π-MESONS AS THE QUANTA OF NON-NEWTONIAN HADRONIC FLUID

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Abstract

It occurs that when we attempt to melt the nucleon in order to obtain the free quark gas, or QGP fluid the mass of the heat quantum (heaton) is equal to the π—meson mass.

Key words: Heat quanta; Quantum heat transport; Quantum diffusion coefficient.
1 Introduction

As is well known Nelson \[1\] in 1966 succeed in deriving the Schrödinger equation from the assumption that quantum particles follow continuous trajectories in a chaotic background. The derivation of the usual linear Schrödinger equation follows only if the diffusion coefficient $D$, associated with quantum brownian motion takes the value $D = \hbar/2m$ as assumed by Nelson.

In the paper we study the transfer process of the quantum particles in the context of the thermal energy transport in highly excited matter. It will be shown that when matter is excited with short thermal perturbation the response of the matter can be well described by quantum hyperbolic heat transfer equation (QHT) which is the generalization of the parabolic quantum heat transport equation (PHT) with the diffusion coefficient $D = \hbar/m$, where $m$ denotes the mass of the diffused particles. The obtained QHT has the form

$$\frac{1}{c^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau c^2} \frac{\partial T}{\partial t} = \frac{\alpha_i^2}{3} \nabla^2 T,$$

where $T$ denotes temperature, $c$ is the velocity of light and $\alpha_i = (1/137, 0.15, 1)$ is the fine structure constant for electromagnetic interaction, strong interaction and strong quark-quark interaction respectively, $\tau_i$ is the relaxation time for scattering process.

When the QHT is applied to the study of the thermal excitation of the matter, the quanta of thermal energy, the heaton can be defined with energies $E_{h^e} = 9 \text{ eV}$, $E_{h^H} = 7 \text{ MeV}$ and $E_{h^q} = 139 \text{ MeV}$ for atomic, nucleon and quark level respectively.

2 The quantum heat transport equation

One of the best models in mathematical physics is Fourier’s model for the heat conduction in matter. Despite the excellent agreement obtained between theory and experiment, the Fourier model contains several inconsistent implications. The most important is that the model implies an infinite speed of propagation for heat. Cattaneo \[2\] was the first to propose a remedy. He formulated new hyperbolic heat diffusion equation for propagation of the heat
waves with finite velocity.

There is an impressive amount of literature on hyperbolic heat transport in matter [3, 4, 5]. In our book [6] we developed the new hyperbolic heat transport equation which generalizes the Fourier heat transport equation for the rapid thermal processes. The hyperbolic heat conduction equation (HHC) for the fermionic system can be written in the form:

$$\frac{1}{\left(\frac{1}{3}v_F^2\right)} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau \left(\frac{1}{3}v_F^2\right)} \frac{\partial T}{\partial t} = \nabla^2 T, \quad (2)$$

where $T$ denotes the temperature, $\tau$ – the relaxation time for the thermal disturbance of the fermionic system and $v_F$ is the Fermi velocity.

In the subsequent we develop the new formulation of the HHC considering the details of the two fermionic systems: electron gas in metals and nucleon gas.

For the electron gas in metals the Fermi energy has the form:

$$E_F^e = (3\pi^2)^2 \frac{n^{2/3}\hbar^2}{2m_e}, \quad (3)$$

where $n$ – density and $m_e$ – electron mass. Considering that

$$n^{-\frac{1}{4}} \sim a_B \sim \frac{\hbar^2}{me^2}, \quad (4)$$

and $a_B$ – Bohr radius, one obtains

$$E_F^e \sim \frac{n^{2/3}\hbar^2}{m_e} \sim \frac{\hbar^2}{ma^2} \sim \alpha^2 m_e c^2, \quad (5)$$

where $c$ – light velocity and $\alpha = 1/137$ is the fine structure constant. For the Fermi momentum, $p_F$ we have

$$p_F^e \sim \frac{\hbar}{a_B} \sim \alpha m_e c \quad (6)$$

and for Fermi velocity, $v_F$

$$v_F^e \sim \frac{p_F}{m_e} \sim \alpha c. \quad (7)$$
Considering formula (7) equation (2) can be written as
\[ \frac{1}{c^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{c^2 \tau} \frac{\partial T}{\partial t} = \frac{\alpha^2}{3} \nabla^2 T. \] (8)

