THE EVIDENCE FOR THE LONG DISTANCE SCALE WITH $H_0 < 65$

Allan Sandage
The Observatories of the Carnegie Institution of Washington
and
G.A.Tammann
Astronomisches Institut der Universitat Basel

Abstract. The status of the determination of the Hubble constant is reviewed, setting out the evidence for the long distance scale with $H_0 = 55 \pm 5$. In parallel, various precepts used by others, said to favor the short distance scale with $H_0 > 70$ are discussed. The strongest evidence for the long scale are (1) the calibration of the peak absolute magnitude of type Ia supernovae with their Hubble diagram tied to the remote cosmic kinematic frame, (2) the distance to the Virgo cluster by six largely independent methods including Tully-Fisher using a complete cluster sample and a new calibration using recent HST Cepheid data, and (3) field spirals binned by luminosity class, also calibrated using Cepheid distances. The three methods give $H_0 = 56 \pm 3.55 \pm 2$, and $53 \pm 3$ (internal errors). $H_0$ does not vary significantly over scales from 10-500 Mpc. $H_0$ does not increase outward, as appearances using field galaxies would give if the raw data were not corrected for observational selection bias.

Higher values of $H_0$ still in the literature are based on (1) an untenably small distance to the Virgo cluster claimed by equating (against newly available evidence) the Cepheid distance of M100 with the mean distance of the cluster, (2) an untenably large Virgo cluster velocity tied to the remote cosmic kinematic frame, (3) a questionable route through the Coma cluster on the assumption that its random motion can be neglected at its assumed distance, (4) an incorrect precept that the Cepheid distance to NGC 1365, a possible member of the Fornax cluster, gives the distance to NGC 1613, parent to two SNe Ia, calibrating them, (5) an unjustified reliance on planetary nebulae and surface brightness fluctuations as distance indicators at the present stage of their calibration, and (6) either an underestimation or a neglect of the importance of observational selection bias in flux-limited samples, both for cluster galaxies (the Teerikorpi cluster incompleteness bias), or for field galaxies (the Malmquist bias). There is no valid evidence for $H_0 > 70$.

The status of the time scale test is reviewed using recent discussions of the age of globular clusters based on seven studies since 1993. The result is $13-14 (\pm 2)$ Gyr for the age of the Galactic globular cluster system. Even with a gestation period of the Galaxy of 1 Gyr, there is no time scale crisis in cosmology provided that $q_0 < 0.3$, $H_0 = 55$, and $\Lambda = 0$.

1. The Controversy

It is written that practical cosmology reduces to the “search for two numbers”. This simplicity was pronounced before the marriage had occurred between high-energy particle physics in the free quark era and the actual cosmological observations made at the telescope. Nevertheless, one of the few premises still agreed to by the current debaters is that the value of the Hubble constant remains central to the subject.

The most important reason now, even as in Hubble’s time, concerns the time scale. If the inverse Hubble constant were, for real, smaller than a “known age of the universe”
(significantly outside the errors), then the standard model, sans cosmological constant, fails.

Since 1978 our critics have espoused Hubble constants that began with values larger than 100 km s$^{-1}$ Mpc$^{-1}$ (de Vaucouleurs & Bollinger 1979 with earlier references therein), found by using (1) what are now known to be incorrect local distance calibrations, (2) by not correcting for selection bias, or (3) by claiming the absence of bias altogether and a Hubble constant that increases outward (de Vaucouleurs & Peters 1986). This is a sure signature that bias in flux-limited samples exists (Sandage 1988a, 1994a).

Although the values of $H_0$ by the proponents of the short distance scale have gradually decreased over time, yet most claims even by commentators that themselves have not been in the arena (Fukugita et al. 1993; Hogan 1994; Bolte & Hogan 1995) are for $H_0$ between 70 and 85, giving $H_0^{-1}$ between 14 and 11.5 Gyr. Recall that the free-expansion age is $H_0^{-1} = 9.8$ Gyr for $H_0 = 100$. These numbers would, of course, support a time-scale crisis even for an empty Universe with objects of ages $> 14$ Gyr.

We began a series of investigations on the distance scale in 1963 with the Palomar Hale telescope, first measuring the Cepheid distance to NGC 2403 (Tammann & Sandage 1968), and continuing in a series of papers called “Steps Toward the Hubble Constant” (Sandage & Tammann 1974 for Paper I; 1995a for Paper X). We are now proceeding again (cf. Saha et al. 1997 for Paper VIII of a new series, with earlier references therein) using Cepheid distances to type Ia supernovae measured with HST. Our value of the Hubble constant has consistently been steady near $H_0 = 55 \pm 10$ since 1974.

Nevertheless, our low value of $H_0$ has been generally discounted. The principal reason is the quite astounding apparent internal agreement of the several new and independent methods amongst themselves (Tully-Fisher, planetary nebulae, surface brightness fluctuations, globular clusters) used by others beginning in the mid 1980’s, generally giving $H_0$ between 80 and 100. Part of the agreement is, of course, a lemming effect, where, when a choice between precepts giving different final values must be made, that choice has often gone to the high $H_0$ value because other methods appeared to be in support. But that support could be shown either to be soft, or in fact, incorrect.

To that point, we have often reviewed the subject, both in conference reports or monographs (Tammann 1986, 1987, 1988, 1992, 1993, 1996a,b; Tammann & Sandage 1996; Sandage & Tammann, 1982, 1984, 1986, 1995b; Sandage 1993, 1995b, 1996) and in Journal papers (Tammann & Sandage 1995; Sandage & Tammann 1995a, 1996; Sandage 1988a,b, 1994a,b; 1996a,b; Federspiel, Sandage, & Tammann 1994; Sandage, Tammann, & Federspiel 1995), setting out the reasons why each of the methods said to give the high values of $H_0$ contain the same types of error, generally traced either to (1) neglect of the pernicious effect of observational selection bias, (2) incorrect panegyrics of why particular samples and/or methods are immune from, and therefore need not be corrected for, bias, or (3) a misunderstanding of methods to correct for the bias even when the need for correction is clear.

In each case, we have shown that the application of bias corrections, and/or a more proper calibration of the methods themselves (in particular Tully-Fisher and globular clusters), reduce the high values of $H_0$ to less than 65.

The same conclusions, for nearly the same reasons, have also been reached by Bottinelli et al. (1986a,b; 1987), Teerikorpi (1987), and now by Theureau et al. (1996) in which they obtain $H_0 = 55$ using their large sample of field galaxies with the Tully-Fisher method, calibrated with the recent Cepheid distances, carefully corrected for selection bias.

The purpose of this report is to update the current state of the debate. The plan of the paper is to (1) set out in Section 2 the fact that the Hubble constant exists and that its rate can be determined beyond all local perturbations of the velocity field by tying
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$H_0$ to the remote cosmic kinematic frame, (2) to present in Section 3 and Section 4 the evidence based on methods using type Ia supernovae, 21 cm line widths, and globular clusters. All three give $m - M = 31.7$ for the distance modulus of the Virgo cluster. This distance ($D = 22$ Mpc), when tied into the proper Virgo cluster cosmic velocity frame, gives $H_0 = 54 \pm 5$. (3) In Section 3 we review the method used by Freedman et al. (1994), Tanvir et al. (1994), and Whitmore et al. (1995) in their adoption of a demonstrably incorrect distance modulus to the Virgo cluster core, and that by stepping this distance to Coma, obtain an incorrectly high value of $H_0$. (4) We discuss in Section 5 the recent route through the Fornax cluster (Freedman et al. 1996) that assumes that the modulus of NGC 1365 defines the distance to NGC 1316, parent galaxy to two normal SNe Ia, thereby incorrectly calibrating $\langle M_{\text{max}} \rangle$ for SNe Ia. (5) We set out in Section 6 the independent evidence using field galaxies, corrected for selection effects and calibrated using Cepheids in local galaxies, that $H_0 = 53 \pm 3$. (6) In Section 7 we comment on physical methods that are independent of the distance scale ladder, also leading to $H_0 \approx 50 - 65$, and (7) finally set out in Section 10 the current position on the time scale test of the standard model.

