Towards Dynamic Pricing for Shared Mobility on Demand using Markov Decision Processes and Dynamic Programming

Yue Guan\textsuperscript{*}, Anuradha M. Annaswamy\textsuperscript{1} and H. Eric Tseng\textsuperscript{2}

\textsuperscript{1}Department of Mechanical Engineering, Massachusetts Institute of Technology
\textsuperscript{2}Research and Advanced Engineering, Ford Motor Company

October 7, 2019

Abstract

In a Shared Mobility on Demand Service (SMoDS), dynamic pricing plays an important role in the form of an incentive for the decision of the empowered passenger on the ride offer. Strategies for determining the dynamic tariff should be suitably designed so that the incurred demand and supply are balanced and therefore economic efficiency is achieved. In this manuscript, we formulate a discrete time Markov Decision Process (MDP) to determine the probability desired by the SMoDS platform corresponding to the acceptance rate of each empowered passenger at each state of the system. We use Estimated Waiting Time (EWT) as the metric for the balance between demand and supply, with the goal that EWT be regulated around a target value. We then develop a Dynamic Programming (DP) algorithm to derive the optimal policy of the MDP that regulates EWT around the target value. Computational experiments are conducted that demonstrate the regulation of EWT is effective, through various scenarios. The overall demonstration is carried out offline. The MDP formulation together with the DP algorithm can be utilized to an online determination of the dynamic tariff by integrating with our earlier works on Cumulative Prospect Theory based passenger behavioral modeling and the AltMin dynamic routing algorithm, and form the subject of future works.

Index terms: Shared Mobility on Demand, Dynamic Pricing, Estimated Waiting Time, Markov Decision Process, Dynamic Programming, Forward Search, Smart Cities.

1 Introduction

Shared Mobility on Demand Service (SMoDS) has transformed urban mobility and introduced a continuum of solutions between the traditionally binary modes of private individual vehicles and public mass transit, so as to lead to a range of services with different degrees of cost, flexibility, and carbon footprint. This manuscript pertains to an SMoDS solution that consists of customized dynamic routing and dynamic pricing. We build on our earlier works in [6, 7, 1] and propose an Markov Decision Process (MDP) formulation and a Dynamic Programming (DP) planning algorithm towards dynamic pricing for the SMoDS platform.

\textsuperscript{*}Corresponding author. Email: guany@mit.edu.
Dynamic pricing has achieved remarkable successes in emerging ride sharing platforms such as Uber, Lyft, and Didi Chuxing, where passengers are empowered to have the option to decide whether to accept or decline the ride offers. Dynamic tariffs therefore provide the incentive signals that affect the decisions of the passengers, more specifically, tune the probability with which the passengers accept the SMoDS ride offers. The objective of dynamic pricing is to balance the demand and supply of the SMoDS platform, and hence to further achieve economic efficiency. The balance could be measured via several Key Performance Indicators, of which we use Estimated Waiting Time (EWT) in our formulation [5]. EWT approximates the average time that an upcoming passenger would wait until being picked up. A large EWT(t) at time t indicates that demand exceeds supply and vice versa. The goal is to regulate EWT(t) around a target value EWT* by suitably designing the dynamic pricing strategies.

The main contributions of this manuscript include an MDP formulation that serves as the underlying tool to determine the desired probability of acceptance p* for each passenger and a DP planning algorithm to derive the optimal policy of the MDP in an offline context. Computational experiments are conducted to demonstrate effective regulation of EWT(t) around EWT* for various EWT* values and for a time-varying EWT*. The extension of the DP algorithm to online scenarios is also discussed. With the desired probability of acceptance addressed in this manuscript, and by integrating with the passenger behavioral model from [7], the dynamic tariff that nudges the passenger towards p* can be derived.

Dynamic pricing for the SMoDS has been quite a popular research topic during recently years. [9] proposes the spatio-temporal pricing mechanism that has prices be smooth in space and time hence drivers will not decline the dispatched rides to seek ones with higher returns nearby. [2] develops a queueing-theoretic economic model for dynamic pricing and proves that dynamic pricing is more robust than static pricing. [8] utilizes the internal data from Uber and calibrates a steady-state model that verifies increased total welfare via dynamic pricing. However, to the best of our knowledge, no prior work has been reported related to the applications of MDP and DP in dynamic pricing for the SMoDS platform.

2 Preliminaries - Dynamic Routing and Dynamic Pricing

The SMoDS solution we are developing consists of dynamic routing and dynamic pricing, which delivers a customized dynamic route and dynamic tariff to each passenger [6] [1] [7]. The overall schematic is illustrated in Fig. 1 with three building blocks functioning in the following manner. When receiving a ride request, the first block derives an optimized dynamic route. The second and third blocks ensure dynamic pricing that accommodates the possibility that the empowered passenger may either accept or reject the ride offer and still generates an overall performance desired by the SMoDS platform. Of these, the second concerns a passenger behavioral model that derives the actual probability of acceptance for a specified dynamic tariff. The third and final block is the construction of a desired probability of acceptance from each passenger that will ensure the desired performances by the SMoDS platform. Using this third block, one can then design the actual dynamic tariff by solving the inverse problem. We focus on determining desired probability of acceptance in this manuscript while only briefly introduce dynamic routing and passenger behavioral modeling in this section, and refer the readers to [6] and [7] for more details.
2.1 Dynamic Routing

An Alternating Minimization (AltMin) based optimization algorithm is developed for dynamic routing with added spatial flexibility enabled by space window [6]. AltMin derives dynamic routes for passengers given their requested pickup, drop-off locations, willingness to walk and other service requirements that minimize a weighted sum of various travel time cost terms. AltMin has been demonstrated via various computational experiments to outperform classic constrained optimization formulations that are solved by standard solvers in terms of both computational complexity and optimality.

