$U_e(1) \times U_g(1)$ Actions in $2 + 1$–Dimensions: Full Vectorial Electric and Magnetic Fields

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Abstract

It is considered a dimensional reduction of $U_e(1) \times U_g(1)$ 3 + 1–dimensional electromagnetism with a gauge field (photon) and a pseudo-vector gauge field (pseudo-photon) to 2 + 1–dimensions. In the absence of boundary effects, the quantum structure is maintained, while when boundary effects are considered, as have been previously studied, a cross Chern-Simons term between both gauge fields is present, which accounts for topological effects and changes the quantum structure of the theory. Our construction maintains the dimensional reduced action invariant under parity ($P$) and time-inversion ($T$). We show that the theory has two massive degrees of freedom, corresponding to the longitudinal modes of the photon and of the pseudo-photon and briefly discuss the quantization procedures of the theory in the topological limit (wave functional quantization) and perturbative limit (an effective dynamical current theory), pointing out directions to solve the constraints and deal with the negative energy contributions from pseudo-photons. We recall that the physical interpretation of the fields in the planar system is new and is only meaningful in the context of $U_e(1) \times U_g(1)$ electromagnetism. In this work it is shown that all the six electromagnetic vectorial fields components are present in the dimensional reduced theory and that, independently of the embedding of the planar system, can be described in terms of the two gauge fields only. As far as the author is aware it is the first time that such a construction is fully justified, thus allowing a full vectorial treatment at variational level of electromagnetism in planar systems.

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1 Introduction

Several physical phenomena are effectively two dimensional, for example Bloch electrons [1], superconductivity [2] and the Hall effect [3], to name a few. Describing electromagnetism at variational level in 2 + 1–dimensions lead us to the respective Maxwell action and electromagnetic field definitions

$$S_{\text{Maxwell}} = \int dx^3 F_{\mu\nu} F^{\mu\nu},$$

$$E^i = F^0_{\; i} \quad (i = 1, 2), \quad B = F_{12},$$

such that in relation to 3 + 1–dimensional electromagnetism, only the planar electric field components and the orthogonal magnetic field component are present, and the theory has no propagating degrees of freedom. For many models and theories involving electromagnetic fields in planar or embedded systems these are enough to describe the physics in question. Also, for some application where boundary effects are relevant, topological effects are considered through the inclusion of a topological Chern-Simons term

$$S_{\text{CS}} = m \int dx^3 \epsilon^{\mu\nu\lambda} A_\mu F_{\nu\lambda}.$$  

This term is of lower order than the Maxwell term (it has only one derivative in the fields) and in some applications it is dominant in relation to the Maxwell term. It changes the quantum structure of the
theory and gives a topological mass \( m \) to the gauge field \( A \) \(^4\) which implies that in the perturbative regime the theory has one propagating degrees of freedom (longitudinal massive mode), as opposed to the Maxwell theory. However it is, generally non-renormalizable \(^5\) and explicitly breaks parity \( P \) and time-inversion \( T \) symmetries. In addition to having propagating degrees of freedom, it is often justified theoretically, as a quantum correction to the Maxwell action \(^5\). From a more phenomenological perspective it has been used as an effective description of the fractional hall effect \(^6\) through a collective gauge field and it also describes boundary conformal field theories \(^7\). However a relevant question can be asked, can we have a description of planar electromagnetism containing the full vectorial electromagnetic fields as we do in standard electromagnetism? Possible approaches to answer this question are to consider four dimensional descriptions of the planar physics based in extensions of the Maxwell action including Kalb-Ramon form fields \(^8\) or to consider dimensional reduction of standard electromagnetism such that the gauge field component orthogonal to the planar system is described effectively by a scalar field \(^9\). The physical meaning of a Kalb-Ramon form field \(^10\) is to effectively describe either dynamical vortexes \(^11\) or dynamical currents \(^12\). Usually imply chiral symmetry breaking and can also be described effectively as planar Chern-Simons theories \(^13\), which take us back to the original Maxwell Chern-Simons theories already discussed, hence our question remains pertinent. The scalar field obtained upon dimensional reduction, depending in the boundary conditions and symmetries of the embedding in the four dimensional system, may be null (this is the case, for example, for standard Neumann boundary conditions). In what follows we will consider another possible construction by deriving a dimensional reduction of electromagnetism with one vector gauge field (the standard photon) and one pseudo-vector gauge field (pseudo-photon). We will obtain directly a boundary cross Chern-Simons term that accounts for topological boundary effects and simultaneously preserves \( P \) and \( T \) symmetries, this feature is not new in our work and have been studied in detail in \(^15\) \(^14\). Also this theory accommodates the six components of the physical electric and magnetic fields defined in terms of the gauge fields and independently of the boundary conditions and embedding. This feature is, in principle, physically appealing. Even being in a planar system, we are still dealing with the same physics, in particular electromagnetism. Also we are considering here a dimensional reduced theory which is theoretically consistent with the four dimensional Maxwell equations. We note that originally \( U_e(1) \times U_g(1) \) has been justified by the inclusion of magnetic monopoles \(^16\) \(^18\) \(^19\) \(^17\) maintaining the field configurations regular, i.e. free of extended singularities as the Dirac String and Wu-Yang fiber-bundle \(^20\). Although the existence of magnetic monopoles is still the best justification for electric charge quantization \(^20\), so far they have not been experimentally observed. However we remark that it is enough to consider non-trivial background electric fields (\( \nabla \cdot E \neq 0 \)) or magnetic fields (\( \dot{B} \neq 0 \)) to justify the existence, at functional level, of pseudo-photons \(^21\) and justify extended \( U_e(1) \times U_g(1) \) electromagnetism. There will be new problems associated to this theory that we shortly discuss by the end of this work, namely due to the pseudo-photon being a ghost (or phantom, it has a negative kinetic term in the Hamiltonian), hence the quantization of the theory is not a straight forward task. Nevertheless, this apparent nuisance, may turn out to justify the low energy of the Laughlin’s wave functions due to the negative contributions of pseudo-photon excitations, as was put forward in \(^22\) using a semi-classical model. Also the model of \(^22\) proofs the equivalence between Dirac’s quantization condition and the, experimentally verified, quantization of magnetic flux. Moreover a more fundamental description of any theory underlines a better understanding of the theory, therefore more reliable prediction and control of physical systems.

