Heavy Quark Potentials and Quarkonia Binding

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Abstract. I review recent progress in studying in-medium modification of inter-quark forces at finite temperature in lattice QCD. Some applications to the problem of quarkonium binding in potential models is also discussed.

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1 Introduction

The study of in-medium modifications of inter-quark forces at high temperatures is important for detailed theoretical understanding of the properties of Quark Gluon Plasma as well to detect its formation in heavy ion collisions. In particular, it was suggested by Matsui and Satz that color screening at high temperature will result in dissolution of quarkonium state and the corresponding quarkonium suppression could be a signal of Quark Gluon Plasma formation [1].

Usually the problem of in-medium modification of inter-quark forces is studied in terms of so-called finite temperature heavy quark potentials, which are, in fact, the differences in the free energies of the system with static quark anti-quark pair and the same system without static charges. Alternatively this problem can be studied in terms of finite temperature quarkonium spectral functions [2, 3, 4] which were also discussed during this conference by Karsch, Hatsuda and Petrov [5]. Recently substantial progress has been made in studying the free energy of static quark anti-quark pair which I am going to review in the present paper. An important question is what can we learn about the quarkonium properties from the free energy of static charges which will be discussed at the end of the paper.

2 The free energy of static charges

Following McLerran and Svetitsky the partition function of the system with static quark anti-quark ($Q\bar{Q}$) pair at finite temperature $T$ can be written as

$$Z_{Q\bar{Q}}(r, T) = \langle W(r) W^\dagger(0) \rangle Z(T),$$

with $Z(T)$ being the partition function of the system without static charges and

$$W(x) = \mathcal{P} \exp(ig \int_0^{1/T} d\tau A_0(\tau, x))$$

is the temporal Wilson line. $L(x) = \text{Tr} W(x)$ is also known as Polyakov loop and in the case of pure gauge theory it is an order parameter of the deconfinement transition. As the $Q\bar{Q}$ pair can be either in color singlet or octet state one should separate these irreducible contributions to the partition function. This can be done using the projection operators $P_1$ and $P_8$ onto color singlet and octet states introduced in Refs. [7, 8]. Applying $P_1$ and $P_8$ to $Z_{Q\bar{Q}}(r, T)$ we get the following expression for the singlet and octet free energies of the static $Q\bar{Q}$ pair

$$\exp(-F_1(r, T)/T) = \frac{1}{Z(T)} \frac{\text{Tr} P_1 Z_{Q\bar{Q}}(r, T)}{\text{Tr} P_1} = \frac{1}{4} \text{Tr} \langle W(r) W^\dagger(0) \rangle$$

$$\exp(-F_8(r, T)/T) = \frac{1}{Z(T)} \frac{\text{Tr} P_8 Z_{Q\bar{Q}}(r, T)}{\text{Tr} P_8} = \frac{1}{8} \text{Tr} W(r) W^\dagger(0) - \frac{1}{24} \text{Tr} (W(r) W^\dagger(0)).$$

Although usually $F_{1,8}$ is referred to as the free energy of the static $Q\bar{Q}$ pair, it is important to keep in mind that it refers to the difference between the free energy of the system with static quark anti-quark pair and the free energy of the system without static charges.

As $W(x)$ is a not gauge invariant operator we have to fix a gauge in order to define $F_1$ and $F_8$. As we want that $F_1$ and $F_8$ have a meaningful zero temperature limit we better to fix the Coulomb gauge because in this gauge a transfer matrix can be defined and the free energy difference can be related to the interaction energy of a static $Q\bar{Q}$ pair at zero temperature ($T = 0$). Another possibility discussed in Ref. [9] is to replace the Wilson line by a gauge invariant Wilson line using the eigenvector of the spatial covariant Laplacian [10]. For the singlet free energy both methods were tested and they were shown to give numerically indistinguishable results, which in the zero temperature limit are the same as the canonical results obtained from Wilson loops. The interpretation of the color octet free energy at small temperatures is less obvious and
averaged free energy is a thermal average over the free energies of static pair were (mis)interpreted as potentials. At next to leading order which is expressed entirely in terms of gauge invariant Polyakov loops. This is the reason why it was extensively studied on lattice during the last two decades. The color averaged free energy is a thermal average over the free energies in color singlet and color octet states

\[
\exp(-F_{\text{av}}(r, T)/T) = \frac{1}{Z(T)} \frac{\text{Tr}(P_1 + P_8) Z_{QQ}(r, T)}{\text{Tr}(P_1 + P_8)} = \exp(-F_1(r, T)/T) + \frac{g^2}{4\pi T} \exp(-F_8(r, T)/T).
\]

