Dilaton gravity approach to three dimensional Lifshitz black hole

Yun Soo Myung$^{1,a}$, Yong-Wan Kim$^{1,b}$, and Young-Jai Park$^{2,c}$

$^1$Institute of Basic Science and School of Computer Aided Science, Inje University, Gimhae 621-749, Korea
$^2$Department of Physics and Center for Quantum Spacetime, Sogang University, Seoul 121-742, Korea

Abstract

The $z = 3$ Lifshitz black hole is an exact black hole solution to the new massive gravity in three dimensions. In order to understand this black hole clearly, we perform a dimensional reduction to two dimensional dilaton gravity by utilizing the circular symmetry. Considering the linear dilaton, we find the same Lifshitz black hole in two dimensions. This implies that all thermodynamic quantities of the $z = 3$ Lifshitz black hole could be obtained from its corresponding black hole in two dimensions. As a result, we derive the temperature, mass, heat capacity, Bekenstein-Hawking entropy, and free energy.

PACS numbers: 11.25.Tq, 04.70.Dy, 04.60.Kz, 04.70.-s
Keywords: Lifshitz black hole; dimensional reduction

$^a$ysmyung@inje.ac.kr
$^b$ywkim65@gmail.com
$^c$yjpark@sogang.ac.kr
1 Introduction

Recently, the Lifshitz-type black holes \cite{1, 2, 3, 4, 5, 6, 7} have received considerable attentions since these may provide a model of generalizing AdS/CFT correspondence to non-relativistic condensed matter physics \cite{8, 9, 10}. However, although their asymptotic spacetimes are apparently simple, the problem of obtaining an analytic exact solution seems to be a highly nontrivial task. A few examples include a four-dimensional topological black hole which is asymptotically Lifshitz with the dynamical exponent $z = 2$ \cite{11}. An analytic black hole solution with $z = 2$ that asymptotes the planar Lifshitz spacetime was found in four-dimensional spacetimes \cite{12}, and the $z = 3$ Lifshitz black hole \cite{13} was derived from the new massive gravity (NMG) in three-dimensional spacetimes \cite{14, 15, 16, 17, 18, 19, 20}. Numerical solutions were also explored in \cite{21, 22}. However, their complete thermodynamic studies are limited because it is not easy to compute their conserved quantities in asymptotic Lifshitz.

On the other hand, two-dimensional (2D) dilaton gravity has been used in various situations as an effective description of 4D and 3D gravities after a black hole in string theory has appeared \cite{23, 24}. It is known that the 2D dilaton gravity approach completely preserves the thermodynamics of 4D and 3D black holes \cite{25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40}. Hence, it is quite reasonable to apply the 2D dilaton gravity approach to the Lifshitz black holes in order to find their thermodynamic quantities. In this work, first, we check that the $z = 3$ Lifshitz black hole is also a solution to the 2D dilaton gravity. Then, we use the 2D dilaton gravity approach to the $z = 3$ Lifshitz black hole in three-dimensional spacetimes to obtain all thermodynamic quantities. In addition, we wish to point out differences and similarities between the $z = 3$ Lifshitz black hole and the $z = 1$ nonrotating BTZ black hole in the 2D dilaton gravity approach.

2 3D New Massive Gravity

The NMG action \cite{14} composed of the Einstein-Hilbert action with a cosmological constant $\lambda$ and higher order curvature terms is given by

\begin{align*}
S_{NMG}^{(3)} &= S_{EH}^{(3)} + S_{HC}^{(3)}, \\
S_{EH}^{(3)} &= \frac{1}{16\pi G_3} \int d^3x \sqrt{-G} \left( \mathcal{R} - 2\lambda \right), \\
S_{HC}^{(3)} &= -\frac{1}{16\pi G_3 m^2} \int d^3x \sqrt{-G} \left( \mathcal{R}_{MN} \mathcal{R}^{MN} - \frac{3}{8} \mathcal{R}^2 \right),
\end{align*}

where $G_3$ is the Newton constant in 3D, $m$ is a mass parameter, and $\mathcal{R}_{MN}$ is the Ricci tensor in 3D.
where $G_3$ is a three-dimensional Newton constant and $m^2$ a parameter with mass dimension 2. From now on, we set $G_3 = 1/8$ to obtain the same normalization of the refs. \[7, 41\].

