Computing Maximum Fixed Point Solutions over Feasible Paths in Data Flow Analyses

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Abstract

The control flow graph (CFG) representation of a procedure used by virtually all flow-sensitive program analyses, admits a large number of infeasible control flow paths i.e., these paths do not occur in any execution of the program. Hence the information reaching along infeasible paths in an analysis is spurious. This affects the precision of the conventional maximum fixed point (MFP) solution of the data flow analysis, because it includes the information reaching along all control flow paths. The existing approaches for removing this imprecision are either specific to a data flow problem with no straightforward generalization or involve control flow graph restructuring which may exponentially blow up the size of the CFG.

We lift the notion of MFP solution to define the notion of feasible path MFP (FPMFP) solutions that exclude the data flowing along known infeasible paths. The notion of FPMFP is generic and does not involve CFG restructuring. Instead, it takes externally supplied information about infeasible paths and lifts any data flow analysis to an analysis that maintains the distinctions between different paths where these distinctions are beneficial, and ignores them where they are not. Thus it gets the benefit of a path-sensitive analysis where it is useful without performing a conventional path-sensitive analysis. Hence, an FPMFP solution is more precise than the corresponding MFP solution in most cases; it is guaranteed to be sound in each case.

We implemented the proposed computation of feasible path MFP solutions for reaching definitions analysis and potentially uninitialized variable analysis. We evaluated the precision improvement in these two analyses by analyzing 30 benchmark applications selected from open source, industry, and SPEC CPU 2006. The evaluation results indicate that the precision improvement in these two analyses respectively reduce the number def-use pairs by up to 13.6% (average 2.87%, geometric mean 1.75%), and reduce the potentially uninitialized variable alarms by up to 100% (average 18.5%, geo. mean 3%). We found that the FPMFP computation time was 2.9× of the MFP computation time on average.

Keywords: Compilers, Data Flow Analysis, Infeasible Control Flow Paths, Static Program Analysis, Maximum Fixed-Point Solution
1. Introduction

The effectiveness of techniques that optimize, verify, and debug computer programs depends on the availability of accurate information describing the program behavior during various executions. To this end, data flow analysis gathers code level information that could answer either a subset of queries over a program (demand-driven analyses) or all possible queries of a particular type over a program (exhaustive analyses) without executing the program. It is well known that computing highly precise exhaustive data flow analysis solutions (like meet over paths (MOP) \[1\]) is undecidable; even in the instances where it is computable, it does not scale well to practical programs. On the other hand, scalable and widely used solutions (like maximum fixed-point (MFP) \[1\]) are not as precise.

1.1. The Context of This Work

In this work, we focus on exhaustive data flow analyses because they compute information that can be used to answer all queries of a particular type over all program executions. In particular, we compute MFP solutions (details in Section 4) and try to make them more precise. A MFP solution includes the data flow values that reach a program point along all control flow paths (CFPs) \[1\], which are paths in the control flow graph (CFG) \[1\] representation of the program. Experimental evidence \[2\] on Linux kernel code suggests that 9-40% of conditional statements in a program lead to at least one infeasible CFP i.e., these CFPs do not occur in any execution of the program. Additionally, we found in our benchmarks on average 60% (Geometric mean 29%) functions had at least one infeasible CFP in their CFG. Inclusion of data flow values reaching along such infeasible CFPs makes the computed data flow information an over-approximation of the actual data flow information. This dilutes the usefulness of the information for client analyses.

Various approaches reported in the literature detect infeasible CFPs \[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\]. Most of these approaches cater to the needs of program testing, in which the knowledge of infeasible CFPs is used to refine various test coverage criteria (like branch, path coverage) leading to less testing overhead. However, not much attention has been given towards improving the precision of data flow analysis by avoiding the data flow values reaching along known infeasible CFPs. A potential reason being that CFP based data flow approaches like MOP computation (that considers CFPs independently, and hence can inherently exclude infeasible CFPs) are not scalable, because the number of CFPs as well as the number of data flow values may be very large or even unbounded. Hence, scalable approaches like MFP computation do not maintain a mapping between individual CFPs and the corresponding data flow values, thus making it non-trivial to identify (and discard) the data flow values reaching along infeasible CFPs.

We name the MFP solutions computed by restricting the data flow values at each program point to those values that reach along feasible CFPs as feasible path MFP (FPMFP) solutions. Existing approaches that attempt to compute such solutions are either unsuitable because they involve CFG restructuring which can exponentially blow up the size of the CFG or are analysis specific with no straightforward generalization.

1.2. Our Contributions

We observe that infeasible CFPs are a property of programs and not of any particular data flow analysis over programs. We use this observation to separate the identification of infeasible CFPs in the CFG of a program, from computation of FPMFP solutions that exclude data flow values reaching along known infeasible CFPs. These two phases are described below.
1. In the first phase, we use the work done by Bodik et al. [2] to detect infeasible CFPs in the CFG of a program. In their work, they identify minimal infeasible path segments (MIPS) which are minimal length sub-segments of infeasible CFPs such that the following holds: if a CFP \( \sigma \) contains a MIPS as a sub-segment then \( \sigma \) is infeasible. MIPS captures the infeasibility property of CFPs in a concise form (illustrated in Section 3).

Given a set of MIPS as input, we do the following processing over them.

(i) We propose a novel criteria (called contains-prefix-of) to cluster MIPS. This clustering is necessary because treating each MIPS in isolation may lead to an imprecise solution, if the MIPS are intersecting with each other (illustrated in Section 3.3).

(ii) We establish an equivalence relation over the clusters of MIPS, so that the clusters that belong to the same equivalence class can be treated as a single unit during FPMFP computation (illustrated in Section 6). This increases the scalability of FPMFP computation without reducing the precision of the computed solution.

2. In the second phase, we automatically lift the existing MFP specifications of an analysis to its FPMFP specifications using the processed MIPS from the first phase. These specifications are then used to compute the FPMFP solution. The computation of FPMFP solution satisfies the following properties that distinguish FPMFP from existing approaches:

(i) Computation restricts the data flow values at each program point to those values that reach along feasible CFPs (data flow values reaching along known infeasible CFPs are identified and discarded).

(ii) Computation does not involve CFG restructuring and is generically applicable for all MFP based data flow analyses.

Items (i) and (ii) in each of the above phases describe the distinguishing features of FPMFP over existing approaches. Moreover, separating the above two phases is an useful insight that avoids the costly repetition involved in identifying infeasible CFPs for each different data flow analysis over the same underlying program, which happens in analysis specific approaches [13, 14, 15, 16, 17, 18]. Instead, we perform the first phase that involves identification of infeasible CFPs once for each program, and the second phase that involves computation of FPMFP is performed once for each different data flow analysis.

The core ideas of FPMFP have been presented before [19, 20]. This paper enhances the core ideas significantly and provides a complete treatment. In particular, the following contributions are exclusive to this paper.

1. We unify our earlier approaches [19, 20] by proposing a novel formalization for handling infeasible paths in the MFP computations. This formalization is based on the contains-prefix-of relation (explained in Section 3.3) that precisely distills the useful interactions between MIPS from all possible interactions. The formalization is more intuitive to understand because of its declarative nature as opposed to the procedural formalization earlier, and its correctness is easier to prove and verify. Moreover, the new formalization of FPMFP is close to the standard MFP formalization. We believe this will ease the adoption of the FPMFP technique in data flow analysis.

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1We use the acronym MIPS as an irregular noun, whose plural form is same as its singular form.
Table 1: Solutions of a data flow analysis. As we go down the rows, more entries become “Yes” leading to increased efficiency and decreased precision.

2. We formalize the concept of FPMFP for inter-procedural setting. In particular, we extend it to the functional approach \[21\] of inter-procedural data flow analysis. We evaluate our inter-procedural technique of FPMFP computation on an industry strength static program analysis tool called TCS ECA \[22\]. Inter-procedural static analyses are known to be desirable to software developer community \[23\].

3. We improve the practical applicability of the FPMFP by adding novel optimizations that improve the scalability of the FPMFP computation by the factor of 2 without affecting soundness or precision of the computed solution. The approach has now scaled on the codesets of size 150 KLOC.

4. We prove that the FPMFP computes an over-approximation of the most precise solution of a data flow problem. The most precise solution is called the \textit{meet over feasible paths} (MOFP) solution and is explained in Section 2.

1.3. Organization of the Paper

Section 2 describes the limitations of existing approaches of data flow analysis. Section 3 describes the concept of Minimal Infeasible Path Segments that facilitate FPMFP solutions, and illustrates the FPMFP solution using an example. Section 4 explains the computation of FPMFP. Section 5 explains FPMFP computation in inter-procedural setting. Section 6 presents the optimizations that improve the scalability of FPMFP computation and explains the worst-case complexity of the FPMFP computation. Section 7 presents experimental evaluation and Section 8 describes related work. Section 9 concludes the paper. The appendix (Section 10) gives the proof of soundness of FPMFP solution.

2. Existing Approaches and Their Limitations

Table 1 lists the conventional solutions of a data flow analysis and mentions their computability and (relative) precision by describing the nature of paths traversed in computing the solutions. These paths are illustrated in Figure 1. In particular, the paths that are traversed by a data flow analysis consist of CFG paths or spurious paths (described below) or both. CFG paths (CFPs) that do not represent any possible execution of the program are \textit{infeasible}; others are \textit{feasible}. Apart from feasible and infeasible CFPs, a data flow analysis could also traverse \textit{spurious} CFPs which are paths that result from an over-approximation of CFPs—they do not appear in the CFG
but are effectively traversed by a specific method of data flow analysis as shown in Table I and as explained below.

- **Meet Over Feasible Paths (MOFP)** solution captures the information reaching a program point along feasible CFPs only and does not merge information across CFPs. Since a program could have infinitely many CFPs, computing this solution is *undecidable*. The information defined by MOFP precisely represents the possible runtime information at each program point because it does not include information reaching along infeasible and spurious CFPs.

- **Meet Over Paths (MOP)** solution captures the information reaching a program point along feasible and infeasible CFPs. Since the information reaching along infeasible CFPs is also considered, MOP is less precise compared to MOFP. Computing MOP is also *undecidable*.

- **Maximum Fixed Point (MFP)** solution differs from MOP in that the information is effectively merged in the shared segments of CFPs because the solution is inductively computed. Since only one piece of information is stored at a program point regardless of the number of paths passing through it, computing MFP is *decidable*. MFP is less precise than MOP because information across paths is merged and this over-approximation cannot distinguish between feasible and infeasible paths. Spurious CFPs are not traversed in MFP computation.

- **Flow Insensitive (FI)** solution merges information across all program points by ignoring the control flow. Effectively, it over-approximates the set of CFPs and hence includes information along feasible, infeasible, and spurious CFPs.

We propose an approach to compute FPMFP solutions in which the computation at each program point is restricted to those data flow values that reach along feasible CFPs (data flow values reaching along known infeasible CFPs are excluded and spurious CFPs are not traversed).

Our work can be seen in the mould of trace partitioning [24] which tries to maintain some disjunction over data flow values based on path conditions that represent control flow of values instead of merging the values indiscriminately across all paths to compute an MFP solution.
However, in trace partitioning, the desirable disjunctions (over data flow values) are not generic, instead, they need to be specified for each data flow analysis separately which limits the usefulness of trace partitioning.

Towards this end, we use the infeasibility of control flow paths—a property of programs and not of any particular data flow analysis over the program—as the criteria for trace partitioning. This allows us to achieve a generic and practical trace partitioning that can be automatically incorporated in any data flow analysis to obtain a more precise solution.

The main challenge in achieving this automatic approach is that it is difficult to eliminate the effect of an infeasible path when data flow values are merged which happens in MFP computation. We meet this challenge by identifying MIPS (computed using existing approaches [2]) and creating equivalence classes of CFPs; such that all CFPs in one equivalence class overlap with the same set of MIPS. These classes allow us to lift any data flow analysis to a data flow analysis that computes one data flow value for each equivalence class. The resulting solution ignores data flow values corresponding to equivalence classes containing infeasible CFPs and hence is more precise than MFP (and sometimes more precise than MOP and as precise as MOFP like in Figure 2).

Example 1. Figure 2 shows an example of different solutions for an analysis that determines the values of variables. For MOP and MOFP, each data flow value has a constraint that represents the CFPs along which the value reaches the program point. Since MFP, FPMFP, and FI solutions merge the values, they compute a range of values in terms of lower and upper bounds. FPMFP solution is more precise than MOP and MFP because it excludes the data flow value $a \mapsto [0, 0]$ reaching along infeasible path at node $n_6$ (the infeasible path is shown by double lines).

### 3. Feasible Path MFP Solutions

In this section, we define minimal infeasible path segments and show how they allow us to define the FPMFP solution. Section 4 describes how FPMFP solution is computed.

#### 3.1. Background

**Control Flow Paths**

A control flow path is a path in the control flow graph representation of a procedure. The start node of a CFP is always the start node of the CFG, however, the end node of a CFP can be any node in the CFG. We use the term “CFPs that reach node n” to refer to all CFPs that have n as the end node. All CFPs referred henceforth are intra-procedural, unless explicitly stated otherwise.

**Infeasible Control Flow Paths**

We use the following concepts related to infeasible paths [2].

We denote a CFP by $\rho : n_1 \xrightarrow{e_1} n_2 \xrightarrow{e_2} n_3 \xrightarrow{e_3} \ldots \xrightarrow{e_p} n_{p+1}$, $p \geq 2$, where $n_i$ denotes the node at the $i^{th}$ position in the path, and $e_i$ denotes its out edge. CFP $\rho$ is an infeasible control flow path if there is some conditional node $n_k$, $k \leq p$ such that the subpath from $n_1$ to $n_k$ of $\rho$ is a prefix.
of some execution path but the subpath from $n_1$ to $n_{k+1}$ is not a prefix of any execution path\(^3\).

A path segment $\mu : n_i \rightarrow \ldots \rightarrow n_k \rightarrow n_{k+1}$ of CFP $\rho$ above is a minimal infeasible path segment (MIPS) if $\mu$ is not a subpath of any execution path but every subpath of $\mu$ is a subpath of some execution path. Infeasibility occurs when inconsistent conditions are expected to hold at two different edges across a path segment. Hence every infeasible path must have at least two edges.\(^3\)

For MIPS $\mu$ of the previous paragraph, we call all nodes $n_j$, $i < j < k + 1$ as the intermediate nodes of $\mu$. Additionally, we call the edges $e_i$, $e_k$ as the start and the end edge of $\mu$ respectively, while all edges $e_j$, $i < j < k$ are called as the inner edges of MIPS $\mu$. As a rule, the end edge of

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\(^3\)Note that the subscripts used in the nodes and edges in a path segment signify positions of the nodes and edges in the path segment and should not be taken as labels. By abuse of notation, we also use $n_i$, $e_j$ etc. as labels of nodes and edge in a control flow graph and then juxtapose them when we write path segments.

\(^3\)We assume that the infeasible paths arising from the conditional statements of the form $\text{if}(\text{cond})$ where $\text{cond}$ is known to be either definitely zero or definitely non-zero, have been eliminated by the compiler in the CFG.
Let $a$ be an integer variable, then the states representing possible value range for variable $a$ at each edge over all executions are:

- $\text{states}(e_0, [a]) = \{(a \mapsto i) \mid -\infty \leq i \leq \infty\}$
- $\text{states}(e_1, [a]) = \{(a \mapsto 0)\}$
- $\text{states}(e_2, [a]) = \{(a \mapsto i) \mid -\infty \leq i \leq \infty\}$
- $\text{states}(e_3, [a]) = \{(a \mapsto i) \mid 5 < i \leq \infty\}$
- $\text{states}(e_4, [a]) = \{(a \mapsto i) \mid -\infty \leq i \leq 5\}$

Path segment $\mu_1 : n_1 \xrightarrow{e_1} n_3 \xrightarrow{e_3} n_4 \xrightarrow{e_4} n_5$ is a MIPS because $e_4$ cannot be reached from $e_1$ in any execution although it can be reached from $e_3$ in some execution i.e., $\text{states}(e_1, [a]) \cap \text{states}(e_4, [a]) = \emptyset$.

The start, inner, and end edges of $\mu_1$ are given by, start($\mu_1$) = $\{e_1\}$, inner($\mu_1$) = $\{e_3\}$, end($\mu_1$) = $\{e_4\}$

For the FPMFP computation, we only admit MIPS that do not contain cycles. All MIPS referred henceforth are MIPS without cycles.

**Example 1.** Figure 3 illustrates infeasible paths and MIPS. Observe that the CFP $\rho : n_0 \xrightarrow{e_0} n_1 \xrightarrow{e_1} n_3 \xrightarrow{e_3} n_4 \xrightarrow{e_4} n_5$ is infeasible but not minimal. However, its suffix $\mu_1$ is a MIPS, $\mu_1 : n_1 \xrightarrow{e_1} n_3 \xrightarrow{e_3} n_4 \xrightarrow{e_4} n_5$, because $\mu_1$ is infeasible but no sub-segment of $\mu_1$ is infeasible.

### 3.2. Our Key Idea

We define the following notations that allow us to describe the key ideas. For a given MIPS $\mu$, let start($\mu$), inner($\mu$), and end($\mu$) denote the set of start, inner, and end edges of $\mu$. Sets start($\mu$) and end($\mu$) are singleton because each MIPS has exactly one start and one end edge. MIPS exhibit the following interesting property:

**Observation 3.1.** CFPs that contain a MIPS $\mu$ as a sub-segment are infeasible. Consequently, the data flow values computed along CFPs that contain $\mu$ are unreachable at the end edge of $\mu$, so these values should be blocked at the end edge. Thus, if we define a data flow analysis that separates the data flow values that are to be blocked, we can eliminate them thereby avoiding the effect of infeasible paths.
Table 2: Separating the Values Flowing through MIPS $\mu_1$ for example in Figure 3. $\top$ represents top value of the lattice, the last column contains the result of meet of all data flow values present in a row at each edge, for simplicity values of only $z$ are shown.

Example 1. In Figure 3 a CFP $\sigma : n_0 \xrightarrow{e_0} n_1 \xrightarrow{e_1} n_3 \xrightarrow{e_3} n_4 \xrightarrow{e_4} n_5$ is infeasible because it contains MIPS $\mu_1 : n_1 \xrightarrow{e_1} n_3 \xrightarrow{e_3} n_4 \xrightarrow{e_4} n_5$. Hence, the data flow value that reaches along $\sigma$ (e.g., $z \mapsto [-\infty, 0]$) is blocked at $e_4$ (which is end edge of MIPS $\mu_1$).

Observation 3.1 leads us to the following key idea to handle infeasible paths: at each program point, we separate the data flow values reaching along CFPs that contain a MIPS, from the data flow values reaching along CFPs that do not contain any MIPS. This allows us to discard these values at the end of the MIPS (because these values are unreachable as per Observation 3.1).

For a MIPS $\mu$, following is a sufficient condition to block the data flow values that reach along CFPs that contain $\mu$.

$$\mathbb{C}: \text{At the end edge of } \mu, \text{ block the data flow values that } \text{flow through } \mu \text{ (i.e., values that flow along } \mu \text{ — from start edge of } \mu \text{ till the end edge of } \mu).$$

Example 2. For example in Figure 3 the values (of variable $z$) that flow through MIPS $\mu_1$ are shown in Table 2. The value $z \mapsto [-\infty, 0]$ reaches the start edge ($e_1$) of $\mu_1$, and flows through $\mu_1$. This value is separated from the values that do not flow through $\mu_1$ e.g., $z \mapsto [1, \infty]$. This separation allows us to identify and block the value $z \mapsto [-\infty, 0]$ at the end edge ($e_4$) of $\mu_1$.

The final value at each edge, except the end edge, is computed by taking a meet of values that flow through $\mu_1$, with values that do not flow through $\mu_1$. However, the final value at end edge contains only the values that do not flow through $\mu_1$. Thus, we get a precise value range for $z$ at edge $e_4$ (i.e., $z \mapsto [1, \infty]$) because value $z \mapsto [-\infty, 0]$ is blocked (a MFP solution that includes data flow values along infeasible CFPs gives value range $z \mapsto [-\infty, \infty]$ at edge $e_4$).

Note that the data flow values that reach the start edge of a MIPS $\mu$ may change or get killed at intermediate nodes or edges when they flow through $\mu$, depending on the type of data flow analysis. In this case, we block the updated data flow value accordingly, as illustrated in example 3 below.
flows through $\mu$ only. This allows us to block $l$ at edge $e\mu$ally separate the data flow values that flow through MIPS computed by meet of data flow values in each row at each edge —contains
ure 4c. This allows us to block $l$ In particular, at edge $e\mu$ we introduce the necessary changes to handle multiple MIPS in the program. We use the example computation are presented in sections 3.4 and 4.

We first explain the FPMFP computation in presence of single MIPS in the program. Later, we introduce the necessary changes to handle multiple MIPS in the program. We use the example in Figure 4a as a running example for explaining FPMFP computation. Two MIPS in Figure 4a are $\mu_1 : e_3 \rightarrow e_4 \rightarrow e_5$ and $\mu_2 : e_3 \rightarrow e_4 \rightarrow e_6$ (They are marked with double arrows). For simplicity of exposition, we restrict the discussion to computation of value range of variable $l$ only.

In FPMFP computation for the example in Figure 4a, we separate the value $l \mapsto [0, 0]$ that flows through $\mu_1$ from the value $l \mapsto [2, 2]$ that does not flow through $\mu_1$ as shown in Figure 4b. This allows us to block $l \mapsto [0, 0]$ at the end edge ($e_5$) of $\mu_1$. Thus, the FPMFP solution —computed by meet of data flow values in each row at each edge —contains $l \mapsto [2, 2]$ (a precise value) at edge $e_5$.

