Abstract We estimate the scattering amplitude of the process $p\bar{p} \rightarrow D^0\bar{D}^0$ within a double-handbag framework where transition distribution amplitudes, calculated through an overlap representation, factorize from a hard subprocess. This process will be measured in the PANDA experiment at GSI-FAIR.

1 Introduction

The PANDA detector [1] at the Facility for Antiproton and Ion Research (FAIR) in Darmstadt, Germany, will provide the ideal experimental setup to study several exclusive final channels in proton-antiproton collisions. Also the measurement of the production of heavy hadron pairs emerging from $p\bar{p}$-annihilations is planned, for which there is thus the need to have theoretical input. In Ref. [2] we have studied the meson-pair production $p\bar{p} \rightarrow D^0\bar{D}^0$ within a double handbag approach in the same line as the study of the process $p\bar{p} \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ in Ref. [3], where arguments have been given in favor of a generalization of the handbag approach, the validity of which has been demonstrated in the case of deeply virtual Compton scattering and meson production. We argue that, having the heavy $c$-quark mass as large intrinsic scale and assuming restricted parton virtualities and intrinsic transverse momenta, the $p\bar{p} \rightarrow D^0\bar{D}^0$ amplitude factorizes into a hard subprocess amplitude and soft hadronic transition matrix elements. Neglecting intrinsic proton charm contributions, the production of the $\bar{c}c$-pair can only occur in the partonic subprocess. The hadronic transition matrix elements are transition distribution amplitudes [4], which generalize the concept of generalized parton distributions for 3-quark operators. We have developed an overlap representation according to Ref. [5] to model the hadronic transition matrix elements in terms of hadronic light-cone wave functions. In our studies we only consider the valence Fock-state components of the hadrons and we describe the proton as a quark-diquark system, where we only
The assignment of particle momenta and helicities can be seen in Fig. 1. We work in a symmetric center-of-momentum system (CMS) in which the 3-axis is chosen to be along the hadronic diquark picture for the proton, we consider the partonic subprocess. The double handbag mechanism for the hadronic matrix elements of partonic field operators. Specifically, we investigate the process where we have introduced the skewness parameter which parameterizes the relative momentum transfer into longitudinal light-cone plus direction.

We consider the process within a perturbative QCD motivated framework where the hadronic amplitude can be split up into a hard partonic subprocess and into soft hadronic matrix elements of partonic field operators. Specifically, we investigate the process in a double handbag mechanism as shown in Fig. 1. In such a framework only the minimal number of hadronic constituents which are required to convert the initial $p\bar{p}$ into the final $D\bar{D}^0$ pair actively take part in the partonic subprocess. Working in a quark-scalar diquark picture for the proton, we consider the partonic subprocess $S[ud][S[ud]] \rightarrow \bar{c}c$. The remaining partons inside the parent hadrons act as spectators. In order to produce the heavy $\bar{c}c$-pair in the partonic subprocess the gluon has to be a highly virtual one. The $c$-quark mass serves as a natural hard scale, allowing us to treat $S[ud][S[ud]] \rightarrow \bar{c}c$ perturbatively. Thus, the hadronic $p\bar{p} \rightarrow D\bar{D}^0$ amplitude can be written as

$$M_{\mu\nu} = \int d^4k_1 \theta(k_1^+) \int \frac{d^4z_1}{(2\pi)^4} e^{i\bar{k}_1z_1} \int d^4k_2 \theta(k_2^-) \int \frac{d^4z_2}{(2\pi)^4} e^{i\bar{k}_2z_2} \times (D^0: p^\mu | \mathcal{F} \Phi^+(-z_1/2) \Phi^S[ud]^+ (+z_1/2) | p: p, \mu) \mathcal{H}^L(k_1, k_2) \times (\bar{D}^0: q^\nu | \mathcal{F} \Phi^S[ud]^+ (+z_2/2) \mathcal{F}^* (-z_2/2) | \bar{p}: q, v),$$

