Black hole spin inferred from 3:2 epicyclic resonance model of high-frequency quasi-periodic oscillations

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ABSTRACT

Estimations of black hole spin in the three Galactic microquasars GRS 1915+105, GRO J1655-40, and XTE J1550-564 have been carried out based on spectral and timing X-ray measurements and various theoretical concepts. Among others, a non-linear resonance between axisymmetric epicyclic oscillation modes of an accretion disc around a Kerr black hole has been considered as a model for the observed high-frequency quasi-periodic oscillations (HF QPOs). Estimates of spin predicted by this model have been derived based on the geodesic approximation of the accreted fluid motion. Here we assume accretion flow described by the model of a pressure-supported torus and carry out related corrections to the mass-spin estimates. We find that for dimensionless black hole spin $a = cJ/GM^2 \lesssim 0.9$, the resonant eigenfrequencies are very close to those calculated for the geodesic motion. Their values slightly grow with increasing torus thickness. These findings agree well with results of a previous study carried out in the pseudo-Newtonian approximation. The situation becomes different for $a \gtrsim 0.9$, in which case the resonant eigenfrequencies rapidly decrease as the torus thickness increases. We conclude that the assumed non-geodesic effects shift the lower limit of the spin, implied for the three microquasars by the epicyclic model and independently measured masses, from $a \sim 0.7$ to $a \sim 0.6$. Their consideration furthermore confirms compatibility of the model with the rapid spin of GRS 1915+105 and provides highly testable predictions of the QPO frequencies. Individual sources with a moderate spin ($a \lesssim 0.9$) should exhibit a smaller spread of the measured 3:2 QPO frequencies than sources with a near-extreme spin ($a \sim 1$). This should be further examined using the large amount of high-resolution data expected to become available with the next generation of X-ray instruments, such as the proposed Large Observatory for X-ray Timing (LOFT).

Key words. X-rays: binaries – black hole physics – accretion, accretion disks

1. Introduction

Studying the X-ray spectra and variability provides a useful tool for probing strong-field gravity effects and places constraints on the properties of compact objects, such as the mass or spin of a black hole. One of the commonly accepted ways to measure the black hole spin is related to fitting the X-ray spectral continuum or the relativistically broadened Fe K alpha lines. Using the spectral fitting methods, estimations of black hole spin have been carried out by many authors in the past (see e.g., McClintock et al. 2006; Middleton et al. 2006; Miller et al. 1998; Okazaki et al. 1985; Psaltis et al. 1999; Stella & Vietri 1998a,b, 1999; Titarchuk & Wood 2002; Titarchuk et al. 2004; Yaqoob & Reynolds 2005; Yang et al. 2009). Despite the different involved physical mechanisms, several of the HF QPO models assign the observed frequencies to orbital frequencies that can under some approximations be expressed in terms of Keplerian and epicyclic frequencies of perturbed circular geodesic motion.

One of the early models, for instance, the so-called relativistic-precession model (Stella & Vietri 1998a,b, 1999), identifies the kHz QPO frequencies $\nu_1$ and $\nu_2$ with the Keplerian orbital frequency and the periastron-precession frequency, $\nu_1 = \nu_K$, $\nu_2 = \nu_K - \nu_1$ (where $\nu_1$ denotes the radial epicyclic frequency).