2. The Hubble constant exists

The Hubble constant means something only if the expansion is real and if the form of the expansion velocity-field is linear. There is now no question that both of these conditions are met.

Consider first the reality of the expansion. Three tests exist. Each has proved positive. (a) The Tolman surface brightness (SB) test that identical luminous objects will have SB’s that become fainter with redshift as $(1 + z)^4$, appears to have been verified (Sandage & Perlmutter 1991; see also Kjaergaard et al. 1993 for a modified method). (b) The time dilution test, based on the prediction of Wilson (1939) that standard clocks will appear to be slow by the factor $(1 + z)$, has also apparently been verified (Perlmutter et al. 1995) using SNe Ia, (3) the temperature of the relic radiation must increase as $(1 + z)$ (Tolman 1934, eq. 171.6 combined with 171.2). The effect has apparently been measured by Songaila et al. (1994), replacing the upper limits known before (Meyer et al. 1986).

That the form of the Hubble expansion is linear with distance has been proved by using progressively more suitable “standard” objects in the Hubble diagram such as brightest cluster galaxies (BCG), and/or SNe of type Ia. A linear redshift- distance relation is proved by a straight line correlation in that diagram between apparent magnitude and log redshift with a slope of $d\text{mag}/d\log v = 5$ required by the inverse square law for intensity diminution with distance. The scatter will be small only if the spread in absolute luminosity is small. Clearly, in the absence of random motions, the scatter, read as residuals in apparent magnitude at a given redshift, measures the spread in absolute magnitude.

To determine the absolute value of distances in such Hubble diagrams, giving the Hubble constant when the distances are divided into the observed redshifts, requires only that any particular Hubble diagram be calibrated using absolute magnitudes. In what follows we calibrate two such diagrams, one based on normal SNe Ia, and the other using distance ratios to Virgo, plus the absolute distance to Virgo determined by several methods.

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1The term “velocity of expansion” appears to be meaningless in cosmology, the expansion being a time variation of the metric scale factor in the famous Lemaitre equation, not a “Doppler” effect (Harrison 1981). Of course, that fraction of the redshift that reflects peculiar (random and streaming) motions does, undoubtedly, mean real Doppler velocities.
The proof that the expansion is linear began with the first extensive data by Hubble & Humason (1931) that enlarged the original sample of Hubble (1929) many fold. It continued through the Palomar/Stromlo campaign of the 1960-1980’s where many clusters were added around the sky (Sandage 1972, 1975, 1978; Sandage & Hardy 1973) to test for isotropy of the expansion.

Figure 1 shows the Hubble diagram made at an intermediate stage in the Palomar program, using brightest cluster galaxies (BCG) (see also Sandage, Kristian, & Westphal 1976). The line has the forced slope of $d \text{mag} / d \log z = 5$. Clearly this requirement for a linear expansion is satisfied directly from the data.

\[ \text{Figure 1. The Hubble diagram using brightest galaxies in clusters and groups with data determined at Palomar and Stromlo. The abscissa is apparent } V \text{ magnitude corrected to a standard metric size, for K dimming, for Galactic absorption, for Bautz-Morgan contrast effect, and for the population richness effect. (Diagram from Sandage & Hardy 1973).} \]

\[ ^2 \text{The requirement that the form of the expansion must be linear in the standard model, although perhaps obvious, is discussed in some detail elsewhere (Sandage 1995a, Lecture 3).} \]
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Fig. 2 shows the Hubble diagram for a complete sample of 56 supernovae of type Ia at maximum light in $B$ (top) and $V$ (bottom). Again the line is forced to have a slope of $d \text{mag}/d \log v = 5$, and clearly the fit of the data to the line is excellent. The sample is selected from the literature to have reasonably well determined $B(\text{max})$ and $V(\text{max})$ magnitudes, $3 < \log v < 4.5$, and $B(\text{max}) - V(\text{max}) \leq 0.20$. The color restriction is to exclude highly absorbed and intrinsically red SNe Ia, like SN 1992K, which are known to be heavily underluminous. All SNe Ia fulfilling the above color restriction with known spectrum are spectroscopically “Branch-normal” (cf. Branch, Fisher, & Nugent 1993), the only exception being SN 1991T which is also significantly brighter than other SNe Ia in the Virgo cluster. Its exclusion could only decrease the true value of $H_0$ (cf. Section 3.2). As seen in Figure 2 SNe Ia in spirals appear to be brighter by $0.23 \pm 0.08$ in $B$ and $0.26 \pm 0.08$ mag in $V$ then those in E/S0 galaxies.

The second important point from Figs. 1 and 2 is the very small dispersion about the line, showing (1) that the expansion velocity field is extraordinarily quiet, seen by reading the residuals vertically (see Tammann & Sandage 1985; Sandage & Tammann 1995b concerning velocity anomalies on all scales), and (2) that the spread in absolute magnitude for brightest cluster galaxies and SNe Ia is small, seen by reading the residuals horizontally.

Figure 3 emphasizes the last point by combining the data in Figures 1 and 2 with an appropriate absolute magnitude offset. The small sigmas of the residuals, noted in the diagrams, show that BCG and SNe Ia are among the best standard candles known.

The Hubble diagram in Fig. 3 can be read at large enough redshifts (i.e. $v > 20000 \text{ km s}^{-1}$) to define the global value of $H_0$, freed from all velocity anomalies. $H_0$ follows from these diagrams once the absolute magnitude calibration of SNe Ia, or BCGs can be made. This has been accomplished in the manner set out in the next two sections.

3. Route through SNe Ia Calibrated by Cepheids

Our strongest evidence for the long distance scale is the determination of the absolute magnitude of Branch-normal SNe Ia, calibrating thereby Figure 2. The calibration is based on Cepheid distances to the parent galaxies. At this writing, Cepheids have been discovered, measured, and discussed in five nearby parent galaxies that have produced six SNe Ia. The ongoing experiment using Cepheids in galaxies that have produced SNe Ia is by a consortium composed of Sandage, Saha, Labhardt, Macchetto, Panagia and Tammann.

3.1. Reliability of the Cepheid Zero Point

The Cepheids are noncontroversial as being the most reliable extragalactic distance indicators known. Agreement of the zero point of their P-L relation has been achieved to better than 0.1 mag as determined over the past 30 years by a number of authors, including those shown in Table 1.

| Source                        | $<M_V>_{10 \text{ days}}$ |
|-------------------------------|---------------------------|
| Kraft (1961)                  | -4.21                     |
| Sandage & Tammann (1968)      | -4.20                     |
| Feast & Walker (1987)         | -4.13                     |
| Madore & Freedman (1991)      | -4.14                     |

Table 1. The absolute $V$ magnitude of Cepheids with $P = 10$ days according to different P-L relations.
Figure 2. Hubble diagrams in $B$ (top) and $V$ (bottom) for 56 blue SNe Ia with $B(\text{max}) - V(\text{max}) \leq 0.20$ after correction for Galactic absorption. The linear regressions for all SNe Ia give $m_B(\text{max}) = 5 \log v - (3.26 \pm 0.04)$ and $m_V(\text{max}) = 5 \log v - (3.30 \pm 0.04)$. (Data from Hamuy et al. 1996, Riess 1996, Patat 1996, Leibundgut et al. 1991 and some additional sources).
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Figure 3. Combination of Figs. 1 and 2 to emphasize the smallness of the residuals about the line of forced slope 5, proving (1) the linearity of the expansion, (2) the lack of large streaming motions about the cosmic flow (vertical residuals), and (3) the sharply peaked luminosity function both for BCG and SNe Ia (horizontal residuals). The SNe Ia with log$v > 4.5$ were kindly provided by Perlmutter (1996) and Leibundgut et al. 1996). Note that $m_V$ is plotted for BCGs but $m_B$ for SNe Ia.

The zero points of the first three entries have been determined by photometric parallaxes of Galactic clusters and associations containing Cepheids. As such, they ultimately rest on trigonometric parallaxes that define the zero point of the age-zero main sequence of the HR diagram.

When the zero point of the first three entries is applied to Cepheids in the Large Magellanic Cloud, the distance modulus of LMC is determined to be $18.50 \pm 0.10$ (Sandage & Tammann 1968; Feast & Walker 1987). The zero point of Madore & Freedman (1991), which we adopt in the following because these authors also give a calibration of the I band P-L relation, rests on the assumption that $(m - M)_0^{LMC} = 18.5$. Is the assumption correct?

