Triangle Formation Control of Multi-AUVs with Communication Constraints

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Abstract. Aimed at solving the problem of multi-AUVs formation control, an analysis has been made on a triangle formation based on leader-follower construction, with a multi-AUVs formation controller designed with communication constraints taken into account. The desired velocity of the follower is the resultant velocity of three component velocities, each of which has different function. The convergence of the proposed controller is proved by establishing an objective function that can represents the status of the formation. Simulation experiments are carried out to demonstrate the proposed controller is effective.

Introduction

Autonomous Underwater Vehicles (AUVs) have gained steadily increased attention for they are particularly suited for executing underwater missions in environments where global human control cannot arrive. Formation control of multi-AUVs systems has triggered the interests of the research community for it is a fundamental problem for cooperative control of multi-AUVs systems, which has potential benefits for specific applications such as seafloor survey, oceanographic research, underwater archeology, etc. By multi-AUVs formation control we mean a group of AUVs travel according to a prescribed pattern. In some cases, members in the system are expected to form certain geometry according to a real or virtual vehicle, which means agents in the system should keep desired distances and angles relative to each other. This formation is widely used in pipe line inspection [1] or similar tasks.

A variety of formation control schemes have been proposed so far, such as leader-follower method [2], behavior based method [3], virtual structure method [4], artificial potential field method [5] and graph theory based method [6]. Leader-follower method is widely research for it is easy to realize and can be combined with other methods. Formations organized using this method consists of several vehicles acting as leaders and a set of vehicles leaders that travels keep desired position in a coordinate based on the status of the leaders. Kowdiki proposed a formation control technique of a group of vehicles employing artificial potential field navigation and leader-follower formation control scheme [7]. Consolini focus on controlling leader-follower formations with rigidity fixed graph [8]. Ren proposed a unified, distributed formation control architecture that accommodates an arbitrary number of group leasers and arbitrary information flow among vehicles that requires only local neighbor-to neighbor information exchange [9] [10].

In this paper, a research is made on a formation consists of three vehicles, with one of them acting as leader, while the other two as followers. The vehicles should form certain triangle pointing at desired orientation. A novel formation controller is proposed. The desired velocity of each follower is treated as a resultant velocity from three component velocities. An objective function represents the status of the formation is established to prove the convergence of the proposed controller. Simulation experiments are carried out to demonstrate that the proposed method is effective.
Design of the Formation Controller

A formation consists of three AUVs is researched with one being the leader while the other two followers. When the three AUVs travel at the same depth, they are moving in 2-D space and can form a triangle formation, as is shown in Fig. 1. When the leader keeps travelling at constant speed, the purpose of the formation control is to drive the three AUVs to form a desired triangle, whose edges are $|DL_1|$, $|DL_2|$, and $|DD|$, and the triangle should point at certain orientation, which means the direction of the formation should be defined. Here we define the direction of the vector $\overrightarrow{ML}$ as the direction of the formation, which represents a vector perpendicular to line $F_1F_2$ and pointing at the leader. Then a formation controller is proposed, who will give out the desired velocity of every AUV in the formation.

Controller for the leader is easy to design. The desired velocity of the leader $\vec{v}_l$ is a constant value towards the destination of the leader. Then the controller for the followers should drive them to converge to the desired triangle and turn the triangle to the desired orientation. The desired velocity of the follower is then a resultant velocity:

$$\vec{v}_f = \vec{v}_l + \vec{v}_{\text{form}} + \vec{v}_{\text{turn}}$$  \hspace{1cm} (1)

(1) Component velocity $\vec{v}_l$ is the same as the velocity of the leader, which will drive the followers to the destination with the leader.

(2) Component velocity $\vec{v}_{\text{form}}$ can drive the followers to form the desired triangle based on relative distances between AUVs. For follower $F_i$, this component velocity can be given as:

$$\vec{v}_{\text{form}} = \vec{v}_{\text{FF}_i} + \vec{v}_{\text{FL}_i}$$  \hspace{1cm} (2)

$$\vec{v}_{\text{FF}_i} = k \cdot \Delta | \overrightarrow{FF}_i | \vec{e}_{\overrightarrow{FF}_i} = k \cdot (| \overrightarrow{FF}_i | - |DD|) \vec{e}_{\overrightarrow{FF}_i}$$

$$\vec{v}_{\text{FL}_i} = k \cdot \Delta | \overrightarrow{FL}_i | \vec{e}_{\overrightarrow{FL}_i} = k \cdot (| \overrightarrow{FL}_i | - |DL|) \vec{e}_{\overrightarrow{FL}_i}$$  \hspace{1cm} (3)

Where $\vec{e}_{\overrightarrow{FF}_i}(x_{\overrightarrow{FF}_i}, y_{\overrightarrow{FF}_i})$ and $\vec{e}_{\overrightarrow{FL}_i}(x_{\overrightarrow{FL}_i}, y_{\overrightarrow{FL}_i})$ are corresponding unit vectors.

