Nonextensivity in the Solar Neighborhood

D. B. de Freitas\textsuperscript{1,2} and J. R. De Medeiros\textsuperscript{2} \textsuperscript{(b)}

\textsuperscript{1} Instituto de Educação, Ciência e Tecnologia do Rio Grande do Norte, 59550-000 João Câmara, RN, Brazil
\textsuperscript{2} Departamento de Física, Universidade Federal do Rio Grande do Norte, 59072-970 Natal, RN, Brazil

PACS 97.10.Kc – Stellar rotation
PACS 97.10.Yp – Star counts, distribution, and statistics
PACS 05.90.+m – Other topics in statistical physics, thermodynamics, and nonlinear dynamical systems

Abstract. - In the present study, we analyze the radial velocity distribution as a function of different stellar parameters such as stellar age, mass, rotational velocity and distance to the Sun for a sample of 6781 single low–mass field dwarf stars, located in the solar neighborhood. We show that the radial velocity distributions are best fitted by $q$–Gaussians that arise within the Tsallis nonextensive statistics. The obtained distributions cannot be described by the standard Gaussian that emerges within Boltzmann-Gibbs (B–G) statistical mechanics. The results point to the existence of a hierarchical structure in phase space, in contrast to the uniformly occupied phase space of B–G statistical mechanics, driven by the $q$–Central Limit Theorem, consistent with nonextensive statistical mechanics.

Introduction. – The behavior of the distribution of different stellar physical parameters, in particular rotational and radial velocities, seems to be better explained on the basis of a Tsallis maximum entropy distribution function \cite{1} than by standard Gaussian and Maxwell–Boltzmann distributions or analytical functions (Soares \textit{et al.} \cite{2} and Carvalho \textit{et al.} \cite{3,4,5}). Indeed, the nonextensive generalization of Boltzmann-Gibbs statistics, presented by Tsallis \cite{1}, appears now as a powerful parametrization of the statistical mechanics of out-of-equilibrium states in open systems as is the case of stars, planetary and stellar systems.

The Tsallis statistics, is based on $q$–exponential and $q$–logarithm functions defined by,

$$\exp_q(f) = [1 + (1-q)f]^{1/1-q}, \quad (1)$$

$$\ln_q(f) = \frac{f^{1-q} - 1}{1-q}, \quad (2)$$

from where, associated with the $q$–statistics emerges the entropy $S_q$ \cite{6}

$$S_q = 1 - \sum_i p_i^q \left(\frac{q}{q - 1}\right), \quad (q \in R), \quad (3)$$

\textsuperscript{(a)}E-mail: danielbrito@dfte.ufrn.br
\textsuperscript{(b)}E-mail: renan@dfte.ufrn.br

Fig. 1: The spatial distribution of the present working sample of dwarf stars in the solar neighbourhood. The Sun is centered in point (0,0,0).
where the $q$-Gaussian recovers the usual one at $q = 1$. This new distribution function, the $q$-Gaussian, was applied successfully to different astrophysical problems \[16\] in addition of stellar radial and rotational velocities.

In the present work we study the behavior of the distribution of stellar radial velocity in the solar neighborhood, using the Tsallis maximum entropy distribution. This work brings the analysis, for the first time, to our knowledge, of the behavior of the $q$-Gaussian distribution applied to a stellar parameter as a function of other additional relevant stellar parameters. In fact, we study the behavior of the $q$-Gaussian distribution of radial velocity as a function of stellar age, mass, rotation and distance of the stars to the Sun. This new investigation also offers the possibility of check the validity of the $q$–Central Limit Theorem, recently conjectured by Umarov, Tsallis and Steinberg \[17\] for physical sub-systems, such as those associated to stars and stellar systems.

In section 2 we describe the sample and how we selected stars and physical stellar parameters for the present analyses. Section 3 brings the main results of the present analysis. Finally, conclusions are presented in last section.

