GLOBALLY STABLE ADAPTIVE DYNAMIC SURFACE CONTROL FOR COOPERATIVE PATH FOLLOWING OF MULTIPLE UNDERACTUATED AUTONOMOUS UNDERWATER VEHICLES

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ABSTRACT

The cooperative path following problem of multiple underactuated autonomous underwater vehicles (AUVs) involves two tasks. The first one is to force each AUV to converge to the desired parameterized path. The second one is to satisfy the requirement of a cooperative behavior along the paths. In this paper, both of the tasks have been further studied. For the first one, a simplified path following controller is proposed by incorporating the dynamic surface control (DSC) technique to avoid the calculation of derivatives of virtual control signals. Besides, in order to handle the uncertain dynamics, a new type of neural network (NN) adaptive controller is derived, and then an NN based energy-efficient path following controller is firstly proposed, which consists of an adaptive neural controller dominating in the neural active region and an extra robust controller working outside the neural active region. For the second one, in order to reduce the amount of communications between multiple AUVs, a distributed estimator for the reference common speed is firstly proposed as determined by the communications topology adopted, which means the global knowledge of the reference speed is relaxed for the problem of cooperative path following. The overall algorithm ensures that all the signals in the closed-loop system are globally uniformly ultimately bounded (GUUB) and the output of the system converges to a small neighborhood of the reference trajectory by properly choosing the design parameters. Simulation results validate the performance and robustness of the proposed strategy.

Key Words: Cooperative path following, dynamic surface control, autonomous underwater vehicles, neural networks, uncertainties, globally stable.

I. INTRODUCTION

With more and more concerns about the abounding and valuable ocean resources, recent years have witnessed a remarkable growth in a wide range of underwater activities. Autonomous underwater vehicles (AUVs) play an important role in performing underwater tasks, in both civilian and military applications, such as deep sea inspections, long-duration surveys, oceanographic mapping and neutralize undersea mines. For specific applications, a fleet of AUVs dealing with a variety of underwater tasks provides significant advantages, such as flexibility and efficiency, beyond what are possible with single AUV. Recently, there have been many research activities focusing on cooperative motion control for multiple AUVs [1,2]. A typical cooperative motion control problem is cooperative path following, which is concerned with the design of control laws that force a group of vehicles along the parameterized predefined paths while keeping a desired spatial formation [3,4].

The cooperative path following problem of multiple underactuated AUVs has been investigated in a number of publications. In [5], the leader–follower strategy is adopted for cooperative path following control in terms of arbitrary parallel paths. Meanwhile, the amount of information that broadcast from the leader to followers, to fulfill the control requirement, is minimized to one single variable. In [6], a decentralized direct speed adaptation under communication constraints is derived...
for cooperative path following of multiple homogenous underactuated AUVs and a helmsman like behavior is injected into the design. In [7], the cooperative path following problem is considered for underactuated AUVs in the presence of communication constrains. The topology of the communication network is captured in the framework of graph theory and the proposed control scheme is applicable to communication networks with both bidirectional and unidirectional communication links. Yet, a common feature in these works [3–7] is that the cooperative path following controllers are designed based on a backstepping technique [8,9]. The controllers are complicated for the sake of requiring derivative operations at each recursive step and the complexity of the controller grows drastically as the order of the system increases. In [10], a dynamic surface control (DSC) technique is proposed to eliminate this problem by introducing a first-order filtering of the synthetic input. In [11], the neural network (NN) based DSC approach is proposed for adaptive tracking control of strict-feedback systems with arbitrary uncertain nonlinearities. In [12], the NN based DSC technique is employed to solve the leader–follower formation control problem of autonomous marine surface vehicles (MSVs) with uncertain local dynamics and uncertain leader dynamics. In spite of a significant progress in the area, however, the model of autonomous marine surface vehicles considered in [3,5–7,12–15] are three degrees of freedom. For six degrees of freedom AUV with underactuated character moving on three-dimensional space, the controllers will be much more complicated than that due to the increase of system order. Although the DSC technique has been explored for many kinds of cooperative control of multi-agent/robot system [16–19,44,45], there are still no results on the problem of cooperative path following of underactuated AUVs, especially under the condition that the vehicle dynamics are totally unknown. Thus, it is important to develop a simplified cooperative path following control structure for underactuated AUVs. These concerns motivate our study.

Nonlinear uncertain dynamics is one of the main characteristics in AUVs’ control systems since the exact models of them are not easy to establish, which include unknown hydrodynamic parameters, unknown modeling error, and other parametric and functional uncertainties. To overcome the above problem, many NN-based control techniques are proposed to compensate for the highly uncertain, nonlinear and complex plants due to the fact that they have a good approximation ability over a compact domain. In [21], NN approximation capabilities and adaptive techniques are adopted to compensate for unknown AUV parameters, and constant or time-varying disturbances induced by waves and ocean currents. A non-linear controller is designed to make the reference point track a desired trajectory which is generated by an open-loop path planner. In [22], a one-layer NN controller with preprocessed input signals is designed to control the AUV track along a desired trajectory, which is specified in terms of desired position and attitude. In the absence of unknown disturbances and modeling errors, it is shown that the tracking error system is semi-globally uniformly ultimately bounded (SGUUB). In [23], semi on-line neural Q-learning, a new continuous approach of the Q-learning algorithm with a multilayer NN is used to learn behavior state/action mapping online. Experimental results show the feasibility of the presented approach for AUVs.

It is worth noting that all the aforementioned NN-based strategies can only guarantee the closed-loop system to be SGUUB under certain condition, i.e., the neural approximation should remain valid all the time, which is difficult to verify beforehand. The consequence is the possible deterioration of tracking performance or even instability due to either an improper initialization or external disturbances. [24] proposed a method that each virtual and actual controller switches between an adaptive neural controller and a robust controller for a class of strict-feedback systems. The overall controller ensures GUUB. However, [24] requires that both NN controller and robust controller must work together all the time in the neural active region. Thus, the control commands will be inevitably repeatedly imposed on the system and cause a waste of control energy [25,26]. This problem is of great importance in AUV system since AUV works in the harsh underwater environment and energy supply is very limited and comparatively little work of global tracking control has been considered until now. From a practical perspective, it is meaningful to design an energy-efficient control strategy for the problem of cooperative path following of multiple underactuated AUVs.

On the other hand, a major assumption in cooperative path following design of [2,3,5,14,15,27,28] is the global knowledge of the reference velocity. In spite of leading to decentralized control laws whereby only exchange of path variables among the vehicles and the information exchange is kept minimum, they all assume that the common reference speed is available to all vehicles and develop control laws that make use of this information. This may bring more information exchanges and even more communication constraints under the harsh ocean environment for AUVs. In order to eliminate the need for global knowledge of the reference speed, one option is to employ the distributed control strategy [29]. In [30], a leader-follower problem for a multi-agent system under a switching interconnection topology is considered. The distributed observers design
allows an active leader to be followed in an unknown velocity. In [31], an algorithm for distributed estimation of the active leader’s unmeasurable state variables is presented. A neighbor-based local controller together with a neighbor-based state-estimation rule is developed for each agent. Under the proposed control scheme, each agent can follow the leader if the input of the active leader is known.

Inspired by the aforementioned discussions, in this paper we develop a cooperative path following control algorithm for multiple underactuated AUVs with uncertain dynamics and partial knowledge of the reference speed. First, by incorporating the DSC technique, a smoothly switching function based NN adaptive path following controller is designed to force each AUV to follow a predefined path. Second, with the analytic tool of graph theory, the speed and path variables are synchronized to each vehicle owing to the proposed synchronization control law where the distributed speed estimate strategy is incorporated. Under the proposed controllers, all signals in the closed-loop system are guaranteed to be GUUB. Compared with the previous results of the problem of cooperative path following of AUVs/MSVs [2–6,14,15,19,27,28,43], the advantages of the proposed designs are listed as follows:

- For the individual path following design, the contribution of this paper is twofold. First, a simplified control structure is developed for AUVs. The proposed DSC based path following controllers avoid the problem of analytic computation by introducing the first-order filters. Second, the system uncertainties can be handled by the proposed switching function based NN adaptive approximators, then, the energy-efficient path following controllers are firstly proposed, which contains two types of controllers working under different conditions. The whole design ensures the GUUB stability of the closed-loop system and therefore is appealing to practical applications for AUVs.

