Spin-induced black-hole scalarization in Einstein-scalar-Gauss-Bonnet theory

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We construct black hole solutions with spin-induced scalarization in a class of models where a scalar field is quadratically coupled to the topological Gauss-Bonnet term. Starting from the tachyonically unstable Kerr solutions, we obtain families of scalarized black holes such that the scalar field has either even or odd parity, and we investigate their domain of existence. The scalarized black holes can violate the Kerr rotation bound. We identify “critical” families of scalarized black hole solutions such that the expansion of the metric functions and of the scalar field at the horizon no longer allows for real coefficients. For the quadratic coupling considered here, solutions with spin-induced scalarization are entropically favored over Kerr solutions with the same mass and angular momentum.

Introduction. Compact objects in gravity theories involving scalar degrees of freedom can undergo a phase transition induced by a tachyonic instability and known as “spontaneous scalarization.” By now it is clear that this instability comes in different flavors. The possibility of matter-induced spontaneous scalarization was originally proposed for compact neutron stars in scalar-tensor theories [1]. More recently it was shown that spontaneous scalarization is possible also in the absence of matter. Curvature-induced spontaneous scalarization of black holes (BHs) was first proposed in Einstein-scalar-Gauss-Bonnet (EsGB) theories [2–4], and charge-induced scalarization can also occur in Einstein-scalar-Maxwell theories [5].

In this paper we focus on EsGB theories with action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} (\partial_\mu \phi)^2 + f(\phi) R_{\text{GB}}^2 \right],$$

where we use geometrical units ($G = c = 1$), and $\phi$ is a real scalar field coupled to the Gauss-Bonnet (GB) invariant $R_{\text{GB}}^2 = R_{\mu
u\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$. We will focus on the simple quadratic coupling function $f(\phi) = \frac{1}{2} \eta \phi^2$, with $\eta < 0$ [6–8]. Recent work showed that, in these theories, the Kerr BH solutions of general relativity can develop a spin-induced tachyonic instability when their dimensionless spin parameter $j \equiv J/M^2 \gtrsim 0.5$ [6]. Shortly afterwards, this was confirmed analytically [7] and numerically [8].

These works pointed out the existence of a spin-induced instability, but they did not address its end point. Here we show that spin-induced instabilities do indeed give rise to families of scalarized BH solutions.

The instability threshold depends on the Gauss-Bonnet (GB) coupling $\eta/M^2$ and on the (even or odd) symmetry of the scalar field under parity transformation. For small values of the GB coupling and associated large values of $j$, the thresholds for even and odd parity differ, whereas for large values of the GB coupling the two thresholds almost coincide.

We construct these BHs numerically, starting from the respective threshold solutions. We then vary the input parameters to map out the domain of existence of scalarized BHs for both even- and odd-parity scalar fields. The expansions of the metric functions and of the scalar field at the horizon provide us with an analytic criterion to identify critical solutions that form the second boundary of the domain of existence.

We investigate the thermodynamical stability of these BH solutions by computing their entropy. Solutions with curvature-induced scalarization are entropically disfavored with respect to Schwarzschild and Kerr BHs when $f(\phi)$ is quadratic [3, 9], but they become entropically favored when we add a quartic term [10] or for exponential coupling functions [2, 11]. Linear perturbation theory shows that the entropically favored (disfavored) fundamental scalarized solutions are mode stable (unstable) [10, 12, 13]. Here we find that BH solutions with spin-induced scalarization are entropically favored over Kerr solutions with the same mass and angular momentum, but their dynamical stability remains an open question.

General framework. The generalized Einstein and scalar field equations follow by varying the action (1) with respect to the metric $g_{\mu\nu}$ and the scalar field $\phi$:

$$G_{\mu\nu} = T_{\mu\nu}, \quad \nabla^2 \phi + \frac{df}{d\phi} R_{\text{GB}}^2 = 0,$$

where the effective stress-energy tensor

$$T_{\mu\nu} = -\frac{1}{4} g_{\mu\nu} \partial_\rho \phi \partial^\rho \phi + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi$$

$$- \frac{1}{2} (g_{\mu\nu} g_{\lambda\omega} + g_{\lambda\mu} g_{\nu\omega}) \eta^{\lambda\omega\delta\epsilon} \bar{R}^{\delta\epsilon}_{\alpha\beta} \nabla_\gamma \partial_\delta \phi f(\phi),$$

with $\bar{R}^{\delta\epsilon}_{\alpha\beta} = \eta^{\delta\gamma\sigma\tau} R_{\sigma\tau\alpha\beta}$ and $\eta^{\gamma\sigma\tau} = \epsilon^{\gamma\sigma\tau}/\sqrt{-g}$.
To construct stationary, axially symmetric spacetimes with two commuting Killing vector fields ($\xi = \partial_t$ and $\eta = \partial_r$) we employ a Lewis-Papapetrou-type ansatz [14, 15]

$$ds^2 = -be^{F_0}dt^2 + e^{F_1} \left( dr^2 + r^2 d\theta^2 \right) + e^{F_2}r^2 \sin^2 \theta (d\phi + \omega dt)^2,$$

