Beam Splitter Entangler for Light Fields

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We propose an experimentally feasible scheme to generate various types of entangled states of light fields by using beam splitters and single-photon detectors. Two light fields are incident on two beam splitters and are split into strong and weak output modes respectively. A conditional joint measurement on both weak output modes may result in an entanglement between the two strong output modes. The conditions for the maximal entanglement are discussed based on the concurrence. Several specific examples are also examined.

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Quantum entanglement has been identified as a basic resource in achieving tasks of quantum communication and quantum computation. Photons are considered to be the best quantum information carriers over long distances, and those in entangled states have been used to experimentally demonstrate quantum teleportation, quantum dense coding, quantum cryptography, and the generation of GHZ-states of three or four photons.

In comparison with other candidates for engineering quantum entanglement, light fields possess more abundant capacity to create various types of entangled states including the discrete, the continuous variable, and the combination of the both. Almost all of the reported experiments adopted the parametric down-conversion process as a standard source for the entangled photon pairs as well as two-mode squeezed states. Recently it was found that linear optical elements, such as beam-splitters and polarization beam-splitters, can also be used to generate entangled light fields. In Ref. [6], an efficient quantum computation with linear optics is devised, it is undoubtedly an entangler for light fields. On the other hand, a single beam splitter can also act as an entangler for light fields if the input modes are in appropriate nonclassical states. However, the types of the resultant entangled states from the existent schemes are still very limited and cannot satisfy the requirements for quantum information processing and for other applications. Thus how to generate various types of entangled states of light fields is still of great significance.

Very recently several schemes have been proposed to entangle distant atoms and atomic ensembles by means of photon interference. It is natural to ask whether the idea of photon interference can be extended to the generation of the entangled states of light fields. This letter will give a positive answer.

In this letter we present an experimentally feasible scheme for the generation of entangled states of light fields by using beam splitters and the single-photon detectors. The setup we are considering here consists of three lossless beam splitters, BS1, BS2 and BS3, and two single-photon detectors D1 and D2, as shown in Fig. 1. For simplicity, we assume that BS1 and BS2 are of the same amplitude reflection and transmission coefficients R and T with a relation $|R|^2 + |T|^2 = 1$, and that the distance from BS1 to BS3 is the same as that from BS2 to BS3. As an optical element, a beam splitter generally has two input ports and two output ports. In our setup only one input port has a non-zero input field for both BS1 and BS2, the other input port is always left in the vacuum state. In this case the non-zero input field will be split into two output fields. As has been proved, the two output fields may be entangled when the input field is in an appropriate nonclassical state. The entanglement quantity and the properties of the output fields depend to a large extent on the input fields as well as on the amplitude reflection and transmission coefficients R and T of the beam splitter. If R and T of the beam splitter BS1 (and BS2) are largely different, the non-zero input field will be split into two output fields in which one is very strong, and the other is very weak. In fact, we can design BS1 and BS2 in such a way that the weak output field possesses maximally one photon, that is, the weak output mode is in the vacuum state $|0\rangle$ or in the one photon number state $|1\rangle$. Correspondingly the strong output mode will be in a state that keeps the photon number of the input field unchanged or in a state that annihilates one photon from the input field since the lossless beam splitter conserves the photon number of the input fields. In this way we have prepared a specific entangled state of the weak and the strong output fields. Subsequently we let the two weak output fields from BS1 and BS2 be combined at BS3, a 50%:50% beam splitter, and then detected by single-photon detectors D1 and D2. If only one photon is registered by D1 or D2, we successfully generate an entangled state of the two strong output fields of BS1 and BS2. Otherwise, we fail to generate the desired entangled state and should repeat the process again until one photon is registered.

Our scheme is based on a simple optical element, the beam splitter. The perfection of the beam splitter is very high, the requirements for BS1 and BS2 can thus be satisfied with the required precision easily, therefore our scheme is experimentally feasible. In particular, as will be shown below, our scheme can be used to generate various types of entangled states of light fields.
In order to illustrate our method explicitly, we first examine the effect of a beam splitter on its output fields. Let us denote by $\hat{a}_j, \hat{b}_j$ the input mode amplitudes shown in Fig. 1 and by $\hat{c}_j, \hat{d}_j$ the output mode amplitudes, where the subscript $j$ stands for BS$_j (j = 1, 2)$. Suppose the initial quantum state of the input fields for BS$_j$ is a product state, $|0, \psi_j^+\rangle = |0\rangle_{a_j} \otimes |\psi_j^+\rangle_{b_j}$, in which the mode $\hat{a}_j$ is supposed to be always in the vacuum state $|0\rangle_{a_j}$ and the mode $\hat{b}_j$ in a superposition of the photon number states which can be expressed as

