NONLOCAL EFFECTS IN QUANTUM GRAVITY

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Abstract

Recently [2], a new quantum gravity theory was presented in which the quantum effects were represented by the conformal degree of freedom of the space–time metric. In this work we show that in the framework of this theory quantum gravity is nonlocal.

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I. INTRODUCTION

In a recent work [2], it was shown that the quantal behaviour of matter can be understood as a purely geometrical effect. In fact the conformal degree of freedom of the space–time metric would be determined by quantal effects. In this view, the geometry has two physical significances. First, its conformal degree of freedom represents what is usually called the quantal effects. Secondly, its other degrees of freedom determine the causal structure of the space–time. This second part, in the absence of quantal effects (where the conformal factor is a constant), is called classical gravity. These two parts are highly coupled so that the theory is expected to be quantum gravity theory.

The theory primarily rests on the de-Broglie–Bohm quantum theory [1], which is the causal counterpart of quantum mechanics. In Bohmian mechanics, any particle is always accompanied by an objectively real field exerting some force on the particle. This is called the quantum force. In the case of relativistic particles, the quantum potential is nothing but the mass of the particle. So the equation of motion of a relativistic particle is [2]:

\[ \frac{d(Mu_\mu)}{d\tau} = c^2 \nabla_\mu M \]  \hspace{1cm} (1)

where

\[ M^2 = m^2 + \frac{\hbar^2 \Box |\Psi|}{c^2 |\Psi|} \]  \hspace{1cm} (2)

and

\[ \Box \Psi + \frac{m^2 c^2}{\hbar^2} \Psi = 0 \]  \hspace{1cm} (3)

The theory rests also on the de-Broglie ansatz that the presence of quantum force is identical
to having a curved space–time. This fact can be seen simply by writing (1) in the Hamilton–Jacobi form:

$$g^{\mu\nu}\nabla_\mu S\nabla_\nu S = \mathcal{M}^2 c^2; \quad \nabla_\mu S = \mathcal{M} u_\mu$$

Equation (4) can be rewritten as:

$$\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu S\tilde{\nabla}_\nu S = m^2 c^2; \quad \tilde{g}^{\mu\nu} = \frac{\mathcal{M}^2}{m^2} g^{\mu\nu}$$

Accordingly, an appropriate action for quantum gravity is written in [2]. The corresponding equations of motion are:

$$\mathcal{R}\Omega + 6\Box\Omega + \frac{2\kappa}{m}\rho\Omega \left(\nabla_\mu S\nabla^\mu S - 2m^2\Omega^2\right) + 2\kappa\lambda\Omega = 0$$

$$\nabla_\mu \left(\rho\Omega^2\nabla^\mu S\right) = 0$$

$$\left(\nabla_\mu S\nabla^\mu S - m^2\Omega^2\right) \Omega^2 \sqrt{\rho} + \frac{\hbar^2}{2m} \left[\Box \left(\frac{\lambda}{\sqrt{\rho}}\right) - \lambda \frac{\Box\sqrt{\rho}}{\rho}\right] = 0$$

$$G_{\mu\nu} - \frac{[g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu] \Omega^2}{\Omega^2} - 6\nabla_\mu \Omega \nabla_\nu \Omega \omega^2 + 3g_{\mu\nu} \nabla_\alpha \Omega \nabla_\beta \Omega - \frac{2\kappa}{m}\rho\nabla_\mu S\nabla_\nu S - \frac{\kappa}{m}\rho g_{\mu\nu} \nabla_\alpha S\nabla_\beta S$$

$$\quad + \kappa m \rho \Omega^2 g_{\mu\nu} + \frac{\kappa h^2}{m^2} \left[\nabla_\mu \sqrt{\rho} \nabla_\nu \left(\frac{\lambda}{\sqrt{\rho}}\right) + \nabla_\nu \sqrt{\rho} \nabla_\mu \left(\frac{\lambda}{\sqrt{\rho}}\right)\right] - \frac{\kappa h^2}{m^2} g_{\mu\nu} \nabla_\alpha \left(\frac{\nabla_\beta \sqrt{\rho}}{\sqrt{\rho}}\right) = 0$$

$$\Omega^2 = 1 + \frac{\hbar^2}{m^2} \frac{\Box\sqrt{\rho}}{\sqrt{\rho}}$$

where $\Omega$ is the conformal degree of freedom of the metric, $\lambda$ is a lagrange multiplier and $\rho = \Psi^* \Psi$ is the matter density. A special case is when $\lambda$ can be expanded in powers of $\alpha = h^2/m^2$. Then, it can be simply shown [3] that in this case $\lambda = 0$, and the equations of motion are:
\[ \nabla_\mu \left( \rho \Omega^2 \nabla^\mu S \right) = 0 \]  

(11)

\[ \nabla_\mu S \nabla^\mu S = m^2 \Omega^2 \]  

(12)

\[ G_{\mu\nu} = -\kappa T_{\mu\nu}^{(m)} - \kappa T_{\mu\nu}^{(\Omega)} \]  

(13)

\[ T_{\mu\nu}^{(m)} = \frac{\rho}{m} \nabla_\mu S \nabla_\nu S \]  

(14)

\[ \kappa T_{\mu\nu}^{(\Omega)} = \left[ g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu \right] \Omega^2 + 6 \frac{\nabla_\mu \Omega \nabla_\nu \Omega}{\omega^2} - 3 g_{\mu\nu} \frac{\nabla_\alpha \Omega \nabla^\alpha \Omega}{\Omega^2} \]  

(15)

\[ \Omega^2 = 1 + \alpha \frac{\Box \sqrt{\rho}}{\sqrt{\rho}} \]  

(16)

As one can see, there are two contributions to the background metric \((g_{\mu\nu})\). First, we have \(T_{\mu\nu}^{(m)}\) which represents the gravitational effects of matter. Second, there is \(T_{\mu\nu}^{(\Omega)}\) which is a result of the quantal effects of matter. Since in the evaluation of the \(T_{\mu\nu}^{(\Omega)}\) the background metric is used, the gravitational and quantal contributions to the background metric are so highly coupled that no one without the other has any physical significance. In this way the theory is a quantum gravity theory.

