Effect of the Heterogeneity of Metamaterials on Casimir-Lifshitz Interaction

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The Casimir-Lifshitz interaction between metamaterials is studied using a model that takes into account the structural heterogeneity of the dielectric and magnetic properties of the bodies. A recently developed perturbation theory for the Casimir-Lifshitz interaction between arbitrary material bodies is generalized to include non-uniform magnetic permeability profiles, and used to study the interaction between the magneto-dielectric heterostructures within the leading order. The metamaterials are modeled as two dimensional arrays of domains with varying permittivity and permeability. In the case of two semi-infinite bodies with flat boundaries, the patterned structure of the material properties is found to cause the normal Casimir-Lifshitz force to develop an oscillatory behavior when the distance between the two bodies is comparable to the wavelength of the patterned features in the metamaterials. The non-uniformity also leads to the emergence of lateral Casimir-Lifshitz forces, which tend to strengthen as the gap size becomes smaller. Our results suggest that the recent studies on Casimir-Lifshitz forces between metamaterials, which have been performed with the aim of examining the possibility of observing the repulsive force, should be revisited to include the effect of the patterned structure at the wavelength of several hundred nanometers that coincides with the relevant gap size in the experiments.

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I. INTRODUCTION

Despite nearly six decades of research after the original works of Casimir [1] and Lifshitz [2], the dependence of Casimir-Lifshitz force between bodies on their geometrical and material properties is still a subject of ongoing investigation [3]. The effect of geometry has been studied using a variety of techniques, which include perturbative expansion around ideal geometries [4,6] and in dielectric contrast [7,11], semiclassical [12] and classical ray-optics [13] approximations, multiple scattering [14,15] and multipole expansions [16,18], world-line method [19], exact numerical diagonalization methods [20], and the method of numerical calculation of the Green function [21]. These studies have significantly advanced our understanding of the subtle effect of geometry on Casimir-Lifshitz interactions, and have led to proposals for using the knowledge in designing useful nano-scale mechanical devices [22].

The dependence on material properties has also been studied extensively since the work of Dzyaloshinskii, Lifshitz, and Pitaevskii, who pointed out that the force can be attractive or repulsive depending on the relative values of the dielectric constants of the successive layers [23]. The existence of a repulsive mode of the interaction is very interesting, as it explains, for example, why a wetting layer of liquid should form on a solid in equilibrium with vapor [24]. Despite the theoretical possibility, it is not trivial to find a condition where the Casimir-Lifshitz interaction between two solid bodies that are either metallic or dielectric turns repulsive due to the presence of a non-solid medium between them. However, a recent experiment has shown that such a repulsive force can be observed between gold and silica particles that are separated by bromobenzene [24]. Another possibility for repulsive Casimir-Lifshitz interactions was pointed out by Boyer, who showed that the force between a purely dielectric semi-infinite body and a purely magnetic one separated by vacuum, is repulsive [25]. Considering that the Casimir-Lifshitz force will be a key player in the realm of micro/nano-electromechanical systems (the domain that involves length scales of the order of 100 nm to 1 μm) the possibility of producing repulsive forces gives hope for eliminating stiction. However, from the work of Lifshitz we know that at those distances the Casimir-Lifshitz force will be determined by the relatively high frequency part of the permittivity and permeability of the materials in imaginary frequency, and at such high frequencies the response of natural magnetic materials to the electromagnetic field is negligible (μ ∼ 1) [2,23,26]. In other words, while the theoretical possibility for creating a repulsive force exists, natural materials with the required magnetic properties cannot be found.

In recent years, engineered materials—called metamaterials—have been developed based on the proposed concept of negative refractive index [27], and their physical properties have been extensively studied [28,31]. This development has brought about the possibility of designing materials with special permittivity and permeability in a desirable range of frequencies. This could, in turn, result in producing nontrivial magnetic response in a broad range of frequencies, and possibly help achieve the repulsive Casimir-Lifshitz force [32,37]. A main characteristic of metamaterials is...
eralization of the dielectric contrast perturbation theory and its application to dielectric heterostructures. We develop a perturbative scheme for the calculation of the Casimir-Lifshitz force as a series expansion in powers of the contrast in permittivity and permeability profiles. We use the theoretical formulation to calculate the force for a model of metamaterials that is made of a two dimensional periodic structure of varying magneto-dielectric properties, as shown in Fig. 1. We find that the periodicity in the structure of our model metamaterials causes the normal Casimir-Lifshitz force to change as compared to its value when the materials are assumed to be uniform. This change is found to be significant when the distance between the two bodies is comparable to the wavelength of periodic structure of the bodies. The heterogeneity also introduces a lateral component to the Casimir-Lifshitz force, which is analogous to the lateral Casimir force between corrugated surfaces and dielectric heterostructures, and is more significant at smaller separations.

