Semileptonic and leptonic B decays, circa 2016

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Abstract. We summarize the status of semileptonic and leptonic B decays, including $|V_{cb}|$ and $|V_{ub}|$ exclusive and inclusive determinations, decays to excited states of the charm meson spectrum and decays into $\tau$ leptons.

Keywords: QCD; heavy flavour; B decays

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INTRODUCTION

Today accuracy in measurements and theoretical calculations is indispensable to check the Standard Model (SM) and explore the small region of parameters space left to its extensions, at our energies. Semi-leptonic and leptonic B decays are well suited to respond to such necessity. The heavy mass of the $B$ meson allows to exploit simplifications in the limit of infinite quark mass and to better separate low and high energy regimes. Past, present and future $B$ factories have provided and will provide an unparalleled level of precision in branching ratios and related observables, and LHCb is following through. One example is given by the determination of the values of the CKM parameters $|V_{cb}|$ and $|V_{ub}|$, which strongly affects the identification of new physics [1]. The inclusive and exclusive semi-leptonic searches rely on different theoretical calculations and on different experimental techniques which have, to a large extent, uncorrelated statistical and systematic uncertainties. This independence makes the comparison of $|V_{cb}|$ and $|V_{ub}|$ values from inclusive and exclusive decays an interesting test of our physical understanding. Extensive investigations have strongly reduced the uncertainties on $|V_{cb}|$ and $|V_{ub}|$ (amounting to about 5% and 15%, respectively, in the 1999 LEP determinations [11]), but a tension between inclusive and exclusive determination remains to this day.

Here, we summarize the status of semileptonic and leptonic B decays, mediated at lower parton level by tree diagrams, including decays to excited states of the charm meson spectrum and decays into $\tau$ leptons.

HEAVY-TO-HEAVY SEMILEPTONIC DECAYS

Exclusive decays into light leptons

For negligible lepton masses ($l = e, \mu$), the differential ratios for the semi-leptonic CKM favoured decays $B \to D^{(*)} l \nu$ can be written as

\[
\frac{d\Gamma}{d\omega}(B \to D^* l \nu) = \frac{G_F^2}{48\pi^3} (m_B - m_{D^*})^2 m_D^2 \chi(\omega)(\omega^2 - 1)^2 |V_{cb}|^2 |\eta_{EW}|^2 |\mathcal{F}(\omega)|^2
\]

\[
\frac{d\Gamma}{d\omega}(B \to D l \nu) = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^2 |V_{cb}|^2 |\eta_{EW}|^2 |\mathcal{F}(\omega)|^2
\]

in terms of a single form factor $\mathcal{F}(\omega)$ and $\mathcal{G}(\omega)$, for $B \to D^* l \nu$ and $B \to D l \nu$, respectively. In Eq. (1), the differential cross sections are proportional to $|V_{cb}|^2$, $\eta_{EW}$ is a structure-independent one-loop electroweak enhancement factor and

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1 For recent reviews see e.g. R. Kowalewski and T. Mannel in [2], Refs. [3, 4, 5, 6, 7, 8, 9, 10] and references therein.
\( \chi(\omega) \) is a phase space factor which reads

\[
\chi(\omega) = (\omega + 1)^2 \left( 1 + \frac{4\omega}{\omega + 1} \frac{m_B^2 - 2\omega m_B m_\pi^* + m_\pi^2}{(m_B - m_\pi^*)^2} \right)
\]

The recoil parameter \( \omega = p_B \cdot p_{D^*}/m_B m_{p_{D^*}} \) corresponds to the energy transferred to the leptonic pair. In the heavy quark limit both form factors are related to a single Isgur-Wise function, \( \mathcal{F}(\omega) = \mathcal{G}(\omega) = \xi(\omega) \), which is normalized to unity at zero recoil, that is \( \xi(\omega = 1) = 1 \). There are perturbative and nonperturbative corrections to this prediction, the latter suppressed by powers of \( \Lambda_{\text{QCD}}/m \), where \( m = m_c \) and \( m_b \).

At zero recoil, the heavy quark symmetries provide the structure of the symmetry breaking non-perturbative corrections, which start at order \( 1/m^2 \) and \( 1/m \) for the \( \mathcal{F}(\omega = 1) \) and \( \mathcal{G}(\omega = 1) \) form factors, respectively. The downside of zero-recoil analyses is that, since decay rates vanish at zero-recoil, one needs to extrapolate the experimental points to zero recoil, using a parameterization of the momentum dependence. There are several parameterizations for the momentum dependence of the form factors, that generally fall under two categories i) based on a simple pole, as the BZ (Ball-Zwicky) [12] at 4 parameters, or the BK (Becirevic-Kaidalov) at 3 parameters [13] ii) based on a series expansion, where \( \omega \) is mapped onto a complex variable \( z(\omega) \) via the conformal transformation \( z = \frac{\sqrt{\omega + 1} - \sqrt{\omega}}{\sqrt{\omega + 1} + \sqrt{\omega}} \). The expansion in \( z \) converges rapidly in the kinematical region of heavy hadron decays. Common examples are the CLN (Caprini-Lellouch-Neubert) [14], the BGL (Boyd-Grinstein-Lebed) [15] and the BCL (Bourrely-Caprini-Lellouch) [16] parameterizations.

Several computations of form factors have been performed on lattice. The difficulties related to heavy fermions can be naïvely summarized by observing that direct simulation of high mass such \( ma \geq 1 \), where \( a \) represent a lattice spacing, gives discretization errors out of control. As of today \( m_{2l} \sim 1/a \) and no direct simulation is possible. The main way out is the usage of effective theories, as Heavy Quark Effective Theory (HQET) [17] and Non-Relativistic QCD (NRQCD) [18]. In broad terms, they eliminate high degrees of freedom relying on a systematic expansion in \( \Lambda_{\text{QCD}}/m_b \). The downside is the introduction of new sources of errors (matching of HQET to QCD, renormalization, control of extrapolation, etc.) to take care of.

Another common approach to non-perturbative calculations of form factors are QCD sum rules. The sum rules are based on the general idea of calculating a relevant quark-current correlation function and relating it to the hadronic parameters of interest via a dispersion relation. They have reached wide application for calculation of exclusive amplitudes and form factors in the form of light cone sum rules (LCSR), employing light-cone operator product expansion (OPE) of the relevant correlation functions. Uncertainties may originate from the truncation of the expansions, the input parameter uncertainties, and the assumption of quark-hadron duality.

Let us first consider the \( B \to D^*/l^+\nu \) channel, which is less suppressed in the phase space and whose branching fractions are more precise (even twice) in the majority of experimental measurements. On lattice, the progress on the \( B \to D^* \) form factor is slower, since this channel poses greater technical complications than the \( B \to D \), due to the fact that the \( D^* \) is unstable. The FNAL/MILC collaboration has performed the non perturbative determination of the form factor \( \mathcal{F}(1) \) in the lattice unquenched \( N_f = 2 + 1 \) approximation [19, 20]. The FNAL/MILC collaboration uses FNAL \( b \)-quark and asqtad \( u, d, s \) valence quarks. Their latest result at zero recoil exploits the full suite of MILC (2+1)-flavor asqtad ensembles for sea quarks and gives the following estimate [20].

\[
\mathcal{F}(1) = 0.906 \pm 0.004 \pm 0.012
\]

The first error is statistical and the second one is the sum in quadrature of all systematic errors. Using the previous form factor and the 2012 HFAG average [21] the following estimate for \( |V_{cb}| \) has been given [20]

\[
|V_{cb}| = (39.04 \pm 0.49_{\text{exp}} \pm 0.53_{\text{latt}} \pm 0.19_{\text{QED}}) \times 10^{-3}
\]

which is reported in Table 1. The central value is not very different from the central value of the 2009 determination from the same Collaboration [19], but errors are considerably reduced. The analysis strategies are similar, but the lattice-QCD data set is much more extensive, with higher statistics on all ensembles, smaller lattice spacings (as small as \( 0.045 \text{ fm} \)) and light-to-strange-quark mass ratios can be as low as \( 1/20 \). The lattice QCD theoretical error is now commensurate with the experimental error, they contribute respectively for about 1.4% and 1.3%, while the QED error contributes for about 0.5%. Largest QCD errors come from discretization and are estimated taking the difference between HQET description of lattice gauge theory and QCD. The discretization error could be in principle reduced by going to finer lattice spacings or by using a more improved Fermilab action. Subleading errors appearing at the 0.4-0.6% level are nontrivial to improve. Reducing the error from the QED Coulomb correction would require a detailed
study of electromagnetic effects within HQET, and reducing the QCD matching error would require a two-loop lattice perturbation theory calculation or nonperturbative matching [20].

