Reactions with pions and vector mesons in the sector of odd intrinsic parity

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Talk based on: C. Terschlüsen, B. Strandberg, S. Leupold, F. Eichstädt, Reactions with pions and vector mesons in the sector of odd intrinsic parity, Eur. Phys. J. A49, 116 (2013).
Motivation (I)

Running coupling constant in QCD

- high energies: can use perturbation theory
- low energies: cannot use perturbation theory

PDG, J. Phys. G33, 1 (2006)
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- low energies: cannot use perturbation theory

Possible solution:
- effective theories
  - hadrons as relevant degrees of freedom
Motivation (II)

Chiral perturbation theory ($\chi$PT): low-lying pseudoscalar mesons are relevant degrees of freedom, i.e. pion, kaons and $\eta$-meson

→ vector mesons are heavy

⇒ not applicable for energy range of hadronic resonances

$(\rho, \omega, K^*, \varphi, \eta')$
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$\rightarrow$ vector mesons are heavy
$\Rightarrow$ not applicable for energy range of hadronic resonances ($\rho$, $\omega$, $K^*$, $\varphi$, $\eta'$)

$\leftarrow$ • needs effective theories or alternatives for higher energies
(exist sucessful models as vector meson dominance (VMD))
• needs to know transition between $\chi$PT and other theories
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(exist successful models as vector meson dominance (VMD))

$\bullet$ needs to know transition between $\chi$PT and other theories

want to explore transition between $\chi$PT regime and higher energies for odd intrinsic parity and SU(2)
Introduction (I): Wess-Zumino-Witten Lagrangian

- meson with spin $J$, parity $P$: intrinsic parity is given by $P \cdot (-1)^J$
- vector mesons have even, pseudoscalars odd intrinsic parity

\[
L_{\text{WZW}} = -\frac{n_e}{24} \pi^2 f_\pi \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \pi_0 \partial_\nu A_\alpha A_\beta - \frac{i n_e}{24} \pi^2 f_3 \pi \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \pi_0 \partial_\nu \pi - \partial_\alpha \pi_0 A_\beta - \gamma
\]

with $n = N_c = 3$
Introduction (I): Wess-Zumino-Witten Lagrangian

- meson with spin $J$, parity $P$: intrinsic parity is given by $P \cdot (-1)^J$
  $\leftrightarrow$ vector mesons have even, pseudoscalars odd intrinsic parity

- in $\chi$PT:
  reactions of an odd number of pions in leading order described by Wess-Zumino-Witten (WZW) Lagrangian

\[
\mathcal{L}_{\text{WZW}} = -\frac{n e^2}{24\pi^2 f_\pi} \varepsilon_{\mu\nu\alpha\beta} \partial_\mu \pi^0 \partial_\nu A_\alpha A_\beta + i \frac{n e}{24\pi^2 f_\pi^3} \varepsilon_{\mu\nu\alpha\beta} \partial_\mu \pi^+ \partial_\nu \pi^- \partial_\alpha \pi^0 A_\beta
\]

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**Introduction (I): Wess-Zumino-Witten Lagrangian**

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  \[ P \cdot (-1)^J \]
  \[ \rightarrow \text{vector mesons have even, pseudoscalars odd intrinsic parity} \]

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\mathcal{L}_{\text{WZW}} = -\frac{n \, e^2}{24 \pi^2 \, f_\pi} \varepsilon^{\mu \nu \alpha \beta} \partial_\mu \pi^0 \partial_\nu A_\alpha A_\beta
+ i \frac{n \, e}{24 \pi^2 \, f_\pi^3} \varepsilon^{\mu \nu \alpha \beta} \partial_\mu \pi^+ \partial_\nu \pi^- \partial_\alpha \pi^0 A_\beta
\]

with $n = N_c = 3$

\[ \rightarrow \text{* parameter independent (note: sign of } n \text{ pure convention)} \]

\[ \text{* WZW Lagrangian is of order } p^4 \]
Introduction (II): Vector-meson Lagrangian

