Consistency of Planck Data With Power-law Primordial Scalar Power Spectrum

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Abstract

In this work we explore the possibility of variations in the primordial scalar power spectrum around the power-law shape, as predicted by single-field slow-roll inflationary scenarios. We search for a trace of these fluctuations in a semiblind, model-independent way in observations of the cosmic microwave background (CMB) sky. In particular, we use two sets of perturbation patterns, specific patterns with typical features such as oscillations, bumps, and transitions, as well as perturbation modes, constructed from eigenanalysis of the forecasted or measured covariance of perturbation parameters. These modes, in principle, span the parameter space of all possible perturbations to the primordial spectrum and, when rank ordered, the ones with the highest detectability would suffice to explore the constrainable features around the power-law spectrum in a data-driven (and not theoretically biased) manner. With Planck measurements of CMB anisotropies, the amplitudes of all perturbation patterns considered in this work are found to be consistent with zero. This finding confirms, in the absence of theoretical biases, the consistency of the Planck data with the assumption of a power-law inflationary pattern for the primordial spectrum.

Unified Astronomy Thesaurus concepts: Cosmology (343); Cosmic microwave background radiation (322); Cosmological parameters (339); Astronomy data analysis (1858)

1. Introduction

The inflationary paradigm is the most widely accepted scenario to seed fluctuations in the temperature and polarization of the cosmic microwave background (CMB) and the large-scale distribution of matter in the universe. The predictions of the simplest class of inflationary model, i.e., single-field slow-roll inflation, for the primordial perturbations are Gaussian scalar and tensor fluctuations, described by power-law spectra (Starobinskiï 1979; Linde 1982):

\[ P_s(k) = A_s (k/k_p)^{n_s}, \quad P_t(k) = A_t (k/k_p)^{n_t}, \]

with \( A_s \) and \( A_t \) standing for the amplitudes and tilts of scalar (s) and tensor (t) perturbations, and \( k_p \) the corresponding pivot scale. These predictions for the primordial scalar power spectrum (PSPS) are in great agreement with CMB observations, with \( \ln(10^{10} A_s) = 3.044 \pm 0.014 \) and \( n_s = 0.9649 \pm 0.0042 \) at \( k_p = 0.05 \text{Mpc}^{-1} \), whereas the 95% upper bound on the amplitude of the tensor power spectrum, parametrized by the tensor-to-scalar ratio, is \( r \lesssim 0.10 \) at \( k_p = 0.002 \text{Mpc}^{-1} \) (Planck Collaboration et al. 2020a, 2020b). In this work our focus is on the PSPS.

Despite the great agreement, there could still be small deviations around these predictions. Various scenarios of the early universe make clear predictions for specific patterns of the PSPS. For instance, see Danielsson (2002), Martin & Brandenberger (2003), Bozza et al. (2003), Chen (2011), Jackson & Shiu (2013), and Flauger et al. (2017) for models of global oscillation, and Adams et al. (2001), Chen et al. (2007), Achúcarro et al. (2011), Miranda et al. (2012), and Bartolo et al. (2013) for localized oscillatory features. The parameters of the various models of the early universe have been constrained by different cosmological data (e.g., Meerburg et al. 2014; Beutler et al. 2019; Planck Collaboration et al. 2020b). Forecasts have also been made on the detectability of their imprints with future surveys (e.g., Huang et al. 2012; Ballardini et al. 2016; Chen et al. 2016; Xu et al. 2016; Beutler et al. 2019; Li et al. 2021).

A parallel and complementary approach to this theoretically motivated path would be a model-independent analysis. In this semiblind approach, one relaxes general degrees of freedom in the parameter space of all perturbations to the power-law PSPS, as many as numerically feasible (and required), and allow for the data to find and construct the perturbation patterns that are most tightly constrainable. This nonparametric search for deviation around the power-law spectrum in Planck data was investigated in Planck Collaboration et al. (2020b); see also Zhao et al. (2009), Ishida & de Souza (2011), Farhang et al. (2012), Hall et al. (2013), Sapone et al. (2014), Regan & Munshi (2015), Feng & Li (2016), Huang & Wang (2017), Taylor et al. (2018), Farhang & Vafaie Sadr (2019), and Sharma et al. (2020) for examples of the application of this method in different contexts in cosmology. In particular, Esmaeilian et al. (2021) construct the perturbation eigenmodes to the PSPS for future CMB-S4-like and large-scale surveys.

Our goal here is to investigate the consistency of Planck observations with the power-law PSPS in an enhanced parameter space with different sets of degrees of freedom to cover different sorts of fluctuations. The main parameter set would be perturbation eigenmodes constructed for Planck data. The major results of this work are presented in Figure 2, showing the data-driven trajectories of possible perturbations to the PSPS. As is evident from the figure, no significant deviation is detected and the PSPS is found to be consistent with the power-law spectrum.

