave processes; however, these are involved in the plastic deformation and cannot be grouped with the elastic waves.

Kolsky’s waves are similar in a way to the waves of interest, since they would form in a medium by shock loading. Kolsky’s waves propagate at the rate \( 10 \leq V_{aw} \simeq (\theta/\rho)^{1/2} \leq 10^2 \text{ m/s} \) (here \( \rho \) is the medium’s density). However, matching of the values \( V_{aw} \sim \theta^2 \); \( V_{pw} \sim \theta^2 \) and \( V_{aw}/V_{pw} = 10^2 \) reveals that Kolsky’s waves and the waves of interest basically differ. Moreover, the waves of interest have dispersion law \( \omega \sim k^2 \), which is generally employed for addressing self-organization processes in nonlinear media as pointed out by Scott (2003).

It is thus inferred herein that the localized plasticity phenomenon is a specific space-time periodic process occurring in a system far from equilibrium. This appears to be a promising line of observation in view of the fact that the available descriptions of plastic flow dynamics contain no references to such processes occurring spontaneously. The first to discover that the above possibility tends to be overlooked were Glansdorf and Prigogine (1971). They argued that it would be possible to describe adequately the deforming medium’s properties in terms of self-organization in the open system. The deforming medium undergoes self-organization via evolutionary processes termed as autowaves. Mathematically, the autowaves differ radically from the elastic waves in solids. The elastic waves are solutions to hyperbolic differential equations of the type \( \dot{y} = c \cdot y \) which have solutions of the form \( y = A \cos(\omega t - kx) \) (here \( c = \omega/k \) is the wave propagation rate). Where we have to deal with longitudinal elastic waves, the wave propagation rate may be expressed in terms of medium’s characteristics, i.e. \( c = E/\rho \).

The autowaves are solutions to parabolic differential equations of the type \( \dot{y} = \partial(x,y) + Dy^n \). In order to derive such an equation, the nonlinear function \( \partial(x,y) \) is added to the equation of \( \dot{y} = Dy^n \), which is used to describe the kinematical viscosity or diffusion of the medium. Apparently, the coefficient \( D \) has the dimension \( L^2T^{-1} \). This mathematical ambiguity would hamper the analysis of autowave formation by the plastic flow, using the equation \( \dot{y} = \partial(x,y) + Dy^n \). The theory of parabolic differential equations holds that disturbance transport rate is formally considered to be infinitely high.
However, the propagation rate of disturbance fronts is a finite value. This should be taken into account in the studies of the physics and mechanics of solids plasticity.

Using the condition of deformation flow continuity 0000000000000000 introduced by Hill (1998), equation of the type \( \dot{\varepsilon} = \partial(x, y) + D\varepsilon \) can be derived for the plastic flow. Really, thus we obtain

\[
\dot{\varepsilon} = f(\varepsilon, \sigma) + D\varepsilon
\]

(4)

where \( f(\varepsilon, \sigma) \) is a nonlinear function generally called ‘point kinetics for strain’. Scott (2003) proposed the theory of autowave processes, which holds that in order to address the deforming medium, another equation is required. For this purpose, one can write an equation in \( \sigma \), which has the same form as (4), i.e.

\[
\dot{\sigma} = g(\varepsilon, \sigma) + D\sigma
\]

(5)

equation (5) apparently corresponds to the additivity condition \( \dot{\sigma} = \dot{\sigma}_e + \dot{\sigma}_{\text{visc}} \). The first term in the right side of (5), so-called ‘point kinetics’ for stress, describes the relaxation rate of elastic stress and the second term, the relaxation rate of viscous stresses.