Independent confirmation of the Cepheid LMC modulus, and, therefore, of the Cepheid zero point in Table 1, comes from five other methods, summarized elsewhere (Tammann 1996a, his Table 2). They give $(m - M)_0^{LMC} = 18.57 \pm 0.06$, confirming the adopted zero point of the P-L relation within an error of less than 10%.

The evidence is also strong that the zero point is virtually independent of variations in metallicity over the range of $[\text{Fe/H}]$ from 0 to -2 (Freedman & Madore 1990; Chiosi et al. 1993; Sandage 1996). Furthermore, selection bias, caused by the intrinsic spread
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of the P-L relation related to the intrinsic width of the instability strip, can also be avoided, but only if the Cepheids span a sufficient period range and if the data are suitably restricted in period (Sandage 1988c).

3.2. Calibration of SNe Ia Without Second-Parameter Effects

Rather than again setting out the detailed results of the current status of the SNe Ia experiments, summarized elsewhere (Saha 1996; Sandage et al. 1996; Tammann 1996a; Saha et al. 1997), we simply state the result.

The mean absolute magnitudes for Branch-normal SNe Ia, based on either six (in $V$) or seven (in $B$) independent calibrations, are

\[ M_B(\text{max}) = -19.52 \pm 0.07, \]
\[ M_V(\text{max}) = -19.48 \pm 0.07. \]

That these values conflict fundamentally with the short distance scale with $H_0 \approx 85$ is seen by comparing equations (1) and (2) with the statement by de Vaucouleurs (1979, his Table 9) that $<M_V(\text{max})>$ must be $-18.50$ for his distance scale with $H_0 = 88$ to be correct, or the statement by Pierce (1994) that $<M_B(\text{max})>$ must be $-18.74$ for his value of $H_0 = 86$ to be correct.

Equations (1) and (2) show that the scale of de Vaucouleurs would be $H_0 = 56$ and that of Pierce would be $H_0 = 61$ if corrected to the values in equations (1) and (2). Calculated in this way, the average would be $<H_0> = 58$. That value will, however, still contain the random errors of each of the methods used in the determinations by de Vaucouleurs and, independently, by Pierce to obtain their relative distances.

Combining equations (1) and (2) with the equations of the ridge lines in Figure 2 for blue SNe Ia gives Hubble constants of

\[ H_0 = 56 \pm 2 \] (internal error),

\[ H_0 = 58 \pm 2 \] (internal error).

The results are quite robust against various subsamples of both the seven calibrating SNe Ia and the Hubble diagrams if they are divided into spiral and E galaxy groups and further separated by pre and post 1985 data.

An interesting subsample are the 12 blue SNeIa which have occurred in spiral galaxies after 1985 in the distance range $3.8 < \log v < 4.5$. They match best the seven calibrating SNeIa which lie predominantly in spiral galaxies, they have the most reliable photometry, and they are least affected by any peculiar motions. They give with the calibration in equation (1) and (2) $H_0 = 55 \pm 2$ in $B$ and $H_0 = 57 \pm 2$ in $V$. Their rms scatter about the Hubble ridge line is only $\sigma_B = 0.21$ and $\sigma_V = 0.18$ mag emphasizing the power of SNeIa as standard candles.

3.3. Suggested Second-Parameter Correlations for SNe Ia in their Effects on the Determination of $H_0$

The interpretation of the supernova experiments given by equations (1) and (2) has been challenged on the basis that there may be a range of true absolute magnitudes even of Branch-normal SNe Ia, depending on (1) details of the shape of the light curve (Pskovskii 1977, 1984; Phillips 1993; Hamuy et al. 1995; Riess, Press, & Kirshner 1995), (2) intrinsic color of an individual SN (Höflich & Khokhlov 1996), (3) color or Hubble type of the parent galaxy (Branch, Romanishin, & Baron 1996).

Whatever the correlations may eventually be found to be, their total effect on $<M(\text{max})>$ is clearly small. The proof is that the observed dispersion in apparent magnitude at a given redshift in the Hubble diagram of Figure 2 is itself so small.
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The size of the effect suggested by Phillips (1993) was shown to be too large by a factor of three (Tammann & Sandage 1995). We postulated that the problem was due to his use of secondary distance indicators from TF and SBF that were not accurate enough for the purpose. That the slope of the correlation of decay rate of the light curve with $M$(max) derived by Phillips is too large by a factor of between three and four was confirmed by Hamuy et al. (1995, 1996).

The color correlation derived by Branch et al. 1996 is more convincing, but these authors show that the effect on the Hubble constant, based on our six calibrating SNe Ia with Cepheid distances, is nil. They derive $H_0 = 58 \pm 7$ from the totality of the data, closely the same as if no correction had been applied.

Whatever the final outcome will be of these suggestions that second parameters may be needed for a better determination of $<M$(max)$>$, the already tight Hubble diagrams of SNe Ia, without second parameters, show that their effect on $H_0$ will be less than 5%. The only reason for concern would be a systematic luminosity difference between the seven calibrators and the 56 distant SNe Ia. But the former coming from a distance-limited sample and the latter from a flux-limited sample they can differ only in the sense – for basic principles of stellar statistics – that the calibrators are underluminous which causes the results in equation (3) and (4) to be upper limits.

The one possibility to increase the value of $H_0$ is to postulate internal absorption of the distant SNe Ia in spirals although this is against the expectations of a flux-limited sample which is biased against all dimming effects. In spite of this Riess, Press, & Kirshner (1995) have proposed non-negligible amounts of internal absorption for many SNe Ia in Figure 2. However, their results raise more problems than they solve. (1) The already small luminosity scatter of SNe Ia in spirals is closely the same as the scatter of SNe Ia in E/S0 galaxies; if one “corrects” the SNe Ia in spirals for internal absorption and thereby reduces their luminosity scatter, one has to accept the conclusion that SNe Ia in spirals were better standard candles than those in early-type galaxies. (2) The dependence of absolute magnitude on SN color $B$(max) − $V$(max) is the same in spirals and E/S0s [$M_B \propto 1.8 (B-V)$ and $M_V \propto 0.8 (B-V)$ for SNe Ia after 1985] and is already considerably flatter than theoretical models predict for the intrinsic correlation (Höflich & Khokhlov 1996; cf. van den Bergh 1995), leaving no room for any internal absorption. (3) The total absorption in $V$ proposed by Riess, Press, & Kirshner (1996) is less than the Galactic absorption alone (Burstein & Heiles 1984) by as much as $\sim 0.24$ mag for some SNe Ia (SN 1992K, SN 1993ac). For these and other reasons the suggested absorption corrections are highly implausible.

4. The Route based on the Distance to the Virgo cluster

The second method that gives the global value of $H_0$ directly, reading a Hubble-like diagram at large redshifts (i.e. $v \gtrsim 10000 \, \text{km} \, \text{s}^{-1}$), requires knowledge of the actual distance to the Virgo cluster, plus the ratio of distances of remote clusters to the Virgo cluster itself.

Relative distances can be found by a variety of reliable methods, often agreed upon by both the proponents and opponents of both the long and short distance scales. Only the absolute distance of the Virgo cluster is the point of controversy.

Consider first the non-controversial Hubble diagram using distances relative to Virgo.

4.1. The relative Hubble diagram to $v = 10000 \, \text{km} \, \text{s}^{-1}$

The method was proposed and initially applied using distance ratios of 17 “remote” clusters relative to Virgo (Sandage & Tammann 1990; Jerjen & Tammann 1993). The
distance ratios are determined by a variety of methods (Tully-Fisher, $D_n - \sigma$, first ranked cluster members).

