Beyond SM Physics and searches for SUSY at the LHC

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Abstract. This is the written version of a talk given by S.K. at the $10^{th}$ International Conference on High Energy and Astroparticle, Constantine, Algeria. We briefly review the Standard Model (SM) and the major evidences and main direction of physics beyond the SM (BSM). We introduce supersymmetry, as one of the well-motivated BSM. Basic introduction to Minimal Supersymmetric Standard Model (MSSM) is given. We analyze the thermal relic abundance of lightest neutralino, which is the Lightest Supersymmetric Particle (LSP) in the MSSM. We show that the combined Large Hadron Collider (LHC) and relic abundance constraints rule out most of the MSSM parameter space except a very narrow region. We also review non-minimal SUSY model, based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ (BLSSM), where an Inverse Seesaw mechanism of light neutrino mass generation is naturally implemented. The phenomenological implications of this type of model at the Large Hadron Collider (LHC) are analyzed.

1. Introduction

The standard model (SM) of particle physics is proved to be in an excellent agreement with most of the confirmed experimental results. For instances, the success of the SM includes, the discovery of the vector bosons $W$ and $Z$, with masses and decay properties coincide accurately with the SM expectations. While we have now an impressive list of experimental confirmations of the success of the SM, compelling arguments indicate that the SM cannot be the complete theory of Nature. Among the theoretical problems that face the SM and strongly suggest New Physics (NP) beyond the SM are the following. The SM does not include gravity, therefore it cannot be valid at energy scales above $M_{Pl} \sim 10^{19}$ GeV. Also, the SM does not allow for neutrino masses, therefore it cannot be even valid at energy scales above $M_{seesaw} \gtrsim 1$ TeV. Moreover, the SM fails to address other issues such as the naturalness problems of the Higgs sector in the SM, the strength of the charge conjugation-parity (CP) violation in the SM, which is not sufficient to account for the cosmological baryon asymmetry of the universe, and the absence of viable Dark Matter (DM) candidate in the SM. Therefore, it is common a tempt to conclude that the
SM is only an effective low energy limit of a more fundamental underlying theory. However, it
is mandatory for any new fundamental theory to exactly reproduce the SM at the Fermi scale.

The supersymmetric theories are considered as the most promising candidate for the unified
theory beyond the SM. Despite the absence of experimental verification, relevant theoretical
arguments can be given in favor of supersymmetry (SUSY). First of all, supersymmetry ensures
the stability of hierarchy between the weak and the Planck scales. If we believe that SM should
be embedded within a more fundamental theory including gravity with a characteristic scale of
order the Planck mass \( M_P \), then we are faced with the hierarchy problem. There is no symmetry
protecting the masses of the scalar particles against quadratic divergences in the perturbation
theory. This problem of stabilizing the scalar masses is solved in SUSY models since now the
scalar mass and the mass of its superpartner are related. SUSY is also a necessary ingredient
in string theories to avoid tachyons in the spectrum. Further, the evolution of the three gauge
coupling constant of the SM into a single unification scale, only after taking into account SUSY
as we will show below, is considered as a hint that SUSY might be true.

In supersymmetric theories, each particle must have a superpartner (a boson for a fermion
and vice versa). Hence, many new particles must be included into the theory. It is worthwhile
to mention that some physicists find these plethora of new particles is a defect of the supersymmetric
theory. However, we would like to remind that there are several reasons to accept the idea of
existing new particles (even the non-baryonic particles). For instance, in astronomy there is
overwhelming evidence that most of the mass in the universe is some non-luminous (and non-
baryonic) DM of as yet unknown composition. The lightest SUSY particle (which is absolutely
stable) is a natural candidate for solving the DM problem. Moreover, we don’t believe that SM
spectrum (the quarks, leptons, Higgs and gauge bosons) is the complete list of the elementary
particles in nature. We believe that increasing the energy in the accelerators will give us the
chance to discover new particles. Few decades ago we were only familiar with less than one-third
of the SM particles. Hence, the existence of new particles with masses larger than the Fermi
scale could be a prediction, may be verified experimentally over the next years.

It is the aim of this paper to review the Supersymmetric extensions of the SM. The plan of
the paper is as follows. In the next section we give a brief introduction to the SM. In Sect. 3 we
discuss the drawbacks of the SM. Potential directions for physics beyond the SM are described
in Sect. 4. In Sect. 5 Supersymmetry is introduced as one of the best candidates for physics
beyond SM. Supersymmetric DM is analyzed in Sect. 6. In Sect. 7 we review non-minimal
supersymmetric model that accounts for neutrino masses. In Sect. 8 we show that in this class
of models, the lightest right-handed sneutrino is an interesting candidate for scalar DM. We
conclude in Sect. 9.

2. The Standard Model

Most of the available experimental data, up to date, can be explained well in the framework of
the standard model (SM)[1, 2, 3]. Not only this, but the SM successfully predicted the existence
of the third generation of fermions, scalar boson (Higgs) and massive gauge bosons namely,
\( W^\pm, Z^0 \) long time even before being discovered at colliders. The SM is based on a gauge theory
describing three fundamental forces in our universe. These are, the electromagnetic force, the
weak nuclear force, and the strong nuclear force. Unfortunately, gravity cannot be included
in this theory. The theory is based on the gauge symmetry group \( SU(3)_C \times SU(2)_L \times U(1)_Y \),
where \( C \) represents the color, \( L \) denotes left-handed chirality and \( Y \) stands for hypercharge.
Gauge bosons are associated with each gauge symmetry group. These bosons are listed as:

\[
SU(3)_C \rightarrow 8 \ G_\mu^a (a = 1, \ldots, 8), \\
SU(2)_L \rightarrow 3 \ W_\mu^a (a = 1, 2, 3),
\]
The fermionic sector of the SM includes quarks and leptons ordered in three families of left-handed doublets and right-handed singlets. In flavor space they can be written as:

\[ L_1 = \left( \begin{array}{c} \nu_e \\ e^- \end{array} \right)_L, \quad e_{R1} = e^-_R, \quad Q_1 = \left( \begin{array}{c} u \\ d \end{array} \right)_L, \quad u_{R1} = u_R, \quad d_{R1} = d_R, \]

\[ L_2 = \left( \begin{array}{c} \nu_\mu \\ \mu^- \end{array} \right)_L, \quad e_{R2} = \mu^-_R, \quad Q_2 = \left( \begin{array}{c} c \\ s \end{array} \right)_L, \quad u_{R2} = c_R, \quad d_{R2} = s_R, \]

\[ L_3 = \left( \begin{array}{c} \nu_\tau \\ \tau^- \end{array} \right)_L, \quad e_{R3} = \tau^-_R, \quad Q_3 = \left( \begin{array}{c} t \\ b \end{array} \right)_L, \quad u_{R3} = t_R, \quad d_{R3} = b_R. \]

