Mixed Branes at Angle in Compact Spacetime

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Abstract

In this article the interaction of branes at angles with respect to each other with non-zero internal gauge fields are calculated by construction of the boundary states in spacetime in which some of its directions are compact on tori. The interaction depends on both angle and fields.

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1 Introduction

A useful tool for describing branes and their interactions, specially in non-zero back-ground fields is the boundary state [1, 2, 3, 4, 5, 6, 7, 8]. The overlap of boundary states through the closed string propagator gives us the amplitude of interaction of branes. By introducing back-ground fields to the string $\sigma$-model action one obtains mixed boundary conditions (i.e. a combination of Dirichlet and Neumann boundary conditions) for the strings. Previously we obtained the boundary state for a stationary mixed brane (i.e. a brane in back-ground fields). We also observed that when some directions are compactified on tori [6, 8], the winding numbers of the emitted states around the compact directions of the brane are correlated with their momenta along the brane. Also we studied the interaction of two of these mixed branes, in the parallel and perpendicular cases.

In this article we consider more general case of branes with respect to one another. In particular we take two non-intersecting one-branes which make an angle $\phi$ with each other in presence of the non-zero back-ground $B$-field. To keep our analysis general, we keep certain directions compact. The correct result for the non-compact case is recovered when the radii are take to infinity.

In section 2 we obtain the boundary state for the oblique $m_1$-brane and interaction of two non-intersecting angled mixed branes for the bosonic part of the theory. In section 3 we develop these, for the NS-NS and the R-R sectors of superstring theory. Finally in section 4 we extract the contribution of the massless states on the interaction of the branes.

We denote a brane in the back-ground field by “$m_p$-brane”, which is a “mixed brane” with dimension “$p$”.

2 The bosonic part

Boundary state

Previously we obtained the boundary state of a $m_p$-brane [3]. For the $m_1$-brane along the $X^1$-direction with field strength $F_{01} = E$, the boundary state satisfies the following equations

\[(\partial_\tau X^0 - E \partial_\sigma X^1)|_{\tau_0} = 0 \]  
\[(\partial_\tau X^1 - E \partial_\sigma X^0)|_{\tau_0} = 0 \]  
\[[X^i(\sigma, \tau) - y^i]|_{\tau_0} = 0 \]
In these equations the set \( \{ y^i \} \) shows the position of the \( m_1 \)-brane with \( i \in \{ 2, 3, \ldots, d-1 \} \). With equation (3) we have fixed the position of the \( m_1 \)-brane. Also \( \tau_0 \) is the \( \tau \) variable on the boundary of the closed string world sheet. Now consider a \( m_1 \)-brane in the \( X^1X^2 \)-plane, which makes angle \( \theta \) with \( X^1 \)-direction. For this brane, boundary state equations have the form,

\[
\left( \partial_\tau X^0 - E \cos \theta \partial_\sigma X^1 - E \sin \theta \partial_\sigma X^2 \right)_{\tau_0}| \ B_x, \tau_0 \rangle = 0 ,
\]

(4)

\[
\left( \cos \theta \partial_\tau X^1 + \sin \theta \partial_\tau X^2 - E \partial_\sigma X^0 \right)_{\tau_0}| \ B_x, \tau_0 \rangle = 0 ,
\]

(5)

\[
\left( -(X^1 - y^1) \sin \theta + (X^2 - y^2) \cos \theta \right)_{\tau_0}| \ B_x, \tau_0 \rangle = 0 ,
\]

(6)

\[
(X^j - y^j)_{\tau_0}| \ B_x, \tau_0 \rangle = 0 , \quad j \neq 0, 1, 2.
\]

(7)

In terms of the modes \( X^\mu(\sigma, \tau) \) has the form,

\[
X^\mu(\sigma, \tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu \sigma + \frac{i}{\sqrt{2\alpha'}} \sum_{m \neq 0} \frac{1}{m} \left( \alpha_m e^{-2im(\tau-\sigma)} + \tilde{\alpha}_m e^{-2im(\tau+\sigma)} \right),
\]