The rest of the paper is organized as follows. In Sec. II we develop the theoretical formulation of the perturbation theory that can be used for studying Casimir-Lifshitz interaction between magneto-dielectric heterostructures. Section III is devoted to applying the perturbative scheme to the particular problem of semi-infinite patterned magneto-dielectric structures at the leading order of the perturbation theory. In Sec. IV the results of the calculation of the normal and lateral Casimir-Lifshitz forces are shown, and finally, Sec. V concludes the paper with some discussion and remarks.

II. THEORETICAL FORMULATION

We consider an arrangement of magneto-dielectric objects in space with arbitrary shapes and frequency dependent dielectric and magnetic properties, as shown in Fig. 2. The frequency- and space-dependent dielectric function $\varepsilon(\omega, \mathbf{r})$ and magnetic permeability $\mu(\omega, \mathbf{r})$ describe the medium. As shown in Ref. [38], the Casimir-Lifshitz force for a model of metamaterials that is made of a two dimensional periodic structure of varying magneto-dielectric properties, as shown in Fig. 1. We find that the periodicity in the structure of our model metamaterials causes the normal Casimir-Lifshitz force to change as compared to its value when the materials are assumed to be uniform. This change is found to be significant when the distance between the two bodies is comparable to the wavelength of periodic structure of the bodies. The heterogeneity also introduces a lateral component to the Casimir-Lifshitz force, which is analogous to the lateral Casimir force between corrugated surfaces and dielectric heterostructures, and is more significant at smaller separations.

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energy of the system can be written as

$$E_{\text{CL}} = \hbar \int_0^\infty \frac{d\zeta}{2\pi} \text{tr} \ln [K_{ij}(\zeta; \mathbf{r}, \mathbf{r}')] , \quad (1)$$

where

$$K_{ij} = \left[ \frac{\epsilon^2}{c^2} (i \zeta, \mathbf{r}) \delta_{ij} + \frac{1}{\mu (i \zeta, \mathbf{r})} \partial_k - \frac{1}{\mu (i \zeta, \mathbf{r})} \partial_k \delta_{ij} \right] \times \delta^3(\mathbf{r} - \mathbf{r}'), \quad (2)$$

involves the dielectric function and magnetic permeability profiles in imaginary frequency.

**A. Perturbation Theory**

Following [8, 38], we develop a systematic expansion of $E_{\text{CL}}$ in terms of the dielectric contrast $\delta \varepsilon(i \zeta, \mathbf{r}) = \varepsilon(i \zeta, \mathbf{r}) - 1$ and the inverse magnetic permeability contrast $\delta \mu^{-1}(i \zeta, \mathbf{r}) = \mu^{-1}(i \zeta, \mathbf{r}) - 1$. Using the Fourier transforms

$$K_{ij}(\zeta; \mathbf{q}, \mathbf{q}') = \int d^3 \mathbf{r} d^3 \mathbf{r}' K_{ij}(\zeta; \mathbf{r}, \mathbf{r}') e^{i \mathbf{q} \cdot \mathbf{r}} e^{-i \mathbf{q}' \cdot \mathbf{r}'} , \quad (3)$$

$$\delta \varepsilon(i \zeta, \mathbf{q}) = \int d^3 \mathbf{r} [\varepsilon(i \zeta, \mathbf{r}) - 1] e^{i \mathbf{q} \cdot \mathbf{r}} , \quad (4)$$

$$\delta \mu^{-1}(i \zeta, \mathbf{q}) = \int d^3 \mathbf{r} [\mu^{-1}(i \zeta, \mathbf{r}) - 1] e^{i \mathbf{q} \cdot \mathbf{r}} , \quad (5)$$

the kernel in Eq. (2) can be decomposed as

$$K_{ij}(\zeta; \mathbf{q}, \mathbf{q}') = K_{0,ij}(\zeta, \mathbf{q})(2\pi)^3 \delta^3(\mathbf{q} + \mathbf{q}')$$

Here

$$K_{0,ij}(\zeta, \mathbf{q}) = \frac{\partial^3}{c^2} \delta_{ij} + \left( q_i q_j - q_k q_k \delta_{ij} \right) \delta \mu^{-1}(i \zeta, \mathbf{q} + \mathbf{q}'), \quad (6)$$

$$\delta K_{ij}(\zeta; \mathbf{q}, \mathbf{q}') = \frac{\partial^2}{c^2} \delta_{ij} \delta \varepsilon(i \zeta, \mathbf{q} + \mathbf{q}'), \quad (7)$$

$$\delta K_{ij}^m(\zeta; \mathbf{q}, \mathbf{q}') = (q_j q_i' - q_k q_k' \delta_{ij}) \delta \mu^{-1}(i \zeta, \mathbf{q} + \mathbf{q}'), \quad (8)$$

entail the permittivity and permeability profile.

