Backstepping sliding mode control of electric linear load simulator

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Abstract: In order to solve the excess force problem in the electric linear load simulator (ELLS), a backstepping sliding mode controller based on the backstepping control idea is proposed. According to the function, the state of the system is divided into load torque subsystem and PMSM mechanical subsystem. First, the virtual control of the load torque subsystem is designed, and then the controller is designed for the PMSM mechanical subsystem, and the Lyapunov stability is proved. Finally, the simulation test is compared. The simulation results show that, compared with the traditional PID control, the controller based on the backstepping sliding mode design can effectively suppress the excess force and improve the loading accuracy of the system.

1. Introduction

Electric linear servo is a high-precision position control device, which is mainly used in the aerospace to quickly adjust the attitude of the aircraft and accurately change the launch angle of the missile. Its performance directly affects the working characteristics of the object to be controlled. In the manufacture of steering gear, simulation loading test is essential [1]. Electric linear load simulator (ELLS) can be realized in the laboratory environment to reproduce the load of the linear steering gear under actual working conditions [2]. With the development of technology, the performance of the aircraft to be controlled has higher requirements, which means that the steering gear needs to have higher working characteristics, and the load simulator is required to reproduce more complex loads, and have higher accuracy and speed.

As a hardware in the loop simulation loading equipment, ELLS can load the steering gear in a laboratory environment, verify and detect the static and dynamic characteristics of the steering gear under complex working conditions, predict the deficiencies in design and manufacturing, and feed back to the design department. It can shorten the development cycle of the steering gear and reduce the cost. In order to reproduce the working load of the steering gear as realistically as possible, ELLS should have the advantages of wide loading frequency band, high loading accuracy [3-4].

As a strong position disturbance system, the existence of extra force is inevitable, which will seriously affect the loading accuracy [5]. At the same time, non-linear factors such as inertia and friction are indispensable, which affect the dynamic performance and interference suppression ability of ELLS [6]. Therefore, how to design an effective controller is the key to suppress the excess force, which is helpful to improve the loading performance of the load simulator. Scholars at home and abroad have done a lot of research and have been applied in practice, which effectively improves the loading performance of load simulator. Literature [7] uses dual motors in the load simulator. The main motor is used for loading and the auxiliary motor is used as position follower. The original passive
loading is approximately converted to active loading, and the position disturbance is approximately eliminated. This scheme requires the synchronous compensation motor loading frequency width to approach or even exceed the load simulator, the mechanical structure is complex, the volume is large, and the system is not easy to debug. Literature [8] adopts the angle feedforward compensation of the steering gear, and introduces the torque feedforward compensation method, the excess force is suppressed, but the system has poor characteristics in the high frequency link. Literature [9-10-11] proposes a full-state feedback control method based on backstepping control, recombining the system equations into multiple virtual subsystems, using backstepping control ideas to design virtual control variables for each virtual system, and then gradually Inversely, a nonlinear controller containing the loading system and the state of the loaded system. Literature [12] adopts the method based on RBF-PID with small overshoot, fast response speed, strong anti-interference ability, and better tracking accuracy.

In this paper, a controller based on backstepping sliding mode is proposed to solve the problem of excess force in the ells system. By dividing the ELLS system into two subsystems, using the idea of backstepping control, the virtual control input is calculated, and then the appropriate sliding mode controller is designed to reduce the disturbance of the excess force to the system. Lyapunov stability function is used to analysis the stability and asymptotic convergence of the designed control strategy. MATLAB/Simulink software simulation proves the effectiveness of the controller set in this paper.

2. System Components
The structure of ells system used to test the performance of linear steering gear is shown in Figure 1. It consists of: PMSM, bellows coupling, torque and speed sensor, ball screw pair, grating ruler, tension and pressure sensor, NI industrial computer, PXI motion control card, PXIe data acquisition card and PC. Torque speed sensor, grating ruler, tension and pressure sensor are used for signal acquisition, and PC is used for software programming and loading display.

