Modeling of nonlinear properties of casing centralizers equipped with axial thrust

I Shatskyi1, A Velychkovych2, I Vytvytskyi3, and M Senyushkovych3

1Ivano-Frankivsk Branch of Pidstryhach-Institute for Applied Problems in Mechanics and Mathematics, NAS of Ukraine, Department of modelling of damping systems, Mykytynetska 3, Ivano-Frankivsk, Ukraine
2Ivano-Frankivsk National Technical University of Oil and Gas, Department of Construction and Civil Engineering, Karpateka 15; Ivano-Frankivsk, Ukraine
3Ivano-Frankivsk National Technical University of Oil and Gas, Department of Drilling, Karpateka 15; Ivano-Frankivsk, Ukraine

E-mail: ipshatsky@gmail.com, a_velychkovych@ukr.net

Abstract. The issue of qualitative centering of the casing in wells with complex trajectory is considered. When choosing an effective centralizer, two competing requirements are simultaneously assigned to its mechanical properties. The centralizer must have sufficiently low stiffness in order to provide the descent of the casing into wellbore. At the same time, the centralizer must have high stiffness in order to provide proper clearance gap for high-quality cementing of the annular space. Several new designs of rod centralizers are proposed in the paper. Their main feature is that they are equipped with an additional thrust ring, which allows one to change the mechanical characteristics of the centralizer at high contact loadings. The boundary-value problems for the differential equations of mechanics of the arc rods with boundary conditions in the form of inequalities are formulated and solved. Analytical relations between the clamping force and mutual approach of the casing and the well wall, which characterize the radial stiffness of the centralizer equipped with axial thrust, are determined. It is established that the described designs of the centralizers have essentially nonlinear characteristics that contribute to the efficient centering of the casing. At vertical and slightly deviated from the vertical intervals of the wellbore, the centralizer behaves as soft and does not interfere with the descent of the string; at intervals with large curvature of the wellbore the stiffness of the centralizer increases due to the axial thrusts.

1. Introduction
A careful centering of the casing is the key factor providing a high quality well cementing. Equipping of casing with special centering devices is intended to ensure its equidistance from the walls of the well and thereby improve the quality of cementing of the annular space. The deviation of the casing from the correct trajectory depends on a number of factors: the size of the well, the size of the casing, the type and properties of centralizers, their number and location in the well, etc. One of the actual tasks that petroleum engineers and well completion specialists are required to perform on a regular basis is to choose the optimum type and number of centralizers for each casing. Today, service companies have a wide range of technical solutions to improve the reliability of casing strings [1–4], however the problem of effective casing centering under the complex configuration of the well axis remains relevant. This problem is especially important because of the increase in the volume of...
directional and horizontal drilling in the world, the development of modern technologies for the re-entry of abandoned and emergency wells, the predrilling of additional lateral wellbores through the casing windows, etc.

Today, elastic rod centralizers [5, 6] are the most used in the practice of well drilling. To evaluate the efficiency of centering devices of this type, to determine their required number and locations, it is recommended to investigate the mechanical properties of the centering devices and their mechanic behavior under contact loading. In particular, the theoretical analysis of the stiffness and strength characteristics will prevent from their excessive consuming or their insufficient use in the centering interval.

The performance criterion used to evaluate the quality of the centralizers is their ability to concentrically place the casing in the well at any position on its axis. There is a problem in choosing an effective centralizer because its two properties are simultaneously competing with each other. The centralizer must have a sufficiently low stiffness not to slow down the descent of casing in the well. At the same time, the centralizer must have a sufficiently high stiffness to provide the necessary clearance for a high-quality cementing of the annular space [6–8]. In previous studies, the authors tried to solve this problem by giving the centralizer an elastic-rigid characteristic by using radial rigid thrusts [6]. In this work, we aim to evaluate the effects of installing axial thrusts in cast and welded rod centralizers. Such additional thrusts give the non-linear characteristics to the centralizers to allow for the descent of the casing and good centering ability of the devices at high clamping forces.