The SM Lagrangian has the form

\[ \mathcal{L}_{\text{SM}} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4} B_{\mu
u} B^{\mu
u} + \bar{L}_i i D_\mu \gamma^{\mu} L_i \]

\[ + \bar{e}_{Ri} i D_\mu \gamma^{\mu} e_{Ri} + \bar{Q}_i i D_\mu \gamma^{\mu} Q_i + \bar{u}_{Ri} i D_\mu \gamma^{\mu} u_{Ri} + \bar{d}_{Ri} i D_\mu \gamma^{\mu} d_{Ri}, \]

where the covariant derivatives \( D_\mu \) are defined as follows:

\[ D_\mu Q_i = \left( \partial_\mu - ig_a T_a G^{\mu}_a - ig_2 \frac{\tau_a}{2} W^{\mu}_a - ig_1 \frac{Y}{2} B_\mu \right) Q_i, \]

\[ D_\mu L_i = \left( \partial_\mu - ig_2 \frac{\tau_a}{2} W^{\mu}_a - ig_1 \frac{Y}{2} B_\mu \right) L_i, \quad D_\mu f_R = \left( \partial_\mu - ig_1 \frac{Y}{2} B_\mu \right) f_R, \]

with \( T_a, \tau_a, Y, \) and \( g \)'s represent the generators and the couplings of the corresponding gauge symmetry groups. The hypercharge quantum number \( Y \) is related to the electric charge, \( Q \), and the third component of the isospin, \( I_3 \), via the relation \( Q = I_3 + \frac{Y}{2} \).

The scalar sector of the SM consists of only one complex \( SU(2)_L \) doublet \( \Phi \), known as the Higgs field, with hypercharge \( Y = 1 \)

\[ \Phi = \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right), \]

and can be described by the Lagrangian

\[ \mathcal{L}_{\text{scalar}} = (D_\mu \Phi)\dagger (D^\mu \Phi) - V(\Phi), \]

here \( V(\Phi) \) is the Higgs potential and can be defined as

\[ V(\Phi) = -\mu^2 \Phi \dagger \Phi + \lambda (\Phi \dagger \Phi)^2. \]

The parameters \( \lambda \) and \( \mu^2 \) must be positive to ensure that the potential is bounded from below. In this case the potential has a minimum

\[ \langle \Phi \rangle_0 = \langle 0 | \Phi | 0 \rangle = \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right), \]

where \( v/\sqrt{2} \) is the vacuum expectation value (VEV), with \( v = \sqrt{\mu^2/\lambda} \). Only the neutral component of the Higgs doublet can get a VEV so that \( SU(2)_L \) symmetry is broken while other symmetries remain unbroken i.e. \( SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q \). As a consequence, the eight
gluons and the photon remain massless while the gauge bosons, $W^\pm$ and $Z^0$, associated with the $SU(2)_L$, symmetry acquire masses \(4, 5, 6\) given as

\[
M^2_W = \frac{1}{4}g_2^2 v^2, \quad M^2_Z = \frac{1}{4}(g_1^2 + g_2^2) v^2,
\]

The fermion masses can be also generated via the Yukawa interaction of the left and right-handed fermion fields to the Higgs field. The corresponding Lagrangian is given as

\[
L_Y = Y^{eij} \bar{L}_i \Phi e_R^j + Y^{u_{ij}} \bar{Q}_i \Phi u_{Rj} + Y^{d_{ij}} \bar{Q}_i \Phi d_{Rj} + h.c,
\]

where $\tilde{\Phi} = i\tau_2 \Phi^\ast$, $Y_{\tilde{\Phi}} = -1$, and $Y^{r_{ij}}$ are the Yukawa couplings. After electroweak symmetry breaking, the Higgs field $\Phi$ can be expressed as,

\[
\Phi = \begin{pmatrix} 0 \\ (v + h)/\sqrt{2} \end{pmatrix},
\]

after substitution in Eq.(12), one gets the following fermion mass matrices

\[
M^{r_{ij}} = \frac{v}{\sqrt{2}} Y^{r_{ij}}, \quad r = e, u, d.
\]

To obtain the fermion masses, we need to diagonalize the mass matrices via unitary fields transformations, this simplifies to:

\[
m_e = \frac{v}{\sqrt{2}} Y_e, \quad m_u = \frac{v}{\sqrt{2}} Y_u, \quad m_d = \frac{v}{\sqrt{2}} Y_d.
\]

Clearly, only quarks and charged leptons electrons acquire masses while neutrinos remain massless since they do not have right-handed components. As can be seen From Eqs.(12 and 13), the neutral higgs boson, $h$, interactions with quarks and charged leptons can lead to phenomenological predictions that can be tested in colliders. These are also predictions of the SM and thus can be used also as probes for physics beyond the SM.

### 3. Evidence for Physics Beyond the SM

After discovering the only scalar boson of the SM in 2012, at the CERN Large Hadron Collider (LHC) by CMS [7] and ATLAS [8], the SM has been confirmed as being extremely successful in describing the most aspects of the nature with remarkable precision at the 100 GeV
In the SM, quarks and electrons acquire masses through Yukawa couplings as can be seen in Eq.(15). Neutrinos remain massless because there are no RH $\nu$ in the SM. However, it has proven experimentally that $m_\nu \neq 0$ [9]. Neutrino masses can be generated if lepton number is violated by dimension 5 operator [10]. In the literature several mechanisms were proposed to generate neutrino masses. For instances, seesaw mechanism, shown in Fig.1, with three different types[12, 13, 14, 15].

In 1933, Zwicky noticed that the mass of luminous matter in the Coma cluster is much smaller than its total mass [16]. On the other hand, the observation of 1000 spiral galaxies showed that away from the center of galaxies the rotation velocities do not drop off with distance as shown in Fig.2 [17, 18]. This observation is in contradiction with what we expect as the velocity of rotating objects is given by $v(r) = \sqrt{\frac{G M(r)}{r}}$. Dark matter (DM) was proposed as a possible explanation for this observation where the disk galaxies are assumed to be immersed in extended DM halos. However, in the SM, there is no candidate to play the role of the cold dark matter of the universe.

Other issues also include Higgs vacuum stability where quadric coupling evolves to zero or negative values as one can see from Fig.(3) [19]. This turns to be a problem as in the SM $M_H = \sqrt{\lambda v}$. Higgs Mass Hierarchy is also one of the problems in the SM due to the absence of a symmetry to protect Higgs mass that receive contributions from the loop dominated by top quark contributions. The contributions are proportional to the square of $\Lambda$, the cutoff scale of the theory, that can be set to Planck scale ($10^{19}$ GeV).

It is believed that the early universe began as a huge burst of energy known as the big bang. In the short moment after that bang, matter and anti-matter existed in that early phase of
universe in equal amounts (CP symmetry). However, with time evolving and cooling of the 
universe, our present universe is dominated by only matter not their anti-matter. This matter 
asymmetry of the universe is known as Baryon Asymmetry (Matter- Antimatter Asymmetry), 
cannot be explained in the context of the SM since the amount of CP violation in the SM 
is not enough for such explanation. Neither the standard model of particle physics, nor the 
theory of general relativity provides an obvious explanation. In 1967, A. Sakharov showed that 
the generation of the net baryon number in the universe requires: Baryon number violation, 
thermal non-equilibrium and C and CP violation [20]. All of these ingredients were present in 
the early Universe! Do we understand the cause of CP violation in particle interactions? Can 
we calculate the BAU from first principles?

\[
\frac{(n_B - n_{\bar{B}})}{n_\gamma} = 6.1 \times 10^{-10}.
\]

With all issues discussed above, there are also a number of questions we hope will be answered 
in beyond SM physics:

i) Electroweak symmetry breaking, which is not explained within the SM.

ii) Why is the symmetry group is \(SU(3) \times SU(2) \times U(1)\)?

iii) Can forces be unified?

iv) Why are there three families of quarks and leptons?

v) Why do the quarks and leptons have the masses they do?

vi) Can we have a quantum theory of gravity?

vii) Why is the cosmological constant much smaller than simple estimates would suggest?

