Unexpected properties of interactions of high energy protons

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Abstract

Experimental data on proton-proton interactions in high energy collisions show quite a special and unexpected behaviour of the proportion of elastic scattering compared to inelastic processes with increasing energy. It decreases at the beginning (at comparatively low energies) but then starts increasing. From Intersecting Storage Rings (ISR) energies of 23.5 - 62.5 GeV up to higher energies 7 - 13 TeV at the Large Hadron Collider (LHC) it increases by a factor more than 1.5! According to intuitive classical ideas we would expect a stable tendency with increasing proportion of the break-down of protons compared to their survival probability. One can assume that either the asymptotic freedom or the extremely short time of flight of high energy protons through each other are in charge of such a surprising
effect. The unquestionable principle of unitarity combined with the available experimental data on elastic scattering is used to get new conclusions about the shape of the interaction region of colliding protons. Its evolution at present energies is considered. Some predictions about its behaviour at even higher energies are described with different assumptions on relative roles of elastic scattering and inelastic processes. The shape can transform rather drastically if the proportion of elastic processes keeps rising. This unexpected property leads to an unexpected corollary. The possible origin of the effect and its interrelation to the strong interaction dynamics are speculated.

1  Foreword

If a cup falls to the floor, it breaks up to pieces but sometimes stays intact. The harder it hits the floor, the less chance to be unbroken.

If two high energy protons collide, many new particles (mostly pions) are produced, but sometimes they scatter elastically and retain their entity. It is surprising enough that at very high collision energies the proportion of elastic processes increases with increasing energy from the ISR to the LHC.

This unexpected and paradoxical phenomenon and its consequences at present and higher energies are discussed in the review.

2  Introduction

One gets often accustomed to unexpected facts and they become just either the everyday reality or the trivial observation. However sometimes they stay unexplained for a long time.

In the 50-th, the strong interactions of hadrons impressed the physics community by production of resonances in the pion-proton collisions. Afterwards, the resonances filled in all the tables of elementary particles and became the well known phenomenon. This process lasts up to now with the discovery of the famous Higgs-boson or by the ”closure” of the massive two-photon resonance. The phenomenon is described in terms of the dynamical levels of the system.

However not all the discoveries have the required interpretation. In the beginning of the 70-th, it was unexpectedly found that the total cross section of the interaction of positively charged kaons with protons became increasing
with the energy increase already at energies of the Protvino accelerator up to 70 GeV in the rest (laboratory) system of one of the protons or about 12 GeV in the center-of-mass system. Let us remind that up to that time it was commonly believed that hadronic cross sections must either decrease or tend to constant values with energy increase. This belief was first strongly shattered by the so-called ”Serpukhov effect”. Nowadays it is well known that the total cross section of interaction of high energy protons steadily increases with the increase of energy of colliding partners. The elastic scattering cross section as well as the cross section of inelastic processes increase with energy also. Both the larger intensity of the interaction due to the larger number of the actively participating partons (mostly, gluons) and its larger spatial extension can be in charge of that behaviour. Moreover, it happens that all hadronic cross sections increase with energy. Almost half a century has passed since then but no fundamental explanation of such behaviour in the quantum field theory has been proposed. Phenomenologically, it is usually described nowadays by the power-law energy dependence due to exchange by the so-called supercritical Pomeron. Its dynamical origin is yet unclear.

It is less known that experimental data hide another quite surprising and completely unexpected phenomenon of increase of the ratio of elastic to inelastic (or total) cross sections with energy increase in the interval from ISR (20-60 GeV in the center-of-mass system) \[1, 2\] to the highest explored accelerator energies at LHC (7-13 TeV) \[3, 4, 5\]. The share of elastic collisions in the total outcome of all processes used to decrease at lower energies that coincided with our expectations. However, it reversed the tendency at ISR (the corresponding data were analyzed by me and the table with them was demonstrated earlier in Physics-Uspekhi journal \[6, 7\]). Their relative roles evolve drastically. The inelastic cross section is about 5 times larger than the elastic one at ISR while their ratio decreases to 3 at LHC energies. According to the intuitive classical ideas we would expect the opposite behaviour with increasing probability of the break-down of both colliding protons into more and more ”pionic pieces” compared to their survival probability, when protons are scattered purely elastically. Moreover, this increasing proportion of elastic scattering approaches such critical value at LHC energies \[8, 9, 10\] which, probably, indicates the transition to some principally new regime of interactions. Somehow the protons tend to keep their entity while colliding with higher and higher energies. No reliable explanation to this fact exists as well! Some simplest proposals are considered only.

Here, we show the consequences of such an increase at present energies
in the picturesque presentation of the spatial interaction regions of colliding protons. We describe their possible non-trivial evolution at higher energies if this tendency persists. The adopted approach relies only on the unitarity condition and experimental data about elastic scattering of protons. No phenomenological input has been used. That assures the validity of conclusions. The results of some phenomenological models are discussed just to provide additional support to our statements.

The general indubitable principle of conservation of total probability known in particle physics as the unitarity condition relates elastic and inelastic processes. Sum of their ratios to total outcome should be equal 1. Therefrom, some knowledge about inelastic processes can also be gained using the elastic scattering data. The latter ones depend on smaller number of variables. Thus they can be analyzed more easily. Surely, from another side, that leads to the somewhat restricted sample of conclusions about inelastic processes which one gets from the unitarity condition. Nevertheless, one gains some knowledge about the spatial interaction region of protons at present energies and its possible evolution at higher energies.

From the heuristic point of view, the increase of the share of elastic scattering to the critical value attained at LHC can for the first time reveal the transition from the traditionally considered branch of the unitarity condition dominated by inelastic processes (where elastic scattering is treated as the shadow of inelastic collisions) to another branch with the dominance of elastic scattering. That would require the completely new physical interpretation of the mechanism of proton (hadron) interactions and, probably, the formulation and further studies of new dynamical equations.

The increase of the proportion of the elastic scattering processes reveals itself, first of all, in the spatial evolution of the elastic and inelastic interaction regions of colliding protons from ISR to LHC energies. It happened to be instructive to learn that the inelastic interaction region becomes more Black (absorptive) at the center, has steeper Edges (sharper decrease) and enLarges in size due to its periphery (the so-called BEL-scenario [11]) with energy increase in this energy interval. Even though the form of these regions can not be measured directly in experiment, this knowledge has been used, for example, for interpretation of some peculiar features of experimental data on jet production at 7 TeV. Also, it inspires theoretical ideas about possible experimental implications of their further evolution at higher energies. If the noticed tendency persists at higher energies, the profiles of both elastic and inelastic interactions can change drastically and show quite unexpected
features, especially in the case of head-on collisions. Thus the BEL-scenario can be replaced by the absolutely new toroid-like regime with the enlarged role of elastic scattering for central collisions. It could be named as TEH-regime (Toroidal Elastic Hollow).

