Noncommutative Universe and Chameleon Field Dynamics

Nasim Saba∗ and Mehrdad Farhoudi†
Department of Physics, Shahid Beheshti University, G.C., Evin, Tehran 19839, Iran

May 12, 2018

Abstract
We consider a noncommutative standard model with a minimal coupling scalar field and a dynamical deformation between the canonical momenta of its scale factor and scalar field, and a chameleon model with a non-minimally coupling scalar field. We indicate that there is a correspondence between these two models, more specific, actually between the noncommutative parameter and the chameleon coupling strength, and also between the matter density of the chameleon model and the noncommutative geometry. In addition, the analogy constrains the type of the matter field in the chameleon model to be nearly a cosmic string–like during the inflation. Thus, the scenario enables the evolution of the universe being described by one single scalar field. That is, the effects of the chameleon and the inflaton can be described by one single scalar field which plays the role of inflaton in the very early universe and then, acts as a chameleon field. Moreover, the proposed correspondence procedure not only sets some constraints on the noncommutative parameter and the chameleon coupling constant, but also nearly specifies functions of the scalar field and its potential.

PACS numbers: 02.40.Gh, 04.50.Kd, 98.80.-k, 98.80.Cq
Keywords: Noncommutative Geometry; Chameleon Cosmology; Inflationary Universe.

1 Introduction

Alternative theories of gravitation have almost a long history and different motivations; among which the scalar–tensor theories of gravitation, that extend general relativity by introducing a new degree of freedom, become one of the most popular alternatives to the Einstein gravitational theory, see, e.g., Refs. [4]–[7]. However, it is obvious that any proposed theory in this issue will be valid if it satisfies both the current astronomical observations and the laboratory experiments. In this regard, although the scalar–tensor theories are claimed to be successful in explaining the present accelerated expansion of the universe (that has been declared via analyzing the various observed cosmological data [8]–[20]), yet there are some problems in these theories in describing the results of the solar system tests of gravity. This shortcoming is mostly caused due to coupling between the scalar and the matter fields. Actually, due to such a coupling, the detection of a fifth force and also violation of the equivalence principle (EP) are expected [21] [22] that are in contrast with the results of the solar system tests of gravity [23] [25]. In this respect, the chameleon model [20] [40] (originally proposed by Weltman and Khoury in 2004 [26] [27]) is a scalar–tensor theory that, due to interaction of the chameleon field with an ambient matter field, invokes a screening mechanism [41] [43] which uses non–linear effects to suppress deviations from general relativity in the solar system while allowing them to be relevant on larger, i.e. cosmological, scales. Hence, it remains consistent with the tests of gravity both on the terrestrial and the solar system scales with no need to tune free parameters (masses and couplings) to evade solar system tests. In fact, the scalar field in the chameleon cosmology, similar to

∗Electronic address: n_saba@sbu.ac.ir
†Electronic address: m-farhoudi@sbu.ac.ir

1See, e.g., Refs. [1]–[3] and references therein.
the quintessence models [44–49], provides a dynamical alternative to the static cosmological constant. Due to the coupling of the scalar and the matter fields with the gravitational strength, the chameleon field acquires a density-dependent mass and consequently, its property changes depending on the environmental situations. Because of such a dependence, its mass is very light in the cosmological scales, wherein it may play the role of dark energy and causes the cosmic late time acceleration. Although, in the regions of high density, such as on the earth, the chameleon field acquires a large mass that makes its effects being short-ranged and hence, becomes invisible in search for the EP-violation and fifth force in the current experimental and observational tests.

However, in Refs. [50, 51] by proving two theorems, it has been claimed that the effect of chameleon-like scalar fields is negligible on density perturbations considering the linear scales and also, its influence may not be regarded for the observed cosmic acceleration except as some form of dark energy. Also, in Ref. [52], we considered a coupling between the chameleon field and an unknown matter scalar field, and analyzed a possibility of an influence of the chameleon field on a cosmological acceleration of the universe inflation. In the context of the slow-roll approximations, by employing the potential usually used in this issue in the literature, we evaluated the number of e-folding of the model in order to verify its viability during the inflation. We examined the model for different ranges of the free parameters, however, the results of analysis showed that there is not much chance of having the viable chameleonic universe inflation. That is, as expected for the extreme case, the exponential term (entered due to the conformal factor describing the interaction of the chameleon field with an ambient matter) in the resulted effective potential cannot inflate unless one fixes the value of the scalar, which in turn means that the matter density must be constant.

Nevertheless, in probing the role of the chameleon field on the cosmological scales, we noticed it has been shown [53] that, although the cosmological factor would be in principle constant during the last Hubble time, but this point does not keep the chameleon field being accounted for a late-time acceleration of the universe expansion. Also, although the influence of the chameleon field on the universe inflation is insignificant, but it has been indicated [54] that the cosmological constant as well as dark energy density might be induced by torsion in the Einstein–Cartan gravitational theory. Indeed, as the torsion is a natural geometrical quantity in addition to the metric tensor, it supplies a robust geometrical background in the origin of those. Moreover, as mentioned in Ref. [52], even in the limiting case, where the coupling strength in the exponential term of the conformal factor tends to zero (i.e., reducing to the minimal coupling case), still one can trace its effect in the corresponding Klein–Gordon equation. That is, it looks as if the ambient matter fluid somehow acts like an extra field and hence, it would have some effects during the chameleonic inflation.

In this sense, encouraged by these issues, we have proposed in Ref. [52] that if the chameleon model, through some mechanism, reduces to the standard inflationary model during the inflation, then it may cover the whole era of the universe from the inflation up to the late time. In this respect, the task of the present work is to investigate such a scenario at the very early universe.

Besides, it should be mentioned that in Refs. [55, 56], via a supersymmetric potential introduced in Ref. [57], a chameleon scenario has been embedded within the string compactifications, and hence, it has been shown that the volume modulus of the compactification can act as a chameleon field. The late time investigation of this scenario has been presented in Ref. [55], while Ref. [56] describes it during the inflation. Hence, it has been indicated that in order to cover the cosmology of both the late time and the very early universe, there exists a superpotential consisting of two pieces. One piece drives inflation in the very early universe, and the other one is responsible for the chameleon screening at the late times.

Moreover, in recent years, some attempts have been devoted to search for an inflationary solution in the context of the string theory [58–63] owing to the fact that the inflation would typically occur at energies near the string scale. On the other hand, the noncommutativity of the spacetime (first introduced by Snyder [64] and nowadays known as noncommutative geometry [65, 66]) has also attracted some attentions due to the strong motivation of the string and M–theories, see, e.g., Refs. [67–69]. The noncommutativity effects are significant at the Planck scale where quantum gravity effects are prominent. Indeed, during the inflation (that is a period of rapidly accelerating expansion soon after the bing bang [70–77]), the energy scale of the universe is extremely high, hence, it is reasonable to take into consideration the noncommutativity effects, as for instance, such effects have been considered
in some other contexts, see, e.g., Refs. [78]–[91]. In addition, some efforts have been devoted to explain
the standard model of particle physics in the framework of noncommutative spectral geometry and the
cosmological consequences of such an approach [92, 93].

In this work, we study the effects of noncommutativity in the standard scalar–tensor model in which
gravity is minimally coupled with a dynamical scalar field, and then, compare it with the chameleon
mechanism while investigating an analogy between these two models. Actually, as a complementary to
our previous work [52], such an analogy may provide a scenario to overcome the issue of the chameleon
model during the inflationary epoch.

