We review the present status in the determination of the accurate value of $\alpha_s$ from $\tau$-decays, where we discuss in detail the different sources of theoretical errors.
1 INTRODUCTION

Since the original and previous theoretical works in [1]-[5] and the ALEPH measurement [6], there has been an amount of progress both theoretically and experimentally in the determination of $\alpha_s$ from tau decays. The theoretical progress resides mainly in a better understanding of the QCD perturbative and nonperturbative contributions to the inclusive hadronic width, as discussed in recent reviews [7]-[9] while the experimental one can be essentially due to the improved measurement of the leptonic width (see e.g. [10]) and of the $\tau$-lifetime. In this (written) version of my talk, I will only limit to the discussion of the different sources of theoretical errors in the determination of $\alpha_s$, in order to avoid a repetition of the different discussions already done in previous works. In this way, the present paper complements the previous review in [9]. It is now well-known that the inclusive branching ratio:

$$R_\tau \equiv \frac{\Gamma (\tau \rightarrow \nu_\tau + \text{hadrons})}{\Gamma (\tau \rightarrow \nu_\tau e\bar{\nu}_e)},$$

(1)

can be reliably measured through the leptonic branching ratio $B_l$:

$$R^B_\tau \equiv \frac{1 - B_e - B_\mu}{B_e}.$$  

(2)

or/and the $\tau$-lifetime:

$$R^\Gamma_\tau \equiv \frac{\Gamma_\tau - \sum_{e,\mu} \Gamma_{\tau \rightarrow \nu_e}}{\Gamma_{\tau \rightarrow \nu_e}}.$$  

(3)

The PDG94 compilation gives the value [11]:

$$R^\Gamma_\tau = 3.55 \pm 0.06 \quad R^B_\tau = 3.56 \pm 0.04,$$

(4)

leading to the average:

$$R_\tau = 3.56 \pm 0.03.$$  

(5)

However, there is a continuous effort in improving the measurements of the leptonic width in the four LEP experiments as presented in this workshop. The most precise recent preliminary data come from the ALEPH group and lead to [10]:

$$R^B_\tau = 3.645 \pm 0.024,$$

(6)

which are in agreement with the other LEP data [12], but are larger by about 2$\sigma$ than the CLEO II result [13], using PDG values of the leptonic width:

$$R^B_\tau = 3.552 \pm 0.035.$$  

(7)

These data are larger than the naïve parton model prediction $R_\tau = N_c$ by about 20%. This discrepancy is expected to be resolved by the electroweak and QCD corrections. Using the analyticity of the two-point correlator built from the vector and axial-vector currents, which governs the semileptonic process, and using the Cauchy theorem which
transforms the integral over the real axis into the one around the circle of a radius $M_\tau$ in the complex $s$-plane, the QCD expression of the inclusive width can be written as [2]:

$$R_\tau \equiv 3 \left( |V_{ud}|^2 + |V_{us}|^2 \right) S_{EW} \left( 1 + \delta_{EW} + \delta^{(0)} + \sum_{D=2,4,...} \delta^{(D)} \right), \quad (8)$$

where $|V_{ud}| \simeq 0.9744 \pm 0.0010$, and $|V_{us}| \simeq 0.2205 \pm 0.0018$ are the CKM mixing angles [11]; $S_{EW} = 1.0194$ [14] and $\delta_{EW} = 0.0010$ [17] are the electroweak corrections coming respectively from the summation of the leading-log and from the constant term; $\delta^{(0)}$ is the perturbative contribution. Within the standard SVZ-expansion [16], $\delta^{(2)}$ is the quark mass corrections, while $\delta^{(D \geq 4)}$ is the nonperturbative contributions simulated by the operators of dimension $D$.

