WORLD-VOLUME EFFECTIVE ACTIONS OF EXOTIC FIVE-BRANES

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SHIN SASAKI (KITASATO UNIV.)

WITH
TETSUJI KIMURA (TOKYO INSTITUTE OF TECHNOLOGY)
MASAYA YATA (KEK)

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INTRODUCTION
Exotic branes?

M-theory compactified on $T^8$

U-duality $E_8(8)(\mathbb{Z})$ multiplet (particles in 3d)

[Elitzur-Giveon-Kutasov-Rabinovici (1997), Bau-O’Loughlin (1997), Obers-Pioline (1998)]

Particles multiplet in 3d

“ordinary branes” wrapped on cycles

- waves
- F-strings
- D-branes
- NS5-branes
- KK-monopoles

Other states: Exotic branes (Q-branes) wrapped on cycles
Exotic branes are found in a duality chain

**Example**

- **NS5-brane**
  - T-dual (transverse direction) \( R_1 \)
- **KK-monopole**
  - T-dual (non Taub-NUT direction) \( R_2 \)
- **\( 5^2_2 \)-brane** ← *an exotic brane*

**Quadratic dependence on radii**

\[ T \sim g_s^{-2}(R_1 R_2)^2 \]

**String coupling**

**World-volume space dim**

**5^2_2**
Exotic brane as 1/2 BPS state

$5_2^2$-brane is a 1/2 BPS solution in SUGRA
[de Boer-Shigemori (2010)]

\[ ds_{5_2^2}^2 = dx_{012345}^2 + H (d\rho^2 + \rho^2 (d\theta)^2) + \frac{H}{K} ((dx^8)^2 + (dx^9)^2), \]

\[ e^{2\phi} = \frac{H}{K}, \quad B = -\frac{\sigma \theta}{K} dx^8 \wedge dx^9, \]

\[ H = h_0 + \sigma \log \frac{\mu}{\rho}, \quad K = H^2 + (\sigma \theta)^2, \quad \sigma = \frac{R_8 R_9}{2\pi \alpha'}, \]

($\rho, \theta$) : $x^6$-$x^7$ plane
\[
\begin{align*}
\left\{ \begin{array}{ll}
\theta = 0 & : g_{88} = g_{99} = H^{-1}, \\
\theta = 2\pi & : g_{88} = g_{99} = \frac{H}{H^2 + (2\pi\sigma)^2}
\end{array} \right.
\end{align*}
\]

Monodromy \neq \text{ diffeo or gauge transformations} \\
\text{(multi-valued function)}

\textbf{non-geometric}

Monodromy = \text{SO}(2, 2) \ T\text{-duality}

\textbf{T-fold (U-fold)}
Nature of exotic branes?

- **Supergravity viewpoint**
  [Lozano Tellechea-Ortin (2000), Hull (2004), de Boer-Shigemori (2010, 2012), de Boer-Mayerson-Shigemori (2014), and more.]

- **World-sheet viewpoint**
  [Kikuchi-Okada-Sakatani (2012)]
  [Kimura-S.S, NPB876(2013)876, JHEP08(2013)126, JHEP03(2014)128]
  [cf. today’s talk by Kimura and Poster by Yata (Friday)]

- **Double field theory viewpoint**
  [Andriot-Hohm-Larfors-Lust-Patalong (2012), Andriot-Betz (2013), Blumenhagen-Desr-Plauschinn-Rennecke-Schmid (2012, 2013), Geissbuhler-Marques-Nunez-Penas (2013), and more]

- **World-volume viewpoint (This talk)**
  [Chatzistavrakidis-Gautason-Moutsopoulos-Zagermann (2014)]
  [Kimura-Yata-S.S, arXiv:1404.5442]
WORLD-VOLUME THEORIES
World-volume effective theory

We focus on 1/2 BPS exotic five-branes in type IIB string theory

- Brane fluctuation modes
- World-volume gauge fields

Zero-modes associated with the geometry (solution)

should be embedded into the supermultiplets

16 SUSY in 6 dim

- N=(1,1) vector multiplet
- N=(2,0) tensor multiplet

Duality chain is helpful
String duality chains on various five-branes. The numbers in parentheses denote the numbers of supercharges in six dimensions except for the $M$-brane and the $GG$-branes. The subscripts $T$, $V$, and $H$ mean the tensor multiplet, the vector multiplet, and the hypermultiplet, respectively.
IIB $5^2_2$-brane

Duality chain

D5 $\rightarrow$ NS5 $\rightarrow$ KKM $\rightarrow$ $5^2_2$ $\rightarrow$ $5^2_3$