The sample has now been enlarged to 31 relative distances by adding data given by Giovanelli (1996). The complete data, listed elsewhere (Tammann & Federspiel 1996), use redshifts reduced to the Virgocentric kinematic frame for $v < 3000$ km s$^{-1}$ using an infall velocity of 220 km s$^{-1}$ (Tammann & Sandage 1985) and the catalog of corrections by Kraan-Korteweg (1986) of observed redshifts to the Virgocentric frame. The observed redshifts for $v > 3000$ km s$^{-1}$ have been reduced to the kinematic frame of the cosmic microwave background (CMB) using the dipole amplitude of 630 km s$^{-1}$. The rationale for these precepts is that there is a free expansion within the “local bubble” except as decelerated by the Virgo complex (given by the Virgocentric corrections), and that the “local bubble” is falling, in first approximation, in bulk toward the hot CMB pole (cf. Federspiel et al. 1994).

The resulting relative Hubble diagram is shown in Figure 4. The slope of $d \text{mag} / d \log cz = 5$ is forced. As in Figures 1-3, this requirement for a linear expansion is well met. Furthermore, the small scatter of 0.11 mag or, read vertically, of $\Delta v/v = 0.052$ sets an upper limit – banning all other error sources – of $\sim 260$ km s$^{-1}$ at a median velocity of $\sim 5000$ km s$^{-1}$ for the mean value of any (one-dimensional) random or streaming velocities about the ideal Hubble flow (cf. Lauer & Postman 1994).

Figure 4. Hubble diagram of 31 clusters with known relative distances. Asterisks are data from Jerjen & Tammann (1993). Open circles are from Giovanelli (1996). Filled circles are the average of data from both sources. The ordinate is log redshift reduced to the CMB frame. The abscissa are the distance modulus differences of each cluster relative to Virgo.
The equation of the ridge line in Figure 4 is

$$\log v(CMB) = 0.2 \left[ (m - M)_{cl} - (m - M)_{Virgo} \right] + 3.070 \pm 0.024,$$

(5)

if the strongly deviating point for the Eridanus cluster (at delta modulus difference ≈ 1) is removed.

Note that the Virgo cluster is predicted from equation (5) to have a cosmic velocity, freed from all random and streaming motions, of 1175 ± 30 km s\(^{-1}\). Comparing this value with the actually observed mean cluster velocity of \(v_0 = 922 \pm 35\) (Binggeli et al. 1993), reduced to the centroid of the Local group by the precepts in the RSA, on obtains an “infall” (actually retarded expansion) velocity of the Local Group of 253 ± 46 km s\(^{-1}\). We take this as confirmation of \(v_{infall} = 220 \pm 46\) km s\(^{-1}\) adopted above, rather than the much higher values often used in the earlier literature.

The global value of \(H_0\) follows from equation (5) as

$$\log H_0 = (8.070 \pm 0.024) - 0.2 (m - M)_{Virgo}.$$  

(6)

For \(H_0\) we need, therefore, the distance modulus of the Virgo cluster, yet the velocity of the cluster is not needed! We set out in the next subsections four independent methods, discussed in the order of their power, that average to \((m - M)_{Virgo} = 31.7\). Recall that advocates of the short scale \((H_0 \sim 80)\) require \((m - M)_{Virgo} = 30.9\) (Jacoby et al. 1992).

Equation (6) can also be written in the following form

$$H_0 = (50 \pm 3)(23.5/D_{Virgo}),$$

(7)

where the Virgo cluster distance \(D_{Virgo}\) is in Mpc. This may be compared with \(H_0 = (50 \pm 7)(21.7/D_{Virgo})\) derived earlier by tying the Virgo cluster to the relative cluster distances then available (Tammann & Sandage 1985).

4.2. Virgo distance via the Tully-Fisher method

The method using 21 cm line-widths has been applied many times but with variable success. Widely divergent values are in the literature that in some cases favor the short distance scale (e.g. Pierce & Tully 1992 giving \(m - M = 30.9\)) and in others the long scale (Kraan-Korteweg et al. 1988 with \(m - M = 31.6\); Fouqué et al. 1990 for the same value if corrected to the modern local calibrators; Federspiel, Tammann, & Sandage 1997 with \(m - M = 31.7\) set out here).

It has been shown (Federspiel et al. 1994; Sandage, Tammann, & Federspiel 1995) that the reasons for small values of \((m - M)\) for Virgo (the short scale) using TF are two; (1) use of incorrectly small distances to the local calibrators in earlier papers by proponents of the short scale, and (2) neglect of the disastrous effect of the Teerikorpi (1987, 1990) cluster incompleteness bias. It can be shown that this bias produces errors in the modulus up to 1 mag depending on how far one has sampled into the cluster luminosity function regardless how the sample is chosen, if the sample remains incomplete. The modulus error is a strong function of the fraction of the luminosity function that remains unsampled (Kraan-Korteweg et al. 1988; Sandage et al. 1995).

The calibration of the TF relation has been dramatically improved by the advent of Cepheid distances with HST. There are now 14 Cepheid distances available for spirals suitable of the calibration. Detailed data with complete references to the extensive literature are given elsewhere (Tammann & Federspiel 1996; Federspiel et al. 1997) and are not repeated here.

A new study of the TF relation has been completed (Schröder 1996; Tammann & Federspiel 1996; Federspiel et al. 1997) made using a now complete sample of Virgo cluster spirals. Rigid criteria have been invoked in the selection of members over a
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Many subtleties, not seen in earlier studies, have been found. These include a variation of the derived modulus on the wavelength of the observations (covering UBVRI), and a correlation of the derived modulus on the degree of hydrogen depletion for the spirals.

There is space only to quote the result, now based on the new Cepheid calibration of the TF method, account being taken of the two effects just mentioned. The value adopted by Tammann & Federspiel (1996) is

$$(m - M)_{\text{Virgo}} = 31.66 \pm 0.15,$$  \hspace{1cm} (8)

but they warn “The application of the TF relation is considerably more intricate than often realized. It not only takes multicolor information for \textit{complete} cluster samples, but the result is also sensitive to the input parameters. For example, the Virgo modulus in $B$ is too large by 0.07 mag (relative to the adopted mean) and is too short by 0.09 mag in $I$. These values may change from cluster to cluster depending on the color excess and the HI-deficiency of the spiral members.”

4.3. Virgo cluster distance from SNe Ia

Galaxies associated with the Virgo cluster complex have produced at least eight type Ia supernovae (Sandage & Tammann 1995b; Tammann 1996a), including objects with older photometry, giving mean apparent magnitudes at maximum of $B(\text{max}) = 12.10 \pm 0.13$, and $V(\text{max}) = 12.11 \pm 0.16$. Hamuy et al. 1996 have determined the mean apparent peak magnitudes of five blue, particularly well observed SNe Ia in Virgo as $B(\text{max}) = 12.16 \pm 20$, and $V(\text{max}) = 12.07 \pm 0.20$. Taking the larger sample, because it is less sensitive to depth effects in the Virgo cluster, and combining it with the absolute magnitude calibration via Cepheids observed with HST of $M_B = -19.52 \pm 0.07$ and $M_V = -19.48 \pm 0.07$ from equations (1) and (2) gives

$$(m - M)_{\text{Virgo}} = 31.61 \pm 0.16.$$  \hspace{1cm} (9)

4.4. Virgo cluster distance from globular clusters

The peak of the luminosity function (LF) of globular clusters (GC) has frequently been used as a possible standard candle. A new calibration of GCs in the Galaxy and in M31 combined with a compilation of published GCLFs in five Virgo ellipticals has led to a Virgo modulus of $(m - M) = 31.75 \pm 0.11$ (Sandage & Tammann 1995a). Meanwhile, Whitmore et al. (1995) found a very bright peak magnitude in $V$ and $I$ for NGC 4486 from HST observations. Their data with our 1995 precepts gave $(m - M) = 31.41 \pm 0.28$ in a critical discussion of the Whitmore et al. result (Sandage & Tammann 1996). However, later data make it unclear that the GCs in NGC 4486 are suitable for the experiment. The NGC 4486 GC system have a bimodal color distribution in $V - I$, unlike any sample of coeval clusters, suggesting age differences and possible merger effects (Fritze-v. Alvensleben 1995; Elson & Santiago 1996). Turning a blind eye to this problem and averaging over all available data for Virgo cluster GCLFs gives $(m - M) = 31.67 \pm 0.15$. We are aware that the method may still face considerable uncertainties.