### 2 THE SIZE OF DIFFERENT CORRECTIONS

Table 1: QCD predictions for the different components of the $\tau$ hadronic width:

| $\alpha_s(M_\tau^2)$ | $R_{\tau,V}$ | $R_{\tau,A}$ | $R_{\tau,S}$ | $R_\tau$ |
|----------------------|-------------|-------------|-------------|---------|
| 0.20                 | 1.62 ± 0.02 | 1.53 ± 0.03 | 0.145 ± 0.005 | 3.29 ± 0.01 |
| 0.22                 | 1.64 ± 0.02 | 1.54 ± 0.03 | 0.145 ± 0.005 | 3.33 ± 0.02 |
| 0.24                 | 1.66 ± 0.02 | 1.56 ± 0.03 | 0.145 ± 0.005 | 3.37 ± 0.02 |
| 0.26                 | 1.68 ± 0.02 | 1.58 ± 0.03 | 0.145 ± 0.005 | 3.41 ± 0.02 |
| 0.28                 | 1.70 ± 0.02 | 1.61 ± 0.03 | 0.145 ± 0.005 | 3.45 ± 0.02 |
| 0.30                 | 1.72 ± 0.02 | 1.63 ± 0.03 | 0.145 ± 0.006 | 3.50 ± 0.02 |
| 0.32                 | 1.75 ± 0.02 | 1.65 ± 0.03 | 0.145 ± 0.006 | 3.54 ± 0.03 |
| 0.34                 | 1.77 ± 0.02 | 1.67 ± 0.03 | 0.145 ± 0.006 | 3.58 ± 0.03 |
| 0.36                 | 1.79 ± 0.02 | 1.69 ± 0.03 | 0.144 ± 0.006 | 3.63 ± 0.03 |
| 0.38                 | 1.81 ± 0.02 | 1.71 ± 0.03 | 0.144 ± 0.007 | 3.67 ± 0.04 |
| 0.40                 | 1.83 ± 0.03 | 1.73 ± 0.03 | 0.143 ± 0.007 | 3.71 ± 0.04 |
| 0.42                 | 1.85 ± 0.03 | 1.75 ± 0.04 | 0.143 ± 0.007 | 3.75 ± 0.04 |

The size of these different contributions has been discussed in detail in the original paper [2] and in different review talks [7]-[9], so that we will not discuss them in any more detail here. They can be summarized as:
2.1 Perturbative corrections

We use the result of [17] for the $D$-function governing the $e^+e^- \rightarrow$ hadrons total cross-section:

$$D(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \left( \frac{\alpha_s}{\pi} \right)^n,$$

where for the charged vector and axial-vector current and for three flavours:

$$K_0 = K_1 = 1 \quad K_2 = 1.6398 \quad K_3 = 6.3711,$$

from which, we deduce the perturbative corrections to order $\alpha_s^3$ [2]:

$$\delta_{BNP}^{(0)} = \left( \frac{\alpha_s}{\pi} \right) + 5.2023 \left( \frac{\alpha_s}{\pi} \right)^2 + 26.366 \left( \frac{\alpha_s}{\pi} \right)^3$$

which, for a typical value of $\alpha_s(M_\tau)$ gives a correction of about 20%, while each term is respectively 11%, 6% and 3%, indicating that at this low $\tau$-scale the convergence of the calculated perturbative series is quite good, though it can be improved [3] by using another expansion parameter other than $\alpha_s$ as we shall discuss later on.

2.2 Quark masses and nonperturbative corrections

For a typical value of $R_\tau = 3.6$ and within the standard SVZ-expansion, the quark mass and nonperturbative corrections, in units of $10^{-3}$, are [2]:

$$\delta^{(2)} \simeq -(10 \pm 2)$$
$$\delta^{(4)} \simeq -(3.3 \pm 0.5)$$
$$\delta^{(6)} \simeq -(7 \pm 4)$$
$$\delta^{(8)} \simeq 0.01.$$