We can now recast the expression of $E_{\text{CL}}$ into a perturbative series using the identity

$$\text{tr} \ln [K] = \text{tr} \ln [K_0] + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \times \text{tr} \left[ (K_0^{-1} \delta K^n + K_0^{-1} \delta K^m)^n \right] . \quad (10)$$

where the inverse of the kernel $K_0$ is given as

$$K_{0,ij}^{-1}(\zeta, \mathbf{q}) = \frac{\partial^2}{c^2} \delta_{ij} + q_i q_j \left[ \frac{\partial^3}{c^2} + q^2 \right] . \quad (11)$$

The general term for the series expansion in Eq. (10) takes on the form

$$\text{tr} \left[ (K_0^{-1} \delta K^n + K_0^{-1} \delta K^m)^n \right] = \int \frac{d^3 q^{(1)}}{(2\pi)^3} \ldots \frac{d^3 q^{(n)}}{(2\pi)^3}$$

$$\times \left[ \left( \frac{\partial^3}{c^2} \delta_{i_1 i_2} + \frac{1}{\partial^3 c^2} \frac{q^{(1)}}{q^{(1)}} \right) \delta \varepsilon(i \zeta, -\mathbf{q}^{(1)} + \mathbf{q}^{(2)}) + \left( \frac{\mathbf{q}^{(1)} \cdot \mathbf{q}^{(2)}}{c^2} \delta_{i_1 i_2} - q_{i_1} q_{i_2} \right) \right]$$

$$\times \ldots$$

$$\times \left[ \left( \frac{\partial^3}{c^2} \delta_{i_n i_1} + \frac{1}{\partial^3 c^2} \frac{q^{(n)}}{q^{(n)}} \right) \delta \varepsilon(i \zeta, -\mathbf{q}^{(n)} + \mathbf{q}^{(1)}) + \left( \frac{\mathbf{q}^{(n)} \cdot \mathbf{q}^{(1)}}{c^2} \delta_{i_n i_1} - q_{i_n} q_{i_1} \right) \right] . \quad (12)$$

Using the above explicit form, the Casimir-Lifshitz energy can be calculated for an arbitrary assortment of magneto-dielectric materials by following standard diagrammatic methods.

In the rest of this paper, we focus only on the second order and calculate the energy for periodic structures.

**B. Second Order Term**

We consider the leading contribution in the perturbation theory, which comes from the second-order term of the series (or its correction by a so-called Clausius-Mossotti factor using a resummation [8, 38]). We note that at the second order, we can use $\delta \mu^{-1} = -\delta \mu$, which will simplify the calculations that will follow later on. We find the
second order Casimir-Lifshitz energy as

\[
E_2 = -\frac{h}{4\pi^2\varepsilon_0} \int_0^\infty \frac{d\zeta}{\varepsilon_0} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{1}{k^2 + q^2} \left\{ \left( \frac{1}{c^4} + \frac{2}{c^2} (k^2 + q^2) + (k \cdot q)^2 \right) \delta\varepsilon(i\zeta, k + q) \delta\varepsilon(i\zeta, -k - q) \right. \\
+ 4 \frac{2}{c^2} (k \cdot q) \delta\varepsilon(i\zeta, k + q) \delta\mu(i\zeta, -k - q) + \left. \left( k \cdot q \right)^2 + k^2q^2 \right\} \delta\mu(i\zeta, k + q) \delta\mu(i\zeta, -k - q) \right\}.
\]

The above result strongly depends on the relative positioning, geometry, and electromagnetic characteristics of the bodies. Note that the \( \delta\varepsilon\delta\mu \) term manifests the non-additive nature of electric and magnetic contributions to the Casimir-Lifshitz energy.

### III. MAGNETO-DIELECTRIC HETEROSTRUCTURES

We now consider two parallel semi-infinite magneto-dielectric bodies placed at a separation \( H \), such as the one shown in Fig. 1. Introducing the labels “up” and “down” bodies, the permittivity function \( \epsilon \) and the permeability function \( \mu \) can be written as

\[
\epsilon(i\zeta, r) = \begin{cases} 
\epsilon_u(i\zeta, x), & \frac{H}{2} \leq z < +\infty, \\
1, & -\frac{H}{2} < z < \frac{H}{2}, \\
\epsilon_d(i\zeta, x), & -\infty < z < -\frac{H}{2},
\end{cases}
\]

(14)

\[
\mu(i\zeta, r) = \begin{cases} 
\mu_u(i\zeta, x), & \frac{H}{2} \leq z < +\infty, \\
1, & -\frac{H}{2} < z < \frac{H}{2}, \\
\mu_d(i\zeta, x), & -\infty < z < -\frac{H}{2},
\end{cases}
\]

(15)

where \( r = (x, z) \).

