Efficient reliability assessment method for bridges based on Markov Chain Monte Carlo (MCMC) with Metropolis-Hasting Algorithm (MHA)

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Abstract. Reliability assessment plays a vital role in bridge health monitoring (BHM) technique. The analysis results of inspection data and monitoring data, such as numerical data, image data and video data, are not well due to there is no efficient reliability assessment method. This paper analysed the applied effect of Markov Chain Monte Carlo (MCMC) simulation method. The subset simulation method is used to analyse small failure probability events. Furthermore, the reliability assessment process based on Markov Chain Monte Carlo (MCMC) simulation method with Metropolis-Hasting Algorithm (MHA) is proposed. The advantage of this method is to improve the application efficiency and accuracy of reliability assessment based on BHM data.

1. Introduction

There are approximately 850,000 highway bridges, which were made with different materials and constructed at different years in China. In order to ensure the safety and durability of these bridges, many inspection and monitoring actions are adopted according to the standards that Code for Maintenance of Highway Bridges and Culverts and Standards for Technical Condition Evaluation of Highway Bridges. From the actual situation at present of inspection and monitoring, the data acquisition problem has been solved due to development of internet of things (IOT) techniques. A lot of numerical data, image data and video data are obtained from inspection and bridge health monitoring (BHM) system. At present, the key problem is how to analyse these data and assess the reliability of bridge structure. The most important aspect is the efficient reliability assessment method needs be proposed. This is the purpose and motivation of this study.

The current reliability assessment methods based on inspection and monitoring data do not have capability of predicting the future conditions of bridges. The main reason is the deterioration performance and time-dependent reliability can’t be considered. Therefore, the decision-making procedure is usually subjective and does not provide optimization for life-cycle cost (LCC).

As is well known that the suitable deterioration models are vital parts of the highway bridge asset management system and need to be developed based on inspection data, monitoring data and maintenance data. For inspection and monitoring data, the damage and deflection can be detected according to modal parameter method and visual inspection method. For maintenance data, a condition scale of 1 to 5 is used by Standards for Technical Condition Evaluation of Highway Bridges in China. Condition rating 1 represents the best condition state without any visual defect, whereas rating 5
indicates the completely failed condition of the bridge or component. Inspections are normally conducted once every 1 or 2 years.

To solve the problem that the deterioration model and reliability assessment method, many scholars studied in these areas and came to many useful conclusions. Niroshan K et al. maintains that the MCMC-based deterioration model performs better than regression-based nonlinear optimization (RNO) and Bayesian maximum likelihood (BML) in terms of network-level condition prediction accuracy and capture of model uncertainties\textsuperscript{(1)}. In order to better apply bridge health monitoring data, Ni et al.\textsuperscript{(5)} first proposed the concept and method of bridge structure reliability evaluation based on health monitoring information. Zhang Liye et al.\textsuperscript{(3)} proposed a real-time reliability assessment method based on acceleration monitoring information and PSD method, which was applied to the cable-stayed bridge of the main navigation channel of Donghai Bridge. Frangopol et al.\textsuperscript{(4)} proposed a reliability assessment method based on monitoring data and applied it to the existing Lehigh River Bridge. Some scholars have begun to consider how to comprehensively apply test (supervision) data and mechanical model, and the research idea is relatively advanced. Due to the long service life of bridges, the deterioration model of structural performance is an important factor to be considered in reliability evaluation. Zhang Liye et al.\textsuperscript{(5)} obtained the deterioration model of structural vibration frequency through the accelerated carbonization experiment of concrete beams. Agrawal et al.\textsuperscript{(6)} proposed a classification method to calculate the deterioration level of bridge components based on Weibull distribution. Akgul et al.\textsuperscript{(7)} studied the deterioration of structural reliability under time-varying loads. Existing research shows that the reliability assessment based on the inspection and monitoring data has a good application prospect. The key problem is how to consider the deterioration process and how to efficient apply monitoring data.

From above analysis, the better application prospect studies is Markov method. A Markov chain approach is the most popular stochastic deterioration modelling technique, and has been extensively used for predicting the future conditions of infrastructure facilities at the network level\textsuperscript{(5)}. Many advanced bridge management system (BMS) in the world use state-based Markov deterioration models (SBMDM). For example, Pontis used in the United States and other countries; Ontario Bridge Management System (OBMS) used in Ontario, Canada; Quebec Bridge Management System (QBMS) used in Quebec, Canada; KUBA (Kunstbauten in German, or engineering structures) used in Swiss Federal Roads, Switzerland; and the New York State DOT (NYSDOT) BMS used in New York\textsuperscript{(5)} are used for deterioration modelling. The main task of SBMDM is to estimate transition probability matrixes (TPMs) from inspection and monitoring data, which are also known as calibrating Markov models. A TPM describes probabilities of state transitions from one condition state to another in a given inspection cycle. A stationary Markov model applies time-independent TPMs by assuming a homogeneous deterioration pattern for a selected data set. By holding other potential contribution factors, it is possible to form similar characteristic element groups to isolate and analyse the deterioration process with time at the network level. Deterioration patterns of similar characteristic component groups, without repair and treatment work, can be assumed to be homogeneous when limited condition rating data are available and deterioration process of those can be modelled using stationary Markov models considering a single TPM.

