EXPEDITION: A SYSTEM FOR THE UNSUPERVISED LEARNING OF A HIERARCHY OF CONCEPTS

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ABSTRACT

We present a system for bottom-up cumulative learning of myriad concepts corresponding to meaningful character strings (n-grams), and their part-related and prediction edges. The learning is self-supervised in that the concepts discovered are used as predictors as well as targets of prediction. We devise an objective for segmenting with the learned concepts, derived from comparing to a baseline (reference) prediction system, that promotes making and using larger concepts, which in turn allows for predicting larger spans of text, and we describe a simple technique to promote exploration, i.e. trying out newly generated concepts in the segmentation process. We motivate and explain a layering of the concepts, to help separate the (conditional) distributions learnt among concepts. The layering of the concepts roughly corresponds to a part-whole concept hierarchy in this work. With rudimentary segmentation and learning algorithms, the system is promising in that it acquires many concepts (tens of thousands in our small-scale experiments), and it learns to segment text well: when fed with English text with spaces removed, starting at the character level, much of what is learned respects word or phrase boundaries, and over time the average number of "bad" splits within segmentations, i.e. splits inside words, decreases as larger concepts are discovered and the system learns when to use them during segmenting. We also report on promising experiments when the input text is converted to binary and the system begins with only two concepts, "0" and "1". The system is transparent, in the sense that it is easy to tell what the concepts learned correspond to, and which ones are active in a segmentation, or how the system "sees" its input in an episode. We expect this framework to be extensible and we discuss the current limitations and a number of directions for enhancing the learning and inference capabilities.

"Concepts are the glue that hold our mental world together.", G. Murphy, The Big Book of Concepts.

".. to cut up each kind according to its species along its natural joints, ...", Plato, Phaedrus.

1 Introduction

Concepts, such as "water", "chair", and "eat", are fundamental to human intelligence: our mental model(s) of the world and our basic cognition are founded on concepts [43]. What is the nature of concepts, i.e. how are they computationally represented, and how can a system acquire diverse and richly inter-related concepts in an unsupervised manner, i.e. without an explicit teacher, from the low-level perceptual stream? There is evidence that much learning, of numerous concepts, and how they relate and constrain one another, and ultimately a sense of what is probable or common in every day experience, in humans and animals, takes place without explicit teaching, achieved largely through observing [47] [50] [44] [18]. Inspired by considerations of early human learning, and in particular perceptual and category learning,
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Figure 1: (a) The architecture of the Expedition system: Learning provides the (hierarchical) network of concepts to inference, and inference determines, in each episode, which (small) part of this network needs updating. (b) Repeatedly get an episode (a span of text), make "sense" of it (i.e. summon the candidate relevant concepts, via recall/prediction, matching and coherence), then learn from the final selected construction (the final segmentation structure): using the final segmentation, update (co-)occurrence counts, prediction weights, etc. The Expedition prediction system is a realization of this general learning loop.

that likely continue throughout life \[28, 7, 18\], here we propose and explore an approach to efficient unsupervised learning of discrete and discernible "concepts", in a sparse manner in the text domain. The learning is achieved in a cumulative bottom up fashion.

One way to motivate this particular work is as follows: Imagine a system endowed with the capabilities of a human brain, but that has access only to a stream of text, such as all the English language text on the web. The system "lives in text" or has a single modality only, i.e. its percepts or senses are limited to detecting characters and their relative positions in a limited span, such as up to 100 consecutive characters at a time. Natural language is rich and contains numerous types of regularities. The web is vast. How much can the system (sufficiently inclined) learn from this information-rich and practically unlimited stream, and what learning capabilities are activated? Will the system pick up words and phrases? Will it learn, in effect, the rules of grammar and the structural cues that can be derived from the punctuation, and will it pick up certain patterns such as "A" and "a" are often interchangeable, and so are the numerical digits? How would it organize and structure the acquired knowledge? Would internal discernible concepts develop and if so can we establish a correspondence to the input text, such as words and phrases, and perhaps general abstract categories such as the category of verbs? Would multiple such systems trained on sufficiently similar input streams, share enough common concepts to communicate, and in effect compare and adjust their concepts in a useful manner? What would the concept structure(s) and interconnections look like? What kind of changes take place in a single episode, and what guarantees can we make regarding the evolution of long-term learning, as these changes accumulate? Can we build components of such learning mechanisms in an artificial learning system? Our work explores a scalable unsupervised approach to learning simple concepts, involving one inference and two main learning components, driven by the goal of improving prediction in the text stream.

A concept in this work corresponds to a linear pattern, or a string of characters, such as "a", "is", and "school". However, a concept is more than just the string pattern it corresponds to: it has associations to other concepts, and may have parts and may be parts of many other concepts. Thus a concept is a node in a network, and the network contains different types of edges. In particular, the network is hierarchical. We achieve learning of concepts via a system, we call the Expedition system, that contains several (learning/inference) modules. Figure 1 gives a picture of the different modules in the system and their interaction, as well as the basic learning loop.

Each concept maintains and updates statistics such as occurrence and co-occurrence counts, and prediction weights, derived from the stream of text that it processes. The system uses the predictions to segment and interpret its input.

\[1\]It is natural to describe the various computations and datastructures, from the point of view of concepts, specially for the learning.
This process breaks a span of text (a line, or a string of characters) into chunks or subsegments, by simultaneously mapping the subsegments into its existing concepts. Segmentation+interpretation can be thought of as a rudimentary "sense making". We drop the term "interpretation" and refer to this process as segmentation for short. The segmentation of the string of characters and the mapping of the segments into a few internal concepts, which we refer to as the active concepts, provides the 'what' and 'where' of concepts in an episode. This information is in turn used to update various statistics such as the active concepts' prediction weights (predicting one another) and co-occurrences. This main learning loop of segmentation (sense-making) and updating weights repeats ad infinitum, and is a major source of learning.

Another important form of learning is discovering new concepts. In this work, new concepts are built via concatenating (composing) existing concepts based on a few co-occurrence criteria, where the co-occurrence information is collected over the many learning episodes of the main learning loop. As new concepts are discovered and used during segmentation, prediction relations among the concepts are added or updated, and the network grows. In our relatively small-scale experiments involving a few million episodes, tens of thousands of concepts, corresponding to n-grams of characters (words and phrases) are discovered. Connections among concepts are kept relatively sparse: a concept has relatively few edges compared to the totality of concepts. In this work, the edges in the network are prediction edges and part-related edges. Constraints from matching the raw input combined with how well the active concepts predict another, which we refer to as COMA (COherence and MAtch), help determine the final chosen segmentation in an episode, and the chosen segmentation determine which concepts and co-occurrences are observed and updated. The concept co-occurrence counts are used, over time, to compose bigrams of existing concepts (an instance of the AND or the conjunction operation), and thus to build new "larger" concepts (i.e. concepts that correspond to larger patterns in the input). The larger concepts enable the system to be more successful at predicting, in particular for successfully predicting increasingly longer stretches of the input stream.

In summary, the system engages in a self-learning or a so-called self-supervised learning regime, setting up for itself many learning episodes or puzzle solving sessions, to build a large predictive model of its world. Can higher-level concepts, e.g. in our case terms and phrases in natural language, be discovered effectively in a bottom up manner, starting from the level of characters, via repeated segmentation and prediction as we described? There are many challenges. There are several sources of uncertainty, imperfections, and noise, including: 1) limits of the local window used for prediction, 2) finite experience with concepts and tail challenges: at any given point, many concepts are relatively new and may not have reliable statistics or provide reliable predictions, 3) imperfections of the segmentation process: the search algorithms make local moves and can search only a tiny fraction of segmentation possibilities. Thus, the proposed system is composed of several interacting noisy modules. A major question is therefore whether the combination of prediction, composition (bigram creation in this work) and segmentation effectively yields sufficiently good higher-level concepts, since all the modules provide error-prone inputs to one another. Any errors made may persist (e.g. bad compositions that continue to be used) or have the potential to compound and propagate over time and may slow or stop progress (i.e. convergence to and getting stuck in local optima).

Another challenge is devising an appropriate objective to "motivate" the system to build (discover) larger concepts. We propose an objective based in part on roughly how large the concept is, and in part on how well it is predicted (both historically on average, as well as in a particular episode). The objective is derived from measuring how well the system is performing compared to simple prediction at the character level. Since concepts are discovered incrementally, and we define their utility in terms of how well they are predicted, we will also have to address the need for an exploratory period: a newly generated concept needs to be used a few times before we have an adequate estimate of its goodness as well as when to use it.

The system, with rudimentary algorithms that we describe, is promising: When run on natural language English text with blank spaces removed, starting at the character level, we find that, after some learning, the n-grams learned, i.e. the higher level concepts, correspond to words and phrases, and the splitting of the text into concepts continues to improve with more training. Thus we provide evidence that the combination of prediction, composition, and segmentation can be an effective self-supervised learning strategy. In the end, the system is a kin to a language model: it provides both a hierarchical vocabulary (of concepts), as well as a way of segmenting the low level characters and mapping them into this vocabulary. It does not require any preprocessing of the text. The only requirement is providing ample (unlabeled) data.

Modern large language models based on deep neural networks, trained using backpropagation, incorporate a number of advances and have found a diverse range of applications [5,13]. A major question is whether and in what ways they can be extended to provide us the (possibly discrete) concepts needed for advanced cognitive tasks. On the other hand, for our approach, to be competitive with such, a number of enhancements are needed. Throughout the paper, we discuss various challenges, possible alternatives to our current implementation, and potential enhancements. A major open question is how much the learning representation power of this approach can be extended, in particular to achieve some
manner of abstraction (and how this may be combined with composition), to, for example, further address sparsity and tail challenges. We discuss a few ideas in those directions in the paper. We expect that the approach is flexible and the system is extensible, in particular for incorporating additional kinds of learning.

This paper is a snapshot of our project for advancing research on systems that acquire myriad concepts with rich relations in an unsupervised manner. The models we report on are also snapshots in another sense: the learning can keep going, but timing and computation (memory) constraints have limited what we report on. We provide evidence that the learning continues to make good progress, and highlight areas for improving the approach. We only briefly explore some algorithmic and parameter variations (such as choice of learning rate schedule) and interactions among parts, and we leave more extensive comparisons and further explorations of the space to future work.

We begin the paper by an overview of the Expedition system. We then describe concepts and edges (the network structure) in Sect. 3. We present the segmentation and prediction processes, and scoring and objectives in Sect. 4, followed by Sect. 5 on learning (updating edges and learning new concepts). We then present experiments, which includes various plots of trajectories of learning, examples of what is learned and example segmentations, together with some exploration of the effects of parameters, such as search width, on the output, such as the segmentation quality. We then discuss related work and conclude. Appendix A explores edge weight updating and motivates a decay schedule for the learning rate, and Appendix B presents experiments where the lowest level input is converted to binary strings, i.e. what happens when we begin with only two concepts, 0 and 1?

2 Overview of Expedition

The input to Expedition is a stream of text, where the input stream is broken into lines in our experiments, each line of text being the input for a learning episode. We remove blank spaces in the experiments but otherwise we do not do any extra processing. Thus the line "An apple (or 2) a day!" is input to the system as "Anapple(or2)aday!". One reason we remove blank spaces to see whether and how well the system can learn words and phrases without the aid of separators. The only concepts in the system correspond to characters in the beginning, which we refer to as primitive concepts. There is a one-to-one correspondence between the primitive concepts and characters. A concept corresponds to a string or a consecutive sequence of characters, but is more than just a representation however, as it maintains a rich set of connection information, such as information about its associations (prediction edges) as well part-related connections (Sect. 3).

2.1 Episodic Tasks: Segmenting and Updating Weights

The system is repeatedly fed a line of text, which is readily converted to a sequence of primitive concepts. The system then segments this buffer into a small subset of its (current) higher-level concepts. The concepts in the final segmentation are the active concepts. These include the highest level concepts the system 'sees' in an episode. The system updates various statistics, such as concept co-occurrences, and prediction weights among the active concepts. In our implementation, these statistics are kept with the concepts themselves (Section 3). These learned statistics influence future segmentations by the system, and the segmentations determine what (higher-level) concepts are seen in the input, and thus which co-occurrences are updated, which in turn affects which subsequent higher level concepts are ultimately discovered.

2.2 Periodic Phase: Misc. Tasks Such as Adding New Concepts

Periodically, such as every say 1000 episodes, the system performs the periodic functions, such as updating concept priors, and new concept constructions, via composition, using co-occurrence statistics or the prediction weights. The system can also occasionally add a new layer of concepts (explained next in Sec. 2.3).

Initially, the segmentation is trivial or basically given, as each episode is a concatenation of characters, meaning the input buffer is a sequence of the corresponding primitive concepts. Over time, larger concepts are built, by the composition operation, and segmentation becomes a non-trivial search problem. The composition operation in our implemented system is a binary operation, putting two concepts that co-occur sufficiently frequently together (Sect. 5.2).

2.3 Levels, or Layering of Concepts

A concept corresponding to a low level string, such as a single character, for example 'a', occurs with other low level concepts, i.e. other characters, but after some learning, it can co-occur with higher level concepts too. For example, 'a' can co-occur with a concept corresponding to 'b' and also with a concept corresponding to 'book'. To the extent
possible, we do not want to mix (or confuse) co-occurrence statistics for the concept (corresponding to) 'a', i.e. when it occurs with primitives in the lowest layer vs. when it occurs with higher level concepts. Thus, we have found it useful to differentiate and support several levels or a layering of concepts that we describe next. We will use the terms level and layer interchangeably. Each level has the potential to learn a more powerful (conditional) distribution of the input.

Layer zero contains the primitives (the primitive concepts) and only the primitives, and initially only layer 0 exists. Layer \( i, i \geq 1 \), is created after sufficient experience with concepts at layer \( i - 1 \). Each level \( i \geq 1 \), has a clone or a replica of each of the concepts in the lower layer, \( i - 1 \), as well as the newly created bigrams, or holonyms, of concepts from layer \( i - 1 \). Therefore, a concept in level \( i \geq 1 \) has up to \( (i \) maximum of \( i \) composition (concatenation) operations. Crucially, even when a concept in layer \( i \) is a clone of a concept in layer \( i - 1 \), its co-occurrence and prediction connections are only to layer \( i \) concepts and thus they are entirely different (e.g. the concept (corresponding to) 'a' in layer 0 has a different set of prediction weights from its clone, the concept 'a' in layer 1, etc.). This (separation of prediction edges) is the original motivation for the idea of layering. Figure 4 shows a concept with several connection examples. We also note that in this implementation, concepts, in particular the clones, become more specific as we go up the levels (see Sec. 4.3). The segmentation process needs to support turning concepts in the input buffer from layer \( i \) to layer \( i + 1 \), all the way to the current highest layer.

