Quantization of the complex linear superfield

Marc Grisaru 1,*, Antoine Van Proeyen 2,† and Daniela Zanon3

1 Physics Department, Brandeis University, Waltham, MA 02254, USA
2 Instituut voor theoretische fysica, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium
3 Dipartimento di Fisica dell’Università di Milano and INFN, Sezione di Milano, via Celoria 16, I-20133 Milano, Italy.

ABSTRACT

The quantization of the complex linear superfield requires an infinite tower of ghosts. We use the Batalin-Vilkovisky method to obtain a gauge-fixed action. In superspace, the method brings in some novel features

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1 Introduction

Whereas the conventional superspace description of the four-dimensional $N = 1$ scalar multiplet is in terms of a chiral scalar superfield $\Phi$ satisfying $\bar{D}\dot{\alpha}\Phi = 0$ with kinetic lagrangian $\bar{\Phi}\Phi$, an alternative description is by means of a complex linear superfield $\Sigma$ satisfying $\bar{D}^2\Sigma = 0$ with lagrangian $\bar{\Sigma}\Sigma$, the so-called nonminimal scalar multiplet \[1\]. The equivalence between the two descriptions can be exhibited by a duality transformation, or by examining the corresponding component actions; their auxiliary field content is different, but the dynamical scalar and spinor degrees of freedom are represented in the same way.

The complex linear superfield appears in various contexts in the superspace description of supersymmetric systems. It is present as a compensator in the nonminimal, $n \neq -\frac{1}{3}, 0$ formulation of supergravity \[1\]. It has also been used in models describing the supersymmetric extension of the low-energy QCD action, as a substitute for the chiral scalar superfield \[2\]. Finally, it appears as a ghost, if one quantizes the chiral superfield by first solving the chirality constraint in terms of a general superfield, $\Phi = \bar{D}\phi$.

Whereas at the classical level the complex linear superfield presents no problems, its quantization runs into some inherent obstacles because the constraint $\bar{D}^2\Sigma = 0$ is difficult to handle, in contrast to the familiar chirality constraint $\bar{D}\dot{\alpha}\Phi = 0$. It is not clear how to define functional differentiation or integration with respect to $\Sigma$: the chiral superfield functional differentiation formula $\delta\Phi(z)/\delta\Phi(z') = \bar{D}^2\delta(z - z')$ with $z \equiv (x, \theta)$ has no counterpart for $\delta\Sigma(z)/\delta\Sigma(z')$; similarly the procedure defining chiral superfield functional integration (see ref. \[1\] sec. 3.8) has no obvious extension to the linear superfield case. Alternatively, if one solves the linearity constraint by $\Sigma = \bar{D}\sigma^\alpha, \Sigma = \bar{D}\sigma^\alpha, \bar{D}\bar{\sigma}^\dot{\alpha}$, functional integration over $\sigma^\alpha, \bar{\sigma}^\dot{\alpha}$ requires gauge fixing and the quantization leads to an infinite tower of ghosts. (We should emphasize that the situation is quite different for the real linear superfield $G = \bar{G}$ satisfying $\bar{D}^2G = \bar{\bar{D}}^2G = 0$ which can be presented as the field strength of a chiral spinor superfield (tensor multiplet) $G = D\phi^\alpha + \bar{D}\phi^\dot{\alpha}$ whose quantization presents no difficulties, see \[1\], sec.6.2.c.)

Although at the classical level the linear superfield describes the same physical degrees of freedom as the chiral superfield, its quantization is not purely academic. For example, in the quantization of nonminimal supergravity, it cannot be avoided. If one considers, for example, quantum effects in $N = 2$ Yang-Mills coupled to $N = 2$ supergravity (whose $N = 1$ description necessarily involves nonminimal, $n = -1$, supergravity) loop effects due to the linear superfield are present \[3\]. Furthermore, since the chiral scalar and linear scalar superfields are related by a duality transformation at the classical level, its quantization could lead to further understanding of how duality is affected by quantum effects.
This paper is devoted to the quantization of the complex linear superfield, viewed as the field strength of the unconstrained gauge superfields $\sigma^\alpha$, $\bar{\sigma}^{\dot{\alpha}}$. As mentioned above, one encounters the problem of ghosts for ghosts and one unavoidably runs into an infinite tower of ghosts whose systematic handling is best done by using the Batalin-Vilkovisky (BV) formalism [4]. Although the final result is difficult to use in applications, we believe it is worthwhile to have a complete solution to the quantization problem. Furthermore, the system under consideration presents a nice illustration of the power of the Batalin-Vilkovisky method in handling an intricate quantization problem. We believe that this application of the BV formalism to superspace quantization is new and has novel features not encountered in ordinary space-time quantization.

Our paper is organized as follows: in section 2 we give a brief description of the classical linear superfield and its component decomposition, and describe qualitatively its quantization. In section 3 we review the BV formalism. In section 4 we begin the quantization procedure by describing the first level gauge fixing. Section 5 presents the full result, while section 6 describes the techniques we have used. Section 7 discusses our results. The Appendices contain various additional tools and ancillary material.

2 The classical theory and its gauge structure

The kinetic action for a complex linear superfield $\Sigma$, $\bar{\Sigma}$, with $\bar{D}^2\Sigma = D^2\Sigma = 0$ is

$$S = -\int d^4x d^4\theta \, \Sigma\bar{\Sigma}$$ (2.1)

The components of $\Sigma$ are given by

$$B = \Sigma|, \quad \rho_\alpha = D_\alpha \Sigma|, \quad \bar{\zeta}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \Sigma|$$
$$H = D^2 \Sigma|, \quad p_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}} D_\alpha \Sigma|, \quad \bar{\beta}_{\dot{\alpha}} = \frac{1}{2} D^\alpha \bar{D}_{\dot{\alpha}} D_\alpha \Sigma|$$ (2.2)

and their complex conjugates. The component action is [3, 1]

$$S = \int d^4x [\bar{B}\Box B - \bar{\zeta}^{\dot{\alpha}} i \partial_{\alpha\dot{\alpha}} \zeta^\alpha - \bar{H} H + \beta^\alpha \rho_\alpha + \bar{\rho}^{\dot{\alpha}} \bar{\beta}_{\dot{\alpha}} - \bar{p}^{\alpha\dot{\alpha}} p_{\alpha\dot{\alpha}}]$$ (2.3)

with propagating complex scalar and spinor degrees of freedom just like for the standard scalar multiplet, but with a different auxiliary field structure. (The minus sign in front of the superspace action was chosen so that the component scalar field has the correct sign for its kinetic term.)
The equivalence of the descriptions of the scalar multiplet by the linear superfield $\Sigma$ and by the chiral superfield $\Phi$ can be exhibited by means of a duality transformation, starting with the action

$$S_D = - \int d^4x d^4\theta [\bar{\Sigma}\Sigma + \Phi\Sigma + \bar{\Phi}\bar{\Sigma}]$$

(2.4)

(with unconstrained $\Sigma$ and chiral $\Phi$) \footnote{1}. Using the equations of motion to eliminate the superfields $\Sigma$, $\bar{\Sigma}$, leads to the usual chiral superfield action. Eliminating instead the superfields $\Phi$, $\bar{\Phi}$ (whose equations of motion impose the linearity constraint $\bar{D}^2\Sigma = D^2\bar{\Sigma} = 0$) leads to the linear superfield action.

