Spin Filter in DVCS amplitudes

Bernard L. G. Bakker\textsuperscript{a} and Chung-Ryong Ji\textsuperscript{b}

\textsuperscript{a} Department of Physics and Astrophysics, Vrije Universiteit, De Boelelaan 1081, NL-1081 HV Amsterdam, The Netherlands
\textsuperscript{b} Department of Physics, North Carolina State University, Raleigh, NC 27695-8202, USA

In deeply virtual Compton scattering (DVCS), it is found that in the kinematics with large transverse photon momenta angular momentum is not conserved if the amplitudes are calculated in terms of widely used reduced operators. Consequently, those kinematics will lead to the wrong analysis of experimental data in terms of generalized parton distributions. Moreover, the contribution of the longitudinally polarized virtual photon in those kinematics should not be neglected in the analysis of DVCS amplitudes.

For some time already, it has been realized that in non-forward kinematics, e.g. deeply virtual Compton scattering (DVCS), the scattering amplitudes, and thus cross sections, can be expressed in terms of objects, generalized parton distributions (GPDs), which complement the knowledge encoded in parton distribution functions \cite{1,2,3}. This idea has inspired many authors, whose work has been summarized in several important review papers \cite{4,5}.

The paramount feature of the treatment of deep inelastic scattering (DIS) and DVCS is factorization, i.e., writing the full scattering amplitude as a convolution of a hard-scattering amplitude to be calculated in perturbation theory, and a soft part embodying the hadronic structure. The use of a hard photon that is far off-shell, say \(-q^2 = Q^2 \gg Q\) any relevant soft mass scale, enables factorization theorems \cite{6} with the identification of the hard scattering amplitude. A further step is the introduction of light-front (LF) variables with the choice of a preferred kinematics in which the amplitudes are calculated and the link between the theoretical quantities, GPDs, and the cross sections can be established. Light-front dynamics (LFD) (see e.g. Ref. \cite{5}) can be invoked to further analyze the physics, as it has the advantage that vacuum diagrams are either rigorously absent or suppressed. In the context of DVCS it means that in a reference frame where the momentum of the incoming photon \(q^\mu\) has vanishing plus component: \(q^+ \equiv (q^0 + q^3)/\sqrt{2} = 0\), it cannot create partons, as their momenta must have positive plus-components and these components are conserved in LFD. This simplification facilitates the partonic interpretation of amplitudes.

This paper is devoted to a number of aspects of the GPD formalism. A soft part embodying the hadronic amplitude: the latter being essential for a correct application of the GPD formalism.

Before we get into the discussion of the GPD formalism, we first report our benchmark calculation of the complete full DVCS amplitude for the scattering of a massless lepton \(\ell\) off a point-like fermion \(f\) of mass \(m\). In the final state, we find the scattered lepton \(\ell'\), the fermion \(f'\) with momentum \(k'\) and a (real) photon \(\gamma'\), viz \(\ell \rightarrow \ell' + \gamma'\), \(\gamma' + f \rightarrow f'\). (‘Complete’ means that the amplitude includes the leptonic part and ‘full’ means that no approximations are made in the calculation of the hadronic amplitude.) The complete amplitude at tree level can be written as

\[
\mathcal{M} = \sum_h \mathcal{L}((\lambda', \lambda) h) \frac{1}{q^2} \mathcal{H}(\{s', s\} \{h', h\}),
\]

where the quantities \(\lambda', \lambda, h', h, s', s\), and \(s\) are the helicities of the outgoing and incoming leptons, outgoing and incoming photons, and the rescattered and target fermions, respectively. Leaving out inessential factors, we may write

\[
\mathcal{L}((\lambda', \lambda) h) = \bar{u}(\ell'; \lambda') \gamma^+(q; h) u(\ell; \lambda),
\]

\[
\mathcal{H}(\{s', s\} \{h', h\}) = \bar{u}(k'; s') (\mathcal{O}_s + \mathcal{O}_u) u(k; s),
\]

where the s- and u-channel operators of the intermediate fermion are given by

\[
\mathcal{O}_s = \frac{\bar{q}'(q; h')(k + \gamma' + m) \gamma^+(q; h)}{(k + q')^2 - m^2},
\]

\[
\mathcal{O}_u = \frac{\bar{q}'(q; h)(k - \gamma' + m) \gamma^+(q; h')}{(k - q')^2 - m^2}.
\]

