Tests on the Hierarchy of Trilinear and Quadrilinear Weak Bosons Couplings at the NLC and Comparison with the LHC/SSC

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Abstract

I first review a few basic guiding principles that lead to the notion of a hierarchy of couplings in searches of New Physics involving weak bosons processes. The hierarchies within a linear and a non-linear realization of symmetry breaking are compared to the usual phenomenological parameterization of the WWV vertex. Limits that one expects to obtain at the NLC(500GeV) and LHC/SSC on the trilinear and quadri-linear anomalous W couplings are compared. The cleanness of an $e^+e^-$ gives the NLC a clear advantage in constraining the tri-linear couplings. However, with “only” 500GeV, the $e^+e^-$ is not competitive with pp colliders in probing the quadri-linear couplings. An interpretation in terms of scalar-like and vector-like models is given and I argue that in the presence of a light (or “not-so-heavy”) Higgs, the New Physics affecting the $W$ sector would be easier to pin-down with a moderate energy $e^+e^-$ machine.

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1 W Physics and the Higgs Connection

The $W$ system as described by the SM (Standard Model) reflects the marriage of two fundamental principles:

** Gauge Principle in its non-Abelian form
** Spontaneous Symmetry Breaking which provides the $W$ and $Z$ bosons with mass.

These two items taken together are a quite unusual combination as you may convince yourself if you go through the Particle Data Book: the $W$ would constitute the only known system of massive and gauge spin-1 particles. Yet, it must be stressed that to date there has been no direct tests of either the local non-Abelian nature nor of the exact realization of SSB (Spontaneous Symmetry Breaking) in the $W$ system.

The gauge principle leads most straightforwardly to the universality of the weak coupling constant, that is, all couplings involving the $W$&$Z$ are the same allowing for the quantum numbers. The non-Abelian local gauge symmetry tells us that the coupling of the $W$ to fermions is the same as the tri-linear $W$ coupling as well as the quadri-linear coupling. The couplings of the $W$ and $Z$ to fermions have been tested to a high degree of accuracy directly. However, these tests can be, in a sense, regarded as direct Abelian tests. Indirect limits on the 3-$W$ and the 4-$W$ self-couplings have been worked out through their effects on quantum corrections, but these limits are plagued with “theoretical error bars”, interpretations and ambiguities.

The tri-linear as well as the quadri-linear couplings of the $W$ come solely from the (generalized) $SU(2)$ kinetic term through the field strength, $W_{\mu\nu}$

$$W_{\mu\nu} = \frac{1}{2} \left( \partial_\mu W_\nu - \partial_\nu W_\mu + i g [W_\mu, W_\nu] \right) = \frac{\tau^i}{2} \left( \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g \epsilon^{ijk} W^j_\mu W^k_\nu \right)$$

(1)

(with $W^i_\mu = W^i_\mu \tau^i$, the normalization for the Pauli matrices is $\text{Tr}(\tau^i \tau^j) = 2 \delta^{ij}$). The Abelian hypercharge field does not generate any tri-linear couplings. Defining

$$B_{\mu\nu} = \frac{1}{2} (\partial_\mu B_\nu - \partial_\nu B_\mu) \tau_3 \quad B_\mu = \tau_3 B_\mu$$

(2)

the kinetic term writes

$$L_{\text{Gauge}} = -\frac{1}{2} \left[ \text{Tr}(W_{\mu\nu} W^{\mu\nu}) + \text{Tr}(B_{\mu\nu} B^{\mu\nu}) \right]$$

(3)

So far, in a sense, this only describes the transverse $W$’s. Save for the quantum numbers and group assignments the construction is as the one used for QCD. The longitudinal $W$’s, as well as the mixing between the left and right fermionic states are to be revealed in the mass terms.

1.1 Longitudinal $W$’s and the inclusion of mass

The reason that the longitudinal degrees of freedom do not efficaciously contribute to the above Lagrangian can be gleaned by recalling that a longitudinal state of polarization for the $Z$, say, of momentum $k$ may be written as

$$\epsilon^L_\mu = \frac{k_\mu}{M_Z} - M_Z \frac{s_\mu}{s.k} \text{ with } s^2 = 0$$

(4)
which exhibits the all-important high-energy leading behaviour \( (k_\mu/M_Z \sim E_Z/M_Z) \). This also shows that a longitudinal \( Z \) could be represented as the gradient of a scalar field \( Z^L_\mu \propto \partial_\mu \phi_3 \). It is clear that \( Z^L_\mu (\phi_3) \), does not contribute to the kinetic term since \( Z^L_\mu (\phi_3) = \partial_\mu Z^L_\nu - \partial_\nu Z^L_\mu = 0 \). However, it contributes to the mass. The mass terms of concern to us here are

\[
\mathcal{L}_M = M^2_W W_\mu^+ W^{-\mu} + \frac{1}{2} M^2_Z Z_\mu Z^\mu
\]

Put by hand, on its own, this term breaks the gauge invariance. Rather, it completely hides it. To introduce the longitudinal modes in a manifestly gauge invariant way, one exploits the fact that the longitudinal mode may be regarded as the gradient of a scalar field. One then has to turn this gradient into a covariant derivative.

### 1.2 Symmetry Breaking: The SM option

For the \( W \)'s one needs three of these (pseudo)-scalars. One then has to “group” them, \( i.e. \), find a representation for them. Here, we are helped by another very well confirmed experimental measurement. The \( \rho \) parameter is to the per-mil level equal to 1. This means that in the absence of mixing with the hypercharge, the \( W^\pm \) and \( W^0 \) have the same mass. This corresponds to an extra global \( O(3) \approx SU(2) \) symmetry, termed \( SU(2)_c \) custodial symmetry, which manifests itself in the scalar sector. It so happens that the most simple representation of the scalars has this symmetry. In the \( SM \) one introduces a complex doublet, \( \Phi \), with hypercharge \( Y = 1 \),

\[
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \equiv \exp\left(\frac{i \omega_i \tau_i}{v} \right) \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \text{and} \quad \mathcal{D}_\mu \Phi = \left( \partial_\mu + i \frac{g W_\mu + g' Y B_\mu}{\sqrt{2}} \right) \Phi
\]

\( \omega_i \) are the Goldstone Bosons, \( \mathcal{D}_\mu \Phi \) is the covariant derivative on \( \Phi \). This gives the most general renormalizable Lagrangian

\[
\mathcal{L}_{H,M} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) - \lambda \left[ \Phi^\dagger \Phi - \frac{\mu^2}{2 \lambda} \right]^2
\]

As is well known, when one goes to the unitary gauge not only do we recover the above mass terms but also the interaction of the Higgs scalars. Therefore the study of the interaction in the \( W \) sector is a window on the mechanism of symmetry breaking.

