Running of Axial Coupling Constant

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We illustrate that both of massless pseudoscalar meson fields and constituent quark fields are dynamical field degrees of freedom in energy region between chiral symmetry spontaneously broken scale and quark confinement scale. This physical configuration yields a renormalization running axial coupling constant $g_A(\mu)$. The one-loop renormalization prediction $g_A(\mu = m_N) = 0.75$ and $g_A(\mu = m_\pi) = 0.96$ agree with $\beta$ decay of neutron and $\pi^0 \rightarrow \gamma \gamma$ respectively. We also calculate $\rho^\pm \rightarrow \gamma \pi$ and $\omega \rightarrow \gamma \pi$ decays up to the next to leading order of $N_c^{-1}$ expansion and including all order effects of vector meson momentum expansion. The results strongly support prediction of renormalization running, $g_A(\mu = m_\rho) = 0.77$.

The asymptotic freedom, chiral symmetry spontaneously broken (CSSB) and quark confinement (QC) are essential features of QCD, which are related to the non-Abelian gauge symmetry structure of QCD. They divide QCD-description of strong interaction into three different energy regions. It is well-known that the dynamical field degrees of freedom in the energy region above CSSB scale are current quarks and gluons, and in the energy region blow QC scale are pseudoscalar meson fields purely. In the energy region between CSSB scale and QC scale, the CSSB implies that the dynamical field degrees of freedom should be constituent quarks (quasi-particle of quarks), gluons and Goldstone bosons (massless pseudoscalar meson fields) associated with CSSB. Nevertheless, it is still debated whether pseudoscalar mesons are double counted when they are regarded as fundamental fields, as well as light bound states of quarks [1]. Theoretically, it is clear that there is not double counting problem, since fundamental pseudoscalar mesons are light bound states of light current quarks instead of heavy constituent quarks. Practically, however, due to lack of essential evidences to confirm this point, the pseudoscalar mesons are still regarded as compositied fields of constituent quarks in many works of low energy QCD in the literatures [2]. The purpose of this present paper is to provide an evidence to confirm that pseudoscalar mesons are fundamental dynamical degrees of freedom in the energy region below CSSB scale indeed.

Our analysis is based on to solve a puzzle which also lies low energy strong interaction. In QCD below CSSB scale, the axial weak current with light flavors can be defined with a non-unit coupling constant,$$ j_5^{a \mu} = g_A \bar{\psi} \gamma_\mu \gamma_5 \tau_a \frac{\tau_a}{2} \psi, \quad (a = 1, 2, 3),$$ where $\bar{\psi} = (\bar{u}, \bar{d})$ and $\tau^a$ are Pauli matrices. If we recognize that the wavefunction of nucleon has $SU(3)_{\text{color}} \times SU(2)_{\text{flavor}}$ symmetry, the beta decay of neutron requires $g_A = 0.75$ [1]. Meanwhile, the PCAC (partially conserved axial current) hypothesis and Adler-Bell-Jackiw anomaly [2] imply axial current divergence$$ \partial_\mu j_5^{a \mu} = \frac{f_\pi}{2} m_\pi^2 \pi^a + \frac{N_c}{24\pi} g_A \epsilon_{\alpha \beta \gamma_0} \epsilon^{\mu_\alpha \mu_\beta} F_{\mu_\alpha} F_{\mu_\beta}, \quad (1)$$ where $f_\pi = 185.2\text{MeV}$ is the pion decay constant and $F_{\mu_\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is photon field strength. Thus in chiral limit, the transition matrix element for the $\pi^0 \rightarrow \gamma \gamma$ reduces to

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The experimental data for $\Gamma(\pi^0 \rightarrow \gamma\gamma)$ yields axial coupling constant $g_A = 1$ for $N_c = 3$. This value is very different from one determined by $\beta$ decay of neutron. Then how to resolve this puzzle is an open question. In this paper, we will provide a possible explanation on this problem in the framework of constituent quark model. In this interpretation, $\beta$ from one determined by

\[ g_A \alpha_{e.m.} e^{\mu \nu \alpha \beta} \epsilon_{1 \mu \epsilon_{2 \alpha \epsilon_{2 \beta}}}. \]  

