Opportunity to assess stability ecosystems based on the dynamic model interaction of biomass and resources

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Abstract. A dynamic model describing the interaction of biomass ecological systems and resources in the conditions of random changes of negative impacts is considered. It is shown that the proposed model depending on the values of its parameters, describes either the steady state of the system or its degradation. The model calculations showed that the increase in the average value and variation of the negative impact increase the risks of degradation of the ecological system. The methods of quantitative assessment of sustainability of ecological systems on the basis of the analysis of data of regime observations characterizing the density of biomass and resources of a particular ecosystem are offered.

1. Introduction
The creation of an enabling environment at the present time considerable attention is being given. Big attention is payed to conductive environment generation. Following [1] “conductive environment – is environment, that provides sustainable activity of natural ecological systems, natural and human-induced objects”. The definition of ecological systems sustainability is widely established nowadays, and can be determined in following propositions.

Ecological system includes positive and negative feedback, that maintain system homeostasis within environment parameters [2]. There are two types of homeostasis: resistant – the ability to save structure and functions despite external impact; resilient – the ability to recover the structure and functions in condition of lack of ecosystem’s components [3].

Ecological system suffers from complex of negative nature and anthropogenic impact. It is necessary and very important to analyze the resistance of ecosystem to this complex of impact. The analysis helps to plan events directed for environment protection. The analysis should include incoming data that describes the nature object and methods of comparison incoming data with the list of valid values.

Most experimental methods, that used for environment research, analyze the potential of this or that natural resources: agricultural, forest areas, water supplying objects etc. Environment quality standards, that are used in Russian Federation, oriented on hygienic and sanitary indicators mostly. Because of that, we can say, that the system is oriented to protect population from the results of economic activity, instead of environment protection from anthropogenic impact.
Traditional method of anthropogenic impact on environment research is analyzing several parameters of investigated area. And comparison parameters with similar parameters, that characterizes the background area.

This method allows to make statistic conclusions about similarity or discrepancy of nature environment around researchable and background areas. At the same time, there is no possibility to analyze risk assessment of ecological system resistance loss in these areas.

In previous work [4] we described the dynamic model, that describes biomass and resources within ecological system. Main parameters, that describes system condition are biomass and resources density. Parameters, that determine the character of evolution of the system are dependence biomass density on resources density and kinetic coefficients, that identify biomass growth rate, replenishment of resources on account of dead biomass growth rate and the loss of resources on account of abiotic process growth rate. In addition, the model allows to consider external influence.

In this article we review diversification of the model, adding parameters of external impact.

2. Based section
The main content of this work were model calculations, systems of differential equations. In the quality of design model were used Euler scheme [5]. The difference scheme was constructed to provide calculation resistance using any step size. Random process modeling was organized according to recommendations, mentioned in [6]. We used average values of biomass and resources density and its variation coefficient to compare the results of calculations of different parameter values of system of equations.

The main equation is presented like (1), According to that, biomass density growth rate is proportional to the multiplication of biomass density and the difference between achieved biomass density and achievable density, according to existing resources in system.

Density resources changes determines by the difference of replenishment resources growth rate, accepted proportional to biomass density and the loss of resources on account of abiotic process, accepted resources density.

The intensity of the negative impact on the system was presented as a random process, given its average value, dispersion and type of autocorrelation function.

\[
\begin{align*}
\frac{dN}{dt} &= a \cdot N \cdot \left( f(R) \cdot g(t) - \frac{N}{N_{\text{max}}} \right) \\
\frac{dR}{dt} &= b \cdot N - c \cdot R
\end{align*}
\]  

(1)

Where is

- \( N \) – the density of biomass,
- \( R \) – the density of resources,
- \( N_{\text{max}} \) – the maximum density of biomass with full security of resources,
- \( A \) – the growth rate of biomass in the absence of limitations,
- \( B \) – rate of replenishment of resources due to dead biomass,
- \( C \) – the rate of loss of resources,
- \( f(R) \) – a function that varies from 0 to 1 as resource density increases from 0 to \( \infty \),
- \( g(t) \) – a function that describes the negative impact (values between 0 and 1).

In the accepted record for the function \( g(t) \) the value 0 corresponds to the maximum, and the value 1 is the minimal influence of negative factors.

