Tests for Cosmological Evolution of a Brane Universe Model

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Abstract

The relativistic Friedmann Lemaitre cosmology model (FLCM) is very sucessful to describe the evolution history of the Universe from the "First three Minutes". Any alternative model should be consistent with the FLCM explanations to the later stage evolutions of the Universe at certain points. An noncompact extra dimension model was recently proposed by Randall and Sundrum. Binetruy et al. obtained the modified Friedmann equation, in which the energy density of the brane appears quadratically in contrast with the linear behavior of the standard Friedmann equation. We investigate kinds of classical cosmological effects of the new models and get a general solution of the cosmic evolution for this extended model, with more detail discussions of the brane tension parameter on these cosmological tests.

1 Introduction

Brane world cosmology seems to provide an alternative explanation for the present accelerating stage of the Universe[1] without introducing either a cosmological constant or an evolutional quintessence-like component. The basic idea in these scenarios is the existence of a higher dimensional bulk in which our Universe is sitting as a hypersurface, named 3-brane. The particle physics standard model matter fields are located in the brane while gravity propagates in all dimensions, namely bulk. The large extra dimensions also can solve the mass hierarchy problem of the standard particle physics model. Previously it has been assumed that the extra dimensions are compact to be unobservable small, as the so-called Kaluza-Klein model[2, 3]. Recently Randall and Sundrum relaxed the constraints of Kaluza-Klein model and proposed a new idea [4, 5] that the extra dimension is noncompact, which was also proposed earlier by Wesson[7]

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(and for the equivalence between them see for example, J.Leon, Mod. Phys. Lett. A16 (2001) 2291), i.e., the extra dimension is large and conceivable.

The relativistic Friedmann-Lemaitre cosmology model (FLCM) is very successful to describe the evolution history of the Universe from the "First three Minutes". Any alternative models should match the FLCM explanations to the later stage evolutions of the Universe at certain points. How to test the Randall-Sundrum model? The best way is to investigate its cosmological effects, which can provide constraints on these type of models. Binetruy et al. have researched the cosmological evolution of the Universe and shown that the equations governing the cosmological evolution of the brane will be different from the analogous Friedmann equations of standard cosmology [8], i.e., the energy density of the brane Cosmology appears quadratically in the new Friedmann equation in contrast with the linear behavior of the usual equation. To reconcile brane cosmology with the standard cosmological scenario, they put a cosmological constant in the bulk and introduce in the brane, in addition to ordinary matter fields, a constant tension that exactly compensates the bulk cosmological constant, so that the quadratic term would be cancelled, to leave the linear term dominated in low energy evolution stages as argued. In this paper we mainly discuss the additional quadratic brane density term of the brane cosmology model which is obviously different from the standard Friedmann one, trying to give more qualitative relations to see the effects from the unconstrained tension parameter, with the hope to give more physics limits to the parameter.

First, we briefly review the solutions presented by Binetruy et al [8], from their generalized Friedmann equation,

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^4}{36} \rho_b^2 + \frac{C}{a_0^4} - \frac{k}{a_0^2} + \frac{\kappa^2}{6} \rho_B$$

where $a_0$ is scale factor in brane when the coordinate of the fifth dimension $y = 0$. The constant $\kappa$ is related to the five-dimensional Newton’s constant $G(5)$ and the five-dimensional reduced Planck mass $M(5)$ by the relations $\kappa^2 = 8\pi G(5) = M(5)^{-3}$. The $\rho_B$ and $\rho_b$ are the energy density in bulk and in brane respectively. The $C$ is an integral constant that may be interpreted as a dark radiation term for the sake of radiation dominated behavior form, and $k$ is the spatial curvature. From Eq.(1), we know that square of Hubble parameter $H^2 = \dot{a}_0^2/a_0^2$ is quadric of the energy density in brane $\rho_b$, while it is linear to $\rho_b$ in the standard cosmology. So it must be able to be reduced to the standard theory, i.e. linear to $\rho_b$, if it is reasonable. Binetruy, et al. assumed that the energy density in the brane could be decomposed in two parts,

$$\rho_b = \rho_\Lambda + \rho (2)$$

where $\rho_\Lambda$ is a constant that represents an intrinsic tension of the brane and $\rho$ stands for the ordinary energy density in cosmology. Substituting in Eq.(1) one gets

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^2}{6} \rho_B + \frac{\kappa^4}{36} \rho_\Lambda^2 + \frac{\kappa^4}{18} \rho_\Lambda \rho_b + \frac{\kappa^4}{36} \rho_b^2 + \frac{C}{a_0^4} - \frac{k}{a_0^2}$$

