Extracting Equivalent Circuit Parameters of a CRLH Transmission Line

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Abstract: A method to determine the inductive and capacitive components of the series impedance and shunt admittance for configuring the equivalent circuit of a composite right/left-handed (CRLH) transmission line is proposed. This method uses Z matrices with multiple frequencies obtained via full-wave analysis of an actual CRLH transmission line unit cell structure. Results are presented for the application of the proposed approach to a CRLH microstrip line.

Keywords: equivalent circuit, circuit parameter, Z matrix, CRLH transmission line

Classification: Antennas and propagation

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1 Introduction

A composite right/left-handed (CRLH) transmission line has received much attention lately because it can realize a negative phase constant, which cannot be achieved by natural materials[1][2]. Several works have been reported on utilization of the CRLH transmission line in a broad range of applications, including wide-angle steering antenna[3], cloaking[4], and small antennas[5].

A CRLH transmission line is composed of a periodically arranged array of unit cells. The unit cell can be equivalently expressed by a two-port network composed of series and parallel R, L, and C elements. The transmission characteristics of the unit cell can be designed by evaluating the values of the R, L, and C elements of the equivalent two-port network that satisfy a target performance. The R, L, and C elements are realized by line or gap structures in an actual CRLH transmission line.

The design of the actual CRLH transmission line structure is often achieved through full-wave analysis using an electromagnetic simulator. The structure of the CRLH transmission line is first configured on the electromagnetic simulator, and the transmission characteristics are calculated. The structure is designed in a trial and error approach by changing the line and gap structures until the required performance is obtained.

However, design of the transmission line using the equivalent circuit is easier than the trial and error approach using an electromagnetic simulator. Therefore, the circuit parameters satisfying the target transmission characteristics are first determined by the equivalent circuit; then, the structure of the actual CRLH transmission line satisfying the derived circuit parameters is designed. This method is efficient compared to designing the structure using only the trial and error approach.

When using the equivalent circuit to design the CRLH transmission line, it is required that the actual structure corresponding to the designed circuit parameters is realized. To achieve this, the relationship between the actual structure and the parameters of the equivalent circuit must be known. The relationship between the actual structure and the series impedance/shunt admittance of the equivalent circuit can be determined using the Z parameters of the unit cell two-port network obtained from full-wave analysis. However, obtaining the inductive and capacitive components separately for both the series impedance and shunt admittance is difficult because only a combination of the inductive and capacitive reactances can be obtained from the Z parameters.

In this report, a method to separately determine the inductive and capacitive components of the series impedance and shunt admittance in the equivalent circuit using Z matrices of multiple frequencies obtained via full-wave analysis of an actual CRLH transmission line unit cell structure is presented.
The effectiveness of the proposed method is evaluated by application to a CRLH microstrip line comprising an interdigital capacitor and a shunt stub line.

2 Extraction of circuit parameters

The equivalent two-port network of a CRLH transmission line is generally expressed using an L-type circuit as shown in Fig. 1(a). However, the Z parameters, \( Z_{12}, Z_{21}, Z_{22} \), of the L-type circuit are identical. Thus, the asymmetric T-type circuit shown in Fig. 1(b) is considered first for the equivalent two-port network, then the circuit parameters will be transformed to the L-type circuit.

\[ Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \]  \hspace{1cm} (1)

\( Z_1, Z_2, \) and \( Z_3 \) in the equivalent circuit are expressed by

\[ Z_1 = R_1 + j(\omega L_{se1} - \frac{1}{\omega C_{se1}}) \]  \hspace{1cm} (2)

\[ Z_2 = R_2 + j(\omega L_{se2} - \frac{1}{\omega C_{se2}}) \]  \hspace{1cm} (3)

\[ Z_3 = \frac{1}{Y_3} \]  \hspace{1cm} (4)

\[ Y_3 = G + j(\omega C_{sh} - \frac{1}{\omega L_{sh}}) \]  \hspace{1cm} (5)