$$|\psi_j^+\rangle = \sum_{n=0}^{\infty} f_n^{(j)} |n\rangle,$$

(1)

with the normalized condition $\sum_{n=0}^{\infty} |f_n^{(j)}|^2 = 1$. The output fields are then of the following form,

$$|\Psi_j\rangle_{\text{out}} = \hat{B}_j|0, \psi_j^+\rangle_{\text{in}} = \sum_{n=0}^{\infty} f_n^{(j)} \hat{B}_j|0, n\rangle_{\text{in}},$$

(2)

where the beam splitter operator is

$$\hat{B}_j = \exp \left[ \frac{\theta}{2} (\hat{a}_j \hat{b}_j e^{i\phi} - \hat{a}_j^\dagger \hat{b}_j^\dagger e^{-i\phi}) \right],$$

(3)

with the amplitude reflection and transmission coefficients $T = \cos \frac{\theta}{2}$, $R = \sin \frac{\theta}{2}$. Here $\phi$ denotes the phase difference between the reflected and transmitted fields.

The terms $\hat{B}_j|0, n\rangle_{\text{in}}$ in Eq. (2) can be evaluated according to Eq. (3),

$$\hat{B}_j|0, n\rangle_{\text{in}} = \sum_{k=0}^{n} c_k^n |k, n - k\rangle_{\text{out}},$$

(4)

where $|k, n - k\rangle_{\text{out}} = |k\rangle_{c} \otimes |n - k\rangle_{d}$, and $c_k^n = \binom{n}{k}^{1/2} \exp (ik\phi) R^k T^{n-k}$.

In the following we assume that the amplitude reflection coefficient $R$ for BS$_j (j = 1, 2)$ is so small ($R \ll T$) that the terms containing $R^2$ in Eq. (4) can be neglected when $k \gg 2$, thus Eq. (4) is simplified as

$$\hat{B}_j|0, n\rangle_{\text{in}} \approx T^n |0, n\rangle_{\text{out}} + \exp (i\phi) R T^{n-1} |1, n-1\rangle_{\text{out}} = T^n |0, n\rangle_{\text{out}} + \exp (i\phi) R T^{n-1} \hat{d}_j|1, n\rangle_{\text{out}}.$$  

(5)

Substituting Eq. (5) into Eq. (2) we get

$$|\Psi_j\rangle_{\text{out}} = |0\rangle_{c_j} \otimes |u_j\rangle_{d_j} + \exp (i\phi) R |1\rangle_{c_j} \otimes |v_j\rangle_{d_j},$$

(6)

where $|u_j\rangle$ and $|v_j\rangle$ take the form

$$|u_j\rangle = \sum_{n=0}^{\infty} f_n^{(j)} T^n |n\rangle,$$  

(7a)

$$|v_j\rangle = \hat{d}_j \sum_{n=0}^{\infty} f_n^{(j)} T^n |n+1\rangle.$$  

(7b)

The state (6) is a specific entangled state in which the mode $\hat{c}_j$ (i.e. the weak mode) possesses maximally one photon.

Now we show how to generate the entangled states of two strong output modes $\hat{d}_1$ and $\hat{d}_2$ by manipulating the two weak modes $\hat{c}_1$ and $\hat{c}_2$. As shown in Fig. 1, we suppose the two input fields are, respectively, incident on BS$_1$ and BS$_2$ simultaneously. Afterwards the two weak output modes $\hat{c}_1$ and $\hat{c}_2$ are combined at BS$_3$ with the output mode amplitudes $A_1 = [i\hat{c}_1 + e^{i\gamma}\hat{c}_2]/\sqrt{2}$, $A_2 = [\hat{c}_1 + i e^{i\gamma}\hat{c}_2]/\sqrt{2}$. Here the factor $i$ denotes a $\pi/2$ phase shift between the reflected and the transmitted modes, and $\gamma$ denotes the phase shift induced by a wave plate (WP) placed in the path of mode $\hat{c}_2$. The detection of a single photon by D$_1$ or D$_2$ is accompanied by the wave function collapse $|\Psi_1\rangle_{\text{out}} \otimes |\Psi_2\rangle_{\text{out}} \rightarrow A_{1,2} (|\Psi_1\rangle_{\text{out}} \otimes |\Psi_2\rangle_{\text{out}})$. Neglecting the high-order terms $o(R^2)$, we find the final state of the two strong modes, conditional on a click of either D$_1$ or D$_2$, takes the following form,

$$|\Phi_{1,2}\rangle = |u_1\rangle|u_2\rangle \mp i \exp (i\gamma) |u_1\rangle|v_2\rangle.$$  

(8)

Here and in what follows the subscripts $d_j$ are omitted to simplify the notation. Note that the state (8) is not normalized. The probability for successfully generating the above state is proportional to $R^2$. The phase factor in state (8) can be controlled through the WP.

