A correlated electromechanical system

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Abstract
A correlation with phonons sustained by a pair of electromechanical resonators that differ both in size and frequency is demonstrated. In spite of the electromechanical resonators being spatially distinct, they can still be strongly dynamically coupled via a classical analogue of the beam splitter interaction with a cooperativity exceeding five, and parametric down-conversion which results in both resonators self-oscillating. This latter regime yields a classical variant of a two-mode squeezed state which is identified as perfectly correlated phase-locked vibrations between the two resonators. The creation of a correlation between two separate mechanical resonators suggests that extending this interaction to vacuum phonon states could enable a macroscopic two-mode squeezed state to be generated. Conversely, the ability to resolve the correlated state via the self-oscillations could be harnessed to build a new class of detector where an external stimulus neutralises the phase-locked vibrations.

1. Introduction
The light from a two-mode squeezed state generated from parametric down-conversion [1–4] has become an indispensable resource for a plethora of quantum enabled technologies including quantum communication [5], optical quantum computing [6] and quantum teleportation [7]. The entangled photons at the heart of this phenomenon stem from the vacuum states of the constituent electromagnetic oscillators [8]. Indeed, entanglement from two-mode squeezing has also been extended to microwave photons in superconducting circuits [9], and even to hybrids composed of photons and phonons in optomechanical systems [10–12] where again the inputs comprise of vacuum states.

Inspired by these advances, parametric down-conversion has recently been executed purely with phonons corresponding to different vibration modes in the same mechanical system [13–15]. The thermal phonon states sustained by these vibration modes yielded, instead of an entanglement, correlated vibrations namely the classical analogue of two-mode squeezing. Crucially, and in contrast to the electromagnetic and hybrid cases where the entanglement was spatially distributed [16, 17], for instance, between the mechanical resonator and the optical cavity, the classical phonon variant was manifested between mechanical vibration modes that were nearly degenerate in frequency and were co-localised. Consequently, a fundamental aspect of two-mode squeezing, in which the resultant state can be spatially separated, remains an open challenge in the classical limit with phonons. The demonstration of a correlation between two different mechanical resonators from parametric down-conversion, even in the classical limit, would indicate that extending this interaction to vacuum phonons should enable entanglements to be generated between two disparate solid state–state objects. Ultimately, access to such a state would be extremely desirable, as it would provide an invaluable platform in which to investigate the absence of quantum mechanical phenomena in our everyday classical world [12, 18–20]. Meanwhile, the ability to decipher classically correlated vibrations between spatially distinct mechanical resonators could be exploited to manipulate the vibrations of one of the resonators by measuring the vibrations of the other, thus further expanding the functionality portfolio of electromechanical resonators, as well as enabling systems with broken time-translation symmetry to be investigated [21].
where phonons are generated in both resonators. The frequency axis is not to scale due to the large spectral difference between the fundamental flexural modes of the micro-resonator and the nano-resonator but the amplitude axis details the motion-induced piezovoltage measured in a voltage-to-current amplifier. (c) Spectroscopy of the parametric coupling on the red sideband via weak (4 mV$_{rms}$) harmonic probing of the nano-resonator with $V$$_{p}$ = 500 mV$_{rms}$ where the probe frequency has been normalised to $\omega$ and the pump frequency to $\omega$ = $\omega$$_{p}$. (d) The corresponding theoretical response extracted from equations (1) - (6), for the red sideband spectroscopy via the nano-resonator, can perfectly reproduce the experimental response with $\Gamma$($\omega$) = 10$^{3}$.$\omega$. The coupling rate (points) and the cooperativity (red line), extracted from the peak splitting in the nano-resonance as a function of $\frac{V_{p}}{\sqrt{\omega_{p}}}$ at $\omega$ = 0.05, where the latter reveals a linear dependence and the former confirms operation deep into the strong coupling regime namely $C$ $\gg$ 1 (grey line) being available. Also shown is the theoretical coupling rate (red line) which is extracted from the calculated peak splitting in the nano-resonance as a function of $\frac{V_{p}}{\sqrt{\omega_{p}}}$ for $\omega$ = $\omega$$_{p}$ below which an extrapolation (dashed blue line) confirms the linear dependence.

