Hybrid RANS/LES of round impinging jets

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Abstract. Fluid flow and heat transfer characteristics are presented for simulations of round impinging jets at two nozzle-plate distances H/D=2 and 10 (D is the nozzle exit diameter) and two Reynolds numbers Re=5000 and 70,000 with hybrid RANS/LES (Reynolds-averaged Navier-Stokes/Large Eddy Simulation), dynamic Smagorinsky LES and RANS k−ω models. Three k−ω based hybrid RANS/LES models are analyzed. With the hybrid RANS/LES models, improved heat transfer results are obtained, when compared to RANS, in the impact region and in the developing wall-jet region. For accurate predictions at low nozzle-plate distance, it is necessary to sufficiently resolve the formation and development of the near-wall vortices in the jet impingement region. At high nozzle-plate distance, it is important to capture the evolution and breakup of the unsteady vortices in the shear layer of the jet, so that realistic mean and fluctuating velocity profiles are obtained in the impact jet region.

1. Introduction

Round impinging jets are difficult to simulate numerically due to the complexity of the flow physics. Close to the nozzle exit, roll-up vortices form by Kelvin-Helmholtz instability. Further downstream of the nozzle exit, the roll-up vortices break up into smaller structures and impact the outer part of the wall-jet. For low nozzle-plate distance, secondary vortices are induced at the wall, as demonstrated in the LES studies of Olsson & Fuchs (1998) and Hadziabdic & Hanjalic (2008). For higher Reynolds number, these near-wall structures have a strong effect on the heat transfer between the fluid and the wall. Two peaks form in the Nusselt number distribution on the plate: a primary peak at the impact point and a secondary peak around r/D=2, with the local minimum around r/D=1. We analyze the role of the secondary vortices on the heat transfer in the developing wall jet and demonstrate that it is necessary to sufficiently resolve them for accurate heat transfer prediction. For lower Reynolds number and for larger nozzle-plate distance, there is no secondary peak in the heat transfer rate. We demonstrate that for correct prediction of the peak value of the Nusselt number in the impact point, at larger nozzle-plate distance, where the impingement plate is beyond the core of the jet, the crucial aspect is to capture the breakup of the turbulent eddies in the jet and in the stagnation flow regions. This is necessary for correct prediction of the mean velocity and temperature gradients at the impingement plate. We use experimental results on heat transfer and flow field by Baughn & Shimizu (1989); Cooper et al. (1993) and Lee & Lee (1998).
2. Hybrid RANS/LES formulations

Three hybrid RANS/LES model variants are studied, that in RANS mode reduce to the newest version of the \(k-\omega\) model by Wilcox (2008). In LES mode, the dissipation term in the \(k\)-equation is modified according to Strelets (2001); Davidson & Peng (2003) and Kok et al. (2004) in all model variants.

The transport equation for turbulent kinetic energy reads

\[
\frac{Dk}{Dt} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \max \left( \beta^* k \omega, \frac{k^{3/2}}{C_{DES} \Delta} \right) + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\sigma^* k}{\omega} \right) \frac{\partial k}{\partial x_j} \right],
\]

where \(\nu\) is the kinematic molecular viscosity, \(k\) is the turbulent kinetic energy and \(\omega\) is the specific dissipation rate. The components of the modelled stress tensor are \(\tau_{ij} = 2\nu \delta_{ij} - \frac{2}{3}k \delta_{ij}\), where \(S_{ij} = 1/2(\partial U_i/\partial x_j + \partial U_j/\partial x_i) - 1/3(\partial U_k/\partial x_k) \delta_{ij}\) are the components of the shear rate tensor. The local grid size \(\Delta\) is defined by \(\Delta = \max(\Delta_x, \Delta_y, \Delta_z)\) where \(\Delta_x, \Delta_y, \Delta_z\) denote the distances between the cell faces in \(x, y\) and \(z\) directions. The grid size is multiplied with a tuning constant \(C_{DES}\), which we derive later. The motivation for the modification of the destruction term in Eq. (1) is that the dissipation in the \(k-\omega\) RANS model is \(\epsilon = k^{3/2}/L_t\), where the turbulent length scale is \(L_t = k^{1/2}/(\beta^* \omega)\). So, it means that in the dissipation term, the turbulent length scale is replaced by the grid size.

In the first hybrid model, M1, the eddy viscosity is defined according to Davidson & Peng (2003); Kok et al. (2004); Sagaut et al. (2006) as

\[
\nu_1 = \min \left( \frac{k}{\omega}, \beta^* C_{DES} \Delta \sqrt{k} \right).
\]

The motivation for this modification is that the RANS eddy viscosity is \(\nu_1 = \beta^* L_t \sqrt{k}\). So, it means that also in the eddy viscosity expression, the turbulent length scale is replaced by the grid size. The grid size is multiplied with the tuning constant \(C_{DES}\).

