The Race between Technological Progress and Female Advancement: Changes in Gender and Skill Premia in OECD Countries

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Abstract

In recent decades, the male–female wage gap has fallen, while the skilled–unskilled wage gap has risen in advanced countries. The rate of decline in the gender wage gap has tended to be greater for unskilled than skilled workers, while the rate of increase in the skill wage gap has tended to be greater for male than female workers. To account for these trends, we develop an aggregate production function extended to allow for gender-specific capital–skill complementarity, and estimate it using shift–share instruments and cross-country panel data from OECD countries. We confirm that information and communication technology (ICT) equipment is not only more complementary to skilled than unskilled workers but also more complementary to female than male workers. Our results show that changes in gender and skill premia are the outcome of the race between progress in ICT and advances in female educational attainment and employment.

KEYWORDS: Gender wage gap; skill premium; capital–skill complementarity; information and communication technology; production function.

JEL CLASSIFICATION: C33, E23, E24, J16, J24, J31, O50.

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1 Introduction

There have been substantial changes in wage inequality worldwide over recent decades. The male–female wage gap (measured by the ratio of average wages of male workers to average wages of female workers) has fallen in many countries, while the skilled–unskilled wage gap (measured by the ratio of average wages of college graduate workers to average wages of non-college graduate workers) has risen in some countries (Krueger, Perri, Pistaferri and Violante, 2010). As described later in Figure 1, both for the United States and the OECD average, the rate of decline in the gender wage gap has tended to be greater for unskilled workers (defined as non-college graduate workers) than skilled workers (defined as college graduate workers), while the rate of increase in the skill wage gap has tended to be greater for male than female workers.

What could account for these trends in advanced countries? Although it is well known that there were a fall in the male–female wage gap and a rise in the skilled–unskilled wage gap in some countries such as the United States, it is perhaps less known that there were differences in the trends of the male–female wage gap between skilled and unskilled workers and of the skilled–unskilled wage gap between male and female workers. When the skilled–unskilled wage gap is viewed as the relative price of skills in the labor market, changes in the premium to skill can be interpreted as the outcome of the race between the supply of and demand for skills (Tinbergen, 1974). The supply of skills varies according to the educational attainment of the workforce, while the demand for skills varies according to skill-biased technological change (Katz and Murphy, 1992; Autor, Katz and Kearney, 2008; Goldin and Katz, 2010). In this context, the difference of changes in the premium to skill between male and female workers could be attributed to a difference in the relative magnitude of shifts in the supply of and demand for skills between male and female workers. From this viewpoint, the rate of increase in the skill wage gap can be greater for male than female workers, if the excess demand for skills is greater for male than female workers. In this study, we examine whether this argument is
consistent with data and whether the same logic applies to the gender wage gap.

When the male–female wage gap is viewed as the relative price of skills possessed by men and women, the relative demand for skills can vary according to the expansion of new technology such as information and communication technology (ICT), while the relative supply of skills can vary according to advances in female educational attainment and employment. We consider formulating the demand shift due to technological progress in terms of capital–skill complementarity (Griliches, 1969). If capital equipment is more complementary to skilled than unskilled labor, a rise in capital equipment results in a rise in the demand for skilled relative to unskilled labor, and hence, a rise in the skilled–unskilled wage gap (Krusell, Ohanian, Rios-Rull and Violante, 2000). We examine whether the same logic applies to substitution between ICT equipment, male labor, and female labor. If ICT equipment is more complementary to female than male labor, a rise in ICT equipment would result in a fall in the demand for male relative to female labor, and hence, a fall in the male–female wage gap. The rationale behind this hypothesis is that men and women have different comparative advantages (Welch, 2000), and the increased importance of interpersonal skills, in which women have a comparative advantage, is associated with the increased use of computers (Borghans, ter Weel and Weinberg, 2014). In addition, a rise in the educational attainment and employment of women relative to men has brought substantial changes to the composition of the workforce. Ceteris paribus, the more the relative supply, the lower the relative wages. Although the supply shift cannot, by itself, account for a fall in the male–female wage gap or a rise in the skilled–unskilled wage gap, it may contribute to making a difference in the trends of the gender wage gap between skilled and unskilled workers and of the skill wage gap between male and female workers.

In this study, we decompose changes in gender and skill premia into the effect attributable to the demand shift (i.e., the capital–skill complementarity effect) and the effect attributable to the supply shift (i.e., the relative labor quantity effect), as a step towards understanding the sources and mechanisms of changes in wage inequality. We develop an aggregate production function extended to allow for gender-specific capital–skill complementarity, which forms the basis
for the decomposition analysis, and then estimate it using shift–share instruments and cross-country panel data from OECD countries for the years 1980 to 2005. The main contributions of this paper to the literature are twofold. First, we provide the first estimates for the elasticities of substitution between ICT capital and four types of labor (male skilled, female skilled, male unskilled, and female unskilled labor). We demonstrate that ICT capital is not only more complementary to skilled than unskilled labor but also more complementary to female than male labor. The estimated elasticities of substitution between ICT capital and these four types of labor are large in the order of male unskilled, female unskilled, male skilled, and female skilled labor. Second, we provide a greater understanding of trends in the male–female wage gap and the skilled–unskilled wage gap. Changes in gender and skill premia in OECD countries can be accounted for by the relative magnitude of the capital–skill complementarity effect and the relative labor quantity effect that work in the opposite direction. Differences in the gender wage gap between skilled and unskilled workers and in the skill wage gap between male and female workers can be understood in terms of race between progress in ICT and advances in female educational attainment and employment.

The rest of this paper proceeds as follows. The next section reviews the literature. Section 3 describes data used in the analysis. Section 4 presents the model and considers the estimation procedure. Section 5 discusses the results. The final section provides a summary and conclusion.

2 Related Literature

This paper is related to three strands of literature. First, the paper is related to the literature on the skill wage gap. Katz and Murphy (1992), Goldin and Katz (2010), and Autor, Katz and Kearney (2008), among others, estimate a constant-elasticity-of-substitution (CES) aggregate production function with two types of labor (skilled and unskilled labor) in the United States. In this setup, given the fact that the relative supply of skilled labor has increased over
time, widening skill wage gap is attributed to the unobserved demand shift referred to as skill-biased technological change. Krusell, Ohanian, Rios-Rull and Violante (2000) estimate a nested CES aggregate production function, in which capital equipment can be more complementary to skilled than unskilled labor, and attribute widening skill wage gap in the United States mainly to a rise in capital equipment.

Second, this paper is related to the literature on the gender wage gap. Acemoglu, Autor and Lyle (2004) estimate a CES aggregate production function with two types of labor (male and female labor) in the United States. In this setup, given that the relative supply of female labor has increased over time, narrowing gender wage gap is attributed to the unobserved demand shift referred to as gender-biased technological change (Heathcote, Storesletten and Violante, 2010). Ngai and Petrongolo (2017) develop a multi-sector general equilibrium model, calibrate it to the U.S. economy, and attribute narrowing gender wage gap in the United States mainly to changes in industrial structure (i.e., the shift in employment from goods to service industries).

Finally, this paper is related to the literature on the labor market impact of computerization. The literature points to computerization as a cause for the increased demand for skilled and female labor. Autor, Katz and Krueger (1998) show that the wage-bill share of skilled labor is higher in computer-intensive industries in the United States. Machin and Van Reenen (1998) and Michaels, Natraj and Van Reenen (2014) show that an increase in the wage-bill share of skilled labor is associated with the increased use of computers in OECD countries. Weinberg (2000) shows that the female shares of employment and hours worked are higher in computer-intensive occupations and industries in the United States. Raveh (2015) shows that an increase in the wage-bill share of female labor is associated with the increased use of ICT in OECD countries. The literature also points to computerization as a source of changes in occupational skill requirements contributing to changes in wage inequality. Autor, Levy and Murnane (2003) and Spitz–Oener (2006) show that the relative increase in nonroutine analytical and interactive tasks, which contributes to increasing demand for skilled labor in the United States and West Germany, is associated with computerization. Borghans, ter Weel and Weinberg (2014) show
that the increased importance of interpersonal skills, which contributes to narrowing the gender wage gap in Britain, Germany, and the United States, is associated with the increased use of computers. Black and Spitz–Oener (2010) show that the relative decrease in routine cognitive and manual tasks, which contributes to narrowing the gender wage gap in West Germany, is associated with computerization.

The closest study to ours is by Beaudry and Lewis (2014), who examine the impact of personal computer adoption on gender and skill premia using cross-city data for the years 1980 to 2010 in the United States. Beaudry and Lewis (2014) show that computerization contributes not only to widening the skill wage gap but also to narrowing the gender wage gap in the United States. Apart from obvious differences in the sample and variables, our study differs from theirs in important ways. First of all, we aim to account for differences in the trends of the male–female wage gap between skilled and unskilled workers and of the skilled–unskilled wage gap between male and female workers in advanced countries. For this purpose, we allow for the heterogeneous impact of ICT on the gender wage gap by skill and on the skill wage gap by gender. Second, we consider a more general production function and estimate the elasticities of substitution among capital and labor inputs. Finally, we conduct the decomposition analysis to deepen understanding on the sources and mechanisms of changes in gender and skill premia.

3 Data

We start this section by describing the sample and variables used in the analysis. We, then, document trends in the male–female wage gap and the skilled–unskilled wage gap in OECD countries. We end this section by providing the preliminary analysis of ICT and wage inequality.

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1See also Burstein, Morales and Vogel (2019), who show that, using a general equilibrium assignment model, computerization plays a major role in explaining changes in skill and gender premia between the years 1984 and 2003 in the United States.
3.1 Sample and variables

The data used in our analysis are from the EU KLEMS database, which collects detailed and internationally comparable information on the prices and quantities of capital and labor inputs in major OECD countries. Our analysis uses the March 2008 version because it contains the longest time series covering the 1980s, during which there were dramatic changes in technology and inequality. For the years 1980 to 2005, we include in the sample all countries, industries, and years, for which data needed for estimation are available. Our sample comprises 11 OECD countries: Australia, Austria, the Czech Republic, Denmark, Finland, Germany, Italy, Japan, the Netherlands, the United Kingdom, and the United States. Each country is composed of 30 industries. The sample includes 260 country-year observations.

