Thermodynamics and weak cosmic censorship conjecture in (2 + 1)-dimensional regular black hole with nonlinear electrodynamics sources

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Received: 19 May 2019 / Accepted: 19 November 2019 / Published online: 1 February 2020
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Abstract We study the dynamical behavior of spinor particles and get the energy–momentum relation of charged particles by solving the Dirac equation. Based on the energy–momentum relation, we investigate the laws of thermodynamics and the weak cosmic censorship conjecture for the (2 + 1)-dimensional regular black hole with nonlinear electrodynamics sources in the normal phase space and extended phase space. Our results show that the first law of thermodynamics as well as the weak cosmic censorship conjecture are valid in both the phase spaces. However, the second law of thermodynamics is violated in the extended phase space, implying that the first law and weak cosmic censorship conjecture do not depend on the phase space while the second law depends. In addition, in the extended phase space, we find the configurations of the extremal and near-extremal black holes does not change for the final states and initial states are the same.

1 Introduction

Based on the ideas of Henneaux et al. [1], Caldarelli et al. [2] took the lead in regarding the cosmological constant as a state parameter of the thermodynamic systems. For the Kerr-Newman-AdS black hole, it was shown that the first law of thermodynamics was \(dM = TdS + \Omega dJ + \Phi dQ + \Theta d\Lambda\), in which \(\Lambda\) is the generalized force conjugating to the state parameter. The variable cosmological parameter can enrich the thermodynamic state functions [3–9], and the phase spaces of the thermodynamic system thus are extended, which is different from previous treatments where the cosmological parameter is a constant [10–15]. Recently, some authors claimed that the cosmological constant can be used as the pressure of the thermodynamic system [16,17], and its thermodynamic conjugate can be defined naturally as volume. In this framework, the mass is not the internal energy but the enthalpy. On this premise, the thermodynamics and phase transitions of a class of black holes with cosmological constants have been studied extensively in the extended phase space [18–26]. However, most of them focused on only the first law of thermodynamics. There was little work to check whether the second law of thermodynamics as well as weak cosmic censorship conjecture are valid in the extended phase space.

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Recently, Gwak [27,28] suggested that the laws of thermodynamics can be checked by throwing charged particles into black holes in the framework of extended phase space. Their results showed that the second law of thermodynamics was violated as the contributions of pressure and volume are considered. They also investigated the weak cosmic censorship conjecture and found that it was valid. However, different from the case in the normal phase space, the configurations of the black hole are found to be not changed for both the extremal and near-extremal black holes. Subsequently, this work was extended to Born-Infeld-anti-de Sitter black hole, in which the contribution of the vacuum polarization energy was considered besides the pressure [29]. In addition, [27] has also been extended to the case that the absorbed particles are fermions, for which the Dirac equation was used to obtain the energy-momentum relation [30–32]. In this paper, we intend to extend the idea in [27] to the (2 + 1)-dimensional regular black hole with nonlinear electrodynamics sources. Unlike previous studies, this space-time is coupled by Einstein’s gravity and nonlinear electrodynamics. We want to explore whether nonlinear electrodynamics affect the thermodynamics and the weak cosmic censorship conjecture, which have not been reported so far. We will pay attention to the case that the absorbed particles are fermions so that the Dirac equation will be used to investigate the dynamical behavior of the particles.

The present work is organized as follows. In Sect. 2, the relation between the energy and momentum of absorbed fermions is obtained by solving the Dirac equation. In Sect. 3, we will study thermodynamic laws and weak cosmic censorship conjecture in the normal phase space. Section 4 will extend the investigations on thermodynamic laws and weak cosmic censorship conjecture to the extended phase space. Finally, the conclusion is presented in Sect. 5. Throughout the paper, the units $\pi = \hbar = c = 1$ are used.

