Dual Superconductivity from Yang-Mills Theory via Connection Decomposition

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Abstract

We derive an Abelian-Higgs-like action from $SU(2)$ Yang-Mills theory via monopole-condensation assumption. Abelian projection as well as chromo-‘electric-magnetic’ duality are naturally realized by separating the small off-diagonal gluon part from diagonal gluon field according to the order of inverse coupling constant $(1/g)$. It is shown that Abelian dominance can follow from infrared behavior of running coupling constant and the mass generation of chromo-electric field as well as off-diagonal gluon is due to the quantum fluctuation of orientation of Abelian direction. Dual superconductivity of theory vacuum is confirmed by deriving dual London equation for chromo-electronic field.

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1 Introduction

Although quantum chromodynamics(QCD) has been recognized to be the standard theory of strong interactions, it fails to apply perturbatively in low-energy regime where the effective coupling constant is expected to be large and nonperturbative methods are needed. Till now, lattice QCD seems to be the only effective approach to low-energy(nonperturbative) aspects of QCD including the quark confinement[1], except for various phenomenological methods or semi-phenomenological methods (such as QCD sum rules[2]). Recently, an appealing proposal[3] was...
made in Yang-Mills (YM) theory by Faddeev and Niemi so as to separate collective infrared variables from gauge field degrees of freedom via decomposing gauge connection into an Abelian part, a unit color-vector $n$ as well as dual variables, which manifests the pure YM theory as an effective Abelian theory with duality structure between chromo-electric and chromo-magnetic field. This proposal is also called Faddeev-Niemi decomposition (FND). Owing to absence of quark sources, this proposal was usually adopted in revealing the knotted-vortex structure of QCD vacuum [4] or the gluonball spectrum [5], rather than the dual-superconductor-oriented confining mechanism.

On the other hand, the close link of FND to the Abelian Projection (AP) proposed by 't Hooft [6], and the underlying dual structure inherent in FND made it very suggestive and remarkable, and motivate ones to inquire how FND bears on the dual-superconductor picture [7, 8] of quark confinement as well as the supposed monopole condensation. As shown by lattice simulations [9, 10] in the maximal Abelian gauge (MAG), dual Abelian dynamics of QCD dominates in infrared regime, where monopole degrees of freedom forms a condensate responsible for the needed dual Meissner effect. Furthermore, the lattice simulations on center vortices and monopoles (see [11, 12] for a review) revive the interests of continuum-theory analysis [15, 16, 17] of nonperturbative QCD.

In this Letter we present a calculation procedure for effective Abelian-Higgs-like action from $SU(2)$ Yang-Mills theory. This action is derived by reformulating YM theory via FND of the gluon field. In this reformation, Abelian projection as well as chromo-‘electric-magnetic’ duality are naturally realized by separating the small off-diagonal gluon part from diagonal gluon field according to the order of inverse coupling constant $(1/g)$. It is shown that Abelian dominance in confining regime of low-energy QCD can follow from infrared behavior of running coupling-constant. To second order of $1/g$, we confirm dual superconductivity of theory vacuum by deriving dual London equation for chromo-electric field.

2 Connection decomposition and Abelian projection

QCD has a crucial feature that it almost has no free parameter and asymptotic freedom. If we ignore quark energy we left with pure YM interaction energy in which only one parameter, namely, the coupling constant $(g)$, is available. This can be a good approximation to QCD in low-energy limit and can find its analogy in the quantum theory of ultra-cold trapped atomic gas [18] where kinetic energy of the gas system is, via Thomas-Fermi approximation, neglected in contrast with inter-atomic interaction in zero-temperature limit. We note that this analogy seems also work for weak-interaction case of them as we know that quarks with high energy tend to freedom and the particles in gas are almost free at high-temperature. This makes the link of FND to the AP for QCD confinement, the dual structure of reformulated YM theory
to dual-superconductor vacuum of QCD physically relevant. Motivated by these link, we study
dual-superconductor picture of QCD from the viewpoint of FND-based reformulation of YM
theory.

