$T_{cc}^+$ and its partners

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Inspired by the $T_{cc}^+$ signal discovered by the LHCb Collaboration, we systematically investigate the doubly heavy tetraquark states with the molecule configuration $[Q_1q_2ar{q}_1]_1[Q_2ar{q}_4]_1$, ($Q = c$ and $b$, $q = u, d$ and $s$) in a nonrelativistic quark model. The model involves a color screening confinement potential, meson-exchange interactions and one-gluon-exchange interactions. The state $T_{cc}^+$ with $IJ^P = 0^+$ is a very loosely bound deuteron-like state with a binding energy around 0.34 MeV and a huge size of 4.32 fm. Both the meson exchange force and the coupled channel effect play a pivotal role. Without the meson exchange force, there does not exist the $T_{cc}^+$ molecular state. In strong contrast, the QCD valence bond forms clearly in the $T_{bb}^-$ system when we turn off the meson-exchange force, which is very similar to the hydrogen molecule in QED. Moreover, the $T_{bb}^-$ becomes a helium-like QCD-atom if we increase the bottom quark mass by a factor of three. Especially, the states $T_{bb}^-$ with $0^+$, $T_{cc}^+$ with $0^+$ and $0^+$ and the $V$-spin antisymmetric states $T_{bsb}$ with $1^+$, $T_{bcs}$ with $1^0$ and $2^1$ can form a compact, hydrogen molecule-like or deuteron-like bound state with different binding dynamics. The high-spin states $T_{bc}^0$ with $02^+$ and $T_{bcs}^0$ with $12^+$ can decay into $D$-wave $BD$ and $B_iD$ although they are below the thresholds $B^*D^*$ and $B^*_iD^*$, respectively. The isospin and $V$-spin symmetric states are unbound. We also calculate their magnetic moments and axial charges.

I. INTRODUCTION

The theoretical explorations on the possible stable doubly heavy tetraquark states were pioneered in the early 1980s [1]. These states were investigated with various formalisms such as the MIT bag model [2], constituent quark models [3,4], chiral perturbation theory [5], string model [6], lattice QCD [7], and QCD sum rule approach [8,9]. Although the state $T_{bb}^-$ with $0^+$ seems stable in various theoretical frameworks, its production turns out to be very challenging. The discovery of the doubly charmed baryon $Ξ_{cc}^{++}$ by the LHCb Collaboration [10] has stimulated the enthusiasm on the doubly heavy tetraquark states [11,24].

Recently, the LHCb Collaboration discovered the doubly charmed state $T_{cc}^+$ with $IJ^P = 0^+$ by analyzing the $D^0D^0π^+$ invariant mass spectrum [25], which has a minimal quark configuration of $ccar{u}ar{d}$. Its binding energy relative to the $DD^*$ threshold and width are

$$E_b = -273 ± 61 ± 5^{+14}_{-14} \text{ keV},$$

$$\Gamma = 410 ± 165 ± 43^{+18}_{-38} \text{ keV}.$$  

The LHCb Collaboration also released a more profound decay analysis, in which the unitarized Breit-Wigner profile was used [26]. Its binding energy and decay width were updated as

$$E_b = -361 ± 40 \text{ keV}, \Gamma = 47.8 ± 1.9 \text{ keV}.$$  

The binding energy and decay width of the $T_{cc}^+$ signal match very well with the prediction of the $DD^*$ molecular state [27,29]. The discovery of the $X(3872)$ [29] pioneered the observation of a family of hidden-charm and hidden-bottom tetraquark and pentaquark states in the past decades. Similarly, the discovery of the $T_{cc}^+$ shall open a new gate for a family of the $T_{cc}^+$-like doubly heavy tetraquark, pentaquark and hexaquark states.

The discovery of the state $T_{cc}^+$ has inspired a large amount of investigations on its properties and structure within the different theoretical frameworks [28,30-40]. Many $T_{cc}^+$-like doubly heavy tetraquark candidates were proposed, such as the states $T_{cc}^1$, $T_{bb}^1$, $T_{cc}^0$, $T_{bc}^0$, $T_{bcs}^0$, $S$-wave $D_1D_1$, $D_1D_2^*$ and $D_2^*D_2^*$ states [30], doubly charmed $P_{cc}$ and triply charmed $H_{cc}$ states [51,53].

In this work, we will analyze the underlying dynamics in the formation of the loosely bound $T_{cc}^+$ state very carefully. We will exhaust the most promising partners of the $T_{cc}^+$ and compare the different binding mechanisms in the $T_{cc}^+$ and $T_{bb}^-$ systems.

This paper is organized as follows. After the introduction, the details of the quark model are given in Sec. II. The construction of the wavefunctions of the doubly heavy tetraquark states with the molecule configuration is shown in Sec. III. The numerical results and discussions of the stable doubly heavy tetraquark states are presented in the following sections. The last section is a brief summary.

II. QUARK MODEL

The underlying theory of strong interaction is quantum Chromodynamics (QCD). At the hadronic scale, QCD is highly non-perturbative due to the complicated infrared behavior of the non-Abelian $SU(3)$ gauge group. At present it is still impossible to derive the hadron spectrum analytically from the QCD Lagrangian. The QCD-
inspired constituent quark model remains a powerful tool in obtaining physical insight for the complicated strong interaction systems although the connection between the light current quarks in QCD and the light constituent quarks in the quark model is not established clearly.

The constituent quark model was formulated under the assumption that the hadrons are color singlet non-relativistic bound states of constituent quarks with phenomenological effective masses and various effective interactions. The model Hamiltonian used here can be written as

\[ H_n = \sum_{i=1}^{n} \left( m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_c + \sum_{i>j}^{n} V_{ij}, \]

where \( m_i \) and \( \mathbf{p}_i \) are the mass and momentum of the i-th quark or antiquark, respectively. \( T_c \) is the center-of-mass kinetic energy of the states and should be deducted. \( V_{ij}^{\text{OGE}} \), \( V_{ij}^{\text{OGE}} \), \( V_{ij}^{\text{con}} \) are the one-gluon-exchange interaction, one-boson-exchange interaction \((\pi, K \text{ and } \eta)\), \( \sigma \)-meson exchange interaction and color confinement potential between the particles \( i \) and \( j \), respectively.

The origin of the constituent quark mass can be traced back to the spontaneous breaking of \( SU(3)_L \otimes SU(3)_R \) chiral symmetry and consequently constituent quarks should interact through the exchange of Goldstone bosons \([54]\). Chiral symmetry breaking suggests dividing quarks into two different sectors: light quarks \((u, d \text{ and } s)\) where the chiral symmetry is spontaneously broken and heavy quarks \((c \text{ and } b)\) where the symmetry is explicitly broken. The meson exchange interactions only occur in the light quark sector. The central parts of the interactions originating from chiral symmetry breaking can be resumed as follows \([55]\),

\[ V_{ij}^{\text{OGE}} = V_{ij}^\sigma \sum_{k=1}^{3} \mathbf{F}_i \cdot \mathbf{F}_k + V_{ij}^\rho \sum_{k=4}^{7} \mathbf{F}_i \cdot \mathbf{F}_k + V_{ij}^{\chi}(\mathbf{F}_i \cdot \mathbf{F}_j \cos \theta_P - \sin \theta_P), \]

\[ V_{ij}^{\chi} = \frac{g_{ch}^2}{4\pi} \frac{m_\pi^3}{12m_im_j} \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} \sigma_i \cdot \sigma_j \]

\[ \times \left( Y(m_\chi r_{ij}) - \frac{\Lambda^2}{m_\chi^2} Y(\Lambda \chi r_{ij}) \right), \]

\[ V_{ij}^{\sigma} = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda^2 m_\sigma}{\Lambda^2 - m_\sigma^2} \left( Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right). \]

