New observations in the BRST analysis of dynamical non-Abelian 2-form gauge theory

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Abstract – We generalize the \textit{usual} gauge transformations connected with the 1-form gauge potential to the Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST symmetry transformations for the four (3+1)-dimensional (4D) topologically massive non-Abelian gauge theory that incorporates the famous \((B \wedge F)\) term where there is an explicit topological coupling between 1-form and 2-form gauge fields. A novel feature of our present investigation is the observation that the (anti-)BRST symmetry transformations for the auxiliary 1-form field \((K_{\mu})\) and 2-form gauge potential \((B_{0i})\) are \textit{not} generated by the (anti-)BRST charges that are derived by exploiting \textit{all} the relevant (anti-)BRST symmetry transformations corresponding to \textit{all} the fields of the present theory. This observation is a new result because it is drastically different from the application of the BRST formalism to (non-)Abelian 1-form and Abelian 2-form as well as 3-form gauge theories.

Introduction. – In recent years, there has been a great deal of interest in the study of higher \(p\)-form \((p = 2, 3, 4, \ldots)\) gauge theories because of their relevance in the context of (super)string theories and related extended objects (see, \textit{e.g.}, \cite{1,2}). The merging of the 1-form and 2-form gauge fields has provided us with the topological massive gauge theories in 4D. In such (non-)Abelian theories, the 1-form gauge field acquires mass in a very natural fashion \cite{3}. As a consequence, it provides an alternative to the method of mass generation by Higgs mechanism in the context of the standard model of high-energy physics.

In view of the fact that the Higgs particles of the standard model have not yet been observed experimentally, the above 4D topologically massive (non-)Abelian theories \cite{3–7} have attracted a renewed interest in the recent past. In this context, it is pertinent to point out that we have studied the 4D topologically massive \textit{Abelian} 2-form gauge theories within the frameworks of superfield and BRST formalisms \cite{8,9} and derived the absolutely anti-commuting (anti-)BRST symmetry transformations. We have also considered the dynamical non-Abelian 2-form theory within the superfield scheme \cite{10} where we have exploited its “scalar” and “vector” gauge symmetry transformations to derive the proper (anti-)BRST symmetry transformations. In a very recent publication \cite{11}, we have derived the coupled Lagrangian densities that respect the above off-shell nilpotent and absolutely anti-commuting (anti-)BRST transformations corresponding to the “scalar” gauge symmetry.

The purpose of our present letter is to derive the off-shell nilpotent symmetry generators \textit{(i.e.} conserved charges) for the above off-shell nilpotent (anti-)BRST symmetry transformations \cite{11} and derive their corresponding BRST algebra. One of the novel observations of our present endeavor is the finding that the generators of the above nilpotent symmetry transformations do \textit{not} generate the symmetry transformations corresponding to the auxiliary vector field \(K_{\mu}\) and the \(B_{0i}\) component of the anti-symmetric tensor gauge field \(B_{\mu\nu}\). We provide the possible reasons behind this novel observation in the language of the constraints of the theory. We would like to lay emphasis on the fact that our present novel observation, in the context of the dynamical non-Abelian 2-form theory, is a new result and it is drastically different from the

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application of the BRST formalism to (non-)Abelian 1-form [12,13] and Abelian 2-form as well as 3-form gauge theories in 4D [8,9,14].

Our present paper is organized as follows. In the second section, we discuss the local gauge symmetry transformations and their generator corresponding to the 1-form non-Abelian gauge field. Our section three is devoted to the up-gradation of the above gauge symmetry transformations to the off-shell nilpotent BRST symmetries and derivation of the corresponding conserved charge. Section four deals with the anti-BRST symmetries and their generator. The ghost symmetries and corresponding generator are discussed and derived in our section five. We also deduce BRST algebra, in this section, in a simple manner. Finally, in section six, we make some concluding remarks.