2. General concepts of reliability
The reliability is used to assessment safety of existing or newly built bridges requires that the resistance effects is greater than the load effects in probabilistic sense. The degree of reliability is described using the limit state function such as following.

\[
Z(t) = R(t) - S(t)
\]

where \( R(t) \) and \( S(t) \) denotes the stochastic processes of resistance effects and load effects, respectively.
For the existing bridges that have been used for $T_0$ years, the reliability of service life in the subsequent $T_n$ should be expressed as

$$P_s(T_n) = P_s\left\{ Z(t) > 0, t \in (T_0, T_n) \right\}$$

(2)

Therefore, the failure probability can be determined as

$$P_f(T_n) = 1 - P_s(T_n)$$

(3)

The failure probability reflects the possibility of failure of the bridge structure. The key problem of reliability analysis is the establishment of limit state function. A useful method is to obtain reliability analysis parameters from BHM data.

3. Markov Chain Monte Carlo (MCMC) Simulation Method with Metropolis-Hastings Algorithm (MHA)

The Metropolis-hasting Algorithm (MHA) is often applied to MCMC simulation and has been increasingly used in the last 30-40 years for simulating complex, nonstandard multivariate distributions. It was one of the top 10 algorithms used in the twentieth century and has recently been used in many civil engineering scenarios [9, 10]. The MCMC models are developed in a Bayesian framework, which has been widely applied to estimating unknown parameters or posterior distributions in complex statistical models including deterioration models [11-14]. Many scholars proposed the Metropolis-Hastings algorithm based on MCMC simulation technology to updating the Markov deterioration model of highway and railway bridges. Furthermore, other studies have also been applied to the deterioration model of bridge network management level. In this paper, this technology is used to update and predict the bridge structure resistance model.

The essentially of MCMC simulation method is Monte Carlo integral of The Markov chain. The basic idea is to establish the Markov chain to sample the unknown variables and obtain the posterior distribution when the chain reaches the steady-state distribution. The MCMC method based on Bayes inference principle is mainly used to generate the samples of posterior distribution, calculate the edge distribution and the moment of posterior distribution. Different sampling methods will composite different MCMC methods.

MCMC simulation technology with Metropolis-Hastings algorithm based on Bayes inference principle analyses the BHM data, and predicts and updates the time-dependent deterioration model of bridge structure resistance. In the random truncation test of bridge reliability, the regression model of reliability is constructed based on Bayes analysis theory. Taking the commonly used exponential distribution and Weibull distribution as examples, the MCMC method based on Metropolis-Hastings sampling is used to dynamically simulate the Markov chain of parameter posterior distribution, and the Bayes estimation of parameters under random truncation condition is given.

The core of MCMC simulation technology with Metropolis-Hastings Algorithm is to determine the rules of structure transfer from the current value to the next value. The general method of establishing a Markov chain is metropolis-Hastings sampling method.

The description of the Markov chain based on MCMC as follows: a list of random variables $\{X_n\}_{n \geq 0}$ is called the Markov chain, if given the current value $X_n$ of any $n$, the past value $\{X_k, k \leq n-1\}$ and the future value $\{X_k, k \geq n+1\}$ are independent of each other, for any $i_0, i_1, \ldots, i_{n-1}, i_n$ and any $n \geq 0$, which is given by

$$P\left( X_{n+1} = j \mid X_0 = i_0, \ldots, X_{n-1} = i_{n-1}, X_n = i \right) = P\left( X_{n+1} = j \mid X_n = i \right)$$

(4)

The above process is called Markov process or Markov chain.
Suppose we want to sample from a posterior distribution \( f(x) \), and the Metropolis-Hastings sampling method, starting from the initial value \( x_0 \), specifies a rule for transforming from the current value \( x_t \) to the next value \( x_{t+1} \), a Markov chain \( \{x_0, x_1, \ldots, x_n, \ldots\} \) is obtained. Specifically, at a given current value \( x_t \), a random number \( x' \) is generated from a distribution \( g(x | x_t) \), and an accepted probability is calculated to determine whether this random number \( x' \) will be the next value in the sequence. Specifically as follows,

(a) Generate a candidate value \( x' \) from the proposed distribution \( g(x | x_t) \);

\[
\alpha(x_t, x') = \min \left\{ 1, \frac{f(x') g(x | x_t)}{f(x_t) g(x_t | x')} \right\}
\]

(b) Calculate the acceptance probability

(c) According to the probability \( \alpha(x_t, x') \), in accordance with the transfer rules, accept \( x_{t+1} = x' \), otherwise \( x_{t+1} = x_t \).

