Non-tachyonic open descendants
of the 0B string theory

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Abstract

We use the crosscap constraint to construct open descendants of the 0B string compactified on $T^6/Z_3$ and on $T^4/Z_2$ free of tachyons both in the closed and in the open unoriented sectors. In four dimensions the construction results in a Chan-Paton gauge group $U(8) \otimes U(12) \otimes U(12)$ with three generations of chiral fermions in the representations $(8,1,12) \oplus (8,12,1) \oplus (1,66,1) \oplus (1,1,66)$.

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1 Introduction

It has long been known that, besides the five supersymmetric strings, there are a number of non-supersymmetric theories in ten dimensions [1]. These can be obtained as $Z_2$ orbifolds of the supersymmetric ones, where the $Z_2$ generator $(-)^F$ is related to the total fermion number $F$, and is accompanied by an action on the gauge group in the case of the heterotic strings. Particularly interesting for what concerns our purposes are the two tachyonic 0A and 0B theories, that descend from the type IIA and IIB superstrings. Using the characters of affine SO(8) at level one, their partition functions read

$$T_{0A} = |O_8|^2 + |V_8|^2 + S_8 C_8 + C_8 S_8,$$

$$T_{0B} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2.$$  

The NS-NS sector, common to both theories, includes a tachyon, together with a metric tensor, a dilaton and a rank 2 antisymmetric tensor at the massless level. The R-R sectors are different. At the massless level the 0A string contains two vectors and two 3-forms, thus simply doubling the type IIA R-R massless states, while the 0B theory contains two scalars, two 2-forms and one 4-form without definite chirality. Being non chiral, both theories are anomaly free in ten dimensions.

One common and important feature of the 0A and 0B string theories is that they are symmetric under the interchange of left and right modes. We can then apply the procedure developed in [2, 3] to mod out both theories by the world-sheet parity $\Omega$ and construct open descendants. In ten dimensions this has been done long ago by Bianchi and Sagnotti [3] (see also [4]). If we choose a “standard” action for the $\Omega$ projection that symmetrises the NS-NS sector and antisymmetrises the R-R one, typically we get open descendants tachyonic both in the closed and open unoriented sectors. But this is not the only choice available for the action of the world-sheet parity. In [5] it has been shown that one can suitably modify the Klein bottle projection in a way consistent with what has been called the crosscap constraint. Typically, for supersymmetric theories these “exotic” $\Omega$ projections result in open descendants without the open unoriented sector [4], since the Klein bottle amplitude in the transverse channel does not contain any massless states, and
thus does not contribute to tadpoles. As a result, one is forced to set to zero the Chan-Paton multiplicities in order to get a consistent model. Fortunately, this is not the case for the non-supersymmetric theories. In the 0B string one can use the sign ambiguities in the Klein bottle projection to remove the tachyons from the spectrum. In ten dimensions this has been shown by Sagnotti \[7\] (see also \[8\]). The final result is a chiral closed unoriented massless spectrum consisting of a metric tensor, a dilaton, a rank 2 antisymmetric tensor, a scalar and a (anti)self-dual 4-form, and in the open unoriented sector a U(32) gauge group with Majorana-Weyl fermions in the 496 ⊕ 496 representations. In this case the anomaly cancellation involves the R-R (anti)self-dual 4-form and the 8-form dual to the R-R scalar, in addition to the R-R 2-form, in a generalized Green-Schwarz mechanism \[7\]. In the 0A string theory it is not possible to modify the Klein bottle projection, and thus one can not remove the tachyons from the spectrum.

At this point one may ask if this result can be generalized lower dimensions. This is precisely the aim of this letter. We will study orbifold compactifications of the 0B string theory to six and four dimensions and will look for “exotic” Klein bottle projections in order to remove tachyons whenever this is possible. In section 2 we discuss compactifications of the 0B string to four dimensions and construct its open descendants, both tachyonic and non-tachyonic, in the \(T^6/Z_3\) case. Section 3 is devoted to the six-dimensional \(T^4/Z_2\) case.

