Towards a unified description of metallic transport

Qikai Guo,1 César Magén,2,3 Marcelo J. Rozenberg,4 and Beatriz Noheda1,5

1Zernike Institute for Advanced Materials, University of Groningen, The Netherlands
2Instituto de Nanociencia y Materiales de Aragón (INMA), CSIC-Universidad de Zaragoza, 50009 Zaragoza, Spain
3Laboratorio de Microscopías Avanzadas (LMA), Universidad de Zaragoza, 50018 Zaragoza, Spain
4Université Paris-Saclay, CNRS, Laboratoire de Physique des Solides, 91405 Orsay, France
5CogniGron center, University of Groningen, The Netherlands

Understanding metallic behaviour is still one of the central tasks in Condensed Matter Physics. Recent developments have energized the interest in several modern concepts, such as strange metal [1, 2], bad metal [3, 4], and Planckian metal [5, 7]. However, a unified description of metallic resistivity applicable to the existing diversity of materials is still missing. Herein we present an empirical analysis of a large variety of metals, from normal metals to strongly correlated metals, using the same phenomenological approach. The electrical resistivity in all the cases follows a parallel resistor formalism [8], which takes both $T$-linear and $T$-quadratic dependence of the scattering rates into account. The results reveal the significance of the model by showing that the different metallic classes are determined by the relative magnitude of these two components. Importantly, our analysis shows that the $T$-linear term arises from the Planckian dissipation limit and it is present in all considered systems. This formalism extends previous reports on strange and normal metals [6, 9], facilitating the classification of materials with non-linear resistivity curves, an important step towards the experimental confirmation of the universal character of the Planckian dissipation bound.

Interactions of electrons with other (quasi-)particles (e.g. phonons, magnons or electrons themselves) are responsible for the electrical transport of metallic systems. In simple metals, electron-electron interactions lead to a Fermi liquid description [10] of the resistivity at low-temperature ($T$) as a $T^2$ dependence; while a linear increase of resistivity is usually observed at high-$T$ because of the boosted scattering strength between electrons and phonons. However, this well-defined regime meets with problems in strongly correlated metals, either when the metallic system is driven close to a Quantum Critical Point, which gives rise to a "strange metal" with a $T$-dependence of resistivity of the type $T^n$, with $1 \leq n < 2$ at low-$T$ [11]; or when the increased scattering drives the mean-free path ($l$) to approach the Mott-Ioffe-Regel (MIR) limit [12], which compels the resistivity to show saturation at high-$T$ [13]. However, in some so-called incoherent or bad metals, the resistivity overcomes this upper limit, implying such large scattering rates that, according to Heisenberg’s principle, the uncertainty in the quasi-particles energy prevents their coherence, thus, disqualifying the quasi-particle description altogether [11, 13].

Different conduction mechanisms become dominant at different temperatures and, thus, an overall description of metallic resistivity over a wide temperature range requires considering the combined effect of the various contributions in a phenomenological manner. $T$-linear resistivity has been typically associated to electron-phonon scattering and, thus, such a dependence was not expected at low temperatures. However, it is now well established that $T$-linear resistivity can emerge well below the Debye temperature in various systems, from simple metals to strongly correlated metals, as long as the scattering rate ($1/\tau$) per Kelvin of the charge carriers reaches a universal bound, $k_B/h$. This so-called "Planckian dissipation" limit (PDL) is independent of the distinct behaviour and conduction mechanism [6]. These findings challenge the significance of a specific scattering mechanism in determining "strange" metallic transport and motivates the search for a phenomenological description that applies to a large variety of metals.

In Fig. 1a, we put together the $\rho(T)$ curves of a wide diversity of metallic systems. These systems include cuprates with different doping levels, ruthenates, heavy fermions, alkali-doped $C_{60}$, iron pnictides, transition metals and monovalent metals. These materials have been classified as simple metals, correlated metals, strange metals, bad metals or Planckian metals. Compared with the slowly increasing resistivity of simple metals (in the yellow region), the slope of $\rho(T)$ at around 300 K of the systems shown in the blue region (which in most of cases is the maximum slope), is large, as expected from strong electron scattering. Among them, under-doped cuprates and Rb$_3$C$_{60}$ are well-established bad metals [13]. The resistivity in these systems can cross the $\rho_{\text{MIR}}$ limit at relatively low temperatures and approach a value far beyond it at high temperatures, violating the quasi-particle scenario.