In the second hybrid RANS/LES model, M2, and third model, M3, the eddy viscosities are

\[
\nu_2 = \min \left( \frac{k}{\omega}, (C_s \Delta_{LES})^2 S \right), \quad \nu_3 = \min \left( \frac{k}{\omega}, \beta^* C_{DES} \Delta_{LES} \sqrt{k} \right).
\]

where \(\Delta_{LES} = (\Delta_x \Delta_y \Delta_z)^{1/3}\). With model M2, we follow the Limited Numerical Scales approach of Batten et al. (2004), which means that the eddy viscosity from the turbulence model is limited by comparison with an LES subgrid scale model. \(C_s\) is the Smagorinsky constant \((C_s = 0.1)\) and \(S = \sqrt{2S_{ij} S_{ij}}\) is the magnitude of the strain rate tensor. Model M3 uses the eddy viscosity formula of model M1, but the length scale \(\Delta\) (Eq. 2) is replaced by \(\Delta_{LES}\).

Under local equilibrium (production of \(k\) equal to dissipation of \(k\)), the eddy viscosities of models M1 and M3 reduce in LES mode to a Smagorinsky subgrid viscosity

\[
\nu_1 = \left( (\beta^*)^{3/4} C_{DES} \Delta \right)^2 S \quad \text{and} \quad \nu_3 = \left( (\beta^*)^{3/4} C_{DES} \Delta_{LES} \left( \frac{\Delta}{\Delta_{LES}} \right)^{1/4} \right)^2 S.
\]

The role of the term \((\Delta/\Delta_{LES})^{1/4}\) with model M3 is to increase the eddy viscosity on high aspect ratio cells, as in the model by Scotti et al. (1993), which improves the predictive qualities of LES on anisotropic grids. On an isotropic grid, the subgrid viscosity is the same for all models, provided that \(C_{DES}\) is appropriately chosen. In Eq. (4), the Smagorinsky constant \(C_s = (\beta^*)^{3/4} C_{DES}\) is set to the usual value 0.1, which gives \(C_{DES} = 0.6086\).

For the RANS simulations (Wilcox, 2008), a stress limiter is applied. This stress limiter is not present in the hybrid RANS/LES models.
3. Results

Figure 1(a) shows profiles of the mean velocity magnitude, obtained with the $k-\omega$ RANS and the hybrid RANS/LES models, compared with experimental data at radial distance $r/D=1$ from jet axis, for $H/D=2$ and $Re=70,000$. The data are normalized with the bulk velocity $V_0$ at the jet exit. Good agreement between predictions and experiments is obtained. For RANS, the good quality is due to the stress-limiter, which controls the production of the turbulent kinetic energy in the stagnation flow region. Figure 1(b) and (c) show the total fluctuating velocity in radial and axial directions. The RANS fluctuating velocities correspond reasonably well with the measured fluctuating velocity components with, however, overprediction of the turbulent kinetic energy near the impingement plate (at $(H-y)/D < 0.05$). The M1 and M3 hybrid models produce much too large fluctuating velocities on the coarse grid (1.4M). Better results are obtained with M2, which further improve on the fine mesh (14M). We conclude that for the mean velocity prediction, the resolution on the coarse grid (1.4M) is sufficient. However, a much finer grid is needed for good prediction of the second-order moments in the developing wall jet. Figure 1(d) shows the Nusselt number along the plate. RANS overpredicts the stagnation point Nusselt number. This is due to overestimation of the turbulent kinetic energy in the near-wall region in the impact zone. This can be understood from figures 1(b) and 1(c). The overprediction is much larger at the impact point (not shown). So, the stress limiter does not limit enough. With the hybrid models, the stagnation point Nusselt number is slightly overpredicted on both coarse and fine meshes. The minimum in the Nusselt number profile is not captured on the coarse mesh and some improvement is obtained on the fine mesh. The explanation for the secondary peak in the Nusselt number profile is the breakup of the near-wall structures into fine scale turbulence (see fig. 2 hereafter). But, even the finest grid is not fine enough to fully resolve the breakup of the vortices in the shear layer of the jet.

