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1-1-2005

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Hong, Qi; Wu, Thomas X.; Zhu, Xinyu; Lu, Ruibo; and Wu, Shin-Tson, "Designs of wide-view and broadband circular polarizers" (2005). Faculty Bibliography 2000s. 5276.
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Designs of wide-view and broadband circular polarizers

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Abstract: A novel methodology for designing wide view circular polarizers is proposed. Both single wavelength and broadband wide-view circular polarizers are discussed. Over the ±85° viewing cone, the light leakage from the crossed circular polarizers is less than 2.87×10^{-4} using the proposed single wavelength circular polarizers (λ=550 nm) and less than 1.7×10^{-3} using the proposed broadband circular polarizer (λ=450~650 nm). An example of using the designed broadband, wide-view circular polarizers for enhancing the optical efficiency of a direct-view liquid crystal display is elucidated.

OCIS codes: (260.5430) Polarization; (260.3720) Liquid crystal devices

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1. Introduction

Circular polarizer is an important optical component with many useful applications, such as optical communications, optical remote sensors, and liquid crystal displays (LCDs) [1-4]. Two methods have been commonly used to generate a circularly polarized light: Bragg reflection using a cholesteric liquid crystal (CLC) film and a linear polarizer laminated with a quarter-wave film. In the former approach, a right-handed CLC film would reflect the right-handed circularly polarized light and transmit the left-handed component. A drawback is that blue shift occurs at oblique angles [1, 5]. In the latter approach a quarter-wave film is laminated to a linear polarizer [6]. In the normal incidence, a very good circular polarization is produced. However, at oblique angles the produced state of polarization becomes elliptical resulting in light leakage through the crossed circular polarizers. Wide-view circular polarizers using biaxial retardation films have been proposed for improving the light efficiency of LCDs [2-4]. However, the reported contrast ratio is limited to ~10:1 at 60° viewing cone because of the still large light leakage. For direct-view LCDs, broad bandwidth is as important as wide viewing angle [1-4, 7-11].

Phase compensation methods have been widely applied in LCDs for reducing the dark state light leakage and thus increasing the contrast ratio at wide viewing angles [2-4, 7-11]. To obtain a wide-view circular polarizer, a straightforward approach is to combine a wide-view linear polarizer [7-11] with a wide-view quarter-wave film [2-4]. However, this approach is difficult to obtain a pure circular polarization state, especially at a large incident angle. In this paper, we apply the phase compensation methods to develop wide-view circular polarizers for both single wavelength and broadband white light. The produced state of polarization is very close to the ideal circular state of polarization over a wide range of incident angles. Over the entire ±85° viewing cone, after reducing the air-interface surface reflection, the light leakage from the crossed single-wavelength circular polarizers is <2.87×10⁻⁴ at λ=550 nm and <1.7×10⁻³ over the 450~650 nm spectrum for the crossed broadband circular polarizers. This device is particularly useful for enhancing the optical efficiency of direct-view LCDs.

2. Stokes parameters

The state of polarization can be represented by Stokes parameters \((S_1, S_2, \text{ and } S_3)\) and plotted on Poincaré sphere [6] after the parallel and perpendicular components of the electric field are solved using the 4-by-4 matrix method [12]. If the state of polarization is represented by vector \(P = (S_1, S_2, S_3)\), then the polarization difference between two states of polarization \(P_{(1)}\) and \(P_{(2)}\) can be described by

\[
\Delta P_{(1)-(2)} = \sqrt{(S_{1,(1)} - S_{1,(2)})^2 + (S_{2,(1)} - S_{2,(2)})^2 + (S_{3,(1)} - S_{3,(2)})^2},
\]

where \(S_{1,(1)}, S_{2,(1)}, S_{3,(1)}\), \(S_{1,(2)}, S_{2,(2)}, \text{ and } S_{3,(2)}\) are the Stokes parameters of \(P_{(1)}\) and \(P_{(2)}\), respectively. \(P_{(LCP)} = (0, 0, 1)\) denotes the left-handed circular polarization and \(P_{(RCP)} = (0, 0, -1)\) gives the right-handed circular polarization. \(S_3\) equals to zero for the linear polarization and \(|S_3|\) is neither zero nor one for the elliptical polarization. Since \(S_1, S_2\) and \(S_3\) satisfy the relationship that \(S_1^2 + S_2^2 + S_3^2 = 1\), Eq. (1) can be simplified so that the polarization difference between \(P_{(X)}\) and \(P_{(RCP)}\) is related to the \(S_3\) of \(P_{(X)}\) by

\[
\Delta P_{(X)-(RCP)} = P_{(X)} - P_{(RCP)} = \sqrt{2(1 + S_{3,(X)})},
\]

where \(P_{(X)} = (S_{1,(X)}, S_{2,(X)}, S_{3,(X)})\). Once \(S_{3,(X)}\) descents to \(-1\), \(\Delta P_{(X)-(RCP)}\) approaches zero and
$P_X$ becomes $P_{(RCP)}$.

