Single Level String Theory

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The \(\mathbf{O}(D,D)\) covariant generalized metric, postulated as a truly fundamental variable, can describe novel geometries where the notion of Riemannian metric ceases to exist. Here we quantize a closed string upon such backgrounds and identify flat, anomaly-free, non-Riemannian string vacua in the familiar critical dimension, \(D=26\) (or \(D=10\)). Remarkably, the whole BRST closed string spectrum is restricted to just one level with no tachyon, by means of its inverse, lowers and raises the fundamental entities (instead of \(g\)).

Motivation: absence of tachyon kinetic term in DFT

Ever since the adoption of Riemannian geometry into the formulation of General Relativity, the metric, \(g_{\mu\nu}\), has been privileged to be the fundamental variable that provides a concrete mathematical tool to address the notion of spacetime. In particular, the ‘flat’ spacetime where the gravitational effect is negligible is simply given by a constant metric of Minkowskian signature. Needless to say, the Standard Model upon this background is arguably the perfect model, \(\psi = x^M = (\tilde{x}_\mu, x^\nu)\). With it, the closed string effective action [1] can be reformulated as a DFT coupled to the tachyon,

\[
\int e^{-2d} \left[ S_{DFT} - \frac{2(D-26)}{4\sigma} - \mathcal{H}_{MN} \partial_M T \partial_N T + \frac{1}{2} \mathcal{O}(T^3) \right].
\]

Here \(d\) is the \(\mathbf{O}(D,D)\) singlet DFT-dilaton related to the conventional dilaton through \(e^{-2d} = \sqrt{-g} e^{-2\phi}\), and \(S_{DFT}\) denotes the scalar curvature in DFT which can be spelled out explicitly in terms of \(d\), \(\mathcal{H}_{MN}\), and \(\mathcal{J}_{MN}\) [20]:

\[
S_{DFT} = \frac{1}{2} \mathcal{H}_{MN} \partial_M \mathcal{H}_{KL} \partial_N \mathcal{H}^{KL} + \frac{1}{4} \mathcal{H}_{MN} \partial^K \mathcal{H}_{ML} \partial^K \mathcal{H}^{NL} - \partial_M \mathcal{H}_{MN} \partial^N + 4 \partial_M (\mathcal{H}_{MN} \partial^N d) - 4 \mathcal{H}_{MN} \partial_M d \partial_N d.
\]

The so-called section condition should be imposed on the doubled coordinates, \(\partial_M = (\partial^\nu, \partial)_\mu\),

\[
\partial_M \partial^{\mu} = \partial^\nu \partial^{\mu} \partial^{\nu} \partial^{\nu} \partial_{\mu} \equiv 0,
\]

such that all quantities have \(D\)-dimensional halved dependence. After solving the section condition by letting \(\partial^\nu \equiv 0\) and assuming the Riemannian parametrization [3], the DFT action [1] reduces to [1].

Crucially, [3] is not the most general solution to the defining relations [2]. It only becomes so, if the upper left \(D\times D\) block, \(i.e., \mathcal{H}^{\mu\nu}\), is invertible. DFT works perfectly fine with any generalized metric that satisfies [2]. The \(D\times D\) block can be degenerate, hence non-Riemannian by nature [21, 26] (see also [25, 32] for supersymmetric or exceptional examples). Most general parametrizations of a generalized metric have been classified by two non-negative integers, \((n, \bar{n})\), rendering string chiral and anti-chiral over \(n\) and \(\bar{n}\) directions [23], see also [13] later. The Riemannian geometry of [3] is of \((0, 0)\) type and non-relativistic/ultra-relativistic strings [34, 40] belong to \((1, 1)\) or other types [22, 23, 25, 32].
Postulating \( \{ J_{MN}, H_{MN}, d \} \) as the only geometric quantities available to contrive connections and curvatures, one can construct a compatible covariant derivative, \( \nabla_L = \partial_L + \Gamma_{L} \), with the unique choice of the ‘Christoffel’ symbols, \( \Gamma_{LMN} \). This in turn gives the scalar curvature \( S_{(0)} \) and two-index ‘Ricci’ curvature which further form the off-shell conserved ‘Einstein’ tensor, \( \nabla_M G^{MN} = 0 \). This is all analogous to General Relativity, though there seems no four-index Riemann tensor \( \bar{\Gamma} \). Using them, one can concisely express all the equations of motion of the DFT action \( \mathcal{H} \):

