Preheating with higher dimensional interaction

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Particle production caused by the oscillation after inflation is important since it explains reheating after inflation. On the particle theory side, we know that effective action may have additional higher dimensional terms (usually called non-renormalizable terms) suppressed by the cut-off scale. Moreover, interaction between inflaton and so-called moduli field will be higher dimensional. Therefore, if such higher dimensional interaction is significant for resonant particle production, one cannot avoid the effect in preheating study. We explicitly calculated the required number of oscillation for the energy transfer. Consequently, cosmological history of an oscillating field and the moduli problem can be reconsidered.

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I. INTRODUCTION

If the Universe starts with inflation, elementary particles of the Universe should be created after inflation. According to the inflationary theory, reheating after inflation, which converts the vacuum energy into radiation, is responsible for the creation of those particles. At the beginning of that process, inflaton field is supposed to start oscillation, and then reheating occurs due to particle production caused by the oscillating field. The oscillation causes production of particles, which interact with each other and finally they come to be in thermal equilibrium. The mechanism of particle production after inflation has been discussed by many authors [1]. In this paper we consider a case in which reheating starts with a regime called preheating [2]. In Ref.[2], assuming a renormalizable interaction, it has been shown that during that period the energy transfer from the inflaton oscillation to other Bose fields and particles is extremely efficient. In this paper we are focusing on the initial stage, where preheating occurs and particles are produced. Cosmological evolution of the Universe after preheating could be different from the one after perturbative reheating.

In contrast to the generality of the mechanism, the original scenario of preheating is based on renormalizable interaction, which restricts the application of the idea only to the fields that have renormalizable interaction. Therefore, although many theoretical models of particle physics may have fields that do not have renormalizable interaction with inflaton, preheating has been applied only to those “special” cases. Although not mandatory, inflationary model may expect amplitude as large as the Planck scale [3, 4] or the scale of Grand Unified Theory (GUT).

Before discussing the calculational details, it will be helpful to set out naive questions that may arise when one considers the case. An important point is that there has been no paper discussing higher dimensional interaction for preheating. Obviously such interaction has been neglected, but the reason is not quite obvious.

1. One might claim that large amplitude cancels the suppression of the cutoff. On the other hand, one might wonder whether such speculation can be applied when particle production occurs near the origin.

2. One might think that preheating could be negligible when the coupling is very weak. Since higher dimensional interaction is suppressed by the cut-off, it is usually “very weak” compared with renormalizable interaction. Then it could be natural to conclude that higher dimensional interaction is negligible.

3. In contrast to 1, one might have an intuition that particle production will be trivial and there will be no significant difference from the standard scenario. Then, one might simply assume that only the critical frequency where the infrared instability band ends will change.

Also, if one focuses on particle production at the enhanced symmetric point (ESP), one might have an impression that preheating might not work since higher dimensional interaction gives $\delta m \approx 0$, where $\chi$ is the scalar field that is supposed to be produced. This could not be a problem since the adiabatic condition is broken when the inflaton moves a small distance away from the ESP, but obviously the situation is not trivial.

In this paper we carefully consider these naive speculations and intuitions. We also evaluate numerical co-
II. WHAT ARE THE DIFFERENCES?

In this section we carefully review the standard scenario of preheating to show which equations have to be modified. The starting point of the “standard” preheating scenario is the Lagrangian

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} g^2 \phi^2 \chi^2. \]  

(1)

Assuming homogeneous background, classical equation of \( \phi \) obeys

\[ \ddot{\phi} + 3H \dot{\phi} + (m^2 + g^2 \langle \chi^2 \rangle) \phi = 0. \]  

(2)

Here \( \langle \chi \rangle \) is initially negligible if \( \phi \)-oscillation starts with large amplitude \( \Phi_0 \). Then, interaction between the classical field \( \phi \) and the quantum scalar field \( \chi \) is considered. One can write the equation for modes with physical momentum \( k \) in the form

\[ \ddot{\chi}_k + (k^2 + g^2 \Phi^2 \sin^2(mt)) \chi_k = 0 \]  