### 1.3 Symmetry Breaking: The Non-Linear Realization [1]

One can also uncover the gauge invariance of the mass terms even in the eventuality that the Higgs does not exist. Instead of using the dim-1 field \( \Phi \), one appeals to the (dim-0) matrix \( \Sigma \) which only describes the Goldstone Bosons with the built-in custodial \( SU(2)_c \) symmetry:

\[
\Sigma = \exp\left(\frac{i \omega^i \tau^i}{v} \right) \quad (v = 246 \text{ GeV is the vev}) \quad \text{and} \quad \mathcal{D}_\mu \Sigma = \partial_\mu \Sigma + i \frac{g W_\mu + g' Y B_\mu}{\sqrt{2}} \tau_3
\]
The gauge invariant form of the mass terms is made explicit by the use of the covariant derivative and the Σ-“field” through the operator of order $\mathcal{O}(p^2)$ (2-because it involves two derivatives)

$$L_M = \frac{v^2}{4} \text{Tr}(D^\mu \Sigma^\dagger D_\mu \Sigma) \equiv -\frac{v^2}{4} \text{Tr}(V_\mu V^\mu) \quad \text{with} \quad V_\mu = (D_\mu \Sigma) \Sigma^\dagger$$

The unitary gauge is obtained by formally setting $\Sigma \rightarrow 1$. Of course, with the non-linear realization one ends up with a non-renormalizable model. At one-loop the ensuing divergences are only logarithmic and can be associated with the Higgs mass dependence of the SM low-energy observables. This construction is important because it shows that seemingly non-invariant operators can be made gauge invariant without recourse to the Higgs particle, a point which has been stressed some time ago [1] and has been revived recently [2]. For the record I would like to quote a sentence from a lecture given by Appelquist 13 years ago [3]:

"The massive Yang Mills theory is formally equivalent to the non-linear Lagrangian .......with the advantage of being straightforward to analyze dimensionally and being easily regularized by the linear model".

This important reminder should still be kept in mind when trying to criticize phenomenological parameterization of New Physics, $\mathcal{N}P$, in the bosonic sector. The use of the covariant derivative or the field strength, which is nothing else but the commutator of two covariant derivatives, will give a gauge invariant description, even with operators beyond the $\mathcal{SM}$. We only have to decide about the Higgs content in order to choose between a linear or a non-linear realization of $\mathcal{SSB}$. What should also transpire from these considerations, is that the probing of the self-interactions of the $W$ and the search for any departure from the minimal structure is a test of the symmetry breaking especially if the Higgs persists to be elusive.

## 2 Anomalous Weak Bosons Self-Couplings: Parameterizations and Classifications

### 2.1 The standard “phenomenological” parameterization of the tri-linear coupling

One knows [4] that a particle of spin-$J$ which is not its own anti-particle can have, at most, $(6J+1)$ electromagnetic form-factors including $C$, $\mathcal{P}$ and $\mathcal{CP}$ violating terms. The same argument tells us [4] that if the “scalar”-part of a massive spin-1 particle does not contribute, as is the case for the $Z$ in $e^+e^- \rightarrow W^+W^-$, then there is also the same number of invariant form-factors for the spin-1 coupling to a charged spin-$J$ particle. This means that there are 7 independent $WWZ$ form factors and 6 independent $WW\gamma$ form-factors
beside the electric charge of the W. This number of invariants is derived by appealing
to angular momentum conservation and to the conservation of the Abelian U(1) current:
\(i.e., \text{two}\) utterly established symmetry principles one would, at no cost, dare to tamper
with. Although one can not be more general than this, if all these 13 couplings were si-
multaneously allowed on the same footing, in an experimental fitting procedure and most
critically to the best probe \(e^+e^- \rightarrow W^+W^-\), it will be a formidable task to disentangle
between all the effects, or to extract good limits on all.

One then asks whether other symmetries, though not as inviolable as the two previous
ones, may be invoked to reduce the set of permitted extra parameters. One expects that
the more contrived a symmetry has thus far been verified, the less likely a parameter which
breaks this symmetry is to occur, compared to a parameter which respects these symme-
tries. For instance, in view of the null results on the electric dipole mo-
mements of fermions and other \(CP\) violating observables pointing to almost no \(CP\) violation, \(CP\) violating
terms, and especially the electromagnetic ones, are very unlikely to have any detectable
impact on W-pair production. Therefore, in a first analysis they should not be fitted.

The same goes for the \(C\) violating \(WW\gamma\) couplings. Additional symmetry principles and
then theoretical “plausibility arguments” can be invoked to further reduce the parameter
space of the anomalous couplings. However, before invoking any additional criteria other
than angular momentum conservation, conservation of the Abelian current, unobservable
\(CP\) and electromagnetic \(C\) violation, we should give the most
general phenomenological parameterization of the \(WWV\) vertex. This parameterization is to be used at tree-level in
processes describing vector boson pair production by light fermions (or any other crossed
channels of these). It assumes the vector bosons to be either on-shell or associated to a
conserved current. With this warning....

2.1.1 \(C\) and \(P\) conserving \(WWV\) couplings

There are now two parameterizations on the market and some confusion between the
defining parameters has, unfortunately, arisen, especially as concerns the parameter \(\kappa_Z\).
Below, I give the two parameterizations and the conversion between the two. The oft-used
parameterization of Hagiwara et al. [5], (the HPZH parameterization) is

\[\mathcal{L}_1 = -ie \left\{ A_\mu \left( W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^- \right) + \left( 1 + \Delta \kappa_\gamma \right) F_{\mu\nu} W^{+\mu}W^{-\nu} \right\} \]

\[+ \cot \theta_w \left[ \frac{g_1^2}{1 + \Delta g_1^2} Z_\mu \left( W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^- \right) + \left( 1 + \Delta \kappa_Z \right) Z_{\mu\nu} W^{+\mu}W^{-\nu} \right] \]

\[+ \frac{1}{M_W^2} \left( \lambda_\gamma F^{\nu\lambda} + \lambda_Z \cot \theta_w Z^{\nu\lambda} \right) W^{+\nu}_\mu W^{-\mu}_\nu \]  

The BMT Collaboration [6] has preferred the use of the following couplings

\[\mathcal{L}_1 = -ie \left\{ A_\mu \left( W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^- \right) + \left( 1 + \Delta \kappa_\gamma \right) F_{\mu\nu} W^{+\mu}W^{-\nu} \right\} \]
\[ + \left( \cot g \theta_w + \delta_Z \right) \left[ Z_\mu \left( W^{-\mu \nu} W^+_{\nu} - W^{+\mu \nu} W^-_{\nu} \right) + \left( 1 + \frac{\kappa_Z}{g_{WWZ}} \right) Z_{\mu \nu} W^{+\mu} W^{-\nu} \right] \]

\[ + \frac{1}{M_W^2} \left( y_\gamma F^{\nu \lambda} + y_Z Z^{\nu \lambda} \right) W^+_{\nu \lambda} W^{-\nu} \]

The conversion is given by

\[ x_\gamma = \Delta \kappa_\gamma ; \quad \delta_Z = \frac{c_w}{s_w} \Delta g^1_Z ; \quad x_Z = \frac{c_w}{s_w} (\Delta \kappa_Z - \Delta g^1_Z) ; \quad y_\gamma = \lambda_\gamma ; \quad y_Z = \frac{c_w}{s_w} \lambda_Z \]

(11)

Coming back to the warning about the use of this phenomenological parameterization outside its context, for instance to vector boson scattering. Even at tree-level it should be modified/extended to include appropriate accompanying “anomalous” quartic couplings. This is especially acute for \( \lambda \) and \( g_Z^1 \), to restore \( U(1)_{em} \) gauge invariance, at least...

