Five-quark components in $\Delta(1232) \rightarrow N\pi$ decay

Q. B. Li

*Helsinki Institute of Physics POB 64,
00014 University of Helsinki, Finland

D. O. Riska

†Helsinki Institute of Physics and Department of Physical Sciences,
POB 64, 00014 University of Helsinki, Finland

(Dated: January 14, 2022)

Abstract

Five-quark $qqqq\bar{q}$ components in the $\Delta(1232)$ are shown to contribute significantly to $\Delta(1232) \rightarrow N\pi$ decay through quark-antiquark annihilation transitions. These involve the overlap between the $qqq$ and the $qqqq\bar{q}$ components and may be triggered by the confining interaction between the quarks. With a $\sim 10\%$ admixture of five-quark components in the $\Delta(1232)$ the decay width can be larger by factors $\sim 2 – 3$ over that calculated in the non-relativistic quark model with 3 valence quarks, depending on the details of the confining interaction. The effect of transitions between the $qqqq\bar{q}$ components themselves on the calculated decay width is however small. The large contribution of the quark-antiquark annihilation transitions thus may compensate the underprediction of the width of the $\Delta(1232)$ by the valence quark model, once the $\Delta(1232)$ contains $qqqq\bar{q}$ components with $\sim 10\%$ probability.
I. INTRODUCTION

While the constituent valence quark model provides a simple and almost quantitative phenomenological description of the magnetic moments of the octet baryons, it does in its simplest versions, where the pions couple directly to the quarks, lead to values for the decay width of the $\Delta(1232)$, which are only about one half of the empirical value. More sophisticated covariant versions of the quark model with realistic wave functions for the 3-quark system fail to overcome this deficiency \cite{1, 2}. Coupled channel treatments of the hadronic interacting $\pi N\Delta$ system suggest that the problem may be cured by the “pion cloud” contribution, which is automatically included in that approach \cite{3, 4}.

Here this question is addressed by an extension of the non-relativistic valence quark model to include explicitly those 5-quark $qqqq\bar{q}$ configurations in the proton and the $\Delta(1232)$, which are expected to have the lowest energy. The presence of such “sea-quark” contributions in the proton has been demonstrated in several experiments \cite{5, 6, 7, 8}. Given the presence of explicit $qqqq\bar{q}$ components in both the nucleon and (the expected presence in) the $\Delta(1232)$ resonance, pion decay of the latter may take place in the form of transitions between the respective $qqqq\bar{q}$ components but also as annihilation transitions of the form $qqqq\bar{q} \rightarrow qqq\pi$ in addition to the conventional quark model transitions between the pure $qqq$ states. Here both cases are considered, with the result that only the annihilation transitions contribute significantly. The amplitude for these are - at least on the basis of a qualitative estimate based on simple wave function models - strong enough to increase the calculated width in the quark model by factors 2 – 3 if there is a 10% probability for $qqqq\bar{q}$ components in the $\Delta(1232)$, the magnitude depending on the model for the confining interaction. The transitions between the explicit $qqqq\bar{q}$ components themselves are however found to be of minor significance, mainly because of the small amplitude of the $qqqq\bar{q}$ component in the proton.

In section II the $qqqq\bar{q}$ configurations in the nucleon and the $\Delta(1232)$, which are expected to have the lowest energy, and the pionic transitions between these are considered. In section III the $qqqq\bar{q} \rightarrow qqq\pi$ transitions are treated along with a numerical estimate of their significance. Finally section IV contains a summarizing discussion.
II. FIVE-QUARK COMPONENTS IN THE PROTON AND THE $\Delta(1232)$

A. Low lying $qqqq\bar{q}$ configurations

Positive parity demands that in a $qqqq\bar{q}$ component in a baryon either one of the 4 quarks or the antiquark $\bar{q}$ is orbitally excited to the $P$–shell. If the 4 quarks are in the ground state, the corresponding spatial wave function is completely symmetric, $[4]_X$, and overall antisymmetry demands that the flavor-spin state have mixed symmetry $[31]_{FS}$, which can combine with the color state with the conjugate mixed symmetry $[211]_C$ to total antisymmetry $[14]$. If the antiquark is in its ground state, the spatial state of the orbitally excited $qqqq$ system has to have the mixed symmetry $[31]_X$. In this case overall antisymmetry allows the flavor-spin state to have the following symmetries: $[4]_{FS}$, $[31]_{FS}$ or $[211]_{FS}$. The possible symmetry configurations of the $qqqq$ system that meets these conditions have been classified in ref. [9]. The $qqqq\bar{q}$ configurations that are most likely to have appreciable probabilities in the proton and the $\Delta(1232)$ are those, which have the lowest energy and (or) the strongest coupling to the main $qqq$ configuration.

