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A crashworthy problem on composite structures using a mathematical approach

Simonetta Boria\textsuperscript{a,\*}, Giovanni Belingardi\textsuperscript{b}, Fabio Giannoni\textsuperscript{a}

\textsuperscript{a}School of Science and Technology, Mathematics division, University of Camerino, Camerino, Italy
\textsuperscript{b}Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Torino, Italy

Abstract

During the last decades the attention given to vehicle crash energy management has been centred on composite structures. The use of fibre-reinforced plastic composite materials in automotive structures, in fact, may result in many potential economic and functional benefits due to their improved properties respect to metal ones, ranging from weight reduction to increased strength and durability features. Although significant experimental work on the collapse of fibre-reinforced composite shells has been carried out, studies on the theoretical modelling of the crushing process are quite limited since the complex and brittle fracture mechanisms of composite materials. Moreover most of the studies have been directed towards the axial crush analysis, because it represents more or less the most efficient design.

A mathematical approach on the failure mechanisms, pertaining to the stable mode of collapse of thin-walled composite structures subjected to axial loading, is investigated. The analysis is conducted from an energetic point of view. The main energy contributions to the absorption (bending, petal formation, circumferential delamination, friction) are identifies and then the total internal energy is equated to the work done by the external load. The total crushing process can be seen as a succession of deformation states, each responsible for a partial absorption. The minimum configuration for each impact force in the specific state of deformation, function of several variables and dependent on geometric and material parameters is obtained. A comparison between theory and experiments concerning crushing loads and total displacements is presented, showing how the proposed analytical model is effective in predicting the energy absorption capability of axially collapsing composite shells.

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* Corresponding author. Tel.: +39-0737-402503.
E-mail address: simonetta.boria@unicam.it
1. Introduction

The survivability of driver and passengers in an accident is achieved by the use of thin-walled impact attenuators placed in specific vehicle zones, able to absorb as much as possible the kinetic energy through a progressive deformation. The crashworthiness design requires to know the crushing force and the final deformation of such structures when subjected to axial loading in order to pass specific homologation tests.

It is generally accepted that thin-walled tubes offer the most weight efficient solution for crashworthy aspects. Composite materials are now carefully considered in automobiles because they promise to be far more efficient than conventional materials [1]. A representative pioneering work to study the crush behavior of composite tubes submitted to axial load has been done by Thornton et al. [2]. They studied the behavior of various composite tubes, taking into account different fiber types, lay-ups and thickness to diameter ratio. Their experimental results showed that rectangular and square sections are less effective in energy absorption than circular ones. Hull et al. [3] discussed failure mechanisms for composite tubes in detail and commented upon the influence of geometry and material composition on structural performance. Farley and Jones [4] studied also the effect of crushing velocity on the energy absorbing characteristic of composite tubes with different lay-ups. In the Mamalis et al. book [5] many of their results, done on composite tubes with different sections under different loading conditions, are summarized.

Based on experimental observations by Mamalis et al. [5,18] thin-walled structures under axial loading can deform in four different modes: deformation confined at impact wall (Mode I), longitudinal crack progression (Mode II), centrally confined circumference crack (Mode III), large hinge progressive folding (Mode IV). The challenge of design is to arrange the column of material such that the destructive zone can progress in a stable manner (Mode I) due to the large amount of crush energy absorption. The mean load of crushing becomes an important factor to estimate while choosing a material and geometry for an impact energy absorbing application. Recently many experimental studies have been combined with numerical analysis to predict the final deformation of such structure for various applications: progressive damage in braided composite tubes [6], structural components of a Formula 1 racing car [7], crash-boxes for automotive application based on advanced thermoplastic composite [8], BAR Honda rear impact structure [9], certification of the composite rimp energy absorber for a star Mazda series [10], composite frontal crash-box for a Formula SAE car [11]. Very few authors analyzed the collapse mechanism of composite shells from the theoretical point of view [5, 12,13], due to the difficulty to model analytically the brittle behavior and heterogeneity of these composite structures. Mamalis et al. model the crumpling and bending process of thin-walled components of fiberglass materials, taking into account the energies involved in total axial crushing. Velmurugan et al. [12] adopt a simpler analytical approach considering only the first load cycle. Each analysis simplifies the energy formulation using experimental evidences.

