Orbital motions and the conservation-law/preferred-frame $\alpha_3$ parameter

L. Iorio
Ministero dell’Istruzione, dell’Università e della Ricerca (M.I.U.R.)-Istruzione
Fellow of the Royal Astronomical Society (F.R.A.S.)
Viale Unità di Italia 68, 70125, Bari (BA), Italy

May 22, 2014

Abstract

We analytically calculate some orbital effects induced by the Lorentz-invariance/momentum-conservation PPN parameter $\alpha_3$ in a gravitationally bound binary system made of a compact primary orbited by a test particle. We neither restrict ourselves to any particular orbital configuration nor to specific orientations of the primary’s spin axis $\hat{\psi}$. We use our results to put preliminary upper bounds on $\alpha_3$ in the weak-field regime by using the latest data from Solar System’s planetary dynamics. By linearly combining the supplementary perihelion precessions $\Delta \varpi$ of the Earth, Mars and Saturn, determined with the EPM2011 ephemerides, we infer $|\alpha_3| \lesssim 6 \times 10^{-10}$. Our result is about 3 orders of magnitude better than the previous weak-field constraints existing in the literature, and of the same order of magnitude of the constraint expected from the future BepiColombo mission to Mercury. It is, by construction, independent of the other preferred-frame PPN parameters $\alpha_1, \alpha_2$, both preliminarily constrained down to a $\approx 10^{-6}$ level. Future analyses should be performed by explicitly including $\alpha_3$ and other related PPN parameters in the models fitted by the astronomers to the observations, and estimating them in dedicated covariance analyses. The wide pulsar-white dwarf binary PSR J0407+1607 yields a preliminary upper limit on the strong-field version $\hat{\alpha}_3$ of the Lorentz-invariance/momentum-conservation PPN parameter of the order of $3 \times 10^{-17}$. It relies upon certain assumptions on the unknown values of the pulsar’s spin axis orientation $\hat{\psi}$, the orbital node $\Omega$ and the inclination $I$. Neither the pulsar’s proper motion, still undetected, nor a possible value of the pulsar’s mass $m_p$ up to two solar masses substantially affect our result.
1 Introduction

Looking at the equations of motion of massive objects within the framework of the parameterized post-Newtonian (PPN) formalism [1–4], it turns out that, in general, the parameter \( \alpha_3 \) [4–6] enters both preferred-frame accelerations (see Eq. (6.34) of [4]) and terms depending on the body’s internal structure which, thus, represent “self-accelerations” of the body’s center of mass (see Eq. (6.32) of [4]). The latter ones arise from violations of the total momentum conservation since they generally depend on the PPN conservation-law parameters \( \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4 \) which are zero in any semiconservative theory such as general relativity. It turns out [4] that, for both spherically symmetric bodies and binary systems in circular motions, almost all of the self-accelerations vanish independently of the theory of gravity adopted. An exception is represented by a self-acceleration involving also a preferred-frame effect through the body’s motion with respect to the Universe rest frame: it depends only on \( \alpha_3 \) (see eq. (1) below). The aim of the paper is to work out in detail some orbital effects of such a preferred-frame self-acceleration, and to preliminarily infer upper bounds on \( \alpha_3 \) from latest observations in different astronomical and astrophysical scenarios. As a by-product of the use of the latest data from Solar System’s dynamics, we will able to bound the other preferred-frame PPN parameters \( \alpha_1, \alpha_2 \) as well.

The plan of the paper is as follows. In Section 2 the long-term orbital precessions for a test particle are analytically worked out without any a-priori assumptions on both the primary’s spin axis and the orbital configuration of the test particle. Section 3 deals with the confrontation of our theoretical predictions with the observations. The constraints on \( \alpha_3 \) in the existing literature are critically reviewed in Section 3.1 while new upper bounds are inferred in Section 3.2 in the weak-field regime by using the latest results from Solar System’s planetary motions. The effects of eq. (1) on pulsar timing are analytically worked out in Section 4 where some constraints on the strong-field version \( \hat{\alpha}_3 \) are obtained for the wide pulsar-white dwarf binary PSR J0407+1607. Section 5 summarizes our findings.
2 Orbital precessions

Let us consider a binary system made of two nearly spherical bodies whose barycenter moves relative to the Universe rest frame with velocity $\mathbf{w}$. Let us assume that one of the two bodies of mass $M$ has a gravitational self-energy much larger than the other one, as in a typical main sequence star-planet scenario. It turns out that a relative conservation-law/preferred-frame acceleration due to $\alpha_3$ arises\(^1\): it is [4, 5, 7]

$$A_{\alpha_3} = \frac{\alpha_3 \Theta}{3} \mathbf{w} \times \mathbf{\psi},$$

(1)

where $\mathbf{\psi}$ is the angular velocity vector of the primary, assumed rotating uniformly, and

$$\Theta \equiv \frac{\mathcal{E}}{M c^2},$$

(2)

is its fractional content of gravitational energy measuring its compactness; $c$ is the speed of light in vacuum. In eq. (2),

$$\mathcal{E} = -\frac{G}{2} \int_V \rho(r) \rho(r') \frac{d^3r \, d^3r'}{|r - r'|}$$

(3)

is the (negative) gravitational self-energy of the primary occupying the volume $V$ with mass density $\rho$, and $M c^2$ is its total mass-energy. For a spherical body of radius $R$ and uniform density, it is [8]

$$\Theta = -\frac{3GM}{5Rc^2}.$$  

(4)

The acceleration of eq. (1) can be formally obtained from the following perturbing potential

$$U_{\alpha_3} = -\frac{\alpha_3 \Theta \mathbf{w} \psi}{3} (\mathbf{u} \cdot \mathbf{r})$$

(5)

as

$$A_{\alpha_3} = -\nabla U_{\alpha_3}.$$  

(6)

In eq. (5), we defined

$$\mathbf{u} \equiv \mathbf{\dot{w}} \times \mathbf{\dot{\psi}}.$$  

(7)

\(^1\)The other purely (i.e. $\Theta$–independent) preferred-frame accelerations proportional to $\alpha_3$ in Eq. (8.72) of [4] either cancel out in taking the two-body relative acceleration or are absorbed into the Newtonian acceleration by redefining the gravitational constant $G$. See Chap. 8.3 of [4] and Eq. (2.5) of [7].
Note that, in general, \( \hat{w} \) and \( \hat{\psi} \) are not mutually perpendicular, so that \( \mathbf{u} \) is not an unit vector. For example, in the case of the Sun, the north pole of rotation at the epoch J2000.0 is characterized by \( \alpha_\odot = 286.13^\circ \), \( \delta_\odot = 63.87^\circ \), \( \hat{\psi}_x = 0.122 \), \( \hat{\psi}_y = -0.423 \), \( \hat{\psi}_z = 0.897 \).

As far as \( w \) is concerned, in the literature on preferred-frame effects it is common to adopt the Cosmic Microwave Background (CMB) as preferred frame. In this case, it is determined by the global matter distribution of the Universe. Latest results from the Wilkinson Microwave Anisotropy Probe (WMAP) yield a peculiar velocity of the Solar System Barycenter (SSB) of \( w_{SSB} = 369.0 \pm 0.9 \text{ km s}^{-1} \), \( l_{SSB} = 263.99^\circ \pm 0.14^\circ \), \( b_{SSB} = 48.26^\circ \pm 0.03^\circ \), where \( l \) and \( b \) are the Galactic longitude and latitude, respectively. Thus, in Celestial coordinates, it is

\[
\hat{w}^\text{SSB}_x = -0.970, \\
\hat{w}^\text{SSB}_y = 0.207, \\
\hat{w}^\text{SSB}_z = -0.120.
\]
Thus, the components of $u$ are

$$u_x = 0.135,$$  \(19\)

$$u_y = 0.856,$$  \(20\)

$$u_z = 0.385,$$  \(21\)

with

$$u = 0.949,$$  \(22\)

$$\vartheta = 71.16^\circ,$$  \(23\)

where $\vartheta$ is the angle between $\hat{w}_{\text{SSB}}$ and $\hat{\psi}$. As far as the solar rotation $\psi$ is concerned, it is not uniform since it depends on the latitude $\varphi$. Its differential rotation rate is usually described as $[17, 18]$

$$\psi = A + B \sin^2 \varphi + C \sin^4 \varphi,$$  \(24\)

where $A$ is the equatorial rotation rate, while $B, C$ set the differential rate. The values of $A, B, C$ depend on the measurement techniques adopted and on the time period examined $[17]$; currently accepted average values are $[18]$

$$A = 2.972 \pm 0.009 \mu \text{rad s}^{-1},$$  \(25\)

$$B = -0.48 \pm 0.04 \mu \text{rad s}^{-1},$$  \(26\)

$$C = -0.36 \pm 0.05 \mu \text{rad s}^{-1}.$$  \(27\)

As a measure for the Sun’s rotation rate, we take the average of eq. \(24\) over the latitude

$$\langle \psi \rangle = A + \frac{B}{2} + \frac{3}{8} C = 2.59 \pm 0.03 \mu \text{rad s}^{-1},$$  \(28\)

where the quoted uncertainty comes from an error propagation.

