SCREENING CORECTIONS IN DIS

AT LOW $Q^2$ and $x$

E. Gotsman$^a$, E. Ferreira$^b$, E. Levin$^c$, U. Maor$^d$ and E. Naftali$^a$

$^a$HEP Department
School of Physics and Astronomy,
Raymond and Beverly Sackler Faculty of Exact Science,
Tel-Aviv University, Ramat Aviv, 69978, Israel

$^b$Instituto de Fisica, Universidade Federal do Rio de Janeiro
Rio de Janeiro RJ21945-970, BRASIL

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*E-mail: gotsman@post.tau.ac.il
†E-mail: erasmo@if.ufrj.br
‡E-mail: leving@post.tau.ac.il, elevin@quark.phy.bnl.gov
§E-mail: maor@post.tau.ac.il
¶E-mail: erann@post.tau.ac. 
Abstract

We expect that s-channel unitarity should materialize in hard DIS reactions through screening corrections (SC) indicating that the gluon distribution function is approaching saturation, it is not as yet clear what the kinematical scales are at which these effects become important. While the global DIS $\gamma^* p$ total cross section, or $F_2(x,Q^2)$, data are well reproduced by DGLAP evolution without substantial SC, there exists experimental data from HERA which suggests deviations from DGLAP predictions in the small $Q^2$ and $x$ limits. These signatures are observed in both the fine details of $F_2(x,Q^2)$ provided $Q^2$ and $x$ are small enough, as well as in the diffractive channels. In this investigation we present a detailed study of $\partial F_2/\partial \ln Q^2$ which is supported by a coupled analysis of $J/\Psi$ photoproduction and DIS production. Both channels are directly proportional to $xG(x,Q^2)$, and as such serve as excellent discriminators between different approaches and models. In the first phase of our investigation we have found that none of the latest editions of the parton distribution functions (GRV98, MRS99, CTEQ5) provides an adequate and simultaneous reproduction of $Q^2$ logarithmic slope of $F_2$ at small $Q^2$ values as well as $J/\Psi$ photoproduction (Details of this will be published separately \[\text{[1]}\]). We then show that taking GRV98NLO as input and correcting it for SC, we can reproduce the recent HERA data well. The calculation depends on one parameter $R^2 = 8.5 GeV^{-2}$ which is directly determined from the $J/\Psi$ photoproduction forward differential slope. With this input we obtain an excellent fit to the $J/\Psi$ photo and DIS production data. Our calculations made in the LLA of pQCD take into account the corrections implied by the real part of the production amplitude, off diagonal (skewed) gluon distributions and the Fermi motion of the charm quarks within the bound Charmonium system. The SC are consistently calculated for both the percolation of a $q\bar{q}$ through the target and the screening of the gluon parton distribution which forms the base of our calculation. Our main conclusion is that, whereas we find strong support for the need for SC in the small $Q^2$ and $x$ limits of the channels we have investigated, the latest HERA data is not sufficiently precise to directly determine the gluon saturation scale.
1 Introduction

Over the past few years we have been witness to vigorous experimental, phenomenological and
theoretical investigations of the proton deep inelastic scattering (DIS) structure functions and
some exclusive channels with small $Q^2 < 5 GeV^2$ and $x < 10^{-2}$. These comprehensive studies
aim at establishing the applicability and possible need for a re-formulation of pQCD, as we
know it, when approaching the kinematic interface with the less understood npQCD dominated
domain. The standard procedure for the pQCD analysis of DIS on a nucleon target has been
to utilize the DGLAP evolution equations for the structure functions as the key ingredient for
fixing the parton distribution functions (p.d.f.). These p.d.f. are then used as input for the
calculations of exclusive DIS channels, usually executed in the color dipole approximation.

The physics of small $Q^2$ and small $x$ is associated with the search for the scale of gluon
saturation implied by s-channel unitarity \([2]\). One should remember, though, that gluon sat-
suration signals the transition from perturbative to non perturbative QCD. This transition is
preceded by SC signatures which are expected to be experimentally visible even though the
relevant scattering amplitude has not yet reached the unitarity black limit. Moreover, from
our experience with soft Pomeron physics, we know that different channels have different scales
at which unitarity corrections become appreciable. Specifically, the scale associated with the
diffractive channels are considerably smaller than those associated with the elastic channel \([4]\).

