Effect of Yarn Cross Section on Air Drag for Spandex Yarn
By Kiyoshi Hatta*, Toshiyasu Kinari**, Sukenori Shintaku** and Nobuo Iwaki**, Members, TMSJ

* Ishikawa National College of Technology, Tsubata, Ishikawa
** Faculty of Engineering, Kanazawa University, Kodatsuno, Kanazawa

Based on the Journal of The Textile Machinery Society of Japan, Vol.50, No.8, 7216- 7223(1997 8)

Abstract
The effect of yarn cross section on the air drag coefficient and the characteristic length is examined for spandex yarn with non-circular cross section.

Methods for estimating the air drag of spandex yarns in axisymmetric air flow are investigated. Results obtained are as follows:
1) Although the drag coefficient for thin yarns is similar to that of a cylinder, the air drag coefficient of spandex yarn shows similarity to that of a flat plate because of remarkable irregularities in its cross section.
2) The drag coefficient for a fixed-end yarn is the same as that for a flat plate. The drag coefficient for a free-end yarn is twice that of the fixed-end yarn. This value is useful in calculating yarn air drag.
3) It is efficient and reasonable to use the effective diameter converted from the yarn perimeter length—without any consideration to its irregularity—as the characteristic length of the spandex yarn.

1. Introduction
Empirical estimates on the relation of the yarn and the air flow exist from taking note of the inevitably-occurring air drag forces; for example, the increase in the stress on the melt spinning process, and the ballooning phenomenon on the twisting process. In addition, these forces are already used to positive effect; for example, in the reservoir of yarn and the control of yarn tension. In particular, the picking on the jet loom is a sample of the most positive use of the airflow. However, the yarns dealt with in the weaving factory have recently become incredibly varied with the varying needs of the consumer market, and air drag forces have not yet been examined for special yarns (such as spandex yarn) with stretch qualities or irregular cross sections. On the experimental apparatus for drawing core yarn into hollow spindles manufactured by Hori and others[11], the air drag force is reasonable to blow off the spandex core yarn resisting the spindle contact friction force, yarn elastic force, yarn dead weight, etc.

In this paper, a method for calculating the air drag force for spandex yarn in the free-end case is investigated, which is considered more useful in many textile machinery applications that use the generated drag force for their operations.

2. Drag Coefficient and Conventional Studies
Equation (1) expresses the air drag force of yarn with a circular cross section in axisymmetric flow.

\[ D_f = \frac{1}{2} C_f \rho \nu^2 \pi d L \]  
\[ D_f: \text{air drag force} \]

\[ C_f: \text{air drag coefficient} \]
\[ \nu: \text{relative velocity between filament and air} \]
\[ \rho: \text{density of air} \]
\[ d: \text{filament diameter (a: filament radius)} \]
\[ L: \text{length of filament exposed to airflow} \]

Historically, many investigators have investigated the air drag coefficient \( C_f \) in relation to the length, the thickness, and the structure of the fiber. They have expressed the form of an effective equation, arranged using the radius Reynolds number, \( Re_a=\frac{\nu a}{\nu} \) (adopting the yarn radius as the representative length), the diameter Reynolds number, \( Re_d=\frac{\nu d}{\nu} \), or the length Reynolds number, \( Re_x=\frac{\nu L}{\nu} \) (adopting the length of filament exposed to the air flow as the representative length), where \( \nu \) is the kinematic viscosity of air.

Gould and Smith[2] expressed an experimental equation for the drag coefficient \( C_f \) for mono-filament yarn under fixed-end conditions using a suction wind tunnel at air velocities of up to 100m/s.

\[ C_f = 0.27 \ Re_a^{-0.61} \]  
\[ (20<Re_a<200, \text{yarn less than 300} \mu \text{m in diameter}) \]  

Gould and Smith reported that the drag coefficient in the region over \( Re_a =200 \) approached that predicted by the theory of complete turbulent flow.

