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Finite-temperature holographic QCD
in the Veneziano limit

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Outline

1. Introduction and motivation
2. Holographic V-QCD models at finite temperature
   [MJ, Kiritsis arXiv:1112.1261]
   [Alho, MJ, Kajantie, Kiritsis, Tuominen arXiv:1210.4516]
3. Finite (temperature and) chemical potential
   [Alho, MJ, Kajantie, Kiritsis, Rosen, Tuominen, work in progress]
Motivation

QCD: \( SU(N_c) \) gauge theory with \( N_f \) quark flavors (fundamental)

- Often useful: “quenched” or “probe” approximation, \( N_f \ll N_c \)
- Here Veneziano limit: large \( N_f, N_c \) with \( x = N_f/N_c \) fixed ⇒ backreaction

Veneziano limit, backreaction ⇒ better modeling of QCD?
Important new features, mostly not captured by probe limit:

- QCD at finite \( T, \mu \), as a function of \( x \)
- Already at zero \( T, \mu \), nontrivial structure varying \( x = N_f/N_c \)

Particularly interesting regime: large \( \mu \) and/or \( x \), where lattice computations have issues
Motivation: expected phase diagram

- Conformal transition at $x = N_f/N_c \simeq 4$
- Walking/quasi-conformal regime + Miransky scaling below the transition
The fusion:

1. IHQCD: bottom-up model for pure glue by using dilaton gravity
   [Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon brane actions
   [Klebanov, Maldacena; Bigazzi, Casero, Cotrone, Iatrakis, Kiritsis, Paredes]

Consider 1 + 2 in the Veneziano limit with full backreaction
⇒ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]
Defining V-QCD

Degrees of freedom are two scalar fields:

- The tachyon $\tau \leftrightarrow \bar{q}q$, and the dilaton $\lambda \leftrightarrow \text{Tr} F^2$
- $\lambda$ is identified as the 't Hooft coupling $g^2 N_c$

$$S_{V-QCD} = N_c^2 M^3 \int d^5x \sqrt{g} \left[ R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

$$- N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau)}$$

$$V_f(\lambda, \tau) = V_{f0}(\lambda) e^{-a(\lambda) \tau^2} ; \quad ds^2 = e^{2A(r)} \left( \frac{dr^2}{f(r)} - f(r) dt^2 + dx^2 \right)$$

- Need to choose $V_g$, $V_{f0}$, $a$, and $\kappa$ ... 
  - Match with QCD behavior at qualitative level
  - The simplest and most reasonable choices do the job!
Various solutions

Solve EoMs, with fifth coordinate ↔ energy scale

- UV boundary: contact to field theory
- IR structure: several solutions, leading to phase structure

Two classes of IR geometries:
1. Black hole solutions
   - \( f'(r_h) = -4\pi T \); \( s = 4\pi M^3 N_c^2 e^{3A(r_h)} \)
2. Thermal gas solutions (\( f \equiv 1 \))
   - Any \( T \), zero \( s \)

Two types of tachyon behavior
(quark mass and condensate from UV boundary conditions):
1. Vanishing tachyon – chirally symmetric
2. Nontrivial tachyon – chirally broken

⇒ four types of background solutions

Calculate free energy or pressure in each case, determine the dominant solution
Thermodynamics

Pressure and interaction measure

Low $T$: thermal gas $\rightarrow$ High $T$: black hole

Zero pressure at low $T$ due to missing pion loops
Phase diagram: example

Phases on the \((x, T)\)-plane

- \(\chi^S\) Black Hole
- \(\chi^{SB}\) Thermal Gas

\(x_c\)
Turning on finite chemical potential

Work in progress!