In the current SMoDS design, AltMin on the one hand derives dynamic routes that are provided in the ride offer, on the other hand contributes to the MDP formulation towards dynamic pricing, by means of the functions $F_{DR}(\cdot)$, $F_{EWT}(\cdot)$, and $F_{wait}(\cdot,\cdot)$ which we will formally introduce in Section 3.

2.2 Dynamic Pricing

Dynamic pricing is divided into two phases as follows.

2.2.1 Passenger Behavioral Modeling

The first phase is passenger behavioral modeling. A passenger behavioral model is one that inputs the specifications of the SMoDS ride offer and alternative transportation options, and outputs the actual probability with which the passenger takes the SMoDS. According to discrete choice model [3], the actual probability of acceptance of the $\ell$th transportation option given $Q \in \mathbb{Z}_{>0}$ options to choose from is given by

$$p^\ell = \frac{e^{U_\ell}}{\sum_{q=1}^{Q} e^{U_q}}, \quad \forall \ell \in \{1, \cdots, Q\}$$

Here we apply Cumulative Prospect Theory (CPT) in passenger behavioral modeling to capture the subjective decision making of the passengers when facing risk or uncertainty. This is because...
the SMoDS is exposed to uncertainty since the vehicles need to accommodate new passengers at anytime during the route therefore the service quality is to some extent stochastic. $U_\ell$ denotes the subjective utility of taking the $\ell^{th}$ option perceived by the passenger, which can be computed via the framework we have developed in [7].

### 2.2.2 Desired Probability of Acceptance

Given the ride specifications, the passenger is empowered to take the SMoDS with certain probability. The probability of acceptance impacts the expected performances of the SMoDS platform. With the dynamic routes derived via AltMin, dynamic pricing serves as an incentive to tune this probability. As discussed in Section 1, the goal is to regulate $EWT(t)$ around the target value $EWT^*$. Therefore, the probability of acceptance desired by the SMoDS platform should minimize the following quantity which essentially measures the average deviation of the incurred $EWT(t)$ from $EWT^*$

$$
\frac{1}{T} \int_0^T |EWT(t) - EXT^*| \, dt
$$

Here $T > 0$ denotes the evaluation horizon. Note that how to choose an appropriate $EWT^*$ value is beyond the scope of this manuscript, which requires the knowledge of the demand and supply of the SMoDS platform, i.e., request pattern and fleet portfolio, and explicitly stated objectives, e.g., some combination of revenue and ridership. This is one of our main future directions. We assume that $EWT^*$ is explicitly given for the rest of the manuscript if not otherwise clarified.

From (1), we obtain $p$ the actual probability of acceptance from each passenger. And the desired value $p^*$ is to be derived via the methodology developed in this manuscript. We then input $p$ and $p^*$ to the transactive controller to derive the dynamic tariff that nudges the passenger towards $p^*$ by simply setting $p = p^*$ and solving the inverse problem.

### 3 A Markov Decision Process Formulation

In this section, we propose a discrete time MDP formulation for determining the desired probability of acceptance.

In the MDP formulation, the agent is the SMoDS server, while the fleet as well as the passengers including both existing and future ones make up the environment. The goal is to derive the optimal policy that maximizes the expected total discounted rewards. The MDP is defined using the tuple $(S,A,P,R,\gamma)$ [12], which denote the state space, action space, state transition function, discount factor, and reward function, respectively. Several clarifications are made as follows before elaborating each element in the tuple.

- The agent receives ride requests in discrete time which are $\Delta t_r \in \mathbb{R}_{>0}$ apart, and $t_r = k\Delta t_r, k \in \mathbb{Z}_{>0}$ is denoted as a general notion of the discrete timestamps. Denotes $f_{\Omega_{t_r}}(\omega_{t_r})$ or just $f(\omega_{t_r})$ as the distribution of $\Omega_{t_r}$, which we assume is independent on the state.

- The action of the agent is to design the desired probability of acceptance for each new passenger at $t_r$, given the state. We assume that by suitably designing the SMoDS ride offer, the passenger will accept the offer with the desired chance. Hence we do not consider the passenger behavioral model in this manuscript, the current MDP formulation is therefore fully observable.

- A policy $\pi(\cdot)$ is a distribution over actions given the state, and an optimal policy $\pi^*(\cdot)$ is one that maximizes the expected total discounted rewards.
The actions are taken sequentially, one at a time, and are dependent on the decisions from previous passengers in response to the previous actions.

The SMoDS fleet portfolio is assumed to be fixed.

We adopt most of the notation in Sutton [12] and Silver [10]. We use uppercases as general terms to represent states, actions, rewards, ride requests, and decisions from passengers, and lowercases to represent the corresponding realizations. In addition, calligraphic fonts are utilized to represent the state and action spaces as well as the reward and state transition functions.

3.1 State Space $\mathcal{S}$

We denote $S_t \in \mathcal{S}$ as the state at $t \in \mathbb{R}_{\geq 0}$. In addition, at the timestamps $t_r$ of any new ride request received by the SMoDS server, $S_{t_r^-}$ and $S_{t_r^+}$ denote the states right before and after the server processes the request and the passenger decides whether to take the ride offer or not at $t_r$, respectively. Although the proposed MDP formulation operates in discrete time since actions can only be taken at discrete $t_r$ when receiving new requests, the states can be defined in continuous time.

