Diffractive deep inelastic scattering from multiple soft gluon exchange in QCD

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Diffractive hard scattering is interpreted as the effect of soft gluon exchanges between the emerging energetic quarks and the nucleon’s color field, resulting in an overall color singlet exchange. Summing multiple gluon exchanges to all orders leads to exponentiation and an amplitude in analytic form. Numerical evaluation reproduces the precise HERA data and gives new insights on the density of gluons in the proton.

Diffractive deep inelastic scattering (DDIS) in lepton–proton collisions involves hard scattering events where, in spite of the large momentum transfer $Q^2$ from the electron, the proton emerges essentially unscathed with only a very small transverse momentum, keeping almost all of its original longitudinal beam momentum. The leading proton is well separated in momentum space, or rapidity $y = \ln(E + p_z)/(E - p_z)$, from the central hadronic system produced from the exchanged virtual photon’s interaction with the proton. Thus, this new class of events is characterized by a large rapidity gap (LRG) void of final state particles.

Diffractive deep inelastic scattering (DIS) was discovered by the ZEUS and H1 experiments at HERA [1], but the first discovery of such a hard diffraction process was in $p\bar{p}$ collisions by the UA8 experiment [2]. These processes had actually been predicted [3] by combining Regge phenomenology for low-momentum transfer (soft) processes in strong interactions via pomeron exchange, with large-momentum transfer (hard) processes based on perturbative QCD. By parametrizing the parton content of an exchanged pomeron (or alternatively diffractive parton density functions) it is possible to describe the HERA data. However, the extracted parton densities are not universal, since when used to calculate diffractive hard scattering processes in $p\bar{p}$ collisions at the Tevatron one obtains cross sections an order of magnitude larger than observed.

As an alternative dynamical interpretation the Soft Color Interaction (SCI) model was developed in Ref. [4], based on the assumption that the hard perturbative part of the interaction is the same as in ordinary DIS. The large momentum transfer means that the hard subprocess occurs on a spacetime scale much smaller than the bound state proton and is thus “embedded” in the proton. The emerging hard-scattered partons, therefore, propagate through the proton’s color field and may interact with it. Soft exchanges will dominate, due to the large coupling and the lack of suppression from hard gluon propagators. Therefore, the momenta of the hard partons are essentially undisturbed, which is consistent with the fact that soft long distance interactions do not affect hard short distance ones. However, the exchange of color may change the color charges of the emerging partons such that the confining string-like field between them will have a different topology, resulting in a different distribution of the final state of hadrons produced from the string hadronization [5]. In particular, a region in rapidity without a string will result in an absence of hadrons there, i.e. a rapidity gap. This SCI model is very successful in describing data [6], but lacks a solid theoretical basis.

Here, we present a new QCD-based model, which leads to effective color singlet exchange and thereby to diffractive scattering. The model is inspired by the success of the SCI model, and may be seen as an explicit realization of the earlier attempt [7] to understand this soft gluon exchange in terms of QCD rescattering.

As depicted in Fig. 1, the exchanged photon fluctuates into a quark–antiquark color dipole which interacts with the first gluon carrying a longitudinal momentum fraction $x_P$. This defines the “hard” part of the process. Both the $q\bar{q}$ dipole and the proton remnant then have overall color octet charges and may interact through the exchange of a number of soft gluons with longitudinal momentum fractions $x'_i \ll x_P$, $\sum x'_i = x'$. This multiple gluon exchange constitutes the “soft” part of the process and includes at least one exchange. Thus the overall exchange from both the hard and soft parts contains two gluons or more. The color factor of the total

FIG. 1: $\gamma^* p \rightarrow Xp$ process with resummed gluon exchanges, and illustration of the factorization in Eq. (4) of the amplitude into a hard and a soft part connected via an unintegrated gluon density function (UGDF).
soft exchange may combine to form an overall color singlet together with the first, hard gluon. In this case both the $q\bar{q}$-system and the proton remnant emerge as color singlets and hadronize independently with a large separation in rapidity due to the dominance of small-$x$ gluons in the proton. Lacking a large momentum transfer to the proton, its remnant can recombine into a leading proton with close to the full beam momentum. The photon with a space-like virtuality $Q^2$ has through the momentum exchange $x_p$ been turned into a time-like hadronic system $X$ of invariant mass $M_X$.

