Active dynamic damping of vibrations of a mechanical system with a finite number of degrees of freedom

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Abstract: Increased machine efficiency, increased speeds of working bodies, reduced material consumption, increased load, the need to ensure reliable operation of equipment and safe working conditions are the main factors that determine attention to vibration protection problems. This constitutes the current trend in the modern dynamics of machines. In this work, we propose a method for the dynamic synthesis of dynamic vibration dampers of a viscoelastic mechanical system having the structure of several (two) bodies elastically attached to the object. The aim of the work is to develop methods of mathematical and computer modeling of the processes of dynamics of active and passive vibration protection systems, as well as to increase the efficiency of using passive and active vibration protection of objects. When developing a system for protecting radio-electronic devices (RED) at resonant frequencies, elements of the theory of automatic control and mathematical modeling were used. An algorithm for generating feedback signals for the information-measuring system for active control of vibration-protective radio-electronic devices is proposed. A method is developed for numerically-analytical study of oscillatory processes of nonlinear mechanical systems consisting of spatial rigid bodies interconnected by massless viscoelastic elements.

1. INTRODUCTION

Increased productivity and efficiency of machines, increased speeds of motion of working bodies, reduced material consumption, increased load due to vibrations and shocks, the need to ensure reliable operation of equipment and safe working conditions are factors that determine attention to vibration protection tasks. The latter is a fairly developed and relevant direction of the modern dynamics of machines. Modern machines are equipped with complex automatic control systems, which makes it possible to use external energy sources in monitoring the dynamic state of technical objects, and consider the tasks of ensuring the necessary level of vibrational motions as tasks of ensuring technological quality. Fundamental results on resolving these issues are presented in numerous works of scientists: [1-6]. The most effective among them are active shock absorbers, in which in addition to damping elements, there are elements with an additional energy source, which allows you to change the stiffness of the suspensions, and thereby reduce the effect of vibration loads on the RED. Such shock absorbers are designed to reduce the amplitude of vibrations not only at resonant frequencies, but also in all the required frequency range, which entails complication design by introducing additional means of measuring vibration, therefore, the use of such vibration protection means is...
justified only in exceptional cases, responsible RED. In addition, each of them must be tuned to a specific operating frequency (as a rule - the first natural frequency of the device), the value of which for each particular product varies in a certain range, so the introduction of adaptation into the vibration protection system is important [7-9]. A feature of the work of many machines and mechanisms in comparison with control objects in the theory of automatic control lies in the actual coincidence of physical and informational representations. This is due to the fact that at a certain level of consideration, mechanical systems are perceived at the level of conversion of specific power or kinematic parameters of the dynamic state, which is understandable and follows from the accepted form of perception, defined by the laws of mechanics. Self-organization of the established relative motion caused by disturbances, and the reaction of systems to various types of influences, are associated with various forms of introducing feedbacks.

The presented work reflected the results of studies related to the development of methods for assessing the possibilities and forms of implementing dynamic modes of damping oscillations of mechanical systems based on the feedback concept. In practice, vibration protection is sufficient only at resonant frequencies, as other frequencies on the RED do not have a strong effect. Thus, the task of creating a structurally simple information-measuring system for controlling the active vibration protection of the RED, which implements a highly effective way to reduce vibration loads at resonant frequencies, is relevant. With the development of technology, the problem of protection (aircraft, moving objects, etc.) of devices from harmful vibration and shock effects becomes more and more urgent. The severity of the problem is due to the fact that on the one hand, there is an increase in the power of machines and power plants per unit of weight, and, as a result of this, the proportion of power dissipated in the form of vibration and noise increases. On the other hand, the sensitivity of devices increases, the complexity of precise mechanisms increases, and the requirements for their trouble-free operation are tightened. The effect of vibration on the elements of devices (unique objects, for example, electronic or biological) causes various phenomena in them related to nonlinear deformation, depending on the physic-mechanical properties of the material of the elements. To protect objects from vibrational effects, linearly active vibro-protective systems have been increasingly used recently, in which control forces are applied that are applied directly to the protected object in order to compensate for vibrational disturbances. However, existing systems are only effective in compensating for narrow-band processes. When compensating for broadband vibration effects, difficulties arise associated with ensuring the required quality of functioning, stability and the occurrence of various nonlinear phenomena during vibration processes. Purpose of work is the development of methods of mathematical and computer modeling of the dynamics of active and passive vibration protection systems in order to increase and evaluate the effectiveness of their use in nonlinear mechanical systems. And also, in increasing the efficiency of using passive and active vibration protection of objects (for example, in electronic devices) due to the optimal selection of their parameters and taking into account the structurally dissipative properties of the system. To achieve this goal, the following tasks are solved: - development of a mathematical (and computer) model for the simultaneous use of the dynamics of active and passive vibration protection systems in order to increase the efficiency of their use in mechanical systems.