(2)

The pseudoscalar mesons must be regarded as independent dynamical field degrees of freedom in this energy region. The renormalization of pseudoscalar-constituent quarks loop effects will lead to axial coupling constant $g_A$ running with renormalization scale. Our results for one-loop renormalization are that taking $g_A(\mu = m_N) = 0.75$ as input, we predict $g_A(\mu = m_\pi) = 0.96$ which matches with requirement of $\pi^0 \rightarrow \gamma\gamma$ decay well. This result is an evidence that pseudoscalar mesons are fundamental fields in energy scale below CSSB. In order to confirm this conclusion further, we will calculate the anomol decays $\rho^\pm \rightarrow \gamma\pi^\pm$ and $\omega \rightarrow \gamma\pi^0$. In this type of decays, effects of $g_A$ are the leading order. Our calculations will be up to the next to leading order of $N_c^{-1}$ expansion and including all order effects of vector meson momentum expansion. The results strongly support prediction of one-loop renormalization for axial coupling constant, $g_A(\mu = m_\rho) = 0.77$.

The simplest version of chiral quark model was originated by Weinberg $^{[8]}$, and developed by Manohar and Georgi $^{[9]}$ provides a QCD-inspired description on the constituent quark model. At chiral limit, it is parameterized by the following $SU(3)_c$ invariant chiral constituent quark lagrangian

\[ \mathcal{L}_X = i\bar{\psi}(\partial + F + g_A \Delta \gamma_5 - )\psi - m_\psi \psi + \frac{F^2}{16} Tr_f \{\nabla_\mu U \nabla^\mu U^\dagger\}. \]  

(3)

Here $Tr_f$ denotes trace in $SU(3)$ flavour space, $\bar{\psi} = (\bar{u}, \bar{d}, \bar{s})$ are constituent quark fields. The $\Delta_\mu$ and $\Gamma_\mu$ are defined as follows,

\[ \Delta_\mu = \frac{1}{2}\{\xi^\dagger(\partial_\mu - ir_\mu)\xi - \xi(\partial_\mu - il_\mu)\xi^\dagger\}, \]

\[ \Gamma_\mu = \frac{1}{2}\{\xi^\dagger(\partial_\mu - ir_\mu)\xi + \xi(\partial_\mu - il_\mu)\xi^\dagger\}, \]  

(4)

and covariant derivative are defined as follows

\[ \nabla_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu = 2\xi^\dagger \Delta_\mu \xi, \]

\[ \nabla_\mu U^\dagger = \partial_\mu U^\dagger - il_\mu U^\dagger + iU^\dagger r_\mu = -2\xi^\dagger \Delta_\mu \xi, \]  

(5)

where $l_\mu = v_\mu + a_\mu$ and $r_\mu = v_\mu - a_\mu$ are linear combinations of external vector field $v_\mu$ and axial-vector field $a_\mu$. $\xi$ associates with non-linear realization of spontaneously broken global chiral symmetry $G = SU(3)_L \times SU(3)_R$ introduced by Weinberg $^{[8]}$.

\[ \xi(\Phi) \rightarrow g_R \xi(\Phi) h^\dagger(\Phi) = h(\Phi)\xi(\Phi)g_L^\dagger, \quad g_L, g_R \in G, \quad h(\Phi) \in H = SU(3)_c. \]  

(6)

Explicit form of $\xi(\Phi)$ is usual taken

\[ \xi(\Phi) = \exp \{i\lambda^a \Phi^a(x)/2\}, \quad U(\Phi) = \xi^2(\Phi), \]  

(7)

where the Goldstone boson $\Phi^a$ are treated as pseudoscalar meson octet. The constituent quark fields transform as matter fields of $SU(3)_c$,

\[ \psi \rightarrow h(\Phi)\psi, \quad \bar{\psi} \rightarrow \bar{\psi}h^\dagger(\Phi). \]  

