A weak, attractive, long-range force in Higgs condensates

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Abstract

Due to the peculiar nature of the underlying medium, density fluctuations in a ‘Higgs condensate’ are predicted to propagate for infinitely long wavelengths with a group velocity \( c_s \to \infty \). On the other hand, for any large but finite \( c_s \) there is a weak, attractive \( 1/r \) potential of strength \( \frac{1}{c_s^2} \) and the energy spectrum deviates from the purely massive form \( \sqrt{p^2 + M_h^2} \) at momenta smaller than \( \delta \sim \frac{M_h}{c_s} \). Physically, the length scale \( \delta^{-1} \) corresponds to the mean free-path for the elementary constituents in the condensate and would naturally be placed in the millimeter range.
In this Letter, I shall discuss some phenomenological aspects of the ground state of spontaneously broken theories: the ‘Higgs condensate’. The name itself (as for the closely related gluon, chiral,...condensates) indicates that, in our view, this represents a kind of medium made up by the physical condensation process of some elementary quanta. If this were true, such a vacuum should support long-wavelength density fluctuations. In fact, the existence of density fluctuations in any known medium is a basic experimental fact depending on the coherent response of the elementary constituents to disturbances whose wavelength is much larger than their mean free path. This leads to an universal description, the ‘hydrodynamical regime’, that does not depend on the details of the underlying molecular dynamics. By accepting this argument, and quite independently of the Goldstone phenomenon, the energy spectrum of a Higgs condensate should terminate with an ‘acoustic branch’, say \( \tilde{E}(p) = c_s|p| \) for \( p \to 0 \), as for the propagation of sound waves in ordinary media.

Some arguments suggest that, indeed, the vacuum of a ‘pro forma’ Lorentz-invariant quantum field theory may be such kind of medium. For instance, a fundamental phenomenon as the macroscopic occupation of the same quantum state (say \( p = 0 \) in some frame) may represent the operative construction of a ‘quantum aether’ [1, 2]. This would be quite distinct from the aether of classical physics, considered a truly preferred reference frame and whose constituents were assumed to follow definite space-time trajectories. However, it would also be different from the empty space-time of special relativity, assumed at the base of axiomatic quantum field theory to deduce the exact Lorentz-covariance of the energy spectrum.

In addition, one should take into account the approximate nature of locality in cutoff-dependent quantum field theories. In this picture, the elementary quanta are treated as ‘hard spheres’, as for the molecules of ordinary matter. Thus, the notion of the vacuum as a ‘condensate’ acquires an intuitive physical meaning. For the same reason, however, the simple idea that all deviations from Lorentz-covariance take place at the cutoff scale may be incorrect. In particular, non-perturbative vacuum condensation may give rise to a hierarchy of scales such that the region of Lorentz-covariance is sandwiched both in the high- and low-energy region.

In fact, in general, an ultraviolet cutoff induces vacuum-dependent reentrant violations of special relativity in the low-energy corner [3]. In the simplest possible case, these extend over a small shell of momenta, say \( |p| < \delta \), where the energy spectrum \( \tilde{E}(p) \) deviates from a Lorentz-covariant form. Since Lorentz-covariance becomes an exact symmetry in the local...
limit, for very large $\Lambda$, the scale $\delta$ is naturally infinitesimal in units of the energy scale associated with the Lorentz-covariant part of the energy spectrum, say $M$. By introducing dimensionless quantities, this means $\epsilon \equiv \frac{\delta}{M} \to 0$ when $t \equiv \frac{\Lambda}{M} \to \infty$ so that the continuum limit can equivalently be defined either as $t \to \infty$ or $\epsilon \to 0$. Notice that, formally, $O(\frac{\delta}{M})$ vacuum-dependent corrections represent $O(M\Lambda)$ effects which are always neglected when discussing how Lorentz-covariance emerges at scales much smaller than the ultraviolet cutoff. In this sense, Lorentz-covariance is formally valid in the local limit but, for finite $\Lambda$, there are infinitesimal deviations in an infinitesimal region of momenta that cannot be understood without exploring the physical properties of the vacuum.

