Possible Puzzles in Nonleptonic $B$ Decays

Adam F. Falk *

Department of Physics and Astronomy
The Johns Hopkins University
3400 North Charles Street
Baltimore, Maryland 21218 U.S.A.

ABSTRACT

I discuss the recent controversy over the semileptonic branching ratio of the $B$ meson, pointing out that it is only the combination of this quantity with the reported charm multiplicity in $B$ decays which is in serious conflict with theoretical calculations. I consider possible solutions to the problem.

1. Introduction

There has been considerable recent interest in the question of whether there is a serious conflict between the measured semileptonic branching ratio of the $B$ meson and its theoretical calculation. While a straightforward analysis of this quantity, based on the operator product expansion, certainly yields a result which is in conflict with experiment, is this discrepancy worth worrying about? In particular, is it pointing us toward new physics beyond the Standard Model, or telling us something interesting about QCD, or neither? What is the most conservative, and hence most plausible, explanation of the puzzle? Here I will argue that its resolution lies most likely within QCD itself and is probably quite mundane, namely that the calculation of inclusive nonleptonic decay rates is considerably less trustworthy than the calculation of inclusive semileptonic decays and fails even for total energy releases as large as $m_b$.

I will concentrate on the theoretical foundation of the calculation, explaining the roles which are played by the Operator Product Expansion (OPE) and the two versions of the duality hypothesis, “local” and “global”. We will see that if we combine the OPE and the assumption of local duality with the reported semileptonic branching ratio of the $B$ meson, then we are led to the expectation of excess charm production in inclusive $B$ decays, as much as 1.3 charm quarks per decay. The charm data, as reported, do not support such an excess; if anything, they indicate a small deficit of charm. What is more, the most plausible and least “damaging” emendations to the OPE do not solve this problem, as they also would imply excess charm. The real puzzle is not the semileptonic branching ratio alone, but rather the combination of it with the reported charm multiplicity. We are led to the conclusion that, if the data are confirmed, the key assumption of local duality may not be justified in nonleptonic $B$ decays.

*Invited talk presented at the Eighteenth Johns Hopkins Workshop on Current Problems in Particle Theory, Florence, Italy, August 31–September 2, 1994.
2. Theoretical Analysis of Inclusive \( B \) Decays

The inclusive decay rate of the \( B \) meson may be organized by the flavor quantum numbers of the final state,

\[
\Gamma_{\text{TOT}} = \Gamma(b \rightarrow c \ell \bar{\nu}) + \Gamma(b \rightarrow c\bar{u}d') + \Gamma(b \rightarrow c\bar{c}s') . \tag{1}
\]

We neglect rare processes, such as those mediated by an underlying \( b \rightarrow u \) transition or penguin-induced decays. We denote by \( d' \) and \( s' \) the approximate flavor eigenstates \( (d' = d \cos \theta_1 - s \sin \theta_1, \ s' = d \sin \theta_1 + s \cos \theta_1) \) which couple to \( u \) and \( c \), respectively, and we ignore the effect of the strange quark mass. It is convenient to normalize the inclusive partial rates to the semielectronic rate, defining

\[
R_{ud} = \frac{\Gamma(b \rightarrow c\bar{u}d')} {3\Gamma(b \rightarrow c\bar{e}\nu)} , \quad R_{cs} = \frac{\Gamma(b \rightarrow c\bar{c}s')} {3\Gamma(b \rightarrow c\bar{e}\nu)} . \tag{2}
\]

The full semileptonic width may be written in terms of the semielectronic width as

\[
\Gamma(b \rightarrow c \ell \bar{\nu}) = 3f(m_\tau)\Gamma(b \rightarrow c\bar{e}\nu) , \tag{3}
\]

where the factor \( 3f(m_\tau) \) accounts for the three flavors of lepton, with a phase space suppression which takes into account the \( \tau \) mass. Then, since the semileptonic branching ratio is given by \( Br(b \rightarrow c \ell \bar{\nu}) = \Gamma(b \rightarrow c \ell \bar{\nu})/\Gamma_{\text{TOT}} \), we may rewrite Eq. (3) in the form

\[
R_{ud} + R_{cs} = f(m_\tau) \frac{1 - Br(b \rightarrow c \ell \bar{\nu})} {Br(b \rightarrow c \ell \bar{\nu})} . \tag{4}
\]

The measured partial semileptonic branching fractions are\( ^{3,4} \)

\[
Br(\bar{B} \rightarrow Xe\bar{\nu}) = 10.7 \pm 0.5 \%,
Br(\bar{B} \rightarrow Xm\bar{\nu}) = 10.3 \pm 0.5 \% , 
Br(\bar{B} \rightarrow X\tau\bar{\nu}) = 2.8 \pm 0.6 \% , \tag{5}
\]

leading to a total semileptonic branching fraction \( Br(b \rightarrow c \ell \bar{\nu}) \) of 23.8 \pm 0.9\%, with the experimental errors added in quadrature. Of the semileptonic rate, 11\% comes from decays to \( \tau \), corresponding to a phase space suppression factor \( f(m_\tau) = 0.74 \), consistent with what one would expect in free quark decay\( ^{5,6} \) If we substitute the measured branching fractions into the right-hand side of Eq. (4), we find

\[
R_{ud} + R_{cs} = 2.37 \pm 0.12 . \tag{6}
\]

This is the result which we must try to reproduce theoretically.