In this paper, the linear polarizer is modeled as a lossy uniaxial material. The employed refractive indices of the polarizer, positive birefringence uniaxial A-plate and C-plate, and negative birefringence uniaxial A-plate and C-plate are as follows: $n_{e,pol} = 1.5 + i \times 3.251 \times 10^{-3}$, $n_{o,pol} = 1.5 + i \times 2.86 \times 10^{-5}$, $n_{e,p,A-plate} = 1.5124$, $n_{o,p,A-plate} = 1.5089$, $n_{e,p,C-plate} = 1.5124$, $n_{o,p,C-plate} = 1.5089$. $n_{e,n,A-plate} = 1.5089$, $n_{o,n,A-plate} = 1.5124$, $n_{e,n,C-plate} = 1.5089$, and $n_{o,n,C-plate} = 1.5124$. The thickness of the polarizer is 210 $\mu$m. The A-plate and C-plate with negative $d\Delta n$ can be realized by negative birefringence A-plate and C-plate. We assume the color dispersions of linear polarizer, A-plate, and C-plate are negligible. On both sides of the absorptive polarizer, the protective Tri-Acetyl-Cellulose (TAC) films exhibit a small birefringence and act as negative birefringence C-plates. The phase change due to the TAC film can be minimized if we laminate a positive birefringence C-plate to the exit protective film. The phase retardation of this C-plate compensates for the adjacent protective film so that the C-plate effect of the linear polarizer is negligible.

3. Single-wavelength wide-view circular polarizers

A conventional circular polarizer consists of a linear polarizer and a quarter-wave plate. The quarter-wave plate is laminated on the light emerging side of the linear polarizer and its slow axis is oriented at 45° with respect to the absorption direction of the polarizer. At normal incidence, the light emerging from the linear polarizer sustains $\pi/2$ phase change from the quarter-wave plate so that it becomes circularly polarized light. However, at oblique angles, the phase change contributed by the $\lambda/4$ plate is different from $\pi/2$ [2-4] so that the produced polarization state becomes elliptical as Fig. 1(a) illustrates.

![Fig. 1.](image)

(a) State of polarization produced by a conventional circular polarizer. The red lines show the states of polarization for $\theta = 0°$ to $85°$ at each fixed $\phi$, where $\phi = 0° ~ 360°$ with 10° interval. (b) $S_3$ of the produced state of polarization at different view angles. $S_3 = -1$ at normal incidence angle and reaches its maximum of $-0.829$ at $\theta = 85°$, $\phi = 130°$ and $310°$. In both figures, $\lambda = 550$ nm.

In Fig. 1(a), the $S_3$ of the produced polarization increases from $-1$ to $-0.829$ ($\Delta P_{(RCP)}$ = $0.585$) when the incident angle $\theta$ increases from $0°$ to $85°$ at $\phi = 130°$ and $320°$. Figure 1(b) plots the variation in the produced $S_3$ with respect to the incident angle $\theta$ and the azimuth of incident plane $\phi$. The variation in $S_3$ is relatively small when the incident angle is within $30°$. Above $45°$, $S_3$ increases drastically. The peaks of $S_3$ occur at $\phi = 40°$, $130°$, $220°$, and $320°$.

If a pair of crossed circular polarizers is constructed as Fig. 2(a) depicts, the analyzer and the second quarter-wave plate form a crossed circular polarizer. Figure 2(b) plots the iso-transmittance contour of light leakage. Although the light leakage is almost zero at normal viewing direction, it increases to 0.098 at $\theta = 85°$ because of the resulted elliptical polarization. The light leakage...
is the strongest at near bisectors ($\phi_i = 40^\circ, 130^\circ, 220^\circ, \text{and} 320^\circ$) since the produced $S_3$ peaks at these angles, as depicted in Fig. 1(b).

During simulations, an ideal anti-reflection (AR) film is assumed in order to reduce the interference of the air-polarizer surface reflection. The ten-layer anti-reflection film is coated on the air interface of both polarizers. This AR film is designed using genetic algorithm [13] and the gradient refractive indices profile is illustrated in Fig. 3(a). The origin represents the air-AR interface. The transmittance of this ten-layer AR film is greater than 0.97 over the $\pm 85^\circ$ incident cone for $\lambda = 450 \sim 650$ nm as Fig. 3(b) illustrates.

