SHOCK CORRUPTION BY RAYLEIGH–TAYLOR INSTABILITY IN GAMMA-RAY BURST AFTERGLOW JETS

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ABSTRACT
Afterglow jets are Rayleigh–Taylor unstable and therefore turbulent during the early part of their deceleration. There are also several processes which actively cool the jet. In this Letter, we demonstrate that if cooling significantly increases the compressibility of the flow, the turbulence collides with the forward shock, destabilizing and corrugating it. In this case, the forward shock is turbulent enough to produce the magnetic fields responsible for synchrotron emission via small-scale turbulent dynamo. We calculate light curves assuming the magnetic field is in energy equipartition with the turbulent kinetic energy and discover that dynamic magnetic fields are well approximated by a constant magnetic-to-thermal energy ratio of 1%, though there is a sizeable delay in the time of peak flux as the magnetic field turns on only after the turbulence has activated. The reverse shock is found to be significantly more magnetized than the forward shock, with a magnetic-to-thermal energy ratio of the order of 10%. This work motivates future Rayleigh–Taylor calculations using more physical cooling models.

Key words: gamma-ray burst: general – hydrodynamics – ISM: jets and outflows – radiation mechanisms: non-thermal – shock waves – turbulence

Online-only material: color figure

1. INTRODUCTION
Magnetized relativistic jets are important astrophysical phenomena, most notably in the context of gamma-ray bursts (GRBs), but also in active galactic nuclei and tidal disruption events. As a result, the dynamics of relativistic jets have been studied extensively, often in terms of the GRB central engine (MacFadyen & Woosley 1999; Aloy et al. 2000; Morsony et al. 2007; Komissarov & Barkov 2009; McKinney et al. 2012; López-Cámara et al. 2013), but also in the largely engine-independent afterglow phase when ejecta accelerated by the central engine has transferred its energy to a collimated blast wave (Rhoads 1999; Kumar & Panaitescu 2000; Granot et al. 2001; Panaitescu & Kumar 2002; Lütyikov & Blandford 2002; Zhang et al. 2003; Peng et al. 2005; Granot 2007; van Eerten et al. 2010; De Colle et al. 2012). Given these extensive studies, there are still many fundamental questions which remain unanswered. For example, afterglow jets are thought to be magnetized, as synchrotron emission necessitates a strong magnetic field, yet no clear mechanism has been demonstrated which robustly generates such a field. Additionally, current jet models are parameterized by a small handful of parameters (van Eerten & MacFadyen 2012), which would seem to suggest a straightforward standardization of GRB afterglow light curves. However, GRB afterglows display a great deal of variety and variability, especially at early times, hence there likely exist additional important elements missing from simplified hydrodynamical models.

One avenue which potentially addresses these issues is vorticity generation behind the forward shock. Vorticity could both amplify magnetic fields via turbulent dynamo and produce variability in GRB light curves. Understanding where vorticity comes from and how much is present will help to complete the picture of how relativistic jets generate afterglow emission.

The source of vorticity is still unclear, but several mechanisms have been suggested. One possibility is small-scale Weibel instabilities in the plasma particles making up the shock itself (Spitkovsky 2008). However, such instabilities may have a short range of influence. Alternatively, vorticity can be generated when a shock overtakes high-density clumps in the interstellar medium (ISM; Sironi & Goodman 2007; Goodman & MacFadyen 2008), but it is unclear whether large enough clumps exist to make this a robust mechanism.

In this work, we consider the vorticity generated by Rayleigh–Taylor (RT) instability, as first suggested by Levinson (2009). After a GRB ejects a relativistic flow (ejecta), it expands and its thermal energy drops adiabatically until it is subdominant to the kinetic energy. The ejecta then coasts and becomes a very thin shell with width $\Delta r \sim 1/G^2$, where $G$ is the Lorentz factor (Kobayashi et al. 1999). When deceleration finally occurs, shocks are generated at the interface between ejecta and ISM. A forward shock pushes its way into the ISM, and a reverse shock pushes its way back into the ejecta. In the heated region between these two shocks resides the contact discontinuity, separating ejecta from ISM. This contact discontinuity is RT unstable.

