Dynamical quantum phase transitions in a mesoscopic superconducting system

K. Wrześniowski, I. Weymann, N. Sedlmayr, and T. Domański

1Institute of Spintronics and Quantum Information, Faculty of Physics, A. Mickiewicz University, 61-614 Poznań, Poland
2Institute of Physics, M. Curie-Skłodowska University, 20-031 Lublin, Poland

(Dated: December 21, 2021)

We inspect signatures of dynamical quantum phase transitions driven by two types of quenches acting on a correlated quantum dot embedded between superconducting and metallic reservoirs. Under stationary conditions the proximity induced on-dot pairing, combined with strong Coulomb repulsion, prefers the quantum dot to be either in the singly occupied (spinful) or BCS-type (spinless) ground state configuration. We study the time evolution upon traversing such a phase boundary due to quantum quenches by means of the time-dependent numerical renormalization group approach, revealing non-analytic features in the low-energy return rate. Quench protocols can be realized in a controllable manner and we are confident that detection of this dynamical singlet-doublet phase transition would be feasible by charge tunnelling spectroscopy.

Motivation. – Understanding the evolution of quantum many-body systems away from equilibrium is currently a topic of intensive investigations in condensed matter and ultracold atom physics, where the dynamical critical phenomena are driven by time-dependent perturbations [1]. In particular, dynamical quantum phase transitions (DQPTs) [2] can appear at critical times $t_c$ following a quantum quench. Usually, this occurs for quenches across equilibrium phase transition boundaries, although this is neither a necessary nor sufficient condition [3]. DQPTs manifest themselves by non-analytic (cusp-like) features appearing in the return rate function [4], in analogy to a static (classical or quantum) phase transitions revealing itself in the free energy at a critical temperature, magnetic field, pressure, etc. DQPTs have also been generalised to non ground state initial conditions with mixed results [5–12], and in driven systems [13–21]. So far, the studies of critical properties [22, 23], dynamical order parameters [24, 25], and spontaneously broken symmetries [26, 27] have been mostly addressed in bulk systems, often in either spin chains [28–34] or topological insulators and superconductors [35–44]. Some studies have also addressed the superconducting transition [45, 46]. On the other hand, empirical evidence for DQPTs has been reported only in a few cases, mainly using trapped ion systems [47], although alternative suggestions for measuring DQPTs exist [22, 26, 48–51].

While clear-cut signatures of static and dynamical phase transitions are observable solely in the thermodynamic limit, it has been recently shown [52] that DQPTs can also be realized in systems comprising of a limited (finite) number of constituents. Indeed such a result is supported by the appearance of signatures of DQPTs in finite-size analyses [53]. Here, we propose another realm for a feasible dynamical quantum phase transition in a finite system, which could be realized in hybrid nanostructures. As a specific example, we consider a correlated quantum dot (QD) at the interface between superconducting (S) and normal (N) leads. A dynamical singlet-doublet transition can originate from a quench imposed either on the coupling strength $\Gamma_S(t)$ or the energy level position $\epsilon_d(t)$ when crossing through the phase boundary, which is marked with a solid line in (b). The dashed [dotted] arrow in (b) indicates the direction of the quench performed in $\Gamma_S(t)$ [$\epsilon_d(t)$] studied in this paper.

FIG. 1. (a) Schematic of a correlated quantum dot sandwiched between superconducting (S) and normal (N) leads. A dynamical singlet-doublet transition can originate from a quench imposed either on the coupling strength $\Gamma_S(t)$ or the energy level position $\epsilon_d(t)$ when crossing through the phase boundary, which is marked with a solid line in (b). The dashed [dotted] arrow in (b) indicates the direction of the quench performed in $\Gamma_S(t)$ [$\epsilon_d(t)$] studied in this paper.
by the Hamiltonian $\hat{H}_0$, whose ground state obeys the Schrödinger equation, $H_0 |\Psi_0\rangle = E_0 |\Psi_0\rangle$. At time $t = 0$, the Hamiltonian is suddenly changed, $\hat{H}_0 \to \hat{H}$, causing the evolution $i\frac{d}{dt} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$, which results in the time-dependent state $|\Psi(t)\rangle = e^{-it\hat{H}} |\Psi_0\rangle$. Fidelity (similarity) of the states $|\Psi_0\rangle$ and $|\Psi(t)\rangle$ at an arbitrary time $t \geq 0$ can be characterized by the Loschmidt amplitude, 

$$\langle \Psi_0 |\Psi(t)\rangle = \langle \Psi_0 |e^{-it\hat{H}} |\Psi_0\rangle.$$ 

