I discuss the applicability of classical techniques to the study of the dynamics of infrared, bosonic fields at the electroweak phase transition. I present the lattice as a natural means of cutting off hard, nonclassical modes, and discuss the problem of integrating out the excluded modes perturbatively. I then apply the classical lattice technique to study the dynamics of the phase interface (bubble wall) and the generation of Chern-Simons number in each phase and on the bubble wall.

1 Applicability of the classical approximation

Understanding electroweak baryogenesis requires not only understanding the thermodynamics of the electroweak phase transition but also the detailed dynamics of the phase transition, in particular the behavior of the out of equilibrium plasma in and around the phase interface (bubble wall) during the transition. In particular we are interested in understanding the dynamics of the Higgs condensate and of the infrared gauge fields responsible for changing Chern-Simons number, which through the anomaly is proportional to the creation of baryon number. The evolution of these infrared fields cannot be understood perturbatively because of the infrared problem of thermal Yang-Mills theory, which has already been discussed in earlier talks. However there are nonperturbative (numerical) tools available if the behavior of these fields is well approximated by the classical theory, as originally suggested by Grigoriev and Rubakov. But before using this approximation to study the infrared dynamics, we should examine in at least some detail whether the approximation is actually justified.

An absolutely necessary, though not sufficient, condition is that the thermodynamics are classical. After all, the thermodynamics are just the equal time special case of the dynamics. So I will explore that question first.

The thermodynamics of the full Standard Model at finite temperature are governed by the path integral

\[ Z = \int DA_\mu D\Phi \ldots e^{-S/\hbar}, \]  
\[ S = \int_0^{\beta \hbar} d\tau \int d^3x \left[ \frac{1}{4g^2} F_{\mu \nu} F^{\mu \nu} + (D_\mu \Phi)^\dagger (D_\mu \Phi) + V(\Phi^\dagger \Phi + \ldots) \right], \]  

where...
where here and throughout, when I remember, I will write factors of $\hbar$ explicitly (which seems appropriate if one is studying the relation between classical and quantum theories), and where $\ldots$ means fermions, gluons, hypercharge (which I leave out for convenience and because $\Theta_W$ is small), and any other species such as supersymmetric partners. These extra species can very easily be included in what I discuss.

Now the time integral has periodic boundary conditions for bosons and antiperiodic boundary conditions for fermions, so we are considering a theory with one compactified dimension. Like any weakly coupled Kaluza-Klein theory, we can construct the effective infrared theory by finding the zero modes of the compact subspace and writing a theory in the noncompact dimensions with these degrees of freedom. The matching procedure between the full and reduced theories has been performed by Farakos et. al. as Laine discussed in his talk. The resulting partition function is, in a particularly convenient notation,

$$ Z_{DR} = \int D\!A_\mu D\Phi e^{-\beta H},$$

$$ H = \int g^2 \frac{1}{4g^2} F_{ij}^a F_{ij}^a + \frac{1}{2} (D_1 A_0)^2 + \frac{m^2}{2} A_0^2 +$$

$$ + (D_\mu \Phi)^\dagger (D_\mu \Phi) + V(\Phi^\dagger \Phi) + \frac{g^2}{4} A_0^2 \Phi^\dagger \Phi \right], $$

which is exactly the partition function of 3+1 dimensional, classical Yang-Mills Higgs theory as first noted by Ambjørn and Krasnitz. Why did that happen?

Perturbation theory is an expansion in nonlinearity, in $g^2$ or $\lambda$ (though in what follows I will always write $g^2$ in parametric arguments and estimates). But $g^2$ has dimensions of inverse length times inverse energy. Since perturbation theory must be an expansion in a dimensionless quantity (so we can add together the terms), $g^2$ must always appear with some dimensional quantity which balances its dimensions. In the vacuum theory only one quantity has dimensions involving energy, $\hbar$. (One frequently thinks of particle masses, but the classical field theorist would say that the fields have natural oscillation frequencies and you only get a mass scale by multiplying by $\hbar$.) So in vacuum the perturbation theory is an expansion in $g^2\hbar$, generally with a $16\pi^2$ in the denominator from phase space factors, and $g^2\hbar/16\pi^2 \ll 1$. Perturbation theory works brilliantly in vacuum, at least for the electroweak sector.