Standard method: add a gauge field $A_\mu$ dual to $\bar{q}\gamma^\mu q$

$$S_{V-QCD} = N_c^2 M^3 \int d^5x \sqrt{g} \left[ R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

$$-N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \times \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau + w(\lambda) F_{ab})}$$

$$A_0 = \mu - nr^2 + \cdots$$

$A_0$ can be integrated out $\Rightarrow$ one integration constant, which can be mapped to $\mu$
Computation of pressure

Three phases (plots by Timo Alho)

Phases mapped to $(\mu, T)$-plane
  - Tachyonic Thermal gas (ChSB), all $\mu, T$ (not shown)
  - Tachyonless BH (red)
  - Tachyonic BH (blue)

Integrate

$$ dp = s \, dT + n \, d\mu $$

along the lines shown
Path independence verified numerically

Highly nontrivial check of the consistency of the model and the numerics
First attempt: \( x = N_f/N_c = 1 \), Veneziano limit, zero quark mass
V-QCD models meet expectations from QCD at qualitative level

Analysis of structure at finite chemical potential in progress – first results obtained

Future work: quantitative fit to QCD (lattice + experiments) – towards a more realistic model
Extra slides
QCD phases in the Veneziano limit

Expected structure at zero $T$, $\mu$, and quark mass; finite $x = N_f/N_c$

- Phases:
  - $0 < x < x_c$: QCD-like IR, chiral symmetry broken
  - $x_c \leq x < 11/2$: Conformal window, chirally symmetric

- Conformal transition at $x = x_c$

- RG flow of the coupling: running, walking, or fixed point
Fixing the potentials reasonably, at zero quark mass, after some analysis:

- Meets standard expectations from QCD!
- Conformal transition at $x \sim 4$

[Kaplan, Son, Stephanov; Kutasov, Lin, Parnachev]
Matching to QCD

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions \( \Rightarrow \)

\[
\lambda(r) \approx -\frac{\beta_0}{\log r} \quad \tau(r) \approx m(-\log r)^{-\gamma_0/\beta_0} r + \sigma(-\log r)^{\gamma_0/\beta_0} r^3
\]

with \( r \approx 1/\mu \to 0 \)

In the IR ($\lambda \to \infty$):

- \( V_g(\lambda) \) chosen as for Yang-Mills, so that a “good” IR singularity exists

- \( V_{f0}(\lambda), a(\lambda), \) and \( \kappa(\lambda) \) chosen to produce tachyon divergence: several possibilities (\( \to \) Potentials I and II)

- Extra constraints from the asymptotics of the meson spectra
Other important features

\[
\langle \bar{q}q \rangle \sim \sigma \sim \exp \left( - \frac{\kappa}{\sqrt{x_c - x}} \right)
\]

1. Miransky/BKT scaling as \( x \to x_c \) from below
   - E.g., The chiral condensate \( \langle \bar{q}q \rangle \propto \sigma \)
     (From tachyon UV \( \tau(r) \sim m_q (\log r) r + \sigma (\log r) r^3 \))
2. Unstable Efimov vacua observed for \( x < x_c \)
3. Turning on the quark mass possible
Finite temperature – definitions

Lagrangian as before

\[ S_{V-QCD} = N_c^2 M^3 \int d^5 x \sqrt{g} \left[ R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V_g(\lambda) \right] \]

\[ - N_f N_c M^3 \int d^5 x V_f(\lambda, \tau) \sqrt{- \det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau)} \]

A more general metric, \( A \) and \( f \) solved from EoMs

\[ ds^2 = e^{2A(r)} \left( \frac{dr^2}{f(r)} - f(r) dt^2 + d\mathbf{x}^2 \right) \]

Black hole thermodynamics:

\[ f(r) = 4\pi T (r_h - r) + \mathcal{O} \left( (r - r_h)^2 \right) ; \quad s = 4\pi M^3 N_c^2 e^{3A(r_h)} \]

Also: Thermal gas solutions \((f \equiv 1, \sim \text{zero } T \text{ solutions})\)
Scalar singlet masses

Scalar singlet spectrum (PotII):

