On the Electric Fields Produced by Dipolar Coulomb Charges of an Individual Thundercloud in the Ionosphere

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In this paper we study the transmission of the electrostatic field due to coulomb charges of an individual thundercloud into the midlatitude ionosphere, taking into account the total geomagnetic field integrated Pedersen conductivity of the ionosphere. It is shown that at ionospheric altitudes, a typical thundercloud produces an insignificant electrostatic field whereas a giant thundercloud can drive the horizontal electrostatic field with a magnitude of ~270 μV/m for nighttime conditions.

Keywords: thundercloud, ionosphere, electrostatic field

1. INTRODUCTION

Thunderclouds are tropospheric sources of intense electrostatic fields and electromagnetic radiation. It is known that lightning-associated electric fields penetrate into the ionosphere; they have been observed in the E and F regions as transient electric fields with a typical duration of 10-20 ms and a magnitude of 1-50 mV/m (e.g., Kelley et al. 1985, 1990; Vlasov & Kelley 2009). According to the theoretical model of global atmospheric electricity developed by Hays & Roble (1979), the African array of multiple thunderclouds is responsible for the steady state electrostatic field of ~300 μV/m at ionospheric altitudes for nighttime conditions. The calculations by Park & Dejnakarintra (1973) showed that an isolated giant thundercloud could produce electrostatic fields of ~700 μV/m in the nighttime midlatitude ionosphere. However, Park & Dejnakarintra (1973) neglected the ionospheric Pedersen conductivity above 150 km. The purpose of this study is to theoretically examine the mapping of electrostatic fields of coulomb charges of an individual thundercloud into the midlatitude ionosphere, taking into account the height-integrated Pedersen conductivities of both hemispheres.

2. BASIC EQUATIONS

In the simplest thundercloud model, the electrical structure of a thundercloud is represented by two volume Coulomb charges of the same absolute value Q but opposite signs, with a positive charge in the upper part of the thundercloud and a negative charge in the lower part of the thundercloud (e.g., Chalmers 1967). Typical thunderclouds extend from 2-3 km to 8-12 km in altitude, and so-called giant thunderclouds extend above an altitude of 20 km (e.g., Uman 1969; Weisberg 1976). The magnitude of Q is estimated to range from 5 to 25 coulombs for the typical thunderclouds, whereas in giant thunderclouds, Q may exceed 50 coulombs (e.g., Malan 1963; Kasemir 1965).

We use a cylindrical coordinate system \((r, \phi, z)\), in which the origin is placed at the earth's surface and the \(z\) axis points upward and passes through the centers of thundercloud positive and negative volume charges.
The mapping of thundercloud electrostatic field into the ionosphere is studied following a similar formalism to that used by Park & Dejnakarintra (1973). In the steady state case, the electrostatic field distribution above the thundercloud is described by the following equations:

\[ \nabla \cdot \mathbf{J} = 0 \]  
(1)

\[ \mathbf{J} = \sigma \mathbf{E} \]  
(2)

\[ \mathbf{E} = -\nabla \Phi \]  
(3)

where \( \mathbf{J} \) is the electric current density, \( \sigma \) is the electrical conductivity tensor, and \( \mathbf{E} \) and \( \Phi \) are the electrostatic field and potential, respectively. If we assume that the geomagnetic field \( \mathbf{B} \) is vertical and the electrical conductivity tensor depends only on \( z \), the following equation for the electrostatic potential \( \Phi \) can be obtained from (1), (2), and (3):

\[ \delta^2 \Phi / \delta r^2 + (1/r) \delta \Phi / \delta r + (1/\sigma_r) \delta (\sigma_r \delta \Phi / \delta z) / \delta z = 0 \]  
(4)

where \( \sigma_r \) is the Pedersen conductivity and \( \sigma_\theta \) is the specific conductivity. The atmospheric conductivity below 70 km is isotropic since drifts of charged particles are not affected by the geomagnetic field. Equation (4) can be solved analytically if the conductivities \( \sigma_\theta \) and \( \sigma_r \) are exponential functions of \( z \). In the case of isotropic conductivity (setting \( \sigma_\theta = \sigma_r = b \exp(z/h) \), where \( b \) and \( h \) are constants), we obtain

\[ \Phi = \int_0^\infty J_0(kr) [ A_1(k) \exp(c_1z) + B_1(k) \exp(c_2z) ] dk \]  
(5)

where \( J_0 \) is the zero-order Bessel function of the first kind, \( A_1 \) and \( B_1 \) are coefficients, and \( c_1 = -l/(2h) - [l/(4h^2) + k^2]^{1/2} \), \( c_2 = -l/(2h) + [l/(4h^2) + k^2]^{1/2} \). For the anisotropic region, where we let \( \sigma_\theta = b_0 \exp(z/h_\theta) \) and \( \sigma_r = b_r \exp(z/h_r) \), the solution to Equation (4) is

\[ \Phi = \int_0^\infty J_0(kr) [ A_2(k) J_\nu(kf) + B_2(k) K_\nu(kf) ] f^r dk \]  
(6)

where \( J_\nu \) and \( K_\nu \) are the \( \nu \)-order modified Bessel functions of the first and the second kind, respectively, and \( A_2 \) and \( B_2 \) are coefficients, \( \nu = h_r(h_r - h_\theta) \), \( f = 2v_h(b_r/h_r)^{1/2} \exp[-z/(2v_h)] \). The coefficients \( A_2, B_2, A_\nu \), and \( B_\nu \) are determined from the boundary conditions.

