The Increase of Critical Thinking Skills through Mathematical Investigation Approach

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Abstract. Some research findings on critical thinking skills of prospective elementary teachers, showed a response that is not optimal. On the other hand, critical thinking skills will lead a student in the process of analysis, evaluation and synthesis in solving a mathematical problem. This study attempts to perform an alternative solution with a focus on mathematics learning conditions that is held in the lecture room through mathematical investigation approach. This research method was Quasi-Experimental design with pre-test post-test design. Data analysis using a mixed method with Embedded design. Subjects were regular students enrolled in 2014 at the study program of education of primary school teachers. The number of research subjects were 111 students consisting of 56 students in the experimental group and 55 students in the control group. The results of the study showed that (1) there is a significant difference in the improvement of critical thinking ability of students who receive learning through mathematical investigation approach when compared with students studying through expository approach, and (2) there is no interaction effect between prior knowledge of mathematics and learning factors (mathematical investigation and expository) to increase of critical thinking skills of students.

1. Introduction
Critical thinking has been an important issue for many years. The word critical means to understand the people and things around us and analyses our own thinking processes. Unfortunately, criticizing ability is used just as limiting the thinking process of a person [1]. Construction of learning more teacher-centered response to the condition where students only receive the information without considering their ability to think. It has undercut the ability of students in an effort to do the analysis and synthesis. The main principle of learning is to provide a great opportunity so that students can develop the ability to think and manage this capability so that there is a meaningful learning. One type of thinking skills and become the focus of this research is the ability to think critically. In this context, [2] pointed out that "Critical thinking is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. This statement emphasizes that the ability to think critically is an intellectual process that guides a person in action. [3] views critical thinking as a mental process that involves a high quality and high level of thinking for problem-solving and decision-making. Terms such as higher level thinking and reflective thinking have often been used interchangeably with the term critical thinking throughout the literature ([4], [5], [6], [7]). However, while there are many definitions of the term...
critical thinking, [8] suggested that critical thinking resulted from the interaction of a set of dispositive toward critical thinking: seeking a clear statement of the questions, seeking reasons, trying to be well-informed, and trying to remain relevant the main point.

Expert opinion given above shows how important the role of critical thinking skills in a person, in particular for the development of the capacity of a student, as a subject as well as in a learning process. While different conceptualizations of critical thinking have shed considerable light on the nature and meaning of critical thinking, at the same time, it is often acknowledged that critical thinking skills require time to develop. In fact, [9] also emphasizes that the students should not only employ critical thinking skills in class, but also be able to activate them in real-life situations and to recognize situations when these skills should be used. But the condition of students' critical thinking skills, looks very varied

Some studies indicate it. For example, [10] found that the critical thinking skills of teacher candidates of elementary mathematics were medium level but not high enough. Learning only a content-based curriculum, children cannot become better thinkers able to give reasons for their conclusions, to think flexibly and creatively, to solve problems and make good decisions [11]. On the other hand, [12] conducted a study through the selection of learning content and found that students' critical thinking can be successfully encouraged and developed in elementary mathematics education by the selection of content (tasks), the solution of which emphasizes skills of critical thinking in all students, and that we should not make any limitations regarding the classification of students by their overall achievement in school.

On the other hand, mathematical investigation is an approach to learning that is focused on the exploration of a topic in depth and make connections between different representations. These efforts provide great opportunities, based on the ability of each student, to be able to solve the problems of open-ended, finding patterns, self-discovery, reducing the role of the teacher, using its own methods, divergent thinking, and so as an attempt to solve the problem. The Cockcroft report (1982) cited by [13] stated that the idea of investigation is fundamental both to the study of mathematics itself and Also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very manyfields.

The investigative approach is broken down into steps in Maths Charter Working Mathematically (n.d.) and Kanasa (n.d.) (cited in [14]) has illustrated different methods of investigating and problem solving in a visual way which will add meaning for many students. The ability to pose questions, collect data, investigate and discover answers, solve problems, describe, share and elaborate on findings, evaluate, reflect on and judge the accuracy of answers, draw conclusions, revise and test models of investigating and reject or defend ideas are skills that should be stressed in the modern mathematics classroom. These processes need to take place in small groups and in the classroom as a whole. Furthermore, [15] explains that the stages of mathematical investigation consists of five phases: entry, goal setting (or problem posing), attack (or problem solving), review and extension. At the entry phase, the students should understand the task. There are generally two approaches to goal setting: students may pose a specific problem or they may decide to search for any pattern. The latter approach usually leads to specializing, formulating and testing of conjectures, and generalizing, in the attack phase. This is the process of investigation.

