Scaling of Elliptic Flow, Recombination and Sequential Freeze-Out of Hadrons in Heavy-Ion Collisions

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The scaling properties of elliptic flow of hadrons produced in ultrarelativistic heavy-ion collisions are investigated at low transverse momenta, \( p_T \lesssim 2 \text{ GeV} \). Utilizing empirical parameterizations of a thermalized fireball with collective-flow fields, Resonance Recombination Model (RRM) is employed to describe hadronization via quark coalescence at the hadronization transition. We reconfirm that RRM converts equilibrium quark distribution functions into equilibrated hadron spectra including the effects of space-momentum correlations on elliptic flow. This provides the basis for a controlled extraction of quark distributions of the bulk matter at hadronization from spectra of multi-strange hadrons which are believed to decouple close to the critical temperature. The resulting elliptic flow from empirical fits at RHIC exhibits transverse kinetic-energy and valence-quark scaling. Utilizing the well-established concept of sequential freeze-out we find that the scaling at low momenta can be extended to bulk hadrons (\( \pi, K, p \)) at thermal freeze-out, and thus result in an overall description compatible with both equilibrium hydrodynamics and quark recombination.

I. INTRODUCTION

Ultrarelativistic heavy-ion collisions (URHICs) enable the creation and study of superdense strongly interacting matter, possibly associated with the formation of novel phases where partons deconfine and chiral symmetry is restored [1]. One major finding by the experimental program at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory is the large azimuthal anisotropy of hadron transverse-momentum \( p_T \) spectra in non-central nuclear collisions, the so-called elliptic flow [2]. It is quantified by the second harmonic coefficient, \( v_2(p_T) \), in the Fourier expansion of the azimuthal angle, \( \phi \), for the \( p_T \)-spectra of hadrons.

Experimental measurements suggest that the behavior of \( v_2(p_T) \) may be classified into three regimes. At high transverse momentum, \( p_T \gtrsim 6 \text{ GeV} \), the azimuthal anisotropy is believed to be due to the path-length dependence of high-energy partons as they traverse the hot medium and lose energy. We will not be concerned any further with this mechanism in the present work. At low transverse momentum, \( p_T \lesssim 2 \text{ GeV} \), \( v_2 \) increases with \( p_T \) essentially linearly but with a delayed onset for hadrons with increasing mass, giving rise to the so-called mass ordering of elliptic flow [3]. This regime is well described by hydrodynamic simulations and thus indicates a large degree of equilibration, encompassing more than 90\% of the observed particles [4-8]. At intermediate transverse momenta, \( 2 \text{ GeV} \lesssim p_T \lesssim 6 \text{ GeV} \), \( v_2(p_T) \) saturates and exhibits a remarkable scaling between baryons and mesons [9],

\[
v_2(p_T) = n_q v_2^{(q)} (p_T/n_q), \tag{1}
\]

where \( n_q \) denotes the number of valence quarks contained in hadron \( h \). Equation (1), also known as “constituent-quark number scaling” (CQNS), has been successfully described in the framework of quark coalescence models, where quarks recombine into hadrons at the transition from partonic to hadronic matter [10]. In line with Eq. (1), these models thus imply that the observed hadron elliptic flow can be reduced to a universal quark elliptic flow at hadronization. This assertion, however, requires several simplifying assumptions. First, interactions in the subsequent hadronic evolution (between hadronization at \( T_c \approx 170 \text{ MeV} \) and kinetic freeze-out at \( T_k \approx 110 \text{ MeV} \)) are neglected. Furthermore, Eq. (1) can only be derived in instantaneous quark coalescence models where energy conservation is violated and narrow hadron wave functions in momentum space are utilized. In addition, the underlying quark distribution functions are assumed to factorize in coordinate and momentum space, implying that the elliptic flow of the system is implemented point-by-point (i.e., locally), rather than through a space-dependent flow field which is a consequence of a collectively expanding source. The implementation of realistic space-momentum correlations into quark coalescence models has indeed been proved difficult [11]. Finally, quark coalescence models typically do not comply with the principle of detailed balance, implying that the proper equilibrium limit cannot be established. This hampers attempts to extend the description into the low-\( p_T \) regime where thermalization is believed to prevail. This has become a more pressing issue once it was realized that CQNS can be generalized to encompass both low and intermediate \( p_T \) if \( v_2 \) is plotted versus transverse kinetic energy, \( KE_T \equiv m_T - m \), where \( m_T = (p_T^2 + m^2)^{1/2} \) is the hadron’s transverse mass [12-17]. In this representation, elliptic-flow data for all ob-
served hadrons fall onto one universal curve for $v_2/n_q$ versus $KE_T/n_q$ \[12, 13\].

Progress on the above issues has been recently reported by reformulating the quark-coalescence process at hadronization in terms of resonant $q\bar{q} \rightarrow M$ scattering ($M$: meson) \[12, 13\]. Implementing the underlying cross section into a Boltzmann transport equation (e.g., in Breit-Wigner approximation) not only guarantees energy-momentum conservation but also satisfies detailed balance and thus leads to the correct thermal-equilibrium limit. It has been verified \[13\] that the meson spectra resulting from the Resonance Recombination Model (RRM) are compatible with CQNS, Eq. (1), in the factorized (local) approximation to $v_2(p_T)$. A further step has been taken in Ref. \[20\] where the RRM has been applied to more realistic quark distribution functions as generated by relativistic Langevin simulations within an expanding QGP fireball \[21\] for semi-central Au-Au collisions at RHIC. Since the Brownian-motion approximation underlying the Langevin process is only reliable for relatively massive and/or high-momentum particles, $m_T \gg T$, the results have been restricted to charm and strange quarks. Under inclusion of the full space-momentum correlations imprinted on the quark distributions by the hydro-like flow fields of the expanding fireball, the elliptic flow of $\phi$ and $J/\psi$ mesons was found to exhibit CQNS in $KE_T$ from 0 to $\approx 3$ GeV, which encompasses both low and intermediate $p_T$, i.e., the quasi-thermal and kinetic regimes. However, several important questions remain, e.g., the manifestation of CQNS and $KE_T$ scaling for bulk (light) particles in the low-$p_T$ (thermal) regime, the robustness and generality of the flow fields utilized in the fireball simulations or the role of reinteractions in the hadronic phase. A thorough understanding of these issues, in connection with a quantitative description of hadron data, is hoped to ultimately enable the extraction of quark distribution functions just prior to the conversion to hadronic degrees of freedom, and thus to quantitatively establish the presence of a collectively expanding partonic source in URHICs.

