Warm-Intermediate inflationary universe model

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Abstract

Warm inflationary universe models in the context of intermediate expansion, between power law and exponential, are studied. General conditions required for these models to be realizable are derived and discussed. This study is done in the weak and strong dissipative regimes. The inflaton potentials considered in this study are negative-power-law and powers of logarithms, respectively. The parameters of our models are constrained from the WMAP three and five year data.

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I. INTRODUCTION

It is well known that warm inflation, as opposed to the conventional cool inflation, presents the attractive feature that it avoids the reheating period [1]. In these kind of models dissipative effects are important during the inflationary period, so that radiation production occurs concurrently together with the inflationary expansion. If the radiation field is in a highly excited state during inflation, and this has a strong damping effect on the inflaton dynamics, then, it is found a strong regimen of warm inflation. Also, the dissipating effect arises from a friction term which describes the processes of the scalar field dissipating into a thermal bath via its interaction with other fields. Warm inflation shows how thermal fluctuations during inflation may play a dominant role in producing the initial fluctuations necessary for Large-Scale Structure (LSS) formation. In this way, density fluctuations arise from thermal rather than quantum fluctuations [2]. These fluctuations have their origin in the hot radiation and influence the inflaton through a friction term in the equation of motion of the inflaton scalar field [3]. Among the most attractive features of these models, warm inflation end at the epoch when the universe stops inflating and ”smoothly” enters in a radiation dominated Big-Bang phase [1]. The matter components of the universe are created by the decay of either the remaining inflationary field or the dominant radiation field [4].

A possible evolution during inflation is the particular scenario of intermediate inflation, in which the scale factor, $a(t)$, evolves as $a = \exp(Atf)$, where $A$ and $f$ are two constants, where $0 < f < 1$; the expansion of this universe is slower than standard de Sitter inflation ($a = \exp(HT)$), but faster than power law inflation ($a = t^p; p > 1$), this is the reason why it is called “intermediate”. This model was introduced as an exact solution for a particular scalar field potential of the type $V(\phi) \propto \phi^{-4(f^{-1} - 1)}$ [5]. In the slow-roll approximation, and with this sort of potential, it is possible to have a spectrum of density perturbations which presents a scale-invariant spectral index, i.e. $n_s = 1$, the so-called Harrison-Zel’довich spectrum provided that $f$ takes the value of $2/3$ [6]. Even though this kind of spectrum is disfavored by the current WMAP data [7, 8], the inclusion of tensor perturbations, which could be present at some point by inflation and parametrized by the tensor-to-scalar ratio $r$, the conclusion that $n_s \geq 1$ is allowed providing that the value of $r$ is significantly nonzero [9]. In fact, in Ref. [10] was shown that the combination $n_s = 1$ and $r > 0$ is given by a version of
the intermediate inflation in which the scale factor varies as $a(t) \propto e^{t^{2/3}}$ within the slow-roll approximation.

The main motivation to study this sort of model becomes from string/M-theory. This theory suggests that in order to have a ghost-free action high order curvature invariant corrections to the Einstein-Hilbert action must be proportional to the Gauss-Bonnet (GB) term \[11\]. GB terms arise naturally as the leading order of the $\alpha$ expansion to the low-energy string effective action, where $\alpha$ is the inverse string tension \[12\]. This kind of theory has been applied to possible resolution of the initial singularity problem \[13\], to the study of Black-Hole solutions \[14\], accelerated cosmological solutions \[15\]. In particular, very recently, it has been found \[16\] that for a dark energy model the GB interaction in four dimensions with a dynamical dilatonic scalar field coupling leads to a solution of the form $a = a_0 \exp A t^f$, where the universe starts evolving with a decelerated exponential expansion. Here, the constant $A$ becomes given by $A = \frac{2}{3n}$ and $f = \frac{1}{2}$, with $\kappa^2 = 8\pi G$ and $n$ is a constant. In this way, the idea that inflation, or specifically, intermediate inflation, comes from an effective theory at low dimension of a more fundamental string theory is itself very appealing.

Thus, our aim in this paper is to study an evolving intermediate scale factor in the warm inflationary universe scenario. We will do this for two regimes; the weak and the strong dissipative regimes.

