Kinetics, Pseudo-Kinetics, Uncertainty Principle and Quantum 1/f Noise

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1/f noise at arbitrary low frequencies is the way of existence of irreversibility in thermal motion governed by reversible laws of mechanics. This statement not once was confirmed in statistical mechanics beyond its traditional kinetical roughenings. Here we point out that in case of quantum statistical mechanics in principle it is sufficient to avoid such the roughening as the “Fermi golden rule”. This means taking into account the time-energy uncertainty principle (time-frequency one in classical limit) and thus uncertainties in characteristics of real collisions and scatterings of particles and/or quanta. We consider the resulting “pseudo-kinetics” and demonstrate how it produces quantum 1/f-noise.

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I. INTRODUCTION

1. Not long ago in [1] we have reviewed the idea for the first time pronounced many years ago in [2–5]: any physical random process gives a place for 1/f noise. Indeed, mere logics prompts, for instance, that if a process produces random events but constantly forgets their previous numbers then it is definitely unable to control their production rate at large time intervals, hence, producing also arbitrary long deviations of the rate from its past values, that is flicker type rate fluctuations.

In classical statistical mechanics (SM) such the way of thinking gets support [6–11] when one can spy upon correlations due to determinism of mechanical motion and respectively discard statistical hypotheses principally superfluous in view of the determinism. The forgetting of the past here is the same as complexity of mechanical motion. Because of its time reversibility its future also gets forgotten, in the sense of its unpredictability, thus acquiring features of irreversibility: relaxation, dissipation, diffusion and noise. At that, rates of relaxation, dissipation, diffusion and noise are not attributes of mechanics’ laws and therefore can appear from these laws in the form of fluctuations only.

Moreover, these fluctuations always are of flicker type. Indeed, since the rates are time-non-local characteristics, their fluctuations say about prehistory of present system’s state and therefore have no influence on the future as fully determined by present instant state. Hence, the rates’ fluctuations do not cause a feedback reaction. Hence, they have no definite relaxation times.

We see that 1/f-noise arises as another side of usual (“white”) thermal noise and hides no specific physical “mechanisms”. Instead of them, its theory needs in adequate mathematical approaches to SM equations. So, the story of 1/f-noise strongly resembles the ancient story of the phlogiston which eventually was understood as thermal agitation of particles of the matter.

2. In quantum SM both the past and the future are in a fog of quantum uncertainty, so that connection between them is not fatal and allows some “free will”. At the same time the quantum uncertainty obeys completely deterministic laws of evolution, and therefore all the aforesaid about 1/f noise covers quantum case as well.

This was demonstrated for tunnel transport noise in many-electron systems [12] and quantum random walk of a particle interacting with thermal bath [13], in particular, electron in phonon bath (thermally oscillating crystal lattice) [14, 16]. These investigations confirmed title of the work [12], “Lattice scattering causes 1/f noise”, and showed how it arises just from electron-lattice (particle-bath) interaction by itself, without any external driving.

The above claimed verbal proof of inevitability of 1/f-noise in irreversible phenomena was formally fortified by proving theorems [13, 14] which do connect statistics of particle’s random walk in bath medium to exact density matrix evolution of the system and state that diffusivity of the particle must possess flicker fluctuations because their absence is in contradiction to unitarity of the evolution in SM.

Such principal results stimulate search for more light approximate methods simplifying SM as much as possible but without loss of 1/f-noise, in contrast to usual kinetics always losing it. Interestingly, this requirement can be satisfied if one takes into account factual duration (of quantum amplitude formation) of quantum transitions. The works [13] and [16] gave clear illustrations of usefulness of this recipe, and this aspect of the theory will be focus of our consideration below.

II. 1/F-NOISE AS SURPRISE OF THEORETICAL PHYSICS

A. The golden rule of kinetics

1. The word “surprise” is taken from the R. Peierls’
book [17] and describes typical relations between theoreticians and theories in physics under usual mix of findings and mistakes. Theoreticians hope to meet pleasant surprises, as is indicated by W. Pauli’s “law of conservation of sloppiness” cited in [17].

We will dispute this optimism at least in the case discussed in section 5.3 in [17], namely, the question of applicability of the famous “Fermi golden rule” [18, 19]. The golden rule (GR) is usual artificial addition of applicability of the famous “Fermi golden rule” [18, 19]. The golden rule (GR) is usual artificial addition of applicability of the famous “Fermi golden rule” [18, 19].

Let us recall general expression

\[ p_{\tau}(2 \leftarrow 1) = \frac{2}{\hbar} \left[ 1 - \cos ((E_2 - E_1) / \hbar \tau) \right] |\Phi_{21}|^2 \]

of the lowest order of QNTP [21] for probability (square of modulus of quantum amplitude) of that during time \( \tau \) under influence of perturbation \( \Phi \) with matrix elements \( \Phi_{21} = \langle 2 | \Phi | 1 \rangle \) a quantum system makes transition between stationary (in absence of the perturbation) states \( |1\rangle \) and \( |2\rangle \) with energies \( E_1 \) and \( E_2 \). It will be comfortable to write (1) as

\[ p_{\tau}(2 \leftarrow 1) = \tau \gamma_{\tau}(2 \leftarrow 1) \]

with probability of transition per unit of time of transition \( \gamma_{\tau}(2 \leftarrow 1) \) and normalized to unit sharp peak-like quasi-delta function (QDF)

\[ \delta_{1/\tau}(\epsilon) \equiv \frac{\hbar}{2 \pi \tau} \left[ 2 \sin (\epsilon \tau / 2 \hbar) / \epsilon \right]^2 \]

which turns into usual Dirac’s delta-function (DDF) in the limit \( \tau \to \infty : \delta_0(\epsilon) = \delta (\epsilon) \).

Formally the GR is performing in (2) the QDF replacement with DDF,

\[ \delta_{1/\tau}(\epsilon) \Rightarrow \delta (\epsilon) \]

under assumption that energies of the states belong to continuous spectrum. Or, what is the same, replacement

\[ \gamma_{\tau}(2 \leftarrow 1) \Rightarrow \gamma_{\infty}(2 \leftarrow 1) \]

which enforces time-dependent “probabilities per unit time” to become constants independent on duration of transitions.

2. If full Hamiltonian of system under consideration is

\[ H = H_0 + \Phi \]

then \( |1\rangle \) and \( |2\rangle \) can be thought eigen-states of \( H_0 \): \( H_0 |1\rangle = E_1 |1\rangle \) and so on. For definiteness we may keep in mind the “Brownian” particle (BP) in a medium (“thermostat”) with Hamiltonian (9) from [1], with

\[ H_0 = \frac{p^2}{2M} + H_{th} \]

representing system without interaction between its parts (BP and medium) while \( \Phi \) represents their interaction.

The kinetic theory treats quantum evolution like jump-like transitions between states of a given set as if next time jumps realize or not dependently on outcomes of imaginary chance trials with chances established from above (“dice tossings” by god of mathematicians). Following this concept, dividing (long enough) time of system’s evolution into (not too long) intervals with duration \( \tau \), each almost surely containing not more then one transition, and looking through all such series of transitions “by the probability theory” [22], one can conclude that (density of) probability \( \rho \) of finding our system to be in one or another state (eigen-state of unperturbed Hamiltonian) is given by

\[ \rho(t) \approx (1 + \tau \tilde{K}_{1/\tau})^{1/\tau} \rho(0) \approx \exp (t \tilde{K}_{1/\tau}) \rho(0) \]

with kinetic operator (KO) \( \tilde{K}_{1/\tau} \) which acts by formula

\[ (\tilde{K}_{1/\tau} \rho)_1 \equiv \sum_{2} \left[ \gamma_{\tau}(1 \leftarrow 2) \rho_2 - \gamma_{\tau}(2 \leftarrow 1) \rho_1 \right] \]

with ciphers in the role of indices of possible states.

Such the theory is very uncomfortable because of arbitrariness of the quantity \( \tau \) about which one only can say that it certainly is bounded both from above and from below. Kinetics disposes of this inconvenience, like frequently in mathematical physics, by means of physically absurd idealization when the factor \( \tau \) before the KO symbol and in the exponent (number of intervals \( t/\tau \)) in (3) is tended to zero while inside the KO (in QDF) the same factor is tended to infinity. As the result of such wild arbitrariness, one creates formally unambiguous evolution law

\[ \rho(t) = \exp (t \tilde{K}_0) \rho(0) \]

or, in differential form,

\[ \frac{\partial \rho}{\partial t} = \tilde{K}_0 \rho \]

with KO \( \tilde{K}_0 \) acting by formula

\[ (\tilde{K}_0 \rho)_1 \equiv \sum_{2} \left[ \gamma_{\infty}(1 \leftarrow 2) \rho_2 - \gamma_{\infty}(2 \leftarrow 1) \rho_1 \right] \]

with the sum symbol presuming at once integration over continuously varying indices and summation over discrete ones.

Equation (11) with right-hand side constructed like (12) is termed “master equation” [18]. In the context of quantum theory an equation of such kind for the first time was suggested by Pauli [23] already in 1928. At that, Pauli considered transitions between not individual states but groups of states with close energies, assuming complete randomness of their phases and applying averaging over them. This trick eventually is equivalent to application of GR.

Unfortunately, kinetic equations are so much extremal roughenings of SM that they categorically lose flier
noise. In order to make sure of this, first let us compare
defects of the approximations (8) and (10) to recognize
another approximation which is able to catch flicker
noise.

B. Beyond the golden rule:
failure of kinetics and advances of pseudo-kinetics

1. From viewpoint of quantum mechanics, the golden
rule approximation (GRA) (10)-(12) seems more rough
than (8) but much more simple. Therefore the
attempt in [17] to justify just the GRA by comparing KO
and PKO (12) looks very natural.

