Bending analysis of laminated plate via polynomial RBF

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Abstract: The present novel work deals with the investigation of bending response of laminated plate using five variable higher order shear deformation theory (HSDT). Governing differential equation (GDE) of the plate is acquired by energy principle. Polynomial radial basic function based on Meshfree method is used for discretizing the GDE. A MATLAB code is developed to obtain the results. Convergence and validation for the code is obtained using already published results. The effect of orthotropy and span to thickness ratio on the response of the plate is studied.

Keywords: Laminated, RBF, HSDT, bending, Meshfree method

1. Introduction

In the past decades, composite material assumes a key role in numerous fields of designing and engineering applications, such as aerospace, automotive, defense, submarines, sport and health instrument applications because of its craving properties, e.g as high strength, light weight and low maintenance cost, and so forth. As applications constantly develop, the field of researches in academia and industry and attractive to investigate better arrangements on a very basic level. It incorporates more precise demonstrating of composite material as well as new thoughts. Several laminate theories have been connected to the investigation of laminates plate in the past. HSDT represents enhanced in-plane and out-plane with parabolic variations of transverse shear stresses than CLPTs and FSDTs. Also, it satisfies the surface free boundary conditions.

Recently laminated plates have been of keen interest to many researchers. Mahi et al., 2015 [1] introduced a new hyperbolic shear deformation theory for the response of composite plates. Alipour, 2016 [2], studied laminated plates under various loads using analytical approach. Thai et al., 2016 [3] implemented isogeometric analysis of laminated plates using layerwise HSDT. Singh et al. [4] introduced an inverse hyperbolic HSDT for analysis of composite plates. Reddy, 1984 [5] developed an hsdt which predicts stresses and deflections more accurately than FSDT. Mantari et al., 2014 [6]
obtained static analysis of composite plates using Navier closed-form solution. Aydogdu, 2009, [7] determined an new hsdt from 3-D solutions using inverse method.

2. Mathematical Formulation

A laminated composite (X–Y–Z) plate is taken with $\theta$ angle orientation of fibers as shown in Fig. 1. The plate has length $a$ and width $b$ and total thickness $h$. The displacement field at any point in the sandwich plate is expressed as Kumar R et al 2018 [8].

\[ U, V \text{ and } W \text{ are the transverse displacements in x, y and z directions, respectively.} \]

\[ u_0, v_0, w_0, x, y \text{ are the five unknows at mid plane of the plate.} \]

The stress-strain relations for kth layer of the laminated plate is expressed as:

\[ \sigma_{xx} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ \sigma_{yy} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ \sigma_{xy} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ \sigma_{xz} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

Where, $\bar{Q}_{ij}$ are represent as the transformed reduced stiffness coefficients.

The governing differential equations of the rectangular plate along with variationally admissible boundary conditions are derived using the energy principle, which is expressed as Singh et al. [9].
Boundary Conditions
The boundary conditions for an arbitrary edge with simply supported and are:

\[ x = 0, a: N_{xx} = 0, v_0 = 0, w_0 = 0, M_{xx} = 0, \phi_y = 0 \]
\[ y = 0, b: u_0 = 0, N_{yy} = 0, w_0 = 0, \phi_x = 0, M_{yy} = 0 \]  

3. Solution Methodology

Governing differential equations Eqn. (3) and boundary conditions Eqn. (4) are discretized using radial basis function (RBF). RBF based meshless formulation performs on the principle of interpolation of scattered data over the whole domain.

The solution of the linear governing differential equations (3) is assumed in terms of polynomial RBF for nodes 1: N, as;

\[
u_0, v_0, w_0, \phi_x, \phi_y = \sum_{j=1}^{N} \left( a_j^x, a_j^y, a_j^w, \alpha_j^x, \alpha_j^y \right) g\left( \|X - X_j\|c \right) \]

(5)

polynomial function \( g = r^c \) where, \( c = 7 \)

Where, \( N \) is total numbers of nodes, which are equal to the summation of boundary nodes NB and domain interior nodes ND. \( g\left( \|X - X_j\| \right) \) is polynomial RBF expressed as \( g = r^c, \alpha_j^x, \alpha_j^y, \alpha_j^w, \alpha_j^x, \alpha_j^y, \alpha_j^w \) are unknown coefficients. \( \|X - X_j\| \) is the radial distance between two nodes.

The displacements in terms of RBF are expressed as:
Similarly, other coefficients can be written.