To compute \( R_{ud} \) and \( R_{cs} \), it is necessary to compute individually the partial widths \( \Gamma(b \rightarrow c\bar{e}\nu) \), \( \Gamma(b \rightarrow c\bar{u}d') \) and \( \Gamma(b \rightarrow c\bar{c}s') \). Because the computation is conceptually more straightforward for this case, we begin with the nonleptonic widths.
2.1. Nonleptonic decays

The nonleptonic quark decay \( b \rightarrow c \bar{q} q_1 q_2 \) is mediated by four-quark operators of the form

\[
\mathcal{O} = \bar{c} \Gamma_1 b \bar{q}_2 \Gamma_2 q_1,
\]

where the \( \Gamma_i \) denote generic Dirac matrices. Such operators are induced by the exchange of \( W \) bosons which have been integrated out the theory at the scale \( \mu = M_W \), where \( \Gamma_1 = \Gamma_2 = \gamma_\mu (1 - \gamma_5) \) and the coefficient of \( \mathcal{O} \) in the effective Hamiltonian is \( G_F / \sqrt{2} \). As the theory is evolved down to the scale \( \mu = m_b \), this coefficient changes, and additional operators of the form (7) are induced with coefficients which depend on \( \log(\mu/m_b) \). In this schematic discussion of the inclusive decay rate, we will ignore the interference between these operators and treat only the generic operator \( \mathcal{O} \) above.

The inclusive decay rate of the \( \overline{B} \) meson mediated by the operator \( \mathcal{O} \) is given by

\[
\Gamma_{\mathcal{O}} \sim \sum_X \langle \overline{B} | \mathcal{O}^\dagger | X \rangle \langle X | \mathcal{O} | \overline{B} \rangle = \sum_X |\langle X | \mathcal{O} | \overline{B} \rangle|^2,
\]

where the sum runs over all states \( X \) with the appropriate quantum numbers. This expression may be rewritten via the optical theorem in terms of the imaginary part of a forward scattering amplitude,

\[
\Gamma_{\mathcal{O}} \sim \text{Im} \langle \overline{B} | T\{\mathcal{O}^\dagger, \mathcal{O}\} | \overline{B} \rangle.
\]

The next step is to compute the time-ordered product \( T\{\mathcal{O}^\dagger, \mathcal{O}\} \) in perturbative QCD.

Before we proceed with this step, it is useful to examine the conditions under which it is sensible to apply a perturbative analysis. In Fig. 1, I present a cartoon of the quantity

\[
f(m_Q) \equiv \text{Im} \langle M_Q | T\{\mathcal{O}^\dagger, \mathcal{O}\} | M_Q \rangle,
\]

where the \( b \) quark has been replaced with a quark \( Q \) of mass \( m_Q \) and the \( \overline{B} \) meson by the ground state meson \( M_Q \). The expectation value of the correlator is given as a function of \( m_Q \). We see in Fig. 1 that the behavior of \( f(m_Q) \) is qualitatively different for \( m_Q \) large and for \( m_Q \) small compared to the scale \( \Lambda_{\text{QCD}} \) which characterizes the nonperturbative regime of QCD. For small \( m_Q \), the correlator is dominated by a few resonances whose position and strength depends on nonperturbative strong dynamics. In this “resonance regime”, no perturbative calculation can reproduce the rapid variations in the shape of \( f(m_Q) \). For large \( m_Q \), however, these rapid variations damp out, as in this region the correlator is dominated by multiparticle states. While even here there are new thresholds associated with the production of additional pions, their effect is small compared to the smooth background of states to which they are being added. In this regime, one might expect the smooth output of a perturbative calculation of the correlator to approach the physical \( f(m_Q) \). This property, that at large enough energies one may compute accurately within perturbative QCD, is known as “local duality”.
Fig. 1. Cartoon of \( f(m_Q) \equiv \text{Im} \langle M_Q | T\{\mathcal{O}^1, \mathcal{O}\} | M_Q \rangle \).

