THREE-HAIR RELATIONS FOR ROTATING STARS: NONRELATIVISTIC LIMIT

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ABSTRACT

The gravitational field outside of astrophysical black holes is completely described by their mass and spin frequency, as expressed by the no-hair theorems. These theorems assume vacuum spacetimes, and thus they apply only to black holes and not to stars. Despite this, we analytically find that the gravitational potential of arbitrarily rapid, rigidly rotating stars can still be described completely by only their mass, spin angular momentum, and quadrupole moment. Although these results are obtained in the nonrelativistic limit (to leading order in a weak-field expansion of general relativity, GR), they are also consistent with fully relativistic numerical calculations of rotating neutron stars. This description of the gravitational potential outside the source in terms of just three quantities is approximately universal (independent of equation of state). Such universality may be used to break degeneracies in pulsar and future gravitational wave observations to extract more physics and test GR in the strong-field regime.

Key words: equation of state – gravitation – methods: analytical – stars: rotation

Online-only material: color figures

1. INTRODUCTION

Neutron stars (NSs) are excellent laboratories to study extreme, relativistic astrophysics at supra-nuclear densities (Lattimer & Prakash 2007). NS observations can also help us carry out more stringent tests of general relativity (GR) in the strong gravity regime (Will 2006). These observations are currently limited to binary pulsar measurements, but soon gravitational wave (GW) observations of NSs may allow astrophysical and fundamental physics studies (Yunes & Siemens 2013).

To fully exploit NS observations, one needs to understand how the NS’s interior imprints onto observables. The nuclear equation of state (EoS; the relation between the fluid’s state variables, e.g., pressure and energy density) determines the radial density profile and the compressibility of the star. Thus, the EoS connects microscopic physics to macroscopic observables, like the mass and radius.

The EoS also controls the NS’s shape, which is aspherical due to rotation. The stellar shape affects the gravitational field outside the source, which controls various NS observables. The exterior gravitational field can be described through a multipolar decomposition (Backdahl & Herberthson 2005; Backdahl 2007), just as when describing the exterior electromagnetic field of a charged object with multipole moments.

The NS’s multipole moments enter directly into astrophysical observables. Specifically, the \( \ell = 0 \) moment (monopole) corresponds to the mass. The \( \ell = 1 \) moment (mass-current dipole) corresponds to the star’s spin angular momentum, the product of the star’s moment of inertia \( I \) about its spin axis and its spin angular frequency \( \Omega \). The \( \ell = 2 \) moment (mass quadrupole \( Q \)) can be mapped to the quadrupolar deformation of the star (Hartle 1967; Baubock et al. 2013a). NS observations, such as X-ray atomic line profiles from NS surfaces (Cottam et al. 2002), depend on the star’s EoS-dependent multipole moments (Baubock et al. 2013b).

Naively, one may be discouraged by needing to know an infinite number of EoS-dependent multipole moments to describe the gravitational field outside NSs. Recently, however, approximately EoS-independent relations were discovered between certain quantities related to the star’s multipole moments, i.e., between the moment of inertia \( I \), the tidal Love number, and the quadrupole moment \( Q \) of slowly rotating NSs (Yagi & Yunes 2013a, 2013b). Both \( I \) and \( Q \) depend strongly on the EoS when written as functions of NS mass or compactness, but \( I \) as a function of \( Q \) is approximately EoS-independent.

Such approximately EoS-independent relations are of widespread interest to a variety of communities because they describe the gravitational field outside a NS without knowledge of the star’s EoS (Yagi & Yunes 2013a, 2013b). The I-Love-Q relations can be used in X-ray observations of millisecond pulsars (Psaltis et al. 2013), as well as in GW observations of NS mergers (Yagi & Yunes 2013a, 2013b). In the latter, these relations may break degeneracies between the star’s spin angular momentum and its quadrupolar deformation, allowing for better measurements of both, and for EoS independent tests of GR (Kramer et al. 2006; Yagi & Yunes 2013a, 2013b).