$S_t$ describes the status of the SMoDS platform at $t$, which consists of the statuses of both the fleet and passengers, as well as unprocessed requests if any. Denote $m_t$ and $n_t$ as the numbers of active vehicles and passengers at $t$, respectively, $m_t, n_t \in \mathbb{Z}_{\geq 0}$. $S_t$ simply concatenates the states of each vehicle, passenger, and any unprocessed request as follows

$$S_t = \left[\{S^{v_i}_t\}_{i \in [m_t]}, \{S^{p_j}_t\}_{j \in [n_t]}, \Omega_t\right]$$

where $S^{v_i}_t$ and $S^{p_j}_t$ denote the state of the $i^{th}$ vehicle and $j^{th}$ passenger, respectively. We denote $[m_t] = \{1, 2, \cdots, m_t\}$ and $[n_t]$ similarly for ease of notation. $\Omega_t = \emptyset$ if $t \neq t_r$. $S^{v_i}_t$ and $S^{p_j}_t$ are defined in (4) and (5), respectively.

$$S^{v_i}_t = [L^{v_i}_t, G^{v_i}_t, O^{v_i}_t]$$

$S^{v_i}_t$ contains i) $L^{v_i}_t$ the location, ii) $G^{v_i}_t$ a set of consecutive routing points representing the route, and iii) $O^{v_i}_t$ other status indicators of the $i^{th}$ vehicle at $t$. $O^{v_i}_t$ may contain any status indicator of that is required by the dynamic routing algorithm, for example, the number of passengers onboard should be included if a capacity constraint is imposed.

$$S^{p_j}_t = [L^{p_j}_t, G^{p_j}_t, O^{p_j}_t]$$

$S^{p_j}_t$ contains i) $L^{p_j}_t$ the location, ii) $G^{p_j}_t$ the route, including the requested and negotiated pickup and drop-off locations, and iii) $O^{p_j}_t$ other status indicators of passenger $j$ at $t$. Similarly, $O^{p_j}_t$ may contain any status indicator that is required for dynamic routing, for example, the positions in the request, pickup, and drop-off queues should be included if the maximum position shift constraints are imposed [6].

3.2 Action Space $\mathcal{A}$

We denote $A_{t_r} \in \mathcal{A}$ as the action taken at $t_r$, apparently $A_{t_r} \in [0, 1]$. More specifically, since the desired probability of acceptance is realized through the design of the SMoDS ride offer and ultimately equates the actual probability of acceptance defined in [1], and the dynamic tariff should
be reasonably charged, for example, it should not be negative nor unrealistically high compared with that of the alternative transportation options, hence $A_{t_r}$ does actually lie in a strict subset of $[0,1]$, i.e.,

$$A_{t_r} \in [A_{t_r}, \overline{A}_{t_r}], \quad 0 < A_{t_r} < \overline{A}_{t_r} < 1$$

(6)

where $A_{t_r}$ and $\overline{A}_{t_r}$ denote the lower and upper bounds of $A_{t_r}$ respectively, both of which are functions of the state, and the specifications of the alternatives, denoted as

$$\{A_{t_r}, \overline{A}_{t_r}\} = F_b(S_{t_r}, A_{t_r})$$

(7)

$S_{t_r}$ is utilized to derive the specifications of the SMoDS ride offer via the dynamic routing algorithm, which together with $A_{t_r}$ the specifications of the alternatives determine the space of action $A_{t_r}$. In fact, the optimal action $a^*_{t_r}$ only takes the value at either end point of the corresponding action space $[a_{t_r}, \overline{a}_{t_r}]$, which we formally state in Theorem 3.1.

**Theorem 3.1.** \(\forall t_r\), we have the optimal action

$$a^*_{t_r} \in \{a_{t_r}, \overline{a}_{t_r}\}$$

(8)

The proof is provided in Appendix.

Once the action is taken by the agent and delivered to the passenger, the passenger would respond with the decision on the ride offer. We denote $D_{t_r}$ as the decision of the passenger at $t_r$, where $d_{t_r} = 1$ denotes acceptance and $d_{t_r} = 0$ denotes rejection. Apparently $D_{t_r}$ obeys the Bernoulli distribution as

$$D_{t_r} \sim B(1, a_{t_r})$$

(9)

### 3.3 State Transition Function $\mathcal{P}$

Consider $S_t, t \in \mathbb{Z}_{\geq 0}$ defined in the continuous time domain, there are three distinct scenarios where the state might transit from one to another as follows.

#### 3.3.1 State Transition due to Internal Dynamics

When there is no new ride request received nor existing request processed, the transition is purely due to the internal dynamics of the state, i.e., movements of the vehicles and passengers following the routes. This type of state transition is deterministic and can be described as follows

$$\mathcal{P}_{s_t s_{t+\tau}} = \begin{cases} 1, & \text{if } s_{t+\tau} = F_D(s_t, \tau) \\ 0, & \text{otherwise} \end{cases}$$

(10)

(10) is valid \(\forall t, \tau \geq 0\) and when no ride request is received during $[t, t + \tau]$. $F_D(\cdot, \cdot)$ essentially captures the internal dynamics when there is no external disturbance, i.e., request received or processed. We omit the superscript that represents the action since there is no action taken here.
3.3.2 State Transition due to Receiving New Requests

When there is a new request received at $t_r$, state transition occurs via augmenting the current state by the request specifications

$$
\mathcal{P}_{s_{t_r}} = \begin{cases} 
  f(\omega_{t_r}), & \text{if } s_{t_r} = [s_{t_r}, \omega_{t_r}] \\
  0, & \text{otherwise}
\end{cases}
$$

(recall that $f(\omega_{t_r})$ denotes the distribution of $\Omega_{t_r}$). Similar as in (10), we omit the representation of actions in the superscripts due to absence of actions in this scenario.