### 2 Energy and momentum of an absorbed fermion

The action of the (2 + 1)-Einstein gravity coupled with nonlinear electrodynamics is given by [33–35]

$$S = \int d^3x \sqrt{-g} \left[ \frac{R - 2\Lambda}{16\pi} + L(F) \right], \quad (1)$$

where $g$ is the determinant of the metric tensor, $R$ is the Ricci scalar, $\Lambda = -1/l^2$ is the cosmological constant, $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$, and $L(F)$ is the Lagrangian of the nonlinear electrodynamics with $F = F^{\mu\nu} F_{\mu\nu}$ [36]. From Eq. (1), we can obtain the solutions of the (2 + 1)-dimensional regular black hole with nonlinear electrodynamics sources, that is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\phi^2, \quad (2)$$

where

$$f(r) = r^2/l^2 - m - q^2 \log(q^2/a^2l^2 + r^2/l^2) \quad (3)$$

in which $m$ and $q$ are the parameters that relate to the mass and charge of the black hole, and $a$ is a parameter that relates to the nonlinear electrodynamics. The strength of electric field is

$$E(r) = \frac{a^4qr^3}{16\pi \left(q^2 + a^2r^2\right)^2}. \quad (4)$$
After integration with the relation
\[ A_t(r) = \int_r^\infty E(x) \, dx \] [34], we can get the non-vanishing component of the vector potential
\[ A_t(r) = -a^2q r^2 - q^3 \log \left[ q^2 + a^2 r^2 \right] \frac{32 \pi}{r^2}. \] (5)

By using the formula given in [37], we can get the electric charge of the black hole, \( Q = 8q \).
Comparing with the static, \((2 + 1)\)-dimensional black hole, we also can easily obtain the mass of the regular black holes, \( M = m/8 \).

In this case, the metric function can be rewritten as
\[ F(r) = -8M + \frac{r^2}{l^2} - \frac{1}{64} Q^2 \log \left( \frac{Q^2}{64a^2 l^2} + \frac{r^2}{l^2} \right). \] (6)

Now, we investigate the dynamical of a charged fermion as it is swallowed by the \((2 + 1)\)-dimensional regular black hole with nonlinear electrodynamics sources. The Dirac equations for electromagnetic field in curved space-time can be expressed as
\[ i \gamma^\mu \left( \partial_\mu + \Omega_\mu - i \frac{e}{\hbar} A_\mu \right) \Psi - \frac{\mu_0}{\hbar} \Psi = 0, \] (7)

where \( u_0 \) and \( e \) are, respectively, the rest mass and charge of a fermion, \( \Omega_\mu = i \gamma^\mu \Gamma_{\mu \tau} \sum_{\tau}, \) \( \sum_{\tau} = \frac{i}{4} \left[ \gamma^\rho, \gamma^\tau \right], \gamma^\mu \) matrices satisfy \( \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu \nu} I, \) in which \( I \) is the unit matrix.

In order to get the solution of the Dirac equation, we have to choose matrices \( \gamma^\mu \). Here, we set
\[ \gamma^\mu = \left( -iF - \frac{1}{2} \gamma^2, F \gamma^1, \frac{1}{r} \gamma^3 \right), \] (8)

where \( \sigma^\mu \) is the Pauli matrix
\[ \sigma^1 = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \sigma^2 = \left( \begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right), \sigma^3 = \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right). \] (9)

A fermion have spin up state and a spin down state with spin 1/2. For the sake of simplicity, we only investigate the spin up case for the case of spin down is similar.

We use the ansatz for the two-component spinor \( \Psi \) as
\[ \Psi = \left( \begin{array}{c} C(t, r, \phi) \\ D(t, r, \phi) \end{array} \right) \exp \left( \frac{i}{\hbar} G(t, r, \phi) \right). \] (10)

Substituting Eq. (10) into Eq. (7), we get the following two simplified equations
\[ C \left( \mu_0 + \frac{1}{r} \partial_\phi G \right) + D \left[ \sqrt{F} \partial_r G - \left( \frac{1}{\sqrt{F}} \partial_t G - \frac{1}{\sqrt{F}} e A_t \right) \right] = 0, \] (11)
\[ C \left[ \sqrt{F} \partial_r G + \left( \frac{1}{\sqrt{F}} \partial_t G - \frac{1}{\sqrt{F}} e A_t \right) \right] + D \left( \mu_0 - \frac{1}{r} \partial_\phi G \right) = 0. \] (12)

