Improved Measurement of the Form Factors in the Decay
Λ⁺ → Λe⁺νₑ

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Abstract

Using the CLEO detector at the Cornell Electron Storage Ring, we have studied the distribution of kinematic variables in the decay \( \Lambda_c^+ \to \Lambda e^+ \nu_e \). By performing a four-dimensional maximum likelihood fit, we determine the form factor ratio, \( R = f_2/f_1 = -0.31 \pm 0.05 \text{(stat)} \pm 0.04 \text{(syst)} \), the pole mass, \( M_{\text{pole}} = (2.21 \pm 0.08 \text{(stat)} \pm 0.14 \text{(syst)}) \text{ GeV}/c^2 \), and the decay asymmetry parameter of the \( \Lambda_c^+ \), \( \alpha_{\Lambda_c} = -0.86 \pm 0.03 \text{(stat)} \pm 0.02 \text{(syst)} \), for \( \langle q^2 \rangle = 0.67 \text{ (GeV}/c^2)^2 \). We compare the angular distributions of the \( \Lambda_c^+ \) and \( \bar{\Lambda}_c \) and find no evidence for CP-violation: \( A_{\Lambda_c} = \frac{(\alpha_{\Lambda_c} + \alpha_{\bar{\Lambda}_c})}{(\alpha_{\Lambda_c} - \alpha_{\bar{\Lambda}_c})} = 0.00 \pm 0.03 \text{(stat)} \pm 0.01 \text{(syst)} \pm 0.02 \), where the third error is from the uncertainty in the world average of the CP-violating parameter, \( A_{\Lambda} \), for \( \Lambda \to p\pi^- \).
The charm quark is unstable and decays via a first order weak interaction. In semileptonic decays, which are analogs of neutron $\beta$ decay, the charm quark disintegrates predominantly into a strange quark, a positron and a neutrino. The rate depends on the weak quark mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cs}|$ and strong interaction effects, parameterized by form factors, which come into play because the charm quark is bound with light quarks to form a meson or baryon. Charm semileptonic decays allow a measurement of the form factors because $|V_{cs}|$ is tightly constrained by the unitarity of the CKM matrix $|V_{cb}|$.

Within the framework of Heavy Quark Effective Theory (HQET), semileptonic ($J^P = 1/2^+ \rightarrow 1/2^+$) transitions of $\Lambda$-type baryons are simpler than mesons as they consist of a heavy quark and a spin and isospin zero light diquark. This simplicity leads to more reliable predictions for form factors in heavy-to-light transitions. The measurement of form factors in the $\Lambda_c^+ \rightarrow \Lambda e^+\nu_e$ transition provides a test of HQET predictions in the charm baryon sector, a test of Lattice QCD, and information for the determination of the CKM matrix parameter $|V_{cb}|$.

In terms of the helicity amplitudes, the decay $\Lambda_{c^+} \rightarrow \Lambda^+ \rightarrow \Lambda^+ e^+\nu_e$ can be described in terms of four form factors: two axial form factors $F^A_1$ and $F^A_2$, and two vector form factors $F^V_1$ and $F^V_2$. These form factors are functions of $q^2$, the invariant mass squared of the virtual $W^+$. The decay can be described in terms of the helicity amplitudes $H_{\lambda\lambda'\lambda''\lambda'''}$, where $\lambda, \lambda'$ and $\lambda''$ are the helicities of the $\Lambda$ and $W^+$. The helicity amplitudes are related to the form factors in the following way:

\[
\begin{align*}
\sqrt{q^2}H^V_{\frac{1}{2}0} &= \sqrt{Q_-}[(M_{\Lambda c} + M_\Lambda)F^V_1 - q^2 F^V_2], \\
H^V_{\frac{1}{2}1} &= \sqrt{2Q_-}[ -F^V_1 + (M_{\Lambda c} + M_\Lambda)F^V_2], \\
\sqrt{q^2}H^A_{\frac{1}{2}0} &= \sqrt{Q_+}[(M_{\Lambda c} - M_\Lambda)F^A_1 + q^2 F^A_2], \\
H^A_{\frac{1}{2}1} &= \sqrt{2Q_+}[ -F^A_1 - (M_{\Lambda c} - M_\Lambda)F^A_2],
\end{align*}
\]

