A Feedback Linearization Based Nonlinear Control Approach for Variable Speed Wind Turbines

Afef Fekih and Abdullah Al Shehri

University of Louisiana at Lafayette, USA

Abstract: This paper describes the design and implementation of a nonlinear control strategy for the control of the shaft speed of wind turbine systems. The proposed approach is based on input-output linearization techniques. Because wind turbine systems are highly nonlinear, feedback linearization constitutes a suitable optimal control design for those systems. Further, Electromechanical systems in general are good candidates for nonlinear control applications because the nonlinearities, being modeled on the basis of physical principles, are often significant and exactly known.

The underlying design objective is to endow the wind turbine with high performance dynamics while maximizing power extraction when the wind turbine operates in the partial load regime. In addition to fulfilling the aforementioned control objectives, our control design aims to reduce the complexity of the control scheme, saving thereby the computation time of the control algorithm, which is an improvement over previous work found in the technical literature.

Application of the proposed approach to an induction generator based variable speed wind turbine has led to optimum operations and maximization of power extraction when the wind turbine operates in the partial load regime.

Keywords: Wind turbine, feedback linearization, induction motors, variable speed.

I. INTRODUCTION

Wind energy has been the fastest growing energy technology since the 1990, in terms of percentage of yearly growth of installed capacity per technology source [1]. According to 2012 half year report recently released by the Global Wind Energy Council, the global wind industry installed more than 16,000 MW of wind power capacity in the first six months of 2012, a 7% increase in global installed capacity. This brings the total installed capacity globally to more than 254,000 MW by the end of June 2012 [2]. About 75 countries worldwide have commercial wind power installations, with 22 of them already passing the 1 GW levels [3].

The growth of wind energy, however, has not been evenly distributed around the world, with the five leading countries, China, USA, Germany, Spain and India, representing together a total share of 74 % of the

This rapid growth in wind energy installations worldwide has led to a growing demand for better modeling and control of wind turbines. The uncertainties and difficulties in measuring the wind inflow to wind turbines, coupled with the complex design of the wind energy systems makes the control design challenging. Moreover, the goals and strategies of wind turbine control are affected by the turbine configuration as well as the conditions under which the wind turbine produces the power.

When controlling a wind turbine, the overall target is to minimize the operational cost while maximizing the generated power. Advanced control technology can improve the performance of wind turbines by increasing turbine efficiency, and thus energy capture, and by reducing structural load, which increases the life span of the structures and components. This requires a design that continuously monitors the trade-off between energy efficiency and increased service lifetime of wind energy systems by alleviating fatigue loads.

Wind turbines can operate at fixed or variable speed. Fixed speed wind turbines are the pioneers of the wind turbine industry. They are simple, reliable and use low cost electrical parts [4]. They use induction generators and they are connected directly to the grid, giving them an almost constant speed stuck to the grid frequency, regardless of the wind speed [5, 6]. Variable speed wind turbines are currently the most used wind energy systems. Their advantages compared to fixed speed wind turbines are numerous [7].

In wind turbine technology, the doubly-fed induction generators are widely used in variable speed wind turbines due to their reliability, ruggedness and relatively low cost. However, from the control point of view, induction motors constitute a class of highly coupled and multivariable systems with two control inputs (stator voltages) and two output variables (rotor speed and rotor flux modulus), required to track desired reference signals. One particular approach for the control of induction motors is the Field Oriented Control [8]. Partial feedback linearization together with a proportional integral (PI) controller is used to regulate
the motor states. This control strategy achieves the required control objective asymptotically. A disadvantage of the Field Oriented Controller is that the method assumes the magnitude of the rotor flux to be regulated to a constant value. Therefore, the dynamics of the speed and flux may interfere in the high speed ranges. Eliminating this coupling and achieving high performance dynamics for all speed ranges can be realized by considering an input-output linearization technique [8, 9].

In this paper, a new controller based on the theory of feedback linearization is proposed for a doubly-fed induction motor driven by a variable speed wind turbine. The contributions of this paper are two-fold:

- The $d$-$q$-axis stator voltages are optimally controlled to achieve the maximum wind power generation.
- The effect of magnetic saturation at high speed ranges is considered in the control scheme design.
- The controller adjusts the turbine speed to the optimal speed, thus optimizing the power efficiency coefficient.
- Although exact input-output decoupling controls for variable speed wind turbines were proposed in [10, 11], they used the full-order model, which resulted in a highly complex control strategy that required powerful and expensive digital controllers. In our approach, we use a reduced order model which produces considerable simplification and reduced complexity of the control scheme.

