Configurational Entropy can disentangle conventional hadrons from exotica

P. Colangelo\textsuperscript{a} and F. Loparco\textsuperscript{a,b}

\textsuperscript{a}Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Via Orabona 4, I-70126 Bari, Italy
\textsuperscript{b}Dipartimento Interateneo di Fisica "M. Merlin", Università e Politecnico di Bari, via Orabona 4, 70126 Bari, Italy

Abstract

We evaluate the Configurational Entropy (CE) for scalar mesons and for $J^P = \frac{1}{2}^+$ baryons in a holographic approach, varying the dimension of boundary theory operators and using the soft-wall dual model of QCD. We find that hybrid and multiquark mesons are characterized by an increasingly large CE. A similar behavior is observed for $J^P = \frac{1}{2}^+$ baryons, where the CE of pentaquarks is larger than for three-quark baryons, for same radial number. Configurational Entropy seems relevant in disentangling conventional hadrons from exotica.

1 Introduction

Color neutral multiquark/multigluon hadrons can exist in QCD, in principle \cite{1}. Nevertheless, by far the largest part of the observed hadrons can be interpreted as $\bar{q}q$ and $qqq$ "conventional" configurations. The observed meson spectrum can be constructed in terms of $\bar{q}q$ constituents with quantum numbers $J^{PC}$ attributed according to the simple rules of the quark model, and the few exceptions, candidates of $\bar{q}Gq$ hybrids, $\bar{q}qqq$ tetraquarks and $GG$ glueballs, are the subject of disputes based on different interpretations \cite{2}. The same situation characterizes the baryon sector, where the observation of five-quark states has been recently reported only for systems comprising a heavy quark-antiquark pair \cite{3}. This must be contrasted with the plethora of observed conventional three-quark baryons and with the non-observation of pentaquarks made of light quarks \cite{4}.

The prevalence of conventional ("ordinary") quark configurations over non-conventional ("exotic") multiquark/multigluon configurations has, of course, the origin rooted in the
nonperturbative dynamics of QCD. Hints about its emergence come from large $N_c$ arguments\footnote{Recent discussions can be found in \cite{6} and in references therein.}. Here we explore the possibility that this regularity in the spectrum can find an interpretation using the notion of Configurational Entropy (CE) for hadrons\footnote{Here we use the name Configurational Entropy usually adopted in the literature, although Configurational Complexity has been recently proposed as the most appropriate name for the quantity we analyze \cite{7}.}

The use of Configurational Entropy to characterize the energy density profile of spatially localized systems and to measure the stored information \cite{8,9} has been inspired by the information theory, where the Shannon Entropy can be attributed to a sequence of bits in a binary string \cite{10}. This notion has allowed to describe the properties of various systems, finding that those with lower CE are dominant \cite{11}. In particular, in applications to the hadron spectroscopy, the properties of high-spin light meson production \cite{12}, the properties of the lowest spin glueballs and of the meson Regge trajectories \cite{13}, and the features of lowest-lying quarkonia at $T = 0$ and at finite temperature \cite{14} have been scrutinized evaluating CE in the framework of AdS/QCD models. Here we wish to compute CE in a holographic model, considering fields dual to QCD operators interpolating multiquark/multigluon hadronic states, to scrutinize the regularities emerging when the number of constituent quarks and gluons increases.

A definition of Configurational Entropy has been proposed in \cite{8} for a system characterized by a property described by a square-integrable bounded function, the profile function $F(x) \in L^2(\mathbb{R})$, having Fourier transform

$$F(k) = \int_{-\infty}^{+\infty} dx \ e^{-ikx} \mathcal{F}(x)$$

that satisfies the Plancherel relation

$$\int_{-\infty}^{+\infty} dk \ |F(k)|^2 = \int_{-\infty}^{+\infty} dx \ |\mathcal{F}(x)|^2.$$  

Starting from $F(k)$, the function

$$f(k) = \frac{|F(k)|^2}{\int_{-\infty}^{+\infty} dq \ |F(q)|^2}$$

represents the relative weights of the various $k$ modes. If $f(k)$ is non-periodic, the modal fraction can be further defined,

$$\hat{f}(k) = \frac{f(k)}{f(k)_{\text{max}}}.$$
which is bounded, \(0 \leq \hat{f}(k) \leq 1\). \(f(k)_{\text{max}}\) is the maximum weight for the mode \(k\). The system Configurational Entropy is defined as \[S_{CE} = -\int_{-\infty}^{+\infty} dk \, \hat{f}(k) \ln \hat{f}(k).\] (5)

