Radiative peristaltic transport of Ree-Eyring fluid through porous medium in asymmetric channel subjected to combined effect of inclined MHD and convective conditions

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Abstract. This study emphasizes the flow phenomenon of trapped bolus traveling along the interior walls of asymmetric inclined channels contains a non-Newtonian Ree-Eyring. The flow was exposed to influenced by inclined MHD field, thermal heat radiative, and porous media. Further, no slip and convective thermal conditions are considered. Mathematical expression for governing equations are reformulated and in accordance with lubrication approximations, nonlinear partial differential equations of the flow reduced into a system of ordinary differential equations associated with boundary conditions an approximate solution is deduced by implementing perturbation strategy for tiny A Ree-Eyring fluid parameter. Finally, a graphical description is presented to figure out the elevation behavior of flow quantities i.e. velocity profile, temperature distribution, pressure rise, and streamlines formulation due to variation of emerging involved parameters. The study analyzed that the velocity profile reveals mixed behavior via increment of Ree-Eyring parameters $\eta, A$ as well as Hartman number $H$ and Darcy number $Da$. whereas the thermal radiative parameter $Rn$ accelerates the temperature distribution profile. The study calculations are made by the “Mathematica 11.3” package.

1. Introduction
Peristaltic pumping is a type of fluid motion that appears when a progressive wave of area clasping and compressing propagates along the wall of a distensible channel. Bolus development through the esophagus, movement of blood in small vessels, intrauterine fluid motion, lymph transport, and embryo motion through the uterine cavity are the biological applications of the peristaltic mechanism. Moreover, it is an inherent property of many modern industrial applicable such as the ceramic and porcelain industry, nuclear industry, pharmaceutical industry, and heart-lung machines [1]. After the first attempt on peristaltic transport given by Latham [2]. Fung and Yih [3] introduced the peristaltic mechanism in the laboratory frame. Nowadays many pieces of literature are available on peristalsis considering different fluids models and flow configurations see Refs. [4,5,6].

Incited by the fact that enormous fluids in nature have an immense impact in modern industry, science, and technology is non-Newtonian like paints, cosmetic products, colloidal fluids, certain oils, and shampoo known by their behavior cannot be predicted by a single constitutive equation [7,6]. Inspired by the aforementioned applicable literature survey, many researchers presented various models to visualize the non-Newtonian fluid models. Recently the most considerable non-Newtonian fluid
gained the attraction of many researchers due to its wide medical and scientific application. Furthermore, its constitutive equation is derived from the kinetic theory of liquids, not from empirical relations alike the power-law model, and it properly reduced into the Newtonian fluid model at high shear stress known by Ree-Eyring fluid [8,9]. Numerous scientific literature outlined the peristaltic transport of Ree-Eyring with different effects and flow situations in Refs. [10-12].

On other hand, a flow through magneto hydrodynamic (MHD) field has vigorous attention in recent years via its vastly applicable in metallurgy processes, engineering, geophysics, and industry. Plasma confinement, nuclear reactor, bleeding reduction during surgeries, stirring, crystal growth process, and MRI (magnetic resonance imaging) to diagnose the disease are several utilization of electrically fluid flows in the presence of (MHD) field [13]. In view of these diverse, many studied considering the peristaltic transport of MHD various fluid models are mentioned through the studies [7,14-16].

Thermal radiation can control the excess heat generation inside the body since high temperatures pose serious stresses for the human body which leads to injury or even death. This effect occurs in many physiological, technological, and industrial processes like plasma tissue heat conduction, MHD generators, laser surgery, destruction of cancer tumors, polymer processing industry, etc. However, the rate of heat or energy transfer between two bodies via thermal radiation essentially depends on the absolute temperature difference this means that the influence of thermal radiation on fluid flow has major coverage at high temperatures. Motivated by these developments some modern attempts have been devoted to investigating the influence of thermal radiation on the peristaltic mechanism for different non-Newtonian fluid. Nikiforov [17] elevate an electromagnetic hyperthermia technique in which the undesired cancer cells are exposed to a higher temperature field of more than 41 °C with aid of thermal radiation. In such a technique, a magnetic liquid is injected into the malignant tissues and then exposing the framework to an alternating current. So, the temperature is produced in the infused magnetic fluid and hence the cancer cells will be destroyed see also [14]. Hayat et al. [18] analyzed numerically heat transfer due to viscous dissipation and radiation on peristaltic flow of Sutter by fluid in a vertical channel. While Bhatti et al. [19] explored theoretically nonlinear thermal radiation impact on EMHD peristaltic propulsion of non-Newtonian fluid-particle (dusty) suspensions in a planar channel including a homogenous porous media. Naveed Imrana et al. [20] discuss the influence of heterogeneous-homogenous effect in the peristaltic flow of non-Newtonian Rabinowitsch fluid considering thermal radiation effects and viscous dissipation for more knowledge in this aspect see Refs.[21-26].

