Evidence for neutrino oscillations is pointing to the existence of tiny but finite neutrino masses. Such masses may be naturally generated via radiative corrections in models such as the Zee model where a singlet Zee-scalar plays a key role. We minimally extend the Zee model by including a right-handed singlet neutrino $\nu_R$. The radiative Zee-mechanism can be protected by a simple $U(1)_X$ symmetry involving only the $\nu_R$ and a Zee-scalar. We further construct a class of models with a single horizontal $U(1)_{FN}$ (à la Froggatt-Nielsen) such that the mass patterns of the neutrinos and leptons are naturally explained. We then analyze the muon anomalous magnetic moment ($g_\mu - 2$) and the flavor changing $\mu \to e\gamma$ decay. The $\nu_R$ interaction in our minimal extension is found to induce the BNL $g_\mu - 2$ anomaly, with a light charged Zee-scalar of mass $100 - 300$ GeV.

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The minimal Zee model [3] introduces one extra singlet charged scalar ($S_1^\pm$) together with the usual two-Higgs-doublet sector. By assuming no right-handed $\nu_R$, as in the SM, this scalar only interacts with left-handed neutrinos and leptons. Thus, the Zee model contains the following additional Lagrangian,

$$\Delta L_1 = \sum_{j,j'} \frac{f_{jj'}}{2} \varepsilon_{ab} \bar{L}_{aj} L_{aj'} S_1^+ + m_3 \varepsilon_{ab} \tilde{H}_1^+ \tilde{H}_2^+ S_1^+ + h.c.$$  

(1)

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Neutrino-Lepton Masses, Zee Scalars and Muon $g - 2$

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where $\ell_j, \ell_j' \in \{\mu, \tau\}$ and $L_j = (\nu_j, \ell_j)^T$ is the left-handed doublet of the $j$th family. The $(H_1, H_2)$ are the usual two-Higgs-doublets with hypercharge $(1/2, -1/2)$, where $H_1 = (H_1^+, H_0)\,^T$, $H_2 = (H_2^+, H_0)\,^T$, and $H_3 = i\tau_2 H_3^*$. The Yukawa sector can conserve total lepton number by assigning to $S_2^\pm$ the lepton numbers $\mp 2$. Thus, the total lepton number is only softly violated by the dimension-3 trilinear Higgs operator in Eq. (1). As such, the small Majorana neutrino masses are radiatively generated at one-loop and are automatically finite.

We minimally extend the Zee model by including a single right-handed Dirac neutrino $\nu^R$ with following Yukawa interactions,

$$\Delta L_2 = \left[ f_1 \bar{\nu}^R e_R + f_2 \bar{\nu}^R H_R + f_3 \bar{\nu}^R H_R \right] S_2^+ + \text{h.c.}$$

where $S_2^\pm$ is a second singlet Zee-scalar. The nontrivial issue with embedding $\nu^R$ is to avoid arbitrary tree-level Dirac mass terms generated by the Yukawa interactions $\lambda_j \nu^R H_R$ and $\lambda_j \nu^R H_R$ (which mix $\nu_j$ and $\nu_{j'}$), so that the predictive power of the radiative Zee-mechanism can be effectively protected. We achieve this goal by noting that the Yukawa sector (2) of $\nu^R$ possesses a global $U(1)_X$ symmetry, which can properly forbid the neutrino Dirac mass terms once a Zee-scalar $S_2^\pm$ is included together with $\nu^R$. It can be shown, by assigning the most general $U(1)_X$ quantum numbers for the Zee-model with $\nu^R$, that the unwanted tree-level neutrino Dirac masses cannot be removed without $S_2^\pm$. We define our simplest Type-I models with $U(1)_X$ in Table 1, where only $\nu^R$ and $S_2^\pm$ carry $U(1)_X$ charges while all other fields are singlets of $U(1)_X$. Hence, the Type-I extension gives a truly minimal embedding of $\nu^R$ into the Zee model.

