Reduction of Modal Vibration Effect in Load Identification of a Train Bogie Frame

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Abstract. In order to identify the external applied load on train bogie frame, the load response measured by sensor, e.g. strain gauges, is used in engineering practice. However, it is difficult to model the load-response correlation precisely since the bogie frame system characteristic is complicated, especially, it contains many modal vibration effects. The commonly used linear load-response correlation assumption thus deviates from the truth especially at resonant frequencies. A new approach to reduce the modal vibration effect from the load response based on single degree of freedom system is proposed in this article. The approach is applied on real measurement of Beijing Subway train bogie frame and the reduction performance is verified.

Keywords: Modal Vibration Effect; Load Identification; Train Bogie Frame; Single degree of Freedom System.

1. Introduction
In engineering practice, to measure the load amplitude applied on train bogie frame, strain gauges is often mounted as sensor on train bogie frame. Engineers usually assume the measured strain change reflects the external load linearly [1-6]. However, this is not the case both in principle and in practice. The measurement includes the effect of not only the interested external load but also the bogie frame internal modal vibration effect (MVE).

Sometime the MVE is quite large so that if it is omitted in the analysis, the estimated external applied load deviate significantly from the truth. See the two response amplitude peaks caused by MVE shown in Fig. 1.

On the other hand, the root cause of MVE is complicated and unclear. Different external stimulation at different time stamp continuously appear during the measurement. The result vibration is thus not always only in steady state but also often in transient state. Simply using finite element simulation is not able to solve the problem.
In this article, a new approach is proposed to model the MVE so that it could be significantly reduced in analysis. With this approach, the following pre-processing is necessary:

a) Filter the measured response signal using multiple narrow band-pass filter near the system resonant frequency;
b) Transform the frequency domain load response obtained in a) into time domain;
c) Divide the time domain load response into sequential time frames according to a pre-defined rule. See Fig. 2.

The main process is based on each divided time frame. The following assumptions are made:
1) Assume in each time frame, the MVE is only caused by a simple harmonic stimulation.
2) Inside each time frame, assume the bogie frame mechanical system at each sensor position has single degree of freedom (Single-DOF);
3) Assume the load response in each time frame is independent and not overlapped with load response in other time frames.

The following steps of the approach based on above assumptions are:
i. single-DOF vibration equation could be used to model the measured response in each time frame.
ii. the parameters of the vibration equation, e.g. vibration frequency, could be calculated during model fitting.
iii. In vibration equation, the maximum steady status vibration amplitude which is the function of vibration frequency could be calculated.

iv. Replace the vibration frequency with a critical frequency in maximum steady status vibration amplitude function. The definition of critical frequency is the start or end point of the frequency range where MVE becomes significant. This definition is raised first in this article.

The load response excluding MVE in a single time frame is approximately considered as the outcome of iv.

The entire reduced load response could be obtained by reversing the pre-processing steps. Firstly combine the time frames in time domain in each filtered narrow frequency band; then combine the different narrow frequency band in MVE frequency range plus the unprocessed non-MVE frequency range.

The theoretical derivation as well as the application example of measurements obtained from Beijing Subway train bogie frame is presented in following chapters. The result shows the impressive reduction performance of the MVE.

2. Theoretical Background and Derivation

2.1. Load-strain Correlation.

As discussed above, while placing strain gauge sensors on train bogie frame, the correlation of the strain response and the real external applied load is expressed by a coefficient \( \gamma(\omega, \xi) \) when considering the MVE:

\[
\gamma(\omega, \xi) = \frac{\varepsilon(\omega, \xi)}{F(\omega)} / \gamma \quad (1)
\]

Where \( \varepsilon(\omega, \xi) \) is the strain response, which is also called load response in this article; \( F(\omega) \) is the real external applied load; \( \omega \) is the frequency; \( \xi \) is the modal damping ratio.

\[\text{Figure 3. Load-strain coefficient in frequency domain.}\]

Fig.3 shows the MVE in Load-strain correlation in frequency domain. \( \omega_d \) is the resonant frequency. \( \gamma_{\text{static}} \) is the coefficient representing the linear correlation. It is obvious that the common used linear assumption cannot be applied in the frequency range \( [\omega_d - \Delta \omega, \omega_d + \Delta \omega] \). Furthermore, there might be more than one resonant frequency of the real mechanical system, as shown in Fig. 1. Multi-peaks might appear in real measurement.