Stellar Working Sample and Observational Data. – The present working sample is composed of 6781 single F and G spectral types dwarf stars, with precise radial velocity computed by Nordstrom et al. \[18\]. According these authors the sample is complete to the magnitude limit of 7.8 and should also be volume complete for the F– and G–type dwarfs stars to a distance of $\sim 40$ pc. The referred sample is defined in a volume around the Sun of $\sim 200$ pc, as shown in fig. 1

For our analysis we taken the radial velocities, mass, age and projected rotational velocities from Holmberg et al. \[19\], which gives an improved version of the work by Nordstrom et al. \[18\], with a new calibration for all the relevant stellar parameters. All the selected objects are low-mass stars with effective temperature ranging from 4500 K to 7800 K ($T_{\odot} \approx 5778$ K)\footnote{$\odot \equiv$ Sun} presenting a mass range of 0.65$M_{\odot}$ to 2.0$M_{\odot}$ with error estimation of about 0.05$M_{\odot}$ and metallicity range [Fe/H] between −0.75 and 0.5. Nordstrom et al. \[18\] show that the distribution of their estimated metallicities obeys a Gaussian distribution, with a mean of $−0.14$ and dispersion of 0.19 dex. For futher details on the observational procedure, calibration, and error analysis, the reader is referred to \[18\] and \[19\].

Results and Discussion. – In the following, we analyze the behavior of the distribution of the radial velocity in the context of the Tsallis statistics, taking into account, in particular, the dependence of such a behavior on different stellar parameters, namely age, mass, projected rotational velocity and distance of the stars to the Sun. The generalized $q$-Gaussian distribution function for radial velocity $v_r$ can be written as (e.g., \[3\])

$$
\Phi_q(v_r) = A_q \left[ 1 - (1 - q) \left( \frac{v_r}{\sigma_q} \right)^2 \right]^{\frac{1}{1-q}},
$$

(4)

where $\sigma_q$ is the characteristic width and $q$ is a free parameter that denotes the entropic index. The parameter $q$ is related to the size of the tail in the distribution (e.g., \[16\]). At the limit, when $q \to 1$, the above distribution reproduces the usual Gaussian distribution, given by

$$
\phi(v_r) = A \exp \left( -\frac{v_r^2}{\sigma^2} \right).
$$

(5)

All $q$-Gaussian distributions are two-parameters ($q$–$\sigma_q$) nonlinear functions given by eq. (4). The result of our computation for these two parameters, for well defined stellar age intervals is presented in tables \[1\] and \[2\] and \[3\] which brings also the total number $N_t$ of stars in each age interval, $\sigma_{Gauss}$ of the Gaussian distribution from eq. (4) and $R^2$–parameter obtained by a nonlinear regression method based on the Levenberg-Marquardt algorithm \[20\] and \[21\]. This method was used to compute the $q$-Gaussian with symmetric Tsallis distribution from eq. (4). The obtained values of the parameter $q$, strongly suggest that the distribution of radial velocity for the present sample of field dwarf stars is far from being in agreement with a standard Gaussian, since the values of this parameter are significantly different from 1, independent of the stellar age considered, as well as the considered spectral types. The error bars in fig. 3 correspond to a 0.05 confidence limit, as given in tables \[1\] and \[2\] and \[3\].

To illustrate the results listed in the referred tables, we present in figs. \[2\] and \[3\] a few examples of the obtained fits for the distribution of stellar age and distance to the Sun, once we consider the usual and the $q$-Gaussian. As we can see, the $q$-Gaussian is the more suitable distribution fitting the obtained functions, irrespective of the region of the distributions, in contrast to the standard gaussian that fits very well only the central regions of the observed distributions. Such a behavior is the same, irrespective of the stellar parameter here considered.

For instance, Carvalho et al. \[3\] have found that for stellar clusters older than 1 Gyr exists a positive gradient between the parameter $q$ of radial velocities distributions

| Age (Gyr) | $N_t$ | $\sigma_{Gauss}$ | $q$ | $\sigma_q$ | $R^2$ |
|----------|-------|------------------|---|------------|-------|
| 0–1      | 386   | 32.78            | 1.37±0.37 | 28.07 | 0.97  |
| 1–2      | 1701  | 28.91            | 1.21±0.22 | 25.87 | 0.99  |
| 2–3      | 1391  | 31.24            | 1.26±0.21 | 28.10 | 0.99  |
| 3–4      | 902   | 34.59            | 1.15±0.18 | 33.39 | 0.99  |
| 4–5      | 452   | 32.22            | 1.15±0.23 | 25.99 | 0.96  |
| 5–6      | 402   | 32.04            | 0.75±0.24 | 44.98 | 0.97  |
| 6–7      | 325   | 33.37            | 1.05±0.27 | 38.89 | 0.95  |
| 7–8      | 291   | 33.74            | 1.46±0.39 | 32.76 | 0.97  |
| 8–9      | 228   | 34.74            | 1.17±0.62 | 43.18 | 0.92  |
| 9–10     | 155   | 36.45            | 1.65±0.75 | 35.95 | 0.91  |

Table 1: Best $q$ and $\sigma_q$ values determined using non-linear regression L–M method for all the stars of the present working sample.
Fig. 2: Comparison between the semi-log plot of the observed (dots) and fitted Gaussian (dashed line) and $q$-Gaussian (full line) distribution of radial velocity of F-type stars. Upper and lower panels represent, respectively, examples of the distributions segregated by age and distance to the Sun.

