Probing as Quantifying the Inductive Bias of Pre-trained Representations

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Abstract

Pre-trained contextual representations have led to dramatic performance improvements on a range of downstream tasks. This has motivated researchers to quantify and understand the linguistic information encoded in them. In general, this is done by probing, which consists of training a supervised model to predict a linguistic property from said representations. Unfortunately, this definition of probing has been subject to extensive criticism, and can lead to paradoxical or counter-intuitive results. In this work, we present a novel framework for probing where the goal is to evaluate the inductive bias of representations for a particular task, and provide a practical avenue to do this using Bayesian inference. We apply our framework to a series of token-, arc-, and sentence-level tasks. Our results suggest that our framework solves problems of previous approaches and that fastText can offer a better inductive bias than BERT in certain situations.

1 Introduction

Improved pre-trained representations have continuously led to new performance heights on NLP applications. This has prompted researchers to analyze these representations in an attempt to determine which linguistic properties they encode. Probing is one of these methods, which consists of training a supervised model—called a probe—to predict a property directly from the representations. Presumably, the existence of a high-performing probe suggests that the representation encodes the property of interest (Alain and Bengio, 2016; Belinkov and Glass, 2019). Despite the apparent simplicity of probing, the community has yet to find consensus on several important problems about the endeavor:

Problem I. Counter-intuitively, probing may underplay the quality of representations that are presumed to be effective. In some extreme cases, probing suggests that random representations are equally good or better than trained ones (Zhang and Bowman, 2018; Pimentel et al., 2020a). This is certainly nonsensical as random representations, by construction, cannot encode any knowledge but only reliably predict the most frequent label.

Problem II. There is an ongoing debate on the choice of probes: Initially, linear probes were proposed to test linear separability of learned representations (Montavon et al., 2011; Alain and Bengio, 2016; Liu et al., 2019). However, more recently, neural networks are applied with the goal to extract as much information as possible from the representations (Adi et al., 2017; Conneau et al., 2018; Pimentel et al., 2020b). Not surprisingly, it has been found that more complex tasks often require more complex models (Belinkov and Glass, 2019). To reduce the risk of overfitting, recent methods aim at trading off probing performance with the probe’s complexity (Hewitt and Liang, 2019; Pimentel et al., 2020a; Voita and Titov, 2020).

Problem III. The formal goals of probing and its relation to regular NLP tasks are unclear. This manifests in several ways. Firstly, while some argue that probing should focus on “simple” tasks (Conneau et al., 2018), others argue that probing should focus on complex tasks to be informative (Pimentel et al., 2020a). However, this leaves it unclear where the boundary between probing and regular NLP tasks lies and if there even should be a distinction. Secondly, it is unclear what experimental probing results tell us. Knowing that BERT excels at text generation, is it really surprising that we can predict the tense of a word from its representations? Indeed, the NLP community is still in search of how probing can be of service to them.

To address these problems, we propose to compare representations in terms of the inductive bias they provide for a particular task. While classical machine learning often focuses on the inductive biases of models alone, and not representations, we pro-
pose to instead think of models as representation–probe pairs. Such a paired model takes raw text as input, converts it into a representation, e.g., using BERT (Devlin et al., 2019), and predicts a property of interest using a probe. We formalize the notion of the inductive bias of this model using the Bayesian model evidence. The evidence naturally trades off performance and complexity (Rasmussen and Ghahramani, 2001; MacKay, 2003; Bishop, 2006), therefore, it is well-suited to quantify the quality of the inductive bias that a representation–probe pair provides for a particular task.

By formulating probing as quantifying inductive biases using the evidence, our formulation naturally addresses the problems mentioned above. The evidence inherently penalizes random representations (Problem I) and allows us to automatically select probes that have the right complexity for the given task and representation (Problem II). In turn, automatically controlling probe complexity leads to an apples-to-apples comparison between representations, since every representation has access to the probe best suited for it. For example, this provides a fair basis for comparison between simpler fastText representations that might require a more complex probe than an already complex BERT representation. Finally, probing based on the evidence unifies probing and task-driven NLP (Problem III): the goal of the experimenter should be to identify the representation–probe pair with the best inductive bias for a particular problem. When considering probing tasks, the evidence simultaneously measures how well a representation captures the property being probed, taking into account the inherent complexity of its encoding in the representation. But when more traditional NLP tasks are considered (e.g., question answering), the evidence can be used to pick which representation should serve as the basis of a downstream model.

