Phase coherence and extreme self phase modulation

P. Kinsler

Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2AZ, United Kingdom.

(Dated: February 1, 2008)

PHSPM Phase coherence in XSPM Dr.Paul.Kinsler@physics.org

I study how pulse to pulse phase coherence in a pulse train can survive super-broadening by extreme self phase modulation (SPM). Such pulse trains have been used in phase self-stabilizing schemes as an alternative to using a feedback process. However, such super-broadened pulses have undergone considerable distortion, and it is far from obvious that they necessarily retain any useful phase information. I propose measures of phase coherence applicable to such pulse trains, and use them to analyze numerical simulations comparable to self-stabilization experiments.

I. INTRODUCTION

In some recent experiments (e.g. [1, 2]), carrier-envelope phase (CEP) stabilized idler pulses were generated in an optical parametric amplifier (OPA) by combining a pump pulse with its super-broadened replica as the signal pulse. The phase relationships in the optical parametric interaction ensure that the phase of the idler pulse is independent of that of the pump. This principle has been tested experimentally using a train of pump pulses with arbitrary CEP [3], and a train of idler pulses with stabilized phase has been achieved. The distinguishing feature of this technique is that it acts on each pulse in isolation, and does not rely on feedback. The idler phase is stabilized to an unknown value determined by the details of the spectral broadening process.

Fang et al. [1] broadened their signal pulses using extreme self-phase modulation (SPM) in sapphire, then selected suitable spectral components from the super-broadened spectrum to form the signal pulse for their OPA stage (see fig. 1). The success of the experiment raises interesting questions about how the severely distorted temporal profile of the signal pulse can contain sufficient sensible phase information to achieve the desired result, especially given that small differences between input pulses can be turned into large differences by the strong nonlinearity.

In this paper I investigate how (inter-pulse) phase coherence can survive in a train of pulses broadened by a third-order nonlinearity. In most pulse propagation models based on a complex envelope $A(t)$, the nonlinear interaction term governing self-phase modulation (SPM) appears as $\chi^{(3)}|A|^2A$. This is linear in phase (of $A$), and is usually expected to be well-behaved, although intensity fluctuations in the pulse train lead to CEP fluctuations in the output [2, 3]. However, the true form of the nonlinear interaction is $\chi^{(3)}E^3$. In the envelope picture, this adds an extra (non resonant and CEP sensitive) term to the nonlinearity, i.e. $\chi^{(3)}A^3$. This initially leads to third harmonic generation (THG) which can disrupt the phase stabilization process; higher (odd) harmonics can also be generated. Even more importantly, the THG is generated from the whole broadened spectrum, not just the initial central peak. Thus we cannot always assume that all of the THG contribution will remain safely out of range of our SPM broadened part: we might well expect that fluctuations will be strongly mixed into the nonlinear process, destroying any pulse-to-pulse repeatability. Since the true nonlinearity includes this wide range of multi-wave interactions, we need to be aware that all have the potential to obscure the phase coherence of the train of broadened pulses.

I define criteria characterizing the effect of fluctuations in intensity, pulse width, and CEP on the phase of the broadened pulse. Applying these to simulations comparable to the experiment of Fang et al., I show the importance of minimising fluctuations in the input pulse train: even though SPM is insensitive to CEP variation, intensity and width fluctuations can play a significant role. The wide bandwidth of these pulses, along with the need to retain the full $\chi^{(3)}$ interaction, places significant demands on the required numerical resolution. Therefore simulations of pulse propagation were done using the PSSD method [2], which evolve the carrier oscillations directly.

Because the strong nonlinearity means that the results will vary significantly from one set of parameters to another, the data presented in this paper serves largely to indicate the various issues involved in extreme SPM broadening, rather than being a set of specific predictions. Nevertheless, the simulation results indicate that the phase coherence in the broadened pulse train is maintained only up to a point, and that sensitivity to pulse intensity or width fluctuations is much stronger than for CEP variation. Further, SPM can only broaden pulses by so much before multi-wave cross talk and/or THG destroy the pulse-to-pulse phase coherence.

In section II the basic phase properties relevant for this analysis are described, and in section III the measures of phase coherence are introduced. Section IV describes the simulation methods used, and then in section V the effects of fluctuations in the input pulse train (intensity, pulse width, and CEP) are described. Finally, section VI comprises a brief discussion, and section VII contains my conclusions.

