Presumable applications of cellular automates with strong anticipation in quantum physics

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Abstract. The paper discusses the use of models with discrete geometry in field theory problems. Approaches of cellular automata in problems of quantum field theory are considered. The equations of the model of cellular automata are given taking into account the property of strong anticipation (according Daniel M. Dubois) for such objects and the possible consequences in the interpretation of such solutions are discussed. The illustrations of the behavior of such systems and the possible directions of subsequent research are given.

1. Introduction
Geometry, beginning with the development of modern science, and especially since the time of Newton enters theoretical physics in one way or another. One of the key questions is the choice of spaces, in terms of which they formulate the corresponding equations and sets of values that can take physical quantities and variables.

Up to now, usually the basic equations are partial differential equations. It is important to emphasize that with this description it is assumed that the independent variables vary continuously (spatial, temporal). Further, unknown functions, as a rule, were applied continuous or smooth. This made it possible to use the whole arsenal of mathematical analysis, the theory of differential equations, including their symmetry analysis. The standard reference can be considered the book of W. Tirring [1].

However, especially recently, changes in the methods of describing the corresponding mathematical objects began to appear. Examples include p-adic physics (I.Volovich, A.Khrennikov), non-standard analysis (A.Robinson), superphysics (E.Witten), application of category theory [2], physical structures (Yu.Kulakov, Yu.Vladimirov) and others. However, among all the various generalizations, it is necessary to note a direction that combines approaches that take into account both continual and discrete aspects. As is well known [3,4], the discreteness in quantum mechanics has various aspects, beginning with the fact that there exists a minimal (Planck) length ($10^{-35}$ m). Using the presence of this scale of length, models are constructed on a lattice with certain limiting transitions with a decrease in the lattice step (for example, [5,6]).

But relatively recently (around the 1980s and 1990s), a new direction in the theoretical physics of the micro and macro world has emerged, connected with so-called cellular automata. The very idea of cellular automata was born at the intersection of computer science, biology, physics (see [7–11]). We recall briefly the idea of classical cellular automata (see [7–9]). The space is divided into many identical cells in the case of a plane, cubes in three-dimensional space, and so on. Each of the elements
has a finite set of states. In the simplest case, each cell has 2 states. States can change with time in discrete steps, and the changes in states themselves follow rules that are based on knowing the state of a given cell and cells in some neighborhood of it (for example, for a plane: the Neumann neighborhood is 4 cells or 8 cells in the Moore neighborhood). One of the main examples is the game "life" [7–9]. Following the work on cellular automata, we can formally write this as follows:

2. Description of classical cellular automata

Here we give a formal description of cellular automata with anticipation. First, we describe the classical cellular automata (hereafter abbreviated CA) from articles and books on spacecraft (see for example [7–14]). For simplicity, we give here only a description in the case of a one-dimensional spacecraft from [14, p. 2], but the description can easily be adapted to the multidimensional case.

One-dimensional spacecraft is an array of cells (cells) $x_i$ where (integers) $i \in \Sigma$ and each $x$ takes a value from a finite alphabet $\Sigma$. Thus, a sequence of cells $(x_i)$ of finite length $n$ represents a string or global configuration $c$ on $\Sigma$. In this way, many finite configurations will be represented as $\Sigma^n$. Evolution is represented by a sequence of configurations $\{c^t\}$ that leads to the mapping $\Phi: \Sigma^n \rightarrow \Sigma^n$. Thus, their global map has the form

$$\Phi(c^t) \rightarrow c^{t+1},$$

(1)

Where $t$ the time step and each global state is determined by a sequence of cell states. Also, the state of the cells in the configuration $c^t$ is updated to the next configuration $c^{t+1}$ simultaneously using the local function $\varphi$ as follows

$$\varphi(x_{i-r}^t, \ldots, x_i^t, \ldots, x_{i+r}^t) \rightarrow x_i^{t+1}.$$  

(2)

