Rare Z Decays *

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Motivated by the well known impact of rare decays of hadrons and leptons on the evolution of the Standard Model and on limits for new physics, as well as by the proposal for Giga-Z option at TESLA, we investigate the rare decay $Z \to b \bar{s}$ in various extensions of the Standard Model.

1. INTRODUCTION

The central role played by rare decays on our understanding of elementary particle physics, is well known, where “rare” stands here for Flavor Changing Neutral Currents (FCNC), which are either small or practically vanishing in the SM.

Some highlights are:

1. In K physics: The first appearance of charm in loops from which $m_c \approx 1.5$ GeV was predicted [2].
2. In B physics: The importance of $b \to s\gamma$ in the SM and for extracting limits on Beyond the SM (BSM) scenarios [3].
3. The top quark FCNC provide an excellent tool to investigate various extensions of the SM [4].
4. The experimental upper limit of the decay $\mu \to e\gamma$ [5], places severe limits on extensions of the SM.

In the following sections we will discuss two variants of 2 Higgs Doublet Models (2HDM) and two of Supersymmetry (SUSY). Of the latter the first one will be: SUSY with squark mixing, while in the second one FCNC will result from SUSY with R Parity Violation (denoted by RPV, or $R_P$). As we will see, $\text{BR}(Z \to b \bar{s})$ can be either smaller, the same or above the SM with a maximal value of $\text{BR}(Z \to b \bar{s}) \approx 10^{-6}$.

Experimentally, the attention devoted to FCNC in hadronic $Z$ decays at LEP and SLD has been close to nil. The best upper limit is [9] $\sum_{q=d,s} \text{BR}(Z \to b \bar{q}) \leq 1.8 \times 10^{-3}$ @ 90% CL. This is a preliminary DELPHI limit (which will probably remain as such forever...) based on about $3.5 \times 10^6$ hadronic decays. Experimentalists who are privy to LEP and SLD data should be encouraged to look in their data and improve the above limit.

Due to space limitations, the following discussion of various BSM models and their predictions for $\text{Br}(Z \to b \bar{s})$, will be sketchy. Many more details and a more complete set of references can be found in [1]. In fact, almost each reference should start with: “See e.g....” and end with: “... and references therein.”

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†Based on work done in collaboration with D. Atwood, S. Bar-Shalom and A. Soni [1].
2. GENERIC CALCULATION

We start with a generic calculation of the diagrams which modify (at one loop) the $Vd_I\bar{d}_J$ vertex, due to charged or neutral scalar, as depicted in Fig. 1. In our case $V = Z, d_I = b$ and $\bar{d}_J = \bar{s}$. The indices $i, j$ and $\alpha, \beta$ indicate which fermions and scalars we are considering, respectively. The Feynman rules are:

$$V_{\mu}f_if_j = i\gamma_{\mu} \left( a^J_{L(V'f)} L + a^J_{R(V'f)} \right)$$
$$V_{\mu}S_{\alpha}S_{\beta} = ig\gamma_{\mu} (p_\alpha - p_\beta)_{\mu}$$
$$S_{\alpha}d_if_i = i \left( b^{ij}_{L(\alpha)} L + b^{ij}_{R(\alpha)} R \right),$$
where $L(R) = [1 - (+)\gamma_5]/2$.

There are 4 one-loop amplitudes, each corresponding to one of the 4 one-loop diagrams. Each amplitude is proportional to $\epsilon_{\mu}(q)$ times $\bar{u}(p_i) [\gamma^\mu (A_L L + A_R R) + (B_L L + B_R R) p_\mu] u(p_s)$. $A_{L,R}$, $B_{L,R}$ are momentum dependent form factors, calculable from the diagrams. There are 4 per diagram, thus we have 16 form factors. $A_L$ for diagram (1) is:

$$A_L = -2\sum_{\alpha,\beta} g_2 a^{\alpha\beta} b_{L(\alpha)} b_{L(\beta)} C_{24},$$
and similarly for the other 15 form factors. $C_{24}$ is one of the usual one-loop scalar functions [10] at $m^2, m^2, m^2, m^2, m^2, m^2, q^2, m^2$.