The work is organized as follows. In the next section, we briefly derive the cosmological equations
of the standard scalar–tensor model in the presence of a specified noncommutativity parameter. In
Sect. III, we revisit the chameleon model and concisely derive the equations governing its evolution.
However, we furnish these two sections with almost different expression and in a way to provide our
requirements for the proposal of the work. The main purpose of this study is to compare these two
models, where this goal is performed in Sect. IV. Finally, we conclude the summary of the results in the
last section. Throughout the work, we take the signature of spacetime to be \((-,+,+,+), \) \(c = 1 = \hbar\)
and the reduced Planck mass \(M_{\text{Pl}} \equiv (8\pi G)^{-1/2}\).

2 Noncommutative Standard Scalar–Tensor Model

In this section, we outline the standard scalar–tensor model with a minimal coupling between
gravity and the scalar field in the context of noncommutativity described by the action

\[
S = \int d^4x\sqrt{-g}\left(\frac{M_{\text{Pl}}^2 R}{2} - \frac{1}{2}\partial_{\mu}\phi\partial^\mu\phi - V^{[\text{NC}]}(\phi)\right),
\]

where \(R\) is the Ricci scalar constructed from the metric \(g_{\mu\nu}\), \(g\) is the determinant of the metric
and the lowercase Greek indices run from zero to three. Also, \(\phi\) is a scalar field and \(V^{[\text{NC}]}(\phi)\) is a potential
for this model.

In addition, we consider a homogeneous scalar field, that is \(\phi = \phi(t)\), and also assume that the
background spacetime is spatially flat, homogeneous and isotropic given by the Friedmann–Lemaître–
Robertson–Walker (FLRW) metric

\[
ds^2 = -N^2(t)dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right),
\]

where \(N(t)\) is a lapse function, \(a(t)\) is the scale factor describing the cosmological expansion, and \(t\) is
the cosmic time. It is straightforward to show that the Hamiltonian of the model is

\[
\mathcal{H}_0 = -\frac{1}{12M_{\text{Pl}}^2}Na^{-1}P_a^2 + \frac{1}{2}Na^{-3}P_{\phi}^2 + Na^3V^{[\text{NC}]}(\phi),
\]

where \(P_a\) and \(P_{\phi}\) are the conjugate momenta associated to the scale factor and the scalar field, respectively.
However, as the conjugate momentum of \(N(t)\) vanishes, one has to add it as a constraint to the
Hamiltonian and hence, the corresponding Dirac Hamiltonian is

\[
\mathcal{H} = \mathcal{H}_0 + \lambda P_N,
\]

where \(\lambda\) is a Lagrange multiplier.

First, let us derive the equations of motion corresponding to the commutative phase space coordinates,
\(\{a, \phi, N; P_a, P_{\phi}, P_N\}\), in which the ordinary phase space structure is described by the usual
Poisson brackets

\[
\{a, P_{\phi}\} = 1 = \{\phi, P_{\phi}\} \quad \text{and} \quad \{N, P_N\} = 1,
\]

while the other brackets vanish. Applying these brackets and using Hamiltonian (1), the equations of
achieved as also (14) and (15), after performing some manipulations, the noncommutative equations of motion are obtained

\[ \dot{a} = \{a, \mathcal{H}\} = -\frac{1}{6M_{Pl}^2} Na^{-1}P_a, \]  

\[ \dot{P}_a = \{P_a, \mathcal{H}\} = -\frac{1}{12M_{Pl}^2} Na^{-2}P_a^2 + \frac{3}{2}Na^{-4}P_\phi^2 - 3Na^2V^{[NC]}(\phi), \]  

\[ \dot{\phi} = \{\phi, \mathcal{H}\} = Na^{-3}P_\phi, \]  

\[ \dot{P}_\phi = \{P_\phi, \mathcal{H}\} = -Na^3V^{[NC]}(\phi), \]  

\[ N = \{N, \mathcal{H}\} = \lambda, \]  

\[ \dot{P}_N = \{P_N, \mathcal{H}\} = \frac{1}{12M_{Pl}^2} a^{-1}P_a^2 - \frac{1}{2}a^{-3}P_\phi^2 - a^3V^{[NC]}(\phi), \] 

where dot and prime denote the derivative with respect to the cosmic time and the scalar field, respectively. Furthermore, to satisfy the constraint \( P_N = 0 \) at all times, the secondary constraint \( \dot{P}_N = 0 \) should also be satisfied.

Then, in order to investigate the proposed effect of the noncommutativity, inspired by the motivations mentioned in Ref. [89], we leave all the Poisson brackets for the noncommutative variables (shown with the tilde sign) unchanged, except the following plausible dynamical deformation between the canonical conjugate momenta of the scale factor and of the scalar field. That is, we deliberately choose

\[ \{\tilde{P}_a, \tilde{P}_\phi\} = \ell M_{Pl}^2 \tilde{\phi}, \] 

where \( \ell \) is a constant length indicator parameter that can present and trace the quantum behaviors (see, e.g., Ref. [89]), and if it vanishes, the standard (classical) counterpart relations will be recovered. Of course this choice satisfies the dimensionality requirements\(^1\) is restrictive, but we limit ourselves to it for reasons of simplicity which easily leads to the desired results (see Sect. IV).

In this case, the minimally deformed (noncommutative) version of the Dirac Hamiltonian is achieved by replacing the undilute variables with the tilde ones in relations (3) and (4), namely

\[ \tilde{\mathcal{H}} = -\frac{1}{12M_{Pl}^2} \tilde{N}a^{-1}\tilde{P}_a^2 + \frac{1}{2}Na^{-3}\tilde{P}_\phi^2 + \tilde{N}a^3V^{[NC]}(\tilde{\phi}) + \tilde{\lambda}\tilde{P}_N, \] 

where we have kept the noncommutative Hamiltonian with the same functional form as in (3). However, from now on, for simplicity and less confusion, we drop the tilde sign for the noncommutative variables, and also work in the comoving gauge, namely, we set \( N(t) = 1 \).

At this stage, by applying the new bracket (12), the equations of motion associated to the new variables \( a, \phi, N \) and \( P_N \) are left unchanged while the equations of motion for the new momenta sectors are deformed as

\[ \dot{\tilde{P}}_a = \{P_a, \mathcal{H}\} = -\frac{1}{12M_{Pl}^2} a^{-2}P_a^2 + \frac{3}{2}a^{-4}P_\phi^2 - 3a^2V^{[NC]}(\phi) + \ell a^{-3}M_{Pl}^3 P_\phi e^{\ell \phi}, \]  

\[ \dot{\tilde{P}}_\phi = \{P_\phi, \mathcal{H}\} = -a^3V^{[NC]}(\phi) + \frac{1}{6}a^{-1}M_{Pl}^4 P_\phi e^{\ell \phi}. \] 

As mentioned, the usual brackets are restored by setting \( \ell = 0 \). With the aid of Eqs. (6), (8), (11) and also (14) and (15), after performing some manipulations, the noncommutative equations of motion are achieved as

\[ 3M_{Pl}^2H^2 = \frac{1}{2} \dot{\phi}^2 + V^{[NC]}(\phi) = \rho_\phi^{[NC]} \equiv \rho_{tot}, \]  

\[ M_{Pl}^2 \left( \frac{2}{a} \ddot{a} + H^2 \right) = -\left( \frac{1}{2} \dot{\phi}^2 - V^{[NC]}(\phi) + \frac{1}{3} \ell a^{-2} \dot{\phi} M_{Pl}^3 e^{\ell \phi} \right) \equiv -\left( p_\phi^{[NC]} + \rho_{tot} \right), \]  

\[ \ddot{\phi} + 3H \dot{\phi} + V^{[NC]}(\phi) = -\ell H a^{-2} M_{Pl}^3 e^{\ell \phi} \equiv Q/\dot{\phi}, \] 

