Elliptic and Triangular flow in asymmetric heavy-ion collisions

Md. Rihan Haque, Md. Nasim, and Bedangadas Mohanty
Variable Energy Cyclotron Centre, Kolkata 700064, India
(Dated: November 23, 2011)

We present a study of the elliptic ($v_2$) and triangular ($v_3$) flow and their corresponding eccentricity fluctuations for asymmetric (Au+Ag, Au+Cu and Au+Si) collisions at $\sqrt{s_{NN}} = 200$ GeV. These are compared to the corresponding results from symmetric (Au+Au and Cu+Cu) collisions at the same energy. The study which is carried out using a multi-phase transport (AMPT) model shows that triangularity ($v_3$), fluctuations in triangularity and $v_3$ do not show much variation for the different colliding ion sizes studied. However the eccentricity ($v_2$), fluctuations in eccentricity and $v_2$ shows a strong dependence on colliding ion size for a given number of participating nucleons. Our study thus indicates that asymmetric heavy-ion collisions could be used to constrain models dealing with flow fluctuations in heavy-ion collisions.

PACS numbers: 25.75.Ld

I. INTRODUCTION

Knowing the initial geometry and fluctuations in heavy-ion collisions has recently been shown to have important consequences on interpreting the data from the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) experiments [1, 2]. For example, the long standing physical interpretation of dihadron correlations in azimuthal angle $\phi$ and pseudorapidity $\eta$ to be due to propagation of shock waves (due to high momentum quarks or gluons) in the medium and jet-medium interactions are being revisited [3, 4]. The contribution from the odd harmonics associated with the particle azimuthal angle distribution (originally thought to be zero due the left-right symmetry in the transverse plane of symmetric heavy-ion collisions) to dihadron correlations are found to be important. Now it is being widely discussed that non zero odd harmonic contributions could arise from fluctuations in the transverse positions of nucleons undergoing hadronic scattering. Further it is now known that even the ideal hydrodynamics calculations under predicts the measured elliptic flow in central heavy-ion collisions at RHIC [5]. Only inclusion of flow fluctuations arising due to the eccentricity fluctuations can provide some satisfactory explanation of the data. It is also believed that measurement of higher order flow coefficients and their fluctuations can substantially improve the constraints on the transport properties of the system formed in high energy heavy-ion collisions.

Experimentally it has not been possible to separate the contribution from elliptic flow ($v_2$), fluctuations and non-flow from the data [6]. This is due to the lack of knowledge of the probability distributions for flow fluctuations. The fluctuations in $v_2$ can arise due to fluctuations in the eccentricity of the overlap region of the two colliding nuclei or deviations of the participant plane from the reaction plane. While non-flow effects are those correlations among particles that are not related to the reaction plane (e.g due to resonance decay and jets).

Asymmetric heavy-ion collisions could provide density profiles that are different or not accessible through symmetric heavy-ion collisions. For example a mid-central Cu on Au collision could lead to the Cu nucleus being occluded in the Au, leading to non zero odd harmonic of flow and being highly sensitive to early time dynamics. In addition asymmetric heavy-ion collisions could provide opportunity to disentangle the path length and energy dependence of parton energy loss. While in symmetric systems both initial energy density and transverse size of the medium increases with collision centrality, for asymmetric heavy-ion collisions it is possible to encounter situations where the transverse dimension of the medium is same but one can get variation in energy density. Keeping these aspects in mind, RHIC has now proposed to carry out asymmetric heavy-ion collisions program [8]. In this report we only concentrate on the study of the variation of $v_2, v_3$, their fluctuations, $v_2$ and $v_3$ for Au+Au, Au+Ag, Au+Cu, Cu+Cu and Au+Si collisions at $\sqrt{s_{NN}} = 200$ GeV using a multi-phase transport model (AMPT) with default settings. The goal being to simulate the expectation of above observables in asymmetric heavy-ion collisions relative to symmetric heavy-ion collisions.