In order to reduce the number of parameters, the system (1) was replaced by a variable converted to a non-dimensional view (2).
\[
\begin{align*}
\frac{du}{dt} &= u \cdot (f(v) \cdot g(t) - u) \\
\frac{dv}{dt} &= \beta \cdot (u - \gamma \cdot v)
\end{align*}
\]  

Where is
\[
\begin{align*}
u &= N / N_{\text{max}} \quad &- \text{non-dimensional biomass density}, \\
v &= R / R_{0.5} \quad &- \text{non-dimensional resource density}, \\
\tau &= t \cdot a \quad &- \text{non-dimensional time}, \\
R_{0.5} &= \text{resource density corresponding to the condition } f(R) = 0.5, \\
\beta &= b \cdot N_{\text{max}} / a \cdot R_{0.5} \quad &- \text{resource recovery rate setting}, \\
\gamma &= c \cdot R_{0.5} / b \cdot N_{\text{max}} \quad &- \text{resource loss rate setting}.
\end{align*}
\]

Note that the factors that determine the transition to non-dimensional parameters have different physical nature. So the maximum density of biomass \(N_{\text{max}}\), density of resources, providing biomass density equal to half of the maximum \(R_{0.5}\), resource regeneration rate setting \(\beta\) and characteristic define the dependency \(f(R)\). Are determined mainly by the biological characteristics of the ecosystem. Function \(g(t)\) reflects the joint negative impact of natural and anthropogenic factors and depends mainly on Natural-climatic conditions and anthropogenic load. Resource loss rate setting \(\gamma\) associated with the characteristics of soil cover, in particular with the filtration coefficient and the water regime.

Function \(f(R)\) in the equation (1) determines the maximum biomass density that the system can support at a given resource density value. An essential feature of this function is that its main changes occur in a relatively narrow interval of values. Indeed, low resource densities may not be sufficient to sustain the environmental system, and with a high resource density, its impact on biomass density will be negligible. For example, in work [7] the scale of plant availability by easily digestible forms of nitrogen for soils of Western Siberia is given, according to which very low degree of availability by easily hydrolyzed nitrogen is its maintenance in soil less than 30, and high – 90 and more mg/kg.

Based on these considerations, the dependence of \(f(R)\) represented the integral function of lognormal distribution (3).

\[
f(v) = \text{lognorm}(v, \mu, \sigma)
\]

Where is
\[
\mu, \sigma \quad - \text{parameters of the lognormal distribution}.
\]

Given the fact that variable \(v\) is defined as the ratio of the current resource density to the density of resources, providing biomass density equal to half of the maximum possible biomass density, in the calculations, the parameter \(\mu\) in the expression (3) was assumed to be 0, and the parameter \(\sigma\) defined the width lognormal distribution.

Note that dependence (3) allows for a twofold interpretation of the. It can be interpreted as a reaction of one of the biological species forming part of the ecological system and as a reaction of the total biomass of the system to changes in resource density. In the second case the change of the total biomass of the system can take place both by changing the productivity of individual species, and by changing the species diversity.

The approximate approximation was tested by us in relation to the dependence of crop yields depending on the humus content presented in the work [8]. Estimation of approximation parameters in Table 1.
Table 1. Estimation of parameters of approximation of productivity of some crops from humus content in soil

| Agricultural culture                  | Distribution options M | σ    | Maximum productivity (c/ha) | Median distribution (% of Humus) |
|--------------------------------------|------------------------|------|-----------------------------|----------------------------------|
| Green mass of lupine                 | 0.006                  | 0.371| 170.9                       | 1.01                             |
| Potatoes                             | 0.254                  | 0.231| 159.2                       | 1.29                             |
| Oats                                 | 0.213                  | 0.323| 15.9                        | 1.24                             |
| Green mass of the Semelella          | 0.257                  | 0.292| 185.8                       | 1.29                             |
| Winter rye                           | 0.169                  | 0.213| 18.4                        | 1.18                             |
| Barley                               | 0.095                  | 0.292| 9.0                         | 1.10                             |

Note that the maximum productivity and median distribution in Table 1 correspond to the values of $N_{max}$, $u$ and $R_{0.5}$ in the expression (1). As can be seen from the data presented in Table 1, expression (3) results in close values of $\sigma$ parameters and $R_{0.5}$ for all crops while estimates of maximum productivity vary greatly, as they various measures were used.

In work [4] it was shown that under the conditions of the constant level of external influence the system described by the expression (2) when used as $f(v)$ expressions (3) can have up to three stationary states, two of which may be characterized by a non-zero density of biomass and resources.

The values of the variables $u$ and $v$ corresponding to this stationary state are determined by the system of algebraic equations (4).

\[
\begin{align*}
g \cdot \lognorm(v, 0, \sigma) - u &= 0 \\
u - g \cdot v &= 0
\end{align*}
\]  
(4)

$g$ — a time-constant external negative impacts.

Stationary states with non-zero biomass and resource density are possible only with a certain combination of parameters $g$, $\sigma$ and $\gamma$ included in the system of equations (4). This condition is determined by inequality (5).

\[
\frac{\gamma}{g} \leq \max \left( \frac{\lognorm(v, 0, \sigma)}{v} \right)
\]  
(5)

Note that if the left and right sides of the expression (5) are equal, the system (2) has one stationary state with a non-zero density of biomass and resources, in case the left side is less than the right — two such states.

