Spectrum Sensing with Multiple Primary Users over Fading Channels

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Abstract—We investigate the impact of multiple primary users (PUs) and fading on the spectrum sensing of a classical energy detector (ED). Specifically, we present novel closed-form expressions for the false-alarm and detection probabilities in a multiple PUs environment, assuming Nakagami-m fading and complex Gaussian PUs transmitted signals. The results reveal the importance of taking into consideration the wireless environment, when evaluating the ED spectrum sensing performance and selecting the ED threshold.

Index Terms—Cognitive radio, Energy detector, Fading channels, Spectrum sensing.

I. INTRODUCTION

The rapid growth of wireless communications and the foreseen spectrum occupancy problems, due to the exponentially increasing consumer demands on mobile traffic and data, inspired the evolution of the concept of cognitive radio (CR). One fundamental task in CR that allows the exploitation of the under-utilized spectrum, is spectrum sensing. As a result, great amount of effort has been put to derive optimal, sub-optimal and ad-hoc solutions to the spectrum sensing problem and investigate their performance [1]–[6].

Scanning the open literature, most of the related works have neglected the impact of multiple primary users (PUs) and fading on the spectrum sensing performance of the CR’s energy detector (ED). However, in several widely used wireless communication standards, such as long term evolution advanced (LTE-A), WiFi and WiMAX, where code-division multiple-access (CDMA) is used, users simultaneously operate in the same frequency band. These applications motivated a general investigation of the effect of PU traffic on the sensing performance, when multiple PUs are present. To the best of the authors’ knowledge, there is only one published work in the open literature [7], where the effect of multiple PUs on spectrum-sensing performance was investigated, considering only the impact of additive white Gaussian noise (AWGN) channels. Moreover, in [8], the authors derived the sensing time and the transmission time that jointly maximize the sensing efficiency and the sensing accuracy in a multiple mobile PU network. However, in [8], the spectrum sensing method and the effect of fading channels was neglected.

In this letter, we present an analytical framework to evaluate and quantify the effects of multiple PUs and fading on the spectrum sensing performance of a classical ED. In particular, we present novel closed-form expressions for the false-alarm and detection probabilities in a multiple PUs environment, assuming Nakagami-m channels and complex Gaussian transmitted signals.

Notations: Unless otherwise stated, \( \Re \{x\} \) and \( \Im \{x\} \) represent the real and imaginary part of \( x \), operators \( E[\cdot] \) and \( |\cdot| \) denote the statistical expectation and the absolute value, respectively, while the operator \( \exp (\cdot) \) denotes the exponential function. The operator \( \text{card} \{A\} \) returns the cardinality of the set \( A \). The lower [9, Eq. (8.350/1)] and upper incomplete Gamma functions [9, Eq. (8.350/2)] are represented by \( \gamma (\cdot, \cdot) \) and \( \Gamma (\cdot, \cdot) \), respectively, while the Gamma function [9, Eq. (8.310)] is denoted by \( \Gamma (\cdot) \). Finally, \( \gamma (\cdot, \cdot, \cdot, \cdot) \) is the extended incomplete Gamma function defined in [10, Eq. (6.2)].

II. SYSTEM AND SIGNAL MODEL

We consider a multiple PUs/secondary user (SU) environment, where \( M \) static PUs operate in the same frequency band, which is sensed by a single CR device. The two possible states, i.e., busy or idle, of the \( i \)-th PU are denoted with the parameters \( \theta_i \in \{0, 1\} \), where \( i = 1, 2, \ldots, M \). Suppose that the \( n \)-th sample of the transmitted signal of the \( i \)-th PU, \( s_i(n) \), is conveyed over a flat-fading wireless channel, with channel gain, \( h_i(n) \). Hence, at the SU detector the \( n \)-th sample of the baseband equivalent received signal can be expressed as

\[
y(n) = \sum_{i=1}^{M} \theta_i h_i(n) d_i^{-\xi_i/2} s_i(n) + w(n),
\]

where \( d_i \) and \( \xi_i \) stand for the distance between the \( i \)-th PU and the SU, and the corresponding link path-loss exponent, respectively, while \( w(n) \) represents the AWGN. We assume that \( s_i \) and \( w \) are zero-mean circular symmetric complex white Gaussian processes with variances \( \sigma_s^2 \) and \( \sigma_w^2 \). Furthermore, \( h_i \) is a zero mean complex random variable (RV) with variance \( \sigma_{h_i}^2 \) and \( |h_i| \) follows Nakagami-\( m \) distribution. Without loss of generality, it is assumed that SU is located in the first \((i = 1)\) PU cell.