Of course, these KOs differ to the extent of narrowness
of the QDF $\delta_{1/\tau}(\epsilon)$’s peak, i.e. smallness of QDF’s width
$\Delta\epsilon \approx 2\pi\hbar/\tau$, in comparison with width $\epsilon_0$ of factor
$\{\Phi_{21}\}^2\rho_2$ as function of difference $E_2 - E_1$. According
to the reasonings exposed in [17] in case of interactions
- collisions - of a free charge carrier (let electron) with
phonons and defects of crystal lattice in semiconductors
and even metals practically always $\Delta\epsilon/\epsilon_0 \ll 1$, even if
assuming that $\tau$ is less than mean time between
collisions, $\tau_0$. This statement serves in [17] as base for
conclusion that the “sloppiness” of GRA is negligible,
thus excusable, i.e. GRA has no significant alternatives.

From our viewpoint, however, that is wrong inclusion,
for one should compare not KOs (12) and (9) in
themselves but corresponding evolution operators, that
is exponentials in (10) and (8). All the more because
these operators have qualitative differences, at least in
their relations to “unperturbed energy” $E$.

2. Namely, evolution due to (8) is accompanied with
unbounded spreading, or diffusion, of the distribution $\rho(t)$ over the energy $E$. It is so merely because at
any fixed $\tau$’s value each particular transition gives
random increase or decrease of $E$ by a step of order of
QDF’s width $\sim \Delta\epsilon \approx 2\pi\hbar/\tau$, and these steps (as
well as successive transitions themselves) are mutually
statistically independent (incoherent). It is in obvious
contradiction to (quantum-mechanical) reality.

In opposite, evolution under GRA (10) due to $\Delta\epsilon = 0$
precisely conserves the unperturbed energy $E$ and thus
all time passes not leaving some hyper-surface $E = \text{const}$
in the space of indices ($H_0 = \text{const}$ in the space of states).
This behavior also is in contradiction to physical reality,
since in fact $H_0$ is not integral of motion.

Thus, kinetics in any variant is physically inadequate
theory. Fortunately, our comparison of evolutions with
$\tau = \text{const} < \infty$ and $\tau = \infty$ prompts third, original,
possibility when duration of ripening of transitions $\tau$ is
not fixed but grows together with duration of system’s
evolution and observation $t$ and takes the only finite
value which is free of theoretical arbitrariness, that is
$\tau = t$, so that

$$\rho(t) = \exp (t\hat{K}_{1/\tau}) \rho(0).$$

(13)

In such way one removes fundamental defect of GRA,
i.e. replacing of actual transitions, which are squeezed in
time inside frames of system observations, by fantastic
transitions stretched over infinite time. Formally the
value $\tau = t$ lies between $\tau = 0$ and $\tau = \infty$, therefore the corresponding evolution law (13) can be
named “golden mean approximation” (GMA).

Clearly, evolution in GMA does not conserve the
unperturbed energy $E$. But its probable departure from
its initial value all the time keeps closed between nearly
constant finite bounds, since QDF’s width $\sim \Delta\epsilon \approx 2\pi\hbar/t$
and thus magnitude of steps of random walk over $E$
do decrease as evolution (observation) time increases.
This is physically plausible picture which makes GMA
reasonable alternative to GRA and thus surprise in the
sense of [17].

At the same time, evidently, this picture no more
belongs to the kinetics, because now “transition probabilities (per unit time)” have no definite values.
Instead, they become very significantly varying with
evolution time, so that, in essence, they lose naive
probabilistic meaning at all. In this way the theory
gives back at least a part of coherence of (quantum-)
mechanical motion. Therefore such approximations can
be named also “pseudo-kinetic”, while any operator like
$\hat{K}_{1/t}$, containing QDF $\delta_{1/t}(\epsilon)$, “pseudo-kinetic operator”
(PKO).

Importantly, the PKO automatically introduces
unrestrictedly low frequencies $\sim 1/t$ as mathematical
building material for flicker noise. Next, consider how principally it happens.

3 Now let us compare KO $\hat{K}_0$ and PKO $\hat{K}_{1/t}$ from
viewpoint of their stationary probability distributions
$\rho_{st}$, i.e. solutions to equations $\hat{K}_0 \rho = 0$ and $\hat{K}_{1/t} \rho = 0$.
Since KO conserves the unperturbed energy, it possesses infinitely many linearly independent stationary
distributions, for instance, microcanonical $\rho_{st} \propto \delta(E - E_0)$, that is uniform on some given hyper-surface $E = E_0 = \text{const}$ in index space. In opposite, since PKO does
not conserving the energy, it has only one stationary
distribution $\rho_{st} = \text{const}$, uniform in the whole index
space.

In the language of spectral theory of linear operators,
both KO and PKO have zero eigen-value, but in case of
KO it is infinitely degenerated while in case of PKO non-degenerated.

In other words, change of KO to PKO fully kills the
degeneration. Taking into account essential cognation of KO and PKO, at $\Delta\epsilon/\epsilon_0 \approx 2\pi\hbar/\epsilon_0 t \ll 1$ we can consider this change as weak perturbation of KO in the
sense of the abstract perturbation theory (PT) [25]. If
so, the removal of degeneration of zero eigen-value (EV)
means that the corresponding eigen-functions (EF), after
transforming under perturbation, do acquire new EVs
which (all except one) are non-zero though small to the
extent of perturbation parameter $2\pi\hbar/\epsilon_0 t \times 1/t$.

Here just such CVs slipping to zero $\propto 1/t$ are
responsible for $1/f$-noise, for they imply presence of
arbitrary slow relaxation modes in evolution by PKO and
thus arbitrary long statistical correlations. Namely, that are modes connected to the additional (activated) degree of freedom along $\mathcal{E}$. In [10] we presented the example: fourth-order correlations of electron velocity (in phonon medium) which represent $1/f$-noise in electron’s rate of diffusion (diffusivity).

In principle, there is nothing except trivial uncertainty in energy transfer under transitions (or in precision of resonances in the classical limit) due to finiteness of interaction duration. Next, consider it in more detail.

III. FROM UNCERTAINTY PRINCIPLE TO $1/f$ INTERACTION FLUCTUATIONS

1. Preliminarily, notice that because of symmetry $\gamma_\tau(2 \leftarrow 1) = \gamma_\tau(1 \leftarrow 2)$ PKO is symmetric operator. Therefore all its EVs are real and its action can be represented in the form

$$\langle \hat{K}_{1/t} f \rangle_1 = \sum_2 \gamma_t(1 \leftarrow 2) [f_2 - f_1] . \quad (14)$$

From here for any function $f$ of state indices one can easy derive inequality

$$\sum_1 f_1^*(\hat{K}_{1/t} f) = \frac{1}{2} \sum_1 \gamma_t(1 \leftarrow 2) |f_1 - f_2|^2 \leq 0 ,$$

and from here deduce (physically doubtful) statement that all EVs of PKO are non-positive.

2. Let us focus at random walk, or fluctuations, of unperturbed energy $\mathcal{E}$. When taken with opposite sign, these fluctuations can be interpreted, in view of conservation of the total energy $H_0 + \Phi$, also as fluctuations of interaction energy. We are interested in their marginal probability distribution

$$W(\mathcal{E}) = \sum_1 \delta(\mathcal{E} - \mathcal{E}_1) \rho_1 , \quad (15)$$

its time evolution and their temporal correlations. For simplicity let us assume that conditions of $\mathcal{E}$’s walk are uniform in the state space: $\rho$ is uniform on any equal $\mathcal{E}$ hyper-surface, and quantity

$$g(\mathcal{E} - \mathcal{E}_2) \equiv \sum_1 \delta(\mathcal{E} - \mathcal{E}_1) \frac{2\pi}{\hbar} |\Phi_{12}|^2 \quad (16)$$

is a function of only difference of energies before and after transitions. Evidently, it then is non-negative and even function: $g(-\epsilon) = g(\epsilon) \geq 0$. Then PKO reduces to an operator acting in the set of functions $f(\mathcal{E})$ of one variable $\mathcal{E}$ by formula

$$\hat{K}_{1/t} f(\mathcal{E}) = \int \delta_{1/t}(\mathcal{E} - \mathcal{E}') \times \quad (17)$$

$$\times g(\mathcal{E} - \mathcal{E}') [f(\mathcal{E}') - f(\mathcal{E})] d\mathcal{E}' .$$

This operator possesses clear continuous spectrum of EFs $(2\pi \hbar)^{-1/2} \exp(i\theta \mathcal{E}/\hbar)$. Their CVs are

$$\lambda_{\epsilon}(\theta) = \int \delta_{1/t}(\epsilon) g(\epsilon) [\cos(\theta\epsilon/\hbar) - 1] d\epsilon \quad (18)$$

and all belong to interval $-g_t < \lambda_{\epsilon}(\theta) \leq 0$ with lower bound $-g_t = -\int \delta_{1/t}(\epsilon) g(\epsilon) d\epsilon$. Correspondingly,

$$\exp(t\hat{K}_{1/t}) f(\mathcal{E}) = \int S_{1/t}(t, \mathcal{E} - \mathcal{E}') f(\mathcal{E}') d\mathcal{E}' \quad (19)$$

with propagator - or, in terms of theory of random processes, transient probability (density) of the energy, - what may be expressed by

$$S_{1/t}(t', \mathcal{E} - \mathcal{E}') = \int e^{\lambda_{\epsilon}(\theta) t'} e^{i\theta (\mathcal{E} - \mathcal{E}')/\hbar} \frac{d\theta}{2\pi} . \quad (20)$$

Alternatively, by dividing PKO (17) into two obvios parts, this propagator can be represented in the form of series:

$$S_{1/t}(t', \mathcal{E} - \mathcal{E}_0) = e^{-g_t t'} \sum_{n=0}^{\infty} (g_t t')^n (D_t(\epsilon) \otimes)^n \delta(\epsilon) , \quad (21)$$

where $\epsilon = \mathcal{E} - \mathcal{E}_0$, the symbol $\otimes$ denotes operation of convolution over variable $\epsilon$, and we introduced function

$$D_t(\epsilon) = \frac{\delta_{1/t}(\epsilon) g(\epsilon)}{\int \delta_{1/t}(x) g(x) dx} = \frac{\delta_{1/t}(\epsilon) g(\epsilon)}{g_t} . \quad (22)$$

We want to trace in this model how vanishingly small uncertainties $\Delta \epsilon \sim 2\pi \hbar / t$ of energy transfers in particular transitions do summation into non-vanishing fluctuations of rate of the interaction.

