Stock price prediction using geometric Brownian motion

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Abstract. Geometric Brownian motion is a mathematical model for predicting the future price of stock. The phase that done before stock price prediction is determine stock expected price formulation and determine the confidence level of 95%. On stock price prediction using geometric Brownian Motion model, the algorithm starts from calculating the value of return, followed by estimating value of volatility and drift, obtain the stock price forecast, calculating the forecast MAPE, calculating the stock expected price and calculating the confidence level of 95%. Based on the research, the output analysis shows that geometric Brownian motion model is the prediction technique with high rate of accuracy. It is proven with forecast MAPE value ≤ 20%.

1. Introduction
Capital market are markets for various long term financial instruments that can be traded. Capital markets are vital for function of country economy since capital markets generate two functions, first as decision variable for investor or as a facility for companies to get funding from investors, second as capital markets become facility for society to invest on financial instruments and one of them is stock [1].

Stock price index is indicator for stock price movement. The index is one of the references for investor to invest in capital markets, especially stock. One of the stock price indexes in Indonesia Stock Exchange (IDX) is Indeks Harga Saham Gabungan (IHSG) or Jakarta Composite Index. To be able to describe reasonable market condition, IHSG use all companies registered as index calculation components [2].

Stock is one of the popular instruments in financial market. It is because stock able to give high rate of profit return which interest the investors. However, stock trading has high rate of risk. The fluctuation of stock price affect the investor’s decision on investing their capital. Mistake in decision making will result loss for investor. Thus, to minimize the high risk investor needs information as a reference for decision making of which stock they should buy, sell, and maintain[3].

There are 2 factors that has significant influence in stock price modelling, which previous state of stock that influence the current stock price and stock response towards latest information of stock [4]. Based on those factors, it can be concluded that the change of stock price follows the Markov chain. The Markov chain process is a stochastic process which make current price has influence to forecast the future stock price.

Forecasting is the best method to predict the future of stock price [5]. However, the rate of loss risk using this method is relatively still high due to the inaccurate forecast output. The forecasting method used for forecast the future stock closing price for short term investment with low rate of. One of the model that can be used for forecasting stock price is geometric Brownian motion model or as known
as Wiener process. Geometric Brownian motion model is stochastic model with continuous time, where the random variable follows the Brownian motion [5].

On the previous research the concept of geometric Brownian motion has been described by Dmouj [4]. Based on [4] it is described the concept of random walk, Brownian motion and analytical solution of model geometric Brownian motion model. In review [4] stated that the forecast of stock close price is develop using confident level and mean function of lognormal distribution. Other research that implement geometric Brownian motion model is research conducted by Omar dan Jaffar [5]. Their research forecasted the stock close price for several small companies registered in Malaysia stock exchange. The forecast is limited to short term investment. In their research it is proven that geometric Brownian motion model is accurate in forecasting the stock close price for two weeks period. It is proven by the small value of Mean Absolute Percentage Error (MAPE).

Based on that background, this research forecast the stock price of Jakarta Composite Index using the geometric Brownian motion. With utilizing the daily stock close price during January 2014 to December 2014 to forecast the stock price of January 2015. The stages for forecasting the stock price are calculating return value, Estimating the parameter, result collection of stock price forecast, then calculating the MAPE value. In this research 4 forecasts are obtained using geometric Brownian motion. Based on analysis and discussion, the MAPE value $\leq 20\%$.

2. Research Methodology

The methodology in conducting this research as follows:

2.1. Data Collection

At this stage stock price data collection is conducted by using the source of yahoo finance. The stock data that is used are several stock prices under the Jakarta Composite Index, which are: Charoen Pokphand Indonesia Tbk, Harum Energy Tbk, Media Nusantara Citra Tbk, PP London Sumatra Indonesia Tbk, Vale Indonesia Tbk, Indo Tambangraya Megah Tbk, Indocement Tunggal Prakasa Tbk. The stock price data that is used are daily stock close price during January 2014 to December 2014. Then the normality test is conducted on the stock close price data using SPSS software with Kolmogorov-Smirnov test.