Where is the transition between the resonance and perturbative regimes, or in other words, above what value of \( m_Q \) does one expect local duality to hold? In particular, is \( m_b \) sufficiently large for perturbative QCD to be applicable? Since we cannot vary \( m_b \) experimentally, we cannot answer this question by tracing out \( f(m_Q) \) and looking for where it becomes smooth. We can only argue on dimensional grounds. Naively, we might expect to be well in the perturbative regime, since \( m_b \approx 5 \text{ GeV} \) is large compared to typical QCD scales of hundreds of MeV. However, the situation is not necessarily so clear, since the nonleptonic decay is into three colored particles which must divide the available energy between them, and charm quark(s) in the final state carry away substantial rest energy as well. We also note that the transition point between the resonance and perturbative regimes undoubtedly depends on the operator \( \mathcal{O} \), as well as on the external states \( M_Q \).

While we are free to have our prejudices, it is probably best to leave this question open for the time being. We may plunge ahead with the perturbative calculation of the total width, while bearing in mind that the assumption of local duality is made by necessity but cannot rigorously be justified. Later, it will be necessary to evaluate any discrepancies between experiment and the theoretical results in light of their sensitivity to this crucial assumption.

It is also true that local duality may well fail at different \( p^2 \) in the channels \( b \rightarrow c\bar{u}d' \) and \( b \rightarrow c\bar{s}s' \). In fact, we might expect it to fail first (at higher energy) in the channel with two charm quarks in the final state, since, due to the charm quark rest energy, the kinetic energy released per particle is lower in this channel. For example, in a simple free quark decay model the \( s \) quark has an average energy of only 1 GeV.
This intuitive expectation is our first hint, with more to come, that the quantity $R_{ud}$ is more reliably calculable than $R_{cs}$.

### 2.2. Semileptonic decays

The situation is considerably better for semileptonic decays. Here the decay is mediated by a product of currents,

$$ O = \bar{c}\gamma^\mu(1 - \gamma^5)b\bar{\ell}\gamma^\mu(1 - \gamma^5)\nu, \quad (11) $$

and the matrix element factorizes into hadronic and leptonic pieces,

$$ \langle X\ell(p_\ell)\bar{\nu}(p_\bar{\nu})|J^\mu_hJ^\nu_h|B\rangle = \langle X|J^\mu_h|B\rangle \cdot \langle \ell(p_\ell)\bar{\nu}(p_\bar{\nu})|J^\nu_h|0\rangle. \quad (12) $$

We then find an expression in which the integral over the momenta of the leptons is explicit,

$$ \Gamma \sim \int dy\, dv\cdot\hat{q}\, d\hat{q}^2 \, L_{\mu\nu}(v\cdot\hat{q},\hat{q}^2, y) \, W_{\mu\nu}(v\cdot\hat{q}, \hat{q}^2), \quad (13) $$

where $L_{\mu\nu}$ is the lepton tensor and $W_{\mu\nu}$ the hadron tensor. Here the momentum of the external $b$ quark is written as $p_b^\mu = m_b v^\mu$. The other independent kinematic variables are $\hat{q}^\mu = p_\ell^\mu + p_{\bar{\nu}}^\mu$ and $y = 2v\cdot p_\ell/m_b$. It is convenient to scale all momenta by $m_b$, so $\hat{q} = q/m_b$. The hadronic tensor is given by

$$ W_{\mu\nu} = \sum_X \langle B|J^\mu_h|X\rangle \langle X|J^\nu_h|B\rangle = -2\text{ Im} \langle B|\int dx\, e^{iq\cdot x} \, T \{ J^\mu_h(x), J^\nu_h(0) \} |B\rangle = -2\text{ Im} T_{\mu\nu}. \quad (14) $$

We may perform the integrals in $y$, $v\cdot\hat{q}$ and $\hat{q}^2$ in Eq. (13) to compute the total semileptonic decay rate, or leave some of them unintegrated to obtain various differential distributions.

