Dilatons in Dense Baryonic Matter

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We discuss the role of dilaton, which is supposed to be representing a special feature of scale symmetry of QCD, trace anomaly, in dense baryonic matter. The idea that the scale symmetry breaking of QCD is responsible for the spontaneous breaking of chiral symmetry is presented along the similar spirit of Freund-Nambu model. The incorporation of dilaton field in the hidden local symmetric parity doublet model is briefly sketched with the possible role of dilaton at high density baryonic matter, the emergence of linear sigma model in dilaton limit.

Keywords: dilaton, QCD, scale anomaly, hidden local symmetry, baryonic matter

1. Introduction

The recent discovery of Higgs-like particles at LHC activates many interesting research not only on phenomenology but also on the origin of Higgs particle. In the framework of Standard Model the masses of leptons and quarks are explained by Higgs mechanism. But the question on how masses of hadrons are generated has not been satisfactorily answered yet. Since the light quark masses are too small to explain the hadron mass, the proton mass, for example, it is believed that the masses of hadrons are generated dynamically in the framework of quantum chromodynamics (QCD).

If we ignore quark masses, a very good approximation for the hadrons consisting of light quarks), QCD has no scale at the classical level, but a scale emerges at the quantum level in the form of trace anomaly, an explicit breaking of scale symmetry. On the other hand, the spontaneous symmetry breaking of chiral symmetry ($\chi_{SB}$) gives rise to another important scale determined by the quark condensate, $\langle \bar{q}q \rangle$, which is supposed to generate most of the hadron masses. Thus it is natural to ask whether the chiral symmetry breaking and the scale symmetry breaking (SSB) are linked to each other and if so, how intricately.

The idea as to how $\chi_{SB}$ is tied to SSB has been discussed recently in connection to the discovery of Higgs-like particles in LHC. In the technicolor scheme, the explicit $\chi_{SB}$ in heavy quark sector triggers the spontaneous scale-symmetry breaking (SSB),
of which a Nambu-Goldstone boson is identified as a Higgs-like particle. It turns out
that, in this walking technicolor scheme, the phenomenology of Goldston boson of
SSB can be differentiated from Higgs in the standard model in more detailed analysis
of LHC data. And this development may shed new light on new mechanism of mass
generation for hadrons.

In hadronic sector, the idea that the scale symmetry and chiral symmetry are tied
to each other has been explored in the line of thought that the explicit breaking of
scale symmetry due to the QCD anomaly triggers the spontaneous $\chi$SB. Assuming
that the QCD anomaly can be decomposed into two parts, "hard" and "soft," the
explicit symmetry breaking due to the soft part can be treated as small enough so
that there is an approximate scale symmetry at low energy scale. Then the dilaton
emerges as a Nambu-Goldstone boson as argued in the work of Freund and Nambu.

We can construct a phenomenological Lagrangian which implements the above
idea by introducing dilaton field into the Lagrangian of pions and vector mesons
constructed under the hidden local gauge symmetry principle. One of the interesting
consequences is the scaling of Lagrangian parameters: masses and coupling constants
are dialed by the change of vacuum expectation value of dilaton field, i.e., BR
scaling. The possible forms of scaling laws have been explored in various contexts.
One of them is the observation of a half-skyrmion phase when skyrmions are put
on the lattice which can be implemented into a new scaling law giving a stiffer
equation of state. Another interesting feature of dilaton field is the dilaton limit,
that leads to the condition that a particular combination of pions and dilaton lead
to the linear sigma model giving rise to a nontrivial scaling behavior.

In section 2, the work of Freund and Nambu is briefly sketched to develop
a model in which pions and dilaton are introduced to take care of spontaneous
breaking of both chiral symmetry and approximate scale symmetry. We introduce
the phenomenological Lagrangian, denoted dHLS, which implements the above idea
by introducing dilaton field into the Lagrangian constructed under the hidden local
gauge symmetry principle. In section 3, we elaborate on how a dilaton field can
be applied to the entangled transformation of pions in nonlinear realization of $\chi$SB
at lower scale such that the chiral symmetry is linearly realized in the form of linear
sigma model. The summary is given in section 4.

2. Dilaton and Chiral Symmetry Breaking
The first example which demonstrates the possibility of spontaneous symmetry
breaking of the scale symmetry and its physical application of the dilaton field
is the Freund-Nambu model. It has two real scalar fields, $\psi$ and $\phi$, with the
potential

$$V(\psi, \phi) = V_a + V_b,$$

where

$$V_a = \frac{1}{2} f^2 \psi^2 \phi^2, \quad V_b = \frac{\tau}{4} \left( \frac{\phi^2}{g^2} - \frac{1}{2} \phi^4 - \frac{1}{2g^2} \right) = \frac{\tau}{8g^4} (g^2 \phi^2 - 1)^2.$$
\( V_a \) is scale invariant but \( V_b \) includes scale symmetry breaking terms \( \frac{\phi^2}{2g^2} \) and \(-\frac{1}{2g^4}\).