In the present paper we conduct investigations of scaling properties of hadron spectra by focusing on the low-$p_T$ regime coupled with the assumption of complete kinetic equilibration. We will utilize the RRM to convert locally equilibrated quark distribution functions into hadron spectra using a blast-wave type quark source with realistic flow fields at the hadronization transition (including space-momentum correlations characteristic of hydrodynamic simulations). Contrary to earlier work \[20\], we do not attempt to generate the flow fields from a dynamic evolution, but rather extract them from empirical fits to hadron $p_T$-spectra and elliptic flow. In fact, after verifying that resonance recombination converts collective, locally equilibrated quark distribution functions into equilibrium hadron distributions with identical (space-momentum dependent) collective properties, one can adopt the source parametrization at the hadron level and thus readily extend the description to baryons. For this part of the investigation we focus on multi-strange hadrons (i.e., hadrons containing at least 2 strange and/or anti-strange quarks) which are believed to kinetically decouple close to the hadronization transition. This notion is supported by empirical blast-wave fits to RHIC and SPS data \[1, 22, 23\], as well as by the putative absence of resonances that these hadrons could form in interactions with bulk particles such as pions, kaons or nucleons (hadronic resonances drive the collective expansion of the hadronic phases in URHICs). We refer to hadrons decoupling close to $T_c$ as “group-I” particles, which are thus the prime candidates to infer properties of the quark phase just above the critical temperature. On the other hand, the most prominent bulk particles ($\pi$, $K$, $p$, etc.) are well-known to interact quasi-elastically via strong hadronic resonances, such as $\rho(770)$, $K^*(892)$ and $\Delta(1232)$, which renders their interactions operative until a kinetic freeze-out temperature, $T_{fo}$, which is significantly lower than the one of group-I particles, $T_{fo} \approx 110$ MeV $< T_c \approx 170$ MeV. We refer to hadrons decoupling at $T_{fo}$ as “group-II” particles. Clearly, a description of $KE_T$ scaling in the low-$p_T$ regime would be incomplete (to say the least) without $\pi$, $K$, and $p$. In an attempt to include these particles, we adopt the simplest possible scenario of introducing a second source for group-II particles by fitting their spectra and $v_2$ with a space-dependent elliptic flow field at $T_{fo}$. We then evaluate in how far such a minimalistic (but in our opinion well-motivated) sequential freeze-out can be compatible with CQNS and $KE_T$ scaling, and which mechanisms could be responsible for them. In doing so we also revisit the question of resonance feed-down contributions to the pion $v_2$ for which differing results seem to exist in previous works.

Our article is organized as follows. In Sec. \[II\] we recapitulate basic features of the RRM, specifically its equilibrium limit in the presence of anisotropic flow fields. In Sec. \[III\] we quantitatively construct realistic flow fields for semi-central Au-Au collisions at RHIC using group-I particle freeze-out close to the hadronization transition. In Sec. \[IV\] we reiterate this procedure for group-II particles at kinetic freeze-out. In Sec. \[V\] we examine our results for group-I and -II particles with respect to CQNS and $KE_T$ scaling. The effects of feed-down on the inclusive pion $v_2$ are revisited in App. \[A\]. We summarize and conclude in Sec. \[VI\].

II. RESONANCE RECOMBINATION MODEL AND EQUILIBRIUM LIMIT

Early quark coalescence models which employed instantaneous approximations in the recombination of quarks \[10, 24\] were quite successful in providing an explanation for two rather unexpected phenomena in hadron spectra at intermediate $p_T$ in Au-Au collisions...
at RHIC. Specifically, these were the enhancement of baryon-to-meson ratios \(B/M = p/\pi, \Lambda/K\) over their values measured in \(p-p\) collisions, as well as CQNS of \(v_2(p_T)\). While an enhanced \(B/M\) ratio is also a natural outcome of hydrodynamic flow, the latter is not expected to generate enough yield to dominate hadron production at \(p_T > 2-3\) GeV. The instantaneous projection of parton states onto hadron states conserves 3-momentum by construction but energy conservation is violated due to the sudden approximation \[11\]. This approximation is believed to be viable at intermediate \(p_T\) where corrections are expected to be of order \(O(Q/p_T)\) with \(Q = m - m_q - m_{\bar{q}}\) \((m\): meson mass, \(m_{\bar{q}, q}\): antiquark/quark mass), albeit with unknown coefficient \[18\]. However, this poses a significant problem at low \(p_T\) and/or for hadrons (resonances) with large binding energy \(Q\)-value).

In Ref. \[13\] quark coalescence in an interacting medium in the vicinity of the hadronization transition was reinterpreted as a process akin to the formation of resonances, \(q + \bar{q} = M\), and implemented via a Boltzmann equation

\[
p^\mu \partial_\mu f_M(t, \vec{x}, \vec{p}) = -m\Gamma f_M(t, \vec{x}, \vec{p}) + \rho^0 \beta(\vec{x}, \vec{p}),
\]

(2)

where \(f_M(t, \vec{x}, \vec{p})\) denotes the phase space density of the meson, and the gain term is given by

\[
\beta(\vec{x}, \vec{p}) = \int \frac{d^3p_1d^3p_2}{(2\pi)^6} f_q(\vec{x}, \vec{p})f_{\bar{q}}(\vec{x}, \vec{p}) \times \sigma(s)v_{rel}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2).
\]

(3)

The use of an explicit (resonant) cross section automatically satisfies energy-momentum conservation. For simplicity it has been modeled by a relativistic Breit-Wigner form,

\[
\sigma(s) = g^0 \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2) + (\Gamma m)^2},
\]

(4)

where the same reaction rate \(\Gamma\) is used as in the loss term, the first term on the right-hand-side \((rhs)\) of Eq. (2), and \(g^0\) is the degeneracy factor. This guarantees detailed balance, which is essential for recovering thermodynamic equilibrium in the long-time limit, \(\Delta\tau \gg 1/\Gamma\).

If hadronization is rapid enough to produce hadrons in equilibrium, as it seems to be the case for the bulk of hadrons in URHICs, this limit is applicable and the resulting hadron-momentum distribution is given by

\[
f_M^g(\vec{p}) = \frac{E_M(\vec{p})}{m\Gamma} \int d^3x \beta(\vec{x}, \vec{p})
\]

(5)

For more details, we refer the reader to Ref. \[18\]. It is well known that the unique equilibrium solution of a Boltzmann transport equation is a Boltzmann distribution if and only if energy conservation as well as detailed balance are satisfied. The RRM complies with these requirements. In the remainder of this section we will verify numerically that Eq. (4) recovers the thermal Boltzmann distribution for mesons in the presence of a spatially dependent anisotropic flow field at the quark level (in previous work \[18\], this has only been exemplified for \(p_T\) spectra, not for \(v_2\)).