No explanation of this phenomenon at present energies has yet been proposed. What concerns our attempts to extrapolate it to higher energies, we hope that experimental studies of elastic scattering of polarised protons or charge asymmetries of pions produced in inelastic collisions (or other yet unexploited observations) could help in the proper choice of different possibilities. From the theoretical side, one can try to use more traditional QCD approach with enlarged fluctuations of gluon fields at collisions or revolutionary speculate on peculiar properties of solitons and instantons using the corresponding equations in attempts to find a reasonable explanation.

Let us stress once again that the approximations adopted in the considered approach are completely justified so that one can claim that all results are obtained directly from combination of the two well-grounded sources - the unitarity condition and experimental data about elastic scattering and do not require any phenomenological input and modelling. Therefore the derived conclusions are very reliable. Their extrapolation to ever higher energy regions relies on the only assumption that the tendency of the increase of the share of elastic scattering experimentally observed in the energy interval from ISR to LHC will persist there as well.

The structure of this review is as follows. In section 3 we start with the description of general features of experimental results on elastic scattering of protons. Then the effective theoretical tool of the unitarity condition is introduced in section 4. There we discuss the accuracy of main approximations for elastic scattering amplitude which will be necessary for reliable estimates in the framework of the unitarity condition. It is applied further in section 5 to the special case of central head-on collisions of protons which allows to demonstrate typical features of unitarity constraints. Then in section 6 the transverse spatial shapes of the inelastic and elastic interaction regions at current energies are demonstrated and their energy evolution is discussed. Possible extrapolations of the profiles of interactions beyond modern (LHC) energies to asymptotics are presented in section 7 for different assumptions on the energy behaviour of the proportion of elastic scattering. Finally, some conclusions are given at the very end of the paper. Some assumptions about possible dynamical origin of the observed effect are discussed as well.
3 Elastic scattering

The information about elastic scattering of protons comes from the measurement of the differential cross section $d\sigma/dt$ at some energy $s$ as a function of the transferred momentum $t$ at its experimentally accessible values. It is related to the scattering amplitude $f(s, t)$ in the following way

$$\frac{d\sigma}{dt} = |f(s, t)|^2 \equiv (\text{Re}f(s, t))^2 + (\text{Im}f(s, t))^2. \quad (1)$$

The variables $s$ and $-t$ are the squared total energy $2E$ and the squared transferred momentum of the two colliding protons in the center-of-mass system $s = 4E^2 = 4(p^2 + m^2)$ ($p$ is the proton’ momentum) and $-t = 2p^2(1 - \cos \theta)$ at the scattering angle $\theta$. From this measurement one gets the knowledge only about the modulus of the amplitude, i.e. about the sum of the squared values of its real and imaginary parts but not about their signs. The Coulomb scattering contribution to it can be neglected everywhere except small angles. However, namely there the Coulomb scattering of the electrically charged protons appears to be comparable to their nuclear interaction. The interference between the nuclear and Coulomb contributions to the amplitude $f$ becomes quite large and allows to find out from the shape of the experimental differential cross section the ratio of the real and imaginary parts of the elastic scattering amplitude $\rho(s, t) = \text{Re}f(s, t)/\text{Im}f(s, t)$. This can be done just in forward direction $t = 0 \rho(s, 0) = \rho_0$ (to be more precise, extremely close to it) but not at any other values of $t$.

The typical shape of the experimentally measured differential cross section at high energies shown in Figures contains some characteristic features. Those are the above mentioned interference region at extremely small values of $|t|$, almost invisible in Fig. 1, the exponentially decreasing (with increase of $|t|$) diffraction cone with energy dependent slope $B(s)$ (Fig. 1), the dip (Fig. 2) and more slowly decreasing tail at larger transferred momenta with much smaller values of the cross section compared to the diffraction cone (Fig. 2).

3.1 The diffraction cone

The diffraction cone is shown in Fig. 1. Protons scatter mainly at processes with small transferred momenta. The differential cross section is much larger
Fig. 1. The differential cross section of elastic proton-proton scattering at $\sqrt{s}=7$ TeV measured by the TOTEM collaboration (Fig. 4 in [3]). The region of the diffraction cone with the $|t|$-exponential decrease is shown.

there than at larger transferred momenta. Its exponential parameterization is demonstrated by the straight line at the logarithmic scale.

There are several tiny features of this plot. In the very narrow region of extremely small transferred momenta the amplitude is represented by the sum of the nuclear and Coulomb amplitudes. Their interference produces some increase of the differential cross section in there. It has been used for estimates of the real part of the amplitude. Moreover small deviations of the order of 1 per cent from the exponential shape (invisible in Fig. 1) were noticed at the extremely precise measurements at 8 TeV [4]. Also one can see the somewhat steepened shape at the very end of the diffraction cone approximated there by another exponent (the dashed line) which differs from the leading one albeit not very strongly and the whole effect is noticeable only in a very small interval of transferred momenta. Let us note that at lower energies the shape was slightly flattened but not steepend. The impact of all these specific features on our further calculations is easily estimated. It will be shown very small because we will use the averaged integrated parameters. Therefore in what follows we adopt the simple exponential parameterization of
Fig. 2. The differential cross section of elastic proton-proton scattering at $\sqrt{s}=7$ TeV measured by the TOTEM collaboration (Fig. 4 in [3]). The region beyond the diffraction peak is shown. The predictions of five phenomenological models are demonstrated.
the diffraction cone which is precise enough for the corresponding transferred momenta and has been used by experimentalists:

\[ |f(s,t)| \approx \frac{\sigma_{\text{tot}}(s)}{4\sqrt{\pi}} \exp\left[B(s)t/2\right] \]

(2)

where \(\sigma_{\text{tot}}(s)\) is the total cross section and \(B(s)\) is the energy dependent slope of the diffraction cone.

3.2 The real part of the elastic scattering amplitude

Some theoretical information about the energy behaviour of the real part of the forward scattering amplitude can be obtained from the dispersion relations which follow from the analyticity property of the amplitude. They relate it to the integral of the imaginary part at zero angle, i.e. to the total cross section according to the optical theorem (see Eq. (5) below). Using reasonable extrapolations of the total cross section to higher energies it was predicted long ago \[12, 13, 14\] that at high energies the real part is small compared to the imaginary part and their ratio is about 0.12 - 0.15 with slow decrease at asymptotic energies. Both real and imaginary parts are positive at \(t = 0\) due to positivity of the latter. These predictions were confirmed by experiment. At LHC energies the measured ratios range is 0.12 - 0.145 \[3, 5, 15\]. Thus the real part only contributes about 1 - 2% to the differential cross section (1) at \(t = 0\).

What concerns the behaviour of the real part as a function of the transferred momentum, some general theoretical guesses \[16, 17\] indicated that it can become zero somewhere within the diffraction cone. Therefore its decrease inside the diffraction cone should be steeper than for the imaginary part, and, consequently, its integral contribution from this region to the elastic cross section must be even smaller. No definite position was ascribed in the papers \[16, 17\] to the point where it crosses the abscissa axis. Recently, some possibilities to use the analytical properties of the elastic scattering amplitude for getting some knowledge about its real part were considered in Ref. \[18\].