The one-instanton and instanton-anti-instanton manifest as operators of dimensions larger or equal than 9. Their respective contributions $\delta_I \simeq 10^{-6}$ [18] and $\delta_{I-I} \simeq 10^{-3}$ [19] are negligible. However, there is an internal inconsistency of the approach for large value of $\alpha_s$. A recent phenomenological fit [20] has shown that the instanton effect to $R_\tau$ cannot exceed $0.5 \times 10^{-3}$. It is clear from the previous discussions that these nonperturbative corrections are small compared with the typical perturbative corrections $\delta^{(0)} \simeq 20\%$. This is due to the fact that the largest quark mass effect due to the strange quark is suppressed by the $\sin^2\theta_c$ factor. Charm and beauty quark masses only contribute via higher order $\alpha_s^2$ virtual loops, whose effects $\delta_c^{(2)} \simeq 0.4 \times 10^{-3}$ [21] are negligible, while the dimension-four condensate only contributes through radiative corrections due to the particular $s$-structure of $R_\tau$ and the Cauchy theorem. Higher dimension operators of dimension $D$, including instantons effects are suppressed by $1/M_D^2$ and are clearly suppressed. Moreover, recent experimental measurements [10, 13] of the dimension-four, -six and -eight condensates are consistent with the QCD spectral sum rules estimates used to get the theoretical estimate.
in Eq. (12) and which indicate the internal consistency of the SVZ-approach. The sum of the previous nonperturbative effects has been also measured by the ALEPH group [3]:

\[ \delta_{SVZ} \equiv \sum_{D=4,6,...} \delta^{(D)} \simeq (3 \pm 5) \times 10^{-3}, \] (13)

which confirms the smallness of the nonperturbative contributions estimated previously. This tiny nonperturbative effect, which is of the same size as the \( \alpha_s^3 \) perturbative one, then permits the extraction of \( \alpha_s \) with a high accuracy from \( R_\tau \). Comparatively to the other low-energy and deep inelastic processes used to extract \( \alpha_s \) [22, 11], tau decays is the only place (at present), where the different sources of theoretical contributions have been discussed in detail and pushed so far (perturbatively and nonperturbatively), such that one can have a solid control of different theoretical errors.

### 3 THE VALUE OF \( \alpha_s \)

Using the different corrections discussed in detail in the previous works and summarized in the previous section, and using the recent PDG94 [11] published experimental value quoted previously, one obtains from the inclusive mode the value:

\[ \alpha_s(M_\tau) = 0.33 \pm 0.03, \] (14)

as can be deduced from Table 1 [4]. The determination from the recent preliminary data gives:

\[
\begin{align*}
\alpha_s(M_\tau) & = 0.355 \pm 0.021 & \text{ALEPH} \\
& = 0.309 \pm 0.024 & \text{CLEO II},
\end{align*}
\] (15)

with a preliminary average consistent with the PDG94 value. We shall adopt in the following discussion the PDG94 value. One can also extract the value of \( \alpha_s \) from the separate axial-vector \( R_{\tau,A} \) and vector \( R_{\tau,V} \) channels and from the sum of the exclusive modes \( R_{\tau,excl} \equiv R_{\tau,A} + R_{\tau,V} + R_{\tau,S} \), where \( R_{\tau,S} \) is the sum of the Cabibbo suppressed channel. The compilations of these rates have been done in [4], while their updated values are given in Table 2 from [9]. It is interesting to notice that these values are about the same as the ones from the latest data [7]. We have used here \( R_{\tau,S} = 0.145 \pm 0.020 \) from [9] which agree within the error with the recent data 0.16±0.02 of the direct sum of the exclusive modes [23]. We have retained in the average the most accurate error from \( R_{\tau} \). One should notice that the relative inaccuracy of \( \alpha_s \) from the vector and axial-vector channels is mainly due to the larger effects of the nonperturbative \( D = 6 \) condensates in these channels, which are smaller for \( R_{\tau} \).

The previous precision in the determination of \( \alpha_s \) has been possible by the very inclusive nature of the \( \tau \)-decay process and by the fact that \( M_\tau \) is in compromise energy region where the process is low enough in energy for being very sensitive to \( \alpha_s \) but still high enough for being less sensitive to the nonperturbative terms which contribute as high powers in \( 1/M_\tau \), and which are exceptionnally small due to the particular \( s \)-structure of the
Table 2: Values of $\alpha_s$ from different observables in $\tau$-decays.

