Research on Stock Returns Forecast of the Four Major Banks Based on ARMA and GARCH Model

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Abstract. This paper uses the prediction method of time series analysis to fit and predict the stock daily logarithmic return series of the four major banks. The ARMA model and the GARCH model are constructed and compared empirically. The results show that the GARCH model is better than that of the ARMA model in the fitting effect. In the prediction effect, The ARMA model is the best, followed by the GARCH model.

1. Introduction

In the stock market, many scholars are concerned about the changes in stock prices and the prediction of stock prices. Uncertainty changes in stock prices often manifest themselves as market fluctuations. Since the stock return rate series can better reflect the volatility of the stock market than the stock price series, in the stock market, modeling and predicting the stock return rate has important research value compared with the stock price prediction. There are various theoretical methods for forecasting. Some scholars have studied artificial neural network prediction models, gray prediction models, and support vector machine prediction models.

In recent years, there have been many achievements in China using the ARMA model, ARCH model and GARCH model to study time series data changes and fluctuations. For example, Wu YX (2016) [1] used the ARIM model to predict the law and trend of stock price changes, and the prediction effect is ideal. Lu WB (2006) [2] applied the nonparametric GARCH model to predict the volatility of the Chinese stock market. Wang JF (2011) [3] used the GARCH model to fit and predict the volatility and yield of the Shanghai and Shenzhen 300 index series, and achieved the desired results. Wei HY (2014) [4] used the GARCH model to study the exchange rate prediction of RMB against the US dollar, and the prediction results were quite satisfactory. Luo YH (2013) [5] applied the ARMA model to better reflect the dynamic changes of China's agricultural product price index. Fu Y (2012) [6] used the ARMA model to predict and analyze China's sports stock prices. He BL(2008) [7] applied the ARIM and ARCH models to predict stock prices. The results show that the ARCH model is more satisfactory than the ARIM model in predicting results.

However, in terms of predicting the power of stock price volatility, there are few research results comparing the ARMA model, and the GARCH model. Therefore, it is of great significance to compare the prediction effects of the ARMA model and the GARCH model.

2. Time series model

2.1. ARMA model
The ARMA model [8] is a commonly used stochastic time series model, created by the US statisticians Box and Jenkins, also known as the Box-Jenkins method. The general expression of the ARMA \((p, q)\) model is shown in below.

\[
    r_t = \varphi_1 r_{t-1} + \varphi_2 r_{t-2} + \cdots + \varphi_p r_{t-p} + u_t - \theta_1 u_{t-1} - \cdots - \theta_q
\]  

where \(p\) is the order of the autoregressive part and \(q\) is the order of the moving average part, so it is recorded as ARMA \((p, q)\).

The precondition of using the ARMA model is that the time series as the analysis object is a set of zero mean stationary series. But in reality, there are a lot of unstable time series. Therefore, before the ARMA model is established, the sequence needs to be stabilized.

### 2.2. GARCH model

In the financial market, the stock price or the market has the characteristics of volatility aggregation. In 1982, Engle first proposed the use of the ARCH model to characterize the conditional heteroscedasticity of financial market fluctuations. The main idea of the ARCH model is that the perturbation term is uncorrelated, and its conditional variance depends on the size of its previous value. The general expression of the ARCH \((q)\) model is shown in below.

\[
    r_t = \mu + a_t, a_t = \sigma_t \varepsilon_t \\
    \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\]  

where \(\varepsilon_t\) is assumed to be a standard normal distribution with independent and identical distribution, \(\alpha_0 \geq 0, \alpha_i \geq 0 (i = 1, 2, \cdots, q)\).

Regarding the test of ARCH effect, that is, the conditional heteroscedasticity test is performed on the squared residual sequence of the mean equation. If a significant ARCH effect is found, a wave rate model can be established. However, the ARCH model has a significant feature, that is, the order of the model usually fitted is larger. In order to solve this problem, in 1986, Boller-Slav proposed a generalized autoregressive conditional heteroscedasticity model -GARCH model [9] based on Engle's ARCH model.