One can introduce the new field, \( \chi \), defined by

\[
\chi = \frac{(g^2 \phi^2 - 1)/(2g)}{(2g^2)}
\]

of which the scale transformation

\[
\delta \chi = \epsilon (x \cdot \partial + 2) \chi + \frac{\epsilon}{g}
\]

manifests one of the characteristics of Nambu-Goldstone bosons. In terms of the condensate \( \phi_0 \) given by the minimum of the potential, \( \phi_0 = 1/g \), the masses take the form

\[
m_\psi^2 = f^2 \phi_0^2, \quad m_\chi^2 = \tau \phi_0^2.
\]

It should be noted that the mass of \( \psi \) comes from the scale invariant term \( V_a \), so is independent of \( \tau \). Thus it can have an arbitrary value. However \( m_\chi \) depends on \( \tau \), going to zero linearly as \( \tau \to 0 \). This is known to be a characteristic of an approximate spontaneously-broken scale symmetry.

Let us consider the Lagrangian of linear sigma model with \( \Phi = \sigma + i \tau \cdot \vec{\phi} \) with a potential of the following form

\[
V(\phi) = \lambda \left( \frac{1}{2} Tr \Phi \Phi^\dagger - v^2 \right)^2 = \lambda (\sigma^2 + \sum_i \phi_i^2 - v^2)^2.
\]

The scale invariance is broken due to nonvanishing \( v^2 \). But it is also this term which breaks chiral symmetry spontaneously. When we assign the vacuum expectation only to \( \sigma, \phi_i, i = 1, 2, 3 \) become Nambu-Goldstone bosons related to the spontaneous chiral symmetry breaking and we call them pions, \( \pi \). For \( \lambda = 0 \), there is no explicit breaking of scale symmetry regardless of the value of \( v^2 \). We introduce a scalar field, \( \chi \), and reformulate the above potential in the following form,

\[
V(\phi, \chi) = \lambda \left( \frac{1}{2} Tr \Phi \Phi^\dagger - \chi^2 \right)^2 + V(\chi)
\]

The first term is analogous to \( V_a \) in eq.(2), which is scale invariant. The second term is to break scale symmetry. In general, \( V_\chi \) gives nonvanishing vacuum expectation value for \( \chi \), which can be of the following forms

\[
V_\chi = \frac{\tau}{8} (\chi^2 - v^2)^2, \quad \text{or} \quad V_\chi = \tau \chi^4 \ln \frac{\chi}{e^{1/4} v}.
\]

One can see that the scale symmetry breaking in eq.(6) is transferred to the second term of eq.(7), which plays a similar role as eq.(2). If the scale symmetry breaking term with \( \tau \) is small enough such that there is an approximate scale symmetry, then it can be considered as spontaneously broken and associate a Nambu-Goldstone boson of the form

\[
\eta = (\chi^2 - v^2)/2v, \quad \text{or} \quad \chi = ve^{\eta/v}.
\]

The origin of scale symmetry breaking in hadron physics can be traced back to the QCD trace anomaly and a dilaton field can be associated with the process.
eq. (7) reduces to eq. (6). It is now clear in this formulation that the spontaneous chiral symmetry breaking is not possible without scale symmetry breaking. Or put differently, scale symmetry breaking is responsible for the chiral symmetry breaking. This indicates one of the possible ways that the trace anomaly of QCD can be linked to chiral symmetry breaking.

The chiral symmetry, which is broken spontaneously, can be realized non-linearly by introducing \( U(\pi) = e^{i \vec{\pi} \cdot \vec{\tau}/F_\pi} \) instead of four-component scalar fields, \( \Phi \). The pions \( \pi_i, \ i = 1, 2, 3 \) transform non-linearly under the broken sector of chiral transformation, with the pions derivatively coupled. The lowest derivative term of pion dynamics (namely the current algebra term) can be written

\[
\mathcal{L} = \frac{F_\pi^2}{4} \text{tr} \left[ \partial_\mu U \partial^\mu U^\dagger \right].
\]  

(10)

This term breaks scale symmetry explicitly because of the explicit scale \( F_\pi \). This scale can be traced back to the vacuum expectation value of the scalar field \( \chi \) as in eq. (7) with \( F_\pi = v \) and the scale symmetry breaking can be traded in the potential \( V(\chi) \)

\[
\mathcal{L}(U, \chi) = \frac{\chi^2}{4} \text{tr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] + V(\chi) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi.
\]  

