Dark matter and a suppression mechanism for neutrino masses
in the Higgs triplet model

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Abstract

We extend the Higgs triplet model so as to include dark matter candidates and a simple suppression mechanism for the vacuum expectation value ($v_\Delta$) of the triplet scalar field. The smallness of neutrino masses can be naturally explained with the suppressed value of $v_\Delta$ even when the triplet fields are at the TeV scale. The Higgs sector is extended by introducing $Z_2$-odd scalars (an SU(2)$_L$ doublet $\eta$ and a real singlet $s^0_2$) in addition to a $Z_2$-even complex singlet scalar $s^0_1$ whose vacuum expectation value violates the lepton number conservation by a unit. In our model, $v_\Delta$ is generated by the one-loop diagram to which $Z_2$-odd particles contribute. The lightest $Z_2$-odd scalar boson can be a candidate for the dark matter. We briefly discuss a characteristic signal of our model at the LHC.

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I. INTRODUCTION

Existence of dark matter (DM) has been established, and its thermal relic abundance has been determined by the WMAP experiment [1, 2]. If the essence of DM is an elementary particle, the weakly interacting massive particle (WIMP) would be a promising candidate. It is desired to have a viable candidate for the dark matter in models beyond the standard model (SM). The WIMP dark matter candidate can be accommodated economically by introducing only an inert scalar field [3–5], where we use “inert” for the $Z_2$-odd property. The imposed $Z_2$ parity ensures the stability of the DM candidate. Phenomenology in such models have been studied in, e.g., Refs. [6–12].

On the other hand, it has been confirmed by neutrino oscillation measurements that neutrinos have nonzero but tiny masses as compared to the electroweak scale [13–17]. The different flavor structure of neutrinos from that of quarks and leptons may indicate that neutrino masses are of Majorana type. In order to explain tiny neutrino masses, many models have been proposed. The seesaw mechanism is the simplest way to explain tiny neutrino masses, in which right-handed neutrinos are introduced with large Majorana masses [18, 19].

Another simple model for generating neutrino masses is the Higgs Triplet Model (HTM) [19, 20]. However, these scenarios do not contain dark matter candidate in themselves.

In a class of models where tiny neutrino masses are generated by higher orders of perturbation, the DM candidate can be naturally contained [21–26]. In models in Refs. [21–25], the Yukawa couplings of neutrinos with the SM Higgs boson are forbidden at the tree level by imposing a $Z_2$ parity. The same $Z_2$ parity also guarantees the stability of the lightest $Z_2$-odd particle in the model which can be the candidate of the DM as long as it is electrically neutral.

In this paper, we consider an extension of the HTM in which by introducing the $Z_2$ parity $m_\nu$ is generated at the one-loop level and the DM candidate appears. In the HTM, Majorana masses for neutrinos are generated via the Yukawa interaction $h_{\ell\nu} \overline{L_i} i\sigma_2 \Delta L_\nu$ with a nonzero vacuum expectation value (VEV) of an SU(2)$_L$ triplet scalar field $\Delta$ with the hypercharge of $Y = 1$. The VEV of $\Delta$ is described by $v_\Delta \sim \sqrt{2} \mu v^2/(2M_\Delta^2)$, where $v$ is the VEV of the Higgs doublet field $\Phi$ and $M_\Delta$ is the typical mass scale of the triplet field; the dimensionful parameter $\mu$ breaks lepton number conservation at the trilinear term $\mu \Phi^T i\sigma_2 \Delta^T \Phi$ which we refer to as the $\mu$-term. As the simplest explanation for the smallness of neutrino masses, the
mass of the triplet field is assumed to be much larger than the electroweak scale. On the other hand a characteristic feature of the HTM is the fact that the structure of the neutrino mass matrix \((m_\nu)_{\ell\ell}'\) is given by that of the Yukawa matrix, \(h_{\ell\ell}' \propto (m_\nu)_{\ell\ell}'\). The direct information on \((m_\nu)_{\ell\ell}'\) would be extracted from the decay \(H^{\pm\pm} \rightarrow \ell^\pm \ell'^\pm\) if \(H^{++}\) is light enough to be produced at collider experiments, where \(H^{++}\) is the doubly charged component of the triplet field \(\Delta\). At hadron colliders, the \(H^{\pm\pm}\) can be produced via \(q\bar{q} \rightarrow Z^*(\gamma^*) \rightarrow H^{++}H^{--}\) and \(q\bar{q} \rightarrow W^* \rightarrow H^{\pm\pm}H^{\mp}\). The \(H^{\pm\pm}\) searches at the LHC put lower bound on its mass as \(m_{H^{\pm\pm}} \gtrsim 300\text{ GeV}\) [30, 31], assuming that the main decay mode is \(H^{\pm\pm} \rightarrow \ell^\pm \ell'^\pm\).