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*Well, not quite exactly, since in the classical theory the bare Debye mass is zero, so the value is forced to equal the counterterm from divergent 1 and 2 loop graphs.*
But at finite temperature there is a natural energy scale in the problem, the temperature $T$; if there is a length scale $l$ or frequency $\omega$ either in the question being asked or naturally in the theory, then $g^2$ can appear in the perturbative expansion in the combinations $g^2 l T$ or $g^2 T / \omega$, which need not be $\ll 1$. So perturbation theory can be very badly behaved. However, these combinations do not contain $\hbar$, so the reason for the breakdown of perturbation theory is classical. The expansion in $g^2 \hbar$ works brilliantly, in vacuum and at finite temperature (the dimensional reduction approximation is an expansion in this combination), so while perturbation theory may go to hell, it will go to hell in a classical way, and we can hope to use classical physics to extract the (nonperturbative) leading order in $g^2 \hbar$ behavior of the system.

The same argument, more or less, works for the dynamics. If one writes the real time perturbation theory value for any graph, drops all subleading terms in the Bose-Einstein population function $f_b$, and replaces $f_b(\omega) = T / (\hbar \omega)$ then one will recover the classical value for that diagram. This is a very good approximation in the infrared, since generally it is $f_b + 1/2$ and not $f_b$ which appears and since

$$f_b(\omega) + \frac{1}{2} = \frac{T}{\hbar \omega} + \frac{1}{12} \frac{\hbar \omega}{T} + \ldots .$$

The series has a radius of convergence of $\hbar \omega / T = 2\pi$, and for small $\hbar \omega / T$ it is approximated extremely well by its leading term.

However there is obviously a complication. The classical approximation is not valid when $\hbar \omega \sim T$, and such frequencies will inevitably appear in loop corrections even if the question being considered only involves lower frequencies. However these “hard” degrees of freedom should behave perturbatively and it should be possible at least in principle to integrate them out and make a classical theory of the soft modes only.

For the thermodynamics this can be done without too many complications. Rotational and gauge invariance severely restrict the form of equal time operator insertions which can appear, and dimensional reduction works. However, for unequal time dynamics, while vacuum corrections (which contain all the ultraviolet divergences) are restricted by Lorentz invariance, the effects of hard thermal particles need not be Lorentz invariant, since the plasma chooses a special rest frame. The form of these corrections is not so simple. The dominant terms (in an expansion in $g^2 \hbar$, which should be well behaved) are the hard thermal loops. Physically what they account for is that the infrared gauge fields at one spacetime point disturb the hard modes propagating through that point, and that the hard modes carry information about that disturbance with them to some lightlike separated point in the future, where they interact again with the soft modes in a way which conveys information about the fields at
the previous location. This effect is nonlocal, which could be a complication to its implementation in a regulated classical simulation. I will come back to this point later.

2 Lattice regulation

We want to study the real time dynamics of the classical theory nonperturbatively. The only reliable tools I know of are numerical. Classical real time lattice gauge theory is particularly well suited to our needs because it preserves exact gauge invariance. The lattice implementation restricts the theory to include only a finite number of degrees of freedom; in real space, the lattice, and in momentum space, the first Brillouin zone. We are thus obliged, in order to study the classical theory nonperturbatively, to regulate it in a way which drops precisely those degrees of freedom which do not behave classically, namely the large momentum or “hard” degrees of freedom. The problems of separating the soft and hard modes, eliminating the hard ones, and nonperturbatively treating the resulting soft classical modes are thus dealt with simultaneously in an extremely convenient way.

Ambjørn et. al. have developed a numerical implementation of the classical lattice theory\cite{Ambjorn} and both Alex Krasnitz and I have developed thermalization algorithms\cite{Krasnitz,Meade}. I will not discuss the details of the implementation here, because it has already been presented at this conference\cite{Ambjorn} and because you either already know, don’t want to know, or would be best advised to learn directly from the literature.

I should also note that the classical evolution probes a system with the same thermodynamics as the quantum theory in the dimensional reduction approximation, and because the classical evolution is ergodic, it can be viewed as a microcanonical monte-carlo of the dimensionally reduced theory, which has some advantages over conventional canonical approaches— in particular, mixed phase configurations are stable and naturally adjust to find the equilibrium temperature\cite{Laine}. But I will not go into detail on this matter here.

