An Unsupervised Bayesian Neural Network for Truth Discovery

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Abstract—The problem of estimating event truths from conflicting agent opinions is investigated. An autoencoder learns the complex relationships between event truths, agent reliabilities and agent observations. A Bayesian network model is proposed to guide the learning of the autoencoder by modeling the dependence of agent reliabilities corresponding to different data samples. At the same time, it also models the social relationships between agents in the network. The proposed approach is unsupervised and is applicable when ground truth labels of events are unavailable. A variational inference method is used to jointly estimate the hidden variables in the Bayesian network and the parameters in the autoencoder. Simulations and experiments on real data suggest that the proposed method performs better than several other inference methods, including majority voting, the Bayesian Classifier Combination (BCC) method, the Community BCC method, and the recently proposed VISIT method.

Index Terms—truth discovery, unsupervised learning, autoencoder, Bayesian network, social network.

1 INTRODUCTION

It is common for agents in the same crowdsourcing or social sensing network to report conflicting opinions [1], [2], [3], [4], [5], [6], [7]. Some of the agents are unreliable and maybe biased. The majority voting method fuses the agents’ opinions together by treating the opinions from a majority of agents as the estimated truth. This is based on the assumption that all agents have the same reliability [8]. This assumption may not be reasonable when agents are from different backgrounds and their reliabilities or biases vary widely. Truth discovery methods have been proposed to estimate event truths in consideration of agent reliabilities.

The relationships between event truths, agent reliabilities and agent observations are complex. To model these relationships, various assumptions of agent reliabilities are adopted in the literature. The papers [6], [9], [10], [11], [12] developed probabilistic models for truth discovery from binary observations with an agent’s reliability being the probability an event is true given that the agent reports it to be true. Multi-ary observations are considered in [13], [14], [15], [16]. A Bayesian method named TruthFinder was proposed by [13] to iteratively estimate the probability that each agent is correct and the event truths. In [14], a Bayesian method named AccuSim was developed to learn if agents copy opinions from others, their reliabilities and the event truths. The CATD method was proposed in [15] for the case where most agents provide only limited opinions. In [16], a hidden Markov model is used to infer event truths and agent reliabilities that evolve over time, and [17] used a maximum likelihood estimation approach to estimate event truths when each agent’s reliability may vary across events.

In [18], the authors proposed a method called Bayesian Classifier Combination (BCC) for truth discovery using a confusion matrix to represent the reliability of each agent. The use of confusion matrices generally outperforms models that use scalar agent reliabilities, as demonstrated by [19]. In this paper, BCC was shown to be amongst the best methods in decision making and single-label tasks. However, it is difficult to infer accurately an agent’s reliability if it observes only a small subset of events. To mitigate this problem, the Community BCC (CBCC) model was proposed by [20], which grouped agents with similar backgrounds into communities and assumed that the confusion matrix of an agent is a perturbation of the confusion matrix of its community. The papers [5] and [21] showed that agents in a crowd are related through social ties and are influenced by each other. In [22], the authors adopted a model in which an agent can be influenced by another agent to change its observation to match that of the influencer, while [11] assumed that agents’ dependency graphs are disjoint trees. This was extended to general dependency graphs in [6], [12].

In our previous work [23], we considered the use of social network information and community detection to aid in truth discovery. We called our approach VISIT. The relationships among event truths, agent reliabilities and agent observations are modeled by categorical distributions with confusion matrices as parameters. This assumption limits the flexibility of the model to fit complex real data.

Neural networks have shown good promise in modeling nonlinear relationships in many applications [24], [25]. Neural networks are effective in the truth discovery problem only if the following issues are properly resolved:

(a) Unsupervised learning: For the truth discovery problem, it is often difficult to obtain enough ground truth labels of events to learn a supervised model. Thus, developing an unsupervised model is important and meaningful.

(b) Modeling interpretable structures: In the truth discovery problem, the observations from agents often imply hidden structures. For instance, agents having similar
background, culture, and socio-economic standing may form communities and share similar reliabilities [20], [23]. Successfully discovering the hidden structures can improve the performance of the truth discovery model. However, neural networks are not good at modeling interpretable structures [26].

(c) **Dealing with dependency**: The interpretable structures in item (b) result in different data samples to have dependencies, which violates the independence assumption that is used in many neural network based methods [27].

In this paper, to solve the first issue, an autoencoder is used to learn the complex relationship among event truths, agent reliabilities and agent observations. Autoencoders [28] are a kind of unsupervised artificial neural networks widely used to learn data features. However, the optimization process of an autoencoder is easily stuck in less attractive local optima [29], thus proper model constraints are required to obtain better performance. The constraints are introduced by Bayesian networks in our model. Bayesian network models provide a natural way to characterize the relationship among variables in an unsupervised way [30], [31], [32]. In this paper, a Bayesian network model is proposed to model communities and further constrain the learning of the autoencoder by modeling the dependencies between agent reliabilities (a variable in our autoencoder) corresponding to different data samples. In summary, our approach combines the strengths of neural networks in modeling nonlinear relationships and the strengths of Bayesian networks in characterizing hidden interpretable structures.

**Notations**: We use boldfaced characters to represent vectors and matrices. Suppose that $M$ is a matrix, then $M(m, \cdot)$, $M(\cdot, m)$, and $M(m, n)$ denote its $m$-th row, $m$-th column, and $(m, n)$-th element, respectively. The vector $(x_1, \ldots, x_N)$ is abbreviated as $(x_i)_{i=1}^N$ or $(x_i)$ if the index set that $i$ runs over is clear from the context. The $i$-th element of a vector $x$ is $x(i)$. Let $1$ denote a column vector of all $1$'s and $I$ be the identity matrix. We use $\text{Cat}(p_1, \ldots, p_K)$ and $\mathcal{N}(\mu, \Sigma)$ to represent the categorical distribution with category probabilities $p_1, \ldots, p_K$ and the normal distribution with mean $\mu$ and covariance $\Sigma$, respectively. The notation $\sim$ means equality in distribution. The notation $y \mid x$ denotes a random variable $y$ conditioned on $x$, and $p(y \mid x)$ denotes its conditional probability density function. $\mathbb{E}$ is the expectation operator and $\mathbb{E}_q$ is expectation with respect to the probability distribution $q$. We use $I(a, b)$ to denote the indicator function, which equals $1$ if $a = b$ and $0$ otherwise. We use $|S|$ to represent the cardinality of the set $S$. The vectorized version of $M$ with the columns of $M$ stacked together as a single column vector is denoted as $\text{vec}(M)$, while $\text{one\_hot}(x)$ is the one-hot representation of $x$ in a given set that will be clear from the context.

