I. INTRODUCTION

The Nonlinear Schrödinger equation (NLSE),

\[ i \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0 \]  \quad (1)

is arguably the most studied partial differential equation in nonlinear physics. Its significance is best expressed by D.K. Campbell who famously said, 'Live by Nonlinear Schrödinger, die by Nonlinear Schrödinger!' [1]. The NLSE has been used to model various nonlinear phenomena in optics [2], atomic physics, condensed matter physics [3], plasma physics [4], quantum field theory [5] and optomechanics [6]. Its applications is not limited to physics only, rather the NLSE has been used in a wide variety of diverse research areas such as: oceanography [7], molecular biology [8], meteorology [9] and even, geology [10]. The NLSE, i.e. Eq. (1), admits many solutions such as bright and dark-soliton solutions etc. which are successfully used to model various experimental observations in physics [1], and in particular, nonlinear fiber optics [11]. However, recently the rogue wave solutions admitted by the NLSE and its variants has become a topic of extensive research, since the experimental observation of optical rogue waves in 2007 [12], in the context of nonlinear optics. A rogue wave (RW) is a spatiotemporally localized single wave which emerges suddenly and dies out rapidly leaving no trace to follow. Since its first appearance recorded via satellite surveillance at Draupner oil platform in 1995 [13], it became an area of immense theoretical research in oceanography. It was first demonstrated by Benjamin and Feir that, due to random infinitesimal perturbation, progressive wave of finite amplitude in deep water, fluctuates [14]. Benjamin-Feir instability, mostly known as the modulation instability (MI) has been the well accepted theory behind rogue wave generation on the surface of ocean. This MI dynamics, exhibited by many nonlinear dispersive systems, is associated with the exponential growth of the initial infinitesimal perturbation on the continuous-wave (CW) background [15]. On further exploration, Zakharov first showed that the equations for propagation of deep water waves can be reduced to a single nonlinear equation, the so-called self-focusing integrable nonlinear Schrödinger equation (NLSE) [16, 17]. Soon after that, using Inverse Scattering Transformation, Zakharov and Shabat presented an exact analytical solution of the self-focusing NLSE, which describes the splitting of a continuous wave into definite number of single envelope solitons [15]. Since then, the study of analytical solution of the NLSE has revealed different ‘solitons on zero background’ such as the envelope soliton, the bi-soliton [15, 19] and various types of ‘solitons on finite background (SFB)’ or breathers [20].

Over the years, SFB solutions, which are also the exact solutions of the NLSE, are determined by several methods [15, 21, 22]. In 1979, as asymptotic solution of NLSE, Ma found out ‘a series of solitons’ breathing temporally with a specific period, via the Inverse Scattering Transformation technique [22]. Four years after Ma soliton, considering temporal period to be infinite, Peregrine presented a localized solution in both space and time [10]. Soon after the realization of Peregrine soliton (PS) on plane wave background, Akhmediev found ‘a family of solution’ of integrable NLSE which breathes spatially [23]. Further, taking spatial period of AB as infinite, the ratio of two polynomials (both as a function of space and time) appears as another class of SFB solution, known as the Rational Soliton (RS) solution. Thereafter, this RS solutions can be considered as the limiting case of either the Ma solitons or the Akhmediev breathers. Whereas the first order RS solution was already given by Pere-
grine, the higher-order RS solutions now can describe the intense accumulation of energy for the emergence of higher amplitude localized waves. \cite{25}. Here it is worthwhile to note that these solutions are not rogue waves, until they satisfy the statistical criterion for rogue waves generated as result of noise-seeded MI present in the system \cite{25, 26}. A recent remarkable achievement includes the observation of RS upto fifth-order in water tank \cite{27, 28, 29}. Further, the detection of AB \cite{30}, PS \cite{31} and Kuznetsov-Ma (KM) \cite{32} breather in table-top experiments attracted significant interest to understand RW dynamics. Apart from hydrodynamics, as mentioned earlier, rogue waves first appeared as massive intensity fluctuations near soliton-fission supercontinuum generation in a highly nonlinear fiber \cite{12}, aptly termed as optical rogue waves. This discovery in nonlinear fiber optics, encouraged researchers to explore rogue wave dynamics in other domains of physics such as, matter waves in Bose Einstein Condensation (BEC) \cite{33, 34}, plasma \cite{35, 36} etc. Because of the wide range of applications, the demand for realistic optical fiber, the effect of anisotropy and inhomogeneity should be included. In order to resolve the issue, it is necessary to incorporate space-dependency in the dispersion and nonlinear coefficients of the NLSE. Previous studies have revealed that the mathematical models depicting extreme wave events are mostly based on the higher-order solutions of the standard NLSE or its variants such as the variable coefficient NLSE (Vc-NLSE), the Sasa-Satsuma equation, the Hirota equation, the coupled NLSE, the Davey-Stewartson equation etc. \cite{12, 49}. In this work, we propose to obtain an exact analytical solution of such variable coefficient NLSE (Vc-NLSE) by reducing it to standard NLSE via self-similarity transformation. This paper is organized in the following manner: Section II provides analytical model of 1-D Vc-NLSE and its solution. Section III portrays the general classification of RS and their dynamics. Section IV yields the evolution of the first, second and the third-order RS solution as managed-breathers. And section V illustrates modulation of those managed-breathers. We conclude the effect of optical wave propagation under regulation of coefficients.