Labor inputs are divided into skilled and unskilled labor, each of which is further divided into male and female labor. Skilled labor consists of workers who completed college, and unskilled labor consists of workers who did not enter or complete college. Wages are calculated by gender and skill by dividing total labor compensation by total hours worked for all workers (including part-time, self-employed, and family workers without age restrictions) in all industries (except private households with employed persons) and all jobs (including side jobs). We adjust for changes in the labor composition and efficiency according to demographic changes in a way similar to Autor et al. (2008). Appendix A.1 provides details of the adjustment procedure.

Capital inputs are divided into ICT and non-ICT capital. ICT capital consists of computing equipment, communications equipment, and software, while Non-ICT capital consists of transport equipment, other machinery and equipment, and non-residential structures and infrastructures. The rental price of capital is calculated in the standard way as described in Jorgenson (1963) and O’Mahony and Timmer (2009). Appendix A.2 provides details of the calculation procedure.

All variables measured in monetary values are converted into U.S. dollars using the purchasing power parity index and deflated using the gross value added deflator as described in
3.2 Trends in the relative input prices and quantities

During the period between the years 1980 and 2005, both for the United States and the OECD average, there was a stark difference in the trends of the male–female wage gap between skilled and unskilled workers (Figures 1a and 1b). The male–female wage gap fell among unskilled workers, while it did not among skilled workers. This is perhaps surprising but not inconsistent with the fact documented by Blau and Kahn (2017) that the rate of decline in the gender wage gap in the United States was greater at the middle or the bottom than at the top of the wage distribution from the 1980s to the 2000s. At the same time, the skilled–unskilled wage gap increased significantly both among male and female workers in the United States, but the rate of increase was much greater among male than female workers (Figure 1c). In OECD countries, on average, the skilled–unskilled wage gap increased moderately among male workers after a slight drop in the early 1980s, while it did not increase among female workers except for the period between the years 1987 and 1995 (Figure 1d). In summary, from the 1980s to the 2000s, both for the United States and the OECD average, the rate of decline in the male–female wage gap was greater among unskilled than skilled workers, while the rate of increase in the skilled–unskilled wage gap was greater among male than female workers.

Turning to the relative quantities of labor, there were a fall in the supply of male relative to female labor and a rise in the supply of skilled relative to unskilled labor during the same period. It is interesting to note that the rate of decline in the relative supply of male labor differs by skill, and the rate of increase in the relative supply of skilled labor differs by gender. Both for the United States and the OECD average, the relative supply of male labor fell a lot more among skilled than unskilled workers (Figures 2a and 2b), while the relative supply of skilled labor increased a lot more among female than male workers (Figures 2c and 2d).
Figure 1: Gender and skill premia

(a) Male vs. female, United States

(b) Male vs. female, OECD

(c) Skilled vs. unskilled, United States

(d) Skilled vs. unskilled, OECD

Notes: The wages of male skilled, female skilled, male unskilled, and female unskilled labor are denoted by $w_{mh}$, $w_{fh}$, $w_{mu}$, and $w_{fu}$, respectively. All series are logged and normalized to zero in the year 1980.

During the period between the years 1980 and 2005, trends in the rental price of capital are quite different between ICT and non-ICT capital both for the United States and the OECD average (Figures 3a and 3b). The rental price of ICT capital fell dramatically, while that of non-ICT capital remained almost unchanged. Turning to the quantities of ICT and non-ICT capital, there was a rise in both quantities during the same period. However, the rate of increase in ICT capital was far greater than that in non-ICT capital both for the United States and the OECD average (Figures 4a and 4b).
3.3 Preliminary Analysis

We estimate the elasticities of substitution among capital and labor inputs and decompose changes in gender and skill premia in the next section and onwards. Before proceeding with the analysis, we are concerned about whether a rise in ICT capital influences the gender wage gap and the skill wage gap in OECD countries. To address this concern, we start our analysis by estimating the simplest possible models relating gender and skill premia to ICT capital. For the sake of preliminary analysis, we assume for the moment, as in Beaudry and Lewis (2014), that there is no difference in the impact of ICT capital on the gender wage gap between skilled and unskilled workers and on the skill wage gap between male and female workers.
Let $\Delta x$ denote a change in $x$, and let $c$ and $t$ denote indices for countries and years. We first consider the model relating the male–female wage gap ($w_m/w_f$) to ICT capital ($k_i$).

$$\Delta \ln \left( \frac{w_{m,ct}}{w_{f,ct}} \right) = \beta_g \Delta \left( \frac{k_{i,ct}}{\ell_{m,ct}} \right) - \Delta \ln \left( \frac{\ell_{m,ct}}{\ell_{f,ct}} \right) + \Delta u_{g,ct}, \quad (1)$$

where $\ell_m$ and $\ell_f$ are the quantities of male and female labor, respectively, and $u_g$ is the error term. It is worth noting that equation (1) can be derived from an aggregate production function of the form: $y = A k_o^\alpha [\mu k_i + (1 - \mu) \ell_m]^{1-\alpha-\lambda}$, where $y$ is the output; $A$ is factor-neutral technology; $k_o$ is non-ICT capital; and the share parameters $\alpha$, $\lambda$, and $\mu$ lie between zero and one. We refer to the first term as the capital–skill complementarity effect and the second term as the relative labor quantity effect. This specification, which is borrowed from Autor, Levy and Murnane (2003), assumes that ICT capital is more complementary to female than male labor. In such a case, since a rise in ICT capital should result in a decline in the male–female wage gap, we would expect the parameter of interest, $\beta_g$, to be negative.

We also consider the model relating the skilled–unskilled wage gap ($w_h/w_u$) to ICT capital.

$$\Delta \ln \left( \frac{w_{h,ct}}{w_{u,ct}} \right) = \beta_s \Delta \left( \frac{k_{i,ct}}{\ell_{u,ct}} \right) - \Delta \ln \left( \frac{\ell_{h,ct}}{\ell_{u,ct}} \right) + \Delta u_{s,ct}, \quad (2)$$
Figure 4: Capital quantities

(a) ICT vs. non-ICT, United States
(b) ICT vs. non-ICT, OECD

Notes: The quantities of ICT and non-ICT capital are denoted by $k_i$ and $k_o$, respectively. All series are logged and normalized to zero in the year 1980.

where $\ell_h$ and $\ell_u$ are the quantities of skilled and unskilled labor, respectively, and $u_s$ is the error term. Equation (2) can be derived from an aggregate production function of the form: $y = A k_i^{\alpha} (1 - \mu) \ell_u^{1 - \alpha - \lambda}$. Given this functional form, ICT capital is more complementary to skilled than unskilled labor. In such a case, since a rise in ICT capital should result in an increase in the skilled–unskilled wage gap, we would expect the parameter of interest, $\beta_s$, to be positive.

The parameters, $\beta_g$ and $\beta_s$, can be estimated by regressing $w_m \ell_m / w_f \ell_f$ on $k_i / \ell_m$ and regressing $w_h \ell_h / w_u \ell_u$ on $k_i / \ell_u$, respectively. In doing so, we use shift–share instruments, also known as Bartik (1991) instruments, to take into account the endogeneity of capital and labor quantities. Shift–share instruments are constructed by interacting industry shares and growth rates as follows:

$$
\Delta \ln z_{ct}^{b} = \sum_{d \in \mathcal{D}} \sum_{d' \in \mathcal{D} \setminus \mathcal{C}} \frac{z_{c,d,t-\tau}}{z_{c',d,t-\tau}} \Delta \ln \left( \sum_{c' \notin c \in \mathcal{C}} z_{c',d,t} \right)
$$

for $z \in \{k_i, \ell_m, \ell_f, \ell_h, \ell_u\}$ and $\tau \in \{5, 10, 20\}$, where $c$, $d$, and $t$ are indices for countries, industries, and years, respectively, and $\mathcal{C}$ and $\mathcal{D}$ are sets of countries and industries, respectively. Shift–share instruments consist of shift, which measures global shocks to industries, and share, which measures local exposure to global shocks. Borusyak, Hull and Jaravel (2019) show that
shift–share instruments are valid if either shift or share is exogenous. We use the leave-one-out measure of global industry growth rates to ensure the exogeneity of global shocks.\textsuperscript{2}

The results from the preliminary analysis indicate that the male–female wage gap declines with a rise in ICT capital, while the skilled–unskilled wage gap increases with a rise in ICT capital (Table 1). These results are consistent with those of Beaudry and Lewis (2014) in the United States. The results remain almost unchanged, whether looking at the 5-, 10-, or 20-year differences. A 10 percent increase in ICT capital results in a 1.3 to 1.6 percent decrease in the gender wage gap and a 3.4 to 3.9 percent increase in the skill wage gap.\textsuperscript{3}

Table 1: Impact of ICT capital on gender and skill premia

|           | 5-yr diff. | 10-yr diff. | 20-yr diff. |
|-----------|------------|-------------|-------------|
| $\beta_g$| -0.028     | -0.033      | -0.031      |
|           | (0.005)    | (0.007)     | (0.006)     |
|           | {65.1}     | {62.5}      | {50.3}      |
|           | -0.134     | -0.156      | -0.149      |
|           | (0.025)    | (0.031)     | (0.030)     |
| $\beta_s$| 0.081      | 0.092       | 0.089       |
|           | (0.012)    | (0.013)     | (0.014)     |
|           | {56.9}     | {53.5}      | {43.2}      |
|           | 0.340      | 0.387       | 0.373       |
|           | (0.049)    | (0.057)     | (0.058)     |

Notes: Standard errors in parentheses are clustered at the country level. First-stage $F$ statistics in curly brackets are obtained under the null hypothesis that shift–share instruments are irrelevant. Elasticities in italics are $\beta_g \left( \frac{k_i}{\ell_m} \right)$ or $\beta_s \left( \frac{k_i}{\ell_u} \right)$, where capital and labor quantities are evaluated at sample means across all countries and years.