To produce circular state of polarization at a large incidence angle, we laminate one uniaxial C-plate to the quarter-wave plate as Fig. 4 drafts. This positive birefringence C-plate contributes phase retardation at oblique angles [1] so that the produced polarization is closer to an ideal circular polarization, while the normal incidence angle performance of conventional circular polarizer is not compromised.

Figure 5(a) uses Poincaré sphere to demonstrate how the C-plate with $d\Delta n = 59.9$ nm reduces the $S_3$ of the produced polarization to $-0.952$ at $\theta_i = 85^\circ$. Although the state of polarization emerging from the quarter-wave plate is deviated from an ideal circular polarization, the C-plate reduces the difference by modifying the transmitted $S_2$ and $S_3$. 
Fig. 5. (a) States of polarization inside a wide-view circular polarizer when $d\Delta n$ of C-plate equals to 59.9 nm, where $\theta = 85^\circ$, $\phi = 130^\circ$, and $\lambda = 550$nm. (b) Variations in the produced $S_3$ with respect to the $d\Delta n$ of C-plate when $\theta = 85^\circ$. The configuration of this circular polarizer is shown in Fig. 4.

To find the $d\Delta n$ of this C-plate for minimizing $S_3$ over the $\pm 85^\circ$ viewing cone, Fig. 5(b) illustrates that the produced $S_3$ decreases to its minimum when the $d\Delta n$ of the C-plate is gradually increased from 0 to 59.9 nm. Further increasing the $d\Delta n$ of the C-plate increases the produced $S_3$. By exhaustive search we can find when the $d\Delta n$ of the C-plate equals 59.90 nm, the $S_3$ of the produced state of polarization is less than $-0.952$ ($\Delta P_{(550\mu m+1C)} \leq 0.31$) over the entire $\pm 85^\circ$ viewing cone as Fig. 6(a) shows. Due to this additional C-plate, the produced $S_3$ remains $-1$ at normal incidence and slowly increases to $-0.952$ as the viewing angle increases to $85^\circ$, which is significantly reduced in contrast to a conventional circular polarizer. This decreases the light leakage of the crossed circular polarizers to 0.027 over the $\pm 85^\circ$ viewing cone, as demonstrates in Fig. 6(b). The peaks of light leakage shift to $\phi = 55^\circ$, $145^\circ$, $235^\circ$, and $325^\circ$ due to the presence of the C-plate. Ten-layer ideal anti-reflection film in Fig. 3(a) is assumed and coated on the air interface of both polarizers.

Since the C-plate does not change the $S_1$ of the polarization state [8], the produced $S_3$ remains as high as $-0.952$ at $85^\circ$ viewing angle and cannot be further reduced as we observed in Figs. 5 and 6. Whereas all of the three Stokes parameters are changed inside an A-plate [8], to further improve the viewing angle performance we could laminate an extra A-plate to the circular polarizer shown in Fig. 4 and obtain a new design shown in Fig. 7. The C-plate is laminated between two A-plates. Over wide incident angles, the combination of these A-
plates and C-plate is expected to be equivalent to the quarter-wave plate in the conventional circular polarizer at normal incidence. Due to the presence of the additional A-plate, the azimuthal angle and the $d\Delta n$ of the quarter-wave plate must be redesigned so that the produced state of polarization remains circular at normal incidence. The azimuthal angles of both A-plates as well as the $d\Delta n$ of all A-plates and C-plate are the subject of design. Using genetic algorithm [13], by minimizing the cost function

$$\cos t = \max \left\{ \left[ \frac{2}{N} \left( S_{1, (2A+1C)} + 1 \right) \right] \left( \theta = 0^\circ \sim 85^\circ, \phi = 0^\circ \sim 360^\circ \right) \right\},$$  

where $S_{1, (2A+1C)}$ is the $S_1$ of the produced state of polarization $P_{2A+1C}$, we obtain the parameters of the phase retardation films. For this design, the azimuthal angles of A-plates are: $\phi_{\text{A, 1st}} = 72.36^\circ$ and $\phi_{\text{A, 2nd}} = 36.84^\circ$; the $d\Delta n$ of all retardation films are: $d\Delta n_{\text{A, 1st}} = 89.77$ nm, $d\Delta n_{\text{A, 2nd}} = 89.77$ nm, and $d\Delta n_{\text{C}} = 106.08$ nm.

