BUOYANCY INSTABILITIES IN WEAKLY MAGNETIZED LOW-COLLISIONALITY PLASMAS

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ABSTRACT

I calculate the linear stability of a stratified low-collisionality plasma in the presence of a weak magnetic field. Heat is assumed to flow only along magnetic field lines. In the absence of a heat flux in the background plasma, Balbus demonstrated that plasmas in which the temperature increases in the direction of gravity, a result of considerable importance to the theory of stellar structure (Schwarzschild 1958). Remarkably, however, this well-known result changes in 2008. The American Astronomical Society. All rights reserved. Printed in U.S.A. The Astrophysical Journal

1. INTRODUCTION

Thermally stratified fluids are buoyantly unstable when the entropy increases in the direction of gravity, a result of considerable importance to the theory of stellar structure (Schwarzschild 1958). Remarkably, however, this well-known result changes in a low-collisionality plasma in which (1) the collisional mean free path of electrons is larger than the electron Larmor radius and (2) thermal conduction is the dominant mode of heat transport (Balbus 2000). In such a plasma, heat is transported primarily along magnetic field lines. For the simple problem of a horizontal magnetic field in a vertically stratified plasma, Balbus (2000) showed that the condition for the plasma to be buoyantly unstable becomes that the temperature (not entropy) increase in the direction of gravity. The resulting “magnetothermal instability” (MTI) has been studied with nonlinear simulations by Parrish & Stone (2005, 2007).

In a subsequent paper, Balbus (2001) generalized his initial result to rotating flows and magnetic fields of arbitrary orientation, but still under the assumption that there is no heat flux in the background plasma (i.e., that the field lines are initially isothermal). This latter assumption is unlikely to hold in many low-collisionality astrophysical plasmas such as clusters of galaxies and hot accretion flows onto black holes.

In this paper I extend Balbus’s calculation and study the stability of weakly magnetized plasmas in the presence of a background heat flux. I show that the presence of a heat flux drives a buoyancy instability analogous to the MTI when the temperature decreases in the direction of gravity (a situation that is MTI stable according to Balbus’s analysis). This instability is distinct from the heat flux–driven overstabilities described in Socrates et al. (2008). In the next two sections I summarize the equations and assumptions used in my analysis (§2) and the results of the linear stability calculation (§3). I then discuss possible applications of the heat flux–driven version of the MTI, in particular to the intercluster plasma in clusters of galaxies (§4).

2. BASIC EQUATIONS AND LINEAR PERTURBATIONS

The equations used for my analysis are those of ideal magnetohydrodynamics, supplemented by a heat flux along magnetic field lines; they are identical to those given in Balbus (2001) and Socrates et al. (2008). The equations are the conservation of mass, momentum, magnetic flux, and an internal energy equation,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{\nabla \times \mathbf{B} \times \mathbf{B}}{4\pi} - \nabla P + \rho \mathbf{g}, \\
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \\
\rho T \frac{ds}{dt} = -\nabla \cdot \mathbf{Q} = \nabla \cdot \left[ \chi \mathbf{b} (\mathbf{b} \cdot \nabla) T \right],
\]

where \( \rho \) is the mass density, \( \mathbf{v} \) is the fluid velocity, \( \mathbf{B} \) is the magnetic field, \( \mathbf{g} \) is the gravitational acceleration, \( P \) is the pressure, \( T \) is the temperature, \( s \) is the entropy per unit mass, \( \mathbf{b} = \mathbf{B} / B \) is a unit vector in the direction of the magnetic field, and \( d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla \) is a Lagrangian time derivative. I consider an ideal gas with an adiabatic index of 5/3 throughout this paper.

The internal energy equation (eq. [4]) accounts for the fact that the heat flux \( \mathbf{Q} \) in a plasma is primarily along magnetic field lines when the electron Larmor radius is small compared to the electron mean free path (e.g., Braginskii 1965). In this limit, the heat flux is given by

\[
\mathbf{Q} = -\chi \mathbf{b} (\mathbf{b} \cdot \nabla) T,
\]

where the thermal diffusivity due to electrons is (Spitzer 1962)

\[
\chi \simeq 6 \times 10^{-7} T^{5/2} \text{ ergs cm}^{-1} \text{ K}^{-1}.
\]

I often use \( \kappa = \chi T / P \) in place of \( \chi \) for convenience (\( \kappa \) has units of cm\(^2\) s\(^{-1}\), i.e., of a diffusion coefficient).
2.1. Background Plasma