Notably, most correlated metals are located in an intermediate region between the good metals and the bad metals, as shown in the blow-up plot of Fig. 1b.
Interestingly, a number of these intermediate systems remain unclassified, such as Sr$_2$RuO$_4$ [21] and CrO$_2$ [29], which have been reported to possess properties of both conventional and bad metals. Other systems, like Sr$_3$Ru$_2$O$_7$, Nd-LSCO (p=0.24), Bi2212 (p=0.23), and Ba(Fe$_{1/3}$Co$_{1/3}$Ni$_{1/3}$)$_2$As$_2$, have been discussed as Planckian metals [6, 9, 27]. However, regardless of their different origin and classification, the resistivity of all the metallic systems in the intermediate region show many comparable features. For instance, the maximum slope of resistivity in most of materials is well below an upper limit of 1 $\mu$Ω cm/K. The same limit has been reported in high-$T_c$ superconductors, other electron-correlated systems and even simple metals [6].

In an effort to unify the behaviour of the different metallic systems, we follow the seminal work of N. E. Hussey et al. in high-$T_c$ cuprates [8], showing that the resistivity of La$_{2-x}$Sr$_x$CuO$_4$ at various doping levels can be successfully described by a parallel resistor formalism [20]:

$$\rho(T)^{-1} = \rho_{\text{ideal}}(T)^{-1} + \rho_{\text{sat}}^{-1}$$

where $\rho_{\text{ideal}}$ is the resistivity in the absence of saturation, which is shunted by the large value of $\rho_{\text{sat}}$ at high temperatures. An adequate definition of $\rho_{\text{ideal}}$ is then needed in order to describe $\rho(T)$ in a wide temperature range. A dual-component model, with linear and quadratic terms, has been used in the cuprates [8, 30] as:

$$\rho_{\text{ideal}}(T) = \rho_0 + A_1T + A_2T^2$$

where $\rho_0$ represents the residual resistivity. Generalizing this to other types of metals implies that $A_1$ takes either electron-phonon or the Planckian limit behaviour into account and that $A_2$ (approximately) describes higher order contributions (Fermi liquid or others).

Here we show that this dual-component parallel-resistor formalism (DC-PRF) can describe the metallic behaviour of very distinct systems independently of the dominant scattering mechanism. The DC-PRF has been used to fit the remarkable variety of different resistivity data shown in Fig. 1, from the bad metals to the good metals. As shown in Fig. 2-c and Supplementary information, the electrical resistivity of all these materials can be well described by Eqs. 1-2.

This analysis provides us with a unified view of metallic behaviour. As shown in Fig. 3-a, b, the fitting of resistivity to all those metallic systems, reveals a clear evolution of coefficient $A_1$ and $A_2$ as a function of their room temperature resistivity ($\rho_{300K}$). Interestingly, the data includes NdNiO$_3$ (NNO), which we consider in the present work as both, a test case and illustration of the utility of the formalism. In fact, this compound is attracting significant attention, since superconductivity was reported in the infinite layer system Nd$_{1-x}$Sr$_x$NiO$_2$ [31], and is a remarkable example of a material whose metallic behavior has been particularly difficult to classify [25, 32, 33].

FIG. 1: Resistivity of various metallic systems. a, Compilation plot showing the resistivity of various metallic systems. The dashed line indicates the linear-$T$-resistivity slope of 1 $\mu$Ω cm; the arrow shows the MIR limit (0.7 mΩ cm) of La$_{2-x}$Sr$_x$CuO$_4$. (Data source: Cu, Nb [15], Al, Co, and Pd [14]; La$_{2-x}$Sr$_x$CuO$_4$ with $x$=0.17-0.33 [8], with $x$=0.04 and 0.07 [17]; Bi$_2$Sr$_2$Ca$_{0.88}$Y$_{0.12}$Cu$_2$O$_y$ [18]; Rb$_3$CuO [19]; YBa$_2$Cu$_3$O$_{6.45}$ [20]; Sr$_2$RuO$_4$ [21]; CeRu$_2$Si$_2$ [22]; CrO$_2$ [23]; VO$_2$ [24]; SmNiO$_3$ [25]; Nd$_{0.825}$Sr$_{0.175}$NiO$_2$ [26]; Nd-LSCO (p=0.24) and Bi2212 (p=0.23) [9]; Ba(Fe$_{1/3}$Co$_{1/3}$Ni$_{1/3}$)$_2$As$_2$ [27]; Sr$_3$Ru$_2$O$_7$ [6].)
FIG. 2: Fitting of the electrical resistivity of various metallic systems using Eq. [1]. a, Underdoped cuprate; b, Ruthenates; c, Simple metals; d, Nickelate (NdNiO$_3$). The triangle in d indicates the temperature above which the data deviates from a $T$-linear dependence. Data source: resistivity of NdNiO$_3$ is measured in the present work; while data of other systems are extracted from Refs. [6, 16, 17, 21], respectively.)

