GALACTIC KINEMATICS TOWARD THE SOUTH GALACTIC POLE: FIRST RESULTS FROM THE YALE–SAN JUAN SOUTHERN PROPER MOTION PROGRAM

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ABSTRACT

The predictions from a Galactic structure and kinematic model code are compared with the color counts and absolute proper motions derived from the Southern Proper Motion survey, covering more than 700 deg² toward the south Galactic pole in the range 9 < B_J < 19. The theoretical assumptions and associated computational procedures, the geometry for the kinematic model, and the adopted parameters are presented in detail and compared with other Galactic kinematic models of its kind. The data with which the model is compared consist of more than 30,000 randomly selected stars, and they are best fitted by models with a solar peculiar motion of +5 km s⁻¹ in the V-component (pointing in the direction of Galactic rotation), a high LSR speed of 270 km s⁻¹, and a (disk) velocity ellipsoid that always points toward the Galactic center. The absolute proper motions in the U-component indicate a solar peculiar motion of 11.0 ± 1.5 km s⁻¹, with no need for a local expansion or contraction term. The fainter absolute motions show an indication that the thick disk must exhibit a rather steep velocity gradient of about -36 km s⁻¹ kpc⁻¹ with respect to the LSR. We are not able to set constraints on the overall rotation for the halo or on the thick disk or halo velocity dispersions. Some substructure in the U and V proper motions could be present in the brighter bins, 10 < B_J < 13, and it might be indicative of (disk) moving groups.

Key words: astrometry — Galaxy: fundamental parameters — Galaxy: kinematics and dynamics — Galaxy: structure — stars: fundamental parameters — stars: kinematics

1. INTRODUCTION

Proper motions are one of the fundamental observational quantities in astronomy, as they provide an estimate of the distribution of stellar velocities within a few kiloparsecs from the Sun. This information provides constraints on the various theories of the structure and dynamics of the Galaxy, and hence it is an important observational quantity.

One of the limitations of the available proper motions for the study of the large-scale kinematic properties of stars in our Galaxy has been the relative lack of absolute proper motions, as opposed to the more common relative motions giving only the transverse component of the motion with respect to, e.g., the mean motion of a group of stars. The solution to the problem of calculating absolute proper motions came with the possibility of using galaxies to define an inertial (i.e., nonrotating) reference frame for the evaluation of the proper motions of stars. The concept is based on the fact that, since galaxies are so far away, their proper motions are extremely small and could be considered zero. Even if the transverse velocities of galaxies were comparable to their radial velocities, this would amount to proper motions of only 0.02 mas yr⁻¹ for a Hubble constant of 50 km s⁻¹ Mpc⁻¹.

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The idea of using galaxies to determine the zero point of the motions probably first originated at Lick Observatory, with W. H. Wright in about 1919 (Klemola, Jones, & Hanson 1987), before the extragalactic nature of these nebulae was widely accepted. Wright realized that a wide-field telescope was required since a large number of galaxies would be needed to establish the inertial frame to sufficient accuracy. In 1947 the Lick Northern Proper Motion program was started. By 1986 all the second-epoch plates had been taken, and the first interpretative results were published soon thereafter (Hanson 1987). A similar program (the Southern Proper Motion program, or SPM), aimed at determining proper motions for stars in the southern hemisphere, was started jointly by Yale and Columbia University in 1965 in Argentina, and the first results have been released recently (Platais et al. 1998).

At the same time, our knowledge of stellar populations and Galactic structure in the Milky Way has increased enormously in the last few decades (for a review, see Majewski 1993). In particular, Galactic structure models have been developed that allow the prediction of the observed kinematic properties of stars by making a number of assumptions based on our current knowledge about the local stellar system. The predictions of these models can then be directly compared with observations, from which broad properties of the stellar velocity distribution can indeed be inferred (Méndez & van Altena 1996). The use of models is necessary because the observed proper-motion distribution is, in itself, the product of the convolution of stars located at different distances from the Sun having dif-
different tangential velocities, and representing different Galactic components; the disentanglement of these various contributions when no additional information is available must be done, then, in a statistical fashion.

In this paper we present the first systematic analysis using a Galactic kinematic model of the data gathered in the course of the SPM survey in a large area (more than 700 deg$^2$) toward the south Galactic pole (SGP), down to an apparent magnitude of $B_J \sim 19$. A number of issues related to the velocity distribution function of stars from the disk, thick disk, and halo are addressed.

The arrangement of this paper is as follows: In § 2, we present an overview of the SPM photometric and proper-motion data and their errors. In § 3, the model used to compare the observed proper-motion distributions with those predicted is described in detail, while §§ 4 and 5 present the model predictions as compared with the magnitude/color and proper-motion data, respectively. Finally, § 6 presents the main conclusions of the paper.

2. THE SOUTHERN PROPER MOTION KINEMATIC AND COLOR CATALOG

We analyze the kinematic and color data gathered in the context of a massive proper-motion survey, the SPM, which is the southern-sky counterpart of the Lick Observatory Northern Proper Motion (NPM) program (Klemola et al. 1987). The main goal of both programs is the measurement of absolute proper motions relative to external galaxies. A description of the scientific motivation for the SPM can be found in Wesselink (1974) and van Altena et al. (1990). When finished, the SPM will produce absolute proper motions, positions, and $BV$ photographic photometry for approximately 1 million stars south of $\delta = -17^\circ$.

In this paper we present an analysis of the SPM Catalog 1.1, an improved version of the SPM Catalog 1.0 described by Platais et al. (1998). This catalog provides positions, absolute proper motions, and $BV$ photometry for 58,887 objects at the south Galactic pole. The sky coverage of the SPM Catalog 1.0 is about 720 deg$^2$ in the magnitude range $5 < V < 18.5$. Boundaries of the SGP area are indicated in Figure 1 of Platais et al. (1998). In this paper we utilize only about 31,000 stars that have been randomly chosen. The accuracy of individual absolute proper motions is $3-8$ mas yr$^{-1}$, depending on the star’s magnitude. The mean motion as a function of magnitude has random errors below 1 mas yr$^{-1}$, and various comparisons with Hipparcos motions at the bright end, and other independent measurements at the faint end, indicate systematic errors also smaller than 1 mas yr$^{-1}$ (see § 5.1). A great effort has been put into correcting positions and proper motions for magnitude-dependent systematic errors (see § 5.1). For further details about the catalog structure, contents, and plate measurement and other astrometric/photometric details, the reader is referred to Paper I (Girard et al. 1998) and Paper II (Platais et al. 1998).

In the SGP region, the mean number of stars per field down to the plate limit $B_J \sim 19$ is about 35,000. As a consequence of the scan-time limitations imposed by the plate-measuring machine, we could measure only $\sim 10\%$ of that number of stars per field. Therefore, an effort was made to predict a minimal number of anonymous, randomly selected stars that had to be measured in order to ensure statistically well-sampled components of the Galactic disk, thick disk, and halo, over the $B_J$ magnitude range from 9 to 19. This was done using an earlier version of our Galactic structure and kinematic model code (see Méndez et al. 1993) that also allowed for the incorporation of the expected accuracy of the SPM proper motions. For instance, the thick disk stars can be sampled properly starting from $B_J = 14$, whereas halo stars appear in statistically significant numbers only at $B_J > 17$ (see § 4.1). The initial working numbers of how many stars had to be measured in each magnitude bin were given in van Altena et al. (1994). Later, more stars with $V > 15$ were added in order to increase the density of a secondary reference frame; thus, on average at the SGP there are 46 anonymous stars per square degree.

2.1. The Randomly Selected Sample

In this section we provide details of the basic observational material analyzed in this paper. We focus on the randomly selected sample mentioned in the previous section. Out of a total 33,498 randomly selected stars, 31,023 have complete $B-V$ color and absolute proper motion information—and we only utilize these. Most stars missing colors are extremely faint ($B, V > 19$) and could not be measured in one bandpass.

The random sample has been chosen such that, at a given $B_J$ magnitude interval, a fixed number of randomly selected stars were extracted from the COSMOS/UKST Object Catalog (Yentis et al. 1992). We note that the COSMOS catalog is known to be complete to magnitudes much fainter ($B_J \sim 22$) than the limit imposed by our astrometric plates. Even though the SPM catalog presents calibrated $B$ and $V$ photographic photometry, the initial selection of random objects was performed using the COSMOS $B_J$ magnitudes. Therefore, for the random SPM sample, 1 mag intervals have been chosen in the range $9 \leq B_J \leq 19$, where we have adopted the relation $B_J = B - 0.28(B-V)$ from Blair & Gilmore (1982), valid for the UKST plate passband (see also Bertin & Dennufeld 1997). Correspondingly, the analysis below partitions the whole catalog into these magnitude bins. Indeed, lumping the SPM-SPM data into two or more $B_J$ magnitude bins will lead to selectively incomplete samples—proper statistical corrections must be applied when doing this. However, at a given magnitude interval, the complete sample and the randomly selected sample differ only by a scale factor, because the observed sample has been drawn randomly from all the stars available in that particular magnitude bin. Therefore, color and kinematic properties binned in the proper magnitude intervals should be the same as those derived from a complete sample, except for the increased uncertainty on the derived values because of the smaller sample—an effect that is fully taken into account in the analysis below.

Similarly to Méndez & van Altena (1996), the kinematic comparisons with the model predictions take the form of histograms of proper motions along the Galactocentric direction and along Galactic rotation. Both the observed and the model histograms are fully convolved with observational errors, so that the model comparisons can be carried out directly. Rather than making the comparisons graphically, we have used statistical descriptors of the proper-motion distribution: the median proper motion and the proper-motion dispersion (however, the full shape of the...
proper-motion histograms is also used for an assessment of the overall fit to the model predictions; see § 5.3).

Table 1 presents the basic data concerning uncertainties in the photographic colors as a function of $B_J$ magnitude derived from the catalog itself. The second and third columns indicate the mean and median value of the error in color respectively, while the fourth and fifth columns indicate the actual number of stars used in the computations and the total number available in that particular magnitude range. We have used an iterative procedure to determine robust estimates of the values quoted using the procedure presented in Méndez & van Altena (1996). Basically, the method trims outliers in an iterative fashion by computing preliminary values for the median and dispersion. Then, a window of semiwidith 3 times the dispersion centered on the median is used to recompute the median and dispersion, until convergence. This method is similar to using the robust technique of probability plots (Daniel & Wood 1980) to estimate dispersions and properly account for outliers (Lutz & Upgren 1980), and indeed from Table 1 we can see that, in all cases, the number of stars excluded amounts to less than 5%, i.e., smaller than the fraction of excluded objects that are known to bias the derived values for the dispersion (± 10% at both extremes of the distribution of values). The same method has been applied to obtain mean, median, and dispersion values for the absolute proper motions.