3. Let at the beginning of observation the energy is definitely known: $W(t = 0, \mathcal{E}) = \delta(\mathcal{E} - \mathcal{E}_0)$. We will consider, in the frameworks of GMA [13] and PKO [17], probability distribution $W(t, \mathcal{E}) = \exp(t\hat{K}_{1/t}) W(0, \mathcal{E})$ of following energy fluctuations.

First, calculate their variance by using the series (21). Obviously, it has meaning of expansion over number $n$ of elementary transitions and prescribes them Poissonian statistics with mean number of transitions $\langle n \rangle = g_t t$, while non-negative, and normalized to unit, function (22) plays role of probability distribution of discrepancy (non-conservation, uncertainty) of energy in separate transition. At that $n$-fold convolution of $D(\epsilon)$ represents sum of $n$ mutually statistically independent contributions. Since variance of such sum equals to sum of variances of the contributions, one finds

$$\int (\mathcal{E} - \mathcal{E}_0)^2 W(t, \mathcal{E}) d\mathcal{E} = \langle n \rangle \int \epsilon^2 D_t(\epsilon) d\epsilon = \quad (23)$$

$$= t \int \epsilon^2 \delta_{1/t}(\epsilon) g(\epsilon) d\epsilon \approx \frac{\hbar}{\pi} \int g(\epsilon) d\epsilon .$$

At the end here we neglected QDF’s oscillations, since at large enough time the function $g(\epsilon)$ may be thought much more smooth than $\cos(\epsilon t/\hbar)$. As one can see, under $\int g(\epsilon) d\epsilon < \infty$ the variance tends with time to a finite value which is fully determined by the inversely-quadratic “wings”, or “long tails” $\propto \epsilon^{-2}$, of QDF, that is just by what is discarded in GRA.

4. All the more, the QDF’s tails determine statistics of energy fluctuations in case when $\int g(\epsilon) d\epsilon = \infty$ and the
variance turns into infinity. According to (16), this is the case if \((2\pi/\hbar)\sum_i |\Phi_{12}|^2 = \infty\), i.e. the matrix elements (ME) of transitions by themselves, without additional requirement of energy conservation, allows for infinitely many variants of transitions from a given state. This is both theoretically thinkable and physically realistic situation. Let us discuss it with representation (19)-(20).

We may start it from simple special case when \(g(\epsilon) = g = \text{const}\) (by themselves ME equally connect states with any energy difference). Subjecting QDF to Fourier transform, one has

\[
\int \delta_{1/\ell}(\epsilon) \cos(\theta \epsilon / \hbar) d\epsilon = \left[1 - \frac{\theta}{\ell}(t - |\theta|)\right], \quad (24)
\]

where \(1(\cdot)\) is the Heaviside step function. Therefore, if \(|\theta| < t\) then EVs (15) are equal to \(\lambda_i(\theta) = -g|\theta|/t\), so that the exponentials in (20) become time-independent. Hence, in the region of not too small energy fluctuations, at \(|\epsilon| = |E - E_0| \gtrsim \hbar / t\), their distribution takes (quasi-) stationary form

\[
W(t, E) \approx \int e^{-g|\theta|} \cos(\theta \epsilon / \hbar) \frac{d\theta}{2\pi \hbar} = \Delta_g(\epsilon) = \frac{1}{\pi} \frac{gh}{g^2 \hbar^2 + \epsilon^2}, \quad (25)
\]

that is the form of Cauchy distribution.

5. Now recall the qualitative specificity of Cauchy distributions (CD) what differs CD from distributions with finite statistical moments (for example, Gaussian ones, \(G_g(\epsilon) = \frac{1}{\sqrt{2\pi} gh} \exp(-\epsilon^2 / 2gh^2)\)). Namely, under convolutions of CDs summation of their widths takes place: \(\Delta_g \otimes \Delta_g = \Delta_{2g}\), instead of squares of widths (as under convolutions of Gaussians: \(G_g \otimes G_g = G_{\sqrt{2}g}\)). Such specificity of Cauchy statistics may be more brightly expressed by saying that mutually independent random quantities distributed “by Caucgy” are added as if they were completely dependent (e.g. literally equal). So paradoxical property explains why distributions close to CD involuntarily appear in both phenomenological and microscopic theories of flicker noise with the origin characterized in Introduction I. 3. 4. 6. 8. 11.

Formally the mentioned property of CD is due to its quadratic tails, and therefore it extents to other distributions with such tails, including our QDF. Just by this reason the sum of widths of \(\langle n \rangle = gt\) convolutions of QDF in the expansion (21) gives fixed finite width of CD in (25). It shows that under neglect of QDF’s oscillations its approximation by CD,

\[
\delta_{1/\ell}(\epsilon) \approx \Delta_{1/\ell}(\epsilon), \quad (26)
\]

seems reasonable.

6. Returning to the general case \(g(\epsilon)\), at sufficiently large observation time, when \(\langle n \rangle \gg 1\), and EVs are dominated by QDF’s tails, we can write

\[
\lambda_i(\theta) \to \frac{\chi(\theta/\hbar)}{t}, \quad (27)
\]

\[
\chi(x) = \frac{\hbar}{\pi} \int g(\epsilon) \frac{\cos(x \epsilon) - 1}{\epsilon^2} d\epsilon,
\]

and for propagator (20) respectively

\[
S_{1/\ell}(t', \epsilon) \to \int e^{(t'/t)x} \chi(x) e^{ix\epsilon} \frac{dx}{2\pi}, \quad (28)
\]

Thus, according to (19) and (28), in most general case too the departure \(E - E_0\) of energy from its initial value becomes frozen with (quasi-) stationary distribution possessing finite width and characteristic function \(\langle exp[ix(E - E_0)] \rangle \to \exp(\chi(x))\).

All that means that the remarkable property of summation of independent energy's increments (discrepancies, uncertainties) like dependent ones survives when QDF’s tails are cut off by factor \(g(\epsilon)\) (though may produce less tailed distributions than CD).

Naturally, time evolvent of such summation law for \(E\)'s increments looks as \(E\)’s fluctuations looks of flicker type, with infinitely propagating correlations.

7. At last, demonstrate these correlations, in the meantime confining ourselves by two-time ones.

Consider the fluctuation \(\epsilon = E - E_0\) as random process \(\epsilon(t)\). Exploiting PKO in our GMA similarly to use of KO in GRA, introduce joint characteristic function of \(\epsilon(t)\) and \(\epsilon(t_0)\) \((0 < t_0 < t)\) by expression

\[
\langle e^{ix\epsilon(t)} e^{iy\epsilon(t_0)} \rangle = \int \text{d}e e^{ix\epsilon(t)} e^{iy\epsilon(t_0)} \hat{\kappa}_{1/\ell} e^{ix\epsilon(t)} e^{iy\epsilon(t)} \delta(\epsilon), \quad (29)
\]

while operator exponentials (propagators) there we express like in (15) through integral operator (20). Besides, being interested in large time scales, we apply to (29) the approximation (28). This implies

\[
\langle e^{ix\epsilon(t)} e^{iy\epsilon(t_0)} \rangle \approx e^{(1-t_0/t)\chi(x)} e^{(t_0/t)\chi(x+y)}, \quad (30)
\]

It is not hard to deduce from here that

\[
\langle \epsilon(t) \epsilon(t_0) \rangle = \frac{t_0}{t} \langle \epsilon^2 \rangle, \quad (31)
\]

where the variance \(\langle \epsilon^2 \rangle\) must be taken from (23). This formula shows that correlation function of energy fluctuations always has flicker type (slow, non-integrable) dependence on difference of its time arguments, and that its flicker behavior may be manifold, from inverse proportionality to constancy, corresponding to quasi-static fluctuations.
IV. STATISTICAL-MECHANICAL APPROACH TO PSEUDO-KINETICS

A. The Liouville equation and evolution of quantum state’s probability distribution

The formally exact description of our system with Hamiltonian (6), $H = H_0 + \Phi$, requires density matrix (DM) $\varrho$ and quantum Liouville equation (von Neumann equation)

$$\frac{\partial \varrho}{\partial t} = \hat{L}_\varrho = (\hat{L}_0 + \hat{L}_\Phi) \varrho$$

(32)

where $\hat{L}$ is Liouville super-operator whose action onto operators (‘matrices’) is defined by

$$\hat{L}_\varrho = -\frac{i}{\hbar} [H, \varrho] = -\frac{i}{\hbar} (H\varrho - \varrho H)$$

(33)

with $\hat{L}_0$ and $\hat{L}_\Phi$ representing two parts of $H$.