3.3.3 State Transition due to Processing Existing Requests

When the agent takes an action to process the new request, and the passenger responds with the decision on the offer, the state transit by possibly updating the status of the vehicles and passengers according to the decision from the passenger, as well as eliminating the existing request.

$$
P_{a_{t_r}}^{s_{t_r}} = \begin{cases} 
  a_{t_r}, & \text{if } s_{t_r} = F_{DR}(s_{t_r}) \\
  1 - a_{t_r}, & \text{if } s_{t_r} = s_{t_r} \\
  0, & \text{otherwise}
\end{cases}
$$

(12) essentially states that the passenger accepts the SMoDS ride offer with the probability of $a_{t_r}$ and as a result the state $s_{t_r}$ transits to $s_{t_r} = F_{DR}(s_{t_r})$. $F_{DR}(\cdot)$ is essentially the dynamic routing algorithm, i.e., the AltMin algorithm in the SMoDS solution we are developing [6], which is a function of $s_{t_r}$ the state when the request is received at $t_r$. While the passenger might decline the offer with the probability of $(1 - a_{t_r})$ and hence the state transits back to that right before receiving the request by eliminating the exiting request $\omega_{t_r}$, i.e., $s_{t_r} = s_{t_r}$. The superscripts of $a_{t_r}$ indicate that this type of state transition is dependent on the actions.

Combining (10) through (12), we derive the state transition function for $S_{t_r}$ the states defined in the discrete time domain when actions are taken as follows

$$
P_{a_k}^{s_{k+1}} = \begin{cases} 
  a_k f(\omega_{k+1}), & \text{if } s_{k+1} = [F_D(F_{DR}(s_k), \Delta t_r), \omega_{k+1}] \\
  (1 - a_k) f(\omega_{k+1}), & \text{if } s_{k+1} = [F_D(s_k, \Delta t_r), \omega_{k+1}] \\
  0, & \text{otherwise}
\end{cases}
$$

(13) where we omit $\Delta t_r$ in the subscripts and use $k \in \mathbb{Z}_{>0}$ to represent $k\Delta t_r$ for ease of notation.

**Theorem 3.2.** The states $S_{t_r}$ defined in (3) through (5) with the transition function derived in (13) are Markov.

The proof is provided in Appendix.

3.4 Reward Function $\mathcal{R}$

As has been discussed in Section 2, the objective of the MDP formulation is to derive the optimal policy $\pi^*(\cdot)$ such that EWT$(t)$ is well regulated around EWT*$. We break down the average deviation defined in (2) into segments by timestamps of requests, and define the reward as the direct contribution to (2) of taking action $a_{t_r}$ at state $s_{t_r}$ as follows

$$
\mathcal{R}_{a_{t_r}}^{s_{t_r}} = -E_{D_{t_r}} \left[ \frac{1}{\Delta t_r} \int_{t_r}^{(t_r + \Delta t_r)^-} |EWT(\tau) - EWT^*| \ d\tau \right]
$$

(14)
where the expectation is taken over the decision from the passenger in response to \(a_{t_r}\). Essentially measures the average closeness, i.e., the opposite of average deviation, of \(\text{EWT}(t)\) from \(\text{EWT}^*\) in response to \(a_{t_r}\), from \(t_r^+\) till \((t_r + \Delta t_r)^-\) right before when the next action is about to take place. Following the discussions in Section 2, \(\text{EWT}(t)\) is a panel parameter of the SMoDS platform measuring the expected waiting time if an upcoming request is received, where the expectation is taken over \(f_{\Omega_{t_r}}(\omega_{t_r})\). Hence \(\text{EWT}(t)\) is a function of the state \(s_t\) following

\[
\text{EWT}(t) = \mathbb{E}_{\Omega_{t_r}}[F_{\text{wait}}(s_t, \omega_{t_r})] = F_{\text{EWT}}(s_t) \tag{15}
\]

where \(F_{\text{wait}}(\cdot, \cdot)\) denotes the waiting time derived via the dynamic routing algorithm given \(s_t\) and \(\omega_{t_r}\). However, (15) might be hard to derive even if \(f_{\Omega_{t_r}}(\omega_{t_r})\) is known. One could therefore choose appropriate approaches to approximate \(\text{EWT}(t)\) instead of deriving it exactly if necessary.

### 3.5 Discount Factor \(\gamma\)

The discount factor \(\gamma \in [0, 1]\). Typically \(\gamma < 1\), while if the episode is guaranteed to terminate, we let \(\gamma = 1\).

### 3.6 Value Function

With the discrete time MDP formulation elaborated in Sections 3.1 through 3.5, we rewrite the objective function from (2) in the form of expected total discounted rewards

\[
\mathbb{E} \left[ \sum_{k=0}^{\infty} \gamma^k R_{A_{t_r+k\Delta t_r}}^{s_{t_r+k\Delta t_r}} \right] \tag{16}
\]

where we let \(A_0 = \emptyset\) for ease of notation. \(\pi^*(\cdot)\) can be derived either directly via policy based approaches or indirectly via value based ones, or both. For value based approaches, the optimal value function of any state \(S_{t_r} = s_{t_r}\) is defined as

\[
v^*(s_{t_r}) = \max_{\pi(\cdot)} \mathbb{E} \left[ \sum_{k=0}^{\infty} \gamma^k R_{A_{t_r+k\Delta t_r}}^{\pi(s_{t_r+k\Delta t_r})} \mid s_{t_r} \right] \tag{17}
\]

where the expectation is taken over the upcoming requests, and the decisions from the passengers in response to the actions (including \(D_{t_r}\)). Using Bellman Optimality Equation, we rewrite (17) as

\[
v^*(s_{t_r}) = \max_{\pi(\cdot)} \mathbb{E} \left[ R_{A_{t_r}}^{s_{t_r}} + \gamma \mathbb{E} \left[ v^*(s_{t_r+k\Delta t_r}) \mid s_{t_r} \right] \right] \tag{18}
\]

where the expectation is taken over \(D_{t_r}\) and \(\Omega_{t_r+k\Delta t_r}\).