When dividing by \( \gamma_{\text{static}} \) on both sides of Eq. 1,

\[
F_{\varepsilon}(\omega, \xi) = \gamma_{\text{normalized}}(\omega, \xi)F(\omega) \quad (2)
\]

Where \( \gamma_{\text{normalized}}(\omega, \xi) = \gamma(\omega, \xi) / \gamma_{\text{static}} \); \( F_{\varepsilon}(\omega, \xi) = \varepsilon(\omega, \xi) / \gamma_{\text{static}} \), which is the estimated external applied load based on linear assumption; \( F(\omega) \) is the real applied load.
\( \gamma_{\text{normalized}}(\omega, \xi) \) thus indicates the distortion of the estimated load by using linear load-strain correlation assumption. It is called load amplification coefficient in this article since it reflects the amplified effect. See Fig. 4.

**Figure 4.** Load amplification coefficient

### 2.2. Forced Vibration of a Single-DOF System with Damping.

As mentioned in introduction chapter, the MVE reduction approach first define the modal vibration frequency range, then narrow-band filter the load response (strain signal). After filtering, the time domain load response is divided into sequential time frames, the load response in each time frame is assumed as simulated caused by a simple harmonic stimulation on a single-DOF system. In this section, the vibration model in each divided time frame is discussed.

Since the bogie frame is a steel structure, its undamped modal frequency and damped modal frequency is assumed to be equivalent. The mathematical model of the displacement of the forced vibration of a Single-DOF system with damping could be thus simplified as Eq. 3. Furthermore, the displacement and strain response of Single-DOF elastic system is assumed to be linear, as \( \varepsilon(t) = cx(t) \).

The strain vibration equation is Eq. 4.

\[
x(t) = e^{-\xi \omega t} \left( \frac{\dot{x}_0 + \frac{\xi \omega x_0}{\omega_d}}{\omega_d} \sin \omega_d t + x_0 \cos \omega_d t \right) + \frac{\omega_d}{k} \int_0^t F_{\text{1sdof}}(\tau)e^{-\xi \omega (t-\tau)} \sin \omega_d (t - \tau) d\tau \quad (3)
\]

Where \( \omega \) is the vibration frequency, which equals the load input frequency; \( x_0 \) is the initial value of displacement; \( \dot{x}_0 \) the initial value of the first derivative of the displacement; \( \omega_d \) is the system (damped and undamped) modal frequency which equals the resonant frequency under current modal; \( \xi \) is modal damping ratio; \( k \) is the system stiffness. \( F_{\text{1sdof}} \) is the external applied load on the system.

\[
\varepsilon(t) = e^{-\xi \omega t} \left( \frac{\dot{\varepsilon}_0 + \frac{\xi \omega \varepsilon_0}{\omega_d}}{\omega_d} \sin \omega_d t + \varepsilon_0 \cos \omega_d t \right) + \frac{\omega_d}{k} \int_0^t F_{\text{1sdof}}(\tau)e^{-\xi \omega (t-\tau)} \sin \omega_d (t - \tau) d\tau \quad (4)
\]

Where \( \varepsilon(t) \) is the load response; \( \varepsilon_0 \) is the initial value of load response; \( \dot{\varepsilon}_0 \) is the initial value of the first derivative of load response.

Take the previous assumption which the external applied load is a simple harmonic stimulation, i.e.

\[
F_{\text{1sdof}}(t) = a \sin \omega t
\]

The vibration equation becomes:
\[ e(t) = e^{-\xi_0 t} \left( \dot{\epsilon}_0 + \frac{\xi_0 \omega_0}{\omega_d} \sin \omega_d t + \epsilon_0 \cos \omega_d t \right) + B e^{-\omega_0 t} \left( \sin \varphi \cos \omega_d t + \frac{\xi_0 \omega_0}{\omega_d} \sin \varphi - \omega_0 \cos \omega_0 \sin \omega_d t \right) + B \sin(\omega t - \varphi) \]  

(6)

Where

\[ B = \frac{ca}{k \sqrt{1 - \nu^2} + (2 \xi_0)^2} \]  

(7)

\[ \varphi = \tan^{-1}(2 \nu / (1 - \nu^2)) \]  

(8)

Figure 5. Typical load response of simple harmonic stimulation in the MVE frequency range \((\omega \in [\omega_0 - \Delta \omega, \omega_0 + \Delta \omega])\).

The typical load response of simple harmonic stimulation with different initial value and frequency is illustrated in Fig. 5.

