GLOBAL VS LOCAL COSMIC STRINGS FROM PSEUDO-ANOMALOUS U(1).

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We study the structure of cosmic strings produced at the breaking of an anomalous U(1) gauge symmetry present in many superstring compactification models. We show that their coupling with the axion necessary in order to cancel the anomalies does not prevent them from being local, even though their energy per unit length is found to diverge logarithmically. We discuss the formation of such strings and the phenomenological constraints that apply to their parameters.

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INTRODUCTION

Most classes of superstring compactification lead to a spontaneous breaking of a pseudo-anomalous U(1) gauge symmetry\textsuperscript{[1]} whose possible cosmological implications in terms of inflation scenarios were investigated\textsuperscript{[2]–[3]}. Anomalies are cancelled through a mechanism which is a four-dimensional version of the famous anomaly cancellation mechanism of Green and Schwarz\textsuperscript{[4]} in the 10-dimensional underlying theory; the coupling of the dilaton-axion field to the gauge fields plays a central role. Cosmic strings\textsuperscript{[5]} might also be formed in this framework; because of their being coupled to the axion field, such strings were thought to be of the global kind\textsuperscript{[2,6–8]}. We show indeed that the axion field configuration can be made to wind around the strings so that any divergence must come from the region near the core instead of being the gauge field strength. Compactifying six dimensions to get to four spacetime dimensions an antisymmetric tensor is dual to a pseudoscalar; one has in particular:

\[ B_{\mu \nu} \]

This antisymmetric tensor field is obtained by placing three of the field strength with compact indices by their background values yields a term proportional to \( B \wedge F \). It is well-known that in four spacetime dimensions an antisymmetric tensor is dual to a pseudoscalar \( \alpha \):

\[ \varepsilon_{\mu \lambda \tau \nu} \partial^\tau B^{\lambda \nu} \sim \partial^\tau a. \]

The coupling is then simply \( A^\mu \partial_\mu a \).

In this formulation, the pseudoscalar belongs to the same (chiral) supermultiplet as the dilaton field \( s \) and they form a complex scalar field \( S = s + ia \). This superfield couples in a model independent way to the gauge fields present in the theory; one has in particular:

\[ \mathcal{L} = -\frac{s}{4M_P} \sum_a F^a_{\mu \nu} F^{a}_{\mu \nu} + \frac{a}{4M_P} \sum_a \tilde{F}^a_{\mu \nu} \tilde{F}^a_{\mu \nu}, \]  

where \( M_P \) is the reduced Planck scale, the index \( a \) runs over all gauge groups and

\[ \tilde{F}^a_{\mu \nu} = \frac{1}{2} \varepsilon^{\rho \sigma \mu \nu} F^a_{\rho \sigma}. \]

Thus \( a \) has axion-like couplings and is indeed called the string axion. And the vacuum expectation value of the dilaton \( s \) yields the gauge coupling \( 1/g_s^2 \).

An abelian symmetry with gauge field \( A_\mu \) may seem to have (mixed) anomalies: under \( A_\mu \rightarrow A_\mu + \partial_\mu a \)

\[ \delta \mathcal{L} = -\frac{1}{2} \delta_{GS} a \sum_a F^a_{\mu \nu} \tilde{F}^a_{\mu \nu}. \]

But this can be cancelled by an appropriate shift of the string axion \( a \). Since there is a single model-independent axion, only one abelian symmetry, henceforth referred to as \( U(1)_X \), may be pseudo-anomalous.

The kinetic term for the dilaton-axion fields is described at the supersymmetric level by the D-term of the Kähler function \( K = -\ln(S + \bar{S}) \). This may be modified to include the Green-Schwarz term \( A^\mu \partial_\mu a \)\textsuperscript{[1]}:

\[ K = -\ln(S + \bar{S} - 4\delta_{GS} V) \]

where \( V \) is a vector superfield describing the anomalous \( U(1)_X \) vector supermultiplet. This D-term includes other terms, such as a mass term for the \( A_\mu \) gauge field. The constant \( \delta_{GS} \) may be computed in the framework of the weakly coupled string and is found to be 0.
\[ \delta_{GS} = \frac{1}{192\pi^2} \sum_i X_i, \tag{3} \]

where \( X_i \) are the charges of the different fields under \( U(1)_X \).

