Identifying and resolving the degeneracies in neutrino oscillation
parameters in current experiments

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Abstract

The three major unknown neutrino oscillation parameters at the present juncture are the mass hierarchy, the octant of the mixing angle $\theta_{23}$ and the CP phase $\delta_{CP}$. It is well known that the presence of hierarchy $-\delta_{CP}$ and octant degeneracies affects the unambiguous determination of these parameters. In this paper we show a comprehensive way to study the remaining parameter degeneracies is in the form of generalized hierarchy $-\theta_{23} - \delta_{CP}$ degeneracy. This is best depicted as contours in the test ($\theta_{23} - \delta_{CP}$) plane for different representative true values of parameters. We show that depending on whether the wrong-hierarchy and/or wrong-octant solutions occur in this plane with wrong or right value of $\delta_{CP}$, a total of eight different possibilities can exist. These multiple solutions, apart from affecting the determination of the true hierarchy and octant, also affect the accurate estimation of $\delta_{CP}$. We identify which of these eight different degenerate solutions can occur in the test ($\theta_{23} - \delta_{CP}$) parameter space, taking the long-baseline experiment NO$\nu$A running in the neutrino mode as an example. Inclusion of the NO$\nu$A antineutrino run removes the wrong-octant solutions appearing with both right and wrong hierarchy. Adding T2K data to this resolve the wrong hierarchy – right octant solutions to a large extent. The remaining wrong hierarchy solutions can be removed by combining NO$\nu$A + T2K with atmospheric neutrino data. We demonstrate this using ICAL@INO as the prototype atmospheric neutrino detector. We find that the degeneracies can be resolved at the 2$\sigma$ level by the combined data set, for the true parameter space considered in the study.

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I. INTRODUCTION

The standard 3-flavor neutrino oscillation probability is described by 6 parameters, namely three mixing angles – $\theta_{12}$, $\theta_{23}$, $\theta_{13}$, two mass squared differences – $\Delta m_{31}^2$, $\Delta m_{21}^2$ ($\Delta m_{ij}^2 = m_i^2 - m_j^2$) and the Dirac CP phase $\delta_{CP}$. The neutrino oscillation data from solar, atmospheric, reactor and accelerator experiments have so far given information about each of these oscillation parameters except $\delta_{CP}$ [1–3]. At present, the unknowns in neutrino oscillation physics are: (i) the sign of $\Delta m_{31}^2$ ($\Delta m_{31}^2 > 0$ known as Normal Hierarchy (NH) and $\Delta m_{31}^2 < 0$ known as Inverted Hierarchy (IH)), (ii) the octant of $\theta_{23}$ ($\theta_{23} > 45^\circ$ known as Higher Octant (HO) and $\theta_{23} < 45^\circ$ known as Lower Octant (LO)). (iii) the CP phase $\delta_{CP}$; any value of this parameter other than $0^\circ$ and $\pm 180^\circ$ would signal CP violation in the lepton sector. In this case it is often useful to talk in terms of lower half-plane (LHP) with $-180^\circ < \delta_{CP} < 0^\circ$ and upper half-plane (UHP) with $0^\circ < \delta_{CP} < 180^\circ$.

The appearance channel $P_{\mu e}$ often known as the ‘golden channel’ can measure all the three unknown parameters described above\(^1\). However the measurement is complicated by the fact that different sets of values of parameters can give the same oscillation probability. This gives rise to degeneracies that render an unambiguous determination of true parameters difficult. It was discussed in [4] that there can be eight-fold degeneracies in neutrino oscillation probabilities which are: (a) the intrinsic or $\theta_{13} - \delta_{CP}$ degeneracy [5], (b) the hierarchy–$\delta_{CP}$ degeneracy [6] and (c) the intrinsic octant degeneracy [7]. The intrinsic degeneracy refers to clone solutions occurring due to a different $\theta_{13}$ and $\delta_{CP}$ value. This degeneracy can be removed to a large extent by using spectral information [8]. Moreover the current precision determination of $\theta_{13}$ [9–12] has removed this degeneracy to a great extent. The hierarchy–$\delta_{CP}$ degeneracy leads to wrong hierarchy solutions occurring for a different value of $\delta_{CP}$ other than the true value. The intrinsic octant degeneracy refers to duplicate solutions occurring for $\theta_{23}$ and $\pi/2 - \theta_{23}$.

Many papers have discussed possibilities of resolution of these degeneracies by using different detectors in the same experiment [13–15]. The synergistic combination of data from different experiments was also discussed as an effective means of removing such degeneracies by virtue of the fact that the oscillation probabilities offer different combinations of

\(^1\) Originally $P_{\mu e}$ was termed as the golden channel because of its sensitivity to $\theta_{13}$, hierarchy and $\delta_{CP}$. 
parameters at varying baselines and energies [8, 16–23]. In particular, the synergy between long-baseline (LBL) experiments NOνA and T2K in resolving the hierarchy–$\delta_{CP}$ degeneracy has been discussed recently in [24–27].

It has been shown in [21, 28, 29] that a precise measurement of the mixing angle $\theta_{13}$ is helpful for removal of octant degeneracy. Octant sensitivity in the T2K and NOνA experiments has been studied recently in [30, 31] in view of the measurement of a non-zero $\theta_{13}$. The octant degeneracy is different for neutrinos and antineutrinos and hence a combination of these two data sets can be conducive for removal of this degeneracy for most values of $\delta_{CP}$ [32–34].

Since atmospheric neutrino baselines experience strong earth matter effects, these effectively remove the overlap between right and wrong-hierarchy solutions [35–38]. In particular, atmospheric neutrino experiments capable of distinguishing neutrinos and antineutrinos can be very useful in resolving degeneracies related to the mass hierarchy [38–48]. The octant sensitivity of the atmospheric neutrinos comes from both appearance [49] and disappearance channels [50], and also benefits from significant matter effects, especially facilitated by the large value of $\theta_{13}$ measured by reactor experiments. Atmospheric neutrinos also provide a synergy with LBL experiments in terms of probability behaviour with respect to parameters, so that the combination of atmospheric neutrino data with LBL data exhibits reduced effect of the hierarchy and octant degeneracies [31, 45, 51–53].

Recently it has been realized that for the appearance channel the octant degeneracy can be generalized to the octant–$\delta_{CP}$ degeneracy corresponding to any value of $\theta_{23}$ in the opposite octant [31, 54]. A continuous generalized degeneracy in the three dimensional $\theta_{23} - \theta_{13} - \delta_{CP}$ plane has been studied in [54]. In this work, we show that with the high precision measurement of $\theta_{13}$ by reactor experiments, the remaining degeneracies can be discussed in an integrated manner in terms of a generalized hierarchy–$\theta_{23} - \delta_{CP}$ degeneracy.

