The implementation of the Type III seesaw mechanism for neutrino masses in the context of left-right theories where parity is broken spontaneously is investigated. We propose a simple left-right symmetric theory where the neutrinos masses are generated through a double seesaw mechanism which is a combination of Type I and Type III seesaw. In this context we find a possible candidate for the cold dark matter in the Universe and discuss the Baryogenesis via Leptogenesis mechanisms. The spectrum of the theory, the phenomenological constraints and the possibility to test the theory at the Large Hadron Collider are investigated.

I. INTRODUCTION

In the LHC Era we hope to test the theory beyond the Standard Model (SM) which describes physics at the TeV scale. The existence of massive neutrinos is a strong motivation for physics beyond the SM. There are only three simple mechanisms to generate Majorana neutrino masses at tree level. In the case of Type I seesaw mechanism [1, 2], one can add at least two fermionic singlets $N_i$ (right-handed neutrinos) and the neutrino masses read as $m_{
u} \sim h_{\nu}^2 v^2 / M_N$, where $h_{\nu}$ is the Dirac Yukawa coupling, $v = 246$ GeV is the SM Higgs vacuum expectation value (vev) and $M_N$ is the right-handed neutrino mass. If $h_{\nu} \approx 1$ and $M_N \approx 10^{14-15}$ GeV, one obtains the natural value for the neutrino masses $m_{\nu} \approx 1$ eV.

In the so-called Type II seesaw mechanism [3] for neutrino masses the Higgs sector of the SM is extended by adding an $SU(2)_L$ Higgs triplet $\Delta$. In this scenario the neutrino masses are given by $m_{\nu}^{II} \approx Y_{\nu} v_{\Delta}$, where $v_{\Delta}$ is the vev of the neutral component of the triplet and $Y_{\nu}$ is the Yukawa coupling. $v_{\Delta} \approx \mu v^2 / M_{\Delta}$, where $M_{\Delta}$ is the mass of the triplet and $\mu$ defines the mixing between the SM Higgs and the triplet. A natural setting would be $Y_{\nu} \approx 1$ and $\mu \sim M_{\Delta} \approx 10^{14-15}$ GeV.

Recently, several groups have investigated the implementation of the Type III seesaw mechanism [4, 5, 6, 7] in the context of grand unified theories (GUTs). In this case adding at least two extra matter fields in the adjoint representation of $SU(5)$, one can generate neutrino masses, $m_{\nu}^{III} \approx \Gamma_{\nu} v^2 / M_{\nu}$. Here $M_{\nu}$ stands for the mass of the fermionic triplets and $\Gamma_{\nu}$ is the Dirac Yukawa coupling. The implementation of this mechanism [4, 5, 6, 7] has been studied in the context of $SU(5)$ GUT unified theories, where once we realize Type III seesaw, one gets Type I as a bonus since both fields responsible for seesaw live in the adjoint representation of $SU(5)$.

Parity is considered as a fundamental symmetry and is explicitly broken in the SM by the asymmetry between the left and right handed multiplets. Therefore, as is well known the SM does not explain the $V - A$ character of the $\beta$ and $\mu$ decays. One could say that the existence of massive neutrinos and the unknown origin of parity violation in the SM are probably one of the main physical motivations for physics beyond the SM. In the context of the so-called left-right symmetric theories [8, 9, 10] one has the appealing possibility to understand the origin of parity violation and its strong connection to the generation of neutrino masses [2]. In these theories the observed $V-A$ structure of weak interactions is only a low-energy phenomenon which should disappear when one reaches the TeV scale or higher. Left-right symmetric theories where the neutrino masses are generated through the Type I and Type II seesaw mechanisms have been investigated in detail [10, 11, 12, 13, 14, 15].

In this Letter we study for the first time the implementation of the Type III seesaw mechanism in the context of left-right theories. We propose a simple renormalizable left-right symmetric theory where the neutrino masses are generated through a double seesaw mechanism which is a combination of the Type I and Type III seesaw mechanisms. We find a cold dark matter candidate which is like the wino in the minimal supersymmetric SM and discuss the leptogenesis mechanism. We investigate the spectrum of the theory and the possible signals at the LHC. We refer to this theory as “Type III-LR”.

This work is organized as follows: In section II we discuss the main features of left-right theories and the different mechanisms to generate neutrino masses. We show for the first time the implementation of Type III seesaw. In section III we discuss the possible dark matter candidates and the Baryogenesis via Leptogenesis mechanisms, while in section IV we investigate the spectrum of the theory and the main signals at future colliders. In section V we summarize our main results.

