A New Four-Wing Chaotic System Generated by Sign Function

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Abstract
In the paper, a new chaotic system is obtained by adding a Sign Function to a three-dimensional chaotic system. Its some basic characteristics including the equilibrium point, phase trajectory, bifurcation diagram, Lyapunov exponents and so on, are subsequently calculated. Moreover, the dynamic characteristics of the new chaotic system is also analyzed with the variation of the system parameters. In the end, the paper design an analog circuit to implement the chaotic system, the results from the circuit are consistent with those from the numerical analysis, and thus the chaotic characteristics of the new system is verified physically. The new chaotic system can provide a new model for engineering applications.

Keywords: four-wing chaotic system; sign function; dynamic characteristics

1. Introduction
Chaos is a new discipline developed over the past half a century, the phenomenon of chaos is ubiquitous in our nature. In 1963, Lorenz, a famous meteorologist, found a phenomenon of chaos when he studied meteorological changes. Then, he published a paper named "Deterministic nonperiodic flow" [1]. And the curtain of the chaotic phenomenon slowly began to open. After more than 10 years of development, many landmark papers have been published. For example, the concept of strange attractors was proposed by French physicist Ruell and Dutch mathematician Takens[2]. The theory of "Period three implies chaos" has also been proposed by the Chinese-American Tianyan Li and American mathematician Yorke[3]. The scalability and universal constants in the phenomenon of double-periodic bifurcations are found by Feigenbaum[4]. All these important discoveries have made important contributions to the development of chaos research.

With the deepening of the research on chaotic theory, it is gradually realized that the characteristics of the multi-wing chaotic systems are more complicated than ones of the single-wing or two-wing chaotic systems, and multi-wing chaotic systems have a wide range of applications. So, there are many different research which dedicate to construct some multi-wing chaotic systems, and they present some different methods such as the absolute value [5], the triangular wave sequence [6] and so on.

The multi-wing chaotic systems may be obtained by adding the multi-pulse function with the sign function to some one-wing or two-wing chaotic systems. In the paper, a new multi-wing chaotic system is firstly constructed by adding the multi-pulse function to a three-dimensional chaotic system. Then chaotic characteristics of the new system is showed by analyzing phase trajectory diagram, bifurcation diagram, equilibrium point and Lyapunov exponent approach. Finally, the new chaotic system is studied by the real analog circuit. All the results verify that the chaotic characteristics exist in the new system.

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2 A New Four-Wing Chaotic System

2.1 Sign Function

Generally, sign function can be described by

\[
f(x) = \begin{cases} 
1 & x > 0 \\
0 & x = 0 \\
-1 & x < 0 
\end{cases}
\] (1)

where \( x \in \mathbb{R} \).

The sign function is a continuous function except for the only jump break point \( x = 0 \). Because \( \text{sgn}(-x) = -\text{sgn}(x) \), the sign function is an odd function.

The multi-pulse function \( p(x) \) adopted in the paper is

\[
p(x) = m \ast (\text{sgn}(x + B_1) - \text{sgn}(x - B_2) + \text{sgn}(x + B_2) - \text{sgn}(x - B_2) - 4)
\] (2)

where \( m, B_1 = 0.8 \) and \( B_2 = 1.17 \) is a positive parameter. It will be used to generate multi-wing chaotic attractors in this paper.

2.2 A New Chaotic System

The fourth term of the 4D hyper-chaos system in Ref [8] is firstly removed to constitute a 3D chaotic system. As follows:

\[
\begin{align*}
\dot{x} &= ax + byz \\
\dot{y} &= cy + dxz \\
\dot{z} &= k z + exy
\end{align*}
\] (3)

And then add the multi-pulse function \( p(x) \) in the second term of system (3), and finally a new chaotic system with sign functions is obtained. As follows:

\[
\begin{align*}
\dot{x} &= ax + byz \\
\dot{y} &= cy + dxz + p(x) \\
\dot{z} &= k z + exy
\end{align*}
\] (4)

where \( a, b, c, d, e \) and \( k \) are system parameters, \( x, y \) and \( z \) are variables, and \( p(x) \) is the multi-pulse function(2). the new system exhibits the characteristics of the four-wing chaotic attractor, when the initial value is \((1,1,1)\) and \( a = 8, b = -1, c = -40, d = 1, e = 1, k = -14 \).

The corresponding changes of multi-pulse function \( p(x) \) will happen when selecting different free variables \( m \). So, in the paper, some characteristics of the new chaotic system will be discussed when free variable \( m \) changes.

