MIRRORS AND PHASES OF N=4 IN D=3

ZHENG YIN
Department of Physics, University of California
Berkeley, CA 94720-7300, U.S.A.
and
Mail Stop 50A-5101 (zyin@thsrv.lbl.gov)
Lawrence Berkeley National Laboratory
1 Cyclotron Road, Berkeley, CA 94720, U.S.A.

Abstract. We review brane engineering of mirror pairs of 3d N=4 theories. It reveals aspects of 3d physics not known from previous field theoretic studies: novel QFT’s without Lagrangian description and transitions to them from conventional QFT’s.

1. Introduction

3d mirror symmetry for was first proposed in [1] as a duality between certain pairs of generally different 3d N=4 theories at the infrared limit. Infinite sequences of new mirror pairs and strong field theoretic evidence for them were found in [2]. It is a nonperturbative duality that in particular equates certain quantities receiving large quantum corrections with some that are determined entirely classically. Naturally one asks whether this can be the consequence of some string dualities. There are several different approaches [1, 2, 3, 4, 5, 6, 7], each of which has its own advantage and is related to the others by some sequences of dualities. In this talk I will review one that is particularly intuitive [4, 5]. The global R-symmetry of N=4 theories appears as geometric rotations; R-symmetry breaking part of the moduli space of vacua and the parameters of the field theory are realized as the moduli space of D-branes configurations; mirror symmetry itself is implemented by the S duality of type IIB string theory [4]. This construction allows us to engineer a large class of theories and find their mirror duals [5]. Since then it has been generalized to 3d N=2 theories [8, 9]. Similar ideas of constructing field theories in 4d, reviewed in Eguchi’s
lecture at this school [10], have also been very fruitful*. In this talk, I will focus on 3d N=4 theories. After reviewing the rules for “model building” via “brane engineering,” I will show how the mirror pairs emerge from this prescription. As an unexpected reward we can predict an infinite number of 3d field theories without conventional Lagrangian descriptions. Some of them are dual to ordinary Lagrangian theories via mirror symmetry, but the rest are not; yet they can be smoothly connected in the moduli space of brane configurations.

2. What is 3d Mirror Symmetry?

The structure of N=4 supersymmetric gauge theories in three dimensions can be easily obtained by dimensionally reducing the minimally supersymmetric 6d Yang-Mills Lagrangian [16]. The global R-symmetry of the 3d theory is $SU(2)_{345} \times SU(2)_{789}$. The reason for choosing these subscripts will become clear in the brane realization described later. The field content consists of vector multiplets and hypermultiplets. For each vector multiplet associated with a $U(1)$ factor of the gauge group there are 3 real Fayet-Iliopoulos parameters, $\vec{\zeta}$, which can be thought as coming from the VEV’s of a background hypermultiplet. For each hypermultiplet, there are 3 real mass parameters, $\vec{m}$. They are the VEV’s of a background vector multiplet. There are also gauge coupling constants, which also come from background vector multiplets. The transformation properties of the parameters and VEV’s under R-symmetry are summarized in table 1.

Note that usually the scalars in a hypermultiplet are written as a doublet under $SU(2)_{789}$. However, it is convenient, by a change of variables, to rearrange them into a singlet $b$ and a triplet $\vec{r}$. On the other hand, an interesting feature peculiar to three dimensions is that a vector potential is dual to a scalar by the usual electro-magnetic duality. Of course this duality transformation can be precisely formulated only for a free $U(1)$ gauge fields, but this is what is available for the low energy effective theory at generic points of the vector multiplet branch of moduli space. Therefore it is meaningful to include the dualized scalar in considering the moduli space of vacua. By supersymmetry, the moduli space must be hyper-Kähler for both the hypermultiplets and the (dualized) vector multiplets. Because of their different patterns of global R-symmetry breaking, however, the VEV’s of the vector and hyper multiplets respectively are distinct order parameters of the theory, even after taking into account of quantum fluctuations. Vacua of N=4, D=3 gauge theories always contain a vector multiplet branch in which the gauge group is generically broken down to $U(1)^N$ where $N$ is the total rank of the gauge group. This branch is parameterized by the

*See, for example, [11, 12, 13, 14, 15].
4\(N\) real scalars from the \(N\) corresponding vector multiplets. If it has a sufficient number of hypermultiplets, there can also be a hypermultiplet branch and/or mixed branches.