**Preliminaries: usual local gauge symmetry transformations and their generator.** – We begin with the Lagrangian density of the 4D topologically massive non-Abelian gauge theory\(^1\) that incorporates the topological mass parameter \(m\) through the celebrated \((B \wedge F)\) term. This is given by [6,7]

\[
\mathcal{L}_0 = -\frac{1}{4} F_{\mu \nu} \cdot F_{\mu \nu} + \frac{1}{12} H_{\mu \nu \eta} \cdot H_{\mu \nu \eta} + \frac{m}{4} \varepsilon^{\mu \nu \eta \kappa} B_{\mu \nu} \cdot F_{\eta \kappa},
\]

(1)

where the 2-form curvature \(F^{(2)} = dA^{(1)} + iA^{(1)} \wedge A^{(1)}\) defines the curvature tensor \(F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - (A_{\mu} \times A_{\nu})\). In the above, the 2-form \(F^{(2)} = \frac{1}{2} (dx^\mu \wedge dx^\nu) F_{\mu \nu}\) and the 1-form \(A^{(1)} = dx^\mu A_\mu\) define the \(SU(N)\) valued curvature tensor \(F_{\mu \nu}\) and gauge potential \(A_{\mu}\), respectively. Similarly, the 3-form \(H^{(3)} = \frac{1}{3} (dx^\mu \wedge dx^\nu \wedge dx^\eta) H_{\mu \nu \eta}\) defines the curvature tensor

\[
H_{\mu \nu \eta} = (\partial_\mu B_{\nu \eta} + \partial_\nu B_{\mu \eta} + \partial_\eta B_{\mu \nu}) - [(A_{\mu} \times B_{\nu \eta}) + (A_{\nu} \times B_{\mu \eta})] - [(K_\mu \times F_{\nu \eta}) + (K_\nu \times F_{\mu \eta})],
\]

(2)

in terms of the compensating non-Abelian 1-form \((K^{(1)} = dx^\mu K_{\mu} \cdot T)\) auxiliary field \(K_{\mu} = K_{\mu} \cdot T\), the non-Abelian 2-form \([B^{(2)} = \frac{1}{2} (dx^\mu \wedge dx^\nu) B_{\mu \nu} \cdot T]\) gauge potential \(B_{\mu \nu} = B_{\mu \nu} \cdot T\) and the non-Abelian 2-form curvature tensor \(F_{\mu \nu} = F_{\mu \nu} \cdot T\) for the non-Abelian 1-form gauge field \(A_{\mu} = A_{\mu} \cdot T\). Here the \(SU(N)\) generators \(T_a\) satisfy the Lie-algebra \([T^a, T^b] = i f^{abc} T^c\) where \(f^{abc}\) are the structure constants that have been chosen to be totally anti-symmetric in indices \(a, b, c\) for the semi-simple Lie group \(SU(N)\) [13]. The above Lagrangian density (1) respects \((\delta_{\mathcal{L}_0} = 0)\) the usual local gauge symmetry transformations \((\delta_{gt})\) corresponding to the 1-form gauge field as [6,7]

\[
\begin{align*}
\delta_{gt} A_\mu &= D_\mu \Omega, & \delta_{gt} B_{\mu \nu} &= -(B_{\mu \nu} \times \Omega), \\
\delta_{gt} K_\mu &= -(K_\mu \times \Omega), & \delta_{gt} F_{\mu \nu} &= -(F_{\mu \nu} \times \Omega), \\
\delta_{gt} H_{\mu \nu \eta} &= -(H_{\mu \nu \eta} \times \Omega),
\end{align*}
\]

(3)

where \(\Omega = \cdot T \equiv \Omega^a T^a\) is the \(SU(N)\) valued infinitesimal “scalar” gauge parameter\(^2\) and the covariant derivative \(D_\mu \Omega = \partial_\mu \Omega - (A_\mu \times \Omega)\).