\(^1\)It is easy to show that the dimensions of \( P_a, P_\phi, \phi, \ell \) and consequently \( \{P_a, P_\phi\} \) are \( L_P^{-3}, L_P^{-2}, L_P^{-1}, L_P \) and \( L_P^{-2} \), respectively, where \( L_P = \sqrt{\hbar G/c^3} \) is the Planck length, however we have used the units in which \( c = 1 = \hbar \).
where $H(t) \equiv \dot{a}/a$ is the Hubble expansion rate of the universe and we have defined a deformed pressure density as
\begin{equation}
    p_\ell \equiv \frac{1}{3} \ell a^{-2} \dot{\phi} M_{Pl}^3 e^{\ell \phi} \tag{19}
\end{equation}
and the $Q$ term as
\begin{equation}
    Q \equiv -\ell H a^{-2} \dot{\phi} M_{Pl}^3 e^{\ell \phi}. \tag{20}
\end{equation}
Moreover, the time derivation of the Hubble parameter is
\begin{equation}
    -2M_{Pl}^2 \dot{H} = \dot{\phi}^2 + \frac{1}{3} \ell a^{-2} \dot{\phi} M_{Pl}^3 e^{\ell \phi}. \tag{21}
\end{equation}

As it is obvious, by taking the noncommutativity effect into account, the Klein–Gordon equation and the total pressure density have been obtained different from the corresponding usual commutative ones. However, if one sets $\ell = 0$, then each equation will have the same form as its corresponding commutative one that usually is applied for the inflationary scenario. Nevertheless, in a similar manner of methodology employed in Ref. [90] and since a conservative force is obtained from the derivative of a potential, the right hand side of Eq. (18) can be interpreted as an additional force. Also, definitions $p_\ell$ and $Q$ (relations (19) and (20)) yield
\begin{equation}
    3H p_\ell = -Q. \tag{22}
\end{equation}

On the other hand, as the conservation equation of the model is
\begin{equation}
    \dot{\rho}_{tot} + 3H (\rho_{tot} + p_{tot}) = 0, \tag{23}
\end{equation}
with the aid of relation (22), it leads to
\begin{equation}
    \dot{\rho}_\phi^{[NC]} + 3H (\rho_\phi^{[NC]} + p_\phi^{[NC]}) = Q. \tag{24}
\end{equation}
That is, while the effects of noncommutativity manifest itself in the pressure term $p_\ell$, the results dictate that, even though the total energy density is conserved, but the corresponding one for the scalar field and the one for the noncommutativity part (i.e., Eqs. (24) and (22), respectively) are not separately conserved. Indeed, their conservation equations are not independent and the $Q$ term stands as an interacting term among them. Such a similar interacting term has been employed in the literature, see, e.g., Refs. [95, 96].

### 3 Chameleon Model

In this section, we give a brief review of the chameleon scalar field model, and derive the equations governing the cosmological evolution of the universe. In this regard, we start with the well–known action that governs the dynamics of the chameleon scalar field model in 4–dimensions, i.e.\footnote{In the next section, we discuss on equality and/or differentness of those parameters that appear in this section and the previous one.}
\begin{equation}
    S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2 R}{2} - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V^{[CH]}(\phi) \right] + \sum_i \int d^4x \sqrt{-\tilde{g}^{(i)}} L_m^{(i)} \left( \psi^{(i)}, \tilde{g}_{\mu\nu}^{(i)} \right), \tag{25}
\end{equation}
where $V^{[CH]}(\phi)$ is a potential in the chameleon model, $L_m^{(i)}$’s are the Lagrangians of the matter fields and $\psi^{(i)}$’s are various matter scalar fields. Also, $\tilde{g}_{\mu\nu}^{(i)}$’s are the matter field metrics that are conformally related to the Einstein frame metric $g_{\mu\nu}$ via
\begin{equation}
    \tilde{g}_{\mu\nu}^{(i)} = e^{2 \frac{\delta \phi}{M_{Pl}}} g_{\mu\nu}. \tag{26}
\end{equation}
where $\beta_i$'s are dimensionless constants representing different non-minimal coupling constants between the scalar field and each one of the matter species as the strength of the matter couplings. However, for simplicity, we just focus on a single matter component, and henceforth, we drop the index $i$. Besides, the potential shown in action (25) is an arbitrary function, whereas the chameleon effect is a mechanism to hide the fifth force mediated by the scalar field, for which certain conditions on potential are needed. Nevertheless in the following analysis, in order to match the two models, we end up on some necessary conditions on the potential that are mostly compatible with the chameleon model.

Moreover, we consider the FLRW metric (2) in the comoving gauge and a homogeneous scalar field. In this case, the corresponding metric in the Jordan frame is

$$dz^2 = -e^{2\frac{\phi}{M_{Pl}}} dt^2 + \tilde{a}(t) \left( dx^2 + dy^2 + dz^2 \right),$$

where $\tilde{a}(t)$ is the scale factor in this frame, i.e. $\tilde{a}(t) \equiv a(t) \exp(\beta \phi/M_{Pl})$. Varying action (25) with respect to the scalar field yields the field equation of motion

$$\Box \phi = V^{[\text{CH}]}(\phi) - \frac{\beta}{M_{Pl}^2} e^{\frac{\phi}{M_{Pl}}} \tilde{T}_{\mu \nu},$$

where $\Box \equiv \nabla^\alpha \nabla_\alpha$ corresponding to the metric $g_{\mu \nu}$ and $\tilde{T}_{\mu \nu} = -(2/\sqrt{-g})\delta(\sqrt{-g}L_m)/\delta g^{\mu \nu}$ is the energy–momentum tensor that is conserved in the Jordan frame, i.e. $\nabla_\mu \tilde{T}^{\mu \nu} = 0$. We also assume the matter field as a perfect fluid with the equation of state $\tilde{p} = \tilde{\rho} \tilde{w}$, where, in the FLRW background, one has

$$\tilde{g}^{\mu \nu} \tilde{T}_{\mu \nu} = -(1 - 3\tilde{w}) \tilde{\rho}$$

with $\tilde{\rho}$ as the matter density in the Jordan frame.

