Current Modulation Induced Stability in Laser Diode Under High Optical Feedback Strength

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ABSTRACT The back-reflection of emitted laser beam (optical feedback, also know as selfmixing) from various external interfaces are sufficient to cause instability, and prohibiting its use in various fields such as communication, spectroscopy, imaging to name a few. So it is desirable to study the laser dynamics and the conditions causing it to be stable in spite of strong optical feedback. With the aid of mathematical formulation, simulation and backed by experimental evidences, it is demonstrated that the frequency deviation of the laser emission due to current (intensity) modulation alters the dynamic state and boundary conditions of the system such that even under large optical feedback strength, the laser may attain stability and retain single modal state. The frequency deviation resulting from former is shown to modify the phase of the system in opposite direction to that induced by the later, showing that there exists an optimal modulation current which compensates the effect of optical feedback and may be used to retain the laser in single modal stationary state. The method thus provides a methodology to avoid optical feedback-induced instability in semiconductor lasers by using the proper amplitude of current (intensity) modulation.

INDEX TERMS Optical feedback, self-mixing interferometry, frequency coefficient, intensity modulation.

I. INTRODUCTION

Semiconductor lasers are use in diverse fields from communication to spectroscopy to medical imaging, surgery and health to name a few. Even though it used in wide range of applications, it suffers sufficient degradation in performance; instability and chaos in case of external perturbation such as optical feedback (OF), current modulation causing intensity modulation (IM), and/or combination both. These phenomenons have shown that laser maintain multiple stable states depending upon the feedback strength ($C$). OF, in which the fraction of electromagnetic radiation emitted from laser is re-injected to its cavity, has gained considerable attention because it can cause a very rich dynamical behaviour which may modify the laser properties significantly. Authors in [1], [2] demonstrated comprehensive analysis of the time-delayed feedback control of steady states for large delay. Tkach and Chaplyvy [3] classified laser diodes subjected to optical feedback in five regimes. Lasers in regimes I-II is sensitive to the feedback strength and the distance; in regime III it is independent upon the distance, however is sensitive to the feedback strength; regime IV corresponds to coherence collapse and regime V is insensitive to external perturbation. So the model presents here is applicable to regime I-III. Further, Donati in [4] revised the regimes depending upon the electric field and the feedback factor. Depending upon the latter two factors and external distance of perturbation, the author defined the region from quasi-unperturbed, to period 1 oscillation, multi periodicity to chaos.

The properties of the laser depend upon the feedback level, characterized using the feedback factor $C$, and the phase of the returned field ($\phi_0$) [5]. These two factors determine the operational conditions of the laser, its spectral characteristics,
the number of allowed modes and ultimately the stability of the emission [6]. On one hand, OF introduces improvements in the performance of the laser (as a decrease in linewidth), while, on the other hand, it may severely degrade the laser performance (by e.g. linewidth broadening and coherence collapse) depending upon the C value [7]. A better understanding of the dynamical behaviour of the semiconductor laser would be beneficial to avoid, or take advantage of such instabilities in real world applications where OF is involved, such as displacement, distance and velocity measurement [8]–[16], imaging [17], [18], surface profiling [19], strain measurement [20], or characterization of the reflectivity of materials [21], increase its sensitivity [22]. A comprehensive tutorial including theory, applications and future road map on OF is presented in [8], [23].

In parallel, OF combined with the current modulation of the laser has also shown a rich phenomenology, including quasi periodicity, period doubling, tripling and chaotic output depending upon the modulation frequency ($f_m$) and amplitude of current modulation ($A_m$) [24]–[26]. Further, the modulated laser under feedback has been shown to present interesting phenomena such as enhancement of the modulation bandwidth [27]–[30], modal instability [30]–[32], mode-locked pulses [33]–[38], frequency locked state [39], or low frequency fluctuations [40], [41].