Using the identity

\[
\left( \frac{\varepsilon_0}{\varepsilon_0^2 + k^2} \right)^{-1} = \int_0^\infty dt_1 \exp[-t_1(\varepsilon_0^2/k^2 + k^2)],
\]

and introducing the new variables \( P \equiv \frac{1}{2}(k - q) \) and \( Q \equiv k + q \), we can simplify Eq. (13) by performing the Gaussian integrations over the variable \( P \). This yields

\[
E_2 = -\frac{h}{4\pi^2\varepsilon_0} \int_0^\infty d\zeta \frac{\zeta^2}{2} \int d^2x \cdot d^2x' \int \frac{d^3Q}{(2\pi)^3} \int \frac{d^3Q'}{(2\pi)^3} e^{i\mathbf{Q} \cdot (\mathbf{x} - \mathbf{x}')} \times \int_1^\infty dp \frac{e^{-\frac{2\pi}{\varepsilon_0^2}(\mathbf{Q}^2 + (\mathbf{Q}'^2))}}{[4p^2 + (\mathbf{Q}^2 + \varepsilon_0^2)]^{3/2}} [\mathcal{E}_{ee} + \mathcal{E}_{em} + \mathcal{E}_{mm}],
\]

(16)

where

\[
\mathcal{E}_{ee} = (2p^4 - 2p^2 + 1)\delta\varepsilon_u(i\zeta, x)\delta\varepsilon_d(i\zeta, x'),
\]

(17)

\[
\mathcal{E}_{em} = (-2p^2 + 1)\delta\varepsilon_u(i\zeta, x)\delta\mu_d(i\zeta, x'),
\]

(18)

This result can now be used to study the Casimir-Lifshitz interaction between two macroscopic bodies with any magneto-dielectric profiles at the second order in perturbation theory.

### A. Two homogenous semi-infinite bodies

Let us first consider two homogenous semi-infinite bodies of area \( A \) and separation \( H \). Assuming that the permittivity and permeability are frequency-independent, the Casimir-Lifshitz force \( F = -\partial E_2/\partial H \) takes on a simple form

\[
F = -\frac{hcA}{64\pi^2H^4} \times \left[ 23 (\delta\varepsilon_u\delta\mu_d + \delta\mu_u\delta\varepsilon_d) - 7 (\delta\varepsilon_u\delta\mu_u + \delta\varepsilon_d\delta\mu_d) \right],
\]

(20)

as found previously in the literature [41-43]. We can now consider patterned magneto-dielectric objects, and study how the structural heterogeneity affects both the normal and lateral Casimir forces.

### B. Patterned magneto-dielectric objects

Let us now consider the configuration shown in Fig. 1. The magneto-dielectric “chessboard” heterostructure can be characterized with wavelengths \( \lambda_x \) and \( \lambda_y \) along the \( x \) and \( y \) directions. In a repeat unit of the material along each direction \( \alpha (\alpha = x, y) \), a fraction \( f_\alpha \) of the material has permittivity \( \varepsilon_2 (\omega) \) and permeability \( \mu_2 (\omega) \), and the remaining fraction (1 – \( f_\alpha \)) has permittivity \( \varepsilon_1 (\omega) \) and permeability \( \mu_1 (\omega) \). Below, we will consider two possibilities; one with the domains with higher permittivity and permeability coinciding and another where they are in a staggered configuration. The vector (\( a_\alpha x, b_\alpha y \)) denotes the displacement of the upper object relative to the lower one, as shown in Fig. 1.

We use the Clausius-Mossotti resummation of the perturbation theory for the permittivity contribution [38, 39], which amounts to replacing \( \delta\varepsilon(i\zeta, x) \) in Eq. (16) by

\[
\overline{\delta\varepsilon}(i\zeta, x) \equiv \left[ \frac{\delta\varepsilon(i\zeta, x)}{1 + \frac{1}{3}\delta\varepsilon(i\zeta, x)} \right].
\]
Due to the periodicity of the magneto-dielectric structures, it is natural to use the Fourier series expansion of the permittivity and permeability profiles. We have

\[
\delta \varepsilon_d(i\zeta, \mathbf{x}) = \sum_{n,m=-\infty}^{\infty} A_{nm}(i\zeta) e^{i2\pi n \frac{x}{\lambda c} + i2\pi m \frac{y}{\lambda c}},
\]

\[
\delta \varepsilon_o(i\zeta, \mathbf{x}) = \sum_{n,m=-\infty}^{\infty} A_{nm}(i\zeta) e^{i2\pi n \frac{x}{\lambda o} + i2\pi m \frac{y}{\lambda o}},
\]

\[
\delta \mu_d(i\zeta, \mathbf{x}) = \sum_{n,m=-\infty}^{\infty} B_{nm}(i\zeta) e^{i2\pi n \frac{x}{\lambda d} + i2\pi m \frac{y}{\lambda d}},
\]

\[
\delta \mu_o(i\zeta, \mathbf{x}) = \sum_{n,m=-\infty}^{\infty} B_{nm}(i\zeta) e^{i2\pi n \frac{x}{\lambda o} + i2\pi m \frac{y}{\lambda o}}.
\]