(8)

where \( L^\mu \) is zero for non-compact directions. For compact directions we have \( L^\mu = N^\mu R^\mu \) and \( p^\mu = \frac{M^\mu}{R^\mu} \), in which \( N^\mu \) is the winding number and \( M^\mu \) is the momentum number of the closed string state, and \( R^\mu \) is the radius of compactification of \( X^\mu \)-direction. Suppose \( X^0, X^1 \) and \( X^2 \) to be in the set of compact directions \( \{ X^{\mu c} \} \), therefore boundary state equations (4)-(7) in terms of the modes have the following forms,

\[
\left( p^0 - \frac{1}{\alpha'} E(L_1 \cos \theta + L_2 \sin \theta) \right)_{op}| \ B_x, \tau_0 \rangle = 0 ,
\]

(9)

\[
(p^1 \cos \theta + p^2 \sin \theta - \frac{1}{\alpha'} E L^0)_{op}| \ B_x, \tau_0 \rangle = 0 ,
\]

(10)

\[
\left( -(x^1 + 2\alpha' \tau_0 p^1 - y^1) \sin \theta + (x^2 + 2\alpha' \tau_0 p^2 - y^2) \cos \theta \right)_{op}| \ B_x, \tau_0 \rangle = 0 ,
\]

(11)

\[
(L_1 \sin \theta - L_2 \cos \theta)_{op}| \ B_x, \tau_0 \rangle = 0 ,
\]

(12)

\[
(x^j + 2\alpha' \tau_0 p^j - y^j)_{op}| \ B_x, \tau_0 \rangle = 0 ,
\]

(13)
The matrix $\Omega$ is orthogonal, therefore
\[ L^2_{op} | B_x, \tau_0 \rangle = 0 , \] (14)
for the zero modes, and
\[
\left(\left[\alpha_m^0 + E(\alpha_m^1 \cos \theta + \alpha_m^2 \sin \theta)\right]e^{-2im\tau_0} + \left[\bar{\alpha}_m^0 - E(\bar{\alpha}_m^1 \cos \theta + \bar{\alpha}_m^2 \sin \theta)\right]e^{2im\tau_0}\right) | B_x, \tau_0 \rangle = 0 ,
\] (15)
\[
\left(\left(\alpha_m^1 \cos \theta + \alpha_m^2 \sin \theta + E\alpha_m^0\right)e^{-2im\tau_0} + \left(\bar{\alpha}_m^1 \cos \theta + \bar{\alpha}_m^2 \sin \theta - E\bar{\alpha}_m^0\right)e^{2im\tau_0}\right) | B_x, \tau_0 \rangle = 0 ,
\] (16)
\[
\left((-\alpha_m^1 \sin \theta + \alpha_m^2 \cos \theta)e^{-2im\tau_0} - (-\bar{\alpha}_m^1 \sin \theta + \bar{\alpha}_m^2 \cos \theta)e^{2im\tau_0}\right) | B_x, \tau_0 \rangle = 0 ,
\] (17)
\[
\left(\alpha_m^j e^{-2im\tau_0} - \bar{\alpha}_m^j e^{2im\tau_0}\right) | B_x, \tau_0 \rangle = 0 ,
\] (18)
for the oscillatory modes. Note that $j \in \{3, 4, ..., d-1\}$. The oscillating parts can be written as
\[
\left(\alpha_m^\mu e^{-2im\tau_0} + S_{\mu \nu}^{\alpha} \bar{\alpha}_m^{-\nu} e^{2im\tau_0}\right) | B_x, \tau_0 \rangle = 0 ,
\] (19)
where the matrix $S$, which depends on angle $\theta$ and the electric field $E$ is,
\[
S_{\mu \nu}^{\alpha}(\theta, E) = (\Omega^\alpha_{\beta}, -\delta^i_k) , \quad \alpha, \beta = 0, 1, 2 ,
\] (20)
\[
\Omega^\alpha_{\beta} = \frac{1}{1 - E^2} \begin{pmatrix}
1 + E^2 & -2E \cos \theta & -2E \sin \theta \\
-2E \cos \theta & E^2 + \cos 2\theta & \sin 2\theta \\
-2E \sin \theta & \sin 2\theta & E^2 - \cos 2\theta
\end{pmatrix} .
\] (21)
The matrix $\Omega$ is orthogonal, therefore $S$ is also an orthogonal matrix, (note that $(\Omega^T)^{\alpha \beta} = \eta^{\alpha \gamma} \eta_{\beta \gamma} \Omega^{\beta \alpha}$ with $\eta_{\mu \nu} = diag(-1, 1, ..., 1)$).