The energy levels of these $qqqq\bar{q}$ configurations are split by the hyperfine interaction between the quarks. The configuration with the lowest energy depends on the form of this interaction. If the hyperfine interaction is spin-dependent, as usually assumed, the $qqqq$ configurations that have the lowest energy are those with the most antisymmetric spin state, which is the the mixed symmetry state [22]. This is the case if the hyperfine interaction is described by the colormagnetic interaction and also if the interaction is described by the schematic flavor and spin dependent interaction $-C_{\chi} \sum_{i<j} \vec{X}_F \cdot \vec{X}_F \cdot \vec{\sigma}_i \cdot \vec{\sigma}_j$, which leads to the empirical ordering of the baryon spectrum if $C_{\chi} \sim 20 – 30$ MeV [10]. In both cases the flavor-spin symmetry of the $qqqq$ part of the the lowest energy $qqqq\bar{q}$ component in the proton is likely to be $[4]_{FS}[22]_F[22]_S$. Because the total isospin of this $qqqq$ configuration is 0, it cannot be a component in the $\Delta(1232)$, however. The lowest energy $qqqq$ configuration in the $\Delta$ has the flavor-spin symmetry $[4]_{FS}[31]_F[31]_S$. In these configurations the antiquark $\bar{q}$ is in its ground state. The other $qqqq\bar{q}$ configurations in the proton and the $\Delta(1232)$ are expected to have a much higher energy [9].
B. Transitions between five-quark configurations

In the “chiral quark” model pions couple directly to constituent quarks. The transition operator for a transition of the type $\Delta^{++}(s_z = 3/2) \rightarrow p(s_z = 1/2)\pi^+$ is then in the non-relativistic approximation:

$$T_\pi = -ig_A^q f_\pi \sum_i \tau_+^i \sigma_+^i q_{\pi^+}. \tag{1}$$

Here the sum runs over the quarks and $g_A^q$ and $f_\pi$ are the axial vector coupling constant of the constituent quarks, and $f_\pi$ is the pion decay constant. The pion momentum component $q_{\pi^+}$ is defined as $q_{\pi^+} = -(q_{\pi x} + iq_{\pi y})/\sqrt{2}$.

The matrix element of (1) in the valence quark model with conventional 3-quark flavor and spin wave functions \cite{11} for the transition $\Delta^{++}(s_z = 3/2) \rightarrow p(s_z = 1/2)\pi^+$ is:

$$\langle p, 1/2|T_\pi|\Delta^{++}, 3/2 \rangle = -ig_A^q f_\pi \sqrt{2}q_{\pi^+} \left(1 - \frac{\vec{r}^2}{6\omega_3^2}\right). \tag{2}$$

The last factor accounts for the spatial extent of the $qqq\bar{q}$ component of the baryon in the harmonic oscillator model. The parameter $\omega_3$ may be determined from the empirical radius of the proton as $\omega_3 = 1/r_p \approx 225$ MeV.

Consider then the corresponding matrix element for the $qqqq\bar{q}$ components in the proton and the $\Delta(1232)$. If the amplitudes of the $[4]_{FS}[22]_F[22]_S$ configuration of the proton is denoted $A_{p5}$, the corresponding wave function is

$$\psi_p(s_z = 1/2) = A_{p5} \sqrt{6} \sum_{a,b} \sum_{m,s} (1, 1/2, m, s| 1/2, 1/2) \ [211]_C(a) [31]_X(m(a)) [22]_F(b) [22]_S(b) \bar{\chi}_s \varphi(\{r_i\}). \tag{3}$$

Here the color, space and flavor-spin wave functions of the $qqqq$ subsystem have been denoted by their Young patterns respectively, and the sum over $a$ runs over the 3 configurations of the $[211]_C$ and $[31]_X$ representations of $S_4$, and the sum over $b$ runs over the 2 configurations of the $[22]$ representation of $S_4$ respectively \cite{12}. Note that as the isospin of the $qqqq$ of the $[22]_F$ configuration is 0, the antiquark can only be a $\bar{d}$ quark.

The wave function for the $[4]_{FS}[31]_F[31]_S$ configuration in the $\Delta^{++}$ is

$$\psi_{\Delta}(3/2)^J = \frac{A_{\Delta 5}^{(J)}}{3} \sum_{a,b} \sum_{m,s,M,j} (1, 1, m, M| J, j)(J, 1/2, j, s| 3/2, 3/2) \ [211]_C(a) [31]_X(m(a)) [31]_F(b) [31]_S(b) \bar{\chi}_s \varphi(\{r_i\}). \tag{4}$$
Here $J$ denotes the total angular momentum of the $qqqq$ system, which takes the values 1 and 2, and $A_{\Delta_5}^{(J)}$ is the amplitude of the configuration in the $\Delta(1232)$. The sum over $a$ again runs over the 3 configurations of the $[211]_C$ and $[31]_X$ representations of $S_4$. Here the sum over $b$ runs over the 3 configurations of the $[31]$ representation.

