Non-radial strong curvature naked singularities in five dimensional perfect fluid self-similar space-time

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Abstract

We study five dimensional (5D) spherically symmetric self-similar perfect fluid space-time with adiabatic equation of state, considering all the families of future directed non-spacelike geodesics. The space-time admits globally strong curvature naked singularities in the sense of Tipler and thus violates the cosmic censorship conjecture provided a certain algebraic equation has real positive roots. We further show that it is the weak energy condition (WEC) that is necessary for visibility of singularities for a finite period of time and for singularities to be gravitationally strong. We, also, match the solution to 5D Schwarzschild solution using the junction conditions.

keywords: gravitational collapse, causal structure, naked singularity, strength, junction condition.

1 Introduction

The singularity theorems of Hawking and Ellis [1] have established the fact that the gravitational collapse of sufficiently massive star under fairly general conditions will result in a singularity. However, these theorems do not indicate the nature of singularity. The cosmic censorship conjecture (CCC) articulated by Penrose [2], in its strong version forbids the visibility of singularity to anyone even if one is infinitesimally close to it, whereas in the weak form, it essentially states that gravitational collapse from a regular initial data never creates the space-time singularity visible to distant observers.

The development of superstring and other field theories has led to proliferation of several articles on higher-dimensional (HD) space-time from the viewpoint of both cosmology [3] and gravitational collapse [4]. Therefore, an important question that arises: will the examples of naked singularities in four-dimensional (4D) spherical gravitational collapse goes over to HD space-time or not? If yes, then a related question is whether the dimensionality of space-time has any effect on the formation and nature of the singularity.

In recent development, it has been shown that, in null radiation collapse [5] and in dust collapse [6], the increase in dimension leads to monotonic shrinkage of naked singularity window, i.e., it favours occurrence of black holes.
In a recent work [7], we have studied gravitational collapse in adiabatic self-similar fluid in 5D space-time. It turns out that the WEC is a necessary and sufficient condition for a singularity to be globally naked and gravitationally strong. Ghosh and Deshkar has extended these studies to n dimensional space-time by considering null geodesics only [8]. Our purpose now is to further examine the structure and strength of a naked singularity in the 5D perfect fluid collapse by considering all the families of future directed non-spacelike geodesics.

In Sec. 2, we determine the field equations of 5D self-similar spherically symmetric space-time. These field equations are used to study the occurrence of globally naked strong curvature singularities for all the families of future directed non-spacelike geodesics in Sec. 3. To represent a stellar solution, the junction conditions are also discussed in Sec. 4. The fluid solution is matched to 5D Schwarzschild solution in retarded Eddington-Finkelstein coordinates. Conclusions are given in Sec. 5.

2 Self-similar perfect fluid in five dimensional space-time

5D self-similar spherically symmetric space-time in comoving coordinates is given by

\[
ds^2 = -e^{2\nu} dt^2 + e^{2\psi} dr^2 + r^2 S^2 d\Omega^2.
\]

Self similarity implies that all variables of physical interest may be expressed in terms of the self-similarity parameter \(X = t/r\). Therefore, \(\nu, \psi\) and \(S\) are functions of \(X = t/r\) only and

\[
d\Omega^2 = d\theta^2 + \sin^2 \theta \, d\phi^2 + \sin^2 \theta \, \sin^2 \phi \, d\vartheta^2
\]

is the metric on an 3-sphere. The pressure and energy density can be put in the form:

\[
P = \frac{p(X)}{8\pi r^2}, \quad \rho = \frac{\eta(X)}{8\pi r^2}.
\]