Phenomenological analyses for \(H^{\pm\pm}\) in the HTM at the LHC have also been performed in Ref. [32]. Triplet scalars can contribute to lepton flavor violation (LFV) in decays of charged leptons, e.g., \(\mu \rightarrow \bar{e}ee\) and \(\tau \rightarrow \ell\ell'\ell''\) at the tree level and \(\ell \rightarrow \ell\gamma\) at the one-loop level. Relation between these LFV decays and neutrino mass matrix constrained by oscillation data was discussed in Refs. [33, 34]. In order to explain the small \(v_\Delta\) with such a detectable light \(H^{++}\), the \(\mu\) parameter has to be taken to be unnaturally much lower than the electroweak scale. Therefore, it would be interesting to extend the HTM in order to include a natural suppression mechanism of the \(\mu\) parameter (therefore \(v_\Delta\)) in addition to the DM candidate.

In our model, lepton number conservation is imposed to the Lagrangian in order to forbid the \(\mu\)-term in the HTM at the tree level while the triplet Yukawa term \(h_{\ell\ell}'\overline{T}_{\ell}'i\sigma_2\Delta L\) exists. The VEV of a \(Z_2\)-even complex singlet scalar \(s_1^0\) breaks the lepton number conservation by a unit. An SU(2)_L doublet \(\eta\) and a real singlet \(s_2^0\) are also introduced as \(Z_2\)-odd scalars in order to accommodate the DM candidate. Then, the \(\mu\)-term is generated at the one-loop level by the diagram in which the \(Z_2\)-odd scalars are in the loop. By this mechanism, the smallness of \(v_\Delta \ll v\) is realized, and the tiny neutrino masses are naturally explained without assuming the triplet fields to be heavy. The Yukawa sector is then the same as the one in the HTM, so that its predictions for the LFV processes are not changed. See Refs. [33, 35] for some discussions about two-loop realization of the \(\mu\)-term\(^1\).

This paper is organized as follows. In Sec. II we give a quick review for the HTM to define notation. In Sec. III the model for radiatively generating the \(\mu\) parameter with the dark matter candidate is presented. Some phenomenological implications are discussed in

\(^1\) The two-loop \(\mu\)-term in Ref. [35] is given with softly-broken \(Z_4\) symmetry, but the tree level \(\mu\)-term would be also accepted as a soft breaking term. The two-loop \(\mu\)-term in Ref. [33] is given with \(Z_3\) symmetry which is broken by a VEV of a scalar \(S\), but the tree level \(\Phi^T i\sigma_2\Delta^\dagger \Phi S^*\) seems allowed by the \(Z_3\).
II. HIGGS TRIPLET MODEL

In the HTM, an SU(2)$_L$ triplet of complex scalar fields with hypercharge $Y = 1$ is introduced to the SM. The triplet $\Delta$ can be expressed as

$$\Delta = \begin{pmatrix} \Delta^+ / \sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+ / \sqrt{2} \end{pmatrix},$$

(1)

where $\Delta^0 = (\Delta^0 + i \Delta_i^0) / \sqrt{2}$. The triplet has a new Yukawa interaction term with leptons as

$$L_{\text{triplet-Yukawa}} = h_{\ell \ell'} L_\ell^c \sigma_2 \Delta L_{\ell'} + \text{h.c.},$$

(2)

where $h_{\ell \ell'} (\ell, \ell' = e, \mu, \tau)$ are the new Yukawa coupling constants, $L_\ell = (\nu_L, \ell)^T$ are lepton doublet fields, a superscript $c$ means the charge conjugation, and $\sigma_i (i = 1-3)$ denote the Pauli matrices. Lepton number ($L#$) of $\Delta$ is assigned to be $-2$ as a convention such that the Yukawa term does not break the conservation. A vacuum expectation value $v_\Delta = \sqrt{2} \langle \Delta^0 \rangle$ breaks lepton number conservation by two units. The new Yukawa interaction then yields the Majorana neutrino mass term $(m_\nu)_{\ell \ell'} (\nu_L^c)^c \nu_{\ell'} L / 2$ where $(m_\nu)_{\ell \ell'} = \sqrt{2} v_\Delta h_{\ell \ell'}$.