where $Q_{\pm} = (M_{\Lambda c} \pm M_\Lambda)^2 - q^2$. The remaining helicity amplitudes can be obtained using the parity relations $H^{V(A)}_{\lambda'\lambda''\lambda'''} = \pm H^{V(A)}_{-\lambda\lambda'\lambda''\lambda'''}$. In terms of the helicity amplitudes, the decay angular distribution can be written as:

\[
\Gamma_S = \frac{d\Gamma}{dq^2 d\cos \theta_A d\cos \theta_W d\chi} = B(\Lambda \rightarrow p\pi^-) \frac{1}{2} \frac{G_F^2}{(2\pi)^4} |V_{cs}|^2 \frac{q^2 P}{24M^2_{\Lambda c}} \times
\]

\[
\left\{ \frac{3}{8} (1 - \cos \theta_W)^2 |H^V_{\frac{1}{2}1}|^2 (1 + \alpha_\Lambda \cos \theta_\Lambda) + \frac{3}{8} (1 + \cos \theta_W)^2 |H^{-\frac{1}{2}0}|^2 (1 - \alpha_\Lambda \cos \theta_\Lambda) \right\} + \frac{3}{4} \sin^2 \theta_W \left[ |H^A_{\frac{1}{2}0}|^2 (1 + \alpha_\Lambda \cos \theta_\Lambda) + |H^{-\frac{1}{2}1}|^2 (1 - \alpha_\Lambda \cos \theta_\Lambda) \right]
\]

\[- \frac{3}{2\sqrt{2}} \alpha_\Lambda \cos \chi \sin \theta_W \sin \theta_\Lambda \left( (1 - \cos \theta_W) Re(H^{-\frac{1}{2}0}H^V_{\frac{1}{2}1}) + (1 + \cos \theta_W) Re(H^{V}_{\frac{1}{2}0}H^{-\frac{1}{2}1}) \right),
\]

where $G_F$ is the Fermi coupling constant, $|V_{cs}|$ is a CKM matrix element, $P$ is the magnitude of the $\Lambda$ momentum in the $\Lambda_c^+$ rest frame, $\theta_\Lambda$ is the angle between the momentum vector of the proton in the $\Lambda$ rest frame and the $\Lambda$ momentum in the $\Lambda_c^+$ rest frame, $\theta_W$ is the angle between the momentum vector of the positron in the $W^+$ rest frame and the $\Lambda$ momentum.
in the $\Lambda_c^+$ rest frame, $\chi$ is the angle between the decay planes of the $\Lambda$ and $W^+$, and $\alpha_\Lambda$ is the $\Lambda \rightarrow p\pi^-$ decay asymmetry parameter measured to be $0.642 \pm 0.013$ \cite{1}.

In HQET, the heavy flavor and spin symmetries imply relations among the form factors and reduce their number to one when the decay involves only heavy quarks. For heavy-to-light transitions, two form factors are needed to describe the hadronic current. In this Letter, we follow Ref. \cite{4}, in which the $c$ quark is treated as heavy and the $s$ quark as light. Two independent form factors $f_1$ and $f_2$ are related to the standard form factors in the following way $F_1^V(q^2) = -F_2^V(q^2) = f_1(q^2) + \frac{M_{\Lambda_c}}{m_c} f_2(q^2)$ and $F_2^V(q^2) = -F_1^V(q^2) = \frac{M_{\Lambda_c}}{m_c} f_2(q^2)$. In general, $f_2$ is expected to be negative and smaller in magnitude than $f_1$. If the $s$ quark is treated as heavy, $f_2$ is zero.