We cannot see from the analysis here the elasticities of substitution among capital and labor inputs, while we find a significant impact of ICT capital on the gender wage gap and the skill

\textsuperscript{2}Borusyak et al. (2019) show that the estimates obtained from the industry-level IV regression are equivalent to those obtained from the region-level IV regression, and standard errors obtained from the industry-level IV regression are valid. The equivalence result does not strictly hold here for two reasons. First, we make the leave-one-out adjustment. Second, the endogenous regressor is the ratio of capital to labor inputs, in which the numerator and the denominator differ in industry shares. When we adjust the variables in such a way that the equivalence result holds, parameter estimates remain essentially unchanged, and standard errors obtained from the industry-level IV regression are smaller than those obtained from the country-level IV regression. Throughout the paper, we choose to report more conservative standard errors that are obtained from the country-level IV regression.

\textsuperscript{3}We exclude from the sample the Czech Republic when using the 10-year difference and the Czech Republic and Germany when using the 20-year difference because data needed for estimation are not available in these countries for a sufficient number of periods.
wage gap. The reason for this is that the elasticity of substitution is fixed by assumption at infinity between $k_i$ and $\ell_m$ (or $\ell_u$) and one between the $k_i-\ell_m$ (or $k_i-\ell_u$) composite and $\ell_f$ (or $\ell_h$) in the production functions described above. In the next section and onwards, we consider a more general form of production function, and estimate the elasticities of substitution between ICT capital and four types of labor.

## 4 Model and Estimation

In this section, we present the aggregate production function, which we use to account for trends in the gender wage gap and the skill wage gap. Moreover, we describe ways to identify and estimate parameters in the aggregate production function and calculate the elasticities of substitution among capital and labor inputs.

### 4.1 Production function

We assume that the output ($y$) is produced from two types of capital (ICT capital, $k_i$, and non-ICT capital, $k_o$) and four types of labor (male skilled labor, $\ell_{mh}$, female skilled labor, $\ell_{fh}$, male unskilled labor, $\ell_{mu}$, and female unskilled labor, $\ell_{fu}$) using a constant-returns-to-scale technology in competitive markets. We assume that young, middle-aged, and old workers are perfect substitutes in production; however, we adjust for changes in the labor composition and efficiency according to demographic changes, as detailed in Appendix A.1. Let $A$ denote factor-neutral technology. Building upon the seminal work of Krusell, Ohanian, Rios-Rull and Violante (2000), who estimate the aggregate production function with two types of capital (capital equipment and structure) and two types of labor (skilled and unskilled labor), the six-factor production function is specified as:

$$y = A k_o^\alpha [\lambda B^\sigma + (1 - \lambda) \ell_{mu}^\sigma]^{1/\sigma}, \quad (4)$$
where

\[ B = \left[ \mu C^\rho + (1 - \mu) \ell_{fu}^\rho \right]^{\frac{1}{\rho}}, \]  
\[ C = \left[ \gamma D^\eta + (1 - \gamma) \ell_{mh}^\eta \right]^{\frac{1}{\eta}}, \]  
\[ D = \left[ \psi k_i^\xi + (1 - \psi) \ell_{fh}^\xi \right]^{\frac{1}{\xi}}. \]  

This production function involves four substitution parameters (\(\sigma, \rho, \eta, \text{ and } \xi\)) that are less than one and four share parameters (\(\lambda, \mu, \gamma, \text{ and } \psi\)) that lie between zero and one. As in Krusell et al. (2000), skilled labor is placed in the lower nest than unskilled labor. For each skill type, female labor is placed in the lower nest than male labor. The specification is chosen to be consistent with the data. Appendix A.3 provides a further discussion on the alternative specifications.

The production technology exhibits capital–skill complementarity for male labor if \(\sigma > \eta\) and for female labor if \(\rho > \xi\). The production function allows for gender-specific capital–skill complementarity. To put it another way, the production technology exhibits capital–gender complementarity for unskilled labor when \(\sigma > \rho\) and for skilled labor when \(\eta > \xi\). In this sense, the production function allows for skill-specific capital–gender complementarity.

Profit maximization implies that the marginal rate of technical substitution equals the ratio of input prices. For the purpose of our analysis, we look at the marginal-rate-of-technical-substitution conditions concerning the gender wage gap and the skill wage gap. The male–female wage gap among skilled workers is:

\[ \frac{w_{mh}}{w_{fh}} = \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{1}{1 - \psi} \right) D^{-(\eta - \xi)} \ell_{mh}^{-\gamma(1 - \eta)} \ell_{fh}^{1 - \xi}, \]  

The male–female wage gap among unskilled workers is:

\[ \frac{w_{mu}}{w_{fu}} = \left( \frac{1 - \lambda}{\lambda} \right) \left( \frac{1}{1 - \mu} \right) B^{-(\sigma - \rho)} \ell_{mu}^{-\lambda(1 - \sigma)} \ell_{fu}^{1 - \rho}. \]
The skilled–unskilled wage gap among male workers is:

\[
\frac{w_{mh}}{w_{mu}} = \left( \frac{\lambda}{1 - \lambda} \right) \mu (1 - \gamma) B^{\sigma - \rho} C^{\rho - \eta} \ell_{mh}^{1 - \sigma} \ell_{mu}^{1 - \sigma}.
\] (10)

The skilled–unskilled wage gap among female workers is:

\[
\frac{w_{fh}}{w_{fu}} = \left( \frac{\mu}{1 - \mu} \right) \gamma (1 - \psi) D^{\eta - \xi} \ell_{fh}^{1 - \xi} \ell_{fu}^{1 - \rho}.
\] (11)

To measure the degree of capital–labor substitution, we additionally look at the marginal-rate-of-technical-substitution condition relating female skilled labor to ICT capital. The wage to rental price ratio is:

\[
\frac{w_{fh}}{r_i} = \left( \frac{1 - \psi}{\psi} \right) \left( \frac{\ell_{fh}}{k_i} \right)^{(1 - \xi)}.
\] (12)

The level equations (8)–(12) are used to pin down the share parameters.

### 4.2 Identification, estimation, and decomposition

We now describe equations used to estimate the substitution parameters. After taking logs and differences over time and adding error terms (\(u\)) in equations (8)–(12), the estimating equations are obtained as follows. The log change of the male–female wage gap among skilled workers is:

\[
\Delta \ln \left( \frac{w_{mh}}{w_{fh}} \right) = (\eta - \xi) \Delta \ln D - (1 - \eta) \Delta \ln \ell_{mh} + (1 - \xi) \Delta \ln \ell_{fh} + \Delta u_1.
\] (13)

The log change of the male–female wage gap among unskilled workers is:

\[
\Delta \ln \left( \frac{w_{mu}}{w_{fu}} \right) = (\sigma - \rho) \Delta \ln B - (1 - \sigma) \Delta \ln \ell_{mu} + (1 - \rho) \Delta \ln \ell_{fu} + \Delta u_2.
\] (14)
The log change of the skilled–unskilled wage gap among male workers is:

$$\Delta \ln \left( \frac{w_{mh}}{w_{mu}} \right) = (\sigma - \rho) \Delta \ln B + (\rho - \eta) \Delta \ln C - (1 - \eta) \Delta \ln \ell_{mh} + (1 - \sigma) \Delta \ln \ell_{mu} + \Delta u_3. \quad (15)$$

The log change of the skilled–unskilled wage gap among female workers is:

$$\Delta \ln \left( \frac{w_{fh}}{w_{fu}} \right) = (\rho - \eta) \Delta \ln C + (\eta - \xi) \Delta \ln D - (1 - \xi) \Delta \ln \ell_{fh} + (1 - \rho) \Delta \ln \ell_{fu} + \Delta u_4. \quad (16)$$

The log change of the wage to rental price ratio is:

$$\Delta \ln \left( \frac{w_{fh}}{r_i} \right) = - (1 - \xi) \Delta \ln \left( \frac{\ell_{fh}}{k_i} \right) + \Delta u_5. \quad (17)$$

All time-invariant factors are eliminated in the first-difference equations (13)–(17). Therefore, the estimates of substitution parameters are robust to any time-invariant country-specific effects. There are four unknowns \((\sigma, \rho, \eta, \xi)\) in the system of five first-difference equations (13)–(17). This implies that the substitution parameters can be identified from these equations and estimated using variation in capital and labor quantities across countries over time. In doing so, shift–share instruments are used to deal with the endogeneity of input changes, thereby exploiting variation in global industry-specific input changes with the lagged industrial composition of each country. For given substitution parameters, there are four unknowns \((\lambda, \mu, \gamma, \psi)\) in the system of five level equations (8)–(12). This implies that the share parameters can be identified from the level equations.

As the number of equations exceeds the number of parameters, the share and substitution parameters can be over-identified in the system of the level and first-difference equations. More specifically, the substitution parameter \(\sigma\) and the share parameter \(\lambda\) can be over-identified from the first-difference equations (14) and (15) and the level equations (9) and (10). In theory, the parameters \(\sigma\) and \(\lambda\) estimated from equations (9) and (14) should be equal to those estimated
from equations (10) and (15). We test whether these over-identifying restrictions hold to validate the model.

Estimation proceeds in four steps. To put it briefly, we estimate production function parameters in the order of those in equations (7), (6), (5), and (4) from the lowest to the highest nest. In each step, we estimate the substitution parameters from the first-difference equations (13)–(17) using shift–share instruments, and then estimate the share parameters from the level equations (8)–(12). We compute standard errors using block bootstrap, clustered at the country level, with 500 replications. Appendix A.4 provides further details of the estimation procedure.

