How neutrinos get mass and what other things may happen besides oscillations?

ERNEST MA
Department of Physics, University of California, Riverside CA 92521, USA

Abstract. In this talk I address the theoretical issue of why new physics is required to obtain a nonzero neutrino mass. I then discuss what other things may happen besides neutrino oscillations. In particular I consider a possible new scenario of leptogenesis in R-parity nonconserving supersymmetry.

Keywords. Neutrino masses; leptogenesis; supersymmetry.

PACS Nos 14.60; 12.60; 98.80

1. Introduction

In the minimal Standard Model, under the gauge group SU(3)$_C$ × SU(2)$_L$ × U(1)$_Y$, the leptons transform as:

\[
\left( \begin{array}{c}
\nu_i \\
l_i \\
\end{array} \right)_L \sim (1, 2, -1/2), \quad l_i R \sim (1, 1, -1),
\]

and the one Higgs doublet transforms as: \( \Phi = (\phi^+, \phi^0) \sim (1, 2, 1/2) \). Without additional particles at or below the electroweak energy scale, i.e. \( 10^2 \) GeV, \( m_{\nu} \) must come from the following effective dimension-5 operator [1],

\[
\Lambda^{-1}(\nu_i \phi^0 - l_i \phi^+)(\nu_j \phi^0 - l_j \phi^+).
\]

All theoretical models of neutrino mass differ only in its specific realization [2].

2. Canonical, minimal, and next-to-minimal seesaw

Add 3 heavy singlet right-handed neutrinos to the minimal Standard Model: 1 \( \nu_R \) for each \( \nu_L \). Then the operator of eq. (2) is realized because each heavy \( \nu_R \) is linked to \( \nu_L \phi^0 \) with a Yukawa coupling \( f \); and since \( \nu_R \) is allowed to have a large Majorana mass \( M_R \), the famous seesaw relationship \( m_{\nu} = m_D^2/M_R \) is obtained [3], where \( m_D = f(\phi^0) \). This mechanism dominates the literature and is usually implied when a particular pattern of neutrino mass and mixing is proposed.
Actually, it is not necessary to have 3 $\nu_R$’s to get 3 nonzero neutrino masses. Add just 1 $\nu_R$. Then only 1 linear combination of $\nu_e, \nu_\mu, \nu_\tau$ gets a seesaw mass. The other 2 neutrino masses are zero at tree level, but since there is in general no more symmetry to protect their masslessness, they must become massive through radiative corrections. As it turns out, this happens in two loops through double $W$ exchange and the result [4] is doubly suppressed by the charged-lepton masses. Hence it is not a realistic representation of the present data for neutrino oscillations.

Add 1 $\nu_R$ and 1 extra Higgs doublet [5]. Then 1 neutrino gets a seesaw mass. Another gets a one-loop mass through its coupling to $\phi^0_1$, where $\langle \phi^0_1 \rangle = 0$. This second mass is proportional to the coupling of the term $(\phi^0_2 \phi^0_1)$ times $\langle \phi^0_1 \rangle^2$ divided by $M_R$. The third neutrino gets a two-loop mass as in the minimal case. This scheme is able to fit the present data.

3. Heavy Higgs triplet

Add 1 heavy Higgs triplet $(\xi^{++}, \xi^+, \xi^0)$. Then the dimension-4 term

$$\nu_i \nu_j \xi^0 - (\nu_i \nu_j + \nu_i \nu_j) \xi^+ / \sqrt{2} + l_i l_j \xi^{++}$$

is present, and $m_\nu \propto \langle \xi^0 \rangle$. If $m_\xi \sim 10^2$ GeV, this would require extreme fine tuning to make $\langle \xi^0 \rangle$ small [6]. But if $m_\xi > > 10^2$ GeV, the dimension-4 term should be integrated out, and again only the dimension-5 term

$$(\nu_i \phi^0 - \nu_i \phi^+) (\nu_j \phi^0 - \nu_j \phi^+) = \nu_i \nu_j (\phi^0 \phi^0) - (\nu_i \nu_j + \nu_i \nu_j) (\phi^0 \phi^+) + l_i l_j (\phi^0 \phi^+),$$

remains, so that [7] $m_\nu = 2 f \mu \langle \phi^0 \rangle^2 / m_\xi^2$, where $f$ and $\mu$ are the couplings of the terms $\nu_i \nu_j \xi^0$ and $\phi^0 \phi^0 \xi^0$ respectively. This shows the interesting result that $\xi$ has a very small vacuum expectation value inversely proportional to the square of its mass [8].

$$\langle \xi^0 \rangle = \mu \langle \phi^0 \rangle^2 / m_\xi^2 \ll m_\xi.$$  (5)

The SU(2)$_L \times$ SU(2)$_R \times$ U(1)$_{B-L}$ version of this relationship is $\nu_L \sim \langle \phi^0 \rangle^2 / v_R$ [9].