The organization of the paper is as follows. In Section 2 the data set and simulations are briefly introduced. We then discuss details of the analysis and introduce the patterns of perturbations explored in this work in Section 3. The results are presented in Section 4. Section 5 closes the paper with our final words and a discussion of the results.
2. Simulations and Data

In this work we use Planck observations of fluctuations in CMB temperature and polarization (Planck Collaboration et al. 2020c) to probe the physics of the early universe through its impact on the CMB power spectra. In parts we also use simulations of the CMB power spectra, generated by the publicly available Boltzmann Code for Anisotropies in the Microwave Background (CAMB) and Planck noise for comparison with results from real data, as will be discussed in Section 4.

3. Analysis

We assume the underlying PSPS of the universe is close to the slow-roll power-law inflationary prediction, \( \mathcal{P}_0(k) \), with possible small deviations, \( \Delta \mathcal{P}(k) \), around it:

\[
\mathcal{P}(k) = \mathcal{P}_0(k) + \Delta \mathcal{P}(k) = \mathcal{P}_0(k)[1 + \delta \mathcal{P}(k)].
\]

We follow the search for deviations along two different paths: a search for specific, though quite general, patterns (Section 3.1), and a semiblind search (Section 3.2). The goal is then to measure the amplitudes and other free parameters of these patterns, as will be discussed below. For parameter estimation we sample the parameter space using the Cosmological Monte Carlo code (CosmoMC; Lewis & Bridle 2002). This space consists of parameters characterizing perturbations to PSPS, along with standard cosmological parameters and the experimental nuisance parameters. In the following \( k_{\text{min}} \) and \( k_{\text{max}} \) are the minimum and maximum wavenumbers considered in the analysis, and we take \( (k_{\text{min}}, k_{\text{max}}) \sim (0.004, 1) \) h/Mpc.

3.1. Specific Patterns

The search for features around the primordial power-law spectrum was also done in Planck Collaboration et al. (2020b). The main models used there for the reconstruction of perturbations were cubic splines, a sum of several top hats, a penalized likelihood method, and several specific patterns. Our method differs from their work in the reconstruction scheme based on the eigenanalysis of the covariance matrix of perturbations. We also search for different specific features in the spectrum except for the oscillatory pattern, which we keep for completeness.

Figure 1. Top: deviations from the power-law inflationary prediction in the form of three specific patterns: an oscillation, or OSC, a transition, or TR, and a Gbump (left). Three Fisher-based eigenmodes, or FEMs (middle), and three data-driven eigenmodes, or dEMs (right). Bottom: response in the CMB temperature power spectrum due to small changes in the amplitude of the above perturbations.
where $A$ and $n$ (positive integer) are the amplitude and frequency of the oscillation and $y = (\ln k - \ln k_{\min})/\Delta \ln k$ and $\Delta \ln k = \ln(k_{\max}/k_{\min})$.

### 3.2. Semiblind Patterns

Alongside the search for the several specific patterns in the primordial spectrum, we also investigate the possibility of unknown deviations from the power-law PSPS, not properly expressible by the above patterns. Any general deviations can in principle be expanded using a complete set of base functions that span the full $k$-range of interest (see, e.g., Farhang et al. 2012, for more details). We use three sets with different motivations as discussed below.

**Gaussian bumps (Gbumps).** As the first approximation to the expansion base functions, we use $N$ Gbumps (Equation (2)). The bump centers are logarithmically spaced in the $k$-range, and their widths are taken to be $\sigma = \delta \ln k/3$ with $\delta \ln k = (\ln k_{\max} - \ln k_{\min})/N$. We refer to this case as the multi-Gaussian expansion. The Gbumps can be considered as naive, however tame, approximations to Dirac delta functions. We treat these Gbumps, in the limit of very large $N$, as the base functions of the $N$-dimensional parameter space of all possible deviations to the scalar primordial power spectrum. Nevertheless, they have their numerical limitations. For instance, to cover all points in the $k$-space, the widths need to be chosen so that there is a nonvanishing overlap between neighboring Gbumps. This overlap, on the other hand, destroys the orthogonality of the bumps and can result in a correlation of the final eigenmodes constructed from the Gbumps. One could in principle correct for this error. However, as will be discussed in Section 5, such high precision is not required here.

**Data-driven eigenmodes of the covariance matrix (dEMs).** The large number of Gbumps, and the correlations between their amplitudes, in particular for neighbors, are expected to lead to relatively high uncertainties in the measurements. These large errors would render possible deviations hard to detect. We therefore go one step further and construct linear combinations of Gbumps with vanishing linear correlations. We call these combinations eigenmodes (EMs), and rank order them based on their estimated uncertainties. We then keep only the EMs with the lowest errors and the rest are discarded.