We consider as starting point the 3 + 1–dimensional action for \( U_e(1) \times U_g(1) \) electromagnetism introduced in \(^13\) \(^19\) given by \( S_4 = \int d^4x \mathcal{L}_4 \) and Lagrangian density

\[
\mathcal{L}_4 = -\frac{1}{4} F_{IJKL} F^{IJKL} + \frac{1}{4} G_{IJ} G^{IJ} + \frac{1}{4} \epsilon^{IJKL} F_{IJK} G_{KL} + A_e^I J_e^I,
\]

where the gauge connections are \( F_{IJK} = \partial_I A_J - \partial_J A_I \) and \( G_{IJ} = \partial_I C_J - \partial_J C_I \) with the space-time indexes \( I = \perp, \mu \) such that \( \perp \) stands for the spatial direction orthogonal to the planar system and \( \mu = 0, 1, 2 \) correspond to the 2 + 1–dimensional space-time indexes.

2 Dimensional Reduction

In this section we address a possible dimensional reduction scheme for action \(^3\). We consider a planar system of a certain thickness \( \delta \), with two boundaries \( \Sigma_1 \) and \( \Sigma_2 \) as represented in figure \(^11\). Let us take in consideration the regularity of the each of the gauge fields, such that the constants \( r_A = 0, 1 \) and
Figure 1: 2 + 1–dimensional system of thickness $\delta_\perp$, with boundaries $\Sigma_1$ and $\Sigma_2$, embedded in a 3 + 1–dimensional system.

$r_C = 0, 1$ are set accordingly to:

$$r_{A(C)} = 0, 1 : A(C) \text{ is regular, non−regular}$$  \hspace{1cm} (4)

Then taking a decomposition of the Maxwell and current terms in action (3) into the 2 + 1-dimensional components $\mu$ and the orthogonal direction $x_\perp$ and integrate by parts the Hopf term containing derivatives along $x_\perp$, we obtain

$$L_4 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + A_\perp J_\perp^\perp + A_\mu J_\mu^\perp$$

$$-\frac{1}{2} \partial_{\perp} A_\mu \partial_\perp A^\mu + \partial_\perp A_\mu \partial^\mu A_\perp - \frac{1}{2} \partial_{\mu} A_\perp \partial^\mu A_\perp$$

$$+ \frac{1}{2} \partial_\mu C_\perp \partial_\mu C^\perp - \partial_\perp C_\mu \partial^\mu C_\perp + \frac{1}{2} \partial_{\mu} C_\perp \partial^\mu C_\perp$$

