COHERENCE EFFECTS IN CHARMONIUM PRODUCTION OFF NUCLEI: CONSEQUENCES FOR $J/\psi$ SUPPRESSION

A. Capella

Laboratoire de Physique Théorique
Université de Paris XI, Bâtiment 210, F-91405 Orsay Cedex, France

Abstract

The probabilistic Glauber formula for nuclear absorption used in the literature is only valid at low energies and $x_+ \simeq 0$. Due to energy conservation, $\sigma_{abs}$ is replaced by an effective cross-section $\sigma_{abs} + x_+ (\sigma_{c\bar{c}-N} - \sigma_{abs})$ which increases with $x_+$ and tends to the total $c\bar{c} - N$ cross-section $\sigma_{c\bar{c}-N}$. Experimental data can be described with $\sigma_{abs} \sim 4 \div 5$ mb and $\sigma_{c\bar{c}-N} \sim 15 \div 20$ mb. At high energies, due to the increase of the coherence length, this formula changes. The main change is the replacement of $\sigma_{abs}$ by $\sigma_{c\bar{c}-N}$ for all values of $x_+$, as $s \to \infty$. Thus, if $\sigma_{c\bar{c}-N} > \sigma_{abs}$ the $J/\psi$ suppression due to nuclear interaction will increase with energy.

LPT Orsay 02-63
June 2002

Talk presented at the International Workshop on Charm Production, ECT*, Trento, June 2002

1Unité Mixte de Recherche UMR n° 8627 - CNRS
LOW ENERGY PROBABILISTIC FORMULA. Let us consider a proton-nucleus collision. The probabilistic Glauber formula for the $J/\psi$ survival is well known. A $c\bar{c}$ pre-resonant state is produced inside the nucleus at some value $Z$ of the longitudinal coordinate, and interacts with the nuclei on its path. If the interaction is inelastic, the $c\bar{c}$ pair can transform into another pair which has vanishing projection into $J/\psi$. The corresponding cross-section is called absorptive cross-section $\sigma_{abs}$.

We can write

$$\sigma_{abs} = (1 - \varepsilon)\sigma_{c\bar{c} - N},$$

where $\sigma_{c\bar{c} - N}$ is the total $c\bar{c} - N$ cross-section and $\varepsilon\sigma_{c\bar{c} - N}$ is the contribution to $\sigma_{c\bar{c} - N}$ of those intermediate states which have a non-zero projection into $J/\psi$. In the Glauber model one has

$$\sigma_{\psi pA}(b) = \sigma_{\psi pN} A \int_{-\infty}^{+\infty} dZ \rho_A(b, Z) \exp \left[ -\sigma_{abs} A \int_{Z}^{\infty} dZ' \rho_A(b, Z') \right],$$

$$= \left( \sigma_{\psi pN}/\sigma_{abs} \right) \left[ 1 - \exp \left( -\sigma_{abs} A T_A(b) \right) \right]. \quad (1)$$

Note that for open charm production $\sigma_{abs} = 0$ and $\sigma_{pA} = A\sigma_{pN}$.

ASYMPTOTIC FIELD THEORY FORMULA. At high energies the coherence length increases and the projectile interacts with the nucleus as a whole [2]. Thus, it is no longer possible to consider its collisions as “successive” and expression (1) is changed. As we shall see below this change consists in the replacement [1]

$$(1/\sigma_{abs}) \left[ 1 - \exp \left( -\sigma_{abs} A T_A(b) \right) \right] \Rightarrow A T_A(b) \exp \left[ -\frac{1}{2} \tilde{\sigma} A T_A(b) \right] \quad (2)$$

where $\tilde{\sigma} \equiv \sigma_{c\bar{c} - N}$. The change is twofold. There is a change in the form of the expression and $\sigma_{abs}$ has been replaced by $\sigma_{c\bar{c} - N}$. If $\sigma_{abs} \sim \sigma_{c\bar{c} - N}$ the change (2) is not important. Indeed the two expressions coincide at the first and second order in $\tilde{\sigma}$ and, since $\sigma_{abs}$ is not large, the result will not be significantly changed. However, if $\sigma_{c\bar{c} - N} \gg \sigma_{abs}$, there will be a very important increase of the so-called normal $J/\psi$ suppression (which can no longer be called nuclear absorption since it is due to the total cross-section $c\bar{c} - N$, rather than to its absorptive part). This can be the case if the $c\bar{c}$ pair is produced in a color state and is accompanied by light quarks in order to have a colorless system. In this case, $\sigma_{c\bar{c} - N}$ can be a typical hadronic cross-section, due to the presence of the light quarks [3]. Furthermore, the probability for this $c\bar{c}$
system to keep its projection into $J/\psi$ after interacting inelastically has to be large, i.e. $\varepsilon \approx 1$.

