Bimaximal Neutrino Mixing From a Local $SU(2)$ Horizontal Symmetry

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(July, 2002)

Freedom from global anomalies in the presence of a local $SU(2)_H$ horizontal symmetry under which right handed charged leptons transform nontrivially requires that there be at least two right handed neutrinos with masses of order of the horizontal symmetry breaking scale. If a third right handed neutrino is introduced to satisfy quark lepton symmetry, it is unprotected by the horizontal symmetry, becomes superheavy with a Planck scale mass and decouples from lower energy physics. The resulting seesaw mechanism with two right handed neutrinos in combination with the horizontal symmetry leads naturally to a near bimaximal pattern for the neutrino mixing with an inverted mass hierarchy and is compatible with all data. It predicts a correlation between the solar mixing angle and $\theta_{13}$, that is testable in the proposed long baseline experiments.

UMD-PP-02-60

1. INTRODUCTION

It is well known that the standard model predicts massless neutrinos due to the presence of an exact global $B-L$ symmetry. Since it is generally believed that global symmetries are broken by nonperturbative Planck scale effects, nonvanishing neutrino masses \(^{[1]}\) of order \(m_\nu \approx \frac{\phi^2}{M_{Pl}} \approx 10^{-5} \text{ eV}\) can arise within the standard model framework from gravitational effects. Neutrino masses of such magnitudes are however much too small to account for observed neutrino oscillations. In fact atmospheric neutrino data requires that there must at least be one neutrino with considerably larger mass (of order 0.05 eV), implying that the scale of new physics responsible for breaking the $B-L$ symmetry of the standard model must be much lower scale than the Planck scale \((\leq 10^{-3} M_{Pl})\). In the context of the seesaw model for understanding the small neutrino masses \(^{[2]}\) where heavy right handed neutrinos are responsible for breaking $B-L$ symmetry and hence for small neutrino masses, atmospheric neutrino data implies that their typical masses should be around \(\leq 10^{-4} M_{Pl}\). Thus the right handed neutrino masses must be protected by additional symmetries. In the SO(10) or left-right models which provide the conventional venues for implementing the seesaw mechanism, the relevant symmetry is the local $B-L$ symmetry. Requiring that an extension of the standard model respect the $B-L$ symmetry implies that there must three right handed neutrinos \((\nu_e^R, \nu_\mu^R, \nu_\tau^R)\) to cancel the gauge anomalies. All the \(\nu_R^R\)'s are expected to have masses much lower than the Planck scale and of order of the $B-L$ symmetry breaking scale.

In this note we explore an alternative approach to keeping the right handed neutrinos lighter than the Planck scale. Instead of the local $B-L$ symmetry (or perhaps in addition to it) we propose that there be a local $SU(2)_H$ horizontal symmetry at the seesaw scale under which the right handed charged leptons transform nontrivially. There are then two possibilities which are of interest for our considerations: (i) $SU(2)_H$ acts on both right handed quarks and leptons or (ii) it acts only on leptons from considerations of gauge anomalies. Freedom from global Witten anomaly on the other hand requires that in both cases there must be two right handed neutrinos that trasform as a doublet of $SU(2)_H$. The local $SU(2)_H$ symmetry then implies that the masses of those two right handed neutrinos are protected and must be at the scale of $SU(2)_H$ breaking. If there is a third right handed neutrino for reasons of quark lepton symmetry, then it will acquire a mass of order of the Planck or string scale and decouple from neutrino physics at lower energies. This therefore provides a physically distinct way of implementing the seesaw mechanism. It leads to a neutrino mass pattern that is inverted. Furthermore, it has the interesting property that it leads to the near bimaximal mixing pattern in a more natural manner than the $B-L$ seesaw approach. The $SU(2)_H$ symmetry plays a crucial role in generating the near bimaximal pattern. This mixing pattern seems to be favored by the detailed analyses of present solar and atmospheric neutrino data \(^{[3]}\).

The fact that dominance of two right handed neutrinos in a seesaw model under certain circumstances can lead to bimaximal mixing needs was noted in a different context in \(^{[4]}\). $SU(2)_H$ horizontal symmetry not only provides a rational for this hypothesis but also naturally leads to a near bimaximal mixing. We will also see that it provides a natural understanding of why $\Delta m^2_{23}/\Delta m^2_{12} \ll 1$.

