Model-independent test of spatial variations of the Newtonian gravitational constant in some extrasolar planetary systems

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Abstract

In this paper we directly constrain possible spatial variations of the Newtonian gravitational constant $G$ over ranges $\approx 0.01 - 5$ AU in various extrasolar multi-planet systems. By means of the third Kepler's law we determine the quantity $\Gamma_{XY} = G_X/G_Y$ for each couple of planets $X$ and $Y$ located at different distances from their parent star: deviations of the measured values of $\Gamma$ from unity would signal variations of $G$. The obtained results for $\eta = 1 - \Gamma$ are found to be well compatible with zero within the experimental errors ($\eta/\delta\eta \approx 0.2 - 0.3$). A comparison with an analogous test previously performed in our Solar System is made.

Key words: stars: planetary systems-stars: exoplanets-celestial mechanics-ephemerides-gravitation

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1 Introduction

More than 200 exoplanets have been so far discovered with different techniques such as radial velocity$^1$, microlensing$^2$, direct imaging$^3$. For an update list see on the WEB http://exoplanet.eu/. One of the most important orbital parameters phenomenologically measured in such systems with usually great accuracy is the sidereal orbital period: since it is affected by many

$^1$It allows for the most accurate ephemerides.
$^2$Until now there are only four planets discovered with such method: their orbital parameters are rather poorly determined for our purposes.
$^3$Until now there are only four planets discovered with such method: only their masses and semimajor axes are available for them.
Table 1: Relevant parameters [7] of the three planets υ And b, υ And c and υ And d. a is the semimajor axis, in AU, e is the eccentricity, P is the sidereal orbital period, in days, and m sin i is the minimum planet’s mass, in Jovian masses. The stellar mass and radius are $M = 1.32 M_\odot$ [36] and $R = (1.15 \pm 0.15) R_\odot$ [30], respectively.

| Planet | a (AU) | e     | P (d)           | m sin i (m Jup) |
|--------|--------|-------|----------------|----------------|
| b      | 0.0595 ± 0.0034 | 0.023 ± 0.018 | 4.617113 ± 0.000082 | 0.687 ± 0.058 |
| c      | 0.832 ± 0.048  | 0.262 ± 0.021 | 241.23 ± 0.30       | 1.98 ± 0.17    |
| d      | 2.54 ± 0.15    | 0.258 ± 0.032 | 1290.1 ± 8.4        | 3.95 ± 0.33    |

Newtonian and non-Newtonian effects, it may be used, in principle, as an important probe to measure or constrain such features within the experimental errors. It is our intention to use some of the best determined multiple planetary systems in order to put on the test a key feature of the gravitational interaction, i.e. the possibility that the Newtonian gravitational constant $G$ may vary with distance.

1.1 Overview of the considered multi-planet systems

υ And (HD 9826) is a 2.41 Gyr old main sequence star of spectral type F8V located at a distance of 13.47 pc from us with RA (J2000) 01 36 4 7.843 and DEC (J2000) +41 24 19.65 [36]. Its mass and radius are $M = 1.32 M_\odot$ [36] and $R = (1.15 \pm 0.15) R_\odot$ [30], respectively. It harbors a planetary system [29, 7] composed by three Jupiter-type planets: the closest one orbits υ And at a 0.0595 AU distance, while the farthest one is at 2.54 AU from its parent star. The relevant orbital parameters are in Table 1.

Another triple system [33, 7] is hosted by the 6.41 Gyr old main sequence star μ Ara (HD 160691) at 15.28 pc from us with RA (J2000) 17 44 08.703 and DEC (J2000) -51 50 02.59 [36]. Its spectral type is G3 IV-V, its mass is $M = 1.15 M_\odot$ [36] and its radius is $R = (1.245 \pm 0.255) R_\odot$ [30]. The relevant orbital parameters of such a system, consisting of two Jupiter-type bodies and one Neptune-like planet, are in Table 2. The closest planet orbits at 0.0924 AU from its parent star, while the farthest one is at 3.78 AU from μ Ara.