The paper is organized as follows. The structure of a variable speed wind turbine is briefly reviewed in section II. The proposed control approach is developed in section III. Computer simulation studies are conducted in section IV to examine the effectiveness of the proposed approach under steady and variable wind conditions. Finally, some concluding remarks are given in section V.

II. VARIABLE SPEED WIND TURBINES

Wind turbines are structures that transform the kinetic energy of the incoming air stream into electrical energy. There are two basic configurations, namely vertical axis wind turbines (VAWT) and horizontal axis wind turbines (HWAT). Most modern wind turbines are horizontal-axis wind turbines (HAWTs) with three (rotor blades usually placed upwind. The main components of a HAWT are the turbine tower which carries the nacelle, and the wind turbine rotor, consisting of rotor blades and hub, as shown in Figure 1 [12]. The airfoil-
shaped blades capture the kinetic energy of the wind and transform it into the rotational kinetic energy of the wind turbine’s rotor. The rotor drives the low-speed shaft, which in turn drives the gearbox.

Wind turbines can operate at fixed or variable speed. Fixed speed wind turbines are the pioneers of the wind turbine industry. They are simple, reliable and use low cost electrical parts. They use induction generators and they are connected directly to the grid, giving them an almost constant speed stuck to the grid frequency, regardless of the wind speed.

Variable speed wind turbines are currently the most used wind energy systems. As their name indicates, variable-speed generator and power electronic drives enable the wind turbine control system to adapt the rotational speed of the wind turbine rotor to the varying wind speed over a relatively wide speed range [4, 6]. The variable speed operation is possible due to the power electronic converters interface, allowing a partial to full decoupling from the grid. Variable speed operations yield 20 to 30% more energy than fixed speed operation since they tend to operate closer to their maximum aerodynamic efficiency for a higher fraction of the time. Variable speed operation can also reduce turbine loads, since sudden increase in wind energy due to gusts can be absorbed by an increase in rotor speed rather than by component bending [1].

In wind turbine technology, the doubly-fed induction generators are widely used in variable speed wind turbines due to their reliability, ruggedness and relatively low cost. However, from the control point of view, induction motors constitute a class of highly coupled and multivariable systems with two control inputs (stator voltages) and two output variables (rotor speed and rotor flux modulus), required to track desired reference signals.

One particular approach for the control of induction motors is the Field Oriented Control [7]. Partial feedback linearization together with a proportional integral (PI) controller is used to regulate the motor states. This control strategy achieves the required control objective asymptotically. A disadvantage of the Field Oriented Controller is that the method assumes the magnitude of the rotor flux to be regulated to a constant value. Therefore, the dynamics of the speed and flux may interfere in the high speed ranges. Eliminating this coupling and achieving high performance dynamics for all speed ranges can be realized by considering an input-output linearization technique [8, 9].

III. MODELING AND PROBLEM FORMULATION

Consider a variable-speed wind turbine with a Doubly Fed Induction Generator (DFIG). The wind turbine consists of an electric part and a mechanical part, the dynamics of which may be modeled as follows:

The dynamics of the electrical part in the (dq) frame are described by a fourth-order state space model [13].

\[
\begin{bmatrix}
\frac{\dot{\phi}_{sd}}{\tau_r} + \frac{1}{\tau_r} \phi_{sd} - (\omega_r - \omega) \phi_{sq} + \frac{M_{sr}}{\tau_r} i_{sd} \\
\frac{\dot{\phi}_{sq}}{\tau_r} + \frac{1}{\tau_r} \phi_{sq} - (\omega_r - \omega) \phi_{sd} + \frac{M_{sr}}{\tau_r} i_{sq} \\
\frac{\dot{i}_{sd}}{\tau_s} + \frac{1}{\tau_s} i_{sd} - \frac{1}{\tau_s} i_{sq} + \omega_s i_{sq} \\
\frac{\dot{i}_{sq}}{\tau_s} + \frac{1}{\tau_s} i_{sq} - \frac{1}{\tau_s} i_{sd} - \omega_s i_{sd}
\end{bmatrix}
\begin{bmatrix}
\phi_{sd} \\
\phi_{sq} \\
i_{sd} \\
i_{sq}
\end{bmatrix}
= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\frac{1}{L_s} & 0 \\
0 & \frac{1}{L_s}
\end{bmatrix}
\begin{bmatrix}
V_{sd} \\
V_{sq}
\end{bmatrix}
\tag{1}
\]