The Fourier series coefficients can be easily found as

\[
A_{nm}(i\zeta) = 2 \left[ \delta \varepsilon_d(i\zeta) - \delta \varepsilon_o(i\zeta) \right] \frac{\sin(n\pi f_x) \sin(n\pi f_y)}{mn\pi^2},
\]

and

\[
B_{nm}(i\zeta) = 2 \left[ \delta \mu_d(i\zeta) - \delta \mu_o(i\zeta) \right] \frac{\sin(n\pi f_x) \sin(n\pi f_y)}{mn\pi^2},
\]

for \( n \neq 0 \) and \( m \neq 0 \). Using Eq. (16), one obtains the Casimir-Lifshitz energy \( E_C \) of the chessboard magneto-dielectric heterostructure as

\[
E_C = -\frac{\hbar A}{2\pi^2 \epsilon_0^2} \sum_{m,n=0}^{\infty} \cos(2\pi na + 2\pi mb)
\times \int_0^\infty d\zeta \int_1^\infty dp \zeta^2 \frac{e^{-\zeta^2}}{\sqrt{4p^2 + (2\pi m c)^2 + (2\pi n c)^2}}
\times \left[ (2p^4 - 2p^2 + 1) \left( A_{nm}^2(i\zeta) + B_{nm}^2(i\zeta) \right) - 2(2p^2 - 1) A_{nm}(i\zeta) B_{nm}(i\zeta) \right],
\]

where the prime on the summation indicates that the \( m = n = 0 \) term comes with a prefactor of 1/2.

**C. Material Properties**

While the experimental realizations of metamaterials involve a multitude of complex structures, in the present study we consider a simplified model where the heterostructure could have two types of effective permittivities—corresponding to metallic and dielectric materials—and two types of permeabilities—corresponding to magnetic and non-magnetic materials.
\[
\frac{F_{\text{lat}}}{A} \quad \text{(Nm}^{-2}\text{)}
\]

(\text{a})

\[
\frac{F_{\text{lat}}}{A} \quad \text{(Nm}^{-2}\text{)}
\]

(\text{b})

FIG. 5: (Color online). The lateral Casimir force for the case of \(\text{EhMh} - \text{EIM}l\) as a function of the dimensionless parameter \(a\) for various distances \(H\). Here \(\lambda_x = \lambda_y = 500\) nm, and the two panels correspond to (a) \(f_x = f_y = 0.5\), and (b) \(f_x = 0.75\) and \(f_y = 0.25\).

For the metal, which has a relatively higher dielectric constant especially at lower frequencies, we use the Drude model that in imaginary frequency reads

\[
\epsilon_h(i\zeta) = 1 + \frac{\Omega_p^2}{\zeta^2 + \gamma_D^2}. \tag{24}
\]

Alternatively, for the dielectric medium with relatively lower dielectric function we use the Drude-Lorentz model

\[
\epsilon_l(i\zeta) = 1 + \frac{\Omega_p^2}{\zeta^2 + \omega_c^2 + \gamma_e^2}. \tag{25}
\]

Similarly, we choose a simple Drude-Lorentz model for the permeability of the magnetic material, namely

\[
\mu_h(i\zeta) = 1 + \frac{\Omega_m^2}{\zeta^2 + \omega_m^2 + \gamma_m^2}. \tag{26}
\]

while for the non-magnetic material we have

\[
\mu_l(i\zeta) = 1. \tag{27}
\]

In the above equations, \(\omega_c\) (\(\omega_m\)) is the electric (magnetic) resonance frequency, and \(\gamma_e\) (\(\gamma_m\)) is the electric (magnetic) dissipation parameter. Using the plasma frequency of gold \(\omega_p(Au) \equiv \omega_p = 1.37 \times 10^{16}\) rad/s as a frequency scale, the numerical values of the magneto-dielectric characteristic parameters are chosen as: \(\Omega_D/\omega_p = 1.0\), \(\gamma_D/\omega_p = 0.004\), \(\Omega_e/\omega_p = 0.04\), \(\Omega_m/\omega_p = 0.1\), \(\omega_c/\omega_p = \omega_m/\omega_p = 0.1\), and \(\gamma_e/\omega_p = \gamma_m/\omega_p = 0.005\) [34].