3. Elastic-plastic Strain Invariant

Equations (1) through (3) also hold for so-called phase autowaves of localized plastic deformation, which form at the stage of linear work hardening. This type of autowaves can be described as a system of equidistant localized plasticity nuclei, which travel synchronously at a constant velocity over the tensile sample (see Figure 1). The study was made for a range of pure metals and alloys in single-crystal or polycrystalline state, which had FCC, BCC or HCP lattice. All the diagrams \( \sigma(\varepsilon) \) obtained for the test samples show up a linear hardening stage (see Table 1).

| Cu  | Mg  | Cd  | Zn  | Al  | In  | Zr  | Ti  | Pb  | Sn  | V   | Nb  | γ-Fe | α-Fe | Ni  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|-----|
| 1.5 | 0.25| 0.55 | 0.6 | 2.2 | 2.3 | 0.6 | 0.6 | 3.2 | 1.3 | 1.3 | 0.9  | 0.7  | 1.1  | 0.7  | 0.7 |

We matched the products \( \lambda \cdot V_{aw} \) and \( \chi \cdot V_i \) (here \( \chi \) is interplanar spacing of crystal lattice and \( V_i \) is transverse ultrasound wave rate). These values are quantitative characteristics of the processes involved in the elastic and plastic deformation, which are assumed to characterize, respectively, the autowaves propagating in the deforming solid and the processes involved in crystal lattice straining. Numerical analysis suggests that \( 0.25(\text{po}) \leq 2\lambda V_{aw}/\chi V_i \leq 2.30(\text{po}) \), the mean being \( \langle 2\lambda V_{aw}/\chi V_i \rangle = 1.15 \approx 1 \).

Hence,

\[
\lambda \cdot V_{aw} \approx 1/2 \chi \cdot V_i
\]

(6)

This result testifies that the processes involved in the elastic and plastic deformation are closely related. Hence, equation (6) is termed ‘elastic-plastic strain invariant’.

According to Madelung (1996), \( V_i = \chi \omega_0 \) (here \( \omega_0 \) is the Debye frequency). Hence,

\[
\lambda V_{aw} \approx \frac{1}{2} \left( \frac{\chi V_i}{2\omega_0} \right)^2 \approx \frac{1}{2} \frac{\chi V_i}{\omega_0} \approx \frac{1}{2} \frac{G}{\omega_0 \chi} \rho \approx \frac{1}{2} \frac{\partial W/\partial \varepsilon}{\xi_i}
\]

(7)

where \( \nu \ll \chi \) is the atomic displacement near interparticle potential minimum and the elastic modulus from (7) is expressed in terms of this potential as \( G \approx \left( \partial^2 W/\partial \varepsilon^2 \right) \cdot \chi^4 \). In this case, the value \( \xi_i = (\omega_0 \chi) \cdot \rho = \rho \cdot V_i \) is the specific acoustic resistance of the medium, which might be related according Nettel (2009) to crystal lattice perturbation due to external actions. The interparticle potential from (7) can be given as
\[ W(v) \approx \frac{1}{2} \left[ \sigma^2 \frac{\partial W}{\partial \sigma^2} \right] \cdot \sigma^2 + \frac{1}{6} \left[ \sigma^2 \frac{\partial W}{\partial \sigma^2} \right] \cdot \sigma^2 = \frac{1}{2} f_2 \cdot \sigma^2 - \frac{1}{3} f_1 \cdot \sigma^2 \] (8)

where \( f_2 = \frac{\partial^2 W}{\partial \sigma^2} \) is a quasi-elastic coupling coefficient and \( f_1 = 0.5 \cdot \frac{\partial W}{\partial \sigma^2} \) is an anharmonicity coefficient. With the proviso that \( \frac{1}{2} f_2 \cdot \sigma^2 \gg \left| \frac{1}{3} f_1 \cdot \sigma^2 \right| \), equation (6) assumes the form

\[ \lambda V_\omega \approx \frac{f_2}{\xi} \approx \frac{f_2}{V_\rho} = Z \approx 10^{-5} \text{ m}^2/\text{s} \] (9)

where \( Z \) is apparently a criterion of plasticity.

