Optimal subreflector position determination of shaped dual-reflector antennas based on the parameters iteration approach

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Abstract A new method based on the parameters iteration technique has been developed to determine the optimal subreflector position for shaped Cassegrain antennas, that are distorted by gravity, to improve their electromagnetic (EM) performance. Both the features of shaped surface and the relationship between optical path difference (OPD) and far field beam pattern are employed. By describing the shaped dual-reflector surface as a standard discrete parabola set, we can utilize the optical features of the standard Cassegrain system in the classical OPD relationship. Then, the actual far field beam pattern is expressed as the synthesis of ideal beam and error beam by decomposing subreflector adjustment parameters using a mechanical-electromagnetic-field-coupling-model (MEFCM). Furthermore, a numerical method for determining optimal subreflector position is presented. The proposed method is based on the iteration technique of subreflector adjustment parameters, and the optimal far field pattern is used for the iteration. The numerical solution of optimal adjustment parameters can be obtained rapidly. Results for a 25 m shaped Cassegrain antenna demonstrate that the adjustment of the subreflector to the optimal position as determined by the proposed method can improve the EM performance effectively.

Key words: methods: analytical — methods: numerical — telescope — techniques: radar astronomy

1 INTRODUCTION

Structural deformation in a large reflector antenna is inevitable in operating conditions due to exterior loads such as gravity, temperature and wind, which will result in antenna gain loss and pointing error caused by beam distortions. The mechanical-electromagnetic-field-coupling-model (MEFCM) (Duan & Wang 2009; Lian et al. 2015; Wang et al. 2018) is widely applied to analyze the influence of structural deformation on the electromagnetic (EM) performance of reflector antennas. It can be used to rapidly analyze the influence of different types of errors on a far field beam pattern, such as surface random errors and structural deformation errors (Wang et al. 2007). In MEFCM, a key step is to obtain the aperture field phase error (PE) or optical path difference (OPD), especially for dual-reflector antennas (Ban et al. 2017).

Many researchers have investigated the influence of structural errors on OPD and EM performance of reflector antennas. Duan & Wang (2009) studied the influence of surface random errors and systematic errors on EM performance and established the optimization model for the integrated mechanical-electromagnetic performance. Baars (2007) examined the influence of different types of structural deformation on OPD for primary-focus antennas, such as axial and lateral feed defocus errors. Ruze (1969) explored the influence of different types of structural deformation on the EM performance for dual-reflector antennas and derived the relationship between different types of structural errors and OPD for a dual-reflector, such as feed displacement, subreflector translation, rotation offsets, etc.

To realize high gain and favorable beam pattern for large-aperture and high-frequency reflector antennas, many researchers have focused on the active compensation technique through feed or subreflector adjustment. Song and Zhang (Song et al. 2009; Zhang et al. 2018) proposed a computational model for analyzing the effect of both reflector errors and phase center errors on a far field pattern.
to find the optimal phase center. A method for describing the relationship between far field and aperture field by the aperture field integration method was presented in Lian et al. (2014) and the amount of feed adjustment was determined from the far pattern by the proposed method. In these studies, the researches mainly focused on the correction of the feed position and pose based on deformation of the primary reflector surface. However, for dual reflector antennas, the feeds are difficult to adjust frequently because they are located on the secondary focus near the primary reflector vertex, which is large and heavy. In fact, adjustment of the subreflector position would be an easier way to improve aperture efficiency. For the problem of EM performance degradations caused by primary reflector deformation, the group of approximate paraboloids was used to fit the deformed surface in Wang et al. (2013). The new subreflector position was obtained by utilizing an optimal geometric match to compensate the primary surface deformation. In Doyle (2009), the subreflector or feed was moved to compensate the effect of antenna structural deformation. The common premise in Wang et al. (2013) and Doyle (2009) was information on deformation of the primary reflector in most working conditions. Nevertheless, the environment of antenna work was very complicated, including gravity, thermal and wind loads, and it was not easy to obtain the actual shape of the antenna surface quickly. Generally, the surface deformation of the antenna can be measured by industrial photogrammetry, but the measurement needs to use a crane for assistance and it only works at night, and the operation is difficult and needs a long time to complete the measurement, so the deformed surface shape within the scope of all elevations cannot be obtained in a short time. Therefore, it is desirable to correct the antenna deformation by adjusting the subreflector, which is quick and simple. In this way, the EM performance of the antenna is acquired directly from the receiver and terminal of the radio telescope antenna, and the real-time subreflector position adjustment can be realized based on electromechanical coupling theory to improve the antenna efficiency.