The above two equations have a nontrivial solution for \( C \) and \( D \) if and only if the determinant of the matrix coefficients is zero. So we get
\[ \frac{1}{r^2} \left( \partial_\phi G \right)^2 - \mu_0^2 + \left( \sqrt{F} \partial_r G \right)^2 - \left( \frac{1}{\sqrt{F}} \partial_t G - \frac{1}{\sqrt{F}} e A_t \right)^2 = 0. \] (13)

There are two Killing vectors in the \((2 + 1)\)-dimensional space-time, so \( G(t, r, \phi) \) in Eq. (13) can be separated into
\[ G = -\omega t + L\phi + R(r) + \lambda. \] (14)
where $\omega$ and $L$ are Dirac particle’s energy and angular momentum, respectively, and $\lambda$ is a complex constant. Substituting Eq. (14) into Eq. (13), we obtain

$$\partial_r R(r) = \pm \frac{1}{F} \sqrt{(\omega + eA_t)^2 + F \left( \mu_0^2 - \frac{L^2}{r^2} \right)}.$$  

(15)

In order to study thermodynamics, we focus on only the radial momentum of particles near the horizon, i.e., $p^r \equiv g^{rr} p_r = \partial_r R(r)$. From Eq. (15), we get

$$\omega = |p^r_h| - eA_t(r_h).$$  

(16)

As done in [27,38–44], we choose the positive sign in front of $|p^r_h|$ thereafter.

3 Thermodynamics and weak cosmic censorship conjecture of $(2 + 1)$-dimensional black hole in the normal phase space

The electrostatic potential between the black hole horizon and the infinity is defined by $\Phi = A_t(\infty) - A_t(r_h)$. After integration, we can obtain

$$\Phi_h = -\frac{Q \left( l^2 Q^2 + (l^2 Q^2 + 64a^2r_h^2) \log \left( \frac{Q^2}{64a^2} + \frac{r_h^2}{l^2} \right) \right)}{256 \left( l^2 Q^2 + 64a^2r_h^2 \right)},$$

(17)

where $r_h$ is the event horizon of the $(2 + 1)$-dimensional regular black hole with nonlinear electrodynamics sources, which is determined by $F(r_h) = 0$. According to the definition of surface gravity, the Hawking temperature can be written as

$$T_h = \frac{r_h}{128l^2\pi} \left( 64 - \frac{64a^2Q^2r_h^2}{l^2 Q^2 + 64a^2r_h^2} \right).$$  

(18)

By using the Bekenstein–Hawking entropy area relation, the black hole entropy can be written as

$$S_h = \frac{1}{2} \pi r_h.$$  

(19)

From Eq. (6), we can get the mass of the $(2+1)$-dimensional regular black hole with nonlinear electrodynamics sources, which can be expressed as

$$M = \frac{r_h^2}{8l^2} - \frac{Q^2}{512} \log \left[ \frac{1}{l^2} \left( \frac{Q^2}{64a^2} + r_h^2 \right) \right].$$  

(20)

As a charged fermion is swallowed by the regular black holes, the variations of the internal energy and charge of the black hole system should satisfy

$$\omega = dM, \quad e = dQ,$$

(21)

where the conservations of energy and charge have been considered. Now, Eq. (16) can be rewritten as

$$dM = \Phi_h dQ + p^r_h.$$  

(22)
Similarly, the event horizon of a black hole changes as it absorbs a particle, leading to change in $F(r)$. In the new horizon, there is also a relation, $F(r_h + dr_h) = 0$. In other words, the change of the horizon should satisfy

$$dF_h = F(r_h + dr_h) - F(r_h) = \frac{\partial F_h}{\partial M} dM + \frac{\partial F_h}{\partial Q} dQ + \frac{\partial F_h}{\partial r_h} dr_h = 0. \quad (23)$$

Inserting Eq. (22) into Eq. (23), $dM$ will be deleted. Interestingly, $dQ$ will be deleted too. We can get $dr_h$ directly

$$dr_h = \frac{4p^r_h \left(l^4 Q^2 + 64a^2 l^2 r^2_h\right)}{r_h \left(1 - a^2 \right) l^2 Q^2 + 64a^2 r^2_h}. \quad (24)$$

With Eqs. (19) and (24), the variation of entropy can be expressed as

$$dS_h = \frac{2\pi p^r_h \left(l^4 Q^2 + 64a^2 l^2 r^2_h\right)}{r_h \left(1 - a^2 \right) l^2 Q^2 + 64a^2 r^2_h}. \quad (25)$$

From Eqs. (18) and (25), we get

$$T_h dS_h = p^r_h. \quad (26)$$

So, the internal energy in Eq. (22) can be rewritten as

$$dM = TdS + \Phi dQ, \quad (27)$$

which is the first law of the $(2 + 1)$-dimensional regular black hole with nonlinear electrodynamics sources in the normal phase space.

