Interference phenomena in radiation of a charged particle moving in a system with one-dimensional randomness

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Abstract

The contribution of interference effects to the radiation of a charged particle moving in a medium of randomly spaced plates is considered. In the angular dependent radiation intensity a peak appears at angles $\theta \sim \pi - \gamma^{-1}$, where $\gamma$ is the Lorentz factor of the charged particle.
I. INTRODUCTION

It is well known that a charged particle passing through a stack of randomly spaced plates radiates electromagnetic waves (see, for example, [1]). The radiation is caused by the scattering of the electromagnetic field (pseudo-photon) of the charged particle from the inhomogeneities in the dielectric constant. In an earlier study one of us has shown [2] that, in analogy with three-dimensional random media [3], the multiple scattering of the electromagnetic field plays an important role. However, in the multiple scattering approach only the diffusion contribution was taken into account. At this level the approach is equivalent to the radiative transfer theory for light transport in e.g. slab geometries, see [4] for a recent review.

On the other hand, interference effects are important when waves propagate in random inhomogeneous media. Anderson localization [5] and the enhanced backscattering peak [6] are manifestations of these effects. Other interference effects show up in correlations and higher moments of the transmitted intensity. They were also reviewed in [4].

In the present paper we want to investigate interference effects for radiation of a charged particle moving in a system with one-dimensional randomness.

II. FORMULATION OF THE PROBLEM

The system which we want to study consists of a stack of plates randomly spaced in a homogeneous medium. The dielectric constant of the system can be represented in the form

\[ \varepsilon(z, \omega) = \varepsilon_0(\omega) + \sum_i [b(\omega) - \varepsilon_0(\omega)] \left( |\Theta(z - z_i - a/2) - \Theta(z - z_i + a/2)| \right), \]  

(1)

where \( z_i \) are the random coordinates of the plates, \( a \) is their thickness, and \( \varepsilon_0(\omega) \) and \( b(\omega) \) are dielectric permeabilities of the homogeneous medium and the plate, respectively. It is convenient to represent the dielectric permeability as the sum of an average and a fluctuating part

\[ \varepsilon(z, \omega) = \varepsilon + \varepsilon_r(z, \omega), \quad <\varepsilon_r(z, \omega)> = 0, \]  

(2)

where \( \varepsilon = <\varepsilon(z, \omega)> \) and averaging over the random coordinates of plates is determined as follows

\[ <f(z, \omega)> = \int \prod_i \frac{dz_i}{L_z} f(z, z_i, \omega), \]  

(3)

where \( L_z \) is the system size in the \( z \)-direction. The vector potential of the electromagnetic field created by a moving charged particle satisfies the equations.
\[
\text{div} \vec{A} - \frac{i\omega}{c} \varepsilon(\vec{r}, \omega) \varphi(\vec{r}, \omega) = 0,
\]
\[
\nabla^2 \vec{A} + \frac{\omega^2}{c^2} \varepsilon(\vec{r}, \omega) \vec{A}(\vec{r}, \omega) = j(\vec{r}, \omega)
\] (4)

where \( j(\vec{r}, \omega) \) is the current associated with the moving charged particle

\[
j(\vec{r}, \omega) = -\frac{4\pi e}{c} \frac{\vec{v}}{v} \delta(x) \delta(y) e^{i\omega z/v}, \vec{v} \parallel z
\] (5)

The symmetry of the problem allows us to choose the vector potential along the \( z \)-axis, \( A_i = \delta_{z} A \). The electric field is expressed through the potentials in the following way

\[
\vec{E}(\vec{r}, \omega) = \frac{i\omega}{c} \vec{A}(\vec{r}, \omega) - \text{grad}\varphi(\vec{r}, \omega)
\] (6)

