Quantum computation with cold bosonic atoms in an optical lattice

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We analyse an implementation of a quantum computer using bosonic atoms in an optical lattice. We show that, even though the number of atoms per site and the tunneling rate between neighbouring sites is unknown, one may perform a universal set of gates by means of adiabatic passage.

Keywords: Quantum computing, adiabatic, cold atoms, optical lattice.

1. Introduction

Some tasks in Quantum Information require the implementation of quantum gates with a very high fidelity (A. M. Steane 2002; E. Knill 1998; D. Aharonov 1999). This implies that all parameters describing the physical system on which the computer is implemented have to be controlled with a very high precision, something which it is very hard to achieve in practice.

For example one can imagine an implementation in which qubits are stored in atoms and are manipulated using Raman transitions. It may happen that the relative phase of the lasers driving a Raman transition can be controlled very precisely, whereas the corresponding Rabi frequency $\Omega$ has a larger uncertainty $\Delta\Omega$. If we denote by $T$ the time required to execute a local gate (of the order of $\Omega^{-1}$), then a high gate fidelity requires $T\Delta\Omega \ll 1$ (equivalently, $\Delta\Omega/\Omega \ll 1$), which may very hard to achieve, at least to reach the above mentioned threshold.

In this paper we analyse an implementation of quantum computing using atoms confined in optical lattices. These systems have interesting features for quantum computing. Namely, a large number of atoms can be trapped in the lattice at a very low temperature, which provides a large number of qubits. Also, neutral atoms interact weakly with the environment, which leads to a relatively slow decoherence. However, the same setups pose also important experimental challenges, such as being able to load the lattice with one atom per site or being able to measure the interaction and tunneling constants of these systems with high accuracy. These obstacles, together with the uncertainties in the atom-laser interaction, must be overcome to implement current proposals for quantum computing with neutral atoms (D. Jaksch 1999; R. Ionicioiu 2002).

In this paper we analyse a way that solves the above mentioned problems and show how to achieve a very high gate fidelity even when most of the parameters describing the atomic ensemble (number of atoms per lattice site, tunneling rate, Rabi frequencies, etc) cannot be adjusted to precise values, and even have uncertainties of the order of the parameters themselves.
Our method combines the technique of adiabatic passage with ideas of quantum control theory. The use of adiabatic passage to implement quantum gates is not a new idea, and indeed several methods based on Berry phases have been proposed recently (P. Zanardi 1999; J. Pachos 2000; J. A. Jones 1999; G. Falci 2000; L.-M. Duan 2001). Furthermore, adiabatic passage techniques have been suggested as a way of implementing a universal set of holonomies (L.-M. Duan 2001), i.e. quantum gates which are carried out by varying certain parameters and whose outcome only depends on geometrical properties of the paths in parameter space (P. Zanardi 1999; J. Pachos 2000). However, all these proposals are based on the existence of holonomies in the system, which in turn implies a huge degeneracy in the system. This will not be the case in our scheme.

The outline of the paper is as follows. In §2 we introduce the requirements for quantum computing and show which tools are available in current experiments with cold atoms in optical lattices. We will demonstrate that, due to imperfections in the loading of the lattice, the Hamiltonian of the system is not known with enough accuracy to perform quantum computing in a ‘traditional’ way. In §3 we develop a technique to circumvent our ignorance about the Hamiltonian. Performing adiabatic passage with the different parameters of our problem, we show how to produce a universal set of gates (Hadamard, phase, and CNOT). In §4 we quantify the errors of our proposal, studying the influence of the speed of the adiabatic process, and of other imperfections. In §5 we summarize our results and offer some conclusions.