As a first test we study \(\phi\) mesons in an azimuthally symmetric and longitudinally boost-invariant fireball. The phase space-densities of the input strange and antistrange quarks \((m_s = 0.45\) GeV\) are parameterized through a blast-wave with radial flow velocity \(\vec{v}(\vec{r}) = v_0\vec{r}/R_0\). We compare the result for \(\phi\) mesons \((m_\phi = 1.02\) GeV, \(\Gamma_\phi = 0.05\) GeV\) calculated from resonance recombination of \(s\) and \(\bar{s}\), Eq. (3), with the direct blast-wave expression for the \(\phi\) meson with identical flow field and temperature as for the quark input. Fig. 1 shows that both ways of computing the \(\phi\) spectra are in excellent agreement. We have reconfirmed that the results are insensitive to variations of the resonance width \(\Gamma\) when ensuring \(\Gamma \lesssim Q\) with \(Q = m_\phi - (m_s + m_{\bar{s}})\).

FIG. 1: (Color online) Comparison of \(\phi\)-meson \(p_T\) spectra computed from (i) resonance recombination of strange quarks using blast-wave flow fields for strange quarks (rectangles), and (ii) a direct application of an equilibrium blast-wave distribution (solid line) with the same flow field as for quarks (solid line). The hadronization temperature has been chosen at \(T_e = 180\) MeV and the radial flow at the surface as \(v_0 = 0.55\).

To account for anisotropic flow fields which are suitable for generating configurations reminiscent of hydrodynamic simulations for fireballs in non-central (azimuthally asymmetric) nuclear collisions, we adopt in this work the parameterization introduced by Retière and Lisa (RL) \[22\]. It assumes longitudinal boost-invariance \[26\] and parameterizes the transverse flow rapidity as a function of radius \(r\) and spatial azimuthal

\[1\] Throughout this manuscript, we will use \(p_T (p_c)\) to denote hadron (quark) transverse momenta.
where $T_\ell$ is the freeze-out temperature at constant longitudinal proper time $\tau_f$, $\mu_i$ is the chemical potential of particle $i$, $g_i$ is the spin-isospin degeneracy factor, $K_1$ is a modified Bessel function and $\alpha_T = p_T/T_\gamma \sinh \rho(r, \phi_s)\alpha_T$, $\beta_T = m_T/T_\gamma \cosh \rho(r, \phi_s)$. Equation (9) has been written in Boltzmann approximation which works well for sufficiently heavy particles; we will, however, use Bose distributions for pions. From Eq. (10) we can calculate the elliptic flow of particle $i$ as

$$v_2^i(p_T) = \frac{\int_{0}^{2\pi} d\phi_p \cos(2\phi_p)}{\int_{0}^{2\pi} d\phi_p} \frac{\int_{-1}^{1} d\cos(\phi_\rho) dN_i}{\int_{-1}^{1} d\cos(\phi_\rho) dN_i}.$$

FIG. 2: (Color online) Comparison of elliptic flow for $\phi$, $D$ and $J/\psi$ mesons (top to bottom) computed from: (i) resonance recombination of strange quarks (symbols), and (ii) a direct application of the blast-wave expression at the meson level (lines). Both quark and meson distributions are based on the same anisotropic flow field constructed from the RL parameterization with $T_c = 180$ MeV, $R_s = 7.5$ fm, $R_T = 8.5$ fm, $\rho_0 = 0.66$ and $\rho_2 = 0.07$.

angle $\phi_s$ as

$$\rho(r, \phi_s) = \overline{\rho}[\rho_0 + \rho_2 \cos(2\phi_b)],$$

where $\rho_0$ represents the average radial flow rapidity and $\rho_2$ the anisotropy of transverse flow. The angle $\phi_b$ of the flow vector is generally tilted from the position angle $\phi_s$ (both angles are measured with respect to the reaction plane) which can be reflected by the relation

$$\tan \phi_b = \left(\frac{R_s}{R_T}\right)^2 \tan \phi_s.$$

This ansatz is motivated by the picture that the transverse boost is locally perpendicular to elliptical subshells. The transverse flow rapidity, $\rho(r, \phi_s)$, is assumed to increase linearly with the normalized “elliptic radius”, defined as

$$\overline{\rho} = \sqrt{r^2 \cos^2 \phi_s/R_s^2 + r^2 \sin^2 \phi_s/R_T^2}.$$

We emphasize this that flow field incorporates space-momentum correlations that are considered a much more realistic representation of a collectively expanding source than the factorized ansatz of a “local” $v_2$ (independent of position) as often adopted in quark coalescence model calculations.

Using the RL parameterization the differential momentum spectrum for a particle $i$ directly emitted from the source takes the form [16, 22, 27] (feed-down contributions are evaluated in App. A)

$$\frac{dN_i}{p_T dp_T d\phi_p d\phi_\rho dy} = \frac{2g_i}{(2\pi)^3} \tau_f m_T e^{\mu_i/T_f} \times \int r dr \int d\phi_\rho K_1(m_T, T, \beta_T) e^{\alpha_T \cos(\phi_\rho - \phi_s)}.$$

With the RL parameterization for an anisotropic flow field we are now in position to compare the elliptic flow generated for coalesced mesons in the RRM for locally equilibrated quark distributions, Eq. (5), with the one directly obtained from applying the blast-wave expression [9] at the meson level. We fix the parameters as quoted in the caption of Fig. 2 noting that at this point our aim is not to fit experimental data but to scrutinize generic properties of the RRM. We emphasize again the nontrivial spatial dependence of the flow field figuring into the quark distributions in Eq. (10). To extend the scope of our comparison we consider, in addition to $\phi$ mesons, also $D$ mesons ($m_D = 1.9$ GeV, $\Gamma_D = 0.1$ GeV) and $J/\psi$ mesons ($m_{J/\psi} = 3.1$ GeV, $\Gamma_{J/\psi} = 0.1$ GeV), as bound states of charm ($m_c = 1.5$ GeV) and light quarks ($m_{u,d} = 0.3$, GeV) and of charm and anticharm quarks, respectively. Fig. 2 illustrates the comparison of the meson-$v_2(p_T)$ for RRM and their direct blast-wave counterparts, showing again excellent agreement within numerical accuracy. It is quite remarkable that RRM generates negative $v_2$ at low $p_T$ for the $J/\psi$ starting from strictly positive $v_2(p_T)$ for the underlying charm-quark distributions. A possibly negative $v_2$ is a well-known mass effect caused by the depletion of the heavy-particle yield at low momenta in the presence of sufficiently large collective flow [7]. However, we are not aware of any quark coalescence model calculation that has reproduced this result. It demonstrates that the meson distributions resulting from RRM have reached local equilibrium and follow the collective flow of the source. We have checked that the negative $v_2$ is rather robust against variations of blast-wave parameters. By increasing $\rho_0$ or $\rho_2$ the negative value can be amplified.

