Study on ISAR imaging for forward-looking missile-borne millimetre wave radar

Cai Wen¹, Jiang Zhu², Yan Zhou¹, Jinye Peng¹

¹School of Information Science and Technology, Northwest University, Xi’an, People’s Republic of China
²Xi’an Institute of Space Radio Technology, People’s Republic of China

E-mail: pjy@nwu.edu.cn

Abstract: Target recognition is a challenging problem in air-to-surface terminal guidance for missile-borne millimetre wave radar (MWR). The class of high range resolution profile (HRRP)-based automatic target recognition approaches is sensitive to the attitude of target. One effective way to improve the performance of target recognition is acquisition of multidimensional information from target’s image. Here, 2D ISAR imaging of the ground static target for the forward-looking missile-borne MWR is studied. The relative signal model and some key points are analysed. The effectiveness of the proposed approach is demonstrated by simulation and real data processing results.

1 Introduction

For air-to-surface terminal guidance application, the target recognition is a challenging problem in missile-borne millimetre wave radar (MWR). The class of one-dimensional (1D) high range resolution profile (HRRP)-based automatic target recognition approaches is popular and widely studied [1]. These approaches differentiate targets according to characteristics of target’s HRRP. However, HRRP is sensitive to the attitude of target, especially at millimetre wave band, which is much more sensitive due to the short wave length. The characteristics of target’s HRRP may be different from one aspect angle to another aspect angle. In this case, the performance of 1D HRRP-based automatic target recognition algorithm degrades rapidly. High-resolution imaging of the target and acquisition of abundant electromagnetic scattering information from the target’s image are the effective way to improve the performance of target recognition.

Synthetic aperture radar (SAR) imaging and inverse SAR (ISAR) imaging are two preferable techniques for moving platform high-resolution imaging [2]. The range high-resolution and cross-range high resolution of SAR are realised by transmitting wideband signal and forming a large virtual aperture utilising the motion of platform, respectively. Compared with SAR imaging, ISAR imaging does not need to design a special trajectory and can be applied in forward-looking radar. Furthermore, ISAR imaging can be applied in non-cooperative target imaging and has been widely studied for ground-based radar [3], shipborne radar [4], and airborne radar [5]. In this paper, 2D ISAR imaging of the ground static target is studied for the forward-looking missile-borne MWR.

The relative imaging geometry and signal model are established, and main steps of ISAR imaging are given. In addition, the problem of cross-range scaling is discussed. Finally, some real data processing results are given to demonstrate the effectiveness of the proposed approach.

2 Geometry and principle of ISAR imaging for forward-looking missile-borne MWR

2.1 Geometry of ISAR imaging for forward-looking missile-borne MWR

The geometry of relative motion between the radar and the target is depicted in Fig. 1. The radar is usually mounted in the nose of the missile in order to detect and track the target ahead of the missile conveniently. Assuming the missile moving along the straight line from point A to point B with a constant velocity in the Cartesian coordinate system OXYZ. The height of the radar to the ground is \( h \), and the velocity vector of the missile is \( \mathbf{v}(t) = [0, v_y(t), 0]^T \); hence, the coordinate of the radar can be expressed as

\[
\mathbf{r}_m(t) = [0, 0, h]^T + \mathbf{v}(t) \cdot t = [0, v_y(t), 0]^T t
\]

where \( t (0 \leq t \leq T_o) \) is the coherent integration time, and \( T_o \) is the total integration time. The ground static target is at point P, and the corresponding coordinate \( \mathbf{r}_t = [x_0, y_0, 0]^T \). The distance between radar and target can be expressed as

\[
R(t) = ||\mathbf{r}_t - \mathbf{r}_m(t)|| = \sqrt{h^2 + (y_0 - v_y(t) t)^2 + h^2}
\]

From Fig. 1, it can be seen that the aspect change of the target is \( \Delta \theta \). This is equivalent to assume that radar is static, and the rotation angle of target with respect to the radar is \( \Delta \theta \). In addition, from (2), it can be seen that the distance between radar and target is varied with time. That is to say, there exist not only rotational motion but also translational motion between the radar and target. The translational motion will be compensated before imaging, and the rotational motion will be retained for Doppler spectrum analysis. The principle of ISAR imaging can be explained based on rotation model when the size of target is much less than the distance between the radar and target [6]. In the next section, we will briefly introduce the principle of ISAR imaging for forward-looking missile-borne MWR.