Inspite of significant theoretical progress in recent years \([3]\), it is still not clear what the
saturation scale in the present experimentally accessible kinematic region is. We recall that,
while the global analysis of $F_2(x, Q^2)$ (or $\sigma_{tot}^\gamma p(W, Q^2)$) data shows no conclusive deviations
from DGLAP, there are dedicated HERA investigations suggesting deviations from the DGLAP
expectations in the small $Q^2$ and $x$ limits. These signatures are observed in both the fine details
of $F_2(x, Q^2)$ as well as in the diffractive channels, provided $Q^2$ and $x$ are small enough.

In the following we present a detailed study of $\partial F_2/\partial \ln Q^2$ which is supported by a coupled
analysis of $J/\Psi$ photo and DIS production. The strategy of our investigation is based on the
observation that these observables ( in LLA of pQCD ) are linear in $xG(x, Q^2)$ and $(xG(x, Q^2))^2$
respectively, and being relatively well measured may serve as effective discriminators when we
compare their detailed features with existing relevant theoretical approaches and models. In
the first phase of our investigation, we found that none of the latest p.d.f. \([3][4][5]\) provides an
adequate simultaneous reproduction, at small pQCD scales, of the recent HERA data on the
logarithmic slope of $F_2$ \([3][4][5]\) at small $Q^2$ and $x$, as well as the abundant high energy data on
$J/\Psi$ photo and DIS production \([10][11][12]\). We then proceed to show that when GRV98NLO is
corrected for SC, as suggested in our previous publications \([13][14]\), it gives a good description
of these data. The SC calculation depends on one parameter, $R^2 = 8.5 GeV^{-2}$, which is directly
deduced from the $J/\Psi$ photoproduction forward differential slope. We elaborate on a recent
suggestion \([15]\) that the gluon saturation scale may be determined by examining the behaviour of
$\partial F_2(x, Q^2)/\partial \ln Q^2$ against $Q^2$ and $x$ at fixed $W$ values. In our opinion, this suggestion which was
discussed also in a recent presentation of the ZEUS new data \([1]\), reflects the particular kinematic
relationship between $Q^2, W^2$ and $x$ and does not provide an unambiguous determination of the
desired scale.

Our approach can be tested in the high energy analysis of photo and DIS exclusive produc-
tion of $J/\Psi$. This is seemingly a straightforward procedure as the LLA pQCD calculation of
this cross section is proportional to $(xG(x, Q^2))^2$, where $x$ and $Q^2$ are determined from $m_c$, the
c-quark mass, and $W$, the c.m. energy. A realistic calculation depends on a few corrections to
the original pQCD estimate: $C_R^2$ due to the amplitude real part, $C_g^2$ due to the contribution of
the off diagonal (skewed) gluon distribution \([16]\) and $C_F^2$ due to the Fermi motion deviations
from the simple static non relativistic estimate of the $J/\Psi$ wave function \([17]\). All of these cor-
rections contribute to the final (amplitude squared) estimate. We present a detailed analysis of
this channel which is consistent with our approach and input choices made in the analysis.
of $\partial F_2/\partial \ln Q^2$.

We conclude with some general comments and observations on the outstanding problems of gluon saturation and screening corrections.

2 The small $Q^2$ and $x$ behavior of $\partial F_2/\partial \ln Q^2$

Checking for unitarity corrections in $F_2$ studies is not simple. As is well known, a global DGLAP analysis of the data with the recent p.d.f. \[6\] is adequate. A study \[18\] comparing the screened and non screened DGLAP calculations of $F_2(x, Q^2)$ showed only a small difference due to SC even in the small $Q^2$ and $x$ attained by present HERA measurements. Clearly, a unitarity study in the above kinematic limit requires a dedicated investigation, confined to small $Q^2$ and $x$, which can magnify the presumed experimental signatures. We recall that in the small $x$ limit of DGLAP we have

$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} = \frac{2\alpha_S}{9\pi} x G^{DGLAP}(x, Q^2).$$

(1)