Other, preliminary, values for the $B \to D^*$ form factor at zero recoil, in agreement with the value reported in (3), were also obtained at $N_f = 2$, by using charmed quarks having a realistic finite mass and two ensembles of gauge configurations produced by the European Twisted Mass Collaboration (ETM) [22].

The 2016 FLAG $N_f = 2 + 1$ $|V_{cb}|$ average value yields [23]

$$|V_{cb}| = (39.27 \pm 0.49 \exp \pm 0.56_{\text{latt}}) \times 10^{-3}$$

(5)

This average, reported in Table 1, employs the latest HFAG experimental average [24] $\mathcal{F}(1) \eta_{EW} |V_{cb}| = (35.81 \pm 0.45) \times 10^{-3}$, the value $\eta_{EW} = 1.00662$, and the value $\mathcal{F}(1)$ given in Eq. (3).

At the current level of precision, it would be important to extend form factor unquenched calculations for $B \to D^*$ semileptonic decays to nonzero recoil, in order to reduce the uncertainty due to the extrapolation to $\omega = 1$. Indeed, at finite momentum transfer, only old quenched lattice results are available [25] which, combined with 2008 BaBar data [26], give $|V_{cb}| = 37.4 \pm 0.5_{\exp} \pm 0.8_{\text{th}}$.

Older form factor estimates are available via zero recoil sum rules, giving [27, 28]

$$\mathcal{F}(1) = 0.86 \pm 0.02$$

(6)

in good agreement with the lattice value in Eq. (3), but slightly lower in the central value. That implies a relatively higher value of $|V_{cb}|$, that is

$$|V_{cb}| = (41.6 \pm 0.6_{\exp} \pm 1.9_{\text{th}}) \times 10^{-3}$$

(7)

where the HFAG averages [21] have been used. The theoretical error is more than twice the error in the lattice determination (4).

For $B \to D l \nu$ decay, unquenched lattice-QCD calculation of the hadronic form factors at nonzero recoil have become available in 2015, due to the FNAL/MILC collaboration [29]. Prior results at non-zero recoil were only available in the quenched approximation [30, 31] 2.

In [29] the FNAL/MILC collaboration has calculated the form factors for a range of recoil momenta and parameterized their dependence on momentum transfer using the BGL z-expansion, determining $|V_{cb}|$ from the relative normalization over the entire range of recoil momenta. Their estimate gives [29]

$$|V_{cb}| = (39.6 \pm 1.7_{\exp + \text{QCD}} \pm 0.2_{\text{QED}}) \times 10^{-3}$$

(8)

The average value is almost the same than the one inferred from $B \to D^* l \nu$ decay by the same collaboration, see Eq. (4) and Table 1.

Two months later, new results on $B \to D l \nu$ form factors at non-zero recoil were announced by the HPQCD Collaboration [33]. Their results are based on the non-relativistic QCD (NRQCD) action for bottom and the Highly Improved Staggered Quark (HISQ) action for charm quarks, together with $N_f = 2 + 1$ MILC gauge configuration. A joint fit to lattice and 2009 BaBar experimental data [32] allows the extraction of the CKM matrix element $|V_{cb}|$, which reads

$$|V_{cb}| = (40.2 \pm 1.7_{\text{latt + stat}} \pm 1.3_{\text{syst}}) \times 10^{-3}$$

(9)

The first error consists of the lattice simulation errors and the experimental statistical error and the second error is the experimental systematic error. The dominant error is the discretization error, followed by higher order current matching uncertainties. The former error can be reduced by adding simulation data from further ensembles with finer lattice spacings.

The same year, in order to interpret the $\Delta \Gamma/\Delta \omega$ distribution, the Belle collaboration [34] has performed a fit to the CLN parameterization, which has two free parameters, the form factor at zero recoil $\mathcal{F}(1)$ and the linear slope $\rho^2$. The fit has been used to determine $\eta_{EW} \mathcal{F}(1) |V_{cb}|$, that, divided by the form-factor normalization $\mathcal{F}(1)$ found by the FNAL/MILC Collaboration [29], gives $\eta_{EW} |V_{cb}| = (40.12 \pm 1.34) \times 10^{-3}$ [34]. Assuming $\eta_{EW} \simeq 1.0066$, it translates into [34]

$$|V_{cb}| = (39.86 \pm 1.33) \times 10^{-3}$$

(10)

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2 By using the step scaling approach and the 2009 data from BaBar Collaboration, for $B \to D l \nu$ decays [32], the value $|V_{cb}| = 37.4 \pm 0.5_{\exp} \pm 0.8_{\text{th}}$ was obtained. The errors are statistical, systematic and due to the theoretical uncertainty in the form factor $\mathcal{F}$, respectively.
The Belle Collaboration also obtain a slightly more precise result (2.8% vs. 3.3%) by exploiting lattice data at non-zero recoil and performing a combined fit to the BGL form factor. It yields $\eta_{EW} |V_{cb}| = (41.10 \pm 1.14) \times 10^{-3}$ which translates into \[34\]

$$|V_{cb}| = (40.83 \pm 1.13) \times 10^{-3}$$ \hspace{1cm} (11)

assuming once again $\eta_{EW} \simeq 1.0066$.

A very recent $|V_{cb}|$ determination is in agreement with previous results, being \[35\]

$$|V_{cb}| = (40.49 \pm 0.97) \times 10^{-3}$$ \hspace{1cm} (12)

It makes use of latest lattice \[29,33\], Belle \[34\] and Babar \[32\] results, and consider the BGL, CLN, and BCL parameterizations, checking that they all yield consistent results.

Semileptonic $B_d$ decays can also probe CKM matrix elements. Moreover, semileptonic $B^0_d$ decays are also used as a normalization mode for various searches for new physics at hadron colliders and at Belle-II. On lattice, the valence strange quark needs less of a chiral extrapolation and is better accessible in numerical simulations with respect to the physical $u(=d)$-quark. Zero-recoil form factors for $B_d \rightarrow D_s\ell\nu$ decays have now been computed at $N_f = 2$, for the first time, using the ETM (European twisted mass) approach \[36\]. They employ lattice spacings within the range $a \simeq 0.054$ to 0.098 fm, using the maximally twisted Wilson quark action and obtain the result $\mathcal{F}(1)^{B_d \rightarrow D_s} = 1.052 \pm 0.046$, which has an uncertainty of 4%.

### Exclusive decays into heavy leptons

The $B \rightarrow D^{(*)}\tau\nu_\tau$ decays are more difficult to measure, since decays into the heaviest $\tau$ lepton are phase space suppressed and there are multiple neutrinos in the final state, following the $\tau$ decay, which stand in the way of the reconstruction of the invariant mass of $B$ meson.