2.1.2 \( \mathcal{CP} \) preserving but \( \mathcal{P} \) violating operators

The inclusion of the other operators assumes violation of \( \mathcal{C} \) and/or \( \mathcal{P} \). These may be searched for only if one reaches excellent statistics. Therefore the next operator which may be added is the \( \mathcal{CP} \) conserving but \( \mathcal{P} \)-violating Z coupling. In the HPZH parameterization [5] this coupling is introduced through \( g_5^{\gamma Z} \)

\[ \mathcal{L}_2 = -e \left( \frac{c_w}{s_w} g_5^Z \right) \epsilon^{\mu \nu \rho \sigma} \left( W^{+\mu} (\partial_\rho W_\nu) - (\partial_\rho W^{+\mu}) W_\nu \right) Z_\sigma \]

(13)

2.2 A natural hierarchy of couplings through gauge invariance and scaling

Recently this general parameterization has been fiercely attacked on the ground that it does not respect the full local \( SU(2) \times U(1) \) gauge invariance [4]. By now, recalling the introductory remarks, the parry to the criticism should be immediate. It is the same as for the mass term, i.e, \( 2W \) coupling: the general Lagrangian written above is but a particular parameterization written in a specific gauge where only the physical fields are kept and only those parts describing tri-linear couplings are exhibited [3]. As what has been done for the mass term we can always rewrite any of the above parameters within a gauge invariant operator [2, 3]. This is achieved by extensively using the covariant derivative and specifying how to represent the Goldstone Bosons. For the latter specification one would, essentially, be making an assumption about the “lightness” of the Higgs. Unless, of course, the Higgs has already been discovered. At the 500GeV NLC one will not wait too long to know....

The important point about a gauge-invariant formulation is that it greatly extends the domain of application of the anomalous parameters, ....even at the quantum level. However,
imposition of local symmetries alone is not sufficient to reduce the number of parameters. What has been making the success of present-day physical theories, which apart from masses have only a few parameters, is not just their built-in gauge invariance. It is also because they are characterized by the lowest dimension of all possible gauge invariant operators. This makes them renormalizable and predictive. Higher dimension operators destroy the power of unequivocal predictive calculability as one needs more and more inputs from experiments. However, the very fact that these higher dimension operators are, necessarily, inversely proportional to the scale ($\Lambda$) of $\mathcal{NP}$, that they parameterize, means that their effect at low energy is small. They contribute with a penalising factor $((E/\Lambda)^{n-4})$, where $E$ is the typical low energy of the particular process and $n$ is the dimension of the operator. Therefore, due to the limited accuracy in our experiments one can only hope to see the effect of the next dimension operators which will be referred to as next-to-leading or sub-leading operators. The leading being, of course, those of the $\mathcal{SM}$ (with or without the Higgs). Higher order, or sub-sub-leading operators, (with even larger $n$) are even less likely to have any impact. There is a tacit assumption here, namely that the coefficients of the operators are not too large so that an expansion in energy is possible. This is the scaling argument augmented in the case of spin-1’s with the gauge principle. This is really “Wilsonian” in spirit [10]:

“The couplings should have an order of importance, and for any desired but given degree of accuracy only a finite subset of the couplings would be needed”.

The most straightforward illustration of these notions is provided by a very simple example. This is the Lagrangian describing photons at energies much below the electron mass. It is also interesting because in a sense it describes anomalous self-couplings of the photon (bilinear and quartic).

### 2.2.1 Interlude: The Effective Lagrangian for Photons Below the “Electron Threshold”

Imagine a world with just massless photons and that we want to write the most general Lagrangian with the only information or rather stricture being the local $U(1)$ gauge invariance. If we require the Lagrangian to be renormalizable then the only operator possible is the kinetic term below, it is the marginal operator[^1]. The only problem with this example is that this term does not represent any interaction, it is a free-field trivial theory. Interactions between photons is possible through the introduction of higher order effective operators. These would be the impact the “heavy” unobservable electrons will leave at these lilliputian energies. Because the symmetry we have at these energies is the $U(1)$ local symmetry the only possibility to describe any of these interactions is to use the electromagnetic field strength (or its dual). These are dimension-2 objects and scale as the energy (or frequency) of the photon. The first terms in (in principle infinite) set of operators is $[\mathcal{L}^{\text{QED}}_{\text{eff.}}]$

$$
\mathcal{L}^{\text{QED}}_{\text{eff.}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\beta_1}{m^2} \frac{e^2}{16\pi^2} \left( F_{\mu\nu} \Box F^{\mu\nu} + \frac{e^2}{m^2} F_{\mu\nu} \Box^2 F^{\mu\nu} \right)
$$

[^1]: In the sense of being equally important at all energies.
\[ + \frac{1}{m^4 16\pi^2} \left( \beta_2 (F_{\mu\nu} F^{\mu\nu})^2 + \beta_3 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right) + \ldots + \mathcal{L}_{\text{gauge fixing}} \quad (14) \]

(Note that we have added a gauge-fixing term so that we can invert the photon propagator.)

The first anomalous operator, characterized by $\beta_1$, is a correction to the two-point function. If one were to make an analogy with $W$ physics this kind of self-energy operators can be extremely contrived from LEP1 measurements. I have not included any tri-linear anomalous couplings. This is not forbidden by gauge invariance, in fact I have taken a theoretical bias: for the three-neutral particles one needs to break $\mathcal{C}$ invariance which is not possible also in the fundamental theory: QED with electrons only. The first genuine interaction is a quartic coupling which describes the scattering of light by light.

These operators scale as the inverse of the “scale of New Physics”. In this particular case this must be related to the electron mass. At energies much lower than this scale the effect of these operators is small since the presence of higher and higher derivatives means that the corrections are of order $(\omega/m_e)^n$ for a dim-n operator, where $\omega$ is a typical photon frequency. Therefore only very few of the next to lowest operators are necessary to have a good enough precision. Since we know that QED is the fundamental theory which gives rise to this effective Lagrangian we have definite predictions for the values of all the $\beta_i$. These low-energy values are obtained by taking the full QED lagrangian and considering one-loop diagrams. One then expands the results in the limit of a very large electron mass, $m_e$. It should be kept in mind that in the process, a renormalization procedure has been carried out, which among other things defines and specifies the value of $\alpha$.