This is a continuum generalization of the Shannon information entropy,

\[S_{SH} = -\sum_{\ell=1}^{n} p_{\ell} \ln p_{\ell}\] (6)

defined for a system characterized by a discrete set of events \(e_{\ell}\) (\(\ell = 1, \ldots, n\)), each one occurring with probability \(p_{\ell}\) (with \(\sum_{\ell=1}^{n} p_{\ell} = 1\)) \[10\].

To determine CE of meson and baryons, a suitable square-integrable function \(F(x)\) for the various hadrons must be selected. Using the soft-wall holographic model of QCD such a function can be recognized, and a computation for each hadron species can be carried out, as done in Sect. 2 for \(J^{PC} = 0^{++}\) mesons and in Sect. 3 for \(J^{P} = \frac{1}{2}^{+}\) baryons.

2 Scalar mesons

To compute CE for scalar mesons we use an approach based on a holographic model of QCD. AdS/QCD is a theoretical framework inspired by the AdS/CFT duality \[15\] aimed, in the so-called bottom-up version \[3\] at constructing higher dimensional models sharing properties with QCD, namely in the hadron sector \[17\]. We use the soft-wall model in which linear Regge trajectories for light mesons are recovered \[18\]. In Poincaré coordinates \(x_{M} = (x_{0}, x_{1}, x_{2}, x_{3}, z)\), an AdS\(_{5}\) manifold is described using the metric tensor

\[g_{MN} = e^{2A(z)} \text{diag}(+1, -1, -1, -1, -1) = e^{2A(z)} \eta_{MN},\] (7)

with warp factor

\[A(z) = \ln \left(\frac{L}{z}\right)\] (8)

and indices \(M, N = 0, 1, 2, 3, 5\). \(L\) is the AdS\(_{5}\) radius. To break the conformal symmetry and to construct 5D models for hadrons, in the soft-wall model a quadratic background dilaton \(\varphi\), depending only on the bulk coordinate \(z\), is included in the 5D actions,

\[\varphi(z) = c^{2} z^{2}\] (9)

whith \(c\) a dimensionful parameter.

\[\text{A review of AdS/CFT principles and applications can be found in [16].}\]
Following gauge/gravity duality, a scalar field \( \phi(x, z) \) is associated to each QCD gauge invariant operator \( O(x) \) with \( J^{PC} = 0^{++} \). It is described by the 5D action

\[
S_{5S} = \frac{1}{\kappa} \int d^5x \sqrt{|g|} e^{-\varphi(z)} L(\phi, \partial \phi, g),
\]

(10)

with

\[
L = \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - \frac{1}{2} m^2 \phi^2
\]

(11)

and \( \kappa \) a constant making (10) adimensional. The resulting equation of motion, in the general case of \( d + 1 \)-dimensions with \( \sqrt{|g|} = e^{(d+1)A(z)} \), reads:

\[
\phi'' + [(d-1)A' - \varphi'] \phi' - [m^2 e^{2A} + \eta^{\mu \nu} \partial_\mu \partial_\nu] \phi = 0 .
\]

(12)

The primes indicate derivatives with respect to the bulk coordinate \( z \), and the indices \( \mu, \nu \) run in \( 0, 1, \ldots, d-1 \).

According to the holographic dictionary [16], \( m^2 \) is related to the dimension \( \Delta \) of the boundary theory operator \( O(x) \) dual to \( \phi(x, z) \),

\[
m^2 L^2 = \Delta(\Delta - d) .
\]

(13)

Hence, scalar operators of increasing \( \Delta \) (\( O = \bar{q} q, \bar{q} \bar{q} q q, \text{etc.} \)) are dual to fields \( \phi \) of increasing mass \( m \).