Next, \( \Theta = [\theta_1, \theta_2, \ldots, \theta_M] \) represents the set of \( M \) PUs (busy and idle) located at distances \( \bar{d} = [d_1, d_2, \ldots, d_M] \) from the SU, while \( \hat{\Theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_L] \subseteq \Theta \) denotes the set of the \( L \leq M \) active PU located at distances \( \bar{\hat{d}} = [\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_L] \). Additionally, \( \Theta_0 = [0, 0, \ldots, 0] \) stands for the all idle PU occupancy set. \( \Theta_1 = [1, \theta_2, \ldots, \theta_M] \), with \( \theta_j \in \{0, 1\}, j = 2, \ldots, M \), represents the PU occupancy set, in which the first
The energy test statistic, $T$, is compared against a threshold $T > \gamma$ or idle, otherwise.

For a given channel realization set $H = \{h_1, h_2, \cdots, h_M\}$ and PUs occupation set $\Theta = \{\theta_1, \theta_2, \cdots, \theta_M\}$, the real and imaginary parts of the received signals are uncorrelated, i.e., $E[R \{y\} \Im \{y\}] = 0$, with variances

$$E[R \{y\}^2] = E[\Im \{y\}^2] = \sigma^2 = \frac{\sum_{i=1}^{M} \theta_i |h_i|^2 d_i \xi_i \sigma^2_i}{2} + \frac{\sigma^2_w}{2}.$$  

(3)

The received energy test statistic follows chi-square distribution with $2N_s$ degrees of freedom and cumulative distribution function (CDF) given by

$$F_T(x|H, \Theta) = \frac{\Gamma(N_s, \frac{N_s x}{2\sigma^2})}{\Gamma(N_s)}.$$  

(4)

Furthermore, since $N_s$ is an integer, (4) can be re-written as (9) [Eq. (8.352/2)]

$$F_T(x|H, \Theta) = 1 - \sum_{n=0}^{N_s-1} \frac{1}{n!} \left( \frac{N_s x}{\sigma^2} \right)^n \exp \left( -\frac{N_s x}{\sigma^2} \right).$$  

(5)

**Theorem 1.** The CDF of the energy test statistics for a given channel set, $\Theta \subseteq \Theta_r$, with $L \in [1, M]$ active PUs, can be evaluated by (5), given at the top of the next page, where $a = \{\tilde{m}_1, \tilde{m}_2, \cdots, \tilde{m}_L\}$, $b = \{\frac{d_1 \xi_1 \sigma^2_1}{2m_1}, \frac{d_2 \xi_2 \sigma^2_2}{2m_2}, \cdots, \frac{d_L \xi_L \sigma^2_L}{2m_L}\}$, and $c = \frac{\sigma^2_w}{2}$.

Furthermore, $d_i$ and $\xi_i$ represent the distance and the corresponding path-loss exponent between the $i$-th active PU and the ED, whereas $\sigma^2_i$ and $\sigma^2_L$ stand for the variances of the $i$-th active PU’s transmitted signal and $i$-th fading channel. The shape factor of the fading channel between the $i$-th active PU and the CR device is denoted as $\tilde{m}_i$. Moreover, note that in (6), $\Xi(i, k)$ is defined in (7).

**Proof:** Since $|h_i|$ is a zero mean Nakagami-$m$ distributed RV, the variance of the received signal, given by (3), is a sum of squared Nakagami-$m$ RV with probability density function (PDF) given by (11)

$$f_{\sigma^2}(y) = \sum_{i=1}^{L} \sum_{k=1}^{a_i} \Xi(i, k) \frac{(y - c)^k - 1}{b_i^k (k - 1)!} \exp \left( -\frac{y - c}{b_i} \right).$$  

(7)

with $y \in [c, \infty)$. Hence, the unconditional CDF of the energy test statistic, $T$, can be expressed as

$$F_T(x|\Theta) = \int_c^\infty f_{\sigma^2}(y) dy - \sum_{n=0}^{N_s-1} \frac{1}{n!} \left( \frac{N_s x}{2} \right)^n \times \int_c^\infty y^{-n} \exp \left( -\frac{N_s x}{2y} \right) f_{\sigma^2}(y) dy.$$  