3. ELLS mathematical model
As a passive torque loading system, the hardware structure of ELLS is rigidly connected with the steering gear, resulting in strong position disturbance of ELLS. In the loading test, the steering gear is actively moving, and ELLS does position tracking on the one hand, and force loading on the other. It is unavoidable that there is excess force. In addition, there are also nonlinear factors, which seriously affect the accuracy of the output force of ells. Therefore, the mathematical model is very important.

Compared with the traditional electro-hydraulic load simulator, ELLS has the advantages of simple structure, small volume, high speed and high reliability. As the ELLS drive component, PMSM is connected with the steering gear to be tested through coupling, torque speed sensor, ball screw pair and tension pressure sensor. The tension pressure sensor can monitor the loading force in real time and realize the force closed-loop control. PMSM is a nonlinear system with time-varying parameters and strong coupling, which is often simplified in actual models. Through coordinate transformation, the three-phase coupling model of the motor is simplified to d-q axis. Generally, the dynamic performance
and saturation interval of PMSM drives are ignored. The study found that the voltage $V_{in}$ input by the ELLS controller to the PMSM driver and the electromagnetic torque $T_e$ output by the PMSM can be approximately replaced by a proportional link, which has the following equation:

$$T_e = K_m V_{in}$$  \(1\)

Among them: $K_m$ is the conversion coefficient of the controller output voltage $V_{in}$ into electromagnetic torque $T_e$; $V_{in}$ is the voltage input of the driver. The torque balance equation of the ELLS system is:

$$T_e = J_m \frac{d w_m}{dt} + B_m w_m + \frac{P}{2\pi} F$$  \(2\)

Among them: $J_m$ is the moment of inertia of each part of the system converted to the motor shaft; $P$ is the lead of the ball screw pair; $B_m$ is the reduced damping coefficient; $w_m$ is the system rotation speed; $F$ is the load force. After connecting with the steering gear.

$$F = \frac{2\pi}{P} K_A (\theta_m - \theta_i)$$  \(3\)

Among them: $K_A$ is the stiffness coefficient; $\theta_m$ is the rotation angle of the ELLS end; $\theta_i$ is the equivalent rotation angle of the connecting steering gear end. Take $K_w$ as:

$$K_w = \frac{2\pi}{P}$$  \(4\)

Combined with equation (3) and equation (4), the output force of ELLS is:

$$F = K_w K_A (w_m - w_i)$$  \(5\)

Among them: $w_m$ is the speed of the ELLS; $w_i$ is the equivalent speed of the end connected to the steering gear.

Because ELLS is a strong position coupling system and combined with equations (3), (4) and (5), it is easy to know that the motion of the actuator will directly affect the dynamic loading characteristics of the ELLS system and reduce the accuracy of its output force. Establish the state equation of the ELLS system, and take the state variable $x = [x_1, x_2, x_3]^T$, the variables are as follows:

$$x_1 = F; x_2 = w_m; u = V_{in}$$  \(6\)

By substituting equation (1), equation (2), equation (4) and equation (5) into equation (6), the equation of state of ELLS is as follows:

$$\begin{align*}
\dot{x}_1 &= K_w K_A (x_2 - w_i) \\
\dot{x}_2 &= \frac{K_m}{J_m} u - \frac{1}{J_m K_w} x_1 - \frac{B_m}{J_m} x_2 \\
y &= x_1
\end{align*}$$  \(7\)

In summary, the open-loop control chart of the ELLS system is shown in Figure 2:
The loading force of the system can be obtained through the tension and pressure sensor installed in the ELLS. The displacement measured by the grating ruler can be simply converted to obtain the rotation angle. That is, state variables can be detected in real time. The detection value can be used to set an appropriate controller to make the ELLS output force $F$ accurately track the loading signal $F_d(t)$. This paper makes the following assumptions:

Hypothesis 1: $F_d(t)$ is continuous, and the first two derivatives are consistent and continuous.

Hypothesis 2: $\theta$ is continuous, and the first two derivatives are consistent and continuous.

