Qubit rotation in QHE

Dipti Banerjee

Department of Physics, Rishi Bankim Chandra College, Naihati, 24-Parganas(N), Pin-743165, West Bengal, India
E-mail: deepbancu@homail.com and dbanerje@ictp.it

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Abstract
In the quantum Hall effect, the ground state wavefunction at \( \nu = 1 \) is the building block of all other states at different filling factors. It is developed by the entanglement of two spinors forming a singlet state. The inherent frustration visualized by the non-Abelian matrix Berry phase is responsible for the quantum pumped charge to flow in the Hall surface. The physics behind the quantum Hall states is studied here from the viewpoint of topological quantum computation.

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1. Introduction

Entanglement is one of the basic aspects of quantum mechanics. It was known long ago that quantum mechanics exhibits very peculiar correlations between two physically distant parts of the total system. Afterwards, the discovery of Bell’s inequality (BI) [1] showed that BI can be violated by quantum mechanics but has to be satisfied by all local realistic theories. The violation of BI demonstrates the presence of entanglement [2]. The theorem of BI may be interpreted as incompatibility of the requirement of locality with the statistical predictions of quantum mechanics. So to study the Bell state, the role of local spatial observations, apart from spin correlations, should also be taken into account [3]. This indicates that the spatial variation of a quantum mechanical state would carry its memory through some geometric phase known as the Berry phase (BP) [4]. It is expected that the influence of the BP on an entangled state could be linked up with the local observations of spins.

To have a comprehensive view of the quantum mechanical correlation between the two spin-1/2 particles in an entangled state, we should take into account the role of the BP related to a spinor. This study belongs to the field of geometric quantum computation where noticeably the geometrical and topological gates are resistant to local disturbances. In quantum mechanical entanglement of two spin-1/2 particles, the BP plays an important role during the spin echo method [5]. Kitaev [6] described the topological and quantum computer as a device in which quantum numbers carried by quasiparticles residing in a two-dimensional (2D) electron gas have long-range Aharonov–Bohm (AB) interactions between them. These AB interactions are responsible for nontrivial phase values during the interwinding of the quasiparticle’s trajectories in the course of time evolution of qubits in quantum Hall effects (QHE). Quantization plays an important role in realizing the physics behind different states of QHE from the viewpoint of the BP [7]. Recently, we have studied the rotation of a quantized spinor identified as a qubit in the presence of a magnetic field under the spin echo method [8]. We will aim to understand the rotation of QHE qubits specially in the lowest Landau level (LLL) \( \nu = 1 \) and then the parent states \( \nu = 1/m \) from the viewpoint of geometric quantum computation.

2. Quantization of the Fermi field and qubits of singlet states

The quantization of the Fermi field can be achieved assuming anisotropy in the internal space through the introduction of the direction vector as an internal variable at each space-time point [9]. The opposite orientations of the direction vector correspond to particle and antiparticle. Incorporation of spinorial variables \( \theta (\bar{\theta}) \) in the coordinate result in the enlargement of the manifold from \( S^2 \) to \( S^3 \). This helps us to consider a relativistic quantum particle as an extended one, where the extension involves gauge degrees of freedom. As a result the position and momentum variables of a quantized particle becomes

\[
Q_\mu = i \left( \frac{\partial}{\partial p_\mu} + A_\mu \right), \quad P_\mu = i \left( \frac{\partial}{\partial q_\mu} + \bar{A}_\mu \right),
\]

where \( q_\mu \) and \( p_\mu \) are related to the position and momentum coordinates in the sharp point limit and \( A_\mu (\bar{A}_\mu) \) are
non-Abelian matrix valued gauge fields belonging to the group \( SL(2C) \).

