Combining probes of large-scale structure with COSMOLIKE

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ABSTRACT

Developing accurate analysis techniques to combine various probes of cosmology is essential to tighten constraints on cosmological parameters and to check for inconsistencies in our model of the Universe. In this paper we develop a joint analysis framework for six different second-order statistics calculated from three tracers of the dark matter density field, namely galaxy position, shear, and magnification. We extend a data compression scheme developed in the context of shear–shear statistics (the so-called COSEBIs) to the other five second-order statistics, thereby significantly reducing the number of data points in the joint data vector. We use COSMOLIKE, a newly developed software framework for joint likelihood analyses, to forecast parameter constraints for the Dark Energy Survey. The simulated Monte Carlo Markov Chains cover a five-dimensional cosmological parameter space comparing the information content of the individual probes to several combined probes (CP) data vectors. Given the significant correlations of these second-order statistics we model all cross-terms in the covariance matrix; furthermore, we go beyond the Gaussian covariance approximation and use the halo model to include higher order correlations of the density field. We find that adding magnification information (including cross-probes with shear and clustering) noticeably increases the information content and that the correct modelling of the covariance (i.e. accounting for non-Gaussianity and cross-terms) is essential for accurate likelihood contours from the CP data vector. We also identify several null tests based on the degeneracy of magnification and shear statistics which can be used to quantify the contamination of data sets by astrophysical systematics and/or calibration issues.

Key words: cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

High-quality data sets from near-term wide-field imaging surveys, e.g. Kilo-Degree Survey (KiDS),¹ Hyper Suprime-Cam (HSC),² Dark Energy Survey (DES),¹ allow for tight constraints on cosmological parameters from the large-scale structure (LSS) of the Universe, being complementary with cosmic microwave background (CMB) constraints from the Wilkinson Microwave Anisotropy Probe (WMAP)⁴ (see Hinshaw et al. 2013, and references therein) and Planck.⁵ The improved data quality and the small statistical uncertainties (as a result of the increased survey volume) pose new challenges for the data analysis; the development and refinement of LSS data analysis methods is crucial for the success of even larger, future data sets from the Large Synoptic Survey Telescope (LSST)⁶, and from future satellite missions Euclid⁷ (Laureijs et al. 2011) and the Wide-Field Infrared Survey Telescope (WFIRST).⁸ For all of the aforementioned surveys the tightest constraints on cosmology will be obtained from a joint analysis of all probes that can be extracted from the data (e.g. cluster mass function, shear peak statistics, baryon acoustic oscillation (BAO) peak fitting, and various second-order statistics derived from clustering, shear, and magnification). Combining LSS with supernovae and CMB

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⁵http://www.esa.int/Our_Activities/Space_Science/Planck
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constraints is straightforward; due to the fact that these probes have very little correlation a joint likelihood analysis frequently comes down to multiplying the corresponding posteriors (e.g. Kilbinger et al. 2013). Combining the various probes of LSS themselves is complicated for several reasons. First, the cosmological information of various LSS probes is highly correlated, which prohibits a joint analysis on the level of posterior probabilities. Instead the analysis requires a joint likelihood using a covariance matrix that includes all cross-correlation terms between the individual probes. Secondly, not only is the cosmological information correlated, even more problematic are the correlations of various systematic effects originating from astrophysics and the measurements themselves.

COSMOlike, a new analysis framework for high-accuracy combined probes (CP) analyses, includes the covariance matrix’s cross-terms in the likelihood analysis, moreover it consistently models the CP model vector as a function of cosmology and also as a function of the uncertainties in the nuisance parameters. Developing such a CP prediction code is challenging given that modelling the individual probes already requires refined knowledge and high-level expertise on the corresponding astrophysics and systematics. Although this knowledge is present in the corresponding communities, even the individual analysis methods are under constant development in order to meet the new data quality, and unfortunately these methods are largely independent from each other. Phrasing the problem differently: the large correlation of the LSS probes is not reflected in the correlation of the development of the individual analysis techniques.

For example, probably the most important astrophysical uncertainty for clustering-based measurements is the relation of dark and luminous matter, modelled through various bias parametrizations and/or halo occupation distribution (HOD) models. Constraints on these models come from measuring cross-correlations of shear and clustering (sometimes called galaxy–galaxy lensing). Cosmic shear uses the same cross-terms to offset uncertainties due to intrinsic alignment; simply combining both methods uses the galaxy–galaxy lensing information twice. Similar problems occur when modelling shear calibration which affects cluster masses calibrated through cosmic shear which affects cluster masses calibrated through weak lensing, cosmic shear, galaxy–galaxy lensing, and shear peak statistics, all at the same time. Other examples are the modelling of baryonic uncertainties and photo-z calibration which affect all probes but in different ways.

Whereas solving all these problems is beyond the scope of this paper, it is our intention nevertheless to take first steps towards a coherent analysis framework of LSS probes. We limit our problem to second-order statistics (power spectra, correlation functions, or linear transformations thereof) that can be derived from a galaxy catalogue containing measurements of galaxy shear, galaxy position, and galaxy magnification. As we explain further in Section 2 we obtain six different second-order statistics from these measurements corresponding to six different probes of the density field. We exclude galaxy clusters and shear peak statistics for now, since these are first-order number count statistics which cause additional complications in the sense that they require a different likelihood function (Poisson distribution instead of Gaussian) in their analysis. Incorporating these probes at the level of the covariance matrix in a Gaussian likelihood together with second-order statistics is questionable.

Regarding nuisance parameters we account for uncertainties from modelling bias and correlation parameters that affect all probes involving clustering (see Section 5.1), however, the extension to other astrophysical contaminations, e.g. intrinsic alignment and baryonic effects, is straightforward, and although not being part of the analysis we address it in the discussion.

Within the aforementioned restrictions COSMOlike v1.0, which we use in this paper, advances the existing state of the art of simulated likelihood analysis.

(i) We simulate an actual likelihood analysis in a five-dimensional cosmological parameter space (plus 10 parameters for modelling bias and correlation parameter).

(ii) We use a full non-Gaussian covariance which includes correlations between all probes and account for higher order correlations of the density field (see Section 4).

(iii) We develop a data compression scheme for the joint likelihood analysis which simultaneously solves the shear–shear E/B-mode problem.

