Light fermions in composite models

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Abstract

In preon models based on chiral gauge theories, we show that light composite fermions can ensue as a result of gauging a subset of preons in a vector-like manner. After demonstrating how this mechanism works in a toy example, we construct a one generation model of quarks which admits a hierarchy between the up and down quark masses as well as between these masses and the compositeness scale. In simple extensions of this model to more generations we discuss the challenges of obtaining any quark mixing. Some possible phenomenological implications of scenarios where quarks and leptons which are heavier are also less pointlike are also considered.
1 Introduction

The idea that quarks and leptons are composite (for a review, see ref.[1]) has been pursued as an explanation for the observed fermionic masses and mixings. There are many problems, however, met in implementing this idea. One of the main problems is to understand why quarks and leptons are so light, compared to the inverse of their 'size' set by the compositeness scale. In vector-like gauge theories like QCD, the masses of the composite states are of the same order as the compositeness scale. Because of this, QCD-like theories appear ill suited for building sensible composite models. It has been suggested [2] that perhaps in nature the masses of quarks and leptons are protected from being of the order of the compositeness scale by some approximate chiral symmetry. Again, this idea argues against vector-like theories, for we know that in these theories all global chiral symmetries are spontaneously broken.

In view of the above considerations, it appears that chiral gauge theories are much more natural theories to consider for preon models of quarks and leptons. With chiral gauge theories, however, one typically runs into a converse problem. Namely, unbroken chiral symmetries lead to massless fermions and it is difficult to break these symmetries just slightly, so as to make the resulting bound state fermion masses small but non-zero. In the literature models have been proposed which break the chiral symmetries of the preon theory by explicit mass terms and/or four-fermion interactions introduced at the fundamental level. Although some of these models are interesting, introducing such seed breaking undermines to a great extent the original motivation to make quarks and leptons composite.

One may arrive at the idea of compositeness from a completely different perspective. An attractive, and well-known, way of breaking the electroweak symmetry dynamically is provided by technicolor interactions [3]. However, technicolor interactions themselves cannot generate fermion masses and one is forced to introduce yet further interactions, extended technicolor (ETC), to accomplish this task [4]. If the known fermions and some technifermions were to be composite, the preonic theory would invariably produce some effective four-fermion interactions among these states. These interactions could hopefully serve as the seeds for fermion mass generation, without the need of having to introduce new ETC forces (for early work in this direction see, for example, ref.[5]). Thus, the idea of compositeness nicely combines with that of technicolor. It is, however, not clear a priori whether compositeness can
cure some of the familiar technicolor diseases, such as having light pseudoGoldstone bosons in the physical spectrum and sizable flavor-changing neutral currents (FCNC).

The objective of this paper is two-fold. First, we want to show that in composite models based on chiral gauge theories, it is possible in principle to generate light but not massless fermionic bound states. This mass generation does not necessitate bare preonic masses (which are, in fact, forbidden!) or non-renormalizable interactions introduced by hand, but it is entirely dynamical coming as a result of additional vector-like interactions acting on a subset of the preons. After demonstrating how this mechanism works in a simple context we broach the second objective of this paper, which is to study whether the observed mass pattern of quarks and leptons can be accounted for by such a scenario, perhaps by incorporating as well some version of the technicolor idea. Although we have not been totally successful in our second goal, the semirealistic model which we construct suggests interesting generic features which may have important phenomenological consequences.

The most challenging point in trying to construct a realistic model of this type is related to the issue of quark mixing. Because in our model preons corresponding to different generations carry different quantum numbers, it is not possible to introduce a Cabibbo-Kobayashi-Maskawa (CKM) matrix directly at the preonic level. Thus, the CKM matrix must ensue as a low-energy phenomenon and be in principle calculable in terms of the fundamental parameters of the theory. As will be seen, it is relatively easy to introduce a hierarchy of masses for the up- and down-type quarks. However, it is difficult to actually break all the vestiges of residual family symmetries in the model considered, so as to actually generate a CKM matrix. Nevertheless, if a non-trivial CKM matrix were to ensue, it is very natural in these scenarios that the concomitant FCNC would dominantly affect the heavy quark sector. Since FCNC effects involving heavy quarks are not thoroughly studied experimentally, these considerations suggest that the banishing of all FCNC may not necessarily be the most sensible strategy to adopt in model building. In this respect, our philosophy differs from that of recent attempts to incorporate FCNC suppressing mechanisms in composite and non-composite technicolor models.

In the analysis of composite models based on chiral gauge theories, which we will present, there are several dynamical assumptions involved. First, it will turn out that in the model including three generations (in which mixing, unfortunately,
is difficult to obtain), because of the plethora of fermionic species, the weak, color and technicolor interactions are not asymptotically free. Thus, although the weak and strong couplings behave exactly as in the standard model at low energy, they start to grow above energies of around 1 TeV when technifermions become relevant. Asymptotically non-free non-abelian gauge theories have been invoked previously [8] in the context of extended technicolor in attempts to alter the naive relation between the ETC scale and fermionic masses and thus suppress FCNC effects. We have nothing to add here as far as the dynamics of such theories is concerned, nor do we rely on this FCNC suppression in our further discussion. We merely will assume that such theories can be made consistent, at least in the presence of a cutoff, and that for vector-like theories chirality is spontaneously broken, essentially in the same way as it happens in ordinary QCD. We note, however, that in the one-generation model described in Sect.3 color and technicolor are asymptotically free.

