Normal State Property of the t-J Model

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Abstract

Using the spin-hole coherent state representation and taking a long range antiferromagnetic Néel order as a background of the localized spin degree part, we have studied the normal state behavior of the t-J model, and shown that a strongly short-range antiferromagnetic correlation of the localized spin degree part is responsible for the anomalous non-Korringa-like relaxation behavior of the planar copper spin, the Korringa-like behavior of the planar oxygen spin may derive from the charge degree part describing a Zhang-Rice spin-singlet; The charge degree part feels a strongly staggered magnetic field induced by this short-range antiferromagnetic correlation as a doping hole hopping, this staggered magnetic field enforces the charge degrees to have different responses to external magnetic and electric fields and to show two relaxation rate behaviors corresponding to the planar resistivity and Hall angle, respectively. We have found that the temperature dependence of magnetoresistance is $T^{-n}$, $n \simeq 3$, near the optimal doping, $n \simeq 4$, in the underdoping region, violating Kohler’s rule, the transport relaxation rate is of the order of $2k_B T$, all that are consistent with the normal state of the cuprate superconductors.

74.20.Mn, 75.10.Jm, 75.40.Gb.
I. INTRODUCTION

Recently, the significant progress has been made in the understanding of the low energy spin dynamics of the normal state of the cuprate superconducting materials in both theoretical and experimental aspects. In the undoping case, the spin dynamics of the cuprates, such as \( \text{La}_2\text{CuO}_4 \), is well described by the quantum Heisenberg model on a square lattice of \( \text{Cu} \) sites. The authors of Refs.[1,2] have extensively studied it by using the scaling and renormalization group theory and/or large-N expansion methods, and have given some valuable results which are in good agreement with the current experimental data. However, in the doped case, up to now there is not a general consensus on choosing a microscopic theory qualitatively to describe the unusually magnetic and transport properties of the normal state over the entire doping range from insulator to high doped compounds, although many models have been proposed to describe them.

In all hole-type cuprates near optimal doping, the mostest important properties of the normal state are that: a). The linear dependence of the in-plane resistivity \( \rho \) on temperature (T) has been confirmed from \( T_c \) up to temperature as high as 1000K [12]. b). The in-plane Hall resistivity \( \rho_{xy} = R_H B \) varies strongly over a wide range of T (\( B \), external magnetic field; \( R_H \), the Hall coefficient), it falls as \( \sim T^{-1} \) between \( T_c \) and temperature as high as 500K. Anderson [13] predicted that the cotangent of the Hall angle should vary with impurity content \( n_i \) as \( \cot \theta_H = \alpha T^2 + \beta n_i \), which was immediately confirmed by Ong [14] and others [13] [16]. c). The magnetoresistance \( \Delta \rho/\rho \propto B^2 T^{-n} \), \( n \simeq 3 \sim 4 \), strongly violates Kohler’s rule [17] [18]. d). The relaxation behaviors at various sites show sharply contrast. The relaxation behavior of the planar oxygen obeys Korringa-like behavior, while the planar copper sharply shows non-Korringa behavior. e). Dynamical antiferromagnetic correlations persist in all the metallic state and the superconducting state. These properties provide significant constraints on candidate descriptions of their anomalous normal state behavior. The items a),b),c) strongly indicate that there exist two different relaxation times in the system [13] \( 1/\tau_{tr} \sim T \), and \( 1/\tau_H \sim T^2 \). While the items d),e) strongly show that among the planar
copper spins there exists a strongly commensurate antiferromagnetic correlation, the planar oxygen, residing at the middle point between two nearest neighbor coppers, is not affected by this commensurate antiferromagnetic correlation and shows the Korringa-like relaxation behavior, at least it is true for the $YBa_2Cu_3O_{6+x}$ materials. For the $La_{2-x}Sr_xCuO_4$ system, although the neutron scattering experiments show at low temperature four incommensurate peaks in the spin fluctuation spectrum, whose position depends on the level of $Sr$ doping, the nuclear magnetic resonance experiment shows that the property d) is remaining invariance.

The unusually physical properties of the normal state of the cuprate superconducting materials may originate from their strongly antiferromagnetic correlation. The doping will destroy the long range antiferromagnetic correlation, but the system still maintains a strongly short range antiferromagnetic correlation. In Refs. 4-6, we have given a detail study following this idea, and obtained some results which can qualitatively explain the unusually physical properties of the normal state. In this paper, using the similar method as in Refs. 4-6, we study the normal state behavior of the t-J model. The t-J model carries on the important electronic strongly correlated property of the cuprate superconducting materials, through completing studying its property we hope to get more understanding of the strongly correlated electronic system and the cuprate superconducting materials. It is well-known that the gauge theory of the t-J model gives a better description to the temperature dependence of the in-plane resistivity of the normal state, but up to now one has not known whether it can also give a reasonable description to the temperature dependence of the Hall coefficient and the magnetic behavior of the normal state, i.e., can it show not only the two relaxation time behaviors but also the strongly antiferromagnetic correlation behavior? In this paper, we show that the unusual magnetic behavior of the normal state is induced by a strongly short-range antiferromagnetic correlation among the localized spin degrees on the copper sites, while because of there existing this strongly short-range antiferromagnetic correlation in the localized spin degree part, the charge degree part will feel a strong staggered magnetic field as the doping hole hopping, this staggered magnetic field drastically influ-
ences the behavior of the charge degree part, and enforces the charge degree part to have different responses to external magnetic field and electric field and to show two relaxation time behaviors corresponding to the in-plane resistivity and Hall angle, respectively. In the usual slave boson(or fermion) description of the t-J model, the spin degree and the charge degree of electrons are separated, the spin degree part effectively describes the localized spins on the copper sites, while the charge degree part effectively describes the spin-singlet (or Zhang-Rice singlet) of the doping hole spin and the copper spin. However, because the strongly dynamical antiferromagnetic correlations persist in all the metal state, it is reasonable that we use the long range antiferromagnetic Néel order of the localized spin degrees as our starting point to study the normal state behavior of the t-J model.