*Electronic address: Dr.Paul.Kinsler@physics.org
II. PHASE PROPERTIES

Here I focus on the spectral phase $\phi_s(\omega)$, since a broadened pulse will suffer chirp and other distortion, removing any straightforward way to characterize the phase in the time domain. This spectral phase will depend on the phase of the input pulse $\phi_p(\omega)$, on its other properties (intensity, width, wavelength, etc), and on the propagation medium (e.g. sapphire). Considering just intensity $I$ and input CEP $\phi_p$, we have that the (broadened) signal pulse has a phase

$$\phi_s(I, \phi; \omega) = \phi_p(\omega) + \Phi(I, \phi; \omega)$$

$$= \phi_p(\omega) + \Phi(I_0, \phi_0; \omega) + \Delta_1(I, \phi_0)(I - I_0) + \delta_1(\omega)$$

$$+ \Delta_\phi(I_0, \phi_0)(\phi - \phi_0) + \delta_\phi(\omega), \quad (2)$$

where $I_0$ is the average intensity of pulses in the train, and $\phi_0$ the average CEP; further terms including the effect of pulse width ($\tau$) variation can easily be added. The function $\Phi$ encodes the full effect of the complicated nonlinear interactions on the phase. Here the linear responses of the phase about $I_0$ and $\phi_0$ are given by $\Delta_1$ and $\Delta_\phi$, while the remaining nonlinear response is contained in $\delta_1$, $\delta_\phi$.

The broadened signal pulse is then mixed with the original pulse in an OPA. The difference-frequency idler pulse that results has a spectral phase which is the difference of the pump phase $\phi_p(\omega)$ and the signal phase $\phi_s(\omega)$,

$$\phi_I(I, \phi; \omega) = \phi_p(\omega) - \phi_s(I, \phi; \omega)$$

$$= -\Phi(I_0, \phi_0; \omega)$$

$$- \Delta_1(I, \phi_0)(I - I_0) - \delta_1(I, \phi; \omega)$$

$$- \Delta_\phi(I_0, \phi_0)(\phi - \phi_0) - \delta_\phi(I, \phi; \omega). \quad (3)$$

We can see that each idler pulse output from the OPA will have a phase $\phi_I$ stabilized to a value set by the nonlinear propagation through the sapphire – the unknown CEP $\phi_p(\omega)$ of the input pump pulse has canceled out. However, this phase $\phi_I$ will vary from pulse to pulse depending on how the fluctuations in the input pulse train alter each instance of the linear and nonlinear evolution. Although I consider only the (three-wave) difference-frequency generating term, note that the full $\chi^{(2)}$ OPA interaction contains potentially phase sensitive terms [6].

III. PHASE COHERENCE

In order to determine whether there is sufficient useful pulse-to-pulse phase coherence after the SPM broadening, I analyze the last four terms in eqn. (2) to define the necessary measures of phase coherence.

I first consider the linear response of the output pulse to a variation in the input pulse; e.g. how the spectral phase of the pulse varies due to small changes in the intensity. Knowing the linear response enables us to set tolerances on the input pulse train to guarantee a certain level of stability between pulses in the output train. We also need to know how reliable this estimated linear response is: since we are considering a complex nonlinear interaction, we cannot simply the response to input variations will be linear. What we hope is that the dominant response will be linear, and that the nonlinear corrections will remain small.

The quantities defined below in eqns. (5) and (6) are expressed for simplicity only w.r.t. intensity fluctuations; but by swapping the intensity arguments $I_1, I_2$ for (e.g.) CEP’s $\phi_{p1}, \phi_{p2}$, we can equally well define a $\Delta_\phi, \sigma^2_\phi$, expressing the input CEP variation; just as by swapping the intensity $I_1, I_2$ for pulse widths $\tau_1, \tau_2$ we could define $\Delta_\tau, \sigma^2_\tau$.

Since our simulations provide data (such as spectral phases) not easily accessible in experiments, we can use this to our advantage in determining the likely coherence properties of the broadened pulse train, and even suggest changes to improve performance.