## 2 Four dimensional compactifications

Let us start by considering four-dimensional compactifications on orbifolds \(T^6/Z_N\) (\(N = 3, 4, 6, 7, 8, 12\)) \[9\]. For a generic \(N\), the 0B string partition function associated to the geometrical action of the orbifold generators may be written as

\[
\mathcal{T} = \frac{1}{N} \sum_{\alpha,\beta=0}^{N-1} n_{\alpha,\beta} \{ \gamma_{\alpha,\beta} \bar{\gamma}_{-\alpha,-\beta} + \delta_{\alpha,\beta} \bar{\delta}_{-\alpha,-\beta} + \eta_{\alpha,\beta} \bar{\eta}_{-\alpha,-\beta} + \epsilon_{\alpha,\beta} \bar{\epsilon}_{-\alpha,-\beta} \} \times \]
\[
\times \prod_{i=1}^{3} (\phi^i_{\alpha,\beta} \bar{\phi}^i_{-\alpha,-\beta}) \left[ A^i_2(\tau, \bar{\tau}) \right]_{(\alpha,\beta)=(0,0)}, \tag{2.1}
\]

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For a given twist \((t_1, t_2, t_3)\) on the internal torus \(T^6 = T^2 \times T^2 \times T^2\), the contribution of the world-sheet fermions is

\[
\gamma_{\alpha,\beta} = \frac{1}{2\eta^3} \left\{ (O_2 + V_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \right] + (O_2 - V_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right] \right\},
\]

\[
\delta_{\alpha,\beta} = \frac{1}{2\eta^3} \left\{ (O_2 + V_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \right] - (O_2 - V_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right] \right\},
\]

\[
\eta_{\alpha,\beta} = \frac{1}{2\eta^3} \left\{ (S_2 + C_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right] + i(S_2 - C_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right] \right\},
\]

\[
\epsilon_{\alpha,\beta} = \frac{1}{2\eta^3} \left\{ (S_2 + C_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right] - i(S_2 - C_2) \prod_{i=1}^{3} \theta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right] \right\},
\]

while the contribution of the bosonic coordinates is

\[
\phi_{\alpha,\beta}^i = 2 \sin(\beta t_i \pi) \frac{\eta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right]}{\theta^\left[\alpha t_i \left(1/2 + \beta t_i\right)\right]} \quad \text{and} \quad \phi_{\alpha,\beta}^i = \frac{1}{\eta^2} \quad \text{if} \ \alpha t_i, \beta t_i \in \mathbb{Z},
\]

(2.2)

with \(\eta\) the Dedekind function. The notation \([\Lambda^2_2(\alpha,\beta) = (0,0)]\) means that the lattice sums are present only when the orbifold generator acts trivially on a particular direction. Finally, the numbers \(n_{\alpha,\beta}\) represent the fixed point multiplicities as given by the Lefschetz theorem.

From the partition function (2.1) one can then extract the spectrum of the theory. For generic \(N\), the twisted sector includes a tachyon, directly related to the ten dimensional tachyon of the 0B theory, that is invariant with respect to the action of the orbifold. For \(N \neq 3, 4\) one also finds tachyons in the twisted sectors. Their masses, however, are smaller in absolute value than that of the untwisted tachyon. The orbifolds \(Z_{3,4}\) are an exception, since rather than twisted tachyons the spectrum contains additional massive (for the \(Z_3\) case) or massless (for the \(Z_4\) case) scalars. These states are Kaluza-Klein excitations of the untwisted tachyon. The contribution of the internal compact scalar Laplacian to the four-dimensional field equations simply cancels the negative tachyonic “mass”, leading to massless or massive states. For these twisted states, the eigenvalues of the internal Laplacian are independent of any moduli and thus the mechanism takes place at every point in moduli space.

Since the theory is still left-right symmetric, one can mod out by \(\Omega\) and derive open descendants [11, 12]. Typically, for the “standard” Klein bottle projection the final result is a Chan-Paton gauge group that somehow doubles the one associated with the same
orientifold of the type IIB superstring, while the low-lying excitations inevitably contain tachyons both in the closed and in the open unoriented sectors. What happens if we use the sign ambiguities left over from the crosscap constraint to change the Klein bottle projection? For \( Z_N \) orbifolds with \( N \neq 3, 4 \) nothing interesting happens. The orthogonal and/or symplectic gauge groups become unitary, the untwisted open and closed tachyons are thrown away, but the twisted closed ones still survive. This is due to the fact that for a geometrical action of the orbifold generators, the associated partition function belongs to the family of charge conjugation modular invariants. This means that states in the \( \theta \)-twisted sector are paired with states in the \( \theta^{-1} \)-twisted one. As a result, they do not contribute to the Klein bottle projection for any choice of signs unless \( \theta = \theta^{-1} \). The net number of tachyons is then simply halved in the open descendant and there is no way to eliminate them. The case is completely different for the \( Z_{3,4} \) orbifolds. As we have said in the previous paragraphs, they contain only the untwisted tachyon, and therefore one can hope to eliminate it using the “exotic” Klein bottle projection. For this reason we will now consider the parameter space orbifold of the 0B theory on the \( T^6/Z_3 \) orbifold in some detail.

