A PRODUCT-TO-SUM FORMULA FOR THE QUANTUM GROUP OF $SL(2,\mathbb{C})$

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This paper exhibits a product-to-sum formula for the observables of a certain quantization of the moduli space of flat $SU(2)$-connections on the torus. This quantization was defined using the topological quantum field theory that was developed by Reshetikhin and Turaev from the quantum group of $SL(2,\mathbb{C})$ at roots of unity. As a corollary it is shown that the algebra of quantum observables is a subalgebra of the non-commutative torus with rational rotation angle. The proof uses topological quantum field theory with corners, and is based on the description of the matrices of the observables in a canonical basis of the Hilbert space of the quantization.

1. Introduction

In this paper we prove a product-to-sum formula that holds for the quantization of the moduli space of flat $SU(2)$-connections on the torus. A quantization of this moduli space was given by Witten in [12] in conjunction with the Jones polynomial, using a path integral on the space of connections. Witten’s path integrals were put into rigorous framework by Reshetikhin and Turaev in [9] using quantum groups. This paper is based on their construction.

The algebra of observables is densely generated by the functions of the form $\cos 2\pi(px + qy)$, $p, q$ integers. The quantization of such a function consists of the coloring of the curve of slope $p'/q'$ on the torus by the virtual representation $V^{n+1} - V^{n-1}$, where $n$ is the greatest common divisor of $p$ and $q$, $p' = p/n$, $q' = q/n$, and we denote by $V^k$ the $k$-dimensional irreducible representation of the quantum group of $SL(2,\mathbb{C})$. In the work Reshetikhin and Turaev $V^k$ is only defined for $k = 1, 2, \ldots, r-1$, but we will see below how this definition is extended to all integers. Let us denote by $C(p, q)$ the operator that is associated by the quantization to $\cos 2\pi(px + qy)$.

A product-to-sum formula for the version of these operators defined using the Kauffman bracket was discovered in [3] and that is what inspired the following result.
**Theorem 1.1.** In any level \( r \) and for any integers \( m, n, p, q \) the following product-to-sum formula holds

\[
C(m, n) \ast C(p, q) = t^{|mn|} C(m + p, n + q) + t^{-|pq|} C(m - p, n - q).
\]

So we see that the quantization of the algebra of the observables on the moduli space of flat \( SU(2) \)-connections on the torus is isomorphic to a subalgebra of the noncommutative torus defined by Rieffel [10]. To be more rigorous, the above product-to-sum formula induces a \( \ast \)-product on the algebra of functions on the character variety on the torus. This \( \ast \)-algebra is a subalgebra of the noncommutative torus.

The noncommutative torus is a \( C^\star \)-algebra generated by \( U \), \( V \), \( U^{-1} \), \( V^{-1} \) where \( U \) and \( V \) are two unitary operators satisfying the exponential form of the Heisenberg noncommutation relations \( UV = t^2 VU \). The inclusion is given by

\[
C(p, q) \rightarrow t^{-pq}(U^pV^q + U^{-p}V^{-q}).
\]

The elements \( \frac{1}{2}t^{-pq}(U^pV^q + U^{-p}V^{-q}) \) are the noncommutative cosines, so Theorem 1.1 is nothing but the product-to-sum formula for noncommutative cosines. Of course, \( t \) root of unity is the situation Rieffel did not consider, for here the rotation angle in the definition of the noncommutative torus is rational.

What we consider to be a nice feature of this paper is that the proof of this result about the quantization of moduli spaces uses topological cut-and-paste techniques.

**2. The background of the problem**

Let us briefly discuss the case of the torus and then state the main result. The moduli space of flat \( SU(2) \)-connections on the torus is the “pillow case”, the quotient of the complex plane by the lattice \( \mathbb{Z}[i] \) and by the symmetry with respect to the origin. This space is the same as the character variety of \( SU(2) \)-representations of the fundamental group of the torus.