About the fractional gravitational energy of the Sun, a numerical integration of eq. \(3\) with the standard solar model, yields for our star \([19]\)$

$$|\Theta| \approx 3.52 \times 10^{-6}.$$  \(29\)
The long-term rates of change of the Keplerian orbital elements of a test particle can be straightforwardly worked out with a first order calculation within the Lagrange perturbative scheme [20, 21]. To this aim, eq. (5), assumed as a perturbing correction to the usual Newtonian monopole \( U_N = -GMr^{-1} \), must be averaged out over a full orbital revolution of the test particle. After evaluating eq. (5) onto the Keplerian ellipse, assumed as unperturbed reference trajectory, and using the eccentric anomaly \( E \) as fast variable of integration, one has

\[
\langle U_{\alpha_3} \rangle_{P_b} = \frac{\alpha_3 \Theta w \psi e}{2} \left\{ \cos \omega (u_x \cos \Omega + u_y \sin \Omega) + \ight.
\]

\[
+ \sin \omega [u_z \sin I + \cos I (u_y \cos \Omega - u_x \sin \Omega)] \right\},
\]

where \( a, e, I, \Omega, \omega \) are the semimajor axis, the eccentricity, the inclination to the reference \( \{x, y\} \) plane adopted, the longitude of the ascending node, and the argument of pericenter, respectively, of the test particle.

In obtaining eq. (30), we computed eq. (5) onto the Keplerian ellipse, assumed as unperturbed reference trajectory. In fact, one could adopt, in principle, a different reference path as unperturbed orbit which includes also general relativity at the 1PN level, and use, e.g., the so-called Post-Newtonian (PN) Lagrange planetary equations [22, 23]. As explained in [22], in order to consistently apply the PN Lagrange planetary equations to eq. (5), its effects should be greater than the 2PN ones; in principle, such a condition could be satisfied, as shown later in Section 3.2. However, in the specific case of eq. (5), in addition to the first order precessions of order \( \mathcal{O}(\alpha_3) \), other “mixed” \( \alpha_3 c^{-2} \) precessions of higher order would arise specifying the influence of \( \alpha_3 \) on the 1PN orbital motion assumed as unperturbed. From the point of view of constraining \( \alpha_3 \) from observations, they are practically negligible since their magnitude is much smaller than the first order terms and the present-day observational accuracy, as it will become clear in Section 3.2.

In integrating eq. (5) over one orbital period \( P_b = 2\pi n_b^{-1} = 2\pi \sqrt{a^3 G^{-1} M^{-1}} \) of the test particle, we kept both \( w \) and \( \psi \) constant. In principle, the validity of such an assumption, especially as far as \( \psi \) is concerned, should be checked for the specific system one is interested in. For example, the standard torques which may affect the Sun’s spin axis \( \hat{\psi} \) are so weak that it changes over timescales of Myr or so [12, 24]. Shorter timescales occur for certain binary pulsars due to the general relativistic geodetic precession [25, 27] which induces relatively fast precessions of the proper spins of
the system’s components provided that they are misaligned with respect to
the orbital angular momentum. However, for the double pulsar PSR J0737-
3039A/B \cite{28,29}, whose orbital period is as little as \( P_b = 0.1 \text{ d} \), the period
of the recently measured general relativistic precession of the B’s spin is
\( P_{\psi^B} \approx 71 \text{ yr} \) \cite{30}. In principle, also the time variations of the rotation rate
\( \psi \) should be taken into account. Indeed, in the case of the Sun, both the
equatorial rate \( A \) \cite{31} and the differential rates \( B,C \) \cite{32} vary with different
timescales which may be comparable with the orbital frequencies of the
planets used to constrain \( \alpha_3 \). However, we will neglect them since they are
at the level of \( \approx 0.01 \mu\text{rad s}^{-1} \) \cite{32}. Also the orbital elements were kept fixed
in the integration which yielded eq. (30). It is a good approximation in most
of the systems which could likely be adopted to constrain \( \alpha_3 \) such as, e.g.,
the planets of our Solar System and binary pulsars. Indeed, \( I, \Omega, \omega \) may
experience secular precessions caused by several standard effects (oblateness
of the primary, N-body perturbations in multiplanetary systems, 1PN grav-
itoelectric and gravitomagnetic precessions à la Schwarzschild and Lense-
Thirring). Nonetheless, their characteristic timescales are quite longer than
the orbital frequencies; suffice it to say that the period of the 1PN periastron
precession of PSR J0737-3039A/B is of the order of \( P_{\omega\text{1PN}} \approx 21 \text{ yr} \) \cite{33}. In
the case of our Solar System, the classical N-body precessions of the planets
for which accurate data are currently available may have timescales as large
as \( P_{\omega\text{N-body}} \approx 10^4 \text{ yr} \), while the orbital periods are at most \( P_b \lesssim 30 \text{ yr} \).

From eq. (30), the Lagrange planetary equations \cite{20} yield \cite{4}

\[
\left\langle \frac{da}{dt} \right\rangle = 0, \quad \tag{31}
\]

\[
\left\langle \frac{de}{dt} \right\rangle = \frac{\alpha_3 \Theta w \psi \sqrt{1 - e^2}}{2n_b a} \left[ u_z \sin I \cos \omega + \right.
\]

\[
+ \cos I \cos \omega \left( u_y \cos \Omega - u_x \sin \Omega \right) - \sin \omega \left( u_x \cos \Omega + u_y \sin \Omega \right) \right], \quad \tag{32}
\]

\textsuperscript{2}That figures hold for Saturn. See http://ssd.jpl.nasa.gov/txt/p\_elem.txt on the
WEB.

\textsuperscript{3}Ashby et al. \cite{7}, using the true anomaly \( f \) as fast variable of integration, calculated
the shifts of the Keplerian orbital elements corresponding to a generic time interval from
\( f \) to \( f_0 \).
\[
\langle \frac{dI}{dt} \rangle = -\frac{\alpha_3 \Theta \omega \psi e \cos \omega}{2n_1 a \sqrt{1 - e^2}} \left[ u_z \cos I + \sin I \left( u_x \sin \Omega - u_y \cos \Omega \right) \right], \quad (33)
\]

\[
\langle \frac{d\Omega}{dt} \rangle = -\frac{\alpha_3 \Theta \omega \psi e \sin \omega}{2n_1 a \sqrt{1 - e^2}} \left[ u_z \cot I + u_x \sin \Omega - u_y \cos \Omega \right], \quad (34)
\]

\[
\langle \frac{d\omega}{dt} \rangle = \frac{\alpha_3 \Theta \omega \psi}{2n_1 a e \sqrt{1 - e^2}} \left\{ (-1 + e^2) \cos \omega \left( u_x \cos \Omega + u_y \sin \Omega \right) +
\right.
\]
\[
\left. + \sin \omega \left( -u_y \cos I \cos \Omega + u_z e^2 \csc I - u_z \sin I + u_x \cos I \sin \Omega \right) \right\}, \quad (35)
\]

\[
\langle \frac{d\varpi}{dt} \rangle = \frac{\alpha_3 \Theta \omega \psi}{2n_1 a e \sqrt{1 - e^2}} \left\{ (-1 + e^2) \cos \omega \left( u_x \cos \Omega + u_y \sin \Omega \right) +
\right.
\]
\[
\left. + \sin \omega \left[ -u_z \sin I + (e^2 - \cos I) \left( u_y \cos \Omega - u_x \sin \Omega \right) +
\right.
\]
\[
\left. + e^2 u_z \tan \left( \frac{I}{2} \right) \right\}, \quad (36)
\]

where the angular brackets \( \langle \ldots \rangle \) denote the temporal averages. It is important to note that, because of the factor \( n_1^{-1} a^{-1} \propto \sqrt{a} \) in eq. (31)-eq. (36), it turns out that the wider the system is, the larger the effects due to \( \alpha_3 \) are. We also stress that the long-term variations of eq. (31)-eq. (36) were obtained without any a-prori assumption concerning either the orbital geometry of the test particle or the spatial orientation of \( \psi \) and \( \omega \). In this sense, eq. (31)-eq. (36) are exact; due to their generality, they can be used in a variety of different specific astronomical and astrophysical systems for which accurate data are or will be available in the future.

