Description of paramagnetic–spin glass transition in Edwards-Anderson model in terms of critical dynamics

M. G. Vasin

March 23, 2022

Abstract

Possibility of description of the glass transition in terms of critical dynamics considering a hierarchy of the intermodal relaxation time is shown. The generalized Vogel-Fulcher law for the system relaxation time is derived in terms of this approach. It is shown that the system satisfies the fluctuating–dissipative theorem in case of the absence of the intermodal relaxation time hierarchy.

1 INTRODUCTION

Despite of significant success in description of the dynamics of glass-transition, which they managed to attain by now, in particular in the framework of the mode coupling theory, the idea of description of glass-transition within the scaling theory of phase transitions stays as very attractive one. At first sight from the point of view of technical approaches such task does not present any complexity [1]. However, attempts of description of the phase transitions in disordered systems within the framework of the scaling approach faced serious difficulties connected with the necessity of a deviation from the framework of usual representations of the fluctuation theory of phase transitions [2].

It is possible that part of these difficulties is caused by using of the static fluctuation theory of phase transitions and by the complexity of averaging over configurations of the exchange integrals. In this work an attempt to consider this problem within dynamic approach, which is an alternative to the replica method and in which such difficulties are absent, is made. It

*Corresponding author
is assumed, what the ultrametric valleys space has a hierarchical structure and is self-similar. Therefore it seems to be possible that we can formulate a dynamic scaling theory, in which, on the one hand, the fundamentals would not be impaired, and, on the other hand, the system ultrametricity would be taken into account.

2 THEORETICAL MODEL

We will consider the Edwards-Anderson model close to the paramagnetic–spin glass transition temperature. Let us assume that the states space of the frustrated system is decomposed into a set of the valleys divided between each other by energy barriers with a complex relief, and describe the states of every valley with functions $\varphi_a(t, \bar{r})$, where $a$ is the valley number. We will emanate from the standard formulation of the stochastic dynamics problem, and let us suppose that the states, corresponding to various valleys, interact with each other. Then, according to the mode coupling theory, the generalization of this problem to the case of the fields $\{\varphi_a(t, \bar{r})\}$ system has the following form:

$$\partial_t \varphi_a(t, \bar{r}) = -\sum_b \alpha_{ab} \frac{\delta F\{\varphi\}}{\delta \varphi_b(t, \bar{r})} + \xi_a(t, \bar{r}), \quad \langle \xi_a(t, \bar{r})\xi_b(t', \bar{r}') \rangle = 2\alpha_{ab} \delta(\bar{r} - \bar{r}') \delta(t - t').$$

Here the indexes $a$ and $b$ mark the valleys (modes), $\alpha_{ab}$ are the inter-mode (inter-valley) coupling coefficients,

$$F\{\varphi_a(t)\} = \int d^d r \left[ \frac{1}{2} (\nabla \varphi_a(t, \bar{r}))^2 + \frac{1}{2} (\beta + \rho J_a(\bar{r})) \varphi_a^2(t, \bar{r}) + \frac{1}{4} \varphi_a^4(t, \bar{r}) \right],$$

where $J_a(\bar{r})$ are random fields ($\langle J \rangle = 0$). For the convenience let us turn from the operator $\alpha_{ab}$ to the inverse operator $\bar{\alpha}_{ab}$, then the problem has the form:

$$\sum_b \bar{\alpha}_{ab} \partial_t \varphi_b(t, \bar{r}) = -\frac{\delta F\{\varphi\}}{\delta \varphi_a(t, \bar{r})} + \zeta_a(t, \bar{r}), \quad \langle \zeta_a(t, \bar{r})\zeta_b(t', \bar{r}') \rangle = 2\bar{\alpha}_{ab} \delta(\bar{r} - \bar{r}') \delta(t - t').$$