In principle, the scheme we proposed here is closely related to the quantum entanglement swapping [15] in which the particles that have never interacted directly are entangled, and can be regarded as a specific version of entanglement concentration protocol by using entanglement swapping [13]. The entanglement between the weak and the strong output fields for both BS$_1$ and BS$_2$ is a necessary condition for the successful generation of the desired entangled state (8). Accordingly the requirement that both input fields of BS$_1$ and BS$_2$ are in nonclassical states should be satisfied [14, 11].

Now let us examine the entanglement properties of the resultant state (8). To this end, we first transform it to the normalized basis,

$$|\Phi_{1,2}\rangle = \nu_1\nu_2 |U_1\rangle|U_2\rangle \mp i \exp (i\gamma) \mu_1\nu_2 |U_1\rangle|V_2\rangle,$$  

(9)

where $|U_j\rangle = |u_j\rangle/\mu_j$, $|V_j\rangle = |v_j\rangle/\nu_j$ are normalized states for system $j$ with the normalized constants

$$\mu_j = \left( \sum_{n=0}^{\infty} |T^n f_n^{(j)}|^2 \right)^{1/2},$$  

(10a)

$$\nu_j = \left( \sum_{n=0}^{\infty} (n+1) |T^n f_{n+1}^{(j)}|^2 \right)^{1/2}.$$  

(10b)

Considering $|U_j\rangle$ and $|V_j\rangle$ may be nonorthogonal, the state (9) is actually a general bipartite entangled state. The concurrence [17] has been proved to be a convenient...
entanglement measure for such states and has been evaluated by Wang [13]. We find the concurrence of the entangled state (9) takes the form

\[
C_{1,2} = \frac{2\mu_1\nu_1\nu_2}{\mu_1^2\nu_2 + \nu_1^2\mu_2 + \mu_1\nu_1\nu_2 (\pm e^{i\gamma} (U_1 V_1) + e^{i\gamma} (V_1 U_1) (U_2 V_2))}.
\]

The concurrence ranges from 0 to 1 with the value 1 corresponding to a maximally entangled state (MES). In a special case where the input field of BS1 is the same as that of BS2, i.e., \( f_n^{(1)} = f_n^{(2)} \), we have \( |u_1⟩ = |u_2⟩ \equiv |u⟩ \), \( |v_1⟩ = |v_2⟩ \equiv |v⟩ \), \( \mu_1 = \mu_2 \), \( \nu_1 = \nu_2 \), and therefore \( |U_1⟩ = |U_2⟩ \equiv |U⟩ \), \( |V_1⟩ = |V_2⟩ \equiv |V⟩ \), the concurrence is thus simplified as

\[
C_{1,2} = \frac{1 - |⟨V|U⟩|^2}{1 \pm \sin γ |⟨V|U⟩|^2}.
\]

From Eq. (12) one can readily deduce the following conclusions: (i) If \( |U⟩ \) and \( |V⟩ \) are orthogonal, i.e., \( ⟨V|U⟩ = ⟨v|u⟩ = 0 \), we always have \( C_{1,2} = 1 \), and the resultant entangled state (8) is an MES. (ii) If, however, \( |U⟩ \) and \( |V⟩ \) are nonorthogonal, the condition that state (8) is an MES is \( γ = 3π/2 \) when the photon is detected by \( D_1 \) or \( γ = π/2 \) when the photon is detected by \( D_2 \).

Finally we give several examples to demonstrate the power of our scheme as an entangler for the generation of various types of entangled states of light fields.

