Incomplete Information Game and Simulation of Logistics Supervision

Yi-cheng GONG*, Yan-na ZHANG, Juan ZHAO, Li YU and Ting PAN
Wuhan University of Science and Technology, China
*Corresponding author

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Abstract. This article aims to improve incomplete information game of logistics supervision and its practice feasibility. For the uncertainty of logistics enterprises’ type, firstly we established an incomplete information game of logistics supervision. Second we carried on Harsanyi transformation to convert it into a complete but imperfect information game. Meanwhile we use Trigonometric distribution to solve uncertainty of logistics enterprises’ type. By investigation, the most likely value is 0.77 in Trigonometric distribution. Lastly, Monte Carlo (MC) simulation is adopted to simulate players’ behavior strategy and expected benefit. The expected benefits of logistics demand enterprises and 3PLs are 33.5667 and 16.5375 respectively. The simulation results show that the randomizing coincides with the probability distribution of the Mix Nash equilibrium above 98.7%.

Introduction

One of the most outstanding achievements of logistics industry in the 1990s was the broad rise of third-party logistics service providers (3PLs) as in [1]. Under the situation of global economic integration, more enterprises contract logistics to 3PLs. As in [2] there are many reasons for outsourcing institute in New York, such as reducing their cost and developing their core business.

Many scholars study logistics by supervision game. In 2016 Z.Q Wang and S.R Sun analyze the cooperation relationship of the third logistics from the demand, supply and the total social income as in [3]. As in [4] H. Liu and L.M. Li use principal-agent theory to study the game between enterprises and 3PLs. They established dynamic game tree without actual risk factor. As in [5] T. Feng indicated the close cooperation can bring many benefits to enterprises, and main reason of risk is information asymmetry. The more asymmetric the information is, the more uncertain factors the game has, and the more risks client will bear. As in [6] S.H. Ghodspour and C.O. Brien in 2001 established a model of mixed integer programming with the information asymmetry and game theory.

The scholars discussing supervision game just converted principal-agent model to deduction of mathematical symbols and gave the algebraic solutions. Or they put forward some suggestions for implementation of supervision qualitatively. This paper use the Harsanyi transformation to convert incomplete information game to a complete but imperfect information game, then we get a Harsanyi game tree. While in fact, it is difficult for game players to randomize their selections according to the probability distribution of mixed strategy Nash equilibrium. And it is still stiff to let Nature select the uncertain types of game players randomly. To solve the first difficulty, we adopt the method as in [7], and for the second difficulty, this paper chooses trigonometric distribution to simulate the probability distribution of the types. And the parameters can be estimated by the prior knowledge of the game.

Harsanyi Transformation of Incomplete Information Games and Trigonometric Distribution

Harsanyi Transformation of Incomplete Information Games

An incomplete information game means at least one player’s information is insufficient. It cannot be analyzed due to its structural uncertainty for a long time. In 1967-1968, Harsanyi propose a transformation to convert an incomplete information game into a complete information dynamic game as in [6]. In which “Nature” is a virtual game player without payoff and objective function.
Harsanyi transformation, firstly *Nature* decide players’ feature. Then all the players move at the same time to select except *Nature*. After Harsanyi transformation, an incomplete information game can be analyzed by theories and methods of the complete information game.

After Harsanyi transformation, Bayesian Nash equilibriums can be attained. A Bayesian Nash equilibrium is a strategy combination \((s_1^*(\theta_1), \ldots, s_n^*(\theta_n))\). Each player maximizes his expectation utility function according to his type \(\theta_i\) and other player’s strategy \(s_j^*(\theta_j)\) as Eq.1.

\[
s_i^*(\theta_i) = \arg\max_{s_i} \sum_{\theta_j} p(\theta_j|\theta_i)\cdot u(s_i, s_j^*(\theta_j), (\theta_i, \theta_j))
\]

where \(u(s_i) = \sum [\prod_{j=1}^n \delta_j(s_j)] u(s_i)\), which is an income function of type combination. And \(p(\theta_j|\theta_i)\) is the probability judgment of the player.

**Trigonometric Distribution**

Trigonometric distribution is a continuous probability distribution. It is suitable for variables which is lack of historical statistical data. If we could obtain the most pessimistic value \((a)\), optimistic value \((b)\) and likely value \((c)\). Then trigonometric distribution has distribution function as Eq.2.

\[
F(x) = \begin{cases} 
0, & x \leq a \\
\frac{(x-a)^3}{(b-a)(c-a)}, & a \leq x \leq c \\
1 - \frac{(b-x)^2}{(b-a)(b-c)}, & c \leq x \leq b \\
1, & \text{others}
\end{cases}
\]

By inverse transformation, we can obtain random trigonometric sample \(u\) as Eq.3.

\[
u = \begin{cases} 
\frac{a + \sqrt{(b-a)(c-a)r}}{(b-a)}, & 0 \leq r \leq (c-a)/(b-a) \\
\frac{b - \sqrt{(b-a)(b-c)(1-r)}}{(b-a)(b-c)}, & (c-a)/(b-a) \leq r \leq 1
\end{cases}
\]

Where \(r\) is a uniform random number in \([0,1]\). Trigonometric distribution can be simulated.

