Collective radiance of giant atoms in non-Markovian regime

Qing-Yang Qiu, Ying Wu, and Xin-You Lü*

School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China

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We investigate the non-Markovian dynamics of two giant artificial atoms interacting with a continuum of bosonic modes in a one-dimensional (1D) waveguide. Based on the diagrammatic method, we present the exact analytical solutions, which predict the rich phenomena of collective radiance. For the certain collective states, the decay rates are found to be far beyond that predicted in the Dicke model and standard Markovian framework, which indicates the occurrence of super-superradiance. The superadiance-to-subradiance transition could be realized by adjusting the exchange symmetry of giant atoms. Moreover, there exist multiple bound states in continuum (BICs), with photons/phonons bouncing back and forth in the cavity-like geometries formed by the coupling points. The trapped photons/phonons in the BICs can also be re-released conveniently by changing the energy level splitting of giant atoms. The mechanism relies on the joint effects of the coherent time-delayed feedback and the interference between the coupling points of giant atoms. This work fundamentally broadens the fields of giant atom collective radiance by introducing non-Markovianity. It also paves the way for a clean analytical description of the nonlinear open quantum system with more complex retardation.

Collective radiance, giant atoms, non-Markovianity, bound states in continuum

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1 Introduction

Collective radiance [1-3] is a prototypical topic in the theoretical study of indirect interaction among emitters mediated by photons or phonons. Dicke [4] revealed that the radiation emitted from an assembly of two-level atoms would be enhanced, and the total radiated power is proportional to square of the number of participating dipoles. Generally speaking, the collective enhancement of radiative decay usually comes up when the emitters are very close to each other. However, there is still a case when the separation between neighboring emitters becomes comparable to the coherence length of an independently emitted photon. In this regime, the traveling time of a photon, normally ignored under the Markov approximation, is no longer negligible, since it will form a link among the emitters and then distinctly affect the interference properties of the electromagnetic field. To avoid the paradox caused by the Markovian approximation [5], the exact theoretical description of non-Markov dynamics becomes very necessary.

It has been shown that the informational back-flow of the non-Markovian environment commonly originates from structured bath spectral densities [6-9] or strong system-bath couplings [10]. Non-Markovian retarded effects have been previously shown in scores of systems, including the geometrically large atomic system [11], the single atom in front of a mirror [12], the double giant cavities system [13], two...
macroscopically separated emitters [14], and a linear chain of $N$ separated qubits [15]. In recent years, a rich variety of numerical and analytical methods, for instance, matrix product states, space-discretized waveguide [16-19], and the diagrammatic method have been proposed to have an entire grasp of the counterintuitive phenomena induced by non-Markovianity [20].

In conventional formalism of light-matter interaction, atoms are generally treated as point-like dipoles. Recently, following the tremendous progress in experimental physics for superconducting circuits [21-24], atomic size in the platforms with surface acoustic waves [25-27] or meandering coplanar waveguide [28] can be comparable to the wavelength of microwave photons they interact with, and thus invalidates the dipole approximation. Artificial atoms with such a novel structure are called superconducting giant atoms. The nature of non-local coupling of giant atoms has led to many interesting effects, including frequency-dependent decays and Lamb shifts [29], single-photon scattering [30, 31], tunable chiral bound states [32], oscillating bound states [33], unconventional electromagnetically induced transparency [34], and decoherence-free atomic interaction [35]. While the non-Markovian dynamics of a single two-legged giant atom can be well described analytically through the Lambert W function [36], the collective performance of many giant atoms with retarded feedback is yet perplexing.

Here we study the collective radiance of two giant artificial atoms in the 1D waveguide quantum electrodynamics (QED) system [37-41]. We find the analytical solutions of the atomic excitation probability amplitudes by solving the relevant multi-delay differential equations with the diagrammatic method. For the purpose of further insights of collective behavior in the non-Markovian regime, we also present the exact atomic collective decay rates of the considered model, where super-superradiance [42] can be observed for certain critical atomic separation. The complex interference effects of delayed photons induce a wealth of dynamical behaviors, including the multiple enhanced radiation bursts and long-lived BICs. Interestingly, those collective radiance dynamics, e.g., the superradiance-to-subradiance transition and the change of BIC areas, are tunable with respect to the initial states, atomic spacing and coupling arrangement of the two giant atoms with the waveguide. Moreover, we also show that the trapped photons/phonons within BIC can be re-released by applying a time-dependent energy level splitting on the giant atoms.

