Mini Little Higgs and Dark Matter

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We construct a little Higgs model with the most minimal extension of the standard model gauge group by an extra $U(1)$ gauge symmetry. For specific charge assignments of scalars, an approximate $U(3)$ global symmetry appears in the cutoff-squared scalar mass terms generated from gauge bosons at one-loop level. Hence, the Higgs boson, identified as a pseudo-Goldstone boson of the broken global symmetry, has its mass radiatively protected up to scales of $5-10$ TeV. In this model, a $Z_2$ symmetry, ensuring the two $U(1)$ gauge groups to be identical, also makes the extra massive neutral gauge boson stable and a viable dark matter candidate with a promising prospect of direct detection.

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\[ \Lambda \approx 4\pi f. \] Among the 5 Goldstone bosons in the effective field theory, one is eaten by the massive neutral gauge boson \( B' \equiv (B_1 - B_2)/\sqrt{2} \), while the other 4 become pseudo-Goldstone bosons and identified as the Higgs doublet \( h \). We parametrize the triplet in terms of \( h \) as

\[ \phi^T = f (i \sin \frac{h}{f}, \cos \frac{h}{f}) = (i h, f - \frac{h^2}{2f}) + \cdots. \] (3)

From the kinetic terms of scalars, the field dependent masses of gauge bosons are derived as

\[ M_W^2(h) = \frac{c_w^2 M_Z^2(h) = \frac{1}{2} g^2 f^2 \sin^2 \frac{h}{f},} \]

\[ M_{B'}^2(h) = \frac{50}{9} g'^2 f^2 \cos^2 \frac{h}{f}, \] (4)

and used to calculate the one-loop Coleman-Weinberg effective potential \([3]\), from which we obtain the leading terms of the Higgs potential as \( V(h) = m_h^2 h^4 + \lambda h (h^2)^2 + \cdots \) \([3]\). The Higgs mass contributions from the gauge sector at one-loop level are

\[ m_h^2|_g = \frac{3g'^2\Lambda^2}{32\pi^2 \left( \frac{27 - 118 s_w^2}{9 s_w^2} \right)} + \frac{3M_t^4}{32\pi^2 f^2} (\log \frac{\Lambda^2}{m_t^2} + 1). \] (5)

with \( M_{B'} = 5\sqrt{2}g' f/3 \approx 0.8 f \). The contributions to the Higgs quartic term from the gauge sector are smaller than from the fermion sector, and are neglected here. For \( s_w^2 \) in the range \((0.22,0.24)\) \([4]\) and \( \Lambda \) in the range of 5 to 10 TeV, the cutoff-squared contributions to the Higgs mass are cancelled to be smaller than the logarithmically divergent part. This cancellation comes from the \( U(1) \) charge assignments of the \( SU(2)_w \) singlet \( S \). Assuming some grand unified theories can provide us this set of simple discrete choices of \( U(1) \) charges, then this accidental cancellation is free from fine-tuning, since we can not continuously change the quantized charges of fields.

In the fermionic sector of our model, all standard model fermions have identical charges under these two \( U(1)'s \) and have their charges to be one half of their hypercharges. For example, we have \((t_L, b_L)^T = (2, \frac{1}{6}, \frac{1}{6}),\)

\( t_R \) as \((1, \frac{1}{3}, \frac{1}{3}) \) and \( b_R \) as \((1, -\frac{1}{3}, -\frac{1}{3}) \) under the gauge symmetries of our model. Hence our model is free of gauge anomaly. To cancel the cutoff-squared contribution to the Higgs boson mass from the top quark, there are two ways to introduce additional vector-like fermions. The simplest way with only one vector-like fermion manifestly breaks the \( Z_2 \) symmetry, while the other way with more vector-like fermions keeps the \( Z_2 \) symmetry exact. We will consider both cases in turn.

