THE GRAVO-MAGNETO LIMIT CYCLE IN ACCRETION DISKS

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ABSTRACT

Previous theoretical studies have found that repeating outbursts can occur in certain regions of an accretion disk due to sudden transitions in time from gravitationally produced turbulence to magnetically produced turbulence. We analyze the disk evolution in a state diagram that plots the mass accretion rate versus disk surface density. We determine steady state accretion branches that involve gravitational and magnetic sources of turbulence. Using time-dependent numerical disk simulations, we show that cases having outbursts track along a nonsteady “dead zone” branch and some steady state accretion branches. The outburst is the result of a rapid inter-branch transition. The gravo-magneto outbursts are then explained on this diagram as a limit cycle that is analogous to the well-known S-curve that has been applied to dwarf nova outbursts. The diagram and limit cycle provide a conceptual framework for understanding the nature of the outbursts that may occur in accretion disks of all scales, from circumplanetary to protoplanetary to active galactic nuclei accretion disks.

Key words: accretion, accretion disks – galaxies: nuclei – magnetohydrodynamics (MHD) – planets and satellites: formation – protoplanetary disks – stars: pre-main sequence

1. INTRODUCTION

Disk turbulence plays a key role in the outward transport of angular momentum that permits disk accretion. Two main sources of disk turbulence are gravitational instability (Paczynski 1978; Lodato & Rice 2004) and magnetic instability due to the magnetorotational instability (MRI; Balbus & Hawley 1991). For the magnetic instability to operate, a critical level of ionization is required for the gas to couple strongly enough to the magnetic field. In certain situations, a nonturbulent, high density, cool, weakly ionized disk layer known as a “dead zone” can become gravitationally unstable, while remaining magnetically stable (Gammie 1996). The gravitational instability can lead to turbulent heating, resulting in increased ionization of the gas. The ionization can then trigger the magnetic instability, causing a much higher level of disk turbulence and accretion, an outburst. After an outburst, the remaining disk gas cools and is replenished by accreting gas from larger radii (e.g., Armitage et al. 2001; Zhu et al. 2009b). The outburst then repeats at later times in an approximately periodic manner. Such behavior provides a possible model for FU Ori outbursts in young stars.

The gravitational instability is thought to be a natural outcome of dead zones in a layered disk, regions where the disk is nonturbulent near the disk mid-plane, but turbulent due to the MRI near the disk surface (Gammie 1996). The surface disk turbulence occurs as a consequence of surface ionization by external sources of ionization, such as cosmic rays or X-rays, that then permit the MRI to operate (Glassgold et al. 2004). In a layered region containing a dead zone, steady state accretion is generally not possible and the dead zone gains mass from some of the accretion flow near the disk surface. As mass builds up in the dead zone, it can become self-gravitating. As discussed above, the self-gravitating state is turbulent and can trigger a disk outburst.

The purpose of this Letter is to describe the disk outburst cycle in terms of transitions between steady state configurations of an accretion disk. We determine the limit cycle for the outbursts in a diagram that plots the mass accretion rate $\dot{M}$ versus disk surface density $\Sigma$. A similar approach was taken to explain the disk instability in dwarf novae (Bath & Pringle 1982; Faulkner et al. 1983). The dwarf nova thermal-viscous instability can be understood by the S-shaped curve of steady state configurations in such a diagram. In this Letter, we determine an analogous curve for outbursts in layered disks with gravitational and magnetic sources of turbulence. Our results do not conform to an S-curve because the middle portion of the S is missing. That is, unlike the S-curve case, there is a range of accretion rates for which there are no steady state accretion configurations.

In Section 2, we describe the equations for the disk model. In Section 3, we find steady state solutions to the disk equations and the resulting $M$ versus $\Sigma$ curves. In Section 4, we describe the results of numerical simulations and show that the instability cycle can be understood in terms of transitions between the stable solutions in the $\Sigma$–$M$ diagram. Section 5 contains the discussion and conclusions.

2. DISK MODEL

The surface density evolution in an accretion disk is determined by mass and angular momentum conservation (Pringle 1981). The disk model we adopt is essentially the same as Armitage et al. (2001). The main difference is that we explicitly include the effects of both MRI and self-gravity together in the disk evolution equations, while they treated these effects in separate equations. We have verified that the numerical results that they obtained can be recovered using the equations we describe below.