When applying with real measurement data, the \(w_d\) could be obtained from the peak load response frequency; the parameters \(ca/k\) and \(\omega_0\) in Eq. 6 are calculated while fitting the Eq. 6 with the measurement using global optimization method which minimizes the Euclidean distance.

2.3. Modal Vibration Effect Reduction.

Since the time frame after pre-processing (narrow band filtering and time frame dividing) is the basic analyzing and processing unit in this article, in order to have a perspective of entire load response signal, index is introduced in the following discussion. \(P_{\omega j}(t)\) is defined as equivalent to \(e(t)\) in each divided time frame, where \(\omega\) is the modal index. For example, 1 indicates the first peak while 2 indicates the second peak in Fig. 1; \(j\) is the narrow frequency band index during pre-process filtering; \(ij\) is the divided time frame index in frequency band \(j\).

The same index is defined for the external load in Eq. 4:

\[ F_{\text{load}} = a_{\omega j} \sin \omega_{\omega j} t \]  

(9)

\(P_{\omega j}(t)\) is thus the load response including MVE.

Define \(P_{\omega j}(t)\) as the load response excluding MVE, which is the target variable in this article.

\[ P_{\omega j}(t) = A_{\omega j} \sin(\omega_{\omega j} t + \varphi_{\omega j}), i_j = 1, 2, ..., n_j, j = 1, 2, ..., q_w \]  

(10)

where \(A_{\omega j}\) is the unknown maximum amplitude of the load response excluding MVE, which is the essential target of the whole article; \(\omega_{\omega j}\) is the vibration frequency, which equals the load input
frequency; \( \phi_{nj} \) is the phase; \( q_{w} \) is the number of narrow frequency band used for filtering; \( n_{j} \) is the number of divided time frames in the frequency band \( j \).

The first two elements of the Eq. 6 represent the transient state while the last element represents the steady state of modal vibration. Comparing steady state part of Eq. 6 with Eq. 10 in frequency domain, the maximum steady state load response \( P_{wji} (\omega) \) changes over different load input frequency \( \omega \) inside the MVE range \([\omega_{w_{min}} - \Delta \omega_{w_{min}}, \omega_{w_{max}} + \Delta \omega_{w_{max}}]\). On the other hand, \( P_{wji} (\omega) \) always equals the constant value \( A_{w_{ji}} \). \( \omega_{w_{die}} \) is the modal frequency of \( w \)-th modal, \( \Delta \omega_{w_{die}} \) defines the half width of the \( w \)-th modal vibration frequency range. See Fig. 6.

![Figure 6. Steady state \( P_{wji} (\omega) \) and \( P_{wji} (\omega) \) in MVE range.](image)

Define both \( \omega_{w_{die}} - \Delta \omega_{w_{die}} = \omega_{w_{die}^{-}} \) and \( \omega_{w_{die}} + \Delta \omega_{w_{die}} = \omega_{w_{die}^{+}} \) as the critical frequencies for \( w \)-th modal vibration, \( A_{w_{ji}} \) could be obtained by:

\[
A_{w_{ji}} = P_{w_{ji}} (\omega) \bigg|_{\omega = \omega_{w_{die}^{-}} \text{ or } \omega_{w_{die}^{+}}} \quad (11)
\]

Where \( P_{w_{ji}} (\omega) \) is assume to be the load response of steady state, i.e. \( P_{w_{ji}} (\omega) \) equals the fourier transform of the last element in Eq. 6.

After model vibration effect reduction on load response, the external applied load could be estimated using the classical linear load-response assumption:

\[
\hat{F}(t) = \gamma_{\text{static}} \left( \varepsilon_{\text{static}}(t) + \sum_{j=1}^{q_{w}} \varepsilon_{j}(t) + \sum_{j=1}^{q_{w}} \varepsilon_{2j}(t) + \ldots + \sum_{j=1}^{q_{w}} \varepsilon_{wj}(t) + \ldots + \sum_{j=1}^{q_{w}} \varepsilon_{nj}(t) \right) \quad (12)
\]

Where \( \varepsilon_{\text{static}}(t) \) is the load response in non-MVE range;

\[
\varepsilon_{wj}(t) =
\begin{cases}
P_{wj}(t), 0 \leq t \leq t_{wj1} \\
P_{wj}(t), t_{wj1} < t \leq t_{wj2} \\
\vdots \\
P_{wj}(t), t_{wji-1} < t \leq t_{wji} \\
P_{wj}(t), t_{wji} < t \leq t_{wji+1} \\
\vdots \\
P_{wj}(t), t_{wnj-1} < t \leq t_{wnj} 
\end{cases}
\quad (13)
\]

3. Application
The motor vertical load response measurement data obtained on the bogie frame of a Beijing Subway vehicle is used to verify the proposed approach. The measurement lasts 155s, including load response
of two motors separately, namely load 1 and load 2. The estimated load based on full frequency range linear load-strain assumption is already existed. The application is focusing on reducing the MVE directly from the existing load estimate. Thus the y-axis of all following figures represents the existing load estimated based on full frequency range linear load-strain assumption.