Nevertheless, one can easily estimate from the data presented in Fig. 1 and Fig. 2 the upper limit of the real part of the amplitude at the dip. Its ratio to the imaginary part at \(t = 0\) is calculated as the square root of the ratio of the differential cross sections at those points and, surely, is very small \(\leq 0.006\). This estimate supports our intention to neglect the contribution
of the real part of the amplitude in further calculations where its integrally averaged characteristics are only used.

Further guides about its behaviour can only be obtained from particular models of proton interactions. Those of them which pretend to make precise fits of a wide variety of present experimental data are surely preferred. Even then they should not be absolutely trusted because we have some experience that several details got wrong even at present energies and could become worse at extrapolations to new energy fields. Nevertheless, as such an example, we show in Fig. 3 borrowed from Ref. [19] the behaviour of the real and imaginary parts of the elastic scattering amplitude at energy 7 TeV within the large interval of the transferred momenta. Its shape is derived with the help of a particular phenomenological model [19] which happened to be very successful in fits of many experimental characteristics in a wide range of energies up to LHC.

In particular, one can see that the real part at 7 TeV is much smaller than the imaginary part everywhere within the diffraction cone and crosses the abscissa axis in accordance with theoretical expectations [16, 17]. Its relative contribution to the differential cross section (1) is given by the term $\rho^2(s,t)$ where $\rho(s,t) = \text{Re} f(s,t)/\text{Im} f(s,t)$. It can be neglected in the model considered. The accuracy of experimental data is not yet high enough for such small contributions to be taken into account. That corresponds well with our prejudice that the diffraction cone is somehow a shadow of inelastic processes because the elastic amplitude is substantially imaginary there. It is interesting to note that according to the model [19] the imaginary part dominates everywhere besides the dip interval which is very short. However, the differential cross section is already very small there compared to the diffraction cone. Thus in our analytical estimates we will neglect the real part of the amplitude but sometimes come back to it to show once again how irrelevant for our conclusions is its contribution.

The steep exponential decrease of differential cross sections in the diffraction cone implies that namely this region contributes mostly to Eq. (7. The integral contribution of the real part of the amplitude $f$ in there must even be noticeably smaller than its overestimated value $\rho_0 \text{Im} f$. That is why it is possible to neglect it further in analytical calculations.
Fig. 3. Real $Re f$ and imaginary $Im f$ parts of the proton-proton amplitude at 7 TeV according to a particular phenomenological model [19]. Note that the contribution of the real part to $d\sigma/dt$ becomes noticeable only near the dip where $d\sigma/dt$ is small. It can be completely neglected inside the diffraction cone. Moreover, it becomes equal zero inside it as was predicted [16] [17]. It is quite interesting that the imaginary part dominates in the Orear region of intermediate transferred momenta as well.
3.3 The differential cross section outside the diffraction cone

In what follows we will need to estimate the contribution of the region outside the diffraction peak to some analyzed variables. Comparison of Fig. 2 with Fig. 1 shows that the differential cross section is much lower (by more than 4 orders of magnitude!) at the dip and at the tail compared to its values at the beginning of the diffraction cone. Moreover, it decreases approximately as \( \exp(-c(s)\sqrt{|t|}) \) in this region. It is usually called as the Orear region by the name of its first observer and can be explained (see \[20\]) by subsequent iterations (rescattering) in the solution of the unitarity equation in the \((s,t)\)-representation. Surely, the \(t\)-exponential parameterization \[2\] used for the diffraction cone underestimates the contribution of the tail with \(-\sqrt{|t|}\)-exponent at high transferred momenta. However, the integral contribution to our variables of the excess in the tail region over our approximation \(2\) is easily estimated. We show below that it is negligibly small. The interplay of the real and imaginary parts of the amplitude \(f\) can be more complicated there as seen, e.g., from Fig. 3. However, the smallness of the modulus, i.e., of \(\sqrt{d\sigma/dt}\), implies the smallness of both of them in this region even though their ratio \(\rho\) becomes infinitely large if the imaginary part becomes equal to zero.

4 The unitarity condition

Our main goal here is to get some knowledge about the spatial region of interactions of high energy protons at current energies, to draw a pictorial view of its evolution with increasing energy and to discuss possible theoretical and experimental implications of these findings.

The most stringent and reliable information (albeit rather limited!) about the interrelation of elastic and inelastic processes comes from the unitarity of the \(S\)-matrix

\[ S S^+ = 1 \]  \hspace{1cm} (3)

or for the scattering matrix \(T\) \((S = 1 + iT)\)

\[ 2\text{Im}T_{ab} = \Sigma_n \int T_{an}T_{nb}^* d\Phi_n, \]  \hspace{1cm} (4)

where \(a, b, n\) denote the number of particles. The whole \(n\)-particle phase space \(\Phi_n\) is integrated over. For the elastic scattering amplitude \(a = b = 2\),
the unitarity condition relates the amplitude of elastic scattering $f \propto T_{22}$ to the amplitudes of $n$-particle inelastic processes $T_{2n}$, declaring that the total probability of all outcomes of the interaction (elastic and inelastic ones) must be equal to $1$. 

In the $s$-channel this unquestionable condition is usually expressed in the form of the well known integral relation (for more details see, e.g., [21, 20, 6]). This relation is quite complicated for arbitrary values of the transferred momentum, $t$. However, for forward scattering at $t = 0$ it leads to the widely used optical theorem showing the normalization of the imaginary part of the amplitude $\text{Im} f(s)$ by its direct connection with the total cross section $\sigma_{\text{tot}}$:

$$\text{Im} f(s, 0) = \frac{\sigma_{\text{tot}}(s)}{4\sqrt{\pi}}$$

and to the general statement that the total cross section is the sum of cross sections of elastic and inelastic processes

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}},$$

i.e., that the total probability of all processes equals 1.

One can use the Fourier – Bessel transform of the amplitude $f$ to reduce the integral relation to the more simple algebraic one. This transformation retranslates the momentum data to the shortest transverse distance between the trajectories of the centers of colliding protons called the impact parameter, $b$, and is written as

$$i\Gamma(s, b) = \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| f(s, t) J_0(b\sqrt{|t|}).$$

(7)

Then the unitarity condition in the $b$-representation reads

$$G(s, b) = 2\text{Re}\Gamma(s, b) - |\Gamma(s, b)|^2.$$  

(8)

(for reviews see, e.g., Refs [6, 7]). This relation establishes the connection between the distributions of the intensity of all processes in the transverse configuration space:

$$\frac{d^2\sigma_{\text{inel}}}{db^2} = \frac{d^2\sigma_{\text{tot}}}{db^2} - \frac{d^2\sigma_{\text{el}}}{db^2}$$

(9)

The non-linear contribution from the elastic amplitude appears in the right-hand side for $n = 2$.  

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The left-hand side in Eqs (8), (9) describes the transverse impact-parameter profile of inelastic collisions of protons. It satisfies the inequalities $0 \leq G(s, b) \leq 1$ and determines how absorptive is the interaction region at the given impact parameter (with $G = 1$ for the full absorption and $G = 0$ for the complete dominance of elastic scattering). The profile of elastic processes is determined by the subtrahend in Eqs (8), (9). Thus we get a spatial view of the whole process if the elastic scattering amplitude $f$ is integrated in Eq. (7).