In log scale  Normalized to the lowest state

No light dilaton?
Meson mass ratios

Mass ratios (PotII): Lowest states normalized to $\rho$

All ratios tend to constants as $x \rightarrow x_c$: indeed no dilaton
Matching to QCD

Similar strategy as in IHQCD

Matching in the UV ($\lambda \to 0$):

- Take analytic potentials at $\lambda = 0$
  ⇒ RG flow consistent with QCD (when $A \leftrightarrow \log \mu$)
- Require correct (naive) operator dimensions in the deep UV
- Match expansions of potentials with perturbative QCD beta functions
  - $V_g(\lambda)$ with (two-loop) Yang-Mills beta function
  - $V_g(\lambda) - xV_{f0}(\lambda)$ with QCD beta function
  - $a(\lambda)/\kappa(\lambda)$ with the anomalous dimension of the quark mass/chiral condensate (⇒ properly running quark mass!)
- After this, a single undetermined parameter in the UV: $W_0$

$$V_{f0}(\lambda) = W_0 + W_1 \lambda + \mathcal{O}(\lambda^2)$$
In the IR ($\lambda \to \infty$), there must be a solution where the tachyon action $\propto e^{-a(\lambda)\tau^2} \to 0$

- $V_g(\lambda)$ chosen as for Yang-Mills, so that a “good” IR singularity exists
- $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities ($\to$ Potentials I and II)
- Extra constraints from the asymptotics of the meson spectra

Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (!)
Analysis of the backgrounds ($r$-dependent solutions of EoMs) at zero temperature

- Expect two kinds of solutions, with
  1. Nontrivial tachyon profile (chirally broken)
  2. Identically vanishing tachyon (chirally symmetric)

- Identify the dominant vacua

- Fully backreacted system $\Rightarrow$ rich dynamics, complicated numerical analysis . . .
Backgrounds at zero quark mass

Sketch of behavior in the conformal window ($x > x_c$):

- Tachyon vanishes (no ChSB)
- Similar to IHQCD, different potential  
  \( \Rightarrow \) IR fixed point
- Dilaton flows between UV/IR fixed points

Here UV: \( r \to 0 \), IR: \( r \to \infty \)

As $x$ goes below $x_c$, this solution becomes unstable (tachyon BF bound)
Right below the conformal window \((x < x_c; \ |x - x_c| \ll 1)\)

- Dilaton flows very close to the IR fixed point
- “Small” nonzero tachyon induces an IR singularity

Result: “walking”
Actual solutions

UV: \( r = 0 \)
IR: \( r = \infty \)
\( A \sim \log \mu \sim -\log r \)
\( x_c \approx 3.9959 \)

\( x = 4 \) (IR fixed point)

\( x = 3.9 \) (walking)

\( x = 2 \) (running)
The BF bound and $x_c$

At an fixed point

$$\tau(r) \sim C_1 r^\Delta + C_2 r^{4-\Delta}$$

with

$$-m^2 \ell^2 = \Delta(4 - \Delta)$$

Requiring real $\Delta$ gives the Breitenlohner-Freedman bound for the tachyon (Starinets’ lectures)

$$-m^2 \ell^2 = \Delta(4 - \Delta) \leq 4$$

- Saturated for $\Delta = 2$, then $\tau(r) \sim C_1 r^2 + C_2 r^2 \log r$
- Violation of BF bound $\Rightarrow$ instability
Analysis of this instability of the tachyon $\Rightarrow x_c$

Dependence on the UV parameter $W_0$ and IR choices for the potentials

Resulting variation of the edge of conformal window $x_c = 3.7 \ldots 4.2$

Agrees with most of the other estimates
Potentials I

\[ V_g(\lambda) = 12 + \frac{44}{9\pi^2} \lambda + \frac{4619}{3888\pi^4} \frac{\lambda^2}{(1 + \lambda/(8\pi^2))^{2/3}} \sqrt{1 + \log(1 + \lambda/(8\pi^2))} \]

\[ V_f(\lambda, \tau) = V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \]

\[ V_{f0}(\lambda) = \frac{12}{11} + \frac{4(33 - 2x)}{99\pi^2} \lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4} \lambda^2 \]

\[ a(\lambda) = \frac{3}{22} (11 - x) \]

\[ \kappa(\lambda) = \frac{1}{\left(1 + \frac{115 - 16x}{288\pi^2} \lambda\right)^{4/3}} \]

