The US 2000-2003 Market Descent: Clarifications

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Abstract

In a recent comment [Johansen A 2003 An alternative view, Quant. Finance 3: C6-C7, cond-mat/0302141], Anders Johansen has criticized our methodology and has questioned several of our results published in [Sornette D and Zhou W-X 2002 The US 2000-2002 market descent: how much longer and deeper? Quant. Finance 2: 468-81, cond-mat/0209065] and in our two consequent preprints [cond-mat/0212010, physics/0301023]. In the present reply, we clarify the issues on (i) the analogy between rupture and crash, (ii) the Landau expansion, “double cosine” and Weierstrass-type solutions, (iii) the symmetry between bubbles and anti-bubbles and universality, (iv) the condition of criticality, (v) the meaning of “bullish anti-bubbles”, (vi) the absolute value of $t_c - t$, (vii) the fractal log-periodic power law patterns, (viii) the similarity between the Nikkei index in 1990-2000 and the S&P500 in 2000-2002 and (ix) the present status of our prediction.

In a recent comment [5], Anders Johansen has criticized our methodology and has questioned several of our results published in this journal [17] and in our two consequent preprints [19, 20]. We regret the controversial tone adopted in [5] but welcome this opportunity to clarify our work further.

In a series of works starting with [16] (see [13] and references therein), financial bubbles have been defined as regimes in which the stock market exhibits an unsustainable super-exponential growth, that can be characterized quantitatively as a genuine critical phenomenon with specific log-periodic power-law (LPPL) signatures. The underlying mechanism is proposed to be found in imitation between investors and their herding behavior, which lead to self-reinforcement positive feedbacks.

In [7], Johansen and Sornette introduced the concept of “anti-bubbles” to describe decaying LPPL price trajectories that are sometimes found to follow very large market highs. Based on models of imitation between investors and their cooperative herding behavior [7, 4, 14], it was realized that speculation and imitation also occur during bearish markets, leading to price trajectories that seem approximately symmetric to the accelerating speculative bubbles ending in crashes, under a time reversal transformation ($t_c - t \rightarrow t - t_c$ where $t_c$ is a critical time corresponding to the end of the bubble or the start of the anti-bubble). The two first examples of anti-bubbles were found [7] in the Japanese Nikkei stock index from 1990 to 1998, whose analysis led to the successful prediction of two trend reversals [9, 13], and in the Gold future prices after 1980, both after their
all-time highs. Several other examples have been described in the Russian stock market [12] and in emergent and western markets [8]. Our recent work [17, 19, 20] adds many other cases that all started in the summer of 2000.

**Status of the rupture analogy.** More precise and probably more relevant than the analogy with material rupture is the concept of a finite-time singularity as developed in [4, 14, 10, 2], which emerges from positive feedbacks. The concept of a finite-time singularity is the counterpart in the time-domain of the concept of criticality. The fight between positive and negative feedbacks is the key concept underlying the proposal of LPPL signatures of speculative bubbles and anti-bubbles in stock markets [13].

**Landau expansion, “double cosine” and Weierstrass-type solutions.** A. Johansen criticizes our use of the “double cosine” function on the basis that a sound theoretical justification is lacking, while he puts his faith in the Landau expansion introduced in [15] and extended up to third order in [7]. Actually, the full solution of the simplest renormalization group equation for a critical point has been analyzed in depth in [3] and provides an improvement of these approaches in the form of Weierstrass-type functions of the form

\[
\ln[p(t)] = A + B\tau^m + \Re\left(\sum_{n=1}^{N} C_n e^{i\psi_n \tau - s_n}\right),
\]

where \(\tau = |t - t_c|\), \(s_n = -m + in\omega\) and \(\Re()\) is the real part operator. The existence of different phases \(\psi_n\) incriminated by A. Johansen can be seen to derive naturally from the Mellin transform of the regular part of the renormalization group equation. In simple words, the different phases \(\psi_n\) embody an information on the mechanisms of interactions between investors. There is thus a sound theoretical justification for such a phase shift (understand \(\psi_2 - \psi_1\)) between the first and the second harmonics (understand the first two terms \(n = 1\) and \(2\) of the expansion (1)). When the phases have certain relationships (phase locking), a discrete hierarchy of critical times emerge, which has been found to describe very well the US stock market since the summer of 2000 [20]. A. Johansen’s misconception can probably be traced to the incorrect idea that the phase of the simple cosine formula (case \(N = 1\)) has no financial meaning because it can be gauged away in a redefinition of the time scale.

