An exploration of double diffusive convection in Jupiter as a result of hydrogen–helium phase separation

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ABSTRACT

Jupiter’s atmosphere has been observed to be depleted in helium ($Y_{\text{atm}} \sim 0.24$), suggesting active helium sedimentation in the interior. This is accounted for in standard Jupiter structure and evolution models through the assumption of an outer, He-depleted envelope that is separated from the He-enriched deep interior by a sharp boundary. Here we aim to develop a model for Jupiter’s inhomogeneous thermal evolution that relies on a more self-consistent description of the internal profiles of He abundance, temperature, and heat flux. We make use of recent numerical simulations on H/He demixing, and on layered double diffusive (LDD) and oscillatory double diffusive (ODD) convection, and assume an idealized planet model composed of an H/He envelope and a massive core. A general framework for the construction of interior models with He rain is described. Despite, or perhaps because of, our simplifications made we find that self-consistent models are rare. For instance, no model for ODD convection is found. We modify the H/He phase diagram of Lorenzen et al. to reproduce Jupiter’s atmospheric He abundance and examine evolution models as a function of the LDD layer height, from those that prolong Jupiter’s cooling time to those that actually shorten it. Resulting models that meet the luminosity constraint have layer heights of $\approx 0.1–1$ km, corresponding to $\approx 10 000–20 000$ layers in the rain zone between $\sim 1$ and $3–4.5$ Mbar. Present limitations and directions for future work are discussed, such as the formation and sinking of He droplets.

Key words: convection – planets and satellites: individual: Jupiter – planets and satellites: interiors – planets and satellites: physical evolution.

1 INTRODUCTION

Helium abundance measurements in Jupiter’s atmosphere, beginning with ground based, aircraft, and Pioneer 10 and 11 spacecraft observations and culminating in the Galileo Entry Probe experiment, exhibit a remarkable agreement about a depletion in helium compared to the protosolar value (Orton & Ingersoll 1976; Gautier et al. 1981; Niemann, Atreya & Carignan 1998; von Zahn, Hunten & Lehacher 1998). The Galileo in situ measurement also revealed a significantly subprotosolar neon abundance. Both the He and Ne abundances are thought to result from phase separation of helium from hydrogen under high pressures and of downward rain of He–Ne rich droplets (Stevenson 1998; Wilson & Militzer 2010).

A helium rain region in Jovian planets, long predicted to occur (Salpeter 1973; Stevenson 1975) is likely accompanied by a composition gradient and superadiabatic temperatures (Stevenson & Salpeter 1977). The precious in situ observation of Jupiter’s atmospheric He abundance of $Y := M_{\text{He}}/(M_{\text{He}} + M_{\text{H}}) = 0.238 \pm 0.005$ by mass (von Zahn et al. 1998) thus indicates a more exotic Jovian interior than so far described by standard models. Those commonly represent Jupiter by few, sharply separated homogeneous and adiabatic layers (Chabrier et al. 1992; Saumon et al. 1992; Guillot, Gautier & Hubbard 1997; Gudkova & Zharkov 1999; Saumon & Guillot 2004; Nettelmann et al. 2008, 2012), even if He rain is explicitly accounted for in the planet’s thermal evolution (Hubbard et al. 1999). These simplifications have of course been quite valid, as neither H/He phase diagrams with predictive power existed, nor was a theory for heat transport in an inhomogeneous medium under Jovian interior conditions available. Both are important, but not necessarily sufficient, for determining the gradients in He abundance and in temperature in Jupiter’s interior.

Thanks to growing computer power, this situation has changed in recent years. Using ab initio simulations, Morales et al. (2009), Lorenzen, Holst & Redmer (2009, 2011) and Morales et al. (2013a) have studied the demixing behaviour of He from H under Jupiter and Saturn interior conditions, where hydrogen undergoes a
transition from non-metallic to metallic fluid. Semiconvection, a fluid instability that can occur in the presence of a destabilizing temperature gradient and a stabilizing composition gradient, has recently been investigated by Rosenblum et al. (2011), Mirouh et al. (2012), Wood, Garaud & Stellmach (2013) using 3D numerical simulations. They observe semiconvection, also called double diffusive (DD) convection, to occur in two forms: as layered double diffusive (LDD) convection characterized by convective layers and dynamic, turbulent interfaces where composition and temperature change drastically, and as oscillatory double diffusive (ODD) convection, where density perturbations oscillate around an equilibrium position. Moreover, they have developed a prescription for the heat flux as a function of the gradients in density and temperature, which is crucial for determining the resulting temperature gradient ($\nabla T := d\ln T/d\ln P$) in the planet.

At same heat flux, the superadiabaticity $\nabla_T - \nabla_{ad}$, where $\nabla_{ad}$ is the adiabatic temperature gradient, is enhanced in a semiconvective region compared to the case of full, overturning convection (Chabrier & Baraffe 2007; Leconte & Chabrier 2012). A warmer-than-adiabatic interior as a result of semiconvection has been demonstrated to be able to prolong the cooling time of exoplanets (Chabrier & Baraffe 2007) and of Saturn (Leconte & Chabrier 2013) by several Gyr; it also allows one to add more heavy elements into the planet. In particular, Leconte & Chabrier (2012, hereafter LC12) find that if LDD convection occurs throughout the interior of Jupiter, its heavy-element content may be two times larger than derived from standard models.

In just a few years, Juno is expected to deliver new observational data on Jupiter. Properties of interest (here the core mass, heavy-element content and depth of zonal flows) can often not be measured directly but are inferred from model calculations that match the data. Now that an accurate He abundance measurement, comprehensive H/He demixing calculations, as well as semiconvective heat flux models all are at hand, we feel it is time to start to apply these three ingredients to begin to develop more advanced Jupiter models. While this is a clear advance over previous work, we also caution that there is a fourth leg to this ‘chair’ that is missing in this work: we do not employ a theory for the formation, growth, and rain-out of He droplets here.

With this fundamental caveat in mind, we apply in this paper a theory of DD convection as a result of assumed He rain, and investigate its effect on Jupiter’s thermal evolution. We explore whether Jupiter’s luminosity can be explained by the assumptions of Section 1.1, which we think is a more self-consistent set of assumptions than conventional models rely on. This paper more aims at providing and discussing illustrative examples, rather than an evolved description of the physical processes inside the planet. We hope this paper will initiate the development of the theoretical framework for the case of sedimentation in a giant planet. That task would vastly exceed the scope of this paper.

Outline. The computation of DD convection due to He rain is performed within a double iterative procedure. In Section 3, we describe the inner loop, through which we ensure consistency between the temperature gradient and the heat flux. The theory of semiconvection (Sections 3.5–3.6) provides the superadiabaticity profile in the demixing region for given characteristic material parameters (Section 2), given a heat flux profile (Section 3.2), and a given He gradient profile (Section 4). The Lorenzen et al. (2009, 2011) H/He phase diagram that we use to compute the He abundance profile is described in Section 4.1. In Section 4.2, we describe two modifications to it, and in Section 4.3 the outer loop that yields the consistency between the He profile and the temperature profile. Section 5 contains our results for the application of the slightly modified H/He phase diagram (‘modified-1’), and Section 6 for the more severely (‘modified-2’) H/He phase diagram. In Section 7, we discuss the results and suggest future steps. Section 8 contains a summary.

1.1 Fundamental assumptions

Our method and results rely heavily on the following assumptions made in this work.

(i) Jupiter’s observed atmospheric He depletion is a result of He rain-out.

(ii) The internal He abundance profile is dictated by the H/He phase diagram.

(iii) The exchange of He droplets between vertically moving eddies and the ambient fluid is negligible. This is the standard assumption in the Ledoux criterion.

(iv) The internal temperature–pressure profile is not affected by the heavy elements.

(v) Throughout the evolution, either LDD or ODD convection occurs in the demixing region.

(vi) Jupiter’s homogeneous deep interior below the rain zone remains adiabatic.

1.2 Fundamental caveats

Conventional adiabatic models can well explain Jupiter’s observed luminosity. Therefore, the additional energy source implied by our assumptions (i) and (ii) requires a compensating process for the total energy balance. The assumption (v) of semiconvection serves that purpose. However, one could in principle imagine a different scenario; for instance, core erosion (Guillot et al. 2004) could influence the energy balance as well. In fact, the applicability of the theory of semiconvection to the case of demixing and sedimentation has not been proven yet. This theory requires the diffusivity of solute to be less efficient than that of heat in order to maintain a composition gradient, while demixing and sedimentation imply an efficient albeit non-diffusive redistribution of solute. We nevertheless assume its applicability here on the grounds that a stabilizing compositional difference should exist between rising fluid elements and the surrounding medium. This is because an adiabatically evolving fluid element losing solute to condensation gains latent heat, stays warmer, and hence can hold a higher equilibrium abundance of solute than the surrounding. In other words, the non-diffusive nature of the condensation and rainfall leads to a reduction of the stabilizing composition gradient, although this reduction does not nullify it ($\beta > 0$). As an approximation, we use $\beta = 1$, where $0 \leq \beta \leq 1$ is a scaling factor for the full predicted mean molecular weight gradient, and examine the results.

On the other hand, there is no known analogue. For instance, rain-forming water in the Earth is a minor constituent without stabilizing effect; demixing and sedimentation of Fe–Ni in the young Earth occurred down to the centre without leaving behind a composition gradient in the mantle; and semiconvective regions in stars are often treated as zones of enhanced diffusion (Langer, El Eid & Frick 1985; Ding & Li 2014), while here we assume diffusion to be negligible compared to sedimentation. We return to these points in Sections 7.4–7.8.
2 MATERIAL PROPERTIES

The dimensionless Prandtl number
\[
\operatorname{Pr} = \frac{\nu}{\kappa_T}
\]  
(1)
is the ratio of kinematic shear viscosity \(\nu\) to thermal diffusivity \(\kappa_T\), which have SI units of m\(^2\) s\(^{-1}\). In stars, \(\operatorname{Pr} \ll 1\), while in the water-rich interiors of Uranus and Neptune \(\operatorname{Pr} > 1\) might be possible (Soderlund et al. 2013). The dimensionless diffusivity ratio
\[
\tau = \frac{D}{\kappa_T}
\]  
(2)
measures the ionic particle diffusivity \(D\) in relation to \(\kappa_T\). In stars and gas giants, \(\tau \ll 1\) because the ions are slower than the electrons and photons, and more heat is transported by electrons and photons than by the ions. For Jupiter, we neglect energy transport by photons. The thermal diffusivity \(\kappa_T\) is related to the thermal conductivity \(\lambda\) through
\[
\lambda = \rho c_p \kappa_T,
\]  
(3)
where \(\rho\) is mass density, \(c_p\) specific heat, and \(\lambda\) has SI units of W K\(^{-1}\) m\(^{-1}\). Both \(\operatorname{Pr}\) and \(\tau\) are important parameters because they define the transition between the DD (i.e. semiconvective) and the stable regime. In fact, the transition occurs at the critical value
\[
R^{*-1} = \frac{1 + \operatorname{Pr}}{\tau + \operatorname{Pr}},
\]  
(4)
as initially discussed by Walin (1964) for the case of a stabilizing salt gradient in water heated from below, see equation 13 therein. The inverse density ratio is defined as
\[
R_0^{-1} = \frac{\alpha_T}{\alpha_\mu} \frac{V_T - V_{\text{ad}}}{V_\mu - V_{\text{ad}}}
\]  
(5)
(LC12; Mirosh et al. 2012). \(R_0^{-1}\) includes the partial derivatives
\[
\alpha_T = \frac{\mu}{\rho} \frac{\partial \rho}{\partial T}, \quad \alpha_\mu = -\frac{T}{\rho} \frac{\partial \rho}{\partial \mu}.
\]  
(6)
While \(\alpha_T\) can directly be calculated from the EOS, \(\alpha_\mu\) is calculated using
\[
\frac{\partial \rho}{\partial \mu} = \frac{\rho^2}{\mu^2} \left( \frac{\mu H}{M_H} - \frac{\rho L}{M_L} \right).
\]  
(7)
As a ratio of composition gradient \((\nabla_\mu)\) to superadiabaticity \((\nabla_T - \nabla_{\text{ad}})\), \(R_0^{-1}\) is basically a density ratio between the differences in density due to different compositions and due to different temperatures that occur between a vertically moving parcel and its surrounding, respectively. The range \(R_0^{-1} \in (1, R_{\text{crit}}^{-1})\) precisely defines the region of parameter space unstable to semiconvection. Ledoux instability implies \(R_0^{-1} < 1\), which marks the boundary between the overturning convective and the DD regime. Thus, \(R_0^{-1}\) is the central quantity for determining whether a medium is in a state of DD convection or not. An overview of these different regimes is given in Fig. 1. In the stable regime, \(\nabla_T = \nabla_{\text{rad}}\), where \(\nabla_{\text{rad}}\) is the temperature gradient needed to transport the heat by conduction and radiation. Section 3 deals with the method of deriving \((\nabla_T - \nabla_{\text{rad}})\), while Section 4 refers to \(\nabla_\mu\).

