On the dynamics of the torus around the kicked black hole

O. Donmez¹, Anwar Al-Kandari¹, and Ahlam Abu Seedou¹

College of Engineering and Technology, American University of the Middle East, Kuwait

ABSTRACT

There is a special interest to understand the dynamical properties of the accretion disk created around the newly formed black hole due to the supermassive black hole binaries which merge inside the gaseous disk. The newly formed black hole would have a kick velocity at up to thousands of km/s that drives a perturbation on a newly accreted torus around the black hole. Some of the observed supermassive black holes at the center of the Active Galactic Nucleus (AGN) moves with a certain velocity relative to its broader accretion disk. In this paper, the effects of the kicked black holes onto the infinitesimally thin accreted torus are studied by using the general relativistic hydrodynamical code, focusing on changing the dynamics of the accretion disk during the accretion disk-black hole interaction. We have found the non-axisymmetric global mode $m = 1$ inhomogeneity, which causes a spiral-wave-structure, is excited on the torus due to kicked black hole. The higher the perturbation velocity produced by the kicked black hole, the longer time the torus takes to reach the saturation point. The created spiral density waves which rapidly evolve into the spiral shocks are also observed from the numerical simulations. The spiral shock is responsible for accreting matter toward the black hole. Firstly, the spiral-wave-structure is developed and the accretion through the spiral arms is stopped around the black hole. At the later time of simulation, the formed spiral shocks partly causes the angular momentum loss across the torus.

Subject headings: general relativistic hydrodynamics: numerical relativity: spiral wave: torus: instability: kicked black hole

¹College of Engineering and Technology, American University of the Middle East, Kuwait
1. Introduction

In recent years, number of numerical simulations (Baker et. al. 2008; Kornreich & Lovelace 2008; Gerosa et. al. 2018) and astronomical observations (Webb et. al. 2018; Bustillo et. al. 2018; O'Shaughnessy et. al. 2017; Atri et. al. 2019) have revealed the kicked black hole at the center of the black hole X-ray binaries and the AGN. This has occurred due to the asymmetric momentum loss of the black hole, either in a case of black hole mergers or in the release of the gravitation radiation during this process. It was found from the numerical simulation that the gravitational wave recoil velocities range from the smallest possible value up to thousands of $km/s$ (Lousto & Zlochower 2011; Ponce et. al. 2012). The supermassive black hole kick velocities were also measured from the smallest possible value up to $500 km/s$ (Gerosa & Moore 2016). It is also expected to be a kicked velocity from the binary coalescence in the galaxy mergers. Kick velocities up to a few hundred $km/s$ of the stellar mass black holes formed in the core-collapse supernovae were first described by Bekenstein (1973) which might account for their relatively small branching ratio to long GRBs (van Putten 2004).

A smoothed particle hydrodynamics numerical simulation of the impulsive kick transmitted to a black hole on the dynamical evolution of the accretion disk has been done by Ponce et. al. (2012). They have found the higher potential luminosity considering the region $0.4 pc$ from the galactic center for more oblique kick angle. The occurrence of the kicked black hole inside the gaseous disks perturbs the surrounding gas and it could create a signature of the electromagnetic radiation which gives an evidence of merging. The kick directed into the equatorial plane was studied by Corrales et. al. (2010) using the 2D FLASH code. The one-armed spiral shock wave appeared as a characteristic respond and it produced a total luminosity up to $10\%$ of the Eddington luminosity. The generated shocks’ energy can produce electromagnetic flares that could last up to hundreds of thousands years (Shields & Bonning 2008; Schnittman & Krolik 2008).

One of the new observations was made in a distant galaxy which is known as quasar 3C 186 (Chiaberge et. al. 2017). It is the most massive black hole which is kicked out of its central home. The merging two black holes produces gravitational waves which carry energy and angular momentum of the system. Finally, when they merge in a violent collision, and the released energy is enough, the newly formed black hole would be kicked away from the center of the galaxy or even out of the entire galaxy in the opposite side of the location with the highest gravitational waves. The range of the kick can vary depending on the properties of these two colliding black holes, their separation, and inclination angles.

The more observations showed that the broad-line emission system has a different red-shift when it is compared with the narrow-line emission system in several AGNs, Chiaberge1. such as $SDSS J092712.65 + 294344.0$ with a blueshift of $2650 km/s$ (Komossa et. al. 2008), $SDSS J105041.35 + 345631.3$ with a kicked velocity $3500 km/s$ (Shields et. al. 2009b), $CXOC$
$J100043.1 + 020637$ with a kicked velocity $1200\, km/s$ (Civano et al. 2010), and $E1821 + 643$ with a kicked velocity $2100\, km/s$ (Rabinson et al. 2010). When the black hole is kicked, broad-line emitting region is dragged out along with the kicked black hole while the narrow-line region is left behind (Chiaberge et al. 2017). It is believed that Radio-loud AGN with strong relativistic jets could contain the rapidly spinning black holes. Fastly spinning black holes could lead the accreted matter getting close to the black hole horizon and create a strong interaction between the matter and black hole.