After estimation, we decompose changes in gender and skill premia into the capital–skill complementarity effect and the relative labor quantity effect, as described in the first-difference equations (13)–(16). Suppose that the production technology exhibits capital–skill–gender complementarity (i.e., $\sigma > \rho > \eta > \xi$). Given that the rate of increase in ICT capital is greater than that in all types of labor, the capital–skill complementarity effects are negative for the male–female wage gap and positive for the skilled–unskilled wage gap. Meanwhile, suppose that the production technology is concave (i.e., $\sigma, \rho, \eta, \xi < 1$). Given that the rate of increase in female labor is greater than that in male labor and that the rate of increase in skilled labor is greater than that in unskilled labor, the relative labor quantity effects are positive for the male–female wage gap and negative for the skilled–unskilled wage gap. Consequently, we would expect that the capital–skill complementarity effect and the relative labor quantity effect work in the opposite direction. We further decompose these two effects into the effects due to specific capital and labor inputs. Because of non-linearity, the results of such detailed decomposition depend on the order of decomposition. To address this issue, we implement the Shapley decomposition (Shorrocks, 2013).
4.3 Factor demands

We measure the degree of substitution among capital and labor inputs in terms of McFadden’s (1963) shadow elasticity of substitution. We denote a set of factor demands by \( x \in \{ \ell_{mh}, \ell_{fh}, \ell_{mu}, \ell_{fu}, k_i, k_o \} \), a set of factor prices by \( p \in \{ w_{mh}, w_{fh}, w_{mu}, w_{fu}, r_i, r_o \} \), and the total cost of production by \( c \). McFadden’s shadow elasticity of substitution between inputs \( x \) and \( x' \) is defined as:

\[
\varepsilon = - \left. \frac{\partial \ln \left( \frac{x}{x'} \right)}{\partial \ln \left( \frac{p}{p'} \right)} \right|_c .
\]  

(18)

McFadden’s shadow elasticity of substitution, which represents the curvature of the cost function along the isocost curve, can be expressed as the weighted sum of Morishima elasticities (Chambers, 1988). The elasticities of substitution can be calculated from the factor demand function. The factor demand function can be derived from the production function and the marginal-rate-of-technical-substitution conditions. Appendix A.5 provides the exact form of factor demand function.

The relative factor demand is of the simpler form:

\[
\frac{x}{x'} = f \left( w_{mh}, w_{fh}, w_{mu}, w_{fu}, r_i \right),
\]

(19)

where \( x/x' \in \{ \ell_{mh}/\ell_{fh}, \ell_{mu}/\ell_{fu}, \ell_{mh}/\ell_{mu}, \ell_{fh}/\ell_{fu} \} \). We use this equation to measure the contribution of factor prices to changes in the relative labor demand.

5 Results

We start this section by presenting the estimates of parameters in the production function. We, then, consider the sources and mechanisms of changes in gender and skill premia. We end this section by discussing the implications for the impact of factor prices on the relative labor
5.1 Production function estimates

Table 2 presents the estimates of production function parameters when using the 5- and 10-year differences separately. In both cases, the estimated substitution parameters are significantly different. The relative magnitude of the estimated substitution parameters (i.e., $\sigma > \rho > \eta > \xi$) is consistent with the capital–skill–gender complementarity hypothesis that ICT capital is not only more complementary to skilled than unskilled labor but also more complementary to female than male labor. Even when the null hypotheses of $\sigma = \rho$, $\rho = \eta$, and $\eta = \xi$ are individually tested, all the hypotheses can be strongly rejected at the 1 percent significance level. The capital–skill complementarity effects on the gender wage gap and the skill wage gap are proportional in magnitude to differences in substitution parameters, as can be seen from equations (13)–(16). The estimated difference between $\eta$ and $\xi$ is greater than that between $\sigma$ and $\rho$. This result implies that the capital–skill complementarity effect on the gender wage gap would be greater for skilled than unskilled labor, while the capital–skill complementarity effect on the skill wage gap would be greater for female than male labor. The relative labor quantity effects are inversely proportional in magnitude to the substitution parameters, as can also be seen from equations (13)–(16). The effects of labor quantities would be large in the order of female skilled, male skilled, female unskilled, and male unskilled labor.

We confirm that the over-identifying restrictions cannot be rejected with a Wald statistic of 0.800 (a $p$-value of 0.670). In fact, the estimates of $(\lambda, \sigma)$ are very close, at (0.639, 0.812), (0.638, 0.802), and (0.643, 0.836), regardless of whether the parameters are estimated using equations (9), (10), (14), and (15); equations (9) and (14) only; or equations (10) and (15) only.

The results described here remain virtually unchanged, whether using the 5- or 10-year differences. Therefore, we focus on parameter estimates when using the 5-year difference in the subsequent analysis.
Table 2: Production function estimates

|                  | 5-yr diff. | 10-yr diff. |
|------------------|------------|-------------|
| $k_i$ vs. $\ell_{fh}$ | $\xi$=-0.442, $\eta=0.324, \rho=0.529, \sigma=0.812$ | $\xi$=-0.538, $\eta=0.352, \rho=0.546, \sigma=0.821$ |
|                  | (0.158)    | (0.226)     |
| $\{k_i, \ell_{fh}\}$ vs. $\ell_{mh}$ | $\psi=0.880, \gamma=0.277, \mu=0.575, \lambda=0.639$ | $\psi=0.915, \gamma=0.260, \mu=0.573, \lambda=0.638$ |
|                  | (0.099)    | (0.100)     |
| $\{k_i, \ell_{fh}, \ell_{mh}\}$ vs. $\ell_{fu}$ | $F$ statistics: 5-yr diff. = 42.0, 10-yr diff. = 29.2 | Wald test statistics [p-values]: 5-yr diff. = 0.800 [0.670], 10-yr diff. = 0.312 [0.856] |

Notes: Standard errors in parentheses are computed using block bootstrap, clustered at the country level, with 500 replications. First-stage $F$ statistics are obtained under the null hypothesis that shift–share instruments are irrelevant. Wald test statistics are obtained under the null hypothesis that over-identifying restrictions are valid.

5.2 Robustness checks

In the analysis so far, we have included all industries and ignored the influence of institutional changes. We are concerned about whether the results presented above are driven by changes in industrial structure, international trade, or labor market institutions. To address this concern, we conduct two types of robustness checks. First, we estimate the production function parameters separately for goods and service industries and for traded and non-traded industries, and see

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4Goods industries include five broad categories of industries: agriculture, hunting, forestry, and fishing; mining and quarrying; manufacturing; electricity, gas and water supply; and construction. Service industries include nine broad categories of industries: wholesale and retail trade; hotels and restaurants; transport and storage, and communication; financial intermediation; real estate, renting, and business activities; public administration and defense, and compulsory social security; education; health and social work; and other community, and social and personal services. Traded industries include two broad categories of industries: agriculture, hunting, forestry, and fishing; and manufacturing. Non-traded industries include ten broad categories of industries: construction; wholesale and retail trade; hotels and restaurants; transport and storage, and communication; financial intermediation; real estate, renting, and business activities; public administration and defense, and compulsory social security;
whether the estimated parameters differ across industries. Second, we estimate the production function parameters after adjusting for changes in institutional factors. Basically, we consider the possibility that the actual wage may deviate from the competitive wage according to institutional factors. We focus on the influence of collective bargaining in light of the results by Blau and Kahn (2003), who attribute international differences in the gender wage gap to the collective bargaining coverage. We, thus, estimate the production function parameters after including the collective bargaining coverage in log-change form as an additional regressor in the first-difference equations,\(^5\) and see whether the estimated parameters change.

Table 3: Robustness checks

| Substitution parameters | \(\xi\) | \(\eta\) | \(\rho\) | \(\sigma\) |
|-------------------------|--------|--------|--------|--------|
| **All industries**      | -0.442 | 0.324  | 0.529  | 0.812  |
|                         | (0.158)| (0.075)| (0.060)| (0.051)|
| **Goods industries**    | -0.827 | 0.216  | 0.485  | 0.703  |
|                         | (0.454)| (0.086)| (0.095)| (0.071)|
| **Service industries**  | -0.347 | 0.334  | 0.521  | 0.830  |
|                         | (0.132)| (0.084)| (0.056)| (0.059)|
| **Traded industries**   | -0.826 | 0.205  | 0.533  | 0.725  |
|                         | (0.403)| (0.083)| (0.090)| (0.065)|
| **Non-traded industries** | -0.349 | 0.342  | 0.516  | 0.824  |
|                         | (0.137)| (0.082)| (0.057)| (0.054)|
| **Collective bargaining coverage** | -0.442 | 0.293  | 0.481  | 0.659  |
|                         | (0.158)| (0.076)| (0.087)| (0.148)|

Notes: Standard errors in parentheses are computed using block bootstrap, clustered at the country level, with 500 replications.

The first five rows of Table 3 present the estimates of substitution parameters for all, goods, service, traded, and non-traded industries. Goods industries consist of traded, construction, and energy-related industries, while service industries consist of non-traded industries other than construction industries. Not surprisingly, the results for goods industries are almost the same as education; health and social work; and other community, and social and personal services.

\(^5\)The data on the collective bargaining coverage (the percentage of employees with the right to bargain) can be obtained from OECD.Stat.
those for traded industries, while the results for service industries are almost the same as those for non-traded industries. The majority of industries are classified into service and non-traded industries. The results for service and non-traded industries are similar to those for all industries. The estimated substitution parameters are slightly different between goods and service industries and between traded and non-traded industries. The capital–skill complementarity effect, which is proportional in magnitude to differences in substitution parameters, appears to be stronger in goods or traded industries than in service or non-traded industries. Nonetheless, the relative magnitude of the estimated substitution parameters is the same in all cases. Hence, we confirm the capital–skill–gender complementarity hypothesis across industries.

