Medium polarization and finite size effects on the superfluidity of the inner crust of neutron stars

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(Dated: June 3, 2008)

\begin{abstract}
The $^{1}$S\(_0\) pairing gap $\Delta$ associated with the inner crust of a neutron star is calculated, taking into account the coexistence of the nuclear lattice with the sea of free neutrons (finite size effects), as well as medium polarization effects associated with the exchange of density and spin fluctuations. Both effects are found to be important and to lead to an overall quenching of the pairing gap. This result, whose quantitative value is dependent on the effective interaction used to generate the single-particle levels, is a consequence of the balance between the attractive (repulsive) induced interaction arising from the exchange of density (spin) modes, balance which in turn is influenced by the presence of the protons and depends on the single-particle structure of the system.
\end{abstract}

PACS numbers:

\section{I. INTRODUCTION}

Neutron stars are possible remnants of supernova, an explosion signaling the death of a massive star when it has run out of nuclear fuel, displaying a spectacular rapid increase in brightness.

Theoretical models consistent with the experimental findings testify to the fact that a neutron star has a thin atmosphere and three internal regions referred to as: the outer crust, the inner crust, the core. In the outer crust, matter consists of spherical atomic nuclei and electrons. At the bottom of this region, neutrons begin to drip out of nuclei, thereby producing a neutron gas between nuclei. It is generally accepted that in the density range $0.001\rho_0 < \rho < 0.5\rho_0$, where $\rho_0 = 2.8 \cdot 10^{14}$ g cm\(^{-3}\) corresponds to nuclear saturation density, neutrons stars display a superfluid inner crust, consisting of a Coulomb lattice of nuclei permeated by a gas of free neutrons. The superfluidity of the inner crust has important consequences concerning different aspects of the physics of neutron stars, such as its heat capacity and thus its cooling rate. It is also responsible for a number of macroscopic quantum phenomena, such as quantized vortices (in rotating neutron stars) and quantized magnetic flux tubes (in magnetized neutron star cores). The interaction of vortices with nuclei forming the Coulomb lattice in the inner crust is thought to be connected with the presence of glitches in the pulsation periods of neutron stars and the postglitch relaxation of the associated period (cf. e.g. ref. \cite{1}).

For a consistent description of these phenomena it is necessary to take into account finite size effects arising from the coexistence of finite nuclei and free neutrons. Moreover, polarization processes can renormalize the interaction between particles in an important way, and thus strongly influence the superfluid properties of the system.

For example, in uniform neutron matter a strong quenching of the $^{1}$S\(_0\) pairing gap arising from the polarization of the medium is generally predicted \cite{2}, although no consensus has been reached concerning the actual intensity of this effect. Hence, superfluid properties can be, in principle, correctly described only if medium polarization processes (e.g. self-energy, induced interaction, vertex corrections) are properly taken into account \cite{3}.

The present paper represents a step along the path which leads to this ambitious goal. We divide the inner crust into spherical Wigner-Seitz (WS) cells of different densities, and take into account both finite size effects (arising from the presence of a nucleus at the center of the cell) and, among all possible medium polarization effects, those associated with the interaction induced by the exchange of medium fluctuations, which are expected to give the largest contribution at the low densities typical of the system under consideration. We shall use a theoretical scheme which can be applied both to finite nuclei \cite{4} and to the inner crust of a neutron star, the only essential difference between these two systems being the value of the Fermi energy. A preliminary study along these lines, limited to a single value of the neutron density, was published in ref. \cite{5}.

\section{II. THE MEAN FIELD}

We consider five regions of different densities in the inner crust previously studied by Negele and Vautherin in the WS cell approximation \cite{6}. The number of protons, $Z$, and the radius of the cell, $R_{WS}$, have been taken from their study, and are listed in Table \ref{tab1}.

