A typo in the Paterson-Wegman-de Champeaux algorithm

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Abstract

We investigate the Paterson-Wegman-de Champeaux linear-time unification algorithm. We show that there is a small mistake in the de Champeaux presentation of the algorithm and we provide a fix.

1 Introduction

In this paper we investigate the Paterson-Wegman algorithm \cite{Paterson1971}, as improved by de Champeaux \cite{deChampeaux1994}. The algorithm has linear-time complexity. In Figure 1 we present the pseudo-code proposed by de Champeaux. We add line numbers and we make some cosmetic changes, which do not affect the algorithm logic. For example, we omit the else branch of an if statement whose then branch ends with an exit statement. The de Champeaux presentation of the algorithm ends with a post-processing step, described in Figure 6.

The issue we identify is that the post-processing step enters an infinite loop. The infinite loop is caused by a bug in the occurs-check test. We give an input producing an infinite loop in the next section. The bug can be fixed syntactically by indenting an assignment statement, i.e., moving it inside the inner code block.

This issue was noticed and fixed by Erik Jacobsen \cite{Jacobsen1995} (see footnote on Page 34). However, in this paper we present and analyze an troublesome input in detail.

2 Troublesome example

We show how the de Champeaux algorithm works when trying to unify the terms $X$ and $f(X)$. The algorithm starts with the DAG representation of the two terms, which we show in Figure 2. As the two terms have maximal sharing between them, there is only one node labeled $X$. There are two roots, each corresponding to one of the terms to be unified. We use simple arrows to denote the relation between parent and child nodes of the DAG.

The algorithm creates links (undirected edges) between nodes that should be in the same equivalence relation. We use dashed lines to denote the links created by the algorithm. The algorithm also maintains stacks (shown graphically on the right) and a set of pointers from nodes to nodes, which are represented by two-headed arrows.

The algorithm starts by creating a link between $X$ and $f(X)$ (Figure 3).

The next step is to call Finish on all functional nodes (line 3). In this example we have only one functional node, $f$. At this step, we have $r = f(X)$. Because $\text{complete}(r)$ is marked as false and $\text{pointer}(r)$ is NIL, we jump straight to line 12, where we set $\text{pointer}(r)$ to $r$ and push it to the stack (Figure 4).

At the first iteration of the while loop, at line 15, we have $s = r$. As $s$ and $r$ have the same function symbol, we do not enter the if statement at line 16. As $s$ does not have any parent, we do not enter the if statement at line 18. The variable $s$ has a link to $X$ and, as a result, at line 21 we have $r = f(X), s = f(X), t = X$. The variable $t$ is not marked complete and is
Procedure Solver($u, v$):

1. Create link $(u, v)$
2. While there is a function node $r$, Finish($r$)
3. While there is a variable node $r$, Finish($r$)
4. BUILD-SIGMA(SIGMA)

Procedure Finish($r$):

1. if complete($r$) then
2. Exit
3. if pointer($r$) ≠ NIL then
4. Exit with failure
5. Create new pushdown stack with operations Push(*) and Pop
6. pointer($r$) := $r$
7. Push($r$)
8. while stack ≠ NIL do
9. $s$ := Pop
10. if $r, s$ have different function symbols then
11. Exit with failure
12. FOR-EACH parent $t$ of $s$ do
13. Finish($t$)
14. FOR-EACH link $(s, t)$ do
15. if Complete($t$) or $t = r$ then
16. Ignore $t$
17. else if pointer($t$) = NIL then
18. pointer($t$) := $r$
19. Push($t$)
20. else if pointer($t$) ≠ $r$ then
21. Exit with failure
22. else
23. Ignore $t$ // (since $t$ is already on STACK)
24. if $s ≠ r$ then
25. if Variable($s$) then
26. Subs($s$) := $r$
27. Add $s$ to SIGMA (input to BUILD-SIGMA)
28. else
29. Create links{$j$th son($r$), $j$th son($s$) | $1 ≤ j ≤ q$}
30. Complete($s$) := true
31. Complete($r$) := true

Figure 1: Paterson-Wegman algorithm as presented by de Champeaux. We add line numbers and we make some cosmetic changes.

Figure 2: The data structures at the start of the algorithm.
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Figure 3: The data structures representation after adding the first link.

Figure 4: The data structures after pushing the first functional node to the stack.

not equal to $r$, so we enter the if statement at line 24, set $pointer(t)$ to be $r$ and push it on the stack (Figure 5).

After this step, we jump straight to line 30, because there is only one link. We do not enter the if statement at line 30 because $s$ equals $r$. Then we set $complete(s)$ to true at line 36. Note that $s$ is still $f$. In the next iteration of the while loop at line 14 we have $s = X$. Because of the shared structure of common variables, we call $Finish(f)$ at line 19, but $complete(f)$ is true, so we exit this function call at line 8. Next follows the loop at line 20. We have the initial link $X$ and $f(X)$, so in this case $t = f(X)$, but $complete(t)$ is true and the node $t$ is ignored (line 22). Moving on, on line 30, we enter the if statement and jump to line 32, because $s = X$, which is a variable. At line 32 we set $subs(X) = f(X)$ and at line 33 we add $X$ to $SIGMA$. Then, at line 36, we set $complete(s)$ to true. The stack is now empty, so we go to the line 38 where we set $complete(r)$ to true (this is the second time we set $complete(s)$ to true). The execution of $Finish$ is done and we call $Finish$ on all variable nodes. We have only one variable, $X$, which has $complete(X)$ set to true, so we immediately return. Now we call $BUILD-SIGMA$. One important observation is that we finished the main algorithm and the occurs-check at line 9 did not happen.