Finally:

$$\Gamma(Z \rightarrow b\bar{s}) = 2 \frac{N_C}{3} \left( \frac{1}{16\pi^2} \right)^2 \frac{M_Z}{16\pi} \times \left[ 2 \left| A_L^T \right|^2 + \left| A_R^T \right|^2 + \left| B_L^T \right|^2 + \left| B_R^T \right|^2 \right],$$
where $A_L^T$ is the Total sum of $A_{LS}$ from the 4 diagrams, and similarly for $A_R^T$, $B_L^T$ and $B_R^T$.

3. MODELS AND PREDICTIONS

The stage is now ready for identifying, for each model, the relevant scalars $S_i$, fermions $f_i$ and the couplings $a, b$ and $g$ (with the appropriate indices), as expressed in the Feynman rules above. Then, the route for obtaining $\Gamma(Z \rightarrow b\bar{s})$ using the generic equation in the previous section is clear.

3.1. Two Higgs doublet models

In 2HDM with flavor diagonal couplings of the neutral Higgs to down-quarks, the FCNC $Z \rightarrow b\bar{s}$ go through the one-loop diagrams in Fig. 1. The scalars are the charged Higgs bosons, $S_{\alpha = 1} = H^\pm$ and the fermions are $f_i = u_i$, $i = 1, 2, 3$. The couplings are: $Z_{\mu}u_i\bar{u}_j$ is as in the SM (therefore only $i = j$ survives), $Z_{\mu}H^+ H^-$ is derived from the kinetic energy part of the Lagrangian $L$ and $H^\pm \bar{u}_id_j$ is obtained from the Yukawa part which, in common notation is [11]:

$$\mathcal{L}_Y = -\sum_{i,j} Q^1_{L_i} \left( U^1_{ij} \Phi_1 + U^2_{ij} \Phi_2 \right) u_R^j$$
$$+ \left( D^1_{ij} \Phi_1 + D^2_{ij} \Phi_2 \right) d_R^j.$$
3.1.1. Two Higgs doublet model of type II
In this model, called 2HDMII, $U^1 = D^1 = 0$, $Z \to b\bar{s}$ was considered before [12]. Using realistic values in the tan$\beta - m_{H^+}$ plane, we obtain: $\text{BR}(Z \to b\bar{s}) \lesssim 10^{-10}$, two orders of magnitude below the SM.

3.1.2. Two Higgs doublet model “for top”
In this variant [13], named T2HDM, the top is rewarded for its “fatness” by having its mass proportional to the large $v_2$, while all other masses are proportional to $v_1$. It therefore makes sense to consider here only tan$\beta$ $> 1$. Using T2HDM parameters consistent with data we find: $\text{BR}(Z \to b\bar{s}) \lesssim 10^{-8}$.

3.2. Supersymmetry with squark mixing

FCNC in SUSY can emanate from squark mixing in:

$$\mathcal{L}_{\text{soft}} = -\tilde{Q}_i^\dagger (M'^2_Q)_{ij} \tilde{Q}_j - \tilde{U}_i^\dagger (M'^2_U)_{ij} \tilde{U}_j - \tilde{D}_i^\dagger (M'^2_D)_{ij} \tilde{D}_j + A^u_i \tilde{Q}_i H_u \tilde{U}_j + A^d_i \tilde{Q}_i H_d \tilde{D}_j,$$

with the usual notation for the squark fields [1] and where $i, j$ are generation indices. Furthermore,

$$M'^2_{U,D} = \begin{pmatrix} (m'^2_{U,D})_{LL} & (m'^2_{U,D})_{LR} \\ (m'^2_{U,D})_{LR}^\dagger & (m'^2_{U,D})_{RR} \end{pmatrix},$$

where $(m'^2_{U,D})_{LL,RR}$ are $3 \times 3$ matrices. Under certain assumptions [14] and taking only $\tilde{b} - \tilde{s}$ or $\tilde{t} - \tilde{c}$ mixing into account:

$$(m'^2_{U,D})_{LL,RR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta'_{LL,RR} \\ 0 & \delta'_{LL,RR}^\dagger & 1 \end{pmatrix} m'^2_0.$$