For a logarithmic rate series \(r_t\), let \(a_t = r_t - \mu\) be the innovation at time \(t\). It is said that \(a_t\) conforms to the GARCH \((m, s)\) model, if \(a_t\) satisfies:

\[
    \begin{align*}
    r_t &= \mu + a_t, a_t = \sigma_t \varepsilon_t \\
    \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2
    \end{align*}
\]  

where \(\varepsilon_t\) is an independent and identically distributed sequence with mean 0 and variance 1, \(\alpha_0 > 0\), \(\alpha_i \geq 0, \beta_j \geq 0\), and \(\sum_{j=1}^{\max(m,s)} (\alpha_j + \beta_j) \leq 1\) \((m, s)\). In most applications, \(m, s\) are not very large, only the low-order GARCH model is used. In fact, in many cases, the GARCH \((1, 1)\) model is sufficient.

### 3. Empirical Analysis-Take the Big Four Banks of China as an Example

#### 3.1. Data selection and statistical analysis

This paper selects the daily closing prices of the shares of Bank of China, Industrial and Commercial Bank of China and China Construction Bank from October 1, 2007 to October 1, 2019, as well as the Agricultural Bank of China July 15, 2010-October 1, 2019. The stock daily closing price is used as sample data. This paper studies the fluctuation of the stock market, taking the stock market's daily rate of return as the inspection variable. The stock market's daily rate of return is expressed by the logarithmic first-order difference of the closing prices of the two adjacent days, and \(R_t\) is used as the
stock day of the t day Logarithmic rate of return. The daily logarithmic rate of return constitutes a new sample time series, and a basic statistical analysis is performed on the series to obtain its basic description statistical analysis results and its timing chart, as shown in Table 1 and Figure 1, respectively.

![Time series of daily logarithmic returns of the four largest banks in China](image1)

**FIG. 1 Time series of daily logarithmic returns of the four largest banks in China**

It can be seen from the timing chart of the returns of the four major banks in Figure 1 that the returns of the banks fluctuate around zero mean, and it can be preliminarily judged that the daily returns of the four major banks are stable. The P values obtained by the normality test are close to 0, which means that the zero hypothesis that the series is normally distributed can be rejected at a confidence level of at least 99%.

3.2. Analysis of the solution of ARMA model

To identify, fit and estimate the model, the sequence autocorrelation coefficient (ACF) and partial autocorrelation coefficient (PACF) are used to identify the order of the ARMA model, and combine the AIC information criteria to select the best model. The higher, the better the fit.

| Each Model | Construction Bank | ICBC Bank | Agricultural Bank | Bank of China |
|------------|--------------------|-----------|-------------------|---------------|
|            | ARMA(2,2)          | ARMA(2,4) | ARMA(2,2)         | ARMA(3,1)     |
| ar1        | -0.24              | 1.173     | -0.249            | -0.577        |
| ar2        | -0.582             | -0.443    | -0.727            | -0.062        |
| ar3        | 0                  | 0         | 0                 | -0.051        |
| ma1        | 0.237              | -1.186    | 0.249             | 0.558         |
| ma2        | 0.516              | 0.397     | 0.624             | 0             |
| ma3        | 0                  | 0.075     | 0                 | 0             |
| ma4        | 0                  | -0.067    | 0                 | 0             |

Table 1. Parameter estimation results of ARMA model
It can be seen from Table 3 that taking the Bank of China’s daily logarithmic return rate as an example, the ARMA (3, 1) model is finally selected, namely:

$$r_t = -0.577r_{t-1} - 0.062r_{t-2} - 0.051r_{t-3} + a_t + 0.558a_{t-1}$$  \hspace{1cm} (4)$$

The results of the randomness test of the residual sequence of the ARMA models of the four major banks are shown in Table 2.