Based on the above discussion we can conclude that, Standard Model is defined by 4- 
dimension QFT (Invariant under Poincare group). The symmetry governs the SM is local 
\(SU(3)_C \times SU(2)_L \times U(1)_Y\) and the particle content (Point particles): 3 fermion (quark and 
Lepton) Generations with no Right-handed neutrinos resulting in massless neutrinos. In the 
SM, symmetry breaking can be achieved via one Higgs doublet. Moreover, in the SM, no 
candidate for Dark Matter and formalism of the SM does not allow to include gravity.

4. Directions Beyond the Standard Model

The failure of the SM to address the problems discussed above motivates going beyond SM. 
Directions Beyond the Standard Model can be achieved upon extending some sectors or
symmetries in the SM in several ways. These directions can be summaries as follows:

- Extension of gauge symmetry
- Extension of Higgs Sector
- Extension of matter content
- Extension with flavor symmetry
- Extension of Space-time dimensions (Extra-dimensions)
- Extension of Lorentz Symmetry (Supersymmetry)
- Incorporate Gravity (Supergravity)
- Adopt the concept of one dimension object, instead of zero-point particle. (Superstring).

One of the most popular extension of the SM is Supersymmetry (SUSY) which is based on a symmetry linking fermions and bosons and thus enlarge the usual space-time to include fermionic components. Beyond the SM physics include also string theory and extra dimensions where the dimensionality of the space-time increase to include extra dimensions which result in consequence that can not be seen else where.

5. Supersymmetry

Historically, Supersymmetry was introduced in 1973 as a part of an extension of the special relativity. Supersymmetric theories are promising candidates for unified theory beyond the SM. Moreover, in SUSY, the mechanism of the electroweak symmetry breaking is natural. Supersymmetry is an extension of the space time symmetry that relates bosons and fermions. As a consequence, contributions from the new scalar bosons can ensure the stability of hierarchy between the weak and the Planck scales through avoiding the fine tuning in the renormalization of the Higgs boson mass at the level of $O(10^{34})$.

Hierarchy problem is one of the naturalness problems of the Higgs sector in the SM. In the SM, the Higgs mass receives contribution from one-loop radiative corrections. This contribution is proportional to the square of the momentum running in the loop and can be set to the cut-off scale or larger than that. With the cut-off scale of the order of the GUT scale, $M_G \approx 10^{16}$ GeV, Higgs mass is not protected to be of $O(100 \text{ GeV})$ to break the electroweak symmetry. In SUSY, the loop diagrams, shown in Fig.4, that are quadratically divergent cancel, term by term against the equivalent diagrams involving superpartners:

\[
m_h^2 = m_{h, \text{tree}}^2 + c \frac{g^2}{4\pi^2} M_{\text{pl}}^2, \quad \text{without SUSY}
\]

\[
m_h^2 = m_{h, \text{tree}}^2 \left(1 + c' \frac{g^2}{4\pi^2} \ln \left( \frac{M_{\text{pl}}}{M_W} \right) \right), \quad \text{with SUSY}
\]
If \( m_h \sim O(100) \text{ GeV} \), the masses of superpartners should be \( \lesssim O(1) \text{ TeV} \). Thus, some of the superpartners will be detected at the LHC.

Additional support for low scale (\( \sim 1 \text{ TeV} \)) SUSY follows from gauge coupling unification. Within SM, the gauge coupling constants describing the strengths of the electroweak force, the weak and strong nuclear forces do not unify if they run to high energies using the renormalization group equations of these coupling constants while within SUSY they do as can be seen in Fig. 5.

As known, the formalism of SM does not allow proper incorporation of the gravitational interactions in its gauge group symmetry. On the other hand, the Poincaré group corresponds to the basic symmetries of the special relativity. It turns out that, in order to unify gravity with the gauge interactions, we need to combine Poincaré and internal symmetries [21]. It should be noted also that, according to the Coleman-Mandula theorem, the most general symmetry which quantum field theory can have is a tensor product of the Poincaré group and an internal group [22]. In this context, SUSY is an extension of the spacetime symmetry reflected in the Poincaré group. Thus, upon space-time \( (x^\mu) \rightarrow \) Superspace \( (x^\mu, \theta^\alpha) \), SUSY is a translation in Superspace given as:

\[
\begin{align*}
  x^\mu & \rightarrow x'^\mu = x^\mu + \frac{i}{2} \bar{\epsilon} \gamma^\mu \theta \\
  \theta & \rightarrow \theta' = \theta + \epsilon
\end{align*}
\]

Linking bosonic degrees of freedom to fermionic ones can be generated in SUSY by an operator \( Q \) that carries spin-1/2 acting as

\[
Q |\text{Boson}\rangle = |\text{Fermion}\rangle, \quad Q |\text{Fermion}\rangle = |\text{Boson}\rangle. \quad (16)
\]

The simplest and most useful supersymmetry algebra in four dimension Minkowski space is is obtained by adding to Poincaré algebra a Majorana spinor charge \( Q_{\alpha}, \hat{\alpha} = 1, 2 \), satisfying the property

\[
\{Q_{\alpha}, Q_{\hat{\alpha}}\} = 2\sigma^\mu_{\alpha\hat{\alpha}} P_\mu, \quad (17)
\]

where \( P_\mu \) are the generators of translation and \( \sigma^\mu \) are the Pauli and unit matrices. Superfield \( \Phi(x, \theta, \bar{\theta}) \), as function of the Superspace coordinates, can be defined as

\[
\begin{align*}
\Phi(x, \theta, \bar{\theta}) &= \phi(x) + \theta \psi(x) + \bar{\theta} \bar{\chi} + \theta \theta \ m(x) + \bar{\theta} \theta \ n(x) + \theta \sigma^\mu \bar{\theta} \ v_\mu(x) \\
&\quad + \bar{\theta} \theta \theta \ \lambda(x) + \bar{\theta} \theta \theta \ \eta(x) + \theta \theta \bar{\theta} \ d(x),
\end{align*}
\]
Chiral Superfields, corresponding to a Weyl fermion and a complex scalar, must satisfy the condition $\bar{D}\Phi = 0$ where $D$ is a derivative operator in the Superspace. As a consequence, chiral Superfield can be expressed as

$$\Phi(x, \theta) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta F(x)$$

(18)

where $\phi(x), \psi(x)$ and $F(x)$ denote a complex scalar, a Weyl fermion and auxiliary fields respectively. The infinitesimal SUSY transformation of chiral superfield yields $\Phi \rightarrow \Phi + \delta \Phi$ with

$$\delta \Phi = i(\xi Q + \bar{\xi} \bar{Q}) \Phi$$

and implies

$$\delta \xi \phi = \sqrt{2}\xi \psi,$$

$$\delta \xi \psi = \sqrt{2}\xi F - \sqrt{2}i\sigma^n \bar{\xi}D_\mu \phi,$$

$$\delta \xi F = \sqrt{2}i\psi \sigma^n \bar{\xi}D_\mu.$$  

where $\delta F$ is a total derivative. Thus, if a Lagrangian is made out of the highest component of a superfield, it is SUSY invariant. In SUSY, vector Superfields corresponds to a gauge boson (massless vector) and a Weyl fermion and defined via the requirement

$$V = V^+,$$

(19)

leading to

$$V(x, \theta, \bar{\theta}) = -\theta \sigma^n \bar{\theta}v^\mu + i\bar{\theta}^2 \bar{\theta}^\alpha \lambda^\alpha - i\theta^2 \theta^\alpha \lambda^\alpha + \frac{1}{2} \theta^2 \bar{\theta}^2 D(x)$$

(20)

here $D(x)$ is a non-propagating auxiliary field, it transforms under a SUSY transformation into a total derivative.