A. Linear response

The linear response of the phase to intensity variation can be estimated by taking an ensemble average over a large number of simulations. Each simulation starts with a pulse selected at random from the distribution of all possible pulses in the input pulse train. I define the linear response as an ensemble average over pairs of broadened signal pulses:

$$\Delta_1(\omega) = \left\langle \frac{\phi_s(I_1; \omega) - \phi_s(I_2; \omega)}{|I_1 - I_2|} \right\rangle. \quad (5)$$

More simply, we might calculate $\Delta_1$ by simply taking an average of phase differences over a suitable range of pulse variation. If this $\Delta_1$ is small, phase is little affected by input variation – at least to a linear approximation. For the case of intensity variation, this linear response has been measured experimentally, as reported by Li et al. [7].
B. Nonlinear response

I estimate the contribution of the nonlinear part of the phase response using

\[ \sigma_\phi^2(\omega) = \langle \delta_\phi^2(\omega) \rangle = \left \langle \frac{|\phi(I_1; \omega) - \phi(I_2; \omega)|^2}{(I_1 - I_2)^2} - \Delta I(\omega) \right \rangle. \] (6)

Although \( \sigma_\phi^2(\omega) \) is calculated in the same manner as a variance, it does not mean that \( \phi \), varies randomly as the intensity (or some other parameter) shifts. This \( \sigma_\phi^2(\omega) \) is simply a measure of how nonlinear the response of \( \phi \) is to pulse variation.

If this \( \sigma_\phi^2(\omega) \) is small (\( \sigma \ll \Delta_I \)), then the phase changes due to pulse fluctuations is predominately linear, and \( \Delta_I \) is a useful quantity. Note that the size of \( \sigma_\phi^2(\omega) \) is strongly dependent on the range in \( I \); if that range is altered then \( \sigma_\phi^2(\omega) \) should be recalculated.

C. Other measures of coherence

The measures I introduce above could not be easily measured in an experiment: although the spectral phase of pulses can be measured (e.g. [3]), doing this for each individual pulse in a train of (dissimilar) broadened pulses would be a rather challenging task. Here they are intended primarily as a theoretical construct useful for analyzing simulation results. In contrast, Dudley and Coen [3] have proposed a measure of “shot to shot” coherence \( g_{12}^{(1)}(\omega) \), which can be calculated by taking an ensemble average of pairs of results taken from a set of simulations. The measure is

\[ \left| g_{12}^{(1)}(\omega, t_1 - t_2) \right| = \frac{\langle E_1^*(\omega, t_1) E_2(\omega, t_2) \rangle}{\sqrt{\langle |E_1(\omega, t_1)|^2 \rangle \langle |E_2(\omega, t_2)|^2 \rangle}}. \] (7)

Although this is easy to calculate from a simulation, it includes contributions from intensity variation in the broadened pulses, a complication which we need to avoid in this work.

IV. SIMULATIONS

The simulations were done using the PSSD method[4, 10], which offers significant advantages over the traditional FDTD and Pseudospectral Time-Domain (PSTD) techniques (see e.g. [11]) for modeling the propagation and interaction of few-cycle pulses. Run times are generally faster, and the PSSD method also offers far greater flexibility in the handling of dispersion. Whereas FDTD & PSTD propagate fields \( E(z), H(z) \) forward in time, PSSD propagates fields \( E(t), H(t) \) forward in space. Under PSSD, the entire time-history (and therefore frequency content) of the pulse is known at any point in space, so arbitrary dispersion incurs no extra computational penalty. In contrast, the FDTD or PSTD approaches must use convolutions or time-response models for dispersion.

However, since \( z \)-propagated simulations do not handle either reflections or backward waves easily, we need to ensure we remain in the uni-directional propagation limit, where for an \( n \)-th order perturbative nonlinearity, \( n\chi^{(n)} E^{n-1} \) is small (see e.g. [12]). Note also that it is also possible to do these simulations using explicitly direction fields [13, 14, 15] or even wideband envelopes [15, 16].

I apply the PSSD algorithm to the source-free wave equation in non-magnetic media, with an instantaneous \( \chi^{(3)} \) nonlinearity, so that the equations for \( E \) and \( H \) in the 1D (plane wave) limit are

\[ \frac{dH_y(t; z)}{dz} = -\frac{d}{dt} [\epsilon_0 c \epsilon_r(t) \ast E_x(t; z) + \epsilon_0 \chi^{(3)} E_x(t; z)] \] (8)

\[ \frac{dE_x(t; z)}{dz} = -\frac{d}{dt} [\mu_0 H_y(t; z)], \] (9)

where the * denotes the convolution necessary to allow for the dispersion of the medium.

