DIAGRAMMATIC METHOD FOR WIDE CORRELATORS IN GAUSSIAN ORTHOGONAL AND SYMPLECTIC RANDOM MATRIX ENSEMBLES

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Abstract

We calculate connected correlators in time dependent Gaussian orthogonal and symplectic random matrix ensembles by a diagrammatic method. We obtain averaged one-point Green’s functions in the leading order $O(N^0)$ and wide two-level and three-level correlators in the first nontrivial order by summing over twisted and untwisted planer diagrams.

PACS: 05.40.+j; 05.45+b

Keywords: Random matrix; Disordered system; Diagrammatic expansion; Universal correlator

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Introduction  Recently, Brézin and Zee pointed out the universality of wide connected correlators in random matrix theories [1]. They recognized its importance from the viewpoint of physics in disordered systems, though this universality in random matrix theories for two dimensional quantum gravity was already shown by Ambjørn, Jurkiewicz and Makeenko [2]. This universal feature of the wide connected correlator is shown by explicit calculations in quite extensive classes of random matrix ensembles in various ways [4, 5, 6], while the well-known short distance correlator can be calculated in limited narrow classes [1, 3]. We have to take into account these purely mathematical properties of random matrix theories themselves in order to recognize the essentially universal nature of level statistics of physical systems.

In this letter, we calculate the wide connected two level and three level correlators in gaussian orthogonal (GOE) and symplectic (GSE) ensembles with time dependence as discussed in some unitary ensembles [7, 9, 10, 11]. The diagrammatic method is known to be useful to calculate wide connected correlators in some unitary invariant ensembles [7, 12] and in GOE [8]. We are going to calculate the explicit form of the two level and three level correlator in GOE and GSE by this method. The results in GOE is consistent with that already obtained by Verbaarschot, Weidenmüller and Zirnbauer. We review their method for correlators in GOE and extend it to those in GSE. For these ensembles, we have to sum over both orientable and non-orientable diagrams. The results can be checked to compare those calculated by solving functional equations [4, 13] or by replica method [14], when time dependence is suppressed.

Gaussian orthogonal ensemble  To begin with, we explore the case of the GOE which is defined by the ensemble of real symmetric matrices obeying the probability distribution

\[ P(H) = \frac{1}{Z} \exp \left( -\frac{N}{2} \int_{-\infty}^{\infty} dt \, \text{Tr} \left[ \left( \frac{dH}{dt} \right)^2 + m^2 H^2 \right] \right), \]

where \( H \) is an \( N \times N \) matrix for GOE. The measure \( DH \) is explicitly written as

\[ DH = \prod_{k=1}^{N} dH_{kk} \prod_{i<j} dH_{ij}. \]

A dressed line is defined by

\[ G_{ij}(z) \equiv \left\langle \left( \frac{1}{z - H(0)} \right)_{ij} \right\rangle \equiv \int DH \, P(H) \left( \frac{1}{z - H(t)} \right)_{ij}, \]

which is independent of time \( t \) because of the transrational invariance of the distribution function \( P(H) \). The averaged one-point Green’s function \( G(z) \) is calculated from this one-point function \( G_{ij}(z) \)

\[ G(z) \equiv \frac{1}{N} \left\langle \text{Tr} \frac{1}{z - H(0)} \right\rangle = \frac{1}{N} \sum_{i=1}^{N} G_{ii}(z). \]
The one-point function $G_{ij}(z)$ is expanded in the power series of $H$

$$G_{ij}(z) = \frac{1}{z} \sum_{n=0}^{\infty} \frac{1}{z^n} \langle (H^n)_{ij} \rangle \tag{5}$$