(8)

$\Delta_\mu$ is $SU(3)_c$ invariant field gradients and $\Gamma_\mu$ transforms as field connection of $SU(3)_c$.

\[ \Delta_\mu \rightarrow h(\Phi)\Delta_\mu h^\dagger(\Phi), \quad \Gamma_\mu \rightarrow h(\Phi)\Gamma_\mu h^\dagger(\Phi) + h(\Phi)\partial_\mu h^\dagger(\Phi). \]  

(9)

Thus the lagrangian $^{[8]}$ is invariant under $G_{global} \times G_{local}$.

Let us consider one-loop effects renormalization of lagrangian $^{[8]}$. In terms of defining the following “renormalization” quantities,

$^1$It should be pointed out that, this model as low energy effective model of QCD is not completely renormalizable. The reason is that more and more new divergent terms will appear when loop effects are included. Fortunately, the terms in lagrangian $^{[8]}$ are still renormalizable.
the divergences yielded by loop-effects of constituent quarks and massless Goldstone fields can be cancelled by divergent constants $Z_{\psi}, Z_{g}, \delta m, Z_{F}$ and $Z_{\xi}$. In this paper, we focus our attention on renormalization of axial constant $g_{A}$. The diagrams in fig. 1 concern our purpose. In $\overline{\text{MS}}$ scheme, explicit calculation gives

$$Z_{\psi} = 1 - \frac{2m_{R}^{2}}{3\Lambda_{\chi}^{2}} g_{R_{A}} N_{e}, \quad Z_{\psi} \delta m = -\frac{2m_{R}^{2}}{3\Lambda_{\chi}^{2}} g_{R_{A}} N_{e}, \quad Z_{\psi} g_{R_{A}} = 1 - \frac{m_{R}^{2}}{6\Lambda_{\chi}^{2}} (3 + \frac{5}{12} g_{R_{A}}) N_{e}, \quad Z_{\psi} Z_{\xi} = 1 - \frac{m_{R}^{2}}{4\Lambda_{\chi}^{2}} (3 + \frac{5}{12} g_{R_{A}}) N_{e},$$

(11)

where $\Lambda_{\chi} = 2\pi f_{\pi} \simeq 1.2$ GeV is CSSB scale and

$$N_{e} = \lim_{\epsilon \to 0} \frac{2}{\epsilon} - \gamma_{E} + \ln 4\pi.$$

(12)

Then renomalization lagrangian is written as follow

$$\mathcal{L}_{R} = i \bar{\psi}_{R} (\mathbb{D} + \Gamma_{R}) \psi_{R} - \left\{ 1 - \frac{2m_{R}^{2}}{3\Lambda_{\chi}^{2}} g_{R_{A}} N_{e} \right\} m_{R} \bar{\psi}_{R} \psi_{R} + \left\{ 1 - \frac{m_{R}^{2}}{6\Lambda_{\chi}^{2}} (3 + \frac{5}{12} g_{R_{A}}) N_{e} \right\} g_{R_{A}} \bar{\psi}_{R} \gamma_{5} \psi_{R} + \frac{F^{2}}{16} Tr \{ \nabla_{\mu} U_{R} \nabla^{\mu} U_{R}^{\dagger} \},$$

(13)

where $\mu$ is renomalization scale. The $\mu$-independence of renormalization requires the constituent quark mass $m_{R}$ and axial constant $g_{R_{A}}$ should be $\mu$-dependent, and should satisfy(for sake of convenience, we omit the subscript “$R$” in $m_{R}$ and $g_{R_{A}}$ hereafter)

$$\frac{m(\mu)}{m(\mu')} = 1 - \frac{2\bar{m}^{2}}{3\Lambda_{\chi}^{2}} \bar{g}_{A} \ln \frac{\mu'}{\mu} + O\left( \frac{m_{4}}{\Lambda_{\chi}^{4}} \right),$$

$$\frac{g_{A}(\mu)}{g_{A}(\mu')} = 1 + \frac{\bar{m}^{2}}{\Lambda_{\chi}^{2}} (3 - \frac{4}{3} \bar{g}_{A}^{2}) \ln \frac{\mu'}{\mu} + O\left( \frac{m_{4}}{\Lambda_{\chi}^{4}} \right),$$

