Rapidity Gaps in $e^+e^-$ Annihilation and Parton-Hadron Duality

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Abstract

We calculate the probability for rapidity gaps in the parton cascade for different approximations within the perturbative QCD and compare the results with recent measurements. The aim is to find out whether the dual connection between the parton and hadron final states – observed so far in various inclusive measurements – holds as well for the extreme kinematic configurations with colour sources separated by large rapidity gaps. A description of the data is possible indeed choosing the parameters of the cascade in the range suggested by recent analyses of the energy spectra (the $k_\perp$ cutoff $Q_0 \gtrsim \Lambda_{\text{QCD}} \sim 250$ MeV).

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1 Introduction

Recently the studies of $p\bar{p}$ collisions with high $p_T$ jets at the Tevatron and of dijet events in $e^+e^-$ collisions at HERA showed that a considerable fraction of these hard scattering events (of order 1-10%) contain large gaps in the rapidity distribution of particles. Such phenomena had been expected from the exchange of colour neutral objects.

In $e^+e^-$ annihilation the primary parton process is the production of $q\bar{q}$ pairs followed by gluon bremsstrahlung and quark pair production. In this case the suppression of central gluon radiation would produce a rapidity gap for the partons but the recoiling $q\bar{q}$ like systems separated by the gap may become the source of nonperturbative hadron production to neutralize the color field; then no gap would remain between the observed hadrons.

One way to generate large rapidity gaps between the final hadrons in $e^+e^-$ annihilation is through the production of colour singlet clusters separated by large rapidity gaps. Such configurations can be obtained perturbatively in hard processes leading to at least four partons in the final state, like $e^+e^- \rightarrow q\bar{q}q\bar{q}$ or $q\bar{q}g g$. In this case the hadrons are produced within each colour neutral $q\bar{q}$ or $gg$ cluster and the large gap persists. As these processes involve a highly
virtual intermediate quark or gluon the rate for these large rapidity gap events is rather small though. One finds that the rate keeps decreasing with increasing gap size, contrary to the case of \( p\bar{p} \) or \( ep \) collision.

Such an unlimited decrease of the gap probability has been seen already some time ago. The recent analysis at the \( Z^0 \) resonance by the SLD Collaboration has shown the fall of the gap probability over five orders of magnitude (see also the first results from ALEPH). At the same time the observed absolute size of the gap probability exceeds the expectations from the above calculations by about two orders of magnitude. This observation excludes this type of hard colour neutralization to be the dominant mechanism for the production of large rapidity gaps.

In this paper we investigate how the fraction of rapidity gap events \( f(\Delta y) \) for the parton cascade (without hadronization) compares with the experimental results on hadrons. At first sight one might expect that the decrease of the gap fraction for hadrons and therefore in the experimental data is steeper than the one of the parton cascade because of the additional suppression from removing the hadrons emitted into the gap during the hadronization phase. We will see however that this effect strongly depends on the scale \( Q_0 \) for which the parton cascade is terminated.

There are many phenomena in multiparticle production which are in favour of a very simple hadronization picture, called local parton hadron duality (LPHD), which compares the observed sufficiently inclusive particle densities and correlations directly with the calculations at the parton level, assuming in effect the production of a hadron close to the parton in phase space (for reviews, see [12,13]). In this approach the parton cascade is evolved down to the low scales \( Q_0 \approx \Lambda \) of a few hundred MeV; \( Q_0 \) denotes here the cutoff in the transverse momentum of the emitted partons and \( \Lambda \) the QCD scale for the running coupling.

This calculation of the gap rate is similar in spirit to the “clan” model analysis within a simplified parton shower interpretation: the primary independent emission of hadronic objects leads to a Poisson distribution with the gap rate related to the probability for no emission by an exponential.