Numerical values of the material properties in the 1–10 Mbar region along the Jupiter adiabat are given in Table 1. The values for \(\lambda, \kappa_T, \nu\), and the particle diffusivities are taken from French et al. (2012), who computed the transport properties along the Jupiter adiabat using ab initio simulations. For the shear viscosity \(\nu\) they found the dominant contribution to be the motions of the nuclei; other contributions are neglected in our applied values for \(\nu\).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Parameter} & \textbf{Value} \\
\hline
\hline
\lambda & 1.34 \times 10^{-5} \\
\kappa_T & 4.4 \times 10^{-5} \\
\nu & 2.5 \times 10^{-6} \\
\hline
\end{tabular}
\caption{Material properties along the Jupiter adiabat.}
\end{table}

\section{Modeling Double Diffusive Convection}

We aim to determine the local temperature gradient in a non-adiabatic planetary interior. For that purpose, we make use of reference models (Section 3.1), and relations between the temperature gradient and the heat flux that can locally be transported along that gradient (Sections 3.4–3.6). As the heat flux is constrained by the luminosity at the planet’s photosphere, and as the energy loss of a planet ultimately leads to cooling and contraction, we also re-visit planetary cooling (Section 3.2).

\subsection{Reference Jupiter models}

To compute the demixing region in Jupiter, we define two types of reference Jupiter models (two-layer, 2L, and three-layer models). Furthermore, we make simplifying assumptions about its internal structure by putting all heavy elements of mass \(M_Z\) as inferred from standard structure models into the core and assuming a pure H/He envelope of mean protosolar H/He ratio. In particular, we use reference models with a core mass of 28 or 32 M\(_\oplus\), with \(M_Z = 28\) M\(_\oplus\) being a typical value for SCVH EOS based models (Saumon & Guillot 2004), while a 32 M\(_\oplus\) core is found to best reproduce Jupiter’s observed mean radius under the assumption of LDD convection.

Before demixing begins, Jupiter is described by a 2L model with a rock core and one homogeneous adiabatic H/He envelope. For instance, our reference model ‘2L-ha-T180’ for that case has a 1-bar temperature of 180 K and a 28 M\(_\oplus\) core mass. Our homogeneous, adiabatic 2L reference model for the case that demixing does not occur in present Jupiter of surface temperature \(T_{1\text{ bar}} = 169\) K is labelled ‘2L-ha-T169’. Finally, our quasi-homogeneous, adiabatic reference model for the case that demixing does occur but in the form of a sharp layer boundary at 1 Mbar between the depleted outer and the enriched inner envelope is a three-layer model and labelled ‘3L-qha-T169’.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Illustration of the four different regimes in the presence of a stabilizing composition gradient \(\nabla_\mu\), which increases from left to right, while \(\nabla_T\) increases from right to left.}
\end{figure}
### 3.2 Planetary cooling and luminosity profile

The heat loss due to cooling and contraction of a planetary mass shell \( dm \) at mass level \( m \) per time interval \( dt \) is given by

\[
\delta Q(m) = T(m) \delta s(m) \, dm,
\]

where \( \delta s(m) \) is the specific entropy of that mass shell, and \( \delta s \) is the change of the specific entropy during \( dt > 0 \). This heat loss increases the planet’s total luminosity by \( dq = -\delta Q/dt \). With the specific energy of heat, \( \epsilon_q := \delta Q/dm = T(d) \) the heat released from below the sphere of radius \( r(m) \) is \( I_q(m(r)) = -\int_0^{r(m)} dm \, dq/dr \). Beside \( I_q \), there can be further contributions to the luminosity of a giant planet or star such as sources from the decay of radioactive elements \( (\epsilon_{\text{radio}}) \), nuclear reactions \( (\epsilon_{\text{nuc}} \cdot \text{stars}) \), or neutrino loss \( (\epsilon_{\nu} \cdot \text{neutron stars}) \), so that in general

\[
\frac{dL}{dt} = -\frac{\delta q}{dt} + \frac{dI_{\text{radio}}}{dm} + \epsilon_{\text{nuc}} - \epsilon_{\nu} + \cdots \tag{9}
\]

For the majority of stars, the first two terms can be neglected, and the local luminosity be computed, in parallel with the temperature and compositional profiles, as an integral over the nuclear reaction rates. For giant planets however, only the first two terms in equation (9) play a role, so that the luminosity becomes

\[
L(m(r)) = -\int_0^{r(m)} dm \left[ \frac{\delta q}{dr} - \frac{dI_{\text{radio}}}{dm'} \right]. \tag{10}
\]

To obtain Jupiter’s current luminosity profile, we evolve the planets’ internal structure down from the state before demixing began. The time interval \( dt \) between two subsequent internal states appears in equation (10) as a scaling factor. As in Nettelmann et al. (2012) we use \( dt \) to adjust the known intrinsic luminosity \( L_{\text{int}} \),

\[
\frac{L_{\text{int}}}{L_{\text{int}}} = \int_0^{r_{\text{int}}} dm' T \, ds,
\]

where \( L_{\text{int}} = L_{\text{eff}} - L_{\text{eq}} \) and \( L_{\text{eff}} \) is either the observed luminosity at present time \( (F_{\text{eff}} = L_{\text{eff}}/4\pi R_j^2 = 13.6 \text{ W m}^{-2} \) for Jupiter), or the predicted one of a model atmosphere, as required for instance for the evolving planet at earlier times. \( L_{\text{eq}} \) describes the incident flux \( (F_{\text{eq}} = 8.2 \text{ W m}^{-2} \) for Jupiter) that is derived from the stellar luminosity, orbital distance, and Bond albedo.

To conclude, for given temperature and entropy profiles, equations (10) and (11) provide us with the internal luminosity and heat flux profiles, \( F(m) = l(m)/4\pi r^2(m) \).

### 3.3 Thermal evolution

To compute Jupiter’s thermal evolution with H/He phase separation and DD convection we generate interior models for different surface temperature down to \( T_{\text{surf}} = 169 \text{ K} \). These models provide the internal profiles of temperature and entropy, which are needed to compute the inhomogeneous evolution, i.e. the evolution when the composition changes with depth. Note that for homogeneous evolution it suffices to know the entropy only up to a constant offset value which may depend on composition, as that offset value cancels out when taking the difference \( T \, ds \).

For sufficiently high surface temperatures, interior temperature is too high for demixing to occur. Thus, we represent Jupiter’s evolution prior to the onset of demixing by a series of adiabatic, homogeneous 2L models with a rock core. To compare the evolution with and without He rain we also expand that series down to \( T_{\text{surf}} = 169 \text{ K} \).

The cooling of the planet is then computed as described in Nettelmann et al. (2012), but here we neglect angular momentum conservation. For the outer boundary condition, we use either the Graboske et al. (1975) model atmosphere grid, or the non-grey atmosphere model of Fortney et al. (2011), which these authors found to yield a \( \sim 500 \text{ myr} \) longer cooling time for Jupiter.

### 3.4 Conductive heat transport

The local temperature gradient \( dT/dr \) depends on the processes through which the heat is transported. Possible heat transport mechanisms in giant planets are radiation, conduction, ODD convection, and LDD convection, and overturning convection. The relation between heat flux and temperature gradient in the 1D conductive case reads

\[
F_{\text{cond}} = -\lambda \, dT/dr. \tag{12}
\]

Equation (12) yields the heat flux that is transported by conduction along a known temperature gradient. In the case of predominantly conductive heat transport we could invert equation (12) to obtain the temperature gradient. However, conductive heat transport, and also radiative heat transport, is usually inefficient in giant planets so that the temperature gradient needs to be determined by other means.

### 3.5 Relation between heat flux and temperature gradient in LDD convection

We derive an expression for the relation between the heat flux in case of LDD convection, \( F_{\text{LDD}} \), and the temperature gradient, following closely the description of Wood et al. (2013), their equations 16–18. In LDD convection, convective layers are separated by interfaces, and thin adjacent boundary layers, of strongly varying temperature and composition gradients. Thus, the temperature gradient is not continuous on the scale of individual ‘steps’ in the staircase. However, it is possible to consider an average temperature gradient, if

**Table 1.** Material properties along the Jupiter adiabat. Data taken from French et al. (2012).

| \( r \) (\( R_J \)) | \( T \) (K) | \( P \) (GPa) | \( \lambda \) (W/K/m) | \( \kappa \) (m²/s⁻¹) | \( v \) (m²/s⁻¹) | \( \text{Pr} \) | \( D_H \) (m²/s⁻¹) | \( \tau_{\text{max}} = (D_H/\kappa) \) | \( \tau_{\text{min}} = (D_H/\kappa) \) | \( R_{\text{crit}}^{-1} \) |
|-----------------|---------|--------|----------------|-----------------|-------|-------|----------------|----------------|----------------|----------------|
| 0.196           | 18000   | 3410   | 1470           | 2.70e-05        | 0.26e-06 | 0.01  | 0.428e-06     | 0.0159          | 0.0101          | 40.4            |
| 0.350           | 16000   | 2460   | 1040           | 2.26e-05        | 0.282e-06    | 0.012 | 0.436e-06     | 0.0193          | 0.0111          | 32.6            |
| 0.478           | 14000   | 1640   | 721            | 1.89e-05        | 0.296e-06    | 0.0157| 0.450e-06     | 0.0238          | 0.0134          | 26.7            |
| 0.584           | 12000   | 1030   | 465            | 1.50e-05        | 0.295e-06    | 0.0197| 0.458e-06     | 0.0312          | 0.0165          | 20.4            |
| 0.680           | 10000   | 600    | 283            | 1.19e-05        | 0.313e-06    | 0.0263| 0.468e-06     | 0.0393          | 0.0192          | 15.7            |
| 0.770           | 8000    | 300    | 153            | 8.56e-06        | 0.342e-06    | 0.04  | 0.481e-06     | 0.0562          | 0.0233          | 11.5            |
| 0.852           | 6000    | 120    | 59.6           | 4.99e-06        | 0.366e-06    | 0.072 | 0.471e-06     | 0.0944          | 0.0359          | 6.7             |
| 0.890           | 5000    | 64     | 20.2           | 2.16e-06        | 0.367e-06    | 0.17  | 0.369e-06     | 0.1708          | 0.0796          | 3.4             |
| 0.930           | 4500    | 23     | 3              | 3.55e-07        | 0.368e-06    | 1.036 | 0.274e-06     | 0.7718          | 0.73            | 1.2             |

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taken over at least one subsequent pair of layers plus interfaces, as they are found to occur in computer simulations (Mirouh et al. 2012; Wood et al. 2013). It is this average temperature gradient we are interested in.

In a convective medium, where heat is transported by both conduction and turbulent motions the total heat flux $F$ is given by $F = F_{\text{cond}} + F_{\text{conv}}$. Usually, a mixing length theory (MLT) based expression for $F_{\text{conv}}$ is used. In case of LDD convection, the turbulent heat flux reduces to $F_{\text{LDD}}$. Thus, we have

$$F = F_{\text{cond}} + F_{\text{LDD}},$$

(13)

using the notation of Wood et al. (2013), $F_{\text{cond}} = -\rho c_p \kappa (dT/dr)$ where $\rho$ is the local density, and

$$F_{\text{LDD}} = \rho c_p \kappa_T (1 - N_{\text{T}}) \left( \frac{dT}{dr} - \frac{dT_{\text{ad}}}{dr} \right),$$

(14)

where $N_{\text{T}}$ is the thermal Nusselt number, whose expression is discussed below. Inserting equations (14) and (12) into equation (13) and using equation (3) we obtain

$$F = -\lambda \frac{dT}{dr} - \lambda (N_{\text{T}} - 1) \left( \frac{dT}{dr} - \frac{dT_{\text{ad}}}{dr} \right).$$

(15)

Next we seek to express $N_{\text{T}}$ in terms of parameters that can be evaluated, namely of those introduced in Section 2. Wood et al. (2013) found that the expression

$$N_{\text{T}} = 1 = f_t(R_0^{-1}; \tau) \text{Ra}^a \text{Pr}^b,$$

(16)

with $a = 0.34 \pm 0.01$ and $b = 0.34 \pm 0.03$ provided a reasonable fit to their numerical experiments. The function $f_t(R_0^{-1}; \tau)$ remained poorly constrained but was found to take values between 0 and 0.2 for $R_0^{-1} = 1.1 - 2$ and $\tau = 0.01 - 0.3$, with $f_t$ decreasing with $R_0^{-1}$, but fairly independent on $\tau$. Various functional formula for $f_t$ will be tested. Inserting equation (16) into (15) gives

$$F = -\lambda \frac{dT}{dr} - \lambda f_t(R_0^{-1}, \tau) \left( \text{Ra} \text{Pr} \right)^{1/3} \left( \frac{dT}{dr} - \frac{dT_{\text{ad}}}{dr} \right).$$

(17)

With the product $\text{Ra} \text{Pr} = g \alpha l_H^3 \times (dT/dr - dT_{\text{ad}}/dr)/\kappa_T^2$

equation (17) then becomes

$$F = -\lambda \frac{dT}{dr} - \lambda f_t \left( \frac{g \alpha l_H^3}{\kappa_T^2} \right)^{1/3} \left( \frac{dT}{dr} - \frac{dT_{\text{ad}}}{dr} \right)^{4/3},$$

(18)

which can be evaluated and solved for the temperature gradient $dT/dr$ numerically. Note that the first term in equation (18) is $F_{\text{cond}}$ and the second one is $F_{\text{LDD}}$. Equation (18) is equivalent to equation 7 in LC12.