Combining the results from Figs. 1 and 2 we conclude that hadron phase-space distributions obtained from resonance recombination precisely reflect the collective properties of a source in local equilibrium, explicitly documenting the fact that the RRM achieves this through energy conservation and detailed balance. These insights also imply that the intermediate-$p_T$ regime, where $v_2$ saturates and shows an explicit dependence on quark number, can not be in full equilibrium.
TABLE I: Parameter values for kinetic freeze-out temperature, $T_f$ in MeV, radial-flow rapidity $\rho$, rapidity-asymmetry, $\rho_2$, and elliptic radii, $R_x$ and $R_y$ in fm, for group-I and -II hadrons using the RL blast-wave expression for mid-central Au-Au collisions at RHIC.

| Group | $T_f$  | $\rho_0$ | $\rho_2$ | $R_x$ | $R_y$ |
|-------|--------|----------|----------|-------|-------|
| I     | 180.0  | 0.75     | 0.072    | 7.5   | 8.9   |
| II    | 110.0  | 0.93     | 0.055    | 11.0  | 12.4  |

III. RECOMBINATION AND QUARK DISTRIBUTIONS AT HADRONIZATION

Based on the verification that resonance recombination maps the collective local-equilibrium properties of the quark source into hadron spectra, we can reverse the strategy and use this as a tool to extract quark distributions by a quantitative study of experimental data on hadrons which arise from quark coalescence. The generality of this argument even allows to extend the analysis to baryons without having to explicitly calculate their spectra in RRM (which would be more involved than for mesons as it requires to recombine 3 quarks). As alluded to in the introduction, this procedure can only give access to quark distributions if the subsequent hadronic phase exerts a negligible distortion of the hadron spectra right after their formation around $T_c$. We label such hadrons as belonging to “group I”, and identify multi-strange hadrons ($\phi$, $\Xi$, $\Omega$) as the prime candidates currently available from experiment. These hadrons have no well-established resonance excitations on the most abundant bulk particles ($\pi$, $K$ and $N$) in the hadronic medium, while elastic $t$-channel exchange processes are suppressed by the OZI rule. One can therefore expect that multi-strange hadrons do not suffer significant rescattering in the hadronic phase, which is indeed supported by blast-wave fits to experimental data favoring kinetic decoupling not far from $T_c$. The flow fields extracted for group-I hadrons therefore reflect the collective properties of the quark phase.

We first determine the flow parameters in the RL parameterization by performing a blast-wave fit to the transverse-momentum spectra and elliptic flow of group-I hadrons to experimental data from STAR in mid-central (20-40%) Au-Au collisions. For definiteness (and easier comparison to the RRM calculations in Ref. [20]), we fix the temperature to $T_c = 180$ MeV; we do not utilize it as a parameter but obtain fits of satisfactory quality with this value. The transverse-flow rapidity, $\rho_0$, is mostly governed by fits to the $p_T$-spectra of $\phi$, $\Xi$ and $\Omega$ hadrons, while the flow-asymmetry parameter, $\rho_2$, and the fireball’s spatial eccentricity, $R_y/R_x$, are driven by fitting the elliptic flow of $\Xi$ and $\phi$. The resulting RL blast-wave curves are shown with data in Fig. 3, and the central values of the parameters are collected in Tab. II. As to be expected, the slopes of the spectra can be fitted well. The absolute normalizations are also reproduced reasonably as well, which is confirmed by the $\Omega$-to-$\phi$ ratio on a linear scale, compared to STAR data [29].

In semi-central Au-Au$(\sqrt{sNN}=200$ GeV) collisions for transverse-momentum spectra (upper panel) and elliptic flow (middle panel) of $\phi$, $\Xi^+$ and $(\Omega + \bar{\Omega})/2$ using the RL ansatz with parameters specified in Tab. II. The lower panel shows the resulting $\Omega$-to-$\phi$ ratio on a linear scale, compared to STAR data [28].

FIG. 3: (Color online) Blast-wave fits to STAR data [13, 29, 31] in semi-central Au-Au$(\sqrt{sNN}=200$ GeV) collisions for transverse-momentum spectra (upper panel) and elliptic flow (middle panel) of $\phi$, $\Xi^+$ and $(\Omega + \bar{\Omega})/2$ using the RL ansatz with parameters specified in Tab. II. The lower panel shows the resulting $\Omega$-to-$\phi$ ratio on a linear scale, compared to STAR data [28].

Degeneracy factors for spin and isospin are taken into account, but baryon chemical potential and
FIG. 4: (Color online) Upper panel: Equilibrated transverse momentum distributions of quarks on a hypersurface with temperature just above momentum distributions of quarks on a hypersurface with temperature just above the freeze-out temperature of 180 MeV, and (ii) group-I hadrons, has been attempted by in Ref. [32] based on a 1-dimensional instantaneous quark coalescence model. We reiterate here that such an extraction cannot be reliably applied at low $p_T$ due to the simplifying assumptions inherent to such models, but it would certainly be interesting to make a comparison between both approaches.

The strangeness suppression factor, $\gamma_s$, are neglected in these fits (they contribute at the 10% level which can easily be absorbed by fine-tuning the freeze-out temperature). Note that the $\Omega$-to-$\phi$ ratio is quite different from linear for $p_T$ up to $\sim 1.5$ GeV, and that this ratio does not go to zero for $p_T \to 0$ as predicted by quark coalescence models discussed in Ref. [28, 29]. This reiterates once more the importance of energy conservation and the thermal-equilibrium limit if on attempts to describe hadron spectra through quark coalescence in the low-$p_T$ regime.

We are now in a position to extract the quark distribution functions at hadronization by assuming: (i) the flow field for group-I hadrons is indeed the flow field for the hadronic phase in equilibrium just below $T_c$ (which is further supported by the fact that it is possible to use a freeze-out temperature of 180 MeV), and (ii) group-I hadron formation occurs via coalescence from a partonic phase (which is supported by the general CQNS characteristics of RHIC data). In Fig. 4 we not only include the extracted strange- and light-quark spectra and elliptic flow (which are directly involved in the RRM-based fits) but also those implied for the heavier charm and bottom quarks. Equilibration of the latter 2 quark flavors is currently an open issue, but this may rather be a question of how far up in $p_T$ it applies. Qualitatively, the same limitation applies to light and strange quarks, which presumably enter the kinetic regime at $p_T \simeq 1$ GeV, where the data for the quark-number scaled $v_2$ level off at a value of about 7-8%. Once available, $D$- and $B$-meson data will allow for similar estimates, and we provide here the underlying heavy-quark spectra as a reference for such an analysis.