The ratio of branching fractions

$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\nu_\ell)} \hspace{1cm} (l = e, \mu)$$ \hspace{1cm} (13)

is typically used instead of the absolute branching fraction of $B \rightarrow D^{(*)}\tau\nu_\tau$ decays, to reduce several systematic uncertainties such as those on the experimental efficiency and on the form factors, and to eliminate the dependence on the value of $|V_{cb}|$. In 2012, FNAL/MILC has published the first unquenched lattice determination of $\mathcal{R}(D)_{SM}$ \[37\], whose update in 2015 \[29\] is in excellent agreement with the results presented in 2016 by the HPQCD Collaboration \[33\]. A SM phenomenological prediction \[38\] of $\mathcal{R}(D^*)$ is currently available, but no lattice-based computations. Summarizing, the most recently determinations are

$$\begin{align*}
\mathcal{R}(D)_{SM} &= 0.299 \pm 0.011 \quad \text{FNAL/MILC} \ [29] \\
\mathcal{R}(D)_{SM} &= 0.300 \pm 0.008 \quad \text{HPQCD} \ [33] \\
\mathcal{R}(D^*)_{SM} &= 0.299 \pm 0.003 \ [35] \\
\mathcal{R}(D^*)_{SM} &= 0.252 \pm 0.003 \ [38]
\end{align*}$$ \hspace{1cm} (14)

Older $\mathcal{R}(D)_{SM}$ determinations \[39,40\] are in agreement as well.

Exclusive semi-tauonic $B$ decays were first observed by the Belle Collaboration in 2007 \[41\]. Subsequent analysis by Babar and Belle \[42,43,44\], performed using an hadronic or an inclusive tagging method, measured branching fractions above–yet consistent with–the SM predictions. In 2012-2013 Babar has measured $\mathcal{R}(D^{(*)})$ by using its full data sample \[45,46\], and reported a significant excess over the SM expectation, confirmed in 2015 by LHCb \[47\] and in 2016 by Belle \[48\], the latter performing the first measurement of $\mathcal{R}(D^{(*)})$ at the $B$ factories using the semileptonic tagging method. By averaging the recent measurements \[44,45,46,47,48\], the HFAG Collaboration has found \[49\]

$$\begin{align*}
\mathcal{R}(D) &= 0.397 \pm 0.040 \pm 0.028 \\
\mathcal{R}(D^*) &= 0.316 \pm 0.016 \pm 0.010
\end{align*}$$ \hspace{1cm} (15)

where the first uncertainty is statistical and the second is systematic. $\mathcal{R}(D^*)$ and $\mathcal{R}(D)$ exceed the SM predictions in Refs \[38\] and \[33\] by $3.3\sigma$ and $1.9\sigma$, respectively. The combined analysis of $\mathcal{R}(D^*)$ and $\mathcal{R}(D)$, taking into account
measurement correlations, finds that the deviation is 4σ from the SM prediction. The first measurement of $R(D^*)$ using the semileptonic tagging method has been reported this year by the Belle Collaboration, giving

$$R(D^*) = 0.302 \pm 0.030 \pm 0.011$$  \hspace{1cm} (16)$$

which is within 1.6 σ of the SM expectation, where the standard deviation σ includes systematic uncertainties.

Very recently, the Belle collaboration has presented the first measurement of the τ lepton polarization in the decay $B \rightarrow D^* + \tau + \bar{\nu}$ as well as a new measurement of in the hadronic τ decay modes which is statistically independent of the previous Belle measurements, with a different background composition [50]. The preliminary results are consistent with the theoretical predictions of the SM within 0.6σ standard deviations, in particular [50] $R(D^*) = 0.276 \pm 0.034^{+0.026}_{-0.036}$.

The alleged breaking of lepton-flavour universality suggested by most of the data is quite large, and several theoretical models have been tested against the experimental results: minimal flavor violating models, right-right vector and right-left scalar quark currents, leptoquarks, quark and lepton compositeness models [51, 52, 53, 54], modified couplings [55, 56], new sources of CP violation [59], quantum effects [60], type III two-Higgs-doublet [61], R-parity violating Susy [62] and lepton-flavour non universal SU(2) × SU(2) × U(1) models [63], just to quote some. A welcome feature of measurements in the τ sector is the capacity of putting stringent limits on new physics models (see e.g. [64, 65, 66, 67]).

At Belle II, with more data, there will be a better understanding of backgrounds tails under the signal. At 5 ab$^{-1}$ the expected uncertainty is of 3% for $R(D^*)$ and 5% for $R(D)$. Data from Belle II may in principle be used for the inclusive $B \rightarrow X_c \tau \nu$ decays, where predictions for the dilepton invariant mass and the τ lepton energy distributions already exist [68].

In the context of leptonic non-universality, it is worth mentioning the violation observed at LHCb in the $B \rightarrow K^+ \ell^- \ell^-$ channel [69]. Across the dilepton invariant-mass-squared range $1 \text{ GeV}^2 < m^2_{\ell\ell} < 6 \text{ GeV}^2$, the ratio $R(K) = \text{Br}(B \rightarrow K\mu^+\mu^-)/\text{Br}(B \rightarrow Ke^+e^-) = 0.745^{+0.090}_{-0.074} \pm 0.036$ disagrees with the theoretically clean SM prediction $R(K) = 1.0003 \pm 0.0001$ by 2.6σ [70].

### B-Mesons Decays to Excited D-Meson States

The increased interest in semi-leptonic B decays to excited states of the charm meson spectrum derives by the fact that they contribute as a background to the direct decay $B \rightarrow D^{(*)}/\ell\nu$ at the B factories, and, as a consequence, as a source of systematic error in the $|V_{cb}|$ measurements.

The spectrum of mesons consisting of a charm and an up or a down anti-quark (open charm mesons) is poorly known. A QCD framework for their analysis can be set up by using HQET. In the limit of infinite heavy quark mass, the spin of the heavy quark $s_h$, is conserved and decouples from the total angular momentum of the light degrees of freedom $j_f$, which becomes a conserved quantity as well. The separate conservation in strong interaction processes of $j_f = j_f + 1/2s_h$ and parity $P = (-1)^{L+1}$, since $j_f \equiv L + s_h$, where $L$ is the orbital angular momentum and $s_h$ the spin of the light degrees of freedom. Within each doublet the two states are degenerate in the limit of infinite heavy quark mass.

The low-mass spectrum includes the ground states, with principal (radial) quantum number $n = 1$ and $L = 0$ (1S, in the spectroscopic notation), which implies $j_f^P = \frac{1}{2}^-$. The ground state doublet consists of two states with $J^P = (0^-, 1^-)$, that is $D$ and $D^*$ mesons.

When $L = 1$, there are four states ($1P$ states), which are generically referred to as $D^{**}$ 3. The doublet having $j_f^P = \frac{1}{2}^+$ is named $(D^0, D^+_1)$ and corresponds to $J^P = (0^+, 1^+)$. The doublet having $j_f^P = \frac{1}{2}^+$ is named $(D_1, D_2^*)$ and corresponds to $J^P = (1^+, 2^+)$. These states are generally identified with $D^0(2400)$, $D^+_1(2430)$, $D_1(2420)$ and $D^*_2(2460)$ 4. $D_1(2420)$ and $D^*_2(2460)$ have relatively narrow widths, about 20-30 MeV, and have been observed and studied by a number of

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3 Sometimes in literature this term is extended to include all particles in the low-mass spectrum except the ground states.

4 The naming convention followed is to use $D^{(*)}$ (mass) to denote the states having $P = (-1)^f$, that is $J^P = 0^-, 1^-, 2^+, \ldots$, (natural spin-parity)and with $D^{(*)}$ (mass) all the others (unnatural spin-parity); the prime is used to distinguish between the two doublets.
experiments since the nineties (see Ref. [71] and references therein). The other two states, $D_0^*(2400), D_1^*(2430)$, are more difficult to detect due to the large width, about 200-400 MeV, and their observation has started more recently [72, 73, 74, 75, 76].