Since $\tilde{\rho}$ is not conserved in the Einstein frame, we propose to have a conserved matter density that is independent of $\phi$ and obeys the relation

$$\rho \equiv \rho_0 a^{-3(1+w)}$$

in the Einstein frame, i.e. with the fluid equation $\dot{\rho} + 3H(1+w)\rho = 0$. In relation (30), $\rho_0$ is a constant for the matter density that acts as a cosmological constant with $w = -1$. For this purpose, as the continuity equation for $\tilde{\rho}$ in the Jordan frame is

$$\dot{\tilde{\rho}} + 3\frac{\tilde{a}}{a} (1 + w) \tilde{\rho} = 0,$$

i.e. $(\tilde{a}^{3(1+w)} \tilde{\rho})_{,0} = 0$, hence in order to have relation (30), one gets

$$\tilde{\rho} = e^{-3(1+w) \frac{\phi}{M_{Pl}}} \rho.$$

Therefore, by relations (29) and (32), Eq. (28) reads

$$\Box \phi = V^{[\text{CH}]}(\phi) + \rho(1 - 3\tilde{w}) \frac{\beta}{M_{Pl}^2} e^{(1-3w) \frac{\phi}{M_{Pl}}} \equiv V_{\text{eff}}'(\phi).$$

This implies that the dynamic of the scalar field is not ruled only by the self-interacting potential $V^{[\text{CH}]}(\phi)$, but it is actually governed by an effective potential defined as

$$V_{\text{eff}}(\phi) \equiv V^{[\text{CH}]}(\phi) + \rho e^{(1-3w) \frac{\phi}{M_{Pl}}} ,$$

which depends on the background matter density $\rho$ of the environment. Consequently, the value of $\phi$ at the minimum of $V_{\text{eff}}$ and the mass fluctuation about the minimum depend on the matter density which can give a chance to the chameleon field to be hidden from local experiments.

Now, by the FLRW metric (2), the field equation (33) reads

$$\ddot{\phi} + 3H \dot{\phi} + V^{[\text{CH}]}(\phi) = -\rho(1 - 3\tilde{w}) \frac{\beta}{M_{Pl}^2} e^{(1-3w) \frac{\phi}{M_{Pl}}} \equiv X/\phi,$$

where $X$ is an arbitrary function of $\phi$. This equation allows to study the chameleon effect on the scalar field and find the mass function.
where
\[ X \equiv -\rho(1-3w) \frac{\beta}{M_{Pl}} \phi e^{(1-3w) \frac{2\phi}{M_{Pl}}}. \] (36)

In addition, the variation of action (25) with respect to the metric tensor \( g_{\mu\nu} \) associated to the Einstein frame gives the field equations
\[ G_{\mu\nu} = \frac{1}{M_{Pl}^2} \left( T_{\mu\nu}^{[\beta]} + T_{\mu\nu}^{[\phi]} \right), \] (37)
where \( T_{\mu\nu}^{[\beta]} \) and \( T_{\mu\nu}^{[\phi]} \) are the energy–momentum tensors of the coupling term and the scalar field, respectively, defined as
\[ T_{\mu\nu}^{[\phi]} \equiv -\frac{1}{2} g_{\mu\nu} \partial^\alpha \phi \partial_\alpha \phi - g_{\mu\nu} V^{[\phi]}(\phi) + \partial_\mu \phi \partial_\nu \phi, \] (38)
\[ T_{\mu\nu}^{[\beta]} \equiv -\frac{2}{\sqrt{-g}} \delta L_m/\delta g^{\mu\nu}. \] (39)

One can easily verify that
\[ T_{\mu\nu}^{[\beta]} = e^{2\beta \phi} M_{Pl} \tilde{T}_{\mu\nu}, \] (40)
then by the conservation of \( \tilde{T}_{\mu\nu} \) in the Jordan frame, we obtain
\[ \nabla_\mu T_{\mu\nu}^{[\beta]} = X. \] (41)

That is, \( T_{\mu\nu}^{[\beta]} \) is not conserved in the Einstein frame and the geodesic equation is affected by the \( X \) term that can be interpreted as an additional force. Also, the Friedmann–like equations for the model in the context of the perfect fluid can be obtained as
\[ 3M_{Pl}^2 H^2 = \frac{1}{2} \dot{a}^2 + \frac{\rho_{\phi}^{[\phi]} + \rho_{\phi}^{[CH]} + \rho_{\phi}^{[\beta]} \equiv \rho_{\phi}^{tot}}{\rho_{\phi}^{[\beta]} + \rho_{\phi}^{[CH]} = -p_{\phi}^{tot}}, \] (42)
and
\[ M_{Pl}^2 \left( \frac{\ddot{a}}{a} + H^2 \right) = -\left( \frac{1}{2} \dot{a}^2 - V^{[\phi]}(\phi) + w \rho_{\phi}^{(1-3w) \frac{2\phi}{M_{Pl}}} \right) \equiv -\left( p_{\phi}^{[\phi]} + p_{\phi}^{[CH]} \right) \equiv -p_{\phi}^{tot}, \] (43)
where we have defined a coupled mass density and a coupled pressure density
\[ \rho_{\phi}^{[\beta]} \equiv \rho e^{(1-3w) \frac{2\phi}{M_{Pl}}} \] and \[ p_{\phi}^{[\beta]} \equiv w \rho e^{(1-3w) \frac{2\phi}{M_{Pl}}}, \] (44)
that has the same equation of state as the matter field. Also, the time derivative of the Hubble parameter in this case is
\[ -2M_{Pl}^2 \dot{H} = \dot{\phi}^2 + (1 + w) \rho e^{(1-3w) \frac{2\phi}{M_{Pl}}}. \] (45)

Therefore, by taking the non–minimal coupling effect into account, the Klein–Gordon equation, the total mass and the total pressure densities have been achieved different from the corresponding ones without coupling. However, if one sets \( \beta = 0 \), then each equation will have the same form as its corresponding standard one.

Moreover, definitions (36) and (44), with the fluid equation for the matter density \( \rho \), give
\[ \dot{\rho} + 3H(\rho + \rho) = -X, \] (46)
that is consistent with Eq. (41). On the other hand, as the conservation equation of this model is also the same as Eq. (23). Eq. (46) leads to
\[ \dot{\rho}_{\phi}^{[CH]} + 3H(\rho_{\phi}^{[CH]} + p_{\phi}^{[CH]}) = X. \] (47)
That is, the effects of non–minimal coupling manifest itself in both the energy and the pressure densities (i.e., \( \rho_{\phi} \) and \( p_{\phi} \)). Furthermore, the results show that the total energy density is conserved however, it is not conserved separately for the scalar field and its coupling to the matter field. In another word, their conservation equations are not independent and the \( X \) term, that stands as an interacting term among them, manifests itself as a deviation term into the geodesic equation and are interpreted as an additional force.
4 Analogy of the Models

In this section, as we are interested in having one scalar field which plays the role of both inflaton and chameleon, hence, we have considered the scalar field to be the same in the noncommutative standard and the chameleon models. On the other hand, since the Hubble constant is an observational quantity, we have preferred to choose the Hubble parameter also to be the same in both models. Nevertheless, in this case, it is not necessary that the corresponding scale factors being exactly the same, however those are proportional to each other, namely

$$a_{\text{CH}}(t) = A a_{\text{NC}}(t), \quad (48)$$

where $A$ is a constant, and $a_{\text{CH}}$ and $a_{\text{NC}}$ are the scale factors in the chameleon model and in the noncommutative standard one, respectively, while we have shown those without any distinguished label in the previous sections. Moreover, it is not also necessary that potentials being the same in the both models, as we have considered them to be different from the beginning.