For instance, for large but finite $t$, does the group velocity $c_s = \frac{dE}{dp}$ remain below unity (in units of the light velocity $c$) when $|p| \to 0$? Of course, a group velocity $c_s > 1$ does not correspond to any ‘classical’ velocity being faster than light. Moreover, when $t \to \infty$, the relevant wavelengths are $O(1/\epsilon)$ and thus infinitely long on the physical length scale defined by $1/M$. As a consequence, in a strict continuum limit there would be no way to form the sharp wave fronts needed to transfer any type of information. However, what about the cutoff theory? For instance, if we just play with the numbers, and choose a mass unit $M = O(10^2)$ GeV with values of $t$ as large as $O(10^{16})$, we find conceivable to observe deviations from Lorentz-covariance at scales $O(10^{-33})$ cm. On the other hand, if such deviations were reentrant with an $\epsilon \sim 1/t$, what about vacuum fluctuations whose wavelengths were larger than a few millimeters? Should we consider them as ‘infinitely’ long? In this sense, a non-trivial structure of the vacuum raises delicate issues for which no general answers exist ‘a priori’.

2. After this preliminary discussion, our analysis will start after having understood that, quite independently of the Goldstone phenomenon, there is a gap-less mode of the singlet Higgs field in the spontaneously broken phase of $\lambda\Phi^4$ theories.

The presence of such a gap-less mode reflects the quantum nature of the scalar condensate that cannot be treated as a purely classical c-number field. In fact, either considering the re-summation of the one-particle reducible zero-momentum tadpole graphs in a given background field or performing explicitly the last functional integration over the strength of the zero-momentum mode of the singlet Higgs field, one finds two different solutions for the inverse zero-4-momentum propagator in the spontaneously broken phase of a (one component) $\lambda\Phi^4$ theory: a) $G_a^{-1}(p_\mu = 0) = M_h^2$ and b) $G_b^{-1}(p_\mu = 0) = 0$. For the convenience of the reader, we shall briefly repeat the main argument of ref. 3.
When discussing spontaneous symmetry breaking, the starting point is the separation of the scalar field into a constant background and a shifted fluctuation field, namely

$$\Phi(x) = \phi + h(x)$$  \hspace{2cm} (1)

In order Eq.(1) to be unambiguous, $\phi$ denotes the spatial average in a large 4-volume $\Omega$

$$\phi = \frac{1}{\Omega} \int d^4x \Phi(x)$$  \hspace{2cm} (2)

and the limit $\Omega \to \infty$ has to be taken at the end.

In this way, the full functional measure can be expressed as

$$\int [d\Phi(x)]... = \int_{-\infty}^{+\infty} d\phi \int [dh(x)]...$$  \hspace{2cm} (3)

and the functional integration on the r.h.s. of Eq.(3) is over all quantum modes for $p_\mu \neq 0$.

After integrating out all non-zero quantum modes, the generating functional in the presence of a space-time constant source $J$ is given by

$$Z(J) = \int_{-\infty}^{+\infty} d\phi \exp[-\Omega(V_{NC}(\phi) - J\phi)]$$  \hspace{2cm} (4)

and $V_{NC}(\phi)$ denotes the usual non-convex ('NC') effective potential obtained order by order in the loop expansion. Finally, by introducing the generating functional for connected Green’s functions $w(J)$ through

$$\Omega \ w(J) = \ln \frac{Z(J)}{Z(0)}$$  \hspace{2cm} (5)

one can compute the field expectation value

$$\varphi(J) = \frac{dw}{dJ}$$  \hspace{2cm} (6)

and the zero-momentum propagator

$$G_J(p_\mu = 0) = \frac{d^2w}{dJ^2}$$  \hspace{2cm} (7)

In this framework, spontaneous symmetry breaking corresponds to non-zero values of Eq.(4) in the double limit $J \to \pm 0$ and $\Omega \to \infty$.

Now, by denoting $\pm v$ the absolute minima of $V_{NC}$ and $M_h^2 = V_{NC}'$ its quadratic shape there, one usually assumes

$$\lim_{\Omega \to \infty} \lim_{J \to \pm 0} \varphi(J) = \pm v$$  \hspace{2cm} (8)
and
\[ \lim_{\Omega \to \infty} \lim_{J \to \pm 0} G_J(p_\mu = 0) = \frac{1}{M_h^2} \] (9)

In this case, the excitations in the broken phase would be massive particles (the conventional Higgs bosons) whose mass \( M_h \) is determined by the positive curvature of \( V_{\text{NC}} \) at its absolute minima.