Figure 2 shows the general learning loop. We flesh out the important functions in the following sections.

![Figure 2: The main loop: Repeatedly perform the episodic leaning. And once in a while, do the periodic tasks.](image)

**3 The Network Structure: Concepts and Edges**

In this work, a concept corresponds to a string of one or more characters. Initially, the system only contains the primitive concepts, each primitive corresponding to a single character. Over time the system acquires higher level concepts, corresponding to larger strings by concatenating (composing) lower level concepts. The system generates a tiny subset of all possible strings of length \( k \) (a diminishing fraction of all possible strings, as \( k \) grows), i.e. those that are (likely) meaningful. To do this, each concept keeps a number of connections, the 'horizontal' or left and right connections (prediction weights and co-occurrences), as well as the 'vertical' or up and down edges (parts and part-of connections). These connections allow for predictions and are also used for segmentation. We next go over the notation we use to refer to concepts and then describe in further detail what each concept keeps track of.

![Figure 3: The learning inside each episode involves segmenting first.](image)

**3.1 A Notation for Concepts With Some Examples**

We denote the concept corresponding to a string \( x \) in level \( i \), when the concept exists, by \( \text{con}_{i}(x) \), for example \( \text{con}_3(\text{ther}') \). This notation is specially useful when explaining segmentation for multiple layers in Sect. 4.1. In layer 0, every character seen in input has a corresponding concept. In our implementation, a new primitive concept is allocated in layer 0 the first time a character is seen (and if there already exist higher layers, the appropriate clones for that character are generated too). Thus we can have \( \text{con}_0(\text{r}') \), and once layer 1 is created, we'll have \( \text{con}_1(\text{r}') \), where \( \text{con}_1(\text{r}') \) is the (unique) clone of \( \text{con}_0(\text{r}') \). The prediction connections and co-occurrence statistics of \( \text{con}_1(\text{r}') \) involve only concepts in layer 1, while its part-of connections go to layer 2, and its parts connections, if any (none for \( \text{con}_1(\text{r}') \), since it’s a clone), go to layer 0. Similarly, \( \text{con}_2(\text{r}') \) is the clone of \( \text{con}_1(\text{r}') \).

We note that \( \text{con}_0(\text{r}') \) is always undefined or meaningless, since only single character concepts, the primitives, exist in layer 0. More generally, no concept corresponding to \( k > 1 \) or more character-long string can exist in a layer below level \( \log k \) due to our binary composition constraints.
3.2 Associative Connections ("Horizontal")

Each concept keeps and updates edges with weights, the prediction edges or the ‘horizontal’ connections (imagining text is written/read horizontally), to other concepts that occur with it, in segmentations, within a window of size at most some $k$ concepts in our experiments. In the implementation of this paper, $k$ is set to 3 and the left and right occurrences are distinguished.

These weights are implemented via hasmaps, and hard or soft constraints on the size of the hasmaps are imposed so the memory consumption is kept in check and the processing is efficient (e.g. predicting and updating) [37]. Let $\Delta$ denote the set of relative positions. Our context size is 3 in the experiments, thus the set of (relative) positions is $\Delta = \{\pm 1, \pm 2, \pm 3\}$ ($|\Delta| = 6$ possibilities). Each concept keeps a separate weight map for each position $i \in \Delta$. Example of (horizontal) edges for positions -1 and 1 is shown in Fig. 4.

We will next describe a few properties and semantics of the weights. Let $w_{c_1,c_2,i}$ denote the weight of prediction edge from $c_1$ to $c_2$ for position $i \in \Delta$. The weights are non-negative, and absence of an edge means 0 weight. When a concept is first seen, it has no edges (empty maps). The system begins with basically a tabula rasa. The weights are updated using exponentiated moving average (EMA) updates in our experiments [37], shown in Fig. 7. One can verify that the weights remain in $[0, 1]$ and with the manner of updating, for a specific position sum to no more than 1.0 ($\forall c, i, \sum_j w_{c,j,i} \leq 1$, or the weights of a position, at any time point, form a semi- or sub-distribution). Moreover, under fairly general assumptions (e.g. taking into account the learning rate $r$ of EMA and the budget on number of edges), for each position, the weights converge to approximate conditional probabilities, i.e. for instance the weight $w_{c_1,c_2,1}$ converges to the probability of observing $c_2$ immediately in the next position, given $c_1$ is observed in current position. See Appendix [A] for a review of additional properties of EMA.

3.2.1 Sparse EMA and Rate Decay

Note that the weight updates, which are linear operations, can be carried out relatively efficiently if the size of corresponding data structures (e.g. hasmaps) are kept in check. Letting the degree be $d$ (size of a map), then a weight update takes $O(d)$, while the totality of concepts can be orders of magnitude larger than $d$. When a concept $c_1$ is not already connected to target $c_2$, the edge is added, the weight of the newly added edge being set to the learning rate $r$ with the EMA update. The learning rate should be set so that it is adequately less than the minimum probabilities that we want to model (e.g. a tenth). For instance, if the minimum probability of interest [2] is 0.01, we want it to be say 10x lower, i.e. 0.001, otherwise, if the rate is too large, the error in the conditional probability estimation can be too large. A rate that’s too low may slow down learning substantially. To speed up convergence, for each concept $c$ we can start the rate from a high value, and lower it with each update for that position, or each time the concept $c$ is seen, to some minimum positive rate $r_{min}$ such as $10^{-3}$ or $10^{-4}$. Appendix [A] motivates this frequency-based rate decay schedule.

3.3 Cross-Layer (or "Vertical") Connections

Concepts keep track of their part-related edges, edges to their parts, and edges to the concepts they are part of, which we can imagine as vertical, in contrast to the horizontal edges. See Fig. [4]

Each concept keeps a list of bottom-up connections to compositions in the next layer (its immediate holonyms) that it is a part of, as well as its clone, in the next layer. These connections are used during (bottom to top) segmentation. The number of such connections is kept manageable. We posited that a concept need only keep 100 to 1000s of such connections. For instance, while the character ‘a’ may be part of tens of thousands of words and phrases (concepts) in English, the primitive that corresponds to ‘a’ will be a part of only 10s to 100s of significant bigrams. Thus, the layering and significance tests when composing reduces the connection possibilities.

Similarly, each concept in a layer $i \geq 1$ keeps a list of top-down connections to its part concepts in layer $i - 1$. Note that a concept corresponding to a string of $k$ characters can in principle be split into two subconcepts (substrings) in $k$ many ways. However, many such possibilities will be insignificant in lower layer and will not be generated. Still, a concept may have more than 2 parts, e.g. $con_2$ (‘new’) can have the pair of parts $(con_1 (‘n’), con_1 (‘ew’))$, as well as $(con_1 (‘ne’), con_1 (‘w’))$ (the parts will always be paired).

The top-down connections are used during matching a candidate composition during (top-down) segmentation. Note that, in addition to segmentation, these vertical connections are also useful in understanding which string pattern a composition concept corresponds to.

[2]More generally, one could also keep connections that are position or direction insensitive, but we have not experimented with such.

[3]The minimum probability leading to good performance depends on the domain and some experimentation is in general required.
3.4 Other Information Kept with Concepts

We also keep a few (scalar) fields with each concept \( c \), including the historical predicted probability (probability received when \( c \) occurs in a selected segmentation), denoted \( c.hpp \), and \( c.freq \), i.e. the frequency the number of episodes seen so far, and various other statistics and counters (first-seen, last-seen, prior, \( \cdots \)). A few of these, such as the \( c.hpp \), are used in the algorithms, for example during the segmentation process for guiding the search towards a good segmentation. Others are for reporting only, to get insights into the trajectory of the learning.

4 Inference: Segmenting & Predicting

The segmentation process ultimately generates a mapping from stretches of raw characters in the input to internal concepts, and in that sense it is “interpretation” too. We refer to it simply as segmentation. First we formalize what valid segmentations are, then we present a segmentation algorithm, a beam search, and describe the scorings of candidate concepts and segmentations that guide the search.

4.1 Segmentation Structure

The input to the segmentation process at any given layer \( i \), is a sequence of concepts at layer \( i \), and the output is also a sequence of concepts (a segmentation), at layer \( i + 1 \). A generic sequence at a layer \( i \), is denoted \( c_{i,1} c_{i,2} \cdots c_{i,k_i} \), first index, \( i \), referring to the level, the second goes over the consecutive positions: if the sequence is \( k \) concepts long, then there are \( k \) positions. We use the shorthand \( [c_{i,j}] \) for a sequence (at level \( i \)).

Thus, a segmentation is simply a sequence of concepts in our implementation. An example, segmentation of the string "new", is described below. In an episode, given is an input character sequence \( [c_j] \), \( [c_j] = c_1 c_2 \cdots c_{k_0} \), of length \( k_0 \geq 1 \), which is readily converted to the corresponding primitives concept sequence in layer 0, \( con_0(c_1) \cdots con_0(c_{k_0}) = [c_{0,j}], 1 \leq j \leq k_0 \) (with \( k_0 \) positions). Segmentation of this sequence (into concepts of next layer) yields a concept sequence at layer 1, \( [c_{i,j}], 1 \leq j \leq k_1 \), where \( k_1 \leq k_0 \). This process is repeated until we get a segmentation, i.e. a concept sequence, at the highest layer, at the concept sequence from one layer forms the input sequence for the next segmentation process. The constraint that ties the segmentations across layers, in this work, is that the concept sequence in layer \( i, i \geq 1 \), must exhaustively and partitionally cover the concept sequence at layer \( i - 1 \). Next we describe what we mean by covering.

In our implementation, each concept in layer \( i + 1 \) is either a clone of a layer \( i \) concept or is composed of two parts, i.e. it is the concatenation of two (consecutive) layer \( i \) concepts. Thus there are two possibilities for (a valid) covering: 1) Concept \( c_{i+1,j} \) covers the single position \( j' \), or matches the concept at that position, in the segmentation for layer \( i \)
iff \( c_{i+1,j} \) is the clone of \( c_{i,j} \) (the concept in position \( j' \)), or 2) \( c_{i+1,j} \) covers the consecutive positions \( j' \) and \( j'+1 \), or matches \( c_{i,j} \) and \( c_{i,j'+1} \), iff \( c_{i,j} \) and \( c_{i,j'+1} \) are paired parts of \( c_{i+1,j} \).

A concept sequence in layer \( i + 1 \), \([c_{i+1,j}]=c_{i+1,1}c_{i+1,2}\cdots c_{i+1,k_{i+1}}\) is a segmentation of the concept sequence in layer \( i \) if every concept in layer \( i + 1 \) covers one or two positions in layer \( i \) as defined above, with the following additional constraints: 1) consecutive concepts cover consecutive and non-overlapping positions, i.e. if \( c_{i+1,j} \) covers up to position \( j' \) (of layer \( i \) segmentation), then \( c_{i+1,j'+1} \) covers starting from position \( j'+1 \), and 2) all concepts in layer \( i \) are covered (by exactly one concept in the \( i+1 \) sequence).

As an example, if the input character sequence is \( c_1 = \text{‘n’}, \ c_2 = \text{‘e’}, \ c_3 = \text{‘w’}, \) thus \( k_0 = 3 \), then the primitives sequence is \([c_{0,j}]=[con_0(\text{‘n’}), con_0(\text{‘e’}), con_0(\text{‘w’})]\) and we have 3 positions to cover. A (valid) segmentation in layer 1 would be \([c_{1,j}]=[con_1(\text{‘ne’}), con_1(\text{‘w’})]\), where \( con_1(\text{‘ne’}) \) covers positions 0 and 1 in layer 0, or matches \( c_{0,1} \) and \( c_{0,2} \), and \( c_{1,2} = con_1(\text{‘w’}) \) is a clone of (and matches) \( c_{0,3} \) and covers position 2. This segmentation is partitional, i.e. the same position in layer 1 is not covered by more than one concept in layer 2, and exhaustive or complete, in that every position in layer 1 is covered. A segmentation at layer 2 would be \([con_2(\text{‘new’})]\), covering positions 0 and 1 in the previous layer. Another segmentation in layer 2 would be \([con_2(\text{‘ne’}), con_2(\text{‘w’})]\) (use clones to cover both positions).

In the future, we may want to relax either conditions of partitionality or completeness, e.g. allow segmentations that overlap to some extent and/or do not accurately cover every lower level position, allowing some mismatches or approximate matches, possibly with some penalization for mismatches, in order to train robust models that can handle noise/corruption and occlusion in the input.

### 4.2 The Segmentation Algorithm

In the segmentation algorithm presented here, segmentation proceeds one layer at a time, a segmentation at layer \( i \) yields one or more candidate (valid) segmentations at layer \( i + 1 \). Figure 5 shows the segmentation algorithm. The process of segmenting a layer \( i \) sequence to get one or more candidate segmentations at layer \( i + 1 \) is the same for all layers \( i \), and we explain the process next.

Briefly at a high level, segmentation (search) goes as follows. Given a layer \( i \) sequence, the input layer, initially all its concepts (positions) are marked uncovered. Pick a remaining uncovered concept at random or by some quality score (Sections 4.2.1 and thereafter). The quality score could be the maximum score over the concept’s holonyms in the next layer. The concept’s holonyms, as well as its clone, are then matched against the buffer. The clone always matches, and a few holonyms may match too. One of the matches is picked, either purely at random or by a function of the concept quality score (see Sect. 4.2.4). The matched one or two concepts in the input layer (layer \( i \)) are marked covered, and we repeat the process for remaining uncovered concepts until all are marked covered. Once all are covered, we have a candidate segmentation at layer \( i + 1 \).

Not all segmentations are equal, for instance, a good segmentation for “anewbike” is, “a”, “new”, “bike”, but if the system initially joins “a” and “n” together to get the concept “an”, and commits to it, we get a poor segmentation for the rest of the string ”ewbook”. The system performs a beam search to pick the most promising segmentation at the highest levels. In order to select the most promising at a given layer and guide the search, it assigns a score to each candidate segmentation generated. Scoring a segmentation is in turn a function of the concepts in the sequence, in particular how well the concepts in the segmentation “fit” or cohere with one another, or what we may refer to as coherence, as well as how long the concepts are, as we want the system to learn longer concepts. A third factor is how well a concept matches the input, but in this implementation we assume all matches are perfect (a match is all or nothing). The handling of noise and approximate matching, leading to a more relaxed and potentially more powerful segmentation, is an important direction that we leave to future work.