Now, we compare the behavior of the other parameters for any probable correspondence between the models during the inflation where the Hubble parameter is nearly constant. In this respect, a direct match of the corresponding Friedmann Eqs. (16) and (42) leads to

$$V_{\text{NC}}(\phi) - V_{\text{CH}}(\phi) = \rho e^{(1-3w)\frac{H}{c}} \beta M_{\text{Pl}}, \quad (49)$$

for any $w$ and $\beta$. To proceed further, we have presumed the following considerations. It is obvious that, in the onset of the inflation when the matter density of the universe is extremely high, the effect of coupling term being inefficient in the effective potential of the chameleon model, and it cannot inflate unless the matter density approaches a constant value with $w = -1$ (i.e., a cosmological constant). Besides, when in addition $\beta = 0$, the chameleon model reduces to the standard inflationary model, and in this case, the right hand side of Eq. (49), by relation (30), is $\rho_0$. On the other hand, as it is unlikely that the constant values $\beta$ and $w$ being appeared in the functionality of the potentials, thus we likely assume

$$V_{\text{NC}}(\phi) - V_{\text{CH}}(\phi) = \rho_0 \quad (50)$$

to be valid for any $w$ and $\beta$ values. Hence, in turn, one has

$$V^{\prime}_{\text{NC}} = V^{\prime}_{\text{CH}} \equiv V^{\prime}. \quad (51)$$

In this situation, by matching the Klein–Gordon equations of motion (18) and (35), we plausibly obtain

$$\ell \leftrightarrow (1 - 3w) \frac{\beta}{M_{\text{Pl}}} \quad (52)$$

and

$$\ell H a_{\text{NC}}^{-2} M_{\text{Pl}}^3 \leftrightarrow \rho (1 - 3w) \frac{\beta}{M_{\text{Pl}}}. \quad (53)$$

Relation (52) implies that the noncommutative parameter $\ell$ in the standard model is related to $\beta$ as the coupling strength of the scalar field to the matter field in the chameleon model. Inserting this relation into relation (53) also leads to

$$H a_{\text{NC}}^{-2} M_{\text{Pl}}^3 \leftrightarrow \rho, \quad (54)$$

that indicates the matter density of the environment in the chameleon model also relates to influence of geometry in the noncommutative standard model. Besides, from relation (49), one has

$$\rho e^{\phi} = \rho_0, \quad (55)$$

\footnote{However, we have also examined having different Hubble parameters between two models, but from mathematical point of view, this choice was not fruitful.}

\footnote{A constant Hubble parameter (i.e., say $H_c$) gives $a_{\text{NC}}(t) = a_{0\text{NC}} e^{H_c t}$ and $a_{\text{CH}}(t) = a_{0\text{CH}} e^{H_c t}$ where $a_{0\text{NC}}$ and $a_{0\text{CH}}$ are initial constants for the noncommutative and the chameleon models, respectively.}
that in turn, by relation (30), gives
\[ a_{\text{CH}} = e^{\ell \phi / 3(1+w)}. \] (56)

The comparison looks enough if one also matches Eqs. (17) and (43) (or equivalently, Eqs. (21) and (45)) that, by relations (52) and (54), lead to
\[ \dot{\phi} = 3(1+w)H. \] (57)

This result can also be obtained by differentiating relation (55) with respect to time while using relation (30). Besides, relations (52) and (54) obviously show that definitions \( Q \) and \( X \) terms are exactly equal as expected, that is, the exchange of energy between geometry and the scalar field in the noncommutative model are the same as the exchange of energy between the matter and the scalar field in the chameleon model. Also, as Eq. (16) indicates Eq. (41) in the chameleon model, one can similarly have \( \nabla \mu T^{(\mu)}_{\nu} = Q \) from Eq. (22), where \( T^{(\mu)}_{\nu} \) can be interpreted as an energy–momentum tensor associated to the noncommutativity effect.

Furthermore, assuming the Hubble parameter to be a constant during the inflation, relations (30), (48) and (54) actually yield
\[ a_{\text{CH}}^{-2} \propto a_{\text{CH}}^{-3(1+w)} \Rightarrow w = -1/3. \] (58)

That is, for a nearly constant \( H \), the type of matter field is constrained to be nearly a cosmic string–like during the inflation. Also, these relations give \( A = \pm \sqrt{\rho_0 / (H M_{\text{Pl}}^2)} \), where the positive sign is acceptable for the inflationary expansion. In this case, integrating result (57) roughly gives
\[ \phi \sim 2H_{\text{in}} t + \phi_0, \] (59)
where \( \phi_0 \) is an initial constant that, by relations (30) and (55) with \( w = -1/3 \), is \( \phi_0 \sim (2/\ell) \ln a_{\phi,\text{CH}} \).

Solution (59) implies that the scalar field is roughly an increasing function of time during the inflation. On the other hand, using Eq. (42) with relations (52) and (55), and result (57), we get the chameleon potential to be
\[ V_{\text{CH}} = \left[ 3M_{\text{Pl}}^2 - \frac{9(1+w)^2}{2H^2} \right] H^2 - \rho_0, \] (60)
with constraint
\[ V_{\text{CH}}(\phi) > \frac{3(1+w) \rho_0}{2}. \] (62)

Hence, from equality of \( V_{\text{CH}} \) from solutions (60) and (61), one can obtain the value of \( \epsilon \) in terms of the other parameters as
\[ \epsilon = \frac{1 + w}{2M_{\text{Pl}}^2} \left[ \frac{\rho_0}{H^2} + \frac{9(1+w)}{\ell^2} \right]. \] (63)
Moreover, by differentiating potential (60) while using result (57), we obtain
\[ V'_{\text{eff}} = V' = -\frac{2 \epsilon \ell}{3(1 + w)} V^{[\text{NC}]}, \]  
(64)
that can also be achieved by inserting relation (52) and result (57) into Eq. (35) while using relations (55) and (63). Hence, the price of our proposal for matching the models is that the derivative of the chameleon effective potential is a nearly negative constant value in the inflationary epoch (for \( w > -1 \)), however, it can be made to be almost zero as being required in the chameleon models.

Obviously, Eq. (45) indicates that, onset of the inflation where \( \epsilon \ll 1 \) (that is, \( \dot{H} \sim 0 \)), the equation of state parameter should be \( w \sim -1 \), and thus, the scalar field acts as a constant in this case. Hence, the cosmological constant starts the extremely expansion of the universe. Then, while the expansion is occurring, the matter field being diluted and acts as a cosmic string-like. In this situation, \( V^{[\text{NC}]} \) from solution (60) with constraint (62) leads to a lower bound on the noncommutative parameter, namely \( \ell > \sqrt{2/3}/M_{\text{Pl}} \), where we have considered \( \ell \) as a positive number. However, \( V^{[\text{NC}]} \) from solutions (60) and (61) together, with constraint (62), gives a more restricted lower bound on \( \ell \), namely
\[ \ell > \sqrt{2} \frac{1}{\epsilon} \frac{1}{M_{\text{Pl}}}. \]  
(65)
This constraint, via relation (52) with \( w = -1/3 \), also leads to a lower bound on the chameleon coupling constant as
\[ \beta > \sqrt{\frac{1}{2 \epsilon}} \]  
(66)
that, even considering \( \epsilon \) to have an infinitesimal value in the inflationary epoch (say, e.g., of the order \( 10^{-5} \)), it still yields an acceptable estimate for \( \beta \) consistent with the recent experimental constraint obtained by Ref. [97], namely \( \beta < 3.7 \times 10^2 \). In turn, the recent experimental constraint on \( \beta \) also sets an upper bound on the value of noncommutative parameter as \( \ell < 7.2 \times 10^2 \) by relation (52) in an unit where \( M_{\text{Pl}} = 1 \).

