On the Estimation of Variance of Calibration Regression Estimators with Multiple Auxiliary Information

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Author’s contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

This paper introduces the concept of calibration estimators to Statistical Regression Estimation and proposes a multivariate calibration regression (M-REG) estimator of population mean in stratified random sampling. It develops a new approach to variance estimation that is more efficient in estimating populations with multiple auxiliary variables using the principle of analysis of variance (ANOVA). The relative performance of the new variance estimation method with respect to the estimation of variance of the proposed M-REG estimator is compared empirically with a corresponding global variance estimation method. Analysis and evaluation presented, proved the dominance of the suggested new approach to variance estimation.

Keywords: Analysis of variance; calibration estimation; efficiency; optimality conditions; stratified random sampling; variance estimation.

Mathematics Subject Classification: 62D05; 62G05; 62H12

1 Introduction

The concept of calibration estimator was introduced by [1] in survey sampling. Calibration estimation is a method that uses auxiliary variable(s) to adjust the original design weights to improve the precision of
survey estimates of population or subpopulation parameters. The calibration weights are chosen to minimize a given distance measure (or loss function) and these weights satisfy the constraints related auxiliary variable information. Calibration estimation has been studied by many survey Statisticians. A few key references are [2-21].

The large sample approximation (LASAP) method has been the most dominant approach to variance estimation in survey sampling. However, it has been observed that this approach always depends on certain optimality conditions that need to be satisfied to guarantee a better and efficient estimator. Again, it is well established in sample surveys that incorporate auxiliary information, that the precision of survey estimates is always improved when multiple auxiliary information are available. Keeping this in view, this paper introduces calibration weightings to statistical regression estimation, develops a new variance estimation method that is more efficient for estimating populations with multiple auxiliary variables using the principle of analysis of variance (ANOVA) and proposes a multivariate calibration regression estimator of population mean in stratified random sampling. The efficiency of the proposed variance estimation method is compared with the conventional large sample approximation (LASAP) method.

2 The Suggested Multivariate Calibration Regression (M-REG) Estimator

The calibration estimator for the stratified random sampling is defined by [22] as given by:

$$\bar{y}_{sr}(Tr) = \sum_{h=1}^{H} W_h^* \bar{y}_h$$  \hspace{1cm} (1)

where $W_h^*$ is the calibration weights which minimizes given calibration constraints.

Motivated by [22], this paper introduces a multivariate calibration regression (M-REG) estimator in stratified random sampling as given by

$$\bar{y}_{sr}(MREG) = \sum_{h=1}^{H} \varphi_h^* \bar{y}_h$$  \hspace{1cm} (2)

with the new weights $\varphi_h^*$ called the multivariate calibration weights. The multivariate calibration weights $\varphi_h^*$ are chosen such that a chi-square-type loss functions of the form:

$$L(\varphi_h^*, W_h) = \sum_{h=1}^{H} \frac{(\varphi_h^* - W_h)^2}{W_h Q_h}$$  \hspace{1cm} (3)

is minimized while satisfying the calibration constraints

$$\sum_{h=1}^{H} \varphi_h^* \bar{x}_{1h} = \sum_{h=1}^{H} W_h \bar{x}_{1h}$$  \hspace{1cm} (4)

$$\sum_{h=1}^{H} \varphi_h^* \bar{x}_{2h} = \sum_{h=1}^{H} W_h \bar{x}_{2h}$$  \hspace{1cm} (5)
By substituting (8) in (2) gives

\[
\sum_{h=1}^{H} \varphi_i^* \bar{X}_{ih} = \sum_{h=1}^{H} W_h \bar{X}_h
\]

(6)

So that

\[
\Delta = \sum_{h=1}^{H} \left( \varphi_i^* - W_h \right)^2 - 2 \lambda_1 \left( \sum_{h=1}^{H} \varphi_i^* \bar{X}_{ih} - \sum_{h=1}^{H} W_h \bar{X}_h \right) - 2 \lambda_2 \left( \sum_{h=1}^{H} \varphi_i^* \bar{X}_{2ih} - \sum_{h=1}^{H} W_h \bar{X}_{2h} \right)
\]

\[
-2 \lambda_3 \left( \sum_{h=1}^{H} \varphi_i^* \bar{X}_{3ih} - \sum_{h=1}^{H} W_h \bar{X}_{3h} \right)
\]