- For the cooperative design, distributed speed estimator (54) is firstly developed building on graph theory and Lyapunov theory, by which the restriction on the common reference speed being available to all cooperating AUVs are explicitly relaxed. The desired cooperative behavior can be achieved under the condition that the common speed information is available only to one or partial fleet of AUVs.

**Notation.** In this paper, $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean Space. $\lambda_{\min}(M)$, $\lambda_2(M)$ and $\lambda_{\max}(M)$ denote the smallest, the second smallest, and the biggest eigenvalue of a matrix $M$, respectively. $\|\cdot\|$, $\|\cdot\|_F$, and $tr(\cdot)$ represent the Euclidean norm, the Frobenius norm, and the trace of a matrix. $\text{diag}[b_1,\ldots,b_n]$ represents a diagonal matrix with scalars $b_1,\ldots,b_n$ on the diagonal. $\mathbf{1}_n$ denotes a column vector with all entries equal to one.

## II. PRELIMINARIES AND PROBLEM FORMULATION

### 2.1 Stability results

In this subsection, we review the following stability results here for ease of reference. Consider a general nonlinear system [27]:

$$\dot{x} = f(x,t),$$

with state $x(t) \in \mathbb{R}^n$. We say that the solution of (1) is UUB if there exists a compact set $\Omega \in \mathbb{R}^n$ such that for all $x(t_0) = x_0 \in \Omega$, there exists a $\varepsilon_0 > 0$ and a number $T(\varepsilon_0,x_0)$ such that $\|x(t)\| < \varepsilon_0$ for all $t \geq T(\varepsilon_0,x_0) + t_0$. Especially, if the compact set $\Omega = \mathbb{R}^n$, then the solution of system (1) is GUUB.

### 2.2 Graph theory

A undirected graph $G = (\mathcal{V}, \mathcal{E})$ consists of a finite set $\mathcal{V} = \{1,2,\ldots,n\}$ of $n$ vertices and a finite set $\mathcal{E}$ of $m$ pairs of vertices $\{i,j\} \in \mathcal{E}$ named edges. If $\{i,j\}$ belongs to $\mathcal{E}$ then $i$ and $j$ are said to be adjacent. A path from $i$ to $j$ is a sequence of distinct vertices called adjacent. If there is a path in $\mathcal{G}$ between any two vertices, then the graph $\mathcal{G}$ is said to be connected. The adjacency matrix of the graph $\mathcal{G}$, denoted by $A = [a_{ij}] \in \mathbb{R}^{n \times n}$, is a square matrix with rows and columns indexed by the vertices, such that $a_{ij}$ equals one if $\{i,j\} \in \mathcal{E}$ and zero otherwise. If the graph $\mathcal{G}$ is connected, then zero is an eigenvalue of $L$ and all nonzero eigenvalues are positive. This implies that for a connected undirected graph, there exists a matrix $G = G G^T$ such that $L = GG^T$, where rank $G = n - 1$. If each vehicle can be represented by a vertex, then the communication relationship between any two vehicles can be described by an edge between the corresponding vertices.

**Lemma 1 ([18]).** If $\mathcal{G}$ is a connected undirected graph, then there exist a positive definite matrix $P$ such that $\theta^T L \theta = s^T P s$, where $\theta = [\theta_1,\ldots,\theta_n]^T \in \mathbb{R}^n$, $s = [s_1,\ldots,s_n]^T \in \mathbb{R}^n$, $s_i = \sum_{i=1}^n a_{ij}(\theta_i - \theta_j)$.

### 2.3 Neural networks

In this paper, linear-in-parameter NN is used to approximate the uncertainties in the system. Before introducing our control design method, let us first recall the
approximation property of the NNs [8,11,32–38]. The NNs take the form of $W^T\sigma(\xi)$ where $W \in \mathbb{R}^{c \times m}$ is called weight matrix, with $c$ being the number of NN nodes; and $\sigma(\xi) \in \mathbb{R}^{c}$ is a vector valued function defined in $\mathbb{R}^{c}$, with $\xi \in \mathbb{R}^{q}$ being the NN input vector. Denote the components of $\sigma(\xi)$ by $\rho_{l}(\xi), l = 1, \ldots, c$, and $\rho_{l}(\xi)$ is a basis function. In this work, $\rho_{l}(\xi)$ is chosen as the commonly used hyper tangent function, which have the following form:

$$\rho_{l}(\xi) = \frac{1 - \exp(-p\xi_{l})}{1 + \exp(-p\xi_{l})},$$

where $p \in \mathbb{R}$ is a positive constant. According to the approximation property of the NNs [8,19,21], given a continuous real-valued function $f(\xi) : \Omega \rightarrow \mathbb{R}^{m}$, with $\Omega \subseteq \mathbb{R}^{q}$ a compact set, and any $\epsilon_{M} > 0$, for some sufficiently large integer $q$, there exists $\|f\|_{L} \leq W_{M}$, such that the NN $W_{M} \sigma(\xi)$ can approximate the given function $f(\xi)$ as

$$f(\xi) = W_{M}^{T} \sigma(\xi) + \epsilon,$$

where $\|\epsilon\| \leq \epsilon_{M}$ represents the network reconstruction error.

2.4 Key definition

In this subsection, a switching function is introduced below, and then a property of this function is given by Lemma 1 which is crucial to the cooperative path following design.

**Definition 1.** For all $\sigma_{l} \in \mathbb{R}^{c}$, and given constants $0 < c_{1l} < c_{2l}$, function $F(\sigma_{l})$ is called $l$th-order smoothly switching function, if it satisfies the following conditions:

(a) $\|\sigma_{l}\| \leq c_{1l}, F(\sigma_{l}) = 0$;

(b) $\|\sigma_{l}\| \geq c_{2l} > 0, F(\sigma_{l}) = 1$;

(c) $F(\sigma_{l})$ is the $l$th-order continuous differentiable.

In particular, for all $\xi_{i} \in \mathbb{R}^{q}$, the following switching function is incorporated into the design.

$$m(\xi_{i}) = \begin{cases} 
0, & \|\xi_{i}\| \leq c_{1i} \\
1 - \cos\left(\frac{\pi}{2} \sin^{n}\left(\frac{\pi}{2} \frac{\|\xi_{i}\| - c_{1i}}{c_{2i} - c_{1i}}\right)\right), & \text{otherwise} \\
1, & \|\xi_{i}\| \geq c_{2i}
\end{cases}$$

**Lemma 2** ([25]). The function $m(\xi_{i})$ is an $l$th-order smoothly switching function for all $\xi_{i} \in \mathbb{R}^{q}$.

2.5 Problem formulation

Consider a group of $n$ underactuated AUVs with the $i$th AUV dynamics described by a 6-degree-of-freedom model as [39]

$$\dot{\eta}_{i} = R_{i}\nu_{i},$$

$$\dot{R}_{i} = R_{i}S(\omega_{i})$$

$$M_{i}\dot{\nu}_{i} = -S(\omega_{i})M_{i}\nu_{i} + f_{n}(\cdot) + b_{1}u_{i},$$

$$J_{i}\dot{\omega}_{i} = -S(\nu_{i})J_{i}\omega_{i} + f_{m}(\cdot) + b_{2}u_{m},$$

