Can Future Observation of the Living Partner Post-tag the Past Decayed State in Entangled Neutral K-Mesons?

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Entangled neutral K-mesons allow the study of their correlated dynamics at interference and decoherence times not accessible in any other system. We find novel quantum phenomena associated to a correlation-in-time between the two partners: the past state of the first decayed kaon, when it was entangled before its decay, is post-tagged by the result and the time of the future observation of the second decay channel. This surprising “from future to past” effect is fully observable and leads to the unique experimental tag of the KS-state, an unsolved problem since the discovery of CP violation.

I. INTRODUCTION

Since long ago, several authors have stated the crucial role that the neutral kaon system has played for understanding the intricacies of the quantum world. In particular the words of R. Feynman [1], T.D. Lee [2] and L. B. Okun [3] are all emphasizing the uniqueness of this system as a jewel donated to us by nature. They were referring to the peculiar properties of single neutral kaon states, which display several rare phenomena like the strangeness oscillation, the tiny mass splitting and the large difference in lifetimes of the physical states, the violation of the fundamental discrete symmetries Charge-Parity (CP) and Time-Reversal (T), the regeneration when traversing a slab of material.

The present research is related to another peculiar character of neutral kaons: the “strange entanglement”, i.e. the entanglement which is specific to two neutral kaon systems with all the interconnections with the above properties. It is worth reminding here that the entanglement is one of the most striking feature of quantum mechanics, as stressed by E. Schrödinger [4], in reply to the famous argument by A. Einstein, B. Podolsky and N. Rosen [5] (EPR) based on local realism.

Several tests of Quantum Mechanics and searches for possible decoherence and CP violation effects that can exploit strange entanglement of neutral kaons have been proposed [6–38]. The experimental investigation of strange entanglement started with the CPLEAR experiment [39], and continued with the KLOE and KLOE-2 experiments [40–42] at DAΦNE [43–45], yielding several precision results [46–54].

The characteristic behaviour of strange entanglement, with the peculiar properties of neutral kaons not found in any other system, makes possible the exploration of novel phenomena: the surviving correlation-in-time from the observation of the future decay of the living partner at a given time to the identification of the past kaon state leading to the first decay. This from the future to the past information in a system with non-trivial time evolution, entering into times in which the system was still entangled, could contribute to unveil the kind of reality to be associated to each part of the system.

The methodology that we follow consists in comparing the description of the double decay distribution at times $t_1, t_2$ with $\Delta t = t_2 - t_1 > 0$ using (i) the formalism of the two decay times state first introduced by Lee and Yang (LY) [55–58] with (ii) the time history (TH) of the entangled state from the coherent correlated neutral kaon system until its fate. The quantum consistency of the two approaches and the $t_1, t_2$ symmetry of the first approach, with no special role of one of the two decay times, naturally demand the study of a novel problem: is it possible to infer the initial kaon state previous to the first decay $t_1$ from the observation of the second decay at time $t_2 > t_1$, i.e. a correlation able to provide information from the future to the past? Contrary to the information from the past to the future, i.e., the prediction of the kaon state at time $t_2$ from the observation of the first decay $t_1$, the question formulated in this paper involves information on a part of the system at times in which the state was still fully entangled, i.e. before the first decay, when asking which is which is considered to be unspeakable in John Bell’s terminology [59].

In the following, first we analyze the correlation from past to future, i.e. which is the state before the second decay at $t_2$, from the observation of the first decay channel at $t_1$. Then we infer the correlation from future to past, i.e. which is the state before the first decay at $t_1$, from the observation of the second decay channel at $t_2$. We identify the decoherence region in $\Delta t$ in which the surviving correlation tells us $K_L$ at $t_2$ and $K_S$ at $t_1$, providing the unique way to tag a $K_S$ experimentally. We summarize the results presenting some final remarks and
our conclusion.

II. FROM PAST TO FUTURE

We consider an entangled two body neutral kaon system, as actually realised at DAΦNE, with $\phi \rightarrow K^0K^0$ decays, the source of EPR coherent $K^0\bar{K}^0$ pairs in the $C = -1$ antisymmetric state: $|i\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \}$.