6d N=(1,1) vector multiplet

$U(1)^2$ isometries $\rightarrow k_1^\mu, k_2^\mu$ Killing vectors

Space-time covariant expression of T-duality rule
Covariant Buscher rule

T-duality in NS-NS sector

\[ g_{\mu \nu} \longrightarrow g'_{\mu \nu} = g_{\mu \nu} - \frac{(i_k g)_\mu (i_k g)_\nu - ((i_k B)_\mu - \lambda \partial_\mu \varphi)((i_k B)_\nu - \lambda \partial_\nu \varphi)}{k^2}, \]

\[ B_{\mu \nu} \longrightarrow B'_{\mu \nu} = B_{\mu \nu} - \frac{(i_k g)_\mu ((i_k B)_\nu - \lambda \partial_\nu \varphi) - ((i_k B)_\mu - \lambda \partial_\mu \varphi)(i_k g)_\nu}{k^2}, \]

\[ e^{2\phi} \longrightarrow e^{2\phi'} = \frac{1}{k^2} e^{2\phi} \]

winding zero-modes \( \varphi \)
Covariant Buscher rule

non-covariant Buscher rule in R-R sector \[\text{[Meessen-Ortin (1998)]}\]

\[
C''(n) = (-)^n i_k C^{(n)} - C^{(n-1)} \wedge (i_k B - \lambda d\varphi) \\
- (-)^{n-2} k^{-2} i_k C^{(n-1)} \wedge (i_k B - \lambda d\varphi) \wedge i_k g
\]

Available only inside the pull-back
After long calculations ....
\[ S_{\text{DBI}}^{5_2} = -T_{5_2} \int d^6 \xi \ e^{-2\phi} (\det h_{IJ}) \left(1 + \frac{e^{2\phi} (i_{k_1} i_{k_2} (C^{(2)} + C^{(0)} B^{(2)}))^2}{\det h_{IJ}}\right)^{1/2} \]

\[ \times \sqrt{\det (\Pi_{\mu\nu}^2 \partial_a X^\mu \partial_b X^\nu) + \frac{K_a^{(1)} K_b^{(1)}}{(k_2)^2} - \frac{(k_2)^2 (K_a^{(2)} K_b^{(2)} - K_a^{(3)} K_b^{(3)})}{\det h_{IJ}} + \lambda F_{ab})} \]

\[ h_{IJ} = k_I^\mu k_J^\nu (g_{\mu\nu} + B_{\mu\nu}), \quad \Pi_{\mu\nu}^2 = g_{\mu\nu} - \frac{1}{(k_2)^2} (i_{k_2} g)_\mu (i_{k_2} g)_\nu, \]

\[ K^{(1)}_\mu = (i_{k_2} B - \lambda d\varphi')_\mu, \]

\[ K^{(2)}_\mu = \left( i_{k_1} g - \frac{k_1 \cdot k_2}{(k_2)^2} (i_{k_2} g) + \frac{1}{(k_2)^2} (i_{k_1} i_{k_2} B) (i_{k_2} B - \lambda d\varphi') \right)_\mu, \]

\[ K^{(3)}_\mu = \left( (i_{k_1} B - \lambda d\varphi) - \frac{k_1 \cdot k_2}{(k_2)^2} (i_{k_2} B - \lambda d\varphi') + \frac{1}{(k_2)^2} (i_{k_1} i_{k_2} B) i_{k_2} g \right)_\mu \]
Four transverse directions? $X, X', X^1, X^2 \rightarrow \text{NO}$

* Two geometric modes $X, X'$ are projected out

$$- \det(\Pi_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \cdots)$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - h^{IJ} g_{\mu\rho} g_{\nu\sigma} k_I^\rho k_J^\sigma \text{ projection operator}$$

* Two winding zero-modes $\varphi, \varphi'$ appear

$$(i_{k_1} B - d\varphi), \quad (i_{k_2} B - d\varphi')$$

$$\delta B = d\Lambda^{(1)}, \quad \delta \varphi = -i_{k_1} \Lambda^{(1)}, \quad \delta \varphi' = -i_{k_2} \Lambda^{(1)}$$
Appropriate tension

\[ T \sim e^{-2\phi} \det h_{IJ} \sim e^{-2\phi} R_1^2 R_2^2 \]

\[ 5^2 \]

solitonic volume of fibered torus

World-volume gauge symmetry

\[ \delta A_a = \partial_a \chi \]