Among the various types of OF, the two main configuration employed in this paper are (a) C-OF where the external target is vibrated and injection current is not modulated (b) CWFM-OF, where the external target is kept stationary and the injection current is modulated [8], [17], [42]. In this paper, we introduce the use of CWFM-OF to induce laser stability even for large C values i.e. high feedback strength. The current modulation results in modulation of carrier density and photon density both. The modulation of carrier density modulates the refractive index of the material forming the laser cavity (plasma loading), that in turn changes the emission frequency [43], [44]. While IM follows the light-current (L-I) curve of laser, Frequency modulation (FM) varies linearly with the modulation current. FM results in a frequency spectrum that consists of the central modulation frequency ($f_m$) and a series of side-bands separated by $f_m$. The amplitudes of the side-band pairs are given by a Bessel function of the first kind $J_0(β)$, where $β$ is the frequency modulation index defined by the ratio of maximum frequency deviation ($Δf$) to the modulation frequency i.e. $β = Δf / f_m$. Since $Δf = Ω/2A_m$, the amplitude of such side-bands depends upon the amplitude of the current modulation ($A_m$), and the frequency coefficient of laser ($Ω/2f_m$) [45].

In addition to IM, modulation current also the temperature of the material forming the active layer of laser. Since, refractive index of the material that form the cavity of the laser changes with the temperature, there is frequency modulation due to thermal affect as well. In addition, the changes in temperature change the bandgap of the junction causing changes in the gain profile and emission frequency. This change is different for different type of laser (e.g. AlGaAs has different shift as compared to InGaAs and so forth). Fukuda et al. [46] has already studied various physical processes that affect the emission frequency due to heating. Among the other factors, they concluded the following that - (a) Joule’s heating and (b) plasma loading (c) type of modulating waveform (either pulsed or other) and (d) measurement time affect the emission frequency. In the context presented in this paper in regards to the continuous wave current modulation,

- **Plasma Loading:** It is not applicable as this affect is frequency dependent observed about frequency above 1 GHz [46]
- **Type of Waveform:** Pulsed waveform (of short duration) cause significant changes in temperature instantly and hence the emission frequency. However, in our case, due to triangular waveform, the abrupt changes in temperature is also not the case here [46]. The change in temperature with the continuous wave and compared against the pulsed wave is comprehensively presented by Agnew et al. in [47]. It is demonstrated that the latter induce greater heating effect and thus greater frequency deviation as compared to former.

However, Joule’s heating is applicable to the given context of OF. In this context, it was observed that temperature in the active layer depends on the heat conductance between the active layer and package. They quickly spreads within the laser chip and then gradually dissipate to the surrounding region. Roumy et al. in [48] presents a detailed investigation on how the controlled and joint effect of amplitude modulation and temperature affect the laser behaviour such as threshold current, slope efficiency, and output power subjected to in presence of OF. However the dynamic effect of the heat introduced due to current modulation was not investigated. Agnew et al. in [47] carried the detailed investigation on variation in temperature and emission frequency due to thermal effect; Bertling in [49] describes variation of temperature with measurement time; and the thermal coefficient variations with the monitoring time interval after changing the magnitude of injected current is described in [46].

Based on these extensive evidences, continuous wave current modulation, advanced laser package to dissipate heat and the fact that temperature stabilizer is used in most of the experimental setup, thermal heating is unlikely to effect the measurements. The main contributions of this research work are listed below.

- Demonstrate by the aid of mathematical formulation and simulations that C act as frequency deviation parameter in the case of CWFM-OF.
- Detailed comparison between classical optical feedback C-OF2 [6] and CWFM-OF3 [9] is made. Based on several comparisons, it is demonstrated that the latter is less susceptible to C. This is because of the current

2 in the case of C-OF, the external target is swept and the current to the laser is constant
3 in the case of CWFM-OF, the external target is stationary and current to the laser is modulated
modulation induced frequency deviation (explained by term $\Omega_f A_m$ in Eq. (2), Sec. II) act in opposite direction to that of C-OF. It is also demonstrated that a particular amplitude of modulating current pulls back the laser to single mode state, even in presence of strong feedback.

