SIMILARITY SOLUTION FOR THE FLOW BEHIND A MAGNETOGASDYNAMIC EXPONENTIAL SHOCK WAVE IN A PERFECT GAS WITH VARYING DENSITY, HEAT CONDUCTION, AND RADIATION HEAT FLUX

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Similarity solutions are obtained for the propagation of a shock wave driven by a piston moving with time dependence according to an exponential law in a perfect gas with azimuthal magnetic field as well as with conduction and radiation heat fluxes. Heat conduction is described by the Fourier law, and radiation is considered to be of diffusion type for the optically thick grey gas model. The thermal conductivity and absorption coefficient are assumed to vary with temperature and density. The density and magnetic field strength ahead of the shock front are assumed to vary exponentially. The effects of the variations in the strength of the ambient magnetic field, heat transfer parameters, adiabatic exponent, and in the ambient density variation index on the flow field characteristics are studied. The shock strength is shown to be independent of the heat transfer parameters. The medium compressibility increases in the absence of a magnetic field.

Keywords: exponential shock wave, self-similar solution, magnetic field, conduction heat flux, radiation heat flux.

Introduction. Radiation plays a significant role in a number of hydrodynamic processes related to shock waves. Several researchers extended the self-similar approach of Sedov [1] to the problems of blast waves under the effect of radiation [2–4]. Recently self-similar solutions for a shock wave in an ideal, nonideal, and dusty gas with heat conduction and radiation heat fluxes have been obtained by many authors [5–12].

Shock waves in the presence of a magnetic field in a conducting perfect gas are important for the interpretation of shocks in the cases of explosions in supernovae and in the ionosphere. The industrial applications are drag reduction in duct flows, control of turbulence of immersed jets in the steel casting process, advanced propulsion and flow control schemes for hypersonic vehicles, and design of efficient coolant blankets in tokamak fusion reactors involving applied external magnetic fields [13, 14].

The limiting case of a self-similar flow field with a power-law shock is the flow field formed by an exponential shock described by Sedov [1] (see [15–17]). Ranga Rao and Ramana [15] obtained approximate analytic solutions for unsteady self-similar motion of a perfect gas induced by a piston moving exponentially. Several authors extended the problem of Ranga Rao and Ramana, considering different media, i.e., nonideal and dusty gases under the effects of a magnetic field, gravitation field, heat flux, and of rotation [8, 16–20]. In all of these works, self-similar solutions for the flow behind a magnetogasdynamic exponential shock wave in a perfect gas under the effect of conduction and radiation heat fluxes along with variable density have not been obtained. The present work is also the extension of the work of Ranga Rao and Ramana with regard to conduction and radiation heat fluxes, azimuthal magnetic field, and variable density.

The aim of this study is to obtain similarity solutions for motion of an exponential shock wave which is driven by a piston or explosion moving exponentially with time in a perfect gas with the effect of an azimuthal magnetic field as well as of conduction and radiation heat fluxes and variable density. Ahead of the shock front, the density is taken to be decreasing and the azimuthal magnetic field strength is assumed constant, increasing or decreasing according to an exponential law. The equilibrium flow conditions are assumed to be maintained. The viscosity, radiation pressure, and the radiation energy are presumed negligible. The assumption of an optically thick grey gas is physically consistent with neglect of the radiation pressure and radiation energy. The shock is assumed to be isothermal. The effects of various parameters on the solution are studied in detail. Motion of the piston or explosion is assumed to follow an exponential law [15–17]:

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\[ r_p = B^* \exp (\lambda t), \quad \lambda > 0, \quad (1) \]

where \( r_p \) is the piston (or explosion) radius, \( \lambda \) is a dimensional constant, and \( B^* \) is the piston radius at \( t = 0 \). The law of piston motion (1) implies a boundary condition for the gas velocity at the piston. It is also assumed that the shock propagation obeys an exponential law:

\[ R = B \exp (\lambda t), \quad (2) \]

where \( R \) is the shock radius and \( B \) is a dimensional constant which depends on \( B^* \) and on the nondimensional position of the piston. It is more convenient to consider piston motion in terms of shock motion. Therefore, we will consider \( B \) rather than \( B^* \) as the known parameter of the problem.

**Equations of Motion and Boundary Conditions.** The fundamental system of equations for a one-dimensional, unsteady, adiabatic fluid flow behind an exponential shock wave in a perfect gas with radiation and heat conduction in the presence of an azimuthal magnetic field can be written as [9, 11, 21–23]

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{iu \rho}{r} = 0, \quad (3)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \left( \frac{\partial \rho}{\partial r} + \rho h \frac{\partial h}{\partial r} + \frac{\mu h^2}{r} \right) = 0, \quad (4)
\]

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + (i - 1) \frac{hu}{r} = 0, \quad (5)
\]

\[
\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho r^i} \frac{\partial (r^i F)}{\partial r} = 0, \quad (6)
\]

where \( i = 0, 1, 2 \) for planar, cylindrical, and spherical symmetry of the flow field, respectively.