(7)

Minimizing the chi-square-type loss functions (3) subject to the calibration constraints [(4),(5), (6)] gives the multivariate calibration weights for stratified random sampling as:

\[
\varphi_i^* = W_h + W_h Q_h (\lambda_1 \bar{X}_{1h} + \lambda_2 \bar{X}_{2h} + \lambda_3 \bar{X}_{3h})
\]

(8)

Substituting (8) into [(4),(5), (6)] respectively gives the following system of equations as:

\[
\begin{bmatrix}
\mu_{11} & \mu_{12} & \mu_{13} \\
\mu_{12} & \mu_{22} & \mu_{23} \\
\mu_{13} & \mu_{23} & \mu_{33}
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{bmatrix}
= 
\begin{bmatrix}
\mu_{10} \\
\mu_{20} \\
\mu_{30}
\end{bmatrix}
\]

(9)

Where

\[
\begin{align*}
\mu_{11} & = \sum_{h=1}^{H} W_h Q_h \bar{X}_{1h}^2 \\
\mu_{12} & = \sum_{h=1}^{H} W_h Q_h \bar{X}_{1h} \bar{X}_{2h} \\
\mu_{13} & = \sum_{h=1}^{H} W_h Q_h \bar{X}_{1h} \bar{X}_{3h} \\
\mu_{22} & = \sum_{h=1}^{H} W_h Q_h \bar{X}_{2h}^2 \\
\mu_{23} & = \sum_{h=1}^{H} W_h Q_h \bar{X}_{2h} \bar{X}_{3h} \\
\mu_{33} & = \sum_{h=1}^{H} W_h Q_h \bar{X}_{3h}^2 \\
\mu_{10} & = \sum_{h=1}^{H} W_h (\bar{X}_{1h} - \bar{X}_{1h}) \\
\mu_{20} & = \sum_{h=1}^{H} W_h (\bar{X}_{2h} - \bar{X}_{2h}) \\
\mu_{30} & = \sum_{h=1}^{H} W_h (\bar{X}_{3h} - \bar{X}_{3h})
\end{align*}
\]

Solving the system of equations in (9) for \(\lambda\) gives

\[
\lambda_1 = \frac{(\mu_{13}\mu_{23} - \mu_{12}\mu_{33})(\mu_{12}\mu_{30} - \mu_{13}\mu_{20}) - (\mu_{13}\mu_{22} - \mu_{13}\mu_{22})(\mu_{12}\mu_{30} - \mu_{12}\mu_{30})}{(\mu_{12} - \mu_{12}\mu_{33})(\mu_{12}\mu_{23} - \mu_{23}\mu_{33}) - (\mu_{13}\mu_{22} - \mu_{13}\mu_{22})(\mu_{12}\mu_{33} - \mu_{13}\mu_{23})}
\]

\[
\lambda_2 = \frac{(\mu_{13}\mu_{23} - \mu_{12}\mu_{33})(\mu_{12}\mu_{10} - \mu_{11}\mu_{30}) - (\mu_{12}\mu_{12} - \mu_{12}\mu_{12})(\mu_{13}\mu_{20} - \mu_{12}\mu_{30})}{(\mu_{12} - \mu_{12}\mu_{33})(\mu_{12}\mu_{23} - \mu_{23}\mu_{33}) - (\mu_{13}\mu_{22} - \mu_{13}\mu_{22})(\mu_{12}\mu_{33} - \mu_{13}\mu_{23})}
\]

\[
\lambda_3 = \frac{(\mu_{12}\mu_{12} - \mu_{12}\mu_{12})(\mu_{10}\mu_{30} - \mu_{10}\mu_{30}) - (\mu_{13}\mu_{22} - \mu_{12}\mu_{22})(\mu_{12}\mu_{10} - \mu_{11}\mu_{20})}{(\mu_{12} - \mu_{12}\mu_{33})(\mu_{12}\mu_{23} - \mu_{23}\mu_{33}) - (\mu_{13}\mu_{22} - \mu_{12}\mu_{22})(\mu_{12}\mu_{33} - \mu_{11}\mu_{23})}
\]