The outline of the paper is as follows. The next section presents a short review of the modified Friedmann equation and the warm-intermediate inflationary phase. In the Sections \[III\] and \[IV\] we discuss the weak and strong dissipative regimens, respectively. Here, we give explicit expressions for the dissipative coefficient, the scalar power spectrum and the tensor-scalar ratio. Finally, our conclusions are presented in Section \[V\]. We chose units so that $c = \hbar = 1$.

II. THE WARM-INTERMEDIATE INFLATIONARY PHASE.

We start by writing down the modified Friedmann equation, by using the FRW metric. In particular, we assume that the gravitational dynamics give rise to a Friedmann equation of the form

$$H^2 = \frac{\kappa}{3} [\rho_\phi + \rho_\gamma],$$

(1)
where \( \kappa = 8\pi G = 8\pi/m_p^2 \) (here \( m_p \) represents the Planck mass), \( \rho_\phi = \dot{\phi}^2/2 + V(\phi) \), \( V(\phi) = V \) is the scalar potential and \( \rho_\gamma \) represents the radiation energy density.

The dynamics of the cosmological model, for \( \rho_\phi \) and \( \rho_\gamma \) in the warm inflationary scenario is described by the equations

\[
\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -\Gamma \dot{\phi}^2, \tag{2}
\]

and

\[
\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma \dot{\phi}^2. \tag{3}
\]

Here \( \Gamma \) is the dissipation coefficient and it is responsible of the decay of the scalar field into radiation during the inflationary era. \( \Gamma \) can be assumed to be a constant or a function of the scalar field \( \phi \), or the temperature \( T \), or both \cite{1}. On the other hand, \( \Gamma \) must satisfy \( \Gamma > 0 \) by the Second Law of Thermodynamics. Dots mean derivatives with respect to time.

During the inflationary epoch the energy density associated to the scalar field dominates over the energy density associated to the radiation field \cite{1, 2}, i.e. \( \rho_\phi > \rho_\gamma \), the Friedmann equation (1) reduces to

\[
H^2 \approx \frac{\kappa}{3} \rho_\phi, \tag{4}
\]

and from Eqs. (2) and (4), we can write

\[
\dot{\phi}^2 = -\frac{2\dot{H}}{\kappa (1 + R)}, \tag{5}
\]

where \( R \) is the rate defined as

\[
R = \frac{\Gamma}{3H}. \tag{6}
\]

For the weak (strong) dissipation regime, we have \( R < 1 \) (\( R > 1 \)).

We also consider that during warm inflation the radiation production is quasi-stable \cite{1, 2}, i.e. \( \dot{\rho}_\gamma \ll 4H\rho_\gamma \) and \( \dot{\rho}_\gamma \ll \Gamma \dot{\phi}^2 \). From Eq. (3) we obtained that the energy density of the radiation field becomes

\[
\rho_\gamma = \frac{\Gamma \dot{\phi}^2}{4H} = -\frac{\Gamma \dot{H}}{2\kappa H (1 + R)}, \tag{7}
\]

which could be written as \( \rho_\gamma = C_\gamma T^4 \), where \( C_\gamma = \pi^2 g_* / 30 \) and \( g_* \) is the number of relativistic degrees of freedom. Here \( T \) is the temperature of the thermal bath.

From Eqs. (5) and (7) we get that

\[
T = \left[ \frac{\Gamma \dot{H}}{2\kappa C_\gamma H (1 + R)} \right]^{1/4}. \tag{8}
\]
From first principles in quantum field theory the dissipation coefficient $\Gamma$ is computed for models in cases of low-temperature regimes [17] (see also Ref. [18]). Here, was developed the dissipation coefficients in supersymmetric models which have an inflaton together with multiplets of heavy and light fields. In this approach, it was used an interacting supersymmetric theory, which has three superfields $\Phi$, $X$ and $Y$ with a superpotential, $W = g\Phi X^2 + hXY^2$, where $\phi$, $\chi$ and $y$ refer to their bosonic component. The inflaton field couples to heavy bosonic field $\chi$ and fermions $\psi_\chi$, obtain their masses through couplings to $\phi$, where $m_{\psi_\chi} = m_\chi = g\phi$. In the low-temperature regime, i.e. $m_\chi, m_{\psi_\chi} > T > H$, the dissipation coefficient, when $X$ and $Y$ are singlets, becomes [17]

$$\Gamma \simeq 0.64 g^2 h^4 \left( \frac{g\phi}{m_\chi} \right)^4 \frac{T^3}{m_\chi^2}. \quad (9)$$