In order to extract the form factor ratio $R = f_2/f_1$ from a fit to the decay rate, $\Gamma_S$, an assumption must be made about the $q^2$ dependence of the form factors. The model of Körner and Krämer (KK) \cite{4} uses the dipole form $f(q^2) = \frac{f(q^2_{pole})}{(1-q^2/M_{pole}^2)(1-q^2/M_{pole}^2)}$ for both form factors, where the pole mass is taken from the naive pole dominance model: $M_{pole} = m_{D_s^*} = 2.11 \text{ GeV}/c^2$.

In this Letter, we perform, for the first time, a simultaneous fit for the form factor ratio and pole mass in the decay $\Lambda_c^+ \rightarrow \Lambda e^+\nu_e$, and we make a first search for $CP$-violation in this decay. The data sample used in this study was collected with the CLEO II \cite{6} and upgraded CLEO II V \cite{7} detectors operating at the Cornell Electron Storage Ring (CESR). The integrated luminosity consists of $13.7 \text{ fb}^{-1}$ taken at and just below the $\Upsilon(4S)$ resonance, corresponding to approximately $18 \times 10^6 e^+e^- \rightarrow c\bar{c}$ events. Throughout this paper charge conjugate states are implicitly included, unless otherwise indicated, and the symbol $e$ is used to denote an electron or positron.

The analysis is an extension of the technique described in \cite{8,9}. The decay $\Lambda_c^+ \rightarrow \Lambda e^+\nu_e$ is reconstructed by detecting a $\Lambda e^+$ pair with invariant mass in the range $m_{\Lambda e^+} < m_{\Lambda_c}$. The positron is required to come from the region of the event vertex. To reduce the background from $B$ decays, we require $R_2 = H_2/H_0 > 0.2$, where $H_i$ are Fox-Wolfram event shape variables \cite{10}. Positrons are identified using a likelihood function, which incorporates information from the calorimeter and $dE/dx$ systems. The minimum allowed momentum for positron candidates is 0.7 GeV/c, as the positron fake rates are much higher at lower momentum. Positrons must be detected in the region: $|\cos \theta| < 0.7$, where $\theta$ is the angle between the positron momentum and the beam line. The $\Lambda$ is reconstructed in the decay mode $\Lambda \rightarrow p\pi^-$. The $\Lambda$ baryon is long-lived $(c\tau = 7.89 \text{ cm})$, accordingly, the $\Lambda$ vertex is required to be greater than 5 mm from the primary vertex in the $r-\phi$ plane, but the $\Lambda$ momentum must extrapolate to the primary vertex. The $dE/dx$ measurement of the proton is required to be consistent with the expected value. Combinations that satisfy interpretation as a $K^0_S$ are rejected. The magnitude of the $\Lambda$ momentum is required to be greater than 0.8 GeV/c in order to reduce combinatorial background. These $\Lambda$ candidates are then combined with Right Sign (RS) tracks consistent with positrons, and the sum of the $\Lambda$ and $e^+$ momenta is required to be greater than 1.4 GeV/c in order to reduce the background from $B$ decays.

The above selection criteria permit the isolation of signal events with low background. The number of events passing the selection is 4060, of which 123 $\pm$ 12 are consistent with fake $\Lambda$ background, 338 $\pm$ 67 with $\Xi_c \rightarrow \Xi e^+\nu$ feed through and 398 $\pm$ 58 with $e$ fake background. The sidebands of the $p\pi^-$ invariant mass distribution are used to estimate the fake $\Lambda$ background. The background from $\Xi_c \rightarrow \Xi e^+\nu$ decays is estimated using the result of a previous CLEO analysis \cite{11}. 