The linearity constraint can be solved in terms of an unconstrained spinor superfield and its complex conjugate by

$$\Sigma = \bar{D}_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}} \quad , \quad \bar{\Sigma} = D_\alpha\sigma^\alpha ,$$

(2.5)

and the action becomes

$$S_{cl} = - \int d^4x d^4\theta \ D_\alpha\sigma^\alpha \cdot D_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}} = - \int d^4x d^4\theta \ \sigma^\alpha D_\alpha D_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}} .$$

(2.6)

The solution of the constraint has introduced some gauge invariance, since clearly the general spinor superfield $\sigma^\alpha$ has more components than $\Sigma$; the operator $D_\alpha D_{\dot{\alpha}}$ is not invertible. We view $\Sigma$ as the field strength of the gauge field $\sigma^\alpha$.

The above action is invariant under the variation $\delta\sigma^\alpha = D_\beta\sigma^{(\alpha\beta)}$ with unconstrained symmetric (as indicated by the brackets) bispinor gauge parameter, since $D_\alpha D_{\dot{\alpha}}$ is antisymmetric in the indices. However, this variation has zero modes, $\delta\sigma^{(\alpha\beta)} = D_\gamma\sigma^{(\alpha\beta\gamma)}$ with the new parameter symmetric in its indices. Similarly, we find zero modes $\delta\sigma^{(\alpha\beta\gamma)} = D_\delta\sigma^{(\alpha\beta\gamma\delta)}$, and so on. Proceeding in this manner one discovers an infinite chain of transformations with zero modes which, upon quantization, leads to an infinite tower of ghosts. This comes about because at every step the ghosts have more components than are necessary to remove gauge degrees of freedom, an apparently unavoidable situation if one wants to maintain manifest Lorentz invariance. It is this feature which makes the quantization of the complex linear superfield difficult.

This superspace situation is somewhat analogous to that of a bosonic theory for a vector field $V^\mu$ with lagrangian $(\partial_\mu V^\mu)^2$ and gauge invariance $\delta V^\mu = \partial_\rho V^{\mu\rho}$ in terms of a second rank antisymmetric tensor gauge parameter. Again one has zero modes $\partial_\mu V^{\mu\rho\sigma}$, etc., because the gauge parameter has too many components. However in this case, at least in 4 dimensions, the tower of ghosts ends with the fourth rank antisymmetric tensor $V^{\mu\nu\rho\sigma}$. 

3
3 Brief account of the BV formalism

We review in this section the Batalin-Vilkovisky quantization \[4\], with emphasis on the case where the gauge transformation of the original gauge field has zero modes, requiring second, and possibly further generations of ghosts. One assigns ghost number \(g\) to the \(g\)'th generation ghost, with the physical field having \(g = 0\). Generically we denote the physical and ghost fields by \(\Phi^A\). For each field one introduces an antifield \(\Phi^*_A\) of opposite statistics, and for functionals \(F(\Phi, \Phi^*)\), \(G(\Phi, \Phi^*)\) one introduces an antibracket

\[
(F, G) = F \frac{\delta}{\delta \Phi^A} \cdot \frac{\delta}{\delta \Phi^*_A} G - F \frac{\delta}{\delta \Phi^*_A} \cdot \frac{\delta}{\delta \Phi^A} G
\]

(summed over \(A\)). One assigns ghost number \(g(\Phi^*_A) = -g(\Phi^A) - 1\).

One defines the minimal extended action \(S_{\text{min}}\) by

\[
S_{\text{min}} = S_{\text{cl}} + \Phi^*_A \delta \Phi^A ,
\]

which contains, apart from the classical action, terms involving the transformations of all the fields, with the gauge parameters replaced by the corresponding ghosts. The minimal extended action has ghost number zero, and satisfies the master equation \((S_{\text{min}}, S_{\text{min}}) = 0\). Moreover it has to satisfy the ‘properness condition’. That is essentially the statement that for each gauge invariance (or zero mode of gauge invariances) one has introduced a ghost, (or a ghost for ghosts).

Gauge fixing is performed by canonical transformations on the set of fields and antifields \[4, 7\]. Such canonical transformations from the set \{\(\Phi^A, \Phi^*_A\)\} to a new basis \{\(\tilde{\Phi}^A, \tilde{\Phi}^*_A\)\} can be determined by a generating fermion \(F(\Phi, \tilde{\Phi}^*)\) of ghost number \(-1\), through

\[
\tilde{\Phi}^A = \frac{\delta F(\Phi, \tilde{\Phi}^*)}{\delta \Phi^*_A} \quad \quad \Phi^*_A = \frac{\delta F(\Phi, \tilde{\Phi}^*)}{\delta \Phi^A} .
\]

Generally, we use generating fermions of the form

\[
F(\Phi, \tilde{\Phi}^*) = \Phi^A \tilde{\Phi}^*_A + \Psi(\Phi) ;
\]

\(\Psi\), also of ghost number \(-1\), is called the gauge fermion.

So far we had not yet introduced fields with negative ghost numbers. This is the first place where we see the necessity of adding nonminimal fields (and their antifields). The usual antighost is the first of these. They should be introduced in the action without changing its physical content, and maintaining the master equation. For example, one introduces an antighost \(b\), with ghost number \(-1\) (with its antifield \(b^*\) of ghost number
0), and adds the term \((b^*)^2\) to the extended action. In this basis \(b\) itself does not yet appear in the action, but it will after the canonical transformation. There are several reasons for introducing non-minimal fields, as we will explain below. One may also be led to introducing Nielsen-Kallosh ghosts \([3]\), as we will explain in section 6.2.

In the simplest case, when there are no zero-modes, one introduces for each gauge invariance one such field \(b^i\) as nonminimal field, and gauge-fixing functions \(F_i(\sigma)\), where \(\sigma\) stands here for the classical fields \((\text{of ghost number } 0)\). The gauge fermion is then

\[
\Psi = b^i F_i(\sigma) .
\]

In summary, we start with the minimal extended action, and add ‘trivial’ (usually quadratic) terms with non-minimal fields (or antifields).

\[
S_{\text{tot}} = S_{\text{min}} + S_{\text{nm}}
\]

This is the extended action in \textit{classical basis}. Then one performs a canonical transformation on the fields and antifields to go to the \textit{gauge-fixed basis}. By definition, in such a basis, the part of the action without antifields, denoted as the \textit{gauge-fixed action}

\[
S_{\text{Q}} = S_{\text{tot}}|_{\Phi^*_A = 0} ,
\]

has invertible kinetic terms. Note that, if we are only interested in the gauge-fixed action and not in its BRST transformations, the canonical transformation amounts to the substitution

\[
\Phi^*_A \rightarrow \bar{\Phi}^*_A + \frac{\delta \Psi}{\delta \Phi^*_A} .
\]

However, after the canonical transformation, we omit the tildes.

