Numerical Study on Aging Dynamics in the 3D Ising Spin-Glass Model. II. Quasi-Equilibrium Regime of Spin Auto-Correlation Function

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Using Monte Carlo simulations, we have studied isothermal aging of three-dimensional Ising spin-glass model focusing on quasi-equilibrium behavior of the spin auto-correlation function. Weak violation of the time translational invariance in the quasi-equilibrium regime is analyzed in terms of effective stiffness for droplet excitations in the presence of domain walls. Within the range of computational time window, we have confirmed that the effective stiffness follows the expected scaling behavior with respect to the characteristic length scales associated with droplet excitations and domain walls, whose growth law has been extracted from our simulated data. Implication of the results are discussed in relation to experimental works on ac susceptibilities.

KEYWORDS: spin glass, slow dynamics, aging, droplet theory

§1. Introduction

In recent years aging dynamics in spin glasses has been extensively studied. A basic experimental protocol is isothermal aging which is relaxation after a spin-glass (SG) system is quenched from a high temperature above the SG temperature $T_c$ to a low temperature below $T_c$. Experimental studies of the relaxation of dc and ac susceptibilities during isothermal aging have revealed remarkable waiting time effects. The experiments have stimulated much theoretical interests and many theoretical ideas have been proposed most of which could account for qualitative features of the aging effects.

Real tests on the theories are possible by investigating quantitative features of the aging effects, especially scaling properties. Progress in this direction, which may shed light on the controversial issues on nature of the SG phase, is still left to be done to a large extent.

The droplet theory in particular the one proposed by Fisher and Huse contains concrete ansatz on possible scaling laws of the aging effects which are well amenable to be tested quantitatively by experiments and numerical simulations. In our previous paper (hereafter referred to as I), we have studied the relaxation of energy of the three-dimensional (3D) Gaussian SG model with nearest neighbor interactions below $T_c$ using Monte Carlo (MC) simulations. We analyzed the data comparing with the droplet theory. A key quantity in the droplet theory is the typical separation between domain walls $R(t)$ which grows irreversibly by isothermal aging. Most of the scaling laws involve this characteristic length. Indeed in I we confirmed that the relaxation of excessive energy per spin $\delta e_T(t)$ at time $t$ with respect to the equilibrium value follows the expected scaling law,

$$\delta e_T(t) \sim \tilde{\Upsilon}(R(t)/L_0)^\theta/(R(t)/L_0)^d,$$

when we used $R(t)$ obtained by our independent simulation (see below). In the above equation $\tilde{\Upsilon}$ and $L_0$ are characteristic energy and length scales respectively and $d$ is the dimension of the space which is 3 here. The energy exponent obtained is $\theta = 0.20 \pm 0.03$ which agrees with the result of the defect energy analysis at $T = 0$.

The simulated $R(t)$ mentioned above was obtained by analyzing the replica-overlap function and found, in agreement with the previous works (but see also [2]), that the growth law of $R(t)$ is well approximated by the form,

$$R(t) \sim L_0(t/\tau_0)^{1/z(T)},$$

where $\tau_0$ is a certain characteristic unit of time. More explicitly, our data of $R(t)$ at each $T$ are well fitted to a function $bt^{1/2}$ with adjustable parameters $b$ and $z$. The latter depends on $T$ and is expressed as

$$1/z(T) \simeq 0.17T,$$

at $T < 0.7$. Here and in the following, the Boltzmann constant $k_B$ is set to 1, and temperature $T$ is measured by the unit of variance of the distribution of the interaction bonds $J$ which is set to 1. As Kisker et al. already pointed out, however, the simulated $R(t)$ can be fitted also to a logarithmic function $R_0 + b[\ln t]^{1/\psi}$ with adjustable parameters $R_0$, $b$ and $\psi$, where $\psi$ is independent of temperature. This form, in the limit $R(t) \gg R_0$, is compatible with the asymptotic form proposed by the droplet theory,

$$R(t) \sim L_0[(T/\Delta)\ln(t/\tau_0)]^{1/\psi},$$

where $\Delta$ is a characteristic unit for energy barriers associated with thermal activation of droplet excitations. $L_0$ and $\tau_0$ in eqs. (1.2) and (1.4) are different sets of characteristic length and time scales.
The above-mentioned results may be summarized as

1) Various quantities (such as $\delta \chi(t)$) simulated the isothermal aging and the characteristic length scales (such as $R(t)$) obtained by the independent simulation satisfy the scaling forms (such as eq.(1.1)) which are predicted by the droplet theory.

2) The growth law of $R(t)$ is well fitted to the power-law form eq.(1.2), but it can be also fitted to a certain logarithmic form which is compatible with the logarithmic growth law eq.(1.4).

Feature 2) suggests that the computational time window available by modern computers is not enough large for us to determine definitely a true asymptotic behavior of $R(t)$ in the limit $t \rightarrow \infty$. Therefore we call the above results seen in the present computational time window the pre-asymptotic behavior of aging. A similar situation was already encountered in the pioneering numerical work by Huse [14] who could confirm the density of domain walls follows the expected scaling form $1/L_0^d(R(t)/L_0)^d$ with reasonable value of the fractal dimension $d$, but with the growth law being in some pre-asymptotic form.

In the present paper [12] we study isothermal aging behavior of the spin auto-correlation function in the so-called quasi-equilibrium regime. Here we focus our study on its scaling properties, or more explicitly, whether the simulated correlation function can be described by a certain scaling form which is derived from the droplet theory, i.e., feature 1) above. To this end, we have performed extensive MC simulations on the 3D Gaussian Ising SG model in the same time window as that in our previous work I.