Typical array sizes used were \( N = 2^{15} \), with time resolution of 0.1fs (hence a time window of \( T = 3.27689ps \)). Spatial propagation steps were chosen to be \( dz = 0.4cT/N \approx 12\text{nm} \) in order to ensure numerical stability. The pulse profile used as an initial condition was

\[ E(t) = E_0 \sin(\omega_p t + \phi) \text{sech}(t/\tau). \] (10)

Pulses were propagated through 2mm of sapphire modeled using the dispersion parameters from DeFranzo & Pazol [17], and nonlinearity data from Major et al. [18]. The pulse wavelength was \( \lambda_p = 786\text{nm} \), which corresponds to \( \omega_p \approx 2.4 \times 10^{13}\text{rad/s} \). The reference peak intensity \( I_{ref} \) for the part of the pulse being SPM broadened was chosen to be compatible with the pulse energy of 4% of 1.5mJ (length 130fs) reported by Fang et al. [2], being \( I_{ref} = 0.33 \times 10^{14}\text{W/cm}^2 \). In [1], their OPA stage selected a signal wavelength \( \lambda_s \sim 1400\text{nm} \) by angle tuning, generating an idler \( \lambda_i \sim 1600\text{nm} \). Here, however, we simulate only the important SPM broadening stage.

It is worth noting that in these simulations, a transform-limited 130fs pulse is seen to be too narrowband to undergo sufficient broadening before multi-wave cross-talk destroyed the phase coherence. Consequently, we shortened our default pulse length to 30fs, as the simpler alternative to adding a strong chirp.

The simulations did not incorporate transverse effects in order to keep computation times down. Each 1D simulation through 2mm of sapphire takes about one hour on a 2.4GHz PC, so individual simulations involving transverse effects would be a rather challenging task. Here they also possible to do these simulations using explicitly direction fields [13, 14, 15] or even wideband envelopes [15, 16].

In conclusion, we address the concerns of this paper.
V. FLUCTUATIONS IN THE PULSE TRAIN

Some previous work\[^{19,20}\] showed how pulses broadened in experiments like those of Fang et al.\[^{1}\] might preserve useful phase information. However, these simulations used artificial nonlinear strengths and pulse widths to ensure the presence of a strong SPM lobe directly at the desired signal frequency in order to guarantee good performance. They also included the OPA stage, demonstrating that it could preserve the phase information supplied by the SPM lobe. These indicated a phase stability at the output of the OPA stage similar to that seen by Fang et al., even though the parameters used were not strictly comparable.

I now present simulation results that match the pulse intensity and material dispersion more closely. Fang et al. were unlikely to have been operating in the regime where a strong SPM lobe sat at their chosen signal frequency, since that was far into the wings of the broadened spectra. Unfortunately, the match is not perfect since the experimental parameters quoted by Fang et al. are insufficient to fully characterize a simulation. Fang et al. used a near-degenerate OPA setup, so were looking to broaden their input pulses from 2.4rad/fs all the way down to 1.2rad/fs; thus we are most interested in the results in the nearby frequency range $\omega \sim 1.3$ rad/fs.

It is therefore difficult to be sure whether a phase-stable train of broadened pulses will arise, since we need the wing of the broadened spectra to be stronger than the underlying background of complicated phasesensitive nonlinear processes. When doing the large set of simulations representing the variability of the input pulse train, three different pulse properties were altered in turn: intensity, pulse width, and CEP. We chose variations of the order of a few percent for intensity and pulse width: specifically a range of 5% in steps of 0.5%. For CEP, we obtained results for a $\pi/2$ range in CEP in steps of $\pi/16$, which provides results applicable for the full $2\pi$.

The spectral intensities at our chosen input intensities and propagation distances are shown on fig. 2. We see that the lowest input intensity pulse has only minimal power present in the range of interest, even for the full 2mm propagation, whereas the highest intensity pulse broadens rapidly. Further, we see (in the high intensity case) the usual series of SPM lobes pushing down to lower frequencies as the pulse propagates. We need to bear these spectra in mind when analyzing the following results, since a well behaved phase response will be of little practical use if those frequency components are undetectable. This caveat is particularly relevant in the lowest pulse intensity cases.

Note that in all the following figures, the central frequency of the input pulse is off the right hand side of the frame, since $\omega_p \simeq 2.4$rad/fs.

![FIG. 2: Spectral intensities for (a) $I_0 = I_{ref} = 0.33 \times 10^{14}$W/cm$^2$, (b) $I_0 = 3I_{ref}$; at (solid line, red, medium) 0.85mm, (dashed line, green, light) 1.43mm, (dot-dashed line, blue, dark) 2.00mm. Peak intensity values at $z = 0$mm are about $10^3$.](image)

### A. Intensity fluctuations

Pure SPM causes a phase shift proportional to intensity, but here I am instead interested in the unavoidable and potentially significant complications arising because the nonlinearity also includes THG effects.