For the analysis of the vector-like pieces of our models a second set of dynamical assumptions enters. If all the preons were massive we could use mass inequalities [9] to argue that the vectorial global symmetries are preserved by vector-like gauge theories, while chiral global symmetries are spontaneously broken. It then would follow that in the limiting case when the bare masses are taken to zero, the vacuum with unbroken vectorial symmetries and broken chiral ones either remains the true ground state or is degenerate with it (see Vafa and Witten in ref.[9]). Even though in our models we cannot contemplate taking this limit, we shall assume that the former applies, thus neglecting the possibility of accidental degeneracy. Note that the mass inequalities hold irrespectively of whether the vector-like gauge theory is asymptotically free or not. In sect.4 we will also discuss the relevance of departures of technicolor theory from vector-like behavior due to additional interactions, remnants from the preonic theory. It is interesting to understand if such interactions may lead to the breakdown of vectorial symmetries, in particular those associated with family numbers.

Finally, in the analysis of the chiral gauge components of our models, we rely heavily on the complementarity principle [10] to ascertain the pattern of symmetry breakdown. Because the idea of complementarity may not be as well known, we will describe it briefly in the following section.

The outline of the paper is as follows. In sect.2 we describe how, by gauging vector-like subgroups, one can actually generate light fermions in a chiral gauge preon theory, illustrating the mechanism with a simple but unrealistic model. In sect.3 we
describe a semirealistic, one generation, composite model of quarks which can easily encompass a mass hierarchy. Generations, the issue of quark mixing and the concomitant appearance of FCNC in such models are broached in Sect.4. Here, even though no realistic models are actually constructed, some of the possible phenomenological implications of this type of scenarios are noted. Finally, Sect.5 contains our conclusions.

2 Mass generation in a chiral gauge theory

The chiral gauge component of the theory discussed in this section was studied long ago by Bars and Yankielowicz [11] and is described in some detail in connection with the problem of mass generation in ref.[1]. The model is based on the gauge group $SU_{gauge}(N)$ and has $N + 4$ copies of massless chiral fermions $F_{ia}$ ($i = 1, ..., N$, $a = 1, ..., N + 4$) in the fundamental representation and a single copy $S^{ij}$ in the conjugate symmetric representation. This content is free from gauge anomalies. The model may be analyzed by using the complementarity principle [10]. The most attractive channel [12] (that is the one with the largest relative Casimir operator) favors a condensate

$$\langle F_{ia} S^{ij} \rangle \equiv \langle \Phi_a^i \rangle = \Lambda^3 \delta_a^i, \quad a, i, j = 1, ..., N, \quad (1)$$

which is in the fundamental representation of the gauge group. In this case, one expects that there is no phase boundary between confining and Higgs phases [10]. Thus, as far as symmetry realization and massless composites are concerned, one may as well study the Higgs phase and consider the effective Higgs field $\Phi_a^i$ as fundamental. In the Higgs picture the v.e.v. $\langle \Phi_a^i \rangle$ breaks both the gauge group and the global $SU(N + 4)$ symmetry of the $F$’s. However, a certain global subgroup $SU(N) \times SU(4)$ remains unbroken. Here $SU(N)$ is the diagonal part of $SU_{gauge}(N)$ and the $SU(N)$ subgroup of global $SU(N + 4)$, operating on the first $N$ of the $F$ fermions, while $SU(4)$ is the subgroup of $SU(N + 4)$ operating on the last four $F$’s which do not participate in the condensation. In addition, there is also a global $U(1)$ symmetry which survives the formation of the condensate $\langle \Phi_a^i \rangle$ and will be discussed further below. In the Higgs phase, the only possible fermionic mass term can originate from a coupling of the form $\Phi_a^i F^{ia} S^{ij} + h.c.$ Upon diagonalization, this interaction leaves massless two sets of fermions, transforming according to $SU(N) \times SU(4)$ as $((N \times N)_{asym}, 1)$ and $(N, 4)$, respectively. In terms of the original preons $F$ and $S$ these fermions may be
constructed schematically as
\[
    f_{[ab]} = F_{[a}F_{b]}S; \quad f'_{aA} = F_aF_AS, \quad a, b = 1, \ldots, N; \quad A = 1, \ldots, 4.
\]  

(2)

One can check that these fermions satisfy 'tHooft’s anomaly matching conditions [2] for all anomalies of the unbroken global group $SU(N) \times SU(4) \times U(1)$.

At the lagrangian level, there are two particle number $U(1)$ symmetries associated with the numbers of $F$’s and $S$’s, respectively. However, only a certain linear combination of these two is anomaly-free with respect to the $SU(N)$ gauge fields. The anomaly-free fermion number is
\[
    q = n_S(N + 4)/N - n_F(N + 2)/N,
\]
where $n_S$ and $n_F$ are the $S$ and $F$ particle numbers. A linear combination of this anomaly-free generator and a diagonal generator of the global $SU(N + 4)$,
\[
    I_{N+4} = N^{-1} \text{diag}(1, \ldots, 1, -N/4, \ldots, -N/4),
\]
gives the generator $q' = q - 2I_{N+4}$ of the $U(1)$ symmetry that is unbroken by the condensate (1) and acts on the composite states. Indeed, the $q'$ charges of the preons: $F_{ia} (q' = -(N + 4)/N); \quad F_{iA} (q' = -(N + 4)/(2N)); \quad S^{ij} (q' = (N + 4)/N)$ guarantee that $\Phi^j_a$ has $q' = 0$.