Let us make use of the method of reduction of the stochastic problem to the quantum-field model and write the full dynamic generating functional of the function $\varphi_a(t, \bar{r})$ in the form:

$$\Phi\{\varphi\} = \prod_{t, \bar{r}, a} \delta \left( \sum_b \bar{\alpha}_{ab} \partial_t \varphi_b(t, \bar{r}) + \frac{\delta F\{\varphi\}}{\delta \varphi_a(t, \bar{r})} - \zeta_a(t, \bar{r}) \right) \times \left| \det \left( \bar{\alpha}_{ab} \delta(\bar{r} - \bar{r}') \partial_t + \frac{\delta^2 F\{\varphi\}}{\delta \varphi_a(t, \bar{r}) \delta \varphi_b(t', \bar{r}')} \right) \right|. $$

Using the Grassman variable algebra and the diagram technique it can show that in case of a purely dissipative problem the stochastic equation has only one solution, therefore one can
eliminate the absolute value of the determinant. This functional is the distribution function analog and cuts out from the continual integral over fields \( \varphi_a(t, \bar{r}) \) only the configurations which satisfies the original stochastic problem. Integrating over the random field \( \zeta \) one gets the statistical sum in the following form:

\[
Z = \left\langle \int \left( \prod_a D\varphi_a D\varphi'_a \right) \exp \left[ S\{\varphi, \varphi', J\} \right] \right\rangle_J,
\]

\[
S\{\varphi, \varphi', J\} = \sum_{ab} \int dt d^4r \left[ \bar{\alpha}_{ab} \varphi'_a(t, \bar{r})\varphi'_b(t, \bar{r}) - \bar{\alpha}_{ab} \varphi'_a(t, \bar{r}) \partial_t \varphi_b(t, \bar{r}) + \varphi'_a(t, \bar{r}) \nabla^2 \varphi_a(t, \bar{r}) - \beta \varphi'_a(t, \bar{r}) \varphi_a(t, \bar{r}) - \rho J_a(\bar{r}) \varphi'_a(t, \bar{r}) \varphi_a(t, \bar{r}) - v \varphi'_a(t, \bar{r}) \varphi_a(t, \bar{r}) \right].
\]

Ultrapometricity of the valleys space (and the time scale accordingly) is an important stage of our model definition. We do it by means of the random \( \text{impurity} \) field \( J \): Let us assume that the valley states are independent of one another, therefore the averaging over fields \( J \) is carried out for every valley independently — the correlation of \( J_a \) and \( J_b \) is absent. However, upon expiration of \( \tau_{ab} \) time, which is necessary for the valleys \( a \) and \( b \) to come to equilibrium, their states cease to be independent and the averaging over \( J \) becomes to be common. Thus, let us assume that a correlator \( \langle J_a(t, \bar{r}) J_b(t', \bar{r}') \rangle = 0 \), if the time gap between the replica states is smaller than some characteristic time \( \tau_{ab} \), which is necessary for the valleys \( a \) and \( b \) to come to equilibrium \( |t - t'| < \tau_{ab} \), and, otherwise, \( \langle J_a(t, \bar{r}) J_b(t', \bar{r}') \rangle = \delta(\bar{r} - \bar{r}') \) for \( |t - t'| > \tau_{ab} \) (Fig.1). For that we represent the correlator of the random bonds as follows:

![Figure 1: Graphic expression of \( \langle J_a(t) J_b(t') \rangle \) correlator.](image)

\[
\langle J(0) J(0) \rangle
\]

\[
\langle J_a(t, \bar{r}) J_b(t', \bar{r}') \rangle = \theta(|t - t'| - \tau_{ab}) \delta(\bar{r} - \bar{r}').
\]

The relaxation time \( \tau_{ab} \) depends on the distance apart the valleys \( a \) and \( b \) in the ultrametric space \( x_{ab} \): \( \tau_{ab} = \tau_0 e^{x_{ab}} \).
3 RENORMALIZATION AND ANALYSIS