(3)

This equation describes an oscillator with a frequency \( \omega^2 = k^2 + g^2 \Phi^2 \sin^2(mt) \). If \( \Phi \) is not time-dependent (i.e., when \( \Phi = \Phi_0 \)), the equation can be written as a Mathieu equation with \( A_k \equiv k^2 + 2q, q \equiv g^2 \Phi^2 \sin^2(mt) \). Mathieu equation gives so-called “broad resonance” when \( q \gg 1 \). For very small \( \phi \) the change in \( \omega \) ceases to be adiabatic. The standard condition necessary for particle production is

\[ \frac{\dot{\omega}}{\omega^2} > 1. \]  

(4)

Therefore, the typical momenta for the resonance is estimated as \( g^2 \dot{\phi} < \omega^3 \) for \( \phi \approx m \Phi \), or

\[ k^2 < k_*^2 \equiv (g^2 \phi m \Phi)^{2/3} - g^2 \phi^2. \]  

(5)

The maximum \( \phi \) is (which appears when \( k = 0 \))

\[ \phi < \phi_* = \frac{1}{2} \sqrt{\frac{m \Phi}{g}}. \]  

(6)

Thinking about higher dimensional interaction, which (in the simplest case) can be written as \( \sim g^2 \dot{\phi}^4 \chi^2 / \Lambda^2 \), it is easy to find that Eq. (3) will be replaced by

\[ \ddot{\chi}_k + \left( k^2 + g^2 \Phi^4 \sin^4(mt) \right) \chi_k = 0. \]  

(7)

Although \( \sin^2(mt) \) has been replaced by \( \sin^4(mt) \), Floquet theory predicts (rather naively) resonance and exponential growth. Therefore, intuitive argument suggests that broad resonance will happen for higher dimensional interaction if the Universe is not expanding. On the other hand, quantitative calculation of the resonant particle production in an expanding Universe is not obvious. Our paper aims to find both analytical and numerical estimation of the effect in the expanding Universe.

III. MODEL AND CALCULATION

In this section we carefully follow the calculation in Ref. [2, 8] so that the reader can easily compare our result with the usual calculation. Our formulation is carefully prepared so that one can easily figure out why preheating is efficient even though the interaction is higher dimensional. For that purpose we start with an obvious case using a rather special approximation, which is called “quadratic approximation” in this paper.

Numerical coefficients are carefully evaluated so that it does not ruin the accuracy of the estimation. More detailed argument on application to the effective action of GUT will be found in Ref. [8]. Note that our calculation has direct application to those phenomenological models.

A. Rather trivial example: quadratic approximation

Our aim in this section is to find an obvious example of higher dimensional interaction that can be examined using the standard method of preheating. Although the approximation is valid only in a very narrow range of the parameter space, the result gives a convincing lower limit of the particle production. We are going to extend the analysis in the next section, using more rigorous calculation of Ref. [8].

First, consider a model of parametric resonance described by an oscillating field \( \phi \) and a particle \( \chi \) with the potential and the interaction:

\[ V(\phi, \chi) = \frac{1}{2} m_\phi^2 |\phi|^2 + \frac{g^2}{2} |\phi|^4 \chi^2. \]  

(8)

Around \( j \)-th zero crossing at \( t = t_j \), we assume sinusoidal oscillation \( \phi(t) = \Phi_j \sin[m_\phi(t-t_j)] + i \mu_j \), which leads to

\[ |\phi(t)|^2 = \Phi_j^2 \sin^2[m_\phi(t-t_j)] + \mu_j^2, \]  