3.6 Relation between heat flux and temperature gradient in ODD convection

The heat flux in the case of ODD convection can be expressed in terms of the Nusselt number in the same way as in the case of LDD convection (equation 14). For $N_{\text{T}}$ we adopt the fit to the simulation data of Mirouh et al. (2012),

$$N_{\text{T}} \sim 0.75 \left( \frac{\text{Pr}}{\tau} \right)^{0.25 \pm 0.15} \left( 1 - \frac{\tau}{R_0^{-1}} \right) \left( 1 - \frac{R_0^{-1} - 1}{R_{\text{ad}}^{-1} - 1} \right).$$

(19)

The procedure then is the same as described in Section 3.5, only that equation (19) instead of equation (16) is inserted into equation (15). The resulting values of $R_0^{-1}$ can then be used as a self-consistency check for our fundamental assumption ($v$).

3.7 Academic exercises

In order to understand the behaviour of the possible superadiabaticity in Jupiter as a function of layer height and composition gradient, we investigate toy models first. In Section 3.7.1, we assume $f_t$ to be constant. In Section 3.7.2, we will account for the dependence of $f_t$ on $R_0^{-1}$ explicitly.

3.7.1 Constant $f_t$ values

Fig. 2 displays the relation (18) between the total flux $F$, scaled by Jupiter’s intrinsic flux at the surface, $F_{\text{surf}} = 5.44 \text{ W m}^{-2}$, and the relative superadiabaticity ($dT/dr - dT_{\text{ad}}/dr = \nabla T/\nabla_{\text{ad}} T - 1$) using values of the material parameters of Table 1 that are typical for the 1–2 Mbar region in Jupiter, where H/He demixing is supposed to occur. The solid lines are for constant $f_t$ values. Other lines will be explained in Section 3.7.2.

According to Fig. 2, the flux $F_{\text{LDD}}$ increases with $\nabla T$. For small $\nabla T/\nabla_{\text{ad}} T - 1 \ll 10^{-4}$, $F_{\text{LDD}} \ll F_{\text{cond}}$ for all layer heights so that $F \approx F_{\text{cond}} \approx 10^4 F_{\text{surf}}$. With increasing relative superadiabaticity, $F \approx F_{\text{LDD}} \sim f_t(R_0^{-1})^3$; the smaller the layer height, and the smaller $f_t$, the lower the heat flux. Through layer heights below 1 m the heat flux is as inefficient as conductive heat transport and would require high relative superadiabaticities of 10–100 to allow Jupiter’s observed heat flux be transported. On the other hand, layer heights larger than 1000 m would imply $\nabla T \approx \nabla_{\text{ad}} T$ and thus are expected to have little effect on Jupiter’s temperature profile compared to adiabatic standard models.

In principle, the range in possible layer heights is further restricted by the requirement that layers can form at all, which is seen to...
occur in simulations not below a minimum length-scale of about $5/3 \times 20 \times 10^2$ the instability length-scale parameter $d$ (Wood et al. 2013),
\[
d = \left( \frac{\kappa_T v}{\alpha g|dT/dr - dT_{ad}/dr|} \right)^{1/4}.
\] (20)

Wood et al. (2013) point out that the value of $L_H$ is just slightly larger than the wavelength of the fastest growing linear mode (20d), which is the most important one because it rapidly dominates the dynamics of the system due to the exponential amplification of the initial state, at least within linear instability analysis. With $d \approx 5-50$ cm, this lower limit on $L_H$ of 1.5–15 m agrees well with the minimum layer height of 1 m found in Fig. 2. For illustration however, we present here the relations for a wider range of layer heights, and also of $R_0^{-1}$ values, than actually allowed.

### 3.7.2 $f_T$ as a function of superadiabaticity for constant composition gradient

We replace the formerly constant $f_T$ values by a function $f_T(R_0^{-1}, \tau)$ which is obtained by fitting the simulation data of Wood et al. (2013), their fig. 6, within $\tau = 0.03-0.3$ and $R_0^{-1} = 1.1-1.5$. We choose a functional form that guarantees $f_T \to 0$ for $R_0^{-1} \to \infty$ and $f_T \to 1$ for $R_0^{-1} \to 1$. $R_0^{-1}$ values lower than 1 are not of interest because the medium would then be in the overturning convection state, while for high $R_0^{-1}$ values semiconvection ceases in favour of a diffusive heat transport ($f_T = 0$). Thus, we use the function
\[
f_T(R_0^{-1}, \tau) := \frac{c_1(\tau)}{\left[R_0^{-1} - (1 - \epsilon)\right]^{c_2}}
\] (21)

and adjust $c_1(\tau)$ and $c_2$ to match the simulation data. The small parameter $\epsilon = 10^{-3}$ ensures that $f_T(R_0^{-1} = 1)$ is well behaved and close to 1 as in the usual MLT, although even for $f_T = 1$ equation (18) would not exactly describe the MLT case because of the different exponents of $R_0$. We find $c_1 = 0.3$ and $c_1(\tau) = c_{14} + c_{12} \tau + c_{13} \tau^{-1}$, with $c_{14} = 0.06348$, $c_{12} = -0.06746$, $c_{13} = 0.0008262$. Our fit function $f_T$ is shown in Fig. 3. In the demixing region, $f_T$ typically decreases weakly by a factor of 2 and adopts values in the extrapolated regime at $R_0^{-1} > 1.5$. As $F_{LD}$ scales with $f_T \times \nu_{ad}^2$, any uncertainty in $f_T$ can be expressed as an uncertainty in $L_H$ and thus should not affect the resulting possible range of superadiabaticities. By using our fit formula $f_T(R_0^{-1}, \tau)$, we already include the dependence of $f_T$ on the composition gradient. For our academic exercise, we simplify this dependence by setting
\[
R_0^{-1} = \frac{c_3}{\nu_T - \nu_{ad}},
\] (22)

with a constant toy composition gradient $c_3$ for which we assume $c_3 = 0.01$ and 0.1, respectively, in close agreement to the values that we calculate for the demixing region in Jupiter. Here, we investigate the effect of a given constant composition gradient on the intrinsic flux (equation 18) and on the possible superadiabaticity.

We go back to Fig. 2 to examine the dashed and dotted curves therein. In addition, Fig. 4 shows a zoom-in for $c_3 = 0.1$. First, the dashed and dotted curves have a steeper slope than those for constant $f_T$ values, probably because of $f_T \sim (R_0^{-1})^{-0.3}$, and thus $F_{LD} \sim (\nu_T - \nu_{ad})^{3/5}$ instead of $4/3$ (compare equation 18). Secondly, the smaller the composition gradient (smaller $c_3$ value), the higher the flux at fixed superadiabaticity.

Thirdly, the smaller $c_3$ (dashed $\to$ dotted) and the larger the superadiabaticity ($\sigma$ $\to$ right), the smaller becomes the $R_0^{-1}$ value. Eventually $R_0^{-1} \to 1$ happens. Because of our functional choice for $f_T$, which prohibits $R_0^{-1} < 1$ by letting $f_T$ rise to infinity, the slope of $\sigma$ then tends to infinity. This behaviour tells us that the medium wants to transition to overturning convection regime. We recover the well-known fact that for a given composition gradient, there is an upper limit to the superadiabaticity in LDD convection. The lower $c_3$, the lower the maximum possible superadiabaticity. For small layer heights (here for $L_H < 100$ m), this upper limit on the

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**Figure 3.** Fit function $f_T(R_0^{-1}, \tau)$ Upper panel: function $f_T(R_0^{-1}, \tau)$ (dashed) and the data points that $f_T$ is designed to fit from fig. 6 in Wood et al. (2013) (symbols). Lower panel: fit coefficient $c_1(\tau)$, see equation (21).

**Figure 4.** Similar to Fig. 2 but for $c_3 = 0.1$ only. The thin solid black curve is for ODD convection assuming $Nu_T - 1 = 0.5$. The left vertical line marks the transition between the stable and the ODD regime ($R_0^{-1} = R_{ad}^{-1}$) while the right vertical line marks the transition between ODD and LDD convection ($R_0^{-1} = 3$). Thus, only the cyan curve displays the physically allowed relation, which depends on the assumed layer height in the LDD regime.
superadiabaticity implies that the desired flux cannot be transported by semiconvection, which implies a lower limit on $l_{\text{th}}$.

Fourthly, for $l_{\text{th}} \geq 100\text{ m}$ and $F \approx F_{\text{Jup}}$, Fig. 2 shows $F_{\text{cond}} \ll F \approx F_{\text{LDD}}$. In that case, $l_{\text{th}}^2 \sim (\nabla_T - \nabla_{\text{ad}})^{-3}$. With increasing layer height at a given composition gradient, the needed superadiabaticity might eventually become so small that $R_{\text{crit}}^{-1}$ gets larger than $R_{\text{cond}}^{-1}$, which contradicts the assumption of LDD convection. In particular, $R_0^{-1}$ close to $R_{\text{crit}}^{-1}$ is usually associated with ODD convection (LC12; Mirosh et al. 2012). We therefore include ODD convection in our considerations.

Fifthly, Fig. 4 indicates that layer heights above 1 km may yield $R_{\text{cond}}^{-1} > R_{\text{crit}}^{-1}$ if $F \approx F_{\text{Jup}}$ is to be transported while ODD convection appears to be too inefficient to transport $F_{\text{Jup}}$. The cyan curve highlights the heat flux–superadiabaticity relation for which the $R_{\text{crit}}^{-1}$ value would be consistent with the respective regime (stable, ODD, or LDD). In our toy models, a narrow range of $l_{\text{th}} \approx 100-300\text{ m}$ emerges for which $F_{\text{Jup}}$ can be transported.

From here on, we can methodically proceed in two different directions: we could only use those relations like the cyan curve in Fig. 4 that guarantees $R_{\text{cond}}^{-1}$ values consistent with the assumed regime (stable, LDD, and ODD). That approach narrows down the $l_{\text{th}}$ value before any self-consistent, converged model is found. Here, we decide to trod a different way: we first construct models with He rain for a wide range of $l_{\text{th}}$ values, and then ask whether the fully converged model satisfies the consistency criterion for $R_{\text{crit}}^{-1}$. We think that this approach makes it easier to understand the behaviour of the solutions, as they smoothly transition from already explored territory, where the effect of He rain on the planet’s thermal evolution is dominated by the gravitational energy release (Hubbard et al. 1999; Fortney & Hubbard 2003), to new territory, where we will see the effect to be dominated by the internal temperature profile.

3.8 Consistency between $F(T)$ and $T(F)$: the inner loop

In Sections 3.5 and 3.6, we have presented the relation between heat flux and temperature gradient in LDD and ODD convection, respectively (for a given $\nabla_v$). One can walk the trail in either direction and use these relations to derive the heat flux from a given temperature gradient (and composition gradient), or the temperature gradient for a given heat flux (and $\nabla_v$), depending on what is known a priori. Both the heat flux and the temperature profiles are a priori unknown in Jupiter, unless the interior is adiabatic (ignoring here any uncertainty due to the EOS).

We showed in Section 3.2 how we can determine the heat flux profile for given profiles of temperature and entropy. We compute a first guess on $F(m)$ by using the reference model 3L-qha-T169 and equation (10). Then, an iterative procedure is performed that iterates between the computation of the temperature gradient from the heat flux profile (equation 15) and the computation of the heat flux profile from the temperature and entropy profiles (equation 10), until a converged solution is obtained. For this inner loop the helium abundance profile is kept constant.\(^2\)

\(^1\) In detail, $T(F)$ is computed by a fourth-order Runge–Kutta integration of $\nabla_v$ and then $s(T, P)$ is derived from the EOS.