Let us consider as an example the doubly differential distribution $d\Gamma/dy\, d\hat{q}^2$, for which we must perform the integral over $v\cdot\hat{q}$, for $y$ and $\hat{q}^2$ fixed. The range of integration is given by $(y + \hat{q}^2/y)/2 \leq v\cdot\hat{q} \leq (1 + \hat{q}^2 - m_q^2)/2$, where $m_q$ is the mass of the quark to which $b$ decays, and $\hat{m}_q = m_q/m_b$. The integration is pictured as the solid contour in Fig. 2, along with the analytic structure of $T_{\mu\nu}$; the cut extends along the real axis except in the region $(1 + \hat{q}^2 - \hat{m}_q^2)/2 < v\cdot\hat{q} < ((2 + \hat{m}_q)^2 - \hat{q}^2 - 1)/2$. Note that we have already included only the imaginary part of $T_{\mu\nu}$ by integrating over the top of the cut and then back underneath. In general, along the cut $T_{\mu\nu}$ will look qualitatively like the cartoon shown in Fig. 1 depending on $v\cdot\hat{q}$ in a complicated nonperturbative way. However, we may now use Cauchy’s Theorem to deform the contour of integration to the dashed contour in Fig. 2 which lies far away from the cut everywhere but at its endpoints. Along this new contour, we are far from any physical intermediate states, and it is safe to perform the operator product expansion in perturbative QCD.
The additional integrals over the lepton phase space allow us to determine the physical rate in terms of certain smooth integrals of $T^{\mu\nu}$ rather than in terms of the time-ordered product evaluated at a single point. These smooth integrals may be computed reliably by deforming the contour into the unphysical region. That we we can compute integrals of $T^{\mu\nu}$ in this way is the property of “global duality”. In principle, it rests on a much firmer theoretical foundation than does local duality, which asserts the computability of QCD correlators directly on the physical cut rather than off in some unphysical region of the complex plane.

Unfortunately, the dashed contour must still approach the physical cut near the endpoints of the integration, introducing an unavoidable uncertainty into the calculation. There is a two-pronged argument that this uncertainty is small. First, for large $m_b$, the proportion of the contour which is within $\Lambda_{\text{QCD}}$ of the physical cut scales as $\Lambda_{\text{QCD}}/m_b$ and thus makes a small contribution to the total integral. Second, in this small region one may invoke local duality, as in the case of nonleptonic decays. The endpoints of the integration lie conveniently at the point of maximum recoil of the final state quark, where this property is most likely to hold. In addition, since there are fewer colored particles in the final state than in nonleptonic decays, one would expect this assumption to be more reasonable in the semileptonic than in the nonleptonic case.

The point is that semileptonic decays, because they rely on global rather than local duality, may be calculated much more reliably in perturbative QCD than may nonleptonic decays. This is why we have chosen, in defining $R_{ud}$ and $R_{cs}$, to normalize the nonleptonic widths to the semielectronic width. (Of course, this also eliminates...
the dependence on the CKM parameter \( V_{cb} \) and some of the dependence on \( m_b \).) The theoretical uncertainties in \( R_{ud} \) and \( R_{cs} \) will be dominated entirely by the uncertainties in the numerators.

2.3. The perturbative calculation

The computation of \( T\{O^\dagger, O\} \) in perturbation theory follows well-known techniques for both semileptonic\(^7\)–\(^1\) and nonleptonic\(^2\) decays. Since the quarks in the intermediate state are assumed to be far from the mass shell by an amount of order \( m_b \), we may develop the operator product in a simultaneous expansion in \( 1/m_b \) and \( \alpha_s \). Graphically, the expansion proceeds as in Fig. 3 in the nonleptonic case, to whatever accuracy is desired. The Operator Product Expansion takes the form of an infinite series of local operators,

\[
T\{O^\dagger, O\} = \sum_{n=0}^{\infty} \frac{c_n(\alpha_s)}{m_b^{n+1}} O_n ,
\]

where the operator \( O_n \) has dimension \( n + 3 \) and the \( c_n \) are calculable functions of \( \alpha_s \). The notation is somewhat schematic, in that more than one operator may appear for a given \( n \). For example, the operators of lowest dimension which appear in the expansion are of the form

\[
\begin{align*}
O_0 &= \bar{b} \Gamma b , \\
O_1 &= \bar{b} \Gamma^\mu D_\mu b , \\
O_2 &= \bar{b} \Gamma^{\mu\nu} D_\mu D_\nu b ,
\end{align*}
\]

where \( D_\mu \) is the covariant derivative, and \( \Gamma^{\mu\cdots} \) are combinations of Dirac matrices.
Once the OPE has been performed, we must compute the expectation values of the local operators $O_n$ in the $\overline{B}$ state. Such expectation values involve the structure of the meson as determined by nonperturbative QCD, and typically are incalculable. However, here we may use the additional symmetries of the heavy quark limit to simplify the description of these matrix elements. In this limit, we construct an effective theory in which the production of heavy quark-antiquark pairs is eliminated, at the price of introducing into the Lagrangian an infinite tower of nonrenormalizable operators suppressed by powers of $1/m_b$. In the effective theory, we may compute the matrix elements which arise at a given order in $n$ in terms of a few nonperturbative parameters. In fact, the matrix element of $O_0$ is determined exactly by the conservation of heavy quark number; if the expansion is truncated at this point, then we reproduce precisely the result of the free quark decay model. Since all matrix elements of $O_1$ vanish, the leading corrections to free quark decay come only at relative order $1/m_b^2$, not at order $1/m_b$. These corrections come from initial state binding effects of the $b$ quark in the $B$ meson, the energy of the heavy quark being altered from its “free” value by its interactions with the cloud of light quarks and gluons which surrounds it. They are governed by only two parameters, defined by the expectation values

$$
\langle \overline{B} | \overline{b} (iD)^2 b | \overline{B} \rangle \equiv 2M_B \lambda_1, \\
\langle \overline{B} | \overline{b} ( -i \sigma^{\mu\nu}) G_{\mu\nu} | \overline{B} \rangle \equiv 6M_B \lambda_2.
$$