4. Some generic consequences

Once neutrinos have mass and mix with one another, the radiative decay $\nu_2 \rightarrow \nu_1 \gamma$ happens in all models, but is usually harmless as long as $m_\nu < \text{few eV}$, in which case it will have an extremely long lifetime, many many orders of magnitude greater than the age of the Universe. The present astrophysical limit [10] is $10^{14}$ years.

The analogous radiative decay $\mu \rightarrow e \gamma$ also happens in all models, but is only a constraint for some models where $m_\mu$ is radiative in origin. The present experimental limit [11] on this branching fraction is $1.2 \times 10^{-11}$.

Neutrinoless double $\beta$ decay occurs, but is sensitive only to the $\nu_e - \nu_e$ entry of $M_\nu$, which may be assumed to be zero in many models. The present experimental limit [12] is 0.2 eV.
5. Leptogenesis in the 2 simplest models of neutrino mass

Leptogenesis is possible in either the canonical seesaw or Higgs triplet models of neutrino mass. In the canonical seesaw scenario, $\nu_R$ may decay into both $l^- \phi^+$ and $l^+ \phi^-$. In the Higgs triplet scenario, $\xi^{++}$ may decay into both $l^+ l^+$ and $\phi^+ \phi^-$. The lepton asymmetry thus generated may be converted into the present observed baryon asymmetry of the Universe through the electroweak sphalerons [13].

The decay amplitude of $\nu_R$ into $l^- \phi^+$ is the sum of tree-level and one-loop contributions, where the intermediate state $l^- \phi^-$ may appear as a vertex correction through $\nu_R'$ exchange [14]. The interference between them allows a decay asymmetry of $l^- \phi^+ - l^+ \phi^-$ to be produced, provided that CP is violated. This requires $\nu_R' \neq \nu_R$ and is analogous to having direct CP violation in $K$ decay, i.e. $\epsilon' \neq 0$.

There is also CP violation in the self-energy correction [15] to the mass matrix spanning $\nu_R$ and $\nu_R'$, which is analogous to having indirect CP violation in the $K^0 - \bar{K}^0$ system, i.e. $\epsilon \neq 0$. This effect has a $(m - m')^{-1}$ enhancement, but the limit $m' = m$ is not singular [16].

Similarly, the decay amplitude of $\xi^{++}$ into $l^+ l^+$ has a self-energy (but no vertex) correction involving the intermediate state $\phi^+ \phi^+$. This generates a decay asymmetry given by [8]

$$
\delta_i \approx \frac{\text{Im}[\mu_i \mu_i^* \sum k \hat{f}_{ik} f_{kj}^{*}]}{8 \pi^2 (M_1^2 - M_2^2)} \left( \frac{M_i}{\Gamma_i} \right).
$$

Again, CP violation requires 2 different $\xi$'s.

6. Radiative neutrino mass

The generic expression of a Majorana neutrino mass is given by $m_\nu \sim f^2\langle \phi^0 \rangle^2/\Lambda$, hence $\Lambda > 10^{18}\text{GeV} (1 \text{eV}/m_\nu)^2$, i.e. the scale of lepton number violation is very large (and directly unobservable) unless $f < 10^{-5}$ or so.

If $m_\nu$ is radiative in origin, $f$ is suppressed first by the loop factor of $(4\pi)^{-1}$, then by other naturally occurring factors such as $m_1/M_W$ or $m_\nu/M_W$. In that case, $\Lambda$ may be small enough to be observable directly (or indirectly through lepton flavor violating processes).