For the EM construction, we first generate the correlation matrix, $C$, of the amplitudes of the $N$ Gbumps through post-processing the CosmoMC results. The goal is to linearly transform the expansion basis, here the $N$ Gbumps, so that the representation of $C$ can be diagonal in the new basis. These new basis functions are constructed from the eigenvectors of $C$,

$$E_i(k) = \sum_{n=1}^{N} X_{ni} g_n(k), \quad k = 1, \ldots, N,$$

where $X$ is a matrix whose columns are the eigenvectors of $C$, $g_n(k)$ refers to the Gbumps, and $E_i(k)$ is any of the $N$ perturbation eigenmodes. The diagonalization of the covariance matrix in the new basis and the (close to) orthogonality of the Gbumps guarantee the linear uncorrelation of the eigenmodes. Any function representing possible deviations in the power spectrum can be expanded in terms of this new basis. The expansion coefficients yield a measure of the contribution of each mode to the perturbation. Moreover, the eigenvalues of $C$ represent the (squares of the) estimated errors of the eigenmodes as measured by the data in hand. The modes and the $C_{l}$ response to changes in the PSPS in the form of these modes are shown in the right panel of Figure 1.

**Fisher-based eigenmodes (FEMs).** For comparison, we also use eigenmodes of perturbations to the PSPS constructed from a Fisher matrix. In our Fisher matrix, $\mathcal{F}(\vec{q})$, the parameter set, $\vec{q}$, are the amplitudes of the Gbumps, and the simulations for the Planck power spectrum represent the data, $d$,

$$[\mathcal{F}(\vec{q})]_{\alpha\beta} = -\left< \frac{\partial^2 \ln P_l}{\partial q_{\alpha} \partial q_{\beta}} \right>, \quad (6)$$

where $P_l = P_l(\vec{q}|d)$ is the Bayesian posterior distribution of the parameter set, $\vec{q}$, for the data set, $d$, and $\left< (...) \right>$ is the ensemble average. In the limit of a Gaussian distribution for the parameters, one has $\mathcal{F}^{-1} = C$, and therefore the two matrices share eigenvectors. The uncorrelated modes of perturbations can thus be constructed from Fisher eigenvectors. The details of this approach are described in Esmaeilian et al. (2021). The only important difference is that the eigenmodes here are marginalized over the standard cosmological parameters, while the eigenmodes in Esmaeilian et al. (2021) were constructed with fixed standard parameters. See Farhang et al. (2012) for a detailed description of marginalized mode construction.

The modes from this approach are expected to differ (although not hugely) from the dEMs in several ways. The covariance matrix in the latter case was marginalized on numerous nuisance parameters (as well as standard parameters), while in the former the marginalization was only performed on the standard parameters. Moreover, the dEM covariance matrix was based on the sampling of the parameter space while for the FEMs the assumption of the Gaussianity of $C$ was assumed. This Gaussianity assumption of the covariance matrix should be treated with care as the likelihood surface for the many correlated, poorly constrainable amplitudes of the Gbumps is probably far from a perfect Gaussian.

### 4. Results

We use Planck data to measure the free parameters of the perturbation patterns, introduced in Sections 3.1 and 3.2. The results are presented in Table 1. As is evident from the table, the measured amplitudes for the three specific patterns are consistent with zero. For the blind search we use 60 Gbumps with their 60 amplitudes as the free parameters, along with the standard cosmological and observational nuisance parameters. All the amplitudes are found to be consistent with zero, as expected, since we do not see any hint (Table 1) for a deviating bump in the PSPS in the single Gbump scenario with varying $k_c$. From these 60 Gbumps, we construct the dEMs, i.e., the $E_i(k)$’s in Equation (5). The measured amplitudes of these eigenmodes are presented in Table 2, and are compared to the measurements of the Fisher-based eigenmodes. The analysis is performed with the first three modes in both cases, and no deviation from the power-law PSPS is observed. The result of the analysis with four eigenmodes was also null. It is interesting to note the huge gap between the estimated uncertainties of the amplitudes for the two sets of eigenmodes (Table 2) and the specific patterns of perturbations (Table 1).

Figure 2 illustrates the 1σ trajectories in the $k$-space of perturbations to the PSPS spanned by the various values taken by the parameters in each scenario. The reconstructed...
The estimated 1σ uncertainty is marginalized over all other parameters included in the analysis, including the bump width and position in the Gbump, the width inverse and wavelength of the transition in the TR case and the frequency in the OSC scenario.