The field equation is given by
\[
\mathcal{R}_{MN} - \frac{1}{2}g_{MN}\mathcal{R} + \lambda g_{MN} - \frac{1}{2m^2}K_{MN} = 0, \tag{4}
\]
where
\[
K_{MN} = 2\Box\mathcal{R}_{MN} - \frac{1}{2}\nabla_M\nabla_N\mathcal{R} - \frac{1}{2}\Box g_{MN} \\
+ 4\mathcal{R}_{MNPQ}\mathcal{R}^{PQ} - \frac{3}{2}\mathcal{R}\mathcal{R}_{MN} - \mathcal{R}_{PQ}\mathcal{R}^{PQ} g_{MN} + \frac{3}{8}\mathcal{R}^2 g_{MN}. \tag{5}
\]

In order to have Lifshitz black hole solution with dynamical exponent $z$, it is convenient to introduce dimensionless parameters
\[
y = m^2 \ell^2, \quad w = \lambda \ell^2, \tag{6}
\]
where $y$ and $w$ are proposed to take
\[
y = -\frac{z^2 - 3z + 1}{2}, \quad w = -\frac{z^2 + z + 1}{2}. \tag{7}
\]
For the $z = 1$ nonrotating BTZ black hole, one has $y = \frac{1}{2}$ and $w = -\frac{3}{2}$, while $y = -\frac{1}{2}$ and $w = -\frac{13}{2}$ are chosen for $z = 3$ Lifshitz black hole.

Now, let us consider the Achucarro-Ortiz type of dimensional reduction \[41\] by introducing the dilaton $\Phi$ as
\[
ds_{(3)}^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + \ell^2\Phi^2(x)d\theta^2. \tag{8}
\]
After integration over $\theta$ on $S^1$, the action \[\text{(1)}\] is reduced to give a 2D effective dilaton action
\[
S_{NMG} = S_{EH} + S_{HC}, \tag{9}
\]
\[
S_{EH} \quad = \quad \ell \int d^2x \sqrt{-g} \quad \Phi \left( R - 2\lambda \right), \tag{10}
\]
\[
S_{HC} \quad = \quad -\frac{\ell}{2m^2} \int d^2x \sqrt{-g} \quad \Phi \left[ \frac{1}{4} R^2 + \frac{1}{\Phi} R \nabla^2 \Phi + \frac{2}{\Phi^2} \nabla_\rho \nabla_\sigma \Phi \nabla^\rho \nabla^\sigma \Phi \right] \frac{1}{\Phi^2} \left( \nabla^2 \Phi \right)^2. \tag{11}
\]
We note that higher derivatives containing $\mathcal{R}_{MN}\mathcal{R}^{MN}$ and $\mathcal{R}^2$ are partially realized into the dilaton field. At this stage, we wish to distinguish 3D curvature $\mathcal{R}$ from 2D curvature $R.$
After a lengthy calculation, we obtain the equations of motion for 2D metric tensor $g^{\mu \nu}$ and dilaton $\Phi$ as

$$
\lambda \Phi g_{\mu \nu} + g_{\mu \nu} \nabla^2 \Phi - \nabla_\mu \nabla_\nu \Phi - \frac{1}{2m^2} \left[ - \frac{1}{2} g_{\mu \nu} \Phi \left( \frac{1}{4} R^2 + \frac{1}{\Phi} R \nabla^2 \Phi + \frac{2}{\Phi^2} \nabla_\rho \Phi \nabla^\rho \nabla^\sigma \Phi - \frac{1}{\Phi^2} (\nabla^2 \Phi)^2 \right) 
+ \frac{1}{2} \Phi R R_{\mu \nu} + \frac{1}{2} g_{\mu \nu} \nabla^2 (\Phi R) - \frac{1}{2} \nabla_\mu \nabla_\nu (\Phi R) + R_{\mu \nu} \nabla^2 \Phi 
+ g_{\mu \nu} \nabla^4 \Phi - \nabla_\mu \nabla_\nu (\nabla^2 \Phi) + R \nabla^2 \Phi + 2 \nabla_\mu (R \nabla_\nu \Phi) 
- g_{\mu \nu} \nabla_\rho (R \nabla^\rho \Phi) + \frac{2}{\Phi} \left( \nabla_\mu \nabla_\rho \Phi \nabla^\rho \nabla^\sigma \Phi + \nabla_\rho \nabla_\mu \Phi \nabla^\rho \nabla_\nu \Phi \right) 
+ 2 \nabla_\rho \left( \frac{1}{\Phi} \nabla_\mu \nabla_\nu \Phi \nabla^\rho \Phi - \frac{2}{\Phi} \nabla^\rho \Phi \nabla_\mu \Phi \nabla_\nu \Phi \right) - \frac{2}{\Phi} \nabla^2 \Phi \nabla_\mu \nabla_\nu \Phi 
+ 2 \nabla_\mu \left( \frac{1}{\Phi} \nabla^2 \Phi \nabla_\nu \Phi \right) - g_{\mu \nu} \nabla_\rho \left( \frac{1}{\Phi} \nabla^2 \Phi \nabla^\rho \Phi \right) \right] = 0, \tag{12}
$$

