Original Article

Optical Back-Propagation for Nonlinear Compensation in OFDM-Based Long Range-Passive Optical Networks

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Abstract: In direct-detection optical OFDM system, the nonlinear impairment is the key factor that limits the system performance. The back-propagation techniques in digital and optical domains have been proposed to compensate the nonlinear effects, however they can be unsuitable for long-range passive optical networks (LR-PONs) due to their implementation at receiver. In this study, we propose an optical back propagation (OBP) approach for compensation of the nonlinear and dispersion distortions in direct-detection optical OFDM system. The proposed OBP using split-step Fourier method is implemented at transmitter that is suitable for high-rate OFDM-based LR-PONs applications. In this OBP, the fiber Bragg grating (FBG) is used as a step for dispersion compensation and the high-nonlinear fiber (HNLF) with a short length is used as a step for nonlinear compensation. The performance improvement based on our proposed approach has been demonstrated via Monte-Carlo simulations of the 100 Gbit/s direct-detection optical OFDM system with 80 km of standard single mode fiber link. The influence of optical conjugation process and launching conditions has been investigated. The obtained results show that the proposed OBP can improve remarkably the performance of system with the launched power range from -2 dBm to 6 dBm.

Keywords: OFDM, direct detection, optical transmission, nonlinear compensation, optical back propagation.

1. Introduction

The orthogonal frequency division multiplexing (OFDM) has become the promising solution of long-range passive optical networks (LR-PONs) due to its high spectral efficiency and high chromatic
dispersion tolerance. OFDM-based PONs can be easily compatible with the recent electrical wire/wireless networks such as DABs, DVBs, 4G/5G mobile networks, ... [1]. Moreover, by splitting the high data rate channel into several subcarriers with smaller bandwidth and separated by small guard-band offers multiple advantages in comparison with using single carrier. It has lower requirements in terms of optical signal-to-noise ratio (OSNR), analog-to-digital/ digital-to-analog converters (AD/DAC) bandwidth and narrow optical filter [2].

The OFDM is a cost-effective and practical technique that can be applied in the next generation PONs. However, the nonlinear impairment is one of the main drawbacks to limit the performance of OFDM-based LR-PONs. Several nonlinearity compensation techniques proposed recently have dealt with the nonlinear effects. The solution for PAPR suppression of OFDM signal based on companding algorithms can improve remarkably the systems’ BER performance [3, 4]. The digital back propagation (DBP) implemented at the receiver by solving the inverse nonlinear Schrodinger equation (NLSE) can compensate perfectly both dispersion and nonlinear effects of the systems [5]. These techniques are off-line signal processing methods that has a trade-off between their complexity and performance. The mid-span spectrum inversion (MSSI) method based on the principle of optical phase conjugation (OPC) compensates the fiber transmission impairments in optical domain [6]. By placing the OPC in the middle of the link, all the accumulated spectral phase distortions arisen in the first half of fiber link are reversed in the second half of it. The optical back propagation (OBP) technique, proposed by Kumar et al. [7], is implemented by backward propagation in optical domain. In the receiver site, the linear compensation is realized by using dispersion compensation fibers (DCFs) and nonlinear compensation is realized by using high nonlinear fibers (HNLFs). These all-optical methods perform the good improvement in the systems’ BER performance but their position is not suitable for PONs, whose ODNs and ONUs need to be cost-effective and simple design.

In this paper, we propose and demonstrate a new model of optical back-propagation technique that is located at the OLTs of OFDM-based LR PONs. This OBP consists of HNLFs for nonlinear compensation, fiber Bragg gratings (FBGs) for dispersion compensation and an OPC for conjugating the signal. The results show that there is optimum launched power range where the performance of the system at very high bitrate of 100 Gbit/s using 64 QAM is minimum when using the OBP.

The rest of this paper is organized as follows. Section 2 describes the proposed method in detail. Simulation results are discussed in section 3. Finally, section 4 concludes this paper.

2. Proposed method

2.1. Optical back propagation at transmitter

Back-propagation method performs a reversed propagation in either digital or optical domain to recover the signals that are impaired by dispersion and nonlinear distortions. However these methods including DBP and OBP are often implemented at the receiver that can be unsuitable for LR-PONs applications. In a LR-PON where an OLT delivers the signal to many ONUs, the impairments occur more in downlink due to its higher rates, therefore the implementation at receiver of each ONU becomes infeasible in practice. By using real photonic devices, OBP handles with the computational complexity and less-flexible configuration which are the cons of DBP [7-10]. Although, the OBP in the receiver site provides a good performance improvement, it also causes ONUs of the LR-PONs to become expensive and complicated.

