Analysis of autonomous coordination between McKibben pneumatic actuators in the antagonist musculoskeletal model

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Abstract: Cooperative relationships exist among various physical phenomena. Several cooperative phenomena have been analyzed theoretically by considering them as synchronous phenomena. We focus on the cooperative motion obtained in the antagonistic structures by considering the McKibben pneumatic actuator (MPA) as one of the synchronous phenomena. This study aims to realize the various movements of a robot with MPAs based on the autonomous coordination between MPAs. In a previous study, we proposed a novel tension feedback control inspired by animal motion and confirmed that this control law generates some cooperative relationship between the MPAs. In this paper, we mathematically analyze the realized cooperative relationships as a synchronous phenomenon, and clarify the mechanism by which the delay in the change in the MPA length with the change in the MPA pressure generates the transition of phase difference. This result suggests that multiple targeted motions can be generated by a single control law.

Key Words: antagonistic structure, McKibben pneumatic actuator, autonomous motion, synchronization phenomenon, coordination between actuators

1. Introduction

Cooperative relationships exist among various physical phenomena, such as metronomes and plastic bottle oscillators [1, 2]. Cooperative phenomena related to animals, such as the luminescence pattern of fireflies, activation of neurons, and cries of frogs, have been also observed [3–5]. In addition, several nonlinear physical phenomena have been analyzed theoretically by considering them as synchronous phenomena with the aim of revealing the coordination mechanism [6, 7]. In this study, we focus on
the motion of the robot driven by the McKibben pneumatic actuator (MPA). It has been verified that multiple MPAs in an antagonistic structure can generate interesting cooperative motions [8, 9]. We analyze one of the cooperative motions as a synchronous phenomenon between MPAs.

As depicted in Fig. 1, the MPA, which was developed as a type of artificial muscle, contracts along the long axis by applying air pressure and generating tension. The MPA consists of a silicone rubber tube covered with a nylon mesh. It exhibits a high back-drivability owing to its soft materials and high output per mass. Therefore, it is utilized in rehabilitation equipment and musculoskeletal robots that imitate animals [10, 11]. In a previous study, robots with MPAs realized several motions, such as jumping with a simple input [12]. However, the MPA model is complicated because it includes nonlinear elements in its driving principle. Hence, it is not easy to control a robot with MPAs based on this model. The input design of an MPA to drive a robot is often performed through trial-and-error parameter tuning. To address this limitation, robots with MPAs were developed to realize motion through input design using reinforcement learning [13]. However, in this method, each motion requires a separate input design.

To this end, this study aims to realize the autonomous motions of the robots with MPAs by controlling the coordination between the MPAs. This control method allows various movements of the robots with MPAs by combining the autonomous motions generated using a single control law. To realize such autonomous movement, we focus on the movement of animals. As both MPA and muscles generate tension in the direction of compression alone, it is necessary to construct an antagonistic structure to actively drive joints. Animals realize movements by controlling and coordinating their muscles [14]. Experiments with animals suggest that the brain does not always control movement in animals; instead, animals use autonomous motion generation. For example, the periodic movements of flexors and extensors for walking emerge at the spinal cord due to a neural circuit called the central pattern generator (CPG) [15]. In addition, the elongation reflex approximately adjusts the muscle length using information from sensory organs, such as muscle spindles [16]. Recently, several studies have realized the movements of robots inspired by animal models. Certain CPG models expressed as neural oscillator systems enables walking without a target trajectory [17]. Moreover, the implementation of reflexes realizes a robust gait [18, 19]. Owaki et al. proposed the “TEGOTAE” control method, which uses force information obtained from the surroundings [20]. Although this control method does not include the direct coordination between the legs, it can reproduce the gait transitions according to the walking speed. In summary, the knowledge obtained from animals can be used for the autonomous movement of robots.

To realize the coordination between MPAs, we focus on the fact that animals use sensory information, such as muscle length and tension, for the autonomous movements. In our previous study, we proposed an oscillator-based control law that adjusted the motion of the MPA using its tension and applied it to a monoarticular leg model with antagonistic MPAs [8, 9]. Through several simulations and experiments, we confirmed that tension feedback control generated some autonomous motions, and the cooperative relationships between MPAs were nonlinearly bifurcated. However, this coordination generation mechanism has not yet been clarified. Thus, the application of autonomous cooperation between MPAs to control the motions of a robot requires a theoretical analysis. In this study, we analyzed the cooperative relationships between antagonistic MPAs by considering it as the
synchronization between two phase oscillators. As a first step of the mathematical analysis, we ana-
lyzed the two characteristic modes that were identified. In the first mode, the tension was balanced,
and the link was immobile. The stability of this model was analyzed based on the control theory.
In the second mode, the pressures of the MPAs were periodic, generating a roughly anti-phase wave.
By applying the analytical methods to synchronization phenomena, we obtained a simple model and
clarified the transition mechanism between MPAs. These analyses of the motion mechanisms are
critical for realizing various motions of a multi-degree of freedom (multi-DOF) robot with MPAs by
utilizing modes obtained at a single joint.

2. Leg model and control law

2.1 Model of monoarticular leg driven by antagonist muscles and tension model of MPA

For the theoretical analysis, our study focused on a monoarticular leg model with a simple antagonistic
structure. Figure 2a illustrates the model to be analyzed. This model can be considered as a 1-DOF
symmetric pendulum driven by antagonized MPAs. As depicted in Fig. 2a, the component supporting
the pendulum corresponds to the upper body, the center of rotation of the pendulum corresponds to
the hip joint, and the pendulum corresponds to the leg.

Figure 2b depicts the parameters of the rod pendulum model and the details of each parameter
are listed in Table I. The rod pendulum is assumed to have a constant density and the vertically
downward direction is assumed to be 0 [rad].

The angular equation of motion around the joint can be derived as follows:

\[ I \ddot{\theta} = f_{m1}L_3 \cos \alpha - f_{m2}L_3 \cos \beta - \frac{1}{2} mgL_0 \sin \theta \]  (1)

| Symbols | Definitions                          | values       |
|---------|--------------------------------------|--------------|
| \(L_0\) | Pendulum length                      | 550 [mm]     |
| \(\theta_0\) | Initial pendulum angle               | 0 [rad]      |
| \(m\)   | Pendulum mass                        | 1 [kg]       |
| \(g\)   | Gravitational acceleration           | 9.80665 [m/s²]|
| \(L_1, L_2\) | Offset between the rotation center | 30 [mm]      |
| \(L_3\) | Length to MPA attachment position in | 500 [mm]     |
|         | rod pendulum                        |              |
| \(l_{m1,0}, l_{m2,0}\) | Natural length of MPA | 300 [mm]    |
| \(D_{1,0}, D_{2,0}\) | Natural diameter of MPA | 15 [mm]      |
| \(L_{w1}, L_{w2}\) | Wire length of MPA                  | 200.9 [mm]   |
| \(\phi_{1,0}, \phi_{2,0}\) | Initial phase of oscillator          | π/2, 0 [rad] |

Fig. 2. Model of 1-DOF pendulum driven by antagonistic actuator.

