Standard Model and Gravity from Spinors

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Abstract. We propose to unify the Gravity and Standard Model gauge groups by using algebraic spinors of the standard four-dimensional Clifford algebra, in left-right symmetric fashion. This generates exactly a Standard Model family of fermions, and a Pati-Salam unification group emerges, at the Planck scale, where (chiral) self-dual gravity decouples. As a remnant of the unification, isospin-triplets spin-two particles may naturally appear at the weak scale, providing a striking signal at the LHC.

The set of quantum numbers of the fermions in a family of the Standard Model (SM) is one of the crucial hints that Nature has given us for understanding the fundamental interactions. While this hint has been extensively used to guess the fundamental symmetries, it is not excluded that it could still guide us to surprisingly new structures. A traditional approach has been to embed a SM family in partial unification groups like the Left-Right symmetric one SU(2)_L × SU(2)_R × U(1)_{B−L} × SU(3) and the Pati-Salam one SU(2)_L × SU(2)_R × SU(4) [1,2], or into Grand-Unification ones like SU(5), SO(10). All these approaches consider the gauge groups as internal symmetries, direct product with the spacetime Lorentz symmetry, and spinors appear in multiplets of the gauge group.

On the other hand, there is a different way to group fermions in multiplets, appearing in algebraic spinor theory [3]. This is based on the fact that the Clifford algebra is isomorphic to the algebra of inhomogeneous differential forms, i.e. combinations of zero, one, ..., up to d-forms, so that spinors, built out of these objects, have algebraic as well as geometrical meaning. While these spinors still satisfy the Dirac equation, they are not in the minimal representation of the Lorentz group: they are generic elements of the Clifford algebra, and thus objects of dimension 2^d. As such, they contain naturally more particles, that appear in multiplets and can accommodate various sets of quantum numbers, including the SM ones.

There is a fairly long history of such attempts [4,5,6,7,8], and all provide some evidence of the emergence of the SM gauge group. In addition they often gauge together the internal and Lorentz symmetries, and thus represent a promising setup where also gravity could be reformulated and unified with the other interactions.

However, most of these approaches end up in postulating extra dimensions. The reason for this is basically that one SM family contains 16 Weyl spinors, that is 32 complex components, and thus 2^4 = 16 is not enough to describe even one family. A larger algebra is necessary, and while it is always possible to enlarge an internal symmetry, to maintain contact with spacetime geometry this requires extra dimensions. By doing so, one can easily accommodate a family, and even motivate the emergence of more families (an even number usually). This was the strategy pursued in [3] (Cl_2,6), [6] (Cl_7), [7] (Cl_{9,1}).

In this letter we suggest that within this framework one can avoid the use of extra dimensions and accommodate a whole SM family in a left-right symmetric approach, by using the standard Clifford algebra of four-dimensional gamma matrices. Breaking the large underlying symmetry group leads to a form of Pati-Salam (PS) unification, and the SM gauge group can then be reached by a standard breaking chain.

The main outcomes of this approach are that: (i) the existence of the color SU(4) and SU(2)_L × SU(2)_R groups is connected with the fact that spacetime is four-dimensional; (ii) gravity is uni-
fied with the weak isospin groups in chiral way, so that independent selfdual spin-connections appear; (iii) the reality of spacetime leads to a unique Lorentz group while the two weak groups remain split in left and right copies; (iv) the metric and its signature emerge from the symmetry breaking, and (v) this breaking also predicts a spin-two isospin-triplet field that may have natural mass at the electroweak scale.