It is important to understand how the $U(1)$ anomaly of the preon theory manifests itself at the level of composites. Due to complementarity, we may again analyze the theory in the Higgs phase where the relevant fluctuations are instantons. At very short distances, much shorter than $\Lambda^{-1}$, where $\Lambda$ is the dynamical scale of the theory, the Higgs v.e.v. is inoperative, and instantons of $SU_{\text{gauge}}(N)$ give rise to a 'tHooft effective interaction [13] involving $N + 4$ $F$’s and $N + 2$ $S$’s, corresponding to the numbers of zero modes of these representations. As we go to longer distances, the v.e.v. turns on and the zero modes coupled to the condensate (1) are lifted. These correspond to the first $N$ $F$’s and $N$ linear combinations of $S$’s. The remaining six zero modes become those of composites, four of them corresponding to states $f'$ and two to $\bar{f}$. That is, the 'tHooft interaction at the preon level leads to an effective interaction at the bound state level of the form
\[
    \epsilon^{ABCD} f'_{aA} f'_{bB} f'_{cC} f'_{dD} \bar{f}_{[ab]} \bar{f}_{[cd]},
\]
where for notational simplicity we have suppressed Lorentz indices. This dimension nine effective vertex is accompanied by a coupling of order $1/\Lambda^5$ in the effective
lagrangian. Thus, in chiral gauge theories, at least where complementarity is applicable, the $U(1)$ anomaly is reflected in the low-energy effective lagrangian by a multi-leg fermionic vertex suppressed by a high power of the compositeness scale.

In the model described above, the composite fermions (2) are strictly massless and the global chiral symmetry $SU(N) \times SU(4) \times U(1)$ is exact. A possible way to break this group, so that some fermions receive small masses is to gauge an appropriate part of it [14]. At the preonic level, this corresponds to assigning to some preons gauge charges of an additional gauge group. A useful way to proceed is to let the four preons, which we have denoted as $F_A, A = 1, ..., 4$, form two doublets with respect to a vector-like $SU(2)$ gauge group characterized by a confining scale $\Lambda' \ll \Lambda$.

$$F_A = \{F_{1\alpha}, F_{2\alpha}\}, \quad \alpha = 1, 2.$$  \hspace{1cm} (5)

Since $\Lambda' \ll \Lambda$, the effect of the new gauge interaction is best understood at the level of composite states. The states labelled by $f$'s, which were $SU(4)$ singlets, are now $SU(2)$ singlets, while the states denoted by $f'$ decompose analogously to eq.(2), $f'_A = \{f'_{1\alpha}, f'_{2\alpha}\}$. Because the $SU(2)$ gauge theory is vector-like, it produces chirality-breaking condensates similar to those of QCD. Assuming the pattern of condensation

$$\langle \epsilon^{\alpha\beta} f'_{1\alpha} f'_{2\beta} \rangle = \Lambda^3 \delta_{ab},$$  \hspace{1cm} (6)

the global $SU(N)$ symmetry acting on composite states is broken spontaneously to the orthogonal group $O(N)$. Moreover, the low-energy $U(1)$ symmetry, besides being broken by (6), is now also broken explicitly by the anomaly associated with the $SU(2)$ gauge fields. The $O(N)$ symmetry is thus the only symmetry remaining which acts on the $f$ states, and it allows for a mass term $m_f \langle f_{[ab]} f_{[ab]} \rangle$. Since the $f$'s do not have $SU(2)$ quantum numbers themselves, the only way by which the symmetry-breaking pattern is communicated to them is through the contact interactions with the $f'$ states, suppressed by the compositeness scale. In particular, the anomalous interaction of eq.(4) is necessary because all non-anomalous interactions preserve a separate particle number of the $f$'s. The condensates of eq.(6), in conjunction with the anomalous vertex (4), give precisely a mass term for the $f$'s with

$$m_f \sim \frac{\Lambda^6}{\Lambda^5}.$$  \hspace{1cm} (7)

\[If the condensates $\langle \epsilon^{\alpha\beta} f'_{1\alpha} f'_{1\beta} \rangle, \langle \epsilon^{\alpha\beta} f'_{2\alpha} f'_{2\beta} \rangle$ form too, the $SU(N)$ symmetry is broken to a symplectic group which also allows mass terms for the states $f$.\]
Due to the difference between mass scales $\Lambda$ and $\Lambda'$, the mass (7) can be made arbitrarily small compared to either of these scales.

An unfortunate feature of this particular model, which needs to be avoided in realistic model building, is that it typically produces light fermions in real representations of orthogonal or symplectic groups, leaving no room for the weak interaction group $SU_W(2)$. In other words, since the observed quarks and leptons are chiral with respect to weak interactions, care should be taken that this possibility remains open for the bound states produced by the preon theory. This suggests that one wants condensates, analogous to those of eq.(8), not to give masses to some of the bound state fermions, but only to give rise to appropriate $SU_W(2)$ conserving four-fermion effective interactions tying left- and right-handed fermions together. Masses could then be generated by another set of condensates (technicolor), with these effective interactions playing the role of ETC interactions. We will see how this works in a model of one generation of quarks in the next section.

Even though the models discussed in the sequel are in many respects different from the above simple model, some features of it will remain relevant. In particular, the multileg interactions produced by the instantons of the preonic theory will be an important ingredient in our attempts to generate quark mixing in Sect.4. Besides, one can think of a physical context where the mass pattern produced by our toy model may be sufficient, namely giving Majorana masses to right-handed neutrinos. Since the generation of such masses does not involve $SU_W(2)$ breaking, which has a relatively low energy scale, such Majorana masses can be arbitrarily large, precisely as is needed to make the observable neutrinos naturally light.

3 A one generation model of quarks

More realistic composite models for quarks and leptons can be constructed by making use of multiple repetitions of the preonic model discussed in the last section. These models are not economical in their structure, but they do provide a very nice theoretical laboratory to test ideas. Furthermore, we note that the complicated structures which are introduced not only produce the "quasi-elementary" fermions wanted, but also all the necessary symmetry breaking dynamics to generate their masses. In the simplest version of these models, quarks and leptons are made by different preon theories. Thus, for illustrative purposes it will suffice to consider, to begin with, just a
model for one generation of quarks. In doing so, we will not run into a problem with the hypercharge anomaly, because in our model the hypercharge anomaly of quarks is cancelled by that of techniquarks.