**Example 1, photon number state inputs.** When the input field of BS1 and that of BS2 are in the Fock states \( |n⟩ \) and \( |m⟩ \) respectively, the resultant entangled state is

\[
|ϕ⟩_{1,2} = \frac{1}{\sqrt{n+m}} \left[ ∑_{n=0}^{∞} \frac{(-\Gamma)^n}{n!2^n} |2n⟩ + i \exp (iγ) \sqrt{m|n⟩|m-1⟩}\right].
\]

If \( n = m \), the above state becomes an MES

\[
|ϕ⟩_{1,2} = \frac{1}{\sqrt{2}} \left[ |n-1⟩|n⟩ + i \exp (iγ) |n⟩|n-1⟩\right].
\]

**Example 2, even (or odd) coherent state inputs.** Even and odd coherent states are superposition of coherent states, i.e., Shrōdinger cat states, and take the following form

\[
|α⟩^c = N_e(|α⟩ + |−α⟩) = \cosh |α|^2 \frac{1}{2} ∑_{n=0}^{∞} \frac{α^{2n}}{\sqrt{(2n)!}} |2n⟩,
\]

\[
|α⟩^o = N_o(|α⟩ − |−α⟩) = \sinh |α|^2 \frac{1}{2} ∑_{n=0}^{∞} \frac{α^{2n+1}}{\sqrt{(2n+1)!}} |2n+1⟩.
\]

where \(|α⟩\) and \(|−α⟩\) are coherent states, \(N_e\) and \(N_o\) are normalized coefficients. If both input field of BS1 and that of BS2 are in the same even coherent states, \(|α⟩^e\), the resultant entangled state is

\[
|ϕ⟩_{1,2} = \frac{1}{\sqrt{2}} [⟨Tα⟩^o |Tα⟩^e + i \exp (iγ) ⟨Tα⟩^e |Tα⟩^o].
\]

It is well known that even and odd coherent states are orthogonal, the resultant state (16) is thus an MES. A similar result can be worked out if both input field of BS1 and that of BS2 are in the same odd coherent states, \(|α⟩^o\).

**Example 3, squeezed vacuum state inputs.** A single mode squeezed vacuum state is a superposition of even photon number states,

\[
|SV(r)⟩ = (\cosh r)^{-1/2} ∑_{n=0}^{∞} \frac{(-\Gamma)^n}{n!2^n} |2n⟩,
\]

where \(Γ = \exp (iθ) \tanh r\), \(r\) is the squeeze parameter. If both input field of BS1 and that of BS2 are in the same squeezed vacuum states, \(|SV(r)⟩\), the resultant entangled state is

\[
|ϕ⟩_{1,2} = d_1 |SV(\bar{r})⟩_{d_1} ⊗ |SV(\bar{r})⟩_{d_2} + i \exp (iγ) |SV(\bar{r})⟩_{d_1} ⊗ d_2 |SV(\bar{r})⟩_{d_2}.
\]

In the above examples the resultant entangled states are either a discrete entanglement (example 1) or continuous variable entanglement (examples 2 and 3). In practice, our scheme can be used to generate a combination entanglement of the discrete and the continuous variable, see the following example.

**Example 4, Fock state and even coherent state inputs.** If one input field, say, the input of BS1, is in a Fock state \(|n⟩\), the other input field, the input of BS2, is in an even coherent states, \(|α⟩^e\), we may obtain the following entangled state

\[
|ϕ⟩_{1,2} = [n \cosh (|Tα|^2)]^{1/2} |n-1⟩|Tα⟩^e + i \exp (iγ) \times Tα \sinh (|Tα|^2) |n⟩|Tα⟩^o.
\]
The concurrence of the entangled state (19) is
\[
C = \frac{2|\alpha| \left[ n \sinh (|\alpha|^2) \cosh (|\alpha|^2)^{1/2} \right]}{|\alpha|^2 \sinh (|\alpha|^2) + n \cosh (|\alpha|^2)}. \tag{20}
\]
If $|\alpha|^2 \tanh (|\alpha|^2) = n$, we have $C = 1$, and the entangled state (19) is an MES.

In summary, we have presented an experimentally feasible scheme to generate various types of entangled states of light fields by using beam splitters and single-photon detectors. Our scheme is experimentally feasible because the basic elements in our scheme are accessible to experimental investigation with current technology. It has been shown that various types of light fields, such as the discrete, the continuous variable light fields and the combination of the both, can be entangled using our scheme.

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FIG. 1: Experimental setup. Two input light fields in modes $b_1$ and $b_2$ are incident on BS$_1$ and BS$_2$, and split into weak outputs in modes $c_1$ and $c_2$ and strong outputs in modes $d_1$ and $d_2$. Photons in weak outputs are then mixed on BS$_3$ and subsequently detected by single-photon detectors D$_1$ and D$_2$. 