There are two possible scenarios that arise in these models depending on how charged leptons are aligned. We discuss only one of them in detail in this paper. This model has the following predictions:

(i) The sign of the $\Delta m^2_{23}$ is opposite to the case of normal hierarchy for neutrinos.

(ii) There is a correlation between the value of $U_{e3}$ and the solar mixing angle $\sin^2 2\theta_\odot$ as in a class of softly broken $L_e - L_\mu - L_\tau$ models \(^{[5]}\) i.e. a lower $\sin^2 2\theta_\odot$ requires
a higher $U_{e3}$. This correlation is testable in future long baseline experiments such as NUMI Offaxis proposal or proposed JHF facilities.

(iii) The effective neutrino mass $m_{\beta\beta}$ that can be measured in neutrinoless double beta decay experiments is related to the atmospheric mass difference squared $\Delta m^2_{\text{atm}}$ and $U_{e3}$ i.e. $m_{\beta\beta} \simeq 2U_{e3} \cdot \sqrt{2\Delta m^2_{\text{atm}}}$. While in this paper, we have only considered the $SU(2)_H$ symmetry to act on the leptonic sector, aesthetic reasons suggest that the $SU(2)_H$ could act simultaneously on quarks and leptons. We note that in a model of this type, our conclusion regarding only two RH neutrinos at the $SU(2)_H$ scale still remains valid as long as the right handed charged leptons transform as a doublet. The details implications of this model are currently under investigation.

II. THE $SU(2)_H$ MODEL

Horizontal symmetries have often been invoked to understand family replication and flavor structure of the quarks and leptons in extensions of the standard models. They can either be $U(1)_H$, $SU(2)_H$ or $SU(3)_c$ type. We will consider $SU(2)_H$ type models, which seem to have interesting consequences for neutrino mixings.

Gauge anomaly constraints can be satisfied in four distinct ways for an $SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_H \equiv G_{\text{STD}} \times SU(2)_H$ model if we consider only the known fermions: (i) only left handed fermions of the standard model transform as $SU(2)_H$ doublets; (ii) only quarks (both left and right handed) transform as the $SU(2)_H$ doublets; (iii) only right handed fermions (both quarks and leptons) transform as $SU(2)_H$ doublets and (iv) only leptons transform as doublets. In cases (iii) and (iv), freedom from global $SU(2)_H$ Witten anomaly implies that there must be at least two right handed neutrinos (we will call them $(\nu_{e,R}, \nu_{\mu,R})$, transforming as a doublet of $SU(2)_H$. We show below that this model leads naturally to the near bimaximal pattern for neutrino mixings.

We will focus in this paper on the possibility that only leptons transform under $SU(2)_H$ and quarks are singlets. This is the minimal model with an $SU(2)_H$. Our considerations can be extended to the other case with small modifications. We give below in Table I the assignment of fermions and Higgs bosons under the gauge group $G_{\text{STD}} \times SU(2)_H$

| Particles | $G_{\text{STD}} \times SU(2)_H$ | Quantum numbers |
|-----------|---------------------------------|-----------------|
| $\Psi \equiv (\psi_e, \psi_\mu)$ | (1,2,-1,2) | |
| $\psi_e$ | (1,2,-1,1) | |
| $E_R \equiv (e_R, \mu_R)$ | (1,1,-2,2) | |
| $\tau_R$ | (1,1,-2,1) | |
| $N_R \equiv (\nu_{e,R}, \nu_{\mu,R})$ | (1,1,0,2) | |
| $\nu_{\tau,R}$ | (1,1,0,1) | |
| $\phi \equiv (\phi_1^0, \phi_2^0) \phi_1^0 / \phi_2^0$ | (1, 2, -1, 2) | |
| $\chi_H$ | (1,1,0,2) | |
| $\phi_0$ | (1,2,-1,1) | |
| $\Delta_H$ | (1,1,0,3) | |

Table caption: Representation content of the various fields in the model under the gauge group $G_{\text{STD}} \times SU(2)_H$.