For the present study we have used high quality epitaxial NNO films grown on LaAlO$_3$ substrates, which have been characterized in detail in our previous work [34, 35]. High-angle annular dark field (HAADF) STEM image shown in Supplementary Section 1 demonstrates the high crystalline quality of the NNO films with an atomically sharp interface with the substrate. In this 10 nm film, a first-order metal-to-insulator transition happens below 150 K. Measurements of resistivity in an extended temperature range allow for clear determination of the $T$-dependence in the metallic state.

As shown in Fig. 2d, a cuprate-like linear-$T$ resistivity is observed in NNO in a ultra-wide temperature range (about 400 K). In our previous works, we showed that this $T$-linear behaviour of resistivity can be achieved in optimized NNO films with low epitaxial strain and low oxygen vacancy content [34] and, more interestingly, it has signatures of Planckian dissipation [35]. With the further increase of $T$, the rise of $\rho(T)$ shows an obvious deviation from the linear dependence, which is caused by the addition of a parallel saturation resistance that takes over the behaviour in the high temperature regime [4, 13, 29]. Moreover, as in NdNiO$_3$ strong electron-electron interactions are expected, the combined effect of all these contributions should be considered. Indeed, we can show that the metallic resistivity of the NdNiO$_3$ film over a temperature range of 600 K can be well fit with the DC-PRF of Eqs. [1, 2].

One of the interesting features unveiled in Fig. 3 is that the increase of $A_1$ saturates at a maximum value $\sim 1 \mu\Omega$ cm/K, which we have previously discussed in relation to the definition of the intermediate region of Fig. 1a. However, the DC-PRF allows to extract the linear contribution to resistivity in a wider variety of metallic systems and in a wider temperature range. In this way, we find that the upper limit is, actually, obeyed by all the correlated systems, even in those well-established bad metals.

The same $A_1 \sim 1 \mu\Omega$ cm/K limit has been reported in high-$T_c$ cuprates [28] and is associated with the PDL. Indeed, we notice that the extracted $A_1$ from those strange metals (inside the orange-encircled region) approximately approaches this upper limit. Despite being derived for simple and isotropic Fermi surfaces, one can use the Drude formula of conduction to estimate that the universal Planckian bound on dissipation $(1/\tau = k_B T/h)$ to estimate $A_1 = (m^*/n)(k_B/e^2 h)$, which includes the carrier density ($n$) and carrier effective mass ($m^*$) and, thus, it is system-specific. Therefore, due to their lower $m^*/n$ ratio (see Fig. 1b and Supplementary Information section...
FIG. 3: Evolution of extracted coefficients from the fit of $\rho(T)$ in various metals. From the fit of $\rho(T)$ in various metals to the DC-PRF model of Eqs. [12], the coefficients $a$, $A_1$ and $b$, $A_2$ are obtained and plotted as a function of the corresponding room temperature resistivity ($\rho_{300K}$). The blue dashed line in $a$, indicates the maximum value obtained for $A_1 \sim 1 \mu\Omega$ cm/K. Error bars are also obtained from the fitting results. Shadows are a guide-to-the-eye showing the general evolution of the coefficients. $c$, the $m^*/n$ ratio is plotted as a function of $\rho_{300K}$, showing a trend similar to $A_1$. In the inset $A_1 n/m^*$ is shown to remain at, or slightly below, $k_B/(e^2\hbar)$, which corresponds the Planckian dissipation limit, also for the normal metals and bad metals. $d$, ($\rho_{sat}$) versus $A_2$, representing the electron correlations. Bad metals, recognized by their large $\rho_{sat}$, are shown to display the largest $A_2$ (the quadratic term dominates the scattering in the widest temperature range); while in strange metals (encircled in orange in all the figures) is the linear term the one dominating at most temperatures (see section 5 in Supplementary Information).

4), the simple metals are expected to display smaller $A_1$ values than the correlated metals. Indeed, as shown in the inset of Fig. $[12]$, the product $A_1 n/m^*$, which characterizes the PDL, confirms that such a limit is generally obeyed [6]. Thus, our analysis shows that the Planckian bound is a significant contribution to the $\rho(T)$ in all investigated metals. Until now, the relevance of this limit in the energy dissipation of carriers has not been reported to exist in systems that show non-linear-$T$ resistivity.