### TABLE 1. R charges of the VEV’s and parameters

| Multiplets/Parameters | Notation | \(SU(2)_{345} \times SU(2)_{789}\) |
|-----------------------|----------|----------------------------------|
| Vector                | \(\vec{a}: a^3, a^4, a^5\) | (3,1)                            |
|                       | \(A_\mu \to \sigma\) | (1,1)                            |
| Hyper                 | \(\vec{r}: r^7, r^8, r^9\) | (1,3)                            |
|                       | \(b\) | (1,1)                            |
| Fayet-Iliopoulos      | \(\vec{\zeta}: \zeta^7, \zeta^8, \zeta^9\) | (1,3)                            |
| mass                  | \(\vec{m}: m^3, m^4, m^5\) | (3,1)                            |
| (electric) coupling   | \(e\) | (1,1)                            |

The low energy effective action up to two derivatives and four fermions is controlled by the metric of the moduli space, which depends on the parameters of the theory as well as the position on the moduli space. The dependence is constrained by extended supersymmetry: the Kähler potential is the sum of a term that depends only on the hypermultiplet scalar and one only on vector multiplet scalars \([17, 18]\). So a mixed branch is the direct product of a vector branch \(M_V\) and a “hyper” branch \(M_H\). By reinterpreting the parameters of the theory as the VEV’s of background superfields, one can further deduce the effects of tuning them on the metric. The gauge coupling \(\frac{1}{g^2}\) lives in a vector multiplet, so it can continuously deform the metric of \(M_V\) but not that of \(M_H\). Since \(g^2\) also plays the role of \(h\), one concludes that the the metric of \(M_H\) is determined purely classically whereas that of \(M_V\) in general receives quantum corrections. Mass parameters also live in a vector multiplet, so they also can continuously deform \(M_V\) but not \(M_H\) - they can only affect \(M_H\)’s dimensionality by changing the number of massless hypermultiplets. Fayet-Iliopoulos parameters, on the other hand, live in hypermultiplet, and therefore can only deform \(M_H\) and change the dimensions of \(M_V\). This is summarized in table 2.

Looking at the above two tables, the pattern for a possible duality emerges. Starting with some theory that we call A model, if we switch what we mean by \(SU(2)_{345}\) and \(SU(2)_{789}\), and at the same time exchange masses with Fayet-Iliopoulos parameters, will we end up with an apparently different theory, model B, that is nonetheless equivalent to A? \(M_V\) of one
theory would have to be mapped to $\mathcal{M}_H$ of the other. Classically this is definitely not true: $\mathcal{M}_H$ is a complicated space obtained via the hyper-Kähler quotient construction [19] while $\mathcal{M}_V$ is just a flat space quotiented by the Weyl group for the gauge symmetry. Furthermore, when some of the mass terms $\vec{m}_A$ of, say, A, vanish, enhanced global (flavor) symmetries emerge and act on $\mathcal{M}_H^A$. For B there would have to be corresponding global symmetries emerging at vanishing Fayet-Iliopoulos parameters $\vec{\zeta}_B$ and acting on $\mathcal{M}_V^B$. Classically there is no such symmetry. Therefore this hypothetical duality, named mirror symmetry in three dimensions in [1], must be a quantum equivalence. However, a glance at tables 1 and 2 reveals a missing dual of the gauge coupling constant $e$, whose property as predicted by mirror symmetry is listed in table 2 with question marks. Brane interpretations of this parameter, known as the magnetic coupling $m$ (without an arrow), have been proposed [4, 6], but it has yet to be found in any Lagrangian formulation. Given such, mirror symmetry can only manifest for ordinary field theory when $e$ approach a particular value*. Being dimensionful ($e^2$ has the dimension of mass in 3d), the only natural candidates are 0 and $\infty$. $e = 0$, the classical limit, is already ruled out. So we are left with the strong coupling limit, which, since it is in 3d, is also the infrared limit. This also leads us to one of the most striking aspects of this proposed mirror symmetry: it maps from one theory the metric for $\mathcal{M}_V$, a quantity that receives very large quantum corrections, to, in the dual theory, the metric for $\mathcal{M}_H$, which is given by purely classical expressions. As many physicists have suggested, this may imply the line between quantum and classical physics is more blurred than previously thought.