II. TYPE III SEESAW IN LEFT-RIGHT THEORIES

The so-called left-right symmetric models are one of the most appealing extensions of the SM where one can understand the origin of parity violation in a simple way and we can generate neutrino masses. The simplest theories are based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. Here $B$ and $L$ stand for baryon and lepton number, respectively. The matter multiples for quarks and leptons are given by

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (2, 1, 1/3), \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim (1, 2, 1/3),$$

(1)
\[ \mathcal{L}_\text{Y}^{\text{quarks}} = \tilde{Q}_L \left( Y_1 \Phi + Y_2 \tilde{\Phi} \right) Q_R + \text{h.c.,} \]

where the bidoublet Higgs is given by
\[ \Phi = \begin{pmatrix} \phi^0_1 & \phi^+_2 \\ \phi^-_1 & \phi^0_2 \end{pmatrix} \sim (2, 2, 0), \quad \text{and} \quad \tilde{\Phi} = \sigma_2 \phi^+ \sigma_2. \]

Once the bidoublet gets the vev the quark mass matrices read
\[ M_U = Y_1 v_1 + Y_2 v^*_2, \quad \text{and} \quad M_D = Y_1 v_2 + Y_2 v^*_1, \]
with \( v_1 = \langle \phi^0_1 \rangle \) and \( v_2 = \langle \phi^0_2 \rangle \). In the case of the bidoublet one has the following transformation under the left-right parity
\[ \Phi \rightarrow \Phi^\dagger. \]

and \( Y_1 = Y^\dagger_1 \) and \( Y_2 = Y^\dagger_2 \). These aspects of the model have been studied in detail \[1, \overline{1}, \overline{2}]\). Now, as it has been noticed by Mohapatra and Senjanović, in this context one can have a deep relation between the origin of neutrino masses and parity violation \[2\)]. In the most popular renormalizable left-right models, the neutrino masses are generated through the Type I \[1, \overline{2}\] and Type II \[3\] seesaw mechanisms once the Higgs triplets, \( \Delta_L \sim (3, 1, 2) \) and \( \Delta_R \sim (1, 3, 2) \) are introduced. Predicting the existence of doubly charged Higgses which could be discovered at the LHC \[16\]. Before study the implementation of Type III seesaw in this context we discuss briefly the different well known possibilities to generate neutrino masses.

II. A. Dirac Neutrinos

In this context the charged lepton masses are generated through the interactions
\[ \mathcal{L}_l = \bar{l}_L \left( Y'_3 \Phi + Y'_4 \tilde{\Phi} \right) l_R + \text{h.c.,} \]
and the relevant mass matrix is given by
\[ M_\nu = Y_3 v_2 + Y_4 v^*_1. \]

At the same time one gets a Dirac mass matrix for the neutrinos
\[ M_\nu^D = Y_3 v_1 + Y_4 v^*_2. \]

In the limit \( v_2 \ll v_1 \) and \( Y_3 \ll Y_4 \),
\[ M_\nu \approx Y_4 v^*_1, \quad \text{and} \quad M_\nu^D = v_1 \left( Y_3 + \frac{v^*_2}{|v_1|^2} \right). \]

Therefore, assuming that \( Y_3 \) is very small one can have Dirac-neutrinos. However, in this case one has the same situation as in the SM plus right-handed neutrinos where we can assume a small Dirac Yukawa coupling for neutrinos. As is well-known in this scenario one has to introduce extra Higgses in order to break parity and the left-right symmetry \[11, \overline{12}\].

II. B. Majorana Neutrinos: Type I plus Type II

In the so-called minimal left-right theories it is assumed that the neutrino masses are generated through the Type I and Type II seesaw mechanisms introducing a pair of Higgs triplets, \( \Delta_L \sim (3, 1, 2) \) and \( \Delta_R \sim (1, 3, 2) \) \[3\]. In this case the relevant interactions are given by
\[ \mathcal{L}_\nu = \mathcal{L}_l + h \left( \bar{l}^T_L C i \sigma_2 \Delta_L l_L + \bar{l}^T_R C i \sigma_2 \Delta_R l_R \right) + \text{h.c.,} \]
with \( h = h^T \), and
\[ \Delta_{L,R} = \begin{pmatrix} \sqrt{2} \delta_{L,R}^+ & \delta_{L,R}^+ \delta_{L,R}^* \\ \delta_{L,R}^0 & -\sqrt{2} \delta_{L,R}^+ \end{pmatrix}. \]

Under the left-right parity transformation one has the following relation
\[ \Delta_L \rightarrow \Delta_R. \]

