Geometrical Diagnostic for Generalized Chaplygin Gas Model

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A new diagnostic method, $O_m$ is applied to generalized Chaplygin gas (GCG) model as the unification of dark matter and dark energy. On the basis of the recently observed data: the Union supernovae, the observational Hubble data, the SDSS baryon acoustic peak and the five-year WMAP shift parameter, we show the discriminations between GCG and $\Lambda$CDM model. Furthermore, it is calculated that the current equation of state of dark energy $w_{de} = -0.964$ according to GCG model.

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1. Introduction

The type Ia supernovae (SNe Ia) investigations, the cosmic microwave background(CMB) results from WMAP observations, and surveys of galaxies all suggest that the expansion of present universe is speeding up rather than slowing down. The accelerated expansion of the present universe is usually attributed to the fact that dark energy (DE) is an exotic component with negative pressure. Many kinds of DE models have already been constructed such as $\Lambda$CDM, quintessence, phantom, quintom, generalized Chaplygin gas (GCG), modified Chaplygin gas, holographic dark energy, agegraphic dark energy, and so forth. In addition, model-independent method and modified gravity theories (such as scalar-tensor cosmology, braneworld models) to interpret accelerating universe have also been discussed. So, a general and model-independent manner to distinguish these models introduced by different theories or methods is necessary. Statefinder diagnostic method is presented in Refs. and it has been applied to a large number of DE models. Recently, another geometrical diagnostic $O_m$ is also introduced in Ref. to differentiate $\Lambda$CDM with other models. An important property for $O_m$ diagnostic is that it can be used to distinguish DE models with small influence from density parameter $\Omega_0$, though the current observations suggest an uncertainties of at least 25% in the value of current matter density $\Omega_0$. In this paper, we apply $O_m$ diagnostic to GCG model.

The paper is organized as follows. In section 2, the GCG model as the unification of dark matter and dark energy is introduced briefly. Based on the recently observed data: the Union SNe Ia, the observational Hubble data (OHD), the baryon acoustic oscillation (BAO) peak from Sloan Digital Sky Survey (SDSS) and the five-year

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1 For example, using mathematical fundament one expands equation of state of DE $w_{de}$ or deceleration parameter $q$ with respect to scale factor $a$ or redshift $z$, such as $w_{de}(z) = \omega_0 = \text{const}$, $w_{de}(z) = \omega_0 + \omega_1 z$, $w_{de}(z) = \omega_0 + \omega_1 \ln(1 + z)$, $w_{de}(z) = \omega_0 + \frac{\omega_1}{1+z}$, $q(z) = q_0 + q_1 z$, $q(z) = q_0 + \frac{q_1}{1+z}$, and so forth. Where $\omega_0$, $\omega_1$, or $q_0$, $q_1$ are model parameters. For more information about model-independent method, please see review paper.
WMAP CMB shift parameter $[27]$, $Om$ diagnostic is used to GCG model in section 3. Section 4 is the conclusions.

2. generalized Chaplygin gas model

In the GCG approach, the most interesting property is that the unknown dark sections in universe–dark energy and dark matter, can be unified by using an exotic equation of state. The energy density $\rho$ and pressure $p$ are related by the equation of state (EOS) $[8]$

$$p = -\frac{A}{\rho^\alpha}, \quad (1)$$

where $A$ and $\alpha$ are parameters in the model.

By using the energy conservation equation: $d(\rho a^3) = -pd(a^3)$, the energy density of GCG is expressed as

$$\rho_{\text{GCG}} = \rho_{0\text{GCG}}[A_s + (1 - A_s)(1 + z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}}, \quad (2)$$

where $a$ is the scale factor, $A_s = \frac{\rho_{de}}{\rho_{0\text{GCG}}}$. For the GCG model, as a scenario of the unification of dark matter and dark energy, the GCG fluid is decomposed into two components: the dark energy component and the dark matter component, i.e., $\rho_{\text{GCG}} = \rho_{\text{de}} + \rho_{\text{dm}}$, $p_{\text{GCG}} = p_{\text{de}}$. Then according to the general recognition about dark matter, $\rho_{\text{dm}} = \rho_{0\text{dm}}(1 + z)^3$, the energy density of the DE in the GCG model is given by

$$\rho_{\text{de}} = \rho_{\text{GCG}} - \rho_{\text{dm}} = \rho_{0\text{GCG}}[A_s + (1 - A_s)(1 + z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}} - \rho_{0\text{dm}}(1 + z)^3. \quad (3)$$

