On the Neutrino Mass Spectrum and Neutrinoless Double-Beta Decay

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Abstract

Assuming 3-$\nu$ mixing, neutrino oscillation explanation of the solar and atmospheric neutrino data and of the first KamLAND results, massive Majorana neutrinos and neutrinoless double-beta (($\beta\beta$)$_{0\nu}$-) decay generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos, we analyze in detail the possibility of determining the type of the neutrino mass spectrum by measuring of the effective Majorana mass $|<m>|$ in ($\beta\beta$)$_{0\nu}$-decay. The three possible types of neutrino mass spectrum are considered: i) normal hierarchical (NH) $m_1 \ll m_2 \ll m_3$, ii) inverted hierarchical (IH), $m_1 \ll m_2 \cong m_3$, and iii) quasi-degenerate (QD), $m_1 \cong m_2 \cong m_3$, $m_{1,2,3} > \sim 0.20$ eV. The uncertainty in the measured value of $|<m>|$ due to the imprecise knowledge of the relevant nuclear matrix elements is taken into account in the analysis. We derive the ranges of values of $\tan^2 \theta_\odot$, $\theta_\odot$ being the mixing angle which controls the solar neutrino oscillations, and of the nuclear matrix element uncertainty factor, for which the measurement of $|<m>|$ would allow one to discriminate between the NH and IH, NH and QD and IH and QD spectra.

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1 Introduction

The solar neutrino experiments Homestake, Kamiokande, SAGE, GALLEX/GNO, Super-Kamiokande (SK) and SNO \[2\],\[3\],\[4\] the data on atmospheric neutrinos obtained by the Super-Kamiokande (SK) experiment \[3\] and the results from the KamLAND reactor antineutrino experiment \[4\], provide very strong evidences for oscillations of flavour neutrinos. The evidences for solar $\nu_e$ oscillations into active neutrinos $\nu_{\mu,\tau}$, in particular, were spectacularly reinforced by the first data from the SNO experiment \[3\] when combined with the data from the SK experiment \[4\], by the more recent SNO data \[4\], and by the just published first results of the KamLAND \[4\] experiment. Under the rather plausible assumption of CPT-invariance, the KamLAND data practically establishes \[4\] the large mixing angle (LMA) MSW solution as unique solution of the solar neutrino problem. This remarkable result brings us, after more than 30 years of research, initiated by the pioneer works of B. Pontecorvo \[7\] and the experiment of R. Davis et al. \[8\], very close to a complete understanding of the true cause of the solar neutrino problem.

The interpretation of the solar and atmospheric neutrino, and of the KamLAND data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current (see, e.g., \[3\],\[10\]):

$$\nu_{\ell L} = \sum_{j=1}^{3} U_{\ell j} \nu_{\ell L}. \quad (1)$$

Here $\nu_{\ell L}, \ell = e, \mu, \tau$, are the three left-handed flavor neutrino fields, $\nu_{\ell L}$ is the left-handed field of the neutrino $\nu_{\ell}$ having a mass $m_j$ and $U$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \[11\]. If the neutrinos with definite mass $\nu_{\ell}$ are Majorana particles, the process of neutrinoless double-beta ($\bar{\beta}\beta$) decay will be allowed (for reviews see, e.g., \[12\],\[13\]). If the $(\beta\beta)_0$-decay is generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos $\nu_{\ell}$ and the latter have masses not exceeding few MeV, the dependence of the $(\beta\beta)_0$-decay amplitude on the neutrino mass and mixing parameters factorizes in the effective Majorana mass $|<m>|$ (see, e.g., \[12\]):

$$|<m>| = |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}} |, \quad (2)$$

where $\alpha_{21}$ and $\alpha_{31}$ are two Majorana CP-violating phases \[14\],\[15\]. If CP-invariance holds, one has \[16\] $\alpha_{21} = k\pi$, $\alpha_{31} = k'\pi$, where $k, k' = 0, 1, 2, ...$. In this case

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1 \quad (3)$$

represent the relative CP-parities of the neutrinos $\nu_1$ and $\nu_2$, and $\nu_1$ and $\nu_3$, respectively. The oscillations between flavour neutrino are insensitive to the Majorana CP-violating phases $\alpha_{21}$, $\alpha_{31}$ \[14\],\[17\] - information about these phases can be obtained in the $(\beta\beta)_0$-decay experiments \[18\],\[19\],\[20\],\[21\],\[22\],\[23\] (see also \[24\]). Majorana CP-violating phases, and in particular, the phases $\alpha_{21}$ and/or $\alpha_{31}$, might be at the origin of the baryon asymmetry of the Universe \[24\].

One can express \[24\] (see also, e.g., \[20\],\[27\],\[19\]) the masses $m_{2,3}$ and the elements of the lepton mixing matrix entering into eq. \[4\] for $|<m>|$, in terms of the neutrino oscillation parameters measured in the solar and atmospheric neutrino and KamLAND experiments: $m_{2,3}$ — in terms of the neutrino mass squared differences $\Delta m^2_\odot$ and $\Delta m^2_A$, driving the solar and atmospheric neutrino oscillations, and the mass $m_1$, and $|U_{e j}|^2, j = 1, 2, 3,$ — in terms of the mixing angle which controls the solar $\nu_e$ transitions $\theta_\odot$, and of the lepton mixing parameter $\sin^2 \theta_\odot$ limited by the data from the CHOOZ and Palo Verde experiments \[28\],\[29\].

\[2\] We assume that $m_j > 0$ and that the fields of the Majorana neutrinos $\nu_{\ell}$ satisfy the Majorana condition: $C(\bar{\nu}_{\ell})^\dagger = \nu_{\ell}, \quad j = 1, 2, 3$, where $C$ is the charge conjugation matrix.
The observation of $\beta\beta$ decay will have fundamental implications for our understanding of the symmetries of the elementary particle interactions (see, e.g., \cite{12}). Under the general and plausible assumptions of 3-$\nu$ mixing, neutrino oscillation explanation of the solar and atmospheric neutrino data, massive Majorana neutrinos and $\beta\beta$ decay generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos, which will be assumed to hold throughout this study, the observation of $\beta\beta$ decay can provide unique information on \cite{27, 19, 21, 22, 23, 33} i) the type of neutrino mass spectrum which can be normal hierarchical (NH), inverted hierarchical (IH), or quasi-degenerate (QD), ii) on the absolute scale of neutrino masses, and \cite{13, 14, 20, 21, 22, 23} iii) on the Majorana CP-violating phases $\alpha_{21}$ and $\alpha_{31}$.

A measured value of $|<m>| \sim$ few $\times 10^{-2}$ eV can provide, in particular, unique constraints on, or even can allow one to determine, the type of the neutrino mass spectrum in the case $\nu_{1,2,3}$ are Majorana particles \cite{33}. The solar neutrino data and the first KamLAND results \cite{4} favor relatively large value of $\cos 2\theta_{12} \sim 0.40$ \cite{34, 35, 36, 37, 38}. A value of $\cos 2\theta_{12} \gtrsim 0.25$ would imply \cite{33} the existence of significant lower bounds on $|<m>|$ (exceeding 0.01 eV) in the cases of IH and QD neutrino mass spectrum, and of a stringent upper bound (smaller than 0.01 eV) if the spectrum is of the NH type. The indicated lower bounds are in the range of the sensitivity of currently operating and planned $(\beta\beta)$ decay experiments.

Information on the absolute values of neutrino masses in the range of interest can also be derived in the $^3$H $\beta$-decay neutrino mass experiment KATRIN \cite{39}, and from cosmological and astrophysical data (see, e.g., ref. \cite{33}).

Rather stringent upper bounds on $|<m>|$ have been obtained in the $^{76}$Ge experiments by the Heidelberg-Moscow collaboration \cite{11}, $|<m>| < 0.35$ eV (90% C.L.), and by the IGEX collaboration \cite{42}, $|<m>| < (0.33 - 1.35)$ eV (90% C.L.). Taking into account a factor of 3 uncertainty in the calculated value of the corresponding nuclear matrix element \cite{13}, we get for the upper limit found in \cite{11}: $|<m>| < 1.05$ eV. Higher sensitivity to $|<m>|$ is planned to be reached in several $(\beta\beta)$ decay experiments of a new generation. The NEMO3 experiment \cite{43}, which began to take data in July of 2002, and the cryogenics detector CUORICINO \cite{44} to be operative in 2003, are expected to reach a sensitivity to values of $|<m>| \sim 0.2$ eV. Up to an order of magnitude better sensitivity, i.e., to $|<m>| \simeq 2.7 \times 10^{-2}$ eV, $1.5 \times 10^{-2}$ eV, $5.0 \times 10^{-2}$ eV, $2.5 \times 10^{-2}$ eV and $3.6 \times 10^{-2}$ eV is planned to be achieved in the CUORE, GENIUS, EXO, MAJORANA and MOON experiments \cite{14} respectively.