In contrast, the quadratic $A_2$ coefficient does not display a bound and continues increasing to reach the largest values in bad metals (see Fig. 3b). The behaviour of $A_2$ follows clear trends consistent with the general knowledge of the increasing correlations across different materials classes, further supporting the significance of the approach. The different behaviour of $A_1$ and $A_2$ indicates two independent scattering processes. Two distinct inelastic scattering channels have been reported in high-$T_c$ cuprates [8,35], being one contribution conjectured to arise from conventional transport theory, while the other one is correlated with the Planckian dissipation [2]. Our analysis is consistent with that and suggests that it is straightforward to generalize such a picture to a large variety metallic systems.

The different behaviour of $\rho(T)$ in various systems is mainly determined by the relative strength of these two processes. The relative importance of these two terms can be assessed by the magnitude of $T^* = A_1/A_2$, that is the temperature at which the linear and quadratic terms in $\rho(T)$ become equal [37]. Section 5 of the Supplementary Information shows $\rho_{sat}$ as a function of $T^*$ evidencing that the bad metals show the smallest values of $T^*$; while the strange metals show the largest $T^*$. A low (high) $T^*$ corresponds to a highly enhanced (depressed) $A_2$, since $A_1$ is of the same order for correlated metals.

Interestingly, in most of the investigated bad metals, $\rho_0$ (obtained from the DC-PRF fits) is also significantly larger than in other metals (see Supplementary Information).
tion Section 6), confirming the widespread notion that bad metals are dirty metals.33 This fact, next to the low $T^*$ values, is responsible for the increased $\rho_{\text{sat}}$ values that define bad metals, see Fig. 3d. On the other hand, the strange metals have a significantly smaller $A_2$, compared to bad metals, and a similar $A_1$, which gives rise to a strong decrease in the leading to a $\rho(T)$ action of the quadratic contribution, appearing linear in a wide temperature range.

Fig. 2 shows that metals can be primarily sorted in three main classes according to their $A_2$ and $\rho_{\text{sat}}$ values (low, intermediate and large) corresponding to normal, correlated and bad metals, respectively. The strange metals belong to the intermediate $A_2$ regime, which has this parameter in the narrow range of $[5 \times 10^{-7} - 5 \times 10^{-6}]$ mΩ cm/K². We have discussed the difficulties in the classification of some system such as Sr$_2$RuO$_4$, CrO$_2$ or NdNiO$_3$. Our analysis shows that these materials belong to the intermediate $A_2$ regime of correlated materials and are quite far from the values of bad and good metals. Interestingly, these three materials happen to be unexpectedly clean, according to the $\rho_0$ obtained from the DC-PRF analysis, which are comparable to those of the simple metals, as shown in section 5 of the Supplementary information.

To conclude, the customary classification of metals as normal, bad or strange, runs short to describe the complexity of electron correlated systems, often leading to controversial conclusions. The Hussey formalism consisting of $T$-linear ($A_1$) and $T$-quadratic ($A_2$) components added to the residual resistivity, in parallel with a saturation term, can phenomenologically describe all observed behaviours and provide a general framework to classify metals in accordance to the relative magnitude of $A_2$ and $\rho_{\text{sat}}$. Generally, the saturation term represents the MIR limit, while $A_2$ describes the electron interactions. The exception are the bad metals, for which the saturation term overshoots the MIR limit, due to the combined effect of a large $\rho_0$ and an largely enhanced $A_2$. However, regardless of their relative differences in these parameters, $A_1$ is found to reach an upper bound, for sufficiently large ($m^*/n$) ratios. The clear link of this bound with the Planckian dissipation limit supports its proposed universality 34, extending its scope to a larger number of metals and evidencing that all metals obey the Planckian constrain.

I. ACKNOWLEDGEMENTS

We are indebted to Nigel Hussey, Jan Zaanen, Thom Palstra and Francisco Rivadulla for insightful discussions. We are grateful to Jacob Baas, Arjun Joshua and Henk Bonder for their invaluable technical support. Qikai Guo acknowledges financial support from a China Scholarship Council (CSC) grant and we both acknowledge financial support from the Ubbo Emmins Funds (University of Groningen).

II. SUPPLEMENTARY MATERIAL

A. Methods

High quality epitaxial NdNiO$_3$ thin films were deposited on single-crystal LaAlO$_3$ (LAO) substrates by pulsed laser ablation of a single-phase target (Toshima Manufacturing Co., Ltd.). Before deposition, the LAO substrates were thermally annealed at 1050 °C under flowing O$_2$ and etched by DI water to obtain an atomically flat surface with single-terminated terraces. During the thin film deposition, the substrates were heated to a temperature of 700 °C and then the ablated material from the plume was uniformly deposited on the substrates. The oxygen pressure in the chamber during deposition was 0.2 mbar and the laser fluence on the target was 2 J/cm² with a laser frequency of 1 Hz. The distance between target and substrate was kept at 52.5 mm during the deposition. After deposition, the samples were cooled down to room temperature at 5 °C/min under a high oxygen pressure (900 mbar) to avoid the formation of oxygen vacancies in the lattice.