Inside this circular polarizer, all of the $S_1$, $S_2$ and $S_3$ are modified so that the compensations between the retardation films are further improved as Fig. 8(a) demonstrates. The two A-plates not only reduce $S_3$ to $-1$ but also reduce the transmitted $S_1$ and $S_2$ to zero. On the other hand, the C-plate tempers the transmitted state of polarization to further reduce the viewing angle sensitivity. Therefore, the produced $S_3$ is only slightly increased to $-0.991$ when the viewing angle increases to $85^\circ$ as depicted in Fig. 8(b). This is equivalent to having the polarization difference $\Delta P_{(2A+1C)-(RCP)}$ less than 0.134 over the $\pm 85^\circ$ viewing cone. The produced $S_3$ remains at $-1$ at normal angle as in the conventional right-handed circular polarizer. For the left-handed circular polarizer using the configuration shown in Fig. 7, the $d\Delta n$ of all A-plates and C-plate are not changed but the azimuthal angles of the A-plates are the negative of their counterparts in the right-handed circular polarizer.

From above discussions, an extra phase retardation film gives an extra degree of freedom to improve the viewing angle of a circular polarizer. By laminating an additional A-plate and a C-plate to the above circular polarizer as Fig. 9 depicts, the polarization difference between
the produced polarization and the desired circular polarization can be further reduced. In this configuration, A-plates are interlaced with C-plates. By minimizing the cost $\sqrt{2(\Delta P_{3(A+C)} + 1)}$ over the $\pm 85^\circ$ viewing cone using genetic algorithm [13], we obtain the design parameters of A-plates and C-plates, where $\Delta P_{3(A+C)}$ is the $S_3$ of the produced state of polarization $P_{3(A+C)}$. From this design, the azimuthal angles of A-plates are: $\phi_{\text{ne}_A,1\text{st}} = 78.55^\circ$, $\phi_{\text{ne}_A,2\text{nd}} = -28.71^\circ$, and $\phi_{\text{ne}_A,3\text{rd}} = 42.46^\circ$; the $d\Delta n$ of all retardation films are: $d\Delta n_{A,1\text{st}} = 75.69$ nm, $d\Delta n_{A,2\text{nd}} = 24.30$ nm, $d\Delta n_{A,3\text{rd}} = 128.76$ nm, $d\Delta n_{C,1\text{st}} = 106.56$ nm, and $d\Delta n_{C,2\text{nd}} = -21.08$ nm.

The additional A-plate and C-plate significantly reduce the viewing angle sensitivity of the circular polarizer because of the extra compensations between retardation films. As illustrated in Fig. 10(a), all of the $S_1$, $S_2$ and $S_3$ are subtly modified in the retardation films so that the produced $S_3$ remains less than $-0.999$ ($\Delta P_{3(A+C)-(RCP)} \leq 0.045$) over the entire $\pm 85^\circ$ viewing cone, which can be seen in Fig. 10(b). Variation in the produced $S_3$ is further reduced so that the produced polarization is nearly circular at any incident angle within $\pm 85^\circ$.

Since the produced polarization approaches the ideal circular polarization, the light leakage of the crossed circular polarizers is less than $2.87 \times 10^{-4}$ over the $\pm 85^\circ$ viewing cone as Fig. 11(a) shows. Although the light leakage is more pronounced at $\phi_{\text{i}} \approx 10^\circ$, $100^\circ$, $190^\circ$, and $280^\circ$, it is still less than $1.72 \times 10^{-2}$ at other azimuthal angles when the incident angle is within $\pm 85^\circ$. As compared to the case of using conventional circular polarizer, as shown in Fig. 2(b), our results are significantly improved despite the increased cost.

Figure 11(b) depicts the configuration of the crossed circular polarizers. The polarizer and the first three A-plates together with the first two C-plates form a wide-view right-handed circular polarizer. The analyzer and the last three A-plates together with the last two C-plates form a second circular polarizer crossed to the first one. The arrangement of the A-plates and C-plates are reversed for the crossed polarizers so that the state of polarization emerging from
the last A-plate is linear along the absorption direction of the analyzer, thus the light leakage is small. The ideal anti-reflection film in Fig. 3(a) is assumed and coated on the air interface of both polarizers. For a left-handed circular polarizer using the configuration in Fig. 9, the $d\Delta n$ of all A-plates and C-plates are not changed but the azimuthal angles of the A-plates are the negative of their counterparts in the right-handed circular polarizer.

![Fig. 11](image)

**Fig. 11.** Crossed wide-view circular polarizers: (a) iso-transmittance contour showing the light leakage at $\lambda = 550$ nm; (b) device configuration. The ten-layer anti-reflection film is assumed.