The squared absolute value of the Loschmidt amplitude, $L(t) = |\langle \Psi_0 |e^{-it\hat{H}} |\Psi_0\rangle|^2$, referred to as the Loschmidt echo, can be regarded as dual to the partition function $Z = \langle e^{-\beta\hat{H}} \rangle$ in statistical physics, where the inverse temperature $\beta = 1/k_BT$ is replaced by the imaginary time it. Furthermore, the free energy, $F(T) = -k_BT \ln Z$, is equivalent to the return rate $\lambda(t)$ defined via $L(t) \equiv e^{-N\lambda(t)}$, where $N$ stands for the number of degrees of freedom. The usual critical temperature $T_c$, referring to a discontinuity of the free energy or its derivatives $\lim_{T \to T_c} F(T)$, is hence equivalent to the critical time $t_c$, at which the non-analyticities occur in the return rate $\lim_{t \to t_c} \lambda(t)$. For simple systems a sequence of critical points equally spaced along the time axis occurs, however, non-periodic behavior is also possible [4, 44], and likely generic.

**Microscopic model.** – Let us present the microscopic scenario, in which the quantum quench $H_0 \to \hat{H}$ qualitatively affects the properties of our heterostructure. The correlated quantum dot contacted with the superconducting and normal leads (Fig. 1) can be described by the Hamiltonian

$$\hat{H} = \hat{H}_{QD} + \sum_{\beta} (\hat{H}_\beta + \hat{V}_{\beta-QD}),$$

with $\beta \in \{N, S\}$ referring to the metallic and superconducting reservoirs respectively. The correlated quantum dot is described by $\hat{H}_{QD} = \sum_\sigma \varepsilon_\sigma(t) \hat{d}_\sigma^\dagger \hat{d}_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow$, where $\hat{d}_\sigma (\hat{d}_\sigma^\dagger)$ stands for the annihilation (creation) operator of spin $\sigma = \uparrow, \downarrow$ electrons whose energy $\varepsilon_\sigma(t)$ depends on time, and $U > 0$ is the Coulomb potential of repulsive interactions in the dot. The macroscopic superconductor is assumed to be of BCS type: $H_S = \sum_{k, \sigma} \xi_S \hat{c}_{\sigma k}^\dagger \hat{c}_{\sigma k} - \sum_{k} \Delta (\hat{c}_{\uparrow k}^\dagger \hat{c}_{\downarrow k}^\dagger + \hat{c}_{\downarrow k} \hat{c}_{\uparrow k})$, with an isotropic pairing gap $\Delta$ and dispersion relation $\xi_S k$. The normal lead will be simply treated as a free fermion gas, $H_N = \sum_{k, \sigma} \xi_N \hat{c}_{N, \sigma k}^\dagger \hat{c}_{N, \sigma k}$. The hybridization of the localized (QD) and itinerant (N,S) electrons is given by, $\hat{V}_{\beta-QD} = \sum_{k, \sigma} (V_{\beta k} \hat{d}_\sigma^\dagger \hat{c}_{\sigma k} + V_{\beta k} \hat{c}_{\sigma k} \hat{d}_\sigma^\dagger)$, where $\hat{c}_{\sigma k}$ is the corresponding annihilation operator and $V_{\beta k}$ denotes the tunnelling matrix elements.

