On the short distance nonperturbative corrections in heavy quark expansion

S. Arunagiri

Department of Nuclear Physics, University of Madras, Guindy Campus, Chennai 600 025, Tamil Nadu, INDIA

We study the corrections due to renormalons to the heavy hadron decay width. The renormalons contribution estimated in terms of finite gluon mass based on the assumption that the gluon mass represents the short distance nonperturbative effects in the standard OPE (and hence in the heavy quark expansion). We found that the corrections are about 10% of the leading decay rate. We point out the implications for the assumption of quark-hadron duality in heavy quark expansion.

1. Introduction

The divergence of the perturbation theory at large order brings in an ambiguity to physical quantities specified at short distances. According to the present understanding, the ambiguity is given by a class of renormalon diagrams which are chain of $n$-loops in a gluon line. The phenomenon is deeply connected with the operator product expansion (OPE). The perturbative part of the OPE receives the renormalon corrections. Since in the OPE the first power-suppressed nonperturbative term is absent and the renormalon corrections constitute short distance nonperturbative effect, they are more significant than large order corrections.

The phenomenology of the power corrections is thus significant also for the heavy quark expansion (HQE) which describes the inclusive decays of heavy hadrons by an expansion in the inverse powers of the heavy quark mass, $m_Q$. As the inclusive decay rate of heavy hadrons scales like the fifth power of the heavy quark mass, the power corrections arise due to momenta smaller than the heavy quark mass. However, these IR renormalons would, being nonperturbative effect, have greater influence in the HQE prediction of quantities of interest. These short distance nonperturbative effects can be sought for explaining the smaller lifetime of $\Lambda_b$. We should note that these power corrections to heavy quark decay rate represents the breakdown of the quark-hadron duality. Therefore, it may shed light on the working of the assumption of quark-hadron duality in the heavy quark expansion.

In this talk, we present the study on the renormalon corrections to heavy hadron decay rate at the leading order, assuming that the nonperturbative short distance corrections given by the gluon mass that is much larger than the QCD scale, $\lambda^2 \gg \Lambda_{QCD}^2$. We carry out the analysis for both $B$ meson and $\Lambda_b$ heavy baryon. Our study shows that the short distance nonperturbative corrections to the baryon and the meson differ by a small amount which is significant for the smaller lifetime of the $\Lambda_b$. In both the cases, these duality violating corrections are of the order of 10%. In the next section, the significance of the renormalons contribution is elucidated. The estimation of the $\lambda^2$ value for $B$ and $\Lambda_b$, as renormalons corrections, using QCD sum rules is presented in section 3. In view of the predicted $\lambda^2$ values, the inclusive decay widths and the implications for quark-hadron duality are discussed in section 4, followed by concluding remarks in section 5.

2. Power Corrections

For the correlator of hadronic currents $J$:

$$\Pi(Q^2) = i \int d^4xe^{iQx} \langle 0 | T \{ J(x) J(0) \} | 0 \rangle$$  (1)
where \( Q^2 = -q^2 \), the standard OPE is expressed as

\[
\Pi(Q^2) \approx [\text{parton model}](1 + a_1 \alpha + a_2 \alpha^2 + \ldots) + O(1/Q^4)
\]

(2)

where the power suppressed terms are quark and gluon operators. The perturbative series in the above equation can be rewritten as

\[
D(\alpha) = 1 + a_0 \alpha \sum_{n=1}^{\infty} a_n \alpha^n
\]

(3)

where the term in the sum is considered to be the nonperturbative short distance quantity and is given by a set of renormalon graphs. It is studied by Chetyrkin et al. [2] assuming that the short distance tachyonic gluon mass, \( \lambda^2 \), imitates the nonperturbative physics of the QCD. This, for the gluon propagator, means:

\[
D_{\mu\nu}(k^2) = \frac{\delta_{\mu\nu}}{k^2} \to \delta_{\mu\nu} \left( \frac{1}{k^2} + \frac{\lambda^2}{k^4} \right)
\]

(4)

On one hand, the nonperturbative short distance corrections are argued to be the \( 1/Q^2 \) correction in the OPE. On the other hand, the may have deep insight of the confining configuration of the QCD vacuum.