Because of its strong coupling nature, proving mirror symmetry within the context of field theory will be difficult. Embedding the field theory in the dynamics of branes [20, 21, 22, 23, 24] renders many aspects of mirror symmetry manifest [4, 5], if one assumes the S duality of type IIB string. This will be reviewed extensively in the rest of this talk. Before that, I want to give some example of mirror pairs and one of the many pieces of field

*However, it is possible, and the brane realization discussed later strongly suggests, that $m$ can be a field theory parameter that has no Lagrangian representation.
theoretic evidence supporting it [2]. They are logically independent of any conjectures about string theory.

A model has gauge group $U(K)$ with $N$ fundamentals and 1 adjoint hypermultiplets. B models has gauge group $U(K)^N$, which we label as $U(K)_\alpha$, $\alpha = 0, \ldots, N - 1$. Its hypermultiplets consist of one fundamental charged under $U(K)_1$ and $N$ bi-fundamentals. The latters are each charged respectively under $U(K)_\alpha \times U(K)_{\alpha + 1}$ in representation $(\bar{K}, K)$, with cyclic identification $\alpha \sim \alpha + N$. These field contents are nicely encoded in the “quiver” diagrams[25] of figure 1. Each inner node with a number $K$ represents a $U(K)$ gauge group. Each link connecting a pair of them represents a bi-fundamental charged under the pair as (fundamental, fundamental*). An outer node with number $N$ represents fundamentals with multiplicity $N$ charged under the gauge group associated with the inner node to which it is attached.

**Figure 1.** Quiver for A and B models.

Using the notation given in table 1, the metric for $\mathcal{M}^A_V$ takes the form

$$ds^2 = g_{ij}d\vec{a}_i d\vec{a}_j + (g^{-1})_{ij}(d\sigma_i + \bar{\omega}_{i\ell} \cdot d\vec{a}_\ell)(d\sigma_j + \bar{\omega}_{j\ell} \cdot d\vec{a}_\ell),$$

under the constraint

$$\bar{\nabla} g_{ij} = \bar{\nabla} g_{ij},$$

$$\frac{\partial}{\partial a_i^x} \omega^y_{j\ell} - \frac{\partial}{\partial a_j^x} \omega^y_{i\ell} = \epsilon_{xyz} \frac{\partial}{\partial a_i^z} g_{j\ell}.$$  

Here $i, \ldots = 0, \ldots, k - 1$ index the Cartan of $U(k)$; $x, \ldots = 3, 4, 5$. In [2], $g_{ij}$ is computed. Perturbatively it is one-loop exact:

$$g_{i \neq j} = \frac{2}{|\vec{a}_i - \vec{a}_j|} - \frac{1}{|\vec{a}_i - \vec{a}_j + m_{\text{adj}}|} - \frac{1}{|\vec{a}_i - \vec{a}_j - m_{\text{adj}}|}.$$
\[ g_{ii} = \frac{1}{e^2} + \sum_{\alpha=0}^{N-1} \frac{1}{|\bar{a}_i - \bar{m}_\alpha|} \]
\[ + \sum_{j=1 \ldots K} \delta_{ij} \begin{pmatrix} \frac{-2}{|\bar{a}_i - \bar{a}_j|} + \frac{1}{|\bar{a}_i - \bar{a}_j + \bar{m}_\text{adj}|} + \frac{1}{|\bar{a}_i - \bar{a}_j - \bar{m}_\text{adj}|} \end{pmatrix}. \]