In what regards the $^3$H $\beta$-decay experiments, the currently existing most stringent upper bounds on the electron (anti-)neutrino mass $m_{\nu_e}$ were obtained in the Troitzk \cite{40} and Mainz \cite{41} experiments and read $m_{\nu_e} < 2.2$ eV. The KATRIN $^3$H $\beta$-decay experiment \cite{39} is planned to reach a sensitivity to $m_{\nu_e} \sim 0.35$ eV.

In the present article we study in detail the possibility of determining the type, or excluding one or more types, of neutrino mass spectrum by measuring of $|<m>|$ in the next generation of $(\beta\beta)$ decay experiments. The three possible types of spectra are considered \cite{1}: i) hierarchical (NH) $m_1 \ll m_2 \ll m_3$, ii) inverted hierarchical (IH), $m_1 \ll m_2 \simeq m_3$, and iii) quasi-degenerate (QD), $m_1 \simeq m_2 \simeq m_3 \equiv m_0$, $m_{12,3}^2 \gg \Delta m^2_{\alpha}$. In our analysis we take into account, in particular, the uncertainty in the determination of $|<m>|$ due to the imprecise knowledge of the relevant nuclear matrix elements. This permits us to determine the requirements which the possibility of distinguishing between i) the NH and IH, ii) the NH and QD, and iii) the IH and QD spectra, imposes on the uncertainty in the values of the $(\beta\beta)$ decay nuclear matrix elements. We derive

\footnote{Evidences for $(\beta\beta)$ decay taking place with a rate corresponding to 0.11 eV $\lesssim |<m>| \lesssim 0.56$ eV (95% C.L.) are claimed to have been obtained in \cite{34}. The results announced in \cite{35} have been criticized in \cite{33}.}

\footnote{We assume throughout this study that CPT-invariance holds in the lepton sector.}

\footnote{The quoted sensitivities correspond to values of the relevant nuclear matrix elements from ref. \cite{43}.}

\footnote{We work with the convention $m_1 < m_2 < m_3$ and use the term “spectrum with normal (inverted) hierarchy” for the spectra with $\Delta m_{10}^2 \equiv \Delta m_{30}^2$ ($\Delta m_{20}^2 \equiv \Delta m_{02}^2$), while we call “normal hierarchical (NH)” (“inverted hierarchical (IH)” the neutrino mass spectrum with normal (inverted) hierarchy and $m_1 \ll m_2, m_3$.}

\[\Delta m_{\alpha}^2 \equiv \Delta m_{20}^2 = m_{12}^2 - m_{20}^2 = m_{12}^2 - m_{02}^2 = \Delta m_{10}^2 - \Delta m_{02}^2,\]

\[\Delta m_{\alpha}^2 \equiv \Delta m_{20}^2 = m_{12}^2 - m_{20}^2 = m_{12}^2 - m_{02}^2 = \Delta m_{10}^2 - \Delta m_{02}^2,\]

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also the maximal values of $\tan^2 \theta_{\odot}$ for which the measurement of $|<m>|$ would allow one to discriminate between the NH and IH, NH and QD and IH and QD spectra, for different given values of the nuclear matrix element uncertainty factor. An upper limit $|<m>| < \times 10^{-2}$ eV would imply a significant constraint on the type of neutrino mass spectrum in the case the massive neutrinos are Majorana particles, e.g., on the theories in which the neutrino masses are generated via the see-saw mechanism.

It should be noted that the determination of the type of neutrino mass spectrum, based on the measured value of $|<m>|$, would provide simultaneously unique information on the absolute neutrino mass scale [21, 22, 32, 33]. Similar information cannot be obtained in the neutrino oscillation experiments, in which the sign of $\Delta m^2_{\nu}$ can be determined (see, e.g., [18, 19]) since neutrino oscillations depend on neutrino mass squared differences and are insensitive to the absolute neutrino mass scale. The sign of $\Delta m^2_{\nu}$ can be determined in very long base-line neutrino oscillation experiments at neutrino factories (see, e.g., [18]), and, e.g., using combined data from long base-line oscillation experiments at the JHF facility and with off-axis neutrino beams [49]. Under certain rather special conditions it might be determined also in experiments with reactor $\bar{\nu}_e$ [50].

2 Neutrino Oscillation Data and the Effective Majorana Mass

The predicted value of $|<m>|$ depends in the case of $3-\nu$ mixing on: i) $\Delta m^2_{\nu}$, ii) $\theta_{\odot}$ and $\Delta m^2_{\nu}$, iii) the lightest neutrino mass, and on iv) the mixing angle $\theta$. Using the convention $m^{}_{1} < m^{}_{2} < m^{}_{3}$, one has $\Delta m^2_{\nu} = \Delta m^2_{31}$, where $\Delta m^2_{jk} \equiv m^2_{j} - m^2_{k}$, and $m^{}_{3} = \sqrt{m^2_{1} + 2 \Delta m^2_{31}}$, while either $\Delta m^2_{\nu} \equiv \Delta m^2_{31}$ or $\Delta m^2_{\nu} \equiv \Delta m^2_{32}$. The two possibilities for $\Delta m^2_{\nu}$ correspond respectively to the two different types of neutrino mass spectrum — with normal and with inverted hierarchy. In the first case we have $m^{}_{2} = \sqrt{m^2_{1} + \Delta m^2_{31}}$, $|U_{e1}|^2 = \cos^2 \theta_{\odot}(1 - |U_{\odot}|^2)$, $|U_{e2}|^2 = \sin^2 \theta_{\odot}(1 - |U_{\odot}|^2)$, and $|U_{e3}|^2 \equiv \sin^2 \theta$, while in the second $m^{}_{2} = \sqrt{m^2_{1} + \Delta m^2_{31} - \Delta m^2_{32}}$, $|U_{e2}|^2 = \cos^2 \theta_{\odot}(1 - |U_{e1}|^2)$, $|U_{e3}|^2 = \sin^2 \theta_{\odot}(1 - |U_{e1}|^2)$, and $|U_{e2}|^2 \equiv \sin^2 \theta$.

Given $\Delta m^2_{\nu}$, $\Delta m^2_{\nu}$, $\theta_{\odot}$ and $\sin^2 \theta$, the value of $|<m>|$ depends strongly on the type of the neutrino mass spectrum, as well as on the values of the two Majorana CP-violating phases, $\alpha_{21}$ and $\alpha_{31}$ (see eq. (2)), present in the lepton mixing matrix. Let us note that in the case of QD spectrum, $m^{}_{1} \cong m^{}_{2} \cong m^{}_{3}$, $m^{}_{1,2,3} \gg \Delta m^2_{\nu}$, $|<m>|$ is essentially independent on $\Delta m^2_{\nu}$ and $\Delta m^2_{\nu}$, and the two possibilities, $\Delta m^2_{\nu} \equiv \Delta m^2_{21}$ and $\Delta m^2_{\nu} \equiv \Delta m^2_{32}$, lead effectively to the same predictions for $|<m>|$.

The possibility of determining the type of the neutrino mass spectrum if $|<m>|$ is found to be nonzero in the $(\beta \beta)_{0\nu}$-decay experiments of the next generation, depends crucially on the precision with which $\Delta m^2_{\nu}$, $\theta_{\odot}$, $\Delta m^2_{\nu}$, $\sin^2 \theta$ and $|<m>|$ will be measured. It depends also crucially on the values of $\theta_{\odot}$ and of $|<m>|$. The precision itself of the measurement of $|<m>|$ in the next generation of $(\beta \beta)_{0\nu}$-decay experiments, given the latter sensitivity limits of $\sim (1.5-5.0) \times 10^{-2}$ eV, depends on the value of $|<m>|$.

The KATRIN experiment [39] can test the hypothesis of a QD spectrum [4] provided $m^{}_{1,2,3} \cong m^{}_{\nu} \gtrsim (0.35 - 0.40)$ eV. The KATRIN detector is designed to have a 1 s.d. error of $0.08$ eV$^2$ on a measured value of $m^2_{\nu}$. This experiment is expected to start in 2007.