respectively. Moreover, the trace part of the equation of motion (12) is given by

$$
\Phi (R - 2 \lambda) - \frac{1}{2m^2} \left[ \frac{1}{4} \Phi R^2 + \Phi \nabla^2 R - \frac{2}{\Phi} \nabla_\rho \Phi \nabla^\rho \nabla^\sigma \Phi 
+ 4 \Phi \nabla_\rho \nabla_\sigma \left( \frac{1}{\Phi} \nabla^\rho \nabla^\sigma \Phi \right) + \frac{1}{\Phi} \left( \nabla^2 \Phi \right)^2 - 2 \Phi \nabla^2 \left( \frac{1}{\Phi} \nabla^2 \Phi \right) \right] = 0, \tag{13}
$$

In deriving the above equations, we use $R_{\mu \nu} = R g_{\mu \nu} / 2$.

Considering the linear dilaton background,

$$
\Phi = \frac{r}{\ell}, \tag{15}
$$

we find the $z = 3$ Lifshitz black hole in two spacetimes

$$
ds^2_{(2), z=3} = g_{\mu \nu} dx^\mu dx^\nu = - \left( \frac{r^2}{\ell^2} \right)^3 \left( 1 - \frac{M \ell^2}{r^2} \right) dt^2 + \frac{dr^2}{r^2 (\frac{r^2}{\ell^2} - M)}, \tag{16}
$$

where $M$ is an integration constant related to the the ADM mass of black hole. For $z = 1$, the ADM mass is determined to be $M = \frac{r^2}{2 \ell^2}$, while for $z = 3$, the ADM mass is proportional
to $M^2$ as discussed in the next section. The curvature invariants of the $z = 3$ Lifshitz black hole solutions in two-dimensional spacetimes are given by

$$R = -\frac{18}{\ell^2} + \frac{4M}{r^2}, \quad R_{\mu\nu}R^{\mu\nu} = \frac{R^2}{2} = \frac{162}{\ell^4} - \frac{72M}{r^2\ell^2} + \frac{8M^2}{r^4},$$

which show the curvature singularity at the origin. We observe that the singular behaviors of $\mathcal{R}$ and $\mathcal{R}_{MN}\mathcal{R}^{MN}$ persist in the 2D dilation gravity.

Furthermore, the above metric could be expressed in terms of the dilaton $\Phi$ as

$$ds^2_{(2),z} = -\Phi^{2z}\left(1 - \frac{M\ell^2}{r^2}\right)dt^2 + \frac{dr^2}{\left(\frac{r^2}{\ell^2} - M\right)}.$$  

(18)

For the $z = 1$ case, which corresponds to the particular point $y = 1/2$, $w = -3/2$, we find the nonrotating BTZ black hole, while for the $z = 3$ case, which corresponds to the particular point $y = -1/2$, $w = -13/2$, Lifshitz black hole is recovered. However, the $z = 2$ case is not a solution to the 2D dilation gravity [9].

We note that for the $z = 1$ case, $M$ is the ADM mass because it could be calculated in asymptotically AdS spacetimes using the Hamiltonian formalism. However, for the $z = 3$ case, we could not identify $M$ as the ADM mass because it should be calculated in asymptotically Lifshitz spacetimes. According to the information on Hořava-Lifshitz black holes [42, 43], which are also Lifshitz black holes with $0 \leq z \leq 4$ [44], it is conjectured that the ADM mass $\mathcal{M}$ is given by

$$M \propto \sqrt{\mathcal{M}}.$$  

(19)

