Toward a principle of quantumness

Cristinel Stoica*
Department of Theoretical Physics,
National Institute of Physics and Nuclear Engineering – Horia Hulubei,
Bucharest, Romania.
(Dated: February 17, 2014)

Quantum correlations and other phenomena characteristic to a quantum world can be understood as simply consequences of a principle derived from the projection postulate. This principle states that these specifically quantum phenomena are caused by the tension between the constraints imposed by incompatible observations. This tension is found to be at the root of Bohr’s complementarity, Heisenberg’s uncertainty, results concerning non-locality, contextuality, quantum correlations in time and space.

PACS numbers: 03.65.-w, 03.65.Ta, 03.65.Ud

CONTENTS

I. Introduction
1

II. The essence of quantumness
1
A. The postulates of Quantum Mechanics
1
B. Entanglement
2
C. The Tension Principle
2

III. Applications of the Tension Principle
3
A. Complementarity and uncertainty
3
B. Quantum contextuality
4
C. Quantum correlations
4
D. A unified view on correlations
4

IV. Conclusion
5

Acknowledgments
5

References
5

I. INTRODUCTION

In the following, I will call “quantumness” those distinctive characteristics of Quantum Mechanics (QM) which make it different from classical physics. Although these phenomena are predicted by the theory itself, there is a spread opinion that they lack an explanation. But the mere fact that Quantum Mechanics predicts them, means that it explains them in terms of its fundamental postulates. They appear unexplained because their analysis is too often obfuscated by discussions about non-locality, realism, faster than light signaling, ontic vs. epistemic etc, even when it is not the case. By focusing the discussion of these phenomena only on the essential, we can see that indeed there is no need to appeal to additional explanations or supplementation with hidden variables or other concepts, since QM is able to clean its own mess.

However, many physicists consider desirable to be able to isolate the source of quantumness in the form of a principle, more like how Special Relativity is founded on two simple principles that have a physical meaning. I emphasize that this doesn’t mean to search for a principle outside what QM already tells us, but only to isolate the essence, the root of all phenomena that make QM so different from classical physics.

With respect to this, J.A. Wheeler wrote [1]

... if one really understood the central point and its necessity in the construction of the world, one ought to be able to state it in one clear, simple sentence.

According to Fuchs and Stacey [2],

Can we find some axiomatic system that really goes after the weird part of quantum theory? [...] What I would like as a goal is a way to push quantum theory’s specific form of contextuality all into one corner.

The aim of this article is to make more explicit the fact that quantumness is simply a consequence of a principle which I will call the tension principle, and which follows directly from the projection postulate. This principle states that quantumness is the consequence of the tension between the constraints imposed by incompatible observations.

II. THE ESSENCE OF QUANTUMNESS

A. The postulates of Quantum Mechanics

For the purpose of this article it is enough to rely on a well known formulation of Quantum Mechanics [3, 4], which I remind briefly.

A quantum system has associated a Hilbert space $\mathcal{H}$. The state of the system is represented by a vector $|\psi\rangle$ in

* cristi.stoica@theory.nipne.ro
The Hilbert space \( \mathcal{H} \). Its time evolution is governed by a unitary operator \( U(t) \in U(\mathcal{H}) \), by
\[
|\psi_t\rangle = U(t)|\psi_0\rangle.
\] (1)

An observable is a Hermitian operator \( \hat{O} \), and can be written as
\[
\hat{O} = \sum_{\lambda} \lambda \mathcal{P}_\lambda,
\] (2)
where \( \mathcal{P}_\lambda \) are the projection operators onto the eigenspaces \( \mathcal{H}_\lambda \), indexed by the eigenvalues \( \lambda \). The Hilbert space \( \mathcal{H} \) admits the orthogonal decomposition
\[
\mathcal{H} = \bigoplus_{\lambda} \mathcal{H}_\lambda,
\] (3)
and \( \mathcal{H}_\lambda = \mathcal{P}_\lambda \mathcal{H} \). Any observation or measurement of the observable \( \hat{O} \) of the system whose state is \( |\psi\rangle \) is governed by the following postulates, which were distilled from the original Born rule \([3, 4]\):

**PROJ.** The projection postulate. An observation finds the system in a state obtained by projecting \( |\psi\rangle \) on one of the eigenspaces of \( \hat{O} \), and the outcome is the corresponding eigenvalue \( \lambda \).