As a further check of the validity of eq. (31)-eq. (36), we re-obtained them by projecting the perturbing acceleration of eq. (1) onto the radial, transverse and normal directions of a trihedron comoving with the particle, and using the standard Gauss equations [20].
3 Confrontation with the observations

3.1 Discussion of the existing constraints

Under certain simplifying assumptions, Will [4] used the perihelion precessions of Mercury and the Earth to infer
\[ |\alpha_3| \lesssim 2 \times 10^{-7}. \]  (37)

More precisely, he assumed that \( \hat{\psi}_\odot \) is perpendicular to the orbital plane, and used an expression for the precession of the longitude of perihelion \( \varpi \) approximated to zeroth order in \( e \). Then, he compared his theoretical formulas to figures for the measured perihelion precessions which were accurate to a \( \approx 200 - 400 \) milliarcseconds per century (mas cty\(^{-1}\)) level. Previous bounds inferred with the same approach were at the level [5]
\[ |\alpha_3| \lesssim 2 \times 10^{-5}. \]  (38)

A modified worst-case error analysis of simulated data of the future spacecraft-based BepiColombo mission to Mercury allowed Ashby et al. [7] to infer a bound of the order of \( |\alpha_3| \lesssim 10^{-10} \).

Strong field constraints were obtained from the slowing down of the pulse periods of some isolated pulsars assumed as rotating neutron stars; for an overview, see [34]. In particular, Will [4], from the impact of eq. (1) on the rotation rate of the neutron stars and using statistical arguments concerning the randomness of the orientation of the pulsars' spins, inferred
\[ |\hat{\alpha}_3| \lesssim 2 \times 10^{-10}, \]  (39)

where \( \hat{\alpha}_3 \) is the strong field equivalent of the conservation-law/preferred-frame PPN parameter. This approach was followed by Bell [35] with a set of millisecond pulsars obtaining [35,36]
\[ |\hat{\alpha}_3| \lesssim 10^{-15}. \]  (40)

Tighter bounds on \( |\hat{\alpha}_3| \) were put from wide-orbit binary millisecond pulsars as well [34]. They rely upon the formalism of the time-dependent eccentricity vector \( \mathbf{e}(t) = \mathbf{e}_F + \mathbf{e}_R(t) \) by Damour and Schaefer [37], where \( \mathbf{e}_R(t) \) is the part of the eccentricity vector rotating due to the periastron precession, while \( \mathbf{e}_F \) is the forced component. Wex [38] inferred
\[ |\hat{\alpha}_3| \leq 1.5 \times 10^{-19} \]  (41)
at 95% confidence level, while Stairs et al. [39] obtained

\[ |\hat{\alpha}_3| \leq 4 \times 10^{-20}, \]  

(42)

at 95% confidence level. Such strong-field constraints are much tighter than the weak-field ones by Will [4]. Nonetheless, it is important to stress that their validity should not be straightforwardly extrapolated to the weak-field regime for the reasons discussed in [15], contrary to what often done in the literature (see, e.g., [7]). More specifically, Shao and Wex [15] warn that it is always recommendable to specify the particular binary system used to infer given constraints. Indeed, using different pulsars implies a potential compactness-dependence (or mass-dependence) because of certain peculiar phenomena, such as spontaneous scalarization [40], which may take place. Moreover, they heavily rely upon statistical considerations to cope with the partial knowledge of some key systems’ parameters such as the longitude of the ascending nodes and the pulsars’ spin axes. Also the inclinations are often either unknown or sometimes determined modulo the ambiguity of \( I \rightarrow 180^\circ - I \). Finally, assumptions on the evolutionary history of the systems considered come into play as well.

A general remark valid for almost all the upper bounds on \( \alpha_3/\hat{\alpha}_3 \) just reviewed is, now, in order before offering to the reader our own ones. We stress that the following arguments are not limited merely to the PPN parameter considered in this study, being, instead, applicable to other non-standard\(^4\) effects as well. Strictly speaking, the tests existing in the literature did not yield genuine “constraints” on either \( \alpha_3 \) or its strong-field version \( \hat{\alpha}_3 \). Indeed, they were never explicitly determined in a least square sense as solved-for parameters in dedicated analyses in which ad-hoc modified models including their effects were fit to observations. Instead, a somewhat “opportunistic” and indirect approach has always been adopted so far by exploiting already existing observation-based determinations of some quantities such as, e.g., perihelion precessions, pulsar spin period derivatives, etc. Theoretical predictions for \( \alpha_3 \)-driven effects were, then, compared with more or less elaborated arguments to such observation-based quantities to infer the bounds previously quoted. In the aforementioned sense, they should rather be seen as an indication of acceptable values. For example, think about the pulsar spin period derivative due to \( \hat{\alpha}_3 \). In [34] it is possible to read: “Young pulsars in the field of the Galaxy […] all show positive period derivatives, typically around \( 10^{-14} \) s/s. Thus, the maximum possible

\(^4\)With such a denomination we refer to any possible dynamical feature of motion, included in the PPN formalism or not, departing from general relativity.
contribution from $\hat{\alpha}_3$ must also be considered to be of this size, and the limit is given by $\hat{\alpha}_3 < 2 \times 10^{-10}$ \cite{4}. In principle, a putative unmodelled signature such as the one due to $\alpha_3/\hat{\alpha}_3$ could be removed to some extent in the data reduction procedure, being partly “absorbed” in the estimated values of other explicitly solved-for parameters. That is, there could be still room, in principle, for larger values of the parameters of the unmodelled effect one is interested in with respect to their upper bounds indirectly inferred as previously outlined. On the other hand, it must also be remarked that, even in a formal covariance analysis, there is the lingering possibility that some still unmodelled/exotic competing physical phenomenon, not even conceived, may somewhat lurk into the explicitly estimated parameters of interest. Another possible drawback of the indirect approach could consist in that one looks at just one PPN parameter at a time, by more or less tacitly assuming that all the other ones are set to their standard general relativistic values. This fact would drastically limit the meaningfulness of the resulting bounds, especially when it seems unlikely that other parameters, closely related to the one which is allowed to depart from its standard value, can, instead, simultaneously assume just their general relativistic values. It may be the case here with $\alpha_3$ and, e.g., the other Lorentz-violating preferred-frame PPN parameters $\alpha_1, \alpha_2$. Actually, even in a full covariance analysis targeted to a specific effect, it is not conceivable to estimate all the parameters one wants; a compromise is always necessarily implemented by making a selection of the parameters which can be practically determined. However, in Section 3.2 we will show how to cope with such an issue in the case of the preferred-frame parameters $\alpha_1, \alpha_2, \alpha_3$ by suitably using the planetary perihelia. Moreover, the upper bounds coming from the aforementioned “opportunistic” approach should not be considered as unrealistically tight because they were obtained in a worst possible case, i.e. by attributing to the unmodelled effect of interest the whole experimental range of variation of the observationally determined quantities used. Last but not least, at present, it seems unlikely, although certainly desirable, that the astronomers will reprocess observational data records several decades long by purposely modifying their models to include this or that non-standard effect every time. It is beyond the scopes of the present work.