Substituting (8) in (2) gives

\[
\bar{y}_{ih}^{(MREG)} = \sum_{h=1}^{H} W_h \bar{y}_h + \sum_{h=1}^{H} W_h Q_h [\lambda_1 \bar{X}_{1h} + \lambda_2 \bar{X}_{2h} + \lambda_3 \bar{X}_{3h}] \bar{y}_h
\]

(10)

By substituting the \(\lambda\) in (10) and setting \(Q_h = 1\) gives the proposed multivariate calibration regression (M-REG) estimator of population mean in stratified random sampling as given by:

\[
\bar{y}_{ih}^{(MREG)} = \bar{y}_{ih} + \beta_1 \mu_{10} + \beta_2 \mu_{20} + \beta_3 \mu_{30}
\]

(11)
Expressing (11) in terms of the e’s gives

\[ \begin{align*}
\beta_1 &= \frac{b_{12}[(b_{12}b_{23} - b_{23}) + b_{24}(b_{13}b_{23} - b_{12}b_{23}) + b_{34}(b_{12}b_{23} - b_{13}b_{22})]}{(b_{12} - b_{11}b_{22})(b_{13}b_{23} - b_{12}b_{23}) - (b_{12}b_{22} - b_{12}b_{13})(b_{12}b_{13} - b_{11}b_{23})} \\
\beta_2 &= \frac{b_{12}[(b_{12}b_{23} - b_{12}b_{23}) + b_{24}(b_{11}b_{23} - b_{11}b_{12}) + b_{34}(b_{11}b_{22} - b_{11}b_{22})]}{(b_{12} - b_{11}b_{22})(b_{13}b_{23} - b_{12}b_{23}) - (b_{12}b_{22} - b_{12}b_{13})(b_{12}b_{13} - b_{11}b_{23})} \\
\beta_3 &= \frac{b_{12}[(b_{12}b_{23} - b_{12}b_{23}) + b_{24}(b_{11}b_{23} - b_{11}b_{12}) + b_{34}(b_{11}b_{22} - b_{11}b_{22})]}{(b_{12} - b_{11}b_{22})(b_{13}b_{23} - b_{12}b_{23}) - (b_{12}b_{22} - b_{12}b_{13})(b_{12}b_{13} - b_{11}b_{23})}
\end{align*} \]

Where

\[ \begin{align*}
b_{11} &= \sum_{h=1}^{H} W_h \bar{x}_{1h}^2 \\
b_{22} &= \sum_{h=1}^{H} W_h \bar{x}_{2h} \\
b_{33} &= \sum_{h=1}^{H} W_h \bar{x}_{3h} \\
b_{12} &= \sum_{h=1}^{H} W_h \bar{x}_{1h} \bar{x}_{2h} \\
b_{13} &= \sum_{h=1}^{H} W_h \bar{x}_{1h} \bar{x}_{3h} \\
b_{23} &= \sum_{h=1}^{H} W_h \bar{x}_{2h} \bar{x}_{3h} \\
b_{14} &= \sum_{h=1}^{H} W_h \bar{x}_{1h} \bar{y}_h \\
b_{24} &= \sum_{h=1}^{H} W_h \bar{x}_{2h} \bar{y}_h \\
b_{34} &= \sum_{h=1}^{H} W_h \bar{x}_{3h} \bar{y}_h
\end{align*} \]

3 Estimation of Variance for the Proposed Estimator

This section attempts to derive the estimator of variance of the proposed multivariate calibration regression (M-REG) estimator under the large sample approximation (LASAP) method and the suggested analysis of variance (ANOVA) method as discussed in sections 4.1 and 4.2 respectively.