4.5. Virgo cluster distance from resolved Cepheids

We now must approach the most controversial aspect of the disagreement between us and our critics. The first HST Cepheid distance of a galaxy associated with the Virgo complex was for NGC 4321 (M 100) (Freedman et al. 1994). Amidst unprecedented publicity with its subsequent major influence in the archive literature, the surprisingly small distance of $(m - M) = 31.2 \pm 0.2$ ($D = 17.1 \pm 1.7$ Mpc) was precipitately interpreted as the distance to the Virgo cluster E galaxy core itself (Freedman et al. 1994; Mould et
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al. 1995; Kennicutt et al. 1995). These authors further adopted the large value of the cosmic velocity of the Virgo core of $1404 \text{ km s}^{-1}$, unsupported by equation (2), to obtain $H_0 = 82 \pm 17$.

Their precept that M100 itself defines the distance to the cluster core was of course the only way the authors could proceed; it was the only distance they had. However, the well known wide spatial spread of the spirals as an envelope surrounding the more compact E cluster core already signaled a back-to-front ratio in Virgo that eventually must be, and now has begun to be, accounted for.

Proof that a large cluster depth effect exists came with the Cepheid distance (Sandage et al. 1996) to NGC 4639, parent galaxy to the type Ia supernova 1990N. The distance modulus of this low-velocity (and hence certain cluster member) galaxy was determined to be $(m - M)_0 = 32.03 \ (D = 25 \pm 2.5 \text{ Mpc})$, which is 0.8 mag fainter than for M100. Two galaxies, each assumed to be a member of the cluster, cannot both define the distance to the cluster core when their distances are in the ratio of 1.5. The important distance difference between the two cluster members is also supported by their TF distances (Federspiel et al. 1997).

Because spirals clearly form an extended envelope surrounding the condensed E galaxy core, distances to many more spirals in the Virgo region must be determined before a “Cepheid distance” to Virgo can be determined by this direct assault.\footnote{It should be noted that the route through SNe Ia in Section 3 where, to be sure, the spirals NGC 4496A, NGC 4536, and NGC 4639 are used in the SNe Ia calibration of $<M(\text{max})>_{\text{SNeIa}}$, no use is made of their probable Virgo cluster connection, either as to the distance to the cluster core itself, or any redshift data therefrom.}

There is however an indirect method via a Cepheid distance to a spiral in the Leo group where the back-to-front ratio may be more favorable. Tanvir et al. (1995) have determined a Cepheid distance to NGC 3368 (M96) and Graham et al. (1996) have a Cepheid distance to NGC 3351 (M95). The mean of the two is $(m - M) = 30.22 \pm 0.12$ (increased by 0.05 mag for a zeropoint offset of HST photometry for bright stars, following Saha et al. 1996). If also the red-giant tip distance of NGC 3379 (Sakai et al. 1996) is considered the mean group distance becomes $(m - M) = 30.28$.\footnote{Cepheid observations in an other Leo group member, NGC 3627, parent of SN 1989, by our consortium are scheduled for cycle 6 of HST.} The relative distance between the Leo group and the Virgo core is moderately well determined to be a modulus difference of $1.25 \pm 0.15$ mag, based on five indicators (see Table 3 of Tammann & Federspiel 1996). Adding this difference to the adopted mean modulus of the Leo group gives

\begin{equation}
(m - M)_{\text{Virgo}} = 31.53 \pm 0.21,
\end{equation}

which we adopt in Table 2 below to be the (provisionally determined) “Cepheid modulus to Virgo”.

4.6. Other Methods

Other methods summarized elsewhere (Tammann & Sandage 1996; Sandage & Tammann 1995b; Tammann 1996a) can only be mentioned here for lack of space. We simply list the results for the $D_n - \sigma$ recalibration of Dressler’s (1987) result by Tammann (1988) and the measurement of normal novae in three Virgo E galaxies (Pritchet & van den Bergh 1987) discussed in the same summaries just cited.

4.7. Conclusion on $H_0$ by going through Virgo

The summary of the above methods to the distance of the Virgo cluster is in Table 2.

The six methods give very consistent results. This is remarkable in two respects. First, the methods include independent scales. The TF, SNe Ia, and Cepheid methods depend on the zero point of the P-L relation of Cepheids, the globular clusters rest on
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Table 2. The Virgo cluster modulus from various methods

| Method          | $(m - M)_{\text{Virgo}}$ | Hubble type |
|-----------------|--------------------------|-------------|
| TF              | 31.60 ± 0.15             | S           |
| SNe Ia          | 31.61 ± 0.16             | E, S0, S    |
| Globular Cl     | 31.67 ± 0.15             | E, S0       |
| Cepheids        | 31.53 ± 0.21             | (E), S      |
| $D_n - \sigma$  | 31.85 ± 0.19             | S0, S       |
| Novae           | 31.46 ± 0.40             | E           |
| mean            | 31.66 ± 0.08             |             |

the absolute magnitude calibration of the RR Lyrae (controversial to be sure, but only at the 0.2 mag level; we have used that of Sandage 1993b), the $D_n - \sigma$ method depends on the Cepheid distances to M31 and M81 but also on the independent size of the Galactic bulge, and the novae rely on Cohen’s (1985) Galactic calibration. Second, the different distance determinations comprise both spiral, S0 and E galaxies.

Adopting $(m - M)_{\text{Virgo}} = 31.66$ and using equation (6) of Section 4.1 gives

$$H_0(\text{global}) = 55 \pm 2 \quad \text{(internal error)},$$

for the direct method through Virgo but tied to the Machian frame at distances of 10000 km s$^{-1}$ through Figure 4.

5. Does going to Coma help?

5.1. Improper Precepts

Both the Freedman et al. (1994) Key Project consortium and the Tanvir et al. (1995) astronomers attempt to pass by the question of the correct Virgo or Leo cosmic redshift (i.e in the Machian kinematic frame). Rather they adopt an assumed distance modulus difference between the Coma cluster and either the Virgo core or the Leo group. They then determine their Coma distance by adding the assumed modulus differences to their Virgo or Leo distance and then use the observed or somehow corrected redshift of Coma to divide into their Coma distance to obtain $H_0$. This assumes no random motion of Coma relative to the cosmic flow. Whitmore et al. (1996) follows the same procedure using a Virgo modulus from the globular clusters in M87, obtaining, as does Freedman et al. and Tanvir et al., high values of $H_0$ near 80.

It is of course a circular exercise to go through Coma if their assumed Virgo moduli via M100 and the GC in NGC4486 are wrong for the reasons discussed earlier. Furthermore, one need not assume that Coma has no random motion relative to the cosmic flow; any random velocity can be determined with high precision from equation (5) once any particular modulus difference between Coma and Virgo is assumed. The method is discussed elsewhere (Sandage & Tammann 1996, Section 4).

For example, if the modulus difference between Coma and Virgo is 3.72 ± 0.09 mag, which is the mean of the value used by Jerjen & Tammann (1993) and the two values given by Dekel (1995) using the “Potent” formalism for systematic motion, then the predicted cosmic redshift (in the Machian kinematic frame) of Coma from equation (5) is 6516 km s$^{-1}$. With $(m - M) = 31.66 \pm 0.08$ for Virgo, the Coma distance modulus is then 35.38 ± 0.12 (or D = 119 ± 7 Mpc), and the Hubble constant becomes

$$H_0 = \frac{6516}{119} = 55 \pm 4.$$  (12)
The difference between the observed and cosmic redshift of \( 7188 \, \text{km\,s}^{-1} \) and \( 6516 \, \text{km\,s}^{-1} \) implies a one-dimensional peculiar motion of the Coma cluster of \( 672 \, \text{km\,s}^{-1} \) with respect to the CMB. This value is high in comparison with the mean peculiar motion of clusters (Section 4.1), but it compares well in size with the local CMB motion.

On the other hand, if the modulus difference was \( 3.80 \, \text{mag} \) from Jerjen & Tammann, the cosmic Coma redshift would be \( 6761 \, \text{km\,s}^{-1} \), the distance would be 124 Mpc, the Hubble constant would again be 55, and the peculiar motion would be smaller at \( 427 \, \text{km\,s}^{-1} \). Of course the Hubble constant is identically the same by both procedures because, by adopting the same Virgo modulus of 31.66, we simply move along the correlation line in Figure 4. This is the line for constant Hubble constant (Sandage & Tammann 1996, Section 4).