where $R_{i}$ is defined in [4], $\eta_{i} = [x_{i}, y_{i}, z_{i}]^{T} \in \mathbb{R}^{3}$ represents the position vector in the earth-fixed reference frame. $\nu_{i} = [u_{i}, v_{i}, w_{i}]^{T} \in \mathbb{R}^{3}$ and $\omega_{i} = [p_{i}, q_{i}, r_{i}]^{T} \in \mathbb{R}^{3}$ denote the linear and angular velocities in the body-fixed reference frame, respectively. $M_{i} = M_{i}^{T} \in \mathbb{R}^{3 \times 3}$ and $J_{i} = J_{i}^{T} \in \mathbb{R}^{3 \times 3}$ denote constant symmetric positive definite mass and inertia matrices, respectively. $f_{n}(\cdot) = f_{n}(\nu_{i}, \eta_{i}, R_{i})$ and $f_{m}(\cdot) = f_{m}(\nu_{i}, \omega_{i}, \eta_{i}, R_{i})$ represent all the remaining forces and torques acting on the body including parametric and functional uncertainties. $u_{i} \in \mathbb{R}$ and $u_{m} \in \mathbb{R}^{3}$ denote the control inputs, which act upon the system through $b_{1} = [1, 0, 0]^{T} \in \mathbb{R}^{3}$ and $b_{2} = \text{diag}[1] \in \mathbb{R}^{3 \times 3}$, respectively. $R_{i} \in \mathbb{R}^{3 \times 3}$ is a rotation matrix that describes the orientation (i.e. Euler angles, $\varphi_{i}, \theta_{i}, \psi_{i}$, denoted by $\bar{\eta}_{i} = [\varphi_{i}, \theta_{i}, \psi_{i}]^{T}$) of the vehicle by mapping body coordinates into inertial coordinates. $S(\cdot) \in \mathbb{R}^{3 \times 3}$ is a skew-symmetric matrix defined by

$$S(x) = \begin{bmatrix} 0 & -x_{3} & x_{2} \\
x_{3} & 0 & -x_{1} \\
-x_{2} & x_{1} & 0 \end{bmatrix}, \forall x = [x_{1}, x_{2}, x_{3}]^{T}.$$
Assumption 1 ([4,25,27]). For $i = 1, \ldots, n$, suppose there exist known positive smooth functions $F(\xi_i)$ such that

$$|f(\xi_i)| \leq F(\xi_i)f_{1M}, \quad (9)$$

where $f_{1M}$ is an unknown constant.

**Remark 1.** Noting that $f_{in}(\cdot)$ and $f_{iM}(\cdot)$ represent the forces and torques generated by external ocean environment and the forces and torques generated by the body, and the forces and torques imposed on the control system are also finite and $|f(\xi_i)| \leq F(\xi_i)f_{1M}$ (when $\xi_i = [v_i, \eta_i, R_i, \omega_i]$) can be satisfied in practical systems. The control objective of this paper is stated as follows.

Cooperative path-following problem. Let $\eta_{id}(\theta_i) = [x_{id}(\theta_i), y_{id}(\theta_i), z_{id}(\theta_i)]^T \in \mathbb{R}^3$, $i = 1, \ldots, n$, be a series of desired paths parameterized by continuous variables $\theta_i \in \mathbb{R}$. Suppose that each $\eta_{id}(\theta_i)$ is sufficiently smooth and its second derivative $\eta_{id}^{\theta \theta}$ is bounded, i.e., given any positive number $\delta_i$, the set $\Omega_i = \{[\eta_{id}^T, \eta_{id}^{\theta T}, \eta_{id}^{\theta \theta T}]^T : ||\eta_{id}||^2 + ||\eta_{id}^{\theta}||^2 + ||\eta_{id}^{\theta \theta}||^2 \leq \delta_i \}$ is compact, where $(\cdot)^{\theta} = (\partial(\cdot)/\partial \theta_i)$ and $(\cdot)^{\theta \theta} = (\partial^2(\cdot)/\partial \theta_i^2)$, $||\eta_{id}^{\theta \theta}|| \leq \eta_{id}^{\theta \theta}$ with $\eta_{id}^{\theta \theta}$ a positive constant. Design the feedback control laws for $u_i$ and $u_{id}$ such that all signals in the closed-loop control network system are GUUB, and the path following error $\eta_i - \eta_{id}$, along-path speed tracking error $\dot{\theta}_i - v_{id}$, and path variable coordination error $\theta_i - \theta_j$ satisfy

$$\lim_{t \to \infty} ||\eta_i - \eta_{id}|| \leq \epsilon_{i1},$$

$$\lim_{t \to \infty} ||\theta_i - v_{id}|| \leq \epsilon_{i2},$$

$$\lim_{t \to \infty} \theta_i - \theta_j \leq \epsilon_{i3},$$

where $v_{id}$ is a constant reference speed which assigned by the virtual leader; $\epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3} \in \mathbb{R}$ are some small constants.

**III. STATE FEEDBACK CONTROLLER DESIGN**

3.1 Individual path following design

In this subsection, NN based DSC technique is employed to yield individual path following control strategy. The controller design procedure on the $i$th AUV contains three steps. The synchronization control law for synchronized path following will be derived in the next subsection.

**Step 1.** The error variables are defined as

$$z_{i1} = R_i^T(\eta_i - \eta_{id}), \quad (13)$$

$$\dot{\theta}_i = \dot{\theta}_i - v_{id}, \quad (14)$$

$$\gamma_i = \dot{\theta}_i - v_{id}, \quad (15)$$

where $v_{id} \in \mathbb{R}$ is the estimate of common reference speed $v_0$, $z_{i1}$ and $\gamma_i$ denote the path following error and along-path tracking error, respectively. Differentiating $z_{i1}$ with respect to time and using (5) (15) yields

$$\dot{z}_{i1} = -S(\alpha_i)z_{i1} + v_i - R_i^T[\eta_{id}(v_{id} + \gamma_i)]. \quad (16)$$

To stabilize (16), choose a virtual control law $\alpha_{i1}$ as

$$\alpha_{i1} = -K_{i1}z_{i1} + R_i^T[\eta_{id}(v_{id} + \gamma_i)], \quad (17)$$

where $K_{i1} \in \mathbb{R}^{3 \times 3}$ is a diagonal matrix and its diagonal elements are positive constants. The update law for $\dot{v}_{id}$ will be specified in the next subsection, since it is supported by the information exchanges between its neighbors.

Consider a scalar function

$$V_{i1} = \frac{1}{2}z_{i1}^2, \quad (18)$$

whose time derivative along (16) (17) is

$$\dot{V}_{i1} = -z_{i1}^2K_{i1}z_{i1} + z_{i1}^2(\alpha_i - \alpha_{i1}) - \mu_i\gamma_i, \quad (19)$$

where $\mu_i = z_{i1}^2R_i^T[\eta_{id}]$. Introduce a new state variable $\dot{v}_{id} \in \mathbb{R}$ and let $\alpha_{i1}$ pass through a first-order filter with a time constant $\chi_{i1} \in \mathbb{R}$

$$\chi_{i1}\dot{\alpha}_{i1} + \dot{\alpha}_{i1} = \alpha_{i1}. \quad (20)$$

Let $\epsilon_{i1} = \dot{v}_{id} - \alpha_{i1}$ and $z_{i2} = v_i - \dot{v}_{id}$. Then, define the second scalar function

$$V_{i2} = V_{i1} + \frac{1}{2}\epsilon_{i1}^2, \quad (21)$$

Invoking (19), the time derivative of $V_{i2}$ is given by

$$\dot{V}_{i2} = -z_{i1}^2K_{i1}z_{i1} + z_{i1}^2(z_{i2} + \epsilon_{i1}) - \mu_i\gamma_i + \epsilon_{i1}^2. \quad (22)$$
Step 2. Taking the time derivative of $z_{i2}$ along (7) gives

$$M_f \dot{z}_{i2} = S(M_f z_{i2})\omega_i + f_i(\cdot) + b_1 u_{i}\alpha_i - S(\omega_i) M_f \tilde{v}_{i\theta} - M_f \dot{\tilde{v}}_{i\theta}. \quad (23)$$