In terms of the superfields components the most general renormalisable non-Abelian gauge invariant Lagrangian is given as:

$$L = \sum_i (|D\phi_i|^2 + i\psi_i \sigma^\alpha D_\mu \psi_i^* + |F_i|^2) - \sum_a \frac{1}{4g_a^2} \left[ (F_{\mu\nu}^a)^2 - i\lambda^a \bar{\phi} \lambda^a - \frac{1}{2} (D^a)^2 \right]$$

$$+ \frac{1}{2} \sum_{ia} g^a \psi_i T^a \lambda^\alpha \phi_i^* + h.c. + \sum_{ij} \frac{1}{2} \partial^2 W \bar{\psi}_i \psi_j,$$

(21)

with $D_\mu, F_{\mu\nu}, \lambda, \phi$ and $\psi$ represent the gauge covariant derivative, the field strengths, the gaugino fields, the scalar and fermionic fields, respectively. $T^a$ and $g^a$ are being the generators
and coupling constants of the corresponding groups. Eliminating the auxiliary fields \( F^i \) and \( D^a \), through equations of motion, gives rise to the scalar potential of the form

\[
V_{\text{SUSY}} = \frac{1}{2} |D^a|^2 + |F^i|^2,
\]

with \( F^i = \partial W / \partial \phi_i \) and \( D^a = g^a \sum_i \phi_i^* T^a \phi_i \).

Up to date, we did not observe squarks and selectrons. If Supersymmetry is an exact symmetry then all particles in the same supersymmetric multiplet would have the same mass. This indicates that SUSY must be broken symmetry or else SUSY particles should have been observed with same mass as SM-partners. On the other hand, the cancellation of quadratic divergences requires SUSY partners not to be heavier than \( \sim \) TeV. Several ways have been discussed in the literature to break SUSY. From the definition of the SUSY algebra:

\[
H = \frac{1}{4} (\bar{Q}_1 Q_1 + \bar{Q}_1 Q_1 + \bar{Q}_2 Q_2 + \bar{Q}_2 Q_2) \geq 0,
\]

If the vacuum is supersymmetric the \( E_{\text{vac}} = \langle 0 | H | 0 \rangle = 0 \) and if SUSY is broken then \( E_{\text{vac}} > 0 \). Hence SUSY is broken if \( \langle 0 | F_1 | 0 \rangle \neq 0 \) or \( \langle 0 | D | 0 \rangle \neq 0 \). One may introduce terms in the Lagrangian which break SUSY softly i.e. these terms do not lead to quadratic divergences. The general structure for the SUSY breaking includes three sectors:

i) Observable sector: which comprises all the ordinary particle and their SUSY particles,
ii) Hidden sector: where the breaking of SUSY occurs,
iii) The messengers of the SUSY breaking from hidden to observable sector.

The soft SUSY breaking terms are: masses for the scalars, masses for the gauginos, cubic couplings for scalars.

The Minimal Supersymmetric Standard Model (MSSM) is a straightforward supersymmetrization of the SM with minimal number of new parameters. The particle content of the MSSM, see Table 1, consists of two Higgs doublet SM, scalar SUSY partners and fermionic SUSY partners. The superpotential of the MSSM is given by

\[
W_{\text{MSSM}} = Y^u_{ij} Q_L^i U^C_{Lj} H_u + Y^d_{ij} Q_L^i D^C_{Lj} H_d + Y^e_{ij} L_L^i E^C_{Lj} H_d + \mu H_d H_u,
\]

the indices \( i \) and \( j \) refer to quark and lepton families. The parameters \( Y^u_{ij}, Y^d_{ij} \) and \( Y^e_{ij} \) correspond to the Yukawa couplings present in the SM, which are non-diagonal \( 3 \times 3 \) matrices in flavor space. The \( \mu \) parameter has mass dimension. It should be noted that, a new symmetry, R-symmetry has been introduced to forbid \( B - L \) violating interactions in the superpotential, i.e,
no proton decay). In the MSSM, universal soft SUSY breaking terms includes Universal scalar mass $m_0$, Universal gauging mass $M_1/2$ and Universal trilinear coupling $A_0$. These terms induce about 100 free parameters which reduce the predictivity of the MSSM. However, at a specific high scale models, these parameters can be reduced through the relations among them as in the constrained MSSM (mSUGRA).

SUSY can provide a natural mechanism for understanding Higgs physics and electroweak symmetry breaking (EWSB). In the MSSM, $H_u$ couples to a $t$ ($s$)quark with a large Yukawa coupling, unlike $H_d$, which couples to a $b$ ($s$)quark and a $\tau$ ($s$)lepton. The Yukawa coupling gives a negative contribution to the squared masses $m^2_{H_u,d}$. The running from $M_X$ down to the EW scale, shown in Fig. 6, reduces the squared Higgs masses until, eventually, conditions satisfied and the gauge symmetry is broken. This is an appealing feature in SUSY models that generally explains the mechanism of the EWSB dynamically.

The Higgs sector of the MSSM consists of two complex Higgs doublets, $H_u$ and $H_d$. After EWSB, three of the eight degrees of freedom are eaten by $W^\pm$ and $Z$. The five physical degrees of freedom that remain form a neutral pseudoscalar (or CP-odd) Higgs boson $A$, two neutral scalars (or CP-even) $h$, and $H$ and a charged Higgs boson pair (with mixed CP quantum numbers) $H^\pm$. The mass of the lightest CP-even (SM-like) Higgs, at the one-loop level, is given by

$$m_h^2 \leq M_Z^2 + \frac{3g^2}{16\pi^2 M_W^2} \frac{m_t^4}{\sin^2\beta} \log \left( \frac{m_t^2}{m_{\tilde{t}_1}^2} \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right).$$

MSSM predicts an upper bound for the Higgs mass: $m_h \lesssim 130$ GeV, which was consistent with the measured value of Higgs mass (of order 125 GeV) at the LHC. This mass of lightest Higgs boson implies that the SUSY particles are quite heavy. This may justify the negative searches for SUSY at the LHC-run I. In the MSSM, the gluino mass $m_{\tilde{g}} \simeq 2.5 m_{1/2}$. Thus, due to the LHC constraint shown in Fig. 6, we conclude that $m_{\tilde{g}} \geq 1.5$ TeV for a value $m_{1/2} \geq 600$ GeV.