From here onward we analyze the low energy properties of such a N-QD-S hybrid structure, restricted to the region $|\omega| \ll \Delta$, safely inside the pairing gap $\Delta$, which in conventional superconductors is usually of the order of meV [56]. Under such circumstances, we can impose the auxiliary couplings $\Gamma_\beta = \pi \sum_{\beta} |V_{\beta k}|^2 \delta(\omega - \xi_{\beta k})$, assuming them to be energy-independent. The mobile Cooper pairs leaking onto the quantum dot will induce on-dot pairing. By integrating out the fermionic degrees of freedom from outside the pairing gap, such a superconducting proximity effect can be modeled by

$$\hat{H}_S + \hat{V}_{S-QD} \approx (\Gamma_S \hat{d}_d \hat{d}_d^\dagger + \text{H.c.}),$$

so that $\Gamma_S$ effectively plays the role of a pairing potential. The term (2) competes with the repulsive Coulomb interaction, leading to different characteristic signatures. We shall discuss them briefly in the following. The coupling to the normal lead $\Gamma_N$ will be assumed to be much smaller than $\Gamma_S$, to guarantee a sufficiently long life-time of the in-gap quasiparticles. Such a situation is customarily encountered in scanning tunneling microscope (STM) measurements, where various nanoscopic objects deposited on surfaces of superconductors are probed by a normal or superconducting tip placed at a secure distance from the impurities.

**Equilibrium quantum phase transition.** – Interplay between the superconducting proximity effect and electron correlations leads to qualitative changes of the QD ground state [57]. True eigenstates of the quantum dot are represented either by the singly occupied configurations $|\sigma\rangle$ (which are degenerate in the absence of a magnetic field) or BCS-type superpositions of the empty and doubly occupied states $u |0\rangle - v |\uparrow\downarrow\rangle$, $v |0\rangle + u |\uparrow\downarrow\rangle$. Upon changing the ratio of $\Gamma_S/U$ (or the QD level $\varepsilon_d$), the ground state can evolve between these spinful and spinless configurations. Such a parity changeover is manifested by the energy crossing of the in-gap quasiparticles, known as Andreev or Yu-Shiba-Rusinov bound states [58–60]. In the superconducting atomic limit (when the pairing gap $\Delta$ is the largest energy scale) the quantum phase transition between the doublet and the BCS-type singlet occurs at

$$\left(\varepsilon_d + \frac{U}{2}\right)^2 + \Gamma_S^2 = \left(\frac{U}{2}\right)^2.$$
Quench protocol for the singlet-doublet transition. – We now propose two possible scenarios of the quantum quench $H_0 \rightarrow H$, in which an abrupt change of our setup would enforce a transition between the Hamiltonians with singlet and doublet QD ground state configurations. In the first approach, we impose a variation of the coupling to the superconductor

$$\Gamma_S(t) = \begin{cases} \Gamma_{S0} & \text{for } t \leq 0, \\ \Gamma_S & \text{for } t > 0, \end{cases}$$  \tag{4}

assuming that the initial ($\Gamma_{S0}$) and final ($\Gamma_S$) values are on the opposite sides of the phase transition boundary [indicated with a dashed line in Fig. 1(b)]. The second type of quantum quench affects the energy level of the QD [marked with a dotted line in Fig. 1(b)]

$$\varepsilon_d(t) = \begin{cases} \varepsilon_d & \text{for } t \leq 0, \\ \varepsilon_d + V_G & \text{for } t > 0. \end{cases}$$  \tag{5}

In both cases (4,5) we choose such parameters which guarantee the quantum dot to be initially in the doublet and finally in the singlet configurations. Evolution between these different states (via a sequence of critical times) in a long-time limit terminates in the steady state, owing to relaxation processes in the continuous spectrum of the normal electrode [65]. Additional sources of such relaxation phenomena could be provided by electronic states existing outside the superconducting gap [66], but we neglect their influence here.

In STM spectroscopy the coupling of an impurity to a superconducting substrate could be varied by manipulating the tip-impurity distance [54]. This allows us to traverse the quantum phase transition by changing the effective exchange interaction [55]. As regards the ballistic charge tunneling via metal - quantum dot - superconductor hybrid nanostructures, there is a fairly large flexibility in a controllable variation of both the coupling $\Gamma_S$ and the quantum dot energy level $\varepsilon_d$. Evidence for the resulting parity (singlet-doublet) crossings has been reported by several groups, for instance using InAs nanowires placed between superconducting Al and metallic Au electrodes [62, 67] and in a carbon nanotube contacted to superconducting Nb and weakly coupled to a normal metal [68].