In terms of the four-momenta $q$ of the photon, $P$ and $P'$ of the initial and final proton, the important kinematical variables are

$$x_B = \frac{Q^2}{Q^2 + W^2}, \quad \beta = \frac{Q^2}{Q^2 + M_X^2}, \quad x_p = \frac{x_B}{\beta},$$

where $Q^2 = -q^2$ and

$$M_X^2 = \frac{1 - \beta}{\beta} Q^2, \quad W^2 \equiv (P + q)^2 = \frac{Q^2}{x_B} (1 - x_B).$$

In the forward limit of interest here, the momentum transfer $t = (P' - P)^2$ along the proton line is small, $|t| \ll Q^2, M_X^2$.

Let us now outline the model and the calculation of the diffractive structure function (see [8] for details). The involved momenta are specified in Fig. 1. We consider the asymmetric case where one hard gluon carries most of the longitudinal momentum transfer $x_p$. Using cutting rules, we derive a factorization of the amplitude into a convolution of a hard part and a soft part. The hard part is treated in normal perturbative QCD. The soft part consists of any number of soft gluon exchanges, collectively in a color octet with total $x' \ll x_p$. The soft exchanges are resummed, in the large $N_c$ limit, to all orders in $\alpha_s$. These soft gluons are not perturbative, since the strong coupling becomes large. We model them as interacting with quarks as perturbative gluons but with a non-perturbatively small momentum transfer.

In the center-of-mass frame of the final state, i.e. the outgoing proton with momentum $P'$, and the diffractive system $X$ with momentum $q' = k_1 + k_2$, where $k_{1,2} = -k_{2,1}$, the quark virtuality $k^2$ is the hard scale of the process, $\mu_F^2 \equiv k^2 = \varepsilon^2 + k_1^2$ and is expressed in terms of its energy $\varepsilon$ and transverse momentum $k_1$ given by

$$\varepsilon^2 = z(1 - z)Q^2 + m_q^2, \quad k_1^2 = z(1 - z)M_X^2 - m_q^2,$$

where $z$ is the fraction of the longitudinal momentum carried by the quark, and we consider the light quark mass limit $m_q \ll Q^2$.

In impact parameter space, with $b$ conjugate to $\Delta_\perp$, the total amplitude for the $\gamma^* p \rightarrow X p$ process can be written as a convolution of the hard and soft subprocess amplitudes and a function $\mathcal{V}$, which describes the distribution of gluons in the proton,

$$M(\delta) \sim \int d^2 b e^{-i b \cdot \delta} \hat{M}_{hard} \hat{M}_{soft} \mathcal{V},$$

where $\delta \equiv \sqrt{-t} = |\Delta_\perp + \Delta'_\perp|$. The function $\mathcal{V}$ is the Fourier transform of the unintegrated gluon distribution function (UGDF), and will be specified below. This factorization is schematically illustrated in Fig. 1.

The amplitude for the hard subprocess $\gamma^* g \rightarrow q\bar{q}$ is decomposed into its longitudinal ($L$) and transverse ($T$) parts depending on the photon polarization. It includes the two possible couplings of the gluon to the $q\bar{q}$ pair, and can be Fourier transformed to impact parameter space with $r$ conjugate to $k_1$, the transverse momentum of a quark in the intermediate state. The hard amplitudes are then given by

$$\hat{M}_{L}^{hard} = iC \alpha_s(\mu_F^2) \sqrt{\beta} W^3 z^{3/2}(1 - z)^{3/2} K_0(\varepsilon r), \quad (5)$$

$$\hat{M}_{T,\pm}^{hard} = iC \alpha_s(\mu_F^2) \sqrt{\frac{2\beta}{1 - \beta}} \frac{1}{\sqrt{x_p}} W^2 z^{1/2}(1 - z)^{3/2}$$

$$\times \varepsilon K_1(\varepsilon r) \frac{r_x + i r_y}{r},$$

where $C = 8\pi e_q \sqrt{\pi \alpha_{em}/N_c^2}$ and $K_{0,1}$ are Bessel functions.

