Electromagnetic Nucleon-to-Delta Transition in Chiral Effective-Field Theory

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We perform a relativistic chiral effective-field theory calculation of pion electroproduction off the nucleon ($e^- N \to e^- N \pi$) in the $\Delta(1232)$-resonance region. After fixing the three low-energy constants, corresponding to the magnetic (M1), electric (E2), and Coulomb (C2) $\gamma N \Delta$ couplings, our calculation provides a prediction for the momentum-transfer and pion-mass dependence of the $\gamma N \Delta$ form factors. The prediction for the pion-mass dependence resolves the discrepancy between the recent lattice QCD results and the experimental value for the “C2/M1 ratio” at low $Q^2$.

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The $\Delta(1232)$-resonance, the first excited state of the nucleon, dominates many nuclear phenomena at energies between the one- and two-pion production thresholds. The electromagnetic excitation of the $\Delta$-resonance, the $\gamma N \Delta$ transition, has recently received a lot of attention. At low momentum-transfer ($Q^2$) it highlights the role of the pion cloud, whereas at larger $Q^2$ it probes the onset of the perturbative QCD regime.

The $\gamma N \Delta$ transition is predominantly of the magnetic dipole (M1) type which, in a simple quark-model picture, is described by a spin flip of a quark in the $s$-wave state. Any $d$-wave admixture in the nucleon or the $\Delta$ wave-functions allows for the electric- (E2) and Coulomb- (C2) quadrupole transitions. Therefore by measuring these one is able to assess the presence of the $d$-wave components and hence quantify to which extent the nucleon or the $\Delta$ wave-function deviates from the spherical shape (“hadron deformation”).

The $\gamma N \Delta$ transition has been accurately measured in the pion photo- and electro-production reactions. The E2 and C2 are found to be relatively small, the ratios $R_{EM} = E2/M1$ and $R_{SM} = C2/M1$ are at the level of a few percent. On the theoretical side, the most recent state-of-the-art lattice QCD study obtained a puzzling result: the computed ratio $R_{SM}$ at low momentum-transfer appears to be significantly different from the observed value. It is important to note that the lattice calculations were done at larger pion masses, while the result compared with experiment was obtained by a linear extrapolation to the physical pion mass.

In this Letter we present a first chiral effective-field theory ($\chi$EFT) calculation of pion photo- and electro-production on the nucleon in the $\Delta$-resonance region. Besides finding a good agreement of our calculation with observables, we are able to study the chiral behavior ($m_\pi$-dependence) of the $\gamma N \Delta$ transition. Our results show that there is no apparent discrepancy between the lattice data and the experimental result for $R_{SM}$.

Our starting point is the relativistic chiral Lagrangian of pion and nucleon fields supplemented with the relativistic $\Delta$-isobar fields. We organize the Lagrangian $\mathcal{L}^{(i)}$, such that superscript $i$ stands for the power of electromagnetic coupling $e$ plus the number of derivatives of pion and photon fields. Writing here only the relevant terms involving the spin-3/2 isospin-3/2 field $\gamma^{\nu a}$, $\gamma^{\nu a \beta \gamma_5}$:

\[
\mathcal{L}^{(1)}_\Delta = \bar{\psi}_\mu (i \gamma^{\nu a} D_\alpha - M_\Delta \gamma^{\mu \nu}) \psi_\nu + \frac{i e A}{2 f_\pi M_\Delta} \{N T_\alpha \gamma^{\mu \nu} \partial_\alpha \psi_\nu \partial_\beta \pi^a + H.c\}. (1a)
\]

\[
\mathcal{L}^{(2)}_\Delta = \frac{3i e g_{EM}}{2f_\pi M_\Delta} \bar{\psi}_\mu \gamma^{\mu \nu} A_\mu \psi_\nu \partial_\beta \pi^a + H.c., \quad (1b)
\]

\[
\mathcal{L}^{(3)}_\Delta = \frac{3e}{2f_\pi M_\Delta} \bar{\psi}_\mu \gamma^{\mu \nu} \partial_\nu \pi^a \partial_\alpha \gamma_5 + \bar{\psi}_\mu \gamma^{\mu \nu} \partial_\nu \pi^a \partial_\alpha \gamma_5 + H.c., \quad (1c)
\]

