Quark confinement in a random background Gross-Neveu model

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Abstract

We study an extended Gross-Neveu model with $N_f$ quark flavors and with an additional SU(2) (global color) degree of freedom of the quarks. The four fermion interaction in the color channel is mediated by a random color matrix with fixed strength. The effective potential for the quark condensate and the scalar correlation function is investigated in the large $N_f$-limit. Quark anti-quark thresholds are absent in the scalar correlation function implying that the decay of the scalar meson in free quarks is avoided. The transition from the low energy to the high energy perturbative regime is found to be smooth. Perturbation theory provides a good description of Green’s functions at high energies, although there is no quark liberation.

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1 Introduction

One of the most challenging problems nowadays in hadron physics is to understand why mesons and baryons do not decay into free quarks (quark confinement), although these are the fundamental constituents of Quantumchromodynamics (QCD), the right theory of strong interactions. Lattice simulations verify the quark confinement from a theoretical point of view [1, 2]. More and more evidence arose in the recent past, that the quark confinement is due to a condensation of monopoles which subsequently expels electric flux from the vacuum [3]. This provides a linear rising potential between static quarks implying confinement. A recent success by Seilberg and Witten [4, 5] shows that monopole condensation is indeed the mechanism of quark confinement in certain super-symmetric Yang-Mills theories.

Despite the increasing knowledge of the low energy properties of QCD, hadron physics is still only feasible by effective quark models [6, 7]. They successfully describe the mechanism of the spontaneous breakdown of chiral symmetry and the physics of the light mesons, but inherently suffer from a non-confinement of the quarks [7]. Recently, an effective quark model was proposed which incorporates the quark confinement in a sense that quark anti-quark thresholds are absent in (mesonic) Green’s functions [8]. The main ingredient in this model is an effective low energy four quark interaction mediated by a random gluonic background field. The ground state properties of this model were investigated under extreme conditions, i.e. high temperature and/or density. The results were found to be in agreement with the phenomenological expectations.

In order to study further the impact of random background fields on the quark confinement, we here study an extended version of the Gross-Neveu model [9]. As in QCD, the effective (running) coupling strength of this model becomes small at high energies (asymptotic freedom). Here, we will illustrate a mechanism which serves as a possible answer to the question how perturbation theory at high energies provides a good description of Green’s functions, although there is no quark liberation.

2 Ground state properties

2.1 Model description and effective potential

The model under investigations is described by the Euclidean generating functional in two dimensions for mesonic Green’s functions, i.e.

\[ Z[j] = \left\langle \ln \int \mathcal{D}q \mathcal{D}\bar{q} \exp \left\{ - \int d^2x \left[ L - j(x) g\bar{q}(x)q(x) \right] \right\} \right\rangle_O, \quad (1) \]

\[ L = \bar{q}(x) \left( i\slashed{\partial} + im \right) q(x) + \frac{g}{2N_f} (\bar{q}(x)q(x))^2 + \frac{1}{2N_f} \bar{q}\tau^a q(x) \bar{q}\tau^b q(x) G^{ab}, \quad (2) \]
where $m$ is the current quark mass. In the limit $m = 0$, the theory is invariant under global chiral rotations. The quarks fields $q(x)$ transform under a global $O(N_f)$ flavor and a global $SU(2)$ (color) symmetry. $g$ and $G^{ab}$ are dimensionless coupling constants in the color singlet and the color triplet channel respectively. An average of all orientations $O$ of the background field $G^{ab}$, transforming as $G' = OTGO$ with $O$ being a $3 \times 3$ orthogonal matrix, is understood in (1) to restore global $SU(2)$ color symmetry. From [8] we expect that these randomness leads to the confinement of quarks.

