FPAA-based implementation of fractional-order chaotic oscillators using first-order active filter blocks

Alejandro Silva-Juárez a, Esteban Tlelo-Cuautle a,⇑, Luis Gerardo de la Fraga b, Rui Li c

INAOE, Luis Enrique Errro No. 1. Tonantzintla, Puebla 72840, Mexico
b CINVESTAV, Av. Instituto Politécnico Nacional No. 2508, San Pedro Zacatenco 07360, Mexico
c UESTC, Qingshuihe Campus, Xiyuan Ave No.2006, West Hi-Tech Zone, 611731, China

HIGHLIGHTS

• A detailed procedure to implement fractional-order chaotic oscillators using analog electronics in the frequency domain.
• Design of fractional-order integrator using first-order active filters implemented with amplifiers.
• Details on the design of fractional-order chaotic oscillators using a fieldprogrammable analog array.

GRAPHICAL ABSTRACT

Implementation of the fractional-order Chen’s chaotic oscillator using a field-programmable analog array (FPAA). Charef’s method is applied to approximate the fractional-orders as ratios of two polynomials in the Laplace domain, which are implemented by first-order all-pass and low-pass filters in the FPAA.

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ABSTRACT

Fractional-order chaotic oscillators (FOCOs) have been widely studied during the last decade, and some of them have been implemented on embedded hardware like field-programmable gate arrays, which is a good option for fast prototyping and verification of the desired behavior. However, the hardware resources are dependent on the length of the digital word that is used, and this can degrade the desired response due to the finite number of bits to perform computer arithmetic. In this manner, this paper shows the implementation of FOCOs using analog electronics to generate continuous-time chaotic behavior. Charef’s method is applied to approximate the fractional-order derivatives as a ratio of two polynomials in the Laplace domain. For instance, two commensurate FOCOs are the cases of study herein, for which we show their dynamical analysis by evaluating their equilibrium points and eigenvalues that are used to estimate the minimum fractional-order that guarantees their chaotic behavior. We propose the use of first-order all-pass and low-pass filters to design the ratio of the polynomials that approximate the fractional-order. The filters are implemented using amplifiers and synthesized on a field-programmable analog array (FPAA) device. Experimental results are in good agreement with simulation results thus demonstrating the usefulness of FPAs to generate continuous-time chaotic behavior, and to allow reprogramming of the parameters of the FOCOs.

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⇑Corresponding author.
E-mail address: etlelo@inaoep.mx (E. Tlelo-Cuautle).

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Introduction

Fractional-order chaotic oscillators (FOCOs) can be designed using analog or digital hardware resources [1], and in both cases the exactness depends on the numerical method or approximation to solve the fractional-order derivatives. This paper shows the implementation of FOCOs using analog electronic devices that can be reprogrammed, as the field-programmable analog arrays (FPAA). The inspiration comes from the successful implementation of integer-order chaotic oscillators on FPAA as the design of a true random source introduced in [2]. Nowadays, still some research is done on the implementation of integer-order chaotic oscillators using FPAA, as shown in [3–5]. The main advantage of an FPAA is its capability to perform reprogrammability and dynamic reconfiguration of the parameters of the chaotic oscillators [6,7].

The FPAA includes analog blocks to synthesize filters and also includes multipliers and amplifiers. The FPAA can be used for fast verification of a fractional-order dynamical system, and in a further step those blocks can be designed using integrated circuit technology, as it was done in designing fractional-order filters [8], and fractional-order elements [9]. However, the possibility of success when using an FPAA, depends on the number of analog blocks that are required to implement a fractional-order dynamical system, which is a challenge as a problem to reduce the number of amplifiers in a FOCO. On this direction, the authors in [10] show the approximation of the fractional-order differentiator and integrator using active filter transfer functions, and then the resulting circuit can be fully integrated using complementary metal–oxide–semi conductor (CMOS) technology [11].