![Figure 1](image_url)

**Figure 1.** (a) mean velocity, (b) total radial and (c) total axial fluctuating velocity profiles for $H/D=2$, $Re=70,000$ at distance $r/D=1$ from the jet axis. (d) Nusselt number along the plate.
Figure 2(a), (b) and (c) show instantaneous values in the xy-plane of the ratio of the eddy viscosity to the RANS value, $\nu_t/\nu_{RANS}$ for the hybrid models on the coarse grid at $H/D=2$ and $Re=70,000$. We have to take into account that the RANS value is reduced with respect to a true RANS due to the modified destruction term in the $k$-equation. The strongest reduction of the eddy viscosity is obtained with the model M2 due to the Smagorinsky definition of the eddy viscosity with the smallest grid size measure. The eddy viscosity is significantly reduced in the jet shear layer and the centre of the wall-jet layer. Near to the wall, the wall-jet layer is in RANS mode, with a much bigger zone by the M1 than for the other models. Figure 2(d) shows the contour plots of the $Q$-criterion ($Q = 1/2(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$) at distance $(H-y)/D=0.05$ from the impingement plate for model M2. The inner circle denotes $r/D=1$ and the outer $r/D=2$.

At $r/D=1$ the broken ring vortices are visible, which further breakup into smaller streamwise oriented structures. At $r/D > 2.5$, the vortex structures become too small to be resolved. In principle, the hybrid RANS/LES models function appropriately. The flow structures responsible for the turbulence production in the shear layer of the jet and in the impingement zone are captured in LES mode. However, referring to figure 1(d), we have to conclude that the breakup process is not followed deeply enough.

Figure 3(a) shows the Nusselt number profile for $H/D=2$, $Re=5000$. Small differences are visible between the LES results on the basic (1.4M) and fine (5M) grids in the stagnation flow region. With RANS, a slightly too high stagnation point Nusselt number is obtained with respect to experiments and LES. Again, this is due to a small overestimation of the turbulent kinetic energy in the impact zone. The stagnation point Nusselt number is very well reproduced by the M2 and M3 hybrid models. All numerical results underestimate the experimental Nusselt number for $r/D > 1$. For the Reynolds number 5000, the local minimum in the heat transfer around $r/D=1$ is only very weak. Clearly, the resolution requirements are not as strong as for the much higher Reynolds number, discussed earlier.
Figure 3(b) shows the heat transfer rates predicted by RANS and by the three hybrid RANS/LES models for H/D=10 and Re=5000. The hybrid results are all good here, while RANS overpredicts very much in the impact zone. To understand this behaviour, figure 4 shows the mean and fluctuating velocity profiles along the symmetry axis. The nozzle exit is placed at y/D=0 and the impingement plate is at y/D=10. The RANS $k-\omega$ model predicts a too slow decay of the mean velocity profile along the jet axis. This is due to underprediction of the turbulence mixing in the shear layer of the jet, as visible from the fluctuating velocity. Much improved results are obtained with the hybrid models, very near to the results of an LES with the dynamic Smagorinsky model on a fine grid. Figure 4(b) shows that somewhat larger levels of fluctuating velocities are reproduced with LES than with the hybrid models. The difference might be explained by the finer grid applied for LES than for the hybrid models. This results in a somewhat smaller level of the modelled energy reproduced by LES close to the jet exit, which enhances the flow unsteadiness there. That the decay of the mean velocity profile along the jet axis is incorrectly predicted by the RANS, has big consequences for the heat transfer rate prediction in the impact zone (figure 3 b). All hybrid models give heat transfer rates which agree very well with measured values and computed values using LES. This is due to the ability of the hybrid RANS/LES models to reproduce the vortex breakup process in the shear layer of the jet and in the near-wall region.

![Figure 3](image-url)

**Figure 3.** Nusselt number profiles along the impingement plate for (a) H/D=2, Re=5000, (b) H/D=10, Re=5000.

![Figure 4](image-url)

**Figure 4.** Profiles of (a) mean velocity magnitude (b) axial total fluctuating velocity components along the jet axis for H/D=10, Re=5000.
4. Summary

Simulations of round impinging jets at low and high nozzle-plate distances (H/D=2 and 10) and at low and high Reynolds numbers (Re=5000 and 70,000) were presented using the $k-\omega$ RANS and three hybrid RANS/LES $k-\omega$ based models. For low nozzle-plate distance, the mean velocity profiles are well reproduced by all models. The fluctuating velocities are well reproduced by the hybrid models. RANS overpredicts the fluctuating velocity magnitude in the impact region. This leads to overprediction of the heat transfer in the impact zone. The hybrid models cure this deficiency. For large nozzle-plate distance, again, the hybrid models reproduce the mean and fluctuating velocity profiles very well in the jet flow region. The RANS results for velocity and heat transfer are very much in error due to incorrect prediction of the mixing in the shear layers of the jet.

Acknowledgments

The first author acknowledges an international cooperation grant of Ghent University and the research program 'Iuventus Plus' supported by the Polish Ministry of Science and Higher Education from the national budget funds for science in 2010-2011.

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