2 Model and multi-delay differential equations

In Figure 1, we show two types of theoretical models and the corresponding experimental implementations for the giant atoms. By neglecting counter-rotating terms and apply-
ing electric-dipole approximation, the total Hamiltonian of the system reads ($h = 1$)

$$\hat{H} = \frac{\omega_0}{2} \sum_{m=0}^{\infty} \hat{\sigma}_m^{(+)} \hat{\sigma}_m^{(-)} + \sum_{a = R, L} \int_0^{\infty} \frac{\omega}{2} \hat{a}_a^{\dagger}(\omega) \hat{a}_a(\omega) d\omega + \sum_{a = R, L} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \int_0^{\infty} g(\omega) \hat{a}_n^{\dagger}(\omega) e^{i \epsilon_n x_m / v_g} d\omega + H.c.$$,  

(1)

where $\hat{a}_a(\omega)$ is the annihilation operator of the right- ($\alpha = R$) or left- ($\alpha = L$) moving photons (or phonons) satisfying $[\hat{a}_\alpha(\omega), \hat{a}_\beta^{\dagger}(\omega')] = \delta_{\alpha\beta}\delta(\omega - \omega')$. $x_m$ denotes the $m$th coupling point of $m$th giant atom, and $v_g$ is the group velocity of the field in the waveguide. The first and second terms on the right hand represent the free Hamiltonians of giant atoms and 1D bosonic modes, respectively. The resonant frequency of emitters $\omega_0$ is assumed to be far away from the cut-off frequency of the waveguide, which confirms that a perfect linearized dispersion can be applied. Moreover, $\hat{\sigma}_m^{(+)} = |e_m^{(m)} g|$ is the raising operator of $m$th emitter, and the notations $\epsilon_L = -1$ and $\epsilon_R = 1$ have been introduced in the third term.

To obtain the non-Markov dynamical property of the system, we proceed by analytically determining the excitation probability amplitudes of two giant atoms. The following discussion has been limited into a single-excitation subspace, and then the instantaneous state is

$$|\psi(t)\rangle = \sum_{m=0}^{\infty} c_m(t) \hat{\sigma}_m^{(+)} |g, g, 0\rangle + \sum_{a = R, L} \int_0^{\infty} d\omega \phi_a(\omega, t) \hat{a}_a^{\dagger}(\omega) |g, g, 0\rangle,$$

(2)

where $|g, g, 0\rangle$ is the ground state of system and $|0\rangle$ denotes the vacuum state of 1D waveguide modes. Moreover, $\phi_L(\omega, t), \phi_R(\omega, t)$ denotes the amplitude of probability to find a single photon propagating to left/right at time $t$ with mode $\omega$ in the waveguide. In the interaction picture, the system dynamics is decided by the Schrödinger equation $i \frac{d}{dt} |\psi(t)\rangle = \hat{H}_{\text{int}}|\psi(t)\rangle$, where $\hat{H}_{\text{int}}$ is the interaction Hamiltonian of the system. Tracing out the bosonic modes, then we obtain the equations of motion (EOMs) of separate giant atoms, and it can be described as (see Supporting Information S1 for more details)

$$\dot{c}_m(t) = -\gamma c_m(t) - F_{m, 1}(t) - F_{m, 2}(t) - \frac{1}{2} \left(F_{m, 1}(t) + F_{m, 2}(t)\right)$$

(3)

for $m \neq n$. Similarly, the atomic evolution of excitation amplitudes of braided giant atoms is governed by [43]

$$\dot{c}_m(t) = -\gamma c_m(t) - F_{m, 2}(t) - \frac{1}{2} \left[3F_{m, 1}(t) + F_{m, 3}(t)\right],$$