Here $\psi_{e,\mu,\tau}$ denote the left handed lepton doublets. We arrange the Higgs potential in such a way that the $SU(2)_H$ symmetry is broken by $< \chi_1 > = v_H; < \chi_2 > = v_H$ and $< \Delta_{H_{3,2}} > = v_H$, where $v_H, v_H > v_{\text{ckm}}$. Note that we have used the $SU(2)_H$ symmetry to align the $\Delta_H$ vev along the $H_{1,3}$ direction. At the weak scale, all the neutral components of the fields $\Phi$ and $\phi_0$ acquire nonzero vev’s and break the standard model symmetry down to $SU(3)_c \times U(1)_{em}$. We denote these vev’s as follows: $< \phi_0^0 > = \kappa_0; < \phi_1^0 > = \kappa_1$ and $< \phi_2^0 > = \kappa_2$. Clearly $\kappa$’s have values in few to 100 GeV range. As we discuss later, we expect a hierarchy between the two vev’s $\kappa_1$ and $\kappa_2$, which is important in our discussion of neutrino mixings.

Note that $< \Delta_H > \neq 0$ breaks the $SU(2)_H$ group down to the $U(1)_{L_e-L_{\mu}}$ group which is further broken down by the $\chi_H$ vev. Since the renormalizable Yukawa interactions do not involve the $\chi_H$ field, this symmetry ($L_e-L_{\mu}$) is also reflected in the right handed neutrino mass matrix.

To study the pattern of neutrino masses and mixings, let us first note that a bare mass for the $\nu_{\tau,R}$ field is allowed at the tree level unconstrained by any symmetries. This mass can therefore be arbitrarily large and $\nu_{\tau,R}$ will decouple from the low energy spectrum. We will work in this limit of decoupled $\nu_{\tau,R}$ and write down the gauge invariant Yukawa couplings involving the remaining leptonic fields.

$$\mathcal{L}_Y = h_1 \bar{\psi}_e \Phi N_R + h_0 T r(\bar{\Psi} \phi_0 N_R^T)$$

$$- i f N_R^T \bar{\tau}_{2} \Delta_H N_R$$

$$h_1^T T r(\bar{\Psi} \Phi \bar{\tau}_{2} \tau_{2}) \tau_R + h_2^T \bar{\Psi} \phi_0 E_R^T$$

$$h_3^T \bar{\psi}_e \phi_0 \tau_R + h_4 \bar{\psi}_e \Phi E_R + h.c.$$
be some small contributions from the $\nu_{\tau-R}$ sector if we did not decouple it completely. We ignore these small contributions in our analysis. As we will see below, this feature of the right handed neutrino sector is crucial to the light neutrino mass matrix that leads in the zeroth order to bimaximal mixing. To see this, let us write down the $5 \times 5$ seesaw matrix for neutrinos:

$$M_{\nu_L,\nu_R} = \begin{pmatrix} 0 & 0 & 0 & h_{0K_0} & 0 \\ 0 & 0 & 0 & 0 & h_{0K_0} \\ h_{0K_0} & 0 & h_{1K_1} & h_{1K_2} \\ 0 & h_{0K_0} & h_{1K_1} & h_{1K_2} & f\nu_H \\ 0 & 0 & 0 & f\nu_H & 0 \end{pmatrix}$$ (2)

After seesaw diagonalization, it leads to the light neutrino mass matrix of the form:

$$M_\nu = -M_D M_R^{-1} M_D^T$$ (3)

where $M_D = \begin{pmatrix} h_{0K_0} & 0 \\ 0 & h_{0K_0} \\ h_{1K_1} & h_{1K_2} \end{pmatrix}$; $M_R^{-1} = \frac{1}{f\nu_H} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The resulting light Majorana neutrino mass matrix $M_\nu$ is given by:

$$M_\nu = -\frac{1}{f\nu_H} \begin{pmatrix} 0 & (h_{0K_0})^2 & h_{0K_1}h_{0K_1}k_2 \\ (h_{0K_0})^2 & 0 & h_{0K_1}h_{0K_1}k_2 \\ h_{0K_1}h_{0K_1}k_2 & h_{0K_1}h_{0K_1}k_2 & 2h_{1K_1}^2 \end{pmatrix}$$ (4)

To get the physical neutrino mixings, we also need the charged lepton mass matrix defined by $\psi_L M_\nu \psi_R$. This is given in our model by:

$$M_e = \begin{pmatrix} h_{2K_0} & 0 & h_{1K_0} \\ 0 & h_{2K_0} & h_{1K_0} \\ -h_{2K_0} & h_{1K_0} & h_{0K_0} \end{pmatrix}$$ (5)

Note that in the limit of $\kappa_1 = 0$, the neutrino mass matrix, have exact ($L_e - L_\mu - L_\tau$) symmetry [3] where the charged lepton mass matrix breaks this symmetry.