(11)

Defining a new field

\[
\Sigma \equiv \chi U(\pi),
\]  

(12)

we can see that

\[
\mathcal{L}(U, \chi) \rightarrow \mathcal{L}(\Sigma) = \frac{1}{4} \text{tr} \left[ \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] + V(\Sigma \Sigma^\dagger),
\]  

(13)

where we have used \( \chi = \frac{1}{7} \text{Tr} \Sigma \Sigma^\dagger \). If we can identify \( \Sigma \) as

\[
\Sigma = \sigma + i \vec{\pi} \cdot \vec{\tau},
\]  

(14)

then it is nothing but the linear sigma model. However it is not clear how the linear representation of nonlinear field can be achieved by adding a dilaton, which is chiral singlet. Apart from the theoretical justification, we may expect this can be achieved physically in a certain limit, for example in baryonic matter with higher density. This is the issue that concerns the “dilaton limit” discussed in\(^9\,^\text{10}\).

3. Dilaton Limit of dHLS

It has been observed that the dilaton limit leads to a very interesting consequence when nucleons are included in the chiral Lagrangian. The vector coupling constant is found to have a limiting value \( g \rightarrow 1 \) in the dilaton limit. This idea has been extended\(^\text{13}\) to the effective theory where dilaton is incorporated into the hidden local symmetry theory, where vector mesons are present as explicit degrees of freedom. In this section the results are briefly sketched.
The dilaton field which is responsible for the spontaneous symmetry breaking of chiral symmetry, thus generating pions, can be naturally included in the effective Lagrangian at low baryon density. The redundant symmetry, i.e., hidden local symmetry (HLS), appearing in the formalism of nonlinear realization of spontaneous chiral symmetry breaking has been used to incorporate the vector mesons systematically. However the Lagrangian of HLS is, by construction, noninvariant under scale symmetry. The pattern of symmetry breaking is thus not consistent with the scale symmetry breaking due to QCD trace anomaly, which is believed to be the only reason why the effective theory of QCD should have scale symmetry breaking term modulo the light quark mass. The dilated HLS is constructed such that all terms in the Lagrangian are scale invariant except for the potential of dilaton, that reproduces the relevant scale symmetry breaking for QCD trace anomaly. One of the simplest ways to construct the dilated HLS is to follow the standard trick of inserting the dilaton field \( \chi \) as “conformal compensator” into the Lagrangian to obtain scale symmetric Lagrangian, given by

\[
\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_\chi, 
\]

\[
\mathcal{L}_N = \bar{Q} i \gamma^\mu D_\mu Q - g_1 F_\pi \frac{\chi}{F_\chi} \bar{Q} Q + g_2 F_\pi \frac{\chi}{F_\chi} \bar{Q} \rho_3 Q - i m_0 \bar{Q} \rho_2 \gamma_5 Q + g_\pi \bar{Q} \gamma^\mu \hat{\alpha}_{\parallel \mu} Q + g_\rho \bar{Q} \rho_3 \gamma^\mu \hat{\alpha}_{\perp \mu} Q,
\]

\[
\mathcal{L}_M = \frac{F_\pi^2}{F_\chi^2} \chi^2 \left[ \hat{\alpha}_{\perp \mu} \hat{\alpha}_{\perp \mu}^\dagger \right] + \frac{F_\sigma^2}{F_\chi^2} \chi^2 \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \mu}^\dagger \right] + \frac{F_\sigma^2 - F_\pi^2}{2F_\chi^2} \chi^2 \left[ \hat{\alpha}_{\parallel \mu} \right] \left[ \hat{\alpha}_{\parallel \mu}^\dagger \right] - \frac{1}{2} \left[ \rho_{\mu \nu} \rho_{\mu \nu}^\dagger \right] - \frac{1}{2} \left[ \omega_{\mu \nu} \omega_{\mu \nu}^\dagger \right],
\]

\[
\mathcal{L}_\chi = \frac{1}{2} \partial_\mu \chi \cdot \partial^\mu \chi - V(\chi)
\]