We evaluate this function with the diagrammatic decomposition by using the free propagator of the real symmetric matrix

$$\langle H_{ij}(s)H_{kl}(t) \rangle = \frac{1}{2N} (\delta_{ij} \delta_{jk} + \delta_{ik} \delta_{jl}) K(s-t), \tag{6}$$

where $K(t) = \frac{1}{2m} e^{-m|t|}$. We can extend the time dependence of the probability distribution as the time dependent factor $K(t) = \sigma^2 e^{-u(t)}$ with an arbitrary suitable function $u(t)$. The corresponding diagram to this free propagator which has a twisted part is shown in fig 1. The one-point function can be calculated by summing over all oriented planer diagrams in eq(5). Any diagram with twisted propagators does not contribute to the leading order in the $1/N$ expansion, and therefore the same diagrams contribute to the leading order as in the unitary ensemble. When the series eq(5) is written in

$$G_{ij}(z) = \delta_{ij} \frac{\sigma^2}{2z^2} \sum_{n=0}^{\infty} g_n, \tag{7}$$

gn represents the number of the oriented planer diagrams with n free propagators. The number $g_n$ obeys the following recursion relation

$$g_{n+1} = g_n + \sum_{m=0}^{n-1} g_m g_{n-m}, \quad n > 1, \tag{8}$$

where $g_0 = 1$. The corresponding diagrammatic representation to the recursion relation (8) is depicted by fig 2. On the other hand, the vacuum polarization function

$$\Sigma(z) \equiv z - G(z)^{-1} = z \sum_{n=1}^{\infty} s_n \left( \frac{\sigma^2}{2z^2} \right)^n. \tag{9}$$

satisfies the equation

$$G(z)(z - \Sigma(z)) = 1. \tag{10}$$

This gives the relation among the expansion coefficients $g_n$ and $s_n$

$$\sum_{m=0}^{n} s_n g_{n-m} = 0, \quad n > 1, \tag{11}$$

where the coefficient $s_0 \equiv -1$. We can show the following relation from the eq(11) by the mathematical inductivity

$$s_n = g_{n+1} - \sum_{m=0}^{n-1} g_m g_{n-m}. \tag{12}$$
This equation indicates the vacuum polarization function consists of the one-particle irreducible diagrams. This equation (12) and the recursion relation (8) give

\[ s_n = g_n, \quad n > 1, \]

and therefore

\[ \Sigma(z) = \frac{\sigma^2}{2} G(z) = \frac{\sigma^2}{2(z - \Sigma(z))} \quad \text{(13)} \]

The vacuum polarization function \( \Sigma(z) \) is obtained by solving the quadratic equation

\[ \Sigma(z) = \frac{1}{2} \left( z - \sqrt{\frac{1}{2} - \Sigma(z)^2} \right) \quad \text{(13)} \]

Then we obtain the averaged one-point Green’s function

\[ G(z) = \frac{1}{\sigma^2} \left( z - \sqrt{z^2 - 2\sigma^2} \right). \quad \text{(14)} \]

The obtained function \( G(z) \) satisfies the boundary condition \( G(z) \to 1/z \) as \( z \to \infty \).

Now we turn to the connected two level correlator

\[ G(z_1, z_2; t) \equiv \left\langle \frac{1}{N} \text{Tr} \frac{1}{z_1 - H(t)} \frac{1}{N} \text{Tr} \frac{1}{z_2 - H(0)} \right\rangle_c. \quad \text{(15)} \]

We calculate this function in the following expression of the resolvent operator where we have to only sum over simpler diagrams following Brézin and Zee [12]

\[ G(z_1, z_2; t) = \frac{1}{N^2} \frac{\partial}{\partial z_1} \frac{\partial}{\partial z_2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \frac{1}{z_1^n z_2^m} \left\langle \text{Tr} H(t)^m \text{Tr} H(0)^n \right\rangle_c. \quad \text{(16)} \]

First, we ignore contractions within the same trace, in which we take into account only \( m = n \) parts

\[ \sum_{n=1}^{\infty} \frac{1}{n^2 (z_1 z_2)^n} \left\langle \text{Tr} H(t)^n \text{Tr} H(0)^n \right\rangle_c. \quad \text{(17)} \]