We now turn to the soft subprocess amplitude, which can be calculated order-by-order and then resummed. It is important to realize that these soft gluons carry non-perturbatively small momentum transfers. We deal with this by taking $\alpha_s(\mu)$ at very small scales as a parameter, which we fix using the infrared-stable analytic perturbation theory (APT) [9]. In the limit that $\mu \rightarrow 0$, we use $\alpha_s^{soft} = \alpha_s^{APT}(\mu \rightarrow \Lambda_{QCD}) \approx 0.7$. The softness of the color-screening gluons with $x' \ll x_p$ implies that intermediate particles are on-shell, and the dipole size $r$ is frozen. Cutting the intermediate propagators, we pick up phase shifts originating from the hard amplitude, which depend on the soft momentum exchanges $\Delta'_\perp$. The diagram for one soft gluon exchange is a tree-level diagram, while two-gluon exchange leads to a loop integral. We calculate these contributions and perform the Fourier transforms with respect to $\Delta'_\perp$, where remarkably the second order diagram turns out to be the second term in a series that will exponentiate. This relies on the large $N_c$ limit, where the color factors simplify to $C_F \simeq T_F N_c$. We get

$$e^{-i r k'_1} \hat{M}_{1}^{soft} = e^{-i r k'_1} A \mathcal{W}(b, r),$$

$$e^{-i r k'_2} \hat{M}_{2}^{soft} = e^{-i r k'_2} \frac{A^2 \mathcal{W}(b, r)^2}{2!}, \quad \ldots \quad (7)$$

where $A = 2\pi i C_F \alpha_s(\mu_F^2)$, $\mu_F^2 \sim \Delta_\perp^2$ is the gluon virtuality, and we have defined

$$\mathcal{W}(b, r) = \frac{1}{2\pi} \ln \frac{|b - r|}{|b|}. \quad (8)$$

Summing over the number of soft gluons in the final state leads to exponentiation in impact parameter space, so that for the total soft subprocess amplitude we finally get

$$e^{-i r k'_1} \hat{M}_{soft}(b, r) = -e^{-i r k'_1} (1 - e^{A\mathcal{W}(b, r)}). \quad (9)$$
A similar expression was previously derived in the case of scalar Abelian gauge theory in Ref. [10]. Note, that \( M_{\text{soft}}(b, r) \) is independent of the photon polarization in the soft limit of small \( \Delta_{\perp}^2 \).

To describe the coupling of the gluons to the proton, we use the framework of \( k_i \)-factorization and generalized (off-diagonal) UGDFs, which contain all information about the non-perturbative coupling of the gluons to the proton, and is based on a well-defined formal procedure for the transition from the parton level to the hadron level (see e.g. Ref. [11]). The coupling of a gluon to a quark is thus given by an off-diagonal UGDF \( F_{g}^{\text{off}}(x, x', \Delta_{\perp}^2, \mu_F^2) \), absorbing a factor \( C_F \alpha_s(\mu_F^2)/\pi \), and by convention a gluon propagator \( \Delta_{\perp}^{-2} \) into the UGDF in order to keep it regular as \( \Delta_{\perp}^2 \to 0 \). The absorbed coupling \( \alpha_s(\mu_{\text{soft}}^2) \) corresponds to the coupling of a screening gluon with virtuality \( \mu_{\text{soft}}^2 \sim \Delta_{\perp}^2 \) to a quark in the proton, whereas the coupling of the hard gluon to the \( qq \) dipole and to a quark in the proton is treated perturbatively at the hard scale \( \mu_F \).