Where \((\phi_{sd}, \phi_{sq})\) are the rotor fluxes projections on the \( (d,q) \) axis reference frame. \((i_{sd}, i_{sq})\), are the stator currents projections on the \( (d,q) \) axis reference frame and \((V_{sd}, V_{sq})\) are the stator voltages. \(L_s, L_r\) are the stator and rotor self-inductances and \(M_{sr}\) is the mutual inductance. \(\tau_r = \frac{L_r}{R_s}; \quad \tau_s = \frac{L_s}{R_s}\); \(L_i = L_s - \frac{M_{sr}^2}{L_r};\)

\(R_i = R_s + R_r \left(\frac{M_{sr}}{L_r}\right)^2; \quad \beta = \frac{M_{sr}}{L_s L_i}; \quad \mu = \frac{1}{2} \frac{M_{sr}}{J L_r}; \quad \tau_i = \frac{L_i}{R_i}\).

The dynamics of the mechanical part are described by a first-order state space:

\[
J \frac{d\omega}{dt} = T_m - T_em - C_r \omega_r 
\tag{2}
\]

where \(J\) is the moment of inertia of the rotor and \(\omega_r\) is the rotor angular velocity, \(C_r\) is the friction coefficient, \(T_m\) is the mechanical torque, and \(T_em\) is the electromagnetic torque developed by the motor.

The electromagnetic torque is expressed in terms of rotor fluxes and stator currents as:

\[
T_em = n p \frac{M_{sr}}{L_s} (i_{sd} \phi_{sd} - i_{sq} \phi_{sq}) 
\tag{3}
\]

The mechanical power captured by the wind turbine is [14]:

\[
P_m = T_m \omega_r = \frac{1}{2} p A C_p (\lambda, \beta)|V_{in}^2|
\tag{4}
\]
The tip speed ratio is the air density, \( \beta \) is the blade pitch angle, \( A = \pi R^2 \) is the area swept by the rotor blades of radius \( R \), \( V_w \) is the wind speed.

\[
C_p = fC_p(\lambda, \beta)
\]

is the performance coefficient of the wind turbine, a highly nonlinear power function of \( \beta \) and the tip speed ratio \( \lambda \in (0, \infty) \), defined as:

\[
\lambda = \frac{\omega R}{V_w} \tag{5}
\]

By adding the dynamics of the electrical part (1) to the mechanical part (2) and neglecting the friction, the induction motor can be modeled as a fifth-order, nonlinear system with the following state space model:

\[
\dot{x} = f(x) + gu
\]

with:

\[
x = [\omega \phi_\alpha \phi_\beta \phi_{iq} \phi_{is} i_{dq} i_{ds}]^T, \quad u = [V_{id} V_{iq}]^T
\]

the state vector

\[
u = [V_{id} V_{iq}]^T, \quad \text{the control vector}
\]

\[
\begin{bmatrix}
\mu(\phi_{iq} - \omega i_{id}) - \frac{npT_e}{J} \\
-\frac{1}{\tau_e} \phi_{\alpha} + (\omega_e - \omega) \phi_{\beta} + \frac{M}{\tau_e} i_{id} \\
-\frac{1}{\tau_e} \phi_{\beta} + (\omega - \omega_e) \phi_{\alpha} + \frac{M}{\tau_e} i_{id} \\
\frac{\beta}{\tau_e} \phi_{\beta} + \beta \omega \phi_{\beta} + \frac{1}{\tau_e} i_{id} + \omega i_{iq} \\
\frac{\beta}{\tau_e} \phi_{\alpha} - \beta \omega \phi_{\alpha} + \frac{1}{\tau_e} i_{id} - \omega i_{iq}
\end{bmatrix}
\]

The manipulated quantities are differentiated with respect to time until the input appears and the derivatives of the state variables are eliminated using the directional or Lie derivative of a state function along a vector field.

\[
\begin{bmatrix}
\dot{x} = f(x) + \sum_{i=1}^{n} g_i(x) u_i \\
y = h(x)
\end{bmatrix}, \quad (x \in \mathbb{R}^n, u \in \mathbb{R}^m)
\]

into a linear and controllable one

\[
z = Az + Bv (z \in \mathbb{R}^n, v \in \mathbb{R}^m)
\]

by means of nonlinear state feedback: \( u = \alpha(x) + \beta(x)v \), with \( \beta(x) \) a nonsingular \( (m \times m) \) matrix and nonlinear state space change of coordinates \( z = h(x) \). Linear techniques can then be applied in the design of the control \( v \).