Accordingly, a significant deviation of the data from Eq. (1), where $x G^{DGLAP}$ is obtained from the global $F_2$ analysis, may serve as an experimental signature indicating the growing importance of unitarity corrections. This was first suggested by Caldwell \[19\], showing a rather complicated plot of $\partial F_2/\partial \ln Q^2$ in which each point had different $Q^2$ and $x$ values. The Caldwell plot suggested a dramatic turn over of $\partial F_2/\partial \ln Q^2$ corresponding to $Q^2$ of about $3 GeV^2$ and $x < 5 \cdot 10^{-3}$ in contrast to the behavior expected from GRV94 \[20\] at sufficiently small $Q^2$ and $x$. The problem with this presentation is that, as suggestive as it may seem, it does not discriminate between different dynamical interpretations \[13\][14][15][21][22]. It is actually compatible with an overall data generator \[23\] as well as the latest p.d.f. which were re-adjusted to account for this observation. We conclude that, where as a reproduction of the Caldwell plot is required as a pre condition for a serious consideration of any suggested model, the plot on its own can not serve as an effective discriminator between models even in the extreme case when they are fundamentally different.

A far better discrimination is obtained if we carefully study the small $Q^2$ and $x$ dependences of $\partial F_2/\partial \ln Q^2$ at either fixed $Q^2$ or fixed $x$ values being free from the kinematic correlation between $Q^2$ and $x$ that plagued the Caldwell plot. Such preliminary HERA data have recently became available \[8\][9]. As we shall show, a pQCD analysis of these data is consistent with a SC interpretation \[14\].

We follow the eikonal SC formalism presented in Ref.\[4\], where screening is calculated in both the quark sector, to account for the percolation of a $q\bar{q}$ through the target, and the gluon sector, to account for the screening of $x G(x, Q^2)$. The factorizable result that we obtain is

$$\frac{\partial F_2^{SC}(x, Q^2)}{\partial \ln Q^2} = D_q(x, Q^2) D_g(x, Q^2) \frac{\partial F_2^{DGLAP}(x, Q^2)}{\partial \ln Q^2}. $$

(2)

SC in the quark section are given by

$$D_q(x, Q^2) \frac{\partial F_2^{DGLAP}(x, Q^2)}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2} \int dB^2 \left(1 - e^{-\kappa_q(x, Q^2; b^2)}\right),$$

(3)

$$\kappa_q = \frac{2\pi \alpha_S}{3Q^2} x G^{DGLAP}(x, Q^2) \Gamma(b^2).$$

(4)

The calculation is significantly simplified if we assume a Gaussian parameterization for the two gluon non perturbative form factor,

$$\Gamma(b^2) = \frac{1}{R^2} e^{-b^2/R^2}.$$ 

(5)
SC in the gluon sector are given by

$$xG^{SC}(x, Q^2) = D_g(x, Q^2)xG^{DGLAP}(x, Q^2),$$

where

$$xG^{SC}(x, Q^2) = \frac{2}{\pi^2} \int_x^1 \frac{dx'}{x'} \int_0^{Q^2} dQ' \int db^2 \left(1 - e^{-\kappa_g(x', Q^2; b^2)}\right).$$

Note that $$\kappa_g(x', Q^2; b^2) = \frac{\kappa_g(x', Q^2; b^2)}{2}$$ defined in Eq. (4). An obvious difficulty in the above calculation of $$xG^{SC}$$ stems from the fact that the $$Q^2$$ integration spans not only the short (pQCD), but also the long (npQCD) distances. To overcome this difficulty we assume that

$$xG \left(x, Q^2 < \mu^2\right) = \frac{Q^2}{\mu^2} xG \left(x, Q^2\right),$$

where $$\mu^2 \approx Q_0^2$$. Our choice of the above interpolation is motivated by the gauge invariance requirement that $$xG \propto Q^2$$ when $$Q^2 \to 0$$.

The SC calculation described above can be applied to any given input p.d.f. where the only adjusted parameters are $$R^2$$ and $$\mu^2$$. As we shall see in the next section, $$R^2 = 8.5 GeV^{-2}$$ is determined directly from the forward slope of $$J/\Psi$$ photoproduction. $$\mu^2$$ is conveniently fixed at $$Q_0^2$$, the lowest $$Q^2$$ value accessible for the input p.d.f. we use. Once we have chosen our p.d.f., our SC calculation is essentially parameter free. We have checked that our output results are not sensitive to the fine tuning of these fixed parameters.