The use of traditional active filter transfer functions [12–14], is a good option to approximate fractional-order differentiators or integrators, and one can use first-order [15], or second-order structures [16]. In this manner, this paper is inspired on using first-order active filter transfer functions to implement FOCOs on an FPAA, and therefore, the resulting design can be implemented in CMOS technology as it has already shown in [17], for the implementation of the fractional-order FitzHugh-Nagumo neuron model. In addition, the design of FOCOs using first-order active filters, can be transformed to circuits based on operational transconductance amplifiers [18], well-known as GaM-C filters [19], that benefit from the advantages of good amplifier-RC structures and allow monolithic integration using CMOS technology [20].

Two decades ago, the authors in [21] summarized some methods that transform a fractional-order operator into an integer-order system of an order higher than one, and which holds a constant phase within a bandwidth of a chosen frequency. Basically, an irrational function is approximated to a rational function defined by the quotient of two polynomials in the Laplace variable s. The number of poles and zeros in the transfer function is related to the desired bandwidth and the error criteria that can be approached by applying the methods introduced by Oustaloup [22], Carlson [23], Matsuda [24], Krishna [25], Charef [26], among others [27]. These methods are classified in the frequency domain, where the majority of research on implementing a FOCO with analog electronics has been oriented to use arrays of resistors and capacitors to implement a fractor or fracant, as done in [28–30]. Reference [31] provides details of the generation of approximated transfer functions, \( H(s) \) for different fractional-orders, in increments of 0.1, assuming errors around 2 dB and 3 dB.

Different to the designs of the FOCOs in [32–35], the goal of this paper is proposing the use of first-order active filter transfer functions to approximate the fractional-orders of the FOCOs. It is worth mentioning that this work presents the first FPAA-based implementation of FOCOs. Further, the implementation of FOCOs into an FPAA will make possible the development of practical applications such as the synchronization of two FOCOs, as already shown in [36–39]. It can also be extended to implement fractional derivatives of arbitrary order [40], and fractional Proportional-Integral-Derivative controllers [41,42].

The rest of this paper is organized as follows: Section “Simulation of fractional-order chaotic oscillators” shows the dynamical analysis of the two FOCOs that are the cases of study herein. Their minimum fractional-order is estimated from their eigenvalues and their phase-space portraits are simulated using FDE12 [43] within MatLab. We use the transfer functions that approximate fractional-order integrators given in [31], and Section “Approximation of 1/\( s^0 \) using first-order active filters” shows their implementation using first-order active filters. Section “Design of FOCOs using first-order active filters” shows the simulation of the two FOCOs given in Section “Simulation of fractional-order chaotic oscillators” using first-order active filters designed by voltage amplifiers. Section “FPAA-based implementation of FOCOs” shows the complete design of the two FOCOs using an FPAA and their experimental attractors are observed in an oscilloscope. It is highlighted that both FOCOs require multipliers and amplifiers, and the first-order active filters can easily be implemented using the embedded blocks into the FPAA, which also require amplifiers. Finally, Section “Conclusions” gives the conclusions.

Simulation of fractional-order chaotic oscillators

This section shows the dynamical analysis and simulation of two FOCOs. The first one is called FOCO1 and it is based on Chen’s oscillator [44,45], which fractional-order mathematical model is given as [46],

\[
\begin{align*}
\frac{d^q y}{dt^q} &= a(y(t) - x(t)), \\
\frac{d^q x}{dt^q} &= (c - a)x(t) - x(t)z(t) + cy(t), \\
\frac{d^q z}{dt^q} &= x(t)y(t) - bz(t).
\end{align*}
\]

where \((a, b, c) \in \mathbb{R}^3\), and their values to generate chaotic behavior are set to \((a, b, c) = (35, 3, 28)\). This FOCO1 has three equilibrium points: \(E_P = (0.0, 0.0, 0.0)\), \(E_{P1} = (7.9373, 7.9373, 21)\), and \(E_{P2} = (-7.9373, -7.9373, 21)\). The Jacobian of the FOCO1 is given in (2), and must be evaluated at the three equilibrium points \(EP = (x', y', z')\), which provide the eigenvalues listed in (3).