where $M \simeq 0.939$ and $M_\Delta \simeq 1.232 \text{ GeV}$ are, respectively, the nucleon and $\Delta$-isobar masses, $N$ and $\pi^a$ ($a = 1, 2, 3$) stand for the nucleon and pion fields, $D_\mu$ is the covariant derivative ensuring the electromagnetic gauge-invariance, $F^{\mu \nu}$ and $\tilde{F}^{\mu \nu}$ are the electromagnetic field strength and its dual, $T_\alpha$ are the isospin 1/2 to 3/2 transition matrices, $f_\pi \simeq 92.4 \text{ MeV}$ is the pion decay constant. $\mathcal{L}^{(1)}_\Delta$ contains the Rarita-Schwinger Lagrangian of a free spin-3/2 field formulated such that the number of spin degrees of freedom is constrained to the physical number. The couplings in Eq. (1) are consistent with these constraints because of a spin-3/2 gauge symmetry.

We next turn to the power-counting for the pion electroproduction amplitude using the “$\delta$-expansion” scheme. In this scheme the excitation energy of the $\Delta$-resonance: $\Delta \equiv M_\Delta - M \simeq 0.3 \text{ GeV}$ is treated as a light scale, so that for $\Lambda \sim 1 \text{ GeV}$ representing the heavy scales in the theory, we can use a small parameter $\delta = \Delta/\Lambda$. The other typical light scale of the theory, the pion mass, is counted as two powers of the small parameter: $m_\pi/\Lambda \sim \delta^2$. The latter rule is the main distinction
of this scheme from the previous power countings which count $\Delta$ and $m_\pi$ at the same order. This difference plays a crucial role in separating the low-energy and resonance regimes, as well as in approaching the chiral limit where $m_\pi$ vanishes while $\Delta$ remains finite. Because of the distinction of $m_\pi$ and $\Delta$ the counting of a given diagram depends on whether the characteristic momentum $p$ is in the low-energy region ($p \sim m_\pi$) or in the resonance region ($p \sim \Delta$). In the resonance region, one distinguishes the one-$\Delta$-reducible (OAR) graphs, see, e.g., graph (a) in Fig. 1. Such graphs contain $\Delta$ propagators which go as $1/(p - \Delta)$ and hence for $p \sim \Delta$ they are large and all need to be included. Their resummation amounts to dressing the $\Delta$ propagators so that they behave as $1/(p - \Delta - \Sigma)$. The self-energy $\Sigma$ begins at order $p^3$ and thus a dressed OAR propagator counts as $1/\delta^3$.

The pion electroproduction amplitude to next-to-leading order (NLO) in the $\delta$-expansion, in the resonance region, is thus given by graphs in Fig. 1(a) and (b), where the shaded blobs in graph (a) include correction terms depicted in Fig. 1(c)-f). The hadronic part of graph (a) begins at $O(\delta^0)$ which here is the leading order. The Born graphs Fig. 1(b) contribute at $O(\delta)$. We note that at NLO there are also vertex corrections of the type (e) and (f) with nucleon propagators in the loop replaced by the $\Delta$-propagators. However, adopting the on-mass shell renormalizations and $Q^2 \ll \Lambda\Delta$, these graphs start to contribute at next-to-next-to-leading order (NNLO).

We have not shown the $\gamma N\Delta$-vertex correction graph where the photon couples into the $\pi NN$ vertex, because at this order the effect of this graph can fully be absorbed in the graphs Fig. 1(e) and (f) by a field redefinition relating the pseudovector and pseudoscalar $\pi NN$ couplings. Having done that, we compute graphs Fig. 1(e) and (f) using the pseudoscalar coupling.

The self-energy correction, Fig. 1(c), was computed previously. In that calculation, the experimental value for the $\Delta$-resonance width fixes $h_A \simeq 2.85$. To present the results for the vertex corrections we first consider the general form of the $\gamma N\Delta$ vertex:

$$
\bar{u}_\alpha(p') \Gamma^{\mu}_{\gamma N\Delta} u(p) = \sqrt{\frac{3}{2 M(M_\Delta + M)^2 + Q^2}} M_\Delta + M \\
\times \bar{u}_\alpha(p') \left\{ g_M(Q^2) \varepsilon^{\mu\nu\rho\lambda} p'_\nu q_\lambda + g_E(Q^2) (q^\alpha p'^\mu - q \cdot p' g^{\mu\nu}) i \gamma^\nu \\
+ g_C(Q^2) (q^\alpha q'^\mu - q^2 g^{\mu\nu}) i \gamma^\nu \right\} u(p),
$$