This data compression scheme was developed by Schneider, Eifler & Krause (2010, hereafter SEK10) to solve the problem of calculating a shear E-mode two-point statistics (which contains the cosmological information) from a given shear–shear correlation function on a finite interval. Since information from shear data is limited to angular scales [θ_min; θ_max] any E/B-mode statistic which requires information on larger or smaller scales suffers from so-called E/B-mode mixing or leakage. This problem is examined in Kilbinger, Schneider & Eifler (2006) for configuration space quantities, finding a significant (scale-dependent) bias for formerly used shear statistics, e.g. aperture mass dispersion or shear dispersion. The issue has been addressed in even greater detail for Fourier space quantities, mostly in the context of CMB polarization experiments; several groups developed and refined a pseudo-Cl technique (Hivon et al. 2002; Brown, Castro & Taylor 2005) that has been applied to simulated shear data in Hikage et al. (2011). In Fourier space E/B-leakage largely depends on the mask of the survey; several mitigation schemes have been developed (Lewis 2003; Smith 2006; Kim & Naselsky 2010).

Except for shear–shear none of the other five second-order statistics suffers from the E/B-mode problem; nevertheless, the data compression aspects of the Complete Orthogonal Sets of E-/B-Integrals (COSEBIs) are highly desirable for these probes as well. Furthermore, the extension of the COSEBIs scheme allows for a joint cosmological analysis that involves a clean separation of the cosmic shear signal into E- and B-modes.

2 BASIC CONCEPTS

We consider the observables shear γ, magnification μ, and galaxy position g. From these observables the following second-order statistics can be obtained: shear–shear (γγ), magnification–magnification (μμ), galaxy position–galaxy position (gg), shear–magnification (γμ), shear–position (γg), and magnification–position (μg). We want to comprise this second-order cosmological information into a COSEBI data vector:

\[ E = (E^{\gamma\gamma}, E^{\mu\mu}, E^{gg}, E^{\gamma\mu}, E^{\gamma g}, E^{\mu g})^T, \]

where each \( E^{kk} \) contains five COSEBI modes (see SEK10; Eifler 2011; Asgari, Schneider & Simon 2012, for justification of the number of modes). The goal of this paper is to simulate a multi-dimensional likelihood analysis, where ‘simulated’ means that \( E \) is computed from a fiducial cosmological model (see Table 1) using our prediction code; we will refer to this data vector as the fiducial data vector from now on.
We assume that the errors of the input data vector $E$ are described by a multivariate Gaussian:

$$L(E|p_{co}) = \frac{1}{(2\pi)^{N/2}\sqrt{|C|}} \exp\left[ -\frac{1}{2} (E - M)^T C^{-1} (E - M) \right],$$

(2)

where $p_{co}$ denotes the cosmological parameter vector that is assumed in the model vector $M$, hence $M = M(p_{co})$.

The posterior probability in cosmological parameter space is obtained via Bayes’ theorem:

$$P(p_{co}|E) = \frac{P(E|p_{co}) L(E|p_{co})}{P(E)},$$

(3)

where $P(E|p_{co})$ denotes the prior probability (we assume non-informative priors) and the evidence $P(E)$ can be calculated as an integral over the likelihood $P(E) = \int dp_{co} P(E|p_{co}) L(E|p_{co})$ providing a normalization constant for the posterior probability.

Given the functional form of the likelihood as in equation (2) the error bars are fully determined by the covariance of the COSEBIs data vector, which correspondingly to the definition in equation (1) reads

$$C = \begin{pmatrix} \gamma_{\gamma\gamma} & \gamma_{\gamma\mu} & \gamma_{\gamma\mu\mu} & \gamma_{\gamma\nu\nu} & \gamma_{\gamma\nu} & \gamma_{\gamma\rho\rho} \\ \gamma_{\gamma\mu} & \gamma_{\mu\mu} & \gamma_{\mu\nu\nu} & \gamma_{\mu\nu} & \gamma_{\mu\rho\rho} \\ \gamma_{\gamma\mu\mu} & \gamma_{\mu\nu\nu} & \gamma_{\mu\nu\nu\nu} & \gamma_{\mu\nu\nu\rho} & \gamma_{\mu\nu\nu\rho} \\ \gamma_{\gamma\nu\nu} & \gamma_{\mu\nu\nu} & \gamma_{\mu\nu\nu\nu} & \gamma_{\mu\nu\nu\rho} & \gamma_{\mu\nu\nu\rho} \\ \gamma_{\gamma\nu} & \gamma_{\mu\nu} & \gamma_{\mu\nu\nu} & \gamma_{\mu\nu\nu} & \gamma_{\mu\nu\nu\rho} \\ \gamma_{\gamma\rho\rho} & \gamma_{\mu\rho\rho} & \gamma_{\mu\rho\rho} & \gamma_{\mu\rho\rho} & \gamma_{\mu\rho\rho} \end{pmatrix},$$

(4)

with $C$ being symmetric. While postponing a detailed description of the covariance’s modelling to Section 4, we note that we assume the covariance to be constant with respect to the point in parameter space where the likelihood is evaluated. As shown in Eifler, Schneider & Hartlap (2009) (for Gaussian shear–shear covariances) this assumption is problematic and, depending on the survey parameters, can have significant impact on the parameter constraints. We acknowledge that the covariance matrix, since predicted from a cosmological model, in principle has to vary with respect to cosmology (see Kilbinger et al. 2013, for corresponding application to shear data) and we will pursue a corresponding extension of this work in the future.

In practice the COSEBIs are calculated from the correlation functions of the three observables. The corresponding power spectra are related to these correlation functions as

$$\xi_{\gamma\gamma}(\theta) = \frac{1}{2\pi} \int dl J_0(\theta l) C_{\gamma\gamma}(l),$$

(5)

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$$\xi_{\gamma\gamma}(\theta) = \frac{1}{2\pi} \int dl J_0(\theta l) C_{\gamma\gamma}(l),$$

(6)

$$\xi_{\gamma\gamma}(\theta) = \frac{1}{2\pi} \int dl J_0(\theta l) C_{\gamma\gamma}(l),$$

(7)

$$\xi_{\gamma\gamma}(\theta) = \frac{1}{2\pi} \int dl J_0(\theta l) C_{\gamma\gamma}(l),$$

(8)

$$\xi_{\gamma\gamma}(\theta) = \frac{1}{2\pi} \int dl J_0(\theta l) C_{\gamma\gamma}(l),$$

(9)

$$\xi_{\gamma\gamma}(\theta) = \frac{1}{2\pi} \int dl J_0(\theta l) C_{\gamma\gamma}(l),$$

(10)

where we point out the $J_2$ in the polar–scalar correlation functions $\xi_{\gamma\gamma}$ and $\xi_{\gamma\gamma}$ (see e.g. Bartelmann & Schneider 2001, for a derivation). We will return to the filter functions in Section 5.2.