With $m = m_e$ one finds a specific pattern between the “anomalous couplings” emerging from QED:

\[ \beta_1 = 6\beta_2 = \frac{24}{7} \beta_3 = \frac{1}{15} ; \quad \epsilon_2 = \frac{3}{28} \quad (15) \]

For further reference one should note that in this particular example the heavy particle (or $\mathcal{N}P$ ) has been “integrated out” at one-loop. One can think of other effective field theories where the non-renormalizable terms result from integrating out a heavy particle at tree-level, the Fermi four-point interaction is one notable example. This is the reason I have chosen to pull out, from the definition of the $\beta_i$’s, factors of $1/16\pi^2$ which betray their (one-) loop origin. The factors of $e^2$ comes from the observation that each photon field contributes a factor of $e$ and is a reflection of the fact that in the fundamental theory charged particles couple to the photon with the “universal” strength $e$. For effective operators not describing gauge particles universal coupling factors are not contained in the coefficients. Moreover, for operators describing heavy particle tree-level exchanges, there is no reason to include the factor $1/16\pi^2$ with the expectation that these operators have a more significant impact than those corresponding to loop effects.

The term $\epsilon_2$ can be considered as the first order term in the expansion of the “form factor”, $\beta_1$.

The Question of Loops

This is an aside. One might wonder whether one should use the higher dimension operators inside loop diagrams. The worry is that since these are high-derivative operators, power counting indicates that they have a high degree of divergence such that the positive
power in the cut-off (proportional to the scale) introduced to regularize these diagrams “overcomes” the (inverse) power of the mass which scales the operators. One might conclude that these operators do not decouple. This would be very unnatural, in fact, as shown and argued by many [3, 12], any such divergence can be absorbed in the definition of the parameters of the Lagrangian. For the example at hand, $\beta_2$ contributes to the $q^2$ part of the self-energy and $\beta_2$ is used to define a running of $\alpha$. The quartic couplings $\beta_{2,3}$ could through one loop (by joining two of the photon lines) be turned into a two-point function. It is easy to see that the induced vertex (a tadpole-type) is quartically divergent and contributes to the kinetic term. This divergence can be easily disposed off by redefining (rescaling) the electromagnetic field. The regularization procedure in this very simple example does not turn out to be so crucial. In more complex situations it is more practical to use a regularization which respects the symmetries of the Lagrangian, otherwise the regularization procedure can introduce spurious terms which destroy the original symmetries. In any case, what is more important is to have all the symmetries and the ensuing Ward (BRST) identities. We could then use any regularization. The application of the Ward identities will show which of the divergences are an artifact of the regularization. These terms can then be removed by the introduction of (additional) counterterms.

2.2.2 Back to the W system: The Ranking of the Gauge Invariant Operators

There are two important concepts in the ranking of the operators in the two approaches of SSB. In the linear approach the classification is done according to the dimension of the operator, i.e., to the power of the scale, $\Lambda$, of the $\mathcal{NP}$. In the non-linear scenario this is done on the basis of a momentum expansion. Therefore, for the next-to-leading (or most “probable”) operators, of order $\mathcal{O}(p^4)$, a new scale does not necessarily appear. One expects the scale of $\mathcal{NP}$ which “weighs” the anomalous operators to be larger than $\sim TeV$. Therefore the “sub-sub-leading” operators should not be considered. If the NLC(500) is to run after the LHC(SSC) one would, by then, know whether this is correct.... There is another symmetry to be included when listing the most likely operators: the custodial $SU(2)_C$ global symmetry.

On the basis of the above symmetries, one can not help it, but there are operators which contribute to the tri-linear couplings and have a part which corresponds to bi-linear anomalous $W$ self-couplings. Because of the latter and of the unsurpassed precision of LEP1, these operators are already very much unambiguously constrained. I will not list them. I will only list the ones which we have not had direct access to as they have no bi-linear part. These are the operators which in the parlance of [4] are referred to as “blind directions”. I do this with a pervading feeling of uneasiness since one must admit that it is very hard to come up with theories which only give rise to the latter or where the former are very much suppressed. With these few points spelled out, we arrive at the most probable set of yet-untested operators, within a linear [13, 7] or a non-linear [1, 14] realization of SSB.
Table 1: The Next-to-leading Operators describing the W Self-Interactions which do not contribute to the 2-point function.

| Linear Realization , Light Higgs | Non Linear-Realization , No Higgs |
|----------------------------------|----------------------------------|
| $\mathcal{L}_B = ig'_B (D_\mu \Phi) \dagger B^{\mu\nu} D_\nu \Phi$ | $\mathcal{L}_{9R} = -ig' \frac{L_{9R}}{16\pi^2} \text{Tr}(B^{\mu\nu} D_\mu \Sigma \dagger D_\nu \Sigma)$ |
| $\mathcal{L}_W = ig'_W (D_\mu \Phi) \dagger (2 \times W^{\mu\nu}) (D_\nu \Phi)$ | $\mathcal{L}_{9L} = -ig' \frac{L_{9L}}{16\pi^2} \text{Tr}(W^{\mu\nu} D_\mu \Sigma \dagger D_\nu \Sigma)$ |
| $\mathcal{L}_\lambda = \frac{2i}{\Lambda} g^3 \text{Tr}(W_\mu W^{\nu\rho} W^\rho_\mu)$ | $\mathcal{L}_1 = \frac{L_1}{16\pi^2}(\text{Tr}(D^\mu \Sigma \dagger D_\mu \Sigma))^2 \equiv \frac{L_1}{16\pi^2} O_1$ |
|                                  | $\mathcal{L}_2 = \frac{L_2}{16\pi^2}(\text{Tr}(D^\mu \Sigma \dagger D_\nu \Sigma))^2 \equiv \frac{L_2}{16\pi^2} O_2$ |

We see that the combination of gauge invariance, $SU(2)_c$ global symmetry (only broken by mixing) and the principle of “minimality” keeping the leading terms in the energy expansion does not give any $C$ or $P$ violation. Excluding electromagnetic $CP$ violation, $SU(2)_c$ suffices to forbid $CP$ violation also for the $Z$. This is a strong argument for assuming $SU(2)_c$. Moreover, in the non-linear realization the counterpart of $\mathcal{L}_\lambda$ is relegated to a lower cast as it is counted as $O(p^6)$: $\mathcal{L}_\lambda \propto \text{Tr}([D_\mu, D_\nu] [D^\mu, D^\nu] [D_\rho, D_\rho])$. This operator as we will see contributes to $\lambda$ in the phenomenological parameterization. Another view is that this operator is not really telling us much about symmetry breaking. It involves in a sense only transverse $W$’s. If there were no $SSB$, i.e. if the $W$ had no mass, $L_\lambda$ would be the only operator that we would write. Although, for the tri-linear couplings, there are more parameters in the linear realization, one can, in principle, perform more tests (in Higgs production) with $\mathcal{L}_{B,W}$ than with $\mathcal{L}_{9L,9R}$ as anomalous Higgs-$W$ vertices are also induced by $\mathcal{L}_{B,W}$. On the other hand, the operators $L_{1,2}$ which represent genuine quartic couplings (they do not contribute to the tri-linear couplings) and involve a maximum number of longitudinal modes are sub-sub-dominant in the light Higgs scenario. When relinquishing the Higgs, $L_{1,2}$ would be the most important manifestation of alternative symmetry breaking scenarios. Unfortunately, they can not be probed in $e^+e^- \rightarrow W^+W^-$. Note that $L_{9L,W}$ contributes a part to the $4V$ vertex. We will come back to these quartic couplings later.