\[
J(x', y', z') = \begin{bmatrix}
-a & a & 0 \\
-c & -a - z' & c & -x' \\
y' & x' & -b
\end{bmatrix}
\]

\[
\begin{align*}
E_{P1} \Rightarrow \lambda_{(1,2,3)} &= (-3.23.8359, -3.8359) \\
E_{P2} \Rightarrow \lambda_{(1,2,3)} &= (-18.4280, 4.2140 \pm 14.8846) \\
E_{P3} \Rightarrow \lambda_{(1,2,3)} &= (-18.4280, 4.2140 \pm 14.8846)
\end{align*}
\]

A FOCO guarantees chaotic behavior if its eigenvalues accomplish the relationship given in (4), where \(q\) denotes the fractional-order [47–50]. In the case of the FOCO1 and setting \((a, b, c) = (35, 3, 28)\), the minimum commensurate \((q_1 = q_2 = q_3 = q)\) fractional-order is \(q \geq 0.8244\). This means that the FOCO1 given in (1) can generate chaotic behavior if \(q_1 = q_2 = q_3 = 0.9\). By applying FDE12 [43], the phase-space portraits of the FOCO1 are shown in Fig. 1.

\[
q \geq \frac{2}{3} \arctan \left( \frac{|m|}{|n|} \right)
\]

The second case of study is named FOCO2, its mathematical model was introduced in [51], and its fractional-order description is given by (5). It has one quadratic term and three positive real constants that are set to \((a, b, c) = (2.05, 1.12, 0.4)\). This FOCO2...
has two equilibrium points: \( EP_1 = (0, 0, 0) \) and \( EP_2 = (-1, 0, 0) \). The Jacobian matrix is given in (6). For the \( EP_1 = (0, 0, 0) \), the eigenvalues are: \( \lambda_1 = -0.745 \) and \( \lambda_2 = -0.162 \pm j1.147 \). For the equilibrium point \( EP_2 = (-1, 0, 0) \), and for \( (a, b, c) = (1, 1.1, 0.42) \), the eigenvalues are: \( \lambda_1 = -0.589 \) and \( \lambda_2 = 0.504 \pm j1.2 \), this implies chaotic behavior [52]. The minimum fractional-order for (5) when \( (a, b, c) = (2.05, 1.12, 0.4) \), and according to (4), is equivalent to \( q \geq 0.879 \). In this paper we set \( q = 0.9 \) so that the phase-space portraits of (5) are shown in Fig. 2.

\[
\begin{align*}
\alpha D_q^0 x(t) & = y(t), \\
\alpha D_q^0 y(t) & = z(t), \\
\alpha D_q^0 z(t) & = -\alpha x(t) - by(t) - cz(t) - \lambda^2(t).
\end{align*}
\]

For the equilibria \( EP \) considered, the Jacobian matrix is given in (6). For \( EP_1 \), the eigenvalues are: \( \lambda_1 = -0.589 \) and \( \lambda_2 = 0.504 \pm j1.2 \), therefore the phase portraits are shown in Fig. 2.

Amplitude scaling of FOCO1

The following sections show the implementation of the FOCOs using voltage amplifiers and the FPAA. However, they have ranges of operation below the amplitudes shown in Fig. 1. For example, the FPAA AN231E04 QuadApex from Anadigm processes signals within \( \pm 3 \) V. In this manner, the FOCO2 can directly be implemented using this FPAA because according to Fig. 2, the ranges for \( x, y, \) and \( z \) are within \( \pm 3 \). However, the chaotic time series of the state variables of FOCO1 are within the ranges \( x = [-25, 25], y = [-27, 27], \) and \( z = [7, 45] \), so that they must be down-scaled to be within \( \pm 3 \). This is done by scaling the amplitude of the state variables of FOCO1 by \( k = 1/18 \), as follows:

\[
\begin{align*}
x_1 = kx, \quad x_2 = ky, \quad x_3 = kz.
\end{align*}
\]

Therefore, the scaled FOCO1 is updated to

\[
\begin{align*}
\alpha D_q^0 x_1(t) & = a(x_2 - x_1), \\
\alpha D_q^0 x_2(t) & = c - ax_1 - 18x_1x_3 + cx_2, \\
\alpha D_q^0 x_3(t) & = 18x_1x_2 - bx_3.
\end{align*}
\]

