Toward linearization of nonlinear supersymmetric
Einstein-Hilbert-type action of superon-graviton model (SGM)

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Abstract

We attempt the linearization of the nonlinear supersymmetry encoded in the
Einstein-Hilbert-type action superon-graviton model (SGM) describing
the nonlinear supersymmetric gravitational interaction of Nambu-Goldstone
fermion superon. We discuss the linearization procedure in detail by
the heuristic arguments referring to supergravity, in which particular
attentions are paid to the local Lorentz invariance in the minimal
interaction. We show explicitly up to the leading order that 80 (bosons) +
80 (fermions) may be the minimal off-shell supermultiplet of the linearized
theory.

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1. Introduction

In the previous paper[1] we have proposed a new Einstein-Hilbert(E-H)-type superon-graviton model(SGM) action describing the nonlinear supersymmetrically(NLSUSY) [2] invariant gravitational interaction of Nambu-Goldstone(N-G) fermion superon in Riemann spacetime. SGM is obtained by extending the geometrical arguments of Einstein general relativity theory(EGRT) in Riemann spacetime to new (SGM) spacetime, where in addition to the ordinary Minkowski coordinates the coset space coordinates of $\text{super}\text{GL}(4,R)/\text{GL}(4,R)$ turning to the N-G fermion degrees of freedom(d.o.f.) are attached at every spacetime point[1]. We have shown group theoretically that the SGM action with global SO(10) symmetry decomposed as $10 = 5 + 5^*$ with respect to SU(5) may give a unified description of spacetime and matter and proposed superon(-quintet)-graviton model(SGM)[3]. In SGM scenario all observed elementary particles are accomodated in a single irreducible representation of SO(10) super-Poincaré(SP) algebra. And except graviton they are regarded as the superon-composite eigenstates described in the low energy by the local fields of the linear representation of supersymmetry(SUSY)[4] with (broken) SO(10) SP symmetry[3] corresponding to the observed (low energy) nature. Considering that nature has $1 \times 2 \times 3$ gauge symmetry and SUSY GUTs usually contain more than 160 so many particles, we are tempted to suspect the elementariness and to imagine the specific internal structure of spacetime and/or the compositeness of these (observed) particles. Due to the high nonlinear (self) couplings of the superon fields, as depicted in [5], it is inevitable to linearize SGM for obtaining the equivalent low energy effective theory.

In this work we would like to perform explicitly the linearization of N=1 SGM action of E-H-type to obtain the equivalent linear supersymmetric(LSUSY)[4] theory in the low energy, which is renormalizable.

Considering the abovementioned phenomenological potential of SGM, though qualitative and group theoretical so far, and the recent interest in NLSUSY in super-string(membrane) world, the linearization of NLSUSY in curved spacetime may be of some general interest.

This paper is organized as follows. In Sec. 2 we briefly review the E-H-type action of SGM and the symmetries of the action. In Sec. 3 by heuristic arguments we carry out the linearizaton of NLSUSY in SGM referring the supermultiplet of $N = 1$ SUGRA and we show explicitly at the leading order that 80(bosons)+80(fermions) may be the minimal off-shell supermultiplet of the linearized theory. The general discussion of the linearization procedure and the algebraic aspect is also described in Sec. 3. The conclusions are given in Sec. 4.
2. New Einstein-Hilbert-type action of SGM

In this section, for the self-contained arguments we review N=1 SGM action briefly. Extending the geometrical arguments of EGRT on Riemann spacetime to new (SGM) spacetime where besides the Minkowski coordinate $x^a$ the coset space coordinates $\psi$ of $^{\text{superGL}(4,R)}/\text{GL}(4,R)$ turning to the N-G fermion d.o.f. are attached at every Riemann spacetime point, we obtain the following N=10 SGM action\[1];

$$L_{SGM} = -\frac{c^3}{16\pi G}|w| (\Omega + \Lambda), \quad (2.1)$$

where $w^a_\mu$ and $e^a_\mu$ are the vierbeins of unified SGM spacetime * and Riemann spacetime of EGRT respectively, $\psi^i$ ($i, j, ... = 1, 2, ..., 10$) is N-G fermion(superon) originating from the coset space coordinates of $^{N=10 \text{superGL}(4,R)}/\text{GL}(4,R)$, $G$ is the gravitational constant, $\kappa^4 = (\frac{c^3\Lambda}{16\pi G})^{-1}$ is a fundamental volume of four dimensional spacetime of Volkov-Akulov(V-A) model[2], and $\Lambda$ is a small cosmological constant related to the strength of the superon-vacuum coupling constant. Therefore SGM contains two mass scales $\frac{1}{G}$ (Planck scale) and $\kappa \sim \frac{\Lambda}{G} (O(1))$. $\Omega$ is a new scalar curvature analogous to the Ricci scalar curvature $R$ of EGRT, whose explicit expression is obtained by just replacing $e^a_\mu(x)$ by $w^a_\mu(x)$ in Ricci scalar $R$[5]. The SGM action (2.1) is invariant at least under the following symmetries\[6]; global SO(10), ordinary local GL(4R), the following new NL SUSY transformation

$$\delta_{\text{NL}} \psi^i(x) = \frac{1}{\kappa^4} \zeta^i + i\kappa^2 (\bar{\zeta}^i \gamma^\rho \psi^j(x)) \partial_\rho \psi^i(x), \quad \delta_{\text{NL}} e^a_\mu(x) = i\kappa^2 (\bar{\zeta}^i \gamma^\rho \psi^j(x)) \partial_\rho e^a_\mu(x), \quad (2.3)$$

where $\zeta^i$ is a constant spinor and $\partial_\rho e^a_\mu(x) = \partial_\rho e^a_\mu - \partial_\mu e^a_\rho$,

the following GL(4R) transformations due to (2.3)

$$\delta_{\zeta} w^a_\mu = \xi^\nu \partial_\nu w^a_\mu + \partial_\mu \xi^\nu w^a_\nu, \quad \delta_{\zeta} s_\mu = \xi^\kappa \partial_\kappa s_\mu + \partial_\mu \xi^\kappa s_\kappa + \partial_\nu \xi^\kappa s_\mu, \quad (2.4)$$

where $\xi^\rho = i\kappa^2 \bar{\zeta}^i \gamma^\rho \psi^j(x)$ and $s_\mu = w^a_\mu w_\mu$,

and the following local Lorentz transformation on $w^a_\mu$

$$\delta_{L} w^a_\mu = \xi^a_\mu w_\mu \quad (2.5)$$

*The SGM spacetime[1, 5], on which the unified vierbein $w^a_\mu$ is defined, is the curved spacetime whose tangent spacetime have the d.o.f. of $\psi$ as a Grassmann coordinate of the basic manifold besides the Minkowski coordinates $x^a$ (i.e., SO(3,1) × SL(2,C) d.o.f.). The unified vierbein $w^a_\mu$ is defined through $\omega^a = dx^a + i\kappa^4 \bar{\psi}^a d\psi = w^a_\mu dx^\mu$, where $\omega^a$ is the NLSUSY invarinat differential one-form of Volkov-Akulov(V-A)[2].
with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$ or equivalently on $\psi^i$ and $e^a_\mu$

$$\delta_L \psi^i(x) = \frac{i}{2} \epsilon_{ab} \sigma^{ab} \psi^i, \quad \delta_L e^a_\mu(x) = e^a_\nu e^\nu_\mu + \frac{K^4}{4} \varepsilon^{abcd} \bar{\psi}^j \gamma_5 \gamma^j (\partial_\mu \epsilon_{bc}).$$