The equation of state and the equation for the internal energy in the case of a perfect gas are given by

\[ p = \Gamma \rho T, \quad U_m = \frac{p}{(\gamma - 1)\rho}, \quad (7) \]

The total heat flux \( F \) in the energy equation (6) can be written as

\[ F = F_c + F_r, \quad (8) \]

where \( F_c \) and \( F_r \) are the conduction and radiation heat fluxes. According to the Fourier law of heat conduction, the heat conduction flux \( F_c \) can be expressed as

\[ F_c = -K \frac{\partial T}{\partial r}, \quad (9) \]

where \( K \) is the thermal conductivity. The radiation heat flux \( F_r \) can be obtained from the differential approximation of the radiation transport equation in the diffusion limit by assuming a local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas [24]. Thus, \( F_r \) can be written as follows:

\[ F_r = -\frac{4}{3} \sigma \frac{\partial T^4}{\partial r}, \quad (10) \]

where \( \sigma \) is the Stefan–Boltzmann constant and \( \alpha_R \) is the Rosseland mean absorption coefficient.

The thermal conductivity \( K \) and the absorption coefficient \( \alpha_R \) are assumed to vary with density and temperature. According to power laws, they can be written as [10–12, 21]
\[ K = K_0 \left( \frac{T}{T_0} \right)^{\beta_c} \left( \frac{\rho}{\rho_0} \right)^{\delta_c}, \quad \alpha_R = \alpha_R_0 \left( \frac{T}{T_0} \right)^{\beta_r} \left( \frac{\rho}{\rho_0} \right)^{\delta_r}, \]  

(11)

where the subscript zero relates to a reference state. For similarity solutions, the exponents \( \beta_c, \delta_c, \delta_r, \) and \( \beta_r \) must satisfy the similarity requirements. The flow variables immediately ahead of the shock front are given by [25]

\[ u = u_a = 0, \]  

(12)

\[ \rho = \rho_a = \rho^* \exp (-\alpha t), \quad \alpha > 0, \]  

(13)

\[ h = h_a = h^* \exp (-\delta t), \]  

(14)

\[ F = F_a = 0, \]  

(15)

where \( \rho^*, h^*, \alpha, \) and \( \delta \) are dimensional constants and the subscript "a" refers to the conditions immediately ahead of the shock front. From Eqs. (4), (13), and (14) we obtain

\[ p_a = \frac{\alpha h^*}{2} \exp (-2\delta t) \left( \frac{\lambda}{\delta} - 1 \right), \quad \lambda > 0. \]

The jump conditions across the isothermal shock front are given by the laws of conservation of mass, momentum, and energy across the shock. The assumption of such a shock excludes the possibility of a temperature jump (see, for example, [10, 26–28]). Thus we have

\[ \rho_a V = \rho_n (V - u_n), \quad h_a V = h_n (V - u_n), \quad T_a = T_n, \]

\[ p_a + \frac{\mu h_n^2}{2} + \rho_a V^2 = p_n + \frac{\mu h_n^2}{2} + \rho_n (V - u_n)^2, \]  

(16)

\[ \frac{p_a}{(\gamma - 1)p_n} + \frac{p_a}{P_a} \frac{V^2}{2} + \frac{\mu h_n^2}{\rho_n} \frac{F_a}{P_a} V = \frac{p_n}{(\gamma - 1)p_n} + \frac{p_n}{P_n} \frac{(V - u_n)^2}{2} + \frac{\mu h_n^2}{P_n}, \]

where \( V = \lambda R \) is the velocity of the shock front and the subscript "n" relates to the conditions immediately behind the shock front. The shock conditions (16) reduce to

\[ u_n = (1 - \beta)V, \quad \rho_n = \frac{\rho_a}{\beta}, \quad h_n = \frac{h_a}{\beta}, \]

\[ p_n = \rho_a V^2 \left[ 1 - \beta + \frac{1}{\gamma M^2} + \frac{1}{2M^2} \left( 1 - \frac{1}{\beta^2} \right) \right], \]  

(17)

\[ F_n = (1 - \beta)\rho_a V^3 \left[ \beta (\gamma + 1) \right] \frac{1}{2(\gamma - 1)} - \frac{1}{2} + \frac{1}{M^2(\gamma - 1)} + \frac{M_a^{-2}}{2} \frac{\gamma}{\gamma - 1} + \frac{M_a^{-2}}{2} \frac{\gamma - 2}{\gamma - 1}. \]

Here \( M = (\rho_a V^2/\rho_a)^{1/2} \) is the shock Mach number, where the frozen speed of sound is \( (\gamma \rho_a/\rho_a)^{1/2} \), and \( M_A = (\rho_a V^2/\mu h_a)^{1/2} \) is the Alfvén–Mach number. It is found that \( M \) and \( M_A \) are constant for \( \alpha = 2(\lambda + \delta) \). The quantity \( \beta \), such that \( 0 < \beta < 1 \), is obtained from the relation

\[ \beta^3 - \beta^2 \left( 1 + \frac{1}{\gamma M^2} + \frac{M_A^{-2}}{2} \right) + \frac{\beta}{\gamma M^2} + \frac{M_A^{-2}}{2} = 0. \]  