So far, we have derived the first law of thermodynamics. Next, we are going to discuss the second law of the black hole thermodynamics. For the extreme black holes, the temperature is zero, which can produce the radius of the extremal black hole

$$r_e = \frac{lQ}{\sqrt{-64a^2 + a^2 Q^2}}. \quad (28)$$

The second law for the extremal black holes is meaningless strictly for the temperature of the system is zero. So we are interested only the non-extremal black hole, where the temperature is larger then zero. The event horizon thus satisfy $r_h > r_e$. In this case, $dS_h$ is positive in Eq. (25), which shows that the second law of thermodynamics is also valid.

Furthermore, we can investigate the weak cosmic censorship conjecture, which states that the singularity should be hidden by the event horizon of the black hole for an observer located at infinity. To ensure the validity of the weak cosmic censorship conjecture, the existence of an event horizon is needed. To do this, we try to test whether an event horizon exists when a fermion is absorbed by the black hole. That is, whether there are solutions for the equation $F(r_h) = 0$.

For the $(2 + 1)$-dimensional regular black hole with nonlinear electrodynamics sources, there is a minimum value for $F(r)$ with the radial coordinate $r_{min}$. For the case $F(r_{min}) > 0$, there is not a horizon while for the case $F(r_{min}) \leq 0$, there are horizons always. At $r_{min}$, the following relations should be satisfied, yielding

$$F|_{r=r_{min}} = F_{min} = \alpha \leq 0,$$
$$\partial_r F|_{r=r_{min}} = F'_{min} = 0,$$
$$(\partial_r)^2 F|_{r=r_{min}} = F''_{min} > 0. \quad (29)$$
For the extreme black hole, \( \alpha = 0 \), and for the near extreme black hole, \( \alpha \) is a small quantity. As a fermion is swallowed into the black hole, the mass and charge of the black hole change into \( (M + dM, Q + dQ) \). The final locations of minimum value and the event horizon are \( r_{\text{min}} + dr_{\text{min}}, r_h + dr_h \) correspondingly. Here, we have a transformation of \( F(r) \), which is labeled as \( dF_{\text{min}} \). By using condition \( F'_{\text{min}} = 0 \), at \( r_{\text{min}} + dr_{\text{min}} \), \( F(r) \) can be expressed as

\[
F|_{r=r_{\text{min}}+dr_{\text{min}}} = F_{\text{min}} + dF_{\text{min}} = \alpha + \left( \frac{\partial F_{\text{min}}}{\partial M} dM + \frac{\partial F_{\text{min}}}{\partial Q} dQ \right),
\]

(30)

For the extremal black hole, the horizon is located at \( r_{\text{min}} \), Eq. (22) is valid at \( r_{\text{min}} \). Substituting Eq. (22) into Eq. (30), \( dQ \) will be deleted at the same time. Therefore, Eq. (30) can be reduced to

\[
F_{\text{min}} + dF_{\text{min}} = -8 p_h^r.
\]

(31)

Obviously, \( F(r_{\text{min}} + dr_{\text{min}}) \) is smaller than \( F(r_{\text{min}}) \) when a charged fermion is engulfed by the black hole. The extremal black holes will become into non-extremal black holes. In other words, the weak cosmic censorship conjecture is valid in the normal phase space.