An extraction of strange- and light-quark spectra in a similar spirit, i.e., from data of what we refer to as group-I hadrons, has been attempted by in Ref. [32] based on a 1-dimensional instantaneous quark coalescence model. We reiterate here that such an extraction cannot be reliably applied at low $p_T$ due to the simplifying assumptions inherent to such models, but it would certainly be interesting to make a comparison between both approaches.

IV. SEQUENTIAL FREEZE-OUT AND BULK PARTICLES

In URHICs the most abundant bulk particles, like pions, kaons and protons (which we refer to as group-II particles), undergo significant re-scattering in the hadronic phase. This is borne out of experimental blast-wave fits which require kinetic-freeze-out temperatures (e.g., $T_{fo} \simeq 100$ MeV for central Au-Au collisions at RHIC [1]) which are significantly lower than the chemical freeze-out temperatures ($T_{ch} \simeq 160 - 180$ MeV, extracted from observed ratios of different hadron species). Note that the difference in the kinetic and chemical freeze-out temperatures translates into a factor 5-10 different energy densities [33]. Theoretically, the sequential freeze-out systematics can be readily understood from large resonant cross sections operative for group-II particles in the hadronic phase, e.g., $\pi\pi \to \rho \to \pi\pi$, $\pi K \to K^* \to \pi K$ or $\pi N \to \Delta \to \pi N$. The values of these elastic cross sections reach up to 100-200 mb and are thus 1-2 orders of magnitude larger than chemistry-changing inelastic ones (e.g., $\pi\pi \to K^0 K^0$). Further direct evidence for an extended duration of hadronic reinteractions follows from the well-established low-mass dilepton enhancement in URHICs whose magnitude and spectral shape requires several generations of $\rho$ mesons to form with a largely broadened spectral shape due to a strong coupling to the medium [34].

In order to include group-II particles in an analysis of the thermal regime of anisotropic flow fields there is thus no choice but to introduce an additional source parameterization at a much lower temperature than for group-I particles. While this generally detaches group-II particles from coalescence mechanisms at hadronization, we remark that their production through resonance recom-
with earlier blast-wave extractions, at\[\rho_0, T_0]\) of group-II particles. As in the group-I case, we decide results in a good fit to the ansatz to parameterize an anisotropic flow field which is a kinetic regime.

As a result, a relatively large \(\rho_0, T_0\) can be disentangled by fits to hadrons with different masses. With these 2 parameters fixed, an increase in \(\rho_0\) leads to a rapid increase in \(v_2(p_T)\) at relatively large \(p_T\) but to rather small changes at low \(p_T\). On the contrary, increasing \(R_y/R_x\) results in a more uniform increase of \(v_2(p_T)\) over a wide range of \(p_T\). As a result, a relatively large \(\rho_0\), combined with a relatively small eccentricity, i.e., \(R_y/R_x\) close to unity, are favored in fitting the data. The rather tight correlation between these two parameters can be exhibited in pertinent contour plots indicating the “allowed regions” for obtaining an acceptable fit to data given their error bars, see Fig. 6. The central values of the contours for the two particle groups correspond to the values listed in pertinent contour plots indicating the “allowed regions” for obtaining an acceptable fit to data given their error bars, see Fig. 6. The central values of the contours for the two particle groups correspond to the values listed. Experimental data are taken from Refs. [3, 35, 36].

Then we determine \(\rho_0, p_0\) and the eccentricity of the fireball from fits to the measured spectra and \(v_2\) of protons and kaons for the same centrality selection of Au-Au collisions. The results are displayed in Fig. 6 corresponding to the parameter values listed in Tab. II. At kinetic freeze-out, effective chemical potentials for stable particles (pion, kaon, nucleon) are required to correctly predict the absolute normalization of their yields [33]. Neglecting chemical potentials therefore implies that we can only fit the shape of the spectra which is sufficient for our purposes here. In Boltzmann approximation, the chemical potential appears as a constant overall fugacity factor which does not affect the \(v_2\) either. The pions are treated with Bose statistics which entails an appreciable (moderate) enhancement of the \(p_T\) spectra (\(v_2\)) at \(p_T \lesssim 0.5\) GeV. At the same time, feed-down contributions are also significant in this regime. A more detailed discussion of the latter is given in App. A.

It is instructive to elaborate on the qualitative influence of the fit parameters on the observables. As is well-known from more simple blast-wave fits, the interplay of radial flow \((\rho_0)\) and temperature \((T)\) can be disentangled by fits to hadrons with different masses. With these 2 parameters fixed, an increase in \(\rho_0\) leads to a rapid increase in \(v_2(p_T)\) at relatively large \(p_T\) but to rather small changes at low \(p_T\). On the contrary, increasing \(R_y/R_x\) results in a more uniform increase of \(v_2(p_T)\) over a wide range of \(p_T\). As a result, a relatively large \(\rho_0\), combined with a relatively small eccentricity, i.e., \(R_y/R_x\) close to unity, are favored in fitting the data. The rather tight correlation between these two parameters can be exhibited in pertinent contour plots indicating the “allowed regions” for obtaining an acceptable fit to data given their error bars, see Fig. 6. The central values of the contours for the two particle groups correspond to the values listed in pertinent contour plots indicating the “allowed regions” for obtaining an acceptable fit to data given their error bars, see Fig. 6. The central values of the contours for the two particle groups correspond to the values listed.
in Tab. I. The contour for group-II hadrons is tighter due to the larger number of data points for bulk particles. We first note that the contours clearly exclude each other corroborating the significant evolution that the collective flow field is undergoing in the hadronic phase. More specifically, the anisotropy parameter $R_y/R_x$ for group-II particles is smaller than for group-I, which is in line with the general expectation that the fireball becomes more spherical in the expansion. However, the flow anisotropy $p_2$ extracted from the RL blast-wave fits favors a decrease from hadronization (group-I) to kinetic decoupling of the bulk (group-II). This is contrary to the expectation from hydrodynamic expansion that $p_2 = (\rho_x^2 - \rho_y^2)/2$ should increase if the in-plane acceleration is always larger than the out-of-plane acceleration ($\rho_x^2$ and $\rho_y^2$ are the surface-flow rapidities along the $x$- and $y$-direction, respectively; recall Eq. (6)). We cannot resolve this question within the present blast-wave model. A full hydrodynamic simulation could give more insight, possibly requiring substantial viscosity effects in the later stages of the hadronic evolution.