This latter equation can be rewritten as

$$\Gamma \simeq C_{\phi} \frac{T^3}{\phi^2}, \quad (10)$$

where $C_{\phi} = 0.64 h^4 \mathcal{N}$. Here $\mathcal{N} = N_\chi N_{\text{decay}}^2$, where $N_\chi$ is the multiplicity of the $X$ superfield and $N_{\text{decay}}$ is the number of decay channels available in $X$’s decay [17, 19].

From Eq.(8) the above equation becomes

$$\Gamma^{1/4} (1 + R)^{3/4} \simeq \left[ \frac{-2 \dot{H}}{9 \kappa C_\chi H} \right]^{3/4} \frac{C_{\phi}}{\phi^2}, \quad (11)$$

which determines the dissipation coefficient in the weak (or strong) dissipative regime in terms of scalar field $\phi$ and the parameters of the model.

In general the scalar potential can be obtained from Eqs.(11) and (7)

$$V(\phi) = \frac{1}{\kappa} \left[ 3H^2 + \frac{\dot{H}}{(1 + R)} \left( 1 + \frac{3}{2} R \right) \right], \quad (12)$$

which could be expressed explicitly in terms of the scalar field, $\phi$, by using Eqs.(5) and (11), in the weak (or strong) dissipative regime.

Solutions can be found for warm-intermediate inflationary universe models where the scale factor, $a(t)$, expands as follows

$$a(t) = \exp( A t^f). \quad (13)$$

Recalled, that $f$ is a dimensionless constant parameter with range $0 < f < 1$, and $A > 0$ has dimension of $m_f$. In the following, we develop models for a variable dissipation coefficient.
\[\Gamma, \text{ and we will restrict ourselves to the weak (or strong) dissipation regime, i.e. } R < 1 \text{ (or } R > 1).\]

III. THE WEAK DISSIPATIVE REGIME.

Assuming that, once the system evolves according to the weak dissipative regime, i.e. \( \Gamma < 3H \), it remains in such limit for the rest of the evolution. From Eqs. (5) and (13), we obtained a relation between the scalar field and cosmological times given by

\[\phi(t) = \phi_0 + \sqrt{\frac{8A(1-f)}{f \kappa}} t^{f/2}, \quad (14)\]

where \( \phi(t = 0) = \phi_0 \). The Hubble parameter as a function of the inflaton field, \( \phi \), results in

\[H(\phi) = A^{1/f} f^{(2f-1)/f} \left[ \frac{\kappa}{8(1-f)} \right]^{(f-1)/f} (\phi - \phi_0)^{2(f-1)/f}. \quad (15)\]

Without loss of generality \( \phi_0 \) can be taken to be zero.

From Eq.(11) we obtain for the dissipation coefficient as function of scalar field

\[\Gamma(\phi) = B \phi^{-\beta_1}, \quad (16)\]

where

\[B = \frac{8C_\phi^4(1-f)^3}{27^2 \kappa^3 C_\gamma^3} \left[ \frac{8A(1-f)}{\kappa f} \right]^{3/f}, \quad \text{and} \quad \beta_1 = \frac{2(4f + 3)}{f}. \]

Using the slow-roll approximation, \( \dot{\phi}^2/2 < V(\phi) \), and \( V(\phi) > \rho_\gamma \), the scalar potential given by Eq.(12) reduces to

\[V(\phi) \simeq \frac{3H^2}{\kappa} = C\phi^{-\beta_2}, \quad (17)\]

where

\[C = \frac{3f^2 A^2}{\kappa} \left[ \frac{f \kappa}{8A(1-f)} \right]^{2(f-1)/f}, \quad \beta_2 = \frac{4(1-f)}{f}. \]

Note that this kind of potential does not present a minimum. Note also that the scalar field \( \phi \), the Hubble parameter \( H \), and the potential \( V(\phi) \) become independent of the parameters \( C_\phi \) and \( C_\gamma \).