From the aforementioned survey, we conclude that no attempt has been made to examine the combined impact of heat radiative, and inclined (MHD) on peristaltic transport of Non-Newtonian Ree-Eyring fluid yet and the current study will fill this gap. In this article, we explore the peristaltic transport of Ree-Eyring fluid past through inclined asymmetric porous channel subjected to the combined influence of thermal radiation and inclined (MHD) field. Velocity no-slip and convective boundary conditions are taken into account. The flow is illustrated in the laboratory frame which is transformed into a wave frame. The flow mathematical system was simplified by considering the assumption of infinite wave length and low Reynolds number. Analytical solution for stream function and temperature distribution function is evaluated by implementing the regular perturbation method. The impact of impeded physical parameters on the flow quantities is discussed in detail through figures.

2. Mathematical Model
Reconsidering the peristaltic transport of an incompressible electrically conducting Ree-Eyring fluid in asymmetric an inclined channel at an angle \((\alpha_1)\) and width \((d_1 + d_2)\) through porous media see Figure 1. The flow is incident to inclined uniform magnetic field of strength \(\vec{B} = (\beta_0 \sin \alpha_2, \beta_0 \cos \alpha_2)\). The induced magnetic field is neglected by taken a small magnetic Reynolds number. The flow is propagating by the sinusoidal waves of length \(\lambda\) with different Reynolds number. The flow is moving with a constant speed \((c)\) along the walls of the channels.

The geometry of the walls surfaces is given by [4]
\[ Y_1 = H_1(X, t) = d_1 + a_1 \cos \left( \frac{2\pi(X - ct)}{\lambda} \right), \]
\[ Y_2 = H_2(X, t) = -d_2 - a_2 \cos \left( \frac{2\pi(X - ct)}{\lambda} + \phi \right). \]

Where \( Y_1, Y_2 \) are the upper and lower wall respectively, \( a_1, a_2 \) are the wave amplitudes, \( t \) is the time, and \((X, Y)\) the Cartesian coordinates in a fixed frame. \( \phi \) is the phase difference and \( \phi \in [0, \pi] \) such that when \( (\phi = 0) \) corresponds to symmetric channel with waves out of phase, while \( (\phi = \pi) \), the waves in phase. Further \( a_1, a_2, d_1, d_2 \) and \( \phi \) satisfy the condition:
\[ a_1^2 + a_2^2 + a_1 a_2 d_1 d_2 \cos \phi \leq (d_1 + d_2)^2, \]

**Figure 1.** The geometry of the Inclined Asymmetric Channel [4]

To calculate the Lorentz force applying the following Ohm’s formula [27] as below
\[ \vec{F} = \sigma (\vec{V} \times \vec{B}) \times \vec{B}, \]
we have
\[ \vec{F} = (-\sigma \beta_0^2 \cos \alpha_1 \overline{UCos} \alpha_1 - \overline{V} \sin \alpha_1, \sigma \beta_0^2 \sin \alpha_1 \overline{UCos} \alpha_1 - \overline{V} \sin \alpha_1). \]

\( \vec{F} \) is the magnetic force, \( \vec{j} \) is the current density vector, \( \overline{V} = (\overline{U}, \overline{V}, 0) \) the velocity field, \( \sigma \) the electrical conductivity.

The governing equations of continuity, motion, and energy can be constructed as:
\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho \overline{U}) + \frac{\partial}{\partial y} (\rho \overline{V}) = 0, \]
\[ \rho \left( \frac{\partial \overline{U}}{\partial t} + \overline{U} \frac{\partial \overline{U}}{\partial x} + \overline{V} \frac{\partial \overline{U}}{\partial y} \right) = -\frac{\partial \rho}{\partial x} + \frac{\partial \delta_{xx}}{\partial x} + \frac{\partial \delta_{xy}}{\partial y} - \sigma \beta_0^2 \cos \alpha_1 \overline{UCos} \alpha_1 - \overline{V} \sin \alpha_1 - \frac{\mu}{\kappa_0} \overline{U} + \rho g \cos \alpha_2, \]
\[ \rho \left( \frac{\partial \overline{V}}{\partial t} + \overline{U} \frac{\partial \overline{V}}{\partial x} + \overline{V} \frac{\partial \overline{V}}{\partial y} \right) = -\frac{\partial P}{\partial y} + \frac{\partial \delta_{yy}}{\partial y} + \frac{\partial \delta_{xy}}{\partial y} + \sigma \beta_0^2 \sin \alpha_1 \overline{UCos} \alpha_1 - \overline{V} \sin \alpha_1 + \frac{\mu}{\kappa_0} \overline{V} - \rho g \cos \alpha_2, \]
\[ \rho c_p \left( \frac{\partial T}{\partial t} + \overline{U} \frac{\partial T}{\partial x} + \overline{V} \frac{\partial T}{\partial y} \right) = K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \delta_{xx} \frac{\partial \overline{U}}{\partial x} + \delta_{yy} \frac{\partial \overline{V}}{\partial y} + \delta_{xy} \left( \frac{\partial \overline{U}}{\partial y} + \frac{\partial \overline{V}}{\partial x} \right) - \frac{\sigma \beta_0^2 (\overline{UCos} \alpha_1 - \overline{V} \sin \alpha_1)}{2}. \]