The normalization and momentum spectrum of the $e$ fake background is estimated using the Wrong Sign (WS) $h^+\bar{\Lambda}$ data sample (no charge conjugation is implied), where $h^+$ and $\bar{\Lambda}$ satisfy all analysis selection criteria. The $h^+$ tracks in this sample are mostly fakes as there are few processes contributing $e^+\bar{\Lambda}$ pairs after the selection criteria are applied. If no particle identification is used for $h^+$, the probability to find a $(h^+\bar{\Lambda}$ or $h^-\Lambda$) WS or $(h^+\Lambda$ or $h^-\bar{\Lambda}$) RS pair is approximately equal because the net charge of the event is zero. The equality is not perfect due to (a) baryon conservation: a $\Lambda$ is more likely to be produced with an antiproton in WS rather than RS combinations, and (b) associated strangeness production: there is a higher fraction of kaons in RS rather than WS combinations. When the particle identification requirements for $h^+$ are applied, the importance of these correlations is magnified by the high $e$ fake rates of antiprotons and kaons. Therefore, antiprotons are excluded by using only one WS charge conjugate state $(h^+\bar{\Lambda})$, and the momentum region where the $e$ fake rate from kaons is high is excluded by requiring $|\vec{p}_e| > 0.7$ GeV/c. Differences that remain between the momentum spectra and particle species of hadronic tracks $h^+\bar{\Lambda}$ and $h^+\Lambda$ are second order and are accounted for by a systematic uncertainty.

Calculating kinematic variables requires knowledge of the $\Lambda^+_c$ momentum, which is unknown due to the undetected neutrino. The direction of the $\Lambda^+_c$ is approximated using the thrust axis of the event and the kinematic constraints of the decay. The magnitude of the $\Lambda^+_c$ momentum is obtained as a weighted average of the roots of the quadratic equation $\vec{p}_{\Lambda^+_c}^2 = (\vec{p}_{\Lambda} + \vec{p}_e + \vec{p}_\nu)^2$. The weights are assigned based on the measured fragmentation function of $\Lambda^+_c$. After the $\Lambda^+_c$ momentum is estimated, the four kinematic variables are easily obtained. The kinematic variables $t \equiv q^2 / q^2_{\text{max}}$, $\cos \theta_{\Lambda}$, $\cos \theta_W$ and $\chi$ achieve resolutions of 0.2, 0.3, 0.2 and 45°, respectively.

A four-dimensional maximum likelihood fit in a manner similar to Ref. [12] is performed in the space of $t$, $\cos \theta_{\Lambda}$, $\cos \theta_W$ and $\chi$. The technique makes possible a multidimensional fit to variables modified by experimental acceptance and resolution taking into account correlations among the variables. We have performed two types of fit. The first fit is unbinned in all four dimensions. The second fit is unbinned in $\cos \theta_{\Lambda}$, $\cos \theta_W$ and $\chi$, and binned in $t$. While both fits produce consistent results, the second fit is used for the main results.

The signal probability density function for the likelihood function is estimated at each data point using signal Monte Carlo (MC) events, generated according to the HQET consistent KK model with a GEANT based simulation [13], by sampling the MC distribution at the reconstructed level in a search volume around the data point. The volume size is chosen so that the systematic effect from finite search volumes is small and the required number of MC events is not prohibitively high. The background probability density functions are modeled similarly using samples of events for each background component. The $e$ fake background is modeled using a sample of events collected for that purpose from the data. Feed through background from $\Xi_c \rightarrow \Xi e^+\nu$ is modeled by the MC sample generated according to the HQET consistent KK model. Fake $\Lambda$ background is modeled using the data events in the sidebands of the $p\pi^-$ invariant mass distribution. For the binned part of the fit, the above distributions are projected onto $t$ and binned. The background normalizations are fixed in the fits to the measured values.

