Phenomena of Stability Loss of Rotor Rotation at Tilting Pad Bearings

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cAbstract

It is experimentally observed and studied the phenomena of stability loss of rotor rotation in tilting pad bearings of different designs. The trends of changing by stability limiting rotational speed of rotor depending on its unbalance and type of journal bearing are obtained. The results of spectral analysis and phase paths of rotor motion are presented, features of rotor dynamics in range of unstable rotational speed are analyzed. The methods of building and identification of discrete non-linear mathematical models of rotor are developed, permitting to analyze polyharmonic vibrations.

Type of journal bearing selected when turbo-compressor designed largely determines its further vibration reliability. For example, application of tilting pad bearings in 70s permitted to extend range of stable rotational speeds of turbo-compressor rotors rotation and it was considered that the problem of their stability was solved. Occurred increased vibration sometimes by mistake referred to unbalance. In reality subharmonic components occurred resulted in stability loss. This fact could not be established by steady-state measuring devices of vibration on mounting assemblies.

Further development of oil and gas industry requires designing of turbo-compressors with higher pressure ratio that is reached as a rule by increasing number of rotor impellers as well as increasing working rotational speeds. The greater part of such turbo-compressors have comparatively inconsiderable

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weight, and respectively they provide little specific load to bearing and in this case they operate in close proximity to second critical speed. That is why rational selection of type of used bearing providing reliable operation of such compressors in wide range of rotational speed is of current concern.

Taking into consideration the abovementioned, testing of model rotor with mass 150 kg with diameter of bearing journals 90 mm was performed for analysis of phenomena of stability loss at different types of bearings. Testing was performed at balancing rig with Schenck vacuum chamber. General view of rig with model rotor is given in fig. 1.

![Fig. 1. Model rotor at balancing rig](image1)

Four types of bearings were used when tested: 5-pad bearing with self-aligning pads (fig.2.a), 4 pad (fig.2.b) and 3 pad (fig.2.c) damper bearings, as well as 3 centre bearing (fig.2.d).

![Fig. 2. Types of tested bearings](image2)

Four rotor speedups were performed for each type of bearing. First start was performed for rotor balanced with high accuracy (vibration speed of rig supports in all range of rotation speeds did not exceed 1.0 mm/sec). At sequent three starts different balance was applied to nearest disk to front support. Amplitude of rotor vibration displacement near bearing journals as well as in central part was recorded in horizontal and vertical directions. Vibration measuring was performed by eddy current sensors SD 052. Conversion of analog signal to digit signal was performed by analog to digital converter L-791 Lcard, which provides continuous collection of analog data at sampling frequencies from 0.005 HZ to 400.0 kHz.
Upon test results values of limiting rotational speed of rotor $\omega_{gr}$ are determined depending on unbalance. Results are shown in fig. 3 as dependence diagrams of limiting relative rotational speed $\bar{\omega} = \omega_{gr} / \Omega$ on relative unbalance $\varepsilon$.

Figure 3 shows that for all types of bearings increasing of unbalance results in reducing rotor stability and restriction of stable rotational speeds intervals that confirms earlier conducted theoretical analysis [1,2].

3 pad damper bearing was found to be more stable to self-induced vibration. It shows its highest damper properties for quenching nonsynchronous oscillations. The minimum interval of stable rotational speeds was for 4 pad damper bearing. Evidently it is connected with conditions of pads loading. In List of Reference Literature [3] it is noted that only two opposite pads are loaded in 4 pad bearing, while uniform hydrostatic film for all liners was provided in 3 pad bearing. For 3 centre bearing and 5 pad bearing with self-aligning pads limits of stable frequencies are close, but with increasing unbalance for 3 center bearing more sharp breakdown of stable frequencies range occurs.