(18)
Similarity Transformations. For self-similar motion, the system of the fundamental partial differential equations (3)–(6) reduces to the system of ordinary differential equations in new unknown functions of the similarity variable \( \xi \) [28] given as \( \xi = r/R \), where \( R = R(t) \). Thus we present the solution of the partial differential equations (3)–(6) in terms of the products of scale functions and new unknown functions of \( \xi \) [15, 29]:

\[
U = V U(\xi) , \quad \rho = \rho_{s} D(\xi) , \quad p = \rho_{s} V^{2} P(\xi) , \quad h = \left( \frac{\rho_{s}}{\mu} \right)^{1/2} V H(\xi) , \quad F = \rho_{s} V^{3} Q(\xi) ,
\]

where \( U, D, P, H, \) and \( Q \) are functions of \( \xi \) only. At the shock front \( \xi = 1 \), and at the piston \( \xi = \xi_{p} \). With using the similarity transformations (19), the system of the partial differential equations (3)–(6) reduces to

\[
(U - \xi) \frac{dD}{d\xi} + D(\xi) \frac{dU}{d\xi} + i \frac{DU}{\xi} - \frac{\alpha D}{\lambda} = 0 ,
\]

\[
(U - \xi) \frac{dU}{d\xi} + U(\xi) + \frac{1}{D(\xi)} \left( \frac{dP}{d\xi} + H(\xi) \frac{dH}{d\xi} + \frac{H^{2}(\xi)}{\xi} \right) = 0 ,
\]

\[
(U - \xi) \frac{dH}{d\xi} + \left( 1 - \frac{\alpha}{2\lambda} \right) H(\xi) + H(\xi) \frac{dU}{d\xi} + (i - 1) \frac{H(\xi)U(\xi)}{\xi} = 0 ,
\]

\[
(U - \xi) \frac{dP}{d\xi} - \gamma(U - \xi) \frac{P(\xi)}{D(\xi)} \frac{dD}{d\xi} + \left( 2 + \frac{\alpha(\gamma - 1)}{\lambda} \right) P(\xi) + \frac{i(\gamma - 1)}{\xi} Q(\xi) + (\gamma - 1) \frac{dQ}{d\xi} = 0 .
\]

Substituting Eqs. (9)–(11) into Eq. (8), we get the total heat flux as

\[
F = - \frac{K_{0}}{T_{0}^{\beta_{c}} \rho_{0}^{\delta_{c}}} \rho^{\delta_{c}} T^{\beta_{c}} \frac{\partial T}{\partial r} - \frac{16\sigma T_{0}^{\beta_{c}} \rho_{0}^{\delta_{c}}}{3\alpha_{R_{0}}} T^{3-\beta_{c}} \rho^{-\delta_{c}} \frac{\partial T}{\partial r} .
\]

Substituting Eqs. (7) and (19) into Eq. (24), we obtain the nondimensional total heat flux \( Q \) as

\[
Q = \left[ - \frac{K_{0} \lambda(\rho^{*})^{\delta_{c}} (\lambda B)^{2(\beta_{c}-1)}}{T_{0}^{\beta_{c}} \rho_{0}^{\delta_{c}} \Gamma^{\beta_{c} + 1}} \exp \left\{ (\alpha (\delta_{c} - 1) + 2\lambda (\beta_{c} - 1)) t_{r} \right\} \right]^{\beta_{c} - \beta_{c}}
\]

\[
- \frac{16\sigma T_{0}^{\beta_{c}} \rho_{0}^{\delta_{c}} \lambda(\rho^{*})^{-(\delta_{c} + 1)} (\lambda B)^{2(2-\beta_{c})}}{3\alpha_{R_{0}} \Gamma^{4-\beta_{c}}} \exp \left\{ (\alpha (\delta_{c} + 1) + 2\lambda (2 - \beta_{c})) t_{r} \right\} \right]^{\beta_{c} - \beta_{c}}.
\]

Equation (25) shows that the similarity solution of the present problem exists only when

\[
\beta_{c} = 1 + \frac{\alpha}{2\lambda} (\delta_{c} - 1) , \quad \beta_{r} = 2 + \frac{\alpha}{2\lambda} (\delta_{r} + 1) .
\]

These relations show that the thermal conductivity \( K \) and the absorption coefficient \( \alpha_{R} \) depend on the index of the ambient density variation \( \alpha/\lambda \). For the case of constant density, relations (26) are similar to ones from [7] and [8]. Under the above condition (26), Eq. (25) becomes

\[
Q = -X \left( \frac{1}{D} \frac{dP}{d\xi} - \frac{P}{D^{2}} \frac{dD}{d\xi} \right) ,
\]

where \( X = \Gamma_{c} P^{\beta_{c}} D^{\delta_{c} - \beta_{c}} + \Gamma_{r} D^{\beta_{r} - \delta_{r} - 3} P^{3 - \beta_{r}} \) and \( \Gamma_{c} \) and \( \Gamma_{r} \) are the nondimensional conductive and radiative heat transfer parameters given as

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\[ \Gamma_c = \frac{K_0 \lambda (\rho^*)^{s_e-1} (\lambda B)^2(\beta_e-1)}{T_0^{\beta_e} P_0^{\beta_e-1} \Gamma^{\beta_e-1}} , \quad \Gamma_r = \frac{16\sigma T_0^{\beta_e} P_0^{\gamma_e} \lambda (\rho^*)^{(\delta_e+1)} (\lambda B)^2(\beta_e-1)}{3\alpha R_0^{1-\beta_e}} . \] (28)

