Nonlinear stochastic estimation of altitude and vertical flight speed in real time

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Abstract. The optimal algorithm for calculating the altitude and vertical flight speed of an aircraft, which is based on complex processing of information from static pressure sensors, outdoor air temperature, and vertical acceleration is described. Information processing is performed using nonlinear estimation methods. Adaptive properties of the estimation algorithm are achieved by including into the state vector the parameters that characterize the properties of sensors and static pressure transmission paths.

1. Introduction

Various approaches are known to determine the altitude and speed parameters of an aircraft, and above all, the altitude and vertical flight speed [1, 2]. Based on the kinematic relations connecting height and vertical acceleration [3], the main relations of linear filters are obtained [4, 5], and complex processing for the identification of aerodynamic coefficients is described in [6].

However, none of these approaches solves the problem of accurately determining the altitude and vertical flight speed because they do not take into account the properties of the sensors and the pressure transmission paths. Implementation of these features into estimation algorithm leads to the problem of nonlinear estimation [6, 7]. The present paper deals with the synthesis of an optimal adaptive algorithm for processing signals from static pressure, air temperature, and vertical accelerometer sensors based on nonlinear estimation methods. Adaptive properties of the algorithm are achieved by including into the state vector of the estimation system the parameters that characterize the properties of sensors and static pressure transmission paths.

2. Problem statement

It is known that the flight altitude of the aircraft is determined by the measured values of static pressure, which is converted to the altitude value [8]. The receiver and static pressure sensor are connected by a pressure transmission air duct of finite known length. The static pressure at the output of the air duct is related to the actual air pressure by the transfer function of the form

$$W(P) = \frac{1}{\tau_p S + 1} e^{-\beta S},$$  \hspace{1cm} (1)
where, $\tau_p$ — the time constant of the air duct, $\beta$ — the time shift equal to the ratio of the length of the air duct to the speed of sound, $S$ — the variable of the Laplace operator.

In general, the static pressure at the output of the converter is determined by the expression

$$P_p = P_o + \Delta P.$$  

Here $\Delta P$ — a random process of the "white" noise type with the intensity $Q_p$, $P_o$ — the measured static pressure at the current height, $P_p$ — the static pressure at the outlet of the air duct determined by the solution (1), $\Delta P$ — a slowly changing random error.

Assuming that air is an ideal gas, the kinematic relations between vertical acceleration and static pressure $P$ are determined by the following equations:

$$\frac{dP}{dt} = -\frac{gP}{RT}V_y, \quad \frac{dV_y}{dt} = a_y,$$  

where, $R$ — the gas constant, $g$ — the acceleration due to gravity, $a_y$ — the vertical acceleration, $T$ — the outside air temperature at a given altitude.

According to the ideal standard atmosphere model the outdoor air temperature has a rather complex dependence on the flight altitude. For example, in the surface layer of the atmosphere up to altitude of $H=11000$ m the function $T(f(H))$ may be represented by a typical form:

$$T = T_0 + \beta_T (H - H_o),$$

where, $H_o, T_o$ — the altitude and temperature corresponding to the bottom of the interval, that is to the zero altitude, $\beta_T$ — the temperature gradient, the value of which for this altitude interval in an ideal atmosphere is $-0.0065$ K/m. In real conditions, this value may change in one direction or another. On board of an aircraft, the outdoor air temperature is measured through the temperature of the air flow, which is related to the actual temperature by the function $T_a = KT$, where $K$ is the proportionality coefficient. Then for the aircraft, it can be written that the measured value of the temperature $T_a$ with the outdoor air temperature $T$ is related by the equation:

$$\frac{dT}{dt} = \beta_T V_y, \quad \frac{dT_a}{dt} = \frac{KT - T_a}{\tau_T},$$

where, $\tau_T$ — the time constant of the temperature sensor.

The values of vertical acceleration, static pressure, and outdoor air temperature at the current flight altitude are measured by appropriate sensors, which have both instrumental and methodological errors. Models of errors of the accelerometer, pressure and temperature sensors (1), (2) and (3) can be presented as,

$$\Delta a_y = a_i + \xi_y, \quad \Delta p = p_i + \xi_p; \quad \Delta T = T_i + \xi_T,$$

where, $a_i, p_i, T_i$ — the slowly changing components of the corresponding errors, $\xi_y, \xi_p, \xi_T$ — the stochastic components of the errors, which are random processes with correlation functions

$$K_y(\tau) = D_y e^{-\lambda_y \tau}, \quad K_p(\tau) = D_p e^{-\lambda_p \tau}, \quad K_T(\tau) = D_T e^{-\lambda_T \tau}.$$  

Here, $D_y, D_p, D_T$ — the variances, $\lambda_y, \lambda_p, \lambda_T$ — the correlation radius of the corresponding random processes. Evidently, the assumption (5) means that the sensors stochastic errors are the «coloured» noises.