The bottom row of Table 3 presents the estimates of substitution parameters when adjusting for changes in the collective bargaining coverage. The estimated elasticities of $w_{mh}/w_{fh}$, $w_{mu}/w_{fu}$, $w_{mh}/w_{mu}$, and $w_{fh}/w_{fu}$ with respect to the collective bargaining coverage are 0.186, 0.152, −0.128, and −0.175, with bootstrapped clustered standard errors of 0.410, 0.071, 0.071, and 0.360, respectively. The collective bargaining coverage has a positive effect for the gender wage gap and a negative effect for the skill wage gap, suggesting that male unskilled workers benefit more from the collective bargaining. The effects are, however, statistically insignificant at the 5 percent significance level, except for the male–female wage gap among unskilled workers. More importantly, the estimated substitution parameters remain essentially unchanged regardless of whether the collective bargaining coverage is controlled for. Hence, the results presented above are robust to changes in the collective bargaining coverage.

5.3 Elasticities of substitution

Table 4 presents the estimates for McFadden’s shadow elasticities of substitution evaluated at the sample means across all countries and years. The estimated elasticities of substitution between ICT capital and the four types of labor are large in the order of male unskilled, female unskilled, male skilled, and female skilled labor. This result reassures the capital–skill–gender
complementarity hypothesis. Moreover, the estimated elasticities of labor–labor substitution range from 1.2 to 4.3, which is broadly consistent with those of Acemoglu et al. (2004) in the United States.

Table 4: McFadden’s shadow elasticities of substitution

|       | \(k_i\) | \(\ell_{fh}\) | \(\ell_{mh}\) | \(\ell_{fu}\) | \(\ell_{mu}\) |
|-------|---------|----------------|----------------|----------------|----------------|
| \(k_i\) | 0.694   | 1.231          | 1.445          | 2.040          |
|       | (0.075) | (0.111)        | (0.116)        | (0.438)        |
| \(\ell_{fh}\) | 0.694   | 1.165          | 1.345          | 1.864          |
|       | (0.075) | (0.147)        | (0.196)        | (0.476)        |
| \(\ell_{mh}\) | 1.231   | 1.165          | 1.949          | 3.317          |
|       | (0.111) | (0.147)        | (0.247)        | (1.020)        |
| \(\ell_{fu}\) | 1.445   | 1.345          | 1.949          | 4.288          |
|       | (0.116) | (0.196)        | (0.247)        | (1.317)        |
| \(\ell_{mu}\) | 2.040   | 1.864          | 3.317          | 4.288          |
|       | (0.438) | (0.476)        | (1.020)        | (1.317)        |

Notes: Standard errors in parentheses are computed using block bootstrap, clustered at the country level, with 500 replications.

5.4 Changes in gender and skill premia

Using the estimated production function parameters described above, we conduct a decomposition analysis to understand the sources and mechanisms of changes in gender and skill premia. We first measure the extent to which changes in gender and skill premia are attributable to the capital–skill complementarity effect and the relative labor quantity effect. We, then, evaluate the quantitative contribution of specific capital and labor inputs to changes in gender and skill premia.

Table 5 presents the results on the decomposition of changes in gender and skill premia into the capital–skill complementarity effect and the relative labor quantity effect. The results are reported separately for the 1980–2005 and 1985–2005 periods because the average trend of the skilled–unskilled wage gap appears to change around the year 1985, as seen in Figure 1d. The two effects work in the opposite direction. The male–female wage gap can either decrease due to the capital–skill complementarity effect or increase due to the relative labor quantity effect.
Table 5: Decomposition of changes in gender and skill premia

|                      | 1980–2005 |            |            | 1985–2005 |            |            |
|----------------------|-----------|-----------|-----------|-----------|-----------|-----------|
|                      | Data      | Model     | CSC       | RLQ       | Data      | Model     | CSC       | RLQ       |
| $\Delta \ln \left( \frac{w_{mh}}{w_{fh}} \right)$ | 0.033     | -0.065    | -1.774    | 1.710     | -0.015    | -0.045    | -1.435    | 1.389     |
|                      | (0.183)   | (0.401)   | (0.227)   |           | (0.142)   | (0.319)   | (0.186)   |           |
| $\Delta \ln \left( \frac{w_{mu}}{w_{fu}} \right)$ | -0.077    | -0.130    | -0.203    | 0.073     | -0.086    | -0.109    | -0.184    | 0.075     |
|                      | (0.044)   | (0.045)   | (0.011)   |           | (0.036)   | (0.040)   | (0.010)   |           |
| $\Delta \ln \left( \frac{w_{mh}}{w_{mu}} \right)$ | 0.043     | -0.009    | 0.488     | -0.497    | 0.073     | 0.012     | 0.422     | -0.410    |
|                      | (0.042)   | (0.071)   | (0.057)   |           | (0.033)   | (0.059)   | (0.046)   |           |
| $\Delta \ln \left( \frac{w_{fh}}{w_{fu}} \right)$ | -0.067    | -0.074    | 2.060     | -2.133    | 0.003     | -0.052    | 1.673     | -1.724    |
|                      | (0.158)   | (0.377)   | (0.236)   |           | (0.121)   | (0.299)   | (0.192)   |           |

Notes: Actual and predicted values for the 1980–2005 period are reported in the first and second columns, respectively, and those for the 1985–2005 period are in the fifth and sixth columns, respectively. The capital–skill complementarity effect (CSC) and the relative labor quantity effect (RLQ) for the 1980–2005 period are reported in the third and fourth columns, respectively, and those for the 1985–2005 period are in the seventh and eighth columns, respectively. Standard errors in parentheses are computed using block bootstrap, clustered at the country level, with 500 replications.

The skilled–unskilled wage gap can either increase due to the capital–skill complementarity effect or decrease due to the relative labor quantity effect. Whether the gender wage gap and the skill wage gap eventually increase or decrease depends on the relative magnitude of these two effects.

The capital–skill complementarity effect is proportional in magnitude to differences in substitution parameters. Given the estimates of substitution parameters, the capital–skill complementarity effect on the gender wage gap is greater for skilled than unskilled workers, and that on the skill wage gap is greater for female than male workers. Meanwhile, the relative labor quantity effect is proportional in magnitude to changes in the relative quantities of labor, as well as inversely proportional in magnitude to the substitution parameters. Given that there have been greater increases in skilled and female labor, the relative labor quantity effect on the gender wage gap is greater for skilled than unskilled workers, and that on the skill wage gap is greater for female than male workers. Consequently, the male–female wage gap declined among unskilled workers since the capital–skill complementarity effect exceeds the relative labor quantity effect, while the male–female wage gap did not decline among skilled workers since the two large effects are almost canceled out. The skilled–unskilled wage gap increased among male
workers since the capital–skill complementarity effect exceeds the relative labor quantity effect, while the skilled–unskilled wage gap did not increase much among female workers since the two effects are almost canceled out.

The results described above remain essentially unchanged, whether looking at the 1980–2005 period or the 1985–2005 period. Therefore, we focus on the results for the 1980–2005 period in the subsequent analysis.

Table 6: Contribution of factor quantities to changes in gender and skill premia, 1980–2005

| Data | Model | $k_i$ | $\ell_{fh}$ | $\ell_{mh}$ | $\ell_{fu}$ | $\ell_{mu}$ |
|------|-------|------|-------------|-------------|-------------|-------------|
| 0.033 | -0.065 | -1.213 | 1.623 | -0.475 | 0.000 | 0.000 |
| (0.183) | (0.346) | (0.212) | (0.053) | |
| $\Delta \ln \left( \frac{w_{mh}}{w_{fh}} \right)$ | -0.077 | -0.130 | -0.091 | -0.044 | -0.058 | 0.041 | 0.022 |
| (0.044) | (0.025) | (0.008) | (0.013) | (0.005) | (0.006) | |
| $\Delta \ln \left( \frac{w_{mu}}{w_{fu}} \right)$ | 0.043 | -0.009 | 0.225 | 0.107 | -0.330 | 0.011 | -0.022 |
| (0.042) | (0.046) | (0.019) | (0.043) | (0.002) | (0.006) | |
| $\Delta \ln \left( \frac{w_{mh}}{w_{mu}} \right)$ | -0.067 | -0.074 | 1.347 | -1.560 | 0.087 | 0.051 | 0.000 |
| (0.158) | (0.344) | (0.219) | (0.021) | (0.007) | |

Notes: Actual and predicted values are reported in the first and second columns, respectively. The Shapley decomposition results are reported in the third to seventh columns. Standard errors in parentheses are computed using block bootstrap, clustered at the country level, with 500 replications.

Table 6 presents the results on the quantitative contribution of capital and labor inputs to changes in gender and skill premia. The impact of ICT capital is as large in absolute value as that of labor inputs. The male–female wage gap decreases with a rise in ICT capital and increases with a fall in the supply of male relative to female labor, while the skilled–unskilled wage gap increases with a rise in ICT capital and decreases with a rise in the supply of skilled relative to unskilled labor. Thus, changes in gender and skill premia can be interpreted as the outcome of the race between progress in ICT and advances in female educational attainment and employment.
5.5 Changes in the relative labor demand

Table 7 presents the results on the quantitative contribution of factor prices to changes in the relative labor demand. During the period between the years 1980 and 2005, there were a fall in the quantity of male relative to female labor and a rise in the quantity of skilled relative to unskilled labor. The rate of decrease in the relative quantity of male labor was greater among skilled than unskilled labor, while the rate of increase in the relative quantity of skilled labor was greater for female than male labor. The pattern of changes in the relative quantities of labor is consistent with that of changes predicted from the model. The first two columns of Table 7 show that the actual changes in the relative quantities of labor line up with the predicted changes, even though the production function parameters are not chosen to fit the data on the relative quantities of labor. The third to seventh columns show that changes in the relative labor demand are attributable to changes in not only the relative wages but also the rental price of ICT capital. During this period, there was a significant fall in the rental price of ICT capital due to technological progress. A large part of changes in the relative labor demand is attributable to a fall in the rental price of ICT capital.