Instead I should discuss at a little more length the problem of integrating out the hard modes which are present in the continuum theory, rather than simply dropping them. Again, the first step should be to demand that the thermodynamics of the full system are recovered with the smallest possible error. This is best done in a two step process. First, one goes from the full quantum theory to the dimensionally reduced one, which is the same as studying the thermodynamics of the continuum classical system. This step has been carried out at one loop by Farakos et. al\cite{Farakos} and discussed in Laine’s talk. Next, one should make contact between the continuum 3-D theory and the lattice one.

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This problem has also been studied by Farakos et. al.\cite{Farakos}, who show that to make the continuum theory and the small $a$ (lattice spacing) limit of the lattice theory match, one need only make one and two loop renormalizations of the Higgs and Debye mass squared and of divergent operator insertions such as the $\Phi^2$ insertion. They compute these corrections, yielding a system for which the small $a$ limit rigorously exists and matches the continuum theory.

This does not address the question of how quickly the theory approaches this small $a$ limit, which is of some practical importance in numerical lattice studies. In particular we do not want there to be any $O(a)$ differences between the lattice and continuum theories. Super-renormalizability is very useful here as well. The continuum and lattice theories differ, at tree level, at $O(a^2)$ in the infrared, and so $O(a)$ errors in infrared phenomena can only arise from diagrams with hard loops, i.e. where some propagators carry momenta on order $1/a$, since only such propagators (and the vertices they go into) differ substantially between the lattice and continuum theories. The superficial degree of divergence of any 3-D Feynman diagram is smaller by 1 for each loop than the corresponding 4-D graph, which is why divergences could only happen in 1 and 2 loop corrections to the mass. It also means that corrections to couplings and wave functions, which are normally logarithmically divergent, are now convergent by one power for each loop order. Order $a$ corrections will occur due to hard momenta in isolated loops which, if contracted, would appear as coupling or wave function corrections. Overlapping hard loops will differ between the theories at $O(a^2)$ (except for mass squared corrections, as I mentioned) and need not be considered; and corrections other than wave functions and super-renormalizable couplings only correct nonrenormalizable operators which are already $O(a^2)$. Hence a 1 loop integration over the modes missing on the lattice can remove all $O(a)$ errors, except in the mass squared and in divergent operator insertions. Neither of these exceptions are very important, since we tune the mass to find the phase transition, and since we are generally interested in the difference of operator insertions between the two phases, and these are convergent.

A picture here is worth several words. Figure 1 shows how the lattice only contains some of the degrees of freedom in momentum space. Since we are interested in the IR degrees of freedom around $p = 0$, this is only important in that the UV degrees of freedom interact with them via hard loops. Because the theory is super-renormalizable, the coupling is weaker and weaker as we go further from the origin; the degrees of freedom we drop are weakly coupled. The finer the lattice, the larger the box in the figure and the fewer and more weakly coupled the modes we lose; so a small $a$ limit exists. But the convergence to the limit is much faster by performing a one loop calculation of the
Figure 1: Illustration of the difference between the lattice and continuum theories, in terms of the degrees of freedom in momentum space. The lattice theory only contains degrees of freedom inside a Brillouin zone; the degrees of freedom outside are dropped, and the behavior of the degrees of freedom near the edges are distorted. The interesting degrees of freedom are the infrared modes around the origin.

influence of the hard modes, on the lattice and in the continuum, and applying counterterms equal to the difference to make up for the absence of the hard degrees of freedom. The matching has been performed and an algebraic error in the original draft has been corrected.

What about integrating out the hard modes in a way which will be correct dynamically as well? As I mentioned earlier, these modes propagate at the speed of light and cause interactions between lightlike separated points. Reproducing those effects correctly on the lattice is tricky. There are now three proposals which I know of but none has been implemented and used in the existing literature and I think it would be fair to say that the subject is immature.

However, there are already “hard” modes, in the sense of short wavelength excitations which interact weakly with the infrared modes and carry almost all of the energy in the system, present on the lattice; namely, the high frequency lattice excitations. They propagate in the background of the soft lattice modes
and lead to interactions quite similar to the hard thermal loops. For instance, they lead to damping of the same sort as the hard loops. However, since their dispersion relations are not ultrarelativistic and their population cuts off in a way which is not rotationally invariant, the hard thermal loops they contribute look different from the correct ones.