4. Controller design

The key to improve the loading accuracy of ELLS is to design a suitable controller to ensure that the error between the state variable $x_i$ and the expected output force $F_d(t)$ can converge to zero quickly, and the system has certain robustness and can resist external uncertain disturbance. Based on the idea of backstepping control, this paper designs a backstepping sliding mode controller. According to the functional structure of ELLS, the state equation (7) of ELLS is decomposed into load torque subsystem and PMSM mechanical subsystem. Load torque subsystem is shown in equation 1 of state equation (7); and the mechanical subsystem of PMSM is shown in equation (2) of equation of state (7). Based on the backstepping idea, the load torque subsystem is first analysis: the task is to set the virtual control quantity $x_{2d}$ and realize the gradual follow of the system output $y$ to the system’s desired trajectory $x_{2d}$. Then, for the PMSM mechanical subsystem, a terminal sliding mode controller is designed so that the tracking error of $x_2$ to $x_{2d}$ can quickly converge to zero, and the control input $u$ of the system is introduced, that is, the force control voltage $V_{in}$ input to the PMSM driver. The designed controller needs to ensure that the system is stable in the sense of Lyapunov.

4.1 Load torque subsystem design

ELLS load torque subsystem is:

$$12 (\omega A l x - K x w)$$

Taking the system state variable $x_2$ as its virtual control input, the design goal is to make $x_{2d}$ realize error free tracking for its expected value of $x_{2d}$. Set the error of the load torque subsystem:

$$e_1 = x_1 - F_d$$

$$\dot{e}_1 = \dot{x}_1 - \dot{F}_d$$

Among them: $F_d = x_{id}$. In order to obtain the virtual input $x_{2d}$ and ensure that the system state variable $x_i$ accurately tracks the system expectation $x_{id}$, the sliding surface is designed as follows:

$$S_i = c_1 e_i + \dot{e}_i, c_1 > 0$$
The convergence rate of the selected index is as follows:
\[ \dot{S}_i = -k_i S_i, \quad k_i > 0 \]  
(11)

Obtained: \( S_i \dot{S}_i = -k_i S_i^2 \), it can be obtained when \( S_i \neq 0 \), \( S_i \dot{S}_i < 0 \) is established. The sliding surface \( S_i \) can be reached. Combining equations (10) and (11), we can get:
\[ \dot{S}_i = \dot{e}_i + c_i \dot{e}_i = -k_i S_i \]  
(12)

Substituting equations (7) into the above equation, the virtual input \( x_{2d} \) of the load torque subsystem can be obtained as:
\[ x_{2d} = \frac{-k_i S_i - \ddot{F} + \ddot{F}_d + c_1 \ddot{F}_d + w_f}{c_1 K_w K_d} \]  
(13)

Construct the Lyapunov equation as:
\[ V_1 = \frac{1}{2} S_i^2 \]  
(14)

The derivative can be obtained: \( \dot{V}_1 = -k_i S_i^2 \leq 0 \) holds, and the designed \( x_{2d} \) can satisfy \( e_1 \) to converge to zero in a finite time.

4.2 PMSM mechanical subsystem design

The PMSM mechanical subsystem is:
\[ x_2 = \frac{K_m u}{J_m} - \frac{1}{J_m K_w} x_1 - \frac{B_m}{J_m} x_2 \]  
(15)

Define the error of this step as:
\[ e_2 = x_2 - x_{2d} \]
\[ \dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} \]  
(16)

Among them: \( x_{2d} \) is the virtual input of ELLS load torque subsystem obtained in the previous step. For the PMSM mechanical subsystem, a terminal sliding mode controller is designed to ensure that the error \( x_{2d} \) converges to zero in a finite time.

The design terminal sliding surface is:
\[ S_2 = e_2 + \frac{1}{\gamma^{p/q}} \dot{e}_2 e_2^{p/q}, \quad c_2 > 0 \]  
(17)

Among them: \( \gamma \) is a positive real number; \( p \) and \( q \) are odd numbers.