The scoring of the segmentation will also be used as a measure of the (learning) progress of the system. Table 7 in the experiments section shows a few example segmentations.

#### 4.2.1 Scoring Concepts

Ideally, we want a smooth measure that improves as the system constructs larger and larger concepts. We will use the measure to guide segmentation, as an objective, and we also report it as one reflection of the overall progress of the system. Measures such as average concept length (number of characters) segmented can be brittle, and also insufficiently sensitive to the steady but small progress in prediction. Perplexity (or equivalently entropy) is widely used in language modeling [25, 46] but perplexity goes down in general with larger vocabularies, and requires extension to handle vocabularies where multiple terms (e.g. “b”, “ba”, “bat” and “bath”) can occupy the same location of the input. Probability loss measures such as quadratic loss are smooth too, but also decrease (degrade) in general the more items to predict with. They do not appear to be suitable as measures of “progress”. In the experiments, we do report on several
We will be using in part how each candidate concept "fits" with others in a candidate segmentation, and our measure for whether covered, for each position.

Thus, longer concepts (concepts with more primitives) and concepts with more infrequent primitives have higher intrinsic reward. The priors of the primitive (their occurrence probability) are updated in the periodic tasks. In a segmentation, the intrinsic or matching reward of a concept $c$ is balanced against how much probability the rest of the

\[
\text{MatchReward}(c) = -\log\left( \prod_{1 \leq i \leq k} \text{prior}(c_i) \right) = -\sum_{i} \log(\text{prior}(c_i)), \text{where } c = c_1 \cdots c_k (k \geq 1)
\] (1)

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Some period of exploration is required for (relatively) new concepts, achieved via some regime of randomization as well as other exploration techniques. We describe how we promote exploration below.

Formally, we define the **match (intrinsic) reward** of a concept $c = [c_i]$ as:

\[
\text{MatchReward}(c) = -\log\left( \prod_{1 \leq i \leq k} \text{prior}(c_i) \right) = -\sum_{i} \log(\text{prior}(c_i)), \text{where } c = c_1 \cdots c_k (k \geq 1)
\] (1)

Figure 5: Pseudocode for segmenting an episode, via a beam search. The segmentation datastructures, $s_i$ (at level $i$) and $s_{i+1}$ primarily contain a sequence of concepts (initially empty) as well as auxiliary fields, such as boolean flag, whether covered, for each position.

of these measures as well (quadratic loss, average concept length, etc.). But for guiding the segmentation search, we seek a score that improves as the system expands its vocabulary of concepts. In this and next section, we develop a measure that improves (increases) in general as the number and extent of concepts grow over time.

We will be using in part how each candidate concept "fits" with others in a candidate segmentation, and our measure for this fitness is how well a concept is predicted, i.e. the probability that it attains, from the local context (the predicting concepts within $\Delta$ positions). However, a major challenge in scoring a segmentation is that the concepts can be relatively new, and in general, a segmentation will contain concepts with widely different frequencies or occurrence counts. We posited that a concept needs to be seen 100s of time before its own weights and probabilities to it from concepts that it co-occurs with begin converging to a stable range.® Some period of exploration is required for (relatively) new concepts, achieved via some regime of randomization as well as other exploration techniques. We describe how we promote exploration below.

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Thus, longer concepts (concepts with more primitives) and concepts with more infrequent primitives have higher intrinsic reward. The priors of the primitive (their occurrence probability) are updated in the periodic tasks. In a segmentation, the intrinsic or matching reward of a concept $c$ is balanced against how much probability the rest of the concepts within the probability that it attains, from the local context (the predicting context). However, a major challenge in scoring a segmentation is that the concepts can be relatively new, and in general, a segmentation will contain concepts with widely different frequencies or occurrence counts. We posited that a concept needs to be seen 100s of time before its own weights and probabilities to it from concepts that it co-occurs with begin converging to a stable range. Some period of exploration is required for (relatively) new concepts, achieved via some regime of randomization as well as other exploration techniques. We describe how we promote exploration below.

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Thus, longer concepts (concepts with more primitives) and concepts with more infrequent primitives have higher intrinsic reward. The priors of the primitive (their occurrence probability) are updated in the periodic tasks. In a segmentation, the intrinsic or matching reward of a concept $c$ is balanced against how much probability the rest of the
where probabilities are (initially) prevalent.

For any concept \( c \), the prediction probability \( \text{prob}(c) \) derived from the predictors in the context, will take time to learn (i.e. the larger the ratio), the higher the system’s score.

\[
\text{score}(c) = \log \frac{\text{prob}(c)}{\prod_i \text{prior}(c_i)} = \log \frac{\text{prob}(c)}{-\text{MatchReward}(c)} = \text{MatchReward}(c) + \log(\text{prob}(c)) \quad \text{(concept (COMA) score)},
\]

where \( \text{prob}(c) \) denotes the (prediction) probability assigned to \( c \) by the context concepts in the segmentation (note: \( \log(\text{prob}(c)) \leq 0 \)). See GetProb() and Predict() in Fig. 6. To motivate this set up, we imagine comparing the prediction system against a baseline system that predicts at the character level (never learns bigger patterns). The baseline makes the independence assumption when predicting and does not use any context. Therefore, it assigns \( \prod_i \text{prior}(c_i) \) to a concept \( c = [c_i] \) (irrespective of the segmentation). The log of the ratio,

\[
\log\left(\frac{\text{prob}(c)}{\prod_i \text{prior}(c_i)}\right),
\]

of the probabilities assigned by Expedition and the baseline systems to \( c \), is the score or the reward of the system for predicting concept \( c \) with probability \( \text{prob}(c) \). The farther the system gets from the baseline in the above sense (i.e. the larger the ratio), the higher the system’s score.\(^5\)

Another way to see how the above objective promotes using composition (larger) concepts: consider whether to join concept \( c_1 \), that is predicted on average with probability \( p_1 \), with a primitive concept \( c_2 \) with prior \( p_2 \). Say the joining, or the composition \( c_1 c_2 \) would be predicted on average with probability \( p_3 \). Then the joining is beneficial (increases the score on average) if \( \frac{p_2}{p_1} > p_2 \), i.e. even though in general we have \( p_3 < p_2 \) (the probability derived from prediction of \( c_1 c_2 \) in one shot will be in general less than probability of \( p_1 \) of just predicting \( c_1 \)), as long as the reduction is not more than \( p_2 \), it pays to join the two and predict them together.

4.2.2 Promoting Exploration

For any concept \( c \), the prediction probability \( \text{prob}(c) \) derived from the predictors in the context, will take time to learn and requires using (‘seeing’) concept \( c \) in segmentations. To promote exploration, i.e. the use of newly generated concepts or concepts that are relatively infrequent, a concept that has been seen less than \( t \) times so far (in any

\(^5\)Another way to look at it is that it is a balance between horizontal or side-ways fit vs. vertical or top-down fit (match).

\(^6\)Note that the score can be negative when the ratio is below 1, and this does happen in our experiments, even on an average basis for some concepts. For instance, the concept may be inferior, or due to the poor quality of the probabilities computed by the system, or the difficulty of the task. For example, see Appendix B where negative scores are (initially) prevalent.
We make the following notes regarding our current implementation: 1) The clone always matches (there is always at least one match). 2) There are at most 3 candidate matches to pick from, one is the clone, and up to 2 holonyms, one of which is marked covered, and the remaining uncovered position 2, or concept at position 1, is picked for multiple positions. Simple list and array datastructures are used to implement this search.

We will use the terminology and the notation introduced in Sect. 4.1. To generate a single candidate segmentation of a concept sequence at layer \( i \) (that we describe next), thus we get up to the product of Eq. 5) is picked. For example, assume "new" is in the primitives layer represented as \([\text{con}_0(\text{'n'}), \text{con}_0(\text{'e'}), \text{con}_0(\text{'w'})]\), and assume location 2, i.e. \( \text{con}_0(\text{'w'}) \), is selected, out of the three positions, to be covered first. Assume the holonyms of \( \text{con}_0(\text{'w'}) \) include \( \{\text{con}_1(\text{'w'})\}, \text{con}_1(\text{'wu'}), \text{con}_1(\text{'ew'}), \text{con}_1(\text{'ow'}), \cdots\}. \) Then we get two matching candidates \( \text{con}_1(\text{'ew'}) \) and \( \text{con}_1(\text{'w'}) \), and whichever has higher (historical) score is picked. Assume \( \text{con}_1(\text{'ew'}) \) is picked. Then positions 0 and 1 are marked covered, and the remaining uncovered position 2, or concept \( \text{con}_0(\text{'n'}) \), is next covered by a clone, yielding \( s_1 = [\text{con}_1(\text{'n'}), \text{con}_1(\text{'ew'})] \).

We make the following notes regarding our current implementation: 1) The clone always matches (there is always at least one match). 2) There are at most 3 candidate matches to pick from, one is the clone, and up to 2 holonyms, one matching to the left (covering positions \( j \) and \( j - 1 \)) and one to the right of \( j \), when those positions exist and not yet covered (e.g. three is no left-matching holonym at \( j = 0 \)). 3) The main source of randomization here is in picking an uncovered position. We leave sampling from the matches (to promote more exploration) to future work (see discussions below on interaction of segmentation and learning). 4) Each picked (matching) concept in layer \( i + 1 \) can keep track of the position \( j \) it matched (one of the up to 2 positions suffice), and once all positions in \( i \) are covered, a sort by \( j \) of the picked gives a complete candidate segmentation at layer \( i + 1 \). Note also that the same concept can match and be picked for multiple positions. Simple list and array datastructures are used to implement this search.

The concepts in the context update their prediction weights to a target concept in the usual manner. Note that when \( \text{prob}_{\text{opt}}(c) = 1.0 \), \( \text{score}_{\text{opt}}(c) = \text{MatchReward}(c) \). We have experimented with thresholds \( t = 20 \) and \( t = 50 \), and \( t = 50 \) appears to provide adequate time for learning for our experiments, but a longer period (bigger \( t_{\text{opt}} \)) may be needed in practice. After this period, the moving average of the actual (predicted) probabilities are used.

### 4.2.3 Using Historical Averages of Concept Scores

To make local segmentation moves, \( i.e. \) selecting among matching concepts at the next layer, which eventually creates a complete segmentation candidate at the next layer (see SegmentationMove() in Fig.\[\ref{fig:segmentation_move}\] and the next section), as well as to quickly score an entire candidate segmentation (Sect. \[\ref{sec:candidate_search}\]), we use a historical version of the COMA score, \( i.e. \) a combination of average of probability and MatchReward() of a concept (defined next) to select among matching alternatives, as there is no full segmentation yet available.

For keeping an average we use the EMA function. When a concept is new \( (c.\text{freq} \leq t_{\text{opt}}) \), its hpp, historical-predicted-probability \( (c.\text{hpp}) \) is 1.0. Once \( c.\text{freq} > t_{\text{opt}} \) the hpp of the concept \( c.\text{hpp} \) is updated via EMA whenever the concept is active, \( i.e. \) it occurs in a final selected segmentation, as shown in Fig.\[\ref{fig:historical_average_coma}\] (in function UpdateActiveConcepts()) in Sec. \[\ref{sec:concept_learning}\]. The hpp is thus the moving average of \( \text{prob}_{\text{opt}}(c) \) from Eq. \[\ref{eq:optimistic_probability}\].

The historical score of a concept, \( \text{score}_{\text{hist}}(c) \), is thus:

$$\text{score}_{\text{hist}}(c) = \text{MatchReward}(c) + \log(c.\text{hpp}) \quad (\text{Historical concept score})$$ \label{eq:historical_scores}

### 4.2.4 Beam Search and Making Local Moves

At each layer, we keep up to \( w \geq 1 \) candidate segmentations. Each such concept sequence at layer \( i \) is tried \( b \geq 1 \) times to generate \( b \) candidates (that we describe next), thus we get up to the product \( wb \) segmentation candidates at \( i + 1 \). Some may be duplicates, so we may get fewer unique segmentations. Of these, (up to) \( w \) highest scoring are kept. The next section describes scoring entire segmentations. The process is repeated, at each next (higher) layer, until the current highest layer is reached. Fig.\[\ref{fig:segmentation_move}\] presents pseudocode for a few of the search functions.

We will use the terminology and the notation introduced in Sect. \[\ref{sec:concept_search}\]. To generate a single candidate segmentation at layer \( i + 1 \), from \( s_i \) of level \( i \) (function Generate_A_Segmentation() in Fig.\[\ref{fig:segmentation_move}\]), local matching and covering moves are repeatedly made until all positions of \( s_i \) are covered: a random uncovered position \( j \) at level \( i \) is picked, the concept at position \( j \) tries its clone and its holonym concepts (which belong to layer \( i + 1 \)) and the match with the highest historical score \( (\text{score}_{\text{hist}}(c) \text{ of Eq.}\[\ref{eq:historical_scores}\]) \text{ is picked.\footnote{In our implementation, we keep one candidate segmentation from those that have identical score.}}\)
4.2.5 Scoring and Selecting Candidate Segmentations

When candidate complete segmentations at a given layer are computed, we still use the historical averages, with the possible exception of the top layer, to score candidates and select candidates (the fast score):

\[
fast_{\text{coma}}(s) = \frac{1}{|s|} \sum_{c \in s} \text{score}_{\text{hist}}(c) \quad \text{(fast score of a segmentation, via historical concept scores).} \tag{6}
\]

However, we do report on the actual and optimistic COMA scores of the selected segmentation from the top layer:

\[
\text{coma}(s) = \frac{1}{|s|} \sum_{c \in s} \text{score}(c) \quad \text{(actual, non-optimistic, COMA, for a segmentation.)} \tag{7}
\]

\[
\text{coma}_{\text{opt}}(s) = \frac{1}{|s|} \sum_{c \in s} \text{score}_{\text{opt}}(c) \quad \text{(optimistic COMA, for a segmentation.)} \tag{8}
\]

The COMA score incorporates the (match) reward of each concept with the probability it attains from the context (the rest of the segmentation), using Equation 2 for the score of each concept. However, during the beam search for a good segmentation, across multiple layers, computing the prediction probability for each concept in every candidate segmentation can take a long time. This is a main reason we rely on the fast score. Another consideration is that the goal during segmentation across layers is to get a good segmentation at the top layer, and computing COMA may be unnecessary for the intermediate layers (any layer that is not the top layer). For the top layer, we note that the fast choice for scoring a segmentation is less accurate and uses a concept’s average score instead of its current score in a segmentation. The average score reflects how well a concept in general fits within its context (combined with its match reward) instead of how it fits within its context in the current episode. For the top layer, we can use the (optimistic) COMA, and in a main set of experiments we do that (See Sect. 6.1). However, we leave the question of the extent to which such distinctions make a difference on the learning trajectory of the system to future work. Sect. 6.3 presents graphs that include the evolution of scores of all the scoring techniques.