II. SPIN-HOLE COHERENT STATE REPRESENTATION AND SPIN-CHARGE SEPARATION

We adopt an usual method to deal with the single occupation condition by introducing a slave fermion, so the Hamiltonian of the t-J model can be written as in a hole representation

\[ H = t \sum_{<ij>} (f_j^+ f_i b_{i\sigma}^+ b_{j\sigma} + h.c) + J \sum_{<ij>} (1 - f_i^+ f_i) \hat{S}_i \cdot \hat{S}_j (1 - f_j^+ f_j) + \sum_i \lambda_i (1 - f_i^+ f_i - b_{i\sigma}^+ b_{i\sigma}) \]

(1)

where \( \hat{S}_i = \frac{1}{2} b_{i\alpha}^+ \sigma_{\alpha\beta} b_{i\beta} \), \( b_{i\sigma} \) is a hard-core boson operator which describes the spin degree of the electron, and \( f_i \) is a fermion operator which describes the charge degree of the electron. The electron operator is \( c_{i\sigma} = f_i^+ b_{i\sigma} \), \( \lambda_i \) is a Lagrangian multiplier which ensures the single occupation condition of the electrons. In the spin-hole coherent state representation introduced by Auerbach

\[ |\hat{\Omega}, \xi >_{S} \equiv |\hat{\Omega} >_{S} \otimes |0 >_{f} + |\hat{\Omega} >_{S-\frac{1}{2}} \otimes \xi f^+ |0 >_{f} \]

(2)

where \( |\hat{\Omega} >_{S} \) is a spin coherent state and \( \xi \) is an anticommuting Grassmann variable, the partition functional of the Hamiltonian (1) can be written as
\[ Z = \int D\hat{\Omega}D\xi^*D\xi \exp\left\{-\int_0^\beta [L_\Omega + L_\xi] \right\} \]  

\[ L_\Omega = -i \sum_i 2S\omega_i + JS^2 \sum_{<ij>} (1 - \xi_i^*\xi_j)\hat{\Omega}_i \cdot \hat{\Omega}_j(1 - \xi_j^*\xi_j) \]  

\[ L_\xi = \sum_i \xi_i^*(\partial_\tau + i\omega_i + \mu_i)\xi_i + \sqrt{2tS} \sum_{<ij>} (\xi_j^*\xi_i e^{i\gamma_{ij}} \sqrt{1 + \hat{\Omega}_i \cdot \hat{\Omega}_j + h.c}) \]

where the Berry phase \( \omega \) is a functional of the spin order parameter \( \hat{\Omega}(\tau) \). It is ambiguous modulo \( 4\pi \), and its functional derivative is quite well-behaved \[25\]

\[ \int d\tau \delta \omega = \int d\tau \hat{\Omega} \cdot (\partial_\tau \hat{\Omega} \times \delta \hat{\Omega}) \]  

The parameter \( \mu_i \) is a chemical potential of the slave fermion \( \xi \), \( \gamma_{ij} \) is the phase factor of \( s < \hat{\Omega}\vert b_{i\sigma}^+ b_{j\sigma} \vert \hat{\Omega} >_s \). The Lagrangian \( L_\xi \) is invariant under following gauge transformations

\[ \xi_i \to \xi_i e^{i\theta_i}, \quad \gamma_{ij} \to \gamma_{ij} - \theta_i + \theta_j, \quad \mu_i \to \mu_i + i\partial_\tau \theta_i \]

which derives from the slave fermion representation of the electron operator \( c_{i\sigma} = f_{i\sigma}^+ b_{i\sigma} \).

The single occupation condition in (1) disappears in (4) and (5), because in the spin-hole coherent state representation the term \( (1 - f_i^+ f_i - b_{i\sigma}^+ b_{i\sigma} ) \) is equal to zero at each site. From the equations (4) and (5), we see that the Lagrangian \( L_\Omega \) dominates the antiferromagnetic behavior of the system, then the Lagrangian \( L_\xi \) dominates the ferromagnetic behavior (or destroys the antiferromagnetic behavior) of the system because the factor \( \sqrt{1 + \hat{\Omega}_i \cdot \hat{\Omega}_j} \) is zero for antiferromagnetic order and is biggest for ferromagnetic order. According to the current experimental data of the cuprate superconducting materials, almost all of them show a strongly short range antiferromagnetic behavior in the normal state, even in the superconducting state, the short range antiferromagnetic behavior also appears. Therefore, according to this fact, we take a long range antiferromagnetic Néel order as a background of the spin order parameter

\[ \hbar S\hat{\Omega}_i \simeq h\eta_i \hat{\Omega}(x_i) + a^2 \hat{L}(x_i) \]  

\[ 5 \]
where \( a^2 \) is the unit cell volume, \( \hat{\Omega}(x_i) \) is the slowly varying Néel unit vector order, i.e., spin parameter field \(|\hat{\Omega}(x_i)| = 1\), and \( \hat{L}(x_i) \) is the slowly varying magnetization density field, \( \hat{\Omega}(x_i) \cdot \hat{L}(x_i) = 0 \). The Berry phase term may be separated into two parts

\[
S \sum_i \omega_i \simeq S \sum_i \eta_i \omega(x_i) + \frac{1}{\hbar} \int d^2 x \hat{\Omega} \cdot \left( \frac{\partial \hat{\Omega}}{\partial \tau} \times \hat{L} \right) \tag{9}
\]

where \( \omega(x) \) is the solid angle subtended on the unit sphere by the closed curve \( \hat{\Omega}(x, \tau) \) (parametrized by \( \tau \)). Because of in the long range antiferromagnetic Néel order approximation, the electron hoping must be accompanied with a \( \pi - \text{phase} \) rotation in spin space to match with the nextest neighbor spin orientations, so the t-term in (1) must be changed as

\[
f_i^+ f_j^+ b_i^\sigma b_j^\sigma = e^{-2i \sum_{l \neq i,j} \theta_{ij}(l) S_l^z} f_i^+ f_j^+ e^{2i \sum_{l \neq i,j} \theta_{ij}(l) S_l^z} b_i^\sigma b_j^\sigma = e^{-2i \sum_{l \neq i,j} \theta_{ij}(l) S_l^z} f_i^+ f_j^+ b_i^\sigma b_j^\sigma
\]