The most minimal way to cancel cutoff-squared contribution to the Higgs mass from the top quark is to introduce a colored vector-like quark \( \psi_{L,R} \) charged as \((1, \frac{2}{3}, -1) \). The Yukawa couplings in the top sector are

\[ L_t = y_1 (\bar{q}_L \tilde{H} + \bar{\psi}_L S) t_R + y_2 f \bar{\psi}_L \psi_{R} + h.c., \] (6)

with the first term \( U(3) \) invariant and \( y_2 \) breaking this global symmetry. These two Yukawa couplings in the top sector manifestly break the \( Z_2 \) symmetry, so \( B' \) directly couples to the mass eigenstate of the top quark through the mixing between the top quark and the top partner. Therefore, there is no dark matter candidate in this case. The one-loop contributions to the Higgs boson mass in the top sector are free from cutoff-squared terms, since both couplings \( y_1 \) and \( y_2 \) are necessary to generate a potential for the Higgs doublet \( h \). We calculate the Higgs doublet mass and quartic coupling as

\[ m_h^2|_t = -\frac{3}{8\pi^2} y_t^2 m_t^2 (\log \frac{\Lambda^2}{m_t^2} + 1), \] (7)

\[ \lambda_h|_t = \frac{-m_h^2|_t}{3f^2} + \frac{3y_t^4}{16\pi^2} (\log \frac{\Lambda^2}{m_t^2} + \log \frac{\Lambda^2}{m_t^2} + \frac{3}{2}). \] (8)

Here \( m_t = y_t h_t \) is the top quark mass after \( h \) takes its VEV and \( y_t = y_t y_2 / \sqrt{y_t^2 + y_2^2} \) is the top quark Yukawa coupling. To the leading order in \( v/f \), \( m_t^\prime = \sqrt{y_t^2 + y_2^2} f \) is the mass of the top partner.

From Eq. \([7]\), the Higgs mass from the top sector is still too large, although only logarithmically divergent terms exist. Additional operators are needed to have the Higgs boson mass below 200 GeV. The simplest way is to include a soft \( U(3) \) symmetry breaking operator \( \mu^2 H^H \) \([1, 3]\), which does not reintroduce quadratically divergent contributions to the Higgs mass. Expanding this operator in terms of \( h \), we have

\[ m_h^2|_\mu = \mu^2, \quad \lambda_h|_\mu = -\frac{\mu^2}{3f^2}. \] (9)

Minimizing the Higgs potential from all contributions in Eqs. \([5,7,8,9]\), the electroweak symmetry is broken with the weak scale \( v = 246 \) GeV and the Higgs boson mass \( m_h = 168 \) GeV by choosing \( f = 500 \) GeV, \( \Lambda = 4\pi f \approx 6 \) TeV, \( y_2 = 1.56 \) and \( \mu = 351 \) GeV. Defining the amount of fine-tuning as a variation of the weak scale in terms of \( \mu \) as \( \partial \log v^2 / \partial \log \mu^2 \), we have the fine-tuning to be 1 to 8 for this set of numbers. For \( \Lambda \) in the range \((2\pi f, 4\pi f)\), the Higgs boson mass can vary from 150 to 170 GeV.

The corrections to electroweak precision observables first appear at one-loop level, since at tree level only experimentally unmeasured top quark couplings to \( W \) and \( Z \) bosons are changed. In our model, the strongest constraint on \( f \) comes from the \( T \) parameter defined in \([3]\), which is calculated to be positive from the top sector at one-loop level as

\[ \alpha T = \frac{3y_t^2 y_2^2 m_t^2}{16\pi^2 f^2 m_t^2} (\log \frac{m_t^2}{m_t^2} - 1 + \frac{y_t^2}{2y_2^2}), \] (10)

for \( m_t \ll m_t^\prime \). The current bounds from PDG \([10]\) have approximately \( \alpha T < 1.2 \times 10^{-3} \) at 95% confidence level for the Higgs mass less than 300 GeV. For \( y_t / y_2 < 3/4 \), there is no bound on the symmetry breaking scale \( f \) from the \( T \) parameter. Hence, \( f \) can be as low as 400 GeV.

For this minimal little Higgs model, only two new fields, \( B' \) and \( t' \), exist in the effective field theory below
5-10 TeV. They can have masses as light as 300 GeV and 800 GeV respectively, and are to be discovered at LHC. In the top sector, the “collective symmetry breaking mechanism” in the traditional little Higgs models protects the Higgs mass from receiving cutoff-squared contributions at one-loop level. However, different from previous little Higgs models, collective symmetry breaking mechanism is missing in the gauge boson sector of our model. Fortunately, for the special charge assignments of scalars and the experimental value of the weak mixing angle, the cutoff-squared contributions to the Higgs mass from $W$, $Z$ and $B'$ are approximately cancelled to be even less than the cutoff-logarithmically dependent contributions. Thus this minimal little Higgs model also stabilizes the weak scale up to 5-10 TeV.