The algebra of observables, i.e. the algebra of functions on this space, is generated for example by the functions \( \sin 2\pi n(px + qy)/\sin 2\pi(px + qy) \), where \( n, p, q \) are integer numbers with \( n \geq 0 \) and \( p, q \) coprime. A function of this form has a geometric interpretation, namely it associates to a conjugacy class of flat connections the trace of the holonomy of the connection along the curve of slope \( p/q \), where the trace is computed in the \( n \)-dimensional irreducible representation of \( SU(2) \). Witten defined the path integral only for functions of this form, but the definition can be extended to any (smooth) function on the character variety to yield an operator

\[
op(f) = \int e^{i\mathcal{L}(A)} f(A) DA
\]
where \( f(A) \) is defined using approximations of \( f \) by sums of traces of holonomies of the type describe above,

\[
\mathcal{L}(A) = \frac{k}{4\pi} \text{Tr} \int_{\mathbb{T}^2 \times I} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)
\]

is the Chern-Simmons functional for connections on the cylinder over the torus, \( k \) being the level of quantization, and the path integral is taken over all connections \( A \) that interpolate between two flat connections on the boundary components of the cylinder over the torus.

Witten’s construction was made rigorous by Reshetikhin and Turaev using the quantum group of \( SL(2, \mathbb{C}) \) at roots of unity. Here is how this is done for the torus. Fix a level \( r \geq 3 \), and let \( t = e^{\pi i \frac{1}{2r}} \). For an integer \( n \) define

\[
[n] = \frac{t^{2n} - t^{-2n}}{t^2 - t^{-2}}.
\]

The quantum analogue of the universal enveloping algebra of \( sl(2, \mathbb{C}) \) is the algebra \( U_t \) with generators \( X, Y, K, \bar{K} \) satisfying

\[
\bar{K} = K^{-1}, \quad KX = t^2 XK, \quad KY = t^{-2} YK, \quad XY - YX = \frac{K^2 - K^{-2}}{t^2 - t^{-2}}, \quad X^r = Y^r = 0, \quad K^{4r} = 1.
\]

This is a Hopf algebra, thus its representations form a ring under direct sum and tensor product. Modulo a small technicality where one factors by the part of quantum trace zero, this ring contains a subring generated by finitely many irreducible representations \( V^1, V^2, \ldots, V^{r-1} \). The representation \( V^k \) has the basis \( e_{-\frac{k-1}{2}}, e_{-\frac{k-3}{2}}, \ldots, e_{\frac{1-k}{2}} \) and \( U_t \) acts on this basis by

\[
Xe_j = [m+j+1]e_{j+1}, \quad Ye_j = [m-j+1]e_{j-1}, \quad Ke_j = t^{2j}e_j.
\]

Using the quantum version of the Clebsch-Gordon theorem, these irreducible representations can be defined recursively by

\[
V^{n+1} = V^2 \otimes V^n - V^{n-1}.
\]

This definition works only for \( 2 \leq n \leq r - 2 \).

With these representations we color any knot \( K \) in a 3-dimensional manifold, and the coloring is denoted by \( V^n(K) \). Let \( \alpha \) be the core \( S^1 \times \{0\} \) of the solid torus \( S^1 \times \mathbb{D}, \mathbb{D} = \{z, |z| \leq 1\} \). The Hilbert space of the torus \( V(\mathbb{T}^2) \) has an orthonormal basis given by \( V^k(\alpha), k = 1, 2, \ldots, r - 1 \). Now the convention is made that \( V^r = 0 \) and with this in mind we can extend the definition of \( V^n(\alpha) \) to all \( n \) via the recursion

\[
V^{n+1}(\alpha) = V^2(\alpha) \otimes V^n(\alpha) - V^{n-1}(\alpha).
\]

Here \( V^2(\alpha) \otimes V^n(\alpha) \) is just \( \alpha \) colored by the representation \( V^2 \otimes V^n \).