(9)

where \( \Phi_j \) is the amplitude of the oscillation and \( \mu_j \) is the impact parameter (\( \mu_j \ll \Phi_j \)). Then one can linearize the
interaction using \(\sin[m_\phi(t-t_j)] \simeq m_\phi(t-t_j) \ll 1\), and disregard higher terms. More explicitly, one will find

\[
|\phi|^4 = \left[ \Phi^2 \left\{ m_\phi(t-t_j) - \frac{1}{3!} m_\phi^3(t-t_j)^3 + \ldots \right\}^2 + \mu_j^2 \right]^2 = \mu_j^4 + 2 \mu_j^2 \Phi^2 m_\phi^2(t-t_j)^2 + \Phi^4 m_\phi^4(t-t_j)^4 + \ldots (10)
\]

where quartic term \((t-t_j)^4\) can be neglected when \(\phi_\pm(t-t_j) < \sqrt{2} \mu_j/\Phi_j\). Since the velocity at the bottom of the potential is \(\phi_\pm \simeq m_\phi \Phi_j\), the condition \(m_\phi(t-t_j) \ll \sqrt{2} \mu_j/\Phi_j\) is equivalent to \(|\Re \phi_\pm| < \sqrt{2} \mu_j\). For our estimation we will take \(\mu_j\) modestly large so that the quadratic approximation will be conceivable. Since a large \(\mu_j\) will give an exponential suppression of the number density, \(\mu_j\) has to be chosen carefully. Our choice of \(\mu_j\) will be discussed in the last part of this section. Explicit form of the higher dimensional interaction term is

\[
\frac{g^2}{2} |\phi(t)|^4 \chi^2 \simeq \frac{g^2}{2} \Phi^2 m_\phi^2(t-t_j)^2 + \mu_\phi^4 \chi^2. (11)
\]

Remember that for a renormalizable coupling \((g^2\phi^4/\chi^2)/2\), one will find

\[
\frac{g^2}{2} |\phi(t)|^2 \chi^2 \simeq \frac{g^2}{2} \left\{ \Phi^2 m_\phi^2(t-t_j)^2 + \mu_\phi^2 \right\} \chi^2, (12)
\]

which does not have the suppression \((\sim \mu^2/\Lambda^2)\) in front of the effective mass term.

For a flat Friedmann background with cosmological scale factor \(a(t)\), the equation of motion of the field \(\chi(x, t)\) in Fourier space of comoving momentum \(k\) is written for \(\chi_k(t)\) as

\[
\ddot{\chi}_k + 3H \dot{\chi}_k + \left( \frac{2g^2 \mu^2 \Phi^2 m_\phi^2(t-t_j)^2 + g^2 \mu^4 \chi^2}{\Lambda^2} + \frac{k^2}{a^2(t)} \right) \chi_k = 0. (13)
\]

The physical momentum \(p = k/a(t)\) coincides with \(k\) for Minkowski space. We can eliminate the friction term \(3H \dot{\chi}_k\) by defining \(X_k \equiv a^{3/2} \chi_k\). Then, one can rewrite the equation in a simpler form

\[
\dddot{X}_k + \left( P_j^2(t-t_j)^2 + A_j + \frac{k^2}{a^2(t)} \right) X_k = 0, (14)
\]

where \(P_j^2 \equiv \sqrt{2} g \mu \Phi_j m_\phi / \Lambda\). Changing the time variable as \(\tau \equiv P_j(t-t_j)\), the equation gives

\[
X_k'' + (\kappa_j^2 + \tau^2) X_k = 0, (15)
\]

where prime denotes derivatives with respect to \(\tau\). \(\kappa_j\) is defined by

\[
\kappa_j^2 \equiv \left( \frac{k}{k_{*, j}} \right)^2 + \frac{a(t_j)^2}{k_{*, j}^2} A_j, (16)
\]

where \(k_{*, j} \equiv a(t_j) P_j\). The equation of motion contains terms proportional to \((H^2 X_k + \frac{\dot{\phi}}{\phi} X_k)\), which are included in \(A_j\). One will see that these terms can be neglected after all, since \(k^2/a^2 \gg H^2, \dot{a}/a\) at subhorizon scales \(\mathbb{E} \mathbb{R}\). The remaining term will be \(A_j \simeq g^2 \mu^4 / \Lambda^2\).