**PROB.** The probability rule. The probability that the outcome is the eigenvalue \( \lambda \) is
\[
p_\lambda = \langle \psi | \mathcal{P}_\lambda | \psi \rangle.
\] (4)

Although these postulates are often referred together as “the Born rule”, I prefer to keep them distinct, because they have distinct roles in the understanding of quantumness.

The expectation value of \( \hat{O} \) is
\[
\langle \mathcal{O} \rangle_\psi := \sum_{\lambda} \lambda p_\lambda,
\] (5)
which can be written as
\[
\langle \mathcal{O} \rangle_\psi = \langle \psi | \hat{O} | \psi \rangle.
\] (6)

**B. Entanglement**

If a system consists of two or more subsystems, its Hilbert space \( \mathcal{H} \) is the tensor product of the Hilbert spaces \( \mathcal{H}_i \) of the subsystems,
\[
\mathcal{H} = \bigotimes_i \mathcal{H}_i.
\] (7)

When the Hilbert space can be represented as a tensor product, any vector \( |\psi\rangle \in \mathcal{H} \) can be represented as a linear superposition of tensor products of vectors from the spaces \( \mathcal{H}_i \). If the state \( |\psi\rangle = \bigotimes_i |\psi_i\rangle \), where \( |\psi_i\rangle \in \mathcal{H}_i \), it is called separable, otherwise, it is entangled.

This may happen when the system represented on the Hilbert space \( \mathcal{H} \) is composed by two or more subsystems, represented on \( \mathcal{H}_i \). One example is when a system is composed of more particles. Another example is when a particle has spin – in this case, the particle’s Hilbert space is a tensor product containing the Hilbert space of the spin. Similarly in the case of internal degrees of freedom responsible for the gauge forces etc.

We see that entanglement has nothing intrinsically related to non-locality or even to position. Entanglement can be present in quantum systems which don’t have as degrees of freedom the position, which is not intrinsically quantum mechanical. Understanding this can clarify part of the usual confusion concerning Quantum Mechanics.

**C. The Tension Principle**

The postulates recalled in §IIA have as consequences the plethora of phenomena which make QM so different from the classical world. Quantum correlations, non-locality, contextuality, all these points of tension between QM and classical physics, are obviously consequences of the postulates of QM itself.

In the following I want to propose a principle which aims to concentrate the essence of the quantumness in “one clear, simple sentence”.

According to PROJ, an observation constraints the system to be in an eigenstate of the observable. But more observables impose different constraints on the state of the system, which may be incompatible, in the sense that the same state can’t satisfy all of them simultaneously. The aim of this article is to show that this tension between the different constraints is at the root of major typically quantum phenomena.

**TENS.** The tension principle. Quantumness is caused by the tension between the constraints imposed by incompatible observations.

This tension is essential, and follows directly from PROJ. The “tension”, the degree of incompatibility between two observables \( \hat{A} \) and \( \hat{B} \), is given by the commutator
\[
[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}.
\] (8)

This tension is in fact at the root of the probability rule PROB as well. Assume that we first performed the observation \( \hat{A} \), and found the system in an eigenstate \( |\psi\rangle \) of \( \hat{A} \). If the second observation \( \hat{B} \) doesn’t commute with \( \hat{A} \), then how can the system be also an eigenstate of \( \hat{B} \)? In order to be, it has to be projected. But on which eigenspace of \( \hat{B} \) should it be projected? It will project on any of these eigenspaces, with a given probability. The probability is given by PROB, and is often considered to follow from PROJ by Gleason’s theorem \([5]\).

The probability rule PROB seems to apply to a single observation, so where is the tension? The tension is between the preparation of the system in an eigenstate of the observable \( \hat{A} \), and the measurement of the observable \( \hat{B} \). While at this point PROB doesn’t seem to be very
III. APPLICATIONS OF THE TENSION PRINCIPLE

A. Complementarity and uncertainty

Probably the first known non-classical feature of Quantum Mechanics was the point-particle–wave duality. A particle behaves sometimes like a wave, and sometimes like a classical material point. The point-particle aspect is most manifest when we observe the position $(x, y, z)$, while the wave aspect, in particular interference, when we observe the momentum $(\hat{p}_x, \hat{p}_y, \hat{p}_z)$.

Bohr’s complementarity principle states that the wave aspect and the point-particle aspect cannot be observed simultaneously. This corresponds to the fact that the two observables $\hat{x}$ and $\hat{p}_x$ decompose differently the Hilbert space. Hence, Bohr’s complementarity is a direct consequence of the tension principle TENS.

