Relevance of final state interactions in $\eta' \to \eta\pi\pi$ decays

J.J. Sanz-Cillero

$^a$ Istituto Nazionale di Fisica Nucleare INFN, Sezione di Bari,
Via Orabona 4, I-70126 Bari, Italy

A study of the $\eta' \to \eta\pi\pi$ Dalitz plot distribution is presented in this talk. The size of the branching ratio is properly understood within $U(3)$ Chiral Perturbation Theory and Resonance Chiral Theory, in the framework of the $1/N_C$ expansion. Nonetheless, unitarity effects must be incorporated in order to achieve an appropriate description of the Dalitz slope parameters. After taking the final state interactions into account, our predictions become now in agreement with the available experimental measurements, although some clear differences show up with respect to previous theoretical estimates.

11.15.Pg, 12.39.Fe

Chiral Lagrangians, $1/N_C$ expansion, eta(958)

1. Introduction

In this talk we present the results obtained for the $\eta' \to \eta\pi\pi$ decay within the framework of chiral Lagrangians and the $1/N_C$ expansion. Since $G$-parity prevents intermediate vector mesons to contribute, the scalars play a crucial role in this decay, specially the $f_0(600)$ (or $\sigma$) resonance even though the $a_0(980)$ is also present and, indeed, dominant for the branching ratio.

This decay is used here to test Chiral Perturbation Theory ($\chi$PT) and its extensions such as large-$N_C$ $U(3)$-$\chi$PT and Resonance Chiral Theory ($R\chi$T), eventually providing predictions for some relevant hadronic parameters. Recently, the GAMS-4$\pi$ and VES Collaborations have measured the related Dalitz plot parameters which characterize the shape of the decay, complementing older results [8]. New improved measurements are foreseen at KLOE-2, Crystal Ball, Crystal Barrel and maybe WASA.

On the theory side, the $\eta' \to \eta\pi\pi$ decays have been studied within an effective chiral Lagrangian approach in which the lowest lying scalar mesons are combined into a nonet [9] and, more recently, within the framework of $U(3)$ chiral effective field theory in combination with a coupled-channels approach. Other older analyses based on chiral symmetry can be found in [11].

In the isospin limit considered all along the work the charged and neutral decay amplitudes coincide, although the neutral decay rate has an extra $1/2$ factor due to phase-space symmetry. The Dalitz plot distribution for the charged decay can be described by the two kinematic variables $X = \sqrt{2} (T_{\pi^+} - T_{\pi^-})$ and $Y = \frac{m_{\pi^+}^2 + 2m_{\pi^-}^2 - T_{\pi^+}}{m_{\pi^-}} - 1,$
where \( T_{\pi^+} \) denote the kinetic energies of mesons in the \( \eta' \) rest frame: \( T_\eta = \frac{(m_\eta - m_\pi^2)^2 - s}{2m_\eta} \), \( T_{\pi^+} = \frac{(m_\pi - m_\pi^2)^2 - u}{2m_\pi} \), \( T_{\pi^-} = \frac{(m_\pi - m_\pi^2)^2 - t}{2m_\pi} \) and \( Q = T_\eta + T_{\pi^+} + T_{\pi^-} = m_\eta - m_\pi - 2m_\pi \). The Mandelstam variables \( s \equiv (p_{\pi^+} + p_{\pi^-})^2 \), \( t \equiv (p_{\eta'} + p_{\pi^-})^2 \) and \( u \equiv (p_{\eta'} + p_{\pi^+})^2 \) have been employed here, which obey the relation \( s + t + u = m_\eta^2 + m_\pi^2 + 2m_\pi^2 \). The squared modulus of the decay amplitude can be then expanded around the center of the Dalitz plot [1]:

\[
|M(X, Y)|^2 = |N|^2[1 + (aY + dX^2)] + (bY^2 + \kappa_{21}X^2Y + \kappa_{40}X^4) + \cdots
\]

Odd terms in \( X \) are forbidden due to charge conjugation and the symmetry of the wave function.

2. Large-\( N_C \) \( \chi \)PT

Large-\( N_C \) Chiral Perturbation Theory is an effective field theory where, due to the large-\( N_C \) limit \( (N_C \to \infty) \) [2], the singlet axial current is also conserved, the chiral symmetry is enlarged to \( U(3) \) and the \( \eta' \) becomes the ninth Goldstone boson [1]. A simultaneous expansion in powers of momenta, quark masses and \( 1/N_C \) is devised, such that \( p^2, m_{u,d,s}, 1/N_C = \mathcal{O}(\delta) \) [4]. At NLO, one finds the prediction [1]

\[
\mathcal{M}_{\eta' \to \eta \pi^+ \pi^-} = c_{qq} \times \frac{1}{F_\pi^2} \left[ \frac{m_\eta^2}{2} + \frac{4c_d c_m m_\pi^4}{F_\pi^2 M_S^2} \right] + \frac{1}{F_\pi^2} \left[ c_d(t - m_\eta^2 - m_\pi^2 + 2c_m m_\pi^2) \right] \times \frac{M_S^2 - t}{M_S^2 - u} + \frac{1}{F_\pi^2} \left[ c_d(u - m_\eta^2 - m_\pi^2 + 2c_m m_\pi^2) \right] \times \frac{M_S^2 - u}{M_S^2 - s} + \frac{1}{F_\pi^2} \left[ c_d(s - m_\eta^2 - m_\pi^2 + 2c_m m_\pi^2) \right] \times \frac{M_S^2 - s}{M_S^2 - t} + \cdots
\]