In this case the tachyon diverges exponentially:

\[ \tau(r) \sim \tau_0 \exp \left[ \frac{81 3^{5/6} (115 - 16x)^{4/3} (11 - x)}{812944 2^{1/6}} \frac{r}{R} \right] \]
\[ V_g(\lambda) = 12 + \frac{44}{9\pi^2} \lambda + \frac{4619}{3888\pi^4} \frac{\lambda^2}{(1 + \lambda/(8\pi^2))^{2/3}} \sqrt{1 + \log(1 + \lambda/(8\pi^2))} \]

\[ V_f(\lambda, \tau) = V_{f0}(\lambda) e^{-a(\lambda)\tau^2} \]

\[ V_{f0}(\lambda) = \frac{12}{11} + \frac{4(33 - 2x)}{99\pi^2} \lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4} \lambda^2 \]

\[ a(\lambda) = \frac{3}{22}(11 - x) \frac{1 + \frac{115 - 16x}{216\pi^2} \lambda + \lambda^2/(8\pi^2)^2}{(1 + \lambda/(8\pi^2))^{4/3}} \]

\[ \kappa(\lambda) = \frac{1}{(1 + \lambda/(8\pi^2))^{4/3}} \]

In this case the tachyon diverges as

\[ \tau(r) \sim \frac{27 \ 2^{3/4} 3^{1/4}}{\sqrt{4619}} \sqrt{\frac{r - r_1}{R}} \]
For solutions with $\tau = \tau_* = \text{const}$

$$S = M^3 N_c^2 \int d^5 x \sqrt{g} \left[ R - \frac{4}{3} \frac{\left( \partial \lambda \right)^2}{\lambda^2} + V_g(\lambda) - x V_f(\lambda, \tau_*) \right]$$

IHQCD with an effective potential

$$V_{\text{eff}}(\lambda) = V_g(\lambda) - x V_f(\lambda, \tau_*) = V_g(\lambda) - x V_{f0}(\lambda) \exp(-a(\lambda) \tau_*^2)$$

Minimizing for $\tau_*$ we obtain $\tau_* = 0$ and $\tau_* = \infty$

- $\tau_* = 0$: $V_{\text{eff}}(\lambda) = V_g(\lambda) - x V_{f0}(\lambda)$; fixed point with $V'_{\text{eff}}(\lambda_*) = 0$

- $\tau_* \to \infty$: $V_{\text{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)
Ongoing work: the dependence $\sigma(m)$ of the chiral condensate on the quark mass

- For $x < x_c$ spiral structure
- Dots: numerical data
- Continuous line: (semi-)analytic prediction

Allows to study the effect of double-trace deformations
Black hole branches

Example: PotII at $x = 3$, $W_0 = 12/11$

Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH
More complicated cases:

PotII at $x = 3$, $W_0$ SB

PotI at $x = 3.5$, $W_0 = 12/11$

- Left: chiral symmetry restored at 2nd order transition with $T = T_{\text{end}} > T_h$
- Right: Additional first order transition between BH phases with broken chiral symmetry

Also other cases ...
Phase diagrams on the $(x, T)$-plane

PotI* $\mathcal{W}_0$ SB

No chiral symmetry breaking phase here

PotII* $\mathcal{W}_0$ SB

Conformal window

$T_{\text{crossover}}$

$T_{\text{end}}$

$T_h$

$\frac{T}{\Lambda}$

$x_f$

$0.30$

$0.50$

$0.70$

$1.00$

$1.50$

$2.00$

$0$

$1$

$2$

$3$

$4$

$0.5$

$1.0$

$2.0$

$5.0$

$10.0$
Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ ($\lambda$, $A$, $\tau$)