Take for example the Zee model [17], which adds to the minimal Standard Model 1 extra Higgs doublet $\Phi_2$ and 1 charged singlet $\chi^\pm$. Then the coexistence of the terms $g_{ij} (\nu_i l_j - \nu_j l_i) \chi^\pm$ and $\mu (\phi_2^+ \phi_2^- - \phi_1^+ \phi_1^-) \chi^\pm$ allows the following radiative mass matrix to be obtained:

$$
\mathcal{M}_\nu = \begin{bmatrix}
0 & f_{\mu e}(m_{\mu}^2 - m_e^2) & f_{\tau e}(m_{\tau}^2 - m_e^2) \\
\frac{f_{\mu} (m_{\mu}^2 - m_e^2)}{f_{\tau} (m_{\tau}^2 - m_e^2)} & 0 & f_{\tau \mu}(m_{\tau}^2 - m_{\mu}^2) \\
f_{\tau \mu}(m_{\tau}^2 - m_{\mu}^2) & f_{\tau \mu}(m_{\tau}^2 - m_{\mu}^2) & 0
\end{bmatrix},
$$

where $f_{ij} \sim (g_{ij}/16\pi^2)(\mu(\phi_0^0)/(\phi_1^0)m_\nu^2)$. This model has been revived in recent years and may be used to fit the neutrino-oscillation data.

In the above, the mass of the charged scalar $\chi$ may be light enough to allow observable contributions to $\Gamma(\mu \rightarrow e\nu\bar{\nu})$ at tree level, and to $\Gamma(\mu \rightarrow eee)$ in one loop. Hence lepton flavor violating processes may reveal the presence of such a new particle.
7. R-parity nonconserving supersymmetry

In the minimal supersymmetric Standard Model, $R \equiv (-1)^{3B+L+2J}$ is assumed conserved so that the superpotential is given by

$$W = \mu H_1 H_2 + f_{ij}^L H_1 L_i e_j^c + f_{ij}^Q H_1 Q_i d_j^c + f_{ij}^H H_2 Q_i u_j^c,$$  

(8)

where $L_i$ and $Q_i$ are the usual lepton and quark doublets, and $H_1 = (h_1^0, h_1^-), \ H_2 = (h_2^0, h_2^0)$ are the 2 Higgs doublets. If only $B$ is assumed to be conserved but not $L$, then the superpotential also contains the terms

$$\mu_i L_i H_2 + \lambda_{ijk} L_i L_j e_k^c + \lambda_{ijk} L_i Q_j d_k^c,$$  

(9)

and violates $R$. As a result, a radiative neutrino mass $m_\nu \simeq \lambda^2 (\langle m_\nu^2 \rangle)/16\pi^2 m_{\tilde{\nu}}^2$ may be obtained [18]. Furthermore, from the mixing of $\nu_i$ with the neutralino mass matrix through the bilinear term $L_i H_2$ and the induced vacuum expectation value of $\tilde{\nu}_i$, a tree-level mass $m_\nu \simeq (\mu_i/\mu - \langle \tilde{\nu}_i \rangle/\langle h_1^0 \rangle)^2 m_{\text{eff}}$ is also obtained [19].

8. Leptogenesis from R-parity nonconservation

Whereas lepton-number violating trilinear couplings are able to generate neutrino masses radiatively, they also wash out any pre-existing $B$ or $L$ asymmetry during the electroweak phase transition [20,21]. On the other hand, successful leptogenesis may still be possible as shown recently [22].

Assume the lightest and 2nd lightest supersymmetric particles to be

$$\tilde{W}_3' = \tilde{W}_3 - \epsilon \tilde{B}, \quad \tilde{B}' = \tilde{B} + \epsilon \tilde{W}_3,$$  

(10)

where $\tilde{W}_3$ and $\tilde{B}$ are the SU(2) and U(1) neutral gauginos, and $\epsilon$ is a very small number. Note that $\tilde{B}$ couples to $\tilde{\tau}_L^\pm, \tilde{\tau}_L^\mp$, but $\tilde{W}_3$ does not, because $\tau_L^\pm$ is trivial under SU(2). Assume $\tilde{\tau}_L^1 - h^-$ mixing to be negligible but $\tilde{\tau}_L^2 - h^+$ mixing to be significant and denoted by $\xi$. Obviously, $\tilde{\tau}$ may be replaced by $\tilde{\nu}$ or $\tilde{e}$ in this discussion.