## 2 Learning Architecture

Suppose that $N$ agents with different reliabilities observe $J$ events and each event can be in $R$ possible states. We consider the problem of estimating event truths from conflicted agent opinions when each agent only observes a subset of the events. The symbols used in this paper are summarized in Table 1.

In our model, the observation matrix $M$ is an $N \times J$ matrix and $M(n, j)$ represents the opinion of agent $n$ about event $j$. An entry $M(n, j)$ is null if the agent $n$ does not observe event $j$. For non-null $M(n, j)$, we assume that it is generated from $C_n$, which represents the reliability of agent $n$’s opinion about the ground truth state of event $j$, denoted as $\theta_j$. The reliability matrix $C_n$ is a $R_1 \times R_2$ matrix, where $R_1$ and $R_2$ are two known hyper parameters. In this paper, we use a general matrix to represent an agent’s reliability, and this includes both the reliability concepts of [10], [13], [14], [15], [16] and confusion matrix of [18], [19], [20]. We also assume that a social network connecting the agents is known and its graph adjacency matrix is given by $A$, where $A(n, m) = 1$ iff agent $n$ and agent $m$ are connected. Our target is to estimate $\theta \triangleq (\theta_j)_{j=1}^J$ from $M$ and $A$.

### 2.1 Observation Model

Let $C = (C_n)_{n=1}^N$. The relationship between $M$ and $(C, \theta)$ is complex and nonlinear, and thus it is challenging to find a proper analytical model for it. Moreover, a data set with a large number of accurate labels is usually unavailable, which hinder the application of supervised learning in the truth discovery problem. To solve these two issues, we model the relationship between $(C, \theta)$ and $M$ with a multi-layer neural network and perform inference using an unsupervised autoencoder in Section 3.

We represent the observation model for $M$ by a neural network that learns to decode the agent reliabilities and event states back to the observations. The input to the observation model is $(C_n, \theta_j)$ for $n = 1, \ldots, N$ and $j = 1, \ldots, J$. All the layers of the observation model are fully connected. Let its parameters be $w_D$. From the output layer of the observation model, we obtain an $R \times 1$ vector $d_{n,j}$. We assume

$$\text{one\_hot}(M(n, j))(r) \sim \text{Bern}(d_{n,j}(r)), \quad (1)$$

for all $r \in [1, R]$, where Bern($\cdot$) denotes the Bernoulli distribution.

### 2.2 Bayesian Model Constraints

One problem of learning the observation model is that optimizing its neural network weights can become stuck in less attractive local optima [29]. To mitigate this, proper constraints on key latent variables $C$ and $\theta$ need to be introduced. In this paper, we use a Bayesian network model (see Fig. 1) to construct interpretable constraints. The Bayesian network model not only guides the learning process but also enables to use the community information of the social network linking the agents together. We explain each component of our Bayesian network model below.

#### 2.2.1 Community reliability matrix $\tilde{C}_n$

We assume that a social network connecting the $N$ agents is known. Agents in a social network tend to form communities [33], where a community consists of agents with similar opinion. The community that an agent belongs to is stochastic and unknown a priori but the maximum possible number of communities in the network is known to be $K$. We can thus assign an index $1, 2, \ldots, K$ in an arbitrary
We model the community index autoencoder [34], [35], The Bayesian network model guides the learning process of agent $n$ and assume that for $r$ representing the reliability of community $C$. Let $\pi = (\pi_{n,k})_{1 \leq n \leq N, 1 \leq k \leq K}$ be the distribution of $s_n$ and $z_{n,m}$, which are defined in (4) and (6).

$s = (s_n)_{n=1}^N$ is the community index of agent $n$. Known hyper-parameters defined in (3), (5), (6), (8), (7), (17), and (25), respectively.

$(U_k, V_k), \alpha, (g_0, h_0), p_{MV}^j, \beta, \theta, \tau$ are known hyper-parameters. We now discuss the probability model governing $s_n$ below. Here, we describe how $C_n$ depends on $s_n$. For each $k = 1, \ldots, K$, let $\tilde{C}_k$ be a matrix of the same size as $C_n$ representing the reliability of community $k$. We assume $C_n$ to be a perturbed version of $\tilde{C}_s$, with

$$C_n(r_1, \cdot) | \tilde{C}_s(r_1, \cdot), s_n = k \sim \mathcal{N} \left( \tilde{C}_k(r_1, \cdot), b' \mathbf{1} \right),$$

for $r_1 \in [1, R_1]$, where $b'$ is a known hyperparameter. We assume that for $k = 1, \ldots, K$, and $r_1 \in [1, R_1]$,

$$\tilde{C}_k(r_1, \cdot) \sim \mathcal{N} (U_k(r_1, \cdot), V_k),$$

where $U_k$ and $V_k$ are known hyperparameters.

From Fig. 2, we see that $\{C_n\}$ are learned from different inputs of the reliability encoder. Different from a traditional autoencoder [34], [35], $\{C_n\}$ are not independent and their relationship is modeled by the Bayesian network in Fig. 1. The Bayesian network model guides the learning process of the autoencoder.