II. ANALYTICAL MODEL: VC-NLSE UNDER SIMILARITY TRANSFORMATION

The pulse propagation in an inhomogeneous nonlinear medium, say an optical fiber, is best described by the generalized Nonlinear Schrödinger equation with variable coefficients (Vc-NLSE). So, we consider the following Vc-NLSE with variable dispersion and Kerr nonlinearity:

\[
i\frac{\partial u}{\partial z} + \frac{\beta_2(z)}{2} \frac{\partial^2 u}{\partial x^2} + \chi(z)|u|^2 u = 0
\]  

(2) where \(u(z, x)\) denotes the complex envelope of the optical field; \(z\) and \(x\) are the propagation distance along the medium and the retarded time in a frame moving with the pulse group velocity; \(\beta_2(z)\) and \(\chi(z)\) are the dispersion and the nonlinearity coefficients respectively. Triggered by previous studies \cite{50, 51, 52}, we use the following similarity transformation:

\[
u(z, x) = A(z)V(T, X) e^{iB(z, x)}
\]  

(3)

This similarity transformation reduces Eq.(2) to the known standard self-focusing NLSE:

\[
i\frac{\partial V}{\partial T} + \frac{1}{2} \frac{\partial^2 V}{\partial X^2} + |V|^2 V = 0
\]  

(4) where \(V(T, X)\) represents the complex envelope of the optical field; \(T(z)\) is the dimensionless propagation distance and \(X(z, x)\) denotes the similarity variable which needs to be determined. Also \(A(z)\) and \(B(z, x)\), both being real, are the amplitude and the phase function respectively. Therefore, substitution of Eq.(3) into Eq.(2) will lead to the exact known solution of Eq.(4), provided following set of relations and PDEs are satisfied:

\[
AT_z = 1 \quad (5a)
\]

\[
\beta_2 A X_x^2 = 1 \quad (5b)
\]

\[
\chi A^3 = 1 \quad (5c)
\]

\[
B_z + \frac{\beta_2}{2} B_x^2 = 0 \quad (5d)
\]

\[
\frac{A_z}{A} + \frac{\beta_2}{2} B_{xx} = 0 \quad (5e)
\]

\[
\frac{\beta_2 A}{2} X_{xx} = 0 \quad (5f)
\]

\[
B_x = -1 \frac{X_z}{\beta_2 X_x} \quad (5g)
\]

Here, the subscripts denote partial derivative with respect to \(z\) or \(x\), respectively. It turns out that the variable coefficients of Eq. (2) spontaneously emerges in the parameters of similarity transformation. Solving Eqs. (5) we obtain:

\[
T(z) = \int_0^z \frac{\beta_2(s)}{w^2(s)} ds \quad (6a)
\]

\[
X(z, x) = \frac{x}{w(z)} + \theta(z) \quad (6b)
\]

\[
A(z) = \frac{1}{w(z)} \sqrt{\frac{\beta_2(z)}{\chi(z)}} \quad (6c)
\]

\[
w(z) = \left( \frac{\chi_0}{\beta_2 0} \right) w_0 \frac{\beta_2(z)}{\chi(z)} \quad (6d)
\]

\[
B(z, x) = \frac{w_x z^2}{\beta_2 w} + B_0(z) \quad (6e)
\]

with a condition:

\[
\beta_2 w_{zz} = \beta_2 z w_z \quad (7)
\]
where, \( w(z) \) is the width of rational soliton, \( w_0 \) being the initial width. On the other hand, \( \beta_0 \) and \( \chi_0 \) denote the initial dispersion and the nonlinearity coefficient respectively. \( \theta(z) \), \( B_0(z) \) are real integration constants, chosen to be \( \theta(z) = 0 \), \( B_0(z) = 1 \) for rest of the calculations. Finally, assembling all the solutions from Eqs. (6), the exact solution of Eq.(2) is given by:

\[
u_n(z, x) = \frac{1}{w(z)} \sqrt{\frac{\beta_2(z)}{\chi(z)}} V_n(T, X) e^{i \left( \frac{\pi}{2} \frac{z^2}{w^2} + B_n(z) \right)},
\]

where \( n \) denotes the order of the solution.