2. METHODS

2.1 Statement of the problem and methods of solution

The dynamic behavior of the stress-strain state of a dissipative mechanical system consisting of deformable and non-deformable bodies is considered. Together with the assumption of the oscillatory nature of the motion, this will allow us to apply the freezing procedure [10], which leads to the following complex physical relations for deformable elements of zero volume:

$$F_e = -c_e \Delta e = -c_e \left[1 - \Gamma_e^s(\omega_R) - i\Gamma_e^s(\omega_R)\right] \Delta e.$$  

(1)
\[
\Gamma_{\lambda,m}^{c}(\omega) = \int_{0}^{\infty} R_{\lambda,m}(\tau) \cdot \cos \omega \tau \, d\tau ;
\]
\[
\Gamma_{\lambda,m}^{s}(\omega) = \int_{0}^{\infty} R_{\lambda,m}(\tau) \cdot \sin \omega \tau \, d\tau.
\]

\(F_e\) – effort in i-om concentrated element, \(\Delta e\) - elongation of this element, \(E\) - instant modulus, \(A, \alpha\) and \(\beta\) - dimensionless parameters. The parameters of the relaxation core and the instantaneous elastic modulus are determined from quasistatic experiments by the technique described in [11]. When stating the problem of the natural and forced oscillations of the system, the principle of possible displacements is used, according to which the sum of all active forces acting on the system, including inertia forces, is zero:

\[
\delta A = \delta A_\sigma + \delta A_u + \delta A_F = 0 \quad ,
\]

here

\[
\delta A_F = -\sum_{n=1}^{S_l} \int_{V_n} \rho_n \partial^2 \tau \partial \Delta u \, dV - \sum_{c=1}^{S_c} \Gamma_c \partial \Delta e ;
\]
\[
\delta A_u = -\sum_{n=1}^{S_l} \int_{V_n} \rho_u \frac{\partial^2 \tau}{\partial t^2} \partial \Delta u \, dV - \sum_{k=1}^{m_k} \frac{d^2 \tau}{dt^2} \partial \Delta u_k - \sum_{k=1}^{n_k} \frac{d^2 \tau}{dt^2} \partial \rho_k ;
\]
\[
\delta A_f = -\sum_{n=1}^{S_l} \int_{V_n} \rho_n \partial \Delta u \, dV + \sum_{n=1}^{S_l} \int_{V_n} \partial \Delta u \, dV + \sum_{n=1}^{N_f} F \partial \Delta u_n + \sum_{k=1}^{N_k} m_k \partial \rho_k ,
\]

\(\Delta e\) - strain variations of distributed and linear lumped elements. \(\rho_n\) - material density of the nth lumped element; \(m_k\) - weight \(\kappa\) – go hard body ; \(u, u_k, \partial u, \partial u_k\) - vectors of displacements of points of distributed elements and centers of mass of rigid bodies and their variations; \(f, \rho\) - densities of mass and surface forces applied to distributed elements; \(V_n, E_n\) - volume and surface of the nth distributed element; \(I_n\) - tensor of the central moments of inertia of the nth solid; \(F_m, M_k\) - the main vector and the main moment of forces applied to the solid body. As an example, we consider a body mounted on deformable supports (Figure 1) or an object is considered a dissipative mechanical

Figure 1. Calculation scheme
System (2) is written in matrix form with respect to the matrix–column \( \{X\} = \text{colon}(x_1, \ldots, x_n) \) in the following way:

\[
[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = \{f\} ,
\]

where the inertia matrix \([M]\), damping matrix \([C]\) and stiffness matrix \([K]\) are \(n\)-order symmetric matrices. The perturbation is described by a column matrix \(\{f\}\). The physical meaning of matrix coefficients is as follows:

- \(M_{jk}\) - component of the momentum along \(j\) at unit speed \(n_0\).
- \(C_{jk}\) - damping force over \(j\) at a unit speed in \(k\).
- \(K_{jk}\) - elastic force due to a single movement along \(k\).

If the excitation matrix \(\{f\} = 0\), then equation (3) describes the free oscillations of the system, and if \(\{f\} \neq 0\) - then forced. The solution to equation (3) can be sought in the form

\[
\{x(t)\} = [W]e^{\lambda t} ,
\]

where \(\lambda\) - complex number, \(W\) - complex numerical matrix - column. The numbers \(\lambda\) called characteristic indicators, and numbers \(\dot{\lambda}\) (or \(-i\dot{\lambda}\)) - complex frequencies. Characteristic indicators must be the roots of the characteristic equation

\[
\det([M]\lambda^2 + [C] \lambda + [K]) = 0 .
\]

A system with \(n\) degrees of freedom has \(2n\) characteristic indicators \(\lambda_1, \ldots, \lambda_{2n}\). If all characteristic exponents are the simple roots of equation (5), then the general solution of equation (5) will be equal to the sum of \(2n\) partial solutions of the form

\[
\{X(t)\} = \sum_{k=1}^{2n} C_k \{W_k\} e^{-i\lambda_k t} .
\]

Here \(C_k\) - arbitrary complex constants, and \(W_k\) - numeric matrices are columns.