Given the above assumptions, $\tilde{B}'$ decays into $\tilde{\tau}^\pm h^\pm$ through $\xi$, whereas $\tilde{W}_3'$ decays (also into $\tilde{\tau}^\pm h^\mp$) are further suppressed by $\epsilon$. This allows $\tilde{W}_3'$ decay to be slow enough to be out of equilibrium with the expansion of the Universe at a temperature $\sim 2$ TeV, and yet have a large enough asymmetry $(\tau^- h^+ - \tau^+ h^-)$ in its decay to obtain $n_B/n_\nu \sim 10^{-10}$.

This unique scenario requires $\tilde{W}_3'$ to be lighter than $\tilde{B}'$ and that both be a few TeV in mass so that the electroweak sphalerons are still very effective in converting the $L$ asymmetry into a $B$ asymmetry. It also requires very small mixing between $\tilde{\tau}_L$ with $h^-$, which is consistent with the smallness of the neutrino mass required in the phenomenology of neutrino oscillations. On the other hand, the mixing of $\tilde{\tau}_L^1$ with $h^+$, i.e. $\xi$, should be of order $10^{-3}$ which is too large to be consistent with the usual terms of soft supersymmetry breaking. For successful leptogenesis, the non-holomorphic term $H_2^2 H_1 \tilde{\tau}_L^c$ is required.
9. Conclusion and outlook

Models of neutrino mass and mixing invariably lead to other possible physical consequences which are important for our overall understanding of the Universe, as well as other possible experimentally verifiable predictions.

Acknowledgement

I thank Rahul Basu and the other organizers of WHEPP-6 for their great hospitality and a stimulating meeting. This work was supported in part by the US Department of Energy under Grant No. DE-FG03-94ER40837.

References

[1] S Weinberg, Phys. Rev. Lett. 43, 1566 (1979)
[2] E Ma, Phys. Rev. Lett. 81, 1171 (1998)
[3] M Gell-Mann, P Ramond and R Slansky, in Supergravity edited by P Van Nieuwenhuizen and D Z Freedman (North-Holland, Amsterdam, 1979) p. 315
T Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe edited by O Sawada and A Sugamoto, KEK Report No. 79-18 (KEK, Tsukuba, Japan, 1979) p. 95
R N Mohapatra and G Senjanovic, Phys. Rev. Lett. 44, 912 (1980)
[4] K S Babu and E Ma, Phys. Rev. Lett. 61, 674 (1988); Phys. Lett. B228, 508 (1989)
S T Petcov and S T Toshev, Phys. Lett. B143, 175 (1984)
[5] W Grimus and H Neufeld, Nucl. Phys. B325, 18 (1989); hep-ph/9911465
[6] G B Gelmini and M Roncadelli, Phys. Lett. B99, 411 (1981)
J Schechter and J W F Valle, Phys. Rev. D22, 2227 (1980)
[7] C Wetterich, Nucl. Phys. B187, 343 (1981)
[8] E Ma and U Sarkar, Phys. Rev. Lett. 80, 5716 (1998)
[9] R N Mohapatra and G Senjanovic, Phys. Rev. D23, 165 (1981)
[10] S D Biller et al, Phys. Rev. Lett. 80, 2992 (1998)
[11] M L Brooks et al, Phys. Rev. Lett. 83, 1521 (1999)
[12] L Baudis et al, Phys. Rev. Lett. 83, 41 (1999)
[13] V A Kuzmin, V A Rubakov and M E Shaposhnikov, Phys. Lett. B155, 36 (1985)
[14] M Fukugita and T Yanagida, Phys. Lett. B174, 45 (1986)
[15] M Flanz, E A Paschos, and U Sarkar, Phys. Lett. B345, 248 (1995)
[16] J M Frere et al, Phys. Rev. D60, 016005 (1999)
[17] A Zee, Phys. Lett. B93, 389 (1980); B155, 36 (1985)
[18] L Hall and M Suzuki, Nucl. Phys. B231, 419 (1984)
[19] M A Diaz, J C Romao, and J W F Valle, Nucl. Phys. B524, 23 (1998)
[20] B A Campbell et al, Phys. Lett. B256, 457 (1991)
W Fischler et al, Phys. Lett. B258, 45 (1991)
[21] E Ma, M Raidal and U Sarkar, Phys. Lett. B460, 359 (1999)
[22] T Hambye, E Ma and U Sarkar, Phys. Rev. D62, 015010 (2000)