**Algebraic Spinors.** We will mainly be concerned with flat space $R^{1,3}$, and use the standard Clifford algebra $Cl_{1,3}$ given by Dirac gamma matrices, $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, in Weyl representation with $\gamma_5 = \text{diag}(1,1,-1,-1)$, $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$. For $Cl_{1,3}$ we use the basis $\{\gamma_{A=1...16}\} = \{1_4, \gamma_0, \gamma_i, \gamma_0\gamma_i, -\gamma_i\gamma_4, \gamma_5\gamma_0, \gamma_5\gamma_4, \gamma_5\}$. The real $Cl_{1,3}$ does not contain the imaginary unit, so it can be taken over the complex space. Then it is also the algebra $gl(4, \mathbb{C})$ of all $4 \times 4$ complex matrices, and as such it contains different subalgebras, generically non commuting. For instance one finds $sl(4, \mathbb{C}) \supset sl(3, \mathbb{C}) \supset sl(2, \mathbb{C})$ or $sl(4, \mathbb{C}) \supset sl(2, \mathbb{C}) \times sl(2, \mathbb{C})$.

While usual spinors are column objects transforming from the left under Lorentz transformations, algebraic spinors are objects that themselves belong to the Clifford algebra, $\Psi = \psi^A \gamma_A$, with $\psi^A \in \mathbb{C}$, and transform from the left under algebra-valued transformations. They can be represented as $4 \times 4$ complex matrices,

$$
\Psi = \begin{pmatrix}
\psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\
\psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\
\psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\
\psi_{41} & \psi_{42} & \psi_{43} & \psi_{44}
\end{pmatrix}
$$

and $\Psi \rightarrow e^A \Psi$, where also $\Lambda = \Lambda^A \gamma_A$. Note that $\Lambda$ are generic $gl(4, \mathbb{C})$ transformations, that is a noncompact algebra.

Evidently such transformations act on each column separately, therefore the four columns inside the algebraic spinor represent four invariant subspaces (the four left ideals of the algebra). If we give to $\mu$ the standard meaning of spacetime index, and restrict to a Lorentz transformation $\Lambda = i\omega^{\mu\nu} \gamma_\mu \gamma_\nu$, we can see that we must identify each column with a Dirac spinor, and in the Weyl representation of gamma matrices, the upper and lower halves of each column are *left* and *right* Weyl spinors.

A further useful property of algebraic spinors is that they naturally admit transformations also from the right; in general thus:

$$
\Psi \rightarrow e^A \Psi e^{-\tilde{A}}, \quad \Lambda = \Lambda_A \gamma_A, \quad \tilde{\Lambda} = \tilde{\Lambda}_A \gamma_A,
$$

and the $\tilde{\Lambda}$ transformations recombine the four columns among them, commuting with transformations from the left. They belong to an other $gl(4, \mathbb{C})$, that can accommodate an internal symmetry up to rank-four, for example $U(4)$. Summarizing, an algebraic spinor contains 4 Dirac spinors and these can be related by some internal symmetry of rank at most four.

Unfortunately, a SM family contains 8 Dirac spinors, therefore this algebraic spinor is too small. Moreover, also the internal symmetry is too small to accommodate the SM group in unified way. One easy way out is to introduce extra dimensions, enlarging the Clifford algebra by successive factors of two. Naturally this enlarges also both the transformations from the left and from the right, introducing larger symmetry groups and extra particles.

Instead of following this approach, we suggest here to give up a different assumption, namely that the Clifford algebra transformations $\Lambda$ act on both chiralities, or in other words that the spinor $\Psi$ contains Dirac spinors in its columns. We will then introduce two spinors of opposite chiralities, and each one will be subject to its own transformations.

**Left and Right Spinors.** The main point is that a Clifford transformation contains two commuting sets as $gl(4, \mathbb{C}) \supset sl(2, \mathbb{C}) \times sl(2, \mathbb{C})$, that usually are the two complex-conjugate copies of Lorentz, for the left and right chiralities. If we restrict to left chiral objects, we can choose to assign different meanings to them [9]: one can be used for one chiral copy of Lorentz, and the other for the weak isospin, $sl(2, \mathbb{C})_{\text{lorentzL}} \times sl(2, \mathbb{C})_{\text{weakL}}$. (One can think $sl(2, \mathbb{C})$ and $su(2)$ as having the same generators, $su(2)$ restricted to real parameters.)