Where \( \sigma \) is the electrical conductivity, \( K \) is thermal conductivity, \( \kappa_0 \) positon parameter, \( \rho \) density, \( c_p \) specific heat, \( T \) is the temperature vector, \( P \) the pressure.

Using Rosseland approximation for radiation, the radiative heat flux \( q_r \) is given by
\[ q_r = -\frac{4 \sigma_1 \theta^4}{3 k_1 \frac{\partial \theta}{\partial y}}, \]

\( \sigma_1 \) is the Stefan-Boltzman constant and \( k_1 \) is the mean absorption coefficient. By assuming a sufficiently small temperature difference in the flow, the term the Taylor series approximation for \( T^4 \) in terms of \( T_0 \) take the following form
placement of equation (10) and equation (11) into equation (9), the energy equation will take the form

$$
\rho c_p \left( \frac{\partial T}{\partial t} + \bar{U} \frac{\partial T}{\partial x} + \bar{V} \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial \bar{u}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\rho} \frac{\partial \bar{v}}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\rho} \frac{\partial \bar{v}}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial \bar{u}}{\partial x} \right) + \frac{16 \sigma^2}{3 \kappa_i} \frac{\partial T}{\partial y} + \sigma \beta_0^2 (\bar{U} \cos \alpha_1 - \bar{V} \sin \alpha_1)^2,
$$

(12)

With the boundary conditions

$$
\bar{U} = 0, \quad \psi = \frac{\bar{p}}{2}, \quad K \frac{\partial T}{\partial y} = -\beta_1(T - T_0), \text{at}\ Y = H_1
$$

$$
\bar{U} = 0, \quad \psi = -\frac{\bar{p}}{2}, \quad K \frac{\partial T}{\partial y} = -\beta_1(T - T_1), \text{at}\ Y = H_2
$$

(13)

The stress tensor for Re-Cryst fluid is given as follows [28]:

$$
\tilde{S}_{ij} = \mu \frac{\partial \bar{v}_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1} \left( \frac{1}{\beta} \frac{\partial \bar{v}_i}{\partial x_j} \right), \; i, j = 1, 2
$$

(14)

$$
\sinh^{-1} \left( \frac{1}{\beta} \frac{\partial \bar{v}_i}{\partial x_j} \right) \approx \frac{1}{6} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3,
$$

(15)

Then

$$
\tilde{S}_{ij} = \mu \frac{\partial \bar{v}_i}{\partial x_j} + \frac{1}{\beta} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3 - \frac{1}{6 \beta c_1^2} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3
$$

(16)

Where \( \tilde{V}_1 = \bar{U}, \tilde{V}_2 = \bar{V}, x_1 = X, x_2 = Y \).

\( \tilde{S}_{ij} \) represents the extra stress tensor, \( \beta, c_1 \) are the fluid parameter, and \( \mu \) is the dynamic viscosity.

In view of equation (16), the laboratory frame components of extra stress tensor become

$$
\tilde{S}_{XX} = \mu \frac{\partial \bar{v}_i}{\partial x_j} + \frac{1}{\beta} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3 - \frac{1}{6 \beta c_1^2} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3
$$

$$
\tilde{S}_{XY} = \mu \frac{\partial \bar{v}_i}{\partial x_j} + \frac{1}{\beta} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3 - \frac{1}{6 \beta c_1^2} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3
$$

$$
\tilde{S}_{YY} = \mu \frac{\partial \bar{v}_i}{\partial x_j} + \frac{1}{\beta} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3 - \frac{1}{6 \beta c_1^2} \left( \frac{\partial \bar{v}_i}{\partial x_j} \right)^3
$$

(17)

By the following transformations, we furnished the relationship between laboratory frame \((X, Y)\) and wave frame \((\bar{x}, \bar{y})\) coordinates

$$
\bar{x} = X - c t, \bar{y} = Y, \tilde{u} = \bar{U} - c, \tilde{v} = \bar{V}, \tilde{p} = \tilde{p}(X,Y,t).
$$

(18)