In three-space dimension, in an axis-symmetric system where the anisotropy is introduced along a particular direction, the components of the linear momentum satisfy a commutation relation of the form \[ [p_l, p_t] = i\mu e_{l/t} \] (2)

Here, \( \mu \) corresponds to the measure of anisotropy and behaves like the strength of a magnetic monopole. Indeed, in this anisotropic space the conserved angular momentum is given by \[ \mathcal{J} = \hat{r} \times \vec{p} - \mu \hat{r}, \] (3)

with \( \mu = 0, \pm 1/2, \pm 1, \ldots \). This corresponds to the motion of a charged particle in the field of a magnetic monopole. For the specific case of \( l = 1/2 \) and \( |m| = |\mu| = 1/2 \) for half orbital/spin angular momentum, we can construct from the spherical harmonics \( Y_{1/1}^{\mu\mu} \), the instantaneous eigenstates \(|\uparrow, t\rangle\rangle\), representing the two component up spinor as

\[
\begin{align*}
|\uparrow, t\rangle = & \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \\
& = \begin{pmatrix} Y_{1/2,2/2}^{1/1/2} \\ Y_{1/2,-2/2}^{1/1/2} \end{pmatrix} \\
& = \left( \sin \frac{\theta}{2} \exp i(\phi - \chi)/2 \right) \cos \frac{\theta}{2} \exp -i(\phi + \chi)/2 
\end{align*}
\]

and the conjugate state is a down spinor given by

\[
|\downarrow, t\rangle = \left( -Y_{1/2,-2/2}^{1/1/2} \right) = \left( -\cos \frac{\theta}{2} \exp i(\phi + \chi)/2 \right) \sin \frac{\theta}{2} \exp -i(\phi - \chi)/2.
\]

These two spinors (up/down) represent quantized Fermi field originated by an arbitrary superposition of elementary qubits \(|0\rangle\) and \(|1\rangle\) as for the up spinor

\[
|\uparrow, t\rangle = \left( \sin \frac{\theta}{2} \exp i\phi \right)|0\rangle + \cos \frac{\theta}{2}|1\rangle \right) \exp -i(\phi + \chi)/2
\]

and the down spinor becomes

\[
|\downarrow, t\rangle = \left( -\cos \frac{\theta}{2} \right) \left| 0 \right> + \sin \frac{\theta}{2} \exp -i\phi \left| 1 \right> \right) \exp i(\phi + \chi)/2.
\]

The states \(|\uparrow, t\rangle\) and \(|\downarrow, t\rangle\) can be generated by the unitary transformation matrix \( U(\theta, \phi, \chi) \) [11]

\[
U(\theta, \phi, \chi) = \begin{pmatrix} \sin \frac{\theta}{2} \exp i(\phi - \chi) \\ -\cos \frac{\theta}{2} \exp i(\phi + \chi) \end{pmatrix} \begin{pmatrix} \cos \frac{\theta}{2} \exp -i(\phi - \chi) \\ \sin \frac{\theta}{2} \exp i(\phi + \chi) \end{pmatrix}
\]

in association with the basic qubits \(|0\rangle\) and \(|1\rangle\)

\[
|\uparrow, t\rangle = U(\theta, \phi, \chi) |0\rangle, \quad |\downarrow, t\rangle = U(\theta, \phi, \chi) |1\rangle.
\]

Over a closed path, the single quantized up spinor acquires the geometrical phase [8]

\[
\gamma_1 = i \oint (|\uparrow, t\rangle \nabla \rangle |\uparrow, t\rangle \rmd \lambda = i \oint |0\rangle \rmd U |0\rangle \rmd \lambda = i \oint A_1(\lambda) \rmd \lambda
\]

representing a solid angle subtended about the quantization axis. For the conjugate state, the BP over the closed path becomes

\[
\gamma_1 = -\pi (1 - \cos \theta).
\]