Correlation functions of the fields, available in the model, have the following form, if $\bar{\alpha}_{aa} = \alpha_{aa} = 1$: 

$$
\langle \phi'_a \phi_b \rangle = \frac{\delta_{ab}}{i\omega + \varepsilon_k}, \quad \langle \phi_a \phi_b \rangle = \frac{2\bar{\alpha}_{ab}}{\omega^2 + \varepsilon_k^2}, \quad \langle J_a J_b \rangle = \frac{e^{-i\omega \tau_{ab}}}{i\omega},
$$

where $\varepsilon_k = k^2 + \beta$, and it was assumed that the inoculating operator, $\bar{\alpha}_{ab}$, had the form $\bar{\alpha}_{ab} \simeq \delta_{ab}$. In this case the contribution of $\phi'_a(t, \vec{r})\partial_t \phi_b(t, \vec{r})$ ($a \neq b$) is unrenormalizable. Let us represent these correlation functions in form of the graphs a, b, and c accordingly (Fig.2). It can be checked that the formulated model is multiplicatively renormalizable, and the renormalized effective action has the following form:

$$
S^{(R)}\{\varphi, \varphi', J\} = \sum_{ab} \int d\omega d^dk \left[ Z_1 \varphi'_a \varphi'_b + Z_2 \omega \varphi'_a \varphi_b + Z_3 k^2 \varphi'_a \varphi_a - Z_4 \varphi'_a \varphi_a - Z_5 J_a \varphi'_a \varphi_a - Z_6 \varphi'_a \varphi_a^3 - Z_7 i\omega e^{i\omega \tau_{ab}} J_a J_b \right],
$$

where $Z_1 = Z_{\bar{\alpha}_{ab}}Z^2_{\varphi'}$, $Z_2 = Z_{\bar{\alpha}_{ab}}Z_{\varphi'}Z_{\varphi}$, $Z_3 Z_{\varphi} Z_{\varphi'}$, $Z_4 = Z_{\beta} Z_{\varphi'} Z_{\varphi}$, $Z_5 = Z_{\varphi} Z_{\bar{\alpha}_{ab}} Z_{\varphi}$, $Z_6 = Z_{\varphi} Z_{\varphi'} Z^3_{\varphi}$, $Z_7 = Z_{\chi} Z^2_{\varphi'}$ are corresponding renormalization constants. The canonical dimensions of the fields and parameters of this model are given in the table:

| $F$  | $k, \nabla$ | $\omega, \partial_t$ | $\bar{\alpha}_{ab}$ | $\varphi$  | $\varphi'$ | $J$  | $\beta$ | $\varphi$ | $\varphi'$ | $J$  |
|------|-------------|-----------------------|-------------------|-------------|-------------|-----|---------|-------------|-------------|-----|
| $d^k_F$ | 1 | 0 | 2 | $- (1 + d/2)$ | $- (1 + d/2)$ | $- d/2$ | 2 | $2 - d/2$ | 4 | $- d$ |
| $d^p_F$ | 0 | 1 | -1 | -1 | 0 | -1 | 0 | 0 | 0 | 0 |
| $d_F$ | 1 | $z=2$ | 0 | $- (3 + d/2)$ | $- (1 + d/2)$ | $- (2 + d/2)$ | 2 | $2 - d/2$ | 4 | $- d$ |

Figure 2: Graph representation of the correlators.

Figure 3: Graph which contributes to the renormalization of the term $\bar{\alpha}_{ab}$ (one-loop approximation).
The renormalization procedure is of fundamental importance. The renormalization of the parameter $\bar{\alpha}_{ab}$ is a specially important one, which looks as follows (Fig. 3):

$$Z_{\bar{\alpha}_{ab}} = \bar{\alpha}_{ab} - \frac{g^2}{2} \int_{\lambda k_0 < |k| < k_0} \left( \frac{d^dk}{(2\pi)^d} \right) \frac{\alpha_{ab} \omega^2 + \bar{\alpha}_{ab} \varepsilon_k^2}{\left( \omega^2 + \varepsilon_k^2 \right)^2} \frac{e^{-i\omega \tau_{ab}}}{i\omega}. \tag{1}$$