According to Noether’s theorem, the above infinitesimal continuous symmetry transformations lead to the following conserved current:

\[
J^\mu_{gt} = \frac{m}{2} \varepsilon^{\mu \nu \eta \kappa} B_{\nu \eta} - F_{\mu \nu} - (H_{\mu \nu \eta} \times K_\eta) \cdot (D_\nu \Omega) - \frac{1}{2} (H_{\mu \nu \eta} \times B_{\nu \eta}) \cdot \Omega.
\]

(4)

To prove the conservation law of the above current, it is convenient to re-express the above Noether current as given below

\[
J^\mu_{gt} = \partial_\nu \left[ \frac{m}{2} \varepsilon^{\mu \nu \eta \kappa} B_{\eta \kappa} - F_{\mu \nu} - (H_{\mu \nu \eta} \times K_\eta) \cdot \Omega - F_{\mu \nu} \cdot \Omega \right] + D_\nu \left[ F_{\mu \nu} + (H_{\mu \nu \eta} \times K_\eta) - \frac{m}{2} \varepsilon^{\mu \nu \eta \kappa} B_{\eta \kappa} \right] \cdot \Omega - \frac{1}{2} (H_{\mu \nu \eta} \times B_{\nu \eta}) \cdot \Omega.
\]

(5)

It can be checked that \(\partial_\mu J^\mu_{gt} = 0\) if we use the following Euler-Lagrange equations of motion derived from the starting Lagrangian density (1):

\[
D_\mu \left[ F_{\mu \nu} + (H_{\mu \nu \eta} \times K_\eta) - \frac{m}{2} \varepsilon^{\mu \nu \eta \kappa} B_{\eta \kappa} \right] = \frac{1}{2} (H_{\mu \nu \eta} \times B_{\nu \eta}),
\]

(6)

\[
D_\mu H_{\mu \nu \eta} = \frac{m}{2} \varepsilon^{\mu \nu \eta \kappa} F_{\rho \sigma}, \quad (H_{\mu \nu \eta} \times F_{\nu \eta}) = 0.
\]

The above conserved current leads to the derivation of the conserved charge that turns out to be the generator of a part of the gauge transformations (3). To corroborate the above statement, it can be checked that (5) leads to the derivation of the generator \((Q_{gt}) = \int d^3 x J^3_{gt}\) of gauge transformations, as

\[
Q_{gt} = \int d^3 x \left[ \frac{m}{2} \varepsilon^{0 ij k} B_{i j k} - F_{0 i j} - (H_{0 i j} \times K_j) \right] \cdot (D_\nu \Omega) - \frac{1}{2} (H_{0 i j} \times B_{i j}) \cdot \Omega.
\]

(7)

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\(^1\)We adopt the convention and notations such that the background 4D spacetime metric is flat with signature \((+1, -1, -1, -1)\) so that the dot product between two non-null vectors is \(\mathbf{A} \cdot \mathbf{B} = A_\mu B^\mu = A_\mu B_{-\mu}\). Here the Greek indices \(\mu, \nu, \eta, \ldots = 0, 1, 2, 3\) and the Latin indices \(i, j, k, \ldots = 1, 2, 3\). In the algebraic space, for the sake of brevity, we choose dot and cross products between two vectors as \(\mathbf{P} \cdot \mathbf{Q} = P^\mu Q_\mu\) and \((\mathbf{P} \times \mathbf{Q})^a = f^{abc} P^b Q^c\) where \(a, b, c, \ldots = 1, 2, \ldots (N^2 - 1)\) for the \(SU(N)\) Lie algebra.