As usually, we decompose electric field into two parts: \( \vec{E} = \vec{E}_0 + \vec{E}_r \). Here \( \vec{E}_0 \) is the field originated by the charged particle moving in the homogeneous medium with dielectric constant \( \varepsilon \) and \( \vec{E}_r \) is the radiation field associated with the fluctuations of dielectric constant. The radiation tensor is determined as follows

\[
I_{ij}(\vec{R}) = E_{ri}(\vec{R}) E_{rj}^*(\vec{R})
\] (7)

where \( \vec{R} \) is the radius-vector of the observation point which is far away from the system, \( R \gg L \). For expressing the radiation intensity through the radiation potential \( \vec{A}_r \), we decompose the vector potential analogous to the decomposition of electric field \( \vec{A} = \vec{A}_0 + \vec{A}_r \). The fields \( \vec{A}_0 \) and \( \vec{A}_r \) satisfy the equations

\[
\nabla^2 \vec{A}_0 + \frac{\omega^2}{c^2} \varepsilon \vec{A}_0 = j(\vec{r}, \omega)
\]
\[
\nabla^2 \vec{A}_r + \frac{\omega^2}{c^2} \varepsilon \vec{A}_r + \frac{\omega^2}{c^2} \varepsilon_r \vec{A}_r = -\frac{\omega^2}{c^2} \varepsilon_c \vec{A}_0
\] (8)

Using (4) and (6) one can express the radiation tensor in terms of radiation potential

\[
< I_{ij}(\vec{R}) > = \frac{\omega^2}{c^2} \delta_{z} \delta_{z} < A_r(\vec{R}, \omega) A^*_r(\vec{R}, \omega) > + \frac{\delta_{z} \delta_{z}^*}{\varepsilon} < A_r(\vec{R}, \omega) \frac{\partial^2}{\partial R_i \partial R_j} A^*_r(\vec{R}, \omega) >
\]
\[
+ \frac{\delta_{z} \delta_{z}^*}{\varepsilon} < A^*_r(\vec{R}, \omega) \frac{\partial^2}{\partial R_i \partial R_j} A_r(\vec{R}, \omega) > + \frac{\omega^2}{c^2} \varepsilon_r < A_r(\vec{R}, \omega) \frac{\partial^2}{\partial R_i \partial R_j} A^*_r(\vec{R}, \omega) >
\] (9)

For obtaining (4) we supposed that \( \varepsilon_r \ll \varepsilon \). In order to carry out the averaging over the random coordinates of the plates it is convenient to express the radiation potential in terms of the Green’s function of equation (8)

\[
A_r(\vec{R}) = -\frac{\omega^2}{c^2} \int \varepsilon_r(\vec{r}) A_0(\vec{r}) G(\vec{R}, \vec{r}) d\vec{r}
\]
\[
\left[ \nabla^2 + k^2 + \frac{\omega^2}{c^2} \varepsilon_r(z) \right] G(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}')
\] (10)

where \( k = \omega \sqrt{\varepsilon}/c \).
III. GREEN’S FUNCTION

In this chapter we follow the approach of our previous paper [2]. For the bare Green’s function in momentum representation one has from (10)

\[ G_0(\vec{q}) = \frac{1}{k^2 - q^2 + i\delta} \]  

(11)

In the coordinate representation one has

\[ G_0(r) = -\frac{1}{4\pi r} e^{ikr} \]  

(12)

The average Green’s function in the independent scatterer approximation has the form

\[ G(\vec{q}) = \frac{1}{k^2 - q^2 + i\text{Im}\Sigma(\vec{q})} \]  

(13)

where the imaginary part of the self-energy \text{Im}\Sigma(\vec{q}) is determined self-consistently by the Ward identity

\[ \text{Im}\Sigma(\vec{q}) = \int \frac{d\vec{p}}{(2\pi)^3} B(\vec{p}) \text{Im}G_0(\vec{q} - \vec{p}) = \frac{1}{4\sqrt{k^2 - q^2}} \left[ B(|q_z - \sqrt{k^2 - q^2}|) + B(|q_z + \sqrt{k^2 - q^2}|) \right], \quad |q_\rho| < k \]  

(14)

Here \( B(\vec{p}) = (2\pi)^3 \delta(\vec{p}_\rho)B(|p_z|) \), where \( \vec{p}_\rho \) is the transverse component of the \( \vec{p} \) and \( B(|p_z|) \) is the correlation function of the one-dimensional random field

\[ B(|z - z'|) = \frac{\omega^4}{c^4} < \varepsilon_r(z)\varepsilon_r(z') > \]  