Simple mathematical reasoning shows that the insertion of plasticity criterion is justified for the case of deformation initiated by a chaotic dislocation arrangement. Let \( \rho_{md} \) be mobile dislocation density. Then the average distance between dislocations is approximately equal to dislocation path and is given as \( \langle l \rangle = \rho_{md}^{\frac{1}{2}}. \) According to Friedel (1964), \( \sigma \approx \frac{G b}{2\pi} \rho_{md}^{\frac{1}{2}} \); hence, \( \rho_{md}^{\frac{1}{2}} / \sigma = \langle l \rangle = \frac{G b}{2\pi} = \sigma^{-l}. \) The velocity of quasi-viscous dislocation motion, \( V_{aw} = (b/B) \cdot \sigma \) (here \( B \) is a coefficient of viscous drag of dislocations by the phonon and electron gases in the crystal). Then we can write

\[ l \cdot V_{aw} = const = \frac{G b^3}{2\pi \cdot B} = Z \] (10)

The modulus \( G \) and the coefficient \( B \) are conventionally employed in dislocation motion descriptions. Following Al’chits and Indenbom (1986), we use the values \( G \approx 40 \text{ GPa} \) and \( B \approx 10^{-4} \text{ Pa} \cdot \text{s} \) and thus obtain \( Z \approx 10^{-7} \text{ m}^2/\text{s} \), which is close to the product \( l/2 \chi \cdot V_i \) calculated for studied materials. The above suggests that we have established a reliable quantitative criterion, which might be useful for analysis of the interaction of elastic and plastic deformation on the macro- and micro-scale levels. In its universal form the above criterion is also appropriate for the description of elastic and dislocation deformation. The same criterion can apparently be used to address autowave processes as well; hence, it is taken to be a more general form of the elastic-plastic strain invariant:

\[ \lambda \cdot V_{aw} = l \cdot V_{dist} = l/2 \chi \cdot V_i = Z \] (11)

Thus equation (11) applies to both localized plasticity autowaves and plastic deformation via dislocation glide; it might be used for description of lattice straining due to elastic wave propagation. Hence, equation (11) can be regarded as a more general version of invariant (6).

Now we shall provide a theoretical basis for validation of invariant (6). Special emphasis is laid upon the fact that the products \( \lambda \cdot V_{aw} \) and \( \chi \cdot V_i \) from equation (6) have the dimension of transport coefficient \( L^2 \cdot T^{-1} \) and so do the coefficients \( D_e \) and \( D_\sigma \) from the right sides of equations (4) and (5), respectively.

Due to simultaneously occurring interrelated flow processes, the action of thermodynamic driving force of one flow will be reciprocated by the action of thermodynamic driving force of another flow. The interference coefficients are the same for both flows. This rule is known in the thermodynamics of irreversible processes as the Onsager reciprocity principle, which was discussed, among others, by Landau and Lifshitz (1980). The plastic deformation is regarded as an irreversible non-equilibrium process; hence, the flow terms \( \dot{\varepsilon} \) and \( \dot{\sigma} \) in equations (4) and (5), respectively, will be interrelated. Then we rewrite equations (4) and (5) as

\[ \partial \varepsilon / \partial t = \partial \varepsilon / \partial x \cdot \partial D_e / \partial x + D_e \partial^2 \varepsilon / \partial x^2 = \partial \sigma + D_e \partial^2 \varepsilon / \partial x^2 \] (12)

and

\[ \partial \sigma / \partial t = \phi (\varepsilon, \sigma) + D_\sigma \partial^2 \sigma / \partial x^2 = D_\sigma \partial^2 \sigma / \partial x^2 + \zeta \varepsilon \] (13)
where the terms $\partial\sigma$ and $\zeta\epsilon$ are apparently ‘hydrodynamic’ parts of the strain and stress flows; the terms $D_\sigma \partial^2\epsilon/\partial x^2$ and $D_\zeta \partial^2\sigma/\partial x^2$, respectively, describe interference of these flows.