Generalized parton distributions (GPDs) are not very constrained by data. We use a prescription for the generalized UGDF, which was introduced in Ref. [12], motivated by positivity constraints for GPDs [13]. This prescription works well in the description of recent CDF data on central exclusive charmonium production [14], and allows incorporating the dependence on the longitudinal momentum fraction and transverse momentum of the soft gluons in an explicitly symmetric way,

\[
F_{g}^{\text{off}} \simeq \sqrt{F_{g}(x, \Delta_{\perp}^2, \mu_F^2)} F_{g}(x', \Delta_{\perp}^2, \mu_F^2),
\]

which explicitly involves the soft \( x' \) dependence. Here \( F_{g} \) is the normal diagonal UGDF, which depends on the gluon virtuality, and which when integrated over this virtuality reduces to the well-known collinear gluon PDF \( g(x, \mu_F^2) \). The dependence of \( F_{g} \) on the virtuality is not theoretically well-known for small virtualities, and the UGDF is here modeled using the collinear gluon PDF together with a simple Gaussian Ansatz for the intrinsic transverse momentum dependence,

\[
\sqrt{x F_{g}^{\text{off}}} \simeq \sqrt{x g(x, \mu_F^2)} x g(x', \mu_{\text{soft}}^2) f_{G}(\Delta_{\perp}^2),
\]

where the factor \( \sqrt{x F_{g}} \) is absorbed from the hard subprocess, and the Gaussian width \( \rho_0 \) is the soft hadronic scale, corresponding to the transverse proton size \( r_p \sim 1/\rho_0 \). Note that this leads to an exponential \( t \)-dependence of the cross section \( \sim \exp(B_{\text{PT}} t) \) with the diffractive slope \( B_{\text{PT}} = 1/\rho_0^2 \approx 6.9 \pm 0.2 \text{ GeV}^2 \) [15].

The second PDF in Eq. (11), associated with the soft gluon, is evaluated at very low scale and very small \( x' \). For this PDF we can here introduce a function \( R_{g}(x', \mu_{\text{soft}}^2) \) which is assumed to be slowly dependent on \( x' \) in the case \( x' \ll x_p \):

\[
\sqrt{x F_{g}^{\text{off}}} \simeq R_{g}(x', \mu_{\text{soft}}^2) \sqrt{x p g(x, \mu_F^2)} f_{G}(\Delta_{\perp}^2). \tag{12}
\]

The factor \( R_{g} \), therefore, contains all the soft physics related with soft gluon couplings to the proton. It is interpreted as the square root of the gluon PDF at very small \( x' \ll x_p \) and some soft scale \( \mu_{\text{soft}}^2 \). This is a non-perturbative object, which contributes to the overall normalization and can be determined from data. The factor \( R_{g} \) in Eq. (12) is analogous to the skewedness parameter \( R_{g} \sim 1.2 - 1.3 \), which accounts for the single log \( Q^2 \) skewedness effect in off-diagonal UGDFs [16]. As we will see below, the prescription [12] is consistent with the HERA data for all available \( M_X^2 \) and \( Q^2 \).

The model (11) will lead to a linear dependence of the diffractive structure function on the gluon PDF, as compared to the quadratic dependence often encountered in two-gluon exchange calculations of DDIS [17]. This linear dependence is the same as in the SCI model, where a linear dependence describes both diffractive and non-diffractive events, and indicates a continuous transition between the two types of events.

In terms of the UGDF in Eq. (12), the factor \( V = V(b, r) \) of Eq. (4) is given by

\[
V(b, r) = \frac{1}{\alpha_s(\mu_{\text{soft}}^2)} \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} \sqrt{x p} F_{g}^{\text{off}} \times \left\{ e^{-i r \Delta_{\perp}} - e^{i r \Delta_{\perp}} \right\} e^{i b \Delta_{\perp}}. \tag{13}
\]

