Loop Quantum effects on $O_m$-diagnostic and its Cosmological Implications

Prabir Rudra

Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711 103, India.
Department of Mathematics, Pailan College of Management and Technology, Bengal Pailan Park, Kolkata-700 104, India.

Abstract

In this paper we study the Loop quantum effects on the $O_m$ diagnostic and subsequently on the universe. We reconstruct the $O_m$ diagnostic in the background of Loop quantum gravity and then study the behaviour of various Chaplygin gas dark energy models using the modified diagnostic in a comparative scenario. The trajectories discriminate the various dark energy models from each other both in the Einstein gravity as well as Loop quantum gravity. The Loop quantum effects are also clearly noticeable from the trajectories in past, present and future universe. We see that the Loop quantum deviations are highly pronounced in the early universe, but alleviate as we tend towards the present universe and continue to decay in future. Thus it puts a big question on the effectiveness and consequently the suitability of loop quantum cosmology to explain the future universe.

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1 Introduction

With the discovery of the accelerated expansion of the universe\cite{1,2}, the concept of dark energy (DE)\cite{3} have gained prime status in cosmology during the last decade. Numerous models of DE have been proposed over the years in order to justify the recent cosmic acceleration. Some of the well-known models being Chaplygin gas, scalar field, phantom, etc. Here we consider the Chaplygin gas DE models. The oldest candidate of the Chaplygin gas cosmology was pure Chaplygin gas (CG)\cite{4,5}. Subsequently Generalized Chaplygin gas (GCG)\cite{6,7} was constructed. The equation of state (EoS) for GCG is given by,

$$p_{ch} = -\frac{B}{\rho_{ch}^\alpha}$$  \hspace{1cm} (1)

where $\alpha$ is a constant in the range of $0 < \alpha \leq 1$ (obviously $\alpha = 1$ corresponds to pure Chaplygin gas) and $B$ is a positive constant. Although GCG was a successful candidate in playing the role of DE, satisfying almost all the solar system tests, yet it was plagued by certain cosmological problems like fine tuning and cosmic coincidence problem. So in the quest of a better model Modified Chaplygin gas (MCG)\cite{8,9} came into existence. The EoS of MCG is given by,

$$p_m = A\rho_m - \frac{B}{\rho_m^\alpha}$$  \hspace{1cm} (2)

where $0 \leq \alpha \leq 1$, $A$ and $B$ are positive constants. In due course MCG was modified into Variable modified Chaplygin gas (VMCG) and finally into New variable modified Chaplygin gas (NVMCG). The EoS of VMCG and NVMCG are respectively given as,

$$p_v = A\rho_v - \frac{B(a)}{\rho_v^\alpha}$$  \hspace{1cm} (3)

and

$$p_n = A(a)\rho_n - \frac{B(a)}{\rho_n^\alpha}$$  \hspace{1cm} (4)
where $A(a)$ and $B(a)$ are positive functions of the cosmological scale factor $a'$. In 2003, P. F. Gonzalez-Diaz \[10\] introduced the generalized cosmic Chaplygin gas (GCCG) model. The EoS of GCCG is given by,

$$p_g = -\rho_g^{-\alpha} \left[ C + \left\{ \rho_g^{(1+\alpha)} - C \right\}^{-\omega} \right]$$

(5)

where $C = \frac{A}{1+\omega} - 1$, with $A$ being a constant that can take on both positive and negative values, and $-\mathcal{L} < \omega < 0$, $\mathcal{L}$ being a positive definite constant, which can take on values larger than unity.

In order to complement the statefinder diagnostic \[11\], a new diagnostic called $Om$ \[12\], was proposed in 2008, which helped to distinguish between the energy densities of various DE models. The $Om$ diagnostic is defined as,

$$Om(z) = \left( \frac{H(z+1)}{H_0} \right)^2 - 1 - \left( \frac{z+1}{z+1} \right)^3 - 1$$

(6)

Here $H_0$ is the present value of Hubble parameter, and $z$ is the redshift parameter. The advantage of $Om$ over the statefinder parameters is that, $Om$ involves only the first derivative of scale factor, and so it is easier to reconstruct it from the observational data. For the $\Lambda$CDM model $Om$ diagnostic turns out to be a constant, since $\Lambda$CDM is independent of redshift $z$. This is the reason why we prefer $Om$ diagnostic over statefinder parameters for the present study.