Note that due to the lack of actuation, (23) cannot be stabilized by designing $u_{i\alpha}$ directly. Consequently, let $\Phi_i = z_{i2} - \beta_i$ be a new error variable, where $\beta_i \in \mathbb{R}^3$. Consider the third scalar function

$$V_{i3} = V_{i2} + \frac{1}{2} \Phi_i^T M_f \Phi_i, \quad (24)$$

whose time derivative along (22) (23) is

$$\dot{V}_{i3} = -z_{i2}^2 K_i z_{i2} + z_{i2}^2 e_{i1} + z_{i2}^2 \beta_i - \mu_i \gamma_i + e_i^T \dot{e}_i + \Phi_i^T [B_i \tau_i + z_{i2} - S(\omega_i) M_f \Phi_i - f_i(\cdot)]. \quad (25)$$

where $f_i(\xi_i) = -f_i(\cdot) + S(\omega_i) M_f \tilde{v}_{i\theta} + M_f \dot{\tilde{v}}_{i\theta}$ and satisfies $|f_i(\xi_i)| \leq F(\xi_i) \|\epsilon_i\|_M$ with $F(\xi_i)$ being a known positive smooth function and $f_{i\alpha}$ being an unknown positive constant. $B_i = [b_1, S(M_f \beta_i)]$, $\tau_i = [u_{i\alpha}, \omega_i]^T$. Note that the matrix $B_i$ can always be made full-rank by choosing a suitable $\beta_i$. The control command $u_{i\alpha}$ will be designed next. Note that $\tau_i$ can be regarded as the control law (actually its second component can be treated as a virtual control) to make (25) negative define. To this effect, choose the following desired indirect control law

$$\Lambda_i = B_i^T (B_i B_i^T)^{-1} [-z_{i2} - K_i \Phi_i - S(\omega_i) M_f \Phi_i] + f_i(\xi_i), \quad (26)$$

where $K_i \in \mathbb{R}^{3 \times 3}$ is a diagonal matrix and its diagonal elements are positive constants. Consider a desired control $u_{i\alpha}$ to be equal to the first entry of $\Lambda_i$, and the second virtual control to be equal to the last three entry of $\Lambda_i$

$$u_{i\alpha} = b_3 \Lambda_i, \quad (27)$$

$$a_{i2} = b_4 \Lambda_i, \quad (28)$$

where $b_3 = [b_3^T, 0]$, $b_4 = [0, b_4^T]$. In practice, $f_i(\xi_i)$ in (27) is very hard to obtain accurately. Hence, the controller (27) cannot be implemented. To overcome this problem, an NN is employed to approximate $f_i(\xi_i)$. According to the approximation property of the NNs, we have

$$f_i(\xi_i) = W_i^T \sigma(\xi_i) + \epsilon_i, \quad (29)$$

where $\xi_i = [1, v_i^T, \omega_i^T, n_i, \tilde{v}_{i\theta}, \hat{\nu}_i]^T \in \mathbb{R}^{19}$ is the NN input. $W_i$ is the NN weight. $\epsilon_i$ is the approximation error satisfying $\|\epsilon_i\| \leq \epsilon_{i1M}$ with $\epsilon_{i1M}$ being a positive constant. Then, (26) can be rewritten as

$$\Lambda_i = B_i^T (B_i B_i^T)^{-1} [-z_{i2} - K_i \Phi_i - S(\omega_i) M_f \Phi_i] + (1 - m(\xi_i)) \Lambda_{im} + \epsilon_i \Lambda_{i1M}, \quad (30)$$

where $\Lambda_{im}$ and $\Lambda_i'$ denote the adaptive neural controller dominating in the neural active region and the robust controller works outside the neural active region, respectively. They are designed as follows.

$$\Lambda_{im} = -\hat{W}_{i1M}^T \sigma(\xi_i) - \tanh \left( \frac{\Phi_i^T}{\rho_i} \right) \hat{\nu}_{i1M}, \quad (31)$$

$$\Lambda_i' = -F(\xi_i) \tanh \left( \frac{\Phi_i^T F(\xi_i)}{\rho_i} \right) \hat{f}_{i1M}, \quad (32)$$

where $\rho_i \in \mathbb{R}$ is a positive constant, $\hat{W}_{i1M}, \hat{\nu}_{i1M}$ and $\hat{f}_{i1M}$ are the estimates of $W_i$, $\epsilon_{i1M}$ and $f_{i1M}$, respectively. Their update laws will be specified later.

Substituting (27) (28) with (30) into (25) yields

$$\dot{V}_{i3} \leq -z_{i2}^2 K_i z_{i2} - \Phi_i^T K_i \Phi_i + z_{i2}^2 e_{i1} + z_{i2}^2 \beta_i - \mu_i \gamma_i + e_i^T \dot{e}_i - (1 - m(\xi_i)) \Phi_i^T \hat{W}_{i1M} \sigma(\xi_i) - \hat{\eta}_{i2}^T \Xi_i + (1 - m(\xi_i)) \left[ \Phi_i^T F(\xi_i) \right] \hat{\nu}_{i1M} + m(\xi_i) \left[ \Phi_i^T F(\xi_i) \tanh \left( \frac{\Phi_i^T F(\xi_i)}{\rho_i} \right) \right] \hat{f}_{i1M} + \Phi_i^T S(M_f \beta_i) (\omega_i - a_2). \quad (33)$$

where $\hat{f}_{i1M} = \hat{f}_{i1M} - f_{i1M}$, $\hat{\nu}_{i1M} = \hat{\nu}_{i1M} - \epsilon_{i1M}$, $\hat{W}_{i1M} = \hat{W}_{i1M} - W_i$, $\hat{\eta}_{i2} = \hat{\eta}_{i2} - \Theta_i$ with $\hat{\eta}_{i2} = \left[ \hat{\eta}_{i2} \hat{f}_{i1M} \right]$, $\Xi_i = \left[ \begin{array}{c} (1 - m(\xi_i)) \tanh \left( \frac{\sigma_i}{\rho_i} \right) \Phi_i^T \\ m(\xi_i) F(\xi_i) \tanh \left( \frac{\sigma_i}{\rho_i} \right) \Phi_i^T \end{array} \right]$. The adaptation laws are chosen as

$$\dot{\hat{W}}_{i1M} = \Gamma_{i1M} \left[ (1 - m(\xi_i)) \sigma(\xi_i) F_i \Phi_i - k_{i1W} \hat{W}_{i1M} \right], \quad (34)$$

$$\dot{\hat{\eta}}_{i2} = \Gamma_{i2} \left[ \Xi_i - k_{i1\theta} \hat{\eta}_{i2} \right], \quad (35)$$

where $\Gamma_i \in \mathbb{R}$, $k_{i1W} \in \mathbb{R}$, $\Gamma_{i2} \in \mathbb{R}$ and $k_{i1\theta} \in \mathbb{R}$ are positive constants.
Define the fourth scalar function as

\[ V_{i4} = V_{i3} + \frac{1}{2} \text{tr}(\dot{W}_{i1}^T \Gamma_i^{-1} \dot{W}_{i1}) + \frac{1}{2} \Theta_i^T \Gamma_i^{-1} \Theta_i, \]  

whose time derivative along (33) (34) and (35) is

\[
V_{i4} \leq -z_i^T K_i z_i - \Phi_i^T K_2 \Phi_i + z_i^T e_i + z_i^T \beta_i - \mu_i \dot{y}_i \\
+ e_i^T \hat{e}_i - k_{iiw} \text{tr}(\dot{W}_{i1}^T \dot{W}_{i1}) - k_{iiw} \Theta_i^T \Theta_i + (1 - m(z_i)) \left[ \Phi_i^T - \Phi_i^T \tanh \left( \frac{\Phi_i^T}{\rho_i} \right) \right] \varepsilon_iM + m(z_i) \\
\times \left[ \Phi_i^T F(z_i) - \Phi_i^T F(z_i) \tanh \left( \frac{\Phi_i^T F(z_i)}{\rho_i} \right) \right] f_{iM} \\
+ \Phi_i^T S(M_i \beta_i)(\omega_i - \alpha_i). 
\]  

(37)

Introduce a new state variable \( \tilde{\omega}_{id} \in \mathbb{R}^3 \) and let \( \alpha_2 \) pass through a first-order filter with a time constant \( \chi_{i2} \in \mathbb{R} \)

\[
\chi_{i2} \dot{\tilde{\omega}}_{id} + \tilde{\omega}_{id} = \alpha_{i2}. 
\]  

(38)

Let \( e_{i2} = \tilde{\omega}_{id} - \alpha_{i2} \) and \( z_{i3} = \omega_i - \tilde{\omega}_{id} \). Then, define the fifth scalar function

\[
V_{i5} = V_{i4} + \frac{1}{2} e_{i2}^T e_{i2}. 
\]  

(39)