We consider a disk in Keplerian rotation with angular velocity $\Omega = \sqrt{GM/R^3}$ around a mass $M$ at radius $R$. The disk turbulence is modeled by the effective viscosities due to MRI and self-gravity. The surface density evolution equation is

$$ \frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left\{ \frac{\partial}{\partial R} \left[ (\nu_m \Sigma_m + \nu_g \Sigma_g) R^2 \right] \right\}, $$

where $\nu_m$ is the kinematic viscosity in the magnetic layers and $\nu_g$ is the kinematic viscosity due to turbulence associated with self-gravity of the gas that may act outside the MRI layers.
\[ \Sigma_m \text{ is the surface density in the MRI active layers and } \]
\[ \Sigma_y = \Sigma - \Sigma_m \]  
(2)
is the surface density outside the MRI active layers. \( \Sigma_y \) can contribute to the mass flux if \( v_y > 0 \).

We assume that the disk surface layers are ionized by external sources to a maximum disk density depth of \( \Sigma_{\text{crit}}/2 \) on the upper and lower disk surfaces. Therefore, the disk surface layers always contain MRI turbulence. At a given radius, we assume the disk is sufficiently thermally ionized for MRI to operate throughout the vertical extent of the disk if the mid-plane temperature is above some critical value \( T_{\text{crit}} \). Therefore, if either \( T_e > T_{\text{crit}} \) or if \( \Sigma < \Sigma_{\text{crit}} \) at some radius, then the disk is MRI-turbulent (active) at all heights, where \( T_e \) is the temperature at the disk mid-plane (central temperature). Otherwise, there is either a dead zone layer or the disk is self-gravitating in the presence of magnetic surface layers.

The temperature at the disk mid-plane \( T_e \) evolves according to the simplified energy equation (Pringle et al. 1986; Cannizzo 1993),
\[ \frac{\partial T_e}{\partial t} = \frac{2(Q_+ - Q_-)}{c_p \Sigma}. \]  
(3)
The disk specific heat for temperatures around \( 10^3 \) K is \( c_p = 2.7R/\mu \), where \( R \) is the gas constant and \( \mu \) is the gas mean molecular weight. The local heating owing to viscous dissipation is taken as
\[ Q_+ = \frac{9}{8} \Omega^2 (v_m \Sigma_m + v_y \Sigma_y). \]  
(4)
To determine the local cooling rate, we assume that each annulus of the disk radiates as a black body so that
\[ Q_- = \sigma T_e^4, \]  
(5)
where \( T_e \) is the temperature at the surface of the disk and \( \sigma \) is the Stefan–Boltzmann constant.

The kinematic turbulent viscosity in the magnetic layer is taken to be
\[ v_m = \alpha_m \frac{c_m^2}{\Omega}, \]  
(6)
where the sound speed is \( c_m = \sqrt{RT_m/\mu} \) with temperature in the magnetic layer \( T_m \). The disk is self-gravitating if the Toomre parameter \( Q < Q_{\text{crit}} \), where
\[ Q = \frac{c_s^2 \Omega}{\pi G \Sigma}, \]  
(7)
and the sound speed at the disk mid-plane is given by \( c_s = \sqrt{RT_e/\mu} \), where we approximate the temperature of the self-gravitating region that extends to the disk mid-plane as \( \approx T_e \).

The effective kinematic viscosity from the turbulence induced by the self-gravitational instability is approximated by
\[ v_y = \alpha_y \frac{c_s^2}{\Omega} \left( \frac{Q_{\text{crit}}}{Q} \right)^2 - 1 \]  
(8)
for \( Q < Q_{\text{crit}} \) and zero otherwise (Lin & Pringle 1987, 1990).

