High Frequency Oscillations in Quantum Plasma

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Abstract. The lowest order fast oscillating quantum plasma electron velocity and density perturbations arising from interaction of linearly polarized laser beam with quantum plasma, using perturbative technique involving orders of the incident laser beam. Dispersion relation for the interaction has been established.

1. Introduction

Quantum plasmas have attracted a great deal of attention in recent years due to their wide-ranging applications in different environments, such as in ultrasmall electronic devices [1], in superdense astrophysical systems (particularly, in the interior of Jupiter, white dwarfs and superdense neutron stars)[2], in high-intensity laser-produced plasmas [3,4], in metallic nanostructures, in non-linear quantum optics [5,6] and in dusty plasmas [7,8], etc. It is well-known that a quantum plasma, whose constituents are the electrons, positrons and ions has the properties of high plasma particle number densities and low-temperatures, in contrast to the classical plasma. The latter has the characteristics of high-temperatures and low-particle number densities. The typical quantum scales, such as the time, the length and the thermal speeds of the charged particles are significantly different from those in the classical plasma.

In a one-dimensional Fermi-Dirac plasma, the pressure law [9] is \( p_s = m_s V_{FS}^2 n_s^2 / 3 n_{so}^2 \), where \( m_s \) is the mass of the particle species \( s \) (\( s \) equals \( e \) for the electrons, \( p \) for the positrons and \( i \) for the ions), \( V_{FS} = (2E_{FS} / m_s)^{1/2} \) is the Fermi thermal speed, \( E_{FS} = (\hbar^2 / 2m_s)(3\pi^2 n_{so})^{2/3} \) is the Fermi energy, \( \hbar \) is the Planck constant divided by \( 2\pi \), \( n_s \) is the number density with an equilibrium value...
It is noted that the plasma frequency \( \omega_{ps} = \left( \frac{4\pi e^2}{m_i} \right)^{1/2} \) is significantly high in quantum plasmas, because of the higher values of the equilibrium number density. Accordingly, the quantum plasma effects become important in dense plasmas, when the electron thermal de-Broglie wavelength \( \lambda_{Be} = \frac{\hbar}{m_e V_{Te}} \) approaches the electron Fermi wavelength \( \lambda_{Fe} \) and exceeds the electron Debye radius \( \lambda_{De} \) (viz., \( \lambda_{Be} \approx \lambda_{Fe} > \lambda_{De} \)). Furthermore, the quantum effects associated with the strong density correlation start playing a significant role when the de-Broglie wavelength is larger than the average inter particle distance \( d = n_{eo}^{-1/3} \), i.e., \( n_{eo} \lambda_{pe}^3 \geq 1 \).

The well-known mathematical approaches used for the quantum plasmas are the Wigner-Poisson, the Schrodinger-Poisson and the Dirac-Maxwell, which describe the statistical and hydrodynamic behaviour of the plasma particles at quantum scales. They are the quantum analogue of the kinetic and fluid models in classical plasma physics. Recently, Manfredi [9] has reviewed the Wigner-Poisson and the Schrodinger-Poisson systems in a collisionless quantum plasma. The quantum hydrodynamic (QHD) model basically deals with the transport of charge, momentum and energy in plasmas, and has always been applied to semiconductor physics [10]. This (QHD) model consists of a set of equations describing the transport of charge density, momentum (including the Bohm potential) and energy in a charged particle system interacting through a self-consistent electrostatic potential.

Haas, Manfredi and Felix [11] presented a quantum multistream model by using a nonlinear Schrodinger-Poisson system and derived the dispersion relations for one-and two-stream plasma instability. Later, Anderson et al. [12], did the same by employing the Wigner-Poisson formulation to examine the statistical behaviour of quantum plasma. Haas et al. [13] investigated the linear and nonlinear quantum ion-acoustic waves using the one-dimensional QHD model. The interaction of linearly polarized laser beam with quantum plasma has been studied [14]. Ren et al. [15] studied the dispersion of linear waves in a uniform cold quantum plasma using QHD equations with the magnetic field of Wigner-Poisson systems. Ion-acoustic waves have been studied by Ali et al. [16].