It is believed that general relativity is accurate at small scales only, and therefore needs modifications at cosmological distances. Based on the above concept modified gravity evolved as an alternative to dark energy in order to explain the recent cosmic acceleration. Here, instead of the matter content of the universe, the geometry of space-time itself drives the cosmic acceleration. Some of the well known modified gravity theories are Brane gravity \[13\], Galileon gravity \[14\], Brans-Dicke gravity \[15\], Horava-Lifshitz gravity \[16\], etc. Loop quantum gravity \[17, 18, 19, 20, 21\] evolved a strong counterpart to the above theories, dealing with the quantum effects of the universe. This property makes the theory unique and very interesting, since our ultimate goal is to find a unified theory of general relativity and quantum mechanics.

In recent years Loop quantum cosmology (LQC) has evolved as a major candidate for modified gravity models consistent with recent observational data. It is a non-perturbative and background independent theory trying to describe the quantum effects of the universe \[17, 18, 19, 20, 21\]. Here a discrete quantum theory replaces the classical space-time continuum of General Relativity and hence this theory is identified as a major effort to unify Einstein’s General Relativity with Planck’s Quantum theory. Due to this LQC has attained a prime status in modern cosmology. Extensive research has been carried out over the past few years in order to develop the theory and remove the possible shortcomings.

Our motivation is to study the Loop quantum effects on $Om$ diagnostic and subsequently on the universe. It is obvious that we will need to employ dark energy models for the study. We will reconstruct the $Om$ diagnostic in the background of Loop quantum gravity (LQG) and then study the behaviour of various Chaplygin gas DE models using the modified diagnostic. The trajectories will discriminate the various DE models from each other. The Loop quantum effects on the universe will be realized in a comparative scenario. The paper is organized as follows: In section 2, we study the loop quantum effects on $Om$ diagnostic for different DE models. In section 3, the plots are analyzed and their cosmological implications are discussed. Finally the paper ends with a short conclusion in section 4.

## 2 Loop quantum effects on the $Om$ diagnostic for various Chaplygin gas dark energy models

Einstein’s equations for flat homogeneous and isotropic universe are given by,

$$H^2 = \frac{k^2}{3} \rho$$

(7)

and

$$\dot{H} = -\frac{k^2}{2} (\rho + p)$$

(8)

The modified Friedmann equations for Loop Quantum Cosmology is given by \[22, 23\]

$$H^2 = \frac{\rho}{3} \left( 1 - \frac{\rho}{\rho_0} \right)$$

(9)
\[ \dot{H} = -\frac{1}{2} (\rho + p) \left( 1 - \frac{\rho}{\rho_1} \right) \]  

(10)

Here \( \rho \) is the matter density, \( p \) is the pressure and \( \rho_1 = \sqrt{3} \pi^{3/2} \xi^3 \xi \) is the critical loop quantum density. With the inclusion of this term, the universe bounces quantum mechanically as the matter energy density reaches the level of \( \rho_1 \) (order of Plank density). Here \( \gamma \) is the dimensionless Barbero-Immirzi parameter.

Using the above field equations in the eqn. (6), we get the \( Om \) diagnostic in Einstein gravity as given below,

\[ Om_E(z) = -1 + \frac{\left( \kappa \sqrt{\rho + \rho_M} \right) \left( 1 - \frac{\rho + \rho_M}{\rho_1} \right) + \sqrt{3} \kappa \left( 1 - \frac{2(\rho + \rho_M)}{\rho_1} \right) \left( \rho E + \rho M + \omega \rho_E \right)}{\left( \kappa \sqrt{\rho + \rho_M} \right) \left( 1 - \frac{\rho + \rho_M}{\rho_1} \right) + \sqrt{3} \kappa \left( 1 - \frac{2(\rho + \rho_M)}{\rho_1} \right) \left( \rho E + \rho M + \omega \rho_E \right)}^{2} \frac{2(1+z)}{H_0^2} \]

(11)

where \( \rho = \rho_E + \rho_M \) is the sum of the energy densities for dark energy and dark matter. \( \omega = \frac{\rho_E}{\rho_E} \) is the Equation of State (EoS) parameter of DE.

