Dynamics modelling and analysis of a large-load swinging platform

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Abstract. Swing platform is widely used to simulate the motion attitude of vehicles, ships and aircraft while carrying large loads. Aiming at the excessive driving force and output power of the driving parts caused by the large load swing platform, a new swing platform which can bear the large load was established. The swing platform is equipped with four spring branch chains between the moving platform and the static platform of the 6-UPS parallel mechanism, so as to offset the gravity of the large load and the inertia force of the large load in the process of motion, and the driving force of each branch chain is reduced. In this paper, the structure of the swing platform is introduced, and the dynamics of the swing platform is modeled using the Newton-Euler dynamics equation. Finally, the driving force of each branch chain of the swing platform is obtained by simulation of the dynamics of the swing platform. The simulation results show that the swing platform with four spring branch chains can effectively reduce the driving force of each branch chain compared with the traditional 6-UPS parallel mechanism swing platform.

1. Introduction

Swing platform is a simulation device that can realize specific motion. It can provide a nearly actual vibration environment for experimental subjects. Owing to the reasons above, it is widely used to simulate the motion of vehicles, ships and aircraft, which has high application value in both national defense and civil application [1]. Compared with serial mechanism, the swing platform in the form of parallel mechanism has the advantages of strong position and attitude adjustment ability, clear structural form, high stiffness and strong bearing capacity. Therefore, the parallel mechanism is often used as the main mechanical structure of the swing platform, when simulating the conditions with small amplitude and large load. [2].

In the research of swing platform, the establishment of dynamic model is the basis of dynamic analysis and motion control. Common methods for establishing dynamic equations include Newton-Euler method, Lagrange method, Kane equation, virtual work principle, etc. [3]. Compared with other methods, the Newton-Euler method can find the force (moment) on each joint, and has a fast calculation speed, which is conducive to the realization of real-time control of the mechanism. In reference [4], the reverse dynamics model of Stewart platform mechanism was established by Newton-Euler method and virtual work principle. In references [5,6], the Newton-Euler equation was applied to model the inverse dynamics of the Stewart platform and the closed-form dynamic equations, taking into account the inertial forces of the components, gravity, and the frictional forces of the hinges. The reference [7] studied the general process of dynamical problems of parallel mechanisms based on the Newton-Euler equation. In reference [8], the dynamics of a 6-PUS parallel mechanism was modeled using the Newton-
Euler method. A new 3 DOF spatial parallel mechanism was investigated in reference [9], and a dynamics model was developed using the Newton-Euler method.

For large load swing platforms, the excessive loads and the inertial forces generated during swinging require large driving forces from each branch chain. In this paper, in order to reduce the driving force of each branch chain during the swinging process and reduce the output power of the motor, a new swing platform is formed by adding spring branch chains to the 6-UPS parallel mechanism. Using it as the research object, the dynamics of the swing platform is modeled by using Newton-Euler method, and the effectiveness of adding spring branch chains to reduce the driving force of each branch chain is verified by MATLAB simulation.

2. Modelling

2.1. Description

The swing platform studied in this paper is used in a large load motion simulator, where the load and its own dimensions are large, so the mass of structures such as drive branch chains, spherical hinges, and Hooke hinges has relatively little effect. In the process of building the dynamics model, only the mass of the moving platform and the load is considered, and the rest of the smaller components are ignored; at the same time, without considering the elastic deformation of the components, the resulting model is a singly rigid body dynamics model. In addition, the hinges are considered to be ideal hinges. The swing platform is a 6-UPS parallel mechanism with spring branch chains, as shown in Figure 1. 6-UPS parallel mechanism is composed of moving platform, static platform and 6 UPS branch chains. U, P and S means Hooke hinge, sliding pair and spherical hinge, respectively. The spring branch chain consists of two Hook hinges at each end of the branch chain and springs in the between. Six of the sliding pairs are driving parts, which can be realized by using electric cylinders.

As shown in Figure 1, $A_1, A_2, A_3, A_4, A_5, A_6$ denote the center points of the spherical hinge of the parallel mechanism; $B_1, B_2, B_3, B_4, B_5, B_6$ denote the intersection of two mutually perpendicular axes in the Hooke hinge of the parallel mechanism. The hinge points $A_i$ and $B_i$ are distributed on the vertices of the hexagon and their radius of circumcircle are $r$ and $R$, respectively. On the moving platform, the major side of the hexagon has a center angle of $\alpha$ and the minor side has a center angle of $\alpha'$; On the static platform, the major side of the hexagon has a center angle of $\beta$ and the minor side has a center angle of $\beta'$.