$$+ \epsilon^{\mu\nu\lambda} \partial_\perp (A_\mu \partial_\nu C_\lambda) + \epsilon^{\mu\perp\lambda} \partial_\perp (\partial_\mu A_\lambda C_\perp)$$

$$- r_C \epsilon^{\mu\perp\lambda} A_\mu \partial_\nu C_\lambda - r_A \epsilon^{\mu\perp\lambda} (\partial_\mu A_\lambda) C_\perp$$

$$+ r_C r_A \epsilon^{\mu\perp\lambda} \partial_\mu A_\perp \partial_\nu C_\lambda + r_A r_C \epsilon^{\mu\perp\lambda} \partial_\mu A_\perp \partial_\lambda C_\perp. \hspace{1cm} (5)$$

We are also taking the following assumptions:

1. The fields are localized in the range

$$x_\perp \in [-\delta_\perp/2, +\delta_\perp/2], \hspace{1cm} (6)$$

and are slowly varying over the orthogonal direction such that the integral over $x_\perp$ can be performed in the above range.

2. Gauge transformations are fixed along the orthogonal direction, such that

$$\partial_\perp \Lambda = 0. \hspace{1cm} (7)$$

3. The orthogonal derivatives of the fields constitute boundary conditions of the system such that their effects are manifested at the level of the action through the external currents

$$J_\perp^\mu = \partial_\perp A^\mu, \hspace{0.5cm} J_\perp^\mu = \partial_\perp C^\mu. \hspace{1cm} (8)$$

4. The field components $A_\perp$ and $C_\perp$ are, in the 2 + 1-dimensional system, not charged under any of the $U(1)'s$ and are identified to scalar fields $\phi$

$$\phi = A_\perp, \hspace{0.5cm} \varphi = C_\perp. \hspace{1cm} (9)$$

5. We are taking in account the boundary terms due to the integration by parts of the Hopf term (here we are referring to the last terms in equation (5)). In order to do so we identify $\Sigma(x_\perp = 0) \cong \Sigma_1 \cong \Sigma_2$ and considering the boundary action [15]

$$\int_{\Sigma_1 - \Sigma_2} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu C_\lambda \equiv \frac{k}{2} \int_{\Sigma} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu C_\lambda, \hspace{1cm} (10)$$

where the value of $k = +1, 0, −1$ depends on the specifications of the system, in particular of the embedding and symmetries of the 2 + 1–dimensional system in relation to the 3 + 1–dimensional system [15].
We can now perform the integration over the orthogonal coordinate \( \int_{\delta \perp/2}^{1/2} dx^\perp = \delta \perp \) obtaining the dimensional reduced action \( S_3 = \delta \perp \int dx^3 \mathcal{L}_3 \) with Lagrangian density

\[
\mathcal{L}_3 = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{4} G_{\mu \nu} G^{\mu \nu} + \frac{1}{2} J^\mu_A J_{A \mu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} J^\mu_C J_{C \mu} - \frac{T_C}{2} \epsilon^{\mu \nu \lambda} F_{\mu \nu} J_{C \lambda} + \frac{T_A}{2} \epsilon^{\mu \nu \lambda} G_{\mu \nu} J_{A \lambda} - \frac{r_{AC}}{2} \epsilon^{\mu \nu \lambda} \partial_\mu \varphi F_{\nu \lambda}
\]

We have performed an integration by parts in writing the current terms for \( J^\mu_A \) and \( J^\mu_C \) and the terms containing the 2 + 1-dimensional Levi-Civita symbol swap sign due to the relation \( \epsilon^{\mu \nu \lambda} = - \epsilon^{\nu \mu \lambda} \). We note that the square terms in the currents \( J^\mu_A J_{A \mu} = (\partial^2 A^\mu)^2 \) and \( J^\mu_C J_{C \mu} = (\partial^2 C^\mu)^2 \) are non-dynamical constants which stand for boundary conditions as given by \( \delta \). These are relevant only as a contribution to the vacuum energy being relevant, for example, when one considers gravity \([23]\), such that they play the role of an effective 2 + 1-dimensional cosmological constant \( 4 \Lambda_\text{eff} = (\partial^2 A^\mu)^2 - (\partial^2 C^\mu)^2 \). In this work these terms play no role and we may simply consider that its effect corresponds to a vacuum energy shift. We further note that our 2 + 1-dimensional action is, as it stands, gauge invariant as long as \( \phi \) and \( \varphi \) are not charged under any of the gauge groups \( U(1) \)'s. We recall that we gauge fixed along the orthogonal direction \( \pi^\perp \) and that the coupling between the scalar fields \( \phi \) and \( \varphi \) and the gauge fields \( A \) and \( C \) is done only through the currents \( J_A \) and \( J_C \). Extensions of this construction can be implemented such that the scalar fields are charged under the gauge groups, in such case it is necessary to consider gauge covariant derivatives such that the action remains gauge invariant \([24]\). We do not develop this topic here. We also remark that the middle terms in the second and third line of equation \( \delta \) could be integrated by parts, however this would hold boundary terms that are not gauge invariant.