*In* **χPT**: low-energy constants saturated by vector-meson exchange

→ **χPT** at next-to-leading order is equivalent to a Lagrangian with both pseudoscalar and vector mesons
Introduction (II): Vector-meson Lagrangian

In $\chi$PT: low-energy constants saturated by vector-meson exchange

$\chi$PT at next-to-leading order is equivalent to a Lagrangian with both pseudoscalar and vector mesons

important for us

$$L_{\text{vec}} = - \frac{e f_V}{3} \left( 3 \rho^0_{\mu \nu} + \omega_{\mu \nu} \right) \partial^\mu A^\nu + \frac{f_V h_P}{2 f_\pi} \text{tr} \left( V_{\mu \nu} \partial^\mu \Phi \partial^\nu \Phi \right)$$

$$- \frac{h_A}{8 f_\pi} \varepsilon^{\mu \nu \alpha \beta} \text{tr} \left( \{ V_{\mu \nu}, \partial^\tau V_{\tau \alpha} \} \partial_\beta \Phi \right)$$

$V_{\mu \nu} = \begin{pmatrix} \rho^0_{\mu \nu} + \omega_{\mu \nu} & \sqrt{2} \rho^+_{\mu \nu} \\ \sqrt{2} \rho^-_{\mu \nu} & -\rho^0_{\mu \nu} + \omega_{\mu \nu} \end{pmatrix}$, $\Phi = \begin{pmatrix} \pi^0 & \sqrt{2} \pi^+ \\ \sqrt{2} \pi^- & -\pi^0 \end{pmatrix}$
Introduction (III)

In the sector of odd intrinsic parity, one gets for the Lagrangians:

|       | $\mathcal{L}_{WZW}$ | $\mathcal{L}_{vec}$ |
|-------|---------------------|---------------------|
| $p \ll m_V$ | $p^4$               | $p^6$               |
| $p \approx m_V$ | $p^4$               | $p^2$               |

Both $\mathcal{L}_{WZW}$ and $\mathcal{L}_{vec}$ are needed to describe experimental data.
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\(\hookrightarrow \) \(\mathcal{L}_{WZW}\) and \(\mathcal{L}_{vec}\) formally never of same order
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$L_{\text{WZW}}$ and $L_{\text{vec}}$ formally never of same order

3 possibilities to calculate reactions with odd number of pions:

1. use only the pure $\chi$PT contribution $L_{\text{WZW}},$
2. use only the vector meson contribution $L_{\text{vec}} \ (\sim \text{ VMD}),$
3. use both $L_{\text{WZW}}$ and $L_{\text{vec}}$ (more phenomenological approach)
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& L_{WZW} & L_{vec} \\
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\(\rightarrow\) \(L_{WZW}\) and \(L_{vec}\) formally never of same order

\(\rightarrow\) 3 possibilities to calculate reactions with odd number of pions:

(1) use only the pure \(\chi PT\) contribution \(L_{WZW}\),

(2) use only the vector meson contribution \(L_{vec}\) (\(\sim VMD\)),

(3) use both \(L_{WZW}\) and \(L_{vec}\) (more phenomenological approach)

\(\rightarrow\) will show in the following: both \(L_{WZW}\) and \(L_{vec}\) are needed to describe experimental data
Coupling constants for even intrinsic parity (I)

Use reaction $e^+e^- \rightarrow \pi^+\pi^-$ and pion form factor to determine parameters $f_V$ and $h_P$ in $\mathcal{L}_{\text{vec}}$
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$\rightarrow$ reaction can happen direct ($\chi$PT)
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$\gamma$ reaction can happen direct ($\chi$PT) or via a virtual $\rho^0$ meson
Coupling constants for even intrinsic parity (II)

Use reaction $e^+e^- \rightarrow \pi^+\pi^-$ and pion form factor to determine parameters $f_V$ and $h_P$ in $\mathcal{L}_{\text{vec}}$

→ reaction can happen direct (χPT) or via a virtual $\rho^0$ meson

→ pion-pion rescattering in the final channel

$$|F_\pi|^2$$

best agreement for $f_V = 150$ MeV, $h_P = 1.64$

Data:
Barkov et. al., Nucl. Phys. B256, 365 (1985).
R. Akhmetshin et. al., Phys. Lett. B527, 161 (2002).
R. Akhmetshin et. al., Phys. Lett. B578, 285 (2004).
Coupling constant for odd intrinsic parity (I)