In the case of a multi-filament yarn with filaments held close enough together, the experimenters were able to treat it as a mono-filament of similar surface area. However, they reported that the air drag increases where yarns are held insufficiently close together, or are permitted to vibrate. Similarly, Shimizu and others[3] investigated the drag coefficient \( C_f \) on melt spinning, and showed the value K=0.23 \sim 0.49 using the relation \( C_f =K Re_a^{-0.61} \), but that K=0.77
when vibration or filament spacing affected the results.

On the other hand, Glauert and Lighthill\cite{141} calculated the drag coefficient in the case of laminar flow on boundary layers, as a theoretical analysis of a long, thin cylinder. White\cite{5} reported results of this theoretical analysis, including the effect of curvature in the turbulent boundary layer. He proposed an approximate equation to determine $C_f$ from the length and radius of a cylinder based on the theory. White reported that if arranged by Red and Rex, $C_f$ approaches the value for a flat plate in the range of large Reynolds numbers, but the effect of the cylindrical shape appears in $C_f$ as the Reynolds number becomes small.

Finally, Higuchi and Kasahara\cite{6} analyzed the relation $C_f=K\cdot Rex^n$ for cotton spinning yarn and reported that the value of $K$ is changed by the yarn "closed" condition, or changes in the apparent diameter, while the exponent $n$ shows the effects of the condition of yarn "fuzz". It was reported that $C_f$ is a constant, with no relation to the apparent diameter or the condition of fuzz, at large Reynolds numbers.

### 3. Experimental

#### 3.1 Experimental Apparatus

Fig. 1 shows a sketch of the experimental apparatus. A transparent vinyl chloride (PVC) pipe, of 30mm inside diameter and 925mm length, is used as the wind tunnel. It is connected to the suction blower (Hitachi VAL- 030- E, rated flow 3.6m³/min) by a circular pipe 50mm in diameter. The air flow in the wind tunnel is controlled by an inverter to regulate the suction flow of the blower. The air velocity is confirmed by a hot-wire anemometer (Nihon Kagaku Kogyo MODEL- 6164) mounted in the 50mm pipe. Samples are suspended from the point of a U-gauge (Shinko Tsushin Kogyo UL- 50gf), and the detected tension is output to the pen recorder (To- A Denpa Kogyo EPR- 200A) through the dynamic strain gauge (the same DAS- 405B).

In the experiment, samples are suspended from the U-gauge out to the measurement length (800mm maximum length from the suction entrance). The air drag force is measured at 10m/s air velocity increments up to 100m/s, and again at the maximum air velocity of 105m/s. A stainless pipe 4mm in diameter is inserted into the wind tunnel for the fixed end condition tests, and the samples threaded into it to avoid affecting the air flow. In addition, a weight of 0.02N is hung at the end of yarn to fix it with initial tension $P$.

Cross sections of the spandex yarns are observed under five hundred-fold magnification using the microscope (Keyence VH5900), while they are displayed on the monitor and output concurrently to the video printer (Sharp VP- ED100).

#### 3.2 Samples Tested

For measuring the air drag of spandex yarns, standard yarns are used as follows to compare the effects of thickness or structure. Seven nylon yarns from No.02 to No.8 (5.8 ~ 230tex) are prepared as the mono-filament yarn; four polyester yarns from 50D to 250D (5.6 ~ 27.8tex) that are twisted from zero to 1000t/m and heat-set as the multi-filament yarn; and six cotton yarns from Ne30 to Ne100 (19.7 ~ 5.9tex) as spun yarn. Because all yarns are relatively thick—at 80 ~ 460 µm ($Rex=27 \times 1600$) in apparent diameter—and the standard sampling length is 500mm ($Rex=3.3 \times 10^5 \sim 3.5 \times 10^6$), the expected flow is in the transition zone or the completely turbulent flow zone. On the other hands, four spandex yarns are used as the multi-filament yarn with non-circular cross section: Toyobo Espa 70D (7.8tex, 41 µm/7 filaments); 140D (15.6tex, 32 µm/20 filaments); 280D (31.1tex, 32 µm/40 filaments); and Asahi Kasei Kogyo Roica 420D (46.7tex, 42 µm/36 filament). The 20D spandex yarn is omitted, because from previous observations it acts as a mono-filament yarn, and can be treated similarly to the nylon mono-filament.