![Figure 1. Structure and coordinate system of swinging platform.](image1)

On the static platform, with the center of the hexagon as the origin, the positive direction of the $X$-axis is that $O_B$ pointing in the direction of $B_1$ and $B_6$. The direction passing through $O_B$ and perpendicular to the plane of the static platform is the positive direction of the $Z$-axis. The positive direction of the $Y$-axis is determined by the right-hand rule and the static coordinate system $O_B - X_BY_BZ_B$ is established. In the same way, the dynamic coordinate system $O_R - X_RY_RZ_R$ is established.
Since the swing platform carries a large load and the inertial force generated by the rapid movement, the six electric cylinders have to provide a large driving force. In order to reduce the driving force of electric cylinders as well as reduce the motor power, the 6-UPS parallel mechanism is equipped with four spring branch chains with the branch springs placed vertically and the hinge points on the connecting dynamic and static platforms are arranged symmetrically on the X and Y axes of the coordinate system, as shown in Figure 1.

When the moving platform swings around the Y-axis of the dynamic coordinate system, the two spring branch chains arranged on the X-axis are under tension and pressure respectively, and the center of mass of the dynamic platform and the load is biased to the side of the pressurized spring, so that the two springs can jointly offset part of the load gravity. In addition, when the moving platform swings to the limit position of both sides, the speed is minimum, the acceleration is maximum, the inertia force is maximum, and the two spring branch chains are subject to the maximum pressure (pull) force, which can also offset part of the inertia force of the load.

Usually springs are designed as compression or tension springs only, but spring branch chains need to be both tensioned and compressed. This can be achieved by designing the structure of the spring branch chain. As shown in Figure 2, there are two compression springs arranged in the spring branch chain cylinder. When the push rod moves outward, the right side compression spring is pressed and the spring branch chain is pulled, and when the push rod moves inward, the left side compression spring is pressed and the spring branch chain is pressed.

2.2. Newton-Euler dynamic equation
The general motion of a rigid body often has to be represented by two equations, namely the Newton-Euler equation.

The Newton's equation for a rigid body in general motion is shown in equation (1)

\[ m\ddot{c} = F \]  

Newton's equations describe the translation of a rigid body doing general motion. To describe rotation, it is also necessary to resort to Euler's equations.

\[ I_c \ddot{\omega} + \omega \times I_c \omega = M_c \]  

Since the load is solidly connected to the moving platform, the moving platform and the load are treated as a whole, and the load center of mass is chosen as the object of study. During the swing, the forces (moments) on the center of mass are the driving force of the six branched chains, the gravity of the load and the moving platform, the elastic force of the four spring branch chains, and the moment of the driving force of the six branch chains on the center of mass, the moment of the elastic force of the four spring branch chains on the center of mass. To list the Newton-Euler equation, it is also necessary to find the platform kinematic parameters, including the acceleration of the center of mass and the angular acceleration of the moving platform.

2.3. Solving for velocity and acceleration
Since the swing platform has only three degrees of freedom of rotation around three coordinate axes, and the three rotations have a regularity of periodic motion, the RPY angle (RPY angle represents the angle of rotation of the moving platform around the fixed system) represents the angular velocity of the moving platform.

\[
\omega = \begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{bmatrix} = \begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
\]  

Where A, B, and C are the angles of rotation of the moving platform around the X, Y, and Z axes, respectively.

The angular acceleration of the moving platform is obtained by deriving the angular velocity.
The velocity and acceleration of any point \( q \) on the moving platform in the static coordinate system are shown in equation (5) and equation (6).

\[
\dot{\mathbf{v}}_q = \dot{\mathbf{v}}_{o_p} + \dot{\mathbf{\omega}}_p \times \mathbf{q} \\
\ddot{\mathbf{v}}_q = \ddot{\mathbf{v}}_{o_p} + \dot{\mathbf{\omega}}_p \times \mathbf{q} + \dot{\mathbf{\omega}}_p \times (\dot{\mathbf{\omega}}_p \times \mathbf{q})
\]

where \( \dot{\mathbf{\omega}}_p \) and \( \dot{\mathbf{\omega}}_p \) denote the angular velocity and angular acceleration of the moving platform in the static coordinate system, respectively; \( \dot{\mathbf{v}}_{o_p} \) denotes the velocity of the origin of the moving coordinate system; and \( \mathbf{q} \) denotes the radius vector of point \( q \) in the static coordinate system.