3.1 Large sample approximation (LASAP) method

Let define the following equations

\[ \begin{align*}
e_{hy} &= \left( \frac{\bar{y}_h - \bar{y}_h}{\bar{y}_h} \right), e_{hx1} = \left( \frac{\bar{x}_1 - \bar{x}_{1h}}{\bar{x}_1} \right), e_{hx2} = \left( \frac{\bar{x}_2 - \bar{x}_{2h}}{\bar{x}_2} \right), e_{hx3} = \left( \frac{\bar{x}_3 - \bar{x}_{3h}}{\bar{x}_3} \right)
\end{align*} \]

where \( \bar{y}_h \) and \( \bar{y}_h \) denote respectively, the sample stratum mean and population stratum mean of the study variable \( Y \) while \( \bar{x}_{1h} \) and \( \bar{x}_i \) denote respectively, the population stratum mean and population mean of the \( i \)th auxiliary variable \( X_i \).

So that

\[ \begin{align*}
\bar{y}_h &= \bar{y}_h(1 + e_{hy}), \bar{x}_{1h} = \bar{x}_1(1 - e_{hx1}), \bar{x}_{2h} = \bar{x}_2(1 - e_{hx2}), \bar{x}_{3h} = \bar{x}_3(1 - e_{hx3}) \\
E(e_{hy}) &= \gamma_h C_{hy}, E(e_{hx1}) = \gamma_h C_{hx1}, E(e_{hx2}) = \gamma_h C_{hx2}, E(e_{hx3}) = \gamma_h C_{hx3} \\
E(e_{hx1}e_{hx2}) &= \gamma_h C_{hx1} C_{hx2}, E(e_{hx1}e_{hx3}) = \gamma_h C_{hx1} C_{hx3} \\
E(e_{hx2}e_{hx3}) &= \gamma_h C_{hx2} C_{hx3}
\end{align*} \]

Expressing (11) in terms of the e’s gives

\[ \bar{y}_{st}^{MREG} - \bar{y} = \sum_{h=1}^{H} W_h \left[ \bar{y}_h e_{hy} + \beta_1 \bar{x}_{1h} e_{hx1} + \beta_2 \bar{x}_{2h} e_{hx2} + \beta_3 \bar{x}_{3h} e_{hx3} \right] \] (12)
Solving the system of equations in (15) gives

\[
[y_{st}^2(M\text{REG}) - \bar{y}]^2 = \sum_{h=1}^{H} W_h^2 [\bar{y}_{st}^2 e^2_h + \beta_{1h}^2 \bar{x}_{hx1}^2 e^2_{hx1} + \beta_{2h}^2 \bar{x}_{hx2}^2 e^2_{hx2} + \beta_{3h}^2 \bar{x}_{hx3}^2 e^2_{hx3} \\
+ 2\bar{y}_h \beta_{1h} \bar{x}_{hx1} e_{hx1} + 2\bar{y}_h \beta_{2h} \bar{x}_{hx2} e_{hx2} + 2\bar{y}_h \beta_{3h} \bar{x}_{hx3} e_{hx3} \\
+ 2\beta_{1h} \beta_{2h} \bar{x}_{hx1} \bar{x}_{hx2} e_{hx1} e_{hx2} + 2\beta_{1h} \beta_{3h} \bar{x}_{hx1} \bar{x}_{hx3} e_{hx1} e_{hx3} \\
+ 2\beta_{2h} \beta_{3h} \bar{x}_{hx2} \bar{x}_{hx3} e_{hx2} e_{hx3}] \tag{13}
\]

Taking expectation of both sides of (13) gives

\[
V[y_{st}^2(M\text{REG})] = \sum_{h=1}^{H} W_h^2 \left[ \bar{y}_{st}^2 C_{xy}^2 + \beta_{1h}^2 \bar{x}_{hx1}^2 C_{hx1}^2 + \beta_{2h}^2 \bar{x}_{hx2}^2 C_{hx2}^2 \\
+ 2\bar{y}_h \beta_{1h} \bar{x}_{hx1} \rho_{hx13} C_{hx1} C_{hx3} \\
+ 2\bar{y}_h \beta_{2h} \bar{x}_{hx2} \rho_{hx23} C_{hx2} C_{hx3} \\
+ 2\beta_{1h} \beta_{2h} \bar{x}_{hx1} \bar{x}_{hx2} \rho_{hx12} C_{hx1} C_{hx2} \\
+ 2\beta_{1h} \beta_{3h} \bar{x}_{hx1} \bar{x}_{hx3} \rho_{hx13} C_{hx1} C_{hx3} \\
+ 2\beta_{2h} \beta_{3h} \bar{x}_{hx2} \bar{x}_{hx3} \rho_{hx23} C_{hx2} C_{hx3} \right] \tag{14}
\]

