Vacuum Alignment of the Top-Mode Pseudo-Nambu-Goldstone Boson Higgs Model

Hidenori S. Fukano,1,2 Masafumi Kurachi,1 and Shinya Matsuzaki2,3

1 Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI) Nagoya University, Nagoya 464-8602, Japan.
2 Institute for Advanced Research, Nagoya University, Nagoya 464-8602, Japan.
3 Department of Physics, Nagoya University, Nagoya 464-8602, Japan.

We study the vacuum alignment of the top-mode pseudo-Nambu-Goldstone boson Higgs (TMpNGBH) model, which has recently been proposed as a variant of the top quark condensate model in light of the 126 GeV Higgs boson discovered at the LHC. It is shown that the vacuum of the model, determined from the one-loop effective potential with all the explicit breaking effects included, realizes the electroweak symmetry breaking with the appropriate breaking scale. Phenomenologies of two characteristic particles in the TMpNGBH model, namely the $CP$-odd partner of the Higgs ($A_0'$) and the vectorlike partner of the top quark ($t'$) are also studied based on the newly identified vacuum.

I. INTRODUCTION

The discovery of a 126 GeV Higgs boson at the LHC implies that the era to reveal the origin of mass of the elementary particles has come. Preceding the discovery of the Higgs boson by about two decades the top quark has been discovered at the Tevatron [3, 4]. The top quark is the heaviest particle among the observed particles and its mass is $m_t \simeq 173$ GeV [5], which is coincidently on the order of the Higgs mass and the electroweak symmetry breaking (EWSB) scale ($v_{ew} \simeq 246$ GeV). Considering such observed coincidence, it is worth considering a scenario in which the top quark plays a crucial role to explain the dynamical origin for both the EWSB and the Higgs boson.

The top quark condensate model [6–11] is one of such scenarios. However, the original top quark condensate model is somewhat far from a realistic situation: the predicted value of the top quark mass is too large compared with the experimental value. In addition, a Higgs boson predicted as a $t\bar{t}$ bound state has the mass in a range of $m_t \lesssim m_H \lesssim 2m_t$, which cannot be identified with the 126 GeV Higgs boson at the LHC.

Recently, a variant class of the top quark condensate model was proposed [12, 13]. In these models the realistic top quark mass is obtained by the top-seesaw mechanism as in the literature [14–18], while a composite Higgs boson emerges as a pseudo Nambu–Goldstone boson (pNGB) associated with the spontaneous breaking of a global symmetry, therefore it is light to be identified as the LHC Higgs boson. The model in [12] is called the Top-Mode pseudo Nambu-Goldstone Boson Higgs (TMpNGBH) model, which has recently been proposed as a variant of the top quark condensate model in light of the 126 GeV Higgs boson discovered at the LHC. It is shown that the vacuum of the model, determined from the one-loop effective potential with all the explicit breaking effects included, realizes the electroweak symmetry breaking with the appropriate breaking scale. Phenomenologies of two characteristic particles in the TMpNGBH model, namely the $CP$-odd partner of the Higgs ($A_0'$) and the vectorlike partner of the top quark ($t'$) are also studied based on the newly identified vacuum.

\[
\mathcal{L}_{4f} = G_{4f} \langle \bar{\psi}^i_L \chi R \rangle (\bar{\chi}^j_R \psi^j_L),
\]

(1)

where $\psi^i_L = (t_L, b_L, \chi_L)^T$ (i = 1, 2, 3). This four-fermion interaction possesses the global symmetry $G = U(3)_L \times U(1)_R$. When the value of $G_{4f}$ is large enough to form a fermion-bilinear condensate, namely $G_{4f} > G_{\text{crit}} = 8\pi^2/(N_c \Lambda^2)$ with $N_c$ being the number of QCD color and $\Lambda$ the cutoff scale of the theory, the global symmetry is spontaneously broken down to $H = U(2)_L \times U(1)_V$. In association with the symmetry breaking, the five NGBs emerge as bound states of the $t$ and $\chi$ quarks, in addition to a composite heavy scalar boson, corresponding to the $\sigma$ mode of the usual Nambu-Jona-Lasinio (NJL) model [19]. Three of these five NGBs are eaten by the electroweak gauge bosons when the subgroup of $G$ is gauged by the electroweak symmetry (and if the condensate is formed in a direction where the electroweak symmetry is broken). The other two remain as physical states, and they obtain their masses by additional interaction terms which explicitly break the global $G = U(3)_L \times U(1)_R$ symmetry:

\[
\mathcal{L}_h = - [\Delta_{\chi\chi} \bar{\chi} \chi_L + \text{h.c.}] - G' (\bar{\chi} \chi_R) (\bar{\chi} \chi_R). \]

(2)

Then two NGBs become pNGBs, dubbed as top-mode pNGBs (TMpNGBs). One of the TMpNGBs, which is the $CP$-even scalar ($h_0^0$), is identified as the 126 GeV Higgs boson discovered at the LHC, while the other is the $CP$-odd
scalar \( A_0^L \), which is similar to \( CP \)-odd Higgs in many models like the minimal supersymmetric standard model, the two-Higgs doublet model, etc.

Furthermore, the model includes another four-fermion interaction term,

\[
L_t = G'' (\bar{H}LXR) (\bar{iRXL}) + \text{h.c.}
\]

This, combined with Eq. (1), generates the top quark mass via the top-seesaw mechanism. Note that this term also explicitly breaks the \( G \)-symmetry, but does not contribute to the TmpNGBs’ masses \( (m_{\phi}^L \) and \( m_{\phi}^R) \) at the leading order. However, it was shown that at the next-to-leading order, the term in Eq. (3) gives large corrections to the masses of \( h_0^L \) and \( A_0^L \) via the top and \( \chi \)-quark loops \[12\]. This, namely the fact that even a small explicit breaking term causes large correction to physical quantities at the loop level, poses a question: is the vacuum alignment stable at the loop level? This is the main question we address in this paper.

If there was no explicit breaking term, the vacuum associated with the global symmetry breaking by the four-fermion interaction in Eq. (1) is infinitely degenerate. The question is, which specific point in the degenerate vacua is chosen as the true vacuum after all the explicit breaking terms \((L_h, L_t, \text{electroweak gauge interactions})\) are turned on. In \[12\], the vacuum alignment problem was discussed simply by looking at the tree level Lagrangian: in that case, only the relevant term is \( L_h \), and therefore a proper choice of values of \( \Delta \chi \chi \) and \( G' \) gives a vacuum which breaks the electroweak symmetry appropriately. However, at the one-loop level, all the explicit breaking terms will participate in determining the effective potential, and it could potentially destabilize the EWSB vacuum which was fixed at the leading order. To see whether the EWSB vacuum is chosen as desired even at the loop level, in this paper, we derive the effective potential of the TmpNGBs at the one-loop level with all the explicit breaking effects included. We show that the vacuum alignment is controlled by a single parameter, \( \theta_h \), which is expressed in term of model parameters, and there actually exist parameter choices where various phenomenological requirements are satisfied.

This paper is organized as follows. In Sec. [11] we first derive a low-energy effective Lagrangian induced from the TmpNGB model, then the one-loop effective potential is derived. In Sec. [11] we discuss the vacuum alignment problem based on the effective potential which includes contributions from the one-loop diagrams of all the SM gauge bosons, fermions and the TmpNGBs. Then, we show an example of a set of parameter choice which reproduces various physical quantities, including the EWSB scale, top quark mass, electroweak precision parameters, and Higgs mass. In Sec. [11] we discuss implications for collider phenomenology based on the newly identified true vacuum. Sec. [11] is devoted to the summary of the paper. In appendix [11] we present the detailed derivation of the one-loop effective Lagrangian based on the background field method, and in appendix [11] the coupling property and the partial decay widths of \( t' \) quark are summarized.

II. EFFECTIVE LAGRANGIAN OF THE TMPNBG MODEL

For the purpose of discussing the vacuum alignment of the TmpNGB model \[12\], in this section we derive an effective Lagrangian described by the TmpNGBs \((h_0^L \) and \( A_0^L)\), the \( t' \) quark, the SM gauge bosons and fermions, including terms explicitly breaking the global \( U(3)_L \times U(1)_R \) symmetry.