| Observables | $\alpha_s(M_\tau^2)$ |
|-------------|----------------------|
| $R_\tau = 3.56 \pm 0.03$ | $0.33 \pm 0.03$ |
| $R_{\tau,V} = 1.78 \pm 0.03$ | $0.35 \pm 0.05$ |
| $R_{\tau,A} = 1.67 \pm 0.03$ | $0.34 \pm 0.05$ |
| $R_{\tau,\text{excl}} = 3.58 \pm 0.05$ | $0.34 \pm 0.04$ |
| Average | $0.337 \pm 0.030$ |

decay rate and of the Cauchy theorem. In addition, complications related to hadronization...are not present here \[1\]. In this respect $\tau$-decay is an unique process. Running the previous average value of $\alpha_s$ until $M_Z$, one obtains:

$$\alpha_s(M_Z) = 0.121 \pm 0.003.$$ (16)

This result indicates that a modest accuracy at the $\tau$-mass translates into a high-precision measurement at the $Z^0$-scale as the error shrinks as $\alpha_s^2$ when the scale increases. The extraordinary agreement with the direct measurement of $\alpha_s$ at the $Z^0$-mass is a strong indication of the validity of the scale-dependence of the QCD coupling and of the asymptotic freedom property of QCD. More speculatively, it can also indicates that there are no sizable contributions beyond the standard model (gluino,...) in the energy region below the $Z^0$-mass.

## 4 THE DIFFERENT SOURCES OF ERRORS TO $\alpha_s(M_Z)$

Let us now discuss in detail the different theoretical contributions to the error 0.003 of $\alpha_s(M_Z)$:

### 4.1 Running from $M_\tau$ to $M_Z$

Running the determined value of $\alpha_s$ from the $\tau$ to the $Z$-masses and taking into account the crossing of heavy flavours à la Bernreuther and Wetzel \[24\], by setting $\alpha_f = \alpha_s^{f-1}$, when one crosses the threshold of the $f$-th quark, one induces an error of 0.0008. We have

\[1\] Tau decay is a lucky process as stated by G. Veneziano.
used the recent determination of the running $b$ and $c$ quark masses in [25]:

$$m_b(4.62 \text{ GeV}) = 4.23^{+0.03}_{-0.04} \pm 0.02 \text{ GeV}$$

$$m_c(1.42 \text{ GeV}) = 1.23^{+0.02}_{-0.04} \pm 0.03 \text{ GeV},$$

(17)

from the bottomium and charmonium sum rules. However, a more conservative estimate [9, 24, 26], using quark mass values with errors as large as $\pm 0.3 \text{ GeV}$, leads to an error on $\alpha_s(M_Z)$ of 0.001, indicating that the error $\pm 0.002$ quoted in [27] is unrealistic.

4.2 Quark masses and nonperturbative effects

The error induced by the nonperturbative effects is dominated by the one due to the dimension-6 condensates. With the previous value of $\delta^{(2)}$ and $\delta_{SVZ}$ estimated in Eq. (12), one induces an error of 0.0008 in $\alpha_s(M_Z)$.

4.3 RS- and $\mu$-dependence

In [3], Pich and Le Diberder have shown that the perturbative QCD series is more convergent than the original one in [2], if one uses the expansion in terms of the contour coupling $A^{(n)}$:

$$A^{(n)}(a_\zeta) = \frac{1}{2i\pi} \oint_{|s|=M_T^2} \frac{ds}{s} \left(1 - 2\frac{s}{M_T^2} + 2\frac{s^3}{M_T^6} - \frac{s^4}{M_T^8}\right) a^n_\zeta(s),$$

(18)

instead of the usual QCD coupling $a_\zeta(s) \equiv \alpha_s(\mu)/\pi$, where $\mu \equiv \zeta M_T$, is the introduced subtraction scale. In terms of this coupling, the perturbative series becomes:

$$\delta^{(0)}_{B\!N\!P} \equiv \sum_{n=1} K_n A^{(n)}(a_\zeta)$$

$$\equiv \sum_{n=1} (K_n + g_n) a^n_\zeta,$$

(19)