The scalar potential in the HTM can be written as

$$V_{\text{HTM}} = -m_\Phi^2 \Phi^\dagger \Phi + m_\Delta^2 \text{tr}(\Delta^\dagger \Delta) + \left\{ \mu \Phi^T i \sigma_2 \Delta^\dagger \Phi + \text{h.c.} \right\}$$

$$+ \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 \left[ \text{tr}(\Delta^\dagger \Delta) \right]^2 + \lambda_3 \text{tr}[(\Delta^\dagger \Delta)^2]$$

$$+ \lambda_4 (\Phi^\dagger \Phi) \text{tr}(\Delta^\dagger \Delta) + \lambda_5 \Phi^\dagger \Delta^\dagger \Phi,$$

(3)

where $\Phi = (\phi^+, \phi^0)^T$ [$\phi^0 = (\phi^0_i + i \phi^0_i) / \sqrt{2}$] is the Higgs doublet field in the SM. The $\mu$ parameter can be real by using rephasing of $\Delta$. Because we take $m_\Delta^2 > 0$, there is no Nambu-Goldstone boson for spontaneous breaking of lepton number conservation. The small triplet VEV $v_\Delta$ is generated by an explicit breaking parameter $\mu$ of the lepton number conservation as

$$v_\Delta \simeq \frac{\sqrt{2} \mu v^2}{2m_\Delta^2 + (\lambda_4 + \lambda_5)v^2},$$

(4)

where $v (\simeq 246 \text{ GeV})$ is the doublet VEV defined by $v = \sqrt{2} \langle \phi^0 \rangle$. 

Sec. IV and the conclusion is given in Sec. V. The full expressions of the Higgs potential and mass formulae for scalar bosons in our model are given in Appendix.
In order to obtain small neutrino masses in the HTM, at least one of \( v^2/m_\Delta^2, h_{\ell\ell'}, \mu/v \) should be tiny. A small \( \mu \) is an attractive option because \( m_\Delta \) can be small (\( \lesssim 1 \text{ TeV} \)) so that triplet scalars can be produced at the LHC. Furthermore, large \( h_{\ell\ell'} \) can be taken, which have direct information on the flavor structure of \( (m_\nu)_{\ell\ell'} \). There is, however, no reason why the \( \mu \) parameter is tiny in the HTM. In our model presented below, the \( \mu \) parameter is naturally small because it arises at the one-loop level.

### III. AN EXTENSION OF THE HIGGS TRIPLET MODEL

Since we try to generate the \( \mu \)-term in the HTM radiatively, the term must be forbidden at the tree level. The simplest way would be to impose lepton number conservation to the Lagrangian. The conservation is assumed to be broken by the VEV of a new scalar field \( s_{1}^0 \) which is singlet under the SM gauge symmetry. Notice that \( s_{1}^0 \equiv (s_{1r}^0 + is_{1i}^0)/\sqrt{2} \) is a complex ("charged") field with non-zero lepton number although it is electrically neutral. One might think that the VEV of \( s_{1}^0 \) could be generated by using soft breaking terms of \( L\# \). However, the \( \mu \)-term is also a soft breaking term. Therefore lepton number must be broken spontaneously in our scenario. One may worry about Nambu-Goldstone boson corresponds to the spontaneous breaking of the lepton number conservation (the so-called Majoron, \( J^0 \)). However the Majoron which comes from gauge singlet field can evade experimental searches (constraints) because it interacts very weakly with matter fields. It is also possible to make it absorbed by a gauge boson by introducing the \( U(1)_{B-L} \) gauge symmetry to the model (See, e.g., Ref. [37]). In this paper we just accept the Majoron without assuming the \( U(1)_{B-L} \) gauge symmetry for simplicity.

If \( s_{1}^0 \) has \( L\# = -2 \), we can have a dimension-4 operator \( \lambda s_{1}^0 \Phi^T i\sigma_2 \Delta^\dagger \Phi \). This gives a trivial result \( \mu = \lambda \langle s_{1}^0 \rangle \) at the tree level. Although the dim.-4 operator could be forbidden by some extra global symmetries with extra scalars to break them, we do not take such a possibility in this paper. We just assume the \( s_{1}^0 \) has \( L\# = -1 \). Then the lepton number conserving operator which results in the \( \mu \)-term is of dimension-5 as

\[
(s_{1}^0)^2 \Phi^T i\sigma_2 \Delta^\dagger \Phi. \tag{5}
\]

We consider below how to obtain the dim.-5 operator at the loop level by using renormal-
izable interactions\(^2\). We restrict ourselves to extend only the SU(3)\(_c\)-singlet scalar sector in the HTM because it seems a kind of beauty that the HTM does not extend the fermion sector and colored sector in the SM. An unbroken \(Z_2\) symmetry is introduced in order to obtain dark matter candidates, and new scalars which appear in the loop diagram for the \(\mu\)-term are aligned to be \(Z_2\)-odd particles. We emphasize that the unbroken \(A_2\) symmetry is not for a single purpose to introduce dark matter candidates but utilized also for our radiative mechanism for the \(\mu\)-term.