(2.4)

(2.5)

Here \( \bar{m}_\text{adj} \) is the triplet mass for the adjoint hypermultiplet; \( \bar{m}_\alpha \) those of the fundamentals, indexed by \( \alpha = 1, \ldots, N \). When \( \bar{m}_\text{adj} = 0 \), there is no instanton correction to the metric and (eq. 2.4) is also nonperturbatively exact. For illustration, consider this simpler case, so that \( g_{i \neq j} \) vanishes. (eq. 2.1) and (eq. 2.4) state that \( \mathcal{M}_V^A \) is the direct product of \( K \) multi-Taub-NUT space with charge \( N \). After quotiented by the Weyl group of \( U(K) \), the direct product becomes a symmetric product.

It turns out that setting \( \bar{m}_\text{adj} = 0 \) for A model is mapped by mirror symmetry to a condition on the Fayet-Iliopoulos parameters \( \bar{\zeta}_\alpha \) of B model:

\[ \sum_\alpha \bar{\zeta}_\alpha = 0. \]

In this case, \( \mathcal{M}_H^B \) is given by the symmetric product of \( K \) ALE spaces with \( A_{N-1} \) singularity [26]. The metric of each ALE space is given by

\[ ds^2 = g d\bar{r}^2 + \frac{1}{g} (db + \bar{\omega}(\bar{r}) \cdot d\bar{r})^2 \]

with

\[ g(\bar{r}) = \sum_{\alpha=0}^{N-1} \frac{1}{|\bar{r} - \sum_{\beta=0}^\alpha \bar{\zeta}_\beta|}. \]

Now the metric for each Taub-NUT factor of \( \mathcal{M}_V^A \) can be read from (eq. 2.4) after setting \( \bar{m}_\text{adj} = 0 \).

\[ g(\bar{a}) = \frac{1}{e^2} + \sum_{\alpha=0}^{N-1} \frac{1}{|\bar{a} - \bar{m}_\alpha|}. \]

It is clear that \( \mathcal{M}_V^A = \mathcal{M}_H^B \) if and only if \( e = \infty \) and one makes the following identification\(^1\)

\[ \bar{\zeta}_\alpha = \bar{m}_\alpha - \bar{m}_{\alpha-1}. \]

\(^1\)To see the counting of parameters matches, note that the “center” of the mass parameters can be absorbed by shifting the origin of \( \mathcal{M}_V \) on both sides. Here this is used to set \( \bar{m}_{N-1} = 0 \) for A model.
3. Set the Branes to work

Consider in type IIB string theory, a configuration that includes 3 types of branes, whose worldvolume configurations are given in table 3 [4]. Such a configuration can preserve up to 8 supercharges, corresponding to N=4 in 3d. Of the original Spin(1, 9) Lorentz symmetry, only the (1+2)d Lorentz group $SL(2, R)_{012}$ and the R-symmetry group $SU(2)_{345} \times SU(2)_{789}$ remain manifest. I will now review some basic rules for “model building” from branes and their justifications. Many of them first appeared in [4].

- By sending $M_{\text{planck}}$ to $\infty$, the worldvolume theory on the branes decouple from the bulk fields such as graviton.
- D3-branes can break and end on D5 or NS5-branes without violating RR charge conservation [27, 28] by becoming a magnetic “monopole” on the 5-branes, as depicted on the left in figure 2. There the horizontal lines represent D3-branes; solid and dashed vertical lines represent NS and D5-branes respectively. Conversely, two D3-branes ending on the same 5-brane from opposite sides can rejoin.