Table I. Parameters of 1-DOF pendulum.
where $I$ is the moment of inertia, $\alpha$ and $\beta$ are the angles formed by the pendulum and the tension direction of MPA1 and MPA2, respectively. $\alpha$ and $\beta$ are calculated as follows:

$$\cos \alpha = \frac{L_1 \cos \theta}{l_{m1} + L_{w1}}, \quad \cos \beta = \frac{L_2 \cos \theta}{l_{m2} + L_{w2}}.$$  

The length of each MPA $l_{mi}$ ($i = 1, 2$) is determined by the pendulum angle $\theta$ according to the geometric relationship as follows:

$$l_{m1} = \sqrt{L_1^2 + L_3^2 - 2L_1L_3\sin \theta - L_{w1}},$$

$$l_{m2} = \sqrt{L_2^2 + L_3^2 + 2L_2L_3\sin \theta - L_{w2}}.$$  

The MPA contraction velocities $v_i = \dot{l}_{mi}$ ($i = 1, 2$) are obtained from the pendulum angular velocity $\dot{\theta}$ as follows:

$$v_1 = \frac{L_1L_3 \cos \theta}{\sqrt{L_1^2 + L_3^2 - 2L_1L_3\sin \theta}} \dot{\theta},$$

$$v_2 = -\frac{L_2L_3 \cos \theta}{\sqrt{L_2^2 + L_3^2 + 2L_2L_3\sin \theta}} \dot{\theta}.$$  

Several studies have been conducted on tension models for MPA [21, 22]. In this study, to facilitate the theoretical analysis of the movement, we adopted the linear approximation model, which was proposed in our previous study [23]. It has been confirmed that this model can appropriately express the tension of an MPA despite its simplistic form, in comparison with other models. The linear approximation model is expressed as follows:

$$f_m(P', l_m, v) = S_1P' + S_2P'l_m + S_3l_m + S_4 - \gamma v$$  

where $S_{1,2,3,4}$ and $\gamma$ are constants. The MPA pressure $P'$ is the input value, and the MPA length $l_m$ and contraction velocity $v$ are determined by the pendulum angle $\theta$ and pendulum angular velocity $\dot{\theta}$, respectively. The tension of the MPA is calculated by substituting the MPA pressure $P'$, length $l_m$ and contraction velocity $v$ in Eq. (5).

### 2.2 Tension feedback control method

To generate autonomous movements of robot joints using MPA, the “TEGOTAE” control law is considered [20], which is defined as follows:

$$\dot{\phi}_i = \omega - \sigma N_i \cos \phi_i$$  

where $\phi_i$ is the phase oscillator assigned to each leg, $N_i$ is the ground reaction force, $\omega$ is the intrinsic angular velocity, and $\sigma$ is the feedback gain. This control law is a phase oscillator-based control method that uses sensory information of the ground reaction force $N_i$ of each leg to modify the velocity of the oscillator, to reproduce the cooperative movement between the legs. An important feature of the “TEGOTAE” control law is its ability to produce autonomous coordination between the legs without implementing direct inter-leg coordination. Autonomous coordination is indirectly generated by the coupling of the phase oscillator through the ground reaction force $N_i$.

In this study, two phase oscillators corresponding to the two antagonistic MPAs are prepared. Based on the state of each oscillator, the pressure input to each MPA is determined. To realize autonomous coordination between the MPAs and various pendulum motions, the oscillators are controlled by a tension feedback control. The tension feedback control law applicable to the MPAs is defined as follows:

$$\dot{\phi}_i = \omega - \sigma f_{mi} \cos \theta$$  

where $\phi_i$ is the phase oscillator assigned to each MPA, $f_{mi}$ ($i = 1, 2$) denotes the tension of the MPAs, which corresponds to the sensory information obtained in the model, $\theta$ is the pendulum angle.
Fig. 3. Effect of tension feedback to the phase oscillator: the black arrow indicates that the phase oscillator oscillates at the intrinsic angular velocity without tension feedback; the red arrow indicates that the angular velocity of the oscillator increases from the intrinsic angular velocity with a negative feedback gain; the blue arrow indicates that the angular velocity decreases with a positive feedback gain.

representing the state of the pendulum, \( \omega \) is the intrinsic angular velocity and \( \sigma \) is the feedback gain. The second term expresses the tension feedback. This term modifies the periodic motion with the intrinsic angular velocity when the MPA exerts tension. Furthermore, the angular velocity of the phase oscillator is modified depending on the feedback gain sign, as shown in Fig. 3. When the pendulum angle is large, the effect of feedback is reduced because the tension feedback is directly proportional to \( \cos \theta \). The tension feedback term results in a difference in the angular velocity of the two phase oscillators. This difference causes a change in the relationship between the actuators. Consequently, autonomous coordination between antagonistic MPAs is generated.

Input pressure \( P'_i \) \((i = 1, 2)\) to MPA1 and MPA2 is determined by substituting \( \phi \) from Eq. (7) into the following equation:

\[
P'_i = A(1 - \sin \phi_i)
\]  

where \( A \) is the pressure amplitude. One rotation of the phase oscillator on the circle is equivalent to one contraction and extension of the MPA. When the pressure input in Eq. (8) is applied, the tensions of the MPA are determined using Eq. (5). The pendulum is driven by the tensions of the antagonized MPAs, according to Eq. (1).

2.3 Oscillation modes generated by tension feedback control

From our previous studies, it can be confirmed through simulations and experimental results that, the tension feedback control generates coordination between antagonistic MPAs [8, 9]. In addition, some oscillation modes of the pendulum and the phase oscillator are confirmed. To explain the different movements, we obtained simulations by varying the intrinsic angular velocity \( \omega \) [rad/s] and the feedback gain \( \sigma \) parameters in the tension feedback control law. The parameter range of the simulation was \(-2.00\pi \) [rad/s] \( \leq \omega \leq 2.00\pi \) [rad/s], \(-0.200 \leq \sigma \leq 0.200\). Table II shows the value of \( S_{1,2,3,4,} \gamma \) and \( A \) used in the simulation. From the simulation results, the phase difference and the amplitude were obtained. Here, the phase difference is defined as follows:

\[
\phi = \phi_1 - \phi_2.
\]

| Symbols | Definitions | values              |
|---------|-------------|---------------------|
| \( S_1 \) | MPA pressure term | -0.0036            |
| \( S_2 \) | Cross term       | 0.0172              |
| \( S_3 \) | MPA length term  | 6.5083 \times 10^4 |
| \( S_4 \) | Constant term    | -1.9525 \times 10^4 |
| \( \gamma \) | Velocity term    | 35                  |
| \( A \)   | MPA pressure amplitude | 0.5 [MPa]          |
To obtain a steady-state result, the average phase difference $\bar{\phi}$ and amplitude between 70 [s]–80 [s] are considered. The average amplitude is defined as the average peak value of the simulation result between 70 [s]–80 [s]. Figure 4 depicts the simulation results for the initial phase difference $\pi/2$ [rad]. In Fig. 4, the feedback gain $\sigma$ is plotted on the x-axis, and the intrinsic angular velocity $\omega$ is plotted on the y-axis. The average phase difference $\bar{\phi}$ or amplitude is represented on the $\sigma$–$\omega$ plane by the color shown in the color bar.