Tables 2 and 3 indicate the mean, median, and uncertainties of the mean for the absolute proper motions in the $U$ and $V$ Galactic directions, respectively. In this paper, $U$ is oriented toward the Galactic center, while $V$ points in the direction of Galactic rotation. Since we are looking down to

### Table 1

| $B_J$ Range (mag) | Mean $\sigma_{B-V}$ (mag) | Median $\sigma_{B-V}$ (mag) | Standard Deviation of $\sigma_{B-V}$ (mag) | Stars Used | Total Stars |
|------------------|---------------------------|-----------------------------|------------------------------------------|------------|-------------|
| 9-10             | 0.056                     | 0.054                       | 0.017                                    | 190        | 197         |
| 10-11            | 0.040                     | 0.036                       | 0.013                                    | 486        | 528         |
| 11-12            | 0.043                     | 0.042                       | 0.012                                    | 972        | 1021        |
| 12-13            | 0.054                     | 0.054                       | 0.014                                    | 4319       | 4415        |
| 13-14            | 0.068                     | 0.064                       | 0.017                                    | 3426       | 3524        |
| 14-15            | 0.084                     | 0.081                       | 0.022                                    | 2897       | 3007        |
| 15-16            | 0.095                     | 0.092                       | 0.029                                    | 4132       | 4334        |
| 16-17            | 0.125                     | 0.120                       | 0.038                                    | 5951       | 6170        |
| 17-18            | 0.200                     | 0.192                       | 0.060                                    | 5878       | 6176        |
| 18-19            | 0.290                     | 0.283                       | 0.079                                    | 1589       | 1651        |

**TABLE 2**

| $B_J$ Range (mag) | Mean $\mu_U$ (mas yr$^{-1}$) | Median $\mu_U$ (mas yr$^{-1}$) | Standard Deviation of the Mean $\mu_U$ (mas yr$^{-1}$) | Stars Used | Total Stars |
|------------------|------------------------------|--------------------------------|--------------------------------------------------------|------------|-------------|
| 9-10             | -12.44                       | -12.50                         | 2.68                                                   | 193        | 197         |
| 10-11            | -8.71                        | -7.95                          | 1.36                                                   | 508        | 528         |
| 11-12            | -6.54                        | -6.90                          | 0.77                                                   | 971        | 1021        |
| 12-13            | -5.89                        | -5.30                          | 0.30                                                   | 4216       | 4415        |
| 13-14            | -4.73                        | -4.20                          | 0.29                                                   | 3397       | 3524        |
| 14-15            | -4.68                        | -4.00                          | 0.26                                                   | 2890       | 3007        |
| 15-16            | -3.96                        | -3.50                          | 0.19                                                   | 4152       | 4334        |
| 16-17            | -2.85                        | -2.60                          | 0.14                                                   | 5843       | 6170        |
| 17-18            | -2.27                        | -1.90                          | 0.16                                                   | 5909       | 6176        |
| 18-19            | -2.74                        | -2.20                          | 0.44                                                   | 1581       | 1651        |

**TABLE 3**

| $B_J$ Range (mag) | Mean $\mu_V$ (mas yr$^{-1}$) | Median $\mu_V$ (mas yr$^{-1}$) | Standard Deviation of the Mean $\mu_V$ (mas yr$^{-1}$) | Stars Used | Total Stars |
|------------------|------------------------------|--------------------------------|--------------------------------------------------------|------------|-------------|
| 9-10             | -15.12                       | -12.80                         | 1.35                                                   | 177        | 197         |
| 10-11            | -14.25                       | -12.10                         | 0.82                                                   | 492        | 528         |
| 11-12            | -11.16                       | -9.65                          | 0.48                                                   | 944        | 1021        |
| 12-13            | -10.84                       | -8.85                          | 0.20                                                   | 4162       | 4415        |
| 13-14            | -10.07                       | -8.70                          | 0.20                                                   | 3344       | 3524        |
| 14-15            | -9.49                        | -8.60                          | 0.18                                                   | 2819       | 3007        |
| 15-16            | -8.75                        | -8.10                          | 0.14                                                   | 4099       | 4334        |
| 16-17            | -8.35                        | -7.60                          | 0.11                                                   | 5819       | 6170        |
| 17-18            | -9.34                        | -8.60                          | 0.14                                                   | 5911       | 6176        |
| 18-19            | -9.61                        | -8.60                          | 0.39                                                   | 1574       | 1651        |
the SGP, the derived tangential motions decouple nicely into these two physically meaningful quantities that are easier to model and interpret. From the observed absolute \( \mu_a \) and \( \mu_b \) proper motions, we derived \( \mu_l \) and \( \mu_b \), proper motions in Galactic longitude and latitude, respectively. Then, these motions are projected into \( \mu_V \) and \( \mu_V \). The first conversion is straightforward. The last step, however, deserves further comment, because of the lack of radial velocities. Basically, to convert from the observed and we would need to apply the following equations:

\[
\mu_V = -\mu_b \sin b \cos l - \mu_l \sin l + (V_r/r) \cos b \cos l, \tag{1}
\]

\[
\mu_V = -\mu_b \sin b \sin l + \mu_l \cos l + (V_r/r) \cos b \sin l. \tag{2}
\]

As can be seen from the above equations, the transformation necessarily involves the ratio \( V_r/r \) between each star’s radial velocity and its heliocentric distance, but we have neither. We bypass this by just dropping this last term from equations (1) and (2). Obviously, a similar procedure should be adopted in the model computations; this is further discussed in § 3.2 in this context. The correction term is, however, small, since for these data \( \cos b \approx 0 \). Errors have also been properly propagated from \( \sigma_{\mu_b} \) and \( \sigma_{\mu_b} \) to \( \sigma_{\mu_V} \) and \( \sigma_{\mu_V} \) using equations similar to equations (1) and (2). Note that, near the SGP, the value of \( l \) can take any value from \( 0^\circ \) to \( 360^\circ \), and equations (1) and (2) imply that \( \mu_V \) and \( \mu_V \) have a contribution from both \( \mu_l \) and \( \mu_b \) and, in turn, from \( \mu_b \) and \( \mu_b \). Therefore, it is unlikely that, e.g., a systematic effect on the proper motions in \( \mu_b \) would propagate to affect only \( \mu_V \). We have, however, tested the effects of any remaining systematic effect on our equatorial absolute proper motions upon the derived motions along the \( U \) and \( V \) Galactic components (see § 5.1).

**TABLE 4**

| \( B_J \) Range (mag) | \( \Sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Standard Deviation in \( \Sigma_{\mu_V} \) (mas yr\(^{-1}\)) | \( \Sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Standard Deviation in \( \Sigma_{\mu_V} \) (mas yr\(^{-1}\)) |
|------------------------|---------------------------------|---------------------------------|------------------------|---------------------------------|
| 9–10……… | 37.18 | 12.24 | 17.98 | 4.01 |
| 10–11……… | 30.63 | 6.17 | 18.27 | 2.61 |
| 11–12……… | 23.95 | 2.98 | 14.65 | 1.38 |
| 12–13……… | 19.33 | 1.02 | 12.92 | 0.53 |
| 13–14……… | 17.13 | 0.95 | 11.39 | 0.50 |
| 14–15……… | 13.92 | 0.75 | 9.61 | 0.42 |
| 15–16……… | 11.91 | 0.49 | 8.68 | 0.30 |
| 16–17……… | 10.35 | 0.33 | 8.27 | 0.23 |
| 17–18……… | 12.12 | 0.43 | 10.41 | 0.33 |
| 18–19……… | 17.68 | 1.46 | 15.48 | 1.17 |

**TABLE 5**

| \( B_J \) Range (mag) | Mean \( \sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Median \( \sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Standard Deviation in \( \sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Stars Used | Total Stars |
|------------------------|---------------------------------|---------------------------------|---------------------------------|------------|-----------|
| 9–10……… | 2.32 | 2.20 | 0.57 | 188 | 197 |
| 10–11……… | 2.02 | 1.90 | 0.45 | 498 | 528 |
| 11–12……… | 1.85 | 1.80 | 0.38 | 965 | 1021 |
| 12–13……… | 2.10 | 2.00 | 0.48 | 4247 | 4415 |
| 13–14……… | 2.62 | 2.50 | 0.65 | 3353 | 3524 |
| 14–15……… | 3.50 | 3.10 | 1.27 | 2822 | 3007 |
| 15–16……… | 2.58 | 2.50 | 0.50 | 3816 | 4334 |
| 16–17……… | 3.40 | 3.20 | 0.94 | 5845 | 6170 |
| 17–18……… | 5.88 | 5.70 | 1.82 | 5848 | 6176 |
| 18–19……… | 7.98 | 7.90 | 1.68 | 1522 | 1651 |

**TABLE 6**

| \( B_J \) Range (mag) | Mean \( \sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Median \( \sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Standard Deviation in \( \sigma_{\mu_V} \) (mas yr\(^{-1}\)) | Stars Used | Total Stars |
|------------------------|---------------------------------|---------------------------------|---------------------------------|------------|-----------|
| 9–10……… | 2.30 | 2.20 | 0.54 | 188 | 197 |
| 10–11……… | 2.00 | 1.90 | 0.44 | 494 | 528 |
| 11–12……… | 1.85 | 1.80 | 0.38 | 963 | 1021 |
| 12–13……… | 2.10 | 2.00 | 0.47 | 4230 | 4415 |
| 13–14……… | 2.61 | 2.50 | 0.64 | 3343 | 3524 |
| 14–15……… | 3.52 | 3.10 | 1.31 | 2837 | 3007 |
| 15–16……… | 2.59 | 2.50 | 0.50 | 3811 | 4334 |
| 16–17……… | 3.41 | 3.20 | 0.94 | 5820 | 6170 |
| 17–18……… | 5.91 | 5.70 | 1.82 | 5829 | 6176 |
| 18–19……… | 7.95 | 7.90 | 1.60 | 1506 | 1651 |
Table 4 presents the values for the proper-motion dispersions and its corresponding uncertainty for $U$ and $V$ as a function of apparent magnitude. It is evident that the dispersions are not the same along $U$ and $V$. This fact reveals the intrinsically different kinematic behavior of stars along the Galactocentric and the Galactic rotation directions, a property fully accounted for by our kinematic model (see § 5).

Tables 5 and 6 indicate the mean and median proper motion errors, along with the dispersion (standard deviation) of the mean. These values, along with those presented in Table 1, will be used for convolving the model predictions with the appropriate errors in both color and proper motion (see § 4).

3. THE STAR-COUNTS AND KINEMATIC MODEL

In the following analysis we model the star counts concurrently with the kinematics by using the model presented by Méndez & van Altena (1996). The star-counts model employed here has been tested under many different circumstances and has proved to be able to predict star counts that match the observed magnitude and color counts (in both shape and number) to better than 10%, and in many cases to better than 1%. In particular, the model has been recently shown to provide an excellent match to the overall magnitude and color counts at the SGP and other lines of sight from the deep ESO Imaging Survey (da Costa et al. 1998). The kinematic model presented by Méndez & van Altena (1996) has been also shown to be able to reproduce the kinematics of disk stars in two intermediate Galactic latitude fields, providing for the first time constraints on the expected run of velocity dispersion for disk stars as a function of distance from the Galactic plane, in agreement with theoretical (dynamical) expectations (Fuchs & Wielen 1987; Kuijken & Gilmore 1989).

Even though we adopt the model described by Méndez & van Altena (1996), the fainter magnitude limits now available from the SPM data allow us, for the first time, to study the contribution of other Galactic components. This makes it necessary to provide an outline of the kinematic model parameters, as the star-counts parameters have already been described extensively elsewhere (Méndez & van Altena 1996; Méndez et al. 1996; Méndez & Guzmán 1998).

3.1. Basic Assumptions

The magnitude and color model is based on the fundamental equation of stellar statistics (Trumpler & Weaver 1962; Mihalas & Binney 1981). This equation can be easily extended to include kinematics. If we call $N_j$ the number of stars per unit of solid angle, per unit of apparent magnitude, and per unit of apparent color for Galactic component $j$, then the number of stars per unit of velocity, per unit of solid angle, per unit of apparent magnitude, and per unit of apparent color for component $j$ at position $r$ and velocity $V$ is given by

$$\frac{dN_j}{dV} = N_j f_j(r, V),$$

where $N_j$ is given by the fundamental equation of stellar statistics and $f_j(r, V)$ is the velocity distribution function for component $j$ at position $r$ and velocity $V$. For the velocity distribution we have adopted generalized Schwarzschild (1907, 1908) velocity ellipsoids, represented by orthogonal three-dimensional Gaussian functions with (in general) different velocity dispersions along the principal axes of the velocity ellipsoid. For a detailed justification of the use of this function, the reader is referred to Méndez & van Altena (1996).

The velocity distribution functions adopted in the model are completely specified by three velocity dispersions along the principal axes of the velocity ellipsoid, as well as by the orientation of the velocity ellipsoid in a given coordinate system. If $V = (U', V', W')$ are the velocities along the principal axes of the velocity ellipsoid relative to an inertial reference frame moving with the instantaneous mean speed for Galactic component $j$, then the (normalized) function is given by

$$f_j(r, V) = f_{j0}(r) \exp \left( -\frac{1}{2} \left\{ \frac{U'}{\Sigma_{Uj}(r)} \right\}^2 + \frac{V'}{\Sigma_{Vj}(r)} \right)^2 + \frac{W'}{\Sigma_{Wj}(r)} \right)^2 \right\},$$

$$f_{j0} = \frac{1}{(2\pi)^{\frac{3}{2}} \Sigma_{Uj}(r) \Sigma_{Vj}(r) \Sigma_{Wj}(r)}.$$

where $\Sigma_{Uj}(r), \Sigma_{Vj}(r), \Sigma_{Wj}(r)$ are the velocity dispersions for component $j$, evaluated at position $r$.