The mid-plane disk temperature for an optically thick disk in thermal equilibrium is obtained by considering the energy balance in a layered model above the disk mid-plane. One layer contains the surface density \( \Sigma_m/2 \). The other layer contains the complementary surface density \( \Sigma_y/2 \). The results are that
\[ \sigma T_e^4 = \frac{9}{8} \Omega^2 (v_m \Sigma_m + v_y \Sigma_y) \]  
(9)
and
\[ T_m^4 = \frac{3}{8} \sigma T_e^4. \]  
(10)
The optical depth to the magnetic region is
\[ \tau_m = \frac{3}{8} \kappa(T_m) \frac{\Sigma_m}{2} \]  
(11)
and the optical depth within the complementary region is
\[ \tau_y = \frac{3}{8} \kappa(T_e) \frac{\Sigma_y}{2} \]  
(12)
with
\[ \tau = \tau_m + \tau_y. \]  
(13)
Note \( \tau_y \) is defined even in a dead zone layer (\( v_y = 0 \)) with \( \Sigma_y \) defined by Equation (2). We adopt the simplified opacity of Armitage et al. (2001):
\[ \kappa(T) = 0.02 T^{0.8} \text{ cm}^2 \text{ g}^{-1}. \]  
(14)
The energy equation (3) in a steady state has the solution
\[ \sigma T_e^4 = \frac{9}{8} \Omega^2 (v_m \Sigma_m + v_y \Sigma_y). \]  
(15)
From Equations (9)–(15), we obtain an expression for the cooling function
\[ Q_- = \sigma T_e^4 = \tau^{-1} \left( \sigma T_e^4 \left( \frac{9}{8} \Omega^2 (v_m \Sigma_m + v_y \Sigma_y) \right) \right). \]  
(16)
We apply this cooling function to Equation (3), even when the disk is not in thermal equilibrium. This means that we do not attempt to treat the cooling during the transitions consistently.

3. STEADY STATE DISKS AND STATE TRANSITIONS

We consider the disk to be supplied by material at a constant mass input rate into an outer region. In a steady state, the mass flux through the disk is constant in radius interior to the mass input region. We determine the relationship between the mass flux \( \dot{M} \) and the disk surface density \( \Sigma \) at some radius.

In a steady state, far from radius of the central object, the surface density equation (1) has the solution
\[ \dot{M} = 3\pi (v_m \Sigma_m + v_y \Sigma_y). \]  
(17)
From Equations (15) and (17), we find the steady state surface temperature of the disk:
\[ T_e = \left( \frac{3\dot{M} \Omega^2}{8\pi \sigma} \right)^{\frac{1}{4}}. \]  
(18)
Note that \( T_e \) is a function of the total accretion rate \( \dot{M} \).

We sketch typical solutions in the \( \Sigma-M \) plane at a given radius in Figure 1 to illustrate their principal properties. We show the various branches of disk solutions, as described below. In Section 4, we consider particular numerical solutions.
3.1. MRI Disk Branches

If $T_c > T_{\text{crit}}$ or $\Sigma < \Sigma_{\text{crit}}$ at a given radius, then the disk is assumed to be fully MRI-turbulent, that is, magnetically turbulent at all heights ($\Sigma_g = 0$). The disk surface density in this case simply follows from Equation (17):

$$\dot{M} = 3\pi v_m \Sigma_m. \quad (19)$$

At a fixed radius, Equations (6), (10), (18), and (19) imply that $\dot{M} \propto \Sigma_m^{21/11}$.

3.2. Gravo-Magneto (GM) Disk Branch

Over a certain range of accretion rates, there exists a branch of steady state solutions where the disk is gravitationally unstable and has MRI active surface layers with $\Sigma_m = \Sigma_{\text{crit}}$. We numerically determine $M(\Sigma)$ over a range of $\Sigma$ for which $Q < Q_{\text{crit}}$ and $T < T_{\text{crit}}$. Equations (9) and (17) are numerically solved together with auxiliary equations (6)–(8), (10)–(14), (15), and (18) to obtain this function.

3.3. Dead Zone Branch

Between the lower MRI and the GM steady state branches there are disk configurations that involve dead zones. The dead zone is a nonturbulent mid-plane layer that lies vertically between MRI actively accreting surface layers. Unlike the other branches we have considered, this branch is not in steady state and therefore not described by steady state solutions to the disk equations. It is plotted as a dashed line in the lower portion of Figure 1. If a disk lies along this line, it will evolve to the right, as the dead zone gains mass, while the disk accretion rate is pinned to the rate provided by MRI turbulence that resides in surface layers of fixed density $\Sigma_{\text{crit}}$. The mass gain of the dead zone is produced by some of the accretion flow in the disk surface layers that becomes incorporated into the dead zone. Over time the dead zone gains sufficient mass for the disk to become self-gravitating. The disk state then enters the GM branch.
3.4. Accretion Rates with No Steady State

There exists a range of mass accretion rates for which no steady state solution exists. This range is indicated by the lower shaded region in Figure 1. This gap comes about because the GM branch terminates at an \( \dot{M} \) for which \( T_c = T_{\text{crit}} \), while the higher MRI branch starts at a larger value of \( \dot{M} \) that is also at \( T_c = T_{\text{crit}} \). As we will see in Section 4, a disk that has an accretion rate lying in this range will quickly transition to either of these steady state branches. As a consequence of such transitions, the disk undergoes outbursts.