(15)

Using explicit form (14) of the random field and carrying out the averaging with the help of (14), we obtain

\[ B(q_z) = \frac{4(b - \varepsilon)^2 n \sin^2 q_z a/2}{q_z^2} \omega^4 \]  

(16)

where \( n \) is the concentration of plates in the system. The photon mean free path in the \( z \) direction is determined by

\[ l(\vec{q}) = \frac{\sqrt{k^2 - q_\rho^2}}{\text{Im}\Sigma(\vec{q})} \]  

(17)

As one could expect, the photon mean free path depends on the momentum direction. This is principally different from the isotropic case. In the case when the momentum is directed along \( z \), using (14) and (17), one obtains

\[ l(\theta = 0) = \frac{4k^2}{B(0) + B(2E)} \]  

(18)
From this point we shall call this quantity the pseudo-photon mean free path. It follows from (16) that \( B(0) = (b - \varepsilon)^2 \omega^4 n a^2 / c^4 \) and \( B(2k) / B(0) \sim \sin^2 ak / a^2 k^2 \). Therefore we have for the mean free path

\[
l(l(\theta) = 0) \approx \begin{cases} 
4k^2 / B(0), & ka \gg 1 \\
2k^2 / B(0), & ka \ll 1
\end{cases}
\] (19)

Note that the expressions (19) are obtained in the Born approximation which holds when \(|\sqrt{b/\varepsilon} - 1|ka \ll 1\). Notice also the restriction imposed on the angles. As was mentioned in [2] our consideration is correct up to angles \( \theta \approx \pi / 2 - \delta \) (where \(|\delta| \gg (1 / kl)^{1/3}\)).

IV. THE RADIATION INTENSITY

Substituting (10) into (9) we obtain following expression for the averaged radiation tensor

\[
< I_{ij}(\vec{R}) > = \int d\vec{r} d\vec{r}' A_0(\vec{r}) A_0^*(\vec{r}') e^{-ik(\vec{n} \cdot (\vec{r} - \vec{r}'))} \\
\left[ \delta_{zi} \delta_{zj} \frac{\omega^6}{c^6} G(R, r) G^*(r', R) + \frac{\omega^2}{c^2} \frac{\partial^2 G}{\partial R_i \partial z} \frac{\partial^2 G^*}{\partial R_j \partial z} + \frac{\delta_{zi} \omega^4}{c^2} \frac{\partial^2 G^*}{\partial R_i \partial z} + \delta_{zi} \frac{\partial G^*}{\partial R_j \partial z} + \delta_{zi} \frac{\partial G^*}{\partial R_j \partial z} \right]
\] (20)

As mentioned above, the observation point is far away from the radiating system. For this reason, using (12), one can obtain following useful relations

\[
G_0(\vec{R}, \vec{r}) \approx -\frac{1}{4\pi R} e^{ik(\vec{R} - \vec{n} \vec{r})}, \quad \frac{\partial^2 G_0(\vec{R}, \vec{r})}{\partial R_i \partial z} \approx \frac{k^2 n_i n_z}{4\pi R} e^{ik(\vec{R} - \vec{n} \vec{r})}, \quad R \gg r
\] (21)

Here \( \vec{n} \) is the unit vector in the direction of the observation point \( \vec{R} \). The radiation tensor (20) consists of three contributions. Single scattering and diffusion contributions have been studied in a previous paper [2]. Therefore in the present paper we shall give our main attention to the interference contribution.