Eq. (12) becomes an equation only in the variable \( z \) using the ansatz \( \phi(x, z) = e^{-ip \cdot x} a(p) Y(z) \):

\[
Y'' - \left[ \frac{d-1}{z} + 2c^2 z \right] Y' - \left[ \frac{\Delta(\Delta - d)}{z^2} - \omega^2 \right] Y = 0 ,
\]

(14)

with \( p^2 = \omega^2 \). This can be expressed as a Schrödinger-like equations after the (Bogolubov) transformation \( Y(z) = B(z) X(z) \), with \( B(z) = e^{c^2 z^2} z^{d-1} \):

\[
- X'' + V(z) X = \omega^2 X ,
\]

(15)

and \( V(z) \) acting as a potential,

\[
V(z) = (d-2)c^2 + \frac{d^2 - 1}{4z^2} + c^4 z^2 + \frac{\Delta(\Delta - d)}{z^2} .
\]

(16)

The regular solution of Eq. (15) [19]

\[
X(z) = C e^{-c^2 z^2} z^{2\Delta - d + 1} L \left( \frac{1}{4} \left( \frac{\omega^2}{c^2} - 2\Delta \right), \Delta - \frac{d}{2}, c^2 z^2 \right)
\]

(17)
is obtained in terms of the Laguerre generalized function $L$, with spectrum
\[ \omega_{n,\Delta}^2 = 4c^2 \left( n + \frac{\Delta}{2} \right) \] (18)
for the radial quantum number $n = 0, 1, \ldots$. For $d = 4$ we have:
\[ X_{n,\Delta}(z) = C_{n,\Delta} e^{-\frac{c^2 z^2}{4}} z^{\Delta - \frac{3}{2}} L_n^{\Delta-2}(c^2 z^2) \] (19)
and normalized eigenfunctions of (14)
\[ Y_{n,\Delta}(z) = \sqrt{\frac{2 \Gamma(n+1)}{\Gamma(n+\Delta-1)}} z^{\Delta} L_n^{\Delta-2}(c^2 z^2). \] (20)

Following the proposal in [12], the Configurational Entropy for the various meson states can be computed using the energy-momentum tensor obtained from (10), in correspondence to the regular solution $\phi$ of the equation of motion. Applying the definition
\[ T_{MN} = \frac{2}{\sqrt{|g|}} \frac{\partial}{\partial g^{MN}} \left( \sqrt{|g|} e^{-\phi} \mathcal{L}(\phi, \partial \phi, g) \right), \] (21)
which holds since the action does not depend on derivatives of the metric $g$, we have:
\[ T_{MN} = e^{-\phi} \left( \partial_M \phi \partial_N \phi - \frac{1}{2} \eta_{MN} \eta^{KQ} \partial_K \phi \partial_Q \phi + \frac{1}{2} \eta_{MN} m^2 e^{2A} \phi^2 \right). \] (22)
The $T_{00}$ component, computed with the on-shell solutions, reads:
\[ T_{00}^{n,\Delta}(z) = \Omega e^{-c^2 z^2} \left( \frac{\omega_{n,\Delta}^2 Y_{n,\Delta}(z)^2 - Y_{n,\Delta}'(z)^2 - \frac{\Delta(\Delta - 4)}{2} Y_{n,\Delta}(z)^2}{z^2} \right), \] (23)
with $\Omega$ a factor irrelevant for the subsequent computation. This is the profile function $F(z)$ from which the CE is computed for scalar mesons, considering various radial numbers $n$ and different operator dimensions $\Delta$.

3. $J^P = \frac{1}{2}^+$ baryons

There are different ways of describing baryons in a holographic framework [16]. Here, to describe $J^P = \frac{1}{2}^+$ baryons we start from the 5D action for a fermion field $\Psi(x, z)$ in the AdS$_5$ background:
\[ S_{5F} = \frac{1}{k} \int d^4 x \, dz \sqrt{|g|} \left[ \frac{i}{2} \bar{\Psi} e^A M^A D_M \Psi - \frac{i}{2} (D_M \Psi)^i \Gamma^0 e_A^M \Gamma^A \Psi - m \bar{\Psi} \Psi \right]. \] (24)
$e^M_A$ are the 5D inverse vielbein, $\Gamma^M = e^M_A \Gamma^A$, and $\Gamma^A = (\gamma^\mu, -i\gamma^5)$ (with Greek index $\mu = 0, 1, 2, 3$, and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$) the $4 \times 4$ Gamma matrices in 5D. The $\Gamma^M$ matrices obey the algebra
\[
\{\Gamma^M, \Gamma^N\} = \{e^M_A \Gamma^A, e^N_B \Gamma^B\} = 2 g^{MN} .
\] (25)