Context of this study, researchers applied an a mathematical investigation approach in order to develop students' critical thinking skills. The emphasis on the aspect of mathematical investigation will establish a process for students to think mathematically, and supported the process of interaction between students and lecturers as well as a source of learning as a process of communication and cooperation, are expected to have impact on the ability of mathematical understanding and thinking skills of students. The questions were: (1) Is the mathematical investigation approach can improve the ability of Critical thinking skills of prospective elementary teachers?, (2) Is there an interaction effect between the learning factor (mathematical investigation and expository) and prior knowledge of mathematics in improving the Critical thinking skills of prospective elementary teachers?
2. Experimental Method

Research design using Quasi-Experimental type design with pre-posttest design [16]. Based on this type, the researchers apply a pre-and posttest design, which is divided into two study groups: control group and the treatment group. Both groups were subjected to pretests as supporting material to perform the treatment, then do post-test to know the difference between the two groups. The experimental group using investigative approach to learning mathematics (mathematical investigation), while the control group using expository approach. These different approaches are arranged in a construction of instructional design each time the lecture.

Data were collected through a process of pretest and posttest in both study groups (experimental and control). Pre-test and post-test question contains a set of items in which there are indicators of critical thinking. Indicators of critical thinking abilities that are used in this study, are: (1) identify, select, classify into categories and describe attributes (2) organize the parts to form something new or unique whole, (3) integrate ideas, information and theory to design solutions, (4) summing up and explain the meaning to give the meaning of each section.

Number of items in pre-test consist of 4 items and post-test consists of 5 items, whole items which of the measured variables of the study. Measurement of study variables using a score of 0-4 in each category have been determined under the rubric and scoring techniques.

Both these instruments do the testing process to test the validity and reliability of the instrument. Validity test using Pearson correlation test models. While the reliability of the instruments this study refers to internal consistency reliability with the test model used was Cronbach Alpha coefficients. Descriptive analysis of data using descriptive statistical measures such as mean, standard deviation, and curves. ANOVA test was used to test the hypothesis.

3. Result and Discussion

3.1 Result

The increase of critical thinking skills of research subjects is obtained through the ability to analyze, generalization, synthesis, justify, and resolve non-routine which is operationally constructed as an indicator of research and used as the measurement criteria of critical thinking skills. Based on the observation of these indicators, the results obtained in the following Table 1.

| Group       | N | Minimum | Maximum | Mean | Standard Deviation |
|-------------|---|---------|---------|------|--------------------|
| Control     |   |         |         |      |                    |
| Pre-test    | 55| 2       | 8       | 3.42 | 3.01               |
| Post-test   | 55| 4       | 18      | 8.42 | 1.64               |
| Gain value  | 55| 0.0     | 0.89    | 0.28 | 1.93               |
| Experiment  |   |         |         |      |                    |
| Pre-test    | 56| 2       | 7       | 6.78 | 3.74               |
| Post-test   | 56| 16      | 20      | 12.48| 3.98               |
| Gain value  | 56| 0.0     | 1.00    | 0.42 | 0.25               |

The distribution of the data (Table 1) provide a response that observed in the experimental group showed results higher scores on aspects of post-test and the value of the gain. This is an indication that the application of mathematical investigative approach produces positive impact on critical thinking skills of research subjects.
Table 2. Results of the Analysis of Variance (one way)

| Tests of Between-Subjects Effects |
|----------------------------------|
| Source                           | Type III Sum of Squares | df | Mean Square | F       | Sig.  | Partial Eta Squared |
| Corrected Model                  | .513*                   | 1  | .513        | 9.943   | .002  | .084                |
| Intercept                        | 13.978                  | 1  | 13.978      | 271.029 | .000  | .713                |
| learn_factors                    | .513                    | 1  | .513        | 9.943   | .002  | .084                |
| Error                            | 5.622                   | 109| .052        |         |       |                     |
| Total                            | 20.162                  | 111|             |         |       |                     |
| Corrected Total                  | 6.135                   | 110|             |         |       |                     |

a. R Squared = .084 (Adjusted R Squared = .075)

Results from one way ANOVA analysis in the Table 2, shows that the mathematical investigation approach gives a positive response in providing enhanced critical thinking of research subjects, based on the F value of 9.943 with sig. 0.002<0.05 as the recommended value. Partial eta squared value generated by the model were high at 0.084 < 0.14, so it is classified in the category of medium (Cohen and Miles & Shevlin, (2001) cited in [17]). Value partial eta squared explains that the mathematical investigation approach able to deliver effect as 0.084 in contributing to a unique variant of the critical thinking. While the value of Adjusted R Squared generated model of the main effect of 0.075, which means 7.5% of the variance of the total variance in critical thinking described by mathematical investigation. Using partial eta-square value of this, researchers calculated effect size obtained by the size of Cohen's d and generate value 0.302. the value of the effect size of this is in the category small and the equivalent of percentile standing at 62.0 and percent of nonoverlap at 21.3% [18].