The stress-energy tensor for a perfect fluid is

\[
T^{ab} = (\rho + P)u^a u^b + P g^{ab}
\]

where \(u^a = \delta^a_t\) is the 5-dimensional velocity. The self-similarity in general relativity generalizes the classical notion of similarity and implies the existence of constants of motion along \(dX = 0\), which in turn allows the reduction of the Einstein field equations to a set of ordinary differential equations [9]:

\[
G_t^t = -\frac{1}{S^2} + \frac{e^{-2\psi}}{S} \left[ X^2 \ddot{S} - X^2 \dot{\dot{S}} + X \dot{S} \dot{\psi} + \frac{(S - X \dot{\dot{S}})^2}{S} \right] - \frac{e^{-2\nu}}{S} \left( \dot{S} \dot{\psi} + \frac{\dot{\dot{S}}^2}{S} \right) = -\frac{\eta}{3}
\]

\[
G_r^r = -\frac{1}{S^2} + \frac{e^{-2\psi}}{S} \left[ -S X \dot{\dot{S}} + X^2 \dot{\dot{S}} + \frac{(S - X \dot{\dot{S}})^2}{S} \right] - \frac{e^{-2\nu}}{S} \left( \dot{S} + \frac{\dot{\dot{S}}^2}{S} - \dot{\dot{S}} \dot{\dot{S}} \right) = \frac{p}{3}
\]

\[
G_r^t = \ddot{S} - \ddot{\dot{S}} - \dot{\dot{S}} \dot{\dot{S}} + \frac{\dot{S} \dot{\dot{S}}}{X} = 0
\]

where an overdot denotes the derivative with respect to \(X\). We assume that the collapsing fluid obeys an adiabatic equation of state

\[
p(X) = \lambda \eta(X)
\]
where \( 0 \leq \lambda \leq 1 \) is a constant. The conservation of energy momentum tensor

\[
T_{\alpha\beta} = 0
\]

implies that

\[
\dot{p} + \frac{2p}{X} = -(\eta + p)\dot{\nu}
\]

\[
\dot{\eta} = -(\eta + p)\left[\dot{\psi} + \frac{3\dot{S}}{S}\right]
\]

On integrating Eqs. (10) and (11), respectively, we get

\[
e^{2\nu} = \gamma(\eta X^2)^{-2\lambda/(1+\lambda)}
\]

\[
e^{2\psi} = \alpha(\eta)^{-2/(1+\lambda)}S^{-6}
\]

where \( \alpha \) and \( \gamma \) are integration constants. Eliminating \( \ddot{S} \) from Eqs. (5) and (6), we obtain

\[
\left(\frac{\dot{S}}{S}\right)^2 V + \frac{\dot{S}}{S}\left(\frac{\dot{V}}{2} + 3Xe^{2\nu}\right) + e^{2\psi+2\nu}\left(-\frac{\eta}{3} - e^{-2\psi} + \frac{1}{S^2}\right) = 0
\]

and

\[
\dot{V} = 2Xe^{2\nu}\left[\frac{1}{3}(\eta + p)e^{2\psi} - 1\right] = \frac{2}{3}Xe^{2\nu}(H - 3)
\]

where the quantities \( V \) and \( H \) are defined as follows

\[
V(X) = e^{2\psi} - X^2 e^{2\nu}, \quad H = (\eta + p)e^{2\psi}.
\]

We can also put \( H \) as

\[
H = 8\pi r^2 e^{2\psi} \left(T_1^1 - T_0^0\right)
\]

For all nonspacelike vector \( V^a \), the matter satisfy weak energy condition [1] if and only if

\[
T_{\alpha\beta}V^\alpha V^\beta \geq 0.
\]

Hence, for matter satisfying weak energy condition \( H(X) \geq 0 \) for all \( X \).

It is understood that a curvature singularity forms at the center of the cloud \( (t = 0, r = 0) \), where the physical quantities like matter density and pressure diverge. This leads to the divergence of curvature scalars there. A singularity (a naked singularity or a black hole) can be categorized by the existence of non-spacelike geodesics emanating from the singularity at \( (t = 0, r = 0) \). If such geodesics exist then singularity is at least locally naked, otherwise it is a black hole. Further, if the singularity is naked, then there exists a real and positive value of \( X_0 \) as a solution to the algebraic equation [10]. To analyze this, we further study self-similar field equations.