Space-time gauge symmetry

\[ \delta B = d\Lambda^{(1)} \]
\[ \delta C^{(0)} = 0, \quad \delta C^{(2)} = d\lambda^{(1)}, \quad \delta C^{(4)} = d\lambda^{(3)} - B \wedge d\lambda^{(1)}, \]
\[ \delta (d\tilde{A}^{(1)}) = P[\delta \tilde{C}^{(2)}], \quad \delta \varphi = -i_{k_1} \Lambda^{(1)}, \quad \delta \varphi' = -i_{k_2} \Lambda^{(1)}. \]
Wess-Zumino term

**Wess-Zumino term — formal T-duality of $B^{(6)}$**

A mixed-symmetry tensor needs to be introduced

[Bergshoeff-Riccioni (2010), Bergshoeff-Ortin-Riccioni (2011)]

$$B^{(8,2)}_{\hat{\mu}_1 \cdots \hat{\mu}_6, mn} \quad \hat{\mu}_i \neq \text{isometry directions}$$

$$S_{52}^{WZ} = -\mu_5 \int_{\mathcal{M}_6} P[B^{(8,2)}] + \cdots$$

Hodge dual of $k_1^{\mu} k_2^{\nu} B^{(2)}_{\mu \nu}$

cf. dual graviton for KKM

No scalar modes associated with isometries

[Kimura-Yata-S.S. (2014)]
- Geometric zero-modes $X^1, X^2$
- Winding modes $\varphi, \varphi'$
- Gauge 1-form $A_a$

embedded into 6d $\mathcal{N}=(1,1)$ vector multiplet

- World-volume and space-time gauge invariance
IIB $\frac{5^2}{3}$-brane

After long calculations ....
IIB $5_3^2$ -brane

$$S_{5_3^2}^{\text{DBI}} = -T_5^2 \int d^6 \xi \ |\tau| e^{-2\phi} (\det l_{IJ}) \sqrt{1 + \frac{e^{2\phi}}{\det l_{IJ}} \left\{ i_{k_1} i_{k_2} (B + |\tau|^{-2} C^{(0)} C^{(2)}) \right\}}$$

$$\times \sqrt{-\det(\Pi_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \frac{L_{a}^{(1)} L_{b}^{(1)}}{|\tau|^2 k_2^2} - \frac{k_2^2 (L_{a}^{(2)} L_{b}^{(2)} - |\tau|^{-2} L_{a}^{(3)} L_{b}^{(3)})}{\det l_{IJ}} + \lambda F_{ab})}$$

$$\tau = C^{(0)} + ie^{-\phi},$$

$$l_{IJ} = k_1^{I} k_2^{J} (g_{\mu\nu} - |\tau|^{-1} C^{(2)}_{\mu\nu}),$$

$$L_{\mu}^{(1)} = (i_{k_1} C^{(2)} + \lambda d\tilde{\varphi})_{\mu},$$

$$L_{\mu}^{(2)} = \left( i_{k_1} g - \frac{k_1 \cdot k_2}{k_2^2} (i_{k_2} C^{(2)} + \lambda d\tilde{\varphi}) + \frac{1}{k_2^2} (i_{k_1} i_{k_2} C^{(2)}) i_{k_2} g \right)_{\mu},$$

$$L_{\mu}^{(3)} = \left( (i_{k_1} C^{(2)} + \lambda d\tilde{\varphi}) - \frac{k_1 \cdot k_2}{k_2^2} (i_{k_2} C^{(2)} + \lambda d\tilde{\varphi}) + \frac{1}{k_2^2} (i_{k_1} i_{k_2} C^{(2)}) i_{k_2} g \right)_{\mu},$$

$$F_{ab} = \partial_a A_b - \partial_b A_a + (\text{RR forms})$$
\* Two geometric modes \(X, X'\) are projected out
- \(\det(\Pi_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \cdots)\)
\(\Pi_{\mu\nu} = g_{\mu\nu} - h^{IJ} g_{\mu\rho} g_{\nu\sigma} k_I^\rho k_J^\sigma\)

\* Appropriate tension
\(T \sim |\tau| e^{-2\phi} \det l_{IJ} \sim e^{-3\phi} R_1^2 R_2^2\)

\* Mixed-symmetry tensor in R-R sector
\(S_{52}^{WZ} = -\mu_5 \int_{\mathcal{M}_6} P[C^{(8,2)}] + \cdots\) S-dual of \(B^{(8,2)}\)

\* Scalar gauge transformation associated with R-R form
\(\delta C^{(2)} = d\lambda^{(1)}\), \(\delta \tilde{\phi} = -i k_1 \lambda^{(1)}\) \(\delta \tilde{\phi}' = -i k_2 \lambda^{(1)}\)
SUMMARY AND FUTURE WORKS
We construct world-volume effective actions of type IIB $5^2_2$-brane and $5^2_3$-brane.