- Since the frequency deviation induced due to current modulation is proportional to $\Omega_f$, and is the main factor in pulling back the laser to mono modal state, an experiment is devised to measure it experimentally.
- Based on the experimentally obtained $\Omega_f$, an experiment is devised to demonstrate current modulation induced stability in laser diode under high optical feedback strength. Unlike using Lithium Niobate ($\text{LiNbO}_3$) as the external modulator to modulate the intensity of laser emission, we use direct modulation of current giving rise to intensity modulation (IM) of laser emission thus making the set up compact, lightweight, less expensive and above all limiting the stability of the laser to it’s inherent properties and not dependent upon external devices. Hence a novel technique is proposed and demonstrated that the laser can be driven back to mono mode state in presence of strong feedback, opening the door to various applications where otherwise it would not have been possible.

The paper is organized as follows. In Sec. II, the response of laser under the different types of OF is described, including (a) classical optical feedback (C-OF), where the external mirror is swept and the current to laser is constant; and (b) continuous wave frequency modulated optical feedback (CWFM-OF) where the external mirror is kept stationary and current to the laser is modulated. Section II-A shows how $C$ may be understood as a frequency deviation parameter in both cases. However, due to the different boundary conditions for both cases, the role of $C$ in determining the number of solutions in the cavity behaves in a different manner. In Sec. II-B, the effect resulting by the interaction of the frequency deviation introduced due to $C$ and that introduced by the current modulation amplitude $A_m$ is demonstrated. It is shown that they both act in opposite direction, helping the laser to be in single mode state even in presence of large $C$ values. The optimal relationships of those parameters in order to retain the laser in single mode state are investigated for a number of cases. The frequency modulation coefficient of laser ($\Omega_f$), is shown to be one of the key factors in determining the laser stability in presence of high $C$, so it is measured experimentally in Sec. III(III-A) followed by Sec. III-B, that explains the experimental evidences to demonstrate that the frequency deviation caused by the introduction of laser current modulation pulls back the laser from multi modal state to single mode state even under large $C$ values. Finally the paper ends with a conclusion and discussion in Sec. IV.

**II. THEORY AND NUMERICAL SIMULATIONS**

Different approaches have been used in order to explain the dynamics of semiconductor lasers (SLs) in presence of OF, such as the delayed difference equation [50]–[52] or the compound cavity model [53], Lang and Kobayashi (L-K) model [6]. Refined investigation is followed on how the stationary state of the laser with feedback undergoes changes, loses its stability, and with growing feedback gives rise to more complicated behaviour again pushing the laser back to its stable monomode state by introducing the current modulation. The detailed mathematical, and theoretical description of OF is explained in [4], [22], [23], [48], [54], [55], [55]. The phase equations of the laser under C-OF and CWFM-OF are given by Eq. (1) and Eq. (2) respectively [17], [42].

\[
\begin{align*}
\phi_v &= 2\pi(f - f_0)\tau_{\text{ext}} + C\sin(2\pi(f - f_0)\tau_{\text{ext}} + 2\pi f_0 \tau_{\text{ext}} + \tan^{-1}\alpha), \\
\phi_m &= 2\pi(f - f_0)\tau_{\text{ext}} + C\sin(2\pi(f - (f_0 - \Omega_f A_m))\tau_{\text{ext}} + 2\pi f_0 \tau_{\text{ext}} + \tan^{-1}\alpha),
\end{align*}
\]