2. Two variants of the centralizer design with axial thrust
To solve the described problem, the authors propose to use the centralizer designs with variable stiffness (Fig. 1). Both centralizers have a rigid connection of the lower ends of the elastic bows 2 with the fixed rings 1. In the cast centralizer (a) the upper ends of the elastic bows are free, and at some distance from them the centralizer is equipped with a rigid thrust 4. In the welded centralizer (b) the upper ends of the elastic bows are rigidly connected to the axially movable ring 3. At a certain distance ∆ from the movable ring of the welded centralizer there is a restriction in the form of a rigid thrust 4.

![Figure 1](image_url)

**Figure 1.** The cast (a) and welded (b) centralizer with an additional axial thrust:
1 – fixed centralizer ring, 2 – elastic bow, 3 – moving centralizer ring, 4 – stiff axial thrust, 5 – casing.
The presence of axial thrusts is intended to constructively ensure the nonlinearity of the elastic characteristics of the centralizers. As long as the rigid thrust is not engaged in the centralizer’s operation, the radial stiffness of the device is relatively small. When the distance $\Delta$ is out and the thrust comes into play, the stiffness of the centralizer increases.

3. Results of mathematical modeling and analysis

The problems of the interaction of pipe strings with the wall of a well have been the subject of research in [9–15], and more general issues about the interaction of shells with elastic bodies have been considered, in particular, in publications [16–18]. Analytical and numerical approaches to engineering modeling and study of contact phenomena in shell and rod systems using classical theories of rods and cut shells have been proposed in publications [19–24]. In particular, some contact problems for cyclic rod structures were studied in [25–27]. The authors [28] developed a computer model for centering long objects. Experimental and numerical-analytical studies [29, 30] concern the analysis of mechanical properties of centering devices. Analytical models for investigating the contact interaction of elastic linkage of centralizers with well walls are given in [31], where six variants of attaching a centralizer to the casing are considered. In the current paper, we develop a technique for calculating the stiffness of cast and welded centralizers with a thrust, which was proposed earlier in a paper [32] for a hinged centralizer.

For simulation, we used the classical linear theory of hollow arch-shaped rods and consider the deformation of a single link loaded by the contact force (Fig. 2).

![Figure 2. Schemes of contact interaction of the centralizer link with the well wall: (a) – cast centralizer, (b) – welded centralizer.](image)

Let $f$ be the lifting arm of the centralizer’s link, $2l$ is the length of the rod projection on the axis of abscissa, $2\alpha$ is the small slope of the arc and $R$ is the radius of its curvature. The rod is considered flat, it means we assume that the inequilities are realized:

$$\left(\frac{f}{l}\right)^2 << 1, \quad 1/R \approx 2f/l^2, \quad \alpha \approx 2f/l.$$  

The equilibrium equations and physical relationships of the model are as follows [33]:

$$\frac{dN}{dx} = 0, \quad \frac{dQ}{dx} + \frac{N}{R} = -P\delta(x), \quad \frac{dM}{dx} - Q = 0, \quad x \in (-l, l);$$ \hspace{0.5cm} (1)

$$\frac{du}{dx} + \frac{w}{R} = 0, \quad M = EJ\frac{d^2u}{dx^2} = EJ\frac{d^3w}{dx^3}, \quad x \in (-l, l).$$ \hspace{0.5cm} (2)
Here $N$, $Q$ are the axial transverse forces, $M$ is bending moment, $P$ is the contact force, $EJ$ is stiffness of the rod to the bend, $u, w$ are tangential and transverse displacements of the rod, $\vartheta$ is the rotation angle, $x$ is the coordinate, and $\delta(x)$ is Dirac function.