The previous considerations should be kept in mind in evaluating the bounds on $\alpha_3/\hat{\alpha}_3$ offered in the next Sections.
3.2 Preliminary upper bounds from the planetary perihelion precessions

Pitjeva [41] recently processed a huge observational data set of about 680000 positional measurements for the major bodies of the Solar System spanning almost one century (1913-2011) by fitting an almost complete suite of standard models to the observations. They include all the known Newtonian and Einsteinian effects for measurements, propagation of electromagnetic waves and bodies’ orbital dynamics up to the 1PN level, with the exception of the gravitomagnetic field of the rotating Sun. Its impact, which is negligible in the present context, is discussed in the text. In one of the global solutions produced, Pitjeva and Pitjev [42] kept all the PPN parameters fixed to their general relativistic values and, among other things, estimated corrections $\Delta \dot{\varpi}$ to the standard (i.e. Newtonian and Einsteinian) perihelion precessions of some planets: they are quoted in Table 1. By construction, they account,

Table 1: Preliminary upper bounds on $\alpha_3$ obtained from a straightforward comparison of the figures of Table 4 in [42] for the supplementary rates $\Delta \dot{\varpi}$ of the planetary perihelia, reported here in the second column from the left, with the theoretical predictions of eq. (36). Pitjeva and Pitjev [42] used the EPM2011 ephemerides [41]. The supplementary perihelion precessions of Venus and Jupiter are non-zero at the 1.6$\sigma$ and 2$\sigma$ level, respectively. In the solution which yielded the supplementary perihelion precessions listed, the PPN parameters were kept fixed to their general relativistic values. The Earth provides the tightest bound: $|\alpha_3| \leq 9 \times 10^{-11}$. We also report the figures for the 1PN Lense-Thirring and the 2PN perihelion precessions. All the precessions listed in this Table are in milliarcseconds per century (mas ct$^{-1}$).

|                | $\Delta \dot{\varpi}$ [42] | $\dot{\varpi}_{LT}$ | $\dot{\varpi}_{2PN}$ | $|\alpha_3|$ |
|----------------|----------------------------|---------------------|---------------------|------------|
| Mercury        | $-2.0 \pm 3.0$             | $2.0$               | $7 \times 10^{-3}$  | $2.930 \times 10^{-8}$ |
| Venus          | $2.6 \pm 1.6$              | $-0.2$              | $6 \times 10^{-4}$  | $1.10 \times 10^{-9}$ |
| Earth          | $0.19 \pm 0.19$            | $-0.09$             | $2 \times 10^{-4}$  | $9 \times 10^{-11}$ |
| Mars           | $-0.020 \pm 0.037$         | $-0.027$            | $6 \times 10^{-5}$  | $2.8 \times 10^{-10}$ |
| Jupiter        | $58.7 \pm 28.3$            | $-7 \times 10^{-4}$ | $9 \times 10^{-7}$  | $4.388 \times 10^{-8}$ |
| Saturn         | $-0.32 \pm 0.47$           | $-1 \times 10^{-4}$ | $9 \times 10^{-8}$  | $2.4 \times 10^{-10}$ |

in principle, for any mismodeled/unmodeled dynamical effect, along with some mismodeling of the astrometric and tracking data; thus, they are po-
tentially suitable to put preliminary upper bounds on $\alpha_3$ by comparing them with eq. (36). See Section 3.1 for a discussion on potential limitations and strength of such an indirect, opportunistic approach. We stress once again that an examination of the existing literature shows that such a strategy is widely adopted for preliminarily constraining several non-standard effects in the Solar System; see, e.g., the recent works [43]–[47]. Here we recall that, strictly speaking, it allows to test alternative theories of gravity differing from general relativity just for $\alpha_3$, being all the other PPN parameters set to their general relativistic values. If and when the astronomers will include $\alpha_3$ in their dynamical models, then it could be simultaneously estimated along with a selection of other PPN parameters. Similar views can be found in [48].

From Table I, it turns out that the perihelion of the Earth preliminarily yields

$$|\alpha_3| \leq 9 \times 10^{-11},$$

while Mars and Saturn provide bounds of the order of

$$|\alpha_3| \lesssim 2 \times 10^{-10}.$$  

The bound of eq. (43) is about 3 orders of magnitude tighter than the weak-field bound reported in [4]. The use of the individual supplementary precessions $\Delta \dot{\varpi}$ of the Earth, Mars and Saturn is justified since the current level of accuracy in determining them from observations makes other competing unmodelled effects negligible. By restricting ourselves just to the PN contributions, the 1PN Lense-Thirring precessions [49], quoted in Table I, are too small for the aforementioned planets. The 2PN precessions, computed within general relativity from [50, 51] for a binary system made of two bodies A and B with total mass $M_t$

$$\Delta \dot{\varpi}_{2\text{PN}} = \frac{3}{c^4a^{7/2}} \frac{(GM_t^{5/2})}{(1 - e^2)^2} \left[ \frac{13}{2} \left( \frac{m_A^2 + m_B^2}{M_t^2} \right) + \frac{32}{3} \frac{m_A m_B}{M_t^2} \right],$$

are completely negligible (see Table I). As remarked in Section 3.1 the assumption that the other preferred-frame PPN parameters $\alpha_1, \alpha_2$ are zero when a non-zero value for $\alpha_3$ is admitted, seems unlikely. The availability of more than one perihelion extra-precession $\Delta \dot{\varpi}$ allows us to cope with such an issue. Indeed, it is possible to simultaneously infer bounds on $\alpha_1, \alpha_2, \alpha_3$ which are, by construction, mutually independent from each other. From the following linear system of three equations in the three unknowns $\alpha_1, \alpha_2, \alpha_3$

$$\Delta \dot{\varpi}_j = \alpha_1 \dot{\varpi}_{\alpha_1} + \alpha_2 \dot{\varpi}_{\alpha_2} + \alpha_3 \dot{\varpi}_{\alpha_3}, \ j = \text{Earth, Mars, Saturn},$$

13
where the coefficients \( \dot{\omega}_{\alpha_1}, \dot{\omega}_{\alpha_2}, \dot{\omega}_{\alpha_3} \) are the analytical expressions of the pericenter precessions caused by \( \alpha_1, \alpha_2, \alpha_3 \), and by using the figures in Table 1 for \( \Delta \dot{\omega}^j \), one gets

\[
\alpha_1 = (-2 \pm 2) \times 10^{-6}, \tag{47}
\]

\[
\alpha_2 = (3 \pm 4) \times 10^{-6}, \tag{48}
\]

\[
\alpha_3 = (-4 \pm 6) \times 10^{-10}. \tag{49}
\]

It can be noticed that the bound on \( \alpha_3 \) of eq. (49) is slightly weaker than the ones listed in Table 1 obtained individually from each planet; nonetheless, it is free from any potential correlation with \( \alpha_1, \alpha_2 \). It is also interesting to notice how the bounds on \( \alpha_1, \alpha_2 \) of eq. (47)-eq. (48) are similar, or even better in the case of \( \alpha_2 \), than those inferred in [52] in which the INPOP10a ephemerides were used [53]. In it, all the rocky planets of the Solar System were used to separate \( \alpha_1, \alpha_2 \) from the effects due to the unmodelled Sun's gravitomagnetic field and the mismodelled solar quadrupole mass moment, which have an impact on Mercury and, to a lesser extent, Venus. Interestingly, our bounds on \( \alpha_3 \) of eq. (43)-eq. (44) and eq. (49) are roughly of the same order of magnitude of the expected constraint from BepiColombo [7]; the same holds also for eq. (47)-eq. (48). We remark that the approach of eq. (46) can, in principle, be extended also to other planets and/or other orbital elements such as the nodes [54] to separate more PPN parameters and other putative exotic effects. To this aim, it is desirable that the astronomers will release corrections to the standard precessions of more orbital elements for an increasing number of planets in future global solutions.

It may be worthwhile noticing from Table 1 that Pitjeva and Pitjev [42] obtained marginally significant non-zero precessions for Venus and Jupiter. They could be used to test the hypothesis that \( \alpha_3 \neq 0 \) by taking their ratio and confronting it with the corresponding theoretical ratio which, for planets of the same central body such as the Sun, is independent of \( \alpha_3 \) itself. From Table 1 and eq. (36), it is

\[
\frac{\Delta \dot{\omega}_{\text{Ven}}}{\Delta \dot{\omega}_{\text{Jup}}} = 0.044 \pm 0.034, \tag{50}
\]

---

5 As far as \( \alpha_3 \) is concerned, \( \dot{\omega}_{\alpha_3} \) comes from eq. (35), while \( \dot{\omega}_{\alpha_1}, \dot{\omega}_{\alpha_2} \) can be found in [52].

6 The \( \alpha_1, \alpha_2 \) planetary signals are enhanced for close orbits.
\[
\frac{\dot{\alpha}_V}{\dot{\alpha}_J} = 2.251.
\] (51)

Thus, the existence of the \(\alpha_3\)-induced precessions would be ruled out at a 65\(\sigma\) level, independently of the value of \(\alpha_3\) itself. However, caution is in order in accepting the current non-zero precessions of Venus and Jupiter as real; further independent analyses by astronomers are required to confirm or disproof them as genuine physical effects needing explanation.