We start from the Lagrangian defined at a cutoff scale \( \Lambda \). The Lagrangian is constructed from the third-generation quarks in the SM, \( q = (t, b) \), and an \( SU(2)_L \)-singlet vectorlike quark \( \chi \) with the hypercharge \(+2/3\), which are embedded in the \( U(3) \)-flavor multiplets as \( \psi_{i, L,R}^t \equiv (q_{L,R}, \chi_{L,R})^{T,i} \) \((i = 1, 2, 3)\), as well as the electroweak gauge bosons in the SM:

\[
\mathcal{L}(\Lambda) = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{q}_L i\gamma^\mu \partial_\mu q_R + \bar{\chi}_R i\gamma^\mu \partial_\mu \chi_R + \mathcal{L}_{4f} + \mathcal{L}_h + \mathcal{L}_{\text{ew}} + \mathcal{L}_t ,
\]

where

\[
\mathcal{L}_{4f} = G_4 (\bar{\psi}_L^{i} \chi L)(\bar{\chi}_R \psi_L^i),
\]

\[
\mathcal{L}_h = - [\Delta \chi \chi \bar{\chi}_R \chi L + \text{h.c.}] - G' (\bar{\chi}_R \chi L)(\bar{\chi}_R \chi L),
\]

\[
\mathcal{L}_{\text{ew}} = - \frac{1}{4} W_{\mu \nu} W_{\mu \nu} - \frac{1}{4} B_{\mu \nu} B_{\mu \nu} + \bar{\psi}_L \gamma^\mu L_{\mu} \psi_L + \bar{\psi}_R \gamma^\mu R_{\mu} \psi_R ,
\]

\[
\mathcal{L}_t = G'' (\bar{\chi}_R \chi L)(\bar{t}_R XL) + \text{h.c.}
\]

The left- and right-gauge fields \( L_{\mu} \) and \( R_{\mu} \) include the \( SU(2)_L \) and \( U(1)_Y \) gauge fields \( W_{\mu} \) and \( B_{\mu} \) with the gauge
When the four-fermion coupling strength $G$ explicitly break the global $\tau$ with $f$ nonzero VEV, $G$ is determined from the stationary condition, following the procedure in \cite{11}, we integrate out the quantum-fluctuation fields for fermions in the momentum shell at the scale $\Lambda$ as follows:

$$L_{\mu} = g W_{\mu}^a \left( \frac{\tau^a}{2} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + g^2 B_{\mu} \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right), \quad R_{\mu} = g^2 B_{\mu} \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 2/3 \end{pmatrix},$$  

with $\tau^a (a = 1, 2, 3)$ being the Pauli matrices and $W_{\mu}^a$ and $B_{\mu}$ the field strengths of the electroweak gauge boson fields $W_{\mu}^a$ and $B_{\mu}$. The four-fermion interaction term $L_{4f}$ in Eq.\ref{eq:4f} (with the fermion-kinetic terms) is invariant under the transformation of the global symmetry $G = U(3)_{\psi_L} \times U(1)_{t_R} \times U(2)_{\eta_R}$, while the terms in $L_h$, $L_{ew}$ and $L_t$ explicitly break the global $G$-symmetry: the $\Delta$ and $G'$ terms in $L_h$ break the $G$-symmetry down to $U(2)_{\eta_L} \times U(1)_{Y}$, and $U(2)_{\eta_L} \times U(1)_{t_R} \times U(1)_{t_R}$, respectively; the electroweak gauge interactions in $L_{ew}$ only keep the $SU(2)_L \times U(1)_Y$ gauge symmetry which are embedded in the $G$-symmetry gauged as in Eq.\ref{eq:ew}: the $G''$ term in $L_t$ breaks the $G$-symmetry down to $U(2)_{\eta_L} \times U(1)_{t_R} \times U(1)_{t_R}$. We shall momentarily turn off all the explicit breaking terms, i.e., $L_h = L_{ew} = L_t = 0$, and derive an effective Lagrangian generated from the fermion-bubble sum diagrams at the leading order of the $1/N_c$-expansion. For that purpose, we introduce an $U(3)_L$-triplet auxiliary-field, $\Phi \sim \bar{\chi}_R \psi_L^T$, which can be decomposed as

$$\Phi = \frac{1}{\sqrt{2}} U \cdot \vec{\phi},$$  

where $\vec{\phi}$ is a three-component real-vector and $U$ is a $3 \times 3$ unitary matrix. Using Eq.\ref{eq:phi} we rewrite the Lagrangian at the scale $\Lambda$ as follows:

$$L(\Lambda) = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{q}_R i\gamma^\mu \partial_\mu q_R + \bar{\chi}_R i\gamma^\mu \partial_\mu \chi_R - \left[ \bar{\psi}_L \Phi \chi_R + h.c. \right] - \frac{1}{G_{4f}} (\Phi^\dagger \Phi).$$  

Following the procedure in \cite{11}, we integrate out the quantum-fluctuation fields for fermions in the momentum shell between the cutoff $\Lambda$ and an infrared scale $\lambda$. By keeping only the ultraviolet-divergent contributions arising from the fermion loops at the one-loop level, the resultant Lagrangian then takes the form equivalent to the one generated via the fermion-bubble sum diagrams at the leading order of the $1/N_c$-expansion:

$$L_{eff}(\lambda < \Lambda) = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{q}_R i\gamma^\mu \partial_\mu q_R + \bar{\chi}_R i\gamma^\mu \partial_\mu \chi_R + \partial^\mu \Phi^\dagger \partial_\mu \Phi - y [\bar{\psi}_L \Phi \chi_R + h.c.] - V_0(\Phi),$$  

where

$$V_0(\Phi) = \frac{1}{Z} \left[ \frac{1}{G_{4f}} - \frac{N_c}{8\pi^2} \Lambda^2 \right] (\Phi^\dagger \Phi) + \lambda (\Phi^\dagger \Phi)^2, \quad (13)$$  

and

$$Z = \frac{1}{g^2} = \frac{1}{\lambda} = \frac{N_c}{16\pi^2} \ln \frac{\Lambda^2}{\lambda^2}. \quad (14)$$  

When the four-fermion coupling strength $G_{4f}$ satisfies the criticality condition, $G_{4f} > G_{crit} (= 8\pi^2/(N_c \Lambda^2))$, without loss of generality, we may choose the vacuum expectation value (VEV) of the scalar field $\Phi$ as

$$\langle \Phi \rangle = \frac{f}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (15)$$  

where $f$ is determined from the stationary condition, $\delta V/\delta \langle \Phi \rangle = 0$, as

$$1 - \frac{G_{crit}}{G_{4f}} = \frac{8\pi^2 f^2}{N_c \Lambda^2}. \quad (16)$$  

When the four-fermion coupling strength $G_{4f}$ satisfies the criticality condition, $G_{4f} > G_{crit}$, the scalar field acquires nonzero VEV, $f \neq 0$, which triggers the spontaneous symmetry breaking, $G = U(3)_L \times U(1)_R \rightarrow H = U(2)_L \times U(1)_Y$. The real vector $\vec{\phi}$ in Eq.\ref{eq:phi} can then be expressed as

$$\vec{\phi} = \sigma_t \cdot \vec{\varphi} \quad \text{with} \quad \vec{\varphi} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (17)$$
with \( \sigma_i \) being the real scalar field corresponding to the \( \sigma \) mode as in the usual NJL model. The five NGBs emerge in association with the spontaneous breaking of the \( G \)-symmetry, which are parametrized in the unitary matrix \( U \) in Eq. (10) as

\[
U = \exp\left[ \frac{i}{f} \left( \sum_{\alpha=4,5,6,7} \pi^a_\alpha \lambda^\alpha + \pi^4 \Sigma \right) \right],
\]

(18)

where the Gell-Mann matrices \( \lambda^\alpha \) are normalized as \( \text{tr}[\lambda^a \lambda^b] = 2 \delta^{ab} \), and \( \Sigma_0 \) is defined as \( \Sigma_0 \equiv \text{diag}(0,0,1) \).

We may integrate out the \( \sigma_i \) field since its mass generically becomes as large as the cutoff scale \( \Lambda \) by the radiative corrections from the \( \sigma_i \)-potential. In that case, we may take the scalar field \( \Phi \) in Eq. (10) to be \( (f/\sqrt{2}) U \cdot \varphi \) to approximate the effective Lagrangian Eq. (12) as

\[
\mathcal{L}_{\text{eff}}(\Lambda < \Lambda) \approx \bar{\psi} L_i \gamma^\mu \partial_\mu \psi_L + \bar{q}_L i \gamma^\mu \partial_\mu q_R + \bar{\chi}_R i \gamma^\mu \partial_\mu \chi_R + \frac{f^2}{2} \text{tr} \left[ \partial_\mu U^\dagger \partial^\mu U \Sigma_0 \right] - \frac{y f}{\sqrt{2}} \left[ \bar{\psi}_L (U \Sigma_0) \psi_R + \text{h.c.} \right],
\]

(19)

where we have used \( \varphi^T \cdot A \cdot \varphi = \text{tr}[A \Sigma_0] \) for an arbitrary \( 3 \times 3 \) matrix \( A \).