Table 3. Results of the Analysis of Variance (two way)

| Tests of Between-Subjects Effects |
|----------------------------------|
| Source                           | Type III Sum of Squares | df | Mean Square | F       | Sig.  | Partial Eta Squared |
| Corrected Model                  | .749*                   | 5  | .150        | 2.922   | .016  | .122                |
| Intercept                        | 8.255                   | 1  | 8.255       | 160.954 | .000  | .605                |
| learn_factors                    | .335                    | 1  | .335        | 6.534   | .012  | .059                |
| prior_knowledge                  | .105                    | 2  | .053        | 1.025   | .362  | .019                |
| learn_factors * prior_knowledge  | .095                    | 2  | .048        | .929    | .398  | .017                |
| Error                            | 5.385                   | 105| .051        |         |       |                     |
| Total                            | 20.162                  | 111|             |         |       |                     |
| Corrected Total                  | 6.135                   | 110|             |         |       |                     |

a. R Squared = .122 (Adjusted R Squared = .080)

Then, the result of two-way ANOVA in the Table 3 illustrates that there is no interaction effect between the factors of learning as applied in this study and the prior knowledge of mathematics to increase the critical thinking of the study subjects were confirmed by F value of learn_factors * prior_knowledge amounting to 0.929with sig. 0.398>0.05 as the recommended value. Value partial eta squared of 0.017 in the category of small and use a value of partial eta squared, the calculated effect size with the size of Cohen's d and obtained a value of 0.131. Effect size is in the category of small and the equivalent of percentile standing at 54.0 and percent of nonoverlap at 7.7%. Based on these measures, we conclude that the effect of the interaction between learning factors and prior knowledge of mathematics to the improvement of critical thinking is quite small and is considered insignificant. This condition is confirmed by the interaction profile plot "learn_factors * prior_knowledge" below.
In Figure 1a makes clear that the movement of the line of the domain mathematical investigation to the domain ekspository a pattern of positive slope, meaning that there is an increase in the value of the gain of the reasoning abilities of higher mathematics at the mathematical investigation for any prior knowledge of mathematics, when compared to expository. In Figure 1b illustrates that the movement of the two lines that show gain value of the increase in critical thinking generated by factors of learning in each category prior knowledge of mathematics, seen not happen crossovers (intersection of the two lines). Expository lines symbolize movement has always been at the bottom of the movement of the line that symbolizes the mathematical investigation.

Based on the above description, the conclusion that there is no interaction effect that occurs in both the independent variables studied (learning factors and prior knowledge of mathematics) to the dependent variable (the critical thinking).

3.2 Discussion

The results of a one-way ANOVA test showed that there was the increase critical thinking skills significantly of students having learning through a mathematical investigation than learning by learning through expository approach.

The results of this, not in spite of attributes inherent in mathematical investigation. One is as a process of mathematical thinking. In the process of mathematical thinking, identified four basic processes. [19] stated that we identified four fundamental processes, in two pairs, and Showed how thinking mathematically very often proceeds by alternating between them: (a) specializing – trying special cases, looking at examples. (b) generalizing - looking for patterns and relationships, (c) conjecturing – predicting relationships and results, and (d) convincing – finding and communicating reasons why something is true. Viewed from the side of the mathematical investigation, based on the conclusions of various literature, concluded that the mathematical investigation is a process that includes 4 (four) stages of the process of thinking, namely: specialization, filing of alleged (conjecturing), justification and generalization [20].

Thus, critical thinking skills generated in this study are not detached from the learning process with the mathematical investigation approach in increasing critical thinking skills, because in the process of critical thinking will involve the mathematical thinking that is based on four fundamental processes, namely specializing, generalizing, conjecturing, and convincing, which developed from a low level mathematical thinking skills to a high level of mathematical thinking skills. The position of critical thinking is at a high level thinking skills that require the above process. Critical thinking is ... "... the intellectually disciplined process of actively and skillfully conceptualizing, applying, synthesizing or evaluating information gathered from, or generated by observation, experience, reflection, reasoning, or communication as a guide to belief or action [2]."
As a comparison, researchers present a problem which is used to measure students’ critical thinking skills following

A cylinder with a 2-centimeter radius and a height of 10 cm is submerged in a tank of water that is 20 cm wide and 30 cm long (see Figure). How much does the water level rise? (Source: [21], p. 287)

And the answer sheets of students from the experimental and control group, which both have the same initial knowledge of mathematics, which both are in the medium category.

