On modeling runner included into a hydroacoustic system

V Yu Borodulin¹, A A Dekterev¹, P A Kuibin¹, A V Minakov¹, G A Semenov² and A V Zakharov²

¹ Kutateladze Institute of Thermophysics SB RAS, 1 Lavrentyev ave., Novosibirsk, 630090, Russia
² PJSC “Power Machines” LMZ, 18, Sverdlovskaya nab., St.-Petersburg, 195009, Russia

E-mail: kuibin@itp.nsc.ru

Abstract. Francis turbines operating at part load condition experience the development of non-stationary phenomena and there is a risk of resonance in the case when the disturbance frequencies are close to the eigen hydroacoustic frequencies of the waterway. Usual way for analysis of the hydroacoustic phenomena is an one-dimensional approach when the flow duct is divided into elementary parts, which, in turn, have being modeled by elements of an electrical circuit with equivalent resistances, capacitances, and inductances. Such approach was presented in details by Nicolet (2007) in his thesis. The most serious milestone in use of the electric circuit analogy lies in the problem, how the resistance, capacitance, and inductance of the turbine runner can be evaluated. The simplest approach is reduced to determination of resistance only through universal characteristic of the turbine. In the same time, one should understand that in the wave phenomenon both, the amplitude and phase will be changed during pass over the runner. In this paper, we propose an approach for modeling hydroacoustic phenomena, where the data on the passage of an acoustic signal through the system "spiral chamber – stator columns – guide vanes – runner" are taken from the CFD calculations. That is, during calculation of the flow in this system at some operation point we fix relations between the amplitude and phase of the pressure and discharge pulsations at the input into the system and its output and then use them as characteristics of element of hydroacoustic electric-like chain.

1. Introduction

At present, hydroelectric power stations are practically the only quickly operating and highly efficient regulatory body in the electric grid systems. First of all, it necessary to single out high head hydraulic power stations with a power of hundreds of megawatts, which usually are equipped by Francis turbines. Together with the advantages in regulating and the magnitude of unit capacity when using Francis turbines, there are serious problems with non-stationary phenomena in non-optimal operating modes. Along with hydrodynamic perturbations and oscillations caused by separated flow around the elements of the flow path, appearance of instabilities, possible formation of steam or air-vapor cavities or bubbles, there is a risk of resonance in the case when the disturbance frequencies are close to the eigen hydroacoustic frequencies of the waterway, in the electrical network, eigen mechanical vibrations of buildings and structures, etc.

To analyze hydroacoustic phenomena, a one-dimensional approach is widespread, where the flow duct is divided into elementary parts, which are modeled by elements of an electrical circuit with equivalent resistances, capacitances, and inductances. The most complete description of hydroacoustic
models can be found in the thesis by Nicolet [1] (see also monograph by Dörfler, Sick and Coutu [2]). C. Nicolet is also the main developer of the software package "SIMSEN", which allows calculating non-stationary processes in the electrical, hydraulic and mechanical parts of a hydroelectric power station. The disadvantages of the approaches mentioned include, first of all, in a very simplified method for taking into account the interaction of hydroacoustic perturbations directly with the turbine. Most often it is estimated with use of the universal characteristic of the turbine - by the slope of the "head-to-discharge" relationship at a fixed opening of the guide device. In this case, to a certain extent, the change in the amplitude of the acoustic disturbance is taken into account when it passes through the system "spiral chamber – stator columns – guide vanes – runner". However, any wave is characterized by amplitude and phase.

In this paper, we propose an approach for modeling hydroacoustic phenomena, where the data on the passage of an acoustic signal through this system are taken from CFD calculations. That is, an element (from the entrance to the spiral chamber to the inlet to the draft tube) is integrated into the one-dimensional system of elements of the flow path, modeled by a network of electric four-poles, for which the relationship between pressure and flow pulses at the inlet and outlet is obtained by calculation.