(3) Component velocity $\vec{v}_{\text{turn}}$ will drive the triangle to point at the desired orientation. A set of concentric circles are set to obtain this component velocity, as is shown in Fig. 2. Vehicles are put in concentric circles with the leader being the center while the distances between the leader and each follower being radius. When the triangle rotates, followers should move along corresponding circles with a speed perpendicular to radius.

Let the position of the leader be $L(L_x, L_y)$ while that of the ith follower be $F_i(F_{ix}, F_{iy})$, and then the direction of the formation is $(x_{\text{form}}, y_{\text{form}})$:

$$\begin{cases}
x_{\text{form}} = -\frac{(F_{y1} - F_{y2})(F_{x2} - L_x)(F_{y1} - L_y) - (F_{x1} - L_x)(F_{y2} - L_y)}{(F_{x1} - F_{x2})^2 + (F_{y1} - F_{y2})^2} \\
y_{\text{form}} = -\frac{(F_{x1} - F_{x2})(F_{x1} - L_x)(F_{y2} - L_y) - (F_{x2} - L_x)(F_{y1} - L_y)}{(F_{x1} - F_{x2})^2 + (F_{y1} - F_{y2})^2}
\end{cases}$$  \hspace{1cm} (4)

Formation direction in global coordinate is:

$$\varphi_{\text{form}} = \arctan\frac{y_{\text{form}}}{x_{\text{form}}}$$  \hspace{1cm} (5)

The angle of $\vec{v}_{\text{turn}}$ for the ith follower is then:

$$\varphi_{\text{turn}} = \arctan\left(\frac{y_{\text{FL}_i}}{x_{\text{FL}_i}}\right) - \text{sgn}\left(\sin(\varphi_d - \varphi_{\text{form}})\right) \cdot \frac{\pi}{2}$$  \hspace{1cm} (6)

With $\varphi_d$ being the angle of the desired formation direction in global coordinate. The value of this component velocity is:

$$|\vec{v}_{\text{turn}}| = F_{L} \cdot \arcsin(\sin(\varphi_d - \varphi_{\text{form}})) \cdot k_{\text{turn}}$$  \hspace{1cm} (7)
with \( k_{\text{turn}} \) being a parameter. This makes:
\[
\tilde{v}_{\text{turn}} = (|\tilde{v}_{\text{turn}}| \cdot \cos \phi_{\text{turn}}, |\tilde{v}_{\text{turn}}| \cdot \sin \phi_{\text{turn}})
\]  
As communication constraints should be taken into account, both followers should estimate real-time positions of other vehicles with:
\[
\begin{align*}
L_{i,H+\Delta t} & = L_i + \tilde{v}_i \cdot \Delta t \\
F_{i,i+\Delta t} & = F_{i,i} + \tilde{v}_{i,i} \cdot \Delta t \\
F_{i,i+en+\Delta t} & = F_{i,i} + \tilde{v}_{i,i} \cdot \Delta t
\end{align*}
\]  
...(8)

\( L \) and \( F \) denote formation direction and desired direction respectively.

**Proof of Formation Convergence**

The convergence of the proposed controller can be proved. As all vehicles share the same component velocity \( \tilde{v}_i \) as the leader, it can be regarded that the leader keeps still and that the followers move to form the formation. Component velocity \( \tilde{v}_{\text{turn}} \) only affects the direction of the triangle, but have no effect on the shape of the formation. So the convergence of the formation depends on the component velocity \( \tilde{v}_{\text{form}} \), when \( \tilde{v}_{\text{form}} \) can ensure that the vehicles can converge to the desired geometry, the formation can converge, which means:
\[
\begin{align*}
\Delta |FL| & = |FL| - |DL| = 0 \\
\Delta |FL| & = |FL| - |DL| = 0 \\
\Delta |FF| & = |FF| - |DD| = 0
\end{align*}
\]  
...(10)

Then an objective function \( f(t) \) can be established based on that:
\[
f(t) = |FL| + |FL| + |FF||
\]  
...(11)

For follower \( F_i(i=1,2) \):
\[
\tilde{v}_{\text{form}} = \tilde{v}_{fi} + \tilde{v}_{fi}
\]  
...(12)