The spin auto-correlation function is defined as,

$$C(\tau; t_w) = C_i(\tau; t_w), \quad (1.5)$$

with

$$C_i(\tau; t_w) = \langle S_i(\tau + t_w)S_i(t_w) \rangle, \quad (1.6)$$

where $S_i(t)$ is the sign of the Ising spin at site $i$ at time $t$. At time $t = 0$, the system is prepared in a random initial configuration and we let it relax at $T$ below $T_c$ using conventional heat-bath MC dynamics afterwards. Here and in the following time $t$ is measured by a MC step (MCS). By 1 MCS, all Ising spins are updated once. The auto-correlation function over some time intervals $\tau$ at various waiting times $t_w$ are observed during a MC run of isothermal aging. The over-line in eq.(1.3) denotes the averages over sites and different realizations of interactions (samples), and the bracket in eq.(1.6) the average over thermal noises (or different MC runs).

The quasi-equilibrium regime, on which we focus in this study, is the time regime such that the time separation $\tau$ is relatively small compared with the waiting time $t_w$. Although behavior of the correlation function is not extremely different from the equilibrium limit within this regime, it exhibits systematic deviations from the ideal equilibrium behavior, namely, clear waiting time dependence, i.e., weak violation of time translational invariance. It is directly comparable with the measurement of relaxation of the Fourier spectrum of spontaneous magnetic fluctuations [13] in isothermal aging. Note that the Fourier component at a certain frequency $\omega$ is well defined only at large enough waiting times such that $\omega t_w \gg 1$. For the same reason, the ac susceptibility is also considered to measure mainly relaxation of response of the system in this regime. From the point of view of the droplet theory, the quasi-equilibrium regime is defined more precisely as the time regime such that the typical size $L(\tau)$ of droplet excitations which take place in the time scale of $\tau$ is much smaller than the typical separation $R(t_w)$ of domain walls present after waiting time $t_w$. On the other hand, let us recall for clarity that the so-called aging (or off-equilibrium) regime is the time regime such that $\tau$ is as large as or much larger than the waiting time $t_w$. More precisely, the aging regime is defined as the time regime where $R(t = \tau + t_w)$ is larger than $R(t_w)$. The relaxation of the dc susceptibilities, which include the thermoremanent magnetization (TRM) and zero-field cooled susceptibilities (ZFC), can reflect strong non-stationary within the aging regime.

The droplet theory introduced a phenomenological concept called effective stiffness which characterizes reduction of the excitation gap of small scale droplet excitations due to the presence of domain walls. The latter is considered to explain the waiting time effect within the quasi-equilibrium regime. We directly compare our results with this scaling ansatz. As far as we know, no direct quantitative analysis has been performed to test the scaling ansatz except for an experimental study of relaxation of the ac susceptibility in a two-dimensional SG system [18]. The previous numerical simulations have analyzed the non-stationarity of the spin auto-correlation within the aging regime [5, 21, 14], but the analysis of the weak non-stationarity within the quasi-equilibrium regime has not been carried out.

We found that the spin auto-correlation function $C(\tau; t_w)$ in the quasi-equilibrium regime is consistently explained by the droplet theory in the following senses. i) As a function of $L(\tau)/R(t_w)$, $L(\tau)$ being the characteristic length of droplet excitations introduced above, it obeys a scaling law which is consistent with the scenario in terms of the effective stiffness [13]. ii) Result i) is obtained when $L(\tau)$ and $R(t_w)$ are assumed to obey the same growth law. The latter is a fundamental assumption within the droplet theory for spin glasses, while in a simple ferromagnet domain growth and droplet excitations are completely different kinds of processes. iii) We have confirmed result i) explicitly by making use of the growth law of $R(t)$, which is obtained by our simulation and is approximated by the power law of eq.(1.2). Correspondingly, we have used

$$L(\tau) \sim L_0(\tau/\tau_0)^{1/z(T)}, \quad (1.7)$$

for $L(\tau)$. These results add a further support for feature 1) of the aging dynamics mentioned before, though they don’t resolve the ambiguity pointed out as feature 2). We will argue that a similar situation to feature 2) is also encountered in the experimental studies up to now.

The present paper is organized as follows. In the next section we briefly review the scaling ansatz of the
droplet theory. In §3 we present our simulated data on the spin correlation and response functions in isothermal aging. In particular, behavior of $C(\tau; t_w)$ in the quasi-equilibrium regime ($\tau \ll t_w$) is examined in detail and is interpreted by the droplet theory. We discuss implications of the present results with some other numerical and experimental results in §4.

§2. Droplet Picture

Here we review the droplet theory \cite{FH} by Fisher-Huse (FH) concentrating on the influence of domain walls on the droplet excitations during isothermal aging. Let us begin by recalling the ideal equilibrium situation, where there should be no non-stationarity and the behavior of the auto-correlation function defined in eq. \eqref{eq1} becomes only a function of the time difference $\tau$.

Equilibrium thermal fluctuations are considered as droplet excitations \cite{FH} from a ground state, each of which is a global flip of a droplet (cluster) of spins within a distance $L/2$ from a certain given site $i$. The latter can be considered as a simple two-state system with a free-energy excitation gap $F_L(i)$ and a thermal activation time $\tau_L(i)$. The typical value $F_L^{\text{typ}}$ of the gap $F_L(i)$ scales as,

$$F_L^{\text{typ}} \sim \Upsilon(L/L_0)^0,$$

where $\Upsilon$ is the stiffness constant of domain wall on the boundary of the droplet. The thermal activation time $\tau_L(i)$ is related to the energy barrier of the droplet $B_L(i)$ as,

$$B_L(i) = T \ln[\tau_L(i)/\tau_0].$$

The typical energy barrier $B_L^{\text{typ}}$ of the energy barrier scales as,

$$B_L^{\text{typ}} \sim \Delta(L/L_0)^\psi,$$

where exponent $\psi$ satisfies $0 \leq \psi \leq d - 1$.

The probability distribution function $\rho_L(F)$ is assumed to follow a one-parameter scaling form,

$$\rho_L(F) = \frac{1}{F_L^{\text{typ}}} \left( \frac{F}{F_L^{\text{typ}}} \right),$$

where the typical gap $F_L^{\text{typ}}$ is given by eq. \eqref{eq1}. A very important property of the scaling function $\rho(x)$ is that it has finite intensity at $x = 0$, $\rho(0) > 0$, which allows droplet excitations even at very low temperatures. The latter is considered to be responsible for many non-trivial properties of the SG phase in equilibrium \cite{FH}. It is also the case for dynamics in the quasi-equilibrium regime of the present interest. A similar one-parameter scaling form for $\Psi_L(B)$ is assumed as,

$$\Psi_L(B) = \frac{1}{B_L^{\text{typ}}} \left( \frac{B}{B_L^{\text{typ}}} \right)^\eta,$$

where $B_L^{\text{typ}}$ is the typical energy barrier of eq. \eqref{eq2} and $\Psi(x)$ is a scaling function.