Introducing the dimensionless slow-roll parameter \( \varepsilon \), we write

\[\varepsilon = -\frac{\dot{H}}{H^2} = \frac{8(1-f)^2}{\kappa f^2} \frac{1}{\phi^2}, \quad (18)\]
and the second slow-roll parameter $\eta$

$$\eta = - \frac{\ddot{H}}{H \dot{H}} = \frac{8 (1 - f) (2 - f)}{\kappa f^2} \frac{1}{\dot{\phi}^2}. \quad (19)$$

So, the condition for inflation to occur $\ddot{a} > 0$ (or equivalently $\varepsilon < 1$) is only satisfied when

$$\dot{\phi}^2 > \frac{8 (1 - f)^2}{\kappa f^2}.$$  

The number of e-folds between two different values of cosmological times $t_1$ and $t_2$ (or equivalently between two values $\phi_1$ and $\phi_2$ of the scalar field) is given by

$$N = \int_{t_1}^{t_2} H dt = A (t_2^f - t_1^f) = \frac{f \kappa}{8 (1 - f)} (\phi_2^2 - \phi_1^2). \quad (20)$$

Here we have used Eq.(14).

If we assume that inflation begins at the earliest possible stage, that is, at $\varepsilon = 1$ (or equivalently $\ddot{a} = 0$), the scalar field becomes

$$\phi_1 = 2 \sqrt{2} \frac{(1 - f)}{f \sqrt{\kappa}}. \quad (21)$$

As argued in Refs.[1, 20], the density perturbation could be written as $P_R^{1/2} = \frac{H}{\dot{\phi}} \delta \phi$. In particular in the warm inflation regime, a thermalize radiation component is present, therefore, inflation fluctuations are dominantly thermal rather than quantum. In the weak dissipation limit, we have $\delta \phi^2 \approx HT$. From Eqs.(5) and (8), $P_R$ becomes

$$P_R \approx \left[ \frac{\kappa^3 \Gamma}{25 \, C_\gamma} \right]^{1/4} \left[ \frac{H^{11/3}}{-H} \right]^{3/4} \beta_4 \phi^{2(f-2)/f}, \quad (22)$$

where

$$\beta_4 = \left[ \frac{B \kappa^5}{25 \, C_\gamma} \right]^{1/4} B_1, \quad \text{and} \quad B_1 = \frac{f^2 A^2}{(1 - f)^{3/4}} \left[ \frac{\kappa f}{8 A (1 - f)} \right]^{(8f-5)/(4f)}.$$  

The scalar spectral index $n_s$ is given by $n_s - 1 = \frac{d \ln P_R}{d \ln k}$, where the interval in wave number is related to the number of e-folds by the relation $d \ln k(\phi) = dN(\phi) = (H/\dot{\phi}) d\phi$. From Eqs. (14) and (22), we get,

$$n_s = 1 - \frac{8 (1 - f) (2 - f)}{\kappa f^2} \frac{1}{\dot{\phi}^2}. \quad (23)$$

Since $1 > f > 0$, we clearly see that the scalar index in the weak dissipative regime becomes $n_s < 1$. The scalar spectral index can be re-expressed in terms of the number of e-folding, $N$. By using Eqs.(20) and (21) we have

$$n_s = 1 - \frac{(2 - f)}{[1 + f (N - 1)]}. \quad (24)$$
and the value of \( f \) in terms of the \( n_s \) and \( N \) becomes

\[
f = \frac{(1 + n_s)}{N(1 - n_s) + n_s}.
\]

In particular, for \( n_s = 0.96 \) and \( N = 60 \) we obtain that \( f \approx 0.58 \).