World-volume fields are embedded into the 6d $\mathcal{N}=(1,1)$ vector multiplet

$$X^1, X^2, \varphi, \varphi', A_a$$

Source of mixed-symmetry tensors (non-geometric flux)

$$B^{(8,2)}, C^{(8,2)}$$

Type IIA $5^2_2$-brane — 6d $\mathcal{N}=(2,0)$ tensor multiplet

Type I $5^2_3$-brane

$5^2_2$-brane in $SO(32), E_8 \times E_8$ heterotic theories
Future directions

- Physics from the effective theories
- Supertube effect on world-volume of exotic branes?
- Monodromy ?
- Intersecting configurations of exotic branes?
- World-volume action of M-theory exotic branes?
- Treatment in the double field theory (DFT) or generalized geometry?
- R-branes?

Thank you
BACKUP
Duality chain

\[ \text{M5} \rightarrow \text{NS5} \rightarrow \text{KKM} \rightarrow 5^2_2 \]

- Direct dimensional reduction
- T-dual
- T-dual

N=(2,0) tensor multiplet
6d N=(2,0) tensor multiplet

- Geometric zero-modes \( X^1, X^2 \)
- Winding modes \( \varphi, \varphi' \)
- Scalar associated with M-circle \( Y \)
- Self-dual tensor field \( A_{ab} \)
- Auxiliary field (non-dynamical) \( a \)

DBI and WZ terms are given in the gauge invariant form

[Kimura-Yata-S.S (2014)]
M5-brane action

M5-brane action \[\text{[Pasti-Sorokin-Tonin (1997)]}\]

\[
S_{M5} = -T_{M5} \int d^6 \xi \left[ \sqrt{-\text{det}(P[\hat{g}]_{ab} + i\hat{H}^*_{ab})} + \frac{\sqrt{-\hat{g}}}{4g^{ef} \partial_e a \partial_f a} \hat{H}^{abc} \hat{H}_{bcd}(\partial_a \partial_d) \right] + T_{M5} \int_{M_6} \left( P[\hat{C}^{(6)}] - \frac{1}{2} F^{(3)} \wedge P[\hat{C}^{(3)}] \right)
\]

Self-dual 2-form \[F^{(3)} = dA^{(2)}\]

\[
\hat{H}^{(3)} = F^{(3)} + P[\hat{C}^{(3)}] \quad H^{*(3)} = *_6 H^{(3)}
\]

An auxiliary field \[a\] — non dynamical (can be gauged away)
Dimensional reduction along M-circle

A scalar mode $Y$

Introduce two isometries (Killing vectors) along transverse directions

$$k_{1}^{\mu}, \quad k_{2}^{\mu}$$

T-duality transformations by the covariant Buscher rule
IIA $5_2^2$-brane

\[ S_{5_2^2}^{DBI} = -T_{5_2^2} \int d^6 \xi e^{-2\phi} (\det h_{IJ}) \]

\[ \times \sqrt{-\det(\Pi_{\mu\nu}^2 \partial_a X^\mu \partial_b X^\nu + \frac{K^{(1)}_a K^{(1)}_b}{(k_2)^2} - \frac{(k_2)^2 (K^{(2)}_a K^{(2)}_b - K^{(3)}_a K^{(3)}_b)}{\det h_{IJ}} + \frac{\lambda^2 e^{2\phi}}{\det h_{IJ}} \tilde{F}_a \tilde{F}_b) \]}

\[ \times \sqrt{\det(\delta^a \dot{b} + \frac{ie^{i\phi}}{3!\tilde{N} \sqrt{\det h_{IJ}(\tilde{a})^2}} Z_a \dot{b})} \]

\[ - \frac{\lambda^2}{4} T_{5_2^2} \int d^6 \xi \frac{\varepsilon^{abgdab'e'f'} \tilde{H}_{d'e'f'} \tilde{H}_{abc}(\partial_g a \partial_d a)}{3!\tilde{N}^2} \left[ \tilde{g}^{cd} - \frac{\lambda^2 e^{2\phi} \tilde{g}^{ce} \tilde{g}^{df} \tilde{F}_e(1) \tilde{F}_f(1)}{\det h_{IJ} + \lambda^2 e^{2\phi} \tilde{g}^{a'b'} \tilde{F}_a'(1) \tilde{F}_b'(1)} \right] \]

\[ \tilde{H}_{abc} = F_{abc}^{(3)} + (\text{RR-forms}) \]

\[ \tilde{F}_a^{(1)} = \partial_a Y + (\text{RR-forms}) \]