Assuming CPT-invariance, combined $\nu_e \rightarrow \nu_\mu(\tau)$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu(\tau)$ oscillation analyzes of the solar neutrino data and of the just published first KamLAND results [6], have already been performed in

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7In the convention in which the sign of $\Delta m^2_{\nu} \equiv \Delta m^2_{31}$ is not fixed, the latter determines the ordering of the neutrino masses: $\Delta m^2_{\nu} > 0$ corresponds to $m^{}_{1} < m^{}_{2} < m^{}_{3}$, while $\Delta m^2_{\nu} < 0$ implies $m^{}_{3} < m^{}_{1} < m^{}_{2}$.

8This statement is valid, within the convention $m^{}_{1} < m^{}_{2} < m^{}_{3}$ we are using, as long as there are no independent constraints on the CP-violating phases $\alpha_{21}$ and $\alpha_{31}$ which enter into the expression for $|<m>|$. In the case of NH spectrum, $|<m>|$ depends primarily on $\alpha_{21}$ ($|U_{e3}|^2 \ll 1$), while if the spectrum is with IH, $|<m>|$ will depend essentially on $\alpha_{31} - \alpha_{21}$ ($|U_{e1}|^2 \ll 1$).

9Given the allowed regions of values of $\Delta m^2_{\nu}$ and $\Delta m^2_{\nu}$ [4], one has a QD spectrum for $m^{}_{1,2,3} \cong m^{}_{\nu} > 0.20$ eV.
All analyzes show that the data favor the LMA MSW solution with $\Delta m^2_{\odot} > 0$ and $\tan^2 \theta_{\odot} < 1$, all the other solutions (LOW, VO, etc.) being essentially ruled out. In Tables 1 and 2 we give the best-fit values and the 90% C.L. allowed ranges of $\Delta m^2_{\odot}$ and $\tan^2 \theta_{\odot}$ in the LMA solution region obtained in [34, 35, 36, 37, 38]. The best fit values are confined to the narrow intervals $(\Delta m^2_{\odot})_{\text{BF}} = (6.9 - 7.3) \times 10^{-5}$ eV$^2$, $(\tan^2 \theta_{\odot})_{\text{BF}} = (0.42 - 0.46)$. The latter corresponds to $(\cos 2\theta_{\odot})_{\text{BF}} = (0.37 - 0.41)$.

In the two-neutrino $\nu_\mu \rightarrow \nu_\tau$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$) oscillation analysis of the SK atmospheric neutrino data performed in [3] the following best-fit value of $\Delta m^2_1$ was obtained: $(\Delta m^2_1)_{\text{BF}} \approx 2.5 \times 10^{-3}$ eV$^2$. At 99.73% C.L., $\Delta m^2_1$ was found to lie in the interval: $(1.5 - 5.0) \times 10^{-3}$ eV$^2$. According to the more recent combined analysis of the data from the SK and K2K experiments [51], one has $\Delta m^2_1 \approx (2.7 \pm 0.4) \times 10^{-3}$ eV$^2$. In certain cases of our analysis we will use as illustrative “best-fit” values $(\Delta m^2_1)_{\text{BF}} = 7.0 \times 10^{-5}$ eV$^2$ and $(\Delta m^2_1)_{\text{BF}} = 3.0 \times 10^{-3}$ eV$^2$.

For the indicated allowed ranges of values of $\Delta m^2_{\odot}$ and $\Delta m^2_1$, the NH (IH) spectrum corresponds to $m_1 \lesssim 10^{-3}$ (2 $\times 10^{-2}$) eV.

A 3-$\nu$ oscillation analysis of the CHOOZ data showed [28] that for $\Delta m^2_{\odot} \lesssim 10^{-4}$ eV$^2$, the limits on $\sin^2 \theta$ practically coincide with those derived in the 2-$\nu$ oscillation analysis in [28]. Combined 3-$\nu$ oscillation analysis of the solar neutrino, CHOOZ and the KamLAND data was performed in [34] under the assumption of $\Delta m^2_2 \ll \Delta m^2_1$ (see, e.g., [1, 10, 53]). For the best-fit value of $\sin^2 \theta$ the authors of [34] obtained: $(\sin^2 \theta)_{\text{BF}} \approx (0.00 - 0.01)$. It was also found in [34] that $\sin^2 \theta < 0.05$ at 99.73% C.L.

The existing solar neutrino and KamLAND data favor values of $\Delta m^2_{\odot} \approx (5.0 - 10.0) \times 10^{-5}$ eV$^2$ [34, 35, 36, 37, 38]. If $\Delta m^2_{\odot}$ lies in this interval, a combined analysis of the future more precise KamLAND results and of the solar neutrino data would permit to determine the values of $\Delta m^2_{\odot}$ and $\tan^2 \theta_{\odot}$ with high precision: the estimated (1 s.d.) errors on $\Delta m^2_{\odot}$ and on $\tan^2 \theta_{\odot}$ can be as small as $\approx (3 - 5)\%$ and $\approx 5\%$ (see, e.g., [54, 55]).

Similarly, if $\Delta m^2_1$ lies in the interval $\Delta m^2_1 \approx (2.0 - 5.0) \times 10^{-3}$ eV$^2$, as is suggested by the current atmospheric neutrino data [3, 51], its value will be determined with a $\approx 10\%$ error (1 s.d.) by the MINOS experiment [53]. Somewhat better limits on $\sin^2 \theta$ than the existing one can be obtained in the MINOS experiment [53] as well. Various options are being currently discussed (experiments with off-axis neutrino beams, more precise reactor antineutrino and long base-line experiments, etc., see, e.g., [29]) of how to improve by at least an order of magnitude, i.e., to values of $\approx 0.005$ or smaller, the sensitivity to $\sin^2 \theta$.

The high precision measurements of $\Delta m^2_1$, $\tan^2 \theta_{\odot}$ and $\Delta m^2_{\odot}$ are expected to take place within the next $\approx (6 - 7)$ years. We will assume in what follows that the problem of measuring or tightly constraining $\sin^2 \theta$ will also be resolved within the indicated period. Under these conditions, the largest uncertainty in the comparison of the theoretically predicted value of $|<m>|$ with that determined in the ($\beta\beta$)$_{0v}$-decay experiments would be associated with the corresponding ($\beta\beta$)$_{0v}$-decay nuclear matrix elements. We will also assume in what follows that by the time one or more ($\beta\beta$)$_{0v}$-decay experiments of the next generation will be operative (2009 – 2010) at least the physical range of variation of the values of the relevant ($\beta\beta$)$_{0v}$-decay nuclear matrix elements will be unambiguously determined.

### 3 Determining the Type of Neutrino Mass Spectrum

The possibility to distinguish between the three different types of neutrino mass spectrum in the 3-neutrino mixing case under discussion depends on the allowed ranges of values of $|<m>|$ for the three spectra. More specifically, it is determined by the maximal values of $|<m>|$ in the cases of NH and IH spectra and by the minimal values of $|<m>|$ for the IH and QD spectra. For the NH neutrino mass spectrum ($m_1 \ll m_2 \ll m_3$), the maximal value of $|<m>|$ is obtained in the
case of CP-conservation and equal CP-parities of $\nu_{1,2,3}$:

$$|<m>|^{\text{NH}}_{\text{max}} \cong \frac{1 - s^2}{1 + s^2} \left( m_1 + s^2 \theta_\odot \sqrt{\Delta m^2_{\odot}} + (1 + s^2 \theta_\odot) \frac{s^2}{1 - s^2} \sqrt{\Delta m^2_{\odot}} \right),$$  \hspace{1cm} (4)

where $s^2 \equiv \sin^2 \theta$ and we have neglected $m_1^2$ with respect to $\Delta m^2_{\odot}$ and $\Delta m^2_{\odot}$.

In the case of IH neutrino mass spectrum ($m_1 \ll m_2 \cong m_3$), the effective Majorana mass lies in the interval [18, 19]

$$|<m>|^{\text{IH}}_{\text{min}} \leq |<m>| \leq |<m>|^{\text{IH}}_{\text{max}} ,$$  \hspace{1cm} (5)

with

$$|<m>|^{\text{IH}}_{\text{min}} \cong (1 - s^2) \cos 2\theta_\odot \sqrt{\Delta m^2_{\odot}} , \quad |<m>|^{\text{IH}}_{\text{max}} \cong (1 - s^2) \sqrt{\Delta m^2_{\odot}} ,$$  \hspace{1cm} (6)

where we have neglected $m_1$. The minimal (maximal) value of $|<m>|$, $|<m>|^{\text{IH}}_{\text{min}}$ ($|<m>|^{\text{IH}}_{\text{max}}$), corresponds to CP-conservation and opposite (equal) CP-parities of the neutrinos $\nu_2$ and $\nu_3$.