Substituting (21) into (20) and using (13) we obtain for the single scattering contribution to the radiation intensity

\[
I_0^\theta(\vec{R}) = \frac{k^2}{16\pi^2 R^2 \varepsilon} \int d\vec{r} d\vec{r}' B(r - r') A_0(\vec{r}) A_0^*(\vec{r}') e^{-ik(\vec{n} \cdot (\vec{r} - \vec{r}'))} \\
\left[ \delta_{zi} \delta_{zj} + n_{iz} n_{jz} - \delta_{zi} n_{i} n_{z} - \delta_{zj} n_{i} n_{z} \right]
\] (22)

Solving (8), one obtains for the background potential

\[
A_0(q) = -\frac{8\pi^2 e \delta(q_z - \omega / v)}{c^2 - q^2}
\] (23)

Substituting (23) into (22) after some straightforward calculations we find for the single scattering contribution to the radiation intensity \( I = \frac{c R^2}{2} I_0^\theta(\vec{R}) \) [2],

\[
I_0^\theta(\theta) = \frac{c^2 L_Z B(\mid k_0 - k \cos \theta \mid) \sin^2 \theta}{2c} \frac{\omega^2}{[\gamma - 2 + \sin^2 \theta k^2 / k_0^2]^2} \frac{\omega^2}{k_0^2 c^2}
\] (24)
where $\gamma = (1 - \varepsilon v^2/c^2)^{-1/2}$ is the Lorentz factor of the relativistic particle in the medium, $k_0 = \frac{\omega}{c}$, $n_z = \cos \theta$ and $L_z$ is the system size in the $z$ direction. As seen from (24) at $ak \ll 1$ ($B \sim const$), the forward and backward intensities are equal. When $ak \gg 1$, for relativistic particles $k_0 \to k$, $\gamma \gg 1$ the forward intensity ($\theta \approx 0$) is significantly larger than the backward intensity ($\theta \approx \pi$) because of the factor $B$. The diffusion contribution can be represented in the form

$$
I_D^{ij}(\vec{R}) = \frac{k^2}{16\pi^2 R^2 \varepsilon} \int d\vec{r}d\vec{r}' B(r-r') A_0(\vec{r}) A_0^*(\vec{r}') \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 e^{-ik\vec{n}(\vec{r}_1 - \vec{r}_2)} P(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) G(\vec{r}_3, \vec{r}) G^*(\vec{r}', \vec{r}_4) \left[ \delta_{zi}\delta_{zj} + n_i n_j n_z^2 - \delta_{zi} n_j n_z - \delta_{zj} n_i n_z \right] \quad (25)
$$

where $P$ is the diffusion propagator

$$
P(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \sum_{\vec{r}_2} \sum_{\vec{r}_4} \quad (26)
$$

Here the solid line denotes the averaged Green’s function and the dotted one denotes the correlation function of random field. Respectively, the interference contribution has the form

$$
I_C^{ij}(\vec{R}) = \frac{k^2}{16\pi^2 R^2 \varepsilon} \int d\vec{r}d\vec{r}' B(r-r') A_0(\vec{r}) A_0^*(\vec{r}') \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 e^{-ik\vec{n}(\vec{r}_1 - \vec{r}_2)} P_C(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) G(\vec{r}_3, \vec{r}) G^*(\vec{r}', \vec{r}_4) \left[ \delta_{zi}\delta_{zj} + n_i n_j n_z^2 - \delta_{zi} n_j n_z - \delta_{zj} n_i n_z \right] \quad (27)
$$

where $P_C$ is related to the diffusion propagator in the following manner

$$
P_C(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \sum_{\vec{r}_2} \sum_{\vec{r}_4} \quad (28)
$$

As follows from (28), due to time-reversal invariance (see, for example [7]) $P_C$ is related to the diffusion propagator in the following manner

$$
P_C(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = P(\vec{r}_1, \vec{r}_4, \vec{r}_3, \vec{r}_2) \quad (29)
$$

Thus the interference contribution as well as the diffusion one is determined by the diffusion propagator $P$. We now turn to finding it. Our consideration is similar to the three dimensional case [3]. It follows from (26) that $P$ can be represented in the form
where \( \bar{R} = \frac{1}{4}(\bar{r}_3 + \bar{r}_4 - \bar{r}_1 - \bar{r}_2) \) and \( P \) satisfies the equation

\[
\int \frac{d\vec{q}}{(2\pi)^3} \left[ 1 - \int \frac{d\vec{q}'}{(2\pi)^3} f(\vec{q}, \vec{K}) B(\vec{p} - \vec{q}) \right] P(\vec{K}, \vec{p}, \vec{q}') = f(\vec{q}', \vec{K})
\]

where

\[
f(\vec{q}, \vec{K}) = G(\vec{q} + \vec{K}/2) G^*(\vec{q} - \vec{K}/2)
\]

