The charged $Z_c$ states from rescattering in conventional $B$ decays

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Abstract. In this paper we discuss a possible interpretation of the charged charmonium-like states observed in $B$ decays as a rescattering effect. This approach allows to avoid introduction of exotic hadrons into the theory, while explains the unusual peaking structures observed recently in three body $B$ decays. In particular, the peaks in the $\psi(2S)\pi^+$ mass spectrum are ascribed to the rescattering process with kinematics governed by the presence of conventional radial excitations, the $D^{(*)}$ resonances, in the hidden intermediate state. Our predictions are compared to the LHCb results.

The Belle discovery in 2003 [1] of an unexpected narrow structure in $J/\psi\pi\pi$ invariant mass established a new era of seemingly well understood quarkonium physics. This resonance, called the $X(3872)$, is the first charmonium-like state, that was found not to fit into the conventional quarkonium model. Since the observation of the $X(3872)$ the list of exotic charmonium is updated with few dozens of $XYZ$ states [2]. None of the various either exotic (tetraquarks, molecular states, charmonium hybrids and hadrocharmonium) or conventional (potential models revision etc.) explanations of these states can adequately describe the full range of the observed phenomena.

First charged charmonium-like state, the $Z^+(4430)$, was observed by Belle [3] in 2007 as a resonance structure in the $\psi(2S)\pi^+$ spectrum from $B \to \psi(2S)\pi^+K^-$ decay. While BaBar has not confirmed this observation [5], it also could not rule out the $Z^+(4430)$ existence. Recent LHCb analysis puts an end to the controversy by observation of the $Z^+(4430)$ with extremely high significance and study of the resonant-like properties of the peak [6], including the complex phase motion. Being interpreted as a real resonance containing a $c\bar{c}$ pair, this state necessarily consists of at least an extra pair of light quark-antiquark due to its non-zero charge. The most popular interpretations of the $Z^+(4430)$ are tetraquark [7], hadrocharmonium [8], and $DD^*$ molecules [9], and non-resonant interpretations such as the “cusp effect” [10], rescattering via the chain $\bar{B} \to D^{*-}D_1(2420)K \to \psi(2S)\pi^+K$ [11], and the initial single pion emission mechanism [12].

Recently we have suggested another interpretation of the $Z^+(4430)$ [13, 14] as an effect of rescattering from the open charm channel into a final state with a charmonium and a pion. The peaking structures in $\psi(2S)\pi^+$ spectrum are explained by the presence of the conventional resonance ($D_s^{*-}$ meson) in the hidden intermediate state (conventional $B$ decay into a pair of anti-charm and charm-strange mesons). The Feynman diagrams illustrating this process are shown in figure 1. The main feature of the proposed model is a non-vanishing rescattering
$D^{*0}D^{*+}$ amplitude over a wide range of $(D^{*0}D^{*+})$ masses. It turns out, that the suggested approach is able to explain the Breit-Wigner-like shape of the peak in $\psi(2S)\pi^+$ spectrum, giving a similar complex phase motion. We note that the quantum numbers of $Z^+(4430)$, $J^P = 1^+$, were predicted within this approach before they were measured [13] and then have been confirmed by Belle [15, 16] and LHCb [6]. Other structures in $\psi(2S)\pi^+$ spectrum, giving a similar complex phase motion.

We note that the quantum numbers of $Z^+(4430)$, $J^P = 1^+$, were predicted within this approach before they were measured [13] and then have been confirmed by Belle [15, 16] and LHCb [6]. Other structures in $\psi(2S)\pi^+$ spectrum, in particular a broad bump near $M \sim 4200$ MeV, discovered by the LHCb collaboration and interpreted as another $Z^+$ resonance, were also predicted.

$D^{*0}D^{*+}$ amplitude over a wide range of $(D^{*0}D^{*+})$ masses. It turns out, that the suggested approach is able to explain the Breit-Wigner-like shape of the peak in $\psi(2S)\pi^+$ spectrum, giving a similar complex phase motion. We note that the quantum numbers of $Z^+(4430)$, $J^P = 1^+$, were predicted within this approach before they were measured [13] and then have been confirmed by Belle [15, 16] and LHCb [6]. Other structures in $\psi(2S)\pi^+$ spectrum, giving a similar complex phase motion. We note that the quantum numbers of $Z^+(4430)$, $J^P = 1^+$, were predicted within this approach before they were measured [13] and then have been confirmed by Belle [15, 16] and LHCb [6]. Other structures in $\psi(2S)\pi^+$ spectrum, giving a similar complex phase motion.