The model to be considered is constructed as follows. A doublet of left-handed quarks and each of the two right-handed quarks descend from their own $SU(N)$ preonc gauge theories, which here we choose to have $N = 6$. Each of these preon theories produces massless composite fermions in the $(15,1)$ and $(6,4)$ representations of their respective global $SU(6) \times SU(4) \times U(1)$ groups. Because now we have both left- and right-handed particles at our disposal, we can gauge a common vectorial $SU(4)$, which we will call metacolor, as a whole, rather than gauging only an $SU(2)$ subgroup of it as we did with the toy model of the last section. This $SU(4)$ gauge interaction does not produce masses directly but gives rise to four-fermion interactions between left- and right-handed composites. Furthermore, out of a common vectorial $SU(6)$, two $SU(3)$ subgroups are gauged, one becoming color and the other acting as technicolor. Finally, the $SU_W(2)$ gauge group is built into the model by having a doubled fermionic content in the left-handed preon theory. Within this structure, as we shall discuss, appropriate anomaly-free hypercharge assignments can be made.

Although this model certainly looks like an ugly mechanical aggregate, it has only two parameters more than the standard model. The model has 3 preonc dynamical scales, $\Lambda_L$, $\Lambda^u_R$, and $\Lambda^d_R$, and one additional scale parameter $\Lambda_4$ for the common $SU(4)$ gauge group. The technicolor dynamical scale $\Lambda_{TC}$ is not another parameter, since it is related, as usual, to the W and Z masses. These 4 parameters replace the two Yukawa couplings of a one generation standard model of quarks. We will see, however, that even though there are more parameters, one gains a more dynamical understanding of how mass differences between up- and down-type quarks can arise.

Let us now describe the one-generation model in more detail. As we said above, the model includes three chiral gauge theories, each based on a separate preonc gauge group $SU(6)$. One of this groups has a doubled fermionic content, that is $2 \times 10 = 20$ left-handed fermions in the fundamental representation and two fermions in the conjugate symmetric representation (of dimension 21). This doubling is intended to allow the introduction of the $SU_W(2)$ symmetry. At the preonc level, six out of the ten

‡ Leptons are needed, though, to cancel the global $SU_W(2)$ anomaly - that is to make the total number of $SU_W(2)$ doublets even. With quarks alone, the number of doublets at preonc level in the one-generation model which will be considered is $6 \times 6 + 21 = 57$. 

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pairs of preons belonging to fundamental representation and the pair in the symmetric representation form $SU_W(2)$ doublets, while the remaining $2 \times 4 = 8$ preons in the fundamental representation are taken as $SU_W(2)$ singlets. With these assignments, the order parameter of the complementarity picture $\langle F^{ai} S_{ij} \rangle \propto \delta^a_i$, $a, i, j = 1, ..., 6$ can be made an $SU_W(2)$ singlet, so that $SU_W(2)$ is not broken at this level. Among the six doublets $F^{ai}$, $a = 1, ..., 6$ three form a conjugate triplet under the color group $SU_c(3)$, while three others form a conjugate triplet under a technicolor gauge group which is also an $SU(3)$. So, in total, the left-handed preonic theory has three colored preonic fields $C_L$, three technicolored fields $T_L$, which are in the fundamental representation of the preonic gauge group $SU(6)$ and are doublets with respect to $SU_W(2)$, an $SU_W(2)$ doublet state $S_L$ in the conjugate symmetric representation of preonic $SU(6)$, and $2 \times 4$ fields $M_L^u, M_L^d$ which are in the fundamental representation of the preonic gauge group and are $SU_W(2)$ singlets. There are two other $SU(6)$ preonic theories for the right-handed fermions, one for the up and one for the down quarks. Each has a single copy of the basic fermionic content. So, in addition we have preons $C_R^{u,d}, T_R^{u,d}, S_R^{u,d}$ and $M_R^{u,d}$, where again $C$’s denote conjugate triplets of color and $T$’s denote conjugate triplets of technicolor. All right-handed preons are $SU_W(2)$ singlets.

When we apply complementarity to the left-handed theory, neglecting for the moment the color and technicolor gauge couplings, we are left with composite states which transform as $(15,1)$ and $(6,4)$ under $SU_{diag}(6) \times SU(4) \times U(1)$. Due to the original doubling, each of these states now comes in two varieties, but while the two copies of $(15,1)$ form an $SU_W(2)$ doublet, which we denote as $f_L$, the two $(6,4)$ states, which we denote as $f_L^{u,d}$, are $SU_W(2)$ singlets. The $(6,4)$ states are the only states which have in them the preons $M_L^u, M_L^d$. Analogously, the right handed theories produces composite fermions $f_R^{u,d}$ and $f_R^{u,d}$, all singlets under $SU_W(2)$. If there were no color and technicolor couplings, the states of each theory would transform under their own separate $SU_{diag}(6)$. However, when these couplings are turned on this $[SU_{diag}(6)]^3$ symmetry is broken explicitly to $SU_c(3) \times SU_{TC}(3)$ (times a certain number of $U(1)$’s which we will discuss further below). In addition, we are now going to gauge an $SU(4)$ in such a way that all $M$ preons, both left and right, fall in its fundamental representation. This breaks the global $[SU(4)]^3$ symmetry down to $SU_{gauge}(4)$.