The spectroscopic identification for heavier states is less clear. In 2010 BaBar has observed, for the first time, candidates for the radial excitations of the $D^0, D^{*0}$ and $D^{++}$, as well as the $L=2$ excited states of the $D^0$ and $D^+$ [77]. Resonances in the 2.4-2.8 GeV/c$^2$ region of hadronic masses have also been identified at LHCb [78, 79, 80, 81].

The not completely clear experimental situation is mirrored by two theoretical puzzles. Most calculations, using sum rules [82, 83], quark models [84, 85, 86, 87], OPE [88, 89] (but not constituent quark models [90]), indicate that the narrow width states dominate over the broad $D^{**}$ states, in contrast to experimental results (the “1/2 vs 3/2” puzzle). One possible weakness common to these theoretical approaches is that they are derived in the heavy quark limit and sum rules [82, 83], quark models [84, 85, 86, 87], OPE [88, 89] (but not constituent quark models [90]), indicate that

The other puzzle is that the sum of the measured semi-leptonic exclusive rates having $D^{(*)}$ in the final state is less than the inclusive one ("gap" problem) [76, 92]. Indeed, decays into $D^{(*)}$ make up about 70% of the total inclusive $B \to X_c \ell \nu$ rate and decays into $D^{(*)}\pi$ make up another about 15%, leaving a gap of about 15%. In 2014 the full BABAR data set has been used to improve the precision on decays involving $D^{(*)}\pi\ell\nu$ and to search for decays of the type $D^{(*)}\pi\pi\ell\nu$. Preliminary results assign about 0.7% to $D^{(*)}\pi\pi\ell\nu$, reducing the significance of the gap from 7σ to 3σ [93].

The $B \to D^{**}\ell\nu$ channel has been investigated together with its counterpart with $s$ quark, including the full lepton mass dependence [94]. Lattice studies are in progress with realistic charm mass, and preliminary results on $B \to D^{*+}\ell\nu$ form factors are available [22, 95, 96].

### Inclusive decays

In inclusive $B \to X_c \ell\nu$ decays, the final state $X_c$ is an hadronic state originated by the charm quark. There is no dependence on the details of the final state, and quark-hadron duality is generally assumed. Sufficiently inclusive quantities (typically the width and the first few moments of kinematic distributions) can be expressed as a double series in $\alpha_s$ and $\Lambda_{QCD}/m_b$, in the framework of the Heavy Quark Expansion (HQE), schematically indicated as

$$\Gamma(B \to X_c \ell\nu) = \frac{G_F^2 m_b^5}{192\pi^2} |V_{cb}|^2 \left[ c_3 \langle O_3 \rangle + c_5 \frac{\langle O_5 \rangle}{m_b} + c_6 \frac{\langle O_6 \rangle}{m_b^2} + O \left( \frac{\Lambda_{QCD}^4}{m_b^4}, \frac{\Lambda_{QCD}^5}{m_b^5}, \frac{\Lambda_{QCD}^6}{m_b^6}, \ldots \right) \right]$$

(17)

Here $c_d$ ($d = 3, 5, 6, \ldots$) are short distance coefficients, calculable in perturbation theory as a series in the strong coupling $\alpha_s$ and $O_d$ denote local operators of (scale) dimension $d$. The hadronic expectation values of the operators $\langle O_d \rangle$ encode the nonperturbative corrections and can be parameterized in terms of HQE parameters, whose number grows with powers of $\Lambda_{QCD}/m_b$. These parameters are affected by the particular theoretical framework (scheme) that is used to define the quark masses. Let us observe that the first order in the series corresponds to the parton order, while terms of order $\Lambda_{QCD}/m_b$ are absent. At highest orders in $\Lambda_{QCD}/m_b$, terms including powers of $\Lambda_{QCD}/m_c$, sometimes dubbed intrinsic charm contributions, have to be considered as well [97, 98, 99]. Indeed, roughly speaking, since $m_c^2 \sim O(m_b\Lambda_{QCD})$ and $\alpha_s(m_c) \sim O(\Lambda_{QCD})$, contributions of order $\Lambda_{QCD}^4/m_b^4$, $\Lambda_{QCD}^5/m_b^5$, $\Lambda_{QCD}^6/m_b^6$ are expected comparable in size to contributions of order $\Lambda_{QCD}^4/m_b^4$. At high orders in the HQE the number of new, nonperturbative parameters grows dramatically. At leading order, the matrix elements can be reduced to one, while at dimension-four heavy-quark symmetries and the equations of motion ensure that the forward matrix elements of the operators can be expressed in terms of the matrix elements of higher dimensional operators. The first nontrivial contributions appear at dimension five, where two independent parameters are needed, and two independent parameters are also needed at dimension six. At dimension seven and eight, nine and eighteen independent matrix elements appear, respectively, and for higher orders one has an almost factorial increase of the number of independent parameters.

At order $1/m_b^4$ in the HQE, that is the parton level, the perturbative corrections up to order $\alpha_s^2$ to the width and to the moments of the lepton energy and hadronic mass distributions are known completely (see Refs. [103, 104, 105, 106, 107] and references therein). The terms of order $\alpha_s^{n+1}B_0$, where $B_0$ is the first coefficient of the QCD $\beta$ function, have also been computed following the BLMBrodsky Lepage Mackenzie (BLM) procedure [104, 108].
The next order is $\Lambda_{\text{QCD}}^2/m_b^3$, and at this order the HQE includes two operators, called the kinetic energy and the chromomagnetic operator. Perturbative corrections to the coefficients of the kinetic operator [109, 110] and the chromomagnetic operator [111, 112, 113] have been evaluated at order $\alpha_s^2$.

Neglecting perturbative corrections, i.e. working at tree level, contributions to various observables have been computed at order $1/m_b^3$ [114] and estimated at order $1/m_b^{4.5}$ [115, 116, 117, 118].

A global fit is a simultaneous fit to HQE parameters, quark masses and absolute values of CKM matrix elements obtained by measuring spectra plus all available moments. The HFAG global fit [21] employs as experimental inputs yielding

In the same kinetic scheme, another global fit, including the complete power corrections up to flavor symmetry [125, 126].

was recently measured by Belle and BaBar [123, 124] and found to be in agreement with the expectations from SU(3) flavor symmetry [125, 126].

### Table 1. Status of exclusive and inclusive $|V_{cb}|$ determinations

| Exclusive decays | $|V_{cb}| \times 10^3$ |
|-----------------|-------------------|
| $B \rightarrow D^* l^+ l^-$ |
| FLAG 2016 [23] | $39.27 \pm 0.49_{\exp} \pm 0.56_{\text{latt}}$ |
| FNAL/MILC 2014 (Lattice $\omega = 1$) [20] | $39.04 \pm 0.49_{\exp} \pm 0.53_{\text{latt}} \pm 0.19_{\text{QED}}$ |
| HFAG 2012 (Sum Rules) [27, 28, 21] | $41.6 \pm 0.6_{\exp} \pm 1.9_{\text{fit}}$ |

| $B \rightarrow D l^+ l^-$ |
| Global fit 2016 [35] | $40.49 \pm 0.97$ |
| Belle 2015 (CLN) [34, 29] | $39.86 \pm 1.33$ |
| Belle 2015 (BGL) [34, 29, 33] | $40.83 \pm 1.13$ |
| FNAL/MILC 2015 (Lattice $\omega \neq 1$) [29] | $39.6 \pm 1.7_{\text{exp+QCD}} \pm 0.2_{\text{QED}}$ |
| HPQCD 2015 (Lattice $\omega \neq 1$) [33] | $40.2 \pm 1.7_{\text{latt+stat}} \pm 1.3_{\text{syst}}$ |

| Inclusive decays |
|-----------------|-------------------|
| Gammino et al. 2016 [100] | $42.11 \pm 0.74$ |
| HFAG 2014 [24] | $42.46 \pm 0.88$ |

| Indirect fits |
|-----------------|-------------------|
| UTfit 2016 [101] | $41.7 \pm 1.0$ |
| CKMfitter 2015 (3$\sigma$) [102] | $41.80^{+0.97}_{-1.64}$ |

The results (18) and (19) have practically the same average value, and the uncertainty of about 2% and 1.8%, respectively.