In the following we calculate the gap probability analytically in different approximation schemes of perturbative QCD and compare to the data for the previously determined values of \( Q_0 \) and \( \Lambda \). This will teach us whether the parton hadron duality works in the more extreme, quasi exclusive configurations with large rapidity gaps as well; in this case a confirmation of the duality approach is even less expected than in case of inclusive observables.
2 Rapidity gaps in the parton cascade

A gap in the rapidity distribution occurs if the initial $q\bar{q}$ pair does not radiate into the respective rapidity interval. The probability for no radiation into a certain angular interval is given in field theory by the exponential Sudakov form-factor $\exp(-w_A(P,\Theta,Q_0))$, originally derived in QED. A problem very similar to the rapidity gaps considered here occurs in the calculation of multijet rates in $e^+e^-$ annihilation applying the Durham/$k_\perp$ algorithm, the 2-jet rate is again given by the Sudakov form factor which represents the probability for no-parton-production above a certain $k_\perp$ cutoff (resolution parameter).

In our application we consider the rapidity gap having no gluons inside the angular or rapidity interval above the transverse momentum cutoff $Q_0$ which corresponds to a hadronic scale of a few hundred MeV as outlined in the introduction.

Let us consider specifically the angular interval between $\Theta_1$ and $\Theta_2$ ($\Theta_1 < \Theta_2$) where $\Theta$ is measured with respect to the quark direction. The rapidity is then obtained from $y = -\ln \tan \frac{\Theta}{2}$. Let us further denote the probability for emission of a gluon at an angle $\Theta'$ with the energy $\omega'$ off a parent parton $A$ (either a gluon(g) or a quark(q)) as $\psi_A(\omega',\Theta') = d\ell_A/d\omega'd\Theta'$.

Then, the Sudakov form factor for the angular ordered cascade is given by

$$\Delta_A(P,\Theta,Q_0) = \exp(-w_A(P,\Theta,Q_0)) \quad (1)$$

$$w_A(P,\Theta,Q_0) = \int d\omega' \int_{k_\perp > Q_0} d\Theta' \psi_A(\omega',\Theta') \quad (2)$$

and this represents the probability that no gluon is emitted within the cone of half angle $\Theta$ from the parent parton with the energy $P = Q/2$ with transverse momentum above $Q_0$. In particular,

$$\Delta_A(\Theta_2)/\Delta_A(\Theta_1) = \exp(-w_A(\Theta_2) + w_A(\Theta_1)) \quad (3)$$

represents the probability that there is no emission of a gluon with emission angle between $\Theta_1$ and $\Theta_2$.

These rates have been calculated in different approximations which we discuss in the following.

The double logarithmic approximation (DLA)

The simplest approximation takes into account only the leading contributions from the angle and energy singularities of the gluon emission. In this case the total number of gluons radiated from a primary parton into a forward cone of
half angle $\Theta$ as in (2) is given in a small angle approximation by

$$w_A(P, \Theta, Q_0) = \frac{C_A}{N_C} \int_{\Theta}^{P} \frac{d\omega}{\omega'} \int_{\Theta'}^{\Theta_0} \frac{d\Theta'}{\Theta'} \gamma_0^2(\omega' \Theta')$$

(4)

or, using logarithmic variables

$$w_A(Y, \lambda) = \frac{C_A}{N_C} \int_{0}^{Y} d\eta \int_{0}^{\eta} d\eta' \gamma_0^2(\eta')$$

(5)

with

$$Y = \ln \frac{P \Theta}{Q_0}, \quad \lambda = \ln \frac{Q_0}{\Lambda}$$

(6)

and the DLA anomalous dimension $\gamma_0^2 = 2N_C\alpha_s/\pi$ for running coupling $\alpha_s$,

$$\gamma_0^2(\eta) = \frac{\beta^2}{\eta + \lambda}, \quad \beta^2 = \frac{4N_C}{b}, \quad b = \frac{11}{3}N_C - \frac{2}{3}n_f$$

(7)

where $N_C$ and $n_f$ denote the numbers of colours and flavours respectively. Furthermore $C_A = \frac{4}{3}$ for quarks and $C_A = N_C = 3$ for gluons. One finds

$$w_A(Y, \lambda) = \frac{C_A}{N_C} \beta^2 \{(Y + \lambda) \ln \frac{Y + \lambda}{\lambda} - Y\}$$

(8)

The probability for a gap without primary gluons is then obtained as

$$f_A(\Theta_1, \Theta_2) = e^{-w_A(\Theta_2) - w_A(\Theta_1)}$$

(9)

for two arbitrary angles $\Theta_1 < \Theta_2 < \frac{\pi}{2}$ measured with respect to the quark direction. In this approximation the jets evolve independently in both hemispheres and the respective probabilities factorize. In particular, in the case of a symmetric angular interval $(\Theta_G, \pi - \Theta_G)$ the gap probability is given by

$$f_A(\Theta_G) = e^{-2(w_A(\frac{\pi}{2}) - w_A(\Theta_G))}$$

(10)

