Predictions for masses of bottom baryons

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ABSTRACT

The recent observation of $\Sigma_0^{\pm}$ ($uuub$ and $ddbb$) and $\Xi_0^{-}$ ($dsbb$) baryons at the Tevatron within 2 MeV of our theoretical predictions provides a strong motivation for applying the same theoretical approach, based on modeling the color hyperfine interaction, to predict the masses of other bottom baryons which might be observed in the foreseeable future. For S-wave $qqb$ states we predict $M(\Omega_b) = 6052.1 \pm 5.6$ MeV, $M(\Omega_b^*) = 6082.8 \pm 5.6$ MeV, and $M(\Xi_b^0) = 5786.7 \pm 3.0$ MeV. For states with one unit of orbital angular momentum between the $b$ quark and the two light quarks we predict $M(\Lambda_{b[1/2]}) = 5929 \pm 2$ MeV, $M(\Lambda_{b[3/2]}) = 5940 \pm 2$ MeV, $M(\Xi_{b[1/2]}) = 6106 \pm 4$ MeV, and $M(\Xi_{b[3/2]}) = 6115 \pm 4$ MeV.

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1 Introduction

There has been noteworthy experimental progress recently in the identification of baryons containing a single $b$ quark. The CDF Collaboration has seen the states $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ \cite{1}, while both D0 \cite{2} and CDF \cite{3} have observed the $\Xi_b^-$. The constituent quark model has been remarkably successful in predicting the masses of these states \cite{4}-\cite{9}. Most recently the careful accounting for wave function effects in the hyperfine interaction \cite{10} has permitted the prediction of the $\Xi_b^-$ mass within a few MeV of observation.

All predictions need an input for the mass difference $m_b - m_c$ between the $b$ and $c$ quarks. That the value of $m_b - m_c$ obtained from hadrons containing $b$ and $c$ quarks depends upon the flavors of the spectator quarks was noted in Ref. \cite{6} where Table I shows that the value is the same for mesons and baryons not containing strange quarks but different when obtained from $B_s$ and $D_s$ mesons. Some reasons for this difference were noted and the issue still requires further investigation.

The new CDF mass measurement \cite{3} of the baryon $\Xi_b^-$ confirms the prediction \cite{10} which uses the value of $m_b - m_c$ obtained from $B_s$ and $D_s$ meson masses. Therefore in our present analysis we use this value of of $m_b - m_c$, as well as a very close value obtained from the $\Xi_b^- - \Xi_c^-$ mass difference. Since these values are about 10 MeV lower than the value obtained \cite{6} from nonstrange hadrons, our predictions are lower than other predictions \cite{5,7} which use nonstrange hadron masses as inputs.

In this model the mass of a hadron is given by the sum of the constituent quark masses plus the color-hyperfine (HF) interactions:

$$V_{ij}^{HF} = v \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \langle \delta(r_{ij}) \rangle \tag{1}$$

where the $m_i$ is the mass of the $i$-th constituent quark, $\sigma_i$ its spin, $r_{ij}$ the distance between the quarks and $v$ is the interaction strength. We shall neglect the mass differences between $u$ and $d$ constituent quarks, writing $u$ to stand for either $u$ or $d$. All the hadron masses (except the ones given in Sec. 3) are for isospin-averaged baryons.

Two interesting observations, based on a study of the hadronic spectrum, lead to improved predictions for the $b$ baryons. The first is that the effective mass of the constituent quark depends on the spectator quarks \cite{6,10}, and the second is an effective supersymmetry \cite{9} – a resemblance between mesons and baryons where the anti-quark is replaced by a diquark \cite{11}.