Furthermore, considering spatially flat FRW (Friedmann-Robertson-Walker) universe with baryon matter $\rho_b$ and GCG fluid $\rho_{\text{GCG}}$, the equation of state of DE can be derived as

$$w_{\text{de}} = \frac{p_{\text{de}}}{\rho_{\text{de}}} = \frac{-(1 - \Omega_{0b})A_s[(1 - A_s)(1 + z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}}}{(1 - \Omega_{0b})[A_s + (1 - A_s)(1 + z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}} - \Omega_{0\text{dm}}(1 + z)^3}, \quad (4)$$

where $\Omega_{0\text{dm}}$ and $\Omega_{0b}$ are present values of dimensionless dark matter density and baryon matter component. And Hubble parameter $H$ is

$$H^2 = \frac{8\pi G \rho_b}{3} = H_0^2 E^2 = H_0^2 \{(1 - \Omega_{0b})[A_s + (1 - A_s)(1 + z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}} + \Omega_{0b}(1 + z)^3\}. \quad (5)$$

$H_0$ denotes the current value of Hubble parameter.

3. Om diagnostic for GCG model

It is well known that model-independent quantity $H(z)$ is very important for understanding the properties of DE, since its value can be directly obtained from cosmic observations (for example, the relation between luminosity distance $D_L$ and Hubble parameter is $H(z) = \left(\frac{d}{dz} \left[\frac{D_L(z)}{1+z}\right]\right)^{-1} - 28$ for SNe investigations). Recently, a new diagnostic of dark energy $Om$ is introduced to differentiate $\Lambda$CDM with other dynamical models. The starting point for $Om$ diagnostic is Hubble parameter, and it is defined as $[21]$

$$Om(z) \equiv \frac{E^2(z) - 1}{x^3 - 1}, \quad x = 1 + z. \quad (6)$$
Since $\Omega_m(z)$ only depends upon the scale factor $a$ and its derivative, it is a ”geometrical” diagnostic. For $\Lambda$CDM model, $\Omega_m(z) = \Omega_{0m}$ is a constant, then it provides a null test of this model\(^2\). The benefit of $\Omega_m$ diagnostic is that the quantity $\Omega_m(z)$ can distinguish DE models with less dependence on matter density $\Omega_{0m}$ relative to the EOS of DE $w_{de}(z)$\[^{21}\].

In what follows, we use a combination of the recent standard candle data (Union SNe Ia\[^{24}\]) and the OHD to constrain the evolutions of $\Omega_m(z)$ and $w_{de}(z)$ for GCG model. The Union SNe data includes the SNe samples from the Supernova Legacy Survey\[^{30}\], ESSENCE Surveys\[^{31}\], distant SNe discovered by the Hubble Space Telescope\[^{32}\], nearby SNe\[^{33}\] and several other, small data sets\[^{24}\]. The OHD are given by calculating the differential ages of passively evolving galaxies from the GDDS\[^{34}\] and archival data\[^{35}\]. According to the expression $H(z) = \frac{1}{1+z}\frac{dz}{dt}$, one can see that the value of $H(z)$ can be directly obtained by the determination of the differential age $dz/dt$. Ref.\[^{25}\] get nine values of $H(z)$ in the range of $0 < z < 1.8$ (see Table 1). And these nine observational Hubble data have been used to constrain DE models\[^{37}\].

| $z$   | 0.09 | 0.17 | 0.27 | 0.40 | 0.88 | 1.30 | 1.43 | 1.53 | 1.75 |
|-------|------|------|------|------|------|------|------|------|------|
| $H(z)$ (km$^{-1}$ Mpc)$^{-1}$ | 69   | 83   | 70   | 87   | 117  | 168  | 177  | 140  | 202  |
| 1σ uncertainty | ±12  | ±8.3 | ±14  | ±17.4| ±23.4| ±13.4| ±14.2| ±14  | ±40.4|

Table 1. The observational $H(z)$ data\[^{30,37}\].