4 Thermodynamics and weak cosmic censorship conjecture in the extended phase space

Now, we turn to investigate thermodynamics and weak cosmic censorship conjecture with a variable cosmological parameter, namely \( \Lambda \) is treated as the thermodynamic pressure, and its conjugate variable is the thermodynamic volume, which can be represented as respectively

\[
P = \frac{1}{8\pi l^2},
\]

(32)

\[
V_h = \left( \frac{\partial M}{\partial P} \right)_Q, S_h = \pi r_h^2 \left( \frac{(1 - a^2)l^2Q^2 + 64a^2 r_h^2}{l^2Q^2 + 64a^2 r_h^2} \right).
\]

(33)

From Eqs. (18), (19), (32) and (33), we obtain the Smarr formula

\[
T_h S_h = 2V_h P.
\]

(34)

Considering that \( M \) has the physical significance of enthalpy in the extended phase space [16,17], it has a relation to internal energy as

\[
M = U + PV_h.
\]

(35)

As a charged fermion is swallowed into the black hole, the energy and charge are supposed to be conserved. So, the change in energy and charge of the black hole system should be equal to the energy and charge of the particle. Namely

\[
\omega = dU = d(M - PV_h), \quad e = dQ.
\]

(36)

The energy in Eq. (22) changes correspondingly into

\[
dU = \Phi_h dQ + p_h^r.
\]

(37)

Because of the backreaction, the absorbed fermions will change the event horizon of the black hole. But the event horizon is always determined by the equation \( F(r) = 0 \). In other words, in the new horizon, there is also a relation \( F(r_h + dr_h) = F(r_h) + dF_h = 0 \), namely

\[
dF_h = \frac{\partial F_h}{\partial M} dM + \frac{\partial F_h}{\partial Q} dQ + \frac{\partial F_h}{\partial l} dl + \frac{\partial F_h}{\partial r_h} dr_h = 0.
\]

(38)
With Eq. (35), Eq. (37) can be rewritten as
\[ dM - d(PV_h) = \Phi_h dQ + |p'_h|. \]  \hspace{1cm} (39)

From Eq. (39), we can get \( dl \), and substituting \( dl \) into Eq. (38), we can delete it directly. Interestingly, \( dQ \) and \( dM \) are also eliminated simultaneously. Based on this, there is only a relation between \( |p'_h| \) and \( dr_h \), that is
\[ dr_h = -\frac{p'_h (l^2 Q^2 + 64a^2 r_h^2)^2}{16a^4 Q^2 r_h^3}. \]  \hspace{1cm} (40)

On this basis, the variations of entropy and volume of the black hole can be expressed as
\[ dS_h = -\frac{\pi p'_h (l^2 Q^2 + 64a^2 r_h^2)^2}{32a^4 Q^2 r_h^3}, \]  \hspace{1cm} (41)
\[ dV_h = \frac{\pi p'_h \left(( -1 + a^2) l^4 Q^4 - 128a^2 l^2 Q^2 r_h^2 - 4096a^4 r_h^4 \right)}{8a^4 Q^2 r_h^2}. \]  \hspace{1cm} (42)

According to above formulations, we find
\[ T_h dS_h - P dV_h = p'_h. \]  \hspace{1cm} (43)

The internal energy in Eq. (37) thus would change into
\[ dU = \Phi_h dQ + T_h dS_h - P dV_h. \]  \hspace{1cm} (44)

With Eq. (35), we can obtain the relation between the enthalpy and internal energy in the extended phase space, that is
\[ dM = dU + P dV_h + V_h dP. \]  \hspace{1cm} (45)

Substituting Eq. (44) into Eq. (45), we get
\[ dM = T_h dS_h + \Phi_h dQ + V_h dP, \]  \hspace{1cm} (46)

which is consistent with that in Eq. (27). That is, in the extended phase space, the first law of thermodynamics still holds when charged fermions are absorbed by the black hole.

We are willing to discuss the second law of thermodynamics in extended phase space. From Eq. (41), we find \( dS_h < 0 \). Obviously, in the extended phase space, the change of entropy of the \((2+1)\)-dimensional regular black hole with nonlinear electrodynamics sources is negative. In other words, the entropy of the black hole unavoidably decreases, which is contrary to the second law of thermodynamics. We suspect that the reason might be the contribution of the \( PV_h \) term.