Approximation of \( 1/s^\alpha \) using first-order active filters

The fractional-order operator \( q \) can be approached by a rational transfer function of the form \( H(s) = \frac{1}{s^\alpha} \), as it is introduced by Charef and described in [31]. Let us consider the FOCO1 given in (1), it can be implemented using fractors or fractances as already shown in [1,28,55]. However, they are difficult to implement due to the combinations among RC interconnections, and also, it can result in a huge number of resistors and capacitors that may be difficult to design using CMOS technology. In this manner, we show the approximation of \( H(s) = \frac{1}{s^\alpha} \), which is the fractional-order used to implement (1) and (5). The authors in [31] generate the rational

\[
D_{KY} = k + \sum_{i=1}^{n-k} \frac{\lambda_i}{\lambda_{i+1}}
\]

Table 1 shows the values of the Lyapunov exponents and \( D_{KY} \) of FOCO1 and FOCO2 using the initial conditions \( (x(0), y(0), z(0)) = (-9, -5, 14) \), and \( (x(0), y(0), z(0)) = (0.1, 0.0, 0.0) \), respectively.
transfer functions of the fractional-order integrator \( \frac{1}{s^q} \) for \( 0.1 < q < 0.9 \) and evaluate it at steps of 0.1. The resulting \( H(s) \) in all cases consists of the ratio of two polynomials in the Laplace domain with integer orders, and they guarantee a maximum deviation of 2dB, and a bandwidth of \( \omega_{\text{max}} = 10^3 \text{ rad/s} \). Eq. (10) shows the rational transfer function that approximates \( H(s) = \frac{1}{s^q} \). In electronics, it can be implemented by cascading three first-order active filters, associated to \( H_1(s) \) and \( H_2(s) \) that have one pole and one zero, and \( H_3(s) \) that has only one pole, as it is sketched in (11).

\[
H(s) = \frac{1}{s^q} \approx \frac{2.2675(s + 1.292)(s + 215.4)}{(s + 0.01292)(s + 2.154)(s + 359.4)} \tag{10}
\]

\[
H_1(s) = \frac{s + 215.4}{s + 359.4} \quad H_2(s) = \frac{s + 1.292}{s + 2.154} \quad H_3(s) = \frac{2.2675}{s + 0.01292} \tag{11}
\]

The transfer functions in (11) can be implemented by first-order active filter topologies [56], but one must be aware that the order of the poles and zeros from (10) matters when they are implemented with electronic circuits. For instance, they are already ordered from the higher to the lower pole in (11). In this manner, \( H_1(s) \) and \( H_2(s) \) can be implemented by the active filter topology shown in Fig. 3, having the transfer function given in (12), in which one can derive the design equations given in (13). Therefore, to design \( H_1(s) \), the circuit elements are set to: \( R_1 = R_2 = 1 \text{k} \Omega, C_1 = 0.28 \mu \text{F} \) and \( C_2 = 1 \mu \text{F} \), and to design \( H_2(s) \) they are set to: \( R_1 = R_2 = 10 \text{k} \Omega, C_1 = 0.1 \mu \text{F} \) and \( C_2 = 7.1 \mu \text{F} \).

\[
H_{12}(s) = \frac{V_o}{V_i}(s) = -\frac{1}{K} \frac{s + z_1}{s + p_1} = -\left(\frac{R_2}{R_1}\right) \frac{s + \frac{1}{R_2 C_2}}{s + \frac{1}{R_1 C_1}}, \quad K = \frac{R_2}{R_1} = 1 \tag{12}
\]

\[
C_1 = \frac{1}{R_1 p_1}, \quad C_2 = \frac{1}{R_2 z_1} \tag{13}
\]

The transfer function \( H_2(s) \) from (11) can be designed by using the first-order active filter topology shown in Fig. 4, which has the transfer function given in (14), and the corresponding design equations are given in (15). Therefore, the circuit elements are set to: \( R_1 = 441.14 \text{k} \Omega, R_2 = 77.3 \text{M} \Omega \) and \( C_1 = 1 \mu \text{F} \).