The FPMFP computation explained in previous paragraph becomes imprecise if we additionally separate the data flow values that flow through MIPS $\mu_2$, as explained below. For MIPS $\mu_2$: $e_3 \rightarrow e_4 \rightarrow e_6$, we separate the values that flow through $\mu_2$ (i.e., $l \mapsto [0, 0]$) as shown in Figure 4c. This allows us to block $l \mapsto [0, 0]$ at end edge ($e_6$) of $\mu_2$. However, the FPMFP solution (computed in Figure 4c) is imprecise because it includes the value $l \mapsto [0, 0]$ at edge $e_5$ and $e_6$. In particular, at edge $e_5$, $l \mapsto [0, 0]$ is blocked within $\mu_1$ but it is retained in $\mu_2$ because $e_5$ is not end edge of $\mu_2$. Similarly, at edge $e_6$, $l \mapsto [0, 0]$ is blocked within $\mu_2$ but is retained in $\mu_1$.

We explicate the reasons for this imprecision below: we separate the data flow values that reach along CFPs that contain a MIPS, however, a MIPS can overlap with other MIPS in a way that both MIPS follow the same CFP resulting in the same data flow values flowing through them. In such a case, at the end edge of a MIPS, the data flow along that MIPS will be blocked but the flow along the other overlapping MIPS will be allowed unless the edge is end edge of both the MIPS. This leads to imprecision. Hence we need to handle the overlapping MIPS case explicitly. For this purpose, we define a contains-prefix-of relation between MIPS below.

**Definition 1.** (Contains-prefix-of (CPO)) For a MIPS $\mu$ and an edge $e$ in $\mu$, let $\text{prefix}(\mu, e)$ be the subsegment of $\mu$ from $\text{start}(\mu)$ to $e$. Then, we say that MIPS $\mu_1$ contains-prefix-of MIPS $\mu_2$ at $e$, iff $\mu_1$ contains $\text{prefix}(\mu_2, e)$.

For example in Figure 4 the MIPS $\mu_1$ contains-prefix-of $\mu_2$ at edges $e_3$ and $e_4$. For a MIPS $\mu_1$ and an edge $e$, all MIPS $\mu_2$ that satisfy $\mu_1$ CPO $\mu_2$ are obtained as follows: let $U$ be the set of

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Example 3. Consider a MIPS $\mu$: $n_1 \xrightarrow{e_1} n_2 \xrightarrow{e_2} n_3 \xrightarrow{e_3} n_4$ with $e_1$, $e_2$, $e_3$ as the start, the inner, and the end edge respectively. Let $x$ be an integer variable such that $n_1 : x = 0$, $n_2 : x = x + 1$ be two assignment nodes, and nodes $n_3, n_4$ do not modify the value of the variable $x$.

The data flow value $x \mapsto [0, 0]$ at the start edge ($e_1$) of $\mu$ will be changed to $x \mapsto [1, 1]$ as it flows through $\mu$. Hence $x \mapsto [1, 1]$ is blocked at $e_3$. 

3.3. An Overview of FPMFP computation

We now briefly explain the steps of FPMFP computation that ensure that condition $C$ (from the previous section) is satisfied for all MIPS in the program. The formal details of FPMFP computation are presented in sections 3.4 and 4.

In FPMFP computation for the example in Figure 4a, we separate the value $l \mapsto [0, 0]$ that flows through $\mu_1$ from the value $l \mapsto [2, 2]$ that does not flow through $\mu_1$ as shown in Figure 4b. This allows us to block $l \mapsto [0, 0]$ at the end edge ($e_5$) of $\mu_1$. Thus, the FPMFP solution —computed by meet of data flow values in each row at each edge —contains $l \mapsto [2, 2]$ (a precise value) at edge $e_5$.

Example 3. Consider a MIPS $\mu$: $n_1 \xrightarrow{e_1} n_2 \xrightarrow{e_2} n_3 \xrightarrow{e_3} n_4$ with $e_1$, $e_2$, $e_3$ as the start, the inner, and the end edge respectively. Let $x$ be an integer variable such that $n_1 : x = 0$, $n_2 : x = x + 1$ be two assignment nodes, and nodes $n_3, n_4$ do not modify the value of the variable $x$.

The data flow value $x \mapsto [0, 0]$ at the start edge ($e_1$) of $\mu$ will be changed to $x \mapsto [1, 1]$ as it flows through $\mu$. Hence $x \mapsto [1, 1]$ is blocked at $e_3$. 

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In this case $\mu_2$ also contains-prefix-of $\mu_1$ at $e_3$ and $e_4$, however in general contains-prefix-of may not be a symmetric relation.
all MIPS in the CFG of the program.

\[ cpo_e(\mu_1) = \{ \mu_2 \mid \mu_2 \in \mathcal{U}, \mu_1 \text{ contains prefix}(\mu_2, e) \} \] (1)

We observe the following property of \( cpo_e(\mu_1) \).

**Observation 3.2.** Because of the prefix relation, all MIPS in \( cpo_e(\mu_1) \) essentially follow the same control flow path till edge \( e \) although their start edges may appear at different positions in the path. Next, the values along this path flow through each MIPS in \( cpo_e(\mu_1) \). Hence, these values should be blocked at the end edge of every MIPS in \( cpo_e(\mu_1) \).

For an edge sequence \( e \rightarrow e' \), \( cpo_e(\mu) \) may be different from \( cpo_{e'}(\mu) \) because all MIPS that reach \( e \) may not reach \( e' \), and some new MIPS may start at \( e' \). Hence, we define \( ext \) function below that computes \( cpo_{e'}(\mu) \) from \( cpo_e(\mu) \).

Let \( edges(\mu) \) be the set of all edges in \( \mu \), and \( S = cpo_e(\mu) \) then

\[ ext(S, e') = \{ \mu \mid \mu \in S, e' \in edges(\mu) \} \cup \{ \mu \mid \mu \in \mathcal{U}, e' \in start(\mu) \} \] (2)

At an edge \( e \) of MIPS \( \mu \), we associate the data flow values that flow through \( \mu \) with \( cpo_e(\mu) \) (instead of \( \mu \)). These associations are abstractly illustrated in Figure 5a and instantiated to an example in Figure 5b. We explain each of these below.

In Figure 5a, the data flow values \( d, d', d'' \) reach edges \( e, e', e'' \) respectively, along the following CFP: \( \sigma : n_0 \rightarrow n_1 \rightarrow n_2 \rightarrow n_3 \). The data flow value \( d'' = f_{n_3}(d) \) and \( d'' = f_{n_2}(d') \) where
Table 3: The FPMFP computation for example in Figure 4. A pair \((M, d)\) indicates that the data flow value \(d\) flows through all MIPS in set of MIPS \(M\). The final FPMFP solution is computed by taking meet of values over all pairs at each edge.

An association \((M, d)\) signifies that the data flow value \(d\) flows through all MIPS in set of MIPS \(M\) through all MIPS in set of MIPS \(M\).

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\begin{array}{|c|c|c|c|}
\hline
\text{Edges} & \text{Associations } \langle (M, d) \rangle & \text{FPMFP} & \text{MFP} \\
\hline
\langle \{\}, l \mapsto [2, 2] \rangle & l \mapsto [2, 2] & l \mapsto [2, 2] \\
\langle \{\}, l \mapsto [2, 2] \rangle & l \mapsto [2, 2] & l \mapsto [2, 2] \\
\langle \{\}, l \mapsto [0, 0] \rangle & l \mapsto [0, 0] & l \mapsto [0, 0] \\
\langle \{\}, l \mapsto [0, 2] \rangle & l \mapsto [0, 2] & l \mapsto [0, 2] \\
\langle \{\}, l \mapsto [2, 2] \rangle & l \mapsto [2, 2] & l \mapsto [2, 2] \\
\langle \{\}, l \mapsto [0, 2] \rangle & l \mapsto [0, 2] & l \mapsto [0, 2] \\
\hline
\end{array}
\]

The idea is that a pair \((M, d)\) with sets of MIPS \(M\) through all MIPS in set of MIPS \(M\) associates with each other by CPO relation i.e., there is no MIPS possible subset \(M\) for \(M\), \(\mu\) because \([5, 5]\), \(a\) is created at \(e_5\). Similarly, at edge \(e_4\), data flow value \(a \mapsto [5, 5]\) is mapped with \(\{\}\) i.e., a pair \(\langle \{\}, a \mapsto [5, 5] \rangle\) is created.

Edge \(e_5\) is end edge of \(\mu\), hence, the data flow value \(a \mapsto [0, 0]\) that is mapped with \(\mu\)

\[
f_{m}(x) \text{ computes the effect of node } n_i \text{ on input data flow value } x. \text{ The corresponding associations with sets of MIPS } M, M', M'' \text{ are computed as follows:}
\]

\[
M = \text{ ext}(\{\}, e)
\]

\[
M' = \text{ ext}(M, e')
\]

\[
M'' = \text{ ext}(M', e'')
\]

The idea is that a pair \((M, d)\) at edge \(e\) transforms into another pair \((M', d')\) at a successor edge \(e'\) because

- the CPO relation between MIPS at \(e'\) may be different from that at \(e\), and
- \(d'\) is computed from \(d\) as \(d' = f_{\text{ext}}(d)\).

In Figure 5, the data flow values \(l \mapsto [2, 2]\) at edge \(e_1\) is associated with \(\{\}\) because no MIPS contains \(e\). However, at edges \(e_2, e_3\), the data flow value \(l \mapsto [0, 0]\) is associated with \([\mu_1, \mu_2]\) because \(\mu_1\) CPO \(\mu_2\) at these edges i.e., \(\text{ext}(\{\mu_1\}, e_3) = [\mu_1, \mu_2]\). At \(e_5\), \(l \mapsto [0, 0]\) is associated with \([\mu_1]\) because \(\text{ext}(\{\mu_1, \mu_2\}, e_5) = [\mu_1]\). Since \(e_5\) is end edge of \(\mu_1\), \(l \mapsto [0, 0]\) is blocked at \(e_5\). Similarly, \(l \mapsto [0, 0]\) is blocked at \(e_8\). Rest of the associations at each edge are presented in Table 3.

We now introduce the notations used in the FPMFP computation. We compute a set of pairs of the form \((M, d)\) at each edge \(e\) where \(M\) is a set of MIPS, and \(d\) is the data flow value that flows through all MIPS in \(M\). For completeness, we have a pair \((M', d')\) corresponding to each possible subset \(M'\) of \(U\) at each edge \(e\), where \(d = \top\) if the MIPS in \(M'\) are not related with each other by CPO relation i.e., there is no MIPS \(\mu\) in \(M'\) such that \(\text{cpo}_\mu(\mu) = M'\).

At an edge \(e\), presence of a pair \((M, d)\), \(d \neq \top\) indicates the data flow value \(d\) flows through each MIPS contained in \(M\), and for some \(\mu \in M, M = \text{ cpo}_\mu(\mu)\). Similarly, a pair \(\langle \{\}, d'\rangle\) indicates that the data flow value \(d'\) does not flow through any MIPS at edge \(e\).

Example 1. Figure 6 illustrates the FPMFP computation for the example in Figure 2. Since \(\mu\) is the only MIPS in the example, each data flow value \(d\) is mapped with either \([\mu]\) or \(\{\}\). The data flow value \(a \mapsto [0, 0]\) reaches the start edge \(e_1\) of \(\mu\), hence, a pair \(\langle [\mu], a \mapsto [0, 0] \rangle\) is created at \(e_1\). Similarly, at edge \(e_4\), data flow value \(a \mapsto [5, 5]\) is mapped with \(\{\}\) i.e., a pair \(\langle \{\}, a \mapsto [5, 5] \rangle\) is created.

Edge \(e_5\) is end edge of \(\mu\), hence, the data flow value \(a \mapsto [0, 0]\) that is mapped with \(\mu\)
For a program containing $k$ MIPS, theoretically we need $2^k$ pairs at each program point (one

\[\langle \mu, a \mapsto [0, 0] \rangle\] is discarded i.e., replaced with $\top$ (the resulting pair $\langle \mu, \top \rangle$ at $e_6$ is not shown in figure for brevity).

In Section 6.4, we prove that at any program point at most $k$ pairs can contain the data flow values other than $\top$.\footnote{In Section 6.4 we prove that at any program point at most $k$ pairs can contain the data flow values other than $\top.$}
pair corresponding to each possible subset of set of all MIPS present in the program). The pairs computed in a FPMFP solution satisfy the following property:

At each program point, the meet of data flow values over all pairs gives at least as precise information as in MFP solution, and may give more precise information if some pairs contain information reaching from infeasible CFGs because such information is discarded. The precision is achieved only when the discarded information has weaker value or incomparable value compared to rest of the information reaching a node. Let \( D_n \) be the data flow value in MFP solution at node \( n \), and \( S_n \) be the set of pairs computed in FPMFP analysis at node \( n \) then

\[
D_n \sqsubseteq \bigcap_{(M, d) \in S_n} d
\]

**Example 2.** In Figure [6] at the edge \( e_5 \) the meet of data flow values over all pairs i.e., \( \langle \{\mu\}, a \mapsto [0, 0] \rangle \) and \( \langle \{\}, a \mapsto [5, 5] \rangle \) is \( a \mapsto [0, 5] \) which is same as given by MFP solution in Figure [2]; however, at the edge \( e_6 \) the meet of data flow values over all pairs i.e., \( \langle \{\mu\}, \top \rangle \) and \( \langle \{\}, a \mapsto [5, 5] \rangle \) is \( a \mapsto [5, 5] \) which is more precise compared to \( a \mapsto [0, 5] \) given by MFP solution in Figure [2].

### 3.4. Defining FPMFP solution

We now explain how these ideas are incorporated in a data flow analysis. We compute the FPMFP solution in the following two steps.

1. **Step 1:** we lift a data flow analysis to an analysis that computes separate data flow values for different MIPS (Equation 8 and 9 define the lifted analysis). The lifted analysis blocks the data flow values associated with each MIPS \( \mu \) at the end edge of \( \mu \).

2. **Step 2:** we merge the data flow values that are computed by the lifted analysis (over all pairs at each program point) to obtain the FPMFP solution (Equation 3 and 4 define the merging of data flow values).

We describe each of these steps in detail now. Let \( L \) denote the lattice of data flow values of the underlying data flow analysis. Then the MFP solution of the data flow analysis for a program is a set of values in \( L \), represented by data flow variables \( \text{In}_n / \text{Out}_n \) (Equation 6 and 7) for every node \( n \) of the CFG of the program, when the equations are solved using \( \top \) as the initial value of \( \text{In}_n / \text{Out}_n \). Assume we are given a set \( U \) that contains all MIPS in the CFG of the program, then each \( M \subseteq U \) could be associated with any data flow value \( d \) in \( L \), if \( d \) flows through each MIPS contained in \( M \). These associations are represented by the elements of the lattice \( L : \mathcal{P}(U) \rightarrow L \), where \( \mathcal{P}(U) \) is the powerset of \( U \).

We lift the underlying analysis that computes values \( \text{In}_n / \text{Out}_n \in L \) to an analysis that computes \( \text{In}_n / \text{Out}_n \in L \). Thus \( \text{In}_n / \text{Out}_n \) are sets of pairs \( (M, d) \) such that \( M \subseteq U, d \in L \). Each

where \( k \) is the total number of input MIPS. Further, our empirical evaluation shows that the number of such pairs to be much smaller than \( k \). Moreover, ignoring the pairs that contain \( \top \) value does not affect the FPMFP solution (details are given in Section 5.3).
of these sets represent a function that maps each value in \( \mathcal{P}(\mathcal{U}) \) to a value in \( \mathcal{L} \), i.e., \( \text{In}_n/\text{Out}_n \) cannot contain two pairs \( \langle M, d \rangle \) and \( \langle M', d' \rangle \) where \( M = M' \) but \( d \neq d' \). This collection of pairs gives us the distinctions required to achieve precision by separating the data flow values along infeasible paths from those along feasible paths.

For a data flow value \( \overline{\text{In}}_n \), the meaning of association of a data flow value \( d \) with a set of MIPS \( M \) is as follows:

- **Pair** \( \langle M, d \rangle \in \overline{\text{In}}_n \), indicates that the data flow value \( d \) flows through all MIPS in \( M \). If no such value exists at \( n \), then \( d = \top \).
- **Pair** \( \langle \{\}, d \rangle \in \overline{\text{In}}_n \), indicates that the data flow value \( d \) does not flow through any MIPS at node \( n \). If no such value exists (i.e., every data flow value flows through some or the other MIPS) then \( d = \top \).

**Example 1.** In Figure 5, the data flow value \( a \mapsto [0,0] \) flows through MIPS \( \mu \) at edges \( e_3, e_5, \) and \( e_6 \). On the other hand, the data flow value \( a \mapsto [5,5] \) does not flow through any MIPS. (Recall that flow through a MIPS \( \mu \) means flow along \( \mu \) — from start edge of \( \mu \) till end edge of \( \mu \)).

Section 4 defines the data flow equations to compute \( \overline{\text{In}}_n/\overline{\text{Out}}_n \). The FPMFP solution is a set of values in \( \mathcal{L} \), represented by the data flow variables \( \overline{\text{In}}_n/\overline{\text{Out}}_n \) (Equation 3 and 4) for every node \( n \) in the CFG of the program. They are computed from \( \overline{\text{In}}_n/\overline{\text{Out}}_n \) using the following operations.

\[
\overline{\text{In}}_n = \text{fold}_c(\overline{\text{In}}_n) \quad \text{(3)} \\
\overline{\text{Out}}_n = \text{fold}_c(\overline{\text{Out}}_n) \quad \text{(4)}
\]

where, \( \text{fold}_c(S) = \bigsqcap_{\langle M, d \rangle \in S} d \) \hspace{1cm} \text{(5)}

The \( \overline{\text{In}}_n/\overline{\text{Out}}_n \) values represent the FPMFP solution which represents the data flow information reaching node \( n \) along all CFPs excluding the ones that are known to be infeasible CFPs and are given as input.

### 4. Computing the FPMFP Solution

We now illustrate how the data flow equations for computing the FPMFP solution are derived from that of the MFP specifications of a data flow analysis. Section 4.1 explains this lifting by defining the data flow equations, node flow functions, and the meet operator. A key enabler for this lifting is a set of specially crafted edge flow functions \( \delta \) (Section 4.2) that discard data flow values that reach a program point from infeasible CFPs.

---

*The edge flow function for an edge \( m \to n \) uses \( \text{Out}_m \) to compute \( \text{In}_n \) in a forward analysis, and vice-versa in a backward analysis.*
In this section, we lift Equations 6 and 7 to define a data flow analysis that computes data flow variables.

**Lattices and data flow values**

- \( L = \{[a \mapsto [i, j]] \mid -\infty \leq i \leq j \leq +\infty\} \)
- \( \top = \{a \mapsto [+\infty, -\infty]\} \)
- \( \mathcal{U} = \{\mu\} \)
- \( \mathcal{T} : \mathcal{P}(\mathcal{U}) \rightarrow L \)
- \( \overline{\ln}_{n_{\text{end}}} = \{(\mu), a \mapsto [5, 5]\} \)
- \( \overline{\ln}_{n_{\text{start}}} = \{a \mapsto [5, 5]\} \)
- \( \ln_{n_{\text{start}}} = \{a \mapsto [0, 5]\} \) (Figure 2(b))

**LPMFP Solution Specification**

We lift Equations 6 and 7 to define a data flow analysis that computes data flow variables \( \overline{\ln}_{n}, \overline{\text{Out}}_{n} \) each of which is a set of pairs of the form \( (\mathcal{M}, d) \). As explained in Section 3.4, \( \overline{\ln}_{n}, \overline{\text{Out}}_{n} \) are computed by a data flow analysis at node \( n \) for brevity, we have only shown pairs where \( d \neq \top \).

![Figure 6](https://example.com/figure6.png)

Figure 6: Computing FPMFP solution for the example of Figure 2. The lifted data flow analysis computes a set of pairs \( \overline{\ln}_{n}/\overline{\text{Out}}_{n} \) at each node \( n \) such that a pair \( (\mathcal{M}, d) \), indicates \( d \) flows through each MIPS \( m \in \mathcal{M} \) if \( \mathcal{M} = \{\} \) then \( d \) does not flow through any MIPS at node \( n \). Figure 2 contains single MIPS \( \mu \), hence, two pairs are computed (corresponding to \( \mu \) and \( \emptyset \)) at each program point, for brevity, we have only shown pairs where \( d \neq \top \).

4.1. FPMFP Solution Specification

Let \( \overline{\ln}_{n}, \overline{\text{Out}}_{n} \in L \) be the data flow values computed by a data flow analysis at node \( n \), where \( L \) is a meet semi-lattice satisfying the descending chain condition. Additionally, it contains a \( \top \) element—we add an artificial \( \top \) value, if there is no natural \( \top \).

Equations 6 and 7 (given below) represent the data flow equations for computing MFP solution over the CFG of a procedure, say \( p \). For simplicity we assume a forward analysis. \( B I \) represents the boundary information reaching procedure \( p \) from its callers. The meet operator \( \sqcap \) computes the \( \text{glb} \) (greatest lower bound) of elements in \( L \). Function \( \text{pred}(n) \) returns the predecessor nodes of node \( n \) in the CFG of \( p \). The node flow function \( f_{n} \) and edge flow function \( g_{m \rightarrow n} \) compute the effect of node \( n \) and edge \( m \rightarrow n \) in the CFG respectively. Usually the edge flow functions are identity.

\[
\ ln_{n} = \begin{cases} 
\text{BI} & \text{n = Start}_p \\
\bigcap_{m \in \text{pred}(n)} g_{m \rightarrow n}(\text{Out}_m) & \text{otherwise}
\end{cases} \tag{6}
\]

\[
\text{Out}_n = f_{n}(\ln_{n}) \tag{7}
\]

We lift Equations 6 and 7 to define a data flow analysis that computes data flow variables \( \overline{\ln}_{n}, \overline{\text{Out}}_{n} \) each of which is a set of pairs of the form \( (\mathcal{M}, d) \). As explained in Section 3.4, \( \overline{\ln}_{n}, \overline{\text{Out}}_{n} \) are...
values in \( \mathcal{L} \), where \( \mathcal{L} : \mathcal{P}(\mathcal{U}) \to \mathcal{L} \) such that \( \mathcal{U} \) is the set of all MIPS in the program, and \( \mathcal{L} \) is the lattice of values computed by Equations 6 and 7.

\[
\overline{\text{Begin}}_n = \begin{cases} \mathcal{B} \bigcap_{m \in \text{pred}(n)} \overline{\text{Out}}_m & \text{if } n = \text{Start}_p \\ \overline{\text{Out}}_n & \text{otherwise} \end{cases}
\]  

(8)

\[
\overline{\text{Out}}_n = f_n(\overline{\text{In}}_n)
\]  

(9)

The top value that is used for initialization is \( \{M, \top \} | M \subseteq U \). The node flow function \( f_n \) is pointwise application of \( f_n \) to the pairs in the input set:

\[
f_n(S) = \{ \langle M, d \rangle \in S | \langle M, d \rangle \in S \}
\]  

(10)

The meet operator \( \sqcap \) is pointwise application of \( \sqcap \) to the pairs that contain the same set of MIPS i.e.,

\[
S \sqcap S' = \{ \langle M, d \sqcap d' \rangle | \langle M, d \rangle \in S, \langle M, d' \rangle \in S' \}
\]  

(12)

Note that \( \sqcap \) is defined for all values in \( \overline{\text{In}}/\overline{\text{Out}} \) because \( \overline{\text{In}}/\overline{\text{Out}} \) each contain one pair corresponding to each subset of \( \mathcal{U} \).