In the \( n \)th order, \( n \) untwisted planer diagrams and \( n \) twisted planer diagrams in fig 3 contribute to the leading order

\[ \frac{1}{N^2} \frac{\partial^2}{\partial z_1 \partial z_2} \sum_{n=1}^{\infty} \frac{2n}{n^2} \left( \frac{K(t)}{2z_1 z_2} \right)^n = -\frac{2}{N^2} \frac{\partial^2}{\partial z_1 \partial z_2} \log \left( 1 - \frac{K(t)}{2z_1 z_2} \right). \quad \text{(18)} \]

Next we include contractions within the same trace in \( \left\langle \text{Tr} H(t)^m \text{Tr} H(0)^n \right\rangle \). The following operator product expansion (OPE) formula

\[ \left\langle \text{Tr} \log \left( 1 - H(t)/z_1 \right) H_{ij}(0) \right\rangle = 2 \frac{K(t)}{2} \left( \frac{1}{z_1 - H(t)} \right)_{ij}, \quad \text{(19)} \]

tells us that the two level correlator is obtained by replacing the free one point Green’s function \( \delta_{ij}/z \) to the dressed one \( G_{ij}(z) \) in eq.(18)

\[ G(z_1, z_2; t) = -\frac{2}{N^2} \frac{\partial^2}{\partial z_1 \partial z_2} \log \left( 1 - \frac{K(t)}{2} G(z_1) G(z_2) \right). \quad \text{(20)} \]

This result with respect to the connected part agrees with that obtained by solving functional equations [4,13] and by a replica method [14].
Gaussian symplectic ensemble  The Gaussian symplectic ensemble (GSE) is the ensemble of quaternion real Hermitian matrices. The component $H_{ij}^{\alpha \beta}$ of $2N \times 2N$ quotation real Hermitian matrix $H$ is written as
\begin{equation}
H_{ij}^{\alpha \beta} = H_{ij}^{(0)} \delta^{\alpha \beta} + i \sum_{a=1}^{3} H_{ij}^{(a)} \sigma_k^{\alpha \beta}, \tag{21}
\end{equation}
where $\sigma_k^{\alpha \beta}$ ($k = 1, 2, 3; \alpha, \beta = 1, 2$) is a component of the $2 \times 2$ Pauli matrix, $H_{ij}^{(0)}$ is a real symmetric matrix and $H_{ij}^{(k)}$ ($i, j = 1, \cdots, N$) is real antisymmetric matrix in $H_{ij}^{\alpha \beta}$. The trace of $H^2$ in eq.(23) reads
\begin{equation}
\text{Tr} H^2 = \sum_{i,j} H_{ij}^{\alpha \beta} H_{ji}^{\beta \alpha} = 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{a=0}^{3} (H_{ij}^{(a)})^2. \tag{22}
\end{equation}