\[
\mathcal{H}^{MN} \nabla_M \nabla_N T + \frac{1}{\alpha^2} T + \mathcal{O}(T^2) = 0, \quad G_{MN} = T_{MN}.
\]

(6)

The former is the tachyonic equation of motion and the latter is the ‘Einstein equations’ in DFT. It unifies the equations of motion of \( H_{MN} \) and \( d \) into a single formula, equating the Einstein tensor with a generalised stress-energy tensor. For the tachyon field it reads

\[
T_{MN} = (J + \mathcal{H})_{M} L (J - \mathcal{H})_{N} \partial_{K} T \partial_{L} T - \frac{1}{8} J_{MN} T_{(0)},
\]

where \( T_{(0)} = -\frac{1}{4} T_{M}^{M} \) is the \( O(D, D) \) singlet trace part,

\[
T_{(0)} = H^{MN} \partial_{M} T \partial_{N} T - \frac{1}{\alpha^2} T^2 - \mathcal{O}(T^3) + \frac{2(D-26)}{3\alpha^2}.
\]

In particular, the equation of motion of the DFT-dilaton, or the trace of the Einstein equations, implies \( S_{(0)} = T_{(0)} \). Thus, in \( D = 26 \), if the tachyon potential admits a global minimum away from \( T = 0 \), we have \( T_{(0)} < 0 \) and hence the background cannot be flat, \( S_{(0)} < 0 \).

While we refer the interested readers to section 2 of [42] for a detailed review of the above formalism, for now what suffices us is that the Einstein curvature, \( G_{MN} \), vanishes for constant \( H_{MN} \) and \( d \). Any flat background with vanishing tachyon, \( T = 0 \), solves all the equations of motion \( (7) \) in the critical dimension, \( D = 26 \). Surely this statement is also valid for the Riemannian action \( \mathcal{H} \).

The novelty here is that the DFT action \( \mathcal{H} \) allows non-Riemannian geometries. With the choice of the section by \( \tilde{\partial}_{\mu} \equiv 0 \), the tachyon kinetic term reads \( \nabla_{\mu} \nabla_{\nu} \partial_{\mu} T \tilde{\partial}_{\nu} T \) which obviously vanishes when \( \nabla_{\mu} \equiv 0 \). The vanishing kinetic term then may eliminate the instability of the static configurations: there is no dynamics for the tachyon to roll down (at least classically at linear order). The absence of kinetic term was also discussed for tachyon condensation in open string field theory \( \mathcal{H} \), while it is a generic feature of ‘pregeometrical’ or ‘Purely Cubic’ string field theories \( \mathcal{H} \).

The generalised metric with \( \nabla_{\mu} \equiv 0 \) is in a way ‘maximally’ non-Riemannian. It corresponds to the \( (n, \bar{n}) \) type with \( n + \bar{n} = D \), and assumes the most general form \( \mathcal{H} \),

\[
\mathcal{H}_{MN} = \begin{pmatrix}
0 & Y_{\mu}^{\lambda} \bar{Y}_{\mu}^{\lambda} - Y_{\mu}^{\mu} \bar{X}_{\lambda}^{\lambda} \\
X_{\lambda}^{\mu} Y_{\nu}^{\nu} - \bar{X}_{\lambda}^{\mu} \bar{Y}_{\nu}^{\nu} & 2 \bar{X}_{\lambda}^{\mu} B_{\lambda}^{\nu} \bar{Y}_{\nu}^{\rho} - 2 \bar{X}_{\lambda}^{\mu} B_{\lambda}^{\mu} \bar{Y}_{\nu}^{\rho}
\end{pmatrix},
\]