The noncentral parts, tensor force and spin-orbit coupling, are not given here because we are only interested in the S-wave states here. The function \( Y(x) = \sum_{j} \mathbf{F}_i \cdot \mathbf{F}_j \) and \( \sigma_i \) are the flavor \( SU(3) \) Gell-Mann matrices and spin \( SU(2) \) Pauli matrices, respectively. \( r_{ij} \) is the distance between the particles \( i \) and \( j \). The mass parameters \( m_\pi, m_K \) and \( m_\eta \) take their experimental values. The cutoff parameters \( \Lambda \) and the mixing angle \( \theta_P \) take the values from Ref. \([54]\). The mass parameter \( m_\sigma \) can be determined through the PCAC relation \( m_\sigma^2 \approx m_\pi^2 + 4m_{u,d}^2 \) \([50]\). The chiral coupling constant \( g_{ch} \) can be obtained from the \( \pi NN \) coupling constant through

\[ \frac{g_{ch}^2}{4\pi} = \left( \frac{3}{5} \right)^2 \frac{2g_{\pi NN}}{4\pi} m_\sigma^2. \]

Besides the chiral symmetry breaking, there also exists the one-gluon-exchange (OGE) potential. From the non-relativistic reduction of the OGE diagram in QCD for the point-like quarks, one gets

\[ V_{ij}^{\text{OGE}} = \alpha_s \frac{\Lambda_\pi^2 \cdot \Lambda_\sigma^2}{4} \left( \frac{1}{r_{ij}} - \frac{2\pi \delta(r_{ij}) \sigma_i \cdot \sigma_j}{3m_im_j} \right), \]

\( \lambda_i \) is the color \( SU(3) \) Gell-Mann matrices. The Dirac \( \delta(r_{ij}) \) function, where \( r_{ij} = r_i - r_j \), arises from the interaction between point-like quarks and collapses when not treated perturbatively \([57]\). Therefore, the \( \delta(r_{ij}) \) function is regularized in the form \([55]\)

\[ \delta(r_{ij}) \rightarrow \frac{1}{4\pi r_{ij}^2 \Lambda_0^2 \mu_{ij}} e^{-r_{ij}/r_0 \mu_{ij}}, \]

where \( r_0 \mu_{ij} = r_0 / \mu_{ij} \), \( r_0 \) is an adjustable model parameter and \( \mu_{ij} \) is the reduced mass of two interacting particles \( i \) and \( j \). This regularization is justified based on the finite size of the constituent quarks and should be flavor dependent \([58]\).

The quark-gluon coupling constant \( \alpha_s \) in the perturbative QCD reads \([59]\)

\[ \alpha_s(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2}}. \]

In the present work, we use an effective scale-dependent form given by

\[ \alpha_s(\mu_0^2) = \frac{\alpha_0}{\ln \frac{\mu_0^2}{\Lambda_0^2}}. \]

\( \Lambda_0 \) and \( \alpha_0 \) are adjustable model parameters determined by fitting the ground state meson spectrum.

Finally, any model imitating QCD should incorporate the nonperturbative color confinement effect. We adopt the phenomenological color screening confinement potential,

\[ V_{ij}^{\text{con}} = -\alpha_c \frac{\Lambda_c^2}{4\pi} \frac{m_\sigma}{\Lambda_\sigma^2 - m_\sigma^2} Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \]

The color screening parameter \( \mu_c = 1.0 \text{ fm}^{-2} \) is taken from recent investigations on the doubly heavy tetraquark states \([20][21]\). The adjustable parameter \( \alpha_c \) is determined by fitting the ground state meson spectrum. The color screening parameter \( \mu_c \) is 1.0 fm$^{-2}$ and is taken from
Ref. [60]. The color screening confinement potential can automatically match the quadratic one in the short distance region \( (\mu_r r^2 \ll 1) \). When two mesons are separated to large distances, the confinement potential can guarantee that the energy of the tetraquark system evolves into the sum of the two-meson internal energy calculated by the model Hamiltonian. In the intermediate range region, the hybrid confinement can give a different picture from that given by a single form confinement. Such type of the color screening confinement potential comes from the quark delocalization and color screening model [60], which can describe the nuclear intermediate range attraction and reproduce the \( N-N \) scattering data and the properties of the deuteron. Meanwhile, the model can avoid the spurious van de Walls color force between two color singlets arising from the direct extension of the single-hadron Hamiltonian to the multiquark states [60].

The model has been widely applied to investigate the properties of the baryon-baryon and baryon-meson interactions [61].

III. WAVE FUNCTIONS OF THE DOUBLY HEAVY TETRAQUARK STATES

Our previous systematical investigation on the doubly heavy tetraquark states suggested that the states \([Q_1Q_3][q_2q_4]\) can establish deep compact bound states in the quark models [11], which is obviously contradictory with the properties of the state \( T_{cc}^+ \) reported by the LHCb Collaboration [25, 26].

The research on the states \([Q_1q_2][Q_3q_4]\) indicated that their main color component is the molecule state \([Q_1q_2]^1_1[Q_3q_4]^1_1\). Furthermore, recent study on the states \([c\bar{u}]^1[c\bar{d}]^1\) and \([\bar{b}u]^1[\bar{b}d]^1\) shown that the hidden color components \([c\bar{u}]^1_1[c\bar{d}]^1_1\) and \([\bar{b}u]^1_1[\bar{b}d]^1_1\) can be negligible if all possible color singlet components were taken into accounted [62]. The influence of the hidden color components on the numerical results is very insignificant even if the hidden color configurations are included. In order to reduce the heavy computational work, we therefore omit the hidden color configuration \([Q_1q_2]^1_1[Q_3q_4]^1_1\) in the present work.

Within the framework of the molecule configuration \([c_1\bar{u}_2]^1_1[\bar{c}_1\bar{d}_4]^1_1\), the trial wave function of the \( T_{cc}^+ \) state with \( IJ^P = 0^+ \) can be constructed as a sum of the following direct products of color \( \psi_c \), isospin \( \eta_i \), spin \( \chi_s \) and spatial \( \phi \) terms

\[
\Phi_{IJ}^{T_{cc}^+} = \sum_{\alpha} \mathcal{A} \left\{ \left[ \phi_{i_1,m_1}(r) \chi_{s_1} \right]_{J_1 M_1} \left[ \phi_{i_2,m_2}(R) \chi_{s_2} \right]_{J_2 M_2} \right\} \times \phi_{i,mc}(\rho) \}
\]

Here we assume the magnetic components \( M_I = I \) and \( M_J = J \). The subscripts \( a \) and \( b \) represent the subclusters \([c_1\bar{u}_2]^1_1\) and \([\bar{c}_1\bar{d}_4]^1_1\), respectively. \( \mathcal{A} \) is the antisymmetrization operator and equal to \( 1 - P_{13} - P_{24} + P_{13}P_{24} \) because of the Fermi-Dirac statistic of the identical particles, where \( P_{ij} \) is the permutation operator on the particles \( i \) and \( j \). The summing index \( \alpha \) stands for all possible flavor-spin-color-spatial intermediate quantum numbers.

