Testing the Intrinsic Randomnesses in the Angular Distributions of Gamma-Ray Bursts

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Abstract. The counts-in-cells and the two-point angular correlation function method are used to test the randomnesses in the angular distributions of both the all gamma-ray bursts collected at BATSE Catalog, and also their three subclasses ("short", "intermediate", "long"). The methods eliminate the non-zero sky-exposure function of BATSE instrument. Both tests suggest intrinsic non-randomnesses for the intermediate subclass; for the remaining three cases only the correlation function method. The confidence levels are between 95% and 99.9%. Separating the GRBs into two parts ("dim half" and "bright half", respectively) we obtain the result that the "dim" half shows a non-randomness on the 99.3% confidence level from the counts-in-cells test.

INTRODUCTION

Recently, two different articles [1,2] simultaneously suggest that the earlier separation of gamma-ray bursts (GRBs) - detected by BATSE - into short and long subclasses is incomplete. These articles show that the earlier long subclass alone should be further separated into a new "intermediate" subclass (2 s < $T_{90}$ < 10 s) and into a "truncated long" subclass ($T_{90}$ > 10 s). Therefore, in what follows, the long subclass will contain only the GRBs with $T_{90}$ > 10 s, and the intermediate subclass will be considered as a new subclass (2 s < $T_{90}$ < 10 s). The "short" subclass is defined by $T_{90}$ < 2 s (for definition of $T_{90}$ see [3]).

Fully independently, Balázs et al. [4,5] suggest that GRBs are distributed anisotropically on the sky. In addition, they show that the short subclass shows an anisotropy, but the intermediate + long subclasses do not show. The different behavior is confirmed on 99.7% confidence level [5]. It is difficult to explain such
behavior of subclasses by the instrumental effects alone. Hence, some intrinsic anisotropy should exist. A recent study [6], which is based on the spherical harmonic analysis, shows that just the GRBs of ”intermediate subclass” have an intrinsic anisotropy on confidence level 97%.

Here we shortly describe and summarize the new results of two further tests. The details of these tests will be published elsewhere [7,8].

GRBs will be taken between trigger numbers 0105 and 6963 from Current BATSE Catalog [3] having defined \( T_{90} \) (i.e. all GRBs detected up to August 1996 having measured \( T_{90} \)). From them we exclude, similarly to [9,4,5], the faintest GRBs having a peak flux (on 256 ms trigger) smaller than 0.65 photon/(cm\(^2\)s). The 1284 GRBs obtained in this way define the ”all” class. From them there were 339 GRBs with \( T_{90} < 2 \) s (the ”short” subclass), 181 GRBs with \( 2 < T_{90} < 10 \) s (the ”intermediate” subclass) and 764 GRBs with \( T_{90} > 10 \) s (the ”long” subclass). We will study the all class and the three subclasses separately.

**COUNTS-IN-CELLS TEST**

We separate the sky in declination into \( m_{\text{dec}} > 1 \) stripes having the same ”effective” area (4\( \pi / m_{\text{dec}} \) steradian). This means that the boundaries of stripes are the declinations, which ensure that the probability to have a GRB in a stripe is for any stripe the same (= 1/\( m_{\text{dec}} \)). Because the sky-exposure function of BATSE is not depending on right ascension, this may easily be done by the convenient choices of declinations. We also separate the sky in right ascension \( \alpha \) into \( m_{\text{ra}} > 1 \) stripes. Hence, we separated the sky into \( M = m_{\text{dec}} \times m_{\text{ra}} \) areas (”cells”) having the same ”effective” size 4\( \pi / M \) steradian.

If there are \( N \) GRBs on the sky, then \( n = N/M \) is the mean of GRBs at a cell. Let \( n_i; i = 1, 2, ..., M \) be the observed number of GRBs at the \( i \)th cell (\( \sum_{i=1}^{M} n_i = N \)). Then

\[
\text{var}_M = (M-1)^{-1} \sum_{i=1}^{M} (n_i - n)^2
\]

(1)

defines the observed variance. For the given cell structure with \( M \) cells, the measured variance \( \text{var}_M \) should be identical to the theoretically expected value \( n(1 - 1/M) \).

\( Q \) cell structures may be probed for the same sample of GRBs. We will choose \( m_{\text{dec}} = 2, 3, \ldots, 8 \) and \( m_{\text{ra}} = 2, 3, \ldots, 16 \). I.e. it will be \( Q = 105 \). In the coordinate system with axes \( x = 1/M \) versus \( y = \sqrt{\text{var}_M / n} = (\text{var}/\text{mean})^{1/2} \) the \( Q \) values of \( (\text{var}/\text{mean})^{1/2} \) define \( Q \) points, where \( j = 1, 2, ..., Q \). The theoretical expectation is verified by least squares estimation ( [10], Chapt. 5.3.1.). Our estimator is the dispersion

\[
\sigma_Q = \sum_{j=1}^{Q} (y_j - \sqrt{1 - 1/M})^2
\]

(2)
We throw 1000-times randomly $N$ points on the sphere, and repeat the above calculation leading to $\sigma_Q$ for every simulated sample. Then we compare the size of the $\sigma_Q$ obtained from this simulation with $\sigma_Q$ obtained from the actual GRB positions. Let $\omega$ be the number of simulations, when the obtained $\sigma_Q$ is bigger than the actual value of $\sigma_Q$. Then $(100 - \omega/10)$ is the confidence level in percentage.