A good way to visualize the different degenerate solutions is in terms of contours in the test $(\theta_{23} - \delta_{CP})$ plane for different choices of true values of parameters$^2$. These plots also give an indication regarding the precision of the parameters $\delta_{CP}$ and $\theta_{23}$. Although hierarchy degeneracy is discrete, the $\theta_{23} - \delta_{CP}$ degeneracy is continuous for the appearance channel.

$^2$ Note that prior to discovery of a non-zero value of $\theta_{13}$, the degeneracies were studied mainly in $\theta_{13} - \delta_{CP}$ plane.
probability $P_{\mu e}$. Inclusion of the information from the disappearance channel $P_{\mu\mu}$ restricts $\theta_{23}$ and discrete degenerate solutions are generated. We classify the wrong hierarchy and wrong octant solutions with respect to right or wrong $\delta_{CP}$ values, which also allows us to understand how the hierarchy and octant degeneracy can affect the precision in $\delta_{CP}$. We observe that since the wrong hierarchy and wrong octant solutions can occur for wrong values of $\delta_{CP}$ as well, there can exist, in principle, a total of eight degenerate solutions corresponding to different combinations of hierarchy, octant and $\delta_{CP}$. This is summarized in Table I\(^3\). We identify which degenerate solutions among the eight possibilities listed in Table I exist in the neutrino oscillation probabilities for typical baselines and energies corresponding to the long-baseline (LBL) experiments T2K and NO\(\nu\)A. We analyze the role that can be played by a combination of the two long-baseline experiments NO\(\nu\)A and T2K, in conjunction with an atmospheric neutrino experiment (ICAL@INO), for the resolution of such degeneracies. For representative true values of these parameters, we demonstrate how far the degenerate solutions can be removed by NO\(\nu\)A, NO\(\nu\)A+T2K and NO\(\nu\)A+T2K+ICAL. We also discuss the role of the antineutrino component in resolving these degeneracies.

The paper is organized as follows. In Section II we give the experimental details of the LBL and atmospheric neutrino experiments being considered. In section III first we summarize the parameter degeneracies and identify degenerate solutions at the level of neutrino oscillation probabilities. Then we show their occurrence at the event level considering NO\(\nu\)A and discuss the resolution of the different kinds of degeneracies by combinations of the given experiments. We also present the precision of the parameters $\theta_{23}$ and $\delta_{CP}$ from the combined analysis with NO\(\nu\)A+T2K+ICAL data. The conclusions are presented in Section IV. Appendix A outlines the synergy between the disappearance and appearance channels and the role of antineutrinos.

II. EXPERIMENTAL DETAILS

We use the GLoBES package [55, 56] (along with the required auxiliary files [57, 58]) to simulate the data of the two long-baseline neutrino oscillation experiments T2K (Tokai

\(^3\) It is to be noted in this connection that if the $\delta_{CP}$ precision is not good then there can be continuous regions connecting right and wrong $\delta_{CP}$ solutions and hence it may not always be possible to identify discrete wrong $\delta_{CP}$ solutions.
to Kamioka, Japan) and NOνA (NuMI Off-Axis νe Appearance, Fermilab). The source to detector distance, $L$ for T2K is 295 km. In the T2K experiment [59], muon neutrinos are directed from J-PARC making an off-axis angle of 2.5° towards Super-Kamiokande detector, which is a Water Čerenkov detector of mass 22.5 kt. T2K has been proposed to run for a total luminosity of $7.8 \times 10^{21}$ protons on target (POT) and it has already collected 10% of the total data in the neutrino mode. At present it is running in the antineutrino mode and the results are expected soon [60]. The recently operational NOνA experiment is also sending muon neutrinos through two detectors, one at Fermilab (the near detector) and one in northern Minnesota (the far detector), making an off-axis of 0.8° and traveling a distance of 810 km to reach the far detector, which is a 14 kt TASD (Totally Active Scintillator Detector). The beam power of NOνA is planned to be 700 kW which corresponds to $6 \times 10^{20}$ POT/year which will run for 3($\nu$) + 3($\overline{\nu}$) years. In our simulation of NOνA data, we considered the re-optimized NOνA set up from [26, 61].

For the simulation of an atmospheric neutrino experiment, we consider a magnetized iron calorimeter detector (ICAL) planned by the India-based Neutrino Observatory (INO), whose primary goal is to study atmospheric muon neutrinos. ICAL@INO has an advantage over other detectors because it has a magnetic field which allows charge discrimination, thus providing the facility to distinguish between $\mu^+$ and $\mu^-$. Here we consider the 50 kt detector detector for ICAL@INO with a runtime of 10 years. In our numerical analysis we have used constant detector angular and energy resolutions of 10° and 10% respectively and 85% overall efficiency. The muon resolutions from INO simulations can be found in [62]. We have checked that the constant resolutions used in this paper give results similar to those

| I. RH-RO-R\(\delta_{CP}\) | V. WH-WO-W\(\delta_{CP}\) |
|-----------------------------|-----------------------------|
| II. RH-WO-R\(\delta_{CP}\) | VI. RH-RO-W\(\delta_{CP}\) |
| III. WH-RO-R\(\delta_{CP}\) | VII. RH-WO-W\(\delta_{CP}\) |
| IV. WH-WO-R\(\delta_{CP}\)  | VIII. WH-RO-W\(\delta_{CP}\) |

TABLE I: Various possibilities of degeneracy in the probability $P_{\mu e}$. Here, R=right, W=wrong, H=hierarchy, O=octant.
obtained using resolutions from INO simulation codes [44, 45]. In our analysis of atmospheric neutrinos, we used the Gaussian formula to compute the statistical $\chi^2$. Systematic errors are taken into account using the method of pulls [63, 64] as outlined in [42]. In our simulation, we have added 5% prior on $\sin^2 2\theta_{13}$.

We use the following representative values for the oscillation parameters in our numerical simulation [1–3].

| Oscillation Parameters | True value | Test value          |
|------------------------|------------|---------------------|
| $\sin^2 2\theta_{13}$  | 0.1        | 0.085 – 0.115       |
| $\sin^2 \theta_{12}$   | 0.31       | –                   |
| $\theta_{23}$          | LO=39°, 42°, HO=48°, 51° | 35° – 55°          |
| $\Delta m^2_{21}$      | $7.60 \times 10^{-5} eV^2$ | –                   |
| $\Delta m^2_{31}$      | $2.40 \times 10^{-3} eV^2$ | $(2.15 – 2.65) \times 10^{-3} eV^2$ |
| $\delta_{CP}$         | $90^\circ$, $0^\circ$, $-90^\circ$ | $-180^\circ$ to $+180^\circ$ |

TABLE II: Values of neutrino oscillation parameters used in our simulations. Here the second column gives the true values of the parameters and the third column represents the parameter range over which we have marginalized the test values.