This summarizes the BV quantization procedure. For justification and further details one should consult a standard reference. Suitable short reviews can be found in \([7]\). For longer accounts about BV in a Lagrangian setting, see \([9,10]\).

It is clear that gauge-fixing is not a straightforward procedure. Different choices of non-minimal fields and gauge fermions lead to different propagators. It is a priori not guaranteed that a theory can be gauge-fixed without breaking locality and covariance.

We will see that in our specific system one encounters many features of the BV procedure, which will be explained in a case-by-case manner. However, the only principles that have to be kept in mind are those which were explained in this section.

\section{First steps}

4.1 The minimal extended action

Henceforth, for simplicity of notation, we omit the integration symbols and an overall minus sign, \(- \int d^4x d^4\theta\), in the definition of the action. Thus, the classical action (2.6) is written as

\[ S_{cl} = \bar{\sigma} \dot{\alpha} \bar{D}_\alpha \sigma^{\alpha} . \]  

(4.1)

The quantization of this classical action starts by defining the minimal extended action, (3.2). The minimal zero modes are defined by symmetric multispinors \( \sigma^{(\alpha \beta)} \), \( \sigma^{(\alpha \beta \gamma)} \), ...

\[
\begin{align*}
\delta \sigma^{\alpha} &= D_\beta \sigma^{(\beta \alpha)} \\
\delta \sigma^{(\beta \alpha)} &= D_\gamma \sigma^{(\gamma \beta \alpha)} \\
\delta \sigma^{(\gamma \beta \alpha)} &= D_\delta \sigma^{(\delta \gamma \beta \alpha)} \\
\vdots &= \ldots .
\end{align*}
\]

(4.2)

Therefore, we have one part of the minimal extended action for these transformations (using now for the ghosts the names used above for the parameters)

\[ S_L = \sum_{i=1}^{\infty} \sigma_A^* D_\beta \sigma^{(\beta A_i)} = \sigma_A^* D_\beta \sigma^{(\beta A_i)} + \sigma_A^* D_\gamma \sigma^{(\gamma \beta A_i)} + \ldots , \]

(4.3)

where \( A_i \) is an abbreviation that we will often use for the symmetrized set of indices \((\alpha_1 \ldots \alpha_i)\). The ghost numbers of \((\sigma_A^*, \sigma_A^*)\) are \((i-1, -i)\), and all \( \sigma \) are fermionic, all \( \sigma^* \) are bosonic. (See appendix A for some remarks on the statistics).

Obviously we have the corresponding contribution for the complex conjugates with dotted indices, defining \( S_R \):

\[ S_R = \sum_{i=1}^{\infty} \sigma_A D_\beta \sigma^{(\beta A_i)} = \sigma_A D_\beta \sigma^{(\beta A_i)} + \sigma_A^* D_\gamma \sigma^{(\gamma \beta A_i)} + \ldots . \]

(4.4)

The last line is written (after integration by parts) because it is easier to consider the terms in this form. In fact, the fields and antifields can be divided into either left or right Fields\(^2\). The former will appear only as the left factors of a term, while right Fields appear only at the right; see appendix A. The extended action \( S_{min} = S_{cl} + S_L + S_R \) satisfies the master equation.

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1It is always possible to introduce extra zero modes, as we shall do later on.

2We write Fields, with a capital when it means either a field or an antifield.
4.2 First level gauge fixing

The non-minimal part, a priori arbitrary in the BV approach, should be introduced with an eye towards gauge fixing. To complete the kinetic term in (4.1) to an invertible \(i\partial_{\alpha\dot{\alpha}}\), using (A.1), we introduce gauge fixing functions

\[
F_{\dot{\alpha}} = D_{\alpha} \sigma^{\dot{\alpha}}, \quad \tilde{F}_{\dot{\alpha}} = \tilde{D}_{\dot{\alpha}} \sigma^{\alpha} \tag{4.5}
\]

and corresponding (antighost) fields \(b^\alpha_{\dot{\alpha}}\) and their complex conjugates \(\tilde{b}^\alpha_{\dot{\alpha}}\) (both fermions with ghost number \(-1\)). Note that the antighosts \(b^\alpha_{\dot{\alpha}}\) have four field components, while there were only three gauge symmetries described by \(\sigma^{(\alpha\beta)}\). We will have to compensate for that later on. Indicating the antifields of the antighosts (bosons of ghost number 0) respectively by \(b^*_{\dot{\alpha}}\) and \(\tilde{b}^*_{\dot{\alpha}}\), we add to \(S_{\text{min}}\) the non-minimal term

\[
S_{nm,1} = \bar{b}^{\star \dot{\alpha}} b^\alpha_{\dot{\alpha}}. \tag{4.6}
\]

We can then perform a canonical transformation generated by the gauge fermion

\[
\Psi_1 = b^\alpha_{\dot{\alpha}} \bar{D}_{\dot{\alpha}} \sigma^{\alpha} + \bar{\sigma}^{\dot{\alpha}} D_{\alpha} \bar{b}^\alpha_{\dot{\alpha}}. \tag{4.7}
\]

This implies the substitutions

\[
\begin{align*}
b^{\star \alpha}_{\dot{\alpha}} &\rightarrow b^{\star \alpha}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}} \sigma^{\alpha} \\
b^{\alpha}_{\dot{\alpha}} &\rightarrow b^{\alpha}_{\dot{\alpha}} + \bar{\sigma}^{\dot{\alpha}} D_{\alpha} \\
\sigma^{\star}_{\dot{\alpha}} &\rightarrow \sigma^{\star}_{\dot{\alpha}} + b_{\dot{\alpha}}^\alpha \bar{D}_{\dot{\alpha}} \\
\sigma^{\dot{\alpha}} &\rightarrow \sigma^{\dot{\alpha}} + D_{\alpha} \bar{b}^\alpha_{\dot{\alpha}}.
\end{align*} \tag{4.8}
\]

We keep the superspace derivatives to the right of the 'left' antifields. They act then on unwritten delta functions arising from the functional differentiation of the gauge fermion. In this way we never have to interchange superspace derivatives with fields. We get from the first terms in \(S_L\) and \(S_R\) and from the non-minimal action

\[
\begin{align*}
S_{L,1} &\rightarrow S_{L,1} + b^{\alpha}_{\dot{\alpha}} \bar{D}_{\dot{\alpha}} D_{\beta} \sigma^{(\beta \alpha)} \\
S_{R,1} &\rightarrow S_{R,1} + b^{\dot{\alpha}} D_{\alpha} \bar{D}_{\beta} \sigma^{(\beta \dot{\alpha})} \\
S_{nm,1} &\rightarrow S_{nm,1} + \bar{b}^{\star \dot{\alpha}} \bar{D}_{\dot{\alpha}} \sigma^{\alpha} + \bar{\sigma}^{\dot{\alpha}} D_{\alpha} b^\alpha_{\dot{\alpha}} + \sigma^{\dot{\alpha}} D_{\alpha} \bar{D}_{\dot{\alpha}} \sigma^{\alpha}.
\end{align*} \tag{4.9}
\]