6. SUSY Dark Matter

One of the longstanding problems that maybe considered as a hint to the necessity of existence of physics beyond SM is dark matter in the Universe\cite{16}. The presence of a such matter, which is different from the familiar baryonic matter, is supported by astrophysical observations and cosmological considerations. The relic abundance of dark matter is given by \cite{23}

$$\Omega_{DM}h^2 = 0.1188 \pm 0.0010,$$

here $h$ denotes the reduced Hubble constant and accounts for nearly 25% of the total energy of the Universe. So far, it is believed that DM can be accounted for by a particle that is either stable,
at least on cosmological scales, or has a lifetime much larger than that of the universe. This requirement can be achieved by an appropriate symmetry imposed on the model. Candidates of such particle must have attractive gravitational interactions and their other interactions with the SM states should be very suppressed which can be fulfilled by electrically and color neutral particles. This can be understood as up to date there is no evidence that DM has any other interaction except gravity. Further more, DM candidates have to be non-relativistic, i.e. cold, at the time of matter-radiation equality in the Universe. The possibility of hot dark matter is ruled out by several observations such as gravitational growth of small-scale structure, formation of stars, galaxies, and clusters of galaxies so early and the weak lensing signals we see and the pattern of fluctuations in the cosmic microwave background. In the SM, neutrinos only can be a candidate for DM. However, neutrinos are too light to account for the dark matter that must be present in our Universe. So, we need to look for candidates of DM in beyond SM physics. In the following we explore these possibilities in Supersymmetry as an extension of the SM.

In the SM, the mixing of the gauge bosons $W^i$ and $B^0$, after electroweak symmetry breaking, leads to the physical (mass) states $\gamma, Z^0$ and $W^\pm$. Similarly, in the MSSM, the mixing among SUSY partners of the SM fields results in the neutralinos as new fermionic mass state. Particularly, the neutral gauginos ($\tilde{B}, \tilde{W}_0$) and the neutral higgsinos ($\tilde{H}_0^u, \tilde{H}_0^d$) mixing form four neutral mass-eigenstates called neutralinos. If we define a gauge-eigenstate basis

$$\psi^0 = \begin{pmatrix} \tilde{B} \\ \tilde{W}_0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix},$$

one can write

$$-\frac{1}{2} \psi^{0T} m_{\tilde{\chi}^0} \psi^0 + \text{h.c.}$$

with

$$m_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2} g_1 v_d & \frac{1}{2} g_1 v_u \\ 0 & M_2 & \frac{1}{2} g_2 v_d & -\frac{1}{2} g_2 v_u \\ -\frac{1}{2} g_1 v_d & \frac{1}{2} g_2 v_d & 0 & -\mu \\ \frac{1}{2} g_1 v_u & -\frac{1}{2} g_2 v_u & -\mu & 0 \end{pmatrix},$$

where $v_d = \langle H_d \rangle$ and $v_u = \langle H_u \rangle$. The parameters $M_1, M_2$ and $\mu$ can have arbitrary phases. However, one can redefine the phases of $B$ and $W^0$ to make both $M_1$ and $M_2$ real and positive. Usually, $\mu$ is taken to be real to avoid unacceptably large CP-violating effects including EDM for both the electron and the neutron. Hence, the neutralino mass matrix $m_{\tilde{\chi}^0}$ is real and symmetric and therefore, it can be diagonalized analytically [24] by a single $4 \times 4$ real matrix $N$ such that

$$N^* m_{\tilde{\chi}^0} N^{-1} = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0})$$

The resulting four neutral mass eigenstates are called neutralinos, with the convention that the masses are ordered as $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0}$. The physical Majorana neutralino (mass eigenstates) can be written as [25]

$$\chi_M^0 = N \begin{pmatrix} \tilde{B} \\ \tilde{W}_0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix} + N^* C \begin{pmatrix} \tilde{\chi}_1^0 \\ \tilde{\chi}_2^0 \\ \tilde{\chi}_3^0 \\ \tilde{\chi}_4^0 \end{pmatrix}$$
In the limit $|\mu| \to \infty$, $\tilde{\chi}^0_0$ corresponds to a pure bino with mass $m_{\tilde{\chi}^0_0} \simeq M_1$, $\tilde{\chi}^0_2$ corresponds to pure wino with mass $m_{\tilde{\chi}^0_2} \simeq M_2$ while $\tilde{\chi}^0_3$ and $\tilde{\chi}^0_4$ are pure higgsinos with masses $m_{\tilde{\chi}^0_3} \simeq m_{\tilde{\chi}^0_4} \simeq |\mu|$. In mSUGRA, we have \[ M_1 \approx \frac{5}{3} \tan^2 \theta_W M_2 \approx 0.5 M_2 \] (30)
which indicates that \[ M_1 < M_2 \ll |\mu| \] (31)
and therefore, $\tilde{\chi}^0_1$ is the lightest neutralino and usually it is the lightest SUSY particle. Since, it is also electrically neutral and has no color charge, it is an attractive candidate for non-baryonic dark matter \[27\] if being stable. In fact $R$-parity symmetry can ensure that $\tilde{\chi}^0_1$ is stable. The conservation of the R-parity symmetry implies that SUSY particles can only be produced (destroyed) in pairs form (into) SM particles. Moreover, heavy unstable SUSY particle will decay in a chain until the lightest SUSY particle (‘LSP’), $\tilde{\chi}^0_1$, is produced. Thus, the stability of $\tilde{\chi}^0_1$ is guaranteed by the $R$-parity symmetry. As a result, within SUSY models conserving $R$-parity symmetry, the LSP is a good candidate of DM.

We turn now to discuss the calculations of the cosmological neutralino relic abundance. Generally, all SUSU scalar particles can contribute to $\Omega_{\tilde{\chi}^0_1} h^2$ as they decay until finally LSPs are produced, and all the (co)annihilation processes must be considered. However, the most important contributions to the neutralino relic density come from the LSP. In this case, the most important final states into which the neutralino can annihilate include the two-body final states which occur at tree level. In particular, these states are, SM fermion-antifermion pairs (see Fig.7), SM massive gauge bosons $W^+ W^-$, $Z^0 Z^0$ and $Z^0 h$. Other states includes one SM massive gauge boson in addition to one SUSY Higgs boson, pair of charged SUSY Higgs and a combination of SM Higgs with one of the neutral SUSY Higgs bosons. For detailed calculations and discussion, we refer to Ref.[28].

The MSSM parameter space consists of 100 free parameters. However, they are highly constrained by flavor and CP-violating observables. In fact, with the discovery of the Higgs boson at LHC, the lack of positive signals from direct searches at the LHC, and null results from direct detection experiments, the MSSM turns to be almost ruled out.

