Long range crossed Andreev reflections in high Tc superconductors

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We analyze the non-local transport properties of a \(d\)-wave superconductor coupled to metallic electrodes at nanoscale distances. We show that the non-local conductance exhibits an algebraical decay with distance rather than the exponential behavior which is found in conventional superconductors. Crossed Andreev processes, associated with electronic entanglement, are favored for certain orientations of the symmetry axes of the superconductor with respect to the leads. These properties would allow its experimental detection using present technologies.

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I. INTRODUCTION

Cooper pairs in superconducting nanostructures provide a potential source of entangled electrons \[1, 2, 3, 4, 5\], a possibility that has been recently explored in conventional superconductors both theoretically \[6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\] and experimentally \[17, 18, 19, 20\]. In a typical experimental device, a superconducting region is contacted by several metallic electrodes at nanoscale distances with the aim of analyzing the non-local transport properties at subgap voltages. In the limit of vanishing contact transparency the non-local conductance is controlled by two type of processes yielding opposite contributions: direct elastic tunneling of electrons between two separate leads (elastic cotunneling, EC) and crossed Andreev reflection (CAR) processes in which injected electrons from one lead are reflected as holes in the other lead (see Fig. 1). The time reverse of these last processes involve entangled electron pairs on two separate leads \[21\]. In conventional superconductors the average conductance tends to cancel due to the opposite contribution of EC and CAR processes \[8, 9\]. Several mechanisms have been proposed to avoid a complete cancellation and have been invoked to explain the available experimental results. Among them one can quote the use of ferromagnetic leads \[7, 10, 11, 12\] and the effect of electron-electron interactions in certain experimental geometries \[13\]. On the other hand, the magnitude of these non-local processes decays exponentially with the distance between the leads on a scale fixed by the superconducting coherence length \(\xi_0\). This anisotropy is behind the non-local nature of the electromagnetic response of HTcS \[24, 25\]. In fact, some indirect evidence of CAR processes in HTcS coupled to ferromagnetic leads has been presented \[26, 27\].

In the present work we analyze the non-local transport in \(d\)-wave superconductors and show that in contrast to the conventional \(s\)-wave case, CAR processes are long ranged. Moreover, we show that for certain orientations of the axes of the superconductor with respect to the contacts, CAR processes dominate over EC at low voltages and small contact transparency. We believe that these findings open the possibility of using HTcS as a source of entangled electron pairs.

II. CROSSED ANDREEV REFLECTION AND ELASTIC COTUNNELING IN HIGH Tc SUPERCONDUCTORS

The situation to be analyzed is illustrated in Fig. 1. We consider a semi-infinite \(d\)-wave superconducting re-
region connected to two normal leads, denoted by a and b, and separated a distance d. Our aim is finding the current induced on lead b, I_b, when a voltage V_a is applied on lead a. The two processes contributing to this current are depicted on panels (b) and (c). Being the result of the diffraction of quasiparticles by an anisotropic pair potential, the relative weight of the two processes will be affected by the orientation of the superconductor symmetry axis (angle α in Fig. 1). In fact, EC processes are favored for electron propagation along the nodal lines (where the order parameter vanishes), while CAR processes reach a maximum amplitude along directions where the modulus of the pair potential is maximum. Although on a spatial average the contribution of the two processes should be equal as in the case of isotropic s-wave superconductors, a dominance of one of the two can be found for specific orientations of the symmetry axis with respect to the leads. These qualitative arguments allow to understand the dominance of CAR over EC processes for the α = 0 case (d_{x^2−y^2} symmetry) and the opposite behavior in the case of α = π/4 (d_{xy} symmetry) as discussed in detail below.