Table 1. Quantum number assignments for Type-I and -II models. The hypercharge is defined as $Y = Q - 3$.  

| $L_j$ | $\ell_j R$ | $\nu_R$ | $H_1$ | $H_2$ | $S_1^+$ | $S_2^+$ | $S^0$ |
|-------|------------|---------|-------|-------|--------|--------|------|
| $U(1)_Y$ | $-1/2$ | $-1$ | $0$ | $1/2$ | $-1/2$ | $1$ | $1$ | $0$ |
| $U(1)_X$ | $0$ | $0$ | $x$ | $0$ | $0$ | $0$ | $-x$ | $-$ |
| $U(1)^a_{FN}$ | $0$ | $y_j$ | $x'$ | $0$ | $z$ | $-z$ | $-x-y$ | $-1$ |
| $U(1)^b_{FN}$ | $u_j$ | $y_j$ | $x'$ | $0$ | $z$ | $-z$ | $-x-y$ | $-1$ |

Table 2 classifies all (dis-)allowed operators of Type-I up to dimension-4. It shows that, as long as $x \neq 0$, the radiative Zee-mechanism is protected and the $\nu^R$ remains massless. Such a massless $\nu^R$ does not contribute to the invisible Z-width as it carries no weak charge. A special case of our Type-I is to consider its discrete subgroup $Z_4$ under which $\nu^R$ and $S_2^\pm$ transform as, $\nu^R \to i\nu^R$, $S_2^\pm \to i S_2^\pm$, while all other fields remain invariant. Other non-minimal variations of our Type-I can be easily constructed.

Table 2. Summary of $U(1)$ charges carried by the effective operators in Type-I and -II models. 

| Operators | $U(1)^a_X$ | $U(1)^a_{FN}$ | $U(1)^b_{FN}$ |
|-----------|------------|----------------|----------------|
| $T_y^j H_1 \ell_j R$ | $0$ | $y_j$ | $y_j - u_j$ |
| $T_y^j H_2 \ell_j R$ | $0$ | $y_j - z$ | $y_j - z - u_j$ |
| $T_y^j H_1 \nu_R$ | $x$ | $x'$ | $x' - u_j$ |
| $T_y^j H_2 \nu_R$ | $x$ | $x' + z$ | $x' + z - u_j$ |
| $\bar{\nu}_y^j \ell_j LS_2^+$ | $0$ | $-z$ | $u_j + u_j' - z$ |
| $\bar{\nu}_y^j \ell_j LS_2^+$ | $0$ | $x + y_j - z$ | $x + y_j - y$ |
| $H_1 H_2 S_2^+$ | $0$ | $0$ | $0$ |
| $\bar{\nu}_y^j \ell_j LS_2^+$ | $0$ | $x + y_j - z$ | $x + y_j - y$ |
| $\nu^R \ell_j LS_2^+$ | $2x$ | $2x'$ | $2x'$ |
| $\nu^R \ell_j LS_2^+$ | $0$ | $z$ | $z + x - y$ |
| $\nu^R \ell_j LS_2^+$ | $0$ | $-z$ | $x + y - z$ |

Neutrino Oscillations, Lepton Masses and Horizontal $U(1)_{FN}$ Symmetry

While the above Type-I models give the most economic embedding of $\nu^R$ with all the good features of the original Zee-model retained, they do not provide any insight on two important issues: (i) There is no theory prediction on the size of the Zee-scalar Yukawa couplings $f_{yj}$ in Eq. (1), but the neutrino oscillation data requires the following hierarchy (3):

$$f_{12} \approx \frac{m_2^2}{m_1^2} \approx 3 \times 10^2, \quad f_{13} \approx f_{23} \approx 0.1,$$

where $f_{12}/f_{13} \approx 10^2$ denotes the MSW large angle solution (vacuum oscillation solution). (ii) The small lepton masses and their large hierarchy are not understood. Our goal is to construct this same $U(1)$ group as a horizontal symmetry involving all the leptons so that these two issues can be naturally explained à la Froggatt-Nielsen (FN) (5). [This $U(1)$ will be called $U(1)_{FN}$.] The basic idea is to consider a horizontal $U(1)_{FN}$ spontaneously broken by the vacuum expectation value $\langle S^0 \rangle$ of a singlet scalar $S^0$. We can assign $U(1)_{FN}$ charges for relevant fields such that different mass terms are suppressed by different powers of $\epsilon \equiv \langle S^0 \rangle / \Lambda$ where $\Lambda$ is the scale at which the $U(1)_{FN}$ breaking is mediated to the light fermions. For instance, a low energy effective operator carrying a net $U(1)_{FN}$ charge $q$ (either $\geq 0$ or $< 0$) will
be suppressed by $\epsilon_3^{[q]}$. Though all mass terms are now allowed in the effective theory, we will build a class of FN-type models (called Type-II) in which the arbitrary tree-level neutrino Dirac-mass terms are suppressed to a level much below the one-loop radiative Zee-masses, and thus the predictive power of the Zee-mechanism remains. The role of the FN-scalar $S^0$ is to provide the spontaneous $U(1)_{FN}$ breaking and generate the relevant $U(1)_{FN}$-invariant effective operators that will give the desired neutrino Yukawa couplings and lepton masses at the weak scale. The heavy $S^0$ will be eventually integrated out from the low energy theory and our relevant particle spectrum of Type-II is the same as Type-I.