The amplitude-frequency characteristic of the measurement is shown in Fig. 7.

Figure 7. Amplitude-frequency characteristic of motor vertical load response.

Following conclusion has been made:
- Lower than 25Hz, responses-load correlation could be treated as linear;
- There are dominant frequencies at 42Hz, 50Hz, 74Hz;
- The response amplitude at 74Hz is relatively small. Thus, reduction of MVE is not applied at this dominant frequency.

On the other hand, applying finite element simulation to the bogie frame, 4 modal frequency is obtained: 34.96Hz, 43.31Hz, 50.68Hz, 71.27Hz. It indicates that the measured dominant frequency 42Hz and 50Hz is caused by the second and third mode of the bogie frame. The mechanical deformation of those two modes is illustrated in Fig. 8. By using random decrement method [7, 8], the damping ratio of the second and third mode is 0.009 and 0.015.

Figure 8. Amplitude-frequency characteristic of motor vertical load response.

3.1. Data Pre-processing and Modal Vibration Effect Reduction.

The band-pass width used for filtering is in principle should be as small as possible, however, as the width of band-pass filter becomes smaller, more distortion on original signal is introduced. In this application, the bandwidth is set to 1.5Hz. Three band-pass filtering is made near the second modal frequency 42Hz, i.e. [40.5Hz, 42Hz], [42Hz, 43.5Hz], [43.5Hz, 45Hz]. The critical frequency is thus set as 40.5Hz. Six band-pass filtering is made near the third modal frequency 50Hz, i.e. [46.5Hz, 48Hz], [48Hz, 50Hz], [49.5Hz, 51Hz], [51Hz, 52.5Hz], [52Hz, 53Hz], [53.5Hz, 55Hz].
[48Hz, 49.5Hz], [49.5Hz, 51Hz], [51Hz, 52.5Hz], [52.5Hz, 54Hz], [54.5Hz, 55.5Hz]. The critical frequency is set to 55.5Hz.

How to divide the time domain into separate time frames is crucial but difficult because of the spectrum leakage effect after band-pass filtering. A tentative rule to decide start and end for each time frame is proposed as followings:

- Set the threshold as 14% of the maximum amplitude, neglect small values lower than threshold;
- Divide the time frame if the amplitude envelop varies larger than 14%/1.1;
- For the adjacent time frames, the end of time frame is shared as the start of next time frame.
- For the time frames disconnected with previous ones at start point, if current envelop peak is higher than the threshold but the previous envelop peak is lower than the threshold, the start of current time frame is at the inflection point between previous valley and current peak.
- For the time frames disconnected with following ones at end point, the end of current time frame is at the valley where next peak is lower than the threshold.

Fig. 9 shows an example of a single time frame which has the start and end point placed.

Fig. 10 illustrates a wider range in time domain of how time frames are automatically divided. In both Fig. 9 and Fig. 10, The green curve is the original band-pass signal where the MVE is not reduced. By fitting the mathematic vibration model with the green curve, the modelled signal is illustrated by the orange curve. After replacing vibration frequency with critical frequency in maximum steady status vibration amplitude function, the signal with reduced MVE (blue curve) is obtained.

The final resulted estimated load signal in both time domain and frequency domain is shown in Fig. 11 and Fig. 12. The reduced value at two modal frequencies 42.5Hz and 50Hz is around 5 times less...
than original signal. In the MVE range [40.5Hz, 45Hz] and [46.5Hz, 55.5Hz], the result value is comparative with the ones in non-MVE ranges.

![Figure 11. MVE reduction performance in time domain.](image1)

![Figure 12. MVE reduction performance in frequency domain.](image2)

4. Conclusion
The modal vibration reduction approach based on Single-DOF system assumption is proposed in this article and its performance is verified with the real engineering practice.

Because of limited scope, the data processing in the application is not detail elaborated.

Precise load estimation on the mechanical structure is very much essential for its fatigue analysis and reliability evaluation. Further validation could be executed by estimating the fatigue of mechanical structure using the approach introduced in this article and compare with the real fatigue.

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