The notion of inflation (past or present) in standard cosmological models is shown to be a consequence of a sufficiently high second law entropy production from the internal heating of the universal expansion. The longitudinal viscous internal heating of matter requires neither “inflaton” fields nor “quintessence” fields which in theory may induce a cosmological term into the Einstein equations. The purely thermodynamic principles required to understand inflation within the context of the standard general relativity equations will be discussed in detail.

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I. INTRODUCTION

Under the hypotheses of the standard cosmological model one reduces space and time scales of the history of the universe to a single central function of time $a(t)$. In detail, the standard cosmological space-time metric reads

$$c^2d\tau^2 = c^2dt^2 - a^2(d\chi^2 + \sigma_\kappa(\chi)(d\theta^2 + \sin^2\theta d\phi^2)),$$

where (for $\kappa = -1, 0, 1$)

$$\sigma_1(\chi) = \sin \chi, \quad \sigma_0(\chi) = 1, \quad \text{and} \quad \sigma_{-1}(\chi) = \sinh \chi.$$

The equation of motion for $a(t)$ reads

$$\ddot{a}(t) + \Omega^2(t)a(t) = 0,$$

where

$$\Omega^2 = \left(\frac{4\pi G}{3c^2}\right)(\varepsilon + 3P).$$

In Eqs(3) and (4) one confidently writes $\Omega^2$, very sure in the knowledge that the energy per unit volume $\varepsilon$ and the pressure $P$ are both positive. Actually, all one knows from the relativistic stability of matter is that

$$\varepsilon \geq 3P.$$

When it turns out that energy density $\varepsilon$ and/or the pressure $P$ are not positive, one may either invent new funny sounding quantum fields, e.g. the “inflaton field” or the “quintessence field”, or take comfort in what Einstein himself called his big research blunder, i.e. the cosmological term in the gravitational field equations. Armed with these possible new quantum fields, one finds that $\Omega^2$ can indeed be negative, at least for some time periods in the history of the universe. Thus, inflation could exist in very ancient epochs. Recently it has even been suggested, on the basis of very distant super nova explosions, that presently we live in an ancient epoch, although the inflation turns out to be rather small. The small present inflation is thought to require merely a quintessence field (whatever that may be).

Our purpose is replace the invention of these new quantum fields with a more close scrutiny of the second thermodynamic law. In most of the present treatments of the cosmological thermodynamic equations of state, the entropy in a large expanding sub-volume of the universe remains constant. In other words, in a “big bang” explosion so strong that it staggers the human imagination, not even one small bit of entropy was created. This view is in stark contrast to those very tiny explosions that have existed in human wars which tend to leave an entropy of disorder scattered all over the place.

FIG. 1. The expansion of the universe at a rate $\nu$ is schematically pictured as an expanding sub-volume $V$ of the universe here pictured inside a insulating piston. The total pressure can be driven negative if the viscous heating of the expanding material within the piston is sufficiently large.

Connected with what we suggest is a failure of the notion of an entropy conserving explosion, is the failure of the notion of a completely local effective Lagrangian. In the usual standard cosmological model, one may derive Eqs.(3) and (4) from an effective local Lagrangian, which exists whenever the coordinate of interest is changing slowly compared with all of the other coordinates which are being integrated out of the cosmological dynamics. Since the Hubble expansion rate
\[ \nu = \left( \frac{\dot{a}}{a} \right) \]  

(6)

diverges at the big bang, \( \nu \to \infty \) as \( t \to 0 \), the regime in which \( a(t) \) varies slowly is not abundantly clear. The mechanism for inflation here being discussed is shown above in FIG.1. One follows a material sub-volume \( V \) of the universe, here pictured in a schematic fashion as material inside a piston. Neither heat nor matter flows into or out of the sub-volume \( V \).

In Sec.II the notion of an effective Lagrangian for \( a(t) \) will be derived from the action principle of general relativity. The gravitational part of the effective Lagrangian may be derived from the scalar curvature in the usual manner. The matter part of the effective Lagrangian may be computed from the energy per unit volume \( \varepsilon \) even for the case wherein the effective action is non-local in time and depends on the rate entropy production. In Sec.III, the thermodynamics laws of cosmology will be reviewed. In Sec.IV, the manner in which the second law may induce inflationary epochs into the cosmological evolution will be discussed. In the concluding Sec.V, our mechanism for obtaining inflation without recourse to the cosmological term will be reviewed.