Therefore it gives less direct information about medium modification of inter-quark forces. Given the partition function \( Z_{QQ}(r, T) \) we can calculate not only the free energy but also the entropy as well as the internal energy of the static charges

\[
S_i(r, T) = \frac{\partial}{\partial T} \ln \left( T \frac{Z_{QQ}(r, T)}{Z(T)} \right) = -\frac{\partial F_i(r, T)}{\partial T}, \quad i = 1, 8, \text{av.}
\]

\[
U_i(r, T) = T^2 \frac{\partial}{\partial T} \ln \left( \frac{Z_{QQ}(r, T)}{Z(T)} \right) = F_i(r, T) + TS_i(r, T), \quad i = 1, 8, \text{av.}
\]

### 3 Free energy of a static \( Q\overline{Q} \) pair and screening of inter-quark forces at high temperatures

Perturbatively the quark anti-quark potential can be related to the scattering amplitude corresponding to one gluon exchange and in the non-relativistic limit it is given by

\[
V(r) = \langle T^{a} T^{b}\rangle g^2 \int \frac{d^3k}{(2\pi)^3} e^{ikr} D_{00}(k). \tag{9}
\]

Here \( D_{00}(k) \) is the temporal part of the Coulomb gauge gluon propagator and in general it has the form

\[
D_{00}(k) = (k^2 + \Pi_{00}(k))^{-1}.
\]

Furthermore the averaging over color gives \( \langle T^{a} T^{b}\rangle = -4/3 \) for the color singlet and \( \langle T^{a} T^{b}\rangle = +1/6 \) for the color octet case. At zero temperature the polarization operator \( \Pi_{00} \) gives rise only to running of the coupling constant \( g = g(r) \) (recall that \( \alpha_s = g^2/(4\pi) \)). But at finite temperature \( T \) it has a non-trivial infrared limit \( \Pi_{00}(k \to 0) = m_{D}^2 = gT \sqrt{N_c/3 + N_f/6} \). Therefore at distances \( r \gg 1/T \) the potential has the form

\[
V(r, T) = \langle T^{a} T^{b}\rangle g^2 \frac{1}{4\pi T} \exp(-m_{D}r). \tag{10}
\]

The singlet and octet free energies defined in the previous section can also be easily calculated in leading order perturbation theory. Again because of \( \Pi_{00}(k = 0) = m_{D}^2 \) one has

\[
F_{1,8}(r, T) = \left( -4 \frac{1}{6} \right) g^2 \frac{1}{4\pi T} \exp(-m_{D}r). \tag{11}
\]

At leading order the singlet free energy has exactly the same form as the potential and has no entropy contribution. This is the reason why the free energies of static \( Q\overline{Q} \) pair were (mis)interpreted as potentials. At next to leading order which is \( \mathcal{O}(g^3) \) the free energies have the form

\[
F_{1,8}(r, T) = \left( -4 \frac{1}{6} \right) g^2 \frac{1}{4\pi T} \exp(-m_{D}r) - \frac{g^2 m_{D}}{3\pi}, \tag{12}
\]

and the entropy contribution \(-TS\) appears (recall Eq. 7).