In this study, we propose a new optical back-propagation approach in which the OBP is implemented at transmitter. In other words, the OBP can be located in the OLTs instead of ONUs as
shown in Fig. 1. This makes the LR-PON implementation cost effective and feasible. Thus, the OBP in proposed approach plays a role as pre-compensation in optical domain. The optical signal propagates through the OBP section, then it is phase conjugated before propagating in the fiber transmission link.

The signal propagation in optical fiber can be described by the nonlinear Schrodinger equation (NLSE) that is given as [11]

$$\frac{\partial U}{\partial z} = -\frac{\alpha}{2} U - i \beta_2 \frac{\partial^2 U}{\partial t^2} + iy |U|^2 U$$

(1)

where $U(t, z)$ is the optical field envelope, $\alpha$, $\beta_2$ and $\gamma$ are the loss, dispersion and nonlinear coefficients of the transmission medium, respectively. This equation can be rewritten in a reduced form as

$$\frac{\partial U}{\partial z} = (\hat{D} + \hat{N}) U$$

(2)

where $\hat{D}$ is the linear operator and $\hat{N}$ is the nonlinear operator. These operators are changed into the lossless form as follow

$$\hat{D} = -\frac{\beta_2}{2} \frac{\partial^2}{\partial t^2}, \quad \hat{N} = \gamma |U(t, z)|^2$$

(3)

For the OBP section with the total nonlinear length $L_{OBP}$, the output signal is derived from (2) as

$$U(t, L_{OBP}) = M U(t, 0)$$

(4)

where $M$ is considered as the propagation operator and it is given by

$$M = \exp \left\{ i \int_0^{L_{OBP}} [\hat{D} + \hat{N}] dz \right\}$$

(5)

Then, the signal after the OPC becomes

$$U_{OPC}(t) = U^* (t, L_{OBP}) = M^* U^* (t, 0)$$

(6)

and

$$M^* = \exp \left\{ -i \int_0^{L_{OBP}} [\hat{D} + \hat{N}] dz \right\}$$

(7)

Figure 1. Typical architecture of LR-PON using OBP at the OLT location. Details of the OBP module is shown in lower section.
Next, the output of OPC propagates through a transmission fiber link with length of $L$, dispersion coefficient $\beta'_2$, nonlinear coefficient $\gamma'$ and the attenuation coefficient $\alpha'$. The output signal of this the transmission fiber is

\[
U_{\text{out}}(t) = M'U^*(t, L_{\text{OBP}})
\]

with $M' = \exp\left\{i \int_0^L \left[ \hat{\beta}' + \hat{N}' \right] dz \right\}$

and $\hat{\beta}' = \frac{\beta'_2}{2} \frac{\partial^2}{\partial t^2}$, $\hat{N}' = \gamma' |U|^2 + i \frac{\alpha'}{2}$

Substitute (6), (7), (9) into (8), we obtain

\[
U_{\text{out}}(t, L) = \exp\left\{i \left[ \int_0^L \hat{\beta}' dz - \int_0^{L_{\text{OBP}}} \hat{\beta} dz \right] \right\}
\]

\[
\times \exp\left\{i \left[ \int_0^L \hat{N}' dz - \int_0^{L_{\text{OBP}}} \hat{N} dz \right] \right\}U^*(t, 0)
\]

Equation (11) shows that the signal can be fully recovered as if dispersion and nonlinear distortions of the transmission link and the OBP are exactly the same. In other words, all fiber impairments can be mitigated by this optical back propagation.