We then introduce a left algebraic spinor $\Psi_L$, that is again algebra valued, $\Psi_L = \psi^A_L \gamma_A$ with complex entries, and that again transforms from the left under algebra transformations: $\Psi_L \rightarrow e^{A_L} \Psi_L$. When $\Psi_L$ is represented as a $4 \times 4$ complex matrix, we have again that $e^{A_L}$ acts on each column separately.
However, the $sl(2, \mathbb{C})_{\text{lorentz}}_L$ inside $e^{\Lambda_L}$ acts only on half column, so each column contains two Weyl spinors. These two spinors are mixed by the commuting $su(2)_{\text{weak}}$, so that inside each column we actually find an isospin doublet of Weyl fermions.

As before, $\Psi_L$ can be transformed also with independent transformations $\Lambda_L = \Lambda_L^A \gamma_A$ from the right:

$$
\Psi_L \rightarrow e^{\Lambda_L} \Psi_L e^{-\Lambda_L},
$$

and $\Lambda_L \in \tilde{g}l(4, \mathbb{C})$ act as internal symmetries, of rank at most four, e.g. $U(4)_L$.

Summarizing, $\Psi_L$ contains 4 isospin doublets of Weyl spinors, that we can now identify with the left half of a standard model family. Moreover, since they are related by an internal symmetry of rank 4, it is very suggestive to represent $\Psi_L$ also with lepton and colored quarks indices:

$$
\Psi_L = \begin{pmatrix}
\nu_{L1} & u_{L1,r} & u_{L1,g} & u_{L1,b}
\nu_{L2} & u_{L2,r} & u_{L2,g} & u_{L2,b}
\epsilon_{L1} & d_{L1,r} & d_{L1,g} & d_{L1,b}
\epsilon_{L2} & d_{L2,r} & d_{L2,g} & d_{L2,b}
\end{pmatrix}.
$$

A SM family is described just by a left-right symmetric couple of such spinors: $\Psi_L, \Psi_R$.

Let us now discuss the (global) internal symmetries. First we observe that since we speak now of objects of separate chirality, at this stage everything is duplicated. From the independent $\Lambda_L, \Lambda_R, \Lambda_{L,R}$, we have

$$
\tilde{gl}(4, \mathbb{C})_L \times \tilde{gl}(4, \mathbb{C})_L \times \tilde{gl}(4, \mathbb{C})_R \times \tilde{gl}(4, \mathbb{C})_R.
$$

One should think these as geometric symmetries, dictating the field representations, only partially realized in our low energy world. This is exactly what we will find when trying to write a theory in spacetime.

Let’s restrict first to one chirality. The $\Lambda_L$ transformations, belonging to $\tilde{gl}(4, \mathbb{C})_L$, contain generators that mix Lorentz and weak indices as well as noncompact ones, that are not observed in our world. We will thus need a breaking of $\tilde{gl}(4, \mathbb{C})_L$ to $sl(2, \mathbb{C})_{\text{lorentz}}_L \times su(2)_{\text{weak}}_L$.

The $\Lambda_L$ transformations belong to an other $\tilde{gl}(4, \mathbb{C})$ and should be restricted to be compact, because noncompact internal symmetries are always plagued by ghosts. This minimal requirement leads from $\tilde{gl}(4, \mathbb{C})$ to its maximal compact group $U(4)$, i.e. a group that unifies color and $B - L$ by treating lepton as the fourth color, as in the celebrated Pati-Salam group. The representation $\mathbf{4}$ of $\Psi_L$ explicitly showed this.

In this “broken” phase the symmetries would thus be $sl(2, \mathbb{C})_{\text{lorentz}}_L \times su(2)_{\text{weak}}_L \times u(4)_L$. Then, the right chirality would in principle give rise to a second copy of these, i.e. $sl(2, \mathbb{C})_{\text{lorentz}}_R \times su(2)_{\text{weak}}_R \times u(4)_R$. If on one hand it is nice to have a duplication of the weak isospin group because the SM quantum numbers suggest it, on the other hand especially the doubling of Lorentz should be removed in the real world.