Under particle exchange we call particle-1 the first one to decay at time $t_1$, particle-2 the last to decay at time $t_2$. We remind the reader that Quantum Entanglement is associated to Non-Separability in two aspects: (i) we cannot identify which is which due to indistinguishability, and (ii) we cannot specify the two parts of the system which are not definite, showing that the parts have no local physical reality.

In fact, the antisymmetric state $|i\rangle$ is unique and therefore identically given in terms of any two generic linearly independent neutral kaon states, orthogonal or not. As a particular case, it can be written in terms of the $K_S, K_L$ states with definite time evolution $\{f_1, t_1\}$: $|i\rangle = \frac{N}{\sqrt{2}} \{ |K_S\rangle |K_L\rangle - |K_L\rangle |K_S\rangle \}$ with $|N|^2 = (1 - |\langle K_S|K_L\rangle|^2)^{-1} \approx 1$. As a consequence, the entangled state $|i\rangle$ at any time $t$ after its production remains unaltered, even in presence of $K^0 - \bar{K}^0$ mixing:

$$|i(t)\rangle = \frac{N}{\sqrt{2}} \{ |K_S\rangle e^{-i\lambda_S t} |K_L\rangle e^{-i\lambda_L t} - |K_L\rangle e^{-i\lambda_L t} |K_S\rangle e^{-i\lambda_S t} \} = e^{-i(\lambda_S + \lambda_L)|t|} |i\rangle. \quad (1)$$

If nothing is registered after the observation of the first decay at time $t_1$ (i.e. integrating over all subsequent decays at times $t_2$ of particle-2), the survival probability of the entangled state is necessarily characterised by the total width $\Gamma = \Gamma_S + \Gamma_L$ of the system 62: $P(t_1) = \| |i(t = t_1)\rangle \|^2 = e^{-\Gamma t_1}$. This also holds for any decay channel $t_1$-distribution with no subse-

The two decay times formalism defines in the combined two terms of the entangled state $\{f_1, t_1\}$ what one calls particle-1 – the first one to decay – and particle-2 – the second one to decay. The (formal) use as evolution times is justified because they are disjoint and there is no overlap between them: $t_1$ before, and $t_2$ after, the performed measurement and its associated projection. Accordingly, the decay amplitude of the initial state $|i\rangle$ to channel $f_1$ at time $t_1$ for particle-1 and channel $f_2$ at time $t_2$ for particle-2, and the corresponding observable double differential decay rate $I(f_1, t_1; f_2, t_2)$ can be readily calculated 60 63 65:

$$I(f_1, t_1; f_2, t_2)_{LY} = |\langle f_1(t_1)f_2(t_2)|T|i(t)\rangle|^2 = |\langle f_1 f_2|T|i, t_1, t_2\rangle|^2 = C_{12} \{ |\eta_1|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2} + |\eta_2|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} - 2|\eta_1||\eta_2| e^{-\frac{1}{2}((\Gamma_S + \Gamma_L)(t_1 + t_2))} \} \cos[\Delta m \Delta t + \phi_1 - \phi_2], \quad (3)$$

with $\langle f_1|T|K_S\rangle$ and $\langle f_1|T|K_L\rangle$ the decay amplitudes to the $f_1$ channel of $K_S$ and $K_L$, $\eta \equiv |\eta_i| e^{i\phi_i} = \langle f_i|T|K_{f_i}\rangle / \langle f_i|T|K_{f_i}\rangle$, and $C_{12} = \frac{|N|^2}{2} \{ |\langle f_1|T|K_S\rangle| \langle f_2|T|K_S\rangle|^2 \}$. As a corollary of the above approach one can notice that at an intermediate step of the calculation – after the first decay at time $t_1$ – the state of the surviving kaon (particle-2) immediately before its decay at time $t_2$ is expressed as:
\[ |K^{(2)}(t = t_2)\rangle = \langle f_1|T|i_{t_1,t_2}\rangle = \frac{\mathcal{N}}{\sqrt{2}} \langle f_1|T|K_S\rangle e^{-i(\lambda_S + \lambda_L)t_1} \left[ e^{-i\lambda_L \Delta t}|K_L\rangle - \eta_1 e^{-i\lambda_S \Delta t}|K_S\rangle \right]. \] (4)