As before, we will consider the system’s DM and its evolution within the basis formed by eigen-states of $\hat{H}$, so that the probability distribution of these states $\rho$, which was under our interest, now is the DM’s diagonal: $\rho_1 = \varrho_{11}$. Sometimes we will temporarily identify $\rho$ and matrix produced by turning all non-diagonal elements of $\varrho$ into zero, which may be written in the form $\rho = \hat{D} \varrho$ with $\hat{D}$ being super-operator of this procedure. Obviously, it is projection (super-) operator, that is possessing property $\hat{D}^2 = \hat{D}$. We introduce also the complementary projection (super-) operator $\hat{N} = 1 - \hat{D}$ and notice that $\hat{N} \hat{D} = \hat{D} \hat{N} = 0$.

We assume, as implicitly before, that the Hamiltonian of interaction between system’s parts, $\Phi$, is the same as perturbation Hamiltonian, is purely non-diagonal, therefore operator identity $\hat{D} \hat{L}_\Phi \hat{D} = 0$ takes place. Besides, taking into account one more operator identity $\hat{L}_0 = \hat{N} \hat{L}_0$, we can write

$$\hat{L} = \hat{N} \hat{L}_\varrho + \hat{N} \hat{L}_\Phi \hat{D} + \hat{D} \hat{L}_\Phi \hat{N}.$$  

(34)

Further, let at $t = 0$ the DM is purely diagonal, $\hat{N} \rho(0) = \rho_0$, that is does not contain correlations between the basis states and is fully determined by their probability distribution $\rho(0)$. Then later

$$\rho(t) = \hat{S}(t) \rho(0) \equiv \hat{D} \exp(t \hat{L}) \rho(0) \cdot$$

(35)

With the help of above identities and standard rules of disentangling of operator exponentials the propagator $\hat{S}(t)$ is rewritable as

$$\hat{S}(t) = \hat{D} \exp \left[ \int_0^t \left( e^{-t' \hat{N} \hat{L}_\varrho \hat{D}} - \hat{D} e^{-t' \hat{N} \hat{L}_\Phi \hat{N}} \hat{D} + \hat{D} \hat{L}_\varrho \hat{N} e^{-t' \hat{N} \hat{L}_\varrho \hat{D}} \right) dt' \right] \hat{D}$$

(36)

with left arrow marking chronological ordering of exponentials. On the other hand, separating diagonal and non-diagonal parts of DM in the Liouville equation and applying (31), it is not hard to derive for this propagator an integro-differential equation

$$\frac{\partial \hat{S}(t)}{\partial t} = \int_0^t \hat{Q}(t-t') \hat{S}(t') \, dt'$$

(37)

with operator kernel

$$\hat{Q}(t) \equiv \hat{D} \hat{L}_\Phi \hat{N} e^{t \hat{N} \hat{L}_\varrho \hat{D}} \hat{N} \hat{L}_\varrho \hat{D}.$$  

(38)

These exact formulas must be simplified, - as far as the interaction $\Phi$ allows to manage with lowest order of the quantum non-stationary perturbation theory (QNPT), - by means of replacement

$$\hat{N} \hat{L}_\varrho \hat{D} \Rightarrow \hat{N} \hat{L}_\varrho \hat{N},$$

(39)

which will be named “weak interaction approximation” (WIA). Using it in (38) and uncovering the super-operators, one can verify that

$$(\hat{Q}(t) f)_1 = \sum_2 2 |\Phi_{12}|^2 \cos \frac{E_{12} t}{\hbar} \left[ f_2 - f_1 \right]$$

(40)

with $E_{12} \equiv E_1 - E_2$, i.e. $\hat{Q}(t) = d^2 (\hat{K}_{1/1} / dt^2$.

B. From rigorous statistical mechanics to the pseudo-kinetics

Let us consider the formula (36). There the first of the two (super-) operators in the exponential creates inter-state correlations, i.e. non-diagonal matrix elements (ME) while the second annihilates them. At that, only those terms of the exponential’s series expansion contribute to (36) what include equal numbers of creations and annihilations to combine then into pairs.

It is clear also that the creation-annihilation pairs do follow one after another, with no time intersections, that is dividing the time among themselves and uncorrelated evolution. Nevertheless, any pair may occupy any part of available time, up to its whole value. Therefore in the framework of WIA it seems reasonable to neglect the exclusion of intersections and enumerate the pairs as if they were not feeling one another. In such manner (36) yields approximation

$$\hat{S}(t) \approx \hat{S}_{1/1}(t) = \exp(t \hat{K}_{1/1})$$

(41)

$$\equiv \exp \int \int_{t > t_1, t_2 > 0} \hat{D} \hat{L}_\Phi \hat{N} e^{(t_1-t_2) \hat{N} \hat{L}_\varrho \hat{D}} \hat{N} \hat{L}_\varrho \hat{D} ,$$

which, in view of (38) and (11), is nothing but the golden mean approximation (GMA) with the pseudo-kinetic operator (PKO) $\hat{K}_{1/1}$.

This derivation of GMA shows its tendency to somehow exaggerate a typical time given to transitions and thus underestimate effects of uncertainty principle. Hence, it is interesting to see what things in the pseudo-kinetics may change when accounting for the time repulsion and competition between transitions.
C. Mechanical corrections to pseudo-kinetics

Next, consider equation (27) with kernel (10) and apply to it the Laplace transform:

\[ \tilde{S}(z) = \int_0^\infty e^{-zt} \tilde{S}(t) dt = [z - \tilde{Q}(z)]^{-1}, \]  

(42)

where the tilde in place of hat decorates transforms instead of operator originals, and

\[ \tilde{Q}(z) f_1(z) = \sum_2 \frac{2\pi}{\hbar} |\Phi_{12}|^2 \Delta_2(zE_{12}) [f_2 - f_1]. \]  

(43)

It is visible that action of operator \( \tilde{Q}(z) \) resembles that of PKO, but with another quasi-delta-function (QDF), in the form of the Cauchy distribution (CD) from (25) although with generally complex-valued “width” parameter \( z \). This QDF, like the old one in PKO, serves to improve kinetics of interaction by involving into it the uncertainty principle, but now making it more correctly (even maybe precisely).

Let us consider spectrum of operator \( \tilde{Q}(z) \) as related to the interaction uncertainty in itself, that is to \( E \) dependency, similarly to previous section, reducing \( \tilde{Q}(z) \) to one-dimensional operator:

\[ \tilde{Q}(z) f(E) = \int \Delta_z(E - E') \times \]  

(44)

\[ \times g(E - E') [f(E') - f(E)] dE'. \]

This operator has the same eigen-functions (EF) but with different eigen-values (EV)

\[ \tilde{\lambda}_z(\theta) = \int \Delta_z(\epsilon) g(\epsilon) [\cos(\theta\epsilon/\hbar) - 1] d\epsilon, \]  

(45)

now complex if \( z \) is complex. The corresponding direct analogue of the pseudo-kinetical propagator kernel (20) is the following kernel of the statistical-mechanical propagator \( \tilde{S}(t) \):

\[ S(t, \epsilon) = \int \exp i\phi_{\epsilon}/\hbar \frac{dz}{2\pi i} \frac{d\theta}{2\pi h} \]  

(46)

\[ \int_{-i\infty + 0}^{i\infty + 0} \exp \left[tz - \frac{\lambda_z(\theta)}{2\pi i} \right] \]  

with \( \epsilon = E - E' \). For large enough time, the internal integral here is determined by small \( z \) region, and its asymptotic at \( t/|\theta| \to \infty \) can be calculated by rule

\[ \lim_{t \to \infty} \int_{-i\infty + 0}^{i\infty + 0} \frac{dz}{2\pi i} = \lim_{z \to +0} \frac{z}{z - \lambda_z(\theta)} \]  

(47)

Since \( \lim_{z \to +0} \frac{\lambda_z(\theta)}{z} = \chi(\theta/\hbar) \), - with \( \chi(x) \) defined in (27), - one obtains

\[ S(\infty, \epsilon) = W(\infty, E_0 + \epsilon) \to \int \frac{\cos(\epsilon x)}{1 - \chi(x)} dx \]  

(48)

distribution. But it differs from that in pseudo-kinetics: now its characteristic function (CF) is

\[ \langle \exp[ix(E - E_0)] \rangle \to [1 - \chi(x)]^{-1}, \]  

(49)

instead of \( \langle \exp[ix(E - E_0)] \rangle \to \exp \chi(x) \) there.

Comparing the limit distribution (48) with GMA result (28) (at \( t' = t \)), one can see that their variances and their tails at large \( \epsilon^2 \gtrsim \langle \epsilon^2 \rangle \) are coinciding, and all their differences concentrate at small \( |\epsilon| \ll \epsilon_0 \) being determined there by the asymptotic \( \chi(x \to \infty) = -h\phi(0)|x| \). Under GMA this asymptotic leads to constancy of density of distribution in vicinity of \( \epsilon = 0 \), but now, in (48), it means a logarithmic divergence near \( \epsilon = 0 \). Now

\[ S(\infty, \epsilon) \approx \int \frac{\cos(\epsilon x)}{1 + \epsilon |x|} \approx \frac{1}{2\pi \epsilon} \ln \left( 1 + \frac{2\epsilon^2}{\epsilon^2} \right) \]  

(50)

with \( \epsilon = h\phi(0) \), when \( |\epsilon| \ll \epsilon_0 \). If \( g(\epsilon) = g = \text{const} \) then this expression is valid anywhere, thus replacing CD.

We see that statistical mechanics suggests noticable corrections to intuitive pseudo-kinetics, but seemingly non-principal ones. At last, compare in this respect two-time correlations.