Assuming that the Z parameters \( Z_{11}, Z_{12}, Z_{21}, Z_{22} \) are known based on electromagnetic simulation or measurement, the values of the resistors, inductors,
and capacitors can be determined. Since $Z_3 = Z_{12} = Z_{21}$, $R_1, R_2,$ and $G$ can be determined using the following equations.

\[
R_1 = \text{Re}(Z_1) = \text{Re}(Z_{11} - Z_{12}) \tag{6}
\]

\[
R_2 = \text{Re}(Z_2) = \text{Re}(Z_{22} - Z_{12}) \tag{7}
\]

\[
G = \text{Re}(Y_3) = \text{Re}\left(\frac{1}{Z_{12}}\right) \tag{8}
\]

$L_{se1}, L_{se2}, C_{se1}, C_{se2}, L_{sh},$ and $C_{sh}$ can be determined from the imaginary part of $Z_1, Z_2$ and $Y_3$. However, these values cannot be uniquely specified as the number of unknown quantities exceeds the number of equations. Then, we propose to use the $Z$ matrices of adjacent plural frequencies by assuming that the circuit parameters are stable in adjacent frequencies. In this case, two angular frequencies $\omega_1, \omega_2$ are considered. Therefore, we have:

\[
\text{Im}(Z_1(\omega_1)) = \text{Im}(Z_{11}(\omega_1) - Z_{12}(\omega_1)) = \omega_1 L_{se1} - \frac{1}{\omega_1 C_{se1}} \tag{9}
\]

\[
\text{Im}(Z_{11}(\omega_2) - Z_{12}(\omega_2)) = \omega_2 L_{se1} - \frac{1}{\omega_2 C_{se1}} \tag{10}
\]

\[
\text{Im}(Z_2(\omega_1)) = \text{Im}(Z_{22}(\omega_1) - Z_{12}(\omega_1)) = \omega_1 L_{se2} - \frac{1}{\omega_1 C_{se2}} \tag{11}
\]

\[
\text{Im}(Z_{22}(\omega_2) - Z_{12}(\omega_2)) = \omega_2 L_{se2} - \frac{1}{\omega_2 C_{se2}} \tag{12}
\]

\[
\text{Im}(Y_3(\omega_1)) = \text{Im}(\frac{1}{Z_{12}(\omega_1)}) = \omega_1 C_{sh} - \frac{1}{\omega_1 L_{sh}} \tag{13}
\]

\[
\text{Im}(Y_3(\omega_2)) = \text{Im}(\frac{1}{Z_{12}(\omega_2)}) = \omega_2 C_{sh} - \frac{1}{\omega_2 L_{sh}} \tag{14}
\]

The matrix expression of the above equation is given by

\[
A = \begin{bmatrix}
\text{Im}(Z_{ii}(\omega_2)) \\
\text{Im}(Z_{ii}(\omega_1))
\end{bmatrix}
\tag{15}
\]

\[
B = \begin{bmatrix}
\omega_1 - \frac{1}{\omega_1} \\
\omega_2 - \frac{1}{\omega_2}
\end{bmatrix}
\tag{16}
\]

\[
C = \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix} = \begin{bmatrix}
L_{se1} \\
1
\end{bmatrix}
\tag{17}
\]

\[
A = BC
\tag{18}
\]

where $c_1 = L_{se1}, 1/c_2 = C_{se1}$. $i = 1$ or 2. Then $L_{se1}$ and $C_{se1}$ can be obtained as follows:

\[
C = B^{-1}A
\tag{19}
$L_{sh}, C_{sh}$ can be also obtained from Eq.(19) by determining $A$ and $C$ as follows:

$$A = \begin{bmatrix} \text{Im}(\frac{1}{Z_{12}(\omega_1)}) \\ \text{Im}(\frac{1}{Z_{12}(\omega_2)}) \end{bmatrix}$$ (20)

$$C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} C_{sh} \\ \frac{1}{L_{sh}} \end{bmatrix}$$ (21)