Heisenberg’s uncertainty principle [6–8] states that

$$\sigma_{x,\psi}\sigma_{p_x,\psi} \geq \frac{\hbar}{2}, \quad (9)$$

where $\sigma_{O,\psi}$ denotes the standard deviation of the operator $\hat{O}$, defined as

$$\sigma_{O,\psi} := \sqrt{\langle \hat{O}^2 \rangle_\psi - \langle \hat{O} \rangle_\psi^2}. \quad (10)$$

Robertson [9, 10] generalized the uncertainty principle to any two observables $\hat{A}$ and $\hat{B}$:

$$\sigma_{A,\psi}\sigma_{B,\psi} \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle_\psi \right|. \quad (11)$$

For the observables $\hat{x}$ and $\hat{p}_x$ we recover (9), since

$$[\hat{x}, \hat{p}_x] = i\hbar. \quad (12)$$

From (5) we see that the uncertainty principle follows directly from the tension principle and the probability rule PROB.

B. Bell’s theorem

Consider a system made of two subsystems, $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. Suppose that the state $|\psi\rangle \in \mathcal{H}$ is entangled, and we observe the state of the two systems, where $\hat{O}_A$ and $\hat{O}_B$ are the observables. This is just a particular case of an observable $\hat{O}$ on the Hilbert space $\mathcal{H}$, where $\hat{O} = \hat{O}_A \otimes \hat{O}_B$, and the Born rule (PROJ & PROB) leads to the expectation value (6)

$$\langle O \rangle_\psi = \langle \psi | \hat{O}_A \otimes \hat{O}_B | \psi \rangle. \quad (13)$$

If we interpret the probability distribution determined by PROB in terms of $\mathcal{H}_A$ and $\mathcal{H}_B$, we find that the outcomes of the two observations are correlated, and the correlation is given by the expectation value of the product of the outcomes on the two sides.

The observables $\hat{O}_A$ and $\hat{O}_B$ act on the subsystems $\mathcal{H}_A$ and $\mathcal{H}_B$. They are equivalent to the observables $\hat{O}_A \otimes I_B$ and $I_A \otimes \hat{O}_B$, which act on $\mathcal{H}$ and commute. If they commute, then where is the tension present here?

Let’s take as an example the EPR-Bell experiment [11–13], where we have two qubits entangled in a singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B), \quad (14)$$

and the observables represent the spin along some directions. Suppose the observable $\hat{O}_A$ has the form

$$\hat{O}_A = \frac{1}{2} (|\uparrow\rangle_A \langle \uparrow | + |\downarrow\rangle_A \langle \downarrow |).$$

Its eigenstates are the vectors $|\uparrow\rangle_A$ and $|\downarrow\rangle_A$. Then, the singlet state (14) can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A |\uparrow\rangle_B - |\downarrow\rangle_A |\downarrow\rangle_B). \quad (15)$$

Therefore, the observable $\hat{O}_A$ not only singles out the directions $|\uparrow\rangle_A$ and $|\downarrow\rangle_A$ in $\mathcal{H}_A$, but also the directions $|\uparrow\rangle_B$ and $|\downarrow\rangle_B$ in $\mathcal{H}_B$. Let $\hat{O}_B'$ be the observable having as eigenstates these directions,

$$\hat{O}_B' = \frac{1}{2} (|\uparrow\rangle_B \langle \uparrow | + |\downarrow\rangle_B \langle \downarrow |).$$

Then, $[\hat{O}_A \otimes \hat{I}_B, \hat{I}_A \otimes \hat{O}_B'] = 0$. The observables $\hat{O}_A$ and $\hat{O}_B'$ are incompatible if and only if $[\hat{O}_B, \hat{O}_B'] \neq 0$, and here is where the tension is present.