In a cyclic change of the time period \( T \), the instantaneous basis states \(|\Phi_1(0)\rangle\rangle\) and \(|\Phi_2(0)\rangle\rangle\) might return to their initial states \(|\Phi_1(0)\rangle\rangle\) and \(|\Phi_2(0)\rangle\rangle\), where the coefficients \( c_1(T) \) and \( c_2(T) \) may not. This doubly degenerate energy level, a 2 \times 2 matrix BP \( \Phi_c \), connects the final amplitudes \( c_1(T) \) and \( c_2(T) \) with the initial amplitudes \( c_1(0) \) and \( c_2(0) \)

\[
\begin{pmatrix} c_1(T) \\ c_2(T) \end{pmatrix} = \Phi_c \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix}.
\]

Hwang et al [13] pointed out that this non-Abelian matrix BP is responsible for pumped charges where the charge transport in a cycle of the pump a \( j \)th optimal channel becomes

\[
Q_j = -i/2\pi \oint |\psi_j| \rmd \psi_j.
\]

This charge will be only topological in nature during the transport of qubits, if the influence of the dynamical phase can be eliminated. The spin echo method is a popular technique for this removal of the dynamical phase where the two cyclic evolutions are applied on a spinor with the second application followed by a pair of fast transformations. Vedral et al [14] showed the appearance of spin echo to a spinor (equation (6)).

\[
|\uparrow\rangle \rightarrow C_x e^{i(\delta_j - \gamma)} |\uparrow\rangle \rightarrow \pi \ rmd \theta |\uparrow\rangle \rightarrow C_x e^{i(\delta_j - \gamma)} |\downarrow\rangle
\]

Here, \( \rightarrow C_x \) introduces the dynamical and geometrical phases, \( \delta_j \) and \( \gamma \) through right cyclic evolution of \(|\uparrow\rangle\rangle\) spinor, respectively. Similar phases of opposite orientations are

\[
\begin{align*}
\mathcal{L}_{\text{ex}} &= \oint L_{ex} \rmd t \\
\pi &= \int \left( \oint \left( \rmd \chi - \cos \theta \oint \rmd \phi \right) \right) \\
\gamma_1 &= -\pi (1 - \cos \theta).
\end{align*}
\]
developed by $\rightarrow C_i$. Referring back to equations (10) and (11), we see that $\gamma_1 = \gamma$ and $\gamma_2 = -\gamma$ for $\gamma = \pi(1 - \cos \theta)$. Thus, two cyclic evolutions accompanied by two $\pi$ rotations eliminate the net dynamical phases doubling the geometric phase of the original state (up/down spinor) according to:

$$|\Psi_+ (t = \tau)\rangle = \frac{1}{\sqrt{2}} \left( e^{i\gamma |\uparrow\rangle \otimes |\downarrow\rangle} - e^{-i\gamma |\downarrow\rangle \otimes |\uparrow\rangle} \right)_{\otimes 2},$$

and the symmetric state becomes

$$|\Psi_+ (t = \tau)\rangle = \frac{1}{\sqrt{2}} \left( e^{i\gamma |\uparrow\rangle \otimes |\downarrow\rangle} + e^{i\gamma |\downarrow\rangle \otimes |\uparrow\rangle} \right)_{\otimes 2},$$

where the total effect of the dynamical phase disappear. The spin echo method is very fruitful [15] in the construction of two qubits through the rotation of one qubit (spin-1/2) in the vicinity of another. Incorporating the spin echo for a half period (as in equation (17)), we find the antisymmetric Bell’s state after one cycle ($t = \tau$),

$$|\Psi_- (t = \tau)\rangle = \frac{1}{\sqrt{2}} \left( e^{i\gamma |\uparrow\rangle \otimes |\downarrow\rangle} - e^{-i\gamma |\downarrow\rangle \otimes |\uparrow\rangle} \right)_{\otimes 2},$$

and the symmetric state becomes

$$|\Psi_+ (t = \tau)\rangle = \frac{1}{\sqrt{2}} \left( e^{i\gamma |\uparrow\rangle \otimes |\downarrow\rangle} + e^{i\gamma |\downarrow\rangle \otimes |\uparrow\rangle} \right)_{\otimes 2},$$