The last term combines with \(S_{cl}\) to lead to a good propagator. The full action at this point is

\[
\begin{align*}
S_1 &= S_{cl} + S_L + S_R + S_{nm,1} = S_{Q,1} + S_{0,1} + S_{s,1} + S_L + S_R \\
S_{Q,1} &= \bar{\sigma}^{\dot{\alpha}} i \partial_{\dot{\alpha}} \sigma^{\alpha} \\
S_{0,1} &= b^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} D_{\beta} \sigma^{(\beta \alpha)} + h.c. \\
S_{s,1} &= b^{\star \dot{\alpha}} b^\alpha_{\dot{\alpha}} + \bar{b}^{\star \dot{\alpha}} \bar{D}_{\dot{\alpha}} \sigma^{\alpha} + \sigma^{\dot{\alpha}} D_{\alpha} b^\alpha_{\dot{\alpha}}.
\end{align*} \tag{4.10}
\]
In order to have eventually kinetic terms which do not mix the several generations of ghosts, it is most convenient to eliminate already at this stage the off-diagonal terms in the last line between $\sigma^\alpha$ and $\bar{b}^\dot{\alpha} \dot{\alpha}$. Such a diagonalization can be performed by a redefinition of $\sigma^\alpha$. To make such redefinitions while keeping the canonical conjugacy of the fields and antifields, one has to embed them in a canonical transformation (3.3). In this case we use the generating fermion

$$F(\Phi, \bar{\Phi}^*) = \Phi^A \bar{\Phi}^A - \bar{b}^\dot{\alpha} D_{\dot{\alpha}} \frac{1}{\Box} i \partial^{\alpha\dot{\beta}} \bar{z}^{\star}_{\dot{\beta}} - \bar{\sigma}^*_{\dot{\beta}} \frac{1}{\Box} i \partial^{\beta\dot{\alpha}} D_{\dot{\alpha}} \bar{b}^{\star^\alpha}. \quad (4.11)$$

This leads to

$$S_{*1} = \bar{b}^\dot{\alpha} b^{*^\alpha} + \bar{b}^\dot{\alpha} \bar{b}^{*^\beta} D_{\dot{\beta}} \frac{1}{\Box} i \partial^{\alpha\dot{\beta}} D_{\dot{\beta}} b^{*^\beta} + \text{terms with } \sigma^*_\alpha \text{ or } \bar{\sigma}^*_\dot{\alpha}. \quad (4.12)$$

The $\sigma^*$ terms are not important for what follows.

Now we have to consider the ghost propagators. Again in $S_{01}$ we have to get terms with the other order of the spinor derivatives, but moreover there is the problem that there are 4 antighosts and only 3 ghosts. To obtain an invertible operator, we will thus have to introduce a fourth ghost. We just pretend that there is an extra symmetry in the classical action, whose ghost will be $\sigma^{[\alpha\beta]}$. The fact that it does not appear in the transformation of $\sigma^\alpha$ then just implies that there is an extra zero mode of the transformation. Thus we write, instead of the second line of (4.2), ($C^\beta_{\alpha\dot{\alpha}}$ is $i$ times the Levi-Civita symbol $\epsilon^{\beta\alpha}$)

$$\delta \sigma^{\beta\alpha} = D_{\gamma} \sigma^{(\gamma\beta\alpha)} + C^{\beta\alpha} \lambda, \quad (4.13)$$

with unsymmetrized $\sigma^{\beta\alpha}$. Obviously $\lambda$ can be used to gauge away algebraically $\sigma^{[\alpha\beta]}$ and recover (4.2). Correspondingly the field $\lambda$ is a new ghost for ghosts. Therefore the 'minimal extended action' has to be modified. We will do this below.

5 The full result

Rather than continuing step by step, we give in this section the final form of the action. Its structure, as well as that of the gauge fermion, are encoded in table 1. The arguments which went into the determination of the set of fields, and the explanation of the steps leading to the gauge-fixed action, will be given in the following section.

First, the set of fields, other than the Nielsen-Kallosh ghosts are given schematically in table 1 for the first four levels. We indicate for all fields the (ghost number, ghost number of antifield), and at the left whether these fields are bosonic or fermionic. For all these fields there are corresponding complex conjugates. The arrows are related to
Table 1: Fields up to fourth level.

| Field | Ghosts | Ghost Number |
|-------|--------|--------------|
| $F_{A_2}$ | $b^\alpha_{(1,2)}$ | $(1,2)$ |
| $F_{A_3}$ | $b^\alpha_{(2,3)}$ | $(2,3)$ |
| $F_{A_4}$ | $b^\alpha_{(3,4)}$ | $(3,4)$ |
| $B_{A_2}$ | $\mu_{(1,2)}$ | $(1,2)$ |
| $B_{A_3}$ | $\mu_{(3,4)}$ | $(3,4)$ |
| $B_{A_4}$ | $\mu_{(4,5)}$ | $(4,5)$ |

The construction of the action, and will be explained later. We note that entries in the same row have the same number of field components. For example, for level 1 at the right appears the ghost $\sigma^{\alpha_1\alpha_2}$, not symmetrized, as discussed above, in contradistinction to the original $\sigma^{A_2} \equiv \sigma^{(\alpha_1\alpha_2)}$. Hence, this ghost contains 4 components as does $b^\alpha_{(1,2)}$. Similarly, at the next level the index structure $A_2\alpha_3$ indicates 6 components, as $A_2$ implies a symmetrization in $(\alpha_1\alpha_2)$, but there is no symmetrization with $\alpha_3$.

Also, one can see in the table that for each field there is a partner with which it will appear multiplied in the gauge-fixed action. Indeed, a field of ghost number different from zero has in the table an adjacent field of opposite ghost number (not connected by arrows). See as a first example the first generation ghost and antighost. Those of ghost number zero will appear multiplied by their own complex conjugate. Here the classical field is the first example.

All these features are well known, e.g. from the pyramids one obtains for the quantization of the antisymmetric tensor \[1\] and also the infinite ones for the Green-Schwarz superstring \([12, 13]\) or superparticle \([14]\). The difference is however that in those cases, any entry on the same row has the same Lorentz indices. Here the Lorentz representations vary from left to right, but the total number of components still matches.
As mentioned at the end of section 4.2 we need extra ghosts and ‘fake’ gauge invariances expressing the fact that these ghosts do not appear in the original minimal extended action. We then have

\[ S_{\text{min}} = S_c + S_R + S_{f,L} + S_{f,R} \]

\[ S_{f,L} = \sigma_{\alpha_1 \alpha_2}^{\ast} C^{\alpha_1 \alpha_2} \lambda + \sigma_{A_2 \alpha_3}^{\ast} C^{\alpha_2 \alpha_3} \lambda^{\alpha_1} + \sigma_{A_3 \alpha_4}^{\ast} C^{\alpha_3 \alpha_4} \lambda^{A_2} + \varsigma^{\ast} C^{\alpha_1 \alpha_2} \lambda^{A_2} + \ldots, \]  

(5.1)

where \( S_{f,R} \) is the complex conjugate of \( S_{f,L} \). An alternative way to interpret these additions is to say that these are non-minimal terms, introducing in a trivial way (as auxiliary fields) the fields \( \sigma_{\alpha_1 \alpha_2} \), \( \lambda \), \( \sigma_{A_2 \alpha_3} \), \( \ldots \) , and their antifields.