As it is seen from (8) the HHC equation is the relativistic equation as it takes into account the finite velocity of light. In order to derive the Fourier law from equation (8) we are forced to break the special theory of relativity and put in equation (8) \( c \to \infty, \tau \to 0 \). In addition it was demonstrated from HHC in a natural way, that in electron gas the heat propagation velocity \( v_h \sim v_F \) in the accordance with the results of the pump probe experiments.

For the nucleon gas, Fermi energy is equal:
\[ E_N^F = \frac{(9\pi)^{\frac{3}{2}} \hbar^2}{8mr_0^2}, \] (9)

where \( m \) – nucleon mass and \( r_0 \), which describes the range of strong interaction is equal:
\[ r_0 = \frac{\hbar}{m_\pi c}, \] (10)

\( m_\pi \) – is the pion mass. Considering formula (10) one obtains for the nucleon Fermi energy
\[ E_N^F \sim \left(\frac{m_\pi}{m}\right)^2 mc^2. \] (11)

In the analogy to the equation (5) formula (11) can be written as follows
\[ E_N^F \sim \alpha_s^2 mc^2, \] (12)

where \( \alpha_s = 0.15 \) is fine structure constant for strong interactions. Analogously we obtain for the nucleon Fermi momentum
\[ p_N^F \sim \frac{\hbar}{r_0} \sim \alpha_s mc, \] (13)

and for nucleon Fermi velocity
\[ v_N^F = \frac{p_F}{m} \sim \alpha_s c, \] (14)

and HHC for nucleon gas can be written as follows:
\[ \frac{1}{c^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{c^2 \tau} \frac{\partial T}{\partial t} = \frac{\alpha_s^2}{3} \nabla^2 T. \] (15)
In the following the procedure for the discretization of temperature $T(\vec{r},t)$ in hot fermion gas will be developed. First of all we introduce the reduced de Broglie wavelength

$$\lambda_B^e = \frac{\hbar}{m_e v_h^e}, \quad \lambda_B^N = \frac{\hbar}{m v_h^N},$$

and mean free path, $\lambda^e$, $\lambda^N$:

$$\lambda^e = v_h^e \tau^e, \quad \lambda^N = v_h^N \tau^N.$$ (17)

Considering formulae (17), (18) we obtain HHC for electron and nucleon gases:

$$\frac{\lambda_B^e}{v_h^e} \frac{\partial^2 T^e}{\partial t^2} + \frac{\lambda_B^e}{\lambda^e} \frac{\partial T}{\partial t} = \frac{\hbar}{m_e} \nabla^2 T^e,$$ (18)

$$\frac{\lambda_B^N}{v_h^N} \frac{\partial^2 T^N}{\partial t^2} + \frac{\lambda_B^N}{\lambda^N} \frac{\partial T^N}{\partial t} = \frac{\hbar}{m} \nabla^2 T^N.$$ (19)

Equations (18) and (19) are the hyperbolic partial differential equations which are the master equations for heat propagation in Fermi electron and nucleon gases. In the following we will study the quantum limit of heat transport in the fermionic systems. We define the quantum heat transport limit as follows:

$$\lambda^e = \chi^e_B, \quad \lambda^N = \chi^N_B.$$ (20)

In that case equations (18), (19) have the form:

$$\tau^e \frac{\partial^2 T^e}{\partial t^2} + \frac{\partial T^e}{\partial t} = \frac{\hbar}{m_e} \nabla^2 T^e,$$ (21)

$$\tau^N \frac{\partial^2 T^N}{\partial t^2} + \frac{\partial T^N}{\partial t} = \frac{\hbar}{m} \nabla^2 T^N,$$ (22)

where

$$\tau^e = \frac{\hbar}{m_e (v_h^e)^2}, \quad \tau^N = \frac{\hbar}{m (v_h^N)^2}.$$ (23)