(7)

where \( i = 1, 2, \ldots, n \) and \( \bar{i} = 1, 2, \ldots, \bar{n} \). Viewing \( (X_{\lambda}^{\mu}, \bar{X}_{\lambda}^{\mu}) \) as a \( D \times D \) matrix, \( (Y_{\mu}^{\nu}, \bar{Y}_{\nu}^{\mu}) \) is its inverse satisfying \( X_{\mu}^{\nu} Y_{\nu}^{\nu} + \bar{X}_{\mu}^{\nu} \bar{Y}_{\nu}^{\nu} = \delta_{\mu}^{\nu} \). The underlying coset is

\[
O(D, D) / O(n, \bar{n}) \times O(n, \bar{n}) \]

whose dimension, \( 4n\bar{n} \), matches the number of infinitesimal fluctuation modes, i.e. moduli, around the generalized metric \( \mathcal{H} \). The types of \( (D, 0) \) or \( (0, D) \) are worthy of note. They are uniquely given by \( H_{MN} = \pm J_{MN} \), and correspond to the most symmetric vacua of DFT with no moduli \( \mathcal{H} \). Intriguingly then, Riemannian spacetime may arise in DFT after the spontaneous symmetry breaking of \( O(D, D) \), which identifies \( \bar{\Omega}_{\mu} \) and \( B_{\mu} \) as the massless Nambu–Goldstone bosons \( \mathcal{H} \) (c.f. [51, 52]).

In the remaining of this Letter, we investigate the quantum consistency of the non-Riemannian geometries \( \mathcal{H} \) as for novel string vacua. Through BRST quantization of string, we show that the type of \( (n, \bar{n}) = (13, 13) \) with \( D = 26 \) is anomaly-free. Remarkably, the string spectrum is finite with no tachyon mode, matches the coset underlying \( \mathcal{H} \), and agrees with the linearized DFT equations of motion, i.e. the vacuum Einstein equations, \( G_{MN} = 0 \). We shall conclude with remarks on extension to type II superstring and application as an alternative to string compactification on Riemannian manifolds.

**BRST quantization of doubled-yet-gauged string**

The doubled string action we consider is \( \mathcal{H} \),

\[
S = \frac{1}{4\alpha^2} \int d^2 \sigma \mathcal{L}, \quad \mathcal{L} = -\frac{1}{2} \sqrt{-\bar{\Omega}_{\mu} \partial_{\mu} T \bar{\partial}_{\nu} T} - \mathcal{O}(T^3) + \frac{2(D-26)}{3\alpha^2}.
\]

(8)

\( \bar{\Omega}_{\mu} \) is a covariant derivative with an auxiliary potential that satisfies a section-condition-like constraint,

\[
D_{\alpha} X^{\lambda} = \partial_{\alpha} X^{\lambda} - A^{\lambda}_{\alpha} - A_{\alpha}^{M} \partial_{M} = 0.
\]

(9)

While the action is completely covariant under desired symmetries like \( O(D, D) \) rotations, Weyl symmetry, worldsheet as well as doubled target spacetime diffeomorphisms, it also concretely realizes the idea that the doubled coordinates in DFT are actually gauged and each gauge orbit corresponds to a single physical point \( \mathcal{H} \). The relevant ‘coordinate gauge symmetry’ reads

\[
\delta x^{\lambda} = \Delta^{\lambda}, \quad \delta A_{\alpha} = \partial_{\alpha} \Delta^{\lambda}, \quad \Delta^{\lambda} \partial_{M} = 0,
\]

(10)

which leaves \( \bar{\Omega}_{\mu} X^{\nu}, H_{MN} \) invariant \( \Delta^{\lambda} \partial_{M} H_{MN} = 0 \), and enables us to identify the first term in the Lagrangian \( \mathcal{H} \) as a ‘proper area’ in doubled geometry \( \mathcal{H} \).

