Magnetic phase diagrams of barcode-type nanostructures

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Abstract

The magnetic configurations of barcode-type magnetic nanostructures consisting of alternate ferromagnetic and nonmagnetic layers arranged within a multilayer nanotube structure are investigated as a function of their geometry. Based on a continuum approach we have obtained analytical expressions for the energy which lead us to obtain phase diagrams giving the relative stability of characteristic internal magnetic configurations of the barcode-type nanostructures. (Some figures in this article are in colour only in the electronic version)

1. Introduction

Magnetic nanoparticles are attracting the increasing interest of researchers in various fields due to their promising applications in hard disk drives, magnetic random access memory, and other spintronic devices [1–5]. In addition, these magnetic nanoparticles can be used for potential biomedical applications, such as magnetic resonance imaging (the nanoparticles can be used to trace bioanalytics in the body), cell and DNA separation, and drug delivery [6]. To apply nanoparticles in various potential devices and architectures, it is very important to control their size and shape in order to retain thermal and chemical stability [7].

The trusty sphere remains the preferred shape for nanoparticles, but this geometry leaves only one surface for modification, complicating the generation of multifunctional particles. Thus, a technology that could modify differentially the inner and outer surfaces would be highly desirable [8]. Tubular nanostructures have stimulated extensive research efforts in recent years because of their particular significance for prospective applications. A wide range of materials including semiconductors, polymers, and metals have been prepared in the form of nanotubes [9–13]. Although magnetic nanotubes has been intensively investigated, barcode-type nanostructures have received less attention, in spite of tailoring their multisegmented nanotube structure, along with the functionalization of the inner wall surface of barcode-type nanotubes with various molecules (for example, proteins and DNA). Moreover, they are expected to be particularly useful in the field of catalysis, advanced microfluidics, molecule separation, and biological and magnetic sensors [14–19]. It is worth mentioning that barcode-type magnetic nanostructures consisting of regular arrays of magnetic segments have been considered as providing the basis for extending magnetic storage densities beyond the superparamagnetic limit. In such a system, a single tube with \( n \) magnetic layers might store up to \( 2^n \) bits, with a volume much larger than those of the grains in conventional recording media, beating this way thermal fluctuations and increasing the recording density by a factor \( 2^{n-1} \) [20, 21]. Recently [14], the preparation of metallic nanotubes based on the preferential electrodeposition of metal along the pore walls of an anodic alumina oxide (AAO) membrane, in the presence of metallic nanoparticles on the wall surfaces, has been reported. In the paper by Lee et al [14] multisegmented metallic nanotubes were prepared, with a bimetallic stacking configuration along the tube axis, showing different magnetic behavior as compared with continuous ones, which encourages a study of the possible magnetic configurations and magnetostatic interactions in these barcode-type magnetic nanotubes. Clearly, for the development of magnetic devices based on those arrays, knowledge of the internal magnetic structure of the barcode-type nanostructures is of fundamental importance.
by the dipolar potential)

Under these assumptions, the magnetic energy is just given by the sum of four terms: exchange, dipolar, anisotropy and Zeeman contributions, which are taken from the well known continuum theory of ferromagnetism [22]. As we are interested in the study of the relative stability of the zero-field magnetic ground state, the contribution of the Zeeman energy can be ignored. Under these assumptions, the magnetic energy is just given by the dipolar ($E_{\text{dip}}$), exchange ($E_{\text{ex}}$), and anisotropy ($E_{\text{a}}$) contributions.