The parameter $\lambda_2$, associated with an operator which violates the heavy quark spin symmetry, is determined by the mass difference between the $B$ and $B^*$ mesons,

$$
\lambda_2 = \frac{1}{2} (M_{B^*}^2 - M_B^2) \approx 0.12 \text{GeV}^2.
$$

Unfortunately, $\lambda_1$ cannot at this time be measured experimentally, and for its estimation we must rely on models of QCD. The same is true of the matrix elements of operators of dimension six and higher.

The ratios $R_{ud}$ and $R_{cs}$ receive contributions, then, both from perturbative ($\alpha_s$) and nonperturbative ($\lambda_1, \lambda_2, \ldots$) sources. I will summarize below the analysis of the semileptonic and nonleptonic radiative corrections which exists in the literature. The nonperturbative corrections to $R_{ud}$ and $R_{cs}$ have recently been studied up to operators of dimension six and shown to be quite small, because they turn out to be unimportant compared to the uncertainties in the radiative corrections, I will not include them from here on.

After some manipulation, the ratios $R_{ud}$ and $R_{cs}$ may be written in the form

$$
R_{ud} = P(\mu) + \delta P_{ud}(\mu, m_c), \\
R_{cs} = G(m_c/m_b) \left[ P(\mu) + \delta P_{cs}(\mu, m_c) \right].
$$

Here

$$
P(\mu) = \eta(\mu) \left[ 1 + \frac{\alpha_s(\mu)}{\pi} + J_2(\mu) \right]
$$

and $\eta(\mu)$ is a function of the strong coupling constant $\alpha_s$ at the scale $\mu$. The details of this analysis are given in the next section.
Fig. 4. The radiative correction $P(\mu)$. The upper curve corresponds to $\Lambda^{(5)}_{\text{MS}} = 220 \text{ MeV}$, the middle curve to $\Lambda^{(5)}_{\text{MS}} = 180 \text{ MeV}$, and the lower curve to $\Lambda^{(5)}_{\text{MS}} = 140 \text{ MeV}$. We take $m_b = 4.8 \text{ GeV}$.

is the radiative correction in the massless ($m_c = 0$) limit, with leading logarithmic ($\eta(\mu)$), one loop ($\alpha_s(\mu)/\pi$) and subleading logarithmic ($J_2(\mu)$) contributions. The inclusion of the subleading logarithms softens the dependence of $P(\mu)$ on the renormalization scale, which is shown in Fig. 4 for a range of values of $\Lambda^{(5)}_{\text{MS}}$. The quantities $\delta P_{ud}$ and $\delta P_{cs}$ parameterize the effect on the radiative corrections of the charm quark mass. Recent calculations\textsuperscript{14,15,19} show $\delta P_{ud}$ to be no larger than $\sim 0.02$. There are indications\textsuperscript{18–20} that $\delta P_{cs}$ may be substantially larger than $\delta P_{ud}$, but a full calculation is not yet available; we consider the uncertainty due to $\delta P_{cs}$ below.

While there is certainly some variation in $R_{ud}$ and $R_{cs}$ due to the dependence of the radiative corrections on the renormalization scale, the largest uncertainty in $R_{cs}$ comes from the phase space factor $G(m_c/m_b)$. There is an additional dependence on the charm mass in this channel because of the second charm quark in the final state. The analytic expression for $G(m_c/m_b)$ has been computed\textsuperscript{6}; we present a plot in Fig. 5. Here we have used the fact that within the heavy quark expansion, the difference between $m_c$ and $m_b$ is much more precisely known than either of the masses individually, and is given in terms of the spin-averaged $D$ meson and $B$ meson masses:

$$m_b - m_c = \langle M_B \rangle_{\text{ave.}} - \langle M_D \rangle_{\text{ave.}} = 3.34 \text{ GeV}.$$  \hspace{1cm} (20)

In Fig. 5 we hold $m_b - m_c$ fixed and consider variations of $m_b$ only.