We begin with $SU(2)$ YM theory where (gluon field) connection $A_\mu = A_\mu^a \tau^a \ (\tau^a = \sigma^a/2, a = 1, 2, 3)$ describes 6 transverse UV degrees of freedom. We use inner product $\tau^a \cdot \tau^b \equiv 2\text{Tr}(\tau^a \tau^b) = \delta^{ab}$, $A \cdot B \equiv A^a B^a$, and across product $A \times B = -i[A, B]$ for short. To parameterize $A_\mu$ in terms
of monopole variables, we invoke the infrared 'magnetic' variable $n(=n^a \tau^a)$, an unit vector in
internal(color) space$[19]$. This vector naturally provides an preferred direction, breaking $SU(2)$
to $U(1)$ by leaving residual $U(1)$ symmetry (rotation around $n$) intact, as required by AP.

Solving $A_\mu$ from $D_\mu n - \partial_\mu n = g A_\mu \times n$, where $g$ is coupling constant, one gets

$$A_\mu = A_\mu n + g^{-1} \partial_\mu n \times n + b_\mu$$  \hspace{1cm} (1)

where $A_\mu \equiv A_\mu \cdot n$ transforms as an Abelian connection($A_\mu \rightarrow A_\mu + \partial_\mu \alpha / g$), for $U(1)$ rotation $U(\alpha) = e^{i \alpha \cdot \gamma \cdot \sigma / 2}$ (the rotation around the direction $n$) and $b_\mu = g^{-1} n \times D_\mu (A_\mu) n$ is $SU(2)$
covariant. Here, the first part $A_\mu n$ in RHS of (1), being the diagonal part of gluon field, corresponds to Abelian subgroup $H = U(1)$ while the second and third terms, both of which are orthogonal to $n$ and being off-diagonal gluon parts(or non-Abelian components of gluon field), correspond to non-Abelian group orbit $SU(2)/H$. We note that (1) can be true variable change$[20]$ if one takes $b_\mu$ itself as a gauge vector field and further imposes two constraints on $b_\mu$. This is necessary for getting marginal contribution to the final effective action which we do not consider in this paper.

Note that the second term in RHS of (1) does not depend upon the original degrees of
freedom $A_\mu$, which implies $A_\mu$ may has intrinsic structure, that is, it may serves as monopoles
in some of its components. This idea is due to Duan’s work on multi-monopoles$[19]$ and has
applied to non-Abelian instantons due to defect$[21]$. To find all relevant variables, we further
decompose $b_\mu$ in terms of $n$. Observed that the internal orbit space $SU(2)/H$ can be spanned
by basis $\partial_\mu n$ and $\partial_\mu n \times n$, one can re-parameterize $b_\mu$ as

$$b_\mu = g^{-1} \rho \partial_\mu n + g^{-1} \sigma \partial_\mu n \times n.$$ \hspace{1cm} (2)

Here, the scalar fields $\rho$ and $\sigma$ can be combined to define a complex variable $\phi = \rho + i \sigma$. Substituting (2) into (1) we get the FND$[3]$ for $SU(2)$ connection

$$A_\mu = A_\mu n + g^{-1} \partial_\mu n \times n + g^{-1} \rho \partial_\mu n + g^{-1} \sigma \partial_\mu n \times n,$$ \hspace{1cm} (3)

in which $A_\mu$ (and $A_\mu$) has dimension of [mass], $n$, $\rho$ and $\sigma$ of unit.

We need to know the transformation role of all new variables under the residual symmetry
$U(\alpha)$. The transformation role of $\rho$ and $\sigma$ can be given by covariance of $b_\mu$ under the rotation
\( U(\alpha) \). Noticing that \( [\partial_\mu, n, e^{-i\alpha n}] = \alpha \partial_\mu n \times n, [\partial_\mu n \times n, e^{-i\alpha n}] = -\alpha \partial_\mu n \), one finds

\[
\mathbf{b}_\mu^U = g^{-1}e^{i\alpha n}(\rho \partial_\mu n + \sigma \partial_\mu n \times n)e^{-i\alpha n},
\]

\[
= g^{-1}(\rho - \alpha \sigma)\partial_\mu n + g^{-1}(\sigma + \alpha \rho)\partial_\mu n \times n,
\]

which implies \( \delta \rho = -\alpha \sigma \) and \( \delta \sigma = \alpha \rho \), or

\[
\delta(\rho + i\sigma) = i\alpha(\rho + i\sigma).
\]

Thus, the complex variables \( \phi \) indeed transforms as a charged scalar:

\[
\phi \rightarrow \phi e^{i\alpha}.
\]

However, it can be seen that the second term in RHS of (3) is not \( U(1) \) covariant. In fact, this term has the form of non-Abelian monopole potential[3].