Fig. 3 plots the logarithm of the phase variance $\sigma^2_I$ caused by intensity fluctuations, for three propagation distances at two different intensities. Linear phase response only holds to a level of 0.1rad/% input intensity fluctuations where the log-variance is less than $-2$. In these results, it is important to note that if the intensity fluctuates, the central frequencies of the SPM lobes also shift, and this can cause large changes in the phase variance $\sigma^2_I$.

First consider the results for the reference intensity $I_0 = I_{ref}$, fig. 3(a). By comparing the $\log_{10}\sigma^2_I$ curves at different distances, we see that the trend at higher frequencies is for the phase variances to decrease, a result of the SPM broadened spectral peak pushing outwards. In contrast, for lower frequencies the opposite trend occurs, because the gradually increasing background of phase-sensitive processes still tends to dominate the spectral wings of the pulse. Indeed, for the 2mm propagation distance, the variances have increased to the point where any hope of a linear phase response has been lost. Note
PHSPM Phase coherence in XSPM Dr. Paul.Kinsler@physics.org

FIG. 3: The effect of intensity fluctuations on the phase coherence measure for (a) \( I_0 = I_{\text{ref}} = 0.33 \times 10^{14} \text{ W/cm}^2 \), (c) \( I_0 = 3I_{\text{ref}} \); (solid line, red, medium) 0.85mm, (dashed line, green, light) 1.43mm, (dot-dashed line, blue, dark) 2.00mm. A value of \( \log(\sigma^2) = -2 \) corresponds to a nonlinear phase adjustment \( \delta I \sim \pm 0.1 \text{ rad}\% \). Width \( \tau = 30\text{ fs} \); \( \lambda_p = 786\text{ nm} \); \( \rightarrow \omega_p \approx 2.4\text{ rad/\text{fs}} \).

The spike in \( \sigma^2 \) for 1.3rad/fs at 1.43mm, this corresponds to the dip in intensity seen on fig. 2(a).

Similar trends are shown for the higher intensity \( I_0 = 3I_{\text{ref}} \), fig. 3(c), although the centre of the spectrum rapidly loses phase coherence, largely due to the strong SPM-induced spectral modulation. Note also the significant modulation of the variances caused by the SPM lobes in the intensity spectrum, which are clearly visible early in the propagation at 0.85mm.

For both intensities, there is a window of low phase variance at 1.43mm, although only for the \( I_0 = 3I_{\text{ref}} \) case is there also appreciable spectral intensity in this region. Therefore on fig. 4 we can plot \( \Delta I \) this \( (3I_{\text{ref}}) \) case. For an \( \omega_s \approx 1.3\text{ rad/\text{fs}} \), we see that \( \Delta I \approx -0.1 \text{ rad}/\% \), hence phase stability is maintained to less than 0.1rad if the intensity variation between the train of input pulses is less than 1%. However, in this particular case, the linear phase shift (for a 1% variation) is of the same order as the nonlinear contribution.

B. Width fluctuations

Fig. 5 shows the effect of pulse width fluctuation on the output phase response. Since the width fluctuations correspond to bandwidth fluctuations, they also cause different amounts of SPM-induced broadening, and shifting SPM lobe positions; hence we see similar trends as for intensity fluctuations.

Again we see the spike at \( I_0 = I_{\text{ref}} \), 1.3rad/fs and 1.43mm, caused by the dip in the intensity spectra. Also, the variances tend to increase with propagation distance. Interestingly, the \( I_0 = 3I_{\text{ref}} \) variances are smaller than those for the lower intensity \( I_0 = I_{\text{ref}} \) case, giving an example of how the balance between phase-sensitive nonlinear effects and coherent (SPM) spectral broadening can change. More generally, however, variations of \( \approx 1\% \) in width or intensity have a comparable effect, although there are many differences of detail.

C. CEP fluctuations

The final type of pulse variation we consider is CEP fluctuations. This case differs markedly from those for variations in intensity or width, since those were dominated by the concomitant alterations to the SPM. In contrast, since SPM is CEP insensitive, CEP fluctuations can only act (at least initially) through the THG-like contribution to the nonlinearity.