Let us turn on the explicit breaking terms in \( \mathcal{L}_h, \mathcal{L}_b, \mathcal{L}_e \) and \( \mathcal{L}_t \) in Eqs. (6), (7) and (8). As noted in (12), it turns out that these explicit breaking terms do not affect the criticality and stationary conditions in Eq. (16). Using the same auxiliary field as in Eq. (10) and neglecting the \( \sigma_i \) field, we thus find that the effective Lagrangian is modified as

\[
\mathcal{L}_{\text{eff}}(\Lambda < \Lambda) = \frac{1}{4} W_{\mu \nu} W^{\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu}
+ \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L + \bar{q}_L i \gamma^\mu \partial_\mu q_R + \bar{\chi}_R i \gamma^\mu \partial_\mu \chi_R + \bar{\psi}_L \gamma^\mu \partial_\mu \chi_R + \bar{\phi}_L \gamma^\mu \partial_\mu \chi_R
+ \mathcal{L}_{\text{eff}}(U),
\]

(20)

with

\[
\mathcal{L}_{\text{eff}}(U) = \frac{f^2}{2} \text{tr} \left[ D_\mu U^\dagger D^\mu U \Sigma_0 \right] - \hat{m}_\chi \left[ \bar{\psi}_L \mathcal{M}_f(U) \psi_R + \text{h.c.} \right]
- c_1 f^2 \text{tr} \left[ U^\dagger \Sigma_0 U \Sigma_0 \right] + c_2 f^2 \text{tr} \left[ U \Sigma_0 + \Sigma_0 U^\dagger \right],
\]

(21)

where

\[
D_\mu U = \left( \partial_\mu - ig \hat{W}_\mu + ig' \hat{B}_\mu \right) U,
\]

\[
\hat{W}_\mu = \sum_{\alpha=1}^3 W_{\mu \alpha} \frac{\lambda^\alpha}{2},
\]

\[
\hat{B}_\mu = B_\mu \frac{\lambda^0}{2},
\]

\[
\lambda^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

and

\[
\mathcal{M}_f(U) = U \Sigma_0 + \frac{G''}{G_{4f}} \Sigma_0 U \Sigma_1 \quad \text{with} \quad \Sigma_1 = \Sigma_0 \cdot \lambda_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},
\]

(23)

\[
\hat{m}_\chi = \frac{1}{\sqrt{2}} y f = \sqrt{\frac{8 \pi^2}{N_c \text{ln}(\Lambda^2/\Lambda^2_\chi)}} f.
\]

(24)

In Eq. (24) we have used Eq. (14). The coefficients \( c_1 \) and \( c_2 \) in Eq. (21) are given by

\[
c_1 = \frac{y^2}{2} \frac{G'}{G_{4f}^2}, \quad c_2 = \frac{y}{\sqrt{2} f} \frac{A_{\chi}}{G_{4f}}.
\]

(25)

Note that, at the tree level, the form of the potential term for NGBs, corresponding to the second line of Eq. (21), is determined solely by the \( \mathcal{L}_b \), and the effect of the explicit breaking terms in \( \mathcal{L}_{ew} \) and \( \mathcal{L}_t \) appear only at loop level. Therefore, to see the effect of all the explicit breaking terms, we compute the effective Lagrangian at one-loop level by including all the contributions from the NGBs, electroweak gauge bosons, as well as fermions. The effective Lagrangian is calculated by keeping only the quadratic divergent terms, and the resultant expression becomes as
follows (for the detail of the calculation, see Appendix A):

\[
\mathcal{L}^{1\text{-loop}}_{\text{eff}}(U) = \frac{f^2}{2} \left( 1 - \frac{\Lambda^2}{4\pi^2 f^2} \right) \left[ \frac{3\Lambda^2}{8\pi^2 f^2} - \frac{f^2\Lambda^2}{32\pi^2} \left( \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 2N_c g_f^2 \left( \frac{G''}{G_{4f}} \right)^2 \right) \right] \text{tr} \left[ U^\dagger U_0 \right] + \text{constant},
\]

(26)

where the ultraviolet divergences have been cutoff by the cutoff scale of the effective Lagrangian \( \Lambda_\chi \). The quadratic divergences can be absorbed by redefinitions of the bare coupling \( f \), \( c_1 \) and \( c_2 \):

\[
F^2 = f^2 - \frac{\Lambda^2}{4\pi^2}, \quad C_1 F^2 = c_1 f^2 \left( 1 - \frac{3\Lambda^2}{8\pi^2 f^2} \right) - \frac{f^2\Lambda^2}{32\pi^2} \left( \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 2N_c g_f^2 \left( \frac{G''}{G_{4f}} \right)^2 \right), \quad C_2 F^2 = c_2 f^2 \left( 1 - \frac{5\Lambda^2}{32\pi^2 f^2} \right).
\]

(27)-(29)

where in Eq. (27) we used Eq. (24). Then the one-loop effective Lagrangian Eq. (26) is redefined at the scale \( \Lambda_\chi \) as

\[
\mathcal{L}^{1\text{-loop}}_{\text{eff}}(U; f, c_1, c_2, \bar{m}_\chi; \Lambda_\chi) = \mathcal{L}_{\text{eff}}(U; F, C_1, C_2, \bar{m}_\chi; \Lambda_\chi)
\]

\[
= \frac{f^2}{2} \left[ \frac{3\Lambda^2}{8\pi^2 f^2} - \frac{f^2\Lambda^2}{32\pi^2} \left( \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 2N_c g_f^2 \left( \frac{G''}{G_{4f}} \right)^2 \right) \right] \text{tr} \left[ U^\dagger U_0 \right] + \text{constant}.
\]

(30)

Thus we read off the effective potential including all the explicit breaking effects along with the quadratic divergences at the one-loop level as

\[
V_{\text{eff}}(U) = C_1 F^2 \text{tr} \left[ U^\dagger U_0 \right] - C_2 F^2 \text{tr} \left[ U U_0 + U_0 U \right] + \text{constant}.
\]

(31)

In the next section we will discuss the vacuum alignment based on the effective potential Eq. (31) with the parameters \( F, C_1 \) and \( C_2 \) defined in Eqs. (27), (28) and (29), respectively.

### III. VACUUM ALIGNMENT

In this section, we address the vacuum alignment of the TMpNGBH model based on the effective potential Eq. (31). We first show that the EWSB is realized as the global minimum of the effective potential, then we fix the model parameters of the effective Lagrangian Eq. (30) by inputing physical quantities and imposing phenomenological constraints.

#### A. Searching for the minimum

The vacuum energy can be obtained simply by replacing \( U \) in Eq. (31) with the vacuum expectation value \( \langle U \rangle \). With appropriate chiral \( U(3)_{L,R} \) rotations of fermion fields \( \psi_{L,R} \) and redefinition of the \( \Delta_{\chi \chi} \), the vacuum expectation

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1 Note the same-sign contributions from the gauge and fermion loops in Eq. (26). This counterintuitive result is understood by the fact that the gauge-loop contribution arises as the form of \( \text{tr}[U^\dagger U_0 U_0] = -\text{tr}[U^\dagger U_0 U_0] + \text{constant} \), while the fermion-loop as \(-\text{tr}[U^\dagger U_0 U_0] \), up to the common loop factor. See also Eq. (A15).
value of $U$ is generically parametrized by a single angle parameter $\theta$ as
\[
\langle U \rangle = \begin{pmatrix}
    \cos \theta & 0 & \sin \theta \\
    0 & 1 & 0 \\
    -\sin \theta & 0 & \cos \theta
\end{pmatrix}.
\] (32)

The physical interpretation of the parametrization in Eq. (32) can be obtained by considering the vacuum expectation value of the scalar field $\Phi$ in Eq. (10):
\[
\langle \Phi \rangle = \frac{f}{\sqrt{2}} \cdot \langle U \rangle \varphi = \frac{f}{\sqrt{2}} \cdot \left[ \cos \theta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \sin \theta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right].
\] (33)

This indicates that a generic vacuum is given as a superposition of two vacuums: i) $\langle \Phi \rangle \propto (0,0,1)^T$, and ii) $\langle \Phi \rangle \propto (1,0,0)^T$. These vacuums break the global $U(3)_L \times U(1)_R$ symmetry down to $U(2)_L \times U(1)_V$ in physically different directions: the vacuum i) breaks the global $U(3)_L$ symmetry in a direction where the electroweak gauge symmetry is unbroken ($\langle \chi_R L L \rangle = 0$ and $\langle \bar{\chi} R X L \rangle \neq 0$), while the vacuum ii) breaks it in a way electroweak symmetry is broken ($\langle \chi_R L L \rangle \neq 0$ and $\langle \bar{\chi} R X L \rangle = 0$). Therefore, $\sin \theta \neq 0$ ($\cos \theta \neq 1$) is required as a model which realizes EWSB, and we will show that such an EWSB vacuum is actually realized as a global minimum of the effective potential in the TMpNGBH model below.