The relative spatial coordinates \( r, R \) and \( \rho \) are defined as

\[
r = r_1 - r_2, \quad R = r_3 - r_4, \\
\rho = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} - \frac{m_3 r_3 + m_4 r_4}{m_3 + m_4}.
\]

The corresponding angular excitations of three relative motions are, respectively, \( \lambda_a, \lambda_b \) and \( \lambda_c \). The parity of the state \( T_{cc}^+ \) can therefore be expressed in terms of the relative orbital angular momenta associated with the Jacobi coordinates as \( P = (-1)^{\lambda_a + \lambda_b + \lambda_c} \). It is worth mentioning that this set of coordinate is only a possible choice of many coordinates and however most propositions to describe the correlation of two mesons. In order to obtain a reliable solution of few-body problem, a high precision numerical method is indispensable. The Gaussian Expansion Method (GEM) [63] has been proven to be very powerful to solve few-body problem. Brink et al first applied the GEM for studying heavy tetraquarks states [4]. We also use the GEM to study doubly heavy tetraquark systems in the present work. According to the GEM, the relative motion wave function can be written as

\[
\phi^G_{lm}(\mathbf{x}) = \sum_{\nu=1}^{n_{max}} c_n N_n x^l e^{-\nu_x x^2} Y_{lm}(\hat{x})
\]

Gaussian size parameters are taken as geometric progression

\[
\nu_n = \frac{1}{r_n}, \quad r_n = r_1 a^{n-1}, \quad a = \left( \frac{r_{n_{max}}}{r_1} \right)^{1/n_{max}}.
\]

\( r_1 \) and \( r_{max} \) are the minimum and maximum of the size, respectively. \( n_{max} \) is the number of the Gaussian wave function. More details about the GEM can be found in Ref. [63]. In the present work, we expand the wavefunction \( \phi^G_{lm}(r) \) \( (\phi^G_{im}(R)) \) with \( n_{max} \) \( (n_{max}) \) Gaussian functions with the different width ranging from 0.1 fm to 2.0 fm because the size of the mesons is less than 1 fm. We expand the wavefunction \( \phi^G_{lm}(\rho) \) with \( n_{max} \) Gaussian functions with the different width ranging from 0.1 fm to 5.0 fm because the size of meson-meson molecule is about several fms. In this way, the total number of the trial wave function \( N_{\text{base}} \) is equal to \( n_{csf} \times n_{max} \times n'_{max} \times n''_{max} \), where \( n_{csf} \), the number of the color-spin-flavor wave function, will be given later. \( N_{\text{base}} \) should be increased gradually by increasing \( n_{max} \), \( n'_{max} \) and \( n''_{max} \) until the convergent numerical results are obtained. In addition, a large width Gaussian function \( \phi^G_{lm}(\rho) \) \( (\rho = |\rho| \to \infty) \) should be introduced to guarantee a fast convergence of numerical results when a bound state does not exist.

Taking all degrees of freedom of identical particles, the Pauli principle must be satisfied by imposing a restriction
on the quantum numbers of the mesons $c_1\bar{u}_2$ and $c_3\bar{d}_4$. The quantum numbers must satisfy the relation $s_a + s_b - S + i_a + i_b - I + l_c = \text{even}$ when $s_a = s_b$ because the present boson system should satisfy the Boson-Einstein statistic. According to the restriction, the $S$-wave $(l_c = 0)$ states with $0^+$ and $0^+$ do not exist. The wave function of the $S$-wave state $[c_1\bar{u}_2][c_3\bar{d}_4]$ with $0^+$ has two possible channels,

$$[D\bar{D}]_+ = \frac{1}{\sqrt{2}}(D^0D^* + D^*D^0),$$

$$[D^*\bar{D}]_+ = \frac{1}{\sqrt{2}}(D^0D^* - D^*D^0),$$

In the channel $[D^*\bar{D}]_-$, the spins of the mesons $D^0$ and $D^{*+}$ couple into the total angular momentum $J$. Similarly, the wave function of the state $[c_1\bar{u}_2][c_3\bar{d}_4]$ with $0^+$ also has two possible channels,

$$[DD]_+ = \frac{1}{\sqrt{2}}(D^0D^* + D^*D^0),$$

$$[D^*\bar{D}]_+ = \frac{1}{\sqrt{2}}(D^0D^* + D^{*+}D^{*0}).$$

The wave functions of the states $[c_1\bar{u}_2][c_3\bar{d}_4]$ with $1^+$ and $1^+$ can be written as

$$[DD^*]_+ = \frac{1}{\sqrt{2}}(D^0D^{*+} + D^{*+}D^0),$$

$$[D^*\bar{D}]_+ = \frac{1}{\sqrt{2}}(D^0D^{*+} + D^{*+}D^0).$$

Analogically, there may exist the partners of the $T_{cc}^+$ state with the configurations $[b_1\bar{u}_2][b_3\bar{d}_4]$ and $[b_1\bar{u}_2][c_3\bar{d}_4]$, denoted as $T_{bb}^+$ and $T_{bc}^+$. We can obtain the wave functions of the $S$-wave state $T_{bb}^+$ by solely making a replacement of $c$ with $b$ in those of the state $T_{cc}^+$. In the case of the state $T_{bc}^+$, the wave functions of the states with $0^+$ and $1^+$ read

$$[BD]_\pm = \frac{1}{\sqrt{2}}(B^-D^0 \pm B^0D^0),$$

$$[B^*D]_\pm = \frac{1}{\sqrt{2}}(B^-D^{*+} \pm B^{*+}D^0).$$

The signs + and − stand for the cases of the isospin symmetry ($I = 1$) and antisymmetry ($I = 0$), respectively. The same notations hold for the other states with $I = 1$ and $I = 0$. The wave functions of the states $[b_1\bar{u}_2][c_3\bar{d}_4]$ with $0^+$ and $1^+$ have the following three possible channels,

$$[\bar{B}D^*]_\pm = \frac{1}{\sqrt{2}}(B^-D^{*0} \pm B^0D^{*0}),$$

$$[B^*D]_\pm = \frac{1}{\sqrt{2}}(B^-D^{*0} \pm B^0D^{*0}),$$

$$[B^*D^*]_\pm = \frac{1}{\sqrt{2}}(B^-D^{*0} \pm B^0D^{*0}).$$

The wave function of the states with $0^+$ and $1^+$ has the following one possible channel,

$$[\bar{B}^*D^*]_\pm = \frac{1}{\sqrt{2}}(B^-D^{*0} \pm B^0D^{*0}).$$

We denote the strange partners of the $T_{cc}^+$ state with the configurations $[b_1\bar{u}_2][b_3\bar{s}_4], [c_1\bar{u}_2][c_3\bar{s}_4]$ and $[b_1\bar{u}_2][c_3\bar{s}_4]$ as the $T_{bb}^-$, $T_{cc}^+$ and $T_{bc}^0$, respectively. We can obtain their wave functions by replacing $d$ with $\bar{s}$ in those of the states $T_{bb}^-$, $T_{cc}^+$ and $T_{bc}^0$ because $I$-spin (isospin) and $V$-spin are equivalent. The wave functions of the states $[b_1\bar{s}_2][b_3\bar{s}_4], [c_1\bar{s}_2][c_3\bar{s}_4]$ and $[b_1\bar{s}_2][c_3\bar{s}_4]$, denoted as $T^0_{bb}, T^0_{cc}$ and $T^0_{bc}$, are similar to those of the states $T_{bb}^-, T_{cc}^+$ and $T_{bc}^0$ with $I = 1$ because they have the same flavor symmetry.

### IV. STRUCTURE OF THE $T_{cc}^+$ STATE WITH $0^+$

We reproduce the mass spectrum of the ordinary mesons to determine model parameters as in Ref. [11]. We collect the results of the heavy-light mesons in Table I.