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Equations (21), (22) define the master equation for quantum heat transport. Having the relaxation time $\tau_e$, $\tau_N$ one can define the “pulsations” $\omega_e^h$, $\omega_N^h$

$$\omega_e^h = (\tau_e)^{-1}, \quad \omega_N^h = (\tau_N)^{-1}$$

(24)

or

$$\omega_e^h = \frac{m_e(v_e^h)^2}{\hbar}, \quad \omega_N^h = \frac{m(v_N^h)^2}{\hbar}$$

i.e.

$$\omega_e^h \hbar = m_e(v_e^h)^2 = \frac{m_e \alpha^2}{3} c^2$$

$$\omega_N^h \hbar = m(v_N^h)^2 = \frac{m \alpha^2}{3} c^2.$$ (25)

Formulae (25) define the Planck-Einstein relation for heat quanta $E_e^h$, $E_N^h$

$$E_e^h = \omega_e^h \hbar = m_e(v_e^h)^2$$

$$E_N^h = \omega_N^h \hbar = m(v_N^h)^2.$$ (26)

The heat quantum with energy $E_h = \hbar \omega$ can be named as the heaton in complete analogy to the phonon, magnon, roton, etc. For $\tau_e$, $\tau_N \to 0$ equations (21), (25) are the Fourier equations with quantum diffusion coefficients $D_e$, $D_N$

$$\frac{\partial T_e}{\partial t} = D_e \nabla^2 T_e, \quad D_e = \frac{\hbar}{m_e},$$

(27)

$$\frac{\partial T_N}{\partial t} = D_N \nabla^2 T_N, \quad D_N = \frac{\hbar}{m}.$$ (28)

The quantum diffusion coefficients $D_e$, $D_N$ for the first time were introduced by E. Nelson [1] and discussed in papers [7, 8, 9].

For finite $\tau_e$, $\tau_N$ for $\Delta t < \tau_e$, $\Delta t < \tau_N$ equations (21), (22) can be written as follows

$$\frac{1}{(v_e^h)^2} \frac{\partial^2 T_e}{\partial t^2} = \nabla^2 T_e,$$ (29)

$$\frac{1}{(v_N^h)^2} \frac{\partial^2 T_N}{\partial t^2} = \nabla^2 T_N.$$ (30)

Equations (29), (30) are the wave equations for quantum heat transport (QHT). For $\Delta t > \tau$ one obtains the Fourier equations (27), (28).
3 The possible interpretation of the heaton energies

First of all we consider the electron and nucleon gases. For electron gas we obtain from formula (17), (26) for $m_e = 0.51 \text{ MeV}/c^2$, $v_h = (1/\sqrt{3})\alpha c$

$$E^e_h = 9 \text{ eV},$$

(31)

which is of the order of the Rydberg energy. For nucleon gases one obtains $(m = 938 \text{ MeV}/c^2, \alpha_s = 0.15)$ from formulae (17), (26)

$$E^N_h \sim 7 \text{ MeV}$$

(32)

i.e. average binding energy of the nucleon in the nucleus (“boiling” temperature for the nucleus)

When the ordinary matter (on the atomic level) or nuclear matter (on the nucleus level) is excited with short temperature pulses ($\Delta t \sim \tau$) the response of the matter is discrete. The matter absorbs the thermal energy in the form of the quanta $E^e_h$ or $E^N_h$.