High energetic astrophysical sources such as Gamma Ray Bursts (GRBs) (Kumar & Zhang 2015), X-ray binaries (Remillard & McClintock 2006), and AGNs (Reynolds 2014) are powered by whether the accretion mechanism onto the black holes or the spin energy of the black hole (van Putten 2002). A high accretion rate could be presented during the development of the instability. One of those instability is created by gravitational instability (Rice et al. 2003; Lodato & Rice 2005), which is prone the development of the non-axisymmetric instability, called spiral density waves. The time dependent rest-mass density causes the material to concentrate at certain places in the effective potential around the black hole and generates spiral arms. The arms slowly rotates and allow the the disk angular momentum transform outswards (Blaes 2014). Spiral structure mechanism leads developing a global non-axisymmetric modes in torus. The accreted matter belongs to spiral arms are characterized by the high internal temperatures which cause a high-energy emission from such sources as AGNs, X-ray binary, and GRBs (Reynolds 2014; Remillard & McClintock 2006). During the development of spiral structure, some vigorous dynamics of the torus would be observed. The vigorous phenomena happening on the torus dynamics around the kicked black hole would cause the conversion of the gravitational binding energy into thermal and kinetic energies during the process of the losing the angular momentum.

The oscillating properties of tori around the black hole and neutron stars have been investigated theoretically and numerically by perturbing the system (Rezzolla et al. 2003; Abramowicz & Fragile 2013). The numerical simulations of the radially perturbed accretion torus around the rotating and non-rotating black hole are performed by (Lee et al. 2004). The perturbation of the black hole torus system can be observed in different types of perturbations such as angular velocity perturbation (Donmez 2017), spherical shell accretion (Donmez 2015), radial velocity perturbation (Donmez 2014), and Bondi-Hoyle accretion (Donmez 2014), which are all studied to extract the oscillation properties of the torus’ PPI instabilities, and the quasi-periodic frequencies produced during these oscillations. The non-axisymmetric perturbations trigger not only the PPI instability but also spiral waves and cause the excitation of frequencies due to the non-linear coupling of the modes. Doing numerical simulation enables us to study the response of the accreted torus if the kicked is directed into the equatorial plane of the accretion disk.

The first numerical results of perturbation of the initially stable torus due to the kicked black
hole at the center of the system are reported. To this extent, we solve the general relativistic hydrodynamical equations in a fixed Kerr space-time metric to investigate how a torus reacts to the kicked black hole. The outline of the paper is as follows. The computational framework of the problem with the physical variables of stable torus, properties of the black hole, and the radial velocities assumed created by kicked black holes are given in Section 2. In Section 3, dynamical properties of the kicked torus found from the numerical simulations are provided around the rotating and the non-rotating black holes. The spiral wave mode \( m = 1 \) generated by gravitational instability is reported and the dynamical structure of perturbed torus around the rotating black hole with the non-rotating case is compared. Finally, the summary of our results is given in Section 4. Unless otherwise specified, throughout this paper, the geometrized unit, \( c = G = 1 \) has been used.

2. Computational Framework of the 2D Model

To model the effects of the kicked black hole on the stable torus, we solve the General Relativistic Hydrodynamical (GRH) equations on the equatorial plane. GRH equations are described for the relativistic perfect fluid in the spacetime coordinate which can be written in covariant and conservative forms. The covariant form of GRH equation is

\[
\nabla_a T^{ab} = 0, \quad \nabla_a (\rho u^a) = 0, \quad (1)
\]

\( T^{ab} = \rho h u^a u^b + P g^{ab} \) stress-energy-momentum tensor for perfect fluid. The tensor depends on the specific enthalpy \( h = 1 + \epsilon + \frac{P}{\rho} \), the rest mass density \( \rho \), pressure \( P \), the four-velocity \( u^a \), and the four–matrix of the space-time \( g^{\mu\nu} \). The perfect fluid equation of state \( P = (\Gamma - 1) \rho \epsilon \) is used to constrain the system. After using \( 3 + 1 \) formalism, Eq.1 can be written conservative form. The formulation of GRH equation and its numerical solution were discussed with the full details in Donmez (2004) (see also Donmez 2006, 2012, 2014). GRH equations are solved around the non-rotating and rotating black holes in curved space-time which is described by Kerr line element,

\[
ds^2 = -(\alpha^2 - \beta_i \beta^i) dt^2 + 2 \beta_i dx^i dt + \gamma_{ij} dx^i dx^j, \quad (2)
\]

where \( \alpha \) and \( \beta_i \) are the lapse function and shift vectors, respectively, which are

\[
\alpha = \left( 1 + \frac{4M a^2}{A} - \frac{2M}{r} \right)^{1/2}, \quad (3)
\]
and

$$\beta_r = 0, \quad \beta_\theta = 0, \quad \beta_\phi = -\frac{2Ma}{r},$$

(4)

where $A = r^4 + r^2a^2 + 2Mra^2$. $a$ is dimensionless black hole spin parameter and $M$ is the mass of the black hole.