4. Broadband wide-view circular polarizers

In the above designs, the produced states of polarization are very close to the ideal circular polarization over a wide viewing cone, however, only at a single wavelength. As the incident wavelength deviates from the designed one, the phase retardations of the A-plates and C-plates will walk off from the designed values. As a result, the produced polarization state is no longer circular as Fig. 12 demonstrates. In Fig. 12, the conventional broadband circular polarizer (red line) is indeed quite insensitive to the wavelength in the 450-650 nm spectral range, but only at normal incident angle. All the other three designs (black, blue and green curves) based on a single wavelength are rather sensitive to the wavelength. In this section we will focus on the designs of broadband and wide-view circular polarizers.

![Fig. 12](image)

**Fig. 12.** The calculated $S_3$ as a function of wavelength for the four types of circular polarizers, as described in the insert. The viewing cone is ±85° for the proposed wide-view circular polarizers, and the viewing angle is 0° for the conventional circular polarizers.

A commonly used broadband circular polarizer is comprised of laminating a half-wave plate between the linear polarizer and the quarter-wave plate as illustrated in Fig. 13(a) [14]. When the azimuthal angles of the half-wave plate and the quarter-wave plate satisfy the following relationship
the produced state of polarization is very close to the ideal circular polarization over a broad spectrum at normal incidence as observed from Fig. 12. However, at oblique angles, the relationship in Eq. (4) is no longer satisfied on the wave plane and the phase retardations of the half-wave plate and the quarter-wave plate are changed. Thus, the produced polarization is no longer circular and varies significantly with the incident spectrum [2-4]. As Fig. 14 shows, the produced $S_3$ from a conventional broadband circular polarizer (black curve) is larger than $-0.5$ at $85^\circ$ incident angle over the spectrum of $450 \sim 650$ nm. The red line in Fig. 14 shows the produced $S_3$ from a broadband wide-view circular polarizer that we are going to discuss.

$$2\theta_{\sigma_{\lambda/2}} - 4\phi_{\sigma_{\lambda/2}} = 90^\circ,$$

(4)

![Fig. 13. (a) Configuration of a conventional broadband circular polarizer with one linear polarizer, one half-wave plate and one quarter-wave plate. The azimuthal angle of the half-wave plate is $75^\circ$ with respect to the absorption axis of the polarizer and the azimuthal angle of the quarter-wave plate is $15^\circ$. (b) Device configuration of a wide-view broadband circular polarizer with one linear polarizer, five uniaxial A-plates and three uniaxial C-plates.](image1)

![Fig. 14. The calculated maximum $S_3$ over the $\pm 85^\circ$ viewing cone as a function of wavelength for the four types of circular polarizers, as described in the insert.](image2)

The above designs of single-wavelength circular polarizer show that replacing the quarter-wave plate in the conventional circular polarizer with the combination of A-plates and C-plates significantly reduces the viewing angle sensitivity of the produced state of polarization. Likewise, if both half-wave plate and quarter-wave plate in the conventional broadband circular polarizer can be replaced by multi-layer equivalent plates, then the resulted circular polarizer would be broadband and wide-view. In this case, over wide viewing angle and broad spectrum, the multi-layer equivalent plates should produce similar states of polarization as their single-layer counterparts do at normal incidence.

To design the multi-layer equivalent half-wave plate, we first derive the states of polarization $P_{\lambda/2}$ emerging from the half-wave plate at $\theta = 0^\circ$ and $\phi = 0^\circ \sim 360^\circ$ for $\lambda = 450 \sim 650$ nm. Then we use the combination of two A-plates and one C-plate to replace the half-
wave plate as Fig. 13(b) depicts. Next, by using genetic algorithm to minimize the cost

$$\text{cost} = \max \left\{ \Delta P_{(2A+1C, \lambda/2)} \left| \theta = 0^\circ \sim 85^\circ, \phi = 0^\circ \sim 360^\circ, \lambda = 450 \text{nm} \sim 550 \text{nm} \right. \right\}, \quad (5)$$

we find the phase retardation film parameters, where $\Delta P_{(2A+1C, \lambda/2)}$ is the polarization difference between the state of polarization $P_{(2A+1C, \lambda/2)}$ emerging from the equivalent $\lambda/2$ plate and the state of polarization $P_{\lambda/2}$ emerging from the single-layer half-wave plate. For this multi-layer equivalent half-wave plate, the azimuthal angles of A-plates are: $\phi_{\text{ne}, A, 1\text{st}} = -61.45^\circ, \phi_{\text{ne}, A, 2\text{nd}} = -5.05^\circ$; the $d\Delta n$ of retardation films are: $d\Delta n_{A, 1\text{st}} = -72.95 \text{ nm}, d\Delta n_{A, 2\text{nd}} = -201.90 \text{ nm},$ and $d\Delta n_{C, 1\text{st}} = -103.38 \text{ nm}$. Using this design, $\Delta P_{(2A+1C, \lambda/2)} < 0.217$ over the entire $\pm 85^\circ$ viewing cone in the 450 ~ 650 nm spectrum.

