Optimal modeling analysis and intelligent calculation of a class of stochastic signal systems

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Abstract. Information flow optimization is very important for decision-making problems in various fields. Using an innovative method of information decision-making optimization, this paper makes a thorough study and analysis of the optimization modeling and intelligent calculation of a kind of stochastic signal system in science and technology by using the big data analysis method. Thus, it provides an ideal decision-making analysis method for effectively realizing the information optimization and processing of such problems.

1. Introduction
With the notable emergence of global information and knowledge economy, the information explosion brought by the expeditious growth of science, technology, and social economy, information has become the third largest resource in human society in addition to material and energy. In the face of massive information and big data, the screening and utilization of information are becoming more and more important [1-2]. Therefore, in various fields of science and technology, how to find a set of effective new means and new methods for information receiving, filtering and information processing and analysis has become a new problem that needs to be solved urgently in current and future scientific research [3-4]. In order to further deal with and solve all kinds of new problems in the field of science and technology, by extending the information decision-making research in the general information space to the comprehensive and multi-perspective information space, this paper makes in-depth research and optimization control on the information filtering and information processing analysis of big data[5-8]. It provides an ideal and effective new theory, new means, and new method for further controlling and improving such information systems' efficiency [9-10].

2. System description
The first element of the vector $U_i = (\xi_i, \zeta_i)$ set is unobservable; for each $t = 0, 1, \ldots$, composed $\xi_t$ by observation $(\zeta_0, \ldots, \zeta_t)$ in the optimal linear estimation of the mean square sense.
If $U_i, t = 0, 1, \ldots$, as a Gauss procedure, $S_i = E(\xi_i \mid F_i)$ and $X_i = ([\xi_i - S_i][\xi_i - S_i]')$ is determined by the following equation:

$$S_{t+1} = a_{i}S_{i} + a_{i}\zeta_{t+1} + (aa^* + b_{i}b_{i}^*) \left( AA^* + B_{i}B_{i}^* \right) \left( \xi_{t+1} - B_{i}S_{i} - B_{i}\zeta_{t+1} \right)$$

(1)

$$l_{t+1} = b_{i}b_{i}^* + aa^* - (aa^* + b_{i}b_{i}^*) \left( AA^* + B_{i}B_{i}^* \right) \left( aa^* + b_{i}b_{i}^* \right)$$

(2)

The equation is based on the initial condition:

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\[ S_0 = E(\xi_0|\xi_0), \quad l_0 = E(\xi_0-s_0)[\xi_0-s_0]^T \]

To solve According to normal correlation theorem

\[ s_0 = \text{cov}(\xi_0,\xi_0) \text{cov}^+(\xi_0,\xi_0) \xi_0 \]
\[ l_0 = \text{cov}(\xi_0,\xi_0) - \text{cov}(\xi_0,\xi_0) \text{cov}^+(\xi_0,\xi_0) \text{cov}(\xi_0,\xi_0) \]

Since \( S_i = E(\xi_i|F_i) \) linearly depends \( \xi_0,\cdots,\xi_i \), in the case of Gauss process \( U_i = [\xi_i,\xi_i] \), constitute \( \xi_i \) the best linear estimation problem according to \( \xi_0,\cdots,\xi_i \) the solution by the equation (1), (2) is given.

For the general case, the random vector is provided \( (\beta,\alpha) \) with \( E(\beta^2 + \alpha^2) < \infty \), and \( (\beta,\alpha) \) \( (\alpha,\beta) \) with the same front two moments of Gauss vector,

\[ l(\beta) = E(\alpha|\beta) \]

(5)

\( r(\alpha) \) is the best linear estimation of the amount by \( \beta \) and \( \alpha \) in the mean-square sense, and E \( l(\alpha) = E\beta \).

When the vector is \( \beta = (\beta_1,\cdots,\beta_k), \quad \alpha = (\alpha_1,\cdots,\alpha_k) \), the above conclusion remains valid.

And then in the containable control, the best control \( v = (v_i), 0 \leq t \leq T \) is decided by the formula below:

\[ v_i = -P^{-1}(t)c^*(t)R(t)s_i, \quad 0 \leq t \leq T \]

where the nonnegative definite symmetric matrix \( R(t) = \|R_{ij}(t)\| 0 \leq t \leq T \) of the stage \( (i \times i) \), is the answer to the following Riccati equation:

\[ -\frac{dQ(t)}{dt} = b^*(t)q(t) + q(t)b^*(t) + J(t) - q(t)D(t)T^{-1}(t)D^*(t)Q(t), \]
\[ Q(T) = h \]

while vector \( S_i \) is determined by the equation group below:

\[ d s_i = [d(t)v_i + b(t)s_i]dt \]
\[ + l_iB^*(t)(A(t)A^*(t))^{-1}[d\xi_i - B(t)s_i, dt], \]
\[ S_0 = S_0 = M\xi_0 \]
\[ l_i = b(t)s_i + l_iB^*(t) + a(t)a^*(t) \]
\[ - l_iB^*(t)(A(t)A^*(t))^{-1}B(t)l_i, \]
\[ l_0 = \text{cov}(\xi_0,\xi_0) \]
then
\[ U(v, N) = P(0) + \int_0^r [Q^{1/2}(t)|Q^{1/2}(t) + r^{1/2}F^{1/2}] dt \]

where
\[ Q(t) = \sum_{i,j=1}^k C_{ij}(s)Q_{ij}(s)ds \]

while \( C_{ij}(t) \) is the element of the matrix \( C(t) = t'B(t)[A(t)A'(t)]^{-1}B(t)r_j \).