The present study addresses an analytical and experimental investigation on the failure mechanism, pertaining to the stable mode of collapse (Mode I), of thin-walled composite conical tubes subjected to axial loading in order to predict the mean loads and total displacements during collapse. The analysis is based on previous research results [5, 12-15] with the attempt to eliminate some simplifications dictated by experimental evidence and improve the modeling from the mathematical point of view. The theoretical modeling identifies the main internal absorbing energy contributions and balances the work done by the external load. During the crushing, after the initial peak, the load oscillates around a mean load. This is due to the arise of a main circumferential intrawall crack, the splaying of the material strips, the formation of two lamina bundles bending inwards and outwards and the generation of a triangular debris wedge of pulverized material. Moreover, an experimental campaign was conducted on circular frusta in CFRP material varying some geometrical parameters, such as wall thickness, mean radius and slope. The tests were done under static and dynamic axial loads. Comparison between theory and experiments concerning loads and crushing is good, indicating that the proposed theoretical model is an efficient approach to predict the energy-absorbing capability of the axially collapsing composite shells, despite the complexity of the phenomenon.

### Nomenclature

| Symbol | Description |
|--------|-------------|
| $R$    | top mean radius of tube |
| $H$    | axial length of tube |
| $T$    | wall thickness of tube |
2. Theoretical modelling

During the crushing of a composite conical structure under axial impact, after the initial peak, the load oscillates around a mean load $P$. The first sharp drop in the load is due to the formation of a main circumferential intrawall crack of height $h$ at the top end parallel to the axis of the shell wall. As the deformation proceeds further, the externally formed fronds curl downwards with the simultaneous development, along the circumference of the shell, of a number of axial splits followed by splaying of the material strips. The post crushing regime is characterized by the formation of two lamina bundles bent inwards and outwards due to the flexural damage; they withstand the applied load and buckle when the load or the length of the lamina bundle reaches a critical value. At this stage, a triangular debris wedge of pulverized material starts to form; its formation may be attributed to the friction between the bent bundles and the platen of the hammer mass. The theoretical model takes into account only the first cycle of progressive crushing, because the total deformation is characterized by successive cycles that are repeated with a similar trend.

The idealized model of the crush zone is shown in Figure 1, where $R$ is the mean radius, $H$ the axial height, $T$ the thickness of the shell and $\phi$ the slope in degree of the tube.

In order to simplify the deformation mechanism, the following assumptions have been adopted: the internal and external fronds maintain a constant length equal to $h$; the transition between straighten and bended zone is sudden, so
the central crack is placed in A as shown in Figure 2; the elastic energy associated with the first impact phase was not considered, because very low respect to the other contributions.

Energy is absorbed in four principal modes during the formation of crush zone in progressive crushing of tubes [13]: work required for bending of petals ($W_b$), work required for petal formation ($W_{bh}$), work required for circumferential delamination ($W_c$) and energy dissipated due to friction between the debris wedge and fronds and between fronds and platen ($W_f$). Follow in detail the expressions used for the various energy contributions.

2.1. Bending energy

As the crushing process initiates, fibers bend both inside and outside the shell radius. Let $t_1$ and $t_2$ be the thickness of the fiber layers bending inside and outside the shell radius and $\alpha$, $\beta$ the bending angles. By construction $t_2$ is equal to $T-t_1$ and the angle $\beta$ is $\alpha-2\phi$. Assuming that the fiber layers are perfectly plastic during bending, the work required to bend the fiber inside and outside the shell radius can be expressed as

$$W_b = W_{bh} + W_{bh} = 2\pi d_c (M_1 \alpha + M_2 \beta) = \frac{\pi}{3} \sigma_0 d_c \{ \alpha [t_1^2 + (T - t_1)^2] - 2\phi t_1^2 \}$$

where

$$d_c := R - T/2 + t_1 + h \sin \phi$$

is the frustum radius at the crack tip, $M_1$ and $M_2$ are the bending moment of the internal and external laminate respectively and $\sigma_0$ is the ultimate stress in uni-axial tension of the laminate.