### 4. Results and Discussion

#### 4.1 Comparison of Yarn End Conditions

Fig. 2 shows the air drag on the free end condition (i.e. hanging the yarn naturally from the U-gauge) and the fixed end condition (hanging a weight at the end of yarn) when the initial tension $P$ is varied. On the mono-filament, the value of drag force tends to zero at small $P$, but it measures from one and half times to twice that of the fixed end due to the yarn vibration on the free end. Gould and Smith\cite{21} confirmed that in their experiment the drag force on the free end increased from 10% to 28% against that of the fixed end, though the 0.1N "fixed condition" initial tension of their experiment is considerably larger than the 0.02N of our experiment. In addition, it is found that the value of drag force increases by means of filament expansion when the initial tension on the multi-filament is small. Consequently, it turns out that the value of drag force on the free end appears to be twice that of
the fixed end due to the effects of filament separation or vibration.

4.2 Effect of Yarn Thickness

Drag coefficients are determined from the equation (1) based on the value of drag force, when the fixing initial tension $F$ on the mono-filament or closed thick yarn is 0.02N. Fig. 3 shows these forces arranged by the radius Reynolds number $Re_a$ and plotted on double logarithmic coordinates. They correlate on a single curve at $20<Re_a<500$, and the slope of the curve is gentle at $Re_a>500$. It is thought that they approach the theoretical drag value for a flat plate in the complete turbulent flow regime. The experimental equation is expressed as follows, determined for the region of the plotted experimental data that correlates to a single curve.

$$C_f = 0.22\, Re_a^{-0.58} \quad (20<Re_a<500) \quad (3)$$

The exponent is similar in equations (2) and (3), but the factor is slightly different due to the difference in fixing initial tension described in the last paragraph. In any case, drag coefficients are well arranged by the radius Reynolds number for the fixed end yarn with circular cross section. The effect of yarn thickness—in other words, the yarn curvature—appears remarkably at the small range of $Re_a$.

4.3 Variation of Drag Coefficient with Yarn Structure

Fig. 4 shows the drag coefficient of mono-filament, twisted and heat-set multi-filament, and cotton yarns in the free end condition. When they are arranged by $Re_a$, drag coefficients of each yarn correlate on a single curve similar to the fixed-
end mono-filament yarn at Reynolds number less than 500 for each kind of yarn. It is not shown clearly in this paper, but the slope of the curve is slightly different for the multi-filament yarn, and it is more gentle for a non-twisted or a slightly-twisted yarn. In addition, \( C_f \) itself tends to increase. It is assumed that the \( C_f \) for the multi-filament yarn is different from the mono-filament yarn because separation of the filaments occurs at the yarn free end, and behaves unpredictably due to yarn vibration in the region of high air velocity. It is expected that the surface area of the cotton yarn affected in the airflow is very large due to the existence of fuzz, but in actuality the exponent of the approximate equation is nearly that of the mono-filament, because the fuzz lies in the direction of air flow and filament separation occurs with relative difficulty. In the fixed end condition for the cotton yarn, the fuzz stands out from the yarn more than that of the free end case, due to the higher yarn binding force. As a result, it is confirmed that the value of drag force is as large as that for the free end case.

### 4.4 Spandex Yarn

#### 4.4.1 Yarn Cross Section and Representative Length

The representative length of the mono-filament, the multi-filament with nearly circular cross section, and the spandex yarn with same sectional area are compared and investigated in Fig. 5. The diameter \( d'_i \) is that used in conventional studies[21, and assumes that the multi-filament has a near-circular cross section. However, the cross sectional shape of spandex yarn is extremely non-circular, as shown in Fig. 6, and it changes continuously along the length of the yarn. Table 1 shows the diameter converted to the representative length for the spandex yarn, where the method used to determine the perimeter length of the spandex yarn is as shown in Fig. 5(c).