In this paper we extend the same methodology to obtain predictions for the masses of additional baryonic states containing the $b$ quark that will be experimentally accessible in the foreseeable future.
# $\Omega_b$ mass prediction

Table I: Hadron masses used in the calculation of the $\Omega_b$ mass prediction

| Splitting | Value (MeV) |
|-----------|-------------|
| $M(\Omega_c)$ | 2697.5 ± 2.6 |
| $M(\Omega_c^*)$ | 2768.3 ± 3.0 |
| $M(\Omega_c^*) - M(\Omega_c)$ | 70.8 ± 1.5 |
| $M(D_s)$ | 1968.49 ± 0.34 |
| $M(D_s^*)$ | 2112.3 ± 0.5 |
| $M(B_s)$ | 5366.1 ± 0.6 |
| $M(B_s^*)$ | 5412.0 ± 1.2 |
| $M(B_s^*) - M(B_s)$ | 45.9 ± 1.2 |
| $M(\Xi_c^0)$ | 2471.0 ± 0.4 |
| $M(\Xi_c^-)$ | 5792.9 ± 3.0 |

Taking the approach implemented in [10] for the prediction of the $\Xi_b$ mass, the spin averaged mass of $\Omega_b$ can be obtained by extrapolation from available data for $\Omega_c$ and a correction based on strange meson masses, as listed in Table I:

$$M(\widetilde{\Omega_b}) = \frac{2M(\Omega_c^*) + M(\Omega_b)}{3} + \frac{2M(\Omega_c^*) + M(\Omega_c)}{3} + \frac{3M(B_s^*) + M(B_s)}{4} - \frac{3M(D_s^*) + M(D_s)}{4}$$

$$= 6068.9 \pm 2.4 \text{ MeV}$$

where $M(\widetilde{X})$ denotes the spin-averaged mass that cancels out the hyperfine interaction between the heavy quark and the diquark containing lighter quarks.

The HF splitting can be estimated as follows:

$$M(\Omega_b^*) - M(\Omega_b) = (M(\Omega_c^*) - M(\Omega_c)) \frac{m_c}{m_b} = 24.0 \pm 0.7 \text{ MeV},$$

where we have used the experimental mass difference [13] $M(\Omega_c^*) - M(\Omega_c) = 70.8 \pm 1.0 \pm 1.1 \text{ MeV} = 70.8 \pm 1.5 \text{ MeV}$ with $m_b/m_c$ taken to be $2.95 \pm 0.06$, as discussed in the Appendix. This gives the following mass predictions:

$$\Omega_b^* = 6076.9 \pm 2.4 \text{ MeV}; \quad \Omega_b = 6052.9 \pm 2.4 \text{ MeV}$$

Taking into account the wavefunction correction as described in [12], one must add the following correction to the spin averaged mass:

$$v \left[ \langle \delta(r_{ss}) \rangle_{\Omega_b} \frac{m^2}{m_s^2} - \langle \delta(r_{ss}) \rangle_{\Omega_c} \frac{m^2}{m_s^2} \right] = v \left[ \langle \delta(r_{ss}) \rangle_{\Omega_b} \frac{m^2}{m_s^2} \langle \delta(r_{ss}) \rangle_{\Omega_b} - 1 \right] \approx \frac{50 \pm 10}{\langle \delta(r_{ss}) \rangle_{\Omega_b}} \left[ \langle \delta(r_{ss}) \rangle_{\Omega_c} - 1 \right] = 2.0 \pm 1.1 \text{ MeV}$$
where the contact probability ratio was computed using variational methods

\[
\frac{\langle \delta(r_{ss}) \rangle_{\Omega_b}}{\langle \delta(r_{ss}) \rangle_{\Omega_c}} = 1.04 \pm 0.02 ,
\] (6)

and we used the following calculation to evaluate the strength of the ss HF interaction:

\[
50 \text{ MeV} \approx M(\Omega) + \frac{1}{4}(2M(\Xi_b^0) + M(\Xi_c^0) + M(\Xi_c^0))
- \frac{1}{3}(2M(\Xi^0) + M(\Xi)) - \frac{1}{3}(2M(\Omega_c^0) + M(\Omega_c)) = \\
\approx \left(3m_s + 3v \frac{\langle \delta(r_{ss}) \rangle_{\Omega_b}}{m_s^2}\right) + \left(m_u + m_s + m_c\right) - \left(2m_s + m_u + v \frac{\langle \delta(r_{ss}) \rangle_{\Omega_c}}{m_s^2}\right) - \left(2m_s + m_c + v \frac{\langle \delta(r_{ss}) \rangle_{\Omega_c}}{m_s^2}\right)
\approx 50 \text{ MeV} \approx \frac{\langle \delta(r_{ss}) \rangle_{\Omega_b}}{m_s^2}
\] (7)