We determine the single-particle states inside each cell in the Hartree-Fock (HF) approximation, making use of either the SLy4 or the SkM$^{*}$ Skyrme interactions \cite{7,8}. These HF calcu-
lations are analogous to those previously performed by other groups [9, 10]. We use wavefunctions vanishing at the edge of the cell, working on a grid with a mesh size of 0.1 fm. The boundary condition affects the single-particle density only in a region of \( \approx 3-4 \) fm from the edge of the cell (see Fig. 1). Single-particle levels up to 100 MeV have been computed. We have also performed calculations without protons, choosing the Fermi energy \( E_F \) so as to obtain about the same number of neutrons in the cell. We have verified that in the case where no protons are considered we reproduce, to a good accuracy, the pairing gap found in infinite uniform neutron matter [11]. The resulting densities and the neutron numbers obtained in the various cells without protons are reported in Table II. We also report the value of the effective mass, \( m_{\text{eff}} \), associated with the effective interaction used. The neutron and proton density distributions calculated with the SLy4 interactions turn out to be quite similar to those calculated in ref. [12]. They are shown in Fig. 1 in the case \( \rho_n = 0.01 \text{ fm}^{-3} \).

### Table II: Neutron number \( Z \), neutron number \( N \) and radius \( R_{WS} \) for the WS cells containing a finite nucleus.

| \( Z \) | \( N \) | \( R_{WS} \) (fm) |
|---|---|---|
| 40 | 458 | 42.2 |
| 50 | 1048 | 35.8 |
| 50 | 1314 | 33.2 |
| 50 | 1740 | 27.6 |
| 40 | 1474 | 19.6 |
| 40 | 4385 | 42.2 |

### Table II: Neutron density \( \rho_n \) (and its ratio with the neutron saturation density \( \rho_0 \)), Fermi momentum \( k_F \), Fermi energy \( E_F \), effective mass \( m_{\text{eff}}/m_{\text{bare}} \), neutron number \( N \) and radius \( R_{WS} \) for the WS cells without protons.

| \( \rho_n \) (fm\(^{-3} \)) | \( \rho_n/\rho_0 \) | \( k_F \) (fm\(^{-1} \)) | \( E_F \) (MeV) | \( m_{\text{eff}}/m_{\text{bare}} \) | \( N \) | \( R_{WS} \) (fm) |
|---|---|---|---|---|---|---|
| 0.0020 | 0.025 | 0.39 | 3.2 | 0.99 | 508 | 42.2 |
| 0.007 | 0.08 | 0.58 | 7.1 | 0.97 | 1074 | 35.8 |
| 0.010 | 0.12 | 0.68 | 10.0 | 0.96 | 1314 | 33.2 |
| 0.023 | 0.29 | 0.89 | 17.7 | 0.92 | 1760 | 27.6 |
| 0.052 | 0.65 | 1.16 | 33.5 | 0.83 | 1480 | 19.6 |
| 0.0014 | 0.017 | 0.35 | 2.18 | 1.00 | 508 | 42.2 |
| 0.010 | 0.12 | 0.68 | 9.6 | 1.00 | 1314 | 33.2 |
| 0.023 | 0.29 | 0.89 | 16.1 | 1.00 | 1760 | 33.2 |

### III. COLLECTIVE MODES

We have computed the excitation spectrum of the system within the framework of the Random Phase Approximation (RPA) in configuration space using the particle-hole basis states \( |j\pi_k\rangle, J M \). The residual interaction was determined as the second derivative of the mean field energy functional, that is, it was derived from the same two-body Skyrme force which determines the mean field, except for the two-body Coulomb and the two-body spin-orbit interactions which have not been included in the calculations. RPA phonons are classified according to their total angular momentum \( J \) and parity \( \pi \), while an index \( \kappa = 1, 2, \ldots \) runs over the states having the same \( J^\pi \) values with increasing energy. Because of the spin-orbit interaction, each phonon \( |\kappa, J^\pi \rangle \) is actually an admixture of \( S = 0 \) and \( S = 1 \) states. However, since the spin-orbit effect is very small for free neutrons, which dominate the RPA response, phonons with non-natural parity are almost pure \( S = 1 \) states, while natural parity states are either almost pure \( S = 0 \) or almost pure \( S = 1 \) states.