It is possible to show that, for an axisymmetric system in steady state, if the velocity distribution function is of the type given in equation (4), then one of the axes of the velocity ellipsoid must point toward the direction of Galactic rotation and another axis must be oriented toward the axis of rotation of the Galaxy (Fricke 1952; King 1990). This prevents the velocity ellipsoids from having any vertex deviation, the cause of which is still not well understood. It probably reflects the initial conditions present at the star formation site (Mihalas & Binney 1981), and so it is a consequence of departures from steady state, from axial symmetry, or a combination of both (King 1990). In the current model implementation, we have neglected any vertex deviation.

Similarly, it is possible to show that, at the Galactic plane, one of the axes will point toward the Galactic center and the other axis will be perpendicular to the plane; however, for locations far from the plane, these axes may change their orientation (King 1990). Indeed, numerical computations of star orbits using a realistic Galactic potential (Carlberg & Innanen 1987) show that the envelopes of these orbits have a tendency to tilt toward the Galactic center, thus providing an estimate of the velocity ellipsoid orientation far from the plane (Gilmore 1990). In addition, Statler (1989a, 1989b) has shown that the spatial variation of the velocity ellipsoid dispersions is still unknown, and it can be determined only by velocity observations at intermediate latitudes (King 1993), which are not yet available (but see Méndez & van Altena 1996 for a discussion of this in the context of proper motions).

The velocity ellipsoid yields the relative frequency of stars for a given Galactic component as a function of velocity with respect to the mean rotational motion of the stars at a given Galactic position (note however that this is different from the motion of the local standard of rest [LSR]). The $(U', V', W')$-velocities in equation (4) can be thus viewed as
the peculiar velocities of the stars considered. Consequently, the velocity ellipsoids are centered at zero velocity, unless there is some kind of streaming motion present. Streaming motions, which can be recognized as moving groups, have not been included in the code so far, because of their considerable complexity (see, e.g., Dehnen 1998). The model peculiar velocities are converted to heliocentric velocities as described below, and the relative fraction of stars for a range of velocities is accumulated (via eq. [3]) onto the corresponding heliocentric velocities to be compared with the observations. In this way, marginal distributions (histograms) of proper motion and/or radial velocity can be output, subject to any kind of restrictions on the observables implemented in the model.

As mentioned before, in order to convert equation (4) into a function that describes the distribution of velocities for a heliocentric observer, it is necessary to know the orientation of the \((U', V', W')\) system relative to a (fixed) set of axes. In what follows we thus describe the geometry employed to evaluate the velocity distribution function in the observable space of heliocentric velocities, as well as the assumptions concerning the velocity dispersions and velocity lags of the different Galactic components included in the code. We will also compare, whenever possible, our assumptions with those of the only two existing global models that include kinematics, namely, the model described by Ratnatunga et al. (1989, hereafter RBC89), based on the Bahcall & Soneira (1980) star-counts model, and the model by Robin & Orlabuck (1987, hereafter RO87), based on the model of synthesis of stellar populations by Robin & Crézé (1986). RBC89’s kinematic model has been compared with a sample of stars from the Bright Star Catalogue (BSC, Hoffleit & Jaschek 1982), while RO87’s model has been compared with Chiū’s (1980) proper-motion survey toward Selected Areas SA 51 \((l = 189°, b = +21°)\), SA 57 \((l = 69°, b = +85°)\), and SA 68 \((l = 111°, b = -46°)\), and with Murray’s (1986) proper-motion survey toward the south Galactic pole. Both models have been, broadly speaking, successful in predicting the observed distribution of stars as a function of proper motion.

3.2. Geometry of the Kinematics: Heliocentric Velocities

The observed heliocentric velocity of a star at distance \(R\) from the Galactic center (as measured on the Galactic plane) and distance \(Z\) from the Galactic plane can be computed from

\[
V_{\text{hel}} = \bar{V}(R, Z) - V_{\text{LSR}}(R) + V_{\text{pec}} - V_0 ,
\]

where \(\bar{V}(R, Z)\) is the mean rotational velocity for a star located at distance \(R\) from the Galactic center and distance \(Z\) from the Galactic plane, and \(V_{\text{LSR}}(R)\) is the solar LSR velocity (which is useful to distinguish from the LSR speed at any other location in the Galaxy; see § 3.4), while \(V_{\text{pec}}\) is the peculiar velocity of the object considered with respect to its own mean Galactic rotational speed and \(V_0\) is the solar peculiar velocity with respect to the solar LSR. If we use a right-handed cylindrical coordinate system, oriented so that one of the axes points toward the Galactic center \((U\text{-axis})\), another axis points toward Galactic rotation \((V\text{-axis})\), and another axis points toward the north Galactic pole \((W\text{-axis})\), then \(V_{\text{hel}} = (U_{\text{hel}}, V_{\text{hel}}, W_{\text{hel}})\) and the different terms in equation (6) are given by the following expressions:

\[
\begin{align*}
\bar{V}(R, Z) &= \begin{pmatrix} V(R, Z) \frac{r}{R} \cos b \sin l \\ V(R, Z) \frac{R - r \cos b \cos l}{R} \\ 0 \end{pmatrix}, \\
V_{\text{LSR}}(R) &= \begin{pmatrix} 0 \\ V_{\text{LSR}}(R) \\ 0 \end{pmatrix}, \\
V_{\text{pec}} &= \begin{pmatrix} U' \cos \beta \cos \alpha + V' \sin \alpha + W' \sin \beta \cos \alpha \\
- U' \cos \beta \sin \alpha + V' \cos \alpha - W' \sin \beta \sin \alpha \\
- U' \sin \beta + W' \cos \beta \end{pmatrix}, \\
V_0 &= \begin{pmatrix} U_0 \\ V_0 \\ W_0 \end{pmatrix},
\end{align*}
\]

where \((R, Z)\) are the distances on the plane of the Galaxy from the Galactic center and perpendicular to it for an object located at heliocentric distance \(r\) and Galactic latitude and longitude \((l, b)\), respectively, \(R_0\) being the solar Galactocentric (cylindrical) distance. \(\bar{V}\) is the mean rotation velocity for the particular component in question at distance \(R\) and height \(Z\) (in general, different from the rotation curve, especially for large radial velocity dispersion systems; see Méndez & van Altena 1996 and § 3.4 below), while \(V_{\text{LSR}}(R)\) is the LSR velocity of the stars in the solar neighborhood (the solar LSR). The velocities \((U', V', W')\) are the peculiar velocities with respect to those oriented along the principal axes of the velocity ellipsoid for that particular component (eq. [4]). The angles \((\alpha, \beta)\) correct for the tilt of the velocity ellipsoid with respect to the local \((U, V, W)\) system. Finally, \((U_0, V_0, W_0)\) are the components of the solar peculiar velocity with respect to the solar LSR. The angles \((\alpha, \beta)\) are given by

\[
\sin \alpha = \frac{r}{R} \cos b \sin l , \quad \cos \alpha = \frac{R - r \cos b \cos l}{R} , \\
\sin \beta = \frac{r \sin b}{\sqrt{R^2 + Z^2}} , \quad \cos \beta = \frac{R}{\sqrt{R^2 + Z^2}} .
\]

The heliocentric proper motions along the \((U, V)\) axes would be given by

\[
\begin{pmatrix} \mu_U \\ \mu_V \end{pmatrix} = \frac{K}{r} \begin{pmatrix} U_{\text{hel}} \\ V_{\text{hel}} \end{pmatrix} ,
\]

where \(K\) is a conversion factor between the chosen units for the velocities and the proper motions. For example, if the velocities are in kilometers per second, the distance \(r\) is in parsecs, and the proper motions are in arcseconds per year, then \(K\) is approximately equal to 4.74.

If we wish to express the velocities in a spherical system centered on the Sun, then the heliocentric velocity can be computed from equations (6)–(10) via a rotation matrix, in the following way:

\[
\begin{pmatrix} V_{\text{rad}} \\ V_l \\ V_b \end{pmatrix} = \begin{pmatrix} \cos b \cos l & \cos b \sin l & \sin b \\ - \sin l & \cos l & 0 \\ - \sin b \cos l & - \sin b \sin l & \cos b \end{pmatrix} \begin{pmatrix} U_{\text{hel}} \\ V_{\text{hel}} \\ W_{\text{hel}} \end{pmatrix} ;
\]

where

\[\begin{pmatrix} U_{\text{hel}} \\ V_{\text{hel}} \\ W_{\text{hel}} \end{pmatrix} = \begin{pmatrix} U_0 \\ V_0 \\ W_0 \end{pmatrix} .\]
the proper motions in Galactic longitude ($\mu_l$) and latitude ($\mu_b$) would be similarly given by

$$\begin{align*}
\begin{pmatrix} \mu_l \\ \mu_b \end{pmatrix} &= \frac{K}{r} \begin{pmatrix} V_l \\ V_b \end{pmatrix}.
\end{align*}$$

(14)

Of course, the choice of computing the pair ($\mu_U$, $\mu_V$) or ($\mu_l$, $\mu_b$) depends on the particular application (see, e.g., eqs. [1] and [2] and comments following them).

If the peculiar velocities and the solar peculiar motion are neglected, the equations above yield, for $r/R \ll 1$, the classical Oort result:

$$\begin{align*}
\begin{pmatrix} \mu_l \\ \mu_b \end{pmatrix} &= \left( -\frac{1}{2} A \sin 2l \sin 2b \right) \begin{pmatrix} \cos b \left( A \cos 2l + B \right) \\ -A \sin 2l \sin B \end{pmatrix},
\end{align*}$$

where the Oort constants are given by

$$\begin{align*}
A &= \frac{1}{2} \left[ V_{LSR}(R_0) - \frac{dV_{LSR}}{dR} \right]_{R_0},
\end{align*}$$

(16)

$$\begin{align*}
B &= -\frac{1}{2} \left[ V_{LSR}(R_0) + \frac{dV_{LSR}}{dR} \right]_{R_0}.
\end{align*}$$

(17)

We must emphasize that, in our kinematic model, we have not used the approximation given by equation (15) to compute the proper motions in terms of the Oort constants; rather, we have used the more general expressions described above. However, we show them here because of their usefulness for computing $V_{LSR}(R_0)$ and $dV_{LSR}/dR |_{R_0}$ from published values for $A$ and $B$ (see § 5.1).

### 3.3. The Solar Peculiar Velocity and the Motion of the Solar LSR

As can be seen from equation (6), the solar peculiar motion, $V_\odot$, and the motion of the solar LSR, $V_{LSR}(R_0)$, are fixed vectors for a heliocentric observer, and they are also independent of the Galactic component being considered. Therefore, it makes sense to describe the adopted values for these two vectors before discussing the velocity ellipsoid parameters for the disk, thick disk, and halo.

There have been a number of determinations for the solar peculiar motion. The classical result, quoted in Mihalas & Binney (1981), gives ($U_\odot$, $V_\odot$, $W_\odot$) = (+9.0, +12.0, +7.0) km s$^{-1}$, essentially based upon Delhaye’s (1965) own compilation. From their analysis of the BSC, RBC89 obtained (+11.0, +14.0, +7.5) $\pm$ 0.4 km s$^{-1}$. In their Figure 6, they also show that a value of +11.5 km s$^{-1}$ for $U_\odot$ gives a better fit to the proper-motion position angle distribution than does the value of +9.0 km s$^{-1}$ quoted by Delhaye. On the other hand, RO87 have adopted the value derived by Mayor (1974), namely, ($U_\odot$, $V_\odot$, $W_\odot$) = (+10.3, +6.3, +5.9) km s$^{-1}$. As can be seen, there is a rather large discrepancy with Delhaye’s and RBC89’s results in the $V_\odot$ component.

More recent values for the solar peculiar motion do not seem to have converged to a single value, especially in the $V$-component: Ratnatunga & Upgren (1997) have found a value of ($U_\odot$, $V_\odot$, $W_\odot$) = (+8, +7, +6) km s$^{-1}$ from Vysotsky’s sample of nearby K and M dwarfs, with uncertainties of $\pm 1$ km s$^{-1}$. Chen et al. (1997), on the other hand, find ($U_\odot$, $V_\odot$, $W_\odot$) = (+13.4 $\pm$ 0.4, +11.1 $\pm$ 0.3, +6.9 $\pm$ 0.2) km s$^{-1}$ from a large sample of B, A, and F main-sequence stars. An extreme case of the discrepancies is that of Dehnen & Binney (1998), who find ($U_\odot$, $V_\odot$, $W_\odot$) = (+10.0 $\pm$ 0.36, +5.25 $\pm$ 0.62, +7.17 $\pm$ 0.38) km s$^{-1}$ from a carefully selected unbiased sample of Hipparcos stars, while Miyamoto & Zhu (1998) find ($U_\odot$, $V_\odot$, $W_\odot$) = (+10.62 $\pm$ 0.49, +16.06 $\pm$ 1.14, +8.60 $\pm$ 1.02) km s$^{-1}$ from 159 Cepheids, also from the Hipparcos Catalogue. As suggested by RO87, these differences in the $V_\odot$ component are mainly due to difficulties in separating the asymmetric drift from the intrinsic solar motion (see § 3.4), and also because of the peculiar motions exhibited by the very young OB stars, which are still moving under the influence of the spiral arm kinematics and/or of their parent molecular cloud. We have temporarily adopted in the model the solar peculiar motion derived by RBC89, but generally speaking it is a free vector that can be altered to determine the effect of uncertainties on the solar peculiar motion upon comparison with any kinematic survey (see § 5.1).