The derivation of the asymptotic formula is straightforward. Let us denote by $\sigma$ the cross-section for the interaction of the light quarks in the projectile and $\bar{\sigma} \equiv \sigma_{c\bar{c} - N}$ that of the produced $c\bar{c}$ system (we assume, for simplicity, that the corresponding amplitudes are purely imaginary). Let us single out one of the light quark interactions in which the $c\bar{c}$ pair is produced (for instance via gluon-gluon fusion). We have

$$
\sigma_{\psi pA}^\psi (b) = \sigma_{\psi pN}^\psi [\sigma_A (\sigma + \bar{\sigma}) - \sigma_A (\bar{\sigma})] / \sigma 
$$

where

$$
\sigma_A (\sigma) = 2 \left\{ 1 - \exp \left[ -\frac{1}{2} \sigma \ A \ T_A (b) \right] \right\} 
$$

is the Glauber total cross-section in terms of the cross-section $\sigma$ for the scattering on a single nucleon. In eq. (3) we have substracted the term with no light quark interaction (since we assume that at least one such interaction is needed to produce the $c\bar{c}$ pair). The division by $\sigma$ is due to the fact that one such collision is precisely the one producing the $c\bar{c}$-pair – first factor of (3). From (3) and (4) we get

$$
\sigma_{\psi pA}^\psi (b) = \left( \sigma_{\psi pN}^\psi / \sigma \right) \sigma_A (\sigma) \exp \left[ -\frac{1}{2} \bar{\sigma} \ A \ T_A (b) \right].
$$

We see that the terms containing $\sigma$ and $\bar{\sigma}$ factorize. The former produce nuclear effects (shadowing) in the nucleus vertex function (which is no longer proportional to $A$). The terms containing $\bar{\sigma}$ correspond to the rescattering of the $c\bar{c}$ system and have the form given by (2).

As stated above, the main change between the probabilistic formula and the asymptotic one consists in the replacement of $\sigma_{abs}$ by $\sigma_{c\bar{c} - N}$. This change is entirely due to coherence effects. At low energy, some contributions vanish due to the fact that there is a non-vanishing minimum momentum transfer ($t_{\text{min}} \neq 0$) – and this contribution is suppressed by the nuclear form factor. It turns out [3] that the other contributions cancel with each other, giving rise to an $A^1$ behaviour typical of open charm production. The only case in which this cancellation is avoided is when the $c\bar{c}$
pair is “destroyed”, (i.e. converted into a $c\bar{c}$ system with vanishing projection on the $J/\psi$). For this reason only the absorptive part of the total $c\bar{c} - N$ cross-section plays a role at low energy. At asymptotic energy $t_{\text{min}} = 0$. Thus, the above cancellation does not occur and the total $c\bar{c} - N$ cross-section contributes to the $J/\psi$ suppression.

The expression of the shadowing corrections in eq. (5) is not realistic since the Glauber expression has been used. It is, indeed, well known that shadowing corrections have to be described by triple Pomeron diagrams. In view of that, in the following we shall only consider in eq. (3) the terms linear in $\sigma$, and will use a standard formulation of the shadowing corrections (which correspond to higher order terms in $\sigma$). It is, nevertheless, instructive to see from the complete expression (5) that the shadowing corrections vanish in the low energy limit and are factorizable in the high energy one. In the following we will assume that factorization also holds at intermediate energies – and will denote the corresponding cross-section $\sigma_{pN}^{\psi,\text{shadow}}$.

**INTERPOLATING FORMULA.** Using the AGK cutting rules \[4\] it is possible to obtain the exact formula at finite energies. We have \[1\] instead of (5)

$$
\sigma_{pA}^{\sigma}(b) = \left(\frac{\sigma_{pN}^{\psi,\text{shadow}}}{\sigma}\right) \sum_{n=1}^{A} \binom{A}{n} \sum_{j=1}^{n} T_{n}^{(j)} \sigma_{n}^{(j)}
$$

where

$$
\sigma_{n}^{(j)}(b) = \sigma \left( -\frac{\bar{\sigma}}{2} \right)^{j-1} \left[ -(1 - \varepsilon)\bar{\sigma} \right]^{n-j} - \sigma \left( -\frac{\bar{\sigma}}{2} \right)^{j-1} (j - 1) \left[ \frac{\bar{\sigma}}{2} - (1 - \varepsilon)\bar{\sigma} \right] \left[ -(1 - \varepsilon)\bar{\sigma} \right]^{n-j} = \\
\left( -\frac{\bar{\sigma}}{2} \right)^{j-1} j \left[ -(1 - \varepsilon)\bar{\sigma} \right]^{n-j} - \left( -\frac{\bar{\sigma}}{2} \right)^{j-2} (j - 1) \left[ -(1 - \varepsilon)\bar{\sigma} \right]^{n-j+1}
$$