Invoking (37), the time derivative of \( V_{i5} \) is given by

\[
V_{i5} \leq -z_i^T K_i z_i - \Phi_i^T K_2 \Phi_i + z_i^T e_i + z_i^T \beta_i - \mu_i \dot{y}_i \\
+ e_i^T \hat{e}_i + e_i^T \hat{e}_i - k_{iiw} \text{tr}(\dot{W}_{i1}^T \dot{W}_{i1}) - k_{iiw} \Theta_i^T \Theta_i + (1 - m(z_i)) \left[ \Phi_i^T - \Phi_i^T \tanh \left( \frac{\Phi_i^T}{\rho_i} \right) \right] \varepsilon_iM + m(z_i) \\
\times \left[ \Phi_i^T F(z_i) - \Phi_i^T F(z_i) \tanh \left( \frac{\Phi_i^T F(z_i)}{\rho_i} \right) \right] f_{iM} \\
+ \Phi_i^T S(M_i \beta_i)(z_{i3} + e_{i2}). 
\]  

(40)

**Step 3.** Taking the time derivative of \( z_{i3} \) along (8) gives

\[
J_i \dot{z}_{i3} = -S(v_i) M_i v_i - S(\omega_i) J_i \omega_i + f_{i\omega}(\cdot) \\
+ b_{i2} u_{i\omega} - J_i \dot{\tilde{\omega}}_{id}. 
\]  

(41)

Consider the sixth scalar function as

\[
V_{i6} = V_{i5} + \frac{1}{2} z_i^T J_i z_{i3}, 
\]  

(42)
\[ \times z_{i3}^T \hat{W}_{i2}^T \sigma(\xi_{i2}) - \tilde{\Theta}_{i2}^T z_{i2} + (1 - m(\xi_{i2})) \left[ z_{i3}^T \right] \]
\[ - z_{i3}^T \text{tanh} \left( \frac{z_{i3}}{\rho_2} \right) \varepsilon_{i2M} + m(\xi_{i2}) \left[ z_{i3}^T F(\xi_{i2}) \right] \]
\[ - z_{i3}^T F(\xi_{i2}) \text{tanh} \left( \frac{z_{i3}^T F(\xi_{i2})}{\rho_2} \right) \varepsilon_{i2M}, \]
where \( \varepsilon_{i2M} = \varepsilon_{i2M - i} \) and \( \tilde{W}_{i2} = \hat{W}_{i2} - W_{i2} \),
\[ - W_{i2}, \quad \Xi_{i2} = \left( 1 - m(\xi_{i2}) \right) \text{tanh} \left( \frac{z_{i3}^T F(\xi_{i2})}{\rho_2} \right) \varepsilon_{i2M}, \]
\[ \tilde{\Theta}_{i2} = \hat{\Theta}_{i2} - \Theta_{i2} \text{ with } \hat{\Theta}_{i2} = \left[ \frac{\varepsilon_{i2M}}{J_{i2M}} \right] . \]

The adaptation laws are chosen as
\[ \hat{W}_{i2} = \Gamma_{i3} \left[ 1 - m(\xi_{i2}) \right] \sigma(\xi_{i2}) z_{i3}^T - k_{i2W} \hat{W}_{i2}, \quad (48) \]
\[ \hat{\Theta}_{i2} = \Gamma_{i4} (\Xi_{i2} - k_{i2b} \Theta_{i2}), \quad (49) \]
where \( \Gamma_{i3}, \Gamma_{i4} \in \mathbb{R} \), \( k_{i2W}, \gamma_{i4} \in \mathbb{R} \) and \( k_{i2b} \in \mathbb{R} \) are positive constants.

Define the seventh scalar function as
\[ V_{\gamma} = V_{\theta} + \frac{1}{2} \text{tr} \left( \hat{W}_{i2}^T \Gamma_{i3}^{-1} \hat{W}_{i2} \right) + \frac{1}{2} \hat{\Theta}_{i2}^T \Gamma_{i4}^{-1} \hat{\Theta}_{i2}, \quad (50) \]
whose time derivative along (47) (48) and (49) is given by
\[ \dot{V}_{\gamma} \leq -z_{i3}^T K_{i1} z_{i1} - \Phi_i^T K_{i2} \Phi_i - z_{i3}^T K_{i3} z_{i3} + z_{i3}^T \varepsilon_{i1}
+ z_{i3}^T \beta_i - \mu_i \gamma_i + \varepsilon_{i1}^T \varepsilon_{i1} + \varepsilon_{i2}^T \varepsilon_{i2} - k_{i1W} \text{tr} (\hat{W}_{i2}^T \hat{W}_{i2})
- k_{i2b} \hat{\Theta}_{i2} \Theta_{i2} + (1 - m(\xi_{i2})) \left[ \Phi_i^T - \Phi_i^T \text{tanh} \left( \frac{\Phi_i^T}{\rho_1} \right) \right]
\times \varepsilon_{i1M} + m(\xi_{i2}) \left[ \Phi_i^T F(\xi_{i2}) + \Phi_i^T S(M_i \beta_i) \varepsilon_{i2} \right]
\times \Phi_i^T F(\xi_{i2}) \text{tanh} \left( \frac{\Phi_i^T F(\xi_{i2})}{\rho_1} \right) \varepsilon_{i1M} - k_{i2b} \hat{\Theta}_{i2} \Theta_{i2}
- k_{i2W} \text{tr} (\hat{W}_{i2}^T \hat{W}_{i2}) + (1 - m(\xi_{i2})) \left[ z_{i3}^T \right]
- z_{i3}^T \text{tanh} \left( \frac{z_{i3}}{\rho_2} \right) \varepsilon_{i2M} + m(\xi_{i2}) \left[ z_{i3}^T F(\xi_{i2}) \right]
- z_{i3}^T F(\xi_{i2}) \text{tanh} \left( \frac{z_{i3}^T F(\xi_{i2})}{\rho_2} \right) \varepsilon_{i2M}, \quad (51) \]

\[ \hat{a}_{i1} = K_{i1} z_{i1} + \hat{R}_i^T \eta_{i1}^\theta \dot{\gamma}_{i1} + \hat{R}_i^T \left[ \eta_{i1}^\theta \dot{\gamma}_{i1} + \eta_{i1}^\theta \dot{\gamma}_{i1} \right]
+ \tilde{S}(\omega_i) z_{i1} + z_{i1} \dot{S}(\omega_i), \quad (52) \]
\[ \hat{a}_{i2} = b_i \hat{B}_i^T (\beta_i \hat{B}_i^{\theta - 1}) - S(\omega_i) z_{i1} + v_i \]
\[ - R_i^T \left[ \eta_{i1}^\theta \dot{\gamma}_{i1} + \gamma_i \right] - k_{i2} \left[ \gamma_i - K_{i1} (rS z_{i1} - v_i) \right]
- R_i^T \left[ \eta_{i1}^\theta \dot{\gamma}_{i1} + R_i^T \eta_{i1}^\theta \dot{\gamma}_{i1} - R_i^T \eta_{i1}^\theta \dot{\gamma}_{i1} \right]
- S(\omega_i) M_i \Phi_i + S(\omega_i) M_i \dot{\gamma}_i - K_{i1} \left[ rS z_{i1} - v_i \right]
\]
\[ + [1 - m(\xi_{i1})] \Lambda^m_i + [1 - m(\xi_{i1})] \Lambda^m_i \]
\[ + \dot{m}(\xi_{i1}) \Lambda_{i1}^m + m(\xi_{i1}) \Lambda_{i1}' \]
its neighbors. Furthermore, since not all the AUVs can obtain the value of common reference speed, some of them have to be estimated throughout the process. Let $\mathcal{N}_i$ be the set of labels of those AUVs that are neighbors of AUV $i$, $\mathcal{N}_0$ be the set of labels of those AUVs that are neighbors of the virtual leader. In such a way, the common reference speed assigned by a virtual leader is available to only one or one subset of AUVs, so the speed estimate strategy is distributed. To this end, choose the following cooperative control law with an auxiliary state $\zeta$ as

$$
\theta = \hat{\theta}_d - K_4^{-1}(L\theta + \mu) - \zeta, \tag{53}
$$
$$
\dot{\zeta} = -(K_4 + K_5)\zeta - L\theta - \mu,
$$

where $\zeta = [\zeta_1, \ldots, \zeta_n]^T \in \mathbb{R}^n$, $\mu = [\mu_1, \ldots, \mu_n]^T \in \mathbb{R}^n$, $\gamma = [\gamma_1, \ldots, \gamma_n]^T \in \mathbb{R}^n$, $K_4 = \text{diag}[k_{i4}] \in \mathbb{R}^{n \times n}$, $K_5 = \text{diag}[k_{i5}] \in \mathbb{R}^{n \times n}$, $k_{i4} \in \mathbb{R}$ and $k_{i5} \in \mathbb{R}$ are positive constants.