In the spirit of the Hamiltonian approach of Ref. [28] the differential conductance defined as σ_{ba} = dI_b/dV_a, can be written as [10]

\[ \sigma_{ba} = \frac{8\pi^2 e^2 p_b^2}{h} \rho_{e,a} G_{ba,12}(eV)^2 - \frac{\rho_{e,b} G_{ba,11}(eV)^2}{2}, \]

where \( \rho_{e,a(b)} \) is the local density of states of the electron (hole) in the lead a (b), while \( \rho_a \) and \( \rho_b \) denote the corresponding hopping parameters coupling the superconductor to the leads. The quantities \( G_{ba,11}(eV) \) and \( G_{ba,12}(eV) \) are the non-local propagators in the superconducting region (indexes 1, 2 refer to electrons and holes in Nambu space). The first term on the right hand size of Eq. (1) is due to CAR processes, while the second term corresponds to EC. Therefore the crossed differential conductance is positive if CAR dominates over EC or negative in the opposite case. In order to make contact with possible experiments it also convenient to analyze the non-local resistance \( R_{ba} \), given by

\[ -\sigma_{ba}/(\sigma_{aa} \sigma_{bb} - \sigma_{ab} \sigma_{ba}), \]

where \( \sigma_{aa(bb)} \) are the local conductances which can be obtained within the same formalism [10, 28]. The propagators \( G_{\alpha\beta,ij} \) of the coupled system are then given by \( G^\dagger(E) = (\vec{g}(E)^{-1} + i\vec{f})^{-1} \), where \( \vec{g} \) is the retarded Green function of the uncoupled superconductor and \( \vec{f} = \rho_0^2 \pi \rho_N \delta_{\alpha\beta} \delta_{ij} \). We have assumed that the densities of states of the normal metals are energy independent, i.e. \( \rho_{e,a(b)} = \rho_{h,a(b)} \equiv \rho_N \). The symbol \( \vec{\gamma} \) here denotes \( 4 \times 4 \) matrices defined in the electrodes \( \oplus \) Nambu space, while we reserve the symbol \( \vec{\gamma} \) for the reduced \( 2 \times 2 \) Nambu space. To calculate \( \vec{g} \) we first determine the superconductor surface Green function in momentum representation, \( \vec{g}_S(E, k_y) \), using the asymptotic solutions of the Bogoliubov de Gennes equation [28], which yields

\[ \vec{g}_S(E, k_y) = \frac{-2mi}{\hbar^2 D} \left( \begin{array}{cc} \frac{1}{k_x^2} + \frac{\Gamma^2 e^{-i\phi}}{k_x^2} & e^{i\phi} - \left( \frac{\Gamma}{k_x^2} + \frac{\Gamma}{k_y^2} \right) \frac{k_y}{k_x} \\ e^{-i\phi} & \frac{1}{k_y^2} + \frac{\Gamma^2 e^{-i\phi}}{k_y^2} \end{array} \right) \]

where

\[ k_\pm = \sqrt{k_x^2 + 2m\Omega/\hbar^2}, \quad \Gamma = |\Delta_+|/(|E + \Omega)| \]
\[ \phi_\pm = \arg(\Delta_+), \quad \Delta \phi = \phi_+ - \phi_-, \quad k_+^2 = k_y^2 - k_y^2 \]
\[ D = (1 - \Gamma^2 e^{-i\Delta \phi}), \quad \Omega = \sqrt{E^2 - |\Delta|^2} \]
\[ \delta = D(1 - e^{i\Delta \phi})/(2 - 2\Gamma^2) \]
\[ k_1^{-1} = k_1^{-1} + k_2^{-1}, \quad k_2^{-1} = k_2^{-1} - k_1^{-1} \]

In the above equations \( \Delta \) is the pair potential which depends on the wave vector, taking the values \( \Delta_+ \) and \( \Delta_- \) along the directions \( \theta \) and \( \pi - \theta \) respectively, where \( \theta = \tan^{-1}(k_y/k_x) \). These are given by \( \Delta_\pm (\theta) = \Delta_0 \), for s symmetry and \( \Delta_\pm (\theta) = \Delta_0 \cos(\theta + \alpha) \) for d symmetry. The retarded component is obtained by adding a small positive imaginary part \( i\hbar \) to the energy. From \( \vec{g}_S(E, k_y) \) one then obtains the non-local components \( \vec{g}_\sigma \) by

\[ \vec{g}_\sigma = \int_{-\infty}^{\infty} \vec{g}_S(E, k_y) f(k_y)^2 e^{-ik_\sigma d} dk_y, \]

where the weighting factor \( f(k_y) \), proportional to the perpendicular wave vector \( k_{x2} \), provides the appropriate connection between the continuous model used to describe the superconducting region and the discrete Hamiltonian approach used to obtain Eq. (1) (see Refs. [13, 50]).