where \( \theta_{ij}(l) = \theta_i(l) - \theta_j(l), \theta_i(l) \) is an angle between the direction from site \( i \) to site \( l \) and some fixed direction, the \( x \) axis for example; \( S_l^z = \frac{1}{2} b_{l\alpha}^\dagger \sigma_{\alpha\beta}^z b_{l\beta} \), the z-component of the spin operator; \( b_i^\sigma = e^{2i \sum_{l \neq i} \theta_i(l) S_l^z} b_i^\sigma \), is a fermion operator. Under the approximations (8) and (9), and eliminated the magnetization density field \( \hat{L}(x) \), the Lagrangians in (4) and (5) can be written as, respectively

\[
L_{\Omega} = \frac{1}{2g_0} \int d^2 x [(\hat{\Omega})^2 + \frac{1}{c^2} (\partial_\tau \hat{\Omega})^2] \tag{11}
\]

\[
L_\xi = \sum_i \xi_i^* (\partial_\tau - \mu_i) \xi_i + \sqrt{2t} \sum_{<ij>} \{ \xi_i^* \xi_j e^{i\gamma_{ij}} [1 + \eta_i \eta_j \hat{\Omega}(x_i) \hat{\Omega}(x_j)]^{\frac{1}{2}} + \text{h.c} \} \tag{12}
\]

where \( \gamma_{ij} = \gamma_{ij} + \sum_{l \neq i,j} \theta_{ij}(l) S < \hat{\Omega}|(b_{l\uparrow}^\dagger b_{l\uparrow} - b_{l\downarrow}^\dagger b_{l\downarrow})|\hat{\Omega} >_S, g_0 = (J(1 - \delta)^2S^2)^{-1}, c^2 = 8(aJ(1 - \delta)S)^2 \). For the \( J \)-term in (4), we have replaced the \( f_i^+ f_i \) and \( f_j^+ f_j \) by \( \delta \equiv < f_i^+ f_i > = < f_j^+ f_j >, \) the doping density. We have omitted the terms \( \sum_i \eta_i \omega(x_i) \) and \( \sum_i \eta_i \omega(x_i) \xi_i^* \xi_i \). If \( \omega(x) \) is a slowly varying function of space coordinates \( \vec{x} \) and ”time” \( \tau \) and the occupation number of the quasiparticle \( \xi \) is equal at the even and odd sites, these two terms have a little contribution to the system. However, the quantity \( \omega(x) \) provides an attractive interaction
between the fermions $\xi_i$ and $\xi_{i+\hat{\delta}}$, $\hat{\delta} = (\pm a, \pm a)$, at the even and odd sites, respectively, which may induce the pairing between the slave fermions at the even and odd sites. Here we assume this effect is very small, and do not consider it, or we only consider the normal state of the system.

For the strongly antiferromagnetic correlation among the spin degrees of the system, taking the Hartree-Fock approximation, the Lagrangian (12) can be written as

$$L_\xi = \sum_i \xi_i^* (\partial_\tau - \mu_i) \xi_i + \sqrt{2} t S \chi \sum_{<ij>} \xi_i^* \xi_j e^{i\gamma_{ij}} + 2atS\eta \sum_i \xi_i^* \xi_i |\vec{\partial} \hat{\Omega}(x_i)|$$

where $\chi = \langle [1 + \eta_i \eta_j \hat{\Omega}(x_i) \cdot \hat{\Omega}(x_j)]^{1/2} \rangle$, $\eta = \langle e^{i\gamma_{ij}} \rangle$, $|\vec{\partial} \hat{\Omega}| \equiv |\partial_x \hat{\Omega}| + |\partial_y \hat{\Omega}|$. We have omitted the fluctuation phase of the fields $\chi$ and $\eta$, and taken them as constants. The effective Hamiltonian for the charge part can be written as

$$H_\xi = \bar{t} \sum_{<ij>} \xi_j^+ \xi_i e^{i\gamma'_{ij}} + V \sum_i \xi_i^+ \xi_i |\vec{\partial} \hat{\Omega}(x_i)|$$

where $\bar{t} = \sqrt{2} t S \xi$, $V = 2atS \eta$. Because of the strongly antiferromagnetic correlation among the spin degrees, the spin parameter field $\hat{\Omega}(x)$ is slowly varying in the coordinate space, so the phase factor $\gamma_{ij}$ is very small and can be omitted, the phase factor $\gamma'_{ij}$ is

$$\gamma'_{ij} = \sum_{l \neq i,j} \theta_{ij}(l) s < \hat{\Omega}_l | b_i^+ b_l - b_i^+ b_l | \hat{\Omega}_s >$$

which is a rapid varying quantity of the lattice sites, so generally, we cannot treat it in the continuous limit. Here we omit a gauge field $A$ which describes the interaction between the spin and charge degree parts of the electrons. Under the spin-hole coherent state representation, if taking the long range antiferromagnetic Néel order as a background of the spin degree part, the charge and spin degree parts are only coupled via the rapidly varying phase factor $\gamma'_{ij} = \gamma_{ij} + \sum_{l \neq i,j} \theta_{ij}(l) s < \hat{\Omega}_l | 2S_z | \hat{\Omega}_s > s$, $\gamma_{ij} = (\vec{x}_i - \vec{x}_j) \cdot A(\vec{x}_i - \vec{x}_j)$ being contributed from the localized spin degree part. If the phase factor $\gamma_{ij}$ is a smooth varying function in coordinate space, this gauge field $A$ must be massless, because the current corresponding to $A$ must be conserved, there appears a term $\langle \xi_i^+ \xi_i | \vec{\partial} \hat{\Omega} | \xi_i \rangle \cdot e^{i\gamma_{ij}}$ in equations (13) and (14),...
which provides a massive term to $\vec{A}$. On the other hand, it is reasonable to omit this gauge field $\vec{A}$ that for strongly antiferromagnetic correlation among the localized spin degrees, the phase factor $\sum_{l \neq i,j} \theta_{ij}(l) < |2S_z|^2 >$ is a rapidly varying function in coordinate space, so the phase factor induced by the gauge field $\vec{A}$ can be omitted. Then for a weakly antiferromagnetic correlation case, we must consider the effect produced by this gauge field. Just done as above, we can also adopt the slave boson method to deal with the t-J model, and obtain the similar Lagrangian as (13) or effective Hamiltonian as (14) only if we consider $\xi$ as a hard-core boson field [6]. So we consider the Lagrangian (13) or effective Hamiltonian (14) is valid for slave fermion and boson descriptions, for slave fermion description, $\xi$ is a fermion field, for the slave boson description, $\xi$ is a hard-core boson field.