The IAU adopted in 1985 a value for the motion of the solar LSR of 220 km s$^{-1}$. Kerr & Lynden-Bell (1986) have discussed extensively the determinations of $V(R_\odot)$, as well as $R_\odot$, and the Oort constants $A$ and $B$ available until then. From a straight mean of different determinations they obtained $V_{LSR}(R_\odot) = 222 \pm 20$ km s$^{-1}$ (their Table 4), while from independent determinations of $R_\odot$, $A$, and $B$ (their Tables 3 and 5), we find $V_{LSR}(R_\odot) = 226 \pm 45$ km s$^{-1}$. Although the uncertainties involved in $V(R_\odot)$ are larger than those of $V_\odot$, we shall see that the relative motion of disk stars is more affected by uncertainties in the solar peculiar motion than uncertainties in the motion of the LSR, since the whole nearby disk is moving approximately with the solar LSR.

### 3.4. Disk Kinematics

Méndez & van Altena (1996) have extensively discussed the assumptions employed to describe the kinematics of the disk component in the model, and we will not repeat those here. We shall only mention that velocity dispersions are parameterized in the model as a function of spectral type and luminosity class (Table 2 of Méndez & van Altena 1996). As a result of the model comparisons presented in Méndez & van Altena (1996), a piecewise linear increase of velocity dispersion with distance from the Galactic plane (in the same amount as predicted by theoretical models) has been found to provide a good match to the proper-motion dispersion of disk stars and has been generally adopted here for the disk component in the amounts specified in Table 6 of Méndez & van Altena (1996). In addition, a number of dynamical and observational arguments (see Méndez & van Altena 1996 for details; also Binney & Merrifield 1998) lead to velocity dispersions having the following dependency on Galactocentric distance:

$$\Sigma_{V_0}(R) = \Sigma_{V_0}^0 \exp \left[-(R - R_0)/H_R \right],$$

(18)

$$\Sigma_{V_0}'(R) = \frac{1}{2} \left[ 1 + \frac{d\ln V_{LSR}(R)}{d\ln R} \right] \Sigma_{V_0}(R),$$

(19)

$$\Sigma_{V_0}''(R) = \Sigma_{V_0}''(R),$$

(20)

where $H_R$ is the exponential scale length for the population considered (index $j$) and $\Sigma_{V_0}(R), \Sigma_{V_0}(R), \Sigma_{V_0}(R),$ and $V_{LSR}(R)$ are the velocity dispersions and circular speed (i.e., the rotation curve or, equivalently, the motion of the local standard of rest), respectively, at distance $R$ from the Galactic center for that population.

Méndez & van Altena (1996) have also presented a general equation (their eq. [4], our eq. [21]) that predicts the velocity lag of the disk component in a dynamically
self-consistent way with the adopted velocity dispersions and the adopted rotation curve. In RBC89’s model, the velocity lag was modeled as being proportional to the velocity dispersion, $\Sigma_v$, with the proportionality factor being a free parameter that changed as a function of the kinematic group considered. On the other hand, RO87 tried a more self-consistent approach, by using a simplified version of the asymmetric drift equation to derive the lag for a particular set of stars as a function of the radial density derivative adopted for that particular component, as it follows from the collisionless Boltzmann equation (Mihalas & Binney 1981; Binney & Tremaine 1987). We have fully developed the RO87 procedure, so that the velocity lag for the disk is not a free parameter in itself but is correlated with the adopted density function for the disk and the disk rotation curve, leading to the aforementioned self-consistent expression, which we reproduce here for completeness:

$$V(R, Z) = \sqrt{V_{\text{LSR}}^2(R) - \Sigma_v^2(R) + \left[ 1 - \frac{2R}{H_R} + S(R, Z) \right] \Sigma_w^2(R)},$$

(21)

where $V_{\text{LSR}}(R)$ is the circular speed at Galactocentric distance $R$ (i.e., the motion of the LSR at that position) and $S(R, Z)$ is a function that describes the contribution to the rotational support from the cross term $\Sigma_{\text{uv}}$ (usually referred to as the tilt of the velocity ellipsoid), given by

$$S(R, Z) = \frac{q (\lambda^2 - 1) R^2}{(R^2 + \lambda^2 Z^2)^2} \left( \frac{R^2 - \lambda^2 Z^2}{R^2 + \lambda^2 Z^2} - \frac{|Z|}{H_Z} \right),$$

(22)

where $q$ is 0 if the velocity ellipsoid has cylindrical symmetry or 1 if the velocity ellipsoid has spherical symmetry, $\lambda$ is the (fixed) aspect ratio of the velocity ellipsoid, defined by $\Sigma_v/\Sigma_w$, evaluated at $Z = 0$, and $H_Z$ is the exponential scale height for the population considered.

It is interesting to compare the values derived by this expression with those adopted by other authors. The velocity lag is given by $V_{\text{lag}} = \vec{V}(R, Z) - V_{\text{LSR}}(R)$ (note that, with this definition, the lag is always negative). Representative values for $V_{\text{lag}}$ computed from equations (21) and (22), are listed in Tables 7 and 8 following the (disk) kinematic groups defined by both RBC89 and RO87. The results shown in Tables 7 and 8 have been obtained by evaluating the above equations at $Z = 0$ and $R_\odot = R = 8.5$ kpc and adopting a radial scale length $H_R = 3.5$ kpc, a spherical velocity ellipsoid ($q = 1$), and a locally flat rotation curve with $V_{\text{LSR}}(R_\odot) = 220$ km s$^{-1}$.

From Tables 7 and 8 we see that our derived values for $V_{\text{lag}}$ tend to be slightly larger than those compiled by RBC89, while the agreement with RO87 is good. Since our approach is a refinement of the procedure adopted by RO87, the agreement with their results is not surprising. On the other hand, it is not unreasonable to assume that the uncertainties in Delhaye’s values for $V_{\text{lag}}$ could easily be of the same magnitude as the discrepancies shown in Table 7 (approximately 2 km s$^{-1}$). Figure 1 shows the effect of different assumptions in the evaluation of the mean rotational speed for the old disk ($\Sigma_v = 30$ km s$^{-1}$) in the solar neighborhood. In general, the predicted differences are quite small and would be difficult to detect unless high-accuracy radial velocities ($\sigma_{\text{rad}} < 0.5$ km s$^{-1}$) and/or absolute proper motions ($\sigma_p < 0.1$ mas yr$^{-1}$) were available for samples of old disk stars located at heliocentric distances closer than 1.5 kpc. Future space astrometric missions (e.g., the US FAME project or its European counterpart, GAIA) might actually able to deliver this (or better) proper-motion accuracy for stars down to $V \sim 15$.

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**TABLE 7**

**VELOCITY LAG AS A FUNCTION OF SPECTRAL TYPE COMPARED BY RBC89 AND THOSE COMPUTED FROM OUR MODEL**

| Spectral Type | Luminosity Class | $\Sigma_v$ (km s$^{-1}$) | $\lambda$ | $V_{\text{lag}}$ (RBC89) (km s$^{-1}$) | $V_{\text{lag}}$ (This Paper) (km s$^{-1}$) |
|--------------|------------------|--------------------------|-----------|--------------------------------|---------------------------------|
| sg           | I–II             | 12                       | 1.5       | 0                              | $-1.3$                          |
| OB           | V                | 10                       | 1.7       | 0                              | $-0.9$                          |
| A            | V                | 15                       | 1.7       | 0                              | $-1.9$                          |
| F            | V                | 25                       | 1.9       | 0                              | $-5.0$                          |
| G            | IV               | 25                       | 2.1       | $-1$                          | $-5.0$                          |
| K            | III              | 31                       | 1.8       | $-5$                          | $-8.0$                          |
| M            | III              | 31                       | 1.9       | $-6$                          | $-8.0$                          |

**TABLE 8**

**VELOCITY LAG AS A FUNCTION OF AGE PREDICTED BY RO87 AND THOSE COMPUTED FROM OUR MODEL**

| Age Range (Gyr) | $\Sigma_v$ (km s$^{-1}$) | $\lambda$ | $V_{\text{lag}}$ (RO87) (km s$^{-1}$) | $V_{\text{lag}}$ (This Paper) (km s$^{-1}$) |
|-----------------|--------------------------|-----------|--------------------------------|---------------------------------|
| 0.00–0.15       | 16.7                     | 2.8       | $-1.6$                          | $-2.2$                          |
| 0.15–1.00       | 19.8                     | 2.0       | $-3.6$                          | $-3.2$                          |
| 1–2             | 27.2                     | 2.1       | $-6.7$                          | $-6.1$                          |
| 2–3             | 30.2                     | 1.6       | $-8.5$                          | $-7.9$                          |
| 3–5             | 36.7                     | 1.6       | $-12.6$                         | $-11.8$                         |
| 4–10            | 43.1                     | 1.7       | $-17.2$                         | $-16.2$                         |

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4 See http://www.usno.navy.mil/fame.

5 See http://astro.estec.esa.nl/GAIA/.
that a velocity lag as large as 100 km s$^{-1}$ has to be ruled out (but see § 5.2).

In the field of global kinematic modeling, RO87 used for their thick disk a velocity lag as computed from a simplified version of the asymmetric drift equation (see § 3.4). Even though this approach is able to predict velocity lags as large as $-37$ km s$^{-1}$ (RO87, Table 1), it requires unrealistically large velocity dispersions to obtain velocity lags twice that value (the above quoted lag value was obtained considering a $U$ velocity dispersion for the thick disk of 80 km s$^{-1}$, a value that is already somewhat larger than the values quoted more recently; see § 3.5.1). Furthermore, since the variations of $\Sigma_U(R)$ and $\Sigma_V(R)$ with Galactocentric distance for this component are not known, our self-consistent approach of § 3.4 cannot be applied. We have therefore decided to assume a constant rotation velocity for the thick disk. This approach has been used by Armandroff (1989) in his study of the system of Galactic globular clusters, and by L93 in his study of RR Lyrae stars (this assumption was first presented by Frenk & White 1980 in the context of globular cluster kinematics). Therefore, $V_{\phi}$ has been assumed to have a fixed value for this component, whose possible range goes from around $-100$ to $-40$ km s$^{-1}$. It should be mentioned that it has been suggested (Majewski 1992, 1993, particularly his Fig. 6) that the thick disk exhibits a velocity lag gradient with distance from the Galactic plane amounting to $-36$ km s$^{-1}$ kpc$^{-1}$ to distances from the Galactic plane of up to 7 kpc. The extent and nature of this gradient have however become more elusive from the recent surveys by Soubiran (1993), who found little or no velocity gradient from her north Galactic pole sample, by Ojha et al. (1994), who also found no velocity gradient or even a reverse gradient (i.e., an increase of rotational velocity with height above the Galactic plane) from their anti–Galactic center field, and by Guo (1995), who found little or no velocity gradient from his south Galactic pole sample, although Guo’s sampling of thick disk stars is restricted to distances closer than 4 kpc, as opposed to 7 kpc for Majewski’s sample. In § 5.2, we actually study the effects of adopting a solid-rigid rotation versus models in which we introduce a velocity shear away from the Galactic plane.

3.6. Halo Kinematics
3.6.1. Velocity Dispersion
We have adopted $(\Sigma_U, \Sigma_V, \Sigma_W) = (130, 95, 95)$ km s$^{-1}$, from the review by Gilmore et al. (1989). These values of the velocity dispersion are in agreement with, e.g., L93’s results for the metal-poor ([Fe/H] $<-1.3$) RR Lyrae variables; he found $(\Sigma_U, \Sigma_V, \Sigma_W) = (160 \pm 15, 112 \pm 10, 99 \pm 10)$ km s$^{-1}$. By comparison, RO87 adopted the results of Norris, Bessell, & Pickles (1985), namely, $(\Sigma_U, \Sigma_V, \Sigma_W) = (131, 106, 85)$ km s$^{-1}$.