Next, we also can discuss the weak cosmic censorship conjecture in the extended phase space. Considering the backreaction, the mass \( M \), charge \( Q \), and AdS radius \( l \) of the black hole will change into \((M + dM, Q + dQ, l + dl)\) as a charged fermion is swallowed by the black hole. The locations of the minimum value and the event horizon will change into \( r_{min} + dr_{min}, rh + drh \) correspondingly. At \( r_{min} + dr_{min} \), the change of \( F(r) \) is subject to
\[ F|_{r=r_{min}+dr_{min}} = \alpha + \left( \frac{\partial F_{min}}{\partial M} dM + \frac{\partial F_{min}}{\partial Q} dQ + \frac{\partial F_{min}}{\partial l} dl \right). \]  \hspace{1cm} (47)

By using \( F_{min}' = 0 \) in Eq. (47), there is also a relation at the new lowest point, which can be expressed as
\[ \partial_{r} F|_{r=r_{min}+dr_{min}} = F_{min}' + dF_{min}' = 0, \]  \hspace{1cm} (48)
implying
\[ dF'_{\min} = \frac{\partial F'_{\min}}{\partial Q} dQ + \frac{\partial F'_{\min}}{\partial l} dl + \frac{\partial F'_{\min}}{\partial r_{\min}} dr_{\min} = 0, \] (49)
where we have used the condition \( F'(r_{\min}) = 0 \) in Eq. (29). Solving Eq. (49), we get
\[ dl = \frac{\left((1-a^2) l^5 Q^4 + 64a^2 (2+a^2) l^3 Q^2 r_{\min}^2 + 4096a^4 r_{\min}^4 \right)}{2r_{\min} \left((1-a^2) l^4 Q^4 + 128a^2 l^2 Q^2 r_{\min}^2 + 4096a^4 r_{\min}^4 \right)} dr \]
\[ = \frac{-128a^4 l^3 Q r_{\min}^3}{2r_{\min} \left((1-a^2) l^4 Q^4 + 128a^2 l^2 Q^2 r_{\min}^2 + 4096a^4 r_{\min}^4 \right)} dQ. \] (50)

For the extremal black hole, the horizon is located at \( r_{\min} \). That is, Eq. (37) is applicable at \( r_{\min} \). Substituting Eq. (37) into Eq. (47), we obtain
\[ F_{\min} + dF_{\min} = -8p^r_h + \frac{2drr_h}{l^2} \left(-1 + \frac{a^2 l^4 Q^4}{(l^2 Q^2 + 64a^2 r_{\min}^2)^2} \right). \] (51)
Substituting Eq. (43) into Eq. (51), we find
\[ F_{\min} + dF_{\min} = \frac{2r_{\min} \left((-1 + a^2) l^2 Q^2 - 64a^2 r_{\min}^2 \right)}{l^4 Q^2 + 64a^2 l^2 r_{\min}^2} dr. \] (52)
In addition, according to the condition \( F'_{\min} = 0 \), we can get \( Q \), that is
\[ Q = \frac{8ar}{\sqrt{-l^2 + a^2 l^2}}. \] (53)
Substituting Eq. (53) into Eq. (52), we get lastly
\[ F_{\min} + dF_{\min} = 0. \] (54)
Equation (54) shows that when a charged fermion is absorbed by the black hole, there is not shift for \( F_{\min} \). In other words, the configurations of the black holes are not changed; the extremal black holes are still extremal black holes as fermions are absorbed.
For the near-extremal black hole, Eq. (37) is not applicable at \( r_{\min} \) for it is valid only at \( r_h \). But we can expand it near \( r_{\min} \) with \( r_h = r_{\min} + \epsilon \). Note that \( p^r_h \) should also be expanded for it is a function of the horizon \( r_h \). To the first order, we get
\[ dM = \frac{A + B + E}{256l^3 \left(l^2 Q^2 + 64a^2 r_{\min}^2 \right)} + \frac{(X + Y + Z)\epsilon}{4l^3 \left(l^2 Q^2 + 64a^2 r_{\min}^2 \right)^2} + O(\epsilon)^2, \] (55)
in which
\[ A = l^5 \left(Q^3 dQ + Q^3 \log \left[\frac{Q^2}{64a^2} + \frac{r_{\min}^2}{l^2} \right] \right) dQ, \]
\[ B = -4096a^2 l^3 r_{\min}^3 dr + 4096a^2 l^2 r_{\min}^2 dl + l^2 \left(64 Q^2 r_{\min}^2 dl - 64a^2 Q^2 r_{\min}^2 dl \right), \]
\[ E = l^3 \left(-64 Q^2 r_{\min} dr + 64a^2 Q^2 r_{\min} dr + 64a^2 Q \log \left[\frac{Q^2}{64a^2} + \frac{r_{\min}^2}{l^2} \right] \right) r_{\min}^2 dQ, \]
\[ X = -128a^4 l^3 Q r_{\min}^3 dQ + 4096a^4 l^2 r_{\min}^4 dr - 8192a^4 r_{\min}^5 dl, \]
\[ Y = Q^4 \left(l^5 dr - a^2 l^5 dr - 2l^4 r_{\min} dl + 2a^2 l^4 r_{\min} dl \right), \]
\[ Z = Q^2 \left(128a^2 l^3 r_{\min}^2 dr + 64a^4 l^2 r_{\min}^2 dr - 256a^2 l^2 r_{\min}^2 dl \right). \]
Substituting Eq. (55) into Eq. (49), we can get $F_{\text{min}} + dF_{\text{min}}$. Substituting $dl$ in Eq. (50) into this equation, we find
\[
F_{\text{min}} + dF_{\text{min}} = \alpha + \frac{4r_{\text{min}}^2 ((1 - a^2) l^2 Q^2 + 64a^4r_{\text{min}}^2 K)}{H} + O(\epsilon)^2,
\]
where we have defined
\[
K = -(1 - a^2) l^4 Q^4 dl + 64a^2 l^2 Q (a^2 ld Q + 2dl Q) r_{\text{min}}^2 + 4096a^4 r_{\text{min}}^4 dl,
\]
\[
H = (-1 + a^2) l^9 Q^6 - 192a^2 l^7 Q^4 r_{\text{min}}^2
- 4096a^4 (3 + a^2) l^5 Q^2 r_{\text{min}}^4 - 262144a^6 l^3 r_{\text{min}}^6.
\]
Substituting Eq. (53) into Eq. (56), we find lastly
\[
F_{\text{min}} + dF_{\text{min}} = \alpha + O(\epsilon)^2,
\]
which is the same as that for the extremal black holes in Eq. (54) for $O(\epsilon)^2$ is the high order terms of $\epsilon$, that is, the near-extremal black holes are also stable. So we can conclude that the weak cosmic censorship conjecture holds for both the extremal and near-extremal black holes in the extended phase space as fermions are adsorbed.