In the present small angle approximation the rapidity is related to the angle by $y \approx -\ln \frac{\Theta}{2}$; in case of the symmetric rapidity interval of full width $\Delta y$, the relevant gap angle is $\Theta_G = 2 \exp(-\Delta y/2)$ which yields $\Theta_G = 2$ for the limiting case $\Delta y = 0$. The use of the small angle approximation for such large angles is formally not allowed, but we will see below that the effect is numerically small (radiation occurs dominantly at small angles). The gap probability $f_A(\Delta y)$ following from (10) corresponds to an almost exponential decrease for the relevant parameter range.
It is interesting to note that for fixed coupling there would be a flattening of the gap fraction \( f_A(\Theta_G) \) for large gaps \( \Delta y \) as can be derived from

\[
w_A(Y) = \frac{1}{2} \gamma_0^2 Y^2 \quad (\text{fixed } \alpha_s)
\]  

This difference comes from the strong radiation near the kinematical limit \( k_\perp \sim Q_0 \gg \Lambda \) in case of running \( \alpha_s \) which becomes important for small limiting angles \( \Theta \). In other words, for large gaps the strong radiation near the kinematic limit is kinematically allowed and has to be suppressed which makes the gap probability much smaller than in the case of fixed \( \alpha_s \) where this radiation is not equally strong.

The modified leading logarithmic approximation (MLLA)

Besides the double logarithmic terms one also takes into account in this approximation the leading single logarithmic corrections. In this subsection we first derive the full \( O(\alpha_s) \) emission probability for \( e^+e^- \) annihilation from which the various approximations beyond DLA can be derived.

We denote the energy fractions of \( q, \bar{q} \) and \( g \) as \( x_+, x_- \) and \( x_g \) and take the jet (thrust) axis along the (say, left-moving) \( \bar{q} \). The energy distribution is then given by

\[
dx = C_F \alpha_s \frac{x_+^2 + x_-^2}{2\pi (1 - x_+)(1 - x_-)} dx_+ dx_-.
\]  

(see, for example, Ref. \( ^{18} \) where also useful kinematic relations are given).

In order to evaluate the gluon emission probability from a parent quark

\[
\varphi_q(\omega', \Theta') \equiv \frac{\partial(x_+, x_-)}{\partial(\omega', \Theta')} \frac{dn}{dx_+ dx_-}
\]  

we need to work out the Jacobian factor to move from \( (x_+, x_-) \) to \( (\omega', \Theta') \) using the kinematical relations

\[
x_+ = \frac{1 - x_g(1 - x_g/2)(1 + \cos \Theta')}{1 - x_g(1 + \cos \Theta')/2}
\]

\[
x_- = \frac{1 - x_g}{1 - x_g(1 + \cos \Theta')/2}
\]

with \( x_g = 2\omega'/Q \). This can be done economically by noting \( x_- = 2 - x_+ - x_g \) and we obtain a simple formula

\[
J \equiv \left| \frac{\partial(x_+, x_-)}{\partial(\omega', \Theta')} \right| = \frac{2}{Q} \left( \frac{\partial x_-}{\partial \Theta'} \right)_{x_g} = \frac{x_-^2 x_g \sin \Theta'}{Q(1 - x_g)}.
\]
The final form of $\varphi_q(\omega', \Theta')$ from parent quark is

$$
\varphi_q(\omega', \Theta') = \frac{C_F}{4N_C \gamma_0^2} x_g^2 x_g \sin \Theta' \frac{x_+^2 + x_-^2}{Q(1 - x_g) (1 - x_+)(1 - x_-)}. \tag{16}
$$

Using the kinematical relation $x_T^2 = 4(1 - x_+)(1 - x_-)(1 - x_g)/x^2$ one finds the emission density

$$
\varphi_q(\omega', \Theta') = \frac{1}{\omega' \sin \Theta'} \frac{C_F}{N_C \gamma_0^2} \frac{x_+^2 + x_-^2}{2}. \tag{17}
$$