Semi-inclusive analyses of $B_s \rightarrow D_s^- X l^+ l^-$ and $B_s \rightarrow D_s^{* -} X l^+ l^-$ decays and measurements of their branching fractions have been performed by the DØ [120] and the LHCb [121] experiments, and, more recently, by Belle [122]. Belle has also reported the first measurement of the semi-inclusive branching fractions $\mathcal{B}(B_s \rightarrow D_s X l^+ l^-)$ and $\mathcal{B}(B_s \rightarrow D_s^{*} X l^+ l^-)$ using its entire $t(5S)$ dataset. The inclusive semileptonic branching fraction of $B_s \rightarrow X l^+ l^-$ decays was recently measured by Belle and BaBar [123, 124] and found to be in agreement with the expectations from SU(3) flavor symmetry [125, 126].
**$|V_{cb}|$ determinations: recap**

Inclusive and exclusive determinations of $|V_{cb}|$ are collected in Table 1. The most precise estimates of $|V_{cb}|$ come from lattice determinations in the $B \to D^*$ channel, followed by determinations based on inclusive measurements. They all stay below 2% uncertainty. The uncertainty on the determination from $B \to D$ semileptonic decays has recently decreased, and some determination almost reach the 2% limit from above. The old tension between exclusive and inclusive determinations seems to be confirmed by choosing to compare the determinations which claim the most precise estimates. Indeed, by considering the latest inclusive determination [100] and the latest inclusive determinations seems to be confirmed by choosing to compare the determinations which claim the most precise estimates. Indeed, by considering the latest inclusive determination [100] and the latest inclusive determinations nearly reach the 2% limit from above. The old tension between exclusive and inclusive determinations is confirmed by the UTfit collaboration [101] gives

$$|V_{cb}| = \left(41.7 \pm 1.0 \right) \times 10^{-3}$$

while the CKMfitter collaboration (at 3σ) [102] finds

$$|V_{cb}| = \left(41.80^{+0.97}_{-1.64}\right) \times 10^{-3}$$

Indirect fits prefer a value for $|V_{cb}|$ that is closer to the (higher) inclusive determination.

**HEAVY-TO-LIGHT SEMILEPTONIC DECAYS**

### Exclusive decays

The CKM-suppressed decay $B \to \pi l \nu$ is the most relevant exclusive channel for the determination of $|V_{ub}|$. In the approximation where the leptons are massless, the differential rate for $B \to \pi l \nu$ decay reads

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2 |\bar{\pi} l|^3}{24\pi} |V_{ub}|^2 |f_+(q^2)|^2$$

where $\vec{p}_\pi$ is the momentum of the pion in the $B$ meson rest frame and $q$ is the momentum of the lepton pair, ranging $0 < q^2 < (m_B - m_\pi)^2 \approx 26.4$ GeV. The form factor $f_+(q^2)$ refers to the parameterization of the matrix element between an heavy meson $H$ and a light pseudoscalar $P$ as

$$\langle P(p_P) | H(p_H) \rangle = f_+(q^2) \left( p_P^\mu + p_P^\mu - \frac{m_H^2 - m_P^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_H^2 - m_P^2}{q^2} q^\mu$$

Non perturbative theoretical predictions for form factors are usually confined to particular regions of $q^2$. Complementary regions are spanned by LCSR (low $q^2$) and lattice QCD (high $q^2$).

The process $B \to \pi l \nu$ for light final leptons is well-controlled experimentally; the current experimental dataset includes various BaBar and Belle measurements, be it untagged or with different tagging methods [127, 128, 129, 130, 131, 132, 133].

The first lattice determinations of $f_+(q^2)$ in the $B \to \pi l \nu$ channel, based on unquenched simulations and obtained by the HPQCD [134] and the Fermilab/MILC [135] collaborations, were in substantial agreement. In the quenched approximation, calculations using the $O(\alpha_s)$ improved Wilson fermions and $O(\alpha_s)$ improved currents have been performed on a fine lattice (lattice spacing $a \sim 0.04$ fm) by the QCDSF collaboration [136] and on a coarser one (lattice spacing $a \sim 0.07$ fm) by the APE collaboration [137]. In 2015, the Fermilab/MILC collaboration has presented its latest $|V_{ub}|$ determination, based on the MILC asqtad 2+1-flavor lattice configurations at four different lattice spacings [138]. Light-quark masses have been set down to 1/20 of the physical strange-quark mass and the limit to the continuum has been obtained using staggered chiral perturbation theory in the hard-pion and SU(2) limits. The form factors have been extrapolated from large-recoil momentum (17 GeV$^2 < q^2 < 26$ GeV$^2$) to the full kinematic range by means of a parameterization based on the BCL $z$-expansion. By combining the lattice form factors with recent experimental data from Babar and Belle Collaborations, they obtain [138]

$$|V_{ub}| = (3.72 \pm 0.16) \times 10^{-3}$$
where the error reflects both the lattice and experimental uncertainties, which are now on par with each other.

In 2015, a new determination has also been presented by the RBC/UKQCD collaboration [139], yielding, at $N_f = 2 + 1$

$$|V_{ub}| = (3.61 \pm 0.32) \times 10^{-3}$$  \hspace{1cm} (25)

Preliminary results on form factors by the ALPHA [140, 141] and HPQCD [142] ($N_f = 2 + 1$) collaborations, were updated and published in 2016. The HPQCD collaboration has presented 2+1+1-flavor results [143]; they have given the first lattice QCD calculations of $B \to \pi l l$ decay for $u/d$ quark masses going down to their physical values, calculating the $f_0$ form factor at zero recoil to 3% precision. The ALPHA collaboration has presented a 2-flavor calculation of semi-leptonic form factors for $B_s$-mesons [144], but the challenges faced have implications also for $B$ meson decays. For the first time, they were able to perform a study of the continuum limit of fully non-perturbatively renormalized form factors.

In 2016 the FLAG working group has proposed the $|V_{ub}|$ estimate ($N_f = 2 + 1$) [23]

$$|V_{ub}| = (3.62 \pm 0.14) \times 10^{-3}$$  \hspace{1cm} (26)

As already mentioned, the experimental value of $|V_{ub}| f_+(q^2)$ can be extracted from the measured branching fractions for $B \to \pi^+ l \nu$ and/or $B^\pm \to \pi^0 l \nu$; then $|V_{ub}|$ can be determined by performing fits to the parameterization of the form factor $f_+(q^2)$. The parameterization adopted by FLAG is the constrained BCL $z$ parameterization. The fit can be done in two ways: one option is to perform separate fits to lattice and experimental results, and extract the value of $|V_{ub}|$ from the ratio of the respective coefficients; a second option is to perform a simultaneous fit to lattice and experimental data, leaving their relative normalization $|V_{ub}|$ as a free parameter. FLAG adopts the second strategy, leading to a smaller uncertainty on $|V_{ub}|$ [23].