The main result of [6] is that this conclusion is not true. In fact, at \( \varphi = \pm v \), besides the value \( \frac{1}{M_h^2} \), one also finds
\[ \lim_{\Omega \to \infty} \lim_{J \to \pm 0} G_J(p_\mu = 0) = +\infty \] (10)
a result that has no counterpart in perturbation theory. As discussed in ref. [6], the existence of such divergent behaviour admits a simple geometric interpretation in terms of the Legendre transform of \( w(J) \). Differently from \( V_{\text{NC}} \), this other definition of effective potential does not provide an infinitely differentiable function in the presence of spontaneous symmetry breaking [7]. Therefore, its left- and right- second derivatives at \( \varphi = \pm v \) do not coincide. In this sense, the existence of a singular zero-4-momentum propagator in the broken phase is a genuine quantum-field theoretical effect, quite independent of any physical interpretation. This will be discussed below.

3. ‘A priori’, the existence of two possible values for the zero-4-momentum propagator implies two possible types of excitations with the same quantum numbers but different energies when the 3-momentum \( p \to 0 \): a massive one, with \( \tilde{E}_a(p) \to M_h \), and a gap-less one with \( \tilde{E}_b(p) \to 0 \). However, the latter dominates the exponential decay \( \sim e^{-\tilde{E}_b(p)T} \) of the connected euclidean correlator for \( p \to 0 \). In this sense, the massive excitation is unphysical in the infrared region. Therefore, differently from the simplest perturbative indications, in a (one-component) spontaneously broken \( \lambda \Phi^4 \) theory there is no energy-gap associated with the ‘Higgs mass’ \( M_h \), as it would be for a genuine massive single-particle spectrum where the relation
\[ \tilde{E}_a(p) = \sqrt{p^2 + M_h^2} \] (11)
remains true for \( p \to 0 \). Rather, the infrared region is dominated by gap-less collective excitations
\[ \tilde{E}_b(p) \equiv c_s |p| \] (12)
depending on an unknown parameter \( c_s \) that controls the slope of the spectrum for \( p \to 0 \) and represents the ‘sound velocity’ for the density fluctuations of the scalar condensate.
The massive branch, however, can become important at higher momenta. To understand this point, let us explore the analogy with superfluid $^4\text{He}$. This analogy is based on the observation that, as for the interatomic $^4\text{He}-^4\text{He}$ potential, the low-energy limit of cutoff $\lambda \Phi^4$ is also a theory of quanta with a short-range repulsive core and a long-range attractive tail.

The essential point is that, for superfluid $^4\text{He}$, the existence of two types of excitations was first deduced theoretically by Landau on the basis of very general arguments. According to this original idea, there are phonons with energy $E_{\text{ph}}(p) = v_s |p|$ and rotons with energy $E_{\text{rot}}(p) = \Delta + \frac{p^2}{2\mu}$. Only later, it was experimentally discovered that there is a single energy spectrum $E(p)$ which is made up by a continuous matching of these two different parts. This unique spectrum agrees with the phonon branch for $p \rightarrow 0$ and agrees with the roton branch at higher momenta.

Although these results are well established, all details of the energy spectrum of superfluid $^4\text{He}$ in the matching region are not yet completely understood. For this reason, in our case of the spontaneously broken phase, we shall just extract the main conclusion: the existence of a single energy spectrum $\tilde{E}(p)$ that tends to $\tilde{E}_b(p)$ for $p \rightarrow 0$ and approaches $\tilde{E}_a(p)$ for larger values of $|p|$.

However, for $\Delta = \mu \equiv M_h$, the matching between Eqs.(11) and (12) is only possible for $c_s > 1$, in units of the light velocity $c$ (this can easily be checked looking for a possible intersection between Eqs.(11) and (12)). Only in this case, for sufficiently high momenta where $\tilde{E}_a(p) < \tilde{E}_b(p)$, the gap-less collective modes will become unphysical and the lowest excitations of the vacuum will correspond to the familiar, massive Higgs boson.

Independently of this argument, the idea that $c_s$ is actually infinitely larger than the light velocity is also supported by a semi-classical argument due to Stevenson that we shall briefly report. Stevenson’s argument starts from a perfect-fluid treatment of the Higgs condensate. In this approximation, energy-momentum conservation is equivalent to wave propagation with a squared velocity given by ($c$ is the light velocity)

$$c_s^2 = c^2 \left( \frac{\partial P}{\partial E} \right)$$

where $P$ is the pressure and $E$ the energy density. Introducing the condensate density $n$, and using the energy-pressure relation

$$P = -E + n \frac{\partial E}{\partial n}$$

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we obtain
\[ c_s^2 = c^2 \left( \frac{\partial P}{\partial n} \right) \left( \frac{\partial E}{\partial n} \right)^{-1} = c^2 \left( n \frac{\partial^2 E}{\partial n^2} \right) \left( \frac{\partial E}{\partial n} \right)^{-1} \] (15)

