Sonoluminescence: The Superradiance Paradigm

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Abstract
In sonoluminescence sound waves in the fluid are converted into UV-photons. In this letter we put forward a mechanism based on superradiance that operates this conversion. The predicted energies of emitted light turns out to lie in the range of 1-10 eV, in good agreement with experiment. Furthermore our paradigm hints why the effect hinges so crucially on the cavitation of a noble gas

1 Introduction
Sonoluminescence – the effect whereby a gas cavitating in water whose bubble’s size is driven by ultrasound waves glows – has been puzzling the mind of scientists since the thirties (for a recent review see [1] and for a general account [2]). The bubble starts expanding from an ambient radius of 5µm to 50µm and then collapses at about about 4 times the speed of sound to 0.5µm. Then, within 50ps of the minimum radius a burst of light containing

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about 1 million photons with energies ranging from 6.5 eV down to 2.0 eV is emitted. So far, there is not a satisfactory understanding on the mechanism involved in the conversion of phonons in the liquid into UV-photons [1]. The most viable candidate is the shock wave model [1], whereby at late stages of the bubble’s collapse, its surface moves supersonically, producing a shock [1]-[8]. In this scenario, the fast collapse heats the gas in the interior of the bubble to temperatures of the order $T \sim 10^5$. The gas is ionized and the hot ionized plasma emits via Bremstrahlung. For $M = v/c \sim 4$ the emitted photons lie in the experimental range. Less viable models involve the emission of light through the electronic discharge owing to a charge separation inside the plasma [1], Unruh-Davies radiation from the surface [10, 11], and the bursting of light that follows the sudden cracking of a putative ice produced by the high pressures present inside the bubble [12]. In the Unruh-Davies type calculation too few photons are produced. This difficulty seems to be circumvented by a recent Casimir energy inspired mechanism, based on the modification of the electromagnetic action [13]. It is essential to remark that no sonoluminescence is produced unless the gas inside the bubble is doped with a noble gas [1]. Unfortunately, none of the proposed mechanisms can address this fact.

Superradiance is the amplification of radiation by rotating absorbing media. Zel’dovich was the first to consider this effect in the context of an electromagnetic field bouncing off an ohmic cylinder rotating with angular velocity $\Omega$ [14] and also of a scalar field [15]. Misner considered a similar effect for quanta scattered by a rotating black hole [16]. In both instances the condition $\omega - m\Omega < 0$ must be satisfied, where $\omega$ and $m$ are the frequency and azimuthal angular momentum of the emitted quanta. It was already known by Ginzburg and Frank [17] that superradiance and Cherenkov emission are interchangeable concepts and rely upon the fact that the modulus of the velocity $\vec{v}$ of the source of radiation exceeds the phase velocity of wave in the medium, in which case the frequency and wave number of the wave satisfy $\omega - k \cdot \vec{v} < 0$. Apart from black holes, there are a myriad of instances of superradiant emission: the onset of viscous flow inside superfluid through the excitation of phonons, transition radiation, to name just a few [18]. A particularly enlightening example for our purposes is the spontaneous emission of sound waves by a homogeneous medium that has a tangential discontinuity in the velocity field [19]. The reflection coefficient of a sound wave incident upon the surface of discontinuity in the velocity field has poles for some values of the incidence angle, which are physically interpreted as the angles where sound waves are spontaneously emitted. The onset of emission is governed by the condition $M = v/c > 1$ (actually $M > 2$, but the tighter bound stems from a kinematical rather than dynamical constraint). The above scenario could easily explain the emission of sound waves during the supersonic contraction
of the bubble, but how could this possibly relate to the emission of light which apparently requires superluminal motion?

In the scenario we put forward, the formation of a shock wave inside the bubble is pivotal. Once it is formed, the gas passing through the shock is compressed, that is to say, a density discontinuity is generated across the shock [19] which, in turn, leads to a discontinuity in the dielectric permittivity (we assume that the permeability \( \mu = 1 \)). In analogy with the previous example, the reflection coefficient for originally incoming waves also display poles at the denominator for given frequencies. These are responsible for the (classical) stimulated emission of light. In what follows we consider a somewhat simplified model, a plane shock moving with constant velocity.