Taking $U = \langle U \rangle$ in Eq. (31) and using $\langle U \rangle$ in Eq. (32), we have
\[
V_{\text{eff}}(U = \langle U \rangle) = F^2 \left[ C_1 \cdot \cos^2 \theta - 2C_2 \cdot \cos \theta \right],
\] (34)
where $C_1$ and $C_2$ are given in Eqs. (28) and (29). We now discuss the vacuum alignment by minimizing the above potential energy with respect to the alignment parameter $\cos \theta$. We first consider the case with $\Delta_{\chi h}^L = 0$, namely the explicit breaking term $L_h$ is absent. In this case, $c_1 = c_2 = 0$ (see Eq. (25)) and therefore from Eqs. (28) and (29), $C_1 < 0$ and $C_2 = 0$. In this case the potential energy is minimized at $\cos^2 \theta = 1$ (i.e. $\sin \theta = 0$), which corresponds to the electroweak symmetric vacuum. This tells us that the existence of the explicit breaking term $L_h$ is crucial to achieve the EWSB vacuum in the TMpNGBH model. Then, we next look into the case where $C_1 \neq 0$, $C_2 \neq 0$ with $C_2/C_1 < 1$ (it will be shown later that there actually exists a parameter space in which such conditions are satisfied). In this case the potential energy is minimized at a nonzero $\theta = \theta_h$
\[
\cos \theta \bigg|_{\theta = \theta_h} = \frac{C_2}{C_1},
\] (35)
which is the desired vacuum breaking the electroweak gauge symmetry.

To check the stability of the vacuum in Eq. (35), we examine the coefficient of the second order derivatives of the one-loop effective potential Eq. (31) with respect to $\pi_i^a$ around $\theta = \theta_h$. Taking into account the parameterization of vacuum expectation value in Eq. (32), we reparametrize the field $U$ as
\[
U \rightarrow U \equiv R(\theta_h) U \quad \text{with} \quad R(\theta_h) = \begin{pmatrix}
    \cos \theta_h & 0 & \sin \theta_h \\
    0 & 1 & 0 \\
    -\sin \theta_h & 0 & \cos \theta_h
\end{pmatrix}.
\] (36)
The newly defined $U$ here takes the same form as in Eq. (18), but now VEV of it is diag(1,1,1). For the vacuum in Eq. (35) to be stable, the following condition has to be satisfied:
\[
\text{eigenvalues of } m_{ab}^2 \bigg|_{\sum_{\pi_i^a} = \langle \pi_i^a \rangle} \geq 0,
\] (37)
where $m_{ab}^2$, $(a,b = 4,5,6,7, A)$ corresponds to the $(a,b)$-element of the mass matrix of the NGBs. We find the non-vanishing elements of the NGB mass-squared matrix take the following forms:
\[
\begin{pmatrix}
    m_{44}^2 & m_{4A}^2 \\
    m_{A4}^2 & m_{AA}^2
\end{pmatrix} = 2C_1 \times \begin{pmatrix}
    \cos \theta_h & -\sin \theta_h \\
    \sin \theta_h & \cos \theta_h
\end{pmatrix} \begin{pmatrix}
    0 & 0 \\
    0 & 1
\end{pmatrix} \begin{pmatrix}
    \cos \theta_h & \sin \theta_h \\
    -\sin \theta_h & \cos \theta_h
\end{pmatrix},
\] (38)
and
\[ m_{S}^{2} = 2C_{1} \sin^{2} \theta_{h} \tag{39} \]

The condition in Eq. (37) thus requires \( C_{1} \geq 0 \).

The massive state in Eq. (38) is identified as the CP-odd scalar \( A_{0} \) \((A_{0} \equiv -\pi_{5}^{\dagger} \sin \theta_{h} + \pi_{5}^{A} \cos \theta_{h})\), while that in Eq. (39) is the CP-even scalar \((\pi_{5} \equiv h_{t}^{0})\), dubbed as the “tHiggs” \(12\). These masses are related by the alignment parameter \( \theta_{h} \):

\[ m_{A_{0}}^{2} = 2C_{1}, \tag{40} \]
\[ m_{h_{t}^{0}}^{2} = 2C_{1} \sin^{2} \theta_{h} = m_{A_{0}}^{2} \sin^{2} \theta_{h}. \tag{41} \]

Other three eigenvalues of mass-squared matrix vanish, which corresponds to three massless NGBs \((\pi_{6,7}^{A}, \pi_{5}^{A} \cos \theta_{h} + \pi_{5}^{A} \sin \theta_{h})\). These are the would-be NGBs to be eaten by the electroweak gauge bosons. It should be noted from Eqs. (40) and (41) that the quadratic divergent contributions to masses of TMpNGBs have been fully absorbed into the renormalization of the decay constant \( F \), the coefficient \( C_{1} \) and the alignment parameter \( \theta_{h} \) (or the coefficient \( C_{2} \)): this implies that the Higgs boson mass is controlled by some tuning of the model parameters, \( G_{4f}, \Delta_{\chi \chi}, G' \) and \( G'' \) in the original Lagrangian Eq. (1), as will be discussed later.

To summarize, the TMpNGBH model properly realizes the EWSB in the vacuum characterized by the alignment parameter \( \theta_{h} \) in Eq. (35), and the TMpNGBs, tHiggs \((h_{t}^{0})\) and CP-odd scalar \((A_{0}^{0})\), obtain their masses by the explicit breaking effects, which are related each other as in Eqs. (40) and (41).

Using Eqs. (35), (36), (40) and (41), we rewrite the Lagrangian Eq. (30) as
\[
L_{\text{eff}}(U; \Lambda_{\chi}) = L_{\text{eff}}(\bar{U}; \theta_{h}; \Lambda_{\chi}) = \frac{F^{2}}{2} \text{tr} \left[ \bar{D}_{\mu} \bar{U}^{\dagger} \bar{D}^{\mu} \bar{U} \Sigma_{0} \right] - \bar{m}_{\chi} \left[ \bar{\psi}_{L} \mathcal{M}_{f}(\bar{U}) \psi_{R} + \text{h.c.} \right] + \frac{F^{2} m_{A_{0}}^{2}}{2} \text{tr} \left[ - \left( \bar{U}^{\dagger} \Sigma_{0} \bar{U} \Sigma_{0} \right) + \cos \theta_{h} \left( \bar{U} \Sigma_{0} + \Sigma_{0} \bar{U}^{\dagger} \right) \right]. \tag{42} \]

Below we will fix the model parameters in the Lagrangian Eq. (42) by imposing phenomenological constraints.

**B. Fixing the model parameters**

We fix the model parameters \( F, \bar{m}_{\chi}, G''/G_{4f}, m_{A_{0}}^{2} \) and \( \theta_{h} \) in the effective Lagrangian Eq. (42) by five phenomenological inputs. To this end, we need expressions of physical quantities in terms of model parameters. Those can be obtained from the results shown in Refs. \(12, 20\): all we need to do is to replace the bare parameters \( f, m_{\chi}, \theta \) in the expressions derived in those references with those redefined in the effective Lagrangian Eq. (12), namely, \( F, \bar{m}_{\chi}, \) and \( \theta_{h} \).