2.2. Literature Study

At this stage, reference theories are collected for supporting fundamental, that is regarding the stock price, random walk, Brownian motion, proses stokastik, lemma Ito and geometric Brownian motion.

2.3. Stock Price Forecasting

At this stage the expected stock price formulation is conducted, the formulation of 95% confidence level and stock price forecasting using geometric Brownian motion. In forecasting the stock price the common formula of volatility and log volatility is used.

To obtain expected stock price with 95% confidence level, stages conducted as follow:

a. Determine pdf lognormal.
b. Determine the lognormal distribution mean function.
c. Determine new stock price model.
d. Determine 95% confidence level.

After the new stock price model is obtained then stock close price forecast is conducted on seven stock. The stages in stock price forecasting as follow:

a. Normality test.
b. Calculate stock return.
c. Estimation of drift value ($\mu$) and volatility ($\sigma$).
d. Forecast of stock price.
e. Calculate MAPE value.
f. Calculate 95% confidence level.

2.4. Conclusion

At this stage obtaining conclusion from analysis and discussion is done.

2.5. Simulation and Report Writing
At this stage simulation from the research is done and report writing follows after the simulation.

2.6. A subsection

Some text.

3. Research Output and Discussion

3.1. Pdf lognormal

At this section pdf lognormal distribution is obtained from pdf normal. For example, $q$ is random variable which lognormal distributed then $v = \ln q$ a is random variable which normally distributed with mean $\mu$ and variance $\sigma^2$ thus defining pdf from variable $v$ as follows:

$$f(v) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(v-\mu)^2}{2\sigma^2}}, & \text{for } -\infty < v < \infty, \\ 0, & \text{for other } v. \end{cases}$$

(1)

Using the pdf from variable $v$ pdf from variable $q$ is obtained. Based on one – one randomize variable theorem, result variable $q$ pdf is obtained from the following equation [4]:

$$h(q) = \frac{f(v)dv}{dq}.$$  (2)

With substitution $v = \ln(q)$, obtain

$$dv = d(\ln q),$$

$$= \frac{1}{q}dq.$$  (3)

The with substitution of $dv = \frac{1}{q}dq$ to equation (2), obtain

$$h(q) = \frac{f(v)dv}{dq},$$

$$= \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{1}{2}\left(\frac{\ln q - \mu}{\sigma}\right)^2}\frac{1}{q}dq.$$  (4)

Thus pdf lognormal from variable $q$ as follow [4]:

$$h(q) = \frac{1}{q\sigma\sqrt{2\pi}}e^{\left(\frac{1}{2}\left(\frac{\ln q - \mu}{\sigma}\right)^2\right)}.$$  (5)

where:

$\mu$ : mean distribution of variable $q$ lognormal

$\sigma^2$ : variance of lognormal distribution of variable $q$.

3.2. Lognormal mean distribution

Using the lognormal distribution pdf which obtained from equation (4) then finding the mean of lognormal distribution is conductedsela. Lognormal distribution mean from variable Mean $q$ is defined as [4]:

$$E(q) = \int_{-\infty}^{+\infty} q h(q) dq.$$  (6)

$$= \int_{-\infty}^{+\infty} \frac{1}{q\sigma\sqrt{2\pi}}e^{\left(\frac{1}{2}\left(\frac{\ln q - \mu}{\sigma}\right)^2\right)} dq.$$  (7)

Suppose if $y = \ln q - \mu$ then $dy = \frac{1}{q}dq$. Integral $\ln q = -\infty$ become $y = -\infty$ and $\ln q = +\infty$ become $y = +\infty$. Thus $E(q)$ can be written as:

$$E(e^{y+\mu}) = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}e^{y+\mu} e^{-\frac{y^2}{2\sigma^2}}dy,$$  (8)

$$= e^\mu e^{\frac{\sigma^2}{2}} \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy.$$  (9)
Next, Suppose if \( z = \frac{y - \sigma^2}{\sigma} \) then \( dz = \frac{1}{\sigma} \, dy \). Integral \( y = -\infty \) become \( z = -\infty \) and \( y = +\infty \) become \( z = +\infty \). Thus \( E(e^{y+\mu}) \) can be written as:

\[
E(e^{z\sigma^2+\mu}) = e^{\mu \frac{\sigma^2}{2}} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{z^2}{2}} \, dz.
\] (9)

Integral on equation (9) is pdf from standard normal distribution and has value of 1, Thus mean function of variable \( q \) lognormal distribution is

\[
E(q) = e^{\mu \frac{\sigma^2}{2}},
\] (10)

where:

\( \mu \) : mean distribution of variable \( q \) lognormal

\( \sigma^2 \) : variance of lognormal sitribution of variable \( q \).