Nonrelativistic RT-unstable outflows were first studied by Chevalier et al. (1992), both analytically and numerically. Jun & Norman (1996) later performed a two-dimensional (2D) magnetohydrodynamics calculation which demonstrated how magnetic fields tend to align themselves along RT fingers. More recently, Ferrand et al. (2010) and Wang (2011) have demonstrated the importance of various microphysical processes at the shock front, and Fraschetti et al. (2010) has performed three-dimensional (3D) numerical calculations. To extend the nonrelativistic results into the relativistic regime, Levinson (2010) performed a stability analysis on the two-shock solution (Nakamura & Shigeyama 2006) and found linear growth rates which could potentially be large enough to impact the forward shock.

In the first numerical studies of the relativistic case, Duffell & MacFadyen (2013b) found that RT generates turbulence which could amplify magnetic fields to within a few percent of equipartition with the thermal energy density. However,
In that work we found the turbulence remained confined within a region behind the forward shock and did not impact the forward shock, though turbulence did penetrate part of the energetic post-shock region.

In this Letter, we demonstrate that it is possible for the RT turbulence to collide with the forward shock. As a result, the shock is perturbed and corrugated and significant turbulence is present everywhere behind it. This turbulence persists for a long time, until the shock becomes nonrelativistic, possibly due to the non-universality of the Blandford–McKee solution (Gruzinov 2000).

The key ingredient allowing the turbulence to collide with the forward shock is a softer equation of state. In fact, a softened equation of state has already been invoked in the nonrelativistic case to explain how RT fingers can catch up to the forward shock (Blondin & Ellison 2001; Ferrand et al. 2010; Wang 2011). In this case, the collision of the ejecta with the ISM with density $\rho_0$ was estimated to emit $\sim 10^{51}$ erg in X-rays (Racusin et al. 2008; Bloom et al. 2009), which should be a non-negligible fraction of the energy in the blast wave. If this cooling is responsible for reduced pressure in the shock, though turbulence did penetrate part of the energetic post-shock region and did not impact the forward shock, as a result, the shock is relativistic. Thus, the shock will continue to be corrugated as long as the relevant cooling processes are effective at softening the equation of state. The choice of $\gamma = 1.1$ to represent effects of cooling is a proof-of-concept which motivates further study using a more accurate cooling prescription.

2. NUMERICAL SET-UP

Our study entails numerically integrating the equations of relativistic hydrodynamics,

$$\partial_\mu (\rho u^\mu) = 0,$$  \hspace{1em} (1)

$$\partial_\mu ((\rho + \epsilon) u^\mu u^\nu + P g^{\mu\nu}) = 0,$$  \hspace{1em} (2)

where $\rho$ is proper density, $P$ is pressure, $\epsilon$ is the internal energy density, and $u^\mu$ is the four-velocity. We employ an adiabatic equation of state:

$$P = (\gamma - 1)\epsilon,$$  \hspace{1em} (3)

and we use relativistic units such that $c = 1$.

We write the equations in spherical coordinates, and assume axisymmetry so that our calculation is 2D. Three-dimensional effects may also be important, as in the nonrelativistic case it has been found that the instability’s growth is 30% larger in 3D than in a 2D calculation (Fraschetti et al. 2010). Thus, the 3D case will be an interesting complement to this work which we plan to address in the future.

In order to track the “ejecta” and “ISM” components of the flow, we also evolve a passive scalar, $X$, according to

$$\partial_\mu (\rho Xu^\mu) = 0.$$  \hspace{1em} (4)

Initially, we choose $X = 0$ in the ISM and $X = 1$ in the ejecta. This passive scalar is helpful for visualizing the turbulent mixing of ejecta with the ISM (Figure 1).