The boundary state equations (9)-(14) and (19) have the following solution
\[
| B_x, \tau_0 \rangle = \frac{T}{2} \sqrt{1 - E^2} \exp \left[i \alpha' \tau_0 \left((-p_{op}^1 \sin \theta + p_{op}^2 \cos \theta)^2 + \sum_{j=3}^{d-1} (p_{op}^j)^2\right)\right]
\times \left[\prod_{j=3}^{d-1} \delta(x^j - y^j)\right] \exp \left[-\sum_{\rho \rho'} \sum_{p^1, p^2} \sum_{p^1, p^2} (| p^0 \rangle | p^1 \rangle | p^2 \rangle) \prod_{j=3}^{d-1} | p_{j L}^i = p_{j R}^i = 0\rangle\right]
\times \prod_{m=1}^{\infty} \left(\frac{1}{m} e^{4im\tau_0} \alpha_{m}^{\mu} S_{\mu \nu}(\theta, E) \bar{\alpha}_{m}^{\nu}\right) \left| 0 \right\rangle ,
\] (22)
where $T = \frac{\sqrt{\pi}}{2^{(d-6)/4}}(4\pi^2\alpha')^{(d-6)/4}$ is the tension of $D_1$-brane in $d$-dimension \[5\]. The last line is the solution of equation (19) and other factors are the solutions of equations (9)-(14). The left and right components of the momentum states $|p^\alpha\rangle = |p^\alpha_L\rangle |p^\alpha_R\rangle$ that are appeared in this state have the following relations

\begin{align*}
 p^0 &= \frac{1}{\alpha'} E (\ell_1 \cos \theta + \ell_2 \sin \theta) , \\
 p^1 &= \frac{1}{\alpha'} E \ell_1 \cos \theta , \\
 p^2 &= \frac{1}{\alpha'} E \ell_1 \sin \theta , \\
 \ell_1 \sin \theta &= \ell_2 \cos \theta ,
\end{align*}

where $p^\mu = p^\mu_L + p^\mu_R$ and $\ell^\mu = \alpha' (p^\mu_L - p^\mu_R) = N^\mu R^\mu$. We must consider equations (23)-(26) in summing over $p^0, p^1$ and $p^2$ in (22). Energy of the closed string state depends on its winding numbers around the $X_1$ and $X_2$ directions. According to the equations (24) and (25) for compact time, closed string state has non-zero momentum along the directions $X_1$ and $X_2$. Therefore its momentum numbers, along these directions (i.e. $M_1$ and $M_2$) are proportional to its winding number, around the time direction. Closed string can wind around the directions $X_1$ and $X_2$ if angle $\theta$ and radii of compactification $R_1$ and $R_2$ are such that the quantity $\frac{R_1 \sin \theta}{R_2 \cos \theta}$ is rational, otherwise $N_1 = N_2 = 0$, i.e. closed string has no winding around the $X_1$ and $X_2$, in this case its energy also is zero.

The ghost part of the boundary state is independent of $E$ and angle $\theta$, it is

\begin{equation}
|B_{gh}, \tau_0\rangle = \exp \left[ \sum_{m=1}^{\infty} e^{4i m \tau_0} (c_{-m} \bar{b}_{-m} - b_{-m} \bar{c}_{-m}) \right] \frac{c_0 + \bar{c}_0}{2} \mid q = 1 \rangle \mid \bar{q} = 1 \rangle .
\end{equation}

**Interaction**

Now we can calculate the overlap of the two boundary states to obtain the interaction amplitude of non-intersecting angled $m_1$ and $m_1'$ branes. Let $m_1'$ also be parallel to the $X_1 X_2$-plane with electric field $E'$ on it, therefore boundary state that describes it, is given by equations (22)-(26) with the change $E \rightarrow E'$, $y \rightarrow y'$ and $\theta \rightarrow \theta'$. These mixed branes simply interact via exchange of the closed strings, so that the amplitude is given by

\begin{equation}
A = \langle B_{1'}, \tau_0 = 0 \mid D \mid B_1, \tau_0 = 0 \rangle ,
\end{equation}
where “$D$” is the closed string propagator. In this amplitude we must use the total boundary state, i.e. \(|B, \tau_0\rangle = |B_x, \tau_0\rangle \ |B_y, \tau_0\rangle\). Here we only give the final result;