If the partial order between elements in \( \mathcal{L} \) is \( \sqsubseteq \) then the partial order between elements in \( \mathcal{L} \) is \( \sqsubseteq \) and is given as follows.

\[
S \sqsubseteq S' \iff (\langle M, d \rangle \in S \land \langle M, d' \rangle \in S' \Rightarrow d \sqsubseteq d')
\]  

(13)

4.2. Defining the Edge Flow Function

We now formally define the edge flow function \( \overline{g}(S) \). The edge flow function performs the following two operations over an input pair \( \langle M, d \rangle \) in \( S \).

1. If \( e \) is end edge of some MIPS in \( M \), then the data flow value \( d \) is blocked,

2. Otherwise \( d \) is associated with a set of MIPS \( M' \) where \( M' = \text{ext}(M, e) \) (described in Section 3).

The data flow value \( d \) from multiple pairs in \( S \) can get associated with the same set of MIPS \( M' \). Hence in the output pair \( \langle M', d' \rangle \), \( d' \) is effectively computed by taking meet of values of \( d \) from individual pairs \( \langle M, d \rangle \in S \) where \( M' = \text{ext}(M, e) \), as shown in Equation 15. Let the function \( \text{endof}(M, e) \) be a boolean function that denotes if \( e \) is end edge of some MIPS in \( M \) or not i.e.,

\[
\text{endof}(M, e) = (\exists \mu \in M, e \in \text{end}(\mu))
\]  

(14)

The first term in the right hand side of the following edge flow function corresponds to the Item (1) in the list above, while the second term corresponds to the Item (2) of the list.

\[
\overline{g}_e(S) = \begin{cases} \langle M', \top \rangle | M' \subseteq U, \text{endof}(M', e) \bigcup \{ \langle M', d' \rangle | M' \subseteq U, \neg\text{endof}(M', e), d' = \bigcap_{\langle M, d \rangle \in S, \text{ext}(M, e) = M'} d \} \end{cases}
\]  

(15)
5. Extending the FPMFP Computation to the Inter-procedural Level

In this section, we explain the FPMFP computation for inter-procedural data flow analysis [21] by describing the call node handling. Specifically, we formalize the FPMFP computation for the functional approach [21] of inter-procedural data flow analysis. First, we briefly explain the functional approach below. Next, we describe the class of inter-procedural MIPS (MIPS that span across multiple procedures) that we admit for the FPMFP computation (Section 5.1). Finally, we explain how FPMFP handles call nodes (Section 5.2).

The functional approach consists of two phases. The first phase computes summaries of procedures in a program, by traversing the callgraph in a bottom-up order (i.e., summaries for callee procedures are computed before computing the summaries of the caller procedures). In this phase, a non-recursive procedure is analyzed once, while a recursive procedure is analyzed multiple times until a fix-point of procedure summaries is reached. After the summary computation, a top-down traversal of the call-graph is performed to propagate data flow values from callers to callees. In this phase, a procedure is analyzed in an intra-procedural manner because the effect of call nodes is represented by the summaries of the callee procedures. Therefore, explaining call node handling and procedure summary computation is sufficient to extend the FPMFP computation at the inter-procedural level using the functional approach.

We first state the types of inter-procedural MIPS that we admit (Section 5.1), followed by the description of call node handling (Section 5.2), and procedure summary computation (Section 5.3).

5.1. Inter-procedural MIPS

An inter-procedural MIPS can span across multiple procedures, unlike an intra-procedural MIPS that remains inside a single procedure. We allow all intra-procedural MIPS as input for a FPMFP computation. Additionally, we allow a specific class of inter-procedural MIPS in the FPMFP computation. We describe this class of inter-procedural MIPS below.

We categorize an inter-procedural MIPS as shown in Figure 8. Here, we distinguish between the two types of inter-procedural MIPS depending on if they start and end in the same procedure or not. We admit a subclass of MIPS that belong to the former type; this subclass is defined below.
We refer these MIPS as balanced inter-procedural MIPS, because each call edge is matched by a corresponding call-return edge.

**Definition 2.** (Balanced Inter-procedural MIPS) Let a MIPS $\mu$ be an inter-procedural MIPS and $V$ be the set of variables present in the condition on the end edge of $\mu$. We say $\mu$ is a balanced inter-procedural MIPS if it starts and ends in the same procedure in a non-recursive manner, and the variables in $V$ are not modified inside the procedures called through the intermediate nodes of $\mu$.

For example in Figure 9, MIPS $\mu_1 : n_1 \xrightarrow{e_1} n_2 \xrightarrow{e_3} n_3 \xrightarrow{e_4} n_4 \xrightarrow{e_5} n_5$ is a balanced inter-procedural MIPS. A balanced MIPS has the following interesting property.

**Example 1.** In Figure 9 MIPS $\mu_1 : n_1 \xrightarrow{e_1} n_2 \xrightarrow{e_3} n_3 \xrightarrow{e_4} n_4 \xrightarrow{e_5} n_5$ goes through the call node $n_3$ that calls procedure $q$. However, the control flow inside $q$ is abstracted out during a FPMFP computation of the caller procedure $p$. In particular, the edges in procedure $q$ are not marked as inner edges of $\mu_1$ (even though the call $q()$ is part of $\mu_1$).

Henceforth, we assume all inter-procedural MIPS that are input to our analysis are balanced. We now explain the handling of a call node followed by the procedure summary computation for bit-vector frameworks.

### 5.2. Handling the Call Nodes

We now describe how the effect of a call node is incorporated in a FPMFP computation. For this, we assume summaries of all procedures are already computed. We explain the procedure summary computation for bit-vector frameworks in Section 5.3.

---

5. For each call edge there is a corresponding call return edge in the MIPS.

6. In general, this may not hold for all inter-procedural MIPS that start and end in the same procedure. Specifically, there could exist an inter-procedural MIPS $\mu_1$ such that the variables in the condition on the end edge of $\mu_1$ are modified along some paths inside the procedures called from intermediate nodes of the MIPS.
5.2.1. Effect of a Caller Procedure on its Callee Procedure

The boundary information (\(B_q\)) for a callee procedure \(q\) is computed by taking the meet of data flow values present at all call sites of \(q\). Next, \(B_q\) is used for solving the callee procedure \(q\). If \(C_q\) is the set of all call sites of \(q\), and \(U_q\) is the set containing 1) intra-procedural MIPS of \(q\), and 2) balanced inter-procedural MIPS that start and end in \(q\) then \(B_q\) is computed as follows:

\[
B_q = \{\{\}, x\} \cup \{\{M, T\} | M \subseteq U_q, M \neq \{\}\}, \text{where} \ \\
x = \bigcap_{n \in C_q} \text{fold}_3(\overline{m}_n)
\]
Figure 10: Generic view of call node handling in a FPMFP computation. $C_{pq}$ represents the transfer of control from procedure $p$ to procedure $q$ at a call node and $R_{qp}$ represents the return of control from $q$ to $p$. $U_q$ is the set containing all i) intra-procedural MIPS of $q$, and ii) balanced inter-procedural MIPS that start and end in $q$.

In Figure 9, the data flow information in $\overline{n_{3}}$ is used to compute the boundary information ($\overline{B}_q$) for procedure $q$ i.e., $\overline{B}_q = \{(\{\}, l \mapsto [2, 2])\}$.

Moreover, a single boundary is maintained for each procedure. If there is a change in the data flow information at any call site of a procedure $q$ then the new boundary $\overline{B}_q'$ is merged with the existing boundary information of $q$ ($\overline{B}_q$). If this leads to a change in the resulting boundary information of $q$, the procedure $q$ is added to the worklist for solving. Later, the procedure is solved in an intra-procedural manner using the new boundary information by the worklist algorithm [21].

5.2.2. Effect of a Callee Procedure on its Caller Procedure

The MFP computation (specified by equations 6 and 7 from Section 3.4) defines the node flow functions ($f_n$) for all nodes—including nodes that call other procedures. In the functional approach of inter-procedural analysis [21], the call node flow functions are defined using summaries of the corresponding callee procedures. We use these node flow functions to compute the effect of a procedure call on the caller procedure using equation 11. For example, in Figure 9 the MFP analysis defines the node flow function $f_{n3}$ which incorporates the effect of call $q()$ i.e., $q$ replaces the original value of $l$ by 0. The FPMFP computation uses $f_{n3}$ to compute $\overline{f}_{n3}$ using equation 11 as follows:

$$\overline{f}_{n3}(\{(\{\}, l \mapsto [2, 2]), (\{\mu_1\}, l \mapsto [2, 2])\}) = \{(\{\}, f_{n3}(l \mapsto [2, 2])), (\{\mu_1\}, f_{n3}(l \mapsto [2, 2]))\}$$

We elaborate the procedure summary computation for bit-vector frameworks in Section 5.3.
5.3. Computing the feasible path GEN and KILL summaries for bit-vector frameworks

In this section, we state the MFP specifications used for the computation of procedure summaries for bit-vector frameworks [1]. Next, we lift these specifications to the corresponding FPMFP specifications.

We use the standard notions of a procedure summary computation for bit-vector frameworks from [1]. In these frameworks, the node flow functions are of the following form \( f_n(X) = (X - \text{KILL}_n) \cup \text{GEN}_n \), where \( \text{GEN}_n \) and \( \text{KILL}_n \) are constant for each node. Hence the procedure summaries can be constructed by composing individual node flow functions, independent of the calling context information.

The meet operator in bit-vector frameworks is either the set-union or set-intersection. For simplicity of exposition, we explain the procedure summary computation assuming the meet operator is union, henceforth we refer these data flow problems as union problems. We explain the computation for forward data flow analysis, a dual modeling exists for backward analysis (like live variables analysis [1]).

Let \( \text{GEN}_n, \text{KILL}_n \) be the constant GEN and KILL information at a non-call node \( n \) respectively. Then GEN and KILL summaries for procedures are computed by solving corresponding GEN and KILL data flow problems as described below.

**Solving the KILL data flow problem.** The kill summary for union problems is computed using \( \cap \) as the meet operator, indicating that a value is added to the kill summary of a procedure iff the value is killed along all CFPs that reach the exit of the procedure. Initially, kill summary of all procedures is set to \( \top \) which is the top value of the kill data flow lattice. \( \emptyset \) is used as the boundary value. The MFP specifications of a KILL data flow problem for a procedure \( p \) are as follows.

\[
\text{KIn}_n = \begin{cases} 
\emptyset & n = \text{Start}_p \\
\bigcap_{m \in \text{pred}(n)} \text{KOut}_m & \text{otherwise} 
\end{cases} 
\]

\[
\text{KOut}_n = \text{KIn}_n \cup \text{nodekill}(n) 
\]

\[
\text{nodekill}(n) = \begin{cases} 
\text{KILL}_n & n \text{ is not a call node} \\
\text{KOut}_{\text{Exit}_q} & n \text{ calls a procedure } q 
\end{cases} 
\]

\( \text{nodekill}(n) \) represents the kill information for the node \( n \); the information is constant if \( n \) is not a call node. Otherwise, \( \text{nodekill}(n) \) is the information at the exit the procedure called from \( n \). For simplicity of exposition, we assume at most one procedure can be called from a node (in practice, more than one procedures can be called from a node through function pointers, in which case the kill summary of the node is computed by doing intersection of kill information at exit of all procedures that can be called from \( n \)).

Equations [16] to [18] are solved for each procedure chosen in a bottom up traversal of the call graph of a program. In this, each non-recursive procedure is solved once, while a recursive procedure is solved multiple times until the data flow values at all points saturate.

The final value in KOut at the exit of a procedure \( p \) is the KILL summary for procedure \( p \), as given by the MFP solution. This summary may include the KILL information reaching from infeasible CFPs. Hence, to get a more precise summary, we lift the MFP specifications to the FPMFP specifications for each procedure \( p \) in the program as follows. Let \( \mathcal{U}_p \) be the set containing 1) intra-procedural MIPS of \( p \), and 2) balanced inter-procedural MIPS that start and end in procedure \( p \).
The FPMFP specifications (obtained by lifting MFP specifications from equations 16 to 18):

\[
\begin{align*}
K\text{In}_n &= \begin{cases} 
K\text{ILL}_n & n = \text{Start}_p \\
\bigcap_{m \in \text{pred}(n)} K\text{Out}_m & \text{otherwise}
\end{cases} \\
K\text{ILL}_n &= |\{|\{}\| \cup |\{M, T\} | M \subseteq \mathcal{U}_p, M \neq |\{|\}\}| \\
K\text{Out}_n &= |\{M, D \cup \text{nodekill}(n)\} | \langle M, D \rangle \in \text{KIn}_n| \\
\text{nodekill}(n) &= \bigcap_{\langle M, D \rangle \in \text{KOut}_{\text{Exit}_q}} D \quad \text{n calls a procedure q}
\end{align*}
\]

The \(\bigcap\) does a point-wise intersection of the kill data flow values in the input set of pairs as follows.

\[
S_1 \bigcap S_2 = |\{|M, D_1 \cap D_2\} | \langle M, D_1 \rangle \in S_1, \langle M, D_2 \rangle \in S_2|
\]

The final kill summary of a procedure \(p\) is computed by doing intersection of the data flow values at the exit of \(p\) i.e.,

\[
\text{KSU}M_p = \bigcap_{\langle M, D \rangle \in \text{KOut}_{\text{Exit}_p}} D
\]

We now explain the GEN summary computation of a procedure below.

**Solving the GEN data flow problem.** The GEN summary for union problems is computed using \(\cup\) as the meet operator, indicating that a data flow value is added to GEN summary if it is generated along atleast one CFP that reaches the exit of the procedure. Next, \(|\{|\}|\) is used as the boundary value. The corresponding MFP specifications are as follows.

The MFP specifications:

\[
\begin{align*}
G\text{In}_n &= \begin{cases} 
|\{} & n = \text{Start}_p \\
\cup_{\text{pred}(n)} G\text{Out}_m & \text{otherwise}
\end{cases} \\
G\text{Out}_n &= (G\text{In}_n - \text{nodekill}(n)) \cup \text{nodegen}(n) \\
\text{nodegen}(n) &= \bigcap_{\langle M, D \rangle \in \text{KOut}_{\text{Exit}_q}} D \quad \text{n calls a procedure q}
\end{align*}
\]

Based on these MFP specifications, we derive the FPMFP specifications for each procedure \(p\) in the program as follows. Let \(\mathcal{U}_p\) be the set containing 1) intra-procedural MIPS of \(p\), and 2) balanced inter-procedural MIPS that start and end in \(p\).
The FPMFP specifications:

\[
\overline{\text{GIn}}_n = \begin{cases} 
\{\langle M, \emptyset \rangle \mid M \subseteq U \} & \text{n = Start} \\
\bigcap_{m \in \text{pred}(n)} \overline{s}_{m \to n}(\overline{\text{GOut}}_m) & \text{otherwise}
\end{cases}
\]  

(28)

\[
\overline{\text{GOut}}_n = \{\langle M, (D - \text{nodekill}(n)) \cup \text{nodegen}(n) \rangle \mid \langle M, D \rangle \in \text{GIn}_n\}
\]  

(29)

\[
\text{nodegen}(n) = \begin{cases} 
\text{GEN}_n & \text{n is not a call node} \\
\bigcup_{\langle M, D \rangle \in \overline{\text{GOut}}_\text{Exit}q} D & \text{n calls a procedure q}
\end{cases}
\]  

(30)

\[
\sqcap' \text{ does a point-wise union of data flow values in the set of input pairs as follows.}
\]

\[
S_1 \sqcap' S_2 = \{\langle M, D_1 \cup D_2 \rangle \mid \langle M, D_1 \rangle \in S_1, \langle M, D_2 \rangle \in S_2\}
\]  

(31)

The final gen summary of a procedure \( p \) is computed by doing the union of data flow values at the exit of the procedure \( p \) i.e.,

\[
\text{GSUM}_p = \bigcup_{\langle M, D \rangle \in \overline{\text{GOut}}_\text{Exit}p} D
\]  

(32)

Substituting GEN and KILL summaries to compute the effect of procedure calls.

The GEN and KILL summaries are computed for all procedures using the corresponding constant boundary values. We call this summary computation phase. Later, these GEN and KILL summaries are used to compute the effect of procedure \( p \) at all callsites of \( p \) during the actual FPMFP computation (using actual boundary values) as follows: Let \( n : p() \) be a callsite of \( p \) then

\[
f_n(X) = (X - \text{KSUM}_p) \cup \text{GSUM}_p
\]  

(33)

\[
f_n(S) = \{\langle M, f_n(D) \rangle \mid \langle M, D \rangle \in S\}
\]  

(34)

6. Optimizations for Computing FPMFP Solutions

We now describe few optimizations that improve the scalability of a FPMFP computation. First, we explicate a relation between efficiency of the FPMFP computation and the number of pairs in the FPMFP computation. Next, we propose ideas to reduce the number of pairs. Our empirical data shows the reduction obtained is significant.

As evident from equations\([8]\) to \([15]\) the efficiency of a FPMFP computation is inversely proportional to the number of pairs. Theoretically, the number of pairs needed is exponential in the number of MIPS in \( U \) (set of all MIPS) because a different pair is created corresponding to each subset of \( U \). However, at any program point the number of pairs that are evaluated is bounded by \(|U| + 1\) (explained in Section\([6.3]\)). Below, we describe the ideas to reduce the number of pairs by either merging them or ignoring the ones that have no effect on the final FPMFP solution. These optimizations are shown in Figure\([11]\). We describe each of these optimizations below.

24
Optimizations that involve merging pairs

Optimizations that involve ignoring a pair

Optimization 1 (Section 6.1)
Optimization 2 (Section 6.2)
Optimization 3 (Section 6.3)

Figure 11: Optimizations for scalability of the FPMFP computation: The leaf nodes describe a particular optimization, and rest of the nodes describe the type of the optimization.

Sets of MIPS

| Edges | Sets of MIPS | FPMFP |
|-------|--------------|-------|
|       | [1]          | [µ₁]  | [µ₂]  | [µ₁, µ₂] | ⊓   |
| e₁    | z → [1, 1]   | T      | T      | T        | z → [1, 1] |
| e₂    | T            | z → [0, 0] | T      | T        | z → [0, 0] |
| e₃    | T            | T      | z → [2, 2] | T        | z → [2, 2] |
| e₄    | z → [1, 1]   | T      | T      | T        | z → [1, 1] |
| e₅    | z → [0, 0]   | T      | T      | T        | z → [0, 0] |

Table 4: The FPMFP solution for example in Figure 12a. For brevity, only values of z are shown.

6.1. Merging Pairs

We now describe an optimization that allows us to merge some of the pairs present at a program point. We observed that a large class of MIPS found in our benchmarks were satisfying the following property.

\[ \mathcal{P}: \text{The condition on the end edge of a MIPS } \mu \text{ evaluates to false at the start edge of } \mu, \text{ and the variables present in the condition are not modified at the intermediate nodes of } \mu. \]

We observe the following peculiarity of MIPS that satisfy \( \mathcal{P} \).

**Observation 6.1.** We do not need to distinguish between MIPS that satisfy \( \mathcal{P} \) and have the same end edge, because the condition on the end edge does not hold at the starts of these MIPS and data flow values associated with them are blocked at the same end edge. Hence, the data flow values associated with them can be merged.

We support Observation 6.1 with a proof in Section 10.4. Observation 6.1 allows us to merge pairs at a program point as described below.