## TWO-POINT ANGULAR CORRELATION FUNCTION TEST

Be given $N_D$ GRBs on the sky. There are $N_D(N_D - 1)/2$ angular distances among them. These measured distances are binned into bins with binwidth 4 degree. Then, using a Monte Carlo simulation, there are distributed randomly $N_R$ points on the sky, where $N_R \gg N_D$. (In our case: $N_R = 1000 \times N_D$.) Then the $N_R(N_R - 1)/2$ angular distances are binned in the same manner. In addition, the $N_RN_D$ random-data pairs are also binned in the same manner.

In [11] the following formula is proposed in order to obtain the $w(\theta)$ correlation function:

$$w(\theta) = \frac{< DD >}{< RR >} - 2\frac{< RD >}{< RR >} + 1 ,$$

where $< ... >$ means a normalized mean (see [11]).

The 1σ uncertainty in $w(\theta)$ for the given bin is given by $\delta w(\theta) = n_i^{-1/2}$ [11]. This formula allows to test the zero expectation value of $w(\theta)$. For any bin be calculated the dimensionless value $x$ from the relation $|w(\theta) - x\delta w(\theta)| = 0$. Then $x\sigma$ is the probability that $w = 0$ holds for the given bin. Because $w = 0$ must occur for any bin, simply the biggest value of $x\sigma$ defines the "Poissonian" confidence level [11].

We will verify the confidence levels by Monte Carlo simulations, too. We will take instead of the the actual BATSE positions - randomly generated $N_D$ positions, and then we will repeat the above procedure. This simulation will be done 500 times. If there are $\omega$ such simulations, when the absolute value of $w_{sim}$ is bigger than the absolute value of $w_{act}$, then $(500 - \omega)/5$ is the confidence level in percentage.

As the final confidence level the smaller value will be taken.

We exclude the non-uniform sky exposure function of BATSE instrument as follows. Assume that the Monte Carlo simulation defines a point at a given position. We generate for any such point an additional random number between 0 and 1, too. If this number is bigger than the actual value of sky-exposure function at that point, then this point is omitted from the Monte Carlo sample.

Note that this test is usually more effective than the counts-in-cells one, because usually the "distance-based" tests are more sensitive than "cell-based" tests ([10], Chapt.2.6).
THE RESULTS

The counts-in-cells tests give $\omega = 287$ ($\omega = 80$, $\omega = 36$, $\omega = 440$) for all (short, intermediate, long) GRBs. Hence, the rejection of null-hypothesis of randomness is confirmed for the intermediate subclass on the 96.4% confidence level. For the short and long subclasses, respectively, and also for all GRBs the null-hypothesis cannot be rejected on the > 95% confidence level.

The calculated four correlation functions give the following results:
1. For the short subclass there is an essential departure from zero for $\theta = (14 \pm 2)$ degrees. We have a 99.2% confidence level for the non-randomness.
2. In the case of intermediate subclass the ”suspicious” angles are the values $\theta = (6 \pm 2)$, $\theta = (50 \pm 2)$, and $\theta = (90 \pm 2)$ degrees. For $\theta = (6 \pm 2)$ degrees we have a 99.8% confidence level for non-randomness.
3. In the case of long subclass for the angle $\theta = (94 \pm 2)$ degrees we have a 99.0% confidence level for non-randomness.
4. In the case of all GRBs the following angles are ”suspicious”: $\theta = (50 \pm 2)$, $\theta = (94 \pm 2)$, $\theta = (130 \pm 2)$ and $\theta = (150 \pm 2)$ degrees. For $\theta = (94 \pm 2)$ we have a 99.8% confidence level for non-randomness.

FIRST CONCLUSION: The intrinsic non-randomness is confirmed on the confidence level > 95% for the intermediate subclass by both methods; the angular correlation function (counts-in-cells) method gives a 99.8% (96.4%) confidence level. This supports the results of [6].

SECOND CONCLUSION: For the remaining two sub-classes and for all GRBs the null-hypotheses of intrinsic randomnesses are rejected only by the angular correlation function method on the confidence levels between 99% and 99.9%. The counts-in-cells test, similarly to [6], did not reject the null-hypothesis of randomness for these cases.

The peak flux = 2 photons/(cm$^2$ s) (on 0.256s trigger) is practically identical to the medium. Therefore, we consider the GRBs having smaller (bigger) peak flux than 2 photons/(cm$^2$ s) as the ”dim” (“bright”) subclass (”half”) of the intermediate subclass. There are 92 GRBs (89 GRBs) at the ”dim half” (”bright half”).

The 105 ”var/mean” tests for these two parts give the results that the ”dim half” has an intrinsic non-randomness on the 99.3% confidence level; the ”bright half” can well be random. The sky distribution of 92 dim GRBs is shown on Figure 1.

THIRD CONCLUSION: The intrinsic non-randomness of the ”dim half” of the intermediate subclass of GRBs is confirmed on the 99.3% confidence level by the counts-in-cells method.

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FIGURE 1. Sky distribution of 82 ”dim” GRBs of the intermediate subclass in equatorial coordinates.