Analysis of the $e^+e^- \rightarrow J/\psi D\bar{D}$ reaction close to the threshold concerning claims of a $\chi_{c0}(2P)$ state

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We analyze the $D\bar{D}$ mass distribution from a recent Belle experiment on the $e^+e^- \rightarrow J/\psi D\bar{D}$ reaction using a unitary formalism with coupled channels $D^+D^-, D^0\bar{D}^0, D_s\bar{D}_s$, and $\eta\eta$, with some of the interactions taken from a theoretical model, but with enough freedom to determine the mass and width of a $D\bar{D}$ bound state that comes from a fit to the data. We show that the mass distribution divided by phase space does not have a peak above the $D\bar{D}$ threshold that justifies the experimental claims of a $\chi_{c0}(2P)$ state from those data. Within the experimental precision we also show that the data are compatible with a $D\bar{D}$ bound state. We take advantage to show that a Breit-Wigner amplitude with the same mass and width gives rise to a radically different shape, disavowing the use of Breit-Wigner fits close to threshold.

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I. INTRODUCTION

A recent experiment by the Belle collaboration on the $e^+e^- \rightarrow J/\psi D\bar{D}$ reaction, looking into the $D\bar{D}$ mass distribution, observes a strong peak close to threshold [1]. A fit to the data in terms of a Breit-Wigner amplitude produces a mass and width $M = 3862^{+26+40}_{-32-13}$ MeV, $\Gamma = 201^{+154+88}_{-67-82}$ MeV. The $J^{PC} = 0^{++}$ hypothesis is favoured over the $2^{++}$. The peak is associated to a new charmonium state $X(3860)$ which the authors propose as a candidate for $\chi_{c0}(2P)$. In the present work we will show that these data does not support the existence of a resonance state around 3860 MeV. We also show that the data, within their large errors, can well accommodate a bound $D\bar{D}$ state below threshold. Such state has been predicted in Ref. [2], and also in Refs. [3, 4] using effective field theory that implements heavy quark spin symmetry. This state would be the analogous state to the $f_0(980)$, long advocated as a $K\bar{K}$ bound state [5], and substantiated with the chiral unitary approach [6–9].

The reason to reopen the issue is that the analysis of a structure close to threshold demands techniques that are consistent with analyticity, coupled channels unitarity and threshold properties (cusps in amplitudes) that go beyond the simple representation by a Breit-Wigner amplitude [10, 11]. As an example, in Ref. [12] a peak observed close to threshold of the $\phi\omega$ mass distribution in the $J/\psi \rightarrow \gamma\phi\omega$ reaction [13], associated there to a state with mass around
1795 MeV and width around 95 MeV (with large uncertainties), was found to be compatible with the effect of the $f_0(1710)$, which is below the $\phi\omega$ threshold.

The existence of a $D\bar{D}$ bound state with $J^{PC} = 0^{++}$ and isospin $I = 0$ was predicted in Ref. [2] around 3700 MeV, and a fit to the early data of $e^+e^- \rightarrow J/\psi D\bar{D}$ [14], analyzing the $D\bar{D}$ spectrum close to threshold, was found compatible with the existence of this state with a mass around 3720 MeV [15]. The data in Ref. [1] gives in principle hopes that we could get more accurate information than in Ref. [15]. However, we will show that this is not the case, but we also show that the claims of a $\chi_{c0}(2P)$ state from those data are not founded.

II. FORMALISM

The $e^+e^- \rightarrow J/\psi D\bar{D}$ process is depicted in Fig. 1. Ignoring factors which depend on the energy $\sqrt{s}$ of the $e^+e^-$ system or are constant, we can write the $D\bar{D}$ mass distribution of the reaction as [15]

$$\frac{d\sigma}{dM_{\text{inv}}(D\bar{D})} = C \frac{1}{(2\pi)^3 \sqrt{s}} \frac{m_e^2}{s} |\vec{p}| |\vec{k}| |T|^2,$$

where $\vec{p}$ is the $J/\psi$ momentum in the $e^+e^-$ center of mass frame and $\vec{k}$ the $D$ momentum in the $D\bar{D}$ rest frame,

$$|\vec{p}| = \frac{\lambda^{1/2}(s, M_{J/\psi}^2, M_{\text{inv}}^2(D\bar{D}))}{2\sqrt{s}},$$

$$|\vec{k}| = \frac{\lambda^{1/2}(M_{\text{inv}}^2(D\bar{D}), M^2_D, M^2_{\bar{D}})}{2 M_{\text{inv}}(D\bar{D})}.$$  

The experiment of Ref. [1] is done around the $\Upsilon(1S)$ to $\Upsilon(5S)$ states, hence $\sqrt{s}$ for $e^+e^-$ ranges from 9.46 GeV to 10.87 GeV. The value of $|\vec{p}|$ is smoothly dependent on $M_{\text{inv}}(D\bar{D})$ in this range and we take $\sqrt{s} = 10$ GeV for the calculations. The magnitude $T$ appearing in Eq. (1) is the $D\bar{D} \rightarrow D\bar{D}$ amplitude to which we come below, but before elaborating on it, we find most instructive to show the results for $|T|^2$ obtained from the data, dividing the experimental cross section by the phase space factor of Eq. (1), $|\vec{p}| |\vec{k}|$. The results are shown in Fig. 2, where the experimental data are taken from Fig. 6 of Ref. [1], from where the data of the background shown in Fig. 5 of Ref. [1] has been subtracted.\(^{1}\)