III. CLASSIFICATION OF RS SOLUTION AND THEIR DYNAMICS

As previously described, Eq.(4) has several exact analytical solutions describing different physical phenomena. We will specifically deal with the first three orders of RS solution obtained by the well known Darboux transformation \([53, 55]\). A well established classification of these RS solutions is given by a complex parameter \( s_j \), where \( j \) is a positive integer, \( j = 1, 2, \ldots \). This \( s_j \) parameter corresponds to the eigenvalue in the Darboux transformation and, it appears that the \( n \)-th order solution contains \((n-1)\) parameter \([56, 58]\). Hence, there is no \( s_j \) parameter in the standard first-order RS solution, one \( s_j \) parameter in the second-order and two parameters, \( s_1 \) and \( s_2 \), in the third-order RS solution.

On the basis of the work done by \([56, 58]\), we have worked out the first, the second and the third-order RS solutions as follows:

\[
V_1 = \left[ 1 - \frac{4(1 + 2iT)}{1 + 4X^2 + 4T^2} \right] e^{iT}(9)
\]

\[
V_2 = \left[ 1 - \frac{D_1}{D_2} \right] e^{iT}(10)
\]

where,

\[
D_1 = 18 + 72ib + 180iT - 288bT - 432T^2 - 288ibT^2 - 96iT^3 - 480T^4 - 192iT^5 - 288aX - 576iaTX - 144X^2 + 288ibX^2 + 288iT^2X^2 - 576iT^2X^2 - 384iT^3X^2 - 96X^4 - 192iT^4 X^4(11)
\]

\[
D_2 = -9 - 144a^2 - 144b^2 - 432bT - 396T^2 - 192bT^3 - 432T^4 - 64T^6 - 144aX - 576aTX - 108X^2 + 576bTX^2 + 288iT^2X^2 - 192T^4X^2 + 192aX^3 - 48X^4 - 192T^2X^4 - 64X^6
\]

and,

\[
V_3 = \left[ 1 - \frac{\sum_{j=1}^{12} H_j X^j}{\sum_{j=1}^{12} F_j X^j} \right] e^{iT}(13)
\]

The explicit expressions for \( H_j \) and \( F_j \) are given in the Appendix A and B respectively. In the following, we adopt the classifications as prescribed by Ling and Zhao \([58]\). The second-order solution containing \( s_1 \) parameter, defined as \( s_1 = a + ib \) and \( s_2 = c + id \) parameters are denoted as type \([0, 0]\) when \( s_1 = s_2 = 0 \). For different \( s_1, s_2 \) values they are classified as type \([0, 1] \), \([1, 0]\) and \([1, 1]\). These parameters \( a, b, c \) and \( d \) are known as the free parameters. The detailed illustrations of solutions on the basis of these free parameters will be discussed later in this paper. We will now study the intensity distribution of Eq.(8) choosing a specific functional form of \( \beta_2(z) \) and \( \chi(z) \) for different \( s_1 \) and \( s_2 \) parameters.

IV. SOLITON TO KM-LIKE BREATHER TRANSFORMATION

In this section, we choose the dispersion coefficient to be periodic of the form, \( \beta_2(z) = 1 + A_1 \cos(\omega z) \), while the nonlinearity as a constant parameter, say, \( \chi(z) = 1 \). Here \( A_1 \) is the amplitude of the modulation, with \(-1 < A_1 < 1 \) and \( \omega \neq 0 \) is the spatial frequency. The corresponding pulse width, \( w(z) \), the amplitude function, \( A(z) \), and the similarity variable, \( X(z, x) \), are given as follows:

\[
w(z) = \frac{\sqrt{\chi(1 + A_1 \cos(\omega z))}}{1 + A_1 \cos(\omega z)}, \quad A(z) = \frac{1}{\sqrt{\chi(1 + A_1 \cos(\omega z))}}, \quad X(z, x) = \frac{x(1 + A_1 \cos(\omega z))}{1 + A_1 \cos(\omega z)}.
\]

With these variables and Eqs.(8),(9),(10) and (13), we illustrate the intensity distribution of the first three orders of RS solution for a periodically modulated dispersion.