Three of the physical inputs are chosen to be the electroweak scale, the LHC Higgs mass, which is identified with the tHiggs \((h_{t}^{0})\) mass in the present model, and the top quark mass:
\[
v_{\text{ew}} \simeq 246 \text{ GeV}, \quad m_{h_{t}^{0}} \simeq 126 \text{ GeV}, \quad m_{t} \simeq \max 173 \text{ GeV}. \tag{43} \]

As the fourth physical input, we take the value of the \( T \) parameter \(21, 22\) to be \(23\)
\[ T \simeq 0.08. \tag{44} \]

We should note that the \( S \) parameter is quite insensitive to the model parameters and vanishingly small as shown in \(12\). Therefore, it is not appropriate to use for fixing the model parameters.

Having specified four physical inputs, only one model parameter is remaining to be fixed. To fix its value, we use the Higgs signal strengths data measured at the LHC. The Higgs signal strengths can be written as a function of a single parameter, say \( \theta_{h} \), as in \(24\). Performing the goodness of fit test by using the Higgs signal strengths data for several decay and production categories reported from the LHC experiments as done in \(20, 2 \), we find the 95\% C.L.\(^{2}\)
constraint on $\cos \theta_h$ to be $\cos \theta_h \geq 0.97$. As a reference point, we may take $\cos \theta = 0.97$, which is the choice to make the model as non-SM-like as possible. (In the following section, we study the LHC phenomenology in the parameter space allowed by the 95% C.L. constraint on the Higgs signal strength, $0.97 \leq \cos \theta_h \lesssim 1$.)

The summary of the actual values of five model parameters which realize physical inputs explained above are as follows:

\[
F \simeq 1.0 \text{ TeV}, \quad \bar{m}_\chi \simeq 1.8 \text{ TeV}, \quad \frac{G''}{G_{4f}} \simeq 0.7, \\
m_{A^0_t} \simeq 518 \text{ GeV}, \quad \cos \theta_h \simeq 0.97. \tag{45}
\]

Once these are fixed, all other physical quantities can be predicted. For example, the mass of the $t'$ quark, a vectorlike partner of the top quark arising in the mass basis of the $t$ and $\chi$-quarks [12], to be

\[
m_{t'} \simeq 1.85 \text{ TeV}. \tag{46}
\]

Now that the values of the model parameters in the effective Lagrangian Eq.[42] have been fixed, we shall next discuss how those values can be realized in terms of the parameters in the original four-fermion interaction model in Eq.[4]. As a reference point, we take the value of $A_\chi$ twice the mass of $t'$ quark:

\[
A_\chi \simeq 3.7 \text{ TeV}. \tag{47}
\]

Then the cutoff of the four-fermion dynamics $A$ is determined from Eq.[27] to be

\[
A \simeq 7 \times 10^2 \text{ TeV}, \tag{48}
\]

and the original decay constant $f \simeq 1.2 \text{ TeV}$. From the stationary condition Eq.[16], we also estimate the ratio $G_{4f}/G_{\text{crit}}$ to get

\[
\frac{G_{4f}}{G_{\text{crit}}} - 1 \simeq 7 \times 10^{-4}, \tag{49}
\]

where $G_{\text{crit}} = 8\pi^2/(N_c A^2) \simeq (140 \text{ TeV})^{-2}$. Finally, we determine the size of the explicit breaking parameters $G'$ and $\Delta_{\chi \chi}$, which is the source of the masses of the TMpNGBs. From Eqs.[25], [28], [29], and using the experimental values for the $SU(2)_L$ and $U(1)_Y$ gauge couplings (renormalized at the $Z$ boson mass scale) $g = 0.653, g' = 0.358$ [5], we find

\[
\frac{G'}{G_{4f}} \simeq 2 \times 10^{-5}, \\
\Delta_{\chi \chi} \simeq 4 \times 10^{-3} \text{ GeV}. \left( \frac{\Delta_{\chi \chi}}{f} \simeq 4 \times 10^{-6} \right). \tag{50}
\]

With these choice of parameters, the TMpNGBH model achieves the realistic situation, where the electroweak symmetry is broken with an appropriate scale, it passes the electroweak precision test, and the 126 GeV Higgs arises as the pNGB having the coupling property consistent with the LHC Higgs. In addition to the 126 GeV Higgs, the model has the $CP$-odd scalar $A^0_t$ and the $t'$ quark with the masses in Eqs.[45] and [46], respectively, which are characteristic to the present model. The LHC phenomenologies of these particles will be discussed in the next section.

**IV. IMPLICATIONS FOR COLLIDER PHYSICS**

In this section, we shall discuss LHC phenomenologies of the $CP$-odd TMpNGB ($A^0_t$) and the vectorlike partner of the top quark ($t'$). Though in the previous section, we have fixed all the model parameters as in Eq.[45] by five physical inputs as a reference point, we will relax the parameter choice by allowing the alignment parameter $\cos \theta_h$ taking a value in the range of $0.97 \leq \cos \theta_h \lesssim 1$. We should note again that this is the range where the coupling property of the tHiggs to SM particles is consistent with the LHC data at 95% C.L. For $0.97 \leq \cos \theta_h \lesssim 1$ the masses of $A^0_t$ and $t'$ monotonically increase from $(m_{A^0_t}, m_{t'}) = (518 \text{ GeV}, 1.85 \text{ TeV})$ to infinity as $\cos \theta_h \rightarrow 1$. In this section, we thus study the LHC phenomenologies of $A^0_t$ and $t'$ with their masses from $(m_{A^0_t}, m_{t'}) = (518 \text{ GeV}, 1.85 \text{ TeV})$ to certain heavier mass regions which are considered to be relevant to the LHC.

The couplings of $A^0_t$ to the SM particles, the tHiggs ($h^0_t$) and the $t'$ quark can be read off from the Lagrangian Eq.[12]. The explicit expressions of the partial decay widths relevant to the LHC study can be found in [20] with the replacement, $f \rightarrow F$ and $\theta \rightarrow \theta_h$. In Fig. 1 we plot the branching ratio of $A^0_t$ as a function of $m_{A^0_t}$ in the range of
518 GeV \leq m_{A^0_t} \leq 2 \text{ TeV} in the left panel of Fig. 1. In this plot, we also indicate the corresponding values of \(\cos \theta_h\) in the upper horizontal axis. This plot is supposed to be the same as the right panel of Fig.2 in Ref. [29], though the appearance of the plot looks different, especially the branching ratio to \(Z_{t^0}\) mode. The crucial difference between the analysis in Ref. [29] and the present one is the presence of the mass relation \(m_{h^0_t} = m_{A^0_t} \sin \theta_h\) in Eq. (41), which is derived consistently at the one-loop level.

From the plot we see that, in the smaller mass region, the \(t\bar{t}\) and \(gg\) modes are the dominant decay channels, and therefore the main production process is the gluon-gluon fusion (ggF). As was discussed in [29], the 8 TeV LHC cross sections \(pp \rightarrow A^0_t \rightarrow gg/\bar{t}t\) for \(m_{A^0_t} \geq 1 \text{ TeV}\) have not seriously been limited by the currently available data yet [24] for the \(gg\)-channel and [25][26] for the \(t\bar{t}\)-channel). It is therefore to be expected that more data from the upcoming Run-II would probe the \(A^0_t\) through these channels. The detailed study will be given elsewhere. Another interesting channel would be \(A^0_t \rightarrow Z h^0_t\) as was emphasized in the previous analysis [29]. However, with the updated branching ratio, this channel seems to be rather challenging even at \(\sqrt{s} = 14 \text{ TeV}\) LHC with 3000 fb\(^{-1}\) data due to the small branching ratio in the smaller mass region.

The \(t^0\) quark arises as a mixture of the gauge-eigenstate top and \(\chi\)-quarks through the diagonalization of the fermion mass matrix in the effective Lagrangian Eq.(42). The explicit expressions of the \(t^0\) couplings and the partial decay widths relevant to the LHC study are listed in Appendix B. In the right panel of Fig. 1 we plot the branching ratios of the \(t^0\) quark as a function of \(m_{t^0}\). In the same way as the plot for the branching ratios of \(A^0_t\), the corresponding value of \(\cos \theta_h\) is also shown in the upper horizontal axis. From the figure we read off

\[
\text{Br}(t^0 \rightarrow W^+ b) \simeq 0.42, \quad \text{Br}(t^0 \rightarrow Z t) \simeq 0.21, \quad \text{Br}(t^0 \rightarrow h^0_t t) \simeq 0.35, \quad \text{Br}(t^0 \rightarrow A^0_t t) \simeq 0.02. \quad (51)
\]

It is worth comparing these values with the branching ratios of the “singlet \(t^0\) quark” in a benchmark model of \(t^0\) quark (e.g. [27]), \(\text{Br}(t^0 \rightarrow W^+ b) \simeq 0.5, \text{Br}(t^0 \rightarrow Z t) \simeq 0.25, \text{Br}(t^0 \rightarrow h^0_t t) \simeq 0.25\), for \(m_{t^0} \simeq 2 \text{ TeV}\) [25][26].