Since the kinematic properties of the Galactic halo are only poorly constrained by present observational data, we have assumed, as with the thick disk, that the halo velocity dispersions are isothermal. Indeed, the locally determined velocity ellipsoid for the highest velocity subdwarfs, which spend most of their time at very large distances from the Galactic center, shows no evidence for a different anisotropy from that for lower velocity subdwarfs (Gilmore & Wyse 1987). Furthermore, Hartwick (1983) showed that the velocity dispersion of 52 metal-weak halo giants has a similar anisotropy to that shown by the nearby RR Lyrae stars analyzed by Woolley (1978).
Ratnatunga & Freeman (1985, 1989) have found from their own sample of field K giants, and from a number of previous determinations, that the $W$ velocity dispersion, $\Sigma_W$, for halo stars is constant with height up to distances of 25 kpc above the Galactic plane. They also pointed out that a constant $W$ velocity dispersion in spherical coordinates would be inconsistent with the observations, and therefore one of the velocity ellipsoid axes should remain parallel to the Galactic plane and point toward the Galactic axis of rotation. Finally, they found that their data are well represented by a model in which the velocity dispersions in rotation. Figure 2 shows the expected counts as a function of magnitude for the SPM-SGP region. The expected contribution from the different stellar populations is indicated. The disk dominates the counts for $B_J < 14$, while the thick disk becomes an important contributor at fainter magnitudes. By $B_J \simeq 17$ there is an appreciable contribution from all three major Galactic components—the disk, the thick disk, and the halo—although the disk dominates the total counts at all magnitudes. [See the electronic edition of the Journal for a color version of this figure.]

Also, the solid angle being integrated had the very same borders in right ascension and declination as the observed area, such that projection effects are similar in both the observations and the model. The model predictions in this section have been performed with the standard Galactic and stellar population parameters for the model as described in § 3.1 and by Méndez & van Altena (1996), with the modifications indicated by comparisons to faint magnitude and color counts with the Hubble Deep Field (HDF; Méndez et al. 1996) and with the HDF Flanking Fields (Méndez & Guzmán 1998). We should also point out that the evaluations performed here are used merely as a guide to the type of stellar population mapped at different magnitude intervals, and that no attempt has been made to fit the observed magnitude counts, because of the incomplete nature of the sample, as described in § 2.1 (although the color counts in intervals of 1 mag in $B_J$ are fully accounted for sample incompleteness through a free scale factor; see § 4.2). It has been demonstrated that the model run in this "blind mode" is able to accurately reproduce (with less than 1% uncertainty) the observed magnitude and color counts over more than 10 mag at the SGP (Prandoni et al. 1999), as well as to other lines of sight as derived from comparisons with multicolor data from the ESO Imaging Survey, covering several tens of square degrees at different Galactic positions (da Costa et al. 1998; Nonino et al. 1999; Zaggia et al. 1999).

From Figure 2 we can clearly see that the disk is the dominant source of the counts at all magnitudes. However, for $B_J > 14$ the thick disk starts to be important, with a contribution to the counts larger than 10% of that of the disk, per magnitude interval. Similarly, the halo becomes important, in the same sense, for $B_J > 17$.

In the range where the disk is the dominant source of the counts (i.e., for $B_J < 14$), we have a mixture of main-sequence, giant, and subgiant stars. Figure 3 indicates that...
in this magnitude range, giants are slightly dominant over main-sequence stars for \( B - V > 1.0 \), while for bluer color we basically should observe only main-sequence stars. Therefore, any attempt to look at the kinematics, in a differential way, between disk main-sequence and giant stars should approximately follow these two distinct color intervals. Indeed, for colors bluer than \( B - V < 1.0 \) the model predicts a ratio of \((9.5 \pm 0.2) \times 10^{-2}\) disk giants and sub-giants per main-sequence star, while for redder color this ratio becomes \(1.58 \pm 0.05\) (the uncertainties come from Poisson statistics on the expected counts for a complete sample). This, and all subsequent color plots, have been convolved with the color uncertainties in the respective magnitude intervals as derived from the catalog itself to obtain a better idea of how well we can or cannot separate distinct populations of stars.

In the range where the thick disk becomes important while the contribution from the halo is still minimal, i.e., \(14 \leq B_J < 17\), most disk stars are on the main sequence, while we have a mixture of giants and main-sequence stars from the thick disk (see Fig. 4). For \(B - V > 1.0\) we expect very little contribution from the thick disk at all (these would then be mostly disk main-sequence stars), while for bluer colors we have a mixture of disk main-sequence and thick disk main-sequence and giant stars (for \(B - V < 1.0\) the ratio between thick disk and disk stars is 0.336, while the ratio decreases to 0.048 for redder colors). Unfortunately, as can been seen from Figure 4, the color ranges encompassed by thick disk main-sequence stars and giants mostly overlap. However, since the thick disk is parameterized in the model by a single kinematic population, the distinction between main-sequence and giants from the thick disk is not as critical as for the disk, where the kinematic parameters are assumed to be dependent upon the spectral type and luminosity class. This fact allows us to treat all thick disk stars in a common fashion, irrespective of their luminosities.

Finally, in the faintest range \((17 \leq B_J < 19)\), where all three components do contribute to the stellar counts, the disk still dominates for \(B - V > 1.0\), while a mixture of thick disk and halo stars appears at bluer colors, producing a characteristic double-peaked color distribution (Fig. 5). As can be seen from Figure 5, it is not possible to separate thick disk and halo stars from colors alone, and in principle, a simultaneous fit to the kinematic parameters for both populations has to be performed at these magnitudes. We can also see that giants from both populations fall at approximately the same color interval and at about the same rate.
Similar colors are also expected for halo and thick disk main-sequence stars, although the model predicts more thick disk main-sequence stars than halo main-sequence stars; the predicted overall ratio (for all colors) in this magnitude interval is $n_{\text{halo}}/n_{\text{thick disk}} = 0.447 \pm 0.003$. These considerations are important because they indicate that the derived kinematics for the halo would necessarily be based on relatively few stars from this population falling in our sample.

The mean distance for the different populations sampled as a function of magnitude is shown in Table 9. We see that disk stars are sampled to less than 1 kpc from the plane, even at the faintest magnitude bins. The thick disk is sampled on a range of about 2 kpc, and up to almost 4 kpc, while the halo sample is based on distant stars located at typical distances of 5 kpc from the plane.

4.2. Color Counts

As expressed before, the magnitude counts suffer from a selective incompleteness as a function of magnitude. However, at a given magnitude bin, the color distribution (just as the proper-motion distributions) will differ from the complete sample distribution only by a scale factor. In this section we thus compare the observed color histograms with the model predictions in the preselected $B_J$ magnitude bins of the survey, using a free scale factor to go from the model-predicted to the observed color counts.

Figures 6 and 7 show a comparison of the standard model being used here and the SPM color counts. As shown in § 4.1, for $B_J < 14$ we have mostly disk stars. Figure 6 shows the error-convolved color distributions in the bright-magnitude portion of the survey, while Figure 7 shows the color distributions for the faint portion. In all cases a scale factor has been applied to the model counts to bring them onto the observed counts. At $B_J < 16$ the scale factor was computed by forcing the model to have the same number of stars as observed in the color range $0.0 \leq B-V \leq 1.5$, where the majority of the stars are found, while for fainter magnitudes the scale factor was computed from the extended range $0 \leq B-V \leq 2.0$.

In general, we note a very good fit to the observed color counts at all magnitudes. However, there are several features worth mentioning. At faint magnitudes ($B_J > 14$) the fit is extremely good, apparently without any further refinements needed to the model, at least within the uncertainties of the counts and the photometric errors. At fainter magnitudes ($B_J > 17$), the appearance of the characteristic double-peaked color distribution due to blue halo and thick disk turnoff stars and red M dwarfs from the disk becomes less distinct as a result of the photometric errors. Nevertheless, the important point here is that the fit to the overall counts is very good, with the slight indication of a small systematic effect in our photometry in the range $0.2 \leq B-V \leq 0.8$ on the amount of $-0.05$ mag in $B-V$. We note that, while the effect of changing the scale height of these (bright) main-sequence stars has a minimal impact on the predicted color counts in the range $14 < B_J < 17$, the effect becomes more important in the last two magnitude bins (where photometric errors are quite large), indicating that the currently adopted value of 325 pc for bright M dwarfs provides a better fit to the color counts than does the smaller value of 250 pc suggested from studies of local fainter M dwarfs in the disk (Méndez & Guzmán 1998).

At bright magnitudes, the situation is more confusing: On one hand, it is apparent that the model is predicting slightly more giants than observed (the red peak in the distributions for $B_J < 12$). On the other hand, it seems that our model predictions are bluer than observed for $B-V < 0.7$ in certain magnitude bins, while in others the fit is quite good (see Fig. 6). To explore the origin of these discrepancies, first it is interesting to note that in the magnitude range $13 < B_J < 14$ we expect to see almost no disk giants, and, at the same time, the contribution from thick disk stars is negligible. This magnitude range is therefore ideal to explore the origin of the discrepancy, where the model is actually bluer than the observed counts, even for $B-V < 1.0$. We have run several models to see whether the observed discrepancy can be accounted for in a reasonable way by tuning up some of the model parameters. Since the major contribution to the counts in this magnitude range comes from disk main-sequence stars, we have concentrated on what determines the shape of their expected counts. There is only one overall relevant model parameter, the scale height, determining the contribution of disk main-sequence stars in this magnitude range. We first note that the model predictions indicate that the absolute magnitude range sampled by this color distribution encompasses the range $+3.2 \leq M_V \leq +4.2$, i.e., these are still quite bright main-sequence stars, at a point where their scale height is known to be increasing quickly with (fainter) absolute magnitude. In our model, we have adopted a variable scale height for main-sequence stars to account for the known fact that older stars have diffused to larger distances from the Galactic plane than younger stars (Wielen & Fuchs

| $B_J$ Range | Disk Main-Sequence Stars (pc) | Disk Giant Stars (pc) | Thick Disk Stars (pc) | Halo Stars (pc) |
|-------------|-------------------------------|-----------------------|-----------------------|----------------|
| 9–10........ | 160                           | 380                   | ...                   | ...            |
| 10–11........ | 190                           | 500                   | ...                   | ...            |
| 11–12........ | 250                           | 620                   | ...                   | ...            |
| 12–13........ | 330                           | 790                   | ...                   | ...            |
| 13–14........ | 440                           | 1070                  | ...                   | ...            |
| 14–15........ | 570                           | ...                   | 2070                  | ...            |
| 15–16........ | 710                           | ...                   | 2060                  | ...            |
| 16–17........ | 810                           | ...                   | 2310                  | ...            |
| 17–18........ | 890                           | ...                   | 2920                  | 5290           |
| 18–19........ | 840                           | ...                   | 3600                  | 5600           |
The functional form giving the scale height as a function of absolute visual magnitude described by Miller & Scalo (1979) and Bahcall & Soneira (1980), which seems to be a good representation of the available observational data (see also Gilmore & Reid 1983), has been included in our model in the way indicated by Bahcall (1986).