II. EFFECTIVE COSMOLOGICAL ACTION

The effective action for general relativity with a general metric

\[ -c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu \]  

(7)

reads

\[ W = W_{\text{gravity}} + W_{\text{matter}} \]  

(8)

where

\[ W_{\text{gravity}} = \left( \frac{c^3}{16\pi G} \right) \int R \left( \sqrt{-g} d^4x \right) \]  

(9)

is the gravitational action; i.e. \( R \) is the scalar curvature. For the standard cosmological metric in Eq.(1),

\[ R = \left( \frac{6}{a^2 \varepsilon^2} \right) \left( a \ddot{a} + a^2 + c^2 \kappa \right). \]  

(10)

Furthermore

\[ \sqrt{-g} d^4x = a^3 d\tilde{\nu}, \]  

(11)

where

\[ d\tilde{\nu} = \sigma_r \sin \theta d\chi d\theta d\phi, \]  

(12)

is a time independent spatial volume element. The time dependent expansion of all such cosmological spatial volume elements is described by the scale factor \( a^3 \) on the right hand side of Eq.(11). From Eqs.(9)-(12), the gravitational action of a finite but large sub-volume of the universe is given by

\[ W_{\text{gravity}} = \int L'_{\text{gravity}} dt \]  

(13)

where

\[ L'_{\text{gravity}} = \left( \frac{3c^2}{8\pi G} \right) \int_{\tilde{\nu}} \left( a^2 \ddot{a} + a \dot{a}^2 + c^2 a \kappa \right) d\tilde{\nu}. \]  

(14)

The Lagrangian \( L'_{\text{gravity}} \) depends on the second derivative \( \ddot{a} \). However, an integration by parts in Eq.(13) shows that the gravitational part of the Lagrangian containing at most the first derivative \( \dot{a} \) may be written

\[ L_{\text{gravity}} = \left( \frac{3c^2}{8\pi G} \right) \int_{\tilde{\nu}} \left( c^2 a \kappa - a \dot{a}^2 \right) d\tilde{\nu}, \]  

(15)

or equivalently

\[ W_{\text{gravity}} = \int L_{\text{gravity}} dt \]  

(16)

where

\[ L_{\text{gravity}} = \left( \frac{3c^2}{8\pi G} \right) \left( c^2 a \kappa - a \dot{a}^2 \right). \]  

(17)

Eq.(17) contains a complete description of the purely gravitational part of the Lagrangian under the standard cosmological hypothesis of a locally isotropic universe. From the gravitational part of the Lagrangian one may obtain the gravitational part of the energy

\[ E_{\text{gravity}} = \dot{a} \left( \frac{\partial L_{\text{gravity}}}{\partial \dot{a}} \right) - L_{\text{gravity}}, \]  

(18)

which reads

\[ E_{\text{gravity}} = - \left( \frac{3c^2}{8\pi G} \right) \left( c^2 a \kappa + a \dot{a}^2 \right). \]  

(19)

The matter contribution to the Lagrangian is a more subtle computation.

To begin, one may consider the pressure-energy tensor of the matter \( T_{\mu\nu} \). This second rank tensor has one time-like eigenvector \( v^\mu \); i.e.

\[ T_{\mu \nu} v^\nu = c^2 \rightleftharpoons v \mu v^\mu = -c^2. \]  

(20)

The eigenvalue \( \varepsilon \) represents the energy per unit volume while \( v^\mu \) represents the local “fluid” mechanical velocity field of the matter. The metric may now be written in the form

\[ g_{\mu \nu} = -(v_{\mu} v_{\nu} / c^2) + h_{\mu \nu}, \quad h_{\mu \nu} v^\nu = 0, \]  

(21)

so that an observer moving with the matter at velocity \( v^\mu \) observes a local time interval
\[c^2 \frac{dt^2}{dv} = (v_\mu dx_\mu/c)^2\]  \hfill (22)

and a local space interval
\[dl^2 = h_{\mu \nu} dx^\mu dx^\nu\]  \hfill (23)

or (equivalently) a proper time
\[c^2 dr^2 = c^2 dt^2 - dl^2.\]  \hfill (24)

Eqs.(1) and (24) are made consistent with
\[dl^2 = a(t)^2 (d\chi^2 + \sigma_\kappa(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)).\]  \hfill (25)

Under the standard cosmological hypothesis that the universe is isotropic with pressure \(p\), the pressure-energy tensor must have the form
\[T_{\mu \nu} = \varepsilon(v_\mu v_\nu/c^2) + Ph_{\mu \nu},\]  \hfill (26)

more usually written
\[T_{\mu \nu} = (\varepsilon + P)(v_\mu v_\nu/c^2) + Pg_{\mu \nu}.\]  \hfill (27)