where $K_n$ are nothing else than the coefficients of $a^n(s)$ of the $D$-function given in Eq. (10), while $g_n$ (for $SU(3)_f$ : $g_1 = 1$, $g_2 = 3.563$ and $g_3 = 19.99$), which depends on the coefficients of the $\beta$-function, are induced by the running of the QCD coupling. One should notice that $g_n$ is much larger than $K_n$, which explains the large scheme- and scale-dependences of the original series expanded in terms of $a^n$. One way to study the Renormalization Scheme (RS) dependence of the result is to move the scheme-dependent coefficient $\beta_3$ of the $\beta$-function around its $\overline{MS}$ value. Using the improved series, one finds that the determined value of $\alpha_s$ is stable versus the variation of $\beta_3$ in the range from 0.5 to 2 times its $\overline{MS}$ value, while the effect of the variation of $\mu$ is almost negligible for $\mu$ larger than $1.2 M_T$. The RS- and $\mu$-dependences induce respectively an error of 0.0005 and 0.0009 to $\alpha_s(M_Z)$ for a given value of $\delta^{(0)}$ of about 0.2.
4.4 Higher-order perturbative effects

In the previous paper [2], the size of the perturbative errors have been estimated from a geometric series estimate of the uncalculated $\alpha_s^4$ coefficient, which leads to the estimate of $\pm 130$ of this coefficient. In another work, this estimate has been improved to be [3]:

$$\delta_4^{(0)} \simeq (78 \pm 25) \left(\frac{\alpha_s}{\pi}\right)^4,$$

(20)

where the first term $g_4 \equiv 78$ has been induced by the running of the previous terms via the Cauchy integral. The error has been estimated by assuming a geometric growth of the coefficients of the $D$-function $(K_4 \approx \pm K_3 (K_3/K_2))$, while it has been multiplied by a factor 2 in [4, 9] in order to be conservative. The previous error has been motivated in order to take into account all unknown higher-order effects including the ones induced by the summation of the QCD series at large order (renormalons, ...). With this conservative error estimate, an error of 0.0014 to $\alpha_s (M_Z)$ is then induced. However, one could further improve [28] the estimate of the $\alpha_s^4$ term of the $D$-function by using the PMS [29] and Effective Charge [30] schemes approach or by measuring it directly from the data [31]. In this way, one obtains respectively the value $K_4 \approx 28$ and $29 \pm 4 \pm 2$, which confirm the previous value $\pm 25$. Therefore, one can write:

$$\delta^{(0)} = \delta^{(0)}_{BNP} + 103 \left(\frac{\alpha_s}{\pi}\right)^4 \pm 2 \times 94 \left(\frac{\alpha_s}{\pi}\right)^5,$$

(21)

where the last coefficient is a bold-guess estimate based again on the geometric growth of the series, which we have multiplied by a factor 2 for a conservative estimate. For a typical value of $\alpha_s$, this error is about 0.4 of the one which we have retained previously, and indicates that the previous error $\pm 50 (\alpha_s/\pi)^4$ is a realistic conservative estimate of the perturbative error. Then, we conclude from our previous discussions that we shall adopt, as a conservative estimate of the uncertainties related to the truncation of the perturbative series including the ones induced by its summation at large order, the value:

$$\Delta \delta^{(0)} \simeq \pm 2 \times (K_4 \approx 25) \left(\frac{\alpha_s}{\pi}\right)^4.$$

(22)

4.5 The sum of the errors within the SVZ-expansion

Adding the previous different sources of errors quadratically, a total value of 0.002 of the theoretical errors for $\alpha_s (M_Z)$ is obtained. The different errors to the predicted value of $R_\tau$ for various values of $\alpha_s$ using the previous estimates of the errors are given in Table 1.

5 SOME SPECULATIVE SOURCES OF ERRORS

Now, let me discuss some other possible exotic sources of errors not included into the SVZ-expansion, and, at the same time, let me answer some (unjustified) criticisms raised in the literature. Here, one should notice that, in contrast with the previous true quantitative
estimate of the errors, the discussions are quite speculative and very qualitative. We shall argue that our conservative estimate of the errors in Eq. (22) contains already the following exotic errors.