The covariant derivative $D_M$ involves the spin connection
\[
D_M = \partial_M - \frac{i}{4} \omega_{MAB} \Sigma^{AB}
\] (26)

with $\Sigma^{AB} = \frac{i}{2} [\Gamma^A, \Gamma^B]$; in our case we have
\[
D_M = \partial_M + \frac{i}{2} \lambda' z \Sigma_{M5} .
\] (27)

In the soft-wall model the background dilaton term $e^{-\varphi(z)}$ included in the action (24) can be removed rescaling the spinor field $\Psi(x, z) \rightarrow e^{\pm \frac{1}{2} \varphi(z)} \Psi(x, z)$ [20]. To obtain normalizable solutions of the EOM and a discrete spectrum, it has been proposed to modify the mass term in Eq. (24) [21],
\[
m \rightarrow m_z = m + J(z) ,
\] (28)

with
\[
J(z) = \frac{c^2 z^2}{L}
\] (29)

and $c$ the same coefficient appearing in the quadratic dilaton $\varphi$.

Separating the fermion field in left-handed (L) and right-handed (R) components,
\[
\Psi(x, z) = \Psi_L(x, z) + \Psi_R(x, z) ,
\] (30)

the solutions for the L and R component can be written as
\[
\Psi_{L/R}(x, z) = e^{-ip_x} u_{L/R}(p) F_{L/R}(z) ,
\] (31)

with $u_{L/R}(p)$ Dirac spinors in 4D momentum space and $F_{L/R}$ functions of $z$. Using Eqs. (30) and (31), with $p^\mu = (\lambda, 0)$, two coupled equations for $F_{L/R}$ (for $d = 4$) are obtained,
\[
\left( \partial_z - \frac{1}{2} \left( \frac{d}{z} + 2c^2 z \right) \pm \frac{m_z L}{z} \right) u_{L/R}(p) F_{L/R}(z) = \pm \lambda u_{R/L}(p) F_{R/L}(z) .
\] (32)

Such equations can be decoupled,
\[
\left( \partial_z - \frac{1}{2} \left( \frac{d}{z} + 2c^2 z \right) \mp \frac{m_z L}{z} \right) \left( \partial_z - \frac{1}{2} \left( \frac{d}{z} + 2c^2 z \right) \mp \frac{m_z L}{z} \right) F_{L/R}(z) = -\lambda^2 F_{L/R}(z) ,
\] (33)
obtaining

\[
F''_{L/R}(z) - \left( \frac{d}{z} + 2c^2z \right) F'_{L/R}(z) \\
+ \left( (d-1)c^2 + c^4z^2 + \left( 1 + \frac{d}{2} \right) \frac{dz L}{z^2} \mp \frac{m_z L}{z} - \frac{m^2 L^2}{z^2} \mp \frac{m'_z L}{z} + \lambda^2 \right) F_{L/R}(z) = 0.
\] (34)

The transformation

\[
F_{L/R}(z) = B(z) Y_{L/R}(z),
\] (35)

with \( B(z) = e^{\frac{c^2 z^2}{2} - d/2} \), brings to

\[-Y''_{L/R} + U_{L/R} Y_{L/R} = \lambda^2 Y_{L/R},
\] (36)

with

\[
U_{L/R}(z) = \pm \frac{m_z L}{z^2} + \frac{m^2 L^2}{z^2} \mp \frac{m'_z L}{z},
\] (37)

acting as a potential. Notice that the ansatz \([28,29]\) produces a confining potential in the \( z \to \infty \) IR region.