\(^2\) In detail, $Y$ is kept constant as a function of mass $m$ to ensure helium mass conservation. Intermediate planet models are computed to ensure additional consistency between $m$ and $P$, as it is $Y(P)$ that is provided by the H/He phase diagram, not $Y(m)$.

4 MODELING THE HELIUM ABUNDANCE PROFILE

Helium is predicted to demix from hydrogen at high pressures ($\sim 1\text{ Mbar}$) and sufficiently low temperatures in regions where hydrogen undergoes pressure ionization from the molecular to the metallic state, where He is still non-metallic (Salpeter 1973; Lorenzen et al. 2011). Demixing is also seen at lower pressures where hydrogen is in the molecular phase, both in numerical simulations (Morales et al. 2013a) and in laboratory experiments (Loubeyre, LeTullec & Picanco 1985), although at much lower temperatures. It is even predicted to occur in fully ionized H–He mixtures up to 200 Mbar (Stevenson 1975).

Published H–He phase diagrams largely agree in predicting demixing of H and He at temperatures of several 1000 K and pressures of a few Mbar, the typical $P$–$T$ space of evolved giant planets like Jupiter and Saturn. However, the predictions of the slope and the locations of the phase boundaries for the demixing temperature as a function of pressure and helium abundance have changed considerably over time, and with them the predictions for the presence and extension of demixing zones in Jupiter and Saturn. Results are diverse, and include demixing in both planets within at least 5–20 Mbar (Klepeis et al. 1991), no demixing in either of them (Pfaffenzeller, Hohl & Ballone 1995), demixing in Saturn down to the core with no demixing in Jupiter (Morales et al. 2013a), and demixing in both planets at depths below 1 Mbar (Lorenzen et al. 2011). The more modern calculations (Lorenzen et al.; Morales et al.) agree much better with each other than the older ones.

We here apply the H–He phase diagram of Lorenzen et al. (2009, 2011) for three reasons: it provides a very dense grid of demixing temperatures $T_{\text{dmx}}(P, x_{\text{He}})$, that is the maximum temperature below which H/He phase separation occurs, as a function of He abundances $x_{\text{He}}$ for relevant pressures $P$; it does predict demixing in Jupiter and allows for the computation of the helium abundance profile and atmospheric depletion; finally, it is based on state-of-the-art first-principles simulations using classical molecular dynamics (MD) simulations for the ionic subsystem and density functional theory (DFT) for the electronic subsystem (DFT-MD simulations), a method that has repeatedly yielded data in remarkably good agreement with experiments, such as for pure hydrogen (see, e.g. Becker et al. 2013).

Independently, Morales et al. (2009, 2013a) also computed the H–He phase diagram by using similar ab initio simulation methods to derive the Gibbs free energy, which yields the energetic preference of mixing or demixing. The main two differences between both groups lie in (i) the determination of the entropy of mixing and (ii) in the functional form used to fit the enthalpy of mixing $\Delta H$ as a function of $x_{\text{He}}$ for given $P$, $T$ values. While Lorenzen et al. (2009, 2011) neglect non-ideal contributions to the entropy of mixing but capture the asymmetric shape of $\Delta H(x_{\text{He}})$ through a high-order expansion, Morales et al. (2009, 2013a) include the non-ideal entropy of mixing but approximate $\Delta H(x_{\text{He}})$ by a quadratic fit only, which appears a reasonable match to their sparse data sample but not to the fine Lorenzen data grid. Both approximations affect the double tangent construction of the Gibbs free energy $\Delta G(\Delta H, S_{\text{mix}})$, from which the energetic preference for demixing and the corresponding equilibrium compositions are derived. While experimental efforts are under way (Soubiran et al. 2013), in the absence of experimental constraints on H/He demixing under planetary interior conditions we consider the deviations in $T_{\text{dmx}}(x_{\text{He}}; P)$ between the two theory groups as an indication for the real uncertainty; it amounts to $\sim 500\text{ K}$ at 4 Mbar and even $1000\text{ K}$ at 1 Mbar.
in Stevenson & Salpeter (1977) we here assume that if demixing occurs, He droplets will form and sink to a depth where demixing is no longer predicted to occur, or the core boundary is reached. More specifically, we assume that He droplets will sink as long as \( T(P, x_{\text{He}}) < T_{\text{demix}}(P, x_{\text{He}}(P)) \). Demixing terminates when the phase boundary between mixed and demixed state is reached, i.e. if \( T(P, x_{\text{He}}) = T_{\text{demix}}(P, x_{\text{He}}) \). This describes the equilibrium state that we require our planetary \( P-T-x_{\text{He}} \) profiles to achieve.

To obtain the atmospheric helium mass fraction, \( Y_{\text{atm}} \) due to He rain we compute an initial H/He adiabat for Jupiter’s known I-bar temperature and protosolar H/He ratio, while heavy elements are neglected, as their distribution in response to He rain is unknown and its investigation is beyond the scope of this paper. Since our initial adiabat intersects with the demixing curve \( (T(P, x_{\text{He}}(\text{proto})^2) < T_{\text{demix}}(P, x_{\text{He}}(\text{proto}))) \) over some pressure range, we lower the \( x_{\text{He}} \) value of the adiabat and iterate between the adiabat and demixing curve until the \( P-T \) profile of the adiabat just touches the demixing curve. This provides us with a unique, converged value \( x_{\text{He}} = x_{\text{He}}^{(\text{atm})} \) for a given 1-bar surface temperature. Abundances \( x_{\text{He}} < x_{\text{He}}^{(\text{atm})} \) would lead to no crossover between the adiabat and demixing curve, while higher He abundances to an intersection. This behaviour is a result of the strong decrease in \( T_{\text{demix}} \) towards lower \( x_{\text{He}} \) values in the relevant \( x_{\text{He}} \) range (Fig. 5), which more than compensates the cooling of the adiabat with lower \( x_{\text{He}} \) values. For the unmodified Lorenzen et al. (2009, 2011) data, this touch point (the equilibrium state) occurs at \( P_0 = 1 \) Mbar. As in Stevenson & Salpeter (1977) we assume that all planetary material, atop the onset pressure \( P_0 \) for demixing will be mixed over time into the demixing region through convection. We here make the assumption of instantaneous sedimentation, meaning that the background profile follows the phase diagram as a result of assumed rapid He droplet formation and assumed rapid sinking to a level where they dissolve, before convection could redistribute the droplets upward (but see also Sections 4.4 and 7.8). Therefore, the excess He from outer envelope material that gets mixed into the He rain zone through convection will sink down. Over time, the He abundance in the planetary atmosphere and in the entire envelope down to \( P_0 \) decreases to the value of \( x_{\text{He}}^{(\text{atm})} \). Because of Jupiter’s short convection time-scale of only \( \sim 3 \) yr, this process is supposed to deplete the atmosphere rapidly. This justifies our assumption of a hydrostatic state, where \( Y_{\text{atm}} = Y(x_{\text{He}}^{(\text{atm})}) \). A similar method was used in Fortney & Hubbard (2003). For the most detailed discussion on He sedimentation we refer the reader to Stevenson & Salpeter (1977).

4.1 The Lorenzen et al. H/He demixing diagram

We here describe our semi-analytical fit to the Lorenzen et al. (2009, 2011) data for \( T_{\text{demix}}(x_{\text{He}}; P) \). The published data span a grid of pressures \( \{1, 2, 4, 10, 24 \) Mbar\}], and a very dense grid in helium abundance ranging from pure hydrogen (\( x_{\text{He}} = 0 \)) to pure helium (\( x_{\text{He}} = 1 \)). Applying the raw data with simple interpolation to Jupiter, we find that the adiabat intersects with the demixing region between 1 and about 3.5 Mbar; thus all information on the helium abundance profile \( Y(P) \) is based on only 3–4 simulated pressure grid points. Since we need smooth gradients \( \nabla_y \) and \( R_0^{-1} \), we are forced to develop a semi-analytical fit to the data in terms of \( T_{\text{demix}}(P; x_{\text{He}}) \) as shown in Fig. 5. Fortunately, the highest helium abundances found to occur in present Jupiter are \( x_{\text{He}} < 0.3 \) so that we can ignore the high-\( x_{\text{He}} \) part of the demixing diagram when fitting the data. First, we choose a representative subset in \( x_{\text{He}} \) and display \( T_{\text{demix}}(P; x_{\text{He}}) \) in Fig. 5, upper panel. For each of these \( x_{\text{He}} \) values we then fit \( T_{\text{demix}}(P; x_{\text{He}}) \) by the fit formula

\[
T_{\text{demix}}(P; x_{\text{He}}) = A_0(x_{\text{He}}) + A_1(x_{\text{He}}) \times \arctan(\log(1 + A_2(x_{\text{He}}) P)).
\]  

(23)

Although the arctan function is non-unique (it maps on to itself under variation of the argument), we found a reasonable behaviour of the coefficients \( A_i(x_{\text{He}}) \), see Fig. 5. Indeed, none of the other functional forms we tried yielded a better behaved \( Y(P) \) profile. Other fit formulas we tried might show an almost indistinguishable functional form used but also lends confidence to the one chosen.

4.2 Atmospheric helium depletion

4.2.1 Assumptions

The H/He demixing diagram allows for the calculation of the helium abundance over a planet’s entire internal pressure range. As
4.2.3 Modifications to the Lorenzen H/He data

Modified-1 H/He data. As mentioned above there is considerable uncertainty about the correct demixing diagram, with differences of up to 1000 K obtained by different groups. Therefore, we introduce two modifications of the Lorenzen H/He data. In our modified-1 version, we apply a constant temperature shift \( \Delta T_{\text{dmx}} \) to shift the whole H/He phase diagram according to

\[
T_{\text{dmx}}^{(\text{mod-1})}(P, x_{\text{He}}) = T_{\text{dmx}}^{(\text{Lor})}(P, x_{\text{He}}) + \Delta T_{\text{dmx}}
\]

until the computed \( Y_{\text{atm}} \) value for present Jupiter is within the observational error bars of both \( Y_{\text{atm}} \) and \( T_{\text{atm}} \). As can be read from Fig. 6, a good match is achieved by \( \Delta T_{\text{dmx}} = -200 \) to \(-300\) K, if using the SCvH EOS, implying that perhaps the Lorenzen demixing diagram slightly underestimates the real demixing temperatures. For our modified-1 version to the Lorenzen et al. (2009, 2011) phase diagram data we use \( \Delta T_{\text{dmx}} = -200\) K. The phase diagram then predicts \( Y_{\text{atm}} = 0.2383 \) for \( T_{\text{atm}} = 169 \) K.

Modified-2 H/He data. In our modified-2 version of the Lorenzen et al. (2009, 2011) data for H/He demixing, which was found to be driven by metalization of hydrogen, we apply a modest pressure shift of those data by 0.4 Mbar, in which case demixing would not occur below 1.4 Mbar. Our ad hoc shift is inspired by the recent revision of the predicted coexistence line of the plasma-phase transition of hydrogen towards 1 Mbar higher pressures, with now excellent agreement between the ab initio simulations of Morales et al. (2013b) and the earlier shock compression experiments by Weir, Mitchell & Nellies (1996). In addition, we stretch \( T_{\text{dmx}}(x_{\text{He}}, P) \) for \( x_{\text{He}} > x_{\text{He}}(Y_{\text{atm}}) \) so that demixing already starts right below \( T_{\text{atm}} = 200 \) K and proceeds with shallower gradients in both \( Y_{\text{atm}} \) \((T_{\text{atm}})\) and in \( x_{\text{He}}(P) \). Mathematically, we apply the modification

\[
T_{\text{dmx}}^{(\text{mod-2})}(P, x_{\text{He}}) = \left( T_{\text{dmx}}^{(\text{Lor})}(P - 0.4/\text{Mbar}, x_{\text{He}}) \right) + \Delta T_{\text{dmx}} \times \left[ 1 + \alpha(x_{\text{He}} - x_{\text{He, atm}}) \right],
\]

with \( \alpha = 0.6 \) if \( x_{\text{He}} > x_{\text{He, atm}} \), otherwise \( \alpha = 0 \). We chose \( \Delta T_{\text{dmx}} = +250\) K to obtain \( Y_{\text{atm}} = 0.2344 \) for \( T_{\text{atm}} = 169 \) K. The modified-2 version is displayed in Fig. 7 for relevant He abundances, along with H/He adiabats for the surface temperature of present Jupiter.