As usual, one needs to know \( P \) at \( K \to 0 \). In this limit the diffusion propagator has the form [3]

\[
P(\vec{K} \to 0, \vec{p}, \vec{q}) = \frac{\text{Im}G(\vec{p}) \text{Im}G(\vec{q})}{\text{Im} \Sigma(\vec{q})} A(\vec{K})
\]

where

\[
A(\vec{K}) = \left[ 3 \int \frac{(\vec{q} \vec{K})^2 \text{Im}G(\vec{q})}{\text{Im}^2 \Sigma(\vec{q})} \frac{d\vec{q}}{(2\pi)^3} \right]^{-1}
\]

Substituting (13) and (14) into (34) and calculating the integral in the limit \( ka \gg 1 \), we obtain

\[
A(\vec{K}) = \frac{20\pi}{k} \frac{1}{3 K^2 l^2 + K^2 h^2 l^2}
\]

In our previous paper [2] we investigated the special case \( K_\rho = 0 \), which was sufficient for studying the diffusion contribution. Note that although we considered the case \( ka \gg 1 \) all results are qualitatively correct also in the in the general case. When we know the form of diffusion propagator we can investigate the investigate the diffusion and interference contributions. First we consider the diffusion contribution. Substituting (11) into (24) and using (33), (23) and (14), we obtain [2]

\[
\begin{align*}
I^D(\vec{K}) &= \frac{e^2}{2\varepsilon} (1 - n_z^2) A(\vec{K}) k^2 \text{Im} \Sigma(k\vec{u}) L_z \times \\
&\times \int \frac{d\vec{q}_\rho}{(2\pi)^2} \frac{1}{(q_\rho^2 + k_0^2 - k^2)^2} \left[ B \left( |k_0 + \sqrt{k^2 - q_\rho^2}| \right) + B \left( |k_0 - \sqrt{k^2 - q_\rho^2}| \right) \right]
\end{align*}
\]

As seen from (35), the main contribution to the integral over \( q_\rho \) for relativistic particles \( \gamma \gg 1 \), \( k_0 \to k \) give the values \( q_\rho \approx 0 \). Accounting for this circumstance and using (35) we have from (36)

\[
I^D(\omega, \theta) = \frac{5}{6} \frac{e^2 \gamma^2}{\varepsilon c} \left( \frac{L_z}{l(\omega)} \right)^3 \frac{\sin^2 \theta}{|\cos \theta|}
\]

When obtaining (37) we substitute as usually \( 1/K^2 \) at \( K \to 0 \) by \( L_z^2 \) (there with we assume that \( L_z < L_x, L_y \)). A more precise approach would be to solve the appropriate Schwarzschild-Milne equation for the present problem. This could bring overall prefactors of order unity [8,4]. For our present purpose we shall not be interested in these effects. It follows from (37) and (24), that \( I^D/I^0 \sim (L_z/l)^2 \gg 1 \). Thus when \( k|\cos \theta|l \gg 1 \) and \( l \ll L_z \) the diffusion contribution is superior to the single scattering one.

As one could expect, the forward and backward intensities in the diffusion contribution are equal to each other. Finally, we note the strong dependence of the diffusion contribution on the particle energy.
V. INTERFERENCE CONTRIBUTION