Before describing the broken phase where this happens, it is useful to lay down explicitly the generators of $sl(2, \mathbb{C})_{\text{lorentz}}_L \times su(2)_{\text{weak}}_L$ inside $\Lambda_L, R$:

$$
su(2)_{\text{weak}} : \{\tau_i\} = \{i\gamma_0, -\gamma_0\gamma_5, \gamma_5\} \equiv \{1_2 \otimes \sigma_i\}
$$

$$
sl(2, \mathbb{C})_{\text{lorentz}} : \{\rho_i\} = \{-i\epsilon_{ijk}\gamma_j\gamma_k\} \equiv \{\sigma_i \otimes 1_2\}.
$$

**Broken phase from fermion kinetic terms.** To describe propagating fermions, one needs to introduce the equivalent of spacetime gamma matrices, i.e. a vierbein or soldering form $\Gamma_\mu$ connecting fermion bilinears with the spacetime derivative $\partial_\mu$. $\Gamma_\mu$ is an algebra-valued field that transforms as $\Gamma_\mu \rightarrow e^{-\Lambda} \Gamma_\mu e^{\Lambda}$ under $gl(4, \mathbb{C})$, and can be taken of 16 real components (for each index $\mu$).

While a generic $\Gamma_\mu$ would be needed in a symmetric phase, we assume that it develops a VEV $\bar{\Gamma}_\mu$, defining the broken phase where we live. To have Minkowski spacetime, $\bar{\Gamma}_\mu$ should leave unbroken a global $so(1,3)_{\text{lorentz}}$ symmetry defined as simultaneous transformations $\Lambda_\mu^a$ of 1) a Lorentz subgroup of spacetime diffeomorphisms and 2) internal $\Lambda$ transformations restricted to $sl(2, \mathbb{C})_{\text{lorentz}}$:

$$
\bar{\Gamma}_\mu \equiv e^{-\Lambda_\mu} \Gamma_\mu e^{-\Lambda_\mu},
$$

as it happens for ordinary gamma matrices. This defines a soldering of the spacetime and internal Lorentz symmetry groups, and thus the signature of spacetime emerges from the VEV $\bar{\Gamma}_\mu$.

Two different $\Gamma_\mu_{L,R}$ should actually be defined, that do this job for the two $sl(2, \mathbb{C})_{\text{lorentz}}_{L,R}$, and their
VEVs should also be aligned as $\Gamma^{L,R}_\mu = \eta_{\mu\nu} \Gamma^{L,R}_\nu$ so that they define the same Minkowski metric $\eta_{\mu\nu}$ for L and R spinors. Explicitly:

$$\Gamma^{L,R}_\mu = \{ \pm 1_{4}; -i \epsilon_{ijk} \gamma_j \gamma_k \} = \{ \pm 1_2, \sigma_i \} \otimes 1_2. \tag{5}$$

$\Gamma^{L,R}_\mu, su(2)_{\text{weak},L,R}$ commute as one checks with $[4]$.

We can now build kinetic terms for left and right fermions (tr is the trace in 4x4 representation)

$$\mathcal{L} = \text{tr}[\Psi^\dagger_L \partial^{\mu} \Gamma^{L}_\mu \Psi_L] + \text{tr}[\Psi^\dagger_R \partial^{\mu} \Gamma^{R}_\mu \Psi_R], \tag{6}$$

and look for the remaining global invariances.

These kinetic terms restrict some subgroups to have unitary elements $e^A e^{A^\dagger} = 1$, i.e. to be compact. This happens to the $sl(2,\mathbb{C})_{\text{weak}}$ groups, whose generators commute with $\Gamma^{L}_\mu$, and also to the $gl(4,\mathbb{C})$. Therefore these are reduced respectively to the compact groups $su(2)_{\text{weak}}$ and $u(4) = su(4) \times u(1)$.

The $sl(2,\mathbb{C})_{\text{lorentz}}$ groups on the other hand follow a different fate: they remain non compact because a generic transformation, not necessarily unitary, is compensated by a spacetime Lorentz transformation of $\partial_\mu$; indeed the internal Lorentz is soldered with the spacetime one. In addition, since the derivative $\partial_\mu$ is the same in both the L and R sectors, then $sl(2,\mathbb{C})_{\text{lorentz},L,R}$ are actually forced to transform together

$$\Lambda^{\dagger}_{\text{lorentz},L} = -\Lambda_{\text{lorentz},R} \tag{7}$$

and we remain with just one diagonal $sl(2,\mathbb{C})_{\text{lorentz}}$, identified with the spacetime Lorentz group $so(1,3)$.