Within the proposed approach $B$ meson decays via the tree diagram to the $D_s^{*-}$ radial excitations ($D_s^{(*)-}$) and $D^{(*)+}$ meson. These Cabibbo-favored decays are expected to have a large branching fraction, however, one of them was observed by Belle [17] with $B(B^- \to D_s^+(2700)^-D^0) \times B(D_s^+(2700)^- \to D^0K) \sim 10^{-3}$. Although other modes have not been observed yet, the large inclusive $B \to D^{(*)}D^{(*)}K$ fraction [18] suggest the branching fraction for $B \to D_s^{(*)-}D^{(*)+}$ decays to be large as well. In the three body final state $D^{(*)}D^{(*)}K$ from these modes, two $D$ mesons are produced with a low relative momenta ($v/c \approx 0.3 - 0.5$). Therefore one can expect the non vanishing rescattering of two charmed mesons into charmonium and a light meson. The diagrams, corresponding to the two decay chains, that only could contribute to the $\psi(2S)\pi^+K^-$ final state due to parity conservation,

$$\bar{B} \to D_s^{*-}D^+, \text{ followed by } D_s^{*-} \to \bar{D}^{*0}K^-, \quad (1)$$

and

$$\bar{B} \to D_s^{*-}D^{*+}, \text{ followed by } D_s^{*-} \to \bar{D}^{*0}K^-, \quad (2)$$

are shown in figure 1a,b. Amplitudes, corresponding to these triangle diagrams, are calculated within on-shell approximation taking into account the $D_s^{(*)-}$ Breit-Wigner amplitude. We assume the $s$-wave rescattering dominates. The full decay amplitude has the following form in the helicity formalism:

$$A_{M_{Z^+(4430)} \to M(D,D^{*-})} = \sum \int A_{BW}(M_{D_s^{(*)-}})D_{\lambda,0}(\theta_{\text{dec}}) D_{\lambda,0}(\theta_{\text{rot}}) D_{\lambda,0}(\theta_{\text{form}}) dM_{D_s^{(*)-}}, \quad (3)$$

where $J$ is the $D_s^{(*)-}$ spin; $\theta_{\text{dec}}$ is the decay angle of the $D_s^{(*)-}$ (the angle between the $\bar{B}$ and $D_s^{(*)-}$ in the $D_s^{(*)-}$ rest frame); $\theta_{\text{rot}}$ is the rotation angle of the $\bar{D}^{*0}$ spin from the $D_s^{(*)-}$ frame for the reaction (1) or the $\bar{B}$ frame for the reaction (2) to the $Z^+(4430)$ frame; $\theta_{\text{form}}$ is the formation angle of $Z^+(4430)$, i.e. the angle between the $\bar{B}$ and $D^*$ in the $Z^+(4430)$ rest frame. The first Wigner $D$-function is responsible for the proper angular distribution of the $D_s^{(*)-}$ decay (in the case considered in (2), only the zero helicity projection works). The second function, $D_{\lambda,0}(\theta_{\text{rot}})$, describes the $D^*$ spin rotation from the frame where it is produced to the frame where it is absorbed. Finally, the $D_{\lambda,0}(\theta_{\text{form}})$ corresponds to the proper formation of the spin-1 $Z^+(4430)$ pseudostate from the vector ($D^*$) and the pseudoscalar ($D$). Two variables, $M_{D_s^{(*)-}}$ and $\theta_{\text{dec}},$

![Figure 1. Triangle Feynman diagrams for $\bar{B} \to \psi(2S)\pi^+K^-$ decay with $D_s^{*-}$ (a) and $D_s^{*-}$ (b) mesons in the hidden intermediate state.](attachment:figure1.png)
fully describe the three-body kinematics, thus $M_{(DD)'++}$, $\theta_{rot}$ and $\theta_{form}$ are functions of these two variables.