An alternate derivation of the $\Omega_b$ mass from the $\Xi_b^0 - \Xi_c^0$ mass difference

Thanks to new measurements of the $\Xi_b^0$ mass \[2, 3\], we now have another way of estimating the spin-averaged $\Omega_b$ mass. Following the approach in Ref. \[10\] the $\Xi_b^0 - \Xi_c^0$ mass difference can be schematically written as

\[
M(\Xi_b^0) - M(\Xi_c^0) = (m_b - m_c) + (\text{wavefunction correction}) + (\text{EM correction})
= (m_b - m_c) + (-4 \pm 4) \text{ MeV} + (V_{bsd}^{\text{EM}} - V_{csd}^{\text{EM}})
\] (8)

where the value of the wave function correction is taken from \[10\] and the last term denotes the EM interactions of the relevant quarks.

Similarly, the spin-averaged $\Omega_b - \Omega_c$ mass difference can be written as

\[
M(\Omega_b) - M(\Omega_c) = (m_b - m_c) + (\text{wavefunction correction}) + (\text{EM correction})
= (m_b - m_c) + (2.0 \pm 1.1) \text{ MeV} + (V_{bs}^{EM} - V_{cs}^{EM})
\] (9)

where the wave-function correction is given in Eq. (5).

Since the $b$ and $s$ quarks have the same charge, the EM contribution $V_{bs}^{\text{EM}} - V_{cs}^{\text{EM}}$ to the $\Omega_b - \Omega_c$ mass difference is almost the same as the EM contribution $V_{bsd}^{\text{EM}} - V_{csd}^{\text{EM}}$ to the $\Xi_b^0 - \Xi_c^0$ mass difference, modulo a negligible correction from the change in the mean radius of the relevant baryons. We then immediately obtain

\[
M(\Omega_b) - M(\Omega_c) = M(\Xi_b^0) - M(\Xi_c^0) + (6.0 \pm 4.1) \text{ MeV}
\] (10)
which leads to
\[ M(\bar{\Omega}_b) = 6072.6 \pm 5.6 \text{ MeV} \]  
(11)
to be compared with \( M(\bar{\Omega}_b) = 6070.9 \pm 2.7 \text{ MeV} \) from Eqs. (2) and (5).

The consistency of these two estimates, based on different experimental inputs, is a strong indication that both the central values and the error estimates are reliable. Moreover, the estimate in Eq. (11) includes EM corrections, while the estimate Eqs. (2) does not, thus indicating that the EM corrections are likely to be smaller than our error estimate. Consequently, in the following we use the estimate (11).

**Wave function correction to the hyperfine splitting**

We must also compute the correction to the HF splitting
\[ M(\Omega_b^*) - M(\Omega_b) = (M(\Omega_c^*) - M(\Omega_c)) \frac{m_c \langle \delta(r_{bs}) \rangle_{\Omega_b}}{m_b \langle \delta(r_{cs}) \rangle_{\Omega_c}} = 30.7 \pm 1.3 \text{ MeV} \]  
(12)
where we used
\[ \frac{\langle \delta(r_{bs}) \rangle_{\Omega_b}}{\langle \delta(r_{cs}) \rangle_{\Omega_c}} = 1.28 \pm 0.04 , \]  
(13)
leading to the following predictions:
\[ \Omega_b^* = 6082.8 \pm 5.6 \text{ MeV}; \quad \Omega_b = 6052.1 \pm 5.6 \text{ MeV} \]  
(14)