The mixed Lorentz-weak transformations do not preserve $\Gamma^{L}_\mu$ and are thus broken.

Summarizing, in the broken phase, in place of the large non compact group $[3]$ we have (at most) the symmetry:

$$so(1,3)_{\text{lorentz}} \times su(2)_L \times su(2)_R \times u(4)_L \times u(4)_R.$$

This group can be linked to the Pati-Salam and SM groups by standard breaking chains, after introducing appropriate Higgs fields needed for the symmetry breakings. In particular, $su(4)_L \times su(4)_R \to su(4) \to su(3)_{\text{color}} \times u(1)_B - L$, can be realized by a field $\Phi_{L,R} \in (4_L,4_R)$ of $gl(4,\mathbb{C})_L \times gl(4,\mathbb{C})_R$, i.e. transforming as $\Phi_{L,R} \to e^{\Lambda_\mu} \Phi_{L,R} e^{-\Lambda_\mu}$, with VEV $\Phi = 1_4$.

For the curved and first-order action in the symmetric phase, an inverse soldering should be defined, see $[2][13]$.

The breaking $gl(4,\mathbb{C}) \to u(4)$ may be realized e.g. with a field transforming as $\Phi \to e^{\Lambda} \Phi e^{\Lambda^\dagger}$, with VEV $\Phi = 1_4$.

We can now build kinetic terms for left and right fermions (tr is the trace in 4x4 representation)$[2]$

$$\mathcal{L} = \text{tr}[\Psi^\dagger_L \partial^{\mu} \Gamma^{L}_\mu \Psi_L] + \text{tr}[\Psi^\dagger_R \partial^{\mu} \Gamma^{R}_\mu \Psi_R], \tag{6}$$

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As in left-right symmetric theories, the L and R weak groups may be broken at different scales giving rise to the observed parity-breaking phenomenology $[13]$. Of course, a complete model should also include the mixing matrices, as well as the mechanism for the quark/lepton, $U/D$ and horizontal hierarchies of masses. All these aspects may require additional fields, and will then be constrained by the unified dynamics beyond the planck scale. We leave the full analysis for future model building $[16]$, and proceed to discuss the gauge fields.

**Gauging.** The gauging of symmetries inside $\Lambda$ and $\bar{\Lambda}$ is realized by introducing a covariant derivative with Clifford-algebra valued vector fields, each having 16 complex components:

$$\partial_\mu \to D^{L,R}_\mu = \partial_\mu + V^{L,R}_\mu + \bar{V}^{L,R}_\mu, \tag{8}$$

with $V_\mu, \bar{V}_\mu$ acting from the correct sides of the fields.

While the tilded fields $\bar{V}^{L,R}_\mu$ are just the gauge fields of (complexified) $u(4)_{L,R}$ in Clifford algebra notation, the fields $V^{L,R}_\mu$ are more interesting: they unify gravity and weak-isospin, that are contained in the $gl(4,\mathbb{C})$ algebras.

It must be noted that in $V$ and $\bar{V}$ one may prefer avoid gauging the $u(1)$ factors (the identity $1_4$): in fact the first is a gauging of dilatations that usually leads to unimodular gravity (eating the trace of
the graviton, see below) and the second is anomalous. Therefore we will start from a gauging of four copies of sl(4, C). With this choice, in the broken phase the only non-decoupled gauge fields are the su(4) and the su(2) ones. To be explicit the $V_L^\mu$ can be parametrized in terms of the complex gauge fields $\omega^L$, $W^L$, $Z^L$ (and similarly for $V_R^\mu$):

$$V_L^\mu = i\omega^L_\mu (\sigma_i \otimes 1_2) + iW^L_\mu (1_2 \otimes \sigma_i) + Z^L_\mu ij (\sigma_i \otimes \sigma_j).$$  