2. Coupled electromechanical system

One natural approach to addressing this challenge is to simply implement parametric down-conversion between a pair of spatially distinct mechanical resonators rather than modes in the same system [22, 23]. However, the challenge to realising this in practice emerges from being able to sufficiently strongly couple the resonators so that they can sustain parametric down-conversion. To address this, the coupled electromechanical system detailed in figure 1(a) is developed, which consists of two radically different mechanical elements with a nano-resonator ($n$) placed into the clamping point of a micro-resonator ($\mu$). This geometry is identified from a finite element method (FEM) simulation in order to maximise their inherent elastic coupling $G$, where the motion of one resonator generates strain which modifies the natural frequency of its counterpart and vice versa [24]. If the coupling between the micro-resonator and the nano-resonator can be modulated via strain then it can be parametrically enhanced, leading to a system Hamiltonian given by

$$\mathcal{H}=\sum_{\mu n}(\frac{1}{2}\omega^{2} + \frac{1}{2}\omega^{2} + \omega^{2} \Gamma_{\mu} \cos((\omega_{p} + \Delta_{\mu} + \phi))$$

(1)

where the kinetic and potential energies of both resonators with frequency $\omega$ and mass $m$, are defined by their canonical position $x$ and conjugate momentum $p$ in the summation [25]. The second term describes the parametric coupling between the resonators which is activated by modulating the strain in the electromechanical system with amplitude $\Gamma$ at frequency $\omega$$_{p}$ = $\omega$ ± $\Omega$, with detuning $\Delta_{\mu}$ and phase $\phi$, which dynamically enhances their union [25]. In this configuration, this purely phononic system can emulate cavity-optomechanical systems where this analogy is predicated on the availability of dynamic strain modulation between the resonators [23, 25, 26]. Note that the intramodal coupling terms of the form $\cos((\omega_{p} + \Delta_{\mu} + \phi))$ have been omitted from the Hamiltonian as they are unavailable at the pump frequencies explored in this study.
2.1. Device
The electromechanical system shown in figure 1(a) was synthesised from a GaAs/AlGaAs high electron mobility transistor heterostructure sustaining a two-dimensional electron gas (2DEG) 90 nm below the surface [23]. The micro-resonator and nano-resonator were defined via conventional micromachining processes and had a length, width and thickness of 100 μm, 30 μm, 1.3 μm and 40 μm, 4 μm, 1.3 μm, respectively. In order to dynamically generate strain in the electromechanical system, piezoelectric transducers were located at the positions of maximal strain, namely at the clamping points of both resonators, again with reference to an FEM simulation, and were composed from the 2DEG confined in a 150 nm thick mesa and 150 nm thick gold Schottky electrodes (pink and yellow, respectively, in the micrograph in figure 1(a)) sandwiching a layer of undoped AlGaAs.

In addition to generating strain, the piezoelectric transducers also enabled the motion of the mechanical elements to be both activated, with application of voltage, and measured from the resultant motion-induced piezovoltage [23, 24]. This permitted the fundamental flexural modes of both resonators, whose motional profiles extracted from FEM simulations are shown in the insets to figure 1(a), to be identified, yielding a natural frequency ω/2π = 394.001 kHz with dissipation γ/2π = 2.8 Hz for the micro-resonator and ω/2π = 2.55761 MHz with γ/2π = 11 Hz for the nano-resonator, as shown in figure 1(b).