Using (29) and (30) and changing the variables of integration by formulae
\[ x_1 = r_1' - r_2', x_2 = r_3 - r_2', \theta = \frac{1}{2}(r_3 + r_2 - r_1'), r_4 \equiv r_4 \]
we find from (27)
\[ I_C(n) = \frac{(1 - n_z^2)ck^2}{32\pi^2\varepsilon} \int d\vec{r}d\vec{r}' B(r - r')A_0(\vec{r}')A_0^*(\vec{r}') \int d\vec{x}_1 d\vec{x}_2 d\vec{r}_4^* d\vec{r}_4 \]
\[ e^{ik\vec{n} \cdot (\vec{r}_4 - \vec{x}_1 + \vec{x}_2)} P(\vec{R}', \vec{x}_1, \vec{x}_2)B(\vec{x}_1)B(\vec{x}_2)G(\vec{R} + \vec{x}_1 + \vec{x}_2) + r_4 - r_4)G^*(\vec{r}' - \vec{r}_4) \]

In the Fourier representation one finds from (39)
\[ I_C(n) = \frac{(1 - n_z^2)ck^2}{32\pi^2\varepsilon} \int \frac{d\vec{q}_1 d\vec{q}_2 d\vec{K}}{(2\pi)^3} B(\vec{q})A_0(-k\vec{n} - \vec{K} - \vec{q})|^2 \]
\[ B(\vec{q}_1)B(\vec{q}_2)P(\vec{K}, \vec{k}, \vec{n} + \vec{K} - \vec{q}_1, k\vec{n} + \vec{K} - \vec{q}_2)G(k\vec{n} + \vec{K})|^2 \]

Substituting (13), (23), (33) and (34) into (40), taking into account that the main contribution in the integral over \( \vec{K} \) give the values \( K \rightarrow 0 \) and sequentially integrating (40) using the Ward identity, we find from (40)
\[ I_C(n) = \frac{10\pi^2 L_z|n_z|(1 - n_z^2)^2 B(|k\cos\theta + k_0|)}{\varepsilon cl} \int \frac{d\vec{K}}{(2\pi)^3} \frac{1}{(3K_r^2 + K_p^2)(K_r^2 + k\vec{n} + k_0)^2 + k_0^2 \gamma^{-2} + K_p^2} \]

Note that at \( ak \gg 1, B(2k)/B(0) \sim 1/k^2a^2 \ll 1 \). Therefore, as follows from (41) in the Cherenkov limit the backward intensity (\( \theta \approx \pi \)) is significantly larger than the forward intensity (\( \theta \approx 0 \)). This is the main characteristic feature of the interference contribution. It is analogous to the light back-scattering peak which occurs in propagation of light in the randomly inhomogeneous media [6]. Considering angles \( \sin \theta \gg \lambda/2\pi L_\rho \) and calculating the integral over \( K \) in (41) we have
\[ I_C(\theta) = \frac{5\sqrt{2}}{2\pi} \frac{\arctan \sqrt{2} e^2 L_z B(|k\cos\theta + k_0|) \sin^2\theta \cos\theta}{c l^2} \frac{1}{[k^2 \sin^2\theta + k_0^2 \gamma^{-2} + k_0^2 \gamma^{-2}]^2} \]

When obtaining (42) we cut the integral over \( K \) on the upper limit at 1/\( l \). Note that the ratio \( \lambda/2\pi L_\rho \), in the optical region, is of order \( \sim 10^{-4} - 10^{-5} \). Therefore one can believe that the condition \( \sin \theta \gg \lambda/2\pi L_\rho \) is always satisfied in the optical region. Comparing (42) with the single scattering contribution (24) we see that \( I_C/I^0 \sim 1/(kl)^2 \ll 1 \). Although the interference contribution is small however it has quite different angular dependence. Accounting for the form of correlation function \( B(16) \), it follows from (12) that the maximum of the radiation intensity for relativistic particles \( \gamma \gg 1, k_0 \rightarrow k \), lies in the region of angles \( \theta \sim \pi - \gamma^{-1} \). These angles are very close to the backward direction. On the other hand, the maximum of the single scattering contribution lies in the strongly forward range of angles \( \theta \sim \gamma^{-1} \).
VI. SUMMARY

We have considered the influence of interference effects on the radiation of a charged particle passing through a stack of randomly spaced plates. It appears that interference contribution to the radiation intensity has a peak in the backward to particle motion direction. Though its value is small compared to the single scattering and diffusion contributions, it can be investigated experimentally. This is possible due its specific angular dependence.

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