The constants of integrations \( \alpha \) and \( \gamma \) can be set equal to unity by a suitable scale transformation. Now, we define two new functions \( y = X^\beta \) and \( U^2 = e^{2\psi-2\nu}/X^2 = y^{-3}\eta^{-3\beta}S^{-6} \) where
\(0 \leq \lambda \leq 1, \delta = 1 + \lambda\) and \(\beta = 2(1 - \lambda)/3(1 + \lambda)\). Using these transformations into Eqs. (14) and (15), we obtain
\[
\beta y \frac{\eta'}{\eta} = \frac{1}{U^2 - \lambda} \left[ 2\lambda - 3\delta \beta y U^2 \frac{S'}{S} - \frac{1}{3} \delta^2 y^3 \eta^{3/2} U^2 \right]
\]
and
\[
\left( \frac{S'}{S} \right)^2 \beta^2 y^2 (U^2 - 1) + \left( \frac{S'}{S} \right) \beta y \left[ 2 + \frac{1}{3} \delta y^3 \eta^{3/2} U^2 \right] - \left[ 1 + \left( \frac{3}{\eta S^2} \right) \frac{1}{3} \delta^2 y^3 \eta^{3/2} U^2 \right] = 0
\]
where the prime denotes differentiation with respect to \(y\). The scale invariant quantity \(U\) represents velocity of the fluid relative to the hypersurface \(X = \text{const}\). The case \(U = 1\) is related to occurrence of naked singularity in the space-time. Therefore, we are interested in the values of different parameters for the solution, which take into account the case \(U = 1\) for some \(X > 0\). For this, we write
\[
\eta(X) = \eta_0 + \eta_0 \sum_{k=1}^{\infty} \eta_k (y - y_0)^k
\]
and
\[
S(X) = S_0 + S_0 \sum_{k=1}^{\infty} S_k (y - y_0)^k.
\]
We analyze now the solutions of above differential equations near the point \(y = y_0 = y(X_0)\) with the condition that \(U(y_0) = (\eta_0 X_0)^{-3\beta} S_0^{-6} = 1\). On using (21) and (22), Eqs. (19) and (20) take, respectively, the forms
\[
\eta_1 = \frac{1}{\beta y_0 (1 - \lambda)} \left( 2\lambda - 3\beta \delta y_0 S_1 - \frac{1}{3} \delta^2 y_0^3 \eta_0^{3/2} \right)
\]
and
\[
\beta y_0 S_1 = \frac{3}{6 + \delta y_0^2 \eta_0^{3/2}} \left[ 1 + \frac{1}{3} \left( 1 - \frac{3}{\eta_0 S_0^2} \right) y_0^3 \eta_0^{33/2} \right]
\]
Here \(\eta'(y_0) = \eta_0 \eta_1\) and \(S'(y_0) = S_0 S_1\). Eliminating \(S_1\) and \(S_0\) from the above equations, we obtain:
\[
Y^6 + (mn - 9n)Y^4 + (6\delta - 6\lambda + 9)Y^3 + 6\delta mn Y + 27\delta^2 - 36\lambda \delta = 0
\]
where
\[
Y = \delta^{2/3} \eta_0^{\beta/2} y_0, \quad m = \frac{\beta (1 - \lambda) \eta_1}{\eta_0^{\beta - 1}}, \quad n = \frac{3 \eta_0^{(\beta - 2)/2}}{\delta^{2/3}}
\]
This sixth degree algebraic equation decides the end state of the collapse. The existence of real positive roots of this equation will put a limitation on the physical parameters \(\eta_0\) and \(\eta_1\). It is easy to verify that the above equation can admit at the most four real positive roots. In similar situation in four dimensions, one gets a quartic equation.
3 Causal Structure near the Naked Singularities

In this section, we employ the above solution for an investigation of the formation of a black hole or a naked singularity in a collapsing self-similar adiabatic perfect fluid in a 5D space-time. A self-similar space-time is characterized by the existence of a homothetic Killing vector:

\[
\xi^a = r \frac{\partial}{\partial r} + t \frac{\partial}{\partial t}
\]  

(26)

which is given by the Lie derivative \( \mathcal{L}_{\xi} g_{ab} = \xi_a \partial_b \mathcal{L} + \xi_b \partial_a \mathcal{L} = 2 g_{ab} \) where \( \mathcal{L} \) is a notation for a Lie derivative. Let \( K^i = dx^i/dk \) be the tangent vector to the geodesics, where \( k \) is an affine parameter. For the self similar metric (1) the Lagrange equations immediately give

\[
K^\theta = \frac{\cos \phi l \tan \omega}{r^2 S^2}, \quad K^\phi = \frac{\cos \vartheta l \sin \omega}{r^2 S^2 \sin^2 \theta} \quad \text{and} \quad K^r = \frac{l \cos \omega}{r^2 S^2 \sin^2 \theta \sin^2 \phi}
\]

(27)

where \( \omega \) and \( l \) are the isotropy and impact parameters respectively. Also, we have

\[
g_{ab} K^a K^b = B
\]

(28)

where \( B = 0, B < 0, B > 0 \) correspond to different classes of geodesics, namely, null, timelike and spacelike, respectively.