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\(^2\)In addition to the local “scalar” gauge symmetry transformations (3), there also exists the “vector” gauge symmetry transformations \((\delta_v)\): \(\delta_v A_\mu = 0, \delta_v F_{\mu \nu} = 0, \delta_v K_\mu = -\Lambda_\mu, \delta_v B_{\mu \nu} = -(D_\mu \Lambda_\nu - D_\nu \Lambda_\mu), \delta_v H_{\mu \nu \eta} = 0\) such that \(\delta_v \mathcal{L}_0 = -(m/2) \partial_\nu [\varepsilon^{\mu \nu \eta \kappa} \Lambda_\kappa \cdot F_{\eta \kappa}]\) where \(\Lambda_\mu = \Lambda_\mu \cdot T\) is an infinitesimal vector gauge parameter [6,7]. As a consequence, the action of the theory remains invariant under the “vector” gauge transformations \(\delta_v\).
The above generator, however, generates only the following local and infinitesimal local gauge symmetry transformations of (3), namely:

\[
\begin{align*}
\delta_{gt} A_i(x) &= -i [A_i(x), Q_{(gt)}] = D_i \Omega(x), \\
\delta_{gt} B_{ij}(x) &= -i [B_{ij}(x), Q_{(gt)}] = -(B_{ij} \times \Omega)(x).
\end{align*}
\]

We conclude that \( Q_{(gt)} \) is not a full generator for the transformations (3).

We wrap up this section with a couple of remarks. First, the auxiliary field \( K_\mu \) leads to the constraint equation of motion \((\mathcal{H}^{\mu\nu\eta} \times F_{\nu\eta}) = 0\) which \textit{can not} be easily satisfied. However, we know that the Maurer-Cartan equation \( F^{(2)} = dA^{(1)} + i(A^{(1)} \wedge A^{(1)}) \) (which defines the curvature tensor \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu] \) for the 1-form gauge field) has a solution \( A^{(1)} = -iUdU^{-1} \) (where \( U \) is an \( SU(N) \) valued transformation function) such that the zero curvature condition \( F_{\mu\nu} = 0 \) is very naturally obtained\(^3\). Second, the gauge transformations (3) can be generalized to BRST and anti-BRST symmetry transformations that lead to the derivation of generators that are more general than \( Q_{(gt)} \). This is what we do precisely in our next sections.

**BRST symmetries and their generator.** The starting Lagrangian density (1) can be generalized to the BRST invariant Lagrangian density that incorporates the gauge-fixing and Faddeev-Popov ghost terms (in the Feynman gauge) as given below \([11]\):

\[
\mathcal{L}_b = -\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu} + \frac{1}{12} H^{\mu\nu\eta} \cdot H_{\mu\nu\eta} + \frac{m}{4} \varepsilon^{\mu\nu\eta\rho} B_{\mu\nu} \cdot F_{\eta\rho} + B \cdot (\partial_\mu A^\mu) + \frac{1}{2} (B \cdot B + B \cdot \bar{B}) - i\partial_\mu \bar{C} \cdot D^\mu C.
\]

The above Lagrangian density respects \((i.e. \ s_b \mathcal{L}_b = \partial_\mu [B \cdot D^\mu C])\) the following off-shell nilpotent \((s_b^2 = 0)\) BRST transformations \((s_b)\) \([11]\):

\[
\begin{align*}
s_b A_\mu &= D_\mu C, \\
s_b C &= \frac{1}{2} (C \times C), \\
s_b \bar{C} &= iB, \\
s_b B &= 0, \\
s_b \bar{B} &= -(\bar{B} \times C), \\
s_b F_{\mu\nu} &= -(F_{\mu\nu} \times C), \\
s_b K_\mu &= -(K_\mu \times C), \\
s_b H_{\mu\nu\eta} &= -(H_{\mu\nu\eta} \times C), \\
s_b B_{\mu\nu} &= -(B_{\mu\nu} \times C),
\end{align*}
\]

where fermionic \((C^2 = \bar{C}^2 = 0, C\bar{C} + \bar{C}C = 0, \) etc.) \((anti-)\)ghost fields \((C, \bar{C})\) are required for the unitarity and \((B, \bar{B})\) are the Nakanishi-Lautrup-type auxiliary fields that satisfy the Curci-Ferrari (CF) restriction \([B + \bar{B} = -i(C \times \bar{C})]\) which can be derived by exploiting the equations of motion (see the fourth section below).