**Optimization 1.** We merge two pairs \( \langle \mathcal{M}, d \rangle \) and \( \langle \mathcal{M}', d' \rangle \) into a single pair \( \langle \mathcal{M} \cup \mathcal{M}', d \cap d' \rangle \)
read a, b; z = 1; d = 1

\[ \begin{align*}
&\text{switch(b)} \\
&\begin{array}{ll}
&\text{case 2} & a = 3; z = 0 \\
&\text{default} & a = 5; z = 2 \\
&\end{array}
\end{align*} \]

print a

a > 0

true false

return -1

return a/z

\[ \begin{align*}
&\text{Paths and MIPS:} \\
&\begin{array}{l}
&MIPS \mu_1 : e_5 \rightarrow e_3 \rightarrow e_9 \\
&MIPS \mu_2 : e_6 \rightarrow e_3 \rightarrow e_9 \\
&\{
&\mu_1 \} \equiv \{ \mu_2 \} \equiv \{ \mu_1, \mu_2 \}
&\end{array}
\end{align*} \]

The FPMFP solution: MIPS \( \{ \mu_1 \}, \{ \mu_2 \} \), and \( \{ \mu_1, \mu_2 \} \) are end edge equivalent so the corresponding data flow values are associated with \( \{ \mu_1, \mu_2 \} \) only. For brevity, computation of values for \( z \) is shown only.

\[ \begin{align*}
&\text{Sets of MIPS} \\
&\begin{array}{l}
&\{ \} \\
&\{ \mu_1, \mu_2 \} \\
&\{ \mu_1, \mu_2 \}
&\end{array}
\end{align*} \]

\[ \begin{align*}
&\text{FPFMFP} \\
&\begin{array}{l}
&\{ \} \\
&\{ \mu_1, \mu_2 \} \\
&\{ \mu_1, \mu_2 \}
&\end{array}
\end{align*} \]

\[ \begin{align*}
&\text{Edges} \\
&e_5 & z \mapsto [1, 1] & \top & z \mapsto [1, 1] \\
&e_6 & \top & z \mapsto [0, 0] & z \mapsto [0, 0] \\
&e_7 & \top & z \mapsto [2, 2] & z \mapsto [2, 2] \\
&e_8 & z \mapsto [1, 1] & \top & z \mapsto [0, 2] \\
&e_9 & z \mapsto [1, 1] & \top & z \mapsto [1, 1]
&\end{align*} \]

Figure 12: Equivalent MIPS

\( i f f \) the sets \( M \) and \( M' \) are \textit{end edge equivalent} (denoted by \( \equiv^e \)) as defined below:

\[ M \equiv^e M' \iff \bigcup_{\mu \in M} \text{end}(\mu) = \bigcup_{\mu' \in M'} \text{end}(\mu') \quad (35) \]

The FPMFP computation for the example in Figure 12 is shown in Table 4. Four pairs are computed at each edge corresponding to each subset of \( \{ \mu_1, \mu_2 \} \). However, MIPS \( \mu_1, \mu_2 \) have the same end edge \( e_9 \) and hence the following sets are end edge equivalent: \( \{ \mu_1 \}, \{ \mu_2 \}, \{ \mu_1, \mu_2 \} \). We obtain the FPMFP solution by merging these pairs as shown in Figure 12b where only two pairs are maintained at each edge.

6.2. Removing Duplicate Data Flow Values Across Pairs

In this section, we explain how the same data flow values can appear in more than one pairs at a program point. Further, we propose an idea to remove this duplication of values while retaining the precision and soundness of the FPMFP solution. This optimization has helped us to improve the scalability of our approach to 150KLOC, whereas without this optimization scalability was 73KLOC.

In a FPMFP computation, at an edge \( e \), two pairs can contain the same data flow value \( d \) \( i f f \) \( d \) reaches \( e \) along two or more CFPs (because in this case \( d \) may flow through two different MIPS which are not related by CPO relation). In such case, if one of the CFP is a feasible CFP then the two pairs can be merged into one as illustrated in Example 1 below.
Example 1. For example in Figure 13, the data flow value $d \mapsto [1, 1]$ flows through MIPS $\mu$, hence it is associated with $\{\mu\}$ at edges in $\mu$ i.e., $e_3, e_5, e_7$. Next, the value associated with $\{\mu\}$ is blocked at the end edge $e_7$ of $\mu$, because this value is reaching along the infeasible CFP that contains $\mu$ i.e., $e_0 \rightarrow e_1 \rightarrow e_3 \rightarrow e_5 \rightarrow e_7$.

On the other hand, $d \mapsto [1, 1]$ also reaches $e_7$ along the following CFP that is not infeasible $\sigma : e_0 \rightarrow e_2 \rightarrow e_4 \rightarrow e_6 \rightarrow e_7$, hence $d \mapsto [1, 1]$ is also associated with $\{\}$). Effectively, $d \mapsto [1, 1]$ is included at $e_7$ in the FPMFP solution.

In such case, we can get the same FPMFP solution without associating $d \mapsto [1, 1]$ with $\{\mu\}$ at $e_5, e_7$ as shown in Table 5. Observe that the corresponding FPMFP solution is same to that in Figure 13.

We use the following general criteria to treat the pairs that contain the same data flow value at a program point.

If a data flow value $d$ reaches the end edge of a MIPS $\mu$ along a feasible CFP then $d$ need not be associated with $\mu$, because even if $d$ is blocked within $\mu$, it reaches along the feasible CFP and hence its association with $\mu$ is redundant.

To find out if a data flow value reaches the end edge of a MIPS along a feasible CFP, we define the contains-suffix-of relation below.

**Definition 3.** (Contains-suffix-of (CSO)) For a MIPS $\mu$ and an edge $e$ in $\mu$, let $\text{suffix}(\mu, e)$ be the sub-segment of $\mu$ from $e$ to $\text{end}(\mu)$. Then, we say that “MIPS $\mu_1$ contains-suffix-of MIPS $\mu_2$ at $e$”, if $\mu_1$ contains suffix($\mu_2$, $e$).

$$cso(\mu_1) = \{\mu_2 \mid \mu_2 \in \mathcal{U}, \mu_1 \text{ contains suffix}(\mu_2, e)\}$$

Observe that the CSO is a reflexive relation, and a dual of the CPO relation defined in section 3.3 in the following sense: at an edge $e$, $\mu_1$ CPO $\mu_2$ means $\mu_1$ and $\mu_2$ follow the same CFP till edge $e$ whereas, $\mu_1$ CSO $\mu_2$ means $\mu_1$ and $\mu_2$ follow the same CFP after edge $e$ however they may or may not follow the same CFP before edge $e$.

The CSO relation has the following properties.

- **Property 1:** if a MIPS $\mu_1$ contains suffix of a MIPS $\mu_2$ at an edge $e$, then the data flow value associated with $\mu_2$ is blocked at or before reaching the end of $\mu_1$ (because after $e$, $\mu_1$ and $\mu_2$ follow the same CFP on which $\mu_2$ ends at or before the end of $\mu_1$).
Example 2. For example in Figure 15a, \( \mu_1 \) CSO \( \mu_2 \) at \( e_5 \). Hence, \( l \mapsto [0,0] \) is blocked at \( e_6 \) in pair \( \langle \mu_2 \rangle, l \mapsto [0,0] \) as well as at \( e_7 \) in \( \langle \mu_1 \rangle, l \mapsto [0,0] \). Thus \( l \mapsto [0,0] \) does not reach \( e_7 \).

- Property 2: if two pairs \( \langle M_1, d \rangle \) and \( \langle M_2, d \rangle \) contain the same data flow value \( d \) at an edge \( e \), and \( d \) is not killed along a MIPS \( \mu_2 \) in \( M_2 \) then the following holds.

\[
\text{d reaches the end of } \mu_2 \text{ along atleast one feasible CFP that passes through } e, \text{ provided that } \mu_2 \text{ does not contain suffix of any MIPS in } M_1 \text{ at } e. \text{ Specifically in this case, the data flow value } d \text{ from pair } \langle M_1, d \rangle \text{ reaches the end of } \mu_2 \text{ (proof in Section 10.4). Hence the association of } d \text{ with } \mu_2 \text{ can be discarded.}
\]

Example 3. For example in Figure 14a, \( \mu_2 \) does not contain suffix of \( \mu_1 \). Further, \( l \mapsto [0,0] \) flows through \( \mu_1 \) and \( \mu_2 \) and is blocked at corresponding end edges as shown in Figure 14b. However, \( l \mapsto [0,0] \) is included in the FPMFP solution at the end edge \( e_6 \) of \( \mu_2 \) because it reaches along the following feasible CFP: \( e_0 \to e_3 \to e_5 \to e_6 \to e_9 \).

In such case, we can get the same FPMFP solution without maintaining an association with \( \mu_2 \).

Based on the above properties, we now formally describe the following criteria to merge two pairs that have the same data flow value.

Optimization 2. At an edge \( e \), we shift the common data flow value \( d \) from two pairs \( \langle M_1, d \rangle \) and \( \langle M_2, d \rangle \) to a single pair \( \langle M_3, d \rangle \) where \( M_3 \) contains only those MIPS in \( M_1 \cup M_2 \) that contain suffix of at least one MIPS in \( M_1 \) and at least one MIPS in \( M_2 \) i.e.,

\[
M_3 = \{ \mu_1 : \mu_1 \in M_1, \exists \mu_2 \in M_2, \mu_1 \text{ CSO } \mu_2 \text{ at } e \} \\
\text{or} \\
\{ \mu_2 : \mu_2 \in M_2, \exists \mu_1 \in M_1, \mu_2 \text{ CSO } \mu_1 \text{ at } e \}
\]

Example 4. For example in Figure 14a, at edges \( e_5 \) and \( e_6 \) \( \mu_1 \) CSO \( \mu_2 \), hence we associate \( l \mapsto [0,0] \) with \( \mu_1 \) only as shown in Figure 14a. Observe the corresponding FPMFP solution is same as in Figure 14a.

Example 5. For example in Figure 15a, the data flow value \( l \mapsto [0,0] \) flows though two MIPS \( \mu_1 \) and \( \mu_2 \). Further neither \( \mu_1 \) CSO \( \mu_2 \) nor \( \mu_2 \) CSO \( \mu_1 \), hence \( l \mapsto [0,0] \) reaches \( \text{end}(\mu_1) \) and \( \text{end}(\mu_2) \) along the following feasible CFPs \( \sigma_1 \) and \( \sigma_2 \) respectively, \( \sigma_1 : e_0 \to e_1 \to e_5 \to e_4 \to e_7, \sigma_2 : e_5 \to e_2 \to e_4 \to e_5 \). Hence, \( l \mapsto [0,0] \) is included in the FPMFP solution at edges \( e_7 \) and \( e_3 \) as shown in Figure 15a.

\footnote{Note that CSO is a reflexive relation, hence if \( \mu_1 \in M_1 \) then \( \mu_1 \) contains suffix of at least one MIPS in \( M_1 \) is trivially true.}
We obtain the same FPMFP solution without associating \( l \mapsto [0, 0] \) with \( \mu_1 \) or \( \mu_2 \) as shown in Figure 15c.

6.3. Removing the pairs that contain \( \top \) data flow value

In a pair \( \langle M, d \rangle \) the data flow value \( d \) remains \( \top \) at edges that are outside a MIPS in \( M \). We use the following observation to eliminate such pairs.

**Observation 6.2.** Ignoring the pairs that contain \( \top \) as data flow value does not affect the precision or the soundness of the FPMFP solution because the FPMFP solution is computed by taking meet of the data flow values in each pair at a program point.

**Optimization 3.** We eliminate the pairs that have \( \top \) as the associated data flow value, if the total number of remaining pairs at the program point is more than one.

For example in Figure 12b, the pairs corresponding to \( \emptyset \) at edges \( e_5 \) and \( e_6 \) contain \( \top \) value hence these pairs can be eliminated. Similarly, pairs corresponding to \( \{\mu_1, \mu_2\} \) at edges \( e_3 \) and \( e_7 \) can be eliminated.

With this modification the node and edge flow functions (Equations 11, 15) remain same except that the meet operator (Equation 12) now assumes absence of a pair corresponding to some \( M \subseteq \mathcal{U} \) in the input sets as presence of a pair \( \langle M, \top \rangle \).
6.4. Complexity Analysis

The cost of the FPMFP computation is equivalent to computing $k + 1$ parallel MFP solutions, where $k$ is bounded by the maximum number of pairs at a program point that do not contain $\top$ as the associated data flow value.

At an edge $e$, the maximum number of pairs that do not contain $\top$ value is same as the maximum number of distinct sets given by $\mathit{cpo}_e(\mu)$ over all MIPS $\mu$, plus 1 for the pair corresponding to $\{\}$. In worst case, $\mathit{cpo}_e(\mu)$ set for each $\mu$ is distinct. Hence the total number of such pairs (i.e., $k$ value from previous paragraph) is bounded by the number of MIPS in the CFG. In practice we found the $k$ value to be much smaller where the FPMFP computation time was $2.9 \times$ of the MFP computation time on average. This is because not all MIPS in a program overlap with each other.

7. Experiments and Results

In this section, we empirically compare FPMFP solution with MFP solution with respect to the precision of the solutions and the efficiency of their computation. Indeed, we did not find any other existing technique that is as generic as the MFP computation, and eliminates the effect of infeasible paths from exhaustive data flow analysis (existing techniques are explained in Section 8).
point analysis, alias analysis in flow sensitive and context insensitive manner, followed by constructs CFGs for procedures in the program. Later, it runs some basic inbuilt analyses like analysis. Initially, the tool transforms the source code to an intermediate representation, and from 17 for the size which contained 7 SPEC CPU-2006 benchmarks, 5 industry codesets, and with Intel core i7-5600U processor having a clock speed of 2.6 GHz. We used 30 C programs of in our benchmarks (Section 7.6)

The Section is organized as follows. First, we explain the experimental setup and benchmark characteristics (Section 7.1). Next, we describe two client analyses on which we compared precision of FPMFP and MFP solutions (Section 7.2), followed by comparing the performance of FPMFP and MFP solutions (Section 7.5). Lastly, we discuss infeasible path patterns observed in our benchmarks (Section 7.6).

7.1. Experimental Setup

We performed our experiments on a 64 bit machine with 16GB RAM running Windows 7 with Intel core i7-5600U processor having a clock speed of 2.6 GHz. We used 30 C programs of up to 150KLoC size which contained 7 SPEC CPU-2006 benchmarks, 5 industry codesets, and 18 open source codesets [23, 26, 27]. These programs were randomly chosen to cover a range of application areas with different program characteristics. The number of distinct MIPS range from 17 for the acpid benchmark to 2.1k for the h264ref benchmark. In these benchmarks, up to 61% (geometric mean 29%) functions had at least one MIPS in their CFG. On average 77% MIPS were overlapping with at least one other MIPS.

We used the iterative fix-point based data flow analysis algorithm [23, 1] implemented in Java in a commercial static analysis tool TCS Embedded Code Analyzer [22] for our data flow analysis. Initially, the tool transforms the source code to an intermediate representation, and constructs CFGs for procedures in the program. Later, it runs some basic inbuilt analyses like pointer analysis, alias analysis in flow sensitive and context insensitive manner, followed by
Table 6: Benchmark properties: three types of benchmarks that we used include Open source, Industry, and SPEC CPU 2006 benchmarks.

| Type       | Name          | KLOC | #MIPS | #Overlapping MIPS | #Balanced MIPS | %Funcs Impacted |
|------------|---------------|------|-------|-------------------|----------------|-----------------|
| Open Source| acpid         | 0.3  | 6     | 6                 | 6              | 7               |
|            | polymorph     | 0.4  | 24    | 24                | 17             | 11              |
|            | nlkain        | 0.5  | 33    | 8                 | 3              | 58              |
|            | spell         | 0.6  | 14    | 11                | 12             | 23              |
|            | ncompress     | 1    | 30    | 27                | 16             | 30              |
|            | gzip          | 1.3  | 49    | 46                | 40             | 25              |
|            | stripce       | 2.1  | 107   | 97                | 74             | 30              |
|            | barcode-nc    | 2.3  | 60    | 49                | 22             | 25              |
|            | barcode       | 2.8  | 76    | 66                | 28             | 24              |
|            | archimedes    | 5    | 118   | 86                | 53             | 38              |
|            | combine       | 5.7  | 234   | 210               | 124            | 29              |
|            | httpd         | 5.7  | 226   | 211               | 88             | 16              |
|            | sphinxbase    | 6.6  | 213   | 199               | 109            | 16              |
|            | chess         | 7.5  | 262   | 186               | 112            | 39              |
|            | antiword      | 19.3 | 669   | 570               | 306            | 32              |
|            | sendmail      | 20.3 | 726   | 681               | 349            | 32              |
|            | sudo          | 45.8 | 250   | 223               | 148            | 15              |
|            | ffmpeg        | 80   | 1363  | 1201              | 891            | 9               |
| SPEC       | mcf           | 5.4  | 19    | 17                | 4              | 20              |
|            | bzip2         | 16   | 501   | 485               | 184            | 40              |
|            | hmer          | 16.5 | 595   | 522               | 270            | 20              |
|            | sjeng         | 27   | 438   | 379               | 198            | 35              |
|            | milc          | 30   | 497   | 375               | 141            | 38              |
|            | h264ref       | 35.9 | 2189  | 1696              | 526            | 31              |
|            | gobmk         | 150  | 1834  | 1435              | 756            | 13              |
| Industry   | NZM           | 2.6  | 52    | 43                | 34             | 43              |
|            | IC            | 3.7  | 50    | 29                | 13             | 15              |
|            | AIS           | 6    | 83    | 56                | 21             | 14              |
|            | FRSC          | 6    | 42    | 21                | 1              | 15              |
|            | FX            | 7.5  | 140   | 78                | 39             | 34              |

constructing the program call graph. Since FPMFP just lifts the abstract operations given in the input MFP specification, it remains oblivious to and respects the analysis specific choices like use of aliasing, pointers, and field sensitivity. In our evaluation, we implemented the functional approach of inter-procedural analysis (described in Section 5) for computing MFP and FPMFP solutions for potentially uninitialized variable analysis and reaching definitions analysis. The
details of the same are discussed in Section 7.2.

Table 7: Reduction in the number of potentially uninitialized variable Alarms

| Benchmarks      | MFP | FPMFP | reduction(%) |
|-----------------|-----|-------|--------------|
| Open Source     |     |       |              |
| 1.acpid         | 1   | 1     | 0(0)         |
| 2.polymorph     | 4   | 4     | 0(0)         |
| 3.nilkain       | 4   | 0     | 4(100)       |
| 4.spell         | 3   | 3     | 0(0)         |
| 5.ncompress     | 7   | 7     | 0(0)         |
| 6.zip           | 0   | 0     | -            |
| 7.stripcc       | 27  | 1     | 26(96.30)    |
| 8.barcode-nc    | 0   | 0     | -            |
| 9.barcode       | 2   | 0     | 2(100)       |
| 10.archmedes    | 61  | 56    | 5(8.20)      |
| 11.combine      | 63  | 63    | 0(0)         |
| 12.httpd        | 117 | 117   | 0(0)         |
| 13.sphinxbase   | 46  | 43    | 3(6.52)      |
| 14.chess        | 16  | 16    | 0(0)         |
| 15.antiword     | 18  | 18    | 0(0)         |
| 16.sendmail     | 103 | 102   | 1(0.97)      |
| 17.sudo         | 62  | 58    | 4(6.45)      |
| 18.ffmpeg       | 124 | 112   | 12(9.68)     |
| SPEC            |     |       |              |
| 19.mcf          | 5   | 5     | 0(0)         |
| 20.bzip2        | 72  | 35    | 37(51.39)    |
| 21.hmmer        | 636 | 629   | 7(1.10)      |
| 22.sjeng        | 19  | 19    | 0(0)         |
| 23.milc         | 188 | 168   | 20(10.64)    |
| 24.h264ref      | 52  | 48    | 4(7.69)      |
| 25.gobmk        | 1216| 1199  | 17(1.39)     |
| Industry        |     |       |              |
| 26.NZ           | 3   | 0     | 3(100)       |
| 27.TC           | 5   | 5     | 0(0)         |
| 28.AIS          | 0   | 0     | -            |
| 29.FRSC         | 3   | 3     | 0(0)         |
| 30.FX           | 84  | 84    | 0(0)         |

7.2. Comparing the Precision of FPMFP and MFP Solutions

7.3. Reduction in the Number of Potentially Uninitialized Variable Alarms

A potentially uninitialized variable analysis reports the variables that are used at a program point but are not initialized along at least one path that reaches the program point. We computed the potentially uninitialized variables as follows: first, we implemented the must defined variables analysis that computes the set of non-array variables that are defined along all CFPs reaching a program point. Next, we take the complement of the must defined data flow results to
Table 8: Reduction in the number of def-use pairs: 1) MFP (resp. FPMFP) column shows the sum of number of def-use pairs at all program nodes \( n \) in the CFG of a program for global and local variables.

| Type | Name          | \#def-use pairs | reduction(%) |
|------|---------------|-----------------|--------------|
|      | MFP | FPMFP  |                  |               |
| Open Source |     |     |                  |               |
| 1. acpid | 156 | 156  |                   | 0.00         |
| 2. polymorph | 228 | 228  |                   | 0.00         |
| 3. alkain | 1042| 965   |                   | 77.39        |
| 4. spell   | 516 | 515   |                   | 1.09         |
| 5. ncompress | 1201| 1175 |                   | 26.16        |
| 6. gzip    | 3423| 3401  |                   | 22.04        |
| 7. stripcc | 2703| 2645  |                   | 58.15        |
| 8. barcode-nc | 3051| 3007 |                   | 44.14        |
| 9. barcode | 3709| 3653  |                   | 56.11        |
| 10. archimedes | 44337| 44216| 121(0.27)     |
| 11. combine | 16618| 15859| 759(4.57)      |
| 12. httpd | 10475| 10072 |                   | 403(3.85)    |
| 13. sphinxbase | 9641| 8482 | 1159(12.02)    |
| 14. chess | 31386| 31303 |                   | 83(0.26)     |
| 15. antiword | 68889| 60144| 8745(12.69)    |
| 16. sendmail | 102812| 101470| 1342(1.31)    |
| 17. sudo | 14391| 14211 |                   | 180(1.25)    |
| 18. ffmpeg | 89148| 86607 |                   | 2541(2.85)   |
| SPEC |     |     |                  |               |
| 19. mcf | 3570| 3565 |                   | 5(0.14)      |
| 20. bzip2 | 82548| 77967| 4581(5.55)     |
| 21. hmer | 70597| 70021 |                   | 576(0.82)    |
| 22. sjeng | 77602| 76959 |                   | 643(0.83)    |
| 23. milc | 18133| 17828 |                   | 305(1.68)    |
| 24. h264ref | 426346| 409325| 17021(3.99)   |
| 25. gobmk | 213243| 184142| 29101(13.65) |
| Industry |     |     |                  |               |
| 26. NZ | 2652| 2651 |                   | 1(0.04)      |
| 27. IC | 3111| 3063 |                   | 48(1.54)     |
| 28. AIS | 2475| 2414 |                   | 61(2.46)     |
| 29. FRSC | 1297| 1295 |                   | 2(0.15)      |
| 30. FX | 4971| 4925 |                   | 46(0.93)     |
Table 9: Comparing performance of MFP and FPMFP computation. Prep. represents the MIPS detection phase. Note the analysis times are not comparable with gcc or clang, because our implementation is in Java.