**Incomplete Information Logistics Supervision Game**

When logistics demand enterprises are looking for a new 3PLs, the enterprises don’t know whether the 3PLs has a good reputation or a poor reputation because of information asymmetry. For logistics enterprises of different reputation, logistics demand enterprises will adopt different supervision. Thus the game is an incomplete information game between logistics demand enterprises and 3PLs.

**Incomplete Information Logistics Supervision Game**

Let \(D\) denote the logistics demand enterprises, and 3PLs denote the third party logistics enterprises. We can get a game between \(D\) and 3PLs. \(D\) decides to supervise 3PLs or indulge, and 3PLs decides whether to work actively. \(D\) will decide according to its own understanding about the type of 3PLs, and vice versa. This paper analyzes the choice of \(D\), the game model is shown as Table 1.

| 3PLs     | Benefit matrix in good reputation | Benefit matrix in poor reputation |
|----------|-----------------------------------|----------------------------------|
|          | Work actively                     | Work negatively                  |
|          | Work actively                     | Work negatively                  |
| (D)      | (65,20)                           | (40,25)                          | (30,15) | (25,20) |
| Supervision | (70,15)                           | (30,10)                          | (40,20) | (20,5) |

The incomplete information makes \(D\)’s strategy decision different. Using the method of marking we can get the Nash equilibrium. As 3PLs in good reputation, there are two pure strategy Nash equilibriums: \(D\) indulges and 3PLs works actively, and the benefit is \((70, 15)\). Or \(D\) supervise and 3PLs works negatively, the benefit is \((40, 25)\). And the two pure strategy Nash equilibriums are not
Pareto optimal equilibriums. Similarly in 3PLs’ poor reputation, there are also two pure strategy Nash equilibriums (indulgence, work actively) and (supervision, work negatively). It shows that the Nash equilibriums are unstable. So we need to seek their mixed strategy Nash equilibrium. The type of 3PLs is important because it affects their respective benefits.

As 3PLs in good reputation, let the probability of logistics demand enterprises supervises 3PLs is \( p \), and \( 1-p \) for indulgence. Similarly, the probability of 3PLs works actively is \( q \), and the probability of 3PLs works negatively is \( 1-q \). The expected benefit of the logistics demand enterprises \( D \) is Eq.4.

\[
ED_1 = p(65q+40(1-q))+(1-p)(70q+30(1-q)) = 30+10p+40q-15pq. \quad (4)
\]

The expected benefit of 3PLs is Eq.5.

\[
E_{3PLs1} = q(20p+15(1-p))+(1-q)(25p+10(1-p)) = 10+15p+5q-10pq. \quad (5)
\]

By the maximal principle, let the first partial derivative of \( ED_1 \) with respect to \( p \) is zero as Eq.6. From Eq.6, \( q \) is solved as Eq.7. Let the first partial derivative of \( E_{3PLs1} \) with respect to \( q \) is zero as Eq.8. And from Eq.8, \( p \) is solved as Eq.9.

\[
\frac{\partial ED_1}{\partial p} = 10-15q = 0. \quad (6)
\]

\[
q = 0.6667 \quad 1-q = 0.3333. \quad (7)
\]

\[
\frac{\partial E_{3PLs1}}{\partial q} = 5-10p = 0. \quad (8)
\]

\[
p = 0.5 \quad 1-p = 0.5. \quad (9)
\]

Therefore, when 3PLs has a good reputation, mixed strategy Nash equilibrium is \((0.5, 0.5), (0.6667, 0.3333)\). It means \( D \) supervises 3PLs with the probability 0.5; 3PLs work actively with the probability 0.6667, and work negatively with the probability 0.3333. Thus in this situation, the expected benefits of \( D \) is \( ED_1 = 56.6667 \). And the expected benefits of 3PLs is \( E_{3PLs1} = 17.5 \).

Similarly, in the situation of 3PLs has a poor reputation, the mixed strategy Nash equilibrium is \((0.75, 0.25), (0.3333, 0.6667)\). It means the logistics demand enterprises supervise 3PLs with the probability is 0.75, and indulgence with the probability is 0.25; 3PLs work actively with the probability is 0.3333, and work negatively with the probability is 0.6667. Thus the expected benefits of \( D \) is \( ED_2 = 26.6667 \). And the expected benefits of 3PLs is \( E_{3PLs2} = 16.25 \).