For a non-relativistic Bose condensate of neutral particles with mass \( m \) and scattering length \( a \), where \( \frac{nah^2}{m^2c^2} \ll 1 \) (in this case we explicitly introduce \( \hbar \) and \( c \)) one finds
\[ E = nmc^2 + n^2 \frac{2\pi ah^2}{m} \] (16)

so that
\[ c_s^2 = \frac{4\pi n ah^2}{m^2} \] (17)

which is the well known result for the sound velocity in a dilute hard sphere Bose gas \[15\].

On the other hand, in a fully relativistic case, the additional terms in Eq.(16) are such that the scalar condensate is spontaneously generated from the ‘empty’ vacuum where \( n = 0 \) for that particular equilibrium density where \[8\]
\[ \frac{\partial E}{\partial n} = 0 \] (18)

Therefore, in this approximation, approaching the equilibrium density one finds
\[ c_s^2 \to \infty \] (19)

thus implying that long-wavelength density fluctuations would propagate instantaneously in the spontaneously broken vacuum.

As Stevenson points out \[14\], Eq.(19) neglects all possible corrections to the perfect-fluid approximation. These require to introduce the effects of a mean free path \( R_{\text{mfp}} \) for the elementary constituents. Due to its finite value, in fact, sound waves will stop to propagate at a typical momentum \( |p| \sim \delta \equiv \frac{1}{R_{\text{mfp}}} \) \[14\]. At this value, the collective modes Eq.(12) become unphysical and for higher momenta the spectrum Eq.(11) applies. The transition corresponds to
\[ \sqrt{\delta^2 + M_h^2} \sim c_s\delta \] (20)

so that for \( c_s \to \infty \), \( \frac{\delta}{M_h} \sim \frac{1}{c_s} \to 0 \). Therefore, \( c_s \) represents the inverse of the parameter \( \epsilon \) introduced before to characterize the continuum limit. For this reason, we shall replace the condition \( \epsilon \to 0 \) with \( c_s \to \infty \).

As anticipated, in the continuum limit, superluminal wave propagation is restricted to the region \( p \to 0 \) and, therefore, one cannot use these waves to form a sharp wave front. As such, there would be no possibility to transfer informations giving rise to violations of...
causality \[14\]. These require a group velocity \( \frac{dE}{|p|} \gg 1 \) at finite \(|p|\) that cannot occur due to the change of the energy spectrum from Eq. (12) to Eq. (11).

In this sense, the perfect-fluid value \( c_s = \infty \) simulates an exact Lorentz-covariant limit where the energy spectrum maintains its massive form for \( p \to 0 \). Yet, this is not entirely true due to the subtleties associated with the zero-measure set \( p = 0 \). This set, in fact, belongs to the range of Eq. (12) and therefore the right \( c_s = \infty \) limit is always \( \tilde{E}(p = 0) = 0 \) and not \( \tilde{E}(p = 0) = M_h \). Just for this reason, the correct procedure is to include both branches of the spectrum in the spectral representation of the fluctuation field \( h(x) \). It may be convenient, however, to separate out the long-wavelength modes Eq. (12), say \( \tilde{h}(x) \), from the more conventional massive part of Eq. (11). The observable effects due to \( \tilde{h}(x) \) depend crucially on the value of \( c_s \) and will be discussed below.

4. Let us ignore, for the moment, the previous indication in Eq. (19) and just explore the phenomenological implications of long-wavelength modes in the spectrum as in Eq. (12). Whatever the value of \( c_s \), these dominate the infrared region so that a general yukawa coupling of the Higgs field to fermions will give rise to a long-range attractive potential between any pair of fermion masses \( m_i \) and \( m_j \)

\[
U_\infty(r) = -\frac{1}{4\pi c_s^2 \langle \Phi \rangle^2} \frac{m_i m_j}{r}
\]  

(21)

The above result would have a considerable impact for the Standard Model if we take the value \( \langle \Phi \rangle \sim 246 \) GeV related to the Fermi constant. Unless \( c_s \) be an extremely large number (in units of \( c \)) one is faced with strong long-range forces coupled to the inertial masses of the known elementary fermions that have never been observed. Just to have an idea, for \( c_s = 1 \) the long-range interaction between two electrons in Eq. (21) is \( \mathcal{O}(10^{33}) \) larger than their purely gravitational attraction. On the other hand, invoking a phenomenologically viable strength, as if \( c_s \langle \Phi \rangle \) were of the order of the Planck scale, is equivalent to re-obtain nearly instantaneous interactions transmitted by the scalar condensate as in Eq. (14).