The important inputs into the theoretical calculation, then, are $\mu$, $m_b$ and, to a lesser extent, $\Lambda^{(5)}_{\text{MS}}$. As first emphasized by Bigi et al.,\textsuperscript{1} it is not a trivial task to fix these parameters to satisfy the experimental constraint $R_{ud} + R_{cs} = 2.37 \pm 0.12$. For
example, if one takes the reasonable values $\mu = m_b = 4.8$ GeV and $\Lambda_{\overline{\text{MS}}}^{(5)} = 180$ MeV and neglects $\delta P_{ud}$ and $\delta P_{cs}$, then $P(\mu) = 1.27$, $G(m_c/m_b) = 0.36$ and $R_{ud} + R_{cs}$ is only 1.73, far short of the experimental number. The question, then, is whether there exist values of the inputs for which we can manage to reproduce the data.

It is useful to start by considering $R_{ud}$, for which the calculation is likely to be more reliable. To enhance this quantity we may vary the renormalization scale $\mu$. The choice $\mu = m_b$ is motivated by the fact that the total energy released in the decay $b \rightarrow c\bar{u}d'$ is $m_b$; however, since this energy has to be divided among three colored particles, perhaps the appropriate scale is lower. For $\mu = 1.6$ GeV $\approx m_b/3$, for example, and $\Lambda_{\overline{\text{MS}}}^{(5)} = 220$ MeV, we have $R_{ud} = P(\mu) = 1.52$, a modest enhancement. If we take this value of $\mu$ as a reasonable lower limit, then the data on the semileptonic branching ratio implies $R_{cs} \geq 0.85$. This restriction on $R_{cs}$ follows simply from the assumption that the perturbative calculation of $R_{ud}$ is reliable.

There is more than one way to achieve $R_{cs} \geq 0.85$, both within the present calculation and by going outside it. The easiest, perhaps, is to lower $m_b$ to 4.4 GeV, which corresponds to $m_c = 1.1$ GeV. These masses for the bottom and charm quarks are somewhat low, but probably not unacceptably so. Taking $\mu = 1.6$ GeV, then, we would find $G(m_c/m_b) = 0.58$, and $R_{cs} = 0.89$. Alternatively, we note that recent calculations indicate that $\delta P_{cs}$ may not be negligible at all, and that the inclusion of charm mass effects in the radiative correction in this channel may lead to an enhancement of as much as thirty percent. Finally, we could postulate the failure of local duality in the

---

*This scenario is supported by a recent estimate of $\mu$ for semileptonic decays using the BLM scale-setting criterion in which a scale $\mu$ as much as an order of magnitude below $m_b$ is indicated.*
$b \to c\bar{c}s'$ channel, since this is the channel in which we expect this assumption to be the least reliable. From any of these points of view, or from a combination of them, the $B$ semileptonic branching ratio is surprising, perhaps, but certainly explicable. We simply have to allow for an enhancement of the final state with two charmed quarks relative to the final state with one, as compared to our naïve expectation. At the same time, such an enhancement is *absolutely necessary* if the data are to be reconciled with the theoretical prediction for $R_{ud}$.

### 3. Experimental consequences

This proposal that an enhancement of $R_{cs}$ explains the observed semileptonic branching ratio is not without experimental implications of its own. The most striking of these is a large number of charmed quarks per $B$ decay. The charm multiplicity $n_c$ may be written

$$n_c = 1 + R_{cs} \frac{Br(B \to X_c \ell \bar{\nu})}{f(m_\tau)}.$$  \hspace{1cm} (21)

Using $Br(B \to X_c \ell \bar{\nu}) = 23.8\%$ and $f(m_\tau) = 0.74$ in Eq. (21) yields

$$n_c = 1.00 + 0.32 R_{cs},$$  \hspace{1cm} (22)

which for the values of $R_{cs}$ necessary to explain the semileptonic branching ratio would indicate $n_c \sim 1.3$.

While there has been no direct inclusive measurement of $n_c$, it may be estimated by summing individual partial widths. Among the $B$ decay products, there are contributions to $n_c$ from charmed mesons, charmed baryons, and $c\bar{c}$ resonances. The number of charged and neutral $D$ mesons per decay, summed over $B$ and $\bar{B}$, has been measured to be

$$n_{D^\pm} = 0.246 \pm 0.031 \pm 0.025,$$

$$n_{D^0, D^{'0}} = 0.567 \pm 0.040 \pm 0.023.$$  \hspace{1cm} (23)