Since the connection \( A_\mu \) has 12 field components while the RHS of (3) has 8 degrees of freedom, corresponding to 4 components of \( A_\mu \), 2 independent components of \( n^a \) and 2 components \( (\rho, \sigma) \), the new variables \( (A_\mu, n^a, \phi) \) are still short of 4 degrees for variable change (3). If one would further fixes 2 longitudinal components of \( U(1) \) connection \( A_\mu \), the resulted 6 degrees of freedom of new variables corresponds to fully gauge-fixed degrees(6 on-shell polarization components) of original gluon field \( A_\mu \). Therefore, FND (3) with localized variable \( n(x) \) serves as one example of partial gauge fixing used in AP.

According to the original idea of ’t Hooft[6], Abelian projection is realized by fixing the non-Abelian part of the gauge ambiguity, breaking full gauge symmetry(that is \( SU(2) \) here) into that of maximal Abelian subgroup(\( U(1) \) here). The singularities in gauge condition lead to difference between two group manifolds and were interpreted as magnetic monopoles in the projected \( U(1) \) gauge theory. Notice that (3) eliminates 4 degrees and it does not obey full gauge transformation law, for instance, it fails to transform as connection under gauge rotation around the direction different from \( n \), we know that (3) corresponds to the singular gauge in which \( A_\mu n \) is the un-fixed diagonal variable and the other off-diagonal terms are non-Abelian components whose degrees of freedom have been reduced.

### 3 Chromo-’Electron-magnetic’ duality and Abelian dominance

Corresponding to gauge symmetry breaking \( SU(2) \rightarrow U(1) \), we assume the physical vacuum of infrared theory forms the monopole condensate, sharing the residual symmetry of \( U(1) \) rotation around direction \( n \). To show Abelian dominance and dual structure of infrared YM theory, we use the well-justified energy-dependence of effective coupling constant: \( g_s^2 \sim 1/B \log(Q^2/\Lambda^2) \), where \( B = -\beta_0 > 0 \) is the negative \( \beta \)-function of QCD[13] at loop level. That means, being converse limit of QCD asymptotic freedom, the effective coupling \( g_s \) becomes sufficiently large in
low-energy \( (Q) \) limit (or infrared limit). For simplicity, we use to \( g \) denote the effective coupling hereafter.

With (3), one finds the projection of non-Abelian gauge field \( G_{\mu\nu} \) along \( n \) to be

\[
G_{\mu\nu} \cdot n = F_{\mu\nu} + B_{\mu\nu} + g^{-1} n \cdot (D_\mu n \times D_\nu n),
\]

where \( F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \) and

\[
B_{\mu\nu} \equiv -g^{-1} n \cdot (\partial_\mu n \times \partial_\nu n)
\]

stands for the chromo-electric and chromo-magnetic field strengths, respectively. One can identify magnetic potential \( C_\mu \) by \( B_{\mu\nu} \equiv \partial_\mu C_\nu - \partial_\nu C_\mu \), where parametrization of \( C_\mu \) can not be given in single-valued way. One can calculate the magnetic charge \( G_m \) by surface integral:

\[
G_m = \int_{V(3)} k_0 d^3 \sigma^0
\]

where \( k_0 \) is the time component of the magnetic current

\[
k_\mu = \partial_\nu B_{\mu\nu}, B_{\mu\nu} = \frac{\epsilon^{\mu\nu\rho\lambda}}{2} B_{\rho\lambda}.
\]

With (3) and (6), one can get

\[
G_m = \int_{V(3)} \frac{\epsilon^{\mu\nu\rho\lambda}}{2} \partial_\nu B_{\rho\lambda} d^3 x = -\frac{1}{g} \int_{V(3)} \frac{\epsilon^{\mu\nu\rho\lambda}}{2} \partial_\rho n \cdot (\partial_\mu n \times \partial_\lambda n) d^3 x = -\frac{4\pi}{g} \int_{V(3)} \frac{\epsilon^{\nu\rho\lambda}}{8\pi} \partial_\rho n^a \partial_\mu n^b \partial_\lambda n^c d^3 x,
\]

