Environmental conditions compensation for a length measurement system based in laser interferometry for machine tool volumetric verification

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Abstract: Measurement systems based in laser interferometry have been usually used in industry for volumetric verification. The behaviour of these systems rely on the wavelength of the laser used and so the estimation of the refractive index of the ambient air is a fundamental step in the measurement process. The environmental conditions can be measured with an environmental compensation unit and several equations can be used to estimate the refractive index. When several measurement systems are needed to measure the position of the machine tool head simultaneously during a volumetric verification, the information of the environmental control unit has to be applied to all the measurements in order to compensate the measurement result with the ambience conditions. This can be done estimating an adequate compensation model from the environmental data. In this work the Ciddor equation has been used to adjust compensation models based in regression curves.

Keywords: Laser interferometry, Environmental compensation, Machine tool, Telescopic system, Multilateration.

1. Introduction
Measurement systems based in laser interferometry, such as laser trackers, laser tracers or recently developed High Precision Telescopic Instruments (HPTI) where an interferometer is integrated in a telescopic system, have been commonly used in industry for volumetric verification (VV) [1].

When high precision measurements are needed, laser interferometry is a widely used technique. The measured displacement with a laser interferometer depends on the wavelength of the laser. The wavelength of the laser in the vacuum can be well known, but the environmental conditions will modify this value when the laser is transmitted through the air [2]. The wavelength of the laser in ambient air can be calculated with the refractive index and the vacuum wavelength. Several formulas are available for the estimation of the refractive index, but two of the most used are Ciddor equation [3-5] and Edler equation modified by Birch and Downs [6-8]. In this work Ciddor equation is going to be used because the wavelength of the infrared laser used is in the range of validity of the Ciddor equation and the range of working temperatures should be between 15 and 25°C.

When several measurement systems (for example HPTI) are needed to measure the position of the machine tool head simultaneously during a VV [9,10], the information of the environmental control unit (ECU) has to be applied to all the measurements in order to compensate the measurement result with the ambience conditions. This can be done connecting an ECU to each system or, as it is proposed in
this work, estimating an adequate compensation model from the ECU data.

The main goal of this work is to compare different models of compensation using the Ciddor equation.

2. Materials and methods

2.1. Hardware description and working principle

The system under evaluation (HPTI) measures the distance between a sensor head and a retroreflector using interferometric principles.

The sensor head and the retroreflector are integrated in a telescopic system (C) developed to measure the distance between two spheres. One of the spheres is mounted in the system (A) while the other is fixed to the target whose distance from the first sphere is to be measured (F). This last sphere is joined to the system through a magnetic holder (E) that also supports a corner retroreflector (I). On the other side, the first sphere and the sensor head (H) are attached to a support piece (B). Finally, the optical fiber connects the sensor head and the interferometer through a conduit manufactured in the support piece (figure 1).

A more detailed description of the telescopic system can be found in [10].

Figure 1. Scheme and components of the measurement telescopic system. (A) sphere mounted on the system; (B) support of the sensor head; (C) telescopic system; (D) laser beam; (E) magnetic holder; (F) sphere fixed to the machine tool; (G) Optical fiber (flat, polish end); (H) sensor head; (I) corner retroreflector.

The interferometer sends out a laser beam that is coupled into an optical fiber. The sensor head is connected to the flat polish end of the optical fiber. The flat polish end of the fiber reflects part of the beam generating a reference beam. The remaining laser beam continues to the retroreflector where it is reflected. The reflected beam from the retroreflector is coupled back into the fiber through the sensor head, interfering with the reference beam. The interference signal, whose wavelength is half the wavelength of the laser, is processed by the interferometer. Finally, the retroreflector change in displacement ($\Delta d$) is calculated from the phase change ($\Delta \phi$) between the reference beam and the reflected beam, equation (1).

$$\Delta d = \frac{\lambda}{2} \cdot \frac{\Delta \phi}{2\pi}$$

The displacement measured depends on the wavelength of the laser beam. The laser beam wavelength has its maximum value in vacuum while its wavelength in air will be shorter. When the laser beam travels through the air, the environmental conditions will modify the wavelength of the laser ($\lambda$). Therefore it is needed the true value of the wavelength to obtain a precise value of the ($\Delta d$) value from equation (1). The actual $\lambda$ can be calculated from the refractive index (n) of the air and the vacuum wavelength of the laser beam ($\lambda_0$) as shown in equation (2).

$$\lambda = \frac{\lambda_0}{n}$$
An analysis of the effect of the wavelength on the measurement uncertainty is shown by Yoon-Soo Jang and Seung-Woo Kim in [2] where a comparison of different methods to estimate $n$ are shown too.