In the vector sector: only parameter $h_A$

can only access $|h_A| = 2.02$.

calculation without rescattering differs about 5%.
Coupling constant for odd intrinsic parity (I)

**In the vector sector:** only parameter $h_A$

→ use decay $\omega \rightarrow \pi^+ \pi^- \pi^0$ including pion-pion rescatterings

→ partial decay width $\sim |h_A f_V h_P|^2$

→ can only access $|h_A|$
In the vector sector: only parameter $h_A$

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- cannot choose sign of $h_A$ since already chose sign of $n$ in $\mathcal{L}_{WZW}$
  $\rightarrow$ corresponds to $+\Phi$ producing pions instead of $-\Phi$
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- cannot choose sign of $h_A$ since already chose sign of $n$ in $\mathcal{L}_{WZW}$
  - corresponds to $+\Phi$ producing pions instead of $-\Phi$

- comparison to experimental data yields $|h_A| = 2.02$
  - calculation without rescattering differs about 5%
Coupling constant for odd intrinsic parity (II)

Still undetermined: sign of parameter $h_A$ relativ to $L_{WZW}$

Consider decay $\pi^0 \rightarrow \gamma e^+ e^-$. 
Coupling constant for odd intrinsic parity (II)

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Consider decay $\pi^0 \rightarrow \gamma e^+ e^-$.  
\[ \rightarrow \text{reaction can happen direct (χPT) } \]

\[ \pi^0 \rightarrow n \rightarrow \]
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Still undetermined: sign of parameter $h_A$ relativ to $\mathcal{L}_{WZW}$

Consider decay $\pi^0 \rightarrow \gamma e^+ e^-$. 
→ reaction can happen direct ($\chi$PT) or via a virtual $\omega$ and $\rho^0$ meson

→ compare $\pi^0$-$\gamma$ transition form factor for $h_A \leq 0$ to data,

$$F_{\pi\gamma}(q^2 = m_{e^+e^-}^2) = 1 + \frac{4\pi^2 f_V^2}{n m_{\rho/\omega}^2} \frac{q^2}{m_{\rho/\omega}^2 - q^2} h_A$$
Coupling constant for odd intrinsic parity (III)

(a) $h_A < 0$

\[
|F_{\pi\gamma}|^2
\]

\[
q^2 [\text{GeV}^2]
\]

Data: H. Behrend et. al., Z. Phys. C49, 401 (1991).
Coupling constant for odd intrinsic parity (III)

(a) \( h_A < 0 \)

\[
|F_{\pi\gamma}|^2
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\( h_A < 0 \), CELLO

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(b) \( h_A > 0 \)

\[
|F_{\pi\gamma}|^2
\]

\( h_A > 0 \), VMD

CELLO
Coupling constant for odd intrinsic parity (IV)

Partial decay width:

- \( \Gamma_{\pi^0 \rightarrow \gamma e^+ e^-} = 9.26 \cdot 10^{-11} \text{ GeV} \),
- \( \Gamma_{\pi^0 \rightarrow \gamma e^+ e^-}^{\text{exp.}} = (9.07 \pm 0.33) \cdot 10^{-11} \text{ GeV} \)

(b) \( h_A > 0 \)

Data: H. Behrend et. al., Z. Phys. C49, 401 (1991).
What does this result mean for our approach?

Recall the form factor:

\[ F = 1 + \frac{4\pi^2 f_V^2}{n m_{\rho/\omega}^2} \frac{q^2}{m_{\rho/\omega}^2 - q^2} h_A \]

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- only \( \mathcal{L}_{\text{WZW}} \): \( h_A = 0 \implies F \equiv 1 \)

(b) \( h_A > 0 \)

\[ |F_{\pi\gamma}|^2 \]

\[ q^2 [\text{GeV}^2] \]

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- only \( L_{WZW} \): \( h_A = 0 \) \( \Rightarrow \) \( F \equiv 1 \)
- only \( L_{vec} \): \( f \sim \frac{q^2}{m^2_{\rho/\omega} - q^2} \)

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\( h_A > 0 \), both Lagrangians are needed to describe the experimental data!