The holographic dictionary for fermion operators with spin \( s = \frac{1}{2} \) connects the fermion mass \( m \) and \( \Delta \):

\[
mL = \Delta - s = \sigma.
\] (38)

Solving (36) with this condition we have:

\[
Y_{L/R}(z) = C_{L/R} e^{-\frac{c^2 z^2}{2} z^{\frac{1}{2}(|1+2\sigma|)}} L_n^{\frac{1}{2}(|1+2\sigma|)} (c^2 z^2),
\] (39)

with radial quantum number \( n \) and

\[
\lambda_{n,\sigma}^2 = \begin{cases} 
    c^2 (4n + 1 + 2\sigma + |1 + 2\sigma|) & (L) \\
    c^2 (4n + 3 + 2\sigma + |1 - 2\sigma|) & (R).
\end{cases}
\] (40)

For \( \sigma \geq 1/2 \) the spectrum for L and R fields coincides \([21]\):

\[
\lambda_{n,\sigma}^2 = 4c^2 \left( n + \sigma + \frac{1}{2} \right).
\] (41)

The regular solutions

\[
Y^{n,\sigma}_L(z) = C_L^{n,\sigma} e^{-\frac{c^2 z^2}{2} z^{\sigma+1}} L_n^{\frac{1}{2}(|\sigma+\frac{1}{2}|)} (c^2 z^2)
\]
\[
Y^{n,\sigma}_R(z) = C_R^{n,\sigma} e^{-\frac{c^2 z^2}{2} z^{\sigma}} L_n^{-\frac{1}{2}(|\sigma-\frac{1}{2}|)} (c^2 z^2),
\] (42)
including the normalization factors, correspond to

\[
F_{L}^{n,\sigma}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma\left(n + \sigma + \frac{3}{2}\right)}} z^{\sigma+\frac{1}{2}} L_{n}^{\sigma+\frac{1}{2}}(e^{2}z^{2}) \quad (43)
\]

\[
F_{R}^{n,\sigma}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma\left(n + \sigma + \frac{1}{2}\right)}} z^{\sigma+\frac{3}{2}} L_{n}^{\sigma-\frac{1}{2}}(e^{2}z^{2}) . \quad (44)
\]

We can now compute the baryon CE considering the energy-momentum tensor

\[
T_{MN} = \frac{2}{\sqrt{|g|}} \frac{\partial \left( \sqrt{|g|} e^{-\phi} \mathcal{L}(\Psi, D\Psi, g) \right)}{\partial g^{MN}} \quad (45)
\]

with

\[
\mathcal{L} = \frac{i}{2} \bar{\Psi} e_{A}^{M} \Gamma^{A} D_{M} \Psi - \frac{i}{2} (D_{M} \Psi)^{\dagger} \Gamma^{0} e_{A}^{M} \Gamma^{A} \Psi - m_{z} \bar{\Psi} \Psi , \quad (46)
\]

evaluated in correspondence to the on-shell solutions. Using Eq. (31) we obtain:

\[
T_{00}^{n,\sigma}(z) = \Lambda \frac{e^{-\phi z^{2}}}{z} \lambda_{n,\sigma}^{2} \left[ (F_{L}^{n,\sigma}(z))^{2} + (F_{R}^{n,\sigma}(z))^{2} \right] , \quad (47)
\]

with the indices \(n, \sigma\) in the spectrum and in the solutions, and \(\Lambda\) a constant. This is the profile function \(F(z)\) that we use to compute the baryon CE.

4 Configurational Entropy for \(J^{PC} = 0^{++}\) and \(J^{P} = \frac{1}{2}^{+}\) hadrons

We can now evaluate the Configurational Entropy of \(J^{PC} = 0^{++}\) mesons and \(J^{P} = \frac{1}{2}^{+}\) baryons starting from Eqs. (23) and (47). For scalar mesons, QCD gauge invariant operators with different (canonical) dimension \(\Delta\) and \(J^{PC} = 0^{++}\) can be constructed in terms of quark and gluon fields:

\[
O = \bar{q}q, \quad O = Tr \left[ \frac{\alpha_{s}}{\pi} G_{\mu\nu} G^{\mu\nu} \right], \quad O = \bar{q}g_{s}\sigma_{\mu\nu} G^{\mu\nu} q, \quad O = \bar{q}qq,q , \quad (48)
\]

and so on. Analyses based on, e.g., QCD sum rules show that such operators have large vacuum-particle matrix elements with meson states interpreted in terms of constituent quark and gluons, hence conventional mesons, scalar glueball, hybrid scalar mesons and

\footnote{Since radiative effects are not taken into account in this description, operator anomalous dimensions are not considered.}
Figure 1: Configurational Entropy of $J^{PC} = 0^{++}$ mesons described by the QCD operators in Eq. (48) with different $\Delta$. $n$ is the radial quantum number.

scalar tetraquarks, respectively. Therefore, they can be used to investigate the regularities in CE. The fields dual to the QCD operators are characterized by a different mass $m$, and have already been studied in the holographic framework [19].

