How do students promote mathematical argumentation through guide-redirecting warrant construction?

D Muhtadi¹, Sukirwan², R Hermanto¹, Warsito³ and A Sunendar¹

¹Universitas Siliwangi, Jl. Siliwangi No. 24 Tasikmalaya, Jawa Barat, Indonesia
²Universitas Sultan Ageng Tirtayasa, Jl. Raya Jakarta Km. 4, Serang, Banten, Indonesia
³Universitas Muhammadiyah Tangerang, Jl. Perintis Kemerdekaan I No. 33, Tangerang, Indonesia

Email: dedimuhtadi@unsil.ac.id

Abstract. Argumentation is a mathematical discourse that is currently widely studied by researchers as a stepping stone in constructing mathematical evidence. A variety of argumentation studies display different formats and constructs ranging from Toulmin's argumentation with a formal logic system to a general discourse that presents justifications and explanations for informal arguments with an inductive approach. It is based on the student's difficulties in preparing a search warrant until later gave rise to the idea of reconstruction of the warrant and the importance of guide-redirecting warrant construction (GWC). The results showed that GWC could guide students in constructing statements and reasons or explanations towards the expected evidence. Students also realize that statements are related to reason and explanation in building whole arguments. Even so, the presentation of the statement is still aligned with the evidence submitted, so for reasons that display the axiomatic system, it still faces problems.

1. Introduction

Argumentation is a fundamental mathematical capability that is associated with a logical thought process to make inference or justification for the statement or mathematical solution that solved [1]. This context has historically arisen from a variety of problematic situations in the construction of mathematical evidence in which the rigidity of evidence in axiomatic systems has created skeptical for students and regards evidence as meaningless ritualistic activity [2]. The argument in many expert views becomes a bridge for learning mathematical proofs. It seems reasonable where the argument resulting from a process of construction evidence is part of an effort to produce evidence [3].

Argumentation studies develop in different formats and constructs. There are at least two different formulations regarding this study, which are related to processes and products. The studies related to the product, emphasis on the student's ability in constructing the argument. The typology of this scheme has been widely researched the experts about the structure of the argument [4], taxonomy schema argument [5-9], the justification students with the argument [10], and the type of arguments [11, 12]. Moreover, studies related to the process of emphasizing the construction of argumentation based on the argumentation scheme to be achieved, and the dialogical element becomes essential [13]. It suggests that the ability of argumentation basically cannot grow by itself,
although naturally, students can construct an argument, treatment, or direction from the teacher is still needed [12].

Ubuz, Dincer, and Bulbul [13] conducted a study of Toulmin's argumentation schemes by exploring complex argumentation constructs based on teacher expressions and student considerations, namely about how students construct a construction definition of the distance from a point to a set. By applying the dialogic concept, the argumentation structure is analyzed schematically by giving rise to guide-redirecting and approval guide-backing. Both of these terms help students to build a warrant, which then leads students to a conclusion that is built. The results of the study show that the process of building an argument is how to encourage students to develop ideas about the warrant. It confirms that the difficulty of students in constructing an argument lies in how students construct a warrant. A warrant is interpreted as proof/explanation, but then more is identified with mathematical evidence that must be supported by axiomatic systems. Thus, even though the concept of argumentation is contrary to formal logic, the approach used still uses a formal approach. It then encourages researchers to redefine warrant more openly, primarily to accommodate each student's effort in constructing arguments [14].

Redefine warrant further open up space for an argumentative approach in facilitating the students in the process of argumentation. This context then gives rise to a new framework in the study of argumentation, where students can be guided to construct a warrant (guided warrant construction) through instructional design. Instructional example [15] to develop a discourse of mathematical argument; for example, by submitting a statement: I agree with ... because ...; I pay attention ... when ...; I wonder why ...; I have a question about ...; I do not agree because ...; by ..., I think ..... The specific description of instructional how students work with mathematical argumentation [16] includes activities: (1) making the conjecture, (2) justifying the conjecture, and (3) deciding whether the conjecture is true or false (i.e., inferring). There are three critical terms related to argumentation, namely: conjecture, justification, and conclusion. Conjecture is intended as a process of deliberate guessing or finding patterns to make mathematical statements of mathematical validity that have not been determined (if ... then ...); justification is the process of giving an explanation relating to a person's reason for establishing the mathematical validity of a conjecture (why); while the conclusion is the process of coming to consensus or agreement about the validity of the conjecture and justification (True or False) [16]. The term conjecture and justification is an attempt to build a warrant that functions to bridge the data between the conclusions.

