Naturally light scalar particles: a generic and simple mechanism

F. Léonard, B. Delamotte

CNRS, Sorbonne Université, Laboratoire de Physique Théorique de la Matière Condensée, LPTMC, F-75005 Paris, France

Nicolás Wschebor

Instituto de Física, Facultad de Ingeniería, Universidad de la República, 11000 Montevideo, Uruguay

The hierarchy problem in the Standard Model is usually understood as both a technical problem of stability of the calculation of the quantum corrections to the masses of the Higgs sector and of the unnatural difference between the Planck and gauge breaking scales. Leaving aside the gauge sector, we implement on a purely scalar model a mechanism for generating naturally light scalar particles where both of these issues are solved. In this model, on top of terms invariant under a continuous symmetry, a highly non-renormalizable term is added to the action that explicitly breaks this symmetry down to a discrete one. In the spontaneously broken phase, the mass of the pseudo-Goldstone is then driven by quantum fluctuations to values that are non-vanishing but that are generically, that is, without fine-tuning, orders of magnitude smaller than the UV scale.

Solving the gauge hierarchy problem of the Standard Model (SM) has triggered an enormous amount of works these last forty years (see, for example, 1). It has been, in particular, one of the important motivations for studying supersymmetric extensions of the SM (for a classical review on the topic, see 2). Other hypotheses for solving this problem have been put forward and intensively studied. Among them, the most popular have been the existence of extra dimensions of space-time 3, 4 or of new strong gauge interactions, such as many variants of the Technicolor model 5, 6. Up to now, all these approaches show two rather severe drawbacks: They are not supported by experimental data 7 and they are rather drastic and sometimes \textit{ad hoc} modifications of the original SM which is a very accurate description of the low energy world.

All the mechanisms mentioned above that aim at solving the hierarchy problem, take for granted that the SM cannot be made natural, that is, cannot avoid fine-tuning without drastic modifications. This belief is largely based on an analysis of the renormalization of the mass of the Higgs that shows that it is corrected at the quantum level by terms of the order of the ultraviolet (UV) cutoff of the theory, usually taken as the Planck or grand unification scale. That the Higgs mass be small compared to the UV scale? This is achieved by slightly modifying the SM rule out that it be of the order of the Higgs mass and already require a good deal of precision tests of the SM. The scalar sector of the SM appears therefore as an almost critical model where the fine-tuning remains unexplained at least if the ultraviolet scale is large compared to the EW scale and if the particle content is not tuned. The scalar sector of the SM is what they are and thus cannot be tuned. The scalar sector of the SM appears therefore as a formal viewpoint. The difference, of course, is that it can be performed at will in condensed matter systems while it is not so in particle physics where the parameters of the SM are what they are and thus cannot be tuned. The scalar sector of the SM appears therefore as an almost critical model where the fine-tuning remains unexplained at least if the ultraviolet scale is large compared to the EW scale and if the particle content is not supplemented by some extra degrees of freedom and/or symmetries.

Given the previous discussion, we focus here on the purely scalar sector leaving for future studies the coupling to the gauge and matter sectors. Accordingly, the problem we solve in this Letter is: How can we generate small masses in a purely scalar model without fine-tuning at the UV scale? This is achieved by slightly modifying the standard $\varphi^4$ scalar model in a way that drastically changes the quantum (or statistical) corrections to the masses in the spontaneously broken phase. For the sake of simplicity, we present the mechanism at work on a simple, two-scalar-field model that applies both in high energy physics and for condensed matter systems.

An obvious candidate for generating small masses without fine-tuning is to use spontaneous symmetry breaking of a continuous symmetry that produce Goldstone modes. In this case, the problem is twofold. First,
The masses are not only small, they vanish. Second, to get nonvanishing and small masses, it seems necessary to add small terms in the action that break explicitly the continuous symmetry so as to avoid strictly massless particles. In such a case, the scalar particle is called a pseudo-Goldstone \[^{11}\]. However, whatever small these terms are, they are expected to generate large quantum corrections to the masses. As a result, a fine-tuning in the bare action is necessary to get rid of these large quantum corrections and all the good properties of the initial Goldstone modes are lost (see, for example, chapter 19 of \[^{12}\]).

An important point for our approach is that the SM is an effective field theory valid only below a UV scale \(\Lambda\). There is therefore no reason why its bare action should involve only renormalizable terms (see, for example, chapter 12 of \[^{13}\]). Our idea is thus to use the Goldstone mechanism as a starting point but to explicitly break the initial continuous symmetry by adding to the action \(S\) a (possibly highly) nonrenormalizable term \(\Delta S_{\text{disc.}}\), instead of the typical breaking by renormalizable terms (the index disc. in \(\Delta S_{\text{disc.}}\) is here for future convenience).

