Structural damage detection using Bayesian inference and seismic interferometry

Murat Uzun | Hao Sun | Dirk Smit | Oral Büyükoztürk

Summary
We present a computational methodology for structural identification and damage detection via linking the concepts of seismic interferometry and Bayesian inference. A deconvolution-based seismic interferometry approach is employed to obtain the waveforms that represent the impulse response functions with respect to a reference excitation source. Using the deconvolved waveforms, we study the following two different damage detection methods that utilize shear wave velocity variations: the arrival picking method and the stretching method. We show that variations in the shear wave velocities can be used for qualitative damage detection and that velocity reduction is more evident for more severely damaged states. Second, a hierarchical Bayesian inference framework is used to update a finite element model by minimizing the gap between the predicted and the extracted time histories of the impulse response functions. Through comparison of the model parameter distributions of the damaged structure with the updated baseline model, we demonstrate that damage localization and quantification are possible. The performance of the proposed approach is verified through two shake table test structures. Results indicate that the proposed framework is promising for monitoring structural systems, which allows for noninvasive determination of structural parameters.

KEYWORDS
Bayesian inference, damage detection, probabilistic model updating, seismic interferometry, stretching method, structural health monitoring

1 INTRODUCTION

Vibration-based damage identification methodologies, as a major branch in structural health monitoring (SHM), generally track the changes in dynamical features of a structural system such as natural frequencies, mode shapes, structural damping values, mode shape curvatures, and modal strain energies.1-6 Tremendous efforts have been made over the last two decades.7-13 Recently, wave propagation–based approaches have also started to gain attention. Considering the physics behind the earthquake response of a structure, actual seismic waves that are initiated due to a fault rupture reach the surface, thereafter propagate into the structure, causing structural vibrations.14-17 As the changes in the wave travel time between two observation locations only depend on the changes in the physical properties of the connecting medium, wave methods are considered to be more sensitive to local changes.18-20 However, wave-based monitoring methods generally assume 1D wave propagation within the building and have its own limitations and challenges.21 For more in-depth damage diagnosis, these macroscale dynamical features can be further coupled with computational models through suc-
cessful implementation of finite element model (FEM) updating schemes when design information about the structure is available (e.g., materials, topology, and connection types).\textsuperscript{22,23} Considering the required modeling assumptions and the inherent measurement noise, the Bayesian model updating framework offers a robust and rigorous basis for structural condition assessment and consequent reliability evaluation.\textsuperscript{24-34} Essentially, it specifies how to characterize and quantify the uncertainty of the models as well as the predictions against the measurements.\textsuperscript{35} Even though the framework is conceptually straightforward, learning reliable models from the data leads to computational challenges (e.g., marginalizing).

Deconvolution-based seismic interferometry, proposed by Snieder and Safak,\textsuperscript{36} is one of the first techniques that employs wave propagation as a basis for monitoring structures. It is a method to extract the Green’s function of the medium, which is the impulse response solution of the differential equation that defines the dynamics of the system of interest. Essentially, deconvolving the recorded signal on receivers of interest (e.g., acceleration time series of the \textit{i}th story) with respect to signal recorded on a reference receiver (e.g., acceleration time series of ground story) yields waveforms consisting of wave propagation information given a virtual impulse input at the reference, namely, the impulse response functions (IRFs). This technique has been proven to be successful and improved by many other researchers in the field.\textsuperscript{37-39} Further details are provided in Section 2.

In this paper, we extend the framework previously presented by Sun et al.,\textsuperscript{39} where interferometric data analytics are linked with Bayesian inference, to establish a more holistic approach for identification of structural damage. In particular, we address the problem by utilizing earthquake response data, aiming to achieve damage detection as well as localization and quantification. First, using earthquake response measurements, we apply the deconvolution interferometry technique to extract the IRFs of the structure of interest. These waveforms are analyzed to form a qualitative damage detection strategy, given that the response data from the undamaged case are present. We test two different damage detection methods that use shear wave velocity variations: (a) the arrival picking method (APM) and (b) the stretching method (SM). It is noted that the SM produces higher sensitivity for damage detection. Then, we perform the finite element model calibration, applying the hierarchical Bayesian framework. The extracted time histories of IRFs are utilized to learn the physical parameters of a suitable FEM. We employ Markov chain Monte Carlo (MCMC) sampling in order to obtain the posterior probability distributions (PDFs) of the model parameters. Uncertainty quantification of model parameters (e.g., lateral story stiffness values) is realized inherently through the Bayesian inference framework. We demonstrate that the damage quantification is possible through comparing the model parameter distributions of the damage state with the baseline parameter distributions of the “intact” (referred to healthy or undamaged state) structure. It should be noted that the work by Sun et al.\textsuperscript{39} has only focused on updating the baseline model using ambient measurements. In this paper, we tried to fill the research gap by investigating damage detection and localization capabilities and limitations of the framework, which has not been previously addressed.

This paper is structured as follows. Section 2 describes the deconvolution-based seismic interferometry approach. We elaborate on the dynamic characteristics of a structure acquired through seismic interferometry, that is, shear wave velocities and damping ratios. Section 3 introduces the Bayesian model updating framework. In Section 4, we first evaluate the structural identification performance of the proposed approach on experimental shake table data of a scaled steel structure with different damage scenarios (induced via bolt loosening at the floor joints) in order to validate the proposed methodology. We further deploy our framework on a full-scale seven-story reinforced concrete (RC) building slice, which was progressively damaged via previously recorded historical earthquake records utilizing the University of California, San Diego Network for Earthquake Engineering Simulations (UCSD-NEES) shake table. Finally, Section 5 discusses the implications of the results and scope for further research.

## 2 | EXTRACTING STRUCTURAL WAVES

In the area of vibration-based SHM, the fundamental objective is to determine the intrinsic changes in a structure based on its measured dynamic response. In this study, we extract the structural waves, which are isolated from the soil–structure interaction, applying the deconvolution-based seismic interferometry technique. The extracted waves are also called IRFs, which can be used to identify dynamical parameters, such as shear wave travel velocities, modal frequencies, mode shapes, and intrinsic attenuation (damping) values. Note that deconvolved waveforms satisfy the same wave equation as the original physical system with different boundary conditions.\textsuperscript{40} We extract the IRFs by deconvolving the recorded acceleration time series of a structure with respect to a reference record of choice. The deconvolution operation can be formulated
Estimating wave velocity variations via a stretching technique

where \( f_{\text{cur}} \) is the current wave, \( \delta \) is angular frequency; \( t \) is time; \( y_{\text{ref}} \) is the reference level of the building; * denotes the complex conjugate; and \( F^{-1} \) denotes the inverse Fourier transform. The stabilizing parameter, \( \epsilon \), is conventionally set to some fixed/predetermined percentile of the power of the reference signal, \( y_{\text{ref}}, t \), and it is essential to regularize the operation. \( S(z, t) \) physically represents the response of the structural system at height \( z \) to a unit impulse at the reference level \( z_{\text{ref}} \). This reconstructed structural system has fixed boundary conditions at height \( z_{\text{ref}} \). Therefore, the extracted IRFs through the height of the building show how the virtual pulse (or disturbance) propagates through the system. These functions represent the input–output relationships and provide a complete picture of the characteristics of linear time-invariant systems. It is noted that an IRF is the intrinsic property of a linear and undamaged system. Considering civil structures under earthquake shaking, the deconvolution operation is done with respect to the ground-level record and yields causal waveforms, because the physical input (i.e., incoming seismic waves) to the structural system is at ground level. This means that the motion of the building response follows the base excitation in time. Selecting another reference where there is no real source, for example, receiver at the top floor (in the presence of an earthquake), yields waveforms consisting of both causal and acausal parts. The down-going acausal wave can be physically interpreted as the up-going wave that enters the structural system prior to shaking at the top floor. Nakata et al. have argued that selecting different reference receivers at different stories could be used to investigate the local damages as the deconvolution operation yields a cutoff building system.