Let us assume that the gluon mass \( \lambda^2 \gg \Lambda^2_{\text{QCD}} \) which is not necessarily to be tachyonic one. Such a situation finds similarity in the case of QED as well as QCD. For an \( e^-e^+ \) pair, separated by a distance \( r \), contained in a cage of dimension \( L \), \( L \gg r \), the potential is of the form

\[
V_{e^-e^+}(r) = -\frac{\alpha_e}{r} + \text{const.} \alpha_e \frac{r^2}{L^3}
\]

(5)

The power correction to the leading Coulomb term can be interpreted as the interaction of dipole with its images. This can also be obtained in terms of one photon exchange, with the virtuality \( \sim L^{-1} \). In the case of QCD, the heavy quark potential is given by

\[
V(r) = -\frac{4\alpha(r)}{3r} + kr
\]

(6)

where \( k \approx 0.2 \) GeV\(^2\), representing the string tension. Now, the correction term in (5) can be considered to be due to one gluon exchange. If the gluon happens to be massive one, we get the gluon propagator modified, as given in (4). It has been argued in [3] that the linear term can be replaced by a term of order \( r^2 \). It is equivalent to replace \( k \) by a term describing the ultraviolet region. For the potential in (3),

\[
k \to k' \approx \text{constant} \times \alpha \lambda^2
\]

(7)

In replacing the coefficient of the term of \( O(r) \) by \( \lambda^2 \), we make it consistent by the renormalisation factor. Thus the coefficient \( \sigma(\lambda^2) \) is given by [4]:

\[
\sigma(\lambda^2) = \sigma(k^2) \left( \frac{\alpha(\lambda^2)}{\alpha(k^2)} \right)^{18/11}
\]

(8)

Introduction of \( \lambda^2 \) brings in a small correction to the Coulombic term. By use of (4), we specify the effect at both the ultraviolet region and the region characterised by the QCD scale. Then, we rewrite (4) as

\[
D(\alpha) = 1 + a_0 \alpha \left( 1 + \frac{k^2}{\tau^2} \right)
\]

(9)

where \( \tau \) is some scale relevant to the problem and \( k^2 \) should be read from (8). We would apply this to estimate the power correction in the heavy quark expansion.

We should note that in the QCD sum rules approach, the scale involved in is given by the Borel variable which is about 0.5 GeV. But in the heavy quark expansion the relevant scale is the heavy quark mass, greater than the hadronic scale. Thus, there it turns out to be infrared renormalons effects. But, still it represents the short distance nonperturbative property, by virtue of the gluon mass being as high as the hadronic scale.

3. Heavy-Light Hadrons

3.1. B Meson

For the heavy light current, \( J(x) = \bar{Q}(x)i\gamma_5 q(x) \), the QCD sum rules for \( B \) meson is already known [2]:

\[
\int_0^{\lambda_B} e^{-\lambda_B/\tau} = \frac{3}{\pi^2} \int_0^{\omega_c} d\omega \omega^2 e^{-\omega/\tau} D(\alpha)_B
\]

\[
- \langle \bar{q}q \rangle + \frac{1}{16\pi^2} \langle g\bar{q} \sigma Gq \rangle + ...
\]

(10)
where \( \omega_c \) is the duality interval, \( \tau \) the Borel variable, \( \bar{\Lambda}_B \) the mass gap parameter, the values of condensates given elsewhere below and

\[
D(\alpha)_{B} = 1 + a_B \alpha \left[ 1 + \frac{\lambda^2}{\tau^2} \left( \frac{\alpha(\lambda^2)}{\alpha(\tau^2)} \right)^{-18/11} \right]
\]

where \( a_B = 17/3 + 4\pi^2/9 - 4\log(\omega/\mu) \), with \( \mu \) is chosen to be 1.3 GeV.

