Analytical modelling of the influence of temperature and capillary diameter on the sonocapillary effect for liquids with different density

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Abstract. The paper examines an approach to calculating the sonocapillary effect. The proposed model for calculating the initial sonocapillary pressure considers the temperature and density of a liquid, the capillary diameter, and the ultrasound frequency. The calculations are based on the incubation time criterion of cavitation. This approach allows us to find the threshold cavitation amplitude of ultrasound for a given frequency and to estimate the corresponding sonocapillary pressure. The calculation results show that the sonocapillary pressure is directly related to the liquid density, increasing the capillary diameter reduces the intensity of the sonocapillary effect, and a rising temperature increases the sonocapillary pressure. However, minimal experimental data are available at present to verify the obtained theoretical results. Therefore, a wider range of new experimental results should be obtained for further studies of the sonocapillary effect based on the presented results.

1. Introduction
Ultrasonic vibration of the volume of a liquid near narrow channels, capillaries, pores and cracks can cause the liquid to rise to abnormally high levels in these voids compared to the usual capillary effect. This phenomenon is called the sonocapillary effect [1, 2]. This effect finds broad application— for example, in the impregnation of fabrics and insulation materials for coiled products, in soldering and in ultrasonic cleaning. Despite the widespread use of the sonocapillary effect, many opportunities for its experimental and theoretical study remain to be exploited.

In a number of experiments, it was shown [2-4] that a necessary condition for the sonocapillary effect to occur is cavitation near the capillary entrance (figure 1). Studies of ultrasound-induced cavitation in liquid are discussed in detail in [5, 6]. These studies have shown that acoustic cavitation strongly depends on the amplitude-frequency characteristics of ultrasound and on the temperature and density of the liquid. However, the available literature lacks a detailed presentation of these relationships for the case of the sonocapillary effect. At the same time, some experimental studies [7, 8] note the influence of capillary size and liquid density.

In this paper, we calculate the influence of capillary size and the density and temperature of the liquid on the sonocapillary effect at various ultrasound frequencies. To calculate this effect, we use the incubation time criterion of cavitation [9, 10], which enables us to introduce an integral characteristic of the complex cavitation process, significantly simplifying the calculation. Based on this approach, a model was presented in [11] for determination of the threshold sonocapillary pressure for various
ultrasound frequencies. In addition, it was shown in [12] that the incubation time criterion allows us to consider the effect of temperature on the threshold vibration amplitudes leading to cavitation. Thus, the available results lead us to propose a model for assessing the effect of temperature on the sonocapillary pressure for liquids with different densities, for different ultrasound frequencies and capillary diameters.

![Figure 1. Schematic illustration of the sonocapillary effect with cavitation processes under the capillary edge.](image)

2. Modelling of sonocapillary effect

2.1. Calculation of threshold loading

In the model, the sonocapillary pressure is calculated for the threshold modes of ultrasonic impact; in other words, for a given frequency, we consider the minimum vibration amplitude at which the sonocapillary effect (SCE) is realized. Experimental studies associate the occurrence of SCE with the initiation of cavitation under the edge of the capillary [2, 3, 7, 13] (figure 1). Therefore, the model uses the starting conditions for cavitation in the liquid to determine the threshold loads of the SCE.

Due to high-frequency ultrasonic vibrations, the effect on the liquid is dynamic. Thus, if we consider the formation of cavitation bubbles as a fracture or disturbance in liquid continuity, then to determine the conditions under which cavitation starts, we propose to use the incubation time criterion, which takes into account the time and energy parameters of the process. Under this criterion, cavitation can begin only when the impulse reaches a critical value. This is determined by the incubation time \( \tau \) and the pressure \( P_c \), which under quasi-static (low-frequency or constant) loading will lead to cavitation. The criterion has an integral form [9, 10]:

\[
\frac{1}{\tau} \int_{t^* - \tau}^{t^*} \left( \frac{P(t)}{P_c} \right)^\alpha \, dt \geq 1
\]  

(1)

where \( P(t) \) is the tensile load in the fluid, \( t^* \) is the time of the cavitation generation, and \( \alpha \) is a dimensionless parameter characterizing the fluid sensitivity to the loading history. The parameter \( \alpha \) depends on the liquid’s viscosity, which is derived from the bubble dynamics equation [10]. The liquids considered in this work are non-viscous; therefore, \( \alpha \) is taken to be 0.5. The pressure \( P_c \) determines the strength of the liquid under quasi-static loading; it is calculated using the phase diagram of the liquid [12].

Cavitation occurs as a result of a tensile load, which in the case of ultrasonic action is represented in the form \( P(t) = P_m \sin \omega t, \quad \epsilon(0, \pi/\omega) \) where \( \omega = 2\pi f \) is the angular frequency. By substituting the
load $P(t)$ into the criterion (1), the threshold amplitude of ultrasonic vibrations can be determined for a given frequency:

$$P_m = \begin{cases} P_c \left( \frac{1}{4} \max_{t \in (0, \pi]} \int_{t_0}^{t_0+\pi} |\sin z|^{\lambda+2} \, dz \right)^{\frac{1}{\lambda+2}}, & \lambda \leq \pi \\ P_c \left( \frac{1}{4} \int_0^{\pi} |\sin z|^{\lambda+2} \, dz \right)^{\frac{1}{\lambda+2}}, & \lambda > \pi \end{cases}$$

(2)

where, as a result of the change $z = \omega t$, the following parameters were introduced: $\lambda = \omega t$ and $t_0 = \omega t^\dagger$.