Table 7: Contribution of factor prices to changes in the relative labor demand, 1980–2005

|          | Data  | Model | $r_i$ | $w_{fh}$ | $w_{mh}$ | $w_{fu}$ | $w_{mu}$ |
|----------|-------|-------|-------|----------|----------|----------|----------|
| $\Delta \ln \left( \ell_{mh}/\ell_{fh} \right)$ | -0.812 | -0.952 | -0.783 | 0.259 | -0.428 | 0.000 | 0.000 |
|          | (0.184) | (0.156) | (0.036) | (0.051) |          |          |          |
| $\Delta \ln \left( \ell_{mu}/\ell_{fu} \right)$ | -0.226 | -0.438 | -0.699 | 0.056 | 0.198 | 1.318 | -1.311 |
|          | (0.372) | (0.420) | (0.049) | (0.150) | (0.378) | (0.487) |          |
| $\Delta \ln \left( \ell_{mh}/\ell_{mu} \right)$ | 0.820 | 0.841 | 0.976 | -0.089 | -0.725 | -0.632 | 1.311 |
|          | (0.356) | (0.408) | (0.053) | (0.187) | (0.348) | (0.487) |          |
| $\Delta \ln \left( \ell_{fh}/\ell_{fu} \right)$ | 1.406 | 1.355 | 1.060 | -0.293 | -0.098 | 0.687 | 0.000 |
|          | (0.159) | (0.131) | (0.046) | (0.030) | (0.091) |          |          |

Notes: Actual and predicted values are reported in the first and second columns, respectively. The Shapley decomposition results are reported in the third to seventh columns. Standard errors in parentheses are computed using block bootstrap, clustered at the country level, with 500 replications.
6 Conclusion

In recent decades, the male–female wage gap has fallen, while the skilled–unskilled wage gap has risen in advanced countries. The rate of decline in the gender wage gap has tended to be greater for unskilled than skilled workers, while the rate of increase in the skill wage gap has tended to be greater for male than female workers. To account for these trends, we have developed the aggregate production function extended to allow for gender-specific capital–skill complementarity, and estimated it using shift–share instruments and cross-country panel data from OECD countries. We have provided the first estimates for the elasticities of substitution between ICT capital and four types of labor (male skilled, female skilled, male unskilled, and female unskilled labor). Consequently, we have confirmed that ICT capital is not only more complementary to skilled than unskilled workers but also more complementary to female than male workers. In addition, using the estimated production function parameters, we have measured the relative magnitude of the capital–skill complementarity effect and the relative labor quantity effect and evaluated the quantitative contribution of specific capital and labor inputs to changes in gender and skill premia. Consequently, we have shown that changes in gender and skill premia can be interpreted as the outcome of the race between the demand shift driven by progress in ICT and the supply shift driven by advances in female educational attainment and employment.
References

Acemoglu, Daron, David Autor, and David Lyle (2004) “Women, War, and Wages: the Effect of Female Labor Supply on the Wage Structure at Midcentury,” *Journal of Political Economy*, Vol. 112, No. 3, pp. 497–551.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney (2008) “Trends in U.S. Wage Inequality: Revising the Revisionists,” *Review of Economics and Statistics*, Vol. 90, No. 2, pp. 300–323.

Autor, David H., Lawrence F. Katz, and Alan B. Krueger (1998) “Computing Inequality: Have Computers Changed the Labor Market?” *Quarterly Journal of Economics*, Vol. 113, No. 4, pp. 1169–1213.

Autor, David, Frank Levy, and Richard Murnane (2003) “The Skill Content of Recent Technological Change: An Empirical Exploration,” *Quarterly Journal of Economics*, Vol. 118, No. 4, pp. 1279–1333.

Bartik, Timothy J. (1991) *Who Benefits from State and Local Economic Development Policies?:* Kalamazoo, MI: W.E. Upjohn Institute for Employment Research.

Beaudry, Paul and Ethan Lewis (2014) “Do Male-Female Wage Differentials Reflect Differences in the Return to Skill? Cross-City Evidence from 1980–2000,” *American Economic Journal: Applied Economics*, Vol. 6, No. 2, pp. 178–194.

Black, Sandra E. and Alexandra Spitz–Oener (2010) “Explaining Women’s Success: Technological Change and the Skill Content of Women’s Work,” *Review of Economics and Statistics*, Vol. 92, No. 1, pp. 187–194.

Blau, Francine D. and Lawrence M. Kahn (2003) “Understanding International Differences in the Gender Pay Gap,” *Journal of Labor Economics*, Vol. 21, No. 1, pp. 106–144.
——— (2017) “The Gender Wage Gap: Extent, Trends, and Explanations,” *Journal of Economic Literature*, Vol. 55, No. 3, pp. 789–865.

Borghans, Lex, Bas ter Weel, and Bruce A. Weinberg (2014) “People Skills and the Labor-Market Outcomes of Underrepresented Groups,” *Industrial and Labor Relations Review*, Vol. 67, No. 2, pp. 287–334.

Borusyak, Kirill, Peter Hull, and Xavier Jaravel (2019) “Quasi-Experimental Shift-Share Research Designs,” mimeo.

Burstein, Ariel, Eduardo Morales, and Jonathan Vogel (2019) “Changes in Between-Group Inequality: Computers, Occupations, and International Trade,” *American Economic Journal: Macroeconomics*, Vol. 11, No. 2, pp. 348–400.

Chambers, Robert G. (1988) *Applied Production Analysis: A Dual Approach*: Cambridge University Press.

Goldin, Claudia and Lawrence F. Katz (2010) *The Race between Education and Technology*: Harvard University Press.

Griliches, Zvi (1969) “Capital-Skill Complementarity,” *Review of Economics and Statistics*, Vol. 51, No. 4, pp. 465–468.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2010) “The Macroeconomic Implications of Rising Wage Inequality in the United States,” *Journal of Political Economy*, Vol. 118, No. 4, pp. 681–722.

Jorgenson, Dale W. (1963) “Capital Theory and Investment Behavior,” *American Economic Review Papers and Proceedings*, Vol. 53, No. 2, pp. 247–259.

Katz, Lawrence and Kevin M. Murphy (1992) “Changes in Relative Wages, 1963–1987: Supply and Demand Factors,” *Quarterly Journal of Economics*, Vol. 107, No. 1, pp. 35–78.
Krueger, Dirk, Fabrizio Perri, Luigi Pistaferri, and Giovanni L. Violante (2010) “Cross Sectional Facts for Macroeconomists,” *Review of Economic Dynamics*, Vol. 13, No. 1, pp. 1–14.

Krusell, Per, Lee E. Ohanian, José-Víctor Rios-Rull, and Giovanni L. Violante (2000) “Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis,” *Econometrica*, Vol. 68, No. 5, pp. 1029–1053.

Machin, Stephen and John Van Reenen (1998) “Technology and Changes in Skill Structure: Evidence from Seven OECD Countries,” *Quarterly Journal of Economics*, Vol. 113, No. 4, pp. 1215–1244.

McFadden, Daniel (1963) “Constant Elasticity of Substitution Production Functions,” *Review of Economic Studies*, Vol. 30, No. 2, pp. 73–83.

Michaels, Guy, Ashwini Natraj, and John Van Reenen (2014) “Has ICT Polarized Skill Demand? Evidence from Eleven Countries over Twenty-five Years,” *Review of Economics and Statistics*, Vol. 96, No. 1, pp. 60–77.

Ngai, L. Rachel and Barbara Petrongolo (2017) “Gender Gaps and the Rise of the Service Economy,” *American Economic Journal: Macroeconomics*, Vol. 9, No. 4, pp. 1–44.

O’Mahony, Mary and Marcel P. Timmer (2009) “Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database,” *The Economic Journal*, Vol. 119, No. 538, pp. F374–F403.

Raveh, Ohad (2015) “Capital-Gender Complementarity,” *Economics Bulletin*, Vol. 35, No. 1, pp. 494–506.

Shorrocks, Anthony (2013) “Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value,” *Journal of Economic Inequality*, Vol. 11, No. 1, pp. 99–126.
Spitz-Oener, Alexandra (2006) “Technical Change, Job Tasks, and Rising Educational Demands: Looking outside the Wage Structure,” *Journal of Labor Economics*, Vol. 24, No. 2, pp. 235–270.

Timmer, Marcel, Ton van Moergastel, Edwin Stuivenwold, Gerard Ypma, Mary O’Mahony, and Mari Kangasniemi (2007) “EU KLEMS Growth and Productivity Accounts.”

Tinbergen, Jan (1974) “Substitution of Graduate by Other Labour,” *Kyklos*, Vol. 27, No. 2, pp. 217–226.

Weinberg, Bruce A. (2000) “Computer Use and the Demand for Female Workers,” *Industrial and Labor Relations Review*, Vol. 53, No. 2, pp. 290–308.

Welch, Finis (2000) “Growth in Women’s Relative Wages and in Inequality among Men: One Phenomenon or Two?” *American Economic Review Papers and Proceedings*, Vol. 90, No. 2, pp. 444–449.
A Appendix

A.1 Composition-adjusted measures

We describe in order the way to construct the data on wages and hours worked. Throughout the study, we adjust for changes in the labor composition and efficiency in a similar way to Autor et al. (2008). Each type of labor, $\ell_{gs}$ for $gs \in \{mh, fh, mu, fu\}$, is composed of three age groups: young (aged between 15 and 29 years), middle (aged between 30 and 49 years), and old (aged 50 years and older).