The computation of this integration gives

\[
G_m = -\frac{4\pi}{g} \int_{V(3)} \delta(\Phi) J(\Phi/x) d^3 x = -\sum_i \frac{4\pi w_i(n)}{g},
\]

in which \( \Phi = (\Phi^1, \Phi^2, \Phi^3) \) is defined as a vector field along \( n \), i.e., \( n^a = \Phi^a/\|\Phi\| \) and \( w_i(n) \) is the winding number (topological charge) of \( \Phi \) around its \( i \)-th singularity, with sign determined by that of spacial Jacobian function \( J(\Phi/x) \). Since \( G_m \) stands for the total magnetic charge in \( V(3) \), one can write

\[
G_m = -\sum_i g^{(i)}; g^{(i)} = \frac{4\pi w_i(n)}{g},
\]

where \( g^{(i)} \) is the magnetic charge in \( i \)-th region \( V^{(i)} \).
Let us parameterize \( \mathbf{n} \) in terms of spherical coordinates as \( \mathbf{n}_0 = (\sin \gamma \cos \beta, \sin \gamma \sin \beta, \cos \gamma) \). One has \( C_\mu = g^{-1}(\cos \gamma \partial_\mu \beta + \partial_\mu \alpha) \), which has a degree of freedom of \( U(1) \) gauge transformation \((C_\mu \to C_\mu + g^{-1} \partial_\mu \alpha)\). This \( U(1) \) symmetry is happened to hold simultaneously for Abelian part \( A_\mu \). To see more specifically the link of FND to AP, let us take \( \mathbf{n}_0 \) along \( \sigma^3 \) and write general gauge rotation as \( U = e^{-\sigma^3 \alpha}e^{-\sigma^2 \gamma}e^{-\sigma^3 \beta} \). Choosing a preferred direction \( \mathbf{n}_0 \) at each point implies we keep the partial symmetry under rotation \( e^{-\sigma^3 \alpha} \), with \( e^{-\sigma^2 \gamma}e^{-\sigma^3 \beta} \) symmetry broken (with \( \gamma \) and \( \beta \) fixed). That means, \( A_\mu = A_\mu \cdot \mathbf{n}_0 \) and \( C_\mu \) are both defined up to rotation \( e^{-\sigma^3 \alpha} (\alpha \to \alpha + \alpha_0) \). Therefore, AP responds, in \( SU(2) \) case, to assigning specific direction \( \mathbf{n}(x) \) at each spacetime point \( x \) in FND (3).

To see Abelian dominance and duality in infrared YM theory specifically, we take infinity limit of coupling \( g \). First, we note that as a physical field, the field exhibiting 'electron-magnetic' duality in color space should be gauge-invariant and such a field tensor has ever been given by the 't Hooft [14],

\[
f_{\mu \nu} = G_{\mu \nu} \cdot \mathbf{n} - g^{-1} \mathbf{n} \cdot (D_\mu \mathbf{n} \times D_\nu \mathbf{n}) = F_{\mu \nu} + B_{\mu \nu}.
\]

in which 'electric' and 'magnetic' field are put in the equal foots. The total flux for the 't Hooft tensor \( f_{\mu \nu} \) is

\[
\mathcal{L}_{\text{Flux}} = \frac{1}{2} \left( F_{\mu \nu} + B_{\mu \nu} \right) dx^\mu \wedge dx^\nu = \oint_{\partial \Sigma} A_\mu dx^\mu + \int_{V(\Sigma)} \partial_\nu B_{\mu \nu} d^3 \sigma^\mu = \Phi_e(V(\Sigma)) + G_m(V(\Sigma)),
\]

where (6) was used. This duality is perfect and well established in non-Abelian gauge theory with Higgs field, to which the internal direction for AP is oriented to [14].

In contrast, our reformulated theory appears as an Abelian gauge theory in effective media of off-diagonal gluons, which correct the perfect Maxwell theory with monopoles by a media-factor for 'magnetic' field \( B_{\mu \nu} \). This can be shown by calculating the non-Abelian gauge field \( G_{\mu \nu} \) in large-\( g \) limit (IR limit). To the first order of \( g^{-1} \), one finds

\[
G_{\mu \nu} \to \mathbf{n}[F_{\mu \nu} + (1 - \rho^2 - \sigma^2 - 2\sigma)B_{\mu \nu}],
\]

which means the dominant part of gluon lines is distributed along Abelian component. This transforms the standard YM theory into an Abelian gauge theory with 'electric-magnetic' duality \((A_\mu \leftrightarrow C_\mu)\)