The 0B string partition function is simply given by (2.1) with \( N = 3 \) and \( n_{0,\beta} = 1, n_{1,\beta} = n_{2,\beta} = 27 \). The low-lying excitations comprise a tachyon, a metric tensor, two abelian vectors and 60 scalars from the untwisted sector, while each of the 27 fixed points in the twisted sectors contribute 6 additional scalars, for a total of 162 scalar fields.

In constructing the open descendants, one starts by halving the torus amplitude. Since the \( Z_3 \) action of the target space twist is L-R symmetric, the “standard” Klein bottle amplitude is

\[
\mathcal{K} = \frac{1}{6} \sum_{\beta=0,1,2} (\gamma_{0,\beta} + \delta_{0,\beta} - \eta_{0,\beta} - \epsilon_{0,\beta}) \phi_{0,\beta} [P(2i\tau_2)]_{\beta=0} ,
\]

where \( \phi_{0,\beta} = \phi_{0,\beta}^1 \phi_{0,\beta}^2 \phi_{0,\beta}^3 \), and \( [P]_{\beta=0} \) means that the lattice sum over the KK momenta is present only for \( \beta = 0 \). Eq. (2.3) contains only the conventional sum over the momentum lattice since, for generic values of the moduli of the internal torus, the condition \( p_L = \omega p_R \) (\( \omega = e^{2i\pi/3} \)) does not have any non-trivial solutions [14]. One may thus anticipate that the open sector includes only Neumann charges associated with the D9-branes. This should
be contrasted with the $Z_2$ case that we will consider in the next section, where additional contributions to $K$ signal the appearance of Dirichlet charges in the open spectrum \[11, 12\]. The light states in the projected closed-string spectrum thus comprise the untwisted tachyon, the metric tensor and 30 massless scalars from the untwisted sector, as well as 81 massless scalars from the twisted sectors.

The twisted sector of the world-sheet orbifold, to be identified with the open-string spectrum, starts with the annulus amplitude

$$\mathcal{A} = \frac{1}{6} \sum_{\beta=0,1,2} \left\{ 2(M^{(\beta)}_\chi M^{(\beta)}_\xi + M^{(\beta)}_\sigma M^{(\beta)}_\tau) \gamma_{0,\beta} \phi_{0,\beta} + \left[ (M^{(\beta)}_\chi)^2 + (M^{(\beta)}_\xi)^2 + (M^{(\beta)}_\sigma)^2 + (M^{(\beta)}_\tau)^2 \right] \delta_{0,\beta} \phi_{0,\beta} + 2(M^{(\beta)}_\chi M^{(\beta)}_\xi + M^{(\beta)}_\sigma M^{(\beta)}_\tau) \eta_{0,\beta} \phi_{0,\beta} + 2(M^{(\beta)}_\chi M^{(\beta)}_\xi + M^{(\beta)}_\sigma M^{(\beta)}_\tau) \epsilon_{0,\beta} \phi_{0,\beta} \right\} \left[ P(i\tau_A/2) \right]_{\beta=0}, \tag{2.4}$$

whose $\Omega$ projection is completed by the Möbius strip amplitude

$$\mathcal{M} = \frac{1}{6} \sum_{\beta=0,1,2} (M^{(-\beta)}_\chi + M^{(-\beta)}_\xi + M^{(-\beta)}_\sigma + M^{(-\beta)}_\tau) \hat{\delta}_{0,\alpha} \hat{\phi}_{0,\alpha} \left[ \hat{P}(i\tau_M/2 + 1/2) \right]_{\beta=0}, \tag{2.5}$$

where we have defined a suitable set of real “hatted” characters in the Möbius amplitude \[3\]. The coefficients $M^{(\beta)}_i$ are

$$M^{(\beta)}_i = n_i + \omega^\beta m_i + \bar{\omega}^\beta \bar{m}_i, \quad (i = \chi, \xi, \sigma, \tau), \tag{2.6}$$

where $n_i, m_i, \bar{m}_i$ are Chan Paton (CP) charges.