Response M070 (Figure 2a) looks give the correct response, despite the procedural errors, the errors included figures for the value of l (width) by 30, should have been 20 as an attitude that is less critical. It seems that M070 is able to provide an indicator of critical thinking skills such as being able to identify and describe the attributes; identify the relationship between the parts of the whole piece; arranging the parts to form something new or unique whole; and is able to integrate the ideas, information and theory to design a solution. These results lead to the correct answer.

Answer N034 (Fig 2b) shows incompetence bring up the indicators for the measurement of critical thinking skills, such as identifying the relationship between the parts involved; identify the relationship between the parts of the whole piece; organize the section to form a the flow solutions and integrate all the information into the design of the solution. The answer N034 portray their difficulties in the process of mathematical thinking. This condition is seen as N034 difficult to identify the relationship between the parts that exist, it is actually N034 difficulties in the Specializing and generalizing, especially on the search patterns and relationships to develop a solution. The end result, it is easy to guess there will be an error response

The answer sheet above, is an example put forward by researchers in demonstrating differences in the response of the two-group research. In the experimental group, showed an increase in critical thinking skills more than in the control group, where the influence of the investigation mathematically according to the effect size obtained by the value of partial eta squared of 0.084 and d Cohen’s of 0.302 indicates the level of impact strength both at medium level. In line with this conclusion, [22] have reported a synthesis of the results of quantitative (meta-analysis) of the studies that evaluated the impact of the use of thinking skills program and approach to cognition, achievement and attitudes. Higgins and colleagues examined the results of 29 studies from around the world, mostly from the United States and Britain, which are used in elementary schools and secondary in literacy, math, and science. The results of the analysis of Higgins et al. supports the conclusion that the intervention skills that are effective in improving students' thinking skills, achievements in their fields, and motivation. Furthermore, [23] found that a group of students who receive instruction critical thinking skills (critical thinking skills instruction) explicitly designed in a study, concluded the increase of the value of the gain is significant in the analysis of their arguments than the group do not receive instructions explicitly critical thinking skills.

Other findings in the study is no interaction effect between the factors of learning (mathematical investigation and expository) and prior knowledge of mathematics (in categories: low, medium, high)
to increase the critical thinking. Profile plots (fig.1a & fig.1b) showed no intersection of the lines and the existence of the movement increase the ability of critical thinking of research subjects, which is moving toward the higher, from the domain of expository to the domain of mathematical investigation for all types of prior knowledge of mathematics. As it noted previously, one of the factors considered in the process of improving a person's ability critical thinking is the position of the subject in the context of prior knowledge of mathematics he had.

The context of the absence of these interactions can be identified from the prior knowledge of mathematics of the research subjects. Description of the answer sheet presented earlier and simple statistics, illustrates the significant difference in the increase critical thinking skills between the experimental and control, seen at the beginning of the study subjects were capable of medium and high. But in subjects with lower initial ability, showed no significant differences. In this condition, looks very realistic to see the urgency of giving mathematical investigation. Learning of mathematics like this will load the investigation activity, investigation task, investigation work or investigation process as well as covering also aspects of problem solving, filing problems, reasoning as well as a process of mathematical thinking.

4. Conclusion
The conclusion of this study stated that: (1) Investigation of mathematics as an approach to learning is able to give a positive response to the increasing of critical thinking, and (2) There is no interaction effect of the factors of learning and prior knowledge of mathematics to the increased ability of critical thinking. One of the implications of the implementation of the mathematical investigation is to provide opportunities and demands for teachers to train themselves in mastering the material to be taught, thus indirectly mathematical investigation serves as a “math lab” for teachers. It is as a consequence of other attributes inherent in the investigation of mathematics, that is, as an activity that requires a thought process, ranging from understanding the problem, the process of mathematical thinking (specialization, conjecturing, justify, generalization) and then ends with a process of review and extension. The condition occurs because of the mathematical investigations are confronted with problems of non-routine, where these conditions provide the demand for teachers to practice doing some investigative tasks that contain activity of investigation before it is taught.

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