To summarize this and the previous section, we have produced a space-dependent parameterization of anisotropic flow fields which, when implemented into a rather simple thermal blast-wave description of locally equilibrated matter, provides a good overall fit to a large body of hadron data in semi-central Au-Au data in the low-$p_T$ regime. As a key ingredient we have introduced the well established concept of sequential freeze-out including just two groups but with clearly distinct flow fields.

V. KE$_T$ SCALING OF $v_2$

A signature result of ideal hydrodynamical simulations of URHICs is the mass splitting of the hadron $v_2$ when plotted as a function of hadron $p_T$, with a larger $v_2$ for the lighter particles at a given $p_T$. On the other hand, if the hydrodynamic results for the hadron $v_2$ are plotted versus $KE_T$ (at a given freeze-out temperature), the splitting nearly vanishes, but not entirely. In fact, a small reverse mass ordering of $v_2$ is found instead, as pointed out in Refs. [13, 57]. In other words, the hydro-induced mass splitting of $v_2(p_T)$ between light and heavy particles is not large enough to render their $v_2(KE_T)$ curves degenerate. The sequential freeze-out picture advocated above may hold an explanation to this apparent discrepancy between hydrodynamic freeze-out (at fixed temperature) and experimental data (which exhibit “perfect” KE$_T$ scaling): Random thermal motion tends to isotropize $p_T$ distributions and thus counteracts (“dilutes”) the collective motion in producing nonzero $v_2$. Thus one should expect that a gap in freeze-out temperatures between, say, (light) pions and (heavy) $\Omega$ baryons increases the mass splitting in a $v_2(p_T)$ plot (the thermal motion of pions is suppressed at lower temperatures), just enough to produce degeneracy in a $v_2(KE_T)$ representation. This appears to be a conspiracy of circumstances that leads to the empirically observed KE$_T$-scaling. However, the underlying mechanism (largely different hadronic rescattering cross sections) is general in the sense that it applies to different centralities, system sizes and even collision energies, presumably coupled with a rather general (local) freeze-out criterion. It therefore could be a candidate for the origin of the widely observed KE$_T$ scaling. In the following, we quantitatively test this idea by analyzing the scaling properties within our sequential freeze-out 2-source ansatz.

With the RL flow-field parameterizations and freeze-out temperatures for the two sources of group-I and group-II particles summarized in Tab. II we proceed to display the $n_q$-scaled hadron-$v_2$ as a function of the $n_q$-scaled hadron-KE$_T$ in Fig. 7. We find that all the hadrons-$v_2$ curves follow a generic scaling very well (with slight deviations for the pions, on which we comment below), very similar to experiment. This agreement is, of course, not unexpected given that our results follow from fits to data, together with the notion that the data satisfy the scaling in the first place (or even led to its discovery). The nontrivial insight here is that our “minimalistic” approach of a sequential freeze-out scenario coupled with a rather simple parameterization of an anisotropic collective flow field, is sufficient to establish the scaling. We also recall that $n_q$-scaling in the equilibrated low-$p_T$ region is not a direct consequence of quark recombination, at least not for group-II particles. To better exhibit the significance of the sequential freeze-out we compare in Fig. 5 the generic scaling curve extracted from Fig. 7 with $v_2$ results where the (rather heavy) $\Omega$ baryon (group-I) is frozen out from the source at $T_{fo} = 110$ MeV (as sometimes done in hydrodynamic calculations with a single freeze-out), and the (rather light) pions and kaons are frozen out from the source at $T_c = 180$ MeV (as advo-
increases the pion freeze-out temperature from 110 MeV to, e.g., 100 MeV. It turns out, however, that a slight decrease of the pion temperature is certainly compatible with experimental data, further suppression of the random thermal motion of the baryon (dash-dotted line) is frozen out from the late source at \( T_0 = 110 \) MeV, while pions (dotted line) and kaons (dashed line) are frozen out from the early source at \( T_c = 180 \) MeV (source parameters as quoted in Tab. [I]).

Let us return to the deviations of the pion \( v_2 \) below the scaling curve especially toward higher \( KE_T \). One may wonder whether feed-down corrections from heavier particles (e.g., \( \rho \rightarrow \pi \pi, \omega \rightarrow 3\pi, \Delta \rightarrow N\pi, \) etc.) could improve the situation. This question is addressed in detail in Appendix [A] with the finding that the combined effect of the decay pions increases the pion \( v_2 \) at \( p_T > 0.5 \) GeV only a little (by a few percent). It turns out, however, that a slight decrease of the pion freeze-out temperature from 110 MeV to, e.g., 100 MeV increases the pion \( v_2 \) preferentially at higher \( KE_T \), resulting in good agreement with the general scaling curve. In doing so we have not performed a possibly small re-adjustment of the flow parameters to attempt a more quantitative conclusion. The key mechanism is again a further suppression of the random thermal motion of the pions, thus augmenting the relative importance of the collective motion. A 5-10 MeV smaller freeze-out temperature is certainly compatible with experimental data, and also not unreasonable from the theoretical point of view. Pions have large resonant cross sections on all major particle types of the bulk (pion, kaon and nucleon), and their mass being the smallest makes it plausible that their momentum spectra are frozen the latest (mass ordering of thermal relaxation times).

VI. SUMMARY AND CONCLUSIONS

In this paper we have analyzed the scaling properties of hadron elliptic flow in ultrarelativistic heavy-ion collisions, which have received considerable attention since the experimental data suggest the existence of a universal behavior possibly related to a collectively expanding partonic source. We have focused our investigations on the low-\( p_T \) regime (\( p_T \lesssim 2 \) GeV) where RHIC data (and hydrodynamic analysis thereof) support the assumption of a locally equilibrated medium.

In the first part of this work we have pursued the earlier idea that hadronization of the putative partonic system at RHIC proceeds through coalescence of quarks and antiquarks. Its implementation in the thermal regime requires a recombination model which obeys 4-momentum conservation and detailed balance to encode the correct equilibrium limit. The Resonance Recombination Model (RRM) satisfies these requirements. The flow field has been modeled by a blast-wave ansatz with azimuthal asymmetries which give rise to space-momentum correlations characteristic for a collectively expanding source. Using RRM we have shown that an expanding parton source converts into hadron spectra with the same collective source properties, which, to our knowledge, has not been achieved in existing models of quark coalescence. We have utilized this connection to extract quark distributions just above the critical temperature for Au-Au collisions at RHIC from measured \( p_T \) spectra and elliptic flow of multi-strange hadrons which are believed to undergo negligible final-state interactions in the hadronic phase (group-I particles).