### 2.2.2 Community index $s_n$

We model the community index $s_n$ of agent $n$ as

$$s_n \sim \text{Cat} (\pi_n),$$

where the mixture weights

$$\pi_n = (\pi_{n,k})_{k=1}^K \sim \text{Dir} (\alpha),$$

with $\alpha$ being a concentration hyperparameter and $\text{Dir} (\alpha)$ is the Dirichlet distribution. We use the mixed membership stochastic block model (MMSB) [30] to model the social connection $A(n,m)$ between agents $n$ and $m$. In this model, $z_{n,m}$ is the community whose belief agent $n$ subscribes to due to the social influence from agent $m$. Under the influence of different agents, agent $n$ may subscribe to the beliefs of different communities. If both agents $n$ and $m$ subscribe to the belief of the same community, they are more likely to be connected in the social network. We assume the following:

$$z_{n,m} | \pi_n \sim \text{Cat} (\pi_n),$$

$$z_{m,n} | \pi_m \sim \text{Cat} (\pi_m),$$

$$\beta_k \sim \text{Be} (g_0, h_0),$$

where $\text{Be} (g_0, h_0)$ is the beta distribution with parameters $g_0, h_0 > 0, k = 1, \ldots, K$, and

$$p (A(n,m) = 1 | z_{n,m}, z_{m,n}, \beta_{z_{n,m}}) = \begin{cases} \beta_{z_{n,m}}, & \text{if } z_{n,m} = z_{m,n} \\ \epsilon, & \text{if } z_{n,m} \neq z_{m,n} \end{cases}$$

**Table 1**

| Symbol | Description | Variational Parameter in Section 3 |
|--------|-------------|-----------------------------------|
| $M = (M(n,j))_{1 \leq n \leq N, 1 \leq j \leq J}$ | $M(n,j) \in [1, R]$ is the observation of agent $n$ of event $j$. | N.A. |
| $C = (C_n)_{n=1}^N$ | $C_n$ is the $R_1 \times R_2$ reliability matrix of agent $n$. | Neural network parameters $w_R$ of the reliability encoder network. |
| $\theta = (\theta_j)_{j=1}^J$ | $\theta_j$ is the true state of event $j$. | Neural network parameters $w_E$ of the event encoder network. |
| $\alpha = (\alpha_n)_{n=1}^N$ | $\alpha_n$ are the outputs of the reliability encoder network. | N.A. |
| $u = (u_j)_{j=1}^J$ | $u_j$ and $d_j$ are the outputs of the reliability encoder network, the event encoder network and the decoder network, respectively. | N.A. |
| $d = (d_{n,j})_{1 \leq n \leq N, 1 \leq j \leq J}$ | $d_{n,j}$ represents the reliability of community $C$. | $(\mu_k, \sigma_k)_{k=1}^K$ |
| $\tilde{C} = (\tilde{C}_k)_{k=1}^K$ | $\tilde{C}_k$ is the reliability matrix of community $k$. | N.A. |
| $A(n,m) = 1 \text{ (or 0) for } n, m \in \{1, \ldots, N\}$ | There is a (or no) social connection between agents $n$ and $m$. | N.A. |
| $z = (z_{n,m})_{1 \leq n \leq N, 1 \leq m \leq N, n \neq m}$ | $z_{n,m}$ is the index of the community agent $n$ subscribes to under the social influence of agent $m$. | $\phi = (\phi_{n,m,k})_{1 \leq k \leq K, 1 \leq n \leq N, 1 \leq m \leq N, n \neq m}$ |
| $\beta = (\beta_k)_{k=1}^K$ | $\beta_k$ is the social network parameter defined in (7). | $\lambda = (\lambda_k)_{k=1}^K, \lambda_k = (G_k, H_k)$ |
| $\pi = (\pi_{n,k})_{n=1}^N = (\pi_{n,k})_{1 \leq n \leq N, 1 \leq k \leq K}$ | $\pi_{n,k}$ is the distribution of $s_n$ and $z_{n,m}$, which are defined in (4) and (6). | $\gamma = (\gamma_{n,k})_{1 \leq n \leq N, 1 \leq k \leq K}$ |
| $s = (s_n)_{n=1}^N$ | $s_n$ is the community index of agent $n$. | $\psi = (\psi_{n,k})_{1 \leq n \leq N, 1 \leq k \leq K}$ |
| $(U_k, V_k), \alpha, (g_0, h_0), p_{MV}^j, \beta, \theta, \tau$ | Known hyper-parameters defined in (3), (5), (6), (8), (7), (17), and (25), respectively. | N.A. |
with $\epsilon$ being a small constant. Note that $\mathbf{A}$ is independent of $\pi$ when $z$ is given, as shown in Fig. 1.

### 2.2.3 Event states $\theta$

A direct method to perform truth discovery is majority voting, i.e., selecting the opinion expressed by the most number of agents as the true state of the event. This assumes that all agents have the same reliability, and that agents are more likely to give the correct opinion than not. Without any prior information, this is a reasonable assumption. Therefore, we let the prior of $\theta$ to be given by

$$\theta_j \sim \text{Cat}(p_j^{MV})$$

for each $j = 1, \ldots, J$, where $p_j^{MV}(r)$ for $r = 1, \ldots, R$ is the proportion of agents who thinks that the state of event $j$ is $r$. We assume that $\{\theta_j : j = 1, \ldots, J\}$ are independent.

### 3 ART: Autoencoder Truth Discovery

In this section, we propose an autoencoder based on unsupervised variational inference [36] for the Bayesian model in Fig. 1.

Let $\beta = (\beta_k)$, $z = (z_{n,m})$, $s = (s_n)$, $\pi = (\pi_n)$, $\theta = (\theta_j)$, and $C = (C_k)$. For simplicity, let $\Omega = (\beta, z, \pi, s, C, \theta)$. As the closed-form of the posterior distribution $p(\Omega \mid M, \mathbf{A})$ is not available, the variational inference method uses a proposal or variational distribution $q(\Omega; \Lambda)$ to approximate the posterior distribution, where the parameters in the vector $\Lambda$ are called the variational parameters. Note that $M$ and $\mathbf{A}$ are assumed to be observed throughout and not included explicitly in our notation for $q$. More specifically, the variational parameters are selected to minimize the following cost function:

$$\mathcal{L} = -E_q [\log p(\Omega \mid M, \mathbf{A}) - \log q(\Omega; \Lambda)],$$

where the expectation is over the random variable $\Omega$ with distribution $q(\Omega; \Lambda)$ conditioned on $M$ and $\mathbf{A}$. To simplify the optimization procedure, we use the mean-field assumption that is widely used in the literature [36], [37] by choosing

$$q(\Omega; \Lambda) = q(\beta; \lambda)q(z; \psi)q(\pi; \gamma)q(s; \psi)q(\tilde{C}; \tilde{\mu}, \tilde{\sigma})q(C; \mathbf{w}_R)q(\theta; \mathbf{w}_E),$$

where $\Lambda = (\lambda_k)_{k=1}^K$, $\beta = (\beta_n)_{n=1}^N$, $\phi = (\phi_{n,m})_{n,m}$, $\gamma = (\gamma_{n,k})_{n,k}$, $\psi = (\psi_{n,k})_{n,k}$, $\mu = (\mu_k)_{k=1}^K$, and $\sigma = (\sigma_k)_{k=1}^K$ are the variational parameters.