As a first test of the model one can check that in the case of s-symmetry for \( E < \Delta_0 \) and \( k_{x2} >> 1 \), \( \sigma_{ba} \propto e^{-2d/\xi} (\cos^2(k_{x2}) - \sin^2(k_{x2}))/d \) with \( \xi(E) = \xi_0/Re(\sqrt{1 - E^2/\Delta_0^2}) \) and \( \xi_0 = h\nu_F/(\pi \Delta_0) \), a result which agrees with Refs. [11, 13]. Therefore \( \sigma_{ba} \) exhibits changes in sign on the \( \lambda_F \) scale and its spatial average is zero [8, 12].

A. Results for \( d_{x^2−y^2} \) symmetry

We now consider the \( d_{x^2−y^2} \) symmetry. Due to the anisotropy of the pair potential an incoming electron from lead a is scattered as a quasiparticle in the superconductor, exploring regions where \( E > \Delta(\theta) \) and \( E < \Delta(\theta) \), with an effective coherence length \( \xi(E, \theta) = \xi_0/Re(\sqrt{1 - E^2/\Delta(\theta)^2}) \) which takes values from \( \xi_0 \) to \( \infty \). For this reason one typically finds that the propagators exhibit a slower decay with distance than in the case of s-symmetry. In this paper we have fixed \( \Delta_0 \sim 20\text{meV} \) [31] and \( \Delta_0/\nu_F \sim 10^{-1} \) as typical values for HTCs [32]. We also take \( \eta \sim 0.002\Delta_0 \) to simulate the effect of weak disorder [33]. Figure 2 illustrates the spatial dependence of the Green functions. Due to the dependence on \( k_y \) of...
Notice that the low energy excitations at a clear dominance of CAR over EC in the tunnel limit. The envelope curve, decaying as $1/d^2$ is indicated by a full line. The corresponding curve for s-symmetry with the same choice of parameters, exhibiting an exponential decay, is also represented for comparison. The inset shows the exponents $r_{1,2}$ in the decay laws of Eq. 5 as a function of energy.

The pair potential, it is not possible to obtain an analytical expression of their variation with $d$ as in the case of s symmetry. However, from numerical regressions for $k_F d >> 1$, $|g_{ba,11}|^2$ and $|g_{ba,12}|^2$ can be fitted as

$$|g_{ba,11(12)}(E)|^2 \sim \frac{c_{1(2)} + d_{1(2)} \cos^2(kd)}{|k_F d|^{r_{1(2)}}}.$$  

(5)

The values of the exponents $r_{1(2)}$ fixing the spatial decay are shown in the inset of Fig.2 as a function of energy. For low energies ($E << \Delta_0$) $k \sim k_F, c_{1(2)} << d_{1(2)}$ and $d_2 >> d_1 \rightarrow 0$ for $E \rightarrow 0$, and therefore the propagator $|g_{ba,12}|$ takes a much larger value than $|g_{ba,11}|$, yielding a clear dominance of CAR over EC in the tunnel limit. Notice that the low energy excitations at $\theta \sim \pi/4$ give a negligible contribution to $|g_{ba,11}|^2$ due to the weighting factor in Eq. 4 that is maximum at low angles. In contrast, most of the weight in $|g_{ba,12}|^2$ comes from $\theta \sim 0$ where $\Delta$ reaches a maximum. On the other hand, for energies higher than $E \sim 0.1\Delta_0$ $|g_{ba,12}|$ and $|g_{ba,11}|$ tend to have the same magnitude on average.