$$
\mathcal{A}_{bos} = \frac{T^2 \alpha' L}{4(2\pi)^{d-2} | \sin \phi |} \sqrt{(1 - E^2)(1 - E'^2)} \int_0^\infty dt \{ e^{i\alpha' t} \left( \frac{\pi}{\alpha' t} \right)^{d_{j_n}} \}
$$

\begin{multline}
\times e^{-\frac{i}{\alpha' t} \sum_j (y_j^m - y_j^m)^2} \prod_{j_c} \Theta_3 \left( \frac{y_{j_c}^c - y_{j_c}^c}{2\pi R_{j_c}} \right) \left| \frac{i\alpha' t}{\pi R_{j_c}^2} \right|
\times \prod_{n=1}^\infty \left[ (1 - e^{-4nt})^{5-d} \left| \text{det} (1 - \Omega' \Omega^T e^{-4nt}) \right|^{-1} \right] \Theta_3 (\nu | \tau) \right] \right) ,
\end{multline}

where \( L = 2\pi R_0 \) is time length, \( \phi = \theta - \theta' \). Also \( \nu \) and \( \tau \) have definitions,

$$
\nu = \frac{R_0}{2\pi \alpha' \sin \phi} \left( (E - E' \cos \phi) \bar{y}_2^c + (E' - E \cos \phi) \bar{y}_2^c \right) ,
$$

$$
\tau = \frac{i\alpha' \pi R_0^2}{\sin^2 \phi} \left( \frac{E^2 + E'^2 - 2EE' \cos \phi}{\sin^2 \phi} - 1 \right) ,
$$

and the determinant is

$$
\text{det} (1 - \Omega' \Omega^T e^{-4nt}) = (1 - e^{-4nt}) \left[ 1 - 2e^{-4nt} \left( 1 + \frac{2(EE' - \cos \phi)^2}{(1 - E^2)(1 - E'^2)} \right) + e^{-8nt} \right] .
$$

The set \{\bar{y}_2, y_3, ..., y_{d-1}\} shows the position of \( m_1 \)-brane, with \( \bar{y}_2 = -y_1 \sin \theta + y_2 \cos \theta \) and \( y_1 \cos \theta + y_2 \sin \theta = 0 \), similarly for \( \bar{y}_2' \). The sets \{\bar{j}_n\} and \{\bar{j}_c\} are non-compact and compact part of \{\bar{j}\}, and \( d_{j_n} \) is dimension of \{\bar{X}^j_n\}. Because of the electric fields, this amplitude is not symmetric under the exchange \( \phi \leftrightarrow \pi - \phi \), on the other hand for angled mixed branes \( \phi \) and \( \pi - \phi \) are two different configurations. From (30) we see that electric fields and compactification of time cause \( \bar{y}_2' \) and \( \bar{y}_2 \) to appear in the interaction. For non-compact time these disappear from the interaction, i.e. \( \Theta_3 (\nu | \tau) = 1 \), as expected. The amplitude (29) is symmetric with respect to the \( m_1 \) and \( m_{1'} \) branes,

$$
\mathcal{A}(E, E', y_1, y_1', y_2, y_2', \bar{y}_2, \bar{y}_2', \theta, \theta') = \mathcal{A}^*(E', E, y_1, y_1', y_2, y_2', \bar{y}_2, \bar{y}_2', \theta, \theta)
$$

for complex conjugation see (28). It is also independent of the compactification of directions \( X^1 \) and \( X^2 \). For non-compact spacetime, remove all factors \( \Theta_3 \) from (29) and change \( j_n \rightarrow j \) therefore \( d_{j_n} \rightarrow d - 3 \), in this case interaction depends on the minimal distance between the branes, that is \( \sum_j (y_j^3 - \bar{y}_j^3)^2 \). When all directions are compact, the factors containing \( j_n \) disappear, in this case \( j_c \in \{3, 4, ..., d - 1\} \).