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Application 1: Decay $\pi^0 \to 2e^+ 2e^-$ (I)

- use information from $\pi^0 \to \gamma e^+ e^-$ to calculate $\pi^0 \to 2e^+ 2e^-$
- measured momenta $(q_1, q_3)$ of electrons and $(q_2, q_4)$ of positrons can be produced in two different ways:
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- measured momenta $(q_1, q_3)$ of electrons and $(q_2, q_4)$ of positrons can be produced in two different ways:

\[ q_4 \rightarrow q_3 \rightarrow q_2 \]

$\rightarrow$ width consists of:

- direct contribution $\Gamma_{\text{dir.}}$ depending only on two combinations $q_i + q_j$
- interference contribution $\Gamma_{\text{int.}}$ depending on all possible combinations $q_i + q_j$
Application 1: Decay $\pi^0 \rightarrow 2e^+ 2e^-$ (II)

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- measured momenta $(q_1, q_3)$ of electrons and $(q_2, q_4)$ of positrons can be produced in two different ways:

Partial decay widths:

\[
\Gamma_{\text{dir.}} = 2.70 \cdot 10^{-13} \text{ GeV},
\]

\[
\Gamma_{\text{int.}} = -0.02 \cdot 10^{-13} \text{ GeV},
\]

\[
\Gamma_{\pi^0 \rightarrow 2e^+ 2e^-} = \Gamma_{\text{dir.}} + \Gamma_{\text{int.}} = 2.68 \cdot 10^{-13} \text{ GeV},
\]

\[
\Gamma_{\exp.}^{\pi^0 \rightarrow 2e^+ 2e^-} = (2.58 \pm 0.13) \cdot 10^{-13} \text{ GeV}
\]
Application 2: Scattering $e^+e^- \rightarrow \pi^+\pi^-\pi^0 (I)$

For this scattering process, one gets a \textbf{vector contribution} as for $\omega \rightarrow 3\pi$ decay,

\begin{itemize}
  \item $f_V h_P$
  \item $h_A$
\end{itemize}
Application 2: Scattering \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \) (I)

For this scattering process, one gets a vector contribution as for \( \omega \rightarrow 3\pi \) decay,

\[ \gamma \omega \rightarrow 3\pi \]

and direct contribution from WZW Lagrangian.
Application 2: Scattering $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ (II)

For this scattering process, one gets a \textbf{vector contribution} as for $\omega \rightarrow 3\pi$ decay, and \textbf{direct contribution} from WZW Lagrangian.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{plot.png}
\end{figure}

The cross section for \textbf{vector contribution only} and \textbf{both contributions} shows only small influence of WZW.

\begin{itemize}
  \item Good agreement with data for both calc.
\end{itemize}

Data: R.R. Akhmetshin et. al., Phys. Lett. B476, 33 (2000).
M.N. Achasov et. al., Phys. Rev. D68, 052006 (2003).
Theory: B. Strandberg, Physics Master Thesis, Uppsala University, 2012.
Summary / Outlook

• instead of describing reactions only with WZW or only with vector Lagrangian, we used *both*
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• only 3 open parameters from the vector Lagrangian: \((f_V, h_P, |h_A|)\) can be fixed from pion form factor and \(\omega \rightarrow 3\pi\) → very good agreement of the pion form factor with data

• sign of \(h_A\) can be fixed by comparing \(\pi^0-\gamma\) transition form factor to experimental data → very good description only possible with both Lagrangians
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• use parameter values to describe \(\pi^0 \rightarrow 2e^+2e^-\) and \(e^+e^- \rightarrow 3\pi\) → very good agreement with data
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  \(\rightarrow\) very good agreement of the pion form factor with data

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  \(\rightarrow\) very good agreement with data

• next step: next-to-leading-order Lagrangian for vector mesons
  \(\rightarrow\) full next-to-leading-order calculation
Pion-pion rescattering