(2)
where we have omitted color and spinor labels for the ease of writing. \( \hat{H}(\hat{k}_1, \hat{k}_2) \) denotes the hard scattering kernel of the \( S[ud]S[ud] \rightarrow \bar{c}c \) subprocess. The \( p \rightarrow D^0 \) transition is written as

\[
\int \frac{dz_1}{(2\pi)^2} e^{ik_1z_1} \langle D^0 : p' \mid T\Psi^c(-z_1/2)\Phi^{\bar{u}d}(+z_1/2) \mid p : p, \mu \rangle, \tag{3}
\]

which is a Fourier transform of a hadronic matrix element of a time-ordered, bilocal product of a \( c \)-quark operator and an \( S[ud] \)-diquark operator \( (T \) denotes the time-ordering of the fields). In Eq. (3) \( \Phi^{\bar{u}d}(+z_1/2) \) takes out the \( S[ud] \) diquark that enters the hard subprocess from the proton state \( | p : p, \mu \rangle \) at space-time point \( +z_1/2, \Psi^c(-z_1/2) \) reinserts the produced \( \bar{c} \) quark into the remainders of the proton at space-time point \( -z_1/2 \), which then gives the final \( | D^0 : p' \rangle \) state. The \( \bar{p} \rightarrow D^0 \) transition is treated in an analogous way.

Given the heavy quark mass as a hard scale and taking into account the physically plausible assumption that the partons are almost on mass-shell and their intrinsic transverse momenta are smaller than a typical hadronic scale of the order of 1 GeV the transverse and minus (plus) components of the active (anti)parton momenta are small as compared to their plus (minus) components. The parton momenta are then approximately proportional to the hadron momenta and one can perform the integrations over \( k_1^\perp, k_2^\perp, \vec{k}_{1\perp} \) and \( \vec{k}_{2\perp} \) in the convolution integral. Moreover, the field operators in the hadronic matrix elements are forced to have a light-like distance and thus the time ordering of the fields in the hadronic matrix elements can be dropped. The \( p\bar{p} \rightarrow D^0 D^0 \) amplitude then reads

\[
M_{\mu\nu} = \int \frac{dz_1}{2\pi} \frac{dz_2}{2\pi} e^{ik_1z_1} \int \frac{d\vec{k}_{1\perp}}{2\pi} \theta(\vec{k}_{1\perp}) \int \frac{d\vec{k}_{2\perp}}{2\pi} \theta(\vec{k}_{2\perp}) \int \frac{dz_3}{2\pi} e^{ik_2z_2} \\
\times \langle D^0 : p' \mid T \Psi^c(-z_1/2)\Phi^{\bar{u}d}(+z_1/2) \mid p : p, \mu \rangle \hat{H}(\vec{k}_1, \vec{k}_2) \\
\times \langle D^0 : q' \mid \Phi^{\bar{u}d}(+z_2/2)\bar{\Psi}^c(-z_2/2) \mid p : q, \nu \rangle, \tag{4}
\]

with the \( p \rightarrow D^0 \) transition matrix element

\[
p^+ \int \frac{dz_1}{2\pi} e^{ik_1z_1} \langle D^0 : p' \mid T\Psi^c(-z_1/2)\Phi^{\bar{u}d}(+z_1/2) \mid p : p, \mu \rangle \tag{5}
\]

and an analogous one for the \( \bar{p} \rightarrow D^0 \) transition.

### 3 Hadronic transition matrix elements

Using the same projection techniques as in Ref. [3] we pick out the dominant components of the \( c \)-quark field

\[
\Psi^c(-z_1/2) = \frac{1}{2k_1} \sum_{\lambda_1} \bar{v}(k_1', \lambda_1') \left( \bar{v}(k_1', \lambda_1') \gamma^\mu \Psi^c_{\text{good}}(-z_1'/2) \right). \tag{6}
\]

