Distribution amplitudes of light mesons and photon in the instanton model

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The leading- and higher-twist distribution amplitudes of pion, ρ-meson and real and virtual photons are analyzed in the instanton liquid model.

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I. INTRODUCTION

 Investigations of hard exclusive processes are essential for our understanding of the internal quark-gluon dynamics of hadrons. Theoretically, such studies are based on the assumption of factorization of dynamics at long and short distances. The short-distance physics is well elaborated by perturbative methods of QCD and depends on particular hard subprocesses. The long-distance dynamics is essentially nonperturbative and within the factorization formalism becomes parametrized in terms of hadronic distribution amplitudes (DAs). These nonperturbative quantities are universal and are defined as vacuum-to-hadron matrix elements of particular nonlocal light-cone quark or quark-gluon operators. The evolution of DAs at sufficiently large virtuality $q^2$ is controlled by the renormalization scale dependence of the quark bilinear operators within the QCD perturbation theory. For leading-order DAs this dependence is governed by QCD evolution equations. When the normalization scale goes to infinity the DAs reach an ultraviolet fixed point and are uniquely determined by perturbative QCD. However, the derivation of the DAs themselves at an initial scale $\mu_0$ from first principles is a nonperturbative problem and remains a serious challenge.

Here we present the results [1, 2] of study of the pion, ρ-meson and photon DAs in the leading and higher twists at a low-momentum renormalization scale in the gauged non-local chiral quark model [3, 4, 5] based on the instanton picture of QCD vacuum.

II. DEFINITIONS AND NOTATIONS

The distribution amplitudes of the mesons or the photon are defined via the matrix elements of quark-antiquark bilinear operators taken between the vacuum and the hadronic state $|h(q)\rangle$ of momentum $q$. It is assumed that the quark and antiquark are separated by the distance $2z$ and the light-like limit $z^2 \to 0$ is taken at a fixed scalar product $q \cdot z$. We use the light-cone expansion of the matrix elements in order to define the DAs [2] (only leading twist terms are presented)

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5[z,-z]u(-z)|\pi^+(q)\rangle = i\sqrt{2}\bar{q}q_\mu\int_0^1 dx e^{-ixz}\phi^\lambda_{\pi^+}(x),$$ (1)

$$\langle 0|\bar{q}(z)\gamma_\mu[z,-z]q(-z)|\pi^+(q)\rangle = ie_q\langle 0|\bar{q}q|0\rangle \chi_{\pi^+} \cdot z,$$ (2)

$$f_{\perp}(q^2) = \int_0^1 dx e^{-ixz}\phi_{\perp}(x, q^2),$$

$$\langle 0|\bar{q}(z)\gamma_\mu[z,-z]q(-z)|\gamma^\lambda(q)\rangle = e_q f_3\gamma_{\perp}(q^2) q_\mu,$$ (3)

$$\frac{e^{\lambda}(\xi)}{q \cdot z} \int_0^1 dx e^{-ixz}\phi_{\perp}^{\gamma\lambda}(x, q^2),$$

where $f_\pi$ is the pion decay constant, $\langle 0|\bar{q}q|0\rangle$ is the quark condensate, $\chi_{\pi^+}$ is the magnetic susceptibility of the quark condensate, and $f_3$ is related to the first moment of the magnetic susceptibility. The symbol $[z,-z]$ in the matrix elements denotes the path-ordered gauge link (Wilson line) for the gluon fields between the points $-z$ and $z$. The integration variable $x$ corresponds to the momentum fraction carried by the quark and $\xi = 2x - 1$ for the short-hand notation. For a real photon, due to condition $e^{\lambda}(\xi) \cdot z = 0$, the structure corresponding to $\phi_{\perp}^{\gamma\lambda}$ decouples. The DAs $\phi_{\perp}(x)$ and $\phi_{\perp}^{\gamma\lambda}(x)$ for the ρ-meson state $|\rho^\lambda(q)\rangle$ are defined in analogy with photon case [2] and with mass-shell condition $q^2 = -M_\rho^2$.

III. INSTANTON-MOTIVATED NONLOCAL CHIRAL QUARK MODEL

In the one loop approximation the quark model evaluation of the distribution function $\phi_{h,J}(x)$ of hadron $h$ corresponding to projection $J$ is given schematically as [13]

$$N_{h,J}\phi_{h,J}(x) = -iN_c \int d^4k \delta(k \cdot n - x) \text{Tr}[[\Gamma_h S(k) \Gamma_h S(k-q)]],$$ (4)

where the quark propagator has the form

$$S(p) = \frac{1}{p - M(p) + ie}, \quad M(p) = M_0 f^2(p^2),$$ (5)

with the dynamical quark mass $M(p)$ expressed via the function $f(p)$ defining the nonlocal properties of the QCD vacuum [14]. $\Gamma_h$ are the vertices defining the hadron state

$$\Gamma_{\pi}(k,k') = \gamma_\mu f(k) f(k'), \quad \Gamma_{\rho}(k,k') = \gamma_\mu f_3 f(k) f_3(k'),$$ (6)