\[
\mathcal{L}_{\text{dual}} = \frac{1}{4} (F_{\mu \nu} + H_{\mu \nu})^2, \quad H_{\mu \nu} = Z(\phi)B_{\mu \nu}, Z(\phi) = (1 - |\phi|^2 - 2 \text{Im } \phi).
\]
We see that the infrared approximation of YM theory become the diagonal one with \(N - 1 = 2\) types of Abelian charges, with magnetic charge dressed from the off-diagonal variable \(\phi\). When sources included, (10) is dual to the QED with magnetic monopoles with electric charge vacuum-polarized and the ‘electric-magnetic’ duality takes the form

\[
F_{\mu\nu} \leftrightarrow B_{\mu\nu}; Z(\phi) \leftrightarrow 1/Z(\phi), g \leftrightarrow 1/g.
\] (12)

From (10) and (11) we see that due to the contribution of off-diagonal gluons the duality otherwise manifested by ’t Hooft tensor \(f_{\mu\nu}\) in infrared regime was replaced in our reformulated theory by that with effective magnetic media-factor \(Z(\phi)\). We also see that the duality between the chromo-electric and chromo-magnetic field(EM) strongly depends upon the Abelian dominance in the sense that the duality becomes exact as Abelian gluon part is getting dominant.

Breaking of \(SU(2) \rightarrow U(1)\), or fixing of the direction of \(n\) (quantum operator) at each point \(x\) makes it acquire nonvanishing vacuum expectation value(vev.), \(\langle n^a(x) \rangle = n^a(x)\) (c-number field), and

\[
\langle \partial^\mu n^a(x) \partial_\nu n^a(x) \rangle = \delta_\mu^\nu \langle (\partial n^a)^2 \rangle = -\delta_\mu^\nu m^2
\] (13)
in which \(m\) is a mass scale and the minus sign comes from the fact \(\partial_\mu n\) is space-like for our static case. Here, \(\delta_\mu^\nu\) arises from the requirement of Lorentz invariance of vev. and \(m^2\) is due to that \(\partial_\mu n^a\) may has a normalized factor (quantum fluctuation of direction of \(n\)) and it has dimension of [Mass]. We can interpret such a behavior as the particles associated with \(n\)-field undergo condensation in theory vacuum, or in other words, the \(S^2\) symmetry (rotation of \(n\)-orientation) of the original theory was broken by the QCD vacuum. One then has

\[
\langle C_\mu^2 \rangle = g^{-2} \langle (\partial n)^2 \rangle = -g^{-2} m^2
\]
\[
\langle B_{\mu\nu} \rangle = 0
\]
since \(B\) is anti-symmetric. Furthermore, the vev.’s of all field components with Lorentz indices or color indices explicitly, such as \(\langle A_\mu \rangle\), \(\langle C_\mu \rangle\) and \(\langle \partial_\mu n^a \rangle\), vanish since these vev.’s become physical in condensate and thereby they are Lorentz invariant and gauge invariant.

Taking these consideration into account, one finds

\[
\langle \mathcal{L}_{\text{dual}} \rangle = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} \lambda Z(\phi)^2,
\] (14)

where \(\lambda \equiv \langle B_{\mu\nu}^2 \rangle\) is positive scale and with dimension of 4. One can see that the duality survives in purely-diagonal gluon dynamics (14) but dual-superconductivity does not since this dynamics is short of crucial ingredient, kinetic term of \(\phi\) field, which makes (14) into an effective superconductor model—Abelian Higgs model. In the next section, we will find such a ingredient could present when we include the contribution of off-diagonal gluon.
3.1 Abelian Higgs action in terms of collective dual variables

As a relativistic generalization of effective superconductor model—the Ginzburg-Landau model, the Abelian Higgs model has long been proposed to describe the confining phase of QCD, in which the string-like singularities provide the confining forces between field sources [7, 8]. In addition to the pure duality analysis via complete Abelian dominance, given in last section, we here include the off-diagonal gluon contribution to the order of $g^{-2}$.