To obtain the gauge-fixed action the following choices are made. The non-minimal terms for the Fields which are not at the extreme right of each row in the table, are introduced as terms quadratic in the antifields. The antifields of ghost number different from zero are multiplied by the ones in the same row to which they are connected by the double arrow \( \leftrightarrow \). Those of ghost number zero are introduced multiplied by their complex conjugates. We thus have, to be added to (4.6)

\[ S_{\text{nm,2}} = d_\alpha^{\ast} b_{\alpha}^{\ast} A_2 + \nu^{\ast} \mu + h.c. \]

\[ S_{\text{nm,3}} = e_{\alpha_1}^{A_2} e_{\alpha_2}^{A_3} + \left( d_\alpha^{\ast} b_{A_3 \alpha_4} + \nu^{\ast} \mu^{\ast} + b^{\ast} + h.c. \right) \]

\[ -\bar{\rho}_\alpha^{\ast} \partial^{\ast} \rho^{\ast} - \bar{\rho}_\alpha^{\ast} \partial \rho^{\ast} \]  

(5.2)

The last line needs some extra comments. The previous antifields of ghost number 0 could be introduced in \( S_{\text{nm}} \) as pure ‘auxiliary fields’. This was due to the fact that they had an equal number of dotted and undotted indices, such that these indices match with those of the complex conjugates. For the fields where this is not the case, of which \( \rho \) is the first example, we have to insert a derivative. Therefore to cancel the extra propagating mode, we have to introduce a ‘Nielsen-Kallosh ghost’ \[ \bar{\rho}_\alpha^{\ast} \], here \( \rho^{\ast}_\alpha \). This is, formally, a field of statistics opposite to that of the antifield. Thus our Nielsen-Kallosh ghosts will be formally bosons, to compensate for the fermionic non-minimal antifields. We will return in section 5.2 to a discussion of the Nielsen-Kallosh ghosts and their statistics.

The gauge fermion contains terms corresponding to the diagonal arrows in the table (observe that the ghost numbers of the connected fields add up to \(-1\)). Continuing after (4.7) for the first level, we add

\[ \Psi_2 = b_{\alpha}^{\ast} D_\beta d_\alpha^{(\beta \alpha)} + b_{\alpha}^{\ast} i \partial_{\alpha}^{\ast} \nu + b_{\alpha}^{\ast} \bar{D}_{\alpha}^{\ast} \sigma^{A_2} + \frac{1}{2} \mu C_{\alpha \beta} \sigma^{\beta \alpha} + h.c. \]  

(5.3)

Signs and factors are chosen to obtain simple kinetic terms. Continuing, we have

\[ \Psi_3 = b_{\alpha}^{\ast} \bar{D}_{\alpha}^{A_3} \sigma^{A_3} + \frac{1}{2} \mu_{\alpha_1} C_{\alpha_2 \alpha_3} \sigma^{A_2 \alpha_3} + b_{\alpha}^{\ast} \left( D_{\alpha_1} d_\alpha^{A_3} + i \bar{\partial}_{\alpha}^{\alpha_1} \nu^{A_2} + i \partial_{\alpha}^{\alpha_1} D^{A_2} \right) \]
\[ + c \dot{A}_2 \dot{D}_{\dot{a}_1} \dot{d}^{A_2}_{\dot{a}_2} - 2 \rho \dot{\alpha} \dot{D}_{\alpha} \nu + h.c. \]  

(5.4)

After the corresponding canonical transformations and various diagonalizations, we find then the following kinetic terms:

\[
S_{Q,2} = b_i^\alpha \dot{\alpha} i \partial_{\dot{\alpha}} \sigma^{\beta \dot{\alpha}} + h.c. \\
S_{Q,3} = \bar{d}_i^{(\dot{a} \dot{b})} \dot{\beta} \partial_{\dot{\beta}} \dot{d}_{\dot{a}}^{(\alpha \beta)} - 4 \bar{\nu} \vec{\nu} \nu + \left[ - \mu \dot{\lambda} + b_i^{(\alpha \dot{a})} \dot{\alpha} i \partial_{\gamma \dot{a}} \sigma^{(\alpha \beta) \gamma} + h.c. \right] - \bar{\rho}_i \dot{\rho}^{\alpha \dot{\alpha}} \rho_\alpha. 
\]

(5.5)

The last term describes the Nielsen-Kallosh ghost. These are the first terms of the final gauge-fixed action. The invertibility of all the kinetic operators will be shown in section 5.3.

One uses the following terminology. The fields along the upper right diagonal are called ‘minimal fields’. They are not completely minimal, as in fact one could do with only the symmetric spinors \( \sigma^{(A_1)} \). The others, introduced in (5.1) to obtain suitable kinetic terms, are called ‘catalysts’ [1] (sec. 7.3.c). The ‘classical fields’ are just the \( \sigma^\alpha \) at the top. The minimal ones in the second row are ‘ghosts’. All the other minimal fields are called ‘ghosts for ghosts’. The next diagonal down and to the right are the antighosts, followed by the ‘hidden ghosts’ [15]. However, in the BV formalism all these Fields are treated in the same way, and we will in what follows just denote all of them as ‘ghosts’.

6 The techniques

6.1 The introduction of non-minimal fields

We will now give the arguments that enter in the determination of the fields which occur in table 1. It will be useful to present them also in a schematic way in table 2. In that table all the fields are split into their irreducible Lorentz representations. For instance, the tensor \( \sigma^{\alpha_1 \alpha_2} \) at the first ghost level in table 1 contains a symmetric and an antisymmetric part. The antisymmetric part is equivalent to a scalar. Therefore we represent this field as \( (20 00) \), where the first column symbolizes the symmetric part, and the second column the antisymmetric part. In such a column, the upper/lower number is the number of symmetrized undotted/dotted indices in that representation.

The classical fields and the minimal ghosts introduced in section 4.1 are the columns \( (n 0) \) in the entry at the right of the \( n \)-th row. To introduce the other fields, the following types of arguments are used.
1. Non-minimal Fields can be introduced to obtain appropriate gauge fixing for the fields one level higher. A non-minimal Field enters in the construction of the quadratic term involving the field to which it is connected by a diagonal arrow in table [1].

2. All entries on the same horizontal line should have the same number of components. For those not connected by arrows this is because they get multiplied to each other in the kinetic terms in the gauge-fixed basis, and these kinetic operators should be invertible.

3. The ones which are connected with $\leftrightarrow^*$ should have the same index structure in order that we can build a non-minimal extended action in the classical basis by multiplying their antifields.