Since the motion of the moving platform is only around three axes of rotation, \( \dot{\mathbf{v}}_{o_p} = 0 \), \( \ddot{\mathbf{v}}_{o_p} = 0 \).

Therefore, the velocity and acceleration of the center of mass of the moving platform and the load in the static coordinate system can be expressed as

\[
\dot{\mathbf{v}}_c = \dot{\mathbf{\omega}}_p \times \mathbf{r}_c \\
\ddot{\mathbf{v}}_c = \ddot{\mathbf{\omega}}_p \times \mathbf{r}_c + \dot{\mathbf{\omega}}_p \times (\dot{\mathbf{\omega}}_p \times \mathbf{r}_c)
\]

Where \( \dot{\mathbf{\omega}}_p \) denotes the angular acceleration of the moving platform in the static coordinate system; \( \dot{\mathbf{v}}_c \) denotes the radius vector of the center of mass of the moving platform and the load in the static coordinate system from the center of rotation of the moving platform, which can be obtained from equation (9)

\[
\dot{\mathbf{r}}_c = \dot{\mathbf{r}}_c \\
\left[ \begin{array}{ccc}
\cos C \cos B & \cos C \sin B & \sin C \\
\sin C \cos B & \sin C \sin B & \cos C \\
- \sin B & \cos B & 0
\end{array} \right]
\]

Where \( \dot{\mathbf{R}} \) denotes the rotation matrix of the dynamic coordinate system with respect to the static coordinate system.

\[
\mathbf{F}_{\text{m}} = \mathbf{R} \cdot \mathbf{r}_c
\]

This transformation matrix is represented by the RPY angle.

\( \mathbf{r}_c \) denotes the radius vector of the center of mass of the moving platform and the load in the moving coordinate system, and also the coordinates of the center of mass in the moving system.

2.4. Establishment of the dynamic model

According to the angular velocity and angular acceleration of the moving platform, as well as the velocity and acceleration of the center of mass of the moving platform and the load, the dynamics of the swing platform can be easily modeled using the Newton-Euler equation.

Newton's equation for moving platform and load is

\[
\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \mathbf{F}_5 + \mathbf{F}_6 + \mathbf{F}_7 + \mathbf{F}_8 + \mathbf{F}_9 + \mathbf{F}_{10} + \mathbf{F}_{11} + \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \mathbf{G} = m \ddot{\mathbf{r}}_c
\]

Where \( \mathbf{F}_i \) denotes the driving force of the branch chain, which is the quantity to be sought; \( \mathbf{G} \) denotes the gravity of the moving platform and the load; \( m \) denotes the mass of the moving platform and the load; and \( \mathbf{F}_{si} \) denotes the spring force of the spring.

In order to offset part of the load and the gravity of the moving platform, the spring is added between the moving and static platforms with an amount of compression \( x_i \), the magnitude of the spring force on the moving platform is shown in equation (12).

\[
F_i = k_i \cdot x_i
\]
When the swing platform starts to swing, the spring length changes, and the spring compression (elongation) and the magnitude of the spring force are shown in equations (13) and (14).

\[ x_i = x_0 - (l_i - h) \quad (13) \]
\[ F_i = k_i \cdot x_i \quad (14) \]

Where \( h \) is the height between the moving and static platforms at the initial position; and \( l_i \) is the length of the i-th spring, which can be obtained from the inverse solution of the position of the parallel mechanism.

Under certain pose conditions, the pose parameter \((x, y, z, A, B, C)\) is a known quantity.

The coordinates of the hinge points of the spring on the static platform in the static coordinate system are \( B_i \) \((i=1,2,...6)\), the coordinates of the hinge points of the spring on the moving platform in the dynamic coordinate system are \( A_i \) \((i=1,2,...6)\).