(4)

where $r$ is a quasi-isotropic radial coordinate, $r_H$ is the isotropic horizon radius, $b = (1 - \frac{r_H}{r})^2$. The metric functions $F_i$ ($i = 0, 1, 2$) and $\omega$ depend on the coordinates $r$ and $\theta$, and they are even under parity. The scalar field $\phi = \phi(\theta, \rho)$ can be either even or odd with respect to parity transformation, i.e. $\phi_\pm(r, \pi - \theta) = \pm \phi_\pm(r, \theta)$. A parity-odd scalar field is consistent with the field equations, since the generalized Einstein equations are quadratic and the generalized Klein-Gordon equation is linear in the scalar field (note that parity is a symmetry only when $f(\phi)$ is even in $\phi$). Scalarized BHs with an even scalar field and no radial nodes are the fundamental scalarized solutions, whereas those with an odd scalar field are angularly excited solutions.

The proper set of boundary conditions is obtained by considering symmetry, regularity and asymptotic flatness of the solutions. This implies $F_i(\infty) = 0$ ($i = 0, 1, 2$), $\omega(\infty) = \phi(\infty) = 0$ as $r \rightarrow \infty$. For a massless scalar field one can construct an approximate solution of the field equations as a power series in $1/r$, with the dominant term being of monopole type in the even case and of dipole type in the odd case: $\phi_0 = Q/r + \ldots$ and $\phi_\pm = P \cos \theta/r^2 + \ldots$, where $Q$ and $P$ are interpreted as the scalar charge and the dipole moment of the scalar field, respectively.

The boundary conditions at the event horizon, located at a surface of constant $r = r_H$, are obtained by considering a power-series expansion in terms of $\delta = (r - r_H)/r_H$: $\partial_r F_0(r_H) = 1/r_H$, $\partial_r F_1(r_H) = -2/r_H$, $\partial_r F_2(r_H) = -2/r_H$, $\omega(r_H) = \omega_H$, $\partial_\theta \phi(r_H) = 0$, where $\omega_H$ is a constant. On the symmetry axis ($\theta = 0, \pi$), axial symmetry and regularity impose $\partial_\theta F_i|_{\theta = 0, \pi} = 0$ ($i = 0, 1, 2$), $\partial_\theta \omega|_{\theta = 0, \pi} = 0$. Since all functions are either even or odd, it is sufficient to consider the range $0 \leq \theta \leq \pi/2$ for the angular variable $\theta$ in the numerical calculations. Consequently, we impose the following boundary conditions on the equatorial plane: $\partial_\theta F_i|_{\theta = \pi/2} = 0$ ($i = 0, 1, 2$), $\partial_\theta \omega|_{\theta = \pi/2} = \partial_\theta \phi_+|_{\theta = \pi/2} = \phi_-|_{\theta = \pi/2} = 0$.

From the horizon metric we obtain the Hawking temperature [14]

$$T_H = \frac{1}{2\pi e^{F_0-F_1}}.$$

(5)

In fact, the equation $G^\theta_\theta = T^\theta_\theta$ implies that $F_0/F_1$ (and therefore the Hawking temperature) is constant. This observation can be used to test the numerical accuracy of our solutions.

The horizon area is given by

$$A_H = 2\pi r_H^2 \int_0^\pi d\theta \sin \theta e^{F_0+F_2}/2.$$

(6)

The entropy of Kerr BHs is a quarter of the horizon area [14], but the entropy of EsGB BHs can be computed as an integral over the spatial cross section of the horizon [16] and it acquires an extra contribution:

$$S = \frac{1}{4} \int_{\Sigma_H} d^2 x \sqrt{h} \left( 1 + 2f(\phi) \frac{\partial}{\partial \phi} \right),$$

(7)

where $h$ is the determinant of the induced metric on the horizon and $\hat{R}$ is the corresponding scalar curvature.

The mass $M$ and the angular momentum $J$ can be found from the asymptotic behavior of the metric functions: $g_{tt} = -1 + 2M/r + \ldots$, $g_{\phi t} = -2J \sin^2 \theta/r + \ldots$.

**Numerical Results.** To obtain the EsGB BHs with spin-induced scalarization we need to solve for the functions ($F_0, F_1, F_2; \omega; \phi$) subject to the boundary conditions specified above, that guarantee regularity and asymptotic flatness.

We provide three input parameters ($\eta, r_H$ and $\omega_H$) and we follow the numerical procedure of Refs. [9, 15]. We introduce the radial variable $x = 1 - r_H/r$, mapping the interval $[r_H, \infty]$ to the interval $[0, 1]$, and discretize the equations on a nonequidistant grid in $x$ and $\theta$, covering the integration region $0 \leq x \leq 1$ and $0 \leq \theta \leq \pi/2$. We perform the integrations using the package FIDISOL/CADSOL [17, 18], based on a Newton-Raphson method, and we extract the physical properties of the BHs when convergence is reached within the required accuracy. The numerical error for the functions is estimated to be of the order $10^{-3}$.