3 MODELLING THE DATA VECTOR

In this section we describe the prediction module of COSMOLike v1.0 (see Fig. 1 for an illustration), which is an extended version of the shear–shear prediction code described in Eifler (2011). All projected quantities are computed from the non-linear density power spectrum which we calculate from an initial power spectrum $P_{\delta}(k) \propto k^n$ using the transfer function of Eisenstein & Hu (1998). In order to model the non-linear evolution of the density field we develop a Hybrid approach combining information from halofit (Smith et al. 2003) and the Coyote Universe emulator (Lawrence et al. 2010).

The latter emulates $P_{\delta}$ over the range $k \in [0.002; 3.4] h$ Mpc$^{-1}$ within $z \in [0; 1]$ to an accuracy of 1 per cent for cosmologies within $\Omega_m h^2 \in [0.120; 0.155]$, $\Omega_{\Lambda} h^2 \in [0.015; 0.0235]$, $n_s \in [0.85; 1.05]$, $\sigma_8 \in [0.6; 0.9]$, $w_0 \in [-1.3; -0.7]$. For any cosmology, $z$, and $k$ within the aforementioned range, we solely rely on the output of the emulator. For all other parameters we compute the non-linear part of $P_{\delta}$ from halofit and rescale this solution by a factor

$$f(k, z, p_{co}) = \frac{P_{\delta}^{\text{Coyote}}(k, z, p_{co})}{P_{\delta}^{\text{halo}}(k, z, p_{co})},$$

(11)

where $p_{co}$ is the cosmology parameter vector of interest and $p_{co}^{\text{close}}$ is the closest point in parameter space where the Emulator returns a solution (‘close’ is defined as minimum difference in each parameter separately).

![Figure 1. Schematic illustration of the modelling of the CP COSEBIs data vector for a given cosmology.](https://academic.oup.com/mnras/article-abstract/440/2/1379/1021174)
In order to simulate w cold dark matter (wCDM) models we follow the strategy outlined in ICOSMO (Refregier et al. 2011), which interpolates halofit between flat and open cosmological models to mimic quintessence cosmologies (also see Schrabback et al. 2010, for more details). Outside the parameter range of the Emulator the precision of \( P_{\delta\delta} \) will of course be significantly below 1 per cent; we nevertheless believe that our approach supersedes other implementations of modelling non-linear structure growth for multiple cosmologies. For example, when using halofit alone it has been shown that the lensing power spectrum is substantially underestimated (e.g. Hilbert et al. 2009). Throughout this paper we assume a redshift distribution as expected from DES. More precisely, this is modelled by modifying a redshift distribution measured from the Canada–France–Hawaii Telescope Legacy Survey (see Benjamin et al. 2007, adjusted for the lower mean redshift of DES). The exact parametrization reads

\[
n(z) = N \left( \frac{z}{z_0} \right)^{a} \exp \left[ -\left( \frac{z}{z_0} \right)^{\beta} \right],
\]

with \( a = 1.3 \), \( \beta = 1.5 \), \( z_0 = 0.56 \).

### 3.1 Modelling the projected power spectra

Kaiser (1992, 1998) shows that projected power spectra are related to the 3D power spectrum of density fluctuations \( P_{\delta\delta} \) via a Fourier equivalent of Limber’s equation, i.e.

\[
P_{\mu\nu}(l) = \int \frac{dx}{x} \frac{q_1(\chi) q_2(\chi)}{f_l(\chi)} P_{\delta\delta}(k, \chi),
\]

with \( q_1, q_2\) being weight functions, \( k = l/x \), and \( f_l(\chi) \) being the comoving angular diameter which corresponds to the comoving coordinate \( \chi \) for the case of vanishing curvature. For simplicity, we will assume the latter in our analysis; note that the tools described in this paper are nevertheless independent of this assumption.

In the case of the shear the weight functions \( q \) read

\[
q_l = \frac{3H_0^2}{2\pi^2} \int \frac{dx}{x} \frac{g(\chi)}{a} = \frac{3H_0^2}{2\pi^2} \int \frac{d\chi}{\chi} \frac{p(\chi) (\chi' - \chi)}{\chi'},
\]

where \( a(\chi) \) is the scale factor and \( p(\chi) d\chi = p(\chi) dz \) is the redshift distribution of source galaxies in the \( n \)th tomography bin. We do not consider tomography in this paper and drop the corresponding denotation of different redshift bins from now on.

Using these weight functions the expression for the shear power spectrum reads

\[
C^{\gamma\gamma}(l) = \frac{9}{4} \left( \frac{H_0}{c} \right)^4 \Omega_m^2 \int_0^{\chi_0} dx \frac{g^2(\chi)}{a^2(\chi)} P_{\delta\delta}(k, \chi).
\]

In the weak lensing approximation the magnification \( \mu \) equals twice the convergence \( \kappa \), where the latter equals the shear \( \gamma \) at the level of two-point statistics, hence we can express \( C^{\mu\mu} \) as \( 4 C^{\kappa\kappa} \) (see e.g. Bartelmann & Schneider 2001) and subsequently

\[
C^{\mu\mu}(l) = 9 \left( \frac{H_0}{c} \right)^4 \Omega_m^2 \int_0^{\chi_0} dx \frac{g^2(\chi)}{a^2(\chi)} P_{\delta\delta}(k, \chi).
\]

In the case of the angular galaxy number density power spectrum the weight function reads \( q_l = p_1(\chi) b \) and subsequently we obtain

\[
C^{\ell\ell}(l) = \int_0^{\chi_0} dx \frac{p_1(\chi)}{\chi^2} b^2 P_{\delta\delta}(k, \chi).
\]

Note that we have not yet specified the bias parameter \( b \) and its functional dependence on \( k \) and \( z \); we address this further in Section 5.