The total magnetization can be written as $M(r) = \sum_{i=1}^{n} M_i(r)$, where $M_i(r)$ is the magnetization of the $i$th ferromagnetic segment. In this case, the magnetostatic potential $U(r)$ splits up into $n$ components, $U_i(r)$, associated with the magnetization of each ferromagnetic segment. Then, the total dipolar energy can be written as $E_{\text{dip}}(r) = \sum_{i=1}^{n} E_{\text{dip}}(i) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_{\text{int}}(i,j)$, where

$$E_{\text{dip}}(i) = \frac{\mu_0}{2} \int M_i(r) \cdot \nabla U_i(r) \, dv$$

is the dipolar contribution to the self-energy of the $i$th ferromagnetic segment, and

$$E_{\text{int}}(i,j) = \mu_0 \int M_i(r) \cdot \nabla U_j(r) \, dv$$

is the dipolar interaction between ferromagnetic segments $i$ and $j$.

Usually, the exchange energy $E_{\text{ex}}$ in multilayer nanostructures has contributions both from the direct exchange interaction within the magnetic segments and from the indirect interaction between them, mediated by the conduction electrons in the nonmagnetic layers. Since the indirect interaction decays rapidly with the thickness of the nonmagnetic segment, it can be neglected provided $d$ is large enough. A good estimate of the range of the indirect exchange interaction can be obtained from the results for multilayers [23]. As a general result, the interlayer exchange coupling vanishes for spacer thicknesses greater than a few nanometers, which does not exceed the value of the exchange length $l_x = \sqrt{2A/\mu_0 M_0^2}$ of ferromagnetic metals.

The cubic anisotropy energy of the particle can be added by means of the following expression:

$$E_{\text{c}}(i) = K_e \int \left( m_i^2 r_{iy}^2 + m_i^2 r_{iz}^2 + m_i^2 r_{ix}^2 \right) \, dv,$$

and the uniaxial anisotropy energy is given by

$$E_{\text{a}}(i) = -K_a \int m_i^2 r_{ix} \, dv.$$

On the basis of the above results, the total energy of the barcode-type nanostructure can be written as $E_{\text{tot}} = \sum_{i=1}^{n} E_{\text{ex}}(i) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_{\text{int}}(i,j)$, where $E_{\text{ex}}(i) = E_{\text{dip}}(i) + E_{\text{c}}(i)$ is the self-energy of the ferromagnetic segment $i$, and $E_{\text{int}}$ is the (dipolar) interaction energy between two magnetic segments. We will proceed to describe the magnetization of the different states we are considering here and then we will evaluate the magnetic energy of each configuration. Results will be given in units of $\mu_0 M_0^2 l_x^3$, i.e., $\tilde{E} = E/\mu_0 M_0^2 l_x^3$.

2.1. Magnetic configurations

It has been shown recently that single magnetic nanorings present three basic ground states depending on their geometry.
2.1.1. \( F_z \) state. These configurations are: \( (F_z) \) a quasi-uniform magnetization state oriented in the direction parallel to the cylindrical axis (\( z \) axis); \( (F_x) \) a quasi-uniform magnetization state oriented in the plane perpendicular to the \( z \) axis; and \( (V) \) a flux-closure vortex state. For long nanorings (\( W \gg R \)), the \( F_x \) phase is not present \([24, 25]\), a result that holds for nanotubes \([26, 27]\).

From this equation it is possible to obtain the expression for the magnetostatic field. Thus we write, \( \mathbf{H}(r, z) = -\nabla U(r, z) = H_r(r, z)\hat{r} + H_z(r, z)\hat{z} \) with

\[
H_r(r, z) = \frac{M_0}{2} \int_0^\infty \frac{dk}{k} \left[ RJ_1(kr) - aJ_1(ka) \right]
\times \left( e^{-ki|\hat{x}-\hat{z}|} - e^{-ki|\hat{x}+\hat{z}|} \right),
\]

and

\[
H_z(r, z) = \frac{M_0}{2} \int_0^\infty \frac{dk}{k} \left[ RJ_1(kr) - aJ_1(ka) \right]
\times \left[ RJ_1(kR) - aJ_1(ka) \right] Y(W, z),
\]

where \( Y(W, z) = \frac{\text{sign}(W)}{2} \left( e^{-ki|\hat{x}-\hat{z}|} \right) - \frac{\text{sign}(W)}{2} \left( e^{-ki|\hat{x}+\hat{z}|} \right) \).