2.1 Estimating wave velocity variations via a stretching technique

Shear wave velocity can be estimated based on the extracted IRFs by tracing the first crest of the shear wave pulse that travels upwards from the reference source location. Therefore, by the APM, the wave velocity is determined by fitting a linear line, in a least-square sense, to wave travel times (time of arrivals of the first peaks) and wave travel distances (story elevations). The slope of the fitted line corresponds to the shear wave velocity of the wave that propagates within the structure. Note that this method can be used when deconvolution is calculated with a virtual source at the roof level (story elevations). The slope of the fitted line is done with respect to the ground-level record and yields causal waveforms, because the physical input (i.e., incoming seismic waves) to the structural system is at ground level. This means that the motion of the building response follows the base excitation in time. Selecting another reference where there is no real source, for example, receiver at the top floor (in the presence of an earthquake), yields waveforms consisting of both causal and acausal parts. The down-going acausal wave can be physically interpreted as the up-going wave that enters the structural system prior to shaking at the top floor. Nakata et al. have argued that selecting different reference receivers at different stories could be used to investigate the local damages as the deconvolution operation yields a cutoff building system.

\[
S(z, t) = F^{-1} \left( \frac{y(z, w)y^*(z_{\text{ref}}, w)}{|y^*(z_{\text{ref}}, w)|^2 + \delta} \right),
\]

where \( S(z, t) \) is the IRF at \( z \); \( y(z, w) \) is the frequency domain quantification of the response measurement at \( z \); \(|y^*(z_{\text{ref}}, w)|^2 \) is the power spectrum of \( y_{\text{ref}} \); \( w \) is angular frequency; \( t \) is time; \( y_{\text{ref}} \) is the reference level of the building; * denotes the complex conjugate; \( \delta \) is a stabilizing parameter (water-level parameter, \( \epsilon = 0.1 \% \)); and \( F^{-1} \) denotes the inverse Fourier transform. The stabilizing parameter, \( \epsilon \), is conventionally set to some fixed/predetermined percentile of the power of the reference signal, \( y_{\text{ref}}, t \), and it is essential to regularize the operation. \( S(z, t) \) physically represents the response of the structural system at height \( z \) to a unit impulse at the reference level \( z_{\text{ref}} \). This reconstructed structural system has fixed boundary conditions at height \( z_{\text{ref}} \). Therefore, the extracted IRFs through the height of the building show how the virtual pulse (or disturbance) propagates through the system. These functions represent the input–output relationships and provide a complete picture of the characteristics of linear time-invariant systems. It is noted that an IRF is the intrinsic property of a linear and undamaged system. Considering civil structures under earthquake shaking, the deconvolution operation is done with respect to the ground-level record and yields causal waveforms, because the physical input (i.e., incoming seismic waves) to the structural system is at ground level. This means that the motion of the building response follows the base excitation in time. Selecting another reference where there is no real source, for example, receiver at the top floor (in the presence of an earthquake), yields waveforms consisting of both causal and acausal parts. The down-going acausal wave can be physically interpreted as the up-going wave that enters the structural system prior to shaking at the top floor. Nakata et al. have argued that selecting different reference receivers at different stories could be used to investigate the local damages as the deconvolution operation yields a cutoff building system.

By the SM, instead of calculating the absolute wave velocity, the relative velocity variations are calculated by assuming that a homogeneous velocity changes that takes place in the medium. In seismology, this method is mainly used for measuring the perturbations of coda waves. The underlying mathematical framework can be found in Snieder et al. Essentially, by stretching the current wave that travels in the perturbed medium, along the time axis to maximize its correlation to a reference wave, an optimal stretching coefficient can be determined. This coefficient, \( \epsilon \), represents the velocity variation of the current waveform with respect to the reference wave velocity, as expressed in Equation (3). First, we express the stretched waveform as

\[
f_{\epsilon}(t) = f_{\text{cur}}(t(1 - \epsilon)),
\]

where \( f_{\text{cur}}(t) \) denotes the current wave (e.g., damaged IRF) and \( f_{\epsilon}(t) \) denotes the stretched wave. Then, the correlation coefficient \( C(\epsilon) \) can be calculated as follows:

\[
C(\epsilon) = \frac{\int_{t_1}^{t_2} f_{\epsilon}(t)f_{\text{ref}}(t)dt}{\sqrt{\left(\int_{t_1}^{t_2} f_{\epsilon}(t)^2dt\right)\left(\int_{t_1}^{t_2} f_{\text{ref}}(t)^2dt\right)}}.
\]

where \( f_{\text{ref}}(t) \) is the reference trace (e.g., undamaged IRF) and \( t_1 \) and \( t_2 \) represent the starting and ending points of the time series of interest along the waveform. We find the maximum correlation coefficients and the respective stretching coefficients by grid search for each measurement channel. Note that, because we are working with discrete-time signals, after the stretching operation, interpolation is necessary to directly calculate the inner product of the stretched and reference signals. Estimated values for the maximum correlation coefficients depend on the selected time periods of interest.
along the waves. For example, in the SHM context, the later part of the deconvolved waves, with the excitation source at the ground level, predominantly contains the fundamental mode, as its attenuation is more gradual. The initial part of the waves, however, contains superposition of many overtones. Note that the SM can also be applied to estimate the frequency series from long-term SHM data.43

### 2.2 | Damping parameter estimation

There exist many damping models (e.g., nonviscous), any of which could be favored depending on the nature of the engineering problem.44 Assuming a linear system, each harmonic’s corresponding damping ratio can be calculated by tracing the amplitude decay with time. In this study, we calculate the damping ratios by fitting linear lines to the envelope of the natural logarithm of the IRFs, which are band-pass filtered around the resonant frequencies within the half-power bandwidth.36,37,39

\[
\xi_r = \frac{1}{N_0\omega_r} \sum_{i=1}^{N_0} |\mu_i|
\]  

(4)

In Equation (4), \(\mu_i\) is the slope of the envelope, \(\omega_r\) is the \(r\)th resonance frequency, \(N_0\) is the number of observations (e.g., observed number of stories), and \(i\) is the observation location/story of interest. Knowing the damping ratios, the quality factor, \(Q\), of the building can be estimated. For homogenous materials, the quality factor does not typically depend on the frequency of the oscillation.45,46

Using Rayleigh’s assumption,47 the proportional damping matrix, \(C\), can be assembled as a linear combination of the stiffness matrix, \(K\), and the mass matrix, \(M\). The proportionality constants can be found using any two damping ratio estimates. Note that this assumption yields classical normal modes for linear dynamical systems and performs well enough with experimental data.48 Therefore, in this study, we estimate the damping ratios of the first two resonant peaks of the frequency response functions of the extracted waves using Equation (4) and use them to determine the Rayleigh damping matrix.

### 3 | PROBABILISTIC MODEL UPDATING

#### 3.1 | Bayesian framework

Time histories of the outputs of a system can be used to estimate the physical parameters of a representative finite element model by making use of probabilistic model updating schemes. In our case, the fundamental aim of structural model updating is to minimize the gap between the predicted and the measured IRFs by quantifying posterior probability distributions of a set of model parameters, \(\theta \in \mathbb{R}^{N_\theta \times 1}\). Herein, we consider a linear shear beam model with \(N_\theta\) degrees of freedoms (DOFs), which is suitable to capture the dynamics of mid-rise and high-rise buildings. This model can be portrayed as a chain of lumped masses that are consecutively connected by springs and dashpots. Note that this type of arrangement leads into band-limited structural matrices. The mass matrix, \(M \in \mathbb{R}^{N_\theta \times N_\theta}\), is assumed to be fixed and known as a priori. As previously mentioned in Section 2.2, we construct our proportional damping matrix, \(C\), by using Equation (4). We parameterize our stiffness matrix, \(K \in \mathbb{R}^{N_\theta \times N_\theta}\) by \(\theta = [\theta_1, \theta_2, \ldots, \theta_{N_\theta}]\), where each \(\theta_i\) represents \(i\)th story’s lateral stiffness in the shaking direction.