When the numbers of active vehicles and passengers are large, and considering that the state space is continuous, deriving \(\pi^*(\cdot)\) through \(v^*(\cdot)\) exactly may be computationally impractical. Alternatively, one could apply function approximation [4] to derive a parametrized suboptimal policy as

\[
\tilde{\pi}(s_{t_r}; f_v) = \arg \min_{\pi(\cdot)} \mathbb{E} \left[ R_{A_{t_r}}^{\pi(s_{t_r})} + \gamma \tilde{v}(s_{t_r+k\Delta t_r}; f_v) \mid s_{t_r} \right] \tag{19}
\]

where \(\tilde{v}(\cdot; f_v)\) is an approximation of \(v^*(\cdot)\) parametrized by the feature vector \(f_v\), whose dimension is much smaller than that of the state and therefore results in more efficient computation. The state-action value function and the corresponding function approximation can be defined similarly.

Moreover, the action space at any \(t_r\) is a function of the state \(S_{t_r}\), which can be appropriately parametrized and approximated as well. Denote \(f_{b}\) as the feature vector that parametrizes \(S_{t_r}\).
Similarly, the dimension of \( f_b \) should be relatively small compared with that of the state to ensure efficient computation. The approximation is denoted as

\[
\left[ A_{t_r}, A_{t_r}; f_b \right] = \tilde{f}(S_{t_r}, \lambda_{t_r}; f_b)
\] (20)

In addition, policy or Actor Critic based approaches can also be approximated with appropriate parametrization. With these, one can apply a broad range of learning or planning algorithms to derive \( \pi(\cdot) \) or \( \tilde{\pi}(\cdot; \cdot) \).

4 A Dynamic Programming Algorithm to Determine \( \pi^*(\cdot) \)

In this section, we start with a specific offline scenario of the MDP formulation, develop a DP planning algorithm and demonstrate how the desired probability of acceptance could be designed to regulate \( \text{EWT}(t) \) around \( \text{EWT}^* \). We then proceed to discuss the implications of the DP algorithm for the general online case.

4.1 Problem Setup

In this section, we adopt an offline setup. 12 synthetic ride requests are generated with origins and destinations uniformly distributed in a square of one by one mile, and are served by one single vehicle. The first 4 requests are scheduled at \( t = 0 \) and these passengers are assumed to accept the ride offers, which initialize the simulation episode. The following \( N = 8 \) requests are scheduled one by one and arrive 4 minutes apart over an interval of 28 minutes. Each request is assumed to have one passenger, and the vehicle has a capacity of 6. The offline setup indicates that the agent has the information of all 12 requests beforehand and takes subsequent requests into consideration when designing the policy. However, the requests are still processed sequentially, meaning that the offers will be provided to the passengers sequentially and the policy for the upcoming passengers may adapt depending on the decisions from previous ones. Dynamic routing are conducted using the AltMin algorithm developed in [6]. Since the simulation episodes are guaranteed to terminate in the offline setup, we set \( \gamma = 1 \).

As the first attempt, and without loss of generality, here we adopt simplified approximations of \( \left\{ a_{t_r}, \bar{a}_{t_r} \right\} \) given \( s_{t_r}, \lambda_{t_r} \), and \( \text{EWT}(t) \) given \( s_{t_r} \), respectively. The alternative is considered as the MoD service without sharing, where the waiting time for each request is set as \( \frac{3}{4} \text{EWT}^* \), and the riding time is the direct travel time from the origin to the destination. Given \( s_{t_r} \), the specifications of the SMoDS ride offer are derived using AltMin, if the total travel time, i.e., waiting plus riding times of the SMoDS does not exceed \( \frac{3}{4} \) of that of the alternative, the action space is set as \{0.5, 0.9\}, otherwise \{0.2, 0.6\}. Given \( s_{t_r} \), \( \text{EWT}(t) \) is approximated as the average time that four representative passengers who request pickup locations at the four corners of the one by one mile square would wait. As has been discussed in Section 3.2 and 3.4, accurate derivations of \( \left\{ a_{t_r}, \bar{a}_{t_r} \right\} \) and \( \text{EWT}(t) \) require knowledge of the alternative transportation options, passenger behavioral model and distribution of requests, and are our future directions. Here in this section, we adopt approximation approaches that are sufficiently reasonable to illustrate the central idea of regulating \( \text{EWT}(t) \) via \( \pi^*(\cdot) \).

4.2 Algorithmic Design

Since we deploy an offline setup, all ride requests are known to the SMoDS server beforehand and deterministic. This reduces the cardinality of the space of \( S_{t_r} \) from continuous to finite. In addition,
according to Theorem 3.1, the action space of each \( S_t \) is finite as well, and has a cardinality of 2. These enable the development of the value based DP planning algorithm that derives \( \pi^*(\cdot) \) exactly to be feasible.

Fig. 2 illustrates a directed and rooted tree structure that explains the DP algorithm. Except for the root and leaves, i.e., terminal vertices represented by black squares, each vertex represents a circumstance when the agent takes an action, and has two children representing two distinct scenarios of subsequent states resulted from the decision of the passenger in response to the action. If the passenger decides to accept the ride offer, the child on the top is chosen and a rejection leads to the one on the bottom. The action is labeled inside each nonterminal vertex, and we let that of the root be \( \emptyset \) since no action is taken therein. Each edge represents the transition after the decision of the previous passenger till when the new request is received and a subsequent action is to be taken, and is associated with a realization of the reward. The height of the tree is \( H = N + 1 \) and each longest path with length equal to \( H \) represents a complete scenario as a result of consecutive decisions from all passengers. For ease of notation, we omit \( \Delta t_r \) in the subscripts of states, actions and rewards for the rest of the manuscript if not otherwise clarified, for example, \( s^{d[k-1]}_k \) represents \( s^{d[k-1]}_{k\Delta t_r} \). In addition, \( d_{[k]} = [d_1, \ldots, d_k], \forall k \in [N] \) represents the decisions of the first \( k \) passengers. We have \( d_{[k]} \) in the superscripts explicitly since the decisions of previous passengers determine which vertex and edge that the DP algorithm traverses. Moreover, we denote \( \omega_k, \forall k \in [N] \) as the \( k^{th} \) ride request, and \( \omega_{N+1} = \emptyset \) for ease of notation.