For the singlet case the entropy has the form

\[
S_1(r, T) = \frac{g^2 m_{D}}{3\pi T} (1 - \exp(-m_{D}r)). \tag{13}
\]
It has the asymptotic value of $\frac{2m_D}{3\pi}$ at large distances and vanishes for $rT \ll 1$. Similarly one can calculate the color octet entropy to be

$$S_8(r, T) = \frac{g^2 m_D}{3\pi T} (1 + \frac{1}{8} \exp(-m_D r)).$$

(14)

Contrary to the color singlet case it does not vanish at small distances. We can also calculate the internal energy which for color singlet state, for example, can be written as

$$U_1(r, T) = -\frac{4}{3} \frac{g^2}{4\pi r} \exp(-m_D r) - \frac{g^2 m_D}{3\pi} \exp(-m_D r),$$

(15)

and unlike the free energy vanishes at large distances (at least to order $g^4$). Using Eqs. (14), (15) one can easily get the perturbative result for the color averaged free energy. For $rT > 1$ the exponentials in Eq. (14) can be expanded and we arrive at the well known leading order result

$$F_{av}(r, T) = -\frac{1}{9} \frac{g^4}{(4\pi r)^2 T} \exp(-2m_D r).$$

(16)

### 4 Numerical results on the free energy of static $Q\bar{Q}$

#### 4.1 Color singlet free energy

In this section I am going to review recent lattice results on the free energy of a static quark anti-quark pair. I will start the discussion with the case of the color singlet channel. The color singlet free energy has been extensively studied only during the last three years. Presently results are available for SU(2) and SU(3) gauge theories as well as in two flavor and three flavor QCD. While for pure gauge theories these studies are very systematic and lattice artifacts are under control, for full QCD they are still in the exploratory stage.

In Fig. 3 the singlet free energy for SU(3) gauge theory (QCD without dynamical quarks or quenched QCD) is shown for different temperatures together with the zero temperature quark anti-quark potential. For temperatures below the transition temperature $T_c \approx 270\text{MeV}$ the free energy rises linearly with the distance $r$ signaling confinement. Above deconfinement $T > T_c$ the free energy has a finite value at infinite separation indicating screening. One can also see from the figure that at short distances, $rT \ll 1$, the free energy is temperature independent in the entire temperature range and equal to the zero temperature potential. The singlet free energy for three flavor QCD is also shown in Fig. 1. The main difference compared to the case of SU(3) gauge theory is the fact that the free energy reaches a constant value at all temperatures. At low temperatures this is interpreted as string breaking, the flux tube breaks if enough energy is accumulated to create a light quark anti-quark pair which with the static $Q\bar{Q}$ could form a static-light meson, i.e. when $V(r = r_{scr}) = E_{binding}$ (see e.g. discussion in Ref. 10). The distance $r_{scr}$, where the free energy effectively flattens off depends on the temperature, it becomes smaller as the temperature increases. Therefore it is interpreted as an effective screening radius and is shown in Fig. 2.

At low temperatures, $T < T_c$, it has a value of about 0.9 fm and rapidly decreases near the transition point. While close to $T_c$ the effect of dynamical quarks is important for the value of the screening radius, at high temperatures the value of the screening radius is similar in quenched and full QCD.

#### 4.2 Color octet free energy

The color octet free energy is shown in Fig. 3 for quenched QCD. At short distances it is repulsive as expected from perturbation theory. Above the deconfinement temperature it has strong temperature dependence which presumably comes from the entropy contribution and is present even at short distances. At high temperatures this is also expected from perturbation theory (see previous section). Above deconfinement the color octet free energy has the same large distance asymptotic value as the color singlet free energy $F_8(r \rightarrow \infty, T) = F_1(r \rightarrow \infty, T) = F_\infty(T)$. This is intuitively expected, at large distances the quark and anti-quark are screened by their respective “clouds” and do not know anything about their relative color orientation. One should note that also below $T_c$ the octet and singlet free energies become equal at large distances (}
compare Figs. 1 and 3, though there is no particular physical reason for this. I will discuss this problem at the end of this subsection. These features of the color octet free energy are present also in the case of three flavor QCD [14].

Perturbation theory predicts that at high temperatures we expect for the ratio of \( \delta F_{1,S}(r,T) = (F_{1,S}(r,T) - F_{\infty}(T)) \) the following \( \delta F_{1}(r,T)/F_{\infty}(r,T) \approx -8 \). In Fig. 4 the lattice data in SU(3) gauge theory are confronted with these expectations. As one can see from the figure the data for this ratio are close to \(-8\) for temperatures \( T > 2T_c \).