where $f$, $f_0$, $\tau_{\text{ext}}$, $\alpha$, $A_m$ are the emission frequency of the laser under feedback, the emission frequency of the standalone laser, the external round trip time, the linewidth enhancement factor, and the peak-to-peak amplitude of modulation current, respectively. For the case of C-OF, Eq. (1) defines a nonlinear equation, whose state is dependent upon the feedback parameter $C$ and the initial phase ($\phi_0 = 2\pi f_0 \tau_{\text{ext}}$) [56]. For $C < 1$, there is only one solution to Eq. (1) and only one mode exists in the cavity, but as the value of $C$ becomes greater than 1, multiple solutions to Eq. (1) may appear and the laser behaves as a multimodal system. In the case of CWFM-OF, defined by Eq. (2), the addition of the modulation current changes the boundary conditions and so the modal behaviour of the laser [17], [42].

In both OF cases, the excess phase equation in presence of optical feedback is a non-linear transcendental equation which cannot be solved analytically. Kliese et al. in [55] provides in depth analysis of OF signal for arbitrary feedback strength. In the case of C-OF (Eq. (1)), the phase of the laser is dependent upon parameters $C$ and $\tau_{\text{ext}} (L_{\text{ext}})$. In the case of CWFM-OF (Eq. (2)), we see how the phase of the laser which determines the number of solutions of the equation (modes) is dependent upon $C$, $\Omega_f$, $A_m$ and $\tau_{\text{ext}} (L_{\text{ext}})$. Numerical methods to solve Eq. (1) and Eq. (2) will be used in coming sections.

**A. FEEDBACK STRENGTH AS A FREQUENCY DEVIATION PARAMETER FOR THE CWFM-OF SIGNAL**

Heil et al. in [57] already demonstrated theoretically and experimentally that the frequency deviation of the field emitted by the laser under C-OF is proportional to the $C$ parameter. Similarly, Taimre in [54] also demonstrated that $C$ acts as frequency modulation parameter. In regards to CWFM-OF, in this paper, it is shown that the beating of the time delayed optical field scattered from a fixed, remote target, and the standing wave inside the cavity of the laser also produces a new beat frequency. The magnitude of the deviation from the frequency of the standalone laser is shown to be proportional to $C$ described in Eq. (2). Eq. (2) may be solved numeri-

\[\text{Here (pp) is dropped for brevity, but continued elsewhere.}\]
cases (CWFM-OF and C-OF) is studied next, so comparisons between them regarding how they behave under changing feedback strength $C$, amplitude of current modulation ($A_m$) and external distance ($L_{ext}$) are presented.

The number of modes in the cavity in the case of CWFM-OF is numerically obtained by simulation from Eq. (2) by equating the phase $\phi_v$ to zero. First $L_{ext}$ and $A_m(pp)$ are set to 0.45 m and 0.14 mA respectively. The choice of these parameters is compatible with the experimental conditions and the specification of the laser. Then $C$ is varied from $C = 2$ (Fig. 2 (a)) to $C = 7$ (Fig. 2 (d)) to see the phase profile, the number of modes and its comparison with the C-OF case under equivalent conditions. It is observed how for these conditions the number of modes for C-OF is larger than one ($N_m > 1$) when $C > 1$ (Fig. 2 (a)). However, the CWFM-OF case has a wider range of $C$ values for which a single mode state is attained, enabling it into feedback levels as large as $C = 3$ (Fig. 2 (c)). Further increasing $C$ enables multiple modes for CWFM-OF also (Fig. 2(c-d)). If, under this multimodal state, $A_m(pp)$ is then increased from 0.14 mA to 0.30 mA, under equivalent conditions to the ones in Fig. 2 (c) (for e.g. $C = 6$), the laser is pulled back from a multimode to a single mode state again (shown in Fig. 3). This is associated to the negative frequency coefficient of laser, $\Omega_f$ value of the laser, i.e. to the fact that increasing the current modulation amplitude decreases the emission frequency, acting in opposite direction to $C$ (where, increasing $C$, increases the frequency deviation as shown in Fig. 1).