The above $\delta'$s represent squark mixing from non-diagonal bilinears in $\mathcal{L}$. $m_0$ is a common squark mass scale, obeying: $m_0 \gg M_Z$. Also, $\delta_{LRS}$ will stand for squark mixing from non-diagonal trilinears in $\mathcal{L}$ [1]. For them we adopt the Ansatz of [15], leading to $\delta_{LRS} \propto vA/m^2_0$, where $A$ is a common trilinear soft breaking parameter for both up and down squarks. $M^2_{D,U}$ become $4 \times 4$ matrices in the weak bases $\Phi^{0}_{D,U} = (\tilde{s}_L, \tilde{s}_R, \tilde{b}_L, \tilde{b}_R, \tilde{c}_L, \tilde{c}_R, \tilde{t}_L, \tilde{t}_R)$. They are diagonalized to obtain the mass eigenstates $\Phi^0_{D,U} = (\tilde{s}_1, \tilde{s}_2, \tilde{b}_1, \tilde{b}_2, \tilde{c}_1, \tilde{c}_2, \tilde{t}_1, \tilde{t}_2)$, with the help of $R_{U,D}$ which rotates $\Phi$ to $\Phi^0$.

We can now describe two cases of squark mixing: $\tilde{b} - \tilde{s}$ and $\tilde{t} - \tilde{c}$ mixing.

3.2.1. $\tilde{b} - \tilde{s}$ mixing

The scalars here are $S_\alpha = \Phi_{D,\alpha}$, $\alpha = 1, 2, 3, 4$, since $\tilde{b} - \tilde{s}$ admixture states run in the loops. The gluon is the only fermion in the loops, thus $f_t = g$. The $a$ couplings are 0, since $Zg\bar{g} = 0$. In other words, one diagram (out of the four generic diagrams) vanishes. The $b$ and $g$ couplings are functions of elements of the rotation matrix $R_D$ mentioned above [16]. Since the two $\delta_{L,R} \lesssim 10^{-2}$ [17], we neglect them. For the other four $\delta$s we assume a common value, i.e. $\delta_{D(23)}^{L(32)} = \delta_{L(32)}^{D(23)} = \delta_{R(32)}^{D(23)} = \delta_{D(32)}^{R(32)} = \delta^D$, and vary $0 < \delta^D < 1$.

The parameters needed for masses, mixing and $\Gamma(Z \to b\bar{s})$ are: $m_\tilde{g}$, $\mu$, $A$, $\tan\beta$, $m_\tilde{g}$ and $\delta^D$. We vary them subject to $m_{\text{squarks}} > 150$ GeV and have plots of practically everything as a function of everything [1].

We find $\text{BR}(Z \to b\bar{s}) \lesssim 10^{-8}$, where the highest value is attained for $m_\tilde{g}$ and one $m_j \approx$ the EW scale, while $m_{\tilde{g}}, j \neq 1 \approx$ few TeV. Such splitting requires “heavy” SUSY mass scale with soft breaking parameters, which is consistent with the non-observability of SUSY particles so far.

3.2.2. $\tilde{t} - \tilde{c}$ mixing

In this scenario the scalars are $S_\alpha = \Phi_{U,\alpha}$, $\alpha = 1, 2, 3, 4$, similarly to the previous case, except for $D \to U$. Obviously, $\tilde{t} - \tilde{c}$ admixture states run in the loops. The loop fermions are the two charginos $f_i = \chi_i$, $i = 1, 2$, and all four generic diagrams contribute to $Z \to b\bar{s}$. The Feynman rules [16] involve elements of the rotation matrix $R_U$ mentioned above and the chargino mixing matrices.