What is stated above is a new scheme for students to promote mathematical argumentation. Moreover, this then inspires a pedagogical framework to facilitate students learning argumentation through guided-redirecting warrant construction.

The term Guide-redirecting Warrant Construction adopts the term guide-redirecting [14], where the intervention is given by the teacher indirectly through statements that lead students to construct arguments. These statements can be in the form of mathematical statements and reasons. Related to this, Dunnewold [17] propose the reconstruction of warrants by using the paradigm IRAC (Issue, Rule, Application, and Conclusion). Based on this paradigm, the argument is written in 2 columns, each containing statements on the left and reasons on the right. Statements contain positive statements arising from asentation. While reasons contain the reasons that support each statement in the statement (support). Reasons can be understood as general reasons, but Dunnewold [17] limits them as apparent reasons. However, because warrant reconstruction is very open to formal and informal arguments [18], the reasons presented can be more general. Sukirwan et al. [12] mentioned this as an explanation. Warrant reconstruction that presents statements, reasons, and explanations is then formulated into a Guide-redirecting Warrant Construction with the format presented in Table 1.
2. Method
This study uses a descriptive qualitative approach that aims to uncover the way students construct argumentation. To achieve this goal, as many as 40 Grade VII students from one of the State Junior High Schools in Tasikmalaya City were used as the research sample. The topic chosen in this study is set. Data collection was carried out through 4 stages, namely: giving pretest, giving treatment with GWC, giving posttest, and interview. Pretest and posttest are given in the same format, containing description questions about the set of 5 questions. The aim is to see how students can construct arguments.

Meanwhile, GWC is given when students learn about sets that are specifically directed at how students construct arguments, both when writing statements, reasons, and explanations. GWC is presented in the form of student activity sheets, while students’ understanding of the basic theory of the set is presented in the form of lectures. Furthermore, to know about how students construct arguments, as many as five students selected as respondents by way of sampling theoretically by reference: (1) represent a group of high, medium, and low, (2) represents the complexity of students' answers, (3) represents an abstraction mathematically students.

To promote students' mathematical arguments, GWC is presented in a format of student activity sheets where students are guided indirectly to fill in specific columns in both the statement, reason and explanation columns. There are three different formats on the student activity sheet, namely: filling in the reason/explanation column, filling in the statement column, and mixed; that is, some of the contents are in the statement column, and some are in the reason/explanation column. In the explanation format, support is described by students through sample cases registered by students. While the reason, support format is described by students through an axiomatic system.

3. Results and Discussion
3.1. Promoting mathematical argumentation
The use of GWC in student activity sheets guides students to construct their arguments. An example of a student’s work can be seen in Figure 1.
In Figure 1, it can be seen that explanation construction really helps students to construct statements that lead to correct conclusions. This construction becomes very important where students are guided to determine a series of arguments that direct students to the relationship between premise and conclusion. If the construction is wrong, indeed, students cannot find a logical equivalent relationship between statements \((P \land Q) \lor R\) and \((P \lor Q) \land (Q \lor R)\). Then, are students aware of the construction of statements that relate the logical equivalent relationship between statements \((P \land Q) \lor R\) and \((P \lor Q) \land (Q \lor R)\)? The following are the results of interviews that reveal students’ opinions about the construction of argumentation.