The fastest growing modes described below have very short wavelengths (where thermal conduction has the largest effect). Thus, a simplified model for the background plasma suffices. I assume that the plasma is thermally stratified in the presence of a uniform gravitational field in the vertical direction, \( \mathbf{g} = -g \hat{z} \). Without loss of generality, the magnetic field is taken to be \( \mathbf{B} = B \hat{x} + B \hat{z} \). I also introduce the dimensionless \( x \) and \( z \) magnetic field strengths, \( b_x = B_x/B \) and \( b_z = B_z/B \), where \( B \) is the magnitude of the initial magnetic field (note that \( b_x \) and \( b_z \) can be either positive or negative). The initial magnetic field is assumed to be very weak so that force balance implies \( dP/dz = -\rho g \). Because \( \mathbf{b} \cdot \nabla \mathcal{T} \neq 0 \), there is a heat flux in the background state, given by

\[
\mathbf{Q} = -\chi (b_x b_x \hat{x} + b_z^2 \hat{z}) \frac{dT}{dz}.
\]

In order for the initial equilibrium to be in steady state, \( \nabla \cdot \mathbf{Q} = 0 \), which implies a temperature that varies linearly with height \( z \). Although this steady state assumption is formally required, it is worth noting that as long as the timescale for the evolution of the system is longer than the local dynamical time, the general features of the instabilities described here are unlikely to depend critically on the system actually being in steady state.

2.2. Linear Perturbations

I carry out a standard Wentzel-Kramers-Brillouin (WKB) perturbation analysis on the background described in § 2.1. All dynamical variables are assumed to vary as \( -i \omega t + i \mathbf{k} \cdot \mathbf{x} \), where \( \mathbf{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \) and the WKB assumption requires \( kH \gg 1 \), where \( H \) is the local scale height of the system. I also define \( k_x^2 + k_y^2 = k^2 \), to be the wavevector perpendicular to the local gravitational field (but not perpendicular to the initial magnetic field). The growing modes of interest have growth times much longer than the sound crossing time of the perturbation. As a result, it is sufficient to work in the Boussinesq approximation, as in Balbus (2000, 2001).

With the above assumptions, the linearly perturbed versions of equations (1)–(5) are given by

\[
\mathbf{k} \cdot \delta \mathbf{v} = 0, \tag{8}
\]

\[
-i \omega \delta b = \frac{\delta \rho}{\rho} \nabla P - i \mathbf{k} \frac{\delta P}{\rho} + \frac{i(\mathbf{B} \cdot \mathbf{k}) \delta \mathbf{B}}{4\pi \rho} - \frac{i(\mathbf{k} \cdot \delta \mathbf{B})}{4\pi \rho}, \tag{9}
\]

\[
\omega \delta \mathbf{B} = -(\mathbf{k} \cdot \delta \mathbf{B}), \tag{10}
\]

\[
\frac{5}{2} i \omega P \frac{\delta \rho}{\rho} + \rho T (\delta \mathbf{v} \cdot \nabla s) = -i \mathbf{k} \cdot \delta \mathbf{Q}. \tag{11}
\]

The key equation for understanding the instabilities discussed in this paper is that for the perturbed heat flux, which is given by

\[
\delta \mathbf{Q} = -\chi \delta \mathbf{b} \left( \delta \mathbf{b} \cdot \nabla \mathcal{T} \right) - \chi \delta \mathbf{b} \left( \delta \mathbf{b} \cdot \nabla T \right) - i \chi \delta \mathbf{b} (\mathbf{k} \cdot \delta T), \tag{12}
\]

where \( \delta \mathbf{b} = \delta (B/B) = \delta B \mathbf{B} - \hat{b} (\delta B/B) \) and \( \delta B \) is the perturbation to the magnitude of the magnetic field. A term proportional to \( \delta \mathbf{b} \propto \delta T \) should formally be included in equation (12), but is small in the local WKB \( (kH \gg 1) \) limit considered here. Equations (8)–(12) differ from the nonrotating limit of Balbus’s (2001) corresponding equations only in the first term in equation (12), which is the portion of the linearly perturbed heat flux due to the presence of a background heat flux in the plasma.