In this work, Ciddor formula [3,4,5] has been used to estimate the refractive index from the environmental conditions. An Environmental Compensation Unit (ECU) has been used to measure temperature, pressure and relative humidity; its characteristics are shown in Table 1.

| Uncertainty $(k=2)^a$ | Working range       |
|-----------------------|---------------------|
| Temperature           | $\pm 0.01^\circ C$  | 0 to 50$^\circ C$ |
| Pressure              | $\pm 0.1$mbar        | 300 to 1100mbar   |
| Relative humidity     | $\pm 0.05\%$        | 10 to 90%         |

$a$ According to the European Co-operation for Accreditation document EA-4/02 [11]

### 2.2. Compensation methods

The displacement acquired without compensate the environmental conditions, i.e. estimated with a refractive index of $n = 1$ ($d_0$) is analysed to evaluate the effect of the ambience conditions. The data acquired compensating the ambient conditions ($d_R, n = n_R$; the refractive index estimated with the Ciddor equation) is used as reference to evaluate the former. Comparing both measurements, the error, $e_0$, associated to the change in the transmission medium and the change in the environmental conditions can be estimated, equation (3).

$$e_0 = d_0 - d_R$$ (3)

As the laser wavelength in vacuum is longer than that in air, $d_0 > d_R$ and $e_0 > 0$. Further, from equations (1) to (3) it is possible to write equations (4), (5) and (6) where the error has been written as a dimensionless parameter relative to the measured displacement obtained with $n = 1$ ($d_0$).

$$d_0 = \frac{\lambda_0}{2} \left( N + \frac{\Delta \varphi}{2\pi} \right)$$ (4)

$$d_R = \frac{\lambda_R}{2} \left( N + \frac{\Delta \varphi}{2\pi} \right) = \frac{d_0}{n_R}$$ (5)

$$\frac{e_0}{d_0} = 1 - \frac{1}{n_R} = 1 - \frac{d_R}{d_0}$$ (6)

In a general form, for a different refractive index, $n = n_i$, equations (7) and (8) can be written.

$$d_i = \frac{\lambda_i}{2} \left( N + \frac{\Delta \varphi}{2\pi} \right) = d_R \cdot \frac{n_R}{n_i} = d_0 \cdot \frac{1}{n_i}$$ (7)

$$\frac{e_i}{d_0} = \frac{1}{n_i} - \frac{1}{n_R} = \frac{d_i}{d_0} - \frac{d_R}{d_0}; \text{with } e_i = d_i - d_R$$ (8)

where $N$ is the integer number of half wavelength in the measured length and $d_i$ is the distance measured with $n = n_i$.

From equation (6) the maximum dimensionless error parameter using $n = 1$, $e_0/d_0$, when the environmental conditions are inside the ECU working range, can be estimated and results in 313ppm when temperature and relative humidity are in their lower limit and pressure is in its upper limit. Under these conditions, the maximum difference, when each variable changes between the upper and the lower value and the rest of the variables stay fixed, is 227.7ppm for the pressure, followed by the temperature with 54.2ppm and the relative humidity with 5.2ppm.

