Analysis of dynamic response of steel reinforced concrete bridge piers under impact

Wei Jiang, Nan Zhang, Jiequan Li, Min Gao, Jiajia Chen and Xiaoxiao Sha
College of Civil Engineering, Nanjing Tech University, Nanjing Jiangsu 211800, China
Email: 932276019@qq.com

Abstract. Through the static loading tests and dynamic tests on reinforced concrete piers and steel reinforced concrete piers with different steel profiles, the influence of built-in steel on the displacement response of concrete piers was studied. Using the single-degree-of-freedom system equivalent, a linear attenuation descent triangle is used to simplify the impact load. Based on the energy conservation equations, the mass transfer coefficients of the elastic stage and the plastic stage were calculated. A theoretical formula for the displacement response of the pier is established. The theoretical values are in good agreement with the experimental values. It provides some reference for further study on the anti-collision design of steel concrete bridge piers.

1. Introduction
With the rapid development of China’s bridge industry and the transportation industry, the number of motor vehicles has been increasing day by day. In recent years, accidents that caused bridges to collapse due to the impact of automobile piers have occurred from time to time. In order to enhance the ability of the bridge to resist impact, many scholars have carried out multi-angle research on the impact of bridges, including the calculation of pier impact force[1] and the analysis of shear resistance of bridge piers[2]. Many achievements have been made[1-3]. In order to investigate the dynamic response of bridge piers under impact loads and study the effect of built-in steel skeleton on the improvement of impact resistance of concrete piers, the displacement response of steel reinforced concrete bridge piers under static and impact conditions has been researched. The anti-collision design provides a certain research basis.

In this paper, the static loading and horizontal impact tests are performed on the reinforced concrete piers and steel reinforced concrete bridge piers, and the displacement response formulas are established. The calculated values and experimental values are compared and analyzed.

2. Impact Test Model
In this test, a set of 1 ordinary reinforced concrete pier model and 4 steel reinforced concrete pier models were designed for a total of 2 groups. The first group was subjected to the horizontal static test and the second group was subjected to the horizontal impact test. Ordinary reinforced concrete bridge piers are used as comparison piers. The cross-sectional reinforcement and dynamic test apparatus are shown in Fig.1. The specimens of the first group are numbered SRCP-S0 to SRCP-S4. SRCP-S0 stands for ordinary reinforced concrete bridge pier, SRCP-S1 stands for built-in angle steel, SRCP-S2 stands for built-in channel steel, SRCP-S3 stands for built-in double-circular steel pipe, SRCP-S4 stands for built-in unit steel pipe and vertical steel bar. Built-in steel bone is pre-welded into a steel skeleton embedded in the pier body template, and the test used the same batch of concrete. The specimens of the second group are numbered SRCP-D0 to SRCP-D4. The section and configuration
are the same as those of the first group and they are in one-to-one correspondence.

Before the impact test, a vertical force with an axial compression ratio of 0.1 was applied to the top of the test pier to simulate the effect of the superstructure of the bridge on the pier. Lateral support of the bridge was set to simulate the bridge's horizontal constraint on the pier. The bottom of the bridge piers are fixed with anchors and the sides are directly attached to the reaction walls. By changing the speed of the vehicle to control the impact energy, the total mass of the test trolley is 1606.5 kg. In order to study the collapse resistance of the pier under the impact, a rigid impact head was used for lateral impact with a shear span ratio of 1.89.

![Figure 1. Reinforcement of the pier and Test device](image1)

![Figure 2. Static test device](image2)

![Figure 3. Static load-displacement curve](image3)

For better lateral comparison of tests, both the steel reinforced concrete pier model and the reinforced concrete pier model use the same lateral reinforcement ratio and approximate longitudinal steel ratio. The basic material properties of the steel concrete pier model are shown in Table 1 and Table 2.

### Table 1. Results of concrete test

| Specimen tag | Vertical steel ratio/% | Axial pressure ratio | Compressive strength/MPa | Elastic Modulus/MPa |
|--------------|------------------------|----------------------|--------------------------|---------------------|
| SRCP-S0(D0)  | 3.87                   | 0.1                  | 37.13                    | 3.44×10^4           |
| SRCP-S1(D1)  | 3.53                   | 0.1                  | 33.67                    | 3.35×10^4           |
| SRCP-S2(D2)  | 3.91                   | 0.1                  | 34.44                    | 3.37×10^4           |
| SRCP-S3(D3)  | 3.90                   | 0.1                  | 34.28                    | 3.37×10^4           |
| SRCP-S4(D4)  | 3.89                   | 0.1                  | 37.69                    | 3.45×10^4           |