III. IDENTIFYING DEGENERACIES IN NEUTRINO OSCILLATION PARAMETERS AND THEIR RESOLUTION

For the baselines relevant for the experiments NOνA and T2K, the earth matter density is in the range $(2.3 – 3.0 \text{ g/cc})$, well below the matter resonance. Therefore oscillation probabilities computed assuming constant matter density can be used for these. Such probabilities calculated using perturbative expansion of the small leptonic mixing angle $\theta_{13}$ (in terms of $s_{13}$) and the mass hierarchy parameters $\alpha$ ($\equiv \Delta m^2_{31}/\Delta m^2_{21}$) are given as follows [65–67]

\[
P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \mathcal{O}(\alpha, s_{13})
\]

\[
P_{\mu e} = 4s^2_{13}s^2_{23} \frac{\sin^2(A - 1)\Delta}{(A - 1)^2} + \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2 A\Delta}{A^2}
\]

\[+ \alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{CP}) \frac{\sin(A - 1)\Delta \sin A\Delta}{(A - 1) A}
\]
where $s_{ij}(c_{ij}) = \sin \theta_{ij}(\cos \theta_{ij})$ for $j > i$ ($i, j = 1, 2, 3$). We use the following notation:

$$\Delta \equiv \Delta m^2_{31}L/4E, \quad A \equiv 2EV/\Delta m^2_{31} = V\ell/2\Delta,$$

where $V = \sqrt{2}G_F n_e$ is the Wolfenstein matter term. The antineutrino oscillation probability can be obtained by replacing $\delta_{CP} \rightarrow -\delta_{CP}$ and $V \rightarrow -V$. Hence in the neutrino oscillation probability $A$ is positive for NH and negative for IH and for antineutrinos, the sign of $A$ gets reversed. We observe the following salient features from the probability formulae:

- The leading order term in the muon neutrino survival probability $P_{\mu\mu}$, also known as disappearance channel, is proportional to $\sin^2 \theta_{23} \sin^2 \Delta$. This gives rise to the intrinsic hierarchy and octant degeneracies:

$$P_{\mu\mu}(\Delta) = P_{\mu\mu}(-\Delta) \quad (3)$$

$$P_{\mu\mu}(\theta_{23}) = P_{\mu\mu}(\pi/2 - \theta_{23}). \quad (4)$$

This leads to a loss of sensitivity to the hierarchy and octant, when the measurement is performed using this channel.

- The appearance channel $P_{\mu e}$ does not have intrinsic degeneracies but suffers from the combined effect of different parameters, which leads to the following set of degeneracies:

$$P_{\mu e}(\theta_{13}, \delta_{CP}) = P_{\mu e}(\theta'_{13}, \delta'_{CP}) \quad (5)$$

$$P_{\mu e}(\Delta, \delta_{CP}) = P_{\mu e}(-\Delta, \delta'_{CP}) \quad (6)$$

Eqs.(4, 5, 6) constitute the eight-fold degeneracy discussed in [4].

Recently it has been discussed that in the context of probabilities which are dependent on $\sin^2 \theta_{23}$ the octant degeneracy can be generalized to include all values of $\theta_{23}$ in the second octant [31] and can also be correlated with $\delta_{CP}$ [31, 68]. The pattern of parameter degeneracies in the three dimensional $\theta_{23} - \theta_{13} - \delta_{CP}$ space arising from the appearance probability $P_{\mu e}$ has been discussed in [54]. This is a continuous degeneracy and can be expressed as

$$P_{\mu e}(\theta_{23}, \theta_{13}, \delta_{CP}) = P_{\mu e}(\theta'_{23}, \theta'_{13}, \delta'_{CP}) \Rightarrow \text{generalized octant degeneracy} \quad (7)$$

However the reactor experiments have measured $\sin^2 \theta_{13}$ with a high degree of accuracy and future measurements are expected to improve it further. This has reduced the impact of $\theta_{13}$
uncertainty on octant degeneracy to a large extent [31]. In this paper we consider another
generalized degeneracy which is the hierarchy−$\theta_{23} - \delta_{CP}$ degeneracy:

$$P_{\mu e}(\theta_{23}, \Delta, \delta_{CP}) = P_{\mu e}(\theta_{23}', -\Delta', \delta_{CP}') \Rightarrow \text{generalized hierarchy} - \theta_{23} - \delta_{CP} \text{ degeneracy.} \tag{8}$$

This degeneracy can be observed best in the test $\theta_{23} - \delta_{CP}$ plane. Studying it in this fashion
allows us to view the degeneracies arising out of the remaining unknown parameters in a
comprehensive manner. We note that while the hierarchy degeneracy is always discrete, the
$\theta_{23} - \delta_{CP}$ degeneracy arising out of the appearance channel is continuous. On the other
hand, the intrinsic octant degeneracy arising from the $P_{\mu\mu}$ channel is independent of $\delta_{CP}$
and discrete in $\theta_{23}$ except for $\theta_{23}$ values close to maximal. Thus combining the survival and
conversion probabilities give rise to disconnected degenerate regions in the $\theta_{23} - \delta_{CP}$ plane.
We have elaborated on this point in the Appendix.

In the next subsection, we study the occurrence of the above degeneracies in terms of
probabilities for NO$\nu$A and T2K, and identify the different possible degenerate solutions at
the probability level.

**A. Identifying degeneracies at the probability level**

Fig. 1 shows the probability $P_{\mu e}$ for neutrinos (left panel) and antineutrinos (right panel)
as a function of $\delta_{CP}$ for both NH and IH. The plots in the upper panel correspond to NO$\nu$A
while those in the lower panel are for T2K. These probabilities are plotted for the energy
where the neutrino flux peaks. The hatched area denotes variation over $\theta_{23}$. For lower
(higher) octant we vary $\theta_{23}$ between $39^\circ - 42^\circ$ ($48^\circ - 51^\circ$). This is a good assumption for
$\theta_{23}$ not too close to its maximal value, for the purpose of illustrating the physics, since for
a given $\theta_{23}(true)$, the disappearance channel anyway excludes values away from $\theta_{23}(true)$
and $\pi/2 - \theta_{23}(true)$. Thus these plots implicitly assume information from the disappearance
channel.

From Fig. 1 the following points can be noted.

For neutrinos -

(i) The NH probabilities are higher than the IH probabilities. This is because of enhanced
matter effect for neutrinos for NH in Earth’s matter.