To validate our framework, we apply it to 28 token-, arc-, and sentence-level tasks. Our results suggest that our framework solves the first two practical problems: we uniformly opt for pre-trained representations over random representations and find that different probe architectures and complexities are indeed required for a fair comparison of representations. For example, we thus find that fastText can provide a better inductive bias than BERT for morphosyntactic probing tasks.

2 Probing

In probing, the goal is to predict linguistic properties from the representation of a sequence of tokens. If a representation enables high performance on this prediction task, it is said to encode the linguistic property. For example, a probing task might be to predict part-of-speech (POS) tags from contextual BERT representations.

Formally, we denote linguistic sequences by $\tau$ that are constructed from a vocabulary $\mathcal{V}$. Therefore, a sequence can be represented as $\tau \in \mathcal{V}^+$. For example, $\tau$ could be a word in context, a whole sentence, or simply a single token. We probe for a linguistic property $\pi \in \Pi$ and have a probing dataset of $N$ pairs $\{(\tau_n, \pi_n)\}_{n=1}^N$ of sequences with associated linguistic properties. We abbreviate all sequences and properties collectively by $\tau$ and $\pi$. A representation $h$ under scrutiny is a mapping from a sequence to a $D$-dimensional real vector, i.e., $h : \mathcal{V}^+ \rightarrow \mathbb{R}^D$. Finally, we employ a probe to predict the linguistic property $\pi_n$ of a sequence $\tau_n$ from its representation $h(\tau_n)$. A probe $f$ maps from a vector space to (a distribution over) linguistic properties, i.e., the composition $(f \circ h)(\tau_n)$ provides means to determine the linguistic property $\pi_n$ corresponding to $\tau_n$. In our framework, we treat the composition of $f$ and $h$ jointly as a model that we need to assess. As an example, the representation $h$ is realized by BERT, the probe $f$ is a linear model, $\tau$ are contexts, and $\pi$ are POS tags.

3 Probing by Quantifying Inductive Bias

At the most fundamental level, the NLP community’s interest in pre-trained representations is about reducing the sample complexity of downstream models. The community hopes that pre-trained representations are able to imbue NLP models with enough information about a given language that the models can reach a higher performance with the same or even fewer data. And, indeed, over and over again this has been shown to be the case (Peters et al., 2018; Devlin et al., 2019; Raffel et al., 2020). Another way of phrasing this desire is that the NLP community hopes that pre-trained representations have a suitable inductive bias for downstream tasks. This paper takes the position that, rather than probing the pre-trained representations for how much linguistic structure they

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1Informally, $\mathcal{V}^+$ is the set of all sequences of elements in $\mathcal{V}$ of length at least 1.
Figure 1: Comparison of the inductive biases of representation–probe pairs using the evidence. The evidence below the respective models and representations indicates that the right probe and representation are selected. The probing task is a binary classification of two properties ( vs ). The same colors are used to mark the probe’s decision function. Representations that naturally separate the properties are preferred over random representations in terms of the evidence, since they have a better inductive bias. **Left:** we compare an optimal representation that distinguishes both property classes (a) and a random representation (b). **Right:** we compare a neural probe (c) to a linear probe (d) which is too simplistic. The evidence correctly prefers a neural probe due to the complicated non-linear decision boundary.

contain—an endeavor that has received much attention (Belinkov et al., 2017; Belinkov and Glass, 2019; Conneau et al., 2018; Liu et al., 2019, *inter alia*) but is still contentious (Hewitt and Liang, 2019; Pimentel et al., 2020a,b; Voita and Titov, 2020)—we should, instead, ask how much they improve the inductive bias on tasks of interest.