Here \( \Psi^c_{\text{good}} \) contains the dynamically independent components of the \( c \)-quark field operator. With the help of Eq. (6) and an analogous one for the antiquark the \( p\bar{p} \rightarrow D^0 D^0 \) amplitude
same helicity as the proton itself and the \( \bar{h} \) helicities. The corresponding bound-state (light-cone) wave functions of the proton and the

\[
\text{we assume the bound-state wave functions to be pure s-wave, such that the parton helicities}
\]

\[
\text{resulting hadron momentum. Working out the wave-function overlaps as in Ref. [2] and inserting the}
\]

\[
\text{total hadron momentum but only on the relative parton momenta with respect to the parent}
\]

\[
\text{hadron momentum. Working out the wave-function overlaps as in Ref. [2] and inserting the}
\]

\[
\text{resulting p \to D^0 and } \bar{p} \to D^0 \text{ transition matrix elements into Eq. (7) we obtain}
\]

\[
\begin{align*}
M_{\mu\nu} &= \frac{1}{4(\hat{p}^+)^2} \sum_{\lambda'_1, \lambda'_2} \int d\tilde{x}_1 \int d\tilde{x}_2 H_{k'_1, k'_2}(\tilde{x}_1, \tilde{x}_2) \frac{1}{\tilde{x}_1 - \xi} \frac{1}{\tilde{x}_2 - \xi} \\
&\times \tilde{v}(k'_1, \lambda'_1) \gamma^+ \tilde{p}^+ \int \frac{d\zeta}{2\pi} e^{i\tilde{q}^+ \cdot \zeta} \langle D^0 : p' | \Psi_{\text{good}}(-z_1^+ / 2) \Phi_{\text{had}}^{[ud]}(+z_1^+ / 2) | p : p, \mu \rangle \\
&\times \tilde{q}^- \int \frac{d\zeta}{2\pi} e^{i\tilde{q}^- \cdot \zeta} \langle D^0 : q' | \Phi_{\text{had}}^{[ud]}(+z_2^+ / 2) \bar{\Psi}_{\text{good}}(-z_2^+ / 2) | \bar{p} : q, \nu \rangle \gamma^- u(k_2', \lambda'_2),
\end{align*}
\]

where we have defined \( H_{k'_1, k'_2}(\tilde{x}_1, \tilde{x}_2) := \bar{u}(k'_2, \lambda'_2) \tilde{H}(\tilde{x}_1, \tilde{x}_2) v(k'_1, \lambda'_1) \) and introduced the average momentum fractions \( \tilde{x}_1 = \tilde{k}_1^+ / \tilde{p}^+ \) and \( \tilde{x}_2 = \tilde{k}_2^+ / \tilde{q}^- \). In order to make predictions

\[
\begin{align*}
\text{one has to model the } p \to D^0 \text{ and } \bar{p} \to D^0 \text{ transition matrix elements. We will do that by means of an overlap formalism in terms of light-cone wave functions, which has been}
\end{align*}
\]

first developed in Ref. [5] to represent the generalized parton distributions of the proton. We

\[
\text{consider the proton and the } D^0 \text{ as } | \bar{S}[ud] u \rangle \text{ and } | \bar{c} u \rangle \text{ (bound)states, respectively. In addition}
\]

\[
\text{we assume the bound-state wave functions to be pure s-wave, such that the parton helicities}
\]

\[
\text{have to add up to the total hadron helicity. Thus, the } u \text{ quark inside the proton has to have the}
\]

\[
\text{same helicity as the proton itself and the } \bar{c} \text{ and } u \text{ quark inside the } D^0 \text{ have to have opposite}
\]

\[
\text{helicities. The corresponding bound-state (light-cone) wave functions of the proton and the}
\]

\[
\text{are denoted by } \Psi_p \text{ and } \Psi_D, \text{ respectively. Those wave functions do not depend on the}
\]

\[
\text{total hadron momentum but only on the relative parton momenta with respect to the parent}
\]

\[
\text{hadrorn momentum. Working out the wave-function overlaps as in Ref. [2] and inserting the}
\]

\[
\text{resulting } p \to D^0 \text{ and } \bar{p} \to D^0 \text{ transition matrix elements into Eq. (7) we obtain}
\]