With the duality interval of about 1.2-1.4 GeV which is little smaller than the onset of QCD which corresponds to 2 GeV and \( \bar{\Lambda} \geq 0.6 \) GeV, we get

\[
\lambda^2 = 0.35 \text{ GeV}^2.
\]

### 3.2. \( \Lambda_b \) Baryon

For the heavy baryon current

\[
j(x) = \epsilon^{abc}(\bar{q}_1(x)C\gamma_5tq_2(x))Q(x)
\]

where \( C \) is charge conjugate matrix, \( t \) the antisymmetric flavour matrix and \( a, b, c \) the colour indices, the QCD sum rules is given by

\[
\frac{1}{2}f_{\Lambda_b}^2 e^{\lambda/\tau} = \frac{1}{20\pi^4} \int_0^{\omega_c} d\omega \omega^3 e^{-\omega/\tau} D(\alpha)_{\Lambda_b}
\]

\[
+ \frac{6}{\pi^4} E_G \int_0^{\omega_c} d\omega e^{-\omega/\tau}
\]

\[
+ \frac{6}{\pi^4} F_\rho e^{-m_0^2/8\tau^2}
\]

where

\[
D(\alpha)_{\Lambda_b} = 1 - \frac{\alpha}{4\pi} a_{\Lambda_b} \left( 1 + \frac{\lambda^2}{\tau^2} \right)
\]

As in the meson case, we obtain

\[
\lambda^2 = 0.4 \text{ GeV}^2.
\]

In both the cases above, the gluon mass turn out to be about 0.6 GeV and above. They mean a somewhat large coefficient of the term at large order in the perturbative expansion.

### 4. Inclusive Decays and Quark-Hadron Duality

According to HQE, the inclusive decay rate of a weakly decaying heavy hadron is, at the leading order, given by

\[
\Gamma(B) = \Gamma_0 \left[ 1 - \frac{\alpha}{\pi} \left( \frac{2}{3} g(x) - \xi \right) \right]
\]

where \( \xi \) stands for the renormalons corrections:

\[
\Delta \Gamma(B)_{IR} \approx a_0 \alpha_s \sqrt{\frac{\lambda^2}{m_B Q}} \left( \frac{\alpha(\lambda^2)}{\alpha(m_B^2)} \right)^{-9/11}
\]

which is little smaller than the onset of QCD chosen to be 1.3 GeV. Numerically, the IR renormalon corrections are found to be

\[
\Delta \Gamma(B)_{IR} \approx 0.1 \Gamma_0
\]

\[
\Delta \Gamma(\Lambda_b)_{IR} \approx 0.11 \Gamma_0
\]

where \( \Gamma_0 \) is the b-quark decay rate at the tree level:

\[
\Gamma_0 = \frac{G_f^2 |V_{KM}|^2 m_B^5}{192\pi^3} f(x)
\]

The corrections being about 10% signify that the decay width is perturbatively under control. On the other hand, these corrections arise due to non-perturbative physics at short distance.

The assumption on the gluon mass has hence heuristic meaning. Though it is used to evaluate the renormalons effects, this would mean physics of confining configurations quantitatively. As is well known, quark-hadron duality signify the interplay of confinement and asymptotic freedom at a particular kinematic regime. Thus, the above quantitative measure can be construed to be duality violating effects.

In HQE, it has been pointed out in [3] that the violation of duality in HQE is of exponential/oscillating in nature:

\[
\Pi(Q^2)_{\text{violation}} = e^{-C Q^2 / \Lambda_{2CD}^2}
\]

where \( C \) is constant and \( Q^2 \) is the energy scale. However, this violating effect is not quantified. This violating quantity has been attributed to the discrepancy in the inclusive properties as predicted by HQE.