(4)

where $F_{m, n}(t) \equiv \gamma e^{i\varphi} c_m(t - n\Delta t) \Theta(t - n\Delta t)$ and $\varphi \equiv \omega_0\Delta t = k_0\Delta x$ is the field propagation phase difference between two adjacent legs. Here, the Heaviside step function $\Theta(\bullet)$ is introduced to hold causality and describes the time-delayed feedback of information among the multiple legs. The equidistant distribution of coupling points and the homogeneous decay rate $\gamma$ in each connecting point have been considered for simplicity. We also have assumed a flat spectral density of the waveguide modes around the resonance of the giant atoms, i.e., $g(\omega) \approx g(\omega_0) = \sqrt{\frac{\gamma}{(4\pi)}}$.

The first term on the right-hand side of eq. (4) depicts the spontaneous emission processes due to the Markovian dynamics. The second term stems from the nature of nonlocal coupling of a single giant atom, and the remainder describes the atomic relaxation dynamics controlled by the delayed photons released from another emitter. A differential equation with a series of information feedbacks like eqs. (3) and (4) is mathematically called a multi-delays differential equation. Here, we obtain the exact analytical description of the system’s non-Markovian dynamical evolution via a diagrammatic method [20], which is excellently consistent with the fully numerical simulation. After some algebra, the general expression for the probability amplitude of two-atom-state is given by (see Supporting Information S2 for more details)

$$c(t) = \frac{1}{2} \sum_{l=0}^{\infty} K_l(t) \sum_{s=0}^{l} D_s(t) f_s,$$

(5)

where $K_l(t) \equiv \gamma^{l-t-s\Delta l}$ and $D_s(t) \equiv \sqrt{2c_0(t) \times e^{-i\varphi + i\varphi s(t - \Delta l)}} \Theta(t - l\Delta l)$ are defined for conciseness, and $f_s$ closely relies on both the specific configurations of the giant atoms and the atomic initial states. When the giant atoms are initially in the symmetric and antisymmetric states, i.e., $|\psi(0)\rangle = \pm \frac{1}{\sqrt{2}}(|eg\rangle \pm |ge\rangle)$, the corresponding amplitude $c_L(t)$ can be given by eq. (5). In Figure 2(a) and (b), we plot the atomic amplitude $|c_L(t)|^2$ in the early four basic traveling time (4\Delta t), which clearly shows the consistent analytical (dotted line) and numerical (solid line) results. Then the above analytical expression can be used to simulate other interesting phenomena induced by coherent time-delayed feedback in the later discussion.

3 Collective decay rates

We next study the collective decay rates of two giant atoms that interact with a common radiation field. The interference between emissions from the coupling points of the giant atoms may enhance or suppress the decay for certain states. This phenomenon is famous as super/subradiance [4]. Here, we utilize the real-space approach to describe quantitatively
for more details). Such a nonlinear characteristic equation can be reduced as a polynomial equation in the presence of the negligible delay, which gives the analytical solutions of the Markovian collective decay rates. More concretely, $\Gamma_{+M} = (2 + 3e^{i\varphi} + 2e^{2i\varphi} + e^{3i\varphi})\gamma$, $\Gamma_{-M} = (2 + e^{i\varphi} - 2e^{2i\varphi} - e^{3i\varphi})\gamma$ for separate giant atoms, and $\Gamma_{+NM} = (2 + 3e^{i\varphi} + 2e^{2i\varphi} + e^{3i\varphi})\gamma$, $\Gamma_{-NM} = (2 + 2e^{2i\varphi} - 3e^{i\varphi} - e^{3i\varphi})\gamma$ for braided giant atoms, where $\Gamma_{+/-M}$ denotes the collective decay rate of the symmetric/anti-symmetric state in the Markovian regime.