Employing the above equation, we transform equations (1), (2) and (6) - (17) in wave frame and then normalize the resulting equations by considering the following dimensionless quantities [25,27]

$$
x = \frac{\bar{x}}{\lambda}, y = \frac{\bar{y}}{\lambda}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c}, h_1 = \frac{h_1}{d_2}, h_2 = \frac{h_2}{d_2}, p = \frac{d_2}{\lambda}, \delta = \frac{d_2}{\lambda}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c}, S = \frac{d_2 S(x)}{\mu c}, Re = \frac{\rho c d_2}{\mu}, \theta = \frac{T - T_0}{(T_1 - T_0)}, Fr = \frac{\mu c p}{k}, H = \frac{\beta_0 d_2}{\mu}, E_c = \frac{c^2}{\mu}, Br = \frac{E_c p}{\mu}, Fr = \frac{c}{\sqrt{\beta d_2}}
$$

(19)

Where \( Re \) is Renolds number, \( \delta \) wave number, \( Pr \) Prandtl number, \( H \) Hartman number, \( E_c \) Eckret number, \( Br \) Brinkman number, \( Fr \) Froude number, \( Da \) Darcy number, \( Rn \) is the thermal radiation parameter, \( \eta, A \) are the Re- Eyring fluid parameters, \( \omega_1, a \) are the heat Biot number \( a, b \) the amplitude ratio of the upper and lower wall, and \( \psi(x, y, t) \) is the stream function.

Using the assumptions of long wavelength and low Renolds number approximation and equation (19), the flow equations reduced and simplified to the forms

$$
Re \frac{\delta (\psi_y \psi_{xy} - \psi_x \psi_{yy})}{Fr} = -p_x + \frac{\delta}{\partial x} S_{xx} + \frac{\delta}{\partial y} S_{xy} - H^2 \cos \alpha_1 \psi_x \cos \alpha_1 - \delta \psi_x \sin \alpha_1 - \frac{1}{\partial a} \psi_y + \frac{Re}{Fr} \sin \alpha_2.
$$

(20)
\[ Re \, \delta^3 ( - \psi_y \psi_{xx} - \psi_x \psi_{xy} ) = -p_y + \frac{\delta^2}{\delta \alpha} S_{xx} + \frac{\delta}{\delta \beta} S_{xy} - \delta H^2 \sin(\psi_y \cos \alpha_1 + \delta \psi_x \sin \alpha_1) - \frac{\delta}{\delta a} \psi_x - \frac{\delta}{\delta Fr} \cos \alpha_2. \] (21)

\[ Pr \, Re \, \delta ( \psi_y \theta_x + \psi_x \theta_y ) = \left( \delta^2 \theta_{xx} + \theta_{yy} \right) + Pr \, Re \theta_{yy} + \delta S_{xx} \psi_{yx} + S_{xy} \left( -\delta \psi_{xy} + \psi_{yy} \right) - \delta S_{yy} \psi_{xy} + H^2 Br (\psi_y \cos \alpha_1 + \delta \psi_x \sin \alpha_1)^2. \] (22)

By applying \( Re \ll 0, \delta \to 0 \) we finally get

\[ 0 = -p_x + \frac{\delta}{\delta y} S_{xy} - H^2 \psi_y (\cos \alpha_1)^2 - \frac{1}{\delta a} \psi_y + \frac{\delta}{\delta Fr} \sin \alpha_2, \] (23)

\[ 0 = -p_y, \] (24)

\[ (1 + Pr \, Re) \theta_{yy} + S_{xy} \psi_{yy} + H^2 Br (\psi_y \cos \alpha_1)^2 = 0. \] (25)

Eliminating \( p \) from equations (23) and (24), we have the following equation

\[ 0 = \frac{\delta^2}{\delta y^2} S_{xy} - H^2 \psi_y (\cos \alpha_1)^2 - \frac{1}{\delta a} \psi_{yy}, \] (26)

Where \( S_{xy} = (1 + \eta) \psi_{xy} - A (\psi_{yy})^3 \). Associated with the following dimensionless boundary conditions

\[ \psi_y = 0, \psi = \frac{\nu}{\zeta}, \theta_y = -\omega_1 \theta, \text{ at } y = y_1 \]

\[ \psi_y = 0, \psi = \frac{-F}{\zeta}, \theta_y = -\omega_1 (\theta - 1), \text{ at } y = y_2 \] (27)

In which \( y = y_1 = d + \alpha \cos(x) \) and \( y = y_2 = -1 - b \cos(x + \theta) \) dimensionless upper and lower walls and \( F \) is the non-dimensional mean flow which related to the non-dimensional time- mean flow \( \theta \) by following dimensionless relationship

\[ F = \theta - 1 - d. \] (28)

And the pressure rise per wavelength is

\[ \Delta p_\lambda = \int_0^1 \alpha \, dy \, dx, \] (29)