With the choice of the section, \( (\tilde{\partial}_{\mu}, \partial_{\nu}) \equiv (0, \partial_{\nu}) \), which we henceforth thoroughly assume, the constraints on the gauge potential \( \mathcal{H} \) and parameter \( \mathcal{H} \) are solved by \( A^{\lambda}_{\alpha} = (\alpha_{\mu}, 0) \) and \( \Delta^{\lambda} \equiv (\Delta_{\mu}, 0) \). Clearly then, it is the tilde coordinates \( \tilde{x}_{\mu} \) that are to be gauged:

\[
D_{\alpha} x^{\lambda} = (\partial_{\alpha} \tilde{x}_{\mu} - A_{\alpha}^{\lambda} - \partial_{\alpha} x^{\nu}).
\]

Upon the Riemannian background \( \mathcal{H} \), the potential
where \( \tilde{C}_{\beta \mu} \)'s are arbitrary local parameters. Yet, despite their seemingly free index, i.e. '\( \beta \)'s, since \( h_{\pm \alpha} \)'s are 2 \( \times \) 2 projection matrices with nontrivial kernel, the extra gauge symmetry can be specified simply by the alternative parameter, \( C_{\mu} \), carrying no worldsheet index.

We proceed to fix all the gauges, \( \delta_{0, 14, 15} \):\( \begin{aligned} 
& e \equiv 1, \quad \omega \equiv 0, \quad \bar{x}_{\mu} \equiv 0, \quad p_{-i} \equiv 0, \quad p_{+i} \equiv 0, \\
& \text{which imply, } -\sqrt{h}h^{\alpha \beta} \equiv \begin{pmatrix} 0 & 1 \\
1 & 0 \end{pmatrix} \text{ on the light-cone and}
& \text{the vanishing of the topological term in (13).}
\end{aligned} \)

The full Lagrangian with Faddeev–Popov ghosts is then

\[ L_{\text{full}} = L_0 - i\delta_\alpha \left( \ln c \beta \epsilon + \bar{x}_{\mu} \tilde{B}^{\mu} + p_{-i} B^{i} + p_{+i} B^{i} \right), \]

where \( \{ \beta, \bar{\beta}, B^{\mu}, B^{\mu} \} \) are the anti-ghosts for the gauge symmetries of \( (11) \), \( (14) \), \( (15) \). With the associated ghosts, \( \{ \epsilon^{\sigma}, \bar{C}_\mu, C_\mu \} \), and auxiliary Nakanishi–Lautrup fields, \( \{ \kappa_\epsilon, \bar{\kappa}_{\epsilon}, \kappa_{\mu}, \bar{\kappa}_{\mu} \} \), the BRST transformations are

\[ \begin{aligned}
\delta_\alpha x^\mu &= \epsilon^\sigma \partial_\alpha x^\mu, \\
\delta_\alpha \bar{x}_{\mu} &= c^\sigma \partial_\alpha \bar{x}_{\mu} + \bar{C}_\mu, \\
\delta_\alpha p_{\pm i} &= (\omega - e \pm 1) C_i + c^\sigma \partial_\alpha p_{\pm i} + \partial_\alpha c^\sigma p_{\pm i} \equiv \partial_\alpha c^\sigma p_{\pm i}, \\
\delta_\alpha b_{\epsilon} &= 0, \\
\delta_\alpha b_{\omega} &= i\kappa_{\epsilon}, \\
\delta_\alpha \tilde{B}^{\mu} &= i\kappa_{\mu}, \\
\delta_\alpha B^{\mu} &= i\kappa_{\mu}, \\
\delta_\alpha \kappa_\epsilon &= \delta_\alpha \kappa_{\mu} &= \delta_\alpha \kappa_\epsilon &= \delta_\alpha \kappa_{\mu} &= 0,
\end{aligned} \]

while \( \delta_\alpha e = \delta_\alpha \epsilon = \delta_\alpha \omega = \delta_\omega \) are already given in \( (14) \), promoting the diffeomorphism parameters, \( c^\sigma \), as ghosts. The transformations are off-shell nilpotent, \( \delta_\alpha^2 = 0 \).