3.1.1 Optimality condition

This section deduced the optimality conditions that would guarantee optimum performance of the estimator on satisfaction. Setting

\[
\frac{\partial \tilde{y}_{st}^2(M\text{REG})}{\partial \beta_1} = 0, \quad \frac{\partial \tilde{y}_{st}^2(M\text{REG})}{\partial \beta_2} = 0 \quad \text{and} \quad \frac{\partial \tilde{y}_{st}^2(M\text{REG})}{\partial \beta_3} = 0
\]

respectively, gives the following system of equations

\[
\begin{bmatrix}
\bar{y}_{st}^2 C_{hx1}^2 & \bar{x}_{hx1} \bar{x}_{hx12} C_{hx1} C_{hx2} & \bar{x}_{hx1} \bar{x}_{hx13} C_{hx1} C_{hx3} \\
\bar{x}_{hx1} \bar{x}_{hx12} C_{hx1} C_{hx2} & \bar{x}_{hx2}^2 C_{hx2}^2 & \bar{x}_{hx2} \bar{x}_{hx23} C_{hx2} C_{hx3} \\
\bar{x}_{hx1} \bar{x}_{hx13} C_{hx1} C_{hx3} & \bar{x}_{hx2} \bar{x}_{hx23} C_{hx2} C_{hx3} & \bar{x}_{hx3}^2 C_{hx3}^2 \\
\end{bmatrix} \begin{bmatrix}
\beta_{1h} \\
\beta_{2h} \\
\beta_{3h} \\
\end{bmatrix} = - \begin{bmatrix}
\bar{y}_{st} \rho_{hx13} C_{hx1} \\
\bar{y}_{st} \rho_{hx12} C_{hx2} \\
\bar{y}_{st} \rho_{hx3} C_{hx3} \\
\end{bmatrix} \tag{15}
\]

Solving the system of equations in (15) gives

\[
\beta_{1h} = \frac{\bar{y}_{st} \rho_{hx13}^2 C_{hx1} + 2 \rho_{hx12} \rho_{hx13} \rho_{hx23} - \rho_{hx12}^2 \rho_{hx23} - \rho_{hx13}^2 \rho_{hx12}}{X_{12} C_{hx1} C_{hx2} + \rho_{hx12}^2 \rho_{hx23} - \rho_{hx13}^2 \rho_{hx12}} \tag{16}
\]

\[
\beta_{2h} = \frac{\bar{y}_{st} \rho_{hx13}^2 C_{hx1} + \rho_{hx12} \rho_{hx13} \rho_{hx23} - \rho_{hx12}^2 \rho_{hx23} - \rho_{hx13}^2 \rho_{hx12}}{X_{12} C_{hx1} C_{hx2} + \rho_{hx12}^2 \rho_{hx23} - \rho_{hx13}^2 \rho_{hx12}} \tag{17}
\]

\[
\beta_{3h} = \frac{\bar{y}_{st} \rho_{hx13}^2 C_{hx1} + 2 \rho_{hx12} \rho_{hx13} \rho_{hx23} - \rho_{hx12}^2 \rho_{hx23} - \rho_{hx13}^2 \rho_{hx12}}{X_{12} C_{hx1} C_{hx2} + \rho_{hx12}^2 \rho_{hx23} - \rho_{hx13}^2 \rho_{hx12}} \tag{18}
\]