Each segmentation at layer \(i \geq 1\) keeps track of which segmentation at \(i - 1\) led to it (if there are multiple ones leading to the same, pick one at random), and we thus get chains of segmentations across layers. In the highest layer, the highest scoring segmentation (ties broken at random) and its chain are kept. The various fields of the concepts in the selected chain (such as prediction weights and seen counts), the active concepts are then updated (Sect. 5).

4.3 Concept Specificity Increases with Level

Concepts become more specific as we go up the concept levels in our current implementation, in the following sense: when the character “a” occurs in the input, \(\text{con}_0(‘a’)\) is always activated, i.e. it always occurs in the final selected segmentation, but \(\text{con}_1(‘a’)\) may not be active, instead a holonym of \(\text{con}_1(‘an’)\), for instance \(\text{con}_1(‘an’)\) may be active, depending on the final selected segmentation. On the other hand, whenever \(\text{con}_i(‘a’)\) is active, \(i \geq 1\), \(\text{con}_{i-1}(‘a’)\) is active too.

4.4 Special Predictors

We implemented three special predictors for each level, the begin-buffer, the end-buffer, and the always-active predictors. These were added to provide insights (into the input text), and to possibly improve performance. The begin-buffer predictor is a concept that predicts, and updates for, the beginning of the buffer or the segmentation at that level (the first 3 positions in our experiments), while the end-buffer does the same for the last few positions of the segmentation. The always-active predicts every position, and basically provides a prior for that level. The begin and end predictors make it possible to have a well-defined COMA score at the top level when a single concept may be suitable (e.g. specially for short input lines).

4.5 Discussion

There are a number avenues for improving the segmentation process and making it more powerful, such as exploring other segmentation objectives, extending the search algorithms and relaxing the matching, and improving the prediction probabilities.
4.5.1 Objectives

Currently, we compare performance with respect to a baseline system operating at the lowest primitives level. An alternative, for example, to measure progress, is by comparing prediction performance of each layer to a baseline that is operating at the layer below. One advantage of comparing performance to the lowest layer is that this is the layer that we ultimately care about predicting: predicting "reality". Higher layers are concoctions of the system itself, and scores based on comparing to them remain only indirect measures of performance.

There may also be other baselines to compare to, and our particular definition of what to compare to and how to compare is improvable. In particular, the next section discusses relaxing the perfect match requirement.

4.5.2 Relaxing the Segmentation

We want to allow approximate (imperfect) matches to handle the possibility of additional noise in the input. For instance, imagine typos and spelling errors. We also want to allow partially overlapping concepts, and partial (non-complete) segmentations, as a way of handling white space for example, as well as to better handle cases where the input is partially hidden.

Currently the search completes a segmentation in one layer, before proceeding to the next. An interesting alternative which may fit better with partial and imperfect segmentation is for the search to "cut-across" layers. The segmentation COMA objective needs to be extended to handle such cases too. A general significant challenge is that, as we extend or relax segmentation and/or extend concept representation, keeping the (code) complexity and efficiency of segmentation (inference) in check. This and whether segmentation can provide the effective feedback for learning more sophisticated concept structures are open questions. We also expect that understanding the interaction of segmentation and the learning trajectory, for example, how the parameters for one, or their details of the algorithm for one, affects the other part, and ultimately the progress of the entire system, is an important area for future investigation.

4.5.3 Improving Prediction Probabilities

There exist a number of ideas to improve prediction, and several of the ideas can be fairly readily implemented within the system. The quality of the probabilities assigned are important for scoring concepts and segmentations (the COMA score). We do plain normalization to attain probabilities. We experimented briefly with a plain softmax function \( \frac{\exp(w)}{\sum \exp(w)} \) but that gave inferior results in terms of quadratic loss on probabilities, probably because of too much increase in the probability of those concepts that obtain higher weights. An adaptive or an online learning of a mapping, an online variant of binning \([55]\), significantly improves the predicted probabilities in terms of quadratic loss. However, we do not know the extent of the impact of these enhancements on the learning trajectory. One can also experiment with weighting the prediction of different concepts, taking into account the frequency of a concept and the relative position, e.g. via expert weighting methods, such as \([17]\).

Many concepts are of the same type or behave similarly in many contexts, e.g. the to-be verbs, the digits, punctuations, or the upper and lower case versions of characters. A major direction is ways of discovering and incorporating such to address sparsity, i.e. that is the problem of zero or very low probabilities for infrequent concepts and, more generally, unseen or little-seen co-occurrences. There will always be concepts that the system has limited experience with, and, with a plethora of concepts, not all legitimate co-occurrences can be observed with finite experience, and limits of system memory will always be a hard constraint too. There is a range of techniques in statistical language modeling which could be useful in addressing sparsity. For instance, analysis and use of the common neighborhoods in the graph of prediction edges, during inference, can improve performance in this regard.

5 Learning: Updating Weights and Scores, and Composing

The learning or updating can be classified into two main forms in Expedition. One form is updating weights of edges and various concept-related statistics used to score concepts and candidate segmentations. These updates take place at the end of each episode after a final segmentation is selected. Another learning is making new concepts, or composing.

5.1 Updating

Figure 7 shows the update operations. A number of updates occur, and all are done on the active concepts once a final segmentation chain is selected. The active concepts, i.e. the concepts in each segmentation from every layer, update their individual statistics, in particular \( c.freq \) and \( c.hpp \), as well as edge weights to nearby concepts (for each relative position \( i \in \Delta \)).
We describe our simple concept generation process, then discuss alternatives and enhancements. Appendix A presents several properties of EMA in particular for weight updating and motivates a tail score (the lower the tail bound), the more confident we are in rejecting that averages of proportion by the null model) \[34, 2\]. Therefore, one could use the prediction weights of edges, which are approximate moving averages of \( p(c_2 | c_1) \), instead of keeping an explicit list of co-occurrence counts (saving time and space).

**5.2 Composition**

We describe our simple concept generation process, then discuss alternatives and enhancements. Each concept keeps statistics about its immediate co-occurrences. In our implementation, each concept keeps track of co-occurrence counts on concepts that appear immediately to the right in a segmentation. Thus, in the final selected segmentation, whenever concept B follows concept A, the co-occurrence count entry for concept B, in the list of co-occurrences for concept A, is incremented by 1. If the entry is not there, it is created. Similar to other connections, the size of these lists are kept within a budget. Also, if a bigram concept has already been created, the system need not update the co-occurrence counts of its parts (an efficient lookup check can be performed). We also impose that either A or B must not be a clone so that duplicate concepts are not generated. In one set of experiments, we relax this condition, and we discuss these decisions later below. Each concept also keeps track of its overall occurrence counts, from which concept priors can be computed.

In each periodic phase, the co-location lists are examined, and pairs of concepts that pass the minimum co-occurrence count (10 in our experiments) and a minimum binomial tail score of 5 generate new bigrams, \textit{i.e.} new composition concepts to be used in layer \( i + 1 \). A tail upper bound of say \( x \), bounds the probability to no larger than \( x \), that we observe \( k \) or more co-occurrences \( (c_2 \text{ following } c_1) \) under any model that specifies \( P(c_2 | c_1) \) is no bigger than the prior of \( c_2 \), \textit{i.e.} \( \text{prior}(c_2) \). The score is the negative log of the tail bound \( -\log(x) \), therefore the higher the (tail) score (the lower the tail bound), the more confident we are in rejecting that \( P(c_2 | c_1) \) is any value at or below \( \text{prior}(c_2) \) and if we reject, we are concluding the conditional must be larger (assuming it exists, \textit{i.e.} we assume it is well defined). This check is a gate for generating candidate holonyms, and compared to pointwise mutual information, also used for (meaningful) bigram creation \[9, 40, 31\], performs better for discovering (good) bigrams for frequent concepts \[34\].

There is some possibility that the holonym may already exist, but with a different pair of parts, and in this case, the new pairs are added as additional paired parts, so the concept “new” in layer 2 may have two decompositions in layer 1: “n” and “ew” as well as “ne” and “wv”. We may do holonym generation (and the needed co-occurrence updates) at the top most layer only, as discussed below.

---

\[\Delta = \{±1, ±2, ±3\} \text{ (up to 3 concept positions left and right), } r_{mix} = 0.01.\]
5.3 Adding a New Layer

In the periodic tasks, once in a while we may also add a new layer. When adding a new layer the system performs the following. All existing concept in top layer \( i \) are cloned for new layer \( i + 1 \), and composition criteria are checked for creating new (non-clone) holonym concepts for the new layer \( i + 1 \). All concepts at \( i + 1 \) are appropriately initialized (frequencies initialized to 0, optimistic historical probabilities at 1, and empty lists of prediction edges and part-of edges).

In the experiments of this paper, we manually added a layer when certain criteria of sufficient training or convergence were met. Specifically, in most experiments, unless otherwise stated, we added a layer once the (moving) average over several (100s of) episodes of the minimum frequency of a concept observed in an episode went beyond 500, meaning that most concepts seen or used in segmentations (see next section), have reached adequate (100s) of learning episodes. Other criteria, that we have experimented with or used (e.g. in Appendices A and B) include waiting until optimistic and actual coherence (Sect. 4.2.5) converge (e.g. to near 10% of one another). Because only a few layers need to be added in general, we do not think this problem is critical, although how best to automate it depends on a careful study of how the various choices affect the (long-term) learning performance, and is worth future investigation.

5.4 Discussion

An interesting question is whether learning and updating should continue in lower layers once a new top layer is added, or whether it should be turned off. For instance, assume the current top layer is layer 3. Should learning continue in levels 0 to 2? There are tradeoffs of efficiency (learning takes computational resources), considerations of non-stationarities, and other considerations such as whether adequate time was spent learning overall, and for the individual concepts. This question also depends on the type of learning. For instance, updating the various types of concept scores and perhaps prediction weights among concepts may continue in lower layers, specially if we assume potential non-stationarities, but we may turn off, at the lower levels, the learning that is for the sole purpose of creating new concepts.

5.4.1 Continue Composing At Lower Layers?

Learning new concepts at a lower layer can be problematic because the same concepts may have already been discovered at higher layers, i.e. duplicate concepts may be generated that then may interfere with one another. We have observed that such events can occur sufficiently often and may cause systematic errors if not somehow addressed. If we have an efficient process for (near) duplicate detection and handling duplicate concepts (e.g. effective merging), this issue could be handled well, though we have not pursued this route. As explained above, in one original set of experiments we do not allow two clones to compose at a given layer (one concept needs to be a non-clone), but we allowed concepts composing in lower layers.

However, the above restriction of not composing when both are clones runs the risk of not creating certain compositions: what if two characters (primitives) can pass the test for composition, but only in layer 1 or higher, i.e. after segmenting with existing concepts, that is, certain concepts may be discoverable only after other compositions of the same level are discovered and segmented with. We do not know whether this issue is a sufficiently frequent phenomenon. In a 2nd set of experiments, we allowed composing at top-most layer only, but allowed any pair of concepts (clone or not) to compose in the top layer. This resolves the issue just mentioned, and we do not run the risk of creating duplicate concepts at different levels. Furthermore, co-occurrence count updates can be turned off at lower levels (efficiency, i.e. less work in lower levels). However, this runs the danger of the system getting stuck with (or limited by) how the segmentation at the lower levels lead it to "see" the world at higher levels: if there are errors or imperfections with how the lower levels segment, the system may not be able to recover. For instance, the input stream may change its nature (nonstationarity), e.g. the language may switch in the text stream. The system may remain stuck with how it segments with concepts originally discovered.

Section 6.1 describes the two main settings in our experiments. We currently favor the second approach (see Section 6.5), at least for a non-changing domain and given that we do not have duplicate removal or merging techniques. It is likely that there are inherent tradeoffs and it is impossible to completely satisfy all the desiderata, but understanding this space could be a fruitful future direction.

5.4.2 On Updating Prediction Weights and Concept Scores

Updating prediction weights can be relatively slow, as it relies on 'take-each-out' (or leave-one-out) updating. We don’t expect the number of levels to grow to more than 10s say, because the number of concepts grows at least exponentially in the number of concepts (see Fig. 8), thus updating prediction edges may not be a large extra cost, at least once a final
segmentation chain is selected, and the updates are done only for such. In the experiments of this we do this paper, we follow this strategy.

On the other hand, depending on the goals of the segmentation and the details of the segmentation search, updating prediction weights in lower layers may not be needed. For instance, one may argue that reaching a good segmentation at the top layer is the most important goal, and the task of the concepts at the lower layer is only to facilitate that search. Then one idea in that direction is that the lower layer concepts need only keep a running average of the score of the top layer concept they lead to. The segmentation search can use those scores for finding good segmentations at the top.

5.4.3 Criteria for Composition

The criterion of distribution change for concept creation is an attractive condition for concept generation: i.e. only generate (or keep) a bigram if the distribution sufficiently changes compared to the distribution of the part concepts. We are basically requiring sufficient change in “meaning”. Otherwise, if using the parts to predict together, gets close enough to the holonym (the composition), the composition candidate can be discarded (do not waste further resources on it). One can use the prediction edges to determine distributions.

Questions include which distribution, front or back of the concept, or a combination of all. Moreover, there will be issues of sparsity and surface form (similar to document similarity): two distributions may look different, i.e. different words or concepts, but the concepts may only look different, but have same or similar enough meaning. The precise details need to be specified and investigated, e.g. what is a sufficient change in distribution? and how to do this efficiently: for instance do we need to generate compositions first and then test, i.e. after sufficient experience decide whether we want to keep a generated concept?

There may also exist a more direct way of tying concept generation to the goal of improving the segmentation objective. Currently, we use a statistical test, the binomial tail, to filter candidates. We believe the test is a robust way of reducing the number of candidates (efficiency), and expect that any good concept will pass the test at some point in practice during learning (i.e. with non-adversarial data), but investigating and establishing this would be useful. See also Appendix B where the larger concepts can be harder to discover from a case of the lowest level containing only two primitives (input strings are binary strings).