Eq. (15) can be solved as the well-known problem of wave scattering at a negative parabolic potential, which leads to

\[
n_{k, j} = e^{-\pi \kappa_j^2} + (1 + 2e^{-\pi \kappa_j^2}) n_{k, j-1} - 2e^{-\frac{\kappa_j^2}{4 \Lambda^2}} \sqrt{1 + e^{-\pi \kappa_j^2}} \sqrt{n_{k, j-1}(1 + n_{k, j-1})} \sin \theta_j, (17)
\]

where the phase \(\theta_j\) causes stochastic growth of the occupation number \(n_k\). According to Ref. \(\mathbb{R}\), the stochastic contribution can be averaged to zero after all, since \(\theta_j\) is uniformly distributed. Then one will find the iterative expression for the occupation number \(n_k\) as

\[
\left( n_{k, j} + \frac{1}{2} \right) = (1 + 2e^{-\pi \kappa_j^2}) \left( n_{k, j-1} + \frac{1}{2} \right). (18)
\]

Since the occupation number becomes large soon after \(\phi\) starts oscillation, the equation shows that the mode occupation number \(n_k\) increases exponentially as long as the mode satisfies \(\pi \kappa_j^2 < 1\).

Although the formalism might look completely the same as the conventional calculation, a significant difference appears in the definition of \(k_\ast\), which defines the threshold of the particle production. If the interaction is renormalizable, one will find \(k_{*, j}^2 / a(t_j)^2 \sim g_n \Phi_j \sim g \phi\), while for higher dimensional interaction one will find \(k_{*, j}^2 / a(t_j)^2 \sim \sqrt{2} g \mu / m_\phi \Phi_j / \Lambda\). In contrast to the case with \(\mu \simeq 0\), \(\omega\) does not vanish at the ESP.

Let us examine the validity of the above calculation. The condition that is needed for the quadratic approximation is \(|\Re \phi_\pm| < \sqrt{2} \mu_j\). At the same time, we need to consider the adiabatic condition \((\omega_\phi / \omega_\tau)^2 < 1\), where \(\omega_\phi = \sqrt{\kappa^2 + g^2 |\phi|^4 / \Lambda^2}\), which is violated when non-adiabatic particle production is efficient. One can rewrite the adiabatic condition as

\[
|\phi| > \phi_\ast \equiv \left( \frac{2 \Lambda \dot{\phi}}{g} \right)^{1/3}. (19)
\]

Requiring \(|\Re \phi| < \sqrt{2} \mu_j\) whenever the adiabatic condition is violated, a lower bound for \(\mu_j\) will be obtained, which is given by a parameter

\[
x_j \equiv \mu_j / \phi_\ast > 1 / \sqrt{3} \sim 0.577. (20)
\]

Here we will focus on the first particle production and discuss the validity of the approximation. Then in the next section we will discuss the resonant particle production. In our formalism, the produced number density can be estimated as

\[
n_{\chi, 1} = \frac{1}{2 \pi^2 a(t_1)^3} \int_0^\infty dk k^2 n_{k, 1}
\]
condition on results are calculated for a flat potential \((m_\phi = 0)\) with initial condition \(\phi = 0.1\Lambda^2\).)

\[
\frac{g^2\Phi_1^2 m_\phi^2}{(2\pi)^3 \Lambda} x_{1}^{3/2} e^{-\sqrt{2}\pi x_{1}^{3/2}}. \tag{21}
\]