**Harsanyi Transformation of Incomplete Information Logistics Supervision Game**

In the incomplete information logistics supervision game, there are two types of 3PLs: a good or poor reputation. But logistics demand enterprise only has one type. By Harsanyi transformation, “*Nature*” is introduced into it. *Nature* determines the type of 3PLs based on a certain probability distribution. Then the logistics demand enterprise will play the actual game with its judgment. The incomplete information logistics regulatory game in Table 1 is transformed into the game tree as Figure 1.

![Figure 1. Complete but not perfect information game tree.](image)

The initial node in the game tree is “*Nature*”, denoted by \( N \). \( N \) choose the type of 3PLs with the prior probability of mutual knowledge. The probability of 3PLs with good reputation is \( u \), \( 1-u \) for poor reputation. Then 3PLs and the logistics demand enterprise carry on the actual game. 3PLs choose work actively or work negatively; the logistics enterprise chooses supervision or indulgence.
The Expansion Game of the Incomplete Information Logistics Supervision Game

In the incomplete information logistics supervision game, demand enterprises do not know the type of 3PLs, so the enterprises should choose the strategy to maximize their own expected utility. The expansion game of the incomplete information logistics supervision game as in Table 2.

| 3PLs (good poor) | Benefit matrix in good reputation |
|------------------|----------------------------------|
|                  | Supervision                      | Indulgence                       |
| (actively actively) | (30+35u,15+5u)                  | (40+30u,20-5u)                   |
| (actively negatively) | (25+40u,20)                     | (20+50u,5+10u)                   |
| (negatively actively) | (30+10u,15+10u)                | (40-10u,20-10u)                  |
| (negatively negatively) | (25+15u,20+5u)                  | (20+10u,5+5u)                    |

In Table 2, the expected benefits of the logistics demand and the 3PLs are given under different types. And their benefits are related to the reputation of 3PLs, which is the function of the probability of good reputation u. By analyzing the prior probability u, we can find the Bayesian Nash equilibrium. Monte Carlo method is utilized to achieve the type simulation.

Simulations

The Simulation of Logistics Supervision Game in Players Behaviors

To simulate, we use the method in [6]. To digitalize strategy, for logistics demand enterprises, 1 means supervision, and 0 indulgence. For 3PLs, 1 means work actively and 0 negatively.

As good reputation, mixed Nash equilibrium is (0.5, 0.6667). Theoretically, the logistics demands enterprises choose supervision 500 times; 3PLs choose the number of work actively 667 times and negatively 333 times. The simulation of 500 times is shown as Figure 2. The number of logistics demands enterprises choosing supervision and indulgence are 476 and 524. This coincides with the theoretical result up to 95.4%. 3PLs choose to work actively 658 times and negatively 342 times. This coincides with the theoretical result up to 98.7%.

As poor reputation, mixed Nash equilibrium is (0.75, 0.3333). In theory, demand enterprises choose supervision 750 times and indulgence 250 times; 3PLs choose work actively 333 times and negatively 667 times. The simulation of 500 times is shown as Figure 3. Demand enterprises choose supervision 739 times and indulgence 261 times. This coincides with the theoretical result by 98.5%. 3PLs choose work actively 353 times and work negatively 647 times. This coincides by 97%.

Simulation of Incomplete Information Logistics Supervision Game

To simulate Nature choosing 3PLs’ type with probability u, trigonometric distribution is used. According to the data from the State Post Bureau of The People’s Republic of China, average score of
3PLs’ satisfaction is 77 points. Thus we can think the most likely reputation of 3PLs is 0.77, and the most pessimistic value is 0, optimistic value is 1. For \( u \) in \([0,1]\). If \( u \geq 0.77 \), 3PLs is good reputation. The expected benefits of logistics demand enterprises and 3PLs are shown in Eq.10 and Eq.11. 

\[
E_D = 56.6667 \times 0.23 + 26.6667 \times 0.77 = 33.5667. 
\]  
\[
E_{3PLs} = 17.5 \times 0.23 + 16.25 \times 0.77 = 16.5375. 
\]  
According to Monte Carlo simulation, the average expected benefits are shown in Figure 4.

![Figure 4. Simulation of players’ benefits.](image)

**Conclusions**

To improve the practice feasibility of game theory for logistics supervision game, three important works are done. Firstly, we established a incomplete information game and analyzed logistics demand enterprises and 3PLs’ benefits under different types of 3PLs. Secondly, we carried on Harsanyi transformation. We convert incomplete information game to a complete but imperfect information game. And analyze their own benefits with the method of complete information game. Then we get an expansion game of incomplete information game. Thirdly, Monte Carlo (MC) simulation is adopted to simulate players’ behavior strategy and expected benefit; and *Nature* selecting the game players’ uncertain types randomly. The simulation coincides with the theoretical situation up to 98.7%.

There are also some defects. Firstly, the model has many dynamic games in real life. Secondly, MC simulation can be developed to Markov Chain Monte Carlo or other different ways.

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