The minimal value of $|<m>|$ for the quasi-degenerate (QD) neutrino mass spectrum ($m_1 \cong m_2 \cong m_3 \equiv m_0, m_0^2 \gg \Delta m^2_{\odot}, \Delta m^2_{\odot}$), for fixed value of $m_0$ is given by

$$|<m>|^{\text{QD}}_{\text{min}} \cong \frac{1 - s^2}{1 + s^2 \theta_\odot} \left( 1 - \tan^2 \theta_\odot - \frac{s^2}{1 - s^2 (1 + \tan^2 \theta_\odot)} \right) m_0 ,$$  \hspace{1cm} (7)

where $m_0 \gtrsim 0.20$ eV and we have neglected $\Delta m^2_{\odot}$ and $\Delta m^2_{\odot}$ with respect to $m_0^2$. As eq. (7) shows, $|<m>|^{\text{QD}}_{\text{min}}$ scales to a good approximation with $m_0$. Correspondingly, the minimal allowed value of $|<m>|$ for the QD mass spectrum is obtained for $m_0 = 0.2$ eV.

In Tables 1 and 2 we show the calculated i) maximal predicted value of $|<m>|$ in the case of NH neutrino mass spectrum, ii) the minimal value of $|<m>|$ for the IH spectrum, and iii) the minimal value of $|<m>|$ for the QD spectrum ($m_0 = 0.2$ eV), for the best-fit and the 90% C.L. allowed ranges of values of $\tan^2 \theta_\odot$ and $\Delta m^2_{\odot}$ in the LMA solution region. In Table 3 we give the same quantities, $|<m>|^{\text{NH}}_{\text{max}}, |<m>|^{\text{IH}}_{\text{min}}$ and $|<m>|^{\text{QD}}_{\text{min}}$, calculated using the best-fit values of the neutrino oscillation parameters, including 1 s.d. (3 s.d.) uncertainties of 5% (15%) on $\tan^2 \theta_\odot$ and $\Delta m^2_{\odot}$ and of 10% (30%) on $\Delta m^2_{\odot}$.

The maximal predicted value of $|<m>|$ for the IH spectrum is given by $|<m>|^{\text{IH}}_{\text{max}} \cong \sqrt{(\Delta m^2_{\odot})_{\text{max}}}$. For the best-fit value [3, 51] and the 99.73% C.L. allowed range [3] of $\Delta m^2_{\odot}$ we have, respectively, $|<m>|^{\text{IH}}_{\text{max}} \cong 0.05$ and 0.07 eV.

On the basis of the results shown in Tables 1 – 3, we can conclude, in particular, that the NH spectrum could be ruled out if the measured value of $|<m>|$ exceeds approximately $0.9 \times 10^{-2}$ eV, where we have been rather conservative in choosing the maximal value.

### 3.1 Theoretical and Experimental Uncertainties in $|<m>|$

Following the notation in ref. [23], we will parametrize the uncertainty in $|<m>|$ due to the imprecise knowledge of the relevant nuclear matrix elements — we will use the term “theoretical uncertainty” for the latter — through a parameter $\zeta$, $\zeta \geq 1$, defined as:

$$|<m>| = \zeta \left( |<m>|_{\text{exp}} \right)_{\text{MIN}} \pm \Delta ,$$  \hspace{1cm} (8)

where $(|<m>|_{\text{exp}})_{\text{MIN}}$ is the value of $|<m>|$ obtained from the measured $(\beta\beta)_{0\nu}$-decay half life-time of a given nucleus using the largest nuclear matrix element and $\Delta$ is the experimental error. An experiment measuring a $(\beta\beta)_{0\nu}$-decay half-life time will thus determine a range of $|<m>|$ corresponding to

$$\left( |<m>|_{\text{exp}} \right)_{\text{MIN}} - \Delta \leq |<m>| \leq \zeta \left( |<m>|_{\text{exp}} \right)_{\text{MIN}} + \Delta .$$  \hspace{1cm} (9)
From this inequality, using eqs. (4) and (6), we get:

\[ \zeta |<m>|_{\text{max}} < |<m>|_{\text{IH}} \leq |<m>|_{\text{min}}, \; \zeta \geq 1. \]  

(11)

From this inequality, using eqs. (6) and (7), we get:

\[ \tan^2 \theta_\odot < \frac{1 - \zeta (\beta + t^2)}{1 + \zeta (\alpha + t^2)}, \]  

(12)

where \( t^2 = s^2/(1 - s^2) \), \( \alpha = \sqrt{\Delta m^2_{2}\text{max}/\Delta m^2_{2}\text{min}} \) and \( \beta = \sqrt{m_1/\Delta m^2_{2}\text{min}} \). For our illustrative “best-fit” values \( \Delta m^2_{2}\text{BF}\text{max} = 7.0 \times 10^{-5} \text{eV}^2 \) and \( \Delta m^2_{2}\text{BF}\text{min} \approx 3.0 \times 10^{-3} \text{eV}^2 \), one has \( \alpha \approx 0.153 \); with \( m_1 \lesssim 0.001 \text{eV} \) one also finds \( \beta \lesssim 0.018 \). For \( \zeta = 1 \), the indicated values of \( \alpha \) and \( \beta \) and \( s^2 = 0.05 \) (0), eq. (12) is fulfilled for \( \tan^2 \theta_\odot \lesssim 0.77 \) (0.85). Taking \( \zeta = 2 \), one finds \( \tan^2 \theta_\odot \lesssim 0.61 \) (0.74), while for \( \zeta = 3 \) the result is \( \tan^2 \theta_\odot \lesssim 0.49 \) (0.65).

The smaller \( m_1 \) and/or \( \Delta m^2_{2}\), the closer the upper bound on \( \tan^2 \theta_\odot \) of interest becomes to 1. The above analysis shows also that the upper bound on \( \tan^2 \theta_\odot \) under discussion exhibits relatively strong dependence on the value of \( s^2 \lesssim 0.05 \): it increases by a factor of \( \sim (1.2 - 1.5) \) when \( s^2 \) decreases from 0.05 to 0.

3.2 Requirements on the Solar Neutrino Mixing Angle

We shall derive next the constraints \( \tan^2 \theta_\odot \) must satisfy in order to be possible to distinguish between the three types of neutrino mass spectrum NH, IH and QD.

Case i). Normal Hierarchical and Inverted Hierarchical Spectra. In order to be possible to distinguish between the NH and IH spectra, the following inequality must hold:

\[ \zeta |<m>|_{\text{max}} < |<m>|_{\text{IH}} \leq |<m>|_{\text{min}}, \; \zeta \geq 1. \]  

(13)

From this inequality using eqs. (11) and (13), we get:

\[ \tan^2 \theta_\odot < \frac{1 - \zeta (\beta + t^2)}{1 + \zeta (\alpha + t^2)}, \]  

(14)

where \( \bar{\alpha} = \sqrt{\Delta m^2_{2}\text{max}/m_0}, \; \bar{\beta} = m_1/m_0 \) and \( \bar{\gamma} = \sqrt{\Delta m^2_{2}\text{min}/m_0} \). For our illustrative “best-fit” values of \( \Delta m^2_{2}\) and \( \Delta m^2_{2}\text{min} \) and \( m_0 \gtrsim 0.2 \text{eV} \), we have: \( \bar{\alpha} \lesssim 0.042, \; \bar{\beta} \lesssim 0.000025, \) and \( \bar{\gamma} \lesssim 0.274 \). Using these
upper limits we find that for \( s^2 = 0.05 \) (0) and \( \zeta = 1 \), eq. (14) is satisfied if \( \tan^2 \theta_\odot \lesssim 0.9 \) (1.0).

Taking \( \zeta = 2, 3 \) we get \( \tan^2 \theta_\odot \lesssim 0.8 \) (1.0) for \( s^2 = 0.05 \) (0). If, e.g., \( m_0 = 2.0 \) eV, one finds \( \tilde{\gamma} \lesssim 0.035 \), i.e., the larger the value of \( m_0 \), the smaller \( \tilde{\gamma} \) and the closer is the upper bound on \( \tan^2 \theta_\odot \) to 1, i.e., the less constraining it is. Since \( \tilde{\gamma} \) enters into eq. (14) multiplied by the relatively small quantity \( t^2 \), the deviation of the upper bound on \( \tan^2 \theta_\odot \) under discussion from 1 is determined essentially by the value of \( \tilde{\alpha} \). Correspondingly, the maximal value of \( \tan^2 \theta_\odot \) permitting to distinguish between the NH and QD neutrino mass spectra decreases with increasing of \( \Delta m^2_\odot \).