1.1. Orbital models of HF QPOs

A rich variety of models has been proposed to explain the BH HF QPOs. Many of them are associated with motion of matter accreting onto the central compact object (e.g., Kato & Fukue 1985; Alpar & Shaham 1985; Lamb et al. 1985; Okazaki et al. 1987; Nowak & Wagoner 1991; Perez et al. 1997; Klužniak 1998, 1999; Miller et al. 1998, 1999; Psaltis et al. 1999; Stella & Vietri 1998a, 1999; Wagoner 1999; Wagoner et al. 2001; Silbergleit et al. 2001; Abramowicz & Klužniak 2001; Titarchuk & Wood 2002; Rezzolla et al. 2003; Püri 2005; Zhang 2005; Šrámková et al. 2007; Kato 2007; Čadež et al. 2008; Stuchlík et al. 2008; Horák et al. 2009; Mukhopadhyay 2009; Lai et al. 2012; Ortega et al. 2014, and others).
Within the model framework, it is usually assumed that the observed kHz variability originates in a bright localized spot or blob orbiting the black hole at a slightly eccentric orbit. The photons emitted by the spot, which have a high orbital velocity of the order of percents of the speed of light, are then propagated from the strong gravity region towards the observer. Consequently, the detected X-ray flux is periodically modulated by the relativistic effects (see e.g., Schnittman & Bertschinger 2004; Bakala et al. 2014, and references therein).

A different class of models is based on the assumption that the light curve reflects variations in the emission of an accretion disc. A theory of normal modes of accretion disc oscillation, referred to as discoseismology, has been developed in the papers coauthored by Kato and by Wagoner. For instance, Kato & Fukue (1980) consider the $g$-mode of disc oscillation, whose frequency is close to the highest radial epicyclic frequency, while Silbergleit et al. (2001) developed a theory of $c$-mode (disc corugation) whose frequency is related to the difference between the vertical epicyclic frequency and the orbital frequency.

1.1. Resonance models

Klužniak & Abramowicz (2001) have introduced another concept of orbital models. In these models, the relativistic effects acting on the emitted photons are also essential, but the HF QPOs are attributed to a non-linear resonance between two modes of accretion disc oscillations. This well explains the observed preferred frequency ratio, $\nu_r/\nu_o = 3/2$. Since the concept deals with a collective motion of accreted matter, it has a high potential of the observed flux modulation (Bursa et al. 2004; Mazur et al. 2013; Vincent et al. 2014; Bakala et al. 2014).

The framework of resonance models has been extensively developed (Abramowicz & Klužniak 2001; Abramowicz et al. 2003; Klužniak et al. 2004; Horák & Karas 2006; Horák et al. 2009) and miscellaneous combinations of disc oscillation modes have been considered. In several particular cases, the observed frequencies can be expressed in terms of Keplerian and epicyclic frequencies (see Török et al. 2005; Horák 2008; Klužniak 2008, for details and references).

1.2. Spin estimation and epicyclic resonance model

In Kerr geometry, which describes the spacetime around a rotating black hole, the frequencies of the radial and vertical epicyclic oscillations depend on the mass and spin of the black hole. It is therefore possible to determine the BH mass or spin from the observed oscillation disc frequencies (see Török et al. 2005; Horák 2008; Klužniak 2008, for details and references).

In this work, we consider one of the popular versions of resonance models represented by the epicyclic parametric (or internal) resonance model that assumes a 3:2 non-linear resonance between the axisymmetric radial and vertical epicyclic modes of accretion disc oscillations. All the estimates derived so far that were related to the epicyclic resonance model assume a geodesic approximation of the accreted fluid motion. Within this approximation, the two observable resonant frequencies are represented by the radial and vertical epicyclic frequencies, $\nu_r$ and $\nu_o$, of test particle motion. In more general accretion flows, non-geodesic effects connected for instance to pressure gradients, magnetic fields, or other forces may affect the properties of the considered oscillation modes and consequently the inferred spin predictions.

In the following, we explore black hole spin predictions implied by the epicyclic resonance QPO model considering non-geodesic influence that originates in pressure forces present in accretion flow, which is modelled by a small equilibrium-pressure-supported, perfect-fluid torus. Properties of the epicyclic modes of torus oscillations, such as modications to their frequencies due to pressure gradients present in the torus, were calculated by Blaes et al. (2007) in the pseudo-Newtonian approximation and were later generalised by Straub & Šrámková (2009) for Kerr geometry. Using the results of Straub & Šrámková (2009), we apply the modified epicyclic frequencies to calculate the corresponding estimates of black hole spin based on the epicyclic resonance QPO model.