(14)

where

$$\bar{m} = \frac{m(\mu) + m(\mu')}{2}, \quad \bar{g}_{A} = \frac{g_{A}(\mu) + g_{A}(\mu')}{2}.$$

(15)
Then inputting \( m(\mu = m_\rho) = 480\text{MeV} \) and \( g_\Delta(\mu = m_N) = 0.75 \), we have
\[
\begin{align*}
m(\mu = m_\pi) &= 494\text{MeV}, \\
g_\Delta(\mu = m_\rho) = 0.77, \\
m(\mu = m_\pi) &= 368\text{MeV}, \\
g_\Delta(\mu = m_\pi) = 0.96.
\end{align*}
\]
In particular, \( g_\Delta(\mu = m_\pi) = 0.96 \) is agree with \( \pi^0 \rightarrow \gamma \gamma \) requirement.

Several remarks are necessary here. 1) The renormalization running of \( m \) and \( g_\Delta \) is rather larger. This point can be easily understood, since there is no a rather small parameter to suppress loop effects in this energy region. This is a feature of low energy QCD. For example, a low energy coupling constant of chiral perturbative theory at \( O(p^4) \), \( L_5 \), is equal to \((2.2 \pm 0.5) \times 10^{-3} \) at energy scale \( \mu = m_\rho \) but \((1.4 \pm 0.5) \times 10^{-3} \) at energy scale \( \mu = m_\rho \). 2) If pseudoscalar mesons are not independent dynamical field degrees of freedom, all diagrams in fig.1 will be absent. Then we can not understand why the value axial constant \( g_\pi \) should cause double counting, since triangle diagrams of constituent quarks also generate Adler-Bell-Jackiw anomally. Thus the lagrangian (17) is still invariant under SU(3). Thus, the lagrangian (18) is still invariant under SU(3). It transforms homogeneously under SU(3),
\[
V_\mu \rightarrow h(\Phi)V_\mu h^\dagger(\Phi),
\]
Thus the lagrangian (18) is still invariant under \( G_{\text{global}} \times G_{\text{local}} \). The effective action describing meson interaction can be obtained via integrating over degrees of freedom of fermions in lagrangian (17)
\[
e^{iS_{\text{eff}}} \equiv \int DqD\bar{q}e^{i\int d^4xL_{\text{eff}}(x)} = <\text{vac}, \text{out}|\text{in}, \text{vac}>_{\Delta, \Gamma},
\]
where \( <\text{vac}, \text{out}|\text{in}, \text{vac}>_{\Delta, \Gamma} \) is vacuum expectation value in presence external sources. The above path integral can be performed explicitly, and heat kernel method [10,11] has been used to regulate the result. However, this method is extremely difficult to compute very high order contributions in practice. This difficulty can be overcomed

\[2\text{The } O(p^4) \text{ chiral coupling constant } L_5 = \left( \frac{3}{5} \frac{\pi^2}{16\pi^2} - \frac{N_C}{16\pi^2} \right) \frac{m}{2m_\pi} \text{ is predicted in this model. Here } g \text{ and } B_0 \text{ are two constants absorbing divergence from quark loops. In particular, } g = \pi^{-1} \text{ and } B_0 = m_\pi^2/(m_u + m_d) \approx 2\text{GeV} \text{ is fitted at } \mu = m_\rho \text{ [9]. Thus } m = 480\text{MeV} \text{ is fitted by } L_5(\mu = m_\rho) = (1.4 \pm 0.5) \times 10^{-3}.\]
via calculating one-loop diagrams of constituent quarks directly. This method can capture all high order contributions of the chiral expansion.