Following thin film growth, the structural and transport properties of all films were studied in detail. X-ray diffraction (XRD) have been performed by means of a Panalytical Xpert MRD-Thin film diffractometer. For high-$T$ measurement of XRD, a domed hot stage with thermal control unit (DHS1100) was employed. Cross-sectional specimens of the films were prepared and studied by scanning transmission electron microscopy (STEM) on a probe corrected FEI Titan 60–300 microscope equipped with a high-brightness field emission gun (X-FEG) and a CEOS aberration corrector for the condenser system. This microscope was operated at 300 kV. Z contrast images were collected by High angle annular dark field (HAADF) STEM. A convergence angle of 24 mrad was used to provide a probe size below 1 Å. Annular bright field (ABF) STEM images were acquired to visualize the oxygen lattice of the nickelate film. For this purpose, the same convergence angle of 25 mrad and an annular detector collected the scattered electrons in an approximate angular range of 12-25 mrad.

The temperature dependence of resistivity for temperatures below room temperature were carried out in a van der Pauw geometry using a Quantum Design Physical Property Measurement System (PPMS), while those measurements above room temperature were performed on a probe station with an Instec, Inc. heating stage. AC impedance measurements were performed in the same probe station using a LCR Agilent E4980.

B. Characterization of NdNiO$_3$ films

The crystalline structure of the NdNiO$_3$ film was studied by atomic resolution scanning transmission electron microscopy (STEM). A high-angle annular dark field (HAADF) STEM image shown in Fig. 3d indicates a
high crystalline quality in the NNO films with atomically sharp interface with the substrate. No misfit dislocations or other common defects such as Ruddlesden-Popper (RP) faults, which are known to form in nickelates in the presence of excess A-cations or oxygen vacancies, are observed in the lattice.

The annular bright field (ABF) image displayed in Fig. S1b provides further evidence of the octahedral oxygen rotations of the NNO orthorhombic phase by direct visualization of the oxygen lattice along the (110) orientation.

C. Fit to various metals

In the present work, a parallel resistor formalism [8]: $1/\rho(T) = 1/(\rho_0 + AT + B T^2) + 1/\rho_{\text{sat}}$, was employed to describe the electrical resistivity of NdNiO$_3$ film. For comparison, the resistivity data of various metallic systems from simple metals to strongly correlated metals were extracted from previous works. In the following, we will discuss in detail the fits to all the materials using the same approach.

1. **La$_{2-x}$Sr$_x$CuO$_4$**

A systematical analysis had been performed on the electrical resistivity of La$_{2-x}$Sr$_x$CuO$_4$ by Cooper et al. [8]. Herein, we extracted the resistivity data from ref. [8] and performed the fit in the same way as described in the main text for the NdNiO$_3$ film. It is worth to emphasize that the $\rho_{\text{sat}}$ in our fit is kept as a free parameter; while a fixed value (900 $\mu\Omega$ cm) of $\rho_{\text{sat}}$ is employed in Cooper’s work. As shown in the figures below, the resistivity curves of all the reported doping levels by Cooper et al. are well reproduced by our analysis. The parameters used in the fit are shown in the inset of each figure. Interestingly, we found that the extracted $\rho_{\text{sat}}$ from the fits show a decrease with increasing hole doping levels. For those with overdoping ($p < 0.18$), the values of $\rho_{\text{sat}}$ are almost comparable with the fixed value used by Cooper et al. However, the fits corresponding to the samples with optimized doping ($p=0.17$ and 0.18), which show strange metal-like linear-$T$ resistivity, give rise to a significantly larger $\rho_{\text{sat}}$. The measured resistivity at these doping levels is far below the predicted $\rho_{\text{sat}}$ from the fit, which is well consistent with the behaviour of strange metals.

The extracted $A_1$ and $A_2$ as a function of hole doping levels is shown in Figure S9. Notably, both marked kinks (solid dashed line) in $A_1$ and $A_2$ (corresponding to the $\alpha_1$ and $\alpha_2$ in their work, respectively) and also the evolution of these two coefficients with hole doping, reported by Cooper et al., are well reproduced by our fit. We believe that all these features demonstrate a reliability of our approach.