2.2. Hoop energy

Assume that the hoop strain varies linearly between A and B in Figure 1 that is it is null in A and maximum in B. The expression for hoop strain energy in a single crush is

$$W_h = W_{h} + W_{h} = \int_0^h \sigma_0 |\epsilon_1| \ dv_1 + \int_0^h \sigma_0 |\epsilon_2| \ dv_2 = \sigma_0 \pi h^2 [T (\sin(\alpha - \phi) + \sin \phi) - 2t_1 \sin \phi]$$

where $\epsilon_1$ and $\epsilon_2$ are the hoop strain corresponding to the layers bending inside and outside the shell radius, and $dv_1$, $dv_2$ are the differential volume for the inside and outside layers.

2.3. Crack energy

The energy required for circumferentially delamination in a single stroke is

$$W_c = 2\pi h G d_c$$

where $G$ is the critical strain energy release rate per unit interlaminar delaminated crack area and $d_c$ verifies (2). $G$ is determined experimentally through DCB test as per ASTM D5528-01 [16].

2.4. Friction energy

After the formation of the internal and external fronds, normal stresses develop on the sides of the debris wedge followed by shear stresses along the same sides due to friction at the interface between the wedge and the fronds. Moreover additional normal and shear stresses develop at the interface between the steel plate and the deforming shell as the formed fronds slide along the interface.

The energy dissipated in frictional resistance for a crush distance $\delta$ is

$$W_f = 2\pi d_c \delta [\mu_1 (P_1 + P_2) + \mu_2 (P_3 + P_4)]$$
where $\mu_1$ is the coefficient of friction between frond and platen, $\mu_2$ the coefficient of friction between the wedge and the fronds. $P_1$ and $P_2$ are the normal force per unit length applied by the platen to the fronds. $P_3$ and $P_4$ are the normal force per unit length applied to the sides of the wedge, as shown in Figure 1. Due to feasibility $\mu_1, \mu_2 \in [0, 1]$ and by construction the deformation is given by

$$\delta = h(\cos \phi - \cos(\alpha - \phi)).$$

Static equilibrium at the interface yields

$$P = 2\pi d_c (P_w + P_1 + P_2),$$

where $P_w$ is the normal force per unit length applied by the platen to the debris wedge given by

$$P_w = P_3 \sin(\beta + \phi) + T_3 \cos(\beta + \phi) + P_4 \sin(\alpha - \phi) + T_4 \cos(\alpha - \phi).$$

$T_3$ and $T_4$ are the frictional forces per unit length developed between wedge and fronds. Assuming that Coulomb friction prevails between the debris wedge and fronds, $T_i = \mu_i P_i$ for $i=3,4$. Note that

$$P_i = \sigma_0 h \quad \text{for } i = 3, 4,$$

$$P_w = 2\sigma_0 h[\sin(\alpha - \phi) + \mu_2 \cos(\alpha - \phi)].$$

Therefore the friction energy (5) is given by

$$W_f = \mu_1 h[\cos \phi - \cos(\alpha - \phi)]P + 4\pi \sigma_0 d_c h^2[\cos \phi - \cos(\alpha - \phi)] \cdot \left[\mu_2 - \mu_1 \sin(\alpha - \phi) - \mu_1 \mu_2 \cos(\alpha - \phi)\right].$$

2.5. Mean load $P$

The total energy dissipated for the deformation of the shell is given by the sum of the bending energy (1), the hoop energy (3), the energy required for circumferentially delamination (4) and friction energy (10), i.e.

$$W_i := W_b + W_h + W_c + W_f,$$

and it is equal to the work done by the external load $P$ on the crushing displacement $\delta$ in a single progression, that is

$$W_e := P\delta = Ph[\cos \phi - \cos(\alpha - \phi)].$$

The mean load $P$ is a function of three variables $h, t_1, \alpha$ and depends on three geometric parameters $\phi, R$ and $T$, i.e.

$$P(h, t_1, \alpha; \phi, R, T) = \frac{\pi}{(1 - \mu_1)h(\cos \phi - \cos(\alpha - \phi))} \left[\frac{1}{3} \sigma_0 d_c(\alpha t_1^2 + (T - t_1)^2) - 2\phi t_1^2\right] +$$

$$+ 2hGd_c + \sigma_0 h^2 (T(\sin(\alpha - \phi) + \sin \phi) - 2t_1 \sin \phi) +$$

$$+ 4\sigma_0 h^2 (\cos \phi - \cos(\alpha - \phi)) d_c(\mu_2 - \mu_1 \sin(\alpha - \phi) - \mu_1 \mu_2 \cos(\alpha - \phi))].$$