Finally, we mention that the use of the supplementary perihelion precessions determined by Fienga et al. with the INPOP10a ephemerides would yield less tight bounds on \(|\dot{\alpha}_3|\) because of the lower accuracy of the INPOP10a-based \(\Delta \dot{\alpha}\) with respect to those determined in [42] by a factor \(\approx 1.4 - 4\) for the planets used here. More recent versions of the INPOP ephemerides, i.e. INPOP10c [54] and INPOP13a [55], have been recently produced, but no supplementary orbital precessions have yet been released for them.

4 Effects on pulsar timing

The basic observable in a binary system hosting an emitting radiopulsar \(p\) is the periodic change \(\delta \tau_p\) in its time of arrivals (TOAs) \(\tau_p\) due to its barycentric orbital motion induced by the often unseen companion \(c\). For our purposes, the latter one will be a less compact \((\Theta_c \ll \Theta_p)\) white dwarf. The unperturbed Keplerian expression of \(\delta \tau_p\) is

\[
\delta \tau_p = x_p \left[ (\cos E - e) \sin \omega + \sqrt{1 - e^2} \sin E \cos \omega \right],
\] (52)

where

\[
x_p = \frac{a_p \sin I}{c}
\] (53)

is the projected semimajor axis of the pulsar’s barycentric orbit; in terms of the semimajor axis \(a\) of the relative motion, it is \(a_p = m_c M_t^{-1} a\). Here, we are interested in calculating the shift per orbit \(\Delta \delta \tau_p\) caused by \(\dot{\alpha}_3\) through eq. (1) or, equivalently, eq. (1). Then, in order to say something on \(|\dot{\alpha}_3|\), we will exploit the measured root-mean-square of the TOAs residuals \(\sigma_{\delta \tau_p}\) of some specific binary pulsars in terms of \(\Delta \delta \tau_p\). The approach is basically the same adopted in Section 3.2 and the observations of Section 3.1 should be taken into account to properly interpret our bounds.

The shift per orbit \(\Delta Y\) of a generic observable \(Y\) with respect to its classical expression due to the action of a perturbing acceleration such as
eq. (1) can be computed as

\[ \Delta Y = \int_0^{P_b} \left( \frac{dY}{dt} \right) dt = \int_0^{2\pi} \left[ \frac{\partial Y}{\partial E} \frac{dE}{dt} + \frac{\partial Y}{\partial \xi} \frac{d\xi}{dt} \right] \left( \frac{dt}{dE} \right) dE, \quad (54) \]

where \( \mathcal{M} \) is the mean anomaly and \( \xi \) collectively denotes the remaining Keplerian orbital elements. The rates \( \dot{\mathcal{M}}, \dot{\xi} \) entering eq. (54) are due to the perturbation and are instantaneous. As such, they are obtained by computing from eq. (5) onto the unperturbed Keplerian ellipse without averaging it over \( P_b \). The derivatives \( \partial Y/\partial E, \partial Y/\partial \xi \) in eq. (54) are calculated by using the unperturbed expression for \( Y \).

As a result, the \( \hat{\alpha}_3 \)-induced shift per orbit of a pulsar-white dwarf binary turns out to be

\[ \Delta \delta \tau_p = -\frac{\pi \hat{\alpha}_3 \Theta w_p \psi_p \sqrt{1 - \cfrac{c^2}{v^2} \sin I \left( u_x \cos \Omega + u_y \sin \Omega \right)}}{3cM_1 m_2^b}. \quad (55) \]

Also in this case, the wide systems are favored to tightly constrain \( \hat{\alpha}_3 \) thanks to the \( n_b^{-2} \) factor in eq. (55). To this aim, it must be noticed that, for most of such binaries, the orientation of the pulsar’s spin \( \hat{\psi}_p \) is completely unknown. The same often holds also for the node \( \Omega \) and for the inclination \( I \) to the plane of the sky, assumed as reference \( \{x, y\} \) plane. Concerning the velocity \( \mathbf{w}_p \) of the binary pulsar with respect to the CMB rest frame, it can be generally considered as the relativistic sum of the velocity \( \mathbf{w}_{SSB} \) of the SSB with respect to the CMB rest frame and the velocity \( \mathbf{w}'_p \) of the binary pulsar with respect to the SSB itself. Let us recall that, if \( K' \) is an inertial frame moving with velocity \( \mathbf{V} \) with respect to another inertial frame \( K \), then the Lorentz transformations connecting the two frames are [57]

\[ r = \Gamma \left( r' + V t' \right) + \left( \Gamma - 1 \right) \frac{(r' \times \mathbf{V}) \times \mathbf{V}}{V^2}, \quad (56) \]

\[ t = \Gamma \left( t' + \frac{r' \cdot \mathbf{V}}{c^2} \right), \quad (57) \]

where

\[ \Gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}. \quad (58) \]
and \( v \) and \( v' \) are the velocity vectors of a moving particle with respect to \( K \) and \( K' \), respectively. From eq. (56)-eq. (57) it straightforwardly follows

\[
v = \frac{1}{1 + \frac{\vec{v'} \cdot \vec{V}}{c^2}} \left[ \vec{v'} + \vec{V} \left( \frac{\Gamma - 1}{\Gamma V^2} \right) \left( \vec{v'} \times \vec{V} \right) \times \vec{V} \right]
\]

for the relativistic sum of the velocities. In our case, \( K \) is the CMB rest frame, \( K' \) is the SSB frame, which can be certainly considered as inertial over timescales of the order of the orbital period of the known wide-orbit binary pulsars, \( V \) is \( \vec{w}_{SSB} \), \( v \) is \( \vec{w}_p \), and \( v' \) is \( \vec{w}'_p \). By posing

\[
x'_p = d_p \cos \delta_p \cos \alpha_p,
\]

\[
x'_p = d_p \cos \delta_p \sin \alpha_p,
\]

\[
z'_p = d_p \sin \delta_p,
\]

where \( d_p \) is the distance of the pulsar from us and \( \alpha_p, \delta_p \) are its Celestial coordinates, one has for the components of \( \vec{w}'_p \)

\[
\dot{x}'_p = \left( \frac{\partial x'_p}{\partial d_p} \right) \dot{d}_p + \left( \frac{\partial x'_p}{\partial \alpha_p} \right) \mu_{\alpha_p} + \left( \frac{\partial x'_p}{\partial \delta_p} \right) \mu_{\delta_p} = -y_p \mu_{\alpha_p} + \cos \alpha_p \left( \dot{d}_p \cos \delta_p - z_p \mu_{\delta_p} \right),
\]

\[
\dot{y}'_p = \left( \frac{\partial y'_p}{\partial d_p} \right) \dot{d}_p + \left( \frac{\partial y'_p}{\partial \alpha_p} \right) \mu_{\alpha_p} + \left( \frac{\partial y'_p}{\partial \delta_p} \right) \mu_{\delta_p} = x_p \mu_{\alpha_p} + \sin \alpha_p \left( \dot{d}_p \cos \delta_p - z_p \mu_{\delta_p} \right),
\]

\[
\dot{z}'_p = \left( \frac{\partial z'_p}{\partial d_p} \right) \dot{d}_p + \left( \frac{\partial z'_p}{\partial \alpha_p} \right) \mu_{\alpha_p} + \left( \frac{\partial z'_p}{\partial \delta_p} \right) \mu_{\delta_p} = d_p \cos \delta_p \mu_{\alpha_p} + \dot{d}_p \sin \delta_p,
\]
where \( \dot{d} \) is the radial velocity of the pulsar, while \( \mu_\alpha, \mu_\delta \) are its proper motions in right ascension and declination, respectively. Either the radial velocity or the proper motions are often unknown or poorly known for pulsars at more than 1 kpc from us \[39, 58, 59\]. When \( w_p \) can be considered as known, its magnitude is relatively small with respect to \( c \). Indeed, by considering binary pulsars located at \( d_p \approx 0.7 - 0.8 \) kpc with a proper motion of \( \mu_\alpha, \mu_\delta \approx 5 \) mas yr\(^{-1} \) \[58, 59\], it turns out that \( w'_p \approx 10^4 \) m s\(^{-1} \). This fact, in addition to the relative smallness of \( w_{SSB} \) (cf. eq. (13)), allows us to safely consider the non-relativistic limit of eq. (59). To be more quantitative, let us consider a fictitious binary pulsar located somewhere in the sky at, say, 0.7 kpc from us with a proper motion of the order of \( 5 \) mas yr\(^{-1} \) \[58, 59\]. We compute its speed \( w_p \) with respect to the CMB frame both with the Lorentz transformations of eq. (59) and with the Galilei transformations. Then, we look at their difference, and study it as a function of \( \alpha_p \) and \( \delta_p \). As a result, a difference of just a few cm s\(^{-1} \) at most, which is negligible (cf. the uncertainty in eq. (13)), is obtained.