3.5. Global Steady State

We are considering a disk that undergoes accretion from an external source at a constant accretion rate. The \( \Sigma - \dot{M} \) diagram applies at each disk radius. The range of unstable accretion rates shown as the lower shaded region in Figure 1 varies with radius. For a disk to be globally stable against GM outbursts, there should be no radius at which the disk accretion rate lies within this range.

3.6. Thermal-viscous Instability

The case of outbursts involving the thermal-viscous instability occurs at higher disk temperatures and accretion rates than the GM instability. This regime is sketched as the well-known S-curve in the upper portion of Figure 1. The instability occurs along the middle portion of the S-curve (the upper dotted line within the upper shaded region). This middle portion consists of unstable steady state solutions that occur between the other two stable branches. The situation with the GM outbursts is different because there are no intermediate unstable steady state solutions, i.e., there are no steady state solutions in the lower shaded region. For a given accretion rate, it is possible for the disk to develop the thermal-viscous instability at small radii and the GM instability further out.

4. TIME EVOLUTION

We analyze the time evolution of a disk subject to only the gravo-magnetic instability. We consider a case with \( R = 3 \) AU, \( M = 1 M_\odot \), \( \dot{M} = 10^{-6} M_\odot \text{ yr}^{-1} \), \( Q_{\text{crit}} = 2 \), \( \Sigma_{\text{crit}} = 200 \text{ g cm}^{-2} \), \( T_{\text{crit}} = 800 \text{ K} \), \( \mu = 2.3 \), and \( \alpha = 0.01 \). The steady state solutions plotted in Figure 2 show that no steady state branch exists at this mass accretion rate. Therefore, we expect outbursts in the accretion rate on to the central star.

We time-integrate the evolution equations (1) and (3), together with Equations (4) and (16), for a disk that extends from \( R = 2.33 \times 10^{-3} \) AU to \( R = 40 \) AU around a solar mass star (see, e.g., Armitage et al. 2001; Martin & Lubow 2011). In this calculation, we ignore the effects of the thermal-viscous instability shown by the S-curve in the upper part of Figure 1. The grid consists of 120 points distributed uniformly in \( R^2 \). Material is added to the disk at a constant rate of \( \dot{M} = 10^{-6} M_\odot \text{ yr}^{-1} \) at a radius of 35 AU. At the inner radius, we impose a zero torque boundary condition, so that there is an inward flow of gas out of the grid and toward the central star. The flow is prevented from leaving the outer boundary by means of a zero radial velocity outer boundary condition.

The disk undergoes outbursts that reach a limit cycle after a few outbursts. In Figure 2, we show the disk evolution over a limit cycle. The cycle proceeds in a counterclockwise sense on the diagram. Along the low horizontal section of the cycle,
the disk is in the dead zone state, as discussed in Section 3.3. The surface density increases and the disk evolves to the right. The disk then becomes self-gravitating with \( Q < Q_{\text{crit}} \) as a consequence of this accumulated dead zone mass. The disk state then moves along the GM branch until it reaches the critical temperature for the MRI to act, \( T_{\text{crit}} \). The disk then becomes well enough ionized for MRI to act and the disk state jumps up to the fully active MRI branch. The upward jump corresponds to the start of a disk outburst that proceeds at a higher accretion rate. The mass then drains out of the disk and the disk cools. As a result, the path then moves down and to the left along the MRI branch. Once the temperature \( T_{\text{e}} \) falls below the critical value \( T_{\text{crit}} \), the path moves downward. The disk once again forms a dead zone and the process repeats itself.