It is then a straightforward task to calculate the ratio of the matrix element of the operator (2) for $\Delta^{++}_{3/2} \rightarrow p_{1/2}\pi^+$ process in the $qqqq\bar{q}$ configuration to that in the conventional $qqq$ configuration. The result is:

$$\delta = A_{\rho3} A_{\Delta3} (1 + \sqrt{2} \frac{A_{\rho5}^2}{A_{\rho3}^2 A_{\Delta3}^2} (A_{\Delta5}^{(1)} + \sqrt{5} A_{\Delta5}^{(2)})) = \frac{\sqrt{2}}{3} \frac{A_{\rho5}}{A_{\rho3} A_{\Delta3}} (A_{\Delta5}^{(1)} + \sqrt{5} A_{\Delta5}^{(2)}).$$

(5)

Here $A_{\rho3}$ and $A_{\Delta3}$ are the amplitudes for the $qqq$ component in the proton and the $\Delta(1232)$, respectively. In this expression one should in principle also include the ratio of the momentum dependent factors that account for the spatial extent of the baryon. In the harmonic oscillator model these factors are $1 - \bar{q}^2/6\omega_3^2$ for the $qqq$ configuration and $1 - \bar{q}^2/5\omega_5^2$ for the $qqqq\bar{q}$ configuration. If the spatial extent of the 3- and the 5-quark components is the same, so that $\omega_5 = \sqrt{6/5} \omega_3$, (6)

this ratio is 1. The relative magnitude of the (inverse) size parameters $\omega_3$ and $\omega_5$ will depend on the interaction that couples the $qqq$ and the $qqqq\bar{q}$ components.

The relative difference from the valence quark model result that inclusion of the $qqqq\bar{q}$ configurations brings, is obtained by addition of the product of the amplitudes for the $qqq$ components $A_{\rho5} A_{\Delta3}$ to the ratio (5):

$$\delta = A_{\rho3} A_{\Delta3} (1 + \sqrt{2} \frac{A_{\rho5}^2}{A_{\rho3}^2 A_{\Delta3}^2} (A_{\Delta5}^{(1)} + \sqrt{5} A_{\Delta5}^{(2)})) = \frac{\sqrt{2}}{3} \frac{A_{\rho5}}{A_{\rho3} A_{\Delta3}} (A_{\Delta5}^{(1)} + \sqrt{5} A_{\Delta5}^{(2)}).$$

(7)

While the flavor-spin $qqqq\bar{q}$ component $[4]_{FS}[22]_F[22]_S$ does not contribute to the magnetic moment of the proton, it does contribute an amount $A_{\rho5}^2/3 \mu_N$ to that of the neutron. The ratio of the proton to the neutron magnetic moment in the $qqq$ configuration is $-3/2$ and thus close to the empirical ratio -1.46 in the static quark model. In covariant versions of the valence quark model the calculated ratio varies between -1.46 and -1.66. A large value for the amplitude $A_{\rho5}$ for the $qqqq\bar{q}$ component would bring large deviations from the empirical magnetic moment ratio.

The expression (7) reveals that the $qqqq\bar{q}$ components only in the case $J = 2$ can lead to an increase of the calculated decay rate of $\Delta^{++} \rightarrow p\pi^+$ only when the probabilities of
the $qqqq \bar{q}$ components are fairly large. As an example consider the case in which the $qqqq \bar{q}$ probabilities of the component of the proton and the $\Delta^{++}$ (with $J = 2$) are 10% and 20%, respectively. In this case $A_{p3} = 0.95$ and $A_{\Delta5}^{(2)} = 0.45$ and $\delta = 0.997$ so that there is no net enhancement. If the $qqqq \bar{q}$ component of the proton would be as large as 20% there would be a net enhancement of 2%. The conclusion then follows that the transitions between the $qqqq \bar{q}$ components do at most lead to an enhancement of the calculated decay width by a few percent.

The proton may also have an admixture with the flavor-spin symmetry structure $[4]_{FS}[31]_F[31]_S$, in which case the antiquark may be either a $\bar{u}$ or $\bar{d}$. The empirical evidence for the flavor asymmetry of the $q\bar{q}$ components in the proton [14] suggests that this should have a smaller probability than the component with flavor-spin symmetry $[4]_{FS}[22]_F[22]_S$. This is also consistent with the fact that it is energetically less favorable [9]. The corresponding proton wave function has the form

$$\psi_p(1/2) = \frac{A_{p5}^{(J)}}{3} \sum_{a,b} \sum_{m,s,M,j} (1, 1, m, M | J, j)(J, 1/2, j, s | 1/2, 1/2)$$

$$(1, 1/2, T, t | 1/2, 1/2) [211]_C(a) [31]_{X,m}(a) [31]_{F,T}(b) [31]_{S,M}(b) \bar{\chi}_{T,s} \varphi(\{r_i\}) . \tag{8}$$