With the multi-layer equivalent half-wave plate, the quarter-wave plate in the conventional broadband circular polarizer is replaced by the combination of three A-plates and two C-plates as Fig. 13(b) sketches. By minimizing the cost

$$\text{cost} = \max \left\{ 2S_{3,(3A+2C, \lambda/4)} + 1 \right\} \left| \theta = 0^\circ \sim 85^\circ, \phi = 0^\circ \sim 360^\circ, \lambda = 450 \text{nm} \sim 550 \text{nm} \right. \right\}, \quad (6)$$

using genetic algorithm we have the design of the equivalent quarter-wave plate, where $S_{3,(3A+2C, \lambda/4)}$ is the $S_3$ of the state of polarization $P_{(3A+2C, \lambda/4)}$ emerging from the multi-layer equivalent quarter-wave plate. For this multi-layer equivalent quarter-wave plate, the azimuthal angles of A-plates are: $\phi_{\text{ne}, A, 3\text{rd}} = 37.61^\circ, \phi_{\text{ne}, A, 4\text{th}} = 2.40^\circ,$ and $\phi_{\text{ne}, A, 5\text{th}} = -46.97^\circ$; the $d\Delta n$ of retardation films are: $d\Delta n_{A, 3\text{rd}} = 90.71 \text{ nm}, d\Delta n_{A, 4\text{th}} = 74.71 \text{ nm}, d\Delta n_{A, 5\text{th}} = -33.68 \text{ nm}, d\Delta n_{C, 2\text{nd}} = 48.09 \text{ nm},$ and $d\Delta n_{C, 3\text{rd}} = -6.04 \text{ nm}.$

Unlike above single wavelength circular polarizers, the variation in the produced state of polarization over a broad spectrum is similar to the conventional broadband circular polarizer at normal incidence angle. This can be seen in Fig. 14, in which the red line shows that, over the $\pm 85^\circ$ viewing cone, the produced $S_3$ is less than $-0.963$ at $\lambda = 450 \text{ nm}$ and decreases to $-0.995$ at $\lambda = 530 \sim 650 \text{ nm}$. The wavelength sensitivity is reduced by satisfying the relationship in Eq. (4) and the viewing angle sensitivity is reduced by the above multi-layer equivalent plates. Thus the light leakage of the crossed circular polarizers is suppressed below $1.7 \times 10^{-3}$ over the $\pm 85^\circ$ viewing cone within the 450~650 nm spectral range, as Fig. 15 depicts.

In Fig. 15, with the broadband wide-view circular polarizer, the light leakage of the crossed circular polarizers is not only kept below $1.7 \times 10^{-3}$ in the visual spectrum but also less than $3.79 \times 10^{-3}$ at $\lambda = 550 \text{ nm}$. This is preferred in the liquid crystal displays since human visual system is more sensitive to the green light so that the green color requires a higher contrast ratio. In contrast, using single wavelength circular polarizers the light leakage

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Fig. 15. The calculated maximum light leakage from three-types crossed circular polarizers over the $\pm 85^\circ$ viewing cone as a function of wavelength. The ten-layer anti-reflection film is assumed.
increases dramatically when the incident spectrum deviates from the designed wavelength.

To form the crossed broadband circular polarizer, the polarizer and the first five A-plates together with the first three C-plates compose a right-handed circular polarizer as Fig. 13(b) sketches. The analyzer and the other five A-plates together with the other three C-plates form the second circular polarizer. The arrangement of the A-plates and C-plates laminating to the analyzer are in reverse order and the azimuthal angles of the A-plates are at ninety degree with respect to their counter parts laminating to the polarizer. The ideal anti-reflection film in Fig. 3(a) is assumed and coated on the air interface of both polarizers. For a left-handed circular polarizer using the configuration in Fig. 13(b), the $d\Delta n$ of all A-plates and C-plate are not changed but the azimuthal angles of the A-plates are the negative of their counterparts in the right-handed circular polarizer.