In the following, we move on to the investigation of the $T_{cc}^+$ with $0^+$ and its partners. In order to obtain the lowest states with positive parity, we assume that the three relative motions are in $S$-wave, namely $l_a = l_b = l_c = 0$. We can obtain the eigenvalue and eigenvector by solving the four-body Schrödinger equation

$$(H_4 - E_4)\Phi_{T_{cc}^+}^{I,J} = 0$$

with the Rayleigh-Ritz variational principle. We define the binding energy $E_b$ of the doubly heavy tetraquark states as

$$E_b = E_4 - \lim_{\rho \rightarrow \infty} E_4(\rho)$$

to identify whether or not the tetraquark states are stable against the strong interactions, where $\lim_{\rho \rightarrow \infty} E_4(\rho)$ is the lowest theoretical threshold of the two mesons which can couple into the same quantum numbers with those of the tetraquark states. Such a subtraction procedure can greatly reduce the influence of the inaccurate model parameters and meson spectra on the binding energies. If $E_b \geq 0$, the tetraquark systems can fall apart into two mesons via the strong interactions. If $E_b < 0$, the strong decay into two mesons is forbidden and therefore the decay can only occur via either the weak or electromagnetic interaction.

We can obtain the convergent numerical results of the state $T_{cc}^+$ with $0^+$ with more than 4600 bases in the spin-color-flavor-orbital space, which are presented in Table II. It can be seen from Table III that the binding energy $E_b = -0.342$ MeV predicted by the model is highly

| State | $D$ | $D^*$ | $D_0$ | $D_0^*$ | $B$ | $B^*$ | $B_0$ | $B_0^*$ |
|-------|-----|-------|-------|--------|-----|------|-------|--------|
| Cal.  | 1867 | 2002  | 1972  | 2140   | 5259 | 5301 | 5377  | 5430   |
| PDG   | 1869 | 2007  | 1968  | 2112   | 5280 | 5325 | 5366  | 5416   |

TABLE I: The $\bar{Q}\bar{q}$ meson spectrum in the model where the mass unit is in MeV and $\langle r^2\rangle^{1/2}$ unit in fm.
consistent with the data given by the LHCb Collaboration. Note that no adjustable parameter is introduced to match the experimental data in our calculation. Neither of the single \([DD^+]-\) and \([D^*D^+]-\) channel alone can form a bound state in the model. The stable \(T_{cc}^+\) state arises from the coupling of these two channels. The dominant component of the state \(T_{cc}^+\) is the \(DD^+\) channel. Therefore, the coupled channel effect plays a critical role in the formation of the state \(T_{cc}^+\) under the assumption of the molecule picture.

The spatial configuration of the state \(T_{cc}^+\) can be ascertained by the sizes of the two subclusters \(D (D^+)\) and \(D^+\) and their relative distance, which can be approximately described by the \(r^2\) and \(\langle R^2 \rangle\) determined by the eigenvectors, respectively. The sizes \(\langle r^2 \rangle\) and \(\langle R^2 \rangle\) are approximately those of the individual meson listed in Table II which is far less than the distance between two subclusters \(\langle R^2 \rangle = 4.32\) fm. In other words, the two subclusters are far away from each other. Therefore, the \(T_{cc}^+\) state is a loosely bound deuteron-like state.

In order to illustrate the mechanism of the formation of the \(T_{cc}^+\) state, we calculate and decompose the contribution to the binding energy \(E_b\) from various parts of the Hamiltonian \(E_h^b\) in the following four cases: (a) with meson exchange and color screening effect, \(\mu_e = 1\); (b) with meson exchange while without color screening effect, \(\mu_e = 0\); (c) without meson exchange while with color screening effect, \(\mu_e = 1\); (d) without meson exchange and color screening effect, \(\mu_e = 0\). In the present model, the meson exchange and color screening effect only occur between two subclusters in the \(T_{cc}^+\) state and do not affect the corresponding threshold. We present the numerical results in Table III in which \(\Delta E_b\) is the kinetic energy difference between the tetraquark system and its corresponding threshold. In addition, the binding energy of each single channel and its ratio in the eigenvector are also given in Table III.

The hybrid color screening confinement potential generally gives bigger binding energies than single type of one, see the cases (a) and (b) or (c) and (d) in the states \(T_{bc}, T_{bb}^0, T_{bc}^0\) and \(T_{bc}^0\). However, the order is reversed in the cases (a) and (b) of the states \(T_{cc}^+\) and \(T_{bb}^0\) because of their stronger meson exchange interaction relative to the confinement potential.

From the cases (c) and (d) in Table III the bound state \(T_{cc}^+\) vanishes if we turn off the meson exchange interactions in the model. In other words, the long-range \(\pi\) and intermediate-range \(\sigma\) meson exchange force play a pivotal role in the formation of the loosely bound \(T_{cc}^+\) state.

We further compare the cases (a) and (b) to illustrate the very delicate competition between the kinetic energy and the attraction from various sources in the Hamiltonian. The main factor hindering the formation of the \(D (D^+)\) and \(D^*\) subclusters into the bound state is the relative kinetic energy between two subclusters in the cases (a) and (b). The interactions \(V_{\pi}, V_{\sigma}, V_{con}\) and \(V_{one}\) provide precious attractions. Without the color screening effect, the confinement and one-gluon-exchange color forces between the two subclusters become stronger, which pull them closer to each other. Now all the meson exchange contributions become larger in magnitude. Especially, the one-pion-exchange force is extremely sensitive to the

| \(\phi_{bc}^c (r)\) | \(\phi_{bc}^b (R)\) | \(\phi_{bc}^b (p)\) | \([DD^+]-\) | \([D^*D^+]-\) | Mixing | Rms |
|---|---|---|---|---|---|---|
| \(r_1\) | \(r_{max}\) | \(n_{max}\) | \(r_1'\) | \(r_{max}'\) | \(n_{max}'\) | \(n_{base}\) | \(E_b\) | \(E_b\) | \(E_b\) | \(\langle r^2 \rangle\) | \(\langle R^2 \rangle\) | \(\langle p^2 \rangle\) |
| 0.1 | 2.0 | 7 | 0.1 | 2.0 | 7 | 0.1 | 5.0 | 20 | 1960 | 0.000, 0.000, 0.43% | 0.75 | 0.75 | 4.41 |
| 0.1 | 2.0 | 8 | 0.1 | 2.0 | 8 | 0.1 | 5.0 | 21 | 2688 | 0.000, 0.000, 0.43% | 0.75 | 0.75 | 4.23 |
| 0.1 | 2.0 | 9 | 0.1 | 2.0 | 9 | 0.1 | 5.0 | 22 | 3654 | 0.000, 0.000, 0.43% | 0.75 | 0.75 | 4.30 |
| 0.1 | 2.0 | 10 | 0.1 | 2.0 | 10 | 0.1 | 5.0 | 23 | 4600 | 0.000, 0.000, 0.43% | 0.75 | 0.75 | 4.32 |
| 0.1 | 2.0 | 11 | 0.1 | 2.0 | 11 | 0.1 | 5.0 | 24 | 5808 | 0.000, 0.000, 0.43% | 0.75 | 0.75 | 4.32 |
| 0.1 | 2.0 | 12 | 0.1 | 2.0 | 12 | 0.1 | 5.0 | 25 | 7200 | 0.000, 0.000, 0.43% | 0.75 | 0.75 | 4.32 |