In electromechanical coupling theory, the relationship between antenna structural deformation and OPD should be used in MEFCM to analyze the influence of structural errors on far field beam pattern. However, these relationships (Ruze 1969; Baars 2007; Duan & Wang 2009) are only suitable for standard reflectors, which can be accurately described by a closed-form expression, and they cannot be directly applied for shaped reflectors. In this paper, the shaped surface described by a discrete standard parabola set is adopted and a new method for optimal subreflector position determination based on the iteration technique is proposed, which aims to obtain the optimal EM performance. The optimal parameters of the subreflector position can be calculated rapidly by an efficient numerical iterative algorithm, generating a fast numerical solution. Numerical computation results for a 25 m shaped Cassagrain antenna indicate that the reduction of EM performance caused by gravity deformation can be substantially compensated by adjustment of subreflector position and the adjustment parameters obtained by the iteration approach are effective and appropriate.

2 OPTICAL PATH DIFFERENCE OF A SHAPED DUAL-REFLECTOR ANTENNA

To enable MEFCM to be applied in the analysis of structural deformation influence for shaped reflectors, we present a method for the description of the shaped reflector surface in this paper, which is based on a standard discrete parabola set. The method mainly deals with electromagnetic coupling analysis of shaped dual-reflector antennas. To obtain high aperture efficiency and low sidelobe levels, the reflector shaping design is adopted to obtain the desired aperture field distribution. The shaping design should satisfy three conditions (Milligan 2005): conservation of power, equality of path-length and law of reflection. Shaped reflectors can spread spherical waves into a desired pattern based on geometric optics. A schematic diagram of the shaped Cassagrain dual-reflector antenna is shown in Figure 1, where a spherical wave from the focal point F is changed into an equal phase plane wave from reflection by the primary reflector and subreflector. With the shaping design, generatrices of the primary and secondary reflector are not standard parabola or hyperbola, respectively, and cannot be accurately described by a closed form equation. The shaped surfaces do not satisfy the geometric relationship of classical Cassegrain systems, and thus the classical dual-reflector OPD relationships (Ruze 1969) cannot be directly employed. To apply Ruze’s OPD relationships, we express the shaped dual-reflector surface based on the standard discrete parabola set in this paper.

2.1 Formulation Expression of the Shaped Dual-Reflector

A diagram of the proposed description method for the dual-reflector surface is shown in Figure 2, and the research topic in the paper is the shaped Cassegrain antenna. The point set of the surface generatrix for the shaped primary reflector is written as a discrete set of points from different
2.2 Formulation for Expression of Optical Path Difference

This paper is focused on the influence of antenna structure distortion on OPD and far field beam pattern. Because the errors caused by primary reflector surface distortion and subreflector displacement are generally not large, they belong to the small deformation errors compared with aperture size and focal length. Furthermore, as the displacement errors of reflector surface are always very small, the influence of the errors on the amplitude distribution of the aperture surface can be ignored and only the influences on the phase distribution need to be considered.

\( \delta_p, \delta_s \) and \( \delta_f \) represent OPDs caused by deformation of the primary reflector, offset of subreflector and displacements of the feed, respectively, and \( \delta \) represents their sum. The OPD \( \delta_p \) due to deformation of the primary reflector is expressed as follows

\[
\delta_p(r_i, \phi_i) = c_{pi} \cdot u_{pi},
\]

where \( u_{pi} = (\Delta x_{pi}, \Delta y_{pi}, \Delta z_{pi}) \) is the displacement vector of the primary surface. The components of \( c_{pi} \) are \( c_{pi1} = -2n_{xi}n_{zi}, c_{pi2} = -2n_{yi}n_{zi} \) and \( c_{pi3} = -2n_{zi}^2 \), and \( n_{xi}, n_{yi}, n_{zi} \) signifies the components of the unit normal vector. The OPD \( \delta_f \) of the displacements of the feed is expressed as follows

\[
\delta_f(r_i, \phi_i) = c_{fi} \cdot u_{fi},
\]