Fig. 3: Comparison between the semi-log plot of the observed (dots) and fitted Gaussian (dashed line) and $q$-Gaussian (full line) distribution of radial velocity of G-type stars. Upper and lower panels represent, respectively, examples of the distributions segregated by age and distance to the Sun.
Table 4: Best $q$ and $\sigma_q$ –values determined using non-linear regression L–M method for the F- and G-Type dwarf stars for other stellar parameters.

| Parameters | F-Type | G-Type |
|------------|--------|--------|
| Masses (M$_\odot$) | $q$ | $\sigma_q$ | $R^2$ | $q$ | $\sigma_q$ | $R^2$ |
| 0.50–1.10 | 1.32±0.31 | 31.31 | 0.98 | 1.43±0.63 | 30.16 | 0.99 |
| 1.10–1.30 | 1.24±0.08 | 27.52 | 1.0 | 1.48±0.61 | 31.32 | 0.91 |
| 1.30–1.50 | 1.27±0.13 | 26.95 | 1.0 | 0.66±0.59 | 39.71 | 0.94 |
| 1.50–2.00 | 1.29±0.33 | 26.15 | 0.97 | 1.23±0.1 | 33.38 | 0.88 |
| average | 1.28±0.21 | – | – | 1.22±0.33 | – | – |

$R_{\odot}$ ($pc$) | $q$ | $\sigma_q$ | $R^2$ | $q$ | $\sigma_q$ | $R^2$ |
|-----------------|--------|--------|--------|--------|--------|--------|
| 0–30 | 1.29±0.67 | 24.44 | 0.89 | 2.0±0.32 | 20.72 | 0.99 |
| 30–50 | 1.29±0.30 | 29.11 | 0.96 | 1.23±0.42 | 34.80 | 0.96 |
| 50–70 | 1.46±0.15 | 24.56 | 0.99 | 1.26±0.33 | 36.42 | 0.97 |
| 70–90 | 1.34±0.17 | 25.92 | 0.99 | 1.26±0.01 | 33.50 | 0.97 |
| 90–110 | 1.01±0.91 | 33.91 | 1.00 | 1.42±0.67 | 29.36 | 0.91 |
| 110–130 | 1.40±0.28 | 24.65 | 0.98 | 1.30±0.71 | 30.59 | 0.93 |
| >130 | 1.33±0.27 | 25.73 | 0.98 | 0.80±2.21 | 36.70 | 0.65 |
| average | 1.30±0.26 | – | – | 1.32±0.67 | – | – |

Table 2: Best $q$ and $\sigma_q$ –values determined using non-linear regression L–M method for the F-Type stars of the present working sample.

| Age(Gyr) | $N_t$ | $\sigma_{Gauss}$ | $q$ | $\sigma_q$ | $R^2$ |
|----------|-------|-----------------|--------|--------|--------|
| 0–1 | 200 | 29.82 | 1.43±0.45 | 24.72 | 0.92 |
| 1–2 | 1573 | 27.96 | 1.30±0.27 | 24.10 | 0.97 |
| 2–3 | 1571 | 30.78 | 1.26±0.21 | 27.72 | 0.88 |
| 3–4 | 560 | 33.75 | 1.12±0.08 | 32.50 | 0.97 |
| 4–5 | 217 | 33.01 | 1.57±0.54 | 25.48 | 0.88 |
| 5–6 | 146 | 43.21 | 0.89±0.67 | 44.82 | 0.78 |
| >6 | 156 | 30.20 | 1.29±0.33 | 40.04 | 0.97 |

Table 3: Best $q$ and $\sigma_q$ –values determined using non-linear regression L–M method for the G-Type stars of the present working sample.