FIG. 1: Evolutions of $\Omega_m(z)$ and $w_{de}(z)$ by using a combination of Union SNe data and OHD for GCG model. Here three different values $\Omega_{0b}=0.02, 0.042, 0.07$ for $\Omega_m(z)$ evolution diagram, and $\Omega_{0m} = \Omega_{0b} + \Omega_{0dm}=0.22, 0.27, 0.32$ for $w_{de}(z)$ diagram are assumed. The shaded regions show the 1σ confidence level. The dashed lines show the values of $\Omega_m(z)$ and $w_{de}(z)$ for $\Lambda$CDM model.

From Eq.\[^{4}\], it can be seen that both $\Omega_{0b}$ and $\Omega_{0dm}$ are included in the expression of $w_{de}(z)$ for GCG model. Given

\(^2\) For null test of $\Lambda$CDM model, one can also see Ref.\[^{22}\].
three different values of $\Omega_{0m}$, the evolutions of $w_{de}(z)$ with 1σ confidence level for GCG model are plotted in Fig. 1 (lower) by using the Union SNe data and the OHD. Furthermore according to Eq. (5), we can see that the Hubble parameter $H(z)$ for GCG model is dependent on the baryon density $\Omega_{0b}$ and two model parameters (A, $\alpha$). It does not explicitly include current matter density $\Omega_{0m}$. And one knows that the observational constraints on parameter $\Omega_{0b}$ is more stringent\(^3\), i.e., it has a relatively smaller variable range relative to $\Omega_{0m}$. On the basis of Eq. (6), we plot the evolutions of $Om(z)$ for GCG model in Fig. 1 (upper). From Fig. 1 it can be found that the $Om(z)$ diagram for GCG model as the unification of dark matter and dark energy is almost independent of the variation of $\Omega_{0b}$, but the evolution of $w_{de}(z)$ is sensitive to the variation of matter density.

In Ref. [39], $Om$ diagnostic has been used to distinguish $\Lambda$CDM and Ricci DE model. Assuming the matter density $\Omega_{0m}$ to be a free parameter, based on the recent cosmic observations Ref. [21] plots the evolution diagram of $Om(z)$ in a model-independent CPL scenario\(^4\). In this paper, treating $\Omega_{0b}$ as a free parameter, we apply the $Om$ diagnostic to GCG model. One knows that for the same dark energy model, the different evolutions of cosmological quantity can be obtained from different observational datasets. This is the so-called data-dependent. And in order to diminish systematic uncertainties and get the stringent constraint on cosmological parameters, people often combine many observations to constrain the evolutions of cosmological quantities. Next we use a combination of the recent standard candle data, the standard ruler data (the BAO peak from SDSS and the five-year WMAP CMB shift parameter $R$) and the OHD to constrain the evolution of $Om(z)$ for GCG model.

Because the universe has a fraction of baryons, the acoustic oscillations in the relativistic plasma would be imprinted onto the late-time power spectrum of the non-relativistic matter [41]. Then the observations of acoustic signatures in the large-scale clustering of galaxies can be used to constrain DE models with detection of a peak. The measured data at $z_{BAO} = 0.35$ from SDSS is [26]

$$A = \sqrt{\Omega_{0m}^{eff} E(z_{BAO})^{-1/3}} \left[ \frac{1}{z_{BAO}} \int_0^z \frac{dz'}{E(z')} \right]^{2/3} = 0.469 \pm 0.017, \tag{7}$$

where $\Omega_{0m}^{eff}$ is the effective matter density parameter [42].