We are therefore led to consider the general class of models described by the Lagrangian (restricted here to the bosonic fields)

\[ \mathcal{L} = -(D\mu \Phi_i)(D^\mu \Phi_i) \]
\[ - \frac{1}{4g^2} \left( F_{\mu\nu} F^{\mu\nu} - \frac{a}{M_p} F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \]
\[ - \delta_{GS}^2 M_p^2 A_\mu A^\mu + \delta_{GS} M_p A^\mu \partial_\mu a \]
\[ - \frac{1}{4} \partial_\mu a \partial_\mu a - \frac{1}{4} m^2 a^2 - V(\Phi_i), \tag{4} \]

where we have set the dilaton field to its vacuum expectation value \( \langle s \rangle = 1/g^2 \), we have included a mass term for the axion, without specifying its origin, and we have introduced scalar fields \( \Phi_i \) carrying the integer charge \( X_i \) under the \( U(1)_X \) symmetry; the covariant derivative is defined by

\[ D^\mu \Phi_i \equiv (\partial^\mu - i X_i A^\mu) \Phi_i, \tag{5} \]

and the potential \( V(\Phi_i) \) by

\[ V(\Phi_i) \equiv \frac{g^2}{2} (\Phi_i^\dagger X_i \Phi_i + \delta_{GS} M_p^2)^2, \tag{6} \]

The Green-Schwarz coefficient \( \delta_{GS} \) and the axion field \( a \) have been rescaled by a factor \( g^2 \).

The lagrangian (1) is invariant under the following local gauge transformation with gauge parameter \( \alpha(x^\mu) \)

\[ \Phi_i \rightarrow \Phi_i e^{i X_i \alpha} \]
\[ A_\mu \rightarrow A_\mu + \partial_\mu \alpha \]
\[ a \rightarrow a + 2 M_p \delta_{GS} \alpha, \tag{7} \]

the transformation of the term \( (a/4g^2 M_p) F_{\mu\nu} \tilde{F}^{\mu\nu} \) cancels the variation of the effective lagrangian due to the anomaly, namely \( \delta \mathcal{L} = -(1/2g^2) \delta_{GS} \alpha F_{\mu\nu} \tilde{F}^{\mu\nu} \) (assuming we are also transforming the fermions of the theory not written explicitly in (1)). Making a rigid gauge transformation with parameter \( \alpha = 2\pi \) without changing \( a \) as a first step but transforming the other fields (including the fermions), leads us to interpret \( a \) as a periodic field of period \( 4\pi \delta_{GS} M_p \), through the redefinition \( a \rightarrow a - 4\pi \delta_{GS} M_p \), which leaves the lagrangian invariant. It is also manifest in the following rewriting of the kinetic term and of the axionic \( \theta \)-term in \( \mathcal{L} \) where it is clear that \( a \) behaves like a phase:

\[ \mathcal{L}_{kin,\theta} = - \frac{1}{4g^2} \left( F_{\mu\nu} F^{\mu\nu} - \frac{a}{M_p} F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \]
\[ - \partial^\mu \phi_1 \partial_\mu \phi_1 - \phi_1^2 X_i^2 \left( \frac{\partial^\mu \eta_i}{X_i} - A^\mu \right)^2 \]
\[ - M_p^2 \delta_{GS}^2 \left( \frac{\partial^\mu a}{2 M_p \delta_{GS}} - A^\mu \right)^2, \tag{8} \]

where we have set \( \Phi_i \equiv \phi_i e^{i\eta_i} \) (\( \phi_i \) being the modulus of \( \Phi_i \)).