2. Cold bosonic atoms in optical lattices

We will consider a set of bosonic atoms confined in a periodic optical lattice at sufficiently low temperature such that only the first Bloch band is occupied. The atoms have two relevant internal (ground) levels, $|a\rangle$ and $|b\rangle$, and we wish to use this degree of freedom to store the qubit. This set-up has been studied in D. Jaksch (1999) where it has been shown how single quantum gates can be realized using lasers and two-qubit gates by displacing the atoms that are in a particular internal state to the next neighbour location. The basic ingredients of such a proposal have been recently realized experimentally (I. Bloch 2002, personal communication). However, in this and all other schemes so far (E. Charron 2002; K. Eckert 2002) it is assumed that there is a single atom per lattice site since otherwise even the concept of qubit is no longer valid. In present experiments, in which the optical lattice is loaded with a Bose-Einstein condensate (D. Jaksch 1998; M. Greiner 2002), this only approximately true, since zero temperature is required and the number of atoms must be identical to the number of lattice sites.

(a) Requirements for computation

The uncertainty of the number of atoms per lattice site poses severe problems. Having $n_i$ atoms on the $i$-th cell, the configuration of this lattice size will be given by a combination of $n_i + 1$ possible states

$$\mathcal{H}_i = \text{lin} \left\{ \frac{1}{\sqrt{n_i!(n_i-k)!}} b_i^k a_i^{n_i-k} |\text{vac}\rangle_i, \quad k = 0 \ldots n_i \right\}. \quad (2.1)$$
To do quantum computing with \( m \) qubits, we must find a \( 2^m \)-dimensional subspace \( \mathcal{H}_c \subset \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_m \), which is energetically separated from the rest, so that once we set our computer in a superposition of qubits \( |0\rangle \) and \( |1\rangle \), it does not leave this subspace. A second, and stronger requirement is that our computation space must be an eigenspace of our Hamiltonian, with the same eigenvalue

\[
H(\{|z_1\rangle \otimes \cdots \otimes |z_n\rangle\}) = 0, \quad \forall z_i \in \mathbb{Z}_2,
\]

which we assume 0. Otherwise the trivial evolution of our system would spoil the quantum computation by introducing uncontrollable, unknown phases.

Right from the beginning we foresee several difficulties. First, for arbitrary interactions, the states with different occupation numbers \( (2.1) \) will be regularly spaced and we will not be able to select our qubits. Furthermore, even if we customize the interactions between bosons, given that we basically ignore the number of atoms per site, \( n_i \), a basic requisite of our scheme will be to show that our procedure works independently of the occupation numbers. Both problems cannot be solved in general. We will rather have to impose some restrictions on our physical system, and this is the purpose of the following subsections.

\[\text{(b) Definition of qubit}\]

A crucial assumption which is suggested by the requirement \( (2.2) \), is to impose that the atoms in internal state \( |a\rangle \) do not interact and do not hop to neighbouring sites\(^\dagger\). In the absence of external fields, the Hamiltonian describing our system is

\[
H = \sum_k \left[ -J_{b,k} (b^\dagger_{k+1} b_k + b_k b_{k+1}) + \frac{1}{2} U_{bb} b^\dagger_k b^\dagger_{k+1} b_k b_{k+1} \right].
\]

(2.3)

Here \( U_{bb} \) and \( J_{b,k} \) describe the interactions between and the tunneling of atoms in state \( |b\rangle \). We will assume that \( J_{b,k} \) can be set to zero and increased by adjusting the intensities of the lasers which create the optical lattice.

For us a qubit will be formed by an aggregate of at least one atom per lattice site and the qubit basis will be formed by the states with at most one atom excited to the state \( b \). More precisely, our computation will be performed in the space

\[
\mathcal{H}_c(\vec{n}) = \left\{ |z_1 \cdots z_M\rangle = \prod_{i=1}^{M} \frac{1}{\sqrt{n_i!}} b^\dagger_{i}^{z_i} a_i^{n_i-z_i} |\text{vac}\rangle : \forall z_i \in \mathbb{Z}_2 \right\}.
\]

(2.4)

It is easy to check that for \( J_b = 0 \), all our qubit states \( |b_1 \cdots b_M\rangle \) form degenerate linear eigenspace of our Hamiltonian \( (2.3) \), which is separated by an energy gap of \( U_{bb} \) from any other configuration.