### 3 Electromagnetic Fields, Canonical Momenta and Boundary Conditions

From the definitions of electric and magnetic field in the 3 + 1-dimensional system \([19][17]\) we obtain the physical electric and magnetic fields definitions in the planar system:

\[
E^\perp = \partial^\perp \phi - J^\perp_A - G_{12},
\]

\[
E^i = F^{0i} - \frac{T_C}{2} \epsilon^{ij} (r_A \partial_j \varphi - J_{C j}),
\]

\[
B^\perp = -\partial^\perp \varphi + J^\perp_C + F_{12},
\]

\[
B^i = -G^{0i} + \frac{T_A}{2} \epsilon^{ij} (r_C \partial_j \phi - J_{A j}),
\]

where we have taken in consideration the regularity of each of the gauge fields as given by \( \delta \). Concerning the canonical momenta for our action we obtain six independent ones

\[
\pi^\perp_A = -F^{0i} + \frac{T_{AC}}{2} \epsilon^{ij} \partial_j \varphi - \frac{r_C}{2} \epsilon^{ij} J_{C j} + \frac{k}{4 \delta \perp} \epsilon^{ij} C_j,
\]

\[
\pi^\perp_C = +G^{0i} - \frac{T_{AC}}{2} \epsilon^{ij} \partial_j \phi + \frac{r_A}{2} \epsilon^{ij} J_{A j} + \frac{k}{4 \delta \perp} \epsilon^{ij} A_j,
\]

\[
\pi_\phi = -\partial^\perp \phi + J^\perp_A - r_{AC} G_{12},
\]

\[
\pi_\varphi = +\partial^\perp \varphi - J^\perp_C + r_{AC} F_{12}.
\]

We note that the relations between the canonical conjugate momenta to \( A_i \) and \( C_i \) \([19]\) and the electric and magnetic fields \([12]\) hold a new correction due to the Chern-Simons boundary contribution in
relation to the 3 + 1–dimensional relations obtained in [19]. Here we are referring to the last terms depending on \( k \) in the definitions of \( \pi^A_1 \) and \( \pi_1^{\perp} \) in (13). This is a common feature of Chern-Simons theories [4]. For completeness on the discussion of definition of canonical momenta, it is relevant to point out that, considering directly the 2+1–dimensional theory without any assumptions of the embedding in 3+1–dimensions, may raise an indefiniton of the canonical structure. This is due to the Chern-Simons term to be defined only up to a boundary term, specifically considering an integration by parts we have that \( \int [A \wedge dC + C \wedge dA] = \int [(1 + \xi) A \wedge dC + (1 - \xi) C \wedge dA] + \xi \oint A \wedge C \). This result is already present in the original work by Schwarz [13] and can be traced back to the choice of a quantization polarization for pure Chern-Simons theories [25]. In simple terms which fields are the canonical coordinates and which fields are the canonical momenta. When we consider both the Maxwell and Chern-Simons terms this indefiniton can be solved, in [26] it was shown that when all topological effects are taking in consideration, including the boundary effects due to integration by parts, the Chern-Simons action for several gauge fields is reduced to the form \( S_{CS} = \int G_{ij} A_i^A dA^i + \oint K_{ij} A^i \wedge A^j \) with \( G_{ij} \) a symmetric matrix and \( K_{ij} \) a anti-symmetric matrix. In addition it has recently been showed that, independently of boundary effects, a consistent definition of canonical structure is still possible [27]. To finalize let us stress that, independently of the above discussion, the canonical momenta \( \pi^A_1 \) and \( \pi^C_1 \) as defined in (13) are directly derived from the 3 + 1–dimensional theory, thus being consistent with the higher dimensional canonical momenta as derived in [19]. This is enough to justify the above choice.