Bulk particles with large hadronic cross sections (e.g., pion, kaons and nucleons) cannot be approximated by decoupling at the hadronization transition. We have included these particles (group-II) in our analysis by constructing another flow field at the typical kinetic freeze-out temperature of 110 MeV, and fitting the blast-wave parameters to their \( p_T \) spectra and elliptic flow. Group-II hadrons are therefore not really sensitive to quark recombination. However, we find that the sequential freeze-out picture with just two sources is well compatible with the empirically observed kinetic-energy and constituent-quark number scaling (where the latter is not directly linked to recombination). The sequential freeze-out naturally corrects for a slight breaking of \( KE_T \) scaling present in a hydrodynamic flow description for a uniform freeze-out. In particular, a smaller temperature is instrumental in suppressing the random thermal motion of the light pions and in this way bring up their \( v_2 \) toward the universal curve. In the thermal regime, CQNS of elliptic flow simply follows from the universal linear curve re-
resulting from $K_E T$ scaling (i.e., it is not directly linked to recombination).

An important task of future work will be an extension of our studies to the intermediate-$p_T$ region. Here, the onset of the kinetic regime occurs and thus off-equilibrium effects become important. While this increases the sensitivity to quark degrees of freedom, it also requires a more detailed knowledge of the interactions of particles, both in the QGP and hadronic phase. Initial studies using Langevin simulations for strange and heavy quarks have been reported ~20, but the scaling properties found there clearly require a deeper understanding, including the effects of baryon formation and a more microscopic description of the elastic (as well as radiative) interactions in the QGP and hadronic matter.

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Appendix A: Feed-Down to Pions From Resonance Decays

Hadron-resonance decays are known to increase pion yields significantly for transverse momenta up to $p_T \approx 0.5$-1 GeV ~39,40. Therefore, it is mandatory to evaluate the effect of the corresponding feed-down not only on the $p_T$ spectra but also on the elliptic flow of the pions ~39,40. In particular, we want to understand how feed-down affects the pion $v_2$ in the context of the $K_E T$-scaling for pion freeze-out at a temperature of $T_{fo} = 110$ MeV where some deviation from protons and kaons in group I has been identified (recall Fig. 7). The total differential pion spectrum can be decomposed as

$$\frac{dN_{\pi}^{tot}}{p_T dp_T d\phi_y dy} = \frac{dN_{\pi}^{dir}}{p_T dp_T d\phi_y dy} + \sum_R \frac{dN_{R \rightarrow \pi}}{p_T dp_T d\phi_y dy}. \quad (A1)$$

where the summation is over all resonances $R$ with significant branching fraction into final states containing pions. In our decay simulation, we include $\rho$, $\omega$, $K^*$, $\Delta$, $\eta$ and $K_0^{*0}$ resonances, whose anisotropic momentum spectra, together with that of direct pions, are obtained from the RL blast-wave calculation with the same parameters as those for group II in Tab. 1. The resonance decays which contribute to the final spectra occur at or after freeze-out. However, the absolute number of the feed-down contribution, which is important to determine the relative contribution to the total pion-$v_2$, depends on the chemical potentials of the resonances at kinetic freeze-out. In the interacting hadronic phase effective chemical potential need to be introduced for hadrons stable under strong interactions (pion, kaon, $\eta$, nucleon, etc.), as to conserve their total numbers which are frozen at chemical freeze-out ~41,42. Part of this procedure is to keep the strong resonances in relative chemical equilibrium, e.g., $\mu_\rho = 2\mu_\pi$ or $\mu_\Delta = \mu_N + \mu_\pi$. At RHIC, the conservation of antibaryon number is of particular importance in the overall chemistry of the hadronic phase, and thus for the actual values of the effective chemical potentials ~3,32,44,46. The values of the chemical potentials of the resonances at $T_{fo}$ are quoted in the caption of Fig. 9; they are taken from Ref. 33 where they are computed from an isentropic evolution starting at chemical freeze-out at $T = 180$ MeV (consistent with our parameters for group-1 freeze-out). Note that the $K_0^{*0}$ decays through weak interactions but carries its own chemical potential (as does the $\eta$).

The $p_T$-spectra of pions from various resonance decays are compiled in Fig. 9. The decay pions generally have steeper $p_T$-spectra than directly produced pions, except for pions from $p$ decays, similar to the observations made in Refs. 39,40. This is essentially a phase-space effect: pions from resonances with a small $Q$-value (the difference of the resonance mass and the sum over masses of its constituent quarks) tend to have a steeper $p_T$-spectrum. The phase-space also affects the $v_2$ of decay pions. The momenta of pions from resonances with large $Q$ values (like the $\rho$) tend to dilute the $v_2$ more due to the random orientation of the decay momentum, most notably at low momenta (where the decay momentum in the parent’s rest frame is comparable to the parent momentum). On the other hand, pions from resonances with small $Q$ values (like the $\Delta$) are essentially aligned with the parent and thus preserve the elliptic flow of the parent. For those pions, the $v_2$ could be larger than that of the parent at the same $p_T$, since the momentum of the resonance is shared by several daughters. However, since the dominant source of secondary pions are $p$ decays, the total pion-$v_2$ might not be much affected by resonance decays.