Design control input \( u \) is:
\[ \begin{cases} 
\dot{u} = u_0 + u_1 \\
u_0 = \frac{J_m}{K_w} \left( \frac{1}{J_m K_w} x_1 + \frac{B_m}{J_m} x_2 + \dot{x}_{2d} \right) \\
u_1 = -\frac{J_m}{K_w} \int_{\tau_0}^{\tau} \gamma^{p/q} \frac{q}{p} e_2^{2-(p/q)} + k_z \text{sgn}(S_2) + (k_1 + c_2) \left| \dot{S}_2 \right| d\tau 
\end{cases} \]  
(18)

Among them: \( k_2 > 0, k_1 > 0, c_2 > 0 \). The Lyapunov equation to construct the system is:
\[ V_2 = V_1 + \frac{1}{2} S_2^2 = \frac{1}{2} S_1^2 + \frac{1}{2} S_2^2 \geq 0 \]  
(19)

From equation (15) and equation (16), we can get:
\[ \dot{e}_2 = \ddot{x}_2 - \ddot{x}_{2d} = \frac{K_m}{J_m} u - \frac{1}{J_m K_w} x_1 - \frac{B_m}{J_m} x_2 - \ddot{x}_{2d} \]
\[ = \frac{K_m}{J_m} (u + u_1) - \frac{1}{J_m K_w} x_1 - \frac{B_m}{J_m} x_2 - \ddot{x}_{2d} \]  
(20)

Substituting equation (18) into the above equation to simplify and get:
\[ \dot{e}_2 = \frac{K_m}{J_m} u_t \]  
(21)

Deriving it and substituting it into equation (18), we can get:
\[ \dot{e}_2 = \frac{K_m}{J_m} \dot{u}_t \]
\[ = -\gamma^{p/q} (p/q) e^{2-(p/q)} - k_2 \text{sgn}(S_2) - (k_3 + c_2 |e_2|)S_2 \]  
(22)

Derivation of \( S_2 \) can be obtained:
\[ \dot{S}_2 = \dot{e}_2 + \frac{1}{\gamma^{p/q}} (p/q) (\dot{e}_2)^{\frac{1}{p}} \dot{e}_2 \]
\[ = \frac{1}{\gamma^{p/q}} (p/q) (\dot{e}_2)^{\frac{1}{p}} \left[ \dot{e}_2 + \gamma^{p/q} (q/p) (\dot{e}_2)^{\frac{2}{p}} \right] \]  
(23)

Substituting equation (22) into the above equation:
\[ \dot{S}_2 = \frac{1}{\gamma^{p/q}} (p/q) (\dot{e}_2)^{\frac{1}{p}} \left[ -k_2 \text{sgn}(S_2) - (k_3 + c_2 |e_2|)S_2 \right] \]  
(24)

Derivation of equation (19) can be obtained:
\[ \dot{V}_2 = S_1 \dot{S}_1 + S_2 \dot{S}_2 \]
\[ = -k_1 S_1^2 + \frac{1}{\gamma^{p/q}} (p/q) (\dot{e}_2)^{\frac{1}{p}} \left[ -k_2 |S_2| - (k_3 + c_2 |e_2|)S_2^2 \right] \]  
(25)

\[ \leq 0 \]

The derivative of the constructed Lyapunov function is non-positive, and the sliding surface is asymptotically reachable. The designed controller makes the ELLS asymptotically stable, so as to realize the error-free tracking of the input.