In Ref. [8], we have another method of analytical calculation, which is valid for small \(\mu\). It shows

\[
n_\chi \sim 0.0121 \frac{\beta^{3/2}}{\Lambda} (1 - 3.06x_{1}^{2} + \mathcal{O}(x_{1}^{4})) \tag{22}
\]

where one can substitute \(\dot{\phi} \sim \Phi_1 m_\phi\) if \(\Phi_1 \gg m_\phi\). Our analytical and numerical results are compared in Fig.1 in which one will see clearly that the quadratic approximation is conceivable when \(x_1 > 0.577\), while analytical calculation in Ref. [8] (small \(\mu\) approximation) is conceivable when \(x_1 < 1/\sqrt{3.06} = 0.572\). (Higher terms in Eq.22 start to dominate at \(x_1 = 0.572\).)

**B. Growth of number density and energy transfer in quadratic approximation**

In this section we mainly follow the useful calculational method considered in Ref. [8] and estimate the number of oscillations needed for significant energy transfer.

When the Universe is dominated by matter, or by oscillating field that scales like matter, \(\Phi_j\) and \(a_j\) depend on \(j\) as [8]

\[
\Phi_j \propto j^{-1}, \quad a_j \propto j^{2/3}. \tag{23}
\]

If we take a fixed value \(x_j = x\) (for example, \(x = 0.577\)) for all \(j\), we find

\[
k_{s,j} \propto a_j \mu_j^{1/2} \Phi_j^{1/2} \propto a_j \phi_{s,j}^{1/2} \Phi_j^{1/2} \propto j^{0}, \tag{24}
\]

which leads to \(e^{-\pi \kappa_j^2} \sim e^{-\pi \kappa_1^2} = n_{k,1}\). Then, Eq.18 gives

\[
n_{k,j} \sim (1 + 2n_{k,1})n_{k,j-1} + n_{k,1} \]

\[
\approx \frac{1}{2} \sum_{l=1}^{j} j C_l (2n_{k,1})^l, \tag{25}
\]

where \(j C_l\) is binomial coefficient.

One might suspect that the fixed value \(x_j \sim 0.6\) could be incorrect in reality, since \(x_j \sim 0.6\) cannot be applied for every \(j\) at the same time. In fact, \(x_j\) will be proportional to \(j^{-2/3}\) because \(\mu_j \propto j^{-1}\) (when the quantity \(a(t_j)^3\Phi_j\mu_j\) is conserved), while \(\phi_{s,j}\) will be proportional to \(\Phi_1^{1/3} \propto j^{-1/3}\). This means that \(x_j \sim 0.6\) will be broken soon after the first particle production. If one wants to see the particle production in the inner area \((x_j < x = 0.577)\), Fig.1 will be a useful guide, in which one will see that smaller \(x_j\) enhances particle production. Therefore, although the estimation of Eq.25 is not accurate when \(x \sim 0\), one can expect that the number density calculated for \(x_j \sim 0.6\) will give a conceivable lower bound for the particle production.

Assuming that \(\chi\) does not decay until \(t = t_j\), the total energy transfer to \(\rho_\chi\) is calculated as

\[
\rho_{\chi,j} = \frac{1}{2\pi^2 a(t_j)^3} \int dkk^2 \omega_k (t_j)n_{k,j} \]

\[
\approx \frac{g^2 \Phi_1^2}{\Lambda} \sum_{l=1}^{j} j C_l R_l^f \int_0^\infty dkk^2 e^{-\pi k_{,1}^2} \]

\[
\approx \frac{g^2 \Phi_1^2}{j^2\Lambda^f} \frac{21/4}{8\pi^3} n_{k,1} \sum_{l=1}^{j} j C_l R_l^f \]

\[
\approx \frac{21/4}{j^2\Lambda^f} \frac{21/4}{8\pi^3} \rho_\phi, j \sum_{l=1}^{j} j C_l R_l^f \tag{26}
\]

where \(R \equiv 2 \exp\left(-\pi A_0 (t_j)^3/k_{,1}^2\right) = 2 \exp\left(-\sqrt{2}\pi x_{1}^{3/2}\right)\).

