Learning difficulties of senior high school students based on probability understanding levels

B Anggara¹*, N Priatna² and D Juandi²

¹ Department of Mathematics Education, postgraduate School, Universitas Pendidikan Indonesia, Bandung, Indonesia
² Department of Mathematics Education, Universitas Pendidikan Indonesia, Bandung, Indonesia

*Corresponding author: bennyangkara@gmail.com

Abstract. Identifying students' difficulties in learning concept of probability is important for teachers to prepare the appropriate learning processes and can overcome obstacles that may arise in the next learning processes. This study revealed the level of students' understanding of the concept of probability and identified their difficulties as a part of the epistemological obstacles identification of the concept of probability. This study employed a qualitative approach that tends to be the character of descriptive research involving 55 students of class XII. In this case, the writer used the diagnostic test of probability concept learning difficulty, observation, and interview as the techniques to collect the data needed. The data was used to determine levels of understanding and the learning difficulties experienced by the students. From the result of students' test result and learning observation, it was found that the mean cognitive level was at level 2. The findings indicated that students had appropriate quantitative information of probability concept but it might be incomplete or incorrectly used. The difficulties found are the ones in arranging sample space, events, and mathematical models related to probability problems. Besides, students had difficulties in understanding the principles of events and prerequisite concept.

1. Introduction

One of the most important concepts in mathematical learning is probability since crucial principles in probability and statistics theory can relate to the physical world in which of course will have students to collect, record, interpret, analyze, communicate, and present the data needed to their decision-making processes [1]. Emphasized that misunderstanding over probability affects people's decisions in important situations, such as medical diagnosis, jury verdicts, investments, appraisals, and so on [2]. Therefore, the concept of probability is wide-applied in with few other branches of science that use it to solve problems.

Seen from some crucial things related to the understanding of probability, the concept of probability should be well-known by students. Students' ability of the concept of probability can be the base to learn other mathematical concepts in higher level and apply them in their lives. However, in learning probability concept, students often find difficulties. The study of [3] showed that in learning the concept of probability there is sometimes a didactic problem as a result of an error in explaining the relationship between probability accumulation through a frequency approaches and abstract statistics with classical probability accumulation. It is known as difficulties in understanding the concept of prerequisites by...
[4]. Described students are just facilitated by certain terms in differentiating permutations or combinations without any process of understanding the meaning of permutations and combinations [3]. According to [5] one of the errors made by students in learning the concept of probability comes from their incorrect way to accumulate the possibility. A study conducted by [6] and [7] which was focused on the model assessment and learning approaches on probability materials revealed that some students are mistaken in making and completing mathematical models in everyday problems of probability situations. In addition, students face difficulties in calculating possibilities of an event in an experiment.

From the difficulties in learning the probability, therefore it needs a solution in solving the problems. Outlined some considerable things for teachers to overcome students’ difficulties in understanding the probability [8]. One of them is that teachers must identify students’ difficulties before helping students in overcoming difficulties. The difficulties that students face then become a learning obstacle. Learning obstacle, according to [9], has a lot of forms and one of them is epistemological obstacle. Epistemological obstacle occurs due to the limited context of students in understanding a concept. In other words, it means that if the concept is confronted in a different problem and context then the students will have it difficult to learn. Epistemological obstacle is an inevitable barrier because of its importance in the construction of the concept of knowledge itself. Described that epistemological obstacle is unrelated to the learning approach used by teachers [10]. Otherwise, it appears as a result of the nature of the mathematical concept itself.