It is quite natural to pursue the study of the thermal excitation to the subnucleon level i.e. quark matter. In the following we generalize the QHT equation (8) for quark gas in the form:

$$\frac{1}{c^2} \frac{\partial^2 T^q}{\partial t^2} + \frac{1}{c^2 \tau} \frac{\partial T^q}{\partial t} = \frac{(\alpha_s^q)^2}{3} \nabla^2 T^q$$

(33)

with $\alpha_s^q$ – the fine structure constant for strong quark-quark interaction and $v_h^q$ – thermal velocity:

$$v_h^q = \frac{1}{\sqrt{3}} \alpha_s^q c.$$  

(34)

Analogously as for electron and nucleon gases we obtain for quark heaton

$$E^q_h = \frac{m_q}{3} (\alpha_s^q)^2 c^2,$$

(35)

where $m_q$ denotes the mass of the average quark mass. For quark gas the average quark mass can be calculated according to formula (10)

$$m_q = \frac{m_u + m_d + m_s}{3} = \frac{350 \text{ MeV} + 350 \text{ MeV} + 550 \text{ MeV}}{3} = 417 \text{ MeV},$$

(36)
where $m_u$, $m_d$, $m_s$ denotes the mass of the up, down and strange quark respectively. For the calculation of the $\alpha_s^q$ we consider the decays of the baryon resonances. For strong decay of the $\Sigma^0(1385 \text{ MeV})$ resonance:

$$K^- + p \rightarrow \Sigma^0(1385 \text{ MeV}) \rightarrow \Lambda + \pi^0$$

the width $\Gamma \sim 36 \text{ MeV}$ and lifetime $\tau_s$

$$\tau_s = \frac{\hbar}{\Gamma} \sim 10^{-23} \text{ s.}$$

For electromagnetic decay

$$\Sigma^0(1192 \text{ MeV}) \rightarrow \Lambda + \gamma,$$

$\tau_e \sim 10^{-19} \text{ s.}$ Considering that

$$\left( \frac{\alpha_s^q}{\alpha} \right) \sim \left( \frac{\tau_e}{\tau_s} \right)^{1/2} \sim 100,$$

one obtains for $\alpha_s^q$ the value

$$\alpha_s^q \sim 1.$$  \hspace{1cm} (37)

Substituting formulae (36), (37) to formula (35) one obtains:

$$E^q_h \sim 139 \text{ MeV} \sim m_\pi,$$  \hspace{1cm} (38)

where $m_\pi$ denotes the $\pi$-meson mass. It occurs that when we attempt to “melt” the nucleon in order to obtain the free quark gas the energy of the heaton is equal to the $\pi$-meson mass (which consists of two quarks). It is the simple presentation of quark confinement. Moreover it seems that the standard approaches to the melting of the nucleons into quarks through the heating processes in “splashes” of the chunks of the nuclear matter do not promise the success.

### 4 QGP as non-Newtonian fluid

We argue that at excitation energy of the order of pion mass the hadronic matter undergoes the phase transition to the non-Newtonian fluid with the relaxation time, $\tau$

$$\tau = \frac{\hbar}{m_q c^2},$$  \hspace{1cm} (39)
where $m_q c^2$ is of the order of 400 MeV and the velocity of sound $v_h \sim c$. The thermal energy quanta in non-Newtonian hadronic fluid is of the order of $140 \text{ MeV} \simeq m_\pi$. In this model the $\pi-$meson can be described as the “phonon”- excitations of the non-Newtonian hadronic fluid. With the excitation energy higher then 140 MeV the boiled fluid evaporates the $\pi-$meson copiously. The spectra of the emitted $\pi-$mesons can be described by formula (6)

$$d\eta = \frac{N_0 mT^{-1}}{K_2 \left( \frac{m}{T} \right)} e^{-\frac{3}{2} \gamma^5 u^2} \exp \left[ -m \gamma T^{-1} \right] dV du. \quad (40)$$

In formula (40) $\gamma = \left( 1 - \frac{u^2}{c^2} \right)^{-\frac{1}{2}}$, $T$ is the temperature of the fluid and $dV$ is the volume element. Function $K_2 \left( \frac{m}{T} \right)$ is the modified Bessel function of the second kind.
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