6 Experiments

These experiments are run on the NSF abstracts dataset [14]. The dataset contains about 120k abstracts, or 2.5 million English text lines and over twenty million term occurrences. Each line contains about 55 characters on average. Blank spaces are removed from each line, to test whether words and phrases are eventually discovered, and no other processing is done. Each episode is a random line of a random abstract.

We report on a number of statistics and measures of progress to get a sense of how the learning progresses next (Sections 6.2 to 6.4). We then give specific examples of concepts, holonyms, prediction edges, and segmentations, along with several related statistics, and report on the effects of changing segmentation search width on segmentation score and the quality of the splits in the segmentation (Sections 6.5 to 6.7). Sect. 6.8 reports on training times and other computational costs.

6.1 The Parameters and two Models

In all the experiments, unless otherwise stated, we used a window of size 3 on both sides of a concept (|Δ| = 6) for prediction and updating, frequency-based decay for the learning rate of the prediction edge weights (to a minimum rate of r_{min} = 0.0001), and budget sizes of 200 for edge-weights (each relative position) and co-occurrence lists. We used 10, 3 i.e. try 10 and keep 3 for the beam search of the segmentation algorithm.

We report on two approaches to generating compositions (holonyms). See 5.4.1 for a discussion. In the first approach, we continued the experiments for some time after layer 4 is created, and we call the obtained model Model4. Here we allowed composition generation at any level, thus layer 4 concepts and segmentation may be present but layer 0 can keep creating and adding concepts into layer 1 (at the periodic phase). However, we imposed the condition that at least one concept in the composition of two needs to be non-clone, so that we avoid creating redundant concepts at different layers. In the second approach, where we continued the experiments for some time after layer 3 was created, we allow concept creation only at the highest layer, but any pair of concepts can be composed (both can be clones). We call the

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9The abstracts are split into 11 files. The files are randomly permuted in each pass, then a random sample of a few 100s of lines are processed from each file.
model we obtained Model3. In both models, historical COMA scores were used to pick segmentation moves and entire segmentations in all layers, except that for Model3, in the top layer, we used the optimistic COMA score (eq. 8) to pick a final segmentation in the top-layer (and its chain). In all cases (whether or not historical), we used the optimistic versions of COMA.

Model3 is a snapshot at nearly 2 million episodes, and Model4 is a snapshot at nearly 2.5 million episodes. The progress in COMA (segmentation) scores are similar in the two systems. There are a few differences, e.g. in the number of concepts generated or the number of holonyms of concepts (see Sect. 6.5), and we currently favor the approach that led to Model3.

### 6.2 Number of Concepts and Their Frequencies

Figure 8 shows the number of (candidate) concepts generated at each level, as well as the number of concepts seen for at least 100 times (in a final selected segmentation), vs. the number of episodes. This is during the training that led to Model4, and the plots for Model3 are similar. The number of primitives (unique single characters) quickly reaches 94, and eventually to 98, in this abstracts dataset (Fig. 8(a)). The numbers of non-clone concepts with freq. above 50 for Model4, for levels 0 though 4, were 96, 2.9k, 5.5k, 19.5k and 14k resp., and for Model3 they were resp. 96, 873, 1.9k, and 12k (up to level 3). Thus, we see a big jump (10x or more) from level 0 to level 1, and the increase in number of concepts from one level to next tapers (although non-uniform, and remains above 3x) for subsequent levels. Recall that for Model3, concept (composition) generation is on only for the top layer (stopped at all lower levels). With each additional layer, it appears that we need roughly 10x more episodes to train the system, under a certain notion of convergence (see Section 6.3).

![Number of concepts observed.](image1)

![Number of non-clone concepts with frequency ≥ 100.](image2)

Figure 8: Number of unique concepts observed (in a segmentation) for each level, $l = 0, 1, \ldots$, vs. time (number of episodes, or lines read), during training of Model4, up to episode 700k.

Figure 9 shows the moving averages (mixing rate of 0.01) of the minimum frequency of a concept observed in the final selected segmentation chain for each level, vs. number of episodes. We see much variance in the moving average of the minimum frequency. Fig. 9(b) shows the same plots, but smoothed: for every 3 consecutive points, the median is kept, and the resulting is average of the 5 past kept numbers. Fig. 10 shows the moving averages of the median frequencies as well as the minimum frequencies, for levels 2 and 3. We observe that the median is much higher, perhaps up to two orders of magnitude (100x).

### 6.3 Segmentation Scores

Fig. 11 shows the progress of segmentation scores at each level. As mentioned earlier, we manually triggered creation of a new level and segmentation at a new layer, e.g. after episode around 600k, we incremented the layers to 3 (i.e. allowed segmenting up to layer $l_{\text{max}} = 3$).
Note that for these scores, the highest scoring segmentation, out of several candidates, is picked at the highest level, and the segmentations at the lower levels, i.e. the segmentation chain, are picked based on which led to the highest segmentation at top.

We later added reporting of both actual (non-optimistic) and optimistic segmentation (COMA) scores. See Fig. 12. Optimistic COMA starts higher, but eventually once most concepts are seen frequently enough (some time after \( c.freq > t_{opt} \) for most observed concepts \( c \)), the optimistic and actual (non-optimistic) COMA scores converge. For Model4 (trained up to level 4), we note that the scores at the top, level 3, have not converged yet, after 2 million episodes. Our experience on this dataset, and with our current parameter settings (such as \( t_{opt} = 50 \)), suggests that for the actual and optimistic segmentation scores to converge to say 10% to 20% of one another each level requires roughly 10x more episodes than the level below for convergence (an order of magnitude larger). Thus level 0 requires about 10k to 20k episodes (see also Table 12 in the appendix), level 1 requires 100k to 200k, level 2 about 1 million to 2 million, and we expect level 3 requires about 10million episodes.
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Figure 11: The progress of segmentation scores (COMA, Eq. 6) for various levels ($l = 0, 1, \cdots$) (Model4).

Figure 12: Optimistic (Eq. 8) vs. actual (Eq. 7) vs. max & min historical segmentation scores (Eq. 6) at the top layer during training that led to Model4. Optimistic (for exploration) and actual segmentation scores eventually converge, actual segmentation score improving as the probability estimates (predictions) improve over time. For fast scoring of candidate segmentations, we use the historical (average segmentation) score of each concept in the segmentation, and those values (moving averages) for the picked segmentation (highest scoring segmentation in the top layer) as well the lowest scoring segmentation in the top-layer (to get a picture of the spread in scores) are shown.
6.4 Quadratic Loss and Number of Concepts Per Episode

Fig. 13(a) shows progress in the moving average over episodes of quadratic loss at each level. Once a segmentation chain is picked, and probabilities are computed in take-each-out manner (see GetProb() in Fig. 6), we can compute the loss for each position and average (for each level separately). We are reporting the loss, \((1.0 - p)^2\), on the probability \(p\) assigned to the true target concept (at each level). \(p\) can be as low as 0 here, thus the maximum loss can be 1.0. We see that convergence is fast, and the loss goes to around 0.84 for level 0 and to 0.9 loss for other higher levels. Quadratic loss is only a crude way of measuring progress of the system.

Fig. 13(b) shows the moving average over episodes of the average number of concepts in the selected segmentation chain. There are about 53 primitive concepts (raw characters in a line of an abstract) on average. Again, we see convergence is fairly fast (while COMA keeps steadily improving).

![Figure 13](image)

Figure 13: Progress in quadratic loss and average number of concepts in each level of the selected chain per episode, over time. The number of primitives (level 0 concepts) is about 54 per episode. The convergence is fast and these measures are only crude ways of measuring system progress, compared to the COMA score over time and for various levels (Figs. 12 and 11).

6.5 Example Holonyms

Table 1 for Model3 shows the number of holonyms and some example holonyms, at different levels, ranked by their frequency for a few concepts, corresponding to the frequent "a" and infrequent "z" as well as "there". Recall that for Model3, co-occurrence statistics are only updated and used at the top layer, and holonyms can only be added at the top layer. For Model4, recall that holonyms could be added into any layer, but with the constraint that we did not allow the creating parts to be both clones. We note that the number of holonyms of a concept (at the next layer) can be significantly higher, which can be an issue for efficiency. In the implementation, we limited the list of co-occurrences, but not the number of holonyms of a concept. If a concept is frequent, such as those corresponding to "a", the less frequent concept to the left can make compositions with it, and we see this for \(con_2('a')\) and \(con_3('a')\) in Table 2 for Model4 (i.e. many holonyms are a concept followed by "a", such as "understa", "thatca", and "pathwa"). This appears less so in Table 1 for Model3, where we allowed cloned concepts to join. We also note that the number of holonyms of \(con_1('a')\) and \(con_2('a')\) in Model3 appear lower than in Model4 (nearly half or less): part of the reason is Model4 was trained for several 100k episodes more, but another reason is likely turning off compositions at lower layers for Model3.

In Table 1 also observe that many holonyms at higher layers, such as \(con_3('ca')\) had been discovered before, but they are created again. Looking into it, \(con_2('ca')\) and \(con_1('ca')\), have relatively low scores, and in particular lower than...
Table 1: Examples of the holonyms of a few concepts from Model3 (in the next level), sorted by their frequency. Each holonym’s frequency and its rank order is also shown. The concept corresponding to “a” at level 0 has 31 holonyms in Model3 (at level 1), and its most frequent holonym is “al” (con3(‘al’)) with frequency 805k.

...the parts (the clones) con2(‘a’) and con2(‘c’), and thus are likely not picked during segmentation at their layers. So...
In Model 3, the top predictions of the begin-buffer predictor (Sect. 4.4) in level 3 were, in order, "e", "a", "s", "in", "the", "The", "and", and "The" (from weight of 0.041 to 0.015). While at level 2, they were, "a", "i", "e", "t", "s", "the", "pro", "The" (from weight of 0.049 to 0.026).

| pos=1 | pos=2 | pos=1 | pos=2 |
|-------|-------|-------|-------|
| con0(\(a'\)) \('a' at level 0) | con3(\(a'\)) \('a' at level 3) | con1(\(th'\)) | con3(\(th'\)) |
| "r",0.107 | "n",0.200 | "t",0.155 | "s",0.076 |
| "e",0.101 | "t",0.191 | "e",0.146 | "t",0.063 |
| "i",0.079 | "I",0.143 | "d",0.125 | "w",0.037 |
| "n",0.074 | "r",0.120 | "t",0.087 | "c",0.017 |
| "c",0.073 | "s",0.065 | "a",0.064 | "n",0.019 |
| "h",0.070 | "c",0.059 | "o",0.050 | "e",0.029 |
| "t",0.070 | "b",0.029 | "c",0.048 | "te",0.022 |
| "m",0.065 | "d",0.027 | "s",0.047 | "an",0.017 |
| "s",0.057 | "m",0.027 | "l",0.044 | "to",0.016 |
| "p",0.034 | "p",0.025 | "n",0.026 | "in",0.016 |
| con2(\(t'th'\)) | con3(\(t'th'\)) |
| "o",0.311 | "e",0.100 |
| "whe",0.076 | "e",0.067 |
| "ge",0.063 | "s",0.060 |
| "fur",0.050 | "n",0.048 |
| "u",0.036 | "t",0.042 |
| "i",0.033 | "the",0.036 |
| "a",0.030 | "mal",0.029 |
| "Fur",0.026 | "i",0.027 |
| "ro",0.025 | "th",0.024 |
| "ra",0.025 | "more",0.021 |

Table 3: Top 10 edge weights for a few concepts and positions: the concepts, con0(\(a'\)), are placed in the table so the -1 edges are slightly to the left, and +1 to the right, for each of reading the implied relations. For con0(\(a'\)), i.e. \(a'\) at level 0, we get \(r\) (con0(\(r'\))) occurring before it with highest probability (0.1) and \(n\) (con0(\(n'\))) immediately after it. For con1(\(th'\)), we get "wi" in level 1 at position -1 with high weight, or together they would be "with", and "er" at position 1 with high weight ("ther"). At level 3, we observe "dep" ("depth") and "workwi" among the high weights.

We next explore a few properties of the prediction edges, in particular properties of the probabilities and measures of uncertainty such as average entropies. Table 5 shows average entropies (averaged over concepts)\footnote{Entropy is \(-\sum p \log(p), or \sum_{c_{i,j,k}} w_{c_{i,j,k}} \log(w_{c_{i,j,k}}) \) (where the sum goes over the edges of a concept \(c_i\) for a position \(k\)), where, where for any position \(k \in \Delta, \sum_{c_i} w_{c_{i,j,k}} \leq 1\) (See Appendix A). We return 0 entropy when there are no edges, e.g. for concepts generated but not seen yet in a segmentation.} for concepts that have been observed for some time (under three different freq. thresholds), for Model3. Position 1 (pos=1) is the next (immediate right side) concept. In general, entropy increases as the concept is seen more, i.e. \(\sum_{c_{i,j,k}} w_{c_{i,j,k}} \log(w_{c_{i,j,k}})\).
Table 4: Statistics on a few concepts in Model3: left columns show concepts ("ther") or related ones ("whether") from Table 3 and right columns show a few most frequent. The frequency (number of times seen), how many episodes ago it was last seen in a segmentation, from the time the snapshot was taken (e.g. con3(ther)) was seen 22 episodes ago, and the average historical reward are shown. The right column shows non-clone concepts with highest freq. at level 3, except the last row, con3(ther'), which is the concept with the highest freq. in level 3 (it is a clone).

| concept     | freq. | last seen | score* | concept     | freq. | last seen | score |
|-------------|-------|-----------|--------|-------------|-------|-----------|-------|
| con3(ther') | 57890 | 25        | 14.0   | con3(ther') | 54456 | 24        | 10.1  |
| con3(ther') | 6370  | 106       | 8.0    | con3(ther') | 50353 | 20        | 25.3  |
| con3(ther') | 4023  | 58        | 10.4   | con3(ther') | 42501 | 22        | 28.0  |
| con3(ther') | 84643 | 16        | 17.1   | con3(ther') | 36479 | 101       | 13.4  |
| con3(ther') | 22195 | 48        | 10.8   | con3(ther') | 28092 | 85        | 25.5  |
| con3(ther') | 3383  | 388       | 21.0   | con3(ther') | 966729| 2         | 1.1   |

Table 5: Entropy of edge weights (conditional probabilities) in Model3, averaged over concepts of a level, where N is the number of concepts that remain with ≥ 0, 50 and 100 thresholds on frequency. Thus, for this model at level 3 there are 20.4k concepts generated, while 14.8k concepts remain if we require a concept to have been seen (in a final selected segmentation) at least 50 times. Entropy goes up with position (the more distant, the more uncertain), and often also with level (the more concepts, the more uncertain), and with higher concept freq. (experience with the concept), i.e. more concepts are seen with more experience.