2.2.3 The most likely $WWV$ couplings

By going to the physical gauge, one recovers the phenomenological parameters with the constraints:

$$\kappa_\gamma - 1 = \Delta \kappa_\gamma = x_\gamma = \frac{e^2}{s_w^2} \frac{v^2}{4 \Lambda^2} (\epsilon_W + \epsilon_B) = \frac{e^2}{s_w^2} \frac{1}{32 \pi^2} \left( L_{9L} + L_{9R} \right)$$
\[ \kappa_Z - 1 = \Delta \kappa_Z = \frac{e^2 v^2}{s_w^2 4\Lambda^2} (\epsilon_W - \frac{s_w^2}{c_w} \epsilon_B) = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} \left( L_{9L} - \frac{s_w^2}{c_w} L_{9R} \right) \]

\[ g_1^Z - 1 = \Delta g_1^Z = \frac{e^2 v^2}{s_w^2 4\Lambda^2} (\epsilon_W) = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} \left( \frac{L_{9L}}{c_w^2} \epsilon_W \right) \]

\[ \lambda_\gamma = \lambda_Z = \left( \frac{e^2}{s_w^2} \right) L\lambda M^2 W \Lambda^2 \]

(16)

Note that these couplings, in the BMT \[6\] notation, verify the custodial global symmetry.

\[ x_Z c_w = -x_\gamma s_w. \] Only \( L_{W,9L} \) give \( \delta Z \) \((\Delta g_1^Z)\) with \( \delta Z = \frac{x_w}{c_w} \epsilon_B \). In the numerical applications I will take \( \alpha \) and "s_w" at \( M_Z^2 \), i.e., in Equation (16) \( e \rightarrow e(M_Z^2) \) and \( s_w^2 \rightarrow s_Z^2 = 0.228 \).

Not that there is a one-to-one correspondence \( L_{9L,9R} \leftrightarrow \epsilon_{W,B} \) for the \( WWV \) parts. So, for two bosons production or neglecting Higgs exchanges in \( 3V \) production, the two sets are equivalent (same constraints).

2.2.4 Next-to-best: Breaking the Global Symmetry and Maintaining the “Order”

The order, here, is the order in the energy expansion or the dimensionality of the operators. To make the point, I stick with the non-linear realization. We know that there is a slight breaking of the global \( SU(2)_c \), for instance, as induced by the top. The first effect appears in the \( 2W \) vertex and contributes to the \( \rho \) parameter. Introducing \( X = \Sigma \tau^3 \Sigma^\dagger \), the contribution to \( \Delta \rho \) is through the leading \( O(p^2) \) operator

\[ \mathcal{L}_{\Delta \rho} = \Delta \rho \frac{v^2}{8} (\text{Tr}(\mathcal{V}_\mu X))^2 \]

A typical \( 3W \) \( SU(2) \)-breaking operator is \[17\]

\[ \mathcal{L}_1 = ig \frac{L_1}{16\pi^2} (\text{Tr}(W^{\mu\nu} X)) (\text{Tr}(X[\mathcal{V}_\mu, \mathcal{V}_\nu])) \]

and leads to

\[ \Delta g_1^Z = \delta_Z = 0 ; \Delta \kappa_\gamma = x_\gamma = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} (4 L_1) ; \Delta \kappa_Z = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} (4 L_1) \]

(19)

so that \( \frac{x_w}{c_w} x_\gamma = \frac{x_z}{c_w} x_\gamma \). The inequality is a reflection of the explicit \( SU(2)_c \) breaking (in the language of BMT). A \( WWZ \) \( C \) violating but \( CP \) conserving operator at the same order in the energy expansion is also possible when the custodial symmetry is broken, as first noticed by Feruglio \[17\]. With \( W^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} W_{\alpha\beta} \)

\[ \mathcal{L}' = g \frac{L'_f}{16\pi^2} (\text{Tr}(\bar{W}^{\mu\nu} \mathcal{V}_\mu)) (\text{Tr}(X \mathcal{V}_\nu)) \rightarrow g_5^Z = \frac{e^2}{s_w^2} \frac{1}{32\pi^2} \left( -\frac{L'_f}{c_w^2} \right) \] with \( g_5^Z = 0 \) (20)

I will, in the remainder, assume that the amount of explicit \( SU(2)_c \) breaking beyond that of the \( SM \) is small so that these effects are of second importance. If not, we see
that one has, with these two additional SU(2)\textsubscript{c} breaking operators, the same number of parameters as with the phenomenological parameterization of the CP conserving “dim-4” WWV vertex.

2.2.5 Quartic Couplings

Writing the contribution of the “genuinely quartic” anomalous operators in the physical gauge, we obtain:

\[
\mathcal{L}^{(4)}_Q = \left(\frac{e^2}{s_w^2}\right)^2 \frac{1}{16\pi^2} \left\{ \mathcal{L}_1 \left( W'^{\mu\nu}W^-_{\mu\nu}W'^{-\mu\nu}W^-_{\mu\nu} + \frac{1}{c_w^2} W'^{\mu}_{\nu\mu}W^-_{\mu\nu} + \frac{1}{4c_w^4} Z_{\mu\nu}Z_{\mu\nu}\right) + \mathcal{L}_2 \left( \frac{1}{2}(W'^{\mu}_{\nu\mu}W'^{-\mu}_{\nu\mu} + W'^{\mu}_{\nu\nu}W'^{-\mu}_{\nu\nu}) + \frac{1}{c_w^2} W'^{\mu}_{\nu\nu}Z_{\mu\nu}\right) \right\}
\]

(21)