From the variational principle, setting \( b_{\epsilon} \equiv b_{\epsilon} + b_{\omega} \), \( b_{\omega} \equiv b_{\omega} - b_{\omega} \), and similarly for \( \kappa_\epsilon, \kappa_\omega \), we acquire

\[ \begin{aligned}
p_{+i} \partial_\alpha x^i + 2ib_{++} \partial_\alpha c^+ + i(\partial_+ b_{++})c^+ &= \kappa_{++}, \\
p_{-i} \partial_\alpha x^i + 2ib_{--} \partial_\alpha c^- + i(\partial_- b_{--})c^- &= \kappa_{--}, \\
p_{-i} &= p_{+i} = \kappa_\mu = \kappa_\mu = \tilde{B}^{\mu} = \tilde{C}_\mu = B^{\mu} = C_\mu = 0,
\end{aligned} \]

and the left/right-moving (chiral/anti-chiral) properties,

\[ \begin{aligned}
\partial_- x^i &= 0, \quad \partial_- p_{+i} = 0, \quad \partial_- c^+ = 0, \quad \partial_- b_{++} = 0, \\
\partial_+ x^i &= 0, \quad \partial_+ p_{-i} = 0, \quad \partial_+ c^- = 0, \quad \partial_+ b_{--} = 0,
\end{aligned} \]

which can be also derived from the reduced Lagrangian,

\[ L_{\text{red.}} = 2(p_{+i} \partial_\alpha x^i + p_{-i} \partial_\alpha x^i + ib_{++} \partial_\alpha c^+ + ib_{--} \partial_\alpha c^-).
\]

Naturally, \( \{ p_{+i}, p_{-i} \} \) are identified as the conjugate momenta of \( \{ x^i, x^i \} \), forming \( D \) pairs of '\( \beta \gamma \)' system.
with the conformal weights $1$ and $0$, for $\beta_i \equiv p_{-i}, \bar{\beta}_i \equiv p_{-i}$ and $\gamma_j \equiv x^j, \bar{\gamma_j} \equiv \bar{x}^j$ respectively. Each pair contributes to a central charge by two.

The BRST charge decomposes, $Q_B = Q_L + Q_R$, with

$$Q_L = \int \! d\sigma \beta_i \partial_L \gamma^i \gamma^+ + i b_{++} \partial_L c^+ c^+ = : \sum_{m,n=-\infty}^\infty (i \partial_m \gamma^i + b_m c_n) c_{m-n} : = -ac_0,$$

and mirroring expression for $Q_R$. The quantization is given by $[\gamma^i_m, \beta_n] = i \delta^i_m \delta_{m+n}$ and $\{ b_m, c_n \} = \delta_{m+n}$, which generate the normal ordering constant, $a$. The BRST charges, $Q_L, Q_R$, are nilpotent, if and only if $n = \bar{n} = 13$, implying the usual critical dimension, $D = 26$, since the central charges are $c_L = 2n - 26$ and $c_R = 2\bar{n} - 26$, both of which should vanish.