To obtain convergence in the matrix elements of the induced interaction, one needs to consider angular momenta up to about \( J = 30 \). For a given value of \( J \), the matrix elements converge rather rapidly with the phonon energy, which enters directly into the denominators of such matrix elements, (cf. Eqs. (3) below). The maximum size of the particle-hole space is 4000 particle-hole...
configurations. We have checked in few cases that this is enough to achieve convergence in the calculation of the pairing gap.

The HF+RPA calculation without protons could be tested against a calculation performed in neutron matter. We have checked that in our configuration space we are able to reproduce the response of the infinite uniform neutron matter to the external fields \( V_{\text{ext}}(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}} (S = 0 \text{ channel}) \) and \( V_{\text{ext}}(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}}\sigma_z \) \( (S = 1 \text{ channel}) \). Analytic formulae for such responses have been given in ref. [12]. The response function in the WS cells has been evaluated according to

\[
S(q, E) = \sum_{n=0}^{+\infty} |\langle n|e^{i\mathbf{q}\cdot\mathbf{r}}\Theta_n|0\rangle|^2 L(E, E_n), \tag{1}
\]

where \(|n\rangle\) are the (mean field or RPA) excited states of the system, \(L\) is a Lorentzian function used to smooth out the computed discrete response, and \(\Theta_n = 1 \text{ or } \sigma_z\) for \(S = 0\) and \(S = 1\) respectively. Because of the spherical symmetry adopted, we have expanded the external operator in multipoles and obtained the total response function adding the contributions from each multipolarity. For this comparison the SLy4 parametrization has been used. The analytical response function for uniform neutron matter is compared with the WS cell approximation in Figs. (2a) and (2b) for a neutron gas with a Fermi momentum \(k_F = 0.39\text{ fm}^{-1}\) and \(k_F = 1.16\text{ fm}^{-1}\) respectively. These two systems correspond to the lowest and highest density regions of the inner crust of a neutron star (WS cells with 508 and 1480 neutrons respectively). The comparison has been performed for a transferred momentum \(q\) equal to 0.51 fm\(^{-1}\) for the lower density region and 0.76 fm\(^{-1}\) for the higher density region.

The overall agreement found between the numerical and analytic results testifies to the fact that the WS cell approximation is a reliable tool to calculate the response of the free neutron gas which permeates the nuclear lattice. We assume that the same is also true for the case of a WS cell with a nucleus in the center. Within this context, a more detailed study of the RPA response will be given in a different paper.

### IV. INDUCED INTERACTION MATRIX ELEMENTS

The matrix elements of the induced pairing interaction in the \(^1S_0\) channel

\[
\langle \nu_{\text{ind}} \rangle = \langle ab; 00 | \nu_{\text{ind}} | cd; 00 \rangle, \tag{2}
\]

![Figure 2](image-url)

**FIG. 2:** The response functions \(S\) per unit volume as a function of the energy, computed in the WS cells at \(k_F = 0.39\text{ fm}^{-1}\) \((N = 508)\) (a) and \(k_F = 1.16\text{ fm}^{-1}\) \((N = 1480)\) (b), are compared to the analytical results in uniform neutron matter. Calculations are made with SLy4 interaction. Both unperturbed (HF) and RPA responses are compared. Solid lines correspond to the uniform neutron system. The linear momentum transferred from the external field is \(q = 0.51\text{ fm}^{-1}\) (a) and \(q = 0.76\text{ fm}^{-1}\) (b).

where \(|ab; 00\rangle\) \((a \equiv n\lambda\ell_\lambda)\), indicates a normalized two-particle state coupled to total angular momentum zero, are computed in Bloch-Horowitz perturbation theory \cite{12, 14, 15, 16}:

\[
\langle ab; 00 | \nu_{\text{ind}} | cd; 00 \rangle = \frac{\sum_{\text{int}} \langle ab; 00 | H_{\text{coupl}} | \text{int} \rangle \langle \text{int} | H_{\text{coupl}} | cd; 00 \rangle}{E_0 - E_{\text{int}}}, \tag{3}
\]

where \(E_{\text{int}}\) is the energy of the intermediate state made by two-quasiparticles and one phonon,

\[
E_{\text{int}} = E_b + E_d + E_\lambda. \tag{4}
\]