With (3), one finds

$$
G_{\mu\nu} = n[F_{\mu\nu} + (1 - \rho^2 - \sigma^2 - 2\sigma)B_{\mu\nu}] +
(g^{-1}\nabla_\mu \rho + 2A_\mu)\partial_\nu n - (g^{-1}\nabla_\nu \rho + 2A_\nu)\partial_\mu n
+ g^{-1}\nabla_\mu \sigma \partial_\nu n \times n - g^{-1}\nabla_\nu \sigma \partial_\mu n \times n,
$$

(15)

where $n_{\mu\nu} = \delta_{\mu\nu}(\partial_\rho n)^2 - \partial_\mu n \cdot \partial_\nu n$, $\nabla_\mu \rho = \partial_\mu \rho + gA_\mu \sigma$ and $\nabla_\mu \sigma = \partial_\mu \sigma - gA_\mu \rho$. With (15), one gets

$$
\mathcal{L}_{\text{dual}} = -\frac{1}{4}F_{\mu\nu}^2 + (1 - \rho^2 - \sigma^2 - 2\sigma)^2 B_{\mu\nu}^2 +
2(1 - \rho^2 - \sigma^2 - 2\sigma)F_{\mu\nu}B^{\mu\nu}
+ \frac{2n_{\mu\nu}}{g^2}(\nabla^\mu \rho + 2gA^\mu)(\nabla^\nu \rho + 2gA^\nu)
+ \frac{2n_{\mu\nu}}{g^2}\nabla^\mu \sigma \nabla^\nu \sigma.
$$

(16)

It is useful to define a $U(1)$ covariant derivative

$$
\nabla_\mu \phi = \nabla_\mu \rho + i\nabla_\mu \sigma
= (\partial_\mu - igA_\mu)\phi.
$$

Here, we look $\phi$ as a charged field strongly coupled with 'electric' field $A_\mu$ by strength $g$. Then, by averaging (16) over $n$ with

$$
\langle n_{\mu\nu}^a \rangle = \delta_{\mu\nu}(\langle n^a \rangle^2) = -\delta_{\mu\nu}m^2, \quad (17)
$$

one can get the effective Lagrangian

$$
\mathcal{L}^{\text{eff}} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{m^2}{2g^2}|\nabla_\mu \phi|^2 + 2m^2 A_\mu^2
+ \frac{2m^2}{g}A^\mu \text{Re}(\nabla_\mu \phi) - V(\phi),
$$

(18)

which is Abelian-Higgs like model and

$$
V(\phi) = \frac{\lambda}{4}(|\phi|^2 - 1 + 2 \text{Im} \phi)^2.
$$
Here, \( \lambda \) can be shown to be

\[
\lambda = \frac{2m^4}{g^2}.
\]  

(19)

In fact, from (13) and invariance of the vev., one has

\[
\langle \widetilde{B}_{\mu \nu}^2 \rangle = g^{-2} \epsilon_{\alpha \beta \gamma \delta} \epsilon_{\mu \nu \lambda \rho} \delta^{\lambda \rho} \langle n^\alpha \rangle \langle n^\beta \rangle \langle n^\gamma \rangle \langle n^\delta \rangle \langle n^\mu \rangle \langle n^\nu \rangle \langle n^\lambda \rangle \langle n^\rho \rangle = g^{-2} \epsilon_{\alpha \beta \gamma \delta} \epsilon_{\mu \nu \lambda \rho} \delta^{\lambda \rho} \langle n^\alpha \rangle \langle n^\beta \rangle \delta_{\gamma \delta} \delta_{\mu \nu} \langle n^\gamma \rangle \langle n^\delta \rangle \langle n^\beta \rangle \langle n^\gamma \rangle = g^{-2} \epsilon_{\alpha \beta \gamma \delta} \epsilon_{\mu \nu \lambda \rho} \delta^{\lambda \rho} \langle n^\alpha \rangle \langle n^\beta \rangle m^4 = 2m^4.
\]

The effective Lagrangian becomes

\[
L_{\text{eff}} = -\frac{1}{4} F_{\mu \nu}^2 + \frac{m^2}{2g^2} |\nabla_\mu \phi|^2 - V(\phi) + 2m^2 (1 + \text{Im} \phi) A_{\mu}^2 + \frac{2m^2}{g} A_{\mu} \text{Re}(\partial_{\mu} \phi),
\]

(20)

We see here that, as a consequence of n-field condensation, not only do the off-diagonal gluons gain mass but also has Abelian gluon field \( A_{\mu} \) acquired a mass \( \sim m \). We obtain massive gluon in our effective theory (20) without invoking the Higgs-like spontaneous symmetry-breaking (SSB) mechanism as done in superconductor theory or dual superconductor picture for confinement. In contrast, it is due to SSB of a color direction field \( n(x) \) in the nontrivial QCD vacuum, provided that magnetic-charges Bose condensed. The lattice simulation[10, 22] has confirmed magnetic-charge condensation in MAG. For \( SU(2) \) theory, it is easy to see that change of variables (3) corresponds to MAG, since \( U(1) \) rotation around \( n \) is maximal Abelian subgroup of \( SU(2) \). Therefore, our effective theory (20) has the key feature of dual superconductor and in this theory desired mass generates from the off-gluon field \( n \). In fact, our calculation from (17) to (20) shows that such a mass generation arises from quantum fluctuation of orientation \( n \), see (17) and (19).

