Non-linear optimization of frames with variable section stiffness of columns using Genetic Algorithm

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Abstract. Recently, in structural engineering, many main elements in structural systems such as columns or girders are designed with variable cross-section for certain purposes. In addition, in order to determine the appropriate sizes or suitable shapes of structural elements for optimal volume of material under the effect of certain loading or with the given sizes of elements the structures can be subjected maximum loading, it is required to studied the optimization problem. This paper presents the non-linear weight optimization problem of frames with variable section stiffness of columns according to the strength, stiffness and stability criteria with the help of finite element method. The non-linear optimization problem is solved by the Genetic Algorithm. The calculation procedure of non-linear optimization of plane frames with variable section stiffness of columns is established and implemented using Matlab calculation programming software. From the results of numerical examples, the conclusions and recommendations will be proposed.

1. Introduction

Nowadays, in design process of structural systems, the optimization problem is increasingly concerned and studied in many researches because of its important role and meaning. The aim of optimization problem is to determine the most appropriate element sizes in saving of the weight or material volume according to the strength, stiffness and stability conditions. Determination of suitable sizes for elements under given loadings will result in the lowest weight of material, which not only allows to reduce the construction cost but also influences the element properties in entire system. In addition, the optimization problem allows to determine the sizes and variation rules of cross-section stiffness change for that with the same material volume the structural system can be subjected the maximum loading.

Frames are popular structural systems, in design and analysis process, frames can be considered as plane or spatial structures. In many cases, it is applied plane frames due to their simplicity in calculation. The choice of the shape and sizes of cross section depends on many factors such as calculation scheme, loading...In structural system, the cross section is often chosen to be constant for comfortable fabrication and installment in construction works. However, in some cases structural elements are designed with variable cross section due to the requirements of architecture point of view or material saving. The elements can be designed with variable cross-section stiffness according to a certain rule. In many researches, the optimization problem of frames has been mentioned for simple
cases such as the cross section is variable according to linear rules or variable in some segments [1,2,3,4].

When solving optimization problem of above mentioned frames, it can be based on the analytical method to determine the internal forces in the structure and to find the solution of optimization problems [1,2]. In the case of that the cross-section of frame elements is variable according to random rule, it will face with certain mathematical difficulties to solve the problem if using analytical method. Therefore, with the development of numerical methods and programming tools, it is possible to apply numerical methods to find internal forces in the structural system for establishment of constrained conditions of the optimization problem, then applying the Genetic Algorithm with the help of Matlab programming software helps solve the mentioned problems without any difficulties.

2. Nonlinear optimization of plane frames with variable section stiffness of columns
The optimization problem of frame structure with variable section stiffness of columns implied in this paper is weight optimization.

It is given loading, function of variation of section stiffness of columns, the beam is designed with constant section. It is required to optimize the section sizes of columns for that the weight (or volume) of overall frame system is minimized with constraint conditions of strength, stiffness and stability.

2.1. Establishment of nonlinear optimization of plane frame structure with variable section stiffness of columns

2.1.1. The objective functions for different shapes of column cross-section
The objective functions in optimization problem is the volume V, the design variables are sizes of columns (radius r for the circle cross-section, height h for the rectangular cross-section and size a for square cross-section). The constraint condition systems are the conditions that satisfy the strength, stiffness and stability requirement. The frame elements are made from materials with elastic modulus E. The initial cross-sectional stiffness of columns is (EI0), the function which describes the variation of cross-sectional stiffness is f(x),

The cross-sectional stiffness of columns at coordinate x, illustrated in Fig. 2 is:

\[ EI_x = E \times I_0 \times f(x) \]  

(1)
The objective function is established from cross-section area \( A(x) \) \([5,6]\). 

\[
V(r) = A_s \times l_s + 2 \int_0^h A(x)dx = A_s \times l_s + 2 \pi R^2 \frac{\pi r^2}{2} \int_0^h f(x)dx
\]  
\( (2) \)

\[
V(a_o) = A_s \times l_s + 2 \int_0^h A(x)dx = A_s \times l_s + 2 \int_0^h a_o^2 \int_0^h f(x)dx
\]  
\( (3) \)

\[
V(h_o) = A_s \times l_s + 2 \int_0^h A(x)dx = A_s \times l_s + 2 \int_0^h b h_o^2 \int_0^h f(x)dx
\]  
\( (4) \)

Where, \( A_s, l_s \) – area and length of beam element respectively ; 
\( h \) – height of the column; 
\( r \) – radius of cross-section at the coordinate of which the stiffness equal \( I_0 \); 
\( R \) – radius of cross-section at the point with the coordinate \( x \); 
\( a_s \) – size of square cross-section at coordinate \( x \); 
\( b \) – width of rectangular cross-section at coordinate \( x \); 
\( h_o \) – initial height of rectangular cross-section at the coordinate of which the stiffness equal \( I_0 \); 
\( a_o \) – initial size of square cross-section at the coordinate of which the stiffness equal \( I_0 \); 
\( f(x) \) - the function which describes the variation of cross-sectional stiffness.

2.1.2. The constrained conditions for different shapes of column cross-section

a) Columns with circle cross-section \([7]\):
- Constrained condition in strength
\[
g_1 = M_c \frac{\pi r^3}{8} R_s \times \gamma_x \leq 0
\]  
\( (5) \)

- Constrained condition in stiffness
\[
g_2 = \int_0^1 8 \times M_{mc} \times M_{ki} E \times \pi r^4 - [\Delta_s] \leq 0
\]  
\( (6) \)

- Constrained condition in stability
\[ g_3 = P \Delta h^2 - \beta E \pi r^4 / 4 \leq 0 \]  

(7)

Where, \( \Delta h \) - Differential steps in finite difference method;
\( P_{cr} \) – the value of critical load before instability occurrence;
\( \beta \) - stability parameter determined from the determinant of (6) in [7] for different supports.