4. If ghosts are introduced in the right diagonal of minimal Fields, then they should be compensated by ghosts for these ghosts in the next ghost generation.

Let us now see how this goes in practice. Typically after a certain level of gauge fixing, ghosts at the next level occur with a kinetic term involving $D_\alpha D_\beta$, see e.g. after the first level, $b_\alpha^\dagger D_\alpha D_\beta \sigma^{(\beta\alpha)}$ in $S_{0,1}$ of (4.10). We want to add a term with $D_\beta D_\alpha$ to complete the previous one to a space-time derivative. This we can obtain from a square of non-minimal antifields with index structures with one more undotted index for the left one and one more dotted index for the right one. In our example this is the reason for introducing the

---

Table 2: Fields up to sixth level.

|      | 1    | 20   |
|------|------|------|
| 2    | 0    | 20   |
| 1;0  | 0    | 31;0 |
| 2;0  | 30;1 | 420  |
| 3;1  | 10;0 | 531  |
| 4;2  | 40;2 | 642  |
| 3;2  | 100;0| 753  |

1. Non-minimal Fields can be introduced to obtain appropriate gauge fixing for the fields one level higher. A non-minimal Field enters in the construction of the quadratic term involving the field to which it is connected by a diagonal arrow in table [1].

2. All entries on the same horizontal line should have the same number of components. For those not connected by arrows this is because they get multiplied to each other in the kinetic terms in the gauge-fixed basis, and these kinetic operators should be invertible.

3. The ones which are connected with $\leftrightarrow^*$ should have the same index structure in order that we can build a non-minimal extended action in the classical basis by multiplying their antifields.

4. If ghosts are introduced in the right diagonal of minimal Fields, then they should be compensated by ghosts for these ghosts in the next ghost generation.
first term $d^\gamma_{(\alpha\beta)} b^*(\alpha\beta)$ in $S_{nm,2}$ in (5.2). Subsequently one uses the shift of these antifields as dictated by the first and third terms in (5.3) to replace $d^\gamma_{(\alpha\beta)}$ by $b^\alpha D_\beta$ and $b^*(\alpha\beta)$ by $\bar D_\alpha \sigma^{(\alpha\beta)}$. This is thus an argument of type 1.

In the example we would thus obtain the kinetic term $b^\hat{\alpha} \partial_\beta \sigma_{(\beta\hat{\alpha})}$, which is not invertible because the number of field components do not match. Therefore we still need a term $b^\hat{\alpha} \partial_\beta \sigma^{[\beta\hat{\alpha}}$. This we can obtain from a term $\nu^* \mu^*$ in the non-minimal action $S_{nm,2}$, with a gauge fermion leading to the appropriate shifts of these antifields as obtained from the second and fourth terms in (5.3). This argument accounts for all the terms in $\Psi_2$. We have thus seen how the Lorentz structure of $b^\hat{\alpha}$ forces us to relax the symmetrization of $\sigma^{A_2}$ to $\sigma^{A_1A_2}$. These are arguments of type 2 for non-minimal Fields.

The introduction of $\sigma^{[\alpha_1\alpha_2]}$ is not done by adding squares of antifields to the non-minimal action. As explained at the end of section 4.2 its presence can be seen as a fake gauge invariance, and leads to the introduction of ghosts for it, in this case $\lambda$. Similarly, the structure of $b^\hat{\alpha}_{A_2}$ forces us to relax $\sigma^{A_1}$ to $\sigma^{A_1A_2}$. These are thus arguments of type 4 for introducing new Fields.

Taking the argument of type 3 also into account, one can now reconstruct the tables. Let us start by following the left downward diagonal in table 2, and first for the fermionic fields (those before the semicolon). The argument of type 1 leads in an alternating manner to the addition of a dotted or of an undotted index, which explains that representation. Moving in a row to the right, we either have the argument of type 3 that the index structure should be the same, or of type 2, pairing first a representation \( \binom{n}{n} \) to \( \binom{n+1}{n} \), and in general, pairings of ghosts

\[ \binom{m}{n} \sim \binom{m+1}{n} \frac{m}{(n-1)0} \]  \hspace{1cm} (6.1)

It is this kind of general kinetic term which we will encounter and treat in section 6.3. Following the horizontal row to the right, we end up with ghost structure as e.g. in the fourth line \( \binom{420}{000} \). This has in comparison to the minimal ghost \( \binom{4}{0} \), the extra ghosts of structure \( \binom{20}{00} \). For these, the argument of type 4 leads to the introduction of the bosonic ghosts, after the semicolon on the next ghost level. In this particular case it explains the \( \binom{20}{00} \) after the semicolon of the extreme right entry in the fifth line. And in general, therefore, the index structure of these bosonic ghosts in the extreme right entry are thus the same as the fermionic ones in the previous generation apart from the leading one.
For the bosonic ghosts in the other entries we can then apply the same reasoning as above for the fermionic ones, which completes the arguments leading to table 2.

6.2 Nielsen-Kallosh ghosts and their statistics.

As mentioned in section 5, in some cases we have to introduce the non-minimal antifields in the extended action with a derivative, and therefore have to cancel a contribution to possible loops, by adding Nielsen-Kallosh ghosts \[8\], e.g.

\[\bar{\rho}^{\prime}_{\alpha} i \partial^{\alpha} \rho^{\prime}_{\dot{\alpha}}.\]  

This occurs when these antifields have ghost number zero and an unbalanced number of dotted and undotted spinor indices. One can easily check from the table 2 that this happens only for the bosonic ghosts (with fermionic antifields) at every other level starting with \(\rho^*_{\dot{\alpha}}\).

There is a subtlety here. Interpreting \(\rho^\prime_{\dot{\alpha}}\) as a bosonic field, its contribution to the action, as written in (6.2), is purely imaginary (and adding the hermitian conjugate leads thus to a zero result, or more exactly, to a total derivative). The same problem occurs when one introduces Pauli-Villars \[16, 10\] regulator fields. These should also have statistics opposite to those of the fields whose divergent loop contributions are regulated. However, using the kinetic term \(\phi \Box \phi\) for fermionic fields gives again a total derivative. Here the correct way is to replace the boson \(\rho^\prime_{\dot{\alpha}}\) by two bosons \(\rho^\prime_{\dot{\alpha}}\) and \(\rho^\prime_{\dot{\alpha}}\) and a fermion \(\rho^{\prime \prime \prime}_{\dot{\alpha}}\) (realistic solution). The action for these fields is then

\[\bar{\rho}^{\prime}_{\alpha} i \partial^{\alpha} \rho^{\prime}_{\dot{\alpha}} + \bar{\rho}^{\prime}_{\dot{\alpha}} i \partial^{\dot{\alpha}} \rho^{\prime \prime}_{\alpha} + \bar{\rho}^{\prime \prime \prime}_{\dot{\alpha}} i \partial^{\dot{\alpha}} \rho^{\prime \prime \prime}_{\alpha},\]  

which can be used without any problems in a path integral. The effect is the same as what one obtains from (6.2) (with fermionic \(\rho^\prime\)), adding a minus sign by hand in all loops (a formalistic solution).