**Case iii). Inverted Hierarchical and Quasi-Degenerate Spectrum.** One could distinguish between these two types of spectra if the following inequality is fulfilled:

\[
\zeta |<m>|_{\text{max}}^{\text{IH}} < |<m>|_{\text{min}}^{\text{QD}}, \quad \zeta \geq 1.
\]  

This condition together with eqs. (5) and (6) leads to the constraint

\[
\tan^2 \theta_\odot < \frac{1 - \zeta \tilde{\gamma} - t^2}{1 + \tilde{\gamma} t^2},
\]

where \( \tilde{\gamma} \) was defined earlier. Using our illustrative \( \Delta m^2_\odot \) and \( \Delta m^2_A \) “best-fit” values and \( m_0 \gtrsim 0.2 \) eV, one finds \( \tilde{\gamma} \approx 0.274 \). For \( s^2 = 0.05 \) (0) and \( \zeta = 1 \), the above limit on \( \tilde{\gamma} \) together with eq. (16) leads to \( \tan^2 \theta_\odot \lesssim 0.5 \) (0.6). Larger values of \( \zeta \) lead to stringent restrictions on \( \tan^2 \theta_\odot \); for \( \zeta = 2 \), for instance, we find \( \tan^2 \theta_\odot \lesssim 0.2 \) (0.3) for \( s^2 = 0.05 \) (0). The requirement that the two spectra could be distinguished is less restrictive for larger values of \( m_0 \) in this case as well.

These simple quantitative analyses show that if \( |<m>| \) is found to be non-zero in the future (\( \beta \beta \))_\odot\text{-}decay experiments, it would be easier, in general, to distinguish between the spectrum with NH and those with IH or of QD type using the data on \( |<m>| \neq 0 \), than to distinguish between the IH and the QD spectra. Discriminating between the latter would be less demanding if \( m_0 \) is sufficiently large. The requirement of distinguishing between the NH and the QD spectra leads to the least stringent conditions.

The above analyses also show that the possibility to distinguish between the IH and QD, and NH and QD, spectra depends rather weakly on the value \( s^2 \), satisfying the existing upper limits

\[\begin{align*}
\zeta &= 3; \quad \Delta m^2_\odot = 0.05 \quad (0.0) \quad \text{(NH versus IH)};
\zeta &= 3; \quad \Delta m^2_\odot = 0.05 \quad (0.0) \quad \text{(QD versus IH)};
\zeta &= 2; \quad \Delta m^2_\odot = 0.05 \quad (0.0) \quad \text{(QD versus NH)};
\zeta &= 2; \quad \Delta m^2_\odot = 0.05 \quad (0.0) \quad \text{(IH versus NH)}.
\end{align*}\]

It is worth noting that in contrast to the conditions which would allow one to establish on the basis of a measurement of \( |<m>| \neq 0 \) the presence of CP violation due to the Majorana CP-violating phases, the conditions permitting to distinguish between the three types of neutrino mass spectrum imply an *upper limit* on \( \tan^2 \theta_\odot \).

In Fig. 1 we show the upper bounds on \( \tan^2 \theta_\odot \), for which one can distinguish the NH spectrum from the IH spectrum and from that of QD type, as a function of \( \Delta m^2_\odot \) for different values of \( \zeta \). As is seen from the figure, the dependence of the maximal value of \( \tan^2 \theta_\odot \) of interest on \( m_0 \) in both cases is modest. Obviously, with the increasing of \( \Delta m^2_\odot \) and/or \( s^2 \), \( |<m>|_{\text{max}}^{\text{NH}} \) also increases. As a consequence, the maximal \( \tan^2 \theta_\odot \) under discussion decreases, which means that the corresponding spectra become harder to distinguish.

As we have seen, in order to be possible to distinguish between the IH and the QD spectra eq. (14) should be fulfilled. Fig. 2 shows the upper bound on \( \tan^2 \theta_\odot \) as implied by eq. (16), for \( s^2 = 0.05 \) and 0.0 as a function of \( \Delta m^2_A \). The upper bound on \( \tan^2 \theta_\odot \) of interest depends strongly on the value of \( m_0 \). It decreases with the increasing of \( \Delta m^2_A \), the dependence on \( \Delta m^2_A \) being noticeable for \( m_0 \approx 0.20 \) eV and rather mild for \( m_0 \gtrsim 0.40 \) eV. As it follows from Fig. 2 for the values of \( \Delta m^2_A \) favored by the neutrino oscillation data and for \( \zeta \gtrsim 2 \), distinguishing between the IH and QD spectra in the case of \( m_0 \approx 0.20 \) eV requires too small, from the point of view of the existing data, values of \( \tan^2 \theta_\odot \). For \( m_0 \gtrsim 0.40 \) eV, the values of \( \tan^2 \theta_\odot \) of interest fall in the ranges favored by the existing solar neutrino and KamLAND data even for \( \zeta = 3 \).
3.3 Requirements on \( \Delta \) and \( \zeta \)

We will investigate now the requirements the experimental and theoretical uncertainties \( \Delta \)
and \( \zeta \) should satisfy in order to allow one to discriminate between the three different neutrino mass
spectra if \( |<m>| \) is measured, or a significantly improved bound on \( |<m>| \) is obtained.

3.3.1 Testing the Quasi-Degenerate Neutrino Mass Spectrum

In order to rule out the QD spectrum it is necessary that
\[
\zeta \left( |<m>_{\text{exp}}|_{\text{MIN}} + \Delta \right) < |<m>|_{\text{QD}} \text{,min}
\]
which translates into a condition on the nuclear matrix element uncertainty \( \zeta \)
\[
\zeta < \frac{|<m>|_{\text{QD}} \text{,min}}{|<m>|_{\text{exp}} \text{,MIN}} \left( 1 + \frac{\sigma(|<m>|)}{|<m>|} \right)^{-1}
\]
For the illustrative sensitivities of the future (\( \beta \beta \))\text{,}\( \omega \)-decay experiments, \( |<m>|_{\text{exp}} \text{,MIN} = 0.01; 0.02; 0.04 \text{ eV} \), negligible \( \sigma(|<m>|)/|<m>| \), and the predicted values of \( |<m>|_{\text{QD}} \text{,min} = (0.04 - 0.056) \text{ eV} \)
reported in Table 3 (the 3 s.d. case), we have \( \zeta < (4.8 - 5.6); (2.4 - 2.8); (1.2 - 1.4) \). The better
the sensitivity of the future experiments, the larger is the allowed nuclear matrix element uncertainty.
Including a non-negligible \( \sigma(|<m>|)/|<m>| \) makes even more restrictive the condition on \( \zeta \).
Proving that the neutrino mass spectrum is of the QD type requires that \( |<m>|_{\text{QD}} \text{,MIN} \) - \( \Delta > |<m>|_{\text{MIN}} \)\text{,}, which implies an upper bound on \( \Delta \):
\[
\Delta < (|<m>|_{\text{exp}} \text{,MIN} - |<m>|_{\text{QD}} \text{,MIN})
\]
Using the values of \( |<m>|_{\text{QD}} \text{,MIN} \) reported in Table 3 (the 3 s.d. case), we find the corresponding upper
bounds \( \Delta < (52, 52, 44, 46) \times 10^{-3} \text{ eV} \), for \( |<m>|_{\text{exp}} \text{,MIN} = 0.1 \text{ eV} \), and \( \Delta < (152, 152, 144, 146) \times 10^{-3} \text{ eV} \), for \( |<m>|_{\text{exp}} \text{,MIN} = 0.2 \text{ eV} \). If the above condition is fulfilled, condition (13) with \( \zeta = 1 \),
which guarantees that the QD spectrum is distinguishable from the NH and IH ones, should also be
satisfied. For the illustrative “best-fit” values \( m^2_{\text{MIN}} = 7.0 \times 10^{-5} \text{ eV} \)\text{,} and \( m^2_{\text{MIN}} = 3.0 \times 10^{-3} \text{ eV} \),\text{,}
and for \( s^2 = 0.05(0) \), eq. (13) (\( \zeta = 1 \)) holds if \( \tan^2 \theta_{\odot} < 0.5(0.6) \).