Three Neptune-type planets [21] have recently been discovered around the 7 Gyr old main sequence star HD 69830 [21] at 12.6 pc from us with RA (J2000) 08 18 23 and DEC (J2000) -12 37 55. Its spectral class is K0V, its mass is $M = (0.86 \pm 0.03) M_\odot$ and its radius is $R = (0.895 \pm 0.005) R_\odot$. The orbital parameters of such system are in Table 3.
Table 2: Relevant parameters of the three planets $\mu$ Ara b [7], $\mu$ Ara c [7] and $\mu$ Ara d [33]. $a$ is the semimajor axis, in AU, $e$ is the eccentricity, $P$ is the sidereal orbital period, in days, and $m \sin i$ is the minimum planet’s mass, in Jovian masses. The stellar mass and radius are $M = 1.15M_\odot$ [36] and $R = (1.245 \pm 0.255)R_\odot$ [30], respectively.

| Planet | $a$ (AU)   | $e$     | $P$ (d)      | $m \sin i$ (m$_{\text{Jup}}$) |
|--------|------------|---------|--------------|-------------------------------|
| b      | 1.510 ± 0.088 | 0.271 ± 0.040 | 630.0 ± 6.2  | 1.67 ± 0.17                  |
| c      | 3.78 ± 0.25  | 0.463 ± 0.053 | 2490 ± 100   | 1.18 ± 0.12                  |
| d      | 0.0924 ± 0.0053 | 0.000 ± 0.020  | 9.550 ± 0.030 | 0.0471                       |

Table 3: Relevant parameters [21] of the three planets HD 69830 b, HD 69830 c and HD 69830 d. $a$ is the semimajor axis, in AU, $e$ is the eccentricity, $P$ is the sidereal orbital period, in days, and $m \sin i$ is the minimum planet’s mass, in Jovian masses. The stellar mass and radius are $M = (0.86 \pm 0.03)M_\odot$ and $R = (0.895 \pm 0.005)R_\odot$, respectively.

| Planet | $a$ (AU)   | $e$     | $P$ (d)      | $m \sin i$ (m$_{\text{Jup}}$) |
|--------|------------|---------|--------------|-------------------------------|
| b      | 0.0785     | 0.1 ± 0.04 | 8.667 ± 0.003 | 0.033                         |
| c      | 0.186      | 0.13 ± 0.06 | 31.56 ± 0.04  | 0.038                         |
| d      | 0.63       | 0.07 ± 0.07 | 197 ± 3       | 0.058                         |
Table 4: Relevant parameters [23] of the four planets 55 Cnc b, 55 Cnc c, 55 Cnc d and 55 Cnc e. $a$ is the semimajor axis, in AU, $e$ is the eccentricity, $P$ is the sidereal orbital period, in days, and $m \sin i$ is the minimum planet’s mass, in Jovian masses. The stellar mass and radius are $M = 0.91M_\odot$ [36] and $R = (0.6 \pm 0.3)R_\odot$ [30], respectively.

| Planet | $a$ (AU)       | $e$            | $P$ (d)     | $m \sin i$ (m$_{\text{Jup}}$) |
|--------|----------------|----------------|-------------|-------------------------------|
| b      | 0.1138 ± 0.0066 | 0.01 ± 0.13    | 14.652 ± 0.010 | 0.833 ± 0.069                |
| c      | 0.238 ± 0.014   | 0.071 ± 0.012  | 44.36 ± 0.25 | 0.157 ± 0.020                |
| d      | 5.97 ± 0.35     | 0.091 ± 0.080  | 5552 ± 78    | 3.90 ± 0.33                  |
| e      | 0.0377 ± 0.0022 | 0.09 ± 0.28    | 2.7955 ± 0.0020 | 0.0377 ± 0.0059            |

A tetra-planetary system [23, 29] is hosted by the 5.5 Gyr old main sequence star 55 Cnc ($\rho^1$ Cnc, HD 75732) at 13.4 pc from us with RA (J2000) 08 52 35.811 and DEC (J2000) +28 19 50.95. Its spectral type is G8 V, its mass is $M = 0.91M_\odot$ [36] and its radius is $R = (1.245 \pm 0.255)R_\odot$ [30]. Such a system is formed by three Jupiter-like planets and one Neptune-type planet spanning the range 0.0377 – 5.97 AU from their parent star. See Table 4.