Let us now work out the Higgs mechanism in this context. We consider for the sake of simplicity a single scalar field \( \Phi \) of negative charge \( X \) and we drop consequently the \( i \) indices. \( \mathcal{L}_{kin,\theta} \) can be rewritten

\[ \mathcal{L}_{kin,\theta} = - \left( M_p^2 \delta_{GS}^2 + \phi^2 X^2 \right) \times \]
\[ \left[ A^\mu - \frac{1}{2} M_p \delta_{GS} \partial^\mu a + \phi^2 X \partial^\mu \eta \right]^2 \]
\[ - \frac{\phi^2 M_p^2 \delta_{GS}^2 X^2}{2 M_p \delta_{GS} + \phi^2 X^2} \left[ \partial^\mu a - \frac{\partial^\mu \eta}{X} \right]^2 \]
\[ + \delta_{GS} \left( \frac{\phi^2 X^2}{2 M_p \delta_{GS} + \phi^2 X^2} \left[ \frac{a}{2 M_p \delta_{GS}} - \frac{\eta}{X} \right] \right. \]
\[ + \frac{1}{2} M_p \delta_{GS} \partial^\mu a + \phi^2 X \eta \left( \frac{1}{M_p \delta_{GS}^2} - \phi^2 X^2 \right) F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}. \tag{9} \]

The linear combination appearing in this last equation

\[ \frac{a}{2 M_p \delta_{GS}} - \frac{\eta}{X} \tag{10} \]

is the only gauge invariant linear combination of \( \eta \) and \( a \) (up to a constant). The other one

\[ \ell \equiv \frac{1}{2} M_p \delta_{GS} a + \phi^2 X \eta \]
\[ \frac{M_p^2 \delta_{GS}^2 + \phi^2 X^2}{M_p^2 \delta_{GS}^2 + \phi^2 X^2} \tag{11} \]

has the property of being linearly independent of the previous one and of transforming under a gauge transformation (\( 7 \)) as \( \ell \rightarrow \ell + a \). We now assume explicitly that \( \Phi \) takes its vacuum expectation value \( \langle \Phi \rangle \equiv \rho^2 \) in order to minimize the potential (\( 6 \)):

\[ \rho^2 = - \delta_{GS} M_p^2/X. \tag{12} \]

We are left, among other fields, with a massive scalar Higgs field corresponding to the modulus of \( \Phi \) of mass \( m_X \) given by

\[ m_X^2 = 2g^2 \rho^2 X^2 = - 2 \delta_{GS} X g^2 M_p \]
\[ \text{and we define} \]
\[ \hat{a} \equiv \left[ \frac{a}{2 M_p \delta_{GS}} - \frac{\eta}{X} \right] \frac{\sqrt{2 } M_p \delta_{GS} X}{\sqrt{M_p^2 \delta_{GS}^2 + \rho^2 X^2}} \tag{14} \]

and

\[ F_a^2 = \frac{1}{128\pi^4} \frac{M_p^2 g^4}{\rho^2 X^2} \left( M_p^2 \delta_{GS}^2 + \rho^2 X^2 \right) \]
\[ = \frac{1}{128\pi^4} M_p^2 g^4 \left( 1 + \left( \frac{m_X}{M_p} \right)^2 \right) \tag{15} \]

so that with \( \rho \) being set:
\[ L_{\text{kin},\theta} = -[M_p^2\delta_{GS} + \rho^2X^2] [A^\mu - \partial^\mu \ell]^2 - \frac{1}{2} \partial^\mu \hat{a} \partial_\mu \hat{a} \\
+ \left[ \frac{\hat{a}}{32\pi^2 F_a} + \frac{\delta_{GS}}{2g^2} \right] F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \]

we can now make a gauge transformation to cancel \( \partial^\mu \ell \) by setting \( \alpha = -\ell + \beta \) where \( \beta \) is a constant parameter. This leaves us with

\[
L_{\text{kin},\theta} = -\frac{m_\alpha^2}{2g^2} A^\mu A_\mu - \frac{1}{2} \partial^\mu \hat{a} \partial_\mu \hat{a} \\
+ \frac{\hat{a}}{32\pi^2 F_a} F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu},
\]

where \( m_\alpha \) given by

\[
m_\alpha^2 = 2g^2 \left[ \rho^2X^2 + M_p^2\delta_{GS} \right] \\
= m_X^2 \left[ 1 + \left( \frac{m_X}{M_p} \right)^2 - \frac{1}{2g^2X^2} \right]
\]

is the mass of the gauge field after the symmetry breaking. The remaining symmetry

\[
\hat{a} \rightarrow \hat{a} + \frac{32\pi^2 F_a}{2g^2} \delta_{GS} \beta
\]

is the rigid Peccei-Quinn symmetry which compensates for the anomalous term arising from a rigid phase transformation of parameter \( \beta \).