(9)

In a flat broken phase, $V_{L,R}$ will then contain only the su(2)$_{L,R}$ gauge bosons, i.e. the (real) $W^L,R$ gauge bosons, while in the curved case also a component along the $sl(2,C)_{\text{lorentz}}$ generators $\rho_i$ is present:

$$V_L^\mu = \omega^\mu_i (\sigma_j \otimes 1_2) + iW^R_\mu \tau_i$$

$$V_R^\mu = \omega^\mu_i (\sigma_j \otimes 1_2) + iW^L_\mu \tau_i,$$

(10)

The $\omega^\mu_i$, $\tilde{\omega}^\mu_i$ reproduce the selfdual and antiselfdual spin connections of gravity, conjugate acting on the left or right fields, as in the reality condition (7). In fact one can make contact with Einstein gravity by redefining $\omega^\mu_i$ in terms of the Christoffel symbols as $\omega^\mu_i = \epsilon^{ijk}\Gamma^h_{jk} + i\Gamma^0_\mu$, that is also the selfdual part of a complex Christoffel connection (see e.g. [11, 15, 9]). We expect this to happen based simply on symmetry arguments, since as discussed above, the two internal Lorentz groups are broken (to the global so(1,3)$_{\text{lorentz}}$), therefore also the fluctuations around $\omega^\mu_i$, $\tilde{\omega}^\mu_i$ are massive and decoupled.

This also tells us that the VEVs $\Gamma_\mu$ are fixed to be at $M_{pl}$ in the Palatini spirit. Indeed, in the broken phase, the existence of $\Gamma_\mu$ allows to write a linear Einstein-Hilbert action $L_{EH} = tr(R \Gamma \Gamma)$ only for the gravitational curvature $R_{\mu \nu}$ (and not for the weak gauge field-strength) defining the dimensionful gravitational coupling $M_{pl}^2 \sim \Gamma^2$.

On the other hand a quadratic action can be written for both curvatures, $R_{\mu \nu}$, $W_{\mu \nu}$, whose couplings will be unified in the symmetric phase [9].

Fluctuations. Since $\Gamma_{L,R}$ act as 'Higgs' fields, it is interesting to analyze their fluctuations. Each $\Gamma_{L,R}$ has 64 components, that can be decomposed as:

$$\Gamma_\mu = M_{pl}(\eta_{\mu \nu} + h_{\mu \nu}) (\tilde{\sigma}^\mu \otimes 1_2) + \Delta_{\mu \nu} (\sigma^\mu \otimes \sigma_i)$$

where $\tilde{\sigma}^\mu = \{ \mp \mathbf{1}_2, \sigma_i \}$ for $L, R$. The first term is the background value defining here minkowski space, and $h$ and $\Delta$ are the (real) fluctuations. It is straightforward to identify among them the goldstone fields of the $g(4, C)$ symmetry breaking, that in the unitary gauge are 'eaten' by the corresponding gauge fields.

These are: the antisymmetric part of $h_{\mu \nu}$, eaten by the fluctuations of the spin connection $\omega$; the three antisymmetric parts of $\Delta_{\mu \nu}$, eaten by the fields mixing lorentz and weak symmetry $Z_{ij}^\mu$; and finally the three traces $\Delta_{\mu \nu}^I i$ that give mass to the noncompact (imaginary) isospin gauge fields, $L W^\mu_i$.

Summarizing, after symmetry breaking we find the following low energy field content: two standard gravitons, $h_{\mu \nu}$, $h^R_{\mu \nu}$ (10 components each) and two new traceless gravitons $\Delta_{L \mu \nu}^i$, $\Delta_{R \mu \nu}^i$ that are isospin triplets for the respective su(2)$_{L,R}$ groups (27 components each) [2].