### III. TRANSPORT PROPERTY OF THE NORMAL STATE

Now we study the effective Hamiltonian (14). In Ref. [26], the authors have studied the effect of a strongly fluctuating gauge field on a degenerate hard-core Bose liquid, shown that the gauge fluctuation causes the boson world lines to retrace themselves, and found a transport relaxation rate of the order of $1/\tau_{tr} \sim 2k_B T$, consistent with the normal state of the cuprate superconductors. The results obtained in [26] are also valid for the effective Hamiltonian (14), because the rapidly varying phase factor $\gamma'_{ij}$ provides a strongly staggered magnetic field which enforces the world lines of the slave boson (or fermion) to retrace themselves and induces the charge degrees having the order of $1/\tau_{tr} \sim 2k_B T$ transport relaxation rate. However, we can use this result only to explain the linear dependence of the resistivity $\rho$ on temperature. In order to study the temperature dependence of the Hall coefficient (or more important, the Hall angle), we must introduce an external magnetic field to the phase factor $\gamma'_{ij}$, while because the phase factor $\gamma'_{ij}$ is a rapid varying function of the coordinate space, we cannot treat it in the continuous limit. To get more valid informations, we adopt this scenario that we separate the rapidly varying phase factor $\gamma'_{ij}$ into two parts.
\[
\gamma'_{ij} = \gamma^{(1)}_{ij} + \gamma^{(2)}_{ij}
\]
\[
\gamma^{(1)}_{ij} = \sum_{l \neq i,j} \theta_{ij}(l) S < \hat{\Omega} | b^+_i b_j | \hat{\Omega} > S
\]
\[
\gamma^{(2)}_{ij} = - \sum_{l \neq i,j} \theta_{ij}(l) S < \hat{\Omega} | b^+_i b_j | \hat{\Omega} > S
\]

and introduce three slave particles

\[
\xi = \psi \bar{\chi}, \quad \xi^+ = \bar{\psi} = \bar{\chi} + \bar{\chi} \Rightarrow \chi = \chi^+\chi
\]

to describe the charge degree part. \( \bar{\chi} \) describes a slave fermion moving in a background "magnetic" field produced by the phase factor \( \gamma^{(1)}_{ij} \), \( \chi \) describes a slave fermion moving in a background "magnetic" field produced by the phase factor \( \gamma^{(2)}_{ij} \), \( \psi \) describes a slave boson or a slave fermion only responding to external magnetic and electric fields, or more intuitively, it can be considered as describing the "mass-centre" of the slave fermions \( \bar{\chi} \) and \( \chi \). However, corresponding to these slave boson and slave fermions, there exist two gauge freedoms

\[
\psi \rightarrow e^{i \theta} \psi, \quad \bar{\chi} \rightarrow e^{-i \theta} \bar{\chi}, \quad \chi \rightarrow \chi
\]

\[
\psi \rightarrow \psi, \quad \bar{\chi} \rightarrow e^{i \bar{\theta}} \bar{\chi}, \quad \chi \rightarrow e^{-i \bar{\theta}} \chi
\]

that introduce two gauge fields. While two current conservation equations corresponding to these two gauge fields and the gauge invariances will maintain the freedom of the system being conservative. Substituting equation (17) into equation (14), we have

\[
\bar{H} = \bar{t} \sum_{<ij>} \psi_j^+ \psi_i (\chi_j^+ \bar{\chi}_i e^{i \gamma_{ij}^{(1)}})(\chi_j^+ \chi_i e^{i \gamma_{ij}^{(2)}}) + V \sum_i \psi_i^+ \psi_i \tilde{\partial} \hat{\Omega}
\]

Under the Hartree-Fock approximation, we can have the following Lagrangian corresponding to the Hamiltonian (19)

\[
L = \sum_i \{ \psi_i^* (\partial_\tau - \lambda_i) \psi_i + \bar{\chi}_i^* (\partial_\tau + \lambda_i + \eta_i) \bar{\chi}_i \} + V \sum_i \psi_i^* \psi_i \tilde{\partial} \hat{\Omega}
\]

\[
+ \bar{t} \sum_{<ij>} \{ A_{ij} \psi_j^+ \psi_i + B_{ij} e^{i \gamma_{ij}^{(1)}} \chi_j^+ \bar{\chi}_i + C_{ij} e^{i \gamma_{ij}^{(2)}} \chi_j \chi_i \}
\]

where, \( m_\psi = (A\tilde{\ell})^{-1} \), \( m_\bar{\chi} = (B\tilde{\ell})^{-1} \), \( m_\chi = (C\tilde{\ell})^{-1} \), \( A_{ij} = < \chi_j^+ \bar{\chi}_i e^{i \gamma_{ij}^{(1)}} > \), \( B_{ij} = < \psi_j^+ \psi_i e^{i \gamma_{ij}^{(2)}} > \), \( C_{ij} = < \psi_j^+ \psi_i > \), \( \theta_{ij} \), \( \bar{\theta}_{ij} \), \( \gamma_{ij}^{(1)} \), \( \gamma_{ij}^{(2)} \), \( \tilde{\partial} \hat{\Omega} \)
\[ Ce^{-i\Theta_{ij}}, \Theta_{ij} = (\vec{x}_i - \vec{x}_j) \cdot \vec{a}(\frac{x_i - x_j}{2}), \bar{\Theta}_{ij} = (\vec{x}_i - \vec{x}_j) \cdot \vec{a}(\frac{x_i - x_j}{2}). \]

We introduce two Lagrangian multipliers \( \lambda_i \) and \( \eta_i \) to add the constraints \( \ref{eq:constraint} \) to the system. Under the gauge transformations \( \ref{eq:gauge_transform} \), the Lagrangian \( \ref{eq:Lagrangian} \) remains invariance.

