Integrating spatial and numerical structure in mathematical patterning

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Abstract. This paper reports a study monitoring the integrating spatial and numerical structure in mathematical patterning skills of 30 students grade 7th of junior high school. The purpose of this research is to clarify the processes by which learners construct new knowledge in mathematical patterning. Findings indicate that: (1) students are unable to organize the structure of spatial and numerical, (2) students were only able to organize the spatial structure, but the numerical structure is still incorrect, (3) students were only able to organize numerical structure, but its spatial structure is still incorrect, (4) students were able to organize both of the spatial and numerical structure.

1. Introduction
Mathematics has been referred to as the Science of patterns [1,2]. Knowledge of repeating patterns has been shown to support knowledge of other areas of mathematics, particularly early algebra [3]. Abstracting patterns is the basis of structural knowledge, the goal of mathematics learning in the research literature [4]. Much recent research has focused on children’s early learning about mathematical structure. The importance of patterning skills, analogical reasoning and the development of structural thinking has been confirmed in several studies [3].

Patterning generally involves searching for mathematical regularities and structures [5]. The view that any kind of structural awareness would need to occur first in situations or “environments in which the only way to manipulate and reconstruct [the relevant] objects is to express explicitly the relationships between them [6]. Patterning is fundamental to mathematics [7,8]. A further line of research has shown that data modelling, a developmental process that begins with young children’s inquiries and investigations of meaningful phenomena also requires children to seek structure and recognize patterns. Preliminary findings of a longitudinal study of data modeling indicate that children can successfully collect, represent, interpret, communicate, and argue about the structure of data provided they address familiar themes [9].

Over the past decade a suite of studies has examined how children develop an Awareness of Mathematical Pattern and Structure (AMPS), found to be common across mathematical concepts [11,10]. An assessment interview, the Pattern and Structure Assessment (PASA) and a Pattern and Structure Mathematics Awareness Program (PASMAP) focuses on the development of structural relationships between concepts. Tracking, describing and classifying children’s models,
representations, and explanations of their mathematical ideas—and analysing the structural features of this development—are fundamentally important.

In PASMAP (Pattern and Structure Mathematical Awareness Program), children are encouraged to seek out and represent pattern and structure across different concepts and to transfer this awareness to other concepts. In other words, the aim is to promote generalization in early mathematical thinking. In the PASMAP pedagogy, the teacher uses probing questions to highlight important features of their drawings, to compare them with the model or with other children’s drawings, and to focus their attention on similarities and differences in crucial aspects of spatial and numerical structure. The indicators of the spatial structure are: congruence and similarity. While the indicator on the numerical structure is counting and subitizing [11].

2. Methods

2.1. Subject
Researchers asked 30 students grade 7th of junior high school (SMPN 1 Pace) to complete tiling problem. After experiencing saturation data in the subject, 5 students are unable to organize spatial and numerical structure correctly, 19 student were able to organize the spatial or numerical structure correctly, and then 6 students were able to organize both of spatial and numerical structure correctly.

2.2. Instrument
There are two types of instruments used, main and auxiliary instruments. The main instrument is the researchers themselves who act as planners, data collectors, data analysts, interpreters, and reporters of research results. The auxiliary instrument used in this study is a Tiling Problem (TP) and interviews. The problem given aims to obtain a description of the integrating of spatial and numerical structure of students, while the interview used was unstructured interview.

2.3. Data analysis
This study is a qualitative research with descriptive exploratory approach. At the data analysis stage, the activities conducted by researchers were (1) transcribing the data obtained from interviews, (2) data reduction, including explaining, choosing principal matters, focusing on important things, removing the unnecessary ones, and organizing raw data obtained from the field, (4) describing the integrating of spatial and numerical structure in the solving of tiling problem, and (5) conclusion.

3. Result and Discussion
Based on the results of the analysis of the TP answer sheets and interviews, we obtained data on the integrating of spatial and numerical structure undertaken by students in solving tiling problem of the type I, II, III, and IV (Figure 1).

Type I: Spatial and Numerical Structure Incorrectly

Figure 1. PRC’s Work about white square tiles surrounded by 1x1 black square tiles
Type I, there are 5 students who did spatial and numerical structure incorrectly. Type II, there are 7 students did spatial structure correctly but numerical structure incorrectly. Type III, there are 12 students did spatial structure incorrectly but numerical structure correctly. Type IV, there are 6 students did spatial and numerical structure correctly. The results of the integrating of spatial and numerical structure are presented in Table 1. In Every type, we chose one subject that did integrating spatial and numerical structure, that is the subject PRC, AND, ABI, and HBB.