**An alternative derivation of HF splitting from effective supersymmetry**

An alternative approach to estimate the HF splitting is to use the effective meson-baryon supersymmetry discussed in [9] and apply it to the case of hadrons related by changing a strange antiquark \( \bar{s} \) to a doubly strange \( ss \) diquark coupled to spin \( S = 1 \):
\[ \frac{M(\Omega_b^*) - M(\Omega_b)}{M(\bar{B}_s^*) - M(\bar{B}_s)} = \frac{M(\Omega_c^*) - M(\Omega_c)}{M(D_s^*) - M(D_s)} = \frac{M(\Xi^*) - M(\Xi)}{M(K^*) - M(K)} \]  
(15)
\[ \approx 0.49 \pm 0.01 \quad \approx 0.54 \]
\[ \Omega_b^* - \Omega_b = (\bar{B}_s^* - \bar{B}_s)(0.52 \pm 0.02) = 23.9 \pm 1.1 \text{ MeV} \]  
(16)
This gives
\[ \Omega_b^* = 6076.8 \pm 2.4 \text{ MeV}; \quad \Omega_b = 6053.0 \pm 2.5 \text{ MeV} . \]  
(17)

The main difference between these predictions and the ones given in the past [5, 7] is the use of masses of hadrons containing strange quarks [10], rather than \( \Lambda_b \) and \( \Lambda_c \) masses, to obtain the quark mass difference \( m_b - m_c \). We also take into account wave function corrections which influence the hyperfine splitting between \( \Omega_b^* \) and \( \Omega_b \). The net result is that we predict substantially lower masses for \( \Omega_b \) than both Ref. [5]: \( M(\Omega_b) = 6068.7 \pm 11.1 \text{ MeV} \), and Ref. [7]: \( M(\Omega_b) = 6065 \text{ MeV} \). Our predicted hyperfine splitting \( M(\Omega_b^*) - M(\Omega_b) = 30.7 \pm 1.3 \text{ MeV} \) (when wave function effects are included) is also larger than those of Refs. [5] (14.5 MeV) and [7] (23 MeV).
3 \( \Xi_b \) isospin splitting

The \( \Xi_b^0 \) mass is expected to be measured by the CDF collaboration through the channel \( \Xi_b^0 \rightarrow \Xi^+_c \pi^- \), where \( \Xi_c^+ \rightarrow \Xi^- \pi^+ \pi^- \), \( \Xi^- \rightarrow \Lambda \pi^- \), and \( \Lambda \rightarrow p \pi^- \) [14].

The source for the isospin splitting (\( \Delta I \)) is the difference in the mass and charge of the \( u \) and \( d \) quarks. These differences affect the hadron mass in four ways [15]: they change the constituent quark masses (\( \Delta M = m_d - m_u \)), the Coulomb interaction (\( V^{EM} \)), and the spin-dependent interactions – both magnetic and chromo-magnetic (\( V^{spin} \)). One can obtain a prediction for the \( \Xi_b \) isospin splitting by extrapolation from the \( \Xi \) data, which has similar structure as far as EM interactions are concerned (note that for \( \Xi_b \) there are no spin-dependent interactions between the heavy quark and the \( su \) diquark which is coupled to spin zero):

\[
\Delta I(\Xi^*) = \Delta M + \left[ V^{EM}_{ssd} - V^{EM}_{ssu} \right] + 2 \left[ V^{spin}_{ds} - V^{spin}_{us} \right] = 3.20 \pm 0.68 \text{ MeV} \quad (18)
\]

\[
\Delta I(\Xi) = \Delta M + \left[ V^{EM}_{ssd} - V^{EM}_{ssu} \right] - 4 \left[ V^{spin}_{ds} - V^{spin}_{us} \right] = 6.85 \pm 0.21 \text{ MeV} \quad (19)
\]

\[
\Rightarrow \Delta I(\Xi_b) = \Delta M + \left[ V^{EM}_{ssd} - V^{EM}_{ssu} \right] - 3 \left[ V^{spin}_{ds} - V^{spin}_{us} \right] = 6.24 \pm 0.21 \text{ MeV} \quad (20)
\]