In the sequel, to simplify notations, we omit the variational parameters in our notations, e.g., we write $q(\beta)$ instead of $q(\beta; \lambda)$. We let $q(C, \theta) = q(C)q(\theta)$. From the graphical model in Fig. 1, we obtain

$$p(\Omega \mid M, \mathbf{A}) \propto p(C)p(s \mid \pi)p(z \mid \pi)p(\pi)p(\mathbf{A} \mid \beta, z)p(\beta)p(C \mid \tilde{C}, s) \cdot p(M \mid C, \theta; \mathbf{w}_D)p(\theta),$$

where $p(M \mid C, \theta; \mathbf{w}_D)$ is the conditional distribution of the output of the decoder network in Fig. 2. We have made its dependence on the decoder network parameters $\mathbf{w}_D$ explicit.

To find the variational parameters, we perform an iterative optimization of $\mathcal{L}$ in which the optimal parameter solutions are updated iteratively at each step. We substitute (10) and (11) into (9) to obtain

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3,$$

where the constant term does not contain any variational parameters, and

$$\mathcal{L}_1 \triangleq -E_q(C, \theta, \tilde{C}, s) \left[ \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; \mathbf{w}_D) + \log p(\theta) - \log q(C, \theta) \right],$$

$$\mathcal{L}_2 \triangleq -E_q(C, s, \pi, \beta, z) \left[ \log p(C) + \log p(s \mid \pi) + \log p(\pi) + \log p(\mathbf{A} \mid \beta, z) + \log p(\beta) - \log q(C) - \log q(\beta) - \log q(z) - \log q(\pi) - \log q(s) \right].$$

We update $q(C)$, $q(\theta)$ by minimizing $\mathcal{L}_1$ and update $q(\beta)$, $q(z)$, $q(\pi)$ by minimizing $\mathcal{L}_2$. Furthermore, we update $q(C)$ and $q(s)$ by minimizing

$$\mathcal{L}_3 \triangleq -E_q(C, \tilde{C}, s, \pi) \left[ \log p(C \mid \tilde{C}, s) + \log p(\tilde{C}) + \log p(s \mid \pi) - \log q(\tilde{C}) - \log q(s) \right],$$

$$= -E_q(C, \tilde{C}, s, \pi) \left[ \log p(s \mid C, \tilde{C}, \pi) + \log p(C \mid \tilde{C}) + \log p(\tilde{C}) - \log q(\tilde{C}) - \log q(s) \right].$$

Equations (15) and (16) show $q(s)$ and $q(\tilde{C})$ can be updated by minimizing

$$-E_q(C, \tilde{C}, s, \pi) \left[ \log p(s \mid C, \tilde{C}, \pi) - \log q(s) \right].$$
and

\[ -E_{q(C, \tilde{C}, s, \pi)} \left[ \log p(\tilde{C} \mid C, s) - \log q(C) \right] , \]

respectively.

The variational parameters are optimized and the variational distributions updated iteratively in a procedure that we call Autoencoder Truth (ART) discovery (since we make use of an autoencoder network described below). Its high-level pseudo code for the i-th iteration is shown in Algorithm 1. In the following, we describe how the variational distributions are chosen and how the estimate for the optimal variational parameters are updated in each iteration.

**Algorithm 1 ART (i-th iteration)**

**Input:** Variational parameters in (i – 1)-th iteration, opinions M, social network data A.

**Output:** Variational parameters in i-th iteration.

for each agent n in \{1, \ldots, N\} do

- for each agent pair \( (n, m) \) in \( \{(n, m)\}_{n=1, m\neq n}^N \) do
  - Update \( \phi_{n\rightarrow m} \) and \( \phi_{m\rightarrow n} \) using (30) and (31).

end for

- Update \( \psi_n \) using (37).
- Update \( \gamma_n \) using (42).
- Sample \( C_n \) using (21).
- Sample \( s_n \) from \( q(s_n) = \text{Cat} \left( (\psi_{n,k})_{k=1}^K \right) \).

end for

- Update \( \lambda \) using (26) and (27).
- Update \( \bar{\mu} \) and \( \bar{\sigma} \) using (40a) and (40b).
- Sample \( \tilde{C} \), \( \theta \) and \( \bar{\sigma} \) using (38) and (22).
- Learn the autoencoder (i.e., update \( w_R, w_E \), and \( w_D \) in Section 3.2).

return \( \phi, \psi, \gamma, C, \lambda, \bar{\mu}, \bar{\sigma}, \tilde{C} \), and \( \theta \).