V. FLICKER CORRELATIONS

A. Naive treatment of energy uncertainty correlation function

Now consider two-time correlators of the interaction (energy) fluctuations \( \epsilon(t) = E(t) - E_0 \) (deviations from start value \( E_0 \)). For this purpose one should first introduce a definition of such correlators. Under statistical mechanical approach to pseudo-kinetics, most simple and seemingly natural definition is

\[ \langle e^{ix(t)} e^{iy(t_0)} \rangle \]  

(51)

\[ = \int d\epsilon e^{-ix\epsilon} \tilde{S}(t - t_0) e^{iy\epsilon} \delta(\epsilon) \]

with \( 0 < t_0 < t \). At not too small \( t_0 \gg 1/g(0) \) and \( t - t_0 \gg 1/g(0) \) it yields

\[ \langle e^{ix(t)} e^{iy(t_0)} \rangle \to \frac{1}{1 - \chi(x)} \cdot \frac{1}{1 - \chi(x + y)}, \]  

(52)

and, consequently (after differentiations over test variables \( x, y \) at point \( x = y = 0 \)),

\[ \langle \epsilon(t) \epsilon(t_0) \rangle \to \langle \epsilon^2 \rangle, \]  

(53)

where variance on right-hand side is given by (23).

Thus, the correlation function (CF) of \( \epsilon(t) \) and \( \epsilon(t_0) \) occurs non-decaying at arbitrary growth of \( t - t_0 \), which corresponds to frozen (quasi-static) fluctuations.

In pseudo-kinetics, similar definition is

\[ \langle e^{ix(t)} e^{iy(t_0)} \rangle = \]  

(54)

\[ = \int d\epsilon e^{-ix\epsilon} \tilde{S}_{1/\lambda_1}(t_1) e^{iy\epsilon} \delta(\epsilon) \]

and again, in this form, it differs from that in pseudo-kinetics.
with \( t_1 = t - t_0 \) and results in
\[
(e^{i\mathbf{x}(t)} e^{i\mathbf{y}(t_0)}) \to e^{N(x) e^{(x+y)}}
\]  
(55)
and, evidently, in exactly the same asymptotically frozen CF [53].

This agreement, nevertheless, is not too significant, for quite similar CF behavior characterizes usual Gaussian random walk (Brownian motion). But our random walk \( \epsilon(t) = \mathcal{E}(t) - \mathcal{E}_0 \), in opposite to the usual one, possesses the fast saturating dispersion [28] instead of infinitely growing one. In combination with [53], it means (under not too small \( t_0 \gg \hbar/\epsilon_0 \)) that
\[
\langle |\epsilon(t) - \epsilon(t_0)|^2 \rangle = \langle \epsilon^2(t) \rangle + \langle \epsilon^2(t_0) \rangle - 2 \langle \epsilon(t) \epsilon(t_0) \rangle \to 0 ,
\]  
(56)
as if our walk was equally fast “stopped”. This, however, looks very unnaturally, because the step \( \epsilon(t) - \epsilon(t_0) \) in principle must have the same numerical rights as the previous step \( \epsilon(t_0) - \epsilon(0) = \epsilon(t_0) \).

Hence, in fact, our above “Markovian” definition [31] is incorrect from viewpoint of formally rigorous statistical mechanics and should be corrected with its help.

B. Quantum correlation functions

In quantum statistical mechanics, because of non-commutativity of variables, their multi-time correlators can be introduced in many non-equivalent ways. Here, we accept the one which directly generalizes the unambiguous definition of multi-time correlators in classical statistical mechanics: if \( O \) is operators of quantum variables, then \( n \)-order joint statistical moment for them, taken at time points \( 0 \leq t_1 \leq t_2 \ldots \leq t_n \leq t \), is
\[
\langle \prod_{k=1}^{n} O(t_k) \rangle = \text{Tr} \left\{ e^{(t-t_n)\hat{\mathcal{E}}} \hat{\mathcal{J}}_O e^{(t_n-t_{n-1})\hat{\mathcal{E}}} \ldots \right. \]
\[
\ldots e^{(t_3-t_2)\hat{\mathcal{E}}} \hat{\mathcal{J}}_O e^{(t_2-t_1)\hat{\mathcal{E}}} \hat{\mathcal{J}}_O e^{t_1\hat{\mathcal{E}}} g(0) \right\},
\]  
(57)
where \( \hat{\mathcal{J}}_O \) is super-operator of symmetrized, or Jordan, multiplication by operator \( O: \hat{\mathcal{J}}_O A \equiv (O + AO)/2 \).

Especially we are interested in diagonal \( O \)'s, that is commuting with \( H_0 \), i.e. with the “unperturbed energy” (UE) \( \mathcal{E} \). For such \( O \)'s we have identity
\[
\hat{\mathcal{J}}_O = \hat{D}\hat{\mathcal{J}}_O \hat{D} + \hat{N}\hat{\mathcal{J}}_O \hat{N}.
\]
Applying it in [57] with two variables, \( O(t_1) \Rightarrow A(t_0) \) and \( O(t_2) \Rightarrow B(t) \), using also the decomposition [54], propagator [36], WIA [39] properties of the trace \( \text{Tr} \), and noticing that [\( O,H_0 = 0 \) implies \( \hat{\mathcal{J}}_O, \hat{N}\hat{\mathcal{J}}_O \hat{N} = 0 \)], one obtains
\[
\langle B(t) A(t_0) \rangle = \text{Tr} \left\{ B \hat{S}(t - t_0) A \hat{S}(t_0) \right. \]
\[
+ \int^{t_0}_{t} db \int_{0}^{t_0} da B \hat{S}(t - b) \hat{Q}_{A}(b - a) \hat{S}(a) \rho(0) \right\},
\]  
(58)
where now \( \text{Tr} \cdots = \sum_{1}^{(\ldots)} \cdot \cdot \cdot \), \( A \) and \( B \) are mere operators of multiplication by functions \( A_1 \) and \( B_1 \) of indices “1” of unperturbed states, and new operator in spaces of such functions (similar to [58]) has appeared,
\[
\hat{Q}_{A}(\tau) \equiv \hat{D}\hat{\mathcal{J}}_A \hat{N} e^{\tau \hat{N} \hat{E}_0 \hat{N}} \hat{J}_{A} \hat{N} \hat{L}_D \hat{D}.
\]  
(59)

C. Rigorous statistical-mechanical corrections to the energy correlation function

In particular, for functions depending on the UE only, \( f_1 = f(\mathcal{E}_1) \), operator [60] reduces to
\[
\hat{Q}_{A}(\tau)(f(\mathcal{E})) = \int g(\mathcal{E} - \mathcal{E}') \times
\]
\[
\cos \left( \frac{\mathcal{E} - \mathcal{E}'}{\hbar} \right) \frac{A(\mathcal{E}) + A(\mathcal{E}')}{2} \left[ f(\mathcal{E}') - f(\mathcal{E}) \right] \frac{d\mathcal{E}'}{\pi\hbar}.
\]
(60)
If we take \( B = A = \mathcal{E} - \mathcal{E}_0 \), then [58] with [60] becomes rigorous statistical-mechanical definition for CF \( \langle \epsilon(t) \epsilon(t_0) \rangle \) of random walk \( \epsilon(t) = \mathcal{E}(t) - \mathcal{E}_0 \).

Obviously, the first right-hand term in [58] looks as the definition [51], i.e. is made as for Markovian random processes, and its contribution (when \( B = A = \mathcal{E} - \mathcal{E}_0 \)) must coincide with [31]. It can be easy verified in the light of previous sections. Hence, clearly, the second term of [58], which contains [31], represents non-Markovian correction to [58]. It can be exactly calculated when \( B = A = \mathcal{E} - \mathcal{E}_0 \), if paying attention to parities of the functions there and, besides, to automatic normalization \( \int S(S, \epsilon) d\epsilon = 1 \). The result is
\[
\langle \epsilon(t) \epsilon(t_0) \rangle = \langle \epsilon^2(t_0) \rangle + \int_{t_0}^{t} db \int_{0}^{t_0} da g(\epsilon) \cos \left( \epsilon (b - a)/\hbar \right) \frac{\epsilon^2}{2} \frac{d\epsilon}{\pi\hbar}
\]  
(61)
and finally
\[
\langle \epsilon(t) \epsilon(t_0) \rangle = \frac{1}{2} \frac{\epsilon^2}{\pi\hbar}(t_0),
\]  
(62)
where \( t_1 = t - t_0 \),
\[
\bar{g}(t) = \int g(\epsilon) \cos (\epsilon t/\hbar) d\epsilon,
\]
and the arrow presumes that \( t_0, t - t_0 \gg \hbar/\epsilon_0 \) (i.e. time intervals between energy measurements are much longer than the measurements themselves).

Thus, the main correction to the UE’s CF is that its residual (non-decaying) value (at \( t_1 \to \infty \)) in fact is twice smaller than predicted in [53].

The meaning of this change becomes clear when including it into calculation of variance of UE’s step \( \epsilon(t) - \epsilon(t_0) \). Instead of [61], it yields
\[
\langle |\epsilon(t) - \epsilon(t_0)|^2 \rangle = \frac{1}{\pi} \int g(\epsilon) d\epsilon.
\]  
(62)
We see that now step $\epsilon(t) - \epsilon(t_0)$ acquires the rights exactly equal to that of the preceding step $\epsilon(t_0) - \epsilon(0)$.

Accordingly, the flicker correlations do acquire a more live character: now they are decaying although by half only and independently on time separation of measurements. In other words, relaxation time of the flicker fluctuations has no definite value and may occur arbitrary large.

Thus, once again statistical mechanics significantly corrects a “hand-made” pseudo-kinetics, but at the same time visually confirming validity and importance of its principal claims to canonical kinetics.