3. Methodology of solution

Dimensionless equations (25) and (27) are highly complicated, non-linear and thus the closed form solution for arbitrary values of all parameters is impossible hence we adopt the perturbation method for the analytical solution of the problem. Inserting the following expressions

\[ \psi = \psi_0 + A \psi_1 + O(A)^2, \] (30)

\[ \theta = \theta_0 + A \theta_1 + O(A)^2, \] (31)

\[ F = F_0 + A F_1 + O(A)^2, \] (32)

Substituting equations (30) - (32) into equations (25) - (28) and then comparing the coefficients of the same power of \( A \) up to the first order we obtain the following systems

3.1. Zeroth order system

The general form of the zeroth-order system is

\[ (1 + \eta) \psi_{0yy} - \left( H^2 (\cos \alpha_1)^2 + \frac{1}{\delta a} \right) \psi_{0yy} = 0, \] (33)

\[ (1 + Pr \, Re) \theta_{0yy} + (1 + \eta)(\psi_{0yy})^2 + H^2 (\cos \alpha_1)^2 Br \psi_{0yy}^2 = 0, \] (34)

the boundary conditions are

\[ \psi_{0y} = 0, \psi_0 = \frac{F_0}{\zeta}, \theta_{0y} = -B_1 \theta_0, \text{ at } y = y_1 \]

\[ \psi_{0y} = 0, \psi_0 = \frac{-F_0}{\zeta}, \theta_{0y} = -B_1 (\theta_0 - 1), \text{ at } y = y_2 \] (35)

3.2. First order system

The first- order system is

\[ (1 + \eta) \psi_{1yyy} - \left( H^2 (\cos \alpha_1)^2 + \frac{1}{\delta a} \right) \psi_{1yy} - \frac{\delta^2}{\delta y^2} (\psi_{1yy})^3 = 0, \] (36)
(1 + Pr Rn) \( \psi_{yy} + Br(1 + \eta)(\psi_{yy})^2 - (\psi_{yy})^4 \) + \( BrH^2(\cos \alpha_1)^2 Br \psi_{1y}^2 = 0, \) \( \tag{37} \)

With the respect to the boundary conditions

\[ \begin{align*}
    \psi_{1y} &= 0, \quad \psi_1 = \frac{f_1}{2}, \quad \theta_{1y} = -\omega_1 \theta_1, \quad \text{at } y = h_1 \\
    \psi_{1y} &= 0, \quad \psi_1 = -\frac{f_1}{2}, \quad \theta_{1y} = -\omega_1 (\theta_1 - 1), \quad \text{at } y = h_2 \end{align*} \]

Solving both systems using Mathematica program, the closed form for stream function \( \psi \), and temperature field \( \theta \) will be obtained

\[
\psi = -\frac{v_1}{v^2} + \frac{2y}{v^2} e^{-\sqrt{\frac{1}{1+y}}(1+y/(e^{1+y} + c_1 + c_2))} + c_3 + y c_4 + A \left( \frac{A_1 + A_2 + A_3}{B_0(1+y)(1+y)^2} + c_7 + y c_8 \right),
\]

\[
\theta_0 = r_1 + y r_2 - \frac{Br(A_4 + A_5 + A_6)}{2(1+y + Pr Rn)},
\]

\[
\theta_1 = r_3 + y r_4 + \frac{Br}{12B(1+y + Pr Rn)(1+y)^2} \left( A_7 - A_8 + \frac{6y}{v^2} (-18 c_1^2 c_4^2 v y^2 (1 + \eta)) - \frac{A_{10} + A_{11} + A_{12}}{v} \right) + 4 e^\frac{2y}{v^2} e^{-\sqrt{\frac{1}{1+y}}(1+y) + \sqrt{\frac{2y}{v^2} (-18 c_1^2 c_4^2 v y^2 (1 + \eta)) - \frac{A_{12} + A_{13}}{v} + A_{14} + A_{15} - A_{16}}},
\]

\[
A_1 = c_2^2 e^{1+y} (1+y)^{3/2} + c_3^2 e^{1+y} (1+y)^{3/2},
\]

\[
A_2 = 2 e^{1+y} (6 c_1^2 c_3 y (1 + \eta) + (1+y)^{3/2} (-15 c_1^2 c_2 + 4 c_4 + 4 n c_3)),
\]

\[
A_3 = 2 e^{1+y} (-12 c_1^2 c_2^2 y (1 + \eta) + (1+y)^{3/2} (-15 c_1 c_2^2 + 4 c_4 + 4 n c_3)),
\]

\[
v = \left( \frac{H^2(\cos \alpha_1)^2 + 1}{Da} \right)^{1/2},
\]

\[
A_4 = \frac{c_2^2 e^{1+y} (1+y)^{3/2} + c_1^2 e^{1+y} (1+y)^{3/2}}{2v},
\]

\[
A_5 = -\frac{2 c_2 e^{1+y} H^2 (1+y)^{3/2} \cos[\alpha_1]}{v^2} + \frac{2 c_1^2 e^{1+y} H^2 (1+y)^{3/2} \cos[\alpha_1]}{v^2},
\]

