Comparison of different spatial mathematical models of the extruder screw channel

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Abstract. One of the important problems in the processing of polymeric materials by an extrusion method is the occurrence of local overheating. The aim of this work is to identify these areas and finding ways to reduce their influence on the finished product. The paper discusses the heat and mass transfer processes, which occur in the extruder screw channel. In the work a comparison of the results, obtained by various mathematical models, was made: a flat channel model (full-scale), and the screw channel models with regard to the gap between the screw crest and the inner surface of the body and without it. The investigation of the possibility of using the inverse movement principle in the study of the problems, associated with the cable insulating on the extrusion equipment was made. As a result of research, temperature fields at the outlet of the extruder channel were obtained.

1. Introduction
To date, extrusion is the most common method of cable insulation. Local overheating resulting from the processing can negatively affect the quality of the insulation. As the processable polymer has a high viscosity, friction between particles of the material reaches a large value, whereby a polymer is heated up to high temperatures that can cause thermal degradation of the polymer [1]. The aim of this work was to study flow and heat transfer processes in the extruder screw channel in the production of plastic insulation and comparison of different geometric models.

Currently, there are a number of mathematical models describing the flow and heat transfer in extruders screw channels [2-5]. All known approaches, based on the laws of conservation, use the principle of inverse movement and turns the screw channel on a plane, that is an assumption.

A schematic illustration of the extruder is represented in figure 1. It consists of the screw, body, heating elements and a forming device. The extruder is divided into four zones: the feeding zone, a delay melting zone, a melting and metering zone. In this paper, a study was conducted with the last 5 turns of the extruder, because this is where the greatest overheating takes place.
2. Mathematical statement and initial data

The mathematical representation of the motion and heat transfer processes of polymers in a screw channel of the plasticizing extruder is based on the laws of conservation of mass, momentum and energy [4,6]:

$$\nabla \cdot \mathbf{V} = 0; \quad (1)$$

$$\rho(\nabla \cdot \mathbf{V}) = -\nabla P + (\nabla \cdot \mathbf{t}); \quad (2)$$

$$\rho C(T)(\nabla \cdot \mathbf{V}) T = \nabla \cdot \lambda \nabla T + (\mathbf{t} \cdot \nabla \mathbf{V}), \quad (3)$$

where $\rho$ and $C$ – the density and specific heat capacity of the polymer; $\lambda$ – thermal conductivity; $\mathbf{V}$ – velocity vector; $T$ – temperature; $P$ – hydrostatic pressure; $\mathbf{t}$ – stress deviator tensor; $(\mathbf{t} \cdot \nabla \mathbf{V})$ – An irreversible increase in internal energy per unit volume due to the energy dissipation in the viscous flow.

The connection between the deviator stress tensor and the strain rate tensor is given by:

$$\mathbf{t} = \mu \ddot{\gamma}, \quad (4)$$

where $\ddot{\gamma}$ – deformation rate tensor; $\mu_s$ – effective viscosity which is a function of shear rate and temperature[1,7,6].

The dependence of the effective viscosity on the shear rate of the investigated polymer is determined by the power law [6]:

$$\mu_s = \mu_0 \dot{\gamma}^{n-1} \quad (5)$$

Viscosity dependence on temperature was determined by the Reynolds equation:

$$\mu_0 = \mu_{0a} \exp(-\beta(T - T_a)) \quad (6)$$

To study the processes in the extruder channel, models presented in Figure 2 were built.

![Figure 1. A schematic illustration of the extruder.](image1)

2.1. A geometric model of the investigated screw: a – Flat model; b – Cylindrical model without a gap; c – Cylindrical model with a gap.
The geometry of the channel is presented in Table 1. The body temperature was set at 200 °C, the temperature of the screw is 210°C, the temperature of the input material is 225°C. Calculations were carried out for the screw rotation speed of 60 revolutions / min.

**Table 1. The geometry of the channel**

| Parameters                              | Value  |
|-----------------------------------------|--------|
| Inner diameter of a barrel, mm         | 160,0  |
| Outer diameter of a screw, mm          | 159,4  |
| Twist, mm                              | 160,0  |
| Channel width, mm                      | 137,3  |
| Flight Wight, mm                       | 15,3   |
| Length of the dosing geometric zone, turn | 5      |
| Channel depth in the metering zone, mm | 4      |
| Helix angle, °                         | 17°39' |
| Radial clearance between the flight and the barrel, mm | 0,3    |

The properties of the material are presented in Table 2. The values of coefficients for rheological models have been determined experimentally on rotating rheometer DHR-2.

**Table 2. Polymer properties**

| Material     | Density, \( \rho \) (kg/m³) | Specific heat capacity, \( C \) (J/kg·K) | Thermal conductivity, \( \lambda \) (W/m·K) | \( \mu_0 \), \( B \), \( (I/K) \) | \( n \) | \( T_0 \), (K) |
|--------------|--------------------------------|------------------------------------------|--------------------------------------------|--------------------------------|------|--------------|
| PE 153-02K   | 779                            | 2500                                     | 0,182                                      | 23336,9                       | 0,0124 | 0,34         | 433          |

**3. Results and Discussion**

As a result of the study, temperature fields at the outlet of the extruder were obtained. Since processable polymer has a sufficiently high viscosity, the friction between the particles of the material heats it to a high temperature. Figure 3 shows the temperature field for the flat channel model using the principle of inverse movement and without it. For visualization of the obtained results, there was a 3-time increase of the field.

**Figure 3.** Temperature fields for flat channel models: a – using the principle of reversed motion; b – without using the principle of reversed motion.
When using the principles of inversed motion, the maximum shear rate occurs near the movable wall. Therefore, in investigation of the heat and mass transfer processes in the channel, using this principle, the character of the flow is changing. The maximum temperature is displaced to the movable walls.

Figure 4 shows the temperature field for three different spatial models: the flat channel model, the cylindrical channel model with a gap between the crest of the screw and the inner surface of the body and without it.

![Temperature field for three different spatial models](image)

Figure 4. Changing of the temperature field for the inverse motion using the power law, at the outlet of the channel: a – flat channel; b – helical channel without a gap; c – helical channel with a gap.

In the transition from the flat model to the actual screw channel with a gap, the character of the polymer flow is changing, so that the maximum and average values of temperature decrease.

4. Conclusion
The largest difference in the maximum temperature between the flat model and the helical channel with a gap amounts to 5.1%, and in the average temperature - to 1.45%. As we approach the geometry of the real form (from the flat to the screw channel with a gap), the maximum value of the temperature decreases.

Changing the temperature field, the viscosity and shear rate fields along the channel in all three geometrical models, when using the inverted motion, has the same pattern for investigated rheological law. Even though, the power law operates in a limited range of shear rates.

In all cases of the movable screw model (unconverted movement), the average temperature is higher than in case of inverted motion. This is caused by the changing character of the polymer flow, in which the maximum value of shear rates is displaced closer to the moving surface.

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