2. **Bad metals**

Bad metals are characterized by their large resistivity, which can increase across the predicted Mott-Ioffe-Regel (MIR) limit even at low temperatures. In the present work, resistivity data of four different systems, such as underdoped La$_{2-x}$Sr$_x$CuO$_4$, Bi$_2$Sr$_2$Ca$_{0.88}$Y$_{0.11}$Cu$_2$O$_y$, Rb$_3$Cu$_{60}$, YBa$_2$Cu$_3$O$_{6.45}$, were extracted from the refs. [17–20] respectively. Among them, the resistivity of La$_{1.96}$Sr$_{0.04}$CuO$_4$, La$_{1.93}$Sr$_{0.07}$CuO$_4$, and Bi$_2$Sr$_2$Ca$_{0.88}$Y$_{0.11}$Cu$_2$O$_y$ show obvious saturation with values well above the MIR limit. However, both Rb$_3$Cu$_{60}$ and YBa$_2$Cu$_3$O$_{6.45}$ display a continuously increased resistivity in the whole investigated temperature range with no sign of saturation. Despite of this different performance, the fit with the parallel resistor model shows to describe the experimental data adequately over a wide temperature range.

3. **Other correlated metals**

- **Sr$_2$RuO$_4$**

An extended measurement of resistivity in Sr$_2$RuO$_4$ had been performed by Tyler et al. [21]. This material displays an interesting case among strongly correlated metals. Sr$_2$RuO$_4$ has been proven to be a very good metal at low temperature, following to a Fermi-liquid quasi-particle scenario. However, the increase of resistivity at high-$T$ shows no sign of saturation at the Mott-Ioffe-Regel limit, invaliding the quasi-particle description. Indeed, the fit of $\rho(T)$ extracted from ref. [21] gave a saturation resistivity above 7.4 $\mu\Omega$ cm, which is far beyond its calculated MIR limit ($\sim 0.2 \mu\Omega$ cm).

- **CeRu$_2$Si$_2$**

CeRu$_2$Si$_2$ is well known as a canonical heavy fermion compound. This material is characterized by its rather large value of specific heat at low-$T$ and a metamagnetic-like transition [40]. The resistivity data of CeRu$_2$Si$_2$ analysed here were obtained from the work of Besnus et al. [22].

- **CrO$_2$**

The resistivity data of CrO$_2$ were extracted from Ref. [23]. CrO$_2$ has also been discussed as bad metal. It shows signs of saturation but at higher values than that predicted by the MIR limit. The DC-PRF fit for this material is significantly worse than for other systems. We have also found that different sources in the literature show different behaviour [39–41], so the intrinsic temperature dependence of resistivity in this material still needs to be determined.

- **VO$_2$**
FIG. S1: Characterization of NdNiO$_3$ film. a Cross-sectional HAADF-STEM images of a 10-nm-thick NNO film grown on a LAO (001) substrate observed along a (100) crystal orientation. b Cross-sectional ABF-STEM image with inverted contrast of the same film observed along the (110) crystal orientation where octahedra oxygen rotations of the orthorhombic NNO crystal structure can be observed.

FIG. S2: Fitting of $\rho(T)$ of La$_{1.83}$Sr$_{0.17}$CuO$_4$.

FIG. S3: Fitting of $\rho(T)$ of La$_{1.82}$Sr$_{0.18}$CuO$_4$.

FIG. S4: Fitting of $\rho(T)$ of La$_{1.79}$Sr$_{0.21}$CuO$_4$.

FIG. S5: Fitting of $\rho(T)$ of La$_{1.77}$Sr$_{0.23}$CuO$_4$. 
An extended measurement of resistivity up to 840 K was performed in VO$_2$ by Philip et al. [24]. Above the metal-insulator transition temperature (≃ 350 K), the temperature dependence of resistivity in the metallic phase of VO$_2$ is linear. The calculated mean-free-path at 800 K is only 3.3 Å, manifesting unconventional behaviour.

- **SmNiO$_3$**
  
  In comparison with NdNiO$_3$ studied in this work, SmNiO$_3$ displays a higher metal-insulator transition temperature, above 400 K. An extended measurement of metallic resistivity has been reported by Jaramillo et al. [25]. Moreover, they revealed a bad-metallic behaviour of this material on the basis of electrical and optical conductivity measurements. Indeed, the fit to the resistivity data extracted from the same work gave a saturation resistivity of $\rho_s \sim 1.077 \mu\Omega$ cm, which is obviously larger than the predicted MIR limit (0.5 $\mu\Omega$ cm) of this
FIG. S11: Fitting of $\rho(T)$ of $\text{La}_{1.93}\text{Sr}_{0.07}\text{CuO}_4$. Data from Ref. [17].