In this case the mass matrix for neutrinos is given by
\[ M_{\nu} = \begin{pmatrix} \sqrt{2} h k_L (M_{\nu}^D) \end{pmatrix} \left( \begin{pmatrix} (M_{\nu}^D)^* \end{pmatrix} \right) \]

where \( \langle \delta^0 \rangle = k_{L,R}/\sqrt{2} \) and in the limit when \( M_{\nu}^D \ll h k_L \) one gets
\[ M_{\nu} = \sqrt{2} h k_L - (M_{\nu}^D)^* M_{\nu}^{-1} (M_{\nu}^D)^*, \]
and
\[ M_{\nu} = \sqrt{2} h^* k_R^*. \]

Therefore, one can understand the smallness of the neutrino masses as a consequence of large left-right scale \( k_R \) (or \( M_{W_R} \) since \( k_L = \gamma/k_R \) \[2\]). Even if this possibility is very appealing we do not know which is the mechanism responsible for neutrino masses and one should explore all possibilities, or at least the simplest scenarios at tree level. This is the main goal of this article.
II. C. The Case of Type III Seesaw

The realization of the Type III seesaw mechanism has not been studied in the context of left-right symmetric theories. In order to realize this mechanism one has to introduce fermionic triplets (one for each family):

\[ \rho_L = \frac{1}{2} \begin{pmatrix} \rho_L^0 - \sqrt{2} \rho_L^+ \\ \sqrt{2} \rho_L^- \end{pmatrix} \sim (3, 1, 0), \quad (19) \]

and

\[ \rho_R = \frac{1}{2} \begin{pmatrix} \rho_R^0 - \sqrt{2} \rho_R^+ \\ \sqrt{2} \rho_R^- \end{pmatrix} \sim (1, 3, 0), \quad (20) \]

and Higgses in the fundamental representation of \( SU(2)_L \) and \( SU(2)_R \), respectively.

\[ H_L = \left( \frac{\phi_L^+ + C A_L^0}{\sqrt{2}} \right) \sim (2, 1, 1), \quad (21) \]

and

\[ H_R = \left( \frac{\phi_R^+ + C A_R^0}{\sqrt{2}} \right) \sim (1, 2, 1). \quad (22) \]

In this case the relevant interactions are given by

\[ -\mathcal{L}_{\nu}^{III} = \mathcal{L}_L + Y_5 (i L^T C i \sigma_2 \rho_L H_L + i R^T C i \sigma_2 \rho_R H_R) + M_{\nu} \text{Tr}(\rho_L^T C \rho_L + \rho_R^T C \rho_R) + \text{h.c.} \quad (23) \]

Notice that in this case under left-right parity transformation one has the following relations

\[ \rho_L \leftrightarrow \rho_R \quad \text{and} \quad H_L \leftrightarrow H_R. \quad (24) \]

Therefore, once the Higgses \( H_L \) and \( H_R \) get the vevs, \( v_L \) and \( v_R \), parity is broken spontaneously. In the case when \( v_L = 0 \) and \( v_R \neq 0 \), and integrating out the neutral components of the fermionic triplets one finds that the mass matrix for neutrinos in the basis \( (\nu L, \nu R, \rho_R^0) \) reads as

\[ M^{III}_{\nu} = \begin{pmatrix} 0 & M_{\nu D}^T \rho_L^0 & 0 \\ M_{\nu D} \rho_L^0 & 0 & -\frac{Y_{5\nu L} v_R}{2\sqrt{2}} \\ 0 & -\frac{Y_{5\nu L} v_R}{2\sqrt{2}} & M_{\rho} \end{pmatrix}. \quad (25) \]

and the fermionic triplets, \( \rho_L \), do not mix having a mass matrix equal to \( M_{\rho} \). As one expects the neutrino masses are generated through the Type I and Type III seesaw mechanisms and one has a “double-seesaw” mechanism since the mass of the right-handed neutrinos are generated through the Type III seesaw once we integrate out \( \rho_R^0 \).

Assuming that \( M_{\rho} \gg Y_5 v_R/2\sqrt{2} \gg M_{\nu D}^T \) one gets

\[ M_{\nu}^{(\nu C)_R} = M_{\nu}^D \begin{pmatrix} M_{\nu D}^1 & 0 \\ 0 & -\frac{Y_{5\nu L} v_R}{2\sqrt{2}} \\ 0 & -\frac{Y_{5\nu L} v_R}{2\sqrt{2}} \end{pmatrix} \quad (26) \]

with

\[ M_{\nu R} = \frac{v_R^2}{8} Y_5 (M_{\rho})^{-1} Y_5^T. \quad (27) \]

Notice the double-seesaw mechanism, where the mass of the right-handed neutrinos are generated once the fermionic triplet is integrated out (Type III seesaw), and later the light neutrinos get the mass through the usual Type I mechanism. Here we stick to the case \( v_L = 0 \) since it has been shown in [11] that this solution corresponds to the minimum of the scalar potential. Also as we will show in the next section when \( \rho_L \rightarrow -\rho_L \), which forbids the mixing between \( \rho_L \) and \( \nu_L \), is a symmetry of the theory the neutral component of the fermionic triplets can be a cold dark matter candidate. See Ref. [17] for an early discussion of the double seesaw mechanism in a different context.