The outputs to be controlled are:

\[
\begin{bmatrix}
h_1(x) & \ldots & h_m(x)
\end{bmatrix}^T
\]

The manipulated quantities are differentiated with respect to time until the input appears and the derivatives of the state variables are eliminated using the state space model of the system. This can be done introducing the directional or Lie derivative of a state function along a vector field:

\[
f(x) = (f_1(x), \ldots, f_n(x))
\]

\[
L_i h(x) = \sum_{j=1}^{n} f_j(x) \frac{\partial h(x)}{\partial x_j}(x)
\]

Where \( \lambda_{opt} \) is the optimum tip speed ratio, corresponding to the optimum operational point of the wind turbine at a given wind speed \( V_w \).

Note that for variable speed wind turbines, the shaft speed of the wind turbine needs to be adjusted over a wide range of wind speeds so that the tip speed ratio \( \lambda \) is maintained at \( \lambda_{opt} \).

IV. CONTROLLER DESIGN

Because wind turbines are highly nonlinear systems, but with smooth nonlinearities, a possible optimal control design solution can be the feedback linearization control [8].

A. Feedback Linearization

The feedback linearization or input-output control problem is to find a state feedback such that the transformed system is input-output decoupled that is, one input influence one output only [9]. The technique requires measurements of the state vector \( x \) in order to transform a multi-input nonlinear control system:

\[
\begin{bmatrix}
\dot{x} = f(x) + \sum_{i=1}^{n} g_i(x) u_i \\
y = h(x)
\end{bmatrix}, \quad (x \in \mathbb{R}^n, u \in \mathbb{R}^m)
\]

into a linear and controllable one

\[
z = Az + Bv (z \in \mathbb{R}^n, v \in \mathbb{R}^m)
\]

by means of nonlinear state feedback: \( u = \alpha(x) + \beta(x)v \), with \( \beta(x) \) a nonsingular \( (m \times m) \) matrix and nonlinear state space change of coordinates \( z = h(x) \). Linear techniques can then be applied in the design of the control \( v \).

The outputs to be controlled are:

\[
\begin{bmatrix}
h_1(x) & \ldots & h_m(x)
\end{bmatrix}^T
\]

The manipulated quantities are differentiated with respect to time until the input appears and the derivatives of the state variables are eliminated using the state space model of the system. This can be done introducing the directional or Lie derivative of a state function along a vector field:

\[
f(x) = (f_1(x), \ldots, f_n(x))
\]

\[
L_i h(x) = \sum_{j=1}^{n} f_j(x) \frac{\partial h(x)}{\partial x_j}(x)
\]

Where \( \lambda_{opt} \) is the optimum tip speed ratio, corresponding to the optimum operational point of the wind turbine at a given wind speed \( V_w \).

Note that for variable speed wind turbines, the shaft speed of the wind turbine needs to be adjusted over a wide range of wind speeds so that the tip speed ratio \( \lambda \) is maintained at \( \lambda_{opt} \).

IV. CONTROLLER DESIGN

Because wind turbines are highly nonlinear systems, but with smooth nonlinearities, a possible optimal control design solution can be the feedback linearization control [8].

A. Feedback Linearization

The feedback linearization or input-output control problem is to find a state feedback such that the transformed system is input-output decoupled that is, one input influence one output only [9]. The technique requires measurements of the state vector \( x \) in order to transform a multi-input nonlinear control system:

\[
\begin{bmatrix}
\dot{x} = f(x) + \sum_{i=1}^{n} g_i(x) u_i \\
y = h(x)
\end{bmatrix}, \quad (x \in \mathbb{R}^n, u \in \mathbb{R}^m)
\]

into a linear and controllable one

\[
z = Az + Bv (z \in \mathbb{R}^n, v \in \mathbb{R}^m)
\]

by means of nonlinear state feedback: \( u = \alpha(x) + \beta(x)v \), with \( \beta(x) \) a nonsingular \( (m \times m) \) matrix and nonlinear state space change of coordinates \( z = h(x) \). Linear techniques can then be applied in the design of the control \( v \).

The outputs to be controlled are:

\[
\begin{bmatrix}
h_1(x) & \ldots & h_m(x)
\end{bmatrix}^T
\]

The manipulated quantities are differentiated with respect to time until the input appears and the derivatives of the state variables are eliminated using the state space model of the system. This can be done introducing the directional or Lie derivative of a state function along a vector field:

\[
f(x) = (f_1(x), \ldots, f_n(x))
\]

\[
L_i h(x) = \sum_{j=1}^{n} f_j(x) \frac{\partial h(x)}{\partial x_j}(x)
\]
Iteratively, $L_r^i h(x) = L_r^i (L_r^{-1} h(x))$. That is, once the system is linearized, one can use a linear controller in the design of the control signal input $\nu$ for the system in the new reference.