FIG. S12: Fitting of $\rho(T)$ of $\text{Bi}_{2}\text{Sr}_{2}\text{Ca}_{0.89}\text{Y}_{0.11}\text{Cu}_2\text{O}_y$. Data from Ref. [18].

FIG. S13: Fitting of $\rho(T)$ of $\text{Rb}_3\text{C}_{60}$. Data from Ref. [19].

FIG. S14: Fitting of $\rho(T)$ of $\text{YBa}_2\text{Cu}_3\text{O}_{6.45}$. Data from Ref. [20].

FIG. S15: Fitting of $\rho(T)$ of $\text{Sr}_2\text{RuO}_4$. Data from Ref. [21].

FIG. S16: Fitting of $\rho(T)$ of $\text{CeRu}_2\text{Si}_2$. Data from Ref. [22].

material.

- **Nd$_{0.825}$Sr$_{0.175}$NiO$_2$**
  Rare-earth nickelates have attracted renewed interests since the discovery of superconductivity in the related infinite-layer compound (Nd$_{1-x}$Sr$_x$NiO$_2$) by Li et al. [31]. The data studied in the present work are extracted from the subsequent work from the same authors [26]. In this work, Li et al. reported the phase diagram of Nd$_{1-x}$Sr$_x$NiO$_2$ infinite layer thin films grown on SrTiO$_3$. In our present work,
resistivity of Nd$_{0.825}$Sr$_{0.175}$NiO$_2$ was studied as it displays the best superconductivity and lowest resistivity among different doping levels.

4. Planckian metals

- Nd-LSCO and Bi2212

The linear-$T$ resistivity in high-$T_c$ superconductors with optimized doping has been a major puzzle in condensed matter physics. In a systematically study by Legros et al. [9], they revealed that the origin of the $T$-linear resistivity in many different cuprates is associated with a universal Planckian dissipation. Herein, the resistivity data of two systems: Nd-doped La$_{2-x}$Sr$_x$CuO$_4$ (Nd-LSCO) with $p=0.24$ and Bi$_2$Sr$_2$CaCu$_2$O$_8+\delta$ (Bi2212) with $p=0.23$, measured under high magnetic field were also extracted and fit with the parallel resistor model. By applying a high magnetic field, the superconductivity transition is suppressed and the $T$-linear resistivity towards absolute zero kelvin is obtained. Notably, the comparable parameters obtained from our fit in these two systems manifest a similar origin of $T$-linear resistivity in different cuprates, which is well consistent with the conclusion of Legros et al.
FIG. S22: Fitting of $\rho(T)$ of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi2212) with $p=0.23$ ($H=55$ T). Data from Ref. [9].

\[ \rho = \rho_0 + A_1 T + A_2 T^2 \]
\[ \rho_{0,T} = 0.33 \text{ m\Omega cm} \]
\[ A_1 = 5.46 \times 10^6 \text{ m\Omega cm/K} \]
\[ A_2 = 9.19 \times 10^9 \text{ m\Omega cm/K} \]

FIG. S23: Fitting of $\rho(T)$ of Sr$_3$Ru$_2$O$_7$. Data from Ref. [6].

\[ \rho = \rho_0 + A_1 T + A_2 T^2 \]
\[ \rho_{0,T} = 0.01 \text{ m\Omega cm} \]
\[ A_1 = 7.58 \times 10^6 \text{ m\Omega cm/K} \]
\[ A_2 = 5.87 \times 10^9 \text{ m\Omega cm/K} \]

FIG. S24: Fitting of $\rho(T)$ of Ba(Fe$_{1/3}$Co$_{1/3}$Ni$_{1/3}$)$_2$As$_2$. Data from Ref. [27].

\[ \rho = \rho_0 + A_1 T + A_2 T^2 \]
\[ \rho_{0,T} = 0.024 \text{ m\Omega cm} \]
\[ A_1 = 3.60 \times 10^6 \text{ m\Omega cm/K} \]
\[ A_2 = 2.38 \times 10^9 \text{ m\Omega cm/K} \]

FIG. S25: Fitting of $\rho(T)$ of NdNiO$_3$. Data from present work.

\[ \rho = \rho_0 + A_1 T + A_2 T^2 \]
\[ \rho_{0,T} = 0.002 \text{ m\Omega cm} \]
\[ A_1 = 3.78 \times 10^6 \text{ m\Omega cm/K} \]
\[ A_2 = 1.99 \times 10^9 \text{ m\Omega cm/K} \]

5. Simple metals

For comparison, the resistivity of several simple metals is also included in this work. These simple metals had also been discussed as Planckian metals by Bruin et al. [6] The resistivity data of Cu and Nb was extracted from the work of Gunnarsson et al. [13]; while those of Al, Co, and Pd were obtained from Ref. [16]. Except for Nb, the resistivity of simple metals shows a linear-$T$ dependence at high-$T$. At low-$T$, a Fermi-liquid-like $T^2$ dependence is observed.