As we have seen in this case since one has a double seesaw mechanism we can have an interesting scenario for the LHC where the fermionic triplets are at the TeV scale, \( M_{\rho} \approx 1 \) TeV, and the right-handed neutrinos at the scale, \( M_{\nu R} \approx 10 \) GeV. Therefore, as it is well known in this case one gets small neutrino masses, \( m_{\nu} \approx 1 \) eV, if the Yukawa couplings are very small. See references [18, 19] for the production of fermionic triplets at the LHC. In the case when we assume that the left-right symmetry scale is very large, \( v_R \sim 10^{14-15} \) GeV, one can have a scenario where \( Y_5 < 1, M_{\rho} \gg v_R, M_{\nu D} \sim M_W \) and one gets \( m_{\nu} \sim 1 \) eV. In the next section we will discuss this first scenario in order to understand the possibility to test this theory at future collider experiments.

III. COLD DARK MATTER AND LEPTOGENESIS

In this theory a possible candidate for the cold dark matter in the Universe is the neutral components of \( H_L \) once we impose the symmetry \( H_L \rightarrow -H_L \). This situation is similar to the case of Inert Higgs Doublet Models [20]. At tree level the charged component, \( \phi_L^+ \), and the neutral components, \( \phi_L^0 \) and \( A_L^0 \) in \( H_L \) have the same mass. However, as it has been pointed out in Ref. [21] once the radiative corrections are considered the charged component is heavier and the neutral components could be a good cold dark matter candidate. Unfortunately, in this case since the real and imaginary components have the same mass one cannot satisfy the constraints coming from direct detection. Then, we do not stick to this possibility. It is important to emphasize that the neutral component of \( H_R \) cannot be a CDM candidate since one has to break parity and the left-right symmetry, and in the case of the neutral components of the bidoublet one knows that both vevs should be different from zero since one has to generate fermion masses in agreement with the experiment.

In the fermionic sector one can find a natural cold dark matter once we impose the symmetry \( \rho_L \rightarrow -\rho_L \). In this case the cold dark matter candidate is the neutral component of the lightest fermionic triplet \( \rho_L \). Since in this theory one has to introduce three fermionic triplets, one for each generation, there are three fields which have the same quantum numbers as the winos in the minimal supersymmetric SM. At tree level the charged and neutral components of the lightest triplet have the same mass. However, once the radiative corrections
are included one has the splitting $\Delta M \approx 166 \text{ MeV}$ and the charged component decays into the dark matter and a pion before nucleo-synthesis. A cold dark matter candidate with the same quantum numbers has been studied in [21] where the authors pointed out that in order to explain the CDM relic density the mass has to be $M_{\rho_L} \approx 2.5 \text{ TeV}$. Since in our case one has three fermionic triplets the cold dark matter candidate has to be the lightest neutral component, $\rho_L^0$.

In this theory one could have in principle different leptogenesis mechanisms. In the case of the natural double seesaw mechanism discussed in the previous section where one has $M_{\rho_L} \gg Y e_{\nu R}/2\sqrt{2} \gg M_{\nu R}^0$, the right-handed neutrinos are lighter than the fermionic triplets. Therefore, one has the so-called Type I Leptogenesis [22] with extra vertex corrections where we have the fermionic triplets inside the loops. In Ref. [23] it has been studied the leptogenesis mechanism when the neutrino masses are generated through the Type I and Type III seesaw mechanisms and it has been shown that the fermionic triplets cannot contribute to the self-energy corrections. These issues will be studied in detail in a future publication.