Does this difference matter? Probably yes. As Peter Arnold argued nicely in his talk, the presence of hard thermal loops slows down the dynamics of very infrared magnetic fields in a way which depends on how many such modes there are and on how exactly they modify the self-energy of the infrared modes. The lattice theory at some arbitrary lattice spacing will typically get this wrong, leading to the wrong strength of damping on the infrared magnetic fields, which could change the Sphaleron rate. But for exploratory purposes I will start out relying on the lattice modes to convey the hard thermal loops, and look on the results not as high precision measurements but as qualitative, intended as much to develop techniques as to determine hard numbers. When the problem of including hard thermal loops more accurately has been solved, which I think will be in the relatively near future, then we can apply these methods to get hard answers (when we know the right electroweak physics, which is another reason not to be looking for final answers at this point in the game).

3 Bubble wall friction

With that said, I will turn to an example of computing an interesting, dynamical infrared quantity with the classical field technique. Namely, I will compute the friction on a moving bubble wall, below the phase transition temperature.

At equilibrium, the interface between the two electroweak phases will on average remain at rest. (That is what we mean by equilibrium.) But when the temperature is lowered, the low temperature “broken” phase has a lower free energy density, and thus a higher pressure. There is a pressure difference across the interface which pushes it, allowing the broken phase to expand at the expense of the symmetric phase. The problem of electroweak baryogenesis is to understand in detail what happens on and near this moving bubble wall, and how fast the wall moves is an important attribute.

The velocity is set by a balance of the velocity independent net pressure on the bubble wall and the velocity dependent frictional force as it moves through the plasma. I define a friction coefficient for the bubble wall as the linear response coefficient relating the mean velocity $v_w$ to the net pressure $P$,

$$\eta \equiv \lim_{T \to T_c} \frac{P}{v_w},$$

which should not be confused with the $\eta$ defined in Ignatius’ talk.
The first thing to note about $\eta$ is that there is a one loop perturbative estimate, which is equivalent to the friction in the free scatter approximation derived independently by Dine et. al and Liu et. al and which for the bosons is strongly infrared dominated. This tells us at once that the contribution cannot be found reliably in perturbation theory, but that it is classical. The second thing to note is that we can find this linear response coefficient from an unequal time, equilibrium correlator via a fluctuation dissipation relation, similar to and inspired by one developed by Khlebnikov.

The idea is the following. Consider the bubble wall in a long box of cross section area $A$. Once we choose an unambiguous definition of the wall position, it is a coordinate, and it has a conjugate momentum. Equipartition tells us that it will typically be moving. But as it interacts with the plasma, its momentum will repeatedly receive kicks which will change its direction; so it will diffuse with a diffusion constant

$$D \equiv \lim_{t \to \infty} \frac{\langle (x(t) - x(0))^2 \rangle}{t}.$$  

The more kicks it receives, the more slowly it will diffuse, because the individual steps making up its diffusive motion will be shorter. The kicks it is receiving are precisely the kicks which will absorb the net motion generated by a pressure in the out of equilibrium case, so the friction coefficient and the diffusion constant are inversely related. In fact, a simple argument gives

$$\eta = \frac{2T}{DA}.$$  

Except for the factor of 2 one can guess this equation just on dimensional grounds; $\eta$ must go as energy density over velocity, that is $[\eta] = \text{energy} \times \text{time} \times \text{length}^{-4}$. Now $T$ is the only unit of energy in the classical theory, $A$ has units of length$^2$, and $D$ has units of length$^2$/time. The 2 is the same 2 which appears with $T$ in the noise correlator in Langevin equations and which appears in the fluctuation dissipation relation between Chern-Simons number diffusion and motion under a chemical potential.

The fermionic contribution to the friction was computed, in an approximation which should be correct at least parametrically, by Tomislav Prokopec and me who found $\eta \sim \bar{g}^8 T^4$. This contribution might be relevant because four of the $g$ are top quark Yukawa couplings, but as we will see it is parametrically smaller than the classical bosonic contribution. The naive parametric estimate for the classical contribution is that the only classical length scale is $1/(g^2 T)$,
which is also the only classical time scale; so \( \eta \sim g^6 T^4 \). Peter Arnold might disagree with this, though. The two electroweak phases are distinguished not only in that one has a larger value of \( \phi^2 \), but also that the “symmetric” phase has large IR magnetic fields, while the broken phase does not. For the wall to move, it must create or destroy these large magnetic fields, and as Arnold, Son, and Yaffe argue, the time scale for these to change is \( 1/(\bar{h} g^4 T) \) due to HTL damping effects. That would give an estimate for \( \eta \) of \( \eta \sim g^4 T^4 / \bar{h} \). If this reasoning is right, then the fermionic contribution to the damping is clearly quite irrelevant.