One of the evidence to find the learning difficulties faced by students in understanding the concept of probability is by grouping students in the levels of understanding probability adapted from [11], namely: level 1, in interpreting probability situations no analysis or evidence of use of probability principles is demonstrated. Features may include: the use of irrelevant information, subjective judgements, disregarding quantitative information, guessing at random, belief in control of probability and absence of any reason. Responses that use recent experiences to predict or estimate probabilities, availability, are included in this level. Level 2, some evidence of the use of probability principles and appropriate quantitative information is evident, but they may be incomplete or are incorrectly used. Probabilistic reasoning based on the assumption of equal likelihood when none exists and the use of the representativeness heuristic is considered to be illustrative of this level. Level 3, probability principles are applied correctly used and an awareness of the role of quantification is evident. However, such quantification is precise or numerical. Level 4, probability principles are used correctly and relationships are explained quantitatively.

2. Method
This study employed a qualitative approach that tends to be the character of descriptive research. Descriptive research design is used to analyze phenomena in learning difficulties naturally. It describes the order of levels of understanding probability student. This research was conducted at one of the high schools in Indramayu, West Java, Indonesia. There are 55 students involved as participants. They are 12th grade students in which 21 males and 34 females. In the procedure used in this study, firstly the writer provides the diagnostic test of learning difficulties to the participants. Diagnostic test of learning probability difficulties contains 10 subjective questions. Secondly, the writer collects the data from test results, observation and interview. Thirdly, the writer analyzes the data by grouping students into four levels of understanding probability and identifying the forms of difficulty experienced by students. Fourthly, the writer draws a conclusion and suggestion.

3. Results and Discussion
The researcher observed the students’ difficulties from the concept comprehension aspect by giving the diagnostic tests of learning probability difficulties to 55 participants. The questions were designed to identify the difficulty of respondents’ comprehension of the concept of probability including aspects probability such as terms, definitions, theoretical probability, background to probability, union and intersection, and dependent and independent events. The analysis was focused based on the components
of the difficulty aspects and classification of four levels of probability comprehension. In each question, participants’ results are analyzed based on cognitive abilities into these four levels.

Table 1. Descriptive statistics of participants’ levels of understanding probability.

| Item | Description | Percentage of Respondent’s Cognitive Levels (%) |
|------|-------------|-----------------------------------------------|
| Q1   | Define the following terms: an experiment, experimental results, sample space, an event, and probability of an event | 56.36 41.82 1.82 0.00 |
| Q2   | What is the sample space for choosing a letter from the word “MATEMATIKA”? | 3.64 43.64 29.09 23.64 |
| Q3   | Suppose that the probability of snow is 0.67, what is the probability that it will NOT snow? | 5.45 7.27 16.36 70.91 |
| Q4   | Let S denote the set of whole numbers from 1 to 16, X denote the set of even numbers from 1 to 16 and Y denote the set of prime numbers 1 to. Draw a Venn diagram depicting S, X, and Y. | 32.73 34.55 32.73 0.00 |
| Q5   | A bag contains 6 red, 3 blue, 2 green, and 1 white balls. A ball is picked at random. Determine the probability is a blue. | 12.73 14.55 14.55 58.18 |
| Q6   | A school decided that its uniform needed upgrading. The colours on offer were blue or cream or blue and cream. 40% of the school wanted cream, 55% wanted blue and 15% said a combination would be fine. Are the two events independent? | 52.73 47.27 0.00 0.00 |
| Q7   | A bookshelf contains 5 Mathematics books and 4 Physics books. What are the chances that 3 particular Mathematics books are put together? | 54.55 45.45 0.00 0.00 |
| Q8   | In an experiment throwing two dice randomly by 360 times, determine the expected frequency of eye dice totaling 6 or 8! | 9.09 12.73 25.45 52.73 |
| Q9   | Tono experimented on taking 5 candies at once randomly from inside a bag. Inside the bag there are 10 sweet candy and 8 sour candy. The experiment was conducted 100 times. The taken candy is returned back into the bag. Make a mathematical model of the possibility of being picked up and determine the expected frequency of at least 3 sweet sweets. | 45.45 38.18 16.36 0.00 |
| Q10  | Probability Andi picked up a red ball in one experiment at a box is 1/3. Make a mathematical model to calculate the odds Andi can take twice the red ball out of three experiments (the ball taken at each experiment is returned back into the box)! Then determine his chances. | 52.73 47.27 0.00 0.00 |