The electrostatic field components are given by

\[ E_r = -\partial \Phi / \partial r, \]  
(7)

\[ E_z = -\partial \Phi / \partial z. \]  
(8)

Since the geomagnetic field \( \mathbf{B} \) is assumed to be vertical, \( E_r \) is perpendicular to \( \mathbf{B} \), while \( E_z \) is parallel to \( \mathbf{B} \).

Above 90 km, the geomagnetic field lines are practically equipotential because the geomagnetic field aligned conductivity \( \sigma_\theta \) is much higher than the transverse conductivity \( \sigma_r \). It allows us to consider the ionospheric region above 90 km as a thin conducting layer with a geomagnetic field line integrated Pedersen conductivity \( \Sigma_\rho \), and the continuity equation of electric current at \( z=90 \) km takes the following form:

\[ \sigma_\theta E_z = \nabla \cdot (2 \Sigma_\rho \mathbf{E}_\perp) \]  
(9)

where \( \nabla \cdot \) denotes the gradient operator in the two dimensions transverse to \( \mathbf{B} \), and the factor 2 before \( \Sigma_\rho \) accounts for a contribution of the Pedersen conductivity of the magnetically conjugate ionosphere. Equation (9) is explicitly expressed as

\[ \sigma_\theta \partial \Phi / \partial z = 2 \Sigma_\rho \left( \delta^2 \Phi / \delta r^2 + 1/r \partial \Phi / \partial r \right) \]  
(10)

We use the conductivity model as shown in Fig. 1. Below 70 km, the conductivity is isotropic and varies exponentially with \( z \) as \( \sigma_\theta = \sigma_r = b_0 \exp(z/h) \) from 0 to 40 km, and as \( \sigma_\theta = \sigma_r = b_2 \exp(z/h) \) from 40 to 70 km (where \( z_0 = 40 \) km) with the values of \( b_1 \) and \( h_1 \) to approximately fit the atmospheric conductivity models by Cole & Pierce (1965).

![Fig. 1. Model altitude profiles of specific (\( \sigma_\theta \)) and Pedersen (\( \sigma_r \)) conductivities. The numbers next to the curves indicate conductivity scale heights in kilometers within each altitude section.](http://dx.doi.org/10.5140/JASS.2015.32.2.141)
below 40 km and by Swider (1988) from 40 to 70 km. In the
anisotropic region between 70 and 90 km, $\sigma_0$ and $\sigma_e$ are
exponentially extrapolated from 70 km to their equinoctial
midday and midnight values at $z=90$ km. At $z \geq 90$ km, the
conductivities are found from

$$\sigma_0 = e^2 \left[ \frac{N_e v_e}{m_e v_e} + \sum_{i} \frac{N_i v_i}{m_i v_i} \right]$$

(11)

$$\sigma_p = e^2 \left[ \frac{N_e v_e}{m_e (\omega_e^2 + v_e^2)} + \sum_{i} \frac{N_i v_i}{m_i (\omega_i^2 + v_i^2)} \right]$$

(12)

where subscripts $e$ and $i$ denote electrons and the $i$th ion
species, $N_e$ and $N_i$ are the electron and ion densities, $e$ is the
electron charge, $m_e$ and $m_i$ are the electron and ion masses,
$v_e$ and $v_i$ are the electron and ion momentum transfer
collision frequencies, and $\omega_e$ and $\omega_i$ are the electron and
ion gyrofrequencies, respectively. The frequencies $v_e$ and $v_i$
are from Schunk (1988). The required input parameters are
taken from the empirical ionospheric model IRI-2012
(http://omniweb.gsfc.nasa.gov/vitmo/iri2012_vitmo.html)
and the neutral atmosphere model NRLMSIS-00
(http://
ccmc.gsfc.nasa.gov/modelweb/models/nrlmsise00.php).

Our calculations show that during solar minimum,
in Equinox, the magnitude of $\Sigma_p$ at middle latitudes is
commonly in the ranges of 5.0-8.0 S and 0.1-0.2 S for day
and night, respectively. However, the nighttime $\Sigma_p$ can be
as low as 0.05 S. Under solar maximum conditions, $\Sigma_p$ is
several times larger than in solar minimum.

3. RESULTS AND DISCUSSION

To compute the electrostatic potential above the
thundercloud from (5) and (6), we impose the following
boundary conditions:

1. $\Phi = (Q/4\pi \varepsilon_0) \left[ (r^2 + (z_h - h_p)^2)^{1/2} - (r^2 + (z_h - h_n)^2)^{1/2} \right]$ at $z = z_h$, 
2. $\Phi$ is continuous at $z = 40$ km
3. $\sigma_0 \partial \Phi / \partial z = 2 \Sigma_p (\partial^2 \Phi / \partial r^2 + 1/r \partial \Phi / \partial r)$ at $z = 90$ km

where $\varepsilon_0$ is the vacuum permittivity, $z_h$ is the altitude of
the plane setting directly above the thundercloud top, and
$h_p$ and $h_n$ are the altitudes of positive and negative charge
centers of the thundercloud, respectively. The first boundary
condition follows from the accepted electrical model of the
thundercloud. We assume that the thundercloud does not
affect the atmospheric conductivity at $z \geq z_h$.

Fig. 2 shows the computed electrostatic field component
$E_z$ normalized to $Q$ as a function of $r$ at ionospheric altitudes $z=90$
kilo meters for the typical and giant thundercloud at nighttime
$z=90$ km for the typical and giant thundercloud at night and by day.
4. CONCLUSION

Our computations show that the geomagnetic field line integrated Pedersen conductivity of the ionosphere plays an important role in troposphere-ionosphere electrostatic coupling. Even for nighttime conditions in solar minimum, when the values of $E_z$ are minimal, the electrostatic charges of the individual thundercloud can drive only small electrostatic fields at ionospheric altitudes.

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