To summarize we have seen that in the presence of the axion the gauge boson of the pseudo-anomalous symmetry absorbs a linear combination \( \ell \) of the axion and of the phase of the Higgs field. We are left with a rigid Peccei-Quinn symmetry, the remnant axion being the other linear combination \( \hat{a} \) of the original string axion and of the phase of the Higgs field.

**II. PSEUDO-ANOMALOUS U(1) STRINGS**

Cosmic strings can be found as solutions of the field equations derivable from Eq. [3] provided the underlying U(1) symmetry is indeed broken, which implies that at least one of the eigenvalues \( X_i \) is negative. This is the first case we shall consider here, so we shall in this section assume again only one field \( \Phi \) with charge \( X \), with \( X < 0 \). Assuming a Nielsen-Olesen-like solution along the \( z \)-axis \[4], we set, in cylindrical coordinates,

\[
\Phi = \phi(r)e^{i\eta}, \quad \eta = n\theta,
\]

for a string with winding number \( n \). This yields the following Euler-Lagrange equations

\[
\Box a = 2\delta_{GS}M_p \partial_\mu A^\mu - \frac{1}{2g^2M_p} F_{\mu\nu}\tilde{F}^{\mu\nu} + m^2 a,
\]

and

\[
\Box \phi = \phi(\partial_\mu \eta - X A^\mu)^2 + g^2 X \phi(\phi^2 + \delta_{GS} M_p^2),
\]

\[
\partial_\mu [\phi^2 (\partial^\mu \eta - X A^\mu)] = 0,
\]

from which the string properties can be derived.

Eq. (24) can be greatly simplified: first we make use of Eq. (2), which implies \( \partial_\mu \tilde{F}^{\mu\nu} = 0 \), and then we derive Eq. (2b) with respect to \( x^\nu \). This gives, upon using Eqs. (21) and (24),

\[
F_{\mu\nu}\tilde{F}^{\mu\nu} = 2m^2 M_p g^2 a,
\]

and, with the help of Eq. (24),

\[
\frac{1}{g^2} \partial_\mu F^{\mu\nu} = \frac{1}{M_p} \tilde{F}^{\mu\nu} \partial_\mu a + \mathcal{J}^{\nu} + J^{\nu},
\]

where the currents are defined as

\[
J^{\mu} = -2X \phi^2 (\partial^\mu \eta - X A^\mu),
\]

and

\[
\mathcal{J}^{\mu} = -\delta_{GS} M_p (\partial^\mu a - 2\delta_{GS} M_p A^\mu).
\]

Eqs. (21) and (23) then simply express those two currents conservation \( \partial \cdot J = \partial \cdot \mathcal{J} = 0 \), when account is taken of Eq. (25).

The standard paradigm concerning the strings obtained in this simple model states that the presence of the axion makes the string global in the following sense: even for a vanishing \( a \), \( A_\mu \) behaves asymptotically in such a way as to compensate for the Higgs field energy density (i.e., \( A_\mu \rightarrow -\partial_\mu \eta/X \)) and therefore yields an energy per unit length which diverges asymptotically. It should be clear however that the behaviour of \( a \) could be different; indeed, it could as well compensate for this divergence as we will now show. In this case then, a divergence is still to be found, but this time at a small distance near the string core, so that the total energy is localised in a finite region of space. This is in striking contrast with the case of a global string where the divergent behavior arises because the energy is not localized and a large distance cut-off must be introduced.

In order to examine the behaviour of the fields and the required asymptotics, we need the stress energy tensor

\[
T^{\mu}_{\nu} = -2g^{\mu\gamma} \frac{\delta \mathcal{L}}{\delta g^{\gamma\nu}} + \delta^{\mu}_{\nu} \mathcal{L},
\]

which reads explicitly


\[ T^{\mu\nu} = 2[\partial^\mu \phi \partial^\nu \phi - \frac{1}{2} g^{\mu\nu} (\partial \phi)^2] - \frac{1}{g^2} (F^{\mu\rho} F^\nu_{\rho} - \frac{1}{4} g^{\mu\nu} F \cdot F) \]  

\[ - \frac{1}{2} g^2 g^{\mu\nu} (X \phi^2 + \delta_{GS} M_p^2)^2 \]  