In interaction picture, the equation \([20]\) is rewritten as follow

\[
e^{iS_{\text{eff}}} = <0|T_q e^i \int d^4x L_1^I(x)|0> = \sum_{n=1}^{\infty} i \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_n}{(2\pi)^4} \bar{\Pi}_n(p_1, \cdots, p_n) \delta^4(p_1 - p_2 - \cdots - p_n)
\]

\[
\equiv i \Pi_1(0) + \sum_{n=2}^{\infty} i \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_{n-1}}{(2\pi)^4} \bar{\Pi}_n(p_1, \cdots, p_{n-1}), \tag{21}
\]

where \(T_q\) is time-order product of constituent quark fields, \(L_1^I\) is interaction part of lagrangian(17), \(\bar{\Pi}_n(p_1, \cdots, p_n)\) is one-loop effects of constituent quarks with \(n\) external sources, \(p_1, p_2, \cdots, p_n\) are four-momentas of \(n\) external sources respectively and

\[
\Pi_n(p_1, \cdots, p_{n-1}) = \int \frac{d^4p_n}{(2\pi)^4} \bar{\Pi}_n(p_1, \cdots, p_n) \delta^4(p_1 - p_2 - \cdots - p_n). \tag{22}
\]

To get rid of all disconnected diagrams, we have

\[
S_{\text{eff}} = \sum_{n=1}^{\infty} S_n, \quad S_1 = \Pi_1(0), \quad S_n = \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_{n-1}}{(2\pi)^4} \bar{\Pi}_n(p_1, \cdots, p_{n-1}), \quad (n \geq 2). \tag{23}
\]

Hereafter we will call \(S_n\) as \(n\)-point effective action.

At the leading order of \(N_c^{-1}\) expansion, the 3-point \(\rho\gamma\pi\) and \(\omega\gamma\pi\) effective action is generated by triangle diagram of constituent quarks

\[
\Pi_3(q, k) = \frac{N_c}{3\pi^2 g f_{\pi}} e g_A \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \{1 - (x_1 + x_2)(1 - x_1 - x_2) \frac{q^2}{2m^2}\}^{-1}
\]

\[
\times \epsilon^{\mu\nu\alpha\beta} q_\alpha k_\beta \{\rho^i_\mu(q) A_\nu(k) \pi_i(q - k) + 3\omega_\mu(q) A_\nu(k) \pi^0(q - k)\}, \tag{24}
\]

where \(g\) is an universal coupling constant which absorbs nonrenormalizable logarithmic divergence from constituent quark loops. In the above equation, \(g\) is from vector meson field normalization

\[
\rho^i_\mu \rightarrow \frac{1}{g} \rho^i_\mu, \quad \omega_\mu \rightarrow \frac{1}{g} \omega_\mu.
\]

In ref. [7], \(g = \pi^{-1}\) has been fitted by the first KSRF sum rule [12].

It is rather complicate to calculate contribution from meson one-loop, i.e., the next to leading order contribution of \(N_c^{-1}\) expansion. The diagrams in fig.2 concern to our calculation. Here it is requirement of unitarity of \(S\)-matrix to sum over chain-like approximation in fig.2-b).

**FIG. 2.** Meson loop correction to \(\rho \rightarrow \gamma\pi\) and \(\omega \rightarrow \gamma\pi\) decay. The dot internal lines denote physical pseudoscalar meson fields(massless pion and massive \(K\) and \(\eta\). a) Tadpole loop. b) Chain-like approximation.
For obtain the tadpole correction in fig.2-a), at leading order of $N_c^{-1}$ expansion, vertices of $\rho\gamma\pi - \bar{K}K(\eta\eta)$ and $\omega\gamma\pi - \bar{K}K(\eta\eta)$ are needed if we treat pion as massless particle. In scheme of dimensional regularizasion, only the vertices generated by triangle diagram of constituent quarks yield non-zero contribution,