Since the predicted \(t^0\) quark in the TMpNGBH model is heavy \((m_{t^0} \geq 1.8 \text{ TeV})\), it might be challenging to search for the \(t^0\) quark via the pair production process as studied in the usual top-partner search by the ATLAS [30][34] and the CMS [35][38] collaborations. A more interesting production process of heavy \(t^0\) quark would be the single production, as stressed in [28][29], such as \(gg \rightarrow t^0 \rightarrow h^0_t b\bar{t}\) as depicted in Fig.2. From Ref. [29] we can read off the production cross section of the singlet \(t^0\) quark with \(s_{L^0}^t \simeq 0.1\) (for the definition of this parameter, see Appendix B), \(\sigma(t^0 t^0) \sim 0.1 \text{ fb}\) at \(\sqrt{s} = 13 \text{ TeV}\) for \(m_{t^0} \simeq 2 \text{ TeV}\) in the case of the pair production process, while \(\sigma(t^0 b\bar{t}) \sim 4 \text{ fb}\) at \(\sqrt{s} = 13 \text{ TeV}\) for \(m_{t^0} \simeq 2 \text{ TeV}\) in the case of the single production. By simply quoting these numbers, we may roughly estimate the cross sections times the branching ratio \(\text{Br}(t^0 \rightarrow h^0_t) \simeq 0.35\) in the present model to be

\[
\begin{align*}
\text{pair production} : & \quad \sigma(pp \rightarrow t^0 t^0 \rightarrow t\bar{t} + 2h^0_t) \sim 0.01 \text{ fb}, \\
\text{single production} : & \quad \sigma(pp \rightarrow t^0 b\bar{t} \rightarrow h^0_t + b\bar{t}) \sim 1.4 \text{ fb},
\end{align*}
\]

for \(\sqrt{s} = 13 \text{ TeV}\). More details of the \(t^0\) quark phenomenology at the LHC are to be pursued in the future.
V. SUMMARY

We addressed the vacuum alignment problem of the recently proposed new top quark condensation model \cite{12}, in which the 126 GeV Higgs boson emerges as a CP-even pseudo-Nambu-Goldstone-Boson (TMpNGB) associated with the global symmetry breaking caused by the supercritical NJL dynamics. We calculated the one-loop effective Lagrangian for the NGB sector, Eq. (30), taking into account all the explicit breaking effects, including electroweak gauge interactions and four fermion interactions responsible for the top-seesaw mechanism. The one-loop effective potential Eq. (31) includes all the explicit breaking terms, and therefore the correct vacuum is determined by the configuration which minimizes the one-loop effective potential. It was found that the true vacuum is parameterized by $\cos \theta_h$ defined as Eq. (35), and a non-zero value of $\cos \theta_h$ realizes the EWSB phase with the appropriate breaking scale.

After we clarified relations among physical quantities and model parameters, we showed that the constraints from the electroweak precision tests and the data on the coupling property of the Higgs boson reported by the LHC place lower bonds on the masses of the CP-odd TMpNGB ($A_0^t$) and $t'$ quark: $m_{A_0^t} \gtrsim 520$ GeV and $m_{t'} \gtrsim 1.8$ TeV. We also discussed the collider phenomenologies of $A_0^t$ and $t'$ quark on the vacuum aligned at the one-loop level. We found that the $A_0^t$ search in the $Zh_0^t$ decay channel would be challenging even at the future LHC, though the $t\bar{t}$ decay mode is worth investigating. As for the $t'$ quark, we pointed out that the single production of $t'$ quark, decaying into $h^0t$ at the future LHC could be an interesting discovery channel. More detailed study of these collider phenomenologies will be pursued in the future.

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Appendix A: Background field method

In this appendix, we derive the one-loop effective Lagrangian Eq. (20) based on the background field method. The building blocks constructing the effective Lagrangian in Eq. (20) are

$$U, \quad \hat{W}_\mu, \quad \hat{B}_\mu, \quad \psi_{L,R}.$$ (A1)

For the purpose of performing the background field method, we decompose the above building blocks into the classical fields (denoted by bar fields) plus the quantum fluctuating ones (by check fields):

$$U = \bar{U} \cdot \bar{U} = \bar{U} \cdot \exp \left[ \frac{i}{f} \left( \sum_{a=4,5,6,7} \tilde{\pi}_a^a \lambda^a + \tilde{\pi}_t^3 \Sigma_0 \right) \right],$$ (A2)

$$\hat{W}_\mu = \bar{W}_\mu + \tilde{W}_\mu,$$ (A3)

$$\hat{B}_\mu = \bar{B}_\mu + \tilde{B}_\mu,$$ (A4)

$$\psi_{L,R} = \bar{\psi}_{L,R} + \tilde{\psi}_{L,R}.$$ (A5)
Then the covariant derivative acting on $U$ given in Eq. (22) is decomposed as

$$D_{\mu}U = (\bar{D}_{\mu}U) \cdot U + \bar{U} \cdot (\bar{D}_{\mu}U),$$

(A6)

with

$$\bar{D}_{\mu}U = \partial_{\mu}U - ig\bar{W}_{\mu}U + ig'\bar{B}_{\mu}U,$$

$$\bar{D}_{\mu}U = (\partial_{\mu} - igU^{\dagger}W_{\mu}U + ig'\bar{B}_{\mu})U.$$

(A7)

And, the gauge field strength for $G_{\mu} = (\bar{W}_{\mu}, \bar{B}_{\mu})$ is expressed in terms of the background and fluctuating fields as

$$G_{\mu\nu} = \bar{G}_{\mu\nu} + \bar{D}_{\mu}G_{\nu} - \bar{D}_{\nu}G_{\mu} - ig^{(\Lambda)}[\bar{G}_{\mu}, G_{\nu}],$$

(A8)

with

$$\bar{G}_{\mu\nu} = \partial_{\mu}G_{\nu} - \partial_{\nu}G_{\mu} - ig^{(\Lambda)}[\bar{G}_{\mu}, G_{\nu}],$$

$$\bar{D}_{\mu}G_{\nu} = \partial_{\mu}G_{\nu} - ig^{(\Lambda)}[\bar{G}_{\mu}, G_{\nu}].$$

(A9)

1. NG and gauge-boson loops

Expanding the Lagrangian Eq. (20) in powers of the fluctuating fields up to the quadratic order, we find that the NGB and gauge boson sectors take the form

$$\mathcal{L}_{\text{NGB+EW}} = \mathcal{L}_{\text{NGB+EW}}^{0} + \mathcal{L}_{\text{NGB+EW}}^{\pm} + \mathcal{L}_{\text{NGB+EW}}^{\bar{G}} + \mathcal{L}_{\text{NGB+EW}}^{S} + \cdots,$$

(A10)

where

$$\mathcal{L}_{\text{NGB+EW}}^{0} = -\frac{1}{4} \bar{W}^{\dot{a}\mu\nu} \bar{W}_{\dot{a}\mu\nu} - \frac{1}{4} \bar{B}^{\mu\nu} \bar{B}_{\mu\nu}$$

$$+ \frac{f^{2}}{2} \text{tr} [\bar{D}_{\mu}U^{\dagger}\bar{D}_{\mu}U] - c_{1} f^{2} \text{tr} [\bar{U}^{\dagger}\Sigma_{0}U] + c_{2} f^{2} \text{tr} [\bar{U} \Sigma_{0} + \Sigma_{0}U^{\dagger}],$$

(A11)

$$\mathcal{L}_{\text{NGB+EW}}^{\pm} = f_{\text{tr}} [(g\bar{W}^{\mu} - g'\bar{B}^{\mu}) (\bar{U} \bar{\pi} \Sigma_{0} \bar{D}_{\mu}U + \text{h.c.})] + \frac{f}{2} \text{tr} [(\bar{D}_{\mu}W^{\mu} - \bar{D}_{\mu}B^{\mu}) \bar{U} (\bar{\pi}_{t} \Sigma_{0}U^{\dagger})],$$