### 2.2. Split-step method in optical domain

The proposed OBP scheme requires all distortions in OBP section are the same as that in the transmission link. In practice, the dispersion and nonlinearity interact together along the propagation medium. The nonlinear and dispersion distortions of the OBP section are based on the split-step method in the optical domain that is similar to the split-step Fourier method for solving NLSE in digital domain [11]. The OBP section is divided into several steps where the dispersion and nonlinear effects are assumed to act independently in each step. In our proposal, the FBG is used as dispersive step because of its advantages including its negligible nonlinearity and insertion loss, very compact size, and dispersion tunability. While the HNLF is used as nonlinear step due to its very low dispersion distortion and negligible loss. Figure 1 shows the schematic of the proposed OBP that consists of steps of HNLF and FBG, an optical phase conjugation (OPC) module, and an Erbium Doped Fiber Amplifier (EDFA). The OPC using the nonlinear waveguide produces the conjugated signal by four-wave mixing (FWM) process to transmit via the transmission fiber section. The EDFA is used to amplify the conjugated signal and control the signal input power of the SMF link.

For nonlinear compensation, the parameters of OBP HNLFs can be computed by nonlinear operators. By comparison $\hat{N}$ with $\hat{N}'$ in the case of ignoring the transmission fiber loss, the nonlinear distortion is perfectly compensated if the nonlinear phase shift of the OBP equals to that of the transmission fiber. The nonlinear phase shift of the OBP is mainly caused by HNLFs and OPC, and can be written as

\[
\Phi_{\text{OBP}} = \sum_{j=1}^{N} \Phi_{\text{HNLF},j} + \Phi_{\text{OPC}}
\]

where the nonlinear phase shift of the $j^{\text{th}}$ HNLF $\Phi_{\text{HNLF},j}$ is

\[
\Phi_{\text{HNLF},j} = \gamma_{\text{HNLF}} P_j L_{\text{HNLF,eff},j}
\]

and $P_j = P_{j-1} e^{-i \alpha_x L_{\text{HNLF,eff},j-1}}$ for $j \geq 2$
where $P_j$ is the launched power, $\gamma_{HNLF}$, $\alpha_{HNLF}$ are the nonlinear and loss coefficients respectively, while $L_{HNLF,j}$, $L_{HNLF,eff,j}$ are the length and the effective length of the $j$th HNLNF, $N$ is the number of steps of the OBP. In the OPC, unfortunately the nonlinear waveguide with the length of $L_{NW}$ also causes a nonlinear phase shift

$$\varphi_{OPC} = \gamma_{NW} P_{OPC} L_{NW,eff}$$

with $P_{OPC} = P e^{-\alpha_{NW} \sum_{j=1}^{N} L_{HNLF,j}}$ (16)

where $\gamma_{NW}$ is the nonlinear coefficient and $L_{NW,eff}$ is the effective length of the nonlinear waveguide. All nonlinear effects mainly arise in the effective length where the optical intensity is high enough, $L_{eff} = \frac{1-e^{-\alpha L}}{\alpha}$ where $L$ is the length and $\alpha$ is the attenuation coefficient of the medium, respectively. With given HNLNF and nonlinear waveguide, the nonlinear phase shift can be controlled by the input power of the OBP. However, the optical power varies in the transmission fiber due to attenuation that creates the difference in power profiles of OBP and fiber link. This asymmetry of optical power reduces the efficiency of OBP in nonlinear compensation.

For linear distortion compensation, the total dispersion of OBP should be equal to the total dispersion of transmission link. Because the dispersion caused by the HNLNF and the nonlinear waveguide is negligible, the FBG is almost responsible for the dispersion of OBP. Hence, the transfer function of the FBGs that can moderate the dispersion distortion of the transmission fiber with the length of $L$ and the dispersion coefficient of $\beta_2$, is defined as follow

$$H_{FBG}(f) = \begin{cases} 
2\pi^2 \left( \frac{\beta_2}{\lambda} \right) f^2 & \text{with } |f| \leq B/2 \\
\pi^2 \left( \frac{\beta_2}{\lambda} \right) f & \text{with } |f| > B/2
\end{cases}$$

(17)

where $B$ is the bandwidth of the FBG. In each step of the OBP, the output of the HNLNF is then fed into the FBG. Using the equations (2), (3) and (4) for the $j$th HNLNF, the output signal of the $j$th HNLNF is $U(f, L_{HNLF,j})$. And the output of the $j$th FBG can be obtained as

$$\tilde{U}_{FBG,j}(f) = \tilde{U}(f, L_{HNLF,j}) e^{j\varphi_{FBG}(f)}$$

(18)

where $\tilde{U}(f)$ is Fourier transform of $U(t)$, the representation of signal in frequency domain.