Hence, CWFM-OF and feedback level act as a “push-pull” system, and there exists at least a theoretical value of $A_m$ for each value of $C$ that theoretically compensates each other’s effects, and retains the laser in single mode state. In practice, however, the amplitude of current modulation becomes limited by the specifications of the laser obtained from the manufacturer. As an example, Fig. 4 (a) shows that at $C = 2$, $A_m(pp)$ of 0.14 mA, the laser is in a single mode state; a deviation of $A_m(pp)$ value to 0.03 mA leaves
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FIGURE 3. Modal behaviour of laser under OF. The laser exhibits multiple modes at \( C = 6 \) for C-OF (solid). When current modulation is added, the laser is pulled back to single mode state by choosing the optimal modulation amplitude \( A_m(pp) = 0.3 \) mA (dash).

FIGURE 4. Relationship between \( A_m \) and \( C \). The amplitude of current modulation required to bring the laser into single mode state is dependent upon feedback strength.

FIGURE 5. Number of modes in CWFM-OF case as a function of feedback strength (C), amplitude of modulating current (\( A_m \)) and external cavity length (\( L_{ext} \)).

it in multi modal state. Alternately, at \( C = 5 \), deviating \( A_m(pp) \) from 0.14 mA to 0.2 mA changes from multi modal to single modal state (Fig. 4 (b)). It is to be noted that the same value of \( A_m(pp) = 0.14 \) mA causes the laser to attain single or multimode states depending upon the feedback level \( C \) present in the experiment. Simulations were carried out in MATLAB to study the detailed effect of \( L_{ext} \), \( C \) and \( A_m \) on the number of modes \( N_m \), and results are presented in Fig. 5.

Several applications of lasers under OF require the laser to be in a single mode state to deliver proper results, e.g. its use as sensors in optical feedback interferometry [10], [58]. One of the main applications of this push-pull behaviour is to retain the laser in a single mode state under large feedback strength values, using a parameter of the laser which may be controlled externally. It was thus interesting to analyze in detail the \( A_m(pp) \) values required to retain the laser in single mode state under large feedback strength. Fig. 6 shows the relationship between them for the cases of different \( L_{ext} \) values ranging from 0.4 to 0.8 m. Points show the value for which a given \( C \) value is compensated by a given \( A_m(pp) \) value to produce a single mode condition. From Fig. 6, no clear relationship or pattern appears, as each individual experimental case shows a different trend. For instance, at \( L_{ext} = 0.45 \) m, 0.5 m and 0.55 m, the relationship appears to be linear, while for the \( L_{ext} = 0.80 \) m case it looks stepwise. The optimal value of \( A_m \) required to pull the laser back into single mode state depends upon the combination of \( L_{ext} \) and \( C \). However, in practice, there is a limit on the value of \( A_m(pp) \) determined by the specifications of the laser which limits the maximum feedback strength which may be pulled back.

III. EXPERIMENT

As explained in previous sections, frequency coefficient (\( \Omega_f \)) plays a significant role in pulling the laser back from multi-modal state to single mode state, so at first step, it is desirable to measure it experimentally. Then using this experimental value, the optimal value of amplitude of current modulation is chosen to pull back the laser to single modal state which is explained in following subsections.

A. EXPERIMENTAL CHARACTERIZATION OF THE FREQUENCY COEFFICIENT OF THE LASER

A simple experimental setup based on CWFM-OF was used in order to measure the frequency coefficient of the laser, and
is presented in Fig. 7. A Hitachi HL8337MG AlGaAs laser diode (LD) was used as source. The emission wavelength, measured with an Instrument System’s SPECTRO 320(D) R5 unit was $\lambda = 826.5$ nm. The optical beam emitting from the LD was focused using a Thorlabs lens 352240 (not shown in the block diagram) with focal length of 8 mm and numerical aperture of 0.5 at the target. The target was a piezoelectric linear stage PI-LISA (P-753.3CD) with an embedded capacitive sensor with a resolution of 0.2 nm. The laser was placed at $L_{\text{ext}} = 0.1$ m from the laser. The current injected to the laser was modulated as a triangular waveform using a signal generator, in order to introduce a frequency sweeping effect in the emission of the laser. The number of fringes ($N_f$) formed as ripples in the linear intensity ramp are related to the distance from the laser to the target ($L_{\text{ext}}$) and the frequency modulation coefficient of laser ($\Omega_f$) following [17]