We plot the analytical Markovian collective decay rates $\Gamma_{+M}$ and the numerical non-Markovian collective decay rates $\Gamma_{+NM}$ for both braided and separate giant atoms in Figure 2(c) and (d) by gradually increasing $\Delta x$. Firstly, it is shown that the Markovian collective decay rates guide effectively the real behavior of collective dynamics if the connecting points are slightly separated. When the distance $\Delta x$ is increased to a critical value $\Delta x_c$, the superradiant collective decay rate extends far beyond than that revealed by the Markovian approximation. Secondly, in the large separation limit, both the symmetric and antisymmetric non-Markovian collective decay rates drastically deviate from the Markovian ones, and tend to be subradiant. This suppression of decay can be explained as follows. After a giant atom finishes the initial emission from its coupling points, it can be re-excited by the wavepackets reflected from all other coupling points. The process of looping emission-reexcitation is repeatedly cycled over and over again and then the light needs longer time to struggle to free itself, which prolongs the effective lifetime of the emitters [44]. Lastly, the $2N^2\gamma$ ($N$ is the total connecting legs of a single giant atom) scaling of the maximum Markovian decay rate for two giant atoms surpasses the scaling of $4\gamma$ given by well-known Dicke superradiance for four small atoms. More interestingly, this scaling is further improved by introducing self-consistent coherent time-delayed feedback. Here we obtain that the super-superradiant decay rate reads about $9.51\gamma$ for separate atoms and $17.26\gamma$ for braided atoms. Different from the case of traditional “small” atoms, the condition $\omega_0\Delta x_c = n\pi$ is not necessary to achieve a maximum superradiance.

4 Collective emission dynamics and BICs

For illustration purposes, we further show the time evolution of the atomic excitation probability in Figure 3. Collective emission dynamics can be significantly modified by the geometric structures of the giant atoms, retardation $\eta \equiv \gamma\Delta t$, atomic initial states, and the phase $\varphi$ of the electromagnetic field acquired upon propagation. In Figure 3(a) and (b), we show the dynamics of symmetric state $|+\rangle$ with $\varphi = 2\pi\eta$. By adjusting the distance of adjacent coupling legs from zero ($\eta = 0$) to a slightly separated length (e.g., $\eta = 0.03$), atomic
evolution will go through from an exponential behavior \((\Gamma=8\gamma)\) to a non-exponential behavior, where a given average decay rate may be drastically beyond \(8\gamma\). As the retardation gets a further increase (e.g., \(\eta = 0.2\)), oscillating behavior of the atomic excitation amplitudes can be captured as a remarkable symbol of the non-Markovian recovery phenomenon. Finally, the atomic time evolution tends to become independent radiation \((\Gamma=2\gamma)\) with a large atomic separation \((\eta = \infty)\). Besides, one may note that both the separate and braided giant atoms share a same dynamical process if the atomic initial state is \(|+\rangle\).

Figure 3(c) and (d) shows the subradiant emission dynamics of anti-symmetric state \(|-\rangle\) with \(\varphi = 2\pi n\). In this case, the emitters can be frozen in the excited states as the retardation is set to be zero. For a finite delay (e.g., \(\eta = 0.2\)), the atomic excitation probability may rest on a fixed value in the long time limit and maintain dynamic equilibrium with surrounding trapped photons. Specifically, this probability reaches the asymptotic value \((1 + 3\eta)^{-2}\) and \((1 + \eta)^{-2}\) for separate giant atoms and braided giant atoms, respectively. In Figure 3(e) and (f), we demonstrate that how the atomic evolution is manipulated by both \(\varphi\) and initial states for a given retardation. In the case of separate giant atoms, the symmetric state \(|+\rangle\) will transfer from a non-radiative state to a radiative state, and the anti-symmetric state \(|-\rangle\) is going to jump to a lower populated steady state, while the phase shifts from \(2(\pi + 1)\) to \(2\pi n\). In another case of braided giant atoms, perfect synchronization in the time domain occurs by exchanging both the initial state \((|+\rangle \rightarrow |-\rangle\) and the phase \((\pi \rightarrow 2\pi n)\).

Moreover, it also can be shown from Figure 3 that, for a finite retardation \(\eta > 0\), a purely exponential decay can be observed before \(t = \Delta t\) during which no delayed signals are received. After this independent emission process, one may discover multi-level collective radiation enhancement as soon as \(t = n\Delta t\) due to the constructive multi-path interference between the spontaneous radiation and stimulated radiation. Or conversely, the system would evolve spontaneously into a deterministic BIC if \((\text{BIC}|_{\psi(t = 0)}\rangle \neq 0\). Long-lived bound states in the continuum are waves that remain localized even though they coexist with an extended spectrum of radiating waves that can carry excitations away [45-48].