$x = 3$

$x = 3.5$

$x = 3.9$

$x = 3.97$
Beta functions along the RG flow (evaluated on the background), zero tachyon, YM

$x_c \approx 3.9959$
Generalization of the holographic RG flow of IHQCD

\[ \beta(\lambda, \tau) \equiv \frac{d\lambda}{dA}; \quad \gamma(\lambda, \tau) \equiv \frac{d\tau}{dA} \]

linked to

\[ \frac{dg_{\text{QCD}}}{d \log \mu}; \quad \frac{dm}{d \log \mu} \]

The full equations of motion boil down to two first order partial non-linear differential equations for \( \beta \) and \( \gamma \)
“Good” solutions numerically (unique)
As $x \to x_c$ from below: walking, dominant solution

- BF-bound for the tachyon violated at the IRFP
- $x_c$ fixed by the BF bound:
  $\Delta = 2$ & $\gamma^* = 1$
  at the edge of the conformal window

- $\tau(r) \sim r^2 \sin(\kappa \sqrt{x_c - x} \log r + \phi)$ in the walking region
- “0.5 oscillations” $\Rightarrow$ Miransky/BKT scaling,
  amount of walking $\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa \sqrt{x_c - x}))$
As $x \to x_c$
with known $\kappa$

\[
\langle \bar{q} q \rangle \sim \sigma \sim \exp \left( -\frac{2\pi}{\kappa \sqrt{x_c - x}} \right)
\]

\[
\Lambda_{UV}/\Lambda_{IR} \sim \exp \left( \frac{\pi}{\kappa \sqrt{x_c - x}} \right)
\]
Comparison to other guesses

V-QCD (dashed: variation due to $W_0$)
Dyson-Schwinger
2-loop PQCD
All-orders $\beta$

[Pica, Sannino arXiv:1011.3832]
Understanding the solutions for generic quark masses requires discussing parameters

- YM or QCD with massless quarks: no parameters
- QCD with flavor-independent mass $m$: a single (dimensionless) parameter $m/\Lambda_{\text{QCD}}$
- In this model, after rescalings, this parameter can be mapped to a parameter ($\tau_0$ or $r_1$) that controls the diverging tachyon in the IR
- $x$ has become continuous in the Veneziano limit
Map of all solutions

All "good" solutions ($\tau \neq 0$) obtained varying $x$ and $\tau_0$ or $r_1$

Contouring: quark mass (zero mass is the red contour)
Conformal window ($x > x_c$)

- For $m = 0$, unique solution with $\tau \equiv 0$
- For $m > 0$, unique “standard” solution with $\tau \neq 0$

Low $0 < x < x_c$: Efimov vacua

- Unstable solution with $\tau \equiv 0$ and $m = 0$
- “Standard” stable solution, with $\tau \neq 0$, for all $m \geq 0$
- Tower of unstable Efimov vacua (small $|m|$)
Efimov solutions

- Tachyon oscillates over the walking regime
- $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. “standard” solution

![Graph showing $\lambda$, log|T| vs. $r$ with markers at $1/\Lambda_{UV}$ and $1/\Lambda_{IR}$]
Effective potential: zero tachyon

Start from Banks-Zaks region, \( \tau_* = 0 \), chiral symmetry conserved \((\tau \leftrightarrow \bar{q}q)\), \( V_{\text{eff}}(\lambda) = V_g(\lambda) - x V_{f0}(\lambda) \)

- \( V_{\text{eff}} \) defines a \( \beta \)-function as in IHQCD – Fixed point guaranteed in the BZ region, moves to higher \( \lambda \) with decreasing \( x \)
- Fixed point \( \lambda_* \) runs to \( \infty \) either at finite \( x(<x_c) \) or as \( x \to 0 \)

**Banks-Zaks**
\( x \to \frac{11}{2} \)

**Conformal Window**
\( x > x_c \quad x < x_c \) ??

\[ \begin{align*}
\beta &
\end{align*} \]
Effective potential: what actually happens

**Banks-Zaks**

$x \to 11/2$

**Conformal Window**

$x > x_c$

$x < x_c$

\[ \tau \equiv 0 \]

- For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- The tachyon screens the fixed point
- In the deep IR $\tau$ diverges, $V_{\text{eff}} \to V_g \Rightarrow$ dynamics is YM-like
Where is $x_c$?