As environmental conditions in industrial practice are usually in a closer interval, a second group of intervals has been evaluated with a maximum error estimated in 283.9ppm when temperature and relative humidity are in their lower limit and pressure is in its upper limit. Under these conditions, the maximum difference when each variable changes between the upper and the lower value and the rest of the variables stay fixed is 19.4ppm for the temperature, followed by the pressure with 19.2ppm and the...
relative humidity with 0.5ppm. These results are shown in table 2, where the relation between the effects of each environmental parameter in the error in ppm can be extracted: when the temperature increases the error decreases; when the pressure increases the error increases and when the relative humidity increases the error decreases although the weight of the relative humidity is lower than the weight of the temperature and pressure.

| Environmental conditions in the ECU range | Environmental conditions in an industrial range |
|-----------------------------------------|-----------------------------------------------|
| T [°C] | P [mbar] | R H [%] | $e_0/d_0$ [ppm] | T [°C] | P [mbar] | R H [%] | $e_0/d_0$ [ppm] |
| 0 | 300 | 10 | 85.3 | 10 | 965 | 35 | 264.7 |
| 50 | 300 | 10 | 71.4 | 30 | 965 | 35 | 246.8 |
| 0 | 1100 | 10 | 313.0 | 10 | 1035 | 35 | 283.9 |
| 50 | 1100 | 10 | 263.8 | 30 | 1035 | 35 | 264.8 |
| 0 | 300 | 90 | 85.1 | 10 | 965 | 65 | 264.6 |
| 50 | 300 | 90 | 66.2 | 30 | 965 | 65 | 246.4 |
| 0 | 1100 | 90 | 312.8 | 10 | 1035 | 65 | 283.8 |
| 50 | 1100 | 90 | 258.6 | 30 | 1035 | 65 | 264.3 |

The estimation of the refractive index with the Ciddor formula is used to model an error curve and find the curve that better fits it. Three cases have been studied: the first case estimates the refractive index for a temperature interval using typical values for the other parameters of the Ciddor formula (pressure, relative humidity and CO2 concentration). The second case estimates the refractive index for a pressure interval using typical values for temperature, relative humidity and CO2 concentration. Finally, the third case estimates the refractive index for an interval of variation of temperature and pressure using typical values for relative humidity and CO2 concentration. The typical values for each parameter are shown in table 3.

| Environmental conditions in the ECU range | Environmental conditions in an industrial range |
|-----------------------------------------|-----------------------------------------------|
| T [°C] | P [mbar] | R H [%] | $e_0/d_0$ [ppm] | T [°C] | P [mbar] | R H [%] | $e_0/d_0$ [ppm] |
| 0 | 300 | 10 | 85.3 | 10 | 965 | 35 | 264.7 |
| 50 | 300 | 10 | 71.4 | 30 | 965 | 35 | 246.8 |
| 0 | 1100 | 10 | 313.0 | 10 | 1035 | 35 | 283.9 |
| 50 | 1100 | 10 | 263.8 | 30 | 1035 | 35 | 264.8 |
| 0 | 300 | 90 | 85.1 | 10 | 965 | 65 | 264.6 |
| 50 | 300 | 90 | 66.2 | 30 | 965 | 65 | 246.4 |
| 0 | 1100 | 90 | 312.8 | 10 | 1035 | 65 | 283.8 |
| 50 | 1100 | 90 | 258.6 | 30 | 1035 | 65 | 264.3 |

Table 2. Dimensionless error parameter $e_0/d_0$ [ppm] if $n = 1$.

Table 3. Typical values for the estimation of the fitting curves.

| Typical value         |                  |
|-----------------------|------------------|
| Temperature           | 20°C             |
| Pressure              | 1000 mbar        |
| Relative humidity     | 50%              |
| CO2 Concentration     | 450 ppm mole     |

2.2.1. Fitting curve for the temperature interval. The compensation for an interval from 10 to 30°C with 0.05°C between values, has been estimated with the Ciddor equation, $c_{0,T}$. A dimensionless compensation has been calculated in order to avoid the influence of the measured displacement, $c_{0,T}/d_0$, equation (9) from equation (8).

$$c_{0,T}/d_0 = 1 - \frac{1}{n_T}$$  \hspace{1cm} (9)

If the fitted curve is a polynomial of second order, the adjustment parameters are an R-square of 0.9999 and a mean quadratic error of 3.00E-9. An adjustment with a first order polynomial has been done too. The results can be compared in table 4 and figure 2.