Most of these applications require observations of old NSs, which are expected to have moderate to weak magnetic fields and slow rotation. Nonetheless, the universal I-Love-Q relations have recently been extended to large tidal deformations (Maselli et al. 2013), moderate magnetic fields (Haskell et al. 2014), and rapidly rotating stars (Doneva et al. 2014; Pappas & Apostolatos 2014; Chakrabarti et al. 2013). A minor controversy recently arose regarding rotation. We here analytically resolve this controversy to show that the I-Q relations remain almost EoS-independent for rapidly rotating stars, supporting the numerical work of Pappas & Apostolatos (2014) and Chakrabarti et al. (2013). We henceforth focus on weakly magnetized and cold stars, but with arbitrarily rapid, rigid rotation.

The existence of approximately EoS-independent I-Q relations suggests that more general relations may exist for other multipole moments. Such an idea is reminiscent of the black hole (BH) no-hair theorems, which state that their gravitational field outside the horizon is completely described by their mass (the \( \ell = 0 \) mass moment) and spin (the \( \ell = 1 \) mass-current moment; Israel 1967; Hawking 1972). However, the no-hair
2. MULTIPOLe MOments

In GR, a stationary, asymptotically flat, and axisymmetric spacetime outside a source is completely described by its mass and mass–current multipole moments. To leading “Newtonian” order in a weak-field/slow-motion expansion (Blanchet 2006).

Let us introduce a suitable coordinate system adapted to the isodensity approximation \( x' = \tilde{r} \Theta(\cos \theta) n' \), where \( n' = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) is the unit direction vector, with

\[
\Theta(\cos \theta) \equiv \sqrt{\frac{1 - e^2}{1 - e^2(1 - \cos^2 \theta)}},
\]

where \( e = \sqrt{1 - a_1^2/a_e^2} \) is the star’s eccentricity and \( \tilde{r} = \text{const.} \) are isodensity surfaces. The stellar surface, at \( \tilde{r} = a_1 \), is an oblate ellipsoid with semi-major and semi-minor axes \( a_1 \) and \( a_3 \), and geometric mean radius \( R = (a_1 a_3)^{1/3} = a_1 (1 - e^2)^{1/6} \).

Using this coordinate system, the angular and radial integrals in Equations (1) and (2) can be separated into

\[
M_\ell = 2 \pi I_{\ell,5} R_\ell,
\]

\[
S_\ell = \frac{4 \pi \ell}{\ell + 1} \int_0^\pi \int_0^{R_\ell(\theta)} \rho(r, \theta) \mu_\ell(\cos \theta) \sin^3 \theta d \theta d r r^{\ell+3} dr,
\]

where we have used Legendre polynomial identities and defined

\[
R_\ell \equiv \int_0^{a_1} \rho(\tilde{r}) \tilde{r}^{\ell+2} d \tilde{r}, \quad I_{\ell,5} \equiv \int_{-1}^1 \Theta(\mu) e^{i \phi} P_\ell(\mu) d \mu,
\]

with \( \mu = \cos \theta \).

The integrals \( I_{\ell,3} \) and \( I_{\ell,5} \) can be done in closed form. Using Equation (7.226.1) from Gradshteyn et al. (2007) and deriving a related identity, we find

\[
I_{\ell,3} = (-)^{\ell+1} \frac{2}{\ell + 1} \sqrt{1 - e^2} e^\ell,
\]

\[
I_{\ell-1,5} - I_{\ell+1,5} = (-)^{\ell+1} \frac{2 (2 \ell + 1)}{(\ell + 2)} \sqrt{1 - e^2} e^{\ell-1}.
\]

Equations (7) and (8) are only evaluated for even and odd \( \ell \), respectively, so they are both real.

All of the EoS dependence is in \( \Omega(e) \) and \( R_\ell \), with the latter containing the radial density profile. Realistic EoSs may be parameterized by a piecewise collection of polytropes, each of the form \( P = K_\rho^{1+\gamma/\nu} \) (Read et al. 2009; Lattimer & Prakash 2007). For simplicity, we here consider single polytropes, but our results are extendable to piecewise polytropes. Let us transform to dimensionless variables following the Lane–Emden approach (Hansen et al. 2004). Take \( \rho = \rho_\circ (\theta(\xi))' \), where \( \rho_\circ = M/(4 \pi R^3) \xi_1 / |\theta'(\xi_1)| \) is the central density, with \( M \) the stellar mass, \( \theta(\xi) \) a dimensionless function related to density, and \( \xi = (\xi_1 / a_1) \tilde{r} \) a dimensionless radius, such that \( \xi = \xi_1 \) is the stellar surface. The radial integral then becomes

\[
R_\ell = \rho_\circ (a_1 / \xi_1)^{\ell+3} \mathcal{R}_{n,\ell,\theta},
\]

and we used the elliptical isodensity approximation to replace \( \theta \) by its spherically symmetric counterpart \( \theta_{\text{sph}} \), a Lane–Emden function.