If one needs to calculate the summation one can use \(n! \sim \sqrt{2\pi n}(n/e)^n\) together with saddle point method for the integration \(\sum_{l=1}^{j} \int dI\). For \(x_1 = 0.577\) and \(g^2 \Phi_1 \approx \Lambda\), \(\rho_{\chi,j} \sim \rho_{\phi, j}\) will be achieved when \(j \sim 25\). Note however that \(j \sim 25\) is a modest estimation.

**C. Apart from quadratic approximation**

In reality, one must take \(x \ll 1\) and (intuitively) the energy transfer will be more significant. To confirm our intuition, we show our numerical calculation in Fig.2 which shows that \(j = 5\) is sufficient for the energy transfer. For simplicity we showed our numerical result calculated in Minkowski space. To compare it with our analytical calculation, we have to calculate the same quantity when the expansion of the Universe is neglected. In that case we can easily obtain \(j \sim 13\) from the analytical calculation (with quadratic approximation). Moreover, already
can be shown as \[2\] later calculation. According to the method of Ref. \[6\], the coefficients of the wave function of Eq.(17), which explains resonant particle production, is identified with \(\chi\) in this case. Of course this result is not trivial. See the original paper \[6\] for more details.

In contrast to the standard scenario, higher dimensional chaotic inflationary model suggest \(\Phi\) is not reliable when \(\phi\) is not given by the conventional form \[2\] in Eq.(15). See Ref.\[2\] for more details. In our numerical calculation we obtain \(\rho_{\chi,j} \sim \rho_{\phi,j}\) is achieved when

\[
\rho_{\chi,j} \sim \frac{g\Phi^2}{\Lambda^2} \sum_{j=1}^{\infty} j C_j r^{j-1} \int_0^\infty dk k^2 e^{-K_1 t}
\]

\[
\sim \frac{2}{27 \cdot (2.47)^2 j^2 \Lambda^2} \frac{g^2 \Phi^2}{r^{j+2}} \cdot \left(1 + r^2\right)^{j+2}.
\]

where \(r \equiv 4\pi^2/9\). In the last line, the summation has been calculated using the method mentioned below Eq.(26), which is reliable for large \(j\) but could have small deviation when \(j\) is not large. The last line of Eq.(30) is useful when one wants to apply the result to a non-inflaton field that may have \(\Phi_1 \ll \Lambda\). We can see that

\[
j \sim 3.25 + 2 \log_{1+r} j + 2 \log_{1+r} \left(\frac{j\Lambda}{g\Phi_1}\right).
\]

If we take \(g\Phi_1/\Lambda = 1\), energy transfer becomes significant at \(j \sim 8\). Neglecting the expansion of the Universe, we obtain \(j \sim 5\), which is in good agreement with our numerical calculation in the Minkowski space.

Looking into more details, we found that the numerical calculation is still indicating that the growth factor of the number density is seemingly larger than that of the analytical calculation. In our numerical calculation we found that the number density increases as \(9 \times 10^{-5} \rightarrow 5 \times 10^{-4} \rightarrow 2 \times 10^{-3} \rightarrow 9 \times 10^{-3} \rightarrow 3 \times 10^{-2} \rightarrow \cdots\). The discrepancy could be caused by \(\theta_1\) in Eq.(17) or by Eq.(18). One will see similar excess in Fig.1. However, all these results are suggesting that preheating is efficient for non-renormalizable interaction, even though the oscillation is “decoupled” in the effective action. Therefore, if chaotic inflationary model suggest \(\Phi_1 \sim \Lambda\), preheating after inflation could be quite significant even if the inflaton sector is “decoupled”. The above results do not depend explicitly on the mass of \(\phi\) as long as \(m_\phi \ll \Phi_1\). In contrast to the standard scenario, higher dimensional interaction predicts significant dependence on the amplitude \(\Phi\).