As can be noted, the considered planetary systems, all discovered with the radial velocity method, are hosted by main sequence, Sun-like stars located at about 12 – 15 pc from us.

1.2 Aim of the paper

Such extra-solar systems offer us different laboratories outside our Solar System to perform direct and model-independent tests of possible variations with distance of the Newtonian gravitational constant $G$ over scales ranging from $\approx 0.01$ AU to $\approx 5$ AU.

2 Spatial variations of the Newtonian gravitational constant in the considered planetary systems

The possibility that the Newtonian gravitational constant $G$ may experience spatial variations is envisaged by many theoretical frameworks dealing with generalized theories of gravity and unified theories of basic physical interactions [9, 24, 25, 1]. In, e.g., the extended chaotic inflation scenario proposed by Linde in [19] the values of the effective gravitational constant in different parts of the universe may differ from each other. Also scalar-tensor
theories predict that $G$ may spatially vary [8]. A scalar-tensor-vector-theory which, among other things, predicts spatial variations of $G$ [5] is the one by Moffat [28]. Some non-perturbative studies of quantum gravity suggest that the effective $G$ might slowly increase with distance [13]; in cosmology, this may work as an alternative to dark matter and be related to the expansion acceleration. Spatial variations of $G$ are also predicted in the frameworks of the Yukawa-like modifications of Newtonian gravity [18, 11], and MOND (MODified Newtonian Dynamics) [27, 31, 10, 32, 37].

Earth-based, laboratory-scale investigations of spatial variations of $G$ can be found in [17, 12, 20]. Both laboratory and astronomical tests can be found in [26, 14, 6]. Constraints on variations of $G$ with scale from gravitational lensing and the cosmic virial theorem are reported in [3]. Large (cosmological)-scale bounds on spatial variations of $G$ have recently been placed by Barrow in [2]. Effects of possible spatial variations of $G$ on the cosmic microwave background are reported in [4].

A useful approach to test spatial variations of $G$ in typical astronomical scenarios as the planetary systems considered here is the following one. According to the third Kepler Law,

$$G(a) = \left(\frac{2\pi}{P}\right)^2 \frac{a^3}{M},$$

so that for a generic pair of planets X and Y we can construct the ratio $\Gamma_{XY} = G_X/G_Y$ as

$$\Gamma_{XY} = \left(\frac{P_Y}{P_X}\right)^2 \left(\frac{a_X}{a_Y}\right)^3;$$

deviations of such a quantity from unity would reveal scale variations of $G$. Such an approach was used in our Solar System [35] by comparing $\Gamma$ to unity as $\eta = 1 - \Gamma$ for those planets for which radar-ranging measurements of their orbital radiuses existed. No evidence for any anomalous behavior of the Newtonian gravitational constant from the orbits of Mercury to that of Jupiter, i.e. in the range $0.38 - 5$ AU, was found; all the determined values for $\eta$ were less than 1 sigma from zero, except for Venus, which was a 1.6 sigma result.

Having at our disposal the phenomenologically measured semimajor axes and orbital periods of the $\nu$ And, $\mu$ Ara, HD 69830 and 55 Cnc planets, we can accurately map possible deviations of $\Gamma$ from unity in each of such systems. It is important to note that the precision with which we presently know the planets’ periods allows us to neglect the corrections to the simple Keplerian model of eq. (1) due to both the quadrupole mass moments $J_2$
Table 5: \( \upsilon \) And, \( \mu \) Ara, HD 69830 and 55 Cnc systems: variation of the gravitational constant \( G \) in the spatial regions crossed by the planets.