Figure 4a depicts the average phase difference of the pendulum under each parameter. $\bar{\phi} > 0$ indicates that $\phi_1$ is ahead of $\phi_2$. In contrast, $\bar{\phi} < 0$ indicates that $\phi_2$ is ahead of $\phi_1$. In the region II, the average phase difference $\bar{\phi}$ does not change from the initial value $\pi/2$ [rad]. In regions other than region II, the average phase difference $\bar{\phi}$ transitions from $\pi/2$ [rad] to other values depending on the choice of parameters. The region $\text{III}_a$ and $\text{III}_b$ in Fig. 4a are the regions wherein the average phase difference $\bar{\phi}$ transitions to the neighborhood of $\pi$. A chaotic region exists between the region $\text{III}_a$ and region $\text{III}_b$. In the chaotic region, slight differences in the parameters of the control law cause significant changes in phase difference and pendulum motion. This fact suggests that the oscillation modes in $\text{III}_a$ and $\text{III}_b$ are different. In addition, region I is the region wherein the average phase difference $\bar{\phi}$ is either 0 [rad] or $2\pi$ [rad]. This result indicates that the phase oscillators move in phase. When $\phi$ is 0 [rad], the phase oscillators move such that the phase difference decreases and converges to zero. In contrast, when $\phi$ is $2\pi$, the phase oscillators move such that the phase difference increases and converges to $2\pi$.

Figure 4b depicts the average amplitude of the pendulum. Here, when the pendulum angle diverges, the average amplitude is replaced with a negative value; for example, the region between $\text{iii}_a$ and $\text{iii}_b$ is indicated in dark blue. Focusing on the region of $\omega > 0$ [rad/s], it can be seen that the average amplitude is 0 [rad] in the region i. The amplitude discontinuously transitions to the region ii, wherein the amplitude is approximately $\pi/8$ [rad]. Furthermore, as the feedback gain $\sigma$ decreases, the amplitude increases to $\pi/6$ [rad] in the region $\text{iii}_a$. Beyond the region wherein the pendulum motion diverges, a steady-state is generated again in the region $\text{iii}_b$. In the region $\text{iii}_b$, the average amplitude decreases as the feedback gain $\sigma$ further decreases.

Comparing Fig. 4a with Fig. 4b, it can be seen that the regions in Fig. 4a and Fig. 4b correspond well. In particular, the region i, wherein the amplitude is 0 [rad], matches region I wherein the average phase difference $\bar{\phi}$ is approximately 0 [rad] or $2\pi$ [rad]. The regions II and ii also match well. Moreover, the pendulum amplitude tends to be large in the region where the average phase difference $\bar{\phi}$ is large. However, both Fig. 4a and Fig. 4b are asymmetrical to the origin. This can be attributed to the influence of the initial phase difference.

Hence, based on the simulation results, it was confirmed that the proposed tension feedback generated a cooperative relationship. The average phase difference $\bar{\phi}$ or amplitude varies significantly based on the parameters. Moreover, as depicted in Fig. 4, there exist regions with clear boundaries.
This result suggests that a certain bifurcation phenomenon occurs in the system with antagonistic MPAs.

3. Steady-state stability of pendulum motion

3.1 Resting mode of pendulum

The first point to be analyzed is the oscillation modes that correspond to region I in Fig. 4a and region i in Fig. 4b. Figure 5 depicts the simulation results for 0 [s]–100 [s] with \( \omega = 0.20\pi \) [rad/s], \( \sigma = 0.050 \) and an initial difference \( \phi_0 = \pi/2 \) [rad]. As shown in Fig. 5a, the pendulum stops moving when in region 1. This indicates that the lengths of the MPAs are equal, the MPA contraction velocities are 0 [m/s], and the tension is balanced. In addition, Fig. 5b shows that the input pressures of the MPAs became non-zero constants. This is because the phase oscillators almost stop moving. Based on these facts, we hypothesize that the phase oscillator stops moving at a stable equilibrium point when the average phase difference \( \bar{\phi} \) converges to 0 [rad].

![Fig. 5. Simulation result of 1-DOF pendulum with \( \omega = 0.20\pi \) [rad/s] and \( \sigma = 0.050 \) (t = 0 [s]–100 [s]).](image)

3.2 State equation of pendulum motion

To verify this hypothesis, we analyzed the stability around the equilibrium point. The equilibrium point satisfies the following equation:

\[
\omega = \sigma f_{mi} \cos \theta. \tag{10}
\]

When this equation is satisfied, \( \dot{\phi}_i = 0 \), the phase oscillator stops moving. The MPA tension \( f_{mi} \) is calculated by substituting Eqs. (3), (4) and (8) into Eq. (5). Solving Eq. (10) for \( \phi_i \) by substituting Eq. (5) into Eq. (10), the equilibrium point \( \phi_e \) can be calculated as

\[
\phi_e = \sin^{-1} \left[ \frac{S_1A + S_2Al_m + S_3l_m + S_4 - k}{S_1A + S_2Al_m} \right] \tag{11}
\]

where \( k = \omega/\sigma \) and the contraction velocity of MPA \( v_i \) is 0 [m/s]. In addition, it is assumed that the MPA length \( l_m \) is the initial length \( l_{m,0} \) and the pendulum angle is 0 [rad]. It is noted that there are two equilibrium points with \( -1 < \sin \phi_e < 1 \). Here, we consider the state equation of the system with variables, such as the pendulum angle \( \theta \), pendulum angular velocity \( \dot{\theta} \), and phase oscillator \( \phi_i \). To express as a state equation, the angular equation of motion and the tension feedback law are linearized around the equilibrium point. Here, \( \phi_1^*, \phi_2^* \) are defined as follows:

\[
\phi_i^* = \phi_i - \phi_e. \tag{12}
\]

Detailed calculations are described in Appendix A. The state equation is

\[
\frac{dx}{dt} = Ax + u \tag{13}
\]

where \( x \) is \((\theta, \dot{\theta}, \phi_1^*, \phi_2^*)^T \) and \( u \) is \((0, u_2, u_3, u_4)^T \). Then, matrix \( A \) is defined as
A = \begin{pmatrix}
0 & 1 & 0 & 0 \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & 0 \\
a_{41} & a_{42} & 0 & a_{44}
\end{pmatrix}.