It must be pointed out here that, since the conformal factor is meaningless as \(\rho \to 0\), the geometry loses its meaning at this limit. This is a desired property, because it is in accord with Mach’s principle, which states that for an empty universe the space–time should be meaningless.

A special aspect of the quantum force is that it is highly nonlocal. This property, which can be seen from the equation (3), is an experimental matter of fact [3]. Since the mass field given by (2) represents the conformal degree of freedom of the physical metric, quantum gravity is expected to be highly nonlocal. In the next section, this is shown explicitly for a specific problem.
II. ILLUSTRATION OF NONLOCAL EFFECTS IN QUANTUM GRAVITY

In order to illustrate how nonlocal effects can appear in quantum gravity through quantum potential, suppose that matter distribution is localized and has spherical symmetry. Then, one has:

\[ \rho = \rho(t; r) \quad (17) \]

\[ \Omega = \Omega(t; r) \quad (18) \]

Suppose, furthermore, that matter is at rest:

\[ -\nabla_0 S = E(t; r) \text{ as } r \to \infty \quad (19) \]

\[ \nabla_i S = 0; \quad i = 1, 2, 3 \quad (20) \]

One expects that at large \( r \), where there is no matter, the background metric would be of the Schwartzschild form:

\[ g_{\mu\nu} = \begin{pmatrix} 1 - r_s/r & 0 & 0 & 0 \\ 0 & -1/(1 - r_s/r) & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \theta \end{pmatrix} \text{ as } r \to \infty \quad (21) \]

where \( r_s \) is a constant (the Schwartzschild radius). The validity of this approximation will be examined at the end. The equation of motion \([12]\) relates \( E \) and \( \Omega \):

\[ E = \frac{m\Omega}{\sqrt{1 - r_s/r}} \quad (22) \]
In order to calculate the conformal factor $\Omega$, one needs the specific form of $\rho$. It must be a localized function at $r = 0$. So we choose it as:

$$\rho(t; r) = A^2 \exp[-2\beta(t)r^2] \quad (23)$$

Using the relation (16), the conformal factor can be simply calculated. This leads to:

$$\Omega^2 = 1 + \alpha[\dot{\beta}^2r^4 - \ddot{\beta}r^2 + 4\beta^2r]$$

from which we get:

$$\Omega^2 \approx \alpha \dot{\beta}^2 r^4 \quad \text{as } r \to \infty \quad (24)$$

Now it is a simple task to examine that the continuity equation (11) is satisfied automatically as $r \to \infty$. This solution is an acceptable one, only if the generalized Einstein’s equations (13) are satisfied. This is so if $T^{(\Omega)}_{\mu\nu} \to 0$ as $r \to \infty$. It can be shown that in the limit $r \to \infty$ we have:

$$\frac{\Box \Omega^2}{\Omega^2} = 2(\ddot{\beta}/\dot{\beta})^2 + 2\dot{\beta}/\dot{\beta} - 20/r \quad (25)$$

$$\frac{\nabla_0 \nabla_0 \Omega^2}{\Omega^2} = (\ddot{\beta}/\dot{\beta})^2 + \dot{\beta}/\dot{\beta} \quad (26)$$

$$\frac{\nabla_1 \nabla_1 \Omega^2}{\Omega^2} = 12/r^2 \quad (27)$$

$$\frac{\nabla_1 \nabla_0 \Omega^2}{\Omega^2} = (8\ddot{\beta}/r\dot{\beta}) \quad (28)$$

$$\left(\frac{\nabla_0 \Omega}{\Omega}\right)^2 = (\ddot{\beta}/\dot{\beta})^2 \quad (29)$$

$$\frac{\nabla_1 \Omega \nabla_0 \Omega}{\Omega^2} = (2\ddot{\beta}/r\dot{\beta}) \quad (30)$$
So provided that higher time derivatives of the scale factor of matter density \((\beta)\) are small with respect to its first time derivative, that is:

\[
\frac{\ddot{\beta}}{\dot{\beta}} \simeq 0; \frac{\dddot{\beta}}{\dot{\beta}} \simeq 0 \quad \text{and so on}
\]  

(31)

one has:

\[
\lim_{r \to \infty} T^{(\Omega)\nu}_{\mu} = 0
\]  

(32)

Also we have from (14):

\[
\lim_{r \to \infty} T^{(m)\nu}_{\mu} = 0
\]  

(33)

So at large distances \(g_{\mu \nu}\) satisfies Einstein’s equations in vacuum, \(G_{\mu \nu} = 0\). Therefore, the solution (21) is acceptable. In this way we find a solution to the quantum gravity equations at large distances.

Consequently, if the time variation of \(\beta\) is small, the physical metric \(\tilde{g}_{\mu \nu} = \Omega^2 g_{\mu \nu}\) is given by:

\[
\lim_{r \to \infty} \tilde{g}_{\mu \nu} = \alpha \beta^2 r^4 g^{(\text{Shwarzschild})}_{\mu \nu}
\]  

(34)

An important points must be noted here. As it was shown, a change in matter distribution (due to \(\dot{\beta}\)) instantaneously alters the physical metric. This is because of the appearance of \(\dot{\beta}(t)\) in equation (34) and it comes from the quantum potential term.

We conclude that the specific form of the quantum potential leads to the appearance of nonlocal effects in quantum gravity.
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