\[
\begin{align*}
M_{\mu\nu} &= 2\mu \nu \int d\tilde{x}_1 \int d\tilde{x}_2 H_{-\mu, -\nu}(\tilde{x}_1, \tilde{x}_2) \frac{1}{\sqrt{\tilde{x}_1^2 - \bar{\xi}^2}} \frac{1}{\sqrt{\tilde{x}_2^2 - \xi^2}} \\
&\times \int \frac{d^2 \tilde{k}_1}{16\pi^2} \Psi_D(\tilde{x}_1, \xi) \tilde{K}_1(\tilde{k}_1, \tilde{x}_1, \xi) \psi_\mu(\tilde{x}_1, \xi) \tilde{K}_1(\tilde{k}_1, \tilde{x}_1, \xi) \\
&\times \int \frac{d^2 \tilde{k}_2}{16\pi^2} \Psi_D(\tilde{x}_2, \bar{\xi}) \tilde{K}_2(\tilde{k}_2, \tilde{x}_2, \bar{\xi}) \psi_\mu(\tilde{x}_2, \bar{\xi}) \tilde{K}_2(\tilde{k}_2, \tilde{x}_2, \bar{\xi}).
\end{align*}
\]

Quantities with a tilde (hat) relate to a frame where the incoming (outgoing) hadron has vanishing x- and y-momentum component.\(^1\)

4 “Peaking approximation” and hard scattering amplitude

The wave function for the heavy D-meson is strongly peaked around \( x_0 \approx m_c / M \) with respect to its momentum-fraction dependence [2,3]. This behaviour is also reflected in the overlap representation of the hadronic transition matrix elements. It means that the kinematical regions close to the peak position contribute most to the \( \tilde{x}_1 \) integrations in Eq. (8). Thus it is justified to replace the momentum fractions appearing in the hard scattering amplitude by the value of the peak position. After doing that it is possible to pull the hard subprocess amplitude out of the convolution integral, which is then rendered solely to an integral

\(^1\) With Eqs. (8) and (9) we correct Eqs. (56) and (57) in Ref. [3] by a missing factor 4. Correspondingly the cross sections given in Ref. [3] have to be multiplied with a factor 16.
over the hadronic transition matrix elements. After applying this peaking approximation the \( p\bar{p} \to D\bar{D}^0 \) amplitude simplifies to

\[
M_{\mu\nu} = 2\mu v H_{-\mu,-\nu}(x_0, x_0) \left[ \int d\xi \frac{1}{\sqrt{\xi^2 - \xi_0^2}} \int \frac{d^2 \kappa}{16\pi^3} \times \psi_D(\vec{\kappa}, \vec{\xi}, \vec{\kappa}_{\perp}, \vec{\xi}_{\perp}) \psi_p(\vec{\xi}, \vec{\xi}_0, \vec{\kappa}_{\perp}, \vec{\xi}_{\perp}) \right]^2. \tag{9}
\]

The hard \( S[ud] \bar{S}[ud'] \to \bar{c}c \) amplitudes can now be calculated by applying the usual Feynman rules augmented with the Feynman rules for diquarks \([6,7]\); we obtain

\[
H_{++} = +4\pi a_s (\vec{\kappa}^2 s) F_1(\vec{\kappa}^2 s) \frac{42M}{\sqrt{3}} \cos \theta, \quad H_{+-} = -4\pi a_s (\vec{\kappa}^2 s) F_1(\vec{\kappa}^2 s) \frac{4}{9} \sin \theta, \quad H_{-+} = -4\pi a_s (\vec{\kappa}^2 s) F_1(\vec{\kappa}^2 s) \frac{4}{9} \sin \theta. \tag{10}
\]

The form factor \( F_1(\vec{\kappa}^2 s) \) accounts for the composite nature of the diquark at the \( SgS \)-vertex \([8]\).

