CMPCC: Corridor-based Model Predictive Contouring Control for Aggressive Drone Flight

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1 Motivation, Problem Statement, Related Work

Among the criteria of designing autonomous quadrotors, generating optimized trajectories and tracking the flight paths precisely are two critical components in the action aspect. As shown in our recent work Teach-Repeat-Replan [2], a cascaded planning framework with global trajectory generation and local collision avoidance support agile flights under the user preferable routines. In work [2], though the first criteria is met by the global and local planners, the controller has no guarantees on tracking the generated motion precisely. Also, in industrial applications, the planner and controller of a quadrotor are mostly independently designed, making it hard to tune the joint performance in different applications.

Some works [7, 4] attempt to compensate uncertainties introduced by disturbances, by designing error-tolerated trajectory planning methods based on Hamilton-Jacobi Reachability Analysis [1]. These works set handcrafted disturbance bounds, making it too conservative to find a feasible solution among dense obstacles. Tal et.al. [9] propose control systems for accurate trajectory tracking that improves tracking accuracy. Nevertheless, they still try to track the unreachable trajectory when facing violent disturbance instead of adjusting the primary trajectory. If un-negligible disturbance occurs, local replanners such as [11, 10] can plan motions to rejoin the reference quickly, but they are inferior to give a proper temporal distribution. The closest work to this paper [5] applies Model predictive contouring control (MPCC) [3] as the planner for miniature car racing, where safety constraints are established by modeling linear functions from the boundary of the racing track. However, such constraints are not directly available for a quadrotor in unstructured environments. What’s more, due to the limited planning horizon, MPCC cannot guarantee feasibility and heavily relies on proper parameter tuning.

To bridge this gap, we propose an efficient, receding horizon, local adaptive low-level planner as the middle layer between our original planner and controller. Our method is named as corridor-based model predictive contouring control (CMPCC) since it builds upon MPCC [3] and utilizes the flight corridor as hard safety constraints. It optimizes the flight aggressiveness and tracking accuracy simultaneously, thus improving our system’s robustness by overcoming unmeasured disturbances. Our method features its online flight speed optimization, strict safety and feasibility, and real-time performance, and will be released1 as a low-level plugin for a large variety of quadrotor systems.

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1 https://github.com/ZJU-FAST-Lab/CMPCC
2 Technical Approach

We get the global optimized reference trajectory and the flight corridor from our previous work Teach-Repeat-Replan [2], as shown in Fig. 1(a). Assuming a reference point moves along the global trajectory, we construct a receding horizon MPC with a linearized quadrotor dynamics. The optimization objective at the \( k \)-th time-step in a horizon is shown in Fig. 1(b), where \( s^{(k)} \) is the drone’s position, \( p^{(k)} \) is the moving reference point, and \( v_p^{(k)} \) is the speed of \( p^{(k)} \). The tracking error \( e^{(k)} = |p^{(k)} - s^{(k)}|^2 \). The objective trades off the minimization of \( \{e^{(k)}\} \) and the maximization of \( \{v_p^{(k)}\} \) in the predictive horizon \( N \).

![Fig. 1: (a) Flight corridors (b) Components of the objective of CMPCC](image)

Then we construct linear inequality constraints for \( s^{(k)} \). For a given reference point \( p^{(k)} \) on the global trajectory, we define \( \Omega \) as the intersection of \( v_p^{(k)} \)’s normal plane \( \Phi \) with the corresponding polyhedron. As shown in Fig. 2(a), the resulting \( \Omega \) is a convex polygon. Then each edge of \( \Omega \) expands a plane sweeping along the direction of \( v_p^{(k)} \), which gives a polygon tube, as shown in Fig. 2(b). The inner side of this tube is considered as the safe space near \( p^{(k)} \) and will be modeled as inequality constraints.

![Fig. 2: Sequence of linear inequality constraints for safety](image)

In addition to physical limits for each state, a terminal velocity constraint is added such that the terminal speed should be less than \( v_p^{(N)} \) in the predictive horizon. Thus, feasibility can be guaranteed since the reference trajectory is globally optimal.
3 Results

The global trajectory \( p_\mu(\theta) \) is parameterized by its reference time \( \theta \), where \( \mu \in \{x, y, z\} \). Note that \( v_p = \frac{\partial p}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} \), where \( \frac{\partial p}{\partial \theta} \) is definite according to the global trajectory but \( \frac{\partial \theta}{\partial t} \) denoted by \( v_\theta \) truly indicates the traveling progress. We model the system states with the position, velocity and acceleration of \( x, y, z, \theta \), and inputs as the jerk of \( x, y, z, \theta \). They are shown in (1) and (2) at the \( k \)-th time-step.

\[
\begin{align*}
\mathbf{x}^{(k)} &= [x, v_x, a_x, y, v_y, a_y, z, v_z, a_z, \theta, v_\theta, a_\theta]^T, \\
\mathbf{u}^{(k)} &= [j_x, j_y, j_z, j_\theta]^T,
\end{align*}
\]