The algorithm consists of two steps. The first step is to conduct dynamic routing to process the requests consecutively which essentially traverses the tree from the root to the leaves, therefore to derive the reward using functions \( F_D(\cdot, \cdot), F_{DR}(\cdot), F_{EWT}(\cdot) \) and \[14\] on each edge. The second step is to conduct backward recursion from the leaves to the root, therefore to derive the optimal value function at each state and the associated optimal action at each vertex, via Bellman Optimality.
Equation delineated in [18]. The details of the DP exact algorithm are outlined in Algorithm [1].

| Algorithm 1: DP Exact Algorithm - DP-E(N) |
|------------------------------------------|
| 1 Initialize the route via AltMin and derive \( s_0^+ \) |
| 2 Initialize \( s_1^c \leftarrow [F_D(s_0^+, \Delta t_r), \omega_1], d[]_0, d[]_0 \leftarrow \emptyset \) |
| 3 Initialize \( v^*(s_k^{d[k-1]}) \leftarrow 0, \forall k \in [N+1], d[k-1] \) |
| 4 for \( k = 1 : N \) do \hspace{1cm} // step 1: derive rewards |
| for \( d[k-1] \) do |
| for \( d_k = 0 : 1 \) do |
| \( d[1:k] \leftarrow [d[1:k], d_k] \) |
| if \( d_k = 0 \) then |
| \( s_{k+}^c \leftarrow s_{k-}^c \) |
| else |
| \( s_{k+}^c \leftarrow F_{DR}(s_k^{d[k-1]}) \) |
| end |
| \( r_{k}^{d[k]} \leftarrow \int_{0}^{1} |F_{EWT}(s_{k+1}, d[k]) - EWT^*| d\tau \) |
| \( s_{(k+1)}^c \leftarrow F_{D}(s_{k+1}, \Delta t_r) \) |
| \( s_{k+1} \leftarrow [s_{(k+1)}^c, \omega_{k+1}] \) |
| end |
| end |
| for \( k = N : -1 : 1 \) do \hspace{1cm} // step 2: backups |
| for \( d[k-1] \) do |
| if \( r_{k}^{d[k-1],1} + v^*(s_{k+1}^{d[k-1],1}) \geq r_{k}^{d[k-1],0} + v^*(s_{k+1}^{d[k-1],0}) \) then |
| \( a_{k}^{d[k-1]} \leftarrow a_k \) |
| else |
| \( a_{k}^{d[k-1]} \leftarrow a_k \) |
| end |
| \( v^*(s_k^{d[k-1]}) \leftarrow a_k^{d[k-1]}[r_k^{d[k-1],1} + v^*(s_{k+1}^{d[k-1],1})] + (1 - a_k^{d[k-1]})[r_k^{d[k-1],0} + v^*(s_{k+1}^{d[k-1],0})] \) |
| end |
| end |

Similar idea can be exploited to develop a DP heuristic algorithm that improves computational efficiency at the cost of optimality. Instead of conducting forward search till the end of the future, i.e., number of lookahead steps being \( N \), the DP heuristic algorithm conducts forward search with steps being \( \tilde{N} < N \), which poses as a hyper parameter to tune the trade-off between efficiency and optimality. For example, to derive \( a_k^{d[k-1]} \), the DP-H(\( \tilde{N} \)) algorithm essentially carries out DP-E(\( \tilde{N} \)) on the sub tree rooted at the vertex \( a_k^{d[k-1]} \) with a height of \( \tilde{N} + 1 \), instead of traversing the entire tree in Fig. 2. The pseudo codes are provided in Algorithm 2.

Note that in line 5, if \( 1 < p < N - \tilde{N} + 1 \), only execute when \( k = \tilde{N} \) in line 4 in Algorithm 1 since \( r_q^{d[q-1]} \) with \( q < p + \tilde{N} - 1 \) have been evaluated in previous iterations.
Algorithm 2: DP Heuristic Algorithm - DP-H(\(\hat{N}\))

\begin{algorithm}
\begin{algorithmic}[1]
\State \textbf{for} \(p = 1 : N - \hat{N} + 1\) \textbf{do}
\State \hspace{1em} \textbf{for} \(d[p-1]\) \textbf{do}
\State \hspace{2em} \(s_1^p \leftarrow s_p^{d[p-1]}\)
\State \hspace{2em} \textbf{if} \(1 \leq p < N - \hat{N} + 1\) \textbf{then}
\State \hspace{3em} \text{execute lines 3-28 in Algorithm 1 and derive} \(a_p^{d[p-1]}\)
\State \hspace{2em} \textbf{else}
\State \hspace{3em} \text{execute lines 3-28 in Algorithm 1 and derive} \(a_q^{d[q-1]}, \forall N - \hat{N} + 1 \leq q \leq N\)
\State \hspace{1em} \textbf{end}
\State \textbf{end}
\State \textbf{end}
\end{algorithmic}
\end{algorithm}