4.3 Internal helium abundance profile \( Y(P, T) \)

Once the values of \( Y_{\text{atm}} \) and the onset pressure are known from the procedure described in Section 4.2, we can derive the internal helium profile.

We first compute a three-layer model with outer envelope He abundance \( Y_1 = Y_{\text{atm}} \) and inner envelope He abundance \( Y_2 \), the latter one adjusted to conserve the total mass of helium, where the layer boundary pressure is set equal to the onset pressure of demixing, \( P_0 = 1 \) Mbar. This is our reference model 3L-qha-T169. We then use the inner envelope adiabat of constant He abundance \( Y_2 \) as the background state \( T(P) \) upon which the first inhomogeneous He profile according to the demixing diagram is computed. The local equilibrium abundances \( x_{\text{He}}^{(A)} \) are determined in dependence on the local temperatures and pressures along that adiabat by solving equation (23) for \( x_{\text{He}}(T_{\text{dmx}} = T(P), P) \) using equation (24). At that point, we make use of our analytic fit to the Lorenzen data in order to obtain a smooth He gradient.

The inner edge of the demixing region is found by requiring He mass conservation. Starting at 1 Mbar and working inward, we ask at each pressure level whether the integrated helium mass above, plus the proposed He mass under a constant extension of the local He abundance down to the core, would match the given total He mass. If so, the \( Y \) profile is forced to leave the equilibrium curve at that mass level, \( m_y^1 \), and to continue with that abundance down to the core. The final internal He profile thus requires three layers: an He-poor outer envelope of \( Y_1 = Y_{\text{atm}} \) between 1 and 10\(^6\) bar, a demixing region with inhomogeneous He abundance between \( m_{y2} = m_{y1}(1 \text{ Mbar}) \) and \( m_{y2} \), and an inner envelope with \( Y_3 > Y_{\text{atm}} \) between \( m_{y2} \) and the core. This procedure only works as long as a \( Y_3 \) value can be found. Otherwise, He layer
formation on top of the core would naturally occur; see also Fortney & Hubbard (2003).

The He abundances in the inhomogeneous region, the extent of the demixing region, \([m_{z1} - m_{z2}]\) and the \(Y_1\) value all depend on the \(T(P)\) profile in the demixing region. We account for that dependence by an outer loop which iterates between \(Y(P), m_{z2}, Y_3\) on the one hand and the \(T(P)\) profile on the other hand, see Section 4.5.

4.4 Calculating the \(R\)-parameter

We compute the composition and temperature gradients, defined as

\[
\nabla \mu := \frac{\partial \mu}{\partial P}, \quad \nabla T := \frac{\partial T}{\partial P},
\]

(26)

where none of the thermodynamic variables are kept constant. These gradients are average gradients in a sense that the average is assumed to be taken across several layers (if LDD convection occurs), and local in a sense that the temperature gradient and the composition gradient may change over large distances of \(\sim 1 R_J\) (700 km). However, we never explicitly compute an average over layers (as LC12 do) because we do not explicitly distinguish between diffusive interfaces and convective, advective layers (as LC12 do).

To compute \(\nabla \mu\) we decompose it into the adiabatic gradient, \(\nabla_{\text{ad}}(T, m, \mu)\) plus an analytically added free value. In fact, we choose the local superadiabaticity \(\nabla \mu - \nabla_{\text{ad}}\) as the running free parameter. To obtain \(\nabla_{\text{ad}}(T, m, \mu)\) and the local derivatives \(\alpha_{\mu}\) and \(\alpha_T\) we create local EOS tables around \((P, T, m)\) of local composition \(Y(P)\). To compute \(\alpha_T\) we use equation (43) in Saumon et al. (1995) and for \(\nabla_{\text{ad}}\) we use equations (45)–(46) therein but with the corrections \(S/S_{\text{Li}} \to S_{\text{Li}}/S \) and \(S/S_{\text{He}} \to S_{\text{He}}/S_{\text{Li}}\).

For computing \(\nabla_T\), we assume a given (superadiabatic) temperature profile \(T(P)\) and a given mean molecular weight profile \(\mu(P)\). The latter one is calculated based on \(Y(P)\) as described in Section 4.3, and by using \(\mu^{-1} = X\mu_X^{-1} + Y\mu_Y^{-1}\) where \(X\) and \(Y\) are the mass fractions of H and He, respectively. The computed composition gradient \(\nabla \mu\) describes the average gradient under our assumption of instantaneous He sedimentation. We also apply it to the computation of the density ratio \(R_0^{-1}\), an approximation that leaves room for future explorations of the physical processes involved with He rain, and deserves some discussion.

The density ratio \(R_0\) is thought to express the buoyancy experienced by a vertically displaced parcel in a surrounding that may have temperature \((\nabla T)\) and composition \((\nabla \mu)\) background gradients, like the ones we computed here. In MLT for convection, it is generally assumed that a vertically displaced parcel expands/contracts adiabatically and maintains its composition because diffusion of particles and of heat occur on longer time-scales than convective transport. Here we face a different situation. In the ODD regime, diffusion of heat and particles out of the parcel may occur, see Stevenson & Salpeter (1977) about overstable modes. Moreover, our assumption of instantaneous He sedimentation implies rapid He condensation, so that droplets, if formed in the parcel, may leave it and thus alter its composition during the journey, contrary to our fundamental assumption (iii). This effect would tend to reduce the composition difference between the moving parcel and its ambient fluid. We can account for this possible reduction by introducing a factor \(\beta \in [0, 1]\), so that the relevant density ratio that determines the stability of the system becomes

\[
R_0^{-1} = \frac{\alpha_T}{\alpha_{\mu}} \frac{\beta \nabla_{\mu}}{\nabla_T - \nabla_{\text{ad}}},
\]

(27)

and \(\nabla_{\mu}\) is the background composition gradient as described above.

The end-member case \(\beta = 1\) (our fundamental assumption iii) reflects the usual assumption of conserved composition. A value \(\beta \leq 1\) may apply if the time-scale for the formation of initial He droplets is longer than the eddy lifetime, so that He droplet formation in a vertically moving parcel becomes a rare event at first place. Still, some droplets may form and sediment out, however, so that \(\beta < 1\). In fact, \(\beta = 1\) might be inconsistent with our assumption of instantaneous sedimentation. If applied to a parcel, the He abundance therein would equal that dictated by the phase diagram for the parcels’ own temperature and thus tend to decrease when it moves upward. Stevenson & Salpeter (1977) suggest \(1 - \beta < 0.97\). In fact, \(\beta \ll 1\) will be preferred according to our results. We note that the real composition gradient in the planet is not well known. Heavy elements may contribute to a stabilizing gradient. For instance, the Jupiter models by Nettelmann et al. (2012) predict an increase in heavy-element abundance at \(P \geq 4\) Mbar, and the most recent ones even at \(P \geq 3\) Mbar (Becker et al. 2014), which is located within or near the lower end of the demixing region. For simplicity, we apply \(\beta = 1\) in this work, keeping in mind that we may overestimate the stabilizing He gradient between the rising parcel and its surrounding. A physically self-consistent treatment of the composition gradient is left to future work, see also Section 7.8.

4.5 Consistency between \(Y(T)\) and \(T(Y)\): the outer loop

The temperature profile that is needed to transport the heat flux in the presence of a composition gradient depends on that gradient, in our case the helium abundance profile, while the latter one at the H/He demixing—mixing phase boundary depends on the local temperatures. To ensure consistency between \(T\) and \(Y\) we iterate between the \(T\) profile for a given \(Y\) profile and the \(Y\) profile for given \(T\). After at most five iterations good convergence is achieved. This is illustrated in Fig. 8 for the case of \(h_i = 1\) km. The iteration starts with the 3L-qha-T169 model adiabat (the solid black curve). This adiabat is colder than the fully homogeneous adiabat of \(Y = Y_{\text{proto}}\) (dotted curve) because He-poor regions as at the outer boundary require lower temperatures for maintaining constant entropy. Superadiabatic temperatures (coloured curves) require higher He abundances for consistency with the H/He demixing curve, see Fig. 5. In turn, higher He abundances for a fixed \(Y_{\text{lim}}\) value imply a steeper He gradient, and thus need higher temperatures for transporting the heat flux. Convergence is rapid for the moderate superadiabaticities in LDD convection.

As a test case, we also imposed the constraint of overturning convection \((R_0^{-1} = 1)\), in which case the converged He abundance by the end of the demixing region was found to be higher than allowed by He mass conservation. In other words, the condition \(R_0^{-1} = 1\) can only be satisfied in Jupiter by letting \(\nabla_T\) and \(\nabla_{\mu}\) go to infinity, meaning a step in \(Y\) and \(T\). We here recover the runaway effect between the gradients in \(Y\) and \(T\) as observed by Fortney & Hubbard (2003).

Having put the pieces together, we can compute the effect of LDD and ODD convection in the demixing region on Jupiter’s present structure.

5 RESULTS FOR THE MODIFIED-1 H/HE DEMIXING DIAGRAM

In this section, we apply the modified-1 H/He demixing phase diagram to models of Jupiter and assume that either ODD or LDD convection occurs in the demixing region with the layer height as a free parameter.
5.1 ODD convection

ODD and LDD convection can lead to very different resulting superadiabaticities and density ratio values in Jupiter’s demixing region at a given pressure level, for instance at 2 Mbar as shown in Fig. 9. When the running free parameter $\nabla_T - \nabla_{ad}$ is low ($<0.1$), $R_0^{-1} > 1$. In the case of ODD convection the factor $(R_0^{-1} - 1)^{-1}$ in equation (19) then yields a flux too low, close to the diffusive limit. In fact, in ODD convection the flux reaches the order of $F_{lag}(P)$ only for $(R_0^{-1} - 1)^{-1} \to \infty$, i.e. for $R_0^{-1} \simeq 1$, and this behaviour occurs over the entire demixing region. A value $R_0^{-1} \simeq 1$ indicates the preference for overturning instead of semiconductor. We conclude that in order to transport the given heat flux by ODD convection, the required superadiabaticity would be so high that the system would want to transition to overturning convection, which contradicts the assumption of ODD convection. Therefore, this does not appear to be a viable path towards a Jupiter model.

At a given superadiabaticity, the heat flux that can be transported by LDD convection is one to two orders of magnitude higher than in ODD convection, even if the layer height is quite small ($<1$ km). LDD convection thus requires lower superadiabaticities. Thus, we focus on LDD convection.

Figure 9. Relation between superadiabaticity, density ratio, and total heat flux for parameter values in Jupiter’s demixing region at 2 Mbar. The values of $R_0^{-1}$ and $\nabla_T - \nabla_{ad}$ that will be assigned to the 2 Mbar pressure level for a given layer height of, respectively, 100 m (yellow) or 1000 m (red), or in the case of ODD convection (solid black) are those that cross the black dashed line. For ODD convection, a sufficiently high heat flux can only be obtained for $R_0^{-1} \simeq 1$, which violates the assumption of ODD. The relations are displayed here for the initial iteration step, i.e. for an He gradient along the 3L-qha-T169 adiabat. The curves do not display final models for Jupiter, as discussed in the text.

5.2 LDD convection

Fig. 10 shows the resulting profiles of temperature, specific entropy, luminosity, heat flux, helium abundance, superadiabaticity, and $R$-parameter in Jupiter’s demixing region for various assumed layer heights between 1 km and 1000 km (coloured curves). Fig. 10 also shows three black curves. The black dashed curve is for the 2L-ha-T180 model (see Section 3.1). The black dotted curve is for the 2L-ha-T169 model, and is supposed to describe the present Jupiter if demixing would never have occurred. Finally, the black solid curve is the 3L-qha-T169 model and is used as the initial state in our double iterative procedure. All these models have a 28 M$_\odot$ rock core and a pure H/He envelope.

**Entropy.** In the outer part of the demixing region and in the adiabatic outer envelope, the entropy is seen to rise above the level of the $T_1$ that is 180 K reference state before demixing began. Therefore, that outer part gives a negative contribution to the total luminosity. The rise above the reference state might surprise, in particular as Fortney & Hubbard (2003) find (for Saturn) the entropy in the outer part to decrease steadily with $T_1$, see their fig. 7. We argue that this difference is due to the different H/He phase diagrams used, especially due to the small $T_1$ interval over which demixing occurs in Jupiter (s increases with $T_1$), and the strong He depletion (s decreases with $Y_{encl}$). Here, the He depletion wins over the cooling effect in the time evolution of the outer envelope’s entropy.