3.3.2 Testing the Normal Hierarchical Spectrum

The NH neutrino mass spectrum would be ruled out if
\[
(|<m>|_{\text{exp}} \text{,MIN} - \Delta) > |<m>|_{\text{NH}} \text{,max}
\]
Parametrizing \( |<m>|_{\text{exp}} \text{,MIN} \) as \( |<m>|_{\text{exp}} \text{,MIN} = y^{\text{NH}} |<m>|_{\text{NH}} \text{,max} \), \( y^{\text{NH}} > 1 \), we get
\[
\Delta < (y^{\text{NH}} - 1) |<m>|_{\text{NH}} \text{,max}
\]
How restrictive this condition is depends on the value of \( |<m>|_{\text{NH}} \text{,max} \). Assuming that the more
precise measurements of \( \tan^2 \theta_{\odot}, m^2_{\text{MIN}} \), and \( \sin^2 \theta \) will not produce results very different from their
current best-fit values, we can use the predictions for \( |<m>|_{\text{NH}} \text{,max} \) given in Table 3 (3 s.d. case):
\( |<m>|_{\text{NH}} \text{,max} \approx 0.0066 \text{ eV} \). With this value one finds that for \( y^{\text{NH}} = 40, 30, 20, 10, 7, 5 \), condition (11) is satisfied if \( \Delta < (25.7, 19.1, 12.5, 5.9, 4.0, 2.6) \times 10^{-2} \text{ eV} \),\text{,}
Alternatively, if experimentally \( \Delta = (0.30, 0.20, 0.10, 0.05) \text{ eV} \), condition (11) will hold provided \( y^{\text{NH}} > 46.5, 31.3, 16.2, 8.6 \).

3.3.3 Probing the Inverted Hierarchical Spectrum

For the IH neutrino mass spectrum, \( |<m>|_{\text{IH}} \text{,max} \) is constrained to lie in the interval given
by eqs. (20) and (21). The IH spectrum can be ruled out if the experimentally measured value of
\( |<m>| \), with both the experimental error \( \Delta \) and the nuclear matrix element uncertainty factor \( \zeta \)
taken into account, lies outside the range given in eq. (1). There are two possibilities.
Case i). 

\[ (|<m>|_{\text{exp}})_{\text{MIN}} - \Delta > |<m>|_{\text{exp}}^{\text{IH}} \]  \quad \text{for} \quad |<m>|_{\text{exp}}^{\text{IH}} \text{ depends on the allowed values of } \Delta m_A^2 \text{ and } s^2 \text{ and is given in the captions of Tables 1 – 3. Using the parametrization } (|<m>|_{\text{exp}})_{\text{MIN}} = y^{\text{IH}} |<m>|_{\text{exp}}^{\text{IH}}, y^{\text{IH}} > 1, \text{ we are lead to the condition}  

\[ \Delta < (y^{\text{IH}} - 1)(1 - s^2) \sqrt{(\Delta m_A^2)_{\text{MAX}}}. \]  \quad (23)

For \( y^{\text{IH}} = 5, 4, 3, 2, 1.5 \) this condition is fulfilled if \( \Delta < (0.28, 0.21, 0.14, 0.07, 0.03) (1 - s^2) \) eV. The larger the measured value of \( |<m>| \), the larger is the maximal experimental error which still permits to rule out the IH spectrum. Alternatively, for a value of the experimental error \( \Delta = (0.2, 0.1, 0.05, 0.03) \) eV it would be possible to rule out the IH spectrum provided \( y^{\text{IH}} > 3.9, 2.4, 1.7, 1.4, \) respectively.

Case ii). The spectra with inverted hierarchy can be ruled out also if:

\[ \zeta \left( (|<m>|_{\text{exp}})_{\text{MIN}} + \Delta \right) < |<m>|_{\text{MIN}}^{\text{IH}} \simeq (1 - s^2) (\cos 2\theta_\odot)_{\text{MIN}} \sqrt{\Delta m_A^2}. \]  \quad (24)

Since \( |<m>|_{\text{MIN}}^{\text{IH}} \) is of the order of 0.01 eV, the experimental uncertainty will be required to be even below this value, making it not within reach of the currently planned experiments, except possibly for the 10t version of GENIUS. For instance, for \( (|<m>|_{\text{exp}})_{\text{MIN}} = 0.01 \) eV and \( \Delta = 0.01 \) eV one finds, e.g., \( \zeta < 1.1 \) if \( |<m>|_{\text{MIN}}^{\text{IH}} = 0.022 \) eV.

Probing the IH neutrino mass spectrum requires that the following conditions be fulfilled:

\[ (|<m>|_{\text{exp}})_{\text{MIN}} - \Delta \geq |<m>|_{\text{MIN}}^{\text{IH}}, \quad \zeta \left( (|<m>|_{\text{exp}})_{\text{MIN}} + \Delta \right) \leq |<m>|_{\text{MAX}}^{\text{IH}}. \]  \quad (25)

Using the fact that \( |<m>|_{\text{MIN}}^{\text{IH}} = \cos 2\theta_\odot |<m>|_{\text{MIN}}^{\text{MAX}} \) and the parametrization \( (|<m>|_{\text{exp}})_{\text{MIN}} \equiv y^{\text{IH}} |<m>|_{\text{MIN}}^{\text{IH}} \), the necessary conditions on \( \zeta \) and \( y^{\text{IH}} \) read

\[ y^{\text{IH}} \geq \cos 2\theta_\odot \left( 1 - \frac{\sigma(|<m>|)}{|<m>|} \right)^{-1}, \quad \zeta y^{\text{IH}} \leq \left( 1 + \frac{\sigma(|<m>|)}{|<m>|} \right)^{-1}. \]  \quad (26)

In the most favorable situation in which \( \sigma(|<m>|)/|<m>| \ll 1 \) and \( y^{\text{IH}} = \cos 2\theta_\odot, \zeta \) is required to be \( \zeta < 1/\cos 2\theta_\odot \). For the present best-fit values of \( \tan^2 \theta_\odot \) reported in Table 1, we obtain \( \zeta < 2.7, 2.7, 2.4, 2.5 \). Let us note, however, that from experimental point of view this possibility is rather demanding: as a first approximation, \( \Delta \) has to be of the order of, or smaller than, the difference between the maximal and minimal values of \( |<m>| \) in the IH case. This difference is typically of the order of \( \sim (0.02 - 0.04) \) eV, and does not exceed \( \sim 0.06 \) eV.

If conditions (26) are satisfied, in order to establish the IH spectrum both eqs. (11) and (15) with \( \zeta = 1 \) should also be valid. Taking as illustrative values \( \Delta m_\odot^2 = 7.0 \times 10^{-5} \) eV\(^2\) and \( \Delta m_A^2 = 3.0 \times 10^{-3} \) eV\(^2\), both conditions are satisfied for \( \tan^2 \theta_\odot \lesssim 0.5 \) (0.6) if \( s^2 = 0.05 \) (0).

3.3.4 The Inverted Hierarchical versus the Quasi-Degenerate Spectrum

Let us assume that a value of \( (|<m>|_{\text{exp}})_{\text{MIN}} \) of a few 10 meV has been found, thus ruling out the NH spectrum. The remaining question to ask in this situation would be whether the neutrino mass spectrum is of the IH or QD type. Distinguishing between the two types of spectra might be possible provided

\[ |<m>|_{\text{MIN}}^{\text{IH}} < |<m>|_{\text{MIN}}^{\text{QD}}. \]  \quad (27)

Obviously, one can reach a definite conclusion concerning the type of the spectrum only if the value of \( |<m>|_{\text{exp}} \) is larger than \( |<m>|_{\text{MIN}}^{\text{QD}} \), or is smaller than \( |<m>|_{\text{MAX}}^{\text{IH}} \).
i) \(|<m>|_{\text{exp}} > |<m>|_{\text{QD min}}^{|}\): this is equivalent to ruling out the IH spectrum and thus to the case analyzed in Subsection 3.3.3, see eq. (23) and the discussion thereafter.

ii) \(|<m>|_{\text{exp}} < |<m>|_{\text{QD max}}^{|}\): using eqs. (6) and (8), we find that

\[
1 < \zeta < \frac{(1 - s^2) \sqrt{(|<m>|_{\text{exp}}|_{\text{MIN}}^{|\Delta m^2_A|_{\text{max}}}})}{(1 - s^2) \sqrt{(|<m>|_{\text{QD min}}^{|}\Delta m^2_A|_{\text{max}}}} + \Delta .
\]

This inequality practically coincides with the second condition in (23). It is more restrictive for smaller values of \((|<m>|_{\text{QD min}}^{|}\Delta m^2_A|_{\text{max}}}\) and larger values of \(\Delta\). Equation (28) can hold only for a rather limited range of parameters, since the sum of \((|<m>|_{\text{exp}}|_{\text{MIN}}^{|}\Delta m^2_A|_{\text{max}}}\) and \(\Delta\) has to be smaller than \(\sqrt{(|<m>|_{\text{QD min}}^{|}\Delta m^2_A|_{\text{max}}}} \simeq 0.07\) eV.