\[ + \frac{1}{2 \phi^2} [J^\mu J^\nu - \frac{1}{2} g^{\mu\nu} J^2] \]  

\[ + \frac{1}{2 X^2 \phi^2} [J^\mu J^\nu - \frac{1}{2} g^{\mu\nu} J^2] - \frac{1}{4} m^2 a^2 g^{\mu\nu} \]  

\[ (31) \]

where account has been taken of the field equations. The energy per unit length \( U \) and tension \( T \) will then be defined respectively as

\[ U = \int d\theta \, r \, dr \, T^{tt} \quad \text{and} \quad T = - \int d\theta \, r \, dr \, T^{zz}, \]  

\[ (32) \]

The question as to whether the corresponding string solution is local or global is then equivalent to asking whether these quantities are asymptotically convergent (i.e., at large distances).

It can be seen on Eq. (31) that only the last two terms can be a potential source of divergences. The Nielsen-Olesen [8] solution for the very last term consists in saying that \( A_{\mu} \) is pure gauge, namely \( \lim_{r \to \infty} D_{\mu} \Phi = 0 \), so that, as already argued, \( \lim_{r \to \infty} A_{\mu} = - \partial_{\mu} \eta / X \). With this solution, setting \( a = 0 \) implies that the second to last term in Eq. (31) should diverge logistically for \( r \to \infty \). However, at this point, it should be remembered that \( a \) can be interpreted as a periodic field of period \( 4 \pi \delta_{GS} M_p \) (as long as a cosine-like mass term is not included as is usually the case at very low temperatures if this axion is to solve the strong CP problem of QCD) and therefore can be assigned a variation along \( \eta \). In fact, setting

\[ a = \frac{2 \delta_{GS} M_p}{X} \eta, \]  

\[ (33) \]

a perfectly legitimate choice, regularizes the integrals in Eqs. (32), at least in the \( r \to \infty \) region.

The solution (33) turns out, as can be explicitly checked using Eqs. (24) and (25), to be the only possible non trivial and asymptotically converging solution. In particular, no dependence in the string internal coordinates (\( z \) and \( t \) in our special case) can be obtained. This means that the simple model used here cannot lead to current-carrying cosmic strings [9] [11]. Moreover, the stationary solution (33) shows the axion gradient to be orthogonal to \( \tilde{F}^{\mu\nu} \), i.e., \( \partial_{\mu} a \tilde{F}^{\mu\nu} = 0 \). Therefore, Eqs. (23) (25) reduce to the usual Nielsen-Olesen set of equations [8], with the axion coupling using the string solution as a source term. It is therefore not surprising that the resulting string turns out to be local.

The total energy per unit length (and tension) is however not finite in this simple string model for it contains the term

\[ U = f.p. + 2\pi \int \frac{dr}{r} \left( \frac{\delta_{GS} M_p}{X} a - \delta_{GS} M_p A_\phi \right)^2, \]  

\[ (34) \]

(f.p. denoting the finite part of the integral) so that, since \( A_\mu \) must vanish by symmetry in the string core, one ends up with

\[ U = f.p. + 2\pi (\frac{\delta_{GS} M_p}{X})^2 \ln (\frac{R_A}{r_a}), \]  

\[ (35) \]

where \( R_A \) is the radius at which \( A_{\mu} \) reaches its asymptotic behaviour, i.e., roughly its Compton wavelength \( m_a \) given in [8], while \( r_a \) is defined as the radius at which the effective field theory (4) ceases to be valid, presumably of order \( M_P^{-1} \); the correction factor is therefore expected to be of order unity for most theories. Hence, as claimed, the strings in this model can be made local with a logarithmically divergent energy. The regularization scale \( r_a \) is however a short distance cut-off, solely dependent on the microscopic structure and does not involve neither the interstring distance nor its curvature radius. In particular, the gravitational properties of the corresponding strings are those of a usual Kibble-Vilenkin string [12], given the equation of state is that of the Goto-Nambu string \( U = T = \text{const.} \), and the light deflection is independent of the impact parameter [13].