Besides having anomalous vertices analogous to eq.(4), each of the three preonic
theories produces contact interactions between the respective composites of the form

\[(\bar{f}_{L}^{u,d} \gamma^{\mu} A f_{L}) (\bar{f}_{R}^{u,d} \gamma^{\mu} A f_{R})\]

(8)

where the \(\lambda^{A}\) are global \(SU(6)\) generators. The interactions in eq.(8) preserve all partial fermionic numbers, that is the fermionic numbers of each of the composite states \(f\) and \(f'\) for the left and for both the right theories separately. Through the \(SU_{\text{gauge}}(4)\) interactions, the left- and right-handed sectors finally meet. Because the \(SU_{\text{gauge}}(4)\) theory is vector-like, one expects that vacuum condensates

\[\langle \bar{f}_{L}^{u,d} f_{R}^{u,d} \rangle = \langle \bar{f}_{L}^{u,d} f_{R}^{u,d} \rangle\]

form, and the contact interactions of eq.(8) will give rise to ETC interactions, as sketched in fig.1:

\[(\bar{f}_{L}^{u,d} \gamma^{\mu} A f_{L}) (\bar{f}_{R}^{u,d} \gamma^{\mu} A f_{R})\]

(10)

Note that since both left- and right-handed \(f'\) states are \(SU_{W}(2)\) singlets, the condensates (3) do not break \(SU_{W}(2)\). As we shall show below, they also preserve hypercharge.

After gauging the metacolor \(SU(4)\) and color and technicolor \(SU(3)\)'s in the manner indicated above, one can identify 7 chiral \(U(1)\)'s which are preserved in the binding of the preonic \(SU(6)\) theories. Three of these \(U(1)\)'s correspond to the charges \(q'\) of each preon theory, which we had identified earlier. With a convenient rescaling, the nontrivial preon assignments of these charges are

\[q'_{L} : \quad \{ C_{L} = 1; \ T_{L} = 1; \ M_{L}^{u} = 1/2; \ M_{L}^{d} = 1/2; \ S_{L} = -1 \} ; \]

\[q'_{R}^{u} : \quad \{ C_{R}^{u} = 1; \ T_{R}^{u} = 1; \ M_{R}^{u} = 1/2; \ S_{R}^{u} = -1 \} ; \]

\[q'_{R}^{d} : \quad \{ C_{R}^{d} = 1; \ T_{R}^{d} = 1; \ M_{R}^{d} = 1/2; \ S_{R}^{d} = -1 \} . \]

(11)

In addition, there are \(U(1)\)'s in each of the three theories, also free from preon anomalies, which exploit the fact that the preons belonging to fundamental representations now come in different types. This freedom allows the introduction of 4 more conserved charges, with the nontrivial preon assignments being as follows

\[q'_{R}^{u} : \quad \{ C_{R}^{u} = 1; \ T_{R}^{u} = -1; \ M_{R}^{u} = 0; \ S_{R}^{u} = 0 \} ; \]

\[q'_{R}^{d} : \quad \{ C_{R}^{d} = 1; \ T_{R}^{d} = -1; \ M_{R}^{d} = 0; \ S_{R}^{d} = 0 \} ; \]
Only 4 combinations of the above chiral $U(1)$’s do not have any metacolor, technicolor or color anomalies. Two of these $U(1)$’s can be chosen in a manifestly vector-like fashion already at the preon level, namely those corresponding to the charges

$$q_V' = q_L' + q_R^{u} + q_R^{d} ;$$

$$q_V = q_L + q_R^{u} + q_R^{d} ,$$

while the other two $U(1)$’s are still chiral. The charges for these latter $U(1)$’s can be taken as

$$\tilde{q}_L = q_L - q_L ;$$

$$\tilde{q}_R = q_R^{u} - q_R^{d} .$$

We note that three of these $U(1)$’s also have no $SU_W(2)$ anomaly, but the fourth one, associated with $q'_V$, has such an anomaly.

The charges (12) are not exactly the ones preserved in the preonic binding. One can see this by noticing that the order parameter (11) of the complementarity picture is not neutral with respect to these charges. However, the order parameter is neutral with respect to certain linear combinations of these charges and diagonal generators of the corresponding preonic $SU(6)$ gauge groups,

$$Q_R^u = q_R^u + diag(1, 1, 1, -1, -1, -1)^u_R ; \quad Q_R^d = q_R^d + diag(1, 1, 1, -1, -1, -1)^d_R ;$$

$$Q_L = q_L + diag(1, 1, 1, -1, -1, -1)_L ; \quad Q_L = q_L + diag(1, 1, 1, -1, -1, -1)_L .$$

The vector-like charge $q_V$ of eq.(13) is modified accordingly,

$$Q_V = Q_L + Q_R^u + Q_R^d .$$

At the level of composite states, there is no difference between $q_R^u$ and $Q_R^u$, etc. because the composite states are neutral with respect to the preonic gauge groups.

The metacolor condensates of eq.(9) obviously preserve the two vector-like charges $q'_V$ and $Q_V$. However, they break $\tilde{q}_L$ and $\tilde{q}_R$ individually, preserving a linear combination of them. Since the $f'$ states have the following $\tilde{q}_L$ and $\tilde{q}_R$ assignments

$$\tilde{q}_L : \quad \{ f'^u_L = 1/2; \quad f'^d_L = -1/2; \quad f'^u_R = 0; \quad f'^d_R = 0 \} ;$$

$$\tilde{q}_R : \quad \{ f'^u_L = 1/2; \quad f'^d_L = -1/2; \quad f'^u_R = 0; \quad f'^d_R = 0 \} .$$
\[ q_R : \quad \{ f_{L}^{\prime u} = 0; \quad f_{L}^{\prime d} = 0; \quad f_{R}^{\prime u} = 1/2; \quad f_{R}^{\prime d} = -1/2 \} , \quad (17) \]

what is preserved by (11) is

\[ \tilde{q}_V = \tilde{q}_R + \tilde{q}_L . \quad (18) \]

A linear combination of \( Q_V \) and \( \tilde{q}_V \),

\[ Y = \frac{1}{6} Q_V + \tilde{q}_V , \quad (19) \]

may be gauged without acquiring an anomaly and is identified with the hypercharge.