We present the minimal model where the dim.-5 operator in eq. (5) is generated by a one-loop diagram with dark matter candidates. Table I shows the particle contents. A real singlet scalar field \(s_2^0\) and the second doublet scalar field \(\eta \equiv (\eta^+, \eta^0)^T, \eta^0 = (\eta_0^+ + i\eta_0^-)/\sqrt{2}\) are introduced to the HTM in addition to \(s_1^0\). Lepton numbers of \(s_2^0\) and \(\eta\) are 0 and \(-1\), respectively. Then \(\eta^T i\sigma_2 \Delta^\dagger \eta\) conserves lepton number. In order to forbid the VEV of \(\eta\), we introduce an unbroken \(Z_2\) symmetry for which \(s_2^0\) and \(\eta\) are odd. Other fields are even under the \(Z_2\).

The Yukawa interactions are the same as those in the HTM. The Higgs potential is given as

\[
V = \frac{1}{2} m_{s_2^0}^2 (s_2^0)^2 + \{\mu_\eta \eta^T i\sigma_2 \Delta^\dagger \eta + \text{h.c.}\} + \{\lambda_{s\Phi\eta} s_1^0 s_2^0 (\eta^\dagger \Phi) + \text{h.c.}\} + \cdots. \tag{6}
\]

Here we show only relevant parts for radiative generation of the \(\mu\)-term. See Appendix for the other terms. Vacuum expectation values \(v\) and \(v_s \equiv \sqrt{2} \langle s_1^0 \rangle\) are given by

\[
\begin{pmatrix} v^2 \\ v_s^2 \end{pmatrix} = \frac{2}{4\lambda_1 \phi \lambda_{s1} - \lambda_{s\Phi1}^2} \begin{pmatrix} 2\lambda_{s1} & -\lambda_{s\Phi1} \\ -\lambda_{s\Phi1} & 2\lambda_1 \phi \end{pmatrix} \begin{pmatrix} m_\Phi^2 \\ m_{s1}^2 \end{pmatrix}. \tag{7}
\]

\(^2\) It will not be difficult to do the same consideration for cases of higher dimensional operators, e.g., dim.-6 one with \(s_1^0\) of \(L\# = -2/3\).
The $Z_2$-odd scalars in this model are two CP-even neutral ones ($H_1^0$ and $H_2^0$), a CP-odd neutral one ($A^0 = \eta_0^0$), and a charged pair ($H^\pm = \eta^\pm$). The CP-even scalars are defined as

$$
\begin{pmatrix}
H_1^0 \\
H_2^0
\end{pmatrix} = \begin{pmatrix}
\cos \theta_0' & -\sin \theta_0' \\
\sin \theta_0' & \cos \theta_0'
\end{pmatrix} \begin{pmatrix}
\eta_0^0 \\
\eta_0^0
\end{pmatrix},
\tan 2\theta_0' = \frac{\sqrt{2} \lambda_{s\Phi \eta} v v_s}{(M_0)_{ss}^2 - (M_0)_{ss}^2},
$$

where $(M_0)_{\eta \eta}^2 \equiv m_\eta^2 + (\lambda_{1\Phi \Phi} + \lambda_{1\Phi \eta}) v^2 / 2 + \lambda_{s\eta} v_s^2 / 2$ and $(M_0)_{ss}^2 \equiv m_{s_2}^2 + \lambda_s v_s^2 + \lambda_{s\Phi} v^2$.

Squared masses of these scalars are given by

$$
m^2_{H_1^0} = \frac{1}{2} \left\{ (M_0)_{\eta \eta}^2 + (M_0)_{ss}^2 - \sqrt{\{ (M_0)_{\eta \eta}^2 - (M_0)_{ss}^2 \}^2 + 2 \lambda_{s\Phi \eta} v^2 v_s^2} \right\},
$$

$$
m^2_{H_2^0} = \frac{1}{2} \left\{ (M_0)_{\eta \eta}^2 + (M_0)_{ss}^2 + \sqrt{\{ (M_0)_{\eta \eta}^2 - (M_0)_{ss}^2 \}^2 + 2 \lambda_{s\Phi \eta} v^2 v_s^2} \right\},
$$

$$
m^2_{A^0} = (M_0)_{\eta \eta}^2,
$$

$$
m^2_{H^\pm} = (M_0)_{\eta \eta}^2 - \frac{1}{2} \lambda_{1\Phi \eta} v^2.
$$

Notice that $m_{H_1^0} \leq m_{A^0} \leq m_{H_2^0}$. We assume $m_{H_1^0} < m_{H^\pm}$ and then $H_1^0$ becomes the dark matter candidate. Hereafter it is assumed that the mixing $\theta_0'$ is small.