Here and below we amount to nothing more than the one-loop approximation. The integral over $\omega$ is divided into two parts: the first part is a circulation integral around the pole $\omega = \varepsilon_k$, the second part — around the pole $\omega = 0$. As usual [3], in the first case the integration is carried out over contour bounding of the complex space upper half-plane, whereas in the second integral, as well as when integrating over $k$ we have to introduce the regularizing parameter $\lambda^z$ ($z = 2$ is the dynamics exponent). Thus, after the integration over $\omega$ the expression (1) has the form:

$$Z_{\bar{\alpha}_{ab}} \simeq \bar{\alpha}_{ab} - 2\pi \int_{\lambda k_0 < |k| < k_0} \left( \frac{d^dk}{(2\pi)^d} \right) \left[ \frac{g^2}{2} \left( \alpha_{ab} - \bar{\alpha}_{ab} \right) e^{\varepsilon_k \tau_{ab}} - \frac{g^2}{4} \left( \alpha_{ab} - \bar{\alpha}_{ab} \right) \tau_{ab} e^{\varepsilon_k \tau_{ab}} \right] \cdot$$

Here the last term is the result of expansion in series of the exponential curve, besides the nondimensional time parameter $\bar{\tau}_{ab} = \gamma e^{x_{ab}}$ ($\gamma = \omega_0 T_0$) is introduced. If one keeps in this formula only substantial logarithmically divergent terms, it has the following form:

$$Z_{\bar{\alpha}_{ab}} = \bar{\alpha}_{ab} + \bar{\alpha}_{ab} g^2 \left( \frac{\bar{\tau}_{ab} (1 - \lambda^z)}{\pi} + \frac{1}{2} \right) \frac{1}{2\pi} \ln \left( \frac{1}{\lambda} \right). \tag{2}$$

Time ultrametricity at the integration over $\omega$ leads to the nontrivial dependence of the $\bar{\alpha}_{ab}$ renormalization on the $\lambda$. Therefore, this very parameter has the most apparent features that are caused by the presence of the hierarchy of system relaxation times and have an effect on the system dynamics.

The $\beta$-renormalisation looks in the following form (Fig. 4):

$$Z_{\beta \delta_{ab}} = \beta \delta_{ab} - \Sigma_{1ab} - \Sigma_{2ab},$$

where $\Sigma_{1ab}$, $\Sigma_{2ab}$ are contributions to $\beta$-renormalisation of (a) and (b) graphs, accordingly. The contribution of the first graph has the form:

$$\Sigma_{1ab} = \beta g^2 \frac{2}{2!} \sum_{c} \int_{\lambda^z \omega_0 < |\omega| < \omega_0} \left( \frac{d^dk}{(2\pi)^d} \right) \frac{\alpha_{ac} \alpha_{cb} \omega^2 + \bar{\alpha}_{ac} \bar{\alpha}_{cb} \varepsilon_k^2}{\left( \omega^2 + \varepsilon_k^2 \right)^2} \cdot$$

Since $\bar{\alpha}_{ab}$ ($a \neq b$) parameter is divergent, the parameter $\alpha_{ab}$ will be small, respectively; therefore, only second and third terms remain in the integral. After the integration over $\omega$, assuming that
Figure 4: Graphs which contribute to the renormalization of the term $\beta$ (one-loop approximation).

$\tau_{aa} = 0$, we get the following:

$$\Sigma_{1ab} = -\delta_{ab} \frac{\beta g^2}{2} (1 - \alpha_{ab}) \int_{\lambda k_0 < |k| < k_0} \frac{d^d k}{(2\pi)^d - 1 \varepsilon_k^2} = -\beta g^2 \delta_{ab} (1 - \alpha_{ab}) \frac{1}{4\pi} \ln \left( \frac{1}{\lambda} \right) = 0.$$

I.e. in case of $\alpha_{aa} = 1$, this graph does not contribute to renormalisation. The second cunterterm has the form:

$$\Sigma_{2ab} = \frac{\beta \nu \delta_{ab} (9\bar{\alpha}_{ab} + 3\alpha_{ab})}{8\pi} \ln \left( \frac{1}{\lambda} \right).$$

Thus, when $\alpha_{aa} = \bar{\alpha}_{aa} = 1$, we have

$$Z_{\beta} = \beta - \beta \frac{3\nu}{2\pi} \ln \left( \frac{1}{\lambda} \right).$$