At first sight, \(\Delta S_{\text{disc.}}\) could seem to play no role because a nonrenormalizable term contributes to renormalize the other couplings in the UV but becomes immaterial in the infrared limit \[^{14}\]. This is why nonrenormalizable terms are, in general not considered. However, it is clear that the above argument cannot be fully correct because even when it is nonrenormalizable, a well-chosen term can explicitly break a continuous symmetry and thus prevent the existence of strictly massless Goldstone bosons. Instead, the mass of the boson is proportional to the coupling constant in \(\Delta S_{\text{disc.}}\).

As shown below, when the model is not too far from its massless limit, that is, from criticality in the language of statistical mechanics, the coupling constant in \(\Delta S_{\text{disc.}}\) falls off very rapidly with the renormalization group (RG) scale, that is, not logarithmically but as a power law, and falls off very rapidly with the renormalization group (RG) \(Z\). As a result, instead of being naturally of the order of \(\Lambda\), the scale, that is, not logarithmically but as a power law, and falls off very rapidly with the renormalization group (RG) \(Z\), the field renormalization constant, to \(k\) (the index disc. in \(\Delta S_{\text{disc.}}\)) is here for future convenience).

The NPRG is based on Wilson’s idea of integrating fluctuations step by step. In its modern version, it is implemented on the one-particle irreducible generating functional \(\Gamma\) (the effective action) which is the Gibbs free energy in statistical mechanics (for original references, see \[^{16–18}\]; for a pedagogical introduction, see \[^{19}\] and \[^{20–27}\] for applications). It amounts to regularizing the low-energy modes, that is, the modes of \(\varphi(q)\) with wavenumbers \(|q| < k\), by giving them a mass while keeping unchanged the other’s, that is, \(\varphi(|q| > k)\). A one-parameter family of models indexed by a scale \(k\) is thus defined by

\[
Z_{k}[J] = \int D\varphi \exp(-S[\varphi] - \Delta S_{k}[\varphi] + J \cdot \varphi) \tag{2}
\]

with \(\Delta S_{k}[\varphi] = \frac{1}{2} \int_{q} R_{k}(q^{2})\varphi_{i}(q)\varphi_{i}(-q)\) and, for instance \[^{25}\],

\[
R_{k}(q^{2}) = Z_{k}(k^{2} - q^{2})\theta(k^{2} - q^{2})\tag{3}
\]

\(Z_{k}\) the field renormalization constant, to be defined later, and with \(J \cdot \varphi\) the step function and \(Z_{k}\) the classical field is computed with \(Z_{k}: \phi_{i}(x) = \langle \phi_{i}(x) \rangle\). The classical field is defined as usual as the Legendre transform of \(W_{k}[J] = \log Z_{k}[J]\):

\[
\Gamma_{k}[\phi] + W_{k}[J] = J \cdot \phi - \frac{1}{2} \int_{q} R_{k}(q^{2})\varphi_{i}(q)\varphi_{i}(-q) \tag{3}
\]

which, for convenience, is slightly modified by the inclusion of the last term. When \(k\) is large, all fluctuations in \(Z_{k}\) are frozen by the \(R_{k}\) term and the classical approximation becomes exact for \(k \approx \Lambda\). With the definition above, Eq. (3), it is then possible to show that \(\Gamma_{k=\Lambda}[\phi] \simeq S[\phi]\) if \(\Lambda\) is very large compared to all other momentum scales \[^{16–19}\]. Since \(R_{k=0}(q^{2}) = 0\), \(\Gamma_{k=0}[\phi]\) is the effective action of the original model, that is, the quantity we want to compute. Thus, \(\Gamma_{k}[\phi]\) interpolates
between the bare action when \( k = \Lambda \) and the effective action of the model under study when \( k = 0 \).

The exact RG flow equation of \( \Gamma_k \) reads [10]:

\[
\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[ \partial_t R_k(q^2)(\Gamma^{(2)}_k[q; \phi] + R_k(q)^{-1}) \right]
\]  

(4)

where \( t = \log(k/\Lambda) \), the trace stands for an integral over \( q \) and a trace over group indices and \( \Gamma^{(2)}_k[q; \phi] \) is the matrix of the Fourier transforms of the second functional derivatives of \( \Gamma_k[\phi] \) with respect to \( \phi_i(x) \) and \( \phi_j(y) \). Starting with the initial condition \( \Gamma_k=\Lambda[\phi_i]=S[\phi_i] \), this flow equation leads at \( k = 0 \) to an exact solution of the problem. For the systems we are interested in, it is impossible to solve Eq. (1) exactly and we therefore have recourse to approximations.