In terms of the Choi isomorphism, the state (14), when expressed in the basis $(|\uparrow\rangle_A, |\downarrow\rangle_A)$, determines an isomorphism

$$|\uparrow\rangle_B \langle \uparrow | + \langle \downarrow | B \langle \downarrow |$$

between the spaces $\mathcal{H}_A$ and $\mathcal{H}_B$. The observable $\hat{O}_B'$ can be obtained from $\hat{O}_A$ by the isomorphism (16). It is true that this isomorphism depends on the basis, but if we use another basis, we obtain an observable with the same eigenspaces, so this doesn’t affect the reasoning. This identification of the space $\mathcal{H}_A$ and $\mathcal{H}_B$, leads, in the case when $[\hat{O}_B, \hat{O}_B'] \neq 0$, to a tension between the observables $\hat{O}_A$ and $\hat{O}_B$. If $\hat{O}_B$ and $\hat{O}_B'$ commute, there is no tension, and the correlations are classical.
There is another way to see this tension, as taking place between the preparation and the two observations. To prepare the system in a singlet state, the observable must have as non-degenerate eigenstate the singlet state (14). Any such observable is incompatible with \( \hat{O}_A \otimes \hat{I}_B \) and \( \hat{I}_A \otimes \hat{O}_B \), which don’t admit the singlet state as eigenstate. There is a tension between the preparation in a singlet state and the observables \( \hat{O}_A \) and \( \hat{O}_B \). Also, there is a tension between \( \hat{O}_A \) and \( \hat{O}_B \), but only if \( [\hat{O}_B, \hat{O}_B'] \neq 0 \).

Therefore, the tension principle TENS is at the root of the non-classical behavior manifest in the EPR-Bell experiment.

C. Quantum contextuality

The projection postulate PROJ, stating that the system is found to be an eigenstate of the observable \( \hat{O} \), has a strange feature. If a system is in a definite state before the measurement, it seems to depend on what observable will be measured in the future. While Bohr resolved this problem by suggesting that there is no reality prior to the observation, this was not compelling for everyone. The reason is that one can easily conceive that there is a pre-assignment of outcomes for each possible observation.

The Kochen-Specker theorem [14] shows that the setup can be chosen so that no pre-assignment of outcomes for each possible observation is possible. Or in fact it depends not only on that observation, but also on any other observations performed together with it.

Assume that a particle contains the information to determine the outcome of any possible observation \( \hat{O} \) we can perform on it. Then, the theorem shows, this information should also depend on the context, i.e., on what other observable \( \hat{O}' \) we measure together with \( \hat{O} \), even if they commute. Consequently, if one tries to build a hidden-variables theory in which the particle contains the information needed to determine the outcome of any possible observation, one should actually make sure this depends on the other observations too.

The Kochen-Specker theorem is obtained by gathering enough observables, so that the tension between them prevents the possibility to pre-assign outcomes for each possible observation.

The original Kochen-Specker theorem doesn’t involve the probability rule PROB, but there are variants which rely on correlations [15]. Conversely, there are also Bell-type results which follow solely from PROJ and not from PROB, for example involving the GHZ state [16–18].

D. Quantum correlations

The correlation between two observables \( \hat{A} \) and \( \hat{B} \) is defined as the expectation value of their product,

\[
C(\hat{A}, \hat{B}) = \langle \hat{A} \hat{B} \rangle.
\]

It is interesting how expectation values work in QM. On the one hand, as seen from the equation (10), they lead to the uncertainty principle (9), and were regarded as proving the limitations of QM, as compared to the potentially infinite precision of measurement in classical mechanics. However, by carefully combining observables, the probability rule PROB can lead to correlations that can’t be obtained by classical theories, unless we allow them to violate local realism and independence of context.

The greater the tension between the constraints imposed by the observables, the greater and more non-classical is the resulting correlation.

Classical correlations are calculated assuming that there are some definite values for the variables. The resulting correlations satisfy some inequalities, which may be violated by the corresponding quantum correlations, because the tension principle doesn’t allow the existence of definite values for all the observables. This provides ways to test the predictions of QM as compared to classical theories.

If the quantum observations are supposed to be made simultaneously on the same system in the same place, one obtains Kochen-Specker-type inequalities.

If the observations are sequential, one obtains temporal inequalities, or Leggett-Garg-type inequalities [19]. There are some theories which aim to prove that the classical world emerges from the quantum one. For instance, the objective collapse theories [20], and the decoherence program [21, 22]. But is the classical theory which is supposed to emerge at the macroscopic level realist? The Leggett-Garg theorem ruled out macroscopic realism, similar to how the Bell [13] and Kochen-Specker [14, 23] theorems ruled out local and non-contextual realism. The Leggett-Garg inequality is violated at all quantum levels, and became a prototype for the temporal inequalities.