FIG. 1: The spatial configuration, left: compact state; middle: hydrogen molecule-like state; right: deuteron-like state, big blue ball and small red ball represent \(Q\) and \(q\), respectively. A large yellow ball represents a subcluster.
TABLE III: The properties of the state $T_{cc}^{+}$ and its partners predicted by the model, $E_k$ unit in MeV and rms unit in fm. $\Delta E_k$ is the kinetic energy difference between the tetraquark system and its corresponding threshold. Four cases: (a) with meson exchange and color screening effect; (b) with meson exchange while without color screening effect; (c) without meson exchange while with color screening effect; (d) without meson exchange and color screening effect. The "−" denotes that the single channel is unbound.

| State $IJ^P$ | Channel $E_k$, $\mu_c$ | Mixing $E_b$, $\langle r^2 \rangle^1_2$, $\langle r^2 \rangle^2_2$, $V_s$, $V_y$, $V^K$, $V^\eta$, $V^{coge}$, $\Delta E_k$ |
|---------------|-------------------------|-------------------------------------------------------------|
| $T_{cc}^{+}$ 01$^+$ | $[DD^\ast]_-$, $[D^\ast D]_-$ | (a) 1.0, 99.6% - 0.4%, -0.34 0.75 0.75 4.32 -2.68 -2.19 0.00 0.17 -1.00 -4.29 9.65 |
|               |                         | (b) 0.0, 98.8% - 1.2%, -0.86 0.76 0.76 2.94 -3.72 -5.41 0.00 0.49 -1.87 -7.88 17.51 |
|               |                         | (c) 1.0, 100% - 0.0%, - | |
|               |                         | (d) 0.0, 100% - 0.0%, - |
| $T_{bb}^{0}$ 01$^+$ | $[BB^\ast]_-$, $[B^\ast B^\ast]_-$ | (a) 1.0, -9.0 98.6% - 1.4%, -12.9 0.71 0.66 1.09 -9.9 -1.4 0.0 0.2 -5.1 -28.6 31.9 |
|               |                         | (b) 0.0, -5.7 98.6% -3.5 1.4%, -10.5 0.71 0.67 1.14 -9.9 -5.0 0.0 0.6 -4.9 -34.4 43.1 |
|               |                         | (c) 1.0, -4.8 99.6% - 0.4%, -6.5 0.71 0.66 1.37 -2.3 -9.2 5.0 |
|               |                         | (d) 0.0, -2.1 99.8% - 0.2%, -3.0 0.71 0.66 1.82 -1.2 -13.1 11.4 |
| $T_{bc}^{0}$ 01$^+$ | $[BD^\ast]_-$, $[B^\ast D^\ast]_-$ | (a) 1.0, -6.5 0.5%, -7.7 98.7% - 3.0 0.8%, -10.5 0.75 0.66 1.20 -8.4 0.6 0.0 0.1 -4.0 -15.7 16.9 |
|               |                         | (b) 0.0, -2.7 0.1%, -4.4 97.2% -1.3 2.7%, -7.6 0.77 0.68 1.30 -8.1 -3.6 0.0 0.4 -3.8 -21.7 29.2 |
|               |                         | (c) 1.0, -3.5 0.8%, -3.9 98.7% - 0.5%, -6.3 0.75 0.66 1.40 -2.1 -3.2 -1.1 |
|               |                         | (d) 0.0, -0.6 0.2%, -1.4 99.6% - 0.2%, -2.2 0.76 0.67 2.07 -0.9 -8.0 6.7 |
| $T_{bb}^{0}$ 02$^+$ | $[BD^\ast]_-$, $[B^\ast D^\ast]_-$ | (a) 1.0, -12.0 100%, -12.0 0.74 0.79 1.28 -6.4 7.3 0.0 0.5 -2.8 0.4 -10.0 |
|               |                         | (b) 0.0, -3.6 100%, -3.6 0.75 0.79 1.88 -4.3 4.5 0.0 0.3 -0.8 -3.7 0.9 |
|               |                         | (c) 1.0, -12.4 100%, -12.4 0.74 0.79 1.25 -3.7 -0.6 -8.0 |
|               |                         | (d) 0.0, -3.6 100%, -3.6 0.75 0.79 1.83 -1.3 -4.6 2.0 |
| $T_{bb}^{0}$ 11$^+$ | $[BB^\ast]_-$, $[B^\ast B^\ast]_-$ | (a) 1.0, -4.8 91.1% - 4.2 8.9%, -11.8 0.66 0.66 0.94 -11.2 0.0 0.3 0.1 -3.5 -6.1 8.6 |
|               |                         | (b) 0.0, -3.8 96.0% - 3.4 4.0%, -9.0 0.68 0.68 0.97 -11.4 0.0 -2.3 -0.4 -4.0 -21.4 30.4 |
|               |                         | (c) 1.0, -90.1% - 9.9%, -3.3 0.67 0.67 1.30 -0.6 5.4 -8.1 |
|               |                         | (d) 0.0, -97.6% - 2.4%, -0.5 0.68 0.68 2.74 -0.2 -1.6 1.3 |
| $T_{scs}^{1/2}$ 0$^+$ | $[BD^\ast]_-$, $[B^\ast D^\ast]_-$ | (a) 1.0, -7.4 99.6% - 0.4%, -9.2 0.64 0.60 1.13 -10.3 0.0 0.8 0.1 -3.0 -15.6 18.8 |
|               |                         | (b) 0.0, -5.1 99.6% - 0.4%, -6.7 0.64 0.60 1.25 -9.5 0.0 0.4 0.1 -2.4 -20.2 24.9 |
|               |                         | (c) 1.0, -1.1 99.8% - 0.2%, -1.6 0.65 0.60 2.00 0.6 -4.3 2.1 |
|               |                         | (d) 0.0, 0.1 99.9% - 0.1%, -0.4 0.65 0.61 3.33 -0.2 -4.2 4.0 |
| $T_{scs}^{1/2}$ 1$^+$ | $[BD^\ast]_-$, $[B^\ast D^\ast]_-$ | (a) 1.0, -4.9 0.5%, -6.1 99.1% - 0.4%, -8.0 0.69 0.60 1.21 -8.9 0.0 1.0 0.2 -2.5 -6.7 8.9 |
|               |                         | (b) 0.0, -2.3 0.2%, -3.8 99.6% - 0.2%, -5.0 0.69 0.60 1.42 -7.8 0.0 0.6 0.1 -1.8 -11.9 15.8 |
|               |                         | (c) 1.0, -0.5 0.4%, -0.7 99.4% - 0.2%, -1.8 0.70 0.60 2.04 -0.5 -2.1 0.9 |
|               |                         | (d) 0.0, -0.1% - 99.8%, -0.1%, -0.2 0.70 0.60 5.53 -0.1 -2.2 2.1 |
| $T_{scs}^{1/2}$ 1$^+$ | $[BD^\ast]_-$ | (a) 1.0, -12.2 100%, -12.2 0.68 0.73 1.15 -7.9 0.0 1.7 0.3 -3.2 -0.2 -2.8 |
|               |                         | (b) 0.0, -5.4 100%, -5.4 0.68 0.73 1.48 -6.2 0.0 1.2 0.2 -1.7 -4.6 5.7 |
|               |                         | (c) 1.0, -6.6 100%, -6.6 0.69 0.75 1.36 -1.6 1.3 -6.4 |
|               |                         | (d) 0.0, -1.4 100%, -1.4 0.69 0.75 2.33 -0.4 -1.9 1.0 |
distance and its contribution to the binding energy increases to \(-5.41\) MeV. In contrast, the kinetic energy difference \(\Delta E_k\) increases to \(17.51\) MeV in the case (b). The binding energy \(E_b = -0.86\) MeV in the case (b) is also in good agreement with the experimental data. Therefore, the \(T_{cc}^+\) state always emerges as a loosely bound state so long as there exists the meson exchange interactions.