The paper is organized as follows. The next section deals with the definition of $v_2$, $v_3$, their fluctuations, $v_2$ and $v_3$, along with a brief description of AMPT model. Section III presents the results from the model calculation for the observables mentioned above. Finally we summarize our findings in section IV.
II. AMPT MODEL AND DEFINITIONS

The current work is based on the AMPT model with default settings [9]. It uses the same initial conditions as considered in the Heavy Ion Jet Interaction Generator (HIJING) event generator [10]. HIJING is a perturbative QCD inspired model which produces multiple minijet partons, these later get transformed into string configurations and then fragment to hadrons based on the Lund jet fragmentation model [11]. A Glauber model prescription is followed for obtaining the number of participating nucleons (N_{part}). In AMPT, the minijet partons are made to undergo scattering before they are allowed to fragment into hadrons. These interactions could give rise to anisotropy in particle production along azimuthal direction. The event plane in AMPT is along the x-axis. There exists another version of the model, not used in this paper, called the string melting version where partonic interactions are considered [1].

We have followed the notations for the various observables as studied in Ref [1]. The participant eccentricity is defined as:

\[ \varepsilon_2 = \sqrt{\langle r^2 \cos(2\phi_{part}) \rangle^2 + \langle r^2 \sin(2\phi_{part}) \rangle^2} / \langle r^2 \rangle \]  

where \( r \) and \( \phi_{part} \) are the polar coordinate positions of participating nucleons in the AMPT model. \( \psi_2 \) is the angle of the minor axis of the ellipse defined by this region and is given as

\[ \psi_2 = \frac{\arctan\left(\frac{\langle r^2 \sin(2\phi_{part}) \rangle}{\langle r^2 \cos(2\phi_{part}) \rangle}\right)}{2} + \pi \]  

\( v_2 \) which is the 2nd Fourier coefficient of the particle distribution with respect to \( \psi_2 \) and is given by

\[ v_2 = \langle \cos(2(\phi - \psi_2)) \rangle \]  

Similar to the definition of the eccentricity and elliptic flow, the participant triangularity, \( \varepsilon_3 \), and triangular flow, \( v_3 \) are defined as:

\[ \varepsilon_3 = \sqrt{\langle r^2 \cos(3\phi_{part}) \rangle^2 + \langle r^2 \sin(3\phi_{part}) \rangle^2} / \langle r^2 \rangle \]  

\[ v_3 = \langle \cos(3(\phi - \psi_3)) \rangle \]  

where \( \psi_3 \) is the angle of the minor axis of participant triangularity and is given by

\[ \psi_3 = \frac{\arctan2\left(\frac{\langle r^2 \sin(3\phi_{part}) \rangle}{\langle r^2 \cos(3\phi_{part}) \rangle}\right)}{3} + \pi \]  

III. RESULTS

Figure 1 shows the \( \langle \varepsilon_2 \rangle \) and \( \langle \varepsilon_3 \rangle \) for asymmetric heavy ion collisions (Ag+Au, Cu+Au and Si+Au) compared to symmetric heavy-ion collisions (Au+Au and Cu+Cu) at \( \sqrt{s_{NN}} = 200 \) GeV as a function of \( N_{part} \). We find for a given \( N_{part} \), the \( \langle \varepsilon_2 \rangle \) values are higher for larger colliding ion sizes. However for the low mass number colliding ions (Cu+Cu collisions and smaller) they are similar. No such large differences are observed for \( \langle \varepsilon_3 \rangle \). This suggests that within the framework of the AMPT model, the asymmetric heavy ion collisions can be used to constrain the models dealing with second harmonic flow coefficient and its fluctuations. However, such collisions may not be that sensitive to studies dealing directly with triangularity and triangular flow.

Figure 2 shows the fluctuations in \( \varepsilon_2 \) and \( \varepsilon_3 \) expressed as the ratio of the corresponding root mean square (rms) values to their average values for both asymmetric heavy-ion collisions and symmetric heavy-ion collisions at \( \sqrt{s_{NN}} = 200 \) GeV as a function of \( N_{part} \). We observe that for mid-central collisions (60 < \( N_{part} < 250 \)) the fluctuations in \( \varepsilon_2 \) increases as the system size decreases and then saturates for colliding systems of size comparable or smaller to Cu+Cu. In contrast the variation in fluc-
tuation of \( \varepsilon_3 \) as a function of system size is very small.