4. Bayes classification method for bridge failure mode identification and efficient reliability evaluation method

In the process of bridge use, extreme events are rarely or even impossible to occur, resulting in no or very few super-threshold values in the monitoring and evaluation information. Bridge failure is a typical event with small failure probability. Subset simulation is an efficient reliability analysis for the small failure probability problem method. It is reasonable through the middle of the failure event and could be divided into a series of probability space with a subset of the sequences contain relations. Thus the small failure probability are expressed as a series of larger conditional failure probability in the form of the product. Then using MCMC sampling condition of sample points to estimate large failure probability for improve the sampling probability of failure probability estimates. Compared with the direct Monte Carlo method for nonlinear implicit limit state equation, the subset simulation reliability analysis method has the significant advantage of being suitable for small probability problems.

Bayes classification method is adopted to design the bridge disease feature classifier. With minimum error rate (maximum posterior probability) and minimum average risk as indicators, the discriminant function and decision surface are constructed. When a sample to be identified is given, the probability of its belonging to a certain category is the basis for determining its belonging to each category, specifically as follows:

The prior probability \( P(\omega_j) \) and conditional probability density function \( p(X | \omega_j) \) can be used to obtain the posterior probability of samples \( X \) belonging to various categories, and this probability value can be used as the basis for category attribution. The minimum error rate (the maximum posterior probability) and the minimum average risk can be expressed as follows:

\[
\text{if } p(X | \omega_j) P(\omega_j) = \max \left\{ p(X | \omega_j) P(\omega_j) \right\}, j = 1, 2, \ldots, c, \text{ then, } X \in \omega_j \tag{5}
\]

\[
\text{if } p(\alpha_j | X) = \min_{k=1, 2, \ldots, c} \left\{ R(\alpha_k | X) \right\}, \text{ then, } X \in \omega_j \tag{6}
\]

Subset simulation method is introduced the reasonable middle failure event and could be divided into a series of probability space with a subset of the sequences contain relations. Thus the small failure probability expression for the solution of a series of easy to larger conditional failure probability in the form of the product. And then using MCMC method extracting conditions of sample points to estimate
The conditional failure probability, improves the sampling probability of failure probability estimates. Specific as follows:

If the functional function of reliability is \( Z(x) \), its failure domain is expressed as \( F = \{ x: Z(x) \leq 0 \} \). When the threshold value is set to \( b_1 > b_2 > \cdots > b_m = 0 \), the failure event should be expressed as \( F_k = \{ x: Z(x) \leq b_k \} \), \( (k = 1, 2, \cdots, m) \). At the same time, the relationship of every failure event should be obtained as \( F_1 \supseteq F_2 \supseteq \cdots \supseteq F_m = F \). Therefore, the failure event are written as \( F_k = \bigcap_{i=1}^{k} F_i \), \( (k = 1, 2, \cdots, m) \). According to the multiplication theorem in probability theory and the relationship between events, the failure probability can be expressed as

\[
P_f = P(F) = P(F_1) \cdot \prod_{i=2}^{m} P(F_i | F_{i-1})
\] (7)

From the above equation, it can be seen that when \( m = 4 \), and the magnitude of \( P_i \) is 0.1, the magnitude of \( P_f \) of can reach \( 10^{-4} \). It can be known that, through subset simulation, the small probability can be converted into the product of the larger conditional probability, so as to efficiently evaluate the failure probability of the event with small probability of failure.

The follow chart of reliability assessment process based on Markov Chain Monte Carlo (MCMC) Simulation Method with Metropolis-Hasting Algorithm (MHA) is shown in figure 1.

![Image](image-url)

Figure 1. The follow chart of reliability assessment process based on Markov Chain Monte Carlo (MCMC) Simulation Method with Metropolis-Hasting Algorithm (MHA)
By subset simulation, the small failure probability is converted into the product of the larger conditional probability, which improves the timeliness of the failure probability assessment. Compared with the direct Monte Carlo method which is suitable for nonlinear implicit limit state equation, the significant advantage of this method is that it is suitable for efficient analysis of small probability events. Therefore, the efficiency and accuracy of reliability analysis are improved.

5. Conclusions
In this study, the reliability assessment process based on Markov Chain Monte Carlo (MCMC) Simulation Method with Metropolis-Hasting Algorithm (MHA) is proposed. For bridge deterioration model problem, Markov Chain Monte Carlo (MCMC) Simulation Method with Metropolis-Hasting Algorithm (MHA) is improved based on bridge inspection data and monitoring data. For small failure probability event problem, the Bayes classification and subset method is integrated application. The advantage of proposed method is that the data can be updated over time which is to improve the application efficiency of BHM data and the accuracy of reliability assessment.

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