Tadpole cancellation in the transverse channel leads to the two conditions

$$M^{(0)}_\chi + M^{(0)}_\xi + M^{(0)}_\sigma + M^{(0)}_\tau = 64, \quad M^{(i)}_\chi + M^{(i)}_\xi + M^{(i)}_\sigma + M^{(i)}_\tau = -8, \quad (i = 1, 2) \tag{2.7}$$

related to untwisted and twisted massless exchanges respectively. All other tadpoles are trivially satisfied as a result of the numerical identification of conjugate charges. Together with (2.3) and (2.4), eqs. (2.7) yield a semi-simple CP gauge group

$$G_{\text{CP}} = [\text{SO}(n) \otimes \text{SO}(8-n) \otimes \text{U}(m) \otimes \text{U}(12-m)]^\otimes 2, \tag{2.8}$$
containing 8 factors, as well as

\begin{align*}
1 \text{ tachyon} & \in (\vec{F}_3, F_7) \oplus (F_4, \vec{F}_8) \oplus (F_3, \vec{F}_7) \oplus (\vec{F}_4, F_8) \oplus (F_1, F_5) \oplus (F_2, F_6), \\
3 \text{ scalars} & \in \tilde{A}_3 \oplus \tilde{A}_7 \oplus \tilde{A}_4 \oplus \tilde{A}_8 \oplus (F_1, F_3) \oplus (F_5, F_7) \oplus (F_2, F_4) \oplus (F_6, F_8) \oplus \text{(c.c.)}, \\
1 \text{ L fermion} & \in (F_7, F_8) \oplus (F_3, F_4) \oplus (F_7, F_8) \oplus (F_3, F_4) \oplus (F_1, F_2) \oplus (F_5, F_6) \oplus (\bar{F}_4, F_7) \oplus (F_4, \bar{F}_7) \oplus (F_3, F_8) \oplus (F_3, \bar{F}_8) \oplus (F_2, F_5) \oplus (F_1, F_6), \\
3 \text{ L fermions} & \in (\bar{F}_7, \bar{F}_8) \oplus (\bar{F}_3, \bar{F}_4) \oplus (F_5, F_8) \oplus (F_2, F_3) \oplus (F_6, F_7) \oplus (F_1, F_4) \oplus (\bar{F}_4, \bar{F}_7) \oplus (F_4, \bar{F}_7) \oplus (F_3, F_8) \oplus (F_3, \bar{F}_8) \oplus (F_2, \bar{F}_5) \oplus (F_1, \bar{F}_6),
\end{align*}

where \( F_i \) (\( A_i \)) stands for the fundamental (antisymmetric) representation of the \( i \)-th factor group and the bars refer to the conjugate representations. The model is free of non-abelian anomalies as a consequence of the cancellation of twisted tadpoles.

Now we can use the results of [3] to modify the Klein bottle projection and to eliminate the tachyons. There are two choices that one can make, but they differ by an overall change of chirality for the fermions in the open unoriented sector. We can thus concentrate on one of the two possibilities, say

\begin{equation}
\mathcal{K}' = \frac{1}{6} \sum_{\beta=0,1,2} \left( -\gamma_{0,\beta} + \delta_{0,\beta} - \eta_{0,\beta} + \epsilon_{0,\beta} \right) \phi_{0,\beta} \left[ P(2i\tau_2) \right]_{\beta=0} .
\end{equation}

The massless states in the projected closed-string spectrum comprise the metric tensor, 30 scalars and an abelian vector from the untwisted sector, as well as 81 massless scalars from the twisted sectors.

The open unoriented one-loop amplitudes corresponding to the projection (2.9) are given by

\begin{equation}
\mathcal{A}' = \frac{1}{6} \sum_{\beta=0,1,2} \left\{ 2 \left( M_1^{(\beta)} \bar{M}_2^{(\beta)} + \bar{M}_1^{(\beta)} M_2^{(\beta)} \right) \gamma_{0,\beta} \phi_{0,\beta} + \\
+ 2 \left( M_1^{(\beta)} \bar{M}_1^{(\beta)} + M_2^{(\beta)} \bar{M}_2^{(\beta)} \right) \delta_{0,\beta} \phi_{0,\beta} + \\
- \left[ (M_1^{(\beta)})^2 + (\bar{M}_1^{(\beta)})^2 + (M_2^{(\beta)})^2 + (\bar{M}_2^{(\beta)})^2 \right] \eta_{0,\beta} \phi_{0,\beta} + \\
- 2 \left( M_1^{(\beta)} M_2^{(\beta)} + \bar{M}_1^{(\beta)} \bar{M}_2^{(\beta)} \right) \epsilon_{0,\beta} \phi_{0,\beta} \right\} \left[ P(i\tau_A/2) \right]_{\beta=0} ,
\end{equation}