The function \( \text{sign}(x) \) gives \(-1\), 0 or 1 depending on whether \( x \) is negative, zero, or positive. Figure 2 illustrates the magnetostatic field profile calculated analytically for nanotubes with the same geometrical parameters as the ones investigated experimentally by Lee et al \([14]\). Finally, the reduced interaction energy between two tubular nanostructures
has been calculated in a rather general way by Escrig et al [21, 30] and is given by
\[ \tilde{E}_{\text{int}}^{F_x} = \frac{\pi R^3}{l_1^2} \int_0^\infty \frac{dq}{q^2} e^{-q^2 \frac{R}{2}} (1 - e^{-q^2})^2 \]
\[ \times (J_1(q) - \beta J_1(q\beta))^2. \]
Thus, the reduced total energy for the \( F_x \) state can be expressed as
\[ \tilde{E}_{\text{tot}}^{F_x} = n \tilde{E}_{\text{self}}^{F_x} = \frac{\pi R^3}{l_1^2} \int_0^\infty \frac{dq}{q^2} e^{-q^2 \frac{R}{2}} (1 - e^{-q^2}) \]
\[ \times (J_1(q) - \beta J_1(q\beta))^2 g_z(n, q, \sigma), \]
where
\[ g_z(n, q, \sigma) = \frac{(n-1)e^{\sigma} + e^{-(n-1)\sigma} - n}{(1 - e^{\sigma})^2} \quad \text{and} \quad \sigma = \frac{d + W}{R}. \]

2.1.2. \( F_x \) state. For the \( F_x \) state, \( M(r) \) can be generally considered as \( M_0 \cos[(i - 1)\theta]x + M_0 \sin((i - 1)\theta)y \), which represent a helicoidal magnetic state, with the \( \theta \) the angle between the in-plane magnetization of adjacent segments. For the in-plane state, the exchange and anisotropy contributions to the self-energy vanish and the reduced self-energy takes the form [21]
\[ \tilde{E}_{\text{self}}^{F_x} = \frac{\pi R^3}{2l_1^2} \int_0^\infty \frac{dq}{q^2} \left( e^{-q^2 \frac{R}{2}} + q \frac{W}{R} - 1 \right) \]
\[ \times (J_1(q) - \beta J_1(q\beta))^2, \]
where \( J_1(z) \) is a Bessel function of the first kind. In order to calculate the interaction energy between the ferromagnetic segments, we first need to calculate the magnetostatic potential \( U(r) \) of a single tubular structure. The expression for this potential is given by
\[ U(r, \phi, z) = M_0 \cos \phi \int_0^\infty \frac{dk}{k} j_1(kr) f(k) \]
\[ \left\{ \begin{array}{ll}
\frac{e^{-kz} \sinh(k \frac{W}{2})}{k} & z > \frac{W}{2} \\
(1 - e^{-kz} \cosh(kz)) & -\frac{W}{2} < z < \frac{W}{2} \\
\frac{e^{kz} \sinh(k \frac{W}{2})}{k} & z < -\frac{W}{2},
\end{array} \right. \]
where \( f(k) = (R j_1(kR) - a j_1(ka)) \). Finally, the reduced interaction energy between two tubular nanostructures is given by
\[ \tilde{E}_{\text{int}}^{F_x} = \frac{\pi R^3}{2l_1^2} \cos \theta \int_0^\infty \frac{dq}{q^2} e^{-q^2 \frac{R}{2}} (1 - e^{-q^2})^2 \]
\[ \times (J_1(q) - \beta J_1(q\beta))^2. \]
Thus, the reduced total energy for the \( F_x \) state can be expressed as
\[ \tilde{E}_{\text{tot}}^{F_x} = n \tilde{E}_{\text{self}}^{F_x} + \frac{\pi R^3}{2l_1^2} \int_0^\infty \frac{dq}{q^2} e^{-q^2 \frac{R}{2}} (1 - e^{-q^2}) \]
\[ \times (J_1(q) - \beta J_1(q\beta))^2 g_z(n, q, \sigma, \theta), \]
\[ g_z(n, q, \sigma, \theta) = \sum_{i=1}^{n} \sum_{j=0}^{n} \frac{e^{-\sigma} \cos((j-i)\theta)}{(1 + e^{\sigma})^2}. \]