HFAG has performed a simultaneous fit to the four most precise measurements from BABAR and Belle and the 2009 Fermilab/MILC lattice calculations [135], yielding [24]

$$|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3}$$  \hspace{1cm} (27)

At large recoil (small $q^2$), direct LCSR calculations of the semi-leptonic form factors are available, which have benefited by progress in pion distribution amplitudes, next-to-leading and leading higher order twists and QCD corrections (see e.g. Refs. [145, 146, 147, 148, 149] and references within). The latest HFAG simultaneous fit uses a recent result for $f_+(0)$ from LCSR [146], and gives

$$|V_{ub}| = (3.53 \pm 0.29) \times 10^{-3}$$  \hspace{1cm} (28)

Another LCSR determination of $|V_{ub}|$ uses a Bayesian uncertainty analysis of the $B \to \pi$ form factor and combined BaBar/Belle data to obtain [150]

$$|V_{ub}| = (3.32^{+0.26}_{-0.22}) \times 10^{-3}$$  \hspace{1cm} (29)

Belle has recently found a branching ratio of $\mathcal{B}(B^0 \to \pi^- l^+ \nu) = (1.49 \pm 0.09_{stat} \pm 0.07_{syst}) \times 10^{-4}$ [133], which is competitive with the more precise results from untagged measurements. By employing this measured partial branching fraction, and combining LCSR, lattice points and the BCL [16] description of the $f_+(q^2)$ hadronic form factor, Belle extracts the value

$$|V_{ub}| = (3.52 \pm 0.29) \times 10^{-3}$$  \hspace{1cm} (30)

All the above mentioned exclusive determinations have been reported in Table 2. Other values for exclusive $|V_{ub}|$ have been computed in the relativistic quark model [151]. In 2016, Belle has also presented the first experimental result on $B \to \pi \tau \nu$, with an upper limit similar to the SM [152].

Recently, significantly improved branching ratios of heavy-to-light semi-leptonic decays other than $B \to \pi$ have been reported, that reflect on increased precision for $|V_{ub}|$ values inferred by these decays. In 2010, the Babar collaboration has started investigating the $B \to \rho l l$ channel [130]. By comparing the measured distribution in $q^2$ (with an upper limit at $q^2 = 16$ GeV) and using the 2004 LCSR predictions for the form factors [155], the estimate $|V_{ub}| = (2.75 \pm 0.24) \times 10^{-3}$ is given, and compared with the $|V_{ub}| = (2.83 \pm 0.24) \times 10^{-3}$ estimate obtained using the old ISGW2 quark model [156]. Both values are lower than the ones extracted by $B \to \pi$ decays. More recent results have been provided by the Belle Collaboration [133].

Measurements of $B \to sl l$ decays have been reported by Belle [157, 158, 133] and Babar [132, 159]. In the Belle tagged analysis [133], which includes results for several exclusive channels, an evidence of a broad resonance around
1.3 GeV dominated by the $B^+ \to f_2l\bar{\nu}$ decay has also been reported for the first time. In the Babar analysis with semileptonically tagged B mesons [158], the value of $|V_{ub}|$ has been extracted, yielding $|V_{ub}| = (3.41 \pm 0.31) \times 10^{-3}$, using the LCSR form factor determination [155], and $|V_{ub}| = (3.43 \pm 0.31) \times 10^{-3}$ using the ISGW2 quark model [156]. A major problem is that the quoted uncertainty does not include any uncertainty from theory, since uncertainty estimates of the form-factor integrals are not available.

In 2015, new LCSR computations have been performed for the form factors of $B \to \rho$, $B \to \omega$, $B_s \to K^*$ and $B_s \to \phi$ semileptonic decays from light-cone sum rules using updated hadronic input parameters [153]. By combining the form factor computation for $B \to \omega l\bar{\nu}$ decays at $q^2 < 12$ GeV with 2012 Babar data [132, 159], they found

$$|V_{ub}| = (3.31 \pm 0.19_{\text{exp}} \pm 0.30_{\text{th}}) \times 10^{-3}$$

and combining the form factor computation for $B \to \rho l\bar{\nu}$ decays at the same $q^2 < 12$ GeV with Belle data [133], they found

$$|V_{ub}| = (3.29 \pm 0.09_{\text{exp}} \pm 0.20_{\text{th}}) \times 10^{-3}$$

The branching fractions for $B \to \eta^{(l)} l\bar{\nu}$ decays have been measured in 2007 by the CLEO Collaboration [160] and, more recently, by the BaBar collaboration [161]. The value of the ratio

$$\frac{\mathcal{B}(B^+ \to \eta l^+\nu_l)}{\mathcal{B}(B^+ \to \eta l^+\nu_l)} = 0.67 \pm 0.24_{\text{stat}} \pm 0.11_{\text{syst}}$$

seems to allow an important gluonic singlet contribution to the $\eta^l$ form factor [161, 162]. In 2015 the analysis of all $B, B_s \to \eta^{(l)}$ form factors has been performed in the LCSR framework [163]. At Belle, the branching ratio $\mathcal{B}(B \to \eta l\nu_l) = (0.42 \pm 0.11_{\text{stat}} \pm 0.03_{\text{syst}}) \times 10^{-4}$ has been measured, and an upper limit at 90% to the branching ratio for $B \to \eta^l l\bar{\nu}$ set of $0.76 \times 10^{-4}$ [152].

Another channel that proceeds at the lowest partonic level in the SM from the $b \to ul\nu$ decay is the $B \to \pi l\bar{\nu}$ one, that gives the possibility to explore several angular observables, of extracting $|V_{ub}|$ and involves resonant contributions of vector and scalar mesons [164, 165].

The $B_s \to K^{(*)} l\bar{\nu}$ decays have not been measured yet, though, being expected to be within the reach of future $B$-physics results, can become an additional channel to extract $|V_{ub}|$ [139, 166, 167, 168, 169].

Although this report focuses on meson decays, it is interesting to compare with another decay depending on $|V_{ub}|$, that is the baryonic semileptonic $\Lambda_b^0 \to pl^-\bar{\nu}$ decays, where, at the parton level, the $b$-quark decays into the $u$-quark
emitting a $W^-$ boson [170, 171, 172]. In the Run I of the LHC (2011-2012), the LHCb experiment collected an integrated luminosity of 3 fb$^{-1}$ at center of mass energies of $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV. At the end of Run I, LHCb measured the ratio of decay rates for $\Lambda_b^0 \to p l^- \bar{\nu}$, at high $q^2$, where the background is expected to be reduced and lattice predictions more reliable. This result has been combined with the ratio of form factors computed using lattice QCD with 2+1 flavors of dynamical domain-wall fermions [173], enabling the first determination of the ratio of CKM elements $|V_{ub}|/|V_{cb}|$ from baryonic decays [174]. The channel $\Lambda_b \to \Lambda_c(\to pK\pi)\mu\bar{\nu}$ has been used as a control channel. To compute the form factors $\Lambda_b \to p$ and $\Lambda_b \to \Lambda_c$, the $b$ and $c$-quarks have been implemented with relativistic heavy-quark actions and the lattice computation has been performed for six different pion masses and two different lattice spacings, using gauge-field configurations generated by the RBC and UKQCD collaborations; the form factor results were extrapolated to the physical pion mass and the continuum limit, parameterizing the $q^2$-dependence using $z$ expansions [173]. The 2015 LHCb result reads [174]

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004_{\text{exp}} \pm 0.004_{\text{latt}}$$

(34)

The value of $|V_{ub}|$ depends on the choice of the value of $|V_{cb}|$. By taking the inclusive determination $|V_{cb}|_{\text{incl}} = (42.21 \pm 0.78) \times 10^{-3}$, the value $|V_{ub}| = (3.50 \pm 0.17_{\text{exp}} \pm 0.17_{\text{FF}} \pm 0.06_{|V_{cb}|}) \times 10^{-3}$ is obtained [175], where the errors are from experiment, the form factors, and $|V_{cb}|$, respectively. This result is 1.4σ lower than the determination from leptonic decays. By taking instead the higher value of the exclusive determination $|V_{cb}| = (39.5 \pm 0.8) \times 10^{-3}$, given by PDG 2014 [176], the LHCb reports [154]

$$|V_{ub}| = (3.27 \pm 0.23) \times 10^{-3}$$

(35)

Let us observe the value of $|V_{ub}|/|V_{cb}|$ extracted, from the same LHCb data, by using the relativistic quark model, is about 1.4 higher than Eq. (34)[177]. By using $|V_{cb}|_{\text{incl}}$, it translates into an higher value for $|V_{ub}|$, in better agreement with the inclusive determination rather than the exclusive one favoured by lattice results.