Fig. 3 further illustrates the different behavior of CAR and EC contributions to the non-local conductance for this orientation as one of the leads moves inside the superconductor while the second is located at the surface. These maps clearly correspond to a diffraction pattern for electrons injected at one point in the surface. In spite of its complex structure one can identify the region of low angles from the surface ($\pi/2 > \theta \geq \pi/4$) where CAR processes have a clear dominance and the nodal lines ($\theta \simeq \pi/4$) around which EC processes are favoured.

The results for $\sigma_{ba}$ and $R_{ba}$ in the $d_{x^2-y^2}$ orientation for arbitrary contact transmission are illustrated in Fig. 4. It is found that an increase in transmission leads to a reduction of the CAR contribution while the EC one increases. As a consequence both quantities exhibit a change of sign when the coupling to the leads increases.
This is illustrated for $\sigma_{ba}$ in inset of Fig. 4. We can obtain further insight on this effect at low energies where Eq. (1) can be approximated as

$$\sigma_{ba}(d) \approx \frac{1 - P^4 g_{aa,12}^2 |g_{ba,12}(d)|^2}{1 + |P^4 g_{aa,12}^2|^4}.$$ (6)

In obtaining this expression we have assumed symmetrical contacts ($p_a = p_b = p$) with $P = p \pi \rho_N$ being the normalized hopping parameter, such that the normal transmission for a single contact is $T_N = 4 P^2 / (1 + P^2)^2$.

Within this approximation the dependence with the separation between the leads does not change when the transmission is increased, as it is seen in the inset of Fig. 4. On the other hand, this equation predicts a change in sign of $\sigma_{ba}$ for $P \approx 0.72$ ($T_N \approx 0.9$) in agreement with the numerical results in the inset of Fig. 4. The non-local resistance $R_{ba}$ averaged on a range $\sim \lambda_F$ is shown in Fig. 4 for $d = 10 \xi_0$. We observe that in the low transmission regime this quantity is negative, as it corresponds to the dominance of CAR processes, and exhibits a peak at low bias.

### B. Results for $d_{xy}$ symmetry

The results for $d_{xy}$-symmetry ($\alpha = \pi/4$) are shown in Fig. 5. The main effect for this symmetry is the appearance of a zero energy bound state $34$, which is associated with a $1/E$ dependence in $g_{ba,11}$. On the other hand, the distance dependence for low energy and $k_F d >> 1$ is in this case approximately $1/d^4$ both for CAR and EC processes. In this orientation, the local Andreev reflection is zero because of diffraction of quasiparticles in the contact $35, 36$ and the CAR contribution to $\sigma_{ba}$ is not zero, but is always smaller than the EC one, leading to a negative non-local conductance as shown in Fig. 5. Basically, the dominance of the EC contribution is caused by the suppression of the pair potential along the $\theta = 0$ line. The effect of varying the contact transmission can be understood analytically within a similar approximation as done for the $d_{x^2-y^2}$ case, which allows to obtain the following expression for the crossed differential conductance

$$\sigma_{ba}(d) \approx \frac{|g_{ba,12}(d)|^2 - |g_{ba,11}(d)|^2}{|1 + i P^2 g_{aa,11}^2|^4}.$$ (7)

Notice that CAR and EC contributions are equally affected by the coupling to the leads (through the $P$-dependent common denominator) and therefore EC dominates over CAR for the whole transmission range. The spatially averaged non-local conductance is negative and presents a zero bias peak that decreases with increasing transmission.

### III. CONCLUSIONS

In summary we have analyzed the behavior of the crossed differential conductance in $d$-wave superconductors in a multiterminal configuration. We have shown that correlations between different leads exhibit an algebraical decay instead of the exponential behavior which is typically found in conventional superconductors. In the case of $d_{x^2-y^2}$-oriented crossed Andreev processes are favored at low voltages and contact transmissions, while for the $d_{xy}$ case a zero bias non-local conductance peak appears, which is dominated by elastic-cotunneling. In both cases the spatially averaged non-local conductance is different from zero. These properties would allow to detect non-local transport at distances several times larger than the characteristic coherence length in these systems.