We [19] have already obtained this form by only appealing to SU(2)\textsubscript{c} global. To make contact with that analysis, the correspondence is \( g_{0,c} = \frac{e^2}{16\pi^2} \frac{1}{s_w^2} L_{1,2} \). Neither the WWW nor the WZZZ have a form like that found in the SM. But most importantly there is a ZZZZ coupling which is not present in the SM at tree-level. Also, note that with these genuine quartic couplings, photons do not appear. The first operator parameterizes the exchange of a heavy scalar. This point is also crucial, because while tri-linear couplings could be the residual effect of integrating out heavy particles at one-loop, the quartic couplings can correspond to integrating heavy states at tree-level and therefore one would expect their coefficients to be larger. We also note that in the combination \( \mathcal{O}_1 - \mathcal{O}_2 \), i.e., \( L_1 = -L_2 \), the 4Z vanishes. This corresponds to “vectorial” theories, i.e., integrating out heavy spin-one. This could be of relevance to technicolour models. The quartic part of the \( \mathcal{L}_{9L} \) does induce new WWZ\gamma, WWZZ and WWWW but no 4Z ensues:

\[
\mathcal{L}^{(4)}_{9L} \rightarrow \frac{\Delta_{\gamma}\gamma}{s_w^2} \left( \frac{e^2}{32\pi^2} \frac{c_w^2}{s_w^2} \left\{ A_\mu Z_\nu W_\mu^+ W_\mu^+ - \frac{1}{2} A_\mu Z_\nu (W_\mu^+ W_\mu^- + W_\mu^- W_\mu^-) \right\} \right)
\]

(22)

3 Direct Searches at the Next Colliders

3.1 \( e^+e^- \rightarrow W^+W^- \)

In \( e^+e^- \) colliders the most promising channel to probe the tri-linear couplings is W pair production, due to the large statistics that it offers. Here, I will refer to the excellent extensive study conducted in Europe by the BM2 [6] Collaboration and will translate their results within the effective Lagrangian approach. BM2 can fit many parameters at a time.
beside giving limits on individual parameters of the phenomenological Lagrangian assuming various relations between them. Misha Bilenky has kindly rerun their program for me within the $SU(2)_c$ symmetric chiral Lagrangian approach, i.e. by considering the effects of $L_{9L,9R}$. With the conversion in $L_{W,B}$, BM2 take advantage of the cleanness of the $e^+e^-$ environment to reconstruct a large number of the $W^+W^-$ density matrix elements (DME). No beam polarization is exploited in this analysis. They, conservatively, only take into account the semi-leptonic decays where the leptons are either $e^\pm$ or $\mu^\pm$ (no $\tau$’s) to have an excellent reconstruction of the the scattering angle ($\theta$). Apart from the total differential cross-section, and disregarding any $CP$ violation, the fits are done on 4 independent combinations of the DME which do not rely on any charge identification of the final fermions and 3 additional DME based on the i.d. of the lepton charge only. This gives them 8 independent observables instead of using the full five-fold differential cross-section for the 4 final fermions. Simulated data are generated according to the tree-level SM expectations with an angular coverage of $|\cos \theta| < 0.98$, taking 8 bins in this variable while in the variables of the fermions 6 bins are taken, requiring a minimum of 4 events in each bin. Idealistically though, only statistical errors are taken into account. The extracted limits are at the 95%C.L. When referring to Fig.1, note that LEP2 represents $\sqrt{s} = 190 GeV$ with $\int \mathcal{L} = 500 pb^{-1}$, while the NLC500 is with $\sqrt{s} = 500 GeV$ and $\int \mathcal{L} = 10 fb^{-1}$. We should still keep in mind that, apart from $RC$, the effect of beamstrahlung has not been included yet.

### 3.2 $e^+e^- \rightarrow W^+W^-\gamma, W^+W^-Z$

Triple vector boson production offers the possibility to check for quartic couplings. Of course, the tri-linear couplings also enter in $WW\gamma$ and $WWZ$ but NOT in $ZZZ$ production as would genuine $SU(2)_c$ symmetric quartic couplings. The only exception is the $L_V$ ($L_1 = -L_2$) realization of the quartic coupling, but then this would contribute to $WWZ$ only. Note that all next-to-leading quartic couplings never contribute to $WW\gamma$. Therefore, by looking in all 3V production channels one can easily discriminate between genuine quartic and tri-linear couplings. The table below illustrate this discrimination (the number of stars indicates the sensitivity of the particular channel to the operators). Besides, in case of a signal, the origin of any of the couplings can be unravelled through characteristic energy and angular distributions. This is discussed in [13]. Let me point out also, that if a tri-linear coupling is to give a detectable signal in $WW\gamma, WWZ$ then it would give a more prominent effect in $WW$ production, so that in this case the value of the coupling would be extracted from $W$ pair analysis. It would then be included in $WWZ$ production so that one checks whether there is any additional contribution in this channel.

The interesting aspect about triple vector production, especially $WWZ$ and $ZZZ$, at a moderate CM energy is that it is a substitute to $W$ fusion processes which are ineffective at 500GeV. The 3-V cross-sections are not very large though. For instance, at $\sqrt{s} = 500 GeV$ we have $\sigma(WWZ) \sim 39 fb$. The $ZZZ$ is tiny $\sigma(ZZZ) \sim 1 fb$, but this very fact classifies this reaction as a rare process and hence it is a good testing ground for $\mathcal{N}\mathcal{P}$. Our [13] analysis on $WW\gamma$ production was done with the following cuts, $|\eta_\gamma| < 2$, $p_T^\gamma > 20 GeV$, $|\cos(\angle eW)| < 0.96$, $\cos(\angle W), \cos(\angle WW) < 0.985$. With these cuts the
Table 2: Contributions of the Next-to-Leading Operators of the Chiral Lagrangian to three-vector productions in $e^+e^-$

| Operator       | $e^+e^- \rightarrow W^+W^-$ | $e^+e^- \rightarrow W^+W^-\gamma$ | $e^+e^- \rightarrow W^+W^-Z$ | $e^+e^- \rightarrow ZZZ$ |
|----------------|-----------------------------|-----------------------------------|------------------------------|-------------------------|
| $L_{9L}, L_{W\phi}$ | ***                         | *                                 | *                           | NO                      |
| $L_{9R}, L_{B\phi}$ | ***                         | *                                 | *                           | NO                      |
| $L_{\lambda}$      | ***                         | *                                 | *                           | NO                      |
| $L_1$               | NO                          | NO                                | *                           | *                       |
| $L_2$               | NO                          | NO                                | *                           | *                       |
| $L_V(L_1 = -L_2)$  | NO                          | NO                                | *                           | NO                      |

cross-section amounts to $\sim 112$fb. The branching ratio into $\tau$'s was not considered in any of the $3V$ production. The limits we extract are based on detecting a $3\sigma$ deviation only in the total cross-section (including branching fractions). Due to the low statistics we did not aim at reconstructing the final polarizations and only statistical errors were taken into account. For $WWZ$ (and $ZZZ$!) $3\nu$ final states were not counted. For $ZZZ$ the discovery criterion was an excess of signal events as large as that corresponding to a $3\sigma$ in $WWZ$ since the bulk of the events looks like in $WWZ$ and as we do not expect invariant mass reconstructions ($M_W$ v.s $M_Z$) to be discriminating. For $WW\gamma$ we have considered the effect of $L_{9L,9R} \equiv L_{W,B} \equiv (\delta_z, x_{\gamma}; x_Z = -s_w/c_w x_Z)$ and also $L_{\lambda}(\lambda_{\gamma} = \lambda_Z)$. In $WWZ$ we considered, in addition to the above couplings, the important effect of the novel $L_1, L_2, L_V$. Finally in $ZZZ$ only $L_1, L_2$ take part. For all $3V$ productions we have only taken one parameter at a time and assumed $\int L = 10 fb^{-1}$.

