Measurement of the $\eta - \eta'$ mixing angle in $\pi^-$ and $K^-$ beams with GAMS-4$\pi$ Setup

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The results of mixing angle measurement for $\eta'$, $\eta$ mesons generated in charge exchange reactions with $\pi^-$ and $K^-$ beams are presented. When the $\eta'$, $\eta$ mesons are described in nonstrange(NS)–strange(S) quark basis the $\pi^-$ and $K^-$ beams allow to study $|\eta_q\rangle$ and $|\eta_s\rangle$ parts of the meson wave function. The cross section ratio at $t' = 0 (\text{GeV}/c)^2$ in the $\pi^-$ beam is
$$ R_{\pi}(\eta'/\eta) = 0.56 \pm 0.04 $$ and result in mixing angle $\phi_P = (36.8 \pm 1.1)^\circ$. For $K^-$ beam the ratio is
$$ R_K(\eta'/\eta) = 1.30 \pm 0.16. $$ It was found that gluonium content in $\eta'$ is
$$ \sin^2 \psi_G = 0.15 \pm 0.06. $$ The experiment was carried out with GAMS-4$\pi$ Setup.

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I. INTRODUCTION

The investigation of the meson structure started during the second part of the last century, when in the native quark model frame the experimental consequence of the $SU(3)$ broken symmetry had been measured. In this model the physical states $\eta$  $\eta'$ are the linear combination of $SU(3)$ singlet and nonet:
$$ (\eta \eta') = \left( \begin{array}{c} \cos \theta_P - \sin \theta_P \\ \sin \theta_P \cos \theta_P \end{array} \right) (\eta_8 \eta_0), $$
where
$$ |\eta_0\rangle = \frac{1}{\sqrt{3}}|u \bar{u} + dd + ss\rangle, \quad |\eta_8\rangle = \frac{1}{\sqrt{3}}|u \bar{u} + dd - 2ss\rangle, $$
and $\theta_P$ — the mixing angle in singlet-octet representation. In nonstrange(NS)–strange(S) quark base
$$ |\eta_q\rangle = \frac{|u \bar{u} + dd\rangle}{\sqrt{2}}, \quad |\eta_s\rangle = |ss\rangle $$
the mixing becomes
$$ (\eta \eta') = \left( \begin{array}{c} \cos \phi_P - \sin \phi_P \\ \sin \phi_P \cos \phi_P \end{array} \right) (\eta_q \eta_s), $$
and the angles $\theta_P$ and $\phi_P$ are related through
$$ \theta_P = \phi_P - \arctan \sqrt{2}. $$

But it is appear that the internal structure of the pseudoscalars is not so simple, and the outlined approach is simplified [1]. The experimental properties of the $\eta'$ can be explained by assuming that there is a gluonium component in the meson wave function [2,3]. KLOE used [6] two angles method to describe $\eta$, $\eta'$ wave functions:
$$ \eta' = \cos \psi_G \sin \psi_P |\eta_q\rangle + \cos \psi_G \cos \psi_P |\eta_s\rangle + \sin \psi_G |G\rangle $$
$$ \eta = \cos \psi_P |\eta_q\rangle - \sin \psi_P |\eta_s\rangle, $$
where $\psi_P$ is the $\eta - \eta'$ mixing angle and $\sin^2 \psi_G$ is the gluonium fraction in $\eta'$. The KLOE result is $\sin^2 \psi_G = 0.12 \pm 0.04.$

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As it was being understood in the process of the study of the meson structure, the accuracy of the mixing angle measurement can be improved if to prepare the known initial quark states (|η⟩ or |η′⟩), as in the Bs, Bd meson decays [7], Fig. 1a. The production η, η′ meson in the charge-exchange reaction with π− and K− beam can be represented by planar diagrams, Fig. 1b. These diagrams are dominated, and the initial state d̄d (|η⟩) shall have been prepared in π− beam and s̄s (|η′⟩) state in the K− beam. In this approach the ratio of the η′, η cross sections for π− beam is

\[ R_\pi = \frac{\sigma(\eta')}{\sigma(\eta)} = \tan^2 \phi_P \]  

and

\[ R_K = \frac{\sigma(\eta')}{\sigma(\eta)} = \cot^2 \phi_P \]  

for K− beam. The idea of measuring the mixing angle by cross sections ratio was proposed for the first time in [8]. Experimental results obtained in π− beam were published in [8, 10]. To remove the mixing angle dependencies from kinematic factors and phase space the measurement is performed at \( t' = t - t_{\min} = 0 \).

\[ R = \frac{d\sigma(\eta')}{dt}/\frac{d\sigma(\eta)}{dt} \bigg|_{t=0} . \]  

If the differential cross section parameters are the same, then \( R \) equal (7) or (8) depending from the beam particle type.

The results of the mixing angle measurements in π− and K− beam for pseudoscalar mesons η, η′ are presented in our report. The dependencies \( R_\pi, R_K \) vs \( t \) are shown also. The data analysis had been performed as in simple quark model and with gluon content.