and

\begin{equation}
\mathcal{M}' = \frac{1}{6} \sum_{\beta=0,1,2} (M_1^{(-\beta)} + \bar{M}_1^{(-\beta)} + M_2^{(-\beta)} + \bar{M}_2^{(-\beta)}) \hat{\eta}_{0,\beta} \phi_{0,\beta} \left[ \hat{P}(i\tau_M/2 + 1/2) \right]_{\beta=0} ,
\end{equation}
In this second case, the coefficients that appear in the annulus and Möbius strip amplitudes are given by

\[ M_k^{(\beta)} = n_k + \omega^\beta m_k + \bar{\omega}^\beta p_k, \quad (k = 1, 2), \tag{2.12} \]

and the CP charges are all complex, so that the gauge group is of the form \( U(n) \otimes U(m) \otimes U(p) \). In order to extract the dimensions of each factor and to check the consistency of the model, one has to cancel tadpoles in the transverse channel. In this case, one does not have the option to set to zero the dilaton tadpole, and the overall size of the gauge group is therefore not determined. It has been shown in [13] that this situation may lead to interesting progress in open string model building. However, there is unique choice of gauge group leading to a tachyonic-free open spectrum. Therefore, the only relevant tadpole conditions are given by

\begin{align*}
M_1^{(0)} + \bar{M}_1^{(0)} - M_2^{(0)} - \bar{M}_2^{(0)} &= 64, \\
M_1^{(i)} + \bar{M}_1^{(i)} - M_2^{(i)} - \bar{M}_2^{(i)} &= -8, \quad (i = 1, 2), \\
M_1^{(i)} - \bar{M}_1^{(i)} - M_2^{(i)} + \bar{M}_2^{(i)} &= 0, \quad (i = 1, 2). \tag{2.13}
\end{align*}

Together with (2.10) and (2.11), and putting to zero the \( n_2, m_2, p_2 \) CP charges, one can see that the solution

\[ G_{CP} = U(8) \otimes U(12) \otimes U(12), \tag{2.14} \]

of equations (2.13) results in an open unoriented sector free of tachyons and with additional charged massless matter given by

\begin{align*}
3 \text{ scalars} &\in (\overline{8}, 12, 1) \oplus (12, 1, \overline{12}) \oplus (1, \overline{12}, 12) \oplus (\text{c.c.}), \\
1 \text{ L fermion} &\in (28, 1, 1) \oplus (\overline{28}, 1, 1) \oplus (1, 12, 12) \oplus (1, \overline{12}, \overline{12}), \\
3 \text{ L fermions} &\in (\overline{8}, 1, \overline{12}) \oplus (8, 12, 1) \oplus (1, \overline{66}, 1) \oplus (1, 1, 66).
\end{align*}

It is not difficult to see that the spectrum is anomaly free (aside from \( U(1) \) factors), as a result of the cancellation of twisted tadpoles.

In the construction of the previous models one could have also considered compactifications in the presence of a quantized NS-NS antisymmetric tensor background. It has been
shown that a $B_{\mu\nu}$ of rank $r$ reduces the dimension of the CP gauge group by a factor of $2^{r/2}$. In our case three different choices are allowed, $r = 2, 4, 6$, i.e. one can introduce a non-vanishing tensor background in one, two or three orbifold “planes”. The resulting massless open spectra, free of non-abelian anomalies, correspond to a CP group $U(8) \otimes U(4) \otimes U(4)$, with three real scalars in the representations $(\bar{8}, 4, 1) \oplus (1, 4, \bar{4}) \oplus (1, 4, 4)$ and one left fermion in the representations $(36, 1, 1) \oplus (3, 1, 4)$ for the $r = 2$ case, to a CP group $U(4) \otimes U(4)$, with three real scalars in the representations $(4, 4) \oplus (4, 0)$ and one left fermion in the representations $(4, 1) \oplus (1, 4)$ for the $r = 4$ case, and, finally, to a CP group $U(4)$ with one left fermion in the representations $10 \oplus \bar{10}$ for the $r = 6$ case.