3. Parametric coupling
First, in order to confirm the availability of parametric coupling between the resonators, one of the piezoelectric transducers on the micro-resonator is activated on the red sideband, i.e. ω − ωμ, as schematically depicted in figure 1(a), with pump amplitude Ψp ≈ Ψ/αG, where α is the piezoelectric coupling coefficient. In this configuration a classical analogue of the beam splitter interaction becomes available with phonons being created in one of the resonators at the expense of phonons being annihilated in the counterpart, and if the pump voltage can be sufficiently increased then this process can even be enhanced to the point where the two mechanical resonators can strongly couple and hybridise undergoing parametric normal mode splitting [23, 26–29]. To that end, the nano-resonator is weakly harmonically probed, with the application of voltage to its piezotransducer around its natural frequency, while ω′, is swept around ω, as shown in figure 1(c), which reveals an up-converted sideband from the micro-resonator approaching the nano-resonance and undergoing an avoided-crossing, the characteristic feature of strongly coupled systems [23]. The resultant parametrically enhanced coupling rate can then be extracted from the peak splitting at exactly ω in figure 1(c) as 2 which reveals a linear dependence on Ψp as shown in figure 1(e). The potency of the available coupling can also be evaluated from the cooperativity C = 4 γ/γμ and for Ψp = 500 mVrms yielding ∼ 6Hz, this corresponds to C > 5, which indicates strong dynamic coupling between the two resonators, even though they differ in size, frequency and occupy different spatial volumes [27]. Access to even larger cooperativities is limited by the piezotransducers inability to sustain pump voltages greater than 500 mVrms due to current leakage, between the gold gate and the 2DEG, which neutralises the piezoelectric effect [23].

3.1. Theoretical response
In order to verify the experimental response detailed in figure 1(c), the equations of motion for both resonators can be extracted from the above Hamiltonian which yields

\[
\begin{align*}
\frac{\mu}{\mu} & + \mu \gamma \mu + \mu \omega^2 \mu (1 + \beta \mu) = \Lambda \mu \cos(\omega \mu + \delta \mu) + \gamma \mu \cos(\omega \mu + \gamma \mu + \phi) \\
& + \gamma \mu \omega^2 (1 + \beta \mu) = \Lambda \mu \cos(\omega \mu + \gamma \mu + \phi) + \gamma \mu \cos(\omega \mu + \gamma \mu + \phi)
\end{align*}
\]

where dissipation terms parameterised by γ, the Duffing non-linearities defined by β and harmonic probe terms with amplitude Λ and detuning δ, all of which are not given in the above Hamiltonian, have also been introduced. It should be noted that dispersive coupling terms of the form \( \frac{\mu}{\mu} \) and \( \frac{\mu}{\mu} \) and non-linear dissipation terms of the form \( \frac{\mu}{\mu} \) and \( \frac{\mu}{\mu} \) have not been included in the above equations of motion as they were both found to be negligible in the present study contrast to a larger variant of this device previously investigated [24].

These equations of motion can be solved using the rotating wave approximation at \( \omega \) by decomposing \( X_{\mu}(\omega) = X(\omega) \cos(\omega \mu + \gamma \mu) [\sin(\omega \mu + \gamma \mu) + \frac{\Lambda}{\mu} \mu \sin(\delta \mu)] + \frac{\gamma}{4 \mu} \mu \omega^2 (X(\omega) \cos(\Delta \mu + \phi) + Y(\omega) \sin(\Delta \mu + \phi)) + \frac{\gamma}{4 \mu} \mu \omega^2 (X(\omega) \cos(\Delta \mu + \phi) + Y(\omega) \sin(\Delta \mu + \phi)) \] (3)
\[
\begin{align*}
\dot{Y}_p(t) &= -\frac{i}{2} \gamma Y_p(t) + \frac{j}{8} \beta_p \omega_p \dot{X}_p(t) (2 \dot{Y}_p(t) + \nu_p(t)) + \frac{\Lambda_p}{2 \mu \omega_p} \cos(\delta_p) \\
&\quad + \frac{\Gamma}{4 \mu \omega_p} (\mp \chi_1(\cos(\Delta + \phi) \mp Y(t) \sin(\Delta_p + \phi)))
\end{align*}
\]