3 The Dynamics Of The New Four-Wing Chaotic System

3.1 Equilibrium point analysis

The multi-pulse function (2) with sign function is segmental function. So, the method of segment calculation is used to analyze the equilibrium point of the new chaotic system. The interval of \( x \) is as follows:

\[
D_1 = \{(x, y, z) | x < -1.17\} \\
D_2 = \{(x, y, z) | -1.17 < x < -0.8\} \\
D_3 = \{(x, y, z) | -0.8 < x < 0.8\} \\
D_4 = \{(x, y, z) | 0.8 < x < 1.17\} \\
D_5 = \{(x, y, z) | 1.17 < x\}
\]

The values of multi-pulse function \( p(x) \) in every interval of \( x \) is follow:

\[
p(x) = m \ast \left[ \text{sgn}(x + 0.8) - \text{sgn}(x - 0.8) + \text{sgn}(x + 1.17) - \text{sgn}(x - 1.17) - 4 \right]
\]

\[
= \begin{cases} 
-4m & x < -1.17 \\
-2m & -1.17 \leq x < -0.8 \\
0 & -0.8 \leq x < 0.8 \\
-2m & 0.8 \leq x < 1.17 \\
-4m & 1.17 \leq x 
\end{cases}
\] (5)

And thus equilibrium points of the system (4) are shown in Table 1.

| X | p(x) | Equilibrium point |
|---|---|---|
| D1 | -4m | \( e_1 \left( \frac{m}{30} \right) \) |
| D2 | -2m | \( e_1 \left( \frac{m}{30} \right) \) |

Table 1 The equilibrium point of equation (4)
3.2 Phase trajectory

Here, When $a = 8, b = -1, c = -40, d = 1, e = 1$ and $k = -14$. The phase trajectory of equation (3) is shown in Fig 1.

![Phase trajectory of equation (3)](image)

Fig 1 The phase trajectory of equation (3)

When $a = 8, b = -1, c = -40, d = 1, e = 1, k = -14$ and $m = 8$, the phase trajectory of equation (4) is shown in Fig 2.

![Phase trajectory of equation (4)](image)

Fig 2 The phase trajectory of equation (4)

Comparing chaotic attractors in Fig 1 with ones in Fig 2, a conclusion will be obtained that multi-wing chaos can be generated by adding multi-pulse function with sign function to equation (3).

3.3 Bifurcation diagram

Bifurcation diagram can indicate the performance of chaotic system with the variation of parameter. The bifurcation diagram of equation (4) is shown in Fig 3.

![Bifurcation diagram](image)

Fig 3 The bifurcation diagram of equation (4)

In the Fig 3, the bifurcation diagram start to enter into much sparse condition when $m = 45$. That is, the bifurcation gradually varies from dense points to several lines, from several lines to one line, respectively. Generally, the system will be chaotic when points in bifurcation diagram are dense. To the contrary, the system is quasi periodicity or cycle when points in bifurcation diagram is several lines or one line.

3.4 Lyapunov exponent

The characteristic of chaotic motion is always analyzed by calculating lyapunov exponent. Therefore, The lyapunov exponents of equation (4) is also computed and shown in Fig 4.

![Lyapunov exponent](image)

Fig 4 Lyapunov exponent of equation (4)

If the biggest one of lyapunov exponents is larger than zero, the equation will show the character of chaos. So, in Fig 4, chaotic dynamics of system (4) can be observed when $m \in (0, 44)$, and periodic dynamics of system (4) can be obtained when $m > 44$. The phenomenon can also be described by phase trajectory, as shown in Fig 5-6.

![Chaotic dynamics](image)

Fig 5 Chaotic dynamics of equation (4), $m = 25$

![Periodic dynamics](image)

Fig 6 Periodic dynamics of equation (4), $m = 55$

4 Analog Circuit for The New Chaotic System

The analog circuit for the new chaotic systems is designed by operational amplifier, multiplier, resistor and
capacitance, as showed in Fig 7. The analog circuit for multi-pulse function is consisted of operational amplifier and resistor, as showed in Fig 8. The experiment results is obtained, as showed in Fig 9. The results are in agreement with the results of the numerical analysis, which physically proves the system (4).

Fig 7 The analog circuit for system (4)

Fig 8 The analog circuit for multi-pulse function

(a) $x - z$ phase plane, (b) $y - z$ phase plane.

Fig 9 Experimental results by using the Multisim.

5 Conclusion

In the paper. Firstly, the basic properties of the sign function is introduced. Secondly, a multi-pulse function $p(x)$ is made up of sign function, and the new chaotic system with multi-pulse function is designed. Third, by analyzing the dynamic characteristics of the system such as phase trajectory, bifurcation diagram and Lyapunov exponent, the chaotic characteristics of the new system are verified. Finally, it can be concluded that the multi-pulse function can produce four-wing chaotic attractor in the chaotic system. Because the chaotic system with sign function can generate complex character of chaos, it is meaningful for the chaotic system with sign function to be used in encryption communication.

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