In order to explore ground state properties of the model, it is convenient to study the effective potential $U$ of the (Euclidean) condensate, since a non-vanishing value signals the spontaneous breakdown of chiral symmetry. The effective action $\Gamma([g\bar{q}q]_c)$ of the fermion condensate is obtained by a Legendre transformation of $Z[j]$ with respect to $j$, i.e.

$$\Gamma([g\bar{q}q]_c) := -Z[j] + \int d^2x \ [g\bar{q}q]_c(x)j(x) , \quad [g\bar{q}q]_c(x) := \frac{\delta Z[j]}{\delta j(x)} . \quad (3)$$

The effective potential $U$ results from $\Gamma$ by restricting $[g\bar{q}q]_c(x)$ to constant values, i.e. $U([g\bar{q}q]_c) = \Gamma([g\bar{q}q]_c)/V$, where $V$ is the Euclidean four volume. The potential $U$ is determined up to an unphysical offset, which is usually chosen in order the potential $U$ to be zero at vanishing condensate $[g\bar{q}q]_c$. The value of the quark condensate at which the effective potential attaches its global minimum is the vacuum expectation value $\langle g\bar{q}q \rangle$. The corresponding minimal value of $U$ is the vacuum energy density. Several minima correspond to different phases of the model. The state with the lowest energy density (value of $U$ at the local minimum) forms the true vacuum.

### 2.2 Solution of the model in the large $N_f$-limit

Although an exact solution of the model might be feasible with the powerful techniques developed in the context of the Gross-Neveu model [10], it is sufficient for our purposes to study the model (1) in leading order of a $1/N_f$-expansion. In the context of two-dimensional models, the $1/N$-expansion was cast into doubt at least for models with a continuous symmetry like the chiral Gross-Neveu model, since it predicts a spontaneous breakdown of the symmetry. The occurrence of Goldstone bosons in two dimensions causes infra-red problems implying that the symmetry cannot be broken spontaneously [11]. However, it was subsequently shown by Witten that the symmetry is “almost” spontaneously broken, and that the large $N$-expansion provides a rather good guide to the ground state properties [12].

The $1/N_f$-expansion is most easily derived by rewriting the theory (1) by means of
auxiliary fields, i.e.

\[
Z[j] = \left\langle \ln \int Dq \, D\bar{q} \, D\Sigma_0 \, D\Sigma^a \exp \left\{-\int d^2x \left[ L_M - j(x) g\bar{q}(x)q(x) \right] \right\} \right\rangle_O ,
\]

\[
L_M = \bar{q}(x) \left( i\partial + i\Sigma_0 + i\Sigma^a \tau^a \right) q(x) + \frac{N_f}{2g} (\Sigma_0 - m)^2 + \frac{N_f}{2} \Sigma^a (G^{-1})^{ab} \Sigma^b .
\]

Integrating out the quarks in (4) one obtains an equivalent theory formulated in the fields \( \Sigma_0, \Sigma^a \) only. The non-perturbative approach applied here is an expansion with respect to \( 1/N_f \) for a fixed background field \( G^{ab} \) and an average over all orientations \( O \) of \( G^{ab} \) once the Green's function has been calculated. For this purpose, we decompose the auxiliary fields into a mean-field part and fluctuations, i.e.

\[
\Sigma_0(x) = M_0 + \sigma_0(x) , \quad \Sigma^a(x) = iM^a + \sigma^a(x) ,
\]

where the orientation of the mean-field \( M^a \) depends on the actual choice of \( G^{ab} \). The imaginary unit in front of \( M^a \) in (6) was introduced for convenience. Later it will turn out that the ground state favors an imaginary constituent quark mass (corresponding to real \( M^a \)) in the color triplet channel. This will be the main observation implying the absence of quark anti-quark thresholds in mesonic correlation functions.