(14)

3.3 Stability analysis of equilibrium point

In Sec. 3.2, the angular equation of motion and the tension feedback control law are expressed by the state equation linearized around the equilibrium point. As a result, the stability of the equilibrium point is determined by the eigenvalues of matrix A.

For example, consider the eigenvalues of a point in region I in Fig. 4a. Two equilibrium points exist with \( \omega = 0.2\pi \) and \( \sigma = 0.05 \). One is \( \phi_{e1} = 0.32\pi \), and the other is \( \phi_{e2} = 0.68\pi \). Calculating the eigenvalues by substituting \( \phi_{e1} \) into Eq. (13), we obtain

\[
\lambda_1 = 0.5064 \pm 13.0890i, \ 0.3574, \ 2.1235.
\]

(15)

\( \phi_{e1} \) is an unstable equilibrium point because the real parts of all the eigenvalues are positive. In contrast, the eigenvalues can be calculated by substituting \( \phi_{e2} \) into Eq. (13),

\[
\lambda_2 = -1.2591 \pm 13.0173i, \ -0.3586, \ -2.1235.
\]

(16)

Equation (16) indicates that \( \phi_{e2} \) is a stable equilibrium point because the real parts of all the eigenvalues are negative. Therefore, phase oscillators stop moving at the equilibrium point \( \phi_{e2} \). Figure 6 depicts that the oscillators become stationary between \(-3\pi/2\) [rad] and \(-\pi\) [rad]. Here, \(-3\pi/2\) [rad]—\(-\pi\) [rad] \( \equiv \pi/2 \) [rad]—\( \pi \) [rad] (mod 2\( \pi \)). This results indicates that the phase oscillators converge to a stable equilibrium point \( \phi_{e2} \) in the simulation. Figure 7 depicts the largest of all the real parts of the eigenvalues in the parameter range. In the case of two equilibrium points, the smaller of the two largest eigenvalues is shown. The largest real part of the eigenvalues is negative in the regions I and i. These results indicate that the phase oscillator stops moving at a stable equilibrium point.

Fig. 6. Simulation result of phase oscillator \( \phi_i \) with \( \omega = 0.20\pi \) [rad/s] and \( \sigma = 0.050 \) (\( t = 0 \) [s]—100 [s]).

Fig. 7. Existence condition of the equilibrium point and the largest real part of the eigenvalues at the equilibrium point. (The red region meets the conditions while the yellow region does not. The black line indicates the boundary between region I and II in Fig. 4a.)
In addition, we focus on the boundary between region I and region II in Fig. 4a. To converge at the equilibrium point, the system at the equilibrium point must be stable; moreover, the equilibrium point must exist. In Sec. 3.2, we consider the case with the contraction velocity $v_i = 0 \, [\text{m/s}]$. However, the contraction velocity term of the tension model affects the existence of the equilibrium point. Solving Eq. (10) for the contraction velocity by substituting Eq. (5) with $v_i \neq 0$, we obtain

$$v_i = \frac{(S_1 A + S_2 A l_m)(1 - \sin \phi_i) + S_3 l_m + S_4 - k}{\gamma}. \quad (17)$$

Because the existence condition of the equilibrium point is $-1 \leq \sin \phi_i \leq 1$, the range of the contraction velocity $v_i$ is defined as follows:

$$\frac{S_3 l_m + S_4 - k}{\gamma} \leq v_i \leq \frac{2(S_1 A + S_2 A l_m) + S_3 l_m + S_4 - k}{\gamma}. \quad (18)$$

Equation (18) indicates that the range of the contraction velocity for the existence of the equilibrium points is determined by $k = \omega/\sigma$. For stability at the equilibrium point, $v_i = 0 \, [\text{m/s}]$ must be included in the range in Eq. (18). In other words, if $v_i = 0 \, [\text{m/s}]$ is not included in the conditions of Eq. (18), the phase oscillators will always be moving because the point that meets both conditions $\dot{\phi}_i = 0$ and $v_i(\dot{\theta}) = 0$ does not exist. Figure 7 also shows the region wherein the condition regarding the equilibrium point is satisfied. The boundary at which the condition regarding the equilibrium point is satisfied or not satisfied matches the boundary between region I and region II in Fig. 4a. This result suggests that the boundary between region I and region II is defined by the existence condition of the equilibrium point. However, when the feedback gain is large, the case in which the phase oscillators do not converge to the equilibrium point needs further consideration, for example, by introducing the time concept.

4. Transition analysis of phase difference by synchronization theory

4.1 Phase difference transition and periodic motion

The second aspect to be analyzed is the oscillation modes that correspond to the region IIIa in Fig. 4a and the region IIIb in Fig. 4b. Figures 8 and 9 depict the simulation results with $\omega = 0.20\pi \, [\text{rad/s}]$, $\sigma = -0.075$ and initial difference $\phi_0 = \pi/2 \, [\text{rad}]$. As shown in Fig. 8, the phase difference $\phi$ transitions from the initial difference $\phi_0 = \pi/2 \, [\text{rad}]$ to $\pi \, [\text{rad}]$ in this region. Therefore, Fig. 9b indicates that the pressures of the MPAs become periodic, generating a roughly anti-phase wave. In addition, the pendulum motion also becomes periodic with a large amplitude, as depicted in Fig. 9a. In this study, we regard this phase difference transition as a synchronization phenomenon. Therefore, the tension feedback control law can be treated as a phase equation. Using a mathematical analysis method for the synchronization phenomenon, the tension feedback control law is expressed as a coupling model of two phase oscillators.

4.2 Combined model of the tension feedback law

To treat the tension feedback control law as a phase equation, the variable in the tension feedback control law should be only $\dot{\phi}_i$. First, the term that depends on the contraction velocity $\gamma v_i$ in Eq. (5)
Simulation result of 1-DOF pendulum with $\omega = 0.20\pi$ [rad/s] and $\sigma = -0.075$ ($t = 77$ [s]–$80$ [s]).

is ignored. As the length of the MPA is a function of the pendulum angle $\theta$, the variables in Eq. (7) denote the phase oscillator $\phi_i$ and pendulum angle $\theta$. Therefore, we approximate the pendulum angle $\theta$ as a function of the phase oscillator $\phi_i$. In this paper, the approximate equation for the pendulum angle $\theta$ is given as follows:

$$\theta = \theta_{\text{max}} - \sin(\phi_1 - \mu) + \sin(\phi_2 - \delta)$$  \hspace{1cm} (19)

where $\theta_{\text{max}}$ is the maximum amplitude of the pendulum, and $\mu$ and $\delta$ are constant. $\mu$, $\delta$ represents the difference between the change in MPA pressure and the change in MPA length. The details of this approximation are described in Appendix B. By using the above approximations, the tension feedback equation can be considered as the phase equation with only the phase oscillator $\phi_i$ as a variable.