Dynamical quantum phase transitions. – To accurately describe the dynamical behavior of the system we resort to the numerical renormalization group method (NRG) [69–71]. This method has been successfully used to analyze the stationary singlet-doublet transition [57]. Here, we make use of its time-dependent extension [72–74] to address the problem of dynamical quantum phase transitions. The core of NRG is a logarithmic discretization (with parameter $\Lambda$) of the conduction band, which allows one to map the Hamiltonian to a chain-like form. Such a model can be diagonalized in an iterative fashion by keeping an appropriate number of low-energy states $N_K$. This allows us to find complete many-body eigenbases of the Hamiltonians $H_0$ and $\tilde{H}$,

\begin{align*}
\sum_{nse} |nse\rangle \langle nse|_0 &= \mathbb{1} \\
\sum_{nse} |nse\rangle \langle nse|_D &= \mathbb{1},
\end{align*}

respectively, where $s$ denotes an eigenstate at iteration $n$ belonging to the discarded ($D$) states of the chain, $e$ indicates the environmental subspace, while $n$ stands for the chain index [72]. Using the NRG representation, the

FIG. 2. (a) The time evolution of the Loschmidt echo, (b) the return rate and (c) the on-dot paring after the quench in the coupling to superconductor $\Gamma_S$. The solid lines correspond to the quench across the phase boundary between the doublet and singlet states, as marked with the dashed arrow in Fig. 1(b). The dashed lines present the results for quench within the doublet phase. The other parameters are: $U = 0.1$, $\varepsilon_d = -0.05$ and $\Gamma_N = 0.001$ in units of half the bandwidth.

FIG. 3. The time dependence of the return rate around the first critical time $t_c$, visible in Fig. 2. The inset presents the dependence of $\lambda(t)/\lambda(t_c)$ on the normalized time $|t - t_c|/t_c$. The parameters are the same as in Fig. 2.
Under stationary conditions one can traverse the quantum phase transition also by an appropriate variation of the QD energy level. Figure 4 presents the relevant time-dependent quantities obtained for $\Gamma_d/U = 0.2$ by performing the quench in the dot level $\varepsilon_d$ along the dotted line visible in Fig. 1(b). Again, the dashed lines correspond to the case when the quench is performed in the doublet phase, to illustrate that the DQPT does not occur then. However, when quenching across the phase boundary clear indications of dynamical quantum critical behavior are present. Note that for this type of quench the cusp-like behavior of $\lambda(t)$ is slightly smeared, which can be attributed to the fact that now dynamical changes occur both in the occupation and pairing function.

Summary. – We have studied the dynamical properties of the correlated quantum dot sandwiched between the metallic and superconducting leads. Considering the quantum quenches across the the phase boundary between the singlet and doublet phases, triggered by either an abrupt variation of the coupling of the quantum dot to the superconductor or a sudden change of the dot’s energy level, we have found clear signatures of dynamical quantum phase transitions. Thus, our work paves the way for the exploration of DQPT in mesoscopic systems, in which the microscopic parameters can be tuned in a fully controllable fashion, allowing for exploration of dynamical critical phenomena with contemporary experimental techniques.

Acknowledgments. – This work is supported by the National Science Centre (Poland) under the grants 2017/27/B/ST3/00621 (KW, IW), 2017/27/B/ST3/01911 (TD), and 2019/35/B/ST3/03625 (NS).

[1] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, “Colloquium: Nonequilibrium dynamics of closed interacting quantum systems,” Rev. Mod. Phys. 83, 863–883 (2011).

[2] M. Heyl, A. Polkovnikov, and S. Kehrein, “Dynamical quantum phase transitions in the transverse-field Ising model,” Phys. Rev. Lett. 110, 135704 (2013).