5 Conclusion

In this work, we first studied the dynamics of charged fermions being swallowed by the $(2+1)$-dimensional regular black holes with nonlinear electrodynamics sources. We obtained the energy-momentum relation of the spinor particles by solving the Dirac equation. With this relation, we got the first law, and the second law of thermodynamics in the normal phase space further. We found that the laws of thermodynamics for the regular black holes held. We also discussed the weak cosmic censorship conjecture and found it was valid in the normal phase space for there exist horizons always stopping the singularities to be naked. Especially, the extremal black holes will evolve into the non-extremal black holes.

We also investigated the laws of thermodynamics and weak cosmic censorship conjecture in the extended phase space, where the cosmological parameter is regarded as an extensive variable of the thermodynamic system. We derived the first law of thermodynamics by making use of the conservations of energy and charge. The second law of thermodynamics was checked too by studying the variation of the entropy of the $(2+1)$-dimensional black hole with nonlinear electrodynamic sources. The result showed that the entropy decreased unavoidably. The second law thus is violated in the extended phase space, which is different from the case in the normal phase space. We investigated the weak cosmic censorship conjecture for the extremal and near-extremal black holes in extended phase space and found that it was valid too. But different from that in the normal phase space, the configurations of the regular black holes will not be changed as charged fermions are absorbed for the final states are still extremal and near-extremal black holes.

In addition, from Eq. (3), we know that the nonlinear electrodynamics will affect the structure of the black hole. However, from the results in Sects. 3 and 4, we found the nonlinear electrodynamics dose not affect the laws of thermodynamics and weak cosmic censorship conjecture in both the normal phase space and extended phase space.

Acknowledgements This work is supported by the National Natural Science Foundation of China (Grant No. 11875095) and Basic Research Project of Science and Technology Committee of Chongqing (Grant No. cstc2019jcyj-msxmX0081;cstc2018jcyjA2480).
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