Keeping \( t_1 \) and \( f_1 \) fixed – the observation – and renormalising the state at time \( t_2 = t_1 \), it corresponds to the evolution from time \( t_1 \) to time \( t_2 \) of the pure state

\[ |K^{(2)}(t = t_1)\rangle = \mathcal{N}_2 \left( |K_L\rangle - \eta_1 |K_S\rangle \right), \] (5)

with \( \mathcal{N}_2 \) a suitable normalization factor. This is precisely the state of the living particle-2 which cannot decay to \( f_1 \), as a result of the projection by the decay of particle-1 at \( t_1 \) as a filtering measurement – see eqs. (7) and (8) below.

It is worth noting here that due to \( \Delta \Gamma = \Gamma_S - \Gamma_L \neq 0 \) two regimes can be identified in the time evolution of state (3): (i) the generic interference region and (ii) the decoherence region, with the relative weight of the \( K_S \) component negligible when the following condition is satisfied:

\[ |\eta_1| e^{-\Delta \Gamma \Delta t/2} \ll 1 \quad \text{[K}_L\text{-tag].} \] (6)

At long enough \( \Delta t \) – depending on what \( f_1 \) was – the living partner is always a \( |K_L\rangle \). This property is well understood and it has been used in the past in order to have \( K_L \) beams “for all practical purposes” (FAPP) in Bell’s terminology [29].

### B. Time history (TH)

It is worth to point out that the result for the living partner is in agreement with the EPR instantaneous information due to the first decay when following the time history of strange entanglement, which we are now going to study in detail.

We first notice that in the case of decay processes, any initial state has some probability per unit time to decay to a given decay channel \( f \) except that with zero probability. In particular the linear combination

\[ |K_{\rightarrow f}\rangle = \mathcal{N}_{\rightarrow f} \left( |K_L\rangle - \eta_f |K_S\rangle \right), \] (7)

having a vanishing decay amplitude \( \langle f|T|K_{\rightarrow f}\rangle = 0 \), cannot decay to \( f \). This state is the one tagged for the unmeasured particle as a consequence of the projection imposed by the decay of the observed particle. For the first decay to \( f_1 \) at time \( t_1 \), the tagged state of the surviving partner is given by Eq.(7) with \( f = f_1 \). In other words, the measured decay on one side prepares, in the quantum mechanical sense, its partner on the other side as a single kaon particle at a starting time \( t = t_1 \). Then the \( K_{\rightarrow f} \) state freely evolves in time – and in this sense the information is from past to future – until its decay time at \( t_2 \), see Eq.(8). We may ask whether this information constrains the past state of the decayed particle at \( t_1 \), which was undefined in the entangled system. This is a question that, for different scenarios, is it being debated in the literature – see, for example Refs.[66–69]. In our case, any state linearly independent to Eq.(7), orthogonal or not, leads to the same decay probability. This “filtering identity” [70] is saying that the orthogonal component \( |K_{\perp f}\rangle \) is filtered from the past undefined state by the decay. The decay acts as a filtering measurement and, for calculation purposes, it is convenient to rewrite the entangled state at \( t_1 \), in terms of these two orthogonal states, as:

\[ |i\rangle = \frac{1}{\sqrt{2}} \left( |K_{\perp f}\rangle|K_{\rightarrow f}\rangle - |K_{\rightarrow f}\rangle|K_{\perp f}\rangle \right). \] (8)

In this way, we may use the concept of transition probabilities at the different relevant times in the history of the system.

In summary four sequential steps are present in the time history of the entangled state \( |i\rangle \):

1. the time evolution of the state \( |i\rangle \) from time \( t = 0 \) to time \( t = t_1 \), with definite total width \( \Gamma \);
2. the projection of the state \( |i(t = t_1)\rangle \) onto the orthogonal pair \( |K_{\perp f_1}\rangle|K_{\rightarrow f_1}\rangle \), filtered by the decay \( f_1 \), times the decay amplitude of the state \( |K_{\perp f_1}\rangle \)
   into the \( f_1 \) channel;
3. the time evolution of the surviving (single) kaon state \( |K_{\rightarrow f_1}\rangle \) from time \( t_1 \) to time \( t = t_2 \);
4. the projection at time \( t = t_2 \) of the evolved state \( |K_{\rightarrow f_1}(\Delta t)\rangle \) onto the state \( |K_{\perp f_2}\rangle \) filtered by the decay \( f_2 \), times the decay amplitude of the state \( |K_{\perp f_2}\rangle \)
   into the \( f_2 \) channel.