| Benchmarks       | Analysis Time (Sec) Prep | Analysis Time (Sec) MFP | Analysis Time (Sec) FPMFP | Increase(x) Prep | Increase(x) MFP | Increase(x) FPMFP |
|------------------|--------------------------|-------------------------|---------------------------|------------------|-----------------|------------------|
| Open Source      |                          |                          |                           |                  |                 |                  |
| 1.acpid          | 0                        | 0                       | 0                         | 0                | 0               | 0                |
| 2.polymorph      | 0                        | 0                       | 0                         | 0                | 0               | 0                |
| 3.nlkain         | 0                        | 0                       | 0                         | 1                | 1               | 1                |
| 4.spell          | 0                        | 0                       | 0                         | 0                | 0               | 0                |
| 5.ncompress      | 1                        | 0                       | 1                         | 1                | 2               | 4                |
| 6.gzip           | 1                        | 1                       | 5                         | 5                | 10              | 14(1.4)          |
| 7.stripcc        | 3                        | 1                       | 6                         | 8                | 5               | 21(3.3)          |
| 8.barcode-nc     | 2                        | 1                       | 2                         | 6                | 8               | 6(-1)            |
| 9.barcode        | 2                        | 2                       | 2                         | 7                | 10              | 8(-1)            |
| 10.archimedes    | 6                        | 3                       | 32                        | 24               | 44              | 156(112)         |
| 11.combine       | 5                        | 3                       | 5                         | 16               | 23              | 19(-3)           |
| 12.httpd         | 40                       | 14                      | 19                        | 28               | 39              | 21(-18)          |
| 13.sphinxbase    | 7                        | 3                       | 3                         | 13               | 16              | 7(-9)            |
| 14.chess         | 13                       | 7                       | 30                        | 31               | 93              | 191(98)          |
| 15.antiword      | 13                       | 16                      | 82                        | 47               | 101             | 218(117)         |
| 16/sendmail      | 110                      | 142                     | 2060                      | 81               | 1548            | 1208(3.5)        |
| 17.sudo          | 15                       | 9                       | 17                        | 35               | 71              | 19(-52)          |
| 18.ffmpeg        | 234                      | 51                      | 80                        | 158              | 266             | 90(-175)         |
| 19.mcf           | 1                        | 1                       | 1                         | 0                | 3               | 3                |
| 20.hzip2         | 8                        | 7                       | 69                        | 28               | 44              | 90(46)           |
| 21.hmmer         | 14                       | 9                       | 23                        | 50               | 67              | 58(-8)           |
| 22.sjeng         | 10                       | 16                      | 62                        | 27               | 122             | 215(92)          |
| 23.milc          | 6                        | 4                       | 7                         | 25               | 25              | 14(-11)          |
| 24.h264ref       | 60                       | 115                     | 451                       | 131              | 1003            | 1644(640)        |
| 25.gobmk         | 4218                     | 1696                    | 9818                      | 390             | 9365           | 17392(8032)      |
| SPEC 2006        |                          |                          |                           |                  |                 |                  |
| 26.NZM           | 1                        | 1                       | 2                         | 1                | 3               | 8(-1)            |
| 27.IC            | 5                        | 4                       | 3                         | 11               | 20              | 9(-11)           |
| 28.FRSC          | 3                        | 2                       | 1                         | -15              | 10              | 4(-6)            |
| 29.AIS           | 4                        | 3                       | 4                         | 9                | 20              | 16(-4)           |
| 30.FX            | 7                        | 3                       | 7                         | 20               | 35              | 47(11)           |

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Table 10: Impact of Optimization 1 on the FPMFP computation (in Optimization 1, we consider two MIPS with the same end edge as equivalent, and subsequently merge the pairs that contain equivalent MIPS).

| Benchmarks | Number of Distinct MIPS Before and After Optimization 1 | Reduction(%) |
|------------|---------------------------------------------------------|--------------|
|            | Before | After | Reduction(%) |
| **Open Source** | | | |
| 1.acpid    | 83     | 6     | 77(92.77)    |
| 2.polymorph 0.96 | 65     | 24 | 41(63.08)    |
| 3.nlkain   | 35     | 33 | 2(5.71)      |
| 4.spell    | 99     | 14 | 85(85.86)    |
| 5.ncompress | 14163  | 30 | 14133(99.79) |
| 6.gzip     | 23524624 | 49 | 23524575(99.99) |
| 7.stripcc  | 774751 | 107 | 774644(99.99) |
| 8.barcode-nc | 208 | 60 | 148(71.15) |
| 9.barcode  | 3851 | 76  | 3775(98.03) |
| 10.archimedes | 1054292 | 118 | 1054174(99.99) |
| 11.combine | 27061 | 234 | 26827(99.14) |
| 12.httpd   | 76591463106 | 226 | 76591462880(99.99) |
| 13.sphinxbase | 6463 | 213 | 6250(96.70) |
| 14.chess   | 19701157845 | 262 | 19701157583(99.99) |
| 15.antword | 1446759232593972 | 669 | 1446759232593303(99.99) |
| 16.sendmail | 94082730655046 | 726 | 94082730654320(99.99) |
| 17.sudo    | 549565 | 250 | 549315(99.95) |
| 18.ffmpeg  | 5019442696182 | 1363 | 5019442694819(99.99) |
| **SPEC 2006** | | | |
| 19.mcf     | 44     | 19 | 25(56.82)    |
| 20.bzip2   | 115451494 | 501 | 115450993(99.99) |
| 21.hmmer   | 8606721 | 595 | 8606126(99.99) |
| 22.sjeng   | 1508084297532741 | 438 | 1508084297532303(99.99) |
| 23.milc    | 2322 | 497 | 1825(78.60) |
| 24.h264ref | 51269021916 | 2189 | 51269019727(99.99) |
| 25.gobmk   | > 2^{64} | 1834 | > 2^{64}(99.99) |
| 26.NZM     | 1064 | 52 | 1012(95.11) |
| 27.JC      | 158 | 50 | 108(68.35) |
| 28.AIS     | 176 | 83 | 93(52.84) |
| 29.FRSC    | 89 | 42 | 47(52.81) |
| 30.FX      | 704 | 140 | 564(80.11) |
Table 11: Impact of Optimization 2 on the FPMFP computation. Timeout indicates the analysis did not finish within 10K seconds.

| Benchmarks       | Analysis Time (in Seconds) | Before | After | Reduction(%) |
|------------------|----------------------------|--------|-------|--------------|
|                  | Before and After Optimization 2 |        |       |              |
|                  |                            |        |       |              |
| Open Source      |                            |        |       |              |
| 1.acpid          |                            | 0      | 0     | 0            |
| 2.polymorph 0.96 |                            | 0      | 0     | 0            |
| 3.nklain         |                            | 0      | 0     | 0            |
| 4.spell          |                            | 0      | 0     | 0            |
| 5.ncompress      |                            | 3      | 1     | 2(66)        |
| 6.gzip           |                            | 22     | 5     | 17(77)       |
| 7.stripcc        |                            | 55     | 6     | 49(89)       |
| 8.barcode-nc     |                            | 2      | 2     | 0            |
| 9.barcode        |                            | 2      | 2     | 0            |
| 10.archimedes    |                            | 53     | 32    | 21(39)       |
| 11.combine       |                            | 19     | 5     | 14(73)       |
| 12.httpd         |                            | 290    | 19    | 271(93)      |
| 13.sphinxbase    |                            | 4      | 3     | 1(25)        |
| 14.chess         |                            | 76     | 30    | 46(60)       |
| 15.antiword      |                            | 705    | 82    | 623(88)      |
| 16.sendmail      |                            | Timeout|       |              |
| 17.sudo          |                            | 64     | 17    | 47(73)       |
| 18.ffmpeg        |                            | Timeout|       |              |
| SPEC 2006        |                            |        |       |              |
| 19.mcf           |                            | 1      | 1     | 0            |
| 20.bzip2         |                            | 931    | 69    | 862(92)      |
| 21.hmmer         |                            | 143    | 23    | 120(83)      |
| 22.sjeng         |                            | 501    | 62    | 439(87)      |
| 23.milc          |                            | 11     | 7     | 4(36)        |
| 24.h264ref       |                            | 2977   | 451   | 2526(84)     |
| 25.gobmk         |                            | Timeout|       |              |
| Industry         |                            |        |       |              |
| 26.NZM           |                            | 4      | 2     | 2(50)        |
| 27.IC            |                            | 6      | 3     | 3(50)        |
| 28.AIS           |                            | 7      | 4     | 3(42)        |
| 29.FRSC          |                            | 3      | 1     | 2(66)        |
| 30.FX            |                            | 9      | 7     | 2(22)        |
report the variables that are used at a program point \( p \) but are not initialized along at least one CFP that reaches \( p \). We call such CFPs as witness CFPs.

As shown in Table 7, we computed two sets of potentially uninitialized variable alarms corresponding to the MFP and FPMFP solutions of must defined variable analysis. In particular, the potentially uninitialized variable analysis that used the FPMFP solution of must defined analysis reported up to 100\% (average 18.5\%, geo. mean 3\%) fewer alarms compared to when it used the MFP solution. In the reduced alarm cases, all the witness CFPs were found infeasible.

More specifically, in 14 out of 27 benchmarks the FPMFP solution helped achieve positive reduction in the uninitialized variable alarms, while in 13 benchmarks there was no reduction in the alarms. In the latter case the following two scenarios were observed: in the first scenario, the addresses of uninitialized variables are passed as parameter to library functions whose source code is not available and hence its behavior is conservatively approximated (i.e, the analysis assumed that the variables are not initialized inside the functions), and in the second scenario a variable is conditionally initialized inside the callee procedure where the evaluation of the condition depends on the calling context thereby necessitating a context sensitive analysis to eliminate the false positive alarm (our implementation is context insensitive).

7.4. Reduction in the Number of Def-Use Pairs

We implemented the classic reaching definitions analysis [28, 1] (non SSA based) in which for each program point, we compute the set of definitions that reach the point. We measured the effect of the reduction in the number of reaching definitions on Def-Use pair computation, which is a client analysis of the reaching definitions analysis. In this, we pair the definition of a variable with its reachable use point.

In the FPMFP computation, the def and use points that are connected with each other by only infeasible paths are eliminated. However, we found that most def-use points are connected by at least one feasible CFP, and hence a 100\% reduction is not achievable by any technique.

As shown in Table 8, we found that up to 13.6\% (average 2.87\%, geometric mean 1.75\%) fewer def-use pairs were reported when using FPMFP solution in place of the MFP solution. In particular, in 28 out of 30 benchmarks the FPMFP solution lead to fewer def-use pairs than the MFP solution. In general, we observed that the precision increased with increase in the number of MIPS. For example, the gobmk benchmark contains a large number of MIPS (1.9k), consequently we observed significant (13\%) reduction in the number of def-use pairs. On the other hand, the acpid benchmark had only 17 MIPS, consequently we did not see any reduction in the number of def-use pairs. As an outlier case, on the sphinxbase benchmark that had only 216 MIPS we observed 12\% reduction in def-use pairs. Here, we observed that the end edges of many MIPS are reachable only along infeasible CFPs, hence in FPMFP computation the data flow along these edges is completely blocked, thus all the corresponding def-use pairs (at the program points pre-dominated by the end edges) are eliminated.

The def-use pairs have multifold applications; in particular, they are useful for program slicing, program dependency graph creation, program debugging, and test case generation. For example in program testing, some coverage criteria mandate that each def-use pair should be covered by at least one test case. Here, up to 13.6\% fewer def-use pairs means fewer manual test cases need to be written to cover these pairs, which is a significant reduction in the manual time and efforts.
7.5. Comparing the Performance of FPMFP and MFP Computations

Computation of the FPMFP solution was less efficient than the corresponding MFP solutions. The details of the relative memory and time consumption of both the analyses for reaching definitions computation are presented in Table 9. The Prep. column represents the time and memory consumption for infeasible path detection phase. This is performed once for each program.

The FPMFP analysis took 2.9 times more time than MFP analysis on average (geo. mean 1.6). In general, the analysis time increased with increase in the number of MIPS in the program. Apart from this, the FPMFP analysis consumed 0.5 times more memory than the MFP analysis on average. However, on 50% of the benchmarks the FPMFP analysis consumed less memory than the MFP analysis because the definitions reaching along infeasible CFPs are ignored during the analysis.

The effect of optimizations for scalability on analysis time

Table 10 shows the reduction in the number of distinct MIPS after applying Optimization 1 (Section 6.1). Here, two MIPS are considered equivalent if they have the same end edge. In this case we can merge two pairs if they contain equivalent MIPS. We found that if we count MIPS with same end edge single time, then the total number of MIPS reduces by up to 99% (average 85%, geometric mean 78%). This leads to reduction in number of pairs at various program points.

Table 11 shows the effectiveness of optimization 2 (Section 6.2) on top of optimization 1. In particular, it shows the analysis time before and after applying the optimization 2. We found that 3 out of 30 applications timed out (i.e., did not finish within 10K seconds) when we disabled optimization 2. Similarly, for remaining applications the analysis time increased by up to 99% (average 48.9%, geometric mean 19.8%) after disabling optimization 2.

```
If(...){
  A: x = 5;
  ...
  If(x < 5){
    A: x = 0;
    B: print("%d", &x)
  }
}
Const int N = 100
A: B: If(x < 5){
  ...
  ...
  ...
  ...
  ...
}
```
1. The first type involves assignment of constant value to a variable along some node in a path followed by the variable being used in some conditional expression that performs either equality or less than or greater than check on the variable later in the path. Two typical examples of this type are shown Figure 16a and 16b. In the first example a variable is assigned a constant value inside the if branch (point A) and subsequently used in other branch condition that appears later in the path (point B). The second example contains a loop that executes at least once hence the path that goes from point A to point B without executing the loop is infeasible.

2. The second type involves two syntactically same conditions used in two branches at different places in a path and the variables in the conditions are not modified along the path that connects the branches. For example in Figure 16c the path that goes from point A to point B is infeasible.

7.7. Summary of Empirical Observations

Previous empirical evidence [2] on Linux kernel code shows 9-40% of conditional statements contribute to at least one infeasible CFP. In our benchmarks up to 61% (geometric mean 29%) functions had at least one MIPS in their CFG. Precise elimination of data flow values reaching from infeasible CFPs has allowed us to reduce the number of def-use pairs by up to 13.6% with an average of 2.87% and a geometric mean of 1.75% over MFP solution. Similarly, the reduction in the potentially uninitialized variable alarms was up to 100% (average 18.5%, geo. mean 3%).

8. Related Work

In this section, we compare FPMFP with existing approaches of improving data flow precision. In particular, we begin by classifying the existing approaches that improve the precision of data flow analysis into two categories. The approaches in the first category avoid the effect of infeasible paths from data flow analysis (Section 8.1), while the approaches in second category avoid the join points (of two or more CFPs) that lead to imprecision in the analysis results (Section 8.4). Lastly, we explain how FPMFP differs from approaches in both the categories.

8.1. Approaches that Avoid Infeasible Paths

In general, presence of infeasible paths in programs is well known [29, 30, 12, 31, 32, 33, 34, 35]. Hedley et al. [30] presented a detailed analysis of the causes and effects of infeasible paths in programs. Malevis et al. [12] observed that the greater the number of conditional statements contained in a path, the greater the probability of the path being infeasible. Bodik et al. [2] found that around 9-40% of the conditional statements in programs show statically detectable correlation with infeasible control flow paths.

Many approaches have been proposed to eliminate the effect of infeasible paths from data flow analysis. We classify these approaches in two categories as shown in Figure 17. The first category includes approaches that are analysis dependent (Section 8.2), while the second category includes the approaches that are analysis independent (Section 8.3). We describe and relate approaches in each of these categories with FPMFP below.
Approaches to Improve Data Flow Precision

- Avoid Infeasible Paths
  - Analysis Dependent Approach
    - Selective Path Sensitivity
  - Analysis Independent Approach
    - CFG Restructuring
- Avoid Imprecise Joins
  - Trace Partitioning
  - Our Approach

Figure 17: Categorization of approaches that improve data flow precision by either avoiding infeasible paths or imprecise joins.

8.2. Analysis Dependent Approaches

In this section, we describe approaches that eliminate effect of infeasible paths from data flow analysis in analysis dependent way. In particular, these approaches either depend on analysis specific heuristics, or they detect infeasible paths on the fly during a data flow analysis (and hence they detect infeasible paths separately for each different data flow analysis over the same program thereby increasing the analysis cost). Specifically, these approaches use the idea of path sensitive analysis to distinguish between the data flow values reaching along different classes of CFPs.

In a completely path sensitive analysis each path information is separately tracked along with the corresponding path constraints. If the path constraint is found unsatisfiable, the path information is not propagated further. This approach does not scale to most practical programs due to presence of loops and recursion leading to an unbounded number of program paths combined with data flow lattices containing possibly infinite values.

The above limitations of a completely path sensitive analysis are avoided by adapting a selective approach to path sensitive analysis. Here, they use selective path constraints governed by various heuristics for deciding which path information should be...
kept separately and which path information can be approximated at join points. These heuristics are often aligned with the end goal of the analysis. Our work differs from these approaches in that our work does not depend on analysis specific heuristics for eliminating the effect of infeasible paths, and therefore can be generically applied to all MFP based data flow analyses. Below, we describe the approaches that use selective path sensitive analyses.

Dor et al. [18] proposed a precise path sensitive value flow analysis approach using value bit vectors. Here, they symbolically evaluate a program by generating symbolic states that include both the execution state and the value alias set. At merge points in the control flow graph, if two symbolic states have the same value alias set, they produce a single symbolic state by merging their execution states. Otherwise, they process the symbolic states separately. Like our work, they avoid the cost of full path sensitive analysis, yet capture relevant correlations. Their work differs from our work in that their work is applicable only to separable data flow problems, while our work is applicable to any data flow problem that can be solved using MFP computation.

Chen et al. [5] proposed an algorithm for detecting infeasible paths by incorporating the branch correlations analysis into the data flow analysis technique. Next, the detected infeasible paths are then used to improve the precision of structural testing by ignoring infeasible paths.

Hampapuram et al. [15] performed a symbolic path simulation approach in which they executed each path symbolically and propagated the information along these paths. If a path is found infeasible in simulation its information is not propagated. A lightweight decision procedure is used to check the feasibility of a path during path simulation.

Dillig et al. [17] proposed a selective path sensitive analysis for modular programs. In this, they ignored the unobservable variables that are the variables representing the results of unknown functions (functions unavailable for analysis); hidden system states (e.g., the garbage collector’s free list, operating system’s process queue, etc.), or imprecision in the analysis (e.g., selecting an unknown element of an array). In particular, they observed that unobservable variables are useful within their natural scope, for example, tests on unobservable variables can possibly be proven mutually exclusive (e.g., testing whether the result of a malloc call is null, and then testing whether it is non-null). However, outside of their natural scope unobservable variables provide no additional information and can be eliminated. Ignoring unobservable variables allowed them to reduce the number of distinctions that needs to be maintained in the path sensitive analysis, for example, they did not distinguish between paths that differ only in the values of unobservable variables. They have applied their work for proving a fixed set of program properties. In that, they eliminate the effect of infeasible paths while proving the properties because the analysis is path sensitive. However, they detect infeasible paths separately for each different data flow analysis on the same underlying program, which does not happen in our approach.

Xie et al. [16] focused on memory access errors (like array index out of bounds and null pointer dereferences) because they may lead to non-deterministic and elusive crashes. They proposed a demand-driven path sensitive inter-procedural analysis to bound the values of both variables and memory sizes. For achieving efficiency, they restrict the computation to only those variables and memory sizes that directly or indirectly affect the memory accesses of interest. They use the following observation about C programs: most of the times memory sizes are passed as parameters to functions, along with the pointers to corresponding memory addresses; further, these sizes are used in conditions that guard the memory accesses. Hence, by tracking the actual values of the memory sizes the corresponding memory accesses can be evaluated as safe or unsafe. Xie et al. were able to detect many memory access errors in real code sets using this method. Moreover, they could eliminate the effect of some of the infeasible paths that reach the memory accesses of interest because they used a path sensitive analysis. Their work differs from
our work on two fronts: first, they do a demand driven analysis while our work is exhaustive; second in their work the infeasible paths are detected on the fly during a data flow analysis, so the same path may be detected multiple times for the same underlying program, while in our work infeasible paths are detected only once for a program.

Das et al. [13] proposed a property specific path sensitive analysis. In this at first, a set of program properties of interest are identified. Next, a partial path sensitive analysis is performed where two paths are separated iff they contain different values for the variables related to properties of interest.

On somewhat similar lines, Dhurjati et al. [13] proposed an iterative refinement based approach to path sensitive data flow analysis. Here, at first, a path insensitive analysis is performed. Next, if the analysis is not able to prove the properties of interest then the analysis results are used to compute a set of predicates that are related to the property of interest. In the next iteration of the analysis, these predicates are used to differentiate paths along which predicate evaluation is different. If this partially path sensitive analysis proves the properties of interest then the analysis is terminated, else the analysis is repeated with a different set of predicates discovered in the recent iteration of the analysis. Thus, the analysis continues in multiple iterations.

Infeasible control flow paths is a property of programs instead of any particular data flow analysis. We use this observation to first detect infeasible paths in programs and propose an approach to improve the precision of any data flow analysis over the programs, unlike above approaches that remove the impact of infeasible paths from a particular data flow analysis.

8.3. Analysis Independent Approaches

We now describe approaches that eliminate effect of infeasible paths from data flow analysis in analysis independent way. In particular, these approaches do not depend on analysis specific heuristics, and they identify infeasible paths only once for a program. These approaches are classified into two categories depending on whether they use CFG restructuring or not, as explained below.

8.3.1. Approaches that Use CFG Restructuring

Bodik et al. [3] proposed an inter-procedural version of infeasible path detection and its application to improve the data flow precision. However, they use control flow graph restructuring which is not done in our approach. CFG restructuring may blow up the size of the CFG. Moreover, it does not take the advantage of analysis specific information, unlike FPMFP that uses analysis specific information to dynamically discard the distinctions between MIPS, where they do not lead to precision improvement as described in Optimization 2 (Section 6.2).