We take the following three kinematics for the momenta of the incoming and outgoing particles in the hadronic amplitudes:

1. \(\delta\)-Kinematics \((q^+ \rightarrow 0\) as \(\delta \rightarrow 0\))

\[
q^\mu = \left(\delta p^+, Q, 0, \frac{Q^2}{2(\zeta + \delta)p^+} + \frac{\zeta m^2}{2(x - \zeta)p^+}\right),
\]

\[
q'^\mu = \left((\zeta + \delta)p^+, Q, 0, \frac{Q^2}{2(\zeta + \delta)p^+}\right),
\]

\[
k^\mu = \left(x p^+, 0, 0, \frac{m^2}{2xp^+}\right),
\]

\[
k'^\mu = \left((x - \zeta)p^+, 0, 0, \frac{m^2}{2(x - \zeta)p^+}\right),
\]

(4)
(2) $q'^+ = 0$ Kinematics (effectively, ‘1+1’ dim.)

$$q'^+ = \left(-\zeta p^+, 0, 0, \frac{Q^2}{2\zeta p^+}\right),$$

$$q'^{\mu} = \left(0, 0, 0, \frac{Q^2}{2\zeta p^+} - \frac{\zeta m^2}{x(x - \zeta)p^+}\right).$$

The momenta $k^\mu$ and $k'^\mu$ are the same as in case (1).

(3) Nonvanishing $q^+$ and $q'^+$ Kinematics (with $m = 0$)

$$q'^+ = \left(-\frac{\zeta}{2} p^+, \frac{Q}{\sqrt{2}} 0, 0, \frac{Q^2}{2\zeta x}\right),$$

$$q'^{\mu} = \left(0, 0, 0, \frac{Q^2}{2\zeta x}\right).$$

The momenta $k^\mu$ and $k'^\mu$ are the same as in case (1) if the limit $m \rightarrow 0$ is taken.

These kinematics correspond to the hard-scattering part of a DVCS amplitude where the fermions are the quarks and $p^+$ is the plus-component of the momentum of the parent hadron target. We use the Kogut-Soper spinors $\gamma_{\frac{1}{2}}$ normalized to $2m$ and the polarization vectors

$$\epsilon(q; \pm 1) = \frac{1}{\sqrt{2}} \left(0, \mp i, \mp q_x \mp i q_y, q^2\right),$$

$$\epsilon(q; 0) = \frac{1}{\sqrt{q^2}} \left(q^+, q_x, q_y, \frac{q^2 - q'^2}{2q^2}\right).$$

that correspond to the LF gauge $A^+ = 0$.

All of these three kinematics yield identical kinematical invariants such as $s = \mp \frac{\zeta}{2} Q^2$ and $u = -\frac{\zeta}{2} Q^2$ in the DVCS limit as $\delta \rightarrow 0$ and $m \rightarrow 0$. However, each of them has its own merit of consideration.

In the $\delta \rightarrow 0$ limit, the $\delta$-kinematics coincides with the well-known $q^+ = 0$ frame [10] frequently cited in the discussion of the GPD formalism. Noticing that taking $q^+ = 0$ will lead to singular polarization vectors in the LF gauge $A^+ = 0$ (see e.g. Eq. (7)), we proceed with care: $q^+$ is set to $\delta p^+$ and all amplitudes are expanded in powers of $\delta$, taking the limit $\delta \rightarrow 0$ at the very end of the calculation of the complete, physical amplitude. The $q'^+ = 0$ kinematics without any transverse component (effectively, ‘1+1’ dimensional) avoids the singularity in the polarization vectors of the real photon and consequently provides a convenient framework of calculation without encountering any singularity. Similarly, the nonvanishing $q^+$ and $q'^+$ kinematics also avoids the singularity in the amplitude calculation, while the photon carry the same order of transverse momenta as the ones in the $\delta$-kinematics given by Eq. (4).