$$L_{\text{ext}} = \frac{\lambda^2 N_f}{2 \Delta \lambda} = \frac{\lambda^2 N_f}{2 A_m(pp) \Omega_f} = \frac{N_f c}{2 \Delta f} = \frac{N_f c}{2 A_m(pp) \Omega_f}, \quad (4)$$

where $\lambda$, $c$, $A_m(pp)$, $\Delta \lambda$ ($\Delta f$), are the emission wavelength, the speed of light in the vacuum, the peak to peak modulation current, and the peak to peak change in wavelength (frequency) respectively. The parameters of the experiment are summarized in Table 1.

Keeping $L_{\text{ext}} = 0.1$ m constant and a peak to peak constant modulation voltage of $V_m(pp) = 700$ mV, changes in modulation frequency $f_m = 1 - 7$ kHz were applied, and the frequency coefficient was computed for each frequency value. To ensure repeatability, the same experiment was performed under equivalent conditions for a different voltage $V_m(pp) = 1$ V (Fig. 8). For illustrative purposes, the case when $f_m = 1$ kHz, $V_m(pp) = 700$ mV is shown in Fig. 9. The CWFM-OF signal has $N_f = 20$ fringes (Fig. 9 (a)). The peak to peak voltage change across the diode was $V_m'(pp) = 278$ mV (Fig. 9 (b)) and peak to peak current change was measured to be $A_m(pp) = 12.63$ mA (Fig. 9 (c)). Using the known values of of $N_f = 20$, $c = 3 \times 10^8$ m/s, $A_m(pp) = 12.63$ mA and $L_{\text{ext}} = 0.1$ m in Eq. (4), the single unknown $\Omega_f$ may be calculated. The computed frequency coefficient $\Omega_f = 0.005$ nm/mA, or equivalently $\Omega_f = -2.2 \text{GHz/mA}$ (negative as the emission frequency decreases with an increase in amplitude of current modulation). The measured value is close to the $-3$ GHz/mA value for AlGaAs lasers reported in [58]. The calculated frequency coefficient of laser ($\Omega_f$) for each case is shown in Fig. 8, showing in almost all experimental cases analyzed a very consistent value equal to 0.005 nm/mA.

Further, the linear relationships of the external parameters of the laser enable a slope-based approach for the confirmation of the measured value of $\Omega_f$, while validating the considered theoretical approach. As $N_f$ can be changed (Eq. (4)) by varying the values of (a) $L_{\text{ext}}$ and (b) $A_m(pp)$, first $A_m(pp)$ is kept constant while $L_{\text{ext}}$ is varied. In this context, the modulation voltage and frequency from the signal generator are fixed to $V_m(pp) = 700$ mV and $f_m = 5$ kHz respectively. This gives a modulation voltage at the laser $V_m'(pp) = 210$ mV corresponding to a modulation current

TABLE 1. Experimental parameters.

| Parameters                   | Value                      |
|------------------------------|----------------------------|
| Distance laser-target ($L_{\text{ext}}$) | 0.1 m                      |
| Modulation frequency ($f_m$)  | 1 - 7 kHz                  |
| Modulation voltage ($V_m(pp)$) | 700 mV (1 V)               |
| Emission wavelength of laser ($\lambda$) | 826.5 nm               |
| Speed of light in air        | $3 \times 10^8$ m/s       |

FIGURE 6. Relationship between $C$ and $A_m$ to pull laser back into single mode state.