### 3.2 Projected cross-correlation power spectra

Next we consider the cross-correlation power spectra between our observables starting with the shear–magnification power spectrum:

\[
C^{\gamma\mu}(l) = \frac{9}{2} \left( \frac{H_0}{c} \right)^4 \Omega_m^2 \int_0^{\chi_0} dx \frac{g^2(\chi)}{a^2(\chi)} P_{\delta\delta}(k, \chi).
\]

The corresponding relation for the shear–galaxy position power spectrum reads

\[
C^{\gamma\ell}(l) = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \Omega_m^2 \int_0^{\chi_0} dx \frac{g(\chi) p(\chi)}{a(\chi)} br P_{\delta\delta}(k, \chi),
\]

where \( r \) denotes the correlation parameter for which, similar to the bias, we postpone an exact description to Section 5.

Finally, we obtain

\[
C^{\mu\ell}(l) = \frac{3}{2} \frac{H_0^2}{c^2} \int_0^{\chi_0} dx \frac{g(\chi) p(\chi)}{a(\chi)} br P_{\delta\delta}(k, \chi)
\]

as the expression for the magnification–galaxy position power spectrum.

We note the following interesting relations, which occur as a consequence of the polar–scalar filter functions \( J_2 \):

\[
C^{\mu\mu}(l) = 4 C^{\gamma\gamma}(l) = 2 C^{\gamma\mu}(l),
\]

\[
\xi^{\mu\mu}(\theta) = 4 \xi^{\gamma\gamma}(\theta) \neq 2 \xi^{\gamma\mu}(\theta)
\]

and

\[
C^{\mu\ell}(l) = 2 C^{\ell\ell}(l),
\]

\[
\xi^{\mu\ell}(\theta) \neq 2 \xi^{\ell\ell}(\theta).
\]

We note that these relations can be used to create linear combinations that can assess the impact of systematics on individual probes. We will expand on this in Section 5.2.

### 3.3 COSEBIs formalism

The COSEBIs formalism was developed in SEK10; we refer the reader to this paper for details beyond the brief summary presented in this section.

Throughout this paper we only consider filter functions that are logarithmic in \( \theta \); these filter functions comprise the second-order shear information into significantly fewer COSEBI-modes compared to filter functions that are linear in \( \theta \)

#### 3.3.1 Weak lensing

The COSEBIs shear E-mode, denoted as \( E_n \), can be expressed as an integral over the shear 2PCF \( \xi^\pm_\delta \) as

\[
E^\ell_\nu_n = \frac{1}{2} \int_{\delta_{\min}}^{\delta_{\max}} d\theta \ \left[ T^{\gamma\gamma}_n(\theta) \xi^\lambda_{\kappa\lambda}(\theta) + T^{\gamma\ell}_n(\theta) \xi^\ell\ell(\theta) \right].
\]

Note that for a properly constructed \( T^{\gamma\gamma}_n \) as described below the corresponding \( T^{\ell\ell}_n \) can be readily calculated as

\[
T^{\ell\ell}_n(\theta) = T^{\gamma\ell}_n(\theta) + \int_0^\theta d\theta' T^{\gamma\ell}_n(\theta') \left( \frac{4}{\theta^2} - \frac{12\theta'^2}{\theta^4} \right).
\]
For further details on this the reader is referred to SEK10 and references therein. In the following we only describe the construction of $T_{n+1}^{YY}$.

In order to allow for a proper E/B-modes separation using a 2PCF over only a finite interval the filter functions $T_{n+1}^{YY}$ must meet the requirement

$$\int d\theta \, \partial \, T_{n+1}^{YY}(\theta) = 0 = \int d\theta \, \partial^3 \, T_{n+1}^{YY}(\theta). \quad (27)$$

In addition, the set of filter functions $T_{n+1}^{YY}$ must be orthonormal, i.e.

$$\int_{\beta_{\min}}^{\beta_{\max}} d\theta \, T_{n+1}^{YY}(\theta) \, T_{n+1}^{YY}(\theta) = \delta_{\min}. \quad (28)$$

The explicit construction of the logarithmic $T_{n+1}$ is described in SEK10. The main steps of the construction are the following.

(i) A variable transformation $\hat{\theta} \rightarrow z = \ln(\theta/\Theta_{\min})$.
(ii) Expressing equations (27) and (28) in $z$ with $T_{n+1}^{YY}(\theta) \rightarrow t_{n+1}^{YY}(z)$.
(iii) Expanding each $t_{n+1}^{YY}(z) = \sum_{j=0}^{n+1} c_{nj} z^n$.
(iv) Calculating the coefficients $c_{nj}$ from the conditions (27) and (28).

Given $n$, the filter function $T_{n+1}^{YY}$ will be of order $n + 1$ in $z$ as it needs to fulfil $n + 1$ constraints, i.e. it must fulfil equation (28) for all $T_{n+1}^{YY}$ with $m \leq n - 1$ and additionally it has to meet the two E/B-mode separation constraints in equation (27). This implies that $T_{n+1}^{YY}$ is of order two.

3.3.2 Extension to clustering and magnification

Having determined the weak lensing COSEBIs filter functions $T_{n+1}^{YY}$, we can calculate the other five probe’s COSEBIs similar to equation (25):

$$E_n^{AB} = \int_{\beta_{\min}}^{\beta_{\max}} d\theta \, T_n^{AB}(\theta) \xi^{AB}(\theta). \quad (29)$$

In this paper we assume the same configuration space filter function $T_{n+1}^{YY}$ for all other probes, henceforth neglecting the superscripts.

In addition to equation (29) the COSEBIs can be calculated directly from the power spectrum:

$$E_n^{AB} = \frac{1}{2n} \int dI \, W_n^{AB}(l) C^{AB}(l), \quad (30)$$

which we only use as a consistency check since computing $C^{AB}(l) \rightarrow \xi^{AB}(\theta)$ using a fast Hankel transformation and subsequently carrying out the finite integration in equation (29) is significantly faster.

The Fourier filter functions $W_n^{AB}(l)$ are needed however for the computation of the COSEBIs covariance; they can be obtained from the $T_n$ as

$$W_n^{YY}(l) = W_n^{\mu\mu}(l) = W_n^{SS}(l) = W_n^{\mu S}(l) = \int d\theta \, T_n(\theta) \, J_0(l \theta), \quad W_n^{YY}(l) = W_n^{\mu\mu}(l) = \int d\theta \, T_n(\theta) \, J_0(l \theta), \quad (31)$$

where the $J_{0/2}$ are a consequence of equations (5)–(10).