The invariance of the neutrino mass matrix under $L_e - L_\mu - L_\tau$ symmetry in the limit of $\kappa_1 = 0$ and $\kappa_2 \neq 0$ is an important issue since it leads to a neutrino mixing pattern which is very close to what is apparently observed in neutrino oscillation experiments and as such has been widely considered in the context of gauge models [3]. Further in this limit, we have $\Delta m^{2}_{\odot} = 0$. To generate solar neutrino oscillation however, a small but nonzero $\Delta m^{2}_{\odot}$ is needed. This happens as soon as a small $\kappa_1 \ll \kappa_2$ is turned on. It turns out that $(\Delta m^{2}_{\odot}/\Delta m^{2}_{A})$ is proportional to $\kappa_1/\kappa_2$. Thus the observed smallness of $(\Delta m^{2}_{\odot}/\Delta m^{2}_{A})$ is protected by a symmetry if $\kappa_1/\kappa_2$ is small. We show in the next section that this is indeed the case in our model due to the presence of a discrete $Z_2$ symmetry in combination with the horizontal symmetry. This makes the neutrino mixing pattern with softly broken $L_e - L_\mu - L_\tau$ in the neutrino sector that is responsible for solar neutrino oscillation a natural consequence of our model. The third neutrino has zero mass in our model.

III. NATURALNESS OF SOFTLY BROKEN $L_E - L_\mu - L_\tau$ SYMMETRY

To demonstrate that a softly broken $L_e - L_\mu - L_\tau$ for leptons arises in a natural manner in our model, we have to show that $\kappa_1 \ll \kappa_2$ is natural and does not receive infinite radiative corrections. For this purpose, we write the Higgs potential of the theory as a sum of two parts

$$V(\Phi, \phi_0, \Delta_H, \chi_H) = V_0 + \mu_0 Tr(\Phi^T \cdot \Delta_H \Phi^\dagger) + \phi_0^\dagger \Phi(\mu' \chi_H + \tilde{\mu}' \tilde{\chi}_H) + M^2 Tr(\Phi^T \Phi) + \mu'' \chi_H \cdot \Delta_H \chi_H + h.c.$$ (6)

where $V_0$ contains the standard $\phi^\dagger \phi$ type terms involving all the Higgs fields as well terms that are not relevant to the discussion of the $\kappa_1/\kappa_2$ and $\chi_H$ is defined as $\tau_2 \chi_H$. Note that in the limit when the parameter $\mu', \tilde{\mu}' = 0$, the theory has $Z_2$ symmetry under which only $\chi_H$ field changes sign and all other fields remain unchanged. The $\mu'$ and $\tilde{\mu}'$ break this symmetry softly and can therefore be chosen to be small compared to $\mu_0$. We do not include in the potential the term $\phi_0^\dagger \Phi^\dagger \Delta_H \chi_H$ term which forbidden by $Z_2$ symmetry which could have broken the $Z_2$ symmetry in a “hard” way. It is then easy to show that all radiative corrections to $\mu'$ and $\tilde{\mu}'$ are proportional to $\mu'$ and $\tilde{\mu}'$. The effects of $\mu'$ and $\tilde{\mu}'$ are similar; so henceforth we will only refer to $\mu'$.

Let us now show that $\kappa_1$ owes its origin to $\mu'$. For this purpose, we need to discuss the breaking of the horizontal symmetry in some more detail.