\[
A_6 = \frac{1}{2} v y^2 (c_4 H^2 + 4 c_1 c_2 (1 + \eta) + c_3 H^2 \cos[2 \alpha_1]),
\]

\[
A_7 = (48 c_1^3 c_2^3 v^3 y^4 + 64 c_1 c_2 (c_2 + c_5 - c_1 c_6) v^2 y^3 (1 + \eta)^{3/2} - \frac{6 c_2 e^{1+y} (1+y)^{3/2}}{2v^2} - \frac{9 c_1 c_2 e^{1+y} (1+y)^{3/2}}{2v^2} + c_3^2 e^{1+y} (1 + \eta) (-27 c_1^2 c_2 y \sqrt{1 + \eta} + (1+y)^{3/2} (8 c_1 + 27 c_2 c_2 - 18 c_5 + 8 c_1 \eta - 18 c_5)),
\]

\[
A_8 = c_2^2 e^{1+y} (1+y) (-27 c_1^2 c_2 y \sqrt{1 + \eta} - (1+y)^{3/2} (8 c_2 + 27 c_2 c_2 - 18 c_5 + 8 c_1 \eta - 18 c_5)),
\]

\[
A_9 = (1 + \eta)^2 (-32 c_1^3 c_2 + 63 c_1^4 c_2^2 - 36 c_1^2 c_2 c_5 + 8 c_5^2 + 18 c_1^3 c_6 - 32 c_1^3 c_2),
\]

\[
A_{10} = -36 c_1^2 c_2 c_5 \eta + 16 c_5^2 \eta + 18 c_1^3 c_6 \eta + 8 c_5^2 \eta^2),
\]

\[
A_{11} = 3 y (-27 c_1^4 c_2^2 v \sqrt{1 + \eta} + 8 c_1^2 c_2 c_5 v \sqrt{1 + \eta} - 27 c_1^4 c_2^2 v \eta \sqrt{1 + \eta} + 16 c_1^2 c_2 c_5 \eta \sqrt{1 + \eta} + 8 c_1^2 c_2 c_5 \eta^2 \sqrt{1 + \eta}),
\]

\[
A_{12} = (1 + \eta)^2 (-32 c_1^3 c_2 + 63 c_1^3 c_2^4 + 18 c_3^2 c_5 - 36 c_1 c_2^2 c_6 + 8 c_6^2 - 32 c_1^2 c_3 \eta + 18 c_3^2 c_5 - 36 c_1 c_2^2 c_6 + 16 c_6^2 \eta + 8 c_6^2 \eta^2).
\]
\[ A_{13} = 3y(-27c_1^2c_2^2v\sqrt{1+n} + 8c_1c_2^2c_6v\sqrt{1+n} - 27c_1^2c_2^4\eta v\sqrt{1+n} + 16c_1c_2^2c_6\eta \sqrt{1+n} + 8c_1c_2^2c_6\eta^2 \sqrt{1+n}). \]

\[ A_{14} = \frac{3v^2}{3}\sqrt{\gamma^2 H^2(1+n)^{5/2} Cos[\alpha_1]^2} - \frac{3v^2}{3}\sqrt{\gamma^2 H^2(1+n)^{5/2} Cos[\alpha_1]^2}. \]

\[ A_{15} = -\frac{3e^{-2\gamma^2 H^2(1+n)/(6c_1c_2^3y(1+n)+1(1+n)^{3/2}(-21c_1^2c_2^2+4c_6c_5+4c_5c_6))\cos[\alpha_1]^2}}{v}. \]

\[ A_{16} = \frac{3e^{-2\gamma^2 H^2(1+n)/(6c_1c_2^3y(1+n)+1(1+n)^{3/2}(-21c_1^2c_2^2+4c_6c_5+4c_5c_6))\cos[\alpha_1]^2}}{v}. \]

\[ A_{17} = 2vy^2(1+n)(117c_1^3c_2^3 - 48c_1c_2^2c_5(1+n) - 48c_1^2c_2(4c_2 + c_6)(1+n) + 16(1+n)c_6H^2 + 4c_5c_6(1+n)) + 16c_6H^2(1+n)\cos[2\alpha_1]). \]

Where \( c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_9, r_1, r_2, r_3, r_4 \) can be found using simple calculations.

### 4. Result and discussions

In this part of the work, graphical results are illustrated in order to figure out the impact of various emerging parameters such as \( \kappa, H, \eta, \) and \( A \) on velocity, temperature profiles, pressure rise, and trapping phenomenon.