Two conventional approaches to populate these states in this regard are spontaneous dissociation of initially excited atoms [12, 14, 33] and multi-photon scattering on an unexcited atomic system [49-51]. Here, we adopt the former way to generate system BICs during which the giant atom legs behave equivalently as effective mirrors. Then the giant atoms and surrounding trapped photons ultimately reach the dynamic equilibrium. Physically, such bound states stem from the destructive interference between the spontaneous radiation, reabsorption and stimulated radiation as described in eqs. (3) and (4). When the giant atoms are initially prepared in an anti-symmetric state, the analytical descriptions of BICs as shown in Supporting Information S4 are given by

\[
|\psi^b_{\text{BIC}}\rangle = \frac{1}{\sqrt{1 + \gamma \Delta t}} \left\{ \frac{1}{\sqrt{2}} \left( |\text{eg}\rangle - |\text{ge}\rangle \right) \otimes |\text{vac}\rangle - \frac{\gamma \nu\pi}{2\pi} \int_0^{\infty} \frac{dk}{k} \sin \frac{k d}{\omega - \omega_0} \right\},
\]

\[
|\psi^a_{\text{BIC}}\rangle = \frac{1}{\sqrt{1 + 3\gamma \Delta t}} \left\{ \frac{1}{\sqrt{2}} \left( |\text{eg}\rangle - |\text{ge}\rangle \right) \otimes |\text{vac}\rangle - \frac{\gamma \nu\pi}{2\pi} \int_0^{\infty} \frac{dk}{k} \sin \frac{k d}{\omega - \omega_0} \right\},
\]

where \(|\psi^b_{\text{BIC}}\rangle\) denotes the BIC in a system formed by separate/braided giant atoms coupled to a 1D waveguide, \(d \equiv \Delta x\), and \(|\emptyset\rangle\) is the ground state of the total system. As shown in eqs. (7) and (8), we found quantitatively a significant overlap between BIC and the initial states in the long time limit. Specifically, we obtain that \(c.e.(t \to \infty) = \langle |\psi_{\text{BIC}}\rangle |^2\) with a value \(1/(1 + 3\gamma \Delta t)\) for separate atoms and \(1/(1 + \gamma \Delta t)\) for braided atoms, which are consistent with the calculations given by the final value theorem in the Laplace space (see Supporting Information S1 for more details).
5 Multi-radiation bursts and tunable output field

Until now, we have studied the non-Markovian dynamics of the atomic part in the preceding sections. Synergistic effect of emitted photons is another interesting viewpoint to give a more complete dynamical description of the atom-field quantum system. We thus continued by investigating the interference patterns of radiation field, which can be characterized by the bosonic field density distribution (FDD) with a general form \( I \propto \langle \psi(t)|E(x,t)\psi(t)\rangle \), where \( E(x,t) \) is the electric field operator. When the emitters are initially prepared in the symmetric (+) or antisymmetric (−) states, the analytical expression of FDD (see Supporting Information S5 for more details) is given by

\[
I_k(x,t) = \frac{\gamma_0}{(2\pi \hbar)^2} \sum_{i,j} |(R_{ij} + L_{ij})|^2,
\]

where

\[
R_{ij} = e^{-\eta(t-t_{ij})} \frac{1}{vg} \left[ \Theta(t-t_{ij}) - \Theta(-t_{ij}) \right],
\]

\[
L_{ij} = e^{-\eta(t+t_{ij})} \frac{1}{vg} \frac{1}{\eta} \left[ \Theta(t+t_{ij}) - \Theta(t_{ij}) \right],
\]

with \( t_{ij} \equiv |x + (-1)^{i+1}x_{ai}|/vg \) with \( i, j = 1, 2 \). The influence of atomic geometric structures and retardation \( \eta \) on FDD is reflected by \( t_{ij} \) and \( c_i(t) \). One may extract vivid physical diagrams from eq. (9) through the unidirectional propagators \( L_{ij} \) and \( R_{ij} \), which describe the unidirectional radiation processes. More specifically, \( R_{ij} \) represents the right-moving emission from the leg labeled by \( x_{a1} \) and \( x_{a2} \), and \( L_{12} \) represents the left-moving emission from the leg labeled by \( x_{a1} \). These radiation fields collide each other to form the interference patterns, which behave superradiant or subradiant.