Physical states are annihilated by $Q_L$ and the anti-ghost zero mode $b_0$ (mirrored by the right-moving sector). Their anti-commutator is

$$L_0 = \{ b_0, Q_B \} = N_\beta + N_\gamma + N_b + N_c - a,$$

where

$$N_\beta = \sum_{p=1}^\infty -ip \beta_p \gamma^i_p, \quad N_\gamma = \sum_{p=1}^\infty ip \gamma^i_p \beta_p,$$
$$N_b = \sum_{p=1}^\infty pb_p c_p, \quad N_c = \sum_{p=1}^\infty pc_p b_p,$$

are the level-counting operators for each creation operator with $p > 1$. These are all positive semi-definite. Hence, the vanishing of $L_0$ on physical states means a drastic truncation of the entire string spectrum to just one level. Computing $\langle 0| L_0, L_{-1} |0 \rangle = -2$ with $L_n = \{ Q_L, b_n \}$, we identify the level to be unity, $a = 1$.

Then, from $\beta_0 |p\rangle = p_i |p\rangle, Q_L c_{-1} |p\rangle = Q_L \gamma^i_{-1} |p\rangle = 0$, and

$$Q_L \beta_{-1} |p\rangle = p_i c_{-1} |p\rangle, \quad Q_L b_{-1} |p\rangle = (ip \gamma^i_{-1} + 2c_{-1} b_0) |p\rangle,$$

the physical states consist of four sectors, (with $|p\rangle$ satisfying $b_0 |p\rangle = 0$),

$$\delta \mathcal{H}_{\bar{a}} \gamma^i_{-1} |p_3 \rangle \otimes \bar{\gamma}^i_{-1} |p_3 \rangle, \quad \delta \mathcal{H}_{\bar{a}} \gamma^i_{-1} |p_3 \rangle \otimes \bar{\gamma}^i_{-1} |p_3 \rangle,$$
$$\delta \mathcal{H}_{a} \beta_{-1} |p_3 \rangle \otimes \bar{\gamma}^i_{-1} |p_3 \rangle, \quad \delta \mathcal{H}_{a} \beta_{-1} |p_3 \rangle \otimes \bar{\gamma}^i_{-1} |p_3 \rangle,$$

which should satisfy on-shell relations for $Q_B$-closedness,

$$p_i \delta \mathcal{H}_{\bar{a}} = 0, \quad p_i \delta \mathcal{H}_{a} = 0, \quad p_i \delta \mathcal{H}^i = 0, \quad p_i \delta \mathcal{H}^{\bar{i}} = 0,$$

and equivalence relations as for gauge symmetries,

$$\delta \mathcal{H}_{\bar{a}} \sim \delta \mathcal{H}_{a} + p_i \lambda_i - p_i \bar{\lambda}_i,$$
$$\delta \mathcal{H}_{\bar{a}} \sim \delta \mathcal{H}_{a} + p_i \bar{\epsilon}_i, \quad \delta \mathcal{H}^i \sim \delta \mathcal{H}^{\bar{i}} + p_i \epsilon^i,$$

where $\xi^i, \bar{\xi}^i$ need to be divergenceless, $p_i \epsilon^i = p_i \bar{\epsilon}_i = 0$.

We have a good reason to denote the physical states by the same symbol as the generalized metric: the $4n\bar{n}$ of $\{ \delta \mathcal{H}_{\bar{a}}, \delta \mathcal{H}_{a}, \delta \mathcal{H}^i, \delta \mathcal{H}^{\bar{i}} \}$ are literally the moduli of the generalized metric $\mathcal{H}$ that we have been dealing with, in the diagonal form where the only non-trivial components are $\mathcal{H}_{\bar{a}} = \mathcal{H}_{a} = \delta^i j$ and $\mathcal{H}^{\bar{i}} = \mathcal{H}^i = -\delta^{\bar{i}} j$. They meet the linearized DFT equations (see also $[66]$):

$$\partial_i \partial_j \delta \mathcal{H}^{\bar{i}} = 0, \quad \partial_i \partial_j \delta \mathcal{H}^i = 0, \quad \partial_i \partial_j \delta \mathcal{H}^{\bar{i}} = 0,$$
$$\partial_i \partial_j \delta \mathcal{H}^i = -\partial_i \partial_j \delta \mathcal{H}^{\bar{i}} + 4 \partial_i \partial_j \delta d = 0,$$