The quasiparticle energies are denoted by \(E_b\) and \(E_d\), while \(E_\lambda\) is the energy of the exchanged phonon and \(E_0\) is the binding energy of a Cooper pair. In keeping with the results of previous studies (cf. e.g. \cite{4, 17, 18, 19}) we have set \(E_0\) equal to \(-2\Delta_F\), where \(\Delta_F\) represents the value.
of the state-dependent pairing gap obtained solving the HFB equation, averaged over the single-particle states lying in the energy range \( E_F \pm 2 \text{ MeV} \). We also computed the second time ordering (see Eq. (2)), associated with the intermediate state of energy \( E_{\text{int}} = E_a + E_c + E_{\lambda} \).

\( H_{\text{coup}} \) represents the particle-vibration coupling Hamiltonian. It reads

\[
H_{\text{coup}} = \sum_{m \lambda} f_{m \lambda}^\dagger \left( A_{m \lambda}^\dagger O_{\lambda} + A_{m \lambda} O_{\lambda}^\dagger \right),
\]

where \( A_{m \lambda}^\dagger \) creates the particle-hole excitation \( |m(i)^{-1} \rangle \), \( O_{\lambda}^\dagger \) is the phonon creator operator, \( X_{n_j}^{\lambda} \) and \( Y_{n_j}^{\lambda} \) are the forward and backward amplitudes of the phonon \( |\lambda\rangle \) associated with the particle-hole component \( |n(j)^{-1} \rangle \). \( \bar{v}_{mjin} \) is the RPA symmetrized matrix element of the residual interaction \( v_{\text{res}} \). We treat the latter in the Landau approximation. Thus

\[
v_{\text{res}}(\vec{r}, \vec{r}') = \langle F_0 + F_0'(r) \cdot \vec{r} \rangle \delta(\vec{r} - \vec{r}') + [G_0(r) + G_0'(r) \cdot \vec{r}] \cdot \vec{r}' \delta(\vec{r} - \vec{r}').
\]

(7)

Exchange terms of \( v_{\text{res}} \) have not been considered. Moreover, as far as the computation of the induced interaction matrix elements is concerned, the terms associated with the momentum dependent terms of the NN effective interaction have been neglected \([4]\). An analysis of the influence of these terms will be presented in future work. We shall only consider the \( \tau_z \cdot \tau'_z \) term, in keeping with the fact that we are interested in the neutron-neutron pairing interaction. In uniform neutron matter, RPA phonons are characterized by their spin: density \( (S = 0) \) modes are associated uniquely with the spin-independent part of the interaction, given by \( C_0(r) \equiv F_0(r) + F_0'(r) \), while spin \( (S = 1) \) modes are produced by the spin-dependent part, \( C_0'(r) \equiv G_0(r) + G_0'(r) \). In the WS cell, one has to take into account also the proton degree of freedom \((\tau_z \cdot \tau'_z = -1)\) contribution to the RPA response. Moreover, because of the spin-orbit interaction, each phonon is an admixture of \( S = 0 \) and \( S = 1 \) states. While coupling to non-natural parity modes can take place only through the spin-dependent part of the interaction, coupling to natural parity modes can receive contributions from both channels, although the spin-independent part is usually the dominant one.