(2.6)

The local Lorentz transformation forms a closed algebra, for example, on $e^a_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^a_\mu = \beta^a_b e^b_\mu + \frac{K^4}{4} \varepsilon^{abcd} \bar{\psi}^j \gamma_5 \gamma^j (\partial_\mu \beta_{bc}),$$

(2.7)

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ae} \epsilon^1_r e^r_b - \epsilon_{2be} \epsilon^1_e e^e_a$. The commutators of two new NLSUSY transformations (2.3) on $\psi^i(x)$ and $e^a_\mu(x)$ are GL(4R), i.e. new NLSUSY (2.3) is the square-root of GL(4R);

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi^i = \Xi^a \partial_\mu \psi^i, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a_\mu = \Xi^a \partial_\rho e^a_\mu + e^a_\rho \partial_\mu \Xi^a,$$

(2.8)

where $\Xi^a = 2i\kappa (\bar{\zeta}_2 \gamma^a \zeta_1 - \zeta_1 \bar{\zeta}_2 e^a_\mu (\partial_\rho e^\sigma_\sigma))$. They show the closure of the algebra. SGM action (2.1) is invariant at least under[6]

$$[\text{global NLSUSY}] \otimes [\text{local GL(4, R)}] \otimes [\text{local Lorentz}] \otimes [\text{global SO(10)}],$$

(2.9)

which is isomorphic to SO(10) SP whose single irreducible representation gives the group theoretical description of SGM[3].

Here we just mention the confusive local spinor transformation which leaves SGM action (2.1) invariant. The following local spinor translation with a local parameter $\epsilon(x)$, $\delta \psi = \epsilon$, $\delta e^a_\mu = -i \kappa (\bar{\epsilon}^\gamma \partial_\mu \psi + \bar{\psi} \gamma^\gamma \partial_\mu \epsilon)$, gives $\delta w^a_\mu = 0 = \delta w^\mu_a$. However, it should be noticed that this local spinor transformation cannot transform away the d.o.f. of $\psi$. Indeed, $\psi$ seems to be transformed away if we choose $\delta \psi = \epsilon = -\psi$, but it is restored precisely in the unified vierbein $w^a_\mu$ by simultaneously transforming $e^a_\mu$, i.e., $w(e, \psi) = w(e + \delta e, \psi + \delta \psi) = w(e + t, 0)$ as indicated by $\delta w^a_\mu = 0$. And also the above local spinor transformation is a fake gauge transformation in a sense that, in contrast with the local Lorentz transformation on the coordinates in general relativity, it cannot be eliminate the d.o.f. of $\psi$ since the unified vierbein $w^a_\mu = e^a_\mu + t^a_\mu$ is the only gauge field on SGM spacetime and contains only integer spin. This confusive situation comes from the new geometrical formulation of SGM on unfamiliar SGM spacetime, where besides the Minkowski coordinates $x^a$, $\psi$ is a Grassmann coordinate (i.e. the fundamental d.o.f.) defining the tangential spacetime with SO(3,1) $\times$ SL(2,C) d.o.f. inspired by NLSUSY, and the local spinor transformation ($\delta \psi = \epsilon(x)$) is just a coordinate transformation(redefinition) on SGM spacetime. These situation can be understood easily by observing that the unified vierbein $w^a_\mu = e^a_\mu + t^a_\mu$ is defined by $\omega^a = dx^a + iK^4 \bar{\psi} \gamma^a d\psi = w^a_\mu dx^\mu$, where $\omega^a$ is the NLSUSY invarinat differential one-form of V-A[2] and $(x^a, \psi)$ are coordinates specifying the (SMG) flat spacetime inspired by NLSUSY.
From these geometrical viewpoints (in SGM spacetime) we can understand that $\psi$ is a coordinate and would not be transformed away, and the initial SGM spacetime is preserved. Therefore the action (2.1) is a nontrivial generalization of the E-H action. Eliminating $\psi$ by some arguments regarding the above local spinor translation as a gauge transformation leads to a different theory (ordinary E-H action) with a different vacuum (Minkowski flat spacetime), which is another from SGM scenario considering that the SGM spacetime is an ultimate physical entity.

The linearization of such a theory with a high nonlinearity is an interesting and inevitable to obtain an equivalent local field theory which is renormalizable and describes the observed low energy (SM) physics.

3. Linearization

The linearizations of NLSUSY in flat spacetime have been carried out by many authors. They have proved algebraically that $N=1$ V-A model is equivalent to $N=1$ scalar supermultiplet\cite{7}\cite{8}\cite{9} action or $N=1$ axial vector gauge supermultiplet action of LSUSY\cite{7}\cite{10}. We have also proved by the heuristic arguments that $N=2$ V-A model is equivalent to $N=2$ LSUSY model with SU(2) invariance\cite{11} and vector $J^\mu = 1^-$ gauge supermultiplet action is obtained by the spontaneous breakdown $SU(2) \to U(1)$ in the linear representation. Interestingly the SU(2) gauge structure of the electroweak standard model(SM) may be explained for the first time provided that the electroweak gauge bosons are the composite-eigenstates of these (SGM) types, however, described by asymptotic local fields in the low energy. And the absence of the low energy excited states of observed gauge bosons, quarks and leptons can be explained despite the compositeness.

These equivalent LSUSY actions possess in general Fayet-Iliopoulos\cite{12} terms indicating the spontaneous breakdown of LSUSY and SU(2). These algebraic exact results are favourable to the SGM scenario. In those works of the linearization it is important to find the SUSY invariant relations which express the LSUSY supermultiplets in terms of NL theories and reproduce the LSUSY transformations on the linearized supermultiplet under the NLSUSY transformation expressed by superon fields.

The SUSY invariant relations of the global SUSY in flat spacetime can be obtained straightforwardly, for the supermultiplets structures are well understood and the algebraic SP structures of both L and NL theories are the same. These algebraic exact results are favourable to the SGM scenario. However, it is unsatisfactory that the linearized theory is the free theory of the supermultiplet and that the existing superfield technique would transform V-A action into the free theory by the formulation.

The situation in SGM is rather different from the flat space case, for the supermul-
triplet structure of the linearized theory of SGM is unknown except it is expected
to be a broken SUSY SUGRA-like theory containing graviton and a (massive) spin
3/2 field and the algebraic structure (2.9) is changed into SP.
Especially for N=10 SGM, the supermultiplet of the linearized theory should con-
tain (massive) fields with spin up to 3, which may be beyond the straightforward
application of the existing (superfield) formalism restricted to N < 9 (i.e. spin ≤ 2).
Therefore by the heuristic arguments and by referring to SUGRA we discuss the
linearization for the moment. We focus to N=1 for simplicity.