Our result can be applied to various other cases in which a light scalar field (such as a curvaton or a modulus field) begins oscillation after inflation. It is also possible to identify \(\chi\) as modulus field, and consider resonant production after inflation. This case will be discussed in the next section. Since particle production occurs in the area very close to the ESP, it is possible to consider the case with \(\Phi/\Lambda > 1\). Of course the effective action is not reliable when \(\phi(t)/\Lambda > 1\), but \(\Phi/\Lambda < 1\) is not always required for the calculation as far as the particle production within the small area \(\phi(t) < \phi_s \ll \Lambda\) can be
described using the effective action. In that way, a milder condition could be \( m_\phi^2 \Phi^2 \ll \Lambda^4 \).

IV. INTUITIVE ARGUMENT FOR THE DECAY RATE

The source of higher dimensional interaction could be diverse. In Ref. [2], effective action of supersymmetric GUT model has been discussed in detail. If one focuses on Planck-scale suppressed interaction, it will be helpful to consider terms like \( \sim H^2 x^2 \).

In this section we consider “decay rate” for quadratic and quartic potential to show how terms like \( \sim H^2 x^2 \) works in preheating. To discuss the effective “decay rate”, we are carefully following the discussion of Sec.III and IV in Ref. [2]. Since perturbative decay rate will appear in the \( q \ll 1 \) limit, it is useful to think about perturbative theory versus narrow resonance.

For the quadratic inflaton potential \( V(\phi) \sim m_\phi^2 \phi^2 / 2 \), we find effective interaction \( H^2 x^2 \sim (m_\phi^2 / 6 M_p^2) \phi^2 x^2 \). Although the source of this interaction is higher dimensional, the resultant preheating uses “conventional” interaction \( \sim g^2 x^2 \phi^2 / 2 \). According to the argument in Ref. [2], perturbative decay rate (\( \Gamma_\phi \)) could be significant when \( \Gamma_\phi > q m_\phi \), where \( q \approx \frac{m_\phi^2}{m_\phi^2} \sim \frac{\Phi^2}{M_p^2} \). For a “decoupled” inflaton, considering the above effective interaction \( H^2 x^2 \sim (m_\phi^2 / 6 M_p^2) \phi^2 x^2 \), \( \Gamma_\phi \) can be estimated as [2] \( \Gamma_\phi (\phi \rightarrow \chi \chi) \sim g^4 \frac{\Phi^2}{m_\phi^2} \sim \frac{m_\phi^4}{\Lambda^2} \).

Also, it could be possible to consider light fermion interacting with \( \mathcal{L}_{\text{int}} \sim (m_\phi / M_p) \psi \bar{\psi} \phi \). Then one will find \( \Gamma_\phi (\phi \rightarrow \psi \psi) \sim \frac{m_\phi^2}{M_p^2} \). These decay rates are suggesting that preheating is important as far as \( \Phi > m_\phi \).

Next we consider quartic inflaton potential \( V(\phi) \sim \lambda \phi^4 / 4 \), where \( \lambda \ll 1 \). Again, one can expect higher dimensional effective interaction \( c^2 H^2 x^2 \sim c^2 \lambda \phi^4 x^2 / (12 M_p^2) \). In this case preheating is caused by truly higher dimensional interaction. Since the oscillation occurs on quartic potential, the oscillating solution is given by the elliptic function. This case has been considered in Ref. [9] for renormalizable interaction. Repeating the argument of [9], the elliptic function can be replaced by the sinusoidal function. Here we use the definitions used in Ref. [9]. The mode equation for \( X_k(t) = a(t) \chi_k(t) \) with the dimensionless conformal time \( x \equiv (48 \Lambda M_p^2)^{1/4} t^{1/2} \) (rescaled using the amplitude \( \Phi \)) is

\[
X_k'' + \left[ k^2 + \frac{c^2 \Phi^2}{12 M_p^2} f(x)^4 \right] X_k = 0, \quad (32)
\]