The results in fig. 6 show that even for the high intensity \( (I_0 = 3I_{\text{ref}}) \) case, the response of the output spectral phase to the full range of CEP variation in the input pulse is remarkably linear. This emphasizes that the CEP stabilization scheme can work as expected, producing a (nearly) input-CEP independent result – as long as the intensity and other parameters are sufficiently well stabilized.

It is worth noting that although these results show that CEP effects are negligible in the regimes considered in this paper, we still see that the variances tend to increase for either longer propagation distances or for higher pulse intensities. Indeed, for the high intensity case at \( z = 2.00\text{mm} \), CEP fluctuations increase the log-variance to about \(-3\), and have some non-negligible ef-
The effect of pulse width fluctuations on the phase coherence measure. for (a) $I_0 = I_{ref} = 0.33 \times 10^{14} W/cm^2$, (c) $I_0 = 3I_{ref}$; (solid line, red, medium) 0.85mm, (dashed line, green, light) 1.43mm, (dot-dashed line, blue, dark) 2.00mm. A value of $\log(\sigma_\tau^2) = -2$ corresponds to a nonlinear phase adjustment $\delta_\tau \sim \pm 0.1$ rad / %. Average width $\tau = 30$fs; $\lambda_p = 786nm$; $\omega_p \approx 2.4rad/fs$.

The effect of input phase fluctuations on the phase coherence measure. for (a) $I_0 = I_{ref} = 0.33 \times 10^{14} W/cm^2$, (c) $I_0 = 3I_{ref}$; (solid line, red, medium) 0.85mm, (dashed line, green, light) 1.43mm, (dot-dashed line, blue, dark) 2.00mm. A value of $\log(\sigma_\phi^2) = -2$ corresponds to a nonlinear phase adjustment $\delta_\phi \sim \pm 0.1$ rad / %. Width $\tau = 30$fs; $\lambda_p = 786nm$; $\omega_p \approx 2.4rad/fs$.

Effect on the linear response to CEP fluctuations. Clearly, even a moderate further increase in intensity or propagation distance would move the propagation into a regime where the train of broadened pulses is no longer coherent, so that the Fang et al. scheme could no longer generate phase stabilized pulse trains.

VI. DISCUSSION

This analysis has characterized the significance of CEP sensitive processes in the pulse broadening process. In particular, $\sigma^2$ characterises the strength of these processes, and highlights important features of extreme-SPM propagation. However, since the usual expectation is that the CEP sensitive THG terms have minimal effect and the SPM term is dominant, it is worth considering why the results here do not always return an unambiguously linear phase response.

Firstly, since the desired signal frequency will be in the wings of the broadened spectrum, small nonlinear effects can easily be significant. Secondly, the THG term applies to the entire spectrum, including its low frequency wing. Both these contributions are enhanced by long propagation distances, which provide more than enough opportunity for CEP sensitive effects to accumulate, and then fold themselves back in to the propagating pulse.

One might wish to compare simulations using a full $\chi^{(3)}E^3$ approach against an SPM-only model. Although non-trivial in standard PSSD, it is possible to use wideband envelope techniques [13, 14, 15, 16] which allow the $\chi^{(3)}E^3$ term to be split efficiently into SPM and THG parts. However, this introduces a non-physical dispersion that averages over carrier cycles, which removes the apparent value of such comparisons. A more physical solution is to alter the dispersion above (e.g.) $\omega \approx 2\omega_0$ to guarantee that there is no significant THG from the bulk of the pulse; however this leaves in place CEP sensitive THG from the low-frequency wing of the pulse spectra. Consequently it is not clear how to compare SPM and THG effects in a way that makes physical sense.

VII. CONCLUSIONS

In this paper I have proposed measures to assess the phase coherence of a pulse train subject to propagation through a nonlinear dispersive medium. I then used these to numerically investigate how pulse trains broadened by extreme SPM could retain pulse-to-pulse phase coher-
ence, as required by the Fang et al. scheme. The $\Delta_I$ measure allows us to set tolerances in intensity fluctuations that guarantee a level of output stability, and the $\sigma_I^2$ measure tells us how reliable those tolerances will be. The response to CEP ($\phi \to \Delta \phi, \sigma_{\phi}^2$) and pulse width ($\tau \to \Delta \tau, \sigma_{\tau}^2$) fluctuations was also investigated. Note that such trains of broadened pulses retain remarkably good phase coherence in response to CEP fluctuations, but that there still remains an underlying sensitivity to CEP changes. Pulse with a wider bandwidth give a more robust spectral phase, since extra bandwidth allows for rapid broadening to the desired spectral range before multi-wave cross-talk can degrade (or destroys) the phase coherence.