These steps straightforwardly lead to the calculation of the observable double differential decay rate by factorising the amplitudes as follows:

\[ I(f_1, t_1; f_2, t_2)_{\text{TH}} = \left| \langle f_2|T|K_{\perp f_2}\rangle \langle K_{\perp f_2}|K_{\rightarrow f_1}(\Delta t)\rangle \langle f_1|T|K_{\perp f_1}\rangle \langle K_{\perp f_1}|K_{\rightarrow f_1}|i(t = t_1)\rangle \right|^2. \] (9)

One can easily verify that the TH approach is fully consistent with the LY approach [71]:

\[ I(f_1, t_1; f_2, t_2)_{\text{TH}} = \]
III. FROM FUTURE TO PAST

As already pointed out, the state \( |i⟩ \) in the LY approach coincides with the state \( |K^{(1)}(t = t_1)⟩ \) of the surviving kaon after the first decay in the TH approach. The \( t_1, t_2 \) symmetry of the correlated state in the LY approach – Eq. (2) – with no special role of one of the two decay times, demands the exploration of its implications when projecting it instead onto the \( f_2 \) channel at time \( t_2 \). With this information, the resulting past decayed state at time \( t_1 \) is:

\[
|K^{(1)}(t = t_1)⟩ = |f_2⟩|t_{t_1, t_2}⟩ = \frac{N}{\sqrt{2}} \langle f_2|T|K_\Sigma⟩\{e^{-i\lambda t_{t_1}} |\eta_2⟩ e^{-i\lambda t_{t_2}} |K_\Sigma⟩} - e^{-i\lambda t_{t_1}} [e^{-i\lambda t_{t_2}} |K_\Sigma⟩]. \tag{10}
\]

Expression (10) corresponds to the state of the decayed kaon \( \text{particle-1} \) immediately before its decay at time \( t_1 \), once \( t_2 \) and \( f_2 \) are fixed for the future “fate” of its partner. Keeping \( t_2 \) and \( f_2 \) fixed – the observation – and varying the first decay time \( t_1 \), it corresponds to the single kaon evolved state, before the first decay, from time \( t = 0 \) to time \( t = t_1 \) of the state:

\[
|K^{(1)}(t = 0)⟩ = N_1 |\eta_2⟩ e^{-i\lambda t_{t_2}} |K_{\Sigma}⟩ - e^{-i\lambda t_{t_2}} |K_{L}⟩. \tag{11}
\]

with \( N_1 \) a suitable renormalization factor. Contrary to eq. (5) which is independent on the past \( t_1 \) decay time, eq. (11) shows a dependence not only on the decay channel \( f_2 \), but also on the future \( t_2 \) decay time.

This is a striking result which clearly involves a correlation-in-time from the future observation at time \( t_2 \) to the past, inferring the initial kaon state before its first decay at \( t_1 \). It becomes well defined during the time evolution of the entangled state \( |i⟩ \) described by eq. (1) when the state of \( \text{particle-1} \) (and \( \text{particle-2} \)) should have been undefined in the absence of any observation. We insist that the post-tagging implied by Eq. (10) is not an artefact of the formalism but a factual observable accessible to experimental studies and thus it is fully physical. In a time history from future to past, the future observation at time \( t_2 \) tags particle-1 at the time \( t_1 = t_2 \) into the state proportional to \{\( |\eta_2⟩|K_{\Sigma}⟩ - |K_{L}⟩\}, the state not decaying to \( f_2 \). Keeping \( t_2 \) and \( \eta_2 \) fixed – the observation –, the backward evolution of this tagged unobserved state to \( t_1 < t_2 \) leads to Eq. (10).