We consider two different possibilities: (1) When the metallic patch has magnetic properties and the dielectric patch is non-magnetic. In this case, which we represent it schematically as \(\text{EhMh} - \text{EIM}l\), we have \(\epsilon_2 = \epsilon_\perp, \mu_2 = \mu_\perp, \epsilon_1 = \epsilon_\parallel, \) and \(\mu_1 = \mu_\parallel\). (2) When the dielectric patch has magnetic properties and the metallic patch is non-magnetic. In this case, which we represent it schematically as \(\text{EIM}l - \text{EhM}h\), we have \(\epsilon_2 = \epsilon_\parallel, \mu_2 = \mu_\parallel, \epsilon_1 = \epsilon_\perp, \) and \(\mu_1 = \mu_\perp\). Below, we will study both the normal and lateral Casimir-Lifshitz forces in both of these cases.
FIG. 7: (Color online). Vector field plots of the lateral Casimir-Lifshitz force as a function of the relative displacements $a$ and $b$, along the $x$ and $y$ directions respectively, for the EhMh–ElMl model at $H = 100$ nm. (a) $f_x = f_y = 0.5$ and $\lambda_x = \lambda_y = 500$ nm. (b) $f_x = 0.75$ and $f_y = 0.25$ and $\lambda_x = \lambda_y = 500$ nm. (c) $f_x = 0.75$, $f_y = 0.25$, $\lambda_x = 500$ nm, and $\lambda_y = 200$ nm.

IV. CASIMIR-LIFSHITZ FORCES BETWEEN THE CHESSBOARD STRUCTURES

Using the material properties described in Sec. III C above, we can now calculate the Casimir-Lifshitz energy for the chessboard magneto-dielectric heterostructure shown in Fig. 1 for the two cases denoted as EhMh–ElMl and EhMh–ElMl above. Due to the lateral heterogeneity of the magneto-dielectric properties, the two bodies exert both normal and lateral Casimir-Lifshitz forces on each other.

A. Normal Force

The normal force between the two patterned structures can be calculated as $F_{\text{nor}}^{C} = -\frac{1}{\lambda_x} \frac{\partial E}{\partial a}$. We set $b = 0$, and study the normal force as a function of $a$ for various values of the gap size $H$. We choose to normalize the force using $F_{\text{nor}}^{0}$, which is the contribution of the $m = \alpha = 0$ term in Eq. (23) to the normal force. Figure 3a shows the normal force relative to $F_{\text{nor}}^{0}$ for the EhMh–ElMl case. Figure 3b corresponds to the symmetric configuration where $f_x = f_y = 0.5$ whereas Fig. 3c corresponds to an asymmetric configuration where $f_x = 0.75$ and $f_y = 0.25$, and in both cases $\lambda_x = \lambda_y = 500$ nm. Figure 3 shows that depending on how the different patches with different magneto-dielectric properties are positioned with respect to one another, the normal Casimir-Lifshitz force can change in magnitude relative to the case with uniform magneto-dielectric configuration. When patches with similar properties are opposite one another $(a = 0)$ the attractive normal force is at its maximum, while the force is weakest when dissimilar patches are exactly opposite one another. While this relative change depends strongly on the gap size, we note that it can easily amount to a few percent in the experimentally relevant gaps sizes of a few hundred nanometers, and could even reach the value of 35% for the gap size of $H = 100$ nm for the asymmetric example. Figure 4 shows a similar behavior for the ElMh–EhMl case, which shows a relatively less dramatic change in the asymmetric example.

B. Lateral Force

The lateral force between the two patterned structures is a vector, with its value and direction depending on the relative positioning of the two bodies. For simplicity, we first focus on the case with $b = 0$, and only study the lateral force for unidirectional displacements along the $x$ axis (see Fig. 1). In this case, the force also lies along the $x$ axis for symmetry reasons, and we can find its value using $F^{\text{lat}} = -\frac{1}{\lambda_x} \frac{\partial E}{\partial a}$, as a function of $a$ for different gap sizes $H$. Figure 5a corresponds to the symmetric configuration where $f_x = f_y = 0.5$ whereas Fig. 5b corresponds to an asymmetric configuration where $f_x = 0.75$ and $f_y = 0.25$, and in both cases $\lambda_x = \lambda_y = 500$ nm. Figure 5 shows that the lateral Casimir-Lifshitz force is very sensitive to the value of $H$, and its dependence on the lateral displacement reflects the symmetry or asymmetry of the relative sizes of the two patches. While the form of the lateral force at relatively larger gap sizes tends to a sinusoidal form, at smaller separations higher harmonics contribute as well to reflect more of the details of the heterogeneity. Figure 6 shows the lateral force for the ElMh–EhMl case, which shows a similar behavior as compared to the previous case.