To make sense of these notations recall the pairing \( \langle \cdot, \cdot \rangle \) on \( V(\mathbb{T}^2) \) obtained by gluing two solid tori such that the meridian of the first is identified with the longitude of the second and vice versa, as to obtain a 3-sphere. This
pairing is not an inner product but is nondegenerate. Pairing two elements of \( V(\mathbb{T}^2) \) yields a colored link in the 3-sphere, and Reshetikhin and Turaev described in [9] a way of associating a number to this link using the ribbon algebra structure of \( \mathcal{U}_t \) (see also [8], [11]). This number is the value of the pairing. For example it is well known that
\[
< V^k(\alpha), V^m(\alpha) > = [mn].
\]

Let \( p \) and \( q \) be two integers, \( n \) their greatest common divisor and \( p' = p/n, q' = q/n \). We describe now the operator \( S(p, q) \) that corresponds to the function \( \sin 2\pi n(p'x + q'y)/\sin 2\pi(p'x + q'y) \) (S stands for sine). Since the pairing is nondegenerate, it suffices to describe \( < S(p, q)V^k(\alpha), V^m(\alpha) > \). This number is computed by placing the curve of slope \( p'/q' \) on the boundary of the solid torus, then gluing another torus to it to obtain the 3-sphere, coloring the \( p'/q' \)-curve by \( V^n \), the core of the first torus by \( V^k \) and the core of the second torus by \( V^m \) and then evaluating the colored link diagram as in [9]. We neglected the discussion about the framing, and the invariants of Reshetikhin and Turaev are defined for framed knots and links. The curve \( \alpha \) will always be framed by the annulus \( S^1 \times [0,1] \) while the curve of slope \( p'/q' \) has the vertical framing (i.e. the framing defined by the vector field orthogonal to the torus).

The quantization of the function \( f(x, y) = 2 \cos 2\pi(px+qy) \) is the operator \( C(p, q) = C(np', nq') = S((n+1)p', (n+1)q') - S((n-1)p', (n-1)q') \).

Here \( C \) stands for cosine and the definition is motivated by the trigonometric formula
\[
2 \cos nx = \frac{\sin(n+1)x}{\sin x} - \frac{\sin(n-1)x}{\sin x}.
\]

These are the operators for which we prove the product-to-sum formula.

3. Proof of the main result

Unlike the case of [3], here an easy inductive proof will not work. The proof we give below was inspired by the computation of the colored Jones polynomials of torus knots from [6]. It uses an apparatus called topological quantum field theory with corners, which enables the computation of quantum invariants of 3-manifolds through a successive application of axioms. This system of axioms is in spirit analogous to the Eilenberg-MacLane system of axioms for homology. The fundamental principles of a TQFT with corners were summarized by Walker in [13]. The construction of the basic data for the case of the quantum group \( \mathcal{U}_t \) was initiated in [4] and completed in [5].

Let us discuss just the facts about the \( SL(2,\mathbb{C}) \) TQFT with corners needed in the proof of Theorem 1.1. As explained in [2], a TQFT in dimension three consists of a modular functor \( V \) that associates vector spaces
to surfaces and isomorphisms to homeomorphisms, and a partition function
that associates to each three dimensional manifold a vector in the vector
space of its boundary.

Walker refined this point of view in [13]. He considered decompositions of
surfaces into disks, annuli, and pairs of pants, called DAP-decompositions,
which correspond through the functor to basis’ of the vector space. His
DAP-decompositions (which should probably be called rigid structures) in-
volves more structure than that. This structure consists of some curves, called
seams and drawn like dotted lines in diagrams, which keep track of the twist-
ings that occur and are not detected by the change in DAP-decompositions.
Transformations of DAP-decompositions are called moves and have the vec-
tor space correspondent of the change of basis. They are useful in preparing
the boundary of a 3-manifold for a gluing with corners. Let us point out
that for $U_t$ one needs also an orientation of the decomposition curves, as
explained in [5] where it was encoded using the Klein four group, but this
fact is irrelevant for all computations below, so we do not refer to it again.