The graphs renormalizing the point $\varphi' \varphi J_a$ have the form represented in Fig.5. Using the above results it is simply to show that renormalisation of this point has the following form:

$$Z_{\varphi} = \varphi - \varphi \frac{3\nu}{2\pi} \ln \left( \frac{1}{\lambda} \right).$$

In one-loop approximation the graphs renormalizing the point $\nu \varphi' \varphi'^3$ have the form represented in Fig.6. However, as it was shown above, the first graph (a) makes a zero-order
Thus, we get:

The third graph (c) contribution is which has the following form after being integrated:

\[
\mathcal{Y}_2 = -v g^2 3\pi \int_{\lambda k_0 < |k| < \lambda k_0} \frac{d^d k}{(2\pi)^d} \frac{1}{\varepsilon_k^2} = -v g^2 \frac{3}{4\pi} \ln \left(\frac{1}{\lambda}\right)
\]

The third graph (c) contribution is

\[
\mathcal{Y}_3 = 36v^2 \delta_{ab} \sum_c \int_{\lambda k_0 < |\omega| < \lambda k_0} d\omega \int_{\lambda k_0 < |k| < \lambda k_0} \frac{d^d k}{(2\pi)^d} \frac{\alpha_{ac} \alpha_{cb} \omega^2 + \delta_{ac} \delta_{cb} \varepsilon_k^2}{(\omega^2 + \varepsilon_k^2)^2} \cdot e^{-i\omega \tau_{ab}} \frac{1}{i\omega},
\]

which has the following form after being integrated:

\[
\mathcal{Y}_3 = -v g^2 3\pi \int_{\lambda k_0 < |k| < \lambda k_0} \frac{d^d k}{(2\pi)^d} \frac{1}{\varepsilon_k^2} = -v g^2 \frac{3}{4\pi} \ln \left(\frac{1}{\lambda}\right).
\]

Thus, we get:

\[
Z_v = Z_v' = \mathcal{Y}_1 - \mathcal{Y}_2 - \mathcal{Y}_3 = v + v g^2 \frac{3}{8\pi^2} \ln \left(\frac{1}{\lambda}\right) - v^2 \frac{9}{2\pi} \ln \left(\frac{1}{\lambda}\right).
\]

Assuming that \(Z_\varphi = \theta(\lambda), Z_{\varphi'} = \theta'(\lambda), Z_\tau = \vartheta(\lambda)\), and carrying out the scaling transformation we get the following renormalization group:

\[
\alpha_{ab}^{(R)} = Z_1 \lambda^{d+z} = \theta^2(\lambda) \lambda^{d+z} \left[ \tilde{\alpha}_{ab} + \bar{\alpha}_{ab} \theta^2 \left( \frac{1 - \lambda^2}{\pi} + \frac{1}{2} \right) \frac{1}{2\pi} \ln \left(\frac{1}{\lambda}\right) \right],
\]

\[
\tilde{\alpha}_{ab}^{(R)} = Z_2 \lambda^{d+2z} = \theta'(\lambda) \theta(\lambda) \lambda^{d+2z} \left[ \tilde{\alpha}_{ab} - O(v^2) \right],
\]

\[
\beta^{(R)} = Z_4 \lambda^{d+z} = \theta'(\lambda) \theta(\lambda) \lambda^{d+z} \left[ \beta - \beta \frac{3v}{2\pi} \ln \left(\frac{1}{\lambda}\right) \right],
\]

\[
\vartheta^{(R)} = Z_5 \lambda^{d+2z} = \theta'(\lambda) \theta(\lambda) \vartheta(\lambda) \lambda^{d+2z} \left[ \vartheta - \vartheta \frac{3v}{2\pi} \ln \left(\frac{1}{\lambda}\right) \right],
\]

\[
\nu^{(R)} = Z_6 \lambda^{d+3z} = \theta'(\lambda) \vartheta^2(\lambda) \lambda^{3d+3z} \left[ v + v g^2 \frac{3}{4\pi} \ln \left(\frac{1}{\lambda}\right) - v^2 \frac{9}{2\pi} \ln \left(\frac{1}{\lambda}\right) \right],
\]