What we see in Fig. 2 is that in the region around 3860 MeV where the $\chi_{c0}(2P)$ was claimed (yet, with a large width of about 200 MeV), there is no structure that justifies the existence of a state. We should note that there is

\[\text{FIG. 1: Diagrammatic representation of the } e^+e^- \rightarrow J/\psi D\bar{D} \text{ reaction.}\]

\(^{1}\) We thank K. Chikilin for informing us that this is the appropriate procedure. We should also note that the data presented in this published paper are not corrected by acceptance, hence, some caution must be taken on the conclusions.
an extra experimental point close to threshold, but the bin of 50 MeV does not allow one to get a meaningful value for the phase space. We should also call the attention to the fact that the sharp fall down of the data in Fig. 2, corresponds exactly to the $D_s\bar{D}_s$ threshold, where a cusp should in principle be expected. We shall come back to this point below.

After this observation, let us present our analysis. Following Ref. [2], we construct the $D\bar{D}$ amplitude using the Bethe-Salpeter equation in coupled channels,

$$T = [1 - VG]^{-1}V,$$

(4)

with the channels $D^+D^-, D^0\bar{D}^0, D_s\bar{D}_s, \eta\eta$, where $V_{ij}$ are the transition potentials and $G_i$ the diagonal matrix accounting for the two-meson loop function for each channel. The important channels for the threshold behaviour are $D^+D^-$ and $D^0\bar{D}^0$. In Ref. [2], in addition to the $D\bar{D}, D_s\bar{D}_s$ channels, other light pseudoscalar-pseudoscalar ($PP$) channels are considered, including $\pi\pi, K\bar{K}, \eta\eta$. Their couplings to the $D\bar{D}$ channels are very much suppressed and their roles around the $D\bar{D}$ threshold are negligible. The only effect is to produce some small width for the $D\bar{D}$ state found. Due to this, in Ref. [16] all light channels considered in Refs. [2, 17], that led to a width of the $D\bar{D}$ state of about 36 MeV, were integrated in just one channel, the $\eta\eta$, and the transition potential from $D\bar{D} \to \eta\eta$ was tuned such as to give that width. Here we follow the same strategy but take this transition potential as a free parameter, such that the experimental data provide the width of the state. Then, as in Ref. [16] we take the $V_{ij}$ matrix elements between $D$ and $D_s$ from Ref. [2] and $V_{D^+D^-, \eta\eta} = V_{D^0\bar{D}^0, \eta\eta} = a, V_{D_s\bar{D}_s, \eta\eta} = V_{\eta\eta, \eta\eta} = 0$. Since all we want from the $\eta\eta$ channel is to generate a width, it is sufficient to take the imaginary part of $G_{\eta\eta}$,

$$i \Im G_{\eta\eta}(M_{\inv}) = -\frac{1}{8\pi} \frac{1}{M_{\inv}} q_{\eta},$$

(5)

with $q_{\eta} = \lambda^{1/2}(M_{\inv}^2, m_{\eta}^2, m_{\eta}^2)/2M_{\inv}$.

For the $G$ function of the rest of the channels, we use dimensional regularization as in Ref. [2], but with the scale mass $\mu$ fixed to $\mu = 1500$ MeV, and the subtraction constant $\alpha$, common to the $D^+D^-, D^0\bar{D}^0, D_s\bar{D}_s$ channels, as a free parameter. This parameter determines the position of the resonance.
III. RESULTS

The procedure followed has three free parameters, the constant $C$ in Eq. (1), the transition potential $a$ between $D\bar{D}$ and $\eta\pi$, and the subtraction constant $\alpha$. The amplitudes that our model produces have a limited range of validity and should not be used much above the $D_s\bar{D}_s$ threshold. There are few experimental points in that range, with large errors and furthermore there is the handicap of not having the acceptance corrected data. For all these reasons we renounce to making a fit to the data that can produce confusing results. Instead, we do a very valuable exercise. With a suitable choice of parameters $a = 50$, $\alpha = -1.3$, we find an approximate description of the data with the coupled channel approach, which we show in Fig. 3. In Fig. 4, we show the results for $|T|^2$ for different channels, with the $T$ matrix found in Eq. (4), and we see that the amplitudes corresponds to a $D\bar{D}$ bound state with mass $M_{D\bar{D}} = 3706$ MeV, and width $\Gamma = 50$ MeV.

![Figure 3](image_url)

**FIG. 3**: The differential cross section of the reaction $e^+e^- \rightarrow J/\psi D\bar{D}$. The red solid line is the result obtained in the coupled channel method, and the green dashed curve corresponds to the results of a Breit-Wigner form, where $M_X = 3710$ MeV, and $\Gamma_X = 50$ MeV.