Using equations (7) and (8) in equation (6), we get the \( Om \) diagnostic in Loop quantum gravity as given by,

\[ Om_L(z) = -1 + \frac{\left( \kappa \sqrt{\rho + \rho_M} \right) \left( 1 - \frac{\rho + \rho_M}{\rho_1} \right) + \sqrt{3} \kappa \left( 1 - \frac{2(\rho + \rho_M)}{\rho_1} \right) \left( \rho E + \rho M + \omega \rho_E \right)}{\left( \kappa \sqrt{\rho + \rho_M} \right) \left( 1 - \frac{\rho + \rho_M}{\rho_1} \right) + \sqrt{3} \kappa \left( 1 - \frac{2(\rho + \rho_M)}{\rho_1} \right) \left( \rho E + \rho M + \omega \rho_E \right)}^{2} \frac{2(1+z)}{H_0^2} \]

(12)

### 2.1 Generalized Chaplygin gas

From the EoS of GCG we get the expression for the energy density of GCG as,

\[ \rho_{ch} = \left( B + C_1 (1 + z)^{3(1 + \alpha)} \right)^{\frac{1}{1 + \alpha}} \]

(13)

where \( C_1 \) is the constant of integration. Using equation (13) in equations (11) and (12) we get the expressions for \( Om \) diagnostic in Einstein gravity and Loop quantum gravity respectively as,

\[ Om_{E}^{ch} = \frac{1}{(1 + (1 + z)^{3})} \left[ -1 + \frac{\kappa \sqrt{\rho_{ch} + \rho_M}}{\sqrt{3}} + \frac{\sqrt{3} \kappa (\rho_{ch} + \rho_M + \omega \rho_{ch})}{2(1+z) \sqrt{\rho_{ch} + \rho_M}} \right] \]

(14)

and

\[ Om_{L}^{ch} = \frac{1}{-1 + (1 + z)^{3}} \left[ -1 + \frac{\kappa \sqrt{(\rho_{ch} + \rho_M) \left( 1 - \frac{\rho_{ch} + \rho_M}{\rho_1} \right)}}{\sqrt{3}} + \frac{\sqrt{3} \kappa \left( 1 - \frac{2(\rho_{ch} + \rho_M)}{\rho_1} \right) (\rho_{ch} + \rho_M + \omega \rho_{ch})}{2(1+z) \sqrt{(\rho_{ch} + \rho_M) \left( 1 - \frac{\rho_{ch} + \rho_M}{\rho_1} \right)}} \right]^{2} \]

(15)

where \( \omega_{ch} = \frac{\rho_E}{\rho_{ch}} \)

### 2.2 Modified Chaplygin gas

From the EoS of MCG we get the expression for energy density as,

\[ \rho_{mcg} = \left[ \frac{B}{1 + A} + C_2 (1 + z)^{3(1 + A)(1 + \alpha)} \right]^{\frac{1}{1 + \alpha}} \]

(16)

where \( C_2 \) is the integration constant. Using eqn. (16) is eqns. (11) and (12) we get the \( Om \) diagnostic for MCG in Einstein gravity and LQC respectively as below,

\[ Om_{E}^{mcg} = \frac{1}{-1 + (1 + z)^{3}} \left[ -1 + \frac{\kappa \sqrt{\rho_{mcg} + \rho_M}}{\sqrt{3}} + \frac{\sqrt{3} \kappa (\rho_{mcg} + \rho_M + \rho_{mcg} \omega_m)}{2(1+z) \sqrt{\rho_{mcg} + \rho_M}} \right] \]

(17)
Fig 1: trajectories in Einstein gravity and LQC for GCG. The other parameters are considered as $B = 1.5, \alpha = 0.5, \kappa = 1, \rho_1 = 0.2, H_0 = 72, \omega_{ch} = -1/3, C_1 = 1$.