While at high temperatures the meaning of the notion of color octet free energy is clear, the meaning of this quantity at low temperatures ("confinement region") is less evident. To understand the problem let us first discuss the spectrum of static quark anti-quark free energies at zero temperature. The lowest energy level of a static \( Q\bar{Q} \) pair is the one where the quark and anti-quark are color singlet state. The corresponding energy as a function of the quark anti-quark separation is the singlet static potential or simply the static potential determined in terms of Wilson loop and used extensively in potential models (see Refs. [17][18]). There are also higher energy levels, whose energy functions are called hybrid potentials for which the gluon fields between the static charges are in excited state (or in other words the string formed between the quark and anti-quark is excited) [17][19][20]. The spectrum of hybrid potentials is shown in Fig. 5. The hybrid potentials are labeled by the angular momentum projection of the gluon field configurations on the quark anti-quark axis, \( L = 0, 1, 2 \) (denoted as \( \Sigma, \Pi, \Delta \)), CP (even, g, or odd, u) and the reflections properties with respect to the plane passing through the quark anti-quark axis (even, +, or odd, -) [17]. The most distinct feature of the hybrid potentials is the different slope at small distances, i.e. in contrast to the singlet potential the hybrid potentials are repulsive. They have a shape which is similar to the shape of the octet free energy at low temperatures shown in Fig. 3. It has been shown that at short distances hybrid potentials correspond to the perturbative color octet potential [21]. This means that at short distances hybrid potentials correspond to a state in which the static \( QQ \) pair is in octet state and the net color charge is compensated by soft gluons field which make the whole object color singlet (obviously only singlet objects can exist in the confinement region). At very small distances the soft gluon field is decoupled and the energy is dominantly determined by the large repulsive interaction of the static quark and antiquark [21].

The correlators which enters the definition of the color singlet and octet free energy have the following spectral representation [22]

\[
\langle \text{Tr} W(r) \text{Tr} W^{\dagger}(0) \rangle = \sum_n e^{-E_n(r,T)/T} \tag{17}
\]

\[
\langle \text{Tr} W(r) W^{\dagger}(0) \rangle = \sum_n c_ne^{-E_n(r,T)/T}, \tag{18}
\]

where \( E_n \) denotes the different energy levels of the quark anti-quark system: singlet potential, hybrid potentials, singlet potentials plus glueballs, etc. The weights \( c_n \) in general are different from one and may have non-trivial \( r \)-dependence [22]. Because of asymptotic freedom \( c_1 \) should approach unity at short distances, while \( c_n, n > 1 \) should vanish; at short distances perturbation theory can be applied and the correlator \( \langle \text{Tr} W(r) W^{\dagger}(0) \rangle \) gives the singlet potential. The tendency of \( c_1 \) approaching unity is clearly seen in the lattice data presented in Ref. [22]. Since the gap between the zero temperature (ground state) potential and the lowest hybrid potential is large for not too large distances the color octet free energy is expected to be given by

\[
e^{-F_8(r,T)/T} \approx \frac{1 - c_1(r)}{N^2 - 1} e^{-E_1(r)/T}, \quad E_1(r) = V(r), \tag{19}
\]

i.e. it is determined by the singlet potential \( V(r) \). This is the reason why at very large distances the color singlet and color octet free energies are equal in the low temperature region (compare Fig. 1 and Fig. 3). On the other hand for sufficiently small distances \( 1 - c_1(r) \approx 0 \) and the octet free energy is expected to be determined by the lowest lying hybrid potential \( F_8(r,T) \approx E_2(r) + T \ln 8 \).
cussed in detail in Ref. [13], where it was also pointed out that $\alpha_{eff1} \simeq \alpha_{eff8}$. In Fig. 6 the running coupling constant is shown for the octet case (the results in the singlet case are essentially identical). At short distances the effective coupling constant is temperature independent and coincides with the zero temperature result. At larger distances it deviates from the zero temperature result and after approaching a maximum it drops because of the onset of screening. Note that for temperatures close to $T_c$ the effective running coupling constant follows the zero temperature running coupling up to distances of about 0.3 fm. At such distances the running coupling is not controlled by the perturbation theory but rather by the linear part of the potential which gives the nearly quadratic rise of this quantity. Thus some non-perturbative confining physics survives deconfinement.