We model the torus’ dynamic defined on the equatorial plane in the limit of an infinitesimal thin disk. In fact, the thickness of the disk is finite and increases with the accretion of the matter towards the black hole. The effect of the disk thickness on the non-axisymmetric instability, spectra, and the black hole spin would be small (Zhou et. al. 2020). It is assumed that the black hole is kicked during one of the astrophysical events processes. As a result of this kick, the oscillating black hole transfers the angular momentum to the accreted torus, and the stable torus is perturbed. In the case of kicked black hole, the matter around the black hole receives an additional velocity. It is handled by focusing on perturbing the radial velocity of the torus at every time step at the center of galaxies or AGNs. The radial perturbations of the initially stable relativistic torus on the equatorial plane around the black holes due to the kicked black hole at the center of galaxies are studied by updating the radial velocity of the torus with the following expressions:

$$v_{\text{new}}^r = v_{\text{old}}^r - \frac{\chi}{r^2} \sin(\phi) \quad \text{when} \quad 0 \leq \phi \leq \pi$$

$$v_{\text{new}}^r = v_{\text{old}}^r + \frac{\chi}{r^2} \sin(\phi) \quad \text{when} \quad \pi \leq \phi \leq 2\pi$$

(5)

where $v_{\text{new}}^r$ and $v_{\text{old}}^r$ are the velocities of torus’ matter in the current and previous time steps respectively, and the amplitude of the oscillation is controlled by $\chi$. Hence, low and high velocity kicks are controlled by changing this parameter.

The non-self-gravitating relativistic torus was first studied analytically by Abramowicz et. al. (1978). A sharp cusp was found for marginally stable accretion disk. The disk was located on the equatorial plane around the black hole. General relativistic torus with a perfect fluid equation of state was numerically discussed in Zanotti & Rezzolla (2003); Zanotti et. al. (2003, 2005); Nagar et. al. (2005). They computed the initial values of stable accreting tori around the non-rotating and the rotating black hole using $\Gamma = 4/3$ to mimic a degenerate relativistic electron gas. In order to have a system in a hydrodynamical equilibrium, the centrifugal and gravitational forces are balanced by the internal pressure in the torus. In this paper, we study the non-stable black hole torus system by perturbing the initially stable and non-self-gravitating relativistic tori which were given in Zanotti & Rezzolla (2003); Zanotti et. al. (2005); Donmez (2014). Initial relativistic tori
used in our numerical simulations are almost stable during the evolution without a perturbation as
seen in Figs.9 and 10 in Donmez (2014). Initial physical parameters of the stable torus and kicked
parameters used throughout this paper are summarized in Table 1.

The initial setups of tori around the non-rotating and the rotating black holes are thin, non-
self-gravitating, and non-axisymmetric and defined on the fixed space-time metric using the Kerr
coordinate. The disk-to-black hole mass ratio chosen as $\frac{M_t}{M} = 0.1$ in our initial models makes the
torus-black hole system stable with respect to the runaway instability. Therefore these models only
respond to perturbations with a small amplitude which creates instabilities on the torus (Daigne
& Font 2004; Zanotti et. al. 2005). Although the mass ratios for the real accreting systems are
significantly lower such as X−ray binaries and AGNs (van Putten & Gupta 2009), the instabilities
created after the applied perturbation could show the same behavior due to the following reasons.
First, the self gravity of the torus is neglected. Second, the fixed background metric will not be
affected due to the falling matter into a black hole. Finally, our torus is located very close to the
black hole horizon. The gravitational time scales would be much larger than the other time scales
in our numerical domain. The 2D perturbation of the relativistic torus on the equatorial plane is
modeled in a polar coordinate. The inner and outer boundaries of the computational domain are
extended from $r_{\text{in}} = 2.8M$ to $r_{\text{out}} = 100M$ around the non-rotating black hole. It goes from
$r_{\text{in}} = 1.7M$ to $r_{\text{out}} = 100M$ for the rotating black hole. The uniform grid is used in the all
models along the radial and the angular directions with $N_r = 1024 \times N_\phi = 512$ cells. The outflow
boundary conditions are imposed at inner and outer boundaries of computational domain. The
time evolution analysis of the stable torus on a fixed space-time metric without any perturbation
shows that the structure of initial torus never changes (does not develop a instability) during the
simulation using the Cowling simulation (Cowling 1934). The neglecting the perturbation of the
gravitational field is known as Cowling approximation. The Cowling approximation of the general
relativistic system can be satisfied by setting the perturbation to be zero in the metric (Yoshida &
Kojima 1997). The cowling approximation guarantees having a unperturbed gravitational field.
The negligible mass accretion rates freeze the growth of instability oscillation mode in the Cowling
approximation (Hawley 1991).

Main parameters of the initial stable torus around the black hole and its perturbation are given
in Table 1 including the model name, black hole spin parameter $a$, torus-to-hole mass ratio $M_t/M$,
the maximum density of the torus $\rho_c(\text{geo})$, constant specific angular momentum $\ell_0$, cusp location
$r_{\text{cusp}}(M)$, $r_c$ and $r_{\text{iso}}(M)$ represent the location of the maximum value of density and the inner
radii of the last stable orbit respectively, and $t_{\text{orb}}$ is the orbital period at $r = r_c$. And $\chi(\text{geo})$
is a free parameter to control the perturbation on the torus radial velocity. The adiabatic index
$\Gamma = 4/3$, the polytropic constant $K(\text{geo}) = 4.969 \times 10^{-2}$ is used in all simulations. We have used
some negligible values for the initial density and pressure with a zero radial and angular velocities
outside the torus. Mainly, we have constructed two different initial stable tori. First one is the
torus around the rotating black hole and the second one is around the non-rotating black hole. The constant polytropic index and specific angular momentum are used in all our models.