4.3 Results of Computational Experiments

With the problem setup described in Section 4.1, we exploit the DP exact algorithm delineated in Algorithm 1 to derive \(\pi^*(\cdot)\) under various \(EWT^*\) values. The results are summarized in Fig. 3. The subplots on the left, middle and right demonstrate the results with \(EWT^*\) being 4, 5, and 6 minutes, respectively. In the subplots on the top, the blue curves illustrate how the expected \(EWT(t)\) varies with \(t\) under the optimal policy \(\pi^*(\cdot)\) derived using DP-E(\(N\)), the orange curves illustrate \(EWT(t)\) in the benchmark scenario where passengers are not empowered and hence all get onboard, and the black curves represent \(EWT^*\). The benchmark curve is identical in the three subplots since it does not depend on \(EWT^*\). The subplots on the bottom summarize expected acceptance rate under \(\pi^*(\cdot)\) for each passenger and the mean across all passengers. Fig. 3 demonstrate that the blue curves could be brought closer to the target dashed lines compared with the orange ones, indicating effective regulation of \(EWT(t)\) around \(EWT^*\) through \(\pi^*(\cdot)\) derived via DP-E(\(N\)). The regulation can be achieved with different \(EWT^*\) values that are within a reasonable range. Though how to choose an appropriate \(EWT^*\) value is beyond the scope of this manuscript, we could roughly argue the impacts of the \(EWT^*\) value on the performances of the SMoDS platform. According to Fig. 3, the average acceptance rate, i.e., the ridership, increases with \(EWT^*\) because the tolerance of the agent to having passengers wait gets higher therefore it is more likely to get the passengers onboard. However, on the other hand, the downgrade of the service quality might occur due to the increased actual waiting times as a response to increased \(EWT^*\). In order to maintain relatively high acceptance rates, the dynamic tariff should decrease. Therefore, the overall impacts of increased \(EWT^*\) on the total revenue depend on two competing contributions, one is increased ridership and the other is decreased revenue per ride. Hence \(EWT^*\) should be suitably determined to tune the trade-off in order to achieve desired overall performances.

In addition to Fig. 3 where \(EWT^*\) is held constant within episodes, we also experiment with time-varying \(EWT^*\). The results are demonstrated in Fig. 4. \(EWT^*\) is set as 4 minutes during the first half of the episode, while when the SMoDS platform gets fairly crowded and \(EWT(t)\) is about to increase substantially, we relax the regulation by increasing \(EWT^*\) up to 6 minutes at \(t = 20\) minutes, and the regulation is adapted instantaneously. These results support our argument above that \(EWT^*\) could be actively tuned to adapt to the actual conditions of the system during practical operations.

Moreover, the DP-H(\(\hat{N}\)) algorithm is implemented with various \(\hat{N}\) values for the case where \(EWT^* = 5\) minutes. Fig. 5 plots the average deviation of \(EWT(t)\) from \(EWT^*\), i.e., the negative
Figure 3: Regulation of EWT(t) around EWT* and the incurred expected acceptance rates of each passenger for different EWT* values.

Figure 4: Regulation of EWT(t) around changing EWT*.

of the average rewards defined in (14), with respect to \( \bar{N} \), under the suboptimal policy derived using DP-H(\( \bar{N} \)). \(^1\) Fig. 5 indicate that the optimality of DP-H(\( \bar{N} \)) is fairly comparable to that of DP-E(\( N \)), for example, with just one lookahead step, comparable regulation in terms of the average deviation can be achieved. Therefore, it is promising to apply DP-H(\( \bar{N} \)) in more general cases with sufficient computational efficiency and preserved optimality.

\(^1\)When \( \bar{N} = 0 \), actions are determined by comparing \( F_{\text{EWT}}(s_{t+}) \) values, instead of rewards and value functions in lines 21 through 25 in Algorithm 1 for cases when \( \bar{N} > 0 \).
4.4 Remarks

In this section, we construct a special case of the MDP formulated in Section 3 where the offline setup is deployed. Being offline reduces the state space from continuous to finite, together with relatively small number of requests and vehicles, enabling the development of the DP algorithm. The policies derived via both DP-E(N) and DP-H(˜N) demonstrate effective regulation of EWT(t) around ETW*, under various ETW* values and time-varying ETW* as well.

When the state space gets more complicated, either due to fairly large number of requests and vehicles, or the adoption of an online setup, one can exploit function approximation briefly discussed in Section 3.6 and develop a broad range of planning and/or learning algorithms. Notably, DP-H(˜N) would be quite useful in either scenario. For example, when dealing with online requests in real time, one can integrate learning and planning. A preliminary value network is learned offline via historical or self-play data, and lookahead search is conducted via DP-H(˜N) to refine the value function on the fly and then take actions accordingly. This is very similar to the algorithmic architecture of AlphaGo [11].

5 Concluding Remarks

In this manuscript, we propose a discrete time MDP formulation to determine desired probabilities of acceptance for empowered passengers towards dynamic pricing in the context of SMoDS, and develop a DP algorithm to derive the optimal policy that regulates EWT(t) around EWT* for a specific scenario of the MDP with an offline setup. Computational experiments are carried out that demonstrate effective regulation of EWT(t) around EWT*, for various EWT* values and for a time-varying EWT* as well. The heuristic version of the DP algorithm, DP-H(˜N), could be exploited as the lookahead search algorithm when large state space or online setups are encountered. The MDP formulation together with our previous works of the AltMin dynamic routing algorithm in [6] and CPT based passenger behavioral modeling in [7], provide a complete solution to the SMoDS.

Future works include developing integrated learning and planning algorithms for large state space or online setups, and investigating disciplines that guide the choice of appropriate EWT* values leading to the desired combination of revenue and ridership for the SMoDS platform. Moreover, the integration of passenger behavioral model with the MDP hence directly designing dynamic tariffs is of interest as well.
Acknowledgments

This work was supported by the Ford-MIT Alliance.