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Schematic illustration of the modelling of the joint COSEBIs covariance for a given cosmology.}
\end{figure}

4 MODELLING OF COVARIANCES

In this section we describe the covariance module of COSMOlike v1.0 (see Fig. 2). We start with explaining the modelling of covariances for projected power spectra; the expression for computing the COSEBIs covariance from the power spectrum covariance is straightforward, however, the actual computation is easily affected by numerical uncertainties. We outline our method and cross-checks at the end of this section.

4.1 Power spectrum covariances

Under the assumption that the density field is Gaussian (which means that the four-point functions can be expressed in terms of two-point functions) the covariance of projected power spectra can be expressed as (Hu & Jain 2004)

\begin{equation}
\text{Cov} \left( C^{AB}(l) C^{CD}(l) \right) = \langle \Delta C^{AB}(l) \Delta C^{CD}(l) \rangle = \frac{2\pi \delta_{1l} \delta_{1l}}{\Omega I_{1} I_{1}} \left[ \left( C^{AC}(l) + N^{AC}(l) \right) \left( C^{BD}(l) + N^{BD}(l) \right) + \langle C^{AD}(l) + N^{AD}(l) \rangle \left( C^{BC}(l) + N^{BC}(l) \right) \right],
\end{equation}

where the superscripts are to be replaced with $\gamma, g, \mu$ depending on the probe under consideration and $\Omega$ denotes the survey volume.

The covariance gets contributions from the signal $C(l)$ and a noise term $N$. Note that

$$N_{\gamma} = \frac{\sigma_{\gamma}^2}{2n_{\text{gal}}}, \quad N_{g} = \frac{\sigma_{g}^2}{n_{\text{gal}}}, \quad N_{\mu} = \frac{1}{n_{\text{gal}}}, \quad (34)$$

and all other noise terms are zero. We assume the intrinsic shape noise, $\sigma_{\gamma}^2 = 0.32$, and note that the factor ‘2’ in the denominator results from the fact that the shear has two components. For the magnification noise parameter we follow the arguments in Krause et al. (2013) defining $\sigma_{\gamma} = 2 \sigma_{\gamma} f^{1/2}$ for scaling relation based estimators of magnification (Huff & Graves 2014), where $f$ denotes the fraction of galaxies for which magnification is measured and $\sigma_{\gamma}$ being the scatter of the convergence estimator. Being optimistic that the method in Huff & Graves (2014) can be extended to late-type galaxies we assume $\sigma_{\gamma} = 1.2$, noting that the uncertainty of this noise level is large.
Since non-linear structure growth at late time induces significant non-Gaussianities in the density field, equation (33) underestimates the error on cosmological parameters and needs to be amended by an additional term, i.e. \( \text{Cov} = \text{Cov}_{\text{G}} + \text{Cov}_{\text{NG}} \). We model non-Gaussian covariance as the sum of the trispectrum contributions (Cooray & Hu 2001; Takada & Jain 2009), including a sample variance term which describes the scatter in power spectrum measurements due to large scale density modes (Takada & Bridle 2007; Sato et al. 2009):

\[
\text{Cov}_{\text{NG}}(C^{AB}(l_1), C^{CD}(l_2)) = \frac{1}{\Omega_2} \int_{|l_1|} d^21 \frac{2\pi}{A(l_1)} \times \int_{|l_2|} d^21 \frac{2\pi}{A(l_2)} T^{ABCD}(l_1, l_2, l_1, l_2, -l_1, -l_2),
\]

with \( T^{ABCD}(l_1, l_1, l_2, -l_2) \) defined as

\[
T^{\alpha\beta\gamma\delta}(l_1, l_2) = 2^6 \left( \frac{3 H_0^2}{2 c^2 \Omega_m} \right)^{\alpha+\beta} \int_0^{\infty} dx \left( \frac{g(x)}{a(x)} \right)^{\alpha+\beta} \times \langle p(x) b \rangle^{\delta-\alpha-\beta} x^{-6} T^{\delta\delta\delta\delta}(l_1, l_2, l_1, l_2, -l_1, -l_2, -l_1, -l_2),
\]

where we assume the correlation parameter \( r = 1 \) and \( \alpha, \beta \in [0; 4] \). For example the pure shear trispectrum \( T^{\gamma\gamma\gamma\gamma} \), and the pure galaxy position trispectrum \( T^{\delta\delta\delta\delta} \) read

\[
T^{\alpha\beta\gamma\delta}(l_1, l_2) = \left( \frac{3 H_0^2}{2 c^2 \Omega_m} \right)^4 \int_0^{\infty} dx \left( \frac{g(x)}{a(x)} \right)^2 x^2 T^{\delta\delta\delta\delta}(l_1, l_2, l_1, l_2, -l_1, -l_2, -l_1, -l_2),
\]

\[
T^{\alpha\beta\gamma\delta}(l_1, l_2) = \int_0^{\infty} dx \left( \frac{p(x)}{x^2} \right) x^{-6} T^{\delta\delta\delta\delta}(l_1, l_2, l_1, l_2, -l_1, -l_2, -l_1, -l_2),
\]

respectively.

### 4.2 Halo model trispectrum

We model the dark matter trispectrum using the halo model (Seljak 2000; Cooray & Sheth 2002), which assumes that all matter is bound in virialized structures that are modelled as biased tracers of the density field. Within this model the statistics of the density field can be described by the dark matter distribution within haloes on small scales, and is dominated by the clustering properties of haloes and their abundance on large scales. In this model, the trispectrum splits into five terms describing the four-point correlation within one halo (the one-halo term \( T_1 \),) between two and four haloes (two-, three-, four-halo term), and a so-called halo sample variance term \( T^{\delta\delta\delta\delta} \), caused by fluctuations in the number of massive haloes within the survey area:

\[
T = T_1 + (T_2 + T_{2h,1s}) + T_{3h} + T_{4h} + T^{\delta\delta\delta\delta}. \tag{39}
\]

The two-halo term is split into two parts, representing correlations between two or three points in the first halo and two or one point in the second halo. As haloes are the building blocks of the density field in the halo approach, we need to choose models for their internal structure, abundance, and clustering in order to build a model for the trispectrum. Our implementation of the one-, two-, and four-halo term contributions to the matter trispectrum follows Cooray & Hu (2001), and we neglect the three-halo term as it is subdominant compared to the other terms at the scales of interest for this analysis. Specifically, we assume NFW halo profiles (Navarro, Frenk & White 1997) with the Bullock et al. (2001) fitting formula for the halo mass–concentration relation \( c(M, z) \), and the Sheth & Tormen (1999) fit functions for the halo mass function \( \frac{dn(M)}{dM} \) and linear halo bias \( b(M) \), neglecting terms involving higher order halo biasing.