Substituting \((16), (17), (18)\) in \((15)\), the resulting (optimum) estimator of variance of \(y_{st}^2(M\text{REG})\) is given by:
where

\[ B_{1h} = \frac{[\rho_{hx1x3} + \rho_{hx12}\rho_{hx2x3} - \rho_{hx1x3}\rho_{hx2x3} - \rho_{hx1x3}\rho_{hx2x3}\rho_{hx1x3}]}{[1 + 2\rho_{hx1x2}\rho_{hx1x3}\rho_{hx2x3} - \rho_{hx1x2}^2 - \rho_{hx1x3}^2 - \rho_{hx2x3}^2]} \]

\[ B_{2h} = \frac{[\rho_{hx1x3}^2 + \rho_{hx1x3}\rho_{hx2x3} + \rho_{hx12}\rho_{hx1x3} - \rho_{hx1x3}\rho_{hx2x3}]}{[1 + 2\rho_{hx1x2}\rho_{hx1x3}\rho_{hx2x3} - \rho_{hx1x2}^2 - \rho_{hx1x3}^2 - \rho_{hx2x3}^2]} \]

\[ B_{3h} = \frac{[\rho_{hx1x3}^2 - \rho_{hx1x3}\rho_{hx2x3}]}{[1 + 2\rho_{hx1x2}\rho_{hx1x3}\rho_{hx2x3} - \rho_{hx1x2}^2 - \rho_{hx1x3}^2 - \rho_{hx2x3}^2]} \]

3.2 Analysis of variance (ANOVA) method

In this section, the estimator of variance of the proposed multivariate calibration regression (M-REG) estimator is derived by the analysis of variance (ANOVA) approach.

Let the corrected sum of products be defined by

\[ S_{ij} = \sum_{h=1}^{H} (\bar{x}_{ih} - \bar{x}_{i})(\bar{x}_{jh} - \bar{x}_{j}), \quad i, j = 1, 2, \ldots m \]

So that

\[ S_{0i} = \sum_{h=1}^{H} (\bar{y}_h - \bar{y})(\bar{x}_{ih} - \bar{x}_{i}), \quad i = 1, 2, \ldots m \]

where \( m \) denotes the number of auxiliary variables.

Let the sum of square regression (SSR) be defined by

\[ S_{0R} = \sum_{i=1}^{m} \hat{\beta}_i S_{0i} \quad (20) \]

where \( \hat{\beta}_i = C_{ij}s_{0i} \) and \( C_{ij} \) is the reciprocal of \( S_{ij} \).

Let the sum of square residual be defined by:

\[ S_{00} = \sum_{h=1}^{H} w_h^2(\bar{y}_h - \bar{y})^2 \quad (21) \]

where \( w_h \) are the calibration weights minimizing the Chi-square loss function

\[ L(W^*_h, W_h) = \sum_{h=1}^{H} \frac{(W^*_h - W_h)^2}{W_h Q_h} \quad (22) \]
Subject to calibration constraint defined by
\[
\sum_{h=1}^{H} \sum_{i=1}^{m} W_h^* S_{oi} = \sum_{h=1}^{H} \sum_{i=1}^{m} W_h^* \bar{X}_{ih}^2
\]  
(23)

So that
\[
\Delta_1 = \sum_{h=1}^{H} \frac{(W_h^* - W_h)}{W_h^* W_h} \left( \sum_{h=1}^{H} \sum_{i=1}^{m} W_h^* S_{oi} - \sum_{h=1}^{H} \sum_{i=1}^{m} W_h^* \bar{X}_{ih}^2 \right)
\]

Setting
\[
\frac{\partial \Delta_1}{\partial W_h^*} = 0
\]
\[
W_h^* = W_h + \partial W_h Q_h S_{oi}
\]  
(24)

Substituting (24) in (23) gives
\[
\theta = \frac{\sum_{h=1}^{H} \sum_{i=1}^{m} W_h (\bar{X}_{ih}^2 - S_{oi})}{\sum_{h=1}^{H} \sum_{i=1}^{m} W_h Q_h S_{oi}^2}
\]
(25)

Substituting (25) in (24) gives
\[
W_h^* = W_h + \sum_{h=1}^{H} \sum_{i=1}^{m} W_h (\bar{X}_{ih}^2 - S_{oi}) \frac{W_h Q_h S_{oi}}{\sum_{h=1}^{H} \sum_{i=1}^{m} W_h Q_h S_{oi}^2}
\]
(26)

