Renormalization constants of local operators within the Schrödinger functional scheme

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We define, within the Schrödinger functional (SF) scheme, the matrix elements of the twist-2 operators corresponding to the first two moments of non-singlet parton density, and the first moment of singlet parton densities. We perform a lattice one-loop calculation that fixes the relation between the SF scheme and other common schemes and shows the main source of lattice artefacts. Few remarks on the improvement case are added.

1. Introduction

The accurate knowledge of hadron parton densities is an essential ingredient for the experimental text of QCD at the accelerator energies. Their normalization is usually obtained from a fit to a set of reference experiments and used for predicting the behaviour of hard hadron processes in different energy regimes. The calculation of the normalization needs non-perturbative methods. These calculations, especially for the higher moments, can reduce, for example, the uncertainties on the gluon parton densities at values of the Bjorken $x$ larger than 0.5. These calculations are made mainly by two groups (see for example [2], [3]).

It’s well known that, to match the scheme of the non-perturbative simulation and the scheme where the experiment made the comparison with theory, it is necessary a lattice perturbative computation of the renormalization constants of the operator in the same scheme where the operator is numerically computed.

The perturbative and non-perturbative calculation are made in the Schrödinger functional scheme (SF). The Schrödinger functional has been discussed extensively in the literature (see [5] for reviews). Among the advantages of the method, we only quote the possibility of performing the computations at zero physical quark mass and of using non-local gauge invariant sources for fermions and gluons without need of a gauge-fixing procedure. In our particular case, we exploit both features.

2. Non-singlet

We define the SF correlation function of the first and second moment of the non-singlet parton densities by the observables:

\[
\begin{align*}
&f_{0,2}(x_0; p) = f_2(x_0; p) = -a^6 \sum_{y,z} e^{ip(y-z)} \times \\
&\langle 1 \bar{\psi}(x) \gamma_1 \bar{D}_2 D_2 \frac{1}{2} \Gamma^3 \psi(x) \rangle \Gamma^3 \zeta(z) \\
&f_{0,23}(x_0; p) = f_3(x_0; p) = -a^6 \sum_{y,z} e^{ip(y-z)} \times \\
&\langle 1 \bar{\psi}(x) \gamma_1 \bar{D}_2 D_2 D_3 \frac{1}{2} \Gamma^3 \psi(x) \rangle \Gamma^3 \zeta(z)
\end{align*}
\]

where the contraction of the classical fields is non-vanishing if the matrix $\Gamma$ satisfies: $\Gamma P_\pm = P_\pm \Gamma$, where $P_\pm = \frac{1}{2}(1 \pm \gamma_0)$ and $p$ is the momentum of the classical field sitting on the boundary.

The quantities $\zeta$ are the response to a variation of the classical Fermi field configurations on the boundaries. We take the limit of massless quarks, but some care should be taken so as to ensure this limit at order $g^2$. The breaking of chiral symmetry of the Wilson action entails a non-zero shift of the quark mass from the naive value at order $g^2$.

The matrix element of the operator for the first moment involves two directions and three for the second moment. These directions must be provided by external vectors: we have chosen to ob-
tain one of them from the contraction matrix $\Gamma$, i.e. from the polarization of the vector classical state $\Gamma = \gamma_2$, and the remaining ones from the momentum $p$ of the classical Fermi field at the boundary.

To compute the renormalization constants of the operators is necessary to remove the renormalization constant of the classical boundary sources $\zeta$. Following ref. [4], this is represented by the quantity called $f_1$. Both $f_2$ and $f_1$ are normalized by their tree-level expressions. We define the renormalization constants such that operator matrix elements (here briefly indicated with $O$) is equal to its tree-level value at $\mu = 1/L$:

$$O^R(\mu) = Z(\mu)^{-1}O^{\text{bare}}(a/L)$$

with $Z(a/L)$ defined by

$$O^{\text{bare}}(a/L) = Z(a/L)O^{\text{tree}}$$

At the one loop the observable can be parametrized as:

$$Z(pL, x_0/L, a/L) = 1 + g^2 Z^{(1)}(a/L)$$

with

$$Z^{(1)}(a/L) = b_0 + c_0 \ln(a/L) + \sum_{k=1}^{\infty} a^k b_k + c_k \ln(a/L) L^k$$

The study of the scale dependence of the renormalization constant is then equivalent to that of the dependence upon the lattice size $L$, provided the external variables, upon which the matrix elements depend, scale like the basic length $L$. We made the calculation with the minimum value of the momentum $pL = 2\pi$, and with a "finite size" momentum $\theta = 0.1/L$, which escape the momentum quantization rule. Both the computation are made with $x_0 = L/4$, and $x_0 = L/2$. The fit (the fitting procedure is described in ref. [4]) to the $N = L/a$ dependence of our results has been made by the expression