The distributed speed update law is designed as

$$
\dot{\hat{v}}_d = -\mathcal{L}\hat{v}_d + K_7B\nu_0I_n, \tag{54}
$$

where $\mathcal{L} = K_kL + K_kB$, $B = \text{diag}[b_1, \ldots, b_n] \in \mathbb{R}^{n \times n}$.

Because the fixed undirected graph is connected and at least one $b_1$ is nonzero, so $\mathcal{L}$ is symmetric positive definite. $k_{i6} \in \mathbb{R}$ and $k_{i7} \in \mathbb{R}$ are positive constants. $b_i$ is the connection weight between vehicle $i$ and the virtual leader, $b_i = 1$ if the virtual leader is available to ith vehicle and $b_i = 0$ otherwise. $K_k = \text{diag}[k_{i6}] \in \mathbb{R}^{n \times n}$, $K_7 = \text{diag}[k_{i7}] \in \mathbb{R}^{n \times n}$.

Using the eighth scalar function as $\zeta = [\xi_1, \ldots, \xi_n]^T \in \mathbb{R}^n$, $\mu = [\mu_1, \ldots, \mu_n]^T \in \mathbb{R}^n$, $\nu = [\nu_1, \ldots, \nu_n]^T \in \mathbb{R}^n$.

Noting that $\hat{v}_d = \hat{v}_d - v_0I_n$, then we have

$$
\dot{\hat{v}}_d = -\mathcal{L}\hat{v}_d, \tag{55}
$$

Consider the eighth scalar function as

$$
V_{\delta_2} = \frac{1}{2}\theta^T L\theta + \frac{1}{2}\xi^T \xi + \frac{1}{2}\hat{v}_d^T \hat{v}_d + \sum_{i=1}^{n} V_{i7}, \tag{56}
$$

and its time derivative along (51) (53) and (55) is

$$
\dot{V}_{\delta_2} \leq -\gamma^T K_k\gamma - \xi^T K_5 \xi - \hat{v}_d^T \mathcal{L}\hat{v}_d + \sum_{i=1}^{n} \left[-z_{i1}^T K_{11}z_{i1} - \Phi_i^T K_{i2} \Phi_i - z_{i1}^T K_{i5} \xi_{i1} + z_{i1}^T \xi_{i1} + \xi_{i1}^T \zeta_1 + \xi_{i1}^T \zeta_1 - \frac{1}{2}(1 - m(\xi_{i1})) \left[\frac{\Phi_i^T}{\rho_{i1}} - \Phi_i^T \tanh\left(\frac{\Phi_i^T}{\rho_{i1}}\right)\right] \left[\xi_{i1}M + m(\xi_{i1})\right] \right].
$$

\section{3. Stability analysis}

\textbf{Theorem 1.} Consider a network of underactuated AUVs with the vehicle dynamics given in (5-8). Suppose the communication network is undirected and connected. Design the first-order filters (20) and (38), control laws (27) with (30) and (44), NN adaptive laws (34) and (48), cooperative control law (53) and distributed speed estimator (54). Then, given any positive number $\delta_2$, for all initial conditions satisfying $\Omega_{\delta_2} = \{(z_{i1}, z_{i2}, \Phi_i, z_{i3}, \xi_{d1}, e_{i1}, e_{i2}, W_{i1}, W_{i2}, \Theta_i, \hat{\Theta}_i)^T : \ V_{\delta_2} \leq M \}, \exists \left[ K_{i1}, K_{i2}, K_{i3}, K_{i4}, K_{i5}, K_{i6}, K_{i7}, K_{i8}, K_{i9}, K_{i10}, K_{i11}, K_{i12} \right] \text{ such that all the closed-loop signals remain bounded all the time, and the path following error } \eta_i - \eta_{id}, \text{ along-path speed tracking error } \dot{\theta}_i - \dot{\theta}_0, \text{ and path variable coordination error } \theta_i - \theta_j \text{ satisfy (10) (11) (12), respectively.}

\textbf{Proof.} Taking the time derivative of $e_{i1}$ and $e_{i2}$ along (20) and (38), we have

$$
\dot{e}_{i1} = -\frac{e_{i1}}{K_{i1}} + \Delta_{i1}(z_{i1}, z_{i2}, \Phi_i, z_{i3}, \gamma, e_{i1}, e_{i2}, \eta_{i1}^\theta, \eta_{i1}^\gamma),
$$
$$
\dot{e}_{i2} = -\frac{e_{i2}}{K_{i2}} + \Delta_{i2}(z_{i1}, z_{i2}, \Phi_i, z_{i3}, \gamma, e_{i1}, e_{i2}, \eta_{i2}^\theta, \eta_{i2}^\gamma),
$$

where $\Delta_{i1}(\cdot)$ and $\Delta_{i2}(\cdot)$ are continuous functions. Since for any $\delta_1$ and $\delta_2$, the sets $\Omega_{\delta_1}$ and $\Omega_{\delta_2}$ are compact, $\Omega_{\delta_1} \times \Omega_{\delta_2}$ is also compact. Hence, $\Delta_{i1}(\cdot)$ and $\Delta_{i2}(\cdot)$ have maximum values $\Delta_{i1,M}$ and $\Delta_{i2,M}$ on $\Omega_{\delta_1} \times \Omega_{\delta_2}$. Furthermore, using