Putting it all together, we find

\[
M_{2\ell+2} = (-)^{\ell+1} \frac{e^{2\ell+2}}{2 \ell + 3} \frac{1}{(1 - e^2)^{\ell+2}} \frac{\mathcal{R}_{n,2\ell,\theta}}{\xi_1^{2\ell+1} |\theta'(\xi_1)|} C_{2\ell+2},
\]

\[
S_{2\ell+1} = (-)^\ell \frac{2 \Omega e^{2\ell}}{2 \ell + 3} \frac{1}{(1 - e^2)^{\ell+2}} \frac{\mathcal{R}_{n,2\ell,\theta}}{\xi_1^{2\ell+1} |\theta'(\xi_1)|} C_{2\ell+2}.
\]
These expressions are valid for all rotation periods. The relation between eccentricity and angular frequency is given in the elliptical isodensity approximation by (Lai et al. 1993)

$$\Omega(e) = \frac{3}{2} \frac{e^2}{S_1} \left[ \frac{C|\theta'(\xi_1)|}{(5-n)M^2R_{n,2}} \right]^{1/2} f(e),$$

with

$$f(e) = [-6e^{-2}(1-e^2) + 2e^{-3}(1-e^3)^{3/2}(3-2e^3) \arcsin(e)]^{1/2},$$

where $C = M/R$ is the stellar compactness. In the $n = 0$ and $n = 1$ cases, one can calculate these moments exactly and purely analytically.

3. UNIVERSALITY AND BREAKDOWN

Let us first work with the dimensionless moments

$$\overline{M}_\ell = (-)^{\ell+1} \frac{M_\ell}{M^{\ell+1} \chi^\ell}, \quad \overline{S}_\ell = (-)^{\ell+1} \frac{S_\ell}{M^{\ell+1} \chi^\ell},$$

where $\chi \equiv S_1/M^2$. With this normalization, $\overline{M}_0 = 1$ and $\overline{S}_1 = 1$ always, and BHs have $\overline{M}_{2\ell}^B = 1 = \overline{S}_{2\ell+1}^B$.

Three-hair NS relations can be obtained as follows. First, from Equations (4) and (5), we find

$$\overline{M}_{2\ell+2} = \overline{M}_{2\ell} \overline{S}_{2\ell+1}.$$  

Note that this relation holds independently of the rotation period and the EoS. Second, we eliminate $C$ from Equation (10) by using Equation (11), where $\Omega$ is eliminated by using $S_1$. This gives

$$\overline{M}_{2\ell+2} = \overline{A}_{n,\ell} (\overline{S}_{2\ell+1})^{1+1/\ell} \quad (\ell > 0),$$

which again holds independently of the rotation period, but depends on the EoS through the coefficients

$$\overline{A}_{n,\ell} = \left( \frac{2\ell + 3}{2\ell(1+\ell)} \right)^{1/\ell} \frac{R_{n,2}^{1+1/\ell} S_{n,2+2\ell}^{-1/\ell}}{[\theta'(\xi_1)|\xi_1^2].}$$

These coefficients have a small variability with $n$, as seen in the left panels of Figure 1.

We can now use Equations (15) and (16) to find three-hair relations for all multipoles in terms of the first three:

$$\overline{M}_{2\ell+2} + i \overline{S}_{2\ell+1} = \overline{B}_{n,\ell} \overline{M}_{2\ell+2}^B (\overline{M}_{2\ell} + i \overline{S}_1),$$

where $\overline{B}_{n,\ell} = (\overline{A}_{n,\ell})^{-\ell}$. Note that these relations are independent of the rotation period, which analytically supports the fully numerical results of Pappas & Apostolatos (2014) in GR for the $\ell = 1$ case (see also Figure 2 and its accompanying discussion).