Among the wide pulsar-wide dwarf binaries, a good candidate to constrain \( \dot{\alpha} \) is the pulsar PSR J0407+1607 \[39, 60\] located at about 1.3 kpc from us and with an orbital period of about 1.8 yr. Its relevant physical and orbital parameters are listed in Table 2. It can be noticed that no

| Parameter | Value |
|-----------|-------|
| \( P_b \) (d) | 669.0704 |
| \( P \) (ms) | 25.70173919463 |
| \( x \) (s) | 106.45026 |
| \( e \) | 0.0009368 |
| \( \omega \) (deg) | 192.74 |
| \( m_p \) (M\(_\odot\)) | 1.35 |
| \( m_{WD} \) (M\(_\odot\)) | 0.2 |
| \( \dot{d} \) | — |
| \( \mu_\alpha \) | — |
| \( \mu_\delta \) | — |
| \( \sigma_{\delta_T} \) (\(\mu s\)) | 16 |

\(^7\)We neglect \( \dot{d} \).
phenomenologically estimated post-Keplerian rates of change of any of the orbital elements are available so far for the pulsar PSR J0407+1607. In particular, its periastron precession has not been determined, so that our prediction of eq. (35) cannot be used to constrain \( \hat{\alpha}_3 \) with this particular binary system. Indeed, the dominant 1PN contribution to the periastron rate of PSR J0407+1607 is as little as \( \dot{\omega}_{1\mathrm{PN}} = 18 \) milliarcseconds per year (mas yr\(^{-1}\)). The accuracy in determining a phenomenological post-Keplerian periastron precession for the pulsar PSR J0407+1607 can be evaluated to be of the order of \( \sigma_{\dot{\omega}} \approx 0.01 \) deg yr\(^{-1}\) [60]. The analytical expression of eq. (55) allows us to partly reduce the use of statistical/probabilistic reasonings to constrain \( \hat{\alpha}_3 \), at least to a certain extent which will be discussed later. To this aim, we compare the root-mean-square TOA residuals \( \sigma_{\delta \tau} \) in Table 2 to the theoretical expression for \( \Delta \delta \tau_p \) of eq. (55) computed for the figures of Table 2. Then, the resulting value \( |\hat{\alpha}_3|^* \), considered as a function of the four independent variables \( \alpha_{\psi}, \delta_{\psi}, \Omega, I \), can be extremized with respect to all of them. It must be stressed that our approach is necessarily limited to those values for which eq. (55) does not vanish. Thus, all face-on orbital geometries are excluded, and, for a given direction \( \hat{w}_p \), also those orientations of the spin axis \( \hat{\psi}_p \) parallel to it. Moreover, even if \( \hat{w}_p \) and \( \hat{\psi}_p \) are not aligned, the values of the node \( \Omega \) for which

\[ \tan \Omega = -\frac{u_x}{u_y} \]  

are to be excluded as well. In this specific sense, \( |\hat{\alpha}_3|^* \) cannot be considered, strictly speaking, an upper limit for \( \hat{\alpha}_3 \). Two independent numerical methods to find absolute minima and maxima return

\[ |\hat{\alpha}_3|^* = 3.9 \times 10^{-17} \]  

for

\[ \alpha_{\psi} = 194.47^\circ, \]  

\[ \delta_{\psi} = 88.30^\circ, \]  

\[ \Omega = 84.62^\circ, \]  

\[ I = -88.01^\circ. \]
It can be noticed that \( \sin I \) does not vanish for eq. (71). Moreover, the same holds also for the spin-dependent term in eq. (55), which takes the value
\[
u_x \cos \Omega + \nu_y \sin \Omega = -0.092
\] (72)
when computed for eq. (68)-eq. (70). Incidentally, the angle between \( \hat{w}_p \) and \( \hat{\psi}_p \) amounts to \( \vartheta_p = 16.3^\circ \). The value of eq. (67), obtained for eq. (68)-eq. (71), is about 3 orders of magnitude larger than the bounds obtained in the literature from a statistical Bayesian approach involving several pulsars.

Let us, now, discuss some of the assumptions underlying the previous analysis. In the case of PSR J0407+1607, its proper motion is not known due to its relatively large distance from us and the short timing baseline of just 2 yr [60] adopted to obtain the figures quoted in Table 2. On the basis of the existing data for other binary pulsars located at almost the same distance (cf. PSR J2317+1439 at \( d_p = 1.46 \) kpc [58, 59]), it may not be unrealistic to assume for the proper motion of PSR J0407+1607 a magnitude of the order of \( \approx 1 \) mas yr\(^{-1} \) or so. As such, it turns out that the speed \( w'_p \) of PSR J0407+1607 with respect to the SSB frame should be of the order of a few km s\(^{-1} \) at most. Thus, in computing the speed of PSR J0407+1607 with respect to the CMB frame, the Galilei transformations can be used and \( \nu_p \) can likely be assumed equal to \( \nu_{SSB} \) as well. The latter assumption is confirmed by repeating the previous numerical analysis by allowing for non-zero values of \( \mu_\alpha, \mu_\delta \) in the range \( 1 - 5 \) mas yr\(^{-1} \): it does not yield appreciable changes in our results since we get \( |\hat{\alpha}_3|^* = (3.8 - 4.2) \times 10^{-17} \) (and \( \vartheta_p = 15.4^\circ \)).

Another potential issue is the value assumed for the mass of the neutron star. Actually, the figure quoted in Table 2 for it is just an assumed typical value according to the statistical analysis in [61]. Moreover, there have been found in nature recycled pulsars having up to two solar masses [62, 63]. However, by repeating our analysis with \( m_p = 2 \) M\(_\odot\) does not alter significantly our results; indeed, we obtain \( |\hat{\alpha}_3|^* = (3.8 - 4.0) \times 10^{-17} \).

5 Summary and conclusions

In this paper, we focussed on the Lorentz invariance/momentum-conservation PPN parameter \( \alpha_3 \) and on some of its orbital effects.

We analytically calculated the long-term variations of the standard Keplerian orbital elements of a test particle orbiting a compact primary, and the shift in the periodic change in the time of arrivals of a pulsar orbited by a relatively non-compact companion such as a white dwarf. Our results are exact in the sense that we did not restrict ourselves to any a priori peculiar
orientation of the primary’s spin axis. Also the orbital geometry of the non-compact object was left unconstrained in our calculations. Thus, they have a general validity which may allow one to use them in different astronomical and astrophysical scenarios.

We used the latest results in the field of the planetary ephemerides of the Solar System to preliminarily infer new weak-field bounds on $\alpha_3$. From a linear combination of the current constraints on possible anomalous perihelion precessions of the Earth, Mars and Saturn, recently determined with the EPM2011 ephemerides in global solutions in which all the PPN parameters were kept fixed to their standard general relativistic values, we preliminarily inferred $|\alpha_3| \leq 6 \times 10^{-10}$. It is about 3 orders of magnitude better than previous weak-field constraints existing in the literature. Slightly less accurate bounds could be obtained from the supplementary perihelion precessions determined with the INPOP10a ephemerides. We obtained our limit on $\alpha_3$ by allowing also for possible non-zero values of the other preferred-frame PPN parameter $\alpha_1, \alpha_2$, for which we got $\alpha_1 \leq 2 \times 10^{-6}, \alpha_2 \leq 4 \times 10^{-6}$. All such bounds, by construction, are mutually independent of each other. An alternative strategy, requiring dedicated and time-consuming efforts, would consist in explicitly modeling the effects accounted for by $\alpha_3$ (and, possibly, by other PPN parameters as well), and re-processing the same planetary data set with such ad-hoc modified dynamical models to estimate $\hat{\alpha}_3$ along with other selected parameters in dedicated covariance analyses.