## 3 Thermodynamics of $z = 3$ Lifshitz black hole

First of all, we mention that the Hawking temperature, which is related with the Lifshitz black hole solution [16], can be determined from the metric as

$$T_H = \frac{1}{4\pi}\left[\sqrt{-g^{tt}g^{rr}} |g_{tt}(r)|\right]_{r = r_+} = \frac{r^3}{2\pi\ell^4},$$

(20)

irrespective of knowing other conserved quantities. Here $'$ denotes differentiation with respect to its argument. In Ref. [45], it is well known that all thermodynamic quantities of the 4D Reissner-Nordström black hole and 3D BTZ black hole [16] [47] can be expressed in terms of dilaton, dilaton potential $V(\Phi)$, its integration $\int V(\Phi) d\Phi$, and its derivative $V'(\Phi)$ in its 2D dilaton gravity. Explicitly, their corresponding relations of temperature $T_H$, mass $J$, and heat capacity $C$ are given by as

$$T_H = \frac{V(\Phi)}{4\pi\ell}, \quad J = \int V(\Phi) d\Phi, \quad C = 4\pi \frac{V(\Phi)}{V'(\Phi)}.$$  

(21)
Therefore, expressing the temperature (20) as a function of potential

\[ T_H = \frac{V(\Phi)}{4\pi \ell}, \]  

with potential

\[ V(\Phi) = 2\Phi^3 = \frac{2\ell^3}{r^3}, \]  

the mass \( J \) and heat capacity \( C \) are found to be

\[ J(\Phi) = \frac{\Phi^4}{2} = \frac{r_+^4}{2\ell^4}, \]  

\[ C = \frac{4\pi}{3} \Phi = \frac{4\pi r_+}{3\ell}, \]

respectively. Here we note that the mass \( J = \Phi^4/2 \) of \( z = 3 \) Lifshitz black hole takes a different form, compared to the mass \( M = \Phi^2(r_+) = \frac{r_+^2}{\ell^2} \) of the nonrotating \( z = 1 \) BTZ black hole. From the conjecture of (19) inspired by the Hořava-Lifshitz black holes, we expect that the ADM mass of \( z = 3 \) Lifshitz black hole is determined to be

\[ \mathcal{M} \propto M^2 = \frac{r_+^4}{\ell^4} \]  

up to the constant. Using the dilaton gravity approach to the \( z = 3 \) Lifshitz black hole, however, we determine the ADM mass of \( z = 3 \) Lifshitz black hole exactly as

\[ \mathcal{M} \rightarrow J = \frac{r_+^4}{2\ell^4} \]  

without calculating the mass in asymptotically Lifshitz\[3] Using the first law of thermodynamics,

\[ dJ = T_H dS, \]  

we derive the Bekenstein-Hawking entropy

\[ S = 4\pi r_+, \]  

which satisfies the area-law for a universal entropy of black holes\[3]. Finally, the free energy is given by

\[ F = J - T_H S = \frac{3}{2} \frac{r_+^4}{\ell^4} \rightarrow -\frac{3}{2} \Phi^4. \] 

\[ ^* \text{Recently, a boundary stress-tensor approach has shown that the negative sign Einstein-Hilbert term provides a consistent thermodynamics of the } z = 3 \text{ Lifshitz black hole obtained from the NMG} \[48], \text{ which was the exactly same result derived here.} \]

\[ ^\dagger \text{Note here that when considered the Newton’s constant, the mass } J \text{ in (21) is expressed by } J = \int \Phi V(\Phi) d\Phi/(8G_3), M \rightarrow 8\pi G_3 M, \text{ and the right hand sides of } S, F \text{ are divided by } 8G_3, \text{ respectively.} \]
It seems appropriate to comment on the $z = 1$ case. For this case, the temperature is given by

$$T_H(\Phi) = \frac{\Phi}{2\pi \ell} = \frac{r_+}{2\pi \ell^2},$$

while the mass and specific heat take the forms

$$J(\Phi) = \Phi^2 = \frac{r_+^2}{\ell^2}, \quad C(\Phi) = 4\pi \Phi = \frac{4\pi r_+}{\ell}.$$  