where $u_\alpha$ is the $\Delta$ vector-spinor, $u$ is the nucleon spinor, $q = p' - p$ is the photon 4-momentum, $Q^2 = -q^2$, and $g_M, g_E,$ and $g_C$ are the form factors which at $Q^2 = 0$ are equal to the physical values of corresponding parameters from Lagrangian $\mathcal{L}$. These form factors relate to the conventional magnetic ($G_M^*$), electric ($G_E^*$) and Coulomb ($G_C^*$) form factors of Jones and Scadron as follows:

$$
G_M^* = g_M + \frac{M_\Delta^2}{Q_+^2} (-\beta_\gamma g_E + \bar{Q}^2 g_C),
$$

$$
G_E^* = \frac{M_\Delta^2}{Q_+^2} (-\beta_\gamma g_E + \bar{Q}^2 g_C),
$$

$$
G_C^* = -\frac{2 M_\Delta^2}{Q_+^2} (g_E + \beta_\gamma g_C),
$$

where $Q_\pm = \sqrt{(M_\Delta \pm M)^2 + Q^2}$, $\bar{Q}^2 = Q^2/M_\Delta^2$, $\beta_\gamma = \frac{1}{2}(1 - r^2 - Q^2)$, with $r = M/M_\Delta$. The ratios $E2/M1$ and $C2/M1$ at the resonance position can be expressed in terms of these form factors as:

$$
R_{EM} = -G_E^*/G_M^*, \quad R_{SM} = \frac{Q_+ Q_-}{2 M_\Delta^2} G_C^*/G_M^*. \quad (4)
$$

The one-loop corrections to the $\gamma N\Delta$ form factors are given by the graphs in Fig. 1(e) and (f). For example, the ($M$-subtracted) result for the graph (e) in Fig. 1 can be cast in the form:

$$
g_M^{(e)} = -C_{N\Delta} \int_0^1 dy \int_0^{1-y} dx \ln M^2 \left\{ -2x [x r + (1 - x - y)(1 + r)] M^{-2} \right\},
$$

$$
g_E^{(e)} = -C_{N\Delta} \int_0^1 dy \int_0^{1-y} dx \left\{ \ln M^2 - 2x [x r + (1 - x - y)(1 + r)] M^{-2} \right\},
$$

$$
g_C^{(e)} = -C_{N\Delta} \int_0^1 dy \int_0^{2y - 1} dx \left\{ \ln M^2 - 2x [x r + (1 - x - y)(1 + r)] M^{-2} \right\},
$$

where $M^2 \equiv (x - \beta)^2 - \lambda^2 + 2\beta x y + \bar{Q}^2 y(1 - y) - i\varepsilon$, $\mu = m_\pi/M_\Delta$, $\beta = \frac{1}{2}(1 - r^2 + \mu^2)$, $\lambda^2 = \beta^2 - \mu^2$, $C_{N\Delta} = 4 g_A h_\Delta Q^2/\beta(1 + r)(8\pi f_p^2)$, $g_A \approx 1.26$. Analogous expressions are obtained for the graph Fig. 1(f). Alternatively, we have computed these graphs by using the sideways dispersion relations (see, e.g., [24]), and obtained identical results.
The vector-meson diagram, Fig. 1(d), contributes to NLO for $Q^2 \sim \Lambda$. We include it effectively by giving the $g_M$-term a dipole $Q^2$ dependence (in analogy to how it is usually done for the nucleon isovector form factor): $g_M \to g_M(1+Q^2/0.71\text{GeV}^2)^{-2}$. Analogous effect for the $g_E$ and $g_C$ couplings begins at NNLO and is not included in the present calculation.

We now present the electroproduction observables corresponding to the NLO amplitude of Fig. 1. Denoting the invariant mass of the final $\pi N$ system by $s$, we restrict ourselves to the resonance kinematics: $s = M_\Delta^2$. The $\gamma^*N \to \pi N$ cross section for unpolarized nucleons are expressed in terms of 5 response functions as:

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \varepsilon \frac{d\sigma_L}{d\Omega_\pi} + \varepsilon \cos 2\Phi \frac{d\sigma_{TT}}{d\Omega_\pi}$$

$$+ \sqrt{2\varepsilon(1+\varepsilon)} \cos \Phi \frac{d\sigma_{LT}}{d\Omega_\pi}$$

$$+ h \sqrt{2\varepsilon(1-\varepsilon)} \sin \Phi \frac{d\sigma_{LT}}{d\Omega_\pi},$$

where $\Theta_\pi$ and $\Phi$ are the pion polar and azimuthal c.m. angles, respectively, and $h$ denotes the electron helicity.