With the observed value [3] \( M(\Xi_b^0) = (5792.9 \pm 2.5 \pm 1.7) \text{ MeV} \) (the error from the D0 experiment is considerably larger [2]) and this estimate, we predict \( M(\Xi_b^0) = 5786.7 \pm 3.0 \text{ MeV} \).

Another option is to use \( \Xi_c \), which has the same spin-dependent interactions, as a starting point:

\[
\Delta I(\Xi_c) = \Delta M + \left[ V^{EM}_{csd} - V^{EM}_{csu} \right] - 3 \left[ V^{spin}_{ds} - V^{spin}_{us} \right] = 3.1 \pm 0.5 \text{ MeV} \quad (21)
\]

\[
\Rightarrow \Delta I(\Xi_b) = \Delta M + \left[ V^{EM}_{ssd} - V^{EM}_{ssu} \right] - 3 \left[ V^{spin}_{ds} - V^{spin}_{us} \right] = \frac{\Delta I(\Xi_c) + 2\Delta I(\Xi^*) + \Delta I(\Xi)}{3} - \frac{2\Delta I(\Xi'_c) + \Delta I(\Xi'_c) + \Delta I(\Xi_c)}{4} = 6.4 \pm 1.6 \text{ MeV} \quad (22)
\]

We summarize the isospin splittings which have been used in these calculations in Table II. All masses have been taken from the 2007 updated tables of the Particle Data Group [16], and all values of \( \Delta I \) are defined as \( M(\text{baryon with } d \text{ quark}) - M(\text{baryon with } u \text{ quark}) \).
Table II: Isospin splittings $\Delta I$ used in calculating $\Delta I(\Xi_b) \equiv M(\Xi_b^-) - M(\Xi_b^+)$.  

| Splitting  | Value (MeV) |
|------------|-------------|
| $\Delta I(\Xi)$ | 6.85 ± 0.21 |
| $\Delta I(\Xi^*)$ | 3.20 ± 0.68 |
| $\Delta I(\Xi_c)$ | 3.1 ± 0.5 |
| $\Delta I(\Xi'_c)$ | 2.3 ± 4.24 |
| $\Delta I(\Xi''_c)$ | −0.5 ± 1.84 |

4 $\Lambda_b$ and $\Xi_b$ orbital excitations

Table III: Masses of $\Lambda$ and $\Xi$ baryon ground states and orbital excitations [16].

|          | $\Lambda$       | $\Lambda_c$      | $\Xi_c^+$  | $\Xi_c^-$  |
|----------|-----------------|------------------|------------|------------|
| $M(1/2^+)$ | 1115.683 ± 0.006 | 2286.46 ± 0.14  | 2467.9 ± 0.4 | 2471.0 ± 0.4 |
| $M(1/2^*)$ | 1406.5 ± 4.0    | 2595.4 ± 0.6    | 2789.2 ± 3.2 | 2791.9 ± 3.3 |
| $M(3/2^-)$ | 1519.5 ± 1.0    | 2628.1 ± 0.6    | 2816.5 ± 1.2 | 2818.2 ± 2.1 |

In the heavy quark limit, the $(1/2^−)$ and $(3/2^-)$ $\Lambda^*$ and $\Xi^*$ excitations listed in Table III can be interpreted as a P-wave isospin-0 spinless diquark coupled to the heavy quark. Under this assumption, the difference between the spin averaged mass of the $\Lambda^*$ baryons and the ground state $\Lambda$ is only the orbital excitation energy of the diquark.