Also, Walker considered framed, or extended manifolds, which are pairs
$(M, n)$ with $M$ a manifold and $n$ an integer, necessary to make the partition
function well defined. In this paper we begin the computation with mani-
folds with framing zero. The framing only changes when we perform moves
on boundary tori, and there the change is computed using the Shale-Weil
cocycle. However all relevant changes are performed back and forth, so the
framing cancels. Thus we simply ignore it.

Following [13] we denote $X = \sqrt{\sum_{j=1}^{r-1} [j]^2}$ (not to be confused with
the generator of $U_t$). The vector space of an annulus has basis $\beta^j_n$, $j =
1, 2, \ldots, r-1$, with pairing given by

$$< \beta^j_n, \beta^k_n > = \delta_{j,k} X/|j|.$$  

Gluing the boundaries of an annulus we obtain a torus, with the same basis
for the vector space. The vector space of the torus $V(T^2)$ is the same
as the Hilbert space of the quantization of the moduli space of flat $SU(2)$-
connections on the torus. The pairing on the torus makes $\beta^j_n$ an orthonormal
basis, but we don’t need this fact now. The moves $S$ and $T$ on the torus
are described in Figure 1. The $(m, n)$-entry of the matrix of $S$ is $[mn]$. The
move $T$ is diagonal and its $j$th entry is $\rho^{j-1}.$

The quantum invariant (i.e. partition function) of the cylinder over a
surface is the identity matrix. So the quantum invariant of the cylinder over
an annulus is $\sum_{n=1}^{r-1} [n] \beta^m_n \otimes \beta^n_n \otimes \beta^m_n \otimes \beta^n_n$ (when taking the cylinder over
the annulus the boundary of the solid torus will be canonically decomposed
into four annuli). If the DAP-decomposition of a 3-manifold involves two
disjoint annuli, then its invariant can be written in the form
\[
\sum_{k,j=1}^{r-1} \beta^k \otimes \beta^j \otimes v_{k,j}.
\]
Gluing the two annuli produces a 3-manifold that in the newly obtained DAP-decomposition has the invariant equal to \( \sum_{k=1}^{r-1} X^{[k]} v_{k,k} \).

For the proof of the theorem we need the following formula, which can be checked using the fact that the sum of the roots of unity is zero.

**Lemma 3.1.** Let \( a, b, c, d \) and \( e \) be integers. Then
\[
\sum_{x,y=1}^{r-1} [ax] t^{bx^2} [cy] ([x(y+d)] t^{2xy} + [x(y-d)] t^{-2xy}) = X^2 t^{be^2+be^2-2de} ([a(c+e)] t^{2(be-d)c} + [a(c-e)] t^{-2(be-d)c}).
\]

The proof of the main result is based on a theorem that is of interest in itself. It describes how the observable \( C(p, q) \) acts on the Hilbert space of the quantization. With our convention for defining \( V^k(\alpha) \) for all \( k \) we have

**Theorem 3.2.** In any level \( r \) and for any integers \( p, q \) and \( k \) the following formula holds
\[
C(p, q)V^k(\alpha) = t^{-pq} \left( t^{2pq}V^{k-p}(\alpha) + t^{-2pq}V^{k+p}(\alpha) \right).
\]

**Proof.** It suffices to check that the two sides of the equality yield the same results when paired with all \( V^m(\alpha) \). We must show that
\[
< C(p, q)V^k(\alpha), V^m(\alpha) > = \frac{t^{-pq}}{t^2 - t^{-2}} \times \left( t^{2(qk-pm+km)} - t^{2(qk+pm-km)} + t^{2(-qk+pm+km)} - t^{2(-qk-pm-km)} \right)
\]

Let \( d \) to be the greatest common divisor of \( p \) and \( q \), \( p' = p/d \) and \( q' = q/d \)
We concentrate first on the computation of
\[
< S((d+1)p', (d+1)q')V^k(\alpha), V^m(\alpha) >
\]
and

\[ (2) \quad < S((d - 1)p', (d - 1)q')V^k(\alpha), V^m(\alpha) > \]

Here is the place where we use the topological quantum field theory with corners from [5].