To begin with, it is worth mentioning that, if no adjustment was needed, the wages for labor of type $gs$ in country $c$ and year $t$ could be calculated as:

$$\tilde{w}_{gs,ct} = \theta_{gs, ct}^\text{young} \tilde{w}_{gs, ct}^\text{young} + \theta_{gs, ct}^\text{middle} \tilde{w}_{gs, ct}^\text{middle} + \theta_{gs, ct}^\text{old} \tilde{w}_{gs, ct}^\text{old},$$

where the share of total hours worked by age group $a \in \{\text{young, middle, old}\}$ is defined as:

$$\theta_{gs, ct}^a = \frac{\tilde{\ell}_{gs, ct}^a}{(\tilde{\ell}_{gs, ct}^\text{young} + \tilde{\ell}_{gs, ct}^\text{middle} + \tilde{\ell}_{gs, ct}^\text{old})}.$$}

To adjust for compositional changes, the weight is fixed at its country-specific mean: $\bar{\theta}_{gs,c}^a = \sum_{t=1}^{T_c} \theta_{gs, ct}^a / T_c$, where $T_c$ is the number of years observed for country $c$. The composition-adjusted wages for labor of type $gs$ in country $c$ and year $t$ can be calculated as:

$$w_{gs,ct} = \bar{\theta}_{gs,c}^\text{young} \tilde{w}_{gs, ct}^\text{young} + \bar{\theta}_{gs,c}^\text{middle} \tilde{w}_{gs, ct}^\text{middle} + \bar{\theta}_{gs,c}^\text{old} \tilde{w}_{gs, ct}^\text{old}.$$}

Similarly, if no adjustment was needed, the hours worked by labor of type $gs$ in country $c$ and year $t$ could be calculated as:

$$\tilde{\ell}_{gs,ct} = \tilde{\ell}_{gs, ct}^\text{young} + \tilde{\ell}_{gs, ct}^\text{middle} + \tilde{\ell}_{gs, ct}^\text{old}.$$}

To adjust for compositional changes, the hours worked by each type of labor are weighted according to its time-invariant labor efficiency. The composition-adjusted hours worked by labor of type $gs$ in country $c$ and year $t$ can be calculated as:

$$\ell_{gs,ct} = (\bar{w}_{gs,c}^\text{young} / \bar{w}_{gs,c}^\text{middle}) \tilde{\ell}_{gs, ct}^\text{young} + (\bar{w}_{gs,c}^\text{middle} / \bar{w}_{gs,c}^\text{middle}) \tilde{\ell}_{gs, ct}^\text{middle} + (\bar{w}_{gs,c}^\text{old} / \bar{w}_{gs,c}^\text{middle}) \tilde{\ell}_{gs, ct}^\text{old},$$

where the efficiency unit is measured by the country-specific mean of wages: $\bar{w}_{gs,c}^a = \sum_{t=1}^{T_c} \tilde{w}_{gs, ct}^a / T_c$ and normalized by $\bar{w}_{gs,c}^\text{middle}$. The variables used in the preliminary analysis include the wages and hours worked of male, female, skilled, and unskilled labor. We adjust for changes in the age and skill composition in the same way as described above when we construct the data on the wages and hours worked of male and female labor. Similarly, we adjust for changes in the age and gender composition when...
we construct the data on the wages and hours worked of skilled and unskilled labor. Although we choose middle-aged unskilled labor as the base group for male and female labor and male middle-aged labor for the base group for skilled and unskilled labor, our results do not depend on the choice of the base group.

A.2 Rental price of capital

The rental price of capital \( r_{jt} \) is determined by the price of investment \( q_{jt} \), the depreciation rate \( \delta_j \), and the interest rate \( i_t \). The price of investment is calculated by dividing the nominal value by the real value of investment for each \( j \in \{i, o\} \). The depreciation rate is calculated from the average of depreciation rates of capital components weighted by the share of capital components. Following O’Mahony and Timmer (2009), the rental price of capital is calculated as:

\[
r_{j,t+1} = \delta_j q_{j,t+1} + i_{t+1} q_{jt} - (q_{j,t+1} - q_{jt}),
\]

where the interest rate is calculated as:

\[
i_t = \frac{\sum_j r_{jt} k_{jt} - \sum_j \delta_j q_{jt} k_{jt} + \sum_j (q_{jt} - q_{j,t-1}) k_{jt}}{\sum_j q_{j,t-1} k_{jt}}.
\]

A.3 Alternative specifications

We consider the alternative specifications of the six-factor production function. The first thing to note is that we aim to extend the production function in such a way that the four types of labor differ in the degree of substitution with ICT capital. This requires ICT capital to be placed in the lowest nest. Therefore, the issue here concerns the position of labor inputs. If labor inputs are placed in reverse order, the production function is specified as:

\[
y = A k^{\alpha'} \left[ \psi' B^\zeta' + (1 - \psi') \ell_f^{\zeta_f'} \right]^{\frac{1-\alpha'}{\zeta'}},
\]
where

\[ B = \left[ \gamma' C \eta' + (1 - \gamma') \ell_{mh} \right]^{1/\eta'}, \]
\[ C = \left[ \mu' D \rho' + (1 - \mu') \ell_{fu} \right]^{1/\rho'}, \]
\[ D = \left[ \lambda' k_i' \sigma' + (1 - \lambda') \ell_{mu} \right]^{1/\sigma'}. \]

The first four columns of Table 8 report the estimates of parameters in the production function (20). The main problem with this specification is that the estimates of the substitution parameters \( \xi' \) and \( \eta' \) exceed one. In this sense, the specification, in which the position of skilled and unskilled labor is reversed, is not consistent with the data, as also confirmed by Krusell et al. (2000). Keeping the order of skilled and unskilled labor fixed, there is only one remaining possibility for the order of male and female labor. If the position of male and female labor is changed for each skill type, the production function is specified as:

\[ y = A k_o^{\alpha''} \left[ \mu'' B \rho'' + (1 - \mu'') \ell_{fu}^{\rho''} \right]^{1-\alpha''/\rho''}, \]

where

\[ B = \left[ \lambda'' C \sigma'' + (1 - \lambda'') \ell_{mu}^{\sigma''} \right]^{1/\sigma''}, \]
\[ C = \left[ \psi'' D \xi'' + (1 - \psi'') \ell_{fh}^{\xi''} \right]^{1/\xi''}, \]
\[ D = \left[ \gamma'' k_i'' + (1 - \gamma'') \ell_{mh}^{\eta''} \right]^{1/\eta''}. \]

The next four columns of Table 8 report the estimates of parameters in the production function (21). The main problem with this specification is that the substitution parameters \( \xi'', \eta'', \) and \( \rho'' \) are not precisely estimated, and the share parameter \( \psi'' \) is estimated to be one. The main reason for this is that the estimate of \( \eta'' \), which is the parameter we first estimate in this specification, does not significantly differ from zero. This implies that we cannot reject the null hypothesis.
that $D$ is of the Cobb-Douglas form in the production function (21). In that case, the production function can be specified as:

$$y = A k^{\alpha''} \left[ \mu'' B^{\rho''} + (1 - \mu'') \ell^{\rho''}_{fu} \right]^{\frac{1-\alpha''}{\rho''}},$$  

(22)

where

$$B = \left[ 1 - \lambda'' \right] C^{\sigma''} + \left( 1 - \lambda'' \right) \ell^{\sigma''}_{mu} \frac{1}{\sigma''},$$

$$C = \left[ 1 - \psi'' \right] D^{\xi''} + \left( 1 - \psi'' \right) \ell^{\xi''}_{fh} \frac{1}{\xi''},$$

$$D = k^{\gamma''} \ell^{1-\gamma''}_{mh}.$$  

The final four columns of Table 8 report the estimates of parameters in the production function (22). The relative magnitude of the estimated substitution parameters is still consistent with the capital–skill–gender hypothesis (i.e., $\xi'' < \eta'' < \rho'' < \sigma''$). Obviously, however, this specification is restrictive relative to the one used in the main text. Given these results, we focus on the preferred specification (4) in our analysis.

Table 8: Estimates of the production functions (20), (21), and (22)

| Substitution parameters | Substitution parameters | Substitution parameters |
|-------------------------|-------------------------|-------------------------|
| $\xi'$ | $\eta'$ | $\rho'$ | $\sigma'$ | $\xi''$ | $\eta''$ | $\rho''$ | $\sigma''$ | $\xi'''$ | $\eta'''$ | $\rho'''$ | $\sigma'''$ |
| 1.071 | 1.279 | 0.235 | 0.343 | -13.74 | 0.084 | 0.078 | 0.605 | -0.636 | 0.000 | 0.331 | 0.768 |
| (0.051) | (0.188) | (0.412) | (0.051) | (7.039) | (0.066) | (0.441) | (0.248) | (0.174) | – | (0.188) | (0.037) |

| Share parameters | Share parameters | Share parameters |
|-----------------|-----------------|-----------------|
| $\psi'$ | $\gamma'$ | $\mu'$ | $\lambda'$ | $\psi''$ | $\gamma''$ | $\mu''$ | $\lambda''$ | $\psi'''$ | $\gamma'''$ | $\mu'''$ | $\lambda'''$ |
| 0.719 | 0.378 | 0.664 | 0.064 | 1.000 | 0.240 | 0.735 | 0.403 | 0.834 | 0.293 | 0.745 | 0.534 |
| (0.014) | (0.070) | (0.044) | (0.006) | (0.438) | (0.061) | (0.059) | (0.161) | (0.040) | (0.031) | (0.017) | (0.041) |

| First-stage $F$ statistics | First-stage $F$ statistics | First-stage $F$ statistics |
|-----------------------------|-----------------------------|-----------------------------|
| 44.2 | 29.4 | 10.6 | 383.1 | 0.0 | 151.4 | 19.2 | 155.0 | 35.5 | – | 19.3 | 186.8 |

Notes: Standard errors in parentheses are computed using block bootstrap, clustered at the country level, with 500 replications. First-stage $F$ statistics are obtained under the null hypothesis that shift–share instruments are irrelevant.
A.4 Estimation procedure

We estimate the substitution parameters \((\sigma, \rho, \eta, \xi)\) and the share parameters \((\lambda, \mu, \gamma, \psi)\) in the production function (4) in four steps. In each step, we use shift–share instruments:

\[
\Delta \ln z_{ct}^b = \sum_{d \in D} z_{c, d, t} - \tau \sum_{d' \in D} z_{c, d', t} - \tau \Delta \ln \left( \sum_{c' \neq c \in C} z_{c', d, t} \right)
\]

for \(z \in \{k_i, \ell_{fh}, \ell_{mu}, \ell_{fu}, B, C, D\}\) and \(\tau \in \{5, 10\}\). Below the hat represents the estimate.