\[
\dot{X}(t) = -\frac{i}{2} \gamma X(t) - \frac{j}{8} \beta \omega \dot{Y}(t) (\dot{X} + \nu(t)) + \frac{\Lambda}{2 \omega} \sin(\delta)
\]

\[
\begin{align*}
\dot{Y}(t) &= -\frac{i}{2} \gamma Y(t) + \frac{j}{8} \beta \omega \dot{X}(t) (2 \dot{Y} + \nu(t)) + \frac{\Lambda}{2 \omega} \cos(\delta) \\
&\quad + \frac{\Gamma}{4 \omega} (\mp \chi_1(\cos(\Delta + \phi) \mp Y(t) \sin(\Delta_p + \phi)))
\end{align*}
\]

where the coupling polarities in the final term correspond to \(\begin{pmatrix} \omega \\ \omega \end{pmatrix}\). From the experiment, strong harmonic probing of the micro-resonator and the nano-resonator enables the Duffing non-linearities to be extracted as \(\beta_p = 0.1\) and \(\beta = 1\). On the other hand, the FEM analysis of the electromagnetic system yields \(\mu = 0.024\). Equations (3)–(6) are then simultaneously solved using the Runge-Kutta method in the red sideband case where the nano-resonator is weakly probed (\(\Lambda = 500\)) while the electromagnetic system is strongly pumped and detuned around \(\omega\), as in the experiment detailed in figure 1(c). The theoretical response of the nano-resonator is extracted in the steady state (i.e. \(\gg 2 \pi / \gamma_p\) and \(2 \pi / \gamma\)) via \(\sqrt{\dot{X}^2 + \dot{Y}^2}\) by using \(\dot{Y}(\omega)\) as a fitting parameter, and is shown in figure 1(d). The corresponding coupling rate can also be evaluated from this output, and is detailed in figure 1(e) as a function of \(\dot{Y}(\omega)\). Both numerical simulations perfectly reproduce the experimental results, thus confirming the parametrically enhanced strong coupling between the two different mechanical resonators.

4. Parametric down-conversion

The availability of strong dynamic coupling on the red sideband suggests that the two-mode squeezer operation could also be highly potent in this device when the pump is activated on the blue sideband, namely \(\omega_p \equiv \omega = \omega_p\) [26]. To that end, noise spectroscopy of the micro-resonator and nano-resonator is performed as a function of \(\nu(t)\) at \(\omega\) which reveals the spectral regions around both resonances being white when \(\nu(t) \ll 100 \text{ mV}_{\text{rms}}\) as shown by the black spectra in figures 2(b) and (c). The noise floor in these measurements is limited by the room temperature amplifiers, yielding a displacement sensitivity of \(~\text{pmHz}^{-1.5}\), which is insufficient to resolve the thermal fluctuations from both resonators even as they undergo parametric amplification [30, 31]. However, when \(\nu(t) \geq 100 \text{ mV}_{\text{rms}}\), both resonators can be readily identified via their self-oscillations as seen from the coloured spectra in figures 2(b) and (c), with the resultant maximum amplitudes as a function of \(\nu(t)\) shown in figure 2(a). The self-oscillations arise when the rate at which the phonons are simultaneously generated, from the parametric down-conversion, in both resonators exceeds their rate of loss, thus yielding a threshold with respect to \(\nu(t)\) [13, 21, 23, 28, 29, 31, 32].