Here we define the distribution function of GSE
\begin{equation}
P(H) = \frac{1}{Z} \exp \left( - \frac{N}{4} \int_{-\infty}^{\infty} dt \text{Tr} \left[ \left( \frac{dH}{dt} \right)^2 + m^2 H^2 \right] \right), \tag{23}
\end{equation}
with the measure
\begin{equation}
DH = \prod_{i \leq j} dH_{ij}^{(0)} \prod_{k=1}^{3} \prod_{i<j} dH_{ij}^{(k)}. \tag{24}
\end{equation}
The free propagator of $H$ is
\begin{equation}
\langle H_{ij}^{\alpha \beta}(s) H_{kl}^{\gamma \epsilon}(t) \rangle = \frac{1}{N} K(s-t) \left( \delta_{il} \delta_{jk} \delta^{\alpha \gamma} \delta^{\beta \epsilon} + \delta_{ik} \delta_{jl} A^{\alpha \beta, \gamma \epsilon} \right), \tag{25}
\end{equation}
where the tensor $A^{\alpha \beta, \gamma \epsilon}$ is defined by
\begin{equation}
A^{\alpha \beta, \gamma \epsilon} \equiv \delta^{\alpha \beta} \delta^{\gamma \epsilon} - \delta^{\alpha \gamma} \delta^{\beta \epsilon}. \tag{26}
\end{equation}
The first and second terms in the free propagator are represented by an untwisted and twisted diagrams shown in fig 4 (a) and (b), respectively. The calculation of the dressed line is done by the expansion
\begin{equation}
G_{ij}^{\alpha \beta}(z) = \langle \left( \frac{1}{z - H(0)} \right)^{\alpha \beta}_{ij} \rangle = \frac{1}{z} \sum_{n=0}^{\infty} \frac{1}{z^n} \langle (H^n)_{ij}^{\alpha \beta} \rangle \tag{27}
\end{equation}
The one-point Green’s function is defined as its trace
\begin{equation}
G(z) = \frac{1}{2N} \langle \text{Tr} \frac{1}{z - H(0)} \rangle = \frac{1}{2N} \sum_{i} G_{ii}^{\alpha \alpha}(z). \tag{28}
\end{equation}
As in the GOE case, only untwisted planer diagrams contribute the one-point Green’s function in the large $N$ leading order
\begin{equation}
G_{ij}^{\alpha \beta}(z) = \frac{\delta_{ij} \delta^{\alpha \beta}}{z} \sum_{n=0}^{\infty} \left( \frac{2g^2}{z^2} \right)^n g_n. \tag{29}
\end{equation}
Since the number $g_n$ of the oriented planer diagrams with $n$ free propagators is identical to that in the GOE case, the vacuum polarization function $\Sigma(z)$ is calculated by summing over one-particle irreducible diagrams.

$$\Sigma(z) \equiv z - G(z)^{-1} = 2\sigma^2 G(z) = \frac{2\sigma^2}{(z - \Sigma(z))},$$  \tag{30}$$

The vacuum polarization function $\Sigma(z)$ is obtained by solving the quadratic equation

$$\Sigma(z) = \frac{1}{2} (z - \sqrt{z^2 - 8\sigma^2}).$$

Then we obtain the averaged one-point Green’s function

$$G(z) = \frac{1}{4\sigma^2} \left(z - \sqrt{z^2 - 8\sigma^2}\right).$$  \tag{31}$$

Note that $G(z)$ tends to $1/z$ as $z \to \infty$.

Next we compute the two-point function $G(z_1, z_2)$ defined by

$$G(z_1, z_2; t) \equiv \left\langle \frac{1}{2N} \text{Tr} \frac{1}{z_1 - H(t)} \frac{1}{2N} \text{Tr} \frac{1}{z_2 - H(0)} \right\rangle_c = \frac{1}{4N^2} \frac{\partial}{\partial z_1} \frac{\partial}{\partial z_2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \frac{1}{z_1^{m/2} z_2^{n/2}} \langle \text{Tr} H(t)^m \text{Tr} H(0)^n \rangle_c. \tag{32}$$

Ignoring contractions within the same trace in the following expression

$$\sum_{n=1}^{\infty}$$  \tag{33}$$

It is the same as in the GOE case that in the $n$th order $n$ untwisted planer diagrams and $n$ twisted planer diagrams in fig 3(a)(b) contribute to the leading order. A part of the twisted diagram shown in fig 3(c) is calculated by the following contraction formula for the tensor $A^{\alpha\beta;\gamma\epsilon}$ defined by eq(26)

$$(A^n)^{\alpha_1\alpha_{n+1},\beta_1\beta_{n+1}} \equiv \sum_{\alpha_2, \ldots, \alpha_n, \beta_2, \ldots, \beta_n} A^{\alpha_1\alpha_2,\beta_1\beta_2} A^{\alpha_2\alpha_3,\beta_2\beta_3} \ldots A^{\alpha_n\alpha_{n+1},\beta_n\beta_{n+1}} = 2^{n-1} A^{\alpha_1\alpha_{n+1},\beta_1\beta_{n+1}}. \tag{34}$$