where \( V(\chi) \) is the dilaton potential that breaks scale symmetry spontaneously and \( F_\chi \) is the vacuum expectation value of \( \chi \) at zero temperature and density. The detailed expressions needed in this section can be found in ref.\(^\text{14}\). As in the previous section, we do the field re-parameterizations \( \Sigma_s = U \chi \frac{F_\pi}{F_\chi} = s + i \vec{\tau} \cdot \vec{\pi} \). And using the nonlinearly transforming nucleon field of parity eigenstates, one finds a complicated expression for \( \mathcal{L}_{\text{sing}} \), composed of a part that is regular, \( \mathcal{L}_{\text{reg}} \), and a part that is singular, \( \mathcal{L}_{\text{sing}} \), as \( \text{tr}(\Sigma_s \Sigma_s^\dagger) = 2 \left( s^2 + \pi^2 \right) \to 0 \), where \( s \) is iso-spin index. The singular part that arises solely from the scale invariant part of the original Lagrangian \( \mathcal{L}_{\text{sing}} \) has the form

\[
\mathcal{L}_{\text{sing}} = (g_{V_{\rho} - g_A} A \left( 1/\text{tr} \left[ \Sigma_s \Sigma_s^\dagger \right] \right) + (\alpha - 1) B \left( 1/\text{tr} \left[ \Sigma_s \Sigma_s^\dagger \right] \right),
\]

One can see that the condition \( \alpha \equiv \frac{F_\pi^2}{F_\chi^2} = 1 \) has been already used in getting Eq. \( \text{(13)} \). That \( \mathcal{L}_{\text{sing}} \) be absent also leads to the condition that

\[
g_{V_{\rho} - g_A} \to 0.
\]

Using large \( N_c \) sum-rule arguments\(^\text{9}\) and the RGE\(^\text{10}\), we infer

\[
g_A - 1 \to 0.
\]
It follows then that
\[ g_{\rho NN} = g_{\rho}(g_{V\rho} - 1) \to 0. \] (22)
This set of limits defines what is referred to as “dilaton limit.” We thus find that in the dilaton limit, the \( \rho \) meson decouples from the nucleon. In contrast, the limiting \( \text{tr}(\Sigma_s \Sigma_s^\dagger) \to 0 \) does not give any constraint on \( (g_{V\omega} - 1) \). The \( \omega \)-nucleon coupling remains non-vanishing in the Lagrangian of fluctuations \( \hat{s} \) and \( \hat{\pi} \) around their expectation values, which in terms of the mass-diagonalizing field \( \mathcal{N} \), takes the form
\[ \mathcal{L}_N = \bar{\mathcal{N}} i \partial \mathcal{N} - \bar{\mathcal{N}} \hat{M} \mathcal{N} - g_1 \bar{\mathcal{N}} \left( \hat{G} \hat{s} + \rho_3 \gamma_5 i \vec{\tau} \cdot \vec{\pi} \right) \mathcal{N} \\
+ g_2 \bar{\mathcal{N}} \left( \rho_3 \hat{s} + \bar{G} \gamma_5 i \vec{\tau} \cdot \vec{\pi} \right) \mathcal{N} + (1 - g_{V\omega}) g_\omega \bar{\mathcal{N}} \hat{\omega} \mathcal{N}. \] (23)
This is just the nucleon part of the linear sigma model in which the \( \omega \) is minimally coupled to the nucleon.

4. Summary

The origin of scale symmetry breaking in hadron physics can be traced back to the QCD trace anomaly, and it can be implemented into an effective theory by introducing a dilaton field. The idea that the scale symmetry and chiral symmetry are tied to each other has been explored in a line of thought that the explicit breaking of scale symmetry due to the QCD anomaly triggers the spontaneous \( \chi_S \text{SB} \). We demonstrate that using a model similar to that of Freund-Nambu the spontaneous chiral symmetry breaking is locked to the scale symmetry breaking through dilaton field. This provides one of the possible ways to relate the trace anomaly of QCD to chiral symmetry breaking.

It is possible to construct a phenomenological Lagrangian that encapsulates the above idea by introducing dilaton field into the Lagrangian of pions and vector mesons constructed under the hidden local gauge symmetry principle with parity doubled baryons. One of the interesting consequences is the scaling of parameters of the Lagrangian: masses and coupling constants are dialed by the change of vacuum expectation value of dilaton field as in BR scaling. Another interesting feature of dilaton field is the dilaton limit that gives the condition that a particular combination of pions and dilaton lead to the linear sigma model, giving rise to a non-trivial scaling behavior. Most of the conventional effective actions constructed for low-density phenomena without dilaton available in the literature do not reveal the above feature. To realize the linear realization feature, the coupling constants should satisfy the particular condition. A highly non-trivial support for the dilaton limit properties comes from a renormalization group analysis of hidden local symmetry implemented with baryons. An intriguing consequence of the analysis is that the \( \rho \) meson nucleon coupling vanishes at critical density corresponding to the dilaton limit before reaching chiral restoration.
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