Phenomenology. Let us discuss the phenomenology of these fields. Of the two singlet-gravitons, a parity-even combination $h^+ = h^L + h^R$ will be massless due to diffeomorphism invariance, and will correspond to the standard graviton. It will also couple universally to L and R matter. The parity odd combination $h^- = h^L - h^R$ on the other hand is not protected by diffeomorphisms and may be massive (see [10, 14]). Its mass can not be predicted at this stage, because it depends on the details of the full theory at energy beyond the planck scale. It is clear that if $h^-$ had planck-scale mass it would be unobservable at low energy, while if it were sufficiently light it would give rise to polarization-dependent gravitational effects, among these gravitational waves.

A different situation arises for the $L, R$ triplet-gravitons: they are not protected by diffeomorphisms therefore they both can be massive. It is then interesting to observe that each $\Delta$ can have two kinds of mass terms: one is 'gauge invariant', and the other comes from a coupling with e.g. one doublet higgs $\phi$ responsible for the SU(2) symmetry breaking. Setting $\Delta_{\mu \nu} = \Delta^{i}_{\mu \nu} \sigma_i$:

$$\text{tr}(\Delta_{\mu \nu})^2, \quad |\Delta_{\mu \nu} \phi|^2.$$  

(11)

Now, the first one is originated from terms of the form $\Gamma^2$, that give rise also to the cosmological constant $\Lambda$.

The appearance of 'colored' gravitons was described in [10] [12]. Here however breaking $g(4, C)$ implies that also the antisymmetric part of $\Delta$ is eaten and is thus completely decoupled.
(see e.g. [9, 12] for explicit constructions) therefore one may expect it to be small, with the result that the mass of the $\Delta_{\mu}^{L,R}$ is linked to the scale of breaking of the relative $SU(2)_{L,R}$ group.

The $L$ triplet is then particularly interesting, because it may have natural mass in the weak range, and being charged under isospin, would be easily produced and observed at the LHC (while its other couplings to matter are gravitational, i.e. planck suppressed). The present experimental lower bound on its mass can be estimated to be as low as $\sim 300$ GeV, from Drell-Yan pairwise production at Tevatron. It would be interesting to estimate the background and thus the corresponding discovery reach of LHC.

As already remarked, the precise mass predictions depend on a complete formulation of the symmetric phase, that we can address only partially here.

**Symmetric phase.** The discussion relied up to now on the use of the extended vierbeins $\Gamma_{\mu}^{L,R}$, that were assumed to develop a VEV, spontaneously breaking the large symmetry \[ \mathfrak{g}. \] It is natural to ask whether a theory in the fully symmetric phase could be given to explain such a breaking. The question was answered affirmatively in models of 'graviweak' unification [9] with smaller, orthogonal gauge groups such as $SO(4, \mathbb{C})$ or $SO(7, \mathbb{C})$, also using an extended vierbein. The action in the symmetric phase is simply \[ \mathcal{R} \wedge \Gamma \wedge \mathcal{R}, \] where the $\epsilon$ invariant tensor was derived from the duality operator in the corresponding algebra (acting in the representation of the $\Gamma$'s), and $\mathcal{R}$ is the full curvature of the connection $\mathcal{V}$.

In the present scenario, since $\mathfrak{gl}(4, \mathbb{C})$ has no duality, one can not write an invariant theory using only $\Gamma$ and $\mathcal{R}$, and at least one new field has to be introduced. Indeed one possibility would be to proceed as in Plebanski-inspired constructions [17, 18], where exactly the missing antisymmetric field is introduced.

Also, to understand the predictions in terms of masses of $\Delta_{\mu}^{L,R}$, one should introduce the full set of Higgs fields for the electroweak breaking, or in other words, to complete the full theory. We expect that, as is common in grand-unified theories, and unlike the bottom-up approach described here, such completions will not be unique, and plan to investigate them in a separate work [16].