We have checked the plausibility of our hypothesis using the LHCb data. The distribution of $M^2(\psi(2S)\pi^+)$ for the $1.0 < M^2(K^-\pi^+) < 1.8\text{ GeV}^2$ interval borrowed from [6] is shown in figure 2a. In this spectrum the contributions from $K^*(890)$, $K_2^*(1430)$ and non-resonant three-body decays as determined by the LHCb fit have been subtracted incoherently. This remaining spectrum we attribute to the rescattering contribution and have performed a toy fit with expected rescattering shape to the residual spectrum. Ignoring of the phase-dependent interference effects is not quite correct procedure, but we can not perform a better way as we have only access to the published LHCb one-dimensional plots. We thus use it for illustration of plausibility of our approach only. In our fit the amplitudes of the reactions (1) and (2) (the later consists of two contributions with $\lambda = 0$ and 1) are free parameters. These amplitudes were calculated numerically from the equation 3. The fit results are shown in figure 2a: the solid line represents the fitting function; the dashed, dotted and dashed-dotted curves show the contributions from the process (1) and (2) with $\lambda=1$ and $\lambda=0$, respectively.

![Figure 2. a) The distribution of $M^2(\psi(2S)\pi^+)$ after incoherent subtraction of contributions from $K^*(890)$, $K_2^*(1430)$ and non-resonant three-body decays extracted from the LHCb data [6]. The solid line represents fit to the data points. The dashed, dotted and dashed-dotted curves represent contributions from the process (1) and (2) with $\lambda=1$ and $\lambda=0$, respectively. b) Argand diagram for the toy fit results near the $Z^+(4430)$ region.](image)

The phase of the Breit-Wigner-like structure near $M^2(\psi(2S)\pi^+) \sim M^2_{Z^+(4430)}$, obtained from our fit, is shown as Argand diagram in figure 2b in the same $M^2(\psi(2S)\pi^+)$ bins as in LHCb paper [6]. Initial phase is set to an arbitrary value, while in the LHCb case it is fixed from the 4-dimensional fit. The Breit-Wigner-like variation around the peak comes from the $D_s^{(*)}\rho_-$ decay’s Breit-Wigner amplitude. However, the direction of the phase motion is flipped with respect to those of usual Breit-Wigner: it tends to rotate clockwise in the Argand diagram. Since there is an ambiguity ($A \leftrightarrow \bar{A}$) in the extraction of the $Z^+(4430)$ amplitude from the measured $|A(Z^+) + A(\text{non}-Z^+)|^2$, our result does not contradict to the LHCb data.

Using the toy fit described above, one can extract the parameters of the $D_s^{(*)}$, which turn out to be well statistically constrained: $M = (2614 \pm 4)\text{ MeV}$, $\Gamma = (92 \pm 10)\text{ MeV}$. One of the main sources of the systematic uncertainties in this study comes from the effect of interference with the $K^{*(s)}$ background. This source was estimated by ascribing different phases to the amplitudes of $K^*(890)$, $K_2^*(1430)$ and $s$-wave three-body phase space and performing another
fit with varying $D_0^{*-}$ mass and width. Variations of the best fit $D_0^{*-}$ parameters depending on the $K^{*+}$ phases are estimated to be $\pm 10$ MeV for the $D_0^{*-}$ mass and $\pm 20$ MeV for its width. Thus, parameters of $D_0^{*-}$ meson are: $M = (2614 \pm 4^{+20}_{-13})$ MeV, $\Gamma = (92 \pm 10 \pm 10)$ MeV.

The discussed approach could be ultimately tested by performing 4-dimensional fit by either Belle, BaBar or LHCb for $B \to \psi(2S)\pi K^-$ decays using amplitudes (3) instead of resonance-like $Z^+$'s. While this analysis could be complicated, there is another simple way to test our hypothesis: the $Z^+$-like structures should appear in the distributions of $M(D^+\bar{D}^0) \times \cos^2(\theta_{\text{form}})$ in either $B \to \bar{D}^*0 D^+ K^-$ or $B \to \bar{D}^0 D^+ K^-$ decays, or in both. The $M(D^+\bar{D}^0) \times \cos^2(\theta_{\text{form}})$ is the $(D^+\bar{D}^0)^*$ combination mass spectrum corrected in each bin for the fraction of the $D^*$ transverse component in the $(D\bar{D})^*$ rest frame, and also the $1^+$ formation factor $D^2(\theta_{\text{form}}) = \cos^2(\theta_{\text{form}})$.