Here the isospin-z component of the 4-quark state is denoted $T$ and that of the antiquark $t$. In this configuration the ratio of the amplitudes for the $\Delta^{++}_{3/2} \rightarrow p_{1/2}\pi^+$ process in the $qqqq \bar{q}$ configuration to that in the conventional $qqq$ configuration is:

$$\frac{5\langle p, 1/2 | T_x | \Delta^{++}, 3/2 \rangle_5}{3\langle p, 1/2 | T_x | \Delta^{++}, 3/2 \rangle_3} = \frac{-\sqrt{2}}{6} \frac{A_{p5}^{(1)} A_{\Delta5}^{(1)}}{A_{p3} A_{\Delta3}} + 2\sqrt{2} \frac{A_{p5}^{(0)} A_{\Delta5}^{(1)}}{A_{p3} A_{\Delta3}} + \sqrt{5} \frac{A_{p5}^{(1)} A_{\Delta5}^{(2)}}{A_{p3} A_{\Delta3}} \tag{9}$$

As the magnitude of the numerical coefficients on the right hand side of this expression is less than 1 and the sign of the ratio is negative, the transitions between these $qqqq \bar{q}$ components cannot increase the calculated decay rate for $\Delta^{++} \rightarrow p \pi^+$ over the result obtained in the pure $qqq$ model calculation.
III. FIVE-QUARK TO THREE-QUARK TRANSITIONS

A. Direct quark-antiquark annihilation

The simplest $qqqar{q} 	o qqq + \pi$ decay mechanism that can contribute to the decay of the $\Delta(1232)$ is $q\bar{q} \to \pi$ pair annihilation process in Fig. 1. The corresponding amplitude is

$$T_\pi = i\sqrt{2} \frac{m_q g^A_{A\pi}(p_q)\gamma_5 u(p_\pi)}{f_\pi}.$$  \hfill (10)

Calculation of the matrix element of this amplitude for the decay $\Delta_{3/2}^{++} \to p_{1/2} \pi^+$ requires the calculation of the overlap of the $qqq$ component of the proton with the residual $qqq$ component that is left in the $\Delta^{++}$ after the annihilation of a $u\bar{d}$ pair. It also requires a specification of the spatial part of the $\Delta^{++}$ wave function.

The spatial wave function may be expressed with the help of the following relative coordinates:

$$\vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \quad \vec{\xi}_2 = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3),$$
\[
\tilde{\xi}_3 = \frac{1}{\sqrt{12}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4), \\
\tilde{\xi}_4 = \frac{1}{\sqrt{20}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5).
\]

(11)

Here \( \vec{r}_i \) represents the position operator of the \( i \)th constituent. To form a complete set of basis vectors, the set (\( \Pi \)) may be completed with the center-of-mass vector \( \vec{R} = \sum_i \vec{r}_i / \sqrt{5} \). The three components of the spatial state with \([31]_X\) mixed symmetry may be formed as normalized combinations of a spatially symmetric function that is multiplied by the vectors \( \tilde{\xi}_1, \tilde{\xi}_2 \) and \( \tilde{\xi}_3 \) respectively, times a completely symmetric function of the coordinates.

For the present purposes it suffices to describe the completely symmetric function of the 4 quark coordinates as a product of harmonic oscillator functions:

\[
\varphi(\xi_i) = \left( \frac{\omega_5^2}{\pi} \right)^{3/4} e^{-\xi_i^2 \omega_5^2 / 2},
\]

(12)

where \( \omega_5 \) is the constant parameter (6) and \( i = 1, 2, 3 \). A similar oscillator wave function is employed for the antiquark.

The desired annihilation matrix element will take the form

\[
\langle T \rangle = 4A_{p3}A_{\Delta 5}^d \int \prod_{i=1}^3 d^3 r_i \psi_{p3}(\vec{r}_1, \vec{r}_2, \vec{r}_3) e^{i\vec{q}_\pi \cdot (\vec{r}_4 + \vec{r}_5)/2} \\
\delta(\vec{r}_4 - \vec{r}_5) \langle T_{15} \rangle \psi_{\Delta 5}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_5) \delta(\vec{R}).
\]

(13)

Here the pure three quark proton wave function is denoted \( \psi_{p3} \) and the coordinates of the annihilated quark and antiquark are taken to be \( \vec{r}_4 \) and \( \vec{r}_5 \). The matrix element of the annihilation amplitude (\( \Pi \)) for annihilation of the 4th quark and the antiquark (with coordinate \( \vec{r}_5 \)) is denoted \( \langle T_{45} \rangle \). It is advantageous to express the matrix element in terms of the relative coordinates:

\[
\langle T \rangle = 4(\frac{2}{\sqrt{5}})^3 A_{p3}A_{\Delta 5}^d \int \prod_{i=1}^4 d^3 \xi_i \psi_{p3}(\vec{\xi}_1, \vec{\xi}_2) e^{-i2\sqrt{3}\vec{q}_\pi \cdot \vec{\xi}_3 / 5} \\
\delta(\vec{\xi}_4 - \sqrt{\frac{3}{5}} \vec{\xi}_3) \langle T_{45} \rangle \psi_{\Delta 5}(\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3, \vec{\xi}_4).
\]

(14)

Here note has been taken of the fact that only the component of the spatial part of the \( \Delta(1232) \) wave function with the mixed symmetry \([31]_X\) that corresponds to the Young tableau:

\[
\begin{array}{cccc}
1 & 2 & 3 \\
4
\end{array}
\]