where $\gamma_1 = -\gamma_2 = -\gamma$. Splitting up equations (18) and (19) into the symmetric and antisymmetric states and rearranging we have

$$|\Psi_+\rangle = \cos \gamma |\Psi_+\rangle_0 - i \sin \gamma |\Psi_-\rangle_0,$$

$$|\Psi_-\rangle = i \sin \gamma |\Psi_+\rangle_0 + \cos \gamma |\Psi_-\rangle_0,$$

the doublet acquiring the matrix BP $\Sigma$ as rotated from $t = 0$ to $t = \tau$.

$$\left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{\tau} = \Sigma \left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{0},$$

$$\Sigma = \begin{pmatrix} \cos \gamma & -i \sin \gamma \\ i \sin \gamma & \cos \gamma \end{pmatrix}.$$  

This non-Abelian matrix BP $\Sigma$ is developed from the Abelian BP $\gamma$. For $\gamma = 0$ there is symmetric rotation of states, but for $\gamma = \pi$ the return is antisymmetric as the values of $\Sigma = I$ and $-I$ (where $I$ = identity matrix), respectively.

The instantaneous quantum state can be represented by the linear combination of degenerate symmetric and antisymmetric states. The symmetric state will return to the antisymmetric state over one-half period of spin echo apart from a matrix valued BP [16]. It may be noted that two half period rotations will complete one spin echo resulting in the return of the state to itself apart from a geometrical phase factor.

$$\left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{2\tau} = \left(\begin{array}{c} \cos \gamma & -i \sin \gamma \\ i \sin \gamma & \cos \gamma \end{array} \right) \cdot \left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{0}.$$  

Following the notion of one complete spin echo here, the state $|\Psi_+ \rangle_{\tau = 2\tau}$ also returns to its initial state $|\Psi_+\rangle_0$ apart from the phase $\cos 2\gamma$.

$$\left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{2\tau} = \left(\begin{array}{cc} \cos 2\gamma & 0 \\ 0 & \cos 2\gamma \end{array} \right) \cdot \left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{0}.$$  

In any even number of half period $\tau$, the symmetric state will return to itself apart from the BP factor with increased power of $\cos 2\gamma$.

$$\left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{2\tau n} = \left(\begin{array}{cc} \cos 2\gamma & 0 \\ 0 & \cos 2\gamma \end{array} \right)^n \cdot \left(\begin{array}{c} |\Psi_+\rangle \\ |\Psi_-\rangle \end{array} \right)_{0},$$

where $n = 1, 2, 3, \ldots$ are natural integers. For an odd number of half periods rotations there will be a mixture of both states. On the other hand, with the value of $\gamma = \pi$, the symmetric/antisymmetric state remains the same after one rotation.

In this connection, we have shown recently [8] that the singlet state between the two spinors at a particular instant is connected with the singlet state of elementary qubits [0] and [1], and the BP of the initial antisymmetric Bell’s state is $\gamma_{\text{ent}} = \pi(1 + \cos 2\theta)$, where, if we introduce a spin echo in the two qubits then the topological phase $\Sigma = \cos 2\gamma$ is of matrix valued.

By varying the magnetic field angle $\theta : 0 \rightarrow \pi/3 \rightarrow \pi/2$, the BP of a qubit changes to, $\gamma : 0 \rightarrow \pi/2 \rightarrow \pi$, that in turn changes the two qubit BP, $\Sigma : I \rightarrow \sigma^\dagger \rightarrow -I$. This explains the physics behind the change from the antisymmetric Bell singlet state $\Psi_-$ to the symmetric Bell state $\Psi_+$ and back to $\Psi_-$. We will now proceed to apply the above idea of entanglement in the field of QHE to study the state formation from one filling factor to another in the light of geometric quantum computation.