Definition \( (2.4) \) fulfills some of the requirements for quantum computing. However, \( \mathcal{H}_c \) does depend on the occupation number of the lattice, while a general state will be an incoherent superposition of different occupation numbers. It remains to show that we are able to produce quantum gates which are insensitive to the numbers \( n_i \). More precisely, if we design a protocol to produce the gate \( U_{id} \), and this protocol is implemented by the unitary operation \( U_{\text{real}} \), we must prove that within the required accuracy

\[
U_{\text{real}}|_{\mathcal{H}_c(\vec{n})} \simeq e^{i\phi(\vec{n})} U_{id}.
\]

(2.5)

\(^\dagger\) This may be achieved by tuning the scattering lengths and the optical lattice.

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The phases $\phi(\vec{n}_i)$ are irrelevant, since they are common to each of the possible computation spaces and final measurements will project our state to one of the subspaces $H_c$.

(c) Available tools

The quantum gates will be realized using lasers, switching the tunneling between neighbouring sites, and using the atom–atom interaction. We will now show how these elements introduce enough modifications to the Hamiltonian (2.3) so as to perform general quantum computing.

For a single qubit gate on qubit $k$ we can induce unitary transformations by means of Stark shifts and transitions between internal states. During the whole operation we set $J_{b,k} = 0$ in order to isolate the qubits. The atom-laser interaction is then described by the Hamiltonian

$$H_{\text{las},k} = \frac{\Delta_k}{2}(a_k^\dagger a_k - b_k^\dagger b_k) + \frac{\Omega_k}{2}(e^{i\varphi} a_k^\dagger b_k + e^{-i\varphi} b_k^\dagger a_k). \quad (2.6)$$

For $U_{bb} \gg |\Delta_k|, |\Omega_k|$, we can replace (2.6) by an effective Hamiltonian of the form

$$H_1 = \frac{\Delta}{2}\sigma_z + \frac{\Omega}{2}(\sigma_+ e^{i\varphi} + \sigma_- e^{-i\varphi}), \quad (2.7)$$

with $\Delta = \Delta_k$ and $\Omega = \Omega_k\sqrt{n_k - 1}$.

For the realization of two-qubit operations we tilt the lattice using an electric field, $H_{\text{tilt}} = \sum_k kg(a_k^\dagger a_k + b_k^\dagger b_k)$. The tilting must be weak as to only virtual hopping of atoms of type $|b\rangle$ ($J_b \ll |U_{bb} - g|$). After adiabatic elimination we find that the effective Hamiltonian becomes

$$H_2 = \tilde{\Delta}|11\rangle\langle11|, \quad \tilde{\Delta} = \frac{J_b^2}{g - U_{bb}}. \quad (2.8)$$

The Hamiltonians $H_1$ and $H_2$ pose now two problems. The first one is that $H_1$ depends on the occupation numbers. A traditional approach to quantum computing would be to tune the parameters $\Delta$ and $\Omega$ and let the resulting Hamiltonian operate for a time $T$, $U_{\text{real}} = \exp(-iH_1 T)$. However, since the parameters are unknown, we cannot take this naive route. The second difficulty resides on the magnitude of $J_b$, $g$, and $U_{bb}$; these values are very sensitive to the properties of the lattice and difficult to control. At most, we will be able to assure that $J_b$ and $g$ are zero, or that $g$ is similar to $U_{bb}$; but we will be unable to fix the value of $\tilde{\Delta}$ with enough accuracy that $U_{\text{real}} = \exp(-iH_2 T)$ resembles a controlled-Z gate.