Using the hierarchical Bayesian inference,39,49-52 we formulate the model updating problem as follows:

\[
p(\theta, \sigma^2 | D) \propto p(D|\theta, \sigma^2)p(\theta)p(\sigma^2),
\]

(5)

where \(p(\theta, \sigma^2 | D)\) denotes the posterior probability density function (PDF) of unknown parameters \(\{\theta, \sigma^2\}\); \(p(D|\theta, \sigma^2)\) is the likelihood function conditional on the measurements data \(D\) (e.g., the extracted IRFs); and \(p(\theta)\) and \(p(\sigma^2)\) are the prior PDFs. In particular, the likelihood function and the prior PDFs can be written as

\[
p(D|\theta, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{N_\theta}{2}} \exp \left\{ -\frac{1}{2\sigma^2} J(\theta) \right\},
\]

(6)

\[
p(\theta) = \frac{1}{\sqrt{2\pi \Sigma_\theta}} \exp \left\{ -\frac{1}{2} (\theta - \theta_0)^T \Sigma_\theta^{-1} (\theta - \theta_0) \right\},
\]

(7)

\[
p(\sigma^2) = p(\sigma^2 | a, \beta) = \frac{\beta^a}{\Gamma(a)} \left( \frac{1}{\sigma^2} \right)^{a+1} \exp \left\{ -\frac{\beta}{\sigma^2} \right\},
\]

(8)
where \( J(\theta) = \sum_{j=1}^{N} ||\hat{S}(\theta, t_j) - S(t_j)||^2 \) is the goodness-of-fit function; \( S(t_j) \in \mathbb{R}^{N \times 1} \) denotes the IRFs at time \( t_j \) extracted from the measurement data \( (j = 1, 2, \ldots, N) \), whereas \( \hat{S}(t_j) \in \mathbb{R}^{N \times 1} \) is the model predicted IRFs; \( N_0 \) is the total number of channels of measurement; \( ||.|| \) stands for the Euclidean norm; and \( \sigma^2 \) denotes the prediction error variance, which is also considered as an unknown parameter and should be estimated right along with the model parameters. The prior PDF of the model parameters in Equation (7) is modeled by the multivariate Gaussian distribution, \( \mathcal{N}(\theta, \Sigma_0) \), where the choice of mean parameter vector \( \theta_0 \in \mathbb{R}^{N \times 1} \) and the covariance matrix \( \Sigma_0 \in \mathbb{R}^{N \times N} \) are user defined. Here, mean parameter vector, \( \theta_0 \), is given by \( \theta_0 = [\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_N]^T \) where \( \bar{\theta}_j \) is the mean value of the respective parameter (e.g., \( \bar{\theta}_2 \) is the mean lateral stiffness of the second story). Because the prediction error variance \( \sigma^2 \) is always positive, utilizing the conjugacy concept, we model its prior PDF \( p(\sigma^2) \) in Equation (8) as an inverse gamma distribution, \( p(\sigma^2) \sim IG(\alpha, \beta) \), with positive constant hyperparameters \( \alpha \) and \( \beta \), which are chosen extremely small to retain a noninformative prior (e.g., \( \alpha = 10^{-3} \) and \( \beta = 10^{-6} \)).

Substituting the likelihood function and the priors, Equations (6) to (8) into Equation (5), the final form of the augmented multivariate posterior PDF can be observed as follows:

\[
p(\theta, \sigma^2 | D) \propto \left( \frac{1}{\sigma^2} \right)^{\frac{N_0 N}{2} + \alpha + 1} \exp \left\{ -\frac{1}{\sigma^2} \left( \frac{J(\theta)}{2} + \beta \right) - \frac{1}{2} (\theta - \theta_0)^T \Sigma_0^{-1} (\theta - \theta_0) \right\}.
\] (9)

Note that \( p(\sigma^2|\theta, D) \) is analytically obtainable, given the model parameter vector \( \theta \), and it follows an inverse gamma distribution, \( IG(\alpha, \beta) \), (see Equation 8) according to

\[
p(\sigma^2|\theta, D) \propto \left( \frac{1}{\sigma^2} \right)^{\frac{N_0 N}{2} + \alpha + 1} \exp \left\{ -\frac{1}{\sigma^2} \left( \frac{J(\theta)}{2} + \beta \right) \right\}.
\] (10)

where the parameters \( \alpha = (N_0 N_1)/2 + \alpha \) and \( \beta = J(\theta)/2 + \beta \). Utilizing Equation (10) facilitates our sampling procedure, because most statistical toolboxes provide built-in sampling implementation for inverse gamma distribution. As the implicit analytical expression for the posterior distribution in Equation (9) requires multidimensional integrals of a very complicated expression, in order to calculate the desired marginal parameter distributions, the Markov chain Monte Carlo method is employed. Details regarding the utilized MCMC sampler are described in the following section. On the other hand, for the prediction error variance, samples can be drawn from the expression shown in Equation (10) for a given \( \theta \). Note that utilizing the Markov chains that effectively represent the multivariate posterior distribution, \( p(\theta, \sigma^2|D) \), one can compute the maximum a posteriori estimate (MAP) of each model parameter along with the posterior covariance matrix. The latter provides an insight about the correlation in between model parameters and a quantitative measure of uncertainty related to parameter vector \( \theta \).

### 3.2 Sampling marginal posterior PDFs

The Bayesian learning schemes are known to yield complicated high-dimensional posterior distributions. Therefore, drawing independent samples utilizing typical Monte Carlo methods, such as importance sampling or rejection sampling, is difficult as they require proposal distributions that are similar to the distribution we want to sample. On the other hand, methods based on the Markov chains, such as Gibbs sampling, Metropolis–Hasting algorithm, Hamiltonian Monte Carlo, and slice sampling, effectively allow us to draw dependent samples that eventually represent the target posterior distributions. The stated simulation procedures are named as MCMC methods. In this study, to draw samples that effectively represent the derived posterior \( p(\theta, \sigma^2|D) \) in Equation (9), we implement an MCMC sampler that is described as follows. Markov chains are created employing the Metropolis–Hasting algorithm, where we update our state vector \( \theta \) systematically, one by one. We use Cauchy distribution (CD) as our proposal distribution, with a predefined width/step size \( \kappa \). For each Markov step, CD is centered on the current value of the state component \( \theta_k^{(c−1)} \). If the candidate step resides in a higher probability density region, then it is accepted. If the chain moves into a lower probability density region, however, the candidate state is accepted according to a Bernoulli trial. The value of \( \kappa \) is determined considering the computation time and the ratio between acceptance and rejection through several simulations. Note that we use the same step size, \( \kappa \), for every model parameter. We initialize the sampler by specifying the chain length, burn-in period length, beginning state of the parameter vector (starting point of the random walk), prior distributions, step size \( \kappa \), the hyperparameters (i.e., \( \alpha \) and \( \beta \)) of the IG distribution, the mass matrix, \( \mathbf{M} \), the Rayleigh coefficients \( \alpha_R \) and \( \beta_R \), and the extracted IRFs \( S(z, t) \). In every iteration, the value of prediction error variance, which will be used in the next
iteration, is sampled following Equation (10). For the same chain index, constant $\sigma^2$ is used. This algorithm outputs the structural model parameter samples (i.e., marginal posteriors) together with prediction error variance samples. Because the algorithm is initialized with a user-defined (mostly arbitrary) state, the samples from the nonstationary initial part of the corresponding chain are discarded. The part of the chain that is used to describe the target distribution is called the “retained period,” whereas the discarded portion is called the “burn-in” period. Through a detailed inspection of the histograms of marginal posteriors, quantitative damage detection is achievable.