At the end of the day, running with the parameters over all values consistent with the data, and with $m_\tilde{g} > 150$ GeV and $m_\chi > 100$ GeV we obtain: $\text{BR}(Z \to b\bar{s}) \lesssim 10^{-8}$, which we could have anticipated since $\text{BR}(\tilde{t} - \tilde{c}$ mixing): $\text{BR}(\text{through } \tilde{b} - \tilde{s}$ mixing) $\approx \alpha : \alpha_s$. 
3.3. Supersymmetry with RPV

Since there is no sacred principle which guarantees R-parity conservation, we assume in this part of the talk that $R_P$ is violated. Then, $R_P$ terms in the SUSY superpotential $W$ lead to FCNC. $\lambda$ terms (pure $L$) in $W$ are irrelevant at the 1-loop level. In addition we assume, for the pure $B$ terms, that $\lambda'' << \lambda'$, and also neglect the bilinear term in the $R_P$ part of $W$.

Then: $V_{\text{RPV}} = \epsilon_{ab} \lambda_{ijk} L_i^a \bar{Q}_j^b \bar{D}_k^c$. In addition, if $R_P$ is OK then the $R_P$ conserving soft SUSY breaking is extended. We need only the bilinear: $V_{\text{RPV}} = \epsilon_{ab} \lambda \bar{L}_i^a H_u^i$, where $L$, $H_u$ are the scalar components of the hatted $L$ and $H_u$, respectively.

We therefore have two types of FCNC:

**Type A:** Trilinear-trilinear: $\Gamma(Z \to b\bar{s}) \propto (\lambda' \lambda')^2$.

**Type B:** Trilinear-bilinear: $\Gamma(Z \to b\bar{s}) \propto (b\lambda)^2$.

3.3.1. Type A: Trilinear-trilinear terms

We further sub-divide type A contributions to 6 groups according to the scalars and fermions running in the loops. For instance, in type A1 the scalars are $S_i = \epsilon_{L,\alpha}$, $\alpha = 1, 2, 3$ and the fermions are $f_i = u_i$, $i = 1, 2, 3$. The $a$ couplings are identical to their SM values, $b_a = 0$ (for all $i$, $j$ and $a$), $b_{ij}^b = -\lambda_{ij}^a$ and $g_{ij}^c = -\epsilon(c_{ij}^a - s_{ij}^a)d_{ij}/2s_{ij}c_{ij}$. Unfortunately, going over all type A groups, taking into account the available limits on $\lambda$’s and on the other relevant parameters, we obtain for the trilinear-trilinear case: $\text{BR}(Z \to b\bar{s}) \lesssim 10^{-10}$. Our results are in agreement with the special cases in [18].

3.3.2. Type B: Trilinear-bilinear terms

In this case, a Higgs exchanged in the loop mixes with a slepton, through $\epsilon_{ab} b_3 \bar{L}_3^a H_u^i$, assuming that only $b_3 \neq 0$. We choose to work in the “no VEV” basis $v_3 = 0$ in which: $H_u \equiv \left(h_u^+ \frac{\epsilon_{ij}^a}{\sqrt{2}}, \epsilon_{ij}^a \frac{v_u + i\phi_0^a}{\sqrt{2}} \right)$, $H_d \equiv \left(\epsilon_{ij}^b v_u + i \phi_0^b \frac{\sqrt{2}, h_d^+}{} \right)$, $\bar{L}_3 \equiv \left(\bar{\nu}_3^0 + i \bar{\nu}_3^0 \frac{\sqrt{2}, \tilde{e}_3^-}{} \right)$, where $\bar{\nu}_3^0, \nu_3^0, \tilde{e}_3^-$ are SU(2) CP-even, CP-odd $\tau$-sneutrinos, $\tilde{\tau}_L$, respectively. In the basis $\Phi_C = (h_u^+, h_d^+, \tilde{e}_3^-)$ we wrote the mass matrix in the charged scalar sector, in the basis $\Phi_E = (\epsilon_{ij}^b \nu_u, \nu_3^0, \tilde{\tau}_L)$ we wrote the mass matrix in the CP-even neutral scalar sector, and in the basis $\Phi_O = (\phi_0^b, \phi_0^b, \nu_3^0)$ we wrote the mass matrix in the CP-odd neutral scalar sector.