and

$$
T_{n}^{(j)}(b) = n! \int_{-\infty}^{+\infty} dZ_{1} \int_{Z_{1}}^{+\infty} dZ_{2} \cdots \int_{Z_{n-1}}^{+\infty} dZ_{n} \cos(\Delta(Z_{1} - Z_{j})) \prod_{i=1}^{n} \rho_{A}(b, Z_{i})
$$

Here $\Delta$ is the inverse of the coherence length

$$
\Delta \equiv \frac{1}{\ell_{c}} = m_{p} \frac{M_{\psi}^{2}}{s x_{1}}
$$
with \( x_F = x_1 - x_2 \) and \( x_1 x_2 s = M_{c\bar{c}} \).

In (8) the longitudinal coordinates of the interacting nucleons have been ordered as \( Z_1 \leq Z_2 \cdots \leq Z_n \). The index \( j \) in (7) and (8) denotes the first interaction (i.e. the one with the smallest value of \( Z \)) which has been cut in such a way that \( t_{\text{min}} = 0 \) for the \( n - j \) ones with \( Z > Z_j \). For details see ref. [1]. The damping factor \( \cos(\Delta(Z_1 - Z_j)) \) depends on the difference \( Z_1 - Z_j \). The only case in which \( t_{\text{min}} = 0 \) is when \( j = 1 \), i.e. when the first interaction is cut. In all other cases \( t_{\text{min}} \neq 0 \). Therefore, the term with \( j = 1 \) is the only one that survives in the limit \( s \to 0 (\Delta \to \infty) \). It is easy to verify that one recovers in this way the probabilistic formula (1) – which depends only on \( \sigma_{\text{abs}} \). Likewise, it can be verified\(^2\) that, in the limit \( s \to \infty (\Delta \to 0) \) we recover the asymptotic expression (3) – which depends only on \( \sigma_{c\bar{c} - N} \).

We see from (9) that at fixed \( s \) and \( x_2 \), \( \Delta \) increases with \( x_1 \). Therefore, if \( \sigma_{c\bar{c} - N} > \sigma_{\text{abs}} \), \( J/\psi \) suppression will increase with \( x_1 \) – a feature observed experimentally. In ref. [5] an attempt was made to describe the data in this framework. However, as recognized in [5], there is a caveat. Since \( \Delta \) is a function of \( x_2 \), one obtains a scaling in \( x_2 \), whereas the data indicate rather a scaling in \( x_1 \) (or \( x_F \)). A way out is to take into account the modifications of the AGK rules resulting from energy conservation [3]. Indeed, by energy conservation, the \( x \) distribution of \( J/\psi \) gets softer when the number \( K \) of inelastic collisions increases. This has been taken into account in [3] introducing a factor

\[
F_k(x_1) = (1 - x_1^\gamma)^k
\]

in the contribution to \( \sigma_{pA}^{\psi} \) corresponding to \( k \) inelastic collisions of the \( c\bar{c} \) system. In ref. [3] this has been done in the probabilistic Glauber model. In order to do it in the general framework described above, one has to decompose \( \sigma_n^{(j)} \), eq. (7), into a sum of terms \( \sigma_n^{j,k} \) corresponding to a fixed number \( k \) of inelastic collisions. It turns out that this amounts to replacing \([-(1-\varepsilon)\bar{\sigma}]^{n-j}\) by \( \sum_{k=0}^{n-j} \binom{n-j}{k} (\varepsilon \bar{\sigma})^k (-\bar{\sigma})^{n-j-k} F_k(x_1) \).

With the form of \( F_k \) in eq. (10) the summation in \( k \) can be performed analytically.\(^2\) For \( \Delta = 0, T_n^{(j)} = T_n^A \). Moreover, if one considers the two terms in the last equality of (8), there is a cancellation between the first term of \( \sigma_n^{(j)} \) and the second term of \( \sigma_n^{(j+1)} \). The only term left is thus the first term of \( \sigma_n^{(n)} \).
The final result is

$$\sigma_n^{(j)}(b) = \left(-\frac{\tilde{\sigma}}{2}\right)^{j-1} j \left[-\sigma_{eff}\right]^{n-j} - \left(-\frac{\tilde{\sigma}}{2}\right)^{j-2} (j-1) \left[-\sigma_{eff}\right]^{n-j+1} \quad (11)$$

where

$$\sigma_{eff} = \bar{\sigma}(1 - \varepsilon) + \bar{\sigma} \varepsilon x_1^{\gamma} \quad (12)$$

Thus, as a consequence of the modification of the AGK rules due to energy conservation, we have obtained the same expression \(^{(7)}\) with the absorptive cross-section \(\sigma_{abs} = (1 - \varepsilon)\bar{\sigma}\) replaced by \(\sigma_{eff}\). Therefore, the asymptotic limit, eq. \(^{(6)}\), which does not depend on \(\sigma_{abs}\) is not changed.