Now we consider a less minimal way to extend the top quark sector, which keeps the $Z_2$ or $T$-parity exact and provides a viable dark matter candidate. We introduce the following colored particles: $q_{1L}, t_R, q_{2L}, q'_{R},\psi_{1L,R}$ and $\psi_{2L,R}$, charged as $(2,\frac{3}{2},\frac{1}{2}), (1,\frac{1}{2},\frac{1}{2}), (2,\frac{3}{2},\frac{1}{2}), (1,\frac{1}{2},-1)$ and $(1,-1,\frac{1}{2})$ under $SU(2)_W \times U(1)_L \times U(1)_R$ respectively. Since only three additional vector-like fermions are introduced beyond the standard model, the gauge anomalies are cancelled automatically in this case. To keep the collective symmetry breaking mechanism and to preserve the $T$-parity, the following Yukawa couplings are introduced

\[
\mathcal{L}_t = \frac{y_1}{\sqrt{2}} (\bar{q}_{1L} H + \bar{\psi}_{1L} S) t_R + y_2 f \bar{\psi}_{1L} \psi_{1R} + \frac{y_1}{\sqrt{2}} (\bar{q}_{2L} \tilde{H} + \bar{\psi}_{2L} S^t) t_R + y_2 f \bar{\psi}_{2L} \psi_{2R} + \frac{y_3}{\sqrt{2}} f (\bar{q}_{1L} - \bar{q}_{2L}) q'_R + h.c. \tag{11}
\]

Under the $T$-parity transformation, we have

\[
T : \quad q_{1L} \leftrightarrow q_{2L}, \quad \psi_{1L,R} \leftrightarrow \psi_{2L,R}, \quad q'_R \rightarrow -q'_R, \quad B_1 \leftrightarrow B_2, \quad S \leftrightarrow S^t, \tag{12}
\]

and all other fields are invariant. The Lagrangian $\mathcal{L}_t$ and the covariant kinetic terms of fields are invariant under the $T$-parity [1], and hence all particles in our model are eigenstates of the $T$-parity.

Diagonalizing the fermion mass matrix, we have the masses of the $T$-odd particles $t'_L$ and $q'_R$ to be $y_2 f$ and $y_1 f$ respectively. To the leading order in $v/f$, the mass of the $T$-even top partner $t'_+$ is $\sqrt{y_1^2 + y_2^2} f$. The top quark is also $T$-even with the top Yukawa coupling as $y_t = y_1 y_2/\sqrt{y_1^2 + y_2^2}$. The cutoff-squared contribution to the Higgs mass from the top quark is cancelled by the $T$-even top partner. The analyses of the full one-loop Higgs potential and the corrections to the electroweak precision observables are similar to the $T$-parity violating case, and the symmetry breaking scale $f$ can be as low as 400 GeV for a 5 TeV cutoff.

The $B'$ gauge boson in the $T$-parity invariant model is the lightest $T$-odd particle (LTP) for $y_{23} \geq 1$. It can not decay into $T$-even standard model particles, and can serve as a viable dark matter candidate. Different from the littlest Higgs model with $T$-parity, where the LTP is much lighter than the symmetry breaking scale $f$ (around 0.2f) [12, 13, 14, 15], the $B'$ in our model is only slightly lighter than $f$ (around 0.8f). The coupling of two $B''$'s to the Higgs boson is $50g^2v/9$, which is a factor of 100/9 larger than the coupling of hypercharge-like gauge bosons to the Higgs boson. The present relic abundance of $B'$ is relating to pair-annihilation rates in the non-relativistic limit by the sum of the quantities, $a(X) = v_\gamma \sigma(B'B' \rightarrow X)$, with $v_\gamma$ as the relative speed between $B'$ bosons and $X$ as possible final states. In our model, $B'$ mainly annihilate into pairs of $W$, $Z$, $h_0$ bosons and top quarks. To leading order in $v/f$ and $m_t/M_{B'}$, we have

\[
a(t) = \frac{16\pi\alpha^2}{3\cos^4\theta_w} \frac{5^4 y_1^4}{y_2^2 (M_{B'}^2 + m_t^2)^2}, \tag{13}
a(WW) = 2a(ZZ) = 2a(h_0h_0) = \frac{2\pi\alpha^2}{3\cos^4\theta_w} \frac{5^4}{1} \frac{1}{M_{B'}^2}.
\]

Here $5/3$ is from the coupling among $t'_L$, $t'_R$ and $B'$, and also indicates a relatively large coupling of two $B''$'s to the Higgs boson: $y_t/y_2$ indicates the mixing between $t'_+R$ and $t_R$.