which enjoy local symmetries inherited from the General Covariance of DFT (generalized Lie derivative, $L_\xi \mathcal{H}_{MN}$),

$$\delta (\delta \mathcal{H}^i) = \partial_i \xi^i, \quad \delta (\delta \mathcal{H}^{\bar{i}}) = -\partial_i \bar{\xi}^i,$$
$$\delta (\delta \mathcal{H}_{\bar{a}}) = \partial_i \lambda_i - \partial_i \bar{\lambda}_i, \quad \delta (\delta d) = -\frac{1}{4} (\partial_i \xi^i + \partial_i \bar{\xi}^i).$$

We may choose a gauge, $\delta d = 0$. Remarkably then, $\mathcal{H}_{a}, \mathcal{H}_{\bar{a}}$ imply $\mathcal{H}_n, \mathcal{H}_{\bar{n}}$. Further, restricted to normalizable solutions, the converse appears also true. The first and second in (22) give $\partial_i \delta \mathcal{H}^{\bar{i}} = 0$, $\partial_i \delta \mathcal{H}^i = 0$, which are generically solved by $\delta \mathcal{H}^{\bar{i}} = e^{i(k_1 \cdots k_l \cdots k_1 \cdots k_l \cdots)} \partial_j \delta \mathcal{H}_{K_1 \cdots K_l}$, hence the third holds. The last implies $\partial_i \partial_j \delta \mathcal{H}^{\bar{i}} = \partial_i \partial_j \delta \mathcal{H}^i = \delta \partial_i \phi$ for some $\phi$. Again for normalizable solutions, we get $\partial_i \partial_j \delta \mathcal{H}^i = \delta \partial_i \phi$ and $\partial_i \partial_j \delta \mathcal{H}^{\bar{i}} = \delta \partial_i \phi$, which can be gained away using the remaining (25).

As the spectrum is finite, DFT itself is to be identified as string field theory around the non-Riemannian vacua. Evidently from the position of the indices, it is $\mathcal{H}^{\bar{i}}$ that may condensate and reduce the ‘non-Riemannianity’ $[26]$.

**Comments: supersymmetric extension**

Our BRST charge formula (20) can be easily extended to a generic $(n, \bar{n})$ non-Riemannian background, to include $n$ pairs of chiral $\beta^i, \bar{\beta}_i$ pairs of anti-chiral $\bar{\beta}_i, \beta_i$, and ordinary (left-right combined) $D - n - \bar{n}$ number of $x^i$. The central charges are $c_{L,R} = D \pm (n - \bar{n}) - 26$, and thus necessarily $n = \bar{n} = D = 26$. Non-relativistic string theories $[34, 37]$ are examples of $(n, \bar{n}) = (1, 1)$ $[61]$.

The necessity of putting $n = \bar{n}$ was also noted in $[22]$ as a condition to embed non-Riemannian geometries into type II doubled superstring $[24, 32]$ or supersymmetric DFTs $[62, 64]$, the constructions of which rely on genuine $O(D, D)$ covariant vielbeins rather than the Riemannian zehnbein, $e_{\mu}^a [65]$. The central charges should be $c_{L,R} = D \pm (n - \bar{n}) - 10$, indicating that $n = \bar{n}$ non-Riemannian geometries are consistent superstring vacua in $D = 10$, which enlarges the string theory landscape far beyond the Riemannian paradigm.

Chiral string means $x^i (\tau, \sigma) = x^i (0, \tau + \sigma)$: it is fixed in space and thus hardly interacts with one another. This classical intuition is complementary to the exclusive ‘single level’ property of the string spectrum we have reported, and may suggest to use non-Riemannian backgrounds as an internal space alternative to string compactification traditionally on “small" Riemannian manifolds. In the presence of external four-dimensional Minkowskian spacetime, the truncation of string spectrum to just one level might be no longer the case as...
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