Deceleration Cylindrical Electrostatic Lens Using the Charge Density Method

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Abstract: With the aid of the integrated charge density method, Laplace's equation is solved to calculate the axial potential distribution of the electrostatic immersion lens consisting of equi-diameter cylindrical electrodes separated by a specified distance. The potential and electric fields can be calculated directly outside the poles by the charge density on the poles. The solution is obtained by solving the integral equation using the bisection method. The optical properties of the decelerating lens determined under finite, zero and infinite magnification mode with no space-charging effects.

1. Introduction:
In many scientific instruments, electrostatic lenses are used to maneuver ions or electron rays by accelerating, distorting, focusing, or combining these three procedures. In addition, electrostatic lenses have been used to control electron or ion beams and are required in many different fields, most notably electron microscopy, cathode ray tubes, thermal valves, ion accelerators, and electron effect studies [1]. The immersion lens is made up of a few electrodes and has two different constant voltages on either side, that is, the electrostatic field \( E(z) = \frac{dU(z)}{dz} \) is zero at both ends [2]. Most electrostatic field calculations involve the charge distribution on the surface of conductors only, that is, the effect of the neglected space charge, and therefore, the equation of the field to be solved will be the Laplace equation [3,4]. Solving the Laplace equation \( V^2 \phi = 0 \) under the specified boundary conditions allows the possibility to be determined as a function of the coordinates through which the field strength components can be calculated [5,6].

In the present work the charge density technique is applied for evaluating lens properties. The lens optical properties such as; image and object position, focal length and aberration coefficients, investigated in detail for designing lens systems, with electron-optically acceptable parameters [7]. The results would be used to obtain the focal lengths and aberration coefficients over a range of voltage ratios.

2. The Charge Density Method (CDM):
The first step in such method is compute the surfaces charge density on the contact plates from which the lens was constructed. The voltage at any point in space is determined by free surface charges on plates in space, in the absence of an electrical dielectric [8,9]. From the density of charges calculated on the electrodes cylindrical surface of the lens, the axial potential of any point in space can be calculated taking into account that the thickness of the electrodes is too small to be equal to the surface charge density inside and outside the cylindrical lens [10].
Figure 1 show the cylindrical lens of radius r and length 10r and its ends have a negligible effect on the potential distribution in the neighborhood of the gap. The first cylinders had been divided in to N circular rings, and the superficial charge density is assumed to be uniform over each ring. These rings are of variable width and narrower near the gap, where the charging density changes more quickly [11]. If the surface charge density on the cylinders is $\sigma_i(r)$, and if there is no other charge, the potential at any point is given by:

$$U(r) = \frac{1}{4\pi\epsilon} \sum_{i=1}^{n} \int_{S_i} \frac{\sigma(r)}{|r-r'|} dS_i$$

(1)

So, each $S_i$ is an equipotential surface as follows:

$$V = \frac{1}{4\pi\epsilon} \sum_{i=1}^{n} \int_{S_i} \frac{\sigma(r)}{|r-r'|} dS_i$$

(2)

To find an approximate solution for equation (2) the surface have been divided to the small surfaces $s_i$ ($i=1$ to $N$) and have supposed that each of carries a uniform charge density $\sigma_i$, therefore equation (2) becomes:

$$V = \sum_{i=1}^{n} A_{ji} \cdot \sigma_i$$

(3)

Then the set of equations (3) reduce to the simple matrix equation:

$$V = A \cdot \sigma$$

(4)

By using the above equation one can use the following initial conditions $V_1=100V$ and $V_2=10V$ where $V_1$ and $V_2$ represent the voltage applied to the first and second electrodes respectively. An iterative procedure has been used to solve equation (4) to find the invers matrix based on Gaussian elimination and back substitution numerical program. The number of rows or columns of the square matrix A is the total number N of areas $S_i$ in to which the conducting surfaces of the lens have been divided [12,13]. To calculate the elements of matrix A, we must find the potential on the ring $j$ affected by density of the uniform charge $\sigma_i$ on the ring $i$. So the element of matrix $A_{ji}$ is given by:

$$A_{ji} = \frac{s_i f_{ji}}{4\pi^2 \epsilon \cdot r} K(t_{ij})$$

(5)

Where

$$\left(t_{ij}\right)^2 = \frac{4r^2}{4r^2 + \left(z_j - z_i\right)^2}$$

(6)

and $K(t_{ij})$ is the first kind of elliptic integral [6].
3. Optical Properties:
Three different magnification states were taken into account in this investigation, namely the finite magnification conditions, zero and infinite due to similarity to the lens path. It is important to note that in this work only the possible axial distribution is taken into account. The effects of the space charge were neglected to achieve exactly the Laplace's equation [14,15]. A Computer programs have been written in FORTRAN power station 90 language, which have been sequentially executed. The final result would be the axial potential distribution, which would be generated anywhere in the space for decelerating electrostatic lens. A personal computer has been used for executing these programs, the trajectory of the charged particles has been determined with the aid of the fourth-order Runge-Kutta method [4,16]. The initial conditions depend on the reassigned magnification of the lens. Optical properties such as focal length and magnification are calculated by integrating the paraxial ray equation [7]. The spherical and chromatic aberration coefficients \( C_s \) and \( C_c \) are calculated by using the aberration integral formula equations respectively [7,8]. The integration was performed using Simpson’s rule.