Then, we apply the averaging approximation to the tension feedback control law. The averaging approximation is an analysis method of the synchronization theory that integrates the phase equation for one cycle and takes the average [24, 25]. Equation (7) is approximated as

$$\langle \dot{\phi}_1 \rangle = [\omega - \sigma(S_1A + S_2AB + S_3B + S_4)] + \frac{1}{2}\sigma S_2AC \cos \mu$$

$$- \sigma S_2AC \left( \frac{1}{2} \cos \delta \cos \phi - \frac{1}{2} \sin \delta \sin \phi \right),$$

$$\langle \dot{\phi}_2 \rangle = [\omega - \sigma(S_1A + S_2AB + S_3B + S_4)] + \frac{1}{2}\sigma S_2AC \cos \delta$$

$$- \sigma S_2AC \left( \frac{1}{2} \cos \mu \cos \phi + \frac{1}{2} \sin \mu \sin \phi \right),$$

where $\langle \rangle$ implies that an averaging approximation is applied. Detailed calculations are described in Appendix C. By considering the difference between Eqs. (20), and (21), Eq. (7) is transformed into a combined model with phase difference $\phi$,

$$\langle \dot{\phi} \rangle = \langle \dot{\phi}_1 \rangle - \langle \dot{\phi}_2 \rangle = \frac{1}{2}\sigma S_2AC (\cos \mu - \cos \delta)$$

$$+ \frac{1}{2}\sigma S_2AC \cos \phi (\cos \mu - \cos \delta) + \frac{1}{2}\sigma S_2AC \sin \phi (\sin \mu + \sin \delta).$$  \hspace{1cm} (22)

When $\mu$ is equal to $\delta$, the combined model is expressed in a simple form as follows:

$$\langle \phi \rangle = \frac{1}{2}\sigma S_2AC \sin \phi (\sin \mu + \sin \delta).$$  \hspace{1cm} (23)

In synchronization theory research, the following theory was derived [26]. In one of the synchronization theory models, the combined model was expressed as follows:

$$\langle \dot{\phi} \rangle = \omega + \Gamma(\phi),$$  \hspace{1cm} (24)

the phase difference $\phi$ converges to a value where $\langle \phi \rangle = 0$. The term $\Gamma(\phi)$ is called the combined function. For the derived combined model in Eq. (23), this theory can be applied. Because the
right-hand side of Eq. (23) is a sine function, the phase difference converges to 0 [rad] or \( \pi \) [rad]. Furthermore, the convergence destination is determined by the coefficient of the sine function.

4.3 Verification of phase difference transition

To verify the validity of the coupling model of Eq. (23), we simulated the same parameters using the approximate angles of Eq. (19). Figure 10 depicts the simulation results with \( \omega = 0.20\pi \) [rad/s], \( \sigma = -0.075 \) and initial difference \( \phi_0 = \pi/2 \) [rad]. As shown in Fig. 10a, when \( \mu \) and \( \delta \) are equal to 0, the phase difference does not change from the initial phase difference \( \phi_0 \). However, when \( \mu \) and \( \delta \) are not equal to 0, the phase difference transitions. The phase difference converges to 0 [rad] with \( \mu = \delta < 0 \). In contrast, the phase difference converges to \( \pi \) [rad] with \( \mu = \delta > 0 \). In addition, comparing Fig. 8 to Fig. 10c, the waveforms of the transitions are matched well. These results indicate the validity of the analysis using the synchronization theory. According to Eqs. (8), (C-1) and (C-2), when \( \mu \) and \( \delta \) is positive, there exists a delay in the change in the MPA length with the change in the MPA pressure. Therefore, the difference between the MPA pressure and length during the transition period in Fig. B-2a is essential for the transition of the phase difference \( \phi \). From a practical viewpoint, the MPA length does not instantaneously change with the MPA pressure because of the dynamic properties of the MPA. Thus, it is noteworthy that the delay in the change in MPA length with the change in MPA pressure generates the transition of the phase difference \( \phi \).

5. Conclusion

This study aimed to realize the autonomous motions of robots with MPAs by controlling the coordination between MPAs. We confirmed that the tension feedback control generated the coordination between the MPAs and various movements of a 1-DOF pendulum with antagonistically arranged MPAs through numerical simulations. Herein, we theoretically analyzed these cooperative relationships between the MPAs by considering them as the synchronization phenomena of phase oscillators. Hence, we clarified the mechanisms of certain oscillation modes.

First, we analyzed the mode wherein the pendulum was immobile. As the MPA pressure was constant, the phase oscillator, which determined the MPA pressure, was assumed to converge to the equilibrium point. We discussed the stability of the equilibrium point by describing the motion as a state equation. As a result, one of the equilibrium points was stable, and the stability was verified by simulation. Moreover, we revealed a range of parameters depending on the existing conditions of the equilibrium points.

Next, we analyzed the mode wherein the phase difference transitioned to \( \pi \) [rad], and the pressures of the MPAs generated a roughly anti-phase wave. We applied the analytical method for synchronization phenomena by treating the tension feedback control law as a phase equation. The relationship between the MPAs was expressed as a simple model owing to the averaging of the approximations. We clarified the mechanism by which the delay in the change in the MPA length with the change in the MPA pressure generates the phase difference transition.

The results of these analyses suggest the possibility that the parameter selection of the tension feedback control law allows some targeted motions based on theories, and the various motions of
multi-DOF robots with MPAs are realized by controlling the coordination between the MPAs and the motions of each joint. Theoretical analysis of other oscillation modes, such as similar pendulum motions with a different relationship between the MPAs and the expansion of the tension feedback control method to multi-DOF models are topics for future research.

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Appendix
A. Linear approximation model
In this section, we linearize the angular equation of motion and the tension feedback law around the equilibrium point to express the state equation. Equations to describe the motion of the pendulum angle $\theta$, pendulum angular velocity $\dot{\theta}$, and phase oscillator $\phi_i$ are as follows:

$$\frac{d\theta}{dt} = \dot{\theta}, \tag{A-1}$$

$$I \frac{d\dot{\theta}}{dt} = f_{m1} L_3 \cos \alpha - f_{m2} L_3 \cos \beta - \frac{1}{2} mg L_0 \sin \theta, \tag{A-2}$$

$$\frac{d\phi_1}{dt} = \omega - \sigma f_{m1} \cos \theta, \tag{A-3}$$

$$\frac{d\phi_2}{dt} = \omega - \sigma f_{m2} \cos \theta. \tag{A-4}$$

Linearizing around the equilibrium point, the approximation of the MPA length is described. To simplify the discussion, $L_3$ is considered to be equal to the pendulum length $L_0$. In addition, $L_1$ is equal to $L_2$ and $L_{w1}$ is equal to $L_{w2}$ because the monoarticular model is symmetric. Thus, the approximate length of the MPA is

$$l_{m1} \approx \sqrt{L_1^2 + L_0^2} \left(1 - \frac{L_1 L_0 \sin \theta}{L_1^2 + L_0^2}\right) - L_{w1}, \tag{A-5}$$

$$l_{m2} \approx \sqrt{L_2^2 + L_0^2} \left(1 + \frac{L_1 L_0 \sin \theta}{L_1^2 + L_0^2}\right) - L_{w1}. \tag{A-6}$$