From Eqs.\((20), (21), (22)\) and \((23)\), we can write the parameter \( A \) in terms of the particle physics parameters \( C_\gamma \) and \( C_\phi \), and \( \mathcal{P}_R \), \( N \) and \( n_s \) (since, \( f \) is function of the \( N \) and \( n_s \), as could be seen from the latter equation), in the form

\[
A = \left( \frac{C_\gamma}{C_\phi} \right)^{f/2} \frac{\mathcal{P}_R^{f/2} B_2}{[1 + (N - 1)f]^{(f-2)/2}},
\]

where \( B_2 \) is given by

\[
B_2 = (108)^f \frac{f}{f + 1} \left[ \frac{8 (1 - f)}{\kappa} \right]^{f/2}.
\]

As it was mentioned in Ref.\([22]\) the generation of tensor perturbations during inflation would produce gravitational wave. The corresponding spectrum becomes \( \mathcal{P}_g = 8\kappa(H/2\pi)^2 \).

For \( R < 1 \) and from Eq.\((22)\) we may write the tensor-scalar ratio as

\[
r(k) = \left( \frac{\mathcal{P}_g}{\mathcal{P}_R} \right) \approx \left[ \frac{\beta_5}{\beta_4} \right],
\]

where

\[
\beta_5 = \frac{8\kappa A^2 f^2}{4\pi^2} \left[ \frac{f \kappa}{8 A (1 - f)} \right]^{2(f-1)/f},
\]

and \( r \) in terms of the scalar spectral index, becomes

\[
r \approx \left[ \frac{\beta_5}{\beta_4} \right] \left[ \frac{8 (1 - f) (2 - f)}{\kappa f^2 (1 - n_s)} \right].
\]

Analogously, we can write the tensor-scalar ratio as function of the number of e-folding

\[
r \approx \left[ \frac{8 (1 - f) \beta_5}{\kappa f^2 \beta_4} \right] [1 + f(N - 1)].
\]

In Fig.\((1)\) we show the dependence of the tensor-scalar ratio on the spectral index for the special case in which we fixe \( f = 1/2 \), and we have used three different values of the parameter \( C_\phi \). From left to right \( C_\phi=10^5, 10^6 \) and \( 10^7 \).

The five-year WMAP data places stronger limits on \( r \) (shown in blue) than three-year data (grey)\([23]\). In order to write down values that relate \( n_s \) and \( r \), we used Eq.\((27)\) . Also we have used the values \( C_\gamma = 70 \) and \( \kappa = 1 \). Note that for the value of the parameter \( C_\phi \), (restricted from below, in which \( C_\phi > 10^5 \)), is well supported by the data. From Eqs.\((25)\) and
FIG. 1: Evolution of the tensor-scalar ratio $r$ versus the scalar spectrum index $n_s$ in the weak dissipative regime, for three different values of the parameter $C_\phi$. Here we used $f = 1/2$, $\kappa = 1$, $C_\gamma = 70$, and $P_R = 2.4 \times 10^{-9}$.

(28), we observed that for the special case in which $C_\phi = 10^6$ and $f = \frac{1}{2}$, the curve $r = r(n_s)$ for WMAP 5-year enters the 95% confidence region for $r \simeq 0.26$, which corresponds to the number of e-folds, $N \simeq 146$. For $r \simeq 0.20$ corresponds to $N \simeq 140$, in this way the model is viable for large values of the number of e-folds.

IV. THE STRONG DISSIPATIVE REGIME.

We consider now the case in which $\Gamma$ is large enough for the system to remain in strong dissipation until the end of inflation, i.e. $R > 1$. From Eqs. (5) and (13), we can obtained a
relation between the scalar field and cosmological times given by

$$\ln \left[ \frac{\phi(t)}{\phi_0} \right] = \alpha_1 \frac{t}{5f+2},$$

(29)

where \(\phi(t = 0) = \phi_0\) and \(\alpha_1\) is defined by

$$\alpha_1 = \frac{8}{(5f+2)} (Af)^{5/8} (1-f)^{1/8} \sqrt{\frac{6}{\kappa \alpha}},$$

and \(\alpha = C_\phi \left[ \frac{2}{3 \kappa C_r} \right]^{3/4} \).

The Hubble parameter as a function of the inflaton field, \(\phi\), result as

$$H(\phi) = Af \left[ \frac{1}{\alpha_1} \ln(\phi/\phi_0) \right]^{-8(1-f)/(5f+2)}.$$  

(30)

Without loss of generality we can taken \(\phi_0 = 1\).