The new charged scalar and CP-even and CP-odd neutral scalar mass-eigenstates are obtained by diagonalizing the above-mentioned matrices. They are: $\Phi_C = (H^+, G^+, \tilde{\tau}^+) \Phi_E = (H, h, \tilde{\tau}^+)$, and $\Phi_O = (A, G^0, \tilde{\nu}^-)$. In the limit $b_3 \to 0$: $H, h, A, H^+$ become the usual ones. Rotating with the diagonalizing $R_{C,E,O}$ (for Charged, Even-CP, Odd-CP) matrices, one goes from the $\Phi$s to the $\Phi$0s. All depends on the four parameters $A^0, m^0_{\nu}$ (the masses in the limit $b_3 \to 0$,), $b_3$ and $\tan \beta$.

Let us sub-divide type B into two types according to the scalar and fermion in the loop:

**Type B1:** Here $S_\alpha = \Phi_{C, \alpha}$: $f_i = u_i$ with $\alpha = 1, 3$; $i = 1, 2, 3$. The $a$ couplings are equal to their values in the SM. The $b$ couplings include elements of the rotation matrix (for the charged fields) $R_C$ and $\lambda'$, and $g_a^C = -c \cot \theta W d_{ij}/2s_{ij}c_{ij}$. Its the only case for which our generic form is insufficient. This fact results from the appearance of two new diagrams proportional to a scalar-vector-vector coupling ($ZZ\Phi_F$ in our case). The other eight diagrams are special cases of the generic ones in Fig. 1.

Inserting parameters consistent with the data we found that for type B: $\text{BR}(Z \to b\bar{s}) \lesssim 10^{-6}$.

4. EXPERIMENTAL FEASIBILITY

Let us briefly comment about the feasibility of observing (or limiting) a signal of $\text{BR}(Z \to b\bar{s}) \sim 10^{-6}$, at a Linear Collider producing $10^9$ $Z$-bosons. Such a signal leads to one $b$-jet and one $q$-jet, where $q$ stands for quarks lighter than $b$. The main background is from $Z \to bb$. Using what, we think, are realistic efficiencies we find that a new physics signal $Z \to b\bar{s}$, with a branching ratio of order $10^{-6}$, can reach beyond the 3-sigma level [1]. We can also get a clue about how low one can go in the value (or limit) of $\text{BR}(Z \to b\bar{s})$ with $10^9$ $Z$-bosons, from the fact that the DELPHI preliminary result reached [9] $\text{BR}(Z \to b\bar{s}) \sim \mathcal{O}(10^{-3})$ with $\mathcal{O}(10^6)$ $Z$-bosons. Scaling this limit, especially with the expected advances in $b$-tagging and identification of non-$b$
jets methods, an $\mathcal{O}(10^{-6})$ branching ratios should be easily attained at a Giga-$Z$ factory. One needs realistic simulations as feasibility studies for this important rare $Z$ decay mode.

5. SUMMARY AND CONCLUSIONS

Our results are best summarized in Table 1 which shows the best values for $\text{Br}(Z \rightarrow b\bar{s})$ in extensions of the SM discussed in this talk. The SM result is given for comparison. Note that we have not included interference with the SM as each of the values “stands alone”. In some cases such interference may increase the branching ratio to $\sim 10^{-7}$.

| model       | scalars in loop | Br   |
|-------------|-----------------|------|
| SM          | $W^+$ (no scalars) | $3 \times 10^{-8}$ |
| 2HDMII      | $H^+$           | $10^{-10}$ |
| T2HDM       | $H^+$           | $10^{-8}$ |
| $b\bar{s}$ mix | $b\bar{s}$ admix | $10^{-6}$ |
| $\tilde{t}\tilde{c}$ mix | $\tilde{t}\tilde{c}$ admix | $10^{-8}$ |
| tri-tri $R_p$ | $\tilde{q}, \tilde{b}, \tilde{\ell}$ | $10^{-10}$ |
| tri-bi $R_p$ | $\tilde{b}, H, A$ admix | $10^{-6}$ |

We conclude that Giga-$Z$ experiments will have the opportunity to place significant limits, or hopefully discover the scenario beyond the SM, by searching for hadronic (and leptonic [7]) neutral current flavor changing transitions.

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