Following SGM scenario, we assume that;
(i) the linearized theory possesses the spontaneously broken global (at least) SUSY,
(ii) graviton is an elementary field at least in the leading order (not composite of
superons corresponding to the vacuum of the Clifford algebra) in both NL and L
theories and
(iii) the NLSUSY supermultiplet of SGM (eaµ(x), ψ(x)) should be connected to the
LSUSY composite supermultiplet (˜eµ(e(x), ψ(x)), ˜λµ(e(x), ψ(x))) for the SUGRA-
like linearized theory.

From these assumptions and the arguments spread in the flat space cases we
require that the SUGRA gauge transformation [13] with the global spinor param-
eter ζ should hold for the supermultiplet (˜eµ(e, ψ), ˜λµ(e, ψ)) of the (SUGRA-like)
linearized theory, i.e.,

\[ \delta \tilde{e}^a_\mu(e, \psi) = i \kappa \tilde{\zeta} \gamma^a \tilde{\lambda}_\mu(e, \psi), \]  \hspace{1cm} (3.1)

\[ \delta \tilde{\lambda}_\mu(e, \psi) = \frac{2}{\kappa} D_\mu \tilde{\zeta} = -\frac{i}{\kappa} \tilde{\omega}^{ab}_\mu \sigma_{ab} \zeta, \]  \hspace{1cm} (3.2)

where \( \sigma^{ab} = \frac{i}{4}[\gamma^a, \gamma^b] \), \( D_\mu = \partial_\mu - \frac{i}{2} \omega^{ab}_\mu (e, \psi) \sigma_{ab} \), and \( \zeta \) is a global spinor parameter
and the variations in the left-hand side are computed under NLSUSY (2.3).

**3.1 Case \( \tilde{e}^a_\mu(e, \psi) = e^a_\mu \)**

We put the following SUSY invariant relations which connect \( e^a_\mu \) to \( \tilde{e}^a_\mu(e, \psi) \);

\[ \tilde{e}^a_\mu(e, \psi) = e^a_\mu(x). \]  \hspace{1cm} (3.3)

The relation (3.3) is the assumption (ii) and the metric conditions holds simply.
Consequently the following invariant relation is obtained by substituting (3.3) into
(3.1) and computing the variations under (2.3)[14];

\[ \tilde{\lambda}_\mu(e, \psi) = \kappa \gamma^a \psi(x) \partial_\mu e^a_\mu. \]  \hspace{1cm} (3.4)

(As discussed later these should may be considered as the leading order of the expan-
sions in \( \kappa \) of SUSY invariant relations. The expansions terminate with \( (\psi)^4 \).) Now
we see LSUSY transformation induced by (2.3) on the (composite) supermultiplet
The LSUSY transformation on \( \tilde{e}^a_{\mu} \) becomes as follows. The left-hand side of (3.1) gives
\[
\delta \tilde{e}^a_{\mu}(e, \psi) = \delta^{NL} e^a_{\mu}(x) = i\kappa^2 (\bar{\zeta} \gamma^\rho \psi(x)) \partial_\rho e^a_{\mu}(x).
\] (3.5)

While substituting (3.4) into the right-hand side of (3.1) we obtain
\[
(\bar{\zeta} \gamma^\rho \psi(x)) \partial_\rho e^a_{\mu}(x) + \cdots \text{(extra terms)}.
\] (3.6)

These results show that (3.3) and (3.4) are not SUSY invariant relations and reproduce (3.1) with unwanted extra terms which should be identified with the auxiliary fields. The commutator of the two LSUSY transformations induces \( \text{GL}(4\mathbb{R}) \) with the field dependent parameters as follows;
\[
\begin{align*}
[\delta_{c_1}, \delta_{c_2}] \tilde{e}^a_{\mu}(e, \psi) &= \Xi^\rho \partial_\rho \tilde{e}^a_{\mu}(e, \psi) + \tilde{e}^a_{\rho}(e, \psi) \partial_\mu \Xi^\rho, \\
[\delta_{c_1}, \delta_{c_2}] \tilde{\lambda}^a_{\mu}(e, \psi) &= \Xi^\rho \partial_\rho \tilde{\lambda}^a_{\mu}(e, \psi) + \tilde{\lambda}^a_{\rho}(e, \psi) \partial_\mu \Xi^\rho.
\end{align*}
\] (3.7, 3.9)

These results indicate that the algebra on the linearized field closes and the initial new NLSUSY structure of SGM is maintained on the linearized supermultiplet (i.e. disappointedly the commutators does not induce SP symmetry) provided that the relations (3.3) and (3.4) and SUGRA transformation (3.1) are respected. And due to the complicated expression of LSUSY (3.8) which makes the physical and mathematical structures are obscure, we can hardly guess a linearized invariant action which is equivalent to SGM.

Now we attempt the linearization such that LSUSY transformation on the linearized fields induces SP transformation.

By comparing (3.2) with (3.8) we understand that the local Lorentz transformation plays a crucial role. As for the local Lorentz transformation on the linearized asymptotic fields corresponding to the observed particles (in the low energy), it is natural to take (irrespective of (2.6)) the following forms
\[
\begin{align*}
\delta_L \tilde{\lambda}_\mu(x) &= -\frac{i}{2} \epsilon_{ab} \sigma_{\mu} \tilde{\lambda}_\mu(x), \quad \delta_L \tilde{e}_\mu(x) = e^a_b \tilde{e}_\mu^b, \\
\delta_L \tilde{e}^a_{\mu}(e, \psi) &= \Xi^\rho \partial_\rho \tilde{e}^a_{\mu}(e, \psi) + \tilde{e}^a_{\rho}(e, \psi) \partial_\mu \Xi^\rho.
\end{align*}
\] (3.10)
where $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$ is a local parameter. The relation between (2.6) and (3.10), i.e. the Lorentz invariance encoded geometrically in SGM space-time and (3.10) of the Lorentz invariance defined on the (composite) asymptotic field in Riemann(Minkowski) space-time, is unknown. However this is particularly interesting, for in SGM the local Lorentz transformations (2.5) and (2.6), i.e. the local Lorentz invariant gravitational interaction of superon, are introduced by the geometrical arguments in SGM spacetime[6] following EGRT. While in SUGRA the local Lorentz transformation invariance (3.10) is realized as usual by introducing minimally the Lorentz spin connection $\omega_{\mu}^{ab}$. And the LSUSY transformation is defined successfully by the (Lorentz) covariant derivative containing the spin connection $\tilde{\omega}_{\mu}^{ab}(e, \psi)$ as seen in (3.2), which causes the super-Poincaré algebra on the commutator of SUSY and is convenient for constructing the invariant action. Therefore in the linearized (SUGRA-like) theory the local Lorentz transformation invariance is expected to be realized as usual by defining (3.10) and introducing the Lorentz spin connection $\omega_{\mu}^{ab}$. We investigate how the spin connection $\tilde{\omega}_{\mu}^{ab}(e, \psi)$ appears in the linearized (SUGRA-like) theory through the linearization process. This is also crucial for constructing a nontrivial (interacting) linearized action which has manifest invariances.

We discuss the Lorentz covariance of the transformation by comparing (3.8) with the right-hand side of (3.2). The direct computation of (3.2) by using SUSY invariant relations (3.3) and (3.4) under (2.3) produces complicated redundant terms as read off from (3.8). The local Lorentz invariance of the linearized theory may become ambiguous and lose the manifest invariance.