Solving the set of differential equations (20)–(23) and Eq. (27) for \( \frac{dU}{d\xi}, \frac{dH}{d\xi}, \frac{dP}{d\xi}, \frac{dQ}{d\xi}, \) and \( \frac{dD}{d\xi} \), we have

\[ \frac{dU}{d\xi} = -\frac{U-\xi}{D} \frac{dD}{d\xi} - \frac{iU}{\xi} + \frac{\alpha}{\lambda} , \] (29)

\[ \frac{dH}{d\xi} = \frac{H}{D} \frac{dD}{d\xi} + \frac{HU}{(U-\xi)\xi} \left( 1 + \frac{\alpha}{2\lambda} \right) \frac{H}{U-\xi} , \] (30)

\[ \frac{dP}{d\xi} = \left[ (U-\xi)^2 - \frac{H^2}{D} \right] \frac{dD}{d\xi} + \frac{i(U-\xi)DU}{\xi} - \frac{DU}{\xi} \left( \frac{\alpha}{\lambda} (U-\xi)D + \frac{H^2}{2\lambda(U-\xi) - \frac{2}{\xi}} \right) . \] (31)

\[ \frac{dQ}{d\xi} = -\frac{dD}{d\xi} \left[ (U-\xi)^2 - \frac{H^2}{D} \right] \frac{1}{\gamma - 1} \left[ \frac{U}{\lambda} - \frac{DU(U-\xi)}{(\gamma - 1)\xi} - \frac{iDU(U-\xi)^2}{(\gamma - 1)\xi} + \frac{\alpha(U-\xi)^2D}{\lambda(\gamma - 1)} \right] + \frac{H^2(U-\xi)}{\gamma - 1} \left[ \frac{2}{\xi} - \frac{\alpha}{2\lambda(U-\xi)} \right] - \frac{P}{\gamma - 1} \left[ \frac{2}{\lambda} + \frac{\alpha(\gamma - 1)}{\lambda} \right] - \frac{iQ}{\xi} , \] (32)

\[ \frac{dD}{d\xi} = \frac{D^2}{P - D(U-\xi)^2 + H^2} \left[ \frac{Q}{X} + \frac{iU(U-\xi)}{\xi} - \frac{U}{\lambda} - \frac{\alpha(U-\xi)}{\lambda} + \frac{H^2}{D} \left( \frac{\alpha}{2\lambda(U-\xi) - \frac{2}{\xi}} \right) \right] . \] (33)

With using the similarity transformations (19), the shock conditions (17) are transformed into

\[ U(1) = 1 - \beta , \quad D(1) = \frac{1}{\beta} , \quad H(1) = \frac{M^2}{\beta} , \quad P(1) = 1 - \beta + \frac{1}{\gamma M^2} + \frac{M^2}{2} \left( 1 - \frac{1}{\beta^2} \right) , \] (34)

\[ Q(1) = (1 - \beta) \left[ \frac{\beta(\gamma + 1)}{2(\gamma - 1)} - \frac{1}{2} - \frac{1}{M^2(\gamma - 1)} - \frac{\gamma M^2}{2(\gamma - 1)} + \frac{M^2}{2\beta} \frac{\gamma - 2}{\gamma - 1} \right] . \]

Along with the shock conditions (34), the condition which is to be satisfied at the piston surface is that the fluid velocity is equal to the velocity of the piston itself. From Eq. (19), this kinematic condition can be written as \( U(\xi_p) = \xi_p \). For an isentropic change in state of a perfect gas, we may calculate the isothermal speed of sound in it as follows:

\[ a_{is} = \left( \frac{\partial p}{\partial p} \right)_{T}^{1/2} = \left( \frac{\gamma p}{\rho} \right)^{1/2} , \] (35)

where the subscript \( T \) refers to the process of constant temperature. With using Eq. (19) in Eq. (35), the expression for the reduced isothermal speed of sound is given as \( a_{is}/R = (P/D)^{1/2} \). The adiabatic compressibility of a perfect gas can be calculated as [30]

\[ C_{ad} = \frac{1}{\rho} \left( \frac{\partial p}{\partial p} \right)_{S} = \frac{1}{\gamma p} , \]

where \( S \) is the entropy. The reduced adiabatic compressibility can be written as
\[ \frac{C_{ad}}{C_{ad,n}} = \frac{P(1)}{P(\xi)} . \]

The total energy of the flow field between the piston and the shock wave is given by

\[ E = 2\pi i \int_{r_p}^{R} \rho \left[ U_m + \frac{u^2}{2} + \frac{\mu h^2}{2\rho} \right] r' dr . \]  

(36)

Substituting the similarity transformations (19) and Eq. (7) into Eq. (36), we have

\[ E = 2\pi \rho a_i \lambda^2 R^{3+i} J , \]

where

\[ J = \int \left[ \frac{P(\xi)}{\gamma - 1} + \frac{(U(\xi))^2 D(\xi)}{2} + \frac{H(\xi)^2}{2} \right] \xi^i d\xi . \]