3 Six dimensional compactifications

Six dimensional models display some similarities with the four dimensional case. The orbifolds $T^4/Z_{3,4,6}$ include in the light excitations tachyonic states in the twisted sectors that can not be removed in the open descendants by any choice of the Klein bottle projection. Only for the $Z_2$ case the twisted sector is free from tachyons, and we can then use the freedom left over from the crosscap constraint to find a suitable $\Omega$ projection in the closed unoriented sector in order to project them out. The partition function for the type 0B string theory on the orbifold $T^4/Z_2$ can be written

$$T = \frac{1}{2} \sum_{\alpha,\beta=0,1} n_{\alpha,\beta} \left[ |\gamma_{\alpha,\beta}|^2 + |\delta_{\alpha,\beta}|^2 + |\eta_{\alpha,\beta}|^2 + |\epsilon_{\alpha,\beta}|^2 \right] |\phi_{\alpha,\beta}|^2 [\Lambda_4(\tau, \bar{\tau})]_{(\alpha, \beta) = (0, 0)} , \tag{3.1}$$

where the amplitudes now read

$$\gamma_{\alpha,\beta} = \frac{1}{2\eta^2} \left\{ (O_4 + V_4) \theta^2 \left[ \frac{\alpha^2}{3} \right] + (O_4 - V_4) \theta^2 \left[ \frac{\alpha^2}{1} \right] \right\} ,$$

$$\delta_{\alpha,\beta} = \frac{1}{2\eta^2} \left\{ (O_4 + V_4) \theta^2 \left[ \frac{\alpha^2}{3} \right] - (O_4 - V_4) \theta^2 \left[ \frac{\alpha^2}{1} \right] \right\} ,$$

$$\eta_{\alpha,\beta} = \frac{1}{2\eta^2} \left\{ (S_4 + C_4) \theta^2 \left[ \frac{1}{1} \right] - (S_4 - C_4) \theta^2 \left[ \frac{1}{3} \right] \right\} ,$$

$$\epsilon_{\alpha,\beta} = \frac{1}{2\eta^2} \left\{ (S_4 + C_4) \theta^2 \left[ \frac{1}{3} \right] + (S_4 - C_4) \theta^2 \left[ \frac{1}{1} \right] \right\} ,$$
and
\[ \phi_{0,\beta} = \left( \frac{2 \sin(\beta \pi/2) \eta}{\theta^{1/2 + \alpha/2}} \right)^2. \] (3.2)

In this case the numerical coefficients are \( n_{0,\beta} = 1 \) and \( n_{1,\beta} = 16 \), where 16 is the number of fixed points in the orbifold \( T^4/Z_2 \). The low-lying excitations are then given by a tachyon, and, at the massless level, by a metric tensor, 25 2-forms and 193 scalars.