![Figure 2. D3-brane ending on 5-branes and forbidden configurations.](image)

- To the worldvolume theory on the D3-branes, breaking and ending are tantamount to imposing boundary conditions, reducing N=4, D=4 supermultiplets to N=4, D=3 supermultiplets.
• By taking the appropriate scaling limit, the Kaluza-Klein modes along the 6th direction can be kept massive and integrated out. The effective infrared theory on the D3-brane is therefore a (1+2)d QFT.

• The worldvolume theories on the 5-branes are weakly coupled in the infrared. The fields on them have an infinite volume coefficient in their kinetic terms as compared to D3-brane fields due to their relative sizes. As a result they are frozen as background fields and their VEV’s are parameters of the effective 3d theory. Gauge symmetries on the 5-branes become global symmetries.

• When a NS5-brane crosses a D5-brane, a D3-brane is created and stretched in between. This is a consequence of charge conservation [4]∗.

• Certain configurations are believed to be forbidden. They involve more than one D3 brane stretched between the same NS-D5 pair, such as the one in the right of figure 2.

Following the above rules one can build brane configurations of arbitrary complexity. To read off the contents of the resulting field theory, one also needs to know the following.

• Dynamical fields of the decoupled (1+2)d theory arise out of the lightest excitations of open strings starting and ending on the D-branes. There are three types: open strings connecting between D3-branes in the same “cubicle” (figure 3a) give vector multiplets; those between D3-branes in adjacent “cubicle” give bi-fundamental hypermultiplets (figure 3b); while open strings connecting between D3 and D5 give fundamental hypermultiplets (figure 3c). These can be read off from perturbative open string quantization and the boundary conditions. Their transformation properties under R-symmetry agree with the assignment given in the last section. Their end points are electric sources on the D3-branes.

• Configuration of the D3-branes selects the gauge symmetries and the vacuum, as illustrated in figure 4.

∗For some other treatments of this phenomenon, see [29] and the references therein.
Configuration of the 5-branes determines the parameters (figure 5) and global symmetries of the 3d QFT. Note the identification of the magnetic coupling constant as the inverse square root of the distance between adjacent D5-branes along the 6th direction. The frozen gauge dynamics of the D5-brane gives rise to the flavor global symmetry acting on the fundamental hypermultiplets. Since in conventional Lagrangian field theories, this global symmetry is restored by setting the masses to zero, the magnetic coupling is fixed to be infinite. At the same time, the global symmetry resulting from the NS5-branes is broken unless they coincide, which amounts to setting Fayet-Iliopoulos to zero and $e$ to infinity — it can only appear at a nontrivial infrared fixed point.

- Type IIB string theory has a nonperturbative $S$ duality [30] that inverts the string coupling constant and exchange NS5-branes with D5-branes as well as fundamental strings with D-strings. It leaves invariant D3-branes but acts on their worldvolume theories as the $S$ duality for N=4, D=4 SYM [31].
- We therefore should also consider excitations of open D-strings starting and ending on D3-branes and/or NS5-branes. They are simply obtained from the $S$ dual of figure 3. However, on D3-branes, rather than generating additional degree of freedom, they are related to the open string fields nonlocally — end points of these two types of string on D3-branes are respectively magnetic and electric sources. Each ap-
pear as solitonic excitations of the other and are exchanged by the field theoretic S duality.

- Therefore there are two descriptions of the same 3d theory: one uses open fundamental string fields as the canonical variables while the other uses open D-string fields. Their equivalence is the 3d mirror symmetry, and it follows from the S duality type IIB string theory.
- It is convenient to rephrase this duality as an operation combing an S duality transformation with the interchange of 345 and 789 directions \([4]\). This makes explicit the interchange of the two R-symmetry factors, \(M_V\) with \(M_H\), masses with Fayet-Iliopoulos parameters, and \(e\) with \(m\). For reasons just stated, this is reflected on ordinary Lagrangian field theories only in their infrared limit.