Using the first law of thermodynamics, we also have the Bekenstein-Hawking entropy

$$S = 4\pi r_+. $$

The free energy is given by

$$F = -\Phi^2 = -\frac{r_+^2}{\ell^2}. $$

Before we proceed, we would like to mention that the key observation is the temperature expression (20) and thus, we have derived thermodynamic quantities based on the dilaton gravity approach (21). This approach works well for the BTZ and Reissner-Nordström black holes without higher curvature terms. Here the higher curvature terms appeared and these are necessary to obtain the $z = 3$ Lifshitz black hole like the warped AdS$_3$ solution in the topologically massive gravity [50]. If we know Lifshitz asymptotes well, we may calculate

Figure 1: Figures for temperature $T_H(\Phi)$, mass $J(\Phi)$, free energy $F(\Phi)$, and specific heat $C(\Phi)$, and with $\ell = 1$: Solid lines are for $z = 3$, while dotted lines are for $z = 1$. 
conserved quantities at infinity using the Hamiltonian formalism. However, at this time, one does not know precisely how Lifshitz asymptotes is different from asymptotically A(dS) spacetimes. Hence, we have an intrinsic handicap to determine the thermodynamic quantities of $z=3$ Lifshitz black holes using the conventional approach. Therefore, one has to find an alternative, even it was not proved to be working for the $z=3$ Lifshitz black hole well.

In this work, we might provide one way to determine thermodynamic quantities of $z=3$ Lifshitz black holes by using the 2D dilaton gravity approach. This attempt was supported from the fact that all higher dimensional black holes could be obtained from the 2D dilaton gravity approach after making an appropriate dimensional reduction. In this case, it is important to compare our results with the known results.

At this stage, we compare our thermodynamic quantities with those of ref.[7]. It seems that the temperature (20) is the same, but mass (24) and entropy (33) are consistent with those in [7] except negative signs when choosing $L=2\pi \ell$. As was explained in [7], negative mass and entropy are unfamiliar to black hole physicists and thus, this problem may be resolved when replacing the Newton’s constant $G_3$ by $-G_3$. We insist that this replacement is indeed necessary to find the NMG as a unitary massive gravity [51]. The NMG is equivalent to the Fierz-Pauli massive gravity within the linearized theory. In three dimensions, a massless graviton has no propagating degrees of freedom, while a massive graviton is a physically propagating mode with two helicities. In constructing the NMG with higher curvature terms, the principle was that one can neglect the massless graviton from (2) whatever its norm is positive or negative, in favor of the massive graviton without ghost from (3) [14]. In this sense, the replacement of $G_3 \rightarrow -G_3$ is necessary to obtain the correct thermodynamic quantities. In our work, this replacement should be done on the action (1) to obtain a correct action for a unitary massive gravity. However, this global operation does not change our thermodynamic results because we have derived thermodynamic quantities using the 2D dilaton gravity approach (mainly used the equations of motion) but not the effective action to derive the entropy by Wald’s formula. The latter is sensitive to sign of Ricci scalar and thus, leading to negative entropy and mass using the first law of thermodynamics. Recently, a boundary stress-tensor approach has confirmed that the wrong (negative) sign Einstein-Hilbert term provides a consistent thermodynamics of the $z=3$ Lifshitz black hole obtained from the NMG [48], which was the exactly same result found in our work.

Consequently, there is no difference between our thermodynamic quantities and those of [7] if one considers the NMG for a unitary massive gravity seriously. In this case, we have still found familiar thermodynamic quantities even for $z=3$ Lifshitz black hole in three dimensions.
4 Discussions

First of all, we have derived all thermodynamic quantities of the \( z = 3 \) Lifshitz black hole in three-dimensional spacetimes according to the dilaton gravity approach. This suggests that unknown thermodynamic quantities of higher-dimensional Lifshitz black holes could be obtained when using their 2D dilaton gravity approaches.

Next, we would like to mention differences and similarities between the \( z = 3 \) Lifshitz black hole and the \( z = 1 \) nonrotating BTZ black hole in the 2D dilaton gravity approach. As is shown Fig. 1, temperature, mass, and free energy take different forms as

\[
T_H \propto r_+^z, \quad J \propto r_+^{z+1}, \quad F \propto -r_+^{z+1}, \tag{35}
\]

while the heat capacity takes the nearly same forms as

\[
C \propto r_+. \tag{36}
\]

Importantly, the entropy is exactly the same for \( z = 1, 3 \) black holes.