(15)
contributes. The coordinate vector $\vec{\xi}_3$ realizes this symmetry. With the explicit harmonic oscillator wave functions (12) for the radial wave functions with the arguments $\vec{\xi}_1, \ldots, \vec{\xi}_4$, the matrix element takes the form:

$$
\langle T \rangle = 4 A_{p3} A_{\Delta_5}^{(J)} C_{C} C_{FS} \left( \frac{\omega_3 \omega_5}{\omega^2} \right)^3 \sqrt{2} \omega_5 \left( \frac{2 \omega_5}{\sqrt{3} \pi} \right)^3 \int d^3 \vec{\xi}_3 \frac{i \vec{\xi}_3 \cdot \vec{q}}{\sqrt{3}} e^{-2 \sqrt{3} i \vec{q} \cdot \vec{\xi}_3} e^{-4 \xi_3^2 \omega_5^2 / 5} \langle T_{45} \rangle .
$$

(16)

In this expression oscillator wave functions of the form (12), but with $\omega_3$ in place of $\omega_5$, have been employed of the $qqq$ component of the proton. The product of the factor $(\omega_3 \omega_5 / \omega^2)^3$, where $\omega = \sqrt{(\omega_3^2 + \omega_5^2) / 2}$, and the coefficient $C_C$ is the overlap between the antisymmetric color state $[111]_C$ of the proton in the $qqq$ configuration and the first three components of the mixed symmetry color state $[211]_C$ of the $qqqq\bar{q}$ component of the $\Delta(1232)$ in the color configuration that is conjugate to (15). The coefficient $C_{FS}$ is the corresponding overlap between the mixed symmetry $[21]_{FS}$ flavor-spin state of the proton in the $qqq$ configuration and the corresponding $[4]_{FS}[22]_{F}[22]_{S}$ flavor-spin state of the $qqqq\bar{q}$ component of the $\Delta(1232)$. These coefficients take the values:

$$
C_C = 1, \quad C_{FS} = \frac{1}{\sqrt{6}} (\delta_{J1} + \frac{3}{\sqrt{5}} \delta_{J2}) ,
$$

(17)

when the overall normalization factors of the color, space and flavor-spin wave functions are included.

The radial integral in (16) may be approximately evaluated with a power series expansion in $\vec{q}$ with the result:

$$
\langle T \rangle \simeq 4 i A_{p3} A_{\Delta_5}^{(J)} C_{C} C_{FS} \left( \frac{\omega_3 \omega_5}{\omega^2} \right)^3 \frac{q_+ \sqrt{2}}{4} \left( 1 - \frac{3}{20} \frac{q^2}{\omega_5^2} \right) .
$$

(18)

Consider now the decay $\Delta^{++}(s_z = 3/2) \rightarrow p(s_z = 1/2) \pi^+$, where the $\Delta^{++}$ is in a $uuud\bar{d}$ and the proton in the $uud$ configuration. In this case one of the $u$ quarks annihilates the $\bar{d}$ antiquark in the $\Delta^{++}$ to form the $\pi^+$. The complete amplitude for this annihilation process then becomes:

$$
T = i \frac{\sqrt{6}}{3} A_{p3} (A_{\Delta_5}^{(1)} + \frac{3}{\sqrt{5}} A_{\Delta_5}^{(2)}) \left( \frac{mg_{A}}{f_\pi} \right) \left( \frac{\omega_3 \omega_5}{\omega^2} \right)^3 \frac{q_+}{\omega_5} \left( 1 - \frac{3}{20} \frac{q^2}{\omega_5^2} \right) .
$$

(19)

This magnitude of this amplitude should then be compared to that of the basic decay amplitude in the pure 3-quark configuration (2), multiplied by the factor $A_{p3} A_{\Delta_3}$. Note that in this case the phase of the $qqqq\bar{q}$ components (i.e. the signs of $A_{\Delta_5}^{(J)}$) determines
whether there will be constructive or destructive interference with the decay amplitudes for
transitions between the $qqq$ or the $qqqq\bar{q}$ amplitudes without pair annihilation.

The annihilation mechanism will contribute to the decay width of the $\Delta(1232)$ even in
the absence of $qqqq\bar{q}$ component in the proton. Assume for the sake of an example that
$A_{\Delta_5}^{(2)} = 0.32$, but now that $A_{\Delta_5} = 1$ and $A_{\Delta_3} = 0.95$. This implies a pure $qqq$ proton and a
$\Delta^{++}$ with a 10% probability for the $qqqq\bar{q}$ configuration. With $m = 340$ MeV and $\omega_5 = 245$
MeV, it then follows from these expressions that the direct $qqqq\bar{q} \rightarrow qqq + \pi^+$ annihilation
mechanism, combined with the amplitude from the transition between $qqqq\bar{q}$ components,
increases the calculated decay width by $\sim 69\%$.

The estimate above was based on the assumption that the mean square radii of the
$qqqq\bar{q}$ and $qqq$ components in the proton are equal. If the radius of the $qqqq\bar{q}$ component
is increased by 22% so that $\omega_5$ is reduced to 200 MeV, the contribution from the direct
annihilation process (19) is increased to an enhancement of 81%.