## 2 The model

Our first step is to study the propagation of electromagnetic waves in moving media. The appropriate equations for nonrelativistic motion contains a correction of order \( v/c \) to Maxwell’s equations. Writing the velocity as \( \vec{v} = v\vec{n} \) where \( \vec{n} \) is the normal vector pointing, say, to the right [20]:

\[
\vec{\nabla} \cdot \vec{B} = 0; \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]

\[
\vec{\nabla} \cdot \vec{D} = 0; \quad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}
\]

and

\[
\vec{D} = \epsilon \vec{E} + \alpha \vec{n} \times \vec{H}
\]

\[
\vec{B} = \vec{H} - \alpha \vec{n} \times \vec{E}
\]

where \( \alpha = (\epsilon - 1)v \). Taking a plane wave \( \exp(i(\vec{k} \cdot \vec{x} - \omega t)) \) it follows that,

\[
A \vec{E} + B\vec{n} \times \vec{H} = 0,
\]

\[
B\epsilon \vec{E} + A\vec{n} \times \vec{H} = 0
\]

where \( A = \epsilon \omega^2 - k^2 - \alpha k \omega \) and \( B = \alpha \epsilon \). For compatibility \( A^2 - \epsilon B^2 = 0 \). Thus, to the leading order

\[
\vec{E} = \mp \frac{1}{\sqrt{\epsilon}} \vec{n} \times \vec{H} \quad \text{with} \quad |k| = (\sqrt{\epsilon} \pm \alpha)\omega
\]

The \( \mp/\pm \) signs refer to right and left moving waves, respectively. In the picture below the shock is moving to the right, corresponding to the collapse of the bubble. The above equations must be supplement by the boundary conditions. To the same order in \( v/c \) [20] we have continuity of the normal components at the shock

\[
\Delta \vec{D}_n = 0; \quad \Delta \vec{B}_n = 0,
\]
but
\[ \tilde{n} \times \Delta \tilde{E} = v \Delta \tilde{B}; \quad \tilde{n} \times \Delta \tilde{H} = -v \Delta \tilde{D} \] (7)

Because \( \mu = 1 \), \( \Delta \tilde{B} = \Delta \tilde{H} = O(v/c) \) and \( \Delta \tilde{D} = \Delta(\epsilon \tilde{E}) \). Thus, up to first order in \( v/c \) we have:
\[ \Delta \tilde{E} = 0 \]
\[ n \times \Delta \tilde{H} = -v \Delta(\epsilon \tilde{E}) \] (8)

Let us now consider a (right-moving) wave propagating from the exterior of the shock to its interior. Let \( (\tilde{E}_0, \tilde{H}_0), (\tilde{E}_1, \tilde{H}_1) \) and \( (\tilde{E}_2, \tilde{H}_2) \) represent the incoming, reflected and transmitted waves, respectively. Let us further assume that the incidence is normal to the shock (the effect subsists for non-normal incidence too, but is less intense). Under these circumstances \( \tilde{E}_2 = \tilde{E}_0 + \tilde{E}_1 \), and substituting into eq. (8), it follows that
\[ n \times (\tilde{H}_2 - \tilde{H}_1 - \tilde{H}_0) = -v \left( (\epsilon_2 - \epsilon_1) \tilde{E}_0 + (\epsilon_2 - \epsilon_1) \tilde{E}_1 \right) \] (9)

In the above equation, due care of redshifts of the reflected and transmitted waves was taken into account in the evaluation of the electric permeabilities \( \epsilon_1 = \epsilon_1(\omega), \epsilon_1 = \epsilon_1(\tilde{\omega}), \epsilon_2 = \epsilon(\omega_2) \), where the frequencies are connected via the relations:
\[ \omega - kv = \tilde{\omega} + \tilde{k}v = \omega_2 - k_2v. \] (10)

With the aid of the dispersion relation [eq. (9)], to the leading order \( O(v/c) \), it follows that:
\[ \tilde{\omega} = \omega \left[ 1 - 2v \sqrt{\epsilon_1(\omega)} \right], \quad \omega_2 = \omega \left[ 1 - \frac{v}{\sqrt{\epsilon_1(\omega)} - \sqrt{\epsilon_2(\omega)}} \right]. \] (11)

Further use of eq.(5) yields \( n \times (\tilde{H}_2 - \tilde{H}_1 - \tilde{H}_0) = (-\sqrt{\epsilon_2(\omega_2)} \tilde{E}_2 - \sqrt{\epsilon_1(\tilde{\omega})} \tilde{E}_1 + \sqrt{\epsilon_1 \tilde{E}_0}) \) and the following reflection amplitude:
\[ \frac{E_1}{E_0} = \frac{\sqrt{\epsilon_1(\omega)} - \sqrt{\epsilon_2(\omega_2)} + v(\epsilon_2(\omega_2) - \epsilon_1(\omega))}{\sqrt{\epsilon_2(\omega_2)} + \sqrt{\epsilon_1(\omega_1)} - v(\epsilon_2(\omega_2) - \epsilon_1(\omega_1))}. \] (12)