4.1. Theoretical response

To substantiate this experimental response, equations (3)–(6) are solved in the blue sideband case where \(\dot{Y}(\omega)\) is now used as fitting parameter with \(\delta_p = \delta = \Delta_p = 0\). The results of this calculation, shown in figure 2(d), faithfully captures the threshold to self-oscillations, the larger amplitude of the micro-resonator with respect to the nano-resonator, due to \(\gamma > \gamma_p\) and the change in curvature of both resonators’ amplitudes at large \(\dot{Y}(\omega)\) due to their Duffing non-linearities becoming active [33]. It should be noted that in an earlier related study, self-oscillations of a plate and a beam mode in a torsional resonator were observed, by pumping the system on the blue sideband, where an analytical model was developed to describe the threshold to self-oscillations [21]. A quantitative comparison between the models employed in these studies is unviable due to the different underlying conditions used to construct them, with the dispersive coupling being omitted in the present numerical case in contrast to it being included in the earlier analytical case, where this variation most likely stems from the differing geometries of the two electromechanical systems.

Below threshold parametric amplification can also be seen in both resonators, as shown in figure 2(d), but experimentally this regime is unavailable, as detailed above [31]. Previously, a two-mode thermal squeezed state was observed in this limit, and to that end equations (3)–(6) are instead solved 2000 times on-resonance with four random number generators mimicking the thermal Langevin force driving both resonators’ Brownian
motion, in lieu of the harmonic probe terms in $\Lambda$. The phase portraits of both resonators can then be extracted via the resonators’ in-phase $X_i$ and quadrature $Y_i$ components. The cross-quadratures $X_i : Y_i$ and $X : Y_i$ of the amplified random positions fluctuations, namely $X_i : Y_i$ and $X : Y_i$, yielding filled circular distributions in figure 2(e), indicate that two-mode squeezing can also be expected between the micro-resonator and nano-resonator in the amplification regime, and this is shown in figure 2(e) [13, 14]. Since this region is experimentally inaccessible, it therefore becomes natural to look for features of two-mode squeezing from the self-oscillating states of the resonators instead. In contrast to the filled-circle displacement noise distributions in the amplification regime, corresponding to both random phase and amplitude, the phase portraits from the self-oscillating states yield circular-edged distributions indicating a fixed amplitude but random phase, as shown in figure 2(f). Remarkably, the cross-quadratures from the self-oscillations lead to a variation of two-mode squeezing, also shown in figure 2(f), which physically corresponds to correlated vibrations between the micro-resonator and nano-resonator, whose relative phase becomes locked. Ostensibly, the self-oscillating states operate far above the thermal (and thus vacuum) noise of the two resonators, and even though the squashed distributions captured in the cross-quadratures, shown in figure 2(f), are a manifestation of two-mode squeezing as they originate from the same physical process, namely parametric down-conversion but with large gain, the underlying fundamental noise in the resonators is not suppressed in this limit as in the amplification regime [13–15].

5. Correlated self-oscillations

To experimentally verify this expectation, the self-oscillating states of both resonators are measured at $V_{rms} = 120$ mV via their motion-induced piezovoltage, which is mixed with two local oscillators fixed on their resonance frequencies in an open-loop configuration and then demodulated in two phase-sensitive detectors with a sampling rate of 250 ms for 2000 samples. To ensure that both resonators were simultaneously interrogated, identical measurement circuits were employed, as detailed below. This measurement yields four time series for $X_i$, $Y_i$, $X$, $Y$ from which the phase portraits of the self-oscillating states can be reconstructed, as shown in figure 2.
figures 3(a) and (b). As expected, the self-oscillations yield distributions with a circular-edge corresponding to a fixed amplitude but random phase [34]. Accordingly, the cross-quadratures extracted from this measurement yield squashed distributions originating from a two-mode correlation, as shown in figures 3(c) and (d). This state corresponds to perfectly entwined vibrations between the resonators, which is statistically confirmed by their unity correlation coefficient that is derived from the covariance of the cross-quadratures normalised by the product of their constituent standard deviations [13]. Indeed, mapping the correlation coefficient as a function of \( V_p \) yields the response in figure 3(e) which indicates the absence of a correlation below the self-oscillation threshold with a zero correlation coefficient, and a perfect correlation above threshold with a unity coefficient. This observation is consistent with the spectral measurements in figure 2(a), where below the self-oscillation threshold the measured signals originate from the noise in the amplification chains, and thus the observation of a correlation from parametric amplification is unavailable in this regime.