6.3 General form of propagators

In general we obtain kinetic terms which are formally of the type (6.1). Explicitly, they take the following form:

\[S_k = \Phi_{B_1} \left( i \partial_{\beta_{m+1} \dot{\alpha}_{n}} A_{B_{m+1} \dot{\alpha}_{n}} + i \partial^{\beta_{1} \dot{\alpha}_{1}} \cdots i \partial^{\beta_{n} \dot{\alpha}_{n}} \Gamma^{\beta_{m+1} \cdots \beta_{m}} \right),\]  

\[3\text{The terminology is borrowed from the paper by Pauli and Villars, who used it in a somewhat different sense however.}\]
with \( m \geq n - 1 \). If \( m = n - 1 \), there is no \( \Gamma \) term. A simple case is obtained for \( m = 1, n = 1 \), when one may write \( \Gamma \) as the antisymmetric part of \( \Lambda^{\alpha \beta} \):

\[
\Gamma = \frac{1}{2} C_{\alpha \beta} \Lambda^{\beta \alpha},
\]

(6.5)
such that the two terms combine to

\[
\Phi^{\dagger} \dot{\alpha} \beta \Lambda_{\alpha \beta}^{\beta_1 \beta_2}.
\]

(6.6)

This is how these terms appear in \( S_{Q,2} \), and similarly in \( S_{Q,3} \) in (5.3). At higher levels such a simple rewriting is not always possible (e.g. with \( e^{\dagger} A_2 A_2 \) as \( \Phi \), and \( d^{\dagger} A_3, d \) as \( \Lambda, \Gamma \) with \( m = n = 2 \)).

\( S_k \) is an invertible kinetic term, as we will show by obtaining explicitly the propagator, as the inverse of the kinetic operator. It can be defined as the tensors \( P_1 \) and \( P_2 \), where

\[
\left[ i \partial_{\beta_{m+1}(\dot{\alpha}_n) ; C_{m+1} : D_{n+1}}^{\dagger} + i \partial_{\beta_1}^{\dagger} \ldots i \partial_{\alpha_n}^{\dagger} P_2 \beta_{n+1}^{\beta_{m+1}} ; C_{m} : D_{n} \right] E^C_{D} = E^B_{A},
\]

(6.7)

for an arbitrary (symmetric) tensor \( E \). The tensor \( P_1 \) describes then the propagator between the \( \Phi \) and \( \Lambda \) field, while \( P_2 \) describes the propagator between \( \Phi \) and \( \Gamma \). The solution for these tensors is given by

\[
-P_1 \beta_{m+1}^{\beta_{m+1}} ; C_{m} E^C_{D} = n \frac{1}{\Box} i \partial^{\dagger} \gamma_{m+1} E^B_{A}
\]

\[
+ \left( \frac{n}{2} \right) \frac{1}{\Box^2} i \partial^{\dagger} \gamma_{m+1} \beta_{2} \beta_{2} \beta_{m+1} \partial_{\beta_1} E^{\beta_{1} \beta_{2} \ldots \beta_{m+1}}_{\alpha_{2 \alpha_{n-1}}} + \ldots
\]

\[
+ \left( \frac{n}{n} \right) \frac{1}{\Box^n} i \partial^{\dagger} \gamma_{m+1} \beta_{2} \beta_{2} \partial_{\beta_1} \ldots \partial_{\beta_{n-1}} \partial_{\alpha_{1} \alpha_{1}} \ldots \partial_{\alpha_{n-1}} \alpha_{n-1} E^{\beta_{1} \ldots \beta_{n+1}}_{\gamma_{1} \ldots \gamma_{n}}
\]

\[
-P_2 \beta_{n+1}^{\beta_{n+1}} ; C_{m} E^C_{D} = - \frac{m + 1 - n}{m + 1} \frac{1}{\Box^n} i \partial^{\dagger} \beta_1 \ldots i \partial^{\dagger} \beta_n E^B_{D}
\]

(6.8)

This shows the invertibility of the kinetic terms.

### 6.4 Diagonalizations

We have already given one example of diagonalization of terms in the extended action in section 4.2. It is important to observe that one is not allowed to drop the terms with antifields at intermediate steps. Indeed, since we perform the gauge fixing in several (infinite number of) steps, the terms depending on antifields whose fields will appear in later gauge fermions are still relevant. The final kinetic terms which we get arise often from...
various sources. The $\nu \Box \nu$ term, for instance, in $S_{Q,3}$ in (5.3), gets first contributions from inserting the substitution generated by the gauge fermion $\Psi_2$ into the two terms of (4.12), one of which already arose from a previous diagonalization. The canonical transformation generated by $\Psi_3$ gives rise to other terms. Some of these generate kinetic terms mixing $\nu$ with $d A_2 \dot{\alpha}$. Subsequently we diagonalize the kinetic terms of $d A_2 \dot{\alpha}$ in a manner similar to that for $\sigma^\alpha$ in (4.11). All these contributions at the end lead to the $\nu \Box \nu$ result. This is also the consequence of some choices of signs and factors in the gauge fermions.

To perform these diagonalizations, one uses the propagator obtained in (6.8). Indeed, one can use the following general procedure: the extended actions which we have to diagonalize are of the general form

$$S = \Phi_{B_m} (i \partial_{\beta_{m+1} \dot{\alpha}_n} A_{A_{n-1}}^{B_{m+1}} + i \partial_{\dot{\alpha}_1} \beta_1 \cdots i \partial_{\dot{\alpha}_n} \Gamma_{\dot{\alpha}_n}^{\delta_{n+1} \cdots \delta_m} )$$

$$+ U_{B_{m+1}} A_{A_{n-1}}^{B_{m+1}} + V_{A_{m-n}}^A A_m + \Phi_{A_m} W^A A_n,$$  \hspace{1cm} (6.9)

where $U$, $V$ and $W$ are (symmetric) tensors depending on other Fields. We then perform in principle a canonical transformation with generating fermion (if there are antifields in $U$, $V$ and $W$, one should replace them here with their new (tilde) antifields)

$$F(\Phi, \bar{\Phi}^*) = \Phi^A \bar{\Phi}^* + U_{B_{m+1}} A_{A_{n-1}}^{B_{m+1}} \bar{\Phi}^*_D C_m + V_{A_{m-n}}^A A_m + \Phi_{A_m} W^A A_n,$$  \hspace{1cm} (6.10)

That leads then to (all Fields in the new basis)

$$S = \Phi_{B_m} (i \partial_{\beta_{m+1} \dot{\alpha}_n} A_{A_{n-1}}^{B_{m+1}} + i \partial_{\dot{\alpha}_1} \beta_1 \cdots i \partial_{\dot{\alpha}_n} \Gamma_{\dot{\alpha}_n}^{\delta_{n+1} \cdots \delta_m} )$$

$$- U_{B_{m+1}} A_{A_{n-1}}^{B_{m+1}} \bar{\Phi}^*_D C_m - V_{A_{m-n}}^A A_m + \Phi_{A_m} W^A A_n,$$  \hspace{1cm} (6.11)

This one lemma is sufficient to perform all the diagonalizations.