* * * * * * * * * * *

Up to now we were considering interactions in themselves, like “Cheshire cat’s smile” in absence of the cat. From now we want to go to investigation of “the smiling cat as such”, i.e. from fluctuations of the unperturbed energy (UE) itself to that of actually observable variables connected to interaction-induced irreversible processes and their flicker and 1/f noises. First of all, to wandering of a particle in one or another medium (e.g. in phonon medium like in [14]).

VI. PARTICLE’S WANDERING IN QUANTUM MEDIUM

Next, our system will be a particle inside a medium with “free motion Hamiltonian” (7) and interaction Hamiltonian $\Phi$. We are interested in medium-enforced fluctuations of velocity $V = P/M$ of the particle and its resulting random displacements $R(t, t_0) = \int_{t_0}^{t} V(t') dt'$ which give rights to name it “Brownian’particle’ (BP). Especially interesting are those statistical properties of BP’s wandering which can be characterized as flicker fluctuations of its diffusivity (turning into that of its mobility when adding an external force to its Hamiltonian 2 4 5 10 26).

Let us write full indexes of the unperturbed (interaction-free) states as pairs formed by the velocity $V$ and ciphers which now will be symbolical indexes of medium’s states. Then full UE $E$ becomes a function $E_{V1} = E + E_1$ with $E = E(V) = MV^2/2$ and $E_1$ being BP’s and medium’s energies.

A. Pseudo-kinetics of particle in thermostat

Let $W(V,E)$ denote density of joint probability distribution of the velocity and UE,

$$W(V,E) = \sum_{i} \delta(E - E_{V1}) \rho_{V1}.$$  \hfill (63)

If $\rho_{V1} = \delta(E_{V1} - \Sigma)/N(\Sigma)$ is normalized uniform microcanonical distribution, then

$$W(V,E) = \delta(E - \Sigma) N_{th}(E - E(V))/N(\Sigma)$$

with $N_{th}(\Sigma) = \sum \delta(E_i - \Sigma)$ representing density of states of the medium (thermostat). As far as the latter is large enough, we can use general property of (infinite) microcanonical statistical ensembles [24]:

$$N_{th}(E - E)/N_{th}(E) \Rightarrow e^{-E/T},$$  \hfill (64)

where $T$ is temperature of the ensemble (thermostat) determined by equality $1/T \equiv d\ln N_{th}(E)/dE$ (and indifferent to $E$’s shifts after thermodynamic limit).

Hence, in microcanonical equilibrium (in the sense of the uniformity) $W(V,E) = \delta(E - \Sigma) W_0(V)$, where we introduced $W_0(V) = \exp(-E/T) N_{th}(\Sigma)/N(\Sigma)$ which is (normalized) Maxwell’s velocity distribution.

In pseudo-kinetics, however, this is not true equilibrium, for it does not mean stationarity in respect to UE distribution. Therefore the related velocity Maxwellian also occurs non-stationary and thus formally non-equilibrium (unachievable as such as ending of evolution). Nevertheless, $W_0(V)$ may be reasonable approximation for quasi-stationary ensembles.

To consider actual statistics of BP’s motion, including flicker correlations, it seems natural to constrict scope of the original PKO $\tilde{K}_{1/t}$ (or $\tilde{Q}(t)$, if starting from the Liouville equation) to functions of only $V$ and $E$. This simplification requires to assume identical probabilities of medium’s states with equal energies. It is logistic ansatz least if the medium initially was statistically equilibrium. If so, then $\rho$ looks as

$$\rho_{V1} = \int \frac{\delta(E + E_1 - E)}{N_{th}(E - E)} W(V,E) dE,$$  \hfill (65)

where in view of above equalities we can write

$$N_{th}(E - E) = N(\Sigma) e^{(E - E_0)/T} W_0(V).$$

Inserting these expressions into original PKO and then taking projection (63) of the result onto subspace of functions of the pair $X = \{V, E\}$, one obtains

$$\tilde{K}_{1/t} W(V,E) = \int \delta_{1/t}(E - E') \times$$  \hfill (66)

$$\times w(X|X') \frac{W(X')}{w_0(X')} - \frac{W(X)}{w_0(X)} dX',$$

where $W(X) \equiv W(V,E)$, $X' \equiv \{V', E'\}$,

$$w_0(X) \equiv \frac{N_{th}(E - E(V))}{N(\Sigma)} \Rightarrow e^{(E - E_0)/T} W_0(V),$$

and

$$w(V,E | V', E') \equiv \frac{2\pi}{\hbar} \sum_{i=1}^{1,2} \delta(E - E_{V1}) \times$$  \hfill (67)

$$\times |\Phi_{V1 V2}|^2 \delta(E' - E_{V2})/N(\Sigma)$$

with $E = MV^2/2$ and $E' = MV'^2/2$. The latter function obeys obvious symmetry

$$w(V,E | V', E') = w(V', E'|V,E).$$
Clearly, general distribution \( W_{st}(X) \) satisfying \( \hat{K}_{1/t} W_{st}(X) = 0 \), that is stationary with respect to (66), is \( W_{st}(X) = \text{const} \times w_0(X) \). It represents statistical equilibrium with equiprobability of all system’s states, which, however, never can realize, because cannot be normalized on UE axis.

This fact means that evolution governed by (66) must lead again to “frozen” (infinitely slowing down) regime with such quasi-stationary distribution \( W(t, X) \) that \( \hat{K}_{1/t} W(t, X) \propto W(t, X) \) under \( t \to \infty \). In other words, from spectral point of view, the PKO defined by (66) acts as if it had no zero eigen-value (EV) instead possessing a set of nearly zero EVs \( 1/t \to 0 \) with eigenfunctions (EFs) which tend to degenerated stationary EFs of limit KO \( \hat{K}_0 \).

Undoubtedly, such “glassy” behavior of the joint velocity-UE pseudo-kinetic evolution must be inherited by marginal statistics of velocity and BP’s path. To make it visual, let us consider two characteristic classes of possible “transition density” functions \( w(X|X') \).

### B. Energy shift-invariant interaction and marginal PKO of velocity

First of two interesting classes of \( w(X|X') \) appears when variety (“spectrum”) of possible types of transitions keeps constant under shifts along UE axis while their density changes proportionally to density of states:

\[
w(V, \mathcal{E} + \epsilon|V', \mathcal{E}' + \epsilon) = e^{\epsilon/T} w(V, \mathcal{E}|V', \mathcal{E}') .
\]

(68)

This property takes place, in particular, in case of phonon medium (10). Generally, it implies that

\[
w(X|X') = e^{(\mathcal{E}' - \mathcal{E}_0)/T} w(V, V'|\mathcal{E} - \mathcal{E}') .
\]

(69)

with asymmetric function of three arguments, such that

\[
w(V', V; -\epsilon) = e^{-\epsilon/T} w(V, V'; \epsilon) ,
\]

(70)

and PKO is visually strictly invariant with respect to equal shifts of UE values.

As the consequence, it becomes easy to reduce (like in (29)) the joint velocity-UE PKO to marginal velocity’s PKO for \( W(V) = \int W(V, \mathcal{E}) d\mathcal{E} \), merely by integrating (66) over UE. The result is

\[
\hat{K}_{1/t} W(V) = \int dV' \times \left[ w_{1/t}(V, V') \frac{W(V')}{W_0(V')} - w_{1/t}(V', V) \frac{W(V)}{W_0(V)} \right] .
\]

(71)

with transition’s density

\[
w_{1/t}(V, V') \equiv \int \delta_{1/t}(\epsilon) w(V, V'; \epsilon) d\epsilon .
\]

(72)

The latter, in contrast to (67), is non-symmetric function:

\[
w_{1/t}^{\text{anti}}(V, V') \equiv \frac{1}{2} [w_{1/t}(V, V') - w_{1/t}(V', V)] \neq 0
\]

(73)

at \( V \neq V' \) and \( 1/t \neq 0 \).

In order to make these expressions less abstract, let us notice (as in (29)) that function \( w(V, V'; \epsilon) \) by its definition must be proportional to (relative) medium’s density of states at minimum of two (“left” and “right-hand” in (68)) medium’s energies, \( \Sigma = \mathcal{E} - E(V) \) and \( \Sigma' = \mathcal{E}' - E(V') \). This physically clear statement implies

\[
w(V, V'; \epsilon) = w(V, V'|E - \epsilon - E'|) \times \exp \left( -\frac{E - \epsilon + E' + |E - \epsilon - E'|}{2T} \right)
\]

(74)

\[
(\epsilon = \mathcal{E} - \mathcal{E}') ,
\]

where factor \( w(V', V|\sigma) \) represents (chances or probabilities of) transitions as such, with

\[
\sigma \equiv |E - \epsilon - E'| = |\Sigma - \Sigma'|
\]

being energy given up or taken out by medium in separate transition. This formula expresses that, naturally, chances, or probabilities, of particular transitions can depend on the UE “misclosure” \( \epsilon \) only indirectly through the medium energy changes \( \sigma \).

Thus, combining (71) and (73), we can write

\[
w_{1/t}(V, V') = e^{-E'/T} \int_0^\infty d\sigma w(V, V'|\sigma) \times
\]

\[
\times [\delta_{1/t}(E - E' + \sigma) + e^{-\sigma/T} \delta_{1/t}(E - E' - \sigma)] .
\]

(75)

At that, \( w(V, V'|\sigma) \) is symmetric function of the velocity arguments, while its actual dependence on \( \sigma \) reflects one or another concrete variety of excitations of the medium and energy quanta what can be irradiated or absorbed by it. The PKO from (10) gives such the example.