The expression in (1) is the invariant of the link that has one component equal to the \((p', q')\)-curve on a torus colored by \(V^{d+1}\), and the other two components the cores of the two solid tori that lie on one side and the other of the torus knot, colored by \(V^k\) (the one inside) and \(V^m\) (the one outside). The expression in (2) is the invariant of the same link but with the \((p', q')\)-curve colored by \(V^{d-1}\). It was shown in [6] that this number is equal to \(X^{-1}\) times the coordinate of \(\beta^{d+1}_k \otimes \beta^k_k \otimes \beta^m_m\) of the vector that is the quantum invariant of the link complement.

Let us produce the complement of this link by gluing together two simple 3-manifolds, whose quantum invariants are easy to compute. Consider first the cylinder over an annulus \(A\) and glue its ends to obtain the manifold \(A \times S^1\). In the basis of the vector space of \(V(T^2 \times T^2)\) determined by the DAP-decomposition \(\partial A \times \{1\}\) the invariant of this manifold is \(\sum_k \beta^k_k \otimes \beta^k_k\).

Take another copy of the same manifold. Change the decomposition curves of the exterior torus of the first manifold to the \(p'/q'\)-curve and of the exterior torus of the second manifold to the longitude. Of course, to do this on the second manifold we apply the \(S\)-move and so the invariant of the second manifold changes to

\[ \frac{1}{X} \sum_{\delta, j_{n+1}} [d j_{n+1}] \beta^\delta_\delta \otimes \beta^{j_{n+1}}_{j_{n+1}}. \]

With the first manifold the story is more complicated. Consider the continued fraction expansion

\[ \frac{q'}{p'} = \frac{1}{-a_1 - \frac{1}{-a_2 - \frac{1}{-a_3 - \cdots}}}. \]

The required move on the boundary is then \(ST^{-a_n}ST^{-a_{n-1}}S \cdots ST^{-a_1}S\). So the invariant of the first manifold in the new DAP-decomposition is

\[ X^{-n-1} \sum_{j_1, \ldots, j_{n+1}} [j_{n+1} j_n] t^{-a_n(j^2_n - 1)} [j_n j_{n-1}] \cdots [j_2 j_1] t^{-a_1(j^2_1 - 1)} [j_1 k] \beta^k_k \otimes \beta^{j_{n+1}}_{j_{n+1}}. \]

Now expand one annulus in the exterior tori of each of the two manifolds. Then glue just one annulus from the the first manifold to one annulus from the second. This way we obtain the complement of the link in discussion. One of its boundary tori is decomposed into two annuli. Contract one of them. Since the gluing introduces a factor of \(X/[j_{n+1}]\), the invariant of the
manifold is
\[ X^{-n-1} \sum_{j_1, \ldots, j_{n+1}, \delta} \frac{[\delta j_{n+1}]}{[j_{n+1}]} [j_{n+1}j_n] t^{-a_n(j_n^2 - 1)} [j_nj_{n-1}] \cdots [j_2j_1] t^{-a_1(j_1^2 - 1)} \times [j_1k] \beta_{j_{n+1}}^{n+1} \otimes \beta_{\delta}^{\delta} \otimes \beta_{k}^{k}. \]

At this moment we have the right 3-manifold but with the wrong DAP-decomposition. We need to fix the DAP-decomposition of the torus that corresponds to the basis element \( \beta_{j_{n+1}}^{n+1} \) (the boundary of the regular neighborhood of the \((p', q')\)-curve) such as to transform the decomposition curve into the meridian of the link component. For this we apply the move \((ST^{-a_n}ST^{-a_1}S \cdots ST^{-a_1}S)^{-1}\). We obtain the following expression for the invariant of the extended manifold
\[ X^{-2n-2} \sum_{j_1, \ldots, j_{2n+2}, \delta, k, m} [m_{j_{2n+2}}][j_{2n+2}j_{2n+1}] t^{a_1j_{2n+1}} \cdots [j_{n+1}j_{n}] [j_{n}j_{n-1}] \cdots [j_2j_1] t^{-a_1j_1^2} [j_1k] \beta_{\delta}^{\delta} \otimes \beta_{k}^{k} \otimes \beta_{m}^{m}. \]

(in this formula we already reduced \( t^{a_k} \) and \( t^{-a_k} \), \( 1 \leq k \leq n \)).