Balakrishna et al. [37] presented a technique for detecting infeasible paths in programs using abstract interpretation. The technique uses a sequence of path-insensitive forward and backward runs of an abstract interpreter to infer paths in the control flow graph that cannot be exercised in concrete executions of the program. Next, they refined program CFG by removing detected infeasible paths in successive iterations of a property verification.

Marashdih et al. [38] proposed a CFG restructuring approach to eliminate infeasible paths for PHP programs. In particular, they proposed a methodology for the detection of XSS (Cross Site Scripting) from the PHP web application using static analysis. The methodology eliminates the infeasible paths from the CFG thereby reducing the false positive rate in the outcomes. On similar lines, we have found reduction in false positives in possibly uninitialized variable alarms but without using CFG restructuring.
8.3.2. Our Approach

We used the work done by Bodik et al. [2] for detecting infeasible paths from the CFG of a program. Next, we automatically lift any data flow analysis to an analysis that separates the data flow values reaching along known infeasible CFPs from the values that do not reach along the infeasible CFPs. This allows us to block the data flow values that reach along infeasible CFPs at a program point, thereby eliminating the effect of detected infeasible paths from data flow analyses without using CFG restructuring and in a generic manner.

8.4. Approaches that Avoid Imprecise Joins

In this section, we discuss approaches that improve data flow precision by avoiding imprecise joins i.e., the merging of information at the nodes that are shared by two or more paths, and that may lead to loss of precision.

8.5. Approaches that Use Trace Partitioning

In abstract interpretation, the trace partitioning approaches [39, 40, 24] partition program traces where values of some variable or conditional expression differ in two selected traces. The variables or expressions are selected based on the alarms to be analyzed or some heuristic.

Mauborge et al. [24] used trace partitioning to delay the merging of information at join points in CFG where the join points lead to decrease in precision. They separate sets of traces before the join points and merge traces after the join points. The separation and merge points for traces are specified as input to the program in the form of pre-processing directives. Handjieva et al. [39] proposed a restricted version of trace partitioning where merging of parts in partitions was not allowed (Unlike Mauborge et al.’s work).

The trace partitioning is similar to our approach because it keeps data flow values separately for each part (representing a set of traces) in partition. However, our approach does not rely on alarms, does not need the designer of an analysis to decide a suitable heuristic, and is not restricted to a particular analysis. We lift partitioning from within an analysis to the infeasibility of control flow paths which is a fundamental property of control flow paths independent of an analysis. This allows us to devise an automatic approach to implement a practical trace partitioning. An interesting aspect of our approach is that although it is oblivious to any analysis, it can be seen as dynamic partitioning that uses infeasibility of control flow paths as criteria for partitioning.

8.6. Approaches that Use CFG Restructuring

CFG restructuring has also been used to avoid imprecise joins. In particular, Aditya et al. [36] proposed an approach in which at first they identify join points that lead to loss of precision in data flow analysis. Next, they improve the data flow precision by eliminating these join points from control flow graph through control flow graph restructuring.

Ammon et al. [41, 42] proposed a CFG restructuring for avoiding information merging along hot paths i.e., the paths that are more frequently executed in program compared to other paths. They showed that this improves precision of various data flow analysis and increases the opportunities of program optimizations. Their work differs from our work in that they did not address the imprecision added by infeasible control flow paths.
9. Conclusions and Future Work

An exhaustive data flow analysis answers all queries of a particular type over all possible executions of a program. However, in practice it is hard to see an exhaustive analysis that is efficient and computes a precise result, because of the following issues that often have conflicting solutions.

- Issue 1: to achieve efficiency, the analysis may need to merge the information reaching at a program point along different paths, because the number of paths as well as the corresponding data flow information may be very large or even unbounded.

- Issue 2: to achieve precision, the analysis may need to distinguish between the information reaching from different paths, in order to 1) discard the information reaching along infeasible paths, and 2) avoid potential loss of information because of merging that happens at join points of two or more paths.

These issues have been handled in past as follows: the first issue is handled by merging the information reaching along different paths at their shared path segments in a CFG (instead of keeping the information separate). The approach achieves efficiency because it reduces the amount of information computed. However, the computed information is usually weaker than the actual possible information because of the information merging, and inclusion of information reaching along infeasible paths.

To handle the second issue, two types of approaches have been proposed.

1. The first type of approaches eliminate the effect of infeasible paths from analysis in analysis dependent or analysis independent way. In particular, analysis dependent approaches use selective path sensitive analysis that uses heuristics to selectively distinguish information reaching along some paths, and subsequently discard the information along infeasible paths thereby eliminating effect of infeasible paths from the specific analysis. On the other hand, analysis independent approaches use CFG restructuring to eliminate infeasible paths from CFG itself thereby eliminating the effect of infeasible paths from all analyses performed on restructured CFG.

2. The second type of approaches eliminate the effect of imprecise joins by using either CFG restructuring or trace partitioning. More specifically, in CFG restructuring they eliminate known imprecise joins points from CFG. On the other hand, in trace partitioning they create equivalence classes of program traces using user specified criteria. Next, at imprecise join points, only the information corresponding to traces in same class are merged, and information from traces belonging to different classes is kept separate. Thus, they reduce the effect of imprecise joins to some extent. Both the approaches of removing imprecise joins rely on analysis specific heuristics because imprecise joins are analysis specific.

In this work, we proposed a generic approach to remove the effect of infeasible paths from exhaustive data flow analyses. In particular, we introduced the notion of feasible path MFP solutions that separate and discard the data flow values reaching along infeasible paths. A key insight that enabled FPMFP is the following.

Infeasible paths is a property of programs and not of any particular data flow analysis over the programs. Hence, we can separate the identification of infeasible paths in the CFG of a program from discarding the corresponding data flow values during a data flow analysis.
The above insight allows us to devise a generic technique that intuitively lifts a data flow analysis to multiple parallel and interacting data flow analyses each of which eliminates the effect of a class of known infeasible paths. In particular, this is realized in FPMFP through a two phase approach: in the first phase, we detect minimal infeasible path segments from the input program. In the second phase, we lift the input data flow analysis so as to separate the values that flow through a MIPS from the values that do not flow through the MIPS. Further, the analysis blocks the values that flow through the MIPS at the end of the MIPS thereby eliminating the effect of infeasible paths from data flow analysis.

In our experiments on 30 Benchmarks (selected from Open Source, Industry, and SPEC CPU 2006), we compared precision and performance of FPMFP solutions with that of MFP solutions. Here, we observed up to 13.6% (average 2.87%, geo. mean 1.75%) reduction in the number of def-use pairs in the reaching definitions analysis, and up to 100% (average 18.5%, geo. mean 3%) reduction in the number of alarms in the possibly-uninitialized variables analysis, when using FPMFP solution in place of MFP solution. Further, the FPMFP computation took 2.9× more time compared to the MFP computation. In our experience, this cost-precision trade-off is acceptable considering the corresponding reduction in the manual efforts. Specifically, in program testing testcases need to be written to ensure each def-use pair is covered by at least one testcase. Similarly, for validating the possibly-uninitialized variable alarms requires manual efforts and is error prone.

In future, we see that the FPMFP solutions can be further improved both in terms of scalability and precision. First, the scalability can be improved by identifying more optimizations that can discard the pairs that do not lead to precision improvement. Second, the precision can be improved by adding handling for wider class of inter-procedural MIPS. However, this should be complemented by corresponding optimizations to keep the distinctions to bare minimum to retain the efficiency of the approach.

Moreover, we see a possibility that the precision gain and scalability of FPMFP on a program can be anticipated using a lightweight pre-analysis of the program. This can help decide whether to apply FPMFP to the program or not. Similar approaches have been proved helpful in earlier attempts of precise and scalable context sensitive program analyses [43, 44, 45, 46]. We believe this will help for FPMFP also.

References

[1] U. Khedker, A. Sanyal, B. Sathe, Data flow analysis: theory and practice, CRC Press, 2009.
[2] R. Bodik, R. Gupta, M. L. Sozza, Refining data flow information using infeasible paths, in: Software Engineering—ESSEC/FSE’97, Springer, 1997, pp. 361–377.
[3] R. Bodik, R. Gupta, M. L. Sozza, Interprocedural conditional branch elimination, in: ACM SIGPLAN Notices, Vol. 32, ACM, 1997, pp. 146–158.
[4] M. N. Ngo, H. B. K. Tan, Detecting large number of infeasible paths through recognizing their patterns, in: Proceedings of the 6th joint meeting of the European software engineering conference and the ACM SIGSOFT symposium on The foundations of software engineering, ACM, 2007, pp. 215–224.
[5] C. Rui, Infeasible path identification and its application in structural test, Ph.D. thesis, Beijing: Institute of Computing Technology of Chinese Academy of Sciences (2006).
[6] M. Delahaye, B. Botella, A. Gotlieb, Explanation-based generalization of infeasible path, in: 2010 Third International Conference on Software Testing, Verification and Validation, IEEE, 2010, pp. 215–224.
[7] P. M. S. Bueno, M. Jin, Identification of potentially infeasible program paths by monitoring the search for test data, in: Automated Software Engineering, 2000. Proceedings ASE 2000. The Fifteenth IEEE International Conference on, IEEE, 2000, pp. 209–218.
[8] X. Zhuang, T. Zhang, S. Pande, Using branch correlation to identify infeasible paths for anomaly detection, in: Proceedings of the 39th Annual IEEE/ACM International Symposium on Microarchitecture, IEEE Computer Society, 2006, pp. 113–122.
[9] R. Yang, Z. Chen, B. Xu, W. E. Wong, J. Zhang. Improve the effectiveness of test case generation on efsm via automatic path feasibility analysis, in: High-Assurance Systems Engineering (HASE), 2011 IEEE 13th International Symposium on, IEEE, 2011, pp. 17–24.

[10] A. Bertolino, M. Marré. Automatic generation of path covers based on the control flow analysis of computer programs, IEEE Transactions on Software Engineering 20 (12) (1994) 885–899.

[11] T. Chen, T. Mitra, A. Roychoudhury, V. Suhendra. Exploiting branch constraints without exhaustive path enumeration, in: OASIS-OpenAccess Series in Informatics, Vol. 1, Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2007.

[12] N. Maleviris, D. Yates, A. Veewers. Predictive metric for likely feasibility of program paths, Information and Software Technology 32 (2) (1990) 115–118.

[13] M. Das, S. Lerner, M. Seigle. Esp: Path-sensitive program verification in polynomial time, in: ACM Sigplan Notices, Vol. 37, ACM, 2002, pp. 57–68.

[14] D. Dharjati, M. Das, Y. Yang. Path-sensitive dataflow analysis with iterative refinement, in: International Static Analysis Symposium, Springer, 2006, pp. 425–442.

[15] H. Hampapuram, Y. Yang, M. Das. Symbolic path simulation in path-sensitive dataflow analysis, in: ACM SIGSOFT Software Engineering Notes, Vol. 31, ACM, 2005, pp. 52–58.

[16] Y. Xie, A. Chou, D. Engler. Archer: using symbolic, path-sensitive analysis to detect memory access errors, ACM SIGSOFT Software Engineering Notes 28 (5) (2003) 327–336.

[17] I. Dilig, T. Dilig, A. Aiken. Sound, complete and scalable path-sensitive analysis, in: ACM SIGPLAN Notices, Vol. 43, ACM, 2008, pp. 270–280.

[18] N. Dor, S. Adams, M. Das, Z. Yang. Software validation via scalable path-sensitive value flow analysis, in: ACM SIGSOFT Software Engineering Notes, Vol. 29, ACM, 2004, pp. 12–22.

[19] K. Pathade, U. P. Kholker. Computing partially path-sensitive mfp solutions in data flow analyses, in: Proceedings of the 27th International Conference on Compiler Construction, ACM, 2018, pp. 37–47.

[20] K. Pathade, U. P. Kholker. Path sensitive mfp solutions in presence of intersecting infeasible control flow path segments, in: Proceedings of the 28th International Conference on Compiler Construction, 2019, pp. 159–169.

[21] M. Sharir, A. Pnueli, et al.. Two approaches to interprocedural data flow analysis, New York University. Courant Institute of Mathematical Sciences . . . , 1978.

[22] Tcs embedded code analyzer [2017]. URL: https://www.tcs.com/product-engineering/tcs-embedded-code-analyzer

[23] S. Blachshear, N. Gorogiannis, F. W. O’Hearn, I. Sergey. Racerd: compositional static race detection, Proceedings of the ACM on Programming Languages 2 (OOPSLA) (2018) 1–28.

[24] L. Mauborgne, X. Rival. Trace partitioning in abstract interpretation based static analyzers, in: European Symposium on Programming, Springer, 2005, pp. 5–20.

[25] W. Lee, W. Lee, K. Yi. Sound non-statistical clustering of static analysis alarms, in: International Workshop on Verification, Model Checking, and Abstract Interpretation, Springer, 2012, pp. 299–314.

[26] T. Muske, R. Talluri, A. Serebrenik. Repositioning of static analysis alarms, in: Proceedings of the 27th ACM SIGSOFT international symposium on software testing and analysis, 2018, pp. 187–197.

[27] D. Zhang, D. Jin, Y. Gong, H. Zhang. Diagnosis-oriented alarm correlations, in: 2013 20th Asia-Pacific Software Engineering Conference (APSEC), Vol. 1, IEEE, 2013, pp. 172–179.

[28] A. V. Aho. Compilers: principles, techniques and tools (for Anna University), 2/e, Pearson Education India, 2003.

[29] H. Zhu, D. Jin, Y. Gong, Y. Xing, M. Zhou. Detecting interprocedural infeasible paths based on unsatisfiable path constraint patterns, IEEE Access 7 (2019) 15040–15055.

[30] O. Hedley, M. A. Hennell. The causes and effects of infeasible paths in computer programs, in: Proceedings of the 8th international conference on Software engineering, IEEE Computer Society Press, 1985, pp. 259–266.

[31] E. W. S. Lee. Infeasible path detection and code pattern mining (2019).

[32] S. Jiang, H. Wang, Y. Zhang, M. Xue, J. Quan, M. Zhang. An approach for detecting infeasible paths based on a smt solver, IEEE Access 7 (2019) 69058–69069.

[33] H. Gong, Y. Zhang, Y. Xing, W. Jia. Detecting interprocedural infeasible paths via symbolic propagation and dataflow analysis, in: 2019 IEEE 10th International Conference on Software Engineering and Service Science (ICSESS), IEEE, 2019, pp. 282–285.

[34] T. Sewell, F. Kam, G. Heiser. Complete, high-assurance determination of loop bounds and infeasible paths for weak analysis, in: 2016 IEEE Real-Time and Embedded Technology and Applications Symposium (RTAS), IEEE, 2016, pp. 1–11.

[35] F. Zeng, W. Liu, X. Gou. Type analysis and automatic static detection of infeasible paths, in: International Conference on Geo-Spatial Knowledge and Intelligence, Springer, 2017, pp. 294–304.

[36] A. Thakur, R. Govindarajan. Comprehensive path-sensitive data-flow analysis, in: Proceedings of the 6th annual IEEE/ACM international symposium on Code generation and optimization, ACM, 2008, pp. 55–63.

[37] G. Balakrishnan, S. Sankaranarayanan, F. Ivančić, O. Wei, A. Gupta. Slr: Path-sensitive analysis through infeasible-
path detection and syntactic language refinement, in: International Static Analysis Symposium, Springer, 2008, pp. 238–254.

[38] A. W. Marashdih, Z. F. Zaaba, H. K. Omer, Web security: detection of cross site scripting in php web application using genetic algorithm, International Journal of Advanced Computer Science and Applications (IJACSA) 8 (5) (2017).

[39] M. Handjieva, S. Tzolovski, Refining static analyses by trace-based partitioning using control flow, in: SAS, Vol. 98, Springer, 1998, pp. 200–214.

[40] L. H. Holley, B. K. Rosen, Qualified data flow problems, IEEE Transactions on Software Engineering (1) (1981) 60–78.

[41] G. Ammons, J. R. Larus, Improving data-flow analysis with path profiles, in: Proceedings of the ACM SIGPLAN 1998 conference on Programming language design and implementation, 1998, pp. 72–84.

[42] G. Ammons, J. R. Larus, Improving data-flow analysis with path profiles, ACM SIGPLAN Notices 39 (4) (2004) 568–582.

[43] H. Oh, W. Lee, K. Heo, H. Yang, K. Yi, Selective context-sensitivity guided by impact pre-analysis, ACM SIGPLAN Notices 49 (6) (2014) 475–484.

[44] H. Oh, W. Lee, K. Heo, H. Yang, K. Yi, Selective x-sensitive analysis guided by impact pre-analysis, ACM Transactions on Programming Languages and Systems (TOPLAS) 38 (2) (2015) 1–45.

[45] S. Wei, B. G. Ryder, Adaptive context-sensitive analysis for javascript, in: 29th European Conference on Object-Oriented Programming (ECOOP 2015), Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2015.

[46] B. Hassanshahi, R. K. Ramesh, P. Krishnan, B. Scholz, Y. Lu, An efficient tunable selective points-to analysis for large codebases, in: Proceedings of the 6th ACM SIGPLAN International Workshop on State Of the Art in Program Analysis, 2017, pp. 13–18.

10. Proof of Soundness:

In this section, we formally define the meet over feasible paths (MOFP) solution, and prove that FPMFP computes a sound-approximation of MOFP solution. At first, we state some auxiliary definitions which are useful for the proof of soundness.

10.1. Definitions

**Definition 4.** For an edge \( e : m \rightarrow n \), we refer the data flow value \( g_e(\text{Out}_m) \) obtained by applying the edge flow function \( g_e \) to the data flow value \( \text{Out}_m \) as “the data flow value at \( e \”).

**Definition 5.** Recall that all CFPs referred in our analysis are intra-procedural. We refer CFPs that begin at start node of a CFG and end at a particular node \( n \) as “CFPs that reach \( n \)”. Let \( \mathcal{U} \) be the set of all MIPS that are input to a FPMFP computation, then we create the following classes of CFPs that reach a node \( n \); these CFPs are illustrated in Figure 18.

- **paths(\( n \))**: All CFPs that reach node \( n \). This includes feasible as well as infeasible CFPs that reach \( n \).
- **mpaths(\( n \))**: All CFPs that reach node \( n \) and also contain at least one MIPS of \( \mathcal{U} \) (\( \text{mpaths}(n) \) is a subset of \( \text{paths}(n) \)). All CFPs in \( \text{mpaths}(n) \) are infeasible CFPs because they contain a MIPS.
- **mpaths’(\( n \))**: All CFPs that reach node \( n \) and do not contain any MIPS of \( \mathcal{U} \). This represents a set complement of \( \text{mpaths}(n) \) i.e., \( \text{mpaths’}(n) = \text{paths}(n) - \text{mpaths}(n) \).
- **fpaths(\( n \))**: All CFPs that reach node \( n \) and are feasible. \( \text{fpaths}(n) \) is a subset of \( \text{mpaths’}(n) \).
Definition 6. We refer CFPs that begin at start node of a CFG and end at a particular edge \( e \) as “CFPs that reach \( e \).” These are the same CFPs that reach the destination node of \( e \) and have \( e \) as the last edge.

We create the following classes of CFPs that reach an edge \( e \) namely \( \text{paths}(e) \), \( \text{mpaths}(e) \), \( \text{mpaths}^c(e) \), \( \text{fpaths}(e) \). These classes are defined similar to the corresponding classes of CFPs that reach a node \( n \). Also, if \( n \) is the destination node of an edge \( e \) then the following holds.

\[
\begin{align*}
\text{paths}(e) & \subseteq \text{paths}(n) \\
\text{mpaths}(e) & \subseteq \text{mpaths}(n) \\
\text{mpaths}^c(e) & \subseteq \text{mpaths}^c(n) \\
\text{fpaths}(e) & \subseteq \text{fpaths}(n)
\end{align*}
\]

We defined the notion of contains-prefix-of for a MIPS in definition 1, we now extend this to CFPs as follows.

Definition 7. For a CFP \( \sigma \) that reaches an edge \( e \), \( \text{cpo}_e(\sigma) \) contains all MIPS \( \mu \) such that prefix(\( \mu \), \( e \)) is contained in \( \sigma \), i.e.,

\[
\text{cpo}_e(\sigma) = \{ \mu \mid \mu \in \mathcal{U}, \sigma \text{ contains prefix}(\mu, e) \}
\]

Observation 10.1. Let \( \sigma \), \( \sigma' \) be a path obtained by extending a path \( \sigma \) that reaches an edge \( e \) to its successor edge \( e' \). Then \( \text{cpo}_e(\sigma, \sigma') \) precisely contains two types of MIPS: 1) MIPS present in \( \text{cpo}_e(\sigma) \) which contain edge \( e' \), and 2) MIPS that start at \( e' \) i.e.,

\[
\text{cpo}_e(\sigma, \sigma') = \{ \mu \mid (\mu \in \text{cpo}_e(\sigma), e' \in \text{edges}(\mu)) \lor e' \in \text{start}(\mu) \}
\]

From 2

We now formally state the meet over feasible paths (MOFP) solution. Next, we prove that the FPMFP solution is an sound-approximation of MOFP solution.

10.2. The MOFP solution

Given a path \( \sigma \in \text{paths}(n_k) \) consisting of nodes \( (n_1, n_2, ..., n_{k-1}, n_k) \), let \( f_\sigma \) denote the composition of functions corresponding to the nodes in \( \sigma \) excluding the last node \( n_k \) i.e., \( f_\sigma = \).
\( f_{n-1} \circ f_{n-2} \circ \ldots \circ f_n \). Note that \( f_{\sigma} \) represents the effect of a path that reaches at IN of node \( n_k \), hence it excludes \( f_n \). However, \( f_{\sigma} \) includes the effect of all edges in \( \sigma \) including the edge from \( n_{k-1} \) to \( n_k \). For simplicity, we assume the edge flow functions of the input MFP analysis are identity functions for all edges. Observe that if \( \sigma \) contains a single node then \( f_{\sigma} \) is identity function.