Let us note from the very beginning that these profiles can not be measured directly in experiments because the impact parameters are not the measurable quantities. Nevertheless, their energy behaviour has important heuristic value because it can reveal the evolution of the process dynamics. It will be described below how the knowledge of the spatial extension of the inelastic interaction region has been used for the description of the processes of jet production at the LHC energy 7 TeV. One can use various models of the interactions and confront different assumptions. Also, one can try to relate the impact parameters, for example, with the multiplicities of inelastic collisions as it is done for the interactions of the relativistic nuclei. However, we do not speculate on it in this review.

If $G(s, b)$ is integrated over the impact parameter, it leads to the cross section of inelastic processes. The terms on the right-hand side of Eqs (8), (9) would correspondingly produce the total cross section and the elastic cross section in accordance with Eq. (6).

It follows from the above relations that, strictly speaking, one should know both real and imaginary parts of the elastic scattering amplitude to get results about the impact-parameter profiles of inelastic and elastic processes from the unitarity condition. However, its modulus can only be found from experimental data as follows from Eq. (1) and some very limited knowledge about its real part for forward scattering. Nevertheless, one can easily estimate the accuracy of any assumption in calculations according to Eqs. (7), (8).

In particular, we will use the fact that the modulus of the amplitude decreases approximately exponentially (see Eq. (2)) in the diffraction cone and becomes much smaller at the tail compared to its values at the top of the diffraction peak. The slight decline from a simple exponent inside the cone of the order of 1% noticed recently by TOTEM Collaboration \cite{22} at small transferred momenta as well as somewhat steepened behaviour at the very end of the diffraction cone near the dip seen in Fig. 2 do not
influence its integral contribution to \( \text{(7)} \) within the accuracy of determination of the slopes. In what follows, we use the exponential parametrization of the imaginary part of the amplitude \( f \) to proceed with analytical calculations and argue that it is very precise:

\[
\text{Im}\, f(s, t) = \frac{\sigma_{\text{tot}}(s)}{4\sqrt{\pi}} \exp\left[B(s)t/2\right].
\]

(10)

Formally, this approximation is not valid for differential cross sections at large transferred momenta. However, for our purposes the integral contribution of \( f \) at large \(|t|\) to Eq. \( \text{(7)} \) is only important. It is negligibly small there compared to the peak of the diffraction cone. The approximation \( \text{(10)} \) is justified as will be shown below. In fact, that was clear earlier when it was demonstrated that such approximation and direct integration of experimental data lead to the practically indistinguishable results. The accuracy of calculations is very high. Thus one can claim that the results obtained analytically rely only on the unitarity condition and experimentally measured exponential decrease of the differential cross section in the diffraction cone.

5 Central collisions

Before using the detailed formulae for the spatial extension of the interaction region as a function of the impact parameter \( b \), let us study at the beginning the simpler case of the energy dependence of the intensity of interaction for central (head-on) collisions of impinging protons at \( b = 0 \). We introduce the variable \( \zeta \):

\[
\zeta(s) = \text{Re}\, \Gamma(s, 0).
\]

(11)

For the dominant contribution of the diffraction cone (Eq. \( \text{(10)} \)) one gets that \( \zeta \) is directly related to the share of elastic processes:

\[
\zeta(s) = \frac{4\sigma_{\text{el}}}{\sigma_{\text{tot}}}
\]

(12)

One can also write

\[
|\Gamma|^2 = \zeta^2 + \frac{1}{4\pi} \left( \int_0^\infty d|t| \text{Re}\, f \right)^2.
\]

(13)
The last term here can be neglected compared to the first one. That is easily seen from

$$\int_0^\infty d|t| \text{Re} f \leq \int_0^\infty d|t| |\text{Re} f| = \int_0^\infty d|t| \sqrt{\frac{\rho^2(s,t) d\sigma/dt}{1 + \rho^2(s,t)}}. \quad (14)$$

The factor $\rho^2(s,t)/(1 + \rho^2(s,t))$ is very small in the diffraction cone. It can become of the order 1 at large values of $\rho^2(s,t)$ (say, at the dip) but the cross section is small there already (compare Fig. 2 and Fig. 3). Then the unitarity condition (8) is written as

$$G(s, b = 0) = \zeta(s)(2 - \zeta(s)). \quad (15)$$

Thus, according to the unitarity condition (15) the darkness of the inelastic interaction region for central collisions (absorption) is defined by the only experimentally measured parameter $\zeta(s)$ depending on energy. It has the maximum $G(s, 0) = 1$ for $\zeta = 1$. Any decline of $\zeta$ from 1 ($\zeta = 1 \pm \epsilon$) results in the parabolic decrease of the absorption ($G(s, 0) = 1 - \epsilon^2$), i.e. to an even much smaller decline from 1 for small $\epsilon$. The elastic profile, equal to $\zeta^2$ in central collisions, also reaches the value 1 for $\zeta = 1$.

The unitarity condition imposes the limit $\zeta \leq 2$ on the increase of the share of elastic scattering. It is required by the positivity of the inelastic profile. Then there no inelastic processes for central collisions ($G(s, 0) = 0$ according to Eq. (15)). This limit corresponds to the widely discussed "black disk" picture which asks for the relation

$$\sigma_{el} = \sigma_{inel} = \sigma_{tot}/2. \quad (16)$$

The value of the profile of central ($b = 0$) elastic collisions $\zeta^2$ completely saturates the total profile $2\zeta$ for $\zeta = 2$. Below, we shall discuss physics implications of these findings.

With high enough precision one can describe $\zeta$ by the following formulae:

$$\zeta(s) \approx \frac{\sigma_{tot}(s)}{4\pi B(s)} \approx (4\pi)^{-0.5} \int_0^\infty d|t| \sqrt{\frac{d\sigma/dt}{1 + \rho^2(s,t)}}. \quad (17)$$

One should specially note that all formulae contain only experimentally measurable quantities $\sigma_{tot}(s), \sigma_{el}(s), B(s)$. The most convenient for our further
discussion is its interpretation as a share of elastic processes because, in particular, it is proportional to the experimentally measurable dimensionless ratio of the elastic cross section $\sigma_{el}$ to the total cross section $\sigma_{tot}$ \cite{12}.

From the first formula one gets the conclusion that the increase of $\zeta(s)$ with increasing energy demonstrates that the height of the diffraction cone (the numerator) increases faster than its width shrinks (the denominator).

From the second relation in (17) one can get very definite conclusions about the role of different regions of the differential cross section for the variable $\zeta$ and, consequently, for the unitarity condition. In practice, one should just integrate the squared root of the differential cross section over the corresponding interval of transferred momenta. It is clearly seen that its value is mainly determined by such transferred momenta where the differential cross section is large and the real part of the amplitude is small compared to the imaginary part. This is valid in the diffraction cone. The simplest estimates with constant value $\rho_0(s) \approx 0.02$ in place of $\rho(s, t)$ in Eq. (17) show that this contribution is at the level of 1%. It is greatly reduced if its values from Fig. 3 are used since the values of Re$\rho$ are smaller there and, moreover, their contribution is exponentially weighted within the diffraction cone in (17). Surely, one can neglect by small declines from the simple exponential shape both inside and at the end of the diffraction cone because their contribution becomes very small after integration in Eq. (17). In fact, one can definitely state that the exponential parameterization of the imaginary part of the amplitude \cite{10} can be used for description of experimental data in our formulae. The conclusions of the phenomenological model \cite{19} just support our estimates as shown in \cite{24}.