**Symmetry between bubbles and anti-bubbles and universality.** From a mechanistic view point, we advocate the existence of anti-bubbles from the idea that the fight between positive and negative feedbacks is operative both in bullish as well as in bearish markets [13]. From a descriptive view point, our recent works [17, 19, 20] just follow Johansen and Sornette’s previous works [7, 9, 12, 8], which introduced the concept of an “anti-bubble” from a symmetry perspective. A symmetry may have distinct consequences. It can be used to justify the same functional expressions both for bubbles and anti-bubbles. Thus, in the mathematical expressions, the symmetry between bubbles and anti-bubbles amounts to changing \(t_c - t\) for bubbles to \(t - t_c\) for anti-bubbles. Here, we should stress that, if a LPPL anti-bubble follows a LPPL bubble (which is not the general case), the critical time \(t_c\) is not generally the same. A noteworthy exception is the Russian stock market around 1997 [12]. We report in [17, 19] dozens of anti-bubbles in many different stock markets worldwide which started almost all in August 2000, that is, 4 months later than the end of the “new economy” bubble and its crash in April 2000. Another case is Chile around 1994-1995, where the bubble ended in February 1994 while the anti-bubble started in July 1995 [8].

A. Johansen would like that the symmetry between bubbles and anti-bubbles should be extended so that the same log-periodic angular frequency \(\omega\) describes both cases. He thus invokes more than
just a functional but a numerical symmetry. We think that this belief may be too rigid at the present time when we still have a rather limited understanding of this complex problem. We propose an open-minded approach more adapted to a learning phase. It is correct that, for LPPL bubbles, there is rather well-defined cluster of values for $\omega \approx 6.36 \pm 1.56$ and for $m \approx 0.33 \pm 0.18$ as reported in [11] (see equation (4) of [6]). For anti-bubbles in the USA S&P and in many EU markets, we find almost the same value $\omega \approx 12$. This value is comparable with those obtained for the anti-bubbles in the Latin-American markets and Western markets in the 1990’s [8]. It is interesting that this value $\omega \approx 12$ is approximately twice the most probably value $\omega$ found for LPPL bubbles. Does it correspond to a log-periodicity different from that of bubbles? Probably not for the following reason: we have found in [17, 19] that both $\omega$ and $2\omega$ were quite significant in the anti-bubbles, including the Nikkei case that started in 1990. Due to the probable variation of the strength of nonlinear processes in the stock markets, it can be expected that the amplitudes of the first and second harmonics can be different from one realization to the next. Within the renormalization group framework, the relative strength of the first and second harmonics is controlled by the regular part [3] which describes the specific interactions of the investors that led to a given realization of the market. Let us add that the importance of the role of log-periodic harmonics has been demonstrated for turbulence [18, 21], where the evidence is much stronger. For the emergent markets, the LPPL signatures are not as significant as for the major western markets, as noted already in [8]. A. Johansen also notes the $\omega$’s found for different worldwide markets are not peaked and may be due to noise. Instead, we think that this is due to a possible lack of sufficient robustness of the fits, which does not diminish the evidence for log-periodicity but suggests to interpret with care the specific reported values. This can be seen from the fact that, if we impose the additional condition in our fits that the different worldwide markets exhibit an anti-bubble with the same critical time $t_c$, we find that their angular log-periodic frequency $\omega$ are very close to each other. The quasi-simultaneity of the starting time and the ensuing strong synchronization of the anti-bubbles exhibited by the major stock markets in the world, which has been documented in [19], provides an additional justification for the use of the same critical time $t_c$.

**Criticality.** A. Johansen criticizes our abandoning of the constraint $m < 1$ as a necessary condition to qualify the existence of a bubble or anti-bubble, suggesting that we have renounced the concept of criticality. There are several issues here that need to be distinguished. First, our many tests performed by the present authors and previously by A. Johansen with D. Sornette (reported as the work that Johansen performed with Matt Lee in [5]) show that the condition on the exponent $m$ is much less effective in the detection of bubbles than a condition on $\omega$ for instance (see also discussion in Chapter 9 of [13]). This is one justification for abandoning any constraint on this rather sensitive parameter to “let the data speak.” Second, finding values of $m \geq 1$ does not amount to an absence of criticality, because the equation is still critical (that is, it exhibits a singularity) due to the presence of the theoretically infinite hierarchy of log-periodic oscillations. In other words, criticality remains present due to the imaginary part $\omega$ of the exponent $s_n = -m + in\omega$ of the LPPL (see equation (1)) as long as it is non-zero, whatever the value $m$ of its real part. Third, we can relax the condition $0 < m < 1$ for the present purpose because our LPPL formulas describe only a finite range of the time interval: it is well-known that true singularities do not exist in nature as friction, finite-size effects and other regularization mechanisms come into play close enough to the theoretical mathematical singularity. What is important is the ability of the LPPL formula to describe with good accuracy a large range of the data, not necessarily the very close proximity to the phantom singularity. In this respect, we refer to the rather detailed discussion of the effect of finite-size effects on singularities presented in [10].
“Bullish anti-bubbles”. In our analysis of the largest stock markets in the world, we have identified six examples which give a positive coefficient $B$ in (1). In particular, the statistical significance of this result is very high for Australia, Mexico and Indonesia. This regime $B > 0$ is different from the normal bubble and anti-bubble cases previously reported for which $B < 0$. This regime $B > 0$ has been coined “bullish anti-bubbles” [19] to describe the joint features of decelerating log-periodic oscillations and of an overall increasing price. In contradiction with Johansen’s remark, this regime $B > 0$ does not lead to infinite prices in a finite time but describes a long-term growth which turns out to be slower than standard exponential growth. The same remark applies for $m > 1$.