On the strength of dependency $\sigma(\epsilon)$, it is physically admissible that the rates $\partial\epsilon/\partial t$ and $\partial\sigma/\partial t$ are interrelated; hence, the coefficients in the right sides of equations (12) and (13) must be also related. Thus the coefficients in equations (12) and (13) will form a 2-by-2 matrix

$$\begin{pmatrix} \partial & D_\zeta \\ D_{\sigma} & \zeta \end{pmatrix}$$

Consistent with the Onsager reciprocity principle we conclude here that the interference (nondiagonal) matrix coefficients are equal to each other, i.e. $D_\sigma = D_{\sigma}$. The above suggests that the elastic-plastic strain invariant given by equation (6) follows from the general Onsager reciprocity principle for interrelated deformation and stress flows. It is therefore emphasized by Zuev (2005, 2007, 2012) that equation (1) through (4) describing the localized plastic flow autowaves are corollaries of equation (6).

One additional comment is in order here. The evidence presented herein indicates firmly that the mechanical characteristics of the deforming medium are closely related to its acoustic characteristics. This observation is substantiated by the elastic-plastic strain invariant. An attempt is also made to provide a solid grounding in theory. The existing theories establish a line of demarcation between the elastic and plastic deformation processes. Indeed, the elastic deformation processes are governed by the interparticle potential (phonon spectrum) as well as the ideal lattice properties, while the plastic deformation processes are determined by the behavior of lattice defects, i.e. dislocations and dislocation ensembles.

Thus, when it comes to gaining a physical insight into the plastic flow localization phenomenon, the importance of the elastic-plastic strain invariant cannot be too strongly emphasized. The characteristics of elastic waves are related to those of localized plasticity autowaves by equation (6): hence, plasticity is associated with both crystal defects and ideal lattice properties. Apparently, this finding lends support to plasticity model building.

4. Plastic Deformation Localization at the Pre-failure Stage

The experiments were carried out using various pure metals and alloys (see Table 2) to investigate a possible relation of autowave patterns to plastic flow stages. It was firmly established that similar relation exists. Typical example of the result obtained is presented in Figure 2. Notable that examination of the $X \times t$ diagrams and the profiles of the flow curve has permitted the following work hardening stages to be distinguished:

- linear work hardening stage ($\sigma \sim \epsilon^N$: $V_\omega = dX/dt = \text{const}$),
- parabolic work hardening stage ($\sigma \sim \epsilon^p$: $V_\omega = 0$),
- pre-failure stage or collapse of plastic flow localization wave; ($\sigma \sim \epsilon^N$: $n<\frac{1}{2}$; $V_\omega = 0$).

An intricate pattern of moving nuclei has a significant distinctive feature. Thus at the pre-failure stage each nucleus travels at a constant rate of its own, which depends on the place of its origin: the farther away the failure site is from the nucleus’s origin, the higher is the nucleus’s motion rate. Consider the conditions favoring formation of straight line bundles (see Figure 2). Evidently, this is only possible if the domain’s rate, $V_\omega$, depends linearly on the co-ordinate of its origin, $\xi$, in the instant of time, $t = t_0$, i.e. $V_\omega(\xi) = \alpha_0 + \alpha \xi$, where $\alpha$ and $\alpha_0$ are constants. The co-ordinate $\xi$ can be conveniently defined from the position of the stationary nucleus.