### Table 2. Mechanical properties of steel

| Steel type                  | Yield Strength/MPa | Ultimate strength/MPa | Elastic Modulus/MPa |
|-----------------------------|--------------------|-----------------------|---------------------|
| 10                          | 379                | 472                   | 2.0×10^5            |
| 12                          | 406                | 522                   | 2.0×10^5            |
| 14                          | 412                | 557                   | 2.0×10^5            |
| 16                          | 397                | 519                   | 2.0×10^5            |
| 22                          | 393                | 519                   | 2.0×10^5            |
| Angle Steel 30×3            | 266                | 364                   | 2.0×10^5            |
| Channel steel5              | 256                | 355                   | 2.0×10^5            |
| Steel Pipe                  | 271                | 363                   | 2.0×10^5            |
3. **Analysis of Test Results**

3.1. **Static Test Results Analysis**

By performing horizontal static tests Fig.2 on the piers of the same steel form at the impact point height, the "load-displacement" curve Fig.3, cracking load and ultimate bearing capacity at the impact point of the pier can be obtained. The effective stiffness of the elastic stage can be determined from the "load-displacement" curve and the cracking load, and then the effective stiffness of the plastic stage can be obtained from the ultimate bearing capacity and the cracking load. The result of the calculation is shown in the table 3.

**Table 3. Effective stiffness of pier impact point**

| Specimen | Flexible stage | Plastic stage |
|----------|----------------|---------------|
|          | $R_{cr}$/kN    | $K_e$/(kN/mm) | $R_m$/kN   | $K_{cr}$/(kN/mm) |
| SRCP-S0  | 190            | 26.39         | 427.88     | 23.46           |
| SRCP-S1  | 226.8          | 39.65         | 411.88     | 27.50           |
| SRCP-S2  | 203.9          | 32.95         | 453.87     | 25.61           |
| SRCP-S3  | 239.9          | 48.18         | 409.64     | 29.72           |
| SRCP-S4  | 201.8          | 53.13         | 423.41     | 26.18           |

3.2. **Analysis of Dynamic Test Results**

**Figure 4** Pier cracking mode

**Figure 5** Pier failure mod

**Table 4. Bridge Pier Model Displacement Response**

| Specimen | Flexible stage $y_{d}$/mm | Reduction rate/% | Plastic stage $y_{d}$/mm | Reduction rate/% |
|----------|---------------------------|-----------------|--------------------------|-----------------|
| SRCP-D0  | 12.9                      |                 | 45.3                     |                 |
| SRCP-D1  | 11.3                      | 12.4            | 39.9                     | 11.9            |
| SRCP-D2  | 9.1                       | 29.5            | 36.2                     | 20.1            |
| SRCP-D3  | 11.2                      | 13.2            | 38.2                     | 15.7            |
| SRCP-D4  | 12.5                      | 3.1             | 36.9                     | 18.5            |

From Fig.4, Fig.5 and table 4, we can see that the built-in steel bone delays the appearance of cracks in concrete piers and reduces the lateral deformation of concrete piers.

4. **Dynamic Response Analysis**

4.1. **Impact Load Simplification**

The entire impact process can be described using the simplified model shown in the Fig.6. The bridge pier model consists of equivalent masses and nonlinear springs. $y$ is the displacement of the pier's impact point and $F(t)$ is the impact load. $K$ is the effective stiffness of the spring and $R(y)$ is the resistance function of the test specimen.
For better theoretical calculations, we hereby appropriately simplify the impact load curve according to its characteristics. In literature [4], the impact force is considered as the sine wave and the triangular wave respectively, and the equivalent impact force is calculated by keeping the peak force unchanged. The literature [5] uses the half-sine curve model to simplify the impact load with equal peak and equal impulse principle for numerical analysis. Considering the typical impact force history curve shown in Fig.7, the absolute value of the slope of the impact force rising phase is much greater than that of the descending segment. This article will be based on the principle of equal peak and equal impulse to simplify the impact load curve to linear drop triangle impact force history curve as shown in Fig.8.

\[ F(t) = \begin{cases} \frac{F_{\text{max}}}{t_1}, & 0 \leq t \leq t_1 \\ 0, & t \geq t_1 \end{cases} \]

\[ \int_{0}^{t_1} F_p(t) dt = \int_{0}^{t_2} P(t) dt \]

\[ t_1 = \frac{2\int_{0}^{t_2} P(t) dt}{F_{p0}} \]

In the equations above, \( F(t) \) is the equivalent impact force duration; \( P(t) \) is the actual impact force duration; \( F_{\text{max}} \) is the peak impact force; \( t_1 \) is the equivalent impact force duration; \( t_2 \) is the actual impact force of the test;

4.2. Single Degree of Freedom Dynamic Response

Flexible stage:

According to the structural dynamics, the equation of motion for a single degree of freedom system under arbitrary loading is:

\[ M\ddot{y} + R(y) = F(t) \]
In the equation: M is the quality of the single degree of freedom system; R(y) is the resistance function of single degree of freedom system; y is the displacement of the single degree of freedom architecture; F(t) is the general load expression received by the single degree of freedom system; \( F(t) = F_{\text{max}} f(t) \)

Using the Duhamel integral gives the full solution of (4):

\[
y(t) = \beta y_{st} = y_{st} \omega \int_0^t f(\tau) \sin(\omega(t - \tau))d\tau
\]

\( \beta \) is the ratio of the displacement generated at any time F as a dynamic load to the displacement of F when it is acting as a static load.