Figure 8 shows the results of these runs with extreme parameters for the scale height of main-sequence stars [$H_d$(MS)]. The compilations by Miller & Scalo (1979) and Bahcall & Soneira (1980), as well as the results from Gilmore & Reid (1983), seem to indicate that $H_d$(MS) is approximately constant for $M_v \leq +2$ (~90 pc) and for $M_v > +5$ (~325 pc). Following Bahcall (1986), we have used a linear interpolation between $M_v = +2$ and $M_v = +5$. We have considered two extreme cases, taken from the range allowed by observational data (Gilmore & Reid 1983), by assuming a "lower" envelope and an "upper" envelope for $H_d$(MS). The lower envelope is described by a...
scale height of 50 pc for early-type stars and 300 pc for later-type stars; the upper envelope is described by a scale height of 120 pc for early-type stars and 400 pc for later-type stars. The shapes of the upper and lower envelopes are self-similar, in that the slope of the linear interpolation for $H_Z(\text{MS})$ between the early- and late-type stars was kept constant at the same value adopted by Bahcall (1986), namely, $-84 \text{ pc}/M_\odot$. It is apparent from Figure 8 that we cannot effectively distinguish between a lower $H_Z(\text{MS})$, an increase in reddening of $+0.05 \text{ mag}$ in $E(B - V)$, and a $-0.05 \text{ mag}$ systematic effect on the colors. However, we can rule out the first two alternatives on the grounds of previous studies. For example, Méndez & van Altena (1996) were able to set the overall level for $H_Z(\text{MS})$ in the range $+2 \leq M_V \leq +4$ from comparisons with magnitude and color counts in two intermediate-latitude fields. They found a scale height very similar to the one adopted in the standard model used here—thus ruling out the small scale height solution. Our model runs have adopted a reddening of $E(B - V) = 0.03$ at the SGP, and therefore an increase of

Fig. 7.—Same as Fig. 6, but for the fainter portion of the survey. The solid lines indicate our model predictions for a scale height of 325 pc for M dwarfs, while the dashed lines indicate the model predictions for a smaller scale height of 250 pc. The fit of the model predictions to the observed counts does not suggest any modifications to the standard parameters in the model.
preserving the already existing good fit at all other (brighter and fainter) magnitudes. For this reason, we believe that our colors do have a small, but noticeable, systematic effect at some specific magnitudes, most noticeable at $11 \leq B_J < 14$. Indeed, it is not far-fetched to assume that our photographic photometry could have a systematic error of such amount, especially when considering that a number of internal calibration procedures had to be applied in order to make use of all the grating images on the plates, and that the faint photometric zero points were established only from one or two CCD frames placed arbitrarily in the field, and covering a tiny fraction of a full plate (for details see Platais et al. 1998).

The other point concerns the fit to the red peak at bright magnitudes. It does seem as though the model is overpredicting the contribution of disk giants. Indeed, as shown in Figure 9, the predicted number of red giants is extremely sensitive to the adopted value for its scale height [$H_d(G)$]. We have performed a $\chi^2$ fit to the observed colors in the magnitude range $9 \leq B_J < 12$, where the contribution from giants is most important. The model predictions included a “minimal model” with a scale height of 150 pc and a “maximal model” with a scale height of 250 pc. The minimum $\chi^2$ was computed in the color range $0.7 \leq B-V \leq 1.5$ by interpolating between the two extreme model predictions, and by always scaling to the total observed color counts in the same color range. Separate $\chi^2$ fits were performed in the ranges $9 \leq B_J < 10$, $10 \leq B_J < 11$, and $11 \leq B_J < 12$, and the mean (weighted by the number of stars used in the fit) and its standard deviation value for the giant’s scale height turned out to be $H_d(G) = 172 \pm 7$ pc. Figure 10 shows the resultant models adopting this $H_d(G)$ for disk giants, and where some attempt has been made to correct for slight systematic shifts in the SPM photometry. It is important to note that the fits were performed only in the color range $0.7 \leq B-V \leq 1.5$, and therefore the improved fit outside this range provides an indication of the properness of the parameters adopted to describe the main-sequence disk color counts. As expected, runs with this new scale height render a much better fit to the overall color counts in the whole range $9 \leq B_J < 14$, and we adopt this value throughout.

5. MODEL COMPARISONS WITH THE ABSOLUTE PROPER MOTIONS

In this section we present comparisons between the observed absolute proper motions derived from the SPM-SGP data and our model predictions. The basic standard kinematic parameters employed in the model have been described by Méndez & van Altena (1996), and in § 3 above. Here we present only a summary of the basic assumptions.

For the peculiar solar motion we have adopted the value derived by RBC89, namely, $(U_\odot, V_\odot, W_\odot) = (+11.0, +14.0, +7.5)$ km s$^{-1}$, and a flat rotation curve with $V_{LSR}(R_\odot) = +220$ km s$^{-1}$. It must be emphasized that, since we are analyzing data near the SGP, our model predictions do not span a large range in Galactocentric distance, and therefore the model is not overly sensitive to the value adopted for the slope of the rotation curve near $R_\odot$ (but see § 5.1). As for the disk kinematics, we have adopted the velocity dispersions indicated in § 3.4 with the scale heights adopted in § 4.2, a scale length of 3.5 kpc (again, the model is not sensitive to this last parameter, as it enters as a function of the Galacto-
Fig. 10.—Same as Fig. 6, except that our best \( \chi^2 \) scale height of 172 pc for disk giants has been used. The solid line is the run with the standard model, while the dashed line indicates the run with the modified scale height. In the magnitude ranges 11 \( \leq B_J \leq 12, 12 \leq B_J \leq 13, \) and 13 \( \leq B_J \leq 14, \) systematic shifts of \(-0.05, -0.08, \) and \(-0.05\) mag in \( B-V \), respectively, have been applied to the data.

centric distance, which for the SPM-SGP data is quasi-constant; see eq. [21]), and a value of \( q = 0 \) for the velocity ellipsoid (see eq. [22]; \( q = 0 \) implies a velocity ellipsoid parallel to the Galactic plane at all heights from the Galactic plane, while \( q = 1 \) is for a velocity ellipsoid that points toward the Galactic center).

For the thick disk we assume an isothermal velocity dispersion equal to \( (\Sigma_U, \Sigma_V, \Sigma_W) = (70, 50, 45) \) km s\(^{-1}\), as a compromise value between different determinations (see § 3.5.1), and a constant velocity lag of 40 km s\(^{-1}\) with respect to the motion of the LSR. For the halo, we instead adopt \( (\Sigma_U, \Sigma_V, \Sigma_W) = (130, 130, 95) \) km s\(^{-1}\) from Gilmore et al. (1989), and a zero net rotation velocity about the Galactic center.

The three-dimensional integration of the velocity ellipsoid at all distance shells required by the model (see eq. [4]) is very expensive in terms of CPU cycles. Therefore, some trial runs were needed to select the proper integration
resolution. It was found that a Gaussian-normalized resolution of 0.2 between −3.0 and +3.0 in \( U/\Sigma_U, V/\Sigma_V, \) and \( W/\Sigma_W \) leads to differences in the derived proper-motion median and dispersions smaller than 0.1 mas yr\(^{-1}\), except at the brightest bins, where the differences were in any case smaller than 0.3 mas yr\(^{-1}\).

We should note that, because of the geometry for the SGP data, our model comparisons are most sensitive to the motions in the direction of Galactic rotation and along the Galactic center-anticenter direction, and not to the motion perpendicular to the Galactic plane (see eqs. [1] and [2], and the discussion following them).

Finally, all model predictions have been convolved with the proper-motion errors as found in § 2.1 (Tables 5 and 6) before computing any kinematic parameter to be compared with the observed distributions.

We have investigated the effects of changing the mean reddening at the SGP from \( E(B-V) = 0 \) to \( E(B-V) = 0.06 \) on the predicted kinematic parameters and found that the maximum changes occurred at the brighter magnitudes but were in all cases smaller than 0.2 mas yr\(^{-1}\) in the median \( \mu_U \) and 0.4 mas yr\(^{-1}\) in the median \( \mu_V \). The proper-motion dispersions changed by, at most, 0.4 mas yr\(^{-1}\) in \( \Sigma_{\text{rot}} \) and by 0.3 mas yr\(^{-1}\) in \( \Sigma_{\text{proj}} \). As can be seen from Tables 2, 3, and 4, these changes are smaller than the 1 \( \sigma \) observed uncertainties and therefore do not play an important role in this discussion.

### 5.1. Solar Motion and the LSR Speed

In this subsection we present our model comparisons as a function of apparent magnitude, and the sensitivity of those predictions with respect to the assumed values for the solar peculiar motion and the speed of the LSR.

Figures 11 and 12 show the observed and the model predictions in the median proper motions and the proper-motion dispersions, respectively, while Table 10 indicates the values for the different kinematic parameters derived from the standard model (called run 1). In Table 10, the first two rows of each magnitude entry indicate the values for the median and dispersion on the \( U \)-component of the proper motion, while the last two rows indicate the same parameters for the \( V \)-component. Table 10 lists only model predictions for the most representative runs, as otherwise the table would be too cluttered without adding much information for the reader. Table 11 instead gives a summary description of all simulations presented in this paper. For the standard run (run 1), the model parameters are those described above. Figure 11 clearly shows that while the standard model produces a very good fit to the \( U \)-component of the proper motion (along the Galactocentric direction), the \( V \)-component (along Galactic rotation) is grossly underestimated, especially for \( B_J < 14 \), where the disk component dominates the overall kinematics. A comparison with a model having a scale height for disk giants of 250 pc (run 2) does not resolve the problem. Indeed, this solution also produces a bad fit to the predicted motion in \( U \) at the brightest magnitudes, and therefore it reinforces the value of 172 pc found before from the color counts alone.

We have also tried a run with \( q = 1 \) (see eq. [22]; run 3). In this case, the proper motion in \( U \) is not affected (as expected from the projection effects), while the motion in \( V \) is slightly shifted downward (i.e., more negative lags) in an almost systematic way, in the sense of *increasing the discrepancy* with the observed proper motion. A more radical change in the model predictions occurs when we change the solar peculiar motion from the standard value of \( V_\odot = +14 \) km s\(^{-1}\) to \( V_\odot = +5 \) km s\(^{-1}\) (run 4). The largest effect occurs at the brightest bins, i.e., for nearby disk stars, where the solar peculiar motion dominates the reflex motion, while the change becomes less important (although still noticeable) at fainter magnitudes, where one is sampling objects from the other Galactic components located at larger distances, where the dominant effect is that of the overall rotation of the disk and the relative state of rotation between the different Galactic components. This is clearly shown (Fig. 11) by a run in which we keep the old solar peculiar velocity but change the overall rotation speed for the LSR from the IAU-adopted value of \( +220 \) km s\(^{-1}\) to \( +270 \) km s\(^{-1}\) (run 5), as suggested by recent *Hipparcos* results (Miyamoto & Zhu 1998). At the brightest bins, the effect of changing \( V_{\text{LSR}}(R_\odot) \) also becomes noticeable because of the larger fraction of bright giants, which can be seen to large distances, and where the differential rotation effects become amplified. A run in which we simultaneously change the solar peculiar motion and the LSR rotational...
speed to +5 and +270 km s\(^{-1}\), respectively, is given by run 6. We note that, in this case, the velocity lags for the thick disk and halo are increased in proportion to the increase of the disk's rotational speed, as the net rotation of those two components is kept constant at 180 and 0 km s\(^{-1}\), respectively (see §§ 3.5.2 and 3.6.2). We conclude that run 6 clearly provides a much better fit to the median motion in \(V\) than does run 1.

The median proper motion in the \(U\)-component shows a good fit to the observed values, and the changes in the \(V\)-component described above do not affect this parameter in a major way, because of the orthogonality of the projection effects toward the Galactic poles. These results do show us, though, that the adopted value for the solar peculiar motion in this direction is the correct one. To explore the sensitivity of the model predictions to this parameter, runs 8 and 9 (Fig. 14 below) show the effect of changing the standard \(U_\odot = +11.0 \text{ km s}^{-1}\) by \(\pm 3 \text{ km s}^{-1}\). An eyeball fit from Figure 14 suggests for the \(U\)-component of the solar motion a value \(U_\odot = +11.0 \pm 1.5 \text{ km s}^{-1}\). Also, there is no indication of a local expansion or contraction of the Galactic disk, as also found from the kinematics of local molecular clouds (Belfort & Crovisier 1984).