At tree level and in the broken phase, the field acquires a nonvanishing vacuum expectation value (vev) \( \sqrt{2} \kappa_0 \) in one of the \( q \) minima of the potential, that is, \( \rho_{min} = \kappa_0 \) and \( \sigma_{min} = 0 \). The spectrum of the model consists of two massive modes. One is the pseudo-Goldstone boson of mass \( m_T \) and the other one corresponds to the longitudinal mode of mass \( m_L \). In the \( Z_6 \) case where we parameterize the tree-level potential in the following way:

\[
U(\rho, \sigma) = \frac{u_0}{2}(\rho - \kappa_0)^2 + \lambda_6 \sigma, \quad \text{we find} \quad m_{T,0}^2 = 18\lambda_6 \kappa_0^2 \quad \text{and} \quad m_{L,0}^2 = 2u_0 \kappa_0 \quad \text{(here, again,} \quad \rho = \varphi^2/2). \]

Notice that \( m_{T,0} \) is not zero when \( \kappa_0 = 0 \), as expected.

We now discuss the one-loop RG flow equations. In the model above, which is regularized in the infrared by the \( R_k(q) \) term, the couplings and the masses become \( k \)-dependent and their one-loop flow can be obtained from (4) by making the replacement of the full propagator \( (\Gamma^{(2)}_k[q; \phi] + R_k(q)^{-1}) \) by \( (S^{(2)}_k[q; \phi] + R_k(q))^{-1} \) where \( S_k \) is identical to \( S \) up to the replacement of the bare couplings by \( k \)-dependent ones: \( u_0 \rightarrow u(k) \), \( \kappa_0 \rightarrow \kappa(k) \), \( \lambda_{6,0} \rightarrow \lambda_6(k) \), \( m_{T,0} \rightarrow m_T(k) \), \( m_{L,0} \rightarrow m_L(k) \).

The corresponding flow equations read [29] for the function \( R_k(q) \) given below Eq. (2):

\[
\partial_t \kappa = \alpha \left( 1 + 4 \frac{m_T^2}{m_L^2} \right) k^2 I_2(m_T^2) + 3k^2 I_2(m_L^2) \]  

(5a)

\[
\partial_t u = \alpha \left( -36 \lambda_6 k^2 I_4(m_T^2) + 18u^2 I_3(m_L^2) + 2(u + 36 \kappa \lambda_6)^2 I_3(m_L^2) \right) \]  

(5b)

\[
\partial_t \lambda_6 = 30 \alpha \lambda_6 k^2 (u + 6 \kappa \lambda_6) \frac{I_2(m_L^2) - I_2(m_T^2)}{m_L^2 - m_T^2} \]  

(5c)

with \( I_n(m^2) = (1 + m^2/k^2)^{-n} \) and \( \alpha^{-1} = 32 \pi^2 \). Notice that the flow of \( \lambda_6 \) vanishes when \( \lambda_6 = 0 \) which is expected since the \( Z_6 \) symmetry is enlarged to \( SO(2) \) in this case. This is what changes drastically the flow of this coupling compared to the flow of the coupling of same degree in front of the \( SO(2) \)-invariant term: \( (\rho - \kappa)^3 \) (not included here for simplicity).

For \( k \) of the order of the masses, the flows depend on the precise shape of the function \( R_k \) as expected. However, when \( k \) is very large compared to the masses, \( k \gg m_{T,L} \), the flow is particularly simple and \( \partial_t u \) and \( \partial_t \lambda_6 \) become scheme-independent, that is, is independent of the choice of regulator \( R_k(q) \). In this case, it reads:

\[
\partial_t \kappa = 4 \alpha \frac{m_L^2 + m_T^2}{m_L^2} k^2 \]  

(6a)

\[
\partial_t u = 2 \alpha \left( 9u^2 - 18 \lambda_6 k^2 + (u + 36 \kappa \lambda_6)^2 \right) \]  

(6b)

\[
\partial_t \lambda_6 = 60 \alpha \lambda_6 (u + 6 \kappa \lambda_6). \]  

(6c)

A second important property of the flow is that whenever \( k \) becomes smaller than a given mass, the corresponding degree of freedom decouples from the flow since its fluctuations become negligible at momentum scales smaller than its mass. In practice, Eqs. (5) are convenient because they automatically take into account these two aspects of the flow.