4. Mirror Pairs

Now I will present a few examples of mirror pairs constructed in \([5]\). Their field contents are best described by the type of quiver diagrams introduced earlier. They are obtained using the brane-engineering rules outlined above, but with a compactified 6th direction, i.e. with periodic identification along \(X^6\). As a warm-up, let’s look at the A and B models given in figure 1. The corresponding brane configurations are sketched in figure 6.

![Figure 6. Brane configurations for the quivers in figure 1.](image)

For A model, the \(N\) fundamentals of \(U(K)\) originate from open strings stretched between the \(N\) D5 and the \(K\) D3-branes. A special feature of this configuration is that the bi-fundamental, coming from open string stretching between “nearest-neighbor” D3-branes, become the adjoint of \(U(N)\) because there is only one gauge groups. For B model, more generic situation prevails and there are \(N\) bi-fundamentals. The correspondence between the moduli spaces of vacua of A and B as well as the identification (eq. 2) is evident. As it is, there is one constraint on the field theory parameters,
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namely

$$\bar{m}_{\text{adj}} = 0 = \sum_\alpha \bar{\zeta}_\alpha.$$ 

A way to relax this condition has been given in [12].

Now let’s look at mirror pairs of more complex theories. As depicted by the quiver diagrams in figure 7, model A again has gauge group $U(K)^N$ and $N$ bi-fundamentals, but each $U(K)$ now has fundamentals with an arbitrary number of flavors $w_i$. Its mirror, model B, has gauge group $U(K)^M$ with

$$M = \sum_i w_i.$$ 

Besides the $M$ bi-fundamentals, it has $N$ fundamentals arranged as shown in figure 7. If all $w_i > 0$, each $U(K)$ factor has at most one fundamental. If some $w_i = 0$, the corresponding nodes in B model’s quiver coalesce and give rise to fundamentals of higher flavor. Such mirror pairs are again constructed via the S duality of type IIB string theory [5]. The mirror map relates the moduli spaces of the two theories, as well as their parameters, in the same manner as the simpler case discussed above.

![Figure 7. Arbitrary flavor of fundamentals and the mirror](image_url)

It is natural to generalize to the cases with A model having gauge group $\prod_{i=0}^{N-1} U(K_i)$, $N$ bi-fundamentals, and arbitrary fundamentals. Its quiver is shown in figure 8, along with its brane realization. However, here one encounters an important subtlety. Although one can always perform a S duality transformation and obtain the mirror configuration, the result does not always correspond to a gauge theory. To see this, recall that mirror symmetry exchanges $\mathcal{M}_V$ with $\mathcal{M}_H$. A universal property of $N=4$, $D=3$ super-Yang-Mills theories is the existence of the Coulomb phase, a branch
Figure 8. Quiver and brane realization of $\prod_i U(K_i)$

of $\mathcal{M}_B^r$ with $4r$ dimensions, where $r$ is the total rank of the gauge group. This is mapped under mirror symmetry to a branch of $\mathcal{M}_A^r$: the completely Higgsed phase. Therefore a necessary condition for model $B$ to have an ordinary gauge theoretic Lagrangian description is for model $A$ to admit complete Higgsing. This amounts to requiring $[32, 33, 2, 5]$:

$$2k_i - k_{i-1} - k_{i+1} \leq w_i.$$  \hspace{3cm} (4.1)

When this is satisfied, the mirror gauge theory can be constructed along the same vein as before. The details become complicated and can be found in [5].

5. Phases and Transitions

What happens if (eq. 4.1) is not satisfied? S duality still gives a mirror configuration, but one without a gauge theoretic description. An example of this is shown in figure 9a.