The characterization and analysis of the instability are done by using the Fourier mode analysis explained in Donmez (2014). Mainly, we calculate the power mode by computing the real and imaginary parts in Fourier transfer in order to analyze the azimuthal mode $m$.

3. Numerical Results

In this work, we have concentrated on a black hole-torus system which was perturbed by a kicked black hole, believed to occur at the center of AGN, and have conducted the dynamical calculations of the kicked torus. We neglect the self-gravity of the torus material and magnetic field along with using fixed space-time metric around the rotating and the non-rotating black holes. The immediate change in the kinetic energy of the torus’ matter due to the kick would adjust the new equilibrium configuration. The change would also cause the transferring angular momentum of the torus which liberates the gas falling toward or away from the black hole. The redistribution of the angular momentum of the accreted torus due to the oscillating black hole allows the newly formed structures of the disk to evolve. During the evolution, the mass would be transferred, which can lead to forming an spiral arms and a quasi-periodically oscillation of the disk.

3.1. Rotating Black Hole

In order to study the effect of the kicked black hole onto the resulting torus dynamical evolution, we perform the number of numerical simulations around the rotating black hole for the

| Model  | $a/M$ | $M_t/M$ | $\rho_c(geo)$ | $\ell_0$ | $r_{cusp}(M)$ | $r_{iso} - r_c(M)$ | $t_{orb}(M)$ | $r_{out}(M)$ | $\chi(geo/M^2)$ |
|--------|-------|---------|---------------|---------|---------------|------------------|--------------|--------------|----------------|
| K09A   | 0.9   | 0.1     | $7.981\times10^{-2}$ | 2.60    | 1.78          | $1.78 - 3.40$    | 39.4         | 19.25        | 0.01           |
| K09B   | 0.9   | 0.1     | $7.981\times10^{-2}$ | 2.60    | 1.78          | $1.78 - 3.40$    | 39.4         | 19.25        | 0.001          |
| K09C   | 0.9   | 0.1     | $7.981\times10^{-2}$ | 2.60    | 1.78          | $1.78 - 3.40$    | 39.4         | 19.25        | 0.0001         |
| K00A   | 0.0   | 0.1     | $1.140\times10^{-4}$ | 3.80    | 4.57          | $4.57 - 8.35$    | 151.6        | 15.889       | 0.01           |
| K00B   | 0.0   | 0.1     | $1.140\times10^{-4}$ | 3.80    | 4.57          | $4.57 - 8.35$    | 151.6        | 15.889       | 0.001          |
| K00C   | 0.0   | 0.1     | $1.140\times10^{-4}$ | 3.80    | 4.57          | $4.57 - 8.35$    | 151.6        | 15.889       | 0.0001         |

Table 1: The physical parameters of initially stable accretion torus, black holes, and perturbation parameter used in our numerical simulations.
different values of the perturbation parameter $\chi$. The torus’ matter could escape from the gravity of the black hole or remain the bound depending on the direction of the kick and distance.

Morphology and dynamical evaluation of perturbed torus around the rotating black hole for model $K09C$ is given in Fig.1. This figure represents the evaluation of the logarithmic rest-mass density on the equatorial plane at different snapshots. The last snapshot given in the figure shows the distribution of matter after the matter has almost completed 446 orbital periods around the black hole. The orbital period of the models for each model is given in Table 1. As it is seen from the snapshots, perturbation due to the kicked black hole initially triggers the spiral arms around the equilibrium. The instability is extended all over the torus surface. The oscillating torus, seen in Fig.3 starts filling the region and induces a small accretion process towards the black hole. This process does not create a big shock wave to reduce the rest-mass density of the torus until the snapshot at $t = 6070M$, as seen in Fig.1. Later, the oscillation amplitude of the torus is getting bigger during the time evolution, and it causes the creation of the spiral shock wave. The spiral wave causes the angular momentum transfer and the matter the falling towards or away from the black hole. Having a shock wave also causes kinetic energy to convert to thermal energy. The spiral shock waves can be clearly seen in Fig.1 starting from the snapshot $t = 7324M$.

The spiral density wave traveling with same angular velocity of matter on the torus creates co-rotation radius. Inside this radius, matter travels slowly when it is compared with outside. Therefore, negative angular momentum appears inside the wave which causes the matter to fall towards the black hole.

The position of the maximum rest-mass density in the disk during the evolution is shown in Fig.2 around the black hole for different models. As seen from the figure, the effect of kicked black hole onto the disk dynamic is almost the same for different kick values which are represented with $\chi$ in Table 1. However, the positions of the maximum densities make nonlinear oscillation during the evolution. The oscillation might be appeared due to the growth of the gravitational instability (spiral) mode. On the other hand, the maximum rest-mass density of the stable torus was initially at $3.4M$. After the perturbation is applied, the position of the maximum rest-mass density is pushed out from the black hole horizon with a significant distance $r \sim 9M$ around $t \sim 10000M$.