Dependent & independent events aspect}

| Percentage of Respondent’s Cognitive Levels (%) | 52.73 47.27 0.00 0.00 |
Based on Table 1, high percentage of respondent’s cognitive levels is at level 2 with 33.27%, which according to the rubric indicates that some evidence of use of probability principles and appropriate quantitative information is evident, but they may be incomplete or are incorrectly used. The dependent and independent event aspect becomes the hardest aspect for the students. achievement of the 52.73% students were at level 1. According to the rubric, then some attributes of level 1 are noted about the use of irrelevant information, subjective judgments, disregarding quantitative information, guessing at random, belief in control of probability and absence of any reason. For each item, Q1 is the most difficult problem with achievement of the 56.36% students were at level 1. It appears that students are unable to define precisely the random experiment and the probability of an event in the concept of probability. However, few students are able to define the results of experiments, sample spaces, and events appropriately. The error found is the one in defining basic concepts in the following opportunities.

The error of the student in defining the random experiment, the experimental results, the sample space, the occurrence, and the probability of an event based on the student's assumption that the experiment as an occurrence of objects often mentioned in the examples and problems which students studied. Students did not interpret the experiment as an activity. As a result of these errors, students were mistaken in interpreting the sample space, events, and possibility of an event.

Of the problem of dependent and independent event, Q6 and Q9 became the next most difficult problem faced by students. In Q6, some students were able to mention that the principle of the event is two dependent incidents but the reasons described did not seem to use the terms of two events which are dependent. Students are mistaken to use the condition of two mutual occurrences as shown Figure 1.

![Figure 1](image1.png)

**Figure 1.** The error in providing independent reasons.

In Q9, students tend to answer using their own ways, as follows.

![Figure 2](image2.png)

**Figure 2.** The error in describing mathematical models based on classical probability principles.

From Figure 2, the students tried to elaborate the problem by describing a mathematical model based on classical probability principles. But, students were mistaken for not estimating that to solve the problem using the principle of mutual occurrence. The students might think that to determine the probability value is by always using the formula $P(A) = \frac{n(A)}{n(S)}$. Students could not relate the principle of events in any probability problem to the different ways to solve them.
Students, in responding to Q7, also faced many difficulties caused by their inability to identify the experiments of the problem. Therefore, they seem to be wrong in determining the sample space and the intended event space. Students consider that the experiment was to select 3 books from 9 available books. Therefore, the result was $C(9,3)$ and the events described are selecting 3 books from 5 available books. Otherwise, students’ answer was $C(5,3)$ so they calculate the probability in the way as follows.

$$
\text{Jawab:}\; \frac{n(s)}{n(a)} = \frac{\binom{9}{3}}{\binom{5}{3}} = \frac{84}{10} = \frac{42}{5}
$$

**Figure 3.** The error in accumulating sample and events using combination concept.

From Figure 3 above, it appears that students were incorrect in calculating the value of the intended probability according to student’s error in identifying the form of random experiment. Therefore, they were wrong in determining the sample and its occurrence.

Students also had many difficulties in completing the prerequisite concepts such as completing Q4 related to the venn diagram. Students mistakenly described the venn diagram as a result of errors in members of each set as shown below.

$$
\text{Jawab:}\; a.\; 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13
$$

**Figure 4.** The error in describing member of set.

In Figure 4 shows that students did not comprehend how to describe the set in a venn diagram. The error was based on the students’ inadequate knowledge of about set theory. While, set theory is one of the important prerequisite concept in learning the probability in order to facilitate students to solve problems. Based on the data analysis, it can be concluded the difficulties of learning probability concept reveals into some points as follows.