2. Resonant eigenfrequencies and black hole parameters implied by the epicyclic model

The explicit formulae of the epicyclic frequencies in Kerr geometry were first derived by Aliev & Galtsov (1981) and may be written in the Boyer-Lindquist coordinates $t, r, \theta, \phi$ as (e.g., Silbergleit et al. 2001; Török & Stuchlík 2005)

\[ \nu_r^2 = \alpha_r \nu_K^2, \] (1)
\[ \nu_o^2 = \alpha_o \nu_K^2, \] (2)

where

\[ \nu_K = \left( \frac{GM}{r_G^2} \right)^{1/2} \left( x^{3/2} + a \right)^{-1}, \] (3)

\[ \alpha_r (x, a) \equiv 1 - 6 x^{-1} + 8 a x^{-3/2} - 3 a^2 x^{-2}, \] (4)
\[ \alpha_o (x, a) \equiv 1 - 4 a x^{-3/2} + 3 a^2 x^{-2}, \] (5)

\[ x = r/r_G, \quad r_G = GM/c^2. \] (6)

Here $M$ and $a$ denote the mass and the dimensionless angular momentum (spin) of the black hole.

Within the epicyclic resonance (ER) model, it is assumed that the resonant eigenfrequencies are equal to epicyclic frequencies defined at the $3:2$ orbit where we have

\[ \nu_o = 3/2 \nu_r. \] (7)

Furthermore, it is assumed that the eigenfrequencies are equal to the observed upper and lower 3:2 QPO frequencies, $\nu_r$ and $\nu_o$ (e.g., Török et al. 2005). Using this set of formulae and the observed 3:2 HF QPO frequencies along with the independently estimated black hole masses, the intervals of spin predicted by the ER model were previously calculated for the three Galactic microquasars (GRS 1915+105, GRO J1655-40, and XTE J1550-564 – see Török et al. 2005, 2011). The obtained results and relevant properties of the three sources are summarized in Table 1.
2.1. Epicyclic frequencies in a slightly non-slender torus

We assumed a non-geodesic flow that was modelled by an equilibrium, slightly non-slender, pressure-supported perfect-fluid torus. The torus was assumed to have a constant specific angular momentum distribution and orbit a rotating Kerr black hole. We assumed a non-geodesic flow that was modelled by an equi-

\[ \nu^\text{r}_s = \nu_s + \beta^2 C_r(M, r_c, a), \quad C_r(M, r_c, a) < 0, \]

\[ \nu^\text{s}_\theta = \nu_\theta + \beta^2 C_\theta(M, r_c, a), \quad C_\theta(M, r_c, a) < 0, \]  

where \( C_r(M, r_c, a) \) and \( C_\theta(M, r_c, a) \) denote the negative pressure corrections evaluated at the centre of the torus, \( r = r_c \). Explicit forms of expressions for the epicyclic frequencies (8) and (9) are given in formulae (52) and (56) of Straub & Šrámková (2009).

The dimensionless parameter \( \beta \) determines the torus thickness. It is defined as (Blaes 1985)

\[ \beta^2 \equiv \frac{2nc_s^2}{r_c^2 \kappa_U U^2}, \]