4 | VALIDATION

4.1 | Experimental Case Study 1: Scaled eight-story steel structure

We initially evaluate our procedure utilizing measurements from shake table tests, which were operated by the National Center for Research on Earthquake Engineering (NCREE), in Taiwan. The structure has a story height of 33 cm and accommodates steel (ASTM A36) columns that are connected to the slabs with bolts as shown in Figure 1. Each slab has the dimension of $43 \times 45$ cm, and an additional 50-kg mass was installed at the center. The test structure is fixed to the shake table. The acceleration measurements were carried out along the weak direction (see Figure 1) with a sampling rate of 200 Hz. Different damage scenarios were tested by loosening the bolts at the floor joints, as it is illustrated in Figure 1. In our study, we use the dataset in which the base excitation was rescaled to match the ChiChi earthquake with a peak ground acceleration (PGA) value of 0.07 g. Note that other tests with different earthquake inputs, for example, Kobe and El-Centro, were also performed and we use them to validate the learned structural FEM. Recorded acceleration response of the test structure is shown in Figure 2a. In particular, we worked on damaged scenarios where the bolt loosening occurred at the connection between the first and second stories, denoted with D(1), and between the lowest three stories, denoted with D(2). In particular, for each damage scenario, two out of four bolts are removed for each column as illustrated in Figure 1. Note that the intact/baseline state is hereafter denoted by D(0).

Impulse response function estimates of state D(0) are shown in Figure 2b,c. By plotting the IRFs versus the structural elevation, both up-going and down-going waves can be observed. Using the recorded base excitation as the reference, it is shown that the initial parts of the deconvolved waves consist of many overtones, that is, superposition of traveling waves, whereas the later parts mostly contain the fundamental mode of the test structure because the higher modes die out more quickly. In order to calculate the damping ratios through linear curve fitting using Equation (4), we first compute the PSD...
estimate of the extracted IRFs (see Figure 3a). Utilizing the band-pass–filtered IRF envelopes, we compute the slopes of the fitted lines and calculate the damping ratios, respectively, as $\xi_1 = 2.3\%$ and $\xi_2 = 2.4\%$ (see Figure 3b,c). Note that these damping ratios are used to construct the damping matrix, $C$, of the structural FEM. We then investigate the resulting waveforms by observing the wave velocity variations.

With the SM, the stretching coefficient represents the relative velocity variation, and therefore, we first sample 2,000 stretching coefficients between $-40\%$ and $40\%$. Then, we calculate the correlation coefficient of each stretched waveform with its reference trace. It is noted that the reference trace for each story is selected as the initial 3-s piece of the corresponding stories’ intact IRF, that is, from D(0). For example, Figure 4 demonstrates the deconvolved waves before and after stretching operation. It can be seen that the maximum correlation is achieved through contracting the IRFs from the damaged structure and the stretched waves match the reference waves quite well. Nevertheless, it should be noted that, though the IRFs for the damage case are not simply the stretched version of the undamaged one (as shown in Figure 4), the SM can capture well the phase differences, which are highly related to structural stiffness changes. In the APM, wave velocity is determined by tracking the arrival time of the first peak to each story. Figure 5a,b presents the correlation between the severity of the damage and the estimated shear wave velocities. Deconvolution with respect to the bottom and the top of the building yields different waveforms, and therefore, the estimated wave velocities are slightly different. Note that for the waves that are extracted through the deconvolution with respect to the top floor’s recordings, only the acausal parts are utilized in APM.

It is observed that, given the bolt loosening, both APM and SM can detect the changes/variations in the velocity as an index for damage detection. For both methods, the velocity reduction is more evident for the more severely damaged state, for example, D(2). In general, as it is demonstrated in Figure 5c, the SM yields larger velocity variations compared with the APM for both damage states. Hence, in the sense of damage detection, SM might have a better sensitivity.

In the second step of our proposed method, we create an eight-story shear-type model accounting for the topology of the structure and the mass of each floor (see Figure 1). We update the proposed FEM against the IRF estimates extracted from the measured data utilizing the described hierarchical Bayesian framework. Herein, we use the extracted IRFs, with respect to the base excitation, of the intact experimental structure. For $M \in \mathbb{R}^{8 \times 8}$, mass values of $m_1 = 80$ kg and

---

**FIGURE 3**  (a) PSD estimate of the extracted deconvolved waves of D(0). Each story level is represented by a different color. (b,c) Linear fits for the damping parameter estimations, $\xi_1 = 2.3\%$ in (b) and $\xi_2 = 2.4\%$ in (c) for state D(0). Each curve represents a different floor (from the bottom to the top). Final coefficient values are estimated by taking the average of the slopes of the fitted lines.

**FIGURE 4**  Illustration of the stretching method. Extracted impulse response functions are shown before and after stretching operation. Impulse response functions that represent the intact state are colored blue. Note that the stretching coefficient is selected so that the correlation between the reference trace and the damaged trace is maximized.
FIGURE 5  (a,b) Shear wave velocity calculation using arrival picking method. Note that in (a), the deconvolution functions are calculated using the ground level’s acceleration time series, namely, the earthquake input, as the reference signal, whereas in (b), top floor’s response is used as the reference trace. The slope of the fitted lines corresponds to the estimated intrinsic shear wave velocities. (c) Calculated shear wave velocity variations using the arrival picking method and the stretching method. Error bars represent the standard deviation of the stretching coefficients that are calculated for each floor for a given damage state.

$m_{2.8} = 75$ kg are used. Stiffness parameter $\theta_s$ are assumed to have a Gaussian prior with a mean of $1.25 \times 10^5$ N/m with a 30% coefficient of variation (c.o.v) assuming uncorrelated parameters (i.e., $\Sigma_\theta = \sigma_\theta^2 I$). The prior PDF for the prediction error is decided to have hyperparameters of $\alpha = 10^{-3}$ and $\beta = 10^{-6}$. We calculate our likelihood function calculated using Equation (6), for a given model parameter vector $\theta$ and a predefined simulation duration of 7.5 s. For sampling, CD is used as a proposal distribution with a constant width parameter, $\gamma$, of 0.0032 times the general parameter order of $10^5$. The starting point of the chains, $\hat{\theta}(0)$, is set to be 200 kN/m. The decided chain length, $N_{mc}$, is $2 \times 10^4$, and the burn-in period length, $N_{bi}$, is $0.5 \times 10^4$. We set the lower and upper bounds of the parameters as $0.01 \times 10^5$ and $6 \times 10^5$ N/m, respectively.