The series inductance $L_{se,L}$ and series capacitance $C_{se,L}$ of the L-type equivalent circuit can be obtained as follows:

$$L_{se,L} = L_{se1} + L_{se2}$$ (22)

$$C_{se,L} = \frac{C_{se1}C_{se2}}{C_{se1} + C_{se2}}$$ (23)

3 Examples of the extraction of circuit parameters of an equivalent circuit

A CRLH transmission line that includes a series capacitor and a shunt inductor in the microstrip line (MSL) is considered as an example. The structure is shown in Fig. 2. The series capacitor is configured as an interdigital capacitor located at the center of the stripline. The shunt inductor is configured by adding a shunt stub. The structural parameters are shown in Fig. 2. Two cases are investigated in this instance. In the first case, the stub length $l_s$ is changed (Case 1) to 15mm, 22.5mm, and 30mm. The length of the interdigital capacitor $C_s$ is 6mm. In the second case, the length $C_s$ is changed (Case 2) to 4mm, 6mm, and 8mm. The stub length $l_s$ is 22.5mm in Case 2. The

![Fig. 2. CRLH transmission line structure.](image)

L-type circuit parameters for Case 1 and Case 2 that were calculated from the asymmetric T-type equivalent circuits are shown in Fig.3(a) and (b), respectively. As shown in these figures, $L_{sh,L}$ increases with an increase of $l_s$, and $C_{se}$ increases with an increase of $C_s$ when regions of rapid increase are excluded. The other circuit parameters are mostly stable with a variation of $l_s$ or $C_s$. The rapid increase of the series impedance components occurred in
the bandgap region as described later. Figure 3(c) shows the series resonant frequency $f_{se}$ and shunt resonant frequency $f_{sh}$ for Case 1 as determined by the following equations:

$$f_{se} = \frac{1}{2\pi \sqrt{L_{se,L}C_{se,L}}}$$  \hspace{1cm} (24) \\
$$f_{sh} = \frac{1}{2\pi \sqrt{L_{sh,L}C_{sh,L}}}$$  \hspace{1cm} (25)

Figure 3(d) shows the phase dispersion characteristics for Case 1. It can be determined that the shunt resonant frequencies are in agreement with the lower limit of the bandgap of the phase dispersion characteristics shown in Fig. 3(d). When the series resonant frequencies are varied in the bandgap region it is observed that they become stable when the frequency is increased. The series resonant frequencies in the stable regions are in agreement with the upper limit of the bandgap. Since the resonant frequencies allow the lower and upper limit of the bandgap to be determined [7], the validity of the derived circuit parameters is confirmed.

4 Conclusion

A method is proposed to extract the inductive and capacitive components of both the series impedance and shunt admittance in the equivalent circuit of a transmission line corresponding to a unit cell of an actual CRLH transmission line structure. This method uses Z matrices of multiple frequencies to specify the inductance and capacitance values. Based on the results obtained by applying the proposed method to the unit cell of a CRLH microstrip line, it is confirmed that changing the shunt stub length $l_s$ and the length of the interdigital capacitor $C_s$ correspond to changing the shunt inductance and series capacitance, respectively. Moreover, the series and shunt resonant frequencies calculated from the obtained inductance and capacitance values agree with the lower and upper limits of the bandgap for the phase dispersion characteristics. Thus, it can be considered that the inductive and capacitive components thus obtained are appropriate, and the validity of the proposed approach is verified.

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(a) L-type circuit parameters (Case 1).

(b) L-type circuit parameters (Case 2).
(c) Series and shunt resonant frequencies (Case 1).

(d) Phase dispersion characteristics (Case 1).

Fig. 3. Equivalent circuit parameters and resonant frequencies of the CRLH transmission line.