**B. Controller Design**

The controller design is based on the fourth order dynamic model obtained from the $(d,q)$ axis model of the motor under the field oriented assumptions so that either (or both) speed or flux magnitude control objective can be fulfilled. The underlying design concept is to endow the closed loop system with high performance dynamics for high speed ranges while maximizing power efficiency and keeping the required stator voltage within the inverter ceiling limits.

In addition to fulfilling those control objectives, our control design aims to reduce the complexity of the control scheme, saving thereby the computation time of the control algorithm, which is an improvement over previous work found in the technical literature [10-21].

By orientating the flux vector along the $d$ axis, have:

$$\phi^{(d,q)} = (\phi_d, 0)^T$$

Using (12), we eliminate all the terms with quadratic rotor flux in (6) and reduce the third equation in (6) to this expression of the synchronous angular speed:

$$\omega_s = \omega + \frac{M_w i_{sq}}{\tau_r \phi_r}$$

(13)

The rotor flux orientation leads to the following expression of the electromagnetic torque, which is equivalent to a separately excited DC motor:

$$T_{em} = np \frac{M_w}{L_r} (i_{sd} \phi_r)$$

(14)

With the above assumptions (12) and (13), the fifth order model (6) is reduced to the fourth-order model:

$$\begin{align*}
d\omega_s &= \frac{\mu}{J} \phi_i i_{sq} - \frac{np T_r}{J} \\
d\phi_r &= -\frac{1}{\tau_r} \phi_r + \frac{M_w}{\tau_r} i_{sd} \\
di_{sd} &= \frac{\beta}{\tau_r} \phi_r - \frac{1}{\tau_1} i_{sd} + \omega_s i_{sq} + \frac{1}{L_1} V_{sd} \\
di_{sq} &= -\beta \omega \phi_r - \frac{1}{\tau_1} i_{sq} - \omega_s i_{sd} + \frac{1}{L_1} V_{sq}
\end{align*}$$

(15)

The outputs to be controlled are the speed $\omega$ and the square of the rotor flux magnitude $\phi_r^2$. The output vector is:

$$\begin{bmatrix} h_1(x) \\
h_2(x) \end{bmatrix} = \begin{bmatrix} \omega \\
\phi_r^2 \end{bmatrix}$$

(16)

Define the change of coordinates:

$$\begin{align*}
z_1 &= h_1(x) = \omega \\
z_2 &= L_r h_1(x) = \mu \phi_i i_{sq} - \frac{np T_r}{J} \\
z_3 &= h_2(x) = \phi_r^2 \\
z_4 &= L_r h_2(x) = -\frac{2}{\tau_r} \phi_r^2 + \frac{2 M_w}{\tau_r} \phi_i i_{sd}
\end{align*}$$

(17)

Thus the derivatives of the outputs are given in the new coordinate system by:

$$\begin{align*}
z_1 &= \dot{h}_1(x) = z_3 \\
z_2 &= \dot{h}_1(x) = L_r^d h_1(x) + L_{gs} L_r h_1(x) V_{sd} + L_{gs} L_r h_1(x) V_{sq} \\
z_3 &= \dot{h}_2(x) = z_4 \\
z_4 &= \dot{h}_2(x) = L_r^d h_2(x) + L_{gs} L_r h_2(x) V_{sd} + L_{gs} L_r h_2(x) V_{sq}
\end{align*}$$

(18)

This system can be written as:

$$\begin{bmatrix} z_1 \\
z_3 \end{bmatrix} = \begin{bmatrix} L_r^d h_1(x) \\
L_r^d h_2(x) \end{bmatrix} + \Delta(x) \begin{bmatrix} V_{sd} \\
V_{sq} \end{bmatrix}$$

(19)

with:

$$L_r^d h_1(x) = -\mu \left( \frac{1}{\tau_r} + \frac{1}{\tau_1} \right) (i_{sq} + i_{sd}) \phi_r - \mu \beta \omega \phi_r^2 + \frac{\mu}{\tau_r} M_w i_{sd} i_{sq}$$

$$L_r^d h_2(x) = \frac{2}{\tau_r} (2 + \beta M_w) \phi_r^2 - \frac{2 M_w}{\tau_r} \omega_s i_{sq} \phi_r + \frac{2 M_w}{\tau_r} i_{sq} - \frac{6 M_w}{\tau_r \tau_1} i_{sq} \phi_r$$

$$L_{gs} L_r h_1(x) = 0 \quad L_{gs} L_r h_2(x) = \frac{\mu}{L_1} \phi_r$$

$$L_{gs} L_r h_1(x) = 0 \quad L_{gs} L_r h_2(x) = \frac{2 M_w}{L_1} \phi_r$$

The decoupling matrix $\Delta(x)$ is defined as:

$$\Delta(x) = \begin{bmatrix} L_{gs} L_r h_1(x) & L_{gs} L_r h_1(x) \\
L_{gs} L_r h_2(x) & L_{gs} L_r h_2(x) \end{bmatrix} = \begin{bmatrix} 0 & \frac{\mu}{L_1} \phi_r \\
\frac{2 M_w}{L_1} \phi_r & 0 \end{bmatrix}$$