In the following section we will solve these problems. In §4 develop an abstract protocol which, up from the Hamiltonians (2.7) and (2.8), produces a universal set of gates that can be used for quantum computing. Next in §4 we will study the influence of all the processes which we have neglected in the abstract derivation, such as interaction and hopping of atoms in state $|a\rangle$, sensitivity to occupation numbers, etc.
3. Computation with unknown parameters

(a) Basic ideas

Let us consider a set of qubits that can be manipulated according to the single qubit Hamiltonian (2.7) and the two–qubit Hamiltonian (2.8). We will assume that most of the parameters appearing in these Hamiltonians are basically unknown. On the other hand, we will not consider any randomness in these parameters because the corresponding errors may be corrected with standard error correction methods (M. A. Nielsen 2002), as long as they are small, and in most cases random quick fluctuations of the parameters $I$ will be averaged out in the adiabatic process.

In particular we will assume that only the phase of the laser, $\varphi$, can be precisely controlled. For the other parameters we will impose that: (i) they are given by an unknown (single valued) function of some experimentally controllable parameters, (ii) they can be set to zero, and (iii) they are positive†. For example, we may have $\Omega = f(I)$, where $I$ is a parameter that can be experimentally controlled, and we only know about $f$ that $f(0) = 0$ and that we can reach some value $\Omega_m \equiv f(I_m) \neq 0$ for some $I_m$‡. Outside this, $f(I)$ may change in different experimental realizations.

The physical scenario described in §2 corresponds to this situation, but we want to stress that these conditions can be naturally met in more general scenarios. For example, the qubit states $|0\rangle$ and $|1\rangle$ may correspond to two degenerate atomic (ground state) levels which are driven by two lasers of the same frequency and different polarization. The corresponding Hamiltonian is given by (2.7), where the parameters $\varphi, \Omega, \Delta$ describe the relative phase of the lasers, the Rabi frequency and detuning of the two-photon Raman transition, respectively. The Rabi frequency can be changed by adjusting the intensity of the lasers, and the detuning and the phase difference by using appropriate modulators. In practice, $\Omega$ ($\Delta$) can be set to zero very precisely by switching off the lasers (modulators) and $\varphi$ may be very precisely controlled to any number between 0 and $2\pi$. However, fixing $\Omega$ or $\Delta$ to a precise value can be much more difficult.

† This is just accommodate the physical restrictions of §2. The scheme actually becomes simpler when $\Delta$ or $\tilde{\Delta}$ may take negative values.

‡ Note that, in many realistic implementations it is not possible to measure the dependence of these parameters (function $f$) because measurements are destructive (lead to heating or atom losses). Therefore $f(I)$ is different in different experimental realizations.
The idea of obtaining perfect gates with unknown parameters combines the techniques of adiabatic passage (P. Zanardi 1999; J. Pachos 2000) with ideas of quantum control (L. Viola 1998; L.-M. Duan 1998). Let us briefly recall the adiabatic theorem, which is a fundamental tool in our method. Suppose we have a Hamiltonian that depends parametrically on a set of parameters, denoted by $p$, which are changed adiabatically with time along a given trajectory $p(t)$. After a time $T$, the unitary operator corresponding to the evolution is

$$U(T) = \sum_{\alpha} e^{i(\phi_{\alpha} + \psi_{\alpha})} |\Phi_{\alpha}[p(T)]\rangle \langle \Phi_{\alpha}[p(0)]|.$$  \hspace{1cm} (3.1)

Here, $|\Phi_{\alpha}(p)\rangle$ are the eigenstates of the Hamiltonian for which the parameters take on the values $p$. The phase $\phi_{\alpha}$ is a dynamical phase that explicitly depends on how the parameters $p$ are changed with time, whereas the phase $\psi_{\alpha}$ is a purely geometrical phase and depends on the trajectory described in the parameter space. Our basic idea to perform any given gate is first to design the change of the parameters in the Hamiltonians (2.7)-(2.8) such that the eigenvectors evolve according to the desired gate, and then to repeat the procedure changing the parameters appropriately in order to cancel the geometric and dynamical phases.