$$Z^{(1)}(N) = B_0 + C_0 \ln(N) + \sum_{k=1,2} B_k + C_k \ln(N) N^k$$

excluding higher-order terms. Table 1 contains a summary of the constants $B_0$ for the operators, after removing the external legs renormalization, in the various cases that we have discussed. The lattice result converge much faster in their continuum limit, using a "finite size" momentum, confirming the momentum quantization as a major source of lattice artifacts (see ref. [5]). We made the same computation with clover action [6]. Also in this case lattice artefacts start at order $a$ for both coefficients, because the $O(a)$-improvement of the action should be complemented with the improvement of the operators and of the boundary counterterms in order to lead to a full cancellation of effects appearing linear in $a$. The Feynman rules for this computation can be easily derived using ref. [4]. The calculation is done to the order $a^2$. It’s important to stress out that to extract the finite constant to this order we must do the computation to the same order of $f_1$ and of the mass shift. The fitting procedure is the same that in the Wilson case. The finite part of the renormalization constant now will be defined as

$$B_O = B^{(0)}_O + B^{(1)}_O c_{sw} + B^{(2)}_O c_{sw}^2$$

Doing the subtraction of $f_1$ and of the mass shift we obtain the following results:

$$B^{(1)}_O = -0.0327(6) \quad \bar{B}^{(1)}_O = -0.0327012$$

$$B^{(2)}_O = -0.005725(9) \quad \bar{B}^{(2)}_O = -0.005726859$$

Table 1

| Moment | Definition | Constant |
|--------|------------|----------|
| First  | $x_0 = L/4(p \neq 0)$ | $B_0 = 0.2635(10)$ |
| First  | $x_0 = L/2(p \neq 0)$ | $B_0 = 0.2762(5)$ |
| Second | $x_0 = L/4(p \neq 0)$ | $B_0 = 0.1875(20)$ |
| Second | $x_0 = L/2(p \neq 0)$ | $B_0 = 0.1895(50)$ |
| First  | $x_0 = L/4(\theta \neq 0)$ | $B_0 = 0.12180(15)$ |
| First  | $x_0 = L/2(\theta \neq 0)$ | $B_0 = -0.23340(15)$ |
| Second | $x_0 = L/4(\theta \neq 0)$ | $B_0 = -0.06080(55)$ |
| Second | $x_0 = L/2(\theta \neq 0)$ | $B_0 = -0.5675(20)$ |
Where $\tilde{B}^{(1)}_O$ and $\tilde{B}^{(2)}_O$ are the coefficients computed in a standard lattice [9]. It’s clear that the parts of the finite constant proportional to $c_{sw}$ and to $c^2_{sw}$ are the same, within the errors, in the two schemes. So the difference between the total finite constants in the two different schemes will be independent on the fermion action used.

3. Singlet

We can reproduce the same calculation in the flavour singlet sector. In the singlet case the mixing under renormalization of the operator force to compute 4 correlation functions, involving gluonic and fermionic boundary state. The 2 SF correlation function involving fermionic boundary state are: $f_{qq}(x_0;p)$ which is essentially the same of $f_2$, and

$$f_{qq}(x_0;p) = -a^6 \sum_{y,z} e^{i\rho(y-z)} \langle \text{Tr}\{F_1 F_2\} \bar{\zeta}(y) \Gamma(z) \rangle$$ (9)

To study the other two correlation functions it is necessary to fix the gluonic boundary field. We use again the particular gauge group that leaves invariant the Schrödinger functional. We define the Big Teeth state as follows

$$D_i(y) = \left\{ \prod_{k=0}^{T_i-1} U_0(y + ka\hat{0}) \right\} U_i(y + \frac{T_i}{4} \hat{0}) \times \left( \prod_{k=0}^{T_i-1} U_0^{-1}(y + a\hat{0} + ka\hat{0}) \right)$$ (10)

We define then the gluonic boundary state as

$$G_{12}(y,z) = \text{Tr}[g_1(y)g_2(z)]$$ (11)

where

$$g_i(y) = \frac{1}{2i\alpha g_0} \{ D_i(y) - D_i(y)^{-1} \}$$ (12)

The 2 remaining correlation function are defined as:

$$f_{gg}(x_0) = a^6 \sum_{y,z} \left( \frac{1}{4} \bar{\psi}(x) \gamma_1 \hat{D}_2 \gamma_1 \hat{D}_1 \bar{\psi}(x) G_{12}(y,z) \right)$$ (13)

$$f_{gg}(x_0) = a^6 \sum_{y,z} \left( \frac{1}{4} \bar{\psi}(x) \gamma_1 \hat{D}_2 \gamma_1 \hat{D}_1 \bar{\psi}(x) G_{12}(y,z) \right)$$ (14)

It’s important to note that the SF scheme allow us to construct a gauge invariant gluonic source projected at 0 momentum with a tree-level well defined in the continuum limit. This is achieved by the two different polarization of the gluonic source. This improve the convergence to the continuum limit as it’s been remarked in the non-singlet calculation. The results for $f_{gg}$ and $f_{gg}$ are the following:

$$B^{(0)}_{gg} = 0.0261(3) \quad B^{(0)}_{qg} = 0.02919(1)$$ (15)

It’s currently under way the computation of $f_{gg}$ and of the gluonic source renormalization.

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