The results from these three kinematics are summarized in Tables I, II, and III. A straightforward evaluation of $\mathcal{L}(\{\lambda', \lambda\} h)$ gives the result in Table I, where we have used the corresponding lepton kinematics\(^1\) to

\(^1\) The details of lepton kinematics and spinors will be presented somewhere else.

\(^2\) We have also confirmed the similar interchange of the helicity amplitudes between the kinematics with and without the transverse momentum of the virtual photon in the case of a form-factor calculation.

| $\{h', h\} \{s', s\}$ | Eq. (4) | Eq. (5) | Eq. (6) |
|-----------------|--------|--------|--------|
| $(+1, +1)$ | $\frac{1}{2} \sqrt{\frac{x}{x - \zeta}} (1 + \frac{\zeta}{x})$ | $2 \sqrt{\frac{x}{x - \zeta}} (1 + \frac{\zeta}{x})$ | $-2 \sqrt{\frac{x}{x - \zeta}} (1 + \frac{\zeta}{x})$ |
| $(+1, -1)$ | $\frac{1}{2} \sqrt{\frac{x}{x - \zeta}} (1 - \frac{\zeta}{x})$ | $-2 \sqrt{\frac{x}{x - \zeta}} (1 - \frac{\zeta}{x})$ | $2 \sqrt{\frac{x}{x - \zeta}} (1 - \frac{\zeta}{x})$ |
| $(+1, 0)$ | $i \sqrt{2} \sqrt{\frac{x}{x - \zeta}} (1 + \frac{\zeta}{x} - \frac{1}{2})$ | $0$ | $-i \sqrt{2} \sqrt{\frac{x}{x - \zeta}} (1 + \frac{\zeta}{x} - \frac{1}{2})$ |
| $(+1, -1)$ | $i \sqrt{2} \sqrt{\frac{x}{x - \zeta}} (1 - \frac{\zeta}{x} + \frac{1}{2})$ | $0$ | $i \sqrt{2} \sqrt{\frac{x}{x - \zeta}} (1 - \frac{\zeta}{x} + \frac{1}{2})$ |

$\mathcal{L}(\{\lambda', \lambda\} h) = (-1)^{\lambda' - \lambda + h} \mathcal{L}(\{\lambda', \lambda\} h)$. (8)

The full hadronic amplitudes are shown in Table II where we again presented the results only up to order $\delta$. They obey the rule

$\mathcal{H}(\{h', -h\} \{s', -s\}) = (-1)^{h' - h - s + s'} \mathcal{H}(\{h', h\} \{s', s\})$. (9)

The complete DVCS amplitude $\mathcal{A}$ in Eq. (11) is shown in Table III. Since all the singular terms of orders $\delta^{-2}$ and $\delta^{-4}$ are exactly cancelled out in the complete amplitude, we have taken $\delta = 0$ in Table III. Note in Table III that there is an interchange\(^2\) of the polarization of the final photon in the result of the ‘1+1’ dim. kinematics in comparison with the other kinematics, in which the momenta of photons have transverse components. This is remarkable in view of the LF helicity [11]. To appreciate this point, we draw in Fig. II the spin directions of the outgoing photon with the LF helicity $h'$ for the two different kinematics: one without any transverse momentum such...
as Eq. (6) and the other with the transverse momentum of order $Q$ such as Eq. (4) or Eq. (5). One should realize that the LF helicity states are defined for a momentum $q'$ by taking a state at rest with the spin projection along the $z$ direction equal to the desired helicity, then boosting in the $z$ direction to get the desired $q'^+$, and then doing a LF transverse boost (i.e., $E_1 = K_1 + J_2$) to get the desired transverse momentum $q'_{\perp}$. Whether the kinematics includes the LF transverse boost ($E_1$) or not makes a dramatic difference in the spin direction because $E_1$ rotates the spin direction. Thus, for the l. h. panel of Fig. 1 the spin direction of the LF helicity state is opposite (or antiparallel) to the direction of the photon momentum while for the r. h. panel of Fig. 1 the spin directions of the LF helicity state and the Jacob-Wick helicity state are related by the Wigner function $d_{h',h}^0(\tan^{-1} \frac{2m}{Q})$ in the DVCS limit, which becomes unity as $Q \to \infty$. This illustrates the correspondence between the results of a kinematics with $q'_{\perp} = 0$ and a kinematics with the transverse momentum of order $Q$: e.g. in Table 11 the result of $h' = 1$ in the effective ‘1+1D’ kinematics corresponds to the result of $h' = -1$ in the $\delta$-kinematics or the nonvanishing $q^\perp$ kinematics for $\lambda' = \frac{1}{2}, \frac{1}{2}$ and $s, s' = \frac{1}{2}, \frac{1}{2}$. One should note that the conservation of angular momentum is satisfied in the complete full amplitudes for any kinematics. Therefore, we may take the calculation up to now as a benchmark for the discussion of the GPD formalism as we do below. Rewriting the $s$- and $u$- channel hadronic amplitudes as