### 5.2. Going to Coma Directly

A major HST experiment has been completed by Baum et al. (1995) directly on the globular clusters in the off-center E galaxy, NGC 4481, in the Coma cluster. From the globular cluster luminosity function they obtain a minimum distance of \( 108 \pm 11 \, \text{Mpc} \), or \( (m - M)_{\text{Coma}} > 35.21 \). With the observed redshift of \( 7188 \pm 450 \, \text{km\,s}^{-1} \) corrected for the local CMB motion (Jerjen & Tammann 1993), then

\[
H_0 < 67 \pm 8. \tag{13}
\]

This minimum distance of 108 Mpc is 17% larger than the actual distance of 92.6 Mpc set out by Whitmore et al. (1995).

In this direct way through Coma, independent of Virgo, we of course have no value for the modulus difference with Virgo unless we again use our assumed absolute Virgo modulus from Table 2. To remain independent of Virgo forbids to use equation (13) to determine the cosmic velocity \( v_{\text{CMB}} \) of Coma; the peculiar motion of the cluster must then remain unaccounted for.

### 6. Is there a route through Fornax?

Despite enormous efforts over the past 30 years, even the distance ratio of the Fornax cluster to Virgo is only poorly known. A listing of the 30 investigations, given elsewhere (Tammann & Federspiel 1996), separates the data by Hubble type into distances relative to Virgo for spirals and E/S0 galaxies. The difference in distance moduli varies from -0.40 mag, Fornax being closer than Virgo, to being more distant by +0.70 mag. Many different and independent methods have been used in these 30 investigations (for example TF, brightest cluster galaxies, color/luminosity correlations for early-type galaxies, the surface brightness/absolute magnitude relation for dE galaxies, globular clusters, SNe Ia, SBF, planetary nebulae, etc), and the details of the work are enormous, seen in the summary table by Tammann & Federspiel (1996).

Setting aside questions of systematic errors between the methods (which are largely unknown), the weighted averaged data give the modulus difference between Fornax and Virgo for the Fornax spirals as \( -0.22 \pm 0.06 \, \text{mag} \) (Fornax being closer), and \( +0.13 \pm 0.07 \, \text{mag} \) for the E and S0 early-type galaxies, giving a difference between the types of \( 0.35 \pm 0.09 \). We are moderately convinced that this is not due to a difference at Virgo between the Virgo spirals and E/S0 types because here the two types of galaxies give closely the same mean Virgo distance (cf. Section 4.7).

Because of the spread in the totality of the 30 investigations, it would be premature to take the separation in space between the early-type and late-type Fornax galaxies to be real. Nevertheless it may be a warning that the Fornax complex is complicated, perhaps even elongated along the line of sight.
Velocities do not help much to define the cluster structure. The mean of 41 E/S0/dE galaxies is \( v_{220} = 1323 \pm 48 \, \text{km s}^{-1} \), reduced to the Virgocentric frame by our previous precepts (Tammann & Sandage 1985). The dispersion for this sample is \( \sigma = 307 \, \text{km s}^{-1} \). The mean of 27 S/Im galaxies is \( v_{220} = 1436 \pm 66 \, \text{km s}^{-1} \) with \( \sigma = 343 \, \text{km s}^{-1} \). The statistical agreement of the mean velocities can be interpreted as the two galaxy types being at the same distance, but could also be a result of the S/Im members lying in the foreground and falling toward the more compact E/S0 galaxy cluster core.

The mean over all types is \( v_{LG} = 1366 \pm 50 \, \text{km s}^{-1} \), reduced to the centroid of the Local Group by the precepts used in the RSA, or \( v_{220} = 1338 \pm 50 \, \text{km s}^{-1} \) again reduced to the Virgocentric kinematic frame.

The cluster at a distance of roughly 30 Mpc from Virgo may have a large peculiar motion of its own, signaled by the significantly higher mean cluster velocity than that of Virgo, even though they are at about the same distance from us. If we adopt the mean modulus difference of 0.00 mag, obtained by averaging all data from the 30 determinations mentioned earlier (there are many more determinations for E/S0 galaxies than for spirals), equation (5) predicts a cosmic velocity of 1175 km s\(^{-1}\), significantly different than the observed velocities quoted above.

The conclusion from this is that the Fornax cluster is much less suited for the determination of \( H_0 \) than the Virgo cluster. The possible separation of the Fornax members by Hubble type, its expected non-negligible peculiar motion, and the very low weight of the determination of its \( v_{\text{CMB}} \) velocity, call for great caution.

Four exercises to derive \( H_0 \) from Fornax data illustrate the difficulties. Here we do not carry the errors in \( H_0 \) due to uncertainties in various cluster velocities, but show only errors due to distance uncertainties.

(1) A Cepheid distance of 18.2\(+1.3\) Mpc \((m-M) = 31.30 \pm 0.15 \) has been announced by Silvermann et al. (1996) for the exceptionally large spiral NGC 1365 in the region of the Fornax cluster. If this value is confirmed by later definitive photometry and if it is naively taken as the distance to the compact E/S0 cluster core, and using \( v_{220} = 1338 \, \text{km s}^{-1} \) as the correct velocity within the “local bubble” in the Virgocentric kinematic frame (i.e adopting the kinematic model of the near expansion velocity field in Federspiel et al. 1994), one obtains \( H_0 = 74 \pm 6 \).

(2) The turn-over magnitudes of the GCLFs of seven early-type Fornax members have been compiled by Whitmore (1996). The result of \(<m_{10}>_V = 23.80 \pm 0.08 \) is suspicious because it implies the early-type Fornax galaxies to be nearer by 0.13 \pm 0.09 mag than eight early-type Virgo galaxies which give \(<m_{10}>_V = 23.93 \pm 0.04 \) (cf. Whitmore 1996). If in spite of this, the Fornax value is accepted and combined with the absolute calibration of \(<M_{10}>_V = -7.62 \pm 0.20 \) (realistic external error) from the Galaxy and M 31 (Sandage & Tammann 1995a) one obtains \((m-M)_{\text{Fornax}} = 31.42 \pm 0.22 \) or 19.2 \pm 2.1 Mpc. This gives with \( v_{220} = 1338 \, \text{km s}^{-1} \) a value of \( H_0 = 70 \pm 7 \). Whitmore (1996) derived \( H_0 = 84 \) because he calibrated \(<M_{10}>_V \) on the assumption that the early-type Virgo galaxies with known GCLFs were at the same distance as the spirals NGC 4321 (M 100), NGC 4496A and NGC 4536, all three of which are particularly well resolved and now known to lie on the near side of the cluster, shown by the large distance of NGC 4639.

(3) If one combines the mean modulus difference with Virgo of 0.00 mag with the cosmic velocity inferred from equation (5) of 1175 km s\(^{-1}\), and using \((m-M)_{\text{Virgo}} = 31.66 \) from Table 2, then \( H_0(\text{cosmic}) = 55 \). This, of course is the same for Virgo (equation [1]) because the input numbers have been made the same by assuming the same distance.

(4) Given the very small dispersion in M(max) for SNe Ia demonstrated earlier from Figure 1, then the best determination of the distance to the Fornax cluster is from the three SNe Ia produced by the early-type Fornax galaxies NGC 1380 for SN 1992A and
NGC 1316 for the two SNe Ia 1980N and 1981D. The apparent magnitudes for these prototypical Branch-normal SNe Ia are in Table 3.

Table 3. Data for the three SNe Ia in Fornax cluster galaxies

| SN     | galaxy     | B(max) | V(max) | m − M |
|--------|------------|--------|--------|-------|
| 1980N  | NGC 1316   | 12.49  | 12.44  | 31.76 |
| 1981D  | NGC 1316   | 12.59  | 12.40  | 31.79 |
| 1992A  | NGC 1380   | 12.60  | 12.55  | 31.88 |
| mean   |             | 12.56  | 12.46  | 31.81 |

The apparent magnitudes in B and V are taken from the summary by Hamuy et al. (1996). The distance moduli in the final column of Table 3 are the mean of the moduli in B and V assuming absolute magnitudes of $M_B(\text{max}) = -19.32$ and $M_V(\text{max}) = -19.28$. These values have been adopted in the most conservative manner possible. They are 0.2 mag fainter than set out earlier in Section 3 as equations (1) and (2). If second-parameter corrections are required to SNe Ia (Saha et al. 1997) depending on galaxy type, the data suggest that SNe Ia in early type galaxies are slightly fainter than in spirals. The present calibration of $M(\text{max})$ for SNe Ia (Sandage et al. 1996; Saha 1996) is made via Cepheids only in spirals, whereas the galaxies in Table 3 are early types.