The largest contribution comes from the \( c_d \) terms, which are proportional to the external momenta. Everything else is proportional to \( m_\pi^2 \), being suppressed. If one now performs the chiral expansion of the \( \chi PT \) amplitude at low-energies \( (s, t, u, m_\pi^2 \ll M_S^2) \), the large-\( N_C \) ChPT result [2] is recovered up to contributions subleading in \( 1/N_C \) [15].

We used \( M_S = 980 \text{ MeV} \) for the scalar multiplet mass, the resonance coupling \( c_m \) from the high-energy scalar form-factor constraint \( c_m = \frac{4}{c_d} \) [13] (not very relevant as it always appears multiplied by a \( m_\pi^2 \) factor) and the value \( c_d = \)
28.4 MeV fixed through the experimental branching ratio, \( \mathcal{B}(\eta' \rightarrow \eta\pi^+\pi^-) = (43.2 \pm 0.7)\% \) \[12\].

In addition to the contribution from scalar resonances, one might also consider the impact of \( J = 2 \) resonances. Still, as the mass of the lightest tensor multiplet is roughly \( M_{J^P} = 1.2 \) GeV, one may just consider its leading effect in \( 1/M_{J^P}^2 \) rather than the whole non-local resonance propagator structure. Thus, it induces a contribution that has the form of the \((3L_2 + L_0)\) term from Eq. \[2\] \[1\], where the tensor resonance contributions to the \(\mathcal{O}(p^4)\) LECs \(\left(3L_2^2 + L_0^2 = \varrho_0^2 / 3M_{J^P}^2 = 0.16 \cdot 10^{-3}\right)\) were estimated in Ref. \[14\], after imposing high energy constraints on the \(\pi\pi\)-scattering.

### 4. Unitarization

The narrow width of the \(a_0(980)\) \[12\] and the smallness of the \(\eta\pi\) scattering-length \[15\] seem to point out the little relevance of the rescattering in this channel. Thus, in the elastic region (no other channel opens up in the \(\eta'\)-decay phase-space), one has the approximate \(s\)-channel unitarity relation for the \(\pi\pi\) rescattering,

\[
\text{Im} \mathcal{M}_J(s) = \rho(s) \mathcal{T}_J^0(s)^* \mathcal{M}_J(s),
\]

where the decay amplitude \(\mathcal{M}(s, t, u)\) has been decomposed into partial waves \(\mathcal{M}_J(s)\) in the \(\pi\pi\) angle \(\theta_\pi\) \[1\], \(\rho(s) = \sqrt{1 - 4m_\pi^2 / s}\) and \(\mathcal{T}_J^{l=0}(s)\) is the isoscalar \(\pi\pi\) partial-wave scattering amplitude. The absorptive cuts in the \(t\) and \(u\) \(\eta\pi\)-channels have been neglected in \[4\].

There are various options for the reconstruction of the unitarized amplitude as the optical theorem only refers to the absorptive part of the amplitude. In our opinion, the \(N/D\)-method \[16\] is the most reliable one, as it also incorporates the real part of the logarithm that arises in the two-propagator Feynman integral \(B_0(s, m_\pi^2, m_\pi^2)\) at one loop, not only its imaginary part \(\rho(s)/16\pi\):

\[
\mathcal{M}(s, t, u)^{N/D} = \sum_J 32\pi (2J + 1) P_J(\cos \theta_\pi) \times \frac{\mathcal{M}_J(s)^{\text{tree}}}{1 - 16\pi B_0(s, m_\pi^2, m_\pi^2) \mathcal{T}_J^0(s)^{\text{tree}}},
\]

with the Legendre polynomials \(P_J(x)\), the partial wave decomposition of the previously computed tree-level amplitudes \(\mathcal{M}_J(s)^{\text{tree}}\) and \(\mathcal{T}_J^0(s)^{\text{tree}}\), and \(16\pi^2 B_0(s, m_\pi^2, m_\pi^2) = C - \rho(s) \ln \frac{L(s) + 1}{\rho(s) - 1}\). Actually, the integral \(B_0\) is ultraviolet divergent and has a local indetermination \(C\) (denoted through \(a^{SL}(s_0)\) in the \(N/D\) analysis \[16\]) which requires an extra renormalization condition. We will fix it by means of the experimental range for the Dalitz-parameter \(a = -0.098(48)\) \[16, 17\].