(ii) For both NH and IH the probabilities are higher in the LHP.
(iii) The probabilities for higher octant are higher for both NH and IH. While, for antineutrinos -

i) $'A'$ changes its sign and IH probabilities become higher than NH.

ii) The flip in sign of $\delta_{CP}$ makes both the NH and IH probabilities higher in the UHP.

iii) Like neutrinos, the probabilities for higher octant remains higher for both NH and IH.

FIG. 1: The oscillation probability $P_{\mu e}$ as a function of $\delta_{CP}$. Here upper row represents oscillation probability for NOvA [L = 810 km] and lower row represents probability for T2K [L = 295 km]. The left panel is for neutrinos while the right panel is for antineutrinos.

For both neutrinos and antineutrinos the lowest line in the LO (HO) band corresponds to $39^\circ (48^\circ)$ while the highest point to $42^\circ (51^\circ)$, due to the $\sin^2 \theta_{23}$ dependence of the leading order term.

The overlapping regions between various curves at a specific value of $\delta_{CP}$ indicate the degeneracy occurring for the right value of $\delta_{CP}$, while the same value of probability for
different $\delta_{CP}$ values denotes degeneracy occurring at wrong values of $\delta_{CP}$. Clearly the former would correspond to solutions with wrong hierarchy and/or octant with right $\delta_{CP}$, while the latter would correspond to the solutions with wrong hierarchy and/or octant and wrong $\delta_{CP}$.

The degenerate solutions corresponding to a given true value of $\theta_{23}$ and $\delta_{CP}$ can be obtained from the probability figures by drawing a horizontal line through this point. The different intersection points of this line with the probability bands are degenerate as they share the same value of probability. However, the degenerate solutions occurring for a particular $\theta_{23}$ (true) which is not in the vicinity of $\pi/2 - \theta_{23}$ (true) in the opposite octant will be excluded by the disappearance channel and occurrence of these solutions in the test $\theta_{23} - \delta_{CP}$ plane will depend on the $\theta_{23}$ precision of the disappearance channel.

Below we explain the occurrence of the different degenerate combinations of \{hierarchy, $\theta_{23}$, $\delta_{CP}$\} taking the \textsc{NO$\nu$A} neutrino probabilities (the top-left panel) as reference unless otherwise mentioned.

1. The overlapping regions between the NH-LO (blue) and IH-HO (green) bands around $\delta_{CP} = -120^\circ$ and $90^\circ$ give rise to WH-WO-R$\delta_{CP}$ degenerate solutions in the probability. However, for antineutrinos these bands are well separated. Thus combining NO$\nu$A neutrino and antineutrino data can help in removing these solutions.

2. The probability corresponding to UHP of the NH-LO (blue) band can be same as those for the LHP of the IH-LO (yellow) band for $\theta_{23} = 39^\circ$. This can give rise to WH-RO-W$\delta_{CP}$ solutions. Note that this degeneracy is present in the antineutrinos for the same values of $\delta_{CP}$. Thus for true NH, UHP (i.e $0^\circ < \delta_{CP} < 180^\circ$) is the unfavourable half-plane of $\delta_{CP}$ and this degeneracy can not be resolved by using NO$\nu$A data alone. For T2K the probability exhibits a sharper variation with $\delta_{CP}$ and hence this degeneracy is less pronounced between the UHP and LHP. Hence addition of T2K data to NO$\nu$A can be helpful in removing this degeneracy. For LHP (i.e $-180^\circ < \delta_{CP} < 0^\circ$), which is the favourable half-plane of $\delta_{CP}$ in NH, there is no WH-RO-W$\delta_{CP}$ solution for both NO$\nu$A and T2K.

3. For $\delta_{CP} \in \text{UHP}$, the NH-HO (red) band can share same value of probability with NH-LO (blue) band for $\delta_{CP} \in \text{LHP}$. This is the reason for the RH-WO-W$\delta_{CP}$ solution. For antineutrinos, the degeneracy is seen to be between NH-HO-LHP and NH-LO-UHP. Thus a combination of neutrinos and antineutrinos helps to remove this degeneracy.
4. The WH-WO-Wδ_{CP} solution can be observed along the iso-probability line that intersects the NH-LO (blue) and IH-HO (green) bands at different values of δ_{CP}. This degeneracy can be seen for instance between \{NH, 39°, −180°\} and \{IH, 51°, 0°\}. Again antineutrino probability do not possess this degeneracy and thus combining neutrino and antineutrino data can be helpful in removing these solutions.

5. One can also have RH-RO-Wδ_{CP} solutions as a result of a so called “intrinsic CP degeneracy” that occurs for the same hierarchy and same value of θ_{23} but at a different value of δ_{CP}. This is due to the harmonic dependence of the probability on δ_{CP}. For instance within the NH-HO (blue) band, δ_{CP} = 0° and δ_{CP} ≈ −135° have the same value of probability for θ_{23} = 39°. However for antineutrinos this occurs for δ_{CP} = 0° and δ_{CP} = +135°. Thus combination of neutrino and antineutrino data can help to get rid of this degeneracy. This can also be seen to occur for T2K, for \{NH, 48°, −180°\} and \{NH, 48°, 0°\}. For T2K, since the flux peak coincides with the probability peak, the CP dependent term goes as \sin δ_{CP} and thus this degeneracy occurs for δ_{CP} and π − δ_{CP} [68]. For NOνA, since the flux and the probability peak are not at the same energy, the degeneracy does not correspond exactly to δ_{CP} and π − δ_{CP}. It is interesting to note that this degeneracy does not occur for δ_{CP} = ±90°.

Thus among the eight solutions listed above, only the WH-RO-Rδ_{CP} and RH-WO-Rδ_{CP} solutions do not exist even at the probability level. The above description is in terms of probabilities without including any experimental errors. At the event level, many of these may not appear as discrete degeneracies. In particular, for a C.L. beyond the reach of a particular experiment’s precision, the different discrete degenerate solutions merge and the degeneracy becomes continuous.

B. Identifying degeneracies at the event level

To show the occurrence of the different degeneracies at the event level, in Fig. 2 we present a set of contour plots in the θ_{23} − δ_{CP} test-parameter plane assuming only neutrino run (6 years) of NOνA, which is denoted as [6+0]. These plots are drawn assuming true hierarchy to be NH and different choices of true values of θ_{23} and δ_{CP}. In this and all the other subsequent figures the successive rows are for θ_{23} = 39°, 42°, 48°, 51°. The true δ_{CP}
values chosen are $\pm 90^\circ$ corresponding to maximum CP violation and $0^\circ$ corresponding to CP conservation. The blue contours correspond to the right hierarchy and magenta curves correspond to the wrong hierarchy.