### Table 2

|        | $A_1$       | $A_2$       | $A_3$       |
|--------|-------------|-------------|-------------|
| FEM    | 0.000 ± 0.002 | −0.001 ± 0.002 | −0.006 ± 0.004 |
| dEM    | −0.002 ± 0.016 | −0.008 ± 0.016 | 0.039 ± 0.052 |

5. Discussion

In this work we investigated, in a very general sense, the consistency of CMB data, as observed by Planck, with the predictions of the slow-roll single-field inflationary models. Specifically, we explored whether there are hints for deviations from the power-law spectrum of primordial scalar fluctuations. We first searched for certain, yet general, features in the spectrum, such as a Gaussian bump (with varying position and width), a transition (with varying transition wavelength and width), and an oscillatory pattern (with varying frequency). In parallel we also constructed orthogonal basis functions for the parameter space of all perturbations to the PSPS (up to a certain resolution, determined by the number of used basis functions) and rank ordered them based on their errors. This mode construction was done both with sampling the parameter space of the perturbations and likelihood surface exploration, and from Fisher matrix analysis. We then searched for deviations in the terms of these eigenmodes, focusing mainly on the first few. We found no hints of deviations from the power-law spectrum in any of the above approaches, and the reconstructed PSPS was found to be fully consistent with the scale-invariant scenario. Planck Collaboration et al. (2020b) also found null results in their nonparametric search for features in the primordial power spectrum.

There are two points in order here. First, our method was intended to be as blind as possible to any particular theoretical model of the early universe. The Gaussian bumps, the transitionary patterns, and the oscillations, all with varying parameters, were used as rather typical features characterizing general functions and were not driven by theoretical biases. The eigenmode analysis was data driven in the sense that, by construction, most detectable features would show up as the first few modes and their amplitudes would be measured in the next steps of the analysis. Therefore the null results indicate that data, by themselves, do not imply any fluctuations around the scale-invariant spectrum.

It should, however, be pointed out that the above null results do not rule out the possibility of detection of fluctuations with some certain patterns (different and not constructible from the ones explored here) predicted by given theoretical models. Nevertheless, these possible detections are highly model dependent and theoretically biased. This strong prior imposed by theory requires, in turn, strong theoretical justification for the preference of a certain model over the panoply of many other models of the early universe. One could, however, get an idea on how the theoretical patterns could be recovered in a data-driven search by projecting the theoretically predicted power spectrum onto the inflationary eigenmodes. This was explored in Esmaeilian et al. (2021) by investigating the required number of eigenmodes for the reconstruction of different patterns. The quality of reconstruction was characterized by the reconstruction gain, $g_r$, which quantified the relative contribution of the best constrainable modes in the pattern reconstruction. For example, the main features in the step model (Miranda & Hu 2014), introduced by some abrupt features in the inflation potential (Adams et al. 2001) or the sound speed (Achucarro et al. 2011), were shown to be recoverable with as few as five eigenmodes, and the $g_r$ reached a plateau with ~20 modes included in the analysis. On the other hand, an oscillatory pattern such as the logarithmic oscillations model (Planck Collaboration et al. 2020b) required many more modes, and its $g_r$ kept increasing steadily with the
inclusion of more eigenmodes. Similarly, here the first FEM with a single main bump (see Figure 1) is practically equivalent to the step model of Miranda & Hu (2014), while the higher modes with more ups and downs centered around $k \approx 0.1$ can partially recover the oscillatory patterns of the logarithmic oscillations model of Planck Collaboration et al. (2020).

The second point is the relevance of the physics of the very early (inflationary) universe in relaxing the Hubble tension, as reported in the disagreement between the local universe measurements of the Hubble constant ($H_0 = 67.36 \pm 0.54$ km s$^{-1}$ Mpc$^{-1}$; Riess et al. 2021) and the inferred value from CMB data ($H_0 = 73.3 \pm 0.8$ km s$^{-1}$ Mpc$^{-1}$; Planck Collaboration et al. 2020). For a thorough unbiased analysis, the relevant degrees of freedom for the model of the early universe should be opened along with the standard cosmological parameters, including $H_0$. To appropriately address the tension, the data of the local universe should not be included in the analysis so that it can later be compared with the final (CMB-based) inference. The hope is that the extended parameter space, including the new degrees of freedom, may change the likelihood surface in a way that the local high $H_0$ value would lie in this extended parameter region allowed by CMB data. However, this is not what we find in our semiblind search. In particular, we find $H_0 = 67.4 \pm 0.6$ with the first three dEMs, implying that in the absence of theoretical priors, CMB data do not prefer a high $H_0$ value even in an extended parameter space encompassing new degrees of freedom in the early universe.

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