3. Optimization modeling analysis and Intelligent Computing

(1) Allowed spectrum expression generalized stationary process \( T(t), t = 0, \pm 1, \pm 2, \cdots \), is part of a component of the n-dimensional generalized stationary process \( (T_1(t), \cdots, T_n(t)) \) following recursion equations, \( T_j(t) = T(t) - T_j(t + 1) \)
\[ T_j(t + 1) = T_{j+1}(t) + \alpha_j \sigma(t + 1), j = 1, 2, \cdots, n - 1, \]
\[ = -\sum_{j=0}^{n-1} \beta_j T_{j+1}(t) + \alpha_n \sigma(t + 1) \] (6)

Process \( \sigma(t), t = 0, \pm 1, \pm 2, \cdots \), Available show by (6)
\[ E\sigma(j) = 0, \quad t < s, \quad j = 1, 2, \cdots, n \] (7)

The coefficient \( \alpha_1, \alpha_2, \cdots, \alpha_n \) by equation (6) given.

If \( T(t), t = 0, \pm 1, \pm 2, \cdots \), is a real process, each process \( \sigma(t), T_2(t), \cdots, T_n(t) \) is a real process.

If \( T(t), t = 0, \pm 1, \pm 2, \cdots \), is a Gauss process, \( \sigma(t), t = 0, \pm 1, \pm 2, \cdots \), is also a Gauss sequence comprising independent random variables.

(2) Expression (6) can be exported containing fractional rational spectral density of multidimensional generalized stationary sequence component filter equations.

Set \( U_i = [\xi_i, \zeta_i] = [(\xi_i(t), \cdots, \xi_i(t)), (\zeta_i(t), \cdots, \zeta_i(t))], t = 0, \pm 1, \pm 2, \cdots \) for the real general smooth \( i+s \) dimensional process, can be expressed as
\[ U_i = \int_\pi^{-\pi} e^{i\lambda t} H(e^{i\lambda}) \psi(d\lambda) \] (8)

Which \( H(K) = \|H_{r,q}(K)\| \) is an \( m \times n \) order matrix, \( m = k + 1 \), a fractional rational element
\[ H_{r,q}(K) = \frac{U_{r,q}^{(r,q)}}{W_{n,q}} \]

And \( W(d\lambda) = [\psi_1(d\lambda), \cdots, \psi_m(d\lambda)] \) as a non-related components of the random vector measure,
\[ E\psi_j(d\lambda) = 0, \quad E[\psi_j(d\lambda)]^2 = \frac{1}{2\pi}d\lambda \]

And assuming the root of the equation \( V_{n,q}^{(r,q)}(K) = 0 \) is located in the unit park.

Application process for each of the following lines (1).
\[ U_{p,r,q}(t) = \int_{-\pi}^{\pi} e^{ij\lambda} H_{r,q}(e^{ij\lambda}) \psi_p(d\lambda) \]  (9)

Vector \( \zeta_i = (\zeta_1(t), \cdots, \zeta_n(t)) \) and vector \( \overset{\sim}{\zeta}_i \) [composed of vector \( \xi_i = (\xi_1(t), \cdots, \xi_n(t)) \) and shaped like \( T_2(t), \cdots, T_n(t) \) all those additional components] after the recurrence equation obtained by simple transformation group

\[
\begin{align*}
\overset{\sim}{\zeta}_{t+1} &= b_1\xi_t + b_2\overset{\sim}{\zeta}_t + b\sigma(t+1) \\
\zeta_{t+1} &= B_1\xi_t + B_2\overset{\sim}{\zeta}_t + A\sigma(t+1)
\end{align*}
\]

(10)

wherein \( \sigma(t) = (\sigma_1(t), \cdots, \sigma_m(t)) \) is a sequence of non-correlation component of the non-associated vector, \( E\sigma_j(t) = 0, \ E\sigma_j^2(t) = 1 \)

\[
\sigma_j(t) = \int_{-\pi}^{\pi} e^{ij(t-1)} \psi_j(d\lambda)
\]

and matrix \( b_j, B_j, a \) and \( A,j=1,2 \), that can be obtained directly.