Thus the total energy of the shock wave is not constant and varies as \( R^{3+i} \). The increase in the total energy may be achieved by the pressure exerted by the piston on the fluid. Normalizing the flow variables \( u, \rho, p, h, F, \) and \( C_{ad} \) with the use of their respective values at the shock front, we obtain

\[ \frac{u}{u_n} = \frac{U(\xi)}{U(1)} , \quad \frac{\rho}{\rho_n} = \frac{D(\xi)}{D(1)} , \quad \frac{p}{p_n} = \frac{P(\xi)}{P(1)} , \quad \frac{h}{h_n} = \frac{H(\xi)}{H(1)} , \quad \frac{F}{F_n} = \frac{Q(\xi)}{Q(1)} , \quad \frac{C_{ad}}{C_{ad,n}} = \frac{P(1)}{P(\xi)} . \]

**Results and Discussion.** For the existence of a similarity solution, the shock Mach number \( M \) and the Alfven–Mach number \( M_A \) must be constant. Therefore, the following condition must be satisfied:

\[ \lambda + \delta = \frac{\alpha}{2} > 0 . \]  

(37)

The distribution of the flow variables between the shock front and the inner expanding surface is obtained by numerical integration of Eqs. (29)–(33) with boundary conditions (34) using the Runge–Kutta method of the fourth order. In numerical integration, the values of the constant parameters are taken as \( i = 2; \, \alpha/\lambda = 1.5, 2, 2.5; \, \gamma = 4/3, 5/3; \, M_A^2 = 0, 0.01, 0.1; \, M = 5; \, \delta_c = 1; \, \delta_R = 2; \, \Gamma_c = 0.1, 10, 1000; \, \Gamma_r = 0.5, 10, 500. \) Three chosen values of \( \alpha/\lambda \) correspond to an increasing, constant, and decreasing index of the ambient magnetic field strength \( (\delta/\lambda = -0.25, 0, 0.25) \) ahead of the shock front according to Eqs. (14) and (37).

Figures 1–3 show the variations in the reduced flow variables (velocity \( u/u_n \), density \( \rho/\rho_n \), pressure \( p/p_n \), magnetic field strength \( h/h_n \), total heat flux \( F/F_n \), isothermal speed of sound \( a_{is}/R \), and adiabatic compressibility \( C_{ad}/C_{ad,n} \) with the similarity variable \( \xi \) at different values of the parameters \( M_A^2, \, \Gamma_c, \, \Gamma_r, \, \gamma, \) and \( \alpha/\lambda \). The values of the density ratio \( \beta \), shock strength \((1 - \beta)\), and the piston position \( \xi_p \) for different values of the parameters mentioned are given in Tables 1–3.

**Effect of the strength of the ambient magnetic field \( M_A^2 \).** It is seen from Table 1 that the density ratio \( \beta \) and the distance between the piston and the shock front \((1 - \xi_p)\) increase with \( M_A^2 \). In going from the absence of a magnetic field \((M_A^2 = 0)\) to its presence \((M_A^2 > 0)\), \( u/u_n \), \( \rho/\rho_n \), and \( p/p_n \) decrease, whereas \( F/F_n \), and \( C_{ad}/C_{ad,n} \) increase and \( a_{is}/R \) is characterized by different trends (see Fig. 1). As \( M_A^2 > 0 \), \( \rho/\rho_n \) decreases and \( h/h_n \), \( F/F_n \), and \( a_{is}/R \) increase; for \( \alpha/\lambda = 2 \) and 2.5, \( u/u_n \) and \( p/p_n \) decrease, but for \( \alpha/\lambda = 1.5 \) \( p/p_n \) increases, \( u/u_n \) changes negligibly, and \( C_{ad}/C_{ad,n} \) undergoes different variations behind the shock front (see Fig. 1). This is possible due to the small effects of an increase in the index of the ambient magnetic field variation. It is found that the presence of a magnetic field has a significant effect on the flow field but this effect is negligible for increasing index of the variation in the ambient magnetic field strength. It is observed that the density decreases with increase in the strength of the ambient magnetic field. Physically it means that a gas compressed by a shock wave experiences an increase in the strength of the ambient magnetic field which is inversely proportional to an increase in the gas density.

**Effects of the conductive heat transfer parameter \( \Gamma_c \) and the radiative heat transfer parameter \( \Gamma_r \).** The shock strength \((1 - \beta)\) is independent of the heat transfer parameters (see Tables 2 and 3). The distance between the piston and the shock
Fig. 1. Variations of the reduced velocity (a), density (b), pressure (c), magnetic field strength (d), total heat flux (e), isothermal speed of sound (f), and adiabatic compressibility (g) in the region behind the shock front at $\Gamma_c = 10$ and $\Gamma_r = 5$: 1) $\gamma = 4/3$, $\alpha/\lambda = 1.5$, and $M_{\infty}^2 = 0$; 2) $4/3$, 1.5, and 0.01; 3) $4/3$, 1.5, and 0.1; 4) $4/3$, 2, and 0; 5) $4/3$, 2, and 0.01; 6) $4/3$, 2, and 0.1; 7) $4/3$, 2.5, and 0; 8) $4/3$, 2.5, and 0.01; 9) $4/3$, 2.5, and 0.1; 10) $5/3$, 1.5, and 0; 11) $5/3$, 1.5, and 0.01; 12) $5/3$, 1.5, and 0.1; 13) $5/3$, 2, and 0; 14) $5/3$, 2, and 0.01; 15) $5/3$, 2, and 0.1; 16) $5/3$, 2.5, and 0; 17) $5/3$, 2.5, and 0.01; 18) $5/3$, 2.5, and 0.1.
TABLE 1. Density Ratio across the Shock Front and the Position of the Piston Surface for Different Values of $M_A^{-2}$, $\gamma$, and $\alpha/\lambda$ with $\Gamma_c = 10$ and $\Gamma_r = 0.5$