**Extent.** In terms of pressures, the demixing region extends from 1 Mbar to at most 3.5 Mbar. While by definition the entropy is constant outside the demixing region, it changes steadily within it, mostly due to the steadily changing He abundance. From the entropy panel in Fig. 10 we can derive an extension of the...
demixing region over $dM = 0.1-0.15 \, M_J (dM = 30-47 M_\oplus)$, or $dR = 0.92-1.27 R_\oplus \approx 5900-8100 \, km$, depending somewhat on the layer height, with smaller layer heights yielding thinner demixing regions.

**Density ratio.** For layer heights above 1 km, the values of $R_{\text{He}}^{-1}$ are all higher than $R_{\text{crit}}^{-1}$. We remind ourselves that $R_{\text{crit}}^{-1}$ is an upper limit that marks the transition to the diffusive regime. Values $R_{\text{He}}^{-1} > R_{\text{crit}}^{-1}$ are obtained as a result of low superadiabaticities (see the lower-left panel in Fig. 10). Under the assumption of $\beta = 1$ this implies that LDD convection can transport the heat flux too efficiently if the He abundance gradient obeys the modified-1 H/He data. The larger the layer height, the fewer interfaces are present, and thus the smaller becomes the required superadiabaticity, leading to higher $R_{\text{He}}^{-1}$ values. The largest $R_{\text{He}}^{-1}$ values are then obtained for the largest assumed layer height, which is half the size of the demixing region ($l_\text{H} \approx 3600 \, km$). Clearly, in order to get resulting $R_{\text{He}}^{-1}$ values in agreement with the regime of LDD convection as seen in numerical experiments (Mirosh et al. 2012; Wood et al. 2013), smaller layer heights ($l_\text{H} < 1 \, km$) are required for present Jupiter. This finding is in agreement with what we derived from our toy models in Section 3.7.2. How small can the layer height be?

**Minimum layer height.** The shorter the layer height, the warmer the adiabatic deep interior and the higher its specific entropy; thus, the smaller becomes the entropy difference $ds(m)$ with respect to the internal profile at the previous time step. A minimum layer height is obtained when the summation over $Tds$ on Jupiter’s deep interior is no longer capable of compensating the negative luminosity contribution from the planet’s outer part, so that the total luminosity would become negative. We find this minimum to be 1 km if the layer height is kept at constant value. This value of 1 km is imposed by the condition of a positive total planet luminosity, and neither by the Ledoux instability criterion, which would be violated ($R_{\text{He}}^{-1} < 1$) at $l_\text{H} \approx 10 \, m$, nor by the minimum length-scale criterion (equation 20).

**Thermal evolution.** The energy that can escape from the planet is determined by the atmosphere model. If more (less) energy is released from the interior, it will take longer (less long) to transport it through the atmosphere. Because of additional gravitational energy from sinking He droplets, the effect of He rain is generally thought to prolong the cooling time of a planet (Stevenson & Salpeter 1977; Saumon et al. 1992; Hubbard et al. 1999; Fortney & Hubbard 2003).

Fig. 11 shows the effect of He rain and LDD convection on Jupiter’s cooling time relative to that of homogeneous evolution for different assumed layer heights. All displayed cooling curves have a $T_\text{ad}$ of 124.4 K within the observational error bars, and make use of the Graboske model atmosphere. For large layer heights (1000 km) redistribution of He dominates over the temperature effect, so that we observe the expected prolongation of the cooling time. As for $l_\text{H} = 1000 \, km$ the superadiabaticity is negligibly small, this case can be considered equivalent to the usual assumption of adiabatic cooling ($\beta = 0$), where the cooling behaviour is only influenced by the additional gravitational energy from the He rain (Hubbard et al. 1999; Fortney & Hubbard 2003). For the modified-1 H/He diagram, such a model would yield a cooling time prolongation by $\approx 0.7 \, Gyr$, compare the orange and the black curves in Fig. 11.

Conversely, if the deep interior of a planet is prevented from efficient cooling, less energy can escape and the cooling time will tend to decrease. This case has been suggested to apply to Uranus and to explain its faintness (Hubbard, Podolak & Stevenson 1995). Indeed, we see that the shorter the layer height (e.g. 1 km versus 1000 km), the shorter becomes the cooling time, as expected. For $l_\text{H} = 1 \, km$, the effects of additional gravitational energy and of inhibited heat transport balance each other, and the resulting cooling...
data, the balance occurs for \( l_3 \approx 1 \) km. However, the resulting \( R_{\text{crit}}^{-1} \) values assuming \( \beta = 1 \) lie above the allowed range. This is an important result that we consider to be a clue to \( \beta < 1 \). For \( \beta = 0.05-0.5 \) \((\approx 1/R_{\text{crit}}^{-1}), \) the red modified-1 H/He data based model would satisfy all constraints. This implies a reduced composition difference between a vertically moving parcel and its superadiabatic surrounding compared to the difference \( \nabla s_i \) we compute under our assumption (iii).

Conclusion. Given our fundamental assumptions, all models of this section are ruled out because they violate the \( 1 < R_{\text{crit}}^{-1} < R_{\text{crit}}^{-1} \) criterion. If we drop assumption (iii), the modified-1 H/He diagram based model with \( l_3 = 1 \) km can satisfy all constraints and predicts \( \beta = 0.05-0.5 \).

6 RESULTS FOR THE MODIFIED-2 H/HE DEMIXING DIAGRAM

In the previous section, we have seen that none of the models for the modified-1 H/He phase diagram could yield a consistent solution under our fundamental assumptions (Section 1.1). For LDD convection, we obtained too large values \( R_{\text{crit}}^{-1} > R_{\text{crit}}^{-1} \), partly as a result of assuming \( \beta = 1 \), while for ODD convection too small values \( R_{\text{crit}}^{-1} \approx 1 \) as a result of too large superadiabaticities. In this section, we apply the modified-2 H/He phase diagram. It has been designed to lead to an earlier begin of demixing in time and a deeper onset of demixing within the planet while as well reproducing the Galileo probe observational value of \( Y_{\text{atm}} \) at present. In addition to using the modified-2 H/He data we here assume a time variable layer height. Our reasons why we opt for such modifications will be discussed in the following.

6.1 ODD convection

For the assumption of ODD convection in the demixing region we obtained too large superadiabaticities in Section 5 because ODD convection was too inefficient to transport the heat flux. Of course, whether or not ODD convection is efficient enough depends on the amount of heat to be transported. With the modified-1 phase diagram, the outer envelope on top the demixing region yielded a negative contribution to the total intrinsic luminosity because the increase in entropy there (denoted by \( s_1 \) due to the change in He abundance is a larger effect than the decrease in entropy due to cooling, so that \( s_1 \) increased with time (i.e. with decreasing \( T_{\text{bar}} \)).

Thus, the deep interior had to provide a large internal heat flux, even slightly more than the large, observed flux.

However, if the observed total luminosity would instead result from the cooling of the outer envelope while heat release from the deep interior is strongly impeded due to the presence of an ODD or stable region, a solution with ODD convection may exist. To invert the sign of the luminosity contribution from the outer envelope, He rain would have to proceed more slowly in time so that the effect on the entropy from the cooling of the outer envelope wins over the effect of the He depletion. This is one of the reasons why we have introduced the factor \( \sigma \) in the modified-2 H/He phase diagram, so that the He rain begins already at \( T_{\text{bar}} = 200 \) K, rather than at 175 K as predicted by the modified-1 phase diagram (Fig. 6). Yet, although the modified-2 H/He phase diagram leads to the desired sign change for the luminosity contribution from the outer part, as can be concluded from the decrease of \( s_1 \) with decreasing \( T_{\text{bar}} \) in the entropy panel of Fig. 13, we were not able to find a solution with...
the desired shut-down of the deep internal heat flux. At the current stage, we do not know whether a solution for ODD convection exists at all. Anyway, a nearly stably stratified deep interior would impose a challenge to the generation of the observed magnetic field. In the following, we therefore consider LDD convection.

### 6.2 LDD convection

Contrary to the above described ODD case, in the LDD case we want the superadiabaticity to become higher, namely by a factor of a few compared to the results of Section 5.2. As we have seen in Fig. 10, one way to achieve this is to shrink the layer height; but we have also seen that the minimum possible layer height is limited by the condition \( L_{\text{tot}} > 0 \). In fact, \( L_{\text{tot}} < 0 \) happens if the deep internal temperatures get too high at first place, in which case the deep internal entropy (labelled \( s_3(T, \mu) \)) would increase with time (\( \delta q/m > 0 \) in equation 10), resulting in a huge negative luminosity contribution from the mass-rich deep interior that is impossible to be compensated for by the cooling of the outer envelope. What we therefore need in order to enable higher superadiabaticities is a higher upper limit on the allowed \( s_3 \) value, or almost equivalently (\( s \) is an increasing function of \( T \)), on the core temperature. To a good approximation, the upper limit on the core temperature in the presence of H/He demixing is given by the core temperature before demixing begun to operate.\(^3\) Thus, if we let H/He demixing start earlier in the evolution, the core temperature at that time will have been higher. This is another reason why we have stretched \( T_{\text{mix}}(T_{\text{He}}) \) by inserting the factor \( \alpha \) in the modified-2 H/He phase diagram (equation 25). For our illustrative calculations with the modified-2 data, we assume a core mass of 32 \( M_\oplus \), which yields a present-day planet radius in good agreement with Jupiter’s observed one (Fig. 13).

\(^3\)A tiny enhancement of core temperature with time could still be allowed for because \( s_3 \) also is a decreasing function of He abundance, which increases with time. Stevenson & Salpeter (1977) even suggest a strong heating up of the planet for the case of inhibited heat transport through the He rain region.

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**Figure 13.** Results for the evolution of core temperature, planet radius, and envelope entropies in terms of \( T_{1\text{bar}} \), which decreases with time, using the modified-2 H/He demixing data. The solid black curves are for homogeneous, adiabatic evolution; dashed cyan curves for the modified-2 H/He phase diagram and LDD convection; red curves for a toy composition gradient \( c_3 = 0.1 \) and LDD convection, while dot–dashed cyan curves are for the modified-2 H/He data but imposed zero-superadiabaticity.

**Figure 14.** Gradients and fluxes in the demixing of Jupiter computed using the modified-2 H/He phase diagram. Upper panel: mean molecular weight gradient (magenta), adiabatic gradient (violet), superadiabaticity (orange), and \( c_3 = \nabla_\mu(\alpha_{\mu}/\alpha_T) \). Lower panel: heat flux contribution \( F_{\text{LDD}}/F \) (green), heat flux contribution \( F_{\text{cond}}/F \) (dark green). Between 1.4 and 2.3 Mbar, the Jupiter adiabat runs parallel to the H/He modified-2 phase diagram.

Fig. 13 shows the evolution of \( T_{\text{core}} \) and of the envelope entropies \( s_1 \) and \( s_3 \) in terms of \( T_{1\text{bar}} \), which decreases with time. Indeed, the modified-2 H/He phase diagram allows for \( \sim 2000 \)K higher core temperatures, implying higher possible superadiabaticities in the demixing region, than the modified-1 H/He phase diagram did (Fig. 12). Fig. 14 shows \( \nabla_T - \nabla_\mu \approx 0.2 \) for the modified-2 phase diagram case, while the highest value we could obtain for our models using the modified-1 phase diagram (and constant layer height over time) was \( \approx 0.035 \), see Fig. 10.

In order to maintain high superadiabaticities during the evolution, we adjust at each time step (practically, at each \( T_{1\text{bar}} \) step) the layer height to be the smallest possible one within 20 per cent that does not lead to a violation of \( L_{\text{tot}} > 0 \) or of the minimum length-scale criterion. It turns out that the layer heights would decrease with time starting at about 1 km at the beginning of demixing, and ending up to be 100–200 m at present. This result agrees with what we have learned from our toy models in Section 3.7.2.

### 6.2.1 Consistency with \( R_{\text{eq}}^{-1} \)

We next turn to the question whether our modified-2 H/He diagram based models are consistent with the range of allowed \( R_{\text{eq}}^{-1} \) values of LDD convection.

Mirouh et al. (2012) have in detail investigated the range of \( R_{\text{eq}}^{-1}(\mu, r) \) where layer formation occurs. In their simulations, they see it to develop rapidly for \( R_{\text{eq}}^{-1} \) values close to the overturning instability limit (\( R_{\text{eq}}^{-1} = 1 \)); they also see layers to emerge for larger values of \( R_{\text{eq}}^{-1} = 1.5–2 \) but only after a long simulation time, and never observe layer formation for \( R_{\text{eq}}^{-1} > 2 \). Complementary, Mirouh et al. (2012) also investigate the layer formation regime according to the \( \gamma \)-instability theory of Radko (2003). Transferred to a semiconvective system where the temperature gradient acts
6.2.2 Cooling time

In Fig. 16 we present the cooling times for our two modified-2 H/He phase diagram based Jupiter models with LDD convection and adjusted layer heights over time to yield the low \( R_0^{-1} \) values as described above.