Note that the exclusion of the outlying galaxy NGC 1316 as a reliable cluster member has essentially no effect on the following conclusions, because the distance based on only SN 1992A in NGC 1380 is only slightly larger than the adopted mean distance of Table 3.

Using $(m - M) = 31.81 \pm 0.20$ (or $D = 23.0 \pm 3$ Mpc) based on Table 3 with $v_{\text{CMB}} = 1259$ km s$^{-1}$ calculated from equation (5), a modulus difference of 0.15 mag from Virgo based on Table 3, and $(m - M)_{\text{Virgo}} = 31.66$, gives

$$H_0 = 1259/23.0 = 55 \pm 6$$

from Fornax.

6.1. The Route taken by Freedman et al. (1996) through Fornax

The route to equation (14) is straightforward if we (a) adopt the precepts that SNe Ia have only a small intrinsic scatter in $M(\text{max})$ as proved by Figure 1 and that the calibration is given relative to Cepheids by equations (1) and (2), and (b) that the cosmic velocity of Fornax is given by equation (5) using a modulus difference of 0.15 mag. Freedman et al. (1996) reject both of these precepts, and proceed as follows.

They assume that their Cepheid distance for the spiral NGC 1365 with $(m - M) = 31.3 \pm 0.15$ is the distance to the Fornax cluster core. With this distance defining also the distance to the two parent galaxies of the three SNe Ia in Table 3, they derive absolute magnitudes for these SNe Ia $M_B(\text{max}) = -18.74$ and $M_V(\text{max}) = -18.84$, contradicting the small dispersion relative to equations (1) and (2) and also the direct evidence from the tightness of the Hubble diagram of Figure 2.

The result, although contradicting these external evidences, permits them to dismiss the value of $H_0$ from equations (3) and (4) which were derived needing no assumptions as to pedigree of parenthood for the SNe Ia used there. Six of the calibrating SNe Ia lie in galaxies with proper Cepheid distances. Only the seventh calibrator, SN 1989B in NGC 3627, was assumed to share the distance with three other Leo group members; its omission would have no effect on our adopted $<M(\text{max})>$ SNe Ia (cf. Saha et al. 1997).

Our response is that their precept that the distance to NGC 1365, whatever its final value may be, does not define the distance to NGC 1380 and NGC 1316, just as the
distance of M 100 does not define the distance to the Virgo cluster core, shown by the large Cepheid distance to the similar Virgo spiral NGC 4639 (Sandage et al. 1996; Saha et al. 1997).

7. Methods not used

We must mention in passing the methods based on planetary nebulae and surface brightness fluctuations, both of which have an extensive literature (Jacoby et al. 1992 for a review). We have not used either of these methods for several reasons.

Discussions elsewhere of the planetary nebulae method (Bottinelli et al. 1991; Tammann 1993; Méndez et al. 1993) have considered in a detail, not repeated here, the problem encountered by a sloping bright end of the luminosity function and its effect on the determination of distances. Whatever the difficulties with the method and/or its calibration, we judge the small modulus of the Virgo cluster at $(m - M) = 30.84$ determined with the method (Jacoby et al. 1990), compared with $(m - M) = 31.66$ from Table 2, as unrealistic.

Our basis of not understanding the results of the surface brightness fluctuation method is the same. Detailed discussions have been set out elsewhere (cf. Tammann 1996a) giving reasons why the calibration of the method may need a new evaluation. In particular, the SBF distance to the Virgo cluster of $31.03 \pm 0.05$ (Tonry et al. 1997) is not only $0.63 \pm 0.09$ mag smaller than by all other evidence (Table 2), but it also implies a mean absolute magnitude of $M_B(\text{max}) = -18.93 \pm 0.14$ for the eight Branch-normal SNe Ia in the Virgo cluster discussed earlier. This is excluded by the seven Cepheid-calibrated SNe Ia giving $M_B(\text{max}) = -19.52 \pm 0.07$ as in equation (1) and set out in detail elsewhere (Sandage et al. 1996; Saha 1996) and by all existing type Ia models (see Tammann & Federspiel 1996, their Section 4). The reasons remain a mystery, but the evidence is sufficient to make the method suspect for us.

8. Third route through field galaxies corrected for selection bias

The overriding power of the first two routes to $H_0$ in Sections 3 and 4, is that the calibrations of $M(\text{max})$ SNe Ia, and the absolute distance of the Virgo cluster can be used to calibrate Hubble diagrams that extend far into the cosmic expansion field, independent of any and all local velocity anomalies. The traditional methods using local calibrations and local galaxies (for example to the limit of the RSA at $v < 3000$ km s$^{-1}$) do not have that advantage. They are much more sensitive to the details of the local velocity field and to the effect of observational selection bias on flux-limited local samples in the presence of a much wider intrinsic dispersion of $<M>$ than for SNe Ia and the distance ratios that enter Figure 4.

Nevertheless, the first determinations of $H_0$ (Robertson 1928; Lemaitre 1927, 1931; Hubble & Humason 1931) were made by calibrating $<M>$ for local galaxies and applying that calibration to a general field sample, generally, to be sure, with no discussion of selection bias.

The method was improved fundamentally with the discovery by van den Bergh (1960a,b) of a new sub-classification system based on “luminosity classes” depending on the “beauty”, (or geometrical entropy) of galaxian images. He showed that this subdivision by regularity of the spiral pattern (beauty) narrowed the luminosity function far beyond that which would have applied across the entire wide morphological boxes of the original Hubble sequence, even within a given Hubble class.

Once the calibration of appropriate van den Bergh luminosity classes could be obtained by fundamental (Cepheid) means, and/or by luminosity ratios established between
the classes via relative Hubble diagrams, the local Hubble constant follows immediately if, but only if, the effect of observational bias can be determined and eliminated.

The problem of observational selection bias has been a major stumbling block for every discussion of the $H_0$ problem using local galaxies. The bias has often been ignored, whereas, in fact, it is the reason for the difference between the short and the long distance scale.

The case has been made in a series of papers devoted (a) to the effect of the bias, and (b) to developing methods to correct for it using local samples that are flux-limited rather than distance-limited. In every case, the corrections based either on what we have called “Spaenhauer diagrams” (Sandage 1994a,b; Federspiel et al. 1994), or what Bottenelli et al. (1986a,b) and Theureau et al. (1997) have called the “plateau of non-biased data”, show that bias corrections dominate the answer.

Table 4 summarizes the data now available on this way to $H_0$ using field galaxies, corrected by our methods for observational selection bias. Rather than develop here in extenso again the powerful properties of the Spaenhauer diagram that lead to the detection of selection bias and the consequent methods to correct for same, we simply give the references to these methods papers, to which should be added Federspiel et al. (1994), Sandage (1995a) and Sandage, Tammann, & Federspiel (1995).

Table 4. $H_0$ from bias corrected field galaxies

| Method                                      | $H_0$  | Source      |
|---------------------------------------------|--------|-------------|
| Tully-Fisher, distance limited (local)      | 48 ± 5 | Sandage 1994b |
| Tully-Fisher, flux-limited (distance)       | < 60   | Sandage 1994b |
| M 101 look-alike diameters                  | 43 ± 11 | Sandage 1993c |
| M 31 look-alike diameters                   | 45 ± 12 | Sandage 1993d |
| Luminosity class spirals                    | 56 ± 5  | Sandage 1996a |
| M 101, M 31 look alike luminosities         | 55 ± 5  | Sandage 1996aa |
| Tully-Fisher                                | 55 ± 5  | Theureau et al. 1996 |

9. Fourth route through physical methods

Physicists generally will not believe astronomical methods until they say that they understand the basis of these methods. On the other hand, astronomers, if they can show the viability of conclusions from internal astronomical proofs of the reality of particular correlations, such as (1) the P-L relation of Cepheid variables, (2) the existence of the main sequence in the H-R diagram, (3) the tight luminosity function of first ranked galaxies and SNe Ia before a deep understanding (in the physicists sense) of the correlations is at hand, will use these correlations (sans proof except that they work) to obtain new information.