In the continuous limit, the Lagrangian \( \ref{eq:Lagrangian} \) can be rewritten as

\[
L = \int d^2x \left\{ \psi^\dagger (i \partial_x - i a_0) \psi + \bar{\chi}^\dagger (i \partial_x + i \bar{a}_0) \bar{\chi} + \frac{1}{2m_\psi} \psi^\dagger (\vec{\partial} - i \vec{a})^2 \psi \right. \\
+ \frac{1}{2m_\chi} \bar{\chi}^\dagger (\vec{\partial} + i \vec{a})^2 \bar{\chi} + \frac{1}{2m_\chi} \chi^\dagger (\vec{\partial} - i \vec{a} - i \vec{A})^2 \chi \left. \right\} + V' \int d^2x \psi^\dagger \psi |\vec{\partial} \hat{\Omega}| \\
\text{(21)}
\]

where, \( V' = V/a^2 \), \((\vec{x}_i - \vec{x}_j) \cdot \vec{A}(\frac{x_i - x_j}{2}) = \gamma_{ij}^{(1)}, -(\vec{x}_i - \vec{x}_j) \cdot \vec{A}(\frac{x_i - x_j}{2}) = \gamma_{ij}^{(2)} \). It is reasonable in the continuous limit to study the property of the Lagrangian \( \ref{eq:Lagrangian} \), because the phase factors \( \gamma_{ij}^{(1)} \) and \( \gamma_{ij}^{(2)} \) are slowly varying functions, so we can introduce gauge fields to describe them. However, in thermodynamic limit, we have \( \langle b_{i \uparrow}^\dagger b_{i \downarrow} \rangle = \langle b_{i \downarrow}^\dagger b_{i \uparrow} \rangle \), so the gauge fields \( \vec{A} \) and \( \vec{A}' \) can be generally written as \( \vec{A} = \vec{A}' = \vec{A} + \delta \vec{A}, \nabla \times \vec{A} = \vec{B} = \pi (1 - \delta) \), \( \delta \) is the doping density, while the fluctuation field can be absorbed into \( \vec{a} \). We see that the phase factors \( \gamma_{ij}^{(1)} \) and \( \gamma_{ij}^{(2)} \) only provide uniform ”magnetic” fields to the slave fermions \( \bar{\chi} \) and \( \chi \), respectively. Under these approximations, we can easily treat the Lagrangian \( \ref{eq:Lagrangian} \).

First we show that the gauge field \( \vec{a} \) is massive and the gauge field \( \vec{a} \) enforces the slave fermions \( \bar{\chi} \) and \( \chi \) to be confined. To do so, we consider the current-current correlations of the slave fermions \( \bar{\chi} \) and \( \chi \). Because of appearance of the uniform magnetic field \( \vec{B} \) in the slave fermions \( \bar{\chi} \) and \( \chi \) systems, there exists a zero-field Hall conductance dynamically produced by this field \( \vec{B} \) in their current-current correlations \( \ref{eq:hall_conductance} \), so their current-current correlations can be generally written as

\[
\Pi_{\chi\alpha\beta} = \Pi_{\chi \perp} (\delta_{\alpha\beta} \frac{k_\alpha k_\beta}{k^2}) + \Pi_{\chi \parallel} \frac{k_\alpha k_\beta}{k^2} + i \epsilon_{\alpha\beta} \omega \sigma_{xy} \\
\Pi_{\bar{\chi}\alpha\beta} = \Pi_{\bar{\chi} \perp} (\delta_{\alpha\beta} \frac{k_\alpha k_\beta}{k^2}) + \Pi_{\bar{\chi} \parallel} \frac{k_\alpha k_\beta}{k^2} - i \epsilon_{\alpha\beta} \omega \sigma_{xy} \\
\text{(22)}
\]

where \( \sigma_{xy} \), Hall conductance, is a constant. \( \Pi_{\chi\alpha\beta} \) has opposite sign Hall conductance against \( \Pi_{\bar{\chi}\alpha\beta} \) because the slave fermion \( \chi \) carries negative charge to \( \vec{A} \) while the slave fermion
carries positive charge to $\overline{A}$. In the low energy and long wavelength limit, $\Pi_{a\perp}$ and $\Pi_{a\parallel}$, $a = \chi, \overline{\chi}$, are the quadratic functions of $\omega$ and $k$ \[20\]. Generally, they can be written as

$$\Pi_{a\perp} = \eta_a k^2 - \varepsilon_a \omega^2, \quad \Pi_{a\parallel} = \overline{\eta}_a k^2 - \overline{\varepsilon}_a \omega^2.$$ \[23\]

where $\eta_a, \overline{\eta}_a, \varepsilon_a$ and $\overline{\varepsilon}_a$ are constants. For the gauge field $\vec{a}$, after integrating out the slave fermions $\chi, \overline{\chi}$, and gauge field $\vec{a}$, its propagator is

$$D^{-1} = \Pi_\chi (\Pi_\chi + \Pi_{\overline{\chi}})^{-1} \Pi_{\overline{\chi}}$$ \[24\]

We see that, in the long wavelength limit $k \to 0$, the Hall conductance terms in \[24\] produce a mass term for the gauge field $\vec{a}$, so the gauge field $\vec{a}$ has a little influence on the system although the slave boson (or fermion) $\psi$ dynamically produces an unusual term $\frac{i\omega}{k}$, we can omit it in equation \[21\]. However, the propagator of the gauge field $\vec{a}$ reads

$$\overline{D}^{-1} = \Pi_\chi + \Pi_{\overline{\chi}}$$ \[25\]

the Hall conductance terms in \[25\] are cancelled. After integrating out the slave fermions $\chi$ and $\overline{\chi}$, we obtain an effective action of the gauge field $\vec{a}$ as taking a suitable scalling for "time" $\tau$

$$S[\vec{a}] = \frac{1}{4g^2} \int d^3 x F_{\mu \nu}^2, \quad \frac{1}{g^2} \sim \frac{1}{\sqrt{\delta}}.$$ \[26\]

Here for simplicity we include the $a_0$ term. If we consider the topologically nontrivial hedgehog configurations of the gauge field $\vec{a}$ with integer topological charge $q = \frac{1}{2\pi} \int ds \epsilon_{\mu \nu \lambda} \partial_\nu A_\lambda$, the confinement length of the slave fermions $\chi$ and $\overline{\chi}$ is \[28\] \[29\] \[30\]

$$\xi = \frac{ag}{2\pi} \frac{\text{const.}}{g^2}$$ \[27\]

where $a$ is an in-plane lattice constant. However, we have two basic length parameters, the confinement length $\xi$ and the Landau length $l_B \propto \frac{1}{\sqrt{B}}$. In the half filling limit, the confinement length of the slave fermions $\overline{\chi}$ and $\chi$ is determined by the Landau length $l_B$. On the other hand, in the overdoping limit, their confinement length is determined by $\xi$.  