(20)
and:
\[
\det(\Delta(x)) = \frac{2\mu M_r}{\tau_1 L_1} \phi_i^2
\]  

(20)

The decoupling matrix \( \Delta(x) \) is singular if and only if \( \phi_i \) is zero which only occurs at the start-up of the motor. That is, to fulfill this condition one can use in a practical setting, an open loop controller at the start-up of the motor, and then switch to the nonlinear controller as soon as the flux builds up to a non-zero value.

If the decoupling matrix is not singular, the nonlinear state feedback control input is given by:

\[
\begin{bmatrix}
V_{sd}
V_{sq}
\end{bmatrix} = \Delta(x)^{-1} \begin{bmatrix}
-L_1^2 h_1(x) + V_1
-L_2^2 h_2(x) + V_2
\end{bmatrix}
\]  

(21)

This controller linearizes and decouples the system, resulting in:

\[
\begin{aligned}
\dot{h}_1(x) &= V_1 \\
\dot{h}_2(x) &= V_2
\end{aligned}
\]  

(22)

The closed loop system (22) is input-output decoupled and linear. Note that, the linearization affects only the system dynamics that are responsible for input-output mapping, while the rest of the dynamics are internal and do not influence the input-output mapping.

To ensure perfect tracking of speed and flux references, \( V_1 \) and \( V_2 \) are chosen as follows:

\[
\begin{aligned}
V_1 &= -k_{a1}(\omega - \omega_{ref}) - k_{a2}\omega - \int_0^t \omega(\tau) d\tau \\
V_2 &= -k_{b1}(\Phi_r - \Phi_{ref}) - k_{b2}\Phi_r
\end{aligned}
\]  

(23)

Where \( k_{a1}, k_{a2} \) and \( k_{b1}, k_{b2} \) are positive constants determined using a pole placement technique in order to make the closed loop system (22) stable and to have fast response in variable tracking.

C. Considering Magnetic Saturation in the Control Design

Equation (9) shows that the speed is controlled by the quadrature component of the stator current \( i_{sq} \). In order to increase the speed, \( V_{sq} \) must be chosen such that \( \frac{di_{sq}}{dt} > 0 \), that is:

\[
-\beta \omega \Phi_r - \frac{1}{\tau_1} i_{sq} - \omega_i i_{sq} + \frac{1}{L_1} V_{sq} > 0
\]  

(15)

replacing \( \beta \) and \( \tau_1 \) by their values we have:

\[
V_{sq} > L_1 \left( \frac{\omega + \frac{M_\omega i_{sq}}{\Phi_r} + \frac{M_\omega}{L_r} \omega \Phi_r + R_1 i_{sq}}{\tau_1} \right)
\]  

(16)

As the quantity \( L_1 \) is quite small, the dominant term on the right hand side of this inequality is \( \frac{M_\omega}{L_r} \omega \Phi_r \), which means high speeds require rather large input voltages. In practice, the voltages must be kept within the inverter ceiling limits; so the flux \( \Phi_r \) is decreased as the speed \( \omega \) increases above the rated speed. This

Figure 2: Schematic diagram of the improved optimal control algorithm for an induction generator based fixed-speed wind turbine.
method of reducing the flux at high speeds to avoid magnetic saturation is called flux weakening. That is, the flux is required to reach the nominal value $\phi_n$ for $\omega < \omega_n$, $\omega_n$ denotes the nominal speed, and the rotor flux amplitude has to be weakened according to the rule $\phi_{\text{ref}} = [\phi_n] \frac{\omega}{\omega_n}$ for $\omega > \omega_n$ [13].

Operating in the flux weakening regime will maximize power efficiency so that only the minimum stator input power needed to operate the wind turbine at the desired speed is used. That is, even when the motor is operating below the nominal speed, flux may be varied in order to maximize power efficiency.

The schematic diagram of the proposed nonlinear control algorithm is illustrated in Figure 2.