V. ISOSPIN ANTISYMMETRIC \(T_{bb}^-\) STATES

A. Compact \(T_{bb}^-\)

Both of the \([\bar{B} \bar{B}^*]_\cdots\) and \([\bar{B}^* \bar{B}]_\cdots\) channels with \(01^+\) can form a bound state alone and their binding energies are about \(10\) MeV in the cases (a) and (b). After coupling the two channels, the state \(T_{bb}^-\) becomes a rather deeply bound state with \(E_b = -28.6\) MeV and \(-43.8\) MeV, respectively, which are much less that of the bound state \(T_{bb}^-\) with diquark-antidiquark structure \([bb][\bar{u}\bar{d}]\) because of their different color configurations \([11, 22]\). Our present results are very close to the latest lattice QCD predictions in the range of \(20-40\) MeV \([64]\). However, the earlier lattice QCD results indicated that these states were over \(100\) MeV below the \(BB^*\) threshold \([65, 66]\). The strong attraction comes from the interactions \(V^\sigma, V^\pi, V^{con}\) and \(V^{oge}\) in the model. Especially, the contributions from the \(V^{oge}\) and \(V^\pi\) are quite large, which is due to the rather compact size of the \(T_{bb}^-\) system.

The main component of the \(T_{bb}^-\) is the \([\bar{B} \bar{B}^*]_\cdots\). The two subclusters \(B (B^*)\) and \(B^*\) become obscure and overlap with each other severely because the sizes of the subclusters \(\langle r^2\rangle^{1/2}\) and \(\langle R^2\rangle^{1/2}\) are bigger than the relative distance \(\langle \rho^2 \rangle^{1/2}\) between the two subclusters, see Fig. 1. The large \(b\) quark mass allows the two subclusters to get as close as possible. The \(T_{bb}^-\) state with \(01^+\) looks like a compact tetraquark state if there exists the meson exchanges interaction. If so, the \(T_{bb}^-\) state may not be a pure meson-meson molecule state. Instead, it may be a mixture of the meson-meson molecule and other hidden color states. The specific ratio between two components needs further study. Such a qualitative feature is supported by the lattice QCD computations \([64, 67]\), in which the ratio of the meson-meson molecule component is about \(60\%\).

B. Deuteron-like \(T_{bb}^-\)

If we remove the meson exchange interactions and color screening effect from the model and focus on the case (d) in Table \([11]\) the \(T_{bb}^-\) state with \(01^+\) becomes a shallow bound state with \(E_b = -3.9\) MeV and \(\langle \rho^2 \rangle^{1/2} = 1.65\) fm, where the attraction mainly comes from the residual one-gluon-exchange potential \(V^{oge}\). The two subclusters are separated too far away to overlap each other because the sum of the sizes of the subclusters \(\langle r^2 \rangle^{1/2}\) and \(\langle R^2 \rangle^{1/2}\) is less than their relative distance \(\langle \rho^2 \rangle^{1/2}\). Quarks are only allowed to move in the isolated subclusters. Therefore, the \(T_{bb}^-\) state with \(01^+\) looks like a loosely bound deuteron-like molecular state in the case (d), see Fig. 1.

C. Hydrogen molecule-like \(T_{bb}^-\)

If we consider the color screening effect in the case (c) in Table \(\text{III}\), the \(T_{bb}^-\) state with \(01^+\) forms a bound state with \(E_b = -10\) MeV and \(\langle \rho^2 \rangle^{1/2} = 1.04\) fm, where the attraction mainly arises from the decreasing of the kinetic energy. This novel mechanism is completely different from those in the other three cases, where the \(V^{oge}\) and (or) the meson exchange interactions provide a strong attraction while the kinetic energy prevents the two subclusters to form a bound state.

The two subclusters overlap with each other extremely in the cases (a) and (b) while they do not overlap at all in the case (d). Now in the case (c), the two subclusters \(B (B^*)\) and \(B^*\) moderately overlap with each other, see Fig. 1. Such an appropriate spatial overlapping greatly enlarges the phase space of the light quarks \(\bar{q}_2\) and \(\bar{q}_4\) and allows them to roam into the opposite subcluster freely, which helps to lower the kinetic energy of the \(T_{bb}^-\) system. This is the realization of the uncertainty principle.

The delocalization of the light quarks in the state \(T_{bb}^-\) is extremely similar to the valence bond in the hydrogen molecule, where the electron pair is shared by two protons. Therefore, the \(T_{bb}^-\) state with \(01^+\) is very similar to the hydrogen molecule state, which is formed by the delocalization of the light quarks with the color screening effect in the case (c) in the present model. The idea of the QCD valence bond was proposed and investigated in Ref. \([8]\) in 2013 and discussed extensively in the review \([68]\). Recently, Maiani et al discussed the hydrogen molecule-like \(T_{bb}^-\) state when the \([bb]\) pair is in color \(6\) \([69]\). Richard et al also studied the hydrogen molecule-like doubly heavy tetraquark states \([70]\).

D. Helium-like QCD atom in the limit of a large \(m_Q\)

In order to reveal the dependence of the three configurations on the heavy quark mass, we increase the bottom quark mass from \(m_b\) to \(m_Q\) with the mass ratio \(m_b/m_Q\) and calculate the binding energy \(E_b\) and the average distances. We present numerical results in Table \([IV]\). One can see that the binding energy \(E_b\) in the three configurations is very sensitive to the mass ratio. The deeply bound state appears in the limit of a large \(m_Q\). The large heavy quark mass mass permits them to get as close as possible (see \(\langle r_{QQ}^2 \rangle^{1/2}\) in Table \([IV]\)), therefore their attractive Coulomb interaction becomes dominant. The binding energies of the B (deuteron-like) and C (hydrogen-like) configurations are close to each other. Their absolute values are smaller than that of the A (compact) configuration because of the extra attraction from the meson
exchange interactions.

TABLE IV: Variation of the configurations with the mass ratio $m_Q/m_b$. $(r_{1Q}^2)^{1/2}$ and $(r_{2d}^2)^{1/2}$ are the average distance between $Q$ and $Q$ and $\bar{u}$ and $\bar{d}$, respectively. Others have their original meanings.

| Conf. | $m_Q/m_b$ | $E_b$ | $(r_{1Q}^2)^{1/2}$ | $(r_{2d}^2)^{1/2}$ | $(\rho^2)^{1/2}$ | $(r^2)^{1/2}$ | $\langle R^2 \rangle^{1/2}$ |
|-------|-----------|-------|--------------------|------------------|------------------|------------|-----------------|
| 1     | 1         | −28.6 | 0.62               | 1.13             | 0.59             | 0.79       | 0.79            |
| 2     | −93.5     | 0.26  | 0.97               | 0.26             | 0.81             | 0.81       |
| A     | 3         | −140.6| 0.19               | 0.95             | 0.19             | 0.81       | 0.81            |
| 4     | −190.0    | 0.16  | 0.94               | 0.16             | 0.81             | 0.81       |
| 5     | −226.0    | 0.14  | 0.93               | 0.14             | 0.81             | 0.81       |
| 1     | −3.9      | 1.73  | 1.95               | 1.65             | 0.73             | 0.73       |
| 2     | −19.2     | 0.65  | 1.19               | 0.62             | 0.80             | 0.80       |
| B     | 3         | −58.1 | 0.22               | 1.06             | 0.22             | 0.91       | 0.91            |
| 4     | −97.1     | 0.17  | 1.06               | 0.17             | 0.91             | 0.91       |
| 5     | −131.4    | 0.15  | 1.06               | 0.15             | 0.91             | 0.91       |
| 1     | −10.0     | 1.10  | 1.48               | 1.04             | 0.72             | 0.72       |
| 2     | −25.9     | 0.79  | 1.29               | 0.77             | 0.78             | 0.78       |
| C     | 3         | −37.7 | 0.43               | 1.10             | 0.42             | 0.82       | 0.82            |
| 4     | −75.6     | 0.17  | 1.02               | 0.17             | 0.88             | 0.88       |
| 5     | −110.3    | 0.15  | 1.02               | 0.15             | 0.88             | 0.88       |