Table 6 shows a similar pattern with respect to probability mass. The table gives the sum of probabilities on edges, for those edges above the probability threshold (shown for ≥ 0.01 and 0.1), i.e. how much of the mass is concentrated on the probability ranges we expect are most useful (ratios to total probability mass are similar, a bit larger). Higher levels in general have higher entropy and lower mass on high probability edges, and position 2 is more uncertain than pos 1.

Table 6 also presents the average number of edges for several cases. We note that the number of edges with relatively high weight (above 0.01) are in the 10s, and we expect this has implications with regards to efficiency of prediction and inference, i.e. the numbers of significant edges are relatively small, which can speed up operations (specially whenever learning is turned off). Also, we note that as the average edge numbers do not seem to be increasing with higher levels, while the number of concepts substantially increase with level, the sparsity of connections, the graph sparsity, increases with level. The average number of edges over all concepts (under no freq. threshold on concepts) is not shown, while the number of concepts substantially increase with level, the sparsity of connections, the graph sparsity, increases with level.

Table 6: Average (over concepts) of edge-weight (probability) mass over prediction edges with weight surpassing 0.01 and 0.1 thresholds for pos=1 and pos=2. The average number of such edges is also shown for pos=1, and thresholds 0 (no threshold), 0.01 and 0.1. Thus at level 3, for concepts with freq. ≥ 50, on average there are 87 edges per concept, and about 17 edges for such concepts have a weight of no less than 0.01, and this average is 1.4 for thresh. 0.1 (all for pos=1). We observe the number of edges goes up with growing concept freq. (from 50 to 100) but the number of edges with high weight can slightly go down. Across layers, the total mass of edges does not change much, and much of the mass, around 80% or more, are on edges with weight ≥ 0.01 for pos=1.

| p ≥ 0.0 | p ≥ 0.01 | p ≥ 0.0 | p ≥ 0.01 | p ≥ 0.10 |
|---------|----------|---------|----------|----------|
| freq. ≥ 50 | freq. ≥ 100 | freq. ≥ 50 | freq. ≥ 100 |
| level | pos=1 | pos=2 | pos=1 | pos=2 | pos=1 | pos=2 | pos=1 | pos=2 |
| 3     | 87     | 0.80   | 0.64   | 97     | 0.78   | 0.59   | 0.41   | 0.14   | 0.13   |
| 2     | 129    | 0.81   | 0.65   | 134    | 0.80   | 0.63   | 0.44   | 0.16   | 0.2    |
| 1     | 112    | 0.84   | 0.78   | 139    | 0.79   | 0.72   | 0.31   | 0.14   | 0.16   |
| 0     | 70     | 0.90   | 0.89   | 70     | 0.90   | 0.89   | 0.44   | 0.28   | 0.28   |
6.7 Example Segmentations, and Statistics on Bad splits

Table 7 shows a couple of lines and their selected segmentations and segmentation chains (down to levels 1 and 2), via Model3 and Model4. Each concept in a segmentation corresponds to a consecutive sequence of characters in the input line, and therefore has a corresponding beginning and an end. As concepts are created and segmented with, we hope that these boundaries increasingly correspond to the original blank spaces (separations) in the input line.

Here we count the number of "bad splits" on average. When looking at the sequence of active concepts in the top level, a split is the location of an end of one concept and beginning of the next one, (thus, we don’t count the beginning and end of lines). A bad1 split is a split within a word (or that does not align with the space between two words). For instance in the top level selected segmentation from Table 7, in the prefix "regarding the conservation", there is 1 bad1 split, between "reg" and "arding" and 2 good splits (between "arding" and "the", and between "the" and "conser"). The whole selected segmentation via the level 3 model has 5 bad1 splits. A bad2 split is a bad split on both sides, i.e. it is a bad1 split where the next split, as we scan from left to right, is also bad (thus both ends of a concept is internal to one or two words). Table 8 reports on the number of bad1 and bad2 splits as well as COMA scores, for two models and several beam width parameters, averaged over about 200 lines. The table also includes the average segmentation scores, for different beam widths. Level 1 model was trained at 160k episodes, Model3 and Model4 were trained for nearly 2 million episodes each. We observe increased COMA correlates with fewer bad splits. And with more levels, in general the COMA score goes up and bad splits go down. And of course, the more search trials, i.e. the wider the beam search, the better the results. Our default of 10 tries, keep 3 yields similar to the 5,5 row in the Table. With 10,10 and Model4, bad1s and bad2s go to 6.8 and 3.6 resp., and with 15,15 bad1 and bad2s go to 6.5 and 3.5 resp. with COMA of around 11.5 in both cases. Multiple trials give similar results. With additional training and layers, we have observed number of bad splits to go below 3. We note that on average a line has about 8 separating blank spaces ("true" splits) or 9 space-separated tokens.

| Model, score | Line | Model3 segmentations | Model4 segmentations |
|--------------|------|----------------------|----------------------|
| 11.7 | L=3 | regarding the conservation and management of these magnificent | regarding the conservation and management of these magnificent |
| 11.7 | L=2 | regarding the conservation and management of these magnificent | regarding the conservation and management of these magnificent |
| 11.7 | L=1 | regarding the conservation and management of these magnificent | regarding the conservation and management of these magnificent |
| 17.3 | L=4 | regarding the conservation and management of these magnificent | regarding the conservation and management of these magnificent |
| 17.3 | L=3 | regarding the conservation and management of these magnificent | regarding the conservation and management of these magnificent |
| 17.3 | L=2 | regarding the conservation and management of these magnificent | regarding the conservation and management of these magnificent |
| 6.3 | L=3 | Commercial exploitation over the past two hundred years drove | Commercial exploitation over the past two hundred years drove |
| 6.3 | L=2 | Commercial exploitation over the past two hundred years drove | Commercial exploitation over the past two hundred years drove |
| 6.3 | L=1 | Commercial exploitation over the past two hundred years drove | Commercial exploitation over the past two hundred years drove |
| 11.9 | L=4 | Commercial exploitation over the past two hundred years drove | Commercial exploitation over the past two hundred years drove |
| 11.9 | L=3 | Commercial exploitation over the past two hundred years drove | Commercial exploitation over the past two hundred years drove |
| 11.9 | L=2 | Commercial exploitation over the past two hundred years drove | Commercial exploitation over the past two hundred years drove |

Table 7: Segmentation of a couple of example lines (episodes) via two models, using width 15 (15 tries from each, and keep highest scoring 15 at each level). The segmentation chains covering a few lower levels are also shown.

| Model, score | Line | Model3 segmentations | Model4 segmentations |
|--------------|------|----------------------|----------------------|
| 11.9 | L=3 | Commercial(0.017) e(0.236) xploitatio(0.016) n(0.078) overthe(0.038) passtwo(0.011) hundred(0.000) y(0.076) e(0.148) ar(0.074) s(0.083) dro(0.000) ve(0.002) | Commercial(0.017) e(0.236) xploitatio(0.016) n(0.078) overthe(0.038) passtwo(0.011) hundred(0.000) y(0.076) e(0.148) ar(0.074) s(0.083) dro(0.000) ve(0.002) |
| 6.3 | L=2 | Commercial(0.002) cial(0.07) e(0.157) xploit(0.002) t(0.191) ationo(0.004) verthe(0.002) pac(0.057) st(0.051) t(0.074) w(0.052) o(0.095) h(0.180) undred(0.001) y(0.145) e(0.206) rs(0.040) do(0.004) rove(0.000) | Commercial(0.002) cial(0.07) e(0.157) xploit(0.002) t(0.191) ationo(0.004) verthe(0.002) pac(0.057) st(0.051) t(0.074) w(0.052) o(0.095) h(0.180) undred(0.001) y(0.145) e(0.206) rs(0.040) do(0.004) rove(0.000) |
| 5.9 | L=1 | Commercial(0.037) merci(0.004) ale(0.001) xploit(0.002) t(0.17) ationo(0.004) verthe(0.002) pac(0.057) st(0.051) t(0.074) w(0.052) o(0.095) h(0.180) undred(0.001) y(0.145) e(0.206) rs(0.040) do(0.004) rove(0.000) | Commercial(0.037) merci(0.004) ale(0.001) xploit(0.002) t(0.17) ationo(0.004) verthe(0.002) pac(0.057) st(0.051) t(0.074) w(0.052) o(0.095) h(0.180) undred(0.001) y(0.145) e(0.206) rs(0.040) do(0.004) rove(0.000) |

Table 8: Segmentation examples, along with scores the active concepts received (in the take-each-out fashion). The segmentations are of the same line, one by Model4 and two by Model3 (at the top layer of the model), along with score of the segmentation (11.9 by Model4, 6.3 and 5.9 by Model3).

That the number of bad splits is decreasing with more training is not entirely unexpected however, because over time, the newly discovered concepts correspond to longer strings, and they lead to fewer overall splits in general (also see Sect. 6.4). It is likely that additional inference (with wider width) is more successful at matching larger concepts, which
Table 9: Average COMA and average count of bad splits (over 188 episodes) for several levels and beam search parameters. As expected, COMA improves with additional search. Number of bad splits and COMA are (negatively) correlated too.

Table 10: Bad-split ratio scores for a few models and segmentation beam widths, average over the same 388 episodes. The bad ratios decrease with more training and additional inference (width). Multiple runs gives similar results. See also Fig. 14.

(a) Model3: 2 runs with width 1, 2 runs with width 10 (i.e. 10, 10).
(b) Model3 and a model trained up to level 2.
(c) Model3: Minimum requirement of 5 episodes.

Figure 14: Plots of averages of bad-split ratios for for those split counts where we got at least 20 episodes, (a) and (b), or minimum of 5 for (c), for the total split count, when a model was run over nearly 400 episodes. As we increase the beam width or the training, the plots move to the left and the bad-split ratios decrease.

leads to fewer concepts in an episode, fewer splits and therefore fewer bad splits (in addition to improving COMA). We also added the reporting of average over episodes of the bad-split ratio, i.e. the ratio of bad splits to total number of splits in an episode, and we have observed that this ratio also improves somewhat with more training episodes and layers, as well as with a larger beam width. See Table 10

We also averaged the bad-split ratio for each fixed number of splits separately, and Fig. 14 shows the averages for those splits k for which the number of episodes that yielded k splits was at least 20 cases (14(a) and (b)) or at least 5 cases in (14(c)) (to remove clutter and more easily see the patterns). For instance, in one run we got 3, 5, 20, and 24 episodes resp. for split counts of 11, 12, 13, and 14, and among these we are showing averages for 12, 13, and 14 in 14(c) (average for 11 is not shown as only 3 episodes had that count), and averages for only 13 and 14 are shown (min count of 20). Multiple runs yield very similar results specially with higher beam width (a couple shown in 14(a)). We observe that with higher beam-width and more training the plots shift to the left (episodes have fewer number of splits), and also the plots shift down (reflecting a decrease in the bad-split ratio, similar to Table 10).

A smaller search width for segmentation speeds up inference in each episode, but the quality of the final selected segmentation can be poor, and the poor quality may slow down the learning of good co-occurrences and therefore good compositions in the long run. There is a tradeoff. In particular, a beam width of say 1 (or a bad inference algorithm
more generally) may introduce too much noise, too many misteps along the way and by the time a final segmentation is reached at the highest level (e.g. the joining of "nd", of "and", with "m", from "management", to create "ndm", in level 2 segmentation by Model4, in Table 7). See for example Appendix B where Expedition is started with 0 and 1 as primitives, and many (10s of) levels may be required to discover the level of characters and words. We leave further study of the interaction of the search width with learning to future work. It will also be informative to assess the various measures of progress on heldout data.

6.8 Timings and Computational Costs

All code is written in Python. Each period or sweep of 1500 lines (episodes) took 3 minutes, on a Macbook Pro laptop, when layer 1 was the maximum layer, while it took 30 minutes when the maximum layer was level 4 (e.g. for Model4). We note that one could train models in parallel and periodically aggregate the models. Model sizes also grow with more episodes and layers (additional concepts and edges), from a few Megs (compressed) when layer 1 is the maximum layer, to low 100s of Megs for Model4 in our current experiments. The main time complexity is in the segmentation search, where concepts try their holonyms during the search, and predict, for example during COMA scoring for picking the best segmentation at the top layer. As long as the data structures are kept relatively small, e.g. at most 100s to 1000s of entries (in each of prediction weight maps for each positions, and part related vertical connections), we expect the cost of each episode remains manageable. Let \( d \) be the maximum over size of such connection lists (over all concepts). Cost of an episode grows with product of episode size (number of characters in a line), search width, number of layers, and (degree) \( d \), but we expect that \( d \) need not grow or grow very slowly with the number of concepts. For instance, see Table 6 for the case of prediction weights: we expect, for each concept and position, relatively few edges will have sizeable weights, and we conjecture that only sizeable weights (conditional probabilities) are needed for good performance, in practice. Note that tiny weights also require much more training (samples) to estimate well.

7 Related Work

This work is a continuation of our research on prediction games [32, 31, 33], sharing the goal of learning a hierarchy of concepts in a cumulative unsupervised manner. The motivations and philosophy behind the approach and relations to a few general learning tasks, e.g. distribution or density learning, are discussed in that work [32]. Previously the focus was primarily on prediction and composition [31], and the need for a more sophisticated segmentation was later identified but left to future work [33]. Segmentation there was a simple left to right and greedy process: a largest concept (sequence of words or tokens) was extracted and the resulting segmentation (or tokenization) was used for self training of prediction weights. To handle more complex concept structures, it was deemed that a fairly sophisticated segmentation process would be needed [33]. More elaborate segmentation or inference is also likely needed when noise and uncertainty is increased, e.g. when portions of input are corrupted. Furthermore, an appropriate smooth objective would also probably be required to guide the inference. For this work, therefore, we needed to both devise a segmentation algorithm and develop an effective objective to guide the search. The objective had to promote using larger and possibly recently created (new) concepts, but balanced against other desiderata, such as the fit with other concepts deemed present in an episode. The reliance of the segmentation objective on probabilities and the desire to get better probabilities motivated the use of multiple levels for concepts.