Using the above method, a simultaneous fit for the form factor ratio and the pole mass is made. We find $R = -0.31 \pm 0.05$ (stat) and $M_{\text{pole}} = (2.21 \pm 0.08$ (stat)) GeV/$c^2$. This is the main result of the analysis. Figures 1 and 2 show the $t$, $\cos \theta_{\Lambda}$, $\cos \theta_W$ and $\chi$ projections for the data and fit.
FIG. 1: Projections of the data (points with statistical error bars) and the fit (solid histogram) onto $t$, $\cos \theta_\Lambda$, $\cos \theta_W$ and $\chi$. The dashed lines show the sum of the background distributions.

FIG. 2: Projections of the data (points with statistical error bars) and the fit (solid histogram) onto $\cos \theta_\Lambda$, $\cos \theta_W$ and $\chi$ for two $t$ regions. The plots labeled (a), (b) and (c) are for $t < 0.5$; (d), (e) and (f) are for $t > 0.5$. The dashed lines show the sum of the background distributions.

We have considered the following sources of systematic uncertainty and give our estimate of their magnitude in parentheses for $R$ and $M_{\text{pole}}$, respectively. The uncertainty associated with the size of the search volume is measured from a statistical experiment in which a set of mock data samples, including signal and all background components, was fit in the same way as the data (0.006, 0.048). The uncertainty due to the limited size of the signal MC sample is estimated by dividing the sample into four independent equal subsamples and repeating the fit (0.007, 0.012). The uncertainty due to background normalizations is determined by varying the estimated number of background events by one standard deviation separately.
The uncertainty due to the small background contribution of random Λ events estimated by varying these shapes or by using alternative background samples (0.024, 0.049). The uncertainty due to the small background contribution of random Λ pairs from the continuum (e⁺e⁻ → q̅q) and Υ(4S) → BB events, which are not modeled in the fit, is obtained from a generic MC sample and is estimated by repeating the fit with and without this background (0.013, 0.038). The modes Λ⁺ → ΛXe⁺ν, where X represents additional decay products, have never been observed. The current upper limit is $\frac{B(\Lambda⁺ → \Lambda Xe⁺ν)}{B(\Lambda⁺ → \Lambda e⁺ν)} < 0.15$, where $X \neq 0$, at 90% confidence level. The uncertainty due to the presence of these modes is estimated from a series of fits, each having an additional background component with floating normalization to represent a Λ⁺ → ΛXe⁺ν mode. The uncertainty due to the possible presence of Λ⁺ → ΛXe⁺ν is assigned as the largest deviation from the main result found in these fits (0.022, 0.091). The uncertainty associated with the Λ⁺ fragmentation function is estimated by varying this function (0.003, 0.002). The uncertainty related to MC modeling of the reconstruction efficiency of slow pions produced in Λ decays is obtained by varying this efficiency according to our understanding of the CLEO detector (0.004, 0.003). For approximately 10% of the data, all from runs near the end of the CLEO II.V data taking period, a discrepancy between data and MC was found for $t > 0.8$. The effect of the discrepancy was determined by binning the data in $t$ and performing a series of fits with a variable range of $t$ excluded. The size of the systematic uncertainty is conservatively taken to be the largest difference between the results of these fits and the main result (0.004, 0.072). The simulation used to obtain the main result does not include final state radiation. The systematic uncertainty due to this is determined as the difference between the result of a fit where electroweak radiative corrections have been modeled [14] and the main result (0.005, 0.009).

Adding all sources of systematic uncertainty in quadrature, the final result is $R = -0.31 \pm 0.05$ (stat) $\pm 0.04$ (syst) and $\wpole = (2.21 \pm 0.08$ (stat) $\pm 0.14$ (syst)) GeV/$c^2$, the latter value being consistent with vector dominance. We also find $R = -0.35 \pm 0.04$ (stat) $\pm 0.04$ (syst) from a fit with $\wpole = \wpole$ using the values of $R$ and $\wpole$ obtained in the simultaneous fit and the KK model, the mean value of the decay asymmetry parameter of $\Lambda⁺ → \Lambda e⁺ν_e$ [15] averaged over charge conjugate states is calculated to be $\alpha_{\Lambda_e} = -0.86 \pm 0.03$ (stat) $\pm 0.02$ (syst), for $\langle q^2 \rangle = 0.67$ (GeV/$c^2$)$^2$.