\[
\Delta E_L(\Lambda) \equiv \frac{2\Lambda_{c[3/2]}^* + \Lambda_{c[1/2]}^*}{3} - \Lambda = 366.15 \pm 1.49 \text{ MeV}
\]

\[
\Delta E_L(\Lambda_c) \equiv \frac{2\Lambda_{c[3/2]}^* + \Lambda_{c[1/2]}^*}{3} - \Lambda_c = 330.74 \pm 0.47 \text{ MeV}
\]  (23)

\[
\Delta E_L(\Xi_c) \equiv \frac{2\Xi_{c[3/2]}^* + \Xi_{c[1/2]}^*}{3} - \Xi_c = 339.11 \pm 1.11 \text{ MeV}
\]

The spin-orbit splitting seems to behave like $1/m_Q$:

\[
\Lambda_{c[3/2]}^* - \Lambda_{c[1/2]}^* = 113.0 \pm 4.1 \text{ MeV}
\]

\[
\Lambda_{c[3/2]}^* - \Lambda_{c[1/2]}^* = 32.7 \pm 0.8 \text{ MeV}
\]  (24)

\[
\Xi_{c[3/2]}^* - \Xi_{c[1/2]}^* = 26.9 \pm 2.6 \text{ MeV}
\]

where the $\Xi_c$ entries are isospin averages.

The orbital excitation energies in Eq. (23) may be extrapolated to the case of excited $\Lambda_b$ baryons in the following manner. Energy spacings in a power-law potential $V(r) \sim r^\nu$ behave with reduced mass $\mu$ as $\Delta E \sim \mu^p$, where $p = -\nu/(2 + \nu)$ [17]. For light quarks in the confinement regime, one expects $\nu = 1$ and $p = -1/3$, while for the
$c\bar{c}$ and $b\bar{b}$ quarkonium states, with nearly equal level spacings, an effective power is $\nu \approx 0$ and $p \approx 0$. One should thus expect orbital excitations to scale with some power $-1/3 \leq p \leq 0$. One can narrow this range by comparing the $\Lambda$ and $\Lambda_c$ excitation energies and estimating $p$ with the help of reduced masses $\mu$ for the $\Lambda$ and $\Lambda_c$.

\[
\frac{\mu(\Lambda_c)}{\mu(\Lambda)} = \frac{M[ud] m_c}{M[ud] + m_c} \frac{M[ud] + m_s}{M(\Lambda) m_c} = \frac{M(\Lambda)}{M(\Lambda)} m_c = 1.55
\]  

(25)

Now we use the ratio $\Delta E_L(\Lambda_c)/\Delta E_L(\Lambda) = 0.903 \pm 0.004$ to extract an effective power $p = -0.23 \pm 0.01$ which will be used to extrapolate to the $\Lambda_b$ system:

\[
\Delta E_L(\Lambda_b) = \Delta E_L(\Lambda_c) \left[ \frac{\mu(\Lambda_b)}{\mu(\Lambda_c)} \right]^p = \Delta E_L(\Lambda_c) \left[ \frac{M(\Lambda_c) m_b}{M(\Lambda_b) m_c} \right]^p
\]

\[
= \Delta E_L(\Lambda_c) \left[ \frac{M(\Lambda_c) [M(\Lambda_b) - M(\Lambda) + m_s]}{M(\Lambda_b) [M(\Lambda_c) - M(\Lambda) + m_s]} \right]^p
\]

(26)

\[
= \Delta E_L(\Lambda_c) \left[ \frac{1 - \frac{M(\Lambda) - m_s}{M(\Lambda_b)}}{1 - \frac{M(\Lambda) - m_s}{M(\Lambda_c)}} \right]^p = 317 \pm 1 \text{ MeV}
\]

where the last form of the expression shows the explicit dependence of the result on $m_s$. Using the value $M(\Lambda_b) = (5619.7 \pm 1.2 \pm 1.2)$ MeV observed by the CDF Collaboration [12], and rescaling the fine-structure splittings of Eq. (24) by $1/m_Q$ with $m_b/m_c = 2.95 \pm 0.06$, we find

\[
M(\Lambda^*_b[1/2]) - M(\Lambda_c[1/2]) = \frac{m_c}{m_b} (M(\Lambda^*_b[1/2]) - M(\Lambda^*_c[1/2])) = (11.1 \pm 0.4) \text{ MeV} ,
\]