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Based upon the above discussions, the Lagrangian (21) can be rewritten as

\[
L = \int d^2x \{ \bar{\psi} (\partial_\tau - ia_0 + iA_0^{ex}) \psi + \bar{\chi} (\partial_\tau + ia_0 + i\bar{a}_0) \chi + \bar{\chi} (\partial_\tau - i\bar{a}_0) \chi + \frac{1}{2m_\psi} \bar{\psi} (\bar{\partial} + i\bar{A}^{ex})^2 \psi + \frac{1}{2m_\chi} \bar{\chi} (\bar{\partial} + i\bar{A})^2 \bar{\chi} + \frac{1}{2m_\chi} \chi (\bar{\partial} - i\bar{A})^2 \chi + V' \bar{\psi} \psi |\bar{\delta} \Omega| \} \tag{28}
\]

where we add an external gauge fields $\bar{A}^{ex}$ and $A_0^{ex}$, and omit the gauge field $\bar{a}$ and $\bar{a}$.

Although the slave boson (or fermion) $\psi$ dynamically contributes a term $\left( \chi_F k^2 - \frac{i\omega}{v_F k} \right) (\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}) a_\alpha a_\beta$ to the gauge field $\bar{a}$, the mass term derived from the slave fermions $\bar{\chi}$ and $\chi$ for the gauge field $\bar{a}$ will remove the singular behavior of its propagator, and maintains the Fermi liquid behavior of the slave boson (or fermion) $\psi$ invariance. However, the density constraints in (17) and the current conservation law of the slave particle fields $\psi$, $\chi$ and $\bar{\chi}$ enforce the longitudinal currents to satisfy the following equation

\[
J_\psi^\parallel = J_\chi^\parallel = J_{\bar{\chi}}^\parallel \tag{29}
\]

while for the transversal currents there are not any constraints. We see that the slave particle fields $\psi$, $\chi$ and $\bar{\chi}$ interact on each other only via the scalar gauge fields $a_0$ and $\bar{a}_0$. If we redefine the scalar gauge field, $a_0 + \bar{a}_0 = -a'_0$, then we obtain the similar Lagrangian as that in Ref. [31], so we can use their results about the calculations of relaxation rates. The slave fermions $\bar{\chi}$ and $\chi$ have the same relaxation rate induced by the quasiparticle-scalar-gauge fluctuation scattering

\[
\frac{\hbar}{\tau_\chi} = \frac{\hbar}{\tau_{\bar{\chi}}} \simeq 2\eta(0) k_B T \tag{30}
\]

where $\eta(0) \sim 1$ is a constant, while the slave boson (or fermion) $\psi$ has the relaxation rate

\[
\frac{\hbar}{\tau_\psi} = \eta'(0) (k_B T)^2 \frac{t}{\hbar} \tag{31}
\]

where $\eta'(0)$ is a constant. Because in the real case, we have $k_B T/t \ll 1$, so for external electric field we have the transport relaxation rate $\tau_\psi = \tau_\chi \simeq 2k_B T$, consistent with that one directly calculates it[26] using the effective Hamiltonian (14), it also shows that the
separations in (16) and (17) are reasonable. We see that the scalar gauge fields $a_0$ and $\bar{a}_0$ do not change the Fermi liquid behavior of the slave boson (or fermion) $\psi$, so we find that the charge degree part described by the Lagrangian (13) or effective Hamiltonian (14) has two relaxation rates corresponding to different responses to external magnetic and electric fields, respectively. Here we must give a detail explanation about the equations (30) and (31). First we only turn on an external electric field, so we have gauge fields $\vec{A}_{\parallel}^{ex}$ and $A_0^{ex}$. If we take a gauge transformation to the slave boson (or fermion) $\psi$, we can cancel the gauge field $\vec{A}_{\parallel}^{ex}$, and obtain an effective scalar gauge field $\bar{A}_0^{ex}$, so the response of the external electric field is only the density-density correlations of the slave particle fields $\psi$, $\bar{\chi}$ and $\chi$. Because of the constraints (17) and (29), there exist strongly interactions among the slave particles $\psi$, $\bar{\chi}$ and $\chi$ via the gauge fields $a_0$ and $\bar{a}_0$, which will drastically change this response of the external electric field. In the normal state, the resistivity of the system is

$$\rho(T) \propto 2k_B T (1 + O\left(\frac{k_B T}{t}\right)) + \gamma n_i$$

where the last term in bracket is very small $k_B T / t \ll 1$, $\gamma$ is a constant, $n_i$ is the density of impurity, the last term derives from the impurity scattering. However, if we only switch on an external magnetic field, we have a gauge field $\vec{A}_{\perp}^{ex}$, the response of the external magnetic field is only the current-current correlation of the slave boson (or fermion) $\psi$. Although there exist strongly interactions among the slave particles $\psi$, $\bar{\chi}$ and $\chi$ via the scalar gauge fields $a_0$ and $\bar{a}_0$, the Fermi behavior of the slave boson (or fermion) $\psi$ is not destroyed by these scalar gauge interactions, so for the external magnetic field, the charge degree part only show its Fermi liquid behavior because only the slave boson (or fermion) $\psi$ response for the external magnetic field, the Hall angle of the system is

$$\cot \theta_H = \frac{\rho_{xx}^{\psi}}{\rho_{xy}^{\psi}} = \alpha T^2 + \beta n_i$$

where $\alpha \propto \frac{1}{B}$ and $\beta$ are constants, the last term derives from the impurity scattering. According to the above discussions, the anomalous transverse magnetoresistance is closely related to the temperature dependence of the Hall angle, they are derived from the same
origin, the slave boson (or fermion) \( \psi \) system. Because of the slave boson (or fermion) \( \psi \) system remains the Fermi liquid behavior, according to the Kohler’s rule we should have a temperature dependence of the magnetoresistance of the \( \psi \) system \( \Delta \rho / \rho \propto (\tan \theta_H)^2 \propto B^2 T^{-4} \). Since we have the relation \( \Delta \rho / \rho = \Delta \rho \), so we can obtain the following expression of the magnetoresistance of the charge degree part

\[
\frac{\Delta \rho}{\rho} = \frac{\rho^\psi}{\rho} \frac{\Delta \rho^\psi}{\rho^\psi} \propto B^2 T^{-n}
\]