Therefore, the two anomaly-free charges surviving down to the technicolor scale may be taken as \( Q_V \) (or, at that scale, \( q_V \)) and the hypercharge.

With respect to the gauged \( SU_c(3) \times SU_{TC}(3) \) group, the 15 composite states \( f \) transform as \((3,1) + (\bar{3},\bar{3}) + (1,3)\). The first component corresponds to the observable quarks, while the remaining two are techniquarks. The gauge group \( SU_{TC}(3) \) causes techniquarks to condense, thus breaking \( SU_W(2) \) and giving masses to the quarks. The hypercharge symmetry is also broken at this stage, while the vectorial symmetry associated with \( q_V \) is preserved. Let us estimate the dependence of quark masses on the various mass scales present in the theory. The contact interactions eq.(8) are multiplied by factors of \( \Lambda_{preon}^{-2} \) in the effective lagrangian, \( \Lambda_{preon} \) being the scale of the corresponding preonic theory, \( \Lambda_{preon} = \{ \Lambda_L, \Lambda_{u_R}^{(3)}, \Lambda_{d_R}^{(3)} \} \). Hence, the ETC interactions of eq.(10) and fig.1 are multiplied by factors

\[
\frac{1}{\Lambda_{u_R}^{2,ETC}} = \frac{\Lambda_4^2}{\Lambda_L^2 \Lambda_{u_R}^{2}} ; \quad \frac{1}{\Lambda_{d_R}^{2,ETC}} = \frac{\Lambda_4^2}{\Lambda_L^2 \Lambda_{d_R}^{2}} , \quad (20)
\]

where \( \Lambda_4 \) is the scale of \( SU_{gauge}(4) \) theory. The order of magnitude of quark masses is given by \( \Lambda_{TC}^3/\Lambda_{ETC}^2 \). In virtue of eq.(10) this gives

\[
m_u \simeq \frac{\Lambda_{TC}^3 \Lambda_4^2}{\Lambda_L^2 \Lambda_{u_R}^{2}} ; \quad m_d \simeq \frac{\Lambda_{TC}^3 \Lambda_4^2}{\Lambda_L^2 \Lambda_{d_R}^{2}} . \quad (21)
\]

The characteristic feature of this type of models, therefore, is that the scale of compositeness is in inverse relation to the mass: the heavier the particle, the larger is its 'size'. If we are to apply formulas like (21) to reality, the lowest compositeness scale is that for the top quark. The highest possible value for this scale is achieved if we assume \( \Lambda_4 \sim \Lambda_{L}^{(3)} \sim \Lambda_{R}^{(3)} \), where \( \Lambda_{L}^{(3)} \) is the scale of the left-handed preonic theory for the third generation. Then, for \( m_t \sim 100 \text{ GeV} \), one gets

\[
\Lambda_4 \sim \Lambda_{L}^{(3)} \sim \Lambda_{R}^{(3)} \sim 3 \text{ TeV} . \quad (22)
\]
These estimates are too naive and should be modified in models where the technicolor coupling runs sufficiently slowly (walking technicolor) or has a fixed point. Since the above estimate serves mostly for illustrative purposes, we do not discuss these further refinements here.

The existence of an additional scale $\Lambda_4$, which in general is intermediate between the technicolor scale (1 TeV) and the scales of compositeness of light particles, is very welcome from a phenomenological point of view. In our model there are no global symmetries preserved by chiral binding and $SU_{gauge}(4)$ and broken by technicolor. Hence, $\Lambda_4$ is the scale of 'vacuum alignment', where the gigantic global symmetry of the preonic theory is broken by the condensates, in addition to being broken by the gauge couplings. If $\Lambda_4$ can be made sufficiently large, the resulting pseudo-Goldstone bosons will be harmless phenomenologically. In the absence of such an intermediate scale, the global symmetry would be broken by technicolor condensates resulting in lower masses for the pseudoGoldstones. In addition to relatively heavy pseudoGoldstones, there are several strictly massless particles in our model, associated with exact anomaly-free global $U(1)$ symmetries broken spontaneously at various levels. Again, none of these symmetries are broken exclusively by technicolor, so the scales of derivative couplings of Goldstone particles to ordinary matter are at least of order $\Lambda_4$, which in principle allows to put them beyond experimental detection.

4 Prospects for quark mixing

The simplest way to accommodate three generations of quarks in our model is to enlarge the model in a mechanical way, so that each left-handed quark doublet and each right-handed singlet is prepared by its own theory and has its own compositeness scale. This increases the number of preonic theories to nine. All preons, however, share the same $SU_{gauge}(4)$, technicolor, color, $SU_W(2)$ and hypercharge interactions. The masses for all quarks can be generated by the same mechanism as before, but problems arise when one tries to obtain quark mixing. These problems are related to the existence of certain vectorial symmetries, one per generation, which protect the quarks from mixing. At the simplest level, there are restrictions imposed by the existence of three conserved charges, analogous to $q_V$ of the previous section. Although this is not exactly where the worst part of the problem comes from, it is nevertheless a good prototypical example to begin our discussion. Because mass generation in
our model is due, essentially, to vector-like gauge interactions, these vectorial symmetries might be expected to be preserved in binding and thus to prohibit mixing. Therefore, to understand how quarks mixing can arise in this type of models, we have to consider possible deviations of the $SU_{gauge}(4)$ and technicolor interactions from vector-like behavior.