[3] Szabolcs Vajna and Balázs Dóra, “Disentangling dynami-
cal phase transitions from equilibrium phase transitions,” Physical Review B 89, 161105–161105 (2014).
[4] M. Heyl, “Dynamical quantum phase transitions: a review,” Rep. Prog. Phys. 81, 054001 (2018).
[5] Nils O. Abeling and Stefan Kehrein, “Quantum quench dynamics in the transverse field Ising model at nonzero temperatures,” Physical Review B 93, 104302–104302 (2016).
[6] Utso Bhattacharya, Souvik Bandyopadhyay, and Amit Dutta, “Mixed state dynamical quantum phase transitions,” Physical Review B 96, 180303–180303 (2017).
[7] M. Heyl and J. C. Budich, “Dynamical topological quantum phase transitions for mixed states,” Physical Review B 96, 180304–180304 (2017).
[8] Bruno Mera, Chrysoula Vlachou, Nikola Paunković, Vitor R. Vieira, and Oscar Vinyuela, “Dynamical phase transitions at finite temperature from fidelity and interferometric Loschmidt echo induced metrics,” Physical Review B 97, 094110 (2017).
[9] N. Sedlmayr, M. Fleischhauer, and J. Sirker, “The fate of dynamical phase transitions at finite temperatures and in open systems,” Physical Review B 97, 045147 (2018).
[10] Johannes Lang, Bernhard Frank, and Jad C. Halimeh, “Concurrence of dynamical phase transitions at finite temperature in the fully connected transverse-field Ising model,” Physical Review B 97, 174401 (2018).
[11] Johannes Lang, Bernhard Frank, and Jad C. Halimeh, “Dynamical Quantum Phase Transitions: A Geometric Picture,” Physical Review Letters 121, 130603–130603 (2018).
[12] Xu-Yang Hou, Qu-Cheng Gao, Hao Guo, Yan He, Tong Liu, and Chih-Chun Chien, “Ubiquity of zeros of the Loschmidt amplitude for mixed states in different physical processes and its implication,” Physical Review B 102, 104305 (2020).
[13] Shraddha Sharma, Angelo Russomanno, Giuseppe E. Santoro, and Amit Dutta, “Loschmidt echo and dynamical fidelity in periodically driven quantum systems,” Europhysics Letters 106, 67003–67003 (2014).
[14] Arkadiusz Kosior and Krzysztof Sacha, “Dynamical quantum phase transitions in discrete time crystals,” Physical Review A 97, 053621 (2018).
[15] Moos van Caspel, Sergio Enrique Tapias Arze, and Isaac Pérez Castillo, “Dynamical signatures of topological order in the driven-dissipative Kitaev chain,” SciPost Physics 6, 026 (2019).
[16] Kai Yang, Longwen Zhou, Wenchao Ma, Xi Kong, Pengfei Wang, Xi Qin, Xing Rong, Ya Wang, Fazhan Shi, Jiangbin Gong, and Jiangfeng Du, “Floquet dynamical quantum phase transitions,” Physical Review B 100, 085308 (2019).
[17] Sergio Enrique Tapias Arze, Pieter W. Claeyts, Isaac Pérez Castillo, and Jean-Sébastien Caux, “Out-of-equilibrium phase transitions induced by Floquet resonances in a periodically quench-driven XY spin chain,” arXiv:1804.10226 [cond-mat] (2020), arXiv: 1804.10226.
[18] Ryusuke Hamazaki, “Exceptional Dynamical Phase Transitions in Periodically Driven Quantum Systems,” arXiv:2012.11822 [cond-mat, physics:quant-ph] (2020), arXiv: 2012.11822.
[19] R. Jafari and Alireza Akbari, “Floquet dynamical phase transition and entanglement spectrum,” arXiv:2009.09484 [cond-mat, physics:quant-ph] (2020), arXiv: 2009.09484.