3.3. Expected stock price

At this section expected stock price formulation is conducted. Based on [4], equation

\[
\ln S_t = \ln S_0 + \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma B_t,
\] (11)

is Brownian motion with drift, where \( B_t \) is Brownian motion in time \( t \) with \( \mu = 0 \) and has value of \( \varepsilon \sqrt{t} \) [4]. Whereas Brownian motion definition with drift as follow [4]:

\[
B_t = \mu t + \sigma W_t,
\] (12)

where \( t \) represents time and \( W_t \) adalah is random walk process with scale for big value of n and has value of \( \varepsilon \sqrt{t} \) [4]. Thus equation (11) and (12) can be written as :

\[
\ln S_t = \ln S_0 + \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma B_t.
\] (13)

First step to obtain expected stock price equation is by calculating mean value and variance of Brownian motion with drift. Elaboration of the first step as follow

3.3.1. Calculate mean value of Brownian motion with drift

\[
E(B_t) = E(\mu t + \sigma W_t),
\]

\[
= E(\mu t) + E(\sigma W_t),
\]

\[
= \mu t + E(\sigma W_t).
\] (14)

Because \( E(W_t) = 0 \), then

\[
E(B_t) = \mu t.
\] (15)

3.3.2. Calculate variance value of Brownian motion with drift

\[
Var(B_t) = Var(\mu t + \sigma W_t),
\]

\[
= E((\mu t + \sigma W_t)^2) - [E(\mu t + \sigma W_t)]^2.
\]

\[
E((\mu t + \sigma W_t)^2) = E((\mu t)^2) + 2\mu t \sigma E(W_t) + \sigma^2 E(W_t^2),
\] (16)

Based on definition of variance:

\[
Var(W_t) = E(W_t^2) - [E(W_t)]^2,
\]

\[
E(W_t^2) = Var(W_t) + [E(W_t)]^2,
\] (17)

Then:

\[
E((\mu t + \sigma W_t)^2) = (\mu t)^2 + \sigma^2 t.
\]

\[
[E(\mu t + \sigma W_t)]^2 = [E(\mu t) + E(\sigma W_t)]^2,
\]

\[
= \mu^2 + \sigma^2 E(W_t)^2,
\] (18)
Thus obtain:

$$V_{ar}(B_t) = E[(\mu t + \sigma W_t)^2] - [E(\mu t + \sigma W_t)]^2,$$

$$= (\mu t)^2 + \sigma^2 t - (\mu t)^2,$$

$$= \sigma^2 t. \hspace{1cm} (19)$$

Based on the verification above, Brownian motion with drift is normal distributed equation with mean $\mu t$ and variance $\sigma^2 t$ thus $ln S_t$ normally distributed with mean $ln S_{t-1} + (\mu - \frac{\sigma^2}{2}) t$ and variance $\sigma^2 t$ [4]. Thus expected $E(S_t)$ future stock price when $t$ is

$$E(S_t) = e^{\ln S_0 + (\mu - \frac{\sigma^2}{2}) t + \sigma B_t},$$

$$= e^{\ln S_0 e^{(\mu - \frac{\sigma^2}{2}) t + \sigma^2 t}},$$

$$= S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) t}. \hspace{1cm} (20)$$

where:

$S_0$ : Actual beginning stock price

$\mu$ : drift of stock price

$\sigma$ : volatility of stock price.

3.4. Confidence Level

For testing the forecast accuracy of geometric Brownian motion then 95% confidence level is implemented. Based on the following equation:

$$\ln S_t = \ln S_0 + \left(\mu - \frac{1}{2} \sigma^2\right) t + \sigma B_t. \hspace{1cm} (21)$$

With $S_t$ is actial stock price when $t$, $\mu$ and $\sigma$ are estimated parameter from big sample from population. Thus with 95% confidence level shows the actual stock price in 95% confidence level.