We propose to quantify the inductive bias of our model, i.e., a representation–probe pair, using the principle of Occam’s razor (Blumer et al., 1987). Occam’s razor states that we should choose the simplest model that sufficiently explains our observations. One way to operationalize this principle is Bayesian model selection (Rasmussen and Ghahramani, 2001; MacKay, 2003; Bishop, 2006). Bayesian model selection relies on the evidence, which is a distribution over datasets for a given model—that is, how likely is it that a particular dataset could have been generated by that model. With a probing dataset, the evidence encompasses Occam’s razor because (i) a too simple model would assign low probability to the dataset (e.g., it is very unlikely that we sample a smooth cubic curve from a linear model), and (ii) an overly complex model would assign low probability because it can model that dataset as well as many others (e.g., it is unlikely that we sample a cubic from a deep Transformer). In line with Occam’s razor, the evidence is then highest for the simplest model that sufficiently explains the dataset (e.g., a cubic model is the best explanation for a dataset consisting of a cubic polynomial).

In the following, we outline the probabilistic model for probing and the form of the evidence. This enables us to quantify the inductive bias of representations. Crucially, part of the inference is to select the optimal probe for a representation to enable a fair comparison between representations.

### 3.1 A Probabilistic Model of Probing

Computation of the evidence requires the definition of a probabilistic probing framework. In this framework, we compute the evidence given representation–probe pairs that constitute models for a fixed task. In the following, we introduce the necessary parts of the probabilistic framework: the generative model and the corresponding prior and likelihood function for a given model.\(^2\)

We denote a model, i.e., a representation–probe pair, by a tuple \((R, P)\) where \(R \in \mathcal{R}\) denotes one of the representations under consideration and \(P \in \mathcal{P}\) is a probe specification. For example, \(R\) could specify BERT and \(P\) a neural probe with a particular architecture and regularization. Both of these control the prior over a representation and probe denoted by random variables \(h\) and \(f\), respectively, and the likelihood.

The prior factorizes into a prior over representations \(h\) and probes \(f\). First, the prior over representations is given by \(p(h \mid \tau, R)\): it is conditional on part of the model specification \(R\) and the input sequences \(\tau\) of the dataset. Given that \(R\) prescribes a BERT representation, the distribution on \(h\) given a sequence \(\tau\) is simply given by the Dirac delta

\[^2\text{We note that our formulation has a close connection to the MDL formulation of probing (Voita and Titov, 2020).}\]
We obtain the evidence for our representation–probe pair that could easily generate all sorts of datasets.

\[ \mathbb{E} \] (3.2 Maximizing the Model Evidence

To maximize the inductive bias of a representation, we specify the set of probes, parametrizing the prior on probes. We then maximize the evidence in eq. (4) for each representation individually over the set of probes \( \mathcal{P} \). This gives us a solution to the inductive bias maximization in eq. (3) in form of a representation–probe pair \( (R^*, P^*) \).

While \( \mathcal{R} \) is simply the set of representations that we want to probe, the set \( \mathcal{P} \) that characterizes priors on probes is more complex. It is typically a combination of discrete and continuous choices. We construct our prior on probes by incorporating commonly used probes into the set \( \mathcal{P} \): we consider linear (Alain and Bengio, 2016; Adi et al., 2017; Hewitt and Liang, 2019; Liu et al., 2019; Pimentel et al., 2020a) and more complex neural probes (Pimentel et al., 2020b; Voita and Titov, 2020) paired with weight decay to control complexity (Pimentel et al., 2020a; Hewitt and Liang, 2019).

To maximize the evidence for each representation over probes \( \mathcal{P} \), we follow the evidence framework by MacKay (1995, 2003) using the scalable implementation proposed by Immer et al. (2021). This enables us to quantify the inductive bias of a representation (eq. (4)) and maximize

\[ \mathbb{E} \]
it over the set of potential probes $P$ as required by eq. (3), i.e., for each representation we select argmax$_{P \in P} p(\pi | h_R, \tau, P)$. It also allows us to maximize the integral over a set of infinitely many choices of weight decay strength, to further control the complexity of the probes. As shown in §5, this leads to highly consistent results and alleviates overfitting, which is—despite common belief—a problem that even simple linear probes have.

4 Tackling Probing with Evidence

As outlined in §1, current work in probing faces a series of problems. Here we discuss how these problems are directly addressed by the evidence.