#### 4.1. Velocity profile

The effect of Darcy number \( Da \), Hartman number \( H \), Ree-Eyring fluid parameters \( \eta, A \), phase difference parameter \( \phi \), and lower wall amplitude parameter \( b \) respectively on the velocity curve quantifies in figure 2(a)-(f). We attributed an enhancement in the velocity field at the center of the channel due to the rise in Darcy number \( Da \), whereas it decreases over the rest of the channel cross-section by increasing Darcy number \( Da \) via figure 2(a). However, ascending values of Hartman number \( H \) directly related to Lorentz force that resists the fluid flow and consequently decreases its velocity see figure 2(b). Whereas From figures 2(c) and (d) two opposite behaviors are seen for the higher magnitude of Ree-Eyring fluid parameters \( \eta \) and \( A \) respectively on velocity profile i.e. the magnitude of velocity is an increment in \( \eta \) resulted in an increase of \( u(y) \) in the central part of the channel and decreases toward the walls whereas this result is revers with \( A \) impact. In figures, 2(e) and (f) two opposite reactions are noticed on the velocity profile via increment of \( \phi \) magnitude. It depicts \( u(y) \) decays at the region(0 ≤ \( y \) ≤ 0.22), while it reflects increasing behavior for the rest channel. Larger values of \( b \) parameter lead to a reduction in velocity profile at the central part of the channel while this result is reversed toward the boundaries.
Figure 2. Velocity profile for different values of (a) permeability parameter $\kappa$ (b) Hartman number $H$ (c) Ree- Eyring parameter $\eta$ (d) Ree- Eyring parameter $A$ (e) phase difference parameter $\phi$ (f) lower wall amplitude parameter $b$ and for fixed values of parameters{$a = 0.04, d = 0.3, \vartheta = 1.4, x = 0.3, \alpha_1 = \frac{\pi}{6}$}

4.2. Temperature distribution

In this part, we characterize graphical behavior of temperature distribution profile against variation of Prandtl number $Pr$, Brinkman number $Br$, thermal radiation parameter $Rn$, Hartman number $H$, Darcy number $Da$, heat generation parameter $B_1$. figure 3(a) reveals the direct reaction of $Pr$ on temperature profile $\theta(y)$. Whereas increment of $Br$ tends to drop in $\theta(y)$ field see figure 3(b). The impression of $\theta(y)$ against the ascending value of $Rn$ is considered in figure 3(c). We observed that $\theta(y)$ behaves as an increasing function upon $Rn$ rises since it increases the surface heat flux so $\theta(y)$ is developed. It reveals from figure 3(d) that increment of Hartman number causes markedly resistance to temperature distribution profile. Due to enhancement in Darcy number,$\theta(y)$ tends to raise near the boundary regions whereas fluid temperature remains stable in the central part of the channel see figure 3(e). It is visualized from figure 3(f) that the heat generation parameter $B_1$ is directly proportional to the temperature distribution profile.
Figure 3. Temperature profile for different values of (a) Prandtl number $Pr$, (b) Brinkman number $Br$, (c) Thermal radiative number $Rn$, (d) Hartman number $H$, (e) Darcy number $Da$, (f) Heat generation parameter $B_1$ and for fixed parameters \{ $a = 0.3, b = 0.1, d = 0.3, \phi = \frac{\pi}{2}, x = 0.1, \eta = 0.2, A = 0.02, \vartheta = 0.71, \alpha_1 = \frac{\pi}{2}$ \}.

4.3. Pumping phenomenon

The most interesting phenomenon that is used to visualize the physiological behavior of fluids in the peristaltic mechanism is characterized by pumping phenomena. Figure 4 illustrates the consequence of pressure rise $\Delta p_2$ against the averaged flow rate $\vartheta$ via variation of the following pertinent parameters Hartman number $H$, Darcy number $Da$, Ree- Eyring parameters $\eta$, and $A$. The figures portrayed the parabolic behavior for $\Delta p_2$ profile. Furthermore, we noticed that the whole pumping region varied into four regions, the peristaltic pumping where ($\Delta p_2 > 0, \vartheta > 0$), augmented pumping where ($\Delta p_2 < 0, \vartheta > 0$) and ($\Delta p_2 < 0, \vartheta < 0$) and retrograde pumping where ($\Delta p_2 > 0, \vartheta < 0$) while free pumping
is recognized for ($\Delta p_3 = 0$). Figure 4(a) prepared to visualize the variation of $\Delta p_3$ against $\theta$ for various values of $H$. It is obvious that higher values of $H$ cause the augmented and retrograde pumping regions to decreases whereas growth in peristaltic pumping is witnessed. However, the completely opposite response on pumping flow is appeared due to enhance $Da$ magnitude which means the flow can pass easily without imposing higher pressure shown in figure 4(b). One can contemplate from figures 4(c) and 4(d), the dissimilar action for $\eta$, and $A$ parameters respectively on pumping characteristic i.e. $\eta$ is developing the augmented and retrograde pumping regions while the peristaltic pumping has a depressing function whereas this situation is absolutely reversed for enhancing $A$ parameter further free pumping is recognized where ($\theta > 0.55$).