We choose the mass terms for the $\Delta_H$ and $\chi_H$ fields to have negative sign so that at the minimum of the potential they acquire vevs $v_H$ and $v_H$ respectively, breaking the $SU(2)_H$ symmetry down to $U(1)_{L_e-L_\mu}$ and subsequently no horizontal symmetry. Turning to their effect on the vevs of the standard model doublets ($\phi_2$ in $\Phi$ ($\Phi = (\phi_1, \phi_2)$), we see that for $M^2 > 0$, $< \Phi > = 0$ in the absence of horizontal symmetry breaking. Once horizontal symmetry breaking by $\Delta_H$ is turned on i.e. $v_H \neq 0$, for $\mu' = 0$, the $\mu_0$ term gives contributions to the masses of the two standard model doublets in $\Phi$ with opposite sign so that one has $m_{\phi_1}^2 = M^2 + \mu_0 v_H$ whereas $m_{\phi_2}^2 = M^2 - \mu_0 v_H$. We can choose $\mu_0 v_H$ such that $m_{\phi_2}^2$ becomes negative and of order of the electroweak scale leading to $\kappa_2 \neq 0$; but at the same time since $m_{\phi_2}^2 \geq 0$, it keeps $\kappa_3 = 0$. It is clear from examination of the Higgs potential that $\kappa_1$ remains zero as long as $\mu' = 0$ (i.e. in the limit of exact $Z_2$ symmetry). This is also preserved by radiative corrections due to the presence of the discrete symmetry, $Z_2$.

Once $\mu' \neq 0$, the $\phi_1^\dagger \Phi \chi_H$ term in Eq. (9) induces a nonzero vev for the electrically neutral member of the doublet $\phi_1$ giving $\kappa_1 \approx \mu' v_H k_{0L} / 2M^2$. Since $\mu'$ is a soft symmetry breaking parameter, we can choose it appropriately small to obtain $\kappa_1 \ll \kappa_2$. Thus the presence of the horizontal symmetry is crucial to obtaining the desired pattern for the neutrino mass matrix in our model.
As noted there are two horizontal symmetry breaking scales \( v_H^I \) and \( v_H \) in this model. The first vev \( v_H^I \) corresponds to the seesaw scale and is therefore determined by the neutrino masses. For instance, if we choose the Dirac masses of the neutrinos to be \( \sim 10^{11} \text{ GeV} \), we get \( v_H^I \approx 10^{11} \text{ GeV} \). In general this scale is likely to be anywhere between \( 10^{15} \text{ GeV} \) to \( 10^9 \text{ GeV} \) for Dirac masses between 0.1 GeV to 100 GeV. On the other hand the \( \chi_H \) vev \( v_H \) in principle can be much lower. The \( \chi_H \) vev gives mass to the horizontal gauge boson corresponding to the diagonal generator of \( SU(2)_H \), which couples to both electrons and muons. To fit present neutral current observations involving the electrons and muons, one must have \( \chi_H \) vev at least a few TeV. However to generate \( \kappa_1 \approx \kappa_2/10 \) (see the rational for this choice in the next section), we must have \( v_H \approx v_H^I \).

IV. CHARGED LEPTON SPECTRUM

In order to discuss the physical neutrino mixings, we need to work in a basis where the charged leptons are diagonal. There are two ways to get the right charged lepton mass hierarchy in our model: (i) First way is to choose \( \kappa_1 \ll \kappa_2 \approx \kappa_0 = h_2^0 h_0^2 \) and (ii) a second way is where \( m_e = h_2^0 h_0^2 \). We consider only the first case here. In this case for muon and tau lepton masses, we get roughly \( m_\tau \approx \sqrt{h_2^0 h_0^2} m_\mu \) and \( m_\mu \approx h_2^0 h_0^2 \) and for electron, we get \( m_e \approx h_2^0 h_0^2 / m_e \). We will see in the next section that we get \( m_{e3} \approx h_2^0 h_0^2 / m_e \), which we will demand to be of order 0.1. All these constraints can be satisfied since we have five free parameters in \( M_{\nu} \); however, they must satisfy certain constraints e.g. we must have \( h_2^0 \gg h_0^2 \) in addition to the constraints implied by the mass relations given above.

It is clear that we need a certain degree of fine tuning in the charged lepton sector. This fine tuning is however needed only in this minimal version of the horizontal model. For instance if there are two sets of Higgs doublets, one coupling to charged leptons and another to neutrinos, as would be the case in a supersymmetrized version of the model, the charged leptons and the neutrinos get their masses from different Higgs multiplets. As a result, one can get a realistic charged lepton spectrum without fine tuning while preserving other consequences of horizontal symmetry such as the connection between the \( U_{e3} \) and solar mixing angle. The important point is that the presence of a horizontal symmetry leads to a nonvanishing \( U_{e3} \) which makes the model phenomenologically interesting.