In addition to composition for building higher level concepts (conjunctions), we have posited that discovery of groupings (disjunctions) in concept structure to be important and useful too, and the two concept creation operations, various forms of disjunctions and conjunctions, together would create a hierarchy of larger and/or more abstract concepts. This work provides empirical evidence that creating a composition hierarchy is a practical possibility. However, learning concepts that also involve grouping or abstractions of some kinds, remains a challenge and a major open problem. We are proposing a kind of structure learning, each concept a separate structure, thus we seek to learn many structures, without explicit supervision, and with much sharing of substructure. The current concept structures are simple. Currently, updates are not performed with respect to a concept’s internal structure: updates are (mostly) limited to simple (scalar) fields and a concept’s associations, i.e. the prediction weights. The system learns immutable concepts. In general structure learning is very challenging (such as learning various subclasses of finite-state machines, grammar induction, etc.) [33, 13]. It is an intriguing question how much learning of additional sophistication in structure can be effectively supported in the approach presented.

The self-supervised learning in prediction games requires efficiently predicting and learning prediction weights for a large and a growing set of concepts in an online manner, and our work on sparse index learning (or "recall systems")

11In this implementation, a concept may update its parts to contain more than one part-pair, which is a very limited structure update.
has focused on designing such algorithms and update techniques \cite{36, 37, 35, 38, 34}. In particular, in this work we have focused on simple EMA updates and simple ways of deriving probabilities from the predictions.

The working of the Expedition system has similarities to n-gram models for language modeling \cite{40, 40}, in that explicit n-grams are stored for predicting. A major difference is that we aim to learn a hierarchical vocabulary, while the vocabulary is fixed and flat in n-gram models, and there is no segmentation task that is integrated with learning in such systems, to the best of our knowledge. The prediction in Expedition is also more flexible than common approaches based on n-grams. Early work on segmenting text include \cite{19}, which used character entropy.

In computer vision, semantic segmentation refers to the task of assigning classes to pixels, but the classes are often given \cite{50, 42}. In unsupervised (image) segmentation, pixels are grouped together to form parts of possible objects or object classes, but no labels are induced. There is much recent work in vision in particular that involve various types of self-supervised learning and proxy or ’pretext’ tasks, utilizing various cues such as visual depth, for feature learning \cite{12, 26, 27}. e.g. using convolution neural networks to build a vocabulary of visual features \cite{27}. In the audio domain, coincidences have recently been used, in part for clustering to find useful representations \cite{24}.

Neural networks are universal function approximators \cite{22}, and with advances of the past decades (diverse architectures, development and advancement of backpropagation), and ample data and computation, have become highly powerful for extracting diverse regularities. Following the success of neural networks in the vision and speech domains \cite{29, 21}, large language models via deep neural networks, using a number of techniques such as embeddings, prediction, and attention, have had substantial recent successes in various diverse NLP and related problems \cite{11, 52, 13, 45, 5}. In much of existing work in text, the networks begin with an existing vocabulary and the embeddings of that vocabulary as input, and it is remarkable that much powerful learning is achieved without the need to nor the complexity of segmenting. The meanings of words and patterns can become highly distributed, providing advantages in making connections among similar patterns, but also potentially losing some structure and interpretability. There is also work on character n-gram models based on neural networks \cite{11}. Sparse mixture of experts (MOEs), that attempt to activate a small portion of the neural network on a per example basis (conditional learning or gating), trained via backpropagation, have had considerable success in further scaling and speeding up of neural networks training and inference \cite{16}.

Concepts are on one hand foundational to human cognition \cite{43, 44, 10}, and are on the other hand "maddeningly complex" \cite{43}. Concepts are inter-related in diverse ways (part-whole, taxonomic, spatio-temporal, domain-specific, ...), or put another way, concepts seem to enjoy rich "content" (or attributes, in terms of other concepts). The nature of concepts and how they are acquired and adapted over time, along with their rich relations and flexible use, remains largely a mystery. There is ongoing debate among researchers on the various aspects of conceptual development, for instance, whether concepts develop in a general to specific manner, or vice versa, the relation between language development and conceptual change, and whether there are fundamental differences between say perceptual or more concrete concepts (such as visual objects, e.g. a dog) vs. seemingly more abstract concepts (e.g. animal) \cite{10, 44}. Researchers have gone as far as theorizing that children acquire knowledge in a manner very similar to the progress in science, forming models (theories) that are adapted or revised with new experience and evidence (the so-called "theory theory" \cite{18}). Prediction has been proposed as a primary driver of much of human intelligence \cite{20}. Our work also took inspiration from the neuroidal model of the neocortex and in particular random access tasks, i.e. tasks that may involve pairs or multiplicities of concepts (stored items), such as associating two arbitrary concepts, from a large space of acquired concepts, after observing them in one or a few episodes \cite{51}.

Considering the importance and utility of concepts for solving advanced information processing tasks, or symbolic computation under uncertainty, and the complexity of conceptual phenomena, a diversity of algorithms or (sub)systems, working together, is likely required \cite{41}. It is a major open question whether existing neural network techniques based on backpropagation, which have now substantially advanced many machine learning applications, can be extended (e.g. perhaps in a posthoc manner) to support concepts or provide a basis for reaching human-level cognition. The extensive research work on the interpretability of the models learned \cite{6} and the related issues of model robustness and brittleness (adversarial attacks) \cite{49, 24}, may also be linked to the major question of whether neural networks can learn explicit discernible concepts with some robust internal structure.

\footnote{We have referred to the representations learned as "concepts", vs. "features" (elevated their status), as they are used both as predictors as well as targets of prediction, and they contain considerable internal structure, such as the parts and part-of links, in addition to several scalar fields.}

\footnote{Similar to that work, we have also assumed concepts are more "programmable", than the nodes in the common neural network models.}
8 Summary and Future Directions

We presented and explored a self-supervised learning system, containing several interacting modules, that acquires a hierarchy of larger and more specific concepts, from consuming text. In this approach, concepts are discrete and connections are sparse, and as we described, much of the learning (the updates) can be done locally involving one or two concepts. In particular, concepts do a lot of the "book keeping" in this approach. We developed and motivated a (segmentation) objective, that promotes building and using larger concepts corresponding to larger patterns in the input. In particular, the objective requires (conditional) prediction probabilities which in turn required some exploration or experimentation to estimate well. What the system learns, the concepts and the prediction weights, are interpretable, and one can also see what the system "sees" in a given episode (i.e. the segmentation of an input into current concepts). We explored the design space and several tradeoffs briefly, such as how to change the learning rate and whether and when to turn off the learning of new concepts (compositions), but much work remains in those directions.

An important next extension we hope to develop is in terms of matching and segmentation (inference) algorithms and objectives to support approximate matches and segmentations, in order to allow for additional noise or corruption in the input and to provide more flexibility in general. An important open challenge and longer term area of research is to support some forms of abstraction, internal to concepts and/or during inference. This connects to the question of how much further we can push structure-learning in this framework (e.g. learning concept classes that are somewhat closer to the class of regular expressions or Markov models). Future directions also include comparing to existing approaches, and exploring hybrid techniques, as well as running the system on more diverse data and larger datasets.

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A Learning Rate Schedules to Fast Approximate Probabilities

In this section we explore a few properties of EMA and the choice of learning rate for the prediction edge weights. Several such properties were also reviewed in [37]. The discussion applies to any relative position with respect to a concept, in particular for the edges for (relative) position 1, which we assume in our concrete examples here without loss of generality. We find that starting the rate $r$ of EMA high and decaying it over time, e.g. based on the number of updates so far, can provide certain advantages in terms of speed of convergence, and we explain this observation by noting that EMA reduces to simple averaging (which is the best one can achieve in a sense), when rate $r$ is decreased in the manner we describe.

An EMA update is a convex combination of the past (summary or average $a$) and present (latest observation $o$):

$$a^{(t)} = EMA(a^{(t-1)}, o^{(t)}, r) = (1-r)a^{(t-1)} + ro^{(t)}, t \in \{1, 2, 3, \ldots\}$$

(9)

where $a^{(t)}$ is the (running) average at time $t$ (initially, $a^{(0)} = 0$, in case of prediction weights), $r$ is the learning rate, $0 < r < 1$, and $o^{(t)}$ is the observation at time $t$. For the case of updating weights for a particular target concept, $o^{(t)}$ is either 0 or 1, (either that concept is seen in the relative position, or not), i.e. a Bernoulli random variable. Past observations’ weights in the average reduce exponentially fast as a function of $t$, by $(1-r)^t$, rendering EMA finite memory in effect, i.e. the average of the last $O(1/(1-r))$ observations. The advantage of EMA compared to saying the average is that only a small constant memory is required, and with its simplicity, it can track some level of change or non-stationarity. Assume after a (predicting) concept $c_1$, a concept $c_2$ occurs with target probability of say 0.4. The stream of observations that $c_1$ ‘sees’, with regard to $c_2$ will be a sequence of 0s and 1s, something like...
A first condition or assumption is that there actually exists a target conditional probability \( P(c_2|c_1) \) (i.e. it is well defined). This condition often holds, e.g. in a non-changing fixed corpus of text, and when episodes are sampled at random. It is not hard to see this is the case at the primitives or the character level of Expedition in particular (layer 0). In other cases, \( P(c_2|c_1) \) may only slowly change, and such can be tracked with an appropriate rate. We note that there is some non-stationarity in the Expedition system due to the use of more exploration initially when segmenting (at levels 1 and up), with a gradual change to more exploitation as concepts are observed more. We leave a careful understanding of that (challenging) non-stationarity to future work. Another condition, alluded to earlier, is that the learning rate needs to be sufficiently small compared to the (conditional) probability of interest. If the learning rate is set too high, for example if the true target probability is \( P(c_2|c_1) = 0.01 \) and the learning rate is also set near or higher than 0.01, there will be too many wide oscillations, i.e. the relative error \( \frac{e_{c_1,c_2} - P(c_2|c_1)}{P(c_2|c_1)} \) can be greater than say 100% too often (Fig. 15). Another practical source of noise or inaccuracy is our size budgets or limits on number of a concept’s connections. For space efficiency as well as prediction and update efficiency, we drop edges with small weights whenever the edge list of a concept becomes too large, i.e. surpasses a size threshold. As long as we are interested in relatively high weights, e.g. near or above 0.01, a size constraint of say 200 will not cause great inaccuracy (with high probability, i.e. with most co-occurrences). EMA, being a moving average has the potential to track some non-stationarity in \( P(c_2|c_1) \), but for that, we need the rate to be reasonably large: if the non-stationarity is too rapid compared to the size of the rate, EMA is not able to track it.\(^{14}\)

A.1 Rate decay vs. a fixed low rate

When predicting for a target position, we sum the weights on prediction edges (of all predicting concepts in the context) and normalize to get a final set of probabilities on the possible candidate concepts (Figure 6), thus we posit that the accuracy of the individual edge weights (in the sense described above) can make a difference in the ultimate prediction performance. For instance, if the rate is set too high, e.g. around 0.01, we have observed inferior coherence (e.g. see Fig. 16). As discussed briefly above, setting the rate \( r \) of EMA in updating the prediction edge-weights involves tradeoffs between convergence speed (speed of learning), accuracy, stability, and adaptability to non-stationarities.

While a (relatively) low rate is important for learning relatively small probabilities (the accuracy consideration), the speed of learning is important too: At any time, the Expedition system may have many concepts with low frequencies, e.g. in the 10s to 100s (seen counts), which implies their conditional probabilities would be poor approximations if the learning rates are low. It is important to learn fast for such large tail of concepts that occur infrequently (but each episode may contain a few such). Furthermore, ideally we want the estimates, the prediction weights, to converge fast for those probabilities that are fairly large (e.g. above 0.1), i.e. co-occurrences that are fairly strong. Table 6 indicates that many concepts have edges that have those (high) probability ranges. If we use a low rate, such as 0.001 or 0.0001 (useful for stability), then it will take a long time to converge for such. Note that we have an exploration or optimistic period, set to 50 in the experiments of this paper, and thus our ‘time budget’ for learning probabilities is in the high 10s to low 100s of time steps. We will see next that starting the rate high and lowering gradually, can lead to fast convergence specially for the high probabilities fast (e.g. around 0.1), while enjoying the accuracy and stability benefits of a low rate.

Below, we compare progressively lowering the rate as a function of the frequency of the updating concept, \( c_1 \), frequency-based decay, versus a fixed (low) rate, and we will see that the decay option speeds up convergence to an error-tolerance region around a target probability, compared to keeping a fixed (low) rate. Learning rate decay has been shown beneficial for training neural nets and there is research work at explaining the reasons\(^{54,38}\). Here, we motivate a decay variant in the context of EMA updates and learning good probabilities fast.