2.2. Behaviour of commensurable frequencies for moderately and rapidly rotating Kerr black holes

Using formulae (8) and (9), we investigated the behaviour of epicyclic frequencies in fluid tori when they are in the 3:2 ratio. For Schwarzschild and moderately rotating Kerr black holes, the resonant frequencies behave qualitatively in the same fashion. The resonant radius in this case decreases with growing torus thickness, while the individual epicyclic frequencies defined at this radius increase with growing torus size. This is illustrated in Fig. 3, which for different values of \( a \) and \( \beta \), displays the ratio \( R \) of the vertical to radial epicyclic frequency, and the vertical epicyclic frequency calculated at the resonant radius. Panels a) and c) of Fig. 3 drawn for \( a = 0 \) and \( a = 0.5 \) show that from certain value of \( \beta \), there suddenly appear two different tori whose epicyclic frequencies display the 3:2 commensurability. This behaviour persists as we continue to increase the torus size until at some point the 3:2 resonant radius no longer exists. This phenomenon manifests itself in the form of a small loop in the profile of the resonant vertical epicyclic frequency plotted over \( \beta \) (panels b) and d) of Fig. 3). The same behaviour was found by Blaes et al. (2007), who carried out a similar analysis in the pseudo-Newtonian approximation. We find that for Kerr black holes, two different values of resonant frequency are allowed for the same value of \( \beta \) when \( a \leq 0.86 \).

For higher values of \( a \), the behavioural nature of the resonant frequencies begins to change significantly. The resonant radius
Fig. 2. Epicyclic frequencies calculated at the centre of the torus, \( r = r_c \), plotted for various torus thicknesses (\( \beta \)) and black hole spins \( a \). The curves are drawn solely within the region where the considered value of \( \beta \) does not exceed the critical value corresponding to a torus with a cusp.

Fig. 3. Ratio \( R \) of the vertical to radial epicyclic frequency and the resonant vertical epicyclic frequency \( v_r^* \) calculated at the radius \( x_{3:2} \). The arrows in panels a)–d) indicate the behaviour corresponding to the two resonant radii \( x_{3:2} \). The inner (red) and outer (blue) resonant radius is denoted.

retains its decreasing pattern with growing torus size, but two different tori with the 3:2 commensurability are no longer possible. Rather distinct modifications also appear in the character of the resonant frequency, which at first increases with rising \( \beta \), but at some point starts to decrease rapidly. For a black hole spin approaching \( a \approx 0.89 \), the resonant frequencies implied by the highest allowed values of \( \beta \) become lower than those calculated for the geodesic motion. This is illustrated in panels e) and f) of Fig. 3. For a still higher black hole spin (\( a \gtrsim 0.99 \)), characteristic features of the resonant frequencies continue to change.
Fig. 4. Observational bounds on the quantity $M \times v_\mathcal{E}$ obtained for Galactic microquasars vs. relations predicted by the epicyclic resonance model. The vertical areas denote the observational bounds on $M \times v_\mathcal{E}$ determined for GRS 1915+105, XTE J1550–564, and GRO J1655–40. The areas are drawn assuming the conservative limits from Table 1. The solid black curve shows predictions of the model calculated for the geodesic flow. The shaded region indicates the interval predicted by the model for the family of equilibrium non-slender fluid tori of all thicknesses possible in the particular case. The red curve corresponds to the case of tori with a cusp. The arrow labelled $\Delta a$ indicates the shift of the lower limit on the spin of microquasars implied by considering the non-geodesic flow.

2.3. Comparison of predicted 3:2 epicyclic frequencies with observation

Figure 4 uses the relativistic $1/M$ scaling of the orbital frequencies. It displays the $M \times v_\mathcal{E}(\alpha)$ relation expected from the geodesic epicyclic resonance model marked by the thick black line. The shaded region describes the mass-spin interval predicted by the epicyclic resonance model for a slightly non-slender torus of various thickness. In the figure, we also present the conservative observational bounds on $M \times v_\mathcal{E}$ determined for the three microquasars (see Table 1). These bounds are indicated by the colour-coded areas. Apparently, while the geodesic line enters the observationally determined vertical range of microquasars when $a \geq 0.7$, the non-geodesic line enters this range already for $a \geq 0.6$.