For a simple restoration of the manifest local Lorentz invariance we survey the possibility that such redundant terms may be recasted by the d.o.f. of the auxiliary fields in the linearized supermultiplet. As for the auxiliary fields it is necessary for the closure of the off-shell superalgebra to include the equal number of the fermionic and the bosonic d.o.f. in the linearized supermultiplet. As new NLSUSY is a global symmetry, $\tilde{\lambda}_{\mu}$ has 16 fermionic d.o.f.. Therefore at least 4 bosonic d.o.f. must be added to the off-shell SUGRA supermultiplet with 12 d.o.f.[15] and a vector field may be a simple candidate. However, counting the bosonic d.o.f. present in the redundant terms corresponding to $\tilde{\omega}_{\mu}^{ab}(e, \psi)$, we may need a bigger supermultiplet e.g. $16 + 4 \cdot 16 = 80$ d.o.f., to carry out the linearization.

Now we consider the simple modification of SUGRA transformations(algebra) by adjusting the (composite) structure of the (auxiliary) fields. We take, in stead of (3.1) and (3.2),

$$\delta \tilde{e}_{\mu}^{a}(e, \psi) = i\kappa \bar{\zeta}\gamma^{a}\tilde{\lambda}_{\mu}(x) + \bar{\zeta}\tilde{\Lambda}_{\mu}^{a},$$  \hspace{1cm} (3.11) $$\delta \tilde{\lambda}_{\mu}(e, \psi) = \frac{2}{\kappa} D_{\mu}\zeta + \tilde{\Phi}_{\mu}\zeta = -\frac{i}{\kappa} \tilde{\omega}_{\mu}^{ab}\sigma_{ab}\zeta + \tilde{\Phi}_{\mu}\zeta,$$  \hspace{1cm} (3.12)$$

where $\tilde{\Lambda}_{\mu}^{a}$ and $\tilde{\Phi}_{\mu}$ represent the auxiliary fields to be specified and are functionals of $e_{\mu}^{a}$ and $\psi$. We need $\tilde{\Lambda}_{\mu}^{a}$ term in (3.11) to alter (3.5), (3.7), (3.8) and (3.9)
toward that of super-Poincaré algebra of SUGRA. We attempt the restoration of the manifest local Lorentz invariance order by order by adjusting $\tilde{\Lambda}^a_{\mu}$ and $\tilde{\Phi}_{\mu}$. In fact, the Lorentz spin connection $\omega^{ab}_{\mu}(e)$ (i.e. the leading order terms of $\tilde{\omega}^{ab}_{\mu}(e, \psi)$) of (3.12) is reproduced by taking the following one

$$
\tilde{\Lambda}^a_{\mu} = \frac{\kappa^2}{4} [ie_e^a \partial_{[\rho}e^{\rho}_b}_{\mu]} \gamma^a \psi \partial_{\mu}e^{|b]}_{\sigma]}e^b_{\mu} \gamma^a \sigma \rho \psi],
$$

(3.13)

where (3.7) holds. Accordingly $\tilde{\lambda}^a_{\mu}(e, \psi)$ is determined up to the first order in $\psi$ as follows;

$$
\tilde{\lambda}^a_{\mu}(e, \psi) = \frac{1}{4i\kappa} (ik^2 \gamma_a \gamma^\rho \psi(x) \partial_{[\rho}e^a_{\mu]} - \gamma_a \tilde{\Lambda}^a_{\mu}) = -\frac{i\kappa}{2} \omega^{ab}_{\mu}(e) \sigma_{ab} \psi,
$$

(3.14)

which describes the Lorentz covariant gravitational spin coupling of superon. Substituting (3.14) into (3.12) we obtain the following new LSUSY transformation of $\tilde{\lambda}^a_{\mu}$ (after Fiertz transformations)

$$
\delta \tilde{\lambda}^a_{\mu}(e, \psi) = -\frac{i\kappa}{2} \left\{ \delta^{NL} \omega^{ab}_{\mu}(e) \sigma_{ab} \psi + \omega^{ab}_{\mu}(e) \sigma_{ab} \delta^{NL} \psi \right\}
$$

$$
= -\frac{i}{2\kappa} \omega^{ab}_{\mu}(e) \sigma_{ab} \zeta + \{ O(\psi^2) + O(\psi^4) \}
$$

$$
= -\frac{i}{2\kappa} \omega^{ab}_{\mu}(e) \sigma_{ab} \zeta + \{ \tilde{\epsilon}_{ab}(e, \psi) \sigma^{ab} \cdot \omega^{cd}_{\mu}(e) \sigma_{cd} \psi \cdots + O(\psi^4) \}. (3.15)
$$

The first term is the intended ordinary global SUSY transformation indicating the minimal gravitational interaction of $\tilde{\lambda}^a_{\mu}(e, \psi)$ as in SUGRA. The second term is the redundant term with higher orders of superon and contains the terms recasted as the Lorentz transformation of $\tilde{\lambda}^a_{\mu}(e, \psi)$ with the field dependent parameters. (3.14) is the SUSY invariant relations for $\tilde{\lambda}^a_{\mu}(e, \psi)$, for the SUSY transformation of (3.14) gives the right hand side of (3.12) with the appropriate auxiliary fields as shown later. Interestingly the commutator of the two L SUSY transformations on (3.14) induces GL(4R);

$$
[\delta_{\tilde{G}_1}, \delta_{\tilde{G}_2}] \tilde{\lambda}^a_{\mu}(e, \psi) = \Xi^a \partial_{\mu} \tilde{\lambda}^a_{\mu}(e, \psi) + \partial_{\mu} \Xi^a \tilde{\lambda}^a_{\mu}(e, \psi),
$$

(3.16)

where $\Xi^a$ is the same field dependent parameter as given in (3.7).

As for the redundant higher order terms in (3.15) we can adjust them by considering the modified spin connection $\tilde{\omega}^{ab}_{\mu}(e, \psi)$ particularly with the contorsion terms and by recasting them in terms of (the auxiliary field d.o.f.) $\tilde{\Phi}_{\mu}(e, \psi)$. In fact, we found that the following supermultiplet containing 160 (= 80 bosonic + 80 fermionic) d.o.f. may be the supermultiplet of the SUGRA-like LSUSY theory which is equivalent to SGM;

for 80 bosonic d.o.f.

$$
[ \tilde{\epsilon}^a_{\mu}(e, \psi), a^a_{\mu}(e, \psi), b^a_{\mu}(e, \psi), M(e, \psi), N(e, \psi),
A^a_{\mu}(e, \psi), B^a_{\mu}(e, \psi), A^a_{\mu}(e, \psi), B^a_{\mu}(e, \psi), A^{[ab]}_{\mu}(e, \psi) ]
$$

(3.17)
and for 80 fermionic d.o.f.

\[
[ \tilde{\lambda}_{\mu \alpha}(e, \psi), \tilde{\Lambda}^a_{\mu \alpha}(e, \psi) ],
\]

(3.18)

where \( \alpha = 1, 2, 3, 4 \) are indices for Majorana spinor. The gauge d.o.f. of the local GL(4R) and the local Lorentz of the vierbein are subtracted. Note that the second line of (3.17) is equivalent to an auxiliary field with spin 3.