5.1 IR and UV renormalons

Renormalons effects are associated to the insertion of n bubbles of quark loops into a gluon line exchanged in the $\mathcal{D}$-function given in Eq. (9) built from the quark current. It is well-known that they induce a n! growth into the perturbative series. This disease can be (in principle) cured by working with the Borel transform $\tilde{\mathcal{D}}$ of the correlator $\mathcal{D}(s)$:

$$\mathcal{D}(a \equiv \alpha_s/\pi) = \int_0^\infty db \, \tilde{\mathcal{D}}(b) \, \exp(-b/a),$$

which possesses an explicit 1/n! suppression factor. However, the life is not so simple as $\tilde{\mathcal{D}}$ develops singularities at $b = 2k/(-\beta_1)$ in the real axis, which make the integral ill-defined. The infrared (IR) renormalons which correspond to the singularities at $k = +2, +3, ...$, are generated by the low-energy behaviour of the diagrams, and can be absorbed into the definitions of the QCD condensates \[32\] - \[34\]. It has been also proved \[33\] that there cannot be a $k = +1$ singularity and then, no 1/s-ambiguity can be generated by the IR renormalons; that is mainly related to the absence of the $D = 2$ gauge invariant condensates in QCD. The ultraviolet (UV) renormalon singularities corresponding to $k = -1, -2, ...$, are generated by the high-energy behaviour of the virtual momenta and leads to a Borel-summable series. After a Borel-sum, they cannot limit the applicability of perturbation theory \[32, 33\], though they can induce an uncertainty in the truncated perturbative series when the Borel-sum is not done. Their contributions are dominated by the singularity at $k = -1$, which are largely renormalization-scheme dependent. effects not The existence of this effect is still controversial, as according to \[32, 33\], this effect should not be present. However, if one takes for granted the validity of the theoretical estimate obtained in the limit of infinite numbers of flavours and if one considers the result from one exchanged gluon-chain with large numbers of fermion blobs, one would expect \[36\]:

$$\delta_{UV} \sim A(\mu) \sqrt{\alpha_s(\mu)} \left( \frac{\Lambda M_\tau}{\mu^2} \right)^2,$$

where $\mu$ is the subtraction point and $A$ is a renormalization scheme-dependent coefficient. The result indicates that for $\mu$ larger than $M_\tau$, the effects become negligible. Indeed, rigorously, this new term, which is $\mu$-dependent, should also be taken into account in the minimization of the $\mu$-dependence of the QCD perturbative series discussed previously by \[3\]. However, it is incorrect to say \[27\] that the result of \[3\] is false. Indeed, the effect of this UV term is only relevant in the region of $\mu$ smaller than $M_\tau$, where the stability of the perturbative series in the change of $\mu$ is not yet reached. In fact, for $\mu$ larger than $M_\tau$, the UV-renormalon effect obtained in this way goes quickly to 0 like 1/$\mu^4$, so that it cannot disturb the stability analysis in $\mu$ performed by \[3\]. Moreover, one can also argue

\[2\] However, a recent analysis \[37\] has shown that the effects of two chains of gluons can be of the same order as the one due to one gluon-chain This feature invalidates the previous result within a one gluon-chain approximation.
that for arbitrary values of $\mu^2 = s$, the effect of this term vanishes to leading order, like any other dimension-four terms, as a consequence of the Cauchy theorem and of the $s$-structure of the decay rate. Therefore, it can never introduce a large error in $R_\tau$.

5.2 $1/M_\tau^2$ terms

[27] also argues that this $\mu$-dependence can disappear when new terms are added in the evaluation of the UV renormalon effects, in such a way that the previous contribution transforms into:

$$\delta_2 \sim c \left( \frac{\Lambda}{M_\tau} \right)^2,$$

(25)

where $c$ is an unknown coefficient that can eventually contain a log-dependence. However, this argument is not quite true as the transition from Eq. (24) to Eq. (25) is just a reflection of the renormalization scheme dependence of this UV renormalon effect, which manifests in Eq. (25) through $\Lambda$. competition between the Alternatively, one could proceed phenomenologically, by including an ad hoc $C_2/M_\tau^2$ term in the SVZ-expansion, but still, with the caution that this term may not exist at all. Then, by fitting it, independently of the tau decay process, for instance, from the $e^+e^- \rightarrow I = 1$ hadrons data on the total cross-section, as done, for the first time, in [39], one can deduce the quite inaccurate optimal estimate:

$$|C_2| \simeq (9 \pm 4) \times 10^{-3} \text{ GeV}^2,$$

(26)

from the stability analysis of the Laplace sum rule[3], while the Finite Energy Sum Rule gives a very inaccurate value, which we can translate into the conservative upper bound:

$$|C_2| \leq (0.374 \text{ GeV})^2,$$

(27)