Converting back to dimensional moments through Equation (14), we find

$$M_\ell + i \frac{q}{a} S_\ell = \overline{B}_{n,\ell} \overline{M}_{2\ell+2}^B (M_\ell + i \overline{S}_1)^\ell,$$

where $a \equiv S_1/M$, $q \equiv \sqrt{M_\ell^2 - M_{2\ell}^B}$ and $[x]$ denotes the largest integer not exceeding $x$. Note that although Equation (17) is only valid for $\ell > 0$, the relation in Equation (19) is also valid when $\ell = 0$ and $\ell = 1$ with $\overline{B}_{n,-1} = 1 = \overline{B}_{n,0}$, where the value of $\overline{B}_{n,-1}$ is obtained through the Lane-Emden equation. Equation (19) resembles the BH no-hair relation, $M_{BH}^B + i S_{BH}^B = M(ia)\ell$ (Hansen 1974).

Figure 2 compares our nonrelativistic/weak-field three-hair relations (Equation (18) with $\ell = 1$) to the numerical results in full GR of Pappas & Apostolatos (2014) for an $n = 0.5$ polytropic EoS, as well as to their analytic fit to data for 10 realistic EoSs, with a fitting error of $\mathcal{O}(1\%)$. The results of Pappas & Apostolatos (2014) are obtained by numerically solving the GR equations of structure for NSs. Observe that their relativistic results match our weak-field relation as $C$ decreases ($\overline{M}_{2\ell}$ increases), approximating the weak-field regime. The agreement in the low-compactness regime is better than 3%, which is comparable to the accuracy of the elliptical isodensity approximation to the lowest multipole moment (Lai et al. 1993). Even for very compact stars, as $\overline{M}_{2\ell} \rightarrow 1$, our results differ from the fully relativistic one by roughly 40% at most. Observe also that our single-polytropic results are consistent with the full GR results that use realistic EoSs.
One may believe that the elliptical isodensity approximation reduces the number of degrees of freedom of the stellar models, and this is why the relations between multipoles are approximately EoS-independent. However, Pappas & Apostolatos (2014) found such universality without imposing this approximation, and their results are consistent with ours. The O(1%) EoS-variability found in Pappas & Apostolatos (2014) is consistent with the variation in $A_{n,1}$ in Figure 1.

Let us now take the slow-rotation limit, keeping terms to leading-order in $M\Omega \ll 1$ and $e \ll 1$. Expanding Equations (10)–(12) and eliminating $C$ in favor of $S_{2\ell+1}$, we find

$$ |M_{2\ell+2}| = A_{n,\ell} \left[ \frac{|S_{2\ell+1}|^{5(\ell+1)}}{M^{\ell+1}(M\Omega)^{\ell+1}} \right]^{1/7} \left[ 1 + O(M\Omega^2) \right], \quad (20) $$

with the dimensionless coefficients

$$ A_{n,\ell} = \left\{ \frac{25(5-n)^2}{1152} \right\}^{1/(\ell+1)} \left[ \frac{(2\ell + 3)^3 R_{n,2}^{2\ell+2}}{R_{n,2}^{2\ell+2} \xi_1^{2\ell+4} \theta'(\xi_1)^{2\ell+1}} \right]^{1/7}. \quad (21) $$

These coefficients have even smaller variation with $n$ than $A_{n,\ell}$ (Figure 1). In the slow-rotation limit, $R$ and therefore $C$ are constants independent of $\Omega$, since $R = a_1 + O(M\Omega^2)$. For an $n = 0$ polytrope, Equation (20) reproduces exactly the leading-order, weak-field expansion of the $I$–$Q$ relation of Yagi & Yunes (2013a), obtained without imposing the elliptical isodensity approximation. For an $n = 1$ polytrope, the coefficient of the $I$–$Q$ relation differs by 3%, which is consistent with the validity of the approximation.