We will evaluate the functional integral in (4) in a semi-classical expansion around the mean field values. (It was recently pointed out that mean-field theory might generically provide a good description of the ground state of fermionic theories even at finite temperatures \[13\]). As in the case of the standard Gross-Neveu model \[9\], fluctuations \( \sigma_0, \sigma^a \) are suppressed by a factor \( 1/N_f \) implying that we might confine ourselves to the mean-field level (as long as ground state properties are addressed). For constant external sources \( j \), it is sufficient to consider constant mean field values. A straightforward calculation yields

\[
-\frac{1}{V N_f} Z[j] = -\frac{1}{V} \text{Tr} \ln \left( i\partial + i(M_0 + iM^a \tau^a) \right) + \frac{1}{2g} (M_0 - m - igj)^2 - \frac{1}{2} M^a (G^{-1})^{ab} M^b ,
\]

where the trace extends over internal indices as well as over space time. The master fields \( M_0 \) and \( M^a \) are solutions of the gap equations

\[
\frac{\partial Z[j]}{\partial M_0} = 0 , \quad \frac{\partial Z[j]}{\partial M^a} = 0 .
\]

It is easy to check that the right hand three equations reduce to an eigenvector equation to determine \( M^a \), i.e.

\[
G^{ab} M^b = g_c M^a , \quad M := \sqrt{M^a M^a} ,
\]

4
and an equation to calculate $M$. There are three eigenvalues of the matrix $G^{ab}$. With $g_c$ we define the eigenvalue of $G^{ab}$ which subsequently provides the phase with the lowest action. Using (9), the trace term in (7) can be reduced to

$$-\frac{1}{4\pi} \int_0^{A^2} du \ln \left( u^2 + 2u(M_0^2 - M^2) + (M_0^2 + M^2)^2 \right) = \text{const.}$$

(10)

$$-\frac{1}{4\pi} \left\{ 2A_- + \sqrt{A_+^2 - A_-^2} \left[ \pi - 2\arctan \frac{A_-}{\sqrt{A_+^2 - A_-^2}} \right] - A_- \ln \frac{A_+^2}{\Lambda^4} \right\} ,$$

where $A_\pm = M_0^2 \pm M^2$ and $\Lambda$ is the O(4)-invariant cutoff to regularize the momentum integration.

In order to renormalize the model, one observes that the logarithmic divergence in (10) is proportional to $A_-$ and can be therefore absorbed by a redefinition of the coupling constants, i.e.

$$-\frac{1}{\pi} \ln \frac{\Lambda^2}{\mu^2} - \frac{1}{g} = \frac{1}{g_R} , \hspace{1cm} -\frac{1}{\pi} \ln \frac{\Lambda^2}{\mu^2} + \frac{1}{g_c} = \frac{1}{g_c^R} ,$$

(11)

In order to completely renormalize the theory, we also define the renormalized mass by

$$\frac{m}{g} = \frac{m_R}{g_R} .$$

(12)

Field renormalization is not requested at the present stage of approximation. The $\beta$-function is in either case of the coupling constants $\beta(g_R) = -2/\pi g_R^2$ and signals asymptotic freedom. The renormalized coupling constants $g_R, g_c^R$ decrease by increasing momentum scale $\mu$. Note that in coincidence the bare coupling constants vanish in the infinite cutoff limit. This implies that the terms $m^2/g, gj, mj$ approach zero for $\Lambda \to \infty$.

We are now ready to perform the Legendre transformation (8). The calculation is straightforward. The final result for the effective potential of the quark condensate in the large $N_f$-limit is

$$\frac{1}{N_f} U = \frac{1}{2g_R} M_0^2 - \frac{m_R}{g_R} M_0 - \frac{1}{2g_R^2} M^2 + \frac{M_0^2 - M^2}{2\pi} \left( \ln \frac{M_0^2 + M^2}{\mu^2} - 1 \right)$$

(13)

$$-\frac{M_0 M}{2\pi} \left( \pi - 2\arctan \frac{M_0^2 - M^2}{2M_0 M} \right) ,$$

where $M_0$ is directly related to the quark condensate by

$$M_0 = -\frac{i}{N_f} [g \bar{q} q]_c ,$$

(14)
and $M$ satisfies the gap-equation
\begin{equation}
\frac{\pi}{g_R(\mu)} M + M \left\{ \ln \frac{M_0^2 + M^2}{\mu^2} + \frac{M_0}{M} \left( \frac{\pi}{2} - \arctan \frac{M_0^2 - M^2}{2M_0M} \right) \right\} = 0. \tag{15}
\end{equation}

The equations (13,14,15) are one of our main results. We have obtained a renormalization group invariant result, since a change of the renormalization point $\mu$ can be absorbed by a redefinition of the renormalized coupling constants.