The Markov chains that effectively depict the model parameters’ marginal posterior PDFs are shown in Figure 6. Note that chains become stationary approximately after 3,000 iterations. Histograms that properly describe the updated marginal posterior PDFs are presented in Figure 7. These histograms are constructed using only the retained samples, and the solid red line on each histogram depicts a Gaussian distribution fitted to these samples. The mean values of the updated model parameters are denoted as $\hat{k}_i$ where $i = 1, \ldots, 8$. Figure 7 also reports the posterior samples of the prediction error variance, which have a small standard deviation. Table 1 outlines the MAP estimates of the model parameters along with the respective c.o.v. values. It can be seen that the MAP estimates range between 71 and 220 kN/m. Table 1 additionally highlights that c.o.v. values generally increase with the story height. Figure 8 presents a comparison between the extracted and the simulated IRFs ($S_3$ and $S_7$) of the intact structure. Simulations are carried out using the point MAP estimates of the model parameters, and the illustrated results are interesting in several ways. First, both figures indicate that updated shear-type model is capable of capturing the later parts of deconvolved waves without any phase and amplitude difference. This would appear to prove the accuracy in the identification of the fundamental mode’s damping coefficient (i.e., $\xi_1$). Second, the observed amplitude discrepancies at early times could indicate the limitations of the selected damping model, that is, Rayleigh damping, as we only account for the first two mode’s attenuation characteristics in constructing the proportional damping matrix. The used damping model seems to overly dampens out the impulse-like interferences of the overtones. One possible solution to alleviate this issue is to use a high-order damping model (e.g., the generalized Cauchy damping). Nevertheless, we can still state that the satisfactory agreement is achieved using the point MAP estimates of the parameters. We cannot rule out the fact that our framework yields probability distributions of the model parameters. Therefore, before any critical decision, additional analysis should be performed assessing other plausible parameter values.

4.1.1  Validation of the updated intact model—D(0)

In order to validate our identified intact model, we carry out additional simulations utilizing the fitted Gaussian PDFs. The red lines in Figure 7 depict the utilized PDFs. Herein, we aim to evaluate the learned structural model by comparing it with response measurements of another shake table experiment of the intact state (i.e., D(0)). This time, we make use of the dataset in which the base excitation was rescaled to match the Kobe earthquake with a peak ground acceleration (PGA)
Markov chains that represent the marginal posterior distributions of the stiffness parameters of the intact experimental structure, $D(0)$. Black dashed line indicates the predetermined burn-in period, $N_{bi} = 4,000$

FIGURE 6

Updated posterior PDFs of the model parameters of the intact (i.e., $D(0)$) experimental structure. Each histogram effectively represents the marginal posterior distribution estimate of the corresponding model parameter, for example, story stiffness parameters. The red line on each plot depicts the fitted Gaussian distributions for story stiffnesses and the fitted log-normal distribution for the prediction error variance. The mean value of each model parameter is placed on top of each histogram. Coefficient-of-variation values of the retained samples are indicated in the figure legends.

| $k_i$ | Identified MAP value (N/m) | c.o.v. (%) | $k_i$ | Identified MAP value (N/m) | c.o.v. (%) |
|-------|---------------------------|-----------|-------|---------------------------|-----------|
| $k_1$ | $2.09 \times 10^5$        | 1.57      | $k_5$ | $1.10 \times 10^5$        | 2.03      |
| $k_2$ | $2.25 \times 10^5$        | 1.21      | $k_6$ | $1.98 \times 10^5$        | 3.39      |
| $k_3$ | $0.71 \times 10^5$        | 1.31      | $k_7$ | $2.07 \times 10^5$        | 3.63      |
| $k_4$ | $1.45 \times 10^5$        | 2.26      | $k_8$ | $2.13 \times 10^5$        | 4.29      |

TABLE 1 The most probable values (MAP) of the updated model parameters for $D(0)$

value of 0.09 g. Using the same earthquake base input for our updated model, we run 1,000 finite element simulations in which each story’s stiffness parameter is sampled from its fitted marginal posterior within $\pm 3\sigma_\theta$, where $\sigma_\theta$ represents the standard deviation of the fitted Gaussian PDF. The comparison results, in terms of the displacement response, for the third story and the seventh story are shown in Figure 9. The predicted displacements, depicted with green lines, are in good agreement with the doubly integrated acceleration time series, depicted with a blue line, of the experiment. Note that even though every FEM simulation is performed with different model parameter samples, the displacement response behavior only slightly alters, because the marginal posteriors have small c.o.v. values. This result demonstrates the validity of the initial model class assumption for our identification problem.
4.1.2 Damage localization and quantification

We herein aim to localize and quantify the damage induced by bolt loosening. In particular, we repeat the previously described identification steps utilizing the earthquake response data of the damaged test structure, specifically D(1). It is important to note that the damping coefficients are re-estimated using the damaged waveforms, as \( \xi_1 = 3.1\% \) and \( \xi_2 = 2.2\% \). Preserving the same prior assumptions with the baseline structure (i.e., D(0)), we again learn the posterior parameter distributions of the shear-type FEM as shown in Figure 10.

The most remarkable observation is the evident stiffness reduction in \( k_1 \) and \( k_2 \) as shown in Figure 11. Comparing identified point MAP values from D(0) and D(1), it is observed that the MAP estimates of the first and second story interstory stiffness parameters reduce 65\% and 70\%, respectively. Considering the nature of the damage via bolt loosening, which occurs at the first story for D(1), stiffness reduction in \( k_1 \) and \( k_2 \) is consistent with the expectations because both the first and the second stories are connected to the loosened bolt. In this case, the damage can be quantified based on the shifts of the distributions despite that variation of the distributions of other story stiffness parameters is present. For example, compared with the intact case, point MAP values of \( k_6 \), \( k_7 \), and \( k_8 \) increase 30\%, 25\%, and 12\%, respectively. However, the observed differences in other model parameters are not particularly surprising given the fact that our framework assumes linearity before and after the damage, yet we cannot rule out the nonlinearities at the bolted interface. This also could be the reason of the observed increase in c.o.v. values. Additionally, inspecting the model parameter’s marginal posterior distributions, it is seen that apart from \( k_1 \) and \( k_2 \), only other updated model parameter that indicates reduction in stiffness, that is, a false-positive error, is \( k_4 \). To minimize the discrepancy between the extracted and the modeled impulse response behavior, the presented framework adjusts the stiffness values of the undamaged stories as well. It is also noted that bolt loosening at the column and slab connections induces nonconventional damages (unlike the column cross-section reduction), leading to additional uncertainties in model updating. Apart from this expected discrepancy, the localization capability of our approach appears to be well substantiated by the % changes of the identified MAP values of the model parameters. Significant stiffness reductions in both \( k_1 \) and \( k_2 \) indicate the plausible location of the damage.

4.2 Experimental Case Study 2: Full-scale seven-story RC building slice

The test structure is a full-scale RC building slice that consists of an RC web wall for lateral resistance in the direction of earthquake loads, a flange wall for transverse resistance, concrete slabs that are supported by four gravity columns, and
a precast segmental posttensioned concrete column to maintain torsional strength. More detailed information about the structure and the instrumentation could be found in Panagiotou et al. and Erazo and Hernandez. It is important to note that in this study, we use the recordings from the accelerometers that were placed on the closest slabs to the centroid of the web wall, which were sampled at a rate of 240 Hz. These measurement channels are labeled as “H-1.” The picture and a side view of the test structure with the used accelerometer layout are shown in Figure 12.

Shaking experiments were performed between October 2005 and January 2006 using four historical earthquake records of increasing intensity so that progressive structural damage could develop. The corresponding PGA values of these input excitations ranged from 0.15 to 0.91 g. At each damage level, meaning that after each earthquake shaking, low-amplitude Gaussian and ambient vibration tests were performed (see Figure 13). These tests are named S0, S1, S2, S3, and S4 where S0 indicates the undamaged baseline state and Si indicates the damage state after shaking the test structure with the vibration record of ith earthquake. It should be noted that state S3 was divided into S3.1 and S3.2 because before exciting the test structure with the last earthquake motion, the bracing system between the posttensioned precast column and the slabs was stiffened. Table 2 specifies the dynamic tests that are used in our study. Low-amplitude Gaussian excitations (0.03 and 0.05 g) provide an informative dataset that has been studied by several researchers.