In Figure 6 we show the implementation of $BUILD-SIGMA$. The function $BUILD-SIGMA$ creates a substitution from a ordered substitution in linear time. By running the algorithm, we conclude that it enters an infinite loop. In short, below are order of the function calls.

1. $BUILD-SIGMA(list(X))$ - at line 1
2. $EXPLORE-VARIABLE(X)$ -at line 3
3. $DESCEND(f(X))$ - at line 7
4. $EXPLORE-ARGUMENTS(list(X))$ - at line 21

Figure 5: The data structures after adding the variable $X$ to the stack.
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Procedure \textsc{build-sigma}(\textit{list-of-variables}):\
\begin{algorithmic}
\State \textbf{for-each} \textit{variable} $x_i$ in \textit{list-of-variables} \textbf{do}\
\State \hspace{1em} Add to final substitution $x_i \rightarrow \textsc{explore-variable}(x_i)$
\end{algorithmic}

Function \textsc{explore-variable}($x_i$):
\begin{algorithmic}
\If {$\text{ready}(x_i) \neq \text{NIL}$}
\State \textbf{exit} with $\text{ready}(x_i)$
\EndIf
\State \textbf{out} := $\textsc{descend}(\text{subs}(x_i))$
\If {$\text{out} = \text{NIL}$}
\State \textbf{out} := $x_i$
\State \textbf{ready}(x_i) := \textbf{out}$
\State \textbf{exit} with \textbf{out}$
\EndIf
\end{algorithmic}

Function \textsc{descend}($u_i$):
\begin{algorithmic}
\If {$u_i = \text{NIL}$}
\State \textbf{exit} with \text{NIL}$
\EndIf
\If {$\text{variable}(u_i)$}
\State \textbf{exit} with \textsc{explore-variable}(u_i)$
\EndIf
\If {$\text{constant}(u_i)$}
\State \textbf{exit} with $u_i$
\EndIf
\If {$\text{ready}(u_i)$}
\State \textbf{exit} with $\text{ready}(u_i)$
\EndIf
\State \textbf{out} := $\textsc{explore-arguments}(\text{arguments-of}(u_i))$
\If {$\text{out} \neq \text{arguments-of}(u_i)$ OR \text{out} \neq \text{arguments-of}(u_i)}$
\State \textbf{exit} with Cons($\text{head-of}(u_i)$, out)$
\EndIf
\State \textbf{ready}(u_i) := Cons($\text{head-of}(u_i)$, out)$
\State \textbf{exit} with $\text{ready}(u_i)$
\end{algorithmic}

Function \textsc{explore-arguments}($\textit{list-of-arguments}$):
\begin{algorithmic}
\If {$\text{list-of-arguments} = \text{NIL}$}
\State \textbf{exit} with \text{NIL}$
\EndIf
\State 1st-new := $\textsc{descend}(\text{1st}($\text{list-of-arguments}$))$
\State tail-new := $\textsc{explore-arguments}(\text{tail}($\text{list-of-arguments}$))$
\If {1st-new \neq 1st($\text{list-of-arguments}$) OR tail-new \neq \text{tail}($\text{list-of-arguments}$)}
\State \textbf{exit} with Cons(1st-new, tail-new)$
\EndIf
\State \textbf{exit} with $\text{list-of-arguments}$
\end{algorithmic}

Figure 6: Post-processing step described by de Champeaux.
5. \textsc{Descend}(X) - at line 34
6. \textsc{Explore-Variable}(X) - at line 16

The \textit{Ready} variable is not used. As a result, we enter an infinite loop.

3 Fixing the de Champeaux algorithm

The issue with the pseudo-code presented by de Champeaux is on line 36 in the \textit{Finish} procedure. Based on the pseudo-code by Paterson-Wegman, \textit{Complete(s)} should be set to true inside the \textit{if} statement at line 36. We propose a fixed version in Figure 7. This change fixes the pseudo-code and the algorithm remains linear time and there are no further issues.

4 Conclusion

We investigate the Paterson-Wegman linear-time unification algorithm as improved by de Champeaux. We show an example where the occurs-check test fails to work as expected and results in an infinite loop in the post-processing step. We show that the issue is caused by a misindented statement (line 36) in the pseudo-code. Once the statement is properly indented, the algorithm is correct and works in linear-time as claimed.

References

[1] Dennis de Champeaux. About the Paterson-Wegman linear unification algorithm. \textit{J. Comput. Syst. Sci.}, 32(1):79–90, February 1986.
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[3] M. S. Paterson and M. N. Wegman. Linear unification. In \textit{Proceedings of the Eighth Annual ACM Symposium on Theory of Computing}, STOC ’76, pages 181–186, New York, NY, USA, 1976. ACM.
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6 Procedure Finish(r):
7 if complete(r) then
8 Exit
9 if pointer(r) ≠ NIL then
10 Exit with failure
11 Create new pushdown stack with operations Push(*) and Pop
12 pointer(r) := r
13 Push(r)
14 while stack ≠ NIL do
15 s := Pop
16 if r, s have different function symbols then
17 Exit with failure
18 FOR-EACH parent t of s do
19 Finish(t)
20 FOR-EACH link (s, t) do
21 if Complete(t) or t = r then
22 Ignore t
23 else if pointer(t) = NIL then
24 pointer(t) := r
25 Push(t)
26 else if pointer(t) ≠ r then
27 Exit with failure
28 else
29 Ignor e t
30 if s ≠ r then
31 if Variable(s) then
32 Subs(s) := r
33 Add s to SIGMA
34 else
35 Create links{jth son(r), jth son(s) | 1 ≤ j ≤ q}
36 Complete(s) := true
37 Complete(r) := true

Figure 7: On the left-hand side we present the initial pseudo-code due to de Champeaux. On the right-hand side we propose the corrected version. The only difference is at line 36 (note the indentation level).