3. RESULTS

After some algebraic manipulation, equations (8)–(12) can be combined to yield the following dispersion relation:

\[
0 = \omega \omega^2 + i \omega \omega^2 - N^2 \frac{k^2}{\sqrt{2}}, \tag{13}
\]

\[
- i \omega \omega^2 \left[ \frac{d \ln T}{dz} \right] \left( 1 - 2 b_z^2 \right) \frac{k^2}{2} + 2 b_x b_y b_z \frac{k^2}{2}.\]

where

\[
N^2 = \frac{2 m_p g}{5 k_B^2} \frac{d s}{dz} = -g \left[ \frac{d \ln \rho}{dz} - \frac{3}{5} \frac{d \ln p}{dz} \right]. \tag{14}
\]

is the hydrodynamic Brunt-Väisälä frequency, \( m_p \) is the proton mass, \( k_B \) is Boltzmann’s constant,

\[
\omega^2 = \omega^2 - (k \cdot v_A)^2, \tag{15}
\]

\[
\nu_L = B/(4\pi \rho)^{1/2} \text{ is the Alfvén speed}, k \cdot v_A \text{ is the Alfvén frequency, and}
\]

\[
\omega_{\text{cond}} = \frac{2}{5} \kappa (b \cdot k)^2. \tag{16}
\]

is the characteristic frequency at which conduction acts on a given perturbation. For \( \kappa = B = 0 \), equation (13) reduces to the usual dispersion relation for hydrodynamic convection and internal gravity waves, \( \omega^2 = N^2 k^2 / k^2 \).

I now consider equation (13) under the assumption that the frequencies of interest in the problem can be ordered as \( \omega_{\text{cond}} \ll \omega_{\text{Bou}} \ll \omega_{\text{A}}, \) where \( \omega_{\text{Bou}} \sim (g/H)^{1/2} \) is the local dynamical frequency. This ordering of timescales can always be achieved if the magnetic field is sufficiently weak (see § 3.3). In this limit, magnetic forces are dynamically unimportant. The only role of the magnetic field is to enforce an anisotropic transport of heat. With this timescale ordering, the dispersion relation reduces to

\[
\omega^2 \approx g \left( \frac{d \ln T}{dz} \right) \left[ 1 - 2 b_z^2 \right] \frac{k^2}{2} + 2 b_x b_y b_z \frac{k^2}{2}. \tag{17}
\]

3.1. \( dT/dz < 0 \)

For plasmas in which \( dT/dz < 0 \), i.e., in which the temperature increases in the direction of gravity, equation (17) describes the MTI discovered by Balbus (2000). This is easiest to see if we consider the simple case in which \( B_z = 0 \) in the initial state. In that case,

\[
\omega^2 \approx g \left( \frac{d \ln T}{dz} \right) \frac{k^2}{2}. \tag{18}
\]

Equation (18) implies that the plasma is unstable on the local dynamical time. Physically, the MTI arises because magnetically connected fluid elements remain nearly isothermal as they are

3 The referee pointed out correctly that a more precise implementation of the Boussinesq approximation is to set \( b P + B^2 / 8 \pi \) = 0 in the energy equation, not \( b P = 0 \) as I have done (since it is only the total pressure perturbation that is guaranteed to be small, not just the gas pressure perturbation). This leads to an additional term in eq. (13) given by \( 2 \beta P (k^2 / 4 \pi / \kappa) b \cdot k |b| (b \cdot k) b (b \cdot k) \), where \( \beta = P/(B^2 / 8 \pi) \). This additional term is, however, small compared to the dominant terms in eq. (13) by at least a factor of \( \sim \beta^{1/2} \), and thus, I neglect it throughout this paper.
displaced on a timescale $\sim \omega_\text{dyn}^{-1} \gg \omega_\text{cond}^{-1}$. The buoyancy of a fluid in pressure equilibrium with its surroundings thus depends, not on the entropy gradient in the background plasma, but rather on the temperature gradient. Note that this reasoning predicts that the plasma should be stable if $dT/dz > 0$, a result which we shall see is incorrect if the magnetic field has a nonzero vertical component.