We note that the marginal terms have not included in (18) since new variables \( 3 \) are on-shell degrees of freedom. The marginal-term inclusion can be done by using off-shell field decomposition[4] and then calculating the effective action through quantum partition functional \( Z \sim \int [dn^a] e^{-iS} \). Toward the leading infrared term, however, our model is sufficient for the effective description of low-energy YM theory (18).

### 3.2 Dual London equation

To see the relation between the AP and mass generation of Abelian gluon field, we need dual Meissner effect as a possible signal of the monopole condensation. Here, we show that the
effective model of QCD can yield, in low-energy limit, a dual London equation for Abelized fields. Varying (20) gives

\[ \partial_\mu F^{\mu\nu} = \frac{m^2}{2g^2} [i\phi^* \overleftrightarrow{\partial^\nu} \phi - 2 \text{Re} \partial^\nu \phi] - m^2 |\phi + 2|^2 \]

\[ \nabla_\mu \nabla^\mu \phi = - \frac{\partial V(\phi)}{\partial \phi^*} - \frac{m^2}{g^2} \partial^\mu A_\mu + i m^2 A_\mu^2, \text{ and c.c.,} \]

where

\[ \frac{\partial V(\phi)}{\partial \phi^*} = \frac{m^2}{g^2} |\phi|^2 - 1 - 2 \text{Im} \phi (\phi + i) \]

We see that the chromo-electric field \( A_\mu \) strongly coupled with the charged scalar field \( \phi \) with coupling \( g \) while \( \phi \) is weakly coupled to itself in the effective dynamics.

Taking the \( g \to \infty \) limit in (21) and using Lorenz gauge for \( A_\mu \), we find

\[ \phi \approx \phi_0 = -\frac{im^2}{g^2}, \text{ and c.c.,} \]

\[ \partial_\mu F^{\mu\nu} = j^\nu = -m^2 V A^\nu, \] (22)

in which the second equation (22) takes the form of London’s equation. Here

\[ m_V = m(4 + m^4/g^4)^{1/2} \approx 2m, \]

is the mass scale responsible for dual Meissner effect and its inverse \( \lambda_L = 1/m_V \) determines the transverse dimensions of the chromo-electric field \( A_\mu \) penetrating into the vacuum condensate. As in superconductor, (22) implies that chromo-electric field decays as

\[ A_\mu(d) = A_\mu(0) \exp(-d/\lambda_L) \]

as they depart from the singular vortex tube(string), where \( d \) stands for the distance away from string. This is consistent with the dual superconductor picture[7].

We note that similar argument for deriving an equation (22) was also given by Dzhunushaliev[15] via ordered Abelian components assumption and AP. It should be pointed out that in his approach the ’electron-magnetic’ duality needed (12) for dual-superconductivity is not exhibited explicitly and the link of the generation of the vector field mass \( m_V \) with the magnetic charge condensation is not clarified. We also note that the uniform assumption for scalar field \( \phi \) in Ref.[15] only follows in infrared limit, or large-\( g \) limit.

In conclusion, we calculated an effective Abelian-Higgs-like action based on the Faddeev-Niemi decomposition of \( SU(2) \) gauge field. Abelian projectional and chromo-’electric-magnetic’ duality are realized via the decomposition in which gluon fields are divided into \( U(1) \) diagonal part and small non-Abelian off-diagonal parts with order of inverse coupling-constant(1/g). We have shown that Abelian dominance can follow from infrared behavior of running coupling constant and the mass generation for chromo-electric field as well as off-diagonal gluon can arise
from quantum fluctuation of orientation of the unit iso-vector \( \mathbf{n} \). Furthermore, we have derived a dual London equation for chromo-electric field. This enhances the dual superconductor picture as the possible mechanism of quark confinement.

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