This gives a trace formula

$$\text{Tr} A^n \equiv \sum_{\alpha,\beta} (A^n)^{\alpha,\beta} = 2^n$$  \tag{35}$$

which enables us to obtain the two level correlator in this truncation

$$\frac{1}{4N^2} \frac{\partial^2}{\partial z_1 \partial z_2} \sum_{n=1}^{\infty} \frac{n}{n^2} \left( \frac{K(t)}{z_1 z_2} \right)^n (2^n + \text{Tr} A^n) = -\frac{1}{2N^2} \frac{\partial^2}{\partial z_1 \partial z_2} \log \left( 1 - \frac{2K(t)}{z_1 z_2} \right). \tag{36}$$
From this expression, we obtain the exact two level correlator by replacing the free line \( \delta_{ij} \delta^{\alpha\beta} / z \) to the dressed one \( G_{ij}^{\alpha\beta}(z) \) in eq(36) to incorporate contractions within the same trace

\[
G(z_1, z_2; t) = -\frac{1}{2N^2} \frac{\partial^2}{\partial z_1 \partial z_2} \log (1 - 2K(t)G(z_1)G(z_2)).
\]  

This result with respect to the connected part agrees with that obtained by solving functional equations [1, 13] and by replica method [14].

**Three level correlator** We can also calculate the connected three level correlator both in GOE and GSE. That in the GOE case is defined by

\[
G(z_1, z_2, z_3; t_1, t_2, t_3) \equiv \left\langle \frac{1}{N} \text{Tr} \left( \frac{1}{z_1 - H(t_1)} \right) \text{Tr} \left( \frac{1}{z_2 - H(t_2)} \right) \text{Tr} \left( \frac{1}{z_3 - H(t_3)} \right) \right\rangle_c
\]

\[
= -\frac{1}{N^3} \frac{\partial^3}{\partial z_1 \partial z_2 \partial z_3} \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{n_3=1}^{\infty} \frac{1}{n_1 n_2 n_3} \left\langle \text{Tr} \left( \frac{H(t_1)}{z_1} \right)^{n_1} \text{Tr} \left( \frac{H(t_2)}{z_2} \right)^{n_2} \text{Tr} \left( \frac{H(t_3)}{z_3} \right)^{n_3} \right\rangle_c.
\]  

(38)

For GSE, we have to replace \( N \rightarrow 2N \) in the right hand side of the definition (38). First we discuss the GOE case. Ignoring the contraction within the same trace, a diagrams with \( m_1, m_2 \) and \( m_3 \) propagators is depicted in fig 5 and fig 6. The number \( n_a \) of operators in eq(38) can be expressed in terms of the number \( m_a \) of propagators in a diagram in fig 5 or fig 6

\[
n_1 = m_2 + m_3, \quad n_2 = m_3 + m_1, \quad n_3 = m_1 + m_2.
\]  

(39)

The summation can be calculated in terms of the number \( m_1, m_2 \) and \( m_3 \) of the propagators. In the leading order of the \( 1/N \) expansion, there are \( 4n_1n_2n_3 \) diagrams with \( m_1, m_2 \) and \( m_3 \) propagators which consist of \( n_1 \) untwisted diagrams in fig 5 and \( 3n_1n_2n_3 \) twisted diagrams in fig 6. Therefore the three level correlator is calculated as

\[
\sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{n_3=1}^{\infty} \frac{1}{n_1 n_2 n_3} \left\langle \text{Tr} \left( \frac{H(t_1)}{z_1} \right)^{n_1} \text{Tr} \left( \frac{H(t_2)}{z_2} \right)^{n_2} \text{Tr} \left( \frac{H(t_3)}{z_3} \right)^{n_3} \right\rangle_c
\]

\[
\sim \frac{4}{N} \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} \sum_{m_3=1}^{\infty} \left( \frac{K(t_2 - t_3)}{2z_2 z_3} \right)^{m_1} \left( \frac{K(t_3 - t_1)}{2z_3 z_1} \right)^{m_2} \left( \frac{K(t_1 - t_2)}{2z_1 z_2} \right)^{m_3}
\]

\[
+ \frac{4}{N} \left( \sum_{m_2=1}^{\infty} \sum_{m_3=1}^{\infty} \left( \frac{K(t_3 - t_1)}{2z_3 z_1} \right)^{m_2} \left( \frac{K(t_1 - t_2)}{2z_1 z_2} \right)^{m_3} + 2 \text{ cyclic permutations} \right).
\]  