IV. SPECTRUM OF THE THEORY AND POSSIBLE SIGNALS AT THE LHC

In the previous section we have discussed the main properties of the theory. The properties of a left-right theory where the Higgs sector is composed of $\Phi$, $H_L$ and $H_R$ have been studied in detail by Senjanović [11] in a seminal paper. In this context the mass matrix for the gauge bosons $W_L^{\pm}$ and $W_R^{\pm}$ when $v_L = 0$ is given by [11]:

$$M_{\pm}^2 = \begin{pmatrix}
\frac{g^2}{4} (v_1^2 + v_2^2) & -\frac{g^2}{4} v_1 v_2 \\
-\frac{g^2}{4} v_1 v_2 & \frac{g^2}{4} (v_1^2 + v_2^2 + v_R^2)
\end{pmatrix},
$$

(28)

where $g_L = g_R = g$ and the mass matrix for the neutral gauge bosons, $Z$, $Z'$ and $A$ reads as

$$M_0^2 = \begin{pmatrix}
\frac{g^2}{4} (v_1^2 + v_2^2) & -\frac{g^2}{4} (v_1 + v_2) & 0 \\
-\frac{g^2}{4} (v_1 + v_2) & \frac{g^2}{4} (v_1^2 + v_2^2 + v_R^2) & -\frac{1}{4} g \tilde{g} v_R^2 \\
0 & -\frac{1}{4} g \tilde{g} v_R^2 & \frac{1}{4} \tilde{g}^2 v_R^2
\end{pmatrix},
$$

(29)

with $\tilde{g}$ being the gauge coupling for the $U(1)_{B-L}$ symmetry. In the case of the Higgs bosons when $v_L = 0$ only the Higgses in $\Phi$ and $H_R$ mix once the symmetry is broken. Notice that in these two fields one has 12 degrees of freedom, six Goldstone bosons and six physical Higgs bosons in the theory [11]. Therefore, in total one has ten physical Higgses, four charged Higgses $H_1^\pm$ and $H_2^\pm$, four CP-even neutral Higgses $H_0^0$, $H_1^0$, $H_2^0$ and $H_3^0$, and two CP-odd states $H_1^0$ and $A_0^0$.

Let us discuss the possible signals at the LHC which could help us to identify this theory. As usual in this context one predicts the existence of extra gauge bosons, $W_R^{\pm}$ and $Z'$. As is well-known the discovery of these states is crucial to test any left-right symmetric theory. In the case of $W_R$ one has the mechanism at the LHC, $pp \to W_R^+ \to e_R \nu_R$, where $\nu_R$ decays into $e_R j$ [24]. Then, one has two leptons and two jets with high $p_T$. Defining the invariant mass $M_{inv}(ejj)$ and $M_{inv}(eejj)$ for $\nu_R$ and $W_R$, respectively, one should be able to make the reconstruction and identify these fields. In the case of the $Z$ [25] one has the production mechanism, $pp \to (Z')^* \to e^+e^-$ and one looks for excess of dilepton events.

Once we implement the Type III seesaw mechanism in this context one finds that parity conservation at the left-right scale tells us that the masses of the fermionic triplets $\rho_L$ and $\rho_R$ should be the same. This could be a way to test the model at the LHC if one finds these states and determines their masses. At the same time since the neutrino masses are generated through the Type I and Type III seesaw mechanisms one needs to discover the right-handed neutrinos $\rho_R^0$ and the fermionic triplets to test the theory. Now, in the case of the fermionic triplets $\rho_L^0$ one has the following production mechanisms, $pp \to Z^*, \gamma^*, (Z')^* \to \rho_L^0 \rho_L^0$ and $pp \to W_L^+ \to \rho_L^0 \rho_L^0$. Now, if we stick to the possibility that the neutral component is responsible for the CDM, the charged component will have a decay length of few centimeters and decay into the neutral component and a pion. The fermionic triplets $\rho_R$ can be produced via $pp \to \gamma^*, (Z')^* \to \rho_R^0 \rho_R^0$ or $pp \to W_R^+ \to \rho_R^0 \rho_R^0$. In this case $\rho_R^0$ could decay into a lepton and two jets, while $\rho_R^0$ decays into three leptons. All these issues, and the constraints on the $W_R$ mass and the Higgs masses coming from low-energy processes will be studied in a future publication.

V. SUMMARY AND OUTLOOK

The implementation of the Type III seesaw mechanism for neutrino masses in the context of left-right symmetric theories where parity is broken spontaneously has been investigated. We have presented a simple left-right symmetric theory where the neutrino masses are generated through a “double seesaw” mechanism which is a combination of the Type I and Type III seesaw mechanisms. We have found that the lightest neutral component of the fermionic triplets $\rho_L$ can be a candidate for the cold dark matter in the Universe. In this context one could have Type I leptogenesis with extra vertex contributions due to the existence of the fermionic triplets inside the loops. We have discussed the spectrum of the theory, and the possibilities to realize the test at the LHC. The phenomenological and cosmological aspects of this proposal are very rich and deserve to be investigated.

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