Using the expression (23), the CE can be computed for each value of $\Delta$ and radial number $n$. The result, obtained setting $c = 1$, is found to increase with $n$, as shown in Fig. 1: meson states corresponding to higher radial numbers have larger complexity content, as also observed in [12–14].

We are concerned with the dependence on the mass of the dual field, therefore on the operator dimension $\Delta$. CE increases with the dimension, indicating that states with a larger number of constituents are characterized by a larger Configurational Entropy, a behaviour shown in Fig. 2. For the same value of $n$, the scalar glueball has larger CE than $\bar{q}q$ mesons.

Similar results are obtained for $J^P = \frac{1}{2}^+$ baryons. In this case, the different operator dimensions $\Delta$ correspond to various $\sigma$ values in Eq. (38). Examples of operators interpolating ordinary baryons and pentaquarks are the Ioffe’s currents [22]

$$J^N(x) = \epsilon_{abc} \left( u^a_T(x) C \gamma_{\mu} u^b(x) \right) \gamma_5 \gamma^\mu d^c(x)$$

$$J^{PN}(x) = \epsilon_{abc} \left( u^a_T(x) C \sigma_{\mu\nu} u^b(x) \right) \gamma_5 \sigma^{\mu\nu} d^c(x)$$

(49)

for the proton (with $C$ the charge conjugation matrix and $a, b, c$ color indices), and

$$J^P(x) = \epsilon^{abc} \epsilon^{def} \epsilon^{gf} \left( u^a_T(x) C d^b(x) \right) \left( u^d_T(x) C \gamma_5 d^e(x) \right) C s^f_g(x)$$

$$J^{P}(x) = -\epsilon^{abc} \epsilon^{def} \epsilon^{gf} s^f_g(x) C \left( \bar{d}^e(x) \gamma_5 C \bar{u}^d_T(x) \right) \left( \bar{d}^b(x) C \bar{u}^b_T(x) \right)$$

(50)
Figure 2: Configurational Entropy of scalar mesons as in Fig. 1, plotted versus the QCD operator dimension $\Delta$. The results for the radial number $n = 0$ are magnified in the inset.

for a light pentaquark with strangeness $23$.

The operators involve an increasing number of quark and gluon fields, and their vacuum-particle matrix elements are found to be large in case of three-quark conventional hadrons ($\sigma = 4$) and pentaquarks ($\sigma = 7$). The calculation of CE starting form Eq. (47) gives again an increasing behaviour versus the radial number $n$, Fig. 3. Moreover, increasing $\sigma$, the Configurational Entropy is also found to increase, as shown in Fig. 4.

Figure 3: Configurational Entropy of $J^P = \frac{1}{2}^+$ baryons described by QCD operators with different $\sigma$. $n$ is the radial number.
and takes the minimum value for conventional baryons.

5 Conclusions

The Configurational Entropy accounts for information contained in the profile function. Using the soft-wall holographic model of QCD, it can be evaluated choosing, as a profile function, the function $T_{00}(z)$ computed in correspondence to the regular solutions of the equations of motion for fields dual to QCD operators involving several quark and gluon fields. In addition to the increase of CE with the radial number $n$, we found an increase, both in the scalar meson and in the baryon case, with the operator dimension related to the number and kind of quark and gluon fields the QCD operator is made of. A possible interpretation of this regularity is that smaller complexity characterizes ordinary meson and baryon configurations. Configurational Entropy can be relevant in disentangling hadronic states when the number and kind of their constituents is varied, hence conventional hadrons from exotica.

Acknowledgements. We thank F. De Fazio, F. Giannuzzi and S. Nicotri for discussions. This study has been carried out within the INFN project (Iniziativa Specifica) QFT-HEP.
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