Similar conclusions are obtained using a slightly different observable as proposed in Ref. \[12\]. The results of which from the AMPT model are shown in Fig. 3. It has been shown that the relative magnitude of \( v_n \{4\} \) and \( v_n \{2\} \) depends on the event-by-event fluctuations of \( v_n \) if the non flow effects are small. Where the \( \{4\} \) and \( \{2\} \) represents the four particle and two particle cumulant methods to extract flow coefficients respectively. It has been shown in Ref. \[12\] that

\[
\left( \frac{v_n \{4\}}{v_n \{2\}} \right)^4 = \left( \frac{\langle \varepsilon_n \{4\} \rangle}{\langle \varepsilon_n \{2\} \rangle} \right)^4 = 2 - \langle \varepsilon_n^4 \rangle / \langle \varepsilon_n^2 \rangle^2. \tag{7}
\]

Figure 4 shows the same results as in Fig. 3 but as a function of fraction of collision centrality. Where 0 corresponds to the most central collisions and 1 corresponds to the most peripheral collisions. In addition we also show for comparison results from a Glauber model simulation for Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV from Ref. \[12\]. Since the colliding ion species have different maximum \( N_{\text{part}} \) values, it is desirable to study how the fluctuations in \( \varepsilon_2 \) and \( \varepsilon_3 \) vary for the case of same fraction of collision centrality. From Fig. 4 one observes that although the fluctuations in \( \varepsilon_3 \) is still similar for all the colliding ion species studied, those for \( \varepsilon_2 \) now starts showing the ion size dependence more clearly. For a

![FIG. 2: (Color online) (a) Ratio of root mean square (rms) value of \( \varepsilon_2 \) to \( \langle \varepsilon_2 \rangle \) and (b) rms of \( \varepsilon_3 \) to \( \langle \varepsilon_3 \rangle \) for various heavy ion collisions at \( \sqrt{s_{NN}} = 200 \) GeV using AMPT model.](image)

![FIG. 3: (Color online) \( \langle \varepsilon_n^4 \rangle / \langle \varepsilon_n^2 \rangle^2 \), with \( n = 2, 3 \), versus \( N_{\text{part}} \) for various heavy-ion collisions at \( \sqrt{s_{NN}} = 200 \) GeV using the AMPT model.](image)

![FIG. 4: (Color online) \( \langle \varepsilon_n^4 \rangle / \langle \varepsilon_n^2 \rangle^2 \), with \( n = 2, 3 \), versus centrality for various heavy-ion collisions at \( \sqrt{s_{NN}} = 200 \) GeV. The Pb+Pb results corresponds to Glauber model simulations from Ref. \[12\] at \( \sqrt{s_{NN}} = 2.76 \) TeV.](image)
given fraction of collision centrality the fluctuations in $\varepsilon_2$ starts to increase as the colliding ion species size decreases, except for the most central and peripheral collisions. These results show that a systematic study of heavy-ion collisions with colliding system sizes ranging between Cu+Cu and Au+Au may provide a better handle on understanding $\varepsilon_2$ fluctuations and its relevance to $v_2$ fluctuations in the experimental measurements.

Collective flow is mostly driven by the initial spatial anisotropy, hence it is expected that $v_2$ and $v_3$ should be proportional to $\varepsilon_2$ and $\varepsilon_3$ respectively in high energy heavy-ion collisions. Figure 5 shows $\langle v_2 \rangle$ versus $\langle \varepsilon_2 \rangle$ and $\langle v_3 \rangle$ versus $\langle \varepsilon_3 \rangle$ from the AMPT model at midrapidity for $80 < N_{\text{part}} < 120$ in heavy-ion collisions studied at $\sqrt{s_{NN}} = 200$ GeV. We observe that indeed $\langle v_2 \rangle$ and $\langle v_3 \rangle$ are proportional to $\langle \varepsilon_2 \rangle$ and $\langle \varepsilon_3 \rangle$ respectively for all the collision systems studied. However for a given value of $\langle \varepsilon_2 \rangle$, the observed $\langle v_2 \rangle$ decreases with a decrease in the colliding system size. This could be due to larger fluctuations in $\varepsilon_2$ for the smaller systems compared to the larger colliding systems. However there is no such observed differences for $\langle v_3 \rangle$ versus $\langle \varepsilon_3 \rangle$. This is consistent with not much difference in fluctuations of $\varepsilon_3$ for various colliding ions.