The pulsar-white dwarf binary system PSR J0407+1607, characterized by an orbital period 1.8 yr long and a low eccentricity, was used to put preliminary upper bounds on the strong-field version $\hat{\alpha}_3$ of the Lorentz invariance/momentum-conservation PPN parameter. By looking at the root-mean-square residuals of the time of arrivals of the pulsar and using our analytical expression for their $\hat{\alpha}_3$–induced change, we obtained an upper bound on $\hat{\alpha}_3$ depending on the unknown orbital node and inclination and on the pulsar’s spin axis as well. A numerical search for absolute extrema returned $|\hat{\alpha}_3| \lesssim 3 \times 10^{-17}$. Such bound is less stringent than those existing in the literature which, on the other hand, were inferred by combining several different pulsars in statistical analyses characterized by a number of probabilistic assumptions on the systems’ key parameters. However, also our analysis necessarily relies upon certain assumptions on the position of the pulsar’s spin axis in the sky, and of the system’s orbital orientation. Concerning the pulsar’s proper motion, not yet detected, and the fact that recycled pulsars have been found in nature having up to two solar masses, it turns out that they do not substantially affect our result. As recently explained in the literature, the strong-field
constraints should not be straightforwardly compared to the weak-field ones because of a number of issues.

References

[1] K. Nordtvedt, “Equivalence Principle for Massive Bodies. II. Theory,” Physical Review 169 no. 5, (May, 1968) 1017–1025.

[2] C. M. Will, “Theoretical Frameworks for Testing Relativistic Gravity. II. Parametrized Post-Newtonian Hydrodynamics, and the Nordtvedt Effect,” The Astrophysical Journal 163 (Feb., 1971) 611–628.

[3] C. M. Will and K. Nordtvedt, Jr., “Conservation Laws and Preferred Frames in Relativistic Gravity. I. Preferred-Frame Theories and an Extended PPN Formalism,” The Astrophysical Journal 177 (Nov., 1972) 757–774.

[4] C. M. Will, Theory and Experiment in Gravitational Physics. Cambridge University Press, Mar., 1993.

[5] K. Nordtvedt, Jr. and C. M. Will, “Conservation Laws and Preferred Frames in Relativistic Gravity. II. Experimental Evidence to Rule Out Preferred-Frame Theories of Gravity,” The Astrophysical Journal 177 (Nov., 1972) 775–792.

[6] K. Nordtvedt, “Post-Newtonian Gravitational Effects in Lunar Laser Ranging,” Physical Review D 7 no. 8, (Apr., 1973) 2347–2356.

[7] N. Ashby, P. L. Bender, and J. M. Wahr, “Future gravitational physics tests from ranging to the BepiColombo Mercury planetary orbiter,” Physical Review D 75 no. 2, (Jan., 2007) 022001.

[8] S. G. Turyshev, “Experimental Tests of General Relativity,” Annual Review of Nuclear and Particle Science 58 no. 1, (Nov., 2008) 207–248. arXiv:0806.1731 [gr-qc].

[9] P. K. Seidelmann, B. A. Archinal, M. F. A’Hearn, A. Conrad, G. J. Consolmagno, D. Hestroffer, J. L. Hilton, G. A. Krassinsky, G. Neumann, J. Oberst, P. Stooke, E. F. Tedesco, D. J. Tholen, P. C. Thomas, and I. P. Williams, “Report of the IAU/IAG Working Group on cartographic coordinates and rotational elements: 2006,” Celestial Mechanics and Dynamical Astronomy 98 no. 3, (July, 2007) 155–180.
[10] R. J. Warburton and J. M. Goodkind, “Search for evidence of a preferred reference frame,”
   *The Astrophysical Journal* **208** (Sept., 1976) 881–886

[11] R. W. Hellings, “Testing relativity with solar system dynamics,” in *General Relativity and Gravitation Conference*, B. Bertotti, F. de Felice, and A. Pascolini, eds., pp. 365–385. Reidel, Dordrecht, 1984.

[12] K. Nordtvedt, “Probing gravity to the second post-Newtonian order and to one part in 10 to the 7th using the spin axis of the sun,”
   *The Astrophysical Journal* **320** (Sept., 1987) 871–874

[13] T. Damour and G. Esposito-Farèse, “Testing local Lorentz invariance of gravity with binary-pulsar data,”
   *Physical Review D* **46** no. 10, (Nov., 1992) 4128–4132

[14] T. Damour and G. Esposito-Farèse, “Testing for preferred-frame effects in gravity with artificial Earth satellites,”
   *Physical Review D* **49** no. 4, (Feb., 1994) 1693–1706,
   arXiv:gr-qc/9311034.

[15] L. Shao and N. Wex, “New tests of local Lorentz invariance of gravity with small-eccentricity binary pulsars,”
   *Classical and Quantum Gravity* **29** no. 21, (Oct., 2012) 215018,
   arXiv:1209.4503 [gr-qc].

[16] G. Hinshaw, J. L. Weiland, R. S. Hill, N. Odegard, D. Larson, C. L. Bennett, J. Dunkley, B. Gold, M. R. Greason, N. Jarosik,
   E. Komatsu, M. R. Nolta, L. Page, D. N. Spergel, E. Wollack, M. Halpern, A. Kogut, M. Limon, S. S. Meyer, G. S. Tucker, and
   E. L. Wright, “Five-Year Wilkinson Microwave Anisotropy Probe Observations: Data Processing, Sky Maps, and Basic Results,”
   *The Astrophysical Journal Supplement* **180** no. 2, (Feb., 2009) 225–245,
   arXiv:0803.0732.

[17] J. G. Beck, “A comparison of differential rotation measurements - (Invited Review),” *Solar Physics* **191** no. 1, (Jan., 2000) 47–70

[18] H. B. Snodgrass and R. K. Ulrich, “Rotation of Doppler features in the solar photosphere,”
   *The Astrophysical Journal* **351** (Mar., 1990) 309–316
[19] R. K. Ulrich, “The influence of partial ionization and scattering states on the solar interior structure,”
The Astrophysical Journal 258 (July, 1982) 404–413.

[20] B. Bertotti, P. Farinella, and D. Vokrouhlický, Physics of the Solar System. Kluwer Academic Press, Dordrecht, 2003.

[21] S. Kopeikin, M. Efroimsky, and G. Kaplan, Relativistic Celestial Mechanics of the Solar System. Wiley-VCH, Berlin, Sept., 2011.

[22] M. Calura, P. Fortini, and E. Montanari, “Post-Newtonian Lagrangian planetary equations,”
Physical Review D 56 no. 8, (Oct., 1997) 4782–4788,
arXiv:gr-qc/9708057.

[23] M. Calura, E. Montanari, and P. Fortini, “Lagrangian planetary equations in Schwarzschild spacetime,”
Classical and Quantum Gravity 15 no. 10, (Oct., 1998) 3121–3129,
arXiv:gr-qc/9807007.

[24] D. Souami and J. Souchay, “The solar system’s invariable plane,”
Astronomy & Astrophysics 543 (July, 2012) A133.

[25] T. Damour and R. Ruffini, “Certain new verifications of general relativity made possible by the discovery of a pulsar belonging to a binary system,” Comptes Rendus Mathematique 279 no. 26, (Dec., 1974) 971–973.

[26] L. W. Esposito and E. R. Harrison, “Properties of the Hulse-Taylor binary pulsar system,”
The Astrophysical Journal Letters 196 (Feb., 1975) L1–L42.

[27] B. M. Barker and R. F. O’Connell, “Relativistic effects in the binary pulsar PSR 1913+16,”
The Astrophysical Journal Letters 199 (July, 1975) L25–L26.

[28] M. Burgay, N. D’Amico, A. Possenti, R. N. Manchester, A. G. Lyne, B. C. Joshi, M. A. McLaughlin, M. Kramer, J. M. Sarkissian, F. Camilo, V. Kalogera, C. Kim, and D. R. Lorimer, “An increased estimate of the merger rate of double neutron stars from observations of a highly relativistic system,”
Nature 426 no. 6966, (Dec., 2003) 531–533,
arXiv:astro-ph/0312071.
[29] A. G. Lyne, M. Burgay, M. Kramer, A. Possenti, R. N. Manchester, F. Camilo, M. A. McLaughlin, D. R. Lorimer, N. D’Amico, B. C. Joshi, J. Reynolds, and P. C. C. Freire, “A Double-Pulsar System: A Rare Laboratory for Relativistic Gravity and Plasma Physics,” *Science* 303 no. 5661, (Feb., 2004) 1153–1157, arXiv:astro-ph/0401086.

[30] R. P. Breton, V. M. Kaspi, M. Kramer, M. A. McLaughlin, M. Lyutikov, S. M. Ransom, I. H. Stairs, R. D. Ferdman, F. Camilo, and A. Possenti, “Relativistic Spin Precession in the Double Pulsar,” *Science* 321 no. 5885, (July, 2008) 104–107, arXiv:0807.2644 [astro-ph].