Also, for comparison, we describe the spacecraft with memory from [14, p. 3]: A spacecraft with memory extends the standard description of spacecraft by allowing for each cell $x_i$ to remember some part of the previous evolution. Thus, for the implementation of memory, a memory function is constructed, $\phi$:

$$\phi(x_{i-r}^{t-\tau}, \ldots, x_i^{t-\tau}, x_i^t) \rightarrow s_i.$$

(3)

Such that $\tau < t$ determines the degree of memorization and each cell $s_i \in \Sigma$ is a function of the state of a series of cell states $x_i$ before a given instant of time. To determine the evolution, we apply the original rule of the form:

$$\varphi(\ldots, s_{i-3}^t, s_{i-1}^t, s_{i+1}^t, \ldots) \rightarrow x_i^{t+1}.$$  

(4)

In a spacecraft with memory, if the mapping $\phi$ remains unchanged, the historical memory of all past iterations is contained in the use of all past iterations of the mapping. $\phi$.

3. Cellular automata in quantum physics

Quite recently interest to such concepts was initiated by different authors — (see [15–23]). The basic idea was the use of discreteness at the quantum level (Planck length). Then quantum machines are written in such a way that for limiting transitions with decreasing cell size, the equations of quantum mechanics are obtained in the limit. Let us note that one of the motivations for attracting the concept of cellular automata to the above problems of theoretical physics is the probable possibility of finding ways for combining quantum mechanics and gravitation [18, 24]. The formulations of models of cellular automata for problems of quantum physics have different forms depending on the initial
axioms, the technique used and the purpose of the research. For a description of some ideas, we give here variants of the equations used.

So in the works of t’ Hooft [17,18,24] the following version was adopted

$$\sigma(x,t) = \sigma(x-1,t-1)\sigma(x+1,t-1).$$ (5)

As a result, the unitarity of the evolution of systems was deduced and considerations were raised about the manifestation of probabilistic regularities in deterministic systems.

In [19], cellular automata of the form

$$\varphi(i, j, k, t + \Delta t) = W_{+++}\varphi(i + 1, j + 1, k + 1, t) +$$

$$+ W_{++-}\varphi(i + 1, j + 1, k - 1, t) + \ldots + W_{---}\varphi(i - 1, j - 1, k - 1, t),$$ (6)

where $\varphi(i, j, k, t)$ is the value of the field variable in the cell with the coordinates (indices) $i, j, k$ at the time $t$. As a result, Dirac equations were derived.

In a series of papers (see [21]), cellular automata of the form were used:

$$\psi_a(x,t+1) = \sum_{y \in Z, b \in A} U_{xy}^{ab} \psi_b(y,t),$$

$$U_{xy}^{ab} = 0, \forall y \notin N_x,$$ (7)

where $N_x \subset Z$ is the neighborhood of the cell $x$.

T. Else and co-authors considered approaches using the action functional and the derivation of equations-analogues of the Hamiltonian description [23,25]

$$S := \sum_n [(p^a_n + p^a_{n-1})\Delta x_n^a + (\sigma_n + \sigma_{n-1})\Delta \tau_n - A_n].$$ (8)

In these papers, as well as some others, these and other variants are considered cell automata and limit transitions leading to equations related to quantum mechanics. Undoubtedly, such research will continue in the future. However, taking into account additional properties in cellular automata and posing new mathematical problems can lead to new results and interpretations, which are interesting even for quantum mechanics. The first of such properties is anticipation.

4. The property of anticipation (future accounting) in theory and nature

There are many variants of the description of anticipation. Perhaps, for intuitive understanding, the closest is the following: "Anticipation (from Latin anticipatio - I anticipate) is a representation of the result of a process that occurs before its actual achievement and serves as a means of feedback in constructing an action." [27]. This concept was met, however, without formalization and measurement, many times in the context of philosophy, economics, psychology, medicine. Relatively recently (probably in the last 30-40 years), both experimental studies and new theoretical concepts have appeared in the study of anticipation. In neuroscience, the most famous are the studies of B. Libet [27] and numerous subsequent studies.