| System     | \( \Gamma_{bc} \) | \( \Gamma_{bd} \) | \( \Gamma_{cd} \) | \( \Gamma_{be} \) | \( \Gamma_{ce} \) | \( \Gamma_{de} \) |
|------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| \( \upsilon \) And | 0.9 ± 0.3         | 1.0 ± 0.4         | 1.0 ± 0.4         | -                 | -                 | -                 |
| \( \mu \) Ara     | 0.9 ± 0.4         | 1.0 ± 0.4         | 1.0 ± 0.5         | -                 | -                 | -                 |
| HD 69830       | 0.997 ± 0.003     | 0.99 ± 0.02       | 1.00 ± 0.02       | -                 | -                 | -                 |
| 55 Cnc         | 1.0 ± 0.4         | 0.9 ± 0.4         | 0.9 ± 0.4         | 1.0 ± 0.3         | 1.0 ± 0.4         | 0.9 ± 0.4         |

of the parent stars [16] and the post-Newtonian component of their gravity field of order \( O(c^{-2}) \) [31, 22]. Table 5 shows our results for the variations of \( G \) in the \( \upsilon \) And, \( \mu \) Ara, HD 69830 and 55 Cnc systems. The uncertainty was calculated as

\[
\delta \Gamma_{XY} \leq \delta \Gamma|_{PX} + \delta \Gamma|_{PX} + \delta \Gamma|_{aX} + \delta \Gamma|_{aY},
\]

with

\[
\begin{align*}
\delta \Gamma|_{PX} & \leq 2 \frac{P_X}{P_X} \left( \frac{a_X}{a_Y} \right)^3 \delta P_Y, \\
\delta \Gamma|_{PX} & \leq 2 \frac{P_X}{P_X} \left( \frac{a_X}{a_Y} \right)^3 \delta P_X, \\
\delta \Gamma|_{aX} & \leq 3 \left( \frac{P_X}{P_X} \right)^2 \frac{a_X^3}{a_Y} \delta a_X, \\
\delta \Gamma|_{aY} & \leq 3 \left( \frac{P_X}{P_X} \right)^2 \frac{a_X^3}{a_Y} \delta a_Y.
\end{align*}
\]

In the case of \( \upsilon \) And we note that our result for \( \Gamma \) is accurate at 2.5—3 sigma. For \( \mu \) Ara the obtained precision in determining \( \Gamma \) is 2—2.5 sigma. For HD 69830 the situation is much better because only the errors in the orbital periods were accounted for, being those in the semimajor axes not released in [21]. In the 55 Cnc system \( \Gamma \) was measured at a 2.2—3.3 sigma. Reasoning in terms of deviations of the measured quantity \( \Gamma \) from the expected value 1, i.e. by considering \( \eta \), we see that from Table 2 it is possible to obtain figures for such a quantity which are well compatible with zero differing from it by only 0.2—0.3 sigma or less. We note that in our test we used four different planetary systems involving a total of twelve planets; instead, in the Solar System test of [35] only four planets were considered: for one of them, i.e. Venus, \( \eta \) was found to be compatible with 0 at 1.6 sigma level, while for Mercury, Mars and Jupiter the agreement was at about 0.8, 0.3 and 0.5 sigma level, respectively. Thus, our test can, in general, be considered
more accurate than the one of [35]. Moreover, when more data from the considered planetary systems will become available and will be processed, it will be possible to further improve the precision of such an extra-solar test.

The most general conclusion that can be drawn is that our results for spatial variations of $G$ throughout the extension of the considered planetary systems are compatible with zero within the experimental errors. Our analysis relies only upon the third Kepler law, being independent of any model of modified gravity predicting spatial variations of $G$.

3 Conclusions

In this paper we constrained, in a model-independent way, spatial variations of the Newtonian gravitational constant $G$ throughout different extrasolar multi-planet systems extending from 0.01 AU to $2 - 5$ AU and located at about $12 - 15$ pc from us. Our results are compatible with zero within the experimental errors ($\eta/\delta\eta \approx 0.2 - 0.3$). With respect to what previously done in our Solar System with Mercury, Venus, Mars and Jupiter, here we used a total of 13 planets in four independent planetary systems sampling a wider range of distances from their central stars thanks to $\upsilon$ And b, $\mu$ Ara d, HD 69830 b and 55 Cnc e at orbiting at about 0.01 AU from their parent stars. The precision achieved is better than that of the Solar System test and will be increased when more data from such planetary systems will be collected and processed.

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