3. Qubit formation of the quantum Hall state

The QHE shows a prominent appearance of quantization of Hall particles involving gauge theoretic extension of coordinate by $C_\mu \in SL(2C)$ visualized by the field strength $F_{\mu\nu}$ acting as a background external magnetic field. It is noted that the gauge field theoretic extension for a Fermi field associated with the direction vector $\xi_u$ attached to the space-time point $x_u$ results in the field function $\phi(x_u, \xi_u)$ describing a particle moving in an anisotropic space [7].

The external magnetic field introduces frustration in the Hall system. We have considered a 2D frustrated electron gas of $N$ particles on the spherical surface of a 3D sphere of large radius $R$ in a strong radial (monopole) magnetic field. In such a 3D anisotropic space, we can construct the non-Abelian wavefunction from the spherical harmonics $Y_{l\mu}$ with $l = 1/2, |m| = |\mu| = 1/2$ (when the angular momentum in the anisotropic space is given by equation (3)). With the description of a two component up spinor $|\uparrow\rangle = (\xi^0, \xi^1)$ as in equation (6) we can construct the $N$ particles wavefunction of Hall states

$$\Psi^{(m)}_{N_i} = \prod (u_i v_j - u_j v_i)^m,$$  

for parent states $m = l/\nu$, where $\nu$ is the Landau filling factor and this $m = J_i = J_j + J_{ij}$ is the two particle angular momentum equivalent to $m = \mu_i + \mu_j = 2\mu$ (when $i = j$). In a similar manner the same Hall state with opposite polarization can be constructed by using the down spinor $|\downarrow\rangle = (\tilde{\xi}^0, \tilde{\xi}^1)$

$$\Psi^{(m)}_{N_i} = \prod (\tilde{u}_i \tilde{v}_j - \tilde{u}_j \tilde{v}_i)^m.$$  


Here, the two states $\Psi^{(n)}_{\mu}$ and $\Psi^{(m)}_{\nu}$ belong to the same parent filling factor but with the opposite polarization of the spinors.

The above states are grouped into a family depending on the value of $m$. With $m = 3$ the states are the same family of the Laughlin $v = 1/3$ state, etc. In the light of the statement of Jain and Kamilla [17] that regarding the filling factor the Integral quantum Hall effect (IQHE) of composite fermions are the Fractional quantum Hall effect (FQHE) of fermions, any FQHE state can be expressed in terms of the IQHE state. It seems that for LLL $v = 1$, IQHE state $\Phi_1(z)$

$$\Phi_1(z) = (u_i v_j - u_j v_i)$$

(29)

is the basic building block for constructing any other IQHE/FQHE state. The lowest level Hall state $\Phi_1(z)$ has a similarity with two-qubit singlet states formed by a pair of one qubit states.

There is a deep analogy between FQHE and superfluidity [18]. The ground state of the anti-ferromagnetic Heisenberg model on a lattice introduces frustration giving rise to the resonating valence bond (RVB) states corresponding to spin singlets where two nearest-neighbor bonds are allowed to resonate among themselves. It is suggested that RVB states [6] are a basis of fault tolerant topological quantum computation. Since these spin singlet states forming an RVB gas are equivalent to the fractional quantum Hall fluid, its description through quantum computation will be of ample interest.

This RVB where two nearest-neighbor bonds are allowed to resonate among themselves has equivalence with an entangled state of two single qubits. The antisymmetric Hall state $\Phi_1(z)$ for $v = 1$ is formed as one spinor at the $i$th site rotating with $\Phi_1(z)$ at the $j$th site in the vicinity of another at $j$th site captures the image of spin echo

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2) \left( \begin{array}{c} 0 \\ e^{-i\pi} \end{array} \right) \left( \begin{array}{c} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{array} \right)$$

$$= (|\uparrow\rangle_1 |\downarrow\rangle_2) \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \left( \begin{array}{c} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{array} \right)$$

$$= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)$$

(30)