The a priori gauge invariance for \( \tilde{\lambda}_{\mu \alpha}(e, \psi) \) is not necessary for massive case\[16\] corresponding to the spontaneous SUSY breaking. For it is natural to suppose that the equivalent linear theory may be a coupled system of graviton and massive spin 3/2 with the spontaneous global SUSY breaking, which may be an analogue obtained by the super-Higgs mechanism in the spontaneous local SUSY breaking of N=1 SUGRA\[17\].

By continuing the heuristic and perturbative arguments referring to the familiar SUGRA supermultiplet we find the following SUSY invariant relations\[18\]:

\[
\tilde{e}^a_{\mu}(e, \psi) = e^a_{\mu},
\]

(3.19)

\[
\tilde{\lambda}_{\mu}(e, \psi) = -i\kappa(\sigma_{ab}\psi)\omega^{ab}_{\mu},
\]

(3.20)

\[
\tilde{\Lambda}^a_{\mu}(e, \psi) = \frac{\kappa^2}{2}e^{abcd}(\gamma_5\gamma_d\psi)\omega_{bc\mu},
\]

(3.21)

\[
A_{\mu}(e, \psi) = \frac{i\kappa^2}{4}[(\tilde{\psi}^\rho\gamma_\rho\partial_\mu\tilde{\lambda}) - (\tilde{\psi}^\rho\gamma_\rho\tilde{\lambda}_a)\partial_\mu e^a_\rho - (\tilde{\lambda}_a\gamma_\rho\partial_\mu\psi)]
+ \frac{\kappa^3}{4}[(\tilde{\psi}\sigma^{\rho\gamma_\rho\gamma_\rho}\partial_\mu\psi)(\omega_{\rhoab} + \omega_{\rhoab}) + (\bar{\psi}\sigma^{\rho\gamma_\rho\gamma_\rho}\partial_\mu\psi)\omega_{cab}]
+ \frac{\kappa^2}{8}(\tilde{\lambda}_a\sigma_{ab}\gamma^\rho\psi)\omega^{ab}_\rho,
\]

(3.22)

\[
B_{\mu}(e, \psi) = \frac{i\kappa^2}{4}[-(\tilde{\psi}\gamma_5\gamma_\rho\partial_\mu\tilde{\lambda}) + (\tilde{\psi}\gamma_5\gamma_\rho\tilde{\lambda}_a)\partial_\mu e^a_\rho - (\tilde{\lambda}_a\gamma_5\gamma_\rho\partial_\mu\psi)]
+ \frac{\kappa^3}{4}[(\tilde{\psi}\gamma_5\sigma^{\rho\gamma_\rho\gamma_\rho}\partial_\mu\psi)(\omega_{\rhoab} + \omega_{\rhoab}) + (\gamma_5\sigma^{\rho\gamma_\rho\gamma_\rho}\partial_\mu\psi)\omega_{cab}]
+ \frac{\kappa^2}{8}(\tilde{\lambda}_a\gamma_5\sigma_{ab}\gamma^\rho\psi)\omega^{ab}_\rho,
\]

(3.23)

\[
A^a_{\mu}(e, \psi) = \frac{ik^2}{4}[(\gamma^\rho\gamma^a\partial_\rho\tilde{\lambda}) - (\gamma^\rho\gamma^a\tilde{\lambda}_b)\partial_\mu e^b_\rho + (\tilde{\lambda}_a\gamma^\rho\gamma^a\partial_\mu\psi)]
+ \frac{\kappa^3}{4}[-(\tilde{\psi}\sigma^{\rho\gamma_\rho\gamma_\rho}\partial_\mu\psi)(\omega_{\rhoab} + \omega_{\rhoab}) - (\gamma_5\sigma^{\rho\gamma_\rho\gamma_\rho}\partial_\mu\psi)\omega_{abc}]
- \frac{\kappa^2}{8}(\tilde{\lambda}_a\sigma_{bc}\gamma^\rho\gamma^\rho\psi)\omega^{ab}_\rho,
\]

(3.24)

\[
B^a_{\mu}(e, \psi) = \frac{ik^2}{4}[(\tilde{\psi}\gamma_5\gamma_\rho\gamma^a\partial_\rho\tilde{\lambda}) - (\gamma_5\gamma_\rho\gamma^a\tilde{\lambda}_b)\partial_\mu e^b_\rho + (\tilde{\lambda}_a\gamma_5\gamma_\rho\gamma^a\partial_\mu\psi)]
\]
\[\frac{k^3}{8} \left[ \frac{(\bar{\psi} \gamma^5 \sigma^{bc} \gamma^a \gamma^d \partial_\mu \psi)(\omega_{\mu cb} + \omega_{cb \mu}) - (\bar{\psi} \gamma^5 \sigma^{bc} \gamma^a \gamma^d \partial_\mu \psi)\omega_{dbc}]}{8} \right]
\]

\[-\frac{k^2}{8} (\bar{\lambda}_\mu \gamma_5 \sigma^{bc} \gamma^a \gamma_\rho \psi) \omega^{ab}_\rho,\]

\[A^{(ab)}(\epsilon, \psi) = \frac{ik^2}{2} \left[ (\bar{\psi} \gamma^\rho \sigma^{ab} \partial_\rho \bar{\lambda}_\mu) - (\bar{\psi} \gamma^\rho \sigma^{ab} \bar{\lambda}_c) \partial_\mu \bar{e}^c_\rho + (\bar{\lambda}_\rho \sigma^{ab} \gamma_\rho \partial_\mu \psi) \right]
\]

\[-\frac{k^3}{2} \left[ (\bar{\psi} \sigma^{\rho \sigma} \sigma^{ab} \gamma^d \partial_\rho \psi)(\omega_{\mu dc} + \omega_{c d \mu}) + (\bar{\psi} \sigma^{cd} \gamma^{ab} \gamma_\rho \partial_\mu \psi)\omega_{ecd} \right]
\]

\[-\frac{k^2}{4} (\bar{\lambda}_\mu \sigma^{cd} \sigma^{ab} \gamma_\rho \psi) \omega^{ab}_\rho.\]

(3.25)

In fact we can show that the following LSUSY transformations on (3.17) and (3.18) induced by NLSUSY (2.3) close among them (80+80 linearized multiplet) at least up to the order with $\psi^2$ of superon. The contorsion of SUGRA-type breaks the closure and are excluded, so far. We show the explicit forms of some of the LSUSY transformations up to $O(\psi)$.

\[\delta \bar{e}_\mu^a = i \kappa \bar{\zeta} \gamma^a \bar{\lambda}_\mu - \bar{e}_\mu^a \bar{e}^b_\mu + \bar{\zeta} \bar{\lambda}^a_\mu,\]

(3.27)

\[\delta \bar{\lambda}_\mu = -\frac{i}{\kappa} (\sigma_{ab} \zeta) \omega^{ab}_\mu + \frac{i}{2} \bar{e}^{ab} (\sigma_{ab} \bar{\lambda}_\mu)
\]

\[+ A_\mu \zeta + B_\mu (\gamma_5 \zeta) + A^{a}_\mu (\gamma_5 \zeta) + B^{a}_\mu (\gamma_5 \gamma_\alpha \zeta) + A^{ab}_\mu (\sigma_{ab} \zeta),\]