Setting the calibration tuning parameter \( Q_h = S_{oi}^{-1} \) and substituting in (21) gives
\[
S_{00} = \sum_{h=1}^{H} W_h^2 (\bar{y}_h - \bar{y})^2 \left( \sum_{h=1}^{H} \sum_{i=1}^{m} W_h \bar{X}_{ih}^2 \right)^2 \left( \sum_{h=1}^{H} \sum_{i=1}^{m} W_h S_{oi} \right)^{-2}
\]  
(27)

The variance of \( \bar{y}_{st}(\text{MREG}) \) given \( X_1, X_2, X_3 \) is estimated by:
\[
\hat{\sigma}^2[\bar{y}_{st}(\text{MREG})] = \frac{S_{00}}{n - (m + 1)}
\]
(28)

where \( S_{00} \) is the sum of squares of error, \( n - (m + 1) \) is the degree of freedom associated with the error, \( n \) is the set of measurements and \( m \) is the number of auxiliary variables.

| Source | DF | SS | MS | F-Ratio |
|--------|----|----|----|---------|
| Factor | \( m \) | \( \sum_{i=1}^{m} \hat{\beta}_i S_{oi} \) | \( \frac{\sum_{i=1}^{m} \hat{\beta}_i S_{oi}}{m} \) | \( \frac{\sum_{i=1}^{m} \hat{\beta}_i S_{oi}[n - (m + 1)]}{S_{00}(m)} \) |
| Error | \( n - m - 1 \) | \( S_{00} \) | \( S_{00} \) | \( - \) |
| Total | \( n - 1 \) | \( S_{00} + \sum_{i=1}^{m} \hat{\beta}_i S_{oi} \) | \( \frac{n - m - 1}{n - m - 1} \) | \( - \) |
4 Empirical Study

To judge the relative performances of the proposed ANOVA approach to variance estimation [with respect to the estimation of variance of the proposed (M-REG) estimator] over the LASAP approach, the data set in table 2 was considered. The MSE of the proposed multivariate calibration regression (M-REG) estimator under the LASAP Method and the ANOVA Method of variance estimation are given in Table 4.

4.1 Isaki (1983) multivariate regression estimator

The multivariate regression estimator in stratified random sampling established by [23] is given by

$$S_{MR}^2 = S_{yh}^2 + \sum_{h=1}^{H} B_{ih}(S_{xih}^2 - S_{zih}^2)$$

with variance estimator given by:

$$V(S_{MR}^2) = V(S_{yh}^2) + \sum_{h=1}^{H} B_{ih}^2 V(S_{xih}^2) - 2 \sum_{h=1}^{H} B_{ih} \text{Cov}(S_{yh}^2, S_{xih}^2)$$
$$+ \sum_{(i,j) \neq (h)} B_{ih} B_{jh} \text{Cov}(S_{xih}, S_{zjh})$$

(30)

4.2 The percent relative efficiency (PRE)

The percent relative efficiency (PRE) of an estimator \( \theta \) with respect to the [23] multivariate regression estimator \( S_{MR}^2 \) in stratified sampling is defined by

$$PRE(\theta, S_{MR}^2) = \frac{V(S_{MR}^2)}{V(\theta)} \times 100$$

(31)