The above transformations lead to the following Noether current:

\[
J_b^\mu = -F^{\mu\nu} \cdot D_\nu C + \frac{m}{2} \varepsilon^{\mu\nu\rho\sigma} B_{\nu\rho} \cdot D_\sigma C + B \cdot D^\mu C - (H^{\mu\nu\eta} \times K_\eta) \cdot D_\nu C - \frac{1}{2} (H^{\mu\nu\eta} \times B_{\nu\eta}) \cdot C
+ \frac{i}{2} \partial^\mu \bar{C} \cdot (C \times C),
\]

The conservation law \((\partial_\mu J_b^\mu = 0)\) can be proven by using the following:

\[
\begin{align*}
D_\mu F^{\mu\nu} &= \frac{m}{2} \varepsilon^{\mu\nu\rho\sigma} D_\mu B_{\rho\sigma} + D_\mu (H^{\mu\nu\eta} \times K_\eta) \\
&+ \frac{1}{2} (H^{\mu\nu\eta} \times B_{\rho\eta}) - \partial_\nu B - i(\partial^\rho \bar{C} \times C) = 0, \\
\partial_\mu (D^\mu C) &= 0, D_\mu (\partial^\mu \bar{C}) = 0, \\
D_\mu H^{\mu\nu\eta} &= \frac{m}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 0,
\end{align*}
\]

that emerge from the Lagrangian density (9) due to the Euler-Lagrange equations of motion. The above conserved current leads to the derivation of the conserved charge \( Q_b = \int d^3x J_b^0 \). The latter can be succinctly expressed as

\[
Q_b = \int d^3x \left[ -F^{0i} \cdot D_i C + \frac{m}{2} \varepsilon^{0ijk} B_{ijk} \cdot D_i C \\
+ B \cdot D^0 C - (H^{0ij} \times K_i) \cdot D_j C - \frac{1}{2} (H^{0ij} \times B_{ij}) \cdot C \\
+ \frac{i}{2} \bar{C} \cdot (C \times C) \right].
\]

Using the partial integration as well as the equation of motion corresponding to the 1-form gauge field from (12), it can be checked that the above charge can be re-expressed, in a more compact form, as

\[
Q_b = \int d^3x \left[ B \cdot D^0 C - C \cdot \partial^0 B - \frac{i}{2} \bar{C} \cdot (C \times C) \right],
\]

which is exactly same in appearance as is the form of the BRST charge in the context of non-Abelian 1-form gauge theory (see, e.g., [12]). There is a key difference, however, at the deeper level because in (14), we have

\[
\partial^0 B = D_i \left[ F^{0i} + \frac{m}{2} \varepsilon^{0ijk} B_{ijk} - (H^{0ij} \times K_j) \right]
+ \frac{1}{2} (H^{0ij} \times B_{ij}) - i(\bar{C} \times C),
\]

which reduces to the case of non-Abelian 1-form gauge theory in the limit \( B_{\mu\nu} \to 0 \). This is but natural as is evident from (1). The generator (14) is more general than

\(^4\text{To be precise, one knows that } \partial^0 B = D_i F^{0i} - i(\bar{C} \times C) \text{ in the case of the self-interacting non-Abelian 1-form gauge theory where there is no interaction with matter fields [12].}\)
the generator (7) for the gauge transformation because it can be checked that, for a generic field $\Phi$, we obtain
\[ s_b \Phi = -i [\Phi, Q_b]_{\pm}, \quad \Phi = A_0, A_i, C, \bar{C}, B_{ij}, \tag{16} \]
where $(\pm)$ signs, on the square bracket, stand for the bracket to be (anti-)commutator for the generic field $\Phi$ being (fermionic) bosonic in nature.

We close this section with the remarks that the generator $Q_b$ does not generate the BRST symmetry transformations $s_b K_\mu = -(K_\mu \times C)$ and $s_b B_{0i} = -(i B_{0i} \times C)$ because we have not taken into account the primary constraints in the generalization of the Lagrangian density (1) to the BRST level in (9). Such problems do not arise for the 1-form gauge field $A_b$ because we have exploited fully the usual gauge transformations corresponding to the 1-form gauge field that are generated by the first-class constraints (that also include the primary constraint) associated with the 1-form gauge potential.