(34)

For the resistivity \( \rho \sim T \), we have \( n = 3 \); For the resistivity \( \rho \sim T^2 \), we have \( n = 4 \). Generally, in the underdoping range, the resistivity is \( \rho \sim T^\alpha \), \( 1 < \alpha \leq 2 \) in the low temperature range, the magnetoresistance has the temperature dependence \( \Delta \rho / \rho \propto T^{-n} \), \( 3 < n \leq 4 \), consistent with the experimental data in [17] [18]. In the \( YBa_2Cu_3O_{7-\delta} \) samples between 100 and 375K [17], \( \Delta \rho / \rho \) follows a power law \( T^{-n} \), with \( n = 3.5 \) and 3.9 in the \( (T_c =) \) 90-K and 60-K crystals, respectively. In the \( BSCCO \) \( 2:2:1:2 \) single-crystal samples [18], \( \Delta \rho / \rho \) is shown to vary as \( \sim T^{-3} \) from \( T_c \) up to room temperatures. For the \( YBCO \) samples, there exists a \( Cu-O \) chain which may affect the experimental results, but the changing trend of the exponential \( n \), from the optimal doping to the underdoping cases, is consistent with the equation (34). We need more experimental data to testify the temperature dependence of the magnetoresistance given in (34).

IV. MAGNETIC PROPERTY OF THE NORMAL STATE

Because the slave boson (or fermion) \( \psi \) system remains the Fermi liquid behavior, after integrating out the field \( \psi \), we can obtain an effective term provided by the interaction between the spin parameter field \( \hat{\Omega} \) and the slave boson (or fermion) \( \psi \)

\[
L_{\psi}[\hat{\Omega}] = -\beta \sum_n \int \frac{d^2q}{(2\pi)^2} \frac{[\omega_n]}{\omega_F} |\hat{\Omega}(q, \omega_n)|^2
\]

(35)

where \( \omega_F \propto \frac{1}{V^2k_F} \), a character energy scale describing the damping of the quasiparticle-hole pairing excitation to the spin wave spectrum. We must carefully pay attention on the term
(33) which is not directly derived from an usual the quasiparticle-hole pairing excitation of a magnon, because the interaction term between the spin parameter field $\hat{\Omega}$ and the slave boson (or fermion) $\psi$ is $\psi^{\ast} \psi |\bar{\partial} \hat{\Omega}|$, a complicated interaction. Meanwhile, in the momentum space, we remain in mind that origin point of the momentum for the spin parameter field is at $\vec{q} = \vec{Q} = (\pm \pi/a, \pm \pi/a)$, while origin point of the momentum for the slave field $\psi$ is at $\vec{q} = (0, 0)$.

From equations (11) and (35), we obtain an effective action of the spin parameter field of the t-J model

$$S_{\text{eff}}[\hat{\Omega}] = \beta \sum_{n} \int \frac{d^2q}{(2\pi)^2} \left\{ \frac{1}{2g_0}(q^2 + \frac{1}{c^2} \omega_n^2) - \frac{\omega_n}{\omega_F} \right\}|\hat{\Omega}|^2(q, \omega_n)$$

where $|\hat{\Omega}(x, \tau)| = 1$, the origin points of $\vec{q}$ are in the corner points $\vec{Q} = (\pm \frac{\pi}{a}, \pm \frac{\pi}{a})$. The action (36) is the same as that in Refs. [4] [5] that we obtained from a p-d model or an effective Hamiltonian derived from a three-band Hubbard model. This action has two critical regions: one is a $z = 1$ (where $z$ is a dynamic exponent) region which is consisted of three regimes: a renormalized classical (RC) regime, a quantum critical (QC) regime and a quantum disorder (QD) regime [1]; another one is a $z = 2$ region which maybe is also divided into the same two (QC and QD) regimes as above, but their behavior is completely different from that in the $z = 1$ region. In the undoping case, $\omega_F \to \infty$, the system is in the RC regime [1] [2]. In the underdoping case, $\omega_c < \omega_F < \infty$, the system is in the $z = 1$ QC and/or QD regimes[3][4]. In the optimal doping case, $\omega_F < \omega_c$, the system goes into the $z = 2$ region [3] [4] [32]. $\omega_c$ is a characteristic energy scale which indicases a crossover of the system from the $z = 1$ region to the $z = 2$ region as doping. We see that the $\omega_F$ term in (36) which derives from the damping of the quasiparticle-hole pairing excitation to the spin wave spectrum is very important for determining the doping influence on the system, especially in the optimal doping case, this term is dominant.

Generally, in the $z = 1$ region, the $\omega_F$ term is very small, and can be treated perturbatively, in the low energy limit we can obtain following spin susceptibility
\[ \chi(q, \omega) = \frac{\chi_0}{\xi^{-2} + q^2 - \frac{\omega}{\omega_p} \omega^2 - \frac{i\omega}{\omega_p}} \]  

where \( \xi \) is a coherent length, \( \omega_p^2 \) is a renormalized characteristic energy scale of the spin fluctuation. In the \( (z = 1) \) QC regime \[4\] \[5\], \( \xi \sim \frac{1}{T}, \omega_F^R \sim \frac{\omega_F(l)}{T} \); In the \( (z = 1) \) QD regime, \( \xi \) and \( \omega_F^R \) take constants. In the \( (z = 2) \) region, the \( \omega_F \) term is dominant, the \( \omega^2 \) term is irrelevant and can be omitted, in the low energy limit we can obtain following spin susceptibility