In order to understand the critical solutions, which represent the second boundary of the domain of existence of scalarized BH solutions, we consider higher-order terms of local solutions close to the horizon: $F_0 = F_0|_{r_H} + \delta + f_0 \delta^2/2 + \ldots$, $F_1 = F_1|_{r_H} - 2\delta + f_1 \delta^2/2 + \ldots$ ($i = 1, 2$), $\omega = \omega_H + \omega_2 \delta^2/2 + \omega_3 \delta^3/6 + \ldots$, $\phi = \phi_H + \phi_2 \delta^2/2 + \ldots$.

![FIG. 1. Dimensionless discriminant as a function of $\theta$ for $\omega_H = 0.065$ and selected values of $\eta$. When $\eta < \eta_c \approx -179$ the local maximum becomes positive, and scalarized BH solutions cease to exist.](image-url)
We obtain equations for the coefficients of the higher-order terms $f_0, f_1, f_2, \omega_3$, and $\phi_2$ which allow us to express the higher-order coefficients in terms of $F_0, F_1, F_2, \omega, \phi$, and their first and second derivatives with respect to $\theta$. Solving these equations yields a quartic equation for $\phi_2$. The existence of real solutions of the quartic equation depends on the sign of the discriminant $D = (p/3)^3 + (q/2)^2$ of the reduced cubic resolvent, $v^3 + pv + q = 0$. Real solutions exist if $D(\theta) \leq 0$ for $0 \leq \theta \leq \pi/2$. The numerical calculations show that $D(\theta)$ is always negative for $\omega_H > 0.073$. However, for $\omega_H \leq 0.073$ the function $D(\theta)$ develops a local maximum, which tends to zero when $\eta$ is decreased to some critical value $\eta_{cr}$. Solutions for $\eta < \eta_{cr}$ cease to exist.

This is demonstrated in Fig. 1, where we show the dimensionless discriminant $\Delta = \frac{\omega_H}{8}\frac{\eta^2}{M^2} + 1$ for $\omega_H = 0.065$ and decreasing values of $\eta$. Note however that for symmetric BHs with large angular momentum the condition for real solutions is violated at the equator of the horizon.

To map out the domain of existence, we calculate families of scalarized BH solutions with fixed horizon angular velocity $\omega_H$ while varying the coupling constant $\eta$.

In Fig. 2 we show various BH properties as functions of the dimensionless coupling parameter $\eta/M^2$ for families of solutions with fixed values of $\omega_H$. The various panels show the dimensionless scalar charge $Q/M$ (dipole moment $P/M^2$) for the fundamental even (odd) solutions (top left); the dimensionless angular momentum $J$ (top right); the dimensionless horizon area $A_H/8\pi M^2$ (bottom left); and the dimensionless Hawking temperature $T_H M$ (bottom right).

Figure 2 provides important new insight into the domain of existence of BHs with spin-induced scalarization. Bifurcation from the Kerr solutions takes place at some threshold solutions (“Kerr-thr” in the legend) representing the first boundary of the domain of existence. These thresholds are rather close for even and odd solutions, especially for large values of $|\eta/M^2|$. The second boundary is given by the critical solutions (“crit” in the legend) such that the discriminant $D(\theta)$ vanishes somewhere. For
Conclusions. Starting from even- and odd-parity threshold solutions, we have mapped out the domain of existence of BHs with spin-induced scalarization in EGB theories with a quadratic coupling function. The second boundary of the domain of existence corresponds to critical solutions beyond which the horizon expansion of the metric functions and of the scalar field no longer admit real coefficients. If present, a third boundary should correspond to extremal scalarized BHs, but this regime is hard to explore numerically. Scalarized BHs can violate the Kerr bound when $|\eta/M^2| < 1.53 (|\eta/M^2| < 0.55)$ for odd (even) solutions. This violation seems to occur only in the vicinity of the extremal solutions, and it is of the order of 5% (0.5%) for odd (even) solutions.

Scalarized BHs are entropically favored over Kerr BHs with the same mass and angular momentum. If previous studies of curvature-induced scalarization are a useful analogy, this would suggest that BHs with spin-induced scalarization are (linearly) mode stable under perturbations. This may come as a surprise, since we have employed a simple quadratic coupling function; however we chose a negative coupling constant, in contrast with previous work on curvature-induced scalarization. The dynamical stability of BHs with spin-induced scalarization is an important open question that will require further work. Perturbations of rotating BHs in modified gravity are a notoriously difficult technical problem, because the equations are nonseparable (see e.g. Ref. [23] for recent progress on scalar perturbations in a slow-rotation expansion). Time evolutions may provide a practical way to find out if these solutions are dynamically stable.

Last but not least, the problems of well-posedness, gravitational collapse, and gravitational waveforms from binary BH mergers in EGB theories are very active research areas in analytical and numerical relativity [24–31]. The new solutions discussed in this paper may have important implications in this context.

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