Note that, the controller adjusts the turbine speed to the optimal speed given in equation (7). This allows optimizing the power efficiency coefficient $C_p$.

The proposed algorithm has been applied to an induction generator based variable speed wind turbine and implemented in the MATLAB/Simulink environment. The effectiveness of the proposed control strategy is discussed in the next section.

V. COMPUTER SIMULATIONS

To demonstrate the effectiveness of the proposed control algorithm, we provide a series of computer experiments conducted with a two pole, three phase induction motor driven by a variable-speed wind turbine. The parameters of the induction generator as well as the wind turbine are listed in the appendix.

$k_{a1}$, $k_{a2}$ and $k_{b1}$, $k_{b2}$ were constants determined using a pole placement technique to guarantee a damping factor $\zeta = 0.7$ and a cut off frequency $\omega_b = 25$ rad/s resulting in:

$k_{a1} = 3000$, $k_{a2} = 200$, $k_{b1} = 4000$ and $k_{b2} = 300$.

The considered wind profile is depicted in Figure 3. It varies between 5 and 10 m/s and has an average speed of approximately 7m/s and a turbulence intensity of about 0.2. Figure 4 depicts the corresponding wind torque.

The time histories of the power coefficient and the generator’s speed are presented in Figures 5 and 6, respectively. The tip speed is shown in Figure 7. Note that the proposed controller manages to ensure maximum power extraction for the considered wind profile.
Note that Figure 5 depicts a power coefficient that is very close to the maximum power coefficient (0.47). Hence, the evolution of the power coefficient shows that the controller manages to maximize the energy extraction for the given wind profile.

Figure 6 shows the corresponding shaft speed for the given wind profile. Note that the speed variations remain smooth, while tracking the mean tendency of the wind torque. This will ensure optimal power extraction from the wind.

![Generator speed](image1)

Figure 6: Generator speed.

Figure 7 depicts the time histories of the tip speed/wind speed ratio. Note that this latter is equal to the optimum tip speed since high efficiency 3-blade turbines have tip speed ratios of 6 to 7. Hence, the controller ensures a good dynamic response of the shaft speed for the considered wind speed variation ranges.

![Tip speed ratio](image2)

Figure 7: Tip speed ratio.

The time histories of the power control inputs are reported in Figures 8 and 9, respectively. Note that, only a minimum control effort is required by the proposed control approach.

![Control input Vd](image3)

Figure 8: Control input Vd.

![Control input Vq](image4)

Figure 9: Control input Vq.

Based on the above simulations, as well as the many other simulation runs we performed in the lab, we can conclude that the proposed feedback linearization controller ensures maximum power extraction for the given wind profile while using minimum control effort. Hence, maximizing energy extraction coupled with good dynamic response of the turbine are the main positive features of the proposed approach.

VI. CONCLUSION

In this paper, a new controller based on the theory of feedback linearization was proposed for a doubly-fed
induction motor driven by a variable speed wind turbine. The proposed controller is able to adjust the turbine speed to the optimal speed, thus optimizing the power efficiency coefficient. The effect of magnetic saturation at high speed ranges was considered in the control scheme design.

The proposed approach has the advantage of optimizing the power production, while being simple to implement and using minimum control effort.

APPENDIX

Table 1: Wind Turbine Parameters

| Parameter | Value |
|-----------|-------|
| Number of blades | 3 |
| $\rho$ | Wind density | 1.25 |
| $\nu$ | Nominal wind speed | 10m/s |
| $P_n$ | Wind power at nominal wind speed | 10kW |
| Gearbox ratio | 10 |
| $R$ | Wind turbine blade disk radius | 3m |
| $C_p$ | Max Power coefficient | 0.47 |
| $\lambda_{opt}$ | Optimum tip speed ratio | 7 |

Table 2: Induction Generator Specifications

| Parameter | Value |
|-----------|-------|
| $n_p$ | Number of pair poles | 2 |
| $L_r$ | Rotor inductance | 0.1568H |
| $R_r$ | Rotor phase resistance | 1 $\Omega$ |
| $L_s$ | Stator phase inductance | 0.1554 H |
| $R_s$ | Stator phase resistance | 1.2 $\Omega$ |
| $M_{sr}$ | Mutual inductance | 0.15H |
| $J$ | Moment of inertia | 9.77 Kg m$^2$ |
| Grid frequency | 60 Hz |
| Rated Maximum Power | 100 kW |

ACKNOWLEDGEMENT

This work is partially supported by the Louisiana Board of Regents Support Fund contract number LEQSF (2012-15)-RD-A-26 and by LaSPACE/NASA Grant Number NNX10AI40H.