Our results are presented in Figs. 1, 2 and 3. Throughout this investigation we have used as input the $$MS$$ version of GRV98NLO. Using the GRV98DIS version provides very similar results. Following are some comments relating to our calculations and results:

1) As can be seen, in the limit of small $$Q^2$$ and $$x$$ there is a significant difference between the screened and non screened values of $$\partial F_2/\partial \ln Q^2$$. As expected the SC results are smaller and softer than the non screened input. 2) Visibly, our overall reproduction of the experimental data is very good, in particular when considering that our input is essentially parameter free. A proper $$\chi^2$$ calculation requires the knowledge of the unknown theoretical errors. 3) If we follow the standard procedure and replace the theoretical errors with the experimental ones, we obtain excellent $$\chi^2/ndf = 0.75$$ for 21 H1 data points with the exception of 3 points which are visibly out of line. The ZEUS data has errors which are considerably smaller and as a result our $$\chi^2/ndf$$ is not as good, even though our reproduction of the ZEUS data is reasonable. 4) The ZEUS $$Q^2 = 1.9 GeV^2$$ and H1 $$Q^2 = 3.0 GeV^2$$ data are somewhat softer than our predictions, which do not contain a soft non perturbative background.

Golec-Biernat and Wusthoff have suggested studying the $$Q^2$$ and $$x$$ behaviour of $$\partial F_2/\partial \ln Q^2$$ at fixed $$W$$ as a method to determine the gluon saturation scales from the anticipated turn over in these plots. Recent ZEUS low $$Q^2$$ presentations of these plots show, indeed, the anticipated turn over structure in these figures, seemingly suggesting that gluon saturation is attained at $$Q^2 \simeq 1 GeV^2$$. In our opinion the proper variables to study $$F_2$$ are $$x$$ and $$Q^2$$. Trying to study the structure function by introducing other variables, such as $$W$$, may result in spurious effects which are predominantly kinematic. In the specific procedure suggested by Golec-Biernat and Wusthoff, the combination of the kinematic relation between $$x, Q^2$$ and $$W$$ with the very general behaviour of $$xG(x, Q^2)$$ is sufficient to produce a turn over. Its exact location depends on the details of the numerical input. Consequently, the suggested fixed $$W$$ plots do not convey any new dynamical information even if such information is hidden in the analyzed data. Actually, it seems that any $$F_2$$ (or $$xG$$) parameterization which factorizes the $$Q^2$$ and $$x$$ dependences, such as the Buchmuller- Haidt model, is capable of producing the fixed $$W$$ turn over effects.

To illustrate this point we consider the fixed $$W$$ behaviour in two models which have very different dynamics:
1) Our own GLMN [14], which is a pure pQCD dipole model with SC. As such, our model relates indirectly to gluon saturation, even though it is constructed so as to include unitarity corrections below actual saturation.

2) The DL two Pomeron parametrization [22], which is based on the Regge formalism, and consists of the coherent sum of contributions from a "hard" and a "soft" Pomeron, a normal Reggeon and an additional contribution from the charmed sector which is proportional to the "hard" Pomeron. Each of these fixed j-poles are multiplied by a fitted $Q^2$ form factor. The parameters of the "model" were determined from the requirement of a "best fit" to the experimental $F_2(x, Q^2)$ data, and are not directly associated with the gluon distribution in the proton or "saturation".

The results obtained for the logarithmic slope of $F_2$ at fixed W for both parametrizations are compared to the ZEUS experimental results in Fig.4 and 6. In Figs.5 and 7 we also display the behaviour of $\partial F_2/\partial \ln Q^2$ at fixed $Q^2$ and fixed x. We note that both "models" provide a reasonable description of the ZEUS data including the observed turn over in the fixed W plots. As GRV98 is only applicable for $Q^2_0 \geq 1$ GeV$^2$, we have repeated the GLMN calculation with GRV94 which has $Q^2_0 = 0.4$ GeV$^2$ and again reproduce the ZEUS fixed W turn over effect. From the above it appears that any model which provides a reasonable description of $F_2(x, Q^2)$ will exhibit a turn over in fixed W plots of the logarithmic derivative of $F_2(x, Q^2)$, this occurs due to the relation between the kinematic variables x, $Q^2$ and W, and is not related to "saturation".