So far we have not imposed any particular boundary conditions in our construction. To properly define the field content of the dimensional reduced theory and its embedding into the higher dimensional world it is necessary to do so. The standard types of boundary conditions correspond usually to gauging symmetries of the fields with respect to the orthogonal direction \( x^\perp \) [28]. Specifically when the orthogonal fields swap sign with respect to both sides of the planar system we have Neumann boundary conditions, such that they must vanish in the planar system, i.e. \( A_\perp = C_\perp = 0 \). These conditions correspond to an orbifold under the symmetry \( \mathbb{Z}_2 : x^\perp \to -x^\perp \); usually are applicable to the internal fields (meaning the fields that are not imposed externally), and the factor of \( 1/2 \) in the Chern-Simons boundary contributions to the action in (10) can also be justified by this kind of orbifolds [29]. When the fields do not change across the orthogonal direction to the planar system we have Dirichlet boundary conditions such that the orthogonal derivatives of fields vanish in the planar system, i.e. \( \partial_\perp A = \partial_\perp C = 0 \). This kind of boundary conditions usually applies to the external applied fields. For Neumann boundary conditions for all fields, generally we have \( k \neq 0 \), while for Dirichlet boundary conditions for all fields we always have \( k = 0 \). This is shown by noting that the integrations by parts in the action decomposition (5) are null for \( \partial_\perp A_\mu = 0 \). This is resumed as

\[
\text{Neumann : } \quad A_\perp = C_\perp = 0 \quad \Leftrightarrow \quad \phi = \varphi = 0 , \tag{14}
\]

\[
\text{Dirichlet : } \quad \partial_\perp A_\mu = \partial_\perp C_\mu = 0 \quad \Leftrightarrow \quad J^\mu_A = J^\mu_C = 0 .
\]

However, generally, any boundary conditions or combination of boundary conditions can be considered. In particular for systems where spin polarization effects are not relevant (i.e. considering the magnetic momenta of fermions to be null) and when considering both external and internal fields we should, consistently with the above discussion, consider Neumann boundary conditions for the internal fields and Dirichlet boundary conditions for the external fields. Finally considering Neumann boundary conditions (\( \varphi = \phi = 0 \)), regular fields \( (r_A = r_C = 0) \) and constant orthogonal fields \( (J_A = \partial^A = A = 0 \) and \( J_C = \partial^C = C = 0) \), we obtain the action, electromagnetic field and canonical momenta definitions

\[
S_3 = \delta_\perp \int d^3 x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{8\delta_\perp} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{1}{8\delta_\perp} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda} \right] ,
\]

\[
E^\perp = -G_{12}, \quad E^i = F^{0i}, \quad B^\perp = F_{12}, \quad B^i = -G^{0i},
\]

\[
\pi^A_1 = -F^{0i} + \frac{k}{4\delta_\perp} \epsilon^{ij} C_j , \quad \pi^C_1 = C^{0i} + \frac{k}{4\delta_\perp} \epsilon^{ij} A_j.
\]

Hence we have available the six components of the electric and magnetic fields as we do in the original 3+1–dimensional system and its definitions are fully consistent in the framework of extend \( U_e(1) \times U_o(1) \) electromagnetism. This is a novel feature of our construction which is certainly useful in the description of planar systems where longitudinal magnetic fields and orthogonal electric fields are present.
4 Degrees of Freedom and Quantization

We have pointed out that one of the motivations of considering the usual Chern-Simons action \( \mathcal{S} \), is to have propagating degrees of freedom for the photon. Hence it is relevant to ask which are the propagating degrees of freedom for the action (15). Computing the equations of motion for \( A \) and \( C \) we obtain respectively
\[
\partial_\mu F^{\mu \nu} + \frac{k}{4\delta_\perp^2} \epsilon^{\mu \nu \lambda} G_{\nu \lambda} = 0 ,
\]
\[
\partial_\mu G^{\mu \nu} - \frac{k}{4\delta_\perp^2} \epsilon^{\mu \nu \lambda} F_{\nu \lambda} = 0 .
\]
These equations can be decoupled. By considering the solutions of the above equations for \( G_{\mu \nu} \) and \( F_{\mu \nu} \), respectively, and replacing it in the remaining equation, we obtain
\[
\partial_\mu \partial^\nu A^\nu + 2 \left( \frac{k}{2\delta_\perp^2} \right)^2 A^\nu = 0 ,
\]
\[
\partial_\mu \partial^\nu C^\nu + 2 \left( \frac{k}{2\delta_\perp^2} \right)^2 C^\nu = 0 .
\]
In deriving this result we have used the identity \( \epsilon_{\mu \nu \lambda} \epsilon^{\mu \nu} = -2(\delta_{\nu \lambda} \delta_{\mu} - \delta_{\mu \lambda} \delta_{\nu}) \) and performed an integration such that this result is valid up to a total divergence \( \partial^\nu \Lambda \) which can be offset by an appropriate gauge transformation (or by considering an appropriate gauge choice). This result shows that we have massive propagating degrees of freedom (one longitudinal mode for the photon and another one for the pseudo-photon) with mass
\[
m_A = m_C = \sqrt{2} \frac{|k|}{2\delta_\perp^2} .
\]
This is actually welcome, we manage to obtain propagating degrees of freedom maintaining \( P \) and \( T \) invariance.