where \( u_{fi} = (\Delta x_{fi}, \Delta y_{fi}, \Delta z_{fi}) \) is the displacement vector of the feed. The components of \( c_{fi} \) are \( c_{fi1} = -\sin \theta_i \cos \phi_i, c_{fi2} = -\sin \theta_i \sin \phi_i \) and \( c_{fi3} = 1 - \cos \theta_i \). The OPD \( \delta_s \) for the offset of the subreflector is expressed as follows

\[
\delta_s(r_i, \phi_i) = c_{si} \cdot p,
\]

where \( p = [\Delta x_s, \Delta y_s, \Delta z_s, \Delta \gamma_x, \Delta \gamma_y]^T \) is the offset vector of the subreflector. The components of \( c_{si} \) are \( c_{si1} = (\sin \theta_i - \sin \varphi_i) \cos \phi_i, c_{si2} = -(\sin \theta_i - \sin \varphi_i) \sin \phi_i, c_{si3} = -(\cos \varphi_i + \cos \phi_i), c_{si4} = -(c_i - a_i)(\sin \theta_i + M_i \sin \varphi_i) \sin \phi_i \) and \( c_{si5} = -(c_i - a_i)(\sin \theta_i + M_i \sin \varphi_i) \cos \phi_i \).

3 MECHANICAL-ELECTROMAGNETIC-FIELD-COUPLING-MODEL

In this paper, the aperture field method is adopted to calculate the far field radiation pattern of reflector antennas. This method can obtain the aperture field distribution from radiation field of the feed by geometrical optics. According to the Fourier transform relationship between aperture field...
and far field, the MEFCM of a standard reflector (Fig. 3) is expressed as follows

\[
T_1(\theta', \phi') = \int\int_A F(r, \phi) \exp[jkr \sin \theta' \cos (\phi' - \phi)] \times \exp(j\Delta \varphi_1) r dr d\phi,
\]

and the sum of discrete segments for the shaped surface is written as follows

\[
T_1(\theta', \phi') = \sum_{i=1}^{m} F(r_i, \phi_i) \exp[jkr_i \sin \theta' \cos (\phi' - \phi_i)] \times \exp(jk\delta_i) \Delta s_i,
\]

where \( F(r, \phi) \) is the aperture field distribution function of the standard reflector on the aperture surface \( A \); \((\theta', \phi')\) are coordinates of the observation direction in far field and \((r, \phi)\) are coordinates of the point in aperture plane; \( k \) is the free space wave constant, \( k = 2\pi/\lambda \); \( \lambda \) is the working wavelength. \( \Delta \varphi_1 \) is PE distribution function in the aperture plane and \( \Delta \varphi_1 = k \cdot \delta \) where \( \delta \) is the total aperture OPD. The Gauss integration method can be adopted to solve the MEFCM.

### 4 DETERMINATION OF THE ADJUSTED POSITION OF THE SUBREFLECTOR

#### 4.1 Additional Error Beams

Because the antenna structure deformation is only a small displacement, a low order Taylor series expansion is adopted and Equation (6) can be derived as follows

\[
T_1(\theta', \phi') = \int\int_A F(r, \phi) \exp[jkr \sin \theta' \cos (\phi' - \phi)] \times (1 + j\Delta \varphi_1) r dr d\phi = T_0(\theta', \phi') + T'(\theta', \phi'),
\]

where \( T_0 \) signifies the ideal beam pattern and \( T' \) represents the error beam pattern, and the equations can be expressed as follows

\[
T_0(\theta', \phi') = \int\int_A F(r, \phi) \exp[jkr \sin \theta' \cos (\phi' - \phi)] r dr d\phi,
\]

\[
T'(\theta', \phi') = \int\int_A F(r, \phi) \exp[jkr \sin \theta' \cos (\phi' - \phi)] (j\Delta \varphi_1) r dr d\phi.
\]

According to Equation (8), the beam pattern \( T_1 \) caused by structure deformation could be understood as the synthesis of far field ideal beam \( T_0 \) and error beam \( T' \). For practical antennas, the ideal beam can be calculated by the radiation integral of designed aperture distribution function and expressed as \( \tilde{T}_0 \); \( T_1 \) can be obtained by total power measurement of the strong radiation source, such as an artificial satellite, and expressed as \( \tilde{T}_1 \). Then, Equation (8) can be written as

\[
\tilde{T}_1(\theta', \phi') = \tilde{T}_0(\theta', \phi') + T'(\theta', \phi').
\]