The structure of the anisotropies of the cosmic microwave background radiation depends on two eras in cosmology, i.e., the last scattering era and today. They can also be applied to limit DE models by using the shift parameter [43]

$$R = \sqrt{\Omega_{0m}^{eff}} \int_0^{z_{rec}} \frac{H_0 dz'}{H(z')} = 1.715 \pm 0.021, \tag{8}$$

where $z_{rec} = 1089$ is the redshift of recombination, and the value of $R$ is given by five-year WMAP data [27] [21].

We plot the evolution of $Om(z)$ for GCG model by using the single standard candle data in Fig. 2 (a). From this figure, it is easy to see that the difference between GCG and $\Lambda$CDM model is obvious. Since the $Om$ diagnostic is relatively insensitive to the density parameter, the difference between this figure and Fig. 1 (upper) is caused by using the different datasets to constrain the quantity $Om(z)$, i.e. figure 1 is determined from a combination of Union SNe Ia and OHD data, but figure 2 (a) is plotted by means of Union SNe Ia data alone. Furthermore based on above four observational datasets, the combined constraint on $Om(z)$ is presented in Fig. 2 (b). According to Fig. 2 (b),

\(^3\) Such as $\Omega_{0b} h^2 = 0.0214 \pm 0.0020$ from the observation of the deuterium to hydrogen ratio towards QSO absorption systems [38], and $\Omega_{0b} h^2 = 0.02273 \pm 0.00062$ from the five-year WMAP results for the observation of CMB [27], here $h = H_0/100$.

\(^4\) It is an expansion for EOS of DE relative to scale factor $a$, $w_{de}(a) = w_0 + w_1 (1 - a)$, or $w_{de}(z) = w_0 + \frac{w_1}{1+z}$ [13] [23].
we can see that the best fit evolution of \( \Omega_m(z) \) for GCG model is near to ΛCDM case, and \( \Omega_m(0) \equiv \Omega_m(z = 0) = 0.299^{+0.037}_{-0.037} \) (1σ) for GCG model. In addition, by using above four datasets to ΛCDM model, it is obtained that the best fit value of \( \Omega_m \) with confidence level is \( \Omega_m = 0.273^{+0.016}_{-0.015} \) (1σ). We know \( \Omega_m(z) = \Omega_m \) for ΛCDM, then its best fit evolution is included in the 1σ confidence level of \( \Omega_m(z) \) in GCG scenario. And it can be seen that at 1σ confidence level, these two models can not be clearly distinguished by current observed data according to the \( \Omega_m(z) \) diagram.

At last, according to the expression \( \frac{\Omega_m(z) - \Omega_m}{1 - \Omega_m} \approx 1 + w_{de}(z \ll 1) \) \cite{21}, it can be calculated that the current EOS of DE \( w_{de} \approx -0.964 \) by taking \( \Omega_m = 0.273 \) and the best fit value \( \Omega_m(0) = 0.299 \).

4. Conclusion

On the basis of the recently observed data: the Union SNe Ia data, the nine observational Hubble data, the SDSS baryon acoustic peak and the five-year WMAP result, we apply a geometrical diagnostic \( \Omega_m \) to distinguish GCG model and ΛCDM model. From Fig. 2, it is shown that the larger error for the evolution of \( w_{de} \) may be produced by the erroneous estimation of matter density \( \Omega_m \). And the \( \Omega_m(z) \) is a better quantity than \( w_{de}(z) \) to truly distinguish DE models and to show the properties of DE. According to the \( \Omega_m \) diagram, it is easy to see that for the constraint from the single standard candle data, the difference between GCG model and ΛCDM model is obvious, while for the combined constraint, the best fit evolutions of \( \Omega_m(z) \) for them are similar and the difference between these two models is not clear at 1σ confidence level. In addition, we also calculate the value of current EOS of DE, \( w_{de} = -0.964 \), by using the value of \( \Omega_m(0) \) for GCG model. Here the \( \Omega_m(z) \) diagram is not sensitive to the variation of density parameter.
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