**Absolute value of $t_c - t$.** In complete disagreement with A. Johansen’s remark, our use of $|t_c - t|$ in our fits to locate the critical time $t_c$ does not abandon “another restriction coming from the data.” Rather than adding a degree of freedom, this approach instead removes an arbitrariness previously present in the fitting procedure in choosing the time interval over which the fit is performed. Rather than determining an approximate starting time and/or estimating the critical time $t_c$ by the location of the largest market peak, using $|t_c - t|$ makes the fits almost independent of the chosen starting time. This improved robustness has been documented in details by our many numerical tests presented in [17, 19].

**Fractal LPPL patterns.** As we quoted in [17], Drozdz et al. [1] have reported the existence of LPPL within LPPL within LPPL, using eye-ballling in a single case. As mentioned by A. Johansen [5], he with D. Sornette studied this phenomenon rather systematically about a year earlier but never published due to the rather marginal quality level of the results. In [17], we mentioned that the worldwide anti-bubble started in the summer of 2000 has also left its imprint on the Japanese market, leading to an anti-bubble within the large scale anti-bubble that started in January 1990. This possibility of structures within structures is expected on general grounds from the renormalization group model of LPPL singularities leading to Weierstrass-like solutions (see [20, 13]). The problem is that such observation is not very robust when one goes to small time scales, probably due to the fact that “noise” and idiosyncratic news affect more and more strongly the price time series, the smaller is the time scale of observation. However, we note that our report [17] of a 2.5 year long anti-bubble decorating a 13 year long anti-bubble of the Nikkei index should have a special status because both time scales are sufficiently long to compare with the time span over which previous LPPL have been qualified. A. Johansen himself acknowledges that “the real success was with a LPPL analysis on time scales of one to two years of data.” Our report in [17] passes this criterion and should thus be considered at a level different from the published [1] and unpublished analyses at smaller time scales.

**Similarity between the Nikkei index in 1990-2000 and the S&P500 in 2000-2002.** Johansen downplays the “remarkable similarity” we as well as many observers noticed between these two markets. First, the factor of 2 in the value of the log-periodic frequency is explained by the competition between the two first harmonics $n = 1$ and $n = 2$ in (1), as we explained above. In [17], we stress the remarkable similarity in the two markets with respect to the existence of two harmonics in both cases. Second, the Nikkei did go through a now well-recognized speculative bubble culminating at the end of December 1989, even if its price trajectory does not qualify as a very good LPPL. We note in this vain that an anti-bubble is usually the follow-up of very high prices, not necessarily of a LPPL bubble. Even in the case of the US market, we stressed above that the critical time of the bubble occurred 4 months before the critical time of the following anti-bubble. This again stresses that one should exercise caution in twinning rigidly in time the occurrence of
bubbles and of anti-bubbles. Third, Johansen argues that the analysis of the Nikkei was based on 9 years of data compared with less than 3 years for the US market which, he argues, makes these two cases apart. Johansen forgets to mention is that the 9 years of Nikkei data required the use of a log-periodic formula extended to third-order in the Landau expansion mentioned above while the analysis in [17] of the US market used only the first-order formula and its extension with a second harmonics. Johansen and Sornette’s initial analysis of the Nikkei data in [7] showed that, similarly to the US market, the first three years of the Nikkei time series could be adequately described by the first-order formula. It is by extending to large time horizon that it was necessary to use the higher-order terms in the Landau expansion. It is also interesting to note that there was a global anti-bubble starting in January 1994 in the major western stock markets [8], which also bears similarities to the present worldwide 2000-2003 anti-bubble case [19]. The global anti-bubble in the mid-1990’s lasted less than one year, while the 2000-2003 anti-bubble is still alive on many more markets, resulting in a much higher statistical significance level. There is thus no qualitative nor quantitative difference between the Japanese and USA data sets. We would like to add that the similarity between the Nikkei in 1990-2000 and the S&P500 in 2000-2002 can be further strengthened by paralleling the economic and financial distresses of the two countries, as explained in [17].