5.1. An anti-correlated state
The self-oscillations in both resonators harbour correlated or phase-locked vibrations between the micro-resonator and nano-resonator but the above Hamiltonian also suggests a pump phase \( \phi \) dependence of the correlation relative to the local oscillators used in the detection circuit [35]. To experimentally investigate this trait, the self-oscillating states shown in figure 4(a) are generated and their correlation coefficient matrix, capturing all permutations of \( X_m, Y_m, X, Y \), is then extracted, which reveals unity auto-correlations in the diagonal elements (black) and unity off-diagonal elements (pink and purple) which correspond to the cross-quadratures and it indicates a perfectly correlated state. However, as \( \phi \) is adjusted, the squashed distribution in phase-space captured by the now un-rotated cross-quadratures, is anti-squeezed and it reverts to a circular distribution with fixed amplitude but random phase, as shown in figure 4(b). This implies that these cross-quadratures can no longer measure the correlation and this is statistically confirmed by the correlation coefficient matrix which now reveals that while the auto-correlations remain, the off-diagonal elements have collapsed to zero, as shown in figure 4(b). As the pump phase is further adjusted, the correlation can be re-observed but now with a \( \pi /2 \) phase slip implying an anti-correlation, as shown in figure 4(c). Indeed, the corresponding correlation coefficient matrix confirms the re-appearance of the off-diagonal elements but now
with negative polarity. Consequently, by adjusting \( f \), the self-oscillations in the two resonators can be tuned from a perfectly correlated to an anti-correlated configuration, as shown in figure 4(d). The theoretical cross-quadrature correlation coefficients extracted from equations (3)–(6) as a function of \( f \), again from 2000 solutions with \( V = 120 \text{ mV}_{\text{rms}} \) at \( \omega \), as a function of \( \phi = -150^\circ, -60^\circ \) and \( 60^\circ \), respectively. This measurement reveals that the correlation from the self-oscillating states in (a) with unity off-diagonal elements in the correlation coefficient matrix is first anti-squeezed in (b) yielding zero off-diagonal elements, and then re-squeezed in (c) but now with a \( \pi/2 \) phase slip yielding negative off-diagonal elements. (d) The correlation coefficient extracted from the cross-quadratures (points) as a function of pump phase can be reproduced by the above Hamiltonian (line) with \( \gamma (\omega ) > 10^4 \) and it confirms the correlation between the resonators even with self-oscillating inputs.

6. Discussion and summary

Although a hybrid light-matter two-mode squeezed state has been demonstrated [10, 11], a purely solid-state analogue of this effect is particularly desirable as it could provide deep insights into the nature of the quantum to classical divide and the absence of quantum phenomena in our everyday world [36, 37]. Consequently, the observation of a correlation, from parametric down-conversion, between two mechanical resonators, which differ by one order of magnitude in natural frequency and almost two orders of magnitude in volume, indicates that extending this interaction to mechanical resonators in their ground state will lead to an entanglement composed from disparate macroscopic systems. However, the key to achieving these objectives is the dual requirement of low phonon numbers and the ability to resolve them with low back-action [38]. Recent work has shown that these pre-requisites are now within reach to this architecture with phonon numbers below 1000 and a resolution of 70 phonons being observed [39].