Fig. 1 Geometry of ISAR imaging for missile-borne radar
The plane rotation model is depicted in Fig. 2, where the 3D target in Fig. 1 is projected onto the 2D imaging plane $X' Y'$, i.e. the plane APB in Fig. 1. The motion of the target in the imaging plane can be decomposed as translational motion and rotational motion, where the centre of rotation is $O'$ and $\Delta \theta(t)$ is the rotation angle. When the size of target is much less than the distance between the radar and target, the instantaneous slant range [7] of the scatterer with coordinate $(x, y)$ can be expressed as

$$ r(t) \approx R(t) + x \sin(\Delta \theta(t)) + y \cos(\Delta \theta(t)) $$

where $R(t)$ is the slant range of the target which is expressed as (2), and it represents the translational motion of the target. Assuming that a linear frequency modulation (LFM) signal $s(t)$ is transmitted by the radar, i.e.

$$ s(t) = \text{rect} \left( \frac{t}{T_p} \right) \exp \left( j 2 \pi f_t t + j \frac{1}{2} \gamma t^2 \right) $$

where $\tau$ is the fast time, $T_p$ is the pulse width, $\gamma$ is the LFM slope, $f_t$ is the carrier frequency and $\text{rect}(\cdot)$ is the rectangular function with the duration $T_p$. Assuming that the translational motion has been compensated and the range migration induce by rotation is negligible, after range compression the echo signal [7] can be expressed as

$$ s(\tau, t) = A(x, y) \times \sin \left( B \left( \tau - \frac{2(x_0 + y_0)}{c} \right) \right) \exp \left( - j \frac{4 \pi}{\lambda} r'(t) \right) $$

where $t \leq T_p$ denotes the slow time, $A(x, y)$ denotes the complex amplitude of the scatterer at $(x, y)$, $B$ is the signal bandwidth, $c$ is the velocity of light, $\lambda$ denotes the wavelength of carrier frequency, $x_0 = \sqrt{x_0^2 + y_0^2 + h^2}$ is the initial slant range of the target (see Fig. 1) and $r'(t) = \dot{x} + x \dot{\Delta \theta}(t) + y \dot{\cos}(\Delta \theta(t))$ denotes the slant range after translational motion compensation. In the millimetre wave band, the coherent integration angle for ISAR imaging is usually very small, hence $r'(t)$ can be approximated by

$$ r'(t) \approx x_0 + y + x \Delta \theta(t) $$

Therefore, $s(\tau, t)$ can be expressed as

$$ s(\tau, t) \approx A(x, y) \times \sin \left( B \left( \tau - \frac{2(x_0 + y_0)}{c} \right) \right) \exp \left( - j \frac{4 \pi}{\lambda} x_0 \Delta \theta(t) \right) $$

where $A(x, y) = A(x, y) \exp[jk(t_0 + y)/\lambda]$ denotes the complex amplitude. From (7), it can be seen that the Doppler frequency of the scatterer at $(x, y)$ can be expressed as

$$ f_d = \frac{2 \pi}{\lambda} \times \frac{\Delta \theta(t)}{\Delta t} = \frac{2 \pi v(t)}{\lambda} $$

where $v(t)$ is the rotating velocity of the target.

From (7) and (8), it can be seen that range distribution of the scatterer is mainly related to the radial coordinate $y$, and the Doppler frequency of scatterer is mainly related to the cross-range coordinate $x$ when the rotating velocity of the target is a constant. Therefore, the position of scatterer can be localised by range delay and Doppler spectrum analysis. Where the iso-range lines are a set of parallel lines perpendicular to the line of sight of the radar, i.e. the parallel lines parallel to the $X'$ axis in Fig. 2, and the iso-Doppler plane is parallel to the plane of the rotation axis and the line of sight of the radar, i.e. the parallel lines parallel to the $Y'$ axis in Fig. 2. The plane rotation model is depicted in Fig. 2, where the 3D target in Fig. 1 is projected onto the 2D imaging plane $X' Y'$, i.e. the plane APB in Fig. 1. The motion of the target in the imaging plane can be decomposed as translational motion and rotational motion, where the centre of rotation is $O'$ and $\Delta \theta(t)$ is the rotation angle. When the size of target is much less than the distance between the radar and target, the instantaneous slant range [7] of the scatterer with coordinate $(x, y)$ can be expressed as

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$$ f_d = \frac{2 \pi}{\lambda} \times \frac{\Delta \theta(t)}{\Delta t} = \frac{2 \pi v(t)}{\lambda} $$

where $v(t)$ is the rotating velocity of the target.