**Inclusive decays**

The extraction of $|V_{ub}|$ from inclusive decays requires to address theoretical issues absent in the inclusive $|V_{cb}|$ determination. OPE techniques are not applicable in the so-called endpoint or singularity or threshold phase space region, corresponding to the kinematic region near the limits of both the lepton energy $E_l$ and $q^2$ phase space, where the rate is dominated by the production of low mass final hadronic states. This region is sensitive to the Fermi motion of the $b$ quark inside the $B$ meson. It is also plagued by the presence of large double (Sudakov-like) perturbative logarithms at all orders in the strong coupling. Corrections can be large and need to be resummed at all orders 5. The kinematics cuts due to the large $B \to X_c l\nu$ background enhance the weight of the threshold region with respect to the case of $b \to c$ semi-leptonic decays; moreover, in the latter, corrections are not expected as singular as in the $b \to u$ case, being cutoff by the charm mass.

On the experimental side, efforts have been made to control the background and access to a large part of the phase space, so as to reduce, on the whole, the weight of the endpoint region. Latest results by Belle [186] and BaBar [187] use their complete data sample, 657 x 10$^6$ $B\bar{B}$ pairs for Belle and 467 x 10$^6$ $B\bar{B}$ pairs for BaBar. Although the two analyses differ in the treatment of the background, both collaborations claim to access $\sim 90\%$ of the phase space.

On the theoretical side, several schemes are available. They assume an underlying $b$-quark decaying, since weak annihilation contributions seems to be strongly constrained by semileptonic charm decays [188, 99, 189]. All of the schemes are tailored to analyze data in the threshold region, but differ significantly in their treatment of perturbative corrections and the parametrization of non-perturbative effects.

The analyses from BaBar [187] and Belle [186] collaborations, as well as the latest HFAG averages [24], rely on at least four theoretical different QCD calculations of the inclusive partial decay rate: ADFR by Aglietti, Di Lodovico, Ferrera and Ricciardi [190, 191, 192]; BLNP by Bosch, Lange, Neubert and Paz [193, 194, 195]; DGE, the dressed gluon exponentiation, by Andersen and Gardi [196]; GGOU by Gambino, Giordano, Ossola and Uraltev [197] 6.

5 See e.g. Refs. [178, 179, 180, 181, 182, 183, 184, 185] and references therein.

6 Recently, artificial neural networks have been used to parameterize the shape functions and extract $|V_{ub}|$ in the GGOU framework [198]. The results are in good agreement with the original paper.
TABLE 3. Status of inclusive $|V_{ub}|$ determinations

| Inclusive decays (|$V_{ub}| \times 10^3$) | ADFR [190, 191, 192] | BNLP [193, 194, 195] | DGE [196] | GGOU [197] |
|----------------------------------------|----------------------|----------------------|------------|------------|
| HFAG 2014 [24]                        | 4.05 ± 0.13 ±0.18    | 4.45 ± 0.16 ±0.21    | 4.52 ± 0.16 ±0.15 | 4.51 ± 0.16 ±0.12 |
| BaBar 2011 [187]                      | 4.29 ± 0.24 ±0.18    | 4.28 ± 0.24 ±0.18    | 4.40 ± 0.24 ±0.12 | 4.35 ± 0.24 ±0.09 |
| Belle 2009 [186]                      | 4.48 ± 0.30 ±0.19    | 4.47 ± 0.27 ±0.19    | 4.60 ± 0.27 ±0.13 | 4.54 ± 0.27 ±0.11 |

They can be roughly divided into approaches based on the estimation of the shape function (BLNP, GGOU) and on resummed perturbative QCD (ADFR, DGE). Although conceptually quite different, all the above approaches generally lead to roughly consistent results when the same inputs are used and the theoretical errors are taken into account. The HFAG estimates [24], together with the latest estimates by BaBar [187, 199] and Belle [186], are reported in Table 3. The BaBar and Belle estimates in Table 3 refers to the value extracted by the most inclusive measurement, namely the one based on the two-dimensional fit of the $M_X - q^2$ distribution with no phase space restrictions, except for $p_T^V > 1.0$ GeV. This selection allow to access approximately 90% of the total phase space [199]. The BaBar collaboration also reports measurements of $|V_{ub}|$ in other regions of the phase space [187], but the values reported in Table 3 are the most precise. When averaged, the ADFR value is lower than the one obtained with the other three approaches, and closer to the exclusive values; this difference disappears if we restrict to the BaBar and Belle results quoted in Table 3. By taking the arithmetic average of the results obtained from these four different QCD predictions of the partial rate the Babar collaboration gives [187]

$$|V_{ub}| = (4.33 ± 0.24 \exp ± 0.15_{ub}) \times 10^{-3}$$ (36)

Another HFAG average value is $|V_{ub}| = 4.62 ± 0.20 ± 0.29$ [24], obtained from a global fit in the 1S scheme using the BLL (Bauer, Ligeti and Luke) [200], theoretical approach, which is limited to measurements that use $m_X - q^2$ cut.

$|V_{ub}|$ determination: recap

The parameter $|V_{ub}|$ is the less precisely known among the modules of the CKM matrix elements. The error on the inclusive determinations is around 4-5% in the latest averaged HFAG values, in several theoretical schemes. The uncertainties of the lattice based exclusive determinations of $|V_{ub}|$, in the $B \rightarrow \pi$ channel, have halved in the last two years, ranging now around 4%.

By comparing the value (36) (or results in Table 3) with results in Table 2, we observe a tension between exclusive and inclusive determinations, of the order of 2 – 3σ, according to the chosen values. A lot of theoretical effort has been devoted in the past to clarify the present tension by inclusion of NP effects, but this possibility is strongly limited by electroweak constraints on the effective $Zbb$ vertex [201]. The difficult interpretation of this tension is enhanced by the fact that experimental access to such a large portion of phase space as 90%, once accepted that the difficult background subtraction is trustable, seems to reduce the weight of the shape function region in the inclusive determination, and justify a theoretical description analogous to the one used for the inclusive $B \rightarrow X_c$ semileptonic decay, in terms of local OPE.

Within Table 2, the values obtained for $B \rightarrow \rho \pi l \nu$ appear to be systematically lower than the ones for $B \rightarrow \pi l \nu$. Notwithstanding the large errors, several theoretical possibilities have been already put forward, as e.g. corrections for the $\rho$ lineshape in $B \rightarrow \pi\pi l \nu$ [165]. A possibile S-wave background not separated in the experimental analysis of $B \rightarrow \rho l \nu$ has been estimated to give a modest contribution [166], and also the effect of right-handed currents [202, 203] seems to be disfavoured by data.

It is possible to determine $|V_{ub}|$ indirectly, using the CKM unitarity relations together with CP violation and flavor data, as done for $|V_{cb}|$. The indirect fit provided by the UTfit collaboration [101] gives

$$|V_{ub}| = (3.74 ± 0.21) \times 10^{-3}$$ (37)

while the CKMfitter collaboration (at 3σ) [102] finds

$$|V_{ub}| = (3.71 ±0.27) \times 10^{-3}$$ (38)
At variance with the $|V_{cb}|$ case, the results of the global fit prefer a value for $|V_{ub}|$ that is closer to the (lower) exclusive determination. Belle II is expected, at about 50 ab$^{-1}$, to decrease experimental errors on both inclusive and exclusive $|V_{ub}|$ determinations up to about 2%.