Figures 14 and 15 show the propagation of waves (e.g., the IRFs at different floor elevations) taking the ground and top floor measurement as the reference, respectively. Note that in Figure 15, IRFtop denotes that the deconvolution was performed using the top floor’s acceleration time series as the reference. Figure 14 clearly illustrates that as the imposed
FIGURE 12  Left: seven-story University of California, San Diego Network for Earthquake Engineering Simulations building slice. Right: elevation view of the structure with the used accelerometer layout. Note that the accelerometers (i.e., “H1”) are illustrated as red squares.

TABLE 2  Dynamic tests used in this study (data)

| Test # | Test description | Damage state |
|--------|------------------|--------------|
| 39     | 8-min WN (0.03 g) + 3-min AV | S0           |
| 41     | 8-min WN (0.03 g) + 3-min AV | S1           |
| 46     | 8-min WN (0.03 g) + 3-min AV | S2           |
| 49     | 8-min WN (0.03 g) + 3-min AV | S3.1         |
| 61     | 8-min WN (0.03 g) + 3-min AV | S3.2         |
| 64     | 8-min WN (0.03 g) + 3-min AV | S4           |

Abbreviations: AV, ambient vibration; WN, white-noise table input.

FIGURE 13  Left: Acceleration-time histories of the low-amplitude Gaussian excitation of the intact state, S0. From the basement to the top, each curve represents the acceleration response of the corresponding story. Right: Recordings at ground level.

Table 3 reports the wave velocities estimated by the APM utilizing IRFs and $IRF_{top}$s. The most serious decrease is observed after the last but the most strong earthquake shaking. As presented in Figure 16, the utilized segments of the waves could change/affect the velocity variation estimations by the SM, and therefore, the findings need to be interpreted with caution. It is observed that if the initial parts of the waves are used (i.e., $0 \leq t \leq 3$), then output percent variations from both methods are more comparable. The findings of the SM provide additional support regarding its damage sensitivity compared with the APM. Additionally, our analysis reveals that stiffening the slabs and the posttensioned precast...
column between S3.1 and S3.2 causes an increase in the estimated velocity values. Taking these together, these findings are in line with previous results from the previous case study.

Before deploying our Bayesian model updating scheme to quantify the damage state of the test structure, it is necessary to establish an initial FEM to obtain a reasonable parameter set (i.e., $K$ and $M$). Therefore, a primary linear elastic finite element model is developed using commercially available structural analysis software ETABS (Computers and Structures, Inc.) based on the available structural drawings. The web wall, the flange wall, and the posttensioned column are modeled as concrete wall sections (using thin shell elements) with material properties that were extracted from concrete cylinder tests that were performed for each constructed story of the building slice (reported in Moaveni and Conte\textsuperscript{68}). The posttensioned column is approximated as a wall section with an effective width to account for the total section area and the mass. This assumption decreases the torsional stability of our FEM. Slabs are modeled using thin shell elements, and to account for the slotted connection for the floor sections that are in between the web wall and the flange wall, out-of-plane moments are released. Panagiotou and Restrepo\textsuperscript{58} reported that with this geometry, slabs behave like a near-pinned link.
TABLE 3  University of California, San Diego
Network for Earthquake Engineering Simulations
structure: shear wave velocity estimates and
percent changes using the arrival picking method

| State | Estimated velocity (m/s) | % change | State | Estimated velocity (m/s) | % change |
|-------|--------------------------|----------|-------|--------------------------|----------|
| S0    | 401.7                    | -        | S0    | 377.3                    | -        |
| S1    | 388.7                    | 3.2      | S1    | 362.7                    | 3.9      |
| S2    | 364.5                    | 9.2      | S2    | 346.3                    | 8.2      |
| S3.1  | 333.1                    | 17.1     | S3.1  | 331.1                    | 12.2     |
| S3.2  | 341.8                    | 14.9     | S3.2  | 322.2                    | 14.6     |
| S4    | 204.7                    | 49.7     | S4    | 262.9                    | 30.3     |

FIGURE 16  Shear wave velocity variations
using both the arrival picking method and
stretching method for each damage state, that is,
S0, S1, S2, S3.1, S3.2, and S4. The utilized time
intervals in the SM are indicated on the titles of
each plot. Error bars represent the standard
deviation of the stretching coefficients that are
calculated for each story for a given damage state

FIGURE 17  (a) Modal
analysis results of the ETABS
model. (b) Illustration of the
model condensation scheme.
Note that $m_i$ and $k_i$ represent
lumped floor mass and lateral
stiffness of each story $i$

that transfers the in-plane forces and moments while reducing the out-of-plane ones. Gravity columns within each story
and braces between the posttensioned precast column and the building slice floors are modeled as truss elements with
their corresponding section properties from structural drawings. Note that the structural details of the foundation level
are not considered. The natural frequencies with their corresponding mode shapes of the FEM are computed through
modal analysis. The acquired analytical results are within the acceptable agreement with the modal information of the
accelerometer data that is derived from the damage state S0, that is, Test 39, by state-of-the-art linear system identifica-
tion methods (see Moaveni et al.64 for details). Figure 17a illustrates the first three longitudinal modes, that is, 1.99, 10.89,
and 26.13 Hz, of the model ETABS structure.

Assuming in-plane rigid diaphragms, the earthquake response of the full structure can be characterized by computing
the drifts at the center of mass locations of each diaphragm. Therefore, we condense the full-scale finite element model
into a discretized shear-type model, as it illustrated in Figure 17b. This simplification essentially reduces the number of
parameters so that Bayesian learning procedures become computationally affordable. Primary lateral stiffness matrix,
that is, $K_0$, and the diagonal lumped mass matrix, that is, $M$, are constructed using the procedure described in Sun and
Büyüköztürk.69 and Sun et al.39 The procedure is based on assembling the lateral flexibility matrix by applying unit loads
to each story level while tracing the story drifts. $K_0$ is then computed by taking the inverse of the acquired flexibility
matrix. Having a primary stiffness matrix enables us to define a substructure that could be parameterized and updated.
TABLE 4  Identified median estimates of the model parameters and identified damping coefficients from damage states S0, S1, and S2

| $k_i$ | Identified median value ($\times 10^7$ N/m) | Identified median value ($\times 10^7$ N/m) | Identified median value ($\times 10^7$ N/m) |
|-------|-----------------------------------------|-----------------------------------------|-----------------------------------------|
|       | S0                                      | S1                                      | S2                                      |
| $k_1$ | $-1.02$                                 | $-2.71$                                 | $-6.76$                                 |
| $k_2$ | $-3.95$                                 | $-5.70$                                 | $-15.82$                                |
| $k_3$ | $-5.16$                                 | $-4.20$                                 | $-2.06$                                 |
| $k_4$ | $-1.71$                                 | $-0.29$                                 | $7.93$                                  |
| $k_5$ | $1.20$                                  | $2.92$                                  | $9.59$                                  |
| $k_6$ | $0.20$                                  | $-0.93$                                 | $-2.99$                                 |
| $k_7$ | $-1.57$                                 | $-5.73$                                 | $-14.15$                                |
| $\xi_1$ | $3.69\%$                              | $3.89\%$                               | $5.13\%$                               |
| $\xi_2$ | $4.88\%$                              | $6.30\%$                               | $7.02\%$                               |