\[
\chi^{(R)} = Z_7 \lambda^{d+2z} = \vartheta^2(\lambda) \lambda^{d+2z} \lambda.
\]
Taking into account canonical dimensions of the fields and introducing new denotations: \( \xi = \ln(1/\lambda), \ \varepsilon = 4 - d \), let us rewrite this system in the following form:

\[
\tilde{\alpha}_{ab}^{(R)} = \tilde{\alpha}_{ab} + \tilde{\alpha}_{ab} \xi^2 \left( \tilde{\tau}_{ab}(1 - e^{-z\xi}) + \frac{1}{2} \right) \frac{1}{2\pi \xi},
\]

\[
\beta^{(R)} = e^{2\xi} \left[ \beta - \beta \frac{3\nu}{2\pi \xi} \right],
\]

\[
\varrho^{(R)} = e^{\varepsilon \xi/2} \left[ \varrho - \varrho \frac{3\nu}{2\pi \xi} \right],
\]

\[
\upsilon^{(R)} = e^{\varepsilon \xi} \left[ \upsilon + \upsilon \varrho^2 \frac{3}{4\pi \xi} - \upsilon^2 \frac{9}{2\pi \xi} \right].
\]

Let us expand exponents in series and keep only linear components in respect to \( \xi \)-terms. Supposing the continuity of the renormalization group transformation let us describe the evolution of effective action parameters in the form of differential equations:

\[
\frac{d \ln(\tilde{\alpha}_{ab})}{d\xi} = \frac{\varrho^2}{2\pi \xi} \left( \tilde{\tau}_{ab}(1 - e^{-z\xi}) + \frac{1}{2} \right),
\]

\[
\frac{d \ln(\beta)}{d\xi} = 2 - \frac{3\nu}{2\pi \xi},
\]

\[
\frac{d \ln(\varrho)}{d\xi} = \frac{\varepsilon}{2} - \frac{3\nu}{2\pi \xi},
\]

\[
\frac{d \ln(\upsilon)}{d\xi} = \varepsilon + \varrho^2 \frac{3}{4\pi \xi} - \upsilon^2 \frac{9}{2\pi \xi}.
\]

The fixed point condition is assigned by a set of equations: \( \frac{d \ln(\varrho)}{d\xi} = 0, \frac{d \ln(\upsilon)}{d\xi} = 0 \), from which we get: \( \upsilon^* = \frac{\pi \varepsilon}{3}, \varrho^* = \sqrt{\frac{2\pi \varepsilon}{3}} \). Then

\[
\beta^{1/2} \approx \left( \frac{T - T_c}{\kappa T_c} \right)^{1/2} = e^\xi,
\]

\[
\tilde{\alpha}_{ab} \approx \exp \left( \frac{\varrho^2}{2\pi} \left( \tilde{\tau}_{ab}(1 + \frac{1}{2} \xi) \right) \right) \cdot \exp \left( \frac{\varrho^2 \tilde{\tau}_{ab}}{2\pi^2 z} e^{-z\xi} \right) =
\]

\[
= \left( \frac{T - T_c}{\kappa T_c} \right)^{\frac{\varepsilon}{3}} \left( \frac{\tau_{ab}}{\pi} + \frac{1}{2} \right) \cdot \exp \left( \frac{\varepsilon \tilde{\tau}_{ab}}{6\pi} \left( \frac{\kappa T_c}{T - T_c} \right) \right)
\]

Within continuous limit \((\tilde{\alpha}_{ab} \to \tilde{\alpha}(x))\) we can represent a temporary system evolution as the following functional relation:

\[
S(t) = \int dx \Delta'(x) \exp \left( -\frac{t}{\tilde{\alpha}(x)} \right),
\]

where \( \Delta'(x) \sim j^{-x} \) is the distribution density of the valley pairs with respect to distances, \( x \), between such valleys in the ultrametric space. Hence, we can get the following formula for the
observable time of relaxation of a three-dimensional system:

\[ t_{\text{rel}} \sim \left( \frac{T - T_c}{\kappa T_c} \right)^{1/3} \left( \frac{q}{\pi} + \frac{1}{2} \right) \cdot \exp \left( \frac{q \kappa}{6 \pi} \frac{T_c}{(T - T_c)} \right), \]

where \( q = \gamma e^{\sigma / \ln j} \) is a value which is determined by ultrametric space parameters. Thus, on condition that \((T - T_c) \ll 1\), we come to the Vogel-Fulcher-Tammann law.