B. Confinement triggered annihilation

In addition to the direct annihilation mechanism that is illustrated in Fig. 1, the annihi-
lation process may also be triggered by the interaction between quarks. The most obvious
such triggered annihilation process is that, which is caused by the confining interaction, and
which is illustrated in Fig. 2.

The corresponding annihilation amplitude may be derived in the same way as the ampli-
tude for the direct annihilation process by inserting a quark propagator multiplied by the
confining interaction before and after the pseudovector pion-quark vertex. If only the point
coupling and pair terms are retained after application of the Dirac equation, the confine-
ment triggered annihilation amplitude may, in the case of a simple linear scalar confining
interaction, be derived from the direct annihilation amplitude by making the substitution:

$$m \rightarrow m + \frac{1}{2}(cr_{ij} - b),$$

(20)

where $c$ is the string tension, $r_{ij}$ is the distance between the two quarks that interact by the
confining interaction and $b$ is a positive constant, which makes the effective linear confining
interaction potential $cr - b$ negative at short distances. The presence of the $b$ term is
suggested by the phenomenology of the charm meson spectra [15]. If the confining interaction
FIG. 2: Confinement induced $qqqq\bar{q} \rightarrow qqq\pi$ annihilation diagram

has the color coupling $\vec{\lambda}_i \cdot \vec{\lambda}_j$, the string tension for $qq$ and $q\bar{q}$ pairs in $qqqq\bar{q}$ systems system is the same and equals half of the value of that for $qq$ pairs in three-quark systems [16].

The substitution (20) may be viewed as a mass correction, which is natural in the case of a scalar coupled confining interaction. A related mass shift does in the case of charmonium serve to bring the calculated M1 transition rates into agreement with the empirical values [17].

The amplitude for confinement triggered annihilation involves integration over two of the relative coordinates $\vec{\xi}_i$ (11). By the overall antisymmetry it is sufficient to consider the annihilation amplitude in Fig. 2 in which the confining interaction between the 3rd and 4th quarks triggers the annihilation of the 4th quark against the antiquark. In this case the confining interaction $(c/2|\vec{r}_3 - \vec{r}_4| - b/2)$ enters the integrand in the matrix element. As $|\vec{r}_3 - \vec{r}_4|$ is proportional to $|\vec{\xi}_2 - \sqrt{2}\vec{\xi}_3|$ the integrals over the relative coordinates $\vec{\xi}_2$ and $\vec{\xi}_3$ have to be done numerically. The relative coordinates $\vec{\xi}_2$ and $\vec{\xi}_3$ realize the mixed symmetry configurations

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 4 \\ 3 \end{pmatrix},$$

(21)
respectively. These combine with the corresponding mixed color symmetry configurations

\[
\begin{pmatrix}
1 & 3 \\
2 & 4 \\
4 & 3 \\
\end{pmatrix}, \quad \begin{pmatrix}
1 & 4 \\
2 & 3 \\
\end{pmatrix},
\tag{22}
\]
in the 5-quark component of the wave function of the \(\Delta(1232)\).

The matrix element of the confinement contribution may be expressed as (cf.\(^{(13)}\))

\[
\langle T_{\text{conf}} \rangle = 12 A_{p3} A_{\Delta5}^{(j)} \int \Pi_{i=1}^5 d^3 r_i \psi_{p3}(\vec{r}_1, \vec{r}_2, \vec{r}_3) e^{i\vec{q}_r \cdot (\vec{r}_4 + \vec{r}_5)/2}
\]
\[
\{ c |\vec{r}_3 - \vec{r}_4| - \frac{b}{2} \} \delta(\vec{r}_4 - \vec{r}_5) \langle T_{3,45} \rangle \psi_{\Delta5}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_5) \delta(\vec{R}) . \tag{23}
\]

Here the operator \(T_{3,45}\) describes the annihilation process in Fig. 2. The factor 12 on the rhs is the number of contributing similar processes.

The matrix element (23) may be rewritten in more explicit form as:

\[
\langle T_{\text{conf}} \rangle = 6 A_{p3} A_{\Delta5} C_C C_{FS} \left(\frac{\omega_3 \omega_5}{\omega^2}\right)^{3/2} \frac{\sqrt{6}}{3} \sqrt{2} \omega_5 \left(\frac{2}{\sqrt{5} \pi}\right)^6 \int d^3 \xi_2 d^3 \xi_3
\]
\[
\frac{i\xi_3}{\sqrt{3}} \left\{ \frac{c}{2} |\vec{r}_3 - \vec{r}_4| - \frac{b}{2} \right\} \langle T_{3,45} \rangle e^{-\omega_2 \xi_2^2} e^{-\alpha^2 \xi_3^2} e^{-i\beta \xi_3} . \tag{24}
\]

The coefficients \(\alpha\) and \(\beta\) are defined as

\[
\alpha = \frac{2}{\sqrt{5}} \omega_5 , \quad \beta = \frac{2\sqrt{3}}{5} . \tag{25}
\]