### 3.2 Updating of \( q(C, \theta) \)

The relationship between \( C, \theta \) and \( M \) is complex and non-linear. As discussed before in Sections 2.1 and 3.1, we use neural networks to model \( p(M \mid C, \theta; w_D) \) and we aim to optimize these neural networks to minimize \( \mathcal{L}_1 \). From (13), we have

\[
\mathcal{L}_1 = -E_{q(C)q(\theta)q(C)q(s)} \left[ \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; w_D) + \log p(\theta) - \log q(C) - \log q(\theta) \right] \\
= -E_{q(C)q(\theta)q(C)q(s)} \left[ \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; w_D) - \log q(C) \right] \\
- E_q(\theta) \left[ \log p(\theta) - \log q(\theta) \right].
\]

Recall that \( w_D \), \( w_R \), and \( w_E \) denote the parameters of the decoder network, the reliability encoder network and the event encoder network, respectively. To learn \( w_D \) with the gradient descent method, we need to compute the gradient of \( \mathcal{L}_1 \) with respect to \( w_D \). Denoting the two expectation terms of (19) as \( \mathcal{L}_{11} \) and \( \mathcal{L}_{12} \), respectively, we have

\[
\nabla_{w_D} \mathcal{L}_1 = \nabla_{w_D} \mathcal{L}_{11} + \nabla_{w_D} \mathcal{L}_{12}.
\]

As \( \theta \) is a discrete variable, \( \mathcal{L}_{12} \) is easy to compute. The gradient of \( \mathcal{L}_{11} \) with respect to \( w_D \) is given by

\[
\nabla_{w_D} \mathcal{L}_{11} = -\nabla_{w_D} E_{q(C)q(\theta)q(C)q(s)} \left\{ \{ \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; w_D) - \log q(C) \} \right\} \\
= -E_{q(C)q(\theta)q(C)q(s)} \{ \nabla_{w_D} \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; w_D) - \log q(C) \},
\]

which can be computed using the stochastic gradient descent (SGD) algorithm. In each iteration, we replace \( \nabla_{w_D} \mathcal{L}_{11} \) with its unbiased estimator \( \nabla_{w_D} \tilde{\mathcal{L}}_{11} \), where

\[
\tilde{\mathcal{L}}_{11} \triangleq \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; w_D) - \log q(C)
\]

with \( C, \theta, s \), and \( \tilde{C} \) being sampled from \( q(C), q(\theta), q(s) \), and \( q(\tilde{C}) \) respectively.

We cannot use the same process to deal with \( w_R \) and \( w_E \). This is because

\[
\nabla_{w_R} \mathcal{L}_{11} = -\nabla_{w_R} E_{q(C)q(\theta)q(C)q(s)} \left\{ \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; w_D) - \log q(C) \right\} \\
\neq -E_{q(C)q(\theta)q(C)q(s)} \{ \nabla_{w_R} \log p(C \mid \tilde{C}, s) + \log p(M \mid C, \theta; w_D) - \log q(C) \},
\]

as \( w_R \) is the parameter of \( q(C) \). The same reason applies for \( w_E \). To obtain unbiased estimators for these variational parameters, we need to use the reparameterization trick [38].

From (17), we have \( \text{vec}(C_n)(i) \sim \mathcal{N}(\omega_n(i), b) \), where \( \omega_n \) is generated by the reliability encoder network and \( b \) is a known variance. We can reparameterize \( C_n(i) \) as

\[
\zeta_n \sim \mathcal{N}(0, 1), \quad \text{vec}(C_n)(i) = \omega_n(i) + \zeta_n b.
\]
From (18), one_hot(θ_j) ~ Cat(u_j), where the weight vector u_j = (u_j(r)) is generated by the event encoder network. Then, according to (1) in [39], we can reparameterize θ_j as

\[ \chi_j(r) \sim \text{Gumbel}(0, 1) \]

\[ \theta_j = \arg \max_r [\chi_j(r) + \log u_j(r)] \]

(22)

where the Gumbel(0, 1) distribution can be sampled by first drawing \( \Upsilon \sim \text{Uniform}(0, 1) \) and then computing \( \chi_j(r) = -\log(-\log(\Upsilon)) \). With \( C_n \) and \( \theta_j \) being (21) and (22) respectively, and letting \( \zeta = (C_n), \chi = (\chi_j) \) and \( u = (u_j) \), we then obtain

\[ \nabla_{w_E} L_{11} = -\nabla_{w_E} \mathbb{E}_{q(\zeta)q(\theta)q(C)q(s)} \left[ \log p(C | \tilde{C}, s) + \log p(C, \theta, w_D) - \log q(C) \right] \]

\[ = -\mathbb{E}_{p(\zeta)p(\chi)p(C)q(s)} \left[ \nabla_{w_E} \left\{ \sum_{n=1}^{N} \log N(\zeta_n + \xi_n) \right\} \right] \]

(23)

where \( N(\mu, \sigma) \) is the Gaussian probability density function with mean \( \mu \) and variance \( \sigma \). Here, \( \zeta = (\zeta_n) \) is a function of \( w_R \) and \( u \) is a function of \( w_E \). We also have

\[ \nabla_{w_R} L_{11} = -\mathbb{E}_{p(\zeta)p(\chi)p(C)q(s)} \left[ \nabla_{w_R} \left\{ \log p(C | \tilde{C}, s) \right\} \right] \]

\[ + \log p(C, \theta, w_D) - \log q(C) \]

\[ = -\mathbb{E}_{p(\zeta)p(\chi)} \left[ \nabla_{w_R} \left\{ \log p(M | \zeta, \chi, w_D, o, u) \right\} \right] \].

(24)

Now in each iteration, we can apply SGD to find an unbiased estimator of the gradient of \( L_1 \) with respect to \( w_E \) and \( w_R \).

**Remark 1.** The max function in equation (22) is not differentiable and following [39], we use the softmax function as an approximation to \( \arg \max \), i.e., we let

\[ \text{one_hot}(\theta_j)(r) = \frac{\exp(\chi_j(r) + \log u_j(r)/r)}{\sum_{r'} \exp(\chi_j(r') + \log u_j(r'/r'))} \]

(25)

where \( \tau \) is the temperature parameter.

### 3.3 Updating of \( q(\bar{C}, s, \pi, z, \beta) \)

In this part, we want to find variational parameters corresponding to \( (\bar{C}, s, \pi, z, \beta) \) to minimize \( L_2 \). To achieve this, we iteratively update these variational parameters in ART. The variational parameters corresponding to \( \beta \) and \( z \) are updated in the same way as our previous work [23]. For completeness, we reproduce the results below.