Figure 4 clearly illustrates that for $a \leq 0.9$, the highest possible increase of the predicted resonant frequency due to pressure effects is rather small, reaching about 10%. However, for $a \geq 0.9$, the shaded region in the figure shifts from the area above the $\beta = 0$ curve to the area located mostly below this curve. The strongest possible shift of the resonant frequency is now much greater than it was for $a \leq 0.9$. The coloured curves in

even further. While the resonant radius continues to drop monotonically with growing torus thickness, resonant frequencies are no longer higher than those of free test particles. They now decrease for tori of all sizes ($\beta > 0$), see panels g) and h) of Fig. 3. This is because the nature of epicyclic frequencies for rapidly rotating black holes is such that the decrease of frequencies due to higher $\beta$ overrides the increase implied by lower resonant radius.

3. Discussion and conclusions

Following the previous studies of Blaes et al. (2007), Straub & Šrámková (2009), and Török et al. (2011), we explored here the implications of consideration of pressure forces that originate in a pressure-supported fluid torus for the predictions of black hole spin inferred from the epicyclic resonance QPO model. Our findings for moderately rotating Kerr black holes with $a \lesssim 0.9$ agree well with those carried out in the pseudo-Newtonian study of Blaes et al. (2007). The epicyclic resonance model with a non-geodesic flow predicts the black hole spin to be slightly lower than the spin estimated previously by the same model with a geodesic flow. The lower limit on the spin of Galactic microquasars inferred by the model is then decreased to $a \sim 0.6$. These findings indicate that for up to $a \sim 0.9$, the model has a high predictive power (falsifiability).

Different but very interesting results appear for rapidly rotating black holes with $a \geq 0.9$. In this case, the behaviour of the resonant frequencies begins to markedly change in comparison to the pseudo-Newtonian case. This occurs in such a way that the predicted frequencies can be both higher and lower than those predicted by the geodesic model, with the strongest possible shift being much more significant towards the lower values. Therefore, when the source mass is determined from the 3:2 QPO frequencies, the epicyclic resonance model has a lower predictive power for rapidly rotating black holes than for moderately rotating black holes. On the other hand, our findings confirm the compatibility of the model with the rapid spin of the microquasar GRS 1915+105 (McClintock et al. 2006; Török et al. 2011; Fragos & McClintock 2015), since the expected mass interval almost overlaps with the interval constrained observationally. Furthermore, we provided highly testable predictions of the QPO frequencies. When the torus size ($\beta$) is not determined by some additional physical requirements, a wide range of commensurable frequencies is predicted for near-extreme rotating black holes. Individual sources with a moderate spin ($a \lesssim 0.9$) should therefore exhibit a smaller spread of the measured 3:2 QPO frequencies than sources with a near-extreme spin ($a \sim 1$). At present, there is a rather small amount of data needed for further analysis of this issue. Our predictions, however, can be explored using the large amount of high time-resolution data expected to become available with the next generation of X-ray instruments, such as the proposed Large Observatory for X-ray Timing (LOFT; Feroci et al. 2012).

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According to individual studies based on spectral and timing methods, it would also be possible to include several (vertical) bounds on the spin of microquasars. Within the figure, however, we focus primarily on the epicyclic resonance model of HF QPOs and its predictions. The discussion of various spectral and timing estimates of $a$ can be found for instance in Miller (2007), McClintock et al. (2011, 2014), Steiner et al. (2013), Török et al. (2011), and Kotrllová (2014).

Footnote 2: According to individual studies based on spectral and timing methods, it would also be possible to include several (vertical) bounds on the spin of microquasars. Within the figure, however, we focus primarily on the epicyclic resonance model of HF QPOs and its predictions. The discussion of various spectral and timing estimates of $a$ can be found for instance in Miller (2007), McClintock et al. (2011, 2014), Steiner et al. (2013), Török et al. (2011), and Kotrllová (2014).

Footnote 3: Values of $\beta$ as high as $\beta = 0.5$ considered in panel h) of Fig. 3 may require additional treatment, since these values exceed the limits of full applicability of the adopted approximation. Nevertheless, the illustrated phenomenon is very significant and well apparent already for $\beta \sim 0.3$. 

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