For all considered types of bearings the following rules are determined:
1) After stability loss self-excited sub-harmonic components occur;
2) At frequencies higher limiting one and close to it self-induced vibration amplitude is lower than synchronous one; with increasing rotational speed self-induced vibration increases quicker than synchronous one, and exceeds it;
3) Self-induced vibration speed does not depend on rotor rotational speed and is approximately constant, approximately equal to first critical speed of rotor;
4) Self-induced vibrations are of polyharmonic nature; their spectrum has except basic component with frequency equal to critical frequency of rotor (41-43 Hz), components with multiple frequencies 83-85 Hz and 123-126 Hz.

It was also analyzed the effect of different technical parameters of bearings on changing limit of stable rotational speed. During performed experiments it was determined that optimal temperature is within 25 - 35°C, nature of temperature effect on motion stability for all analyzed types of bearings is the same. Diametrical clearance for 5 pad bearing at which rotor has the highest limit of stability is equal to 0,2 mm that meets design values of optimal clearance 0,16 - 0,22 mm. Increasing of oil pressure applied to
bearing considerably effects on rotor motion stability till obtaining 0,15 MPa lightly increasing limit of stable rotational speeds, practically not changing it.

Comparative analysis of races in unstable range of rotational speed showed their difference depending on bearing type.

Methods of building discrete non-linear models of rotor systems have been developed by calculation for analysis of processes of stability loss and appearance of polyharmonic vibrations. The existing methods and programs for calculation of rotors dynamics (for example, on the basis of finite elements method (FEM)) it is permitted to determine critical speeds and forms of natural and forced vibrations of synchronous precession. At the same time analysis of such complicated phenomena as stability loss, appearance of asynchronous self-excited components is out of calculation as per these programs. These phenomena can be analyzed only by numerical integration of equations for rotor motion. Earlier it was considered models with same mass (more rarely – two mass) models, by means of which it managed to determine some trends [5, 6, 7]. However, one mass model is not enough not only for qualitative but for sufficiently adequate quantitative analysis of rotor dynamics of one mass model. It is required discrete 3, 4 mass models which fully represent dynamic properties of physical design and detect possibility of consideration of nonconservative non-linear forces in bearings as well as effects connected with availability of inner friction [2]. Such models can be efficient when developed control systems for magnetic bearings of turbo-compressors with flexible rotors. Application of these models taking into account effective programs of numerical integration of differential equations systems offers wide opportunities for analysis of rotor dynamics of energy-converting machinery.

For solving abovementioned problems the method of building discrete model of rotors with limited masses number is developed.

Diagram of rotor discrete model is given in fig.4:

![Fig. 4. Rotor Discrete Model](image)

One can choose the number of lumped masses \( n \), where the first and last masses are located at supporting points. To derive the differential equations for oscillations first put down formulae for static rotor bends under some constant forces \( F_1, \ldots, F_n \) applied to relevant masses \( m_1, \ldots, m_n \). Based on conditions for static equilibrium and formulae for strength of materials for beam deflections finally it could be got the following formulae for bending at supporting points as:

\[
R_B = F_n + \sum_{i=1}^{n-1} a_{i-1} F_i, \quad x_1 = \frac{R_A}{c_1}
\]

\[
R_A = F_1 + \sum_{i=2}^{n-1} (l - a_{i-1}) F_i, \quad x_n = \frac{R_B}{c_2}
\]

under intermediate masses:
\[ x_i = x_i + \frac{a_{i-1}}{l} (x_n - x_i) + \sum_{i=2}^{n-1} \delta_{ij} \cdot F_j \quad (i, j = 2, n-1), \quad (2) \]

where \( \delta_{ij} \) is a compliance coefficient which can be easily calculated for real rotor model predicted by FEM as static bends at \( i \) - point from individual force, applied at \( j \) point.