\[
H_2(s) = \frac{V_o}{V_i}(s) = -\left(\frac{R_2}{R_1}\right) \frac{s + \frac{1}{R_2 C_2}}{s + \frac{1}{R_1 C_1}} \tag{14}
\]

\[
R_2 = \frac{K}{p_1}, \quad C_1 = \frac{1}{R} \tag{15}
\]

Fig. 5 shows the connection of the first-order active filters to implement (10). However, a scaling procedure must be applied to generate signals in the range allowed by the commercial operational amplifiers that can be AD712/LM3324/TL082. The FOCO1 has signals in the ranges up to 50 for the state variable \( z \), according to the simulation results shown in Fig. 1, so that the scaling process...
must down the amplitudes to be within ±12 Volts, for example. The FOCO2 does not have any design problem because the amplitudes are below 1, as shown in the simulation results in Fig. 2.

**Design of FOCOs using first-order active filters**

This section shows the block diagram descriptions of both FOCO1 and FOCO2, they are implemented with operational amplifiers and multipliers, and the fractional-order integrator is designed by using the topology shown in Fig. 5. The FOCO1 given in (1) can be described as shown in Fig. 6, where \( H(s) \) is the approximation of \( \frac{1}{s^9} \) given in (10), and which is implemented by first-order active filters that are connected as shown in Fig. 5. This block diagram description in Fig. 6 is associated to the equations in the Laplace domain given in (16). This FOCO1 is implemented as shown in Fig. 7, where the multiplier is AD633, with an output coefficient set to 0.1, biased with \( R \) shown in Fig. (8) by using Multisim 14.2 from National Instruments.

\[
\begin{align*}
s^0X(s) &= a(Y(s) - X(s)), \\
s^0Y(s) &= (c - a)X(s) + Z(s) + cY(s), \\
s^0Z(s) &= X(s) + Y(s) - bZ(s).
\end{align*}
\]

The FOCO2 given in (5) has the block diagram description shown in Fig. 9, with is associated to the equations in the Laplace domain given in (17).

\[
\begin{align*}
s^0X(s) &= Y(s) \\
s^0Y(s) &= Z(s) \\
s^0Z(s) &= -aX(s) - bY(s) - cZ(s) - X(s) + bZ(s)
\end{align*}
\]

The electronic circuit of the FOCO2 is shown in Fig. (10), it also requires the use of the multiplier AD633 with an output coefficient set to 0.1, biased with ±12 V, and the resistances values are set to: \( R_1 = R_2 = R_4 = R_5 = R_6 = R_7 = 2 \) kΩ, \( R_2 = 7.5 \) kΩ, \( R_{15} = R_{21} = 1 \) kΩ, \( R_{14} = 20 \) kΩ, and \( R_{17} = 715\Omega \). The simulation results of the electronic circuit are shown in Fig. (8) by using Multisim 14.2 from National Instruments.

**FPAA-based implementation of FOCOs**

The implementation of the FOCO1 given in (1), and FOCO2 given in (5), using commercial operational amplifiers leads us to a very huge Printed Circuit Board design, and the discrete elements can generate errors due to their tolerances. For this reason, the use of an FPAA is more adequate to implement them. In this section we show their complete circuit design into the FPAA Anadigm QuadA-plex Development Board AN231E04 [57]. In this manner, the approximation of \( \frac{1}{s^9} \) given in (11), can be implemented in the FPAA using Configurable Analog Modules (CAMs) that are known as: CAM Low-Pass Bilinear Filter having the transfer function \( T_{lp}(s) \) given in (18) to synthesize \( H_1(s) \), and the CAM Pole and Zero Bilinear Filter having the transfer function \( T_{pz}(s) \) given in (19), to synthesize \( H_1(s) \) and \( H_2(s) \).

**Fig. 4.** First-order active filter to implement \( H_1(s) \) in [11].

**Fig. 5.** Implementation of \( \frac{1}{s^9} \) that is approached in (11) by the cascade connection of three first-order active filters to implement \( H_1(s)H_2(s)H_3(s) \).