However, the examples (the Buchmueller-Haidt model and Donnachie-Landshoff approach) which have been used to demonstrate that the "saturation" is not a unique mechanism for a turn over at fixed W, have one common feature: the "soft" contributions are concentrated at a rather large typical scale $\geq 2$ GeV$^2$. This observation supports the idea that the so called soft Pomeron stems from rather short distances $\leq 0.5$ GeV$^{-2}$.

In conclusion, whereas we find strong support for the need for SC in the small $Q^2$ and x limits for $\partial F_2/\partial \ln Q^2$ we are unable, as yet, to determine the gluon saturation scale directly from the latest HERA data. The gluon saturation scale may be theoretically estimated from the contours produced at the boundary of $\kappa_g = 1$, as discussed in our papers [13][14].

3) Photo and DIS production of $J/\Psi$

The $t=0$ differential cross section of photo and DIS production can be calculated in the dipole LLA approximation [20][21][28][29][30]. When using the static non relativistic approximation of the vector meson wave function [20][28], the differential cross section has a very simple form, viz.

$$\left(\frac{d\sigma}{dt} (\gamma^* p \rightarrow V p)\right)_0 = \frac{\pi^3 \Gamma_{ee} M_V^3}{48 \alpha} \frac{\alpha_s^2 (Q^2)}{Q^8} \left(x G(x, \bar{Q}^2)\right)^2 \left(1 + \frac{Q^2}{M_V^2}\right),$$

(9)

where in the non relativistic limit we have

$$\bar{Q}^2 = \frac{M_V^2 + Q^2}{4},$$

(10)

and

$$x = \frac{4Q^2}{W^2}.$$  

(11)

In the following we discuss in some detail the photo and DIS production of $J/\Psi$. There is a lot of data available for this channel [10][11][12] for the integrated cross section spanning a
Figure 1: $x$ dependence of $H1$ logarithmic slope data at fixed $Q^2$ compared with our calculations.
Figure 2: \( x \) dependence of ZEUS logarithmic slope data at fixed \( Q^2 \) compared with our calculations.
Figure 3: $Q^2$ dependence of H1 and ZEUS logarithmic slope data at fixed $x$ compared with our calculations.
Figure 4: ZEUS logarithmic slope data at fixed $W$ compared with our SC calculation.
Figure 5: Fixed $W$ properties of our SC calculation.
Figure 6: ZEUS logarithmic slope data at fixed $W$ compared with DL two Pomeron model.
Figure 7: Fixed $W$ properties of the DL two Pomeron model.
relatively wide energy range. From a theoretical point of view, its hardness (or separation) scale is comparable to the scales we have studied in our $\partial F_2/\partial \ln Q^2$ analysis. To relate the integrated cross section to Eq. (9) we need to know $B$, the $J/\Psi$ forward differential cross section slope. For this we may use the experimental values, which are approximately constant. As we shall see, SC account well for the reported moderate energy dependence [10].

The main problem with the theoretical analysis of $J/\Psi$ is the observation that the simple pQCD calculation needs to be corrected for the following reasons:

1) A correction for the contribution of the real part of the production amplitude. This correction is well understood and is given by $C_R^2 = (1 + \rho^2)$, where $\rho = ReA/ImA = tg(\frac{\lambda}{2})$ and $\lambda = \partial \ln(xG)/\partial \ln(\frac{Q^2}{4})$.

2) A correction for the contribution of the skewed (off diagonal) gluon distributions [16]. This correction is calculated to be

$$R_g^2 = \left(\frac{2^{2\lambda+3} \Gamma(\lambda+2.5)}{\sqrt{\pi} \Gamma(\lambda+4)}\right)^2. \quad (12)$$

3) A more controversial issue relates to the non relativistic approximation assumed for the $J/\Psi$ Charmonium. Relativistic effects produced by the Fermi motion of the bound quarks result in a considerable reduction of the calculated pQCD cross section [17]. We denote this correction $C_F^2$ and note that it is very sensitive to the value of $m_c$. Ref. [17] assumes that $m_c \approx 1.5 GeV$ and obtains $C_F^2 \approx 0.25$ with minimal energy dependence. A small change in the input value of $m_c$ changes the above estimate significantly. We suggest, therefore, to consider $C_F^2$ as a free parameter. In our calculations we have used $C_F^2 = 0.66$ which corresponds to a c-quark mass of approximately 1.53 GeV.