The explicit expressions for the vertices \( f \) and \( g \) in the \( S = 0 \) and \( S = 1 \) channels, coupling the neutron single-particle states to the \( \kappa \)-phonon of angular momentum and parity \( J^\pi \), are

\[
\begin{align*}
\langle j^m'|H_{\text{coup}}|jm, \kappa J^\pi M \rangle &= \langle j^m'|H_{\text{coup}}|jm, \kappa J^\pi M \rangle \\
&= \sum_{L=J-1} \langle j^m'|i^L [Y_L(\Omega) \times \sigma]_{JM} |jm \rangle \\
&\quad \int dr \phi_{\nu}(r)\{G_0 + G_0'(r)\delta \rho_{J^\pi L\nu}(r) + (G_0 - G_0'(r))\delta \rho_{J^\pi \kappa \nu}(r)\} \phi_{\nu}(r),
\end{align*}
\]

(9)

(8)

\( \phi_{\nu}(r) / r \) is the radial single-particle wavefunction associated with the quantum numbers \( \nu \equiv \{n \ell j\} \). In the above expressions \( \delta \rho_{J^\pi \kappa \nu} \) and \( \delta \rho_{J^\pi L\nu} \) are the neutron and proton contributions to the transition densities associated with the RPA modes \( |\kappa J^\pi \rangle \). For the \( S = 0 \) channel one finds

\[
\begin{align*}
\delta \rho_{J^\pi \kappa \nu}(r) &= \left[ \sum_{ph} \left( X_{ph}^\kappa(J^\pi) + Y_{ph}^\kappa(J^\pi) \right) \phi_{\nu}(r) / r \right. \\
&\quad \left. \times \phi_{\nu}(r) / r \langle j_p || i^L Y_{j_h} || j_h \rangle \frac{1}{\sqrt{2J + 1}} \right],
\end{align*}
\]

(10)

while in the \( S = 1 \) channel we have

\[
\begin{align*}
\delta \rho_{J^\pi \kappa \nu}(r) &= \left[ \sum_{ph} \left( X_{ph}^\kappa(J^\pi) - Y_{ph}^\kappa(J^\pi) \right) \phi_{\nu}(r) / r \right. \\
&\quad \left. \times \phi_{\nu}(r) / r \langle j_p || i^L [Y_L(\Omega) \times \sigma]_J || j_h \rangle \frac{1}{\sqrt{2J + 1}} \right].
\end{align*}
\]

(11)

The neutron and proton contributions are obtained summing over the neutron or proton particle-hole states.
Since the sign of the denominator in the expression of the induced pairing matrix element is always negative (see eq. 3), and since for scattering vertices there is an additional negative sign in the $S = 1$ channel as compared to the $S = 0$ channel [4], it follows that the exchange of vibrations gives rise to an attractive pairing interaction in the $S = 0$ channel, and to a repulsive interaction in the $S = 1$ channel. Hence, the induced interaction arising from the exchange of normal parity modes involves in general a mixture of attractive and repulsive contributions, while for non-normal parity modes only repulsive terms are present. One can show, however, that the repulsive contribution to the diagonal matrix elements $\langle n\ell j, n\ell j; 00 | v_{ind} | n\ell j, n\ell j; 00 \rangle$ associated with normal parity modes vanishes. These diagonal matrix elements are shown in Fig. 3(a) for the case where protons are considered ($^{136}$Sn), and in Fig. 3(b) for the case without protons, ($^{131}$X), corresponding to the density $\rho_n = 0.01$ fm$^{-3}$. They have been calculated with the SLy4 force. The matrix elements of the two systems are of the same order of magnitude ($\approx 0.01$ MeV), except for a few, much larger matrix elements which appear when protons are present. The large matrix elements connect two-particle states based on resonant single-particle states which have positive energy $\epsilon_{n\ell j}$ and are spatially localized within the nucleus (particularly on the nuclear surface). The largest contributions to these matrix elements arise from collective phonons with a well defined surface character, both for density and spin modes. As an example, let us focus on the diagonal matrix elements associated with the single-particle state $(n = 3, l = 7, j = 15/2; \epsilon_{3,7,15/2} = 2.02$ MeV), whose values are equal to -0.49 MeV for normal parity, and to +0.31 MeV for non-normal parity (see arrows in Fig. 3(a)). The transition densities of the phonons which give rise to the most important contribution to each of these large matrix elements are shown in Fig. 4 together with the single-particle wavefunction $\phi_{3,7,15/2}$.