A possible source of such deviations are the $SU_W(2)$ interactions which, in the presence of multiple fermionic species, may grow strong at the $SU_{gauge}(4)$ dynamical scale. The $SU_W(2)$ interactions are certainly not vector-like because they involve only left-handed particles. However, an additional agent is needed to communicate this information to the states $f'$, which are the only ones with $SU_{gauge}(4)$ quantum numbers, because all these states, both left and right, are $SU_W(2)$ singlets! The only such agent are the preonic interactions, and for these to be effective, the $SU_{gauge}(4)$ scale should be close to that of at least one of the preonic theories, say, that of the right-handed top-quark. If some of the preonic interactions are not completely screened at the scale of $SU_{gauge}(4)$, then these interactions themselves can be a source of non-vector-like behavior. In these circumstances, it is conceivable that non-diagonal, flavor mixing condensates of the states $f'$ can be formed in the $SU_{gauge}(4)$ binding, for example

$$\langle \bar{f}'_L d'_L f'_s R \rangle \neq 0.$$ (23)

If (23) obtains, then the vectorial $U_V(1)$ symmetries are broken dynamically. However, even in this case one still has further difficulties.

The real problem with quark mixing arises when one tries to communicate the breakdown manifested by eq.(23) to the observable fermions which reside in the multiplets $f$. One notices that while the ETC contact interactions between $f$ states of different generations are now possible, they will always be of the form

$$\left( f^{(1)}_L \gamma_\mu \lambda^A f^{(1)}_L \right) \left( \bar{f}'_R \gamma_\mu \lambda^A f'_s R \right) \text{ etc.}$$ (24)

That is, while $f$‘s from different generations now can interact, their flavor numbers are still conserved. There is a symmetry reason for this behavior. Indeed, as in the toy model of the previous section, unless the anomalous interactions generated by instantons of the preonic theories are included, the fermionic numbers of the $f$ and $f'$ states are conserved separately. As long as these interactions are neglected, the

$^8$Henceforth, the $f_L$’s and their compositeness scales will carry a generation superscript.
separate flavor symmetries of the states $f$ will keep the observable fermions from mixing, even in the presence of non-diagonal condensates of the states $f'$, eq.(23). Therefore, to get mixing, one needs to consider both ETC interactions (24) and the instanton-induced vertices. Doing so, however, one if immediately faced with two problems. First, the instanton-induced vertices appear to be too small in magnitude to account for the observable mixing. Second - and this is a more profound problem - though the preonic instantons do break the separate fermionic numbers of the $f$’s, they preserve the $Z_2$ subgroups of the corresponding $U(1)$ symmetries! This follows simply from the fact that, as in the toy model of Sect.2, each instanton has two zero modes of the states $f$ of the corresponding flavor associated with it. Moreover, it is difficult to imagine that vector-like combinations of these $Z_2$ symmetries can be broken spontaneously by technicolor, since the non-vector-like interactions are already weak at the technicolor scale, so they cannot drive the system away from vector-like behavior. Thus, these $Z_2$ symmetries should survive to low energies and are sufficient to prohibit the mixing of quarks. Because of this, the simplest family generalization of our one-generation model is unrealistic. Although there are other ways to introduce families in these models, we shall not discuss them further here.

It seems, however, not entirely out of place to discuss for the remainder of this section some general features of mixing matrices expected in this type of composite models, assuming that eventually a more realistic model of this type can be constructed. By ”this type of models” here we mean models where, as in the one-generation model of Sect.3, the masses of quarks are in inverse relation to their compositeness scales. If a non-trivial mixing can be generated, we presume that this property will hold both for the diagonal and non-diagonal entries of the two mass matrices - for the up- and down-type quarks.

The CKM matrix appears after diagonalization of the two quark mass matrices. There is no reason why these matrices should be symmetric, because the left and right components of the quarks have different substructure. This means that, for example, the mass matrix for up-type quarks

\[
\hat{M}_{RL} = \begin{pmatrix}
M_{uu} & M_{uc} & M_{ut} \\
M_{cu} & M_{cc} & M_{ct} \\
M_{tu} & M_{tc} & M_{tt}
\end{pmatrix}
\] (25)

should be diagonalized with the help of two unitary $3 \times 3$ mixing matrices: one, de-
noted by $\hat{O}_R$, acting on right-handed quarks, another, $\hat{K}_u$, on the left-handed quarks, so that
\[ \hat{O}_R \hat{M}_{RL} \hat{K}_u^\dagger = \text{diag}(m_u, m_c, m_t) \equiv \hat{m}_u. \] (26)

In the standard model, it is a matter of choice whether to associate mixing with up- or down-type quarks, or some linear combination, because the only observable effect is the product of the mixing matrices for left-handed up- and down-type quarks - the CKM matrix $V = K^\dagger u K_u$. The mixing matrices for right-handed quarks, e.g. $\hat{O}_R$, have no observable effect in the standard model.

In the kind of models under discussion, however, in addition to all the standard interactions, there are four-fermion interactions of the type of eqs.(10),(24). When two of the legs in eq.(17) are chosen to be quarks and other two techniquarks, these terms play the role of ETC interactions. When all four legs are quarks, they are the new interactions between observable particles and a potential source of FCNC. Unlike the standard model interactions, these new terms are affected by up- and down-, and left- and right-mixings separately. It is easy to imagine now that due to the strong dependence of the non-diagonal entries of the mass matrices on the corresponding compositeness scales and, hence, on the particle masses, the largest mixing occurs for the heaviest particles (we will be a bit more precise below). That is, the CKM matrix should be dominated by mixing of the up-type quarks and, hence, the up-type sector is where the FCNC effects will be the biggest. Thus, composite models of the type described here suggest that the primary place to search experimentally for FCNC effects is in the $D_1 - D_2$ mass difference. Coincidentally, FCNC effects in the up-type sector (e.g. in $D$-mesons) are less thoroughly studied experimentally than those in the down-type system.