E. A unified view on correlations

Quantum correlations may appear differently: the Kochen-Specker-type correlations refer to the context, the Leggett-Garg-type correlations are temporal, and the Bell-type correlations are spatial. However, they all have something in common: the classical inequalities are obtained by similar algebraic calculations, and all the quantum correlations follow from the tension between the constraints imposed by the non-commuting observables. This suggests that they may be more strongly related.

Suppose we start with a set of observables for the same system, so that there is a tension between the constraints they impose. For some states the quantum correlations violate the inequalities which are obeyed by the corresponding classical correlations. We obtained a Kochen-Specker-type scenario. Now, let’s assume the same observations are performed in a temporal order. We obtain a Leggett-Garg-type scenario.
Assume now that the observed system is composite, and the observables act on one of the subsystems or the other. The observables corresponding to different subsystems commute, so it seems there is no tension. However, as explained in §III B, the state in which the composite system is prepared forces relations between the Hilbert spaces of the subsystems. This means that what we thought to be compatible observables, for entangled systems are in fact incompatible. Alternatively, the tension can be seen as taking place between the preparation of the entangled system and the other observables. In the Bell-type scenario, space doesn’t matter, because is not part of the postulates which led to the Bell correlations. What matters is that a tensor product and entanglement are involved, otherwise there is no difference from a general Leggett-Garg-type scenario.

It is already a known fact that Bell-type correlations can be seen as temporal correlations involving commuting observables, applied to entangled states. While the general temporal correlations are bound easier by using semidefinite programming [24], this is more difficult for the Bell-type inequalities, because the observables of the involved parties commute [25, 26].

We can now summarize the relation between the three types of inequalities:

\[ \text{Bell} \subseteq \text{Leggett-Garg} \subseteq \text{Kochen-Specker}. \quad (18) \]

However, this hierarchy doesn’t prohibit the existence of procedures to convert Kochen-Specker theorems into Bell’s theorems [18, 27–30].

Quantum correlations are stronger than classical correlations. A classical mechanism would allow this amount of correlations work only if the two subsystems would interact or exchange information. Classical multivariate probability distributions are joint probability distributions. But in QM, it is not always possible to assign definite values to all variables simultaneously. Therefore, as shown by A. Fine [31, 32], for Bell-type scenarios there is no joint probability distribution which gives the same correlations as QM. Any joint probability distribution should obey Bell’s inequality, while QM violates it. The idea that quantum correlations are characterized by the nonexistence of a joint probability distribution applies also to Kochen-Specker type correlations [15, 33], and temporal correlations [34, 35]. Because quantum correlations are just expectation values of products of observables, the joint probability distributions exist when the involved observables commute. This suggests that the source of quantumness is the non-commutativity of observables [33, 36].

We see that the tension principle can be viewed as the source of quantum probabilities.

IV. CONCLUSION

We have seen that the many features of Quantum Mechanics that appear counterintuitive and paradoxical, can be seen as various manifestations of a single principle. The tension principle states that they are caused by the tension between the constraints imposed by incompatible observations. The tension principle itself is counterintuitive. We can say that the quantum paradoxes have their root in one single paradox, namely that the constraints appear to be contradictory. But maybe Bohr’s saying applies here too: Contraria non contradictoria sed complemen tata sunt. Even if this principle gathers the essence of other typically quantum phenomena, it would be useful to understand how the tension is resolved.

ACKNOWLEDGMENTS

The author cordially thanks Radu Ionicioiu, for very helpful comments and suggestions.