The boundary conditions for fastening the rod are as follows.
The anchorage conditions for the left end ($x = -l$)
\[
 u_x(-l) = u(-l) - w(-l)\alpha = 0, \quad u_y(-l) = w(-l) + u(-l)\alpha = 0, \quad \vartheta(-l) = 0,
\]
and the axial and radial grip conditions of the right end ($x = l$)
\[
 N_x(l) = N(l) - Q(l)\alpha = 0, \quad u_x(l) = u(l) + w(l)\alpha \leq \Delta \text{ or } u_x(l) = \Delta, \quad N_x(l) < 0,
\]
\[
 u_y(l) = w(l) - u(l)\alpha = 0,
\]
are common for both schemes (a) and (b).

At the same time, the constructions under consideration are notable for the freedom of rotation of the cross-section at the right end of the rod, in particular, it is allowed to rotate in case (a)
\[
 M(l) = 0,
\]
and prohibited in case (b)
\[
 \vartheta(l) = 0.
\]

Thus, Eqs. (1)–(6) and Eqs. (1)–(5), (7) are the boundary problems for the system of differential equations of the 6th order with boundary conditions in the form of inequalities corresponding to one-sided joints for the constructions (a) and (b) respectively.

The solution to these problems $z(x) = (N, Q, M, \vartheta, w, u)^T$ are found in the form:
\[
z(x) = \begin{cases} 
  P_1 z^{(1)}(x), & P < P_1; \\
  P_2 z^{(1)}(x) + (P - P_1)z^{(2)}(x), & P > P_2 
 \end{cases} = P_1 z^{(1)}(x) + (P - P_1)H(P - P_2)(z^{(2)}(x) - z^{(1)}(x)).
\]

Here $H(x)$ is the Heveside’s function, and $P_c$ is the value of the contact force, for which the axial gap $\Delta$ is out.

Moreover, $z^{(i)}(x) = (N^{(i)}, Q^{(i)}, M^{(i)}, \vartheta^{(i)}, w^{(i)}, u^{(i)})^T$ are the solutions of two auxiliary problems:

$i = 1$ – the problem of the action of a single concentrated force $P = 1$ on an arch with a axially-movable fixing (condition in Eq. (4) should be replaced by $N_x(l) = 0$); $i = 2$ – the problem of the action of a single concentrated force on an arch with a axially-immovable fixing (it is necessary to take the condition $u_x(l) = 0$ in Eq. (4)).

Using the results of [31], we decode the components of vectors $z^{(1)}(x)$ and $z^{(2)}(x)$. In particular, the consistent displacements are:

for the scheme (a)
\[
 w^{(1)}(x) = \frac{1 \cdot l^3}{EJ} \left( \frac{3}{8} \frac{(x + l)^2}{2l^2} + \frac{11}{16} \frac{(x + l)^3}{3l^3} - \frac{x^3}{3l^3} H(x) \right),
\]
\[
 u^{(1)}(x) = \frac{1 \cdot 2l^2 f}{EJ} \left( \frac{3}{8} \frac{(x + l)^3}{3l^3} - \frac{11}{16} \frac{(x + l)^4}{4l^4} + \frac{x^4}{4l^4} H(x) \right);
\]
w^{(2)}(x) = \frac{1}{EJ} \left( \frac{1}{24} \frac{(x+l)^2}{4l^2} - \frac{17}{48} \frac{(x+l)^3}{3l^3} + \frac{5}{6} \frac{(x+l)^4}{4l^4} - \frac{x^3}{3l^3} H(x) \right),

u^{(2)}(x) = \frac{1}{EJ} \left( -\frac{1}{24} \frac{(x+l)^3}{3l^3} + \frac{17}{48} \frac{(x+l)^4}{4l^4} + \frac{5}{6} \frac{(x+l)^5}{5l^5} - \frac{x^4}{4l^4} H(x) \right); \quad (10)

and for the scheme (b)

w^{(1)}(x) = \frac{1}{EJ} \left( -\frac{1}{4} \frac{(x+l)^2}{2l^2} + \frac{1}{2} \frac{(x+l)^3}{3l^3} - \frac{x^3}{3l^3} H(x) \right),