If our hypothesis is correct, once the rescattering amplitude is known from $B$ decays, it should be universal in other processes, where a $D\bar{D}$ pair produced. Among such process are the charmed meson production in the $e^+e^-$ annihilation, $\gamma\gamma$ fusion etc. In particular, the process $e^+e^- \to D\bar{D}^*\pi$ can result via rescattering into $e^+e^- \to J/\psi(2S)\pi\pi$ final state, which is observed and remains puzzling for almost a decade [2]. Unfortunately, the former process has not been studied in details experimentally: the cross section of $e^+e^- \to D\bar{D}^*$ has been measured only by Belle [19] with not sufficient precision. Moreover, to check that rescattering hypothesis works to explain the $e^+e^- \to J/\psi(2S)\pi\pi$ cross section, one needs the measured helicity decomposition of the $D\bar{D}^*$ final state.

On the other hand, the recent observation of charmonium-like pentaquark by LHCb [20] suggests the similar explanation of the peaking structure observed in the $J/\psi p$ invariant mass distribution. Indeed, if the rescattering $\Lambda_c D \to J/\psi p$ is not small the underlying process that leads to the peaking structure in $J/\psi p$ mass could be the conventional decay $\Lambda_c \to \Lambda_c D^* \to \Lambda_c DK$.

In summary, we suggest a model, which is able to describe the observed features in the $\psi(2S)\pi^+$ spectrum from the $B \to \psi(2S)\pi^+ K^-$ without introducing of exotic multiquark states. A toy fit to the published LHCb data demonstrates the plausibility of our approach. A critical test for this hypothesis is suggested.

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References
[1] Choi S-K et al. (Belle Collaboration) 2003 Phys. Rev. Lett. 91 262001
[2] Pakhlova G V, Pakhlov P N and Eidelman S I 2010 Phys. Usp. 33 219
[3] Choi S-K et al. (Belle Collaboration) 2008 Phys. Rev. Lett. 100 142001
[4] Mizuk R et al. (Belle Collaboration) 2009 Phys. Rev. D 80 031104
[5] Aubert B et al. (BaBar Collaboration) 2009 Phys. Rev. D 79 112001
[6] Aaij R et al. (LHCb Collaboration) 2014 Phys. Rev. Lett. 112 222002
[7] Liu X-H, Zhao Q and Close F E 2008 Phys. Rev. D 77 094005
[8] Dubynskiy S and Voloshin M B 2008 Phys. Lett. B 666 344
[9] Liu X et al. 2008 Phys. Rev. D 77 034003
[10] Bugg D V 2008 J. Phys. G 35 075005
[11] Rosner J L 2007 Phys. Rev. D 76 114002
[12] Meng G Z et al. 2009 (CLQCD Collaboration) Phys. Rev. D 80 034503
[13] Chen D-Y and Liu X 2011 Phys. Rev. D 84 094003
[14] Pakhlov P 2011 Phys. Lett. B 702 139
[15] Pakhlov P and Uglov T 2015 Phys. Lett. B 748 183
[16] Chilikin K et al. (Belle Collaboration) 2013 Phys. Rev. D 88 074026
[17] Chilikin K et al. (Belle Collaboration) 2014 Phys. Rev. D 90 112009
[17] Brodzicka J et al. (Belle Collaboration) 2008 Phys. Rev. Lett. 100 092001
[18] Olive K A et al. (Particle Data Group) 2014 Chin. Phys. C 38 090001
[19] Pakhlova G et al. (Belle Collaboration) 2009 Phys. Rev. D 80 091101
[20] Aaij R et al. (LHCb Collaboration) 2015 Phys. Rev. Lett. 105 115