V. SOLAR NEUTRINO MIXING ANGLE AND \( U_{e3} \) CONNECTION

We now turn to the discussion of neutrino mixings in our model. As already noted, in the limit of \( \kappa_1 = 0 \), the neutrino mass matrix has exact \( (L_e - L_\mu - L_\tau) \) symmetry. The neutrino mixing matrix in this limit has the form:

\[
U^\nu = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & c \\
0 & -s & -s \alpha
\end{pmatrix}
\] (7)

This mixing pattern is quite close to the observed values. Further more, it corresponds to the inverted mass hierarchy where the solar pair of neutrinos are heavier. Atmospheric neutrino data imply that \( s \approx 1/\sqrt{2} \). For solar neutrino oscillation, it predicts \( \sin^2 2\theta_\odot = 1 \). However, since the two heavy eigenstates are exactly degenerate, it leads to no solar neutrino oscillation (since \( \Delta m^2_\odot = 0 \)). Furthermore, the third neutrino has zero mass. We also note that as soon as \( \kappa_1 \) is turned on, we get \( \Delta m^2_\odot \neq 0 \) and there is departure from the exact bimaximal mixing pattern in Eq. (7). How far the neutrino mixing matrix can depart from the exact bimaximal form in Eq. (7) will determine how viable the model is since after the SNO neutral current results \([3,4]\), combined analyses \([1,2]\) of all solar neutrino data \([3,11]\) disfavor exact bimaximal mixing.

We find that to fit the central value of \( \Delta m^2_\odot \) required by data, we need to have \( h_2^0 h_0^2 / m_e \approx 10^{-3} \text{ eV} \). If we assume the Yukawa couplings to be of order one, this implies that \( \kappa_1/\kappa_2 \approx 1/10 \). Since \( \kappa_1/\kappa_2 \) is related to \( Z_2 \) symmetry breaking, we have been able to relate the smallness of \( \Delta m^2_\odot / \Delta m^2_A \) to a symmetry breaking. Thus in this sense the smallness of \( m^2_\odot / \Delta m^2_A \) is natural in our model.

Turning now to the effect of \( \kappa_1 \neq 0 \) on the mixing pattern, unfortunately due to its smallness, the \( U^\nu \) matrix remains practically the same as in Eq. (7). Luckily however, the mixing matrix has a contribution from the charged lepton sector and from Eq. (5), we see that even in the limit of \( \kappa_1 = 0 \), \( M_{\nu} \) breaks \( L_e - L_\mu - L_\tau \) symmetry. The analysis therefore becomes very similar to the first paper in reference \([3]\).

To calculate the contribution of the charged lepton sector to the neutrino mixing, note that since the charged lepton mass matrix is not symmetric, it is diagonalized by bi-orthogonal transformations: \( U^{\ell L} M_{\nu} U^{\ell R \dagger} \). The \( U^{\ell L} \) is the matrix relevant for neutrino mixing and is given by

\[
U^{\ell L} = \begin{pmatrix}
c_{\alpha} & 0 & s_{\alpha} \\
0 & 1 & 0 \\
-s_{\alpha} & 0 & c_{\alpha}
\end{pmatrix}
\] (8)

where \( tana_\odot \approx h_2^0 h_0^2 / m_e^2 \). The physical neutrino mixing matrix \( U \) can now be written down as: \( U^{\ell L} U^\nu \) where \( U^\nu \) diagonalizes the Majorana mass matrix of the neutrinos and is given above.