$$
\delta \Pi_3(q, k) = \frac{N_c}{16\pi^2g_f^2}\epsilon g_A \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \{1 - (x_1 + x_2)(1 - x_1 - x_2)\frac{q^2}{2m^2}\}^{-1}e^{\mu\nu\alpha\beta}g_{\alpha\beta}k_3\pi^i\phi^a\phi^b \\
Tr\left\{\frac{1}{3}(\rho^a_\mu(q))\lambda^i + 3\omega_\mu(q) + eQA_\mu(q)((\rho^a_\mu(k))\lambda^i + 3\omega_\mu(k))(\lambda^a, \lambda^b)[\lambda^i, \lambda^b]\right\} \\
+ eA_\mu(q)([\lambda^a, Q]\lambda^b + \lambda^a[Q, \lambda^b])(\rho^a_\mu(k))\lambda^i + 3\omega_\mu(k)\pi^i \\
+ e(\rho^a_\mu(q)\lambda^i + 3\omega_\mu(q))A_\mu(k)([\lambda^a, Q]\lambda^b + \lambda^a[Q, \lambda^b])\pi^i\right\}, \quad (i, j, l = 1, 2, 3; a, b = 4, ..., 8) \tag{25}
$$

where $Q = \text{diag}(2/3, -1/3, -1/3)$ is charge operator of quark fields, $\phi^a$ denotes $\bar{K}$ or $\eta$-meson fields in internal line. For sake of convenience, we can assume that the masses of $K$ and $\eta$ are degenerated. Then integral over $\phi$ fields, we have tadpole correction to $\rho\gamma\pi$ and $\omega\gamma\pi$ vertices

$$
\Pi_3^{ad}(q, k) = -\frac{4}{3}\delta \Pi_3(q, k), \quad \zeta = \frac{m_\pi^2}{8\pi^2f_\pi}(\frac{4\pi\mu^2}{m_\pi^2})^{\epsilon/2}\Gamma(1 - \frac{D}{2}) \equiv \frac{m_\pi^2}{8\pi^2f_\pi^2}\lambda, \tag{26}
$$

where $\Pi_3(q, k)$ is defined in eq.(24), $\lambda$ absorb the quadratic divergence from meson loops. $\lambda \simeq 2/3$ has been fitted by Zweig rule $[\text{Z}]$.

FIG. 3. The chain-like approximation in fig.2-b) is equivalent to contribution of fig.3-a).

It is generated through contracting an effective VPP(where “V” denotes $\rho$ or $\omega$ meson, “P” denotes pseudoscalar meson) vertex and $\gamma\pi$-PP vertex. Here the effective VPP vertex includes not only tree level vertex but also meson loop contribution(see fig.3-b).

Next we will calculate the contribution in fig.2-b). It is equivalent to calculate the diagram in fig.3-a). In other words, we need to an effective VPP(where “V” denotes $\rho$ or $\omega$ meson, “P” denotes pseudoscalar meson octet) vertex and $\gamma\pi$-PP vertex. In fig.3-b) we have shown that the effective VPP vertex includes not only tree level vertex but also meson loop contribution.

At leading order of $N_c^{-1}$ expansion, the anomalous $\gamma\pi$-PP vertex is generated by both of triangle and box diagrams of constituent quarks,

$$
\Gamma_{\gamma\rightarrow 3P}(q_2, k_1, k_2, k_3) = \frac{3i}{8f_\pi^2}eB(q^2)e^{\mu\nu\alpha\beta}A_\mu(q_2)k_1, k_2, k_3, k_3 T r f \{Q P(k_1) P(k_2) P(k_3)\}, \\
B(q^2) = \frac{N_c}{3\pi^2 f_\pi^2}g_A \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \{1 - \frac{q^2}{2m^2}(x_1 + x_2)(1 - x_1 - x_2)\}^{-1} - \frac{g_\pi^2}{6}, \tag{27}
$$

where $q^2 = (k_1 + k_2)^2$. The effective $\rho$-PP vertex and $\omega - \bar{K}K$ vertex[here we omit those vertices suppressed by isospin conversation] have been derived in ref. $[\text{Z}]$.