Spatial Structure:
Here are excerpts of interviews with subject PRC in solving the problem number 1:

\[ P \]: See your drawing, what white tile and black tile?
\[ PRC \]: 1x1 ...
\[ P \]: How many white tiles and black tiles are there?
\[ PRC \]: White square tile is one, black square tile is eight
\[ P \]: Okay, what about the others?
\[ PRC \]: The 2x2 square white colored tile is twice this one (while pointing to a 1x1 white square tile)
\[ P \]: Why is there an extra picture of a black square tile on it?
\[ PRC \]: Because the 1x1 sized black tile is unlimited, because it does not say how many there are on the problem

Based on the interview results (Figure 1), the subject did not meet the aspect of the spatial structure appropriately. This is because the subject can not draw the results of the installation of white tiles surrounded by squares of black tiles appropriately. Subjects are subjectively able to draw white square tiles surrounded by black colored square tiles. According to PRC, a 1x1 white tile can be represented by a 2x2 white square tile, 2x2 white tile can be represented by a 4x4 white square tile, 3x3 white tile can be represented by a 6x6 white square tile, 4x4 white tiles can be represented by tiles a 9x9-sized white square, and a 5x5 white tile can be represented by a 11x11 white square tile. According to S1, a 1 × 1 square black tile can be represented by a 2x2 square black square tile. Based on the results of written answers and interviews 1a and 1b, the subject of the PRC does not meet the appropriate aspect of the spatial structure. The subject has not been able to draw the result of the installation of white tiles surrounded by black tiles appropriately.

Numerical Structure:

Figure 2. PRC’s Work about the number of black rectangular tiles

Based on the results of written assignments (Figure 2), the PRC is suspected to have not met the exact numerical structure. Then triangulation (interview result and written assignment result), with in depth interview result with the result as follows:

\[ P \]: What about problem 3a?
\[ PRC \]: Yeah, because on the first picture the number of white square tiles 1x1 in size there is one, in the second picture there is a white tile measuring 2x2, in the third picture there is also a white tile measuring 3x3 ... (while showing the picture)
P : Okay, next no 3b how?
PRC : The number of black tiles there are 9, 16 and 30 (while checking pictures) .. Sorry bu, the number of black tiles there are 8, 13 and 17.
P : Number 3c, how?
PRC : The number of rectangular black tiles is 4, 6, and 9

Based on interview results, the subject did not meet the exact numerical structure aspect. The subject has not been able to determine the number of white square tiles on the 1st, 2nd, and 3rd patterns. According to him, the number of white square tiles on the 1st, 2nd and 3rd patterns is one. Subjectively, the number of black square tiles on the 1st, 2nd and 3rd patterns is interpreted by the number of black rectangular tiles measuring 1x2.

Type II: Only Spatial Structure Correctly (Numerical Structure Incorrectly)

Figure 3. AND’s Work about the number of black rectangular tiles

Based on the results of written assignments (Figure 3), AND allegedly meets the aspect of the numerical structure appropriately. Furthermore, triangulation is done, with the result of in-depth interview with the result as follows:

P : What about the number 3d?
AND : The number of black tiles measuring 1x2 is 4, 6, 8
P : Okay, what about the number 3e?
AND : The number of black tiles that size 1x3 there are 4, 4, 5
P : Number 3f, how?
AND : The size of the black tile on the unity pattern is 1x2 there 4. The size of the black tile on the second pattern is 1x3 there 4. The size of the black tile on the third pattern is 1x4 there are 4

Based on interview results, the subject does not meet the exact numerical structure aspects. Unable to specify the number of black tiles measuring 1x2, 1x3, 1x (n + 1) on the 1st, 2nd, and 3rd patterns.

Type III: Only Numerical Structure Correctly (Spatial Structure Incorrectly)

Figure 4. ABI’s Work about the kind of rectangular black tile sizes

Based on the interview results (Figure 4), the subject did not meet the aspect of the spatial structure appropriately. This fact is supported by the following interview passage:
Q : What do you ask from question 2?
ABI : The size of the black tiles on the first, second, and third patterns are 3x3, 4x4, 5x5
P : 3x3, 4x4, and 5x5 including rectangle or square?
ABI : Square
Q : What square is rectangular asked?
ABI : Rectangle

Based on the interview results, the subject did not meet the aspect of the spatial structure appropriately. Unable to determine the exact size of rectangular tile sizes.

**Type IV: Spatial and Numerical Structure Correctly**

![Figure 5](image1.png)

Figure 5.(a). HBB’s Work about white square, (b) HBB’s Work about the kind of rectangular black tile sizes

Based on Figure 5, the subject meets the aspect of the spatial structure appropriately. Subjects were able to determine the exact size of rectangular tile sizes.

![Figure 6](image2.png)

Figure 6. HBB’s Work about the number of black rectangular tiles

Based on the results of a written assignment (Figure 6), HBB allegedly satisfies the numerical aspect of the structure appropriately. Further triangulation (interviews and results of written tasks), with the results of in-depth interview results are as follows:

P : What do you ask from question 3a?
HBB : The number of white square tiles is 1, 4, 9, 16, 25
P : Next to the number 3b, what’s your answer?
HBB : Number of black square tiles on pattern 1, 2, 3 is 8, 12, 16, 20, 24

Based on the results of the interview, the subject of HBB meets the exact aspect of the numerical structure. Able to determine the number of white square tiles on the 1st, 2nd, and 3rd patterns. Able to determine the number of black tiles 1x1, 1x2, 1x3, 1x (n + 1) and 1x (n + 2) in the 1st pattern, 2, and 3. The subject is able to find the nth pattern of the number of white tiles on the nth pattern ie the square of n.
4. Conclusion
The above examples show the special significance of the square grid structure. Spatially, it is fundamental to early geometry, measurement, and graphical representation. Numerically, it arises whenever a unit is repeated whether this unit be an object or a set of objects, a single shape or a composite shape, or a measurement unit. Algebraically, it lends itself to an early introduction to functional relationships and symbolisation. An early understanding of this structure is therefore vitally important to children’s mathematical development.

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