5. Applications

A pair of crossed polarizers is of key components in many transmissive mode LCDs [1]. If an LC cell is laminated between two crossed linear polarizers, to achieve maximum transmittance in the bright state the LC directors should be reoriented to the bisectors of the crossed linear polarizers [1-4, 15]. Vertical alignment (VA) has been used in many LCDs because of its excellent contrast ratio. In a VA-LCD, in order to have uniform image quality over all the azimuthal angles, four domains are formed along the bisectors of the crossed linear polarizers [15, 16]. However, due to the continuity the LC directors twist continuously from domain to domain so that the boundary areas are formed between domains [1-4, 15]. These boundary areas become dark areas under crossed linear polarizers so that the transmittance of the whole pixel is reduced. Nevertheless, under crossed circular polarizers, the transmittance of LCD only depends on the phase retardation ($\delta$) of the LC layer:

$$T = \sin^2(\delta/2).$$  \hspace{1cm} \hspace{1cm} (7)

Hence, the azimuthal angles of the LC directors are not necessary to be at the bisectors. As a result, the use of circular polarizers greatly enhances the bright state transmittance [2-4].

However, the light leakage from the crossed conventional circular polarizer is large at wide viewing angles so that the contrast ratio of VA-LCDs would be low [2-4]. To improve the light efficiency without sacrificing the contrast ratio at wide viewing angles, we can apply above designed wide-view circular polarizers to the multi-domain VA-LCDs. Figure 16(a) drafts the simplified schematic diagram of such a wide-view VA-LCD using crossed wide-view circular polarizers. Broadband wide-view circular polarizer is applied to cover the entire visual spectrum. The LC director distributions are simplified into eight domains at every 45° from 22.5° to 337.5° in the bright state as Fig. 16(b) sketches.

![Fig. 16. (a) Configuration of a high-contrast wide-view VA-LCD with crossed circular polarizers. For this design, the light entering the VA LC layer is circularly polarized light at normal viewing angle. (b) In the bright state, eight domains of LC director distributions are formed at every 45° from 22.5° to 337.5° with respect to the absorption direction of the polarizer.](image-url)
On each side of LC layer, five A-plates and three C-plates are laminated to the adjacent linear polarizer to form a broadband wide-view circular polarizer as illustrated in Fig. 13(b). The arrangement of the A-plates and C-plates are reversed on two sides and the azimuthal angles of the A-plates are at ninety degree with respect to their counter parts on the other side. Since the vertical alignment LC layer is not considered in the design of above-mentioned wide-view circular polarizers, to compensate the phase retardation of the LC layer in the dark state, two C-plates with equal thickness are laminated on both sides of the LC layer. The summation of the phase retardations of these two C-plates is the negative of the phase retardation of the LC layer. We should emphasize that, in the dark state the light at the center of the LC layer is circularly polarized at wide viewing angles.

Fig. 17. A VA-LCD using crossed broadband wide-view circular polarizers when LC directors form eight domains in the bright state: (a) iso-transmittance contour at $\lambda = 450$ nm; (b) iso-contrast contour at $\lambda = 450$ nm; (c) iso-transmittance contour at $\lambda = 550$ nm; (d) iso-contrast contour at $\lambda = 550$ nm; (e) iso-transmittance contour at $\lambda = 650$ nm; (f) iso-contrast contour at $\lambda = 650$ nm. The LCD configuration is sketched in Fig. 16.
We use the finite difference method to simulate the bright state LC director distributions [17, 18] and then use the 4-by-4 matrix method [12] to calculate the transmittance. The employed refractive indices of the polarizers and LC are as follows: $n_{e, pol} = 1.5 + i \times 3.251 \times 10^{-3}$ and $n_{o, pol} = 1.5 + i \times 2.86 \times 10^{-5}$, $n_{e, LC} = 1.5514$ and $n_{o, LC} = 1.4737$ at $\lambda = 550$ nm. Here the color dispersion is assumed to be weak and not considered. The thickness of the polarizer is 210 μm and LC cell gap is 4 μm. We also assume the backlight is uniform within the ±85° viewing cone. The color filters are not considered during calculations. To reduce air-pol larizer surface reflection, an ideal anti-reflection film in Fig. 3(a) is assumed and coated on the air interface of both polarizers.

Figure 17 depicts the optical characteristics of this LCD at $\lambda = 450, 550$ and 650 nm. The maximum transmittance is higher than 0.34 (c.f. maximum transmittance is 0.37) at normal viewing angle for the green and blue light, but for red it is ~0.30. Over the entire ±85° viewing cone, the minimum bright state transmittance remains ~68% and ~90% of the maximum transmittance for the green and red light, respectively. Further more, in all cases the transmittance is uniform over all the azimuth angles. Among these three colors, the green light has the highest contrast ratio, which is greater than 420:1 over the ±85° viewing cone. Although the contrast ratio is lower for the red light, it is still higher than 115:1 over all the viewing angles. By using the proposed broadband wide-view circular polarizer, the contrast ratio of the multi-domain VA-LCD is greatly improved while the high light efficiency is maintained, and furthermore, high angular uniformity is achieved.