It follows that, along non-spacelike geodesics, we have

\[
\xi^a K_a = C + B \kappa
\]

(29)

where \( C \) is a constant. From the above algebraic equation and the condition (28), we get

\[
re^{2\psi} K^r - te^{2\nu} K^t = C + B \kappa
\]

(30)

\[
-e^{2\nu} (K^t)^2 + e^{2\psi} (K^r)^2 + \frac{l^2}{r^2 S^2 \cos^2 \omega} = B
\]

(31)

Solving the above equations yield the following exact expressions for \( K^t \) and \( K^r \):

\[
K^t = C \left[ 1 + BCk \right] \frac{X \pm e^{2\psi} Q}{rV(X)}
\]

(32)

\[
K^r = C \left[ 1 + BCk \right] \frac{1 \pm X e^{2\nu} Q}{rV(X)}
\]

(33)

where

\[
Q = \sqrt{e^{-2\psi - 2\nu} \left[ 1 + \frac{[L^2 - Br^2 S^2 \cos^2 \omega] V(X)}{S^2 \cos^2 \omega [1 + BCk]^2} \right]}
\]

(34)

The function \( Q \) is chosen positive throughout and we have put \( B = \frac{B}{C^2} \) and \( L = \frac{l}{C^2} \). To study the nature of the singularity, we employ the technique developed by Dwivedi and Joshi [11] by making necessary changes for 5D case. The non-spacelike geodesics, by virtue of Eqs. (32) and (33), satisfy

\[
\frac{dt}{dr} = \frac{X \pm e^{2\psi} Q}{1 \pm X e^{2\nu} Q}
\]

(35)
The point \( t = 0, r = 0 \) is a singular point of the above differential equation. The limiting value of \( X \) reveals the exact nature of the singularity through the analysis of non-spacelike geodesics that terminate at the singularity:

\[
X_0 = \lim_{t \to 0, r \to 0} X = \lim_{t \to 0, r \to 0} \frac{t}{r} = \lim_{t \to 0, r \to 0} \frac{dt}{dr}. \tag{36}
\]

Using (35) and L'Hôpital's rule we get

\[
V(X_0)Q(X_0) = 0 \tag{37}
\]

and this in turn gives

\[
V(X_0) = 0 \tag{38}
\]

or

\[
Q(X_0) = 0 \Rightarrow L^2V(X_0) = -S^2(X_0) \cos^2 \omega. \tag{39}
\]

If Eq. (38) or Eq. (39) have any real positive roots, then geodesics clearly terminate at the singularity with a definite tangent, so that the central shell-focusing singularity is at least locally naked. The smallest value of \( X_0 \), say \( X_0^s \), corresponds to the earliest ray escaping from the singularity which, is called the Cauchy horizon of the space-time, and there is no solution in the region \( X < X_0^s \) [10]. Hence, in the absence of a positive root to Eq. (37), the central singularity is not naked because there is no outgoing future directed non-spacelike geodesics emanating from the singularity. Thus, existence of the real positive roots of \( V(X_0)Q(X_0) = 0 \) is a necessary and sufficient condition for the singularity to be naked, and at least one single null geodesics in the \((t, r)\) plane would escape from the singularity.

**Global visibility**

A naked singularity can be considered to be physically significant singularity if a family of geodesics escape from the singularity to far-away observers for a finite period of time. We have worked out a detail analysis of visibility of naked singularity in [7] and analyzed whether the singularities are globally naked and an infinity of curves would emanate from singularity to reach a distant observer.