If the mixing of down-type quarks is neglected, the matrix $\hat{K}_u$ is precisely the observable CKM matrix $V$. In this case, one can obtain the up-type quark mass matrix $\hat{M}_{RL}$ in terms of the parameters of the standard model by considering the matrix equation following from eq.(26),
\[ \hat{M}_{RL}^\dagger \hat{M}_{RL} = \hat{K}_u^\dagger \hat{m}_u^2 \hat{K}_u. \] (27)

The matrix equation (27) constitutes nine real equations for the nine complex entries of the matrix $\hat{M}_{RL}$. So, in general this matrix is only half-determined by eq.(27) (if we knew the matrix $\hat{O}_R$, another half of the equations would come from there). Combined with our model considerations, however, eq.(27) is sufficient, essentially because we
anticipate a certain hierarchy among the elements of $\hat{M}_{RL}$. First (and unrelated to our model considerations), we use the arbitrariness of phases of the right-handed quarks to make $M_{uu}$, $M_{cc}$ and $M_{tt}$ real and positive. Second, because the terms $M_{uc}$ and $M_{cu}$ involve the two lighter (most elementary) quarks, they should be the smallest non-diagonal entries within our model, so we drop them. Finally, the term $M_{ut}$ appears in $\hat{M}^\dagger_{RL}\hat{M}_{RL}$ in the combinations $M_{uu}^*M_{ut} + M_{tu}^*M_{tt}$ and $|M_{ut}|^2 + M_{tt}^2$, so given that $M_{ut}$ is of the same order as $M_{tu}$ or smaller, its contributions to $\hat{M}^\dagger\hat{M}$ are negligible. So $M_{ut}$ also drops out and we are left with nine equations for nine real parameters.

Among the parameters of the standard model, the most uncertain are the mass of the top-quark, for which we allow the range from 100 to 300 GeV, and the CP-breaking phase $\delta$, for which we allow the range from 0 to $\pi/2$. For the two other up-quark masses we use $m_u = 5$ MeV and $m_c = 1.5$ GeV. For the CKM mixing angles we use the central values suggested in ref. [17]: $\sin \theta_1 = 0.22$, $\sin \theta_2 = 0.95 \sin^2 \theta_1$, $\sin \theta_3 = 0.64 \sin \theta_1 \sin \theta_2$. With these values, it turns out that $M_{uu}$ is essentially independent of the choice of $m_t$ and $\delta$, $M_{uu} = 5$ MeV. Also, with good accuracy $M_{tt} \approx m_t$, almost independently of the value of $\delta$. The absolute values of the other matrix elements are plotted as functions of $m_t$ and $\delta$ in fig.2. These values are consistent with the hierarchy suggested by our model.

5 Concluding remarks

Because our discussion has ranged over both technical issues as well as some rather phenomenological points, it is worthwhile to try to summarize the principal results obtained. The central result of this paper is that it is possible to obtain very light fermions in confining gauge theories ($m_f \ll \Lambda_{\text{comp}}$) by appropriately gauging in a vector-like manner a subset of preons in the theory. Although this result is interesting per se, for this mechanism to be relevant for composite models of quarks and leptons, one must envisage that mass generation occurs through two distinct stages. In the first stage, as a result of the gauging of a vector-like subset of preons, one establishes interactions between the right- and left-handed components of the massless fermionic bound states of the underlying preonic theory. These effective interactions then give rise to masses for the fermions, when a further vector-like technicolor interaction is switched on.

In the text a model for one generation of quarks is developed along these lines. In-
Interestingly, the model exhibits both a hierarchy between the obtained fermion masses and the natural scale of the binding in the theory, as well as a hierarchy between the up and down quark masses. Unfortunately, it appears to be difficult to incorporate a family structure along these lines. Although fermions can ensue as simple mechanical repetitions with quite different masses for the bound state fermions, it is difficult to eliminate all vestiges of natural flavor symmetries in the model. As a result, even in the presence of hierarchical fermionic masses, it is not possible to generate dynamically any CKM mixing. More precisely put, although triggering elements exist for the spontaneous breakdown of the remaining discrete family symmetries, the strength of these interactions seems far too small to guarantee that such a breakdown really happens.

Even though we have not been able to construct any realistic model of quarks and leptons along these lines, the above model considerations suggest the possible telltale signals of this class of composite models. The most immediate of these concerns the presence of flavor-changing neutral currents (FCNC) in the theory. In these scenarios it is not that FCNC never appear, but rather they should primarily affect the heaviest of the quarks and leptons. Because FCNC in the heavy-quark states are not well studied experimentally (if at all!), it is quite possible that rather large violations of flavor conservation could be associated with the top quark. As the exercise at the end of the paper shows, it is possible to envisage hierarchical mass matrices along these lines which give rise to the experimentally observed pattern of masses and mixings for the quarks. Whether one can construct a dynamical model which really produces this kind of mass matrices, however, remains an open challenging problem.

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FIGURE CAPTIONS

FIG.1. Emergence of ETC interactions in the composite model.

FIG.2. Absolute values of the elements of the up-type quark mass matrix as functions of the top-quark mass $m_t$ and the CP non-conserving phase $\delta$. 