2.2. **Calculation of sonocapillary pressure**

In the model it is assumed that in the case of SCE, liquid rises through the capillary due to the collapse of bubbles formed in the cavitation region under it. Therefore, to calculate the sonocapillary pressure we consider the energy of the cavitation region to be the surface energy of $N$ bubbles at the moment of their maximum expansion:

$$A_\sigma = 4\pi \sigma R_m^2 N$$

(3)

Here $\sigma$ is the surface tension coefficient and $R_m$ is the maximum radius of cavitating bubbles, which can be estimated for a liquid with density $\rho$ using the amplitude $P_m$ and frequency $f = \frac{1}{T}$ of the ultrasonic field as follows [5]:

$$R_m \approx \frac{T}{4} \left( \frac{2P_m}{3\rho} \right)^{0.5}$$

(4)

The number of bubbles in the cavitation region is estimated using the solution to the problem of packing equal-diameter circles into a larger circle. For packing density $D_b$ and cavitation-region radius $R_{cav}$, the number of bubbles is calculated as [11]:

$$N = D_b \left( \frac{R_{cav}}{3R_m} \right)^3$$

(5)

The derived energy of the collapsed bubbles of the cavitation cloud lifts the liquid up the capillary. The liquid is carried through the capillary by the acoustic flow at the moment of the compression phase [2]. Flow velocity depends on the speed of sound and density of the liquid; it is determined by the amplitude of the ultrasonic field $u = \frac{P_m}{\rho c}$. From this, the height the liquid rises in the capillary can be calculated by the law of conservation of total mechanical energy as $h = \frac{u^2}{2g}$.

The sonocapillary pressure can then be expressed as the work $A_\sigma$ done by the collapsed bubbles as they raise a liquid column of height $h$ in one cycle of oscillations along a capillary of radius $R_c$:

$$P_{sc} = \frac{A_\sigma}{\pi R_c^2 h}$$

(6)

3. **Results and discussion**

In [11] the modelling of the SCE for water showed good agreement with the experimental data. Therefore, this paper considers the effect of fluid parameters on the SCE; to this end, SCE modeling was carried out for ethanol, heptane and heavy water. In the calculations for each liquid, the threshold parameters of ultrasonic loading were determined. The properties of the liquids are presented in table 1. As can be seen, ethanol and heptane are lighter than water, while deuterium oxide has a higher density.
A comparison of the sonocapillary pressure for the studied liquids is shown in figure 2. The pressure $P_{sc}$ was normalized to the maximum value of the sonocapillary pressure in water. Calculations show that the SCE grows with liquid's density increasing.

Table 1. Properties of liquids.

|                | Water   | Heavy water (D$_2$O) | Ethanol | Heptane |
|----------------|---------|----------------------|---------|---------|
| Density (kg·m$^{-3}$) | 1000    | 1105                 | 789     | 680     |
| Sound speed (m·s$^{-1}$)  | 1485    | 1400                 | 1180    | 1130    |
| Surface tension coefficient (mN·m$^{-1}$)  | 72.5    | 71.87                | 22.39   | 20.86   |

![Figure 2](image.png)

The influence of fluid temperature on the SCE was also considered. It is shown in [12] that the incubation time changes with increasing temperature as $\tau(T) = \tau_0 \exp\left(\frac{G}{RT}\right)$, where $G$ is the energy required for the onset of cavitation on an elementary volume, and $\tau_0$ is a parameter characterizing the strength of the liquid at a given scale level. The strength parameters of the liquids considered in this work were calculated and verified by experimental data in [12].

Figure 3 depicts the results of modeling SCE for temperatures ranging from 10°C to 90°C. The sonocapillary pressure is also normalized to the sonocapillary pressure in water at a temperature of 20°C. Modelling results show that the rise in temperature contributes to an increase in the SCE. The most rapid increase in sonocapillary pressure is observed as the liquid approaches its boiling point, which is clearly apparent in the case of ethanol. Its boiling point at atmospheric pressure is 78.37 °C, and its SCE value at that temperature is the highest among the four liquids.
Figure 3. Dependence of normalized sonocapillary pressure on temperature for liquids with different densities.

In addition, figure 3 shows the influence of frequency on the temperature dependence of the SCE. Solid lines are calculated for a frequency of 41.9 kHz and the corresponding threshold amplitude. For heptane, an additional calculation was performed for an ultrasound frequency of 20 kHz, which is indicated in figure 3 with a dotted line. The modelling shows that lowering the frequency decreases the temperature dependence of the sonocapillary pressure.

The results of the SCE testing show a reduction in the effect’s intensity with an increase in the capillary diameter [2, 13], which was also confirmed by modelling the acoustic capillary pressure for water. Figure 4 reveals the modelling results for two ultrasound frequencies, 18.5 kHz and 41.9 kHz, with experimental data [2, 7, 13]. Experiments conducted at the frequency of 41.9 kHz [7, 13] had a slightly higher amplitude of ultrasound than threshold value corresponding to this frequency. This leads to higher sonocapillary pressure compared to the theoretical calculation. For the SCE at 18.5 kHz, there is good agreement between the modelling result and the experimental data.

Figure 4. Sonocapillary pressure versus capillary diameter. Solid lines are theoretical calculations; points 1, 2, 3 are experimental data [2], [13], [7], respectively.

4. Conclusions
The model presented in this study allows us to calculate the influence of temperature, fluid density and capillary diameter on the sonocapillary effect (SCE). Modelling results show that the rise in liquid's
density contributes to an increase in the SCE at low ultrasound frequencies. However, as frequency rises the sonocapillary pressure declines and the influence of liquid density on SCE levels out.

The temperature analysis of the SCE shows that near the boiling point the sonocapillary effect soars. Additionally, we compared temperature dependencies of sonocapillary pressure at two different frequencies. Modelling the dependence of the SCE on the capillary diameter revealed a good agreement with the experimental data for both of the ultrasonic frequencies considered.

Acknowledgments

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