Due to symmetry, the singlet state can be written on any basis with the same form. We can rotate the spin vector by an arbitrary angle $\theta$ with the following transformation.

$$|\uparrow\rangle = \left( \begin{array}{c} \sin \theta e^{i\phi} \\ -\cos \theta \end{array} \right) |0\rangle$$

$$|\downarrow\rangle = \left( \begin{array}{c} -\sin \theta e^{-i\phi} \\ \cos \theta \end{array} \right) |1\rangle$$

(31)

The quantum Hall systems are so highly frustrated that the ground state $\Phi_1(z)$ is an extremely entangled state visualized by the formation of an antisymmetric singlet state between a pair of $i$th and $j$th spinors in the Landau filling factor ($v = 1$).

$$\Phi_1(z) = \left( \begin{array}{c} u_i \\ v_i \\ u_j \\ v_j \end{array} \right) = (u_i v_j - u_j v_i)$$

$$= (u_i v_j) \left( \begin{array}{c} 0 \\ 1 \\ -1 \\ 0 \end{array} \right)$$

(32)

We identify this two-qubit singlet state as a Hall qubit constructed from the the up spinor shown in the previous section

$$\Phi_1(z) = \langle \uparrow, \uparrow \rangle \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) |\uparrow\rangle_1 |\uparrow\rangle_2$$

$$= \langle \uparrow, \uparrow \rangle \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) |\uparrow\rangle_1 |\downarrow\rangle_2$$

(33)

The down spinor can construct the opposite polarization of the Hall qubit

$$\Phi_1(\bar{z}) = (\bar{u}_i \bar{v}_j - \bar{u}_j \bar{v}_i)$$

(34)

that has a similar representation as equation (33)

$$\Phi_1(\bar{z}) = \langle \downarrow, \uparrow \rangle \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) |\downarrow\rangle_1 |\uparrow\rangle_2$$

(35)

Now, these two Hall qubits of two opposite polarizations representing the state of the same LLL $v = m = 1$ will automatically generate two respective non-Abelian Berry connections. The Hall connection for up spinor becomes

$$B_1 = \Phi_1(z)^* d \Phi_1(z) = \left( u_i^* v_j^* \right) \left( d u_i \quad d v_j \right)$$

$$= \left( \mu_i \quad \mu_j \right)$$

(36)

and similarly for down spinor

$$\bar{B}_1 = \Phi_1(\bar{z})^* d \Phi_1(\bar{z}) = \left( \bar{u}_i^* \bar{v}_j^* \right) \left( d \bar{u}_i \quad d \bar{v}_j \right)$$

$$= \left( \bar{\mu}_i \quad \bar{\mu}_j \right)$$

(37)

This is visualizing the spin conflict during parallel transport leading to matrix BP. In the light of Hwang et al [13] our realization includes that in QHE this non-Abelian matrix BP is responsible for the charge flow by pumping. In this QHE matrix BP

$$\gamma^H_1 = \left( \begin{array}{cc} \gamma_i & \gamma_j \\ \gamma_j & \gamma_i \end{array} \right)$$

(39)

where $\gamma_i$ and $\gamma_j$ are the BPs for the $i$th and $j$th spinor as seen in equation (10) and the off-diagonal BP $\gamma_{ij}$ arises due to local frustration in the spin system. Over a closed period $t = \tau$ the QHE state $\Phi_1(z)$ at $v = 1$ filling factor will acquire the matrix BP.

$$\langle \Phi_1(z) | \tau \rangle = e^{i\Omega(c)} \langle \Phi_1(z) | 0 \rangle$$

(40)