On the other hand, the ability to decipher a correlation between the two resonators from their self-oscillations suggests that this phenomenon could be harnessed for technological applications. For instance, if one of the resonators experiences an external stimulus, for example a force or an increase in temperature, this would lead to a change in its natural frequency, which in turn would result in the correlated state collapsing. This transition could be easily monitored in real-time due to the large output signals associated with the self-oscillations, and thus it suggests a new class of mechanical sensor could be realised, in contrast to the more usual amplitude or frequency modulation based protocols [40]. Furthermore, the large frequency difference between the constituent resonators would yield a wider operation bandwidth to the micro-resonator, while the sensitivity from the narrow bandwidth of the micro-resonance would be imparted to the nano-resonator, thus yielding a more versatile detector than would be possible with either resonator alone [41–43].
Finally, the aforementioned related study of self-oscillations in a plate and beam mode from a torsional resonator, under blue sideband excitation, first revealed features of correlated vibrations with the observation of entwined phase diffusions of the modes stemming from frequency fluctuations [21]. In that case, the correlation was even harnessed by measuring the phase diffusion of one mode to stabilise the other. More fundamentally, that study also asserted that the correlated phases were a consequence of the discrete time-translation symmetry imposed by the periodic modulation from the blue sideband pump. The present work thus not only reveals a different manifestation of this phenomenon, via quadratures of the vibrations, but it also suggests that further consequences of the discrete time-translation symmetry could become accessible with the continued study of self-oscillating systems composed from vastly different constituents extending even to light-matter hybrids [44].

A two-mode squeezed state in the guise of a classical correlation, from parametric down-conversion, between two dissimilar mechanical resonators is demonstrated. This observation suggests that this phenomenon when extended into the quantum regime will yield an entanglement between two distinct macroscopic solid-state objects, thus enabling quantum mechanics to be queried in a regime where gravity cannot be neglected. Conversely, the ability to identify a two-mode correlation from self-oscillations suggests a new means to practically exploiting this phenomenon for technological applications.

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Appendix: Experimental

The measurements were performed in a \(^4\)He flow cryostat at 1.5 K with the device located inside a vacuum chamber at \(10^{-7}\) mbar. Both resonators could be independently probed via signal generators (NF Wavefactory 1974) which activated their flexural motion via their respective piezotransducers. The resultant motion-induced piezovoltage was detected via the counterpart piezotransducer on the element being probed, thus being electronically isolated from the drive, and was amplified via an on-chip silicon nano-transistor with a 30 dB gain [24] followed by a room temperature amplifier (Femto DHPAC-100), and was then demodulated either in a spectrum analyser (Agilent 89410A) or in a homodyne measurement via a phase sensitive detector (Stanford Research Systems SR844).

The correlation measurements were performed by piezoelectrically modulating the strain in the electromechanical system at \(\omega\) via the excitation piezotransducer located on the micro-resonator. The resultant self-oscillations in both resonators were independently monitored, via identical amplification chains, and mixed with two local oscillators (NF Wavefactory 1974) locked onto the vibrations, and demodulated in two phase sensitive detectors. The pump generator and the local oscillators were phase locked and synchronised via an external reference clock.