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### 2.3 Coherent integration time analysis

The relationship between the cross-range resolution $\Delta x$ and coherent integration angle $\Delta \theta$ [7] can be expressed as

$$ \Delta \theta = \frac{\lambda}{2 \Delta x} $$

There is a notable characteristic for MWR that the coherent integration angle for ISAR imaging is usually very small, because the wave length is very short. Taking the W-band ($\lambda = 3\text{ mm}$) for example, if the cross-range resolution is 0.1 m, the requisite coherent integration angle is only $0.861^\circ$. Assuming that the requisite moving distance of missile corresponding to the coherent integration angle $\Delta \theta$ is $R_M$. The relationship between $R_M$ and $\Delta \theta$ can be derived from the triangular geometry of Fig. 1, that is

$$ R_M = |AB| = \frac{r_0 \sin(\Delta \theta)}{\sin(\Delta \theta + \psi_o)} $$

where $\psi_o = \cos^{-1}(\cos(\theta_o)\cos(\psi))$, $\cos(\theta_o) = y_0/\sqrt{x_0^2 + y_0^2}$, $\cos(\theta) = \sqrt{x^2 - h^2}/r_o$. From (9) and (10), it can be seen that the length of synthetic aperture $R_M$ is related to the cross-range resolution, the location of target and the height of missile. The corresponding coherent integration time $T_c$ can be expressed as

$$ T_c = \frac{R_M}{v_y} = \frac{r_0 \sin(\Delta \theta)}{v_y \sin(\Delta \theta + \psi_o)} $$

Taking the typical parameters as an example, $\lambda = 3\text{ mm}$, $\Delta x = 0.3\text{ m}$, $h = 200\text{ m}$, and $v_y = 300\text{ m/s}$. The target is located at different place in front of the missile, such as $y_0 = 3000\text{ m}$ or $y_0 = 2500\text{ m}$ and $x_0$ is varied from 0 m to 100 m. The relationship between target's location and coherent integration time is given in Fig. 3. It can be seen that the coherent integration time is only fraction of a second, and for a fixed $x_0$ the larger the $y_0$, the longer the coherent integration time. It also can be found that, for a fixed $y_0$ the larger the $x_0$, the shorter the coherent integration time. That is to say, the larger the squint angle, the longer the coherent integration time. This phenomenon is similar to the theory of SAR.
motion of target is equivalent to a rotation model. The translational
method [8], the cross-correlation method [6], and the frequency
alignment algorithms include the scatterer point referencing
model can be used to realise high-resolution ISAR imaging. For the non-stationary
rotation, the Doppler frequency of a scatterer is a constant. Hence, the
conventional range-Doppler imaging algorithm can be used to
realise high-resolution ISAR imaging. The main steps
of ISAR imaging are summarised in Fig. 4.

3.2 Cross-range scaling
For better image understanding, it is more preferable to rescale the
range-Doppler image and display it in the homogeneous
cross-range domain. However, different from the range scaling factor,
which is determined by the known system parameters, the
cross-range scaling factor is related to the rotating velocity of the object,
and it is usually unknown for a non-cooperative target. In recent
years, many methods have been proposed to estimate this rotating
velocity either by auxiliary configurations [17] or directly from the
received data [18, 19]. All these methods may be effective to some
extent, as illustrated by numerical simulations or experiments on
some real data. However, they may computationally be inefficient
for the iterative imaging process or less robust to a wide variety of
targets with different scattering properties.

3.3 Processing chain of ISAR imaging
The basic theory of ISAR imaging is the same as the theory of
SAR. Both of them can achieve high cross-range resolution
utilising target rotation with respect to the radar. For the ISAR
imaging, it is necessary to complete translational motion
compensation before cross-range focusing [7].

3.1 Translational motion compensation
The translational motion compensation is used to compensate radial motion of the target with respect to the radar. After that, the
motion of target is equivalent to a rotation model. The translational
motion compensation usually includes range alignment and phase
correction.

Normally the range alignment is performed with respect to the
amplitude of the range profiles and also prior to any other
operation including the phase correction. Classic ISAR range
alignment algorithms include the scatterer point referencing
method [8], the cross-correlation method [6], and the frequency
domain method [6]. These methods are very efficient and easy to
implement. Nevertheless, it is also widely accepted that the
misalignment error may accumulate across the coherent integration
interval, which is very likely to occur in presence of the
background noise and/or scintillation of reflectivities. In this paper,
the global method with minimum entropy [9] is used for range
alignment, which is robust to accumulation error and target
scintillation.

After range alignment, the phase correction (also called
autofocusing) is performed to eliminate the phase term due to
target motion. Many schemes for autofocusing have been studied.