(3.28)

\[\delta \bar{\lambda}^a_\mu = \frac{1}{2} \epsilon^{abcd} (\gamma_5 \gamma_\alpha \zeta) \omega_{bc d \mu},\]

(3.29)

\[\delta A_\mu = -\frac{1}{8} \left[ i(\bar{\zeta} \gamma^\rho \partial_\rho \bar{\lambda}_a) e^a_\mu + 3i(\bar{\zeta} \gamma^a D_\mu \bar{\lambda}_a) + 2(\bar{\zeta} \sigma^{\rho \sigma} \gamma_\mu D_\nu \bar{\lambda}_\rho) \right]
\]

\[-\frac{1}{4 \kappa} \left[ 3(\bar{\zeta} D_\mu \bar{\lambda}^a_\mu) + i(\bar{\zeta} \sigma^{cd} \gamma_\mu \bar{\lambda}^{a}_{\rho b}) + i(\bar{\zeta} \sigma^{cd} \gamma_\mu \bar{\lambda}^{a}_{\rho b}) e^b_\mu \right]
\]

\[+ \frac{1}{16} \left[ 4i(\bar{\zeta} \gamma^\rho \bar{\lambda}_a) \omega^\rho_{\mu b} + 4(\bar{\zeta} \sigma^{bc} \gamma^a \bar{\lambda}_a) \omega_{bc d \mu} - 4(\bar{\zeta} \sigma^{cd} \gamma^b \bar{\lambda}^{a}_{\rho} \omega_{ab d \mu})\right]
\]

\[\frac{1}{8 \kappa} \left[ (\bar{\zeta} \gamma^b \gamma^a \sigma^{cd} \bar{\lambda}_a) + (\bar{\zeta} \sigma^{cd} \gamma^b \gamma^a \bar{\lambda}_a) \right] \omega_{cd \mu},\]

(3.30)

\[\delta B_\mu = -\frac{1}{8} \left[ 5i(\bar{\zeta} \gamma_5 \gamma^\rho D_\rho \bar{\lambda}_a) e^a_\mu + 3i(\bar{\zeta} \gamma_5 \gamma^a D_\mu \bar{\lambda}_a) + 2(\bar{\zeta} \gamma_5 \sigma^{\rho \sigma} \gamma_\mu D_\nu \bar{\lambda}_\rho) \right]
\]

\[-\frac{1}{4 \kappa} \left[ 3(\bar{\zeta} \gamma_5 D_\mu \bar{\lambda}^a_\mu) + i(\bar{\zeta} \gamma_5 \sigma^{ab} \partial_\rho \bar{\lambda}_a) + i(\bar{\zeta} \gamma_5 \sigma^{ab} \partial_\rho \bar{\lambda}^{a}_{\rho b}) e^b_\mu \right]
\]

\[+ \frac{1}{16} \left[ 4i(\bar{\zeta} \gamma_5 \gamma^\rho \bar{\lambda}_a) \omega^\rho_{\mu b} + 4(\bar{\zeta} \gamma_5 \sigma^{cd} \gamma^b \bar{\lambda}^{a}_{\rho} \omega_{ab d \mu}) - 4(\bar{\zeta} \gamma_5 \sigma^{cd} \gamma^b \bar{\lambda}^{a}_{\rho} \omega_{ab d \mu})\right]
\]

\[\frac{1}{8 \kappa} \left[ (\bar{\zeta} \gamma_5 \gamma^a \sigma^{cd} \bar{\lambda}_a) + (\bar{\zeta} \gamma_5 \gamma^a \sigma^{cd} \bar{\lambda}_a) \right] \omega_{cd \mu},\]

11
\[
\delta A^a_{\mu} = \frac{1}{8} \left[ -4i(\bar{c} D_{\mu} \bar{\lambda}^a) + i(\bar{\zeta}_5 \gamma^a \gamma^\rho D_{[\mu \lambda_\rho]} \bar{\lambda}^a) + 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu} + 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu} \right],
\]

\[
\delta A^{a} = \frac{1}{8} \left[ -4i(\bar{c} D_{\mu} \bar{\lambda}^a) + i(\bar{\zeta}_5 \gamma^a \gamma^\rho D_{[\mu \lambda_\rho]} \bar{\lambda}^a) + 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu} + 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu} \right],
\]

\[
\delta B^a_{\mu} = \frac{1}{8} \left[ -4i(\bar{c} D_{\mu} \bar{\lambda}^a) + i(\bar{\zeta}_5 \gamma^a \gamma^\rho D_{[\mu \lambda_\rho]} \bar{\lambda}^a) + 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu} + 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu} \right],
\]

\[
\delta A^{[ab]}_{\mu} = \frac{1}{4} \left[ -2i(\bar{c} D_{[\mu} \bar{\lambda}_{ab}) \bar{c}^{c}_{\nu} + i(\bar{\zeta}_5 \gamma^{ab} \gamma^{\rho} D_{\rho} \bar{\lambda}_{ab}) \bar{c}^{c}_{\nu} + i(\bar{\zeta}_5 \gamma^{ab} \gamma^{\rho} D_{\rho} \bar{\lambda}_{ab}) \bar{c}^{c}_{\nu} \right] - 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu} + 2(\bar{\zeta}_5 \sigma^{cd} \sigma^{ab} \bar{\Lambda}_{ab}) \omega_{cd\mu},
\]
where \( \epsilon^{ab} \) is the Lorentz parameter and we put \( \epsilon^{ab} = \xi_{\rho} \omega^{ab}_{\rho} \). Note that the Lorentz transformations are induced on (3.31) and (3.28). In the right-hand side of (3.32) and (3.33), the last terms contain \( \Lambda^{a}_{\mu \rho} \) which is defined by \( \Lambda^{a}_{\mu \rho} = -\epsilon^{abcd} \gamma^{5} \omega_{bcd} \). Note that \( \Lambda^{a}_{\mu} \) is not the functional of the supermultiplet (3.18), so we may have to treat \( \Lambda^{a}_{\mu} \) as new auxiliary field. However, if we put \( \epsilon^{ab} = \epsilon^{ab}(\tilde{\lambda}^{\mu}, \tilde{\Lambda}^{a}_{\mu}) \), e.g. \( \epsilon^{ab} = \bar{\zeta} \gamma^{[a} \bar{\psi}^{b]} \), \( \Lambda^{a}_{\mu} \) does not appear in the right-hand side of (3.32) and (3.33). As a result, the LSUSY transformation on the supermultiplet (3.17) and (3.18) are written by using the supermultiplet itself at least at the leading order of superon \( \psi \). The higher order terms remain to be studied. However we believe that we can obtain the complete linearized off-shell supermultiplets of the SP algebra by repeating the similar procedures (on the auxiliary fields) order by order which terminates with \( (\psi^{4}) \). It may be favorable that 10 bosonic auxiliary fields, for example \( a_{\mu}(e, \psi), b_{\mu}(e, \psi), M(e, \psi), N(e, \psi) \) are arbitrary up now and available for the closure of the off-shell SP algebra in all orders.

We show some general properties of the new NLSUSY algebra and discuss some systematics of the linearization in the next section, which is complementary for linearizing SGM.