The OFDM signal after passing through HNLNFs and FBGs is phase-conjugated by the FWM process in the OPC. The conjugated signal propagates along the distributed fiber to the receiver to mitigate all impairments. The conversion efficiency of the conjugated signal is also important factor in optical back propagation. The power of the conjugated OFDM signal after OPC can be given by [12]

$$P_{conf} = \kappa \left( \frac{D_a}{2} \gamma_{NW} L_{NW} \right)^2 P_p^2 P_{OPC}$$

(19)

where $D_a$ is the degeneracy factor which can be 6 for non-degenerate FWM components and 3 for degenerate FWM components, $P_p$ is the pump power launched into the nonlinear waveguide. The factor $\kappa$ is represented for the partial power of FWM component, and $0 < \kappa < 1$. Power of the conjugated OFDM signal depends on the pump power, the power of the signal at the input of the OPC and the nonlinear coefficient of the nonlinear waveguide. Because the nonlinear coefficient of the waveguide is very high, it is necessary to carefully adjust the input signal power to avoid unwanted nonlinear effects.
3. Simulation and results

3.1. Simulation setup

We have developed a MATLAB based simulation model of IM-DD optical OFDM system to investigate the performance of proposed OBP method in LR-PON application. Figure 3 shows the block diagram of this system including three main components: optical transmitter, optical receiver and transmission link. The OBP module as pre-compensation solution is located at the transmitter site. The OFDM signal that consists of 190 data subcarriers and 66 zero-padded subcarriers is generated from the OFDM modulator. It is then optically modulated by a MZM before launching into the OBP. After propagating through the transmission link of 80 km standard single mode fiber (SMF), the optical signal is converted back into the electrical signal at the receiver. Then, the data is recovered by the OFDM demodulator for performance evaluation. The important system parameters and constants used in our simulation are shown in Table 1. The performance improvement of the OFDM-based LR PON using our proposed OBP is evaluated by the Monte-Carlo simulations.

![Block diagram of IM-DD OFDM system using OBP as pre-compensation.](image)

**Table 1. Simulation parameters**

| Name               | Symbol | Value               |
|--------------------|--------|---------------------|
| SMF parameters     |        |                     |
| Attenuation coeff. | $\alpha_{SMF}$ | 0.2 dB/km          |
| Dispersion coeff.  | $D_{SMF}$ | 17 ps/nm.km        |
| Nonlinear coeff.   | $\gamma_{SMF}$ | 1.4 W$^{-1}$.km$^{-1}$ |
| Fiber length       | $L_{SMF}$ | 80 km               |
| HNLF parameters    |        |                     |
| Attenuation coeff. | $\alpha_{HNLF}$ | 0.5 dB/km          |
| Dispersion coeff.  | $D_{HNLF}$ | 1.7 ps/nm.km       |
| Nonlinear coeff.   | $\gamma_{HNLF}$ | 6.9 W$^{-1}$.km$^{-1}$ |
| Fiber length       | $L_{HNLF}$ | 150 m              |
| NW parameters      |        |                     |
| Parameter                      | Value                        |
|-------------------------------|------------------------------|
| Attenuation coefficient       | \( \alpha_{NW} \) 50 dB/m   |
| Dispersion coefficient        | \( D_{NW} \) 28 ps/nm.km    |
| Nonlinear coefficient         | \( \gamma_{NW} \) \(10^4\) W\(^{-1}\).km\(^{-1}\) |
| Waveguide length              | \( L_{NW} \) 7 cm           |

### 3.2. Results and discussion

As mentioned above, the quality of the conjugated signal through FWM process plays an important role in the performance of OBP. The efficiency of FWM process considerably depends on the pump power of the OPC as described in Eq. 19 that influences to the performance of the OBP. Figure 3 shows the performance of nonlinear compensation versus the launched power of the OFDM signal at different pump power levels. In this simulation, the optical power at the input of the OBP is fixed to keep the nonlinear phase shift unchanged. By adjusting the EDFA gain properly, the optical power of the SMF is always constrained in the range from -6 dBm to 14 dBm. As can be seen from the figure, there is an optimum launched power where the compensation efficiency of the OBP is maximum at each pump level. When the launched power of the SMF is small, the system performance is improved when the launched power increases because the linear noises from the LD, EDFA and the photo-detector are dominant in the system. But when the launched power increases high enough, the nonlinear distortion becomes dominant noise of the link that degrades the system performance. The performance is improved when the pump power increases because the conversion efficiency is proportional to the square of pump power. However, there are unwanted components generated by nonlinear mixing processes in OPC besides the desirable signal at higher pump power. As shown in Fig. 4, the performance of the OBP the efficiency is slightly degraded by reducing the conversion efficiency at the pump power of 550 mW. Hence, the best performance can be obtained at the pump power level of 450 mW.