(8)

Since $y \in [c, \infty)$, $\int_c^\infty f_{\sigma^2}(y) dy = 1$. Additionally, by substituting (7) into (8) and after some mathematical manipulations, (5) yields

$$F_T(x|\Theta) = 1 - \sum_{n=0}^{N_s-1} \sum_{i=1}^{L} \sum_{k=1}^{a_i} \Xi(i, k) \frac{\Xi(i, k)}{b_i^k (k - 1)!} \left( \frac{N_s x}{2} \right)^n \times \int_c^\infty y^{-n} \exp \left( -\frac{N_s x}{2y} \right) dy.$$  

(9)

Finally, by setting $z = \frac{y}{b}$ into (10) and using [10] Eq. (6.2), (10) yields (6). This concludes the proof. 

**Lemma 1.** The CDF of the energy test statistic assuming all the PUs are idle can be evaluated by

$$F_T(x|\Theta_0) = 1 - \sum_{n=0}^{N_s-1} \frac{1}{n!} \left( \frac{N_s x}{\sigma^2_w} \right)^n \exp \left( -\frac{N_s x}{\sigma^2_w} \right).$$  

(11)

**Proof:** If $\Theta = \Theta_0$, according to (3), $\sigma^2 = \frac{\sigma^2_w}{2}$, which is independent of the channel realization set $H$. Substituting this value into (5), we get (11). This concludes the proof.

Based on the above analysis the detection and false-alarm probabilities can be respectively obtained by

$$P_d(\gamma) = \sum_{i=1}^{\text{card} (\Theta_1)} P_r (\Theta_1) \left( 1 - F_T (\gamma | \Theta_1) \right)$$  

(12)

and

$$P_f(\gamma) = \sum_{i=1}^{\text{card} (\Theta_0, 1)} P_r (\Theta_{0, 1}) \left( 1 - F_T (\gamma | \Theta_{0, 1}) \right) + P_r (\Theta_0) \sum_{n=0}^{N_s-1} \frac{1}{n!} \left( \frac{N_s x}{\sigma^2_w} \right)^n \exp \left( -\frac{N_s x}{\sigma^2_w} \right).$$  

(13)
In the special case of protection and false-alarm probabilities in the single PU scenario next page, the PU is active can be obtained by setting which all the PUs-SU links are Rayleigh distributed, the CDF for the power of the neighbor PUs’ signals, and channels, as well as the probabilities of active channel, but also the variances of the neighbor PUs’ characteristics of the neighbor PUs-SU links. Consequently, in order to select the energy detection threshold and the number of samples that will be used to achieve a detection and/or false alarm probabilities requirements, ED should take into consideration not only the variances of the PU signal, noise and channel, but also the variances of the neighbor PUs’ signals, and channels, as well as the probabilities of active PU existence. Note that in practice, the CR device has certain noise measurements and has only an estimate for the noise variance. However, in our analysis, we assume that the ED has perfect knowledge on the noise variance, which is obtained from calibration measurements. This is a typical assumption (12) and references therein) in order to be able to quantify the performance degradation due to the effects of multiple PUs, independently of the classical noise uncertainty problem. Next, we study two important special cases.

Special Case 1 (Rayleigh fading): In the special case in which all the PUs-SU links are Rayleigh distributed, the CDF of the energy test statistics for the given set Θ can be obtained by setting \( a = \{1, 1, \cdots, 1\} \) as

\[
F_T(x|\Theta) = 1 - \sum_{n=0}^{N_s-1} \sum_{i=1}^{L} \sum_{k=1}^{m} \frac{(-c)^{k-1-j} \Xi(i,k)}{n!b_i^m} \left( \frac{N_s x}{2} \right)^n \exp \left( \frac{c}{b_i} \right) \Gamma \left( -n + 1, \frac{N_s x}{b_i}, 1 \right).
\]

Special Case 2 (single PU scenario): In the special case of a single PU, the CDF of the energy test statistic assuming that the PU is active can be obtained by setting \( L = 1 \), \( a = \{m\} \) and \( b = \{\frac{d \tilde{\sigma}_w^2}{m}\} \) into \( \Theta \), as \( \Theta = 1 \), given at the top of the next page.