**Dialog 1**

Researcher: "What do you know about the statement?"
Maura: "O, yes, I realize this because there is an explanation?"
Researcher: "Does it have anything to do with the evidence you have to show?"
Maura: "Maybe, because I found the evidence requested?"
Researcher: "Can you look more closely at it?"
Maura: "O, yes, I find that it is reconstructing between the left and right statements."
Researcher: "What about the preliminary data?"
Maura: "It only helps me to explain the statement."

Maura’s expression asserts that the evidence sought can be reconstructed from the statement in question. Abductive reasoning is the way Maura understood to reconstruct evidence even though it emerged from the GWC. The exciting thing is that Maura was aware of the statement that led to the evidence, even though he had previously written the statement because of an explanation.

In reason-based GWC, argumentation is constructed by students utilizing known data. The results of reason-based student argument construction can be seen in Figure 2.

![Figure 2. GWC on reason-based student activity sheets](image)

In Figure 2, GWC guides students to determine statements or reasons that build a logical relationship to the evidence presented. The GWC is built to explain a series of arguments from statements or reasons, as well as the relationship between statements and reasons that build a warrant for evidence. There are statements and logical reason to build the equivalent relationship between \((A \land B)^c\) with \(A^c \lor B^c\). The following interview provides an overview of students’ understanding of the relationship.
Dialog 2
Researcher : "Did you make the argument based on the statement?"
Agnia : "I made an argument because other arguments already existed."
Researcher : "Do you know that reason supports statements?"
Agnia : "Sure."
Researcher : "What about the set of arguments in the statement?"
Agnia : "O, yes, I found that statement proved to be back and forth."
Researcher : "Is there perhaps a statement that does not go back and forth?"
Agnia : "Maybe it is because the sign is equal to."

The interview results show that initially, students only realized that the argument was built based on a series of previous arguments. Also, the relationship between statements and reason has been realized by students. Students have also become aware of the relationship of logical equivalence where proof applies back and forth.

3.2. Student ability to construct an argumentation

To see the ability of students in constructing arguments, the following are the results of the pretest and posttest arguments for each problem, as shown in Figure 3.

![Figure 3. Students' Ability to Construct Arguments](image)

In Figure 3, it can be seen that the score of the students' posttest results, especially for questions number 1 and number 2 is higher than the score of the pretest results. It shows that the use of GWC has an impact on students' ability to construct arguments. Even so, in questions, number 4 and number 5, the range between the pretest score and the posttest score is relatively small. It shows that students still experience obstacles in constructing arguments.

In searching for students' answers, more than half of students can understand the set well if its members can be appropriately identified. For example, in problem number 3, when members of the set L, M, N have been registered in full, then students quickly determine that \((L \cup M) \cap N = (L \cap N) \cup (M \cap N)\). Examples of students' answers below show that the construction of the statement of the relationship between the set can be explained by way of registering members of each set and can explain the logical equivalent of the operation between the different sets easily.
In Figure 4, students succeed in constructing an argument by presenting a complete statement that is supported by an adequate explanation. These statements appear from students' ability to describe $L \cup M$, $L \cap N$, $M \cap N$, $(L \cup M) \cap N$, and $(L \cap N) \cup (M \cap N)$, so that they are obtained $(L \cup M) \cap N = (L \cap N) \cup (M \cap N) = \{1, 2, 3, 5, 7, 11, 13\}$.

Conversely, students' failure in constructing arguments appears from students' failure to construct statements. An example of this failure is shown in Figure 5.

In Figure 5, students fail to construct arguments. The failure appears from statements that did not reach the expected conclusion. Students fail in presenting the complement of the set $P (P^c)$, the complement of the set $Q (Q^c)$, the complement of $P^c \cap Q^c$ and $P \cup Q^c$. Understanding complement seems to be still a problem for students, where the explanation of set operations is also presented incorrectly.

Also, students fail to present statements and reasons from arguments that demand the use of axiomatic systems. It is evident from the way students answer questions, as presented in Figure 6.
In Figure 6, the statement of sets A, B, and C by registering any of its members shows that students are still fixated on casuistic explanations. Students fail to present a statement because it is not supported by an understanding of the understanding of complement and slice and joint operations. This understanding should be played as a reason which will further clarify the relationship between statement construction and the conclusions to be built [19, 20].