The droplet excitations at different scales $L$ which differ by more than factor 2 are considered as approximately independent from each other. Then using the two-state model, the equilibrium behavior of the spin auto-correlation function is obtained as,

$$C(\tau) = \lim_{t_w \to \infty} C_i(\tau; t_w) \sim \prod_{L: \tau_L(i) < \tau} \tan^2 \left( \frac{F_L(i)}{2T} \right),$$

where $\Pi_L \ldots$ represents multiplicative contributions of the relaxations of two-state systems at various scales $L = 2^n L_0$ with $n = 0, 1, 2, \ldots \infty$ enclosing the site $i$. The factor $\tan^2 \left( \frac{F_L(i)}{2T} \right)$ is the energy overlap of the two-state system of energy gap $F_L(i)$ with respect to the lower state (ground state). The constraint $\tau_L(i) < \tau$ is due to the fact that the droplets whose relaxation time is shorter than $\tau$ can relax within the period of $\tau$ while others with $\tau_L(i) > \tau$ cannot relax appreciably and just give multiplicative contributions of 1. At low enough temperatures, the product in eq. \eqref{eq2} can be expanded into a sum and the averages over sites (and equivalently over samples) can be performed using the probability distribution functions of the free-energy gaps and free-energy barriers. One obtains,

$$1 - C(\tau) \sim \int_{L_0}^{\infty} \frac{dL}{L} \int_0^{T\ln(\tau/\tau_0)} dF \rho_L(F) e^{-F/T} \int dF \rho_L(F) e^{-F/T},$$

$$\sim \int_{L_0}^{\infty} \frac{dL}{L} \phi \left( \frac{L}{L(\tau)} \right) \int dF \rho_L(F) e^{-F/T},$$

and the characteristic length $L(\tau)$ defined as,

$$L(\tau) = L_0[(T/\Delta) \ln(\tau/\tau_0)]^{1/\psi}.$$
the presence of domain wall at a typical distance $R$ from each other the free energy gap of a droplet of size $L$ becomes on average and typically as,

$$F_{L,R} = \Upsilon_{\text{eff}}[L/R](L/L_0)^\theta,$$  \hfill (2.10)

with the effective stiffness constant given by,

$$\Upsilon_{\text{eff}}[L/R] = \Upsilon \left(1 - c_v \left(\frac{L}{R}\right)^{d-\theta}\right),$$  \hfill (2.11)

with $c_v$ being a numerical constant. So as the system ages, it appears more and more stiff.

Now we consider the behavior of the spin autocorrelation function in the presence of the domain walls. It can be obtained simply by replacing the typical free-energy excitation gap $F_{L,R}^{\text{typ}}$ given in eq.(2.1) by the reduced one $F_{L,R}^{\text{typ}}$ given in eq.(2.10). Particularly, we assume that the probability distribution of the free-energy gap $F_{L,R}(i)$ in the presence of the domain walls, which we denote as $\rho_{L,R}(F)$, follows the same scaling form as $\rho_L(F)$ described in eq.(2.4),

$$\rho_{L,R}(F) = \frac{1}{F_{L,R}^{\text{typ}}} \tilde{\rho} \left(\frac{F}{F_{L,R}^{\text{typ}}}\right).$$  \hfill (2.12)

Replacing the distribution of the gap $\rho_{L}(F)$ in eq.(2.7) by $\rho_{L,R}(F)$ defined above, we can obtain the modified behavior of the spin auto-correlation function in the quasi-equilibrium regime $L(\tau) \ll R(t_w)$ as,

$$1 - C(\tau; t_w)$$

$$\sim T \int_{L_0}^{\infty} \frac{dL}{L} \Phi \left(\frac{L}{L(\tau)}\right) \rho_{L,R}(0)$$

$$\sim 1 - C_{\text{eq}}(\tau) + \tilde{\rho}(0) \frac{T}{(L(\tau)/L_0)^\theta}$$

$$\times \int_{L_0/L(\tau)}^{\infty} \frac{dy \Phi(y)}{y^\theta} \left(\frac{\Upsilon_{\text{eff}}[yL(\tau)/R(t_w)]}{-1}\right)$$

$$\sim 1 - C_{\text{eq}}(\tau) + \frac{c_1 \tilde{\rho}(0) T}{(L(\tau)/R(t_w))^{d-\theta}} + \ldots,$$  \hfill (2.14)

where the numerical constant $c_1$ is given by,

$$c_1 = c_v \int_0^{\infty} dy y^{2(d-1)-\theta} \Phi(y).$$  \hfill (2.15)

In eq.(2.14), we expanded the effective stiffness given in eq.(2.11) assuming that $L(\tau)/R(t_w)$ is small enough in the quasi-equilibrium regime we are considering here. The last term is the major correction within the quasi-equilibrium regime to the equilibrium behavior described by the first two terms. The lower limit of the integration $L_0/L(\tau)$ is put to 0 assuming $L(\tau)$ is larger enough than $L_0$. Note that the integrand in eq.(2.15) vanishes as $y \to 0$ because of the inequality $(d-1)/2 > \theta$. The integral is finite as far as the function $\Phi(x)$ defined in eq.(2.8) decays fast enough for large $x$.