(27)

\[
M(\Lambda^*_b[3/2]) = (5940 \pm 2) \text{ MeV} , \quad M(\Lambda^*_b[3/2]) = (5940 \pm 2) \text{ MeV} .
\]

The observed values of the $\Sigma_b$ masses [13],

\[
M(\Sigma_b^-) = 5815.2\pm1.0(\text{stat.}) \pm 1.7(\text{syst.}) \text{ MeV}
\]

\[
M(\Sigma_b^+) = 5807.8^{+2.0}_{-2.2} (\text{stat.}) \pm 1.7(\text{syst.}) \text{ MeV}
\]

(29)

are sufficiently close to the predicted values of $M(\Lambda^*_b[1/2,3/2])$ that the decays $\Lambda^*_b[1/2,3/2] \to \Sigma_b^\pm \pi^\mp$ are forbidden. The $\Lambda^*_b[1/2,3/2]$ should decay directly to $\Lambda_b \pi^+ \pi^-$.  

A similar calculation may be performed for the orbitally-excited $\Xi_b$ states. Here, to a good approximation [10], one may regard the $[sd]$ diquark in $\Xi_b^-$ or the $[su]$ diquark in $\Xi_b^0$ as having spin zero, so that methods similar to those applied for excited $\Lambda_b$ states should be satisfactory. We find

\[
\Delta E_L(\Xi_b) = \Delta E_L(\Xi_c) \left[ \frac{\mu(\Xi_b)}{\mu(\Xi_c)} \right]^p = \Delta E_L(\Xi_c) \left[ \frac{M(\Xi_c) m_b}{M(\Xi_b) m_c} \right]^p = (322 \pm 2) \text{ MeV} .
\]

(30)
Now we use the observed $\Xi_b^-$ mass \(^3\) \(M(\Xi_b^-) = (5792.9 \pm 2.5 \pm 1.7)\) MeV and our estimate of isospin splitting \(M(\Xi_b^-) - M(\Xi_b^0) = 6.4 \pm 1.6\) MeV to predict the isospin-averaged value \(M(\Xi_b) = 5790 \pm 3\) MeV. We then rescale the fine-structure splitting \(^{24}\) and find
\[
\Xi_b^{*\frac{3}{2}} - \Xi_b^{*\frac{1}{2}} = \frac{m_c}{m_b}(\Xi_c^{*\frac{3}{2}} - \Xi_c^{*\frac{1}{2}}) = (9.1 \pm 0.9)\) MeV ,
\](31)
\[
M(\Xi_b^{*\frac{1}{2}}) = (6106 \pm 4)\) MeV , \(M(\Xi_b^{*\frac{3}{2}}) = (6115 \pm 4)\) MeV .
\](32)
The lower state decays to $\Xi_b \pi$ via an S-wave, while the higher state decays to $\Xi_b \pi$ via a D-wave, and hence should be narrower. Decays to $\Xi_b' \pi$ and $\Xi_b^* \pi$ also appear to be just barely allowed, given the values of $M(\Xi_b'; \Xi_b^*)$ predicted in Ref. \([10]\).

5 Conclusions

We have predicted the masses of several baryons containing $b$ quarks, using descriptions of the color hyperfine interaction which have proved successful for earlier predictions. Correcting for wave function effects, we find \(M(\Omega_b) = 6052.1 \pm 5.6\) MeV and \(M(\Omega_b^*) = 6082.8 \pm 5.6\) MeV. These values are below others which have appeared in the literature as a result of our use of hadrons containing strange quarks to evaluate the effective $b-c$ mass difference, the inclusion of electromagnetic contributions, and because of a different hyperfine splitting.