In the present paper, we have obtained for the first time, the dispersion relation for the propagation of a linearly polarized laser beam through a cold quantum plasma using perturbative techniques. In sec. II, the first order electron velocities and density perturbative have been derived. Dispersion relation using standard techniques have been obtained in Sec. III. Sec. IV is devoted to summary and conclusion.
2. Particle Equations Of Motion

Consider a linearly polarized laser beam, represented by the electric vector $\hat{E} = \hat{e}_z E \cos(kz - \omega t)$ ($\hat{e}_z$ is the unit vector of polarization), propagating in a uniform cold quantum plasma of density $n_o$. The plasma electrons obey the equations

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m}\left[\vec{E} + \frac{1}{c}(\vec{v} \times \vec{B})\right] - \frac{1}{2} \nabla (\vec{v} \cdot \vec{v}) + \vec{v} \times (\nabla \times \vec{v}) - \frac{v_F^2}{3n_o} \frac{\Delta n}{n} + \frac{\hbar^2}{2m} \nabla \left( \frac{1}{\sqrt{n}} \nabla^2 \sqrt{n} \right)$$  (1)

and

$$\frac{\partial n}{\partial t} + \nabla . (n \vec{v}) = 0$$  (2)

Where $v_F$ is the Fermi velocity and all the quantum effects are represented by the $\hbar$-dependent term in eq.(1) which is called the Bohm potential term [9,17-18].

Expanding eqs. (1) and (2) for the order of the electromagnetic field, we get

$$\frac{\partial \vec{v}^{(1)}}{\partial t} = -\frac{e}{m}\left[\vec{E} + \frac{1}{c}(\vec{v}^{(0)} \times \vec{B})\right] - \frac{1}{2} \nabla (\vec{v}^{(0)} \cdot \vec{v}^{(0)}) - \frac{v_F^2}{3n_o} \frac{\Delta n^{(0)}}{n} + \frac{\hbar^2}{2m} \nabla \left( \frac{1}{\sqrt{n^{(0)}}} \nabla^2 \sqrt{n^{(0)}} \right)$$  (3)

and

$$\frac{\partial n^{(1)}}{\partial t} = -\nabla \cdot (n^{(0)} \vec{v}^{(1)})$$  (4)

Where subscript $o$ and $1$ stand for the equilibrium and perturbed quantities, and the last term in eq. (3) representing the Bohm potential has been perturbatively expanded using [19]

$$\nabla \left( \frac{1}{\sqrt{n}} \nabla^2 \sqrt{n} \right) = \frac{1}{n_o} (1 - \frac{n^{(1)}}{n_o}) \left( \frac{1}{2} \nabla \nabla^2 n^{(0)} - \frac{1}{2n_o} \nabla n^{(0)} \nabla^2 n^{(0)} - \frac{1}{2n_o} \nabla n^{(0)} n^{(1)} n^{(0)} - \frac{n^{(1)}}{2n_o^2} \nabla^2 n^{(0)} n^{(0)} \right)$$

$$- \frac{1}{4n_o} \nabla (2 \nabla n_o \nabla n^{(0)}) + \frac{1}{4n_o^2} n^{(1)} \nabla (\nabla n_o)^2 + \frac{1}{2n_o^2} (\nabla n_o)^2 \nabla n^{(0)}$$
Assuming the perturbed density [20] to vary according to $\exp(ikr - \omega t)$, and simultaneously solving eqs. (3) and (4) we get the first order perturbed velocity of electrons as 

$$v^{(1)} = \frac{eE}{m\omega} [1 - \Omega k^2 V_p^2 - \frac{\hbar^2 k^4}{4m^2}] \sin(kz - \omega t) \tag{5}$$

where,

$$\Omega = \left\{ \omega^2 + k^2 V_p^2 + \frac{\hbar^2 k^4}{4m^2} \right\}^{-1}.$$

### III. Dispersion Relation

The wave equation governing the propagation of the laser pulse through the quantum plasma is given by

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \tilde{E} = \frac{4\pi}{c^2} \frac{\partial \tilde{J}}{\partial t}. \tag{6}$$

The plasma current density is given by

$$\tilde{J} = -nev \tag{7}$$

Substituting the expression for quiver velocity from eq.(5) in eq.(7) and simultaneously solving eqs.(6) and (7) gives the dispersion relation as

$$\omega^2 = \omega_p^2 + c^2 k^2 - \omega_p^2 V_p^2 k^2 \Omega - \frac{\hbar^2 k^4}{4m^2} \omega_p^2 \Omega \tag{8}$$

where, $\omega_p^2 = \frac{4m_e e^2}{m}$.

In the absence of quantum effects, eq. (8) reduces to the well known linear dispersion of a laser beam propagating in plasma, as well as in the absence of the magnetic fields and the collective
approximations used in ref [14] the dispersion relation for propagation of the linearly polarized laser beam through magnetized quantum plasma reduces to eq.(8).

IV. SUMMARY AND CONCLUSION

We have investigated the dispersion properties of linearly polarized electromagnetic waves in uniform, cold quantum plasmas. First order electron velocity and density have been obtained. Our analysis is based on the quantum hydrodynamic equations using perturbative techniques of electric component of the laser field, which generates a quiver motion of the plasma electrons. It is found that the dispersion relation for any classical plasma can be modified to include the quantum contribution.

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