The first column of Fig. 2 shows the degenerate solutions for $\delta_{CP} = -90^\circ$ for NO$_{\nu}$A running only in the neutrino mode. For $\theta_{23} = 39^\circ$, apart from the true solution, RH-WO-W$\delta_{CP}$ and WH-WO-R$\delta_{CP}$ solutions are observed in the upper and lower right quadrant respectively. The RH-WO-W$\delta_{CP}$ solution is also seen for $\theta_{23} = 42^\circ$. For this case, at $\delta_{CP} = -90^\circ$, the top most point of the blue band in the NO$_{\nu}$A neutrino probability in Fig. 1 one can see that same value of probability is possible for NH-HO (red band) near $\delta_{CP} = +45^\circ$ and $\pm 180^\circ$. This explains the shape of the allowed zone – wider at these values and narrower at $90^\circ$. The WH-WO-R$\delta_{CP}$ solution is seen only at $2\sigma$ level for $\theta_{23} = 42^\circ$. This can be understood by observing that the points $42^\circ$ (the upper tip of the blue band) and $48^\circ$ (the lower tip of the green band) are more separated as compared to $39^\circ$ (the lower tip of the blue band) and $51^\circ$ (the upper tip of the green band). For $\theta_{23}$ in the higher octant ($48^\circ$ and $51^\circ$) there are no spurious wrong-hierarchy solutions even with only neutrinos. This is because for NH, $\theta_{23}$ in the higher octant and $-90^\circ$ correspond to the maximum probability for neutrinos and this cannot be matched by any other combination of parameters. Hence no degenerate solutions appear and only neutrino run for NO$_{\nu}$A suffices to give allowed area only near the true point. Note that the contours for $48^\circ$ extend to the wrong-octant also. However (here and elsewhere) this is not due to any degenerate behaviour of the $P_{\mu e}$ probability, but due to the poor $\theta_{23}$ precision of the $P_{\mu\mu}$ channel near maximal mixing.

The second column represents $\delta_{CP} = +90^\circ$. In this case we observe WH-WO-R$\delta_{CP}$ solution for both $\theta_{23} = 39^\circ$ and $42^\circ$. This can be understood from the intersection of the blue and green bands in Fig. 1 close to $\delta_{CP} = 90^\circ$ in UHP. We also get a WH-RO-W$\delta_{CP}$ region in the LHP. For $42^\circ$ since the $\theta_{23}$ precision coming from disappearance channel is worse, at $2\sigma$ both these solutions merge and discrete degenerate region is not obtained. For $\theta_{23} = 51^\circ$ from Fig. 1 we see that the point \{NH, $+90^\circ$, $51^\circ$\} in the red band intersects the blue band around \{NH, $-90^\circ$, $39^\circ$\} giving RH-WO-W$\delta_{CP}$ solution. WH-RO-W$\delta_{CP}$ solution is also obtained in this case in the LHP. Similar regions are also obtained for $\theta_{23} = 48^\circ$. However the RH-WO-W$\delta_{CP}$ solution merges with the true solution at $2\sigma$ level.
FIG. 2: Contour plots for NO\nuA[6+0] with true values of $\theta_{23} = 39^\circ, 42^\circ, 48^\circ, 51^\circ$ in successive rows. The three columns correspond to $\delta_{CP} = -90^\circ, 90^\circ, 0^\circ$ respectively.
For $\delta_{CP} = 0^\circ$ a discrete RH-RO-W$\delta_{CP}$ solution is seen to be allowed at $1\sigma$ for $\theta_{23} \in$ LO. This is due to the intrinsic CP degeneracy as discussed in the context of probabilities. But at $2\sigma$, due to the $\delta_{CP}$ precision this degeneracy becomes continuous and the whole LHP becomes allowed. For $\theta_{23}$ belonging to the higher octant larger statistical errors are involved as compared to $\theta_{23} \in$ LO and this degeneracy appears as continuous in LHP even at $1\sigma$ and at $2\sigma$ the full $\delta_{CP}$ range becomes allowed. For $\theta_{23} = 39^\circ$ and $42^\circ$, we also see wrong hierarchy solutions appearing in the wrong octant. From the probability figure we identify that this degeneracy occurs around $\delta_{CP} = -30^\circ, -180^\circ, 180^\circ$ for $\theta_{23} = 39^\circ$ which allows LHP of $\delta_{CP}$ at $1\sigma$ and the whole $\delta_{CP}$ range at $2\sigma$. For $\theta_{23} = 42^\circ$ this degeneracy is seen to occur around $\delta_{CP} = -90^\circ$ giving distinct degenerate solutions at $1\sigma$ and $2\sigma$ level. For $\theta_{23} = 42^\circ$, a discrete RH-WO-W$\delta_{CP}$ solution appears at $1\sigma$. From Fig. 1 it is seen that $\{\text{NH}, 42^\circ, 0^\circ\}$ has the same value of probability corresponding to $\{\text{NH}, 48^\circ, 90^\circ\}$. At $2\sigma$ this merges with the RH-RO-W$\delta_{CP}$ solution. For $\theta_{23} = 39^\circ$ this solution appears as a $2\sigma$ allowed patch around $\{\text{NH}, 51^\circ, 90^\circ\}$. From Fig. 1 it can be seen that the above points are not exactly degenerate in terms of probability but due to lack of precision they become allowed. For a similar reason the $2\sigma$ patch with wrong hierarchy appears in the right-octant for $\theta_{23} = 39^\circ$ and $42^\circ$. For $\theta_{23} = 51^\circ$ a right-hierarchy patch occurs with wrong-octant. For $\theta_{23} = 48^\circ$ because of proximity to maximal mixing the true parameter space also extends to the wrong octant. In general we see that the CP precision is poorer for $\delta_{CP} = 0^\circ$ at this stage. This is due to the unresolved degeneracies for $\delta_{CP} = 0^\circ$ which lead to multiple allowed regions and continuous bands at $2\sigma$.

C. Successive resolution of degeneracies with data from different experiments

In this section first we show the status of the above degenerate regions when NO$\nu$A runs in [3+3] configuration. We then study the combined potential of NO$\nu$A[3+3] and T2K[8+0] in resolving the degeneracies. Finally we add the atmospheric neutrino data from ICAL and show its impact. The advantage offered by the atmospheric neutrino detector ICAL and the synergy between the various experiments in removing the degeneracies can be seen from these plots. It is noteworthy that the allowed area in the test $\theta_{23} - \delta_{CP}$ plane also gives an idea about the precision of these two parameters.

Our results are presented in Figs. 3, 4, 5 which correspond to true $\delta_{CP} = -90^\circ$, 15
90° and 0° respectively. In each figure the successive columns represent NOνA[3+3], NOνA[3+3]+T2K[8+0], and NOνA[3+3]+T2K[8+0]+ICAL respectively. In these figures the following generic features can be noted:

- Comparing with the NOνA[6+0] panels, in all the NOνA[3+3] figures we note that the addition of antineutrino information removes the wrong octant degenerate regions. This also includes the wrong-hierarchy regions occurring with wrong octant. For θ_{23} = 39° and 51°, the wrong octant regions are almost completely removed. But for the true θ_{23} values 42° and 48°, both the right hierarchy and wrong hierarchy solutions extend to the wrong octant region.