\[ \bar{u}(k'; s')O_\sigma u(k; s) = \epsilon_\mu^*(q'; h')\epsilon_\nu(q; h)T_\mu^{s\nu}, \]
\[ \bar{u}(k'; s')O_\sigma u(k; s) = \epsilon_\mu^*(q'; h')\epsilon_\nu(q; h)T_\mu^{u\nu}, \]

we may neglect an inessential fermion mass $m$ to express the tensorial amplitudes $T_\mu^{s\nu}$ and $T_\mu^{u\nu}$ as

\[ T_\mu^{s\nu} = \frac{k_\alpha + q_\alpha}{s} \bar{u}(k'; s')\gamma_\mu \gamma_\alpha \gamma_\nu u(k; s), \]
\[ T_\mu^{u\nu} = \frac{k_\alpha - q_\alpha}{u} \bar{u}(k'; s')\gamma_\mu \gamma_\alpha \gamma_\nu u(k; s), \]

and the Sudakov variables $n^u(+) = (1, 0, 0, 0)$ and $n^u(-) = (0, 0, 0, 1)$, one may expand $T_\mu^{s\nu}$ and $T_\mu^{u\nu}$ to find the terms proportional to $\bar{u}(k'; s')\gamma^\nu u(k; s)$ and $\bar{u}(k'; s')\gamma^\nu u(k; s)$ that correspond to the nucleon GPDs $H(x, \Delta^2, \zeta)$ and $\tilde{H}(x, \Delta^2, \zeta)$ defined e.g. in Ref. 1, respectively (here, $\Delta^2 = (q^\perp)^2$). One should note, however, that a special system of coordinates without involving any large transverse momentum (see e.g. Eq. (5)) was chosen in Ref. 1 to compute the scattering amplitude in terms of GPDs. We realize that Ref. 2 uses essentially the same special system of coordinates as Ref. 1.

In order to cover the more general kinematics involving large transverse momenta such as given in Eqs. (4) and (5), we may expand $q^\mu$ (similarly $q'^\mu$) and $k^\mu$ as $q^\mu = q^\perp n^\mu(+) + q^\perp n^\mu(-) + q^\perp k^\perp$ and $k^\mu = k^\perp n^\mu(+) + k^\perp n^\mu(-)$ with $q^\perp$ representing the transverse momentum corresponding to $q^\mu$. For $m = 0$, $k^\perp = 0$ and $T_\mu^{s\nu}$ (similarly

\[
\sum_{h} \mathcal{L}((\lambda', \lambda) S, h) \frac{1}{z} \mathcal{H}(h')(h)(s', s))
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\{\lambda', \lambda\} & h' & \{s', s\} & \text{Eq. (4)} & \text{Eq. (5)} & \text{Eq. (6)} \\
\hline
\{\frac{1}{2}, \frac{1}{2}\} & 1 & \{\frac{1}{2}, \frac{1}{2}\} & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\
\{\frac{1}{2}, \frac{1}{2}\} & 1 & \{-\frac{1}{2}, -\frac{1}{2}\} & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\
\{-\frac{1}{2}, -\frac{1}{2}\} & 1 & \{\frac{1}{2}, \frac{1}{2}\} & 0 & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 \\
\{-\frac{1}{2}, -\frac{1}{2}\} & 1 & \{-\frac{1}{2}, -\frac{1}{2}\} & 0 & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 \\
\{\frac{1}{2}, \frac{1}{2}\} & -1 & \{\frac{1}{2}, \frac{1}{2}\} & 0 & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 \\
\{\frac{1}{2}, \frac{1}{2}\} & -1 & \{-\frac{1}{2}, -\frac{1}{2}\} & 0 & \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 \\
\{-\frac{1}{2}, -\frac{1}{2}\} & -1 & \{\frac{1}{2}, \frac{1}{2}\} & -\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\
\{-\frac{1}{2}, -\frac{1}{2}\} & -1 & \{-\frac{1}{2}, -\frac{1}{2}\} & -\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \\
\hline
\end{array}
\]
Since $q^-$ has the highest power of $Q$ among the components of momenta, one may just take the terms proportional to $q^-$ as shown in Refs. [1] and [2], i.e.,