REFERENCES

[1] Ackermann T. Wind Power in Power Systems. 2nd ed. John Wiley & Sons Ltd 2012. http://dx.doi.org/10.1002/9781119941842
[2] The World Wind Energy Association. 2012 Half year report [homepage on the Internet]. Available from: http://www.windea.org; 2012.
[3] U.S. Department of Energy Office of Energy Efficiency and Renewable Energy [homepage on the Internet]. http://apps1.eere.energy.gov/news/news_detail.cfm/news_id =18084
[4] Pao LY, Johnson KE. Control of wind turbines, approaches, challenges and recent developments. IEEE Control Systems Magazine 2011; 44-62.
[5] Burton T, Jenkins N, Sharpe D, Bossanyi E. Wind Energy Handbook, John Wiley & Sons 2011. http://dx.doi.org/10.1002/9781119999214
[6] Munteanu J. Bratcu A. Cutululis N, Ceaga E. Optimal Control of Wind Energy Systems: Towards a global Approach. Springer-Verlag Berlin 2008.
[7] Leonard W. Control of Electrical Drives, 3rd ed. Springer 2001.
[8] Fekih A. An Alternative Strategy for Field Oriented Control. Proc. of the 2006 American Control Conference ACC 06, pp.2748-2753, Minneapolis, Minnesota, USA, June 2006.
[9] Isidori A. Nonlinear Control Systems. Communications and Control Engineering Series. Berlin:Springer-Verlag 1989.
[10] Roozbahani S, Abbaszadeh K, Torabi M. Sensorless maximum wind energy capture based on input-output linearization and sliding mode control. IET Conference on Renewable Power Generation 2011; pp. 1-6. http://dx.doi.org/10.1049/cp.2011.0198
[11] Delfino F, Pampararo F, Procopio R, Rossi M. A Feedback Linearization Control Scheme for the Integration of Wind Energy Conversion Systems into Distribution Grids. IEEE Syst J 2012; 6(1).
[12] Molina MG, Mercado PE. Modelling and Control Design of Pitch-Controlled Variable Speed Wind Turbines. In: Wind Turbines, Al-Bahadly I, Ed. 1st ed. InTech, Vienna, Austria 2011.
[13] Fekih A. Effective Fault Tolerant Control Design for a Class of Nonlinear Systems: Application to a Class of Motor Control. IET Control Theory Appl 2008; 2(9): 762-72. http://dx.doi.org/10.1049/iet-cta:20070090
[14] Lei Y, Mullane A, Lightbody G, Yacamini R. Modeling of the wind turbine with a doubly fed induction generator for grid integration studies. IEEE Trans. on Energy Conversion 2006; 21(1): 257-64. http://dx.doi.org/10.1109/60.847958
[15] Estensen T, Sloth C, Niss MO, Thorarins BJ. Joint Power and Speed Control of Wind Turbines. Technical report, Aborg University, Department of Electronic Systems 2008.
[16] De Battista H, Mantz RJ, Christiansen CF. Dynamical sliding mode power control of wind driven induction generators. IEEE Trans. on Energy Conversion 2000; 15(4): 451-57. http://dx.doi.org/10.1109/60.800507
[17] Tan K, Islam S. Optimum control strategies in energy conversion PMSG wind turbine system without mechanical sensors, IEEE Trans. on Energy Conversion 2004; 19(2): 392-99. http://dx.doi.org/10.1109/60.827038
[18] Guo Y, Hussein SH, Jiang JN, Tang C. Voltage/Pitch Control for Maximization and Regulation of Active/Reactive Powers in Wind Turbines with Uncertainties. Proc IEEE Conference Decision Control 2010; 3956-63. http://dx.doi.org/10.1016/MCSE.2006.163631
[19] Johnson KE, Pao LY, Balas MJ, Fingersh LJ. Control of variable speed-speed wind turbines: standard and adaptive techniques for maximizing energy capture. IEEE Control Systems Magazine 2006; 26(3): 70-81.
[20] Fekih A. An Improved Optimal Control Design for Wind Energy Systems, Proc of IEEE Green Technologies Conference 2012; pp.1-5.

[21] Vepa R. N Nonlinear Optimal Control of wind turbine generators. IEEE Trans. on Energy Conversion 2011; 26(2): 468-78.
http://dx.doi.org/10.1109/TEC.2010.2087380

Received on 21-12-2012  Accepted on 28-02-2013  Published on 28-02-2013

DOI: http://dx.doi.org/10.6000/1929-6002.2013.02.01.11