**Fig. 6.** Block diagram description of the FOCO1 given in (1), where \( H(s) \) is the approximation \( \frac{1}{s^9} \) given in (10).

**Fig. 7.** Circuit implementation of the block diagram shown in Fig. 6.
\[ T_p(s) = \frac{w_o G}{(s + w_o)} \]  \hspace{1cm} (18)

\[ T_{pz}(s) = \frac{G_{hi}(s + w_z)}{(s + w_p)} \]  \hspace{1cm} (19)

In the FPAA, the integration constants are of the type $1/RC$, and automatically, the Anadigm development tool associates units in $1/\mu s$, so that one deals with $10^{-6}/RC$. Therefore, combining $T_{pz}(s)$ and $T_p(s)$ to implement (11), one gets (20), where $w = 2\pi f$, and then the associated poles are evaluated in (21), and the zeros in (22). In these equations, $f_p$ is the frequency of the first pole in $H_1(s)$ from (11), $f_{p2}$ is the second pole, and $f_o$ is the frequency of the third pole. The cascade connection of these CAMs is shown in Fig. 12.

\[ T(s) = \frac{G_{hi}(w_{p1} + s)}{(w_{p1} + s)} \cdot \frac{G_{h2}(w_{p2} + s)}{(w_{p2} + s)} \cdot \frac{w_o G}{w_o + s} \]  \hspace{1cm} (20)
As mentioned in the previous Section, the whole implementation of the FOCO1 given in (1) requires a scaling of the amplitudes because the FPAA AN231E04 drives signals in the range \( \pm 3 \) V. This FOCO1 requires the use of 17 CAMs within the FPAA, they are multipliers, adders, inverters and bilinear filters. The FPAA embeds four AN231E04 chips, and each one has eight CAMs. The synthesis process to implement the FOCO1 begins by implementing the fractional-order integrator that is approximated by (11). Afterwards, one calculates the parameters of the multipliers, adders, inverters, bilinear filters, and the clock frequencies that are required by the CAMs. From the circuit diagram shown in (7), one choose the type of input inverter or no-inverter in the CAMs (Sum/Difference). The multipliers to evaluate \( xy \) and \( xz \) in (1) requiere two Clocks (A and B), where the relation is that Clock B is 16 times Clock A. Recall that the design of \( \frac{1}{s} \) is performed as it is shown in Fig. 12.

The whole FPAA-based implementation of the FOCO1 given in (1) is shown in Fig. 13, which generates the experimental attractors shown in Fig. 14.

The FOCO2 given in (5) requieres less amplifiers, and the whole FPAA-based implementation is given in Fig. 15, and the experimental fractional-order chaotic attractors are shown in Fig. 16.

### Conclusions

This paper showed the implementation of fractional-order chaotic oscillators (FOCOs) using operational amplifiers and field-
programmable analog array (FPAA). The design process was performed in the frequency domain, for which the FOCOs were simulated with a fractional-order of the derivatives equal to $q = 0.9$. Two cases of study were chosen and named FOCO1 and FOCO2. The fractional-order integrator was approximated by a rational ratio of polynomials in the Laplace domain, and it resulted in the cascade connection of three first-order blocks, which were implemented by first-order active filter topologies. The filters were designed using operational amplifiers, but nowadays that topologies can be implemented as Gm-C topologies using CMOS technology. The whole design of both FOCO1 and FOCO2 was also performed using an FPAA, which embeds four chips and each one has eight Configurable Analog Modules (CAMs). The experimental observation of the attackers generated by the FPAA-based implementation of both FOCO1 and FOCO2, demonstrates that they can be used in applications like chaotic secure communications systems, and more FOCOs can be designed into an FPAA to take advantage of its dynamic reconfiguration and reprogrammability abilities. In addition, the designed circuits that are based on first-order active filters can also be transformed to topologies using operational transconductance amplifiers (Gm-C first-order active filters) that allow a monolithic integration.

**Compliance with Ethics Requirements**

This article does not contain any studies with human or animal subjects.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Fig. 16. Experimental observation of the chaotic attractors of the FPAA-based implementation of the FOCO2 shown in Fig. 15, with axes: 500 mV/Div.
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