Full stiffness matrix, $K$, can be described as

$$K = K_0 + \sum_{i=1}^{N_\theta} \theta_i K_i.$$  \hspace{1cm} (11)

where $K_i$ denotes the stiffness of the $i$th substructure and $N_\theta$ corresponds to the total number parameters. Figure 18 illustrates the obtained primary stiffness ($K_0$) and mass ($M$) matrices. The value of each matrix element is color coded using the respective color bars. It can be seen that the primary stiffness matrix is not exactly band limited yet the concentration is around the main diagonal with a general order of $10^9$ N/m. Similarly, a diagonal mass matrix is assembled using the story mass values of the ETABS model as $m_1 = 3.40 \times 10^4$ kg, $m_{2,3,4,5} = 2.89 \times 10^4$ kg, $m_6 = 2.91 \times 10^4$ kg, and $m_7 = 2.78 \times 10^4$ kg. These values correlate satisfactorily well with the previously reported story weights of the test structure in Panagiotou and Restrepo.58

By applying the presented Bayesian updating scheme, we determine the posterior distributions of the model parameters using the extracted IRFs from damage states S0 (Test #39), S1 (Test #41), and S2 (Test #46). For each damage state, the damping coefficients, $\xi_{1,2}$, are redetermined and used in constructing our condensed model’s damping matrix $C$. As shown in the bottom part of Table 4, our estimated attenuation coefficients lie within an acceptable range interval of the modal damping ratios that was previously reported in Moaveni et al.64 This similarity is expected as the structural response can be equivalently described by modal superposition. It is observed that the damage accumulation increases the exponential decay rate of the IRF amplitudes. Initial assumptions for FEM updating procedure are as follows: Each model parameter is assumed to have a Gaussian prior PDF with a mean of 0 and a standard deviation of $5 \times 10^6$ N/m. Similarly to the previous case studies, a CD is used as the proposal PDF. Its width parameter $\kappa$ is set to the 0.05% of the general parameter order. Other hyperparameters are selected the same as the previous case study. At each iteration, the full stiffness matrix is computed using Equation (11). Note that the resultant IRFs from the response data are band-pass filtered ([0.5, 25] Hz) and then used in the calculation of the likelihood (Equation 6). We create Markov chains with a length of $2 \times 10^4$ and discard the first $1 \times 10^4$ samples. It should be noted that with the assumptions mentioned earlier, the observed acceptance rate of our MCMC sampling is around 50%.

Figure 19 shows the histograms that effectively portray the posterior PDFs of substructure's first three stories' lateral stiffness parameters, that is, $k_1$, $k_2$, and $k_3$. For the first and the second stories, clear shifts in the distributions can be observed. This is in a good agreement with our expectations as the lower stories of the test structure are subjected to rela-
tively higher shear forces and bending moments during the earthquake loading. However, for $k_3$, the direction of the shift corresponds to an increase in the interstory lateral stiffness of the third story of our shear-type substructure. As illustrated in Figure 20, similar trend can be recognized for other model parameters by inspecting the color-coded band-limited stiffness matrices that are assembled using the median values of the posterior chains. Note that the normalized standard deviations of the respective model parameters are indicated with black circles with changing radiuses. It can be seen that the width of the posterior PDFs generally decreases with the increasing story height. This effect could be due the primary

**FIGURE 19** Posterior samples of $k_1$, $k_2$, and $k_3$ from damage states S0, S1, and S2. Black dashed lines denote the median values, $k^*_i$, of the posterior samples for state S0.

**FIGURE 20** From left to right: substructure's updated stiffness matrices that, respectively, correspond to damage states S0, S1, and S2. Coloring is performed according to the median values of the model parameters' marginal posterior samples. Normalized standard deviations are indicated as black circles.

**FIGURE 21** Comparison of the extracted versus reconstructed impulse response functions of the test structure. From top to bottom, each figure depicts the comparison of third and seventh stories' impulse response behavior for damage states S0, S1, and S2, respectively. Simulated time series are created using the point median estimates of the model parameters.
stiffness matrix $K_0$. Another interesting observation that could also be attributed to the complex coupling between $K_0$ and $K_{\text{substructure}}$ is that the direction of the shift of the marginal posterior PDFs does not change as the damage accumulates. For example, as detailed in Table 4, whereas $k_1$ and $k_2$ decrease as a function of damage, $k_4$ and $k_5$ increase. Additionally, estimated median values for the prediction error variances are $\sigma^2 = 9.32 \times 10^{-5}$ for S0, $\sigma^2 = 7.91 \times 10^{-5}$ for S1, and $\sigma^2 = 7.42 \times 10^{-5}$ for S2. Figure 21 shows, from top to bottom, the simulated IRFs in comparison with the IRFs obtained from low-amplitude Gaussian test for damage states S0, S1, and S2. Simulations are carried out using the median estimates of the interstory stiffness parameters. Clear agreement confirms the capability of our shear-type model and the used condensation scheme.

5 | CONCLUSIONS AND DISCUSSION

5.1 | Discussion

Results indicate that the presented framework is promising for monitoring structural systems. It allows for noninvasive determination of structural parameters. Studying the extracted IRFs in the time domain provides a qualitative information about the state of the damage in the structure. The location of the reference signal (i.e., ground story or top story) for the deconvolution operation does not affect the content of the information; however, both operations are advised as one of them could be easier to analyze, especially if the damage state is severe. Variations in the shear wave velocities are observed to be correlated with severity of the accumulated damage. Our results indicate that the SM could be more favorable for minor damage states. The capability of the method, however, depends on the selected time interval for the stretching operation. In addition, through comparison of the deconvolved waveforms that are extracted from different ground inputs with different strengths (e.g., different earthquakes), potential nonlinearities can be observed and utilized as a damage indicator.

Time series of the extracted IRFs can be coupled with hierarchical Bayesian identification schemes. Using the updated simplified models (i.e., marginal posterior PDFs), damage quantification is possible (e.g., shifts in the parameter histograms). The observed minor discrepancies in the early times of the IRFs could imply the limitations of the used damping model as the updated model partially fails to reproduce the impulse-like/high-amplitude response behavior of the experimental structure. Comparing the representative posterior histograms of different damage states, observed shifts in the distributions that correspond to the stiffness reductions are generally in good agreement with the expected/known damaged regions of the structure. However, other marginal posterior PDFs are also affected. This could be due to the potential nonlinearities and modeling assumptions; that is, our shear-type models may be too simple to fully capture the dynamics of the experimental structures and used model condensation scheme.

It should be noted that, as the complexity of the model increases, Bayesian model updating becomes more challenging. To be able to obtain the correct parameter set (given an identifiable model), mechanically meaningful parameter ranges, and a reasonable step size for MCMC should be chosen. A key realization is that modeling errors as well as poor identification of damping values could yield inconclusive identification results, for example, nonconverging Markov chains. In particular, full-scale structures could require condensed FEMs for the successful parameter identification and damage quantification. A further research in finite element model reduction as well as damping modeling and identification could benefit the presented procedure.

5.2 | Conclusion

We present a computational approach for structural identification and damage detection using the concepts of seismic interferometry and the Bayesian inference. To assess the damage (location and severity), Bayesian inference with MCMC sampling is utilized through quantifying parameter uncertainties of an FEM using the extracted IRFs. We initially analyze the shake table experiment dataset that contains various damage scenarios, which are induced via loosening the bolts. We show that the variations in the shear wave velocity (using both APM and SM) can be used for quick damage detection and that the velocity reduction is more evident for the more severely damaged states. We then, using the extracted IRFs of the shaking experiment, update our shear-type FEM. Using the MAP point estimates of the model parameters, which are determined from the posterior samples, we confirm that the extracted IRFs of the experimental structure and the simulated impulse response behavior of the FEM compare well with each other. Additionally, we validate the updated baseline FEM, by comparing the simulated displacement time series with the doubly integrated acceleration response measurements for another earthquake base input. We demonstrate that the induced damage, that is, bolt loosening on
the first floor, affects the posterior distributions of the model parameters quite noticeably. It is observed that the potential damaged region of the test structure can be identified by inspecting the statistical changes of the posterior samples of the updated model parameters.

