Optimal control on the attitude rotation of a flexible satellite model base on tetrahedral configured reaction wheels

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Abstract. The increase in demand for performance for satellite capabilities has pushed the design of the system to be more power consuming. Currently, most of the satellite design are equipped with large solar panel in order to produce sufficient power. Thus, this translates to a higher flexibility in the satellite which makes the satellite prone to vibratory motion. The primary cause of vibratory motion in satellite is the attitude rotation. Reaction wheel is a widely used attitude rotation actuator for a satellite. Commonly, the reaction wheels are configured in the tetrahedral form. This causes uneven maximum torque at any direction. Hence, an iterative optimal control method is applied using GPOPS toolbox to achieve minimum time of rotation with low level of induced vibratory motion. The optimized control is compared with the Eigen-axis Quaternion Feedback control to observe the performance of the optimal control. Depending on the direction of attitude manoeuvre, the optimal control has shown an improvement between 3.49% to 25.11% in manoeuvre time. Overall, the optimal control outperformed the traditional the Eigen-axis Quaternion Feedback control.

1. Introduction
Artificial satellites are launched into the Earth’s orbit since the year 1957. These satellites are mainly for the purposes of communications, military defence, explorations and the research [1]. As the function of a satellite becomes more complex and advance, this leads the satellite to be designed to equip with deployable booms, solar panels or antennas that are flexible in nature. In a multi-body satellite, when the flexible components make up a significant part, the flexibility must consider the dynamic model [2]. An ill-defined dynamic model inherits inaccuracy and unpredictability to the attitude maneuvering of the satellite system. Highly delicate operation such as imaging are sensitive to vibration because it will leads to a decline in imaging quality of space camera[3].

The attitude of satellites changes to fulfil the mission needs. Generally, there are two types of control, namely the active or passive control. The application of passive control is demonstrated in [4]. Some of the published work on active control are shown in [5] using generalized predictive control method, Lyapunov [6] and linear matrix inequality [7]. This paper focuses on introducing a control function based on the PMP optimal control framework. Through this paper, the control framework is shown to manoeuvre the flexible satellite’s attitude with minimum vibratory motion and time compared to the Eigen-axis Quaternion Feedback (EQF) method.

2. Dynamic of Reaction Wheel
The satellite’s system of attitude manoeuvring consists of an actuator and controller. The controller produces signal for the desired input while the actuator produce the input to the manoeuvring system according to the signals. Commonly, for redundancy purpose, satellite system are equipped with four or
more wheels of Reaction Wheel (RW) [8]. A four RW configured actuator system is shown in Figure 1 [9].

![Figure 1. Configuration of the RW](image)

The dynamics of the RW array of generated is obtained from [10] as shown in Equation 1.

$$
\begin{bmatrix}
    \tau_x \\
    \tau_y \\
    \tau_z 
\end{bmatrix} = \tau_{out,j}^N + \Omega_i I_i \times \Omega_j
$$

Equation 1

$I_i, \Omega_i$ and $C R^w$ are the moment of inertia, the angular velocity of the RW and the transformation matrix that relates the wheel frame to the satellite body frame respectively. $\tau_{out,j}^N$ is the torque generated by the RW while $\tau_X, \tau_Y$ and $\tau_Z$ is the component of generated torque based on the X, Y and Z axis. $\Omega_i$ are the angular velocity of the RW.

3. The Flexible Satellite Model

A satellite system consists of a centre hub with appending panel as shown in Figure 2. The central hub of the satellite is defined as a rigid body and the appending panels are flexible that execution of deflection and bending along the panel.

![Figure 2. Simple model of flexible satellite](image)

A point O is considered within the body of the satellite. The angular rotations are done along the X, Y and Z axis that intersect at point O. The rate of angular rotation of the satellite is given by the matrix $\dot{\theta}_s$ as described in the Equation 2.

$$
\dot{\theta}_s = \begin{bmatrix}
    \dot{\theta}_x \\
    \dot{\theta}_y \\
    \dot{\theta}_z 
\end{bmatrix}
$$

Equation 2
The variable $\dot{\theta}_X$, $\dot{\theta}_Y$ and $\dot{\theta}_Z$ is the satellite’s angular velocity. An alternate point P, is considered on the end of the flexible panel that is fixed to the centre hub as shown in Figure 3.1. The point P, is defined as the origin of the local coordinate of the flexible panel and it is governed by the axis $p$, $q$, and $r$ axis. The subscript i represents the corresponding flexible panel in the satellite model. The model is taken from [9] as shown in Equation 3 to Equation 5.

$$
\begin{align}
I_{RX} &+ \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_X + \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Y + \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Z \\
&- (l_y - l_x) \dot{\theta}_y \dot{\theta}_z = \tau_x \theta_s \\
I_{RY} &+ \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Y + \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Y + \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Z \\
&- (l_y - l_x) \dot{\theta}_z \dot{\theta}_y = \tau_y \\
I_{RZ} &+ \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Z + \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Z + \sum_{i=1}^{n} \left[ \int_{-h_i/2}^{h_i/2} \int_{-w_i/2}^{w_i/2} \rho_i \{H_i \} dx_i dy_i dz_i \right] \dot{\theta}_Z \\
&- (l_x - l_y) \dot{\theta}_z \dot{\theta}_y = \tau_z
\end{align}
$$

The xi, yi and zi are the local coordinate of the flexible panels while h, w and L are the height width and length of the flexible panel. $\theta^0$ and $\theta^e$ is the degree of freedom of the satellite with respect to the total body of the satellite and the flexible panel.