The $\mu$-term is generated by the one-loop diagram. Figure 1 is the dominant one in the case of small $\theta_0'$. Then, the parameter $\mu$ is calculated as

$$
\mu = \frac{\lambda_{s\Phi \eta}^2 \mu_\eta v_s^2}{64\pi^2 \{ (M_0)_{ss}^2 - (M_0)_{\eta \eta}^2 \}^2} \left\{ 1 - \frac{(M_0)_{ss}^2}{(M_0)_{\eta \eta}^2} \ln \frac{(M_0)_{ss}^2}{(M_0)_{\eta \eta}^2} \right\}.
$$

The one-loop induced $\mu$ parameter can be expected to be much smaller than $\mu_\eta$. The suppression factor $|\mu/\mu_\eta|$ is estimated in Sec. IV A.
IV. PHENOMENOLOGY

A. Dark matter

If \((M_0)_{\eta\eta} < (M_0)_{ss}\), the dark matter candidate \(H_0^1\) is given by \(\eta_0^1\) approximately because we assume small mixing. See, e.g., Ref. [8] for studies about the inert doublet scalar. Let us assume \(m_{H_0^1} \simeq 75\) GeV and \(m_{A^0} \gtrsim 125\) GeV. As shown in Ref. [9], these values satisfy constraints from the LEP experiments [39, 40] and the WMAP experiment [2]. The mass splitting \((m_{A^0} - m_{H_0^1} \gtrsim 50\) GeV\) suppresses quasi-elastic scattering on nuclei \((H_0^1 N \rightarrow A^0 N\) mediated by the \(Z\) boson) enough to satisfy constraints from direct search experiments of the DM [41]. By using eqs. (9) and (11), we obtain

\[
\lambda_s^2 \Phi \eta v_s^2 \simeq \frac{2}{v^2} \left( m_{A^0}^2 - m_{H_0^1}^2 \right) \gtrsim 0.3. \tag{14}
\]

In order to be consistent with our assumption of small \(\theta_0'\) (e.g., \(\simeq 0.1\)), \((M_0)_{ss} \gtrsim 3\) TeV is required. The value in eq. (14) results in

\[
\frac{\mu}{\mu_\eta} \gtrsim 10^{-4}. \tag{15}
\]

For the greater value of \(m_{A^0}\), the larger \(\mu/\mu_\eta\) is predicted. In particular, by taking \(m_{A^0}\) to be the TeV scale, we obtain \(\mu/\mu_\eta \sim 10^{-2}\), which yields \(v_\Delta \sim 1\) GeV for \(\mu_\eta\) and \(m_\Delta\) to be at the electroweak scale. Such a value for \(v_\Delta\) is suggested in the recent study of radiative corrections to the electroweak parameters [42].

On the contrary, if we take \(m_{A^0} \simeq 83\) GeV which is allowed in a tiny region [9], values in eqs. (14) and (15) become 10 times smaller. We mention that the WMAP constraint might be changed by a characteristic annihilation process \(H_0^1 H_0^1 \rightarrow \Delta^0 \rightarrow \nu\bar{\nu}\) where \(H_0^1 H_1^0(\Delta^0)\) interaction is governed by \(\mu_\eta\) (not by a tiny \(\mu\)). This additional process could sift allowed value of \(m_{H_0^1}\) to lower one while \(m_{A^0} \gtrsim 100\) GeV due to the LEP constraint. Then, \(\mu/\mu_\eta\) might become larger than the value in eq. (15) because of larger \(m_{A^0} - m_{H_0^1}\). This undesired effect would be easily avoided if \(m_\Delta^0\) is away enough from \(2m_{H_0^1}\).