#### 3.3.1 Social network parameter \( \beta \)

Let \( \lambda_k = (G_k, H_k) \). We choose the variational distribution of \( \beta_k \) to be in the same exponential family as its posterior distribution, namely \( q(\beta_k) = Be(G_k, H_k) \). Similar to (14) and (15) in our previous work [23], we can show that

\[ G_k = \sum_{(n,m)} A(n,m) \phi_{n \rightarrow m,k} \phi_{m \rightarrow n,k} + g_0, \]

(26)

\[ H_k = \sum_{(n,m)} (1 - A(n,m)) \phi_{n \rightarrow m,k} \phi_{m \rightarrow n,k} + h_0, \]

(27)

where \( \phi_{n \rightarrow m,k} = q(z_{n \rightarrow m} = k) \) is defined in Section 3.3.2.

From (10) in [40], we also have

\[ E_{q(\beta_k)} [\log(\beta_k)] = \Psi(G_k) - \Psi(G_k + H_k), \]

(28)

\[ E_{q(\beta_k)} [\log(1 - \beta_k)] = \Psi(H_k) - \Psi(G_k + H_k), \]

(29)

which are used in computing the variational distributions of other parameters in our model. Here, \( \Psi(\cdot) \) is the digamma function.

#### 3.3.2 Community membership indicators \( z \)

We let the variational distribution of \( z_{n \rightarrow m} \) to be in the same exponential family as its posterior distribution, namely a categorical distribution with probabilities \( (\phi_{n \rightarrow m,k})_{k=1}^{K} \). Similar to (19) in our previous work [23], one can show that if \( A(n,m) = 0 \),

\[ \phi_{n \rightarrow m,k} \propto \exp\{\phi_{n \rightarrow m,k} [E_{q(\beta_k)} [\log(\beta_k)] - \log(\epsilon)] + E_{q(\pi_n)} [\log(\pi_{n,k})]\}, \]

(30)
where $\mathbb{E}_q(\beta_k) \log(\beta_k)$ and $\mathbb{E}_q(\pi_n) \log(\pi_n, k)$ are computed using (28) and (43) in the sequel, respectively. On the other hand, if $A(n, 0) = 0$, we have

$$
\phi_{n \to n,k} \propto \exp\left\{ \phi_{n \to n,k} \left( \mathbb{E}_q(\beta_k) \log(1 - \beta_k) - \log(1 - \epsilon) \right) + \mathbb{E}_q(\pi_n) \log(\pi_n, k) \right\},
$$

(31)

where $\mathbb{E}_q(\beta_k) \log(1 - \beta_k)$ is computed in (29) in the sequel.

### 3.3.3 Event community indices

We take $q(s_n) = \text{Cat} \left( (\psi_{n,k})_{k=1}^{K} \right)$, where $(\psi_{n,k})_{k=1}^{K}$ is the variational parameter. Let $\hat{\psi}_{n,k} \equiv \log(\psi_{n,k})$. We have

$$
p(s_n = k \mid C_n, \tilde{C}_k, \pi_n) \propto p(s_n = k \mid \pi_n)p(C_n \mid \tilde{C}_k)
= \pi_{n,k} \prod_{r_1 = 1}^{R_1} \prod_{r_2 = 1}^{R_2} \mathcal{N} \left( C_n(r_1, r_2); \tilde{C}_k(r_1, r_2), b' \right),
$$

Thus, $p(s_n = k \mid C_n, \tilde{C}_k, \pi_n)$ is a categorical distribution and is in the same exponential family as $q(s_n) = \text{Cat} \left( (\psi_{n,k})_{k=1}^{K} \right)$. Let

$$
\hat{\psi}_{n,k} \equiv \log \left( \pi_{n,k} \prod_{r_1 = 1}^{R_1} \prod_{r_2 = 1}^{R_2} \mathcal{N} \left( C_n(r_1, r_2); \tilde{C}_k(r_1, r_2), b' \right) \right).
$$

The natural parameter of $p(s_n \mid C_n, \tilde{C}_k, \pi_n)$ is

$$
(s_{n,k} - \hat{\psi}_{n,k})_{k=1}^{K}.
$$

(32)

The natural parameter of $q(s_n)$ is

$$
(\hat{\psi}_{n,k} - \hat{\psi}_{n,K})_{k=1}^{K}.
$$

(33)

According to the relationship between the natural gradient and and the natural parameters (i.e., (22) in [36]), the natural gradient of $L_3$ in (15) with respect to $\hat{\psi}_{n,k}$ can be derived from (32) and (33) and the result is

$$
\hat{\nabla}_{\hat{\psi}_{n,k}} L_3 = \hat{\psi}_{n,k} - \hat{\psi}_{n,K} - \mathbb{E}_q(C_n, \tilde{C}_k, \pi_n) [s_{n,k} - s_{n,K}].
$$

We sample $C_n$ from $q(C_n)$ and obtain the unbiased estimator of $\hat{\nabla}_{\hat{\psi}_{n,k}} L_3$ as

$$
\hat{\nabla}_{\hat{\psi}_{n,k}} L_3' = \hat{\psi}_{n,k} - \mathbb{E}_q(C_n, \tilde{C}_k, \pi_n) [s_{n,k} - s_{n,K}]
$$

where $\Delta_n \equiv \hat{\psi}_{n,K} - \mathbb{E}_q(C_n, \tilde{C}_k, \pi_n)[s_{n,K}]$ and it is constant for $k = 1, \ldots, K$. Then we update $\hat{\psi}_{n,k}$ using

$$
\hat{\psi}_{n,k}^{(i)} = \hat{\psi}_{n,k}^{(i-1)} - \rho(i) \hat{\nabla}_{\hat{\psi}_{n,k}} L_3',
$$

(35)

where $\rho(i)$ is the known step size at $i$-th iteration. Let $\Xi_{n,k}$ be the first three terms of the right-hand side of (34). We compute exponential function of both sides of (35) and obtain

$$
\psi_{n,k}^{(i)} = \psi_{n,k}^{(i-1)} \exp \left( -\rho(i) \Xi_{n,k} \right) \exp(\rho(i) \Delta_n)
$$

= $\psi_{n,k}^{(i-1)} \exp \left\{ \rho(i) \sum_{r_1 = 1}^{R_1} \sum_{r_2 = 1}^{R_2} \frac{1}{2\beta^2} \left( (\psi_{n,k}^{(i-1)})_1 (r_1, r_2) \right)^2 + \left( \psi_{n,k}^{(i-1)} (r_1, r_2) - C_n (r_1, r_2) \right)^2 \right\}