Set of equations (1), (2), and (5) defines the model of the system, the state vector of which must be estimated based on the samples of pressure, temperature, and acceleration sensors.
3. Basic mathematical relations

It is known that the time constants $\tau_p$ and $\tau_T$ depend on the altitude and speed of flight in accordance with the expressions approximating them:

$$\tau_p = \frac{K_p P_0}{p}, \quad \tau_T = \frac{K_T p \theta(M)}{P},$$

(6)

where, $P_0$ — the static pressure at the altitude $H = 0$, $K_p$, $K_T$ — the coefficients of the "sliding" approximation, $\theta(M)$ is a known function. The slowly changing error of the accelerometer is assumed to be constant with a zero mean and known variance, which will later be included in the state vector in order to clarify it. It should be noted here that the vertical acceleration itself will not enter the state vector and is further treated as a parameter or as an external known signal with its own distribution parameters.

To ensure the adaptive properties of the algorithm for calculating the altitude and speed of the flight, it is proposed to enter the parameters $\tau_p$, $\tau_T$, $K_p$ and $K_T$ into the state vector of the system.

Then the generalized mathematical model of the estimation system, taking into account (1), (2) and (5), will take the form:

$$\frac{dP}{dt} = -\frac{g P V_y}{RT}, \quad \frac{dV_y}{dt} = a_y + \Delta a_y, \quad \frac{dT}{dt} = \beta_y V_y, \quad \frac{dT_a}{dt} = \frac{K_T - T_a}{\tau_T},$$

$$\frac{d\tau_p}{dt} = \frac{K_p g P_0 V_y}{RTP^2}, \quad \frac{d\tau_T}{dt} = \frac{K_T g P_0 V \theta(M)}{RTP^2},$$

$$\frac{dK_p}{dt} = 0, \quad \frac{dK_T}{dt} = 0, \quad \frac{dB_p}{dt} = 0, \quad \frac{da}{dt} = 0,$$

(7)

where, $P$ — the value of the static pressure at the output of the air duct.

Temperature, pressure, and acceleration measurements are performed at discrete time points $t_K$ with a constant step, the mathematical description of which depends on the method of accounting for random errors (5) in the measurement model. The simplest way to represent a measurement model is to approximate (5) with "white" noise with a given variance. Then:

$$Z(t_K) = P_p(t_K) + \xi_p(t_K), \quad Z(t_K) = T_a(t_K) + \xi_T(t_K), \quad a' = a_y + a_i.$$

(8)

Random processes $\xi_p(t_K)$ and $\xi_T(t_K)$ are centered discrete "white" noises with known distributions:

$$M[\xi_p(t_K) \xi_p(t_{K+n})] = D_p \delta(k-n), \quad M[\xi_T(t_K) \xi_T(t_{K+n})] = D_T \delta(k-n).$$

Here, $\delta(k-n)$ — the Kronecker symbol.

Let's write the equations (7) and the measurement system (8) in matrix form,

$$\frac{dX}{dt} = f(X) + B a^* + \omega(t),$$

$$Z(t_K) = CX(t_K) + \vartheta(t_K).$$

(9)

The measurement vector $Z(t_K)$ includes two components: static pressure and outdoor air temperature.
The problem of determining the altitude and vertical flight speed, taking into account the representation (9), is formulated as follows. According to the observations \( Z(t_k) \), \( K = 1, 2, 3, ..., n \) it is required to determine estimates of the state vector of system (9) \( X(t_k) \), assuming that its initial state is normally distributed, \( X(t_0) \in N(\bar{X}_0, P_0) \), where \( (\bar{X}_0, P_0) \) — a priori values of mean and correlation matrix of the vector \( X(t_0) \).

4. The adaptive algorithm of optimal estimation

To solve the nonlinear estimation problem we use a discrete-continuous extended Kalman filter [3], the main feature of which is that the state forecast is determined in accordance with the system equation, and the residual gain and error correlation matrices are calculated at discrete times.

Then the algorithm for determining estimates can be represented as the following sequence.

1. Calculating the forecast \( \bar{X} \) of the state vector \( X(t) \) for time interval \( t \in [t_k, t_{K+1}] \)

\[
\frac{d\bar{X}(t)}{dt} = f(\bar{X}(t) + B \alpha(t), (t), \bar{X} = \bar{X}_K .
\]

2. Calculation of the correlation matrix of the forecast errors:

\[
P(t_{K+1}, t_k) = \Phi(t_{K+1}, t_k) V_K \Phi(t_{K+1}, t_k)^T + W(t_{K+1}),
\]

where

\[
d\Phi(t, t_k)/dt = A(t) d\Phi(t, t_k), \quad A(t) = \frac{\partial f(\bar{X}(t))}{\partial \bar{X}(t)},
\]

\[
\Phi(t_k, t_k) = I, \quad t \in [t_k, t_{K+1}].
\]

The correlation matrix \( W(t_k) \), a discrete white noise that simulates the effect of a "standing" wave when transmitting pressure in the air duct, is determined by solving an equation for \( t \in [t_k, t_{K+1}] \):

\[
\frac{dW(t)}{dt} = AW(t) + W(t) A + S_m, \quad W(t_k) = 0.
\]

The intensity \( S_m \) of white noise is determined from the flight data.