In order to remove the superficial dependence of the effective potential $U$ on the subtraction point $\mu$, we introduce two renormalization group invariant scales $s_0, s_c$ by
\begin{equation}
\frac{1}{g_R(\mu)} + \frac{1}{\pi} \ln \frac{s_0^2}{\mu^2} = 0, \quad \frac{1}{g_R(\mu)} + \frac{1}{\pi} \ln \frac{s_c^2}{\mu^2} = 0. \tag{16}
\end{equation}

With the help of (16), the effective potential becomes
\begin{equation}
\frac{1}{N_f} U = - \frac{m_R}{g_R} M_0 + \frac{M_0^2}{2\pi} \left( \ln \frac{M_0^2 + M^2}{s_0^2} - 1 \right) + \frac{M^2}{2\pi} \left( \ln \frac{M_0^2 + M^2}{s_c^2} + 1 \right) \tag{17}
\end{equation}

where
\begin{equation}
M \left\{ \ln \frac{M_0^2 + M^2}{s_c^2} + \frac{M_0}{M} \left( \frac{\pi}{2} - \arctan \frac{M_0^2 - M^2}{2M_0M} \right) \right\} = 0. \tag{18}
\end{equation}

Thereby equation (18) was used to simplify the expression for the effective potential. The two free parameters $g_R, g_R$ of the original theory are removed in favor of the two renormalization group invariant scales $s_0, s_c$ (dimensional transmutation). One of these scales, e.g. $s_0$, can be used as fundamental unit to express dimensionful quantities. The remaining free parameter is the ratio of these scales, or $\ln s_0^2/s_c^2$.

Equation (18) is numerically solved for given value of $M_0$. The potential $U$ is subtracted in order to yield zero at $M_0 = 0$. The numerical result for the effective potential $U$ in (17) is shown in figure 1. Note that $M = 0$ is always a solution of (18). In this case, the effective potential $U$ (17) coincides with that of the standard Gross-Neveu model (solid line in figure 1). Note, however, that for sufficiently small values of $M_0$ a solution with $M \neq 0$ exists. This solution has lower effective potential (dashed line in figure 1). The global minimum of this branch therefore constitutes the vacuum. The effective potential $U$ is smooth at the transition from the phase with $M \neq 0$ to the phase $M = 0$, although $M$ varies rapidly (dot-dashed line in figure 1).

Ground state properties are described by the values of the quark condensate at which the effective potential attaches its global minimum. A non-zero value of $M_0$ corresponds to a non-vanishing quark condensate (see (14)), whereas $M$ controls the absence of quark anti-quark thresholds in Green’s functions as we will see below. In order the effective potential to have a minimum, $M_0$ and $M$ must satisfy
\begin{equation}
\frac{\pi m_R}{g_R} = M_0 \left\{ \ln \frac{M_0^2 + M^2}{s_0^2} - \frac{M}{M_0} \left( \frac{\pi}{2} - \arctan \frac{M_0^2 - M^2}{2M_0M} \right) \right\}, \tag{19}
\end{equation}
in addition to (18). Figure 2 shows the lowest action solution. If the overall scale is chosen to be $s_0$, the only parameter is $\ln s_0^2/s_c^2$. One observes that if $s_c$ exceeds a certain value, i.e.

$$s_c > 2.71828 \ldots s_0,$$

(20)
a phase with a non-vanishing value of $M$ occurs. For values of $s_c$ less than this critical value, $M$ is zero and the phase coincides with that of the standard Gross-Neveu model. In section 3, we will discuss the remarkable properties of this phase.