It should be noted that physics itself has long indicated the possibility of manifestations of anticipation in one form or another, beginning with R. Feynman [15,28] and even earlier (for example, Tetrode in the 20s of the 20th century), for example, in considering delayed and advanced waves in classical electrodynamics. The necessity of taking into account nonlocality both in time and in space is indicated by the results of experiments to verify the inequalities of J. Bell et al. [4,29]. In quantum mechanics, there is even a transactional interpretation of quantum mechanics by J. Kramers [30], based on microprocesses with delay and lead.
In the theoretical plan in the field of biology and simulation, systems and models with anticipation were described explicitly by R. Rosen (see [31], etc.). A significant development and formalization of the concept of anticipation was introduced in the works of D. Dubois [32,33]. Beginning in the early 1990s, D. Dubois — see [32,33], the idea of strong anticipation was introduced: "The definition of a discrete system with a strong anticipation: it is a system that calculates the current state at a time \( t \) as a function of states in the past times \( \ldots, t-3, t-2, t-1 \), present, and states at future times \( t+1, t+2, t+3, \ldots \)

\[
x(t + 1) = A(\ldots, x(t - 2), x(t - 1), x(t), x(t + 1), x(t + 2), \ldots, p),
\]

where the variable \( x \) in future times is calculated directly from the equation.

Definition of a discrete system with a weak anticipation: it is a discrete system — a system that calculates the current state at a time, as a function of the state in the past, present, and predicted (computed) states at future times

\[
x(t + 1) = A(\ldots, x(t - 2), x(t - 1), x(t), x^*(t + 1), x^*(t + 2), \ldots, p),
\]

where the variable \( x^* \) at future times \( t+1, t+2, t+3, \ldots \) is calculated using the predictive model of the system [32], (Dubois, 2001, p. 447).

Therefore, further research in this area should be carried out in a system with strong anticipation.

5. Cellular machines with a strong anticipation

The key idea is to introduce a strong anticipation the design of the spacecraft. Of course, there are many ways to implement this idea. First of all, we describe one of the simplest. Then we give for illustration the results of one of the computer experiments with such a spacecraft.

For this purpose, we assume that the states of the CA cells can depend on the future (virtual) states of the cells. Then the modified rules for CA, for example, can have the form

\[
\phi(\ldots, x_{t+k}^j, \ldots) \rightarrow x_{t+k}^j,
\]

\[
\overline{\phi}(\ldots, 1_{t+k}, 1_{t+k}, 1_{t+k}, \ldots) \rightarrow (1_{t+k}, 1_{t+k}, 1_{t+k}, \ldots),
\]

where \( k \) (integer) is the horizon of antipathy. In view of the assumed properties of such solutions (multivaluedness, see examples and discussion below), the values of the variables in (11), (12) have different interpretations. This is due to the fact that equations (11), (12) are nonlinear equations for the values of cells at future instants of time. Such equations can have one solution, have no solutions, and have many solutions (D. Dubois named it as the hyperincursion) depending on the parameters and the current and past states of the cells. So, in equations (11), (12) represent all probable values \( x_{t+k}^j \) of cells \( i \) at time \( t + j \) and \( 1_{t+k} \) represents the whole prehistory (multivalued) for the cell \( i \) up to the moment \( t + k \). We note that in certain special cases (single-valued solutions of equations (11), (12)) the results coincide with the solutions for equations (3), (4).

Next, for simplicity, we give a system of such CA without memory and only for one-step anticipation \((k = 1)\). The general form for this case has the form

\[
\phi(\ldots, x_{t+1}^j, 1_{t+1} \rightarrow x_{t+1}^j,
\]

\[
\phi(\ldots, 1_{t+1}, 1_{t+1}, 1_{t+1}, \ldots) \rightarrow x_{t+1}^j.
\]

The basis for the singularity of solutions (13), (14) is the possible many-valued solutions and the existence of many branches. This also implies the existence of many configurations of space at a given
time moment. We note that this leads to the existence of new features of solutions and interpretations in the old problems and the possibilities of posing research problems.