- When T2K data is added to NOνA[3+3], it helps in removing these wrong octant extensions. The wrong hierarchy-right octant regions also get significantly reduced in size by adding T2K data to NOνA[3+3]. This is due to the fact that for T2K these solutions occur at different δ_{CP} values than NOνA. Addition of T2K data also improves the precision of θ_{23} and δ_{CP}.

- When ICAL data is added to T2K and NOνA, the remaining wrong hierarchy regions are resolved for all the true values of θ_{23} considered. The wrong octant extensions of the right hierarchy solutions are also reduced in size and the precision of θ_{23} improves. The combination of T2K+NOνA+ICAL can resolve all the degeneracies at 2σ level for true θ_{23} = 39°, 51° for all the three δ_{CP} values. For the θ_{23} and δ_{CP} combinations of {42°, 0°} and for 48°, ±90°, 0° there are still allowed regions in the wrong octant.

Apart from the above features, the following important points can be observed from the figures:

- For δ_{CP} = −90°, there are no wrong hierarchy solutions in NOνA[3+3], and addition of T2K helps in improving the θ_{23} precision. This feature is particularly prominent for θ_{23} = 42° and 48° where T2K data helps in removing the wrong octant extensions for the right hierarchy solutions. With addition of ICAL data the wrong octant solution is almost resolved for θ_{23} = 42° while for θ_{23} = 48° the same is resolved at 1σ.

- For δ_{CP} = −90° and θ_{23} = 51°, NOνA can already resolve all the degeneracies with 6 years of neutrino run only as can be seen in Fig. 2. However, the precision of θ_{23} is
worse with NOνA[3+3]. This is because splitting the neutrino run into equal neutrino and antineutrino run reduces the statistics and hence the precision becomes worse.

- For $\delta_{CP} = 90^\circ$ we also see that for $\theta_{23} = 48^\circ$, the wrong hierarchy region in NOνA[3+3]+T2K[8+0] is still quite large and this is where ICAL has a remarkable role to play. We see that when ICAL data is added, the large wrong octant region corresponding to $\theta_{23} = 48^\circ$ completely vanishes.

- The small 1σ wrong hierarchy-wrong octant allowed zone for $\delta_{CP} = 90^\circ$ and $\theta_{23} = 42^\circ$ in NOνA[3+3] can be identified as the part of WH-WO-R$\delta_{CP}$ solution of NOνA[6+0] by comparing with Fig. 2.

- For $\delta_{CP} = 0^\circ$, adding T2K data to NOνA[3+3] improves the precision considerably and also removes the wrong hierarchy solutions to a large extent. The precision of $\theta_{23}$ and $\delta_{CP}$ around the true solution also improves. The enhanced precision due to adding T2K is also responsible for reducing the continuous allowed regions to discrete degenerate solutions for $\theta_{23}$ values near 45°. Adding ICAL data removes the remaining wrong hierarchy regions and further help to pinpoint the allowed zones at 2σ level.

- In NOνA[3+3] for $\delta_{CP} = 0^\circ$ and $\theta_{23} = 39^\circ$, 42° comparing with the corresponding figures in Fig. 2, we see that the spurious solution appearing at $-150^\circ$ at 1σ due to intrinsic degeneracy is no longer present with the addition of antineutrino data, since for the antineutrino probability in NOνA the intrinsic degeneracy is between 0° and +150° as discussed earlier. Thus the addition of neutrino and antineutrino data solves the intrinsic degeneracy at 1σ at both these $\delta_{CP}$ values but at 2σ both $\pm150^\circ$ remain allowed. The allowed area near the true value increases in size because replacing half the neutrino run with antineutrinos reduces the statistics and hence the precision becomes worse.

The following additional observations can be made regarding alternative parameter values and running modes:

- In generating the above plots we considered T2K running in neutrino mode with its full beam power. We find that once one includes the antineutrino run from NOνA, running T2K in the antineutrino mode is no-longer necessary for removing spurious
wrong-octant solutions. Rather running in the neutrino mode gives enhanced statistics and hence better precision. If on the other hand NOνA runs in full neutrino mode and the antineutrino component comes from T2K we have verified that we get similar results.

- We have presented the results for the case of true NH. If the true hierarchy is chosen to be IH, one would get a different set of allowed regions based on the degeneracies observed in Fig. 1. For example, for $\delta_{CP} = -90^\circ$ and $\theta_{23} = 39^\circ$ for NOνA[6+0] in the true IH case, apart from the true solution, RH-WO-W$\delta_{CP}$ and WH-RO-W$\delta_{CP}$ solutions would be obtained. This can be predicted from Fig. 1 (top left panel) by drawing a horizontal line from the bottom of the IH-LO band at $\delta_{CP} = -90^\circ$, which cuts both the IH-HO and NH-LO bands near $\delta_{CP} = 90^\circ$.

The situation for NOνA[3+3] and other combinations would be more complicated since the allowed regions and precision for true IH depend not only on the probability behaviour but also on the statistics of neutrino and antineutrino data in the respective experiments.

- The results are significantly dependent on the true value of $\theta_{13}$, chosen here to be $\sin^2 2\theta_{13} = 0.1$. Lower values of $\theta_{13}$ (or worse $\theta_{13}$ precision) would lead to poorer CP precision and more difficulty in removing the degeneracies. This is because $\delta_{CP}$ is coupled with $\theta_{13}$ in the oscillation probability.