We will present characteristic indicators in the form

\[
\lambda_k = \omega_k - i\omega_k ,
\]

\[
\lambda_{n-k} = \omega_k + i\omega_k , \quad (k = 1, \ldots, n)
\]

where \(\omega_k > 0\) and \(\omega_k > 0\) - real numbers called damping coefficients and natural frequencies of the damped system, respectively. If \(\{W_k\}\) and \(\lambda_k\) satisfy equation (5), then complex conjugate \([W_k^*]\) and \(\lambda_k^*\) also satisfy him. When there is no damping, all roots lie on the imaginary axis. When damping, the roots are near the imaginary axis. If the system is dissipative and has complete dissipation, then all characteristic exponents lie in the lower half-plane of the complex variable. All particular solutions are decaying functions and, therefore, the general solution is a decaying function of time. If the system has incomplete dissipation, then part of its indicators lie in the left half-plane, and part on the imaginary axis. Among particular solutions, there are periodic ones, that correspond to non-damped degrees of freedom. If the system has negative dissipation, then among the characteristic indicators there may be those whose real parts are negative. The corresponding particular and general solutions will be functions unlimitedly increasing in time.
3. RESULTS AND DISCUSSIONS

As an example, a block diagram of a dynamic vibration damper according to the scheme corresponding to the first group is shown (Figure 2). Accounting for connectivity in the motions of dynamic dampers \( m_1 \) and \( m_2 \) changes the parameters of the dynamic quenching mode and others, but overall the dynamic properties of the system remain the same, if we take into account the number of resonances and the number of modes of dynamic quenching \([12,13]\). As one of the specific applications, the possibilities of a dynamic vibration damper obtained using the generalized technique for constructing mathematical models of systems are considered (Figure 2). The amplitude-frequency characteristic of a system with a dynamic lever damper is shown in Figure 3. The differential equation of motion of the system (1) and the transfer function are respectively of the form

\[
(M + m_1^2) \ddot{y} + k(1 - \Gamma^2) y = \ddot{z}m(i + 1) + k(1 - \Gamma^2) y, \tag{8}
\]

\[
W(p) = \frac{\Sigma}{z} = \frac{m(i + 1)p^2 + k\Gamma}{(M + m_1^2)p^2 + k\Gamma}.
\]

The main material of the printed circuit board is fiberglass or getinax. Recently, in the electronic industry all fiberglass is becoming very popular, therefore, we will set initial conditions corresponding to this material:

- Young's modulus (E), equal to 105 kgf/m²,
- density (ρ) equal to 1400 kg/m³,
- frequency (f) equal to 200 Hz,
- the amplitude of the external vibrational impact (\( A_0 \)), equal to one.

![Figure 2. The calculated (upper) and structural (lower) schemes of the vibration protection system with lever damper](image-url)
Figure 3. The amplitude-frequency characteristics of the system at different mass ratios: curve $a$ meets the condition $M > m_i$; curve $b$ according $M = m_i$; the curve corresponds to $M < m_i$.

The complex system and differential equations are solved by the Laplace integral transform method [14]. The calculation results are shown in Figure 3 and 4. In Figure 4 it is indicated that: 1. $M^* = 0.01$, 2. $M^* = 0.1$, 3. $M^* = 1.0$. Here $M^* = (m_1 + m_2) / M$. It is seen, what with the increase in mass of the dynamic damper, the oscillation is optimized.

Figure 4. A change in the amplitude of motion depending on the frequency for different $M$. (1. $M^* = 0.01$, 2. $M^* = 0.1$, 3. $M^* = 1.0$)

To consider the features of dynamic damping in systems with linkages, the authors conducted theoretical studies and compared experimental data [15]. The experiment was carried out on a prototype, which vibration-proof systems equipped with a device for converting motion [16]. Based on the research, we can draw a number of conclusions: A general variational mathematical formulation of the problem of the dynamics of dissipatively homogeneous and inhomogeneous mechanical systems, as applied to vibration protection systems, is proposed. The relationship between stresses and strains is taken into account using the Boltzmann – Voltaire integral [17-20].
4. CONCLUSIONS

A method for constructing mathematical models for vibration protection systems based on the use of dynamic dampers with several degrees of freedom has been developed. The dynamic properties of dynamic vibration dampers of various structural and technical variants have been studied. Numerical - analytical study of the oscillatory processes of nonlinear mechanical systems consisting of spatial rigid bodies interconnected without massive viscoelastic elements. A numerical - analytical study of the oscillatory processes of nonlinear mechanical systems consisting of a package of rectangular plates of mass concentrations and interconnected without massive viscoelastic elements and struts. Numerical - analytical study of the oscillatory processes of nonlinear mechanical systems consisting of a package of cylindrical shells of mass concentrations and interconnected without massive viscoelastic elements and struts.

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