| Parameter | Stratum 1 | Stratum 2 | Stratum 3 | Stratum 4 |
|-----------|-----------|-----------|-----------|-----------|
| \( N_h \) | 10        | 9         | 26        | 7         |
| \( n_h \) | 3         | 2         | 5         | 2         |
| \( w_h \) | 0.300     | 0.2222    | 0.1923    | 0.2857    |
| \( \bar{X}_{1h} \) | 11.90    | 10.38     | 12.120    | 11.98     |
| \( \bar{X}_{2h} \) | 9.880    | 8.120     | 9.860     | 9.740     |
| \( \bar{X}_{3h} \) | 10.75    | 10.80     | 9.780     | 10.84     |
| \( \bar{Y}_h \) | 15.72    | 14.84     | 13.46     | 16.32     |
| \( C_{hy} \) | 1.062    | 0.986     | 1.208     | 1.023     |
| \( C_{hx1} \) | 1.234    | 1.306     | 1.032     | 0.926     |
| \( C_{hx2} \) | 1.165    | 0.946     | 1.010     | 1.062     |
| \( C_{hx3} \) | 1.246    | 0.864     | 1.026     | 0.926     |
| \( \rho_{hxy1} \) | 0.940    | 0.900     | 0.840     | 0.890     |
| \( \rho_{hxy2} \) | 0.820    | 0.860     | 0.920     | 0.780     |
| \( \rho_{hxy3} \) | 0.923    | 0.968     | 0.842     | 0.956     |
| \( \rho_{hx1x2} \) | 0.860    | 0.800     | 0.760     | 0.840     |
| \( \rho_{hx1x3} \) | 0.910    | 0.942     | 0.864     | 0.760     |
| \( \rho_{hx2x3} \) | 0.840    | 0.860     | 0.780     | 0.920     |
| Mean \( \bar{X}_1 = 11.04 \) | \( \bar{X}_2 = 9.04 \) | \( \bar{X}_3 = 10.20 \) | \( \bar{Y} = 12.62 \) |
\[ S_{ij} = \begin{pmatrix} 3.2252 & 2.8732 & 0.225 \\ 2.8732 & 2.7144 & 0.0136 \\ 0.225 & 0.0136 & 1.2485 \end{pmatrix}, \quad C_{ij} = \begin{pmatrix} 6.7394 & -7.1279 & -7.1369 \\ -7.1279 & 7.9074 & 1.1984 \\ -7.1369 & 1.1984 & 0.9928 \end{pmatrix} \]

\[ S_{oi} = \begin{pmatrix} 5.586 \\ 3.8404 \\ 5.0522 \end{pmatrix}, \quad \hat{\beta}_{ih} = \begin{pmatrix} 4.5285 \\ -3.3943 \\ 3.2674 \end{pmatrix} \]

\[ \hat{\nu}_{opt} [\bar{y}_{xt} (MREG)] = 513.0802 \]

\[ \hat{\nu} (S_{\text{MR}}^2) = 826.4326 \]

**Table 3. ANOVA Table**

| Source | DF | SS   | MS    | F-Ratio |
|--------|----|------|-------|---------|
| Factor | 3  | 28.7683 | 9.5894 | 0.0626 |
| Error  | 8  | 1,226.3428 | 153.2928 | -- |
| Total  | 11 | -- | -- | -- |

**Table 4. MSE and PREs of the proposed estimator under the LASAP and ANOVA methods**

| S/No | Estimator                  | MSE  | PREs       |
|------|----------------------------|------|------------|
| 1.   | \( S_{\text{MR}}^2 \)     | 826.4326 | 100        |
| 2.   | \( \bar{y}_{xt} (MREG) \) ANOVA Method | 153.2928 | 539.1202   |
| 3.   | \( \bar{y}_{xt} (MREG) \) LASAP Method | 513.0802 | 161.0728   |

### 6 Discussion of Results

Numerical results for the percent relative efficiency (PREs) in table 4 reveals that the proposed ANOVA approach to variance estimation with respect to the estimator of variance of the proposed M-REG estimator has 439 percent gains in efficiency while the LASAP approach to variance estimation with respect to the estimator of variance of the proposed M-REG estimator has 61 percent gains in efficiency; this shows that the proposed ANOVA approach to variance estimation is 378 percent more efficient than the conventional LASAP method of variance estimation. This means that in using our proposed ANOVA approach to variance estimation with respect to the estimator of variance of the proposed M-REG estimator, one will have 378 percent efficiency gain over the conventional LASAP method of variance estimation. Also the proposed multivariate calibration regression (M-REG) estimator is more efficient than the [23] multivariate regression estimator (\( S_{\text{MH}}^2 \)).

### 7 Conclusion

Sequel to the discussion of results above, it is concluded that the proposed ANOVA approach to variance estimation fares better than the conventional LASAP method of variance estimation. The new approach to variance estimation is very efficient in estimating populations with multiple auxiliary variables.

Therefore, the new approach to variance estimation is very attractive to survey researchers as it gives consistent and more precise estimates of the population parameters.

**Competing Interests**

Author has declared that no competing interests exist.
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