Thus at the final flow stage the intermittent behavior of plastic deformation would change significantly. As is pointed out by Zuev (2005, 2007), the localization of plastic deformation would inevitably involve the onset of necking and ductile failure, which might be regarded as transition process of a kind. To gain a better understanding of this process, its kinetics must be studied in detail.
This approach goes toward accounting for the failure of plastic material. As soon as the yield point is attained, a localized plasticity pattern would emerge in the deforming medium. The experimental evidence suggests that the evolution of localization zones (nuclei) occurs in an intricate manner. It was established by Zuev et al. (2008) and Zuev (2012) that the development of nuclei pattern is subject to the work hardening law. In the course of deformation the space-time periodic pattern of nuclei is finally replaced by a single localized plasticity nucleus. Thus, transition from the pre-failure stage to necking and viscous fracture might be regarded as a kind of ‘collapse’ of the ordered distribution of localization nuclei.

The localized plasticity nuclei traveling in concerted manner are illustrated in Figure 2. The changing pattern of nuclei is plotted for the final flow stage in the co-ordinates $X - t$ (here $X$ is nucleus position and $t$ is time). It can be seen that the rectilinear plots converge, thus forming a bundle; the bundle has a common intersection point designated as ‘pole’, which has the co-ordinates $X_{pole}$ and $t_{pole}$. These values can be defined with the help of extrapolation of initial parts of $X - t$ plots to the intersection point, $P$. Figure 2 explains this procedure. A series of experiments performed for a range of alloys having FCC, BCC or HCP lattice. It has been found that the co-ordinates $X_{pole}$ and $t_{pole}$ can be defined for all the test samples within a comparatively small area in the plane $X - t$. To do this, the plots $X(t)$ would sometimes be extrapolated to larger times. It may be claimed that at the beginning of the final (pre-failure) stage the localization nuclei would move in a concerted manner to simultaneously arrive in the point $X_f$ at the time $t_f$.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The diagrams $X-t$ plotted for coarse grained Ti: linear work hardening stage (1); parabolic work hardening stage (2); pre-failure stage (3) and pole $P$.

The rectilinear plots $X - t$ similar to those shown in Figure 2 were examined and the co-ordinates $X_f$ and times $t_f$ were measured. The values $X_{pole}$ and $t_{pole}$ are found to be related to the values $X_f$ and $t_f$, respectively. Of particular note is the fact that sample failure would invariably occur in real time at the point in which the localization nuclei arrive simultaneously and which coincides with the pole of a bundle of plots (Table 2). It can be seen that calculations show close agreement with experiment.

| Materials studied | Al | Fe-Si alloy | High chromium steel | V based alloy | Submicrocrystalline Ti | Coarse grained Ti |
|-------------------|----|-------------|---------------------|-------------|-----------------------|------------------|
| $X'_{pole}/X'_f$ | 1.1| 1.0         | 1.0                 | 1.0         | 1.0                   | 1.1              |
| $t'_{pole}/t'_f$ | 1.0| 1.1         | 1.1                 | 1.4         | 1.1                   | 1.1              |

**Table 2.** The ratio of calculated and experimental co-ordinates and times of specimen failure.

An idea of interest was proposed to develop a new non-destructive method for monitoring natural and technical objects, which would facilitate prediction of the place and time of failure. On the base of available evidence, this proposition was verified experimentally for rock samples of marble, sandstone...
and sylvinite by Zuev et al. (2013). The well-established observational results are found to be consistent with the experimental values of co-ordinates and times, $X_{pole}$, $X_{f}$, and $t_{pole}$, $t_{f}$, respectively.

5. Conclusions

1. The plastic flow and failure are regarded here as individual processes, which involve formation and evolution of autowaves in the deforming medium. The plastic deformation is shown to exhibit an intermittent behavior with a changeover in the flow stages from a steady-state flow to the onset of necking and failure.

2. The parabola exponent $n$ is considered an indication of the loss of plastic flow stability; the value $n$ may vary significantly and reverse sign for each nucleus observed at the pre-failure stage. Hence, the deformation behavior and capability for work hardening of individual material volumes might vary significantly.

3. The examination of localized deformation patterns suggests that the location and time of future fracture can be determined long before a macroscopic neck forms in the test sample.

4. On the base experimental evidence a new method can be developed for predicting the place and time of material failure.

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