4.1 Problem I (Representation Selection)

Clearly, random representations have no suitable inductive bias for linguistic tasks. Nonsensical results, such as that random representations outperform pre-trained ones (Zhang and Bowman, 2018; Hewitt and Liang, 2019) simply indicate overfitting, which is strictly penalized in our framework. Compared to pre-trained representations, random representations have low evidence for linguistic tasks because there is no probe that can reliably predict the properties. In Fig. 1a vs. 1b, we illustrate how a random representation is penalized by the evidence. As we will see in §5, our framework consistently assigns lower evidence to the random representations compared to the pre-trained ones.

4.2 Problem II (Probe Selection)

Current probing results are inextricably bound to the choice of probe, yet for probing to provide us with insights about representations, we must break this dependence. For example, one salient issue in probing is that, while pervasive in the literature, there is a spurious association between linear probes and ease of extraction. This is illustrated in Fig. 1, where we can see a linear probe (Fig. 1d) that offers less ease of extraction than a neural probe (Fig. 1c), as measured by the evidence. This means, if we were to compare two representations in Fig. 1, we would obtain misleading results if we restricted our analysis to linear probes. Conversely, we will later see that linear probes can be too complex for some probing tasks and overfit, though the evidence overcomes this problem (Fig. 4). We avoid the problem of selecting a fixed probe by instead choosing a sufficiently large family of probes $P$ and finding the optimal probe, within that family, for each representation: as we will see later, the optimal probe varies considerably across tasks and representations. Instead of heuristic arguments about which probe to choose, the evidence provides a statistically sound way to select one in line with a likelihood-ratio test (Neyman and Pearson, 1933).

4.3 Problem III (Unclear Goals)

In our opinion, an important issue with probing is that the research program has unclear goals. Like much of task-driven NLP, probing is essentially supervised learning with pre-trained representations. We argue that the goal of quantifying and, in particular, maximizing the inductive bias of representation–probe pairs aligns probing with regular NLP: In both cases, one searches for an optimal model at the lowest possible complexity.

5 Experimental Setup

We evaluate our framework on a series of token, arc, and sentence tasks. Our token and arc tasks are multilingual, whereas our sentence tasks only consider English. We remove any property values that have less than 20 examples in any of the splits. All our probes are trained using the Adam (Kingma and Ba, 2015) optimizer. For details on hyperparameters, see App. A.

Token-level tasks. For our token-level probing tasks, we probe for part-of-speech (POS) tags, tense, number, and case. We use the setup in Torroba Hennigen et al. (2020), which consists of mapping the UD v2.5 (Zeman et al., 2019) treebanks to the UniMorph schema (Kirov et al., 2018) using the converter by McCarthy et al. (2018), and extracting examples of tokens tagged for the relevant properties. Next, we obtain the representations for each of those tokens in their sentential context (Torroba Hennigen et al., 2020). Finally, we split the resulting vocabulary using a 65/35 train/test split, such that no word appears in multiple splits.

Arc-level tasks. For our arc-level tasks, we conduct dependency arc labelling (DAL). This consists of classifying the label for a dependency relation given only the representations for the head and dependent of that relation. These are extracted from the UD v2.5 treebanks using the approach in Pimentel et al. (2020a). We use the default UD splits.

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We consider a small but typologically diverse set of languages: English (eng), Arabic (ara), Turkish (tur), Marathi (mar), German (deu), and Chinese (zho).
Sentence-level tasks. For our sentence-level tasks, we consider four tasks. The first is MultiNLI (Williams et al., 2018), a natural language inference task. The other three are the BoolQ (Clark et al., 2019), Commitment Bank (de Marneffe et al., 2019), recognizing textual entailment (RTE; Dagan et al., 2006; Bar Haim et al., 2006; Giampiccolo et al., 2007; Bentivogli et al., 2009) tasks, which are part of the SuperGLUE benchmark (Wang et al., 2019). If a task requires one or more passages as input, we first obtain a passage-level representations by averaging over all of its tokens.