5. DISCUSSION AND CONCLUSIONS

We have explained the GM outbursts triggered in a layered accretion disk by means of a \( \Sigma - M \) diagram, along the lines of the S-curve explanation for the thermal-viscous instability in dwarf nova outbursts. We determined steady state disk solutions that compose branches in the diagram. At a given radius, there are two types of steady state solutions (see Figure 1). One type, labeled MRI, involves a disk that is magnetically unstable and turbulent at all heights. This type describes two branches of solutions, one at low and the other at high accretion rates. Another type, labeled GM, is gravitationally unstable and turbulent near the mid-plane and is magnetically unstable and turbulent only near the disk surface. This type describes one branch of solutions. At a given radius, there may be a range of disk accretion rates for which there are no steady state solutions, the lower shaded region in Figure 1. This range of accretion rates varies with radius. If there is a radial zone within a disk that has an accretion rate lying in this range, then the disk will undergo outbursts in that zone. The outbursts are understood in terms of a limit cycle in the \( M(\Sigma) \) diagram as shown in Figure 2. The cycle tracks along steady state branches of the disk solutions and along the “dead zone” branch. The cycle jumps between states of low and high accretion rates at the initiation and termination of an outburst. The outburst model may be applied to accretion disks of all size scales from circumplanetary to protoplanetary to active galactic nucleus disks.

We have made several simplifications in our model that will be investigated further in future work. We have ignored the thermal-viscous instability that produces the dwarf nova outbursts that is sketched in the upper portion of Figure 1. We adopted a simplified opacity that does not produce this curve in the numerical models. The thermal-viscous instability could be triggered at smaller radii than the GM instability. The interaction between the two instabilities may produce some interesting behavior applicable to T Tauri stars. The role of the thermal-viscous instability and its S-curve has been considered by Bell & Lin (1994) and Zhu et al. (2009b).

Following previous work (Armitage et al. 2001), we have taken a rather small value for the viscosity in the active regions of \( \alpha_m = 0.01 \) and a large value for the depth of the surface magnetic turbulence of \( \Sigma_{\text{crit}} = 200 \text{ g cm}^{-2} \). Some general considerations suggest that \( \alpha \sim 0.1 \) (King et al. 2007). We note that if the \( \alpha_m \) were higher, then the disk would be more stable against the GM outbursts. Similar results to those plotted in Figure 2 are obtained for \( \alpha_m = 0.1 \), but with \( \Sigma_{\text{crit}} = 20 \text{ g cm}^{-2} \).

We have assumed that the turbulent viscosity associated with self-gravity, \( \alpha_g \), depends on the Toomre \( Q \) parameter. However, it has been shown in several papers that \( \alpha_g \) actually depends on the cooling rate (Gammie 2001; Cossins et al. 2009). The final disk \( Q \) value is found to be self-regulated at a value \( \sim 2 \).

We have neglected external heating of the disk and assumed that the accretional heating dominates. For the parameters associated with the GM instability in Figure 2, this assumption may be valid. However, more generally external heating should be considered.

The use of a fixed value for the maximum depth of nonthermal ionization of the outer magnetic layers, \( \Sigma_{\text{crit}} \), is an approximation. A more accurate approach to apply a critical magnetic Reynolds number is required for the MRI to operate, along the lines of Matsumura & Pudritz (2003). In addition, we have taken a single critical temperature for thermal ionization \( T_{\text{crit}} \) at the disk mid-plane temperature above which the disk becomes MRI turbulent at all heights. Instead, there may be a thin turbulent layer near the disk mid-plane that develops at this mid-plane temperature that increases in thickness at higher temperatures (Zhu et al. 2009a).

Several authors have suggested that there is a small but non-zero turbulent viscosity in the dead zone that develops as a response to the turbulence driven in the magnetic surface layers (Fleming & Stone 2003; Turner & Sano 2008). The range of steady state disk flow solutions is then increased (e.g., Terquem 2008). To some extent, this effect will stabilize the outbursts as material can flow through the (nearly) dead zone. However, there may be issues with the model if the rate of flow through the dead zone approaches that through the magnetic surface layer. Such topics can be explored within the framework we have described.

We thank Jim Pringle for advice and encouragement. R.G.M. thanks the Space Telescope Science Institute for a Giacconi Fellowship. S.H.L. acknowledges support from NASA grant NNX07AI72G. We thank the referee for comments.

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