2. Hydroacoustic model

The lengths of pipe elements in hydro power stations (see schematic in Figure 1) is high enough and in view of low frequency disturbances and small diameter of the channels relative the wave length, it is reasonable to consider wave dynamics in the one-dimensional approach. The system of equations describing the hydroacoustic looks as follows [1]

\[ \frac{1}{\rho} \frac{\partial p}{\partial t} + \frac{\partial C}{\partial t} + g \sin \alpha + \frac{\lambda C[C]}{2D} = 0 \]

\[ \frac{\partial p}{\partial t} + C \frac{\partial p}{\partial x} + \rho a^2 \frac{\partial C}{\partial x} = 0 \]

Here \( p \) is the pressure, \( \rho \) is the water density, \( C \) is the flow velocity, \( g \) is the acceleration due to gravity, \( D \) is the channel diameter, \( \lambda \) is the local loss coefficient, \( a \) is the wave speed. The flow velocity \( C \) is considered as averaged over the channel cross-section, so discharge is \( Q = CA \), where \( A \) is the cross-section area.

Hydroacoustic phenomena are characterized by a high wave speed \( a \) (of order of \( 10^3 \) m/s) and low flow velocities \( C \) (of order of \( 10^1 \) m/s), thus the convective terms \( C \frac{\partial}{\partial x} \) can be neglected when compared with the terms \( \frac{\partial}{\partial t} \) responsible for the disturbances propagation. Thus, passing from \( p \) to piezometric pressure (head) \( h \) one can derive simplified system instead of (1)

\[ \frac{\partial h}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{\lambda Q[Q]}{2gDA^2} = 0 \]

\[ \frac{\partial h}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0 \]

The system of equations (2) is similar to equations describing unsteady phenomena in electric circuits. Thus, for some part of the hydraulic system analogs of resistance, capacitance, and inductance can be introduced

\[ R = \frac{\lambda [Q]}{2gDA^2}, \quad L = \frac{1}{gA}, \quad C = \frac{gA}{a^2} \]

Figure 1. System scheme.
The turbine is often characterized by inclination of the curve \( Q_1 - n_1 \) on the turbine hill chart at a fixed wicket gate opening \( \alpha \) (see for example [3])

\[
Z_t = \frac{2n}{D_0 n_1} \left[ Q_{11} - \left( \frac{\partial Q_{11}}{\partial n_{11}} \right)_{n_1} \right]
\]

Here \( D_0 \) is the turbine outlet diameter, \( Q_{11} = Qh^{1/2}D_0^{-3} \) and \( n_{11} = nDh^{-1/2} \) are the specific discharge and specific rotational speed respectively. Considering the Francis pump-turbine, Nicolet [1] introduced effective inductance of the part of system starting from the inlet to spiral case up to outlet from the draft tube

\[
L_t = \int_{x_{\text{in}}}^{x_{\text{out}}} \frac{dx}{gA(x)}
\]

Nonetheless in reality the process of interaction of the pressure wave with the turbine elements looks more complex. The wave enters a curvilinear spiral chamber, interacts with the cascade of blades of the stator columns and guide vanes, then interacts with blades of the rotating runner. So, the simplified characteristics like (4) and (5) don’t reflect complex acoustic processes in the turbine. Here we develop an approach based on CFD modelling to predict the turbine hydroacoustic characteristics.

3. Numerical model

During numerical modelling of the hydroacoustic phenomena there arise few problems. First, it is necessary to take into account the water compressibility. Also, at modelling we need in the equation of state \( P = f(\rho) \). There exist various kinds of the equation of state [4]. In this research we will use Tait equation

\[
1 + \frac{m}{K}(P - P_0) = \left( \frac{P}{P_0} \right)^m
\]

Here \( \rho \) is the liquid density at pressure \( P, P_0, \rho_0 \) are the pressure and density at normal conditions, \( m \) is the compressibility index. The modulus of elasticity \( K = a^2\rho \).

A progressive and economical enough method of Detached Eddy Simulation [5] together with \( k-\omega \) SST model by Menter for turbulence has been used for calculations.

Another problem relates to the rotor-stator interaction. Here we use approach of “frozen runner” when the problem is considered in the reference frame rotating together with the runner.