Then,

$$\frac{1}{l_{m1} + L_{w1}} \approx \frac{1}{\sqrt{L_1^2 + L_0^2}} \left(1 + \frac{L_1 L_0 \sin \theta}{L_1^2 + L_0^2}\right), \tag{A-7}$$

$$\frac{1}{l_{m2} + L_{w1}} \approx \frac{1}{\sqrt{L_2^2 + L_0^2}} \left(1 - \frac{L_2 L_0 \sin \theta}{L_2^2 + L_0^2}\right). \tag{A-8}$$

Substituting Eqs. (8), (A-5)–(A-8) into Eq. (5),

$$f_{m1,2} = S_1 A (1 - \sin \phi_i) + S_2 A (1 - \sin \phi_i) \left(\sqrt{L_1^2 + L_0^2} - L_{w1} \pm \frac{L_1 L_0 \sin \theta}{\sqrt{L_1^2 + L_0^2}}\right)$$

$$+ S_3 \left(\sqrt{L_1^2 + L_0^2} - L_{w1} \mp \frac{L_1 L_0 \sin \theta}{\sqrt{L_1^2 + L_0^2}}\right) + S_4$$

$$\mp \frac{L_1 L_0 \cos \theta}{\sqrt{L_1^2 + L_0^2}} \left(1 - \frac{L_1 L_0 \sin \theta}{L_1^2 + L_0^2}\right) \dot{\theta}. \tag{A-9}$$

Equation (1) can be transformed as

$$\frac{d}{dt} \dot{\theta} = \left(-\frac{3g}{2L_0} + \frac{6L_1^2 (S_1 A - S_2 A L_{w1} - S_3 L_{w1} + S_4)}{m \sqrt{L_1^2 + L_0^2}}\right) \sin \theta - \frac{6g L_1}{m (L_1^2 + L_0^2)} \dot{\theta}.$$
Substituting Eqs. (A-13) and (A-14) into Eqs. (A-10), (A-11), and (A-12),
\[
\dot{\theta} + \frac{3L_1}{mL_0} \left( -\frac{S_1A + S_2AL_{w1}}{\sqrt{L_1^2 + L_0^2}} - S_2A \right) \sin \phi_1 + \frac{3L_1}{mL_0} \left( \frac{S_1A - S_2AL_{w1}}{\sqrt{L_1^2 + L_0^2}} + S_2A \right) \sin \phi_2 \\
+ \frac{3AL_1^2(-S_1 + S_2AL_{w1})}{m\sqrt{L_1^2 + L_0^2}} \sin(\theta + \sin \phi_1 + \sin \phi_2) - \frac{6\gamma L_1^2}{m(L_1^2 + L_0^2)} \left( \frac{L_1L_0}{L_1^2 + L_0^2} \right)^2 \sin^2 \theta \dot{\theta}. \quad (A-10)
\]
Furthermore, Eq. (7) can be redefined as
\[
\frac{d}{dt} \sin \phi_1 = \frac{\sigma(S_2A - S_1)L_1L_0}{\sqrt{L_1^2 + L_0^2}} \sin \theta + \frac{\sigma \gamma L_1L_0}{\sqrt{L_1^2 + L_0^2}} \dot{\theta} \\
+ \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \sin \phi_1 + \omega - \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \\
+ S_3 \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) + S_4 + \frac{S_2AL_1L_0}{\sqrt{L_1^2 + L_0^2}} \sin \theta \sin \phi_1 - \frac{\gamma(L_1L_0)^2}{\sqrt{L_1^2 + L_0^2}} \sin \theta \dot{\theta}, \quad (A-11)
\]
\[
\frac{d}{dt} \sin \phi_2 = -\frac{\sigma(S_2A - S_3)L_1L_0}{\sqrt{L_1^2 + L_0^2}} \sin \theta - \frac{\sigma \gamma L_1L_0}{\sqrt{L_1^2 + L_0^2}} \dot{\theta} \\
+ \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \sin \phi_2 + \omega - \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \\
+ S_3 \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) + S_4 - \frac{S_2AL_1L_0}{\sqrt{L_1^2 + L_0^2}} \sin \theta \sin \phi_2 - \frac{\gamma(L_1L_0)^2}{\sqrt{L_1^2 + L_0^2}} \sin \theta \dot{\theta}. \quad (A-12)
\]
Substituting Eq. (12) into sin \( \phi_1 \), we obtain
\[
\sin(\phi_1^* + \phi_e) = \cos \phi_e \sin \phi_1^* + \sin \phi_e \cos \phi_1^*. \quad (A-13)
\]
With these changes in variables, the following approximation can be used:
\[
\sin \theta \approx \theta, \quad \cos \theta \approx 1, \\
\sin \phi_1^* \approx \phi_1^*, \quad \cos \phi_1^* \approx 1. \quad (A-14)
\]
Substituting Eqs. (A-13) and (A-14) into Eqs. (A-10), (A-11), and (A-12),
\[
\frac{d}{dt} \theta = \left( -\frac{3g}{2L_0} + \frac{6L_1^2(S_1A(1 - \sin \phi_e) - S_2AL_{w1}(1 - \sin \phi_e) - S_4L_{w1} + S_4)}{m\sqrt{L_1^2 + L_0^2}} \right) \theta - \frac{6\gamma L_1}{m(L_1^2 + L_0^2)} \dot{\theta} \\
+ \frac{3L_1 \cos \phi_e}{mL_0} \left( -\frac{S_1A + S_2AL_{w1}}{\sqrt{L_1^2 + L_0^2}} - S_2A \right) \phi_1^* + \frac{3L_1 \cos \phi_e}{mL_0} \left( \frac{S_1A - S_2AL_{w1}}{\sqrt{L_1^2 + L_0^2}} + S_2A \right) \phi_2^* \\
+ \frac{3 \cos \phi_e AL_1^2(-S_1 + S_2L_{w1})}{m\sqrt{L_1^2 + L_0^2}} \theta(\phi_1^* + \phi_2^*) - \frac{6\gamma L_1^2}{m(L_1^2 + L_0^2)} \left( \frac{L_1L_0}{L_1^2 + L_0^2} \right)^2 \theta^2 \dot{\theta}, \quad (A-15)
\]
\[
\frac{d}{dt} \phi_1^* = \frac{\sigma(S_2A(1 - \sin \phi_e) - S_3)L_1L_0}{\sqrt{L_1^2 + L_0^2}} \theta + \frac{\sigma \gamma L_1L_0}{\sqrt{L_1^2 + L_0^2}} \dot{\theta} \\
+ \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \phi_1^* + \omega - \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \\
+ S_3 \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) + S_4 + \frac{S_2AL_1L_0}{\sqrt{L_1^2 + L_0^2}} \theta \phi_1^* - \frac{\gamma(L_1L_0)^2}{\sqrt{L_1^2 + L_0^2}} \theta \dot{\theta}, \quad (A-16)
\]
\[
\frac{d}{dt} \phi_2^* = -\frac{\sigma(S_2A(1 - \sin \phi_e) - S_3)L_1L_0}{\sqrt{L_1^2 + L_0^2}} \theta - \frac{\sigma \gamma L_1L_0}{\sqrt{L_1^2 + L_0^2}} \dot{\theta} \\
+ \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \phi_2^* + \omega - \sigma(S_1A + S_2A) \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) \\
+ S_3 \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) + S_4 - \frac{S_2AL_1L_0}{\sqrt{L_1^2 + L_0^2}} \theta \phi_2^* - \frac{\gamma(L_1L_0)^2}{\sqrt{L_1^2 + L_0^2}} \theta \dot{\theta}. \quad (A-17)
\]
Equations (A-15), (A-16) and (A-17) are arranged as the state equations as follows:

436
\[
\frac{dx}{dt} = Ax + u
\]

where \( x = (\theta, \dot{\theta}, \phi_1, \phi_2)^T \), \( u = (0, u_3, u_4)^T \) and

\[
A = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & a_{22} & a_{23} & a_{24} \\
0 & a_{32} & a_{33} & 0 \\
a_{41} & a_{42} & 0 & a_{44}
\end{pmatrix}
\]

Each term of matrix \( A \) is as follows:

\[
a_{21} = -\frac{3 \gamma}{2 \gamma}
\]

\[
a_{22} = -\frac{6 \gamma L_1^2}{m(L_1^2 + L_0^2)},
\]

\[
a_{23} = \frac{3 L_1 \cos \phi_c}{m L_0}
\]

\[
a_{24} = \frac{3 L_1 \cos \phi_c}{m L_0}
\]

\[
a_{31} = \frac{\gamma (S_2 A - S_1 A - S_3 L_1 L_0)}{\sqrt{L_1^2 + L_0^2}} = -a_{41},
\]

\[
a_{32} = \frac{\gamma L_1 L_0}{\sqrt{L_1^2 + L_0^2}} = -a_{42},
\]

\[
a_{33} = \sigma \cos \phi_c (S_1 A + S_2 A \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right)) = a_{44}.
\]

Furthermore, each term of matrix \( u \) is as follows:

\[
u_2 = \frac{3 A L_2^2 \cos \phi_c (-S_1 + S_2 L_{w1}) \theta (\phi_1 + \phi_2)}{m \sqrt{L_1^2 + L_0^2}} - \frac{6 \gamma L_1^2}{m \sqrt{L_1^2 + L_0^2}} \left( \frac{L_1 L_0}{L_1^2 + L_0^2} \right)^2 \theta^2 \dot{\theta},
\]

\[
u_3 = \omega - \sigma \left( \frac{S_1 A + S_2 A \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right)}{\sqrt{L_1^2 + L_0^2}} \right) \left( 1 - \sin \phi_c \right)
\]

\[
+ S_3 \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) + S_4 + \frac{S_2 A L_1 L_0}{\sqrt{L_1^2 + L_0^2}} \theta \phi_1 - \frac{\gamma (L_1 L_0)^2}{\sqrt{L_1^2 + L_0^2}} \theta \phi_1,
\]

\[
u_4 = \omega - \sigma \left( \frac{S_1 A + S_2 A \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right)}{\sqrt{L_1^2 + L_0^2}} \right) \left( 1 - \sin \phi_c \right)
\]

\[
+ S_3 \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) + S_4 - \frac{S_2 A L_1 L_0}{\sqrt{L_1^2 + L_0^2}} \theta \phi_2 - \frac{\gamma (L_1 L_0)^2}{\sqrt{L_1^2 + L_0^2}} \theta \phi_2.
\]

### B. Pendulum angle approximation

To derive an approximate pendulum angle, we focus on the relationship between the MPA length and MPA pressure. First, we define the phase of the MPA length as follows:

\[
\psi_i = \sin^{-1} \left( \frac{l_{m_i} - l_{\text{mean}}}{l_{\text{max}}} \right).
\]

where \( l_{\text{mean}} \) is the center of oscillation and \( l_{\text{max}} \) is the amplitude of the MPA length when the pendulum motion is periodic. Figure B-1 depicts that the MPA length phase coincides well with the pressure phase in periodic period of Fig. 9. This leads to the following approximation.

\[
l_{m_i} = l_{\text{mean}} + l_{\text{max}} \sin \psi_i
\]

\[
\approx l_{\text{mean}} + l_{\text{max}} \sin \phi_i.
\]
Fig. B-1. Relationship between length and pressure of MPA with $\omega = 0.20\pi$ [rad/s] and $\sigma = -0.075$ ($t = 77$ [s]–$80$ [s]).

However, before the pendulum motion becomes periodic, the phase of the MPA length, which is calculated geometrically from the pendulum angles, may not coincide well with that of the pressure phase. Accordingly, we approximate each MPA length as follows with $\phi_1$:

$$l_{m1} = l_{\text{mean}} + l_{\text{max}} \sin(\phi_1 - \mu), \quad (B-3)$$

$$l_{m2} = l_{\text{mean}} + l_{\text{max}} \sin(\phi_2 - \delta), \quad (B-4)$$

where $\mu$, $\delta$ represents the difference between the change in MPA pressure and the change in MPA length. Here, we consider $\mu$ and $\delta$ to be constant. Because the displacement of the pendulum is proportional to the difference in the displacements of the MPA length ($l_{m1} - l_{\text{mean}}$) and ($l_{m2} - l_{\text{mean}}$), the following relationship holds.

$$L_0 \theta \propto l_{\text{max}}(-\sin \psi_1 + \sin \psi_2)$$

$$= l_{\text{max}}(-\sin(\phi_1 - \mu) + \sin(\phi_2 - \delta)) \quad (B-5)$$

Hence, the following approximation is derived.

$$\theta = \theta_{\text{max}} \frac{-\sin(\phi_1 - \mu) + \sin(\phi_2 - \delta)}{2} \quad (B-6)$$

Figure B-2 depicts the comparison of the approximation with $\omega = 0.20\pi$ [rad/s], $\sigma = -0.075$, initial difference $\phi_0 = \pi/2$ [rad], and $\mu = \delta = 0$. As depicted in Fig. B-2b, the approximate angle $\theta_{\text{app}}$ coincides with the pendulum angle $\theta$ well in the periodic period. However, the approximate angle does not coincide with the pendulum angle $\theta$ in the transition period in Fig. B-2a.