Concerning quantization of the theory we are faced with new problems that we briefly discuss next, pointing directions for further developments. From the action and the definitions of canonical momenta given in [15] we obtain the Hamiltonian and Gauss’ laws constraints
\[
\mathcal{H} = \frac{1}{2} \left( \pi_A - \frac{k}{4\delta_\perp^2} \epsilon^{ij} \partial_i C_j \right)^2 + \frac{1}{4} F_{ij} F^{ij} ,
\]
\[
- \frac{1}{2} \left( \pi_C - \frac{k}{4\delta_\perp^2} \epsilon^{ij} \partial_i C_j \right)^2 = \frac{1}{4} G_{ij} G^{ij} ,
\]
\[
\mathcal{G}_A = \partial_i \left[ \pi_A + \frac{k}{4\delta_\perp^2} \epsilon^{ij} \partial_j C_i \right] ,
\]
\[
\mathcal{G}_C = \partial_i \left[ \pi_C + \frac{k}{4\delta_\perp^2} \epsilon^{ij} \partial_j A_i \right] .
\]
The Hamiltonian constraint contains two distinct sectors corresponding to the usual photon (the \( A \) field) and the pseudo-photon (the \( C \) field). We readily conclude that, as already expected from the \( 3+1 \)-dimensional theory [19], excitations of photons contribute positively to the energy of the quantum state, while pseudo-photon excitations contribute negatively to the energy of the quantum state. In principle existence of negative energy states are unwelcome since they violate causality. The most straight forward way to solve this problem, although not very elegant, is to postulate positive energy for the quantum state such that the contribution due to excitations of pseudo-photons can never overcome the contribution due to excitations of photons. However we remark that, in addition to the Hamiltonian constraint, consistently with the equations of motion (16), both sectors are related through the Gauss’ laws and, upon quantization, these constraints must also to be taken in consideration. Depending on the way we implement these constraints we can have independent excitations, corresponding to each sector, or have a combination of the degrees of freedom, such that physical excitations correspond to a combination of photon and pseudo-photon excitations. These two possibilities correspond to the topological regime of the theory (low-energy) and the perturbative regime of the theory (high-energy). In the topological limit the appropriate approach is to consider a functional quantization formalism [23] [30] such that the quantum constraints (19) are solved at the level of wave functionals. The ground state solution is known to be
\[
\Phi[A, C] = \exp \left\{ + i \frac{k}{4\delta_\perp^2} \int d^2 x \epsilon^{ij} A_i C_j \right\} ,
\]
which leaves both sectors of the theory unconstrained. Excited topological states for Maxwell Chern-Simons theories have been introduced in [31], however have not been properly consider in the present context, for which are expected to correspond to electric and magnetic vortexes as put forward in [22]. For this case the external fields are expected to drive (control) the excitations of the system such that imposing energy positiveness is, in principle, not necessary. As for the perturbative limit we can consider the usual quantum field theory harmonic operator expansion of the fields that leaves both sectors of the theory unconstrained. Excited topological states for Maxwell Chern-Simons theories have been introduced in [31], however have not been properly consider in the present context, for which are expected to correspond to electric and magnetic vortexes as put forward in [22].

Simons theories have been introduced in [31], however have not been properly consider in the present context, for which are expected to correspond to electric and magnetic vortexes as put forward in [22]. Energy excitations, the Maxwell term is playing the role of mass, while the non-linear correction plays the role of the kinetic term. We cannot avoid to notice some similarities between this effective non-linear massive theory which holds an effective description of fermion effects, chiral symmetry breaking may be expected. We have preliminary indicated chiral theory and the four dimensional chiral dynamical current theories [12]. Also we note that, upon inclusion of fermion effects, chiral symmetry breaking may be expected. We have preliminary indicated the possibilities concerning quantization of the limiting cases for the theory that allow to avoid strictly negative energy quantum states. We will develop these issues in detail somewhere else [32].