The comparison of the total pion-$v_2$ to the direct-pion-$v_2$ confirms this expectation, see Fig. 10. Therefore, feed-down contributions are not sufficient to resolve the discrepancy between the direct-pion $v_2$ and the generic scaling curve in the $K_E T$-dependence of $v_2$. In comparison to previous estimates we note that in Ref. 39 a larger enhancement of the total pion-$v_2$ over the direct-pion $v_2$ was reported. In that work, the resonance decays were evaluated at $T_c$; subsequent hadronic rescattering was neglected and the direct pions were computed from a quark coalescence model which does not conserve energy. Both of these approximations may be unreliable for pions at small $p_T$, due to their large interaction cross section and binding energy, respectively. The influence of resonance decays on pion elliptic flow was also studied in Ref. 14, where the authors assumed that direct pions follow the
et al. [12] A. Adare et al., Nucl. Phys. A749, 285c (2005).
[11] S. Pratt and S. Pal, Nucl. Phys. A749, 268 (2005); D. Molnar, arXiv:nucl-th/0408044v2; V. Greco and C. M. Ko, arXiv:nucl-th/0505061v2.
[10] D. Molnar, S. Pal, and V. Greco, Phys. Rev. Lett. 90, 020403 (2003); V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C 68, 034904 (2003). R. Fries, V. Greco, and S. Voloshin, Phys. Rev. C 69, 054910 (2004).
[9] S. A. Voloshin, Nucl. Phys. A715, 397c (2003).
[8] D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 90, 202303 (2003); R. Fries et al., Phys. Rev. C 68, 044902 (2003); V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C 68, 034904 (2003); R. Fries, V. Greco, and P. Sorensen, Rev. Nucl. Sci. 58, 177 (2008).
[7] T. Hirano and M. Gyulassy, Nucl. Phys. A769, 71 (2006).
[6] P. F. Kolb, J. Solfrank, and U. Heinz, arXiv:nucl-th/0305084v2.
[5] U. Heinz, arXiv:hep-ph/0407360v1; P. F. Kolb and U. Heinz, arXiv:nucl-th/0407360v2.
[4] D. Teaney, J. Lauret, and E. V. Shuryak, Phys. Rev. Lett. 86, 4783 (2001); D. Teaney, J. Lauret, and E. V. Shuryak, arXiv:nucl-th/0110037.
[3] J. Adams et al. (STAR Collaboration and STAR-RHC Collaboration), Phys. Rev. C 72, 014904 (2005).
[2] P. Sorensen, arXiv:0905.0174v3 [nucl-ex]; S. A. Voloshin, A. M. Poskanzer, and P. Snellings, arXiv:0809.2949v2 [nucl-ex].
[1] J. Adams et al. (STAR Collaboration), Nucl. Phys. A757, 102 (2005); K. Adcox et al. (PHENIX Collaboration), Nucl. Phys. A757, 184 (2005); I. Arsene et al. (BRAHMS Collaboration), Nucl. Phys. A757, 1 (2005); B. B. Back et al. (PHOBOS Collaboration), Nucl. Phys. A757, 28 (2005).

FIG. 9: (Color online) Transverse momentum spectra of pions from various resonance decays compared to direct pions (solid line) at $T_0 = 110$ MeV. The chemical potentials for the parent particle are resonances are $\mu_\rho = 180$ MeV, $\mu_\Delta = 460$ MeV, $\mu_{K^*} = 250$ MeV, $\mu_\omega = 270$ MeV, $\mu_\eta = 170$ MeV, resulting from a partial chemical equilibrium scenario in which chemical potentials for stable particles are $\mu_\pi = 90$ MeV, $\mu_K = 160$ MeV and $\mu_N = 370$ MeV.

FIG. 10: (Color online) Total pion-$v_2$ (dashed line) compared to direct pion $v_2$ (solid line) at $T_0 = 0.11$ GeV. $K_T$-scaling and then found the scaling still holds at the 10% level when resonances are included.

[26] J. D. Bjorken, Phys. Rev. D 27, 14 (1983).
[25] F. Retiere and M. A. Lisa, Phys. Rev. C 70, 044907 (2004).
[24] B. Mohanty and N. Xu, J. Phys. G: Nucl. Part. Phys. 36, 064902 (2009).
[23] R. J. Fries et al., Phys. Rev. Lett. 90, 202303 (2003); V. Greco, C. M. Ko, and P. Levai, Phys. Rev. Lett. 90, 202302 (2003); R. C. Hwa and C. B. Yang, Phys. Rev. C 73, 034913 (2006).
[22] O. Barannikova (for STAR collaboration), arXiv:nucl-ex/0403014
[21] H. van Hees, V. Greco, and R. Rapp, Phys. Rev. C 73, 034913 (2006).
[20] R. A. Lacy and A. Taranenko, arXiv:nucl-ex/0610029v3.
[19] W. Cassing and E. L. Bratkovskaya, Phys. Rev. C 75, 034919 (2007).
[18] I. Arsene et al., Nucl. Phys. A757, 184 (2005); I. Arsene et al. (PHENIX Collaboration), Phys. Rev. C 78, 031901 (2008).
[17] H. van Hees, V. Greco, and R. Rapp, Phys. Rev. C 73, 034913 (2006).
[16] R. A. Lacy and A. Taranenko, arXiv:nucl-ex/0610029v3.
[15] W. Cassing and E. L. Bratkovskaya, Phys. Rev. C 75, 034919 (2008).
[14] L. Ravagli, H. van Hees, and R. Rapp, Phys. Rev. C 79, 064902 (2009).
[13] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 77, 054901 (2008).
[12] L. Ravagli and R. Rapp, Phys. Lett. B655, 126 (2007).
[11] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 75, 054906 (2007).
[10] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 77, 054901 (2008).
[9] R. C. Hwa and C. B. Yang, Phys. Rev. C 73, 034913 (2006).
[8] L. Ravagli and R. Rapp, Phys. Rev. C 79, 064902 (2009).
[7] J. Tian et al., Phys. Rev. C 79, 064902 (2009).
[6] J. Jia and C. Zhang, Phys. Rev. C 75, 031901 (R) (2007).
[5] J. Adams et al. (BRAHMS Collaboration), Nucl. Phys. A757, 28 (2005).
[4] P. Sorensen, arXiv:0905.0174v3 [nucl-ex]; S. A. Voloshin, A. M. Poskanzer, and P. Snellings, arXiv:0809.2949v2 [nucl-ex].
[31] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 79, 064903 (2009).

[32] H. Z. Huang, J. Phys. G: Nucl. Part. Phys. 36, 064008 (2009); J. H. Chen et al., Phys. Rev. C 78, 034907 (2008).

[33] R. Rapp, Phys. Rev. C 66, 017901 (2002).

[34] R. Rapp, J. Wambach, and H. van Hees, Landolt-Börnstein (Springer), New Series I/23-A, 4-1 (2010); arXiv:0901.3289 [hep-ph].

[35] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. 97, 152301 (2006).

[36] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. C 69, 034909 (2004).

[37] U. Heinz, arXiv:0901.4355 [nucl-th].

[38] J. Sollfrank, P. Koch, and U. Heinz, Z. Phys. C 52, 593 (1991).

[39] V. Greco and C. M. Ko, Phys. Rev. C 70, 024901 (2004).

[40] X. Dong et al., Phys. Lett. B597, 328 (2004).

[41] H. Bebie et al., Nucl. Phys. B378, 95 (1992).

[42] C. Song and V. Koch, Phys. Rev. C 55, 3026 (1997).

[43] C.M. Hung and E.V. Shuryak, Phys. Rev. C 57, 1891 (1998).

[44] P. F. Kolb and R. Rapp, Phys. Rev. C 67, 044903 (2003).

[45] R. Rapp and E.V. Shuryak, Phys. Rev. Lett. 86, 2980 (2001)

[46] D. Teaney, arXiv:nucl-th/0204023