The above interpretation of the MTI can be formalized by noting that for a purely horizontal initial field, the $x$-component is the only component of the perturbed heat flux; it is given by

$$
\delta Q_x \approx -\chi k_x \xi \frac{dT}{dz} - i \chi k_x \delta T,
$$

(19)

where I have used flux freezing to rewrite $\delta \mathbf{b}$ in terms of the $z$-component of the fluid displacement $\xi = i \omega \mathbf{v}$. The associated divergence of the conductive flux is then given by

$$
-\nabla \cdot \delta Q \approx -\chi k_z^2 \frac{dT}{dz} - \chi k_z^2 \delta T.
$$

(20)

In the limit of rapid conduction, the energy equation (eq. [4]) reduces to

$$
\frac{\delta \rho}{\rho} \approx \xi \frac{d \ln T}{dz},
$$

(21)

where I have used the fact that $\Delta T/T = -\delta \rho/\rho$ in the Boussinesq limit. Equation (21) shows explicitly that, if $dT/dz < 0$, an upwardly displaced fluid element ($\xi > 0$) will have its density decrease relative to the background plasma, and thus, it will buoyantly rise (and vice versa for a downwardly displaced fluid element). Note that equation (21) also implies that the Lagrangian perturbation of the temperature vanishes, i.e., $\Delta T/T = \delta T/T + \xi d \ln T/dz = 0$. Physically, this means that a given fluid element’s temperature does not change as it is displaced from its initial position.

According to Balbus (2001), the growth rate of the MTI is independent of the initial magnetic field geometry, provided that the initial magnetic field lines are isothermal, i.e., that there is no heat flux in the background state. Equation (17) shows that this result is modified in the presence of a background heat flux. Consider the limiting case of a primarily vertical magnetic field in a vertically stratified atmosphere ($b_x \ll 1$ and $b_z \approx 1$). In this case, the growth rates of the MTI are reduced, as one would expect physically. Indeed, there are only growing modes for $k_x/k_z \leq 2 b_x$, and the maximum growth rate is reduced to $|\omega| \approx (g|d \ln T/dz|)^{1/2} b_x$. This reduction in the efficacy of the MTI for primarily vertical fields may partially account for the fact that the nonlinear saturation of the MTI in numerical simulations often involves a rearrangement of the field from primarily horizontal to primarily vertical (Parrish & Stone 2007; P. Sharma & E. Quataert, in preparation).

3.2. $dT/dz > 0$

Previous analyses (Balbus 2001) have found that weakly collisional plasmas are stable when $dT/dz > 0$, i.e., when the temperature decreases in the direction of gravity. Equation (17) shows, however, that this is not the case in the presence of a background heat flux. To begin, consider a purely vertical magnetic field ($b_x = 1$, $b_z = 0$), in which case equation (17) reduces to

$$
\omega^2 \approx -g \left( \frac{d \ln T}{dz} \right) \frac{k_z^2}{k_x^2}.
$$

(22)

Equation (22) shows that the simple equilibrium of a vertical magnetic field in a plasma with $dT/dz > 0$ and a heat flux along the initial magnetic field is dynamically unstable. Moreover, the growth rate is identical to that of the MTI (eq. [18]), which applies for horizontal fields and $dT/dz < 0$.

The physical origin of this instability is illustrated schematically in Figure 1. In the presence of perturbed field lines with nonzero $k_x$ and $k_z$, there are regions where the heat flux—which is forced to follow the perturbed field lines—must converge or diverge. These correspond to regions where the plasma is locally heated and cooled. As a result, when $dT/dz > 0$, a fluid element displaced downward is conductively cooled via the background heat flux, causing it to lose energy and sink further down in the gravitational field. By contrast, an upwardly displaced fluid element gains energy from the background heat flux and thus buoyantly rises. The upshot is a magnetically and heat flux-mediated buoyancy instability analogous to the MTI.

To see this interpretation explicitly, note that the $x$-component of the linearly perturbed heat flux for a purely vertical initial field is given by

$$
\delta Q_x \approx i \chi \frac{k_x^2}{k_z^2} \frac{dT}{dz},
$$

(23)

while the $z$-component is given by

$$
\delta Q_z \approx -i \chi \frac{k_z^2}{k_x^2} \delta T.
$$

(24)

The corresponding contributions to $-\nabla \cdot \delta Q$, i.e., to the conductive “cooling,” on the right-hand side of the energy equation (eq. [4]), are

$$
-\frac{d\delta Q_x}{dx} \approx k_x \frac{k_z^2}{k_x^2} \frac{dT}{dz},
$$

(25)

$$
-\frac{d\delta Q_z}{dz} \approx -k_x \frac{k_z^2}{k_x^2} \delta T.
$$

(26)
Finally, using equations (25) and (26), the density perturbation can be directly computed from the energy equation (eq. [4]) in the limit $\omega_{\text{cond}} \to \infty$, which yields
\[
\frac{\delta \rho}{\rho} \approx -\xi_z \frac{d \ln T}{dz}.
\] (27)

Equation (25) shows that if $dT/dz > 0$, an upwardly displaced fluid element ($\xi_z > 0$) will gain heat from the background flux, causing it to expand to even lower density ($\delta \rho < 0$; eq. [27]) and to thus continue to rise buoyantly upward. By contrast, the nearly isothermal ($\Delta T/T \approx 0$) perturbations present in the case of a horizontal field (the MTI limit) cannot be buoyantly unstable when $dT/dz > 0$.