Now, with nearly constant Hubble parameter, i.e., \( \dot{H} \equiv -\epsilon H^2 \), although still with \( w = -1/3 \), by manipulating result (57) while using relation (52), we finally get an equation for the scalar field, namely
\[ \ddot{\phi} + \frac{\beta \epsilon}{M_{\text{Pl}}} \dot{\phi}^2 = 0. \]  
(67)
This concise equation indicates the behavior of the scalar field with respect to time better than the roughly solution (59) during the inflation (note that, the value of \( \epsilon \) exactly vanishes in solution (59)). The result of numerical calculations of Eq. (67) has been depicted in Fig. (1) for two different values of \( \epsilon \) and the other constants while considering the recent experimental constraint on \( \beta \) and constraint (66) on the corresponding \( \epsilon \). The Figures confirm the roughly solution (59) and indicate that the scalar field linearly increases during the inflation.

Our remaining task is to investigate the viability of the models during the inflation. A successful inflationary model must resolve the problems of standard cosmology. Hence, in order to solve the horizon problem, which is more important than the flatness and monopole problems (since by resolving this one, the other problems will be solved automatically), inflation should last for a sufficiently enough time in a way that its achieved number of e–folding being at least around 50 to 70. In this respect, the number of e–folding quantifies the amount of universe expansion during the inflation, and is defined (76) to be
\[ N \equiv \int_{t_b}^{t_e} H dt = \int_{\phi_b}^{\phi_e} \frac{H}{\dot{\phi}} d\phi, \]  
(68)
where the subscripts “b” and “e” refer to the beginning and the end of inflation, respectively. By using result (57) with \( w = -1/3 \), one gets
\[ N = \frac{\ell}{2} (\phi_e - \phi_b). \]  
(69)
Figure 1: The figures show the time behavior of the scalar field that have been plotted for the numerical values $\beta = 100$, $M_{Pl} = 1$ with initial value $\phi_0 = 0.01$, and (the left plot): $\epsilon = 0.0001$ with initial value $\phi_0 = 1$, (the right plot): $\epsilon = 0.001$ with initial value $\phi_0 = 7$.

As the slow–roll condition is broken at the end of inflation, thus the value of $\phi_e$ can be obtained via Eq. (67) while setting the slow–roll parameter to be $\epsilon \sim 1$. Also, for the value of $\phi_b$, we use relation [98]

$$r \sim 16 \epsilon|\phi = \phi_b,$$

where the parameter $r$ is the ratio of the tensor perturbation amplitude to the scalar perturbation amplitude with an upper bound $r < 0.11$ in 95% confidence level stated in the Planck measurements [15]–[20]. Then, by tuning the free parameters and also the initial values for the scalar field and its derivative, one can achieve an appropriate number of e–folding.

Therefore, through the analogy of the noncommutative standard and chameleon models, it has been shown that the chameleon model can be a successful one during the inflationary epoch. Also, we conclude that one can consider a single scalar field being responsible for both roles of the inflaton and the chameleon.

5 Conclusions

In this work, we have indicated that there is a correspondence between the chameleon model (where the chameleon scalar field non–minimally couples with the matter field) and the noncommutative standard model (in which the inflaton scalar field minimally couples with gravity). On the other hand, in another work [94], we have shown that a noncommutative inflationary model, in the presence of a particular type of dynamical deformation between the canonical momenta of the scale factor and of the scalar field, is a successful model during the inflation. Now, through the mentioned correspondence procedure of this work, we have indicated that there is a relevance between the noncommutative parameter and the chameleon coupling strength. Such a correspondence, in turn, presents that noncommutative effects act as a source term in the Klein–Gordon equation in a similar manner to the matter density in a chameleon theory, wherein the matter density of the environment in the chameleon model is related to influence of geometry in the noncommutative standard model. In fact, the results of the work, as a complementary to our previous work [52], provide a possibility to take into account the noncommutative standard model instead of the chameleon model during the inflation. However, we should emphasize that the specific choice of the dynamical deformation (that has been assumed between the canonical momenta of the scale factor and of the scalar field) is very constructive to reach in such a relevance.

Moreover, this correspondence has represented that the type of the matter field in the chameleon model is constrained to be nearly a cosmic string–like during the inflation. The cosmic string, first introduced by Kibble in the 1970s, is a kind of hypothetical object associated to topological defects. It is claimed to be formed during the very early universe [99] [100] and has been predicted in some field theory models. Also, its formation in the context of string theory has attracted some attentions.
(see, e.g., Ref. [101] and references therein) and few efforts have recently been devoted to search for evidence of its existence.

Nevertheless, through the obtained correspondence, the noncommutative standard model provides a viable chameleonic inflationary model as we proposed in Ref. [52]. In fact, there exist two scenarios for the evolution of the universe. In the first one, it can be described by the noncommutative standard model where the influence of geometry drives the universe expansion, whereas in the second scenario, one considers the evolution of the universe via the chameleon model. In this regard, at the beginning of the inflation, when the energy density of the universe is extremely high, the chameleon field acquires a very large mass that makes the effect of its coupling suppressed. Thus, the matter density acts as the cosmological constant and starts the inflation. However, during the inflation, due to extreme expansion of the universe, the matter field being diluted and acts as a cosmic string–like. Therefore, at the beginning of the inflation, the cosmological constant drives the inflation and then, the scalar field plays the role of inflaton. Thus, the evolution of the universe being described by one single scalar field during the inflation that plays the role of inflaton in the very early universe and then, acts as a chameleon field. Furthermore, our proposed correspondence procedure has set some constraints on the noncommutative parameter and the chameleon coupling constant as well as nearly specifying functions of the scalar field and its potential. Through matching the models, we have also obtained a nearly negative constant value for the derivative of potential in the inflationary epoch that can be set to be as close to zero as required to be consistent with the chameleon model conditions.

By the way, the investigation of the chameleon model, when the universe reheats after inflation, can also be of much interest that we propose to study it in a subsequent work.

Acknowledgement

We thank the Research Council of Shahid Beheshti University for financial support.

References

[1] M. Farhoudi, “Classical trace anomaly”, Int. J. Mod. Phys. D 14, 1233 (2005).

[2] M. Farhoudi, “On higher order gravities, their analogy to GR, and dimensional dependent version of Duff’s trace anomaly relation”, Gen. Rel. Grav. 38, 1261 (2006).

[3] M. Farhoudi, “Lovelock tensor as generalized Einstein tensor”, Gen. Rel. Grav. 41, 117 (2009).

[4] Y. Fujii and K. Maeda, The Scalar–Tensor Theory of Gravitation, (Cambridge University Press, Cambridge, 2004).

[5] V. Faraoni, Cosmology in Scalar–Tensor Gravity, (Kluwer Academic Publishers, Netherlands, 2004).

[6] H. Farajollahi, M. Farhoudi and H. Shojaie, “On dynamics of Brans–Dicke theory of gravitation”, Int. J. Theor. Phys. 49, 2558 (2010).

[7] S. Capozziello and V. Faraoni, Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics, (Springer, Heidelberg, 2011).

[8] A.G. Riess et al., “Observational evidence from supernovae for an accelerating universe and a cosmological constant”, Astron. J. 116, 1009 (1998).

[9] S. Perlmutter et al. [The Supernovae Cosmology Project], “Measurements of Omega and Lambda from 42 high–redshift supernovae”, Astrophys. J. 517, 565 (1999).