Results of computational experiments. For the initial study of the properties of the proposed model, software was created that used computational experiments with different parameter values and different initial conditions. Based on the test results, data presentation graphs were constructed that corresponded to a number of specific solved problems [34]. Below, we give for illustrative purposes only two graphs of the behavior of the decisions of the game "life" with anticipation (details of algorithms and other graphs (see [34,35]).

In Figure 1 below is an example that illustrates the existence of several states simultaneously. The discrete time is plotted along the abscissa axis, and the ordinates are the indices representing the possible configurations. Recall that in the theory of cellular automata configuration means a certain set of states of all cells.

![Figure 1](image1.png)

**Figure 1.** The emergence of several configurations in the "life" model with antipathy. The graph is constructed for the additive transition function $a=1.0$, the initial configuration with the number 0007

Figure 1 that in the first step three solutions are simultaneously obtained; three coexisting configurations. In the second step, two of them bifurcate, while the third solution turns out to be a dead end. In the next steps, the "multiplication" and "extinction" of decisions are similarly obtained. Common to the figures given is that after a certain number of steps the number of solutions (configurations) reaches its maximum value (which depends on the initial data) and does not change in the future (Figure 2).

![Figure 2](image2.png)

**Figure 2.** The large number of configurations that coexist in the system (the additive jump function $a=0.5$, the initial configuration 0000)
The main feature revealed in the simulation of such a system with strong anticipation is the appearance of a multivalued state of such a system. This new behavior allows us to re-examine classical questions, including uncertainty.

6. Modification with strong anticipation of cellular models of the microcosm

All this (and much more) points to the advisability of further consideration of physical models (and, probably, systems) with anticipation. One of the obvious and relatively easy to implement options is to take into account the strong anticipation in the analogues of models (5) - (8) and the study of possible consequences and interpretations. Considering what was said above about taking strong anticipation into account, it is possible to propose some modifications of the cellular models above. We note that anticipation (including strong one) can be introduced in different ways. Here we give only one of the possible variants (let's call it "integral"), when we are only interested in some virtual averaged characteristics at the next step in time.

The equation

\[ \sigma(x, t) = a \sigma(x-1, t-1) \sigma(x+1, t-1) + \]
\[ + (1 - \alpha)(\beta \sigma(x-1, t+1) \sigma(x+1, t+1) + \gamma \sigma^2(x, t+1)) \]

(15)

there is an analog of equation (5), but taking into account strong anticipation (\( \alpha \in [0,1] \) — the coefficient of anticipation, - the coefficients of the CA \( \beta, \gamma \in [0,1] \) template):

\[ \phi(i, j, k, t + \Delta t) = \alpha(W_{++} \phi(i+1, j+1, k+1, t) + W_{+-} \phi(i+1, j+1, k-1, t) + ... + W_{-+} \phi(i-1, j-1, k+1, t) + (1 - \alpha)(W_{++} \phi(i+1, j+1, k+1, t+\Delta t) + W_{+-} \phi(i+1, j+1, k-1, t+\Delta t) + ... + W_{-+} \phi(i-1, j-1, k+1, t+\Delta t)), \]

(16)

— analogues of equations (6); formula

\[ \psi_n(x, t+1) = \alpha \sum_{y \in \mathbb{Z}, b \in A} U^{ab}_{xy} \psi_b(y, t+1) + (1 - \alpha) \sum_{y \in \mathbb{Z}, b \in A} U^{ab}_{xy} \psi_b(y, t+1) \]

(17)

\[ U^{ab}_{xy} = 0, \forall y \notin N_x \]

— analogues of relations (7); functional

\[ S = \gamma \left( \sum_n [(p_n^a + p_{n-1}^a) \Delta x_n^a + (\pi_n + \pi_{n-1}) \Delta r_n - A_n] \right) + \]
\[ + (1 - \gamma) \left( \sum_{n+1} [(p_{n+1}^a + p_n^a) \Delta x_{n+1}^a + (\pi_{n+1} + \pi_n) \Delta r_{n+1} - A_{n+1}] \right) \]

(18)

is a generalization of the functional (8). In (18) \( \gamma \in [0,1] \) — the coefficient of accounting for anticipation.