Independently of phenomenology, the identification \( c_s \langle \Phi \rangle = M_{\text{Planck}} \) is also natural \[14\] noticing that for \( c_s \to \infty \) the energy-spectrum becomes Lorentz-covariant (with the exception of \( p = 0 \)). Therefore, in a picture where the ‘true’ dynamical origin of gravity is searched into long-wavelength deviations from exact Lorentz-covariance \[3\], it would be natural to relate the limit of a vanishing gravitational strength, \( M_{\text{Planck}} \to \infty \), to the limit of an exact Lorentz-covariant spectrum, \( c_s \to \infty \) \[14\].

Returning to more phenomenological aspects, we observe that the potential in Eq. (21)
can also be derived as a static limit from the effective lagrangian (\(\tilde{\sigma} \equiv \frac{\hbar}{(\Phi)}\))

\[
\mathcal{L}_{\text{eff}}(\tilde{\sigma}) = \frac{(\Phi)^2}{2}\tilde{\sigma}\left[c_s^2\Delta - \frac{\partial^2}{\partial t^2}\right]\tilde{\sigma} - \tilde{\sigma}\sum_f m_f \bar{\psi}_f \psi_f
\]

(22)

where the free part takes into account the peculiar nature of the energy spectrum Eq.(12). Eq.(22) is useful to represent the effects of \(\tilde{\sigma}\) over macroscopic scales as required by its long-range nature. The key-ingredient is the replacement of \(m \bar{\psi}_f \psi_f\) with \(T_{\mu}^\mu(x)\), the trace of the energy-momentum tensor of ordinary matter, a result embodied into the well known relation

\[
\langle f | T_{\mu}^\mu | f \rangle = m_f \bar{\psi}_f \psi_f
\]

(23)

This relation allows for an intuitive transition from the quantum to the classical theory. In fact, by introducing a wave-packet corresponding to a particle of momentum \(p\) and normalization \(\int d^3x \bar{\psi}_f \psi_f = \frac{m}{E(p)}\) we obtain

\[
-m \int d^4x \bar{\psi}_f \psi_f = -m \int ds
\]

(24)

where \(ds = dt\sqrt{1 - v^2}\) denotes the infinitesimal element of proper time for a classical particle with velocity \(v\). Therefore, using the relation

\[
\sum_n m_n \int ds_n = \int d^4x T_{\mu}^\mu(x)
\]

(25)

where

\[
T_{\mu}^\mu(x) \equiv \sum_n \frac{E_n^2 - \mathbf{P}_n \cdot \mathbf{P}_n}{E_n} \delta^4(\mathbf{x} - \mathbf{x}_n(t))
\]

(26)

Eq.(22) is finally replaced by

\[
\mathcal{L}_{\text{eff}}(\tilde{\sigma}) = \frac{(\Phi)^2}{2}\tilde{\sigma}\left[c_s^2\Delta - \frac{\partial^2}{\partial t^2}\right]\tilde{\sigma} - \tilde{\sigma} T_{\mu}^\mu
\]

(27)

In this way, we get the equation of motion

\[
\left[c_s^2\Delta - \frac{\partial^2}{\partial t^2}\right]\tilde{\sigma} = \frac{T_{\mu}^\mu}{(\Phi)^2}
\]

(28)

and it is clear that, for those very large values \(c_s \to \infty\) suggested by the properties of the vacuum, the \(\tilde{\sigma}\) Green’s function has practically no retardation effects. In this limit, Eq.(28) reduces to an instantaneous interaction

\[
\Delta \tilde{\sigma} = \frac{T_{\mu}^\mu}{c_s^2(\Phi)^2}
\]

(29)
of vanishingly small strength. Finally for very slow motions, when the trace of the energy-momentum tensor reduces to the mass density

$$\rho(x) \equiv \sum_n m_n \delta^3(x - x_n(t))$$

(30)

one gets, formally, a Poisson equation where, however, the Newton constant \( G_N = M_{\text{Planck}}^{-2} \) is replaced by the product \( c_s^{-2} \left< \Phi \right>^{-2} \)

$$\Delta \delta = \frac{1}{c_s^2 \left< \Phi \right>^2} \rho(x)$$

(31)

For this reason, again, one may be tempted to exploit the identification \( M_{\text{Planck}} = c_s \left< \Phi \right> \) \cite{16, 20}.