where \( f(x) = cn \left( x, \frac{1}{\sqrt{3}} \right) \). Here we are using equations (13) and (18) of Ref. [9]. Denoting the period of the oscillation by \( T \), one can expand

\[
f^4(x) = \left[ F_0 + F_1 \cos \left( \frac{4\pi x}{T} \right) + F_2 \cos \left( \frac{8\pi x}{T} \right) + ... \right]^2
\]

where \( F_0 \simeq 0.46, F_1 \simeq 0.50 \) and \( F_2 \simeq 0.04 \). See also Eq.(42) of Ref. [9]. In units of \( x \), the effective frequency is \( 2\pi / T \simeq 0.8 \). Specific value of \( T \) is discussed below Eq.(14) of Ref. [9]. Note that we are considering the same oscillation of \( \phi(t) \) as Ref. [3]. Then, considering “only” the leading term one can recover the Mathieu equation, in which \( q \) can be estimated as \( q \sim c^2 F_0 F_2 x^2 / (12 M_p^2) \). This estimation is of course not rigorous, but would be useful in finding a sensible estimation. Again, as far as the above approximations are valid, perturbative decay from higher dimensional interaction \( \sim \lambda \phi^4 x^2 / (12 M_p^2) \) is not significant compared with preheating. It is not obvious how fermions interact with \( \phi \), since the source of tiny \( \lambda \) in the inflationary model is not quite obvious. Even though, it could be possible to consider interaction given by \( \mathcal{L}_{\text{int}} \sim \lambda^{1/2} \psi \bar{\psi} \phi^2 / M_p \) to find that perturbative decay is not significant.

Because of fine-tunings of parameters required for inflation (e.g. \( m_\phi \ll M_p \) or \( \lambda \ll 1 \) in the above cases), it is not quite obvious in reality how inflaton interacts with other fields. The strength of the specific interaction could depend on the mechanism or the symmetry that makes those parameters fine-tuned. On the other hand, the interaction considered above \( \sim H^2 x^2 \) could be mandatory. Although intuitively, the above argument is showing importance of higher dimensional interaction in inflationary cosmology.

V. CONCLUSION AND DISCUSSION

In this paper we considered a model of preheating when the oscillation is “decoupled”. Even if the oscillation is “decoupled” in the effective action, higher dimensional interaction could not be avoided. Oscillation in such model is normally a decoupled oscillation, which has not been expected to cause efficient preheating. In contrast to the usual expectation, we found that preheating could be quite efficient for higher dimensional interaction. Using three different approaches (quadratic approximation, steepest descent method [3] and numerical calculation), we confirmed that higher dimensional interaction can cause resonant particle production. Unlike standard scenario of renormalizable preheating, the result depends on the ratio \( \Phi / \Lambda \). Note that \( \Phi / \Lambda > 1 \) is not excluded as far as the particle production near the ESP is well described by the effective action. The energy transfer is quick if the amplitude of the oscillation is \( \Phi_1 \sim M_p \). Let us answer to the “naive questions” given in the first section.

1. The final result contains \( \Phi^2 / \Lambda^2 \), but it is wrong to think that \( \phi(t) / \Lambda \ll 1 \) causes suppression around the ESP.
2. Again, although the result contains $\Phi^2/\Lambda^2$, it is wrong to think that higher dimensional interaction is negligible in preheating.

3. The function of $n_k$ is different. Rigorous calculation is required to determine numerical coefficients. Since there is no exact solution, results have to be backed by numerical calculation.

Our result may also indicate that moduli oscillation could cause resonant particle production even if the moduli is decoupled from other particles. More detailed study including instant preheating (i.e. when $\chi$ decays during oscillation) and the curvaton oscillation in thermal environment will be discussed in forthcoming paper.

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[1] See for instance the introduction of Ref.[2] for a brief historical review.
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