If we increase the ratio $m_Q/m_b$, the sizes of the subclusters are convergent in each configuration, see $(r^2)^{1/2}$ and $\langle R^2 \rangle^{1/2}$ in Table IV. However, the distance between two heavy quarks $(r_{1Q}^2)^{1/2}$ decreases gradually till they shrink into a tiny and compact core eventually. The two subclusters overlap completely. The three configurations will degenerate into a single one. Its size can be approximately described by either the $(r^2)^{1/2}$ or $\langle R^2 \rangle^{1/2}$. The QCD-core contributes to the vast majority of the binding energy of the doubly heavy tetraquark states. The light quarks $\bar{u}$ and $\bar{d}$ move around the QQ-core. Their relative distance $\langle r_{2d}^2 \rangle^{1/2}$ is about 1 fm. In summary, the doubly heavy tetraquark states look like a helium-like QCD-atom in the limit of a large heavy quark mass.

VI. OTHER PARTNER STATES OF THE $T_{bc}^+$

A. Isospin antisymmetric states $T_{bc}^0$

The state $T_{bc}^0$ with $00^+$ can form a shallow bound state with the $E_b$ of several or a dozen MeV in the four cases, in which the dominant component is the channel $BD$. Our conclusion is very close to that of other model calculations [27, 71]. In the cases (a) and (b), the overlapping between two subclusters is very obvious so that the state looks like a compact state because of the strong attraction coming from the $V^{\sigma\gamma}$ and $V^\gamma$. Turning off the meson exchange interactions, the $T_{bc}^0$ state with $00^+$ becomes a loosely bound state in the cases (c) and (d) while a hydrogen molecule-like state does not appear.

The nonidentity of the $b$ and $c$ quark in the $T_{bc}^0$ state with $01^+$ enlarges the Hilbert space comparing with the $T_{bb}^0$ case with $01^+$. Now we have to consider three coupling channels.

From Table III the $01^+$ state has a binding energy of several MeV in the four cases, which is slightly lower than the lattice QCD results in the range of 20-40 MeV below the $B^*D$ threshold [19]. The dominant component of the state is the $B^*D$ channel, which is supported by other model predictions [27, 71]. The $T_{bc}^0$ state with $01^+$ is a shallower bound state than the $T_{bb}^0$ with $01^+$ because of the lighter charm quark mass and thus larger kinetic energy. Moreover, the overlapping between the two subclusters, binding energy and the delocalization effect of the light quarks become weaker. However, the physical picture such as the emergence of the compact state, hydrogen molecule-like state or deuteron-like state in four cases in the $T_{bc}^0$ system resembles that of the $T_{bb}^0$ state with $01^+$.

The mass of the $T_{bc}^0$ state with $02^+$ is about 12 MeV lower than the $B^*D^*$ threshold due to its small kinetic energy $E_b$ in the model, see the cases (a) and (c) in Table III. This state can form a hydrogen-like state similar to the $T_{bb}^0$ with $01^+$ due to the delocalization of the light quarks induced by the color screening effect in the confinement. In the cases (b) and (d), the state has a binding energy of 3.6 MeV relative to the $B^*D^*$ threshold, which is a deuteron-like state because the two subclusters are separated well apart. The $T_{bc}^0$ state with $02^+$ is not stable although it is below the $B^*D^*$ threshold because it can decay into the modes $BD$, $BD\pi$, $B^*D\gamma$, $BD^*\gamma$, $BD\gamma\gamma$ etc. However, the state should be very narrow.

B. V-spin antisymmetric states $T_{bbs}^+$ and $T_{bcs}^0$

The corresponding SU(2) groups of the I-spin, and the so-called V-spin and U-spin are three subgroups of the flavor SU(3) group. Therefore, the V-spin antisymmetric $T_{bbs}^+$ with $1/2^+$ and the state $T_{bb}^-$ with $01^+$ should share the same symmetry in their wave functions so that their behaviors should be analogous from the perspective of quark models. Similar arguments hold for the V-spin antisymmetric state $T_{bcs}^0$ and the state $T_{bc}^0$ with $I = 0$.

From Table III the V-spin antisymmetric state $T_{bbs}^+$ with $1/2^+$ is a shallow bound state with a binding energy about 10 MeV relative to the threshold $BB^*$, where the attraction mainly comes from the $V^\sigma$ in the cases (a) and (b). Our results agree with those in Ref. [27] but are less than the latest lattice QCD results about 80 MeV [72]. Similar to the state $T_{bb}^-$ with $01^+$, the state $T_{bbs}^+$ with $1/2^+$ can also form a compact state in the cases (a) and (b), which should therefore be a compound of color singlet and hidden color states. The lattice result indicated that the meson-meson percentage is about 84% while the hidden color percentage is about 16% [72]. In the case
(c), the state $T_{bbs}^0$ is a hydrogen molecule-like bound state because of the delocalization of the light quark $\bar{u}$ and $\bar{s}$ induced by the color screen effect. In the case (d), the state $T_{bbs}^0$ forms a deuteron-like state because of removing the meson exchange interaction and color screening effect from the model.

Both of the V-spin antisymmetric states $T_{bc}^0$ with $\frac{1}{2} 0^+$ and $\frac{1}{2} 1^+$ appear a bound state in the model, which is qualitatively consistent with the conclusions in Refs. [27, 49, 73]. In the cases (a) and (b), the two states are shallow bound states with a binding energy of several MeV because of the $V^0$. The two subclusters have a slight or even no overlapping. In the cases (c) and (d), the two states are very loosely bound without the meson exchange interactions. The two states are deuteron-like states because the subclusters are completely separated from each other. The hydrogen molecule-like configuration appearing in the state $T_{bbs}^0$ with $0^+$ vanishes in the $T_{bc}^0$ states with $\frac{1}{2} 0^+$ and $\frac{1}{2} 1^+$.

The state $T_{bc}^0$ with $\frac{1}{2} 2^+$ is lower than the corresponding threshold $B_sD_s^+$ in the model. Similar to the state $T_{bc}^0$ with $0^+$, the state $T_{bc}^0$ with $\frac{1}{2} 2^+$ is also a hydrogen molecule-like bound state because of the delocalization of light quarks in the cases (a) and (c). Removing the color screen effect from the model, the state $T_{bc}^0$ with $\frac{1}{2} 2^+$ becomes a deuteron-like bound state in the cases (b) and (d). The state is not stable and can decay into $B_sD, B_sD\pi, B_sD\gamma, B_sD\gamma, B_sD\gamma$.