**Figure 4.** Pressure rise $\Delta P_3$ versus $\theta$ via various values of (a) Hartman number $H$ (b) Darcy number $Da$ (c) Ree- Eyring parameter $\eta$ (d) Ree- Eyring parameter $A$ with $\{a=0.1, b=0.3, d=0.3, \phi=\frac{\pi}{2}, y=0.1, t=0.1, \alpha_1=\frac{\pi}{6}, \alpha_2=\frac{\pi}{2}, Re=0.1, Fr=0.1\}$.

### 4.4. Trapping characteristic

Formation of a circular path or bolus interior the flow streamlines along the peristaltic wave is known by trapping, we highlighted in this subsection upon the influences of some interesting pertinent parameters like Hartman number $H$, Darcy number $Da$, angle inclination parameter $\alpha_1$, phase difference parameter $\phi$, Ree-Eyring parameters $\eta$ and $A$, and time flow rate parameter $\theta$ with respect to fixed parameters $\{a=0.3, b=0.1, d=0.3, t=0.1\}$ on the trapping phenomenon. figure 5 captured the diminished behavior for the streamlines contours in size and number due to a rise in $H$ magnitude which leads to enhance Lorentz force that opposite the flow in return. Whereas figure 6 identifies the direct influences of Darcy’s number on the magnitude and number of moving trapping bolus. No remarkable
changes on the streamlines pattern were seen for ascending variation in $\alpha_3$ values i.e. the trapping bolus developed very slowly with $\alpha_3$ see figure 7. The significant impact of emerging parameter $\phi$ on trapping bolus is outlined in figure 8, it is noticed that the size and number of trapped bolus declines for higher values of $\phi$. Figures 9 and 10 recorded enhancement impact for the two Ree- Eyring embedded parameters $\eta$ and $A$ on trapping bolus phenomenon. An elevation of flow rate parameter $\theta$ produces more bolus as well as their size enhance see figure 11.

![Figure 5](image5.png)

**Figure 5.** Streamlines contours for multiple values of Hartman number $H = \{1, 1.5\}$

![Figure 6](image6.png)

**Figure 6.** Streamlines contours for multiple values of Darcy number $Da = \{0.3, 0.8\}$. 


Figure 7. Streamlines contours for multiple values of angle inclination parameter $\alpha_1 = \{\pi/3, \pi\}$.

Figure 8. Streamlines contours for multiple values of phase difference parameter $\Phi = \{\pi/6, \pi\}$. 
Figure 9. Streamlines contours for multiple values of Ree-Eyring fluid parameter $\eta = \{0.2, 0.6\}$.

Figure 10. Streamlines contours for multiple values of Ree-Eyring fluid parameter $A = \{0.1, 0.3\}$.
5. conclusions
In this investigation, several effects like inclined MHD, convective conditions, porous media, and radiative thermal on the peristaltic transport of Ree-Eyring fluid through the asymmetric channel are presented. The dimensionless nonlinear partial differential equations are simplified by adopting the restriction of infinite wavelength and small Renold numbers and then solved exactly using the perturbation technique. Some major outcomes from the graphical analysis of this study are listed below:

1. The velocity profile showed mixed responses throughout the channel via increasing all pertinent interesting physical parameters $H$, $Da$, $\phi$, $\eta$, $A$, $b$.
2. The Effect of Hartman number $H$, Ree-Eyring fluid parameter $A$, and lower wall amplitude parameter $b$ on the velocity distribution is totally same and completely opposite $Da$, $\phi$, $\eta$ impact.
3. It is noticed from figures that Brinkman number oppositely affected on temperature distribution profile whereas Prandtl number directly affected on $\theta(y)$.
4. Hartman number $H$ shows a decreasing behavior on temperature profile while $\theta(y)$ is increasing function with radiative thermal parameter $Rn$.
5. The temperature distribution axial greatly affected by risen up heat generation parameter $B_1$ however, one can be noticed that $\theta(y)$ enhanced near the walls and remains stable in the central part of the channel due to increasing Darcy number $Da$.
6. The pressure rise profile shows a parabolic nature moreover growth in peristaltic pumping is noticed for $H$ whereas the opposite result is found via ascending magnitude of $Da$.
7. The graphical discussion portrayed totally opposite behavior for Ree-Eyring parameters $\eta$ and $A$ on pressure rise.
8. Two asymmetric regions observed for the trapping phenomenon as well as the figures pretend to increase the trapped bolus magnitude and the number of circulation with $Da, \eta, A, \phi$, and flow rate parameter $\vartheta$, in other hands a diminished function for $H$ on trapped bolus is seen.
9. No remarkable effect of $\alpha_1$ on trapping, phenomena is noticed.

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