3.3 $\gamma\gamma \rightarrow W^+W^-$

If the $e^+e^-$ linear collider is turned into a high-energy/high-luminosity $\gamma\gamma$ collider through Compton back-scattered laser light, this process will constitute the largest cross-section. At an effective $\sqrt{s_{\gamma\gamma}} \sim 500$GeV, $\sigma(WW) \sim 80$pb! The $\gamma\gamma$ mode would be a $W$ factory. Of course, one can exploit this to check for anomalous $WW\gamma$ (and also $WW\gamma\gamma$) couplings without making any assumption about the $Z$ counterparts. Keeping in line with my assumptions I will only consider the case $L_{9L,9R}$ (no $\lambda$ fitting). We know that, in effect, we are measuring $\Delta \kappa_{\gamma} \propto L_{9L} + L_{9R} \propto L_W + L_B$, so this is basically a one parameter fit: the combination $L_{9L} + L_{9R}$. I have reinterpreted the results obtained by Choi and Schrempp [24], where a realistic $\gamma\gamma$ luminosity spectrum was considered but without beam polarization effect. The $WW$ reconstruction efficiency including branching ratios is taken at 15% based on events with $|\cos\theta| < 0.7$. Statistical errors are taken into account and the systematic are estimated. The bounds are at the 90% CL.
Figure 1: Comparison between the expected bounds on the two-parameter space \((L_{9L}, L_{9R}) \equiv (L_W, L_B) \equiv (\Delta g_1^Z, \Delta \kappa_\gamma, x_z c_w = -x_\gamma s_w)\) (see text for the conversions) at the NLC500, SSC/LHC and LEP2. The NLC bounds are from \(e^+e^- \to W^+W^-, W^+W^-\gamma, W^+W^-Z\) (for the latter these are one-parameter fits) and \(\gamma\gamma \to W^+W^-\). The SSC/LHC bounds are from \(pp \to WZ, W\gamma\). We also show (“bars”) the limits on one single parameter.
3.4 \( pp \rightarrow W\gamma \) and \( pp \rightarrow WZ \)

For the comparison with the \( pp \) machines, I refer to the analysis conducted recently with the constrained set \( L_{9L,9R} \) \cite{15}. The \( q\bar{q} \rightarrow W^+W^- \) is either fraught with huge QCD backgrounds or in case of the “all-leptonic” decay will be very difficult to reconstruct, it will thus offer very little chance for the study of SSB. The authors \cite{15} consider \( WZ \) and \( W\gamma \) production. \( WZ \) is a much better channel: both final bosons can be longitudinal and hence the largest deviations are expected here. The authors also find that \( q\bar{q}' \rightarrow WZ \) is more efficient than the \( WZ \) fusion process as far as \( L_{9L,9R} \) are concerned. The maximum deviation coming essentially from \( W_LZ_L \), it is clear that \( L_{9L} \) is overwhelmingly dominating through its \( g_{Z1}^\gamma \) term. The \( \kappa_{\gamma,Z} \) only lead to \( W_TZ_T \). In \( W\gamma \) once again one probes \( \kappa_{\gamma} \), that is, the combination \( (L_{9L} + L_{9R}) \). The samples only contain the decays into e\( \mu \). For the \( WZ \) channel, bounds are set by requiring a doubling of events (with at least an excess of 40 at the SSC and 30 at the LHC) in the high-\( p_T \) range \( 300 < p_T < 750 \)GeV. Almost the same criterion is used for \( W\gamma \) (but with \( 400 < p_T < 750 \)GeV). For both SSC and LHC \( \int \mathcal{L} = 10 fb^{-1} \).

3.5 \( W \) fusion processes at \( pp \) and the quartic couplings

The best channel to look for the effect of the genuine quartic couplings is the like-sign \( W \) pair production: \( W^\pm W^\pm \). Within the parameterization in terms of \( L_{1,2,V} \) a very nice theoretical investigation is carried out in \cite{16} and the one-loop contribution of Goldstone Bosons is also included. The effective \( W \) approximation is employed. The limits are based on the observation at the LHC/SSC of an excess of 50\% in the total \( W^+W^- \) yield for an invariant \( WW \) mass in the range \( 0.5 < M_{WW} < 1 \)TeV. Unfortunately the branching fraction into the 1st and 2nd generation leptons are not included while this is essential to reconstruct these events! Also the irreducible SM background is not taken into account. So as the authors stress, the limits are subject to substantial uncertainties. More realistic limits should be within an order of magnitude of those quoted in \cite{16}.

3.6 Comparisons and Conclusions

In Fig. 1 I show the limits one would obtain at the next colliders on the two parameters \( L_{9L} \) and \( L_{9R} \) or equivalently using the conversion in \cite{12} the parameters \( L_W \) and \( L_B \). These limits can also be interpreted, in the case of the two-body reactions, as limits on the set \( (\delta_Z, \Delta \kappa_{\gamma}) \) with the \( SU(2)_c \) symmetry constraint \( x_Zc_w = -x_\gamma s_w \) on \( \Delta \kappa_Z \). For those who prefer the latter parameterization, the \( L_{9L} \) axis is also the \( \Delta g_{Z1}^\gamma \) axis, while the \( \Delta \kappa_{\gamma} \) axis is shown as \( \Delta \kappa_{\gamma} = 0 \). (\( \Delta \kappa_{\gamma} \) are the isolines \( \Delta \kappa_{\gamma} \sim (L_{9L} + L_{9R}) \times 1.35 \times 10^{-3} \)). Also, in this respect, it is worth pointing out that the limits one gets on \( \Delta \kappa_{\gamma} \) when fitting one parameter at a time, crucially depend on which gauge-invariant operator, that is which model, \( \Delta \kappa_{\gamma} \) originates from. The discrepancy between limits on \( \Delta \kappa_{\gamma} \) due to \( L_{9L} \) and \( L_{9R} \) is even more drastic for the \( pp \) machines. For instance translating the limits on \( L_{9L} (L_{9R}) \) as bounds on \( \Delta \kappa_{\gamma} \equiv \Delta \kappa_{\gamma}(L_{9L}) (\Delta \kappa_{\gamma} \equiv \Delta \kappa_{\gamma}(L_{9R})) \), we have
\[ |\Delta \kappa (L_{9L})| < 3 \times 10^{-3} \quad , \quad -6 \times 10^{-3} < \Delta \kappa (L_{9R}) < 7 \times 10^{-3} \quad \text{(NLC(500))} \]
\[ -2 \times 10^{-2} < \Delta \kappa (L_{9L}) < 10^{-2} \quad , \quad -17 < \Delta \kappa (L_{9R}) < 0.16 \quad \text{(SSC)} \quad (23) \]

| LEP190GeV | NLC(500GeV) | NLC(1TeV) | SSC | LHC |
|-----------|-------------|-----------|-----|-----|
| L = 500pb^{-1} | L = 10fb^{-1} | L = 44fb^{-1} | L = 10fb^{-1} | L = 10fb^{-1} |