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Young's inequality yields
\[
\left| e_j^T e_i \right| \leq \frac{\| e_j \|^2}{\chi_{ji}} + \frac{\Delta_{ji}^2}{2d_{ji}} + \frac{d_{ij}}{2},
\]
\[j = 1, 2 - k_{ji}W \text{tr}(\dot{W}_i^T \dot{W}_j) \leq -\frac{k_{ji}W}{2} \| \dot{W}_i \|^2_F + \frac{k_{ji}W}{2} \| \dot{W}_j \|^2_F.
\]
\[j = 1, 2 - k_{ji} \ddot{\theta}_i \ddot{\theta}_j \leq -\frac{k_{ji}}{2} \| \ddot{\theta}_i \|^2 + \frac{k_{ji}}{2} \| \ddot{\theta}_j \|^2_F.
\]
\[j = 1, 2 - \frac{\ddot{z}_i^T e_i}{2d_{i3}} \| z_i \|^2 + \frac{d_{ij}}{2} \| e_i \|^2,
\]
\[z_i^T e_i = \frac{1}{2d_{i4}} \| z_i \|^2 + \frac{d_{ij}}{2} \| e_i \|^2,
\]
\[\phi_i^T S(M_i) \beta_i e_i \leq \frac{\bar{\sigma}(S(M_i) \beta_i)^2}{2d_{i5}} \| \Phi_i \|^2 + \frac{d_{ij}}{2} \| e_i \|^2,
\]
\[\left[ \phi_i^T - \frac{\phi_i}{\rho_i} \right] z_i \| e_i \|^2 \leq 0.2785 \rho_i \| e_i \|^2.
\]
\[\left[ \phi_i^T F(\xi) - \frac{\phi_i F(\xi)}{\rho_i} \right] z_i \| e_i \|^2 \leq 0.2785 \rho_i \| e_i \|^2.
\]
\[\left[ \phi_i^T \text{tanh} \left( \frac{\phi_i^T F(\xi)}{\rho_i} \right) \right] z_i \| e_i \|^2 \leq 0.2785 \rho_i \| e_i \|^2.
\]
where \( d_{i1}, d_{i2}, d_{i3}, d_{i4}, d_{i5} \in \mathbb{R} \) are positive constants. \( \bar{\sigma}(\cdot) \) denotes the maximum singular value of a matrix. Then, (57) can be rewritten as
\[
V_{ii} \leq -\lambda_{\min}(K_1) \| y \|^2 - \lambda_{\min}(K_3) \| \zeta \|^2 - \lambda_{\min}(C) \| \bar{V}_i \|^2
\]
\[+ \sum_{k=1}^{n} \left[ \left( \Delta_{i1}^2 - \frac{d_{i1}}{2d_{i3}} \right) \| \ddot{z}_i \|^2 - \left( \frac{d_{i2}}{2d_{i4}} - \frac{d_{i3}}{2} \right) \| e_i \|^2
\]
\[\leq \lambda_{\min}(K_1) \| y \|^2 - \lambda_{\min}(K_3) \| \zeta \|^2 - \lambda_{\min}(C) \| \bar{V}_i \|^2
\]
\[+ \sum_{k=1}^{n} \left[ \left( \Delta_{i1}^2 - \frac{d_{i1}}{2d_{i3}} \right) \| \ddot{z}_i \|^2 - \left( \frac{d_{i2}}{2d_{i4}} - \frac{d_{i3}}{2} \right) \| e_i \|^2
\]
\[\leq \lambda_{\min}(K_1) \| y \|^2 - \lambda_{\min}(K_3) \| \zeta \|^2 - \lambda_{\min}(C) \| \bar{V}_i \|^2
\]
\[+ \sum_{k=1}^{n} \left[ \left( \Delta_{i1}^2 - \frac{d_{i1}}{2d_{i3}} \right) \| \ddot{z}_i \|^2 - \left( \frac{d_{i2}}{2d_{i4}} - \frac{d_{i3}}{2} \right) \| e_i \|^2
\]
\[\times \| e_i \|^2 - \frac{k_{i11}W}{2} \| W_i \|^2_F - \frac{k_{i21}W}{2} \| W_i \|^2_F - \frac{k_{i12}}{2} \| \dot{\theta}_i \|^2_F + H_i.
\]
with
\[
H_i = \sum_{k=1}^{n} \left[ \frac{1}{2} \left( d_{i4} \| \beta_i \|^2 + d_{i1} + d_{i2} + k_{i11}W \| W_i \|^2 + k_{i21}W \| W_i \|^2 + k_{i12} \| \dot{\theta}_i \|^2 \right.ight.
\]
\[+ 0.2785 \left( 1 - m(\xi_1) \right) \rho_{i1} \| e_i \|^2 + m(\xi_1) \rho_{i1} \| e_i \|^2 + k_{i21} \| W_i \|^2 F + k_{i12} \| \dot{\theta}_i \|^2 F + (1 - m(\xi_2)) \rho_{i2} \| e_i \|^2 + m(\xi_2) \rho_{i2} \| e_i \|^2 \right]\]
(61)
Choose \( \lambda_{\min}(K_1) - \frac{1}{\Delta_{i1}^2} - \frac{d_{i1}}{2d_{i3}} > 0 \), \( \lambda_{\min}(K_3) - \frac{\bar{\sigma}(S(M_i) \beta_i)^2}{2d_{i5}} > 0 \), \( \frac{d_{i2}}{2d_{i4}} - \frac{d_{i3}}{2} > 0 \), and note that, either \( \| y \| > \frac{H_i}{\lambda_{\min}(K_1)} \), or \( \| \zeta \| > \frac{H_i}{\lambda_{\min}(K_3)} \), or \( \| \Phi_i \| > \frac{H_i}{\lambda_{\min}(K_1)} \), or \( \| \bar{V}_i \| > \frac{H_i}{\lambda_{\min}(K_3)} \), or \( \| e_i \| > \frac{H_i}{\lambda_{\min}(K_1)} \), or \( \| e_i \| > \frac{H_i}{\lambda_{\min}(K_3)} \), or \( \| \bar{V}_i \| > \frac{H_i}{\lambda_{\min}(K_1)} \), or \( \| e_i \| > \frac{H_i}{\lambda_{\min}(K_3)} \), or \( \| \bar{V}_i \| > \frac{H_i}{\lambda_{\min}(K_1)} \), or \( \| e_i \| > \frac{H_i}{\lambda_{\min}(K_3)} \), or \( \| \bar{V}_i \| > \frac{H_i}{\lambda_{\min}(K_1)} \), or \( \| e_i \| > \frac{H_i}{\lambda_{\min}(K_3)} \). Then by Lemma 1 we further obtain
\[
\frac{1}{2} \lambda_{\min}(K_1) \| \dot{\theta}_i \| + 0.2785 \left( 1 - m(\xi_1) \right) \rho_{i1} \| e_i \|^2 + m(\xi_1) \rho_{i1} \| e_i \|^2
\]
\[\| e_i \|^2 \leq \frac{1}{2} s^T Ps.
\]
It follows from (62) that the inequality (12) is satisfied with \( \varepsilon_i \leq \sqrt{\lambda_{\min}(K_1)} \), i.e., \( \theta_i \rightarrow \theta \rightarrow Ave(\theta) \).
This concludes the proof.
Remark 5. Many general adaptive neural controllers ensure the SGUUB tracking stability for marine vehicles [1,12–15,20–23], provided that the trajectory stays within the neural active region for all time. However, such a prerequisite is hard to verify in advance. Although [24] proposed a method that ensures GUUB for a class of strict-feedback systems, the switching function can not exactly achieve one, and therefore the neural controller and the robust controller must work together inside the neural active region to inevitably result in a certain degree of a waste of control energy. This problem is of great importance in AUV system since AUV works in the harsh underwater environment and energy supply is very limited. In this paper, the energy-efficient control strategy is mainly rely on switching function based NN adaptive laws (34) and (48), since that (61) can exactly reach zero, it can be seen that only the neural controller works inside the neural active region as shown in (27) with (30) and (44).

Remark 6. The main constraints between the design parameters are shown below (61), i.e., \( d_{13} \) and \( d_{15} \) should satisfy

\[
\frac{1}{2\lambda_{\text{min}}(K_{M1}) - 1/2d_{14}^2} < d_{13} < \frac{2d_{11} - \Delta_{1M}^2}{\chi_{12}d_{11}},
\]

\[
\frac{\sigma[S(M, \beta)]^2}{2\lambda_{\text{min}}(K_{M2})} < d_{15} < \frac{2d_{12} - \Delta_{2M}^2}{\chi_{12}d_{12}}.
\]

It is not difficult to let \( d_{13} \) and \( d_{15} \) to satisfy the above constraints by choosing \( K_{M1} \), \( d_{14}, d_{15}, \chi_{12}, \beta_k, K_{M2}, d_{12} \) and \( \chi_{12} \). Besides, all the other parameters mentioned in Theorem 1 are independent of the other parameters. Thus, the existence of them can be reasonable guaranteed and there is no contradiction between themselves.

Remark 7. The design parameters can be chosen based on the following principle. Since the boundness of the errors are concluded blow (61), the range of parameters to make the close-loop system GUUB are obtained. Following this principle, we can choose the design parameters to make its corresponding errors be small enough such as by choosing \( K_{M1}, d_{14}, d_{15}, K_{M2}, \xi, \gamma, \zeta \) and \( \zeta \) can be guaranteed converge to a small neighborhood of the origin. In addition, smaller control parameters would imply less power consumption. Regarding \( c_{13} \) and \( c_{12} \), it should be noted that the approximation domains of NNs can be assigned by the designers under the proposed design, which is a new type of adaptive path following control algorithm. \( c_{13} \) and \( c_{12} \) can be determined according to the designers’ experience and the practical requirements before the controllers are designed.

IV. SIMULATION EXAMPLE

This section illustrates the application of the previous results to two type of marine vehicles: AUVs and a MSV.

4.1 Cooperative path following of AUVs

The aim of this subsection is to verify that with the NN based DSC design we can robustly perform the cooperative path following problem for a formation of underactuated AUVs without the accurate knowledge of model. An example along given circle paths in 3D space is implemented with the proposed controllers. For simulation, the model parameters are the same as in [4].