Table 4. Fitted curves when only the temperature varies.

| Compensation | Fit type | R-Square | RMSE       |
|--------------|----------|----------|------------|
| $c_{0,T1}/d_T$ | 1$^{st}$ order Polynomial | 0.99981305 | 7.41E-08 |
| $c_{0,T2}/d_0$ | 2$^{nd}$ order Polynomial | 0.99999969 | 3.00E-09 |
2.2.2. Fitting curve for the pressure interval. The compensation in ppm for an interval from 965 to 1035 mbar with 0.1 mbar between values has been estimated with the Ciddor equation. A dimensionless compensation has been calculated in order to avoid the influence of the measured displacement, \( \frac{c_0 P}{d_0} \), equation (10) from equation (8).

\[
\frac{c_0 P}{d_0} = 1 - \frac{1}{n_P}
\] 

The adjustment results, for a polynomial of second order, are an R-square parameter of 1 and a mean quadratic error of 3.43E-9. The results of the second order adjustment and a first order adjustment can be compared in figure 3 and table 5.

![Figure 2](image1)

**Figure 2.** Fitted curves for a temperature interval with the rest of parameters in their typical values. (a) Fitted curve with a first order polynomial. (b) Fitted curve with a second order polynomial.

![Figure 3](image2)

**Figure 3.** Fitted curves for a pressure interval with the rest of parameters in their typical values. (a) Fitted curve with a first order polynomial. (b) Fitted curve with a second order polynomial.

| Compensation | Fit type          | R-Square | RMSE     |
|--------------|-------------------|----------|----------|
| \( \frac{c_0 P_1}{d_0} \) | 1st order polynomial | 1        | 3.39E-11 |
| \( \frac{c_0 P_2}{d_0} \) | 2nd order polynomial | 1        | 3.46E-15 |

2.2.3. Fitting curve for the temperature and pressure interval. The compensation in ppm has been estimated with the Ciddor equation for an interval from 10 to 30°C with 0.05°C between values for the
temperature and an interval from 965 to 1035 mbar with 0.1 mbar between values for the pressure. A dimensionless compensation has been calculated in order to avoid the influence of the measured displacement, $c_{0,T_P}/d_0$, equation (11) from equation (8).

$$
\frac{c_{0,T_P}}{d_0} = 1 - \frac{1}{n_{TP}}
$$

If the fitted surface, in this case, is a polynomial of second order in x and y, the adjustment parameters are an R-square parameter of 1 and a mean quadratic error of 3.55E-9. The results can be compared in figure 4 and table 6 with those obtained with a first order polynomial adjustment in x and y.

![Figure 4](image)

**Figure 4.** Fitted curves for a temperature and pressure interval with the rest of parameters in their typical values. (a) Fitted curve with a first order polynomial. (b) Fitted curve with a second order polynomial.

| Compensation       | Fit type                        | R-Square  | RMSE     |
|--------------------|---------------------------------|-----------|----------|
| $c_{0,T_P1}/d_0$   | 1st order polynomial in x and y | 0.99971005| 1.30E-07 |
| $c_{0,T_P2}/d_0$   | 2nd order polynomial in x and y | 0.99999978| 3.55E-09 |

Table 6. Fitted curves when temperature and pressure varies.

2.2.4. Summary of the compensation models compared in the experimental test. Six compensation models are going to be studied in the experimental test and their results will be compared with those using the Ciddor equation.

The denomination of each model and the estimation on the error associated can be seen in table 7.