1-Parameter Fit

| L_{9L} | L_{9L} < 30.4 | L_{9L} < 2.2 | L_{9L} < 0.7 | \(-16 \leftrightarrow 7\) | \(-22 \leftrightarrow 12\) |
| L_{9R} | \(-125 \leftrightarrow 155\) | \(-4.4 \leftrightarrow 5.1\) | \(-1.5 \leftrightarrow 1.5\) | \(-119 \leftrightarrow 113\) | \(-152 \leftrightarrow 147\) |

2-Parameter Fit

| L_{9L} | \(-103 \leftrightarrow 65\) | \(-13.4 \leftrightarrow 7.7\) | \(-3.7 < L_{9L} < 3.1\) | \(-16 \leftrightarrow 7\) | \(-22 \leftrightarrow 12\) |
| L_{9R} | \(-260 \leftrightarrow 760\) | \(-8.6 \leftrightarrow 60.\) | \(-4.6 < L_{9R} < 19.2\) | \(-119 \leftrightarrow 113\) | \(-152 \leftrightarrow 147\) |

The \(L_{9L}\) is always much better constrained than \(L_{9R}\) especially in \(pp\) machines. In fact the limits one gets at \(pp\) are almost an order of magnitude worse for \(L_{9R}\). As the figure shows (see also Table 3), in the case of a one-parameter fit, as if one were fitting to a particular model, the NLC500 does much better than the SSC by more than a factor 20 on \(L_{9R}\) and \(\sim 3 \div 5\) on \(L_{9L}\). If two parameters are fitted, one sees that the best combined fit comes from the NLC where the \(\gamma \gamma\) option helps in reducing the range of the allowed \(L_{9L} \leftrightarrow L_{9R}\) parameter space even further. Compared to LEP2, NLC brings an order of magnitude improvement, at least...[6]. The analysis also shows that especially in the case of the two-prameter fit, the LEP2 limits translate into large \(L_{9}\) values (\(-260 < L_{9R} < 760\)). This is meaningless in terms of a chiral expansion and may be that, after all, one should stick with the phenomenological parameterization for such large values. Although this presumes optimistic expectations....

A few words on the coupling \(\lambda\). In the \(e^+e^-\) environment where the final polarizations can be reconstructed and where the \(\lambda(L_{\lambda})\) lead to essentially transverse states, these can be easily disentangled from other couplings. The limits on \(\lambda\) from a one parameter fit or from a three-parameter fit, as the BM2 analysis shows, is not much different: the limits are \(\sim 10^{-2}\) at the NLC500.

As the “bars” in Fig. 1 show the 3V cross-sections do not bring new constraints on the tri-linear couplings: the limits are about an order of magnitude worse than in WW production at 500GeV. Therefore the 3V reactions can be “safely” exploited to look for the quartic couplings.

At the NLC500 we find the limits:

\[
-96.2 < L_1 < 81.4 \quad , \quad |L_2| < 118.4 \quad , \quad 81.4 < L_V < 70.3
\]
\[
44 < L_{1,2} < 48
\] (24)

\[ \Delta \kappa (L_{9L}) \]
The last limit assumes that a 3Z final state has been identified taking a $3\sigma$ deviation, while the first assumes a $3\sigma$ in $WWZ$ or the corresponding equal number of excess events in $ZZZ$. Our preliminary study shows that with a $1TeV \int L = 60 fb^{-1}$, these limits can be pushed to $\sim 6$. They would then compete with the SSC limits where the theoretical (see above) analysis points to values of order 1.

In conclusion, a moderate energy $e^+e^-$ machine such as the NLC500 would bring an invaluable information on the symmetry breaking mechanism as exemplified by the NLC bound in Fig. 1. Reconstruction of the various W helicities is important and the $\gamma\gamma$ mode would be a very welcome addition. In the case of models where the genuine quartic couplings are smaller than or of the same order as the tri-linear couplings (somehow “vector dominated models”) [1], the NLC500 seems to be more constraining than the SSC. With only 500GeV, not allowing $WW$ scattering analyses, the NLC500 cannot compete with the SSC in the case of the “scalar models” which I associate with models with a “preference” for $L_1, L_2$. From another viewpoint, the latter, in case of a light Higgs or a “not-too-heavy” Higgs, are expected to be much much smaller than the tri-linear couplings. This means that, even through these indirect effects, the NLC500 is an excellent machine for a light or not-so-heavy Higgs scenario.

Once again, by far, the best channel is $W$ pair production in $e^+e^-$. Theoretically this channel is also very “clean” when compared to the many uncertainties in the physics of $W$ at the $pp$ colliders. The full radiative corrections are well under control [24] and good Monte-Carlo programs exist [25]. The next step which is now easy to implement, in order to get a more meaningful bound on the parameters of the $N\mathcal{P}$, is to combine the $SM$ radiative corrections and the “anomalous” parameters together with the use of the powerful fitting procedure of the $BMT$ Collaboration[1].

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§These comparative discussions are with the tacit assumption that operators which contribute to the 2-point function are not generated by these models or that they are drastically suppressed! The optimistic conclusions of this section would have to be “watered down” when this is not the case. For instance, technicolour-like models naively mimicking QCD and including heavy vector resonances are of the vector-type. Unfortunately, they also predict a contribution to the $2-W$ coupling at tree-level through the operator $L_{10} = gg' \frac{f_{16\pi}}{16\pi} \text{Tr}(\Sigma B_{\mu\nu}\Sigma W^{\mu\nu})$. This is related to the $S$ parameter [21]: $L_{10} \rightarrow -\pi S$. On the other the relations between the $L_i$ from integrating $\rho$-like heavy vectors are [22, 13, 10]: $L = L_{10} = -L_{0L} = -L_{0R} = 4L_1 = -4L_2$. The limit on $L_{10}(M_Z)$ as extracted at the $M_Z$ scale from the Z data gives $L_{10}(M_Z) \sim -0.2 \pm 1.7$ [23]. Following [10] and assuming that the previous relations between the $L_i$ hold at the scale $1.5TeV$ (mass of the vector) means a present bound: $-1.4 < L_{10} < 2$. The latter bound when compared to the limits on $L_{0L}, L_{0R}$ (see Table 3) and $L_1$ means that these models are already very much constrained by the LEP1 data and that the NLC500 would hardly improve on this limit. Ideally, one needs a 1TeV version of the NLC (see Table 3).
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