Representations. In our token and arc tasks, we compare four representations: (i) m-BERT (Devlin et al., 2019), (ii) fastText (Bojanowski et al., 2017; Grave et al., 2018), (iii) a completely random representation that assigns a unique vector to every training example, so it offers no information (Rand.), and (iv) a representation that assigns a unique random vector to every word in our vocabulary, so the only information it provides is the identity of the word (Word Ident.). The dimensionality of (iii) and (iv) is the same as that of the BERT representation. For the sentence tasks, we consider (i) Random, (ii) fastText, (iii) BERT, (iv) ALBERT (Lan et al., 2020), (v) RoBERTa (Liu et al., 2020), (vi) XLNet (Yang et al., 2019), and (vii) T5 (Raffel et al., 2020). App. B lists details on the exact models and implementations used.

Probe Family. In order to ensure fair comparisons, our framework requires us to define a suitably expressive probe family. In line with most of the probing literature, our probe family includes linear and neural probes with 1 and 2 hidden layers and 100 hidden units and tanh activation.
fair comparison: simpler representations like fastText can profit from a more complex probe and have a superior inductive bias to BERT in some cases. Moreover, we find that fastText has a better inductive bias than BERT on multiple morphosyntactic tasks, and that T5 appears to offer the best inductive bias for all our sentence-level tasks.

6.1 Representation Comparison

In the following, we discuss the results presented in Fig. 2 and Fig. 3 in detail.

**Expected trends.** Our results depict trends that should be expected from probing. For example, random representations perform worse than pre-trained representations, especially in tasks with a larger number of classes, such as POS and dependency arc labelling. Word identity representations are better than random representations, which is to be expected, since the former are at least able to associate certain types to their most frequent properties, whereas the latter offer no information, and thus can only rely on the memorization of inputs. This is also why the optimal probe for random representations is always a linear probe: since all we can do is memorize datapoints, the best performance–complexity effective explanation of the data is to learn a majority class baseline.

**Token and Arc-level tasks.** Fig. 2 contains the results of our token and arc tagging tasks. We find that fastText offers a better inductive bias for tense, while BERT is superior for case across all languages except Turkish (tur). In fact, we find that fastText is a better inductive bias for all Turkish token tasks. We believe that this is due to the agglutinative nature of Turkish, which means that fastText’s bag-of-subword units mechanism provides a high-quality inductive bias. For dependency arc labelling (DAL), we find that BERT has a uniformly better inductive bias. For dependency arc labelling, the difference in evidence between the different representations is generally quite small for BoolQ, RTE, and CB. Indeed, despite these being highly complex tasks, a linear probe is uniformly preferred for BoolQ and RTE, which suggests the representations struggle to inform the classification. This suggests that more complex representations are required, e.g., more complex than averaging to obtain sentence representations.

6.2 Controlling Probe Complexity

Fig. 4 shows linear probes on two tasks and how the evidence and cross-entropy change as a function of their weight decay. The graph shows that insufficient regularization leads to poor generalization using BERT, apparent from the gap between training and test loss that grows larger for too weak regularization. This means that insufficiently regularizing linear probes—and hence allowing them to fully use their parameters—reduces their evidence. This suggests that optimal probes may actually be sub-linear, in the sense that a linear probe may already contain too many parameters.

Our implementation addresses this by automatically identifying which parameters are needed and forcing others towards zero. Fig. 5 illustrates the distribution of per-parameter regularization strengths for linear English POS probes; interestingly, the distribution is bimodal, such that every representation has a set of parameters that is completely zeroed out (rightmost mode). The random representation is regularized more than pre-trained ones, because it can only learn a majority baseline. Note that in practice, we do this for probes with multiple layers too, so that the optimal probe we find may be simultaneously deep and sparse.
7 Related Work

Probing aims to provide insights into what linguistic information is encoded in pre-trained representations. Since the introduction of probing for sentence representations (Adi et al., 2017; Conneau et al., 2018), probing has also been applied to representations of words and tokens (Belinkov and Glass, 2019; Liu et al., 2019; Voita and Titov, 2020; Pimentel et al., 2020b). Nonetheless, comparison of representations, the choice of probe, and even probing tasks have been under scrutiny recently (Belinkov and Glass, 2019; Liu et al., 2019; Hewitt and Liang, 2019; Pimentel et al., 2020b).