\[
U = \begin{pmatrix}
c_{\alpha} + s_{\alpha} & -c_{\alpha} + s_{\alpha} & -s_{\alpha} \\
\frac{c_{\alpha}}{\sqrt{2}} & \frac{s_{\alpha}}{\sqrt{2}} & -s_{\alpha} \\
\frac{s_{\alpha}}{\sqrt{2}} & \frac{c_{\alpha}}{\sqrt{2}} & c
\end{pmatrix}
\] (9)
We see that the effective solar neutrino mixing angle becomes less than its maximal value in the presence of the charged lepton mixing parameter $\alpha$. We also note that $\alpha$ induces an $U_{e3} \approx \sqrt{2}/2$. Present upper limits from CHOOZ-PALO-VERDE\textsuperscript{[12]} experiments imply that $U_{e3} \leq 0.16$, which translates to a limit on $\alpha \leq 0.2$. Using this, we get $\sin^2 2\theta_\odot \approx 0.9$ or higher. We also note that the lower $U_{e3}$, the higher the $\sin^2 2\theta$ required. Hence the model predicts that $U_{e3}$ should be very close to the present upper bound from the CHOOZ-PALO-VERDE experiments.

Note that all the discussions above are done at the seesaw scale. There are radiative corrections as we extrapolate down to the weak scale arising from charged lepton contributions. It turns out however that they change the solar mixing angle slightly over and above that already discussed. It has recently been suggested\textsuperscript{[13]} that in some seesaw models there may also be high scale contributions due to different masses of the right handed neutrinos, that could effect the solar mixing angle. In our model the presence of the horizontal symmetry precludes such corrections. There is also likely to be some effect on $\sin^2 2\theta_\odot$ if the right handed neutrino is not totally decoupled.

Another prediction of this model is a value for the neutrino mass measured in neutrinoless double beta decay and we find $m_{\beta\beta} \approx 2\sqrt{2\Delta m^2_{\odot} U_{e3}^2}$. The maximum value for $m_{\beta\beta}$ is therefore $\approx 0.007$ eV. This can be probed in proposed double beta experiments such as GENIUS\textsuperscript{[14]}. If the recently reported evidence for neutrinoless double beta decay\textsuperscript{[15]} by the Heidelberg-Moscow group is confirmed, this model will be ruled out.

VI. CONCLUSIONS AND OUTLOOK

In summary, we have shown that if seesaw mechanism is to be responsible for the bimaximal mixing pattern for neutrino mixings, then a simple way to derive it from an extension of the standard model is to postulate the existence of a local $SU(2)_H$ horizontal symmetry under which right handed leptons transform nontrivially. First this guarantees the existence of two right handed neutrinos, whose masses are at the scale of horizontal symmetry breaking. Second, this minimal model via the seesaw mechanism leads to a near bimaximal mixing pattern suggested by solar and atmospheric neutrino data. We have found two interesting scenarios but in this note focus only on one. It predicts (i) a direct correlation between the mixing parameter $U_{e3}$ and $\Delta m^2_{\odot}$ with the solar mixing angle $\sin^2 2\theta_\odot$ as well as (ii) a negative sign for $\Delta m^2_{\odot}$ which can be tested in proposed long baseline experiments. The model has the interesting feature that the smallness of the $\Delta m^2_{\odot}/\Delta m^2_\odot$ is related to a discrete symmetry of the model, much like the smallness of the electron mass is related to the presence of a chiral symmetry in QED. Better understanding the origin of this discrete symmetry can therefore perhaps explain why $\Delta m^2_{\odot} \ll \Delta m^2_\odot$. It also makes a prediction for neutrinoless double beta decay, which can be tested.

This model could easily be incorporated into models with local $B-L$ symmetry. One would then need the horizontal symmetry scale to be much lower than the $B-L$ symmetry scale.

The model can also be supersymmetrized in a straightforward manner by promoting all the fields to superfields and duplicating the Higgs fields. Below the horizontal symmetry breaking scale, one then has the MSSM and the model preserves coupling constant unification. All features of the neutrino sector remain unchanged and as noted the charged lepton spectrum then arises from a new set of Higgs fields and no fine tuning is required to get charged lepton masses.

As we remarked in the beginning of the paper, one could also work with a horizontal symmetry that operates on the right handed components of both quarks and leptons, in which case global anomaly freedom would again require the presence of two right handed neutrinos as in the case discussed in the text. We expect our results for the neutrino mixings to remain unaltered whereas the Higgs sector may need to be extended to fit the quark mixing angles. We do not pursue this alternative here.

The work of R. N. M. is supported by the National Science Foundation Grant No. PHY-0099544. We thank Luis Lavoura for comments on the first version of the paper.

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