The cross section for elastic scattering of a neutralinos off ordinary mater can determine the detection rate in the direct-detection experiments. Basically, this scattering depends on the strength of the neutralino quark interaction, the distribution of quarks inside the nucleon and the distribution of nucleons inside the nucleus [1]. The measured nuclear recoil energy resulting from this elastic scattering, in direct-detection experiments, can serve as a direct search of neutralinos as DM particles. At tree-level, the elastic scattering of the neutralino ($\tilde{\chi}^0_1$) off nucleus, mediated by Squarks ($\tilde{q}$), Higgsse ($H$) and $Z$ exchange. The Spin-independent scattering cross section of the LSP with a proton versus the mass of the LSP within the region allowed by all constraints (from the LHC and relic abundance) is shown in Fig.(8). Clearly, we need to go beyond MSSM for a possible candidate of DM.
Figure 8. Spin-independent scattering cross section of the LSP with a proton versus the mass of the LSP within the region allowed by all constraints (from the LHC and relic abundance.

7. Non-Minimal Supersymmetric Standard Model

The solid experimental evidence for neutrino oscillations, pointing towards non-vanishing neutrino masses, is one of the few firm hints for physics beyond the SM. In the SM, the global $(B-L)$ symmetry, where $B$ and $L$ stand for baryon and lepton numbers respectively, is conserved. Extending the MSSM by gauging this symmetry, based on the group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, resulting in the so-called B-L Supersymmetric (SUSY) model, BLSSM, and can be considered as the minimal extension of MSSM with significantly enriched particle content. In fact, gauging $(B-L)$ symmetry requires adding three SM singlet fields to cancel the triangle anomaly diagrams. These singlet fields may be identified as the right handed (RH) neutrinos. Within this model, the light Left-Handed (LH) neutrino masses can be generated through either a low (TeV) scale Type-I see-saw mechanism or inverse seesaw mechanism. Another feature of the model is the possibility to spontaneously break the $(B-L)$ symmetry with new Higgses, usually known as bileptons. In turns, the new $Z'$ gauge boson associated with this group, will acquire a mass. As in the case of MSSM, their will be superpartners of the new particles in the model namely, the RH sneutrinos, the superpartners of bileptinos and the superpartner of the new $B'$ boson, the BLino. Besides, the BLSSM have the same features of the MSSM such as gauge coupling unification, solution of the hierarchy problem and others with several new DM candidates as we will discuss in the following.

Regarding type-I seesaw mechanism, right-handed neutrinos can acquire Majorana masses at the scale of B-L symmetry breaking. On the other hand, in the inverse seesaw mechanism, B-L gauge symmetry does not allow these Majorana masses. As a consequence, another pair of SM gauge singlet fermions with masses of $O(1)$ keV has to be introduced. One of the two singlet fermions plays a role in generating the light neutrino masses through its couplings to the right handed neutrino while the other singlet is completely decoupled and can interacts only through the B-L gauge boson and consequently play the role of warm dark mater as we will see below. In the SUSY B-L Model with Inverse Seesaw (BLSSM-IS), the most general superpotential of the model can be given as

$$W = -\mu_\eta \hat{\chi}_1 \hat{\chi}_2 + \mu \hat{H}_u \hat{H}_d + \mu_S \hat{S}_2 \hat{S}_1 \hat{S}_2 - Y_d \hat{d} \hat{q} \hat{H}_d - Y_e \hat{e} \hat{l} \hat{H}_d + Y_u \hat{u} \hat{d} \hat{H}_u + Y_\nu \hat{\nu} \hat{\nu} \hat{H}_u,$$

where $\hat{\chi}_{1,2}$ are SM singlet chiral superfields with $B-L$ charges $+1$ and $-1$, respectively. The VEVs of the scalar components of these superfields breaks $U(1)_{B-L}$ spontaneously. In the superpotential $\hat{\nu}$ represents three chiral singlet superfields with $U(1)_{B-L}$ charge $= -1$. The three chiral SM singlet superfields $\hat{S}_{1,2}$ with $B-L$ charge $= +2, -2$ are considered to implement
the inverse seesaw mechanism and a $Z_2$ symmetry is assumed to forbid the interactions between $S_1$ and other fields.

The SUSY soft breaking Lagrangian is given by

$$\mathcal{L}_{\text{soft}} = m_0^2 \left[ |\tilde{q}|^2 + |\tilde{u}|^2 + |\tilde{d}|^2 + |\tilde{e}_{R}|^2 + |\nu_{R}|^2 + |\tilde{S}_1|^2 + |\tilde{S}_2|^2 + |H_d|^2 + |H_u|^2 + |\chi_1|^2 \right]
+ |\chi_2|^2 + \left[ Y^A_u \tilde{q} H_u \tilde{u}_R + Y^A_d \tilde{q} H_d \tilde{d}_R + Y^A_e \tilde{l} H_\nu \tilde{e}_R + Y^A_\nu \tilde{H}_u \tilde{\nu}_R + \tilde{Y}^A_S \tilde{\nu}_R \chi_2 \tilde{S}_2 \right]
+ \left[ B(\mu H_1 H_2 + \mu' \chi_1 \chi_2) + h.c. \right] + \frac{1}{2} M_{1/2} |\tilde{\phi}^a \tilde{\phi}^a + \tilde{W}^a \tilde{W}^a + \tilde{B}^2 + \tilde{B}'^2 + h.c. |,$$

where the trilinear terms are defined as $(Y^A_f)_{ij} = (Y_f A)_{ij}$ with $f = u, d, e, \nu, S$.

After the $B-L$ and EW symmetry breaking, the neutrinos mix with the fermionic singlet fields. In the flavor basis, the Lagrangian of neutrino masses, can be expressed as

$$\mathcal{L}'_\nu = \mu_s \tilde{S}_2^\dagger S_2 + (m_D \tilde{\nu}_L \nu_R + M_R \tilde{\nu}_R S_2 + h.c.), \quad (32)$$

where $m_D = \frac{1}{\sqrt{2}} Y_\nu v$ and $M_R = \frac{1}{\sqrt{2}} Y_s v'$. Defining $\psi = (\nu_L^c, \nu_R, S_2)$, the neutrino mass matrix can be written as $\mathcal{M}_\psi \tilde{\psi}\bar{\psi}$ with $\mathcal{M}_\psi$ is given by,

$$\mathcal{M}_\psi = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_s \end{pmatrix}, \quad (33)$$

The diagonalization of the mass matrix in Eq.(33) leads [29] to the following light and heavy neutrino masses under the consideration $m_R, \mu_s \ll m_D, M_R$ [30]

$$m_{\nu_l} = m_D M_R^{-1} \mu_s (M_R^T)^{-1} m_D^T, \quad m_{\nu_R} = m_{\nu_H} = \sqrt{M_R^2 + m_D^2}.$$
Figure 9. Feynman diagrams of the dominant annihilation channels of the $B-L$ lightest neutralino $\tilde{\chi}_1$ into the SM vector bosons ($V = W, Z$) and the SM-like Higgs $h$ mediated by the lightest $B-L$ CP-even Higgs.

If $\tilde{m} \simeq \mathcal{O}(1)$ TeV, $Y_h \simeq \mathcal{O}(1)$ and $M_N \simeq \mathcal{O}(500)$ GeV, one finds that $\delta_{\tilde{\chi}}^2 \simeq \mathcal{O}(100 \text{ GeV})^2$, thus the Higgs mass is of order $\sqrt{(90)^2 + \mathcal{O}(100)^2 + \mathcal{O}(100)^2} \simeq 170$ GeV.