II. GAMS-4\pi SETUP AND EVENT SELECTION

The main detector of GAMS-4π Setup is the lead glass electromagnetic calorimeter [11]. The central part of the calorimeter contains PWO cryatalls in order to improve energetic and spatial resolution [12]. The detailed description of the experiment performance and data processing has been given elsewhere [13]. The charged particle beam of IHEP
U70 accelerator consists of 98% $\pi^-$ and 2% $K^-$ mesons, the admixture of other particles is negligible. The type of the beam particle is defined reliable by two threshold cherenkov counters [14] with quartz optics. The charge-exchange reaction at a 32.5 GeV/c beam momentum was used as the source of the monoenergetic pseudoscalar mesons

$$\pi^-(K^-) p \rightarrow M^0 n(\Lambda)$$  \hspace{1cm} (10)

the $M^0$ state decaying into photons. The study of the mesons production was performed in $2\gamma$ decay mode, that significantly reduced the systematic errose at the ratio cross sections measurements. The experimental mass spectrum

![Mass Spectrum](image)

**FIG. 2: Experimental 2\gamma events mass spectrum in $\eta$ and $\eta'$ regions for $\pi^-$ beam (histos $a$ and $b$ accordingly) and for $K^-$ beam ($c$ and $d$).**

are shown in Fig. 2. The $0.66 \cdot 10^6$ events of the decay $\eta \rightarrow 2\gamma$ and $14.27 \cdot 10^3$ of $\eta' \rightarrow 2\gamma$ in $\pi^-$ beam were detected. For $K^-$ beam the number of events are $2.22 \cdot 10^3$ and $0.21 \cdot 10^3$ accordingly. A 2C fit (the additional constrain $M_{\gamma\gamma} = M_\eta$ or $M_{\gamma\gamma} = M_{\eta'}$) was performed for $t$-dependence study to improve the resolution.

### III. MIXING ANGLE MEASUREMENT

A phenomenological function, according to [15], is used to describe the differential cross section

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \bigg|_{t=0} (1 - gct) e^{ct},$$  \hspace{1cm} (11)

where $g = \sigma_- / \sigma_+$ is the ratio flip and non-flip cross section. The fit results are shown in Tabl. I and in Fig. 3. Its interesting to see that in $K^-$ beam the non-flip amplitude dominates.
FIG. 3: Normalized differential cross section $\eta$ and $\eta'$ mesons with fitted function (11) in the $\pi^-$ beam (a and b) and in the $K^-$ beam (c and d).

TABLE I: The results of differential cross section fit by the function (11) in $\pi^-$ and $K^-$ beams ($a \equiv \frac{d\sigma}{dt} |_{t=0}$).

|       | $\eta$   |       |       |
|-------|----------|-------|-------|
|       | $a$      | $c$   | $g$   |
| $\pi^-$ | $3.61 \pm 0.04$ | $7.9 \pm 0.1$ | $2.5 \pm 0.1$ |
| $K^-$  | $14.2 \pm 0.7$  | $9.8 \pm 1.2$ | $0.15 \pm 0.17$ |

|       | $\eta'$  |       |       |
|-------|----------|-------|-------|
|       | $a$      | $c$   | $g$   |
| $\pi^-$ | $4.58 \pm 0.14$ | $8.9 \pm 0.2$ | $1.9 \pm 0.2$ |
| $K^-$  | $8.3 \pm 0.9$  | $6.1 \pm 2.1$ | $0.0 \pm 0.4$ |

The cross section ratio in $\pi^-$ beam at $t' = 0$ is

$$ R_\pi = \frac{d\sigma(\eta')}{dt} / \frac{d\sigma(\eta)}{dt} \bigg|_{t=0} = 0.56 \pm 0.04, $$

(12)

and the error is due to mainly by the accuracy of the efficiency calculation at small $t$. The obtained result is in agree with NICE experiment $R_\pi(\eta'/\eta) = 0.55 \pm 0.06$ (10). The mixing angle in the quark basis is

$$ \phi_P = (36.8 \pm 1.0)^\circ, $$

(13)

and corresponds $\theta_P = -17.9 \pm 1.0$ in the singlet-octet presentation.
For $K^-$ beam the ratio is

$$R_K = \frac{d\sigma(\eta')}{dt} / \frac{d\sigma(\eta)}{dt} \bigg|_{t=0} = 1.30 \pm 0.16.$$  \hfill (14)

From the native quark model we can expect

$$R_\pi \cdot R_K = \tan^2 \phi_P \cdot \cot \phi_P^2 = 1,$$  \hfill (15)

but the experimental data lead to

$$R_\pi \cdot R_K = 0.72 \pm 0.09.$$  \hfill (16)

This discrepancy can be explained if the internal structure of the pseudoscalars contains a some part, which did not consider in the model, or if the planar diagram has limited application for the investigated processes. If we follow the KLOE approach \[\text{(6)}\] then the next relations are obvious

$$R_\pi = \cos^2 \psi_G \tan^2 \phi_P$$

and

$$R_\pi \cdot R_K = \cos^4 \psi_G.$$  \hfill (17)

The gluonium content of the $\eta'$ mesons from our data is

$$\sin^2 \psi_G = 0.15 \pm 0.06,$$  \hfill (18)

The KLOE result based on radiative meson decays is $0.12 \pm 0.04$.

The $R_\pi$ and $R_K$ as the function of $t$ are shown in Fig. 4. These figures require an additional studies.

![Fig. 4](image)

**FIG. 4**: The differential cross section ratio $R(\eta'/\eta)$ in the $\pi^-$ (a) and in the $K^-$ (b) beams as a transfer momentum function.

**IV. CONCLUSION**

The ratio of the differential cross sections at $t' = 0$ (GeV/c)$^2$ in the $\pi^-$ beam equal $R_\pi(\eta'/\eta) = 0.56 \pm 0.04$ and the mixing angle is $\phi_P = (36.8 \pm 1.0)^\circ$ in quark base. For the $K^-$ beam the ratio $R_K(\eta'/\eta) = 1.30 \pm 0.16$ corresponds the mixing angle $\phi_P = (41.3 \pm 1.8)^\circ$ which is incompatible with pion beam result at $\approx 2.5\sigma$ confidence level. We assume that the reason of this difference is the gluon content in $\eta'$ meson. Two angle mixing scheme with gluon mixing angle $\psi_G$ result in gluon content $\sin^2 \psi_G = 0.15 \pm 0.06$ for $\eta'$ meson.
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