A. The interference and decoherence regimes: the \( K_{\Sigma} \) tag

As a counterpart of the observability of the predicted Eq. (4) through the \( t_2 \) time distribution of the second decay, once the first decay to the \( f_1 \) decay channel at \( t_1 \) is fixed, the \( t_1 \) time distribution of the first decay as post-dicted in Eq. (10) is also observable, once the second decay channel \( f_2 \) and the decay time \( t_2 \) are fixed. As function of \( t_1 \), two different regimes can be identified: the generic interference region, in which the \( t_2 \) dependence of Eq. (10) is apparent, and the decoherence region, in which the relative weight of the \( K_\Sigma \) component is negligible. Decoherence is reached for large \( \Delta t \) satisfying the condition:

\[
e^{-\Delta t^2/2}/|\eta_2| \ll 1 \quad \text{[K}_\Sigma\text{-tag]}, \tag{12}
\]

leading to a pure \( K_{\Sigma} \) beam before the first decay. This consequence of the surviving correlation-in-time is most rewarding. Due to \( C\bar{P} \) violation and the non-orthogonality of the stationary states \( |K_{L}⟩|K_{\Sigma}⟩ \neq 0 \), there is no decay channel able to tag either \( K_{\Sigma} \) or \( K_{L} \) on an event-by-event basis. While it is relatively easy to prepare FAPP pure \( K_{L} \) beams, fulfillment of condition (12) constitutes the only known FAPP method to actually post-pare a \( K_{\Sigma} \) beam (i.e. the short-lived stationary state) with arbitrary high purity (depending on \( \Delta t \) and \( |\eta_2| \)), preparation otherwise impossible with other methods. As illustration of the observables in the two different regimes, Figure 4 shows the decay rate distribution into a generic channel \( f_1 \) of state (10) as a function of \( t_1 \) in two cases: either observed at \( t_2 = 3 \tau_{\Sigma} \) (interference region) or when condition (12) is satisfied (decoherence region), with \( f_2 = f_1 \) to maximize the interference effects and make visible the difference between the two cases. This choice \( f_2 = f_1 \) also emphasizes the differing results as due to the dependence on the time of the future observation. Whereas the decoherence case shows a definite width \( \Gamma_{\Sigma} \), the future observation in the interference region leads to a t₁-distribution with no definite lifetime. In the latter case the \( t_1 \) distribution does depend on the decay channel. All these results differ from the time distribution, given by the total width \( \Gamma \), in the absence of any future observation [78,79].

IV. REMARKS AND CONCLUSIONS

In the case of entangled neutral mesons in the \( C = (-) \) state, the dynamics before the first decay was considered to be trivial, even with mixing, as corresponding to a definite time evolution with the total width of the system. Hence in the past the experimental studies were
The present research has gone a step further and seems to recognize that the correlation between the two partners survives their explicit dynamics, with a transition from the quantum correlation of entanglement to a classical correlation of separable $K_S K_L$ states. The information from a measurement on the living partner – $f_2$ decay channel at $t_2$ – to the state of the decayed meson at $t_1$ is most surprising. Entering into the entangled region, i.e. just before the first decay, is not an artefact of our formalism, but a precise experimental observable through the $t_1$ distribution of decays in any channel, as shown in Fig. 1. This is so for even situations of decoherence, Eq. (12), a physical situation only reachable for strange entanglement with the two very different lifetimes of $K_L$ and $K_S$. The relevance of this result for particle physics is outstanding: the unique way to tag $what a K_S$ is, i.e. the solution of an open problem since the discovery of $CP$ violation.

Our results seem to confirm the counter-intuitive feature of time in quantum mechanics. The surviving correlation-in-time found here goes beyond other phenomena, like delayed choice experiments, quantum erasers or teleportation, discussed for photons [80]-[84]. In the case of delayed choice designs, the system is stationary at all times and the choice of the outcome can be made by either advanced or delayed observation with the result unchanged [85]. In the effect discussed here, the tagged state of the past decayed kaon has a non-trivial time dependence with a result depending on the decay time of the future observation (post-tagging), as depicted in Fig. 1. The result is not symmetric when comparing the outcomes in the two senses of “from past to future”, Eq. (4), and “from future to past”, Eq (10). This is characteristic of the neutral K-meson system with flavor mixing, $\Delta \Gamma > 0$ and $\langle K_L K_S \rangle \neq 0$, leading at decoherence times to the unique experimental $K_S$-tag. These predictions are fully observable through the measurement of $t_1$-distributions as the ones shown in Fig. 1 with no analogue in other physical systems.