What concerns the tail of the differential cross section, the convenient approximation of $d\sigma/dt$ by a pure exponential (in Eq. \cite{10}) is most easily verified by taking directly the published distribution and carrying out the integration directly using the measured data. Numerically we find that the data, when the region above the dip are included, yield values of $\zeta$ which are less than 3.9% higher than obtained with the exponential approximation.

This results in a less than $2 \cdot 10^{-3}$ correction to the calculation of $G(7 \text{ TeV}, 0)$ in Eq. (15). These 2 approximations ($\rho_0^2 \approx 0.02$ and exponential form) allow us to greatly simplify the discussion of the profile function, and are, in any case, not contradicted by known data and experimental uncertainties. The discussion of the accuracy of estimates can be found in \cite{25}.

The more detailed estimates of different contributions according to the phenomenological model \cite{10} are given in Ref. \cite{24}. The imaginary part of
Table. The energy behaviour of $\zeta$, $G(s, 0)$ and $\sigma_{in}/\sigma_{el}$.

| $\sqrt{s}$, GeV | 4.11 | 4.74 | 7.62 | 13.8 | 62.5 | 546 | 1800 | 7000 |
|-----------------|------|------|------|------|------|-----|------|------|
| $\zeta$         | 0.98 | 0.92 | 0.75 | 0.69 | 0.67 | 0.83 | 0.93 | 1.00-1.04 |
| $G(s, 0)$       | 1.00 | 0.993| 0.94 | 0.904| 0.89 | 0.97 | 0.995| 1.00 |
| $\sigma_{in}/\sigma_{el}$ | ISR | SpS | FNAL | LHC | 5 | 3 | | |

the amplitude becomes negative after the dip in this model. The contribution to the definition of $\zeta$ is also negative. Its numerical value becomes lowered but again within several percents only.

The experimentally measured proportion of elastic processes $\sigma_{el}/\sigma_t = 0.25\zeta$ demonstrates the non-trivial dependence on energy shown in the Table. The values of the absorption at central collisions $G(s, 0)$ and the ratios of inelastic to elastic cross sections $\sigma_{in}/\sigma_{el}$ are also shown. All values are derived directly from experimental data at corresponding energies $s$. The change of the tendency in the behaviour of elastic processes with energy increase looks especially surprising. One would naively expect that their proportion would decrease being replaced by inelastic processes with higher multiplicities at higher energies. That happens only at low energies up to ISR where the parameter $\zeta$ decreases from about 1 down to values about 2/3. At higher energies protons reveal unexpected stability. The share of elastic scattering increases with energy. The parameter $\zeta$ reaches the critical value 1 for 7 TeV data at LHC where the elastic cross section is about 4 times less than the total cross section.

That looks even more impressive in terms of the ratio of the inelastic cross section to the elastic one

$$\frac{\sigma_{inel}}{\sigma_{el}} = \frac{4}{\zeta} - 1$$

The ratio decreases from 5 at ISR to 3 at LHC as shown in the Table.

It is intriguing whether this increase of the proportion of elastic scattering will really show up in experiments at higher energies or it will be saturated asymptotically with $\zeta$ tending to 1 from below. The asymptotic saturation would lead to the conservative stable situation on the same branch of the unitarity condition while further increase above 1 will require the transition to another branch of the unitarity equation and new physics interpretation.
To explain the last statement let us rewrite Eq. (15) as

\[ \zeta(s) = 1 \pm \sqrt{1 - G(s, 0)}. \]  

The critical value \( \zeta = 1 \) reveals itself in the usage of different signs in front of the square root term (different branches of the unitarity condition) for \( \zeta < 1 \) and \( \zeta > 1 \). One used to treat elastic scattering as a shadow of inelastic processes. This statement is valid when the branch with negative sign in Eq. (19) is considered because it leads to proportionality of elastic and inelastic contributions (\( \zeta \propto G(s, 0)/2 \)) for small \( G(s, 0) \ll 1 \). That is typical for electrodynamical forces in particle interactions (e.g., for processes like \( ee \to ee\gamma \)) and for optics (photon interactions) where the inelastic cross sections are small and their values are governed by the fine structure constant \( \alpha \). The large value of the inelastic cross sections in hadronic collisions with subsequent increase of the elastic proportion at diminishing role of inelastic production destroys the analogy. That is why the observation of this effect comes as a surprise. For strong interactions, the shares of inelastic and elastic processes are compatible (see the Table). The approach of \( \zeta \) to 1 at 7 TeV corresponds to complete absorption in central collisions. This value is considered as a critical one because from (19) one gets significant conclusion that the excess of \( \zeta \) over 1 implies that the unitary branch with positive sign in front of inelastic processes is at work. This branch was first considered in [26] with application to high energy particle scattering. That changes the interpretation of the role of elastic processes as being a simple "shadow" of inelastic ones.

Present experimental data at LHC can not distinguish definitely between the two possibilities of asymptotic saturation and increase of the elastic share. Some slight trend of \( \zeta \) to increase and become larger than 1 can be noticed from comparison of TOTEM data at 7 TeV [3] where it can be estimated\(^2\) in the limits 1.00 and 1.04 and at 8 TeV [4] where according to the data of the same collaboration it is approximately 1.05 though within the accuracy of experimental data about \( \pm 0.024 \). The data of ATLAS collaboration at 8 TeV do not reveal any increase of the proportion of elastic scattering albeit with approximately the same accuracy. The more precise data at these energies and at 13 TeV are needed.

\(^2\)The experimental values of the ratios of elastic to total cross section and \( \rho_0 \) have been used.
The further increase of the share of elastic scattering with energy is favored by extensive fits of available experimental information for the wide energy range and their extrapolations to ever higher energies done in the phenomenological models of Refs [19, 27] as well as by some theoretical speculations (e.g., see Ref. [28]). The asymptotical values of $\zeta$ are about 1.5 in Refs [19, 27] and 1.8 [28]. They correspond to incomplete but rather noticeable decrease of the absorption at the center of the interaction region. The corresponding values of the attenuation at the center $G(\infty, 0)$ are 0.75 and 0.36. It is discussed in more detail in the next Section.