(40)

To obtain the exact 3 level correlator, we have to take into account the vertex correction as well as the self energy correction to the one point function. The vertex corrections can be done with the untwisted propagators as depicted in fig 7. Then the three level correlator becomes

\[
G(z_1, z_2, z_3; t_1, t_2, t_3) = -\frac{4}{N^4} \frac{\partial^3}{\partial z_1 \partial z_2 \partial z_3} F(z_1, z_2, z_3; t_1, t_2, t_3),
\]  

(41)
where
\[
F(z_1, z_2, z_3; t_1, t_2, t_3) \equiv \frac{X_{23} X_{31} X_{12}}{1 - X_{23} 1 - X_{31} 1 - X_{12}} + \frac{X_{31} X_{12} 1}{1 - X_{31} 1 - X_{12} 1 - X_{11}} + \frac{X_{12} X_{23} 1}{1 - X_{12} 1 - X_{23} 1 - X_{22}} + \frac{X_{23} X_{31} 1}{1 - X_{23} 1 - X_{31} 1 - X_{33}},
\]
with \(X_{ab} \equiv K(t_a - t_b)/(2z_a z_b)\) in this approximation. If we take into account the self-energy correction, the free line \(\delta_{ij}/z_a\) should be replaced by the dressed line \(G_{ij}(z_a)\). Therefore we have to use
\[
X_{ab} = \frac{1}{2}K(t_a - t_b)G(z_a)G(z_b),
\]
where \(G(z)\) is given in eq(14). The exact three level correlator in GOE is given by eq(46) with the function (42). This result agrees with that obtained by Verbaarschot, Weidenmüller and Zirnbauer [8].

Next we calculate the three level correlator in GSE. All the contributing diagrams in this case are identical to those in the GOE case. The contribution from the \(n_1 n_2 n_3\) untwisted diagrams becomes
\[
\frac{1}{N} \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} \sum_{m_3=1}^{\infty} \frac{1}{2} \left( \frac{2K(t_2 - t_3)}{z_2 z_3} \right)^{m_1} \left( \frac{2K(t_3 - t_1)}{z_3 z_1} \right)^{m_2} \left( \frac{2K(t_1 - t_2)}{z_1 z_2} \right)^{m_3} + 2 \text{ cyclic permutations}
\]
\[
+ \frac{1}{N} \left( \sum_{m_2=1}^{\infty} \sum_{m_3=1}^{\infty} \frac{1}{2} \left( \frac{2K(t_3 - t_1)}{z_3 z_1} \right)^{m_2} \left( \frac{2K(t_1 - t_2)}{z_1 z_2} \right)^{m_3} \right) \quad (44)
\]
Note that the number of loops with respect to the spin index in an untwisted diagram with \(m_1, m_2\) and \(m_3\) propagators shown in fig 5 (\(\alpha = 1, 2\)) is always \(m_1 + m_2 + m_3 - 1\). Contribution from remaining \(3n_1 n_2 n_3\) twisted diagrams in fig 6 is
\[
\frac{3}{N} \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} \sum_{m_3=1}^{\infty} \frac{1}{2} \left( \frac{2K(t_2 - t_3)}{z_2 z_3} \right)^{m_1} \left( \frac{K(t_3 - t_1)}{z_3 z_1} \right)^{m_2} \left( A_{m_2}^{\alpha \beta, \gamma \epsilon} \left( \frac{K(t_1 - t_2)}{z_1 z_2} \right)^{m_3} \right)
\]
\[
+ \frac{1}{N} \left( \sum_{m_2=1}^{\infty} \sum_{m_3=1}^{\infty} \frac{1}{2} \left( \frac{K(t_3 - t_1)}{z_3 z_1} \right)^{m_2} \left( A_{m_2}^{\alpha \beta, \gamma \epsilon} \left( \frac{K(t_1 - t_2)}{z_1 z_2} \right)^{m_3} \right) \quad (45)
\]
The contraction formula (34) for the tensor \(A^{\alpha \beta, \gamma \epsilon}\) enables us to calculate these contributions, and then the three level correlator in GSE is obtained as
\[
G(z_1, z_2, z_3; t_1, t_2, t_3) = -\frac{1}{4N^4 \partial_{z_1} \partial_{z_2} \partial_{z_3}} F(z_1, z_2, z_3; t_1, t_2, t_3).
\]