5 Conclusions

We derived a dimensional reduced action for $U_e(1) \times U_g(1)$ electromagnetism containing a vector gauge field (photon) and a pseudo-vector gauge field (pseudo-photon). We obtain a 2 + 1–dimensional action that is $P$ and $T$ invariant, has massive propagating degrees of freedom and, independently of the boundary conditions and embedding of the planar system, accounts for the full six vectorial components of the electromagnetic fields in terms of dynamical massive electric currents $j^\nu = \partial_\mu F^{\mu \nu} = (\partial_i E^i, -E^0 + e/\hbar \partial_i B)$. It has no negative energy excitations, the Maxwell term is playing the role of mass, while the non-linear correction plays the role of the kinetic term. We cannot avoid to notice some similarities between this effective non-chiral theory and the four dimensional chiral dynamical current theories [12]. Also we note that, upon inclusion of fermion effects, chiral symmetry breaking may be expected. We have preliminary indicated the possibilities concerning quantization of the limiting cases for the theory that allow to avoid strictly negative energy quantum states. We will develop these issues in detail somewhere else [32].

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References

[1] F. Bloch, Z. Physik 52 (1928) 555-600.
[2] V.L. Ginzburg and L.D. Landau, Zh. Eksp. Teor. Fiz. 20 (1950) 1064; L. N. Cooper, Phys. Rev. 104 (1956) 1189; J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. 106 (1957) 162; Phys. Rev. 108 (1957) 1175.
[3] K. von Klitzing, G. Dorda and M. Pepper, Phys. Rev. Lett. 45 (1981) 494; R. B. Laughlin, Phys. Rev. B23 (1981) 5632; D. C. Tsui, H. L. Stormer and A. C. Gossard, Phys. Rev. Lett. 48 (1982) 1559; R. B. Laughlin, Phys. Rev. Lett. 50 (1982) 1395;
[4] J. Schonfeld, Nucl. Phys. B185 (1981) 157; S. Deser, R. Jackiw and S. Templeton, Annals Phys. 140 (1982) 372-411; Annals Phys. 185 (1988) 406; Annals Phys. 281 (2000) 409-449.
1. A. N. Redlich, Phys. Rev. **D29** (1984) 2366-2374; Phys. Rev. Lett. **52** (1984) 18; S. R. Coleman, B. R. Hill, Phys. Lett. **B159** (1985) 184.

2. S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. **58** (1987) 1252; S.C. Zhang, T. H. Hansson and S. Kivelson, Phys. Rev. Lett. **62** (1989) 82; J. K. Jain, Phys. Rev. Lett. **63** (1989) 199.

3. P. Di Francesco, P. Mathieu and D. Sénéchal, *Conformal Field Theory*, Springer-Verlag NY 1997.

4. A. P. Balachandran and P. Teotonio-Sobrinho, Int. J. Mod. Phys. **A8** (1993) 723, hep-th/9205116.

5. J. H. Belich, Jr. M. M. Ferreira, J. A. Helayel-Neto and M. T. D. Orlando, Phys. Rev. **D67** (2003) 125011; Erratum-ibid. **D69** (2004) 109903.

6. M. Kalb and P. Ramond, Phys. Rev. **D9** (1974) 2273.

7. A. A. Abrikosov, Sov. Phys. JETP **5** (1957) 1174; Zh. Eksp. Teor. Fiz. **32** (1957) 1442; H. B. Nielsen and P. Olesen, Nucl. Phys. **B61** (1973) 45.

8. V.I. Ogievetsky and I.V. Polubarinov, Sov. J. Nucl. Phys. **4** (1967) 156; Yad. Fiz. **4** (1966) 216; H. Sugawara, Phys. Rev. **170** (1968) 1659; C. M. Sommerfield, Phys. Rev. **176** (1968) 2019; S. Deser, Phys. Rev. **187** (1969) 1931; D. Z. Freedman and P.K. Townsend, Nucl. Phys. **B177** (1981) 282; A.P. Balachandran, V.P. Nair, B.S. Skagerstam and C.G. Trahern, Phys. Rev. **D26** (1982) 1443.