Transferred angular momentum from the kicked black hole to the torus starts to affect the torus dynamics after a few dynamical periods ($\sim 4t_{orb}$). Initial perturbation causes instability on the dynamic of the torus and the smaller the transferred angular momentum, the more instability develops in later time of simulation seen in Fig.2 for the rotating black hole. A higher black hole spin creates a larger centrifugal barrier so that it causes to be less accretions towards to the black hole.

The mass accretion rates in arbitrary unit around the rotating black hole in case of different
Fig. 1.— The rest-mass density of the torus on equatorial plane with linearly spaced isocountours around the rotating black hole for the model $K09C$. The spiral structure seen in snapshot is a mechanism to have a $m = 1$ mode. The domain is $[X_{\text{min}}, Y_{\text{min}}] \to [X_{\text{max}}, Y_{\text{max}}] = [-40M, -40M] \to [40M, 40M]$. 
perturbation velocities are shown in Fig.3. It is computed at inside the torus at $r = 3.8M$ close to the black hole horizon. While the accretion rate $dM/dt > 0$ represents the perturbed torus’ matter falling towards the black hole, the matter moves away from the black hole for $dM/dt < 0$. Fig.3 has a clear indication of the torus’ matter oscillates due to black hole kick. All the three cases in Fig.3 indicate that after significant oscillation phase like sinusoidal functions, the mass accretion rate tends to a fairly constant value. It is found that the amplitude of the perturbation makes some delays in case of having an instability but these perturbations do not strongly responsive to accretion rate in overall.

The torus with a constant specific angular momentum around the black hole is unstable in the non-axisymmetric global mode (Chen et.al. 2021). The non-axisymmetric global mode is produced on the torus due to kicked black hole. In order to reveal the growth of this global mode $m = 1$, we compute the power mode amplitude from the density of torus. Power mode is basically computed by finding the Fourier decomposition of the azimuthal distribution of the torus density during the time evolution by using the following equation Donmez (2014)

$$P_m = \frac{1}{r_{out} - r_{in}} \int_{r_{in}}^{r_{out}} \ln\left(\left[\text{Re}(w_m(r))\right]^2 + \left[\text{Im}(w_m(r))\right]^2\right) dr,$$

where $r_{in}$ and $r_{out}$ are inner and outer boundaries of the computational domain, respectively. The real and imaginary parts are $\text{Re}(w_m(r)) = \int_0^{2\pi} \rho(r,\phi)\cos(m\phi)d\phi$ and $\text{Im}(w_m(r)) = \int_0^{2\pi} \rho(r,\phi)\sin(m\phi)d\phi$.

As seen in Figs.3 and 4, the magnitude of perturbation influences the process of stabilization of the matter and saturation time because the angular momentum of the initially stable torus faces a big influence due to the kicked black hole. It is clearly found that the the density wave seen for $m = 1$ in exponential growth mode is developed around the rotating black hole. It is shown in Fig.4 that the saturation time is slightly different for different perturbation velocities. The higher the perturbation velocity, the more delayed it is to reach the saturation point. However, the amplitudes of mode $m = 1$ for different models are almost the same. The growing in instability can happen just by oscillating the black hole-torus system.

The space time diagrams for $(\phi - t)$ and $(r - t)$ of the torus logarithmic rest-mass density during the time evolution around the rotating black hole to see how and where the spiral arms are created are shown in Figs.5 and 6, respectively. In order see more detailed dynamics of the accreted torus, whole evolutions of the numerical simulations are separated into three pieced. In the early stages of the simulation for model $K09C$, the maximum value of density of torus would not be effected too much from the perturbation due to the kicked black hole, seen in Figs.5 and 6. However, the spiral arms are created during the evolution and this would lead to the collimation of the radiation into beams. These types of phenomena could be used to explain why the super
Fig. 2.— The locations of the maximum rest-mass density for models $K09A$ and $K09B$. It gives the position of the maximum rest-mass density at the radial distance as a function of time.

Fig. 3.— The mass accretion rate at $r = 3.8M$ for the models $K09A$, $K09B$, and $K09C$. 
Fig. 4.— Power mode evolution of $m = 1$ non-axisymmetric mode for the models $K09A$, $K09B$, and $K09C$. It saturates around $t = 1350M$ and the saturation amplitude is almost constant during the time evolution. The quasi periodic frequencies could be determined after the saturation point.
Eddington luminosity is observed close to the black hole horizon. The spiral shocks created the hot corona in the inner region of the accretion disk due to frictional and compressional heating. The accretion due to spiral shock varies highly with time and this type of accretion is responsible for the irregular behavior and the quasi-periodic oscillation observed in black hole-torus systems in galaxies and AGN. As it is seen in the middle panel of Fig.5, the oscillation is excited around the maximum of the epicyclic frequency which is $10M$ in the rotating black hole. Additionally, high energetic interactions inside the spiral arms would scatter emission from the region close to the black hole.