6 The shape of the inelastic interaction region at current energies

The detailed shape of the inelastic interaction region at arbitrary values of the impact parameters can be obtained with the help of relations (7), (8) if the behaviour of the amplitude $f(s,t)$ is known. Its modulus and the $\rho_0$ values are obtained from experiment. The most prominent feature of experimental results at present energies from ISR to LHC is the rapid exponential decrease of $d\sigma/dt$ with increasing transferred momentum $|t|$, especially in the near forward diffraction cone. It is just this region of transferred momenta which contributes mostly to Eqs (17), (7). Inserting the exponential shape of the cone in there one can write

$$i\Gamma(s,b) \approx \frac{\sigma_{tot}(s)}{8\pi} \int_0^\infty d|t| \exp(-B(s)|t|/2)(i + \rho(s,t))J_0(b\sqrt{|t|}).$$

Let us stress that the diffraction cone dominates the contribution to Re$\Gamma$ in Eqs (12), (20) so strongly that the tail of the differential cross section at larger $|t|$ can be completely neglected at the level of some per cents by itself even for central collisions as was estimated in the previous chapter. Besides, it is suppressed additionally by the Bessel function $J_0$ at larger impact parameters. Therefore the accuracy of the approximation increases. It was estimated using fits of the experimental differential cross section outside the diffraction cone by simplest analytical expressions. Moreover, it was shown [23, 29] by computing how well the versions with direct fits of experimental data and with their exponential approximation coincide if used in the unitarity condition. Therefore the expression (10) can be treated as following.
directly from experiment and being very precise. Herefrom, one calculates

$$\text{Re}\Gamma(s, b) = \zeta \exp\left(-\frac{b^2}{2B}\right).$$  

(21)

Correspondingly, the shape of the inelastic profile for small $\rho_0$ is given by

$$G(s, b) = \zeta \exp\left(-\frac{b^2}{2B}\right)[2 - \zeta \exp\left(-\frac{b^2}{2B}\right)].$$  

(22)

It scales as a function of $b/\sqrt{2B}$. Its energy dependence is determined by the two measured quantities - the diffraction cone width $B(s)$ and its ratio to the total cross section, i.e. by the variable $\zeta(s)$. It has the maximum at

$$b^2_m = 2B \ln \zeta$$  

(23)

It is positioned in the unphysical region of impact parameters $b^2_m < 0$ for $\zeta < 1$, i.e. at all energies below LHC. Therefore the absorption is incomplete $G(s, b) < 1$ at any physical impact parameter $b \geq 0$. Its largest value is reached at the very center $b = 0$. The inelastic interaction region has the shape of a disk with absorption strongly diminishing to its edges. The disk is semi-transparent at ISR energies. This is demonstrated by the corresponding line ($\zeta = 0.7$) in Fig. 4 [9] shown below.

At $\zeta = 1$, which is only reached at LHC energy 7 TeV, the maximum is positioned exactly at the center $b = 0$ and the maximum absorption occurs just for central collisions, i.e. $G(s, 0) = 1$. The disk center becomes black. The strongly absorptive core of the inelastic interaction region grows in size compared to ISR energies (see [23]) because of increase of the slope $B(s)$. The enlarged size of the inelastic interaction region can be clearly seen from the Taylor expansion of Eq. (22) at small impact parameters:

$$G(s, b) = \zeta[2 - \zeta - \frac{b^2}{B}(1 - \zeta) - \frac{b^4}{4B^2}(2\zeta - 1)].$$  

(24)

The negative term proportional to $b^2$ vanishes at $\zeta = 1$, and $G(b)$ develops a wide strongly absorbing plateau which extends to the comparatively large values of impact parameters $b$ (up to about 0.5 fm). The plateau is very flat because the last negative term in Eq. (24) which diminishes the absorption starts to play a role at 7 TeV (where $B \approx 20 \text{ GeV}^{-2}$) only for larger values of $b$. Therefore the absorption decrease becomes steeper at the periphery. The
earlier proposed scenarium BEL is therefore realized at present energies in such a way. The two lines in Fig. 4 demonstrate the evolution of the shape of the inelastic interaction region from ISR ($\zeta = 0.7$) to LHC ($\zeta = 1.0$) energies. The larger darkness for central collisions at LHC compared to ISR can be ascribed to the enlarged role of soft gluons in the proton structure function. It is claimed in several papers \cite{31,32,33} that already at the LHC energies the hollowness of the plateau can be seen at $b = 0$. Actually, the accuracy of experiments there is still not enough for the definite conclusions. Only at higher energies (or if higher accuracy at LHC would be achieved) it can be definitely observed as displayed in Fig. 4. We discuss these predictions in the next section.

Before discussing the predictions at higher energies, we would like to point out that the cross sections of inelastic processes are determined not only by the strength of the interaction inside the interaction region but also by the purely geometrical factor. Even though the proton interaction region is very dark at central collisions ($G(s,b) \approx 1$ inside the plateau), the cross sections of processes with small impact parameters $b \leq r$ are very small because the corresponding areas proportional to $r^2$ are small for integrals over $b \leq 0.5$ fm. Integrating the total and elastic terms in Eq. (22) up to impact parameters $b \leq r$ one estimates their roles for different radii $r$.

\begin{align*}
\sigma_{el}(s,b \leq r) &= \sigma_{el}(s)[1 - \exp(-r^2/B(s))], \quad (25) \\
\sigma_{tot}(s,b \leq r) &= \sigma_{tot}(s)[1 - \exp(-r^2/2B(s))]. \quad (26)
\end{align*}

One gets that the contribution of processes at small impact parameters $b^2 \ll 2B$ diminishes quadratically at small $r \to 0$. In particular, inelastic processes contribute at $r \to 0$ as

\begin{equation}
\sigma_{inel}(s,b \leq r) \to \pi r^2 G(s,0) + O(r^4); \quad (r^2 \ll B).
\end{equation}

The maximum intensity of central inelastic collisions equal 1 is at $\zeta = 1$. The high intensity must result in high multiplicities of inelastic events. The integral contribution of the near central region of collisions is small. The cross sections of very high multiplicity events are also small. The estimates show that they are quite comparable to one another.

This property has been used in Ref. \cite{29} for the explanation of jets excess observed for very high multiplicity events at 7 TeV compared to predictions of the well known Monte-Carlo models PYTHIA and HERWIG. This excess
Fig. 4. The energy evolution of the shape of the inelastic interaction region for different values of the survival probability $\zeta/4$. The values $\zeta = 0.7$ and $1.0$ correspond to ISR and LHC energies and agree well with the result of detailed fitting to the elastic scattering data [1, 23, 31]. A further increase of $\zeta$ leads to the toroid-like shape with a dip at $b = 0$. The values $\zeta = 1.5$ are proposed in [19, 27] and $\zeta = 1.8$ in [28] as corresponding to asymptotical regimes. The value $\zeta = 2$ corresponds to the ”black disk” regime ($\sigma_{el} = \sigma_{in} = 0.5\sigma_{tot}$). For more discussion of the black disk and the geometrical scaling see Refs [34, 35, 36].
was interpreted as an indication on the active role of the high density gluonic component of the internal structure of protons at that energy. Therefrom it was concluded that such a component should be more properly accounted in the new versions of the Monte-Carlo models. That demonstrates how the knowledge about the spatial view of the inelastic interaction region helps in getting some conclusions about possible omissions in the models used nowadays for description of experimental data on jet production at LHC energise.