At this stage, we would like to mention the stability issue on the relation between 2D dilaton black hole and \( z = 3 \) Lifshitz black holes. This issue on the rotating BTZ black hole was discussed in [52, 53], showing that taking into account quantum corrections may lead to some instability. Also, some reduction to low dimensions may spoil the equivalence between higher dimensional and lower dimensional objects [35]. It is interesting to investigate the stability issue on the relation between 2D dilaton black hole and \( z = 3 \) Lifshitz black holes. However, we remind the reader that at this time, one does not know precisely how Lifshitz asymptotes is different from asymptotically A(dS) spacetimes. Hence, we have some difficulty to study the stability issue. Further, we could not see whether the equivalence between higher dimensional and lower dimensional objects is spoiled for objects in the Lifshitz spacetimes. An important thing is that the dimensional reduction will not change thermodynamic properties of black holes. Therefore, we have shown that thermodynamics of the 2D dilaton black hole is the same as that of the 3D Lifshitz black hole.

Consequently, it is strongly suggested the 2D dilaton gravity approach may shed light on studying thermodynamic properties of the Lifshitz-type black holes.

Acknowledgement

Two of us (Y. S. Myung and Y.-J. Park) were supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number 2005-0049409.
Y.-W. Kim was supported by the Korea Research Foundation Grant funded by Korea Government (MOEHRD): KRF-2007-359-C00007. Y.-J. Park was also partially supported by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MEST) through WCU Program (No. R31-20002).

References

[1] K. Balasubramanian and J. McGreevy, Phys. Rev. Lett. 101, 061601 (2008) [arXiv:0804.4053 [hep-th]].

[2] S. Kachru, X. Liu and M. Mulligan, Phys. Rev. D 78, 106005 (2008) [arXiv:0808.1725 [hep-th]].

[3] U. H. Danielsson and L. Thorlacius, JHEP 0903, 070 (2009) [arXiv:0812.5088 [hep-th]].

[4] T. Azeyanagi, W. Li and T. Takayanagi, JHEP 0906, 084 (2009) [arXiv:0905.0688 [hep-th]].

[5] D. W. Pang, JHEP 0910, 031 (2009) [arXiv:0908.1272 [hep-th]].

[6] W. Li, T. Nishioka and T. Takayanagi, JHEP 0910, 015 (2009) [arXiv:0908.0363 [hep-th]].

[7] R. G. Cai, Y. Liu and Y. W. Sun, JHEP 0910, 080 (2009) [arXiv:0909.2807 [hep-th]].

[8] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200].

[9] S. A. Hartnoll, Class. Quant. Grav. 26, 224002 (2009) [arXiv:0903.3246 [hep-th]].

[10] D. T. Son, Phys. Rev. D 78, 046003 (2008) [arXiv:0804.3972 [hep-th]].

[11] R. B. Mann, JHEP 0906, 075 (2009) [arXiv:0905.1136 [hep-th]].

[12] K. Balasubramanian and J. McGreevy, Phys. Rev. D 80, 104039 (2009) [arXiv:0909.0263 [hep-th]].

[13] E. Ayon-Beato, A. Garbarz, G. Giribet and M. Hassaine, Phys. Rev. D 80, 104029 (2009) [arXiv:0909.1347 [hep-th]].

[14] E. A. Bergshoeff, O. Hohm and P. K. Townsend, Phys. Rev. Lett. 102, 201301 (2009) [arXiv:0901.1766 [hep-th]].
[15] E. Ayon-Beato, G. Giribet and M. Hassaine, JHEP **0905**, 029 (2009) [arXiv:0904.0668 [hep-th]].

[16] G. Clement, Class. Quant. Grav. **26**, 165002 (2009) [arXiv:0905.0553 [hep-th]].

[17] Y. Liu and Y. W. Sun, JHEP **0904**, 106 (2009) [arXiv:0903.0536 [hep-th]].

[18] Y. Liu and Y. W. Sun, JHEP **0905**, 039 (2009) [arXiv:0903.2933 [hep-th]].

[19] E. A. Bergshoeff, O. Hohm and P. K. Townsend, Phys. Rev. D **79**, 124042 (2009) [arXiv:0905.1259 [hep-th]].

[20] J. Oliva, D. Tempo and R. Troncoso, JHEP **0907**, 011 (2009) [arXiv:0905.1545 [hep-th]].

[21] G. Bertoldi, B. A. Burrington and A. Peet, Phys. Rev. D **80**, 126003 (2009) [arXiv:0905.3183 [hep-th]].

[22] G. Bertoldi, B. A. Burrington and A. W. Peet, Phys. Rev. D **80**, 126004 (2009) [arXiv:0907.4755 [hep-th]].