Figure 9. Brane configurations giving rise to novel field theories

To understand this phenomenon, note that from the field theory perspective, mirror symmetry corresponds to the $Z_2$-wise freedom in labeling
the two $SU(2)$ R-symmetries. While the vector and hypermultiplets transform distinctly though somewhat symmetrically under them, their interactions enter in the Lagrangian in rather different form. The Lagrangian descriptions of a theory and its dual would in general be quite different, as the examples above show. Actually, there is no reason a priori to expect that both sides of a mirror pair have Lagrangian descriptions at all. It is natural to conjecture this is the what is happening here\footnote{In fact, barring an unexpected way to incorporate the mysterious magnetic coupling into a Lagrangian formulation, even for the cases in which both of the dual pair have a Lagrangian description, mirror symmetry is manifest only at the infinite coupling, i.e. infrared limit, as mentioned earlier. Here I shall be referring to that limit implicitly unless otherwise stated.}. 3d N=4 theories with Lagrangian description can therefore be classified into those that have two, related by mirror symmetry in the infrared, and those that have only one\footnote{One should note that the lack of a completely higgsed phase is necessary but not sufficient for a non-Lagrangian dual. For example, a free $U(1)$ vector multiplet, which obviously does not have any Higgs phase, is dual to a free hypermultiplet. However, such free theories cannot help explain interacting fixed points, e.g. when the D3-branes in figure 9a or b coincide.}.

An example of the latter type, in which the non-Lagrangian description is that with $E_n$ tensionless string [34, 35], was conjectured already in [1]. It has recently been explicitly engineered using one of the alternative formulations of mirror symmetry from string theory [6, 7]. Here we have a very simple prescription for engineering an infinite number of such Lagrangian-non-Lagrangian mirror pairs. As noted in [1], these are local quantum field theories, simply because on one side of the mirror there is a Lagrangian description that flows to it. However, experience in 2, 4, 5, and 6 dimensions has shown a Lagrangian description, though convenient in many ways, may not be a necessary condition for a local quantum field theory (see, for example, [36, 37, 38, 39]). Indeed, one can easily engineer using branes a third class of theories that have no Lagrangian description on either side of the mirror. An example is shown in figure 9b. Since the decoupling of bulk as well as Kaluza-Klein modes works just as in the more mundane cases, they should still be interacting local quantum field theories, but with no known Lagrangian description flowing to it.

Such interesting phenomena deserve an explanation from string theory. In that context mirror symmetry is the equivalence of two descriptions of the same physics related by S duality. The degrees of freedom on the D3-brane theory can be captured both by open fundamental string and by open D-string fields. Starting with, say, a description using only open fundamental string fields, one can employ standard string perturbation theory to obtain their interactions and write a Lagrangian for the field theory modes in the decoupling limit. The same prescription goes through for a descrip-
tion based solely on open D-string fields. Suppose, however, that there is no way to capture the full degrees of freedom by using only, say, open D-string fields. The description on the B model side may not be in the form of a local action, as open D-strings and open fundamental strings ending on D3-branes are mutually nonlocal. If neither open fundamental string fields nor open D-string fields can account for the full degrees of freedom by themselves respectively, we end up with the third class of theories.

![Figure 10. Transition between different classes of field theories via brane motion.](image)

Remarkably, with branes one can not only engineer examples of all three types of theories, but also interpolate between them continuously by moving the 5-branes around. Shown in figure 10a, b is the mirror of figure 9a. It is a theory of the second type (Lagrangian-non-Lagrangian mirror pair). By moving one NS5-branes past another, we arrive at the theory depicted in figure 10c, d, which is of the first type (Lagrangian-Lagrangian mirror pair). Similar transition can be engineered between theories of the third class and the first two as well. To appreciate the meaning of such a process, recall that the electric and magnetic coupling constants are inversely proportional to the square root of the distance between adjacent NS5 and adjacent D5-branes respectively. Moving 5-branes of the same type past each other effectively makes coupling constants of the corresponding type imaginary. This is indicative of a change of the effective degrees of freedom describing the system, namely a phase transition. That smooth movements in the moduli space of brane configurations can connect distinct classes of field theories via some type of phase transitions is one of the most important lessons to be learned from this work.

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