From the exact result (17) we can easily obtain the approximation for small gluon transverse energy fraction $x_T$, i.e. for $x_- \to 1$,

$$
\varphi_q(\omega', \Theta') = \frac{C_F}{N_C \gamma_0^2} \frac{1}{\omega' \sin \Theta'} \frac{1 + (1 - \frac{2\omega'}{\bar{Q}})^2}{2}. \tag{18}
$$

This formula contains the DGLAP splitting function $P_{qq}(z) = C_F \frac{1 + z^2}{1 - z}$ as factor and can be seen to correspond to the expression

$$
\frac{dn}{dx_+ dx_T} = \frac{\alpha_s}{\pi} P_{qq}(x_+), \tag{19}
$$

for, with $x_T \equiv 2\omega' \sin \Theta'/Q$, the appropriate Jacobian factor is

$$
\left| \frac{\partial(x_+, x_T)}{\partial(\omega', \Theta')} \right| \approx \frac{2(1 - x_+)}{Q}. \tag{20}
$$

The Eq. (18) differs from the DLA result by the inclusion of the nonsingular DGLAP term and the correct angular dependence. We have computed $w_q$ by integrating the density as in (2) numerically with the bounds

$$
\frac{Q_0}{\sin \Theta} \leq \omega' \leq P, \quad \sin^{-1} \frac{Q_0}{\omega} \leq \Theta' \leq \Theta. \tag{21}
$$

This yields an improved result for the gap probability calculated again as in (9) or (10) which we refer to as “MLLA-n”.

An analytical approximation (“MLLA-a”) can be obtained by simplifying the integral over the nonsingular parts of the splitting function (17). One obtains

$$
w_q = w_q^{DLA} - \frac{3}{4} \left[ \ln \frac{Y + \lambda}{\lambda} - e^\lambda (E_1(\lambda) - E_1(Y + \lambda)) \right]. \tag{22}
$$
where $E_1(z) = \int_z^\infty dte^{-t}/t$ and $w_q^{DLA}$ denotes the expression in Eq. (8). The results from these calculations will be discussed in the next subsection.

Comparison of analytical results and a Monte Carlo calculation

The result of our computations in the different approximations are displayed in Fig. 1. For the parameter $\lambda = 0.1$ at $\Lambda = 0.244$ GeV we show the DLA result for the symmetric gap (10) and the MLLA results in the analytic approximation (22) and in the numerical evaluation of (2) with (18). Also shown as data points are the results from the ARIADNE Monte Carlo at the parton level for the same parameters. This Monte-Carlo generates the quark gluon cascade as a sequence of dipole emissions and it is terminated as in the analytical calculations by a transverse momentum cutoff. Furthermore, there is no limitation to the relative magnitude of the parameters (besides $Q_0 > \Lambda$).

As can be seen from the figure the MLLA corrections are not very large: about a factor or two after a decrease of the gap fraction by 5 orders of magnitude. Of the same order is the deviation from the parton Monte Carlo which takes into account effects not considered here: the correct angular recoil from the primary gluon emission also for large angles and the spillover of secondary gluons into the gap if the primary gluon is produced outside the gap. This effect could be responsible for the difference between the MC and the more accurate MLLA calculations. Surprisingly, the simple DLA and the MC results are quite close to each other.

The figure also shows the very strong dependence on the cutoff $Q_0$ for fixed $\Lambda$. Indeed, decreasing the transverse momentum cutoff from $Q_0 = 1$ GeV down to $Q_0 = 0.27$ the gap rate at $\Delta y = 6$ decreases by 4 orders of magnitude. This comes from the singular emission probability $dk_\perp/k_\perp \alpha_s(k_\perp)$ enhanced by the running $\alpha_s$: the closer we come to the pole at $k_\perp = \Lambda$ the more difficult it is to avoid the gluon bremsstrahlung.