7. Possible statements of problems and consequences for interpretations

Equations (15)-(18) presented in the previous section are completely new objects and represent a broad field for research. But it should be noted that they are a fairly complex object for research (even by approximate methods). The main difficulties bring possible branching and multivalued solutions. On the other hand, it is these features that seem most promising in terms of interpretations. Therefore,
this section briefly summarizes considerations for the direction of further research and possible outcomes.

First, we will describe some pending questions for reflection. First, the visualization of solutions in models with strong anticipation surprisingly resembles the illustrations to Everett's interpretation of quantum mechanics [36]. Perhaps a strong anticipation can provide a branching mechanism and a multiplicity of worlds in the Everett scheme. Also, the multiplicity of possible transitions can be considered as a candidate for considering probabilistic schemes [37]. Also, the palette of properties of systems with strong antiparticle can apparently be useful (or in other words related) with a scheme of continuous measurements in quantum mechanics based on a group of constrained paths (see [38]). Let us note in passing that such a scheme, due to the multivaluedness of the models of society and the economy with strong anticipation (A. Makarenko), gives grounds to raise the question of measuring the parameters of society and of more accurately predicting them.

The next circle of questions arises in connection with the properties of the solutions of the models (15)-(18) and their limits, generalizing equations (5)–(8) [17–25]. The usual requirement in their studies was the requirement of reversibility, which makes it possible to obtain the usual equations of quantum mechanics. However, the equations of the models (15)–(18), especially in the case of simultaneous consideration of the memory effects, lead to irreversible relations. Therefore, it is appropriate to consider the measurement process on the basis of models (15)–(18), which may lead to reversibility in the averaged sense, which can correlate with the usual reversibility. Further, it is known that there is a problem of describing dissipative quantum processes and systems, when irreversibility is assumed initially [39, 40]. It may turn out that the proposed scheme (with multivaluedness) will also enable the consideration of systems with dissipation. Note that the computational processes are actually dissipative [41]. In general, in connection with equations (5)–(8), and then and (15)–(18), questions arise about the use of variational principles (for example, [23,25]). Then an interesting problem is the variational principles for equations with strong antiparticle. It may turn out that one should work with multivalued functionals and their averagings (see [42,43]). Also, worth considering is the consideration from the point of view of various structures (Poisson brackets, noncommutative structures, etc.).

In addition, regardless of whether or not to take into account strong anticipation, there are still new questions about the very structure of cellular automaton models. Usually it is assumed that the structure of the division into elements is regular - cells, cubes, etc. However, there are certain doubts about the complete homogeneity of physical systems, and, accordingly, models. Thus, in experiments [44], a possible heterogeneity (with no matter in the voids (holes) of space) is announced at the cosmic level. Theoretically, models with holes in the space structure were considered in [45]. This firstly suggests the consideration of cellular automata with gifts without cells inside them. In part, this resembles the consideration of percolation problems in porous media. Second, based on the available concepts of the general theory of relativity, one can assume the existence of connections between distant cells (for example, analogs of worms). Assuming that such connections between remote cells are still small, a special network architecture of cellular models (as one of the "SMALL WORLD" variants) and the properties of cellular models and models on such network-cellular structures depending on network parameters [46]. One of the usual questions is the investigation of phase transitions in such models. Another option is space-time foam.

Moreover, it is possible that the form and specific parameters of the fundamental laws can depend on the internal network micro and macro architecture. Then it is also advisable to consider the inverse mathematical problems — to calculate the possible microstructure of space and time and its parameters from the results of physical measurements from experiments.

8. Conclusions
Thus, in this paper we consider the use of models of cellular automata in quantum mechanics and the possible ways of developing such representations. As one of the possible generalizations, we consider the introduction of strong antiparticle in the equations and possible consequences. One of the most
promising opportunities is the emergence of polysemy in the solution. New statements of problems and possibilities of interpretations of similar singularities of solutions are carried out.

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