Further setting according to D’Alembert’s method,

\[ F_i = -m_i \cdot \ddot{x}_i \quad (i = 1, n), \]

and inserting into formulae for bending (1), (2) we can get the following differential equation for oscillations of rotor discrete model:

\[
\begin{align*}
x_i &= \frac{1}{c_1} (-m_i \cdot \ddot{x}_i + \frac{1}{l} \sum_{i=2}^{n-1} (l - a_{i-1})(-m_i \cdot \ddot{x}_i)), \\
x_i &= x_i + \frac{a_{i-1}}{l} (x_n - x_i) - \sum_{j=2}^{n-1} \delta_{ij} \cdot m_j \cdot \ddot{x}_j, (i = 2, n-1) \\
x_n &= \frac{1}{c_2} (-m_n \cdot \ddot{x}_n - \frac{1}{l} \sum_{i=2}^{n-1} a_{i-1} \cdot m_i \cdot \ddot{x}_i).
\end{align*}
\]  

(3)

One sets up an equation for mode shapes setting into (3)

\[ x_i = A_i \sin \omega t. \]

After obvious transformations one can calculate any \( k \) mode shape, setting, for example, \( A_1 = 1 \) and then determining \( A_k (i = 2, n) \), insert into them a value of \( k \) natural frequency \( \omega = \omega_k \) and throwing out one of the equations \( (i = 2, n-1) \).

Here one can write the following formulae taking the first \( s \) of critical frequencies:

\[
\begin{align*}
\frac{m}{c_1} \omega_k^2 + \frac{1}{c_1 \cdot l} \sum_{j=2}^{n-1} (l - a_{j-1}) \omega_k^2 \cdot A_{kj} \cdot m_j &= 1, \\
\sum_{j=2}^{n-1} \delta_{ij} \cdot \omega_k^2 \cdot A_{kj} \cdot m_j &= A_{ki} - \frac{a_{i-1}}{l} (A_{kn} - A_{k1}) - 1, \quad (i = 2, n-1), \\
\frac{1}{c_2 \cdot l} \sum_{j=2}^{n-1} a_{j-1} \cdot \omega_k^2 \cdot A_{kj} \cdot m_j + \frac{m}{c_2} \omega_k^2 \cdot A_{kn} &= A_{kn}
\end{align*}
\]

(4)

where: \( k = 1, s \); \( \omega_k \) - \( k \) is a natural frequency; \( s \) - is a number of critical frequencies.
Or in matrix form:
\[
\overline{B} \cdot \overline{m} = \overline{D},
\]  
(5)

where \(\overline{B}\) is a \((n \cdot s) \times n\) matrix, \(\overline{m}\) - is \(n\) column of equivalent masses vector , \(\overline{D}\) is \(s \cdot n\) column of right members.

The idea to create a discrete model deals with “estimation” of masses \(m_1\),...,\(m_n\) based on linear relations (5), if components of \(\overline{B}\) matrix and \(\overline{D}\) columns are calculated based on calculus of “large” FEM – rotor model, taking from this calculus \(s\) values of the first natural frequencies \(\omega_1\),...,\(\omega_s\) and \(n\) relevant amplitudes of natural modes, identified at points of mass position. Therefore it is applied linear parameter estimation method, where FEM calculus results – model are taken as “experimental data”.

It is worthwhile to say that such approach can be very fruitful when creating simple models (equations, calculus methods) based on calculus data of “large” models realized by PC software.

Using linear regression equation one can put down analysis matrix relation to “estimate” equivalent masses of discrete model:
\[
\overline{m} = [\overline{B}^T \cdot \overline{B}]^{-1} \overline{B}^T \cdot \overline{D}.
\]  
(6)

To verify the obtained discrete models for different types of real rotors the comparison of static deflection, critical frequency values and natural mode shaped has been made. Difference between mode shapes for discrete model and for model with distributed masses can be estimated by calculus of shapes transformation matrix [9], which diagonal components show difference between mode shapes and corresponding to them modal unbalances per each of mode shapes:
\[
b_{ij} = \frac{1}{m_{modi}} \int_0^L \mu_{(x)} \cdot \phi_{i(x)}^2 \cdot \phi_{j(x)}^2 dx
\]

where: \(m_{modi} = \int_0^L \mu_{(x)} \cdot \phi_{i(x)}^2 dx\); \(\mu_{(x)}\) is a mass per meter; \(\phi_{i(x)}\) is a mode shape of FEM model; \(\phi_{j(x)}\) is a mode shape of discrete model.