$$T_s^{\mu\nu} = \frac{g^-}{s} \left[ (n^{\mu}n^{\nu} + n^\nu n^\mu - g^{\mu\nu}) \right. \times \bar{u}(k'; s') \gamma_5 u(k; s) \left. - i\epsilon^\alpha \epsilon^\beta n_\alpha n_\beta \times \bar{u}(k'; s') \gamma_5 u(k; s) \right].$$

(13)

However, one should note that Eq. (13) cannot provide the full result of the hadronic amplitude in the kinematics involving large transverse momenta such as Eq. (4) and Eq. (6), because the polarization vectors $\epsilon^\alpha(q^-; h')$ and $\epsilon^\nu(q; h)$ in Eq. (10) amplify the contributions neglected in the tensorial amplitude $T_u^{\mu\nu}$ (similarly $T_s^{\mu\nu}$) given by Eq. (14). For example, the coefficient of $\bar{u}(k'; s') \gamma_5 u(k; s)$ in the $s$-channel hadronic amplitude $\bar{u}(k'; s') O_s u(k; s)$ is given by the following four terms:

$$\frac{1}{2} \left[ 2(k^+ + q^+) \epsilon^- (q^-; h') \epsilon^-(q; h) 
+ \epsilon^-(q^-; h') q_\perp \cdot \epsilon^-(q; h) 
+ \epsilon^- (q; h) q_\perp \cdot \epsilon^-(q^-; h') 
- q^- \epsilon^-(q^-; h') \cdot \epsilon^-(q; h) \right].$$

(15)

Since all of the above four terms have the same powers of $Q$, one must keep them all. In other words, the factorization in the tensorial amplitude $T_s^{\mu\nu} + T_u^{\mu\nu}$ cannot hold in general because the polarization vectors $\epsilon^\alpha(q^-; h')$ and $\epsilon^\nu(q; h)$ can amplify the terms neglected in the tensorial amplitude unless a special system of coordinates is chosen to avoid the large transverse momenta of initial and final photons such as given by Eq. (4). Thus, we note that the formulation of GPDs on the level of the tensorial amplitude is not general enough to cover the kinematics with large transverse momenta such as given by Eqs. (4) and (6) but is limited to the special system of coordinates without involving large transverse momenta as given by Eq. (5). This is the main point of this paper. In the following, we demonstrate this point explicitly, presenting the consequence of taking the reduced amplitude that keeps only the terms proportional to $q^-$ in the tensorial amplitude as done in the formulation of GPDs. Unless the kinematics is chosen properly to avoid the large transverse momenta of initial and final photons, we find that the reduced amplitude does not agree with the full amplitude but yield the wrong result, not even satisfying the conservation of angular momentum.

Since the $q^+ = 0$ frame is used in the GPD formalism, we utilize the $\delta$-kinematics for our demonstration. We perform an expansion in the hard momentum scale $Q$, which allows us to define reduced hadronic amplitudes. In the expansion, it is important to retain terms of orders $\delta^{-1}, \ldots, \delta^2$ as well as orders $Q^{-1}, \ldots, Q^2$, as it turns out that not only are the order $\delta^{-1}$-terms cancelled by order $\delta$ terms in the convolution of $L$ and $H$, but also that the order $Q^{-1}$-contribution of the longitudinally polarized virtual photon gives a finite contribution in leading order. (We have checked that the two limits, $\delta \to 0$ and $Q \to \infty$ commute.)