Simulations of the type used in this paper along with the $\sigma^2$ measures can provide useful insight into experimental design. However, the simulation results presented here contain a great deal of fine detail, so such simulations need to be carefully customized to the desired experimental parameters. This also suggests the potential for obtaining improved results in experiment by making rather small (but very specific) changes to the operating parameters.

Lastly, the dominant process causing the loss of phase coherence in the broadened pulse train is intensity fluctuations in the input; varying the CEP of each pulse in the input train generally has a much smaller effect.

Acknowledgments

I acknowledge many useful discussions with G.H.C. New, S.B.P. Radnor, and J.C.A. Tyrrell, as well as financial support from the EPSRC.

[1] X. Fang, T. Kobayashi, Opt.Lett. 29, 1282 (2004); erratum 29, 2932 (2004).
[2] A. Baltuska, T. Fuji, T. Kobayashi, Phys. Rev. Lett. 88, 133901 (2002).
[3] A. Baltuska, M. Uiberacker, E. Goulielmakis, R. Kienberger, V. S. Yakovlev, T. Udem, T. W. Hnsch, F. Krausz, IEEE J. Sel. Top. Quantum Electron. 9, 972 (2003).
[4] C. Li, E. Moon, H. Wang, H. Mashiko, C.M. Nakamura, J. Tackett, Z. Chang Opt. Lett. 32, 796 (2007).
[5] J.C.A. Tyrrell, P. Kinsler, G.H.C. New, J. Mod. Opt. 52, 973 (2005).
[6] P. Kinsler, G.H.C. New, J.C.A. Tyrrell, \texttt{arXiv:physics/061213}
[7] C. Li, E. Moon, H. Wang, H. Mashiko, C. M. Nakamura, J. Tackett, Z. Chang, Opt. Lett. 32, 796-798 (2007); also erratum on page 2586.
[8] T. Kobayashi, A. Shirakawa, T. Fuji, IEEE Quant. Elec. 7, 525 (2001).
[9] J.M. Dudley, S. Cohen, Opt. Lett. 27, 1180 (2002).
[10] B. Fornberg, “A Practical Guide to Pseudospectral Methods”, Cambridge University Press (1996).
[11] L. Gilles, S.C. Hagness, L. Vázquez, J. Comp.Phys. 161, 379 (2000).
[12] P. Kinsler, J. Opt. Soc. Am. B 24, 2363 (2007); also \texttt{arXiv:0707.0986}
[13] P. Kinsler, S.B.P. Radnor, G.H.C. New, Phys. Rev. A. 72, 063807 (2005).
[14] M. Kolesik, J.V. Moloney, Phys. Rev. E 72, 036604 (2004).
[15] P. Kinsler, “Pulse propagation methods in nonlinear optics” \texttt{arXiv:0707.0982} (2007).
[16] G. Genty, P. Kinsler, B. Kibler, J.M. Dudley, Opt. Express 15, 5382-5387 (2007).
[17] A.C. DeFranzo, B.G. Pazol, Appl. Opt. 32, 2224 (1993); (EFG Sapphire, at 295K).
[18] A. Major, F. Yoshino, I. Nikolakakos, J.S. Aitchison, P.W.E. Smith, Opt. Lett. 29, 602 (2004).
[19] J.C.A. Tyrrell, “Computational nonlinear optics beyond the envelope approximation”, (PhD thesis, Imperial College London, London, 2007).
[20] P. Kinsler, S.B.P. Radnor, J.C.A. Tyrrell, G.H.C. New, “Phase Retention in SPM Superbroadened Pulses”, CLEO 2006, presentation CW01; see following appendix or URL: http://www.opticsinfobase.org/abstract.cfm?id=100491
[21] P.M. Goorjian, S.T. Cundiff, Opt. Lett. 29, 1363 (2004).
Appendix: CLEO’06 summary, “Phase Retention in SPM Super-broadened Pulses”

P. Kinsler, S.B. Radnor, J.C.A. Tyrrell, G.H.C. New,

Pulses that have been super-broadened by a third-order nonlinearity are frequently used when generating a phase stabilized output. But how can such temporally mangled pulses retain any useful phase information?