The present dark matter abundance from WMAP collaboration [10], $0.996 < \Omega_{B'}h^2 < 0.122$ (2$\sigma$), requires $\Omega_{mâ€™} = 0.81 \pm 0.09$ pb [17], assuming the dark matter candidate $B'$ in our model can make up all the dark matter. In Fig. 1, we plot the allowed region to have $B'$ account for all the dark matter in terms of the parameters $y_2$ and $M_{B'}$ in our model ($y_1$ is not independent and is determined by $y_2$ and $y_t \approx 1$). Here $y_2$ is subject to additional constraints $1.031 < y_2 < 4\pi$ to ensure both Yukawa couplings $y_1$ and $y_2$ to be perturbative and below 4$\pi$. For $y_2 > 1.5$, $B'$'s mainly annihilate into pairs of bosons, and the relic abundance barely depends on $y_2$. In this region, to account for all the dark matter, the mass of $B'$ is around 1.2 TeV.
The direct detection of dark matter is to observe the elastic scattering of dark matter particles with nuclei. In our model, the dark matter candidate $B'$ can have elastic scattering with quarks through the t-channel Higgs boson exchange. Therefore, the spin-independent $B'$-nuclei cross section may be measured in our model. (There is also a spin-dependent cross section through a box diagram coupling two $B'$ to two gluons with top quarks and their partners in the loop. For heavy top partners, the spin-dependent cross section is suppressed, and will not be discussed here.) In the non-relativistic limit, the relevant effective operator for the $B'$-quark interaction is $B'_\mu B'^\mu \bar{q}q/2$ with the coefficient as $50g^2m_q/m_H^2$. Using the matrix elements of quarks in a nucleon state and including the Higgs couplings to gluons mediated by heavy quark loops [18], we have the spin-independent $B'$-nucleon elastic scattering cross section as

$$\sigma_{SI} \approx 1.6 \times 10^{-44} \text{cm}^2 \left( \frac{1 \text{TeV}}{M_{B'}} \right)^2 \left( \frac{100 \text{GeV}}{m_h} \right)^4 . \quad (14)$$

In Fig. 2 we compare the predicted spin-independent $B'$-nucleon elastic scattering cross section in our model with several experiments. From Fig. 2 the cross section of direct detection of $B'$ is one to two order of magnitude smaller than the current constraints from CDMS [19] and XENON [20], and is accessible by future experiments like the early phase of Super-CDMS [21]. Compared to other hypercharge-like heavy gauge boson dark matter candidates, $\sigma_{SI}$ in our model is two order of magnitude larger [22]. This can be understood from the coupling of $B'$'s to the Higgs boson, which is larger than the coupling of hypercharge-like heavy gauge bosons to the Higgs boson by a factor of 100/9. This factor is also the crucial factor to cancel the gauge boson cutoff-squared contributions to the Higgs boson mass, and to provide the approximate $U(3)$ global symmetry for the cutoff-squared mass terms in Eq. (2).

In conclusion, a very simple little Higgs model has been constructed based on the $SU(2)_w \times U(1)^2$ gauge symmetry. The Higgs boson is identified as a pseudo-Goldstone boson, with its mass radiatively protected up to scales of 5-10 TeV. Depending on vector-like fermion choices in the top sector, a $Z_2$ interchanging symmetry between these two $U(1)$ gauge groups can be broken or unbroken. For the broken case, only a new neutral gauge boson $B'$ and a top partner $t'$ appear in the effective field theory. For the unbroken case, the $B'$ gauge boson is protected by the $Z_2$ symmetry from decaying into standard model fields and can serve as a dark matter candidate. Detailed calculations show that this $B'$ can make up all the dark matter in the universe, and is accessible by the early phase of future dark matter direct detection experiments.

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