The Berry connection gets modified as the quantum states differ after one rotation. Usually when any state changes by

$$|\psi'\rangle = |\psi\rangle e^{i\Omega(c)}$$

the corresponding changed gauge becomes

$$A'_\psi = A_\psi + i d \Omega(c)$$

provided $\langle \psi | \psi \rangle = 1$. We have pointed out earlier [19] that each quantum Hall state for a particular filling factor has its distinct BP. Hence, the BP is constant for a filling factor. The rotation shifts the BP from the ground level to the excited
level once. With these ideas we have the topological phase difference between the first excited and the ground state acquired by the rate of change of the BP

$$\Gamma^1 - \Gamma^0 = i \oint (\Phi_1(z)|d\gamma^H/\partial\lambda|\Phi_1(z))_0 d\lambda.$$  \hspace{1cm} (41)$$

The rotation of singlet state by 'n' turns will be

$$\langle \Phi_1(z) \rangle^n = e^{i\gamma^n} \langle \Phi_1(z) \rangle^0,$$  \hspace{1cm} (42)$$

where $n = 1, 2, 3, \ldots$ are the natural numbers associated with the number of rotations of the singlet states. We should point out here that the antisymmetric nature of FQHE states would be visualized through the rotation of singlet states. This automatically imposes the following constraint in the topological phase

$$e^{i\gamma^n} = e^{i\pi}, \hspace{1cm} \text{for} \hspace{0.5cm} \langle \Phi_1(z) \rangle_0 = -\langle \Phi_1(z) \rangle_0.$$  \hspace{1cm} (43)$$

where $m = 1, 3, 5, \ldots$ being the odd numbers to maintain the antisymmetric nature of wavefunction. So any number of rotations of the matrix BP lead to an odd multiple of $\pi$ angles provided every state remains antisymmetric. It seems that the BP acts as a local order parameter of the QHE states.

$$\langle \Phi_1(z) \rangle^{m \pi/n} = e^{i\pi} \langle \Phi_1(z) \rangle^{m \pi/n}.$$  \hspace{1cm} (44)$$

Earlier, we showed [7] that the BP for $\nu = 1/m$ state is $\gamma = m \pi \theta = 2 \pi \mu \theta$, where $\theta$ is a coupling constant. This motivated us to write

$$\langle \Phi_1(z) \rangle_1 = e^{i\mu \theta} \langle \Phi_1(z) \rangle_0.$$  \hspace{1cm} (45)$$

This makes the experimental observation of the parent state in FQHE at $m = \text{odd} (3, 5, 7)$ more transparent. It also shows that the topological phase is responsible for controlling the statistics of the Hall state. In the absence of frustration, the role of the matrix BP is trivial. In other words $\gamma_1$ becomes zero leading to the diagonal matrix BP provided the two particle have identical $\theta$ and $\phi$ values.

$$\gamma_1^H = \pi (1 - \cos \theta_1) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$  \hspace{1cm} (46)$$

The non-Abelian matrix BP in QHE is originated due to the frustration offered by the magnetic field and the disorder of spins. In the absence of local frustration (latter) this complexity of connection will be removed. We would like to mention that the spin echo between the two single qubits has the equivalence of an RVB state in FQHE and topological quantum computation with BP is responsible for the formation of higher states considering the Hall qubit at $\nu = 1$ as a building block of any QHE state.

### 4. Discussion

In this paper, we have studied the physics behind the singlet state entangled by two qubits where one is rotating in the field of the other with the BP only. This image of spin echo has been reflected in the field of the QHE. The Hall state for the LLL at $\nu = 1$ is highly frustrated. They are the singlet states identified as the Hall qubit, the building block of other higher IQHE/FQHE states at different filling factors. These states have matrix BP which are responsible for pumped charge flow. In other words, the BP acts as a local order parameter of singlet states. Further, we pointed out that the antisymmetric nature of $\nu = 1/m$ FQHE states depend on their acquired BP. Since these spin singlet states forming an RVB gas are equivalent to the fractional quantum Hall fluid, the description of background physics through quantum computation will be of ample interest. We will proceed to study the hierarchies of FQHE in the light of quantum communication in the future.

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