The MOP solution is represented by variables \( ln_n/Out_n \) for all nodes \( n \) using the following equation.

\[
ln_n = \bigcap_{\sigma \in paths(n)} f_{\sigma}(BI) \quad (40)
\]

\[
Out_n = \bigcap_{\sigma \in paths(n)} f_n \circ f_{\sigma}(BI) \quad (41)
\]

Similarly, the MOFP solution is obtained by taking meet over feasible paths that reach \( n \) (i.e., \( f_{\text{paths}}(n) \)) as follows.

\[
ln_n = \bigcap_{\sigma \in f_{\text{paths}}(n)} f_{\sigma}(BI) \quad (42)
\]

\[
Out_n = \bigcap_{\sigma \in f_{\text{paths}}(n)} f_n \circ f_{\sigma}(BI) \quad (43)
\]

10.3. Soundness Claim

**Theorem 10.1.** The FPMFP solution is a sound-approximation of the MOFP solution i.e., if \( N \) is the set of all nodes in the CFG of a procedure then

\[
\forall n \in N. \quad ln_n \subseteq \bigcap_{\sigma \in \text{paths}(n)} f_{\sigma}(BI) \quad (44)
\]

**Proof.** The proof outline is illustrated in Figure 19. Below, we state Claim 2 and Claim 3 which together prove Claim 1 i.e.,

\[
\text{Claim 2} \land \text{Claim 3} \implies \text{Claim 1} \quad (45)
\]

where,

- **Claim 2:** at any node \( n \), \( ln_n \) is a sound-approximation of the meet of data flow values reaching along paths in \( m_{\text{paths}}(n) \) i.e.,

\[
\forall n \in N. \quad ln_n \subseteq \bigcap_{\sigma \in \text{paths}(n)} f_{\sigma}(BI) \quad (46)
\]

- **Claim 3:** at any node \( n \), the meet of data flow values reaching along paths in \( m_{\text{paths}}(n) \) is a sound-approximation of the data flow values reaching along paths in \( f_{\text{paths}}(n) \) i.e.,

\[
\forall n \in N. \quad \left( \bigcap_{\sigma \in \text{paths}(n)} f_{\sigma}(BI) \right) \subseteq \left( \bigcap_{\sigma \in f_{\text{paths}}(n)} f_{\sigma}(BI) \right) \quad (47)
\]
Claim 1
(Theorem 10.1)

Claim 2
(Lemma 10.1)

Claim 3
(Lemma 10.2)

Claim 4
(Lemma 10.3)

Figure 19: Proof Outline. The claim at the root of the tree (claim 1) is the main proof obligation for soundness of FPMFP. A claim at each node is derived using the claims at its child nodes.

Claim 2 and Claim 3 together imply Claim 1 by transitivity of $\sqsubseteq$. We prove the Claim 2 and the Claim 3 in Lemma 10.1 and Lemma 10.2 respectively.

We first state Claim 4 that will be used to prove Claim 2.

In the FPMFP computation, the data flow value in each pair $(M, d)$ at an edge $e : m \to n$ is a sound-approximation of the meet of the data flow values reaching along each path $\sigma$ where $\sigma \in mpaths^s(e)$ and $cpo_e(\sigma) = M$ i.e.,

$$\langle M, d \rangle \in \mathfrak{F}_c(Out_m) \implies d \sqsubseteq \left( \bigcap_{\sigma \in mpaths^s(e), \, cpo_e(\sigma) = M} f_\sigma(BI) \right) \quad (48)$$

We prove Claim 4 in Lemma 10.3. We now show that Claim 2 follows from Claim 4 because for a node $n$ that is not the Start node of the CFG, $mpaths^s(n)$ is obtained by union of $mpaths^s(e)$ corresponding to each edge $e$ that is incident on $n$.

**Lemma 10.1.** At any node $n$ in the CFG of a program the following holds

$$\forall n \in N. \quad m_n \sqsubseteq \left( \bigcap_{\sigma \in mpaths^s(n)} f_\sigma(BI) \right) \quad (49)$$

Proof. We split the proof of Claim 2 in the following two cases which are mutually exclusive and exhaust all possibilities.

- Case 1: $n$ is the Start node of the CFG. In this case, $mpaths^s(n)$ contains a single path containing the Start node only. Hence the following holds.

$$\left( \bigcap_{\sigma \in mpaths^s(n)} f_\sigma(BI) \right) = BI \quad (50)$$
\[ \overline{I_n} = BI \] from[8] \hfill (51) 
\[ \overline{I_n} \subseteq \left( \bigcap_{\sigma \in \text{mpaths}(n)} f_\sigma(BI) \right) \] from[51] [50] \hfill (52)

- Case 2: \( n \) is not the \textit{Start} node of the CFG. In this case, the following holds.

\[ \overline{I_n} = \bigcap_{m \in \text{pred}(n)} \overline{G_{m \leftarrow n}(\overline{O_{U_m}})} \text{ ...from[8]} \] \hfill (53) 
\[ \langle M_1, d_1 \rangle \in \overline{I_n} \implies d_1 \subseteq \left( \bigcap_{m \in \text{pred}(n)} d'_1 \right) \text{ ...from[12]} \] \hfill (54) 
\[ \langle M_1, d_1 \rangle \in \overline{I_n} \implies d_1 \subseteq \left( \bigcap_{\sigma \in \text{mpaths}(n)} f_\sigma(BI) \right) \text{ ...from[10.3]} \] \hfill (55) 
\[ \langle M_1, d_1 \rangle \in \overline{I_n} \implies \left( \bigcap_{\sigma \in \text{mpaths}(n)} d_1 \right) \subseteq \left( \bigcap_{\sigma \in \text{mpaths}(n)} f_\sigma(BI) \right) \text{ ...from[10.3]} \] \hfill (56) 
\[ \overline{I_n} \subseteq \left( \bigcap_{\sigma \in \text{mpaths}(n)} f_\sigma(BI) \right) \] \hfill (57)

Equations[52] and [57] prove the lemma.

\[ \square \]

**Lemma 10.2.** At any node \( n \) in the CFG of a program the following holds

\[ \forall n \in N. \left( \bigcap_{\sigma \in \text{mpaths}(n)} f_\sigma(BI) \right) \subseteq \left( \bigcap_{\sigma \in \text{paths}(n)} f_\sigma(BI) \right) \] \hfill (58)

Proof. The following holds from the definition of \( \text{mpaths}(n) \) and \( \text{paths}(n) \)

\[ \forall n \in N. \text{paths}(n) \subseteq \text{mpaths}(n) \text{ ...from[5]} \] \hfill (59)

Claim 3 trivially follows from[59]

\[ \square \]

The following observation states some properties of a path of length 1 starting at \textit{Start} of a CFG. These properties are used for proving claim 4 subsequently.

**Observation 10.2.** Let \( \sigma_1 : n_0 \rightarrow^e n_1 \) be a path of length one that begins at \textit{Start} of a procedure, and \( \mathcal{U} \) be the set of all MIPS in the procedure, then \( \sigma_1 \) has the following properties.

1. \( e \) does not have a predecessor edge. Hence, \( e \) cannot be end edge of any MIPS i.e.,

\[ \forall \mathcal{M} \subseteq \mathcal{U}. \neg \text{endof}(\mathcal{M}, e) \] \hfill (60)
2. \( cpo_e(\sigma_1) \) only contains MIPS that start at \( e \) i.e.,

\[
cpo_e(\sigma_1) = \{ \mu \mid \mu \in U, e \in \text{start}(\mu) \} \quad \text{from (61)}
\]

\[\forall M \subseteq U. \text{ext}(M, e) = \{ \mu \mid \mu \in U, e \in \text{start}(\mu) \} \quad \text{from (61)}\]

\[\forall M \subseteq U. \text{ext}(M, e) = cpo_e(\sigma_1) \quad \text{(63)}\]

From (61) and (63) we get

\[
(M' \subseteq U) \land (M' \neq cpo_e(\sigma_1)) \implies (\forall M \subseteq U. \text{ext}(M, e) \neq M') \quad (64)
\]

3. There exists only one path of length 1 that begins at \( \text{Start} \)

\[
\text{mpaths}_c^1(e) = \{ \sigma_1 \} \quad (65)
\]

\[
\left( \bigcap_{\sigma \in \text{mpaths}_c^1(e)} f_\sigma(BI) \right) = f_{\sigma_1}(BI) \quad \text{from (65)}
\]

\[= f_{\sigma_0}(BI) \quad \text{from (66)}
\]

4. Since \( n_0 \) is the Start node of the procedure, \( \overline{m_{n_0}} \) is the boundary value.

\[
\overline{m_{n_0}} = \overline{B} = \{ (\emptyset, BI) \} \cup \{ (M, \top) \mid M \neq \emptyset, M \subseteq U \} \quad \text{from (8)}
\]

Applying node flow function \( f_{n_0} \) and using (11) we get

\[
\overline{f_{n_0}(\overline{m_{n_0}})} = \{ (\emptyset, f_{n_0}(BI)) \} \cup \{ (M, f_{n_0}(\top)) \mid M \neq \emptyset, M \subseteq U \}
\]

\[\overline{\text{Out}_{n_0}} = \{ (\emptyset, f_{n_0}(BI)) \} \cup \{ (M, f_{n_0}(\top)) \mid M \neq \emptyset, M \subseteq U \} \quad (70)
\]

**Lemma 10.3.** In the FPMFP computation, the data flow value in each pair \( \langle M, d \rangle \) at an edge \( e : m \to n \) is a sound-approximation of the meet of data flow values reaching along each path \( \sigma \) where \( \sigma \in \text{mpaths}_c^l(e) \) and \( cpo_e(\sigma) = M \) i.e.,

\[
\langle M, d \rangle \in \overline{\text{Out}_{n_0}} \implies d \subseteq \left( \bigcap_{\sigma \in \text{mpaths}_c^l(e), cpo_e(\sigma) = M} f_\sigma(BI) \right) \quad (71)
\]

**Proof.**

We prove this by induction on the length of paths reaching from \( \text{Start} \) to the edge \( e \). Let \( \text{mpaths}_c^l(e) \) denote the paths of length \( l \) from \( \text{mpaths}_c^l(e) \) (edges may be repeated in the path).

We re-write the proof obligation in the following equivalent form.

\[
\forall l \geq 1. \langle M, d \rangle \in \overline{\text{Out}_{n_0}} \implies d \subseteq \left( \bigcap_{\sigma \in \text{mpaths}_c^l(e), cpo_e(\sigma) = M} f_\sigma(BI) \right) \quad (72)
\]

**Basis.** \( l = 1 \) There exists only one path of length 1 that begins from \( \text{Start} \) node. Let that path be \( \sigma_1 : n_0 \to n_1 \), where \( n_0 = \text{Start} \). Observe that \( e \) cannot be an inner or end edge of any
MIPS because \(e\) does not have a predecessor edge. Consequently, \(\text{cpo}_e(\sigma_1)\) only contains MIPS that start at \(e\). We have stated these properties in Observation 10.2.

Proof obligation for the base case (obtained by substituting \(l = 1\) and \(m = n_0\) in (72)):

\[
\langle M', d' \rangle \in \overline{g}_e(\overline{\text{Out}}_{n_0}) \implies d' \subseteq \bigcap_{\sigma \in \text{mpath}_1(e), \text{cpo}_e(\sigma) = M'} f_\sigma(\text{BI})
\]

(73)

We split the proof of the base case in the following two cases which are mutually exclusive and exhaustive for each pair \(\langle M', d' \rangle\) in \(\overline{g}_e(\overline{\text{Out}}_{n_0})\).

- Case 1: There exists a path \(\sigma \in \text{mpath}_1(e)\) such that \(M' = \text{cpo}_e(\sigma)\). Since \(\text{mpath}_1(e)\) contains a single path \(\sigma_1\), Case 1 becomes \(M' = \text{cpo}_e(\sigma_1)\).

\[
(\langle M', d' \rangle \in \overline{g}_e(\overline{\text{Out}}_{n_0}) \land (M' = \text{cpo}_e(\sigma_1)))
\]

\[
\implies d' = \bigcap_{\langle M, d \rangle \in \overline{\text{Out}}_{n_0}} d
\]

\[
\implies d' = \bigcap_{\langle M, d \rangle \in \overline{\text{Out}}_{n_0}} \text{exit}(M, e) = M'
\]

\[
\implies d' = \bigcap_{\langle M, d \rangle \in \overline{\text{Out}}_{n_0}} d
\]

\[
\implies d' = \text{cpo}_e(\sigma_1)
\]

(74)

(75)

- Case 2: There does not exist a path \(\sigma \in \text{mpath}_1(e)\) such that \(M' = \text{cpo}_e(\sigma)\). Since \(\text{mpath}_1(e)\) contains a single path \(\sigma_1\), Case 2 becomes \(M' \neq \text{cpo}_e(\sigma_1)\).

\[
(\langle M', d' \rangle \in \overline{g}_e(\overline{\text{Out}}_{n_0}) \land (M' \neq \text{cpo}_e(\sigma_1)))
\]

\[
\implies d' = \bigcap_{\langle M, d \rangle \in \overline{\text{Out}}_{n_0}} d
\]

\[
\implies d' = \text{ext}(M, e) = M'
\]

(76)

(82)

(83)
\[ d' = \bigcap_{\sigma \in \{\varepsilon \}} f_\tau(BI) \]

Since \( \{\sigma \mid \sigma \in \text{mpaths}_1'(e), \text{cpo}_e(\sigma) \neq \text{cpo}_e(\sigma_1)\} = \Phi \) from 65
\[ d' = \bigcap_{\sigma \in \text{mpaths}_1'(e), \text{cpo}_e(\sigma) = M} f_\tau(BI) \]

From Case 1 and 2, we get equation 73

**Induction Hypothesis.** Let \( n \) be any arbitrary node in the CFG of a program such that there exist a CFP of length \( l - 1 \) reaching \( n \), and \( n \) has at least one outgoing edge. We assume that Lemma 10.3 holds at each edge \( e' \) that is incident on \( n \) (of the form \( m \rightarrow e' \rightarrow n \)) and is the last edge of the path of length \( l - 1 \) reaching \( n \).

\[ \langle M, d \rangle \in \overline{G}_e(\overline{Out}_m) \implies d \subseteq \bigcap_{\sigma \in \text{mpaths}_1'(e'), \text{cpo}_{e'}(\sigma) = M} f_\tau(BI) \] (87)

**Inductive Step.** We prove that Lemma 10.3 holds for any successor edge of \( e' \) w.r.t. paths of length \( l \) if \( m \rightarrow e' \rightarrow n \rightarrow o \) then

\[ \langle M', d'' \rangle \in \overline{G}_e(\overline{Out}_m) \implies d'' \subseteq \bigcap_{\sigma \in \text{mpaths}_1'(e''), \text{cpo}_{e''}(\sigma) = M} f_\tau(BI) \] (88)

**Proof of the inductive step.** We build the proof in two steps.

Let the function \( \text{inEdges}(n) \) return the set of incoming edges of a node \( n \) in the CFG, then in step 1, we prove that the following holds for \( n \).

\[ \langle M, d' \rangle \in \overline{Out}_m \implies d' \subseteq \bigcap_{\sigma \in \text{mpaths}_1'(e'), \text{cpo}_{e'}(\sigma) = M} f_\tau(BI) \] (89)

In step 2, we prove that for any successor edge \( e'' \) of \( e' \), equation 88 holds.

**Step 1:**

\[ \overline{m}_n = \bigcap_{m \rightarrow e \in \text{pred}(n)} \overline{G}_m(\overline{Out}_m) \] (90)

\[ \langle M, d \rangle \in \overline{m}_n \implies d \subseteq \bigcap_{\sigma \in \text{mpaths}_1'(e'), \text{cpo}_{e'}(\sigma) = M} f_\tau(BI) \] (91)

**Step 2:**

\[ \langle M, d \rangle \in \overline{m}_n \implies d \subseteq \bigcap_{\sigma \in \text{mpaths}_1'(e'), \text{cpo}_{e'}(\sigma) = M} f_\tau(BI) \] (92)
since $f_n$ is monotone

\[
\langle M, d \rangle \in \overline{\text{In}}_n \implies f_n(d) \sqsubseteq \bigcap f_n \circ f_{\sigma}(BI) \quad (93)
\]

\[
\langle M, d' \rangle \in \overline{\text{Out}}_n \implies \exists \langle M, d \rangle \in \overline{\text{In}}_n. f_n(d) = d' \quad \ldots \text{from } 9 \quad (94)
\]

\[
\langle M, d' \rangle \in \overline{\text{Out}}_n \implies d' \sqsubseteq \bigcap f_n \circ f_{\sigma}(BI) \quad \ldots \text{from } 93 \quad 94 \quad (95)
\]

Step 2: We now prove that at any successor edge $e'' : n \rightarrow o$ of $e'$ the following holds:

\[
\langle M'', d'' \rangle \in \overline{\text{Out}}_n \implies d'' \sqsubseteq \bigcap f_{\sigma}(BI) \quad (96)
\]

We consider the following possibilities for any arbitrary pair $\langle M'', d'' \rangle$ in $\overline{\text{Out}}_n$. These cases are mutually exclusive and exhaustive.

- Case A: $e''$ is end edge of some MIPS $\mu$ contained in $M''$ i.e.,
  \[\text{endof}(M'', e'') = \text{true}.\]

  In this case, we prove that equation (96) holds because the following is true.
  \[d'' = \top \quad (97)\]

  \[\bigcap f_{\sigma}(BI) = \top \quad (98)\]

  \[d'' = \top \text{ follows from the definition of the edge flow function (equation 15) i.e.,} \]

  \[
  \langle M'', d'' \rangle \in \overline{\text{Out}}_n \land \text{endof}(M'', e'') \implies d'' = \top \quad \ldots \text{from } 15 \quad (99)
  \]

Equation (98) follows from the fact that there cannot exist a path $\sigma'$ that satisfies the constraint $\sigma' \in \text{mpaths}^c_\varepsilon(e'') \land \text{endof}(\text{cpo}_e(\sigma'), e'')$ from the definition of $\text{mpaths}^c(e'')$.

Hence the following holds,

\[\left( \bigcap f_{\sigma}(BI) = \top \right) \quad (100)\]

From (99) and (100) we get

\[
\langle M'', d'' \rangle \in \overline{\text{Out}}_n \land \text{endof}(M'', e'') \implies d'' \sqsubseteq \left( \bigcap f_{\sigma}(BI) \right) \quad (101)
\]
• Case B: \( e'' \) is not end edge of any MIPS in \( M' \) i.e., \( \text{endof}(M', e'') = \text{false} \).

\[
\langle M', d'' \rangle \in \mathcal{P}_{\pi}(\text{Out}_n) \land \neg\text{endof}(M', e'')
\]

\[
\Rightarrow d'' = \bigcap_{\langle M, d \rangle \in \text{Out}_n, \text{ext}(M, e'') = M'} d \quad \text{... from [15]}
\]

\[
\Rightarrow d'' \sqsubseteq (\bigcap_{\sigma \in \text{mpaths}_{e'-1}(e'), e' \in \text{inEdges}(n), \text{cpo}_\sigma(\sigma) = M, \text{ext}(M, e') = M''} f_\sigma \circ f_{\sigma}(BI)) \quad \text{... from [95]}
\]

\[
\Rightarrow d'' \sqsubseteq (\bigcap_{\sigma \in \text{mpaths}_{e'-1}(e'), e' \in \text{inEdges}(n), \text{cpo}_\sigma(\sigma, e'') = M''} f_{\sigma}(BI)) \quad \text{... from [10.1]}
\]

\[
\Rightarrow d'' \sqsubseteq (\bigcap_{\sigma \in \text{mpaths}_e(e''), \text{cpo}_\sigma(\sigma) = M'} f_{\sigma}(BI)) \quad (102)
\]

From case A and B, we get equation [88]

\[\square\]

10.4. Soundness of the optimizations of scalability

In this section, we prove that the optimizations proposed in Section 6 retain the soundness and precision of the computed FPMFP solution. Optimization 1 and Optimization 2 are proved in sections 10.4.1 and 10.4.2 respectively.

10.4.1. Proof for Optimization 1

Optimization 1 is applied iff the input MIPS satisfy the property \( \mathcal{P} \) (Section 6.1) which is restated as follows.

\( \mathcal{P} \): for a MIPS \( \mu : e_1 \rightarrow e_2 \rightarrow \ldots \rightarrow e_n \), if the condition on the end edge \( e_n \) is \( \text{cond}(e_n) \) then \( \text{cond}(e_n) \) evaluates to \text{false} at the start edge \( e_1 \) of \( \mu \) and variables in \( \text{cond}(e_n) \) are not modified along \( \mu \).

Formally, if \([p]_e\) represents the evaluation of a condition expression \( p \) at an edge \( e \), and \( \text{modified}(p, \mu) \) is the set of variables in \( p \) that are modified at intermediate nodes of a MIPS \( \mu \) then

\[
\"\mu \text{ is a MIPS}\" \iff (\text{true} \land \text{false} \land \text{modified}(\text{false}, \mu) = \phi) \quad (103)
\]
We now prove that if two MIPS satisfy \( P^{10} \) and have the same end edge then they can be considered equivalent. In particular, we prove that if two MIPS have the same end edge \( e \), then any CFP that extends from start edge of either MIPS till \( e \) by traversing edges in either MIPS is an infeasible path. Hence, the data flow values associated with such MIPS can be merged.