The branch chain vector is \( l_i \), which is obtained from the translation and rotation transformation relations of the coordinate system.

\[ l_i = \frac{1}{6} R A_i + t - B_i \quad (15) \]

Where \( t \) is the translation vector of the dynamic coordinate system with respect to the static coordinate system. Since the swing platform has only three directions of rotation, the translation vector contains only the height of the dynamic coordinate system relative to the static coordinate system, \( t = [0 \quad 0 \quad h]^T \).

The length of the spring branched chain \( l_i \) is shown in equation (16).

\[ l_i = \left\| l_i \right\| = \left\| l_i = \frac{1}{6} R A_i + t - B_i \right\| \quad (16) \]

In addition, the coordinates of the hinge points of the driving branch chain on the static platform in the static coordinate system are \( B_i \) \((i=1,2,...6)\), and the coordinates of the hinge points of the driving branch chain on the moving platform in the dynamic coordinate system are \( A_i \) \((i=1,2,...6)\).

The branch vector and the rod length can be found for each branch at any time.

\[ l_i = \left\| l_i \right\| = \left\| \frac{1}{6} R A_i + t - B_i \right\| \quad (17) \]

\[ \sum_{i=1}^{6} R_i \times F_i + \sum_{i=1}^{6} r_i \times F_i = I_C \times \omega_p + \omega \times I_C \times \omega_p \quad (18) \]

Euler equation is shown in equation (19).

\[ \sum_{i=1}^{6} F_i = \left\| \frac{1}{6} R A_i + t - B_i \right\| \quad (19) \]

Where \( R_i \) is the radius vector from the point of the driving force of the i-th branch chain to the center of mass of the load, \( R_i = l_i - \omega r_i \); \( R_i \) is the radius vector of the load center of mass in the static coordinate system, from the origin of the static coordinate system to the load center of mass, \( \omega r_i \) is the radius vector of the load center of mass in the dynamic coordinate system, from the origin of the dynamic coordinate system to the load center of mass; \( r_i \) is the vector radius from the point of the spring force to the center of mass of the load, \( r_i = l_i - \omega r_i \); \( \omega \times \omega_p \) is respectively the angular velocity and angular acceleration of the load; and \( I_C \) denotes the inertia tensor matrix of the load to its center of mass in the static coordinate system.

The dynamics of the swing platform is modeled by associating equation (11) with equation (19), and the two equations are further deformed to obtain equations (20) and (21).

\[ f_i \cdot e_i + f_2 \cdot e_2 + f_3 \cdot e_3 + f_4 \cdot e_4 + f_5 \cdot e_5 + f_6 \cdot e_6 + \sum_{i=1}^{6} f_{6i} \cdot e_{6i} + \omega \times I_C \cdot \omega = m \cdot \ddot{r}_C \quad (20) \]
\[ \sum_{i=1}^{6} f_i (R_i \times e_i) + \sum_{i=1}^{6} f_{6i} (r_i \times e_{6i}) = I_C \cdot \omega_p + \omega \times I_C \cdot \omega_p \quad (21) \]
Where \( \mathbf{e}_f \) denotes the direction vector of each branch chain force in the static coordinate system, \( \mathbf{e}_f = \frac{\mathbf{F}}{||\mathbf{F}||} \); \( \mathbf{e}_s_i \) denotes the direction vector of each spring branch chain forces in the static coordinate system, \( \mathbf{e}_s = \frac{\mathbf{F}_s}{||\mathbf{F}_s||} \); \( f_i \) denotes the magnitude of each branch chain force; and \( f_{s_i} \) denotes the magnitude of each spring branch chain force.

Combining equation (20) with equation (21), equation (22) can be obtained.

\[
\begin{bmatrix}
    e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\
    R_1 \times e_1 & R_2 \times e_2 & R_3 \times e_3 & R_4 \times e_4 & R_5 \times e_5 & R_6 \times e_6
\end{bmatrix}
\begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3 \\
    f_4 \\
    f_5 \\
    f_6
\end{bmatrix}
= m\mathbf{r}_c \cdot \sum_{i=1}^{4} f_{s_i} \mathbf{e}_{s_i} - \mathbf{G} + \mathbf{I}_c \mathbf{e}_p + \mathbf{I}_c \mathbf{e}_p \times \sum_{i=1}^{4} f_{s_i} (r_i \times \mathbf{e}_{s_i})
\]

Each variable in the equation is expressed in the static coordinate system, so the upper left corner is omitted, and \( f = [f_1, f_2, f_3, f_4, f_5, f_6]^T \).