It is an important result of this work that our only model that we consider consistent with the \( R_0^{-1} \) range of LDD convection, being based on a (modified) first-principles derived H/He phase diagram, our fundamental assumptions, and a state-of-the-art Jupiter model atmosphere (Fortney et al. 2011) has a cooling time of only 3.8 Gyr.

With a cooling time of only 3.8 Gyr, our thermal evolution model appears to indicate room for additional complexities in our understanding of Jupiter’s structure and evolution. Standard quasi-homogeneous, adiabatic models that reproduce all observational constraints already result in a cooling time in good agreement with Jupiter’s known age (Nettelmann et al. 2012). However, those models with (sharp) gradients in the abundance of helium and heavy elements as constructed by Nettelmann et al. (2012) and, more recently, Becker et al. (2014) ignore the heat transport and temperature gradient in the layer boundary zone(s) altogether and thus are physically inconsistent. In this paper, we have instead tried for more self-consistency, but at the expense of sacrificing the previous apparent understanding of Jupiter’s thermal evolution. We note that the only suite of physically self-consistent Jupiter models are the 2L models by Militzer et al. (2008); however, they do not reproduce the gravity field data, neither do they provide an explanation for Jupiter’s atmospheric He depletion.

\[ T_1^{-1} = \text{ad}, \text{ LDD} \]

\[ R_0^{-1} \]

\[ \gamma \]

\[ \alpha \]

\[ \beta \]

\[ \tau \]

\[ \nu \]

\[ \Delta T = \Delta \text{ad} \]

\[ \text{time (Gyr)} \]

\[ \text{hom., adiab. 2L} \]

\[ \text{H/He modif. 2} \]

\[ c_3 = 0.1 \]

\[ \text{1000 K, 700 K} \]

\[ \text{140 K, 120 K} \]

\[ \text{200 K, 170 K} \]

\[ \text{540 K, 570 K} \]

\[ \text{1000 K, 700 K} \]

\[ \text{140 K, 120 K} \]

\[ \text{200 K, 170 K} \]

\[ \text{540 K, 570 K} \]

\[ \text{1000 K, 700 K} \]

\[ \text{140 K, 120 K} \]

\[ \text{200 K, 170 K} \]

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We like to emphasize the importance of including a theory for the heat transport in the computation of inhomogeneous thermal evolution. Omitting such a theory by assuming an adiabatic temperature profile ($\nabla_T = \nabla_{ad}$) would yield a significant cooling time prolongation, see the cyan dot--dashed curve in Fig. 16. Such a model may be applicable to Saturn, where He droplets may rain down to the core (Fortney & Hubbard 2003; Pästow et al., in preparation). Our explorations show, however, that the application of such a theory (here: semiconvection) in combination with an H/He phase diagram does not necessarily directly lead to a balance between the additional gravitational energy and the energy transported through the inhomogeneous zone, that yields the correct cooling time. Moreover, our explorations rule out the case $\beta = 0$, which would imply an adiabatic, convective, inhomogeneous interior that cools too slowly.

On the other hand, relaxation of the fundamental assumption (iii) is represented by the model with toy composition gradient $c_3 = 0.1$. Because of the lower composition gradient ($\beta < 1$), it requires lower superadiabaticities to meet the $R_{\alpha}^{\gamma}$ constraint (Figs 15 and 14). Accidentally, in this case the desired energy balance is exactly reached, see the red dashed curves in Figs 16 and 13: the deep internal entropy decreases a bit slower in time than in the adiabatic homogeneous case but the outward heat flux is maintained by the higher internal temperatures, so that the cooling time remains unchanged compared to the homogeneous, adiabatic case without He rain. We evaluate the $c_3$ model to be our best-case Jupiter model, as it satisfies all constraints and would even be consistent with a magnetic dynamo in the convective interior below the demixing zone. This model has internal layer heights of 500–1000 m in Jupiter, slowly decreasing in time, corresponding to $\sim 20,000$ layers in Jupiter’s current He rain zone. Comparing $c_1 = 0.1$ and the $c_3$ profile for $\beta = 1$ as shown in Fig. 14, and considering $R_{\alpha}^{\gamma} = 1–3$, we derive $\beta = 0.1/(0.45–0.9) \approx 0.05–0.25$, in agreement with our estimate from Section 5.2.

Fig. 17 shows the internal heat flux profiles of our best-case Jupiter models. Due to the additional energy from the He rain, the internal flux is higher than in the absence of He rain (coloured solid) while the heat flux drops in the semiconvective rain zone. It expands over time.

7 DISCUSSION AND OUTLOOK

7.1 Comparison with the theory of LC12

This work shares similarities with the work of LC12. Both groups derive an expression for the total heat flux $F$ as a function of the superadiabaticity and the layer height which allows one to pick the $\nabla_T$ value, for a given $l_H$ value, that results into the desired flux value. A difference lies in the $Nu_{\alpha}–Ra_{\alpha}$ relations used to calculate $F$. For the $Nu_{\alpha}–Ra_{\alpha}$ relation, we use a fit formula (equations 15 and 16) to the heat flux ‘measurements’ from hydrodynamic simulations, where the heat flow $F_{LDD}$ through a small (2–3) number of alternating layers and interfaces is determined through the imposed average vertical temperature and density gradients. As found in the simulations, the heat flow through the layers is reduced compared to the vigorous convective case ($F_{LDD} < F_{cond}$), while the heat flow through the interfaces is enhanced compared to the purely diffusive case ($F_{LDD} > F_{cond}$) for a given temperature gradient. The latter result is also reflected in our Jupiter model with LDD convection, see the lower panel of Fig. 14.

In contrast, LC12 derive separate analytic expressions for the heat flows in the convective layers and the diffusive interfaces, respec-
Unfortunately, Saturn’s atmospheric helium abundance is not well known, with measurement mass fractions ranging from 1–11 per cent from the Voyager radio occultation and Voyager infrared spectroscopy experiments to 18–25 per cent from their later re-analyses (Conrath et al. 1984; Conrath & Gautier 2000), see also Fig. 6. Assuming \( Y_{\text{atm}} = 0.20 \) for present Saturn, Fortney & Hubbard (2003) could explain the observed excess luminosity by a modified Pfaffenzeller et al. H/He phase diagram, which predicts He rain down to the core. Due to the current uncertainties in Saturn’s \( Y_{\text{atm}} \) value and in the H/He phase diagrams, an inhomogeneous region and LDD convection as a result of He rain can, however, not be excluded for Saturn, in particular as a transitional state prior to He-layer formation. We here emphasize the importance of an accurate measurement of Saturn’s He abundance, most accurately done by sending an entry probe (Fortney et al. 2009; Mousis et al. 2014).

7.3 Other planets: Uranus

Uranus is the canonical example of a planet where inhibited heat transport is suggested to occur, in this case to explain its low luminosity (Hubbard et al. 1995). Stable stratification, whether in the form of semiconvection or diffusion, has not been taken into account explicitly yet in Uranus structure and evolution models. Our results suggest that if stable stratification occurs in Uranus, then maybe not in the form of semiconvection, because the \( R_{\text{0.1}} \) range where semiconvection can operate is already small for Jupiter (1–\( \sim \)10) and may even reduce to 1–2 as a result of the large Prandtl number (\( \text{Pr} > 1 \)) of water. Stable stratification with suppression of heat flux from the deep interior may be more likely realized in Uranus than in Jupiter because of the planets’ difference in total mass by a factor of 20.

7.4 Stars

The difficulties we face in developing a semiconvective zone model for Jupiter is somewhat at odds given the long history in the treatment of semiconvection in stars (e.g. Stevenson 1979; Langer et al. 1985). For instance, in massive stars of \( 15–30 m_\odot \), a semiconvective zone is suggested to form between the He-burning convective core and the overlying H-rich envelope. In this case, a composition gradient arises from the formation of heavier C/O in the central core. Several relevant differences can be stated that impede the adoption of a well-studied scheme to planets: first, stars have nuclear reactions as a dominant internal heat source while giant planets get their luminosity mostly from the slow cooling of the ions so that the internal luminosity must collapse if the planet’s deep interior is prevented from cooling, which may explain part of our difficulties in finding a solution for the ODD case with the modified-2 H/He data; secondly, in stars the radiative gradient is of the order of the adiabatic gradient (Gabriel et al. 2014) and thus the actual temperature gradient is often (Gabriel et al. 2014; Vazan & Helled, personal communication) but not always (Stevenson 1979; Langer et al. 1985; Ding & Li 2014) equalled with either one, whereas in planets \( \nabla_{\text{rad}} \gg \nabla_\odot \) and thus \( \nabla_{\text{rad}} \gg \nabla_\odot \) unless the internal heat flux is suppressed. We here applied a description of DD convection to obtain an estimate on \( \nabla_\odot \). A different theory could be applied as well. Thirdly, semiconvection in massive stars can greatly alter the distribution of elements through enhanced diffusion (Langer et al. 1985) despite \( \tau \ll 1 \) in stars, while here we assume that a composition gradient is permanently maintained through steady demixing, and that diffusion plays no role despite of \( \tau \ll 1 \) only. In particular – and here we come back to the fundamental caveats of this work – the combination of \( \tau < 1 \) and of diffusive transport along the composition gradient together built the essence of the theory of semiconvection. Here, we have assumed that the (downward) transport of He through sedimentation and (upward) transport through diffusion or convection happen linearly superimposed so that the essence of semiconvection does not get undermined. The effect of diffusive transport on the helium distribution in a cooling giant planet remains to be investigated.

7.5 Adiabatic, homogeneous models

Adiabatic, homogeneous evolution models yield good, or perhaps slightly too long cooling ages for Jupiter (Saumon & Guillot 2004; Fortney et al. 2011; Nettelmann et al. 2012). Those previous models ignore the finite gradients in composition and temperature between an He-poor outer and an He-rich inner envelope, and/or between a heavy-element poor to a heavy-element rich deep interior by assuming ad hoc layer boundaries with infinite gradients in composition and entropy (Guillot et al. 1997; Gudkova & Zharkov 1999; Saumon & Guillot 2004; Nettelmann et al. 2008, 2012). Such a treatment of the heavy elements has raised suspicion about its physical justification and led Saumon & Guillot (2004) to assume a homogeneous distribution of heavy elements (which restricts the number of H/He EOS that can cope with the reduced degrees of freedom). Militzer et al. (2008) dropped both the discontinuity in He and in heavy elements by assuming a fully adiabatic, homogeneous envelope (although that model was inconsistent with the observed gravity field). Here, we have taken a first step to move beyond the successful but ad hoc picture of Jupiter by treating finite helium and temperature gradients, albeit in a still vastly simplified and perhaps premature manner.

7.6 Z distribution

We ignored the distribution of heavy elements in Jupiter’s mantle. On the other hand, Leconte & Chabrier (2013) have solely considered the Z distribution (for Saturn), with the result of a cooling time prolongation of several Gyr (for Saturn). This suggests that perhaps for Jupiter we have only tackled part of the problem, and both distributions (He and Z) need to be treated simultaneously. According to the models by LC12 and Leconte & Chabrier (2013), Jupiter’s interior could already be superadiabatic when He rain starts to operate. This would reduce our need to modify the H/He phase diagram in the \( \beta = 1 \) case. It remains to be investigated how large an inhomogeneous region could be while still allowing for the generation of a magnetic field.

7.7 The \( \beta \ll 1 \) case

Our model with an enforced lower composition gradient (\( c_3 = 0.1 \)) is the only one to satisfy all considered constraints. A lower composition gradient (\( \beta < 1 \)) than imposed by the H/He phase diagram alone may have a variety of different origins, like contributions from He rain-out from rising parcels, from the temperature and composition dependence of the metallization of hydrogen including dissociation and ionization, and from a change in heavy-element abundance in the stratified fluid background. It is also possible that He rain requires some supercooling before the He droplets can form and fall. Currently, these contributions are essentially unknown. A lower composition gradient (\( \beta < 1 \)) could also help to stop the run-away effect between gradients in T and in Y (Fortney & Hubbard 2003), so that a solution with a fully convective while still superadiabatic demixing zone may become possible.
7.8 The Earth as a guide

Droplet formation and rain in the Earth’s atmosphere are extensively studied processes. We review basic properties and discuss possible implications for He rain in a giant planet.

It is well known from daily experience that rain does not fall from a clear sky. In fact, the formation of clouds or hazes proceeds that of rain. In our study, we have neglected clouds/hazes. Rainless clouds are accumulations of microscopic (~20 µm) water droplets of insufficient size for rain-out. Those droplets form if the partial pressure of dissolved water in a vertically moving air parcel exceeds the water condensation pressure or, if the air parcel is cold enough, the sublimation pressure.