By using the parameter of mean $\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) t$ and variance of $\sigma^2 t$ which obtained form previous discussion of expected stock price sub chapter, so that 95% confidence level and variable $\ln S_t$ as follow:

$$\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) t - 1.96\sigma\sqrt{t} \leq \ln S_t \leq \ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) + 1.96\sigma\sqrt{t},$$

$$e^{\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) t - 1.96\sigma\sqrt{t}} \leq S_t \leq e^{\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) + 1.96\sigma\sqrt{t}}, \hspace{1cm} (22)$$

where:

$S_0$ : beginning stock price

$S_t$ : stock price when $t$

$\mu$ : stock price drift

$\sigma$ : Stock price volatility

3.5. Normality test

At this stage, normality test of stock price data during January 1st 2014 to December 31st 2014 period is conducted. Normality test is conducted to find whether the stock data is normally distributed or not.

Based on Kolmogorov – Smirnov normality test, shown $p \geq 0.05$, so it can be concluded that the stock price is normally distributed and feasible to do stock price forecast on the data.

3.6. Stock price forecasting

At this stage stock price is forecasted on several registered companies, they are stock of Charoen Pokphand Indonesia Tbk, Harum Energy Tbk, Media Nusantara Citra Tbk, PP London Sumatra Indonesia Tbk, Vale Indonesia Tbk, Indo Tambangraya Megah Tbk and Indocement Tunggal Prakasa Tbk. Data used to estimate the is daily stock close price during January 1st 2014 to December 31st 2014 period [6], whereas forecasting is done for January 1st 2015 period.
These are the stages in stock price forecasting.

3.6.1. **Calculating stock return**
At this stage monthly stock return calculation is conducted during January 2014 to December 2014 period using the following stock return equation [7]:

\[ R_t = \frac{S_t - S_{t-1}}{S_{t-1}}, \]  

where:
- \( R_t \) : stock return when \( t \)
- \( S_t \) : stock price when \( t \)
- \( S_{t-1} \) : stock price when \( t - 1 \).

3.6.2. **Drift value (\( \mu \)) and volatility (\( \sigma \)) estimation**
At this stage drift and volatility are estimated. Drift value and volatility are constant parameter of stock that is used for forecasting January 2015 stock price. Drift formulation as defined [7]:

\[ \mu = \frac{1}{M \delta_t} \sum_{t=1}^{M} R_t, \]  

where:
- \( \mu \) : drift
- \( R_t \) : stock return
- \( M \) : amount of stock return.

To obtain drift value, mean calculation of drift value is required monthly from January 2014 to December 2014.

After the drift value is obtained, then volatility value is calculated. The volatility formulation that is used in this research is volatility common formula and log volatility. The equation of volatility and log volatility as defined [7]:

{table}
| Hypothesis Test Summary |
|-------------------------|
| 1. The distribution of change is normal with mean 0.00 and standard deviation 0.02. | One-Sample Kolmogorov-Smirnov Test | .176 | Retain the null hypothesis. |
| 2. The distribution of change is normal with mean -0.00 and standard deviation 0.02. | One-Sample Kolmogorov-Smirnov Test | .007 | Retain the null hypothesis. |
| 3. The distribution of return is normal with mean 0.00 and standard deviation 0.02. | One-Sample Kolmogorov-Smirnov Test | .075 | Retain the null hypothesis. |
| 4. The distribution of return is normal with mean 0.00 and standard deviation 0.02. | One-Sample Kolmogorov-Smirnov Test | .200 | Retain the null hypothesis. |
| 5. The distribution of return is normal with mean 0.00 and standard deviation 0.02. | One-Sample Kolmogorov-Smirnov Test | .115 | Retain the null hypothesis. |
| 6. The distribution of return is normal with mean 0.00 and standard deviation 0.02. | One-Sample Kolmogorov-Smirnov Test | .189 | Retain the null hypothesis. |
| 7. The distribution of return is normal with mean 0.00 and standard deviation 0.02. | One-Sample Kolmogorov-Smirnov Test | .073 | Retain the null hypothesis. |

Asymptotic significances are displayed. The significance level is .05.