The branching ratio to $D_s^\pm$ mesons has not yet been determined, because no absolute $D_s$ branching ratio has been measured. However, it is known that

$$n_{D_s^\pm} = (0.1181 \pm 0.0053 \pm 0.0094) \left[ \frac{3.7\%}{Br(D_s \to \phi\pi)} \right],$$  \hspace{1cm} (24)

and the branching ratio for $D_s \to \phi\pi$ is expected to be about 3.7%. Inclusive $B$ decays to $c\bar{c}$ resonances below $D\bar{D}$ threshold contribute two units to $n_c$. The branching ratio to $\psi$ has been measured to be $(1.11 \pm 0.08)\%$, including feed-down from the excited states $\psi'$ and $\chi_c$. Combined with the measurement $Br(B \to \psi'X) = (0.32 \pm 0.05)\%$, $Br(B \to \chi_{c1}X) = (0.66 \pm 0.20)\%$ and $Br(B \to \eta_cX) < 1\%$, we expect that the inclusive $B$ branching ratio to charmonium states below $D\bar{D}$ threshold is between two and three percent.
Finally, there is the contribution to $n_c$ of $B$ decays to charmed baryons. While the production of charmed baryons may be estimated from the measured inclusive production of all baryons in $B$ decays, such an extraction requires strong assumptions about the underlying production process and the predominant decay chains. Such assumptions introduce considerable uncertainty. Recently, however, the CLEO Collaboration has reported certain exclusive branching ratios of $B$ mesons to charmed baryons:

$$\begin{align*}
n_{\Xi^+} & \sim 1.5 \pm 0.7\%, \\
n_{\Xi^0} & \sim 2.4 \pm 1.3\%,
\end{align*}$$

(25)

with a branching ratio to $\Lambda_c$ roughly twice that to $\Xi_c$. Only statistical errors are included here. (The new CLEO data do not support the recent suggestion that charmed baryon production in $B$ decays is dominated by the underlying process $b \rightarrow c \bar{c} s'$, which would yield final states with two charmed baryons rather than one, somewhat alleviating the “charm deficit”.)

Summing over all the contributions, then, the experimental charm multiplicity is $n_c \approx 1.1 \pm 0.1$, where the error is dominated by the uncertainty on charmed baryon production. Such a value does not provide evidence for the excess charm production indicated by the data on the semileptonic branching ratio.

4. Theoretical implications

What are we to make of this situation? We have seen that given the uncertainties in the calculation, neither the semileptonic branching ratio of the $B$ meson nor the charm multiplicity in $B$ decays is by itself much of a problem. However, the combination of the two measurements is in sharp contradiction with current theoretical calculations. While there is considerable room in the computation of either quantity to make it agree with experiment, the adjustments that make one quantity better make the other worse. When viewed in this light, there are important implications for the theory of inclusive $B$ decays.

Of course, it is certainly possible that in the future the data may move in the direction of the theoretical calculations, and that the discrepancy simply will go away. In particular, this may happen once the contributions to $n_c$ of charmed baryons is better understood. If future measurements give support to the prediction $n_c \sim 1.3$, then the resolution of the semileptonic branching ratio puzzle simply will be that $R_{cs}$ is larger than expected. Whether this is due to a failure of local duality, to a low bottom quark mass, or to a renormalization scale much lower than the naïve choice $\mu = m_b$, may still be unclear, but as we have stressed, such an enhancement is not hard to arrange theoretically.

If, on the other hand, the present measurement of $n_c$ is confirmed, then the consequences are more extreme. The most natural explanation of such a situation would be that, for some reason, the calculation of $R_{ud}$ is also not reliable. Again, this could be due simply to a failure of local duality: perhaps $m_b$ is not large enough to calculate any inclusive nonleptonic decay widths at all. Such a conclusion would be disappointing,
of course, since this is the nonleptonic channel in which local duality was considered more likely to hold. However, disappointing is not the same as unnatural, and it is worthwhile to recall that the invocation of local duality was based essentially only on “educated hope”. There is nothing profound, necessarily, in learning that this time our luck was not so good.

An alternative explanation would be that the initial decomposition of the total width was incomplete, in that charmless final states were neglected. A charmless branching ratio of 20% would be sufficient to solve the semileptonic branching ratio puzzle without inducing an unacceptable charm multiplicity. Of course, the neglect of charmless final states is well justified within the Standard Model, in which $b \rightarrow u$ weak transitions are suppressed relative to $b \rightarrow c$ by a factor $|V_{ub}/V_{cb}|^2 \sim 0.01$, and transitions such as $b \rightarrow s\gamma$ are found at the $10^{-3}$ level or below. However, in extensions of the Standard Model this need not be the case, and it is possible to construct scenarios in which, for example, the rate for $b \rightarrow s\gamma$ is substantially enhanced without inducing dangerous new contributions to $b \rightarrow s\gamma$, for which there are already stringent limits. Models with an enhancement of $b \rightarrow s\gamma$ are also constrained by the known limits on the exclusive decay $B \rightarrow K\pi$, although unknown strong interaction effects preclude one from translating these limits into a firm bound on the strength of the $b \rightarrow s\gamma$ coupling.