and

$$\Omega_m^v = \frac{1}{-1 + (1 + z)^3} \left[ -1 + \frac{1}{H_0^2} \left( \frac{\kappa \sqrt{(\rho_{mcg} + \rho_M) (1 - \rho_{mcg} + \rho_M)}}{\sqrt{3}} + \sqrt{3} \kappa \left(1 - 2 \frac{(\rho_{mcg} + \rho_M)}{\rho_1} \right) \left( \rho_{mcg} + \rho_M + \rho_{mcg} \omega_v \right) \right)^2 \right]$$

where $\omega_m = \frac{\rho_m}{\rho_v}$

### 2.3 Variable Modified Chaplygin gas

The EoS of VMCG is given by eqn. (3). We consider $B(a) = B_0 a^{-m}$, where $B_0$ and $m$ are constants. Using the EoS of VMCG, we get the expression for energy density as,

$$\rho_v = \left[ \frac{B_0 (1 + \alpha) (1 + z)^m}{(A + 1) (\alpha + 1) - m} + C_3 (1 + z)^{(\alpha + 1)(A + 1)} \right]^{\frac{1}{1+m}}$$

(19)

where $C_3$ is the constant of integration. Now using the above equation in eqns. (11) and (12), we get the expressions of $\Omega_m$ diagnostic for VMCG in Einstein gravity and LQC respectively as,

$$\Omega_m^v = \frac{1}{-1 + (1 + z)^3} \left[ -1 + \frac{1}{H_0^2} \left( \frac{\kappa \sqrt{(\rho_v + \rho_M) (1 - \rho_v + \rho_M)}}{\sqrt{3}} + \sqrt{3} \kappa \left(1 - 2 \frac{(\rho_v + \rho_M)}{\rho_1} \right) \left( \rho_v + \rho_M + \rho_v \omega_v \right) \right)^2 \right]$$

(20)

and

$$\Omega_m^v = \frac{1}{-1 + (1 + z)^3} \left[ -1 + \frac{1}{H_0^2} \left( \frac{\kappa \sqrt{(\rho_v + \rho_M) (1 - \rho_v + \rho_M)}}{\sqrt{3}} + \sqrt{3} \kappa \left(1 - 2 \frac{(\rho_v + \rho_M)}{\rho_1} \right) \left( \rho_v + \rho_M + \rho_v \omega_v \right) \right)^2 \right]$$

(21)
Fig 2 : trajectories in Einstein gravity and LQC for MCG. The other parameters are considered as $B = 1.5$, $A = 1/3$, $\alpha = 0.5$, $\kappa = 1$, $\rho_1 = 0.2$, $H_0 = 72$, $\omega_m = -1/3$, $C_2 = 1$.

where $\omega_v = \frac{p_v}{\rho_v}$

### 2.4 New Variable Modified Chaplygin gas

We consider $A(a) = A_0 a^{-n}$ and $B(a) = B_0 a^{-m}$ (where $A_0$, $B_0$, $m$ and $n$ are constants) in the EoS of NVMCG (1) and get the expression for energy density as,

$$\rho_n = (1 + z)^3 \exp \left( \frac{3 A_0}{n (1 + z)} \right) \left[ C_4 + \frac{B_0}{A_0} \left( \frac{3 A_0 (1 + \alpha)}{n} \right)^{\frac{3(1 + \alpha) + n - m}{n}} \times \Gamma \left( \frac{m - 3 (1 + \alpha)}{n} \frac{3 A_0 (1 + \alpha)}{n (1 + z)^n} \right) \right]$$

(22)

where $C_4$ is the integration constant and $\Gamma(s,t)$ is the upper incomplete gamma function. Using the above equation in eqns. (11) and (12), we get the following expressions for $\Omega_m$ diagnostic in Einstein gravity and LQC respectively,

$$\Omega_{mE} = \frac{1}{1 - 1 + (1 + z)^3} \left[ -1 + \frac{1}{H_0^2} \left( \frac{\kappa \sqrt{\rho_M + \rho_n}}{\sqrt{3}} + \frac{\left( \sqrt{3} \kappa (\rho_M + \rho_n + \omega_n \rho_n) \right)}{2(1 + z) \sqrt{\rho_M + \rho_n}} \right) \right]^2$$

(23)

and

$$\Omega_{mL} = \frac{1}{1 - 1 + (1 + z)^3} \left[ -1 + \frac{1}{H_0^2} \left( \frac{\sqrt{3} \kappa (\rho_M + \rho_n + \omega_n \rho_n) \left( 1 - \frac{2(\rho_M + \rho_n)}{\rho_1} \right)}{2(1 + z) \sqrt{\rho_M + \rho_n} \left( 1 - \frac{\rho_M + \rho_n}{\rho_1} \right)} \right) \right]^2$$

(24)

where $\omega_n = \frac{p_n}{\rho_n}$
Fig 3: trajectories in Einstein gravity and LQC for VMCG. The other parameters are considered as
$A = 1/3, B_0 = 2, \alpha = 0.5, \kappa = 1, \rho_1 = 10^{-7}, H_0 = 72, \omega_v = -1/3, C_3 = 1, m = 4$.