The error beam can be expressed as

\[
T'(\theta', \phi') = \tilde{T}_1(\theta', \phi') - \tilde{T}_0(\theta', \phi').
\]

The subreflector adjustment will cause additional OPD and additional error beam. After the subreflector adjustment, the far field beam pattern can be written as

\[
T_2(\theta', \phi') = \tilde{T}_1(\theta', \phi') + \tilde{T}''(\theta', \phi').
\]

Let the parameter vector of subreflector adjustment be expressed as \( p \). Then, the additional error beam is written as \( \tilde{T}''(\theta', \phi', p) \) and can be calculated by Equation (10). According to Equation (5), the additional PE is expressed as

\[
\Delta \varphi_1 = g(p) = k \cdot c_{si} \cdot p.
\]

### 4.2 Iterative Approach for Subreflector Adjustment Parameters

It is obvious that the EM performance of a perfect reflector is optimal; that is to say, the gain is the maximum and the beam is symmetric, representing an ideal beam pattern. For Equation (13), both sides are subtracted by \( \tilde{T}_0 \), resulting in

\[
T_2(\theta', \phi') - \tilde{T}_0(\theta', \phi') = \tilde{T}_1(\theta', \phi') - \tilde{T}_0(\theta', \phi') + \tilde{T}''(\theta', \phi', p),
\]

and let beam deviation be

\[
L = \tilde{T}_1(\theta', \phi') - \tilde{T}_0(\theta', \phi') + \tilde{T}''(\theta', \phi', p).
\]
Note that Equation (15) can be understood as deviation between beam pattern after subreflector adjustment and ideal beam pattern. Consequently, it is considered that numerical iterative techniques can be adopted to determine the optimal parameter of the subreflector position. Thus, the parameter iteration method can be adopted to improve the EM performance and minimize deviation. The deviation can be expressed as

$$V_i = \| \hat{T}_1(\theta', \phi') - \hat{T}_0(\theta', \phi') + \hat{T}''(\theta', \phi', p) \| , \quad (17)$$

where $\| \cdot \|$ is the operator of the pattern vector. Generally, the infinity norm can be used.

When the antenna points at a radio source at an elevated angle, the backup structure will be distorted due to gravity, and then surface errors in the primary reflector and displacement of the subreflector will be produced, which will lead to degradation of the EM performance of the antenna. A schematic diagram of antenna performance improvement based on iteration of subreflector adjustment parameters is shown in Figure 4. The beam pattern could be gradually improved by repeating the adjustment procedure and progressively iterating the process.

The performance improvement process based on subreflector adjustment and parameters iteration is shown in Figure 5, and can be described as follows:

1. Set the initial value for the iterations as $p_0$;
2. According to Equation (14), the additional OPD and PE $\Delta \phi_1$ can be calculated;
3. According to Equation (10), the additional error beam $\hat{T}''(\theta', \phi', p)$ can be calculated;
(4) According to Equation (16), the beam deviation after subreflector adjustment can be obtained;

(5) According to Equation (17), the iterative objective function $L$ can be calculated and a judgment will be made according to the expression $V_i \leq \varepsilon$, where $\varepsilon$ is the relative boundary. Through a large number of simulations, we think that the $\varepsilon$ may be generally less than or equal to 1 dB.

If the expression cannot be satisfied, the subreflector adjustment parameter vector $p$ will be modified by $p_i = p_{i-1} + \Delta p_{i-1}$, and the process returns to step (2).

If the expression can be satisfied, the iteration process terminates and the current parameter $p$ is the optimal parameter $p^*$. 

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**Fig. 5** Flow chart for iteration procedure.

**Fig. 6** The image of a 25 m shaped Cassegrain antenna.
5 EXAMPLE AND DISCUSSION

An example of a 25 m shaped Cassagrain antenna has been analyzed. In the example, the antenna is used for radio astronomical observation with C band as the main operating wave band (wavelength 6 cm, frequency 4.8 GHz). The 25 m shaped Cassagrain antenna is shown in Figure 6, and a Stewart platform with 6 degrees of freedom (DoF) has been installed in the quadrapod to adjust subreflector position, which is shown in Figure 7. For the Stewart platform, 5 DoF were used primarily to adjust the subreflector in real-time and the adjustment ranges are given in Table 2.
The basic normal parameters of the 25 m shaped Cassegrain antenna are given in Table 3 and the aperture field distribution is shown in Figure 8, which will be substituted into the MEFCM as $F(r, \phi)$.