In the Standard Model, CP-violation is expected to be small in semileptonic decays and absent in the decay $\Lambda⁺ → \Lambda e⁺ν_e$. If CP is conserved, the following relation is satisfied

$$\frac{d\Gamma(\Lambda⁺ → \Lambda e⁺ν_e)}{dq^2 d\cos \theta_\Lambda d\cos \theta_W d\chi} = \frac{d\Gamma(\bar{\Lambda}⁺ → \bar{\Lambda} e^−\nu_e)}{dq^2 d\cos \theta_\Lambda d\cos \theta_W d(−\chi)}.$$ 

Following [16] and by extension, a CP-violating asymmetry of the $\Lambda⁺$ is defined as $A_{\Lambda_e} = \frac{\alpha_{\Lambda_e} + \alpha_{\Lambda}}{\alpha_{\Lambda_e} - \alpha_{\Lambda}}$. In the KK model, the angular distributions for $\Lambda⁺$ and $\bar{\Lambda}⁺$ are governed by $R$ and $\wpole$. Using the values of $R$ and $\wpole$ obtained in a simultaneous fit to each charge conjugate state separately and the KK model, we calculate $\alpha_{\Lambda_e} \alpha_{\Lambda} = -0.561 \pm 0.026$ (stat) and $\alpha_{\Lambda_e} \alpha_{\Lambda} = -0.544 \pm 0.024$ (stat). From the following expression for the CP asymmetry:

$$\frac{\alpha_{\Lambda_e} \alpha_{\Lambda} - \alpha_{\Lambda}}{\alpha_{\Lambda_e} \alpha_{\Lambda} + \alpha_{\Lambda}} = A_{\Lambda_e} + A_{\Lambda},$$

which is valid to first order in $A_{\Lambda}$ and $A_{\Lambda_e}$, we obtain $A_{\Lambda_e} = 0.00 \pm 0.03$ (stat) $\pm 0.01$ (syst) $\pm 0.02$, where in the systematic uncertainty the correlations
among the systematic uncertainties for the charge conjugate states are taken into account and the third error is from the uncertainty in $A$.

In conclusion, using a four-dimensional maximum likelihood fit, the angular distributions of $\Lambda^+ \to \Lambda\pi^+\nu_e$ have been studied. The form factor ratio $R = f_2/f_1$ and $M_{\text{pole}}$ are found to be $-0.31 \pm 0.05(\text{stat}) \pm 0.04(\text{syst})$ and $(2.21 \pm 0.08(\text{stat}) \pm 0.14(\text{syst}))$ GeV/$c^2$, respectively. This is the most precise measurement of $R$, and it demonstrates that $f_2$ is non zero with a combined statistical and systematic significance exceeding $4\sigma$. This is also the first measurement of $M_{\text{pole}}$ in a charm baryon semileptonic decay. Our measurement is consistent with vector dominance. These results correspond to $\alpha_{\Lambda_c} = -0.86 \pm 0.03(\text{stat}) \pm 0.02(\text{syst})$, for $\langle q^2 \rangle = 0.67$ (GeV/$c^2)^2$. Comparing the angular distributions for $\Lambda_c^+$ and $\bar{\Lambda}_c$, no evidence for $CP$-violation is found: $A_{\Lambda_c} = 0.00 \pm 0.03(\text{stat}) \pm 0.01(\text{syst}) \pm 0.02$, where the third error is from the uncertainty in the world average of the $CP$-violating parameter, $A_{\Lambda}$, for $\Lambda \to p\pi^-$. We gratefully acknowledge the effort of the CESR staff in providing us with excellent luminosity and running conditions. This work was supported by the National Science Foundation, the U.S. Department of Energy, the Research Corporation, and the Texas Advanced Research Program.

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