![FIG. 3: Induced pairing interaction diagonal matrix elements (in the BCS limit) for the cells $^{136}$Sn (a) and $^{131}$X (b), computed with the SLy4 effective interaction. Positive (negative) matrix elements correspond to abnormal parity (normal parity) modes. Note the different scale of the two figures. The arrows indicate the two matrix elements discussed in the text, and associated with the transition densities shown in Fig. 4](image)

![FIG. 4: (a) Proton and neutron transition densities associated with the $J^\pi = 2^+$ normal parity phonon with energy 4.46 MeV for the cell $^{136}$Sn. The single-particle wavefunction $\phi_{n\ell j} = \phi_{3,7,15/2}$ is also shown. (b) The same, for the $J^\pi = 15^+$ (L=14) non-normal parity phonon at 15.27 MeV. There is no proton contribution to this transition density within our particle-hole space.](image)
solute value, than those associated with the SLy4 force. Moreover, the spin modes have matrix elements larger (in absolute value) than the density modes, while they are about the same for SLy4. This can be understood considering the Landau parameters factors $C_F$ and $C_G$ as a function of density in uniform neutron matter. As it can be seen in Fig. 5, in particular at low density, the parameters associated with the SkM* have much larger values (in absolute value); moreover $C_G$ is larger than $C_F$, while the opposite is true for SLy4.

![Fig. 5: Schematic representation of the BCS diagonal induced interaction matrix elements for the two cells $^{131}X$ (left panels) and $^{136}Sn$ (right panels) obtained making use of SLy4 (upper panels) and SkM* (lower panels) effective interactions. The grey area represents the range of the computed matrix elements (panel (a) corresponds to Fig. (3b), while panel (b) corresponds to Fig. (3a) except for the few, larger and scattered matrix elements).](image)

V. THE PAIRING GAP

The pairing matrix elements of the bare and induced interaction were used in a HFB calculation to study the superfluid properties of the system in the $^1S_0$ channel. The HFB calculation is self-consistent only in the particle-particle channel, while single-particle eigenstates and eigenvalues are kept fixed and equal to those computed in the HF calculation used to produce the mean field discussed previously.

We first consider pairing correlations at the mean field level, employing the Gogny interaction [20] as the bare interaction $v_{bare}$. This is rather well justified as long as the neutron density is smaller than, or of the order of 0.01 fm$^{-3}$ ($k_F = 0.7$ fm$^{-1}$), because the pairing gaps obtained with the Gogny or the bare interaction are quite similar in that density range. Instead for higher neutron densities and in the nuclear interior the Gogny interaction produces somewhat larger gaps. While this difference is important for a quantitative study of the pairing gap both in infinite and in finite systems [21], it may be neglected in the present context, where a qualitative discussion of the role pairing interaction plays in the neutron star crust is the main issue.

The state dependent pairing gap $\Delta_{nn'lj}$ associated with the two-particle state $|nlj, n'l'j\rangle$ is given by

$$\Delta_{n_1n_2lj_1j_2} = -\frac{1}{2} \sum_{n_2n_2'lj_2j_2} \sqrt{\frac{2j_2 + 1}{2j_1 + 1}} \left( \sum_q U^q_{n_2lj_2j_2} V^q_{n_2lj_2j_2} \right) \langle n_1lj_1j_1, n'_2lj_1j_2 | v_{tot} | n_2lj_2j_2, n'_2lj_2j_2 \rangle,$$

where $U^q_{nlj}$ and $V^q_{nlj}$ are the quasiparticle amplitudes obtained solving the HFB equations in the pairing channel, and $v_{tot} = v_{bare} + v_{ind}$. We have taken into account the single-particle levels contained in an energy window going from $E_F \pm 9$ MeV for the lowest density WS cell to
$E_F \pm 20$ MeV for the highest density WS cell. Increasing the window by 1 MeV or including one more multipolarity changes the pairing gap by less than 0.01 MeV in all cases.