(A12)

$$\mathcal{L}_{\text{NGB+EW}}^{\pi} = \frac{1}{2} \partial^{a} \bar{\pi}_{t} \partial_{a} \bar{\pi}_{t} - \frac{1}{2} \bar{\Sigma}_{0} \partial^{a} \bar{\Sigma}_{0} + \frac{1}{2} [\partial^{a} \bar{\Sigma}_{0} \partial_{a} \bar{\Sigma}_{0}] \Gamma^{ab} - \frac{1}{2} [\partial^{a} \bar{\Sigma}_{0} \partial_{a} \bar{\Sigma}_{0}] S_{\mu}^{\Lambda \Lambda} + \frac{1}{2} \partial^{a} \bar{\Sigma}_{0} \partial_{a} \bar{\Sigma}_{0} S_{\mu}^{\Lambda \Lambda},$$

(A13)

$$\mathcal{L}_{\text{NGB+EW}}^{\bar{G}} = -g_{c} \bar{a} \bar{b} \bar{c} \bar{a} \bar{b} \bar{c} \bar{b} \bar{a} \bar{c} + \frac{f^{2}}{2} \text{tr} [(g\bar{W}^{\mu} - g'\bar{B}^{\mu}) (g\bar{W}^{\mu} - g'\bar{B}^{\mu}) \bar{U} \Sigma_{0} U^{\dagger}],$$

(A14)

with

$$\Gamma^{a} = -\Gamma_{b}^{a} \mu \mu = \frac{1}{2} \text{tr} [\bar{U}^{\dagger} \bar{D}_{\mu} \bar{U} (\lambda^{a} \Sigma_{0} \lambda^{b} - \lambda^{b} \Sigma_{0} \lambda^{a})],$$

(A15)

$$S_{\mu}^{\Lambda \Lambda} = S_{\mu}^{\Lambda \Lambda} = \frac{1}{2} \text{tr} [\bar{U}^{\dagger} \bar{D}_{\mu} \bar{U} (\Sigma_{0} \Sigma_{0})],$$

(A16)

$$\sigma^{a} = \sigma^{b} \mu \mu = \frac{1}{2} \text{tr} [\bar{D}_{\mu}U^{\dagger} D_{\mu}U (\Sigma_{0} \Sigma_{0}) + (\Sigma_{0} \Sigma_{0})],$$

(A17)

The quadratic-mixing term between $\bar{W}_{\mu}, \bar{B}_{\mu}$ and $\bar{\pi}_{t}$ in the last term of Eq. (A12) can be eliminated by adding the gauge-fixing term $\mathcal{L}_{\text{GF}},$

$$\mathcal{L}_{\text{GF}} = -\frac{1}{\xi} \text{tr} \left[ \left( \bar{D}^{\mu}W_{\mu} + \xi \frac{gf}{4} \bar{U} (\bar{\pi}_{t} \Sigma_{0}U^{\dagger}) \right)^{2} \right] - \frac{1}{\xi} \text{tr} \left[ \left( \bar{D}^{\mu}B_{\mu} - \xi \frac{gf}{4} \bar{U} (\bar{\pi}_{t} \Sigma_{0}U^{\dagger}) \right)^{2} \right].$$

(A18)
with $\xi$ being the gauge-fixing parameter.

We compute the one-loop corrections arising from Eq. (A10) together with Eq. (A18). We work in the Landau gauge $\xi = 0$ and focus on quadratically divergent contributions. In that case, it turns out that the ghost term corresponding to the gauge-fixing term Eq. (A18) does not contribute to the one-loop order, so we can safely drop the ghost contribution.

From Eqs. (A10) and (A18) we first compute the quadratic-divergent contributions arising from the gauge loops and regularize them by the cutoff $\Lambda$. Evaluating the one-loop diagrams as depicted in Fig. 3, we next calculate the quadratic-divergent contributions from the NGB loops to find

$$\frac{\Lambda^2}{32\pi^2} \left[ -4 \text{tr} \left( \bar{D}^\mu D^\mu U \Sigma_0 \right) + 12 c_1 \text{tr} \left( \bar{U} \Sigma_0 U \Sigma_0 \right) - 5 c_2 \text{tr} \left( \bar{U} \Sigma_0 + \Sigma_0 \bar{U} \right) \right],$$

(A19)

where we have dropped terms independent of $\bar{U}$.

![FIG. 3:](image)

FIG. 3: The NGB-loop diagrams giving rise to the quadratic-divergent corrections to the effective Lagrangian at the one-loop level in the background field method. The dashed line denotes the quantum-fluctuation fields of the NGBs ($\tilde{\pi}$).

Evaluating the one-loop diagrams as depicted in Fig. 3, we next calculate the quadratic-divergent contributions from the NGB loops to find

$$\frac{\Lambda^2}{32\pi^2} \left[ -4 \text{tr} \left( \bar{D}^\mu D^\mu U \Sigma_0 \right) + 12 c_1 \text{tr} \left( \bar{U} \Sigma_0 U \Sigma_0 \right) - 5 c_2 \text{tr} \left( \bar{U} \Sigma_0 + \Sigma_0 \bar{U} \right) \right],$$

(A20)

where we have dropped terms independent of $\bar{U}$. In reaching Eq. (A20), we have used the following formulae:

$$\sum_a \text{tr} \left[ \lambda^a (A \Sigma_0) \lambda^a (B) \right] = \frac{1}{2} \text{tr} \left[ B A \Sigma_0 \right] - \frac{3}{2} \text{tr} \left[ A \Sigma_0 B \Sigma_0 \right] + 2 \text{tr} \left[ A \Sigma_0 \right] \text{tr} \left[ B \right],$$

(A21)

$$\sum_a \text{tr} \left[ \lambda^a (\Sigma_0 A) \lambda^a (B) \right] = \frac{1}{2} \text{tr} \left[ A B \Sigma_0 \right] - \frac{3}{2} \text{tr} \left[ A \Sigma_0 B \Sigma_0 \right] + 2 \text{tr} \left[ A \Sigma_0 \right] \text{tr} \left[ B \right],$$

(A22)

$$\sum_a \text{tr} \left[ \lambda^a \left( A \Sigma_0 \right) \right] \text{tr} \left[ \lambda^a (B) \right] = 2 \text{tr} \left[ B A \Sigma_0 \right] - \frac{3}{2} \text{tr} \left[ A \Sigma_0 B \Sigma_0 \right] + \frac{1}{2} \text{tr} \left[ A \Sigma_0 \right] \text{tr} \left[ B \right],$$

(A23)

$$\sum_a \text{tr} \left[ \lambda^a (\Sigma_0 A) \right] \text{tr} \left[ \lambda^a (B) \right] = 2 \text{tr} \left[ A B \Sigma_0 \right] - \frac{3}{2} \text{tr} \left[ A \Sigma_0 B \Sigma_0 \right] + \frac{1}{2} \text{tr} \left[ A \Sigma_0 \right] \text{tr} \left[ B \right],$$

(A24)

$$\sum_a \text{tr} \left[ \lambda^a \lambda^a (A \Sigma_0 B) \right] = \frac{9}{4} \text{tr} \left[ B A \Sigma_0 \right] + \frac{11}{4} \text{tr} \left[ A \Sigma_0 B \Sigma_0 \right],$$

(A25)

and

$$\text{tr} \left[ A \Sigma_0 \right] \text{tr} \left[ B \Sigma_0 \right] = \text{tr} \left[ A \Sigma_0 B \Sigma_0 \right],$$

(A26)

where $A$ and $B$ are arbitrary $3 \times 3$ matrices.