Appendix: Proofs of Theorems

Proof of Theorem 3.1. Expand (18), we have

\[ v^*(s_{t_r}) = \max_{a_t \in [a_{t-1}, a_t]} R_{s_{t_r}} + \gamma \mathbb{E}\left[ v^*(s_{t_r+\Delta t_r}) \mid s_{t_r}, a_t \right] \]

\[ = \max_{a_t \in [a_{t-1}, a_t]} a_t \left\{ r_{t_r}^{[1]} + \gamma \mathbb{E}\left[ v^*(s_{t_r+\Delta t_r}) \mid s_{t_r}, a_t \right] \right\} + (1-a_t) \left\{ r_{t_r}^{[0]} + \gamma \mathbb{E}\left[ v^*(s_{t_r+\Delta t_r}) \mid s_{t_r}, a_t \right] \right\} \]

(21)

The first equality is essentially Bellman Optimality Equation and holds by definition. The second equality holds due to the expansion of the expectation term over \( D_{t_r} \). The third equality holds because the right-hand side of the second equality is linear in \( a_t \), since the remaining expectations are taken over the distribution of \( \Omega_{t_r+\Delta t_r} \). The second subcase in (13) do not depend on \( a_{t_r} \), therefore the maximum must be reached at either end point. Here \( r_{t_r}^{[1]} = -\frac{1}{\Delta t_r} \int_0^{(\Delta t_r)^-} |F_{EW\mathbb{T}}[F_D(F_{DR}(s_{t_r}), \tau)] - E\mathbb{T}^*| \, d\tau, \]

\( s_{t_r+\Delta t_r} = [F_D(F_{DR}(s_{t_r}), \Delta t_r), \omega_{t_r+\Delta t_r}], \)

\( r_{t_r}^{[0]} = -\frac{1}{\Delta t_r} \int_0^{(\Delta t_r)^-} |F_{EW\mathbb{T}}[F_D(s_{t_r}, \tau)] - E\mathbb{T}^*| \, d\tau, \) and \( s_{t_r+\Delta t_r}^{[0]} = [F_D(s_{t_r}, \Delta t_r), \omega_{t_r+\Delta t_r}] \)

Proof of Theorem 3.2. According to (13), we have

\[ \mathbb{P}\left( S_{k+1} = s_{k+1} \mid S_k = s_k, \cdots, S_1 = s_1 \right) = \mathbb{P}\left( S_{k+1} = s_{k+1} \mid S_k = s_k \right) \]

(22)

\( \forall k \in \mathbb{Z}_{>0} \) and \( \{s_1, \cdots, s_{k+1}\} \subset S \). Obviously, (22) holds for the first and third subcases in (13). (22) holds for the second subcase in (13) because \( \mathbb{P}\left( S_{k+1} = s_{k+1} \mid S_k = s_k \right) = \mathbb{P}\left( S_{k+1} = s_{k+1} \mid S_k = s_k, S_k^- = s_k^- \right) \). Hence \( \forall k \in \mathbb{Z}_{>0} \), \( S_k \) are Markov.

\[ ^2 \text{We omit the decisions from previous passengers in the superscripts as the formulas of } r_{t_r}^{[1]}, r_{t_r}^{[0]}, s_{t_r+\Delta t_r}^{[1]} \text{ and } s_{t_r+\Delta t_r}^{[0]} \text{ hold for any scenario.} \]
References

[1] A. M. Annaswamy, Y. Guan, H. E. Tseng, H. Zhou, T. Phan, and D. Yanakiev, “Transactive control in smart cities,” *Proceedings of the IEEE*, vol. 106, no. 4, pp. 518–537, 2018.

[2] S. Banerjee, C. Riquelme, and R. Johari, “Pricing in ride-share platforms: A queueing-theoretic approach,” *Available at SSRN 2568258*, 2015.

[3] M. E. Ben-Akiva, S. R. Lerman, and S. R. Lerman, *Discrete choice analysis: theory and application to travel demand*. MIT press, 1985, vol. 9.

[4] D. P. Bertsekas and J. N. Tsitsiklis, “Neuro-dynamic programming: an overview,” in *Proceedings of 1995 34th IEEE Conference on Decision and Control*, vol. 1. IEEE, 1995, pp. 560–564.

[5] P. Cohen, R. Hahn, J. Hall, S. Levitt, and R. Metcalfe, “Using big data to estimate consumer surplus: The case of uber,” National Bureau of Economic Research, Tech. Rep., 2016.

[6] Y. Guan, A. M. Annaswamy, and E. Tseng, “A dynamic routing framework for shared mobility services,” *ACM Transactions on Cyber-Physical Systems* (in press) (2019).

[7] Y. Guan, A. M. Annaswamy, and H. E. Tseng, “Cumulative prospect theory based dynamic pricing for shared mobility on demand services,” in *2019 IEEE 58th Annual Conference on Decision and Control (CDC)* (in press). IEEE, 2019.

[8] N. Korolko, D. Woodard, C. Yan, and H. Zhu, “Dynamic pricing and matching in ride-hailing platforms,” *Available at SSRN*, 2018.

[9] H. Ma, F. Fang, and D. C. Parkes, “Spatio-temporal pricing for ridesharing platforms,” *arXiv preprint arXiv:1801.04015*, 2018.

[10] D. Silver, “Reinforcement learning.”

[11] D. Silver, A. Huang, C. J. Maddison, A. Guez, L. Sifre, G. Van Den Driessche, J. Schrittwieser, I. Antonoglou, V. Panneershelvam, M. Lanctot et al., “Mastering the game of go with deep neural networks and tree search,” *Nature*, vol. 529, no. 7587, p. 484, 2016.

[12] R. S. Sutton and A. G. Barto, *Reinforcement learning: An introduction*. MIT press, 2018.