From Eq.(11) the dissipation coefficient, \(\Gamma\), can be expressed as a function of the scalar field, \(\phi\), in the case of the strong dissipation regime, as follows

$$\Gamma(\phi) = \frac{\alpha_2}{\phi^2} [\ln(\phi)]^{-\alpha_3},$$

(31)

where

$$\alpha_2 = \alpha [Af (1-f)]^{3/4} \alpha_1^{\alpha_3}, \text{ and } \alpha_3 = \frac{6(2-f)}{(5f+2)}.$$

Analogously, to the case of the weak dissipative regime, we can used the slow-roll approximation i.e. \(\dot{\phi}^2/2 < V(\phi)\), together with \(V(\phi) > \rho_\gamma\). From Eq.(12) the scalar potential becomes

$$V(\phi) \simeq \left( \frac{3 f^2 A^2}{\kappa} \right) \left[ \frac{1}{\alpha_1} \ln(\phi) \right]^{-16(1-f)/(5f+2)},$$

(32)

and as in the previous case, this kind of potential does not present a minimum.

Introducing the dimensionless slow-roll parameter \(\varepsilon\), we have

$$\varepsilon = -\frac{\dot{H}}{H^2} = \left( \frac{1-f}{f A} \right) \left[ \frac{\alpha_1}{\ln(\phi)} \right]^{8f/(5f+2)},$$

(33)

and the second slow-roll parameter \(\eta\) is given by

$$\eta = -\frac{\ddot{H}}{HH} = \left( \frac{2-f}{f A} \right) \left[ \frac{\alpha_1}{\ln(\phi)} \right]^{8f/(5f+2)},$$

(34)

Imposing the condition \(\varepsilon = 1\) at the beginning of inflation the scalar field, \(\phi\), takes at this time the value

$$\phi_1 = \exp \left( \alpha_1 \left[ \frac{1-f}{f A} \right]^{(5f+2)/8f} \right).$$

(35)
The number of e-folds becomes given by

\[ N = \int_{t_1}^{t_2} H \, dt = A \alpha_1^{-8f/(5f+2)} \left[ \ln(\phi_2)^{8f/(5f+2)} - \ln(\phi_1)^{8f/(5f+2)} \right], \tag{36} \]

where Eq. (29) was used.

In the case of high dissipation, i.e. \( R = T/3H \gg 1 \) and following Taylor and Berera\[24\], we can write \( \delta \phi^2 \simeq k_F^2 T^2 \pi^2 \), where the wave-number \( k_F \) is defined by \( k_F = \sqrt{\Gamma H} = H \sqrt{3R} > H \), and corresponds to the freeze-out scale at which dissipation damps out to the thermally excited fluctuations. The freeze-out time is defined by the condition \( k = a k_F \). From Eqs. (29) and (31) we obtained that

\[ P_R \simeq \frac{1}{2 \pi^2} \left[ \frac{\Gamma^3 H^9}{4 C_\gamma \phi^6} \right]^{1/4} \simeq \frac{1}{4 \pi^2} \left[ \frac{\kappa^3 \Gamma^6 H^6}{54 C_\gamma (-H)^3} \right]^{1/4} \simeq \alpha_4 \frac{\ln(\phi)^{\alpha_5}}{\phi^3}, \tag{37} \]

where

\[ \alpha_4 = \frac{1}{4 \pi^2} \left[ \frac{\kappa^3 \alpha_6 A^6 f^6}{54 C_\gamma [A f (1 - f)]^3} \right]^{1/4} \alpha_1^{-6f/(5f+2)}, \quad \text{and} \quad \alpha_5 = \frac{(15f - 18)}{(5f + 2)}. \]

From Eqs. (29) and (31) the scalar spectral index \( n_s = d P_R / d \ln k \), is given by

\[ n_s \simeq 1 - \frac{3 \ln(\phi) - \alpha_5}{\alpha_6 \ln(\phi)^{\alpha_7}}, \tag{38} \]

where

\[ \alpha_6 = \frac{(A f)^{3/8}}{(1 - f)^{1/8}} \sqrt{\frac{\kappa \alpha}{6}} \alpha_1^{(2-3f)/(5f+2)}, \quad \text{and} \quad \alpha_7 = \frac{8 f}{(5f + 2)}. \]