### Table 2. Model residual correlation and stationarity test

| Each Model | Construction Bank | ICBC Bank | Agricultural Bank | Bank of China |
|------------|-------------------|-----------|-------------------|--------------|
|            | ARMA(2,2)         | ARMA(2,4) | ARMA(2,2)         | ARMA(3,1)    |
| Box-Ljung Q Statistics | 0.0014 | 0.0051 | 0.019 | 0.00034 |
| Corresponding P value | 0.9704 | 0.9869 | 0.8906 | 0.985 |
| ADF statistics | -14.275 | -14.273 | -12.093 | -14.082 |
| Corresponding P value | <0.01 | <0.01 | <0.01 | <0.01 |

It can be seen from Table 2 that the models of the daily return rate series fitted by each bank have P values greater than 0.05 in the corresponding residual sequence of the Box-Ljung test, confirming that the residual sequence of each model has no significant correlation, So the model is reasonable. Therefore, the above ARMA model can be used to predict the daily rate of return of the four major banks.

#### 3.3. Analysis of the solution of GARCH model

This paper chooses to establish the GARCH (1, 1) model to obtain the GARCH (1, 1) model of the daily rate of return of each bank, in which the distribution of residual terms is selected from Gaussian distribution, student t distribution, and biased student t distribution. It can fit the given data well. In this paper, combined with the AIC minimum principle selection model, comprehensive consideration, the final selection of the distribution of residual items has a biased student t distribution to achieve a relatively better. The new information of each yield rate series is subject to the biased student-t distributed GARCH (1, 1) model. The parameter estimation results are shown in Table 3.

### Table 3. The estimation result of GARCH model

| GARCH(1,1) model | Mu | omega | Alpha1 | Beta1 | skew | shape |
|-----------------|----|-------|--------|-------|------|-------|
| Construction Bank | 0.0194 | 0.0357 | 0.1132 | 0.8905 | 1.0232 | 3.6417 |
| ICBC Bank | 0.0319 | 0.0313 | 0.1133 | 0.8901 | 1.0502 | 3.6568 |
| Agricultural Bank | 0.0274 | 0.0495 | 0.1450 | 0.8527 | 1.0353 | 3.3275 |
| Bank of China | 0.0125 | 0.0342 | 0.1123 | 0.8908 | 1.0514 | 3.4353 |

It can be seen from Table 3 that all parameter estimates, except for the constants in the model, are highly significant, indicating that GARCH (1, 1) can fit the data better. Among them, taking the daily logarithmic return series of Bank of China stock as an example, applying the new interest with partial student t distribution, the fitted GARCH (1, 1) model is:

$$r_t = 0.0125 + a_t$$

$$a_t = \sigma_t \epsilon_t, \epsilon_t \sim t_{1.0514,3.4353}^*$$

$$\sigma_t = 0.0342 + 0.1123a_{t-1}^2 + 0.8908\sigma_{t-1}^2$$

Among them, $t_{1.0514,3.4353}^*$ represents the normalized biased student t distribution with skew parameter of 1.0514 and degree of freedom of 3.4353. Except for the constant of the mean value equation, other coefficients are significant at 5% level.
Then test the correlation between the residuals of the GARCH (1, 1) model and the square of the residuals. The Ljung-Box statistics of standardized residuals and their squared sequences cannot reject the model, because their corresponding P values are greater than 0.05, so that the residual sequence of each model does not exist ARCH effect, indicating the above GARCH (1,1) model eliminates the conditional heteroscedasticity of the residual sequence, and the fitting of the model is sufficient.