| Age(Gyr) | $N_t$ | $\sigma_{Gauss}$ | $q$ | $\sigma_q$ | $R^2$ |
|----------|-------|-----------------|--------|--------|--------|
| 0–1 | 186 | 34.65 | 1.51±0.66 | 27.97 | 0.93 |
| 1–2 | 126 | 34.56 | 1.56±0.4 | 31.04 | 0.97 |
| 2–3 | 368 | 33.24 | 1.18±0.43 | 31.04 | 0.96 |
| 3–4 | 342 | 34.31 | 1.37±0.03 | 29.46 | 0.94 |
| 4–5 | 235 | 33.32 | 1.43±0.43 | 27.98 | 0.97 |
| 5–6 | 256 | 38.91 | 0.84±0.71 | 41.23 | 0.92 |
| 6–7 | 247 | 46.1 | 1.16±0.47 | 43.35 | 0.93 |
| 7–8 | 240 | 31.34 | 1.49±0.19 | 30.17 | 0.96 |
| 8–9 | 219 | 34.17 | 1.24±0.66 | 40.67 | 0.91 |
| >9 | 146 | 36.97 | 1.78±0.83 | 34.12 | 0.89 |

Conclusions. – We used $q$–Gaussian distributions to investigate the radial velocity of a sample of 6781 single field dwarf stars of spectral types F and G in the solar neighborhood. These low-mass stars have ages ranging between 1 and 10 Gyr and distance to the Sun from a few to tens of parsecs. In addition to the age and distance to the Sun, we studied the behavior of the distribution of the radial velocity as a function of two other relevant parameters, namely stellar mass and projected rotational velocity. The present analysis shows that radial velocity distribution for these stars is far from being a standard Gaussian model. The $q$–Gaussian distribution described in Tsallis nonextensive statistics is clearly the best distribution fitting the obtained functions.

The relationship observed between the $q$–parameter and stellar age, stellar mass, rotational velocity and distance to the Sun indicates that the significant radial velocity deviation of the stellar sample from a standard Gaussian is a result of statistical dependence between the distributions associated with these variables. On the other hand, this
dependence reveals the effects of long-range interactions and the formation of high-energy tails consistent with the $q$-CLT, where the nonextensive character is observed.

***

Research activity of the Stellar Board of the Federal University of Rio Grande do Norte (UFRN) and at the Federal Institute of Rio Grande do Norte (IFRN) are supported by continuous grants from FNDE (PET-Programa de Educação Tutorial) and FAPERN brazilian agency. We also acknowledge continuous financial support from CNPq brazilian.

REFERENCES

[1] Tsallis C., J. Stat. Phys., 52 (1988) 479.

[2] Soares B. B., Carvalho J. C., do Nascimento Jr J. D., and De Medeiros J. R., Physica A, 364 (2006) 413.

[3] Carvalho J. C., Soares B. B., Canto Martins B. L., do Nascimento Jr J. D., Recio-Blanco A. and De Medeiros J. R., Physica A, 384 (2007) 507.

[4] Carvalho J. C., Silva R., do Nascimento Jr J. D. and De Medeiros J. R., Europhys. Lett., 84 (2008) 59001.

[5] Carvalho J. C., do Nascimento Jr J. D., Silva R. and De Medeiros J. R., ApJ, 696 (2009) 48.

[6] Gell-Mann M. and Tsallis C. (Editors), Nonextensive Entropy-Interdisciplinary Applications (Oxford Univ. Press, New York) 2004.

[7] Hamity V. H. and Barraco D.E., Phys. Rev. Lett., 76 (1996) 4664.

[8] Lavagno A., Kaniadakis G., Rego-Monteiro M., Quarati P. and Tsallis C., Astr. Lett. Commun., 34 (1998) 449.

[9] Taruya A. and Sakagami M., MNRAS, 364 (2005) 990.

[10] Boghosian B., Phys. Rev. E, 53 (1996) 4754.

[11] Zanette D. H. and Alemany P. A., Phys. Rev. Lett., 75 (1995) 366.

[12] Kaniadakis G., Lavagno A. and Quarati P., Phys. Lett. B, 369 (1996) 308.

[13] Rajagopal A. K., Phys. Rev. Lett., 76 (1996) 3469.

[14] Taruya A. and Sakagami M., Physica A, 307 (2002) 185.

[15] Lima J. A. S., Silva R. and Santos J., Astron. Astrophys., 396 (2002) 309.

[16] de Freitas D. B. and De Medeiros J. R., Europhys. Lett., 88 (2009) 19001.

[17] Umarov S., Tsallis C. and Steinberg S., Milan J. Math., 76 (2008) 307.

[18] Nordstrom B., Mayor M., Andersen J., et al., A6A, 418 (2004) 989.

[19] Holmberg J., Nordstrom B. and Andersen J., A6A, 475 (2007) 519.

[20] Levenberg K., Q. Appl. Math., 2 (1944) 164.

[21] Marquardt D., SIAM J. Appl. Math., 11 (1963) 431.