**Strength**

We determine the curvature strength of a naked singularity, which is an important aspect of a singularity [12]. It is widely believed that a space-time does not admit an extension through a singularity if it is a strong curvature singularity in the sense of Tipler [13]. A necessary and sufficient condition (criterion) for a singularity to be strong has been given by Clarke and Królik [14]: that for at least one non-spacelike geodesic with affine parameter \( k \), in the limiting approach to the singularity, we must have

\[
\lim_{k \to 0} k^2 \psi = \lim_{k \to 0} k^2 R_{ab} K^a K^b > 0 \tag{40}
\]

where \( R_{ab} \) is the Ricci tensor. We investigate the above condition along future-directed non-spacelike geodesics that emanate from the naked singularity. Eq. (40) can be expressed as

\[
\lim_{k \to 0} k^2 \psi = \lim_{k \to 0} \left[ k^2 \frac{(\eta + p) C^2 e^{2\nu} [X + e^{2\psi} Q]^2}{r^4 [e^{2\psi} - X^2 e^{2\nu}]^2} + \frac{\eta - 2p}{2r^2} B \right] \tag{41}
\]
Using Eqs. (32), (33), and L’Hôpital’s rule, Eq. (41) for \( V(X_0) = 0 \) turns out to be

\[
\lim_{k \to 0} k^2 \psi = \frac{9H_0}{(H_0 + 3)^2} > 0
\]

while \( Q(X_0) = 0 \) yields

\[
\lim_{k \to 0} k^2 \psi = \frac{H_0 U_0^{-2}}{4} > 0.
\]

The strong curvature condition along geodesics is satisfied if \( H_0 > 0 \), which is also a necessary condition for the energy condition. Thus, it follows that naked singularities are gravitationally strong if the WEC is satisfied.

### 4 Matching with the 5D Schwarzschild Solution

We consider a spherical surface whose motion is described by a timelike four-space \( \Sigma \), which divides space-times into interior and exterior manifolds. We shall first cut the space-time along timelike hypersurface, and then join the internal part with the 5D Schwarzschild solution in retarded Eddington-Finkelstein coordinates. The metric on the whole space-time can be written in the form

\[
ds^2 = \begin{cases} 
-e^{2\nu} dt^2 + e^{2\psi} dr^2 + r^2 S^2 d\Omega^2, & r \leq r_\Sigma, \\
-\left(1 - \frac{m}{r}\right)du^2 - 2dudr + r^2 d\Omega^2, & r \geq r_\Sigma.
\end{cases}
\]

The metric on the hypersurface \( r = r_\Sigma \) is given by

\[
ds^2 = -d\tau^2 + R^2(\tau) d\Omega^2.
\]

We suitably modify the approach of junction conditions given in [16, 17] for our 5D case. Hence, we demand

\[
(ds^2_-)_\Sigma = (ds^2_+) = (ds^2)_\Sigma.
\]

The second junction condition is obtained by requiring the continuity of the extrinsic curvature of \( \Sigma \) across the boundary. This gives

\[
K_{ij}^- = K_{ij}^+
\]

where \( K_{ij}^\pm \) is the extrinsic curvature to \( \Sigma \), given by

\[
K_{ij}^\pm = -n_{\alpha'}^\pm \frac{\partial x_{\alpha'}^j}{\partial \xi^i} - n_{\alpha}^\pm \Gamma_{\beta'\gamma'} \frac{\partial x_{\beta'}^i}{\partial \xi^j} \frac{\partial x_{\gamma'}^j}{\partial \xi^i}
\]

and where \( \Gamma_{\beta'\gamma'} \) are Christoffel symbols, \( n_{\alpha}^\pm \) the unit normal vectors to \( \Sigma \), \( x_{\alpha'}^j \) are the coordinates of the interior and exterior space-time and \( \xi^i \) are the coordinates that define \( \Sigma \). The junction condition (46) yeilds

\[
\frac{dt}{d\tau} = \frac{1}{e^{\nu(r_\Sigma, t)}}
\]

\[
r_\Sigma S(r_\Sigma, t) = r(\tau)
\]
\[
\left( \frac{du}{d\tau} \right)^2 \Sigma = \left( 1 - \frac{m}{r^2} + 2 \frac{dr}{du} \right)
\]