The calculation is performed using a novel moving-mesh code, JET (Duffell & MacFadyen 2011, 2013b). The JET code uses high-resolution shock-capturing methods, and is effectively Lagrangian in the radial dimension due to the radial motion of grid cells. In this study we use a resolution of $N_r = 800$ zones in polar angle, (meaning $\Delta \theta = 1.25 \times 10^{-4}$) and roughly $N_r \sim 8000$ zones radially. Previously we demonstrated accurate convergence of the JET code for the relativistic RT problem (Duffell & MacFadyen 2013b).

2.1. Initial Conditions

The system is parameterized by an explosion energy $E$, ejecta mass $M$, and ISM density $\rho_0$. We define a characteristic Lorentz factor $\Gamma = E/M$. For expediency we choose $\Gamma = 30$, and the constants $E$ and $\rho_0$ simply scale out of the problem, due to the scale invariance of the underlying hydrodynamical field equations. Our initial conditions are of a cold expanding flow with kinetic energy $E$ and mass $M$ pushing its way into an ISM with density $\rho_0$. It begins long before a significant amount of the ISM has been swept up, at time $t_0 = 0.01 l$, where $l \equiv (E/\rho_0)^{1/3}$ is the Sedov length. This almost totally specifies the problem, except for the overall shape of the ejecta density profile, which we prescribe as follows based on one-dimensional (1D) numerical calculations of relativistic fireballs (Duffell & MacFadyen 2013a):

$$\rho(r, t_0) = \begin{cases} \frac{E}{\pi \nu_{l/0}^{3/2} \rho_0} & r < R \\ \rho_0 & \text{otherwise} \end{cases} \quad (5)$$

$$v(r, t_0) = \begin{cases} \frac{r}{t_0} & r < R \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$P(r, t_0) \ll \rho(r, t_0). \quad (7)$$

$$\frac{E}{\pi \nu_{l/0}^{3/2} \rho_0}$$
Rayleigh–Taylor turbulence does not collide with the forward shock in the 4/3 case, where we have defined

\[ R = l_\theta \left( 1 - \frac{1}{4l^2} \right). \]  

Our domain is axisymmetric, and extends from \( \theta = 0 \) to \( \theta = 0.1 \) with a reflecting boundary at \( \theta = 0.1 \). This angular size was chosen to represent a patch of a spherical outflow. During early times in the jet’s evolution, while the Lorentz factor is larger than the inverse of the opening angle, causality prevents this choice of opening angle from making any difference in the dynamics. At late times, it is possible that jet spreading introduces an important dynamic to the turbulence, which we do not attempt to capture here.

3. RESULTS

There is a clear difference in the dynamics between the \( \gamma = 4/3 \) and the \( \gamma = 1.1 \) case in Figure 1. In the \( \gamma = 4/3 \) case, the instability collides with the reverse shock, but does not overtake the forward shock. In the \( \gamma = 1.1 \) case, the softer equation of state results in lower pressures which allow the RT turbulence to collide with the forward shock, corrugating it and pushing it forward. The entire heated region between forward and reverse shocks is turbulent. The corrugated shock front also causes further vorticity generation due to shock obliquity. In our calculation, we did not see any re-stabilization of the forward shock, which suggests that the turbulence should persist for as long as the soft equation of state is valid.

In Figure 2 we plot a snapshot of the 1D profile at \( t = l/\Gamma^{1/3} = 0.316l \). Here we look at the \( \gamma = 1.1 \) case, comparing the turbulent 2D calculation with a 1D calculation performed assuming spherical symmetry. In 1D, we clearly see the forward shock, reverse shock, and contact discontinuity. In 2D, the contact discontinuity is totally disrupted, and the reverse shock has been pushed back further into the ejecta. For the 2D results, we plot a spherically averaged proper density, and additionally we plot the density measured along the radial line at \( \theta = 0.05 \). We see the turbulent variability exists everywhere between the forward and reverse shocks.

Turbulence quickly amplifies magnetic fields to rough equipartition with the turbulent kinetic energy density (Haugen et al. 2003; Schekochihin et al. 2004; Beresnyak 2012; Zrake & MacFadyen 2013). Because this turbulence is present all the way up to the forward shock, magnetic fields amplified by the turbulence will facilitate synchrotron emission by the hot electrons in and behind the shock front.