Note that the domain of the function $P$ is given by

$$D = [0, H] \times [0, T] \times \left[2\phi, \pi / 2 - \phi\right]$$

indeed for feasibility the crush length is strictly positive, the thickness of the plies bending outside belong to the interval $[0, T]$ and the external bending angle $\alpha$ is larger than $2\phi$, because $\beta = \alpha - 2\phi \geq 0$ and the denominator of (13)
has to be strictly positive.

3. Experimental observation

In order to analyze the behavior of composite impact attenuators with carbon fibers embedded in an epoxy resin matrix, analysis on circular frusta specimens under static and dynamic loadings were conducted. Such tests were designed to analyze the effects of the geometric parameters and loading conditions on the energy absorption and on the failure mode.

The tapes of the prepreg, from which the specimens were obtained, were provided by Saati SpA and the production of the same has been made by Vega Srl. In particular, the carbon fiber is high strength type CF290, while the epoxy resin is a ER450. The various geometries have been realized by arranging the prepreg layers by means of a quasi-isotropic lamination, achieved by interleaving the orientation of the tissue from 0 °, 90 ° to +45 °, -45 °. An aeronautical production technology was adopted, using male and female molds and an autoclave process.

As regards specimen geometry, top internal diameters of 25, 35 and 50 mm, wall thicknesses of 1.5, 2.5 and 4 mm, slopes of 0°, 5°, 10° and 15° were analyzed. Each specimen has an axial length of 200 mm.

The static tests, conducted in displacement control, were performed using an Instron 8801 servo-hydraulic testing machine with a maximum load 100 kN (Figure 2), at the Laboratory of the Department of Mechanical and Aerospace Engineering at Politecnico di Torino. The test speed was set equal to 5 mm/s for a maximum axial crushing of 100 mm.

The dynamic experimental tests, instead, were done using a drop weight machine at Picchio S.p.A. with a mass in free fall of 301 kg from a height of about 1 m (Figure 2). Impact accelerations and velocities were acquired by a FA3403 tri-axial accelerometer with ±500g full scale and a E3S-GS3E4 photocell, respectively. Moreover a Mikrotron high-speed camera was used with a sampling of 1000 frames/s.

4. Results and discussion

The present approach aims at determining the critical values of the length \( h \), the thickness \( t_1 \) and the opening angle \( \alpha \) belonging to the domain D in which the mean load is minimum. The nonlinear function (13) is not easy to analytically minimize because the mean load gradient is difficult to nullify, so a numerical optimization method is necessary. In view of generalizing the model discussed in Section 2, considering more than just three independent variables, a numerical algorithm named L-BFGS-B, which stands for Limited memory Broyden-Fletcher-Goldfarb-Shanno method for Bound constrained optimization [17], has been implemented.

In order to evaluate the effectiveness of the idealized model, implemented with this numerical approach, specific cases, available from experimental tests, were analyzed. Table 1 reports the mean crushing loads, the crush length and the errors in percentage between the described method and the experimental results for the geometrical cases taken into account. In particular Table 1 refers to static and dynamic loading conditions. By experimental evidence, also reported in [18], the microfracturing mechanism for the progressive collapse of conical shell subjected to dynamic loading is, in general, similar to that obtained during the axial static collapse; therefore the above proposed
theoretical model can be applied also for dynamic conditions modifying only the frictional parameters, such as $\mu_1$ and $\mu_2$. In particular, the delamination with internal and external fronds formation is present on the specimens with a wall inclination less than 5° and on those at 10° with a thickness of 4 mm. In all other cases a perfect internal inversion was observed (Figure 3).