2.3 Quark propagator and analytic structure

Here, we will discuss the Euclidean quark propagator in the phase characterized by $M \neq 0$. In momentum space, it is

$$S(k) = \frac{1}{k^2 + i(M_0 + iM^a \tau^a)}.$$  

(21)

Introducing the eigenvectors $|\pm\rangle$ of the matrix $M^a \tau^a$, i.e.

$$M^a \tau^a |\pm\rangle = \pm M |\pm\rangle,$$

(22)

the quark propagator (21) decomposes into two parts with conjugate complex masses, i.e.

$$S(k) = |+\rangle \frac{1}{k^2 + i(M_0 + iM)} (+| + |-) \frac{1}{k^2 + i(M_0 - iM)} (-|.  

(23)

The hermiticity and unitarity properties of a theory with a pair of particles with conjugate complex masses were extensively studied in several models [14, 16, 17]. One finds evidence that such a theory does not necessarily violate unitarity.

In order to evade the triviality problem [15] of the scalar $\phi^4$-theory, a Higgs model with two scalar degrees of freedom with complex conjugate masses was proposed [16]. A consistent field theory is obtained by quantizing the scalar modes in a Hilbert space with indefinite metric. The model can be described by an Euclidean path integral, although the Minkowskian functional integral might not exist due to the presence of the so-called ghost particles [16]. It is argued that microscopic acausality effects remain undetectable using realistic wave packets, and that the S-matrix of the model is unitary [16].

Recently, the connection of a confining potential and the occurrence of complex singularities in the propagator of the fundamental degrees of freedom was investigated in QED3 [17]. From the literature, we conclude that a model containing particles with conjugate complex masses does not necessarily violate the phenomenological requirements for causality and unitarity of the S-matrix. In contrast, such a model might be a candidate for a low-energy theory of QCD describing the confinement of quarks.
In this paper, we will not further question the analytic properties of the extended version of the Gross-Neveu model studied here. Although this model lacks an immediate application to hadron physics, we hope it is simple enough to allow for a clarification of the phenomenology of particles with conjugate complex masses. These investigations are left to future work.

3 The scalar correlation function

In order to focus on the physical impact of the imaginary constituent quark mass in the color triplet channel, we here concentrate on the scalar correlation function, i.e.

\[
\Delta(p^2) = \int d^4x \, e^{-ipx} \langle g\bar{q}q(x) \, g\bar{q}q(0) \rangle = \int d^4x \, e^{-ipx} \frac{\delta^2 \ln Z[\phi]}{\delta j(x) \delta j(0)} \bigg|_{j=0} .
\]

Note that \( M_0 \) and therefore \( g\bar{q}q \) is the renormalization group invariant and the physical quantity. Unphysical quark anti-quark thresholds manifest themselves as imaginary part of the correlation function. In the following, we will establish the absence of an imaginary part of \( \Delta(p^2) \) in the phase with \( M \neq 0 \).

In order to obtain \( \Delta(p^2) \) in leading order of the \( 1/N_f \)-expansion, we expand the Lagrangian (5) up to second order in the fluctuations in (6), i.e.

\[
Z[j] \approx \left\langle \ln \int D\sigma_0 \, D\sigma^a \exp\{-S^{(2)}\} \right\rangle_O ,
\]

\[
S^{(2)} = \int \frac{dp}{(2\pi)^2} \left\{ \frac{1}{2} \sigma(p) \Pi_0^a(p^2) \sigma(-p) + \frac{1}{2} \sigma^a(p) \Pi_0^{\alpha\beta}(p^2) \sigma^\beta(-p) + i\sigma^a(p) K^\alpha(p^2) \sigma(-p) - \frac{gN_f}{2} \hat{j}(p) \hat{j}(-p) - iN_f \hat{j}(p) \sigma(-p) \right\} ,
\]

where \( \hat{j}(p) \) is the Fourier transform of the external source \( j(x) \) and