1. The substitution parameter \(\xi\) is estimated from the first-difference equation (17):

\[
\Delta \ln \left( \frac{w_{fh}}{r_i} \right) = - (1 - \hat{\xi}) \Delta \ln \left( \frac{\ell_{fh}}{k_i} \right) + \Delta u_5.
\]

The share parameter \(\psi\) is then estimated from the level equation (12):

\[
\ln \left( \frac{w_{fh}}{r_i} \right) + (1 - \hat{\xi}) \ln \left( \frac{\ell_{fh}}{k_i} \right) = \ln \left( \frac{1 - \hat{\psi}}{\psi} \right) + u_5.
\]

Finally, the CES aggregate of \(k_i\) and \(\ell_{fh}\) is estimated as: \(\hat{D} = \left[ \hat{\psi}k_i^{\hat{\xi}} + (1 - \hat{\psi}) \ell_{fh}^{\hat{\xi}} \right]^{\frac{1}{\hat{\xi}}} \).

2. The substitution parameter \(\eta\) is estimated from the first-difference equation (13):

\[
\Delta \ln \left( \frac{w_{mh}\ell_{mh}}{w_{fh}\ell_{fh}} \right) - \hat{\xi} \Delta \ln \left( \frac{\hat{D}}{\ell_{fh}} \right) = - \eta \Delta \ln \left( \frac{\hat{D}}{\ell_{mh}} \right) + \Delta u_1.
\]

The share parameter \(\gamma\) is estimated from the level equation (8):

\[
\ln \left( \frac{w_{mh}}{w_{fh}} \right) + \ln (1 - \hat{\psi}) + \left( \hat{\gamma} - \hat{\xi} \right) \ln \hat{D} + (1 - \hat{\eta}) \ln \ell_{mh} - \left( 1 - \hat{\xi} \right) \ln \ell_{fh} = \ln \left( \frac{1 - \gamma}{\gamma} \right) + u_1.
\]

Finally, the CES aggregate of \(D\) and \(\ell_{mh}\) is estimated as: \(\hat{C} = \left[ \hat{\gamma} \hat{D}^{\hat{\theta}} + (1 - \hat{\gamma}) \ell_{mh}^{\hat{\theta}} \right]^{\frac{1}{\hat{\theta}}} \).
3. The substitution parameter $\rho$ is estimated from the first-difference equation (16):

$$
\Delta \ln \left( \frac{w_{fh} \ell_{fh}}{w_{fu} \ell_{fu}} \right) + \hat{\eta} \Delta \ln \left( \frac{\hat{C}}{D} \right) + \hat{\xi} \Delta \ln \left( \frac{\hat{D}}{\ell_{fh}} \right) = \rho \Delta \ln \left( \frac{\hat{C}}{\ell_{fu}} \right) + \Delta u_4.
$$

The share parameter $\mu$ is estimated from the level equation (11):

$$
\ln \left( \frac{w_{fh}}{w_{fu}} \right) - \ln \hat{\gamma} - \ln \left( 1 - \hat{\psi} \right) - (\hat{\rho} - \hat{\eta}) \ln \hat{C} - \left( \hat{\eta} - \hat{\xi} \right) \ln \hat{D} + \left( 1 - \hat{\xi} \right) \ln \ell_{fh} - (1 - \hat{\rho}) \ln \ell_{fu}
$$

$$
= \ln \left( \frac{\mu}{1 - \mu} \right) + u_4.
$$

Finally, the CES aggregate of $C$ and $\ell_{fu}$ is estimated as: $\hat{B} = \hat{\mu} \hat{\psi} + (1 - \hat{\mu}) \ell_{fu}^{\hat{\rho}}$.

4. The substitution parameter $\sigma$ is estimated via the generalized method of moments jointly using the first-difference equations (14) and (15):

$$
\Delta \ln \left( \frac{w_{mu} \ell_{mu}}{w_{fu} \ell_{fu}} \right) - \hat{\sigma} \Delta \ln \left( \frac{B}{\ell_{fu}} \right) = -\sigma \Delta \ln \left( \frac{\hat{B}}{\ell_{mu}} \right) + \Delta u_2,
$$

$$
\Delta \ln \left( \frac{w_{mh} \ell_{mh}}{w_{mu} \ell_{mu}} \right) + \hat{\rho} \Delta \ln \left( \frac{\hat{B}}{C} \right) + \hat{\eta} \Delta \ln \left( \frac{\hat{C}}{\ell_{mh}} \right) = \sigma \Delta \ln \left( \frac{\hat{B}}{\ell_{mu}} \right) + \Delta u_3.
$$

The share parameter $\lambda$ is estimated from the level equations (9) and (10):

$$
\ln \left( \frac{w_{mu}}{w_{fu}} \right) - \ln \left( \frac{1}{1 - \hat{\mu}} \right) + (\hat{\sigma} - \hat{\rho}) \ln \hat{B} + (1 - \hat{\sigma}) \ln \ell_{mu} - (1 - \hat{\rho}) \ln \ell_{fu} = \ln \left( \frac{1 - \lambda}{\lambda} \right) + u_2
$$

$$
\ln \left( \frac{w_{mh}}{w_{mu}} \right) - \ln \hat{\mu} - \ln \left( 1 - \hat{\gamma} \right) - (\hat{\sigma} - \hat{\rho}) \ln \hat{B} - (\hat{\rho} - \hat{\eta}) \ln \hat{C} + (1 - \hat{\eta}) \ln \ell_{mh} - (1 - \hat{\sigma}) \ln \ell_{mu}
$$

$$
= \ln \left( \frac{\lambda}{1 - \lambda} \right) + u_3
$$

Let $\left( \hat{\lambda}_1, \hat{\sigma}_1 \right)$ denote a set of share and substitution parameters estimated from equations

38
The factor demand functions can be derived as:

\[ \ell_{fh} = y A^{-1} \alpha^{-\alpha} (1 - \alpha) \lambda \gamma^\frac{1}{1-\gamma} (1 - \psi)^{\frac{1}{1-\gamma}} \frac{1}{\omega_{fh}} - \frac{1}{\alpha} \nu E^{1-\alpha+\omega} \frac{1}{\omega} F^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} G^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} H^{1-\omega}, \]

\[ \ell_{mh} = y A^{-1} \alpha^{-\alpha} (1 - \alpha) \lambda \gamma^\frac{1}{1-\gamma} (1 - \gamma)^{\frac{1}{1-\gamma}} \frac{1}{\omega_{mh}} - \frac{1}{\omega} E^{1-\omega+\alpha} \frac{1}{\omega} F^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} G^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} H^{1-\omega}, \]

\[ \ell_{fu} = y A^{-1} \alpha^{-\alpha} (1 - \alpha) \lambda \gamma^\frac{1}{1-\gamma} (1 - \mu)^{\frac{1}{1-\gamma}} \frac{1}{\omega_{fu}} - \frac{1}{\alpha} \nu E^{1-\alpha+\omega} \frac{1}{\omega} F^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} G^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} H^{1-\omega}, \]

\[ k_i = y A^{-1} \alpha^{-\alpha} (1 - \alpha) \lambda \gamma^\frac{1}{1-\gamma} (1 - \mu)^{\frac{1}{1-\gamma}} \psi^{\frac{1}{1-\gamma}} \frac{1}{\omega_{fu}} - \frac{1}{\alpha} \nu E^{1-\alpha+\omega} \frac{1}{\omega} F^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} G^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} H^{1-\omega}, \]

\[ k_o = y A^{-1} \alpha^{-\alpha} (1 - \alpha) \lambda \gamma^\frac{1}{1-\gamma} (1 - \mu)^{\frac{1}{1-\gamma}} \psi^{\frac{1}{1-\gamma}} \frac{1}{\omega_{fu}} - \frac{1}{\alpha} \nu E^{1-\alpha+\omega} \frac{1}{\omega} F^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} G^{1-\omega} \frac{1-\alpha}{(1-\omega)(1-\alpha)} H^{1-\omega}, \]

where

\[ E = \left[ \gamma^\frac{1}{1-\gamma} H^{\frac{1}{1-\gamma}} + (1 - \gamma)^{\frac{1}{1-\gamma}} \frac{1}{\omega_{mh}} - \frac{1}{\gamma} \right]^{\frac{1-\alpha}{\gamma}}, \]

\[ F = \left[ \mu^{\frac{1}{1-\gamma}} G^{\frac{1}{1-\gamma}} + (1 - \mu)^{\frac{1}{1-\gamma}} \frac{1}{\omega_{fu}} - \frac{1}{\mu} \right]^{\frac{1-\alpha}{\mu}}, \]

\[ G = \left[ \gamma^\frac{1}{1-\gamma} H^{\frac{1}{1-\gamma}} + (1 - \gamma)^{\frac{1}{1-\gamma}} \frac{1}{\omega_{mh}} - \frac{1}{\gamma} \right]^{\frac{1-\alpha}{\gamma}}, \]

\[ H = \left[ \psi^{\frac{1}{1-\gamma}} r_i^{\frac{1}{1-\gamma}} + (1 - \psi)^{\frac{1}{1-\gamma}} \frac{1}{\omega_{fh}} - \frac{1}{\psi} \right]^{\frac{1-\beta}{\psi}}. \]