The non-vanishing components of intrinsic curvature \( K_{ij} \) of \( \Sigma \) are determined as follows:

\[
K_{\tau\tau} = (-e^{-\psi} \nu_r) \Sigma
\]

\[
K_{\theta\theta} = \left[ e^{-\psi} r S (S + r S_r) \right] \Sigma
\]

\[
K_{r\tau} = \left[ \frac{d^2u}{d\tau^2} \left( \frac{du}{d\tau} \right)^{-1} - \left( \frac{du}{d\tau} \right) \frac{m}{r^3} \right] \Sigma
\]

\[
K_{\theta\theta} = \left[ \frac{dr}{d\tau} + \left( \frac{du}{d\tau} \right) \left( 1 - \frac{m}{r^2} \right) r \right] \Sigma
\]

\[
K_{\phi\phi} = \sin^2 \theta K_{\tau\tau}^\pm
\]

\[
K_{\varphi\varphi} = \sin^2 \phi K_{\tau\tau}^\pm
\]

where the subscripts \( r \) and \( t \) denote partial derivative with respect to \( r \) and \( t \) respectively. The unit normals to \( \Sigma \) are given by

\[
n^-_{\alpha'} = (0, e^{\psi(r, t)}, 0, 0, 0)
\]

\[
n^+_{\alpha'} = \left( 1 - \frac{m}{r^2} + 2 \frac{dr}{du} \right)^{-1/2} \left( -\frac{dr}{du} 1, 0, 0, 0 \right)
\]

From Eqs. (47), (53) and (55) we have

\[
\left[ \left( \frac{du}{d\tau} \right) \left( 1 - \frac{m}{r^2} \right) r + r \frac{dr}{d\tau} \right] = \left[ e^{-\psi} r S (S + r S_r) \right] \Sigma
\]

Using Eqs. (49), (50) and (51), we can write Eq. (60) as

\[
m = r^2 S^2 \left[ 1 + \frac{r S_t^2}{e^{2\nu}} - \frac{(S + r S_r)^2}{e^{2\psi}} \right]
\]

which is the total energy entrapped inside the surface \( \Sigma \). From Eqs. (52) and (54), using (49), we obtain

\[
\left[ \frac{d^2u}{d\tau^2} \left( \frac{du}{d\tau} \right)^{-1} - \left( \frac{du}{d\tau} \right) \frac{m}{r^3} \right] \Sigma = -\left( e^{-\psi} \nu_r \right) \Sigma
\]

Substituting Eqs. (49), (50) and (61) into (60), we get

\[
\left( \frac{du}{d\tau} \right) \Sigma = \left[ \frac{S + r S_r}{e^\nu} + \frac{r S_t}{e^{\nu'}} \right]^{-1}
\]

Differentiating (63) with respect to \( \tau \) and using Eqs. (61), we can rewrite (62) as

\[
-\left( \frac{\nu_r}{e^\psi} \right) \Sigma = \left[ -\frac{r}{e^\nu} S_{tr} + \psi_t (S + r S_r) + \frac{r \nu_t S_t}{e^\nu} - \frac{r S_t}{e^{\nu'}} - \frac{r S_r^2}{e^\psi} \frac{S_t}{e^\nu} - \frac{S_{tr}}{e^\psi} \left( \frac{S + r S_r}{e^{2\nu}} - 1 \right) \right] \times \left[ \frac{S + r S_r}{e^\nu} + \frac{r S_t}{e^{\nu'}} \right]^{-1} \left( \frac{1}{e^n} \right) \Sigma.
\]
Now, we translate the above equation in terms of $X = t/r$, which gives

$$\begin{align*}
-\frac{1}{S^2} + \frac{e^{-2\nu}}{S} \left[ -SX\dot{\nu} + X^2\dot{\nu}\dot{S} + \frac{(S - X\dot{S})^2}{S} \right] - \frac{e^{-2\nu}}{S} \times \left( \frac{\ddot{S}}{S} + \frac{\dot{S}^2}{S} - \dot{\nu}\dot{S} \right) \\
eq \frac{e^{-\psi - \nu}(-X)}{S} \left[ \ddot{S} - \dot{\nu}\dot{S} - \dot{\psi} + \frac{S\dot{\psi}}{X} \right]
\end{align*}$$

(65)

Comparing (65) with (6) and (7), we can finally write

$$(P)_\Sigma = 0.$$  \hspace{1cm} (66)

Thus, the pressure vanishes at the boundary of the spherical surface, and hence the interior 5D self-similar perfect fluid is matched with the 5D Schwarzschild solution.