Results of comparison are shown in Table 1.
Table 1.

| No. | Number of masses | Compressor series, (speed, RPM) | Static deflection, μm | Critical speed values rad/s | Difference of diagonal matrix components \(b_{ij}\) |
|-----|-----------------|-------------------------------|-----------------------|-----------------------------|----------------------------------|
|     |                 |                               | Distributed masses    | Discrete model              | Distributed masses    | Discrete model              | Error, %                       |                                |
| 1   | 3               | 291GC2-400/56-76M, (3710-5565) | 473                   | 475                         | 1418 2021              | 7628 2036                  | -1.13 0.74 0.54               | 0.99(1 %) 0.97(3 %)          |
| 2   | 3               | 16GC2-395/53-76C, (3710-5565)  | 19                    | 19                          | 6943 18042             | 7037 18051                  | 1.35 0.05                     | 0.98(2 %) 0.88(12 %)         |
| 3   | 4               | 295GC2-560/10-30, (3640-5460)  | 189                   | 189                         | 2303 7995              | 2307 8188                  | 0.17 2.41                     | 0.99(1%) 0.96(4%)           |
| 4   | 4               | HP C153GC2-21/125-300M12 (8917-13375) | 45                   | 45                          | 4703 18158             | 4690 18224                  | -0.28 0.36                    | 1.00(0%) 0.96(4%)          |

As calculus showed for rotors of both tilting-pad bearing and magnetic bearing designs which operating speed range is within the first critical speed range three-mass model can be used (See No.1,2 in Table 1). Rotors operating between the first and the second critical speeds should be presented by four-mass model (See No. 3,4 in Table 1).

Next it is proposed non-linear method of models creation based on the obtained discrete linear systems using above mentioned procedure, and then nonlinear terms, determined by hydrodynamic processes in the bearings should be inserted into them. For example, for four-mass model of the analyzed rotor the differential equations are as follows:
\[
\begin{align*}
m_1 \ddot{x}_1 + c_1 x_1 + \frac{b + c}{l} m_2 \ddot{x}_2 + \frac{c}{l} m_3 \ddot{x}_3 + d_{II} \ddot{x}_1 + q \cdot y_1 + \alpha \cdot x_1 \cdot |x_1| &= 0 \\
m_1 \ddot{y}_1 + c_1 y_1 + \frac{b + c}{l} m_2 \ddot{y}_2 + \frac{c}{l} m_3 \ddot{y}_3 + d_{II} \ddot{y}_1 - q \cdot x_1 + \alpha \cdot y_1 \cdot |y_1| &= 0 \\
m_2 \ddot{x}_2 + \frac{1}{\delta_{22}} x_2 + \frac{\delta_{23}}{\delta_{22}} m_3 \ddot{x}_3 - \frac{a}{l \cdot \delta_{22}} y_3 = \frac{x_1}{\delta_{22}} \cdot \frac{b + c}{l} = m_2 \cdot e_2 \cdot \omega^2 \cdot \cos \alpha \\
m_2 \ddot{y}_2 + \frac{1}{\delta_{22}} y_2 + \frac{\delta_{23}}{\delta_{22}} m_3 \ddot{y}_3 - \frac{a}{l \cdot \delta_{22}} y_1 = \frac{y_1}{\delta_{22}} \cdot \frac{b + c}{l} = m_2 \cdot e_2 \cdot \omega^2 \cdot \sin \alpha \\
m_3 \ddot{x}_3 + \frac{1}{\delta_{33}} x_3 + \frac{\delta_{32}}{\delta_{33}} m_2 \ddot{x}_2 - \frac{c}{l \cdot \delta_{33}} \cdot \frac{a + b}{l} = 0 \\
m_3 \ddot{y}_3 + \frac{1}{\delta_{33}} y_3 + \frac{\delta_{32}}{\delta_{33}} m_2 \ddot{y}_2 - \frac{c}{l \cdot \delta_{33}} \cdot \frac{a + b}{l} = 0 \\
m_4 \ddot{x}_4 + c_2 \cdot x_4 + \frac{a}{l \cdot m_2} \ddot{x}_2 + \frac{a + b}{l \cdot m_3} \ddot{x}_3 + d_{II} \ddot{x}_4 + q \cdot y_4 + \alpha \cdot x_4 \cdot |x_4| = 0 \\
m_4 \ddot{y}_4 + c_2 \cdot y_4 + \frac{a}{l \cdot m_2} \ddot{y}_2 + \frac{a + b}{l \cdot m_3} \ddot{y}_3 + d_{II} \ddot{y}_4 - q \cdot x_4 + \alpha \cdot y_4 \cdot |y_4| = 0
\end{align*}
\]