On the other hand, \(H_1^0\) comes dominantly from \(s_2^0\) if \((M_0)_{\eta\eta} > (M_0)_{ss}\). See, e.g., Ref. [7] for studies about the real inert singlet scalar. Coupling \(\sqrt{2} \lambda_{s\Phi_1} v\) of the \(H_0^1 H_1^0 h^0\) interaction \((h^0\) is the SM Higgs boson\) determines annihilation cross section of \(H_1^0\) and scattering cross section on nuclei. If we introduce the \(U(1)_{B-L}\) gauge symmetry, the scattering of \(s_2^0\) on nuclei can be mediated also by the gauge boson \(Z'\). Notice that the parameter \(\lambda_{s\Phi_1}\) (and
FIG. 2: The unique process in our model for $H_1^0 \approx \eta_r^0$. The bosonic decay of $H^+$ contains information of $\mu_\eta$ indicated by a red blob.

also the $U(1)_{B-L}$ gauge coupling constant) does not affect on $\mu$ parameter in eq. (13). Let us estimate the magnitude of $\mu/\mu_\eta$. In the usual HTM, $h_{\ell\ell'}$ is expected to be $\lesssim 10^{-2}$ for $m_{H^{\pm \pm}} \sim 100$ GeV in order to suppress LFV processes. Thus, we may accept $\lambda_{s\Phi\eta} \approx 1-10^{-2}$ as a value which is not too small. Assuming $(\mathcal{M}_0)_{ss} \ll (\mathcal{M}_0)_{\eta\eta} \sim v_s \sim 1$ TeV for example, we have a suppression factor as

$$\left| \frac{\mu}{\mu_\eta} \right| \sim \frac{\lambda_{s\Phi\eta}^2 v_s^2}{64\pi^2(\mathcal{M}_0)^2_{\eta\eta}} \sim 10^{-3}-10^{-7}. \quad (16)$$

Thus, even if the value of $\mu_\eta$ is in the TeV scale, we can obtain $\mu \sim 0.1$ MeV although we need further suppression with $h_{\ell\ell'} \lesssim 10^{-5}$ to have $m_\nu \lesssim 1$ eV. If we use $h_{\ell\ell'} \sim \lambda_{s\Phi\eta} \sim 10^{-3}$, we obtain $|\mu/\mu_\eta|h_{\ell\ell'} \sim 10^{-12}$ which can connect the TeV scale $\mu_\eta$ to the eV scale $m_\nu$.

B. Collider

The characteristic feature of our model is that $\mu_\eta$ is much larger than $\mu$. Let us consider possibility to probe the large $\mu_\eta$ in collider experiments.

A favorable process is shown in Fig. 2 for $H_1^0 \approx \eta_r^0$. For simplicity, we take $\lambda_5 = 0$ which results in $m_{H^{\pm \pm}} \simeq m_{H^0} \simeq m_{H^0, A^0}$. Recently, it was found in Ref. [42] that the electroweak precision test prefers $\lambda_5 > 0$ in the HTM where the electroweak sector is described by four input parameters. However, results in Ref. [42] might not be applied directly to our

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3 If we introduce $U(1)_{B-L}$ gauge symmetry in order to eliminate the Majoron, $v_s$ should be a little bit larger (e.g., $\geq 3$ TeV) due to constraint on the mass of $Z'$. 


model because the scalar sector is extended. Since $H^{\pm} \rightarrow \ell^\pm \nu^\pm$ is the most interesting decay in the HTM, we assume $2m_{H^\pm} > m_{H^{\pm\pm}}$ in order to forbid $H^{\pm\mp} \rightarrow H^{\pm}H^{\mp}$. Even in this case, the DM $\mathcal{H}_1^0$ can be light enough ($m_{H^\pm} > m_{H^{\pm\pm}} + m_{\mathcal{H}_1^0}$) so that $Z_2$-even charged scalar $H^\pm$ ($\simeq \Delta^\pm$) can decay into $\mathcal{H}^\pm \mathcal{H}_1^0$ via $\mu_\eta$-term which is indicated by a red blob in Fig. 2. The partial decay width of $H^\pm \rightarrow \mathcal{H}^\pm \mathcal{H}_1^0$ is determined by $(\mu_\eta/\mu)^2v_\Delta^2/m_{H^\pm}$ while the width of $H^\pm \rightarrow \ell^\pm \nu$ is proportional to $m_{H^\pm}m_\nu^2/v_\Delta^2$. Taking $\mu_\eta/\mu \sim 10^4$, $v_\Delta \sim 10$ keV, $m_{H^\pm} \sim 100$ GeV, and $m_\nu \sim 0.1$ eV for example, we have $(\mu_\eta/\mu)^2v_\Delta^2/m_{H^\pm} \sim 10^5$ eV and $m_{H^\pm}m_\nu^2/v_\Delta^2 \sim 10$ eV. Then, $H^\pm$ dominantly decays into $\mathcal{H}^\pm \mathcal{H}_1^0$. Finally, $\mathcal{H}^\pm$ decays into $(W^\pm)^* \mathcal{H}_1^0$. Therefore, from a production mechanism $pp \rightarrow (W^\pm)^* \rightarrow H^{\pm\pm}H^\mp$, we would have $\ell\ell j j E_T$ as a final state for which $\ell\ell$ has the invariant mass $m(\ell\ell)$ at $m_{H^{\pm\pm}}$ assuming that the value of $m_{H^{\pm\pm}}$ has been known already.