We analyse recent experiments (such as [1, 2]) in which phase-stabilized idler pulses are generated in an optical parametric amplifier (OPA) by combining a pump pulse with its super-broadened replica. The success of this self-stabilization experiment raises interesting questions about how the severely distorted temporal profile of the super-broadened signal can contain sufficient sensible phase information to achieve the desired result. Our results demonstrate the mechanism by which phase order survives amid the apparent temporal chaos of the pulse’s electric field $E_{\text{pulse}}$. Our analysis is informed by numerical simulations using our recently-developed Pseudo-Spectral Space Domain (PSSD) technique [5], and some results from one instructive example are shown in fig. 7.

In traditional pulse propagation models based on the complex envelope $A(t)$, the nonlinear interaction term governing SPM appears as $\chi^{(3)}|A|^2 A$. This is linear in the phase, and might therefore be expected to be relatively well-behaved. However, one cannot model super-broadened pulses using envelopes, even with the various corrections to envelope propagation that are available. One must therefore resort to a model capable of evolving the carrier oscillations directly, either the well known FDTD, or other methods like PSSD [5] or $G^3$ variables [13]. Given that the form of the nonlinear interaction is now $\chi^{(3)} E^3$, where $E(t)$ is the complete field, it becomes unclear how the multiplicity of potential three-wave interactions could generate and retain the necessary phase information. The crucial questions we address are:

• How does phase information survive in the presence of severe self-phase modulation and self-steepening?
• Given that it does survive, how sensitive is the process to intensity and phase fluctuations in the pump pulse?
• How does the number of cycles in the pulse affect the behaviour?

Our simulations show firstly that a theoretical model of the SPM-OPA system described in [1, 2] can reproduce the character of the observed phase stability. Close analysis of the results shows that, despite the mangled appearance of the super-broadened pulse, some parts of it retain an ordered (phase) structure, and it is these sections that are used in the experiment (e.g. the 1st lobe in fig. 7).

The investigation has also revealed more general rules about the retention of phase information in strong SPM. The simulations have also enabled us to quantify the sensitivity of the phase-stabilisation process to fluctuations in the pump intensity, a feature that bears directly on the viability of the technique in the laboratory. The pump laser used in [1, 2] has an intensity stability to within 1%, leading to absolute phase fluctuations of $\sim 0.1\pi$ radians. This compares remarkably well with our predicted 0.237 rad (see fig. 5), especially given that our model parameters do not closely match the experiment. Of course this match occurs because the phase stability (w.r.t. intensity) is still linear, and proportional to the broadening of the spectral component phase-matched to the signal frequency selected by the set-up of the OPA stage. Since these features are common to both our model and the experiment, the other differences of detail do little to upset the comparison.

Lastly, we investigate the phase stability in the limit

\[ \chi^{(3)}(E)^3 \]

FIG. 7: Pulses at 786nm super-broadened in a sapphire-like medium to generate a lowest-frequency (1st) spectral lobe at $\sim 1200$nm to be used in the OPA stage of a phase stabilization experiment [1, 2]. TOP: Starting at the bottom, this frame shows the pulse’s electric field ($E_{\text{pulse}}$), a time-domain reconstruction of the electric field using just only the frequency content of this 1st lobe (i.e. $E_{L1}$), and a similar reconstruction ($E_{L2}$) from just the second lobe. The upper curves are the phase structure $\phi_{L1}$ and $\phi_{L2}$ first and second spectral lobes respectively. BOTTOM: Spectra of 30fs (top) and 6fs (bottom) pulses at 786nm after similar amounts of spectral broadening, which require different propagation lengths. The nearly identical full and dotted lines compare the spectra differing by the influence of a 5% intensity variation. The other (offset) full lines shows the difference between those very similar pairs of spectra at the two intensities.
of very few cycle pulses. There are two effects here that can shift the phase variation out of the linear regime seen for many cycle pulses. The first is the nonlinear response time, as modelled in [21], who used $\chi^{(3)}$ with a 1fs response time; however, note that recent work [18] gives a faster response (and thus will lead to smaller phase corrections). The second is the strong non-SPM effects from the $\chi^{(3)}$ interaction. We show how these two nonlinear corrections to the phase properties affect the phase stability in the few-cycle limit, and thus how they might affect experiments such as [1, 2] if pushed to those extreme limits.

FIG. 8: Phase stability as a function of percentage intensity variation for an initial 30fs pulse, as per fig. 7. The $\phi_{SPM}$ data is extracted from the 1st spectral lobe after the SPM broadening process. The $\phi_{idler}$ data uses the original pump pulse and the SPM broadened copy in an OPA stage to generate a phase-stabilized idler.