The spatial region of elastic scattering as derived from the subtrahend in Eq. (22) is strongly peaked at low impact parameters decreasing fast at larger values of $b$ according to the Gaussian exponent law. The contribution to the elastic cross section is, nevertheless, suppressed at small $b$ and comes mainly from impact parameters $b^2 \approx 2B$. The average value of the squared impact parameter for elastic scattering can be estimated as

$$<b^2_{el}> = \frac{\sigma_{el}(s)}{\pi \zeta^2(s)}. \quad (28)$$

Inelastic processes are much more peripheral. The ratio of the corresponding values of squared impact parameters is

$$\frac{<b^2_{inel}>}{<b^2_{el}>} = \frac{8 - \zeta}{4 - \zeta}. \quad (29)$$

This ratio exceeds 2 already at LHC energies and would become equal to 6 for (would be!) $\zeta = 2$. The peripherality of inelastic processes compared to elastic ones increases with increase of the proportion of elastic collisions. Elastic collisions are more effective at the most central interactions.

7 Some predictions at higher energies

What can we expect at higher energies?

The only guesses can be obtained from the extrapolation of experimental results at present energies to new regimes even though our previous experience teaches us how indefinite and even erroneous they can be as it often happened. Nevertheless, let us try to use some assumptions relying on the fact that we have used only such most reliable methods of getting the necessary information as the unitarity condition and the quite precise experimental data on the elastic scattering.
First, one may assume that the share of elastic scattering $\zeta$ will increase but approach 1 asymptotically without crossing it. In principle, such an assumption can be valid because the present accuracy of experimental data at 7 and 8 TeV is not high enough and allows it. That would imply that its precise value at these energies is still slightly lower than 1 within the present experimental errors. This is the only possibility to keep the present status of the shape of the interaction region (BEL) when the inelastic profile stays quite stable with slow approach to the complete blackness at central collisions and steady increase of its range with asymptotical saturation. That is a kind of "the black tube" if one implies rather long longitudinal distances as it is commonly believed for the picture with soft wee partons.

Surely, it is not excluded that the share of elastic scattering will suddenly decrease again. Then we would come to the picture which we dealt with, say, at ISR energies and nothing interesting happens. This possibility looks however quite improbable. In both cases one deals with the same branch of the unitarity condition.

Another, more interesting and intriguing possibility is further increase of the share of elastic processes with increasing energy. One has to consider the values $\zeta > 1$. The transition to another branch of the unitarity condition takes place. The BEL-scenario described above becomes drastically changed. The maximum absorption appears at non-zero impact parameters. It shifts to positive values of impact parameters (23) for $\zeta > 1$. Then the inelastic interaction region inevitably acquires the toroid-like shape TEH with a dip at the very center $b = 0$. Most probably, if the accuracy of experimental data is high enough, one will observe at 13 TeV the increase of $\zeta$ above 1 at approximately the same rate as it happened in the range of ISR to LHC where it changed from 0.67 to 1.0 (with intermediate values of 0.8 at $Sp\bar{p}S$ at 546 GeV and 0.9 at Tevatron at 1.8 TeV if the proton-antiproton data are included). Then the darkness at the very central collisions $G(s, b = 0)$ diminishes with increase of $\zeta$. The center becomes more transparent. The dip at the center of the interaction region with a minimum at $b = 0$ should appear instead of the flat plateau. "The black plateau" described at 7 TeV transforms to the toroid-like structure with somewhat lower darkness at the center and maximum blackness equal 1 at more peripheral impact parameter $b_m$ (see [8, 7, 13, 37]). As it follows from the above formulae, this dependence is very slow near $\zeta = 1$ so that the darkness at the center would only become smaller, e.g., by 6% if $\zeta$ increases to 1.2. Therefore one can hardly expect the immediate drastic changes with increase of LHC energies to 13 TeV. Nev-
ertheless, the forthcoming TOTEM+CMS results on elastic scattering at 13 TeV can be very conclusive about the general trend if the precise values of the diffraction cone slope $B$ and the total cross section $\sigma_{\text{tot}}$ (or equivalently, of the proportion of elastic processes) become available and the corresponding value of $\zeta$ happens to be above 1.

The central dip becomes even deeper at larger $\zeta$. The limiting value $\zeta = 2$ leads to complete dominance of elastic scattering at the center $b = 0$ with $\zeta^2 = 4$. It coincides with the total profile $2\zeta = 4$ there. No inelastic absorption can be observed at the center $G(s,0) = 0$. The maximum absorption is shifted to $b_m = \sqrt{2B\ln 2}$. Such situation can be only reached if the positive sign branch of the unitarity condition is applicable.

All these features are demonstrated in Fig. 4 borrowed from Ref. [9]. Beside the demonstration of the present energy results at $\zeta = 0.7$ and 1.0 and of the limiting plot of the attenuation at $\zeta = 2$ some intermediate values 1.5 and 1.8 are shown. These values illustrate the regimes with further increase and asymptotical saturation of the share of elastic scattering.

Such regimes are predicted by some phenomenological models [19, 27] which favor the situation of the increasing proportion of elastic scattering, i.e. of $\zeta$ becoming steadily larger than 1 at higher energies. They are based on good fits of a large set of experimental data at present energies and provide some extrapolations to ever higher energies. The realistic estimates of their predictions at the energies 13 TeV and 100 TeV [38] show that extremely high accuracy of elastic scattering experiments will be necessary to observe some effects. Both models predict that $\zeta$ will be only 3-4% higher at 13 TeV than that at 7 TeV. In accordance with the above formulae, the darkness decrease at the center of the inelastic interaction region is quadratically small compared to the change of $\zeta$ itself and becomes noticeable at the third digit only. That asks for very high precision of forthcoming TOTEM+CMS results at 13 TeV. At the newly planned 100 TeV collider the value of $\zeta$ can increase by 13-20% from 1. It would imply 3-4% lower value of $G(b = 0)$. The maximum blackness 1 will be reached at the impact parameters about 0.5 fm. The formation of the toroid-like structure proceeds very slowly with energy. No model predicts the fast rise of $\zeta$ to values close to 2. The asymptotical values of $\zeta$ preferred by both models are about 1.5. The corresponding asymptotical profiles of inelastic processes are shown in Fig. 4. The somewhat different asymptopia for $\zeta$ equal 1.8 is favored in the theoretical paper [28]. Its prediction of the deeper dip is also demonstrated in Fig. 4. The whole impact-parameter structure in all these models reminds the toroid
(tube) with absorbing black edges which looks as if being more and more transparent for the elastic component at the very center. The inelastic cross section will be only about 1.5 times larger than the elastic cross section at asymptotics for these models. It is most fascinating in the presented scenario that the density of central inelastic interactions tends to 0 for $\zeta \to 2$ which would lead to the "black disk" limit with equal elastic and inelastic cross sections. However, no models predict such a high increase of the share of elastic scattering even at asymptotically high energies.

What concerns the inelastic processes, these models do not predict any drastic evolution of the interaction region with increasing energy over the LHC range. The (almost) black plateau with small dip at the central part near $b = 0$ will become somewhat enlarged in size. Therefore the jet cross sections due to central collisions will slightly increase as well at the beginning. Step by step the inelastic profile will become even more peripheral and the role of peripheral collisions will increase.