The function \(F(z_1, z_2, z_3; t_1, t_2, t_3)\) is defined by eq(12) with the function \(X_{ab}\) for GSE
\[
X_{ab} \equiv 2K(t_a - t_b)G(z_a)G(z_b),
\]
with the one point Green function defined by eq(31). Three level correlators in GOE and GSE obtained by the diagrammatic method are identical those calculated by the replica method and the functional method [13].

We have calculated the averaged one point Green’s functions and the wide connected two level and three level correlators in Gaussian orthogonal and symplectic random matrix ensembles with time evolution by a diagrammatic method. All results are consistent with those obtained in all other methods [8, 4, 13, 14]

We would like to thank S. Higuchi and H. Mukaida for helpful comments.

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\[
\begin{array}{c}
\frac{i}{j} \frac{l}{k} \equiv \frac{1}{2N} \delta_{il} \delta_{jk} \\
(a)
\end{array}
\]

\[
\begin{array}{c}
\frac{i}{j} \frac{l}{k} \equiv \frac{1}{2N} \delta_{il} \delta_{jk} \\
(b)
\end{array}
\]

Figure 1: The diagram of free propagator in the GOE case. (a) an untwisted free propagator. (b) a twisted one.

\[
\begin{array}{c}
\frac{\delta_{ij}}{z^{2n+1}} g_n = n \\
\end{array}
\]

\[
\begin{array}{c}
(n+1) = n + \sum_m n-m \\
\end{array}
\]

Figure 2: The diagram of \( g_n \).
Figure 3: (a) An untwisted planer diagram in the $n$th order with the definition of $n$ untwisted contraction parts. (b) A twisted one with definition of $n$ twisted contraction parts. In the case of the GOE, the spinor indices $(\alpha_1, \beta_1, \ldots)$ should be neglected.
\[
\begin{align*}
\frac{i\alpha}{j\beta} \frac{l\epsilon}{k\gamma} & \equiv \frac{1}{N} \delta_{i1} \delta_{jk} \delta^{\alpha\beta} \delta^{\gamma\epsilon} \\
\frac{i\alpha}{j\beta} \frac{l\epsilon}{k\gamma} & \equiv \frac{1}{N} (\alpha_{ij} \delta_{jk} \delta^{\alpha\beta} \delta^{\gamma\epsilon})
\end{align*}
\]

Figure 4: The diagram of free propagator in the GSE case. (a) an untwisted free propagator. (b) a twisted one.

Figure 5: An untwisted diagram with \( m_1, m_2 \) and \( m_3 \) propagators. (a) \( m_1, m_2 \) and \( m_3 \neq 0 \) case. (b) \( m_1 = 0, m_2 \) and \( m_3 \neq 0 \) case.
Figure 6: A twisted diagram with $m_1, m_2$ and $m_3$ propagators. (a) $m_1, m_2$ and $m_3 \neq 0$ case. (b), (c) and (d) $m_1 = 0$, $m_2$ and $m_3 \neq 0$ case.
Figure 7: Diagrams which require vertex correction. \((n \geq 1)\)