First difficulty is in describing the sample space. It is characterized by the error of students in uncovering random trials of any given problems and the inability of students in connecting random trials conducted with the sample space formed. The error was caused by the learning process and the teaching materials used. In some teaching materials used, the concept of random experiments is not presented well and there is also sometimes a misrepresentation. Likewise, it happens with concept of sample space. In addition, the basic concept did not involve students in the concept construction process.

Second difficulty is the one in constructing the event space. That is marked by the errors of students in understanding the concept of sample space and the event space. Students were not aware that the
event space is a subset of the sample space. Students mostly assumed that the way in determining the members of the sample space and the event space is different so there is no corresponding relationship between the sample space and the event space. Third is the difficulty in understanding the principles of events. These difficulties are characterized by the inability of students in solving problems related to the principle of certain events such as incidents of mutual release or the occurrence of mutual freedom.

Fourth is the difficulty in making mathematical models related to the concept of probability. This is marked by the fallacy of students considering the mathematical model to be related only to the concept of a linear program. This error is due to the inadequate knowledge of students in defining mathematical models. Students often think that the mathematical model applies only to certain concepts but probability. Fifth is difficulty related to the concepts of prerequisite. This difficulty is characterized by the errors of students in understanding the concept of sets and numbers. In addition, students are still incorrect in the selection of rules enumeration, such as misrepresentation in the occurrence of permutations and combinations. Students made errors in making venn diagrams that can make students understand the probability of multiple events. In addition, students mistakenly mentioned certain numbers such as prime numbers. Then, students were also mistaken in choosing the correct enumeration rules for a particular occurrence.

4. Conclusion

Based on the results, student cognitive level most are on level 2, which indicate that participants had some evidence of the use of probability principles and appropriate quantitative information is proved but they may be incomplete or are incorrectly used. The difficulties which students had are the ones in describing the sample space, in constructing the event forms of an experiment, in making mathematical models, in understanding the principles of events, and in understanding the prerequisite concepts.

References

[1] Wahyudin 2008 Pembelajaran dan Model-model Pembelajaran: Pelengkap untuk Meningkatkan Kompetensi Pedagogis Para Guru dan Calon Guru Profesional (Bandung: FPMIPA UPI)
[2] Borovcnik MG and Kapadia R 2010 Research and developments in probability education internationally. InProceedings of the British Congress for Mathematics Education pp. 41-48
[3] Batanero C and Sanchez E 2005 What is the Nature of High School Students’ Conceptions and Misconceptions About Probability? InExploring probability in school 2005 241-266 (Boston MA: Springer)
[4] Yuniarti T 2014 Desain Didaktis Teori Peluang SMA Journal Science Education
[5] Li J and Mendoza L 2002 Misconceptions in Probability ICOTS 6
[6] Saputri M E 2015 Pengaruh Pembelajaran Peer Lesson terhadap Kecemasan Matematika dan Peningkatan Kemampuan Pemecahan Masalah serta Representasi Matematis Siswa SMA (Bandung: Universitas Pendidikan Indonesia)
[7] Gyantra R 2015 Perbandingan Kemampuan Representasi dan Pemecahan Masalah Matematika antara Siswa yang Mendapat Pembelajaran Berbasis Masalah dengan Siswa yang Mendapat Pembelajaran Penemuan Terbimbing (Bandung: Thesis in Universitas Pendidikan Indonesia)
[8] Garfield J and Ahlgren A 1986 Difficulties in Learning Probability and Statistics ICOTS 2
[9] Brousseau G 2002 Theory of didactical Situation in Mathematics (New York: Kluwer Academic Publishers)
[10] Cornu B 2002 Advanced Mathematical Thinking (Eds) (New York: Kluwer Academic Publishers)
[11] Paul M and Hlanganipai N 2014 The nature of misconceptions and cognitive obstacles faced by secondary school mathematics students in understanding probability: A case study of selected Polokwane Secondary Schools Mediterranean Journal of Social Sciences 5 8 446

Acknowledgments

We thank students and teacher for their help and participation.