Let us now return to the $I$–$Q$ relation for arbitrary spin in order to address the minor controversy between the results of Doneva et al. (2014), Pappas & Apostolatos (2014) and Chakrabarti et al. (2013). Equations (10)–(12) with $\ell = 0$ imply

$$ M_2 = \frac{T^2 e^2}{2 \chi^2}, \quad \chi = \frac{\sqrt{15}}{4 \frac{1}{12}} \frac{T^{1/4}}{A_{n,0}^{1/2}} \frac{f(e)}{(1-e^2)^{1/4}}, \quad (22) $$

where recall that $\chi = S_1/M^2$ and $T = \chi/M\Omega$. Although $\chi$ diverges as $e \to 1$, this is an unphysical limit (ellipsoids become degenerate in that limit), and the currently observed pulsars (Hessels et al. 2006) all have $e \lesssim 0.7$. To find a relation for $M_2$ in terms of $T$ and $\chi$, one needs to invert the expression for $\chi$ to obtain $e(\chi, T, n)$. For any given $\chi$, however, the $M_2$–$T$ relation will depend on the EoS only through $A_{n,0}$, regardless of the magnitude of $\chi$. Since $A_{n,0}$ is approximately EoS-independent, the $M_2$–$T$–$\chi$ relation is as well, confirming the results of Pappas & Apostolatos (2014) and Chakrabarti et al. (2013) in full GR. Moreover, one finds that the $I$–$Q$ relation is approximately EoS-independent both for fixed $\chi$ or fixed $M\Omega$, confirming the results of Chakrabarti et al. (2013), and resolving this controversy.

Finally, although the relations in Equations (15), (16), and (20) hold for arbitrary $\ell$, the decoupling used to obtain Equation (18) breaks for large $\ell$. This is because for large $\ell$, Equation (16) approaches linearity in $S_{2\ell+1}$, and thus the solution becomes degenerate, as shown in Figure 3. Observe how the power-law approaches linearity as $\ell$ increases, and the intersection region with the straight line grows.

4. CONCLUSIONS

We found approximately EoS-independent relations for all weak-field multipole moments of arbitrarily rapidly, rigidly rotating stars in terms of the first three: the stellar mass (monopole), the angular momentum (mass–current dipole), and the quadrupole moment. This universality is found for polytropic EoSs with $n \in (0.5–1.0)$, a range of single-polytrope proxy models which are most like NSs (Lattimer & Prakash 2001; Flanagan & Hinderer 2008). This approximate universality is valid for arbitrary rotation, resolving a minor controversy in favor of the numerical results of Pappas & Apostolatos (2014); Chakrabarti et al. (2013). Our results reproduce and extend the universal relations of (Yagi & Yunes 2013a, 2013b; Pappas & Apostolatos 2014) to an arbitrary multipole number, although they deteriorate as $\ell$ grows.

Our results may point to a deep EoS universality between multipole moments of NSs, which allow a description of their exterior gravitational field in terms of only three numbers. Of course, not all NSs will present such universality, as we have here focused on the subset of old, cold, unmagnetized, and rigidly rotating stars. However, all millisecond pulsar observations today and near-future GW detections are concerned with this subset.

Our work motivates the study of universality among multipole moments in the relativistic regime (Yagi et al. 2014), which has a wide applicability in astrophysics, GWs, and experimental relativity. An observation of the NS mass, rotation period, and moment of inertia would suffice to determine the first 7 moments to within 10% accuracy, irrespective of the NS EoS. In turn, the observations of several low multipole moments can in principle be used to test GR irrespective of the NS EoS. Universal relations between multipole moments may be used to break degeneracies in GW observations, probably with third-generation interferometers such as the Einstein Telescope (Punturo et al. 2010), increasing the accuracy of parameter extraction (Yagi & Yunes 2013a).

Such universal relations may also be used in X-ray observations of NSs by breaking degeneracies in parameter extraction (Baubock et al. 2013a). For example, atomic features from NS surfaces like emission and absorption line profiles (Cottam et al. 2002) have already been used to place constraints
on the EoS (Özel 2006). Baubock et al. (2013b) found that the quadrupole moment significantly affects the X-ray profile. The spin of the fastest-spinning millisecond pulsar J1748+2446ad (Hessels et al. 2006) can be as high as $\chi \sim 0.5$, depending on its mass and the EoS. Given that the hexadecapole moment leads to corrections of roughly $O(\chi^2)$ relative to the quadrupole moment, the former might not be negligible for rapidly rotating NSs. If so, the hexadecapole–quadrupole relation found here should help to reduce the number of parameters and break degeneracies in X-ray observations.

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