In this case the open descendants contain more sectors, related to open strings with Dirichlet boundary conditions in the internal directions, whose ends are stuck to the fixed points of the orbifold. This translates into the inclusion of a new set of charges \([11, 12]\) associated to D5-branes. Limiting our attention to the projection that removes the tachyons both from the closed and the open unoriented sectors, the relevant contributions are the Klein bottle amplitude
\[ K = \frac{1}{4} (\gamma_{0,0} + \delta_{0,0} - \eta_{0,0} + \epsilon_{0,0}) \phi_{0,0}(P + W) + \frac{16}{2} (\gamma_{1,0} - \delta_{1,0} + \eta_{1,0} - \epsilon_{1,0}) \phi_{1,0}, \] (3.3)
the annulus amplitude
\[ A = \frac{1}{4} \sum_{\beta=0,1} \left\{ 2(N_1^{(\beta)} N_2^{(\beta)} + N_1^{(\beta)} N_2^{(\beta)}) \gamma_{0,\beta} + 2(N_1^{(\beta)} N_1^{(\beta)} + N_2^{(\beta)} N_2^{(\beta)}) \delta_{0,\beta} - [(N_1^{(\beta)})^2 + (N_2^{(\beta)})^2] \eta_{0,\beta} 
- 2(N_1^{(\beta)} N_2^{(\beta)} + N_1^{(\beta)} N_2^{(\beta)}) \epsilon_{0,\beta} \right\} \phi_{0,\beta}[P]_{\beta=0}
+ \sum_{i,j=1}^{16} \left[ 2(D_1^{(\beta)i} \bar{D}_2^{(\beta)j} + \bar{D}_1^{(\beta)i} D_2^{(\beta)j}) \gamma_{0,\beta} + 2(D_1^{(\beta)i} \bar{D}_1^{(\beta)j} + D_2^{(\beta)i} \bar{D}_2^{(\beta)j}) \delta_{0,\beta}
- (D_1^{(\beta)i} D_1^{(\beta)j} + \bar{D}_1^{(\beta)i} \bar{D}_1^{(\beta)j} + D_2^{(\beta)i} D_2^{(\beta)j} + \bar{D}_2^{(\beta)i} \bar{D}_2^{(\beta)j}) \eta_{0,\beta}
- 2(D_1^{(\beta)i} D_2^{(\beta)j} + \bar{D}_1^{(\beta)i} \bar{D}_2^{(\beta)j}) \epsilon_{0,\beta} \right\} \phi_{0,\beta}[W^{ij}]_{\beta=0}
+ \sum_{i=1}^{16} \left[ (N_1^{(\beta)} \bar{D}_2^{(\beta)i}) + N_2^{(\beta)} D_1^{(\beta)i} + N_1^{(\beta)} D_2^{(\beta)i} + N_2^{(\beta)} \bar{D}_1^{(\beta)i}) \gamma_{1,\beta}
+ (N_1^{(\beta)} D_1^{(\beta)i} + \bar{N}_1^{(\beta)} D_1^{(\beta)i} + N_2^{(\beta)} D_2^{(\beta)i} + N_2^{(\beta)} \bar{D}_2^{(\beta)i}) \delta_{1,\beta}
+ (N_1^{(\beta)} D_2^{(\beta)i} + \bar{N}_1^{(\beta)} D_2^{(\beta)i} + N_2^{(\beta)} D_1^{(\beta)i} + N_2^{(\beta)} \bar{D}_1^{(\beta)i}) \eta_{1,\beta}
+ (N_1^{(\beta)} D_1^{(\beta)i} + N_2^{(\beta)} D_1^{(\beta)i} + \bar{N}_1^{(\beta)} D_2^{(\beta)i} + \bar{N}_2^{(\beta)} D_2^{(\beta)i}) \phi_{1,\beta} \right\}, \] (3.4)
and the Möbius strip amplitude
\[ M = -\frac{1}{4} \left\{ \left[ (N_1^{(0)} + \bar{N}_1^{(0)} - N_2^{(0)} - \bar{N}_2^{(0)}) \hat{\eta}_{0,0} \hat{\phi}_{0,0} \hat{P} \right] \right\}. \]
\[
- \sum_{i=1}^{16} (D_{1i} - D_{2i}) \hat{\eta}_{0,0} \hat{\phi}_{0,0} W \bigg) \\
+ \left[ \left( N_{1i} - N_{2i} \right) - \sum_{i=1}^{16} (D_{1i} - D_{2i}) \right] \hat{\eta}_{0,1} \hat{\phi}_{0,1} \bigg].
\]

Here \( W \) is the lattice sum restricted to winding states, and \( W^{ij} \) has winding numbers shifted by the distance between the fixed points \( x_i \) and \( x_j \), e.g. \( nR \rightarrow (n + x_i - x_j)R \).

The notation \( [W^{ij}]_{\beta=0} \) means that this term is present for \( \beta = 0 \), while it is simply \( \delta_{ij} \) if \( \beta \neq 0 \). The coefficients in the amplitudes for the open unoriented sector have the following expressions in terms of CP multiplicities

\[
N_{s}^{(\beta)} = n_s + (-)^\beta m_s, \quad D_{s}^{(\beta)i} = p_{s}^{i} + (-)^\beta q_{s}^{i}, \quad (\beta = 0, 1, s = 1, 2, i = 1, \ldots, 16),
\]

(3.6)

where \( n, m \ (p, q) \) are associated to D9(5)-branes. In order to extract the dimensions of the various factors of the CP gauge group, one has to cancel tadpoles in the transverse channel. Again, the dilaton tadpole has to be relaxed. The remaining tadpole conditions are

\[
N_{1}^{(0)} - N_{2}^{(0)} = 64, \quad \sum_{i=1}^{16} (D_{1i} - D_{2i}) = -64,
\]

(3.7)

from the untwisted sector, and

\[
\frac{1}{4} \left( N_{1}^{(1)} - N_{2}^{(1)} + \tilde{N}_{1}^{(1)} - \tilde{N}_{2}^{(1)} \right) + \left( D_{1}^{(1)i} - D_{2}^{(1)i} + \tilde{D}_{1}^{(1)i} - \tilde{D}_{2}^{(1)i} \right) = 0, \quad \forall i = 1, \ldots, 16,
\]

(3.8)

from the twisted sector. Together with (3.4) and (3.5), we can see that the solution

\[
G_{CP} = \left[ U(16) \otimes U(16) \right]^{\otimes 2},
\]

(3.9)

of equations (3.7) and (3.8) results in a model free of tachyons both in the closed and in the open unoriented sectors. The other massless excitations comprise a metric tensor, 4 anti-self-dual 2-forms, 20 self-dual 2-forms and 99 scalars from the closed sector, as well as