When the vehicle impact force takes a linearly decreasing triangular wave,

\[
f(t) = 1 - \frac{t}{t_1}
\]

Among them, \( t_1 \) represents the duration of the equivalent impact force.

\[
\beta = \begin{cases} 
1 - \cos(\omega t) + \frac{1}{t_1} \left( \frac{\sin(\omega t)}{\omega} - t \right), & t \leq t_1 \\
\frac{1}{\omega t_1} [\sin(\omega t) - \sin(\omega(t - t_1))] - \cos(\omega t), & t > t_1 
\end{cases}
\]

From equations (7), we can see that the dynamic amplification factor \( \beta \) is a function of time \( t \). In order to apply it more effectively to engineering practice, here we mainly study the maximum value of \( \beta \).

It is known from literature[7] that if \( t_1/T_0 \) is known, the maximum value of \( \beta \) can be obtained from Fig.9.

In Fig.9, \( \xi = t_1/T_0 = T/T_0 \), which represents the ratio of the equivalent impact force duration \( t_1 \) to the natural period \( T_0 \) of the structure.

Plastic stage:
When \( R(y) \) is the ideal elastoplastic system

\[
R(y) = \begin{cases} 
Ky, & y \leq y_0 \\
R_0, & y > y_0
\end{cases}
\]

When in the plastic stage, replacing \( R(y) \) in equation (4) with a constant \( R_0 \) and performing differential transformation, the equation for the plastic stage is:

\[
\frac{1}{4\pi} \ddot{\beta} + 1 = \frac{F}{R_m} f(\xi)
\]

When the vehicle impact force is taken as a simplified linear descending triangular wave,

\[
f(\xi) = \begin{cases} 
1 - \xi(T/t_1), & \xi \leq t_1/T \\
0, & \xi > t_1/T
\end{cases}
\]

Bring equation (10) into equation (9), and use different \( F/R_0 \) and \( t_1/T \) to find the value of \( \beta \), and use \( t_1/T \) as the abscissa. The maximum value \( \beta \) is the maximum response graph when the vertical coordinate is plotted with different \( F/R_0 \), as shown in Fig.10.

4.3. Quality Conversion Factor
In order to simplify the analysis process, the impact of the continuous mass distribution of the test pier is based on the principle of conservation of energy, which is equivalent to a single degree of freedom system for analysis. Considering that the energy of the two systems before and after equivalence is equal [7], in the elastic stage, the single-degree-of-freedom system equivalent can be expressed as Fig.11(a) [8]. and the kinetic energy of the test pier (\( k_{ed} \)) can be expressed as:

\[
k_{ed} = \frac{1}{2} \int m(x) \dot{y}^2(x, t) dx
\]

\( m(x) \) is the mass of the pier column per unit length and \( \dot{y} \) is the velocity. For the elastic stage,
choose the shape function:

\[ y(t) = \begin{cases} 
  y_0(t) \sin \left( \frac{3\pi}{2L} x \right), & 0 < x \leq \frac{L}{3} \\
  y_0(t) \sin \left( \frac{3\pi}{4L} \left( x + \frac{L}{3} \right) \right), & \frac{L}{3} < x < L 
\end{cases} \]  

(12)

Calculated elastic mass conversion factor \( K_m = 0.5 \).

(a) SDOF elastic equivalence  
(b) SDOF plastic equivalence

**Figure 11.** A schematic diagram of the equivalent single degree of freedom system

In the plastic stage, the system is equivalent to the single degree of freedom shown in Fig.11(b). Then select the shape function:

\[ y(t) = \begin{cases} 
  y_0(t) \frac{x}{L/3}, & (0 \leq x \leq L/3) \\
  y_0(t) \frac{L - x}{2L/3}, & (L/3 < x \leq L) 
\end{cases} \]  

(13)

Computable plastic mass conversion factor \( K_m = 0.33 \). [8]

4.4. *Dynamic Response Solution of Steel Reinforced Concrete Bridge Pier under Impact Load*