\(^{14}\)Satisfying multiple goals can be impossible with plain EMA: consider two concepts \( c_2 \) and \( c_3 \), \( P(c_2|c_1) = 0.01 \) while \( P(c_3|c_1) \) is substantially larger and moreover oscillates between 0.1 and 0.5 fairly fast (a few 100s of time points). Thus, learning \( P(c_2|c_1) \) well requires a relatively low rate (\( r \ll 0.01 \)), while learning and tracking the non-stationary \( P(c_2|c_1) \) may require a relatively larger rate, otherwise the estimate may converge to near the midpoint.
To see why frequency-based decay can perform better than a fixed low rate, we note that if we lower the rate from 1.0 as a direct function of the frequency of the (predicting) concept \( c_1 \), until the minimum rate \( r_{\text{min}} \) is reached, we simply get the average of the observations until \( r_{\text{min}} \) is reached:

\[
a(t) = EMA(a(t-1), o(t), 1/t) = (1 - \frac{1}{t})a(t-1) + \frac{1}{t}o(t) = (\frac{t-1}{t}) \sum_{i<t} o(i) + \frac{o(t)}{t}
\]

where we replaced \( a(t-1) \) by \( \sum_{i<t} o(i) \) using induction (holds for \( t = 1 \)). Once \( r \) reaches and is fixed at \( r_{\text{min}} \), EMA updating becomes essentially averaging with a finite memory. Simple averaging is the best we can achieve for estimating probabilities without any extra assumptions, and the frequency-based decay achieves it. For relatively high probabilities, e.g. \( p \geq 0.1 \), this can be better than a fixed low rate. Let (relative or multiplicative) error can be defined as:

\[
\text{error} = \frac{|a(t) - p|}{p} \quad \text{(relative error with respect to target probability } p),
\]

and we seek this error to be within a tolerance \( \epsilon \), say \( \epsilon = 0.1 \). Note that in general, we require \( \theta(\frac{1}{p}) \) samples to estimate the probability \( p \) well with high confidence via averaging, (under a multiplicative error tolerance), as the expected number of observations to see the first 1 is \( \frac{1}{p} \), thus lower probabilities require longer time to estimate well. And rate decay achieves this whether \( p \) is large or small (as long \( p \) is larger than \( r_{\text{min}} \), but EMA with a low fixed rate unnecessarily delays convergence for the larger \( p \).

| \( \pm \text{errors} \) | \( + \text{errors} \) | 1st time | \( \pm \text{errors} \) | \( + \text{errors} \) | 1st time |
|----------------|----------------|--------|----------------|----------------|--------|
| decay, \( r_{\text{min}} = 0.001 \), tolerance \( \epsilon = 0.5 \) | decay, \( r_{\text{min}} = 0.001 \), tolerance \( \epsilon = 0.1 \) | decay, \( r_{\text{min}} = 0.001 \), tolerance \( \epsilon = 0.5 \) | decay, \( r_{\text{min}} = 0.001 \), tolerance \( \epsilon = 0.1 \) |
| p=0.25 | 11 ±11 | 4 ±9 | 5 ±7 | 387 ±336 | 174 ±257 | 32 ±81 |
| p=0.10 | 39 ±39 | 17 ±35 | 15 ±19 | 1765 ±1022 | 876 ±874 | 86 ±214 |
| p=0.05 | 73 ±72 | 34 ±65 | 33 ±34 | 3492 ±1186 | 1673 ±1221 | 158 ±359 |
| p=0.01 | 579 ±481 | 341 ±437 | 156 ±187 | 6820 ±932 | 3039 ±1559 | 566 ±862 |
| static \( r = r_{\text{min}} = 0.001 \), \( \epsilon = 0.5 \) | static, \( r = r_{\text{min}} = 0.001 \), \( \epsilon = 0.1 \) | static, \( r = r_{\text{min}} = 0.001 \), \( \epsilon = 0.5 \) | static, \( r = r_{\text{min}} = 0.001 \), \( \epsilon = 0.1 \) |
| p=0.25 | 690 ±64 | 0 ±0 | 687 ±63 | 2468 ±843 | 26 ±130 | 2255 ±358 |
| p=0.10 | 706 ±117 | 0 ±0 | 697 ±116 | 3309 ±911 | 363 ±491 | 2135 ±332 |
| p=0.05 | 701 ±101 | 0 ±0 | 683 ±100 | 4506 ±1164 | 1126 ±957 | 2008 ±616 |
| p=0.01 | 1036 ±507 | 157 ±315 | 700 ±347 | 7083 ±858 | 2293 ±1335 | 1797 ±853 |

Table 11: Counts of when relative errors (Eq. 10) are too large under two tolerance settings are shown, both two-sided (\( \pm \)errors) and one-sided violations (\( + \)errors, or overshots), for frequency-based decay vs. static learning rate settings, for several representative target probabilities, averaged over 200 trials. Each trial contains a stream of \( T = 10000 \) iid samples of Bernoulli (Boolean, 0 or 1) random variables (first row is for \( p = 0.25 \) for drawing a 1, 2nd is with \( p=0.1 \), etc.). Learning rate decay to an appropriate minimum rate \( r_{\text{min}} \) converges much faster than a fixed rate at \( r = r_{\text{min}} \), and its errors are symmetric (both over- and under-estimations) while, as expected, the static rate consistently under estimates until convergence (i.e. roughly, when it reaches within tolerance for the first time).

Table 11 shows the results of empirical experiments on the count of violations of tolerance, \( \pm \)error (averages over 200 trials), i.e. when \( \text{error} > \epsilon \), for a couple of settings of \( \epsilon \) and the two approaches. We note that even for a target probability of 0.01 (which is closer to \( r_{\text{min}} = 0.001 \) than to 1.0 in terms of ratio), the number of violation of the decay approach is lower. We are also reporting the first time point when the estimate falls within the tolerance region (the first

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15Using one form of multiplicative Chernoff bound [8], with \( X \) being sum of \( n \) independent random variables each in \( \{0, 1\} \) (Bernoulli), and with \( 0 < \epsilon < 1 \), the probability of large deviation, \( P(|X - \mu| \geq \epsilon \mu) \leq 2e^{-n \mu^2 / 3} \). With \( \mu = np \), as long as \( n \geq \frac{1}{\epsilon^2} \log \left( \frac{1}{2\epsilon} \right) \), with probability at least \( 1 - \delta \) our estimation of \( p \) (and \( \mu \)) will be within tolerance \( \epsilon \). A similar lower bound on probability of deviation can be derived as well.
time point that there is no violation), as a way of assessing the speed of learning/convergence. Note that this first time 
$t_0$ is the same as number of errors until that point when the tolerance region is reached (before that point, every time 
point leads to a violation by definition). After this time $t_0$, there may still be many violations, specially if the rate is 
high relative to the target.

The table also gives the number of over-estimation errors (when $\frac{\epsilon(t)-\epsilon}{p} > \epsilon$), and, as expected, the error of the fixed 
rate approach are mostly under-estimates, while the errors of the decay approach are balanced. We also observe that the 
count of errors go up substantially when we raise $r_{min}$, and the errors continue at a high rate for a target of say 0.01 
after the first time point. However, the speed of learning does improve, specially for higher target probabilities, for the 
fixed rate approach.

Fig. 15 shows similar results in graph format: the convergence for target probability of 0.1, additionally showing the 
10 and 90 percentile curves. We note that at any point in time, some 20% fraction of probability estimates across all 
concepts’ connections will be at the 10 or 90 percentile, thus the $r_{min}$ should be low enough that most probability 
estimates would not be too far-off from their target probabilities.

![Graphs showing convergence for different rate schedules](image)

(a) Rate $r_{min} = 0.001$  
(b) Static with percentiles.  
(c) Dynamic with percentiles.  
(d) Rate $r_{min} = 0.05$.

Figure 15: Convergences for EMA under static (fixed at $r_{min}$) and frequency-based decay (”dynamic” in the legend) 
where target probability is 0.1. Convergence is faster for the decay approach. Increasing $r_{min}$ to 0.05 speeds up the 
convergence of the fixed-rate approach, plot (d), but at the cost of additional error.

Table[12] and Figure[16] show the segmentation COMA scores at levels 0 and 1 under different rate schedules. We can 
see that static at $r_{min} = 0.1$ converges fast, but has inferior performance, even at level 0. At higher levels, with many 
more concepts, smaller probabilities are needed in general and it would perform even worse. We observe segmentation 
scores converge faster for decay compared to static for $r_{min} = 0.001$. After a few thousand episodes, static eventually 
converges to a similar performance. We note that lowering $r_{min}$ to 0.0001 did not improve performance at level 0, 
segmentation performance remaining at an average of 0.65 (not shown), similar to $r_{min} = 0.001$.

We have argued that setting the EMA learning rate as a function of the frequency of the predicting concept, in particular 
starting high for new predictors, can be beneficial. A possible extension is to let the rate be a function of the newness of 
the target (predicted) concept as well, i.e. the rate could be somewhat higher for relatively new concepts. Depending 
on the schedule of when concepts get created and used, this may be a beneficial extension worth further study.

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[16] A concept that is observed for the first time may signify non-stationarity, a new concept that a system will see more of, or the concept may just be infrequent.
Average segmentation performance at a few episodes

|        | level 0 | level 1 |
|--------|---------|---------|
|        | 2000    | 3000    | 5000    |
| static, 0.1 | 0.50±0.03 | 0.50±0.04 | 0.493±0.07 |
| static, 0.001 | 0.47±0.05 | 0.52±0.01 | 0.64±0.06 |
| decay, 0.001  | 0.66 ±0.04 | 0.66 ±0.02 | 0.66 ±0.04 |

|        | 65000   | 70000   | 125000  |
| static, 0.1 | 1.53±0.003 | 1.57±0.03 | 1.68±0.01 |
| static, 0.001 | 1.41±0.03 | 1.42±0.02 | 1.95±0.02 |
| decay, 0.001  | 1.86±0.002 | 1.99±0.03 | 2.07±0.06 |

Table 12: Moving average of actual (non-optimistic) COMA segmentation scores at levels 0 and 1, average over 5 runs, shown at a few time points, with standard deviations (over the 5 runs). For level 1 experiments, the same model trained at level 0 for 60k episodes was used, and performance (COMA scores) at 65k, 70k, and 125k are reported.

Figure 16: Rate experiments for model training in level 0 and level 1 (average of 5 and 3 trials respectively). \( r_{\text{min}} \) of 0.1 is too high and inadequate, even for level 0. Rate decay based on frequency has a faster and overall better convergence of COMA compared to a fixed rate.

## B Binary Primitives

Any learning algorithm and system rests on certain assumptions regarding the input and has limitations and biases. The Expedition system of this paper works primarily bottom up, and if at the lowest levels the local conditional probabilities are sufficiently close to random, while there may be rich patterns at sufficiently higher levels, the system may get stuck, for instance it may stop discovering new concepts or may build relatively inferior concepts, and it may never discover the higher regularities. In this section, we replace each character with an 8 bit binary code as described below\(^{17}\), thus the primitives will be two, 0 and 1. While the text input stream is as before and rich in regularities, the regularities could be at such a high level that the system may not discover them via its search of patterns bottom-up starting from the primitives. The embedded patterns may go above the system’s head, so to speak.

There are several options for conversion to binary. We chose a simplest approach: every time a new character is seen for first time, we assign it the 8-bit binary code of a (next-available) counter, initialized to 0, and increment the counter. Table\(^{13}\) shows the 8-bit codes for some of the characters (shown in order increasing counter, or equivalently, roughly in order of when the system first saw the characters).

Each episode is a single token (one word) for these experiments, picked randomly from the random line read (by first splitting the line by space) so that we can run experiments fast. Each episode is on average just over 6 characters long, or around 49 bits or binary primitives long. The parameters of the system are mostly as rest of paper: window of 3

\(^{17}\)On this NSF corpus, we have seen 98 unique characters, and 7 bits would suffice too.
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| 'g' = '00000000' | 'o' = '00000001' | 'a'= '00000010' | 'l'= '00000010' | 't'= '00000100' | 'd'= '00000101' |
| 'e'= '00000110' | 's'= '00000111' | 'c'= '00001000' | ... | 'y'= '00100111' | '('= '00101001' |

Table 13: The binary encoding of a few characters, so the word "goal" becomes the string "00000000000000010000001000000010" input to the system.

Concepts both sides for prediction, same composition parameters, learning rate for prediction being \( r_{\text{min}} = 0.0001 \) and the learning rate schedule is freq. based decay. We used a larger width of 5 and 15 (5 try and keep 15) for the segmentation search, as we expected we required more search and inference to find a good segmentation. A main difference is guiding the segmentation: during segmentation we use the segmentation credit (a moving average) for lower-level concepts, to generate candidate segmentations, but rerank based on optimistic coherence on top to select a final segmentation from top layer. The segmentation credit is the historical optimistic coherence for top concepts, and for lower ones, they get the moving average of the segmentation credit of concepts they lead to at top layer. We manually added a layer once the lower layer’s optimistic and non-optimistic coherence converged (to say within 10% of each other, when averaged over the least several hundred episodes).

At the lowest primitives level, initially we get slightly positive (non-optimistic) segmentation score, around 0.2 (with plain characters, COMA would surpass 0.6, see for example Fig. [16]). But from then on, as layers 1 and higher are added in subsequent periods, the average COMA segmentation score becomes negative all the way till around level 15, as shown in [17]. While the system appears to be stuck, making no progress in these levels, new concepts continue to get created. Several of these concepts have positive historical COMA scores (Eq. [5]), even though overall, the system gets a negative score per episode. Eventually, the score of the system gets back to the positive region (around level 15). Fig. [17](b) shows the number of active concepts at the top level, per episode, and we note that the number of concepts goes down to about 5, whereas the average number of character is just over 6 per token input, suggesting that we are indeed getting bigrams of characters as well. Fig. [17](c) shows the number of concepts that are sufficiently seen (above 50 times). We observe that once level 12 or 13 is added, there appears an inflection point, and the number of concepts grows rapidly: the system perhaps discovers sufficiently many concepts corresponding to individual characters, as well as the bigrams. Before then, the increase in the number of concepts used appeared slow, and we do not see a large increase in concepts as we added level, similar to the extent that we saw when we run the system on plain text. Note that the ideal minimal system, would discover the encodings of all characters in level 3 (8 bits, about 100 such for our corpus), and level 4 would be for the bigrams of such (1000s).

As of this writing, we have continued the training for over 5 million episodes and through level 19. Actual COMA scores exceed 6 (with try 5, keep 15) and average number of concepts per episode is around 3 in the top layer. When we give the system the bit string for the individual characters, e.g. one of the 26 lower case letters of alphabet, for about half of the letters, the system segments them into a single concept that corresponds to that letter, and segments the rest into two parts almost always (a beam width of say 5, 5). For instance, "t", "00000100", is segmented as a single concept, while "a"="00000010" is often segmented into two parts, "0" and "00000100". However, when segmenting its normal input (i.e. the binary representation of individual terms), the splits imposed by the system, align with the natural splits among the original characters only about 5% of the time, but additional learning or inference (e.g. beam width) does help reduce the misalignments, the "bad" splits. Still, from our observations, we don’t expect the character-level misalignments to reduce substantially in this run, and there may be multiple somewhat equivalent ways of partitioning the stream that lead to similar COMA performance. We leave further investigation, in particular doing multiple runs and experimenting with different beam widths and other parameters, to future work.
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(a) Segmentation scores, binary primitives. (b) Number of concepts per episode in top level. (c) Number of concepts with frequency above 50 at episode 2.3 mil.

Figure 17: (a) COMA scores (actual), when primitives are either 0 or 1 (when characters are converted to 8-bit strings). The (moving average) scores quickly go negative for several initial levels and periods, and it appears the system is making no progress overall or slow progress. Eventually however (about level 14), as levels are added and larger concepts are discovered at those levels, the score becomes positive, and progress similar to that on plain text appears to start. (b) The number of concepts with freq. above 50, at episode 2.3 million, appears to have an inflection point at around level 12 or so, and shoots up for higher levels.