**Off-shell nilpotent anti-BRST symmetry transformations and their generator.** – Corresponding to the BRST-invariant Lagrangian density (9), there exists an equivalent (but coupled) anti-BRST invariant\(^6\) Lagrangian density ($\mathcal{L}_b$)
\[ \mathcal{L}_b = -\frac{1}{4} F_{\mu\nu} \cdot F_{\mu\nu} + \frac{1}{12} H_{\mu\nu\eta} H_{\mu\nu\eta} + \frac{m}{4} \epsilon_{\mu\nu\rho\sigma} B_{\mu\nu} \cdot F_{\rho\sigma} \]
\[ - B \cdot (\partial_\mu A^\mu) + \frac{1}{2} (B \cdot B + \bar{B} \cdot B) - i D_b \bar{C} \cdot \partial^\mu C, \tag{17} \]
that respects the following off-shell nilpotent ($s^2_{ab} = 0$) and anti-commuting ($s_b s_a + s_a s_b = 0$) anti-BRST symmetry transformations\(^7\)
\[ s_{ab} A_\mu = D_\mu \bar{C}, \quad s_{ab} \bar{C} = \frac{1}{2} (C \times \bar{C}), \quad s_{ab} C = i \bar{B}, \]
\[ s_{ab} B = 0, \quad s_{ab} F_{\mu\nu} = -(F_{\mu\nu} \times \bar{C}), \quad s_{ab} H_{\mu\nu\eta} = -(H_{\mu\nu\eta} \times \bar{C}), \quad s_{ab} H_{\mu\nu\eta} = -(H_{\mu\nu\eta} \times \bar{C}), \tag{18} \]
where $s_{ab} \mathcal{L}_b = -\partial_\mu [B \cdot D^\mu \bar{C}]$. As a consequence, the action corresponding to the Lagrangian density $\mathcal{L}_b$ remains invariant.

\(^5\)The primary constraints of the theory are nothing but the vanishing of the canonical momenta corresponding to the compensating auxiliary field $K_\mu$ and the component $B_{0i}$ of the 2-form gauge field $B_{\mu\nu}$. On the other hand, the momenta for the field $B_{ij}$ do exist.

\(^6\)The Lagrangian densities in (9) and (17) are the most general forms that can be obtained by exploiting the basic tenets of BRST formalism [11]. It can be seen that [11] $\mathcal{L}_b = \mathcal{L}_b \equiv s_{ab} \frac{1}{2} B_{\mu\nu} = \frac{1}{4} (B_{\mu\nu} + A_\mu + A_\nu + C \cdot \bar{C})$ and $\mathcal{L}_b = \mathcal{L}_b \equiv s_{ab} \frac{1}{2} B_{\mu\nu} - \frac{1}{4} A_\mu A_\nu + C \cdot \bar{C}$. These Lagrangian densities are unique in the sense that the ghost number consideration and mass dimensions (in 4D) have been taken into account [11].

\(^7\)It will be noted that the (anti-)BRST symmetry transformations ($s_b B = 0, s_b B = -(B \times \bar{C})$, $s_{ab} B = 0, s_{ab} B = -(B \times \bar{C})$) for the auxiliary fields $B$ and $\bar{B}$ in (10) and (18) have been derived by requiring the nilpotency and anti-commutativity properties of $s_{ab}$.