We thus obtain the effective Lagrangian arising from the NGB and gauge boson loops at the one-loop level,

$$\left[ \mathcal{L}_{\text{NGB+EW}} + \mathcal{L}_{\text{GF}} \right]^\text{1-loop}_{\bar{U}} = \frac{f^2}{2} \left( 1 - \frac{\Lambda^2}{4\pi^2 f^2} \right) \text{tr} \left[ \bar{D}^\mu \bar{D}^\mu U \Sigma_0 \right]$$

$$- c_1 f^2 \left( 1 - \frac{3 \Lambda^2}{8\pi^2 f^2} \right) - \frac{f^2 \Lambda^2}{32\pi^2} \left( \frac{9}{4} g^2 + \frac{3}{4} g'^2 \right) \text{tr} \left[ \bar{U} \Sigma_0 U \Sigma_0 \right]$$

$$+ c_2 f^2 \left( 1 - \frac{5 \Lambda^2}{32\pi^2 f^2} \right) \text{tr} \left[ \bar{U} \Sigma_0 + \Sigma_0 \bar{U} \right],$$

(A27)

where we used $\lambda^0 = 1 - \Sigma_0$. 
2. Fermion loops

Expanding Eq.(20) in powers of the fluctuating fields for fermions up to the quadratic order, we find the interaction term relevant to the one-loop computation,

\[ \mathcal{L}_f = \mathcal{L}_{\text{kin}}(f) - \frac{y_f}{\sqrt{2}} \left[ \bar{\psi}_L \mathcal{M}_f(U) \psi_R \right] + \text{h.c.}, \]  

(A28)

where

\[ \mathcal{M}_f(U) = \bar{U} \Sigma_0 + \frac{G''}{G_{4f}} \Sigma_0 \bar{U} \Sigma_1. \]

(A29)

From this yukawa interaction we see that the quadratic-divergent contributions from fermion loops give rise to the effective Lagrangian,

\[ \frac{N_c \Lambda^2}{16\pi^2} y^2 f^2 \text{tr} \left[ \mathcal{M}_f(U) \mathcal{M}_f^\dagger(U) \right] = \frac{N_c \Lambda^2}{16\pi^2} y^2 f^2 \left( \frac{G''}{G_{4f}} \right)^2 \text{tr} \left[ \bar{U} \Sigma_0 \bar{U} \Sigma_0 \right], \]

(A30)

where in the second equality we have used \( \Sigma_1 \Sigma_0 = 0, \Sigma_1^\dagger \Sigma_1 = \Sigma_1 \Sigma_1^\dagger = \Sigma_0 \) and omitted terms independent of \( \bar{U} \).

3. Total

Combining Eqs. (A27) and (A30), we have

\[ \mathcal{L}_{\text{eff}}^{1\text{-loop}}(\bar{U}) = \frac{f^2}{2} \left( 1 - \frac{\Lambda^2}{4\pi^2 f^2} \right) \text{tr} \left[ \bar{D}_\mu \bar{D}^\mu \bar{U} \Sigma_0 \right] - \frac{y_f}{\sqrt{2}} \left[ \bar{\psi}_L \mathcal{M}_f(U) \psi_R + \text{h.c.} \right] 
\]

\[ - \left[ c_1 f^2 \left( 1 - \frac{3\Lambda^2}{8\pi^2 f^2} \right) - \frac{f^2 \Lambda^2}{32\pi^2 f^2} \left( \frac{9}{4} g^2 + \frac{3}{4} g'^2 + 2N_c y^2 \left( \frac{G''}{G_{4f}} \right)^2 \right) \right] \text{tr} \left[ \bar{U} \Sigma_0 \bar{U} \Sigma_0 \right] 
\]

\[ + c_2 f^2 \left( 1 - \frac{5\Lambda^2}{32\pi^2 f^2} \right) \text{tr} \left[ \bar{U} \Sigma_0 + \Sigma_0 \bar{U} \right], \]

(A31)

where the Yukawa terms for the background fields of fermions were added. Thus Eq.(26) has been derived.

Appendix B: The \( t' \) quark couplings and partial decay widths

In this appendix, we shall derive the formulas for the partial decay widths of the \( t' \) quark relevant to the LHC phenomenology described in Sec. IV.

Examining the Lagrangian Eq.(20), we see that the \( t' \) quark in the mass basis, \( (t')_m \), arises as the mixture of the gauge (current) eigenstates \( (t, \chi)_g \) through the orthogonal rotation which diagonalizes the mass matrix of the seesaw type keeping \( m_t, m_\tau' \geq 0 \) [12] [16],

\[ \begin{pmatrix} t_L \\ t'_L \end{pmatrix}_m = \begin{pmatrix} c_L \chi = \begin{pmatrix} c_L & -s_L \\ s_L & c_L \end{pmatrix} \begin{pmatrix} t_L \\ \chi_L \end{pmatrix}_g, \end{pmatrix} \quad \begin{pmatrix} t_R \\ t'_R \end{pmatrix}_m = \begin{pmatrix} -c_R & s_R \\ s_R & c_R \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix}_g. \]

The mixing-angle parameters \( c^i_L(R) \) and \( s^i_L(R) \) can be expanded in powers of \( G''/G_{4f} \) to be given up to \( \mathcal{O}((G''/G_{4f})^2) \) as [12]

\[ c^i_L = \cos \theta_h \left[ 1 + \left( \frac{G''}{G_{4f}} \right)^2 \cos^2 \theta_h \sin^2 \theta_h \right], \quad s^i_L = \sin \theta_h \left[ 1 - \left( \frac{G''}{G_{4f}} \right)^2 \cos^4 \theta_h \right], \]

\[ c^i_R = 1 - \frac{1}{2} \left( \frac{G''}{G_{4f}} \right)^2 \cos^2 \theta_h, \quad s^i_R = \frac{G''}{G_{4f}} \cos^2 \theta_h. \]
Thus the relevant $t'$ quark interaction-terms are read off from Eq.\([20]\) as

\[
\mathcal{L}_\nu = \frac{g}{\sqrt{2}} (s_L^t) (W^+ \bar{t}_L \gamma^\mu t_L + \text{h.c.}) + \frac{g}{2 \cos \theta_W} (c_L^t s_L^t) Z_\mu (\bar{t}_L \gamma^\mu t_L + \text{h.c.})
\]

\[
- \frac{g}{\sqrt{2}} [C_{hL} \bar{t}^0_{tL} t_R + C_{hR} \bar{t}^0_{tL} t_R + i C_{AL} \bar{t}^0_{tL} t_R + i C_{AR} \bar{t}^0_{tL} t_R + \text{h.c.}], \tag{B1}
\]

where $\theta_W$ is the Weinberg angle and the coefficients $C_s$ are given as

\[
C_{hL} = s_L^t (s_L^t \cos \theta_h - c_L^t \sin \theta_h) + \left( \frac{G''}{G_{AF}} \right) c_L^t c_R^t \sin \theta_h,
\]

\[
C_{hR} = c_R^t (c_L^t \cos \theta_h + s_L^t \sin \theta_h) + \left( \frac{G''}{G_{AF}} \right) s_L^t s_R^t \sin \theta_h,
\]

\[
C_{AL} = c_L^t c_R^t - \left( \frac{G''}{G_{AF}} \right) c_L^t c_R^t,
\]

\[
C_{AR} = -s_L^t c_R^t - \left( \frac{G''}{G_{AF}} \right) s_L^t s_R^t.
\]

From Eq.\([B1]\), we thus evaluate the $t'$ quark decay amplitudes to obtain the formulae for the relevant partial decay widths,

\[
\Gamma(t' \to W^+ b) = \frac{g^2}{64\pi} (s_L^t)^2 \frac{m_b^3}{M_W^2} \left( 1 - \frac{M_W^2}{m_t^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{m_T^2} \right),
\]

\[
\Gamma(t' \to Z t) = \frac{g^2}{64\pi c_W^2} (c_L^t s_L^t)^2 \frac{m_t^3}{2M_Z^2} \left( \frac{m_t^2}{m_t^2} - \frac{M_Z^2}{m_T^2} \right) \left( 1 - \frac{m_t^2 - M_Z^2}{m_T^2} + \frac{m_t^2 - 2M_Z^2 + m_T^2}{m_T^2} \right),
\]

\[
\Gamma(t' \to h^0 t) = \frac{g^2}{32\pi} m_t \beta \left( \frac{m_t^2}{m_t^2} - \frac{m_{h^0}^2}{m_{h^0}^2} \right) \left[ (C_{hL}^2 + C_{hR}^2) \left( 1 + \frac{m_t^2 + m_{h^0}^2}{m_t^2} \right) + 4 C_{hL} C_{hR} \frac{m_t}{m_T} \right],
\]

\[
\Gamma(t' \to A^0 t) = \frac{g^2}{32\pi} m_t \beta \left( \frac{m_t^2}{m_t^2} - \frac{m_{A^0}^2}{m_{A^0}^2} \right) \left[ (C_{AL}^2 + C_{AR}^2) \left( 1 + \frac{m_t^2 + m_{A^0}^2}{m_t^2} \right) - 4 C_{AL} C_{AR} \frac{m_t}{m_T} \right],
\]

where $\beta(x, y) = \sqrt{(1 - x - y)^2 - 4xy}$.

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