u^{(1)}(x) = \frac{2l^2 f}{EJ} \left( \frac{1}{4} \frac{(x+l)^3}{3l^3} - \frac{1}{2} \frac{(x+l)^4}{4l^4} + \frac{x^4}{4l^4} H(x) \right); \quad (11)

w^{(2)}(x) = \frac{1}{EJ} \left( \frac{1}{16} \frac{(x+l)^2}{2l^2} - \frac{7}{16} \frac{(x+l)^3}{3l^3} + \frac{15}{16} \frac{(x+l)^4}{4l^4} - \frac{x^3}{3l^3} H(x) \right),

u^{(2)}(x) = \frac{2l^2 f}{EJ} \left( -\frac{1}{16} \frac{(x+l)^3}{3l^3} + \frac{7}{16} \frac{(x+l)^4}{4l^4} - \frac{15}{16} \frac{(x+l)^5}{5l^5} + \frac{x^4}{4l^4} H(x) \right). \quad (12)

We evaluate the stiffness of the considering structures by analyzing the axial displacement of the right end of the rod $\Delta_x = u_x(l) = u(l) + \Delta_x l$, and radial approach of the string with the well wall $\Delta_y = -w(0)$. In accordance with the expressions in Eqs. (8)–(12) we have:

$$
\Delta_x = \frac{P}{C_x^{(1)}} + (P - P_s)H(P - P_s) \left( \frac{1}{C_x^{(2)}} - \frac{1}{C_x^{(1)}} \right),
$$

$$
\Delta_y = \frac{P}{C_y^{(1)}} + (P - P_s)H(P - P_s) \left( \frac{1}{C_y^{(2)}} - \frac{1}{C_y^{(1)}} \right), \quad (13)
$$

where $P_s = C_x^{(1)} \Delta$, and stiffness coefficients are

$$
C_x^{(1)} = 6 \frac{EJ}{f l^3}, \quad C_x^{(2)} = \infty,
$$

$$
C_y^{(1)} = \frac{96}{7} \frac{EJ}{l^3}, \quad C_y^{(2)} = \frac{288}{l^3} \frac{EJ}{l^3},
$$

as well as

$$
C_x^{(1)} = 12 \frac{EJ}{f l^3}, \quad C_x^{(2)} = \infty,
$$

$$
C_y^{(1)} = 24 \frac{EJ}{l^3}, \quad C_y^{(2)} = \frac{384}{l^3} \frac{EJ}{l^3},
$$

in the cases (a) and (b) respectively.
The results obtained by Eq. (13) are illustrated in Fig. 3. For calculations we considered: \( l/f = 10 \) and \( C = EJ/l^3 \).

![Loading diagrams for centralizers with thrust](image)

**Figure 3.** Loading diagrams for centralizers with thrust: solid lines – cast centralizer (a), dashed lines – welded centralizer (b)

The kinks of the lines in the plots are due to changing boundary conditions in Eq. (4). The ordinates of these kinks are consistent with normalized values \( P_c \).

4. Conclusion

Some new designs of a rod centralizer of a casing column are offered. The key feature of those designs is that, an additional axial thrust ring which allows one to operate the device at large contact loads is presented.

We are formulated and solved the boundary-value problems for the differential equations of mechanics of the arc rods with boundary conditions in the form of inequalities are formulated and solved. The designed analytical models with a high degree of adequacy describe the autonomous operation of links of cast centralizer (a) for all contact loadings, as well as the operation of links of welded centralizer (b) with limited axial movement. In the case of low load, the calculation according to scheme (b) should be defined more precisely taking into account the combined work of all the rods connected into the cyclic system via the ring.

Both described designs of the centralizers equipped with axial thrust have essentially nonlinear characteristics, which are regulated by the choice of distance \( \Delta \). This nonlinearity facilitates the attainment of the aim of effective centering of the casing. At slightly deviated domains of the well with small clamping forces, the centralizer behaves like a soft one and does not impede the descent of the column; in areas with a large curvature the centralizer becomes stiff due to the thrust. In both cases, the radial clearance gap required for high-quality cementing is provided. So the goal of our study has been achieved.

References

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