2.1.3. Vortex state. Finally, for the vortex state \( V, M(r) \) can be approximated by \( M_0 \tilde{\phi} \), where \( \tilde{\phi} \) is the azimuthal unit vector. Due to the condition of perfect flux closure in the vortex configuration, one magnetic nanostructure in such a configuration does not interact with the others, independently of their magnetic configuration. Thus, there is no difference between the clockwise and counter-clockwise directions. Finally, the reduced total energy for the vortex state is given just by the \( n \) self-energies [21, 26]
\[ \tilde{E}_{\text{tot}}^{V} = -n \pi W \ln \beta + n \kappa_c \pi W R^2 \left( \frac{1}{16} - \beta \right). \]
Here, \( \kappa_c = 2k_c/\mu_0 M_0^2 \).

2.2. Phase diagram for multisegmented nanorings

We proceed to investigate the relative stability of the configurations. Phase diagrams are shown in figure 3 for \( d = l_1, \beta = 0.5, \) and \( n = 5 \). The anisotropy for four different materials are considered according to the values presented in table 1. The diagrams show three regions, corresponding to configurations \( F_x, F_z, \) and \( V, \) as in the case of a single nanoring (\( n = 1 \)). Notice that for the case of Co, the existence of a strong uniaxial anisotropy favors the \( F_z \) phase, decreasing the other two phases, specially the \( V \) one. In the case of cubic anisotropy, the transition lines are similar to the case of a phase diagram without anisotropy. Because of its very low anisotropy, results for pernmalloy describe reasonably well a material with no anisotropy, as was pointed out in [27].

Since nanostructures are usually polycrystalline, the crystallographic orientations of the crystallites are random and,
as a consequence, the average magnetic anisotropy of the particle is very small. In view of that, it will be neglected in our calculations [32, 33].

For different values of \( n \) we can determine the ranges of values of the dimensionless radius \( R/l_x \) and length \( W/l_x \) within which one of the three configurations is of lowest energy. The boundary line between any two configurations can be obtained by equating the expressions for the corresponding total energies. Figure 4 illustrates phase diagrams for \( d = l_x, \beta = 0.5, \) and \( n = 1 \) (solid lines), 3 (dotted lines), and 5 (dashed lines). It is important to observe that for the \( F_z \) and \( F_x \) states the exchange energy is the same. Then, in the absence of applied magnetic fields and crystalline anisotropies, the dipolar energy is fundamental in obtaining the magnetic configuration of lowest energy. Thus, the dipolar contribution represents the shape anisotropy and for multisegmented nanostructures with a small length (namely nanorings) the low energy state is the quasi-uniform in-plane configuration \( F_x \) [25]. As the length is increased, but keeping the radius small enough, there is a transition to the out-of-plane state \( F_z \) at a critical length whose value depends on \( R, \beta, \) and the exchange length \( l_x \). Finally, by comparing our results we observe differences in the behavior of the triple point as a function of \( n \). The triple point occurs for smaller \( R/l_x \) when \( n \) is decreased.

Similar to the case of a single ring, the phase diagram changes with \( \beta \) [25]. The dependence of the whole diagram on the value of \( n \) can be investigated by looking at the trajectories of the triple point in the \( RW \) plane as functions of \( \beta \). Such trajectories are shown in figure 5 for \( d = l_x \) and different values of \( n \). We remark that the radius \( R_t \) of the triple point represents the smallest value of \( R \) for which the vortex configurations are stable, and \( W_t \) is the largest value of \( W \) for which the in-plane configurations are stable.