Initial torus with a constant specific angular momentum is rotating around the black hole and perturbed by the black hole itself. These types of perturbations would cause a redistribution of the specific angular momentum of the torus. In order to mimic the time evolution of the redistribution of the specific angular momentum and clarify the effects of spiral shocks, we plot the variation of angular velocity in Fig.7. As it is seen in the top panel of the figure, the instability grows early time of simulation $t \sim 300M$, the angular velocity is redistributed and it causes rotating a strong spiral shock which can be seen after $t = 1986M$. From our numerical simulations it is obvious that the influence of perturbation on angular velocity creates a strong spiral shock which is excited in the disk and propagate through the stable circular orbit. The stronger the spiral shock would lead the matter the accreting toward the black hole and might be very important to explain the spectral properties of the black hole candidates. Bottom panel of Fig.7 has a clear evidence about the creation of a strong shock at certain times. The angular velocity is varying between the smallest and the highest values of the color bar at $t \sim 1500M$, $t \sim 2200M$, $t \sim 3100M$, $t \sim 4700M$, etc.

### 3.2. Non-rotating Black Hole

The dynamical evolutions of the rest-mass densities on the equatorial plane after the perturbation show a big difference around the rotating and non-rotating black hole as seen in Figs.1 and 8, respectively. The maximum rest mass density of the accreted torus stays almost below its initial value during the time evolution. Having a rotating or non-rotating black hole causes a drastic change on the dynamics of the accreted matter and spiral waves created on the torus.

The kicked non-rotating black hole also leads some excitations of the specific unstable non-axisymmetric modes. In order to compute the azimuthal number $m$ of the spiral wave mode, the dynamical evolution of the torus is observed for different range of perturbation parameters seen in Eq.5. After the black hole-torus system is perturbed, the new equilibrium stage is almost reached at the saturation time $t = 750M$, seen in Fig.9. It shows that the kicked black hole enhances the strength of the fastest growing mode $m = 1$. Spiral wave instability driven on the torus can be characterized by the presence of the radial pressure gradient. As it is seen in Fig.4, the matter that
Fig. 5.— The maximum value of the logarithmic density of the perturbed torus for model K09C at fixed $\phi = 0.0245$ is plotted as a function of time. The time axis is separated into three snapshots and each snapshot shows a different range of times within the simulation which enables us to see more details about the internal structure of the disk dynamics, especially in later times of the simulation. The kicked black hole creates an instability in the inner region of torus and it causes the torus’ matter either falling into the black hole or going away from the black hole.

Fig. 6.— Same as Fig.5 but it is for fixed $r = 3.42M$. phi ($\phi$) represents the angular direction.
Fig. 7.— Top panel: Angular profile of the three-velocity along the angular direction $\phi$ with different times at $r = 3.42M$ for model K09A. While the black dashed curve shows the angular velocity for the initial torus, the others represent the evolution of the velocity during the time. Bottom panel: The logarithmic angular velocity is represented by a surface whose colors display the variation of the spiral shock along the angular direction.
Fig. 8.— The rest-mass density of the torus on equatorial plane with linearly spaced isocountours around the non-rotating black hole for the model $K_{00}C$. The domain is $[X_{\text{min}}, Y_{\text{min}}] \rightarrow [X_{\text{max}}, Y_{\text{max}}] = [-40M, -40M] \rightarrow [40M, 40M]$. 
is getting closer to the black hole horizon would lead the stronger spiral density wave mode.

### 3.3. Comparison of Rotating and Non-rotating Black Hole Cases

The rest-mass densities for models at different snapshots are given around the rotating and non-rotating black holes in Figs.1 and 8 respectively. As it is seen from these two cases, the effect of black hole spin onto the torus as well as the morphology of torus after perturbation are clearly seen. As it is also seen in Fig.10, the matter is pushed away from the center of the black hole due to the oscillating black hole being connected to the black hole horizon with a spiral wave, even in the last snapshot of simulation for the rotating black hole this can be seen. Similarly, we can also clearly see the structure of the spiral arms close to the non-rotating black hole, shown in Fig.8. We observed significant differences in the morphology of the disk when the black hole is rotating even though we used the same initial perturbation parameters.

Overall, the mass accretion rates computed at the inner boundaries around the non-rotating and the rotating black holes show similar behavior during the evolution seen in Figs.10 and 11, but our numerical simulations of the perturbed torus due to the kicked black hole indicate that transition from unstable state to steady-state shows clear evidence of dependencies on the black hole spin. We find that the accretion rates are consistently bigger for the larger black hole spin. The reason for this is that the maximum density of the initial stable torus around the rotating black hole is closer to the black hole horizon. Kicked black hole makes a big impact on the torus dynamics whenever the black hole is kicked. It is found that a higher spin produces a higher accretion rate, which implies the higher critical temperature, during the evolution. The increased in the mass accretion rate causes the emitting radiation in the high-energy. Therefore, the torus-black hole system may be a source of continues high energetic emissions in several channels, possibly in association with GRBs (van Putten 2002).

The saturation time of the instability around the rotating black hole is $t = 1350$, but it is almost half $t = 750$ around the non-rotating black hole. This means that the saturation times and growth of the instability strongly depend on the black hole spin. After the saturations, the torus still keeps losing the matter either towards or away from the black hole due to redistribution of the angular momentum and we still witness the drastic changes in the torus dynamics during the evolution. Additionally, as it is seen in Figs. 4 and 9 the amplitude of the power of the $m = 1$ mode around the rotating black hole is larger than that for the non-rotating black hole. It is a clear evidence for dependency of the mode amplitude on the black hole spin.