D. Precision of $\theta_{23}$ and $\delta_{CP}$

As stated earlier, the contour plots also give information about the precision of $\theta_{23}$ and $\delta_{CP}$. In general presence of degenerate solutions lead to a worse precision (a larger width of the allowed area) in these parameters. For most values of true $\delta_{CP}$ and $\theta_{23}$, there is negligible difference between the $\delta_{CP}$-precision of NOνA[6+0] and NOνA[3+3] around the true point. While there is a qualitative advantage to including both neutrinos and antineutrinos because of their different dependences on $\delta_{CP}$, this advantage is squandered by the lower cross-section of antineutrinos. The precision in these parameters can be quantified using the following formulae:

$$CP \ precision = \frac{\delta_{Max}^{CP} - \delta_{Min}^{CP}}{360^\circ} \times 100\% \quad (9)$$
\[
\theta_{23} \text{ precision} = \frac{\theta_{23}^{\text{Max}} - \theta_{23}^{\text{Min}}}{\theta_{23}^{\text{Max}} + \theta_{23}^{\text{Min}}} \times 100\% \quad (10)
\]

In Table III, we list the values of the 1σ and 2σ precision of \(\theta_{23}\) and \(\delta_{CP}\) using these expressions for the case of NO\(\nu\)A[3+3] + T2K[8+0] + ICAL. The CP precision is seen to be better for \(\delta_{CP} = 0^\circ\) as compared to \(\delta_{CP} = \pm90^\circ\) for a given true value of \(\theta_{23}\). This is because in the absence of degeneracies, the precision simply follows from the probability expression, where \(dP_{\mu e}/d\delta_{CP}\) is smallest at \(0^\circ\) [25]. On the other hand for a given value of \(\delta_{CP}\), the CP precision is seen to become worse with increasing \(\theta_{23}\). The \(\theta_{23}\) precision is worse near maximal mixing and improves as one moves away.

| True value | LO-Precision | | True value | HO-Precision |
|------------|--------------|--------------|------------|--------------|
| \(\theta_{23}\) | \(\delta_{CP}\) | 1σ | 2σ | \(\theta_{23}\) | \(\delta_{CP}\) | 1σ | 2σ |
| +90\(^\circ\) | 1.02 | 26.63 | 2.17 | 39.50 | \(\theta_{23}\) | +90\(^\circ\) | 3.15 | 28.45 | 7.70 | 48.27 |
| 30\(^\circ\) -90\(^\circ\) | 0.89 | 34.52 | 2.17 | 41.52 | \(\theta_{23}\) | -90\(^\circ\) | 3.15 | 30.00 | 7.35 | 43.22 |
| 0\(^\circ\) | 0.64 | 15.83 | 2.04 | 28.33 | \(\theta_{23}\) | 0\(^\circ\) | 4.03 | 17.50 | 7.59 | 35.80 |
| +90\(^\circ\) | 1.6 | 27.00 | 3.32 | 38.52 | \(\theta_{23}\) | +90\(^\circ\) | 0.88 | 30.32 | 2.16 | 43.33 |
| 42\(^\circ\) -90\(^\circ\) | 1.7 | 29.77 | 3.31 | 41.52 | \(\theta_{23}\) | -90\(^\circ\) | 0.98 | 34.48 | 2.16 | 45.00 |
| 0\(^\circ\) | 1.66 | 15.83 | 3.08 | 29.16 | \(\theta_{23}\) | 0\(^\circ\) | 0.88 | 19.16 | 2.16 | 37.50 |

TABLE III: Percentage precision of parameters \(\theta_{23}\) and \(\delta_{CP}\) around true value using NO\(\nu\)A+T2K+ICAL.
FIG. 3: Contour plots in the test $\theta_{23} - \delta_{CP}$ plane for true $\delta_{CP} = -90^\circ$ and true $\theta_{23} = 39^\circ, 42^\circ, 48^\circ$ and $51^\circ$ in successive rows. The first column is for NO$\nu A[3+3]$. The second and third columns are for NO$\nu A[3+3]$ + T2K [8+0] and NO$\nu A[3+3]$ + T2K[8+0] + ICAL respectively.
FIG. 4: Same as in 3 but for true $\delta_{CP} = +90^\circ$. 

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FIG. 5: Same as in 3 but for true $\delta_{CP} = 0^\circ$. 
IV. CONCLUSION

In the era when the value of $\theta_{13}$ was unknown, an eight-fold degeneracy of neutrino oscillation parameters was identified, which included the intrinsic $\theta_{13}$, hierarchy–$\delta_{CP}$ and octant degeneracies. With the precise measurement of $\theta_{13}$ the intrinsic degeneracy is largely removed and a four-fold degeneracy out of the original eight – involving wrong hierarchy and wrong octant solutions remains to be solved by the current and upcoming experiments. In this paper we study these degeneracies in detail and propose that the remaining degeneracies can be studied in the most comprehensive manner by considering the generalized hierarchy–$\theta_{23} - \delta_{CP}$ degeneracy. This degeneracy is continuous for the $P_{\mu e}$ channel. Addition of information on measurement of $\theta_{23}$ by the $P_{\mu\mu}$ channel gives rise to discrete solutions. These are best visualized by contours in the test ($\theta_{23} - \delta_{CP}$) plane drawn for both right and wrong hierarchy for different representative values of true parameters. We show that depending on whether the wrong-hierarchy and/or wrong-octant solutions occur with right or wrong values of $\delta_{CP}$ there can be a total of eight possibilities. We study these possibilities at the probability level for T2K and NO$\nu$A. At this level the degeneracy is defined as equality of the probabilities for different values of parameters. However at the $\chi^2$ contour level, because of the precision of the experiments, one gets finite allowed regions corresponding to degenerate solutions. We define a degenerate solution to be one which is distinct from the true solution at the 1$\sigma$ level.

Taking only the neutrino run of NO$\nu$A as an illustrative example, we identify which of these degenerate solutions actually occur for different representative choices of true parameters. The sample true values that we consider for obtaining the contours are $\theta_{23} = 39^\circ, 42^\circ, 48^\circ$ and $51^\circ$ and $\delta_{CP} = \pm 90^\circ, 0^\circ$. At the present level of precision, for $\delta_{CP} = \pm 90^\circ$ the right (wrong) $\delta_{CP}$ solutions are those which occur in the same (opposite) half-plane as compared to the true solution. Since $\delta_{CP} = 0^\circ$ is common to both half-planes, for this case the right and wrong $\delta_{CP}$ solutions at a particular C.L. are inferred from the nature of the contours. The different degenerate solutions obtained are (i)WH-WO-R$\delta_{CP}$ (ii) RH-WO-W$\delta_{CP}$ (iii)WH-RO-R$\delta_{CP}$ (iv) RH-RO-W$\delta_{CP}$ (v) WH-WO-W$\delta_{CP}$ regions. Although the options (i)-(iii) have been noticed in the literature earlier, the option (iv) which exists for the same true $\theta_{23}$ but different $\delta_{CP}$ has not been discussed much. A probability level discussion was done in [68], where it was called $\theta_{23} - \delta_{CP}$ degeneracy. However since
it can occur for the same hierarchy and same $\theta_{23}$ we call it "intrinsic CP degeneracy". The WH-WO-W$\delta_{CP}$ solutions often appear as part of (i) given the CP precision of the current experiments. We identify a few points in the true parameter space where this solution appears as a distinct degenerate solution. We find that for a true value of $\theta_{23}$ in the range $48^\circ - 51^\circ$ and $\delta_{CP}$ in the lower half-plane ($-180^\circ < \delta_{CP} < 0^\circ$) the NO$\nu$A neutrino probability being highest cannot be matched by any other combination and hence no degenerate solutions appear. In this case only neutrino run is better as it gives a better precision. In all other cases that we have studied, 3 years of neutrino and 3 years of antineutrino run of NO$\nu$A is helpful in removing the wrong octant solutions (i), (ii) and (v) to a large extent. This also improves the CP precision since the wrong $\delta_{CP}$ solutions occurring with the wrong octant are resolved. Next we present the results combining NO$\nu$A[3+3] with T2K[8+0]. It is seen that the synergy between T2K and NO$\nu$A helps in removing the WH-RO-W$\delta_{CP}$ solutions for true $\delta_{CP} = 0^\circ, 90^\circ$. For true $\delta_{CP} = -90^\circ$, NO$\nu$A itself is sufficient for removing this degeneracy. The precision of both parameters also improve when these two sets of information are compounded together. The remaining degenerate solutions at 2$\sigma$ can be resolved by adding ICAL data. The latter is seen to play an important role in removing the wrong hierarchy solution for $\theta_{23} = 48^\circ$. In conclusion, we show that the combination of data from different LBL and atmospheric neutrino experiments can play a crucial role in removing the degeneracies associated with neutrino oscillation parameters thereby improving the precision of the parameters $\theta_{23}$ and $\delta_{CP}$. This also paves the way towards an unambiguous determination of these parameters.