[1] J.A. Wheeler. The quantum and the universe. In Relativity, Quanta and Cosmology in the Development of the Scientific Thought of Albert Einstein, volume 1, pages 807–825, 1979.
[2] C.A. Fuchs and B.C. Stacey. Some negative remarks on operational approaches to quantum theory. Preprint arXiv:1401.7254, 2014.
[3] P.A.M. Dirac. The Principles of Quantum Mechanics. Oxford University Press, 1958.
[4] J. von Neumann. Mathematical Foundations of Quantum Mechanics. Princeton University Press, 1955.
[5] A.M. Gleason. Measures on the closed subspaces of a Hilbert space. J. Math. Mech, 6(4):885–893, 1957.
[6] W. Heisenberg. Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Zeitschrift für Physik, 43(3–4):172–198, 1927.
[7] E.H. Kennard. Zur Quantenmechanik einfacher Bewegungstypen. Zeitschrift für Physik, 44(4–5):326–352, 1927.
[8] Hermann Weyl. Gruppentheorie und Quantenmechanik. Leipzig: Hirzel, 1928.
[9] H. P. Robertson. The uncertainty principle. Phys. Rev., 34:163–164, Jul 1929.
[10] E. Schrödinger. Zum Heisenbergschen Unschärfeprinzip. Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-mathematische Klasse, 14:296–303, 1930.
[11] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? Physical Review, 47(10):777, 1935.
[12] D. Bohm. Quantum Theory. Prentice-Hall (New York), 1951.
[13] J.S. Bell. On the Einstein-Podolsky-Rosen paradox. Physics, 1(3):195–200, 1964.
[14] S. Kochen and E.P. Specker. The problem of hidden variables in quantum mechanics. J. Math. Mech., 17:59–87, 1967.
[15] A.A. Klyachko, M. A. Can, S. Binicioğlu, and A.S. Shumovsky. Simple test for hidden variables in spin-1 systems. Phys. Rev. Lett., 101(2):020403, 2008.
[16] D.M. Greenberger, M.A. Horne, and A. Zeilinger. Going beyond Bell’s theorem. In Bells theorem, quantum theory and conceptions of the universe, pages 69–72. Springer, 1989.
[17] D.M. Greenberger, M.A. Horne, A. Shimony, and A. Zeilinger. Bells theorem without inequalities. Am. J. Phys., 58:1131, 1990.
[18] N.D. Mermin. Hidden variables and the two theorems of John Bell. Rev. Mod. Phys., 65(3):803, 1993.
[19] A.J. Leggett and A. Garg. Quantum mechanics versus macroscopic realism: Is the flux there when nobody looks? Phys. Rev. Lett., 54(9):857–860, 1985.
[20] G.C. Ghirardi, A. Rimini, and T. Weber. Unified dynamics of microscopic and macroscopic systems. Physical Review D, (34):470–491, 1986.
[21] W.H. Zurek. Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse? Physical Review D, 24(6):1516, 1981.
[22] W.H. Zurek. Decoherence, einselection, and the quantum origins of the classical. Rev. Mod. Phys., 75:715, 2002.
[23] J.S. Bell. On the problem of hidden variables in quantum mechanics. volume 38, pages 447–452, 1966.
[24] C. Budroni, T. Moroder, M. Kleinmann, and O. Gühne. Bounding temporal quantum correlations. Phys. Rev. Lett., 111:020403, Jul 2013.
[25] M. Navascués, S. Pironio, and A. Acín. Bounding the set of quantum correlations. Phys. Rev. Lett., 98(1):010401, 2007.
[26] M. Navascués, S. Pironio, and A. Acín. A convergent hierarchy of semidefinite programs characterizing the set of quantum correlations. New Journal of Physics, 10(7):073013, 2008.
[27] P. Heywood and M.L.G. Redhead. Nonlocality and the Kochen-Specker paradox. Found. Phys., 13(5):481–499, 1983.
[28] N.D. Mermin. Simple unified form for the major no-hidden-variables theorems. Phys. Rev. Lett., 65(27):3373–3376, 1990.
[29] N.D. Mermin. Extreme quantum entanglement in a superposition of macroscopically distinct states. Phys. Rev. Lett., 65:1838–1840, 1990.
[30] A. Cabello. “All versus nothing” inseparability for two observers. Phys. Rev. Lett., 87(1):010403, 2001.
[31] A. Fine. Joint distributions, quantum correlations, and commuting observables. Journal of Mathematical Physics, 23(7):1306–1310, 1982.
[32] A. Fine. Hidden variables, joint probability, and the Bell inequalities. Phys. Rev. Lett., 48:291–295, Feb 1982.
[33] J.D. Malley and A. Fine. Noncommuting observables and local realism. Physics Letters A, 347(1):51–55, 2005.
[34] T. Fritz. Quantum correlations in the temporal Clauser–Horne–Shimony–Holt (CHSH) scenario. New Journal of Physics, 12(8):083055, 2010.
[35] M. Markiewicz, P. Kurzynski, J. Thompson, S.-Y. Lee, A. Soeda, T. Paterek, and D. Kaszlikowski. Unified approach to contextuality, non-locality, and temporal correlations. Preprint arXiv:1302.3502, 2013.
[36] J.D. Malley. All quantum observables in a hidden-variable model must commute simultaneously. Physical Review A, 69(2):022118, 2004.