To test our procedure on a full-scale structure, we study the propagating seismic waves that are derived from the RC building slice shake table experiment. Throughout the experimental program, the test structure was deliberately damaged by increasing the intensity of the induced shaking. Examining the collected structural acceleration response datasets of low-amplitude Gaussian (e.g., 0.03 g) and ambient vibration tests, it is observed that the for each damage state, the propagation characteristics of the seismic shear waves change. For highly damaged states (e.g., S4), these differences could be visually noticed from the IRF versus the structural elevation plots. We further our analysis of the deconvolved waves using the APM and SM and compute the shear wave velocity variations as a function of the damage state. Because our primary aim is to link the obtained IRFs to the model characteristics of the building that could inform us about the current state of the structure, we established a full-scale FEM of the test structure from its structural drawings. Subsequently, a model condensation procedure is implemented to deploy our computationally intensive parameter identification framework. Results indicate that the simulated IRFs of our condensed model agree quite well with the extracted IRFs. In parallel with our initial case study, by comparing the deviations of the model posteriors, imposed damage can be quantified. It should be noted that updated structural models can be used to predict structure's performance in the presence of a possible earthquake and further coupled with seismic fragility assessment framework.

ACKNOWLEDGEMENTS

The authors acknowledge the support provided by Royal Dutch Shell through the MIT Energy Initiative. We would like to thank Professor Babak Moaveni for providing the formatted dataset of the UCSD-NEES structure. We greatly appreciate the insightful comments of Dr. Justin Chen, Dr. James Long, and Dr. Reza Mohammadi Ghazi on our manuscript. We also acknowledge the National Center for Research on Earthquake Engineering (Taiwan) for sharing the shaking table test data used in this paper to validate the proposed algorithm.

ORCID

Hao Sun  https://orcid.org/0000-0002-5145-3259

REFERENCES

1. Salawu OS. Detection of structural damage through changes in frequency: a review. Eng Struct. 1997;19(9):718-723.
2. Toksoy T, Aktan AE. Bridge-condition assessment by modal flexibility. Exp Mech. 1994;34(3):271-278.
3. Tseng KK, Wang L. Impedance-based method for nondestructive damage identification. J Eng Mech. 2005;131(1):58-64.
4. West WM. Illustration of the use of modal assurance criterion to detect structural changes in an orbiter test specimen; 1986.
5. Stubbs N, Kim JT, Topole K. An efficient and robust algorithm for damage localization in offshore platforms. In: Proceedings of the ASCE 10th Structures Congress; 1992:543-546.
6. Montalvão D, Ribeiro AMR, Duarte-Silva J. A method for the localization of damage in a CFRP plate using damping. Mech Syst Sig Process. 2009;23(6):1846-1854.
7. Farrar CR, Worden K. An introduction to structural health monitoring. Philos Tran R Soc Lon A Math Phys Eng Sci. 2007;365(1851):303-315.
8. Worden K, Farrar CR, Manson G, Park G. The fundamental axioms of structural health monitoring. In: Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, Vol. 463; 2007:1639-1664.
9. Farrar CR, Worden K, Todd MD, et al. Nonlinear system identification for damage detection, Los Alamos, NM, Los Alamos National Laboratory (LANL); 2007.
10. Carden EP, Fanning P. Vibration based condition monitoring: a review. Struct Health Monit. 2004;3(4):355-377.
11. Doebling SW, Farrar CR, Prime MB, et al. A summary review of vibration-based damage identification methods. Shock Vib Dig. 1998;30(2):91-105.
12. Fan W, Qiao P. Vibration-based damage identification methods: a review and comparative study. Struct Health Monit. 2011;10(1):83-111.
13. Rytter Anders. Vibration-based banded inspection of civil engineering structures. PhD thesis: Aalborg University; 1993.
14. Şafak E. Wave-propagation formulation of seismic response of multistory buildings. J Struct Eng. 1999;125(4):426-437.
15. Todorovska M. Seismic interferometry of a soil-structure interaction model with coupled horizontal and rocking response. Bull Seismol Soc Am. 2009;99(2A):611-625.
16. Kanai K, Yoshizawa S. Some new problems of seismic vibrations of a structure. part 1; 1963.
17. Iwan WD. Drift spectrum: measure of demand for earthquake ground motions. Aust J Struct Eng. 1997;123(4):397-404.
54. Geman Stuart, Geman Donald. Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. *IEEE Trans Pattern Anal Mach Intell.* 1984;6:721-741.

55. Murray Iain. *Advances in Markov Chain Monte Carlo Methods.* United Kingdom: University of London, University College London; 2007.

56. Nichols JM, Link WA, Murphy KD, Olson CC. A Bayesian approach to identifying structural nonlinearity using free-decay response: application to damage detection in composites. *J Sound Vib.* 2010;329(15):2995-3007.

57. Martinelli P, Filippou FC. Simulation of the shaking table test of a seven-story shear wall building. *Earthq Eng Struct Dyn.* 2009;38(5):587-607.

58. Panagiotou M, Restrepo JI. Displacement-based method of analysis for regular reinforced-concrete wall buildings: application to a full-scale 7-story building slice tested at UC—San Diego. *J Struct Eng.* 2010;137(6):677-690.

59. Panagiotou M, Restrepo JI, Conte JP. Shake-table test of a full-scale 7-story building slice. Phase I: rectangular wall. *J Struct Eng.* 2010;137(6):691-704.

60. Panagiotou M, Restrepo JI, Conte JP. Shake table test of a 7-story full scale reinforced concrete structural wall building slice, phase I: rectangular section. Report No. SSRP-07/07, San Diego, CA, Department of Structural Engineering, University of California; 2007.

61. Panagiotou M, Restrepo JI, Conte JP. Shake table test of a 7-story full scale reinforced concrete structural wall building slice phase II: T-wall. SSRP. 2007:07-08.

62. Erazo K, Hernandez EM. High-resolution seismic monitoring of instrumented buildings using a model-based state observer. *Earthq Eng Struct Dyn.* 2016;45(15):2513-2531.

63. Ebrahimian M, Todorovska MI, Falborski T. Wave method for structural health monitoring: testing using full-scale shake table experiment data. *J Struct Eng.* 2016;143(4):04016217.

64. Moaveni B, He X, Conte JP, Restrepo JI, Panagiotou Marios. System identification study of a 7-story full-scale building slice tested on the UCSD-NEES shake table. *J Struct Eng.* 2010;137(6):705-717.

65. Moaveni B, He X, Conte JP, Restrepo JI. Damage identification study of a seven-story full-scale building slice tested on the UCSD-NEES shake table. *Struct Saf.* 2010;32(5):347-356.

66. Simoen E, Moaveni B, Conte JP, Lombaert G. Uncertainty quantification in the assessment of progressive damage in a 7-story full-scale building slice. *J Eng Mech.* 2013;139(12):1818-1830.

67. Moaveni B, Barbosa AR, Conte JP, Hemez FM. Uncertainty analysis of system identification results obtained for a seven-story building slice tested on the UCSD-NEES shake table. *Struct Control Health Monit.* 2014;21(4):466-483.

68. Moaveni B, Conte JP. *System and Damage Identification of Civil Structures.* Berlin, Heidelberg: Springer; 2007.

69. Sun H, Büyüköztürk O. The MIT Green Building benchmark problem for structural health monitoring of tall buildings. *Struct Control Health Monit.* 2018;25(3):e2115.

**How to cite this article:** Uzun M, Sun H, Smit D, Büyüköztürk O. Structural damage detection using Bayesian inference and seismic interferometry. *Struct Control Health Monit.* 2019;26:e2445. [https://doi.org/10.1002/stc.2445](https://doi.org/10.1002/stc.2445)