Finally in Fig. 5(b) we show the $v_2$ for charged particles at midrapidity for mid-central collisions as a function of transverse momentum ($p_T$) for various values of participating nucleon number ($N_{\text{part}}$) at $\sqrt{s_{NN}} = 200$ GeV from the AMPT model. For the centrality range studied we see a clear dependence of $v_2$ on the colliding system size. It increases with an increase in the colliding ion size. This is consistent with the results on variation of $\langle v_2 \rangle$ with $\langle \varepsilon_2 \rangle$ shown in Fig. 5 and the fluctuations in $\varepsilon_2$ shown in Figs. 3 and 4. In Fig. 5(b) the $v_3$ versus $p_T$ shows a much smaller dependence on colliding system size compared to the above case.

IV. SUMMARY

In view of the proposed asymmetric heavy-ion collision program at RHIC in 2012, we have presented an AMPT model based study of $\langle v_2 \rangle$, $v_2$, $\varepsilon_2$, $v_3$ and $\varepsilon_3$ for various asymmetric colliding ion species of Ag+Au, Cu+Au and Si+Au at $\sqrt{s_{NN}} = 200$ GeV. These results are presented as a function of the number of participating nucleons, fraction of collision centrality and transverse momentum. They are compared to results from symmetric colliding systems of Au+Au and Cu+Cu collisions.

Measurable signals of each of the above observables are found for all the colliding systems studied. We find that while $\varepsilon_2$ and its fluctuations, for a given
number of participating nucleons or fraction of collision centrality, are highly dependent on the colliding ion type those for $\varepsilon_3$ and its fluctuations are very similar for all the colliding species studied. These results are reflected in the experimental observables such as $v_2$ and $v_3$. For the same $\varepsilon_2$ at midrapidity and for mid-central collisions, the proportionality constant between $v_2$ and $\varepsilon_2$ seems to depend on the colliding system size. On the other hand, $v_2$ vs. $p_T$ decreases as the colliding system size decreases for the collision centrality range studied. However, no such large sensitivity to colliding ion type on $v_3$, $\varepsilon_3$ and fluctuations in $\varepsilon_3$ are observed from our AMPT model based study. Our study thus indicates that asymmetric heavy-ion collisions can be used to constrain models dealing with flow fluctuations in heavy-ion collisions but with greater sensitivity for $v_2$ related observables than for $v_3$.

Acknowledgments
This work is supported by the DAE-BRNS project sanction No. 2010/21/15-BRNS/2026.

[1] B. Alver, G. Roland, Phys. Rev. C81, 054905 (2010).
[2] K. Aamodt et al., [ALICE Collaboration], Phys. Rev. Lett. 107, 032301 (2011).
[3] B. I. Abelev et al. [ STAR Collaboration ], Phys. Rev. Lett. 102, 052302 (2009).
[4] B. I. Abelev et al. [ STAR Collaboration ], Phys. Rev. Lett. 105, 022301 (2010) ;B. I. Abelev et al. [ STAR Collaboration ], Phys. Rev. C80, 064912 (2009).
[5] K. Aamodt et al., [ ALICE Collaboration ], arXiv:1109.2501 [nucl-ex].
[6] V. Roy, A. K. Chaudhuri, arXiv:1109.1630 [nucl-th]];B. Schenke, S. Jeon, C. Gale, Phys. Rev. Lett. 106, 042301 (2011).
[7] J. -Y. Ollitrault, A. M. Poskanzer, S. A. Voloshin, Phys. Rev. C80, 014904 (2009) ; S. A. Voloshin, A. M. Poskanzer, A. Tang, G. Wang, Phys. Lett. B659, 537-541 (2008) ; L. Yi, F. Wang, A. Tang, [arXiv:1101.4646 [nucl-ex]].
[8] http://www.bnl.gov/npp/docs/pac0611/Overall%20recommendations%20final.pdf BNL program advisory committee recommendations, June 6-8, 2011 [http://www.bnl.gov/npp/pac.asp].
[9] Zi-Wei Lin, C. M. Ko, Phys. Rev. C 65, 034904 (2002); Zi-Wei Lin et al., Phys. Rev. C 72, 064901 (2005).
[10] X. N. Wang and M. Gyulassy, Phys. Rev. D 44, 3501 (1991).
[11] B. Andersson, G. Gustafson, G. Ingelman and T. Sjostrand, Phys. Rep. 97,31 (1983).
[12] R. S. Bhalerao, M. Luzum, J. -Y. Ollitrault, Phys. Rev. C 84, 054901 (2011).