[20] D. M. Kennes, C. Karrasch, and A. J. Millis, “Loschmidt-amplitude wave function spectroscopy and the physics of dynamically driven phase transitions,” Physical Review B 101, 081106 (2020).
[21] Souvik Bandyopadhyay, Sourav Bhattacharjee, and Dipriman Sen, “Driven quantum many-body systems and out-of-equilibrium topology,” Journal of Physics: Condensed Matter 33, 093001 (2021).
[22] Markus Heyl, “Scaling and Universality at Dynamical Quantum Phase Transitions,” Physical Review Letters 115, 140602–140602 (2015).
[23] Bojan Zunkovic, Markus Heyl, Michael Knap, and Alessandro Silva, “Dynamical Quantum Phase Transitions in Spin Chains with Long-Range Interactions: Merging different concepts of non-equilibrium criticality,” Physical Review Letters 120, 130601–130601 (2018).
[24] Elena Canovi, Philipp Werner, and Martin Eckstein, “First-Order Dynamical Phase Transitions,” Physical Review Letters 113, 265702 (2014).
[25] Jan Carl Budich and Markus Heyl, “Dynamical topological order parameters far from equilibrium,” Physical Review B 93, 85416–85416 (2016).
[26] M. Heyl, “Dynamical quantum phase transitions in systems with broken-symmetry phases,” Physical Review Letters 113, 205701–205701 (2014).
[27] Arkadiusz Kosior, Andrzej Syrwid, and Krzysztof Sacha, “Dynamical quantum phase transitions in systems with broken continuous time and space translation symmetries,” Physical Review A 98, 023612 (2018).
[28] C. Karrasch and D. Schuricht, “Dynamical phase transitions after quenches in nonintegrable models,” Physical Review B 87, 195104–195104 (2013).
[29] F. Andraschko and J. Sirker, “Dynamical quantum phase transitions and the Loschmidt echo: A transfer matrix approach,” Physical Review B 89, 125120–125120 (2014).
[30] M. Azimi, M. Sekania, S. K. Mishra, L. Chotorlishvili, Z. Toklikishvili, and J. Berakdar, “Pulse and quench induced dynamical phase transition in a chiral multiferroic spin chain,” Physical Review B 94, 064423 (2016).
[31] Jad C. Halimeh and Valentin Zauner-Stauber, “Dynamical phase diagram of quantum spin chains with long-range interactions,” Physical Review B 96, 134427–134427 (2017).
[32] Jad C. Halimeh, Valentin Zauner-Stauber, and Laurens Vanderstraeten, “Quasiparticle approach,” Physical Review B 100, 035127 (2019), citation Key Alias: Gurarie2018.
[33] Jad C. Halimeh and Laurens Vanderstraeten, “Quasiparticle Origin of Dynamical Quantum Phase Transitions,” Physical Review Research 2, 033111 (2020).
[34] Szabolcs Vajna and Balázs Dóra, “Topological classification of dynamical phase transitions,” Physical Review B 91, 155127–155127 (2015).
[35] Suraj Hegde, Vasudha Shivamoggi, Smitha Vishveshwara, and Dipriman Sen, “Quench dynamics and parity blocking in Majorana wires,” New Journal of Physics 17, 53036–53036 (2015).
[36] Markus Schmitt and Stefan Kehrein, “Dynamical quantum phase transitions in the Kitaev honeycomb model,” Physical Review B 92, 075114–075114 (2015).
[38] N. Sedlmayr, P. Jäger, M. Maiti, and J. Sirker, “Bulk-boundary correspondence for dynamical phase transitions in one-dimensional topological insulators and superconductors,” Physical Review B 97, 064304 (2018).