4. Results and discussion

The intra-sample comparison method is to use all data to estimate and compare models, such as information criteria (such as AIC and BIC) and residual variance estimation. If one of the criteria is chosen, the smaller its value is, the better the model is. The AIC guidelines adopted in this paper compare the above two models within samples, and their values are shown in Table 4.

| Each Model       | Construction Bank | ICBC Bank | Agricultural Bank | Bank of China |
|------------------|-------------------|-----------|-------------------|---------------|
| ARMA             | 9897.55           | 9422.85   | 6222.05           | 9589.75       |
| GARCH            | 3.4993            | 3.3372    | 2.9463            | 3.3071        |

It can be seen from Table 4 that, according to the AIC standard, the daily logarithmic rate of return for stocks establishes the volatility rate equation in comparison with the GARCH model, which is better than the ARMA model. The AIC value of the GARCH model has a relatively large decrease compared to the ARMA model, which indicates that there are volatility aggregation phenomena in the daily logarithmic return series of the four major banks, and the GARCH model can better explain and eliminate conditional heteroscedasticity.

In addition, the time series model is established for prediction, and then the model comparison must consider the prediction ability of the model. In this paper, two indicators of mean square error (MSFE) and mean absolute error (MAFE) are used to measure the prediction effect of the model. For different models, the smaller the predicted MSFE or MAFE, the higher the prediction accuracy [10]. The model corresponding to the smallest MSFE or the smallest MAFE is taken as the best model for this set of data. Among them, these two indicators reflect the size of the error between the predicted value and the actual value, and their definitions are:

$$MSFE = \frac{1}{N} \sum_{i=1}^{N} \left( y_i - \hat{y}_i \right)^2$$ (6)

$$MAFE = \frac{1}{N} \sum_{i=1}^{N} \left| y_i - \hat{y}_i \right|$$ (7)

where $\hat{y}_i$ represents the predicted value, $y_i$ represents the actual value, and $N$ represents the number of predicted samples.

Based on the predictions of each model for the test set of daily logarithmic returns of stocks, the mean square error and the average absolute error of the predictions of each model are calculated. When the two models are given two steps ahead of the forecast, the corresponding The mean square of prediction error (MSFE) and the mean absolute prediction error (MAFE) are summarized in Table 5.

| Model              | ARMA | GARCH |
|--------------------|------|-------|
| Construction Bank  | MSE  | 0.478 | 0.502 |
|                    | MAE  | 0.554 | 0.568 |
| ICBC Bank          | MSE  | 0.799 | 0.81  |
|                    | MAE  | 1.112 | 1.134 |
| Agricultural Bank  | MSE  | 0.932 | 1.053 |
|                    | MAE  | 0.806 | 1.058 |
| Bank of China      | MSE  | 0.342 | 0.381 |
According to Table 5, it can be obtained that the prediction effect of ARMA model and GARCH model is not much different. From the perspective of prediction accuracy, the prediction effect of the ARMA model is the best.

5. Conclusion
This paper uses the prediction method of time series analysis, through empirical analysis of the stock daily logarithmic return series of the four largest banks in China, we can find that both the ARMA model and the GARCH model can simulate the stock daily logarithmic return series of the four major banks. Combined with predictive analysis, the following conclusions can be obtained specifically: (1) In terms of model fitting effect, it is better to establish the GARCH model for the daily logarithmic return series of the four major bank stocks than to establish the ARMA model. This is due to the volatility aggregation phenomenon of the stock daily logarithmic return series of the four major banks and the GARCH model can better explain and eliminate the conditional heteroscedasticity, which also shows that the GARCH model can better reflect the daily stock prices of the four major banks during this period Of actual fluctuations. (2) In terms of model prediction results, the mean square prediction error and the average absolute prediction error of the ARMA model and the GARCH model are relatively small, and the prediction errors between the models are about 1% different. Overall, the prediction effect of the ARMA model is the best in predicting the stock prices of the four major banks.

To sum up, the ARMA model has a better prediction effect on the volatility and stock returns of the four major banks, but the ARMA model has a much better fit for the prediction of stock yields than the GARCH model. The GARCH model can well eliminate the conditional heteroscedasticity for the sequence of stock returns. In future research, it is worth exploring whether the ARMA-GARCH model combined with the ARMA and GARCH models is more ideal in the fitting effect and prediction ability.

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