### 3 Interaction in superstring theory

Boundary state
The boundary conditions on the fermionic degrees of freedom should be imposed on both \( R \otimes R \) and \( NS \otimes NS \) sectors. World sheet supersymmetry requires the two sectors to satisfy the boundary conditions,

\[
\left( \psi^0 - i\eta \tilde{\psi}^0 \right) + E \left( \psi^1 + i\eta \tilde{\psi}^1 \right) \right|_{\tau_0} | B_\psi, \eta, \tau_0 \rangle = 0 \ ,
\]

(33)

\[
\left( \psi^1 - i\eta \tilde{\psi}^1 \right) + E \left( \psi^0 + i\eta \tilde{\psi}^0 \right) \right|_{\tau_0} | B_\psi, \eta, \tau_0 \rangle = 0 \ ,
\]

(34)

\[
\left( \psi^i + i\eta \tilde{\psi}^i \right) \right|_{\tau_0} | B_\psi, \eta, \tau_0 \rangle = 0 \ ,
\]

(35)

for the \( m_1 \)-brane along \( X^1 \)-direction [8]. The parameter \( \eta = \pm 1 \) is introduced for the GSO projection. This state preserves half of the world sheet supersymmetry. For the rotated \( m_1 \)-brane that makes angle \( \theta \) with \( X^1 \)-direction, we must rotate \( \psi^1 \) and \( \psi^2 \), therefore we find the following boundary states,

\[
| B_\psi, \eta, \tau_0 \rangle_{NS} = \exp \left[ i \eta \sum_{r=1/2}^\infty \left( e^{4ir\tau_0} b^\mu_r S_{\mu\nu}(\theta, E) \tilde{b}^{\nu}_r \right) \right] | 0 \rangle \ ,
\]

(36)

for the NS-NS sector, and

\[
| B_\psi, \eta, \tau_0 \rangle_R = \frac{1}{\sqrt{1 - E^2}} \exp \left[ i \eta \sum_{m=1}^\infty \left( e^{4im\tau_0} d^\mu_m S_{\mu\nu}(\theta, E) \tilde{d}^{\nu}_m \right) \right] | B_\psi, \eta, \rangle^{(0)}_R \ ,
\]

(37)

for the R-R sector. The origin of the factors \( \sqrt{1 - E^2} \) in (22) and (37) is in the path integral with boundary action [9]. Zero mode part of the boundary state satisfies

\[
\left( d^\mu_0 - i\eta S^\mu_{\nu}(\theta, E) \tilde{d}^\nu_0 \right) | B_\psi, \eta, \rangle^{(0)}_R = 0 \ .
\]

(38)

The vacuum for the fermionic zero modes \( d^\mu_0 \) and \( \tilde{d}^\mu_0 \) can be written as [9]

\[
| A \rangle | \tilde{B} \rangle = \lim_{z, \bar{z} \to 0} S^A(z) \tilde{S}^B(\bar{z}) | 0 \rangle \ ,
\]

(39)

where \( S^A \) and \( \tilde{S}^B \) are the spin fields in the 32-dimensional Majorana representation. We use a chiral representation for the \( 32 \times 32 \) \( \Gamma \)-matrices of \( SO(1,9) \) as in reference [9], therefore we consider solution of (38) of the form

\[
| B_\psi, \eta \rangle^{(0)}_R = \mathcal{M}^{(n)}_{AB} | A \rangle | \tilde{B} \rangle \ ,
\]

(40)

therefore the \( 32 \times 32 \) matrix \( \mathcal{M}^{(n)} \) obeys the following equation

\[
\left( \Gamma^\mu \right)^T \mathcal{M}^{(n)} - i\eta S^\mu_{\nu}(\theta, E) \Gamma_{11} \mathcal{M}^{(n)} \Gamma^\nu = 0 \ .
\]

(41)
This equation has the solution
\[ \mathcal{M}^{(n)} = C \Gamma^0 (-E \Gamma^0 + \Gamma^1 \cos \theta + \Gamma^2 \sin \theta) \left( \frac{1 + i \eta \Gamma_{11}}{1 + i \eta} \right), \quad (42) \]
where \( C \) is the charge conjugation matrix.