4. Conclusion
GWC guides students to determine the arguments that lead to proof. Statements are the most critical set of arguments in the proof. The statement constructed by student was a translation of the operation on the evidence presented. Lastly, the student was still fixated on the construction of the explanation presented in the case example.

Acknowledgments
The authors would like to thank Lembaga Penelitian Pengabdian Kepada Masyarakat dan Penjaminan Mutu Pendidikan (LP2M - PMP), Universitas Siliwangi that supported this research.

References
[1] PISA 2015 Draft mathematics framework Retrieved from http://www.oecd.org/pisa/pisaproducts/pisa2015draftframeworks.htm.
[2] Ball D L and Bass H 2003 Making mathematics reasonable in school A research companion to principles and standards for school mathematics, pp. 27-44.
[3] Liu Y 2013 Aspects of mathematical arguments that influence eighth-grade students’ judgment of their validity (Degree Doctor of Philosophy in the Graduate School The Ohio State University).
[4] Toulmin S E 2003 The uses of argument (Cambridge: CUP).
[5] Balacheff N 1988 Aspects of proof in pupils practice of school mathematics, in D Pimm (Eds) Mathematics, Teachers and Children 216–235 (London: Hodder & Stoughton).
[6] Harel G and Sowder L 1998 Students’ proof schemes: Result from explanatory studies. In A H Schoenfeld, J Kaput & E Dubinsky (Eds) Research in Collegiate Mathematics Education 234-283.
[7] Stylianides A J and Stylianides G J 2009 Proof constructions and evaluations Educational Studies Mathematics 72 237-253.
[8] Nguyen D N 2012 *Understanding the development of the proving process within a dynamic geometry environment* (Dissertation of Universität Würzburg: Germany).

[9] Nordby K 2013 *Investigating viable arguments: Pre-service mathematics teachers’ construction and evaluation of arguments* (Montana State University).

[10] Bergqvist T 2005 How students verify conjectures: Teachers’ expectations *Journal of Mathematics Teacher Education* **8** 171–191.

[11] Liu Y, Tague J, and Somayajulu R 2016 What do eighth-grade students look for when determining if a mathematical argument is convincing *International Electronic Journal of Mathematics Education* **11** 2373-2401.

[12] Sukirwan, Darhim, Herman T, and Prahmana R C I 2017 The students’ mathematical argumentation in geometry *Journal of Physics: Conference Series* **943** 1-7.

[13] Labinaz P 2014 Reasoning, argumentation, and rationality *Ethics and Politics* **16** 576-594.

[14] Ubuz B, Dincer S, and Bulbul A 2013 Argumentation in undergraduate math courses: A study on definition construction. In A M Lindmeier & A Heinze (Eds.) *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* **4** 313-320 (Kiel, Germany: PME).

[15] Freeman J B 2005 Systematizing Toulmin’s warrants: An epistemic approach *Argumentation* **19** 331-346.

[16] Rumsey C and Langrall C W 2016 Promoting mathematical argument *Teaching Children Mathematics* **22** 413-419.

[17] Dunnewold M 2006 Using the idea of Mathematical proof to teach argument structure *Perspectives: Teaching Legal Research and Writing* **15** 50-53.

[18] Viholainen A 2011 The view of mathematics and argumentation. In M Pytlak T Rowland & E Swoboda (Eds.) *Proceedings of the 7th Congress of the European Society for Research in Mathematics Education* 243–252 (Polandia: CERME).

[19] Muhtadi D, Wahyudin, Kartasasmita B G, and Prahmana R C I 2018 The Integration of technology in teaching mathematics *Journal of Physics: Conference Series* **943** 012020.

[20] Mumu J, Prahmana R C I, and Tanujaya B 2018 Construction and reconstruction concept in mathematics instruction *Journal of Physics: Conference Series* **943** 012011.