**Measuring representation quality.** Prior work has mostly used probe accuracy as a measure of the quality of a representation. However, if not properly cross-validated, this can lead to nonsensical results which suggest that random representations are as good as learned ones (Zhang and Bowman, 2018; Hewitt and Liang, 2019). To alleviate this problem, control tasks (Hewitt and Liang, 2019), fewer data (Zhang and Bowman, 2018), or simplistic probes (Liu et al., 2019) have been used. Using the evidence can be seen as extensive cross-validation (Fong and Holmes, 2020) and is therefore better suited for comparing representations.

In concurrent work, Lovering et al. (2021) try to predict the inductive bias of representations based on the extractability of pre-defined features that are either spurious or relevant to the probing task. Their method tries to reconcile the results from the probing literature with those of the challenge sets literature. In comparison, our method can be seen as integrating over the entire space of features that a representation offers, and as such makes no assumptions about how a task should be solved.

**Simple or complex probes?** The architecture of probes is still under discussion with a trend towards more complex neural probes (Voita and Titov, 2020; Pimentel et al., 2020b). Initially probes were typically linear models (Alain and Bengio, 2016; Adi et al., 2017; Liu et al., 2019) because complex probes could memorize and overfit (Zhang and Bowman, 2018; Hewitt and Liang, 2019). However, restricting ourselves to linear probes only allows us to ask whether a particular task has a linear decision boundary, which tells us little about the information encoded in representations. Therefore, neural probes have recently been used as well (Pimentel et al., 2020b; Voita and Titov, 2020). In particular, this has spawned a line of work on automatically trading off probe performance and complexity. For example, Pimentel et al. (2020a) achieve this using a so-called Pareto hypervolume. Hewitt and Liang (2019) propose control tasks that mitigate overfitting and find that weight-decay helps generalization in line with our observations in §6.2. Voita and Titov (2020) use the minimum description length principle which is equivalent to the evidence in the case of a probabilistic model (MacKay, 2003). However, their framework does not include explicit comparison of representations but is formulated as a comparison and selection of probes. Moreover, our probabilistic framework does not require additional concepts of model transmission which obscure the underlying evidence (MacKay, 1995).

8 Conclusion

Previous approaches to linguistic probing are plagued by several key problems, namely the issues of nonsensical results, probe selection, and ill-defined goals. To overcome these issues, we have proposed a novel probing framework, which focuses on the inductive bias that pre-trained representations offer for different linguistic tasks. We have shown that the Bayesian evidence, a natural measure for inductive bias, can be used in the context of probing. We have found that our framework empirically does not suffer from the aforementioned problems and yields results that coincide well with our linguistic intuitions. We are hopeful that under this new paradigm, future work in probing will be more principled, comparable, and useful to the NLP community at large.
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A Experimental Details

All our probes are trained using the Adam (Kingma and Ba, 2015) optimizer with hyperparameters \(\beta_1 = 0.9, \beta_2 = 0.999\), learning rate 0.1, batch size 512, and for 500 epochs. For each discrete architecture (linear, MLP-1, MLP-2), we run the evidence framework as suggested by Immer et al. (2021) with the following parameters: frequency \(F = 1\), \(K = 100\) number of steps every epoch, learning rate \(\gamma = 0.1\). We implement our method using the laplace-torch (Daxberger et al., 2021). We use an individual weight-decay parameter per parameter group of the probes, i.e., each set of weights and biases are regularized independently per layer. Only for Fig. 5, we use a weight-decay strength individually per parameter of the linear probe which effectively turns off individual parameters by increasing their weight decay. This is also known as automatic relevance determination (MacKay, 1995).

B Representations

Tab. 1 shows the representations we used. For all transformer models, we use the HuggingFace transformers library (Wolf et al., 2020). Note that for fastText we use the multilingual vectors which are language-dependent, and the official fastText library.\(^5\)

| Representation | Model name               |
|----------------|--------------------------|
| m-BERT         | bert-base-multilingual-cased |
| BERT           | bert-base-uncased         |
| fastText       | Language-specific, see here. |
| T5             | t5-base                   |
| RoBERTa        | roberta-base              |
| XLNet          | xlnet-base-cased          |
| ALBERT         | albert-base-v2            |

Table 1: Representations used. All representation except fastText use the HuggingFace implementations (Wolf et al., 2020).

\(^5\)https://pypi.org/project/fasttext/