5. Simulation analysis
In order to verify the designed backstepping sliding mode controller to suppress the excess force, Simulink is used to simulate and analyze the ELLS system. The values of the system parameters are shown in Table 1.
Table 1. parameter values of the system.

| parameter | numerical value | parameter | numerical value |
|-----------|-----------------|-----------|----------------|
| $J_w$ / (kg \cdot m^2) | 0.0135 | $\gamma$ | 3 |
| $B_m$ / (Nm / rad \cdot s^{-1}) | 0.4966 | $p$ | 3 |
| $K_m$ / (Nm / A) | 27 | $q$ | 5 |
| $K_d$ / (Nm / rad) | 6100 | $k_2$ | 260 |
| $P$ / (mm) | 25 | $k_3$ | 100 |
| $c_1$ | 1 | $c_2$ | 0.5 |
| $k_1$ | 260 | | |

In order to test the suppression effect of the designed controller on the excess force, the simulation analysis of the ells system is carried out. In this paper, PID control is used to compare the loading curve with the designed controller. As shown in Figure 3, set the loading force input of the system to 0, and set the motion of the steering gear to be measured to 1mm/1Hz (amplitude is 1mm, frequency is 1Hz). In Figure 3, (a) is the excess force curve of the system when using PID control; Figure 3(b) is the excess force curve of the system when using the controller of this paper. It can be seen from the excess force curve that the error amplitude reaches 30N when PID control is used. After using the backstepping sliding mode controller, the excess force is reduced to 15N, and the amplitude of the excess force is reduced to 50% of the PID control.

In order to better verify the advantages and disadvantages of the controller designed in the paper, the sine amplitude of the system is 1000N, and the 1Hz and 5Hz loading simulation experiments are carried out, and the results are compared with the traditional PID control. During dynamic loading, the motion of the steering gear to be tested is set to 1mm/1(5) Hz (amplitude is 1mm, frequency is 1(5) Hz) and sinusoidal motion ELLS input 1000N/1(5) Hz (amplitude 1000N, frequency is 1(5) Hz), the simulation results are shown in Figure 4. Figures 4 and 5: (a) is the system loading curve when using...
PID control; (b) is the loading curve of the system when using the controller in this paper. The comparison diagram between PID and the controller designed in this paper is made when 1Hz and 5Hz are loaded respectively, as shown in Figure 6.

(a) Load curve during PID control.

(b) Load curve during Backstepping sliding mode control.

Figure 4. 1Hz dynamic loading curve.

(a) Load curve during PID control.

(b) Load curve during Backstepping sliding mode control.

Figure 5. 5Hz dynamic loading curve.
In order to intuitively compare the effect of the designed controller and PID controller, the excess force quantitative index is introduced here:

$$E = \int_0^t |e| dt$$  \hspace{1cm} (26)

Among them: $e$ is the dynamic loading of the excess force; $t$ is taken as 10s; $E_{EEE}$ is the control effect of the controller, and the smaller the value, the better the suppression of the excess force.

**Table 2. Value of excess force.**

| Controller                             | Input signal                      |
|----------------------------------------|-----------------------------------|
|                                        | steering engine (1mm/1HZ)         |
|                                        | steering engine (1mm/5HZ)         |
|                                        | PMSM (1000N/1HZ)                  |
|                                        | PMSM (1000N/5HZ)                  |
| PID                                    | 182.3                             |
|                                        | 1319.8                            |
| Backstepping sliding mode control      | 94.8                              |
|                                        | 494.3                             |
| Excess force suppression rate           | 52%                               |
|                                        | 63%                               |

From the dynamic loading curve figure 6 (a) and table 2, it can be found that the traditional PID controller, the ELLS system has serious phase lag and amplitude attenuation in the loading. After adopting the controller designed in this paper, the suppression of the residual force at 1Hz and 5Hz is improved by 52% and 63% respectively, which effectively improves the loading accuracy of the system.

6. **Conclusion**

In order to improve the dynamic loading accuracy of the electric linear load simulator, aiming at the problems of phase lag and amplitude attenuation in the system, a backstepping sliding mode controller is proposed.
(1) Aiming at the excess force problem of the ELLS system, based on the backstepping design idea, a backstepping terminal sliding mode controller is designed.

(2) The software simulation results show that the traditional PID controller, the ELLS system has phase lag and amplitude attenuation problems. Using the sliding mode controller designed in this paper, the excess force of the system is significantly reduced, and the phase lag is reduced. This control method can better solve the problem of excess force of the passive torque servo system and has better dynamic tracking performance.

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