The first-order RS or PS solution, starts breathing along the same propagation axis with specific period, owing to the effect of periodic modulation of dispersion along the propagation direction. Such temporally breathing solution has strong resemblance with the so-called Kuznetsov-Ma breather, and could be termed as ‘Controlled KM-like breather’. The amplitude of modulation \( A_1 \) controls the background as well as the breather peak power. Whereas, the spatial frequency \( \omega \) controls the breathing frequency of the KM-like breather. For the first-order solution, the breathing period is greater than the modulating frequency. As a result, there is no exchange of energy between the peak and the background. Fig.1(a)-(h) shows the effect of \( A_1 \) and \( \omega \) on the intensity growth.

Similar controllable KM-like breather could be obtained in the case of second order RS solution as well. As per the previously mentioned classification of second-order RS solution (which is based on \( s_1 \) parameter) \([58]\), type \([0] \) possess single peak, whereas the type\([1] \) possess three peaks (triplets) (Fig.2 (a), (e)). Considering \( A_1 = 0.1 \) and \( \omega = 1 \), it has been observed that both
FIG. 1: Surface and contour plots for the first-order RS solutions (or Peregrine Soliton). (a),(b) Standard first-order RS without regulation ; Controlled KM-like Breather with: (c),(d) $\omega = 1, A_1 = 0.1$ ; (e),(f) $\omega = 1, A_1 = 0.5$ and (g),(h) $\omega = 2, A_1 = 0.1$.

type [0] and type [1] solutions show breathing features under periodic dispersion. Here, we have shown only the surface and the contour dynamics of type [1] solution in fig. 2(a)-(h). As the triplets corresponding to $s_1 \neq 0$ get periodic along z-axis, this kind of solutions could be termed as ‘Type [1] controlled KM-like breather’. Like the second-order triplet RS solution, in each periodic unit of the controlled KM-like breather, triplets are symmetric about the x-axis when $a \neq 0$, $b = 0$. On the other hand, they are symmetric about the z-axis when $a = 0$, $b \neq 0$. The distance between peaks increases with the increase in the values of $a$ and $b$.

Unlike the first and the second order RS solution, the characterization of the basic third-order RS solution requires two parameters, $s_1$ and $s_2$. From previous research [58], we know that the third-order solutions show four different features, namely, the standard one peak [56], or type [0,0], the triangular cascade [25, 57] or type [1,0], the pentagram structure [25, 57] or type [0,1] and the claw-structure [60] or type [1,1]. As shown in the Fig. 3(a), $s_1 = 100, s_2 = 0$ ($a = 100, b = 0, c = d = 0$), the surface plot corresponds to a unit of the triangular structure, which is basically six first-order RS solutions of the same amplitude arranged in a triangular shape. Considering $A_1 = 0.1, \omega = 0.5$, it has been observed

FIG. 2: Surface and contour plots for second-order RS solutions. For $a = 20$: (a),(b) Second-order triplets symmetrical about x-axis and (c),(d) Type [1] controlled KM-like breathers; For $b = 20$: (e),(f) Second-order triplets symmetrical about z-axis and (g),(h) Type [1] controlled KM-like breathers. Modulating parameters are chosen as $A_1 = 0.1, \omega = 1$. 
that for the same $s_1$, $s_2$ parameter, the unit of triangular structure is getting modulated periodically along the $z$-axis. We can see a new aspect, where the extreme-end peaks (the first and the third peak situated at the negative $x$-axis in Fig. 3(a)) are getting merged and their peak amplitudes are getting lowered [Fig. 3(b)]. The reason behind this may be attributed to the fact that energy exchange takes place between the extreme-end peaks of each of the periodic units, with the periodically modulated finite background. As the breathing period of each peak situated at the negative $x$-axis matches with the modulation period of the finite background, the peak amplitude decreases, while the background amplitude increases slightly. This feature could be understood by a careful look at the contour plot in Fig. 3(d). Also, the energy exchange between particular peaks and the background depends on the modulation frequency $\omega$. If the breathing period of the peaks situated at $x = 0$ matches with the modulation period, then their amplitude is found to decrease. Fig. 3(e) shows the intensity distribution of the claw-structured third-order solution corresponding to $s_1 = 31$, $s_2 = 500$ i.e., $a = 31$, $b = 0$, $c = 500$, $d = 0$. This solution contains three first-order and one second-order solution with their highest amplitude. Keeping the same $s_1$, $s_2$ parameter and $A_1 = 0.1$, $\omega = 0.5$, we observe periodic claw-structured KM-like breather [Fig. 3(g)]. In this case, we find that the extreme end peaks situated at the negative $x$-axis have higher amplitude than the middle one. Here, the peaks are distributed in such a way that the breathing period of each unit is less than the modulation period.