The problem concerning $H_0$ is the same. Physicists, suspicious of the somewhat intricate astronomical ladder, seek $H_0$ by purely physical methods.

To this end there now exist several possible “purely” physical methods to $H_0$, some of which are astounding in their near magic and beauty. Table 5 summarizes the bulk of these methods, stating the results to mid 1996.

Note that none of these methods support $H_0 = 100$. This was the center of the argument as late as 1988 (see Paturel 1983 for a telling diagram).
The Evidence for the long distance scale with $H_0 < 65$

Table 5. Distance determinations from purely physical methods

| Method                                                      | $H_0$     | Source |
|-------------------------------------------------------------|-----------|--------|
| Radio remnant of SN 1979C in NGC 4321 (Virgo)               | $54 \pm 20$ | 1      |
| Expanding photosphere and $^{56}$Ni SNe Ia models           | $55 - 70$ | 2      |
| Expanding photosphere models of SNe II                      | $73 \pm 6$ | 3      |
| Expanding photosphere models of SNe II                      | $< 50$    | 4      |
| Sunyaev-Zeldovich effect for cluster A 2218                 | $45 \pm 20$ | 5      |
| for 6 other clusters                                        | $60 \pm 15$ | 6      |
| cluster A 2163                                              | $68 \pm 30$ | 7      |
| 2 clusters                                                  | $42 \pm 10$ | 8      |
| Gravitational lenses QSO 0957 + 561                         | $63 \pm 12$ | 9      |
| B 0218 + 357                                                | $\sim 60$ | 10     |
| MWB fluctuation spectrum                                    | $30 < H_0 < 50(70)$ | 11     |

*Sources: (1) Bartel 1991 (2) Branch et al. 1996; Höflich & Khokhlov 1996; Höflich et al. 1996; Ruiz-Lapuente 1996 (3) Schmidt et al. 1994 (4) Baron et al. 1995 (5) McHardy et al. 1990; Birkinshaw & Hughes 1994; Jones 1994; Lasenby & Hancock 1995 (6) Rephaeli 1995; Herbig et al. 1995 (7) Holzapfel et al. 1996; Lasenby 1996 (9) Turner 1996; Kundic et al. 1996 (10) Corbett et al. 1995; Nair 1995 (11) Lasenby 1996

10. **The age of the standard model cosmology**

Substantial progress has been made since 1990 in the question of an independent determination of “the age of the universe”. The cosmological test is, of course, to compare this “age from the big bang creation” with the inverse Hubble constant.

Here, we simply list in Table 6 the various determinations of experiments that

Table 6. Independent determinations of various ages

| Method                                                      | Age(Gyr)  |
|-------------------------------------------------------------|-----------|
| A. Age of Globular clusters                                 |           |
| (1) Sandage 1993bb                                          | $14.1 \pm 0.3$ |
| (2) Chaboyer 1995                                          | $11 - 21$ (total range) |
| (3) Shi 1995                                               | $10 - 14$ |
| (4) Mazzitelli et al. 1995                                 | $13 (+2, -3)$ |
| (5) Demarque 1996                                          | $14.5 \pm 1.6$ |
| (6) Weiss et al. 1996                                      | $< 13$    |
| (7) Caloi et al. 1996                                      | $11 - 13$ |
| B. Cooling time of white dwarfs in the Galactic bulge       |           |
| (8) Wood 1992                                              | $10 - 12$ |
| (9) Segretain et al. 1994                                  | $11.5 - 14$ |
| C. Age of “first” supernovae making the heavy elements in the solar system |           |
| (10) Cowan, Thielemann, & Truran 1990; Thielemann 1995; Truran 1996 | $14.4 \pm 3$ |
| Minimum Age of the “Creation event”                        | $13.5 (+2, -3)$ |

*See also the contributions by M. Bolte and B. Paczynski in this volume
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“determine” (a) the age of the Galaxy, (b) the age of the chemical elements, and (c) the subsequent “age of the universe”, all independent of any consideration of \( H_0^{-1} \).

For this audience there is no need for further comment, except to note that with \( H_0 = 55, H_0^{-1} = 18 \) Gyr, and \( T_U = 13.5 \) Gyr there is no need to invoke a “crisis in cosmology” (sans consideration of \( q_0 \) – the second of the two numbers; Section 3).

11. Consequences

11.1. \( H_0 = 55 \pm 10 \)

A summary of the four routes to \( H_0 \) discussed in sections 3, 4, 8 and 9 is set out in Table 7, listed in what we believe is the power of each method.

| Method                        | \( H_0 \)      | External error |
|-------------------------------|---------------|---------------|
| Cepheid calibrated            | 56 \pm 3      | +8, −10       |
| SNe Ia tied to 35000 km s\(^{-1}\) | 55 \pm 2      | 8             |
| Virgo distance tied to 10000 km s\(^{-1}\) | 53 \pm 3      | 10            |
| Field galaxies                |               |               |
| out to 3000 km s\(^{-1}\) corrected for bias |               |               |
| Physical methods              | 58 \pm 2      | (15)          |
| **conclusion:**               | **55**        | **10**        |

The three independent routes to the global value of \( H_0 \), namely (1) SNe Ia calibrated with Cepheids (a method that does not depend on the distance to the Virgo cluster) in Section 3, (2) the distance to Virgo with the cluster tied to the remote expansion field by Figure 4 in Section 4, and (3) field galaxies corrected to distance-limited samples and calibrated with Cepheids, and then tied to the Virgocentric kinematic redshift frame in Section 5, give 56 \pm 3, 55 \pm 2, and 53 \pm 3 (internal errors) respectively. Their only interdependence is that they rely on Cepheids (predominantly observed with HST), which are the least controversial distance indicators at present and which are reliable to better than 0.2 mag (±10% in distance) as discussed in Section 2. The three methods together make a strong case for \( H_0 = 55 \pm 10 \) (external error). Values of \( H_0 < 40 \) are equally unlikely as values of \( H_0 > 70 \). Furthermore, the first two methods determine the global (Machian frame) value of \( H_0 \) directly, independent of all local velocity anomalies. None of the methods used by proponents of the short distance scale with \( H_0 > 70 \) have this property. In particular, all galaxies used by the “Key Project” consortium are local (Freedman et al. 1994, 1996; Silvermann et al. 1996; Graham et al. 1996). Our objections to their short distance scale with \( H_0 \sim 70−80 \) concern their precepts used to tie their local data to the remote cosmic frame.

11.2. Why is there still a controversy?

We remain baffled. We see no single reason. At least six real points, (and a seventh as well) carry part of the burden against the argument by the proponents for the short distance scale with \( H_0 > 70 \) that still dominates the literature.

1. An unwarrantly high recession velocity of the Virgo cluster;
2. the unrealistic expectation to fathom the depth of the Virgo cluster with only one resolved spiral galaxy (Freedman et al. 1994);

3. the similar unrealistic expectation that the distance to NGC 1365 (Freedman et al. 1996) in the Fornax cluster can recalibrate the $<M_{\text{max}}>$ of SNe Ia, when the geometrical aspect of the problem argues against the precept, and further violates the small dispersion in $<M_{\text{max}}>$ of SNe Ia proved by external data in the six high-pedigree cases as to parentage of the SNe Ia calibrated directly with Cepheids;

4. the myth of a sharp, dispersionless cutoff of the luminosity function of planetary nebulae shells, independent of sample size and other factors;

5. reliance on the surface brightness fluctuation method that has produced an unrealistically small Virgo modulus in severe conflict with Cepheid distances;

6. ignoring the Malmquist-like biases that always artificially increase the value of $H_0$ from flux-limited samples of field galaxies and also from incomplete cluster samples if they are uncorrected for observation selection bias.

7. the lemming problem of follow the leader.

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