$$\Gamma_{\gamma}(k,k') = \gamma_\mu - (k+k')_\mu M_{k,k'}^{(1)}.$$
and $\Gamma_f$ is the projection operator corresponding to a definite twist. Here and below, the notation

$$M^{(1)}(k, k') = \frac{M(k) - M(k')}{k^2 - k'^2}$$

is used. The nonlocal functions are chosen in gaussian form

$$f(p) = f_V(p) = \exp\left(-\frac{p^2}{\Lambda^2}\right),$$

with $p$ denoting the Euclidean momentum, resembling the fact that the instanton field is convenient to take in the axial gauge. As the model parameters we take the values fixed in\cite{12}

$$M_0 = 240 \text{ MeV}, \quad \Lambda = 1110 \text{ MeV}. \quad (8)$$

The distribution amplitudes of the pion and the real photon calculated in the instanton model in the chiral limit may be cast in a closed form. It is convenient to introduce notations for the integration variables ($\mathbf{\tau} = 1 - x$)

$$u_+ = u - i\lambda x, \quad u_- = u + i\lambda x, \quad M_\pm = M(u_\pm),$$

$$D(u) = u + M^2(u), \quad D_\pm = D(u_\pm).$$

Then one gets the expressions

$$\phi^A_\pi(x) = \frac{1}{f_\pi^2} \frac{N_c}{4\pi^2} \int_0^\infty du \int_{-\infty}^\infty d\lambda f_+ f_- (xM_+ + xM_-), \quad (9)$$

$$\phi_{\perp\gamma}(x, q^2 = 0) = \frac{1}{|\langle \bar{q}q \rangle \chi_\text{inst}|^2} \left[ \Theta(\mathbf{\tau}x) \int_0^\infty du \frac{M(u)}{D(u)} - \int_0^\infty du \int_{-\infty}^\infty d\lambda \frac{M_+ M_- M^{(1)}(u_+, u_-)}{2\pi D_+ D_-} \right]. \quad (10)$$

$$\phi_{\parallel\gamma}(x, q^2 = 0) = \Theta(\mathbf{\tau}x). \quad (11)$$

The DAs are scale dependent quantities and the above expressions correspond to the low momentum scale $\mu_0$ typical for the instanton model. For the instanton model it is estimated as $\mu_0 = 530 \text{ MeV} \cite{11}$. The parameters entering normalization coefficients are given by

$$\langle 0 | \bar{q}q | 0 \rangle|_{1 \text{ GeV}} = -0.24 \text{ GeV}^3, \quad \chi_\text{inst}|_{1 \text{ GeV}} = 2.73 \text{ GeV}^{-2}. \quad (12)$$

The results of calculations are shown in Figs. 1-4. They correspond to low momentum scale $\mu_0$ and need to be evolved to higher momenta scale in order to compare with experimentally available information. The DA at asymptotic scales $\mu_{\text{as}} = \infty$ is also presented.

IV. DISCUSSION

Within the instanton model the leading twist pion DA has been found in\cite{8, 9, 10}. The twist-3 and twist-4 DAs have been found respectively in\cite{8} and\cite{11}. The photon DAs up to twist-4 expansion have been found in\cite{12}. Recently there have been published some papers\cite{13, 18, 19, 20} where in nonlocal models similar to considered above the distribution functions for pion and photon were treated inconsistently. Indeed, the
The corresponding values of the evolution ratio \( r \) defining the distribution functions has two parts. One is coming from soft hadronic vertices \( \gamma \) and another refers to the operators of definite twist which are responsible for the power corrections in the hard subprocess. It is well known that, for example, the leading twist operators entering the pion distribution amplitude and distribution function are given by \( \phi_\gamma \), \( \phi_{\gamma \gamma} \) and \( \phi_\gamma \), correspondingly. Nevertheless, the authors of \([17, 18, 19]\) included additional terms proportional to \( (k + k')_\mu \), where \( k \) and \( k' \) are incoming and outgoing momenta of a quark. These additions in \([17, 18, 19]\) modify the known results of the leading twist calculations. However, it is evident, see for example \([11]\), that these additional terms contribute to twist-4 distributions and do not touch the leading twist-2 distributions. From the other side, the full vertex including the local and nonlocal pieces is important in the soft hadronic part and has not to be neglected as it was done in \([10, 20]\) considering the photon distribution amplitudes.

V. CONCLUSIONS

The instanton model of QCD vacuum is realistic tool to get nonperturbative properties of hadrons in terms of parameters characterizing the vacuum. All hadron DAs are suppressed at the bound of kinematical interval due to localized wave function of hadrons, while photon DAs are not zero there. By applying the QCD evolution the photon DAs become immediately zero at the edge points of \( x \)-interval. Nevertheless, the photon DAs are always wider than asymptotic distribution. The first experimental results on direct measurements of pion and photon DAs are discussed in \([15, 16]\).

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