Although it is certainly exciting to consider such novel possibilities, I must stress that they are not as yet well motivated by the data. The most conservative, natural and plausible resolution of the semileptonic branching ratio puzzle lies within QCD itself. In fact, the entire subject ought best to be thought of as an interesting test of the applicability of the Operator Product Expansion and perturbative techniques to the calculation of inclusive nonleptonic decays in the $B$ system. As of today, the jury remains out on this important question.

Acknowledgments

It is a pleasure to thank the organizers of the Workshop for putting together such an enjoyable and interesting conference. This work was supported by the National Science Foundation under National Young Investigator Award PHY-9457916 and Grant No. PHY-9404057, and by the Department of Energy under Outstanding Junior Investigator Award DE-FG02-094ER40869.

References

1. I.I. Bigi, B. Blok, M.A. Shifman and A. Vainshtein, Phys. Lett. B323, 408 (1994).
2. A.F. Falk, M.B. Wise and I. Dunietz, Johns Hopkins Report No. JHU–TIPAC–940005 (1994), to appear in Phys. Rev. D.
3. K. Hikasa et al. (Particle Data Group), Phys. Rev. D45, S1 (1992).
4. A. Putzer, Heidelberg Report No. HD–IHEP–93–03 (1993).
5. A.F. Falk, Z. Ligeti, M. Neubert and Y. Nir, Phys. Lett. B326, 145 (1994); L. Koyrakh, Phys. Rev. D49, 3379 (1994); S. Balk, J.G. Körner, D. Pirjol and K. Schilcher, Mainz Report No. MZ–TH/93–32 (1993).
6. J.L. Cortes, X.Y. Pham and A. Tounsi, Phys. Rev. D25, 188 (1982).
7. J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B247, 399 (1990).
8. I.I. Bigi, M. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett. 71, 496 (1993); I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B293, 430 (1992); Erratum, Phys. Lett. B297, 477 (1993); B. Blok, L. Koyrakh, M. Shifman and A.I. Vainshtein, Phys. Rev. D49, 3356 (1994).
9. A. Manohar and M.B. Wise, Phys. Rev. D49, 1310 (1994).
10. T. Mannel, Nucl. Phys. B413, 396 (1994).
11. A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D49, 3367 (1994).
12. B. Blok and M. Shifman, Nucl. Phys. B399, 441 (1993); B. Blok and M. Shifman, Nucl. Phys. B399, 459 (1993); N. Bilic, B. Guberina and J. Trampetic, Nucl. Phys. B248, 261 (1984); M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. 41, 120 (1985); M. Voloshin and M. Shifman, Sov. Phys.–JETP 64, 698 (1986); V.A. Khoze, M. Shifman, N.G. Uraltsev and M. Voloshin, Sov. J. Nucl. Phys. 46, 112 (1987).
13. For a recent review of Heavy Quark Effective Theory, see M. Neubert, SLAC Report No. SLAC-PUB-6263 (1993), to appear in Physics Reports. An extensive list of references to the original literature is given therein.
14. M. Ježabek and J.H. Kühn, Nucl. Phys. B314, 1 (1989).
15. Y. Nir, Phys. Lett. B221, 184 (1989).
16. M.K. Gaillard and B. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. B52, 351 (1974).
17. G. Altarelli and S. Petrarca, Phys. Lett. B261, 303 (1991); G. Altarelli et al., Phys. Lett. B99, 141 (1981); G. Altarelli et al., Nucl. Phys. B187, 461 (1981).
18. Q. Hokim and X.Y. Pham, Ann. Phys. 155, 202 (1984).
19. E. Bagan et al., Barcelona Report No. UAB-FT-347 (1994).
20. M.B. Voloshin, Minnesota Report No. TPI-MINN-94/35-T (1994).
21. M. Luke, M. Savage and M.B. Wise, Caltech Report No. CALT-68-1950 (1994).
22. S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys. Rev. D28, 228 (1993).
23. T.E. Browder, K. Honscheid and S. Playfer, Cornell Report No. CLNS 93/1261, to appear in B Decays, second edition, ed. S. Stone, World Scientific.
24. T. Bergfeld et al. (CLEO Collaboration), Cornell Report No. CLEO-CONF 94-09 (1994).
25. I. Dunietz, P.S. Cooper, A.F. Falk and M.B. Wise, Phys. Rev. Lett. 73, 1075 (1994).
26. D. Cinabro et al. (CLEO Collaboration), CLEO Report No. CLEO-CONF-94-08 (1994).
27. A.L. Kagan, SLAC Report No. SLAC-PUB-6626 (1994).