The pressure-energy tensor is related to the matter action via the variational equation
\[\delta W_{\text{matter}} = \left(\frac{1}{2c}\right) \int T_{\mu \nu} \delta g_{\mu \nu} \sqrt{\gamma} d^4 x.\]  \hfill (28)

During the course of cosmological expansion
\[\delta g_{\mu \nu} = \left(\frac{2\delta a}{a}\right) h_{\mu \nu},\]  \hfill (29)

so that
\[\delta W_{\text{matter}} = \left(\frac{1}{c}\right) \int P h_{\mu \nu} h_{\mu \nu} \left(\frac{\delta a}{a}\right) \sqrt{\gamma} d^4 x.\]  \hfill (30)

Employing Eqs.(11) and (30) as well as \(h_{\mu \nu} h_{\mu \nu} = 3\), yields the variational equation for a finite (but large) sub-volume \(\bar{W}_{\text{matter}} = \int L_{\text{matter}} dt\); i.e.
\[\delta L_{\text{matter}} = \int_{\bar{V}_\kappa} \left(\frac{3\delta a}{a}\right) Pa^3 d\bar{V}_\kappa,\]  \hfill (31)

which integrates to
\[\delta L_{\text{matter}} = \left(\frac{3\delta a}{a}\right) Pa^3 \bar{V}_\kappa.\]  \hfill (32)

At this point one may employ the pressure-energy tensor identity \(D_\mu T^{\mu \nu} = 0\), or more simply compute the work done on the sub-volume during the expansion
\[\delta \mathcal{E}_{\text{matter}} = -P \delta V = \delta (V \varepsilon), \quad (V = a^3 \bar{V}_\kappa).\]  \hfill (33)

Hence
\[\delta \varepsilon = -(\varepsilon + P) \left(\frac{3\delta a}{a}\right).\]  \hfill (34)

From Eqs.(32) and (34)
\[\delta L_{\text{matter}} = -\left\{\delta \varepsilon + \left(\frac{3\delta a}{a}\right) \varepsilon\right\} a^3 \bar{V}_\kappa.\]  \hfill (35)

From Eq.(35) one proves the following

**Theorem 1:** The matter Lagrangian for a large sub-volume \(V\) of the universe is the negative of the matter energy contained within that sub-volume,
\[L_{\text{matter}} = -\varepsilon V = -\varepsilon a^3 \bar{V}_\kappa = -\mathcal{E}_{\text{matter}}.\]  \hfill (36)

The theorem holds true for any model with a local or non-local matter Lagrangian and/or when internal heating produces entropy. Finally if one adds the gravitational energy to the matter energy the resulting total energy is zero; i.e.
\[\mathcal{E} = \mathcal{E}_{\text{gravity}} + \mathcal{E}_{\text{matter}} = 0.\]  \hfill (37)

From Eqs.(19) and (36), one obtains the conventional cosmological equation
\[\dot{a}^2 + c^2 \kappa = \left(\frac{8\pi G}{3a^2}\right) a^2 \varepsilon.\]  \hfill (38)

Note that the classical general relativistic condition of zero total energy, for every sub-volume in Eq.(37), is often written in quantum mechanical terms employing the Schrödinger equation \(H \Psi = \mathcal{E} \Psi\) for the wave function \(\Psi\). Since \(\mathcal{E} = 0\) the “wave function of the universe” \(\Psi\) obeys \(H \Psi = 0\). We shall not compute \(\Psi\) in detail.

**III. COSMOLOGICAL THERMODYNAMICS**

In the standard cosmological model it assumed that the total entropy \(S\) in a large sub-volume \(V = a^3 \bar{V}_\kappa\) is zero. For a quasi-static yet irreversible cosmological expansion, the entropy \(S\) increases with time. In thermodynamic terms, the energy of a large sub-volume obeys
\[d\mathcal{E}_{\text{matter}} = T dS - pdV,\]  \hfill (39)

where \(p\) is the thermodynamic pressure. If the expanding sub-volume yields internal heating from viscous pressure, then the thermodynamic pressure \(p\) cannot be identified with the total pressure \(P\) in the pressure-energy tensor. With purely internal heating, there is no heat flow into the expanding sub-volume. One may employ the energy Eq.(33)
\[d\mathcal{E}_{\text{matter}} = -P dV,\]  \hfill (40)

relating the total pressure \(P\) to the thermodynamic pressure \(p\) via the internal heating rate \(\dot{Q} = T \dot{S}\). From Eqs.(39) and (40) one proves the following

**Theorem 2:** The ratio of the heating rate \(\dot{Q}\) to the volume expansion rate \(\dot{V}\) subtracts from the thermodynamic pressure to yield the total pressure,
\[ P = p - \left( \frac{\dot{Q}}{V} \right) \]  

(41)

While the thermodynamic pressure \( p \) may appear to be positive, the total pressure \( P \) may go negative if there is sufficient internal heating. If \( \dot{Q} > pV \), then \( P < 0 \). Note that one need not introduce an inflaton field to achieve a negative total pressure. Simple internal heating can do the job.