Two equations describe the hydroacoustic processes together with (6)

\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{u}) = 0
\]

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla (\rho \mathbf{u} \cdot \mathbf{u}) = -\nabla P + \nabla (\tau^m + \tau^t) + (\rho - \rho_0)\mathbf{g} - \rho(2\boldsymbol{\Omega} \times \mathbf{u} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}))
\]

where \( \mathbf{u} \) is the velocity vector, \( \tau \) is the viscous stress tensor, \( \boldsymbol{\Omega} \) is the angular velocity of runner rotation, \( \mathbf{g} \) is the vector of gravitational force. Components of the viscous stress tensor are defined as:

\[
\tau_{ij}^m = \mu \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right]
\]

where \( \mu \) is the dynamic (molecular) viscosity, \( \delta_{ij} \) is the Kronecker’s delta symbol.
Components of the Reynolds stress tensor $\tau'$ are determined with use of Boussinesq hypothesis on isotropic turbulent viscosity

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho k \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (10)$$

Here $\mu$ is the turbulent viscosity, $k$ is the kinetic energy of the turbulent pulsations.

DES method used in this research was based on $k-\omega$ SST model by Menter with equations

$$\frac{\partial \rho k}{\partial t} + \nabla (\rho k \mathbf{v}) = \nabla \left( \Gamma_k \nabla k \right) + G_k - Y_k,$$

$$\frac{\partial \rho \omega}{\partial t} + \nabla (\rho \omega \mathbf{v}) = \nabla (\Gamma_\omega \nabla \omega) + G_\omega - Y_\omega + D_\omega. \quad (11)$$

The dissipation term of the turbulent kinetic energy is modified for the DES turbulence model as described in [6]. In this model dissipative term in $k$-equation is modified by means of switcher $F_{DES}$

$$\frac{\partial \rho k}{\partial t} + \nabla (\rho k \mathbf{v}) = \nabla \left( \Gamma_k \nabla k \right) + G_k - Y_k - F_{DES},$$

$$F_{DES} = \max \left( \frac{L_t}{C_{DES} \Delta}, 1 \right), \quad L_t = \frac{k^{1/2}}{\beta \omega}, \quad C_{DES} = 0.61, \quad (12)$$

where $L_t$ is the turbulent length scale, $C_{DES}$ is empirical constant, and $\Delta$ is defined as the maximum of three sizes of the control volume, $\Delta_x, \Delta_y, \Delta_z$.

Earlier test calculations had shown that this technique can reliably consider large-scale turbulent fluctuations in water turbines and estimate their frequency [7–9].

Few words about the numerical approach. Discretization of transport equations was carried out by the control volume method on unstructured grids. Coupling of the velocity and pressure fields for incompressible flow was realized using the SIMPLE-like procedure. For the approximation of the convective terms in the equations for momentum components the Quick scheme was used. For the approximation of the convective terms in the equations for turbulent characteristics up-wind scheme was used. The nonstationary terms were approximated by an implicit second-order exact-order scheme. Diffusion terms were approximated by a second-order scheme.

4. Calculation of the pressure field inside the turbine

Calculation of the pressure field was fulfilled for the Francis turbine with specific speed $n_q = 50$. Computational grid consisted of about 5 million cells (see Figure 2). The grid at the runner blades and the guide vanes was fined. The grid detailing was sufficient both for describing the integral characteristics of the hydroturbine and for describing the main nonstationary phenomena in the flow path. The time step was $10^{-5}$ s, that was enough to catch the phenomena under consideration. Calculations of unsteady flow in Francis turbine were performed for a fixed head corresponding to the designed head and for three operation points with power 38 % of the optimum power, 84 % and 100%. To analyze the acoustic oscillations of the flow in computational aeroacoustics, it is customary to use the value of the time derivative of the density $\partial \rho / \partial t$. This quantity visualizes well the acoustic waves. Some examples of the acoustic field in spiral chamber are shown in Figure 3. The figure demonstrates the acoustic wave in successive time moments for the optimum power. In Figure 4 (the same operation point as in Figure 3) one can see generation of cylindrical like waves separating from the edges of the runner blades.