Where, $g|Y-X|$ is radial basis function, $a_i^j$ is unknown coefficient. The static problem in terms of RBF can be expressed as:

$$\begin{bmatrix} [K]_L \\ [K]_B \end{bmatrix}_{2N+4N} \{\delta\}_{2N+1} = \begin{bmatrix} \{F\}_L \\ 0 \end{bmatrix}_{2N+1}$$

Where,

$$[K]_L = \begin{bmatrix} (k^a_{1a})_{11,33} & (k^a_{1a})_{11,55} & (k^a_{1a})_{11,22} & (k^a_{1a})_{11,15} \\ (k^a_{1a})_{11,33} & (k^a_{1a})_{33,55} & (k^a_{1a})_{33,22} & (k^a_{1a})_{33,15} \\ (k^a_{1a})_{11,55} & (k^a_{1a})_{33,55} & (k^a_{1a})_{55,22} & (k^a_{1a})_{55,15} \\ (k^a_{1a})_{11,22} & (k^a_{1a})_{33,22} & (k^a_{1a})_{55,22} & (k^a_{1a})_{22,15} \end{bmatrix}$$

$$[K]_{k_{1a}} = \begin{bmatrix} (k^a_{1a})_{11,33} & (k^a_{1a})_{11,55} & (k^a_{1a})_{11,22} & (k^a_{1a})_{11,15} \\ (k^a_{1a})_{11,33} & (k^a_{1a})_{33,55} & (k^a_{1a})_{33,22} & (k^a_{1a})_{33,15} \\ (k^a_{1a})_{11,55} & (k^a_{1a})_{33,55} & (k^a_{1a})_{55,22} & (k^a_{1a})_{55,15} \\ (k^a_{1a})_{11,22} & (k^a_{1a})_{33,22} & (k^a_{1a})_{55,22} & (k^a_{1a})_{22,15} \end{bmatrix}$$

$$\{F\}_L = \begin{bmatrix} \{0\}_{11,N} \{0\}_{11,N} \{0\}_{11,N} \{0\}_{11,N} \{0\}_{11,N} \end{bmatrix}^T$$

$$\{\delta\} = \begin{bmatrix} \alpha^u \alpha^v \alpha^w \alpha^s \alpha^t \end{bmatrix}^T$$

Similarly, other coefficients can be written.
4. Results and discussion

In order to present the accuracy and efficiency of the polynomial RBF solution methodology, four layered symmetric [0/90/90/0] simply supported square laminated plates under transverse sinusoidal load of intensity of $q_o$ at center of plate is considered. The sinusoidal form of the load is expressed as:

$$q = q_o \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi y}{b} \right)$$

The normalized central deflection is taken as:

$$\bar{w} = \frac{100h^3E_2w}{q_o a^4} \left( \frac{a}{2}, \frac{a}{2}, 0 \right)$$

The results are obtained by varying the number of nodes from (7×7) to (17×17). The maximum percentage difference between the results obtained using last two number of nodes is within 0.7 %. It is clear that the present method shows the excellent convergence for bending response of laminated plate and also good agreement with the Reddy[5] HSDT analytical solution after 13×13 nodes. Considering this, a 15×15 node is used throughout the study.

![Fig. 2. Convergence study of four layered symmetric cross ply (0/90/90/0) square laminated plate under sinusoidal load (a/h=10, E1/E2=25)](image)

![Fig. 3. Normalized central deflection $\bar{w}$ of simply supported laminated (0/90/90/0) plate under sinusoidal load is the convergence study of SS plate under sinusoidal load. The material properties for the lamina are taken as: E1 = 25; E2 = 1; G12 = G13 = 0.5; G23 = 0.2; v12 = 0.25. The a/h is taken as 10. The results are obtained by varying the number of nodes from (7×7) to (17×17). The maximum percentage difference between the results obtained using last two number of nodes is within 0.7 %. It is clear that the present method shows the excellent convergence for bending response of laminated plate and also good agreement with the Reddy[5] HSDT analytical solution. The normalized central deflection decreases by orthotropy ratio increase.](image)
Fig. 3. Normalized central deflection $\bar{w}$ of SS laminated (0/90/90/0) plate under sinusoidal load

Fig. 4. The effect of orthotropy ratio on normalized central deflection. ($a/h=10, a=b$)

Fig. 5. Normalized deflection of different type of laminated plates ($E_2=1; E_1=20*E_2; \nu_{12}=0.25; \nu_{21}=\nu_{12}*E_2/E_1; G_{12}=0.5*E_2; G_{13}=0.5*E_2; G_{23}=0.2*E_2$)

Fig. 5. shows the normalized deflection of different type of symmetric and anti-symmetric laminated plates. It can be observed that all the plates normalized central deflection decrease from thick to thin plate.
5. Conclusion

The polynomial RBF based meshfree method was applied to bending analysis of laminated composite plates. A simply supported laminated plate was investigated and the present model was compared to exact and analytical results. The present method shown highly convergence and good agreement with the published results. The reported results will serve as numerical solutions to validate other solution methodologies.

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