| Model and curve description | Input parameters | Error associated \(^{a}\) | Compensation model |
|-----------------------------|------------------|---------------------------|-------------------|
| 1: 1st order polynomial     | Temperature      | $e_1 = d_1 - d_R$         | $d_1 = d_0 - c_{0,T_1}$ \(^{b}\) |
| 2: 2nd order polynomial     | Temperature      | $e_2 = d_2 - d_R$         | $d_2 = d_0 - e_{0,T2}$ \(^{b}\) |
| 3: 1st order polynomial     | Pressure         | $e_3 = d_3 - d_R$         | $d_3 = d_0 - e_{0,P1}$ \(^{c}\) |
| 4: 2nd order polynomial     | Pressure         | $e_4 = d_4 - d_R$         | $d_4 = d_0 - e_{0,P2}$ \(^{c}\) |
| 5: 1st order polynomial in x and y | Temperature and pressure | $e_5 = d_5 - d_R$ | $d_5 = d_0 - e_{0,T_P1}$ \(^{d}\) |
| 6: 2nd order polynomial in x and y | Temperature and pressure | $e_6 = d_6 - d_R$ | $d_6 = d_0 - e_{0,T_P2}$ \(^{d}\) |

\(^{a}\) From equation (8).
\(^{b}\) From table 4.
\(^{c}\) From table 5.
\(^{d}\) From table 6.
3. Experimental test

Data acquisition is carried out using an ECU to measure the ambient conditions. During the tests, the HPTI was fixed to a granite bench, figure 5c. Two data sets of 24h were captured. The characteristics of the measurement results and the environmental conditions in the experimental cases, A and B, are shown in figure 5a and b.

![Figure 5](image)

(a) Case A, displacement measured, and environmental conditions. (b) Case B, displacement measured and environmental conditions. (c) Experimental set.

The internal compensation of the interferometer, estimating the refractive index from Ciddor formula with the ECU measurement of temperature, pressure and relative humidity (CO₂ concentration is taken as 450ppm mole), is used as reference, \( n_{R_i} \), for the evaluation of the compensation obtained from the regression models explained in section 2.2. Six compensation models are analysed, its main characteristics and the quality parameters for its evaluation are shown in Table 7. The results of applying the compensation model are shown in figure 6.

![Figure 6](image)

(a) Results for case A. (b) Results for case B.

The maximum error \( e_i/d_0 \) has been reduced from 262ppm to values under 3ppm for all the models with the case A (0.1ppm with the first order polynomial in temperature and pressure, fifth model) and from 261 to 3.2ppm for all the models with the case B (0.04ppm with the fifth model). A summary of the results is shown in table 8.7.
Table 8. Compensation models results.

| Model                                          | Case A | Case B |
|------------------------------------------------|--------|--------|
| 1: 1\(^{st}\) order Temperature                | \(e_i/d_0\) [ppm] | Time\(^a\) [\(\mu\)s] | \(e_i/d_0\) [ppm] | Time\(^a\) [\(\mu\)s] |
| 2: 2\(^{nd}\) order Temperature                | 2.9    | 0.036  | 3.2    | 0.033  |
| 3: 1\(^{st}\) order Pressure                   | 2.8    | 0.035  | 3.1    | 0.035  |
| 4: 2\(^{nd}\) order Pressure                   | 1.1    | 0.033  | 1.4    | 0.033  |
| 5: 1\(^{st}\) order Temperature and Pressure   | 1.1    | 0.036  | 1.4    | 0.033  |
| 6: 2\(^{nd}\) order Temperature and Pressure   | 0.1    | 0.033  | 0.04   | 0.032  |

\(^a\) Time per iteration measured with 40,000 iterations (processor frequency of 2.60GHz).

4. Conclusions

Environmental conditions affect the measurement process specially when using a laser interferometer. In this work, the compensation of the environmental conditions with the Ciddor equation has been studied in order to adjust a compensation curve to fit the environmental compensation needed. Six compensation models has been presented, the first and the second model are fitted to compensate the temperature effect when the rest of parameters stay in their typical values (with a first order polynomial the first one and a second order polynomial the second). The third and the fourth models compensate the effect of the pressure, again, with a first order polynomial the third and a second order polynomial the second. Finally, the fifth and the sixth models compensate the variation of temperature and pressure using a first and a second order polynomial. The proposed models compensate the environmental conditions obtaining results close to those obtained with the Ciddor equation (under 4ppm) with a time cost under 50 \(\mu\)s per each 1000 iterations. These results allow the use of the compensation models in a measurement process involving several HPTI with only one ECU maintaining high rates of data acquisition frequency.

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