According to Noether’s theorem, the above continuous symmetry transformations lead to the following expression for the conserved current:
\[ J^\mu_{ab} = -F^\mu\nu \cdot D_b \bar{C} + \frac{m}{2} \epsilon_{\mu\nu\rho\sigma} B_{\nu\rho} \cdot D_\sigma \bar{C} - B \cdot D^\mu \bar{C}, \]
\[ + (H^\mu\nu\eta \times K_\eta) \cdot D_b \bar{C} - \frac{1}{2} (H^\mu\nu\eta \times B_{\nu\eta}) \cdot \bar{C} \]
\[ - \frac{i}{2} \partial^\mu \bar{C} \cdot (\bar{C} \times \bar{C}). \tag{19} \]

The conservation law $\partial_\mu J^\mu_{ab} = 0$ can be proven by exploiting the following Euler-Lagrange equations of motion from $\mathcal{L}_b$, namely:
\[ D_\mu F_{\mu\nu} - \frac{m}{2} \epsilon_{\mu\nu\rho\sigma} D_\rho B_{\sigma\eta} + D_\mu (H^\mu\nu\eta \times K_\eta) \]
\[ + \frac{1}{2} (H^\mu\nu\eta \times B_{\nu\eta}) + \partial^\eta B + i (\partial^\eta C \times \bar{C}) = 0, \quad \partial_\mu (D^\mu \bar{C}) = 0, \]
\[ D_\mu (\partial_\mu C) = 0, \quad D_\mu H^\mu\nu\eta - \frac{m}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} = 0, \]
\[ (H^\mu\nu\eta \times F_{\nu\eta}) = 0. \tag{20} \]

From the equations of motion for the 1-form gauge field and (anti-)ghost fields (cf. (12) and (20)), we obtain the celebrated CF condition $B + \bar{B} = -i (C \times \bar{C})$ and the Lorentz gauge-fixing condition $\partial_\mu A^\mu = 0$.

The above observations show that the Lagrangian densities (9) and (17) i) are coupled because the above CF condition implies that $B \cdot (\partial_\mu A^\mu) - i \partial_\mu C \cdot D^\mu C = -B \cdot (\partial_\mu A^\mu) - i D_\mu C \cdot \partial^\mu C$, and ii) are equivalent in the sense that both of them respect the (anti-)BRST symmetry transformations together because we have:
\[ s_{ab} \mathcal{L}_b = -\partial_\mu [B \cdot D^\mu \bar{C}], \quad s_{ab} \mathcal{L}_b = -\partial_\mu [B \cdot D^\mu \bar{C}] \]
\[ s_{ab} \mathcal{L}_b = -\partial_\mu [B \cdot D^\mu \bar{C}] - D_\mu [B + \bar{B} + i (C \times \bar{C})] \cdot \partial^\mu \bar{C}, \]
\[ s_{ab} \mathcal{L}_b = -\partial_\mu [B \cdot D^\mu \bar{C}] + D_\mu [B + \bar{B} + i (C \times \bar{C})] \cdot \partial^\mu C. \tag{21} \]

This establishes the equivalent and coupled nature of $\mathcal{L}_b$ and $\mathcal{L}_b$. The conserved current in (19) leads to the following conserved charge:
\[ Q_{ab} = -\int d^3 x \left[ B \cdot D^\mu \bar{C} - \frac{i}{2} \partial^\mu \bar{C} \cdot (\bar{C} \times \bar{C}) \right]. \tag{22} \]

The above charge generates the anti-BRST symmetry transformations for all the relevant fields of the theory except $K_\mu$ and $B_{0i}$. This is due to the fact that primary constraints corresponding to these fields have not been taken into account in the anti-BRST invariant Lagrangian density (17).

**Ghost symmetry transformations and BRST algebra from symmetry generators.** – It can be checked that under the following infinitesimal transformations $s_g$:
\[ s_g C = + \Sigma C, \quad s_g \bar{C} = - \Sigma \bar{C}, \quad s_g [A_\mu, B_{\mu\nu}, K_\mu] = 0, \tag{23} \]
where $\Sigma$ is a global parameter, the Lagrangian densities (9) and (17) remain invariant. The above infinitesimal symmetry transformations are derived from the following explicit scale transformations:

$$C \rightarrow e^{\Sigma}C, \quad \bar{C} \rightarrow e^{-\Sigma}\bar{C},$$

$$\left(A_\mu, B_{\mu\nu}, K_\mu\right) \rightarrow \left(A_\mu, B_{\mu\nu}, K_\mu\right),$$