9. Ya.I. Kogan and V.V. Fock, JETP Lett. **51** (1990) 210; Mod. Phys. Lett. **A5** (1990) 1365; I. I. Kogan, JETP Lett. **49** (1989) 225; Pisma Zh. Eksp. Teor. Fiz. **49** (1989) 194.

10. A. S. Schwarz, Commun. Math. Phys. **67** (1979) 1-16.

11. E. Witten, in *Shiftman, M. (ed.) et al.: From fields to strings*, vol. 2* 1173-1200, hep-th/0307041.

12. N. Cabibbo and E. Ferrari, Il Nuovo Cimento **XXIII** No 6 (1962) 1147.

13. D. Singleton, Am. J. Phys. **64** (1996) 452; Int. J. Theor. Phys. **34** (1995) 2453, hep-th/9701040.

14. P. C. R. Cardoso de Mello, S. Carneiro e M. C. Nemes, Phys. Lett. **B384** 197, hep-th/9609218.

15. P. Castelo Ferreira, J. Math. Phys. **47** (2006) 072902, hep-th/0510063.

16. P. A. M. Dirac, Proc. Roy. Soc. **A133** (1931) 60; Phys. Rev. **74** (1948) 817; T. T. Wu and C. N. Yang, Phys. Rev. **D12** (1975) 3845; Phys. Rev. **D14** (1976) 437.

17. P. Castelo Ferreira, hep-ph/0609239.

18. P. Castelo Ferreira, hep-th/0703194.

19. M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69** (1992) 1849, hep-th/9204099; I. I. Kogan, Mod. Phys. Lett. **A7** (1992) 2341, hep-th/9205095; S. Carlip, Class. Quant. Grav. **12** (1995) 2845, gr-qc/9506079; S. Fernando and F. Mansouri, Comm. Math. Theor. Phys. **1** (1998) 14, gr-qc/9705016; T. Dereli and Y. N. Obukhov, Phys. Rev. **D62** (2000) 024013, gr-qc/0001017; P. Castelo Ferreira, Class. Quant. Grav. **23** (2006) 3679, hep-th/0506244.

20. P. Castelo Ferreira and J. T. Mendonça, hep-th/0601171; J. T. Mendonça and P. Castelo Ferreira, Europhys. Lett. **75** 189, hep-th/0601166.

21. E. Witten, Commun. Math. Phys. **121** (1989) 351-399; M. Bos and V. P. Nair, Phys. Lett. **B223** (1989) 61; Int. J. Phys. **A5** (1990) 959.

22. I. I. Kogan, Phys. Lett. **B231** (1989) 377; P. Castelo Ferreira, I. I. Kogan and B. Tekin, Nucl. Phys. **B589** (2000) 167, hep-th/0004078.

23. B. Bertrand and J. Govaerts, arXiv:0704.1512v1.

24. J. D. Jackson, *Classical Electrodynamics*, 2nd. Edition, John Wiley & Sons, 1975.

25. P. Horava, J. Geom. Phys. **21** (1996) 1-33.

26. M. Asorey, F. Falceto and S. Carlip, Phys. Lett. **B312** (1993) 477-485, hep-th/9304081.

27. P. Castelo Ferreira, I. I. Kogan and R. Szabo, Nucl. Phys. **B676** (2004) 243, hep-th/0308101.

28. Work in progress.

29. M. C. Diamantini, P. Sodano and C. A. Trugenberg, Nucl. Phys. **B474** (1996) 641-677, hep-th/9511168; Eur. Phys. J. **B53** (2006) 19, hep-th/0511192; J. Phys. **A39** (2006) L253-258; hep-th/0703140.

30. T. W. B. Kibble, J. Phys. **A9** (1976) 1387; Phys. Rep. **67** (1980) 183; W. H. Zurek, Nature **317** (1985) 505; Acta Phys. Pol. **B24** (1993) 1301; Phys. Rep. **276** (1996) 177; N. D. Antunes, L. M. A. Bettencourt and W. H. Zurek, Phys. Rev. Lett. **82** (1999) 2824; E. Kavoussanaki, R. Monaco and R.J. Rivers, Phys. Rev. Lett. **85** (2000) 3452, cond-mat/0005145; D.-S. Lee, C.-Y. Lin and R. J. Rivers, Phys. Rev. Lett. **98** (2007) 020603, cond-mat/0606243.

31. J. P. Eisenstein and A. H. MacDonald, Nature **432** (2004) 691.