The scalar spectra index \( n_s \) also can be write in terms of the number of e-folds \( N \). Thus, using Eqs. (35) and (36), we get

\[ n_s \simeq 1 - f A \left[ \frac{3 \alpha_1 [1 + f (N - 1)]^{-\alpha_7} (f A)^{\alpha_7} - \alpha_5}{\alpha_6 [1 + f (N - 1)] \alpha_1^{\alpha_7}} \right]. \tag{39} \]

For the strong dissipative regime we may write the tensor-scalar ratio as

\[ r(k) = \left( \frac{P_T}{P_R} \right) \simeq \left[ \frac{2 \kappa (f A)^2}{\pi^2 \alpha_4} \alpha_1^{16(1-f)/(5f+2)} \right] \phi^3 \left[ \ln(\phi) \right]^{(f+2)/(5f+2)}. \tag{40} \]

Here, we have used expressions (30) and (37).

Fig. (2) shows (for the strong dissipative regime) the dependence of the tensor-scalar ratio on the spectral index. Here, we have used different values for the parameter \( C_{\phi} \). The WMAP five-year data places stronger limits on \( r \) (shown in blue) than three-year data (grey)\[23\]. In order to write down values that relate \( n_s \) and \( r \), we used Eqs. (37), (38) and (40). Also we
FIG. 2: Evolution of the tensor-scalar ratio $r$ versus the scalar spectrum index $n_s$ in the strong dissipative regime, for three different values of the parameter $C_\phi$. Here, we have used $f = 1/2$, $\kappa = 1$, $C_\gamma = 70$ and $P_R = 2.4 \times 10^{-9}$.

have used the WMAP value $P_R(k_\star) \simeq 2.4 \times 10^{-9}$, the value $f = 0.5$ and $C_\gamma = 70$. Note that for the values of the parameter $C_\phi$ greater than $5 \times 10^6$, our model is well supported by the data. From Eqs. (35), (36), (37) and (40), we observed numerically that for the special case in which we fix $C_\phi = 10^7$ and $f = 1/2$, the curve $r = r(n_s)$ for WMAP 5-years enters the 95% confidence region for $r \simeq 0.245$, which corresponds to the number of e-folds, $N \simeq 359$. For $r \simeq 0.11$ corresponds to $N \simeq 289$, in this way the model is viable for large values of the number of e-folds.
V. CONCLUSIONS

In this paper we have studied the warm-intermediate inflationary model in the weak and strong dissipative regimes. In the slow-roll approximation we have found a general relation between the scalar potential and its derivative. We have also obtained explicit expressions for the corresponding, power spectrum of the curvature perturbations $P_R$, tensor-scalar ratio $r$, scalar spectrum index $n_s$ and the number of e-folds $N$.

In order to bring some explicit results we have taken the constraint $r - n_s$ plane to first-order in the slow roll approximation. When $\Gamma < 3H$ warm inflation occurs in the so-called weak dissipative regimen. In this case, the dissipation coefficient $\Gamma \propto \phi^{-\beta_1}$ for intermediate inflation. We also noted that the parameter $C_\phi$, which is bounded from bellow, $C_\phi > 10^5$, the model is well supported by the data as could be seen from Fig.(1). Here, we have used the WMAP five year data, where $P_R(k_*) \simeq 2.4 \times 10^{-9}$, and we have taken the value $f = 1/2$. On the other hand, when $\Gamma > 3H$ warm inflation occurs in the so-called strong dissipative regime. In this regime, the dissipation coefficient $\Gamma$ present a dependence proportional to $(\log(\phi))^{-\alpha_3}/\phi^2$ in intermediate inflation. In particular, Fig.(2) shows that for the values of the parameter $C_\phi = 7 \times 10^6$, $10^7$ or $10^8$, the model is well supported by the WMAP data, when the value $f = 1/2$ is taken.

In this paper, we have not addressed the non-Gaussian effects during warm inflation (see e.g., Refs.[19, 25]). A possible calculation from the non-linearity parameter $f_{NL}$, would give new constrains on the parameters of the model. We hope to return to this point in the near future.

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