V. TRIPLET BREATHER TO DOUBLET AND SINGLET BREATHER CONVERSION

In this section, we briefly explore some interesting features of the second-order rational solution under periodic dispersion. In case of the solution for the usual, i.e. the constant coefficient NLSE, if we increase the value of $a$ or $b$, the peak separation increases for $s_1 \neq 0$. But under periodically varying dispersion profile, for a fixed value of the modulating amplitude $A_1$ and frequency $\omega$, depending on free parameters $a$ and $b$, the breather dynamics vary. When the real part of the $s_1$ parameter, i.e., $a$, is increased, the distance between the three peaks is found to increase for $A_1 = 0.1$ and $\omega = 0.6$ [Fig. 4(a)-(d)]. The appearance of the singlet breather is exhibited at $a = 1500$ [Fig. 4(e),(f)]. The controlled KM-like breather appears similar to the standard single peak controlled KM-like breather [Fig. 1(c)], having its origin shifted to the positive $x$-axis.

On the other hand, when the imaginary part of $s_1$ parameter, i.e., $b$, is increased, for the same $A_1$ and $\omega$, the distance between three peaks is again found to increase [Fig. 5(a)-(d)]. At a particular value $b = 1500$, the transformation from the triplet to the doublet could be observed, with peaks shifted at the positive $z$-axis [Fig. 5(e),(f)]. The third peak that disappears, exchanges energy with the background.

VI. CONCLUSION

In summary, we have presented analytical rational soliton (RS) solutions, and investigated their evolution under the presence of a spatially dependent periodic dispersion. It is possible to transform soliton into KM-breather with judicious choice of parameters. Singlet and dou-
FIG. 4: Surface and contour plots for: Type [1] controlled KM-like breather with $A_1 = 0.1$, $\omega = 0.6$, $b = 0$. For: (a),(b) $a = 50$ ; (c),(d) $a = 100$ ; and (e),(f) $a = 1500$.

higher-order dynamics could also be seen as a function of blet breather could be extracted from triplet breather via periodical modulation of the dispersion profile. By modulating the amplitude and the spatial frequency of the dispersion profile, one could realize many interesting RS dynamics. Our study shows how the second-order RS dynamics changes as free parameters change. The third and higher-order dynamics could also be seen as a function of free parameters.

Appendix A: $H_j$ Polynomial Expression

$$H_0 = -\frac{a^4}{576} - \frac{a^2b^2}{288} + \frac{1}{192}a^2b^3 - \frac{1}{256}a^2bT^2 - \frac{1}{2304}a^2dT + \frac{1}{576}a^2d^2T - \frac{1}{2304}a^2T^3 - \frac{1}{2304}aT^5 - \frac{1}{2304}T^6$$

FIG. 5: Surface and contour plots for managed KM-like triplet breather with $A_1 = 0.1$, $\omega = 0.6$ , $a = 0$. For: (a),(b) $b = 50$ ; (c),(d) $b = 100$ ; and (e),(f) $b = 1500$. 

$$H_1 = \frac{a^3T}{112} - \frac{1}{18}aT - \frac{a^2}{288} + \frac{a^2}{288} + \frac{1}{288}aT^2 + \frac{1}{288}aT^3 + \frac{1}{288}aT^4 + \frac{1}{288}aT^5 + \frac{1}{288}aT^6 + \frac{1}{288}aT^7 + \frac{1}{288}aT^8 + \frac{1}{288}aT^9 + \frac{1}{288}aT^{10} + \frac{1}{288}aT^{11} + \frac{1}{288}aT^{12}$$