FIGURE 7. Experimental setup to measure the frequency coefficient of laser using CWFM-OF. LD - laser diode, PD - photodiode.
A \text{pp}(pp) = 9.54 \text{ mA}. The distance of the external target from the laser \( L_{\text{ext}} \) is varied from 14 to 7 cm. The variation of \( N_f \) with the change in \( L_{\text{ext}} \) at constant \( A_m(pp) \) is shown in Fig. 10 (a). The experimental data shows a linear relationship between them with a slope of 0.8 cm/fringe. Equating the slope of Eq. (4), \( \lambda^2/(2A_m(pp)\Omega_\lambda) = 0.8 \) and placing the value of \( A_m(pp) = 9.54 \text{ mA}, \lambda = 826.5 \times 10^{-7} \text{ cm}, \Omega_\lambda \) is found again to be 0.005 nm/mA. The same result may be reached by varying \( A_m(pp) \) and seeing the changes in \( N_f \) (Eq. (4)), provided that the distance to the external target is kept constant at \( L_{\text{ext}} = 7 \text{ cm} \). Fig. 10 (b) shows the linear relationship between them with a slope of 0.9 mA/fringe. As in the previous approach, using the equation of slope in the model \( \lambda^2/(2L_{\text{ext}}\Omega_\lambda) = 0.9 \), the only unknown quantity \( \Omega_\lambda \) is found to be 0.005 nm/mA, as in the previous experiments. Hence, by different approaches we have confirmed the value of the frequency modulation coefficient (0.005 nm/mA) and the linearity of \( L_{\text{ext}} \) and \( A_m(pp) \) against \( N_f \), which confirms the theoretical description proposed.

At this point, it is worth summarizing the results so far. (a) From Fig. 1, it is confirmed that the emission frequency increases with feedback strength. (b) From the experiment (Fig. 8 and Fig. 10) the frequency coefficient \( \Omega_\lambda \) was calculated to be +0.005 nm/mA. This is to say that wavelength increases with increase in current. The change in wavelength and change in frequency is related as \( \Delta f = -\Delta \lambda c/\lambda^2 \), hence if one of them is positive (in this case wavelength change), other is negative (frequency change). Combining (a) and (b) it is conclude that OF and current modulation act in opposite direction - former cause the emission frequency to increase while latter cause it to decrease. Based on these observations, in the following section, experimental demonstration that they can be used to stabilize the laser even in strong feedback is presented.

**B. EXPERIMENTALLY INDUCED SINGLE MODE STABILITY USING CURRENT MODULATION**

An experimental demonstration of current modulation pulling the laser back to a quasi-stable single mode state, even under large feedback conditions, is presented next. The experimental block diagram is similar to that in Fig. 7, although now the target (piezo) has been placed at a distance of 8 cm from the laser. The system is perturbed by setting the piezo target to vibration by applying a triangular voltage of \( V_{\text{c}}(pp) = 0.326 \text{ V}, \) resulting in pp displacement of 1.24 \( \mu \text{m} (3\lambda/2) \). Under these conditions, the C-OF signal is acquired remotely using LabVIEW so that the experiment set-up is unaffected by the data acquisition. (Fig. 11(a)). In the case of C-OF, each fringe corresponds to a displacement of \( \lambda/2 \) so the optical feedback signal (OFS) has three fringes in one period as expected (Fig. 11(a)) [10]. From the triangular fringes in C-OF signal waveform and the non-perceptible fringe-loss condition, it is concluded that \( C \) is close to one [10]. The fringe loss criteria [59] is then taken into account i.e. the strong feedback regime of the laser is determined by the presence of fringe loss in the output signal. The position of lens is tuned to focus the optical power from the laser onto a smaller spot on the target, in order to increase \( C \), and the corresponding C-OF signal is acquired (Fig. 11(b)). It is
observed how increasing $C$ causes fringe loss, as expected, pushing the laser into multi modal behaviour and hysteresis due to a larger feedback value [59]. Further, the lens position may be again adjusted to cause more fringe loss, consequently increasing $C$ value (Fig. 11(c)). It may be observed how the increase of feedback level brings on that no fringes are visible and hence that the laser is under strong feedback conditions [59]. While keeping the experimental setup in exactly the same conditions of Fig. 11(c), a triangular modulation voltage $V_{mod}(pp) = 700$ mV resulting in a current of $A_{lm}(pp) = 12.63$ mA is applied to the LD. The waveform for this case is shown in Fig. 11(d). It is observed that the laser, formerly in strong feedback regime, is now in a weak feedback regime, as the right number of sinusoidal fringes is present over the ramp [17], [60]. This correct number of fringes (14) over the ramp corresponds to $L_{ext} = 7.5$ cm value, in good agreement with the actual 8 cm value. The feedback level which forced the laser to instability in the case of C-OF has thus been pulled back to quasi stationary state under the same feedback conditions with the introduction of current modulation.