+ \rho(i) \mathbb{E}_q(\pi_n) \log(\pi_n, k) \exp(\rho(i) \Delta_n)
$$

(36)

where $\hat{\psi}$ and $\hat{\bar{\sigma}}$ are variational parameters of $\tilde{C}$ defined in Section 3.3.4 and $\mathbb{E}_q(\pi_n) \log(\pi_n, k)$ can be computed by (43) below. Since $\sum_{k=1}^{K} \psi_{n,k}^{(i)} = 1$ and $\exp(\rho(i) \Delta_n)$ is constant for every $\psi_{n,k}^{(i)}$, $k = 1, \ldots, K$, in each iteration, we only need to compute $\psi_{n,k}^{(i-1)} \exp \left( -\rho(i) \Xi_{n,k} \right)$ and then

$$
\psi_{n,k}^{(i)} = \frac{\psi_{n,k}^{(i-1)} \exp \left( -\rho(i) \Xi_{n,k} \right)}{\sum_{k=1}^{K} \psi_{n,k}^{(i-1)} \exp \left( -\rho(i) \Xi_{n,k} \right)}.
$$

(37)

### 3.3.4 Community reliability matrix $\tilde{C}$

We let the variational distribution of $\bar{C}_k$ for each $k = 1, \ldots, K$ to be given by

$$
q(\tilde{C}_k (r_1, r_2)) = \mathcal{N} (\tilde{\mu}_k (r_1, r_2), \tilde{\sigma}_k (r_1, r_2))
$$

(38)

We also have

$$
p(\bar{C}_k (r_1, r_2) \mid s, \{ C_n (r_1, r_2) \}_n)
\propto p(\tilde{C}_k (r_1, r_2)) \prod_{\{ n : s_n = k \}} p(C_n (r_1, r_2) \mid \tilde{C}_k (r_1, r_2)),
$$

(39)

which is a normal distribution with mean (cf. (2) and (3))

$$
\frac{1}{V(r_1, r_2)^{-2} + \frac{1}{\beta} \sum_n I(s_n, k)} \left( U_k (r_1, r_2) \right)^2
$$

$$
+ \frac{1}{\beta} \sum_n C_n (r_1, r_2) I(s_n, k)
$$

and variance

$$
\frac{1}{V(r_1, r_2)^{-2} + \frac{1}{\beta} \sum_n I(s_n, k)}.
$$

Let $\hat{\mu}_k (r_1, r_2) \equiv \frac{\bar{\mu}_k (r_1, r_2)}{\bar{\sigma}_k (r_1, r_2)^2}$ and $\hat{\sigma}_k (r_1, r_2)^2 \equiv \frac{1}{2\bar{\sigma}_k (r_1, r_2)^2}$. As $q(\tilde{C}_k (r_1, r_2))$ is in the same exponential family as $p(\bar{C}_k (r_1, r_2) \mid s, \{ C_n (r_1, r_2) \}_n)$ and its natural parameter is $(\hat{\mu}_k (r_1, r_2), \hat{\sigma}_k (r_1, r_2))$. Similar to Section 3.3.3,
we sample $C_n$ from $q(C_n)$ and update $\hat{\mu}_k(r_1, r_2)$ and $\hat{\sigma}^2_k(r_1, r_2)$ using

$$
\hat{\mu}^{(i)}_k(r_1, r_2) = \hat{\mu}^{(i-1)}_k(r_1, r_2) - \frac{1}{b} \sum_n C^{(i)}_n(r_1, r_2) \psi^{(i)}_{n,k},
$$

$$
\hat{\sigma}^2_k(r_1, r_2) = \hat{\sigma}^{(i-1)}_k(r_1, r_2)^2 - \frac{1}{b} \sum_n C^{(i)}_n(r_1, r_2)\psi^{(i)}_{n,k},
$$

(40a)

Finally, we obtain $\hat{\sigma}_k^{(i)}(r_1, r_2)^2 = -\frac{1}{2}\hat{\sigma}_k^{(i)}(r_1, r_2)^2 + \frac{1}{2} \left( V_k(r_1, r_2)^{-2} + \frac{1}{b} \sum_n \psi^{(i)}_{n,k} \right).$

(40b)

3.3.5 Mixture weights $\pi$

We let $q(\pi_n) = \text{Dir}(\gamma_n)$ and thus $q(\pi_n)$ is an exponential family distribution and its variational parameter $\gamma_n$ is also its natural parameter. To find the variational parameter $\hat{\gamma}$ that minimizes $L_2$, we find the partial derivative

$$
\nabla_\gamma L_2 = -\nabla_\gamma \left\{ \mathbb{E}_{q(\pi_n)q(s|\pi)} [\log p(s \mid \pi) + \log p(z \mid \pi)] + p(\pi) - \log q(\pi) \right\},
$$

$$
= -\nabla_\gamma \left\{ \mathbb{E}_{q(\pi_n)q(s|\pi)} [\log p(s \mid \pi, z) - \log q(\pi)] \right\},
$$

(41)

where

$$
p \left( \pi_n \mid \{s_i\}^{N}_{i=1}, \{z_{n-m}\}^{N}_{m=1, m \neq n} \right) \propto \prod_{i=1}^{N} p(s_i \mid \pi_n) \prod_{m=1, m \neq n}^{N} p(z_{n-m} \mid \pi_n) p(\pi_n)
$$

$$
\propto \text{Dir} \left( \frac{\alpha}{K} + \sum_{m=1, m \neq n}^{N} I(z_{n-m}, k) + \sum_{i=1}^{N} I(s_i, k) \right),
$$

with $K$ representing the maximum number of communities.