\hspace{2cm} (b) Local gates

Using the previous ideas we are able to implement a universal set of gates, which is made of a phase gate, $U = e^{i\theta}\sigma_z/2$, a Hadamard gate and a CNOT gate. To perform the phase gate $U = e^{i\theta}\sigma_z/2$ we work with the single-qubit Hamiltonian (2.7). We set $\Delta = 0$ for all times and change the remaining parameters $(\Omega, \varphi)$ as depicted in figure 1(a):

\begin{equation}
(0, 0) \underset{(i)}{\rightarrow} (\Omega_m, 0) \underset{(ii)}{\rightarrow} (\Omega_m, \theta/2) \underset{(iii)}{\rightarrow} (\Omega_m, \theta/2 + \pi) \underset{(iv)}{\rightarrow} (\Omega_m, \theta + \pi) \underset{(v)}{\rightarrow} (0, \theta + \pi) \hspace{1cm} (3.2)
\end{equation}

All steps are performed adiabatically and require a total time $T$, except for step (iii) whose double arrow indicates a sudden change of parameters. Note that $\Omega(0) = \Omega(2T) = 0$, $\Omega(t) = \Omega(2T - t)$ and $\varphi(t) = \pi + \theta - \varphi(2T - t)$, which does not require the knowledge of the function $f$ but implies a precise control of the phase. A simple analysis shows that (i-v) achieve the desired transformation $|0\rangle \rightarrow e^{i\theta/2}|0\rangle$, $|1\rangle \rightarrow e^{-i\theta/2}|1\rangle$. Note also that the dynamical and geometrical phases acquired in the adiabatic processes (i-v) cancel out.

The Hadamard gate can be performed in a similar way. In the space $[\Delta, \Omega_x = \Omega \cos(\varphi)]$, the protocol is

\begin{equation}
(0, \Omega_m) \underset{(i)}{\rightarrow} (\Delta_m, \Omega_m) \underset{(ii)}{\rightarrow} (\Delta_m, 0) \underset{(iii)}{\rightarrow} (\Delta_m, \Omega_m) \underset{(iv)}{\rightarrow} (0, -\Omega_m) \underset{(v)}{\rightarrow} (\Delta, -\Omega_m) \underset{(vi)}{\rightarrow} (\Delta, 0), \hspace{1cm} (3.3)
\end{equation}

as shown in figure 1(b-c). In order to avoid the dynamical phases, we have to make sure that steps (i-v) are run in half the time as (vi-vii). More precisely, if $t < T$, we must ensure that $\Delta(t) = \Delta(T - t)$, $\Omega_x(t) = \Omega_x(T - t)$, $\Delta(T + t) = \Delta(t/2)$ and $\Omega_x(T + t) = \Omega_x(t/2)$. With this requisite we get $\frac{\sqrt{2}}{2}|0\rangle + |1\rangle \rightarrow |0\rangle$, $\frac{\sqrt{2}}{2}|0\rangle - |1\rangle \rightarrow -|1\rangle$. Again, the whole procedure does not require us to know $\Omega$ or $\Delta$, but rather to control the evolution of the experimental parameters which determine them.
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Figure 2. Log-log plot of the gate error, $E = 1 - F$, for the Hadamard (solid), phase (dashed) and CNOT (dotted) gates, when we use the ideal Hamiltonians (2.7) and (2.8) to simulate them. The parameters for the simulations are $\{\Delta_m/\Omega_m = \Delta_m/\Omega_m = 1/10, \varphi_m = \pi/4\}$, and we change the speed of the process to study the influence of non–adiabaticity.