Let us note that the various conditions discussed in this Section do not require any additional input from $^3$H $\beta$-decay experiments or from cosmological and astrophysical measurements.

4 Distinguishing Between Different Neutrino CP-Parity Configurations

In this Section we will discuss whether a measurement of \(|<m>| \neq 0\) might allow one to distinguish between some of the possible neutrino CP-parity configurations when the Majorana phases take CP-conserving values, $\alpha_{21}, \alpha_{31} = 0, \pm \pi$. We will denote these configurations by $i^{-1}(\eta_{CP}(\nu_1) \eta_{CP}(\nu_2) \eta_{CP}(\nu_3))$, where $\eta_{CP}(\nu_j)$ is the CP-parity of the neutrino $\nu_j$, $\eta_{CP}(\nu_j) = \pm i$. The possibility of determining the values of the Majorana CP-violating phases in the general case of CP-non conservation has been discussed in detail in ref. [23].

Inspecting Tables 1 – 3 leads to the conclusion that it might be relatively easy to distinguish between the $(+ - -)$ and $(+ + -)$ configurations in the case of the IH spectrum (i.e., $|<m>|_{\text{IH max}}^{|}\) and the $(+ - -)$ and $(+ + -)$ configurations for the QD spectrum (i.e., $|<m>|_{\text{QD min}}^{|}\) The more interesting question is whether it might be possible to distinguish between the different CP-parity configurations for a given type of neutrino mass spectrum. We will study it briefly in what follows in the cases of IH and QD spectra [24].

4.1 Inverted Hierarchical Spectrum

Due to the smallness of $m_1 |U_{e1}|^2$, one cannot distinguish the $(+ + +)$ from the $(+ + -)$, as well as the $(+ - -)$ from the $(+ - +)$, configurations [22]. The first pair of CP-parity configurations corresponds to $|<m>|_{\text{IH max}}^{|}$, while the second corresponds to $|<m>|_{\text{IH min}}^{|}$. The CP-parity patterns $(+ + +)$, $(+ - -)$ and $(+ + -)$, $(+ + -)$ would be distinguishable if the following condition holds:

\[
|<m>|_{\text{IH min}}^{|} > |<m>|_{\text{IH max}}^{|} .
\]

This can be translated into a condition on $\zeta$, which reads

\[
\zeta < \frac{2(tan^2 \theta_\odot)_{\text{min}}}{2(tan^2 \theta_\odot)_{\text{max}}} .
\]

The first ratio in the right-hand side of eq. (29) is, for an assumed error on $\Delta m^2_A$ of 10 %, approximately 0.8, while for $(tan^2 \theta_\odot)_{\text{min}} = 0.2, 0.3, 0.4, 0.5, 0.6$, the second ratio reads 1.5, 1.9, 2.3, 3.0 4.0, respectively. If $\zeta \lesssim 1.5, 2.0, 3.0, 4.0$, values of $tan^2 \theta_\odot \gtrsim 0.3, 0.4, 0.5, 0.7$ are required in order

\[\text{For an analysis of this possibility for the NH spectrum without taking into account the nuclear matrix element uncertainty, see [21, 22].}\]
to be possible to distinguish between the two cases under study.

4.2 Quasi-Degenerate Spectrum

In the case of QD spectrum, the (+ −) and (− +), and the (+ +) and (− −), configurations are difficult to distinguish due to the smallness of the mixing parameter $s^2$ limited by the reactor antineutrino experiments [28, 29]: the corresponding differences in the predicted values of $|<m>|$ do not exceed $\sim 10\%$. Therefore we shall analyze again the possibility to discriminate between these two pairs. Taking into account that $|<m>|^{QD}_{(+−)} > |<m>|^{QD}_{(−+)}$ and $|<m>|^{QD}_{(++)} > |<m>|^{QD}_{(−−)}$, the indicated two pairs of CP-parity configurations can be distinguished if the following inequality holds:

$$|<m>|^{QD}_{(+−)} > \zeta |<m>|^{QD}_{(−+)}. \quad (31)$$

The above inequality leads to the condition

$$\zeta < \frac{(m_0)_{\min} 1 + (\tan^2 \theta_\odot)_{\max} (1 − s^2)}{(m_0)_{\max} 1 − (\tan^2 \theta_\odot)_{\min} (1 − s^2)} \quad , \quad (32)$$

which is very similar to eq. (30). Assuming a KATRIN inspired error of 0.28 eV on $m_0$, the first fraction in the right-hand side of eq. (32) is 0.1, 0.3, 0.6, 0.7 for $m_0 = (0.35, 0.5, 1.0, 1.5)$ eV, respectively. If $\sigma(3m_0) = 0.10$ eV, as is expected from combined astrophysical and cosmological measurements, then for the same fraction one gets 0.83, 0.93, 0.94, 0.96 for $m_0 = (0.35, 0.5, 1.0, 1.5)$ eV, respectively. In what regards the second fraction, for $s^2 = 0$ the values from Section 4.3 are valid, while for $s^2 = 0.05$ they read 1.7, 2.1, 2.6, 3.3 for $(\tan^2 \theta_\odot)_{\min} = 0.3, 0.4, 0.5, 0.6$.

Thus, if $m_0$ is measured in tritium $\beta$-decay experiments, relatively large $m_0 \gtrsim 1.5$ eV and $\tan^2 \theta_\odot \gtrsim 0.5$ are required in order to distinguish between the (+ −), (− +), and the (+ +), (− −) CP-parity configurations. If astrophysical and cosmological measurements provide $m_0$, then a value of $\zeta \lesssim 1.5, 2.0, 3.0, 4.0$ would require $\tan^2 \theta_\odot \gtrsim 0.2, 0.4, 0.5, 0.6$.

5 Conclusions

Assuming 3-neutrino mixing and massive Majorana neutrinos, $(\beta\beta)_0\nu$-decay induced only by the (V-A) charged current weak interactions, LMA MSW solution of the solar neutrino problem and neutrino oscillation explanation of the atmospheric neutrino data, we have studied the requirements on the “solar” mixing angle $\theta_\odot$, the normal matrix element uncertainty factor $\zeta$ and the experimental error on the effective Majorana mass $|<m>|$, $\Delta$, which allow one to distinguish between, and/or test, the normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) neutrino mass spectra if $|<m>| \neq 0$ is measured, or a stringent upper bound on $|<m>|$ is obtained. The possibility to discriminate between the three types of spectra depends on the allowed ranges of values of $|<m>|$ for the three spectra: it is determined by the maximal values of $|<m>|$ in the cases of NH and IH spectra, $|<m>|^{NH, IH \max}$, and by the minimal values of $|<m>|$ for the IH and QD spectra, $|<m>|^{IH, QD \min}$. These are reported in Tables 1 – 3. In deriving them we have used the values of the solar and atmospheric neutrino oscillation parameters, $\theta_\odot$, $\Delta m^2_\odot$, $\Delta m^2_\Lambda$, and $\sin^2 \theta$, favored by the existing data [1, 2, 3, 4, 5, 6, 28, 29] (Tables 1 and 2) and assumed prospected precisions of future measurements (Table 3).

For the currently favored values of the neutrino oscillation parameters and $\sin^2 \theta = 0$, the upper bound on $\tan^2 \theta_\odot$ permitting to distinguish the NH from the IH spectrum is satisfied even for $\zeta = 3$. If $\sin^2 \theta$ lies close to its present 99.73\% C.L. upper limit of 0.05 [33] (see also [28, 29]), the upper bound of interest decreases by up to 50\% and values of $\zeta$ slightly lower than 3 might be required (Fig. 1). The possibility to discriminate between the NH and the QD spectra depends weakly on $\sin^2 \theta$ and on the neutrino mass $m_{1,2,3} \approx m_0$, and the respective conditions are satisfied even for values of $\zeta$ exceeding 3 (Fig. 1). Without any additional input from $^3$H beta-decay experiments
and/or cosmological and astrophysical measurements, and given the values of $\tan^2 \theta_\odot$ and $\Delta m^2_\Lambda$ favored by the data, the IH and QD spectra can be distinguished only if $\zeta \lesssim 1.5$ (Fig. 2).