As discussed above, in the low energy limit, \(\Delta \to \infty\), only the term \(j = 1\) survives. We obtain in this case

$$\sigma_{pA}^{\psi}(b) = \left(\sigma_{pA}^{\psi}/\sigma_{eff}\right) \left[1 - \exp\left(-\sigma_{eff} A T_A(b)\right)\right] \quad (13)$$

We see that for \(x_1 \to 0\) we recover the probabilistic expression \(^{(1)}\) with \(\sigma_{eff} \sim \sigma_{abs}\). However, for \(x_1 \to 1\) we have \(\sigma_{eff} \sim \sigma_{cc-N}\). While coherence effects produce this change from \(\sigma_{abs}\) to \(\sigma_{cc-N}\) as \(s \to \infty\), the introduction of the factors \(F_k(x_1)\), eq. \(^{(10)}\), leads to the same effect at low \(s\) as \(x_1\) increases. We recover in this way scaling in \(x_1\) at low energies.

As shown in ref. \(^{(3)}\), eqs. \(^{(12)}\)-\(^{(13)}\) give a good description of experimental data with \(\sigma_{abs} = 5\) mb, \(\sigma_{cc-N} = 20\) mb \((\varepsilon = 0.75)\) and \(\gamma = 2\). It was also shown in \(^{(3)}\) that \(\gamma\) increases with the mass of the heavy-quark pair and thus, the effect of energy conservation via the factors \(F_k\) concentrates to larger values of \(x_1\) when this mass increases.

**CONCLUSIONS AND OUTLOOK.** We have shown that the probabilistic Glau- ber formula for the survival of \(J/\psi\) in a \(pA\) collision changes with energy due to coherence effects. The main change is the replacement of \(\sigma_{abs}\) by \(\sigma_{cc-N}\) – the total \(cc-N\) cross-section. The same change occurs at low energy when \(x_1\) increases from 0 to 1, due to energy conservation. If \(\sigma_{abs} \simeq \sigma_{cc-N}\) the discussion in this paper is of little phenomenological relevance. Nuclear absorption will be similar at RHIC and
SPS energies. Moreover, the increase of the $J/\psi$ suppression with $x_F$ cannot be due to the mechanism described here. If, on the contrary, $\sigma_{c\bar{c}-N}$ is substantially larger than $\sigma_{abs}$, the increase with $x_F$ is well reproduced and a huge increase of the $J/\psi$ suppression in $pA$ collisions at RHIC and LHC is predicted.

Numerical calculations using eq. (11) are in progress [6]. They should provide a more precise determination of the value of $\sigma_{c\bar{c}-N}$ from available data. They should also allow to determine quantitatively how the $x_F$ scaling is converted into a scaling in $x_2$ as energy increases – first at low $x_F$ and then extending to larger values of $x_F$ as the energy increases.

ACKNOWLEDGMENTS

It is a pleasure to thank all my collaborators in the subject of this contribution: N. Armesto, M. Braun, K. Boreskov, A. Kaidalov, C. Pajares, C. Salgado and J. Tran Thanh Van. I also thank A. Polleri and the organizers of the ECT International Workshop on Charm Production for an interesting and stimulating meeting.

REFERENCES

[1] M. A. Braun, C. Pajares, C.A. Salgado, N. Armesto, A. Capella, Nucl. Phys. B 509, 357 (1998).
    M. A. Braun and A. Capella, Nucl. Phys. B 412, 260 (1994).

[2] B. Z. Kopeliovich, A. V. Tarasov and J. Huefner, hep-ph/0104256.

[3] K. Boreskov, A. Capella, A. Kaidalov and J. Tran Thanh Van, Phys. Rev. D 47, 919 (1993).

[4] V. Abramovsky, V. N. Gribov and O. V. Kancheli, Sov. J. Nucl. Phys. 18, 308 (1974).

[5] C. A. Salgado, hep-ph/0105231.

[6] N. Armesto, A. Capella and C. A. Salgado, in preparation.