The BLSSM-IS has more possibilities for candidates of DM compared to the MSSM. These includes, the lightest $B-L$ neutralino ($\tilde{B}'$, $\tilde{\eta}_2$)-like, referred as $\tilde{\chi}_1$, and lightest right-handed sneutrino. In the following, we investigate these possibilities.

In the BLSSM-IS, the neutralinos $\tilde{\chi}_i^0$ (with $i$ running from 1 to 7) are the mass eigen states resulting from the superpositions of three fermionic partners of neutral gauge bosons, $\tilde{B}$ (bino), $\tilde{W}^3$ (wino) and $\tilde{B}'$ ($B'$ino), in addition to the fermionic partners of neutral MSSM Higgs ($\tilde{H}_1^0$, and $\tilde{H}_2^0$) and the fermionic partners of $B-L$ scalar bosons ($\tilde{\eta}_1$, and $\tilde{\eta}_2$). The neutralino mass matrix $M_7$ can be expressed as [33]

$$M_7 \equiv \begin{pmatrix} M_4 & O \\ O^T & M_3 \end{pmatrix},$$

here $M_4$ is the MSSM neutralino mass matrix [34, 35, 24, 36], while $M_3$ is an additional $B-L$ neutralino mass matrix and $O$ is off-diagonal matrix with

$$M_3 = \begin{pmatrix} M_{B'} & -g_{BL}v_1' & g_{BL}v_2' \\ -g_{BL}v_1' & 0 & -\mu' \\ g_{BL}v_2' & -\mu' & 0 \end{pmatrix}, \quad O = \begin{pmatrix} \frac{1}{2}M_{BB'} & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{2}\tilde{g}v_1 & 0 & 0 \\ \frac{1}{2}\tilde{g}v_2 & 0 & 0 \end{pmatrix},$$

here $M_{B'}$ denotes $B'$ino mass and $M_{BB'}$ represents the mass mixing term of $\tilde{B}$ and $\tilde{B}'$. At the GUT scale, $M_{B'} = m_{1/2}$ and $M_{BB'} = 0$. Thus, when $\tilde{g} = 0$, the matrix $O$ turns to a zero matrix and the real matrix $M_7$ can be diagonalized with a symmetric mixing matrix $V$ such as

$$V M_7 V^T = \text{diag}(m_{\tilde{\chi}_i^0}), \quad i = 1, \ldots, 7.$$  \hspace{1cm} (37)

The LSP $\tilde{\chi}_1$, in this case, is given by

$$\tilde{\chi}_1 = V_{11} \tilde{B} + V_{12} \tilde{W}^3 + V_{13} \tilde{H}_1^0 + V_{14} \tilde{H}_2^0 + V_{15} \tilde{B}' + V_{16} \tilde{\eta}_1 + V_{17} \tilde{\eta}_2.$$  \hspace{1cm} (38)

Clearly, the LSP can be either pure $B'$ino ($\tilde{B}'$) if $V_{15} \sim 1$ and $V_{1i} \sim 0$ for $i \neq 5$, or pure $B-L$ higgsino $\tilde{\eta}_{1(2)}$ if $V_{16(7)} \sim 1$ and all the other coefficients are close to zero value. It should be noted that, the off-diagonal elements ($M_3)_{12,13}$ and ($M_3)_{21,31}$ are not suppressed. Consequently, unless $\mu'$ is very large, the lightest $B-L$ neutralino is a mixed state between $B'$ino and $\tilde{\eta}_{1,2}$.

We consider now $\tilde{\chi}_1$ as a DM candidate under the assumption that $\tilde{\chi}_1$ was in thermal equilibrium with the SM particles in the early universe where the decoupling occurred when $\tilde{\chi}_1$ was non-relativistic. In this case, the dominant annihilation channels of $\tilde{\chi}_1$ are those channels with final states $WW$, $ZZ$, $hh$ and are mediated by the lightest $B-L$ CP-even Higgs boson as
shown in figure 9. The resulting constraint from the $\Omega h^2_{\chi_1}$ observed limits as function of $\chi_1$ mass, for some selected regions in the parameter space, using 2$\sigma$ results reported by Planck satellite [23], is presented in figure 10 together with the LHC constraints, in particular, the SM-like Higgs and gluino mass constraints. Clearly from fig. 10, the narrow range of the relic abundance limits severely constrain this kind of DM candidates where only few benchmark points are allowed. However, the allowed points are much larger than the corresponding ones in the MSSM. Recall that, in the MSSM, no point with bino-like is allowed and much less points for higgsino-like at very large tan $\beta$ are allowed [37]. Another remark is that the masses of allowed $\tilde{\eta}_2$ lies in the range 100 – 1000 GeV.

8. Right-handed sneutrino: Scalar Dark Matter

We turn now to a scenario in which the lightest right-handed sneutrino can serve as DM candidate. To discuss this scenario, we need first to show the sneutrino mass matrix and discuss the possibility of having lightest sneutrinos after diagonalization of the mass matrix. To do this, we can write $\tilde{\nu}_L$, $\tilde{\nu}_R$ and $\tilde{S}_2$ as [33]

$$\tilde{\nu}_L = \frac{1}{\sqrt{2}} (\tilde{\nu}_L^+ + i \tilde{\nu}_L^-), \quad \tilde{\nu}_R = \frac{1}{\sqrt{2}} (\tilde{\nu}_R^+ + i \tilde{\nu}_R^-), \quad \tilde{S}_2 = \frac{1}{\sqrt{2}} (\tilde{S}_2^+ + i \tilde{S}_2^-),$$

consequently, the sneutrino mass matrix can be written as

$$M^2_{\tilde{\nu}} = \begin{pmatrix} \mathcal{M}^2_{\tilde{\nu}} & 0 \\ 0 & \mathcal{M}^2_{\tilde{\nu}} \end{pmatrix}$$

where, for $\tilde{g} = 0$, the CP-even/odd (right/left) sneutrino mass matrix is given by

$$\mathcal{M}^2_{\tilde{\nu}} = \begin{pmatrix} m^2_{\tilde{\nu}_L} + m^2_{\tilde{\nu}_R} + \frac{1}{2} (M^2_Z \cos 2\beta + M^2_{Z'} \cos 2\beta') + m^2_{\tilde{\nu}_R} + m^2_{\tilde{\nu}_R} + \frac{1}{2} M^2_{Z'} \cos 2\beta' & \pm m_D (A_\nu + \mu \cot \beta) \\ \pm m_D (A_\nu + \mu \cot \beta) & m^2_{\tilde{\nu}_R} + m^2_{\tilde{\nu}_R} + \frac{1}{2} M^2_{Z'} \cos 2\beta' + m^2_{\tilde{\nu}_R} + m^2_{\tilde{\nu}_R} + \frac{1}{2} M^2_{Z'} \cos 2\beta' \\ m^2_{\tilde{\nu}_L} + m^2_{\tilde{\nu}_R} + \frac{1}{2} M^2_R (A_S + \mu' \cot \beta') & \pm m_R (A_S + \mu' \cot \beta') \\ \pm m_R (A_S + \mu' \cot \beta') & m^2_{\tilde{\nu}_L} + m^2_{\tilde{\nu}_L} + \frac{1}{2} M^2_R (A_S + \mu' \cot \beta') \end{pmatrix}$$