Consider a group of three underactuated AUVs with a communication network that induces a graph with the Laplacian matrix \( L \) and connection weight \( B \)

\[
L = \begin{bmatrix}
1 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 1
\end{bmatrix},
B = \text{diag}[1, 0, 0].
\] (64)

The initial linear and angular velocities are \( u_i = 0, v_i = 0, w_i = 0, p_i = 0, q_i = 0, r_i = 0 \). The controller gains are selected as \( K_{a1} = \text{diag}[0.17, 0.02, 0.01], K_{a2} = \text{diag}[200, 400, 200], K_{a3} = \text{diag}[50, 50, 50], K_4 = \text{diag}[1, 1, 1], K_5 = \text{diag}[1, 1, 1], K_6 = \text{diag}[1, 1, 1], K_7 = \text{diag}[1, 1, 1]. \) The filter time constants are chosen as \( \chi_{1} = 0.5, \chi_{2} = 0.5. \) For simulation, the uncertainty of AUV 1 is given as

\[
f_{1v}(\cdot) = \begin{bmatrix} v_1^3 + 0.1u_1x_1w_1 + 0.02y_1, q_1z_1 + 0.2v_1^2 \end{bmatrix}^T.
\]

\[
f_{1w}(\cdot) = \begin{bmatrix} r_1^2 + 0.1\varphi_1, q_1^2y_1 + 0.02\varphi_1, p_1^2r_1 + 0.2\varphi_1^2 \end{bmatrix}^T.
\]

NN adaptive law parameters are chosen as \( k_{1W} = k_{2W} = 0.1, \Gamma_{1} = \Gamma_{3} = 100. \) Fig. 2 shows the tracking performance of three underactuated AUVs moving in 3D

![Fig. 2. Desired and actual vehicles’ paths. [Color figure can be viewed at wileyonlinelibrary.com]](image_url)
As can be seen, each AUV converges to its desired path after a transient process without explicit knowledge of the model.

Figs 3–5 are given to verify the performance of switching function based NN adaptive control. In Fig. 3, from the enlarged parts at steady stages we can see that the uncertainties are efficiently compensated by NNs. To verify the performance of switching, the initial states are chosen outside the NN approximation region ($\eta_1(0) = [2\cos(-3\pi/4), 2\sin(-3\pi/4) - 1, -9]^T, c_{11} = 20, c_{12} = 21.1$), and from Fig. 4 we can see that the switching function will equal to one when the time range is about 0-12s or 44-45s, which means the NN controllers are not working. The NN controllers will work as soon as they are leaving the stages of 0-12s and 44-45s and this can be observed in Figs 3–5.

Furthermore, in order to better demonstrate the superiority of the obtained control algorithm, a comparative simulation is presented as follows. We adopt traditional NN adaptive control scheme to the cooperative path following design and the control law is given by

$$u_i = b_3 \Lambda_i,$$

with

$$\Lambda_i = B_i^T(B_iB_i^T)^{-1}\left[-z_{i1} - (K_{i2} - S(\omega_i)M)\Phi_i + \dot{W}_{i1}^T \sigma(\xi_{i1})\right],$$

$$\dot{W}_{i1} = \Gamma_{wi}[-\sigma(\xi_{i1})\Phi_i^T - k_{wi} \dot{W}_{i1}],$$

$$u_{iio} = b_5^{-1}\left[-K_{i2}z_{i3} - b_iB_i^T\Phi_i + \dot{W}_{i2}^T \sigma(\xi_{i2})\right],$$

$$\dot{W}_{i2} = \Gamma_{wi}[-\sigma(\xi_{i2})z_{i3}^T - k_{wi} \dot{W}_{i2}].$$

For the purpose of a fair comparison, the design parameters are selected as the same as given in subsection 4.1 (below (63)). The simulation results for cooperative path following tracking errors $\|z_{i1}\|$ are shown in Fig. 6, from which we can see that the closed-loop stability is destroyed and the system output cannot follow the desired reference path since control law (65) and (66) can only guarantee the closed-loop system to be SGUUB, but the controller proposed in this paper can ensure the closed-loop to be GUUB.

The path variables coordination errors and speed tracking errors are shown in Figs 7 and 8, respectively. Fig. 9 shows that the speed information is well estimated by the proposed distributed speed estimate strategy while
AUV 2 and AUV 3 cannot obtain the common reference speed $v_0 = 0.1$. From Figs 2, 7, 8 and 9 we can observe that with the analytic tool of graph theory, the desired cooperative behavior is achieved, i.e., the speed and path variables are synchronized to each AUV owing to the proposed cooperative path following control law where the distributed speed estimate strategy is incorporated.

4.2 Path following control of MSV

It is worth to note that the proposed control algorithm can not only deal with underactuated AUV but also fully-actuated MSV. Consider the following kinematic and kinetic equations of MSV described in [39]

$$\dot{\eta} = J(\psi)v,$$

$$M \ddot{v} = \tau - C(v)v - D(v)v - \Delta(v),$$

where $\eta = [x, y, \psi]^T \in \mathbb{R}^3$ represents the position-altitude vector in the earth-fixed reference frame. $v = [u, v, r]^T \in \mathbb{R}^3$ denotes the velocity vector in the body-fixed reference frame. $\tau = [\tau_u, \tau_v, \tau_r]^T \in \mathbb{R}^3$ is the control input vector with $\tau_u$ being the surge force, $\tau_v$ being the sway force, and $\tau_r$ being the yaw moment. $M = M^T \in \mathbb{R}^{3 \times 3}$ is the system inertia matrix. $C \in \mathbb{R}^{3 \times 3}$ is the skew-symmetric matrix of Coriolis. $D \in \mathbb{R}^{3 \times 3}$ is the nonlinear damping matrix. $\Delta \in \mathbb{R}^3$ represents the unmodeled dynamics. The rotation matrix $J(\psi)$ is given by
The proposed switching based NN adaptive path following controller is applied on the fully-actuated MSV and the control law takes the form of

$$\tau_1 = -z_1 - K_2 z_2 + (1 - m(\xi)) \left[ -\dot{W}_1^T \sigma(\xi) - \tanh \left( \frac{z_2^T}{\rho} \right) \dot{\epsilon}_M \right]$$

$$+ m(\xi) \left[ -F(\xi) \tanh \left( \frac{z_2^T}{\rho} \right) \dot{\epsilon}_M \right]$$

with the adaptive laws being

$$\dot{\hat{\epsilon}}_M = \Gamma \left[ (1 - m(\xi)) \sigma(\xi) z_2^T - k_W \dot{W}_1 \right],$$

$$\dot{\hat{\Theta}} = \Gamma (\Xi - k_\Theta \hat{\Theta}),$$

where

$$\Xi = \left[ \begin{array}{c} (1 - m(\xi)) \tanh(z_2^T) z_2^T \\ m(\xi) F(\xi) \tanh(z_2^T) z_2^T \end{array} \right], \quad \hat{\Theta} = \left[ \begin{array}{c} \dot{\epsilon}_M \\ \dot{f}_M \end{array} \right], \quad z_1$$

and $z_2$ are the errors defined in the recursive design procedure [43].

For the purpose of a fair comparison, the traditional NN adaptive controller proposed in [12,42,43] takes the form of

$$\tau_2 = -z_1 - K_2 z_2 + \dot{W}_2^T \sigma(\xi), \quad (67)$$

with the adaptive law being

$$\dot{\hat{\epsilon}}_M = \Gamma_w \left[ -\sigma(\xi) z_2^T - k_W \dot{W}_2 \right].$$

The control parameters are chosen as $K_2 = \text{diag } [25, 34, 3], k_w = 0.3, \Gamma = 100$. Fig. 10 shows that MSV cannot follow the desired path under control law (67) since traditional NN adaptive controller only can guarantee the closed-loop system to be SGUUB, and when the state variables move outside the neural active region, the control accuracy will be dramatically decreased which even lead to instability of the closed-loop system, but the switching based adaptive controller proposed in this paper can ensure the closed-loop to be GUUB with a better control performance.

V. CONCLUSIONS

This paper addresses the cooperative path following problem of multiple underactuated AUVs with parametric modeling uncertainty and unmodeled dynamics. The cooperative path following controllers are devised basing on the NN-based-DSC technique. A smoothly switching function is originally proposed for the problem of cooperative path following so that an energy-efficient control scheme is developed. Moreover, a distributed speed estimate scheme is developed building on graph theory and Lyapunov theory, by which the restriction on the common reference speed being available to all cooperating AUVs are explicitly relaxed. It has been shown that closed-loop signals under the proposed algorithm are GUUB, and the steady-state compact set to which the error signals converge can be made small through appropriate choice of control design parameters. Simulation results showed the effectiveness of the proposed control strategy.

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