The growth of this buoyancy instability is modified if the field is not entirely vertical, just as the growth rate of the MTI is unstable when $\delta \rho > 0$ and to continue to sink.

Unlike in the MTI, the Lagrangian perturbation of the temperature $\Delta T$ does not vanish for the current instability. Instead, equation (27) implies
\[
\frac{\Delta T}{T} \approx 2\xi_z \frac{d \ln T}{dz}.
\] (28)

This is a key difference between the current instability and the MTI. In the presence of a vertical magnetic field, a downwardly displaced fluid element couples to the background heat flux, cools ($\Delta T/T < 0$), and thus becomes buoyantly unstable. By contrast, the nearly isothermal ($\Delta T/T \approx 0$) perturbations present in the case of a horizontal field (the MTI limit) cannot be buoyantly unstable when $dT/dz > 0$.

The growth of this buoyancy instability is modified if the field is not entirely vertical, just as the growth rate of the MTI is modified if the field is not entirely horizontal ($\xi_z \neq 0$).

3.3. Stabilization by a Strong Magnetic Field

Given the existence of growing modes for either sign of $dT/dz$, it is natural to consider what can in fact stabilize the magnetically mediated buoyancy instabilities described here. Two physical effects can lead to stabilization. The first is if the dominant mode of heat transport is via an isotropic conductivity, rather than anisotropic heat transport along magnetic field lines (Balbus 2000, 2001). This is the reason that convection in stars is governed by the entropy gradient, not the temperature gradient. Second, these buoyancy instabilities are stabilized if the magnetic field is sufficiently strong (Balbus 2000; Parrish & Stone 2005).

To study the effects of magnetic tension explicitly, I generalize the argument of Balbus (2000) and find that the dispersion relation given in equation (13) has unstable solutions provided that
\[
(k \cdot v_A)^2 + g\left(\frac{d \ln T}{dz}\right)\left[1 - 2\beta_2 \frac{k_x^2}{k^2} + 2\beta_1 \frac{k_y^2 + k_z^2}{k^2}\right] < 0.
\] (29)

A rough quantitative criterion for when magnetic tension stabilizes all perturbations can be determined by requiring that unstable modes fit within the system under consideration (of size $\sim H$), i.e., that $kH \gtrsim 1$. Then magnetic tension will stabilize the system for $\beta \lesssim 1$, where $\beta = B^2/(8\pi)$ and where I have neglected factors of order unity (e.g., the value of $B_0/B_1$).

It is also worth reiterating that even if the system is unstable, the growth rates are only given by equation (17) if the timescale ordering $\omega_{\text{cond}} \gg \omega_{\text{dyn}} \gg k \cdot v_A$ can be satisfied. Approximating $\omega_{\text{dyn}} \simeq (gH)^{1/2} \approx c_s/H$, where $c_s$ is the sound speed of the plasma, and writing $\kappa \simeq v_e \ell_e$, where $v_e$ is the electron thermal speed and
\[
\ell_e \simeq 7 \times 10^{18} \left(\frac{T}{10^5 \text{K}}\right)^{1/2} \left(\frac{n}{1 \text{ cm}^{-3}}\right)^{-1} \text{ cm}
\] (30)

is the electron mean free path due to Coulomb collisions (for a Coulomb logarithm of 10), these two conditions become
\[
kH \lesssim \beta^{1/2},
\] (31)

\[
kH \gtrsim 1.5 \left(\frac{m_e}{m_p}\right)^{1/4} \left(\frac{H}{\ell_e}\right)^{1/2} \simeq 0.23 \left(\frac{H}{\ell_e}\right)^{1/2}.
\] (32)

Equations (31) and (32) imply that the magnetic field is sufficiently weak for instability to occur at the maximal growth rate given in equation (17) provided that
\[
\beta \gtrsim 0.05 \left(\frac{H}{\ell_e}\right).
\] (33)

Equation (33) neglects factors of order unity (e.g., $B_0/B_1$), but is nonetheless a useful guide to when the growth of buoyancy instabilities in magnetized dilute plasmas occurs at or order the local dynamical time. For more accurate results, the full dispersion relation (eq. [13]) can readily be solved.