In Figure 4, we plot the FDD of the bosonic field, where each coupling point leads to a light cone expanding outward as a whole. In the case of superradiance, a collective radiation burst occurs while two cones meet each other, leading to the coherent enhancement. It is shown from Figure 4(a) and (c) that the multi-radiation bursts are eventually formed in a region outside the atomic ensemble in the presence of multiple radiation sources along the 1D waveguide. In another case of subradiance, the local behavior of radiation field can be observed where the trapped photons or phonons bounce back and forth between the connecting points in several specific areas (see Figure 4(b) and (d)).

In the above discussion, we find that certain photons may be trapped in specific regions. Interestingly, the excitations limited in such a photon-emitter bound state can be re-released by applying a time-dependent energy level splitting on the giant atoms. As the giant atoms generally modeled by superconducting qubits, this operation can be realized conveniently in a manner of applying an external bias magnetic field. We determine the dynamical manipulation process of the system by putting a detector at \( x = x_{a2} + x_0 \). The corresponding output field amplitudes from two separate/braided giant atoms at time \( t = \bar{t} + x_0/v_g \) read

\[
|\varphi_{\bar{t}}(\bar{t})| = \frac{2}{\gamma_0 v_g} \left[ F_{1,3}(\bar{t}) + F_{1,2}(\bar{t}) + F_{1,3}(\bar{t}) + F_{2,2}(\bar{t}) \right],
\]

\[
|\varphi_{\bar{t}}(\bar{t})| = \frac{2}{\gamma_0 v_g} \left[ F_{1,3}(\bar{t}) + F_{1,3}(\bar{t}) + F_{2,2}(\bar{t}) + F_{2,2}(\bar{t}) \right].
\]

In Figure 5, we plot the output field intensity \( |\varphi_{\bar{t}}(\bar{t})|^2 \) for photons flowing through \( x \) over time \( \bar{t} \). Atomic ensemble reaches the stable equilibrium by decaying a part of excitations to the waveguide (in a region filled by brown). Even then, we change the atomic energy gap so that \( \omega_0 \Delta \tau = n\pi \) no longer holds to meet the conditions of generating system’s BIC. Therefore, the detector will detect the optical flow again (in a region filled by dark-green). In a short summary, the number of photons in the cavity formed by the coupling points could be manipulated based on the above procedure.

6 Conclusions and outlooks

In summary, we have studied the non-Markovian collective emission of two giant atoms coupled to a 1D bosonic environment. Based on the analytical and numerical results, we found that this system allows the occurrence of a strongly
modified decay process outside the scope of normal super/subradiance, which is originally triggered by both the giant atomic effects and coherent time-delayed feedback. The system also supports the localized states embedded in the continuum, with photons or phonons bouncing back and forth between coupling points in certain selective areas.

This work opens up new eyesight in the non-Markovian dynamical system with multi-delay signals. Possible applications of our results in quantum information science include the realizations of entangled emitter-photon states, dynamically adjustable bound states, long-range interaction between distant emitters, and the time-dependent superradiant laser. It then would be interesting to combine the effects of intricate time-delays and topological or chiral quantum optics [52].

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Supporting Information

The supporting information is available online at http://phys.scichina.com and https://link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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Figure 5 (Color online) Real-time detection of output field intensity of two separate (a) and braided (b) giant atoms. The dynamical evolution process of atomic states and corresponding transition frequency \( \gamma_m \) versus \( t/\gamma_m \) are displayed in the insets. The temporal profiles of output field for the first four traveling time 4\( \Delta t \) (filled by the brown area) in (a) and (b) are consistent with the cases shown in Figure 4(b) and (d), respectively. After a proper time, we break the parameter condition for dark modes and then the detector receives the optical flow signals again (filled by the dark-green area).
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