Following Randall and Sundrum[5], Binetruy, et al. by choosing $\rho_\Lambda$ such that

$$\frac{\kappa^2}{6} \rho_B + \frac{\kappa^4}{36} \rho_\Lambda^2 = 0$$

(4)
and we then get
\[
\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho + \frac{\rho^2}{2\lambda}) + \frac{C}{a^2} - \frac{k}{a^2} \tag{5}
\]
where it is assumed that \(8\pi G = \frac{\kappa}{\rho \Lambda^4}\) and \(\lambda = 1/\rho \Lambda\). If the brane tension parameter \(\lambda \to \infty\) and the integral constant \(C = 0\), Eq.(5) will be reduced to the standard Friedmann equation
\[
\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{6}
\]
Throughout this paper we neglect the possibility of the existence of a cosmological constant on the brane for the early universe evolution and for simplicity[6], to focus on the brane tension effects. For the inclusion of a cosmological constant or a quintessence-like scalar field with negative equation of state is now under our consideration in another work to appear.

We have in addition the usual equation of conservation for the energy-momentum tensor of the cosmic fluid given by
\[
\dot{\rho} + 3H(p + \rho) = 0 \tag{7}
\]

In this paper, we assume that the integral constant \(C\) in Eq.(5) is equal to zero for simplicity, or for the later evolutions of the Universe the dark radiation term negligible, and \(k = 0\) as the recent observational data suggest that the Universe is flat[9, 10, 11]. In this framework we mainly discuss several classical observational tests for this enlarged brane cosmology model to see the effects of the additional term.

2 The critical density

It is convenient to discuss dimensionless quantity as we often do in physics investigation. In the standard cosmology, for convenience, the density fraction of the world models is expressed in terms of a critical density, which is defined to be the density when the curvature \(k = 0\), i.e., \(\rho_{c,\text{standard}}^\text{standard} = 3H^2/8\pi G\). It is similar that we can define the critical density in the brane Universe models. By choosing the integral constant \(C = 0\) in Eq.(5), we get
\[
H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho + \frac{\rho^2}{2\lambda}) - \frac{k}{a^2} \tag{8}
\]
where, for convenience, we have omitted the subscript of the scale factor, which would not lead to confusion in this paper. By assuming the curvature \(k = 0\) in Eq.(6), we deduced the critical density
\[
\rho_c = \sqrt{\lambda^2 + \frac{3H^2\lambda}{4\pi G} - \lambda} \tag{9}
\]

For convenience, we define the dimensionless brane tension parameter \(\lambda' = 8\pi G\lambda/3H_0^2\). Note that the critical \(\rho_c\) is a monotone increasing function of brane tension parameter \(\lambda'\), and that when \(\lambda \to \infty\), \(\rho_c\) reduces to \(\rho_{c,\text{standard}}^\text{standard}\), as we expect. More details of the
relationship between the critical density \( \rho_c \) and brane tension parameter are shown in Figure 1. The critical density sharply increases with \( \lambda \) in the lower values of \( \lambda' \), and the curve becomes flat when \( \lambda' > 2 \). When \( \lambda' \) is in larger values, in the Eq.(8), the linear term is the leading term, and the quadratic term gives small correction to the standard Friedmann equation; when \( \lambda \) is in smaller values, the quadratic term becomes dominated, which indicates that the brane tension plays an important role in the energy density of the brane cosmology model.

3 A general evolution solution of the brane Universe

In the standard cosmology, one usually uses the comoving coordinates to describe the dynamical evolution states, in which the coordinates of any place don’t vary with time, and the evolution of the Universe is described by a scale factor that is a function of cosmic time. If one knows the relationship between the scale factor and the cosmic time, then the global dynamical behavior about the evolution of the Universe is clear. For example, one can directly deduce the Hubble parameter and the deceleration parameters as functions of time, and the dependence relation of the luminosity distance
on redshift, and so on. So for a cosmological model to get its dynamical evolution solution, we will be able to obtain kinds of observable quantities to be compared with the experimental data for the model testings.

In this section we will consider a general case, where the state equation of matter and energy is

\[ p = w \rho \]  \hspace{1cm} (10)

and \( w \) is assumed to be a constant for simplicity in this paper. From Eqs.(7) and (10), one gets

\[ \rho = \rho_0 a^{-n} \]  \hspace{1cm} (11)

where \( \rho_0 \) is a constant standing for the energy density of the Universe today, \( n \) is a parameter related to the equation of state parameter \( w \) by \( n = 3(1 + w) \). For recent observational data strongly suggest that the Universe is flat, i.e. \( k = 0 \), so we only investigate the case when \( k = 0 \). Substituting Eq.(11) in Eq. (8) and taking \( k = 0 \), we get

\[ H^2 = H_0^2 \Omega_0 a^{-n} + \frac{H_0^2 \Omega_0^2}{2\lambda'} a^{-2n} \]  \hspace{1cm} (12)