5 Entropy and internal energy of a static quark anti-quark pair

As it was noticed in section 2, given the partition function $Z_{QQ}$ we can calculate the entropy and internal energy difference of the system with static charges and the same system without them, which for the sake of simplicity is called entropy and internal energy of $QQ$. Using Eqs. (7) and (8) the entropy and internal energy has been calculated for infinite separation in quenched [12], two flavor [15] and three flavor QCD [14]. The results are shown in Fig. 7. Both the entropy and the internal energy show a very large increase near $T_c$. For quenched QCD the internal energy has been calculated for any separation $r$ [12] and the results are shown in Fig. 8. One can see that with the exception of the small distance region where the internal energy coincides with the zero temperature potential the internal energy is larger than the free energy. In the high temperature limit the internal energy has no constant piece at large distances proportional to the temperature, therefore it is tempting to interpret the free energies as potentials. However, the large increase of $U(r, T)$ near $T_c$ make such interpretation problematic.

6 Quarkonium binding at finite temperature

Following the suggestion by Matsui and Satz [1] the problem of quarkonium binding at finite temperature has been studied using potential models with some phenomenological screened potential (see e.g. [26,27,28]) which led to the conclusions that $J/\psi$ dissolves in the Quark Gluon Plasma at temperatures close to $T_c$. More recently the free energy has been used as the potential in the Schroedinger equation and it was found that $J/\psi$ can survive only to temperatures 1.1T$_c$ [29]. As the free energy contains an $r$-dependent entropy contribution the validity of this approach is doubtful. Finally, very recently the internal energy calculated in Ref. [12] was used as a potential in Schroedinger equation [30] and it was found that $J/\psi$ can survive till 1.1T$_c$ which is not inconsistent with lattice calculations of $J/\psi$ spectral function [31,32]. However, to test
the validity of potential models it is not sufficient to make statements about the dissolution temperature of a given quarkonium state, one should investigate the change in the properties of heavy quarkonium bound states. For example, potential models with screening predict a decrease in the quarkonium masses. The most convenient way to compare the prediction of potential models with direct calculation of quarkonium spectral functions is to calculate the Euclidean meson correlator at finite temperature (see contribution by Mócsy to these proceedings [32]). This quantity can be reliably calculated on the lattice. The basic idea is to use the model spectral function

$$
\sigma(\omega, T) = \sum_i 2F_i(T)\delta(\omega^2 - M_i^2(T)) + m_0(\omega - s_0(T))\omega^2
$$

containing bound states (resonances) and continuum [31]. For a given screened potential one can solve the Schroedinger equation and determine the radial wave function (or its derivative) at the origin $R(0)$ and the binding energy $E_i$. The parameters of the model spectral functions can be related to these quantities, $F_i(T) \sim |R(0)|^2$, $M_i = 2m_{c,b} + E_i$ and acquire temperature dependence because of the temperature dependence of the potential. Here $m_{c,b}$ is the constituent quark mass of c- and b-quarks. The threshold of the continuum $s_0(T)$ can be related to the asymptotic value of the potential at infinite distance $s_0(T) = 2m_{c,b} + V_\infty(T)$. Having specified completely the spectral function of the model the Euclidean correlator can be calculated

$$
G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}
$$

(20)

and compared with lattice results [32]. The Euclidean correlator can be reliably calculated on lattice while extracting the spectral function from it is quite difficult. Using this approach some of the features of the quarkonium correlators, e.g. the enhancement of the scalar correlator above deconfinement temperature observed in lattice calculation, can be understood. More detailed discussion on this topic is given in Ref. [32].

7 Conclusions

Free energies of static $Q\bar{Q}$ pairs have been extensively studied on lattice and provide useful tool to study in-medium modification of inter-quark forces. Many body effects exert large influence on the free energy thus making the simple picture, where the temperature dependence of the free energy reflects the screening of the two-body potential, not applicable. Many body effects are most prominent close to the transition temperature. For temperatures not too close to the transition temperature free energies of static quark anti-quark pairs could provide useful qualitative (though not quantitative) insights into the problem of quarkonium binding in Quark Gluon Plasma. It is interesting to note in this respect that a new model approach to the problem of heavy quark potentials at finite temperature was recently proposed in Ref. [33].

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