The reduced hadronic operators used in the formulation of GPDs are defined as the limits $Q \to \infty$ of the operators given in Eq. (3) and found to be, as expected:

$$O_s|_{\text{Red}} = \frac{\langle \gamma^+(q^-; h') \gamma^+ \gamma^- \rangle \langle \phi(q^-; h') | 1}{2p^+} x - \zeta,$$

$$O_u|_{\text{Red}} = \frac{\langle \gamma^+(q^-; h') \gamma^+ \gamma^- \rangle \langle \phi(q^-; h') | 1}{2p^+} x,$$

(16)

These reduced propagators contain the nilpotent Dirac matrix $\gamma^+$ only, which kills the singular parts of the polarization vectors, namely $\epsilon^-(q^-; h') \gamma^+$. This is the reason for disregarding the singularities in the polarization vectors in $q^+ = 0$ kinematics, as the reduced hadronic amplitude does not ‘see’ it. However, the leptonic part $L$ of the complete amplitude is also singular. Consequently, the complete amplitude calculated with the reduced hadronic part and taking into account the transverse polarizations only, is wrong, even in the limit $Q \to \infty$.

Table [IV] clearly shows that the reduced amplitudes and the full ones disagree. We have checked that the same disagreement occurs in the nonvanishing $q^+$ and $q^+$ kinematics given by Eq. (6), although for the kinematics without any transverse component, e.g. Eq. (5), the reduced amplitudes and the full ones do agree. Upon convoluting the leptonic and hadronic amplitudes to obtain the complete ones, we find that the singular 1/\(\delta\) terms cancel in $\delta$-kinematics, but the full and reduced hadronic amplitudes do not produce the same complete ones. Moreover, if the contribution of the longitudinal polarization of the virtual photon is neglected, i.e., if its propagator is reduced too, the singular parts do not cancel out. So, the contribution of the longitudinal part, contrary to expectations, is not suppressed by a factor $1/Q$ compared to the contributions of the transversely polarized photons. As such, the contribution of the longitudinal polarization should not be neglected in the kinematics given by Eqs. (4) and (6), where the photons carry transverse momenta of order $Q$.

We see here that summing the complete amplitudes over $h$ gives the same result for the full and the reduced amplitudes, but for the interchange of the polarization of the final photon. As this polarization is an observable, we observe that the reduced amplitude gives the wrong amplitude. Clearly the tree-level hard amplitude plays the role of a spin filter. Using the reduced amplitudes means
TABLE IV: Complete amplitudes in $s$-kinematics

| $\{x',\lambda\}$ | $\{h',h\}$ | $\{s',s\}$ | $L_{\frac{s}{2s}}H_{\text{Full}}$ | $L_{\frac{s}{2s}}H_{\text{Red}}$
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\{\frac{1}{2}, \frac{i}{2}\}$ | $\{+1,+1\}$ | $\{\frac{1}{2}, \frac{i}{2}\}$ | $\frac{1}{s}
\sum_h \left( \frac{4(c^2 + \frac{2c}{x} + \frac{3}{2} - \frac{8}{3} \sqrt{3}}{x}\right)$ | $\frac{2}{s}
\sum_h \left( \frac{4(c^2 + 1 - \frac{8}{3} \sqrt{3}}{x}\right)$
| $\{\frac{1}{2}, \frac{i}{2}\}$ | $\{+1,0\}$ | $\{\frac{1}{2}, \frac{i}{2}\}$ | $\frac{1}{s}
\sum_h \left( \frac{4(c^2 - 2c + \frac{3}{2} - \frac{8}{3} \sqrt{3}}{x}\right)$ | $\frac{2}{s}
\sum_h \left( \frac{4(c^2 - 1 + \frac{8}{3} \sqrt{3}}{x}\right)$
| $\{\frac{1}{2}, \frac{i}{2}\}$ | $\{-1,-1\}$ | $\{\frac{1}{2}, \frac{i}{2}\}$ | $\frac{1}{s}
\sum_h \left( \frac{4(c^2 + 1 - \frac{8}{3} \sqrt{3}}{x}\right)$ | $\frac{2}{s}
\sum_h \left( \frac{4(c^2 - 1 + \frac{8}{3} \sqrt{3}}{x}\right)$