Recall that $\alpha$ is a hyperparameter. As $q(\pi_k) = \text{Dir}(\gamma_k)$ is in the same exponential family as $p \left( \pi_n \mid s_n, \{z_{n-m}\}^{N}_{m=1, m \neq n} \right)$, thus if we let (41) be zero, we obtain

$$
\gamma_{n,k} = \mathbb{E}_{q(s_n, \{z_{n-m}\}^{N}_{m=1, m \neq n})} \left[ \frac{\alpha}{K} + \sum_{m=1, m \neq n}^{N} I(z_{n-m}, k) + \sum_{i=1}^{N} I(s_i, k) \right].
$$

(42)

From (10) in [40], we also have

$$
\mathbb{E}_{q(\pi_n)} [\log (\pi_n)] = \Psi(\gamma_{n,k}) - \Psi \left( \sum_{k=1}^{K} \gamma_{n,k} \right),
$$

(43)

which is used in (37). Recall that $\Psi(\cdot)$ is the digamma function.

4 SIMULATION AND EXPERIMENTAL RESULTS

In this section, both simulations and real data experiments are presented. We adopt majority voting, BCC [18], BCC [20] and VISIT [23] as the baseline methods and compare them with our proposed ART. The results demonstrate that ART has better performance than the baseline methods in estimating the true states.

4.1 Synthetic Data

In this simulation, we generate a synthetic data set so that the dependence of our proposed approach on various parameters can be tested. The event states are selected from a set of $R = 4$ elements. The number of agents $N$, the number of events $J$, the number of communities are set to be 80, 200 and 4, respectively. We define sparsity to be the proportion of null elements in the observation matrix $M$. We generate synthetic datasets for 5 sparsity values: 0.4, 0.5, 0.6, 0.7, and 0.8. We sample the community indices first using (4), and then generate observations $M$ and social network $A$ according to the Bayesian model in Fig. 1. Specifically, for agents $n$ in the index sets $[1, 20], [21, 40], [41, 60]$ and $[61, 80]$, we set $\pi_n$ to be $(0.1, 0.1, 0.1, 0.1), (0.1, 0.1, 0.1, 1.0), (1.0, 0.1, 0.1, 0.1), (0.1, 1.0, 0.1, 0.1)$, and $A(n, m)$, respectively. We sample $\gamma_{n,m}$ and $A(n, m)$ from (6) and (7), respectively.

The validation dataset $M(:, 1 : 30)$ is used to choose hyperparameters and the entire dataset $M$ is used to test our method and the baseline methods. To generate the agents’ observation data, we adopt the confusion matrix framework in [18], [20]. In our ART inference procedure however, reliability matrices, which are different from confusion matrices, are used. The size of the reliability matrices are tuned using a validation data set to achieve the best performance. The $4 \times 4$ community confusion matrices $F_k$ are chosen as $11^T - 0.4I, 11^T - 0.3I, 11^T + 0.1I, 11^T + 0.01I$, respectively. The $(r_1, r_2)$-th element of the confusion matrix $C_n$ of agent $n$ is then generated from $N \left( F_{s_{\alpha}}(r_1, r_2), 0.05^2 \right)$. The state $\theta_j$ of event $j$ is randomly generated from $\{1, 2, 3, 4\}$.

Finally, the $(n, j)$-th element of observation matrix $M$ is generated by $M(n, j) = \text{idmax}(C_n(\theta_j, \cdot))$, where $\text{idmax}(\cdot)$ is a function that returns the index of the maximum value of a vector.

Let $R$ be a matrix with elements randomly drawn from the interval $[0, 1, 0.2]$. The event encoder, reliability encoder, and the decoder are all fully connected neural networks with 3 hidden layers. The rest of the simulation settings are given in Table 2.

Comparison of the different methods is shown in Table 3, where we observe that ART achieves better accuracy than the other benchmark methods when the sparsity is over 0.4. As a community-based method, ART has advantages over the method without considering communities (i.e., BCC) when the number of observations is low.

In our next simulation, using the social network we have generated above as the base case, we randomly remove $\alpha\%$ of the edges from the social network and say that such a network has density $(1 - \alpha)\%$. Since MV, BCC, and CBCC do not make use of the social network information, we perform comparisons only between VISIT and ART. The

1. Code: https://github.com/yitianhoulai/ART
results are shown in Table 4. We observe that in general the performances of both ART and VISIT increase with the network densities. ART also outperforms VISIT for different network densities when the sparsity is higher.

### 4.2 IMDB Dataset

We next apply ART on a real data set collected from IMDB, and Twitter. If a user rates a movie in IMDB and clicks the share button, a Twitter message is generated. We collected movie evaluations from IMDB and social network information from Twitter. We divide the movie evaluations into 4 levels: bad (0-4), moderate (5,6), good (7,8), and excellent (9,10). We treat the ratings on the IMDB website, which are based on the aggregated evaluations from all users, as the event truths whereas our observations come from only a subset of users who share their ratings on Twitter. To better show the influence of social network information on event truth discovery, we delete small subnetworks that have less than 5 agents each. The final dataset consists of 2266 evaluations from 209 individuals on 245 movies (events) and also the social network between these 209 individuals. Similar to [42], [43], we regard the social network to be undirected as both follower or following relationships indicate that the two users have similar taste. The settings of the experiment are given in Table 5. The event encoder, reliability encoder, and the decoder are all fully connected neural networks with 3 hidden layers. The validation dataset \( M(i) \) is used to choose hyperparameters and the entire dataset \( M \) is used to test our method and the baseline methods. From Table 6, we observe that ART performs better than the other benchmark methods.

### 5 Conclusion

In this paper, we have combined the strength of autoencoders in learning nonlinear relationships and the strength of Bayesian networks in characterizing hidden interpretable structures to tackle the truth discovery problem. The Bayesian network model introduces constraints to the autoencoder and at the same time incorporates the community information of the social network into the autoencoder. We developed a variational inference method to estimate the parameters in the autoencoder and infer the hidden variables in the Bayesian network. Results on both synthetic data and real data show the superiority of our proposed method over other benchmark methods when the agent observations are sparse.

In this paper, we have not considered correlations between events in our inference. We have also not incorporated any side information or prior information about the events into our procedure. These are interesting future research directions.
research directions, which may further improve the truth discovery accuracy.

| Method     | Accuracy  |
|------------|-----------|
| Majority Voting | 50.6%     |
| BCC        | 51.4%     |
| CBCC       | 53.4%     |
| VISIT      | 54.6%     |
| ART        | 58.3%     |

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