A. Cells without protons

The values of $\Delta_F$ obtained with and without the induced interaction on top of the Gogny interaction, and calculated in the WS cells without protons, are shown by filled and open triangles in Fig. 7. It is seen that in uniform neutron matter the induced interaction quenches the value of the pairing gap, and that the suppression is stronger at large densities.

B. Cells with protons

We now turn our attention to the calculation including protons. The pairing field obtained for all the computed cells is shown in Fig. 8. The circles refer to the calculation with the nucleus at the center of the cell, while the triangles refer to the same gaps without nucleus already shown in Fig. 7.

In Fig. 8(a) we show the results obtained with the SLy4 interaction. One can see that the presence of the nucleus reduces the pairing gap by about 100-200 keV, both with and without the induced interaction. The result without the induced interaction is similar to that obtained in ref. [11], where it was shown that the pairing gap is reduced by about 200 keV close to the Fermi energy. This effect was attributed to the fact that pairing is quenched inside the nuclear volume, where the local Fermi momentum is larger than in the outer neutron gas. This can be seen in a clear way in Fig. 9(a) where we show the spatial dependence of the pairing gap for the case with nucleus at $\rho_n = 0.01$ fm$^{-3}$. This is obtained from our quantum calculation as explained in ref. [22]: first we construct the non local pairing field $\Delta(\vec{r}_1, \vec{r}_2)$; then we perform a Fourier transform respect to the relative coordinate $\vec{r}_1 - \vec{r}_2$, obtaining the gap as a function of the relative momentum $k$ and of the center of mass coordinate $R_{CM}$, $\Delta(k, R_{CM})$ (we neglect the weak dependence on the angle between $k$ and $R_{CM}$); finally we define a local pairing gap $\Delta_{loc}(R_{CM}) = \Delta(k_F(R_{CM}), R_{CM})$, where $k_F(R_{CM})$ is the local Fermi momentum. From this figure one observes that in fact the local pairing gap is reduced from 2.7 MeV far away from the nucleus, to 2.4 MeV in the interior, and thus the mean value of the gap in the whole cell is also reduced (the reduction would be...
stronger using the bare $v_{14}$ Argonne interaction instead of the Gogny interaction, cf. Figs. 5 and 10 of ref. [11]).

FIG. 8: Pairing gap calculated at the Fermi energy for the different WS cells. Empty and filled triangles refer to the calculation without protons, performed with the Gogny interaction only (empty triangles) and with the Gogny plus the induced interaction (filled triangles), already shown in Fig. 7. Empty and filled circles correspond to calculation with protons (nucleus placed at the center of the cell), performed with the Gogny interaction only (empty circles) and with the Gogny plus the induced interaction (filled circles). Mean field and RPA modes are computed making use of the SLy4 (a) and of the SkM$^*$ (b) interactions.

Turning now to the calculations including the induced interaction, considering the results obtained in neutron matter, where the pairing gap goes to zero for $k_F > 1.2$ fm$^{-1}$, one might expect that the pairing would almost vanish inside the nucleus (cf. crosses in Fig. 9(b)). Instead the actual local pairing gap resulting from the microscopic calculation (solid curve in Fig. 9(b)) shows that the gap inside the nucleus has a value of about 1.3 MeV, which represents a much smaller reduction. This is to be interpreted as a result of the enhancement of the attractive (negative) induced matrix elements due to the exchange of nuclear collective surface modes, as it was discussed in the previous section (cf. also Fig. 3). The gap in the outer neutron gas remains larger than in the interior of the nucleus, so that the resulting average gap in the cell is somewhat smaller than in the case without

FIG. 9: Pairing field $\Delta(k_F, R_{CM})$ for the cell $^{136}$Sn, calculated with the SLy4 mean field and the Gogny pairing interaction (a), with the SLy4 mean field and the Gogny+induced pairing interaction (b), with the SkM$^*$ mean field and the Gogny pairing interaction (c) and with the SkM$^*$ mean field and the Gogny+induced pairing interaction (d). The crosses in (b) and (d) are obtained dividing the pairing field shown in (a) and (c) respectively, by the quenching factor calculated in uniform neutron matter.
VI. CONCLUSIONS

The effect of the induced interaction arising from the exchange of density and spin modes of the system results in a quenching of the neutron pairing gap, both in the case of uniform neutron matter and in the case of the inner crust of a neutron star.