We provide two typical Type-II constructions, called Type-IIa and -IIb, respectively. The Type-IIa is the simplest extension of Type-I by further involving only the right-handed weak-singlet leptons in the $U(1)_{FN}$ (cf. Table 1). In the Type-IIb models, we further assign each lepton doublet $L_j$ a charge $u_j$. So, the lepton masses are determined by $\ell_j R$ charges in Type-IIa, while Type-IIb determines these masses by the charges of both $\ell_j R$ and $L_j$. The low energy effective operators up to dimension-4 (with the heavy $S^0$ integrated out) are classified in Table 2, from which we derive the general conditions for protecting the Zee-mechanism in Type-II,

$$10 > |x^*| \sim |x| \gg 1, \quad \text{and} \quad |x - u_j|, |x + y| \gg 1,$$

with $xx^* > 0$ and $|y|, |z| \sim O(1)$. For the explicit analysis below, we choose a typical value of the suppression factor $\epsilon \simeq 0.1$. Thus, choosing leptons in mass-eigenbasis, we write their mass ratios as,

$$m_e : m_\mu : m_\tau \simeq \epsilon^4 : \epsilon^1 : \epsilon^0,$$

which require,

$$(y_1 - u_1) - (y_3 - u_3) = \pm 4, \quad (y_2 - u_2) - (y_3 - u_3) = \pm 1.$$

The tau Yukawa coupling itself can be estimated as $y_\tau \simeq (m_\tau/m_\mu) \tan \beta \simeq 10^{-2} \tan \beta \sim \epsilon^1$ (with $\tan \beta = \left(H_2/H_1\right)$), in the typical range of $\tan \beta \simeq 10 - 40$, and this restricts the $U(1)_{FN}$ charges of $\tau$ as $y_3 - u_3 = \pm 1$. Table 3 summarizes three explicit realizations of Type-II models. From Table 3 and Eq. (3), the Yukawa couplings of $\nu_R$ are predicted as,

**Type IIa**: $(f_1, f_2, f_3) \sim (\epsilon^3, 1, \epsilon^1);

**Type IIb1**: $(f_1, f_2, f_3) \sim (\epsilon^5, 1, \epsilon^2);$

**Type IIb2**: $(f_1, f_2, f_3) \sim (\epsilon^{10}, 1, \epsilon^3).$

From Table 3 and Eq. (3), we further predict the left-handed Yukawa couplings $f_{j\mu}^\prime$, Type IIa : $(f_{12}, f_{13}, f_{23}) \sim \epsilon |z|;

**Type IIb1**: $(f_{12}, f_{13}, f_{23}) \sim (\epsilon^4 + z, \epsilon^6 + z, \epsilon^8 + z);$

**Type IIb2**: $(f_{12}, f_{13}, f_{23}) \sim (\epsilon^{3 + z}, \epsilon^5 + z, \epsilon^{12 + z});$

where the allowed values of $z$ are defined in Table 3. Thus, Type-IIa suppresses $f_{j\mu}^\prime$ couplings to $O(10^{-2} - 10^{-4})$. The models in Type-IIb1 (-IIb2), however, nicely accommodate the hierarchy for the MSW large angle solution (vacuum oscillation solution), while the predicted size of $f_{12} \sim 10^{-3} - 10^{-6}$ is also of the right order. Finally, it is trivial to extend these models with more than one singlet $\nu_R$ (i.e., $\nu_{Rj}$ with $j = 1, \cdots, N_{\nu_R}$ and $N_{\nu_R} = 3$ for instance), by simply defining them to share the same $U(1)$ charges as in Tables 1 and 3.