The unusual inaccuracy of the sum rules analysis, is mainly due to the sensitivity of the result on the region around 1.4-2 GeV, where the data are quite bad. The previous estimate leads to:

$$\delta^{(2)} \simeq (18 \pm 7) \times 10^{-3}.$$

(28)

The error in this result can induce an uncertainty of about 0.002 for $\alpha_s(M_Z)$. Better data or an independent analysis from other channels than $e^+e^-$ are of course needed for improving the previous value of $C_2$. One should notice that the ALEPH group [6] has also used the tau-decay data in order to fix this term, using spectral moments which are much better that ordinary FESR, but the analysis was not conclusive. The effect in Eq. (28) is still small compared with the one proposed in [27]:

$$\delta_2 \simeq \frac{1}{(3 - 5)} \left( \frac{\Lambda \equiv 0.5 \text{ GeV}}{M_\tau} \right)^2,$$

(29)

3For those who are not familiar with the sum rule analysis, the stability corresponds to the minimum sensitivity of the result with respect to the variation of the unphysical sum rule variables. The meaning of the stability procedure has been tested in quantum mechanics by applying the sum rule for the harmonic oscillator eigenvalue equation [40] (see also [11] for a review), where both the exact and approximate solutions are known. The smooth plateau is only obtained for the exact solution, while for the approximate series, the optimal solution corresponds to the minimum of a parabola or to an inflexion point.
which leads to an overestimated uncertainty of 0.005-0.008 to $\alpha_s(M_Z)$. \cite{27} reinforces his result by comparing it with an alternative derivation of the error from the truncation of the perturbative series at the calculated $\alpha_s^3$-term, and by advocating that the error of the asymptotic series is given by the last known term. In this way, he finds:

$$\Delta \delta^{(0)} \simeq 10 \left( \frac{\alpha_s}{\pi} \right)^3,$$

leading to an error of about 0.005 to $\alpha_s(M_Z)$. There are two amusing issues here: --It is interesting to notice that \cite{27} recognizes that the errors from the $1/M^2 \tau$ in Eq. (29) and from the truncation of the asymptotic series in Eq. (31) are of the same origin, so that one should not add them in the total errors. --The coefficient of the $\alpha_s^3$-term given in Eq. (30) is again an arbitrary overestimate compared to the calculated $K_3 = 6.3711$-coefficient of the $D$-function relevant for the discussion. Indeed, \cite{7} has used a quite similar argument, based on the fact that by definition the UV renormalon effect is smaller than the calculated perturbative error, and that the asymptotic behaviour of the perturbative series is reached at order $\alpha_s^3$, in order to give an estimate of the maximal error due to the truncation of the perturbative series. In this way, he obtains:

$$\Delta \delta^{(0)} \leq K_3 a^3 \simeq 0.010.$$

This result is again consistent with the previous one in Eq. (28). Its consistency with the estimate of the errors in Eqs (21) and (22) indicates that, not only, our conservative error in Eq. (22) is realistic, but also, the asymptotic series may already be reached at the $\alpha_s^3$ order, though we do not yet have alternating signs in the coefficient of the series at this order.

5.3 Freezing mechanism

In \cite{38}, it has been suggested that the freezing of the coupling constant at low scale can also induce a $1/M^2 \tau$ contribution, which can be simply seen by parametrizing the QCD coupling as:

$$\left. \frac{\alpha_s}{\pi} \right|_C \simeq \frac{2}{-\beta_1 \log (M^2 \tau + C^2)/\Lambda^2},$$

where $C$ is an unknown coefficient expected to be smaller than the hadronic scale $M_\rho$. Expanding this formula, indeed leads to:

$$\left. \frac{\alpha_s}{\pi} \right|_C \simeq \frac{\alpha_s}{\pi} \left\{ 1 - \frac{C^2}{M^2 \tau} \left( \frac{-\beta_1}{2} \right) \frac{\alpha_s}{\pi} \right\},$$

which for $C \leq M_\rho$ gives:

$$\delta_C \leq 5 \times 10^{-3},$$

which is quite small. Again, we notice that the quoted error in \cite{27} is a factor two larger than the previous one.
5.4 Sum of the speculative errors