In Fig. 2 we show our $\chi$EFT results for the different cross sections entering Eq. (6). The only free parameters in this calculation are the low-energy constants from $g_M = 2.88$, $g_E = -1.04$, $g_C = -2.36$. Within $\chi$EFT, we can estimate the theoretical uncertainty of the NLO result due to higher-order effects. The NNLO corrections to the amplitudes are expected to be of order of $\delta^2$, $m_\pi/\Lambda$, or $Q^2/\Lambda^2$. Therefore, the theoretical uncertainty $R_{err}$ of an observable $R$, which involves a product of two amplitudes, is estimated as (taking here $\Lambda = M$):

$$R_{err} = 2|R_{av}| \cdot \frac{1}{2} \left( \delta^2 + \frac{m_\pi^2}{M^2} + \frac{Q^2}{M^2} \right),$$

where $R_{av}$ is an average value of $R$. In Fig. 2 the average is taken over the range of $\Theta_\pi$. One sees that the NLO $\chi$EFT calculation, within its accuracy, is consistent with the experimental data for these observables.

In Fig. 3 we show the $Q^2$ dependence of the ratios $R_{EM}$ and $R_{SM}$. Having fixed the low energy constants $g_M$, $g_E$, and $g_C$, the $Q^2$ dependence follows as a prediction. The theoretical uncertainty here (shown by the error bands) is estimated according to Eq. (7) with the average $R_{av}$ taken over the range of $Q^2$ from 0 to 0.2 GeV$^2$. From the figure one sees that the NLO calculations are consistent with the experimental data for both of the ratios.

In Fig. 4 we show the $m_\pi$ dependence of the $\gamma N\Delta$ transition ratios, with the theoretical uncertainty estimated according to Eq. (7) where $R_{av}$ is taken over the range of $m_\pi^2$ from 0 to 0.15 GeV$^2$. The study of the $m_\pi$ dependence is crucial to connect to the lattice QCD results, which at present can only be obtained for larger pion masses. The recent state-of-the-art lattice calculations of these ratios use a linear, in the quark mass ($m_q \propto m_\pi^2$), extrapolation to the physical point, thus assuming that the non-analytic $m_\pi$-dependencies are negligible. The thus obtained value for $R_{SM}$ at the physical $m_\pi$ value displays a large discrepancy with the experimental result, as seen in Fig. 4. However, our calculation demonstrates that the non-analytic dependencies are not negligible. While at larger values of $m_\pi$, where the $\Delta$ is stable, the ratios display a smooth $m_\pi$ dependence, at $m_\pi = \Delta$ there is an inflection point, and for $m_\pi < \Delta$ the non-analytic effects are crucial, as was also observed for the $\Delta$-resonance magnetic moment. The $m_\pi$...
dependence obtained in χEFT clearly shows that the lattice results for $R_{SM}$ may in fact be consistent with experiment.

In conclusion, we have performed a manifestly gauge- and Lorentz-invariant χEFT calculation of the $eN \rightarrow eN\pi$ reaction in the $\Delta(1232)$ resonance region. To NLO the $\delta$-expansion, the only free parameters entering the calculation are the $\gamma N\Delta$ couplings $g_M$, $g_E$, $g_C$ characterizing the $M1$, $E2$, and $C2$ transitions. Our results agree well with recent high-precision data from MAMI and MIT-Bates at low $Q^2$. The χEFT framework plays a dual role, in that it allows for an extraction of resonance parameters from observables and predicts their $m_\pi$ dependence. In this way it may provide a crucial connection of present lattice QCD results obtained at unphysical values of $m_\pi$ to the experiment. We have found that the opening of the $\Delta \rightarrow \pi N$ decay channel at $m_\pi = M_\Delta - M$ induces a pronounced non-analytic behavior of the $R_{EM}$ and $R_{SM}$ ratios. While the linearly-extrapolated lattice QCD results for $R_{SM}$ are in disagreement with experimental data, the χEFT prediction of the non-analytic dependencies has allowed us to reconcile these results with experiment. As the next-generation lattice calculations of these quantities are on the way, the χEFT framework presented here will, hopefully, complement these efforts.

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