Then along \( FL \) and \( FF \):
\[
\begin{align*}
|FL|_{i+j+\Delta t} & = (|FL|_{i+j+\Delta t} - |DL|_i - |FL|_{i+j+\Delta t} - |DL|_i) = \Delta |FL|_{i+j+\Delta t} - \Delta |FL|_{i+j} \\
|FF| & = (|FF|_{i+j+\Delta t} - |DD|/2 - |FF|_{i+j} - |DD|/2) = \Delta |FF|_{i+j+\Delta t} - \Delta |FF|_{i+j}
\end{align*}
\]  
...(13)

Where \( |FF| = |FF|_1 + |FF|_2 \)

When \( \Delta t \rightarrow 0 \):
\[
\begin{align*}
\Delta |FL|_{i+j+\Delta t} & \geq 0 \\
\Delta |FF|_{i+j+\Delta t} & \geq 0
\end{align*}
\]  
...(14)

Then it can be derived that:
\[
\begin{align*}
|\Delta |FL|_{i+j+\Delta t} - |FL|_{i+j}| & = -2(\tilde{v}_{fi} + \tilde{v}_{fi}) \nabla_{FL} \cdot \Delta t \\
|\Delta |FF|_{i+j+\Delta t} - |FF|_{i+j}| & = -2(\tilde{v}_{fi} + \tilde{v}_{fi}) \nabla_{FF} \cdot \Delta t
\end{align*}
\]  
...(15)
With
\[
\begin{align*}
\lambda_i &= 1 \quad \Delta |FL|_{i,t+\Delta} > 0, \Delta |FL|_{i,l} > 0 \\
\mu_i &= 1 \quad \Delta |FF|_{i,t+\Delta} > 0, \Delta |FF|_{i,l} > 0 \\
\lambda_i &= -1 \quad \Delta |FL|_{i,t+\Delta} < 0, \Delta |FL|_{i,l} < 0 \\
\mu_i &= -1 \quad \Delta |FF|_{i,t+\Delta} < 0, \Delta |FF|_{i,l} < 0 
\end{align*}
\]
(16)

Form Equation (15), we can get:
\[
f(t + \Delta t) - f(t) = -\sum_{i=1}^{2} (\lambda_i (\bar{v}_{fi} + \bar{v}_{fi}) \hat{e}_{FLi} + \mu_i (\bar{v}_{fi} + \bar{v}_{fi}) \hat{e}_{FFi}) \cdot \Delta t
\]
(17)

As
\[
f'(t) = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}
\]
(18)

Then:
\[
f'(t) = \sum_{i=1}^{2} (M_i + N_i)
\]
(19)

\[
\begin{align*}
M_i &= -\bar{v}_{fi} (\lambda_i \cdot \hat{e}_{FLi} + \mu_i \cdot \hat{e}_{FFi}) = -(\lambda_i + \mu_i \cdot \cos \theta_i) \cdot |FL|_i \\
N_i &= -\bar{v}_{fi} (\mu_i \cdot \hat{e}_{FFi} + \lambda_i \cdot \hat{e}_{FLi}) = -(\mu_i + \lambda_i \cdot \cos \theta_i) \cdot |FF|_i
\end{align*}
\]
(20)

Where
\[
\cos \theta_i = \hat{e}_{FFi} \cdot \hat{e}_{FLi}
\]

As \(f(t) \geq 0\), when \(f(t) \neq 0\), \(f'(t) < 0\) and when \(f(t) = 0\), \(f'(t) = 0\), so it is obvious that \(f(t)\) can converge to zero, which means the formation can finally converge to the desired triangle.

### Simulation results

This section presents the simulation results of the formation controller derived in MATLAB. A formation consists of three AUVs is simulated, and the AUVs should form a desired formation from a random one, with its direction pointing at the desired orientation. The origin position of the leader is \(L(10, 10)\) and followers are \(F_1(0, 20)\) and \(F_2(0, 0)\), with the desired formation setting as \(D[15, 15, 15]\). The desired orientation of the formation is set the same as the heading of the leader. Parameters are set as \(k = 0.1\) and \(k_{turn} = 0.25\), with communication delay being 10s. The simulation results are shown in Fig. 3.

Trajectories of vehicles are shown in Fig. 3 (a), which shows that AUVs form the desired formation rapidly from random positions. Fig. 3 (b) shows some metrics relate to the performance of the formation controllers. As can be seen that the error along every edges of the triangle formation remain fairly stable and close to zero after a brief initial transient. During 100s to 150s, vibration occurs at the turning point, which is mainly caused by communication constraints. The formation converges again within 100s, with direction pointing at the desired orientation.
Conclusion

This paper aims at solving the problem of multi-AUVs triangle formation control with a leader-follower construction. A novel formation controller that can ensure the vehicles form a certain triangle from random positions is proposed. A set of concentric circles are used to help maintain the triangle formation heading at a desired orientation. Effectiveness of the proposed controller is shown by simulation results. Future work will focus on increasing vehicles in the formation.

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