where \( H_0 \) is the Hubble parameter today, \( \Omega_0 \) and \( \lambda' \) are defined by \( \Omega_0 = 3H_0^2\rho_0/8\pi G \) and \( \lambda' = 8\pi G\lambda/3H_0^2 \) respectively. Assuming the initial condition \( a(t)|_{t=0} = 0 \), we get a general solution of Eq.(12)

\[ a(t) = \left[ \left( \frac{\sqrt{n^2 H_0^2 \Omega_0} t}{2} + \sqrt{\frac{\Omega_0}{2\lambda'}} \right)^2 - \frac{\Omega_0}{2\lambda'} \right]^{1/n} \quad n > 0 \]

\[ a(t) \propto \exp \left( \sqrt{\frac{2\lambda' H_0^2 \Omega_0}{2\lambda' + \Omega_0}} t \right) \quad n = 0 \]  \hspace{1cm} (13)

The last solution is consistent with the inflation explanation if the energy scale is very high or we may use it to describe the Universe later accelerating expanding stage without a cosmological constant when the energy scale is very low, as observed today. Assuming \( \Omega_0 = 1 \), we get the relationship between the scale factor and the cosmic time with different parameters respectively, which are shown in Figure 2. Comparing the two curves about \( n = 2 \), we find that for the brane tension parameter smaller, the scale factor \( a \) will increase faster, and for the case of \( n = 3 \) we get the same conclusion. In another way, the curves with larger state equation parameter \( n \) grow faster in one Hubble time scale in contrast to ones with smaller parameter \( n \) when we fix the value of \( \lambda' \), for example 10.

### 4 The deceleration parameter

Following the fact that our Universe is expanding, one may ask a question that the expansion of the Universe is accelerating or decelerating, or how fast it expands. So, besides the Hubble parameter, the deceleration parameter is another important parameter describing the evolution of the Universe to measure the rate of slowing of the expansion, which is defined by

\[ q = \frac{\dddot{a}}{a^2} \]  \hspace{1cm} (14)
where $q$ is dimensionless. Substituting Eq. (12) and its derivative in Eq. (14), one gets

$$q = \frac{(n - 2)\lambda' a^n + (n - 1)\Omega_0}{2\lambda' a^n + \Omega_0}$$

(15)

In the standard cosmology, the deceleration parameter $q$ is a function of the present density parameter $\Omega_0$ and the state equation parameter $n$. It is here also a function of the brane tension parameter $\lambda'$ in the brane cosmology, which we can see from the above equation. In the present epoch the scale factor can be set $a = 1$, so we can get the deceleration parameter today,

$$q_0 = \frac{(n - 2)\lambda' + (n - 1)\Omega_0}{2\lambda' + \Omega_0}$$

(16)

Figure 3 shows that the relationship between the present deceleration parameter and the brane tension parameter $\lambda'$ for $n = 2, 3, 4$, respectively. The present deceleration parameter is a monotone function of the brane tension parameter. The present deceleration parameter sharply falls for lower values of the brane tension parameter, and the curves become flat when $\lambda' > 2$.

From mainly the recent SN Ia and Cosmic Microwave Background observations[1] a cosmological constant-like dark energy may exist today which powers the Universe
expansion accelerating with the decelerating parameter \(-1 < q < 0\). Therefore this observational constraint is not satisfied in the present model without a cosmological constant-like term. Certainly we need more SN Ia observational data to confirm the important constraint that is just the main mission of the near future SNAP project. When the brane parameter approaches infinite large, as in the low energy scale case in our present discussion, the decelerating parameter goes to the standard cosmological one without the dark energy existence. The brane world cosmology model with a cosmological constant-like term for a possible negative decelerating parameter is under our investigation in the paper to appear[12], but the analytic relations are more complicated than the present ones.

5 The cosmic time-redshift relation and the age of brane Universe

An important result for many of cosmology models is the relation between cosmic time \(t\) and redshift \(z\), from which one can calculate the age of the Universe. So one can compare the calculated age of the Universe and the age of the oldest stars in globular clusters to give constraints on the cosmological models. Because \(a = (1 + z)^{-1}\), it
follows immediately from Eq. (12) that

$$\frac{dz}{dt} = -(1 + z) \left[ H_0^2 \Omega_0(1 + z)^n + \frac{H_0^4 \Omega_0^2}{2 \lambda'} (1 + z)^{2n} \right]^{1/2}$$

(17)

Cosmic time $t$ measured from the Big Bang following by the integration

$$t(z) = \int_0^t dt = -\frac{1}{H_0} \int_{(1 + z)}^\infty \frac{dx}{(1 + x) \left[ \Omega_0(1 + x)^n + \frac{\Omega_0^2}{2 \lambda'}(1 + x)^{2n} \right]^{1/2}}$$

(18)

This integral is difficult to evaluate, so we only work out the numerical solutions for different values of the brane tension parameter, which are shown in Figure 4. The cosmic time $t$ is a decreasing function of the redshift $z$. From the curves for $\lambda' = 10$, we can see that larger state equation parameter implies larger age of the Universe; from the two curves for fixed $n = 3$, $\lambda' = 10$ and 100 respectively, we know that larger brane tension parameter also implies larger age of the Universe.