The complete matrix element finally takes the form

\[
T_{\text{conf}} = i \left( \frac{g_A C}{f_\pi \omega} \right) \left( \frac{q^+}{\omega_5^2} \right) \left( \frac{\omega_3 \omega_5}{\omega^2} \right)^3 \frac{512 \sqrt{5}}{125 \pi} A_{p3} \left( A_{\Delta5}^{(1)} + \frac{3}{\sqrt{5}} A_{\Delta5}^{(2)} \right) K(q) . \tag{26}
\]

Here the function \(K(q)\) is defined as

\[
K(q) = \omega_5^5 \int_0^\infty d\xi_3 \xi_3^4 \frac{j_1(\beta q \xi_3)}{\beta q \xi_3} e^{-\alpha^2 \xi_3^2} k(\omega \xi_3) , \tag{27}
\]
where \(j_1\) is the spherical Bessel function of order 1 and the function \(k(y)\) is defined as:

\[
k(y) = \int_0^\infty dx x^2 e^{-x^2} \int_{-1}^1 dz \{ \sqrt{x^2 - 2\sqrt{2} z y} + 2 y^2 - \frac{\sqrt{6} b \omega}{2 c} \} . \tag{28}
\]

With \(b = 0\) MeV, this function takes the value 1 at \(y = 0\) and approaches the straight line \(\sqrt{2\pi y}/2\) when \(y > 1\) as shown in Fig. 3. For other values of the parameter \(b\) the function is shifted by a constant as also shown in the figure.
The function $k(y)$ with $b = 0$ and $b = 300$ MeV, $\omega_5 = 245$ MeV.

FIG. 3: The function $k(y)$ with $b = 0$ and $b = 300$ MeV, $\omega_5 = 245$ MeV.

The function $K(q)$ with $b = 0$ and 300 MeV is shown in Fig. 4. For $\Delta(1232) \rightarrow N\pi$ $q_\pi = 227$ MeV and $K(q_\pi) = 0.74$, when $b = 0$.

For a numerical estimate of the significance of confinement triggered annihilation the string tension $c$ may be taken to one quarter of that in $q\bar{q}$ systems. With a typical value for that as $c_{q\bar{q}} = 1.12$ GeV/fm, the value for the string tension in the $qqqq\bar{q}$ system would be $c = 280$ MeV. Consider again the previous example, in which $A_{p3} = 1$ and $A^{(2)}_{\Delta 5} = 0.32$, but now with the value 300 MeV for the shift parameter $b$ in the confining potential. This value is chosen so as to cover the range of values that have been used in charm meson spectroscopy. The range of values for $b$ between 0 and 300 MeV when the confinement triggered amplitude is added to the amplitude for direct annihilation and the amplitude for $\Delta^{++} \rightarrow p\pi^+$ decay in the appropriately normalized $qqq$ configuration it is found that the net effect is an increase by a factor 2.5 of the decay width that is obtained in the $qqq$ configuration. This estimate is based on the assumption that the probability of the $qqqq\bar{q}$ component in the $\Delta^{++}$ is 10% and that the proton is a pure $qqq$ state. The dependence of the calculated enhancement on the oscillator parameter of the $qqqq\bar{q}$ component of the $\Delta^{++}$ with $b = 300$ MeV is shown in Fig. 5. The enhancement as a function of the amplitude
of the $qqq$ components in the proton and the $\Delta^{++}$ wave functions is shown in Fig.6. The dependence of the enhancement of the decay width on the values of the shift parameter $b$ in the linear confining potential is given in Table II from where it can be seen that, within a realistic range of the values of $b$ and with a 10% $qqqq\bar{q}$ component in $\Delta(1232)$, the final enhancement falls within the range $2 \sim 3$, which is substantial enough to compensate the underpredicted decay width of the pion decay of $\Delta(1232)$ in the $qqq$ quark model.

To have an estimate of the theoretical uncertainty of the magnitude of the contribution of the confinement triggered annihilation process this estimate may be compared to that, which is obtained if the linear confining interaction between the quarks in the $qqqq\bar{q}$ system is replaced by the harmonic oscillator potential, which corresponds to the wave function model employed above. This is obtained by the substitution:

$$cr - b \rightarrow \frac{1}{2}C r^2 - B , \quad (29)$$

Here $B$ is a constant that shifts the interaction potential to negative values at short range. The oscillator constant $C$ is given as [9]:

$$C = \frac{m \omega_5^2}{5} . \quad (30)$$
FIG. 5: The enhancement of the calculated decay width as a function of the oscillator parameter $\omega_5$ for the $qqqq\bar{q}$ component of the $\Delta(1232)$ wave function with the shift of the linear confining potential $b = 300$ MeV. The amplitudes for this component of the proton and the $\Delta^{++}$ wave function are denoted $A_{p3}$ and $A_{\Delta3}$ respectively.

With $m = 340$ MeV and $\omega_5 = 245$ MeV this gives for $C$ the value 105 MeV/fm².