It is straightforward to measure the diffusion constant for the wall surface numerically. Neil Turok and I did so; our results, applying the \( O(a) \) corrections which I mentioned earlier, are that, for \( 4\lambda / g^2 = 0.159 \) (or \( x \approx 0.04 \)), on a lattice with \( \beta_{L,\text{imp}} = 7.3 \), we find \( \eta = 0.020 \pm 0.003 g^6 T^4 \). At these parameter values, we found a jump in the order parameter of \( \Delta \phi^2 = 2.11 g^4 T^2 \), which is right on the 2 loop perturbative value, and a surface tension of \( \sigma = 0.057 g^4 T^3 \), which is somewhat below the perturbative value, a trend which gets stronger at larger \( x \). If we use the value of \( \Delta \phi^2 \) to compute the “free scattering” estimate of the friction coefficient, we get \( \eta = 0.058 g^6 T^4 \), which is larger. However, the strength of hard thermal loop effects, induced by the hard lattice modes, is much smaller here than in the physical, quantum theory, and if the Arnold Son Yaffe argument is right, the true friction, when this is taken into account, should be larger than the one we measured by a factor of 3 or 4. That is, the friction could be larger than the free scattering estimate, which will definitely mean that the bubble wall moves slowly, \( v_w \sim 0.1 \) at the nucleation temperature (and still slower after the universe starts to reheat towards the equilibrium temperature).

Hence we conclude that bubble walls are slow, although this might be different for extremely strongly first order transitions. But more work, and in particular a better accounting of the hard thermal loop effects, is needed before we can quote a solid number.

4 Dynamics of Baryogenesis

If we can find a way to measure the (topological) quantity \( N_{CS} \) on the lattice then we can study how it changes in circumstances which are relevant to baryogenesis.

A local operator (non-topological, non total time derivative) definition of \( N_{CS} \) has been developed by Ambjorn and Krasnitz and is very convenient for lattice measurements. There are three interesting problems we might try to address with this:
• What is the diffusion constant of $N_{CS}$, or the linear response coefficient to a chemical potential for $N_{CS}$, in the symmetric phase?

• Same, but for the broken phase.

• What about on the bubble wall, say, while it is moving and out of equilibrium?

A nice fluctuation dissipation relation\textsuperscript{23,24} tells us that the diffusion constant,

$$\Gamma_d \equiv \lim_{t \to \infty} \frac{\langle (N_{CS}(t) - N_{CS}(0))^2 \rangle}{V_t}, \quad (9)$$

and the linear response coefficient,

$$\Gamma_\mu \equiv \lim_{\mu \to 0} \frac{T \langle \dot{N}_{CS} \rangle_\mu}{V_\mu}, \quad (10)$$

are related, $\Gamma_d = 2\Gamma_\mu$. So the first two questions can either be solved by the diffusion constant technique developed by Ambjorn et. al\textsuperscript{6} and applied by Ambjorn and Krasnitz\textsuperscript{5} and Smit and Tang\textsuperscript{25} or by the chemical potential technique\textsuperscript{8,26}. The third question can only be addressed by the chemical potential technique, and I will address it next.

We want to study how the system’s response to a chemical potential for $N_{CS}$ changes from the symmetric phase, unsuppressed rate to the broken phase, exponentially suppressed rate at the phase interface, a question which is relevant for “local” mechanisms for baryogenesis. The first thing we did was to get a mixed phase configuration in a very long box and to apply a chemical potential uniformly through space, and measure the generation of $N_{CS}$ as a function of distance from the bubble wall. The result is the picture on the left in Figure 2. We see that the generation of $N_{CS}$ shuts off just past the base of the bubble wall. (There is some slight generation in the broken phase, but this is a lattice artifact associated with this definition of $N_{CS}$; a spurious response in the broken phase of about 10% of the symmetric phase rate is generated by UV lattice artifacts.) The picture for the out of equilibrium case, where the system is cooled and the wall propagates, is about the same.

The other thing to try is to apply a chemical potential only to the interior of the bubble wall. To do this, we first measure the averaged dependence of $\phi^2$ on distance from the bubble wall, which appears already on the lefthand picture in Figure 2. Then, at each point in time, we find the bubble wall surface, and determine the vertical distance of each point in the plasma from the bubble wall, and apply a chemical potential at that point which is proportional to the derivative of the wall profile. The chemical potential is then only nonzero in
the interior of the bubble wall, in a way which approximately duplicates what
would happen in a “local baryogenesis” mechanism.