In Fig. 7 the vector field for the lateral Casimir-Lifshitz force, defined as $\mathbf{F}^{\text{lat}}(a, b) = -\frac{1}{\lambda_x} \frac{\partial E}{\partial a} \hat{x} - \frac{1}{\lambda_y} \frac{\partial E}{\partial b} \hat{y}$, is plotted as a function of $a$ and $b$. One can see that the symmetry of the heterostructure affects the configuration of the lateral Casimir-Lifshitz force as a vector field, and that the numerous parameters involved...
can provide opportunities for a rich variety of engineered patterns for the lateral force.

V. DISCUSSION

We have studied the Casimir-Lifshitz interaction between two metamaterials modeled as periodic arrayed structures containing domains of varying magneto-dielectric properties. We have considered two types of permittivity functions—metallic and dielectric—and two types of permeability functions—magnetic and nonmagnetic—and their corresponding two combinations. For both combinations, we have found significant changes in the value of the normal Casimir-Lifshitz force relative to the value that corresponds to the uniform (macroscopic) model of the materials. The relative change is increased as the gap size is decreased, and reaches a few percent in the experimentally relevant gaps sizes of a few hundred nanometers, while it could even reach 35% when the gap size is 100 nm in the models we studied. Considering how delicate it is to find the condition to achieve the repulsive force in realistic situations as recent studies have revealed [34], our results show that the effect of the structural heterogeneity should be taken into account in determining whether the force is repulsive or not. This is particularly pertinent as the characteristic length scale of the periodic features in realistic metamaterials coincides with the gap sizes at which the Casimir-Lifshitz force is particularly relevant. While this issue has been ignored by all previous studies, we note that our study has been performed within the scope of the magneto-dielectric contrast perturbation theory at its leading order and should be considered more as an indication of the relative significance of the effect rather than a study that could provide numerically accurate results [41]. To that end, one needs to employ more sophisticated numerical methods similar to those developed to study the effect of geometry [20,21].

Another consequence of the structural heterogeneity of the metamaterials is the possibility of the emergence of lateral Casimir-Lifshitz forces, which are stronger for smaller gap sizes. While all previous works on lateral Casimir force have focused only on unidirectional geometrical or material heterostructure features, we have considered a two dimensional pattern and presented the vector field distribution of the lateral force. These forces are very sensitive to the details of the periodic patterns of the magneto-dielectric properties, and their versatility allows them to be amenable to detailed engineering by changing these features.