Following the same strategy as in our previous work (Duffell & MacFadyen 2013b), we estimate the magnetic field strength using \( \epsilon_{\text{turb}} \), the ratio of turbulent energy to thermal energy.

\[ \epsilon_B \sim \epsilon_{\text{turb}}. \]  

where \( \epsilon_B \) is the local ratio of magnetic to thermal energy and \( \epsilon_{\text{turb}} \) is the local ratio of turbulent to thermal energy. We calculate this ratio using essentially the same formula as in the previous work:

\[ \epsilon_{\text{turb}} = \frac{(\gamma - 1)(\rho)_{\text{cons}} - (P)_{\text{cons}} - (P)_{\text{vol}}}{(P)_{\text{vol}}}, \]  

where brackets denote an average over angle, and the subscript “vol” implies a simple volume average, whereas “cons” implies a conservative average (mass, energy, and momentum are averaged, and proper density and pressure are calculated from these conserved quantities).
band, at 1018 Hz. First, the 2D profiles are spherically averaged.

shock is significantly more magnetized than the forward shock, synchrotron emission. At the shocks, the magnetization is the same place where we hot electrons are expected to produce the magnetization are at the forward and reverse shocks, the facilitate synchrotron emission. Third, the largest values of the magnetization are at the forward and reverse shocks, the largest values of the magnetization are at the forward and reverse shocks.

The most remarkable part of Figure 3 is the right-hand side. In the gray curve corresponding to $\epsilon_B = \text{constant}$, the initial rise is due to the transition from a coasting to decelerating shell. This peak occurs at time $t_\gamma \sim \Gamma^{-2/3}$. In the case where $\epsilon_B = \epsilon_{\text{turb}}$, the dynamics are identical, but the magnetic field does not turn on until a later time, about a factor of 5 later in observer time, $t_\gamma^{\text{obs}} = 5t_\gamma^{\text{obs}}$ (we did not check the scaling with $\Gamma$ as we only performed the $\Gamma = 30$ case). This means that radiation from an outflow with initial Lorentz factor $\Gamma_0$ will peak at a time $t_B$, when the shell has decelerated to a Lorentz factor $\Gamma_B < \Gamma_0$ ($\sim 15$ in our case). If this peak were interpreted as occurring at $t_\gamma$, the Lorentz factor of the ejecta will be incorrectly estimated to be $\Gamma_B$ instead of $\Gamma_0$. This point may be important for jet models with a baryon-loaded component (Pedersen et al. 1998; Ramirez-Ruiz et al. 2002), and in general for the interpretation of early-time plateaus in GRB light curves (Nousek et al. 2006; Zhang et al. 2006; van Eerten 2014).

4. DISCUSSION

We demonstrate that RT instabilities can generate vorticity and magnetic fields in GRB afterglow jets. The only ingredient necessary for this mechanism is an equation of state which is softer than the usual $\gamma = 4/3$ model. This soft equation of state represents a mechanism for energy loss which reduces the pressure in the forward shock so that RT fingers can collide with it. Several processes occur at the forward shock which act to cool it; cosmic rays and radiation, for example, may carry significant energy away from the shock. Regardless of what cools the shock front, this seemingly benign change to the dynamics can completely change the structure and magnetization of the blast wave.

We estimate a magnetic energy fraction of $\epsilon_B \sim 1\%$ in the forward shock, and $\sim 10\%$ in the reverse shock. We show that the choice $\epsilon_B = \text{constant} = 0.01$ agrees surprisingly well with late-time afterglow calculated assuming a local value of $\epsilon_B = \epsilon_{\text{turb}}$, although this result may change when more accurate models for cooling are employed in the calculation. Finally, we show that this magnetic field does not turn on until an observer time later than the deceleration time (a factor of five later in our $\Gamma = 30$ case). This occurs at a time when the shock has decelerated to a Lorentz factor lower than its original value. This could be related to the cause of observed early-time plateaus in GRB afterglows.

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