**Theorem 10.2.** If \( \mu_1, \mu_2 \) are two MIPS that have the same end edge \( e_n \), and they intersect at some node \( x_k \), then \( \mu_3, \mu_4 \)—constructed as shown below using split and join of \( \mu_1, \mu_2 \) at the intersecting node—are also MIPS.

\[
\begin{align*}
\mu_1 : & x_1 \xrightarrow{e_1} x_2 \rightarrow \ldots \rightarrow x_k \rightarrow x_{k+1} \rightarrow \ldots \rightarrow e_n \\
\mu_2 : & y_1 \xrightarrow{e_1'} y_2 \rightarrow \ldots \rightarrow x_k \rightarrow y_j \rightarrow \ldots \rightarrow e_n \\
\mu_3 : & x_1 \xrightarrow{e_1} x_2 \rightarrow \ldots \rightarrow x_k \rightarrow y_j \rightarrow \ldots \rightarrow e_n \\
\mu_4 : & y_1 \xrightarrow{e_1'} y_2 \rightarrow \ldots \rightarrow x_k \rightarrow x_{k+1} \rightarrow \ldots \rightarrow e_n
\end{align*}
\]

Claim 5. “\( \mu_1, \mu_2 \) are MIPS” \( \implies \) “\( \mu_3, \mu_4 \) are MIPS”

Proof. We use the following fact to prove the claim 5: \( \mu_3 \) and \( \mu_4 \) are constructed using the edges and the nodes from \( \mu_1 \) and \( \mu_2 \) so the following holds

\[
(modified(\text{cond}(e_n), \mu_1) = \phi) \land (modified(\text{cond}(e_n), \mu_2) = \phi) \implies (modified(\text{cond}(e_n), \mu_3) = \phi) \land (modified(\text{cond}(e_n), \mu_4) = \phi)
\]

\[105\]

\( LHS : \) “\( \mu_1, \mu_2 \) are MIPS” \( \implies \) \( ([\text{cond}(e_n)]_{ex_1} = [\text{cond}(e_n)]_{ey_1} = false) \land \\
(modified(\text{cond}(e_n), \mu_1) = \phi) \land \\
(modified(\text{cond}(e_n), \mu_2) = \phi) \) \( \ldots \) from\[102\]

\( \implies \) \( ([\text{cond}(e_n)]_{ex_1} = [\text{cond}(e_n)]_{ey_1} = false) \land \\
(modified(\text{cond}(e_n), \mu_3) = \phi) \land \\
(modified(\text{cond}(e_n), \mu_4) = \phi) \) \( \ldots \) from\[105\]

\( \implies \) “\( \mu_3, \mu_4 \) are MIPS”

10.4.2. Proof of Optimization 2

At an edge, Optimization 2 (Section 6.2) merges pairs that contain the same data flow value. In this section, we prove that this optimization retains the precision and the soundness of the

\[10\]Note that we use algorithms given by Bodik et al for detecting MIPS which are presented in Appendix; they allow us to track if a MIPS satisfies the property \( \mathcal{P} \) or not.
FPMFP solution. A crucial difference between FPMFP and MFP is that in the FPMFP computation certain data flow values are blocked at end edges of MIPS. We prove that the same set of values are blocked at the same set of end edges with or without Optimization 2.

Theorem 10.3. If \( \overline{\gamma_c}(\overline{\mu}) \) contains two pairs \( \langle M_1, d \rangle \) and \( \langle M_2, d \rangle \) with the same data flow value \( d \) then shifting \( d \) to the pair corresponding to a set of MIPS \( M_3 \) defined below does not affect the soundness or precision of the resulting FPMFP solution. Specifically, it does not affect the blocking of \( d \) at the end edges of MIPS in \( M_1 \) and \( M_2 \).

\[
M_3 = \{ \mu_1 \mid \mu_1 \in M_1, \exists \mu_2 \in M_2, \mu_2 \in \text{cso}_{e}(\mu_1) \} \cup \{ \mu_2 \mid \mu_2 \in M_2, \exists \mu_1 \in M_1, \mu_1 \in \text{cso}_{e}(\mu_2) \} \tag{106}
\]

Proof. We consider the following two operations each indicating Optimization 2 is performed or not. Next, we prove that the FPMFP solution is identical after both the operations:

1. Operation 1: pairs \( \langle M_1, d \rangle \) and \( \langle M_2, d \rangle \) are kept as it is in \( \overline{\gamma_c}(\overline{\mu}) \) (i.e., Optimization 2 is not applied).

2. Operation 2: pairs \( \langle M_1, d \rangle \) and \( \langle M_2, d \rangle \) are replaced with \( \langle M_3, d \rangle \) in \( \overline{\gamma_c}(\overline{\mu}) \) (i.e., Optimization 2 is applied). For simplicity of exposition, we assume there is no value associated with \( M_2 \) initially, otherwise we add \( \langle M_2, d' \cap d \rangle \) where \( d' \) is the value associated with \( M_3 \) in \( \overline{\gamma_c}(\overline{\mu}) \).

The possible cases for MIPS in \( M_1 \) and \( M_2 \) w.r.t. Operation 1 and 2 are shown in Table 12. Let the data flow value \( d \) is not killed along a MIPS \( \mu_1 \) in \( M_1 \) (if it is killed then \( d \) does not reach end(\( \mu_1 \)) after either operation is performed). We prove that both the above operations achieve similar effect in the following two cases which are mutually exclusive and exhaustive for all MIPS in \( M_1 \) (similar argument holds for MIPS in \( M_2 \)):

1. Case 1: if \( \mu_1 \in M_1 \) and \( \mu_1 \in M_2 \), then \( d \) is blocked at end(\( \mu_1 \)) after both the operations.

2. Case 2: if \( \mu_1 \in M_1 \) and \( \mu_1 \not\in M_2 \), then \( d \) reaches end(\( \mu_1 \)) after both the operations.

Proof of Case 1.

If \( \mu_1 \in M_1 \) and \( \mu_1 \in M_2 \), then \( d \) is blocked at end(\( \mu_1 \)) after both the operations, as explained below.

1. Operation 1: \( \langle M_1, d \rangle, \langle M_2, d \rangle \) are kept as it is in \( \overline{\gamma_c}(\overline{\mu}) \).

In this case, \( d \) is blocked within \( \langle M_1, d \rangle \) at end(\( \mu_1 \)) because \( \mu_1 \in M_1 \). Also, \( d \) is blocked within \( \langle M_2, d \rangle \) because there exists a MIPS \( \mu_2 \) in \( M_2 \) whose suffix is contained in \( \mu_1 \) i.e.,

\[
\mu_1 \in M_1 \land \mu \in M_3 \implies \exists \mu_2 \in M_2, \mu_2 \in \text{cso}_{e}(\mu_1) \tag{107}
\]

\[
\mu_2 \in M_2 \land \mu_2 \in \text{cso}_{e}(\mu_1) \implies \text{end}(\mu_2) \in \text{edges}(\mu_1) \text{ and } \tag{108}
\]

\[
d \text{ is blocked within } \langle M_2, d \rangle \text{ at end}(\mu_2) \tag{109}
\]

Hence, in this scenario, \( d \) is blocked within both pairs before reaching end(\( \mu_1 \)) or at end(\( \mu_1 \)).
### Table 12: Equivalence of Operation 1 and Operation 2 w.r.t. cases in Optimization 2.

Optimization 2 involves shifting a data flow value \( d \) from pairs \((M_1, d)\) and \((M_2, d)\) to pair \((M_1, d)\). For simplicity, we assume \( d \) is not killed along \( \mu \).

2. Operation 2: \((M_1, r), (M_2, r), (M_3, d)\) belong to \( \prod (\overrightarrow{I}_e) \) and \( \mu_1 \in M_3 \).

   In this case, \( d \) is blocked within \((M_3, d)\) at \( end(\mu_1) \) because \( \mu_1 \in M_3 \).

#### Proof of Case 2.

If \( \mu_1 \in M_1 \) and \( \mu_1 \notin M_3 \), then \( d \) reaches \( end(\mu_1) \) after both the operations.

1. Operation 1: \((M_1, d), (M_2, d)\) are kept in \( \prod (\overrightarrow{I}_e) \) as it is.

   In this case, \( d \) is blocked within \((M_1, d)\) at \( end(\mu_1) \). However, \( d \) in \((M_2, d)\) is not blocked at or before \( end(\mu_1) \) because \( \mu_1 \) does not contain \( suffix(\mu_2, e) \) for any MIPS \( \mu_2 \) in \( M_2 \) (meaning \( \mu_2 \) either 1) does not follow the same CFP as \( \mu_1 \) after \( e \), or 2) follows the same CFP as \( \mu_1 \) but does not end before \( \mu_1 \) on the CFP). Thus, \( d \) in \((M_2, d)\) reaches \( end(\mu_1) \) along a CFP that goes through \( suffix(\mu_1, e) \).

2. Operation 2: \((M_1, r), (M_2, r), (M_3, d)\) belong to \( \prod (\overrightarrow{I}_e) \).

   In this case, \( \mu_1 \) does not contain \( suffix(\mu_3, e) \) for any MIPS \( \mu_3 \) in \( M_3 \) i.e.,

   \[
   \mu_1 \in M_1 \land \mu_1 \notin M_3 \Rightarrow \forall \mu_3 \in M_3, \mu_3 \notin cso_e(\mu_1)
   \]

   This can be proved by proving that following is a contradiction.

   \[
   \begin{align*}
   \mu_1 \in M_1 \land \mu_1 \notin M_3 \land \exists \mu_3 \in M_3, \mu_3 \in cso_e(\mu_1) \\
   \Rightarrow \mu_3 \notin M_2 & \quad \text{from} \ [106] \\
   \Rightarrow \mu_3 \in M_1 \land \exists \mu_2 \in M_2, \mu_2 \in cso_e(\mu_3) & \quad \text{from} \ [106] \\
   \Rightarrow \exists \mu_2 \in M_2, \mu_2 \in cso_e(\mu_1) & \quad cso_e(\mu_3) \subseteq cso_e(\mu_1) \\
   \Rightarrow \mu_1 \in M_3 & \quad \text{from} \ [106] \\
   \Rightarrow false
   \end{align*}
   \]

   Hence \( d \) in \((M_3, d)\) is not blocked at or before \( end(\mu_1) \). Thus, \( d \) in \((M_3, d)\) reaches \( end(\mu_1) \) along a CFP that goes through \( suffix(\mu_1, e) \).

   Similar cases can be proved for MIPS in \( M_2 \). Thus, the FPMFP solutions with or without Optimization 2 are equivalent. \( \square \)

### 11. Appendix: Bodik’s Approach for Detecting Minimal Infeasible Path Segments

In this section, we present the algorithms proposed by Bodik et al. [2] for detection of MIPS. We have used these algorithms in the pre-processing stage of FPMFP computation to generate a set of MIPS. The section is organized as follows. First, we give an overview of Bodik’s approach of infeasible path detection (Section 11.1). Next, we describe the algorithms for detection of infeasible paths in detail (Section 11.2).
11.1. Overview of Bodik’s Approach

Bodik et al. observed that many infeasible paths are caused by statically detectable correlations between two branch conditions, or between a branch condition and an assignment statement appearing on the paths. Hence, to find infeasible paths arising from branch correlations, they use the following criteria: if the constraint of a conditional edge evaluates to FALSE along any CFP reaching the edge, then the CFP is infeasible.

**Example 1.** In Figure 20a, the branch constraint \( b > 1 = \text{true} \) is in conflict with the assignment statement \( b = 0 \), hence the path marked with double line arrows is infeasible. Similarly, in Figure 20b the constraint \( b > 1 = \text{false} \) of the branch at bottom is in conflict with constraint \( b > 1 = \text{true} \) of the branch at top. Hence the path marked with double line arrows is infeasible.

Intuitively, the idea is to enumerate CFPs that reach a conditional edge and identify CFPs along which the edge constraint evaluates to FALSE, because such CFPs are infeasible. In particular they proceed as follows. They start from a branch node, and backward propagate the corresponding constraint (i.e., branch condition) along the incoming paths that reach the node. Here, they evaluate the branch constraint using predefined rules at nodes encountered in backward propagation along a path. A path is labeled as infeasible if the assertions at a node and the constraint have a conflict that is detectable using predefined rules.

**Example 2.** For example in Figure 21 the branching node \( n_7 : a > 1 \) is correlated with node \( n_1 \) in that the constraint \( a > 1 \) evaluates to FALSE at \( n_1 \). Therefore, the following path \( \sigma \) that connects \( n_1 \) to the TRUE branch of \( n_7 \) is infeasible, \( \sigma : n_1 \rightarrow n_2 \rightarrow n_5 \rightarrow n_6 \rightarrow n_7 \rightarrow n_8 \). Bodik’s approach detects this path by backward propagating the constraint \( (a > 1) \) from node \( n_7 \) to \( n_1 \) along \( \sigma \).
(a) Correlation between assignment statement and branch statement.

(b) Correlation between two branch statements.

Figure 20: Statically detectable correlations between program statements leading to infeasible paths.
Figure 21: Example illustrating detection of MIPS using Bodik’s Approach
Algorithm 1 Step 1 to detect MIPS that end at a conditional edge $e$. Let the condition on $e$ be $(v \leq c) = x$, where $x$ is either [TRUE or FALSE], and $v \leq c$ is the expression at the source node of $e$. The comments are prefixed with //.

1: Initialize $Q[e]$ to {} at each edge $e$; set worklist to {}.
   //raise the initial query $q_e : ((v \leq c) = x)$, at each immediate predecessor
   //of $e$ these are edges incident on source node of $e$, and
   //are represented by predEdge($e$)
2: For each $e_m \in$ predEdge($e$)
3:   raiseQuery($e_m, q_e$)
4: End For
5: While worklist not empty
6:   remove pair ($e, q$) from worklist
   //assume unknown outcome of $q$ at the entry edge
   //(i.e., the outgoing edge of program entry).
7: If $e$ is entry edge
8:   $A[e, q] := \text{UNDEF}$
9: Else //answer $q$ using assertions generated at source node of $e.$
10:   answer := resolve($e, \text{source}(e), q$)
11: If answer $\in \{\text{TRUE, FALSE, UNDEF}\}$
12:   $A[e, q] := \text{answer}$
13: Else
14:   For each $e_m \in$ predEdge($e$)
15:     raiseQuery($e_m, q$)
16: End For
17: End If
18: End If
19: End While

Procedure raiseQuery(Edge $e$, Query $q$)
20: If $q \notin Q[e]$
21:   add $q$ to $Q[e]$
22: add pair ($e, q$) to worklist
23: End If
Algorithm 2 Step 2: mark the edges with MIPS information

```
//begin analysis from the edges where any query was resolved to FALSE
//at lines 5 to 19 of Step1
1: worklist:=\{e | a query was resolved to FALSE at edge e\}
   //raise the initial query at the analyzed edge, to mark end of MIPS.
2: While worklist not empty
3: remove an edge e from the worklist
   //if query resolved to false at this edge then mark
   //the edge as start for corresponding MIPS
4: For each query q_{ex} such that A[e, q_{ex}] = FALSE
5:   add q_{ex} to Start[e]
   //mark e as start and ex as end edge for MIPS resulting from
   //query q_{ex} resolving to FALSE at edge e.
6:   add q_{ex} to End[ex]
7: End For
   //determine answers for each query that was propagated backward
8: For each query q from Q[e] s.t. q was not resolved at e
   //move the MIPS started along all predecessor edges of e
9:   If (for all e_m ∈ predEdge(e), q ∈ Start[e_m])
10:      add q to Start[e]
11:      For each e_m ∈ predEdge(e)
12:         remove q from Start[e_m]
13:     End For
14:   Else
15:     For each e_m ∈ predEdge(e)
16:        copy Start[e_m] to Inner[e]
17:        copy Inner[e_m] to Inner[e]
18:     End For
19: End If
20: End For
21: End While
```

11.2. Detailed Explanation of Bodik’s Approach

The Bodik’s approach involves two steps as described below.

1. In the first step, the constraints from branch nodes are propagated backwards (towards CFG entry) to identify nodes (if any) at which these constraints evaluate to FALSE.

2. In the second step, a forward traversal of CFG is done to mark the infeasible paths using data from Step 1.

We now explain each of these steps in detail below.

11.2.1. Step 1

The results of Step 1 for the example in Figure 21 are marked in blue. The details of Step 1 are given in Algorithm 1 and are described below.

At the beginning of Step 1, we raise a query q_{e} at each conditional edge e such that the query q_{e} represents the constraint on e. Consequently e is visited in an execution only if q_{e} is TRUE.
Example 1. For example, the query for the edge $e_8$ is $q_{e_8} : ((a > 1) = \text{TRUE})$, and for the edge $e_9$ is $q_{e_9} : ((a > 1) = \text{FALSE})$. This means the edge $e_8$ (resp. $e_9$) will be executed only when $q_{e_8}$ (resp. $q_{e_9}$) evaluates to TRUE. Similarly, we raise queries $q_{e_2}$ and $q_{e_3}$ at conditional edges $e_5$ and $e_3$ respectively.

Next, the query raised at an edge $e$ is propagated along each predecessor edge of $e$ say $p_e$. Here, we try to resolve the query at $p_e$ using the assertions generated at source of $p_e$ if the assertions restrict the value of variables in the query, for example, because of assignment of values to the variables, or branching out of a conditional expression that tests value of the variables. In the former case, if there is an assignment to a variable present in the query then the query resolves to either TRUE, FALSE, or UNDEF (meaning details are not sufficient to resolve the query to TRUE or FALSE).

In the latter case, if a variable present in the query is tested in a conditional at the source of $p_e$, then the query may resolve to TRUE or FALSE or remain unresolved. If the query is unresolved at $p_e$, then the query is back propagated to predecessors of $p_e$ and so on.

Example 2. For example, in Figure 21, we propagate queries $q_{e_8}$ and $q_{e_9}$ along edge $e_7$, since source node of $e_7$ is a printf statement, none of the queries are resolved, and hence they are back propagated to edge $e_6$ and so on. Further, at the edge $e_5$ both the queries $q_{e_8}$ and $q_{e_9}$ are resolved to UNDEF because of the statement `scanf("%d", &a)` at the source of $e_5$. Similarly, at the edge $e_1$, the query $q_{e_8}$ evaluates to FALSE, and $q_{e_9}$ evaluates to TRUE because of assignments at the source of $e_1$.

At the end of Step 1 we have two arrays $Q$ and $A$, where $Q$ stores the queries raised or backward propagated at an edge, and $A$ stores queries resolution at an edge.

Recall that we defined the query $q_{e_8}$ such that edge $e_8$ executes only if $q_{e_8}$ is TRUE. Consequently, if in Step 1, a query $q_{e_8}$ is resolved to FALSE at the edge $e$, then this implies there is an infeasible path segment from edge $e$ to the edge $e_8$. For example in Figure 21 at edge $e_1$, the query $q_{e_8} : ((a > 1) = \text{TRUE})$ evaluates to FALSE, hence there is an infeasible path (marked by red edges) that goes from $e_1$ to $e_8$. The infeasible path marking happens in Step 2 (Section 11.2.2).

11.2.2. Step 2

Step 2 marks the edges in the CFG with the corresponding infeasible paths that pass through the edges. In particular, this is achieved by maintaining three sets namely Start, Inner, and End at each edge $e$, indicating the MIPS that contain $e$ as start, inner, or end edge respectively. For our running example, the results of Step 2 obtained by using Algorithm 2 are marked in the green color in Figure 21.

The Step 2 proceeds as follows. For each edge $e$ where any query $q_{e_8}$ was resolved to FALSE (in Step 1), we mark $e$ as the start edge of an infeasible path segment (ips) that ends at $e_8$. We also mark $e_8$ (i.e., the edge at which query $q_{e_8}$ was raised in Step 1) as the end edge of the ips.

For example, in the Figure 21 edge $e_1$ is marked as start edge, and the edge $e_8$ is marked as end edge for ips that ends at $e_8$.

\[1\] In the UNDEF case, Bodik also proposes the idea of query substitution which is not described here.

\[12\] Note that only intra-procedural predecessor edges are considered. In case, the source of $p_e$ is a call node and the variables present in the query are modified inside the callee function, then the query is not propagated to predecessor edges of $p_e$. 

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Figure 22: Detecting a balanced interprocedural MIPS. The variable $a$ is not modified inside the procedure $z$, hence the query $(q_{e5} : a > 5 = TRUE)$ is back propagated from $e_4$ to $e_3$. $S, E$ represent the start and end nodes of a procedure respectively. $C_{zp}$ represents the transfer of control from procedure $p$ to procedure $z$ at a call node, and $R_{zp}$ represents the return of control from $z$ to $p$. For brevity, cases where $Q$, $Start$, $Inner$, or $End$ is empty are not shown.
Next, if at some edge $e$ query $q_{ex}$ was propagated backwards (in Step 1) and all its predecessor edges are marked as start edge for some ips that ends at $ex$ (in Step 2) then $e$ can also be marked as start edge for the ips. This follows from the fact that the path segment from each predecessor edge $p_e$ to $ex$ is infeasible path segment but path segment from $e$ to $ex$ is also ips and is shorter than the ips from $p_e$ to $ex$. Using this justification, we mark edge $e_3$ as the start edge for ips that ends at $e_8$ in our running example; observe that the ips that goes from $e_3$ to $e_8$ is shorter than the one that goes from $e_1$ to $e_8$ (hence we un-mark $e_1$ as start edge).

On the other hand, if at some edge $e$ query $q_{ex}$ was propagated backwards (in Step 1) and only some of its immediate predecessor edges are marked as start edge or at least one of the predecessor edge is marked as the inner edge for a ips that ends at $ex$ then we mark $e$ as inner edge for the ips. For instance, we mark edges $e_6$, $e_7$ as inner edges for ips that ends at $e_8$ in our running example.

Finally, at the end of Step 2, each path segment $\sigma : e_1 \rightarrow e_2 \rightarrow \ldots \rightarrow e_n$ in CFG is a MIPS if there exists a query $q$ present in $Start[e_1]$, $End[e_n]$, and $Inner[e_i]$, $1 < i < n$.

**Example 1.** Figure 22 shows how balanced inter-procedural MIPS (definition 2 from Chapter 5) are detected using Bodik’s approach. In particular, the variables in the query $q_{e5}$ are not modified inside the procedure $z$, hence $q_{e5}$ is propagated from $e_4$ to $e_3$ to $e_2$. Subsequently, in Step 2, $e_2 \rightarrow e_3 \rightarrow e_4 \rightarrow e_5$ is marked as a MIPS.