Such as the Doppler centroid tracking method [10], the phase
gradient autofocus technique [11], the prominent point processing
algorithm [12], the contrast optimisation technique [13], and
the entropy minimisation technique [14, 15]. In this paper, the entropy
minimisation technique with fast implementation [15] is used to
realise phase correction. For the stationary rotating target, the
Doppler frequency of a scatterer is a constant. Hence, the
conventional range-Doppler imaging algorithm can be used to
realise high-resolution ISAR imaging. For the non-stationary
rotating target, the Doppler frequency of a scatterer is time-
varying. In this case, the time-frequency-based approach [16]
can be utilised to realise high-resolution ISAR imaging. The main steps
of ISAR imaging are summarised in Fig. 4.

It should be noted that when \( x_0 = 0 \) m, i.e. the target is right in
front of the missile, the imaging plane APB is perpendicular to the
plane XOY. In this case, the Doppler frequency of a scatterer
depends mainly on the height of the reflecting points.

\[
(12)
\]

\[
(13)
\]

From (13), we can see that \( \omega(t) \) is a function of time \( t \). For
example, \( h = 200 \) m and \( v_1 = 300 \) m/s. The target is located at
different location in front of the missile, such as \( y_0 = 3000 \) m and
\( x_0 = 0 \) m, \( x_0 = 50 \) m or \( x_0 = 100 \) m. The relationship between
rotating velocity and time \( t \) is given in Fig. 5. We can see from
Fig. 5 that the rotating velocity is varied with time (near-linearly).
However, the amount of the rotating velocity change is very small
for a short integration time (from Fig. 3, we can see that the
integration time is \(<0.7\) s when \( x_0 = 0 \) m). The target can be treated
as a uniformly rotating target in the step of imaging processing.
Thus we take the mean of \( \omega(t) \) over coherent processing interval as
the cross-range scaling factor. The relationship between
cross-range \( x \) and Doppler frequency \( f_D \) is

\[
(14)
\]
4 Numerical and experimental results

4.1 Simulation results

The simulation parameters of MWR are given in Table 1. The height of the radar is \( h = 200 \) m and the velocity vector of the missile is \( \mathbf{v}(t) = [0, v_y, 0]^T = [0, 300, 0]^T \). The 3D distribution of the scatterers of a target is depicted in Fig. 6, where the length of the target is 11 m (along the \( Y \)-axis), the width of target is 3 m (along the \( X \)-axis), and the height of target is 2.5 m (along the \( Z \)-axis), respectively.

First, we assume the target is located at \( \mathbf{r}_0(t) = [x_0, y_0, z_0]^T = [0, 2500, 0]^T \), that is to say, the target is right in front of the missile. The coherent integration time is 0.5 s. The corresponding ISAR image is given in Fig. 7. The estimated sizes of the target in down-range domain and cross-range domain are about 11 m (length) and 2.6 m (height), respectively.

Then, the target is located at \( \mathbf{r}_0(t) = [x_0, y_0, z_0]^T = [100, 2500, 0]^T \), in this case, the target is on the right ahead of the missile. The coherent integration time is 0.45 s. The corresponding ISAR image is given in Fig. 8. The estimated size of the target in down-range domain and cross-range domain are about 11 and 3.5 m, respectively.

4.2 Real data processing results

The real data is collected by a W-band radar seeker which is mounted at one side of the helicopter. A much longer integration time is needed for ISAR imaging because the flight speed of the helicopter is much slower than that of the missile. The cooperative target is a model tank whose length is 10 m, width is 3.5 m, and height is 2.5 m, respectively. The positive side of the tank is towards to the radar, and a corner reflector is placed 5 m away from the tank along the direction of radial range. The flight speed of the helicopter is about 45 m/s and the flight height is 150 m. The 1st to 122nd frames are taken for ISAR imaging and the coherent integration time is 1.037 s.

The result of range alignment is given in Fig. 9, and the ISAR imaging and cross-range scaling result is given in Fig. 10. The estimated sizes of the target in down-range domain and cross-range domain are about 3.5 and 4.1 m, respectively.

5 Conclusion

The problem of 2D ISAR imaging of the ground static target for the forward-looking missile-borne MWR is studied in this paper. The relative signal model, the coherent integration time, and the problem of cross-range scaling have been discussed. Simulation and real data processing results show that a high-resolution image of the ground target can be obtained by the proposed ISAR technique.

Future research directions include ISAR imaging under missile manoeuvring flight scenario and feature extraction from the ISAR image.
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