\[
\Pi_0^a(p^2) = \frac{N_f}{g} - \int \frac{dk}{(2\pi)^2} \text{tr} \left\{ S(k+p)S(k) \right\} ,
\]

\[
\Pi^{\alpha\beta}_0(p^2) = N_f (G^{-1})^{\alpha\beta} - \int \frac{dk}{(2\pi)^2} \text{tr} \left\{ \tau^\alpha S(k+p) \tau^\beta S(k) \right\} ,
\]

\[
K^\alpha(p^2) = -i \int \frac{dk}{(2\pi)^2} \text{tr} \left\{ \tau^\alpha S(k+p)S(k) \right\} .
\]

The quark propagator \( S(k) \), in momentum space, is given by (21). The calculation of the functions (27-29) is straightforward and closely follows [8]. It turns out that the quantities (27) and (28) can be expressed in terms of two functions \( H_0(p^2) \), \( H_\nu(p^2) \), i.e.

\[
\Pi^0_s(p^2) = \frac{N_f}{\pi} H_0(p^2) , \quad K^\alpha(p^2) = \frac{N_f}{\pi} M^\alpha H_\nu(p^2) .
\]
One finds
\[ H_0(p^2) = \frac{\pi}{g} + \frac{1}{2} \int_0^1 d\alpha \ln \frac{w^2 + 4M^2 M_0^2}{\Lambda^4} + 2 , \]  
(31)
\[ H_v(p^2) = -\int_0^1 d\alpha \arccos \frac{w}{\sqrt{w^2 + 4M^2 M_0^2}} , \]
where
\[ w = \alpha(1 - \alpha)p^2 + M_0^2 - M^2 . \]  
(32)
Whereas \( H_v \) is finite, \( H_0 \) needs renormalization. Equation (11), which was employed to renormalize the effective potential, absorbs the cutoff dependence in (31) as expected. Removing the superficial dependence of \( H_0(p^2) \) on the subtraction point \( \mu \) with help of the renormalization group invariant scale \( s_0 \) in (16), one finally obtains the finite and explicitly renormalization group invariant result
\[ H_0(p^2) = \frac{1}{2} \int_0^1 d\alpha \ln \frac{w^2 + 4M^2 M_0^2}{s_0^4} + 2 , \]  
(33)
We also find that \( M^\alpha \) is an eigenvector of the polarization matrix \( \Pi \), i.e.
\[ \Pi_{\alpha\beta} M^\beta M^\alpha = \frac{N_f}{\pi} \left( \ln \frac{s_0^2}{s_c^2} + H_0(p^2) \right) \frac{M^\alpha}{M} , \]  
(34)
Equipped with the results (30) and (34), it is now easy to integrate out the meson fields \( \sigma_0 \) and \( \sigma^a \) in (25) and to calculate (24). The result of this procedure only depends on \( M^2 \) (besides \( p^2 \) and \( M_0^2 \)). This implies that average over all orientations of the background field \( G_{ab} \) in (25) is trivial, since \( M \) is a singlet. The final result for the scalar correlation function \( \Delta(p^2) \) in leading order of the \( 1/N_f \)-expansion is therefore
\[ \Delta(p^2) = -\frac{\pi N_f}{H_0(p^2)} \frac{H_v(p^2)}{H_0(p^2) + \ln \frac{s_0^2}{s_c^2}} . \]  
(35)
We are now going to study the occurrence of an imaginary part of the scalar correlation function signaling a quark anti-quark threshold. For this purpose, we first perform the analytic continuation of the Euclidean correlation function to Minkowski space. This continuation follows the lines of [14]. The correlation function in Minkowski-space \( i\Delta_M(p_M^2) \) as a function of the Minkowski four-momentum \( p_M \) is related to the Euclidean counterpart by
\[ i\Delta_M(p_M^2) = \Delta(-p_M^2) . \]  
(36)
It is therefore sufficient to study the Euclidean correlation functions at negative momentum squared.
In order to illustrate the disappearance of the quark anti-quark threshold for \( M \neq 0 \), we first study its occurrence for \( M = 0 \). In this case, our model describes the scalar correlation function in the usual Gross-Neveu model \[9\] with a constituent quark mass \( M_0 \). The term of interest is the integrand of \( H_0(p^2) \) in (33), which essentially becomes \( \ln w/s_0^2 \). This implies that whenever \( w \) becomes negative, the function \( H_0 \), (33), acquires an imaginary part, which subsequently describes the decay of the scalar meson in a free quark anti-quark pair. In order for \( w \) to become negative, the Euclidean momentum \( p^2 \) must satisfy,