5 Modelling the \( p \to D^0 \) transition

In order to end up with an overlap representation of the \( p \to D^0 \) transition we have to specify the valence Fock state light-cone wave functions for the proton and the \( D^0 \). According to Refs. \([2,3]\) we take

\[
\psi_p(\vec{\xi}, \vec{\kappa}_{\perp}) = N_p e^{-a_p^2 \vec{\kappa}_{\perp}^2 / (2\pi)^2} \quad \text{and} \quad \psi_D(\vec{\kappa}, \vec{\xi}, \vec{\kappa}_{\perp}, \vec{\xi}_{\perp}) = N_D e^{-a_D^2 \vec{\kappa}_{\perp}^2 / (2\pi)^2} e^{-a_D^2 M^2 (\vec{x}_0^2 - \xi_0^2) / (2\pi)^2} \tag{11}
\]

as light-cone wave functions for the proton and the \( D^0 \), respectively. The light-cone wave function for the \( D^0 \) generates the peak around \( x_0 \) with the help of the mass exponential. Each of the wave functions has two free model parameters, the normalization constant \( N_p/D \) and the transverse size parameter \( a_p/D \). For the proton we choose \( a_p = 1.1 \text{GeV}^{-1} \) and \( N_p = 61.8 \text{GeV}^{-2} \) which amounts to \( (\kappa_{\perp})^2_p = 280 \text{MeV} \) and the valence-Fock-state probability \( P_0 = 0.5 \). For the \( D^0 \) we take \( N_D = 55.2 \text{GeV}^{-2} \) and \( a_p = 1.1 \text{GeV}^{-1} \), which leads to \( f_D = 206 \text{MeV} \) (cf. Ref. \([10]\)) and the valence-Fock-state probability \( P_0 = 0.9 \). In Fig. 2 we show results for the overlap integral occurring within the square- function parametrization introduced above.

6 Cross Sections

The differential \( p\bar{p} \to D\bar{D}^0 \) cross section reads

\[
\frac{d\sigma_{p\bar{p} \to D\bar{D}^0}}{dt} = \frac{1}{16\pi} \frac{1}{s^{1/2} - 1} \frac{1}{4m^2/s} \sigma_0 \quad \text{with} \quad \sigma_0 := \frac{1}{2} \sum_{\mu,\nu} |M_{\mu\nu}|^2. \tag{12}
\]

We show \( d\sigma_{p\bar{p} \to D\bar{D}^0}/dt \) versus \( |t'| = |t - t_0| \) \( (t_0 = t(\theta = 0)) \) in the left panel of Fig. 3 for Mandelstam \( s = 15 \text{GeV}^2 \). The differential cross section is strongly decreasing with increasing \( |t'| \). This behaviour comes from the decrease of the model overlap with increasing CMS scattering angle \( \theta \), cf. Fig. 2. Its decrease becomes even more pronounced for higher values of Mandelstam \( s \). In the right panel of Fig. 3 we show the integrated cross section \( \sigma_{p\bar{p} \to D\bar{D}^0} \) versus Mandelstam \( s \), whose magnitude is in the range of a few nb.
Fig. 2 The wave function overlap occurring in Eq. (9) versus $\bar{x}$ at CMS scattering angles $\theta = 0^\circ$ (left) and $\theta = 90^\circ$ (right) for $s = 10, 20, 30$ GeV$^2$ (dotted, dashed, solid).

Fig. 3 The differential cross section $d\sigma_{p\bar{p}\rightarrow D_0D^0}/dt$ for $s = 15$ GeV$^2$ (left) as a function of $|t'|$ and the integrated cross section (right) as a function of $s$.

7 Summary

We have investigated the process $p\bar{p}\rightarrow D_0D^0$ within a double handbag approach where the hard scale is given by the heavy $c$-quark mass. We have argued that under physically plausible assumptions the $p\bar{p}\rightarrow D_0D^0$ amplitude factorizes into a hard subprocess on the partonic level and transition distribution amplitudes. To model the latter we have constructed an overlap representation in terms of hadronic light-cone wave functions. Our predictions for the differential and integrated $p\bar{p}\rightarrow D_0D^0$ cross section should now be confronted with future experimental data from the PANDA experiment at FAIR.

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