Studying the angular momentum transfer around the black hole would tell us how the accretion happens through the spiral shock wave. In order to show the effect of the black hole spin
Fig. 9.— The same as Fig. 4 but it is for the kicked torus around the non-rotating black holes.

Fig. 10.— The location of maximum value of density of the torus during the evolution around the non-rotating and rotating black holes.
on angular momentum transfer, we compute the angular momentum flux at the location $r = 6M$ inside the torus along the spherical surface. The azimuthal component of the angular momentum flux in the equatorial plane is

$$\frac{dL}{dt} = -\int_{0}^{2\pi} \tilde{\alpha} \sqrt{\gamma \rho u^r u^\phi} d\phi. \quad (7)$$

Here, we will find out how the perturbed torus transports the angular momentum along the equatorial plane; radially inward or outward and its dependencies to the black hole spin parameter.

The perturbed torus around the black hole creates a spiral shock wave around the rotating and non-rotating black holes. The angular momentum of the rotating gas can play an important role in the moving of gas away from the black hole or towards it. The angular momentum of the initial torus is constant in the begging of simulation. After perturbation is felt by the torus, it is clearly seen in Fig.12 that the angular momentum flux varies depending on the black hole rotation parameter. The strength of oscillating angular momentum flux is at least 100 times bigger in case of torus around the rotating black hole. The higher the black hole spin parameter leads the bigger oscillation in the angular momentum transfer. The angular momentum flux almost oscillates around zero value for the rotating black hole. The spikes of the angular momentum transport has a strong correlation with presence of spiral shock, seen in Fig.7.

In addition to power mode evolutions on the non-axisymmetric modes around the rotating and non-rotating black hole given in Figs.4 and 9, we explore more detail information about the local and non-axisymmetric instability using the rest-mass density. The rest-mass density is one of the perturbed physical variable of torus when the black hole oscillates inside the galaxy. The time evolutions of the maximum value of the rest-mass density in logarithmic scale around the non-rotating and rotating black holes are given in Fig.13. The non-axisymmetric perturbations display a significant growth during certain time interval (until $t \sim 10000M$) in both cases. It almost oscillates around some equilibrium point after this time. The amplitude, however, strongly depends on the black hole spin parameter.

4. Discussion and Conclusion

We have studied the dynamical evolution of the torus around the black hole that undergoes perturbation due to the kick, presumably it is the gravitational radiation from the mergers of the black holes. There is a number of black hole-accretion disk systems that might be considered as candidates for the perturbed torus due to the kicked black hole. The results found in our numerical
Fig. 11.— Mass accretion rates for models computed in the inner boundary of the computational domain for models $K00A$ and $K09A$.

Fig. 12.— The time evolution of the angular momentum flux distribution versus time at the location $r = 6M$ around the rotating and non-rotating black holes.
simulations are given in geometrized units; therefore, they might be used to explain many of the perturbed the kicked black hole-torus systems.

In order to understand the dynamical feature of the torus with respect to the perturbation amplitude produced by the kicked black hole, we have varied the parameter $\chi$, which is a free parameter to control the perturbation on the torus radial velocity, and it is found that different values of $\chi$ produce almost the same behavior for given black hole-torus system. Only some time delays when the spiral density wave instability was reached the saturation point and types of shock waves created during the time evolution were observed. It is clearly found that the instability seen for $m = 1$ in the exponential growth mode is developed around the rotating and non-rotating black holes. The non-axisymmetric growing mode $m = 1$ causes the formation of the spiral shock wave appeared in the rotating matter around the black holes (Mewes et al. 2016).

The mass accretion rates, which appear around rotating black hole almost 100 times bigger than the one around the non-rotating black hole, show that the accretion associated with the saturation of spiral density growth mode occurs slightly earlier in case of the rotating black holes. Additionally, the location of the maximum density of the perturbed torus around the rotating black hole gets closer with the non-rotating case after they reach the saturation point, even though they are initially operated with a distance of $3.40M$.

We have observed the instability in our models which have developed due to the kick that the center of black hole-torus system receives. The kicked black hole triggers the radial non-axisymmetric oscillation on the disk. And these oscillations on the equilibrium stage of the torus lead to the spiral density wave. When the growing spiral azimuthal mode occurs just before or after reaching of the saturation point could trigger it not only in the torus, but also in the black hole. It would cause either an increase or an decrease in the black hole kick velocity (Mewes 2016).

The rest-mass density oscillation, the growth of instability and the resulting spiral shock waves that we have found in our numerical simulations during the time evolution could play an important role in explaining the vigorous X-rays phenomena observed from the galactical nuclei and the quasars.

Acknowledgments

The authors are grateful to the anonymous referee for constructive comments on the original manuscript. All simulations were performed using the Phoenix High Performance Computing facility at the American University of the Middle East (AUM), Kuwait.
REFERENCES

Baker, J. G., Boggs, W. D., & Centrealla, J. 2008, ApJ, 668, 2, 1140.

Kornreich, D. A. & Lovelace, R. V. E. 2008, ApJ, 681, 1, 104-12.

Gerosa, D., Hebert, F., & Stein, L. C., 2018, Phys. Rev. D, 97, 104049.