### C. Non-stationarity, glass-likeness and non-self-adjointness of (equilibrium) particle’s wandering pseudo-kinetics

Because of non-zero anti-symmetric component of \( w_{1/t}(V, V') \), (76),

\[
\hat{K}_{1/t} W_0(V) = 2 \int w_{1/t}^{\text{anti}}(V, V') dV' \neq 0
\]

(76)

thus confirming that Maxwellian velocity distribution is not stationary one. At the same time, with no doubts, it adequately describes statistical equilibrium between BP and medium. This contradiction prompts that in fact PKO (70) has no stationary distribution at all, i.e. has no zero EV!

To prove this, it is sufficient to notice that existence of such distribution, \( W_{st}(V) \), would mean that the joint PKO (66), in addition to above mentioned \( W_{st}(V, \mathcal{E}) \propto w_0(X) \), has one more strictly stationary distribution \( W_{st}(V, \mathcal{E}) \propto W_{st}(V) \). But such duplication is incompatible with UE’s tendency to equipartitioning over system (micro-) states.

For simple illustration, consider special factorized case \( w(V, V'|\sigma) = w(V) g(\sigma) w(V') \). Then from the
stationarity equation \( \hat{W}_{st}(V) = 0 \) and (74) (with bounded \( \sigma \)'s) it would follow that
\[
W_{st}(V) \rightarrow \text{const at } E(V) \rightarrow \infty ,
\]
with const = \( \int w(V') W_{st}(V') \, dV' / \int w(V') \, dV' \), thus demonstrating impossibility of sensible solution.

Nevertheless PKO (70) must ensure stable quasi-stationary ("frozen") asymptotics of \( W(t, V) \)'s evolution. This means that (70), like (69), resets zero by small EVs \( \times 1/t \). Alternatively one may imagine more exotic situation with absence of any EVs.

Simultaneously, the conjugated, in the Sturm-Liouville sense, i.e. transposed, operator \( \hat{K}^{-1}_{1/t} \) acting by
\[
\hat{K}^{-1}_{1/t} F(V) = W^{-1}_0 (V) \int w_{1/t}(V', V) \left[ F(V') - F(V) \right] \, dV',
\]
certainly has zero EV with EF \( F(V) = \text{const} \), which merely reflects the fact that PKO always conserves probability: \( \int dV' \hat{K}^{-1}_{1/t} \cdots = 0 \). Hence, anyway our PKO is essentially non-self-adjoint (can not be symmetrized by a non-singular transformation). These circumstances were not fully realized in [16], although suggested by exact formulas of [16]. But in future just they may direct practical approximations of (pseudo-) kinetics of various interactions. For instance, similar to [29], where we have managed without EVs and EFs in mind. Anyway our considerations develope understanding of that real kinetics, ruled by mechanics, always is more or less "glassy", "smeared", "jaming", "flickering" and so on.

D. Case of energy shift-non-invriant interaction, wavelet eigen-functions, and tau-approximation

Another interesting class of interaction density, or interaction intensity, functions \( w(X|X') \) is characterized by their indifference to density of medium states, so that
\[
w(V, V' + \epsilon|V', V' + \epsilon') = w(V, V'|V', V'; E - \epsilon')
\]
with \( w(V, V'; \epsilon) = w(V', V; -\epsilon) \). Then the joint velocity-UE PKO can be written as product
\[
\hat{K}_{1/t} = \hat{K}^{-1}_{1/t} \eta^{-1}(E - E_0),
\]
where \( \eta(\epsilon) \equiv e^{\epsilon/T} \) and
\[
\hat{K}^{-1}_{1/t} W(X) = \int \delta_{1/t}(E - E') \times (75)
\times w(V, V'; E - E') \left[ \frac{W(X')}{W_0(V')} - \frac{W(X)}{W_0(V)} \right] \, dX'.
\]

In opposite to previous case, structure of this PKO prevents UE's exclusion from consideration. But it allows another simplifications.

By definition of operator \( \hat{K}^{-1}_{1/t} \) it is of difference type with respect to UE. Therefore, clearly, if \( \hat{K}^{-1}_{1/t} \) has EF \( W_\Lambda(V, E) = w_0(X) \Psi_\Lambda(V, E) \) with EV \( \lambda \), then UE-shifted function \( W_\Lambda(V, E + \epsilon) \) is \( \hat{K}^{-1}_{1/t} \)'s EF with exponentially scaled EV \( \lambda \eta(\epsilon) = \lambda e^{\epsilon/T} \). Hence, such EF, if it exists, is a "wavelet" function.

It is interesting picture, but we confine ourselves by its specific model example highlighting how slow UE fluctuations do "modulate" rate (spectral power) of faster BP velocity fluctuations.

To formulate the model, notice that in the GRA limit function (67) retains two of four arguments only, \( w(V, E|V', E') \Rightarrow w(V|V') = w(V, E_0|V', E_0) \), and PKO action reduces to
\[
\hat{K}_0 W(V) = \int w(V|V') \left[ \frac{W(V')}{W_0(V')} - \frac{W(V)}{W_0(V)} \right] \, dV'
\]
with KO \( \hat{K}_0 \) from conventional Marcovian kinetics of particle in thermostat. It formally simplifies as much as possible, if one attracts well known ansayz sometimes named "tau-approximation":
\[
w(V, V') \Rightarrow g \, W_0(V) \, W_0(V'),
\]
so that \( \hat{K}_0 \Rightarrow g (\hat{\Pi} - 1) \), where \( \hat{\Pi} \) is projection operator acting onto velocity argument as follows,
\[
\hat{\Pi} F(V) \equiv W_0(V) \int F(V') \, dV'.
\]

Let us transfer this caricature but useful model to the PKO by replacement
\[
w(V, E|V', E') \Rightarrow g(E - E') W_0(V) W_0(V')
\]
with \( g(\epsilon) \) being an even function (thus, as before, assuming UE shift-invariance of intensity of transitions along with its insensitiveness to density of states). Then
\[
\hat{K}^{-1}_{1/t} = \hat{K}_{1/t} \hat{\Pi} - g_t (1 - \hat{\Pi}),
\]
where \( \hat{K}^{-1}_{1/t} \) is new designation for the operator (17), and
\[
g_t = \int \delta_{1/t}(\epsilon) g(\epsilon) \, d\epsilon.
\]

One can see that second term of (77) is responsible for fast relaxation of velocity distribution to vicinity of Maxwellian equilibrium, while first term for slow fluctuations in UE, then transformed, via factor \( 1/\eta(\epsilon) = \exp(-\epsilon/T) \), to such in BP's wandering rate, i.e. its diffusivity.

Factor \( 1/\eta(\epsilon) \), besides, causes asymmetry of UE fluctuations. It was neglected above in the PKO (17) but now will be taken into account by introducing (normalized to unit) distribution
\[
W_T(t, \epsilon; \epsilon') \equiv \exp(t\hat{K}_{1/t} e^{-\epsilon/T}) \delta(\epsilon - \epsilon'),
\]
which at \( T = \infty \) turns to the above exploited one: \( W_\infty(t, \epsilon; \epsilon') = W(t, \epsilon - \epsilon') \equiv S_{1/t}(t, \epsilon - \epsilon') \).

Keeping in mind large time intervals, we will replace \( g_t \) by \( g \equiv g_\infty = g(0) \) and (78) by quasi-stationary limit \( W_T(\epsilon; \epsilon') = W_T(\infty, \epsilon; \epsilon') \). The latter can be
represented, - with help of function [27] and symbol \( \nabla_x \) for differentiation over \( x \), - in the form

\[
W_T(\epsilon; \epsilon') = \exp \left[ -\chi(\sqrt{-\nabla_x^2}) e^{-t/T} \right] \delta(\epsilon - \epsilon') \tag{79}
\]

underlying that it is distribution of “aborted”, strongly non-Gaussian and non-uniform, UE’s diffusion. Of course, it is localized near \( \epsilon' \), moreover, in such way that

\[
\int \eta^s(\epsilon) W_T(\epsilon|\epsilon') \, d\epsilon < \infty \tag{80}
\]

for integer \( s \geq 0 \). It can be deduced from consideration of eigen-value problem

\[-\chi(\sqrt{-\nabla_x^2}) \Psi_\lambda(\epsilon) = \lambda \epsilon e^{\epsilon/T} \Psi_\lambda(\epsilon),\]

with wavelet-like EFs \( \Psi_\lambda(\epsilon) \), and \( W_T(\epsilon|\epsilon') \)’s expansion over them, but we here merely suppose \( \Psi_\lambda \) satisfied at least for \( s \leq 3 \).

Just formulated model is sufficient for demonstration how principal uncertainty of UE fully materializes in kinetics of BP.

E. Particle’s velocity correlation function and diffusivity

We introduce correlation function (CF) of velocity in analogy with \([28]\):

\[
C_2(\tau; t) \equiv \langle V(t + \tau) V(t) \rangle = \int e^{\tau \hat{K}_{\lambda/\tau}} V e^{t \hat{K}_{\lambda/\tau}} W_0(V) \, dV \tag{81}
\]

Since initial velocity distribution is chosen Maxwellian, we speak about equilibrium fluctuations. For brevity and simplicity we will treat \( V \) as projection of velocity vector onto fixed direction.