**Figure 1.** Normality test
\[
\sigma_1 = \frac{1}{(M-1)\delta_t} \sum_{t=1}^{M} (R_t - \bar{R})^2,
\]
\[
\sigma_2 = \frac{1}{(N-1)\delta_t} \sum_{t=2}^{N} (\log S_t - \log S_{t-1})^2,
\]

where:
- \(\sigma_1\) : volatility
- \(\sigma_2\) : log volatility
- \(\bar{R}\) : mean of stock return
- \(M\) : amount of stock return data
- \(N\) : amount of stock data.

3.6.3. Stock price forecasting

At this stage stock price forecasting is conducted on seven stock close price on January 2015 using geometric Brownian motion model. Forecast 1 and Forecast 2 using geometric Brownian motion model by implementing common volatility and log volatility equation. On forecast 1 and forecast 2 to forecast stock price on \(t\) time use the beginning actual stock price on \(t-1\) time. Equation implemented for forecast 1 and forecast 2 as [4]:
\[
F_t = S_{t-1} + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t},
\]

where:
- \(F_t\) : stock price forecast when \(t\)
- \(S_{t-1}\) : actual stock price when \(t-1\)
- \(\mu\) : drift
- \(\sigma\) : volatility, where \(\sigma = \sigma_1\) or \(\sigma = \sigma_2\)
- \(B_t\) : \(\varepsilon \sqrt{t}\)

Where on forecast 3 and forecast 4 geometric Brownian motion model with common volatility equation and log volatility, however to forecast stock price when \(t\) use the beginning value which taken from forecast \(t-1\). Equation used for forecast 3 and forecast 4 as follow:
\[
F_t = F_{t-1} + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t},
\]
with \(F_0\) is actual stock price.

3.6.4. MAPE calculation

To determine the forecast accuracy can be determined by calculating MAPE of forecast 1, forecast 2, forecast 3, and forecast 4. MAPE equation as defined [10]:
\[
MAPE = \frac{\sum_{t=1}^{N} |S_t - F(S_t)|}{S_t N},
\]
where:
- \(S_t\) : actual stock price when \(t\)
- \(F(S_t)\) : stock price forecast when \(t\)
- \(N\) : amount of stock price data.

On figure 2 and figure 3 show the graph of stock price forecast of Media Nusantara Citra Tbk using geometric Brownian motion.
Figure 2. Forecast graph based on beginning value which taken from previous actual stock price.

Figure 3. Forecast graph based on beginning value which taken from previous forecast.

On table 1 shown the MAPE value of forecasts.

Table 1. Forecast MAPE

| Stock                        | MAPE value of each forecast |
|------------------------------|-----------------------------|
|                              | Forecast 1 | Forecast 2 | Forecast 3 | Forecast 4 |
| Charoen Pokphand Indonesia   | 2.0935%    | 1.1675%    | 2.9320%    | 1.6933%    |
| Harum Energy                 | 2.2033%    | 1.4982%    | 3.9128%    | 3.3974%    |
| Media Nusantara Citra       | 3.0529%    | 1.9875%    | 2.6096%    | 4.6496%    |
| PP London Sumatra Indonesia  | 1.8726%    | 1.6479%    | 3.0614%    | 2.9718%    |
| Vale Indonesia               | 1.7580%    | 1.5921%    | 4.7672%    | 4.3637%    |
| Indo Tambangraya Megah      | 2.7305%    | 2.1767%    | 4.6661%    | 3.5049%    |
| Indocement Tunggal Prakasa  | 2.5349%    | 1.9187%    | 4.1217%    | 4.8249%    |
Based on [4], with equation

\[ F_t = F_{t-1} + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}, \]  

(30)

where \( F_0 = S_0 \) is actual stock price and \( B_t = \varepsilon \sqrt{t} \) [4]. Simulation is made for stock price forecasting with 1000 realization of trajectory that may be from geometric Brownian motion, with every realization of trajectory using the equation of

\[ F_t = F_{t-1} + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t} \]

with 22 iterations. Suppose if \( i \) is every trajectory realization that may be from geometric Brownian motion. Thus for \( i = 1 \)

\[ F_1 = S_0 + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}, \]  