5 conclusions

It has been found that the extra dimension does not affect the occurrence of a naked singularity but rather leads to occurrence of a strong curvature naked singularity. It is the weak energy condition that is necessary for visibility of the singularity for a finite period of time. The 5D spherically symmetric self-similar space-time admits a globally strong curvature naked singularity provided the equation $V(X)Q(X) = 0$ has real positive roots, and singularities are found to be gravitationally strong in the Tipler’s sense.

For the sake of completeness, we have matched the solution to the 5D Schwarzschild solution in retarded Eddington-Finkelstein coordinates, so that the resulting solution will represent the collapse of a star.

This study generalizes the results of spherical gravitational collapse in 4D to 5D space-time for all non-spacelike geodesics. The formation of these naked singularities violates the cosmic censorship conjecture. Finally, the results obtained here would also be relevant in the context of superstring theory and for an interpretation of how critical behaviour depends on the dimensionality of space-time.

References

[1] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of spacetime* (Cambridge University Press, Cambridge, 1973).

[2] R Penrose, Riv Nuova Cimento , 1, 252 (1969); in *General Relativity - An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979) pp 581-638.

[3] L. Randall and R. Sundram, *Phys. Rev. Lett.* 83, 3370 (1999); 83, 4690 (1999).

[4] J. Soda and K. Hirata, *Phys. Lett. B* 387, 271 (1996); A. Ilha and J. P. S. Lemos, *Phys. Rev. D* 55, 1788 (1997); A. Ilha, A. Kleber and J. P. S. Lemos, *J. Math. Phys.* 40, 3509 (1999); A. V. Frolov, *Class. Quantum Grav.* 16, 407 (1999); J. F. V. Rocha and A. Wang, *Class. Quantum Grav.* 17, 2589 (2000).

[5] S. G. Ghosh and N. Dadhich, *Phys. Rev. D* 64, 047501 (2001).
[6] S. G. Ghosh and A. Beesham, *Phys. Rev. D* **64**, 124005 (2001).

[7] S. G. Ghosh, S. B. Sarwe and R. V. Saraykar, *Phys. Rev. D* **66**, 084006 (2002).

[8] S. G. Ghosh and D. W. Deshkar *Int. J. Mod. Phys.* Vol. **12**, number 5, 2003.

[9] A. Ori and T. Piran, *Phys. Rev. Lett.* **59**, 2137 (1987).

[10] P. S. Joshi, *Global Aspects in Gravitation and Cosmology* (Clarendon press, Oxford, 1993); C. J. S. Clarke, *Class. Quantum Grav.* **10**, 1375 (1993); R M Wald, gr-qc/9710068; S. Jhingan and G. Magli, gr-qc/9903103; T P Singh, *J. Astrophys. Astron.* **20**, 221 (1999); P. S. Joshi, *Pramana* **55**, 529 (2000).

[11] P. S. Joshi and I. H. Dwivedi, *Commun. Math. Phys.* **146**, 333 (1992); *Lett. Math Phys.* **27**, 235 (1993).

[12] F. J. Tipler, *Phys. Lett. A* **64**, 8 (1987).

[13] F. J. Tipler, C. J. S. Clarke, and G. F. R. Ellis, in *General Relativity and Gravitation*, edited by A. Held, (Plenum, New York, 1980).

[14] C. J. S. Clarke and A. Królak, *J. Geom. Phys.* **2**, 127 (1986).

[15] M. E. Cahill and A. H. Taub, *Commun. Math. Phys.* **21**, 1 (1971).

[16] N. O. Santos, *Mon. Not. R. Astr. Soc.* **216**, 403 (1985).

[17] J. F. V. Rocha, A. Wang and N. O. Santos, *Phys. Lett. A* **255**, 213 (1999).