[23] E. Witten, Phys. Rev. D **44**, 314 (1991).

[24] G. Mandal, A. M. Sengupta and S. R. Wadia, Mod. Phys. Lett. A **6**, 1685 (1991).

[25] C. G. Callan, S. B. Giddings, J. A. Harvey and A. Strominger, Phys. Rev. D **45**, R1005 (1992), hep-th/9111056.

[26] J. G. Russo, L. Susskind and L. Thorlacius, Phys. Lett. B **292**, 13 (1992).

[27] V. P. Frolov, Phys. Rev. D **46**, 5383 (1992).

[28] S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. A **16**, 1015 (2001) [arXiv:hep-th/0009202].

[29] D. Grumiller, W. Kummer and D. V. Vassilevich, Phys. Rept. **369**, 327 (2002).

[30] R. Jackiw, in *Quantum Theory of Gravity*, ed. S. M. Christensen (Hilger, Bristol, 1984).

[31] C. Teitelboim, in *Quantum Theory of Gravity*, ed. S. M. Christensen (Hilger, Bristol, 1984).

[32] M. Henneaux, Phys. Rev. Lett. **54**, 959 (1985).

[33] R. B. Mann, D. Robbins and T. Ohta, Phys. Rev. Lett. **82**, 3738 (1999), gr-qc/9811061.
[34] A. Fabbri, D. J. Navarro and J. Navarro-Salas, Nucl. Phys. B 595, 381 (2001).

[35] S. Nojiri and S. D. Odintsov, Phys. Lett. B 463, 57 (1999), hep-th/9904146.

[36] Y. S. Myung, Y. W. Kim and Y. J. Park, Gen. Rel. Grav. 41, 1051 (2009) arXiv:0708.3145 [gr-qc].

[37] G. Ruppeiner, arXiv:0711.4328 [gr-qc]; Phys. Rev. D 78 (2008) 024016 arXiv:0802.1326 [gr-qc].

[38] Y. S. Myung, Y. W. Kim and Y. J. Park, Phys. Lett. B 663, 342 (2008) arXiv:0802.2152 [hep-th].

[39] Y. S. Myung, Y. W. Kim and Y. J. Park, Phys. Rev. D 76, 104045 (2007) arXiv:0707.1933 [hep-th];
Y. S. Myung, Y. W. Kim and Y. J. Park, Phys. Rev. D 78, 044020 (2008) arXiv:0804.0301 [gr-qc].

[40] W. Kim and E. J. Son, Phys. Lett. B 678, 107 (2009) arXiv:0904.4538 [hep-th].

[41] A. Achucarro and M. E. Ortiz, Phys. Rev. D 48, 3600 (1993) arXiv:hep-th/9304068.

[42] R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D 80, 024003 (2009) arXiv:0904.3670 [hep-th].

[43] R. G. Cai, L. M. Cao and N. Ohta, Phys. Lett. B 679, 504 (2009) arXiv:0905.0751 [hep-th].

[44] Y. S. Myung and Y. W. Kim, Eur. Phys. J. C. (in press) arXiv:0905.0179 [hep-th].

[45] Y. S. Myung, Y. W. Kim and Y. J. Park, Mod. Phys. Lett. A 23, 91 (2008) arXiv:0707.3314 [gr-qc].

[46] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. 69, 1849 (1992) arXiv:hep-th/9204099.

[47] M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D 48, 1506 (1993) arXiv:gr-qc/9302012.

[48] O. Hohm and E. Tonni, JHEP 1004, 093 (2010) arXiv:1001.3598 [hep-th].

[49] Y. S. Myung, Phys. Lett. B 638, 515 (2006) arXiv:gr-qc/0603051.
[50] D. Anninos, W. Li, M. Padi, W. Song and A. Strominger, JHEP 0903, 130 (2009) [arXiv:0807.3040 [hep-th]].

[51] M. Nakasone and I. Oda, Phys. Rev. D 79, 104012 (2009) [arXiv:0903.1459 [hep-th]].

[52] S. Nojiri and S. D. Odintsov, Phys. Rev. D 59 (1999) 044003 [arXiv:hep-th/9806055].

[53] S. Nojiri and S. D. Odintsov, Mod. Phys. Lett. A 13 (1998) 2695 [arXiv:gr-qc/9806034].