$$
\Gamma_{\rho PP}(q, k_1, k_2) = \frac{1}{2}g_{\rho PP}(q^2)(q^2k_{2\mu} - q_1 q_2 k_2) T r f \{\rho^\mu(q) P(k_1) P(k_2)\}, \\
\Gamma_{\omega KK}(q, k_1, k_2) = \frac{1}{2}g_{\omega KK}(q^2)(q^2k_{2\mu} - q_1 q_2 k_2) o\mu\{(K^+(k_1)K^-(k_2) + K^0(k_1)K^0(k_2)) + (k_1 \leftrightarrow k_2)\}, \tag{28}
$$

with

$$
g_{\rho PP}(q^2) = \frac{A_1(q^2) + g_\pi^2 A_2(q^2)}{g f_\pi^2(1 + 2\zeta)(1 + \Sigma(q^2))}, \\
g_{\omega KK}(q^2) = \frac{A_1(q^2) + g_\pi^2 A_2(q^2)}{g f_\pi^2(1 + 2\zeta)(1 - 2\Sigma_K(q^2))}, \tag{29}
$$

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where

\[ A_1(q^2) = g^2 - \frac{N_c}{\pi^2} \int_0^1 dx \cdot (1-x) \ln \left( 1 - \frac{x(1-x)q^2}{m^2} \right), \]

\[ A_2(q^2) = -g^2 + \frac{N_c}{2\pi^2} \int_0^1 dx_1 \int_0^1 dx_2 (x_1 - x_1^2 x_2) \{ 1 + \frac{m^2}{m^2 - x_1(1-x_1)(1-x_2)q^2} \} \]

\[ + \ln \left( 1 - \frac{x_1(1-x_1)(1-x_2)q^2}{m^2} \right), \]

\[ \Sigma(q^2) = \left\{ 1 + \frac{q^2(A_1(q^2) + 2g_\rho^2 A_2(q^2))}{2f_\pi^2(1 + 11\xi/3)} \right\} (4\Sigma_\pi(q^2) - 2\Sigma_K(q^2)), \]

\[ = \frac{q^2}{16\pi^2 f_\pi^2} \left\{ \frac{\lambda}{6} + \int_0^1 dt \cdot t(1-t) \ln \left( \frac{1-x^2 t}{m_K^2} \right) + \frac{i}{6} \arg(-1)\theta(q^2 - 4m_N^2) \right\}, \]

\[ \Sigma_K(q^2) = \frac{1}{16\pi^2 f_\pi^2} \left\{ \lambda(m_K^2 - \frac{q^2}{6}) + \int_0^1 t(1-t)q^2 \left( \ln \left( \frac{1-x^2 t}{m_K^2} \right) \right) \right\}. \]

Then due to soft-pion theorem, the pseudoscalar meson loops in fig.2-b) contribution to \( \rho \gamma \pi \) and \( \omega \gamma \pi \) vertices as follow

\[ \Pi_3^{1-loop}(q,k) = -e q^2 g_{\rho PP}(q^2)[A(q^2) + 2A(0)][\Sigma_\pi(q^2)]^{-1} \left\{ \frac{1}{2} \Sigma_K(q^2) \right\} \cdot \left( \frac{\rho_{\mu\nu} A_{\nu}(k) A_\mu(q) \pi(q-k)}{2m_K^2} \right), \]

\[ + \frac{1}{2} q^2 g_{\omega KK}(q^2)[A(q^2) + 2A(0)][\Sigma_K(q^2)]^{-1} \left\{ \frac{1}{2} \Sigma_K(q^2) \right\} \cdot \left( \frac{\omega_{\mu\nu} A_{\nu}(k) A_\mu(q) \pi(q-k)}{2m_K^2} \right). \]

Here \( \rho \gamma \pi \) coupling receives contributions from both of pion-loop and K-loop, but \( \omega \gamma \pi \) coupling receives dominant contribution from K-loop only.