The diagonalization of $\mathcal{M}^2_{\tilde{\nu}}$ is not straight forward and can be only be done numerically. It turns out that the mass of the lightest CP-odd sneutrino, $\tilde{\nu}_L^-$, is almost equal to the mass of the lightest CP-even (right) sneutrino, $\tilde{\nu}_R^+$ and can be of order $\mathcal{O}(100)$ GeV when $\mu'$ and/or $A_S$ are

Figure 10. The thermal relic abundance of $B - L$ neutralinos, $\tilde{B}'$-like (green points) and $\tilde{\eta}_2$-like (blue points), LSP as a function of their masses. Horizontal lines correspond to the Planck limits on DM abundance. The gray points indicate to the excluded points by the LHC and LEP constraints.
of order $m_{\tilde{\nu}_R}$ and $M_R$, i.e., $\sim \mathcal{O}(1)$ TeV. The lightest sneutrino $\tilde{\nu}_1$ can be written in terms of $\tilde{\nu}_L^+, \tilde{\nu}_R^+, \tilde{S}_2^+$ (in case of it is CP-even) as

$$\tilde{\nu}_1 = \sum_{i=1}^{3} R_{1i}(\tilde{\nu}_L^+)i + \sum_{j=1}^{3} R_{1j}(\tilde{\nu}_R^+)j + \sum_{k=1}^{3} R_{1k}(\tilde{S}_2^+)k.$$  

(41)

with $R_{1j} = R_{1k} = \frac{1}{\sqrt{2}}$ for $j = k = 1$ and the rest of the R coefficients are zeros which indicates that the lightest sneutrino is a combination of $\tilde{\nu}_R^+$ and $\tilde{S}_2^+$ and hence is mainly right-handed. The dominant annihilation channels of $\tilde{\nu}_1$ are those annihilations to CP-even Higgs bosons, $W^+W^-$, $ZZ$ and three light-neutrinos, $\nu_L^j$. The allowed range of the right-handed sneutrino DM after taking into account the the observed limits on DM abundance, the Higgs mass and gluino mass constraints is shown in figure 11. We note from that figure that, the allowed values of $M_{\tilde{\nu}_1}$ range from 80 GeV to 1.2 TeV. In fact this result leave a possibility of having DM candidates in SUSY due to the stringent restraints imposed on the MSSM and in the BLSSM with neutralino DM candidates.

DM signals such as gamma-rays possibly produced by DM annihilation can be probed using gamma-ray telescopes. With the ability to search in energy range from 20 MeV to 300 GeV, the Large Area Telescope (LAT) on the Fermi Gamma-ray Space Telescope (FGST) mission can be considered a good tool for such probes. Galactic Center (GC) gamma-ray photons excess in the range 3–4 GeV was observed by Fermi-LAT [38, 39, 40]. DM particle with mass $\lesssim \mathcal{O}(100)$ GeV and annihilation cross section of order $\langle \sigma_{\text{ann}} v \rangle \simeq 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ can be one of the possible sources responsible for this observed gamma-ray excess. Regarding the $B - L$ neutralinos as DM candidates, the relic abundance constrained their masses to be larger than 100 GeV. In addition, their annihilation cross sections in the galactic halo are of order $10^{-26} \text{ cm}^3 \text{ s}^{-1}$ [33]. As a consequence, they cannot account for such gamma-ray excess. Tuning now to the lightest right-handed sneutrino as a candidate of DM, based on the discussion in Ref.[33], it was shown that, see figure 11, right-handed sneutrino with mass $\mathcal{O}(100)$ GeV annihilating to $W^+W^-$ bosons can account for the observed gamma-ray excess. This can be explained as the right-handed sneutrino is a scalar DM with s-wave contribution to the annihilation cross section leading to a value in galactic halo, almost equal to its value at the decoupling limit, $\sim 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. 

**Figure 11.** Left, The thermal relic abundance of right-handed sneutrino LSP as a function of its mass. The gray triangles denote to the excluded points due to LUX upper bound. Horizontal lines correspond to the Planck limits on DM abundance. Right, The measured spectrum of gamma-rays within the ROI $2^\circ \leq |b| \leq 20^\circ$ and $|l| \leq 20^\circ$ of the GC. The dashed line shows the backgrounds. (Left panel) The gamma-rays spectrum produced for the lightest sneutrino DM annihilation into $WW$ (91 %) with $M_{\tilde{\nu}_1} \simeq 80.3$ GeV and total annihilation cross section $\langle \sigma_{\text{ann}} v \rangle \simeq 2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ (the dot-dashed green curve). The solid blue curve shows the sum of the signal and its backgrounds.
The effective scalar interactions of the DM, either being $B - L$ neutralino $\tilde{\chi}_1$ or the lightest Right-handed Sneutrino $\tilde{\nu}_1^R$, with the up and down quarks can be expressed as

$$\mathcal{L}_{\text{scalar}} = f_q \tilde{\chi}_1 \bar{\chi}_1 \tilde{q} q,$$  \hspace{1cm} (42)

It is mainly due to $Z'$ exchange in the case of $B - L$ neutralino while in the case of $\tilde{\nu}_1^R$ it is due to CP-even Higgs bosons ($h$ and $h'$) exchanges. The $\tilde{\chi}_1$ coupling to protons and neutrons are proportional to $f_u$ and $f_d$. They are quite suppressed since $f_q \propto 1/M_{Z'}^2$, with $M_{Z'} > 2$ TeV and hence the spin-independent cross section of this scattering is expected to be very small. Regarding $\tilde{\nu}_1^R$ coupling to protons and neutrons, the effective coupling $f_q$ in eq. (42) is given by

$$f_q \simeq \frac{g_{\tilde{\nu}_1 \tilde{\nu}_1 h}}{m_h^2} + \frac{g_{\tilde{\nu}_1 \tilde{\nu}_1 h'}}{m_{h'}^2},$$  \hspace{1cm} (43)

Based on the estimations in Ref.[33], $f_q$ is dominated by $h$ exchange as the effective coupling in this case is of order $\mathcal{O}(10^{-3})$ GeV$^{-1}$ compared to the tiny one, $\mathcal{O}(10^{-7})$ GeV$^{-1}$, in case of $h'$ exchange. It turns out that, the effective coupling of $\tilde{\nu}_1^R$ to proton and neutrino, is about three order of magnitudes larger than the corresponding one in the case of neutralinos. As a consequence, one would expect a larger spin-independent cross section for sneutrino DM that may even exceed the LUX limits as shown in Fig.12.

9. Concluding remarks

In this review, we presented a brief introduction related to the formulation of the SM as the most successful theory, up to date, in describing, explaining and predicting lot of well confirmed experimental results. Due, to the failure of the SM in addressing many issues, discussed in the text before, we highlighted the success of SUSY, as one of the more popular candidates of physics beyond SM, in providing solutions to the addressed problems in the SM. These include, solution for the naturalness problems of the Higgs sector in the SM, unification of the gauge couplings, and providing viable candidate for cold dark matter. We showed also that, SUSY provide a natural mechanism for understanding Higgs physics and electroweak symmetry breaking and the inclusion of gravity. With the running of LHC and with the help of future experiments, different SUSY scenarios and parameter space can be probed with a hope to a better understanding of the theory.
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