### 4.3 COSEBIs covariances

Following Sato et al. (2009) the halo sample variance term can be calculated as

\[
\text{Cov}_{\delta\delta\delta\delta}(C^{AB}(l_1), C^{CD}(l_2)) = 2^6 \left( \frac{3 H_0^2}{2 c^2 \Omega_m} \right)^{\alpha+\beta} \times \int_0^{\infty} dx \left( \frac{d^2 V}{d\delta d\Omega} \right)^2 \left( \frac{g(x)}{a(x)} \right)^2 \times \langle p(x) b \rangle^{\delta-\alpha-\beta} x^{-6} T^{\delta\delta\delta\delta}(l_1, l_2, l_1, l_2, -l_1, -l_2, -l_1, -l_2),
\]

where \( \delta \) is the angular scale and \( \Omega \) the volume.

\[
\delta(x) = \left( \frac{3 H_0^2}{2 c^2 \Omega_m} \right)^4 \int_0^{\infty} dx \left( \frac{g(x)}{a(x)} \right)^2 x^2 T^{\delta\delta\delta\delta}(l_1, l_2, l_1, l_2, -l_1, -l_2, -l_1, -l_2),
\]

\[
\delta(x) = \int_0^{\infty} dx \left( \frac{p(x)}{x^2} \right) x^{-6} T^{\delta\delta\delta\delta}(l_1, l_2, l_1, l_2, -l_1, -l_2, -l_1, -l_2),
\]

respectively.

### 5 Likelihood analysis

Cosmology samples the parameter space using parallel Monte Carlo Markov Chain (MCMC; Goodman & Weare 2010) implemented through the emcee python package.\(^9\) The MCMCs presented in this paper consist of at least 200 000 steps and have been checked for convergence. In the following we simulate various likelihood analyses using the data vector in equation (1) and covariance in equation (4) or subsets thereof for survey parameters close to what is expected for the DES (\( \Omega_m = 0.25 \), \( n_s = 0.7 \)). The range of the cosmological parameter space considered in the

\[^9\] http://dan.iel.fm/emcee/

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Figure 3. Full (Gaussian+non-Gaussian) COSEBIs correlation matrix. Since we assume five modes for each of the six probes our data vector contains 30 data points, hence the covariance is $30 \times 30$. We indicate the corresponding autocovariance block matrices in the plots.

Figure 4. Likelihood analysis in five-dimensional cosmological parameter space as described in the text. We show the 68 per cent credible regions for four different likelihood analyses, i.e. individual probes of cosmic shear, galaxy–galaxy lensing, and galaxy clustering, compared to a joint analysis of all three probes (see legend for details).
contours). We consider a five-dimensional cosmological parameter space ($\Omega_m$, $\sigma_8$, $n_s$, $w_0$, and $w_a$) free of nuisance parameters (see Section 5.1 for bias modelling uncertainties).

From Fig. 4 it is clear (and expected) that our non-tomographic likelihood analysis has little constraining power for time-dependent dark energy models if one considers the probes individually. For the CP analysis, the constraints on $\Omega_m$, $\sigma_8$, $n_s$ are improved substantially, which allowing us to put tight constraints on the combination of $w_0$–$w_a$. We expect these constraints to significantly improve if tomographic information is included.

Adding magnification. We further extend the analysis by adding the magnification auto- and cross-correlation probes to the CP data vector, which means we add three new second-order statistics to the existing three. For this part of the analysis we consider a four-dimensional parameter space only, more precisely we fix $w_a = 0$. Fig. 5 compares two different data vectors, namely $E = (E_{\gamma\gamma}, E_{\mu\mu}, E_{gg}, E_{\gamma\mu}, E_{\gamma g}, E_{\mu g})$ (black/solid and blue/dot-dashed contours) and $E = (E_{\gamma\gamma}, E_{gg}, E_{\gamma g})$ (red/dashed contours); for the first case we further compare analyses using the full non-Gaussian covariance (black) to using the Gaussian approximation (blue).

We find a clear improvement in cosmological information when including the magnification auto- and cross-probes in the data vector compared to using shear and clustering only. Although the inclusion of magnification doubles the number or probes in the analysis, the increase in information is not expected to be larger due to the large degeneracy of shear and magnification as a cosmological probe. The improvement will likely be more substantial when including nuisance parameters to account for uncertainties in shear calibration, photo-$z$, intrinsic alignment, and baryons. These uncertainties affect both probes differently, hence adding magnification to the CP framework is a valuable resource of information to mitigate the impact on parameter constraints.

Comparing Gaussian and non-Gaussian covariances. In Fig. 5 we also show the difference in parameter constraints when using Gaussian instead of non-Gaussian covariances for the six probes CP vector. For likelihood analyses of individual probe such comparisons have been carried out in previous papers (see e.g. Eifler et al. 2009; Takada & Jain 2009, for cosmic shear), this however is the first time that such a comparison (1) is shown for the CP case and (2) includes the halo sample variance term in the covariance. We find that there is a clear difference in parameter constraints when

Figure 5. Likelihood analysis in four-dimensional cosmological parameter space. We show the 68 per cent credible regions and marginalize over the all other parameters not shown in a given panel. We compare CP analyses with and without magnification (black/solid and red/dashed, respectively). For the full six probe data vector $E = (E_{\gamma\gamma}, E_{\mu\mu}, E_{gg}, E_{\gamma\mu}, E_{\gamma g}, E_{\mu g})$ we also show the difference when using Gaussian instead of non-Gaussian covariances (blue/dot-dashed and black/solid, respectively).
neglecting the higher order correlations of the density field in the
model, which indicates that the precise
modelling of these higher order terms is non-negligible for accurate
parameter constraints.

5.1 Uncertainties from bias and correlation parameters

Understanding the relation of galaxies and their dark matter
evolution is an important aspect of any cosmological parameter
estimation that includes clustering information. Constraining and
modelling this relation is an active field of research in theory (e.g.
Zheng et al. 2005; McDonald & Roy 2009) and observations (e.g.
Cacciato et al. 2012; Jullo et al. 2012; Mandelbaum et al. 2013). In
practice, any bias model will have to be fine-tuned to the considered
data set (galaxy population/morphology and redshift distribution).
Guidance on any parametrization from first physical principles is
limited; measurements rely mostly on configuration space quanti-
ties, i.e. a parametrization in \( r, z \), or \( \theta \).