If $\mathcal{H}_1^0 \simeq s_2^0$, then $H^\pm$ decays via $\mu_\eta$-term into $\mathcal{H}^\pm A^0$ or $\mathcal{H}^\pm \mathcal{H}_2^0$ followed by $\mathcal{H}_2^0 \rightarrow A^0 J^0$ where a sizable $\lambda_{s\eta_1}$ is assumed. Because of $A^0 \rightarrow \mathcal{H}_1^0 J^0$ through $\lambda_{s\Phi \eta}$, we have again $\ell\ell j j E_T$ with $m(\ell\ell) = m_{H^{\pm\pm}}$ from $pp \rightarrow (W^\pm)^* \rightarrow H^{\pm\pm}H^\mp$.

In the usual HTM in contrast, the final state with such $\ell\ell$ is likely to include additional charged leptons ($\ell\ell\ell\ell$ from $H^{++}H^{--}$, $\ell\ell j j E_T$ from $H^{\pm\pm}H^\mp$, etc.) if $H^{\pm\pm}$ decay dominantly into $\ell^\pm \ell'^\pm$. Therefore, our model would be supported if experiments observe final states which include jets and only two $\ell$ whose invariant mass gives $m(\ell\ell) = m_{H^{\pm\pm}}$. This potential signature might be disturbed by hadronic decays of $\tau$ because $H^{++}H^{--} \rightarrow \ell\ell\tau\tau$ can result in $\ell\ell j j E_T$ with $m(\ell\ell) = m_{H^{\pm\pm}}$. Realistic simulation is necessary to see the feasibility.

V. CONCLUSIONS AND DISCUSSION

We have presented the simple extension of the HTM by introducing a $Z_2$-even neutral scalar $s_1^0$ of $L# = -1$, a $Z_2$-odd neutral real scalar $s_2^0$ of $L# = 0$, and a $Z_2$-odd doublet scalar field $\eta$ of $L# = -1$. The DM candidate $\mathcal{H}_1^0$ in our model is made from $s_2^0$ and $\eta_r$. The $\mu\Phi^T i\sigma_2 \Delta^I \Phi$ interaction which is the origin of $v_\Delta$ (and neutrino masses) is induced at the one-loop level while the $\mu_\eta \eta^T i\sigma_2 \Delta^I \eta$ interaction exists at the tree level. Because of the

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4 Our model also has four parameters for the electroweak sector although $v_\Delta$ is generated at the 1-loop level.

5 Each of two $\mathcal{H}_1^0$ in Fig. 2 can be replaced with $A^0$ which decays into $Z^* \mathcal{H}_1^0$ for $\mathcal{H}_1^0 \simeq \eta_r^0$.

6 If $\lambda_{s\eta_1}$ is small, $\mathcal{H}_2^0$ ($\simeq \eta_r^0$) decays into $Z^* A^0$. 
loop suppression for $\mu$ parameter, the model gives small neutrino masses naturally without using very heavy particles.

For $H_1^0 \simeq \eta^0_\nu$, the suppression factor $|\mu/\mu_\eta|$ is constrained by the DM relic abundance measured by the WMAP experiment. We have shown that $|\mu/\mu_\eta| \sim 10^{-4}-10^{-5}$ is possible. On the other hand, for $H_1^0 \simeq s_0^2$, the suppression factor is somewhat free from experimental constraints on the DM. In our estimate, $|\mu/\mu_\eta| \sim 10^{-3}-10^{-7}$ can be obtained as an example with $\lambda_{s\Phi\eta} \sim 1-10^{-2}$.

The characteristic feature of the model is that $\mu_\eta$ is not small while $\mu$ can be small. A possible collider signature which depends on $\mu_\eta$ would be $\ell\ell jj E_T$ with the invariant mass $m(\ell\ell) = m_{H^0\pm\pm}$ because more charged leptons are likely to exist in such final states in the usual HTM.