The second term $C_{\text{eq}}(\tau)$ in eq.(2.14) is the correlation function in equilibrium obtained by FH, i.e., $C(\tau)$ in eq.(2.7),

$$C_{\text{eq}}(\tau) = \lim_{t_w \to \infty} C(\tau; t_w) \sim q_{\text{EA}} + c_2 \frac{T \tilde{\rho}(0) T(L(\tau)/L_0)^\theta}{\Upsilon(L(\tau)/L_0)^\theta},$$  \hfill (2.16)

with $q_{\text{EA}}$ being the EA order parameter given by

$$q_{\text{EA}} \equiv \langle S_i \rangle = 1 - c_0 T \tilde{\rho}(0) \Upsilon,$$  \hfill (2.17)

where $c_0$‘s are numerical constants.

Let us next discuss briefly the relation between the correlation function $C(t, t')$ with $t = \tau + t_w$ and $t' = t_w$ above described and magnetic responses, or susceptibilities, frequently measured in experiments. In the quasi-equilibrium regime, even with weak violation of the time-translational invariance, the fluctuation-dissipation theorem (FDT) in the following form is expected to hold at least approximately between $C(t, t')$ and the response function $G(t, t')$,

$$G(t, t') = \frac{1}{T} \frac{\partial C(t, t')}{\partial t'}. \hfill (2.18)$$

From this equation the out-of-phase component of the ac susceptibility $\chi''(\omega; t_w)$ is evaluated as

$$\chi''(\omega; t_w) = \int_{t_w-\pi/\omega}^{t_w+\pi/\omega} dw \sin(\omega t) \int_0^t dt' G(t, t') \cos(\omega t')$$

$$\simeq \frac{\omega}{2T} \tilde{C}(\omega; t_w), \hfill (2.19)$$

where $\tilde{C}(\omega; t_w)$ is the Fourier component of $C(\tau; t_w)$. The latter is estimated, to a good approximation, as

$$\tilde{C}(\omega; t_w) \simeq -\frac{\pi}{|\omega|} \frac{d}{d\ln \tau} C(\tau; t_w) \bigg|_{\tau = \tau_w}, \hfill (2.20)$$

with $\tau_w = 2\pi/\omega$. Hence we obtain

$$\chi''(\omega; t_w) \simeq -\frac{\pi}{2T} \frac{\partial}{\partial \ln \tau} C(\tau; t_w) \bigg|_{\tau = \tau_w}. \hfill (2.21)$$

In the above derivation we have used the fact that $C(\tau; t_w)$, as a function of $\tau$ and $t_w$, varies quite slowly in a time scale of $\tau_w$. The fact that the FDT of eq.(2.19) holds well in the quasi-equilibrium regime, i.e., for $R(t_w) \gg L(\tau_w)$, has already been tested by the careful experimental study of the spontaneous magnetic fluctuation which was directly compared with the relaxation of the ac susceptibility. The latter experiments have concluded that while there are obvious $t_w$-dependences on both the magnetic fluctuations and ac susceptibility within the quasi-equilibrium regime, eq.(2.19) is satisfied in the regime $\omega t_w \gg 1$ within the accuracy of the experiments.

Substituting eqs.(2.7) and (2.14) into eq.(2.21), we obtain,

$$\chi''(\omega; t_w) - \chi''_{\text{eq}}(\omega) \propto \left(\frac{L(\tau_w)}{R(t_w)}\right)^d, \hfill (2.22)$$

where $\chi''_{\text{eq}}(\omega)$ is the equilibrium ac susceptibility given by

$$\chi''_{\text{eq}}(\omega) \sim \frac{\tilde{\rho}(0)}{\Upsilon(L(\tau_w))} \frac{dL(\tau)/d\ln \tau|_{\tau = \tau_w}}{L(\tau_w)}. \hfill (2.23)$$
The above equation can be put into a simple form suggested by FH using eq.(2.11):

$$\frac{\chi''(\omega; t_w) - \chi''_{\text{eq}}(\omega)}{\chi''_{\text{eq}}(\omega)} = \frac{\Upsilon_{\text{eff}}[L(\tau)/R(t_w)]^{-1} - \Upsilon^{-1}}{\Upsilon^{-1}}.$$  

(2.24)

This formula has been used in the experimental analysis of the relaxation of the ac susceptibility in a 2D SG system.

Let us also note that from the FDT eq.(2.13) we obtain the relation between $C(t,t')$ and the zero-field cooled susceptibility $\chi_{\text{ZFC}}(t; t_w)$ as

$$\chi_{\text{ZFC}}(\tau; t_w) = \int_{t_w}^{\tau+t_w} dt' G(\tau + t_w, t') = \frac{1}{T}[1 - C(\tau; t_w)].$$  

(2.25)

Thus $1 - C(\tau; t_w)$, which we will analyze in detail in the next section, is directly related to $\chi_{\text{ZFC}}(\tau; t_w)$ in the quasi-equilibrium regime. In the aging regime, on the other hand, the FDT no longer holds. The relation between $G(t,t')$ and $C(t,t')$ in this regime has been extensively studied, mostly from the view-point of the mean-field theory but this problem is beyond a scope of the present work.

To summarize, correction to the ideal-equilibrium behavior of the spin auto-correlation function is obtained based on the concept of the effective stiffness proposed by FH. The correction is a function of $L(\tau)/R(t_w)$ and remains small in the quasi-equilibrium regime $L(\tau)/R(t_w) \ll 1$. Here $L(\tau)$ is the typical size of droplet excitations which can flip within the time period $\tau$ and $R(t_w)$ is the typical separation between the domain walls at waiting time $t_w$. The time dependence of $L(\tau)$ is given by eq.(2.9). In the droplet theory by FH the growth law of $R(t_w)$ is assumed to be the same (see eq.(1.4) based on the expectation that domain growth in spin glasses are driven by successive droplet excitations. On the time scale such that $L(\tau) \sim R(t_w)$, the above concept which separates the frozen-in domain walls and droplet excitations should break down, and the system crossovers to the aging regime at $R(t = \tau + t_w) \gg R(t_w)$ where much stronger non-stationarity is expected.