Fig 4: trajectories in Einstein gravity and LQC for NVMCG. The other parameters are considered
as $A_0 = 1.2, B_0 = 2, \alpha = 0.5, \kappa = 1, \rho_1 = 0.2, H_0 = 72, \omega_n = -1/3, C_4 = 1, m = 4, n = 2$. 
Fig 5: trajectories in Einstein gravity and LQC for GCCG. The other parameters are considered as $C = 5, \alpha = 0.5, \kappa = 1, \rho_1 = 0.2, H_0 = 72, \omega_g = -1/3, C_5 = 1, \omega = -0.5$.

### 2.5 Generalized Cosmic Chaplygin gas

The energy density of GCCG is given by,

$$\rho_g = \left[ C + \left\{ 1 + C_5 (1 + z)^3(1+\alpha)(1+\omega) \right\}^{\frac{1}{1+\omega}} \right]^{1+\alpha}$$  \hspace{1cm} (25)

where $C_5$ is the integration constant. Using the above equation in eqns. (11) and (12), we get the expressions for $Om$ diagnostic as given below,

$$Om^g_E = \frac{1}{-1 + (1 + z)^3} \left[ -1 + \frac{1}{H_0^2} \left( \frac{\kappa \sqrt{\rho_g + \rho_M}}{\sqrt{3}} + \frac{\left( \sqrt{3} \kappa (\rho_g + \rho_M + \rho_g \omega_g) \right)}{2(1 + z) \sqrt{\rho_g + \rho_M}} \right)^2 \right]$$  \hspace{1cm} (26)

and

$$Om^g_L = \frac{1}{-1 + (1 + z)^3} \left[ -1 + \frac{1}{H_0^2} \left( \frac{\sqrt{3} \kappa (\rho_M + \rho_g + \omega_g \rho_g)\left( 1 - \frac{2(\rho_M + \rho_g)}{\rho_1} \right)}{2(1 + z) \sqrt{(\rho_M + \rho_g)\left( 1 - \frac{\rho_M + \rho_g}{\rho_1} \right)}} + \frac{\kappa}{\sqrt{3}} \left( \rho_M + \rho_g \right) \left( 1 - \frac{\rho_M + \rho_g}{\rho_1} \right) \right)^2 \right]$$  \hspace{1cm} (27)

where $\omega_g = \frac{p_g}{\rho_g}$

### 3 Graphical Analysis and Cosmological Implications

Plots showing the trajectories of $Om$ diagnostic for both Einstein gravity and Loop quantum gravity are given above. The loop quantum deviations are clearly visible in the figures. Fig 1 shows the trajectories for GCG. $\omega$ has been considered as $-1/3$. So we have considered an accelerating universe dominated by dark energy. We can see that for early universe (i.e. for higher redshifts), the loop quantum deviations are more pronounced. It reduces in magnitude as we come close to the present epoch. The portion of the plot $z < -1$ is obviously unphysical, yet it has been retained in the figure in order to give it a complete shape and for a better understanding.
of the $\Omega_m$ dynamics. It is be seen that the two trajectories meet at around $z = -6$, which is totally un-physical as far as our notion of cosmology is concerned (because $z > -1$). Therefore we can say that the trajectory for loop quantum gravity attains an asymptotic behaviour around the present regime and continue to do so in the future universe.

Fig2 shows the plot of $\Omega_m$ diagnostics for MCG. Clear deviations in trajectories are visible, corresponding to the loop quantum effects. But this differs from the case of GCG in the sense that the two trajectories never intersect each other. In fig 2, the trajectories for MCG are obtained both for Einstein gravity and LQC. In this paper, we have actually used the $\Omega_m$- diagnostics, not only as a tool to differentiate between various DE models but also we have analogously used the $\Omega_m$-trajectories to get information about the state of universe at a certain cosmological time. The states when compared between the Einstein gravity and LQC at the same cosmological time gives us the Loop quantum deviations suffered by the universe. From the fig.2 it is visible that the model suffers large quantum deviations in the early universe. The deviations alleviate as we reach the present epoch and they coincide in the present time ($z = 0$), as is expected. As we move into the future universe, i.e. $z < 0$, we can see that the deviations again become large and continue to grow as $z$ decreases. So in terms of deviations, the scenario is oscillatory about the present time ($z = 0$). It is actually an oscillating scenario between the two large deviation regimes for $z > 0$ and $z < 0$, with a no deviation regime at $z = 0$. This can probably be attributed to the barotropic term (first term) in the EoS of MCG.