### 2.3. Phase diagram for multisegmented nanotubes

As the multisegmented tubes that motivate this work [14] satisfy \( W/R \gg 1 \), then the \( F_x \) phase can be left out of consideration. Thus, to obtain an expression for the transition line separating the \( F_z \) phase from the \( V \) phase we match the expressions for the energy of these two configurations. It is important to mention that, for tubes with long radius, a third state, which is a mixture of the other two and has been called bamboo or mixed state [34–37], has been observed. As is well known, the consideration of non uniform magnetic configurations considerably complicates the calculations and for simplicity, we studied multisegment magnetic nanotubes whose radii are not large enough to allow the formation of relevant vortex domains at the extremes of the tube. Figure 6 presents the transition line for \( n = 1 \) and 2. To the left of each line the \( F_z \) state prevails while to the right of the same line the vortex \( V \) configuration is more stable. The
Figure 6. Magnetic phase diagrams of non-interacting multisegmented nanotubes for different values of \( n \). The dimensions of the tube, \( W \) and \( R \), are normalized to the exchange length \( l_x \). Experimental points are discussed in the text.

dots labeled (continuous) and (multisegmented) in figure 6 correspond to the cases of the two hysteresis curves reported in the experimental paper by Lee et al [14] defined by (continuous) \( n = 1 \), \( R = 150 \text{ nm} \), \( W = 16 \mu \text{m} \), \( \beta = 0.75 \), and \( l_x = 8.225 \text{ nm} \); (multisegmented) \( n = 2 \), \( R = 150 \text{ nm} \), \( W = 800 \text{ nm} \), \( d = 4800 \text{ nm} \), \( \beta = 0.75 \), and \( l_x = 8.225 \text{ nm} \). It is important to note that the transition line for \( n = 1 \) is almost equal to the one for \( n = 2 \). This is due to the large average distance between the neighboring Ni segments (\( d = 4.8 \mu \text{m} \)) which avoids interaction effects. From this figure we can conclude that the multisegmented system is well inside the \( V \) phase while the continuous system is inside the \( F_z \) phase. It allows us to understand why the experimental samples show a different magnetic behavior; they simply have substantial differences in the length of their ferromagnetic segments.

The results presented above may be generalized. We now proceed to investigate the transition line separating the \( F_z \) phase from the \( V \) phase. To obtain an expression for this transition line we match the expressions for the energy of these two configurations. This leads to \( W/l_x = \alpha(\beta, n, d) \times R^3/l_x^3 \). Function \( \alpha(\beta, n, d) \) is plotted in figure 7. Care must be applied in the limits of the intervals for \( \beta \). In particular, when \( \beta \) goes to 1 we deal with extremely narrow nanotubes, where eventual surface roughness and thickness irregularities of the nanotubes become important. On the other side, when \( \beta \) goes to zero we are approaching the limit of a solid cylinder, where the core in the vortex phase becomes important and must be considered to get the solution. As the multisegmented nanotubes considered experimentally have \( \beta \approx 0.75 \), we have neglected these two cases.

3. Conclusions
In conclusion, we have studied the relative stability of ideal configurations of magnetic barcode-type tubular nanostructures composed of alternate ferromagnetic and nonmagnetic layers. In such systems we investigated the size range of the geometric parameters for which different configurations are of lowest energy. Results are summarized in phase diagrams which clearly indicate that the magnetic behavior of such structures can be tailored to meet specific requirements provided a judicious choice of such parameters is made. The lines separating the magnetic phases and, in particular, the triple point, are very sensitive to the geometry of the barcode-type nanostructures. The phase diagrams presented can provide guidelines for the production of nanostructures with technological purpose.

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