Appendix A: Synergy between appearance and disappearance channel and role of antineutrinos

In this Appendix we discuss the origin of discrete degenerate regions in the test ($\delta_{CP} - \theta_{23}$) plane from the combination of appearance and disappearance channel for NO$\nu$A. We demonstrate the role of antineutrinos in resolving the degeneracies. The reference true point chosen in generating the data is $\delta_{CP} = -90^\circ$ and $\theta_{23} = 39^\circ$. In the upper row of Fig. 6 we plot the sensitivity of NO$\nu$A[6+0]. The serpentine curves in the top-left panel of Fig. 6 represents the allowed area at 90% C.L. from only appearance channel. The area inside the vertical curves represents the allowed area from only disappearance channel at the same
C.L. The area between the blue dotted (magenta dotted) curves denote the region obtained for the right wrong hierarchy.

FIG. 6: The plots in the upper row are for NO\(\nu\)A running only in neutrino mode i.e. NO\(\nu\)A[6+0]. Those in the lower row are for NO\(\nu\)A running in equal neutrino and antineutrino mode i.e NO\(\nu\)A[3+3]. To generate these plots we have assumed true \(\theta_{23}\) = 39°, true \(\delta_{CP}\) = −90° and true hierarchy = NH whereas, test parameters are marginalized over the range given in Table II. The plots in the middle panel are generated for fixed value of test \(\delta_{CP}\) (= −90°).

For the appearance channel the allowed region is continuous and no discrete degenerate solutions appear. This can be understood in the following manner. In the neutrino appearance channel \(\delta_{CP} = -90^\circ\) corresponds to the maximum value in the probability. As one moves away from −90°, the probability decreases and reaches its minimum value at +90°. On the other hand the probability increases (decreases) as \(\theta_{23}\) increases (decreases). So if we draw an imaginary horizontal line and an imaginary vertical line at the true point then the allowed region is expected to come along the diagonal of the rectangle obtained by the in-
intersection of these two imaginary lines and the X,Y axes for $\theta_{23} > 39^\circ$ and $\delta_{CP} \leq +90^\circ$. For $\delta_{CP} > +90^\circ$, the probability starts to increase, so $\theta_{23}$ has to fall to keep the probability same. This explains the serpentine nature of the allowed area. The width of the band corresponds to the $\theta_{23}$ precision of the experiment. For the disappearance channel, the allowed region is in the vicinity of $\theta_{23}$ and $\pi/2 - \theta_{23}$ and parallel to the $\delta_{CP}$ axis since the $P_{\mu\mu}$ probability has a very weak dependence on $\delta_{CP}$. However the combination of disappearance and appearance channels gives discrete regions in the parameter space due to the excellent $\theta_{23}$ precision of the disappearance channel near $\theta_{23} = 39^\circ$ and $51^\circ$. This helps to exclude the other wrong values of $\theta_{23}$. This is shown in the top right panel of Fig. 6. Apart from the allowed regions around true value, one can identify the distinct degenerate solutions corresponding to wrong hierarchy-wrong octant-right $\delta_{CP}$ (WH-WO-R$\delta_{CP}$) and right hierarchy-wrong octant-wrong $\delta_{CP}$ (RH-WO-W$\delta_{CP}$) regions.

To show the exact synergy between the appearance and disappearance channels, in the middle panel of the top row we plot the $\chi^2$ as a function of $\theta_{23}(\text{test})$ for a fixed $\delta_{CP}$ value of $-90^\circ$ for the same hierarchy (NH). This figure shows that though the disappearance channel suffers from intrinsic octant degeneracy and does not have any octant sensitivity itself ($\chi^2 \sim 0$), when added to the appearance channel the combined octant sensitivity in the wrong octant is higher than the only appearance channel. Note that from these figures it is clear that the disappearance channel is mainly responsible for the precision of $\theta_{23}$ while the appearance channel for that of $\delta_{CP}$.

Next we discuss the role of antineutrino runs in NO$\nu$A. In the bottom row of Fig. 6 we plot the same figures as the top row but for 3 year neutrino + 3 year antineutrino run. In the bottom left panel, we see that when antineutrino information is added to neutrino data, the allowed region from the appearance channel is significantly reduced. The reason is as follows: as $\delta_{CP}$ changes its sign for antineutrinos, the serpentine shape of the allowed region gets flipped with respect to $\delta_{CP}$. This excludes the right hierarchy-wrong octant regions of $\delta_{CP} \in \text{UHP}$ (i.e. RH-WO-W$\delta_{CP}$) and the wrong hierarchy-wrong octant regions of $\delta_{CP} \in \text{LHP}$ (i.e. WH-WO-R$\delta_{CP}$). Thus after adding the antineutrino data only the RH-RO-R$\delta_{CP}$ solution remains, as can be seen from the third panel in the bottom row. From the NO$\nu$A antineutrino probability figure in the top right panel of Fig. 1), it is seen that the probability of the true point cannot be matched by any points in the NH-HO or IH-HO bands. This means that for this true point antineutrinos are free from the degeneracies that appear with
the wrong octant in neutrinos. Thus addition of antineutrino information removes the wrong octant solutions of NOνA[6+0] that appear in the top right panels of Fig. 6.

The nature of the disappearance channel contours are seen to remain unaltered but now the allowed area is slightly broader. This is because of a reduction in the overall statistics due to the smaller cross sections of the antineutrinos. This is also seen in the middle panel where the width of the $\chi^2$-contours increase.

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