\[-p^2 < 4M_0^2, \tag{37}\]

implying that the quark-anti-quark threshold occurs at a (Minkowskian) momentum \( p_M = 2M_0 \), which is the familiar result. For \( M = 0 \) the function \( H_0(p^2) \) does no harm, since it is identically zero.

In contrast, for \( M \neq 0 \) the functions \( H_{0/v}(p^2) \) are real for all momentum \( p^2 \). No quark anti-quark threshold occurs at all. This is our main observation. Figure 3 shows the scalar correlation function as function of \( p^2 \). At negative values of \( p^2 \), a peak occurs near the would-be threshold position. This peak is enhanced, if the system is driven towards the deconfinement phase by a choice of the parameter \( \ln s_0^2/s_c^2 \).

The correlation function smoothly approaches the perturbative result (dashed line in figure 3) at large momentum transfer. Our model therefore represents an example the Green’s functions of which can be accurately calculated in perturbation theory at high momentum, although there is no quark liberation.

4 Conclusions

An extended version of the Gross-Neveu model \[9\] for \( N_f \) quark flavors was studied. In addition to the \( O(N_f) \)-flavor symmetry, the model possesses a global SU(2) (say color) symmetry. The interaction in the color triplet channel possesses a random orientation in color space. The effective potential for the quark condensate was calculated in the large \( N_f \) limit. Two phases were found: in one phase, the constituent quark mass in the color triplet channel is zero \( (M = 0) \), and a non-vanishing dynamical generated mass \( M_0 \) in the color singlet channel occurs. This phase corresponds to the vacuum phase of the standard Gross-Neveu model. However, a second phase constitutes the vacuum, since this phase has lower vacuum energy density. This non-trivial phase is characterized by an imaginary constituent quark mass in the color triplet channel \( (M \neq 0) \). The impact of an imaginary mass on causality of Green’s functions and unitarity of the S-matrix is discussed in the literature \[14, 16, 17\]. It is argued that these models do not contradict the phenomenological requirements. In order to clarify on this issue in the context of the model presented here, further investigations are needed, which are left to future work.
Rather than to investigate these theoretical aspects, we here focus on the remarkable physical consequences of the phase with $M \neq 0$. For this purpose, the scalar correlation function was studied. In the Gross-Neveu phase ($M = 0$) of the model, an imaginary part of the correlator corresponding to the quark anti-quark threshold was found. In contrast, the correlation function of the phase with $M \neq 0$ is real, and the quark anti-quark threshold is absent. The occurrence of an imaginary constituent quark mass in the color triplet channel might therefore serve as a signal for quark confinement (which is defined as the absence of quark thresholds in Green’s functions). At large momentum transfer, the scalar correlation function can be accurately calculated by perturbation theory. The model provides example that asymptotic freedom is perfectly compatible with the non-liberation of quarks.

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Figure captions

Figure 1: The effective potential $U$ and the color triplet constituent quark mass $M$ as function of the quark condensate $M_0$ in the chiral limit $m_R = 0$ for two values of $\ln s_0^2/s_c^2$; all quantities are in units of $s_0$.

Figure 2: The color-singlet and color-triplet constituent quark masses $M_0$ and $M$ in the chiral limit $m_R = 0$.

Figure 3: The scalar correlation function as function of the Euclidean momentum transfer $p^2$. 
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