C. Other unstable states

All of the isospin symmetric states $T_{cc}^+, T_{bb}^-, T_{bc}^0, T_{ccss}^+, T_{bbs}^0$ and $T_{bc}^{0}$ cannot form bound states because the interactions can not provide enough attraction in the model. Other model studies on the states $T_{bc}^{0}$ and $T_{ccss}^+$ also suggest that the isospin symmetric states are unbound and unstable [27, 44]. The lattice QCD investigations on the isospin symmetric states $T_{cc}^+, T_{bb}^-, T_{bc}^0, T_{ccss}^+, T_{bbs}^0$ and $T_{bc}^{0}$ indicated that no clear signal of any level below their respective thresholds can be found [69]. Similarly, the V-spin symmetric states $T_{ccss}^{+}, T_{bbs}^-$ and $T_{bc}^0$ can not form stable bound states in the model. The V-spin antisymmetric state $T_{cc}^+$ with $\frac{1}{2} 1^+$, the strange partner of the state $T_{cc}^+$ with $0^+$, could be a stable bound state in some theoretical frameworks [27, 47]. However, the state is not stable in the present model. The situation may change if the mixing of $S-D$ wave is taken into account in the model, which is left for the future work.

VII. MAGNETIC MOMENTS AND AXIAL CHARGES

The magnetic moments of hadrons encode useful information about the distributions of the charge and magnetization inside the hadrons, which help us to understand their geometric configurations. Ignoring the contributions from the quark orbital angular momentum, the operator for the magnetic moment of the doubly heavy tetraquark system is given simply by

$$\hat{\mu}_m = \sum_{i=1}^{4} \frac{\hat{Q}_i}{2m_i}\sigma_i^z,$$

where $\hat{Q}_i$ is the electric charge operator of the $i$-th quark and the $\sigma_i^z$ is the $z$-component of Pauli matrix. We can obtain the magnetic moments of the doubly heavy tetraquark states below their corresponding threshold by directly calculating the matrix element

$$\mu_m = \langle \Phi_{IJ} | \hat{\mu}_m | \Phi_{IJ} \rangle,$$

where $\Phi_{IJ}$ is the eigenvector of those states.

From Table [V] the spin and magnetic momentums of the states $T_{bc}^0$ with $0^+$ and $T_{bc}^0$ with $\frac{1}{2} 0^+$ vanish. The magnetic momentums of the other states depend on their spatial configurations except for the two high spin states. The magnetic momentum of the $T_{cc}^+$ state with $01^+$ is about 0.18 $\mu_N$ and 0.13 $\mu_N$ in the cases (a) and (b), respectively. The magnetic momentum of the $T_{cc}^+$ state with $01^+$ was also studied using the light-cone QCD sum rule formalism [70]. Its value was roughly $0.66 \mu_N$ and 0.43 $\mu_N$ for the compact diquark-antidiquark and molecule pictures, respectively in Ref. [70].

The axial charge $g_A$ is an important quantity for the understanding of both the electroweak and strong interactions. The nonrelativistic leading order axial charge operator for a point-like Dirac constituent quark is given by the Gamov-Teller operator $\hat{\sigma}^z \hat{\tau}^x$ [77], where $\tau^x$ is the isospin operator. Then the axial charge operator for the doubly heavy tetraquark states is given by

$$\hat{g}_A = \sum_{i=1}^{4} \hat{\tau}_i^x \hat{\sigma}_i^z.$$

In this way, we can achieve the axial charge $g_A$ of the states by directly calculating the matrix element

$$g_A = \langle \Phi_{IJ} | \hat{g}_A | \Phi_{IJ} \rangle,$$

which are presented in Table [V]. The axial charges of the states $T_{cc}^+, T_{bb}$ and $T_{bc}^0$ are zero because their isospin magnetic components are zero while that of $T_{bc}^0$ with $\frac{1}{2} 0^+$ is zero because its spin magnetic component is zero. The axial charges of the other states also depend on their spatial configurations but do not change dramatically in the four cases.

VIII. SUMMARY

In the present work, we have performed a systematical investigation of the doubly heavy tetraquark states with the molecule configuration within the framework of the
TABLE V: Magnetic moment $\mu_m$ unit in $\mu_N$ and axial charge $g_A$ unit in $\mu_N$ in the four cases.

| State | $T_{bc}^0$ | $T_{bc}^+$ | $T_{bc}^0$ | $T_{bs}^+$ | $T_{bs}^0$ | $T_{bs}^+$ | $T_{bs}^0$ |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $IJ^P$ | 01$^+$ | 01$^+$ | 00$^+$ | 01$^+$ | 02$^+$ | $\frac{1}{2}1^+$ | $\frac{1}{2}0^+$ | $\frac{1}{2}1^+$ | $\frac{1}{2}2^+$ |
| $\mu_m^a$ | 0.18 | 0.64 | 0.00 | 0.66 | 0.79 | 1.37 | 0.00 | 0.98 | 1.29 |
| $\mu_m^b$ | 0.13 | 0.49 | 0.00 | 0.59 | 0.79 | 1.29 | 0.00 | 0.94 | 1.29 |
| $\mu_m^c$ | - | 0.97 | 0.00 | 0.72 | 0.79 | 1.40 | 0.00 | 0.95 | 1.29 |
| $\mu_m^d$ | - | 0.98 | 0.00 | 0.67 | 0.79 | 1.37 | 0.00 | 1.13 | 1.29 |

$g_A^a$ 0.00 0.00 0.00 0.00 0.00 0.81 0.00 1.13 2.18
$g_A^b$ 0.00 0.00 0.00 0.00 0.00 0.74 0.00 1.09 2.13
$g_A^c$ 0.00 0.00 0.00 0.00 0.00 0.83 0.00 1.09 2.15
$g_A^d$ 0.00 0.00 0.00 0.00 0.00 0.79 0.00 1.03 2.08

There is a broad theoretical consensus that the tendency to form doubly heavy tetraquark bound states is proportional to the mass ratio $\frac{m_Q}{m_q}$. In the limit of large masses of the heavy quarks the corresponding ground state should be deeply bound. Such a qualitative picture is strengthened by our numerical results. The existence of the shallow bound state $T_{cc}^+$ implies that there should exist many stable doubly heavy tetraquark states.

Our investigations indicate that the I-spin antisymmetric states $T_{bb}^+$ with 01$^+$, $T_{bb}^0$ with 00$^+$ and 01$^+$, the V-spin antisymmetric states $T_{bs}^+$ with $\frac{1}{2}1^+$, $T_{bs}^0$ with $\frac{1}{2}0^+$ and $\frac{1}{2}1^+$ can form a compact, hydrogen molecule-like, or deuteron-like bound state depending on different binding dynamics. The compact spatial size of the $T_{bb}^+$ may require the introduction of the hidden-color configuration from the very beginning, which is the topic of our future work.

The high-spin states $T_{bc}^0$ with 02$^+$ and $T_{bs}^0$ with $\frac{1}{2}2^+$ can decay into D-wave $BD$ and $B_sD$ through the strong interactions although they are below the thresholds $B^*D^*$ and $B_s^*D_s^*$, respectively. The I-spin or V-spin symmetric states $T_{cc}^+$, $T_{bc}^0$, $T_{bc}^+$, $T_{bs}^0$, $T_{bs}^+$, and $T_{bs}^+$, are unbound in the model prediction. The state $T_{cc}^+$ is also not bound in the model no matter its V-spin is symmetric or antisymmetric, which may be due to the omission of the S-D wave mixing in the present work.

The discovery of the $T_{cc}^+$ state opened a new window for hadron physics. More theoretical and experimental efforts are called for in order to understand its underlying structure and nonperturbative QCD dynamics in this region. We sincerely hope that some of the doubly heavy tetraquark candidates may be searched for at LHCb and BelleII in the near future.

Acknowledgments

One of the authors C. Deng thanks Prof. J.L. Ping for helpful discussions. This research is partly supported by the National Science Foundation of China under Contracts No. 11975033 and No. 12070131001, Chongqing Natural Science Foundation under Project No. cstc2019jcyj-msxmX0409 and Fundamental Research Funds for the Central Universities under Contracts No. SWU118111.

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