Eq.(33) together with eqs.(24) and (25) give “complete” \( \rho \gamma \pi \) and \( \omega \gamma \pi \) coupling up to the next to leading of \( N_c^{-1} \) expansion at least,

\[ \Pi_3(q,k) = e q^2 g_{\rho PP}(q^2)[A(q^2) + 2A(0)][\Sigma_\pi(q^2)]^{-1} \left\{ \frac{1}{2} \Sigma_K(q^2) \right\} \cdot \left( \frac{\rho_{\mu\nu} A_{\nu}(k) A_\mu(q) \pi(q-k)}{2m_K^2} \right), \]

\[ g_{\rho PP}(q^2) = \frac{N_c}{\pi^2 f_\pi^2} g_{\rho}(1 - \frac{4}{3}\xi) \int_0^1 dx_1 \int_0^{1-x_1} dx_2 (1 - (x_1 + x_2)(1-x_1-x_2) \frac{q^2}{2m^2})^{-1} \]

\[ - q^2 g_{\rho PP}(q^2)[A(q^2) + 2A(0)][\Sigma_\pi(q^2)]^{-1} \left\{ \frac{1}{2} \Sigma_K(q^2) \right\}, \]

\[ g_{\omega KK}(q^2) = \frac{N_c}{\pi^2 f_\pi^2} g_{\omega}(1) \int_0^1 dx_1 \int_0^{1-x_1} dx_2 (1 - (x_1 + x_2)(1-x_1-x_2) \frac{q^2}{2m^2})^{-1} \]

\[ + \frac{1}{2} q^2 g_{\omega KK}(q^2)[A(q^2) + 2A(0)][\Sigma_K(q^2)]. \]

For \( g_{\rho}(m = m_\rho) = 0.77 \), the above results yield

\[ B(\rho^0 \to \pi^0 \gamma) = 4.88 \times 10^{-4}, \quad B(\omega \to \pi^0 \gamma) = 8.99\%. \]

These results agree with data\[ 13 \] \( B(\rho^0 \to \pi^0 \gamma) = (4.5 \pm 0.5) \times 10^{-4} \) and \( B(\omega \to \pi^0 \gamma) = (8.5 \pm 0.5)\% \) well. If we take \( g_{\rho}(1) = 1 \) as a comparison, the theoretical predictions are

\[ B(\rho^0 \to \pi^0 \gamma) = 8.33 \times 10^{-4}, \quad B(\omega \to \pi^0 \gamma) = 15.0\%. \]

These results obviously disagree with data. Therefore, the theoretical prediction\[ 34 \] provides an evidence to confirm the result in eq.\[ 13 \].

To conclude, we discuss the one-loop renormalization in Manohar-Georgi model. This renormalization leads to running of axial coupling constant and constituent quark mass. Inputting \( g_\rho(\mu = m_N) = 0.75 \) which agree with \( \beta \) decay of neutron, the prediction \( g_{\rho}(\mu = m_\rho) = 0.96 \) agree with requirement of \( \pi^0 \to \gamma \gamma \) decay. We also notice that, if massless pseudoscalar meson fields were not independent degrees of freedom in energy between CSSB scale and QC scale, we could not interpret \( \beta \) decay of neutron and \( \pi^0 \to \gamma \gamma \) decay simultaneously. For example, because axial constant is not unit in Extend Nambu-Jona-Lasinio model\[ 4 \], the model can not yield right prediction for \( \pi^0 \to \gamma \gamma \) decay. The similar problem also exists in the models of ref.\[ 4 \]. Therefore, our result in this paper can be regarded as
an evidence that pseudoscalar meson fields are independent degrees of freedom in energy between CSSB scale and QC scale.

The renormalization of axial constant also predicts $g_A(\mu = m_\rho) = 0.77$. For confirming this prediction, we calculate anomalous vector meson decays, $\rho^\pm \to \gamma \pi$ and $\omega \to \gamma \pi$. In this type of decays, effects of $g_A$ is leading order. Our results are up to the next to leading order of $N_c^{-1}$ expansion and include all order effects of vector meson momentum expansion. The theoretical predictions for branch ratios of these two decays agree with results of renormalization and is against $g_A = 1$.

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