Modulation induced stability can thus have direct significance in attaining stability of laser diode performance under strong optical feedback, thus enabling it to be used even in the case of strong optical feedback. Further, the linewidth of the semiconductor laser is dependent upon the feedback strength. The laser linewidth under feedback ($\nu_f$) relative to the stand alone laser ($\nu_0$) is inversely proportional to $C$, and is given by $\nu_f = \nu_0/[1+C]^2$ [61]. Thus, provided that the laser is stable at high feedback strength, it can potentially be used to reduce line width as well, which has relevant applications in the field of spectroscopy.

IV. CONCLUSION AND DISCUSSION

The dynamics of the semiconductor laser under optical feedback in the two distinct cases of C-OF and CWFM-OF have been studied in depth. It was shown how optical feedback acts as a frequency deviation parameter. In case of larger $C$ values, new frequencies appear in the cavity; the power in each of the frequency components also increases with $C$ and the laser attains a multimodal state which is undesirable for several applications. The introduction of current modulation of laser was shown also to cause the modulation of the emission frequency (and phase), with the frequency deviation in this case being opposite to that of the optical feedback. Taking into account the opposite trends of frequency deviation induced in the laser because of OF and IM, an equilibrium condition that compensates the effects of each other and induces stability in the laser even in case of strong feedback was attained. Due to the relevance of the effects of frequency modulation induced in laser emission because of IM, and its significant role in determining the modal behaviour of the laser in case of CWFM-OF, the frequency coefficient ($\Omega_f$) of the laser being considered was measured experimentally out of fourteen experiments, yielding a consistent value of 0.005 nm/mA, equivalent to $-2.2$ GHz/mA, centred at 826.5 nm. The response of the laser in terms of number of modes in presence of different levels of feedback, external distance and current modulation amplitude was then studied and optimal conditions to retain the laser in monomodal state were numerically demonstrated under different feedback levels. Finally, it was experimentally shown how the introduction of current modulation pulled the laser to stability even in case of strong feedback. The practical implication of this method is that the laser can be used in spite of the strong feedback where the stability and monomodal state is desirable ranging from various applications such as optical communication, spectroscopy, and sensing.

However, the frequency deviation in the laser emission ($\Delta f = \Omega_f/A_{lm}$, explained in Section I) induced due to current modulation is dependent upon the amplitude of current modulation, which in turn is laser specific (the L-I curve). This could be the limiting factor to the extent the laser can be brought back to unimodal state from the multimodal state. Further, it would be desirable to explore the joint effect of temperature along with all the parameters experimentally to broaden the deep understanding of the physics behind laser stability in presence of strong OF. This requires comprehensive experimental, backed by theory to characterize thermal coefficient of laser under test (as there are various parameters explained earlier) and use it in experimental real world scenario. This would be the future work.

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