It is important to observe that after performing the described operations the seams came right, so no further twistings are necessary.

Now fix \( k \) and \( m \), let \( \delta = d \pm 1 \) and focus on the coefficients of \( \beta_{d \pm 1}^{d \pm 1} \otimes \beta_{k}^{k} \otimes \beta_{m}^{m} \). Multiplied by \( X \) these are the colored Jones polynomial of the link \([6]\) with the \((p', q')\)-curve colored by the \(d + 1\)-, respectively \(d - 1\)-dimensional irreducible representation of \( U_t \). Since \( C(p, q) = S((d + 1)p', (d + 1)dq') - S((d - 1)p', (d - 1)dq') \) and also
\[ \frac{[(d + 1)j_{n+1}]}{[j_{n+1}]} - \frac{[(d - 1)j_{n+1}]}{[j_{n+1}]} = t^{2j_{n+1}} + t^{-2j_{n+1}}, \]
we deduce that the value of \( < C(p, q)V^k(\alpha), V^m(\alpha) > \) is equal to
\[ X^{-2n-1} \sum_{j_1, \ldots, j_{2n+2}} [m_{j_{2n+2}}][j_{2n+2}j_{2n+1}] t^{a_1j_{2n+1}} \cdots [j_{n+1}j_{n}] [j_{n}j_{n-1}] \cdots [j_2j_1] t^{-a_1j_1^2} [j_1k] \times [j_{n+1}j_{n}] t^{-a_nj_n^2} [j_{n}j_{n-1}] \cdots [j_2j_1] t^{-a_1j_1^2} [j_1k]. \]

We want to compute these iterated Gauss sums. We apply successively Lemma 3.1 starting with \( x = j_n, y = j_{n+1} \), then \( x = j_{n-1}, y = j_{n+2} \) and so on to obtain
\[ X^{-2n+1} \sum_{j_1, \ldots, j_{2n+2}} t^{-a_n}d^2 [m_{j_{2n+2}}][j_{2n+2}j_{2n+1}] t^{a_1j_{2n+1}} \cdots [j_{n+3}j_{n+2}] \times [j_{n-1}(j_{n+2} + d)] t^{-2a_n}d_{j_{n+2}} + [j_{n-1}(j_{n+2} - d)] t^{2a_n}d_{j_{n+2}} \times t^{-a_n}j_{n-1}^2 [j_{n-1}j_{n-2}] \cdots [j_2j_1] t^{-a_1}j_1^2 [j_1k]. \]
and the theorem is proved.

□

on the one hand, and

\[ C(m - p, n - q) V^k(\alpha) = t^{-(m-p)(n-q)} (t^{2k(n-q)} V^{k-m-p}(\alpha) + t^{2k(n-q)} V^{k+m+p}(\alpha)) \]

on the other. An easy check of coefficients shows that the product-to-sum formula holds on the basis of the Hilbert space of the quantization, and we are done.

**Remark 3.3.** The same method can be applied to prove the product-to-sum formula for the Kauffman bracket skein algebra of the torus (see [3]) using the topological quantum field theory with corners constructed in [7].

We conclude with a question. Is the product-to-sum formula true when the deformation variable \( t \) is not a root of unity? The quantization for
arbitrary $t$ is described in [1], and a proof of the formula should involve the analysis of the quasitriangular $R$ matrix of $\mathcal{U}_t$.

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