Let \( J = \begin{bmatrix}
    e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\
    R_1 \times e_1 & R_2 \times e_2 & R_3 \times e_3 & R_4 \times e_4 & R_5 \times e_5 & R_6 \times e_6
\end{bmatrix}, \quad F = \begin{bmatrix}
    m\mathbf{r}_c \cdot \sum_{i=1}^{4} f_{s_i} \mathbf{e}_{s_i} - \mathbf{G} + \mathbf{I}_c \mathbf{e}_p + \mathbf{I}_c \mathbf{e}_p \times \sum_{i=1}^{4} f_{s_i} (r_i \times \mathbf{e}_{s_i})
\end{bmatrix} \),

the dynamic model is expressed as equation (23).

\[
f = J^T F
\]

In equation (23), \( J \) is the force Jacobian matrix, which reflects the transfer relationship between the driving force and the load, and \( F \) is the external load.

3. Dynamics Simulation
This structure with springs on the 6-UPS parallel mechanism and the dynamics model are suitable for large load swing platforms. The following section will verify the validity of this structural and dynamical model through a specific working condition.

3.1. Working conditions assumption
The motion laws of the swing platform are shown in Table 1.

| direction of rotation | Workspace | Spacing |
|-----------------------|-----------|---------|
| Around X axis         | ±18°      | \( \alpha = \frac{\pi}{10} \times \cos \left( \frac{\pi}{4} - t \right) \) |
| Around Y axis         | ±10°      | \( \beta = \frac{\pi}{18} \times \cos \left( \frac{\pi}{4} - t \right) \) |
| Around Z axis         | ±6°       | \( \gamma = \frac{\pi}{30} \times \cos \left( \frac{\pi}{4} - t \right) \) |

The structural parameters of the 6-UPS parallel mechanism are shown in Table 2.

| Radius of moving platform | Radius of static platform | Height of the moving platform |
|--------------------------|---------------------------|-------------------------------|
| 1900mm                   | 2400mm                    | 3000mm                        |

Suppose the mass sum of the load and the moving platform is 55t, the load is a homogeneous cylinder with a height of 10m and a diameter of 5m. The four springs are arranged vertically, and their hinge points at the static platform are 1400mm, 1270mm, 1400mm and 1270mm away from the origin.
respectively. Set the initial compression of the springs to $x_1=120$, $x_2=105$, $x_3=120$, $x_4=105$ respectively, and the initial force on each spring is about $100000\text{N}$.

Now given the swing platform a motion law for analysis: The moving platform moves from $-18^\circ$, $-10^\circ$, $-6^\circ$ to $+18^\circ$, $+10^\circ$, $+6^\circ$ simultaneously around the $X$, $Y$, and $Z$ axes, and then moves back to the starting point.

### 3.2. Simulation results

By the force of each branch chain after simulation in MATLAB, it can be obtained that the theoretical maximum driving force of the six branch chains is $127590\text{N}$ during the motion of the swing platform, which appears on branch chain II. Further, by simulating the swing platform with the four springs removed, it can be obtained that the theoretical maximum driving force of the six branch chains is $428,000\text{N}$, which appears on branch chain II.

The following is the curve of the driving force of each branch chain in the 6-UPS parallel mechanism with and without spring branch chains as a function of time.

![Figure 3. The curve of the driving force of each branch chain.](image)

From the curves of the driving force of each branch chain along with time, it can be seen that the swing platform with four spring branch chains in the 6-UPS parallel mechanism can effectively reduce the driving force of the six branch chains during the movement of the swing platform compared with the conventional 6-UPS parallel mechanism swing platform.

### 4. Conclusion

With the swing platform of the 6-UPS parallel mechanism with spring branch chains as the research object, the dynamics of the swing platform is modeled by applying the Newton-Euler dynamics equation, and finally the dynamics equation of the swing platform is obtained. MATLAB is used to simulate the dynamics of the swing platform, and it is verified that the swing platform with spring branch chains can effectively reduce the driving force of each branch chain compared with the conventional 6-UPS parallel mechanism swing platform. Since the model of the dynamics simulation is simplified, an experimental prototype will be designed for physical verification in further research to lay the foundation for the design and dynamics analysis as well as application of large-load swing platform.

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