[10] A.G. Riess et al., “BV RI light curves for 22 type Ia supernovae”, Astron. J. 117, 707 (1999).
[11] A.G. Riess et al., “Type Ia supernova discoveries at z > 1 from the Hubble space telescope: Evidence for past deceleration and constraints on dark energy evolution”, Astrophys. J. 607, 665 (2004).
[12] M. Tegmark et al., “Cosmological parameters from SDSS and WMAP”, Phys. Rev. D 69, 103501 (2004).
[13] D.N. Spergel et al., “Wilkinson microwave anisotropy probe (WMAP) three year observations: Implications for cosmology”, Astrophys. J. Suppl. 170, 377 (2007).
[14] N. Benitez et al., “Measuring baryon acoustic oscillations along the line of sight with photometric redshifts: The PAU survey”, Astrophys. J. 691, 241 (2009).
[15] D. Parkinson et al., “Optimizing baryon acoustic oscillation surveys II. Curvature, redshifts and external data sets”, Mon. Not. R. Astron. Soc. 401, 2169 (2010).
[16] G. Hinshaw et al., “Nine–year Wilkinson microwave anisotropy probe (WMAP) observations: Cosmological parameter results”, Astrophys. J. 208, 19 (2013).
[17] C.L. Bennett et al., “Nine–year Wilkinson microwave anisotropy probe (WMAP) observations: Final maps and results”, Astrophys. J. 208, 20 (2013).
[18] P.A.R. Ade et al., “Planck 2013 results. XVI. Cosmological parameters”, Astron. Astrophys. 571, A16 (2014).
[19] R. Adam et al., “Planck 2015 results. I. Overview of products and scientific results”, Astron. Astrophys. 594, A1 (2016).
[20] P.A.R. Ade et al., “Planck 2015 results. XIII. Cosmological parameters”, Astron. Astrophys. 594, A13 (2016).
[21] E. Fischbach and C.L. Talmadge, The Search for Non–Newtonian Gravity, (Springer, New York, 1999).
[22] L. Amendola et al., “Cosmology and fundamental physics with the Euclid satellite”, Living Rev. Rel. 16, 6 (2013).
[23] B. Bertotti, L. Iess and P. Tortora, “A test of general relativity using radio links with the Cassini spacecraft”, Nature 425, 374 (2003).
[24] C.M. Will, “The confrontation between general relativity and experiment”, Living Rev. Rel. 9, 3 (2006).
[25] R.S. Decca, D. López, E. Fischbach, G.L. Klimchitskaya, D.E. Krause and V.M. Mostepanenko, “Tests of new physics from precise measurements of the Casimir pressure between two gold–coated plates”, Phys. Rev. D 75, 077101 (2007).
[26] J. Khoury and A. Weltman, “Chameleon cosmology”, Phys. Rev. D 69, 044026 (2004).
[27] J. Khoury and A. Weltman, “Chameleon fields: Awaiting surprises for tests of gravity in space”, Phys. Rev. Lett. 93, 171104 (2004).
[28] S.S. Gubser and J. Khoury, “Scalar self–interactions loosen constraints from fifth force searches”, Phys. Rev. D 70, 104001 (2004).
[29] P. Brax, C. van de Bruck, A.–C. Davis, J. Khoury and A. Weltman, “Detecting dark energy in orbit: The cosmological chameleon”, Phys. Rev. D 70, 123518 (2004).
[30] D.F. Mota and D.J. Shaw, “Strongly coupled chameleon fields: New horizons in scalar field theory”, Phys. Rev. Lett. 97, 151102, (2006).
[31] A. Upadhye, S.S. Gubser and J. Khoury, “Unveiling chameleon fields in tests of the gravitational inverse–square law”, Phys. Rev. D 74, 104024 (2006).

[32] D.F. Mota and D.J. Shaw, “Evading equivalence principle violations, cosmological, and other experimental constraints in scalar field theories with a strong coupling to matter”, Phys. Rev. D 75, 063501 (2007).

[33] P. Brax, C. van de Bruck, A.–C. Davis, D.F. Mota and D.J. Shaw, “Testing chameleon theories with light propagating through a magnetic field”, Phys. Rev. D 76, 085010 (2007).

[34] P. Brax, C. van de Bruck, A.–C. Davis, D.F. Mota and D.J. Shaw, “Detecting chameleons through Casimir force measurements”, Phys. Rev. D 76, 124034 (2007).

[35] C. Burrage, “Supernova brightening from chameleon–photon mixing”, Phys. Rev. D 77, 043009 (2008).

[36] C. Burrage, A.–C. Davis and D.J. Shaw, “Detecting chameleons: The astronomical polarization produced by chameleon–like scalar fields”, Phys. Rev. D 79, 044028 (2009).

[37] A.–C. Davis, C.A.O. Schelpe and D.J. Shaw, “Effect of a chameleon scalar field on the cosmic microwave background”, Phys. Rev. D 80, 064016 (2009).

[38] A. Upadhye, J.H. Steffen and A. Weltman, “Constraining chameleon field theories using the GammeV afterglow experiments”, Phys. Rev. D 81, 015013 (2010).

[39] H. Farajollahi, M. Farhoudi, A. Salehi and H. Shojai, “Chameleonic generalized Brans–Dicke model and late–time acceleration”, Astrophys. Space Sci. 337, 415 (2012).

[40] P. Brax, G. Pignol and D. Roulier, “Probing strongly coupled chameleons with slow neutrons”, Phys. Rev. D 88, 083004 (2013).

[41] J. Khoury, “Theories of dark energy with screening mechanisms”, [arXiv:1011.5909].

[42] A. Joyce, B. Jain, J. Khoury and M. Trodden, “Beyond the cosmological standard model”, Phys. Rept. 568, 1 (2015).

[43] C. Burrage and J. Sakstein, “Tests of chameleon gravity”, Living Rev. Rel. 21, 1 (2018).

[44] P.J.E. Peebles and B. Ratra, “The cosmological constant and dark energy”, Rev. Mod. Phys. 75, 559 (2003).

[45] T. Padmanabhan, “Cosmological constant–the weight of the vacuum”, Phys. Rep. 380, 235 (2003).

[46] D. Polarski, “Dark energy: Current issues”, Ann. Phys. (Berlin) 15, 342 (2006).

[47] E.J. Copeland, M. Sami and S. Tsujikawa, “Dynamics of dark energy”, Int. J. Mod. Phys. D 15, 1753 (2006).

[48] R. Durrer and R. Maartens, “Dark energy and dark gravity: Theory overview”, Gen. Rel. Grav. 40, 301 (2008).

[49] K. Bamba, S. Capozziello, S. Nojiri and S.D. Odintsov, “Dark energy cosmology: The equivalent description via different theoretical models and cosmography tests”, Astrophys. Space Sci. 342, 155 (2012).

[50] J. Wang, L. Hui and J. Khoury, “No-go theorems for generalized chameleon field theories”, Phys. Rev. Lett. 109, 241301 (2012).

[51] J. Khoury, “Chameleon field theories”, Class. Quant. Grav. 30, 214004 (2013).

[52] N. Saba and M. Farhoudi, “Chameleon field dynamics during inflation”, Int. J. Mod. Phys. D 27, 1850041 (2018).
A.N. Ivanov and M. Wellenzohn, “Can chameleon field be identified with quintessence?”, arXiv:1607.00884.

A.N. Ivanov and M. Wellenzohn, “Einstein–Cartan gravity with torsion field serving as origin for cosmological constant or dark energy density”, Astrophys. J. 829, 47 (2016).