Given that we are a priori interested in couplings that can be large, the accuracy of the one-loop results derived from Eqs. (5) could be questionable. We have checked that they are indeed robust at least at a qualitative and even semi-quantitative level by considering a celebrated and nonperturbative approximation of the exact flow, Eq. (4), called the local potential approximation prime (LPA') [30, 31]. It consists in substituting in Eq. (4) an ansatz for \( \Gamma_k \) under the form:

\[
\Gamma_k[\phi] \rightarrow \Gamma^\text{LPA}_k[\phi] = \int_x \left[ \frac{1}{2} Z_k(\partial_x \phi)^2 + U_k(\rho, \sigma) \right] \]  

(7)

where \( Z_k \) is the field renormalization and \( U_k(\rho, \sigma) \) a general function of \( \rho \) and \( \sigma \). The LPA' above misses of course all derivative terms of orders higher than two. Its accuracy stems from the fact that we are only interested in the flows of the couplings of the potential that are only weakly impacted by neglecting higher derivative terms. This approximation has been shown to work extremely...
well for $\mathbb{Z}_q$-symmetric models in $d = 3$ where the couplings are large and the field renormalization small but not fully negligible [29]. It should even work better in dimension four.

When the potential $U_k(\rho, \sigma)$ is truncated by including only the couplings $\kappa(k), u(k), \lambda_0(k)$, the LPA’ flow boils down to Eqs. (3). We have checked that the flow of the masses converges with the order of the expansion of $U_k(\rho, \sigma)$ around the vev of the field when including more and more powers of $\rho - k$ and $\sigma$. We have also checked that keeping the field renormalization $Z_k$ or approximating it by $Z_k = 1$ for all $k$ changes only slightly our results and does not spoil qualitatively our conclusions. Our analysis shows that keeping only the couplings $\kappa(k), u(k), \lambda_0(k)$ already gives the correct general picture of the flow. The flow equation for the potential $U_k(\rho, \sigma)$ in the $\mathbb{Z}_6$ case is given in the Supplemental Material (and for the $\mathbb{Z}_{12}$ case on request).

We provide in Fig. 1 the flows of the mass $m_T$ of the transverse mode in the $\mathbb{Z}_6$ and $\mathbb{Z}_{12}$ cases. They have been obtained within the LPA’ by taking a natural bare action as initial condition of the RG flow, that is, with $u(k = \Lambda) = u_0 = \alpha^{-1}, \lambda_0(k = \Lambda) = \lambda_{6,0} = \alpha^{-2}\Lambda^2$ and the couplings of the higher order terms equal to 0. The bare mass $m^2_T(k = \Lambda) = 2\lambda_0 u_0$ is chosen either 10% or 1% below the critical value that makes the model massless [22]. It is clear from Fig. 1 that the mass decreases dramatically in the first RG steps and then saturates to a value which is much lower for $\mathbb{Z}_{12}$ than for $\mathbb{Z}_6$. This result is easy to understand from the remark that the more irrelevant $\sigma$, the faster the decrease of its coupling constant. This is due to the fact that when $q$ grows, the symmetry $\mathbb{Z}_q$ of the model becomes closer to the full continuous $SO(2)$ symmetry. It is also clear that the closer to the massless case, the smaller the final value of the mass since its decrease takes place on a longer RG “time” $|t|$. In all cases, we find that it is easy to obtain a mass $m_T$ of the transverse mode orders of magnitude smaller than the UV scale and it can easily be $10^{10}$ times smaller for $\mathbb{Z}_{12}$.

To conclude, let us remark that the idea that the smallness of some physical observables can be associated with some non-renormalizable terms is not new. For example, this is the case for the well-known mechanism that generates small masses for neutrinos from operators of dimension five [33]. This is, of course, also the case for gravity which is supposed to be very small because its interaction terms are not renormalizable [13]. Let us point out, however, that these effects are present already at tree level. The mechanism proposed above for generating small masses for scalar particles is, in this respect, different because it is associated with the fluctuations of the fields and is absent at tree level. More precisely, we have shown that it is easy to generate small masses from spontaneous symmetry breaking if some non-renormalizable terms break a continuous group down to a discrete one. This can be done without fine-tuning of the parameters of the bare action. In a sense, the fluctuations are no longer a problem but a solution to the problem of the smallness of the scalar mass.

It is interesting to point out that the very same mechanism has been studied in statistical mechanics and is responsible for a striking phenomenon: The critical exponents associated with a second order phase transition are not the same in the symmetric and broken phases when non-renormalizable discrete anisotropies are present [29]. In this context such terms are called “dangerously irrelevant” [34].

The present work shed new light on the Standard Model hierarchy problem because it is usually admitted that the main difficulty is to generate small masses for scalar particles without fine-tuning. Even if the present mechanism shows how to generate such small masses, it is not obvious how to employ this idea to generate small masses in the gauge sector. Finally, we notice that the study of the interplay between apparent fine-tunings in various physical theories and fluctuations associated with non-renormalizable terms has not been studied in depth in the literature. We believe that this direction of research could be very fruitful as suggested by the model studied above.

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