The superghost part of the NS-NS sector boundary state in the \((-1, -1)\) picture is
\[ |B_{sgh}, \eta, \tau_0\rangle_{NS} = \exp \left[ i \eta \sum_{r=1/2}^{\infty} e^{4ir\tau_0} (\gamma_{-r} \tilde{\beta}_{-r} - \beta_{-r} \gamma_{-r}) \right] |P = -1, \tilde{P} = -1\rangle, \quad (43) \]
and for the R-R sector boundary state in the \((-1/2, -3/2)\) picture is
\[ |B_{sgh}, \eta, \tau_0\rangle_R = \exp \left[ i \eta \sum_{m=1}^{\infty} e^{4im\tau_0} (\gamma_{-m} \tilde{\beta}_{-m} - \beta_{-m} \gamma_{-m}) + i \eta \gamma_{0\beta_0} \right] |P = -1/2, \tilde{P} = -3/2\rangle, \quad (44) \]
where the superghost vacuum is annihilated by \( \beta_0 \) and \( \tilde{\gamma}_0 \) [10].

For both the NS-NS and the R-R sectors the complete boundary state can be written as the following product
\[ |B, \eta, \tau_0\rangle_{R,NS} = |B_x, \tau_0\rangle |B_{sgh}, \eta, \tau_0\rangle_{R,NS} |B_{sgh}, \eta, \tau_0\rangle_{R,NS} . \quad (45) \]

**Interaction**

For calculation of the interaction amplitude, we must use the GSO projected boundary states
\[ |B, \tau_0\rangle = \frac{1}{2} (|B, +, \tau_0\rangle + |B, -, \tau_0\rangle) , \quad (46) \]
the minus sign is for the NS-NS and plus sign for the R-R sector. In each sector the amplitude is given by (28) in which boundary states must be replaced from (46). Finally the total amplitude, \( \mathcal{A} = \mathcal{A}_{NS-NS} + \mathcal{A}_{R-R} \), becomes
\[
\mathcal{A} = \frac{T^2 \alpha' L}{8(2\pi)^8 |\sin \phi|} \int_0^\infty dt \left\{ \left( \frac{\pi}{\alpha' t} \right)^{d_{ju}} e^{-\frac{\alpha' t}{4\pi} \sum_j (y_j')^2} \right\}
\times \Theta_3(\nu | \tau) \prod_{j_c} \Theta_3 \left( \frac{y_j^j - y_j^j}{2\pi R_{j_c}} \middle| \frac{i\alpha' t}{\pi(R_{j_c})^2} \right) \left( \left( 1 - E^2 \right) \left( 1 - E'^2 \right) \right)
\times \frac{1}{q} \left[ \prod_{n=1}^{\infty} \left[ \left( 1 - q^{2n-1} \right)^{5 \det(1 + \Omega' T q^{2n-1})} \det(1 - \Omega' T q^{2n}) \right] \right]
\times \prod_{n=1}^{\infty} \left[ \left( 1 - q^{2n-1} \right)^{5 \det(1 - \Omega T q^{2n})} \det(1 - \Omega T q^{2n}) \right] \right] \right]
- 16(\cos \phi - EE') \prod_{n=1}^{\infty} \left[ \left( 1 + q^{2n} \right)^{5 \det(1 + \Omega T q^{2n})} \det(1 - \Omega T q^{2n}) \right] \right) \right) . \quad (47)
where \( q = e^{-2t} \). The last line comes from the R-R sector. The factor \((\cos \phi - EE')\) is contribution of the fermionic zero modes, according to the sign and value of this factor, R-R interaction is repulsive, attractive or zero. The other two terms come from the NS-NS sector. Note that determinants in the denominators come from the world sheet bosons and in the numerators from the fermions. These determinants have the expansion like (31), that \( e^{-4nt} \) should be changed to \( \pm q^{2n} \) and \( \pm q^{2n-1} \). This amplitude is symmetric with respect to \( m_1 \) and \( m_1' \) branes. Again for non-compact spacetime, in (47) remove all factors \( \Theta_3 \) and change \( j_n \rightarrow j \).