Acquire R/F, \( t_1/T \), and the effective stiffness of the impact point according to the static test results and dynamic test force time-displacement curves. Calculate the theoretical value of the maximum displacement response of steel concrete piers under impact loads. The calculation results are shown in table 5 and table 6. The calculated values are in good agreement with the experimental values.

| Specimen     | \( \beta_{max} \) | Theoretical \( y_d/mm \) | Test \( y_d/mm \) | error /% |
|--------------|------------------|--------------------------|-----------------|----------|
| SRCP-D0      | 1.68             | 12.1                     | 12.9            | 6.2      |
| SRCP-D1      | 1.84             | 10.5                     | 11.3            | 7.1      |
| SRCP-D2      | 1.49             | 9.2                      | 9.1             | -1.1     |
| SRCP-D3      | 1.73             | 10.2                     | 11.2            | 8.9      |
| SRCP-D4      | 1.95             | 10.9                     | 12.5            | 12.8     |

| Specimen     | \( \beta_{max} \) | Theoretical \( y_d/mm \) | Test \( y_d/mm \) | error /% |
|--------------|------------------|--------------------------|-----------------|----------|
| SRCP-D0      | 2.49             | 43.1                     | 45.3            | 4.9      |
| SRCP-D1      | 2.51             | 37.8                     | 39.9            | 5.3      |
| SRCP-D2      | 2.01             | 32.1                     | 36.2            | 11.3     |
| SRCP-D3      | 2.53             | 33.7                     | 38.2            | 11.8     |
| SRCP-D4      | 2.61             | 34.6                     | 36.9            | 6.2      |

| Specimen     | \( \beta_{max} \) | Theoretical \( y_d/mm \) | Test \( y_d/mm \) | error /% |
|--------------|------------------|--------------------------|-----------------|----------|
| SRCP-D0      | 2.49             | 43.1                     | 45.3            | 4.9      |
| SRCP-D1      | 2.51             | 37.8                     | 39.9            | 5.3      |
| SRCP-D2      | 2.01             | 32.1                     | 36.2            | 11.3     |
| SRCP-D3      | 2.53             | 33.7                     | 38.2            | 11.8     |
| SRCP-D4      | 2.61             | 34.6                     | 36.9            | 6.2      |

5. **Conclusion**

In this paper, the following conclusions are drawn through experiments and calculations:
(1) The results of static test show that under the same steel allocation ratio, the effective elastic stiffness and the effective stiffness of steel reinforced concrete bridge piers are larger than those of ordinary reinforced concrete bridge piers. The dynamic test results show that under the same impact energy, the deformation of steel reinforced concrete piers is smaller than that of ordinary reinforced concrete piers.

(2) The bridge pier specimen is equivalenced by the single degree of freedom system. Based on the law of conservation of energy, the mass conversion coefficients of the piers of the elastic stage and plastic stage piers are 0.5 and 0.33, respectively.

(3) Based on the elasto-plastic deformation of the pier mass-simplified nonlinear spring analysis model, according to the single-degree-of-freedom system dynamic response calculation, the calculation method of the dynamic amplification factor $\beta$ of the elastic stage and the plastic stage is established respectively. Finally, the dynamic response solution are checked separately in the elastic stage and the plastic stage. The theoretical results are in good agreement with the experimental results.

Reference

[1] Wencai Fan, Nan Zhang, Qi Xu. The Calculation of Bearing Capacity of the normal Section of RC circular Pier under compact[J]. Highway Engineering, 2010, 1: 107-112. (in Chinese)

[2] Lijie Zhang, Nan Zhang, Yang Yang, Lu He. Calculation Method Study on Shear Resistance of Steel Reinforced Concrete Bridge Pier[J]. Highway Engineering, 2015, 40(2): 34-39. (in Chinese)

[3] Jiang Wang, Delong Wu. Design Method of the Structures under Shock Loads[J]. Missiles and Space Vehicles, 2007, 3: 33-37. (in Chinese)

[4] Hui Wang, Nan Zhang, Xu Chen, Hongqin Lu. Research on the Calculation Method of Equivalent Impact Forces of Bridge Piers[J]. Science Technology and Engineering. 2016, 32: 120-126. (in Chinese)

[5] Xiaoyu Zhou, Rujin Ma, Airong Chen. Anti-shear reliability analysis for a reinforced concrete column subjected to rockfall impact [J]. Journal of Vibration and Shock. 2017, 7: 262-270. (in Chinese)

[6] Sai Wu. Dynamic Response and Damage Assessment of CFST Columns Subjected to Blast Loading[D]. Chang’an University. 2015. (in Chinese)

[7] Biggs, JohnM. Introduction to structural dynamics[M]. Mc Graw-Hill, 1964.

[8] Yi Meng. Experiment and numerical simulation study on reinforced concrete beam under impact loading[M]. Hunan University. 2012. (in Chinese)