Status of the prediction. Finally, “The proof of the pudding is in the eating.” The ultimate evidence attesting the true nature of something lies in the verification of ex-ante predictions by future data. We have offered the predictions for the future of the US market in [17, 20] and of many worldwide markets in [19] as an important additional step for testing the LPPL hypothesis. These predictions for the S&P500 US market are compared with the realized values and are also updated monthly (go to the URL http://www.ess.ucla.edu/faculty/sornette/ and then click on “The future of the USA stock market”). Recall that the prediction published in [17] was made at the end of August 2002. At the time of the latest comparison, March 18 2003, one can see that we correctly predicted the recovery of the market until the end of 2002 but missed the severe drop that followed, which was probably due to the uncertainties associated with the coming war with Iraq. We should also stress that these last months have exhibited a very large volatility, leading to deviations from our prediction that are however comparable in magnitude with previous deviations in the in-sample period. Our predictions are fundamentally “low-frequency” in nature and cannot obviously capture the detailed idiosyncratic volatility. The comparison between our predictions and the realized price should thus be made at the time scale of the prediction horizon, that is, from August 2002 till summer 2004. We stand by our prediction that the market should appreciate somewhat and then resume its overall bearish anti-bubble descent.

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References

[1] Drozdz S, Ruf F, Speth J and Wojcik M 1999 Imprints of log-periodic self-similarity in the stock market *Eur. Phys. J. B* **10** 589-93

[2] Gluzman S and Sornette D 2002 Classification of possible finite-time singularities by functional renormalization *Phys. Rev. E* **66** 016134

[3] Gluzman S and Sornette D 2002 Log-periodic route to fractal functions *Phys. Rev. E* **65** 036142
[4] Ide K and Sornette D 2002 Oscillatory finite-time singularities in finance, population and rupture Physica A 307 63-106

[5] Johansen A 2003 An alternative view Quant. Finance 3 C6-C7

[6] Johansen A 2003 Characterization of large price variations in financial markets Physica A in press

[7] Johansen A and Sornette D 1999 Financial “anti-bubbles”: log-periodicity in Gold and Nikkei collapses Int. J. Mod. Phys. C 10 563-75

[8] Johansen A and Sornette D 2000 Bubbles and anti-bubbles in Latin-American, Asian and Western stock markets: an empirical study, Int. J. Theor. Appl. Fin. 4 853-920

[9] Johansen A and Sornette D 2000 Evaluation of the quantitative prediction of a trend reversal on the Japanese stock market in 1999 Int. J. Mod. Phys. C 11 359-64

[10] Johansen A and Sornette D 2001 Finite-time singularity in the dynamics of the world population and economic indices Physica A 294 465-502

[11] Johansen A and Sornette D 2002 Endogenous versus exogenous crashes in financial markets preprint http://arXiv.org/abs/cond-mat/0210509

[12] Johansen A, Sornette D and Ledoit O 1999 Predicting financial crashes using discrete scale invariance Journal of Risk 1 5-32

[13] Sornette D 2003 Why Stock Markets Crash? (Critical Events in Complex Financial Systems) (Princeton, NJ: Princeton University Press)

[14] Sornette D and Ide K 2003 Theory of self-similar oscillatory finite-time singularities in Finance, Population and Rupture Int. J. Mod. Phys. C 14 in press

[15] Sornette D and Johansen A 1997 Large financial crashes Physica A 245 411-22

[16] Sornette D, Johansen A and Bouchaud J-P 1996 Stock market crashes, Precursors and Replicas J.Phys.I France 6 167-75

[17] Sornette D and Zhou W-X 2002 The US 2000-2002 market descent: how much longer and deeper? Quant. Finance 2 468-81

[18] Zhou W-X and Sornette D 2002 Evidence of intermittent cascades from discrete hierarchical dissipation in turbulence Physica D 165 94-125

[19] Zhou W-X and Sornette D 2003 Evidence of a worldwide stock market log-periodic anti-bubble since mid-2000 preprint http://arXiv.org/abs/cond-mat/0212010

[20] Zhou W-X and Sornette D 2003 Renormalization group analysis of the 2000-2002 anti-bubble in the US S&P 500 index: explanation of the hierarchy of 5 crashes and prediction preprint http://arXiv.org/abs/physics/0301023

[21] Zhou W-X, Sornette D and Pisarenko V 2003 New evidence of discrete scale invariance in the energy dissipation of three-dimensional turbulence: correlation approach and direct spectral detection Int. J. Modern Phys. C 14 in press