Table 3. $U(1)_{FN}$ quantum number assignments for Type-IIa, -IIb1 and -IIb2 models. [The defined range of $z$ is $|z| \sim 3$ for Type-IIa and $0 \lesssim z \lesssim 3$ for Type-IIb1 and -IIb2]

| $L_1$ | $L_2$ | $L_3$ | $\epsilon_R$ | $\mu_R$ | $\tau_R$ | $\nu_R$ | $H_1$ | $H_2$ | $S_1^+$ | $S_2^+$ |
|-------|-------|-------|-------------|-------|-------|-------|-------|-------|-------|-------|
| IIa   | 0     | 0     | -5         | -2    | -1    | $x + 1$ | 0     | $z - z$ | 1 - $x$ |
| IIb1  | -1    | -3    | -5         | 4     | -1    | -4    | $x$   | 0     | $z - z$ | 1 - $x$ |
| IIb2  | 2     | -5    | -7         | 7     | -3    | -6    | $x$   | 0     | $z - z$ | 3 - $x$ |

Zee Scalars, Muon $\gamma - 2$ and $\mu \rightarrow e \gamma$

The above minimally extended Zee-type models economically incorporate the $\nu_R$ and naturally explain the mass patterns of the neutrinos and leptons. The Zee-scalar Yukawa couplings with the neutrinos/leptons also exhibit an interesting spectrum. Now we are ready to analyze their phenomenological impact. The Brookhaven E821 collaboration has announced a 2.6 standard deviation in the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$, i.e., $\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (42.6 \pm 16.5) \times 10^{-10}$, which gives a 90% C.L. range for new physics,

$$15.5 \times 10^{-10} \leq \Delta a_\mu \leq 69.7 \times 10^{-10}.$$  

Different authors have interpreted this anomaly in terms of supersymmetry, muon compositeness, extra $Z'$, leptoquarks and extended neutrino models. We attempt to explain it from the contribution of the Zee-scalars and the singlet $\nu_R$ in our minimal Type-(I, II) models.

The Zee-scalars $S_1^\pm$ and $S_2^\pm$ in Type-I/II contribute to $g_\mu - 2$ via the Yukawa couplings $f_{12,23}$ with $(\mu_L, \nu_R, \tau)$ and $f_2$ with $(\mu_R, \nu_R)$, respectively. Thus, we have,

$$\Delta a_\mu = \frac{m_\mu^2}{96\pi^2} \left( \frac{|f_{12}|^2 + |f_{23}|^2}{M_1^2} + \frac{|f_{23}|^2}{M_2^2} \right) \simeq 11.8 \times 10^{-10} \times \frac{|f_{23}|^2}{M_2^2} \left( \frac{100 \text{ GeV}}{M_{S_2}} \right)^2,$$

with $M_1^2 = (\cos^2 \phi/M_{S_1}^2 + \sin^2 \phi/M_{H^0}^2)^{-2}$. Here $(M_{S_1}, M_{H^0})$ are the mass-eigenvalues of the two charged scalars in Eq. (1) and $\phi$ is their mixing angle. Our models forbid or highly suppress the mixing between $S_1^\pm$
Comparing this with Eq. (7), we see that Type-IIb1 has $f_2$ just below the current bound while Type-IIb2 is well below it. On the other hand, the $f_1$ coupling in Type-IIa lies slightly above the limit by a factor of $2 - 3$; given the uncertainty of the parameters, it can be easily adjusted to stay within the bound. Also, a much weaker bound on $f_3$ can be derived from $\tau \rightarrow \mu \gamma$ decay, i.e., $|f_3| \lesssim 0.06 - 0.16 \sim O(0.1)$ at 90% C.L., for $1.1 \lesssim |f_2| \lesssim 3$, which is consistent with the Type-II predictions in [1]. Finally, if we include $N_{\nu_R}(\geq 2)$ singlet $\nu_{\nu_R}$ with the same Yukawa coupling $f_2$, the upper [lower] bound in Eqs. (11) and (10) [Eq. (12)] will be relaxed by a factor of $\sqrt{N_{\nu_R}}$.

In summary, the Zee model naturally generates small neutrino Majorana masses by radiative corrections, but it neither predicts the Zee-scalar Yukawa couplings nor provides any insight on the lepton mass hierarchy. We have constructed a class of minimally extended Zee-models with the right-handed neutrino $\nu_R$ embedded, where a $U(1)$ symmetry protects the radiative neutrino masses while generating the lepton mass hierarchy, the hierarchy of the Zee-scalar Yukawa couplings required by the neutrino oscillation data, the hierarchy of Zee-scalar Yukawa couplings necessary for consistency with the $\mu \rightarrow e\gamma$ bound, and the size of the Zee-scalar Yukawa coupling needed for the BNL $g_\mu - 2$ anomaly. Furthermore, a light Zee scalar $S_2^\pm$ is predicted in our models, with a mass around $100 - 300$ GeV.

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