Straightforward calculations [8] lead to the final expressions for the diffractive structure functions

\[
x_p F_{L}^{D(4)} = S Q^4 M_X^2 \int_{z_{\text{min}}}^{1} dz (1 - 2z) z^2 (1 - z)^2 |J_L|^2 \tag{14}
\]

and

\[
x_p F_{T}^{D(4)} = 2 S Q^4 \int_{z_{\text{min}}}^{1} dz (1 - 2z) \left\{ (1 - z)^2 + z^2 \right\} |J_T|^2 \tag{15}
\]

where \( S = \sum_q e_q^2/(2\pi^2 N_c^2) \) sums over light quark charges \( e_q \), and

\[
J_L = i \alpha_s(\mu_F^2) \int d^2 r d^2 b e^{-i b \cdot r} e^{-i r \cdot k_{\perp}} K_0(|r\varepsilon|)
\]

\[
\times V(b, r) \left[ 1 - e^{i 4 \varepsilon W} \right],
\]

and

\[
J_T = i \alpha_s(\mu_F^2) \int d^2 r d^2 b e^{-i b \cdot r} e^{-i r \cdot k_{\perp}} \varepsilon K_1(|r\varepsilon|)
\]

\[
\times \frac{r_x \pm i r_y}{r} V(b, r) \left[ 1 - e^{i 4 \varepsilon W} \right]. \tag{16}
\]

Let us briefly discuss the role of higher-order QCD corrections in the framework of our model. One should distinguish corrections from the hard gluon emission due to radiation from the partons in the hard part, and corrections due to interactions between soft gluons in the soft part.
In the first case, additional s-channel gluons emitted from the hard scattering part can be described by DGLAP evolution. They play an important role for large invariant masses $M_X^2 \gg Q^2$, and will be considered below.

In the second case, all interactions between soft gluons in the soft scattering part are absorbed into the UDGF and thus into the soft $R_g$ factor, which enters the overall normalization. Only the number of soft gluon legs attaching to the proton in the lower part of the diagram, and how they attach to the partons in the upper part are important. All long-distance interactions between the gluons are treated as part of the color background field in the proton, and do not affect the resummation procedure.

One could also imagine interactions between the gluon from the hard part and one of the soft part. Such interactions contribute only in the symmetric case $x' \sim x_P$, when all exchanged gluons are either soft or hard. The first case is unrealistic as it may happen only in the case of very small $M_X$ and $Q$ where QCD factorization does not apply. The second case is suppressed by a small $\alpha_s$. Such exchanges may be enhanced by large logarithms, leading to exchange of a BFKL pomeron at non-zero momentum transfer $t$. This is, however, a process with different kinematics and does not contribute to forward diffraction.

In the large-$M_X$, or $\beta \to 0$, limit the additional emission of a gluon in the final state becomes important. This is dominated by the emission of a collinear gluon from the hard gluon, an emission which is enhanced by a large logarithm. Such a gluon will be well-separated from the $q\bar{q}$ pair in momentum space, and will therefore contribute to building up a large $M_X$. We take this contribution into account through a gluon splitting, described using the DGLAP splitting function $P_{gg}$ as (see e.g. [8])

$$x_P F_{gg}^{D(4)} \simeq \frac{1}{N_c^2} \int \frac{dt_d dz_g}{\beta t_q + m_g^2} P_{gg}(z_g) \frac{\alpha_s(t)}{2\pi} x_P F_{gg}^{D(4)},$$

where $t_g$ is the gluon propagator and the integral is cutoff in the infrared by the effective gluon mass $m_g \simeq \Lambda_{QCD}$. The factor $N_c^{-2}$ appears because the emitted gluon must contribute to the color singlet X system.

The HERA data on the diffractive structure function $F_{X(T)}^{D(3)}$ are given in terms of the reduced cross section

$$x_P \sigma_T^{D(3)} = x_P F_{qq,T}^{D(3)} + \frac{2-2y}{2-2y+y^2} x_P F_{qq,L}^{D(3)} + x_P F_{qqg}^{D(3)},$$

where $F_{L,T}^{D(3)} (x_P, Q^2, \beta)$ is the diffractive structure function integrated over $t$, the kinematical variable $y = Q^2/sx_P \leq 1$, and the center-of-mass energy of ep-collisions at HERA is $\sqrt{s} = 318$ GeV.