(31)

\[ F_2 = F_1 + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}, \]  

(32)

\[ F_{22} = F_{21} + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}. \]  

(33)

For \( i = 2 \)

\[ F_1 = S_0 + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}, \]  

(34)

\[ F_2 = F_1 + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}, \]  

(35)

\[ F_{22} = F_{21} + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}. \]  

(36)

For \( i = 3 \)

\[ F_1 = S_0 + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}, \]  

(37)

\[ F_2 = F_1 + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}, \]  

(38)

\[ F_{22} = F_{21} + e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma \varepsilon \sqrt{t}}. \]  

(39)

Replication above is done until \( i = 1000 \).

On figure 4 and figure 5 the simulation shown simulation output of stock price forecast of Media Nusantara Citra Tbk until \( i = 1000 \) using common volatility equation and log volatility.

Figure 4. Stock price simulation model using common volatility equation
3.7. Simulation Model Analysis

Based on geometric Brownian motion simulation model with 1000 possible trajectory, At this stage calculation of expected stock price and 95% confidence level for each stock price data is conducted. Based on [4], to calculate expected stock price when \( t = 22 \) use the equation (7), as follow:

\[
E(S_t) = S_0 e^{\left( (\mu + \frac{\sigma^2}{2})t \right)}.
\] (40)

Whereas to calculate 95% confidence level from stock price required the following equation (8):

\[
e^{\ln S_0 + (\mu - \frac{\sigma^2}{2})t - 1.96\sigma \sqrt{t}} \leq S_t \leq e^{\ln S_0 + (\mu - \frac{\sigma^2}{2})t + 1.96\sigma \sqrt{t}}.
\] (41)

On table 2 and table 3 shown the confidence level of 95% from seven stock price data.

| Table 2. Confidence level of stock price with common volatility equation |
|---------------------------|---------------------------|---------------------------|---------------------------|
| Stock                          | \( E(S_t) \) | \( L \) | \( S_t \) | \( U \) |
| Charoen Pokphand Indonesia     | 3853          | 3231      | 3955         | 4527         |
| Harum Energy                   | 1606          | 1533      | 1520         | 1722         |
| Media Nusantara Citra          | 2555          | 2097      | 2860         | 3055         |
| PP London Sumatra Indonesia    | 1914          | 1563      | 1840         | 2299         |
| Vale Indonesia                 | 3802          | 3036      | 3450         | 4007         |
| Indo Tambangraya Megah         | 14820         | 12026     | 16750        | 18155        |
| Indocement Tunggal Prakasa     | 25648         | 21442     | 23000        | 30214        |

| Table 3. Confidence level of stock price using log volatility equation |
|---------------------------|---------------------------|---------------------------|---------------------------|
| Stock                          | \( E(S_t) \) | \( L \) | \( S_t \) | \( U \) |
| Charoen Pokphand Indonesia     | 3841          | 3560      | 3955         | 4132         |
| Harum Energy                   | 1601          | 1476      | 1520         | 1730         |
| Media Nusantara Citra          | 2545          | 2336      | 2860         | 2763         |
| PP London Sumatra Indonesia    | 1907          | 1737      | 1840         | 2071         |
| Vale Indonesia                 | 3784          | 3434      | 3450         | 4150         |
| Indo Tambangraya Megah         | 14759         | 13479     | 16750        | 16094        |
| Indocement Tunggal Prakasa     | 25569         | 23673     | 23000        | 27535        |
Based on table 2 and table 3, 95% confidence level using log volatility equation tends to have narrow interval compared to 95% confidence level using common volatility equation. Thus on the simulation there is several actual stock price located outside trajectory realization that may be from geometric Brownian motion.

4. Conclusions
Based on the analysis and discussion, it can be concluded that short term forecasting which are forecast 1 and forecast 2 have MAPE value of \( \leq 20\% \). Whereas long term forecast which are forecast 3 and forecast 4 are dissatisfactory because actual stock price located outside green line or outside the area of forecast value.

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