3.2 Case \( \tilde{e}^{a}_{\mu}(e, \psi) = e^{a}_{\mu} + f^{a}_{\mu}[O(\psi^{2}), ...] \)

In the previous section the linearization has been carried out consistently at least in the lowest order of the superon field under the simplest SUSY invariant relation for graviton \( \tilde{e}^{a}_{\mu}(e, \psi) = e^{a}_{\mu} \) of (3.19). In this section we consider the generalization of (3.19) and for a comparison take another way of thinking. We adopt the following assumption in stead of (3.19)

\[
\tilde{e}^{a}_{\mu}(e, \psi) = e^{a}_{\mu} + f^{a}_{\mu}[O(\psi^{2}), ...].
\] (3.35)

This means that the vierbein of LSUSY, i.e. the asymptotic (low energy) gravitational field, has the contribution from the superon-antisuperon (vacuum) higher order components.

Accordingly, the variation of \( \tilde{e}^{a}_{\mu} \) by means of NLSUSY transformations of \( (e^{a}_{\mu}, \psi) \) becomes

\[
\delta \tilde{e}^{a}_{\mu} = \delta e^{a}_{\mu} + \delta f^{a}_{\mu}[O(\psi^{1}), ...].
\] (3.36)

Substituting \( \delta \tilde{e}^{a}_{\mu} = i\kappa \zeta \gamma^{a}_{\mu} \tilde{\lambda}_{\mu} + \bar{\zeta} \Lambda^{a}_{\mu} - e^{a}_{b} \bar{e}^{b}_{\mu} \) \( (e^{a}_{b} = \bar{\zeta} \Gamma^{a}_{b} \psi, \; \Gamma^{a}_{b} = \bar{\Gamma}^{a}_{b}) \) and \( \delta e^{a}_{\mu} = \xi_{\rho} \partial^{\rho} e^{a}_{\mu} \) \( (\xi_{\rho} = i\kappa^{2} \bar{\zeta} \gamma^{\mu} \psi) \) into Eq.(3.36) produces

\[
i\kappa \bar{\zeta} \gamma^{a}_{\mu} \tilde{\lambda}_{\mu} + \bar{\zeta} (\Lambda^{a}_{\mu} - \Gamma^{a}_{b} \psi \bar{e}^{b}_{\mu}) = i\kappa^{2} \bar{\zeta} \gamma^{\rho} \psi \partial^{\rho} e^{a}_{\mu} + \delta f^{a}_{\mu}[O(\psi^{1}), ...].
\] (3.37)

The \( \tilde{\lambda}_{\mu}, \; \bar{e}^{a}_{\mu} \) (namely, \( f^{a}_{\mu} \)), \( \Lambda^{a}_{\mu} \) and \( e^{a}_{b} \) (namely, \( \Gamma^{a}_{b} \)) are expanded in terms of \( (e^{a}_{\mu}, \psi) \) as they satisfy Eq.(3.37) for all orders of \( \psi \).
For example, we have from Eq.(3.37) for the terms with $O(\psi^1)$,

$$i\kappa\bar{\zeta} \gamma^a \lambda_\mu [O(\psi^1)] + \bar{\zeta} (\bar{A}^a_\mu [O(\psi^1)] - \Gamma^a_{b\mu}[O(\psi^0)]\psi e^b_\mu) = i\kappa^2 \bar{\zeta} \gamma^a \psi \partial_\mu e^a_\mu + \delta f^a_\mu [O(\psi^1)].$$ \hspace{0.5cm} (3.38)

Let us consider the example of $\lambda_\mu [O(\psi^1)] = -i\kappa\omega^{ab}_\mu (e) \sigma_{ab} \psi$ following (3.14) and Lorentz parameter $\epsilon^{ab}_\mu = \xi^\rho \omega^a_{\rho b}(e)$, i.e., $\Gamma^a_{b\mu} = i\kappa^2 \gamma^\rho \omega^a_{\rho b}(e)$. When we substitute into (3.38), we have the relation to decide the form of $\bar{A}^a_\mu [O(\psi^1)]$ and $\delta f^a_\mu [O(\psi^1)]$ as

$$-\frac{1}{2} \kappa^2 \epsilon^{abcd} \bar{\zeta} \gamma^b \psi \omega^{cd}_\mu (e) + \bar{\zeta} \lambda^a_\mu [O(\psi^1)] = \delta f^a_\mu [O(\psi^1)].$$ \hspace{0.5cm} (3.39)

If we take $\delta f^a_\mu [O(\psi^1)] = 0$ in Eq.(3.39), we have the $\epsilon^{a}_\mu = \epsilon^{a}_\mu$ case with $\bar{A}^a_\mu [O(\psi^1)] = (1/2) \kappa^2 \epsilon^{abcd} \bar{\zeta} \gamma^b \psi \omega^{cd}_\mu (e)$ as we have already discussed in the previous section 3.1. On the other hand, if we put $\bar{A}^a_\mu [O(\psi^1)] = 0$ in $\delta \epsilon^a_\mu$, we obtain

$$f^a_\mu [O(\psi^2)] = -\frac{1}{4} \kappa^4 \epsilon^{abcd} \bar{\zeta} \gamma^b \psi \omega^{cd}_\mu (e).$$ \hspace{0.5cm} (3.40)

Here we note that since the commutator of two NLSUSY transformations for (3.40) becomes

$$[\delta_1, \delta_2] f^a_\mu = \Xi^\rho \partial_\mu f^a_\rho + \partial_\mu \Xi^\rho f^a_\rho = \delta_{\text{GL}(4R)} f^a_\mu, \hspace{0.5cm} (3.41)$$

the commutator of two NLSUSY transformations for $\epsilon^a_\mu$ also closes on GL(4R) as

$$[\delta_1, \delta_2] \epsilon^a_\mu = \Xi^\rho \partial_\mu \epsilon^a_\rho + \partial_\mu \Xi^\rho \epsilon^a_\rho = \delta_{\text{GL}(4R)} \epsilon^a_\mu. \hspace{0.5cm} (3.42)$$

The 64 bosonic auxiliary fields $\Phi^a_\mu$ (or $A^a_\mu, B^a_\mu, A^{a\mu}_\mu, B^{a\mu}_\mu, A^{[a\mu]}_\mu$) at the lowest order of $\psi$ are read from $\delta \lambda_\mu [O(\psi^0), O(\psi^2)] = -(i/\kappa) \omega^{ab}_\mu (e) \sigma_{ab} \zeta + \bar{\Phi} [O(\psi^2)] \zeta$ in Eq.(20) up to the contribution of the Lorentz transformation $-(i/2) \epsilon^{ab} \sigma_{ab} \lambda_\mu$ as follows;

$$\Phi^a_\mu [O(\psi^2)] = -\kappa^3 [-\sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu] + \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu - \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu] + \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu]
- \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu]
- \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu]
- \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu]
- \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu]
- \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu]
- \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu]
- \sigma_{ab} \partial_\mu \psi \bar{\psi} \gamma^a \epsilon^{ab}_\mu \partial_\rho e^b_\mu] \hspace{0.5cm} (3.43)$$