4. Results and Discussion:
The charge density distribution due to the different applied voltages varies considerably along the surface of each electrode as shown in figure 2. At the electrodes region each point on the graph represents a uniform charge density for a particular ring. The voltage applied on the electrode positioned at the left-hand side of the air gap is higher than that on the right-hand side; hence as one would expect the charge density on each ring of the former is higher than that of the latter. Within the air gap region there is no charge density. However, it is seen that there is an abrupt change in the charge density due to the rings situated at the two terminals of each cylinder, which are at a close proximity to air. Two-electrode immersion lens have been proposed to evaluate the axial potential distribution by using charge density integral method. To illustrate this, a two-cylinder lens was chosen with a gap between the cylinders of 0.2r.

![Figure 2](image-url)

**Figure 2.** The charge density distribution at different voltage ratio

The result given in figure 3 show the behavior of the axial potential distribution for immersion lens, it is asymptotically approaches the electrode potentials \( V_1 \) and \( V_2 \) at the two sides of the lens. This form represents a decelerating lens because \( V_2 \) is less than \( V_1 \), the potential distribution cannot contain fixed potential areas because you will need additional electrodes to maintain these areas. The distribution has one deflection where the axial component of the field reaches it's the maximum value because we used only two electrodes. This may or may not occur in the lens geometry. So we can say that this lens is symmetric or asymmetric.
The trajectory of electron beam under zero magnification condition at different values of voltage ratio (0.05, 0.1, 0.15 and 0.2) can be shown in figure 4. On the exit side, the beam of radial displacement increases as the voltage ratio increases. The beam turns to all the points on the side of the image and goes outside the center of the gap. At the above voltage ratios, the radial paths will have an intersection within the gap region; this intersection moves towards the center of the lens with increasing voltage ratio.

![Figure 3](image)

**Figure 3.** The axial potential distribution for decelerating immersion lens with a gap g=0.1D.

![Figure 4](image)

**Figure 4.** The trajectory of electron beam under zero magnification condition at different values of $V_2/V_1$.

The aberration coefficients have been given considerable attention in the present work since they are the two most important aberrations in electron optical systems. The present investigation has been focused at their effect on the image side and normalized in terms of the image side focal length, i.e. the relative values of $C_s/f_i$ and $C_c/f_i$ are investigated as figures of merit which are dimensionless. Figure 5 shows the relative spherical and chromatic aberration coefficient $C_s/f_i$, $C_c/f_i$ as a function of the voltage ratio under zero magnification condition. The beam trajectories shown in figure (4) was used for evaluating these coefficients as a function of $V_2/V_1$. The minimum value of $C_s/f_i$ decreases with increasing $V_2/V_1$ until reach $V_2/V_1=0.5$ then the $C_s/f_i$ increasing as voltage ratio increase, while $C_c/f_i$ still decreeing as voltage ratio increase.
The path of the electrons along the field of lens was considered under the finite magnification state and decelerated mode. Figure 6 shows the trajectory beam that crosses the field of electron lenses at different values of $V_2/V_1$. These paths are calculated according to two specific conditions for the path inclination in the object's position; these are $R(1) = -0.5$ and $R'(1) = -1$. The path inclination value in the object position greatly affected on magnification. These paths are similar in general shape. Figure 7 show the behavior of the aberration coefficients of electron lens which calculated with the aid of the corresponding paths of the electron beam. The spherical and chromatic aberration coefficients $C_s$ and $C_c$ respectively have been normalized in terms of the magnification $M$ since in the finite mode of operation $M$ is more important than the focal length $f$ although it is dimensionless.

Figure 6. The trajectories of an electron beam of finite magnification under various values of voltage ratio.
Figure 7. The spherical and chromatic aberration coefficients $C_s$ and $C_c$ respectively in terms of the magnification $M$ for finite magnification.

Figure 8. The trajectories of an electron beam of infinite magnification under various values of voltage ratio.

Figure 9. The relative spherical and chromatic aberration coefficient $C_s/f_0$, $C_c/f_0$ as a function of the voltage ratio under infinite magnification condition.
Figure 8 represent the trajectories of beam of electron that crosses the field of immersion lenses at different values of the voltage ratio $V_2 / V_1$. These paths are calculated under infinite magnification condition. Figure 9 show the aberration coefficients of lens which computed by the corresponding trajectory of the electron beam. The spherical and chromatic aberration coefficients $C_s$ and $C_c$ respectively were normalized in terms of the object side focal length $f_o$.

5. Conclusions:

The CDM appears to be very good tool for designing the electrostatic lenses. Various optical properties depending on different geometrical parameters and different voltage ratios are used in the present work. Three different magnifications are used in determination of the optical properties of the lenses. For example, under zero magnification mode for lens operation, very good properties appear from the perspective of an optical electron. However, in the finite magnification mode of the lens performance operation have been found less than unity for chromatic aberration, while for spherical aberration is higher than unity by several orders. On the other hands the optical properties of infinite magnification are higher than unity for both aberrations. Thus, the charge density method CDM are a perfect method for designing different kinds of electrostatic lenses.

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