In Fig. 2 we show the comparison of the results of our model with the latest HERA data [12] on the reduced cross section $x_P \sigma_T^{D(3)} (x_P, \beta, Q^2)$ as a function of $x_P$ in bins of $\beta$ and $Q^2$. The figure shows our main result, calculated using the CTEQ6L1 gluon PDF [18], and also curves obtained using the older GRV94 PDF [20]. The minimal factorization scale $\mu_F$ is fixed to be $\mu^2_{F,min} = 0.2$ GeV$^2$, which, together with Eq. (18) implies a minimal possible fraction of the quark longitudinal momentum $z_{min}$ in the integrals in Eqs. (14,15).

In these results, transverse polarization dominates in all bins. We find that the $q\bar{q}$ contribution alone is enough to describe all the data for $\beta \geq 0.2$, below which the $q\bar{q}g$ contribution becomes significant.

The fixed parameters in our model, which all take reasonable physical values, are an effective gluon mass, $m_g \simeq \Lambda_{QCD}$, used to regulate the infrared divergence in the $q\bar{q}g$ contribution, Eq. (17), and the soft coupling constant $\alpha_s(\mu^2_{soft}) \simeq 0.7$.

We also fit two physical quantities: the soft factor $R_g$, which absorbs the non-perturbative couplings of the soft gluons to the proton, and the constituent quark mass $m_q$. For our model to be consistent, these two parameters should not depend strongly on the two large scales in the process, $M_X$ and $Q$.

We have found that $R_g$ is close to unity for a wide range of scales. It does not depend at all on $M_X$, and only in the lowest $Q$ bins is there a noticeable increase in the normalization $R_g$ by at most a factor 4. Here, however, QCD factorization becomes questionable and the conventional gluon PDFs are poorly known such that using the GRV94 PDF instead of CTEQ6L1 this increase is essentially removed [8]. Thus $R_g$ is essentially an overall normalization factor close to unity, which contains unknown information on the density of soft gluons in the proton.

The kinematics, e.g. Eq. (3), depends on an effective quark mass corresponding to a dynamic, dressed quark generated dominantly by softer gluon radiation which cannot be calculated theoretically. Therefore, the parameter $m_q$ is fitted to data and found to have only a slow variation with the large scales $M_X$ and $Q$; namely in the range $0.3$–$1.2$ GeV consistent with mainly soft dynamics, as expected.

The uncertainty in the PDFs is illustrated in Fig. 2, where we show, in the bins of small $Q^2$ and $M_X$, the results obtained using the GRV94 gluon PDF. In these bins GRV does better in reproducing the data, because of its larger gluon density at these small $x$ and scales.

In summary, in this Letter, we have presented a new QCD-based model of soft gluon exchanges in the final state, which describes data on the diffractive structure function very well. The model is inspired by the phenomenologically successful Soft Color Interaction model [4] and on the work on such soft rescattering in DIS in Ref. [7,10]. The full details of the model and the calculations are presented in [8].

We have considered diffractive DIS, as this is where the most precise data are available, but the soft gluon exchanges arise due to the proton’s color field and should thus be of a universal nature. Our model should therefore be applied to other processes, for example, diffraction in hadron–hadron collisions or diffractive vector meson pro-
FIG. 2: The reduced cross section $x_P \sigma^{D(3)}_{r}(x_P, \beta, Q^2)$ as a function of $x_P$ for different values of $M_X$ and $Q^2$. The latest ZEUS data [13], from diffractive deep inelastic scattering events with a large rapidity gap, compared with our model using the CTEQ6L1 (full line) and GRV94 (dotted line) parametrizations of the gluon density in the proton.
duction. It should also have effects on other observables, such as the underlying event at LHC, which is important to understand and describe. This is also borne out by the results from the SCI model, which has not only been able to describe all diffractive data from HERA and the Tevatron, but also other, non-diffractive data.

This work was supported by the Swedish Research Council and the Carl Trygger Foundation. We are grateful to Igor Anikin for valuable discussions.

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