(24)

where $(\pm)$ signs in the above exponential correspond to the ghost number of a given field of the theory. The conserved current $(J_\mu)$ and charge $(Q_a)$ corresponding to the above infinitesimal transformations (23) are

$$J_\mu = i[\bar{C} \cdot D^\mu C - \partial^\mu \bar{C} \cdot C],$$

$$Q_a = i \int d^3x [\bar{C} \cdot D^0 C - \bar{C} \cdot C].$$

(25)

It is elementary to check that the above charge is the generator of (23).

One of the simplest ways to derive the BRST algebra is to exploit the idea of symmetry generators amongst all the conserved charges of the theory. This can be elucidated in the following fashion:

$$s_b Q_b = -i \{Q_b, Q_b\} = -Q_b,$$

$$s_{ab} Q_a = -i \{Q_a, Q_{ab}\} = +Q_{ab},$$

$$s_b Q_b = -i \{Q_b, Q_{ab}\} = 0 \Rightarrow Q_b^2 = 0,$$

$$s_{ab} Q_{ab} = -i \{Q_{ab}, Q_{ab}\} = 0 \Rightarrow Q_{ab}^2 = 0,$$

$$s_b Q_{ab} = -i \{Q_b, Q_{ab}\} = 0 \Rightarrow Q_b Q_{ab} = Q_{ab} Q_b = 0,$$

$$s_{ab} Q_b = -i \{Q_a, Q_b\} = 0 \Rightarrow Q_a Q_{ab} + Q_{ab} Q_a = 0,$$

(26)

which finally leads to the derivation of the well-known BRST algebra. In the above, the anti-commutativity of $Q_{(a)b}$ is invoked by invoking the CF condition.

Conclusions. – We have exploited one of the key gauge symmetries of the dynamical 4D non-Abelian 2-form gauge theory to perform the BRST analysis. To be precise, it is the “scalar” gauge symmetry that has been the central symmetry of our present discussion and we have not even touched upon the “vector” gauge symmetry that is also present in the theory (cf. footnote 2). One of the novel features of our present investigation is the fact that, even though there exist off-shell nilpotent and anti-commuting (anti-)BRST symmetry transformations for $K_\mu$ and $B_{\mu\nu}$ fields (cf. (10), (18)), the conserved (anti-)BRST charges (corresponding to these symmetries) are not capable of generating them. The (anti-)BRST transformations for the former are not obtained even by the requirement of nilpotency and anti-commutativity properties of the rest of the (anti-)BRST symmetry transformations of our present theory.

We have provided the possible reasons behind the existence of the above ambiguity which is, in some sense, unique to our present gauge theory because it does not appear in the BRST analysis of (non-)Abelian 1-form (see, e.g., [12,13]) and Abelian 2-form as well as 3-form gauge theories [8,9,14]. It should be noted that our Lagrangian densities (9) and (17) have been obtained in their full generality by exploiting the basic tenets of BRST formalism (see, e.g., [11] for details). However, in the BRST analysis, the canonical momenta corresponding to $K_\mu$ and $B_{\mu\nu}$ do not appear at all in the Lagrangian densities of the theory [11]. This is why, we have the presence of the above ambiguity.

It is worthwhile to mention that if we include the above constraints in the theory, then, the “scalar” and “vector” gauge symmetries mix up together. The corresponding “merged” (anti-)BRST symmetries turn out to be off-shell nilpotent but they do not respect the absolute anti-commutativity property. Hence, these nilpotent symmetries are not proper (see [10] for details).

It would be a very nice endeavor to generalize our present idea to the case of the discussion of the “vector” gauge symmetries (within the framework of the BRST formalism) that also exist in the theory. The Hamiltonian analysis of the 4D dynamical non-Abelian 2-form gauge theory is another direction for further investigation. These are the issues that are being pursued at the moment and our results would be reported in our future publications.

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