(3) Given \( \zeta_t', t = 0, \pm 1, \pm 2, \cdots \) is a generalized stationary process with \( E\zeta_t = 0 \) and the spectral density is \( f(\lambda) = e^{i\lambda t} + \frac{1}{\lambda^2} [e^{2i\lambda} + e^{i\lambda}/2 + 1/2] \), in order to determine \( \zeta_t \) with \( \zeta_0 = \{\zeta_0, \cdots, \zeta_s\} \) in accordance with the best linear estimation (which is in the mean square sense, we need to have structure and spectral density Gauss process. And this process by the equation

\[ \zeta_{t+2} + \frac{1}{2}(\zeta_{t+1} + \zeta_t) = \sigma(t+2) + \sigma(t+1) \]

By the answer obtained, which is a Gauss random sequence:

\[ E\sigma(t) = 0, \ E\sigma(t)\sigma(s) = \delta(s, t) = t \]

Now assume \( \xi_t = \zeta_{t+1} - \sigma(t+1) \), for the random vector \( (\xi_t, \zeta_t), t = 0, \pm 1, \pm 2, \cdots \), can be obtained equations

\[
\begin{align*}
\xi_{t+1} &= -\frac{1}{2}\xi_t - \frac{1}{2}\zeta_t + \frac{1}{2}\sigma(t+1) \\
\zeta_{t+1} &= \xi_t + \sigma(t+1)
\end{align*}
\]

wherein \( n_1(t, s) = E(\theta|\mathcal{F}_t^s) \) and \( n_2(t, s) = E(\xi_t|\mathcal{F}_t^s) \) by having the initial conditions

\[
\begin{align*}
T_1(t, t) &= s_t, \ T_2(t, t) = l_t \\
T_1(s + 1, t) &= -\frac{1}{2} T_1(s, t) - \frac{1}{2} T_2(s, t) \\
T_2(s + 1, t) &= T_1(s, t)
\end{align*}
\]

(12)
definite. then(12) initial condition \( S = E(\xi_t|\mathcal{F}_t^s) \) and \( I_s \) also defined by equation

\[
\begin{align*}
s_{t+1} &= \frac{1}{2} s_t - \frac{1}{2} \zeta_t + \frac{1 - l_t}{2(1 + l_t)} (\xi_{t+1} - s_t) \\
l_{t+1} &= \frac{l_t}{1 + l_t}
\end{align*}
\]

(13)(14)

and it is not difficult to prove \( s_0 = 0, l_0 = 1 \).
Described above, in generalized stationary sequence $\zeta, t = 0, \pm 1, \pm 2, \cdots$, the best linear estimate is made (12) to (14) identified, then wherein $\zeta$ of (13) should be replaced by $\zeta$.

4. Numerical Experimentation

To verify the effectiveness of numerical methods (8) - (9), we solve using the following initial-boundary value case:

$$\begin{align*}
\frac{\partial u(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial x} &= \gamma^2 \left( \frac{\partial^2 u(x,t)}{\partial x^2} - u(x,t) \right) + e^t \left[ (1+\gamma)t^\gamma + t^{2\gamma} \right] \\
O < t \leq 1, \quad O < x < 1
\end{align*}$$

(15)

$$u(x,O) = O, \quad 0 \leq x \leq 1, \quad u(0,t) = t^\gamma, \quad u(1,t) = e^t t^\gamma, \quad 0 \leq t \leq 1,$$

(16)

(17)

The precise solution for the initial-boundary value problems of (15) - (17) is $u(x,t) = e^t t^\gamma$. Let

$$E_\infty = \max_{0 \leq t \leq K} \left\{ \| E \| \right\}$$

Table 1 indicates the $E_\infty$ values, the maximum errors for the numerical solutions to the initial-boundary value problems (15) - (17) by using the numerical methods (8) - (9), at different $\gamma$ and $\Delta_t = \Delta_x^2$ values. From table 1: the numerical experiments support our theoretical analysis.

| $\gamma$ | $\Delta_t = \Delta_x^2 = 1/5$ | $\Delta_t = \Delta_x^2 = 1/25$ | $\Delta_t = \Delta_x^2 = 1/100$ |
|----------|-------------------------------|-------------------------------|-------------------------------|
| 0.5      | $1.5602 \times 10^{-2}$         | $5.4026 \times 10^{-3}$         | $1.2586 \times 10^{-2}$       |
| 0.6      | $1.8751 \times 10^{-2}$         | $5.8898 \times 10^{-3}$         | $1.4836 \times 10^{-3}$       |
| 0.7      | $1.9840 \times 10^{-2}$         | $6.4850 \times 10^{-3}$         | $1.8634 \times 10^{-3}$       |

5. Conclusion

This article expands the information flow research from the normal information space to the all-dimensional, multi-angle random dynamic information flow research in the generalized information space. It discusses the recursive filtering algorithm of the optimal filtering, the estimation of one kind of measurable and stationary process in multi-dimensional and generalized space, the finding of a series of valuable information in such generalized process, the producing of the optimal recursive filtering equation and optimal estimation of random signals in such generalized process. It provides a theoretical base and a mathematic method for further researches of optimal control of such process.

6. Prospection

The continuous development of modern information science, big data analysis and intelligent computing technology will greatly accelerate modern science, technology, and information management intelligence. At present, the global economy and information technology field has entered a new round of great changes and transformation period. With the advent of the global information era and the fast development of modern science, technology, social economy, we will face more and more problems of analysis, mining, using massive data and information in various fields, which makes big data analysis, intelligent computing and cloud computing the current research
hotspot. Therefore, the research and application of big data, data mining, and intelligent computing always occupy a core position.

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