[39] Utkarsh Mishra, R. Jafari, and Alireza Akbari, “Disordered Kitaev chain with long-range pairing: Loschmidt echo revivals and dynamical phase transitions,” arXiv:1810.06236 [cond-mat, physics:quant-ph] (2018).

[40] Wei Sun, Chang-Rui Yi, Bao-Zong Wang, Wei-Wei Zhang, Barry C. Sanders, Xiao-Tian Xu, Zong-Yao Wang, Jörg Schmiedmayer, Youjin Deng, Xiong-Jun Liu, Shuai Chen, and Jian-Wei Pan, “Uncover Topology by Quantum Quench Dynamics,” Physical Review Letters 121, 250403 (2018).

[41] Haiping Hu and Erhai Zhao, “Dynamical topology of quantum quenches in two dimensions,” arXiv:1911.02211 [cond-mat, physics:quant-ph] (2019).

[42] Christian B. Mendl and Jan Carl Budich, “Stability of dynamical phase transitions in topological insulators and connections with entanglement entropy and fidelity susceptibility,” Physical Review B 101, 014301 (2020).

[43] N. Sedlmayr, “Dynamical Phase Transitions in Topological Insulators,” Acta Physica Polonica A 135, 1191 (2019).

[44] T. Maslowski and N. Sedlmayr, “Quasiperiodic dynamical quantum phase transitions in multiband topological insulators and connections with entanglement entropy and fidelity susceptibility,” Physical Review B 101, 014301 (2020).

[45] K. Xu, Z. H. Sun, W. Liu, Y.R. Zhang, H. Li, H. Dong, W. Ren, P. Zhang, F. Nori, D. Zheng, H. Fan, and H. Wang, “Probing dynamical phase transitions with a superconducting quantum simulator,” Sci Adv. eaba4935 (2021).

[46] C. Rylands, E. A. Yuzbashyan, V. Gurarie, A. Zabalo, and V. Galitski, “Loschmidt echo of far-from-equilibrium fermionic superfluids,” (2021), arXiv:2103.03754.

[47] P. Jurcevic, H. Shen, P. Hauke, C. Maier, T. Brydges, C. Rylands, E. A. Yuzbashyan, V. Gurarie, A. Zabalo, and T. Domański, I. Weymann, M. Barańska, and V. Gorški, “Constructive influence of the induced electron-hole pairing in semimetals at t = 0,” Journal of Experimental and Theoretical Physics 38, 991 (1974).

[48] A. Jellinggaard, K. Grove-Rasmussen, M.H. Madsen, and J. Nygård, “Tuning Yu-Shiba-Rusinov states in a quantum dot,” Phys. Rev. B 94, 064520 (2016).

[49] T. Domański, I. Weymann, A. E. Feiguin, A. Martín-Rodero, and J. Nygård, “Temperature induced shifts of Yu-Shiba-Rusinov resonances in nanowire-based hybrid quantum dots,” Commun. Phys. 3, 125 (2020).

[50] R. Zitko, J.S. Lim, R. López, and R. Aguado, “Shiba states and zero-bias anomalies in the hybrid normal-superconductor Anderson model,” Phys. Rev. B 91, 045441 (2015).

[51] T. Domański, I. Weymann, M. Barańska, and G. Görski, “Constructive influence of the induced electron pairing on the Kondo state,” Sci. Rep. 6, 23336 (2016).

[52] R. Puebla, “Finite-component dynamical quantum phase transitions,” Phys. Rev. B 102, 220302 (2020).

[53] J. C. Halimeh, D. Trapin, M. Van Damme, and M. Heyl, “Local measures of dynamical quantum phase transitions,” Phys. Rev. B 104, 075130 (2021).

[54] L. Farinacci, G. Ahmad, G. Reeceh, M. Ruby, N. Bogdanoff, O. Peters, B.W. Heinrich, F. von Oppen, and K.J. Franke, “Tuning the coupling of an individual magnetic impurity to a superconductor: Quantum phase transition and transport,” Phys. Rev. Lett. 121, 196803 (2018).

[55] H. Ding, Y. Hu, M. T. Randeria, S. Hoffman, O. Deb, J. Klinovaja, D. Loss, and A. Yazdani, “Tuning interactions between spins in a superconductor,” Proc. Nat. Acad. Sci. 118 (2021), 10.1073/pnas.2024837118.

[56] S. De Franceschi, L. Kounenhouen, C. Schönnerberger, and W. Wernsdorfer, “Hybrid superconductor–quantum dot devices,” Nat. Nanotechnol. 5, 703 (2010).

[57] J. Bauer, A. Oguri, and A. C. Hewson, “Spectral properties of locally correlated electrons in a Bardeen-Cooper-Schrieffer superconductor,” J. Phys.: Condens. Matter 19, 486211 (2007).