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[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51, 793 (1948).
[2] E. M. Lifshitz, Sov. Phys. JETP 2, 73 (1956).
[3] G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Rev. Mod. Phys. 81, 1827 (2009).
[4] R. Golestanian and M. Kardar, Phys. Rev. Lett. 78, 3421 (1997); Phys. Rev. A 58, 1713 (1998); M. Kardar and R. Golestanian, Rev. Mod. Phys. 71, 1233 (1999).
[5] T. Emig, A. Hanke, R. Golestanian, and M. Kardar, Phys. Rev. Lett. 87, 260402 (2001); Phys. Rev. A 67, 022114 (2003).
[6] P. A. Maia Neto, A. Lambrecht, and S. Reynaud, Phys. Rev. A 72, 012115 (2005); R. B. Rodrigues, P. A. Maia Neto, A. Lambrecht, and S. Reynaud, Phys. Rev. Lett. 96, 100402 (2006).
[7] G. Barton, J. Phys. A 34, 4083 (2001).
[8] R. Golestanian, Phys. Rev. Lett. 95, 230601 (2005).
[9] S.Y. Buhmann and D.-G. Welsch, Appl. Phys. B 82, 2, 189 (2006).
[10] G. Veble and R. Podgornik, Eur. Phys. J. E 23, 275279 (2007).
[11] K.A. Milton, P. Parashar, and J. Wagner, Phys. Rev. Lett. 101, 160402 (2008).
[12] M. Schaden and L. Spruch, Phys. Rev. A 58, 935 (1998).
[13] R.L. Jaffe and A. Scardicchio, Phys. Rev. Lett. 92, 070402 (2004).
[14] R. Balian and B. Duplantier, Ann. Phys. (New York) 104, 300 (1977); 112, 165 (1978).
[15] O. Kenneth and I. Klich, Phys. Rev. Lett. 97, 160401 (2006).
[16] R. Golestanian, Phys. Rev. E 62, 5242 (2000).
[17] T. Emig, N. Graham, R.L. Jaffe, and M. Kardar, Phys. Rev. Lett. 99, 170403 (2007).
[18] T. Emig, N. Graham, R.L. Jaffe, and M. Kardar, Phys. Rev. A 79, 054901 (2009).
[19] H. Gies and K. Klingmuller, Phys. Rev. Lett. 96, 220401 (2006); Phys. Rev. Lett. 97, 220405 (2006).
[20] T. Emig, Europhys. Lett. 62, 466 (2003); R. Büscher and T. Emig, Phys. Rev. Lett. 94, 133901 (2005); A. Lambrecht and V. N. Marachevsky, Phys. Rev. Lett. 101, 160403 (2008).
[21] A. Rodriguez, M. Ibanescu, D. Iannuzzi, F. Capasso, J. D. Joannopoulos, and S.G. Johnson, Phys. Rev. Lett. 99, 080401 (2007); A. Rodriguez, M. Ibanescu, D. Iannuzzi, J.D. Joannopoulos, and S.G. Johnson, Phys. Rev. A 76, 032106 (2007); A. Rodriguez, J. D. Joannopoulos, and S. G. Johnson, Phys. Rev. A 77, 062107 (2008).
[22] A. Ashourvan, M. F. Miri, and R. Golestanian, Phys. Rev. Lett. 98, 140801 (2007); Phys. Rev. E 75, 040103 (R) (2007); T. Emig, Phys. Rev. Lett. 98, 160801 (2007); M. Miri and R. Golestanian, Appl. Phys. Lett. 92, 113103 (2008); F. C. Lombardo et al., J. Phys. A 41 164009 (2008); I. Cavero-Pérez et al., Phys. Rev. D 78 065019 (2008); M. F. Miri, V. Nekouie, and R. Golestanian,
Phys. Rev. E 81, 016104 (2010).
[23] I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, Adv. Phys. 10, 165 (1961).
[24] J. N. Munday, F. Capasso, and V. A. Parsegian, Nature 170, 457 (2009).
[25] T. H. Boyer Phys. Rev. A 9, 2078 (1974).
[26] E. M. Lifshitz and L. P. Pitaevskii, electrodynamics of continuous media, Vol. 8; Statistical Physics, Landau-Lifshitz, Course of Theoretical Physics, Vol. 9, Pt. 2 (Butterworth-Heinemann, Oxford, 2002).
[27] V. G. Veselago, Sov. Phys. Solid State 8, 2854 (1967).
[28] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. 47, 2075 (1999).
[29] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. 84, 4184 (2000); J. B. Pendry, Phys. Rev. Lett. 85, 3966 (2000); R. A. Shelby, D. R. Smith, and S. Schultz, Science 292, 77 (2001); S. A. Ramakrishna, Rep. Prog. Phys. 68, 449 (2005); V. M. Shalaev and A. Boardman, J. Opt. Soc. Am. B 23, 386 (2006).
[30] C. M. Soukoulis, S. Linden, and M. Wegener, Science 315, 47 (2007).
[31] V. M. Shalaev, Nature Photonics 1, 41 (2007); A. Boltasseva and V. M. Shalaev, Metamaterials 2, 1 (2008).
[32] O. Kenneth, I. Klich, A. Mann, and M. Revzen, Phys. Rev. Lett. 89, 033001 (2002).
[33] C. Henkel and K. Joulain, EPL 72, 929 (2005).
[34] F. S. S. Rosa, D. A. R. Dalvit, and P. W. Milonni, Phys. Rev. A 78, 032117 (2008); F. S. S. Rosa, D. A. R. Dalvit, and P. W. Milonni, Phys. Rev. Lett. 100, 183602 (2008).
[35] A. Lambrecht and I. G. Pirozhenko Phys. Rev. A 78, 062102 (2008).
[36] R. Zhao, J. Zhou, Th. Koschny, E. N. Economou, and C. M. Soukoulis, Phys. Rev. Lett. 103, 103602 (2009).
[37] V. Yannopapas and N. V. Vitanov, Phys. Rev. Lett. 103, 120401 (2009).
[38] R. Golestanian, Phys. Rev. A 80, 012519 (2009).
[39] A. Azari, H. S. Samanta, and R. Golestanian, New J. Phys. 11, 093023 (2009).
[40] F. Chen, U. Mohideen, G. L. Klimchitskaya, and V. M. Mostepanenko, Phys. Rev. Lett. 88, 101801 (2002); Phys. Rev. A 66, 032113 (2002).
[41] H. B. G. Casimir and D. Polder, Phys. Rev. 73, 360 (1948).
[42] G. Feinberg and J. Sucher, J. Chem. Phys. 48, 3333 (1968).
[43] E. Lubkin, Phys. Rev. A 4, 416 (1971).
[44] For example, we do not observe a repulsive force in the regime of parameters where Lifshitz theory does show repulsion, as found in Ref. [34].