(c) Nonlocal gates

The C-NOT gate requires the combination of two two-qubit processes using $H_2$ and one local gate. The first process involves changing the parameters $[\hat{\Delta}, \Omega_x = \Omega \cos(\varphi)]$ of equation (2.8) according to

\[
\begin{align*}
(\hat{\Delta}_m, 0) & \overset{(i)}{\rightarrow} (\hat{\Delta}_m, \Omega_m) \overset{(ii)}{\rightarrow} (0, \Omega_m) \overset{(iii)}{\Rightarrow} (0, -\Omega_m) \\
& \overset{(iv)}{\rightarrow} (\hat{\Delta}_m, -\Omega_m) \overset{(v)}{\rightarrow} (\hat{\Delta}_m, 0).
\end{align*}
\]

This procedure gives rise to the transformation

\[
U_1 = |0\rangle\langle 0| \otimes I + e^{i\xi}|1\rangle\langle 1| \otimes i\sigma_y,
\]

where $\xi = \int_0^T \delta(t)dt$ is an unknown dynamical phase. The second operation required is a NOT on the first qubit $U_2 = (|0\rangle\langle 1| + |1\rangle\langle 0|) \otimes I$. Finally, if $\hat{\Delta}^{(1)}(t)$ denotes the evolution of $\hat{\Delta}$ in equation (3.4), we need to follow a path such that $\hat{\Delta}^{(3)}(t) = \Omega^{(3)}(t) = 0$. If the timing is correct, we achieve $U_3 = (|0\rangle\langle 0| + e^{i\xi}|1\rangle\langle 1|) \otimes I$. Everything combined gives us the CNOT up to a global unimportant phase $U_{\text{cnot}} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes i\sigma_y = e^{-i\xi}U_2U_3U_2U_1$.

4. Control of errors

In this section we study whether it is feasible to apply the methods developed in §3 to the physical setup envisioned in §2. First of all we will study how fast the operations from §3 must be performed in order to minimize the deviations from the adiabatic theorem. And second and most important, we have to consider contributions to the energy which escape the terms considered in equations (2.7) and (2.8). We will analyse both sources of error separately, combining analytical estimates with numerical simulations of our techniques for small number of atoms.

(a) Adiabaticity

To study the sensitivity of our method against non–adiabatic processes, we have simulated the protocols (3.2)-(3.4) using the ideal model given by Hamiltonians
Figure 3. Log-log plot of the gate error, \( E = 1 - F \), for the (a) phase, (b) Hadamard and (c) CNOT gates. The parameters for the simulations are \( U_{aa} = U_{ab} = J_m \), \( J_a = J_b, g = U_{bb} + U_{ab}/2 \), and \( T = 100/\Omega_m \). For the local gates we choose \( \Delta_m = 6\Omega_m = 1 \), and for the nonlocal gate \( \Omega_m = J_m^2/6 \). For each simulation we choose different population imbalance (\( |n - m| = 0, 1, 2 \) for solid, dashed and dotted lines), and change the interaction constant \( U_{bb} \).

(2.7) and (2.8). For each of the gates we have fixed all parameters \( \{\Delta_m/\Omega_m = \Delta_m/\Omega_m = 1/10, \varphi_m = \pi/4\} \) except the time, and then we have computed how the error decreases as we decrease the speed of the adiabatic passage. The results are shown in figure 2. As a figure of merit we have chosen the gate fidelity (M. A. Nielsen 2002)

\[
F = 2^{-n} |\text{Tr}\{U_{\text{ideal}}^\dagger U_{\text{real}}\}|^2
\]

where \( n \) is the number of qubits involved in the gate, \( U_{\text{ideal}} \) is the gate that we wish to produce and \( U_{\text{real}} \) is the actual operation performed. As expected, the adiabatic theorem applies when the processes are performed with a sufficiently slow speed. Typically a time \( T \sim 300/\Omega_m \) is required for the desired fidelity \( F = 1 - 10^{-4} \).

It is also worth mentioning, that in figure 2 and 3, strong, rapid oscillations of the error are seen. These oscillations are due to either the nonadiabaticity of the process (figure 2), or to imperfections in the Hamiltonian (figure 3). For a two-level system undergoing adiabatic evolution it is easy to prove that, while the amplitude of the oscillations is proportional to the speed of the adiabatic process, the frequency is instead related to the energy difference between contiguous eigenspaces. This frequency is for us unknown, and consequently, these oscillations may not be used to improve the accuracy of our method by looking for some ‘magic times’.