### III. LOCAL STRING GENESIS

Forming cosmic strings during a phase transition is a very complicated problem involving thermal and quantum phase fluctuations [14]. As it is far from being clear how will \( a \) and \( \eta \) fluctuations be correlated (even though they presumably will), one can consider to begin with the possibility that a network of two different kinds of strings will be formed right after the phase transition, call them \( a \)-strings and \( \eta \)-strings, with the meaning that an \( a \)-string is generated whenever the axion field winds (ordinary axion string) while an \( \eta \)-string appears when the Higgs field \( \phi \) winds. Both kinds of strings are initially global since for both of them, only part of the covariant derivatives can be made to vanish. We however expect the string network to consist, after some time, in only these local strings together with the usual global axionic strings.

Let us consider an axionic string with no Higgs winding: as \( A^\mu \neq 0 \), the vacuum solution \( \Phi = \rho \) [Eq. (12)] is not a solution, and thus the axionic string field configuration is unstable. As a result of Eq. (22), the Higgs field amplitude tends to vanish in the string core. At this point, it becomes, near the core, topologically possible for its phase to start winding around the string, which it will do since this minimizes the total energy while satisfying the topological requirement that \( A_{\mu} \) flux be quantized. Such a winding will propagate away from the string.

Conversely, consider the stability of an \( \eta \)-string with \( a = 0 \). The conservation of \( J \) implies, as one can fix \( \partial_{\mu} A^\mu = 0 \), that \( \Box a = 0 \), whose general time-dependent
solution is \( a = a(|r| \pm t) \). Given the cylindrical symmetry, this solution can be further separated into \( a = f(r - t) e^{\theta} \). This means that having a winding of \( a \) that sets up propagating away from the string is among the solutions. As this configuration ultimately would minimize the total energy, provided \( \lim_{t \to \infty} f = -\delta_{\text{GS}} M_p / 2X \), this means that the original string is again unstable and will evolve into the stationary solution that we derived in the previous section. Note finally that if, instead of considering the variables \( a \) and \( \eta \), one had decided to treat the formation problem by means of the new dynamical variables \( \hat{a} \) and \( \ell \), then it would have been clear from the outset that the resulting string configurations could consist in two different categories, namely global axionic strings with a winding of \( \hat{a} \), and local strings with \( \ell \) winding.

It should be remarked at this point that these time evolution can in fact only be accelerated when one takes into account the coupling between \( a \) and \( \eta \); if any one of them is winding, then the other one will exhibit a tendency to also wind, in order to locally minimize the energy density. Indeed, it is not even really clear whether the string configurations we started with would even be present at the string forming phase transition. What is clear, however, is that after some time, all the string network would consist of local strings having no long distance interactions. This means in particular that the relevant scale, if no inflationary period is to occur after the string formations, should not exceed the GUT scale in order to avoid cosmological contradictions.

**CONCLUSIONS**

Spontaneous breaking of a pseudo-anomalous U(1) gauge symmetry leads to the formation of cosmic strings whose energy per unit length is localized around their cores, contrary to what the presence of the axion field in these theories might have suggested. This happens in the simple case we’ve considered here, namely that of a single scalar field acquiring a VEV at the symmetry breaking. In order to be general, this result should be generalized to the case where more than one field gets a VEV; this we now prove.

The potential we consider is that given by Eq. (31) which, in full generality, can be rewritten in the form

\[
V(\Phi) = \frac{g^2}{2}(\Phi^\dagger X \Phi + \delta_{\text{GS}} M_p^2)^2, 
\]

where \( X \) is an \( N \times N \) hermitian matrix, and \( \Phi \) takes values in an \( N \)-dimensional vector space \( V \). We denote by \( p \) the number of negative eigenvalues of \( X \) and \( \phi \) the restriction of \( \Phi \) to that subspace \( V_p \in V \) spanned by the eigenvectors of \( X \) with negative eigenvalues. Once diagonalized, \( X \) can be written as

\[
X = \begin{pmatrix} M & 0 \\ 0 & P \end{pmatrix}, \tag{37}
\]

with \( M \) and \( P \) containing respectively the negative and positive eigenvalues.