Next, we take a Breit-Wigner amplitude,

$$T = \frac{\beta \sqrt{M_{inv}(D\bar{D}) - M_{X(D\bar{D})}^2} + iM_{X(D\bar{D})}\Gamma_{X(D\bar{D})}}{\sqrt{M_{inv}(D\bar{D}) - M_{X(D\bar{D})}^2 + iM_{X(D\bar{D})}\Gamma_{X(D\bar{D})}}}, \quad (6)$$

the parameter $\beta$ gives the strength, and we take $M_{X(D\bar{D})}$ and $\Gamma_{X(D\bar{D})}$, the mass and width of the $D\bar{D}$ bound state, as determined previously in the coupled channel approach.

We show the results of the Breit-Wigner amplitude in Fig. 3 compared to those of the coupled channel approach. We see that the Breit-Wigner amplitude and the coupled channel approach give rise to very different shapes in spite of sharing the same mass and width of the state. This exercise is very illuminating concerning the use of Breit-Wigner amplitudes close to threshold, something strongly discouraged in Refs. [10, 11] (see also Ref. [18]). We should also note the strong Flatté effect in the coupled channels amplitudes in Fig. 4, due to the opening of the $D\bar{D}$ threshold, which is also missed in a standard Breit-Wigner approach.

The other comment worth making is that the coupled channel approach, that contains the $D_s\bar{D}_s$ channel explicitly, produces a cusp at the $D_s\bar{D}_s$ threshold, and with the crudeness of the data, there seems to be a clear indication of such a cusp in the experiment. The coupling of the $D\bar{D}$ bound state to $D_s\bar{D}_s$ should be found as mostly responsible for the strength of the coupled channel approach close to the $D_s\bar{D}_s$ threshold compared to the Breit-Wigner amplitude.
IV. CONCLUSIONS

We have done a reanalysis of the $e^+e^- \rightarrow \psi D\bar{D}$ data [1], by looking at the $D\bar{D}$ mass distribution, from where the existence of a new charmonium state $X(3860)$ was claimed [1]. This conclusion was based on a fit to the data with a Breit-Wigner structure. However, we argue that structures close to threshold require a more sophisticated treatment, demanding unitarity in coupled channels and the fulfillment of analytical properties that a Breit-Wigner amplitude does not fulfill.

We have performed this work using the channels $D^+D^-$, $D^0\bar{D}^0$, $D_s\bar{D}_s$, and in addition the $\eta\eta$ channel, an important one for the decay, but used solely as a means of determining the width of the state from the experimental data. The Bethe-Salpeter equation in coupled channels is evaluated taking same transition potentials from early work on meson scattering in the charm sector that describes basic phenomenology [2], and roughly fitting three free parameters to the data, one related to the strength of the cross section, another one to the position of the resonance, and a third one to its width.

We can summarize our findings as follows:

1) The data divided by phase space did not show any structure which could justify the claims of a $\chi_{c0}(2P)$ state based on a Breit-Wigner fit to the data. Removing this state with a width of 200 MeV would solve a problem, in principle, which is why the width of the $\chi_{c0}(2P)$ state is 200 MeV, and the one of the $\chi_{c2}(2P)$, claimed at 3930 MeV [19], which has much more phase space for decay, has only 29 MeV.

2) We clearly showed that a Breit-Wigner amplitude with the same mass and width as obtained in a coupled channel unitary approach is drastically different to the one obtained from coupled channels close to the $D\bar{D}$ threshold.
3) The data, with its limited precision, and the caveat of not been acceptance corrected, can be accommodated by the influence of a $D\bar{D}$ bound state below threshold.

4) The study shows the potential of this reaction to extract information on a possible bound $D\bar{D}$ state with more data and more precision around threshold. 

It would be most interesting to observe this state in other reactions that do not have the $D\bar{D}$ final channel, but some $PP$ light channels. In this respect, several reactions have been suggested, the radiative decay $\psi(3770) \to \gamma X(3700)$ [20], $\psi(3770) \to \gamma X(3700) \to \gamma \eta \eta$ [17], $\psi(4040) \to \gamma X(3700) \to \gamma \eta \eta'$ [17], $e^+e^- \to J/\psi X(3770) \to J/\psi \eta \eta'$ [17].

A complementary reaction of the one discussed here, with $D\bar{D}$ in the final state, has also been suggested in Ref. [16] looking for the $B^0 \to D^0 \bar{D}^0 K^0$, $B^+ \to D^0 \bar{D}^0 K^+$ decays. The support of Refs. [3, 4], using arguments of heavy quark symmetry, to the early predictions of Ref. [2], provides extra strength to the existence of this $D\bar{D}$ bound state and efforts to find evidence for it should be most welcome.

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We take advantage to ask the authors of Ref. [1] to provide the corrected data, and in general to all authors of the experimental community. Data should be provided in a way that allows to be contrasted by a theory if we wish to extract valuable scientific information from them.
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