| $\gamma$ | $\alpha/\lambda$ | $M_A^{-2}$ | $\beta$ | $1 - \beta$ | $\xi_{\phi}$ |
|----------|------------------|------------|---------|-------------|-------------|
| $4/3$    | 1.5              | 0          | 0.03    | 0.97        | 0.984010    |
|          |                  | 0.01       | 0.090344| 0.909656    | 0.941590    |
|          |                  | 0.10       | 0.267156| 0.732844    | 0.811154    |
| 2.0      | 0                | 0.03       | 0.97    |             | 0.979118    |
|          |                  | 0.01       | 0.090344| 0.909656    | 0.917987    |
|          |                  | 0.10       | 0.267156| 0.732844    | 0.729790    |
| 2.5      | 0                | 0.03       | 0.97    |             | 0.966509    |
|          |                  | 0.01       | 0.090344| 0.909656    | 0.877710    |
|          |                  | 0.10       | 0.267156| 0.732844    | 0.536279    |

TABLE 2. Density Ratio across the Shock Front and the Position of the Piston Surface for Different Values of $\alpha/\lambda$, $\gamma$, and $\alpha/\lambda$ with $\Gamma_r$, $\gamma$, and $\alpha/\lambda$ at $M_A^{-2} = 0.01$ and $\Gamma_r = 10$

| $\gamma$ | $\alpha/\lambda$ | $M_A^{-2}$ | $\beta$ | $1 - \beta$ | $\xi_{\phi}$ |
|----------|------------------|------------|---------|-------------|-------------|
| $4/3$    | 1.5              | 0          | 0.024   | 0.97        | 0.987802    |
|          |                  | 0.01       | 0.0866821| 0.913317    | 0.942523    |
|          |                  | 0.10       | 0.263647| 0.736353    | 0.812244    |
| 2.0      | 0                | 0.024      | 0.97    |             | 0.979263    |
|          |                  | 0.01       | 0.0866821| 0.913317    | 0.919245    |
|          |                  | 0.10       | 0.263647| 0.736353    | 0.735768    |
| 2.5      | 0                | 0.024      | 0.97    |             | 0.978603    |
|          |                  | 0.01       | 0.0866821| 0.913317    | 0.882101    |
|          |                  | 0.10       | 0.263647| 0.736353    | 0.565598    |

| $\gamma$ | $\alpha/\lambda$ | $M_A^{-2}$ | $\beta$ | $1 - \beta$ | $\xi_{\phi}$ |
|----------|------------------|------------|---------|-------------|-------------|
| $5/3$    | 1.5              | 0.1        | 0.090344| 0.909656    | 0.941053    |
|          |                  | 1000       | 0.090344| 0.909656    | 0.94034     |
| 2.0      | 0.1              | 0.090344   | 0.909656|             | 0.915925    |
|          | 1000             | 0.090344   | 0.909656|             | 0.915042    |
| 2.5      | 0.1              | 0.090344   | 0.909656|             | 0.842946    |
|          | 1000             | 0.090344   | 0.909656|             | 0.839723    |