For yarns that have non uniform shape and that include voids (for example, spandex yarn), the diameter \((d'_i)\) from now onward.

![Fig. 5 Perimeter Length With Same Actual Sectional Area](image)

![Fig. 6 Cross Section Of Spandex Yarns](image)
on) converted from the real cross sectional area determined by the linear density and the specific gravity is the mechanical representative length, but is not in any sense the representative length necessary to determine the drag force. Conversely, the diameter \( d_2 \) (the converted contour length of each filament exposed to the airflow) is extremely large. Therefore, the converted diameter \( d_2 \) (as with \( d_0 \)) is determined from the perimeter length \( l_1 \) that envelops the outline of the yarn, neglecting the roughness of each filament, as shown in Fig. 5(c). As a result, \( d_2 \) is equal to \( 1.8d_0 \) for spandex yarns, and the diameter \( d_2 \) of a great-circle circumscribed to each filament is equal to \( 1.2d_0 \) (from geometric alignment) in the case of the multi-filament yarn. (The values are \( 1.6d_0 \) and \( 1.1d_0 \), respectively, for yarns of ten filaments or less).

The value of drag force for the spandex yarn is plotted, together with the value for the mono-filament (shown by the solid line), in Fig. 7. From this plot it can be seen that, although the changing rate of drag force with respect to air velocity is slightly different, the drag forces for the spandex yarn are near those for mono-filament yarns that have the same length \( l_1 \) as the perimeter length \( l_1 \). From comparisons with mono-filament yarns in Table 1, it can be seen that above-mentioned length \( l_1 \) is available as the yarn representative length in the air flow.

### 4.4.2 Drag Coefficient

The change in drag force with respect to the air velocity of the spandex yarn is slightly different from that of the mono-filament; the cause of this difference is considered to exist in the behavior of the drag coefficient \( C_f \). Fig. 8 is the calculated drag coefficient \( C_f \) for the spandex yarn, and is arranged by the radius Reynolds number, computed using \( d_2 \). The drag coefficient for the spandex yarn nearly conforms at \( Re_a=500 \), but unlike the mono-filament— the increasing tendency is not found for small values of radius Reynolds number. Therefore, the effect of yarn thickness (curvature of cross-sectional shape) is small for the spandex yarn.

This drag coefficient \( C_f \) is compared with that of a flat plate in turbulent flow, considering the cross sectional shape of the spandex yarn. The drag coefficient \( C_f \) is expressed by the equation (4) for the flat plate in complete turbulent flow \((Re_x>5 \times 10^5)\).

\[
C_f = 0.074Re_x^{-0.2}
\]

The drag coefficient \( C_f \) for the spandex yarn is divided by the
value of equation (4), using the parameter $V_a^2/\nu x$ to indicate the property of the flat plate. Results are shown in Fig. 9. The numerical value is larger than 1 on the flat plate (because the yarn end condition is free), but its value is nearly constant at 2, with no dependence on changes in $V_a^2/\nu x$. In short, the effect of curvature which appears for thin yarns (such as mono-filaments) is rare in the drag coefficient $C_1$ for the spandex yarn; in other words, it acts almost as the flat plate does. Though direct results are not obtained because it is too difficult to measure the drag force on the spandex yarn in the fixed end condition, it is known from Fig. 2 that the drag force on the free end condition for other yarns is twice that of the fixed end condition. It is therefore logical to suppose that the same consideration can be applied to the drag coefficient of the spandex yarn, too. It is recommended that the drag coefficient of the flat plate must be used for the spandex yarn in the fixed end condition, and twice that for yarn in the free end condition.