How is the edge of the conformal window stabilized?
Tachyon IR mass at $\lambda = \lambda_* \leftrightarrow$ quark mass dimension

$$-m_{IR}^2 \ell_{IR}^2 = \Delta_{IR} (4 - \Delta_{IR}) = \frac{24 a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

$$\gamma_* = \Delta_{IR} - 1$$

Breitenlohner-Freedman (BF) bound (horizontal line)

$$-m_{IR}^2 \ell_{IR}^2 = 4 \iff \gamma_* = 1$$

defines $x_c$
Why $\gamma_*=1$ at $x=x_c$?

No time to go into details . . . the question boils down to the linearized tachyon solution at the fixed point

- For $\Delta_{IR}(4 - \Delta_{IR}) < 4 \quad (x > x_c)$:

  $$\tau(r) \sim m_q r^{\Delta_{IR}} + \sigma r^{4-\Delta_{IR}}$$

- For $\Delta_{IR}(4 - \Delta_{IR}) > 4 \quad (x < x_c)$:

  $$\tau(r) \sim C r^2 \sin [(\text{Im}\Delta_{IR}) \log r + \phi]$$

Rough analogy:
Tachyon EoM $\leftrightarrow$ Gap equation in Dyson-Schwinger approach
Similar observations have been made in other holographic frameworks

[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]
For $m > 0$ the conformal transition disappears. The ratio of typical UV/IR scales $\Lambda_{\text{UV}}/\Lambda_{\text{IR}}$ varies in a natural way $m/\Lambda_{\text{UV}} = 10^{-6}, 10^{-5}, \ldots, 10$ and $x = 2, 3.5, 3.9, 4.25, 4.5$.
The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with $N_f$ quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- At $x = 0$, the theory has confinement, a mass gap and $N_c$ distinct vacua associated with a spontaneous breaking of the leftover $R$ symmetry $Z_{N_c}$.
- At $0 < x < 1$, the theory has a runaway ground state.
- At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- At $1 + 2/N_c < x < 3/2$, the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- At $x > 3$, the theory is IR free.
Why is the BF bound saturated at the phase transition (massless quarks)?

\[ \Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))} \]

- For \( \Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) < 4 \):
  \[ \tau(r) \sim m_q r^{4 - \Delta_{\text{IR}}} + \sigma r^{\Delta_{\text{IR}}} \]

- For \( \Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) > 4 \):
  \[ \tau(r) \sim Cr^2 \sin [(\text{Im}\Delta_{\text{IR}}) \log r + \phi] \]

- Saturating the BF bound, the tachyon solutions will entangle → required to satisfy boundary conditions

- Nodes in the solution appear through UV → massless solution
Saturating the BF bound (sketch)

Does the nontrivial (ChSB) massless tachyon solution exist? Two possibilities:

- \( x > x_c \): BF bound satisfied at the fixed point \( \Rightarrow \) only trivial massless solution (\( \tau \equiv 0 \), ChS intact, fixed point hit)
- \( x < x_c \): BF bound violated at the fixed point \( \Rightarrow \) a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at \( x = x_c \)
As \( x \to x_c \) from below, need to approach the fixed point to satisfy the boundary conditions \( \Rightarrow \) nearly conformal, “walking” dynamics
Massless backgrounds: gamma functions \( \frac{\gamma}{\tau} = \frac{d \log \tau}{dA} \)

\( x = 2, 3, 3.5, 3.9 \)