3 Comparison with data

Apparently the prediction for the gap probability in the parton cascade depends sensitively on the parameter $Q_0$. A determination of both parameters $\Lambda$ and $Q_0$ within the MLLA-LPHD approach has been performed in the study of particle energy spectra applying the moment analysis. This study has shown that the two parameters are very close to each other and there is an upper limit to the $\lambda$-parameter:

$$\lambda \leq 0.1,$$

(23)
Figure 1: Fraction of rapidity gap events in $e^+e^-$ annihilation as a function of the full width of the symmetric gap. The DLA predictions from eq. (10) for different values of the transverse momentum cutoff $Q_0$, and for one value of $Q_0$ also the analytical (a) and the numerical (n) MLLA results (see text). The data points refer to the calculations of the ARIADNE Monte Carlo at the parton level for the corresponding parameters.
whereas the absolute scale was found to be $Q_0 \simeq 270$ MeV. A lower limit on $\lambda$ could in principle be obtained from the study of the mean multiplicity in a calculation of yet higher accuracy. As central value for our prediction we choose therefore $Q_0 = 270$ MeV and $\lambda = 0.1$ (or $\Lambda = 0.244$ GeV) but smaller values of $\Lambda$ are allowed as well.

In Fig. 2 we compare such results from the DLA (which agree closely with those from the parton MC) with the experimental measurement of the fraction of rapidity gap events from the SLD Collaboration. These data also contain a contribution from the $\tau$-lepton events which is important for the large gaps $\Delta y$. The data are well described by the JETSET Monte Carlo, including the $\tau$-decays. The Monte Carlo predictions without $\tau$-decays are also shown in the figure and indicate a steeper falloff for large $\Delta y$. Furthermore it should be noted that the data refer to rapidity gaps between charged particles. From the Monte Carlo one would expect a slightly steeper dependence if the gap is calculated from all final state particles.

One can conclude from the figure that the parton cascade with the low values for the scale parameters $Q_0$ and $\Lambda$ obtained from previous analyses gives already enough suppression for large rapidity gaps without any additional hadronization; values $\lambda = 0.05$-0.1 provide an adequate description of the gap distribution (our predictions refer to gaps in the final state of all charged and neutral particles).

4 Discussion and further predictions

The data are consistent with a description of the hadronic final state in terms of a parton cascade terminated at a rather low cutoff of the order of the final state hadron masses without any additional hadronization phase. This could be interpreted in favour of a dual correspondence of parton and hadron final states and a soft colour neutralization mechanism.

A good description of the data has been obtained by the JETSET Monte Carlo. In this model the parton cascade is terminated at a cutoff $Q_0 \sim O(1$ GeV$)$ and followed by a hadronization phase. According to Fig. 1 the suppression of large rapidity gaps at a scale $Q_0 \sim 1$ GeV is not so strong and most of the suppression must come from the resonance and particle production in the hadronization phase. In our parton calculation the strong suppression of large gaps again comes mainly from the last stage of the cascade evolution where the radiation becomes stronger because of the infrared enhancement and the increasing coupling constant $\alpha_s$. It appears that the average properties of the hadronization phase can be well represented by the parton bremsstrahlung cascade with running coupling.
Figure 2: Fraction of rapidity gap events in $e^+e^-$ annihilation as a function of the full width of the symmetric gap. The data points refer to the measurement of gaps between charged particles ($\tau$-lepton events included) by the SLD Collaboration. The dashed histogram shows the expectation from the JETSET Monte Carlo without $\tau$-leptons. Also shown are the DLA predictions for the gaps of a parton cascade as in Fig. 1.
In the following we note a few further predictions from the perturbative approach to the gap rate.

1. There is a difference in the gap fraction of quark and gluon jets according to the respective colour factors in (8). This can be tested, for example, by studying gaps in jets at high $p_T$ in $p\bar{p}$ collisions between angles of, say, $\Theta_1 < \Theta_2 < 45^\circ$ using eq. (9). The slope roughly doubles when going from quark to gluon jets. This measurement would test the connection of the gap probability to single gluon emission.

2. In the approximation considered the evolution of the parton cascades in both hemispheres is independent, so the corresponding rates factorize. For example the slope should double when going from the gap $(\Theta_G, \pi)$ to $(\Theta_G, \pi - \Theta_G)$.

3. The eq. (3) also predicts the energy dependence of the slope. With rising jet energy the gap distribution gets slightly steeper: at $Q=200$ GeV the slope is larger by 4% in comparison to $Q=90$ GeV.

It will be interesting to study the gap events further in order to test the proposed perturbative approach with soft colour neutralization.

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