Bias and correlation parameter can also be parametrized (e.g.
Bartelmann & Schneider 2001; Bernstein 2009) as a function of \( k, \)
\( z \) in equations (17) and (19), more precisely

\[
b^2((k), \chi) = \frac{P_{gg}(k, \chi)}{P_{gg}^{\text{fid}}(k, \chi)}
\]

(43)

and

\[
r((k), \chi) = \frac{P_{gg}(k, \chi)}{\sqrt{P_{gg}^{\text{fid}}(k, \chi)P_{gg}^{\text{fid}}(k, \chi)}}.
\]

(44)

with \( P_{gg} \) being the observable galaxy number density power
spectrum and \( P_{gg}^{\text{fid}} \) being the galaxy–dark matter cross-power spectrum.

In contrast, we do not parametrize \( b \) and \( r \) in 3D Fourier space,
but directly as a function of the quantity that enters the likelihood
analysis, i.e. we define the relation between the observable galaxy
number density \( \text{COSBIs} E_n^{gg} \) and the projected dark matter density
COSBIs \( E_n^{mm} \) as

\[
E_n^{gg} = X_n E_n^{mm},
\]

(45)

and similarly

\[
E_n^{yy} = Y_n E_n^{mm}.
\]

(46)

The parameters \( X_n, Y_n \) express the relation between projected dark
matter and galaxy number density; their range of uncertainty reflects
the uncertainty in modelling \( b \) and \( r \) as a function of redshift and scale.

COSBIs mix angular scales and we do not consider a tomographic
analysis our uncertainty in describe the effective uncertainty
of \( b \) and \( r \) averaged over a large mix of scales and redshift.

For the likelihood analysis results shown in Fig. 6 we assume
that all \( X_n \) are uncorrelated; the same holds for \( Y_n \) and also for
combinations of \( Y_n \) and \( X_n \). We introduce 10 (nuisance) parameters
(five \( X_n \) and \( Y_n \)) to model the uncertainty between dark and luminous
matter and allow them to vary independently. More precisely, we
model \( X_n = (b_{id} + \Delta b)_n \) and \( Y_n = (b_{id} + \Delta b)_n(r_{id} + \Delta r)_n \),
where \( \Delta b_n \) and \( \Delta r_n \) are drawn from a Gaussian probability
distribution with \( \sigma^2 = 0.1 \) for a more optimistic and \( \sigma^2 = 0.2 \) for a more
pessimistic scenario (labelled scenario 1 and 2, respectively). Our
method can be interpreted as ‘self-calibration’ with Gaussian priors
centred around the fiducial values of bias and correlation parameter
\( b_{id} = 1.2 \) and \( r_{id} = 1.0 \). We address the importance of the prior
below.

The simulated likelihood analysis in Fig. 6 shows the results in
\( \Omega_m, \sigma_8, \omega_b, n_s, \omega_c \) parameter space assuming perfect knowledge of
bias and correlation parameter (black/solid contours), and using the
parametrization in equations (45) and (46) for various scenarios. As
expected, marginalizing over uncertainties in \( X_n \) and \( Y_n \) significantly
weaks cosmological constraints across all parameters and the
effect is slightly stronger for the pessimistic bias scenario. The small
difference between optimistic and pessimistic scenario indicates
that our priors on \( \Delta b \) and \( \Delta r \) hardly affect the self-calibration
procedure, i.e. choosing a larger \( \sigma^2 \) will not significantly alter the
parameter constraints.

We note that our bias parametrization is conservative since the
scale and redshift dependence of \( b \) and \( r \) induces correlations in
\( X_n \) and \( Y_n \); including these correlations decreases the nuisance parameter
range that we marginalize over. We point out that ideally the
inclusion of clustering information on small scales requires sophis-
ticated HOD modelling and marginalization over the correspond-
ing HOD parameters. Our self-calibration method can be seen as a
lower bound; more information on galaxy formation implemented
via HOD modelling can only improve constraints.

In the following we suggest a measurement framework to make
progress on \( X_n \) and \( Y_n \) and their range of uncertainty observationally.
In the context of the aperture mass dispersion this method has been
suggested in van Waerbeke (1998) to detect scale dependence of
galaxy bias by combining second-order statistics of shear and clus-
tering (see Hoekstra et al. 2002; Cacciato et al. 2012, for application
to data).

For the \( \text{COSBIs} \) the corresponding relations read

\[
X_n = f_X E_n^{gg} E_n^{yy}^{-1},
\]

(47)

and

\[
Y_n = f_Y E_n^{yy} E_n^{yy}^{-1}.
\]

(48)

The functions \( f_X \) and \( f_Y \) depend weakly on cosmology (Hoekstra et al.
2002; Schneider, Kochanek & Wambsganss 2006), which can
be mitigated even further by employing strong priors from indepen-
dent experiments, e.g. Planck. This method allows to measure and
constrain a ‘mode-dependent’ bias for the \( \text{COSBIs} \). We however
note that on cosmological scales galaxy bias has little scale but sig-
nificant redshift dependence and that this method should be applied
within sufficiently small tomography bins.

Analogous relations can be derived using \( E_n^{mm} \) instead of \( E_n^{yy} \)
and/or using \( E_n^{mm} \) instead of \( E_n^{yy} \). We emphasize that magnification
can provide additional information to constrain the relation between
dark and luminous matter; at the very least it provides a valuable
cross-check/null test for the above method.