Fig3, represents the plot for VMCG. In contrast to the previous two plots, this plot shows greater loop quantum deviations around the present regime. Although the trajectories coincide around $z = 0$, yet the region of coincidence is far smaller than the previous plots. In fact there is a sharp point of coincidence around $z = 0$ and then it immediately takes off showing larger deviations compared to the other plots. The reason behind this is obviously the chosen power law form of the function $B(a)$. In Fig4, plot have been generated for NVMCG. The plot almost resembles that of MCG. Finally in Fig5, we witness the plot for GCG, which shows the same trend as that of GCG.

In almost all the plots the Loop quantum effects are substantial in the early universe and gets alleviated to a comparable scenario around the present regime. The early universe was dominated by mass-less radiation. This lack or absence of mass is primarily responsible for the large quantum effect. Although radiation pressure exists which gives rise to momentum, yet the effect is far lesser than that of matter due to the absence of mass. As the universe tends towards the present regime, matter content in the universe increases considerably. Both dark and visible matter come into co-existence. This automatically alleviates the loop quantum effect on the universe and so the trajectory almost coincides with that of Einstein gravity. Moreover, we see that not only in the present epoch but also it preserves its comparable nature in the future universe. Obviously the difference between two gravity theories at present depends on the critical loop quantum density $\rho_1$. $\rho_1$ being an adjustable parameter, it is quite obvious to think that $\rho_1$ can be effectively fine tuned in order to realize different cosmological scenarios. But our analysis has shown that there is not much alteration in the scenario, with alteration in the value of $\rho_1$ for most of the DE models. It is only in the case of MCG, that we have to considerably fine tune the value of $\rho_1$ in order to generate coinciding trajectories for both the gravity theories, which is fundamental. So in a nutshell, in the present dark energy dominated epoch the loop quantum effects are diminished to such extent that it almost coincides with Einstein gravity, which follows fundamentally from the theory (LQC) itself. But the interesting thing that comes out from this study is that in the future regime ($-1 < z < 0$) the loop quantum effects continue to decay showing comparable behaviour to Einstein gravity. In that case the obvious questions that follow: How effective will LQC be in the future regime? Will it be able to describe the future universe as effectively as it has done till now? Any modified gravity theory evolves as a modification to Einstein gravity. If after all these, it cannot show considerable deviations from Einstein gravity, is there any necessity for it? Is it not that the modifications are not satisfactory. Do we really need that theory? These are the basic questions that arise from the behaviour of LQC in the future universe. Although on the basis of the results obtained from the above theoretical study, we cannot make a strong statement or reach a conclusion, yet the study does give us something to think about. Something that may change the present and the future cosmological scenario. Something that may rule out Loop quantum cosmology in near future and subsequently pave the way for alternative theories like string theory gaining prominence.

4 Conclusion

In this study we have basically made an attempt to study the loop quantum effects on the universe by studying its effects on the $\Omega_m$ diagnostics. Modified equations for the diagnostics were obtained in the background of loop quantum gravity. To study the effects, trajectories were generated for different class of Chaplygin gas dark
energy models. The study revealed that for almost all the models the quantum effects are highly pronounced in the early universe, which was dominated by mass-less radiation energy. But the more we moved towards the present regime the scenario completely changed as the trajectories for loop quantum gravity became almost comparable to those of Einstein gravity, thus exhibiting a strange alleviation of the quantum effect. Not only the present epoch, the alleviation continued in the future rendering the quantum gravity ineffective. Now the question arises that does the study really opens a gateway to something substantial in near future? Can loop quantum cosmology be ruled out for the future universe? Or is it that the theory is not consistent enough and requires further modifications! Obviously based on this study, it will not be fair to conclude anything, but it does give us a hope of something new. For the time being we keep it an open question, subject to further extensive research.

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