Before concluding, we comment on the momentum \( \delta \) Eq.\,(20) associated with the transition between the two branches of the spectrum Eqs.\,(12) and (11). For \( r \sim R_{\text{mfp}} = \delta^{-1} \) the interparticle potential is not a simple \( 1/r \), as for asymptotic distances, but has to be computed from the Fourier transform of the \( h \)-field propagator

$$D(r) = \int \frac{d^3p}{(2\pi)^3} \frac{e^{ipr}}{E^2(p)}$$

(32)

and depends on the detailed form of the spectrum that interpolates between Eqs.\,(12) and (11). However, in the Standard Model, for \( M_h = \mathcal{O}(\left< \Phi \right>) \) and \( c_s \left< \Phi \right> = M_{\text{Planck}} \), one would predict in any case a length scale \( R_{\text{mfp}} = \delta^{-1} \) in the millimeter range. In this framework, the tight infrared-ultraviolet connection embodied in the relation \( \delta \sim \frac{\left< \Phi \right>^2}{M_{\text{Planck}}} \) would formally be identical to that occurring in models \cite{21} with extra space-time dimensions compactified at a size \( R_c = R_{\text{mfp}} \).

5. Summarizing: the conventional description of the singlet Higgs field as a purely massive field has to be modified to take into account that the energy spectrum has the form \( c_s |p| \) for \( p \to 0 \). In this sense, the Higgs condensate is a truly physical medium that can support long-wavelength density fluctuations. However, their velocity \( c_s \) becomes infinitely large and their wavelengths become infinitely long when approaching the continuum limit. Physically, this corresponds to treat the Higgs condensate in a perfect-fluid approximation.

In a cutoff theory, \( c_s \) is finite and this amounts to introduce a finite mean free path \( R_{\text{mfp}} \equiv \delta^{-1} \) for the elementary constituents in the condensate. In this situation, one finds a weak, attractive long-range force proportional to \( \frac{1}{c_s^2} \) and the energy spectrum acquires its single-particle form \( \sqrt{p^2 + M_h^2} \) at very small momenta \( |p| \sim \delta \) with \( \delta = \mathcal{O}(\frac{M_h}{c_s}) \). There are
some arguments for the identification \( c_s(\Phi) = M_{\text{Planck}} \), that suggest the possible relevance of our picture in connection with the problem of gravity [16, 20]. In this case, the length \( R_{\text{mfp}} \) would be a fundamental scale placed in the millimeter range.

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[16] Of course, the identification \( c_\mu \langle \Phi \rangle = M_{\text{Planck}} \) implies that gravity is somehow ‘dynamically’ induced by the Higgs vacuum itself, an idea that goes back to the old induced-gravity approaches \[17\]. If this were true, the peculiar ‘geometrical’ properties at the base of the classical space-time picture of General Relativity should also come out naturally from the same vacuum structure. Such a derivation is beyond the scope of the present Letter. However, it may be appropriate to mention that an important ingredient for this derivation may be found in the hydrodynamical analogy \[18, 19, 2\], namely in the low-energy properties of those ‘gravity-analogs’ (moving fluids, condensed matter systems with a refractive index, Bose-Einstein condensates,..) that can simulate and/or reproduce the space-time properties of General Relativity. For these systems, space-time is exactly flat at a fundamental level. However, a curved space-time metric emerges when describing the propagation of low-energy fluctuations. In this sense, what we call ‘Einstein gravity’ seems a kind of universal picture as hydrodynamics that, concentrating on the properties of matter at scales much larger than the mean free path for the elementary constituents, is insensitive to the details of the short-distance molecular dynamics \[14\]. In quantum field theory, this means that, as far as the curved space-time properties are concerned, the dynamical origin of gravity may be equivalently found into a superfluid fermionic vacuum \[4\].
into a spontaneously broken phase,... This point of view should not sound too surprising to the extent that it is consistent with an historical origin of General Relativity before the birth of quantum mechanics. Actually, just the hydrodynamical analogy, allowing for a geometric representation of the *reentrant violations of special relativity in the low-energy corner* \[3\] may represent the essential ingredient to understand the origin of the Equivalence Principle from a particle physics context.

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