Eqs. (39) and (40), with an entropy per unit volume of \( s = (S/V) \), yield

\[ d(V \dot{\varepsilon}) = -PdV = Td(Vs) - pdV, \]  

(42)

which implies

\[ d\varepsilon = -(\varepsilon + P) \left( \frac{dV}{V} \right), \]  

(43)

and

\[ d\varepsilon = -(\varepsilon + p - Ts) \left( \frac{dV}{V} \right) + Tds. \]  

(44)

**IV. ENTROPY PRODUCTION**

As stated in Sec.I, it is unreasonable to assume for the largest explosion ever conceived (i.e. the big bang) that the internal heating (i.e. entropy production) should be strictly zero. During a universal expansion one may define a "longitudinal viscosity" \( \zeta \). In detail, the fact that near by matter moves away from us at a velocity \( v = (\dot{a}/a)r \) yields a viscous pressure

\[ p_{\text{viscous}} = -\zeta \text{div}v = -3\zeta \left( \frac{\dot{a}}{a} \right) = - \left( \frac{\dot{Q}}{V} \right). \]  

(45)

in accordance with Eq.(41). Using the identity \( \dot{V}/V = 3(\dot{a}/a) \) yields the heating rate per unit volume of the universal expansion,

\[ \left( \frac{\dot{Q}}{V} \right) = 9\zeta \left( \frac{\dot{a}}{a} \right)^2. \]  

(46)

Furthermore, from Eqs.(4), (41) and (45) one finds

\[ \Omega^2 = \left( \frac{4\pi G}{3c^2} \right) \left\{ \varepsilon + 3p - 9\zeta \left( \frac{\dot{a}}{a} \right) \right\}. \]  

(47)

The usual parameters employed in the standard cosmology are (i) the Hubble expansion rate \( \nu \) and (ii) the universal "deacceleration" \( \phi \); i.e.

\[ \nu = \left( \frac{\dot{a}}{a} \right), \quad \phi = - \left( \frac{a\ddot{a}}{a^2} \right). \]  

(48)

Alternatively \( \phi = (\Omega/\nu)^2 \), leading to the following

**Theorem 3:** The deacceleration parameter \( \phi \) in a cosmology which includes entropy production may be written

\[ \phi = \left( \frac{4\pi G}{3\nu^2c^2} \right) \left\{ \varepsilon + 3p - 9\zeta \frac{\dot{a}}{a} \right\}. \]  

(49)

Eq.(49) is the central result of this work. An inflationary epoch of a standard cosmology will exist if the longitudinal viscosity is sufficiently large

\[ 9\nu \zeta > (\varepsilon + 3p) \quad \text{implies} \quad \phi < 0, \]  

(50)

where \( p \) is the thermodynamic pressure. The condition \( \phi < 0 \) also defines inflation if \( |\phi| > 1 \) and defines quintessence if \( |\phi| < 1 \). The theory of inflation and quintessence can then be based on the longitudinal viscosity without recourse to the introduction of new fields invented merely for cosmological considerations.

In order to estimate the viscous mechanism for inflation, one may start from the Kubo formula for longitudinal viscosity in terms of pressure fluctuations \( \Delta P \) at temperature \( T \);\n
\[ \zeta = \left( \frac{1}{k_BT} \right) \times \int_0^\infty dt \int_V d^3r \int_V d^3r' \text{Re} \left\{ \Delta P(r,t)\Delta P(r',0) \right\}. \]  

(51)

On the other hand, the thermodynamic adiabatic compressibility,

\[ K_s = -\left( \frac{1}{V} \right) \left( \frac{\partial V}{\partial p} \right)_s, \]  

(52)

is determined by the static pressure fluctuations at a given time;

\[ K_s^{-1} = \left( \frac{1}{k_BT} \right) \int_V d^3r \int_V d^3r' \left\{ \Delta P(r,t)\Delta P(r',0) \right\}. \]  

(53)

Hence, if \( t_r \) denotes the relaxation time for pressure fluctuations, then one obtains

\[ \zeta = \tau_r K_s^{-1}. \]  

(54)

From Eqs.(49) and (54), the final expression for the deacceleration parameter reads

\[ \phi = \left( \frac{4\pi G}{3\nu^2c^2} \right) \left\{ \varepsilon + 3p - \left( \frac{9\nu\tau_r}{K_s} \right) \right\}. \]  

(55)

Eq.(55) is in a form suitable for estimating the conditions for which inflation \( \phi < 0 \) holds true.