### 5. Conclusions and Discussion

In this talk we have presented some new results on the \(\eta' \rightarrow \eta\pi\pi\) decay within the large-\(N_C\) \(\chi PT\) and \(\chi T\) frameworks \[1\]. In both of them, the order of magnitude of the experimental branching ratio is conveniently understood. Furthermore, we obtained successful predictions for the Dalitz slope parameters based on the \(s\)-channel unitarization of our tree-level amplitudes \[1\]. These have been summarized in Table \[1\] and compared to other theoretical predictions \[10\] and experimental measurements \[6, 7\]. Preliminary results from BES-III seem to provide much more precise determinations: \(a = -0.047(12), b = -0.068(21)\) and \(d = -0.073(13)\) \[17\]. This clearly would favour our determinations with respect to previous theoretical studies which predict a very small or positive Dalitz parameter \(d\) \[10\]. Future experiments will be able to discern what is the most convenient framework for the study of this and other \(\eta'\) decays.
\[ U(3) - \chi PT \quad R\chi T \quad Th. \quad Exp. \quad Exp. \]

|   | \[Y\]  | \[Y^2\]  | \[X^2\]  | \[X^2Y\] | \[X^4\]  |
|---|---------|---------|---------|-----------|---------|
| a | -0.098(48) | -0.098(48) | -0.127(9) | -0.116(11) | -0.127(16)(8) |
| b | -0.0497(8) | -0.0332(5) | -0.049(36) | -0.042(34) | -0.106(28)(14) |
| d | -0.092(8) | -0.0718(3) | +0.011(21) | +0.010(19) | -0.082(17)(8) |
| κ₂₁ | 0.003(2) | -0.009(2) | — | — | — |
| κ₄₀ | 0.0022(4) | 0.0013(1) | — | — | — |

Table 1

The results for the \(N/D\)-unitarized \(\chi PT\) amplitude are given in the second column \[1\]. In the third column we consider the \(N/D\)-unitarization of \(R\chi T\), including the contribution from \(J = 2\) resonances \[1\]. In both cases, the \(a\) parameter was taken as input. The fourth and fifth columns provide previous theoretical predictions for, respectively, the \(\eta' \rightarrow \eta\pi^0\pi^0\) and \(\eta' \rightarrow \eta\pi^+\pi^-\) decays \[10\]. The last two columns contain the experimental measurements from GAMS-4\(\pi\) \[6\] and VES \[7\], respectively.

REFERENCES

1. R. Escribano et al., [arXiv:1011.5884 [hep-ph]].
2. G. ’t Hooft, Nucl. Phys. B 72 (1974) 461; Nucl. Phys. B 75 (1974) 461; E. Witten, Nucl. Phys. B 160 (1979) 57.
3. S. Weinberg, Physica A 96 (1979) 327; J. Gasser and H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465.
4. R. Kaiser and H. Leutwyler, Eur. Phys. J. C 17, 623 (2000) hep-ph/0007101; H. Leutwyler, Nucl. Phys. Proc. Suppl. 64 (1998) 223.
5. G. Ecker et al., Nucl. Phys. B 321 (1989) 311.
6. A. M. Blik et al., Phys. Atom. Nucl. 72 (2009) 231 [Yad. Fiz. 72 (2009) 258].
7. V. Dorofeev et al., Phys. Lett. B 651 (2007) 22.
8. D. Alde et al. Phys. Lett. B 177, 115 (1986); R. A. Briere et al. [CLEO Collaboration], Phys. Rev. Lett. 84 (2000) 26.
9. A. H. Fariborz and J. Schechter, Phys. Rev. D 60 (1999) 034002.
10. B. Borasoy and R. Nissler, Eur. Phys. J. A 26 (2005) 383.
11. J. A. Cronin, Phys. Rev. 161 (1967) 1483; J. Schwinger, Phys. Rev. 167 (1968) 1432; P. Di Vecchia et al., Nucl. Phys. B 181 (1981) 318; P. Herrera-Siklody, [arXiv:hep-ph/9902446]; J. Schechter and Y. Ueda, Phys. Rev. D 3 (1971) 2874 [Erratum-ibid. D 8 (1973) 987]; C. A. Singh and J. Pasupathy, Phys. Rev. Lett. 35 (1975) 1193 [Erratum-ibid. 35 (1975) 1748].
12. K. Nakamura et al. [Particle Data Group], J. Phys. G 37 (2010) 075021.
13. M. Jamin et al., Nucl. Phys. B 622 (2002) 279.
14. G. Ecker and C. Zauner, Eur. Phys. J. C 52 (2007) 315-323.
15. B. Kubis, EPJ Web Conf. 3 (2010) 01008.
16. J.A. Oller and E. Oset, Phys. Rev. D 60 (1999) 074023; Z.H. Guo, J.A. Oller and J. Prades, in preparation.
17. C. P. Shen (for the BES Coll.), Xth Intern. Conf. on Heavy Quarks and Leptons, 2010. Private communications.