An illustration of unsteady behavior of the pressure at the penstock wall is shown in Figure 5 by red curve. The observation point is located middle of the penstock height. The blue curve corresponds to the calculation in incompressible problem statement. As seen during calculation with taking into account
the water compressibility the amplitude of pressure oscillations is about four times higher than in the perfect incompressible fluid. The oscillations spectrum differs significantly for these two calculations (Figure 6). One can see high frequency harmonics shown in red in comparison with the incompressible calculation (blue curve).

Figure 2. Elements of the computational grid in the turbine area (left) and inside the runner (right).

Figure 3. Evolution of the density time derivative on the turbine walls.
Figure 4. Isolines of the density time derivative on the runner walls.

Figure 5. The time dependence of the pressure at the penstock wall. Blue curve – incompressible model, red curve – compressible model.
Turning back to main goal of this research let’s consider in next Section the discharge and pressure oscillations at the spiral casing input and runner output.

5. Calculation of the turbine hydroacoustic characteristic
An example of calculated time dependencies of the discharge and pressure oscillations at the spiral casing input and draft tube input for the operation point corresponding to 84 % of the optimum power is presented in Figure 7. The level of disturbances is centred relative the averaged values and made dimensionless by dividing over these values. The main sources of disturbances are located in the turbine area and in the draft tube. In the penstock the level of oscillations is lower than in these two areas. Further we analyse spectra of the disturbances found during the simulation. The main attention should be paid to the phase shift between the signals at input to the system and its output. The phase shift between the pressure oscillation at the draft tube input and spiral chamber input is shown in Figure 8. The curve in graph looks as noisy. The reason is in inappropriate way for the phase determination. There exists the problem of $2\pi$ shift in the analyzing software. We smoothed the phase spectrum manually (see red line on the graph). As seen the process of the pressure wave passing through the system "spiral chamber – stator columns – guide vanes – runner" is highly nonlinear.

6. Conclusion
In this paper we proposed new approach for modelling the hydroturbine when it is considered as an element of the whole hydroacoustic system. To realize the approach, first, we perform pulsation flow calculations in the system "spiral chamber – stator columns – guide vanes – runner" system at the interesting operating point. Then we analyze the amplitude and phase spectra of pressure and discharge pulsations at the input into the system and its output. The information obtained is used to formulate a discrete one-dimensional model of the whole flow path.

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Figure 7. Time dependencies of the discharge and pressure oscillations at the spiral casing input and draft tube input.
Figure 8. Phase shift between the pressure oscillation at the draft tube input and spiral chamber input.

References

[1] Nicolet C 2007 Hydroacoustic modelling and numerical simulation of unsteady operation of hydroelectric systems, Phd Thesis, Lausanne, EPFL, 314
[2] Dörfler P, Sick M and Coutu A 2013 Flow-Induced Pulsation and Vibration in Hydroelectric Machinery. Engineers Guidebook for Planning, Design and Troubleshooting (London: Springer)
[3] Dörfler P 1980 Mathematical model of the pulsations in Francis turbines caused by the vortex core at part load Escher Wyss News 1/2 101–6
[4] Hayward A T J 1967 Compressibility equations for liquids: a comparative study British Journal of Applied Physics 18(7) 965–78
[5] Menter F R 1994 Two Equation Eddy Viscosity Turbulence Models for Engineering Applications AIAA J 32(8) 1598–605
[6] Menter F R, Kuntz M, Langtry R 2003 Ten years of experience with the SST turbulence model Turbulence, Heat and Mass Transfer vol 4, ed K Hanjalic, Y Nagano and M Tummers (New York, Wallingford: Begell House Inc) 625–32
[7] Gavrilov A, Dekterev A, Sentyabov A, Minakov A and Platonov D 2012 Application of hybrid methods to calculations of vortex precession in swirling flows Notes on Numerical Fluid Mechanics 117 449–59
[8] Kuznetsov I, Zakharov A, Orekhov G, Minakov A, Dekterev A and Platonov D 2012 Investigation of free discharge through the hydro units of high head Francis turbine IOP Conference Series: Earth and Environmental Science 15 052002
[9] Sentyabov A V, Gavrilov A A, Dekterev A A and Minakov A V 2013 Analysis of RANS turbulence models by calculating the steady-state flow in the Turbine-99 draft tube Computational Continuum Mechanics 6(1) 86–93