Let us finish the review on the scaling theory with the following remarks. The above scaling form of $C(\tau; t_w)$ written in terms of $R(t_w)$ and $L(\tau)$ is expected to hold more generally than the case described above. Particularly, we expect it to hold for a certain class of the functional form of $L(\tau)$ (and so $R(t_w)$), which is determined by the probability distribution function $\Psi_L(B)$ in the above argument: a different $\Psi_L(B)$ from eq.(2.5) associates with a different $L(\tau)$ from eq.(2.3) which still ends up with the same scaling form of $C(\tau; t_w)$ written in terms of $L(\tau)$ and $R(t_w)$. Actually, in our previous work I we found that $\Psi_L(B)$ extracted from distribution of the largest relaxation time of small systems with sizes $L$ has width which does not grow with $L$ in contradiction to the scaling form of eq.(2.3). However, this discrepancy is not crucial in deriving the scaling form of $C(\tau; t_w)$. In fact, from the restriction $\tau_L(i) < \tau$ in eq.(2.4), one finds the same expression for $1 - C(\tau; t_w)$ as eq.(2.14) with $L(\tau)$ given by eq.(1.7).  

§3. Results of Simulations

3.1 Model and method

We have carried out MC simulations on isothermal aging phenomena in the same 3D Ising SG model as in our previous work I, i.e., the one with Gaussian nearest-neighbor interactions with zero mean and variance $J = 1$. The heat-bath MC method we use here is also the same as in I. The SG transition temperature is numerically determined most recently as $T_c = 0.95 \pm 0.04$. The data we will discuss below are obtained at $T = 0.5 \sim 0.8$ in systems with linear size $L_s = 24$ averaged over 160 samples with one MC run for each sample. In the previous work I, it was confirmed that finite size effects do not appear for these parameters within our time window ($\lesssim 2 \times 10^5$ MCS).

3.2 Spin auto-correlation function

In Fig. 1, we show an overall profile of the raw data of the spin auto-correlation function $C(\tau; t_w)$ obtained by the present MC simulation. As can be seen in the figure, the curves of different waiting time $t_w$ exhibit a characteristic crossover depending on $t_w$ as already found in the previous study. The data contain both the quasi-equilibrium regime and aging regime, and the crossover between the two regimes is expected to occur at around $\tau \sim t_w$.

According to the droplet theory, $1 - C(\tau; t_w)$ in the quasi-equilibrium regime is given by eq.(2.14) with a contribution of the equilibrium part $1 - C_{\text{eq}}(\tau)$ and a correction term which is proportional to $(L(\tau)/R(t_w))^{d-\theta}$. Using eq.(1.3) for $L(\tau)$ in eq.(2.14), the equilibrium part is expected to behave as,

$$C_{\text{eq}}(\tau) \propto q_{\text{EA}} + c(\tau/\tau_0)^{-\theta/2}. \quad (3.1)$$

On the other hand, for the aging regime $R(t = \tau + t_w) \gg R(t_w)$ the droplet theory predicts that $C(\tau; t_w) \sim (R(t_w)/R(t = \tau + t_w))^{\lambda}$ where $\lambda$ is a new dynamical exponent which satisfies $\lambda \geq d/2$. Rieger has pointed
out that with eq. (2.2) for \( R(t) \), one obtains,
\[
C(\tau; t_w) \propto (t/t_w)^{-\tilde{\lambda}(T)},
\]
with \( \tilde{\lambda}(T) = \lambda/z(T) \). This interpretation has allowed one to explain the apparent temperature dependence of the exponent \( \tilde{\lambda}(T) \) of the data in the aging regime obtained by MC simulations on 3D Ising SG models. For instance, using the values of exponent \( \tilde{\lambda}(T) \) reported in Fig. 2 of ref. 13 and our eq. (3.3), \( \lambda \) is evaluated at \( T < 0.7 \) as \( 1.5 \sim 1.6 \) which is rather close to its lower bound \( d/2 = 3/2 = 1.5 \).

Our interest in the present work is the details of \( C(\tau; t_w) \) in the quasi-equilibrium regime. In order to investigate the latter regime, it is convenient to look at the spin auto-correlation function \( C(\tau; t_w) \) as a function of \( t_w \) with \( \tau \) considered as a parameter. In Fig. 2, \( 1 - C(\tau; t_w) \) of some fixed small \( \tau \)'s are plotted against \( t_w/\tau \). Note that in this figure data at larger \( t_w/\tau \) are closer to the equilibrium value as in the same sense in the experimental measurements of the ac susceptibilities during isothermal aging. According to the above criterion, the quasi-equilibrium regime is the part \( t_w/\tau \gg 1 \) where the droplet theory suggests scaling forms such as eqs. (2.14), (2.22) and (2.23).

\[
\alpha(\tau) \equiv 1 - C_{\text{eq}}(\tau), \tag{3.4}
\]
and
\[
\Delta C(\tau; t_w) \equiv c_1 \tilde{\rho}(0) \frac{T}{\bar{T}} \left( \frac{L_0}{L(\tau)} \right)^{\theta/(\tau)} \left( \frac{L(\tau)}{R(t_w)} \right)^{d-\theta}. \tag{3.5}
\]

The correction term \( \Delta C(\tau; t_w) \) contains the factor \( (L_0/L(\tau))^\theta \). For the data we have simulated this factor is proportional to \( (\tau/\tau_0)^{-\theta/z(T)} \). The temperature dependence of exponent \( 1/z(T) \) is given in eq. (3.3). The energy exponent was obtained in our previous work I as \( \theta = 0.20 \pm 0.03 \). Therefore the exponent \( \theta/z(T) \) becomes quite small, and so the ratio of the above factor of the maximum time separation \( \tau \) and to that of the minimum \( \tau \) of the data shown in Fig. 2 is close to unity. Actually with \( \tau_{\text{max}}/\tau_{\text{min}} = 2^7 \) and \( T = 0.6 \), the ratio is estimated as 0.94. This observation indicates that the factor \( (L_0/L(\tau))^\theta \) can be approximated as
\[
\left( \frac{L_0}{L_T(\tau)} \right)^\theta \approx \left( \frac{\tau_0}{\tau} \right)^{\theta/(\tau)} \left( \frac{T}{T_\text{m}} \right)^c \left[ 1 - \frac{\theta}{z(T)} \ln \left( \frac{T}{T_\text{m}} \right) \right], \tag{3.6}
\]
where \( \tau_0 \) is a characteristic time scale corresponding to the length scale \( L_0 \), \( T_\text{m} \approx (\tau_{\text{min}} \tau_{\text{max}})^{1/2} \), and \( c \) a numerical constant \( (\approx (\tau_0/T_\text{m})^{\theta/z(T)}) \). In eq. (3.3), in particular, it can be treated practically as a constant.