\[ \bar{\chi}(q, \omega) = \frac{\bar{\chi}_0}{\bar{\xi}^{-2} + q^2 - \frac{\bar{\omega}}{\bar{\omega}_F}} \]  

where \( \bar{\omega}_F = \frac{\omega_F}{g_0} \) is a renormalization group invariant quantity. In the \( (z = 2) \) QC regime \[4\] \[5\], \( \bar{\xi}^2 \sim \frac{1}{T} \). Using these spin susceptibilities in (37) and (38), we can betterly explain the current experimental data \[7\]- \[11\] of the nuclear magnetic resonance spin-lattice relaxation rate and the spin echo decay rate about the copper spin. The NMR spin lattice relaxation rate \( T_1 \) and the spin echo decay rate \( T_{2G} \) can be written as

\[ \frac{1}{T_1} \propto \lim_{\omega \to 0} \int d^2q |A(q)|^2 \frac{\chi''(q, \omega)}{\omega} \propto \frac{\xi_i^2}{\omega_i} \] 

\[ \frac{1}{T_{2G}} \propto [\int d^2q f(q) \chi'^2(q, 0)]^{1/2} \propto \xi_i \] 

where, \( \omega_i = \omega_F^R \) (for \( z=1 \)) or \( \bar{\omega}_F \) (for \( z=2 \)), \( \xi_i = \xi \) (for \( z=1 \)) or \( \bar{\xi} \) (for \( z=2 \)), \( A(q) \sim A \) is the hyperfine coupling constant and \( f(q) \sim f \) is the form factor originating from the hyperfine interaction between the nuclear spin and the surrounding electron spins. In the QD regime we have

\[ \frac{1}{T_1} \propto \begin{cases} \xi \quad & z=1 \\ \omega_F(l) \quad & z=2 \end{cases} \] 

\[ \frac{1}{T_{2G}} = \text{const.} \]

Similarly, in the QC regime we have

\[ \frac{1}{T_1} \propto \begin{cases} \frac{1}{\omega_F(l)} \quad & z=1 \\ \frac{1}{T \omega_F} \quad & z=2 \end{cases} \] 

\[ \frac{1}{T_2} \propto \begin{cases} \frac{1}{T} \quad & z=1 \\ \frac{1}{\omega_F^{1/2} T^{1/2}} \quad & z=2 \end{cases} \]
We see that the spin lattice relaxation rate $T_1$ is considerably affected by the doping because of the quantity $\omega_F(\hat{l}) \sim 1/k_F$, while the spin echo decay rate $T_{2_G}$ depends upon doping through the correlation length $\xi$. For the spin lattice relaxation rate of the oxygen spin, we need more explanation, because in the t-J model the spin degree of the planar oxygen, composed a Zhang-Rice spin-singlet with the localized planar copper spin, is completely suppressed. The slave fermion (or boson) operator $f_i$ in (1) really expresses a Zhang-Rice spin-singlet, if there only exists a commensurate strongly short-range antiferromagnetic correlation for the localized planar copper spins, at least it is true for the YBCO samples, the planar oxygen spin will be not influenced by this commensurate antiferromagnetic correlation because the planar oxygen resides in the middle point of two nearest neighbor copper sites. So only the slave fermion (or boson) $f$ system, describing the charge degree of electron, can influence the spin lattice relaxation rate of the planar oxygen spins [33], just shown as in Section III, which obeys the Korringa-like rule because the response of the charge degree part shows the Fermi liquid behavior to external magnetic field.

V. DISCUSSION AND CONCLUSION

Using the spin-hole coherent state representation, we have studied the normal state property of the t-J model in the usual slave boson and slave fermion treatment of the single occupation constraint, and shown that we can qualitatively explain the unusually magnetic and transport behaviors of the normal state of the cuprate superconducting materials by the t-J model. We think that the short range antiferromagnetic correlation induces the unusual behavior of the normal state of the cuprate materials, so it is a reasonable approximation that we take a long range antiferromagnetic Néel order as a background of the spin degree part of the system. Although the interaction between the charge degree and spin degree will destroy this long range order, but the system still has the short range antiferromagnetic order. In the undoping case, the system can be described by a non-linear $\sigma$-model (the t-J model reduces to the Heisenberg model). In the doping case, the interaction between
the charge degree and spin degree provides a decay term to the non-linear $\sigma$-model, which describes the damping of the quasiparticle-hole pairing excitation to the spin wave spectrum, but this decay term is not directly derived from the quasiparticle-hole pairing excitation of a magnon because of the complicated interaction term between the spin parameter field $\hat{\Omega}$ and the slave boson (or fermion) field $\psi$. Using this effective Lagrangian, we can betterly explain the unusually magnetic behavior of the planar copper spin of the normal state of the cuprate superconducting materials. For the planar oxygen spin, we think that its normal Korringa-like relaxation behavior is coming from the contribution of the slave particle $f$ in (1), described the Zhang-Rice spin-singlet and charge degree of electron. While because of there existing the strongly short-range antiferromagnetic coreletion in the localized spin degree part, the charge degree part will feel a strongly staggered magnetic field as the doping hole hopping, this staggered magnetic field drastically influences the behavior of the charge degree part, and enforces it to have different responses to external magnetic field and electric field and to show two relaxation rate behaviors corresponding to the planar resistivity and Hall angle, respectively. This character of the charge degree part responded to external magnetic field is compatible with the Korringa-like relaxation behavior of the planar oxygen spin. According to these properties of the responses of the charge degree part to external magnetic and electric fields, we have calculated the temperature dependence of the magnetoresistance, and found that near the optimal doping, it varies as $T^{-n}$, $n \sim 3$, in the underdoping cases, it varies as $T^{-n}$, $n \sim 4$, consistent with the current experimental data. The transport relaxation rate is of the order of $2k_B T$, consistent with the normal state of the cuprate superconductors. Of course, the results we have obtained are invalid in the half doping limit, in that case the doping hole tends to localize due to the strong interaction with the nearest copper spin; they are also invalid in the overdoping limit where the antiferromagnetic correlation is very weak and/or there exists a transition from two-dimensional system to three-dimensional system, because it is not reasonable to take a long range antiferromagnetic Néel order as a background of the spin degree part and the charge and spin degrees of electron are confined.
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