7 Conclusions

In this work we have described the superspace BV quantization of the complex linear superfield $\Sigma$ which provides an alternative description of the component $N = 1$ scalar multiplet. As mentioned in the introduction, and due partly to the presence of superspace derivatives with a single spinor index, the treatment of our system brings in some features not encountered in previous applications of the BV method.

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As indicated in section 2, at the classical level the complex linear superfield (nonminimal scalar multiplet) with superspace lagrangian $\bar{\Sigma}\Sigma$ differs from a chiral superfield with lagrangian $\bar{\Phi}\Phi$ only in their auxiliary field structure - yet the quantization of the former gives rise to difficulties which are not present for the latter, and makes any practical applications rather difficult to envisage. Thus, our quantization reveals more about the BV approach than about the physics described by the nonminimal multiplet. In particular, issues such as whether different sets of auxiliary fields lead to different quantum physics (as they appear to do in supergravity [1]) are not easy to investigate here.

The main problem with the linear superfield is that the gauge transformation $\delta \sigma^\alpha = D_\beta \sigma^{(\beta\alpha)}$ “overshoots” in the gauge manifold. We have considered an alternative way of presenting the gauge transformations in terms of chiral spinor and chiral symmetric bispinor superfields $\Sigma^\alpha$, $\Sigma^{(\alpha\beta)}$ as

$$\delta \sigma^\alpha = D^2 \Sigma^\alpha + D_\beta \Sigma^{(\alpha\beta)} = D_\beta \left[ -\frac{2}{3} D^{(\alpha} \Sigma^{\beta)} + \Sigma^{(\alpha\beta)} \right].$$ (7.1)

However, the chirality of the gauge parameters implies that at the component level some gauge components of $\sigma^\alpha$, absent in the action, transform with space-time derivatives. This introduces extra propagating modes in the ghost action, which have to be cancelled in a manner which may not be manifestly supersymmetric.

We have constructed the gauge-fixed action of the complex linear superfield, which contains an infinite tower of ghosts. The structure of all the fields that enter can be inferred from the part exhibited in tables 1 and 2. The full gauge-fixed action is obtained as follows:

- The fields of non-zero ghost number have partners adjacent to them in the table with opposite ghost number. These fields are multiplied together in the action. The generic pattern is (6.1), which explicitly takes the form (6.4). We obtained their propagator in (6.8). However, some of them enter as non-propagating auxiliary fields according to their dimension and index structure.

- Fields of ghost number zero appear in the action multiplied with their complex conjugates.

- Finally there are Nielsen-Kallosh ghosts not indicated in the table as discussed after (6.2).
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A Conventions

The superspace conventions we use are

\[
D^\beta = C^{\beta \gamma} D_\gamma , \\
C_{\alpha \beta} C^{\gamma \delta} = \delta_\alpha^\gamma \delta_\beta^\delta - \delta_\alpha^\delta \delta_\beta^\gamma ; \quad C^\beta_{\alpha} = \delta_\alpha^\beta \\
2A_{[\alpha} B_{\beta]} = C_{\beta \alpha} A^\gamma B_\gamma \\
[\alpha \beta] = \frac{1}{2} (\alpha \beta - \beta \alpha) \\
i_\alpha \delta_\alpha = D_\alpha \bar{D}_\alpha + \bar{D}_\alpha D_\alpha \\
D^2 = \frac{1}{2} D^\alpha D_\alpha \\
\Box = \frac{1}{2} \partial^\alpha \partial_\alpha = D^2 \bar{D}^2 + \bar{D}^2 D^2 - D^\alpha \bar{D}^\beta D_\beta \\
D_\alpha D_\beta = C_{\beta \alpha} D^2
\]

(A.1)

Left and right fields

In the tables, we consider only half of the fields. There are also the complex conjugates which form a similar table. We have indicated by a diagonal arrow the fields which are connected in the gauge fermion. One could also assign a helicity to all the fields and antifields such that in the extended action terms occur in which only Fields of different helicity are multiplied together. Any term in the action or in the gauge fermion is of the form \( L \ O \ R \), where \( O \) is a possible superspace operator or spacetime derivative. We assign helicity \( R \) to all Fields with upper dotted indices and lower undotted indices\(^4\), and vice versa for \( L \) helicity. Fields and antifields have opposite helicities, and so do Fields and their complex conjugates. In the tables the \( \sigma, \lambda, \varsigma, d, \nu, f, \tau \) fields on the NW-SE diagonals have \( R \) helicity. The other diagonals \( b, \mu, e, \rho \) have \( L \) helicity.

Statistics and superspace rules

A more delicate point is how one treats the commutation or anticommutation of fields and the superspace coordinate \( \theta^\alpha \) and \( D_\alpha \) which are fermionic. From (2.2), \( \Sigma \) is bosonic, and therefore \( \sigma^\alpha \) is fermionic. Since antifields have opposite statistics to fields, \( \sigma^{\star \alpha} \) is bosonic. Then, (4.3) implies that all \( \sigma \)-fields with any index structure are fermionic. The gauge fermion is always fermionic. It follows that \( b_{\alpha}^\beta \) is fermionic and therefore \( \sigma^{[\alpha \beta]} \) is also fermionic. Its ghost \( \lambda \) is bosonic. Proceeding in this manner one arrives at the assignments in the table [\[1\]].

We note that these assignments, which are consistent with ordinary space BV methods, lead to rules which are different from those used in other superspace calculations, where objects with even or odd number of spinor indices are assigned (ab)normal statistics

\(^4\)Throughout this work we do not raise or lower indices on Fields.
depending on their ghost number. Consequently some care is required when transferring spinor derivatives past Fields. We have proved that both assignments of statistics lead to the same results. The proof relies on the helicity structure explained above.

**Dimensions**

In assigning dimensions to the Fields, there is some freedom of choice. A suitable choice can be as follows: One assigns dimension 1 to a spacetime derivative $\partial_{\alpha^\cdot}$ and $\frac{1}{2}$ to a superspace derivative $D_{\alpha}$. The action has dimension 1, and the gauge fermion $\Psi$ has dimension $\frac{1}{2}$. The dimensions of a field and antifield add up to $\frac{1}{2}$. All the fermion fields $\sigma^{A\cdot\alpha_{n+1}}$, $b^{A}_{\alpha_n}$, $d^{A}_{\dot{\alpha}_n}$, $e^{A}_{\dot{\alpha}_n}$, $f^{A}_{\alpha_n}$ have dimension 0, therefore corresponding antifields have dimension $\frac{1}{2}$. The boson fields $\lambda$, $\mu$, $\rho$ have dimension $\frac{1}{2}$, and their antifields dimension 0. However $\nu$ has dimension $-\frac{1}{2}$ and thus $\nu^*$ has dimension 1. The field $\rho'$ has dimension equal to that of $\rho^*$, namely 0.

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