![Figure 4: The age of Universe as a function of redshift z](image-url)
6 The dependence of luminosity distance on redshift

Hubble parameter is one of the best parameters describing the evolution of Universe, to express the expanding rate, but it can’t be measured directly. We have to measure it indirectly, i.e., to deduce it from other parameters which can be measured. In the standard cosmology, one gets its value from Hubble’s law, which is the linear relationship between the “distance” to a galaxy and its observed redshift. So if we know the relationship between the “distance” and the redshift, we will know the value of the Hubble parameter. Usually, one uses the luminosity distance as the “distance” to a galaxy, which is defined by $d_L^2 \equiv \mathcal{L}/4\pi F$, where $\mathcal{L}$ is the absolute luminosity of a galaxy and $F$ is the measured energy flux, i.e., the energy per time per area measured by a detector.

![Figure 5: The luminosity distance as a function of redshift $z$](image)

In order to compare the results of the model with cosmological tests we must deduce the relationship between the luminosity distance and the redshift from other relations, see Refs [13, 14]. The deduced relationship between the luminosity distance and the
redshift from the relationship between Hubble parameter and the redshift, as follows

\[
d_L = (1 + z) \begin{cases} 
\sin(\sqrt{|\Omega_k|}d_C)/\sqrt{|\Omega_k|} & k = 1 \\
\sinh(\sqrt{|\Omega_k|}d_C)/\sqrt{|\Omega_k|} & k = 0 \\
-\sinh(\sqrt{|\Omega_k|}d_C)/\sqrt{|\Omega_k|} & k = -1
\end{cases} \tag{19}
\]

where \(d_C\) is defined by

\[
d_C = \int_0^z H_0 \frac{dx}{H(x)} \tag{20}
\]

Substituting \(a = (1 + z)^{-1}\) in Eq.(12), we get

\[
H(z) = H_0 \left[ \Omega_0 (1 + z)^n + \frac{\Omega_0^2}{2\lambda'}(1 + z)^{2n} \right]^{1/2} \tag{21}
\]

Figure 5 shows the relationship between the luminosity distance and the redshift when brane tension parameter \(\lambda'\) and state equation parameter \(n\) are taken to various values respectively. The curves about \(\lambda' = 10\) show that the luminosity distance increases faster as the redshift increases when the state equation parameter \(n\) is smaller. From the two curves when \(n = 3\) and \(\lambda'\) is equal to 10, 100 respectively, we know how the brane tension parameter affects the dependence of luminosity distance on the redshift, i.e., when \(\lambda'\) is larger the luminosity increases faster as the redshift increases.

7 Conclusions

In this paper, following the framework of Binetruy et al.[8], we investigated kinds of cosmological effects of the brane Universe without the explicit a cosmological constant-like term. First, we deduced the critical density in the brane Universe, which is different from that in the standard cosmology and is a function of brane tension parameter. Then we found a general solution of the brane Universe and discussed the dependence of the scale factor on the brane tension parameter as well as the state equation parameter. We discussed the relationship between the deceleration parameter and the brane tension parameter, and found that the deceleration is a monotone decreasing function of the brane tension parameter \(\lambda'\). We also discussed the cosmic time-redshift relation and the dependence of the age of the brane Universe on the brane tension parameter. Finally, we discussed the dependence of luminosity distance on the redshift for different values of the brane tension parameter \(\lambda'\) and the state equation parameter.

We illustrate that when the brane parameter goes infinite large all the results approach the standard Friedmann cosmology ones without a cosmological constant-like term. The dark energy can be accomodated in the standard Friedmann cosmology based on the relativistic gravitational theory, G.R. with a cosmological constant-like term that may accelerate our deep understanding to the implying physical world as well as the fundamental theories describing it, and it can also be possibly interpreted in the extended gravity scenarios and cosmological models, which attract our further continuous research work.
So far brane cosmology is still under frequent discussions with many theoretical speculations, like the interesting one by using it trying to describe the Universe accelerating observations. Surely, we can argue that for the low energy limit the brane cosmology model can be returned the standard cosmology model. But for trying to explore the matching conditions we need more detail analysis to the universe classical tests with the brane tension parameter.

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