The pairing gap obtained is found to be strongly dependent on the effective nucleon-nucleon interaction employed to determine the single-particle properties of the system and its linear response (we performed the calculation making use of SLy4 and SkM* Skyrme interactions). A much stronger suppression of the gap is obtained in uniform neutron matter when the SkM* interaction is used, due to the stronger residual interaction associated with this NN effective force and to the different balance of spin and density modes.

When a nuclear cluster is present in the cell, the local Fermi momentum increases and the pairing gap calculated with the bare interaction is reduced in the interior of the nucleus. The induced interaction quenches the gap both inside in the nucleus and in the outer neutron sea: the relative amount depends on the interaction, but the reduction inside the nucleus is much smaller than that occurring in neutron matter at the corresponding Fermi momentum.

VII. ACKNOWLEDGEMENTS

We thank G. Gori for fruitful discussions and for the great help he gave us. G. Colb provided the code employed in the RPA calculation. The support of the supercomputing group at the Consorzio Interuniversitario Lombardo Elaborazione Automatica (CILEA), Segrate, Italy, has been essential for the calculations. F.B. acknowledges partial support from the Spanish Education and Science Ministry projects FPA2006-13807-c02-01, FIS2005-01105 and INFN08-33. The work of S.B. is supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC). TRI-UMF receives federal funding via a contribution agreement through the National Research Council of Canada.

VIII. APPENDIX A

The detailed calculations of the pairing gap performed in the WS cell without protons can be checked against a much simpler estimate in neutron matter, using a similar assumption for the residual interaction, which is taken into account within the Landau approximation, neglecting the momentum dependence of the Landau parameters. However, the response function will also be estimated within the Landau approximation, while in the microscopic calculation in the WS cell the full RPA response has been considered. The matrix elements of the induced interaction in infinite neutron matter are obtained summing the contributions $v_{ind}^{S=0}$ and $v_{ind}^{S=1}$ in the $S = 0$ and $S = 1$ channels. They are obtained multiplying the square of the Landau parameters $C_F$ (density modes) and $C_G$ (spin modes) by the integral of the RPA response function $R^{S=0}(q)$ (density modes) and $R^{S=1}(q)$ (spin modes) [23, 24]. For density modes one has

$$v_{ind}^{S=0}(k_1, k_2) = C_F^2(k_F) \int_{q_{min}}^{q_{max}} dq q R^{S=0}(q),$$

where $k_1$ and $k_2$ are the momenta of the states exchanging the vibrations. The integration limits are equal to $q_{min} = |k_1 - k_2|$ and to $q_{max} = k_1 + k_2$. One has

$$R^{S=0}(q) = -\frac{1}{2} \frac{N(0)L(q)}{C_F(k_F)L(q)}$$

where $L(q)$ is the Lindhard function and $N(0)$ is the density of single-particle states at the Fermi energy, $N(0) = k_F m^* / \pi^2$. The contribution from spin modes is correspondingly obtained from $R^{S=1}$, where

$$R^{S=1}(q) = -\frac{1}{2} \frac{N(0)L(q)}{C_G(k_F)L(q)}$$

and from

$$v_{ind}^{S=1}(k_1, k_2) = -\frac{3 C_G^2(k_F)}{k_1 k_2} \int_{q_{min}}^{q_{max}} dq q R^{S=1}(q),$$

where the factor 3 is associated with the three spin projections in the $S = 1$ channel.
The pairing gap is then calculated solving the BCS equations with the complete interaction, \( v_{tot}(k_1, k_2) = v_{bare} + v_{ind}^{S=0} + v_{ind}^{S=1} \). In Fig. 7 we have compared these simple expressions with the microscopic calculations in the Wigner cell without nuclei.

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