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[1] P. Recher, E. V. Sukhorukov, and D. Loss, Phys. Rev. B 63, 165314 (2001).

[2] N. M. Chtchelkatchev, G. Blatter, G. B. Lesovik, and
[3] C. Bena, S. Vishveshwara, L. Balents, and M. P. A. Fisher, Phys. Rev. Lett. 89, 037901 (2002).
[4] P. Recher and D. Loss, Phys. Rev. Lett. 91, 267003 (2003).
[5] P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, Phys. Rev. Lett. 91, 157002 (2003).
[6] J. M. Byers and M. E. Flatté, Phys. Rev. Lett. 74, 306 (1995).
[7] G. Deutscher and D. Feinberg, Appl. Phys. Lett. 76, 487 (2000).
[8] D. F. G. Falci and F. W. J. Hekking, Europhys. Lett. 54, 255 (2001).
[9] D. Feinberg, Eur. Phys. J. B 36, 419 (2003).
[10] R. Mélin and S. Peysson, Phys. Rev. B 68, 174515 (2003).
[11] T. Yamashita, S. Takahashi, and S. Maekawa, Phys. Rev. B 68, 174504 (2003).
[12] R. Mélin and D. Feinberg, Phys. Rev. B 70, 174509 (2004).
[13] E. Prada and F. Sols, Eur. Phys. J. B 40, 379 (2004).
[14] A. Brinkman and A. A. Golubov, Phys. Rev. B 74, 214512 (2006).
[15] A. L. Yeyati, F. S. Bergeret, A. Martin-Rodero, and T. Klapwijk, Nat. Phys. 63, 455 (2007).
[16] M. S. Kalenkov and A. D. Zaikin, Phys. Rev. B 75, 172503 (2007).
[17] D. Beckmann, H. B. Weber, and H. v. Löhneysen, Phys. Rev. Lett. 93, 197003 (2004).
[18] S. Russo, M. Kroug, T. M. Klapwijk, and A. F. Morpurgo, Phys. Rev. Lett. 95, 027002 (2005).
[19] P. C. Zimansky and V. Chandrasekhar, Phys. Rev. Lett. 97, 237003 (2006).
[20] D. Beckmann and H. v. Löhneysen, Appl. Phys. A 89, 603 (2007).

The dominance of CAR over EC processes is a necessary but not a sufficient condition to have an efficient entangler. For a more detailed discussion see Refs. [1, 2, 3, 4].

N. Stefanakis and R. Mélin, J. Phys.: Condens. Matter 15, 4239 (2003).
S. Takahashi, T. Yamashita, and S. Maekawa, J. Phys. Chem. Solids 67, 325 (2006).
I. Kosztin and A. J. Leggett, Phys. Rev. Lett. 79, 135 (1997).
M. R. Li, P. J. Hirschfeld, and P. Wölfle, Phys. Rev. Lett. 81, 5640 (1998).
P. Aronov and G. Koren, Phys. Rev. B 72, 184515 (2005).
I. Asulin, O. Yuli, G. Koren, and O. Millo, Phys. Rev. B 74, 092501 (2006).
J. C. Cuevas, A. Martin-Rodero, and A. L. Yeyati, Phys. Rev. B 54, 7366 (1996).
W. J. Herrera, A. L. Yeyati, and A. Martin-Rodero, unpublished.
J. Bardeen, Phys. Rev. Lett. 6, 57 (1961).
Ö. Fischer, M. Kugler, I. Maggio-Aprile, C. Berthod, and C. Renner, Rev. Mod. Phys. 79, 353 (2007).
A. Golubov and F. Tafuri, Phys. Rev. B 62, 15200 (2000).
This is consistent with the scattering rate at low temperatures extracted from surface impedance measurements in YBaCuO see E. Farber et al. Physica C 317-318, 550 (1999).
S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. 63, 1641 (2000).
Y. Takagaki and K. H. Ploog, Phys. Rev. B 60, 9750 (1999).
W. J. Herrera, J. V. Niño and J. J. Giraldo, Phys. Rev. B 71, 094515 (2005).