[58] L.H Yu, “Bound state in superconductors with paramagnetic impurities,” Acta Physica Sinica 21, 115304 (1965).

[59] H Shiba, “Classical spins in superconductors,” Progress of theoretical Physics 40, 435 (1968).

[60] A I Rusinov, Do Chan Kat, and Yu V Kopaev, “Theory of superconductivity in the presence of electron-hole pairing in semimetals at t = 0,” Journal of Experimental and Theoretical Physics 38, 991 (1974).

[61] A. Jellinggaard, K. Grove-Rasmussen, M.H. Madsen, and J. Nygård, “Tuning Yu-Shiba-Rusinov states in a quantum dot,” Phys. Rev. B 90, 064520 (2016).

[62] J.C. Estrada Saldana, A. Vekris, V. Sosnovtseva, T. Kanne, P. Kroghstrup, K. Grove-Rasmussen, and J. Nygård, “Temperature induced shifts of Yu-Shiba-Rusinov resonances in nanowire-based hybrid quantum dots,” Commun. Phys. 3, 125 (2020).

[63] R. Zitko, J.S. Lim, R. López, and R. Aguado, “Shiba states and zero-bias anomalies in the hybrid normal-superconductor Anderson model,” Phys. Rev. B 91, 045441 (2015).

[64] T. Domański, I. Weymann, M. Barańska, and G. Görski, “Constructive influence of the induced electron pairing on the Kondo state,” Sci. Rep. 6, 23336 (2016).

[65] K. Wrzesiński, B. Baran, R. Taranko, T. Domański, and I. Weymann, “Quench dynamics of a correlated quantum dot sandwiched between normal-metal and superconducting leads,” Phys. Rev. B 103, 155420 (2021).

[66] R. Seoane Souto, A. E. Feiguin, A. Martín-Rodero, and A. Levy Yeyati, “Transient dynamics of a magnetic impurity coupled to superconducting electrodes: Exact numerics versus perturbation theory,” Phys. Rev. B 104, 214506 (2021).

[67] E.J.H. Lee, X. Jiang, R. Žitko, R. Aguado, C.M. Lieber, and S. De Franceschi, “Scaling of subgap excitations in a superconductor-semiconductor nanowire quantum dot,” Phys. Rev. B 95, 180502 (2017).

[68] J. Schindele, A. Baumgartner, R. Maurand, M. Weiss, K. O. Jeckelmann, W. Ketterle, and C. Schönenberger, “Nonlocal spectroscopy of Andreev bound states,” Phys. Rev. B 89, 045422 (2014).

[69] Kenneth G. Wilson, “The renormalization group: Critical phenomena and the Kondo problem,” Rev. Mod. Phys. 47, 773–840 (1975).
[70] R. Bulla, T.A. Costi, and T. Pruschke, “Numerical renormalization group method for quantum impurity systems,” Rev. Mod. Phys. 80, 395–450 (2008).

[71] ¨O. Legeza, C. P. Moca, A. I. T´oth, I. Weymann, and G. Zar´and, “Manual for the Flexible DM-NRG code,” arXiv:0809.3143v1 (2008), (the open access Flexible DM-NRG Budapest code is available at http://www.phy.bme.hu/~dmnrg/).

[72] F.B. Anders and A. Schiller, “Real-time dynamics in quantum-impurity systems: A time-dependent Numerical Renormalization-Group approach,” Phys. Rev. Lett. 95, 196801 (2005).

[73] H. T. M. Nghiem and T. A. Costi, “Time-dependent numerical renormalization group method for multiple quenches: Application to general pulses and periodic driving,” Phys. Rev. B 90, 035129 (2014).

[74] K. Wrze´sniewski and I. Weymann, “Quench dynamics of spin in quantum dots coupled to spin-polarized leads,” Phys. Rev. B 100, 035404 (2019).

[75] J. Bedow, E. Mascot, and D. K. Morr, “Emergence and manipulation of non-equilibrium Yu-Shiba-Rusinov states,” (2021), arXiv:2112.07733.