| $\gamma$ | $\alpha/\lambda$ | $M_A^{-2}$ | $\beta$ | $1 - \beta$ | $\xi_{\phi}$ |
|----------|------------------|------------|---------|-------------|-------------|
| $5/3$    | 1.5              | 0.1        | 0.0866821| 0.913317    | 0.942564    |
|          |                  | 1000       | 0.0866821| 0.913317    | 0.941981    |
| 2.0      | 0.1              | 0.0866821  | 0.913317 |             | 0.918722    |
|          | 1000             | 0.0866821  | 0.913317 |             | 0.918168    |
| 2.5      | 0.1              | 0.0866821  | 0.913317 |             | 0.854114    |
|          | 1000             | 0.0866821  | 0.913317 |             | 0.854008    |
front \(1 - \xi_p\) increases moderately with the parameters mentioned (see Tables 2 and 3). The flow variables are characterized by different behaviors with increase in the values of \(\Gamma_c\) and \(\Gamma_r\), for different values of \(\alpha/\lambda\). With increasing \(\Gamma_c\), \(u/u_n\) and \(h/h_n\) undergo negligible effects; for \(\alpha/\lambda = 1.5\) the flow characteristics \(\rho/\rho_n\) and \(F/F_n\) and \(C_{ad}/C_{ad,n}\) decrease but they are almost constant for other values of \(\alpha/\lambda\); \(p/p_n\) increases for \(\alpha/\lambda = 1.5\) and the effect is negligible for other values of this parameter; \(a_{is}/R\) increases for \(\alpha/\lambda = 1.5\) and 2 and undergoes negligible effect for \(\alpha/\lambda = 2.5\) (see Fig. 2). With increasing \(\Gamma_r\), \(h/h_n\) undergoes negligible effect and \(a_{is}/R\) increases; for \(\alpha/\lambda = 1.5\) and 2, the flow characteristics \(u/u_n\), \(\rho/\rho_n\), \(F/F_n\), and \(C_{ad}/C_{ad,n}\) decrease but they undergo negligible effect for \(\alpha/\lambda = 2.5\); \(p/p_n\) increases for \(\alpha/\lambda = 1.5\) and 2 but is almost constant for \(\alpha/\lambda = 2.5\) (see Fig. 3). The heat transfer parameters exert decaying effects on the velocity, total heat flux, density, magnetic field strength, and the adiabatic compressibility. These effects are more significant for increasing index of the ambient magnetic field variation and are negligible for decreasing index. These decaying effects are due to the increase in the distance between the piston and the shock front. It follows from Eq. (28) that the increase in the values of \(\Gamma_c\) and \(\Gamma_r\) increases \(\lambda\) and hence \((R - r_p)\) increases, which is seen from Eqs. (1) and (2).

**Effect of the adiabatic exponent \(\gamma\).** It is seen that the density ratio \(\beta\) and the distance between the piston and the shock front \((1 - \xi_p)\) decrease with increase in \(\gamma\). In the absence of a magnetic field, the flow characteristics \(u/u_n\) and \(C_{ad}/C_{ad,n}\) undergo negligible effects, \(\rho/\rho_n\), \(p/p_n\), and \(F/F_n\) increase moderately, and \(a_{is}/R\) decreases for \(\alpha/\lambda = 2.5\) and changes negligibly for other values of \(\alpha/\lambda\) (see Fig. 1). In a magnetic field (see Fig. 2), \(u/u_n\), \(\rho/\rho_n\), \(p/p_n\), and \(h/h_n\) decrease moderately, \(F/F_n\) and \(C_{ad}/C_{ad,n}\) increase, and \(a_{is}/R\) undergoes different effects. It is found that the effect of increasing adiabatic exponent \(\gamma\) is more pronounced in the presence of a magnetic field. In addition, the strength of the ambient magnetic field and the adiabatic exponent have opposite effects on the distance between the piston and the shock front and on the shock strength. From Eq. (18) it is seen that the increase in \(\gamma\) decreases the value of \(\beta\), therefore the shock strength \((1 - \beta)\) increases, which leads to the above-mentioned effects.

**Effect of the ambient density variation index \(\alpha/\lambda\).** The shock strength \((1 - \beta)\) is independent of \(\alpha/\lambda\) (see Tables 1–3). The distance between the piston and the shock front \((1 - \xi_p)\) increases with \(\alpha/\lambda\). In the absence of a magnetic field, \(u/u_n\) decreases, \(\rho/\rho_n\), \(p/p_n\), \(F/F_n\), and \(C_{ad}/C_{ad,n}\) undergo negligible effects, and \(a_{is}/R\) also exhibits negligible effect in going from \(\alpha/\lambda = 1.5\) to \(\alpha/\lambda = 2\) but increases in going from \(\alpha/\lambda = 2\) to \(\alpha/\lambda = 2.5\) (see Fig. 1). In the presence of a magnetic field (see Fig. 2), \(u/u_n\) decreases, \(\rho/\rho_n\) decreases near the shock front, \(h/h_n\) decreases away from the piston, \(F/F_n\) decreases moderately, \(a_{is}/R\) increases, and \(p/p_n\) and \(C_{ad}/C_{ad,n}\) increase with \(\alpha/\lambda\) up to a certain distance from the shock front and then behave variously. It is found that the effects of an increase in the value of the ambient density variation index are more pronounced in the presence of a magnetic field. The heat transfer parameters and the ambient density variation index have same effects on the distance between the piston and the shock front and on the shock strength.

### TABLE 3. Density Ratio across the Shock Front and the Position of the Piston Surface for Different Values of \(\Gamma_c\), \(\gamma\), and \(\alpha/\lambda\) with \(M_{A}^{-2} = 0.1\) and \(\Gamma_c = 10\)