Matrix element for elastic pion-pion rescattering can be decomposed according to the angular momentum $l$:

$$
\mathcal{M}_{\pi\pi \rightarrow \pi\pi}(s, \cos \theta) = \sum_{l} (2l + 1) T_{l}(s) P_{l}(\cos \theta) = T_{1}(s) \cos \theta
$$

scatteringle amplitude

Legendre polynomial

$\leftrightarrow$ calculate scattering amplitude with Bethe-Salpeter equation
Dalitz parameters for $\omega \rightarrow 3\pi$ (I)

Reduced matrix element for decay $\omega \rightarrow 3\pi$ can be expressed as

$$|C_{\omega \rightarrow 3\pi}(z, \phi)|^2 \sim 1 + 2\alpha z + 2\beta z^{3/2} \sin 3\phi + 2\gamma z^2 + 2\delta z^{5/2} \sin 3\phi + \mathcal{O}(z^3)$$

with Dalitz-plot parameters $\alpha$, $\beta$, $\gamma$ and $\delta$ and

$$x = \sqrt{z} \cos \phi \sim m_{23}^2 - m_{13}^2, \quad y = \sqrt{z} \sin \phi \sim \frac{1}{3}(m_{\omega}^2 + 3m_{\pi}^2) - m_{12}^2$$

- parameters calculated by minimizing $\sqrt{\chi^2}$ for squared difference between matrix element and polynomial expression (to given order)
- outer parts of Dalitz plot expected to be statistically less important in an experiment
  - use (phase space) $\cdot C_{\omega \rightarrow 3\pi}$ to give less weight to these parts
### Dalitz parameters for $\omega \rightarrow 3\pi$ (II)

Results for tree-level approximation for the $\rho$-propagator (t.l.), Breit-Wigner width (B.W.) and pion-pion rescattering (resc.):

|       | $\alpha \cdot 10^3$ | $\beta \cdot 10^3$ | $\gamma \cdot 10^3$ | $\delta \cdot 10^3$ |
|-------|---------------------|---------------------|---------------------|---------------------|
| t.l.  | 226                 | –                   | –                   | –                   |
| B.W.  | 193                 | –                   | –                   | –                   |
| resc. | 202                 | –                   | –                   | –                   |
| t.l.  | 209                 | 77                  | –                   | –                   |
| B.W.  | 182                 | 49                  | –                   | –                   |
| resc. | 190                 | 54                  | –                   | –                   |
| t.l.  | 180                 | 60                  | 83                  | –                   |
| B.W.  | 166                 | 40                  | 46                  | –                   |
| resc. | 172                 | 43                  | 50                  | –                   |
| t.l.  | 185                 | 40                  | 66                  | 48                  |
| B.W.  | 168                 | 33                  | 40                  | 16                  |
| resc. | 174                 | 35                  | 43                  | 20                  |

- qualitative agreement with Niecknig et al.: same signs, $\alpha$ also dominant
- overall, our parameters are larger

F. Niecknig, B. Kubis, S. P. Schneider, Eur.Phys.J. C72, 2014 (2012).
Further details of the parameter determination

- $f_V$ and $h_P$ could also be determined from direct $\rho$-meson decays
  - not used because the $\rho$-meson resonance is broad

- used also decays $\omega \rightarrow e^+e^-/\mu^+\mu^-$ for $f_V = 140$ MeV

- $|h_A|$ can also be determined from $\omega \rightarrow \pi^0\gamma$ using $f_V$ from pion-form factor ($\rho$-$\gamma$ vertex), $|h_A| = 2.17$
  - different values for $|h_A|$ have less than 1% influence on $\pi^0 \rightarrow \gamma e^+e^-$ and $\pi^0 \rightarrow 2e^+2e^-$
  - for $e^+e^- \rightarrow 3\pi$ it makes sense to use result from $\omega \rightarrow 3\pi$ since this is the final channel
  - $\omega$-$\gamma$ vertex numerically less important

- deviations of parameters is of the order of 10%
  - defines expected accuracy of our calculations