We conclude from our previous discussions that the eventual dominant contribution from the $1/M^2_\tau$ terms and the estimate of the perturbative error based on the truncation of the series are of the same origin and should not be added in the total error. Our conservative estimate of the perturbative error in Eq. (22) compares quite well with the phenomenological error in Eq. (28), which reinforces the previous argument. However, though, *grosso-modo*, we agree with [27] on the philosophical interpretation of the UV renormalon, it appears to us that his handling of the different numerical errors is always an overestimate of the *true* error by about a factor 2-3.

5.5 Hadronic uncertainties and $e^+e^-$ stability test

Let us finally comment on the criticisms raised by [42] about the uncertainties introduced by the hadronic states in connection of the application of QCD near the real axis. Our claim is that these effects, which could, naively, be important are zero, thanks to the presence of the threshold factor $(1 - s/M^2_\tau)^2$, which introduces a double zero at $s = M^2_\tau$. We have tested the validity of our argument by varying the $\tau$-mass. In so doing, we express the vector channel decay rate $R_{\tau,V}$ in terms of the $e^+e^- \rightarrow I=1$ hadrons total cross-section using CVC. Though these data are at present less accurate than the one obtained directly from a measurement of $R_{\tau,V}$, they are interesting as they permit to cover a large range of $M_\tau$-values. For $M_\tau$ between 1.4–2 GeV, the agreement between the theoretical curves and the data on $R_{\tau,V}$ using the previous values of $\alpha_s$ are excellent. For smaller values of $M_\tau$, there is a large departure from the theoretical and experimental curves, indicating that we approach the nonperturbative regime, where higher-dimensions condensates not retained in the OPE show up. For larger values of $M_\tau$, the agreement are still good, but the data are inaccurate and contradict with each other which render the analysis in this region unconclusive. Indeed, as already emphasized in [4, 9], the author in Ref. [42] uses ordinary Finite Energy Sum Rules, (i.e. without threshold factors and so are more inaccurate than the one governing tau decays), in the region above 1.8 GeV, in order to test the stability of our determination of $\alpha_s$, from which he found that the $\alpha_s$-value can move by a factor more than 2. However, by examining carefully his analysis, one can simply realize that the test of [42] is more a test of his approach itself, but is irrelevant for testing the determination of $\alpha_s$ from tau decays.

6 CONCLUSION

We have discussed in detail the different sources of theoretical errors in the determination of $\alpha_s$ from tau decays starting from a typical value of $R_\tau = 3.56 \pm 0.03$. These errors have been classified as follows for $\alpha_s(M_Z)$:

- 0.0003 electroweak
- 0.0010 $M_\tau \rightarrow M_Z$
- 0.0005 $RS$ -dependence

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Adding them in quadrature, we obtain the total _theoretical_ error:

\[ \Delta \alpha_s(M_Z) \simeq 0.0023, \]  

while by replacing the perturbative error 0.0014 from the \( \alpha_s^4 \)-term by the maximal value 0.0022 induced by either the \( \alpha_s^3 \)-term in Eq. (31) or the _phenomenological_ one in Eq. (28), one expects to have the phenomenological upper bound:

\[ \Delta \alpha_s(M_Z) \leq 0.0028. \]  

We remind that the error for \( \alpha_s(M_\tau) \) is approximatively ten times the one of \( \alpha_s(M_Z) \). Our previous detailed analysis has shown that the error of 0.006-0.008 _wished_ by [27] is very unrealistic as it assumes a huge \( 1/M_\tau^2 \) contribution, where its numerical derivation is very inaccurate. Moreover, almost all of the theoretical errors given in [27] have been overestimated by at a factor 2 to 3. However, it is amusing to observe that since his first _attack_ in [28], his estimate of the error has decreased by a factor of about 2, but still remains an overestimate of the _true quantitative error_ estimated in detail here.

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