**V. CONCLUSIONS**

The sign of the deacceleration parameter \( \phi \) is completely determined by Eq.(55). An inflationary period is
described by the condition $\phi < 0$. In the “big bang” limit $t \to 0$, one expects very high particle energies on the scale of the particle rest energy; i.e. $\epsilon(p) >> mc^2$ which is certainly true for massless particles such as photons or neutrinos. In such a big bang high energy limit, the energy density $\varepsilon$ is related to the thermodynamic pressure $p$ via $\varepsilon \approx 3p$. Thermodynamics then yields an inverse adiabatic compressibility of $K_s^{-1} \approx (4\varepsilon/9)$. Thus, near the big bang

$$\phi \to \left( \frac{8\pi G\varepsilon}{3p^2c^2} \right) \left( 1 - 2\nu u_s^2 \right)$$

as $t \to 0$. \hspace{1cm} (56)

The condition for inflation near the big bang then reads

$$\tau_p > \left( \frac{1}{2\nu} \right)$$

as $t \to 0$ \hspace{1cm} (inflation). \hspace{1cm} (57)

To see what is involved, the inverse Hubble expansion rate $\nu^{-1}$ is roughly the age of the new born universe. It is evident that when a violent explosion first begins, the time scale $\tau_p$ for the pressure to reach equilibrium is long compared to a very short time scale for a young explosion. If the pressure disequilibrium of the big bang lasts longer than the age of a new-born universe ($\tau_p >> \nu^{-1}$), then an early time period for inflation would exist. Inflaton fields need not play any role in the process.

A present day inflation, albeit quite small, is somewhat more difficult to explain. In a matter dominated universe $\varepsilon \approx pc^2 >> p$, while the adiabatic compressibility $K_s \approx (\rho u_s^2)^{-1}$ where $\rho$ is the mass density and $u_s$ is the adiabatic sound velocity. In the matter dominated present, Eq.(55) reads

$$\phi \approx \left( \frac{4\pi \rho G}{3p^2} \right) \left( 1 - \left( \frac{u_s^2}{c^2} \right) \right)$$

The present universe is thereby considered to be a “dilute gas” in a sense to be discussed below.

Let us first consider a dilute gas of slowly moving (compared to light speed) molecules in a laboratory volume $V$ controlled by a piston (similar to FIG.1). The mean square center of mass velocity $\langle v^2 \rangle = (3k_BT/M)$ of a molecule of mass $M$ is determined by the temperature $T$. And as volume $V$ of the gas is increased, the mean center of mass kinetic energy of the molecule gets smaller. However, the mean internal energy of the molecules does not necessarily cool as fast as the translational center of mass kinetic energy of the molecules. There must be many collisions between molecules to achieve the local thermal equilibrium equipartition of translational kinetic energy and vibrational or rotational internal energy of the molecules. The delay time for this internal energy equilibrium determines the longitudinal viscosity $\zeta$ of the gas.

In the present universe there exists a “dilute gas of galaxies”, i.e. rather complicated “molecules” with very many degrees of freedom. The galaxies move at velocities of (say) $(u_s/c) \sim 10^{-3}$. Hence, from Eq.(58) we see that there can be a present day negative deacceleration if (say) $\nu \tau_p > 10^{-6}$. In other words, the universal expansion forces the center of mass momentum $\mathbf{p}$ of a galaxy to slow down according to the cosmological friction law $d\mathbf{p}/dt = -\nu \mathbf{p}$. But the internal energies of the galaxies are not in equipartition thermal equilibrium with the translational kinetic energy of a galaxy. (If such equilibrium did exist we would all be dead.) For such equilibrium to exist we would need several collisions between galaxies converting translational energies to internal energies. If less that one in a million galaxies have undergone collisions, then it is feasible that the resulting viscous heating would be sufficient to cause $\phi < 0$. The argument for a small present day inflation is (of course) not as theoretically strong as the argument for inflation at times shortly after the big bang.

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