We would like to note that in a number of works the idea that critical phenomena in disordered systems can be described by assumption of existence of the infinite continuous hierarchy of the correlation length and critical indexes was proposed (see, for example, [2, 4]). In our case the renormalization (2) can be interpreted as the presence of such hierarchy.

Finally, let us write down the dynamic sensitivity of the system:

\[ G(\omega) = \sum_{ab} \frac{\tilde{\alpha}_{ab}^{-1}}{i \omega + \tilde{\alpha}_{ab}^{-1} \varepsilon_k}. \]

In continuous limit this formula can be written in terms of “intervalley transitions” \( j \leftrightarrow i \):

\[ G(\omega) = \int_0^\infty dx \frac{\Gamma(x)}{(x + \Gamma(x)) \varepsilon_k}, \]

where \( \Gamma(x_{ab}) = \tilde{\alpha}_{ab}^{-1} \) is a value which is inverse to transition time between the \( a \) and \( b \) valleys. Within \( \omega \to 0 \) limit it can be written down in a more traditional form [5]:

\[ D(\omega) \simeq \int_0^\infty dx \frac{q'(x)}{(x + q'(x)) \varepsilon_k^2}, \]

where \( q'(x) = \Delta'(x) / \Gamma(x) \). As is known [5], to work the fluctuating–dissipative theorem it is necessary that \( q'(x) = \Delta'(z) \). Thus, this theorem is valid when \( \Gamma(x) = 1 \) at all \( x \)-values, i.e. in case of any whatsoever transition times hierarchy is absent in the system.

4 CONCLUSIONS

The outcomes of this work has pointed out to a critical opportunity of application of the standard methods of critical dynamics in description of glass transition. However, the given approach has some weak-points. First of all, it relates to artificial introduction of the hierarchy of relaxation times, therefore two scales of relaxation time do exist in this model: \( \tilde{\alpha} \) corresponding to relaxation in the \( \varphi \)-fields subsystem; and \( \tau \) corresponding to the \( J \)-fields subsystem, where the second one is entered to the model artificially. In fact these time scales should be interconnected with
each other, since relaxation of the $\varphi$-fields determines relaxation of the $J$-fields. Apparently, a form this interconnection directly depends on a specific concrete physical problem, but in our model this is not taken into account. As a consequence, $q$ does not depend on the temperature, that, apparently, is not exactly correct. Therefore, it is possible to believe, that the obtained Vogel-Fulcher-Tammann formula is true in case of weak temperature dependence of $q$.

5 ACKNOWLEDGMENTS

This study was supported by the RFBR grant (04-03-96020-r2004ural).

References

[1] A.N. Vasil'ev, Quantum-Field Renormalization Group in the Theory of Critical Phenomena and Stochastic Dynamics, CRC Press, Boca Raton, London, New York, Washington: 2004, 704 pp., ISBN: 0415310024;

[2] Vik.S. Dotsenko, “Critical Phenomena and Quenched Disorder,” Uspehki Fizicheskikh Nauk 165, 5, 287–296 (1995);

[3] P.C. Hohenberg and B.I. Halperin, “Theory of dynamic critical phenomena,” Rev. Mod. Phys. 49, 435–479 (1977);

[4] A.W.W. Ludwig, “Infinite hierarchies of exponents in a diluted ferromagnet and their interpretation,” Nucl. Phys. B 330, 639–680 (1990);

[5] S.L. Ginzburg, Irreversible Phenomena in Spin Glasses, (in Russian), Nauka, Moscow: 1989, 152 pp., ISBN 5020141569;