### 4.4.3 Calculation and Simulation of Air Drag

From the above, $2C_{fp}$ was adopted as the estimated value of the free end condition, based on equation (4) for the theory of flat plates in turbulent flow. The conversion diameter of envelope perimeter length $d'_{f}$ ($1.8d_{f}$ in this paper) was used as the representative length of yarn in the air flow. The drag force was calculated. In simulations, the sampling length was divided into a large number of finite elements, and the drag force load on each part was calculated by equation (1). Next, the extension caused by the drag force was computed from the stress-strain relation, and the change in cross sectional area was calculated at the same time. Subsequently, the final values of the convergent drag forces and extensions are computed by iterating equation (1), considering the change of extension and the cross section respectively. Results are shown in Fig. 10. The value computed by the simulation (solid line) agrees very well with the plotted experimental data.

### 4.4.4 Effect of Yarn Extension

If the extension $\epsilon$ deforms under the volume invariance for the spandex yarn, the yarn diameter decreases to $(1 + \epsilon)^{-1/2}$.

---

Fig. 9 Comparison With Drag Coefficient On Flat Plate In Turbulence Flow

Fig. 10 Air Drag And Extension Of Spandex Under Simulation And Experiment

(English Ed.) Vol. 45, No. 1 (1999)
Thus, the drag force increases to $(1+\varepsilon)^{1/2}$ in proportion to the surface area. Therefore equation (1) can be used as a simple calculation method in the case of small extensions. For example, the strain $\varepsilon$ of 70D yarn is at maximum 20 percent, and the drag force by extension increases about 10 percent in the highest air velocity of $105\text{m/s}$. In the case of 420D yarn, the strain $\varepsilon$ is 8 percent and the drag force increases about 4 percent. Stated simply, from the viewpoint of the air drag, the effect of cross sectional irregularity is greater than that of the effect of large extension in the spandex yarn.

5. Conclusion

Until recently, the air drag coefficient for thin yarns that are held close enough together to approximate a circular cross section computed using the radius Reynolds number $Re_a$ or the length Reynolds number $Re_x$, has been quite sufficient in most cases. But for the spandex yarn of non-circular cross section, air drag coefficients include no effect of curvature such as is shown for thin yarns, and instead show the tendency to behave nearly as flat plates by their cross sectional shape.

It is thought that the drag coefficient for the spandex yarn changes with the end conditions in the same way as other yarns, namely that the value of the drag coefficient on the free end is twice of that of the fixed end. Though direct results are not obtained for the fixed end condition of spandex yarn, it is considered acceptable to use the drag coefficient of a flat plate for the fixed end condition, and twice that for the free end.

Prior to this study, the diameter of the circumscribed circle (assuming circular cross section) has been used as the representative length for determining the air drag force. However, for a spandex yarn with non-circular cross section, the diameter converted from the perimeter length of the yarn outline—neglecting the ruggedness of each filament—is an easy-to-arrange and reasonable length. For spandex yarns, this diameter is about 1.8 times the converted diameter based on the real cross sectional area calculated from the tex or den value of the yarn (for yarns over 10 filaments).

Acknowledgements

The authors would like to thank Mr. Greg Joughin for critical reading of the manuscript.

References

[1] J. Hori, T. Kinari, S. Shintaku; J. Text. Mach. Soc., Japan, 50, T24 (1997)
[2] J. Gould, F. S. Smith; J. Text. Inst., 71, 38, (1980)
[3] J. Shimizu, N. Okui, K. Tamai; Sen-i Gakkaishi, 39, T398, (1983)
[4] Glauert, Lighthill; Proc. Royal Soc. London, 230, 188, (1958)
[5] F. M. White; Trans. ASME, 94, Series D, 200, (1972)
[6] K. Higuchi, Y. Kasahara; J. Text. Mach. Soc., Japan, 15, 335, (1962)
[7] JSME, Mechanical Engineers' Handbook, A5 Fluid Mechanics, p.46, Maruzen (1986)