4 Interaction due to the massless states

Now from the interaction amplitude (47), we extract the contributions of the NS-NS and the R-R sectors massless states, to see how distant branes interact. Therefore we have the following limits [3, 8],

\[
\lim_{q \to 0} \frac{1}{q} \left\{ \prod_{n=1}^{\infty} \left[ \left( \frac{1 + q^{2n-1}}{1 - q^{2n}} \right)^5 \frac{\det(1 + \Omega' \Omega^T q^{2n-1})}{\det(1 - \Omega' \Omega^T q^{2n})} \right] \right. \\
- \left. \prod_{n=1}^{\infty} \left[ \left( \frac{1 - q^{2n-1}}{1 - q^{2n}} \right)^5 \frac{\det(1 - \Omega' \Omega^T q^{2n-1})}{\det(1 - \Omega' \Omega^T q^{2n})} \right] \right\} = 2 \left[ Tr(\Omega' \Omega^T) + 5 \right] = 8 (1 - E^2)(1 - E'^2) \]

\((48)\)

for the NS-NS sector, and

\[
\lim_{q \to 0} \prod_{n=1}^{\infty} \left[ \left( \frac{1 + q^{2n}}{1 - q^{2n}} \right)^5 \frac{\det(1 + \Omega' \Omega^T q^{2n})}{\det(1 - \Omega' \Omega^T q^{2n})} \right] = 1 \]

\((49)\)

for the R-R sector.

In the NS-NS sector, exchange of the massless states has the amplitude

\[
A_{NS-NS}^0 = \frac{T^2 \alpha' L}{(2\pi)^8 |\sin \phi|} \frac{1 + \cos^2 \phi + 2 E^2 E'^2 - (E^2 + E'^2 + 2 EE' \cos \phi)}{\sqrt{(1 - E^2)(1 - E'^2)}} G \, ,
\]

\((50)\)

\[
G \equiv \int_0^\infty dt \left\{ \left( \frac{\pi}{\alpha' t} \right)^{d_j} e^{-\frac{\pi}{\alpha' t} \sum_j (y'^j_n - y'^j_n)^2} \Theta_3(\nu | \tau) \prod_{j_c} \Theta_3(\frac{y^{j_c} - y^{j_c}}{2\pi R_{j_c}} | \frac{i \alpha' t}{\pi(R_{j_c})^2}) \right\} .
\]

\((51)\)

According to the (50), the terms \((1 + \cos^2 \phi + 2 E^2 E'^2)\) have attractive and \(-(E^2 + E'^2)\) have repulsive effects, the term \(-EE' \cos \phi\) can have attractive or repulsive effect due to the signs of \(E, E'\) and \(\cos \phi\). But, sum of all these terms is always positive, therefore exchange of the
massless states of the NS-NS sector produces attractive force between these branes. In the $R - R$ sector, massless states have the following contribution on the interaction
\[ A_{0}^{(R-R)} = \frac{T^2 \alpha' L}{(2\pi)^8 \sin \phi} [\cos \phi - E E'] G , \] (52)
according to the factor $(\cos \phi - E E')$, this interaction can be attractive, repulsive or zero. Therefore distant branes have interaction amplitude $A_{0} = A_{0}^{(NS-NS)} + A_{0}^{(R-R)}$. This is proportional to the factor $(\cos \phi - \cos \phi_0)^2$, where
\[ \cos \phi_0 = EE' + \sqrt{(1 - E^2)(1 - E'^2)} , \] (53)
so at $\phi = \phi_0$, attractive force of the NS-NS sector cancels the repulsive force of the R-R sector.

For non-compact spacetime, the function $G$ is proportional to the Green’s function in seven dimensional space, i.e.
\[ G_{(nc)} = \frac{1}{\alpha'} (2\pi)^7 G_7(Y^2) \] (54)
where $Y^2$ is minimal distance between the branes. In this space if $E = E' = 0$, then massless states amplitude $A_0$ reduces to the [11] with $p = 1$.

5 Conclusion
We explicitly determined the boundary state for both the NS-NS and R-R sectors of superstring theory, corresponding to a mixed brane parallel to the $X^1 X^2$-plane. Energy of a closed string state emitted from this brane depends on its winding numbers around $X^1$ and $X^2$ directions, radii of compactification of these directions and back-ground internal electric field. Also, for compact time closed string state can have non-zero momentum along the brane.

Compactification of time and non-zero electric fields imply that interaction should depend on the positions $\bar{y}_2'$ and $\bar{y}_2$ of the branes. Depending on the back-ground fields and angle between the branes, R-R interaction is repulsive, attractive, or zero. For both of the NS-NS and the R-R sectors, we extracted contribution of the massless states on the interaction. For non-compact spacetime, these are proportional to the Green’s function of seven dimensional space.

The formalism can be extended to include mixed branes with arbitrary dimensions $p_1$ and $p_2$ and more than one angle.

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