Table 1. Geometrical and crushing characteristics during static and dynamic loading.

| Specimen geometry | Static | | | Dynamic | | |
|---|---|---|---|---|---|---|
| | $R$ [mm] | $T$ [mm] | $\phi$ [°] | $P$ [kN] | Error % | $s$ [mm] | Error % | $P$ [kN] | Error % | $s$ [mm] | Error % |
| 26.25 | 2.5 | 0 | 57.80 | 1 | 95.6 | 1 | 38.40 | 4 | 62.5 | 4 |
| 13.75 | 5 | 25.29 | 3 | 98.7 | 3 | 19.15 | 16 | 125.3 | 4 |
| 13.75 | 5 | 47.37 | 10 | 87.1 | 9 | 24.15 | 3 | 99.3 | 17 |
| 14.5 | 5 | 62.26 | 8 | 104.8 | 9 | 45.13 | 1 | 53.2 | 6 |
| 18.25 | 1.5 | 5 | 23.89 | 15 | 112.5 | 17 | 22.90 | 18 | 104.8 | 11 |
| 18.75 | 2.5 | 5 | 43.69 | 11 | 107.6 | 12 | 30.71 | 12 | 78.1 | 11 |
| 26.25 | 2.5 | 5 | 57.98 | 9 | 105.9 | 10 | 40.17 | 6 | 59.7 | 8 |
| 13.75 | 1.5 | 10 | 21.35 | 11 | 106.7 | 13 | 18.55 | 12 | 129.4 | 3 |
| 13.75 | 2.5 | 10 | 40.08 | 2 | 98.2 | 2 | 25.47 | 15 | 94.2 | 11 |
| 14.5 | 4 | 10 | 73.88 | 5 | 89.7 | 5 | 47.67 | 1 | 50.3 | 12 |
| 18.25 | 1.5 | 10 | 27.99 | 4 | 92.6 | 4 | 23.99 | 9 | 100.0 | 13 |
| 18.75 | 2.5 | 10 | 51.45 | 1 | 95.1 | 1 | 32.29 | 2 | 74.3 | 9 |
| 19.5 | 4 | 10 | 92.97 | 12 | 54.4 | 11 | 59.32 | 16 | 40.4 | 15 |
| 25.75 | 1.5 | 10 | 33.83 | 13 | 85.1 | 11 | 22.99 | 4 | 104.3 | 1 |
| 26.25 | 2.5 | 10 | 60.85 | 3 | 99.4 | 3 | 42.13 | 8 | 56.9 | 3 |
| 13.25 | 1.5 | 15 | 20.14 | 4 | 100.1 | 4 | 19.52 | 7 | 122.9 | 2 |
| 13.75 | 2.5 | 15 | 37.88 | 2 | 93.7 | 2 | 26.88 | 16 | 89.3 | 12 |
| 14.5 | 4 | 15 | 78.40 | 4 | 75.3 | 4 | 56.49 | 15 | 42.3 | 20 |
| 18.75 | 2.5 | 15 | 48.54 | 13 | 85.0 | 11 | 34.02 | 3 | 70.5 | 1 |
| 19.5 | 4 | 15 | 87.92 | 10 | 50.0 | 9 | 67.65 | 17 | 35.4 | 18 |

Fig. 3. Different failure mechanism: (a) Mode I-a; (b) Mode I-b.

From the tables it is clear as, despite the simplifications adopted, the proposed analysis is able to predict within ±20% the mean load, which is absorbed for about 55% from frictional effects, for about 37% from fronds bending, for about 6% from hoop strain and for only 2% from crack propagation. Also according to Mamalis et al. the distribution of the main energy sources can be estimated with the same order. As mentioned before, the model refers to the first cycle of deformation; therefore the crush length $s$ can be obtained multiplying the minimum displacement $\delta$ for the ratio between the experimental energy to absorb and the minimum energy obtained from the model.
5. Conclusions

A crashworthiness problem was investigated using an analytical, numerical and experimental approach. In particular the energy absorption of composite circular frusta subjected to static and dynamic axial loading was analyzed, defining analytically the external load as a function of three variables and identifying the minimum through a numerical approach. Despite some simplification, the method adopted is able to predict the mean load and the total crushing of conical shell made of composite, once known parameters of the used material. Only the first deformation cycle, as Velmurugan et al., and a more realistic formulation for the friction energy, as Mamalis et al., were considered. Differently from other authors no experimental statements for the presented model are assumed, therefore the methodology can be used as the first approach to follow during the design of specific impact attenuators. Forthcoming analysis will eliminate the assumption of a constant crush length both internally and externally to the axial splits, considering also the curvature of the folding.

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