References

[1] Burnham D C and Weinberg D L 1970 Phys. Rev. Lett. 25 84–7
[2] Reid M D and Drummond P J 1988 Phys. Rev. Lett. 60 2731–3
[3] Ou Z Y and Mandel L 1988 Phys. Rev. Lett. 61 30–3
[4] Shih Y H and Alley C O 1988 Phys. Rev. Lett. 61 2921–4
[5] Keck A K 1991 Phys. Rev. Lett. 67 661–3
[6] Knill E, Laflamme R and Milburn G J 2000 Nature 409 46–52
[7] Bouwmeester D, Pan J, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 Nature 390 575–9
[8] Gerry C C and Knight P L 2005 Introductory Quantum Optics (Cambridge: Cambridge University Press)
[9] Eichler C, Bosigy D, Lang C, Baur M, Steffen L, Fink J M, Filipp S and Wallraff A 2011 Phys. Rev. Lett. 107 113601
[10] Palomaki T, A, Teufel J, D, Simmonds R W and Lehnert K W 2013 Science 342 710–3
[11] Riedinger R, Hong S, Norte R A, Slater J A, Shang J, Krause A G, Anant V, Aspelmeyer M and Gröblacher S 2015 Nature 520 313–6
[12] Ockeloen-Korppi C, Damkänga F, Pirkkalainen J M, Clerk A, Woolley M and Sillanpää M 2016 Phys. Rev. Lett. 117 140401
[13] Xue F, Liu Y, Sun C P and Nori F 2007 Phys. Rev. B 75 180501
[14] Eichler C, Bozyigit D, Lang C, Baur M, Steffen L, Fink J M, Filipp S and Wallraff A 2011 Phys. Rev. Lett. 107 113601
[15] Pontin A, Bonalumi, M, Borrielli A, Marconi L, Marino F, Pandraud G, Prodi G, Sarro P, Serra E and Marin F 2016 Phys. Rev. Lett. 116 103601
[23] Mahboob I, Nishiguchi K, Okamoto H and Yamaguchi H 2012 *Nature Phys.* **8** 387–92
[24] Mahboob I, Perrissin N, Nishiguchi K, Hatanaka D, Okazaki Y, Fujiwara A and Yamaguchi H 2015 *Nano Lett.* **15** 2312–7
[25] Yamaguchi H and Mahboob I 2013 *New J. Phys.* **15** 015023
[26] Aspelmeyer M, Kippenberg T J and Marquardt F 2014 *Rev. Mod. Phys.* **86** 1391
[27] Teufel J D, Li D, Allman M S, Cicak K, Sirois A J, Whittaker J D and Simmonds R W 2011 *Nature* **471** 204–8
[28] Alba R D, Massel F, Storch I R, Abhilash T S, Hui A, McEuen P L, Craighead H G and Parpia J M 2016 *Nature Nanotech* **11** 741–6
[29] Mathew J P, Patel R N, Borah A, Vijay R and Deshmukh M M 2016 *Nature Nanotech* **11** 747–51
[30] Mahboob I, Flurin E, Nishiguchi K, Fujiwara A and Yamaguchi H 2010 *Appl. Phys. Lett.* **97** 253105
[31] Massel F, Heikkinen T T, Pirkkalainen J M, Cho S U, Saloniemi H, Hakone P J and Sillanpää M A 2011 *Nature* **480** 351–4
[32] Fan I, Fong K Y, Poot M and Tang H X 2015 *Nature Commun.* **6** 5850
[33] Ekinci K I and Roukes M L 2005 *Rev. Sci. Instrum.* **76** 061101
[34] Etaki S, Konshelle F, Blanter Y M, Yamaguchi H and van der Zant H S J 2013 *Nature Commun.* **4** 1803
[35] Eichler C, Salathe Y, Mlynek J, Schmidt S and Wallraff A 2014 *Phys. Rev. Lett.* **113** 110502
[36] Zurek W H 1991 *Phys. Today* **44** 36–44
[37] Mancini S, Giovannetti V, Vitali D and Tombesi P 2002 *Phys. Rev. Lett.* **88** 120401
[38] Teufel J, Donner T, Castellanos-Beltran M, Harlow J and Lehnert K 2009 *Nature Nanotech.* **4** 820–3
[39] Okazaki Y, Mahboob I, Onomitsu K, Sasaki S and Yamaguchi H 2015 *Nature Commun.* **6** 11132
[40] Albrecht T R, Grütter P, Home D and Rugar D 1991 *J. Appl. Phys.* **69** 6668
[41] Rugar D, Budakian R, Mamin H J and Chui B W 2004 *Nature* **430** 329
[42] Chaste J, Eichler A, Moser J, Ceballos G, Ruland R and Bachtold A 2012 *Nature Nanotech* **7** 301–4
[43] Hanay M S, Kelber S, Naik A K, Chi D, Hentz S, Bullard E C, Colinet E, Duraffourg I. and Roukes M L 2012 *Nature Nanotech* **7** 602–8
[44] Kippenberg T J and Vahala K J 2008 *Science* **321** 1172–6