Furthermore, the CDF of the energy test statistics assuming that the PU is idle can be derived by \( \Theta = 1 \). Therefore, the detection and false-alarm probabilities in the single PU scenario can be respectively obtained as

\[
\begin{align*}
P_d(\gamma) &= P_T(\gamma > \gamma | \theta_k = 1) \\
&= \frac{2m^n}{(\tilde{\sigma}_w^2 d \tilde{\sigma}_s^2)^m \Gamma(m)} \exp \left( \frac{m \tilde{\sigma}_w^2}{\tilde{\sigma}_s^2 d \tilde{\sigma}_s^2} \right) \\
&\times \sum_{k=0}^{m-1} \sum_{n=0}^{N_s-1} \frac{(m-1) \gamma^n}{n! \left(-\tilde{\sigma}_w^2\right)^m \gamma^{-k} \left(N_s \gamma\right)^n} \\
&\times \left( \frac{\tilde{\sigma}_w^2 d \tilde{\sigma}_s^2}{2m} \right)^{-k+n+1} \Gamma \left( k - n + 1, \frac{m \gamma N_s}{\tilde{\sigma}_w^2 d \tilde{\sigma}_s^2}, \frac{m N_s \gamma}{\tilde{\sigma}_w^2 d \tilde{\sigma}_s^2} \right).
\end{align*}
\]
where 

\[ F_T (x | \theta) = 1 - \frac{2m^m}{(\sigma_h^2 d^{-\xi} \sigma_s^2)^m} \exp \left( \frac{m \sigma_w^2}{\sigma_h^2 d^{-\xi} \sigma_s^2} \right) \times \sum_{k=0}^{m-1} \sum_{n=0}^{N_s-1} \binom{m-1}{k} \frac{(-\sigma_w^2)^n}{n!} (N_s x)^n \frac{\left( \frac{\sigma_h^2 d^{-\xi} \sigma_s^2}{2m} \right)^{k-n+1}}{\Gamma(k-n+1, \frac{m \sigma_w^2}{\sigma_h^2 d^{-\xi} \sigma_s^2} \frac{m N_s x}{\sigma_h^2 d^{-\xi} \sigma_s^2} + 1)} \]  

(15)

\[
\text{Fig. 1: ROCs for systems with a single PU and different values of } m \text{ and SNR.}
\]

\[
\text{Fig. 2: ROCs for systems with 6 PUs and different values of } p.
\]

from the PU, which is located in the same cell with the SU, is equal to 0 dB, while the interference-to-noise ratios (INRs) from the other 5 PUs are 0 dB, −1 dB, −2 dB, −3 dB and −5 dB. Note that we assume the same \( p \) for the 5 interfering PUs. It is observed that as \( p \) increases, the probability of interference of an neighbor PU increases; consequently, the spectrum sensing capabilities of the ED are constrained. For example, for a fixed \( P_{fa} = 0.1 \), the detection probability is decreased about 41.8% for \( p = 0.5 \) in comparison with the case in which \( p = 0 \). Notice that the \( p = 0 \) case corresponds to the single PU scenario.

In Fig. 3 ROCs are illustrated for different number of PUs, \( M \), considering that \( m_i = 1 \), for \( i = 1, \ldots, 6 \), and the probability of existence of the \( j \)-th PU, \( j \in \{2, \ldots, M\} \), is equal to 0.5, i.e., \( p = 0.5 \). We observe that as the number of PUs increases, the interference from neighbor PUs increases; hence the false-alarm probability increases and the spectrum sensing capabilities of the ED are constrained.

\[
\text{Fig. 3: ROCs for systems with } N \text{ PUs with } p = 0.5.
\]

V. CONCLUSIONS

In this letter, we studied the impact of multiple PUs in the spectrum sensing performance of a classical ED, assuming Nakagami-\( m \) channels and complex Gaussian PUs’ transmitted signals. Our results revealed the importance of taking into consideration the fading statistics, especially in the medium to high SNR regime. Furthermore, we observed that the spectrum sensing performance is constrained as the probability of interference from neighbor PUs increases. Therefore, when selecting the operational energy detection threshold, we should not only take into consideration the PU that is located in the same cell as the SU, but also the wireless environment, i.e., interference, as well as the fading characteristics of the PUs-SU links.

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