5.2 Null tests involving shear and magnification

The fact that including magnification contributes only little to the
cosmological constraints is not unexpected given the degeneracy
of shear and magnification. This degeneracy however allows us to
test for and to quantify systematics that affect both probes differently.
As an example we will assume that one of the most important con-
taminations of cosmic shear, i.e. intrinsic alignment, does not affect
magnification. We can express the observed shear power spectrum
as the sum of the true shear power spectrum and the two intrinsic
alignment components II (correlation of the intrinsic ellipticity with
the local density field) and GI (correlation of foreground galaxy ellip-
tsicity with background shear) (Hirata & Seljak 2004; Bernstein
2009; Joachimi & Bridle 2010):

\[
C_{gg}^{yy}(l) = C^{yy}(l) + C^{II}(l) + C^{GI}(l),
\]

(49)
Figure 6. Likelihood analysis in five-dimensional cosmological parameter space using the $E_{\gamma\gamma}$, $E_{\mu\mu}$, $E_{gg}$, $E_{\gamma\mu}$, $E_{\gamma g}$, $E_{\mu g}$ data vector and the corresponding non-Gaussian covariance. We show the 68 per cent credible regions where black/solid contours correspond to a likelihood analysis assuming perfect knowledge of bias and red/dashed and blue/dot–dashed contours correspond to marginalizing over two different bias modelling scenarios (see text for further details).

\[ C_{\text{obs}}^{\mu\mu}(l) = C^{\mu\mu}(l). \]  

Using equation (22) in terms of the COSEBIs we can rewrite equations (49) and (50) as

\[ E_n^{\mu\mu} + E_n^{\mu\mu}(\text{obs}) - 4 E_n^{\gamma\gamma}(\text{obs}), \]  

thereby constraining intrinsic alignment. We note that contaminations similar to IA for shear might exist for magnification as well; correlations of the local density field with the intrinsic size of the galaxies ($\text{II}_{\mu}$), and/or correlations of the foreground galaxy size with the magnification of a background galaxy ($\text{GI}_{\mu}$) are likely. As discussed in Schmidt et al. (2012) magnification estimators which are not based on the excess of number densities but on size measurements have little correlation with their environment (e.g. Croton et al. 2005; Maltby et al. 2010) therefore potentially allowing the above technique to be successful.

In any case the above null test can be used for other types of contaminations which dominate the shear related quantity but do not affect magnification; the best example probably being shear calibration.

The number of null tests is not limited to the magnification and shear autocorrelations but additional constraints can be gained from three other relations similar to equations (21) and (23). As a prerequisite we define new COSEBIs filter functions for $\gamma g$ and $\gamma \mu$,

\[ \int d\vartheta \, \vartheta \, T_n(\vartheta) \, J_0(l\vartheta) = \int d\vartheta \, \vartheta \, T'_n(\vartheta) \, J_2(l\vartheta), \]  

with $T_n$ still being the original filter function $T_n^{\gamma\gamma}$ defined in Section 3.3.1. Calculating the new COSEBIs $E_n^{\gamma g}$ and $E_n^{\gamma \mu}$ as an integral over the corresponding correlation functions $\xi^{\gamma g}$ and $\xi^{\gamma \mu}$ using the $T'_n$ we derive the relations

\[ E_n^{\mu\mu} = 2 E_n^{\gamma\mu}, \]  
\[ E_n^{\mu\mu} = 2 E_n^{\gamma\mu}, \]  
\[ E_n^{\mu\mu} = 2 E_n^{\gamma\mu}, \]  

which do not hold for the correlation functions (see equations 22 and 24).

These relations or linear combinations thereof can be used to define null tests and subsequently constrain astrophysical uncertainties.
6 CONCLUSIONS

In this paper we introduce CosmoLike v1.0, a coherent analysis framework to extract cosmological constraints from all second-order statistics that can be derived from a galaxy position ($g$), shear ($\gamma$), and magnification ($\mu$) catalogue.

The CosmoLike prediction code module allows for fast modelling of multiprobe data vectors consisting of various second-order statistics. These are computed from density power spectra that are generated by the Coyote Universe emulator or a modified halofit implementation. The CosmoLike covariance module utilizes a halo model implementation to computing non-Gaussian covariances for all aforementioned projected quantities and their cross-terms. We then generate a CP data vector from our prediction code assuming a fiducial cosmology and test this data vector (and subsets thereof) in several likelihood analyses, the most extensive one covering five cosmological dimensions $\Omega_m$, $\sigma_8$, $w_0$, $n_s$, $w_a$ and a 10-parameter self-calibration bias model.

The analysis scheme suggested in this paper differs from previous work in several ways. First, we include all second-order cross-statistics of the observables ($g$, $\gamma$, $\mu$) into the data vector (thereby increasing the sources of cosmological information), secondly, we model all cross-terms in the covariance matrix, and thirdly, we include all higher order correlations of the density field in the covariance matrix. Furthermore, we employ the COSEBIs formalism to quantify the information content, thereby solving the cosmic shear E/B-mode problem and introducing a data compression scheme for the other five two-point statistics.

Not surprisingly, we find substantial improvement in parameter constraints when using the CP data vector instead of the individual probes, and a sizable increase in the CP likelihood contours when fitting for galaxy bias instead of assuming it perfectly known.

The most interesting results of this paper are the changes in likelihood contours when including magnification in the CP data vector and when modelling the higher order moments of the density field in the covariance matrix.

We find a noticeable improvement when including magnification and all cross-probes in addition to cosmic shear, galaxy–galaxy lensing, and galaxy clustering. Although the inclusion of magnification doubles the number of second-order statistics in the data vector the strong degeneracy of information from magnification and shear and the large assumed noise level $\sigma_g = 1.2$ prevent this improvement to be more significant. We note that magnification in contrast to shear and clustering is a relatively recent cosmological probe, hence the assumed noise level might be too pessimistic.

The degeneracy of shear and magnification allows for interesting constraints on systematics, e.g. intrinsic alignment, shear calibration errors, photo-$z$ uncertainty, etc. We outline several relations that hold in the absence of these systematics and suggest extensions of these null tests. We also emphasize that in the presence of nuisance parameters describing the uncertainty from these systematics, the information increase when including magnification will likely be more significant.

Regarding covariances we find that neglecting the higher order terms in their modelling leads to a clear underestimation of error bars. We emphasize that forecasting exercises for a CP analysis similar to ours should incorporate non-Gaussian covariances including all cross-terms of probes.

In the future we plan to extend CosmoLike to other second-order statistics whose distribution follow the same likelihood function, e.g. CMB polarization, CMB lensing, CMB temperature correlations, and of course also to tomography for the probes considered in this paper. It is straightforward to apply this analysis scheme to any data set from which can measure projected correlation functions. Before extracting meaningful information from data however, the framework described here needs extensions. For example, implementing a detailed HOD-model approach (van den Bosch et al. 2013), adding the parametrization of nuisance parameters, such as baryons, intrinsic alignment, photo-$z$ calibration, and shear calibration, is required.

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