Next, with the aid of eq.(11), we expand this amplitude to first order in \( v/c \), which yields
\[ \frac{E_1}{E_0} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2} + v \left( \epsilon_2 - \epsilon_1 + \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{2\sqrt{\epsilon_2}} \omega \frac{\partial \epsilon_2}{\partial \omega} \right)}{\sqrt{\epsilon_2} + \sqrt{\epsilon_1} - v \left( \epsilon_2 - \epsilon_1 + \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{2\sqrt{\epsilon_2}} \omega \frac{\partial \epsilon_2}{\partial \omega} + \frac{\omega}{\partial \omega} \right)}. \] (13)

Spontaneous emission is associated with the poles of this reflection amplitude. With \( \epsilon \sim 1 \) and the velocity of the shock of the order of the speed of sound, \( v \sim 10^{-5} \), makes impossible to find a root of the denominator
\[ f(\omega) = \sqrt{\epsilon_2} + \sqrt{\epsilon_1} - v \left( \epsilon_2 - \epsilon_1 + \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{2\sqrt{\epsilon_2}} \omega \frac{\partial \epsilon_2}{\partial \omega} + \frac{\partial \epsilon_1}{\partial \omega} \right). \] (14)
if it were not for the following very fortunate fact. As a consequence of having all electronic shells filled up, noble gases do not show absorption lines for a broad frequency band, that is to say, are transparent for $\omega_1 << \omega << \omega_2$. It follows from the Kramers-Kronig relation that in this interval the medium behaves like a plasma and the corresponding expression for the permittivity is (20):

$$\varepsilon(\omega) = a - \left(\frac{\omega_0}{\omega}\right)^2$$

where $\omega_0 = \frac{4\pi ne^2}{m}$ with $n$ being the electronic density and $e, m$ the electron charge and mass, respectively. Consequently, the permittivity vanishes in this interval for some frequency $\tilde{\omega} = a^{-1/2}\omega_0$ (20). Presuming that we are dealing with a weak shock wave (small discontinuities in pressure $\delta p$, number density $\delta n$ and normal velocity $\delta v$) it follows that:

$$\varepsilon_2 \approx 2a\left(\frac{\omega}{\tilde{\omega}} - 1\right)$$

$$\varepsilon_1 \approx 2\frac{\delta n}{n} + 2a\left(\frac{\omega}{\tilde{\omega}} - 1\right)$$

where we omitted terms of order $\frac{\delta n}{n}(\frac{\omega}{\tilde{\omega}} - 1)$. Inserting these approximations into $f(\omega)$ we plotted $f(\omega)$ for a wide range of values for $\nu \sim 10^{-5}$ and $10^{-2} < \frac{\delta n}{n} < 10^{-4}$. Notice that for all cases the root of $f(\omega)$ lies very close to $\tilde{\omega}$.

Estimating the emission frequency. For a gas at pressures of 1 atm and room temperature, $n \sim 10^{20}$ electrons/cm$^3$ which gives $\omega_0 \sim 10\text{eV}$, in good agreement with the experimental value of 6.5eV for the high energy cutoff. The broad band spectrum could then be due to the change in velocity of the bubble, the change in the density during the collapse and also on the dependence of the poles on the incidence angle (for non-normal incidence). Note that our model does not require high temperatures inside the bubble $T \sim 10^5$. This is a most welcome result because such high temperatures would cause the bubble to emit via bremsstrahlung for any gas. Furthermore, the presence of a ionized plasma would imply on cyclotron emission in the presence of a sufficiently large magnetic field. For $B = 20T$ the cyclotron period is 1ps, which is much shorter than the present bound on the flashwidth and no change in the emitted light was observed [1].

Unfortunately our model does not address the following important issue. Sonoluminescence was originally discovered for air cavitating in water. When trying to reproduce the effect, through bubbles containing a mixture of $N_2$ and $O_2$ null results were obtained. Only after doping this mixture with $Ar$ it was possible to make the bubble glow. It is nowadays known that the effect does not occur for any mixture of diatomic gases. It does occur for noble gases bubbles or for a mixture of diatomic gases doped with some noble gas. The presence of non-noble gases produces absorption of light in
the frequency range of interest and void our argument. So it seems that
the present proposal needs to be supplemented by some mechanism that
keeps the gas surrounding the shock to be transparent in a wide range of
frequencies. Whether the superradiance paradigm is a viable alternative to
the bremsstrahlung model can one be answered after our model is intertwined
with the hydrodynamical theory.

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