Webb, J.J, Leigh, N. W. C., Singh, A., Ford, K E S., McKernan, B., & Bellovary, J. 2018, MNRAS, 474, 3, 3835-3846.

Bustillo, J. C., Clark, J. A., Laguna, P., & Shoemaker, D. 2018, Phys. Rev. Lett., 121, 191102.

O’Shaughnessy, R., Gerosa, D., & Wysocki, D. 2017, Phys. Rev. Lett., 119, 011101.

P Atri, J C A Miller-Jones, A Bahramian, R M Plotkin, P G Jonker, G Nelemans, T J Maccarone, G R Sivakoff, A T Deller, S Chaty, M A P Torres, S Horiuchi, J McCallum, T Natusch, C J Phillips, J Stevens, & S Weston 2019, MNRAS, 389, 3116-3134.

Komossa, S., Zhou, H., & Lu, H. 2008, ApJ, 678, L81.

Shields, G. A., Rosario, D. Smith, K. et. al. 2009b, ApJ, 707, 936.

Civano, F., Elvis M., Lanzuisi, G. et. al. 2010, ApJ, 717, 209.

Rabinson, A., Young, S. Axon, D. J., Kharb, P., & Smith, J. E. 2010, ApJ, 717, L122.

Chiaberge et. al. 2017, A&A, 600, A57. Schnittman, J., & Gezari, S. 2009, ApJ, 695, 363.

Lousto, C. O. & Zlochower, Y. 2011, Phys. Rev. Lett., 107, 231102.

Gerosa, D. & Moore, C. J. 2016 Phys. Rev. Lett. 117, 011101.

Bekenstein, J. D. 1973, ApJ, 183, 657-664.

van Putten, M. H. P. M. 2004, ApJ, 611, L81-L84.

Ponce, M., Faber, J. A., & Lombardi, J. C. 2012, 745, 71.

Corrales, L., R., Haiman, Z., & MacFadyen, A. 2010, MNRAS, 404, 947.

Shields, G. A. & Bonning, E. W. 2008, ApJ, 682, 758.

Schnittman, J. D. & Krolik, J. H. 2008, ApJ, 684, 835.

Kumar, P & Zhang, B. 2015, Physics Reports, 561, 1.
Remillard, R. A. & McClintock, J. E. 2006, ARA&A, 44, 49.
Reynolds, C. S. 2014, Spac. Scienc. Review, 183, 277.
van Putten, M. H. P. M. 2002, ApJ, 575, L71-L74.
Rice, W. K. M., Armitage, P. J., et. al. 2003, MNRAS, 339, 1025.
Lodato, G. & Rice, W. K. M. 2005, MNRAS, 358, 1489.
Blaes, O. 2014, Space. Sci. Rev. 183, 21.
Donmez, O. 2014, MNRAS 438: 846-858.
Daigne, F. & Font, J. A. 2004, MNRAS, 349, 841.
van Putten, M. H. P. M. & Gupta, A. C. 2009, MNRAS, 394, 2238-2246.
Rezzolla L., Yoshida S., Macarone T. J., & Zanotti, O. 2003, MNRAS, 344, L37.
Abramowicz, M. A. & Fragile, P. C. 2013, Living Reviews in Relativity, 16.
Lee W. H., Abramowicz M. A., & Kluzniak W. 2004, ApJL, 603, L93.
Chen, E., Yu, S. Y. , & Ho, L. C. 2021, ApJ, 906, 19.
Donmez, O. 2017, MPLA, 32(20), 1750108.
Donmez, O. 2015, MPLA, 30(14), 1550071.
Donmez, O. 2014, MPLA, 23(5), 1450050.
Donmez, O. 2021, EJPC, 81(2), 1-13.
Donmez, O. 2004, Astrophysics and Space Science, 293, 323-354.
Donmez 2006, O. AM&C 181, 256-270.
Donmez 2012, O. MNRAS, 426, 1533D.
Zhou, M., Abdikamalov, A. B. et. al. 2020, MNRAS, 496, 497-503.
Abramowicz, M. A., Jaroszynski, M. & Sikora, M. 1978, A&A,63, 221-224.
Zanotti, O & Rezzolla, L. 2003 MSAIS, 1, 192.
Zanotti, O., Font, J. A., Rezzolla, L. & Montero, P. J. 2005, MN-RAS, 356, 1371.
Zanotti O, Rezzolla L & Font J A 2003, MNRAS, 341, 832.

Nagar, A., Font, J. A., Zanotti, O. & de Pietri, R. 2005, PRD, 72, 024007.

Cowling T. G. 1934 MNRAS, 94, 39–48.

Yoshida, S. & Kojima, Y. 1997, MNRAS, 289, 117–122

Hawley, J. F. 1991, ApJ, 381, 496.

Bursa, M. 2005, Astron.Nachr., 326, 849-855.

Punsly, B. 1999, ApJ, 527, 609-623.

Mewes, V. 2016, PhD Thesies.

Mewes, V., Galeazzi, F., Font. J. A., Montero, P. J., & Stergioulas, N. 2016, MNRAS, 461, 2480.
Fig. 13.— The logarithmic maximum rest-mass density versus time for Models $K^{00}C$ and $K^{09}C$. 