We have evaluated the isospin splitting of the $\Xi_b$ states and find \(\Delta I(\Xi_b) \equiv M(\Xi_b^-) - M(\Xi_b^0) = 6.24 \pm 0.21\) MeV on the basis of an extrapolation from the $\Xi$ and $\Xi^*$ states. This value is consistent with one which includes information from the $\Xi_c$ states, \(\Delta I(\Xi_b) = 6.4 \pm 1.6\) MeV.

We have also evaluated the orbital excitation energy for $\Lambda_b$ and $\Xi_b$ states in which the light diquark ($ud$ or $us$) remains in a state of $L = S = 0$. Precise predictions have been given for the masses of the states $\Lambda_b^{*\frac{1}{2}, \frac{3}{2}}$ and $\Xi_b^{*\frac{1}{2}, \frac{3}{2}}$.

Our predictions are summarized in Table IV. We look forward to further experimental progress in the tests of these predictions.

Table IV: Summary of predictions for $b$ baryons

| Mass in MeV |
|-------------|
| $M(\Omega_b)$ | 6052.1 ± 5.6 |
| $M(\Omega_b^*)$ | 6082.8 ± 5.6 |
| $M(\Xi_b^0)$ | 5786.7 ± 3.0 |
| $M(\Lambda_b^{*\frac{1}{2}})$ | 5929 ± 2 |
| $M(\Lambda_b^{*\frac{3}{2}})$ | 5940 ± 2 |
| $M(\Xi_b^{*\frac{1}{2}})$ | 6106 ± 4 |
| $M(\Xi_b^{*\frac{3}{2}})$ | 6115 ± 4 |
Appendix: Values of quark masses

In choosing values for the quark masses used in this paper, we note that values of quark mass differences can be taken from the difference in masses of baryons containing spin-zero $ud$ diquarks

$$m_i - m_j = M(\Lambda_i) - M(\Lambda_j)$$  \hspace{1cm} (33)

where $i$ and $j$ can be $b, c$ or $s$. This gives

$$m_c = M(\Lambda_c) - M(\Lambda) + m_s = (2286.5 - 1115.68 + 538) \text{ MeV} = 1709 \text{ MeV}$$

$$m_b = M(\Lambda_b) - M(\Lambda_c) + m_c \pm 10 \text{ MeV} = M(\Lambda_b) - M(\Lambda) + m_s \pm 10 \text{ MeV}$$  \hspace{1cm} (34)

$$= 5619.7 - 1115.68 + 538 \pm 10 \text{ MeV} = 5042 \pm 10 \text{ MeV}$$

where $m_s = 538 \text{ MeV}$ has been taken from the fit of Ref. [18] to light-quark baryon spectra.

We have noted [6] that an uncertainty of 10 MeV arises from the difference between the values of $m_b - m_c$ obtained from hadrons having strange and nonstrange spectators. We have chosen the value obtained from strange spectators following its use in previous successful predictions [10].

Although this difference is crucial in predictions like Eq. (2) which depend on mass differences, its effect on mass ratios is negligible. We therefore use the values obtained from baryons with nonstrange spectators to obtain a value for the mass ratio $m_b/m_c$.

$$\frac{m_b}{m_c} = \frac{[M(\Lambda_b) - M(\Lambda) + m_s + \delta m]}{[M(\Lambda_c) - M(\Lambda) + m_s + \delta m]} = \frac{5042 + \delta m}{1709 + \delta m} \approx 2.95 - \frac{\delta m}{876 \text{ MeV}},$$  \hspace{1cm} (35)

where we have introduced the quantity $\delta m$ to take care of any errors in the assumption that $m_s = 538 \text{ MeV}$ and neglected the 10 MeV uncertainty in $m_b$. Taking $\delta m = \pm 50 \text{ MeV}$ in the calculation of $m_b/m_c$ gives the value $m_b/m_c = 2.95 \pm 0.06$ used in the $\Omega_b$ mass prediction.

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