Let us emphasize that the conditions which would allow one to establish the presence of CP violation due to the Majorana CP-violating phases using a measurement of $|<m>| \neq 0$ lead to a lower bound on $\tan^2 \theta_\odot$ and, in general, require $\zeta < 2$. In contrast, the conditions permitting to distinguish between the three types of neutrino mass spectrum imply an upper limit on $\tan^2 \theta_\odot$ and in most of the cases can be satisfied even for $\zeta \simeq 3$.

We have studied also the conditions on $\zeta$ and $\Delta$ which would permit to rule out, or establish, the NH, IH and the QD mass spectra. Typically, the next generation of $(\beta\beta)_{0\nu}$-decay experiments will be able to rule out the QD mass spectrum if $\zeta \lesssim 3$, and establish it if, e.g., the measured $(|<m>|_{\text{exp}})_{\text{MIN}} \sim 0.2 (0.1) \text{ eV}$ (see eq. (5)) and the experimental error is $\Delta \sim 0.15 (0.05) \text{ eV}$. The NH spectrum can be excluded provided the measured value of $|<m>|$ is, e.g., $\sim 10 (7)$ times larger than $|<m>|_{\text{MIN}}^{\text{IH}}$ and the experimental error is $\Delta \lesssim 0.12 (0.06) \text{ eV}$. The IH spectrum can be ruled out for $\Delta \lesssim 0.07 (0.10) \text{ eV}$ provided $(|<m>|_{\text{exp}})_{\text{MIN}}$ is at least by a factor of $\sim 2.0 (2.5)$ larger than $|<m>|_{\text{MIN}}^{\text{IH}}$. Establishing the IH mass spectrum is quite demanding and requires a measurement of $|<m>|$ with an error $\Delta \lesssim 0.02 - 0.04 \text{ eV}$.

Finally, we have studied the possibility to distinguish between certain neutrino CP-parity configurations in the case of CP-conservation. Due to the smallness of $\sin^2 \theta$, there are two pairs of CP-parities in the cases of QD and IH spectra, the two different CP-parity patterns within each pair being indistinguishable. Given the best-fit values of $\tan^2 \theta_\odot$, one can discriminate between these two pairs for the IH mass spectrum if $\zeta \lesssim 2$. For the QD mass spectrum and if $m_0$ is measured in tritium $\beta$-decay experiments, relatively large $m_0 \gtrsim 1.5 \text{ eV}$ and $\tan^2 \theta_\odot \gtrsim 0.5$ are necessary. If astrophysical and cosmological measurements provide $m_0$, values of $\zeta \lesssim 2$ are required.

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Table 1: The best-fit values of $\tan^2 \theta_\odot$ and $\Delta m^2_\odot$ (in units of $10^{-5} \text{eV}^2$) in the LMA solution region, as reported by different authors. Given are also the calculated maximal values of $|<m>|$ (in units of $10^{-3} \text{eV}$) for the NH spectrum and the minimal values of $|<m>|$ (in units of $10^{-3} \text{eV}$) for the IH and QD spectra. The results for $|<m>|$ in the cases of NH and IH spectra are obtained for $m_1 = 10^{-3} \text{eV}$ and the best-fit value of $\Delta m^2_A$, $\Delta m^2_A = 2.7 \times 10^{-3} \text{eV}^2$ [51], while those for the QD spectrum are derived for $m_0 = 0.2 \text{eV}$. In all cases $\sin^2 \theta = 0.05$ has been used. The chosen value of $\Delta m^2_A$ corresponds to $|<m>|^{\text{IH}}_{\text{max}} = 52.0 \times 10^{-3} \text{eV}$.

| Reference | $\tan^2 \theta_\odot$ | $\Delta m^2_\odot$ | $|<m>|^{\text{NH}}_{\text{max}}$ | $|<m>|^{\text{IH}}_{\text{min}}$ | $|<m>|^{\text{QD}}_{\text{min}}$ |
|-----------|---------------------|-----------------|-------------------|-----------------|------------------|
| 34        | 0.46                | 7.3             | 5.9               | 18.4            | 59.9             |
| 35        | 0.46                | 6.9             | 5.8               | 18.4            | 59.9             |
| 36        | 0.42                | 7.2             | 5.7               | 20.3            | 67.2             |
| 37        | 0.43                | 7.0             | 5.7               | 19.8            | 65.3             |

Table 2: The ranges of allowed values of $\tan^2 \theta_\odot$ and $\Delta m^2_\odot$ (in units of $10^{-5} \text{eV}^2$) in the LMA solution region, obtained at 90% C.L. by different authors. Given are also the corresponding maximal values of $|<m>|$ (in units of $10^{-3} \text{eV}$) for the NH spectrum, and the minimal values of $|<m>|$ (in units of $10^{-3} \text{eV}$) for the IH and QD spectra. The results for the NH and IH spectra are obtained for $m_1 = 10^{-3} \text{eV}$, while those for the QD spectrum correspond to $m_0 = 0.2 \text{eV}$. $\Delta m^2_A$ was assumed to lie in the interval [51] $(2.3 - 3.1) \times 10^{-3} \text{eV}^2$. This implies $|<m>|^{\text{IH}}_{\text{max}} = 55.7 \times 10^{-3} \text{eV}$. As in Table 1, the value $\sin^2 \theta = 0.05$ was used.

| Reference | $\tan^2 \theta_\odot$ | $\Delta m^2_\odot$ | $|<m>|^{\text{NH}}_{\text{max}}$ | $|<m>|^{\text{IH}}_{\text{min}}$ | $|<m>|^{\text{QD}}_{\text{min}}$ |
|-----------|---------------------|-----------------|-------------------|-----------------|------------------|
| 34        | 0.32 – 0.72         | 5.6 – 17        | 8.6               | 7.6             | 20.6             |
| 35        | 0.31 – 0.68         | 5.7 – 15        | 8.1               | 8.9             | 25.8             |
| 36        | 0.31 – 0.56         | 6.0 – 8.7       | 6.6               | 13.0            | 43.2             |
| 37        | 0.31 – 0.66         | 5.9 – 8.9       | 7.0               | 9.5             | 28.6             |

Table 3: The values of $|<m>|^{\text{NH}}_{\text{max}}$, $|<m>|^{\text{IH}}_{\text{min}}$ and $|<m>|^{\text{QD}}_{\text{min}}$ (in units of $10^{-3} \text{eV}$), calculated using the best-fit values of solar and atmospheric neutrino oscillation parameters from Table 1 and including 1 s.d. (3 s.d) uncertainties of 5% (15%) on $\tan^2 \theta_\odot$ and $\Delta m^2_\odot$, and of 10% (30%) on $\Delta m^2_A$. In this case one has: $|<m>|^{\text{IH}}_{\text{max}} = 54.5 (59.2) \times 10^{-3} \text{eV}$.

| Reference | $|<m>|^{\text{NH}}_{\text{max}}$ | $|<m>|^{\text{IH}}_{\text{min}}$ | $|<m>|^{\text{QD}}_{\text{min}}$ |
|-----------|-----------------------------|-----------------------------|-----------------------------|
| 34        | 6.1 (6.7)                   | 16.5 (12.9)                 | 55.9 (48.2)                 |
| 35        | 6.1 (6.6)                   | 16.5 (12.9)                 | 55.9 (48.2)                 |
| 36        | 6.0 (6.5)                   | 18.3 (14.6)                 | 63.3 (55.9)                 |
| 37        | 6.0 (6.5)                   | 17.9 (14.1)                 | 61.4 (54.0)                 |
Figure 1: The upper bound on $\tan^2 \theta_{\odot}$, for which one can distinguish the NH spectrum from the IH spectrum and from that of QD type, as a function of $\Delta m^2_{sol}$ for $\Delta m^2_{\text{sol}} = 3 \times 10^{-3}$ eV$^2$ and different values of $\zeta$ (see eqs. (12) and (14)). The lower (upper) line corresponds to $s^2 = 0.05$ (0).
Figure 2: The upper bound on $\tan^2 \theta_\odot$ allowing one to discriminate between the IH and the QD neutrino mass spectra, as a function of $\Delta m^2_A$ for different values of $\zeta$ (see eq. (16)). The lower (upper) line corresponds to $s^2 = 0.05$ (0).