4. The Eigen-axis Quaternion Feedback

In general, most of the satellite applied the EQF controller. The dynamics of the EQF is obtained from [10] shown in Equation 6.

$$\dot{\theta}_E = -k_c (\theta_s - \theta_d) - c_c \dot{\theta}_s$$

Equation 10 shows that the EQF controller equation is equivalent to the Proportional-Derivative control system. $\theta_s$ and $\theta_d$ is the instantaneous, desired angle of the satellite while $\theta_E$ angle difference between the satellite and desired angle. Kc and Cc is the proportional and derivative gain constant.

5. The Optimal Control

GPOPS toolbox is applied to solve the optimal control problem. The dynamic models, boundary condition and states of the system from Equations 1 to 6 are compiled as shown in Equation 7.

Given 
$$
\begin{align}
x^T &\equiv [q_1, q_2, q_3, q_4, \dot{\theta}_X, \dot{\theta}_Y, \dot{\theta}_Z, \Omega_1, \Omega_2] \\
u^T &\equiv [\tau_r, \tau_f, \cdots, \tau_r, \cdots]
\end{align}
$$

Minimize 
$$J = t_f$$

Subject to 
$$
\begin{align}
\dot{x}_s^T &\equiv \frac{-\theta_s \times I_s \dot{\theta}_s}{I_s} \\
\tau &\equiv \tau_{outf}^N = c R^w \tau_{rf} \\
x_f^T &\equiv [q_{0,1}, q_{0,2}, q_{0,3}, q_{0,4}, 0, 0, 0, 0, 0, 0] \\
x_f^T &\equiv [q_{f,1}, q_{f,2}, q_{f,3}, q_{f,4}, 0, 0, 0, 0, 0, 0] \\
-\tau_{max} < \tau_{rf} < \tau_{max} - \Omega_{max} < \Omega < \Omega_{max}
\end{align}
$$

Where, $x^T$ refers to the states, $x_0^T$ is the initial state and $x_f^T$ is the final state. $J$ is the cost function while $u^T$ is control variable.
6. The Simulation

The parameters RW W45HT by Bradford Engineering is chosen for the simulation shown in Table 1.

Table 1. Simulation input

| Variable | Value |
|----------|-------|
| $I_s$    | $\begin{bmatrix} 200 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{kg} \cdot \text{m}^3$ |
| $\tau_{\text{max}}$ | 0.403 $\text{N} \cdot \text{m}$ |
| $\Omega_{\text{max}}$ | 314 $\text{rad/s}$ |

The simulation for the attitude manoeuvring are shown in Figure 3.

![Figure 3. Quaternion rotation for (a) $q_1$ (b) $q_2$ (c) $q_3$ (d) $q_4$.](image)

The flexible response of the satellite system is shown in Figure 4. As shown, the GPOPS solution makes the flexible motion to converge faster than the EQF control.

![Figure 4. Flexibility comparison (EQF vs GPOPS) for (a) Panel 1, (b) Panel 2 and (c) Panel 3.](image)
Shown in Figure 4, in the case when the control signal is optimized, the flexible motion is almost eliminated while the control that applied the EQF execute vibration on post actuation. The quaternion angle of the satellite is converted into Euler angles as illustrated in Figure 5. It is observable that the EQF manoeuvring time is 74.04 seconds while the GPOPS optimal control took 56.05 seconds to complete the manoeuvre. This shows a 24.3% reduce in the manoeuvring time.

However, due to the uneven maximum torque generation at any direction, the performance of the optimal control varies. Table 2 illustrates the manoeuvre time of the EQF and GPOPS of a 120° rotation on direction. The initial state of all manoeuvre is \([0, 0, 0, 1]\) quaternion. Shown in Table 2, regardless of the direction of rotation, the GPOPS optimal control outperform the EQF manoeuvre.

![Figure 5. Rate of attitude rotation of satellite](image)

| \(q\)       | Eigen Axis           | GPOPS (s) | EQF (s) | Improvement (%) |
|-------------|----------------------|-----------|---------|-----------------|
| \([0, 0, 1, 0.5]\) | \([0, 0, 1.155]\) | 63.5757   | 65.94   | 3.59            |
| \([0.5, 0.5, -0.5, 0.5]\) | \([0.577, 0.577, -0.577]\) | 56.1457   | 61.18   | 8.23            |
| \([-0.612, -0.433, -0.433, 0.5]\) | \([0.7071, 0.5, 0.5]\) | 54.1275   | 60.83   | 11.02           |
| \([0.707, 0, 0.5, 0.5]\) | \([0.8165, 0, 0.577]\) | 57.1254   | 65.11   | 12.26           |

Observable, the optimal control outperforms the EQF because it is able to perform Off-Eigen axis manoeuvre. This enable the actuators to produce higher momentum and torque, thus increases the rate of rotation. On the other hand, the EQF is constrained by the pseudo-inverse limitation which limited the maximum generated momentum and torque.

7. Conclusion

Time optimal control shows promising result compared to the traditional EQF method. The works show an improvement of 3.59% to 24.3% in manoeuvre time for various direction of rotation. In addition to that, the vibratory motion is also eliminated on post actuation in optimal control. It can be concluded that, regardless of the direction of rotation, the GPOPS optimal solution will outperform the EQF control.

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