The 64 components of $\Phi^a_\mu [O(\psi^2)]$, e.g., $A^a_\mu, B^a_\mu, A^{a\mu}_\mu, B^{a\mu}_\mu, A^{[a\mu]}_\mu$ can be obtained by using Fierz transformations. If we define $\Lambda^a_\mu [O(\psi^1)] = \gamma^a \psi \partial_\rho e^{a\rho}_\mu = \gamma^a \psi \omega^{a\rho}_\mu$ as 64 fermionic auxiliary fields $(\bar{\Lambda}^a_\mu [O(\psi^1)] = 0$ in $\delta \epsilon^a_\mu\mu$ in the example now we consider), then the the variation of Eq.(3.43) by means of NLSUSY transformations of $(e^a_\mu, \psi)$
at the lowest order of $\psi$ is written in terms of the fields of the linear supermultiplet, $\tilde{\lambda}_\mu[O(\psi^1)]$ and $\tilde{\Lambda}^a_\mu[O(\psi^1)]$; namely, $\delta \tilde{\Phi}_\mu[O(\psi^1)]$ becomes

$$
\delta \tilde{\Phi}_\mu[O(\psi^1)] = -\kappa \left[ \int \frac{i}{\kappa} (\partial_\nu \tilde{\lambda}_\mu \bar{\zeta} \gamma^\nu + \tilde{\lambda}_\nu \bar{\zeta} \partial_\mu \epsilon^{\nu \gamma} e^c)
\right.
$$

$$
+ \sigma_{ab} \zeta \tilde{\Lambda}^a_\nu e^{a \nu} \epsilon^c \partial_\mu e^b_\mu - \sigma_{ab} \zeta \epsilon^a \partial_\mu \tilde{\Lambda}^b_\mu
$$

$$
+ \sigma^{\nu \sigma} \zeta \tilde{\Lambda}^d_\nu e_d \partial_\mu e^{\sigma \epsilon} e_{\epsilon \mu}
$$

$$
- \sigma^{\nu \sigma} \zeta (\partial_\rho \tilde{\Lambda}^a_\sigma e_{a \mu} + \tilde{\Lambda}^a_{a \mu} \partial_\rho e^{a \epsilon}) \right][O(\psi^1)].
$$

(3.44)

The systematic arguments with the generalized assumption for graviton (3.35) can be continued in principle to higher order terms. And it allows more varieties of the way of constructing the SUSY invariant relations choosing the auxiliary fields at least at the lowest order of $\psi$.

Finally, we discuss the commutators for more general cases. Here we consider a functional of $(e^a_\mu, \psi)$ and their derivatives as

$$
f_A(\psi, \bar{\psi}, e^a_\rho; \psi, \bar{\psi}, e^a_\rho, \sigma), \ (A = \mu, \nu, \ldots \text{etc.}) \quad (3.45)
$$

with $\psi, \bar{\psi} = \partial_\rho \psi, \ldots$, and we suppose that $f_A$ is the functional of $O(\psi^2)$ for simplicity. Then we have the variation of $f_A$,

$$
\delta f_A = \frac{\partial f_A}{\partial \psi} \delta \psi + \frac{\partial f_A}{\partial e^a_\rho} \delta e^a_\rho + \frac{\partial f_A}{\partial \psi_\rho} \delta \psi_\rho + \frac{\partial f_A}{\partial \bar{\psi}_\rho} \delta \bar{\psi}_\rho + \frac{\partial f_A}{\partial e^a_\rho, \sigma} \delta e^a_\rho, \sigma \quad (3.46)
$$

and the commutator for $f_A$ becomes

$$
[\delta_1, \delta_2]f_A = \frac{\partial f_A}{\partial \psi}[\delta_1, \delta_2]\psi + [\delta_1, \delta_2] \frac{\partial f_A}{\partial e^a_\rho} e^a_\rho + \frac{\partial f_A}{\partial \psi_\rho} [\delta_1, \delta_2] \psi_\rho
$$

$$
+ \frac{\partial f_A}{\partial \bar{\psi}_\rho} ([\delta_1, \delta_2] \bar{\psi}_\rho) + \frac{\partial f_A}{\partial e^a_\rho, \sigma} ([\delta_1, \delta_2] e^a_\rho, \sigma \quad (3.47)
$$

If we substitute the commutators for $(e^a_\mu, \psi)$ of Eq.(8) into Eq.(3.47), we obtain

$$
[\delta_1, \delta_2]f_A = \Xi^\lambda \partial_\lambda f_A + G_A, \quad (3.48)
$$

where $G_A$ is defined by

$$
G_A = \partial_\rho \Xi^\lambda \left( \frac{\partial f_A}{\partial e^a_\rho} e^a_\lambda + \frac{\partial f_A}{\partial \psi_\rho} \partial_\lambda \psi + \frac{\partial f_A}{\partial \bar{\psi}_\rho} \partial_\lambda \bar{\psi}_\rho + \frac{\partial f_A}{\partial e^a_\sigma, \rho} \partial_\lambda e^a_\sigma + \frac{\partial f_A}{\partial e^a_\rho, \sigma} \partial_\sigma e^a_\lambda \right)
$$

$$
+ \partial_\rho \partial_\sigma \Xi^\lambda \frac{\partial f_A}{\partial e^a_\rho, \sigma} e^a_\lambda. \quad (3.49)
$$
The first term in r.h.s. of Eq.(3.48) means the translation of $f_A$. Therefore Eq.(3.48) shows that the closure of the commutator algebra on GL(4R) for the various functionals $f_A$ in the previous argument depends on $G_A$ of Eq.(3.49), and these argument reproduces all the previous commutators respectively.

4. Conclusions

Now we summarize the results as follows. Referring to SUGRA transformations on the off-shell SUGRA supermultiplet, particularly to the Lorentz transformation, we have obtained the SUSY invariant relations and carried out the linearization explicitly up to $O(\psi^2)$ in the (SUGRA-like) LSUSY transformations. We presented two different ways of the linearization as the subsection 3.1 and 3.2, but we think that they are complementary for finding the correct way to the linearization of higher order terms. The d.o.f. of the high spin (auxiliary) fields $A^{ab}_\mu$ (spin 3) and $\tilde{\Lambda}^{a \mu \alpha}(e, \psi)$ (spin 5/2), though they appear through the arguments of Lorentz transformation, may reflect the characteristic structure of the tangent space of SGM spacetime, which is unstable. Interestingly the linearization mimicking SUGRA seems excluding the naive composite picture $\tilde{\lambda}_\mu = \gamma_\mu \psi + \cdots$, which may be suggested from the flat space linearization. The LSUSY transformations on the two different types of the linearized supermultiplet are different from SUGRA but close on the algebra isomorphic to SP up to $O(\psi^2)$ in the SUSY invariant relations. The complete linearization in all orders, which can be anticipated by the systematics emerging in the present study, needs specifications of the auxiliary fields and remains to be studied. The subsequent construction of the invariant linear SUSY action is challenging.

The linearization of the NLSUSY E-H type SGM action (2.1) with the extra dimensions gives another unification framework describing the observed particles not only as composites but also as elementary fields. The systematic linearization by using the superfield formalism applied to the coupled system of V-A action with SUGRA [19]-[22] is open but may be inevitable to complete the linearization, especially for $N > 1$. The linearization of SGM action for spin 3/2 N-G fermion field[23] (with extra dimensions) may be in the same scope and gives the deep insight into the structure of SGM.

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