C. Averaging approximation of phase equations

The phase equation describes the motion of the phase oscillator. The averaging approximation is a method of integrating the terms with the phase oscillator in the phase equation for one cycle and
taking the average. This method enables the periodic function to take a simple form. Substituting Eq. (19) into Eq. (A-5), we have

\[
L_{m1} \approx \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) - \frac{L_1 L_0}{\sqrt{L_1^2 + L_0^2}} \sin \theta
\]

\[
\approx \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) - \frac{L_1 L_0}{\sqrt{L_1^2 + L_0^2}} \theta
\]

\[
\approx \left( \sqrt{L_1^2 + L_0^2} - L_{w1} \right) + \frac{L_1 L_0 \theta_{\text{max}}}{2 \sqrt{L_1^2 + L_0^2}} \left( \sin(\phi_1 - \mu) - \sin(\phi_2 - \delta) \right)
\]

\[
= B + C(\sin(\phi_1 - \mu) - \sin(\phi_2 - \delta)),
\]

(C-1)

\[
L_{m2} \approx B - C(\sin(\phi_1 - \mu) - \sin(\phi_2 - \delta)).
\]

(C-2)

Substituting Eqs. (B-1), and (8) into Eq. (5), Eq. (7) can be transformed as

\[
\hat{\phi}_1 = \omega - \sigma(S_1 A(1 - \sin \phi_1) + S_2 A(1 - \sin \phi_1)(B + C(\sin(\phi_1 - \mu) - \sin(\phi_2 - \delta)))
\]

\[
+ S_3(B + C(\sin(\phi_1 - \mu) - \sin(\phi_2 - \delta))) + S_4)
\]

\[
= [\omega - \sigma(S_1 A + S_2 AB + S_3 B + S_4)] + \sigma(S_1 A + S_2 AB) \sin \phi_1
\]

\[
+ \sigma(-S_2 AC + S_3 C) \sin(\phi_1 - \mu) + \sigma(S_2 AC - S_3 C) \sin(\phi_2 - \delta)
\]

\[
+ \sigma S_2 AC \sin \phi_1 \sin(\phi_1 - \mu) - \sigma S_2 AC \sin \phi_1 \sin(\phi_2 - \delta),
\]

(C-3)

\[
\hat{\phi}_2 = [\omega - \sigma(S_1 A + S_2 AB + S_3 B + S_4)] + \sigma(S_1 A + S_2 AB) \sin \phi_2
\]

\[
+ \sigma(-S_2 AC + S_3 C) \sin(\phi_2 - \delta) + \sigma(S_2 AC - S_3 C) \sin(\phi_1 - \mu)
\]

\[
+ \sigma S_2 AC \sin \phi_2 \sin(\phi_2 - \delta) - \sigma S_2 AC \sin \phi_2 \sin(\phi_1 - \mu),
\]

(C-4)

where B and C are constants. Integrating the terms containing \(\sin \phi_1, \sin(\phi_1 - \mu), \sin(\phi_2 - \delta)\) and \(\sin \phi_1 \sin(\phi_1 - \mu)\) for one cycle, the following equations can be derived.

\[
\frac{1}{2\pi} \int_0^{2\pi} \sin \phi_1 d\phi_1 = \int_0^{2\pi} \sin(\phi_1 - \mu) d\phi_1 = 0.
\]

(C-5)

\[
\frac{1}{2\pi} \int_0^{2\pi} \sin(\phi_1 - \mu) d\phi_1 = \frac{1}{2\pi} \cos \mu \int_0^{2\pi} \sin \phi_1 d\phi_1 - \sin \mu \int_0^{2\pi} \cos \phi_1 d\phi_1
\]

\[
= \frac{1}{2\pi} \cos \mu \left[ \sin \phi_1 \left|_0^{2\pi} - \sin \mu \left|_0^{2\pi} \right. \right. \right. = 0,
\]

(C-6)

\[
\frac{1}{2\pi} \int_0^{2\pi} \sin(\phi_2 - \delta) d\phi_2 = \frac{1}{2\pi} \cos \delta \int_0^{2\pi} \sin \phi_2 d\phi_2 - \sin \delta \int_0^{2\pi} \cos \phi_2 d\phi_2
\]

\[
= \frac{1}{2\pi} \cos \delta \left[ \sin \phi_2 \left|_0^{2\pi} - \sin \delta \left|_0^{2\pi} \right. \right. \right. = 0,
\]

(C-7)

\[
\frac{1}{2\pi} \int_0^{2\pi} \sin \phi_1 \sin(\phi_1 - \mu) d\phi_1 = \frac{1}{2\pi} \cos \mu \int_0^{2\pi} \sin^2 \phi_1 d\phi_1 - \sin \mu \int_0^{2\pi} \sin \phi_1 \cos \phi_1 d\phi_1
\]

\[
= \frac{1}{2\pi} \left[ \frac{\eta}{2} \cos \mu - \sin \frac{2\zeta}{4} \cos \mu + \frac{\cos \frac{2\zeta}{4} \sin \mu}{4} \right]_0^{2\pi} = \frac{1}{2} \cos \mu.
\]

(C-8)

In addition,

\[
\sin \phi_1 \sin(\phi_2 - \delta) = \cos \delta \sin \phi_1 \sin \phi_2 - \sin \delta \sin \phi_1 \cos \phi_2.
\]

(C-9)

Let \(\phi\) and \(\eta\) be \(\phi_1 - \phi_2\) and \(\phi_2\). Substituting \(\phi\) and \(\eta\) into Eq. (C-9), we obtain

\[
\frac{1}{2\pi} \int_0^{2\pi} \sin(\phi + \eta) \sin \eta d\eta = \frac{1}{2\pi} \int_0^{2\pi} \left( \sin \phi \cos \eta \sin \eta + \cos \phi \sin^2 \eta \right) d\eta
\]

\[
= \frac{1}{2\pi} \left[ - \cos \frac{2\zeta}{4} \sin \phi + \frac{\eta}{2} \cos \phi - \sin \frac{2\zeta}{4} \cos \phi \right]_0^{2\pi} = \frac{1}{2} \cos \phi,
\]

(C-10)

\[
\frac{1}{2\pi} \int_0^{2\pi} \sin(\phi + \eta) \cos \eta d\eta = \frac{1}{2\pi} \int_0^{2\pi} \left( \sin \phi \cos^2 \eta + \cos \phi \sin \eta \cos \eta \right) d\eta
\]
\[
\int_{2\pi}^{0} \sin(\varphi' + \eta') \sin \eta' d\eta' = \frac{1}{2} \cos \varphi' = \frac{1}{2} \cos \varphi', \\
\int_{2\pi}^{0} \sin(\varphi' + \eta') \cos \eta' d\eta' = \frac{1}{2} \sin \varphi' = -\frac{1}{2} \sin \varphi.
\]

Hence, Eq. (21) is also derived.

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