Because of the above simplification, the correction term \( \Delta C(\tau; t_w) \) practically becomes a function of the scaled length \( L(\tau)/R(t_w) = (\tau/t_w)^{1/z(T)} \) alone. We now try to determine \( \alpha(\tau) \) for each \( \tau \) so that a master curve is obtained when \( \Delta C(\tau; t_w) \) of different \( \tau \) are plotted against the rescaled time \( t_w/\tau \). Postponing the description of details of the analysis till after eq. (3.8) below, we demonstrate a typical result in the inset of Fig. 2. As can be seen in the figure, this one-parameter scaling does work very well.

\[
\alpha(\tau) \equiv 1 - C_{\text{eq}}(\tau), \tag{3.4}
\]

3.3 Scaling analysis
The scaling ansatz eq. (2.14) suggests that the data of \( 1 - C(\tau; t_w) \) can be decomposed into the equilibrium part, which does not depend on the waiting time \( t_w \) and the correction term which depends on both \( \tau \) and \( t_w \). Let us denote the former as \( \alpha(\tau) \) and the latter as \( \Delta C(\tau; t_w) \), i.e.,
\[
1 - C(\tau; t_w) \equiv \Delta C(\tau; t_w) + \alpha(\tau), \tag{3.3}
\]
be understood as the following. Combining eqs. (2.13) and (2.14) in §2, we obtain

\[
\alpha(\tau) = 1 - C_{eq}(\tau) \sim \frac{T\tilde{\rho}(0)}{T} \left[ 1 - \left( \frac{L_0}{L(\tau)} \right)^\theta \right] \\
\sim \frac{cT\tilde{\rho}(0)}{T} \frac{\theta}{\tau} \ln \left( \frac{\tau}{\tau_m} \right) + \text{const.,} 
\]  

(3.7)

where we used eq. (2.16) to derive the last expression. The above formula implies that slopes of \(\alpha(\tau)\) with respect to \(\ln \tau\) depend on temperature as \(T^2\) because of the temperature dependence of the exponent \(1/z(T)\) as given in eq. (1.3). From the data at four temperatures we obtained that \(\partial \alpha(\tau)/\partial \ln \tau \propto T^2.4\). The stronger \(T\)-dependence than \(T^2\) may be attributed to that of \(\tilde{\rho}(0)/\bar{Y}\) in the last expression of eq. (3.7).

It is noted that the second expression of eq. (3.7) implies that \(1 - C_{eq}(\tau)\) is concave in \(\ln \tau\) as far as the EA order parameter \(q_{EA}\) is finite. But such a tendency cannot be seen in time-window of the present analysis. No tendency of saturation to \(q_{EA}\) is seen also in Fig. 1 where an overall profile of the spin auto-correlation function obtained by the present simulation up to \(\tau \lesssim t_w \sim 2 \times 10^5\) is shown. In fact up to now, no numerical data of the spin auto-correlation function which exhibit any tendency of saturation to some \(q_{EA}\) have been reported for the 3D SG models in spite of the enormous computational efforts. This is another pre-asymptotic behavior of the simulated data.

Lastly let us discuss the exponent \(\kappa\) defined by,

\[
\Delta C(\tau; t_w) \sim (L(\tau)/R(t_w))^{\kappa}, 
\]

(3.8)

at the limit \(L(\tau)/R(t_w) \ll 1\). The question is it agrees with the expected behavior given in eq. (3.5), i.e., \(\kappa = d - \theta\). It is noted for clarity that the limitation of the computational time window does not allow us to take equilibrium limit \(L(\tau)/R(t_w) \rightarrow 0\) accurately and thus leaves some ambiguities in the determination of \(\kappa\). In Fig. 4, for examples, we show the double logarithmic plots of \(\Delta C(\tau; t_w)\) at \(T = 0.6\) versus \(L(\tau)/R(t_w) = (\tau/t_w)^{1/z(T)}\) for three different sets of \(\{\alpha(\tau)\}\). Here we used \(1/z(T = 0.6) \approx 0.102\) using eq. (1.3).

![Fig. 5. The double logarithmic plot of \(\Delta C(\tau; t_w)\) versus \(L(\tau)/R(t_w)\). The slope of the dotted line is \(d = 3\).](image)

To draw these figures we have first chosen the value \(\alpha(\tau = 4) (= \alpha_1)\) and then determined \(\alpha(\tau)\) of the larger \(\tau\) subsequently in such a way that the data of \(\Delta C(\tau; t_w)\) lie on a single master curve. When we chose a too small value for \(\alpha_1\), such as the case with \(\alpha_1 = 0.0690\) shown in the figure, we obtain a small \(\kappa\) (\(\approx 2.2\)) but a linear portion of the double logarithmic plot almost disappears. A too large \(\alpha_1 (= 0.0712)\), on the other hand, ends up with a large \(\kappa\) (\(\approx 3.3\)) but the plot deviates from a linear behavior at smaller \((L(\tau)/R(t_w))\). With a median value of \(\alpha_1 (= 0.0702)\), which yields \(\kappa \approx 3.0\), we obtain linear behavior of the plot in a reasonable range of \(L(\tau)/R(t_w)\). Thus the acceptable values for \(\kappa\) from our simulated data are \(\approx 2.3 \sim 3.1\). The latter values are consistent with the expected exponent in eq. (3.5), \(d - \theta\) with \(\theta \approx 0.20\). However, it is hard to extract the value of \(\theta\) by the data of \(\Delta C(\tau; t_w)\) alone. Let us remark however that the above ambiguity in \(\kappa\) little affects the values of \(\{\alpha(\tau)\}\), in particular, their slopes with respect to \(\ln \tau\) are not sensitive. This is demonstrated in Fig. 3 by drawing four sets of \(\{\alpha(\tau)\}\) at \(T = 0.6\) with \(\alpha_1 = 0.0690, 0.0698, 0.0702\) and 0.0712 which yield \(\kappa \approx 2.2, 2.6, 3.0,\) and 3.3, respectively.