On the other hand, in [8], the weak decay of heavy hadrons is studied in the 't Hooft model.
It has been found that the duality holds good with the presence of terms of order $1/m_Q$. Such a term is absent in the HQE. We should note that the first-power-suppressed term is absent in the OPE itself. We should note that the 't Hooft QCD is 1+1 dimensional where confinement is built-in. But, in QCD, the phenomenon of confinement is not understood. Therefore, we cannot expect every aspect of the two-dimensional QCD to agree in toto with the QCD.

Recently, it has been shown in [11] that the four-quark operators are indeed responsible for the discrepancy of lifetimes of $B$ and $\Lambda_b$. If one assumes that the HQE is saturated by the terms up to three in $1/m_b$, then the differences between the hadrons under $SU(3)_f$ symmetry yields the expectation values of the four-quark operators of $B$-hadrons such that the ratio, $\tau(\Lambda_b)/\tau(B)$, close to the experimental value. This would straightforwardly mean that duality indeed holds good in HQE as far as the bottom sector is concerned. On the other hand, the present study shows that if the renormalons corrections are considered to be duality violating effects, then the violation of duality is significantly few per cent.

5. Conclusion

Our assumption that the gluon mass, $\lambda^2 \gg \Lambda_{QCD}^2$, that imitates the nonperturbative physics at short distance. This signifies some unknown confining effects that is given by duality breaking effects in the standard OPE and hence in HQE. We found that these effects are about 10% of the leading decay rate. Our studies show that the inclusive decays of heavy hadrons can be studied within the framework of the HQE, notwithstanding the aspects like exponential violation that have not been quantified.

Use of constraints of the mass gap parameter due to the kinetic energy term and the present value on the difference in the mass gap parameter of $B$ and $\Lambda_b$ would result in more precision of the duality braking effects. Besides, such studies in the charm sector are also relevant.

6. Acknowledgements

The author is grateful to Prof. Y. Okada for discussions and hospitality at the Theory Group, KEK, Japan where a part of this work culminated, Prof. S. Narison for discussions and Prof. V. I. Zakharov for useful communications. He thanks Prof. Apoorva Patel and the symposium organisers for invitation to the wonderful event and warm hospitality during the symposium. The author attains pleasure in dedicating this work to Prof. T. Nagarajan, former Head of the Department of Nuclear Physics, University of Madras, India.

REFERENCES

1. V. I. Zakharov, Prog. Theo. Phys. (PS) 131 (1998) 107.
2. K. G. Chetyrkin, S. Narison, V. I. Zakharov, Nucl. Phys. B 550 (1999) 353.
3. Ya. Ya. Balitsky, Nucl. Phys. B 254 (1983) 166.
4. R. Anishetty, Perturbative QCD with string tension, hep-ph/9804204.
5. M. Beneke and V. M. Braun, Nucl. Phys. B 426 (1994) 301; S. Narison, Phys. Lett. B 352 (1995) 122.
6. Y-B. Dai, C-S. Huang, M-Q. Huang, C. Liu, Phys. Lett. B 387 (1996) 379.
7. M. Shifman, hep-ph/9405240.
8. I. I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, hep-ph/9905241.
9. B. Chibisev, R. D. Dikeman, M. Shifman and N. Uraltsev, Int. J. Mod. Phys. A 12 (1997) 2075.
10. C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. D 54 (1996) 2081; I. I. Bigi, M. A. Shifman, N. G. Uraltsev and A. I. Vainshtein, Phys. Rev. D 50 (1994) 2234; M. Beneke, V. M. Braun and V. I. Zakharov, Phys. Rev. Lett. 73 (1994) 3058; M. Luke, A. V. Manohar and M. J. Savage, Phys. Rev. D 51 (1995) 4924.
11. S. Arunagiri, Phys. Lett. B 489 (2000) 309; S. Arunagiri, Ph. D. Thesis, University of Madras, 2000 (unpublished).