**LEPTONIC DECAYS**

In the SM, the weak decay $B^+ \to l^+ \nu_l$ of a charged $B$ meson occurs, in the parton model at the lowest perturbative order, through the annihilation of the heavy and light quark inside the meson, and it is therefore mediated by a charged current. The branching ratio is given by

$$\mathcal{B}(B^+ \to l^+ \nu_l) = \frac{G_F m_B m_l^2}{8\pi} \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

(39)

The decay constant $f_B$ parameterizes the matrix elements of the axial vector current $<0|\vec{q}\gamma\mu q|B^+(p_B)> = p_B^\mu f_B$ and is calculated on lattice. For heavy-light mesons, it is sometimes convenient to define and study the quantity $\Phi_B = f_B \sqrt{m_B}$ which approaches a constant (up to logarithmic corrections) in the $m_b \to \infty$ limit. All the other inputs in Eq. (39) are measured experimentally. The branching fraction depends strongly on the mass of the lepton due to helicity suppression, and thus the $B^+ \to \tau^+ \nu_\tau$ decay is expected to have the largest leptonic branching fraction of the $B^+$ meson and is the only decay of this kind for which there is experimental evidence. It was observed for the first time by Belle in 2006 [204].

$B$ factories have studied purely leptonic decays with a goal of searching for new physics beyond the SM. The analysis relies on reconstructing a hadronic or semi-leptonic $B$ decay tag, finding a candidate in the remaining track and photon candidates, and examining the extra energy in the event which should be almost zero when a real $\tau^-$ decays into electron and muons. The branching fraction measured by Babar using semileptonic tags [205] and hadronic tags [206] averaged to [206]

$$\mathcal{B}(B^+ \to \tau^+ \nu_\tau) = (1.79 \pm 0.48) \times 10^{-4}$$

(40)

in agreement with oldest Belle result using hadronic [204] and semileptonic tags [207]. The results were more than 2$\sigma$ higher than the SM estimate based on a global fit [102]. However, Belle has recently reanalyzed both samples of their data, using the hadronic [208] and semileptonic tags [209]. A much lower averaged branching fraction is given [209]

$$\mathcal{B}(B^+ \to \tau^+ \nu_\tau) = (0.91 \pm 0.22) \times 10^{-4}$$

(41)

which is now aligned with the SM predictions, even though the error is about 20%. In all measurements there are large statistical and dominant systematical errors, and the significances are less than 5$\sigma$. An higher level of precision is required to explore the possibility of new physics effects.

The measurement of the branching fraction provides a direct experimental determination of the product $f_B|V_{ub}|$, which can be used to determine $|V_{ub}|$ when combined with lattice predictions of $f_B$. Combining the experimental values with the mean $B^+$-meson lifetime $\tau_B = 1.641 \pm 0.008$ [210] and their averages for the $B$ meson decay constant, the FLAG working group obtains a range of values varying with the experimental collaboration and $N_f$. By assuming $N_f = (2,2 + 1,2 + 1 + 1)$, they give [23]

$$|V_{ub}| = (3.83 \pm 0.48 \pm 0.15, 3.75 \pm 0.47 \pm 0.09, 3.87 \pm 0.48 \pm 0.09) \times 10^{-3} \quad \text{Belle}$$

$$|V_{ub}| = (5.37 \pm 0.74 \pm 0.21, 5.26 \pm 0.73 \pm 0.12, 5.43 \pm 0.75 \pm 0.12) \times 10^{-3} \quad \text{Babar}$$

(42)

where the first error comes from experiment and the second comes from the uncertainty in $f_B$. Another recent determination employs averages from Babar and Belle lepton modes and yields [175]

$$|V_{ub}| = (4.12 \pm 0.37 \pm 0.06) \times 10^{-3}$$

(43)

The $|V_{ub}|$ values seem to point towards the higher semileptonic inclusive $|V_{ub}|$ determinations, but there is also consistency with the exclusive values, within the large errors. The accuracy is not yet enough to draw definite conclusions and to make the leptonic channel competitive for $|V_{ub}|$ extraction. At Belle II, the total experimental precision is expected to reduce uncertainty below 5% at 50ab$^{-1}$ [211].

The most stringent upper limits for the charged $B$ decaying into muons have been measured by Babar [212] $\mathcal{B}(B^+ \to \mu^+ \nu_\mu) < 1.0 \times 10^{-6}$, and for decays into electrons by Belle [213], $\mathcal{B}(B^+ \to e^+ \nu_e) < 9.8 \times 10^{-7}$, both with untagged methods.
CONCLUSIONS

We have summarized the current status of semileptonic and leptonic $B$ decays, which presents significant theoretical and experimental progress. Higher precision has been attained in challenging measurements, as for instance the branching fractions of exclusive $B \to \rho / \omega l \nu$ decays or fully leptonic charged $B$ decays. More results are expected, at present from further analyses of data provided by the $B$ factories and LHCb, and in the (approaching) future from Belle II.

On the theoretical side, the perturbative calculations, in general, have reached a phase of maturity, and the larger theoretical errors are due to non-perturbative approaches. Errors have been recently lowered in both lattice and LCSR frameworks; new global fits for inclusive processes also sport further reduced theoretical uncertainties. New physics is always more constrained. Still awaiting firmly established solutions are a few dissonances within the SM, as the long-standing tension between the inclusive and exclusive determination of $|V_{cb}|$ and $|V_{ub}|$.

The most precise estimates of $|V_{cb}|$ come from lattice determinations in the $B \to D^*$ channel [20], which is below 2% uncertainty; by comparing with the latest inclusive determination [100], which has almost the same precision, the tension amounts to about $3 \sigma$. This tension is confirmed by comparing with determinations from $B \to D$ semileptonic decays, whose uncertainty has decreased in some recent computations, bordering the 2% limit.

The parameter $|V_{ub}|$ is the less precisely known among the modules of the CKM matrix elements. The error on the inclusive determinations is around 4-5% in the latest averaged HFAG values, in several theoretical schemes. The uncertainties of the lattice based exclusive determinations of $|V_{ub}|$, in the $B \to \pi$ channel, have halved in the last two years, ranging now around 4%. The inclusive/exclusive determination tension amounts to about 2-3 $\sigma$.

Another tension with the SM, recently recognized, is the apparent excess of $\mathcal{B}(D^{(*)})$ on the SM predictions, which is causing an intense theoretical work to explore the possibility of lepton flavour violation or lepton non-universality.

In contrast to the above, a previous tension within the SM seems to have faded after the Belle recent reanalysis of $\mathcal{B}(B^+ \to \tau^+ \nu_{\tau})$, using both the hadronic [208] and semileptonic tags [209]. The results yield an averaged branching fraction for the $B^+ \to \tau^+ \nu_{\tau}$ channel much lower than in the past, and the branching fraction is now aligned with the SM predictions, even though the error is about 20%. An higher level of precision is required to explore the possibility of new physics effects.

Belle II, which has completed the accelerator commissioning phase and should nominally start in fall 2018, is expected to give a substantial boost to understand tensions within the SM. At the ultimate goal of 50 ab$^{-1}$, precision on fully leptonic charged $B$ decays is foreseen to be reduced below 5%, on $\mathcal{B}(D^{(*)})$ to about 2-3%, and more than halved for $|V_{ub}|$ inclusive and exclusive determinations from the $B \to \omega$ channel [214, 211].

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