Some comments can nevertheless be made on the gravitational sector: in the symmetric phase, prior to symmetry breaking, there is no metric and necessarily propagation is not standard: for example, if $\Gamma_{\mu}$ has no VEV fermion kinetic terms are not gaussian, and if the theory is formulated in first order formalism, the gauge action has a single derivative [9, 16]. This is a well known fact in the Weyl-Cartan-Palatini approach to gravity, and it can be related to the problem of its quantization. Here one can speculate that some help in understanding this phase will come from the underlying geometrical structure. A glimpse of this possibility can already be seen in the unusual emergence of the Lorentz group: to use chiral algebraic spinors, we needed in the symmetric phase two copies of the internal Lorentz group; then the fact that there is only one real spacetime group; then the fact that there is only one real spacetime, forces them to be glued in the broken phase. It is thus the soldering of $\Gamma_{L,R}$ with a real spacetime that leads to a unique Lorentz group for spinors.\[ ^{5} \] At the same time an analogous reality condition does not apply to the weak groups, that thus remain split in L and R parts, nicely predicting the left and right isospin fermion doublets. However, to achieve these results the theory demanded us not only two different extended spin connections, but also two extended vierbeins, or in other words two full (self-dual) gravities living in the L and R sectors, above the Planck scale.

The appearance of selfdual spin-connections in $V_{L,R}$ is a direct consequence of the choice of working with chiral spinors; nevertheless it is also most welcome, since there has been considerable progress in the formulation of gravity at both the classical and quantum level by using these variables [19]. They allow a much easier Hamiltonian formulation, even if there are difficulties related to the reality conditions. In the setup described here the L and R sectors are independent at the fundamental level, and the reality conditions are expected to arise dynamically (like \[ ^{4} \]). We stress indeed that differently

\[ ^{5} \text{This fact can be understood in the known theory of SL}(2,\mathbb{C}) \text{ spinors: } SO(1,3) \text{ is (locally) isomorphic to } SL(2,\mathbb{C}), \text{ and does not factorize as } SL(2,\mathbb{C}) \times SL(2,\mathbb{C}); \text{ this instead is true for its complexification, that may act only on a complex spacetime. Indeed, from a mathematical point of view, doubling the algebraic spinors is like doubling the (co)tangent space, that may be interpreted as a complexification of spacetime, along the lines of } [11, 13, 15]. \text{ It is not clear how to interpret geometrically and dynamically the section to a real spacetime.} \]
from an enlargement of the gauge group of gravity, as in [20], here we were led to its duplication in L and R sectors, and both gauge connections decouple because they ‘eat’ two independent sets of goldstone bosons in the vierbeins $\Gamma^{L,R}$.

We observe incidentally that $\bar{\Gamma}^{L,R}_\mu$ could in general lead to a phase with two different coexisting background metrics. In such a phase one could have observable violations of Lorentz invariance, but also healthy massive gravitons and phenomenologically interesting consequences [14].

We finally want to observe that due to the intrinsic geometric nature of algebraic spinors, it may also be that the symmetric phase described here is the effective geometric description of a completely different microscopic theory, where also fermions may be emergent. Therefore the present work may also be a step forward for a realistic effective description of matter, where the SM group can emerge naturally in Left-Right symmetric setup.

**Conclusions.** What we found is that adopting algebraic spinors, and using them in left-right symmetric fashion, we unified the fermions of a SM family. The framework is by construction endowed with a large non compact group, unifying the gravitational and gauge interactions. Then, the requirement to have a broken phase with a field theory in spacetime reduces the internal symmetries to be compact and also phenomenologically close to the SM, pointing to a Pati-Salam unification.

We identified the fields responsible for this breaking, as extended vierbeins whose VEV gives rise both to the metric and to the chiral weak groups, pointing toward a duplication of the connections, as two copies of self-dual extended gravities.

We discussed only partially the most fundamental level of the symmetric $gl(4, \mathbb{C})$ phase extending beyond the Planck scale, where the symmetries strongly restrict the representations and actions, and thus the dynamics leading to the scales of the breakings. This part of the analysis implies formulating the theory in first order, curved space and $gl(4, \mathbb{C})$-invariant way and will be addressed in a separate work.

Finally, a possible experimental signature of this unification has emerged, because the extended vierbeins also contain isospin-triplet tensor fields, and the left one may naturally have mass at the weak scale. If this is the case, it would strikingly show up at the LHC as weakly interacting spin-two particles.

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