The amplitude for confinement triggered annihilation $\Delta^{++} \to p\pi^+$ in this oscillator confinement model may be obtained directly from the expression for linear confinement above by the substitution

$$cK(q_\pi) \rightarrow \frac{\sqrt{6} C}{6 \omega} L(q_\pi).$$

The function $L(q)$ is defined as the integral

$$L(q) = \sqrt{\pi} \omega_5^5 \int_0^\infty d\xi_3 \xi_3^4 \frac{J_1(\beta q \xi_3)}{\beta q \xi_3} e^{-a^2\xi_3^2} \left\{ \frac{3}{4} + \omega^2(\xi_3^2 - \frac{3B}{2C}) \right\}.$$  

This function is plotted in Fig. [Fig] for $B = 0$ and 100 MeV. For $\Delta(1232) \to N\pi$ decay $q_\pi = 227$ MeV and $L(q_\pi) = 2.1$, when $B = 0$.

With the numerical parameter values given above, the magnitude of the confinement triggered annihilation amplitude is smaller by a factor $\sim 2.7$ in the model with oscillator...
FIG. 6: The enhancement of the calculated decay width as a function of the amplitudes of the $qqq$ components of the proton and the the $\Delta^{++}$ wave functions $A_{p3}$ and $A_{\Delta3}$. Here $b = 300$ MeV and the two oscillator parameters are $\omega_3 = 225$ MeV and $\omega_5 = 245$ MeV.

confinement with $B = 0$ than in the case of the linear confining interaction with $b = 0$ MeV. With the oscillator model with $B = 100$ MeV, the confinement triggered annihilation, when combined with the amplitude for direct annihilation would lead to an enhancement of the total calculated pion decay width by a factor $\sim 2$. In Table I we list the calculated enhancement from the $qqq$ quark model value for different values of $B$ in the harmonic confining potential.

These results show that there is a considerable model dependence in the calculated enhancement of the decay with that arises from annihilation transitions that are triggered by the confining interaction. The results thus have to be viewed as qualitative and of a very exploratory nature.
FIG. 7: The function $L(q)$ with $B = 0$ and $B = 100$ MeV, $\omega_5 = 245$ MeV.

TABLE I: Calculated enhancement of the decay width value in the $qqq$ quark model ($\delta$) for different values of the parameter $b$ in the linear confinement and $B$ in the harmonic confining potential. Here the probability of the $qqqq\bar{q}$ component in the nucleon is taken to be zero and in the $\Delta(1232)$ 10% and the oscillator parameters $\omega_3 = 225$ MeV and $\omega_5 = 245$ MeV.

| $b$ (MeV) | 150 | 200 | 250 | 300 | 350 | 400 | 450 |
|-----------|-----|-----|-----|-----|-----|-----|-----|
| $\delta$  | 3.30| 3.03| 2.76| 2.51| 2.27| 2.24| 1.82|

| $B$ (MeV) | 50 | 75 | 100 | 125 | 150 | 175 | 200 |
|-----------|----|----|-----|-----|-----|-----|-----|
| $\delta$  | 2.23| 2.12| 2.00| 1.90| 1.79| 1.69| 1.59|

IV. DISCUSSION

Above it was shown that quark-antiquark annihilation may contribute significantly to the decay width of the $\Delta(1232)$ resonance calculated in the non-relativistic constituent quark model. This contribution depends on the amplitude of the $qqqq\bar{q}$ admixture of the $\Delta(1232)$ and on the nature of the interaction between the quarks and the antiquark. The enhancement of the decay width calculated in the valence quark model was found to be as large as factors 2
if the \( \Delta(1232) \) contains a \( qqqq \bar{q} \) component with 10% probability. This large contribution was obtained with the assumption that the Lorentz nature of the confining interaction is a scalar interaction. This is sufficient to compensate for the underestimate of the decay width of the \( \Delta(1232) \) in the \( qqq \) valence quark model.

The present estimates relied on a non-relativistic harmonic oscillator model for the quark wave functions, which has previously been shown to provide useful, if qualitative, information on baryon phenomenology. To go beyond this model requires a detailed model for the interaction between the quarks, which should be constrained both by the empirical splitting between the \( \Delta(1232) \) and the nucleon as well as by the electromagnetic form factors of the nucleon. Only with such a Hamiltonian model is it possible to obtain quantitative estimates for the amplitude of the \( qqqq \bar{q} \) components in the baryons. A more quantitative calculation should also require covariant framework. The fact that the covariant quark models with instant form kinematics lead to rather similar results as the non-relativistic quark model \[19\], suggests that the qualitative features of the present results will carry over to covariant descriptions based on instant form kinematics.

It should be instructive to extend this phenomenological analysis to the case of the low lying positive parity resonances \( N(1440) \) and the \( \Delta(1600) \), which are very likely to have substantial sea-quark components. The widths of these are typically underestimated by large factors in calculations based on the 3-valence quark model \[1,2\].

Acknowledgments

Research supported in part by the Academy of Finland grant number 54038

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