### 5.1 Shaped Surface and Structural Analysis

The surface of the shaped Cassegrain antenna is described by a discrete standard parabola set and the generatrices of the main reflector and subreflector are both composed of 7604 discrete points, which are obtained by Equations (1)–(2) and shown in Figure 9. Relative parameters of a discrete normal parabola set are derived by expressions given in Table 1, and OPD and far field beam pattern can be computed by Equations (3)–(5) and Equation (7).

To obtain an antenna’s far field beam due to structural deformation, a finite element (FE) model of the antenna reflector’s structure was established, as shown in Figure 10. In addition, the gravitational-structural deformations at elevation angles of 40° and 70° were analyzed. The node displacements of backup structure and quadrupod structure were obtained and then the reflector surface distortion and subreflector rigid displacement were computed by interpolation and coordinate transformation.

### 5.2 Parameter Iteration Results

A numerical computation has been generated based on the theoretical developments presented above to achieve parameters iteration and beam pattern calculation. To rapidly obtain the global optimum solution, a constraint condition is added to the iteration process; that is, the pointing error should be less than 0.1*HPBW (Half Power Beam Width), and the interior-point method is adopted in the iteration process. The numerical computation has been performed based on the FE model. The iteration processes in the cases of elevation angles of 40° and 70° are shown in Figure 11.

In the cases of elevation angles of 40° and 70°, the optimal values are obtained after 107 and 214 iterations, and the optimal parameters are $p^*=[0.5793, -0.2229, -9.9038, -0.0007, 0.0019]$ and $p^*=[0.1223, -5.6443, -3.0728, -0.0019, 0.0002]$, respectively.

The effect of subreflector position adjustment based on optimal parameter $p^*$ is shown in Figure 12. These figures demonstrate that after the subreflector adjustment, the beam pattern is close to the ideal beam pattern. Furthermore, from Figure 12(a) and (c), representing a 0° cut plane (azimuth scan), the beam patterns, after the adjustment, almost coincide with the ideal beam pattern, indicating that reflector gravitational-structural deformations in the azimuth direction have been fully compensated. In Figure 12(b) and (d), representing a 90° cut plane (elevation scan), the shapes of the beam pattern after adjustment approximate the ideal one, but they are just slightly different in terms of the beam center and sidelobes, which indicate that reflector gravitational-structural deformation in the elevation direction has also been well compensated. The OPD distributions in the aperture plane for subreflector adjustment are displayed in Figure 13. It can be seen that the OPD after subreflector adjustment has been decreased significantly.

The results indicate that for the 25 m shaped Cassegrain antenna, the gravitational-structural deformations of the main reflector are almost homologous, that is, the main reflector is almost deformed into an approx-
Fig. 12 Far field beam patterns for subreflector adjustment. (a) 0° cut plane for 40° elevation angle, (b) 90° cut plane for 40° elevation angle, (c) 0° cut plane for 70° elevation angle and (d) 90° cut plane for 70° elevation angle.

Fig. 13 OPD distributions in aperture plane for subreflector adjustment. (a) pre-adjustment for 40° elevation angle, (b) after-adjustment for 40° elevation angle, (c) pre-adjustment for 70° elevation angle and (d) after-adjustment for 70° elevation angle.
imate parabolic shape. A significant loss of beam pattern may occur as a result of subreflector position misalign-ment. Furthermore, the distortion in beam pattern of the 25 m shaped Cassegrain antenna may be substantially com-pensated by suitable adjustments in the position of the sub-reflector and the parameter is effective and appropriate.

6 CONCLUSIONS

A new method to determine the optimal subreflector po-sition for shaped Cassegrain antennas has been presented. This method is based on parameters iteration of subreflec-tor adjustment to compensate the effect of structural de-formation in the reflector. Considering the particularity of the shaped reflector antenna, we accurately describe the shaped dual-reflector surface as a discrete normal parabola set and utilize the features of a classical Cassegrain system in the OPD relationship. By decomposing the subreflec-tor adjusting parameters with MEFCM, such an iteration method for adjusting parameters can be adapted to improve the antenna EM performance, and the optimal adjustment parameters can be rapidly obtained by numerical compu-tation. An example of a 25 m shaped Cassegrain antenna has been provided, and the results indicate that the meth-ods proposed in this paper are effective and can be used in engineering practice.

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