These circumstances are similar at other temperatures we have examined. Therefore we show the representative results of \(\{\alpha(\tau)\}\) in Fig. 3 which give rise to \(\kappa \approx 2.8 \sim 3.0\). In Fig. 5 we show \(\Delta C(\tau; t_w)\) at \(T = 0.6, 0.8\) with \(\kappa \approx 3.0\). For \(\Delta C(\tau; t_w)\) with a common \(\kappa\) but with different temperatures \(T_1\) and \(T_2\), their relative magnitude in the range \(L(\tau)/R(t_w) \ll 1\) can be estimated by that of \((\tilde{\rho}(0)/\bar{Y})\) through eq. (3.5). The relative magnitude of the two \(\Delta C(\tau; t_w)\) shown in Fig. 5 is about 1.3 which is roughly consistent with this estimation \((= 0.8/0.6)^{1.4} \approx 1.50)\).

### 3.4 Out-of-phase linear ac susceptibility

Let us finally discuss briefly the logarithmic derivative of the auto-correlation function, which is supposed to be almost identical to the out-of-phase linear ac sus-
ceptibility via FDT. Here let us denote the logarithmic derivative of the auto-correlation function simply as out-of-phase linear ac susceptibility $\chi''(\omega; t_w)$. We show in Fig. 6 $\chi''(\omega; t_w)$ at different $\omega$ which are obtained by numerical differentiation of eq. (2.21) using the same data of $1 - C(\tau; t_w)$ shown in Fig. 2.

![Fig. 6](image_url)

Fig. 6. $\chi''(\omega; t_w)$ evaluated through eq. (2.21) for $\tau_w = 2^{(2+n)}$ with $n = 0, 1, \ldots, 7$ from bottom to top. In the inset the same data are plotted against $t_w/\tau_w$. They lie on a universal curve.

Using the growth laws eqs. (1.2) and (1.7) in eqs. (2.22) and (2.23) respectively, the expected scaling behavior is obtained as,

$$\frac{\chi''(\omega; t_w) - \chi''(\omega)}{\chi''(\omega)} \propto \left(\frac{\tau_w}{t_w}\right)^{(d-\theta)/z(T)},$$  \hspace{1cm} (3.9)

with

$$\chi''(\omega) \sim (\tau_0 \omega)^{d/z(T)} \chi''(\omega_0),$$  \hspace{1cm} (3.10)

where $\omega_0 = 2\pi/\tau_0$. Note that the exponent $\theta/z(T)$ in eq. (3.10) originates from the factor $(L_0/L(T))^{\theta}$ in eq. (2.23), and that the range of $\tau_w = 2\pi/\omega$ of interest here is the same as that in the analysis of $1 - C(\tau; t_w)$. Because of the same reason as we noted above eq. (3.6), a master curve for $\chi''(\omega; t_w)$ has been obtained by simply plotting them against $t_w/\tau_w$ as shown in the inset of Fig. 6.

Let us remark that $\chi''(\omega; t_w)$ simulated here are apparently larger for the larger $\tau_w$ (smaller $\omega$). The scaling ansatz suggests that this is not the case for $\chi''(\omega; t_w)$ of much smaller frequencies, whose equilibrium values $\chi''_{eq}(\omega)$ do depend on the frequencies and are the smaller for the frequencies, in their asymptotic approach to the equilibrium values. We also note that, in order to check for the FDT to hold in the quasi-equilibrium regime it is desirable to perform direct simulation on the response against ac magnetic field. Since such an analysis is time consuming, it is left for a future work.

§4. Discussions

We have shown so far that the scaling properties of the simulated auto-correlation function $C(\tau; t_w)$ in the quasi-equilibrium range $L(\tau) \ll R(t_w)$ can be well explained by the scaling ansatz by the droplet theory expressed in terms of $L(\tau)$ and $R(t_w)$. For the growth law of the latter, we have used the power-law form, eqs. (1.2) and (1.7) for $R(t_w)$ and $L(\tau)$, respectively. More precisely, the fundamental exponents $\theta$ and $\kappa (= d - \theta)$ in the droplet theory, combined with the exponent $1/z(T)$ of the growth-law of $R(t_w)$, consistently describe our simulated results: the energy relaxation in I and aging behavior of $C(\tau; t_w)$ in the quasi-equilibrium regime in the present work. However, the time window of the present simulation and the previous one in I are in the range where the growth law of $R(t_w)$ exhibits the pre-asymptotic behavior (as feature 2) in §1. Time scales of laboratory experiments on real spin glasses may also be in a similar range as we will discuss below.

The quantity, which has been studied by experiments most frequently and is of interest from our present point of view, is the ac susceptibility $\chi''(\omega; t)$. The latter is measured continuously in time $t$ while the system is isothermally aged. Since the period $\tau_w = 2\pi/\omega$ is, in general, much smaller than the aging time $t$, it should be regarded as the response in the quasi-equilibrium range ($t = t_w + \tau_w \simeq t_w$) as we noted before. Indeed the scaling analysis based on eq. (2.23) was directly carried out by Schins et al on a 2D Ising SG system. Their results are consistent with the 2D version of the droplet theory and, quite remarkably, the asymptotic growth law eq. (4.4) with a reasonable value of $\psi$.