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appendix

The Higgs potential of our model is given by $V = V_2 + V_3 + V_4$ where

$$V_2 \equiv -m_{s_1}^2 |s_1|^2 + \frac{1}{2} m_{s_2}^2 (s_2^0)^2 - m_\Phi^2 \Phi^\dagger \Phi + m^2_\eta \eta^\dagger \eta + m^2_\Delta \text{tr}(\Delta^\dagger \Delta),$$

(17)

$$V_3 \equiv (\mu_\eta \eta^T i\sigma_2 \Delta^\dagger \eta) + \text{h.c.},$$

(18)
\[ V_4 \equiv \lambda_{1\Phi}(\Phi^\dagger \Phi)^2 + \lambda_{1\eta}(\eta^\dagger \eta)^2 + \lambda_{1\Phi \Phi}(\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_{1\Phi \eta}(\Phi^\dagger \eta)(\eta^\dagger \Phi) + \lambda_2 [\text{tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 [\text{tr}(\Delta^\dagger \Delta)]^2 \]
\[ + \lambda_{4\Phi}(\Phi^\dagger \Phi)\text{tr}(\Delta^\dagger \Delta) + \lambda_{4\eta}(\eta^\dagger \eta)\text{tr}(\Delta^\dagger \Delta) \]
\[ + \lambda_{5\Phi}(\Phi^\dagger \Delta \Delta^\dagger \Phi) + \lambda_{5\eta}(\eta^\dagger \Delta \Delta^\dagger \eta) \]
\[ + \lambda_{s1} |s_{1r}^0|^4 + \lambda_{s2} (s_{2r}^0)^4 + \lambda_{s3} |s_{1i}^0|^2 (s_{2r}^0)^2 \]
\[ + \lambda_{s\Phi 1} |s_{1r}^0|^2 (\Phi^\dagger \Phi) + \lambda_{s\Phi 2} (s_{2r}^0)^2 (\Phi^\dagger \Phi) \]
\[ + \lambda_{s\eta 1} |s_{1r}^0|^2 (\eta^\dagger \eta) + \lambda_{s\eta 2} (s_{2r}^0)^2 (\eta^\dagger \eta) + \{ \lambda_{s\Phi \eta} s_{1r}^0 s_{2r}^0 (\eta^\dagger \Phi) + \text{h.c.} \} \]
\[ + \lambda_{s\Delta 1} |s_{1r}^0|^2 \text{tr}(\Delta^\dagger \Delta) + \lambda_{s\Delta 2} (s_{2r}^0)^2 \text{tr}(\Delta^\dagger \Delta). \] (19)

All coupling constants are real because the phases of \( \mu_{\eta} \) and \( \lambda_{s\Phi \eta} \) can be absorbed by \( \Delta \) and \( s_{1r}^0 \), respectively.

Mass eigenstates of two \( Z_2 \)-even CP-even neutral scalars which are composed of \( s_{1r}^0 \) and \( \phi_{r}^0 \) are obtained as
\[
\begin{pmatrix} h_0^0 \\ H_0^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \phi_{r}^0 \\ s_{1r}^0 \end{pmatrix}, \quad \tan 2\theta_0 = \frac{\lambda_{s\Phi 1} v v_s}{\lambda_{s1} v_s^2 - \lambda_{1\Phi} v^2}. \] (20)

Their masses eigenvalues are given by
\[
m_{h_0}^2 \simeq \lambda_{1\Phi} v^2 + \lambda_{s1} v_s^2 - \sqrt{\left(\lambda_{1\Phi} v^2 - \lambda_{s1} v_s^2\right)^2 + \lambda_{s\Phi 1} v^2 v_s^2}, \] (21)
\[
m_{H_0}^2 \simeq \lambda_{1\Phi} v^2 + \lambda_{s1} v_s^2 + \sqrt{\left(\lambda_{1\Phi} v^2 - \lambda_{s1} v_s^2\right)^2 + \lambda_{s\Phi 1} v^2 v_s^2}, \] (22)
where small contributions from \( v_\Delta \) are neglected. Two \( Z_2 \)-even CP-odd neutral bosons (\( \phi_{i}^0 \) and \( s_{1i}^0 \)) are Nambu-Goldstone bosons; \( \phi_{i}^0 \) is absorbed by the \( Z \) boson, and \( s_{1i}^0 \) is the Majoron (or absorbed by the \( Z' \) boson).

Masses of bosons made dominantly from \( \Delta \) are given by
\[
m_{H_T^0}^2 \simeq m_{A_T^0}^2 \simeq m_{H^0}^2 + \frac{1}{4} \lambda_{5\Phi} v^2, \] (23)
\[
m_{H^0}^2 \simeq m_{\Delta}^2 + \frac{1}{4} (2\lambda_{1\Phi} + \lambda_{5\Phi}) v^2 + \frac{1}{2} \lambda_{s\Delta 1} v_s^2, \] (24)
\[
m_{H^{\pm^2}}^2 \simeq m_{H^0}^2 - \frac{1}{4} \lambda_{5\Phi} v^2. \] (25)

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