The predicted proper-motion dispersions (Fig. 12) are less affected than the median proper motions by the changes described above. In particular, we see from Figure 12 that the greatest change comes from a change of the giant stars’
velocity dispersions in the giants and subgiants are perhaps a bit overestimated. We adopt values of \( \text{km s}^{-1} \) on the small value, these results would mean that our proper-motion dispersion is well matched by the model predictions. Di\( \text{e} \)erent internal and external tests on the observed motions, it is relevant to specify the degree of impact on the predicted proper-motion dispersion in the model version of the catalog. Also, a detailed comparison between the SPM proper motions and the \( \text{Hipparcos} \) motions at the bright end of our sample revealed a small offset between the \( \text{Hipparcos} \) and the SPM 1.1 proper motions, mostly in declination, in the sense \( \mu_{\text{Hp}} - \mu_{\text{SPM}} = -0.73 \pm 0.06 \) (m.e.). This discrepancy is actually quite small if we consider that the SPM motions were tied to the extragalactic system at faint magnitudes, where galaxies are measurable in large numbers, while the \( \text{Hipparcos} \) motions rely on the calibrated observations of bright stars. We have thus to bridge a range of about 10 mag to compare the SPM and the \( \text{Hipparcos} \) proper-motion zero points, and it should not be surprising to find a small offset between the two systems. In order to assess the effect of the small offset observed between the SPM and \( \text{Hipparcos} \) catalogs, we have compared the derived SPM motions from SPM 1.0, SPM 1.1, and SPM 1.1 reduced to the \( \text{Hipparcos} \) system. This is shown in Figure 13 for the median proper motions. The effect of these uncertainties on the \( U \)-component is quite small, but it is slightly larger for the \( V \)-component. Still, the use of any of the catalogs does not invalidate our previous conclusions, or the discussion that follows. The proper-motion dispersions, on the other hand, show a variation of less than 0.02 mas yr\(^{-1}\) and are therefore considered negligible in comparison with the modeling effects discussed here. For definiteness, we adopt version 1.1 of the catalog, without the correction for the declination offset to the \( \text{Hipparcos} \) system. The fact that our overall motions are not affected by the systematic effects discussed previously is important in the context of the velocity lags for the thick disk and halo (see, e.g., § 3.5.2).

We have pointed out that a change in the orientation of the velocity ellipsoids from a cylindrical to a spherical projection tends to produce a slightly higher value for the velocity lag, especially at fainter magnitudes (compare runs 1 and 3 in Fig. 11). The upper solid line in Figure 11 shows that while the fit of the model to the observed data for \( V_\odot = +5 \text{ km s}^{-1} \) and \( V_{\text{LSR}}(R_\odot) = +270 \text{ km s}^{-1} \) is good at bright magnitudes \( (B_H < 14) \), the model underestimates the lag at fainter magnitudes. However, as indicated above, by changing the orientation of the velocity ellipsoid we can actually increase the lag. This is shown in Figure 14 (bottom), where a model with \( V_\odot = +5 \text{ km s}^{-1} \), \( V_{\text{LSR}}(R_\odot) = +270 \text{ km s}^{-1} \), and \( q = 1 \) (run 7) yields a better overall fit to the observed median motion in the \( V \)-component than does a model with \( q = 0 \). The model predictions from run 7 are also indicated in Table 10.

We have also further explored the origin of the slight overestimate of the predicted proper-motion dispersion in the \( V \)-component seen in Figure 12 (bottom). Even though

| Run | Model Parameters | General Comments |
|-----|------------------|-----------------|
| 1    | Standard but \( H_2(G) \approx 170 \text{ pc} \) | Poor fit to \( V \)-component |
| 2    | Standard model* | Poor fit to both \( U \) and \( V \)-components |
| 3    | As run 1 but \( q = 1 \) | Decreases predicted \( V \)-motion, poorer fit |
| 4    | As run 1 but \( V_\odot = +5 \text{ km s}^{-1} \) | Better fit to brighter motions in \( V \) |
| 5    | As run 1 but \( V_{\text{LSR}}(R_\odot) = +270 \text{ km s}^{-1} \) | Better fit to fainter motions in \( V \) |
| 6    | As run 1 but \( V_\odot = +5 \text{ km s}^{-1} \) and \( V_{\text{LSR}}(R_\odot) = +270 \text{ km s}^{-1} \) | Better fit to overall motions in \( V \) |
| 7    | As run 6 but \( q = 1 \) | Improves fit to fainter \( V \)-motion |
| 8-9  | As run 5, but changing \( U_\odot \) by \( \pm 3 \text{ km s}^{-1} \) | No expansion or contraction of disk |
| 10   | No \( [Z] \)-gradient in (disk) velocity dispersion in \( V \) | Inconsistent with \( U \) and \( W \) observed gradients |
| 11   | Large LSR slope \( \left[ \frac{dV_{\text{LSR}}(R)}{dR} = -11.7 \text{ km s}^{-1} \right] \) | Bad fit to \( \Sigma_5 \) |
| 12   | As run 7, but \( dV_{\text{LSR}}(R) = -2.4 \text{ km s}^{-1} \) | Best-fit model from disk |
| 13   | 10% decrease in model \( \Sigma_5 \) | Bad fit to \( \Sigma_5 \) and predicted lag too small |
| 14   | Thick disk rotational velocity of \( +160 \text{ km s}^{-1} \) | Improves fit to fainter motions in \( V \) |
| 15   | Thick disk velocity gradient of \( -36 \text{ km s}^{-1} \text{ kpc}^{-1} \) | Best fit to fainter motions in \( V \) |

* As in Méndez & van Altena 1996, with \( H_2(G) \approx 250 \text{ pc} \).

** Standard model adopted \( V_\odot = +12 \text{ km s}^{-1} \).

*** Standard model adopted \( V_{\text{LSR}}(R_\odot) = 220 \text{ km s}^{-1} \).

* Standard value is \( U_\odot = +11 \text{ km s}^{-1} \).

* Run 7 had a zero slope for \( V_{\text{LSR}}(R_\odot) \).

* This slope is the value derived from the \( \text{Hipparcos} \) and ground-based values for Oort’s constants \( a \) and \( b \).

* For an assumed disk speed of \( \pm 270 \text{ km s}^{-1} \) this implies a velocity lag of \( -110 \text{ km s}^{-1} \) for this Galactic component.

* See eq. (23), from Majewski 1993.
FIG. 13.—Median observed absolute proper motions along $U$ (Galactocentric direction, top) and $V$ (Galactic rotation, bottom). The circles are for version 1.1 of the catalog, the triangles are for version 1.0, and the squares for version 1.1 with a declination offset as found from a comparison with the Hipparcos motions at the bright end of our sample. An offset of $0.15 \text{ mag}$ has been applied to the points in order to avoid crowding. Although the changes are noticeable, the possible systematic effects still present in the SPM-SGP motions are negligible in comparison with the discrepancies to the model seen in Figs. 11 and 12. The proper-motion dispersions are the same for all three catalogs and are therefore not shown.

The overestimate is small, it is clearly systematic and it could, therefore, be due to a (set of) wrong assumptions in the kinematic model. The velocity dispersion in the $V$-component is not an entirely free parameter in the model. It is actually derived from the $U$ velocity dispersion and the slope of the rotation curve, as shown in equation (19), and therefore can be used to explore either of these parameters. In addition to the Galactocentric dependency made explicit by equations (18)–(20), there is also a dependency on distance from the Galactic plane, which has been described in Méndez & van Altena (1996), in the sense that the velocity dispersion for disk stars increases as a function of distance to the plane ($|Z|$). This increase as a function of $|Z|$ has been found to be necessary in the model, through comparisons with intermediate-latitude proper motions, mostly in the $U$- and $W$-components (Méndez & van Altena 1996), but it has not been tested so far for motions in the $V$-component. Therefore, a natural test was to turn off the aforementioned expected increase in $\Sigma_V$ versus $|Z|$, to see what would be the effect upon the derived dispersion in the respective component of the proper motions: this is shown in Figure 15 (top) by the dashed line (run 10). As expected, the predicted dispersions are smaller and agree quite well with the observed ones. However, by decreasing the velocity dispersions in $V$, the circular speed for the disk increases (see eq. [21]), thus producing a smaller lag, and as a result, the fit to the median motion as a function of $B_J$ becomes significantly worse (Fig. 15, bottom; dashed line).

Theoretically, it would be hard to explain why the $U$- and $W$-components of the motion do show this increase while the $V$-component does not (Fuchs & Wielen 1987). Therefore, we have explored whether changes in the slope of the rotation curve, or a straight reduction in the local values of $\Sigma_U$ (from which $\Sigma_V$ is derived; see eq. [19]), could be held responsible for the poor fit. Figure 15 shows the results when leaving the $|Z|$-gradient untouched, but for the case of a very large slope in the local rotation curve, $dV_{LSR}(R)/dR = -11.7 \text{ km s}^{-1}$ (corresponding to the lower $3 \sigma$ value derived from the Oort constants, namely, $A = 14.4 \pm 1.2 \text{ km s}^{-1}$ and $B = -12.0 \pm 2.8 \text{ km s}^{-1}$; Kerr & Lynden-Bell 1986 [run 11]), and for a 10% decrease in the local values for the $U$ velocity dispersion (run 13), respectively. The large LSR slope solution (run 11) is somewhere in between runs 7 and 10, producing a worse fit in
A 10% reduction in \( \sigma \) (run 13) produces an effect on the proper-motion dispersion that is similar to a large gradient in the LSR speed (run 11; Fig. 15, top), but it also produces an undesired large change in the computed mean motion for disk stars, in the sense of increasing the net rotation and therefore decreasing the lag predicted by the asymmetric drift equation, as is indeed expected from equation (21). The overall effect of a 10% reduction in \( \Sigma_V \) is that run 13 does not seem to provide a good representation of the SPM-SGP data. The change in the predicted dispersion in the \( U \)-component from run 13 is actually quite small, and we obtain a fit that is as good as the one from run 1 in Figure 12 (top). Finally, we note that a small change in the slope of the rotation curve, from zero slope in run 7 (Fig. 15, solid line) to \( dV_{LSR}/dR = -2.4 \, \text{km s}^{-1} \cdot \text{kpc}^{-1} \) (as predicted by Oort's constants; Fig. 15, dotted line [run 12]), does have a correspondingly small change in both the median and the dispersions, and we adopt this last run as our best-matching model to the observed motions, from changing the kinematic parameters for the disk component alone (see also Table 10). However, from this discussion it is clear that we do not seem to be able to fit simultaneously, and without a small magnitude-dependent bias (\( \sim 0.5 \, \text{mas yr}^{-1} \)), the proper-motion dispersion and the median motion in the \( V \)-component with the current assumptions in the model.

A possible origin for the discrepancy between the observed and predicted dispersions in the \( V \)-component could be an overestimation of our proper-motion measurement errors. Indeed, as explained before, all of our model predictions are convolved with the uncertainties in the catalog. This convolution, of course, increases the expected width of the proper-motion distributions, in accordance with the scatter introduced by the measurement errors. We have tested whether this could actually have an impact on our derived (model) dispersions. For this, we have decreased all of our measurement errors as given in Tables 5 and 6 by an amount equal to 0.5 mas yr\(^{-1}\). We then convolved our model predictions with these new errors and computed the median and dispersion in exactly the same manner as done before. The result of this test is that our derived values were minimally affected by the decrease in the measurement errors. In particular, the proper-motion dispersions changed (decreased) depending on the magnitude interval, but in the magnitude range \( 8 \leq B_J < 14 \) the change was smaller than 0.05 mas yr\(^{-1}\), and in the range \( 14 \leq B_J < 17 \) smaller than 0.1 mas yr\(^{-1}\). So, while some of the discrepancies could be due to this effect, it is quite possible that a fraction of the effect seen is due to inadequacies in the model. An interesting outcome of this experiment was that the computed median motion is largely insensitive to the error convolution, with variations of less than 0.03 mas yr\(^{-1}\) for a decrease of 0.5 mas yr\(^{-1}\) in the uncertainties of the observed motions. Therefore, given the already good fit to the observed median proper motions in \( U \) and \( V \) as a function of \( B_J \), it seems quite likely that, whatever the inadequacy in the model is, it is mostly affecting the predicted proper-motion dispersions, and not the predicted mean motions.