3. Determining the gain

\[
K_{K+1} = P(t_{K+1}, t_k) C^T [C P(t_{K+1}, t_k) C^T + R]^{-1}, \quad R = \text{diag}(D_p, D_f).
\]

4. Calculation of the correlation matrix of the estimation error

\[
V_{K+1} = [K_{K+1} C - I] P(t_{K+1} / t_k) [K_{K+1} C - I]^T + K_{K+1} R K_{K+1} + Q_{K+1}.
\]

5. Determining the current estimate of the state vector

\[
\hat{X}_{K+1} = \bar{X}_{K+1} + K_{K+1} (Z_{K+1} - C \bar{X}_{K+1}).
\]

The value of the current flight altitude is determined by the static pressure estimate in accordance with the expression:

\[
H(T) = \frac{T_0}{\beta_0} \left[ \frac{P}{P_0} \frac{g_0}{g_0} - 1 \right],
\]

where, \( P_0, T_0 \) — the standard air pressure and temperature at sea level, \( g_0 \) — the gravity acceleration.

The described algorithm works in the range of altitudes no more than 11000 m. For other ranges of altitudes, it is necessary to take into account the nature of changes in the temperature gradient.
5. Research result

The efficiency of the estimation algorithm was evaluated based on the results of numerical simulation for the following values of noise parameters: $D_p = 169 \text{ Pa}^2$, $D_e = 0.04 \text{ m}^2\text{c}^4$, $D_D = 9 \text{ K}^2$. The time constants were assumed to be: $\tau_p = 1.44 \text{ c}$, $\tau_e = 1.96 \text{ c}$, and the temperature gradient $\beta = 0.012 \text{ Km}^{-1}$.

The dynamics of the processes of estimating the altitude and flight speed parameters of the aircraft is shown in figure 1. The figure shows the change in the errors of estimating the altitude $\Delta H$, vertical velocity $\Delta V_y$, and the standard deviation of the vertical velocity estimate at level $3\sigma$.

![Figure 1. Graphs of changes in altitude and vertical flight speed estimation errors.](image)

The flight altitude was determined by solving the first equation of the system (5) under the assumption that the value of $g$ does not depend on the flight altitude, the outdoor air temperature changed according to $T = T_0 + \beta H$, where $T_0 = 288.15 \text{ K}$ is the normal air temperature at zero altitude. Note that the constant error of static pressure measurement for a given structure of the measuring system cannot be estimated and, therefore, it is not included in the state vector of the estimation system (7).

Figure 2 shows the process of estimating the time constant of the air duct. Its dependence on altitude is also shown here. It should be noted that the convergence rate of the identification algorithm is proportional to the vertical flight speed.

![Figure 2. Graph of changes in the estimation of the time constant of the air duct.](image)

6. Conclusion

In conclusion, it is noted that the proposed algorithm allows estimating the altitude and flight speed parameters of the aircraft while simultaneously identifying the internal parameters of the measurement system. The rate of convergence of estimation algorithms depends on the type of maneuver performed, and the more energetic the maneuver, the higher the rate of reducing a priori errors in estimated parameters.

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References

[1] Pushkov S G, Lovitskii L L and Korsun O N 2018 Aerodynamic errors of the systems aimed at measuring the static pressure of an aircraft in the sliding modes of flight Measurement Techniques 61 (2) 140–7
[2] Korsun O N, Tulekbayeva A K and Toktabek A A 2017 Estimation of errors of aircraft air parameters measurements based on satellite navigation system data Proc. 2017 2nd Int. Ural Conf. on Measurements URALCON 2017 (IEEE, Inc.) p 145–8

[3] Chernodarov A V 2017 Monitoring, diagnostics and identifications of avionic systems (Moscow, Nauchtekhizdat) p 299

[4] Arkhipov A S and Semenikhin K V 2019 Confidence analysis of linear unbiased estimates under uncertain unimodal noise distributions Journal of computer and systems sciences international 58 (5) 674–83

[5] Zhirabok A N, Shumskii A E and Zuev A V 2019 Diagnosis of linear systems based on sliding mode observers Journal of computer and systems sciences international 58 (6) 898–914

[6] Korsun O N, Om M H, Latt K Z and Stulovskii A V 2019 Real-time aerodynamic parameter identification for the purpose of aircraft intelligent technical state monitoring Procedia Computer Science 12th Int. Symp. Intelligent Systems, INTELS 2016 p 67–74

[7] Klein V and Morelli E A 2006 Aircraft system identification: Theory and practice (Reston, VA: AIAA, Inc.)

[8] Aerodynamics, stability and controllability of supersonic aircraft 1998 ed G S Bjushgens (Moscow: Nauka) p 16