| \(\gamma\) | \(\alpha/\lambda\) | \(\Gamma_c\) | \(\beta\) | \(1 - \beta\) | \(\xi_p\) |
|---------|------------------|-----------|----------|-------------|---------|
| 4/3     | 1.5              | 0.5       | 0.090344 | 0.909656   | 0.940594 |
|         |                   | 500       | 0.090344 | 0.909656   | 0.940351 |
| 2.0     | 0.5              | 0.090344  | 0.909656 | 0.917933   |
|         |                   | 500       | 0.090344 | 0.909656   | 0.915595 |
| 2.5     | 0.5              | 0.090344  | 0.909656 | 0.84771    |
|         |                   | 500       | 0.090344 | 0.909656   | 0.839681 |
| 5/3     | 1.5              | 0.5       | 0.0866821| 0.9133179  | 0.942523 |
|         |                   | 500       | 0.0866821| 0.9133179  | 0.940978 |
| 2.0     | 0.5              | 0.0866821 | 0.9133179| 0.919245   |
|         |                   | 500       | 0.0866821| 0.9133179  | 0.918796 |
| 2.5     | 0.5              | 0.0866821 | 0.9133179| 0.862101   |
|         |                   | 500       | 0.0866821| 0.9133179  | 0.853966 |
Fig. 2. Same characteristics as in Fig. 1 at $M_{\infty}^2 = 0.01$ and $\Gamma_r = 10$: 1) $\gamma = 4/3$, $\alpha = 1.5$, and $\Gamma_c = 0.1$; 2) $4/3$, 1.5, and 1000; 3) $4/3$, 2, and 0.1; 4) $4/3$, 2, and 1000; 5) $4/3$, 2.5, and 0.1; 6) $4/3$, 2.5, and 1000; 7) $5/3$, 1.5, and 0.1; 8) $5/3$, 1.5, and 1000; 9) $5/3$, 2, and 0.1; 10) $5/3$, 2, and 1000; 11) $5/3$, 2.5, and 0.1; 12) $5/3$, 2.5, and 1000.
Fig. 3. Same characteristics as in Fig. 1 at $M_{\Lambda}^{-2} = 0.1$ and $\Gamma_c = 10$: 1) $\gamma = 4/3$, $\alpha/\lambda = 1.5$, and $\Gamma_r = 0.5$; 2) $4/3$, 1.5, and 500; 3) $4/3$, 2, and 0.5; 4) $4/3$, 2, and 500; 5) $4/3$, 2.5, and 0.5; 6) $4/3$, 2.5, and 500; 7) $5/3$, 1.5, and 0.5; 8) $5/3$, 1.5, and 500; 9) $5/3$, 2, and 0.5; 10) $5/3$, 2, and 500; 11) $5/3$, 2.5, and 0.5; 12) $5/3$, 2.5, and 500.
Conclusions. The present work investigates a one-dimensional unsteady adiabatic self-similar flow behind an exponential shock wave propagating in a perfect gas with an azimuthal magnetic field as well as conduction and radiation heat fluxes. On the basis of the work it may be concluded that

i) A similarity solution of the present problem exists only when the sum of the shock radius exponent and the ambient magnetic field exponent is equal to half the ambient density exponent.

ii) The total energy of the flow field behind the shock wave is not constant but varies as a power of the shock radius.

iii) The shock strength decreases with increase in the strength of the ambient magnetic field and increases with the value of the adiabatic exponent, whereas it is independent of the ambient density variation index and the heat transfer parameters.

iv) The distance between the piston and the shock front increases with the strength of the ambient magnetic field, heat transfer parameters, and with the ambient density variation index and decreases with increase in the adiabatic exponent.

v) The flow variables undergo different effects in the absence and presence of a magnetic field with increase in the values of the strength of the ambient magnetic field, heat transfer parameters, adiabatic exponent, and of the ambient density variation index. These effects are more pronounced in a magnetic field.

vi) The increasing, constant, and decreasing ambient magnetic field variation indices have different effects on the flow field behind the shock front.

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NOTATION

\(a_{\text{is}}\), isothermal speed of sound; \(B^r\), piston radius at \(t = 0\); \(C_{\text{ad}}\), adiabatic compressibility; \(D\), nondimensional density; \(E\), total energy of the flow field behind the shock; \(F\), total heat flux; \(F_c\), conduction heat flux; \(F_r\), radiation heat flux; \(H\), nondimensional azimuthal magnetic field strength; \(h\), azimuthal magnetic field strength; \(i\), symmetry index; \(K\), thermal conductivity; \(M\), shock Mach number; \(M_A\), Alfven–Mach number; \(P\), nondimensional pressure; \(p\), pressure; \(Q\), nondimensional total heat flux; \(R\), shock radius; \(r\), space coordinate; \(r_p\), piston or explosion radius; \(t\), time; \(T\), temperature; \(U\), nondimensional velocity; \(U_{\text{in}}\), internal energy per unit mass; \(u\), fluid velocity; \(V\), shock front velocity; \(\alpha_R\), Rosseland mean absorption coefficient; \(\alpha/\lambda\), ambient density variation index; \(\beta\), density ratio; \(\Gamma\), gas constant; \(\Gamma_c\), nondimensional conductive heat transfer parameter; \(\Gamma_h\), nondimensional radiative heat transfer parameter; \(\gamma\), adiabatic exponent; \(\delta/\lambda\), ambient magnetic field variation index; \(\lambda\), shock radius exponent; \(\mu\), magnetic permeability; \(\xi\), similarity variable; \(\rho\), density; \(\sigma\), Stefan–Boltzmann constant. Indices: 0, reference state; \(a\) and \(n\), conditions just ahead and behind the shock front; \(ad\), adiabatic; \(c\) and \(r\), constants corresponding to convection and radiation; \(is\), isothermal; \(p\), at the piston.

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