In work [4] we have shown that in the case when the system (2) has two stationary states with non-zero biomass, the whole area of possible states of the system is divided into two sub-regions (2) and the boundary between these areas passes through the lower stationary point. In this case, if the initial conditions belong to one of these areas, which in this work we will call the area of sustainability, system solution (2) converge to the upper stationary state (2) In the event that the initial conditions belong to another area (areas of degradation), the evolution of the system is reduced to a total loss of biomass and resources. Schematically this situation is depicted in Figure 1.

The scheme presented in Figure 1 suggests that the negative impact on the environmental system remains constant. In the real world this assumption is not fulfilled already because the variation of weather conditions always takes place.
A random process of changing the level of negative impact generates changes in biomass density and resources in the system. Two fundamentally different scenarios of system behavior are possible, as shown in Figure 2. In the first case, the density of biomass and resources remain in the area of sustainability for an uncertain long time. In the second case, the state parameters are output from this area and the system is subsequently degraded.

The probability of output of system state parameters from the stability area is greater, the greater the amplitude of fluctuations of these parameters and the less the closer the upper stationary state to the boundary of the stability area.
Research on the response of the system described by the expression (2) to changes in the nature of the negative impact and is the main objective of this work.

In the calculations, the dependence of g(t) was represented as a random process, characterized by a predetermined probability distribution and an autocorrelation function. In all cases, the probability distribution used the beta distribution, that provided the interval of values g(t) 0 to 1 and the ability to set specific averages and variances for a simulated random process by varying the parameters of the beta distribution.

At the first stage the influence of the type of autocorrelation function of the random process on the simulation results was investigated.

Expressions were used as model autocorrelation functions, listed in table 2, and the simulation of the random process was performed using the algorithms and recommendations, contained in [6]

| Number in order | Type of auto-correlation function |
|-----------------|-----------------------------------|
| 1               | exp (−α · |τ|)                  |
| 2               | exp (−α · |τ|) · (1 + α · |τ|) |
| 3               | exp (−α · |τ|) · (1 − α · |τ|) |
| 4               | exp (−α · |τ|) · (1 + α · |τ| + α² · τ²/3) |

All the autocorrelation functions listed in Table 2 depend on only one parameter (α), however, for different auto-correlation functions the same values α Correspond to different correlation intervals. Therefore, when performing calculations using these autocorrelation functions value α was chosen to maximum correlation intervals defined as the time at which the absolute value of the auto- correlation function exceeds the value 0.01, were equal to each other.

A series of calculations was performed in which the magnitude of the variation coefficient of impact was varied. The analysis of the obtained results showed that system of equations (2) is not sensitive to the type of auto-correlation function, which specifies a random process of changing the intensity of the negative impact. In the area of moderate fluctuations in the intensity of negative impacts these fluctuations have little impact On the values of average biomass density and resources and lead to linear growth of coefficients of variation of these characteristics.

Calculations were performed to determine the nature of the impact on the results of calculations changes in the average level of negative impact and parameter γ in the system of equations (2).

Recall that the reduction of the parameter g and the growth of the parameter γ correspond to the deterioration of the environment system in the first case by increasing the mean value of the negative Impact, in the second – by reducing the ability of the system to retain resources.

The results of these calculations showed that the average density of biomass and resources almost coincided with the values corresponding to the stationary state, for average impact. At the same time, biomass and resource density variation rates grow as the system's living conditions deteriorate, that is decreasing the values of the g parameter and increasing the parameter values γ.

Thus, the proposed model predicts similar reactions of the system to deteriorating living conditions: reduced productivity and increased coefficient of variation of parameters describing the current state of the system.

The calculations show that the proposed model predicts the existence of a correlation relationship between the parameters characterizing biomass and resources of the ecological system. This opens the possibility to quantify the sustainability of the existence of ecological systems based on the analysis of time series of parameters characterizing biomass and resources.

Thus the array of results of simultaneously measured densities of biomass and resources of ecological system is considered as two-dimensional distribution, characterized by its center and the imbalances of biomass density and Resources. The center of this distribution is the estimation of biomass densities and resources corresponding to the upper stationary state. Parameters of the system of equations allowing to define the boundary of stable system existence, can be obtained by the
same methods used in this work for data processing on productivity of agricultural cultures by any other means.

Quantitative criterion of sustainability of existence environmental system can be determined as the likelihood that the system state vector will go beyond the stability area. To obtain this assessment, you must know the appearance of the two-dimensional probability distribution, characterizing the existing sample.

A different approach to the formulation of the quantitative criterion of stability is possible. You can define the direction of the large scattering ellipsoid axis [9] and in this direction compare the standard deviation with the distance from the distribution center to the stability boundary.

3. Conclusions
The dynamic model describing interaction of biomass and resources in ecological system in conditions of accidental changes of negative influences is offered.

It is shown that increase of variation of external influence increases risk of degradation of ecological system.

Methods of quantitative estimation of stability of ecological systems on the basis of analysis of data of regime observations are suggested, characterizing the density of biomass and the resources of a particular ecosystem.

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