4. DISCUSSION

The above analysis shows that, regardless of the sign of the temperature gradient, a weakly magnetized low-collisionality plasma in which heat flows primarily along magnetic field lines is buoyantly unstable. For $dT/dz < 0$, this instability is the magneto-thermal instability (MTI) derived by Balbus (2000, 2001) and simulated by Parrish & Stone (2005, 2007). Although a plasma with $dT/dz > 0$ is MTI stable according to Balbus (2001), I have shown that an analogous buoyancy instability in fact exists for $dT/dz > 0$ in the presence of a vertical magnetic field and a background heat flux. Physically, this new instability arises because perturbed fluid elements are heated/cooled by the background heat flux in such a way as to become buoyantly unstable (§3.2).

In many astrophysical plasmas, the sign of the temperature gradient is fixed to be $dT/dz < 0$ by basic principles. These systems may be MTI unstable, but they will be stable to the new buoyancy instability discussed in this paper. This is typically the case in cooling white dwarfs and neutron stars, where the flow of heat outward requires $dT/dz < 0$. It is also the case in hot accretion flows onto compact objects, because the inflow of matter and the release of gravitational potential energy drives $dT/dz < 0$.

The heat flux–driven instability described in this paper may act in the transition region between cool dense gas and hot low-density plasma in stellar coronae, accretion disks, and the multiphase interstellar medium. However, these regions tend to be strongly magnetized, which will inhibit the instability (§3.3). A more promising application is to the hot intercluster plasma in galaxy clusters. Plasma in hydrostatic equilibrium in a Navarro et al. (1997) dark matter potential well has a temperature profile which is locally isothermal ($\rho \propto r^{-2}$) at a scale radius $R_s \simeq 100-400$ kpc. The temperature is predicted to decrease for radii both smaller and larger than $\sim R_s$. Such a radial variation in the temperature of the intercluster plasma is directly observed in many systems (e.g., Piffaretti et al. 2005). At radii larger than $\sim R_s$, the intercluster plasma is MTI unstable (e.g., Parrish & Stone 2007), but it is MTI stable in the cores of clusters where the temperature
decreases inward. However, it is precisely these radii that are unstable to the buoyancy instability discussed in this paper. Thus, provided that the field is not too strong (see § 3.3), I conclude that the entire intercluster plasma in galaxy clusters is unstable to magnetically mediated buoyancy instabilities.

The implications of this instability for the intercluster medium will be investigated in future papers using nonlinear simulations. Here I briefly comment on the possible consequences. Given the presence of exponentially growing instabilities that amplify magnetic fields at all radii in galaxy clusters, it is natural to suspect that these instabilities play a significant role in generating the observed (e.g., Govoni & Feretti 2004) μG magnetic fields in clusters from smaller cosmological “seed” fields. In addition, just as the MTI is found to reorient the magnetic field to be largely radial, allowing heat to flow down the temperature gradient (Parrish & Stone 2007; P. Sharma & E. Quataert, in preparation), I suspect that the heat flux–driven buoyancy instability discussed here will generate a significant horizontal magnetic field if one was not present originally. This will act so as to decrease the net heat flux through the plasma (which is the origin of the instability in the first place). Given the close connection between the heat flux and magnetic field described in this paper, it is unclear whether current calculations of the effective conductivity and heat flux in cluster plasmas (e.g., Narayan & Medvedev 2001; Chandran & Maron 2004) are correct, since they do not capture this dynamical coupling. It may thus turn out that thermal conduction from large radii will prove to be less effective than previous authors have suspected (e.g., Bertschinger & Meiksin 1986; Ruszkowski & Begelman 2002; Zakamska & Narayan 2003) at heating “cooling flow” cores in clusters. Regardless of the accuracy of this speculation, the results of this paper highlight the need for a proper treatment of the combined effects of thermal conduction and magnetically mediated buoyancy instabilities on the plasma in galaxy clusters. In future work it will also be interesting to study the dynamics of the heat flux–driven instability in the presence of cosmic rays, which may be energetically significant in cluster cores because of a central black hole and which are known to modify the MTI (Chandran & Dennis 2006).

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