Since the $J/\Psi$ photo and DIS cross sections are proportional to $(xG(x, Q^2))^2$, the study of this channel can serve as a compatibility check supplementing our study of $\partial F_2/\partial \ln Q^2$. As noted in the introduction our SC approach was triggered by the observation that none of the latest p.d.f.'s can provide a good simultaneous reproduction of the two channels under consideration.

Our calculation of SC for $J/\Psi$ photo and DIS production is rather similar to the $Q^2$ logarithmic slope calculation presented in the previous section. We follow our earlier publication [31] and define the damping factors due to the screening in the quark sector i.e. the percolation of the $c\bar{c}$ through the target. This is given by the following expressions for the longitudinal and transverse dampings

$$D_{qL}^2 = \left(\frac{E_1(\frac{1}{\kappa_q})e^{\frac{1}{\kappa_q}}}{\kappa_q^2}\right)^2 \quad (13)$$

and

$$D_{qT}^2 = \left(1 + (1 - \frac{1}{\kappa_q})E_1(\frac{1}{\kappa_q})e^{\frac{1}{\kappa_q}}\right)^2 \quad (14)$$

Our expression for $D_q^2$, the damping in the gluon sector is the square of the gluon damping defined in the previous section.

Our final expression for the forward cross section is

$$\left(\frac{d\sigma(\gamma^*p \rightarrow J/\Psi p)}{dt}\right)_0 = C_R^2 \cdot C_g^2 \cdot C_F^2 \cdot \left(\frac{d\sigma}{dt}\right)^{pQCD}_0 \cdot D_q^2 \cdot D_g^2, \quad (15)$$

where $D_q$ denotes the L and T components as appropriate.
Our calculations as compared with the data are presented in Figs. 8 and 9. As can be seen, our reproduction of the data is excellent with a $\chi^2/n.d.f.$ which is well below 1. These excellent $\chi^2$ values are maintained when calculating over the entire data base as well as limiting ourselves to the high energy HERA data. Note that $R^2 = 8.5 GeV^{-2}$, which is the essential parameter in the SC calculation is determined directly from the $J/\Psi$ photoproduction forward slope. In a model such as ours, we expect a weak dependence of $B_H$ on the energy (see Fig.10).

4 Discussion

We have shown that NLO GRV98 when screened both for the percolation of a $q\bar{q}$ pair through the target, and for multigluon exchange provides an excellent reproduction of both the H1 and ZEUS data for the logarithmic derivative of $F_2(x, Q^2)$ structure function of the proton. The experimental data and our model are consistent with $\partial F_2/\partial \ln Q^2$ (at fixed $Q^2$) being a monotonic increasing function of $1/x$. No deviation of this behaviour has been seen even at the lowest values of $Q^2$ and $x$.

The suggested turn over seen in $\partial F_2/\partial \ln Q^2$ at fixed $W$, as a function of $x$ or $Q^2$, does not appear to be connected with saturation effects, and cannot be used as a discriminator, as it appears in all parametrizations of $F_2$ which provide a reasonable description of the data.

The same screened NLO GRV98 also provides an excellent description of the photo and DIS production of $J/\Psi$, once corrections are made for the real part of the production amplitude, for the skewed (off diagonal) gluon distribution and for the Fermi motion of the bound quarks in the charmonium wave function.

In a separate publication [1] we show that we are unable to obtain a simultaneous good description of $\partial F_2/\partial \ln Q^2$ and the photo and DIS production of $J/\Psi$ using other pdf’s on the market e.g. MRS99, CTEQ4 and CTEQ5 even after including screening corrections.

5 Conclusions

Only screened NLO GRV98 is able to provide a satisfactory simultaneous description of the latest HERA data available for $\partial F_2/\partial \ln Q^2$ and photo and DIS production of $J/\Psi$.

The moral of the paper is that only by a simultaneous analysis of all data sensitive to shadowing (saturation) effects, can one obtain a reliable estimate of their sizes.

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Figure 8: Photo production of $J/\Psi$ as a function of $W$ and $x$. Data and our calculations.
Figure 9: DIS production of $J/\Psi$. Data and our calculations.
Figure 10: The energy dependence of the forward differential slope of $J/\Psi$ photoproduction. Data and $SC$ calculations with several values of $R^2$. 
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