(b) Imperfections

Outside the non–adiabaticity of a real experiment, there are two other sources of error which we must consider. (i) The quotient \( J_b/U_{bb} \) is nonzero, which means that more than one atom per well may be excited. (ii) Atoms in state \( |a\rangle \) interact and hop. This introduces new terms in equation (2.3), which are of the form \( J_{a,k}(a_{k+1}^\dagger a_k + a_k^\dagger a_{k+1}), U_{aa}a_{k}^\dagger a_{k}a_{k}a_{k}, \) and \( U_{ab}a_{k}^\dagger b_k^\dagger a_{k}b_k. \) And finally, (iii) atoms in either state may jump to neighbouring sites, permanently changing the occupation numbers.

The effects (i) and (iii) are suppressed if the coupling between internal levels and the amplitude of hopping are both small compared to the energy gap between our computation space and the undesired excitations. In other words, we require

\[
(\Omega/U_{bb})^2 \ll 1 \quad \text{and} \quad (J_{a,k}/U_{bb})^2 \ll 1.
\]
To analyse the remaining errors we develop an effective Hamiltonian which contains (2.6) and (2.3) plus new terms \((J_a, U_{ab}, U_{aa})\) that we did not consider before. In equation (2.6), the virtual excitation of two atoms increments the parameter \(\Delta\) by an unknown amount, \(\Delta_{\text{eff}} \sim \Delta + 2\Omega^2 n_k/(\Delta + U_{ab} - U_{bb})\). If
\[
U_{ab} \ll U_{bb} \quad \text{and} \quad \Omega^2 n_k T/U_{bb} \ll 1,
\]
this shift may be neglected. In the two-qubit gates the energy shifts are instead due to virtual hopping of all types of atoms, and they are also accompanied by the possibility of swapping both qubits \((|01\rangle \leftrightarrow |10\rangle\)). Both contributions are of the order of \(\max(J_b, J_a)^2/g^2 \sim J^2/U_{bb}\), and for
\[
J^2T/U_{bb} \ll 1
\]
they also may be neglected.

To quantitatively determine the influence of these errors we have simulated the evolution of two atomic ensembles with an effective Hamiltonian which results of applying second order perturbation theory to equation (2.3), and which takes into account all important processes. The results are shown in figures 3. In these pictures we show the error of the gates for simulations in which all parameters are fixed, except for \(U_{bb}\) and the occupation numbers of the wells. The first conclusion is that the stronger the interaction between atoms in state \(|b\rangle\), the smaller the energy shifts. This was already evident from our analytical estimates, because all errors are proportional to \(1/U_{bb}\). Typically, a ratio \(U_{bb} = 10^4 U_{ab}\) is required to make \(\mathcal{F} = 1 - 10^{-4}\). Second, the larger the number of atoms per well, the poorer the fidelity of the local gates [Figure 3(a-b)]. And finally, as figure 3(c) shows, the population imbalance between wells influences very little the fidelity of the two-qubit gate.

5. Conclusions

In this work we have shown that it is possible to perform quantum computation with cold atoms in a tunable optical lattice. Our scheme is based on performing adiabatic passage with one-qubit (2.7) and two-qubit (2.8) Hamiltonians. With selected paths and appropriate timing, it is possible to perform a universal set of gates (Hadamard gate, phase gate and a CNOT). Thanks to the adiabatic passage, the proposal works even when the number of atoms per lattice is unknown or the constants in the governing Hamiltonians have large uncertainties. These procedures can not only be used for quantum computing but also for quantum simulation (E. Jané 2002), and the same ideas can also be applied to other setups like the microtraps demonstrated in R. Dumke (2002).

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