K. Hinterbichler, J. Khoury and H. Nastase, “Towards a UV completion for chameleon scalar theories”, J. High Energy Phys. 1103, 061 (2011).

K. Hinterbichler et al., “Chameleonic inflation”, J. High Energy Phys. 1308, 053 (2015).

S. Kachru, R. Kallosh, A.D. Linde and S.P. Trivedi, “de Sitter vacua in string theory”, Phys. Rev. D 68, 046005 (2003).

S. Sethi, C. Vafa and E. Witten, “Constraints on low–dimensional string compactifications”, Nucl. Phys. B 480, 213 (1996).

G.R. Dvali and S.–H.H. Tye, “Brane inflation”, Phys. Lett. B 450, 72 (1999).

C. Herdeiro, S. Hirano and R. Kallosh, “String theory and hybrid inflation/acceleration”, J. High Energy Phys. 0112, 02 (2001).

S. Kachru et al., “Towards inflation in string theory”, J. Cosmol. Astropart. Phys. 0310, 013 (2003).

H. Firouzjahi and S.–H.H. Tye, “Closer towards inflation in string theory”, Phys. Lett. B 584, 147 (2004).

C.P. Burgess, R. Easther, A. Mazumdar, D.F. Mota and T. Multamaki, “Multiple inflation, cosmic string networks and the string landscape”, J. High Energy Phys. 0505, 067 (2005).

H.S. Snyder, “Quantized space–time”, Phys. Rev 71, 38 (1947).

A. Connes, Noncommutative Geometry, (Academic Press, New York, 1994).

I. Hinchliffe, N. Kersting and Y.L. Ma, “Review of the phenomenology of noncommutative geometry”, Int. J. Mod. Phys. A 19, 179 (2004).

N. Seiberg and E. Witten, “String theory and noncommutative geometry”, J. High Energy Phys. 09, 032 (1999).

D.J. Gross and N.A. Nekrasov, “Dynamics of strings in noncommutative gauge theory”, J. High Energy Phys. 10, 021 (2000).

G. Amelino–Camelia, L. Doplicher, S. Nam and Y.S. Seo, “Phenomenology of particle production and propagation in string–motivated canonical noncommutative spacetime”, Phys. Rev. D 67, 085008 (2003).

A.H. Guth, “The inflationary universe: A possible solution to the horizon and flatness problems”, Phys. Rev. D 23, 347 (1981).

A.D. Linde, “A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems”, Phys. Lett. B 108, 389 (1982).

A.D. Linde, “Coleman–Weinberg theory and a new inflationary universe scenario”, Phys. Lett. B 114, 431 (1982).

A.D. Linde, “Scalar field fluctuations in expanding universe and the new inflationary universe scenario”, Phys. Lett. B 116, 335 (1982).

A. Starobinsky, “Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations”, Phys. Lett. B 117, 175 (1982).
[75] A.D. Linde, “Chaotic inflation”, *Phys. Lett. B* **129**, 177 (1983).
[76] S. Weinberg, *Cosmology*, (Oxford University Press, Oxford, 2008).
[77] S. del Campo, “Approach to exact inflation in modified Friedmann equation”, *J. Cosmol. Astropart. Phys.* **1212**, 005 (2012).
[78] M.R. Douglas and N.A. Nekrasov, “Noncommutative field theory”, *Rev. Mod. Phys.* **73**, 977 (2001).
[79] M. Chaichian, M.M. Sheikh–Jabbari and A. Tureanu, “Hydrogen atom spectrum and the Lamb shift in noncommutative QED”, *Phys. Rev. Lett.* **86**, 2716 (2001).
[80] S.M. Carroll, J.A. Harvey, V.A. Kostelecky, C.D. Lane and T. Okamoto, “Noncommutative field theory and Lorentz violation”, *Phys. Rev. Lett.* **87**, 141601 (2001).
[81] J.M. Carmona, J.L. Cortés, J. Gamboa and F. Méndez, “Noncommutativity in field space and Lorentz invariance violation”, *Phys. Lett. B* **565**, 222 (2003).
[82] R.J. Szabo, “Quantum field theory on noncommutative spaces”, *Phys. Rep.* **378**, 207 (2003).
[83] A.E.F. Djemaï and H. Smail, “On quantum mechanics on noncommutative quantum phase space”, *Commun. Theor. Phys.* **41**, 837 (2004).
[84] O. Bertolami, J.G. Rosa, C.M.L. de Aragao, P. Castorina and D. Zappala, “Noncommutative gravitational quantum well”, *Phys. Rev. D* **72**, 025010 (2005).
[85] S. Samanta, “Noncommutativity from embedding techniques”, *Mod. Phys. Lett. A* **21**, 675 (2006).
[86] B. Malekolkalami and M. Farhoudi, “Noncommutativity effects in FRW scalar field cosmology”, *Phys. Lett. B* **678**, 174 (2009).
[87] B. Malekolkalami and M. Farhoudi, “Noncommutative double scalar fields in FRW cosmology as cosmical oscillators”, *Class. Quant. Grav.* **27**, 245009 (2010).
[88] A. Saha, “Galilean symmetry in noncommutative gravitational quantum well”, *Phys. Rev. D* **81**, 125002 (2010).
[89] S.M.M. Rasouli, M. Farhoudi and N. Khosravi, “Horizon problem remediation via deformed phase space”, *Gen. Rel. Grav.* **43**, 2895 (2011).
[90] B. Malekolkalami and M. Farhoudi, “Gravitomagnetism and non–commutative geometry”, *Int. J. Theor. Phys.* **53**, 815 (2014).
[91] S.M.M. Rasouli, A.H. Ziaie, J. Marto and P.M. Moniz, “Gravitational collapse of a homogeneous scalar field in deformed phase space”, *Phys. Rev. D* **89**, 044028 (2014).
[92] W. Nelson and M. Sakellariadou, “Cosmology and the noncommutative approach to the standard model”, *Phys. Rev. D* **81**, 085038 (2010).
[93] M. Buck, M. Fairbairn and M. Sakellariadou, “Inflation in models with conformally coupled scalar fields: An application to the noncommutative spectral action”, *Phys. Rev. D* **82**, 043509 (2010).
[94] S.M.M. Rasouli, N. Saba, M. Farhoudi, J. Marto and P.V. Moniz, “Inflationary universe in deformed phase space scenario”, *Ann. Phys.* **393**, 288 (2018).
[95] A. Sheykhi and M.S. Movahed, “Interacting ghost dark energy in non–flat universe”, *Gen. Rel. Grav.* **44**, 449 (2012).
[96] A.F. Bahrehbakhsh, M. Farhoudi and H. Vakili, “Dark energy from fifth dimensional Brans–Dicke theory”, *Int. J. Mod. Phys. D* **22**, 1350070 (2013).
[97] Jaffe et al., “Testing sub–gravitational forces on atoms from a miniature, in–vacuum source mass”, *Nature* 13, 938 (2017).

[98] R. Durrer, *The Cosmic Microwave Background*, (Cambridge University Press, Cambridge, 2008).

[99] T.W.B. Kibble “Topology of cosmic domains and strings”, *J. Phys. A: Math. Gen.* 9, 1387 (1976).

[100] M. Hindmarsh and T.W.B. Kibble “Cosmic strings”, *Rept. Prog. Phys.* 58, 477 (1995).

[101] E.J. Copeland and T.W.B. Kibble “Cosmic strings and superstrings”, *Proc. Roy. Soc. Lond. A* 466, 623 (2010).