We realize that the bulk of the GPD discussion refers to a kinematics where the transverse momentum of the virtual photon is not of order $Q$ but small or zero (e.g., to the center-of-mass of virtual photon and target hadron, or to the kinematics given by Eq. (2)). Our concern discussed in this work doesn’t apply to this case and the contribution of longitudinal photon polarization is indeed suppressed by $1/Q$ and can be neglected in DVCS. We stress, however, that for a correct analysis of the experiment data one must limit the reference frame to one where the transverse momenta of the photons are small compared to $Q$. Such frames exclude $q^+ = 0$, which is preferred in the case of form-factor calculations.

Based on these straightforward tree-level calculations of DVCS amplitudes, we conclude:

(i) The formulation of GPDs in the level of tensorial amplitude $T_\mu^\nu + T_\mu^\nu$ cannot be general enough to cover the kinematics with large transverse momenta such as given by Eqs. (1) and (3) but is limited to the special system of coordinates without involving large transverse momenta as given by Eq. (4).

(ii) In kinematics where the transverse components of the momenta are of order $Q$ the full hadronic amplitudes and the reduced ones do not agree, even in the limit $Q \to \infty$, which means that the calculations of the DVCS amplitudes using the GPD cannot be trusted in this kinematics; In addition, the contribution of the longitudinally polarized virtual photon is not down by one order in $Q$ but even plays the role of canceling the singular parts;

(iii) The singularities we have found are in no way connected to the strong-interaction part, but entirely due to the minus components of the photon-polarization vectors, meaning that a calculation beyond tree level will encounter the same singularities.

We have found the same singularities to occur in real Compton scattering using the same kinematics. They turn out to be of equal magnitude but opposite sign in the $s$- and $u$-channel amplitudes and thus cancel out, as expected.

This work is supported in part by the U.S. Department of Energy (No.DE-FG02-03ER41260).

[1] X. Ji, Phys. Rev. Lett. 78, 610 (1997)
[2] A.V. Radyushkin, Phys. Rev. D 56, 5524 (1997)
[3] D. Mueller, D. Robaschik, B. Geyer, F. M. Dittes and J. Horejsi, Fortsch. Phys. 42, 101 (1994)
[4] M. Diehl, Phys. Reports 388, 41 (2003)
[5] A.V. Belitsky and A.V. Radyushkin, Phys. Reports 418, 1 (2005)
[6] S. Boffi and B. Pasquini, Riv. Nuovo Cim. 30, 387 (2007)
[7] J.C. Collins, L. Frankfurt, and M. Strikman, Phys. Rev. D 56, 2982 (1997)
[8] S.J. Brodsky, H.-C. Pauli, and S.S. Pinsky , Phys. Reports 301, 299 (1998)
[9] J.B. Kogut and D.E. Soper, Phys. Rev. D 1, 2901 (1970)
[10] S.J. Brodsky, M. Diehl, and D.-S. Huang, Nucl. Phys. B 596, 99 (2001)
[11] C. Carlson and C.-R. Ji, Phys. Rev. D 67, 116002 (2003).
[12] M. Jacob and G.C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959) [Ann. Phys. (N.Y.) 281, 774 (2000)]; G.C. Wick, Ann. Phys. (N.Y.)
[13] A. V. Belitsky, D. Mueller and A. Kirchner, Nucl. Phys. B 629, 323 (2002); A. V. Belitsky and D. Mueller, Phys. Rev. D 79, 014017 (2009)
[14] J. C. Collins and A. Freund, Phys. Rev. D 59, 074009 (1999)
[15] M. Diehl, T. Gousset, B. Pire and J. P.Ralston, Phys. Lett. B 411, 193 (1997)
[16] B.L.G. Bakker and C.-R. Ji, Proceedings of “Light Cone 2009. Relativistic Hadronic and Particle Physics”, São José dos Campos, Brazil, July 8 - 13. 2009, to be published in Nucl. Phys. B (Proceedings).