In the Earth’s atmosphere, the conditions for droplet formation depend on the water phase diagram, on the abundance of water, on the background $P-T$ profile, also called the environmental lapse rate, and on nucleation seeds. The phase diagram of water and the applied one of H/He share similarities such as phase boundaries $T(P)$ that increase with pressure and an enhanced solubility with increasing temperature, suggesting the Earth’s atmosphere to be a reasonable guide for understanding He rain in giant planets. On the other hand, obvious differences exist as well. In the Earth’s atmosphere, the background profile is considered to be given. It can have a number of origins that to first order do not depend on the vertical distribution of water, such as horizontal winds, surface heating, or particulate pollution. Contrary, in the He rainy case the distribution of helium might determine the background temperature profile to first order through the inhibition of large-scale convection as suggested in this work. This is because helium in an H/He planet is a major constituent, while water in the Earth’s $N_2/O_2$ atmosphere is not. Furthermore, the background He abundance is assumed to follow the phase diagram (Stevenson & Salpeter 1977) while the Earth-atmospheric water abundance is generally undersaturated.

On Earth, cloud formation and eventually rain require vertical motions in the atmosphere (a prominent exception being fog). Through vertical motions, upwelling air parcels can expand adiabatically. As long as humidity, i.e. the ratio between the actual vapour pressure and the saturation pressure, is below 100 per cent, the parcels’ $T$-$P$ profile follows a dry adiabat. Conversely, the wet adiabat of air is characterized by 100 per cent humidity so that the parcels’ water abundance follows the phase diagram while the excess water condenses out and is dispensed to the environment, in form of microscopic droplets. Due to latent heat release upon condensation, the wet adiabat is flatter than the dry one.

In this work, we have for two reasons assumed no rain-out from rising parcels ($\beta = 1$) corresponding to a supersaturated wet adiabat. First, this fundamental assumption was inherited from MLT where the concentration of blobs is supposed to remain conserved because of low particle diffusivities. However, the Earth’s atmosphere tells us that loss of particles from a moving parcel can well happen ($\beta < 1$) if the underlying process is non-diffusive in nature. Secondly, the computation of a wet adiabat requires knowledge of the latent heat, while the latent heat from He droplet formation is unknown at present. Concluding, although our assumption of $\beta = 1$ can be deemed inappropriate, both from our modelling results and the known properties of wet water adiabats, the direct computation of the latter one for a demixing H/He mixture is subject to great uncertainties at present.

In the Earth’s atmosphere, larger droplets can sink faster, thereby colliding with smaller ones. Coalescence then leads to droplet growth and eventually to rain. In addition to this basic hydrodynamic-gravitational process, background turbulent fluctuations can have a major impact on the rate and efficiency of collisions and thus on the initiation of precipitation (Tisler, Zapadinsky & Kulmala 2005; Wang et al. 2005). Rainfall on Earth is also strongly affected by aerosols in a number of ways (Ganguly et al. 2012).

Such gross properties may apply also to He rain in an H/He giant planet polluted with heavy elements. However, at present it is unknown how – and if at all – demixing, cloud formation, and rainfall fit into the picture of LDD convection. One might speculate that interfaces occur as a result of He cloud/He haze formation, and that He rainfall occurs like Virga on Earth. It could also be possible that the formation of sufficiently large He rain droplets from He clouds takes a long time so that the assumption of instantaneous sedimentation becomes irrelevant. In that case, convection may persist in the presence of cumulus-like clouds. A convective demixing zone is not excluded if $\beta \ll 1$.

At the very least, this work demonstrates how little we know about the interior of giant planets that are supposed to be ‘simple’, and that an enormous interdisciplinary effort might be needed to gain reliable insight and, as discussed in Section 1, provide the missing ‘fourth leg’ that could support a complete theory. Such an effort should include numerical simulations.

7.9 Numerical simulations and prospects

Numerical simulations have helped to identify the demixing phase diagram of H/He (e.g. Lorenzen et al. 2011), to quantify the flux of heat and solute through semiconvective layers (e.g. Wood et al. 2013), and to predict the formation of water clouds and rainfall (e.g. Ganguly et al. 2012). While each of them is subject to uncertainties on their own (H/He: the function $T_{\text{mix}}(P, x_{\text{He}})$; LDD: layer merging is seen but with unknown convergent state; rain: the role of nucleation seeds), a combined effort of these three branches at least might be necessary to advance our understanding of the processes at work in a Jovian planet. Future numerical simulations should include clouds/hazes and the time-scale for raindrop formation in a mixture where the solute is a major constituent; studies of (DD) convection with a solute that can move by non-diffusive processes, and simulations of the formation and growth of microscopic droplets in a saturated H/He mixture. Simulations should also address the behaviour of heavy elements in the presence of H/He demixing, as it has been done for neon (Wilson & Militzer 2010).

8 SUMMARY

Our results clearly show that the development of a self-consistent Jupiter model is a complex enterprise. We have presented new Jupiter evolution models that we think are an advance over previous work in that they include helium rain as indicated by Jupiter’s He depleted atmosphere, as well as DD convection as a result of the composition gradient in the rain region. Our goal, started with this paper, is to develop thermal evolution and internal structure models of giant planets that include such properties in a self-consistent manner, which we think could drive a new era of planetary modelling, well timed with new data from Juno, for Jupiter, and Cassini, for Saturn. An equivalent approach for understanding composition gradients in low-mass objects such as Ganymede and Mercury is in progress, where the origin of the observed magnetic field is coupled to stable stratification in an iron core as a result of Fe snow (Rückriemen, Breuer & Spohn 2014).
This paper is dedicated to a thorough description of our applied method to iteratively solve for self-consistent profiles of composition and temperature, given an H/He phase diagram and a prescription for the corresponding superadiabaticity. To determine the He profile, we used the Lorenzen et al. (2009, 2011) phase diagram, and modified it in order to obtain an atmospheric He abundance in agreement with the Galileo probe measurement. To determine the temperature profile, we used a description of semiconvection, either in the form of layered or of ODD convection, which was adapted from numerical simulations by Mirouh et al. (2012) and Wood et al. (2013). Furthermore, we applied the SCvH EOS, as it conveniently provides the entropy and partial derivatives.

We presented a wide range of models that we successively ruled out because of lack of self-consistency. Our main results are that (i) adiabatic models with He rain lead to a ~0.7 Gyr too long cooling time; (ii) ODD convective models are difficult to reconcile with Jupiter’s observed high heat flux, (iii) LDD convective models with prohibited particle exchange between convective eddies and the ambient fluid lead to a shortfall of the cooling time by 0.5–1 Gyr relative to the age of the Solar system, (iv) LDD convective models with allowed (but not explicitly modelled) loss of He droplets from upwelling eddies to the ambient fluid and modified it in order to obtain an atmospheric He abundance in agreement with the Galileo probe measurement. To determine the temperature profile, we used a description of semiconvection, either in the form of layered or of ODD convection, which was adapted from numerical simulations by Mirouh et al. (2012) and Wood et al. (2013). Furthermore, we applied the SCvH EOS, as it conveniently provides the entropy and partial derivatives.

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REFERENCES

Becker A., Nettelmann N., Holst B., Redmer R., 2013, Phys. Rev. B, 88, 045122
Becker A., Lorenzen W., Fortney J. N., Nettelmann N., Schöttler M., Redmer R., 2014, ApJS, 215, 21
Chabrier G., Baraffe I., 2007, ApJ, 661, L81
Chabrier G., Saumon D., Hubbard W., Lunine J., 1992, ApJ, 391, 826
Conrath B., Gautier D., 2000, Icarus, 144, 124
Conrath B. J., Gautier D., Hanel R. A., Horst J. S., 1984, ApJ, 282, 807
Ding C. Y., Li Y., 2014, MNRAS, 438, 1137
Fortney J. N., Hubbard W. B., 2003, Icarus, 164, 228
Fortney J. J. et al., 2009, preprint (arXiv:0911.3699)
Fortney J. J., Ikoma M., Nettelmann N., Guillot T., Marley M. S., 2011, ApJ, 729, 32
French M., Becker A., Lorenzen W., Nettelmann N., Bethkenhagen M., Wicht J., Redmer R., 2012, ApJS, 202, A5
Gabriel M., Noels A., Montalban J., Miglio A., 2014, A&A, 569, A63
Ganguly D., Rasch P., Wang H., Yoon J.-H., 2012, J. Geophys. Res., 117, D13209
Gautier D., Conrath B., Flasar M., Hanel R., Kunde V., Chedin A., Scott N., 1981, J. Geophys. Res., 86, 8713
Graboske H. C., Olness R. J., Pollack J. B., Grossman A. S., 1975, ApJ, 199, 265
Gudkova T., Zharkov V. N., 1999, Planet. Space Sci., 47, 1201
Guillot T., Gautier D., Hubbard B. W., 1997, Icarus, 130, 534
Guillot T., Stevenson D. J., Hubbard B. W., Saumon D., 2004, in Bagengal, Dowling T. E., McKinnon W. B., eds., The Interior of Jupiter. Cambridge University Press, Cambridge, p. 35
Hubbard B. W., Podolak M., Stevenson D. J., 1995, in Cruishank D. P., Matthews M. S., Schumann A. M., eds., Neptune and Triton. Univ. Arizona, Tucson, p. 109
Hubbard B. W., Guillot T., Marley M. S., Burrows A., Lunine J. I., Saumon D. S., 1999, Planet. Space Sci., 47, 1175
Klepeis J. E., Schafer K. J., Barbee T., III, Ross M., 1991, Science, 254, 986
Langer N., El Eid M., Fricke K. J., 1985, A&A, 145, 179
Leconte J., Chabrier G., 2012, A&A, 540, A20 (LC12)
Leconte J., Chabrier G., 2013, Nat. Geosci., 6, 347
Lorenzen W., Holst B., Redmer R., 2009, Phys. Rev. Lett., 102, 5701
Lorenzen W., Holst B., Redmer R., 2011, Phys. Rev. B, 84, 235109
Loubeye P., LeTailleur R., Pineaux J.-P., 1985, Phys. Rev. B, 32, 7611
Militzer B., Hubbard W. B., Vorberger J., Tamblyn I., Bonev S. B., 2008, ApJ, 688, L54
Miroiu G. M., Garaud P., Stellmach S., Traxler A. L., Wood T. S., 2012, ApJ, 750, 61
Morales M. A., Schweger E., Ceperley D., Pierleoni C., Hamel S., Caspersen K., 2009, Proc. Natl. Acad. Sci., 106, 1324
Morales M. A., Hamel S., Caspersen K., Schweger E., 2013a, Phys. Rev. B, 87, 174105
Morales M., McMahon J. M., Pierleoni C., 2013b, Phys. Rev. Rev. Lett., 110, 065702
Mousis O. et al., 2014, Planet. Space Sci., 104, 29
Nettelmann N., Holst B., Kietzmann A., French M., Redmer R., Blaschke D., 2008, ApJ, 683, 1217
Nettelmann N., Becker A., Holst B., Redmer R., 2012, ApJ, 750, A52
Niemann H. et al., 1998, J. Geophys. Res., 103, 22831
Orton G., Ingersoll A. P., 1976, in IAU Colloq. 30: Jupiter: Studies of the Interior, Atmosphere, Magnetosphere and Satellites. Univ. Arizona Press, p. 706
Paffenheimer O., Hohl D., Ballone P., 1995, Phys. Rev. B, 45, 97
Radko M., 2003, J. Fluid Mech., 497, 365
Rosenblum E., Garaud P., Traxler A., Stellmach S., 2011, ApJ, 731, 61
Rücksriemen T., Breuer H., Spohn T., 2014, Lunar Planet. Sci. Conf. Vol. 45, Key Characteristics of the Fe-Snow Regime in Ganymede’s Core. p. 2454
Salpeter E. E., 1973, ApJ, 181, L83
Saumon D., Guillot T., 2004, ApJ, 609, 1170
Saumon D., Hubbard W. B., Chabrier G., van Horn H. M., 1992, ApJ, 391, 827
Saumon D., Chabrier G., van Horn H. M., 1995, ApJ, 99, 713
Soderlund K. M., Heimpel M. H., King E. M., Aurnou J. M., 2013, Icarus, 224, 97
Soubliran F., Mazevet M., Winisdoerffer C., Chabrier G., 2013, Phys. Rev. B, 87, 165114
Stevenson D. J., 1975, Phys Rev B, 12, 3999
Stevenson D. J., 1979, MNRAS, 187, 129

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