![Figure 3. Block diagram of IM-DD OFDM system using OBP as pre-compensation.](image-url)
The efficiency of FWM process also depends on the power of the signal at the input of the OPC. A suitable adjustment of the input power is therefore required. Figure 4 shows the spectra at the OPC output in case of different input powers with the same pump power of 450 mW. The quality of the conjugated signal is low when the input signal power of the OPC is weak as seen in Fig. 4(a). The quality of the conjugated signal is improved when the input signal power of the OPC increases as shown in Fig. 4(b). However, too high intensity of the input signal causes a strong nonlinear phase shift in the nonlinear waveguide of the OPC that is clearly seen in Fig. 4(c) by broadening of the signal spectrum. Hence, the spectrum of the conjugated signal is also widened that not only reduces the efficiency of nonlinear compensation of the OBP but also adds more nonlinear noise into the signal. Consequently, the system performance can be seriously degraded in this condition.

![Figure 4. Spectra at the output of the OPC at different input power levels: (a) -6 dBm, (b) 10 dBm, (c) 30 dBm.](image)

Figure 5 shows the performance of the OFDM system as a function of the launched power of the SMF in the case of using the OBP with different nonlinear phase shifts. The best performance is only obtained with a proper nonlinear phase shift of the OBP. In other words, the compensation efficiency of the OBP is decayed when the nonlinear phase shift that depends on the input power of the OBP is too high. At the input power levels of lower 19 dBm, the best performance of the system can be kept unchanged. However, the performance of the system begins downgrading when the power of the OBP increases higher than 19 dBm. This performance degradation is caused by the nonlinear phase shift in the HNLFs that exceeds the required nonlinear phase shift in the SMF link. Moreover, the high launched power of the OBP can lead to the high input power of the OPC section due to very low...
insertion loss of the OBP that causes a spectral broadening in the nonlinear waveguide of the OPC as above mentioned.

![Figure 5. BER vs launched optical power of SMF with different nonlinear phase shifts.](image)

In order to demonstrate the performance improvement of the OBP, the system performance is evaluated in different schemes. Figure 6 demonstrates the performance of the OFDM-based IM-DD optical system versus the launched optical power in three cases: non-compensation, dispersion-compensation and full-compensation. There is an obvious improvement of the BER performance of the OFDM-based IM-DD optical system using proposed OBP-based compensation scheme with optimal setting of OPB parameters. In the case of no compensation, the transmission impairments over 80 km of the SMF caused by nonlinear and dispersion effects deteriorate strongly the system performance at very high bitrate of 100 Gbit/s. Even though, the increase in launched optical power has almost no improvement in the performance. In the case of dispersion compensation by using only the FBGs in the OBP, the performance of the system is remarkably improved up to many orders of magnitude compared to the case of non-compensation. In the case of full compensation by using the OBP, the BER curve of the OFDM-based IM-DD optical system is extended to higher launched power region compared with that in the case of dispersion compensation. Particularly, the launched power can be at least 2 dB higher at the BER of lower $10^{-4}$. This obtained result shows the significant role of the HLNFs in nonlinear compensation of the OBP.

![Figure 6. BER vs launched optical power of SMF in different compensation schemes.](image)
4. Conclusions

We have proposed an advanced OBP for compensating the nonlinear and dispersion effects in the optical domain. In this proposal the OBP module is placed at the transmitter side instead of the receiver side that is suitable for the OFDM-based LR-PON applications. This OBP consists of compact components such as HNLFs, FBGs, and the nonlinear waveguide. A simulation model of the OFDM-based IM-DD optical system is setup to investigate the efficiency of the proposed compensation method. The obtained results show that the performance of the system can be considerably improved by properly choosing the parameters of the OBP. The best compensation efficiency of the OBP or the best performance of the system is obtained in the power range of the SMF from -2 dBm to 6 dBm. As a result, the implementation of the OFDM-based LR-PONs with very high bitrate of 100 Gbit/s is feasible in real conditions by using the OBP.

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