We formulate the optimization problem as follows:

\[
J = \min_{\mathbf{x}, \mathbf{u}} \sum_{k=1}^{N} \left\{ \sum_{\mu=x, y, z} \left( \mathbf{\mu}^{(k)} - p_\mu(\theta^{(k)}) \right)^2 - q \cdot v_\theta^{(k)} \right\},
\]

s.t. \[
\begin{align*}
\mathbf{x}^{(k+1)} &= A_d \mathbf{x}^{(k)} + B_d \mathbf{u}^{(k)}, k = 1, 2, 3, ..., N - 1, \\
x_0 \leq \mathbf{x}^{(k)} \leq x_0, k = 1, 2, 3, ..., N - 1, \\
u_l \leq \mathbf{u}^{(k)} \leq u_u, k = 1, 2, 3, ..., N - 1, \\
C(k) \cdot [x^{(k)}, y^{(k)}, z^{(k)}]^T \leq \mathbf{b}^{(k)}, k = 1, 2, 3, ..., N - 1, \\
|v_s^{(N)}| \leq v_{ts}, s = x, y, z,
\end{align*}
\]

where \( q \) in (3) is the weight of the progress of the reference point. Four kinds of constraints are introduced in the optimization: state-transfer equations governed by the 3rd-order integral model (4), lower and upper bounds of states and inputs (5 and 6), the polygon-tube constraints in (7), and the terminal velocity constraints in (8).

The optimization problem mentioned above is a typical QP, which is solved by OSQP\[8] with warm start speed-up. In practice, we choose a 1s predictive horizon and the sampling interval \( \Delta t = 0.05s \), which means \( N = 20 \). The performance of our algorithm is tested on an Intel i7-6700 CPU, with average solving time around 5ms.

4 Experiments

4.1 Experiments configuration

We use a self-developed quadrotor with an Intel Realsense D435i stereo camera\[^2\] and a DJI N3 flight controller\[^3\] for state estimation. All modules run solely on a DJI Manifold 2-C onboard computer\[^4\]. Our system inherits the localization, mapping and global planning from the Teach-Repeat-Replan system\[^2\], where readers can

\[^2\] https://www.intelrealSense.com/depth-camera-d435i/
\[^3\] https://www.dji.com/n3
\[^4\] https://www.dji.com/manifold-2
check these modules in detail. The overall hardware and software architecture of our system is shown in Fig. 3(a) and Fig. 3(b).

Fig. 3: The hardware and software architecture of the UAV

4.2 Autonomous flight with contact disturbance

Fig. 4: Circumstance of instant force disturbance

Fig. 5: Spatial and time profile facing instant force disturbance

We apply challenging force disturbance to the drone to test the performance of the proposed CMPCC, as shown in Fig. 4(a). The global trajectory (blue), our locally re-planned trajectory (red), and the geometry constraints (magenta) after the hit are
visualized in Fig. 4(b). As seen in the top-down view of the experiment in Fig. 5(a), oscillation occurs near the hit position, but the local trajectory soon converges to the global one. Also, as shown in Fig. 5(b), the instant force disturbance changes the temporal distribution of the optimal trajectory, resulting in the delaying of the trajectory of cmpcc (red) relative to which without disturbance (magenta) and the global trajectory (blue).

### 4.3 Autonomous flight with wind disturbance

We also test our method with wind disturbance by a fan, as shown in Fig. 6(a). Without the proposed CMPCC, the quadrotor tracks the global trajectory generated by Teach-Repeat-Replan with only a feedback controller, and collides with the nearby obstacle soon. However, thanks to the safety guarantee, the proposed CMPCC re-plans a safe trajectory and rejoins the global reference quickly under the wind disturbance, as shown in Fig. 6(b).

![Fig. 6: The circumstance of wind disturbance](image)

The video of the experiments is available.  

5 Main Experimental Insights

In practice, the flight performance of a quadrotor can be affected by many factors. Among all issues, the unexpected and unmeasurable disturbance is always an essential one for quadrotor autonomous navigation, especially for fast and aggressive flight. Recently, most autonomous quadrotor systems [6, 2] are developed with several independent modules include controller, planner, and perception, with the assumption that a properly designed, smooth, derivative bounded trajectory can be tracked by a controller within high bandwidth. However, this assumption does not always hold. No matter how robust the feedback controller is, it’s noted that it may

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5 [https://www.youtube.com/watch?v=_7CzBh-0wQ0](https://www.youtube.com/watch?v=_7CzBh-0wQ0)
fail when encountering drastic disturbance, such as immediate contact and a gust of wind, which are demonstrated in our experiments. Traditionally, people have to spend tons of time tuning the parameters of the feedback controller until a satisfactory performance. In this work, as validated by our challenging experiments, the proposed intermediate low-level replanner successfully compensates disturbances by planning local safe trajectories and automatically adjusting the flight aggressiveness. Therefore, the robustness of fast autonomous flight is improved significantly. Moreover, thanks to the convex formulation, the proposed CMPCC is solved within 5 ms, which suits onboard usage well.

In experiments, we also observe that the polygon tube now we use heavily depends on the static corridor. Therefore it cannot handle the variation of the environment or dynamic obstacles. In the future, we plan to investigate the way to generate safety constraints for CMPCC online.

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