The result of this endeavor is shown in the righthand picture in Figure 2. Integrating over volume, we find a net production of $N_{CS}$ which is about 20% of the value we would get by applying the same integrated chemical potential in the symmetric phase. Remembering that half of this is the UV lattice artifact, and that the wall has a breathing mode so it is thinner some places than others (resulting in some small part of the chemical potential getting applied directly in the symmetric phase), we can set an upper limit on the efficiency of baryogenesis inside the bubble wall of 10% of the symmetric phase efficiency. This is bad but not fatal for local baryogenesis mechanisms, ie it appears that transport is important to make baryogenesis efficient.

5 Conclusions–What needs doing?

I see three serious drawbacks with the study of $N_{CS}$ violation I have just discussed. The first is with the definition of $N_{CS}$, ie using a local operator which is not a total time derivative and which has UV noise.

This problem has now been solved, as Alex Krasnitz reviewed in his talk. Neil Turok and I have developed a topological means to track $N_{CS}$ based on a
Figure 3: Lattice spacing dependence of the diffusion constant for $N_{CS}$ in pure Yang-Mills theory, using a topological definition for $N_{CS}$ and after applying $O(a)$ corrections in the relation between lattice and physical length scales.

topological definition of lattice $N_{CS}$ due to Woit. With it, we have studied the lattice spacing dependence of the diffusion constant for $N_{CS}$, and find a strong spacing dependence, as shown in Figure 3. (Note that we have already applied the $O(a)$ corrections in the match between lattice and physical length scales to the data in the figure; before the correction the lattice spacing dependence is stronger.) We expect a lattice spacing dependence if Arnold, Son, and Yaffe are correct, because the total size of HTL effects depends linearly on $1/a$ (as more and more hard modes are present on the lattice). The size of the effect we find is not as strong as their prediction, but this could be because the parametric limit of small $a$ has not yet been completely attained.

Another problem, which should be solved rather than applying the above methodology to finer and finer lattices, is the problem of more correctly including hard thermal loop effects on the lattice. I know of three proposals in the existing literature for doing this.
The first proposal is that of Bödeker, McLerran, and Smilga\cite{Smilga1} which consists of having a population function defined on a sphere at each lattice site, representing the population of particles moving in each direction, and to evolve these population functions according to lattice Boltzmann equations in the presence of the classical lattice background. The specifics of the lattice implementation have not been worked out and it is not clear to me if they would be practical.

The second proposal is that of Huet and Son\cite{HuetSon} who propose to treat directly the nonlocal dynamical system resulting from integrating over the hard modes, but to make all available parametric approximations to simplify the system. The result is a set of Langevin equations which are nonlocal in space but not in time. The idea has been formulated in the continuum, but carrying it over onto the lattice is straightforward: the only problem is dealing with a highly nonlocal update rule, which is numerically impractical without either a very clever trick or some additional approximations.

The third proposal is that of Hu and Müller\cite{HuMuller} who propose to include the hard thermal loops in a local way by adding particles to the lattice, with dynamics which reproduce the continuum Wong’s equations in the small spacing limit. A numerical implementation appears feasible, although there are some systematics and orders of limits to be carefully taken care of.

I think that one or the other of these proposals, or perhaps a new proposal, for dealing with the hard thermal loops numerically, will be implemented in the foreseeable future, and I will make a guess that, when one has been, we will find a $N_{CS}$ diffusion constant, for HTL strengths corresponding to the Debye mass of the real quantum theory, of $\Gamma_d = (0.4 - -0.9) \alpha^4 T^4$, with 0.6 being my best bet.

Finally, it would be nice to redo the study of $N_{CS}$ generation “on the bubble wall,” but introducing the $CP$ violating bias in the evolution of $N_{CS}$ through a nonrenormalizable operator self-consistently included in the lattice theory. Implementing this would be a real pain, but there is no problem in principle.

About the computation of the friction on the bubble wall, obviously the most important thing to do here is to apply the hard thermal loops in a more realistic way. It would also be nice to have results for a variety of values of the scalar self coupling. And realistically, we expect that there must be new light bosonic degrees of freedom in the theory if the phase transition is to be strong enough for baryogenesis to work. Friction from these extra degrees of freedom depends on the strength of their coupling to the Higgs field, so in particular we might expect a large contribution to the friction in the case that there are light stops.
And speaking of stop, I think this would be a good place.

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