Our results demonstrate that the correlation-in-time is definite between the outcome at a given time of the observed decay and the state of the unobserved partner. This correlation also survives when the observation is made in the future, when the system is no longer entangled after the first decay, post-tagging the past state of the unobserved decayed partner depending on the result and the time of the future observation. The non-trivial time evolution for the neutral K-meson system leads to a result which is non-symmetric in time. As a consequence, it opens the way to a novel kind of experimental studies, not envisaged before.

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[60] In fact, the antisymmetric state \(|i⟩ = \frac{1}{\sqrt{2}} \{ |K_0⟩ |K_1⟩ - |K_1⟩ |K_0⟩ \} \), with \(|K_0⟩ \) and \(|K_1⟩ \) two arbitrary linearly independent superpositions of \(|K^0⟩ \) and \(|K^2⟩ \) states, and \(\{N_0|α⟩|N_0|β⟩\}^2 = (1 - |⟨K_0|K_0⟩|^2)^{-1} \) .
[61] The eigenstates of the effective Hamiltonian describing a single neutral kaon are the short- and long-lived physical states \(|K_0⟩ \) and \(|K_2⟩ \). They have definite masses \(m_{S,L} \) and lifetimes \(γ_{S,L} \) and evolve as a function of the kaon proper time \(t \) as pure exponentials \(|K_{S,L}(t)⟩ = e^{-iλ_{S,L}t}|K_{S,L}⟩ \) with \(λ_{S,L} = m_{S,L} - iγ_{S,L}/2 \), and \(γ_{S,L} = (γ_{S,L})^{-1} \). They are usually expressed in terms of the flavor eigenstates \(|K^0⟩ \), \(|K^2⟩ \) as: \(|K_{S,L}⟩ \bigotimes [1 + i(ε_{S,L})|K^0⟩ \bigotimes (1 - iε_{S,L})|K^2⟩ \bigotimes ] \) with \(ε_S \) and \(ε_L \) two small complex parameters describing the CP impurity in the physical states, making them non-orthogonal, \(⟨K_0|K_0⟩ \simeq ε_L + iε_S \). One can equivalently define \(ε \equiv (ε_S + ε_L)/2 \), and \(δ \equiv (ε_S - ε_L)/2 \); adopting a suitable phase convention \(ε \neq 0 \) implies \(T \) violation, \(δ \neq 0 \) implies \(CPT \) violation, while \(ε = 0 \) or \(ε \neq 0 \) implies \(CPT \) violation. It is worth noting that one of the most stringent tests of the fundamental CPT symmetry is performed comparing the measured \(CPT \) violation in \(K_S \) and \(K_L \) states in [51].
[62] This property holds only for the \(C = -1 \) antisymmetric state, but not for the \(C = +1 \) symmetric state in which the time evolution would induce \(K^0 \rightarrow K^0 \) and \(K^2 \rightarrow K^2 \) terms due to \(K^0 \rightarrow K^2 \) mixing by weak interactions.
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[78] Even though it would be more challenging experimentally, also the survival probability of state (10), $P^{(1)}(t_1) = \| |K^{(1)}(t = t_1) \|^2$, could be measured as a function of $t_1$, which is also a characteristic of the decaying state and not related to any particular choice of the final state $f_1$.

[79] The KLOE/KLOE-2 experiment has already used the method based on condition (12) to select a pure $K_S$ beam to study its semileptonic decay $\pi^+\pi^-$, to search for its rare $CP$ violating decay $\pi^0$, and to study the $K_S$ decay time distribution and $K_{3\pi}$ interference with $f_1 = f_2 = \pi^+\pi^-$. In light of these results, the KLOE-2 experiment is in a unique position to perform a full experimental verification of the surprising phenomenon discussed in this paper about the information from future to past implied by result (10).

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