Thus, the constrained condition system for the case of that columns with circle cross-section:
\[ g_1 = M_y - \frac{\pi r^4}{8} R_y \times \gamma_c \leq 0; \]
\[ g_2 = \int_0^1 \frac{8 \times M_m \times M_k}{E \times \pi r^4} - [\Delta] \leq 0 \]
\[ g_3 = P \Delta h^2 - \beta E \pi r^4 / 4 \leq 0 \]  

(8)

b) Column with square cross section:

The constrained condition is established in [7], similarly the column with circle cross-section, the constrained condition system can be written as following:
\[ g_1 = M_y - \frac{a^3}{6} R_y \times \gamma_c \leq 0 \]
\[ g_2 = \int_0^1 \frac{12 \times M_m \times M_k}{E \times a^4} - [\Delta] \leq 0 \]
\[ g_3 = P \Delta h^2 - \beta E \frac{a^4}{12} \leq 0; \]  

(9)

c) Column with rectangular cross-section:

The constrained condition is established in [7], similarly the column with circle cross-section, the constrained condition system in this case can be written as following:
\[ g_1 = M_y - \frac{b \times h^3}{6} R_y \times \gamma_c \leq 0 \]
\[ g_2 = \int_0^1 \frac{12 \times M_m \times M_k}{E \times b \times h^3} - [\Delta] \leq 0 \]
\[ g_3 = P \Delta h^2 - \beta E \frac{b h^3}{12} \leq 0; \]  

(10)

2.2. Optimization procedure using Genetic Algorithm method

The nonlinear constrained optimization of plane frame is solved using Genetic Algorithm for variable cross-section stiffness of columns as follows.

GA method is usually used to solve unconstrained optimization problems, it should be transformed the constrained problem to an unconstrained problem by using penalty functions [8,9].

In this study, we use the following penalty function:
\[
\begin{cases}
F(r) = V(r) \cdot (1 + C) \\
F(a_y) = V(a_y) \cdot (1 + C) \\
F(h_y) = V(h_y) \cdot (1 + C)
\end{cases}
\]  

(11)

In which, \( F(r), F(a_y), F(h_y) \) – fitness function for different shapes of cross-section of columns, \( C \) – constraint violation functions, determined as sum of the constraints conditions in (10), (11) and (12).

The GA procedure used in this study for optimization frames with semi-rigid connections can be illustrated as the following steps.
Step 1: Input parameters of the problem. The parameters are population size, string length for individual design variable, crossover, mutation rate;
Step 2: Generate the initial population;
Step 3: Decode the binary design variables and generate input file for Finite element analysis;
Step 4: Implementing finite element analysis using Sap2000 software to determine internal forces, check the given constraints, calculate the value of the penalty function, in contrast the calculation process is continued;
Step 5: Check the convergence of the problem. The optimization process is terminated if it is satisfied;
Step 6: Calculate the penalized fitness function for every individual of the population and generate the next generation through reproduction, crossover and mutation;

3. Numerical example
Requirement: Calculating the value of the initial sizes $r$, $a_0$, $h_0$ of two columns with different shapes (circle, square and rectangular, in which the width of rectangular cross-section $b$ equal 0.3m) of cross-sections of plane frame subjected to loads shown in Fig. 3 for minimization of frame volume material. It is assumed that the variation function of section stiffness of column is $f(x)=4+6x^2$ with initial stiffness $I_0$ in top of column. The elastic modulus of plane frame $E=2\times10^6$ kPa; the beam with constant dimensions $0.25\times0.40$; $l=3m$, $h=4m$; $P=12kN$, $q=3kN/m$.

![Figure 3. Example of plane frame with variable section stiffness of columns](image)

Based on the abovementioned calculation procedure of Genetic Algorithm method for non-linear optimization problem of plane frames with variable cross-section stiffness, the authors have established the subroutine using Matlab programming software for different shape of cross-section of columns.

From the solution of given optimization problem, it can be seen that for different shapes of cross-section of column the optimal initial sizes always will be found to minimize the volume of the frames under the effect of any loading and with any variation of cross-section stiffness of columns.

| Cross-section | $\Delta V$ [%] |
|---------------|----------------|
| Circle        | 17.11          |
| Square        | 15.6           |
| Rectangular   | 15.3           |

| Table 1. Optimal sizes of plane frames with variable section stiffness of column |
|-----------------|-----------------|-----------------|
| Circle section  | $r$ [m]         | $a_0$ [m]       | $h_0$ [m]       |
| 0.310           |                 | 0.342           | 0.425           |
| Square section  |                 |                 |                 |
| Rectangular     |                 |                 |                 |

| Table 2. Comparison between results of optimization of plane frame with variable section stiffness of columns and constant section of column. |
The study results of optimization of plane frames with variable section stiffness of columns were compared with the case of that the section of column is constant and the plane frames is subjected the equivalent load. The comparison is shown in Table 2.

4. Summary
The optimization problem of frames with variable cross-section stiffness of columns is solved for different shapes of cross-section of columns and for random rules of variation of section stiffness. This problem can be solved by many methods but using Genetic Algorithm method it is effectively. The studied problem is implemented with the help of subroutine established using Matlab programming software without any mathematical difficulties. The research results showed that the volume material of frame elements obtained from optimization problem is significantly lower than the volume material of constant cross-section under same loadings.

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