Concerning the the ac susceptibility $\chi''(\omega; t)$ in the 3D spin glasses, the Saclay group has often reported that $\chi''(\omega; t)$ obey the $\omega t$-scaling; if $\chi''(\omega; t)$ with different $\omega$ are vertically shifted by certain amount depending on $\omega$, and are plotted against $\omega t$, they lie on a universal curve. The equilibrium values $\chi''_{eq}(\omega)$ thus obtained obey a power-law scaling with a power-law form of $\chi''(\omega; t)$ as we noted before. Indeed the scaling analysis eq. (2.23) with the power-law form of eq. (3.10). The above equation just corresponds to our eq. (3.10) with $\alpha = \theta/z(T)$. The time window of the experiments are $t_w \lesssim 10^{16}$ and $10^{10} \lesssim \tau_w \lesssim 10^{14}$ in unit of microscopic time ($\sim T_c$), while that of our simulation are $t_w \lesssim 10^{5}$ and $4 \lesssim \tau_w \lesssim 256$. Presumably this difference allows one to extract the exponent $\alpha$ from the experimentally measured $\chi''(\omega; t)$. Interestingly, $\alpha$ of CdIn$_3$Cr$_7$S$_8$ reported in Fig. 2b of Ref. 23 rather is in good agreement with $\theta/z(T)$ evaluated by our simulational studies on both the order of magnitude and the temperature dependence, though this is not the case at temperatures close to $T_c$ nor at low temperatures $T \lesssim 0.5T_c$. In contrast to the above experimental results which are favorable to the power-law form of $R(t_w)$ and $L(\tau)$, the Saclay group also reported that $\chi''_{eq}(\omega)$ observed in a wider range (7 decades) of $\omega$ fits rather well to a function of $\ln \omega$ than to the power-law form of eq. (3.10).

Svedlindh et al. reported a $\ln \omega$-dependence of $\chi''(\omega; t)$ and a power-law dependence on $\omega$ in the log-
arithmetic derivative of $\chi''(\omega ; t)$ as,

$$\frac{\partial \chi''(\omega ; t)}{\partial \ln t} \sim \omega^{-\hat{\beta}},$$  \hspace{1cm} (4.2)

at $T < T_c \ (\hat{\beta} \simeq 0.25 \text{ at } T = 0.985 T_c)$. This feature may also be explained by eq. (2.2) so long as $L(\tau_w)$ obeys the power-law form eq. (1.7). The exponent $\hat{\beta}$ in eq. (4.2) is then evaluated as $\hat{\beta} = (d - 2d)/z$, where $[d/\theta] - 2$ times larger than $\hat{\alpha}$ in eq. (3.10).

From our point of view, another important quantity is the zero-field-cooled susceptibility $\chi_{ZFC}(\tau; t_w)$ or the thermoremanent magnetization $m_{TRM}(\tau; t_w)$ in the quasi-equilibrium regime. The two quantities are related by $\chi_{ZFC}(\tau; t_w) = [m_{FC} - m_{TRM}(\tau; t_w)])/h$, where $m_{FC}$ is the field-cooled magnetization under a field of strength $h$, and $\chi_{ZFC}(\tau; t_w)$ is directly related to $1 - C(\tau; t_w)$ as expressed by eq. (2.23) in §2. Although $\chi_{ZFC}(\tau; t_w)$ (or $m_{TRM}(\tau; t_w)$) has been been measured frequently since the pioneering work by Lundgren et al. [2], most of the measurements have been focused on its crossover behavior between the quasi-equilibrium and aging regimes. Recently we have analyzed the data on $m_{TRM}(\tau; t_w)$ of AgMn spin glass measured by the Saclay group [23] by the same procedure as we did on $1 - C(\tau; t_w)$ in §3. The results are qualitatively similar to those of $\chi''(\omega ; t)$ described just above: for the data at temperatures not close to $T_c$, the orders of magnitude of $\hat{\kappa}$ and $\hat{\alpha}$ as well as their temperature dependence are compatible with those obtained by our simulations, though this is not the case for the data at temperatures close to $T_c$.

Besides the experiments on the linear ac susceptibilities, Joh et al. [1] have recently extracted $R(t)$ from the thermoremanent magnetization they have measured under fields of various strengths. Its growth-law agrees with the simulational result of eq. (1.2) even quantitatively, and the estimated $R(t)$ with $t$ of the order of laboratory time is at most only several tens of lattice distances, i.e., a length scale far from the thermodynamic limit we usually imagine.

Thus concerning the growth law of $R(t)$ and $L(\tau)$ in 3D spin glasses, the experimental studies so far reported have not converged to a unique conclusion: it is either a power-law as eq. (1.2) or a logarithmic form as eq. (1.4). This ambiguous situation seems difficult to be resolved by looking at only $\omega$ dependence of $\chi''(\omega)$ because $\hat{\alpha}$ in eq. (1.1) is quite small partially because of the smallness of $\hat{\theta}$. The latter had already made it difficult to settle serious controversies concerning static properties of the SG phase. In this respect, it is of interest to examine experimentally the $\omega$-dependence of $\chi''(\omega ; t)$ more in detail because $\hat{\beta}$ in eq. (4.2) is relatively large. Also, as mentioned above, $\chi_{ZFC}(\tau; t_w)$ in the quasi-equilibrium regime may be another important quantity to be studied in detail. Such experiments have to be performed in a time window of observations as wide as possible.

Among problems on isothermal aging phenomena to be further explored numerically, a most important one may be aging dynamics at temperatures close to $T_c$, where a crossover from aging dynamics governed by the criticality at $T_c$ to that governed by the $T = 0$ criticality (the droplet theory) is expected to occur. Indeed such a crossover in aging dynamics has been found recently in the 4D Ising SG model by Hukushima et al. [3] A similar analysis on the present 3D model may shed light on peculiar behavior of aging dynamics measured also experimentally at temperatures close to $T_c$ as we have pointed out above.

To conclude, we have simulated isothermal aging in the 3D Ising SG model, and analyzed behaviors of the spin auto-correlation function in the quasi-equilibrium regime. The simulated results are well interpreted by the scaling arguments which associate with the characteristic length scales $R(t_w)$ and $L(\tau)$, i.e., those of domains and droplet excitations in time scales of $t_w$ and $\tau$, respectively. We have conjectured that some aspects of the experimental data on the ac susceptibility and the zero-field-cooled susceptibility can be also explained along the same line.

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