### 5.2. Kinematics of the Thick Disk and Halo

We have run several models under different assumptions regarding the thick disk and halo velocity dispersions and velocity lags. The results are shown in Figures 16 and 17, where we have also included run 12 (the best-fit model from § 5.1). In general, we do not have a sensitivity to the assumed velocity dispersions in the model for either the thick disk or the halo, although we clearly see changes in the model predictions for different assumptions involving the net motion of both components. Changes in the (constant) mean motion of the thick disk and halo are magnitude dependent because of the changing mixture of these populations as a function of apparent brightness. Figure 16 (top) shows that changes in the net rotation of the thick disk act mostly as a zero-point shift in the proper motions at faint magnitudes, without altering the shape of the predicted motions. The best fit in Figure 16 (dashed line) is provided by a model with a thick disk velocity of \(+160 \, \text{km s}^{-1}\) (run 14), while we can clearly see that, for an assumed LSR speed of \(270 \, \text{km s}^{-1}\), a run with a velocity lag of \(-40 \, \text{km s}^{-1}\) (dotted line) does not provide a good fit to the median motion in \( V \). A net rotation for the thick disk of \(+160 \, \text{km s}^{-1}\) would imply then, for our preferred LSR speed, a large velocity lag of \(-110 \, \text{km s}^{-1}\) for this component; this would be in agreement with earlier results by Wyse & Gilmore (1986) from an analysis of Chiu's (1980)
absolute proper motions. One way of decreasing this velocity lag is to compensate it by applying a lag to the thick disk. Indeed, the thick disk has been ascribed a velocity gradient away from the Galactic plane (see § 3.5.2). We have tested this possibility by including a rather steep gradient of 36 km s$^{-1}$ kpc$^{-1}$ in the mean motion for this component as per Majewski (1993; his Fig. 6). In this case, the mean motion for the thick disk $\langle \vec{V}(R, Z) \rangle_{\text{thick disk}}$ is computed from

$$\langle \vec{V}(R, Z) \rangle_{\text{thick disk}} = V_{\text{LSR}}(R) - 10 - 36 |Z|/1000,$$

(23)

where $|Z|$ is in parsecs. The results from this model (run 15; see also Table 10) are shown in Figure 16 (bottom; dashed line). This model does indeed produce a better fit to the median motion in the $V$-component than do runs 12 and 13, by predicting a flatter secular proper motion at bright magnitudes while increasing the lag at the faintest bins. Because equation (23) gives the lag with respect to the currently adopted value for $V_{\text{LSR}}$, and Majewski had adopted 220 km s$^{-1}$, we have also tried a model where we have normalized the mean motion to a similar value by adjusting the zero point above from 10 to 60 km s$^{-1}$. This run (shown also in Fig. 16, dot-dashed line), produces a lag that is too large, incompatible with the data at the intermediate and fainter bins. Similarly, a model in which we keep equation (23) but adopt $V_{\text{LSR}} = 220$ km s$^{-1}$ produces the opposite effect (dotted line); in this case we get a bad fit to the secular proper motions at bright magnitudes. We thus conclude that the SPM data are better fitted by a model (run 15; Table 10) with a thick disk having a velocity lag similar to that proposed by Majewski (1993).

As for the kinematics of the halo, our photometric errors prevent us from using colors to cleanly separate halo stars from disk stars, and therefore our sensitivity to model changes on parameters describing the dispersions and velocity lag for the halo is unfortunately small. This is demonstrated by Figure 17, where we show the model predictions for a halo rotating at $+40$ km s$^{-1}$, and a counterrotating halo at $-40$ km s$^{-1}$. It is clear that we cannot effectively distinguish between these three models, and that fainter proper motions, or better colors to minimize the disk contamination, would help to discriminate between these different model runs.

### 5.3. Final Proper-Motion Histograms

In this section we compare the detailed observed proper-motion histograms with those derived from run 15. This
comparison is mostly intended to show the extent of the discrepancies between the observed and predicted motions, and to advance some possible explanations for these discrepancies. Figures 18, 19, and 20 show the predicted versus observed motions in 1 mag intervals in \( B_J \). Both observations and model predictions have been convolved with the observational errors, and the resultant numbers binned in 2 mas yr\(^{-1}\) bins. Furthermore, model predictions have been scaled to the total 1 mag bin counts in the range \(-80 \text{~mas yr}^{-1} < \mu \leq +80 \text{~mas yr}^{-1}\), independently in the \( U\) and \( V\)-components. In general, we see a very good agreement between the model and predicted motions along the \( U\)- and \( V\)-components. In the range \( 10 < B_J < 13 \), we seem to have more stars at small proper motions than those predicted by the model. The effect of this is that the observed median motion is shifted to smaller (absolute) values in the sense of decreasing the observed lag. This effect can be clearly seen in Figure 14, where the model median proper motion in \( V \) in the range \( 11 < B_J < 13 \) is underestimated. In this magnitude range we can also see some structure in the \( U\)-component of the proper motion (Fig. 18, left). Both of these effects could be due to the presence of moving groups. The extent and characterization of these substructures would require a clustering analysis, which is beyond the scope of this paper. We note however that, whatever the cause of the discrepancies between the observed and the predicted model median proper motion, it goes away at fainter magnitudes. In the range \( 13 < B_J < 17 \) the fit to the observed histograms is remarkably good (see Fig. 19). In the last two magnitude bins, however, the model seems to grossly underestimate the observed dispersions (Fig. 20). The cause for this could be an underestimate in the observational proper-motion errors, or a much larger velocity dispersion for early G to late K dwarfs in the disk than that adopted in the model \([\Sigma_U, \Sigma_V, \Sigma_W = (30, 20, 15) \text{~km s}^{-1}]\). We note that changing the velocity dispersions for either the thick disk or halo components by 10% did not change the predicted dispersions. Indeed, the proper-motion dispersion at these magnitudes is still dominated by disk stars with \(+6 < M_V < +8\) (which also dominate the magnitude counts; see § 4.1). On the other hand, we would need to increase the observational errors by 10% in the range \( 17 < B_J < 18 \), and by 70% in the range \( 18 < B_J < 19 \), if the model is required to match the observed dispersions with the currently adopted velocity dispersions (see Table 12 and Fig. 19). While an increase of 10% in our estimated uncertainties cannot be ruled out, a 70% increase is highly questionable—and in this case an increased velocity dispersion for disk stars might have to be advocated. In any event, a detailed analysis of the fainter SPM data is beyond the scope of this paper, and we will also defer this discussion to a future paper.

6. DISCUSSION AND LIMITATIONS OF OUR MODELING

Figure 11 (bottom) clearly demonstrates that there are two regimes for discussion. One is for magnitudes brighter than about \( B_J \leq 14 \), where the \( V \) proper motion is very far from the expectation, and the other is the region \( 15 \leq B_J \leq 18 \), where the \( V \)-motion is approximately independent of magnitude. We have clearly demonstrated that trying to explain both these phenomena with the change of a single parameter is impossible within the context of our kinematic model. Thus, one is forced to adopt both a small solar peculiar velocity and an extremely high disk rotation, to fit the bright stars, and a very steep velocity shear, to fit the very faint stars in our sample. While it is possible this is the correct situation, other studies are not in complete agreement, and therefore it is important to emphasize that these two results do not necessarily have a single solution.

Bright stars do seem to pose a particular problem in that, e.g., the scaled models of Figure 6 suggest some anomaly in the bright stars, whereas the fainter counts fit very well (Fig. 7). The same effect is seen in the proper-motion distribution functions (Figs. 18 and 19), and of course in the median
Fig. 19.—Same as Fig. 18, but for the range $13 < B_j < 17$. Here we see an excellent agreement with the model predictions.
V-motion. Perhaps these consistent anomalies are correlated and suggest a real astrophysical effect, limited to the brighter disk stars? We are exploring whether the SPM data are good enough to have independently seen the strange phase-space structure evident in Dehnen’s (1998) analysis of Hipparcos data. If that is the case, then fitting a single model to both these stars and the better behaved fainter stars should be taken with caution. Fundamentally, the external constraints from other studies on both the gradient of the rotation curve (found to be zero from Cepheids by Metzger, Caldwell, & Schechter 1998) and its amplitude (270 km s⁻¹ is hard to fit with other constraints, but see Méndez et al. 1999) must induce reservations about the fit to the brighter stars.

At fainter magnitudes, however, the parameters that are required to fit the data are quite reasonable. Fitting to the gradient, the data actually require a mean thick disk rotation of about 180 km s⁻¹ at B_J = 16, and 150 km s⁻¹ at B_J = 18 (see eq. [23] and Table 9), which is certainly plausible. In fact, such gradients are consistent with the (limited) data on vertical shear in observed edge-on spirals. We note that Wyse & Gilmore (1990) also see a rather broad distribution of asymmetric drift in the thick disk, from radial velocities. This (marginal) result seems to be holding up in a more recent radial velocity study (Wyse & Gilmore 1999).

7. CONCLUSIONS

The color counts and secular proper motions for a randomly selected sample of stars derived from the SPM catalog have been fitted to the predictions from a star-counts Galactic structure and kinematic model. In general, a good representation of the data is obtained. Tuning of the model parameters requires a scale height for giant stars of 170 pc in order to reproduce the observed color counts at bright magnitudes, while all other parameters are unchanged from the original model presented by Méndez & van Altena (1996). It is somewhat puzzling that Méndez & van Altena had found from star and color counts toward two intermediate-latitude fields that the scale height for subgiant stars is closer to 250 pc. One would then expect that the scale height for a slightly more evolved population would be correspondingly larger, and yet we find that for giants toward the SGP, the SPM color counts indicate a value for their scale height of 170 pc. This could imply that either our parameterization for the density laws of these populations is not the most appropriate, or that their value is indeed a function of, e.g., Galactic latitude. Alternatively, these discrepancies could imply that the assumed density of stars in the model in the giant and subgiant section of the Hess diagram is incorrect. This issue clearly deserves further attention and could certainly be tackled by model comparisons with unbiased counts derived from current near-infrared surveys (2MASS, DENIS) at low Galactic latitudes, where the bright magnitude counts are dominated by these types of stars. Small systematic shifts are observed in the SPM photographic colors as a function of apparent magnitude when compared with the model predictions, and the catalog is corrected for this effect.

The absolute proper motions in the U-component indicate a solar peculiar motion of 11.0 ± 1.5 km s⁻¹, with no need for a local expansion or contraction term. In the V-component, the absolute proper motions can only be reproduced by the model if we adopt a solar peculiar motion of +5 km s⁻¹, a large LSR speed of 270 km s⁻¹, and a (disk) velocity ellipsoid that always points toward the Galactic center. The fainter secular motions show an indication that
the thick disk must exhibit a rather steep velocity gradient of about \(-36 \text{ km s}^{-1} \text{ kpc}^{-1}\). We are not able to set constraints on the overall rotation of the halo or on the thick disk or halo velocity dispersions. At bright magnitudes, the model shows a slightly larger proper-motion dispersion than observed, while the opposite is true at fainter magnitudes. We show that, at bright magnitudes, these discrepancies in the dispersions are not due to wrong assumptions regarding the SPM proper-motion errors, but that at fainter magnitudes our errors could be underestimated by 10% in the range 17 < \(B_J\) < 18 and by 70% in the range 18 < \(B_J\) < 19, although this issue deserves further attention. Some substructure in the \(U\) and \(V\) proper motions could be present in the brighter bins, 10 < \(B_J\) < 13, and it might be indicative of (disk) moving groups. Their existence could be related to the already known moving groups in the Galactic disk (Dehnen 1998), or it could be due to Galactic bar stars presently passing by the solar neighborhood, as found by Raboud et al. (1998) from a sample of stars in the NLTT Catalogue.

Our derived value for the LSR speed would imply a mass for the Galaxy within the solar circle that is larger, by about a factor of 1.5, than previous values. The implications of this finding, as well as supporting new evidence (coming mainly from a new measurement of the proper motion of the Large Magellanic Cloud by Anguita, Loyola, & Predreros 1999 and from the binary motion of the M31–Milky Way and Leo I–Milky Way pairs by Zasitsky 1999), are further discussed in Ménendez et al. (1999). We also note here the interesting prospects of space astrometry, where, e.g., space interferometric space missions such as SIM\(^6\) will be able to directly measure the disk rotational speed throughout the entire Galaxy.

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We have become aware of a confusing error in the caption to Figure 11 (p. 829), where the description of the different line styles for the bottom panel is inconsistent with the text and with Table 11. The figure and corrected caption are reproduced below.

Fig. 11.—Median observed (filled dots with error bars) and predicted (lines) absolute proper motions along U (Galactocentric direction, top) and V (Galactic rotation, bottom). In the top panel, the solid line indicates the run from the standard model (with a scale height of 172 pc for giants; run 1 in Table 11), while the dashed line indicates a run with a scale height of 250 pc for giants (run 2). In the bottom panel, the lower solid line is from run 1, while the dashed line is from run 2. The dot-dashed line is for a model with $q = 1$ (run 3; see text), while the dotted line is for a model with a solar peculiar motion in the $V$-component of +5 km s$^{-1}$ (run 4) instead of the classical value, +14 km s$^{-1}$, adopted in the standard model (run 1). The triple-dot-dashed line indicates the predictions for a model with a disk having a rotational speed of +270 km s$^{-1}$ (run 5), while the upper solid line shows the run with both $V_\odot = +5$ km s$^{-1}$ and $V_{LSR}(R_\odot) = +270$ km s$^{-1}$ (run 6).