$$H_2 = -\frac{1}{144}a^2bT^2 + \frac{1}{288}a^2b^2 + \frac{1}{576}a^2b^3 + \frac{1}{112}a^2bT^3 + \frac{1}{576}a^2bT^4 + \frac{1}{576}a^2bT^5 + \frac{1}{576}a^2bT^6 + \frac{1}{576}a^2bT^7 + \frac{1}{576}a^2bT^8 + \frac{1}{576}a^2bT^9 + \frac{1}{576}a^2bT^{10} + \frac{1}{576}a^2bT^{11} + \frac{1}{576}a^2bT^{12}$$

$$H_3 = -\frac{1}{144}a^2bT + \frac{1}{288}a^2b^2 + \frac{1}{576}a^2b^3 + \frac{1}{112}a^2bT^3 + \frac{1}{576}a^2bT^4 + \frac{1}{576}a^2bT^5 + \frac{1}{576}a^2bT^6 + \frac{1}{576}a^2bT^7 + \frac{1}{576}a^2bT^8 + \frac{1}{576}a^2bT^9 + \frac{1}{576}a^2bT^{10} + \frac{1}{576}a^2bT^{11} + \frac{1}{576}a^2bT^{12}$$
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Appendix B: $F_j$ Polynomial Expression

\begin{align*}
F_0 &= 9216a^4 + 18432a^3b^2 + 12288a^2b^3 + 36864ab^2T + 18432a^2bT^2 + 28672a^2T^3 + 3072a^2T^4 + 32256a^2T^2 + 35456a^2 - 36864abcT + 12288acT^3 + 27648acT^2 + 4608ac + 9216a^3 - 12288abT^3 + 36864abT^2 - 18432a^2dT + 12288a^2T^4 + 37792aT^5 + 6912aT^6 + 3456a^2T^2 + 27648bdT^2 + 4608bd + 16384d^2T^2 + 5904d^2T^3 - 64512d^2T + 77568d^3T^3 + 9216bT + 9216c^2T^2 + 2304c^2 + 9216d^2T^2 + 2304d^2 + 8192dT^2 + 10444d^2T^2 - 4608dT^3 + 5760dT + 12288d^2T^2 + 57344T^3 + 48992T^3 + 17408T^6 + 40896T^4 + 6624T^4 + 36,
\end{align*}

\begin{align*}
F_1 &= -36864a^3T^2 - 18432a^2c - 36864ab^2T^2 - 36864abd - 98304ab^3T + 122880abT^3 - 24576adT^3 + 55296adT + 16384aT^4 + 4096at^6 - 40608at^4 + 20736aT^2 + 20304a + 18432b^2c + 24576bcT + 55296bcT + 8192cT^6 + 55296cT^4 + 50688T^2 + 1152c^2,
\end{align*}

\begin{align*}
F_2 &= 36864a^2bT - 12288a^2T^4 + 18432a^2T^2 - 4608a^2 - 9216ac + 36864abcT + 36864ab^2T^2 + 165888ab^2T + 1608b^2T + 9216bd + 4096bd^2T + 202752dT + 20736aT + 9216c^2 - 73728T^6 + 38664T^3 - 23040T^4 + 32768T^3 + 24576T^4 + 61440T^6 + 98088T^4 - 10368T^2 + 646c,
\end{align*}

\begin{align*}
F_3 &= 12288a^3 + 12288abc + 65536abT^3 + 73728abT - 24576aT + 131072aT^4 + 102400aT^4 + 67584aT^2 - 384a + 24576abc - 8192cT^2 + 12288cT - 4608cT + 15360T^2 + 960,
\end{align*}

\begin{align*}
F_4 &= 36864a^4T^2 + 15360a^2 - 12288ac - 22288bT^2 + 3072a - 12288bd - 32768dT^2 + 36864adT^3 + 9216bT - 8192T^3 - 18432dT + 16384T^4 + 16384T^4 - 16384T^4 - 6144T^4 + 15360T^2 + 960,
\end{align*}

\begin{align*}
F_5 &= -98304ab^3T + 32768T^5 + 12288T^2 - 27648T^4 - 18432T^4,
\end{align*}

\begin{align*}
F_6 &= 18432a^2T^4 + 131072aT^4 + 4096T^4 + 8192dT + 65536T^4 - 16384T^4 + 12288T^3 + 15312,
\end{align*}

\begin{align*}
F_7 &= 8192c - 4096a^3,
\end{align*}

\begin{align*}
F_8 &= -16384T^2 + 16384T^4 - 8192T^2 + 3072,
\end{align*}

\begin{align*}
F_9 &= -16384T^2 + 32768T^2 + 78192,
\end{align*}

\begin{align*}
F_{11} &= 0, F_{12} = 16384.
\end{align*}
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