However, these typical performance of resistivity is absent in Nb. Among simple transition metals, Nb plays special role due to its high superconducting transition temperature. The resistivity of Nb shows a significant saturation at ultra-high temperature. The fit to the $\rho(T)$ of Nb gave a saturation resistivity about 100 $\mu\Omega$ cm, which is well consistent with that calculated by Gunnarsson et al. [13]. Moreover, as shown in Fig. 3c in main text, the Nb shows a significantly larger values of $\alpha_1$ compared to $\alpha_2$. Interestingly, Nb has also been found to show a more obvious deviation from the Planckian limit.
in comparison with other simple metals [6]. We believe these anomalous behaviours of Nb are all attributed to their large electron-phonon effects [42].

**D. Effective mass and carrier density**

Values of effective mass \( m^* \) and carrier density \( n \) used in Fig. 3c and discussed in the main text are obtained from refs. [9, 25, 39–41] and summarized in Table 1. The last column shows \( A_1 n/m^* \) in units of \( k_B/\hbar e^2 \). For those metals with simple isotropic Fermi surfaces, \( A_1 n/m^* = k_B/\hbar e^2 \) at the Plackian dissipation limit [9].

**E. Crossover temperature**

We define \( T^* = A_1/A_2 \) as the temperature at which the linear term becomes equal to the quadratic term. For small values of \( T^* \) (bad metals), the quadratic term prevails for most temperatures, while for large \( T^* \) (strange metals), the linear term dominates up to (and beyond) room temperature. For the other metals, the crossover temperature typically ranges between 70 K and 300 K.

**F. Saturation resistivity and residual resistivity**
TABLE I: Relevant parameters for different reported metals. Error bars for m* and n are obtained from the refs. [9, 25, 39–41]; while error bars in left two columns are calculated from the uncertainty of related parameters.

| Material            | n(10^27 m^-3) | m* /m₀ | m*/n (m₀/10^27 m^-3) | A₁n/m* (k_B/he²) |
|---------------------|---------------|--------|----------------------|------------------|
| Bi2212 (p=0.23)    | 6.8           | 8.4 ± 1.6 | 1.2 ± 0.2            | 1.2 ± 0.3        |
| Nd-LSCO (p=0.24)   | 7.9           | 12 ± 4   | 1.5 ± 0.5            | 0.8 ± 0.3        |
| LSCO (p=0.26)      | 7.8           | 9.8 ± 1.7 | 1.3 ± 0.2            | 0.7 ± 0.2        |
| Rb₂C₆₀             | 3.9 ± 0.5     | 3.6 ± 0.5 | 0.9 ± 0.3            | 0.3 ± 0.2        |
| La₁₋₀.₆Sr₀.₄CuO₄   | 3 ± 0.5       | 4 ± 0.5   | 1.3 ± 0.4            | 0.7 ± 0.2        |
| NdNiO₃             | 10 ± 8        | 7 ± 1     | 0.7 ± 0.1            | 1.32 ± 1.1       |
| Cu                  | 85            | 1.3      | 0.015                | 0.27 ± 0.1       |
| Nb                  | 52            | 12       | 0.23                 | 0.4 ± 0.2        |
| Al                  | 60            | 1.4      | 0.023                | 0.6 ± 0.2        |

FIG. S31: Saturation resistivity ($\rho_{\text{sat}}$) versus $T^* = A_1/A_2$. Bad metals, recognized by their large $\rho_{\text{sat}}$, are shown to display the lowest $T^*$ (largest $A_2$, representing the largest efficiency of electron scattering). The same symbols as in figure 3 of the main paper have been used.
FIG. S32: Saturation resistivity (ρ_{SAT}) as a function of residual resistivity (ρ₀). Both parameters are extracted from the fit to resistivity data of each material, as discussed in section 3. The bad metals (encircled) show, as it is well known, the largest ρ_{sat}, which seem to originate from large ρ₀ values in combination with the lowest T* (the temperature at which the quadratic term takes over), mentioned in the main text. Interestingly, Sr₂RuO₄, CrO₂ and NdNiO₃, the three systems that have been discussed in the paper as notoriously difficult to classify, all display very low ρ₀ values, of the order of those of simple metals. The same symbols as in figure 3 of the main paper have been used.