In a real display panel, the actual contrast ratio could be lowered because the above-mentioned ideal parameters may not be controlled precisely. Moreover, the lower extinction ratio of linear polarizer, imperfect LC alignment, variation and non-uniformity of the compensation film thickness, color dispersions of optical components, as well as the stress birefringence from films and substrates could also reduce the contrast ratio. Other than the above reasons, the actual angular brightness uniformity could also be lowered because of the non-ideal anti-reflection film. At the same time, some LC directors around the domains’ boundaries are not reoriented in the bright state because of the discontinuities between LC directors in different domains [2-4, 15]. This decreases the actual bright state transmittance. Color filters in a real display panel further reduce the actual bright state transmittance.

6. Discussion

Design tolerance is an important concern for display manufacturing. For the design of a single-wavelength circular polarizer shown in Fig. 9, we calculate the maximum light leakage of the crossed circular polarizers over the ±85° viewing cone if the $d\Delta n$ of the A-plates or C-plates varies by ±5%. As depicted in Fig. 18(a), while the light leakage is insensitive to the errors in the second C-plate, a −5% error in the first C-plate increases the light leakage to

![Fig. 18. Design tolerance of the wide-view single wavelength circular polarizer shown in Fig. 9: (a) variations in the \(d\Delta n\) of A-plates and C-plates; (b) variations in the azimuthal angles of A-plates. The viewing cone is ±85° and \(\lambda = 550\) nm. Ten-layer anti-reflection film is assumed.](image-url)
Figure 18(b) depicts the maximum light leakage if the orientations of A-plates vary by ±5%. The light leakage rises to $1.61 \times 10^{-3}$ with a +5% error in the first A-plate. However, the light leakage is almost invariant when the orientation of the second A-plate varies by ±5%. Thus, for this design, the accuracy in the first A-plate and the first C-plate are more critical.

For the broadband circular polarizer shown in Fig. 13(b), we calculate the maximum light leakage of the crossed circular polarizers if the $d\Delta n$ of the A-plates or C-plates varies by ±5%. Results are plotted in Fig. 19(a). Although the light leakage is increased to $1.21 \times 10^{-3}$ with a +5% error in the first two A-plates and increased to $1.49 \times 10^{-3}$ with a −5% error in the first C-plate, it is less than $5.88 \times 10^{-4}$ in all other cases. Figure 19(b) shows the maximum light leakage if the orientations of the A-plates are deviated by ±5%. Although a +5% error in the first A-plate increases the light leakage to $1.49 \times 10^{-3}$, the light leakage is almost invariant with ±5% errors in the second and the third A-plats. The first two A-plates together with the first C-plate compose the equivalent half-wave plate and the other A-plates and C-plates compose the equivalent quarter-wave plate. This circular polarizer is more sensitive to the errors in the equivalent half-wave plate but insensitive to the errors in the equivalent quarter-wave plate. Comparing Figs. 18 and 19, in the least favorable case, the maximum light leakage is still less than $1.61 \times 10^{-3}$ and $1.49 \times 10^{-3}$ for the single-wavelength and broadband circular polarizers, respectively.

**7. Conclusion**

We demonstrate a novel methodology for designing wide-view circular polarizers. Both single wavelength and broadband circular polarizers are discussed. We use phase compensation techniques to reduce the difference between the produced state of polarization and the desired circular state of polarization over a wide range of viewing angle. The phase retardation film parameters are designed using genetic algorithm. The light leakage from the crossed circular polarizers using the proposed single-wavelength circular polarizer is less than $2.87 \times 10^{-4}$ over the ±85° viewing cone at $\lambda=550$ nm. Using the proposed broadband circular polarizer, the light leakage is predicted to be less than $1.7 \times 10^{-5}$ over the ±85° viewing cone for the visual white light and it is lower than $3.79 \times 10^{-4}$ at $\lambda=550$ nm. To highlight its potential applications, we apply the wide-view circular polarizer to a multi-domain VA LCDs. The maximum transmittance is predicted to be greater than 90% and the contrast ratio is higher than 420:1 for the green light. Over the entire visual spectrum the maximum transmittance is greater than 81% and the contrast ratio is higher than 115:1. The uniformity of better than 68% in the bright state transmittance is achievable over the ±85° viewing cone.

The authors are indebted to Toppoly Optoelectronics for the financial supports.