Notice that \([27]\) implies decomposition

\[
e^{t \hat{K}_{\lambda/\tau}} = \hat{P} e^{t \hat{K}_{\lambda/\tau}} + (1 - \hat{P}) e^{-t \eta g^{-1}} \tag{82}
\]

At interval \((0, t)\) in \([28]\) first term of this decomposition only works, but after multiplying by \( V \) its second term only, so that

\[
C_2(\tau; t) = V_0^2 \int de \, e^{-\tau \eta g^{-1}} e^{t \hat{K}_{\lambda/\tau}} \eta^{-1} \delta(\epsilon) \to \int V_0^2 e^{-\tau \eta g^{-1}} W_T(\epsilon|0) \, d\epsilon ,
\]

where \( V_0^2 = \int V^2 W_0(V) \, dV = T/M \) is equilibrium variance of velocity. Clearly, its CF decays with relaxation time \( \eta / g = \exp(\epsilon/T) / g \) which appears as effectively random since dependent on variable \( \epsilon \). At that, inequality \([28]\) guarantees that BP’s diffusion coefficient (diffusivity)

\[
D \equiv \int_0^\infty C_2(\tau; \infty) \, d\tau = \frac{V_0^2}{g} \int \eta(\epsilon) W_T(\epsilon|0) \, d\epsilon \tag{84}
\]

takes a finite value: \( D < \infty \). In such case, fluctuations of diffusivity (BP’s diffusion rate), relative to its average \( D \), become interesting question.

F. Flicker fluctuations of diffusion rate

Like the diffusion coefficient (DC) is connected to quadratic correlation of velocity fluctuations, a correlation function of diffusion coefficient’s fluctuations (DC CF) naturally connects to fourth-order velocity correlations. More precisely, to fourth-order cumulant

\[
C_4(\tau', t; \tau; t_0) \equiv \langle V(0 + \tau + t + \tau') \times V(0 + \tau + t) V(0 + \tau) V(0) \rangle - C_2(\tau; t_0 + \tau + t) C_2(\tau; t_0) \ldots \tag{85}
\]

where dots are replacement of two more products of two CFs representing two time-crossed parings between four multipliers. We want to know how integral over \( \tau, \tau' \) and \( t \) in this expression do behave when the total time coverage of correlations, \( \tau + \tau' \), is infinitely growing. If the integral also grows to infinity, then BP’s wandering passes as if its DC was undergoing flicker fluctuations. An effective CF of these fluctuations logically \([2, 3, 5, 6]\) can be expressed through the fourth cumulant by formula

\[
K_D(\theta) = \int_0^\infty C_4(\tau', \theta - \tau - \tau', \tau; t_0) \, d\tau \, d\tau' \tag{86}
\]

(increasing reference point \( t_0 \) for stationarity).

To consider this question, let us define the fourth-order statistical moment in \([28]\) similarly to \([28]\) by formula

\[
\langle \cdots \rangle \equiv \int \int dV \, dV' \, V e^{\tau' \hat{K}_{\lambda/\tau'}} V' \times e^{t \hat{K}_{\lambda/\tau}} V e^{t_0 \hat{K}_{\lambda/\tau_0}} W_0(V) \, dV \tag{87}
\]

Using again decomposition \([82]\) at intervals \((0, t_0)\), \( \tau \), \( \tau' \) and \( t \), and then performing integration over velocity, we obtain

\[
\langle \cdots \rangle = \int dV \, e^{-\tau' \eta g^{-1}} \times \int [2 e^{-\tau \eta g^{-1}} + e^{t \hat{K}_{\lambda/\tau}} W_T(t_0, \epsilon|0) \tag{88}
\]

In view of \([28]\) integration of the first term in square brackets here over \( \tau, \tau' \) and \( t \) gives finite result, as well as integration of the dots in \([28]\). Hence, these contributions are insignificant for DC CF \([28]\). What is for contribution from the difference of two other terms, it can be written (making \( t_0 \) large enough) as

\[
K_D(\theta) \to V_0^2 \int_0^\infty \int \frac{d\epsilon'}{d\epsilon} \int d\tau' e^{-\eta g^{-1} \epsilon} \times \int [W_T(t, \epsilon|\epsilon') - W_T(t, \epsilon|0)] e^{-\tau g^{-1} \epsilon'} W_T(\epsilon'|0) \tag{89}
\]

where \( t \equiv \theta - \tau - \tau' \) and \( \tau + \tau' < \theta \). This expression makes it clear that DC CF, together with \( W_T(t, \epsilon|\epsilon') \), possesses frozen asymptotics tending at large \( \theta \gg 1/g \) to constant

\[
K_D(\infty) = \int \int \tilde{D}(\epsilon) [W_T(\epsilon|\epsilon') - W_T(\epsilon|0)] \times \int \tilde{D}(\epsilon') W_T(\epsilon'|0) \, d\epsilon \, d\epsilon' \tag{90}
\]
where \( \tilde{D}(\epsilon) \equiv (V_0^2/g) \exp(\epsilon/T) \) plays role of random diffusion rate varied by UE fluctuations.

In case of weak interaction and “rare BP-medium collisions”, in the sense of \( h g \ll T, \epsilon_0 \), one can write approximately

\[
\tilde{D} \approx \frac{V_0^2}{g}, \quad \tilde{D}(\epsilon) \approx \frac{V_0^2}{g} \left( 1 + \frac{\epsilon}{T} \right)
\]

and \( W_T(\epsilon; \epsilon') \approx W(\infty, \epsilon - \epsilon') \), and come to estimate

\[
K_D(\infty) \equiv \frac{D^2 \sigma^2(\infty)}{T^2} \sim \frac{\epsilon_0}{\hbar g} \left( \frac{\hbar}{M} \right)^2 \gtrsim \left( \frac{\hbar}{M} \right)^2.
\]

The constancy of this DC CF asymptotics naturally reproduces that of the CF \( 61 \) of UE itself. But, in principle, it may get essential corrections, like \( 31 \) was changed to \( 61 \), in the framework of statistical-mechanical improvements of pseudo-kinetics.

**G. Towards full statistics of particle wandering**

Instead of direct calculation of DC CF, possibly, it would be more comfortable to obtain it by considering whole statistics of BP’s wandering, for instance, for beginning, characteristic function of BP’s path,

\[
\mathbb{E}(t-t_0, i\epsilon; t_0) = \langle \exp[i\epsilon R(t, t_0)] \rangle.
\]

Then once again we have to give definition of the angle brackets. Avoiding refinements and fantasies of theories of quantum measurements \( 27 \) (all the more their scholasticism) we will apply the recipe \( 28 \) which directly generalizes \( 57 \) to “continuous” (i.e. arbitrary frequent) observations:

\[
\langle \exp[i \int_0^t \epsilon(t') V(t') dt'] \rangle = \text{Tr} \hat{\exp} \left\{ \int_0^t [i\epsilon(t') \hat{J}_V + \hat{L}] dt' \right\} \varphi(0).
\]

Or we may consider “correlation-characteristic function”

\[
C(t, i\epsilon) \equiv \langle V(t) \exp[i\epsilon \int_0^t V(t') dt'] V(0) \rangle = \text{Tr} V \exp[t (i\epsilon \hat{J}_V + \hat{L})] \hat{J}_V \varphi(0).
\]

Most important task then is careful analysis of time dependencies with their “non-Markovian” peculiarities caused by UE uncertainty (though, at that, other dependencies of interactions’ matrix elements and statistical ensembles allow rough approximations).

For the case of energy shift-invariant interaction (i.e. transitions’ density proportional to states’ density), pointed out in Sections VI.B-VLC, attempt of such analysis was made recently in \( 24 \). It verified inevitability of presence of flicker correlations and fluctuations in actual wandering statistics, qualitatively differing it from ideal Brownian motion, and gave them original quantitative estimate. Or, to be precise, sooner that of their lower bounds, thus stimulating new approaches.

**VII. CONCLUSION**

Physical system being able to generate random events is unable to give them definite probability, or chance, if this system does not ke-averagable ep in memory their past numbers. Therefore their number per unit time in no way is stimulated to tend with time to a certain limit and constantly tries arbitrary long time-non-averagable deviations from expectations, i.e. flicker or 1/f type, fluctuations. This trivial reasoning is regularly confirmed by experiments but hardly find place in scientific minds and stays in radical disagreement with unwritten dogmas and lexicon of physical-chemical kinetics.

We have noticed that this contradiction can arise from disrespects of theoreticians for principle of uncertainty of energy and frequency in events on finite time arena. Kinetics neglects it when appeals to “Fermi golden rule”. But quite elementary mathematics has revealed that very small uncertainties \( \sim \hbar/t \) of energy in such events as quantum transitions are summating with observation time \( t \) in proportion to their number \( \sim gt \) up to non-decreasing uncertainty \( \gtrsim \hbar g \) in energies of system states involved to the transitions. This stochastic process does not obey the “law of large numbers”, so that the non-decreasing uncertainty transforms to flicker fluctuations in intermediate energies and probabilities of transitions and all other related quantities.

This surprise of the theory appears already within own frames of kinetics if only it refuses the golden rule, as inconsistent with rigorous statistical mechanics, and thus turns to “pseudo-kinetics”.

Pseudo-kinetics prompts where theory hides sources of flicker noise and suggests its preliminary description, maybe rough but useful, because it surely finds qualitative confirmations and quantitative improvements in statistical mechanics, to an extent of its mathematical development.

We have demonstrated the said by relatively simple example on flicker fluctuations of transitions’ energies.

Then, we have shown that pseudo-kinetics unambiguously predicts flicker fluctuations in such practically observable physical quantity as rate (coefficient) of diffusion (wandering) of particle interacting with medium (equilibrium thermostat).

Considering such effects of time-energy uncertainty, in pseudo-kinetics and in equations of statistical mechanics, we showed also that other aspects of quantum interactions and transitions can be simplified even to primitive models without loss of flicker noise.

It, nevertheless, leaves non-standard mathematical difficulties to be resolved in future. Now, more important thing is that we already made several steps to first-principle microscopic theory of thermal flicker noise and determined aims of next necessary steps.
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