Eq. (36) admits an accidental U(N) symmetry, of which the anomalous U(1) is part; this is not a simple U(N)×U(1) symmetry as each field component transforms differently under U(1) as indicated on Eq. (3). The vacuum configuration is now given by

\[
\langle \phi^\dagger M \phi \rangle = -\delta_{\text{GS}} M_p^2, \tag{38}
\]

so it would seem that the remnant symmetry would be U(N−p), i.e. a scheme U(N)→U(N−p), and a topologically trivial vacuum manifold \( \tilde{V} \). Hence, one would naively not expect cosmic string formation in such a model. This conclusion is in fact not correct, as only part of the original U(N) is gauged, namely the anomalous U(1) subgroup, leading to a Noether current

\[
J^\mu \propto ig[\phi^\dagger M \partial^\mu \phi - (\partial^\mu \phi^\dagger)M \phi] + 2g^2 A^\mu \phi^\dagger M \phi, \tag{39}
\]

which can be made nonzero by imposing a phase variation for \( \phi \) as \( \sim \exp(i\theta) \). Once set to a nonzero value, this current will remain so for topological reasons [15], being called a semi-local or embedded defect. All the previous discussions concerning the simple \( p = 1 \) model hold also for these vortices, including their coupling to the axion field.

The cosmological evolution of the network of strings formed in these theories also leads to serious constraints on the Green-Schwarz coefficient provided no domain wall form connecting the strings; otherwise, the network is known to rapidly (i.e. in less than a Hubble time) decay into massive radiation and the usual constraint relative to the axion mass would hold [14]. If however the string network is considered essentially stable, then its impact on the microwave background limits the symmetry breaking scale \( \delta_{\text{GS}} M_p \) through the observational requirement that the temperature fluctuations be not too large [17], i.e.

\[
GU \lesssim 10^{-6},
\]

with \( G \) the Newton constant \( G = M_p^{-2}/(8\pi^2) \). Therefore, the cosmological constraint reads

\[
\delta_{\text{GS}} \lesssim 10^{-2}, \tag{40}
\]

a very restrictive constraint indeed. It should be remarked at this point that the usual domain wall formation leading to a rapid evaporation of the network does not seem valid for the string solution we considered. In fact, the axion \( a \), defined through Eq. (14), as the solution (34) set up, vanishes everywhere. Therefore, when the Peccei-Quinn symmetry is broken, the axion itself does not have to wind around these strings (it’ll do so however around the ordinary axionic strings present in the model). Hence, it has no particular reason for taking values in all its allowed vacuum manifold so that no domain wall will form.
The strings that we have discussed here might appear in connection with a scenario of inflation. Indeed, the potential \(\Phi^4\) is used for inflation in the scenario known as \(D\)-term inflation \([7]\): inflation takes place in a direction neutral under \(U(1)\) and the corresponding vacuum energy is simply given by:

\[
V_0 = \frac{1}{2} g^2 \delta_{GS} M^4 .
\]  

(41)

The \(U(1)\)-breaking minimum is reached after inflation, which leads to cosmic strings formation. Such an inflation era cannot therefore dilute the density of cosmic strings and one must study a mixed scenario \([6]\). It is interesting to note that, under the assumption that microwave background anisotropies are predominantly produced by inflation, the experimental data puts a constraint \([3,8,19]\) on the scale \(\xi \equiv \delta_{GS}^1 M_p\) which is stronger than \([40]\). Several ways have been proposed \([3,13]\) in order to lower this scale. They would at the same time ease the constraint \([40]\).

A final remark concerning currents is in order at this point. The strings whose structure we have investigated here are expected to be coupled to many fields, fermionic in particular. It is well known that such couplings yield the possibility for the fermions present in the theory to condense in the string core in the form of so-called zero modes \([20]\) which, upon quantization, give rise to superconducting currents \([13]\). (Note indeed the presence of anomalies along the worldsheet which was suggested to imply current inflows \([6]\).) Besides, currents tend to raise the stress-energy tensor degeneracy in such a way that the energy per unit length and tension become dynamical variables. For loop solutions, this means a whole new class of equilibrium solutions, named vortons, whose stability would imply a cosmological catastrophe \([21]\). If these objects were to form, Eq. \((40)\) would change into a drastically stronger constraint. Issues such as whether supersymmetry breaking might destabilize the currents \([2]\), thereby effectively curing the model from the vorton problem, still deserve investigation.

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