[31] J. Javaraiah, “A Comparison of Solar Cycle Variations in the Equatorial Rotation Rates of the Sun’s Subsurface, Surface, Corona, and Sunspot Groups,” *Solar Physics* 287 no. 1-2, (Oct., 2013) 197–214, arXiv:1306.2151 [astro-ph.SR].

[32] J. Javaraiah, “Long-Term Variations in the Solar Differential Rotation,” *Solar Physics* 212 no. 1, (Jan., 2003) 23–49.

[33] M. Kramer, I. H. Stairs, R. N. Manchester, M. A. McLaughlin, A. G. Lyne, R. D. Ferdman, M. Burgay, D. R. Lorimer, A. Possenti, N. D’Amico, J. M. Sarkissian, G. B. Hobbs, J. E. Reynolds, P. C. C. Freire, and F. Camilo, “Tests of General Relativity from Timing the Double Pulsar,” *Science* 314 no. 5796, (Oct., 2006) 97–102, arXiv:astro-ph/0609417.

[34] I. H. Stairs, “Testing General Relativity with Pulsar Timing,” *Living Reviews in Relativity* 6 (Sept., 2003) 5, arXiv:astro-ph/0307536.

[35] J. F. Bell, “A Tighter Constraint on Post-Newtonian Gravity Using Millisecond Pulsars,” *The Astrophysical Journal* 462 (May, 1996) 287, arXiv:astro-ph/9507086.

[36] J. F. Bell and T. Damour, “A new test of conservation laws and Lorentz invariance in relativistic gravity,” *Classical and Quantum Gravity* 13 (Dec., 1996) 3121–3127, arXiv:gr-qc/9606062.
[37] T. Damour and G. Schaefer, “New tests of the strong equivalence principle using binary-pulsar data,” *Physical Review Letters* **66** no. 20, (May, 1991) 2549–2552.

[38] N. Wex, “Small-eccentricity binary pulsars and relativistic gravity,” in *IAU Colloq. 177: Pulsar Astronomy - 2000 and Beyond*, M. Kramer, N. Wex, and R. Wielebinski, eds., vol. 202 of *Astronomical Society of the Pacific Conference Series*, pp. 113–116. 2000. arXiv:gr-qc/0002032.

[39] I. H. Stairs, A. J. Faulkner, A. G. Lyne, M. Kramer, D. R. Lorimer, M. A. McLaughlin, R. N. Manchester, G. B. Hobbs, F. Camilo, A. Possenti, M. Burgay, N. D’Amico, P. C. Freire, and P. C. Gregory, “Discovery of Three Wide-Orbit Binary Pulsars: Implications for Binary Evolution and Equivalence Principles,” *The Astrophysical Journal* **632** (Oct., 2005) 1060–1068, arXiv:astro-ph/0506188.

[40] T. Damour and G. Esposito-Farèse, “Nonperturbative strong-field effects in tensor-scalar theories of gravitation,” *Physical Review Letters* **70** no. 15, (Apr., 1993) 2220–2223.

[41] E. V. Pitjeva, “Updated IAA RAS Planetary Ephemerides-EPM2011 and Their Use in Scientific Research,” *Solar System Research* **47** no. 5, (Sept., 2013) 386–402, arXiv:1308.6416 [astro-ph.EP].

[42] E. V. Pitjeva and N. P. Pitjev, “Relativistic effects and dark matter in the Solar system from observations of planets and spacecraft,” *Monthly Notices of the Royal Astronomical Society* **432** no. 4, (July, 2013) 3431–3437, arXiv:1306.3043 [astro-ph.EP].

[43] A. Avalos-Vargas and G. Ares de Parga, “The precession of the orbit of a charged body interacting with a massive charged body in General Relativity,” *European Physical Journal Plus* **127** (Dec., 2012) 155.

[44] Y. Xie and X.-M. Deng, “f (T) gravity: effects on astromonical observations and Solar system experiments and upper bounds,” *Monthly Notices of the Royal Astronomical Society* **433** (Aug., 2013) 3584–3589, arXiv:1312.4103 [gr-qc].

[45] Y.-K. E. Cheung and F. Xu, “Constraining the String Gauge Field by Galaxy Rotation Curves and Perihelion Precession of Planets,”
The Astrophysical Journal 774 (Sept., 2013) 65, arXiv:1108.5459 [hep-th].

[46] X.-M. Deng and Y. Xie, “Preliminary limits on a logarithmic correction to the Newtonian gravitational potential in the solar system,” (Dec., 2013)

[47] Z.-W. Li, S.-F. Yuan, C. Lu, and Y. Xie, “New upper limits on deviation from the inverse-square law of gravity in the solar system: a Yukawa parameterization,” Research in Astronomy and Astrophysics 14 (Feb., 2014) 139–143.

[48] K. Nordtvedt, “Improving gravity theory tests with solar system “grand fits”,” Physical Review D 61 no. 12, (June, 2000) 122001.

[49] J. Lense and H. Thirring, “Über den einfluß der eigenrotation der zentralkörper auf die bewegung der planeten und monde nach der einsteinschen gravitationstheorie,” Physikalische Zeitschrift 19 (1918) 156–163.

[50] T. Damour and G. Schafer, “Higher-order relativistic periastron advances and binary pulsars.,” Nuovo Cimento B 101 no. 2, (Feb., 1988) 127–176

[51] N. Wex, “The second post-Newtonian motion of compact binary-star systems with spin,” Classical and Quantum Gravity 12 no. 4, (Apr., 1995) 983–1005

[52] L. Iorio, “Constraints on the Preferred-Frame α_1, α_2 Parameters from Solar System Planetary Precessions,” International Journal of Modern Physics D 23 no. 1, (2014) 1450006, arXiv:1210.3026 [gr-qc].

[53] A. Fienga, J. Laskar, P. Kuchynka, H. Manche, G. Desvignes, M. Gastineau, I. Cognard, and G. Theureau, “The INPOP10a planetary ephemeris and its applications in fundamental physics,” Celestial Mechanics and Dynamical Astronomy 111 no. 3, (Nov., 2011) 363–385, arXiv:1108.5546 [astro-ph.EP]

[54] A. Fienga, H. Manche, J. Laskar, M. Gastineau, and A. Verma, “INPOP new release: INPOP10e,” ArXiv e-prints (Jan., 2013), arXiv:1301.1510 [astro-ph.EP].
[55] A. K. Verma, A. Fienga, J. Laskar, H. Manche, and M. Gastineau, “Use of MESSENGER radioscience data to improve planetary ephemeris and to test general relativity,” *Astronomy & Astrophysics* 561 (Jan., 2014) A115, arXiv:1306.5669 [astro-ph.EP].

[56] M. Konacki, A. J. Maciejewski, and A. Wolszczan, “Improved Timing Formula for the PSR B1257+12 Planetary System,” *The Astrophysical Journal* 544 no. 2, (Dec., 2000) 921–926, arXiv:astro-ph/0007335.

[57] V. A. Ugarov, *Special Theory of Relativity*. Mir Publishers, Moscow, 1979.

[58] S. M. Kopeikin, “On possible implications of orbital parallaxes of wide orbit binary pulsars and their measurability,” *The Astrophysical Journal Letters* 439 (Jan., 1995) L5–L8.

[59] S. M. Kopeikin, “Proper Motion of Binary Pulsars as a Source of Secular Variations of Orbital Parameters,” *The Astrophysical Journal Letters* 467 (Aug., 1996) L93.

[60] D. R. Lorimer, K. M. Xilouris, A. S. Fruchter, I. H. Stairs, F. Camilo, A. M. Vazquez, J. A. Eder, M. A. McLaughlin, M. S. E. Roberts, J. W. T. Hessels, and S. M. Ransom, “Discovery of 10 pulsars in an Arecibo drift-scan survey,” *Monthly Notices of the Royal Astronomical Society* 359 no. 4, (June, 2005) 1524–1530, arXiv:astro-ph/0504019.

[61] S. E. Thorsett and D. Chakrabarty, “Neutron Star Mass Measurements. I. Radio Pulsars,” *The Astrophysical Journal* 512 (Feb., 1999) 288–299, arXiv:astro-ph/9803260.

[62] I. H. Stairs, “Pulsars in Binary Systems: Probing Binary Stellar Evolution and General Relativity,” *Science* 304 (Apr., 2004) 547–552.

[63] D. R. Lorimer, “Binary and Millisecond Pulsars,” *Living Reviews in Relativity* 8 (Nov., 2005) 7, arXiv:astro-ph/0511258.