As was discussed, central collisions are responsible for the rare events with highest multiplicities. The decrease of their intensity at ever higher energies would result in lower tail of the multiplicity distributions and in their more steepened shape. In particular, one would also predict the diminished role of jet production from central collisions with further increase of $\zeta$. Once again, these effects will develop very slowly, unfortunately.

8 Discussion and conclusions

The intriguing purely experimental phenomenon of the increase of the share of elastic processes to the total outcome observed in proton interactions at energies from ISR to LHC attracts much attention nowadays. It has not been explained yet. One of the possibilities can be related to the fact that the larger number of the high energy constituents (quarks, gluons) exchange by high momenta. Due to the QCD property of the asymptotic freedom the role of such processes would decrease, and, correspondingly, the relative role of elastic scattering increases. Let us note that the mutual influence of the smaller number of these processes and larger transferred momenta must lead to some increase of the transverse momenta of created particles as observed in experiment. Another possibility is connected to the fluctuations of the partonic picture of colliding protons. The time of flight of protons through one another becomes shorter with increasing energy. The pointlike partons
have almost no chance to interact during such a short time\(^3\). Therefore, the role of elastic processes can increase.

Inspite of absence of the explanation of the observed effect, the increase of the proportion of elastic processes has been used in this review paper for getting its consequences. In particular, the important information about the spatial regions of proton interactions has been obtained. The share approach to 1/4 at LHC (or, equivalently, of \(\zeta\) to 1) can become a critical sign of the changing character of processes of hadron interactions if the above tendency of increase persists. The concave central part of the inelastic interaction region would be formed. The inelastic interaction region would then look like a toroid (tube) hollowed inside and strongly absorbing in its main body at the edges. The role of elastic scattering in central collision becomes increasing. That is surprising and contradicts somewhat to our everyday experience and theoretical prejudices. Intuitively, we would expect the steady increase of the proportion of inelastic processes with increasing energy as it happened up to ISR. Instead of it, we are posed to the problem that from the formal theoretical point of view the new tendency requires now to consider another branch of the unitarity condition that asks for its physics interpretation.

It is hard to believe that protons become more penetrable at higher energies after being so dark in central collisions with \(G(s, 0) = 1\) at 7 TeV unless some special coherence within the internal region develops. Moreover, it seems somewhat mystifying why the coherence is more significant just for central collisions but not at other impact parameters where inelastic collisions become dominant.

Several very speculative ideas come to the mind and have been proposed but not a single one looks satisfactory. Let us try to describe some of them independently of how fantastic they look like.

For example, the role of the string junction in three-quark hadrons can become crucial. Then this effect would not be observed, say, in the pion-proton interactions. However we have no chances to get any experimental information about these processes. Moreover, the success of the quark-diquark models adds some sceptical attitude to this approach. Probably, the relative strengths of the longitudinal and transverse components of gluon (string) fields can help to explain the new physics of TEH-scenarium of the "hollowed interactions" of protons.

\(^3\)The classical analogy of this effect to the bullet passing through a glass was pointed out to me by B.L. Altshuler.
In classical terms, the transparency at central collisions could reveal itself at collisions of the two toruses with so different radii that one of them penetrates through the hole in another one at $b = 0$. In the more general situation, those can be some stratified objects in which the empty spaces of one of them coincide at the collision with the dense regions in the another one. They overlap at peripheral collisions and therefore lead to inelastic processes. These fluctuations of the size and the structure of high energy protons seem very improbable.

One could also imagine that "black" protons start scattering backward like the billiard balls for head-on collisions. Snell’s law admits such situation for equal reflective indices of colliding bodies. However the forward and backward scattering can not be distinguished for two equivalent colliding objects. That can only be checked if forward and backward scattered protons can be somehow identified in experiment. Then they should wear different labels. One can use the proton spin as such a label. In principle, experiments with oppositely polarised protons can resolve the problem. Unfortunately, no polarized protons are available now even at LHC. Thus it is improbable that the TEH-structure will be observed directly. Moreover, the backward scattering would ask all partons to get coherently large transferred momenta. The asymptotic freedom of QGD claims that the probability of such processes must be extremely low.

Beside the case of the two billiard balls colliding head-on, one could consider the hypothesis that centrally colliding protons at $\zeta = 2$ remind solitons which "pass through one another without losing their identity. Here we have a nonlinear physical process in which interacting localized pulses do not scatter irreversibly". Again, in the case of two identical colliding objects it is impossible to decide whether they scatter forward or backward. In the case of solitons it is known that the non-linearity and dispersive properties (the chromopermittivity) of a medium compete to produce such effect. Then one should understand the dynamics of the whole process. For its description one uses the equations of Korteweg-de-Vries and sine-Gordon, the nonlinear Schrodinger equation, the Skyrme model, instantons. It is not at all clear yet how the QCD-nonlinearity and the properties of the quark-gluon medium could be responsible at the quantum-field level for these new features of proton interactions. Again, the asymptotic freedom of QCD seems to forbid such processes.

Coherence of the parton structures inside the interaction region of colliding hadrons can probably lead to the observed effects. It can reveal itself
in "squeezing" (or complete absorption) of the intermediate created inelastic channels. That would lead to the antishadowing effect with increasing role of the elastic channel which reminds the self-focusing of the laser beams. At the model level of reggeon interactions, these possibilities were considered in Refs [44, 45] with the discussion of different variants of the absorbing disk.

Another more exotic hypothesis [46] which could be used to treat the hollowed internal TEH-region is the formation of cooler disoriented chiral condensate inside it ("baked-alaska" DCC). The signature of this squeezed coherent state would be some disbalance between the production of charged and neutral pions [47], probably, noticed in some high energy cosmic ray experiments. However the cross sections for central collisions seem to be extremely small as discussed above. The failure to find such events at Fermilab is probably connected with too low energies available. It leaves some hope for higher energies in view of discussions above. Total internal reflection of coherent states from dark edges of the toroid can be blamed for enlarged elastic scattering (like transmission of laser beams in optical fibers).

The transition to the deconfined state of quarks and gluons in the central collisions could also be claimed responsible for new effects (see Ref. [48]). The optical analogy with the scattering of light on metallic surface as induced by the presence of free electrons is used. Again, it is hard to explain why that happens for central collisions while peripheral ones with impact parameters near $b_m$ are strongly inelastic.

To conclude, the problem of the increasing proportion of the elastic scattering of high energy protons, asking for its own solution, can be further studied only with the advent of experimental facilities of higher energy accelerators. Cosmic ray studies do not look very promising because of the relatively low accuracy of measurements. However the detailed analyses of the extensive air showers, probably, can say something about "escaping" high energy protons. Only very precise experimental results can lead to definite conclusions since the theoretically predicted energy dependence of the darkness of the interaction region discussed above is very mild. However the heuristic value of the foreseen results should not be underestimated. If the tendency of the increasing proportion of elastic scattering processes persists, it would pose a problem of a new view on mechanisms of proton (hadron) high energy interactions. Then one should invent new ways of explaining the transition to quite uncommon regime of proton interactions with peculiar shapes of the interaction region.
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