where \( q = \delta \cdot d_{II} \cdot \omega \) is a circulating force factor, \( \delta \) - dimensionless factor of circulating force, \( \alpha \) is a coefficient under nonlinear quadratic stiffness, \( d_{II} \) is a bearing damping coefficient, \( m_1, m_2, m_3, m_4 \) are discrete masses of a rotor; \( x_i, y_i (i=1,4) \) are corresponding coordinates of these masses in horizontal and vertical planes, \( c_1, c_2 \) are bearing stiffnesses; \( l = a + b + c \) is a rotor between supports; \( \delta_{22}, \delta_{33}, \delta_{32} \) are compliance coefficients of a rotor at relevant points, calculated on the base of FEM model; \( \omega \) is a rotor speed; \( e_2 \) is a relative unbalance of the disk \( m_1 \).

Numerical experiments based on integration of system of nonlinear differential equations (7) using Runge-Kutta method were carried out in Maple software environment.

Model parameters (7) were estimated in the following order. Masses were calculated using above mentioned method. Bearing stiffnesses as well as a bearing damping coefficient were determined from experimental data of synchronous precessions according the estimation methods specified in [10]. Next supposing that a coefficient value under nonlinear stiffness is equal to zero, one can determine a value of dimensionless factor of circulating force \( \delta \) so that stability boundaries defined by numerical and experimental analysis should coincide. Then having a found out value of \( \delta \) one can try a coefficient value under nonlinear stiffness \( \alpha \) so that amplitudes of self-vibrating components of mathematical and experimental models should coincide.

Using experimental data values of coefficient under nonlinear stiffness and circulating force factor for bearings of different types were the result of numerous computational experiments. For example, for tilting 5-pad bearing they occur within the following range accordingly as:

\[
\alpha = 1 + 2 \cdot 10^6 \frac{N}{m^2}, \quad \delta = 0.3 \pm 0.33.
\]
and for damper 3-pad bearing within the following range:

\[ \alpha = 2 + 3 \cdot 10^6 \frac{N}{m^3}, \ \delta = 0.26 + 0.29. \]

Comparison of dynamic characteristics obtained by mathematical models together with experiment data shows almost high. As an example Figure 5 shows comparison of spectra and orbits of mathematical model (Fig. 5a) and experimental data (Fig. 5b) for model rotor of damper 3-pad bearing design.

![Vibration spectra and motion orbits](image)

Fig. 5. Vibration spectra and motion orbits obtained by using: (a) Mathematical model; (b) Experiment

**Conclusion**

All analyzed bearings types under speed 2.3-4 times higher the first critical one it is observed loss in stability resulting in subharmonic self-vibrating overlaps with the main component having a frequency close to rotor critical speed.

Based on the results it can be concluded that max. range of stable frequencies conforms to the rotor 3-pad bearing design.

It has been developed a method of creation and identification of nonlinear discrete multimass mathematical models of turbo-compressor rotor vibration enabling to study features of rotor dynamics related to nonlinear phenomena in tilting-pad bearings and other elements of rotor systems using calculus.
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