4 scalars \( \in (16, \overline{16}; 1, 1) \oplus (\overline{16}, 16; 1, 1) \oplus (1, 1; 16, \overline{16}) \oplus (1, 1; \overline{16}, 16), \)
2 L fermions \( \in \) (136 + \overline{136}, 1; 1, 1) \( \oplus \) (1, 136 + \overline{136}; 1, 1) 
\( \oplus \) (1, 1; 136 + \overline{136}, 1) \( \oplus \) (1, 1, 136 + \overline{136}),

2 R fermions \( \in \) (16, 16; 1, 1) \( \oplus \) (16, \overline{16}; 1, 1) \( \oplus \) (1, 16, 16) \( \oplus \) (1, 1; \overline{16}, \overline{16}),

from the open untwisted sector, and

2 scalars \( \in \) (16, 1; 16, 1) \( \oplus \) (1, 16; 1, \overline{16}) \( \oplus \) (16, 1, 1) \( \oplus \) (1, 1; \overline{16}, 1),

1 R fermion \( \in \) (16, 1; 1, \overline{16}) \( \oplus \) (1, 16; 16, 1) \( \oplus \) (\overline{16}, 1; 1, \overline{16}) \( \oplus \) (1, \overline{16}; \overline{16}, 1),

from the open twisted sector, where the fermions are symplectic Majorana-Weyl. In this case tadpole conditions guarantee the cancellation of the irreducible part of the anomaly polynomial, while a generalized Green-Schwarz mechanism \[15\] is at work to cancel the residual factorized anomaly:

\[
\mathcal{I}_8 = -\frac{1}{8} \left[ \frac{1}{2} \text{tr} R^2 - (\text{tr} F_1^2 + \text{tr} F_2^2 + \text{tr} F_3^2 + \text{tr} F_4^2) \right]^2 + \\
+ \frac{1}{8} \left[ \frac{1}{4} (\text{tr} F_1^2 + \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2)^2 + \frac{3}{2} (\text{tr} F_1^2 + \text{tr} F_4^2)^2 \\
+ \frac{3}{2} (\text{tr} F_2^2 + \text{tr} F_3^2)^2 + \frac{5}{4} (\text{tr} F_1^2 - \text{tr} F_2^2 + \text{tr} F_3^2 - \text{tr} F_4^2)^2 \right] + \\
- \frac{1}{24} \left\{ -(\text{tr} F_1 + \text{tr} F_2 + \text{tr} F_3 + \text{tr} F_4) \left[ \frac{1}{16} \text{tr} R^2 (\text{tr} F_1 + \text{tr} F_2 + \text{tr} F_3 + \text{tr} F_4) + \\
-(\text{tr} F_3 + \text{tr} F_2 + \text{tr} F_3 + \text{tr} F_4) \right] + \\
+(\text{tr} F_1 + \text{tr} F_2 - \text{tr} F_3 - \text{tr} F_4) \left[ \frac{1}{16} \text{tr} R^2 (\text{tr} F_1 + \text{tr} F_2 - \text{tr} F_3 - \text{tr} F_4) + \\
-(\text{tr} F_1 + \text{tr} F_2 - \text{tr} F_3 - \text{tr} F_4) \right] + \\
+3(\text{tr} F_1 - \text{tr} F_2 - \text{tr} F_3 + \text{tr} F_4) \left[ \frac{1}{16} \text{tr} R^2 (\text{tr} F_1 - \text{tr} F_2 - \text{tr} F_3 + \text{tr} F_4) + \\
-(\text{tr} F_1 - \text{tr} F_2 - \text{tr} F_3 + \text{tr} F_4) \right] + \\
+5(\text{tr} F_1 - \text{tr} F_2 + \text{tr} F_3 - \text{tr} F_4) \left[ \frac{1}{16} \text{tr} R^2 (\text{tr} F_1 - \text{tr} F_2 + \text{tr} F_3 - \text{tr} F_4) + \\
-(\text{tr} F_1 - \text{tr} F_2 + \text{tr} F_3 - \text{tr} F_4) \right] \right\}.
\]

In conclusion, we have shown that the crosscap constraint is a very powerful tool in orbifold compactifications of the 0B string theory as well. The freedom in the choice of the Klein bottle projection allows one to remove both closed and open tachyons from
the spectrum of the open descendants, while maintaining the consistency of the models. Typically, the “exotic” Klein bottle projection yields only unitary gauge groups, and thus in four dimensions leads to chiral vacua.

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