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Phase Stochastic Resonance in a forced nano-electromechanical oscillator

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Abstract

Stochastic resonance is a general phenomenon usually observed in one-dimensional, amplitude modulated, bistable systems. We show experimentally the emergence of phase stochastic resonance in the bidimensional response of a forced nano-electromechanical membrane by evidencing the enhancement of a weak phase modulated signal thanks to the addition of phase noise. Based on a general forced Duffing oscillator model, we demonstrate experimentally and theoretically that phase noise acts multiplicatively inducing important physical consequences. These results may open interesting prospects for phase noise metrology or coherent signal transmission applications in nanomechanical oscillators. Moreover, our approach, due to its general character, may apply to various systems.

Stochastic resonance whereby a small signal gets amplified resonantly by application of external noise has been introduced originally in paleoclimatology to explain the recurrence of ice ages and has then been observed in many other areas including neurobiology, electronics, mesoscopic physics, photonics, atomic physics and more recently mechanics. Implementation of stochastic resonances involves generally three ingredients: (i) the existence of metastable states separated by an activation energy, as in excitable or bistable nonlinear systems, (ii) a coherent excitation, whose amplitude is however too weak to induce deterministic hopping between the states, and (iii) stochastic processes inducing random jumps over the potential barrier. In the classical picture of a bistable system, this corresponds to the motion of a fictive particle in a double-well potential periodically modulated in amplitude by the signal and subjected to noise. When an optimal level of noise is reached, the system’s response power spectrum displays a peak in the signal to noise ratio, unveiling the stochastic resonance phenomenon. The resonance occurs as a 'bona-fide' resonance in a frequency band around a signal frequency approximately given by the time-matching condition, i.e. when the potential modulation period is twice the mean residence time of the noise-driven particle. Experimental works on stochastic resonance are almost exclusively using amplitude modulation going along with additive amplitude noise or multiplicative amplitude noise. In this case, it corresponds to a pure one dimensional effect. Few studies take advantage of a bidimensional phase space by e.g. using phase modulation and/or phase noise (i.e. phase random fluctuations of input signal). Most of them use amplitude noise to demonstrate amplitude stochastic resonance, or introduce noise in the form of the response of a stochastic oscillator. However in the latter scheme, neither the noise nor the modulation are controlled, thus preventing to unveil the specific roles of phase modulation and phase noise in stochastic resonance.

In this Letter, stochastic resonance is implemented in a nonlinear nanomechanical oscillator forced close to its resonant frequency. It enables, in a bidimensional phase space, the implementation of phase stochastic resonance observed simultaneously both on the phase and amplitude response of the oscillator. It is here demonstrated by achieving the stochastic enhancement of a phase modulated signal by phase noise observed on the bidimensional response of the oscillator. This opens new avenues for stochastic resonance in bidimensional systems by allowing for instance stochastic amplification of mixed phase-amplitude modulated signals by complex value noise. We highlight that the system’s response can be projected on any variable in phase space and that the amplification depends on the chosen basis. Finally, we derive a stochastic nonlinear amplitude equation for the forced stochastic Duffing oscillator, which describes qualitatively well our system, and show that phase noise acts multiplicatively inducing important physical consequences.

The forced nanomechanical oscillator consists of a suspended InP photonic crystal membrane which acts as a mirror in one arm of an interferometer fed with an He-Ne laser. The membrane is activated by underneath integrated interdigitated electrodes driven by an AC-bias voltage \( V(t) \) (see Fig.). This voltage induces an electrostatic force on the oscillator which drives its out-of-plane motion as described in [11]. The oscillator is placed in a vacuum chamber with a pressure of about \( 10^{-4} \) mbar at room temperature. The phase \( \Phi \) and the amplitude modulus \( R \) of the oscillator’s motion are retrieved by use of a balance homodyne detection. From the recorded time traces of \( \Phi \) and \( R \), we can reconstruct the polar plots with the two quadratures \( X = R \cos(\Phi) \) and \( Y = R \sin(\Phi) \).
V. Tra of the driven oscillator response in a frequency sweep - the device shows the membrane (thickness of 260 nm and a distance of about 400 nm). (Top right) Finite Element Model (F.E.M.) simulation of the fundamental mechanical mode under study with enhanced out-of-plane displacement for clarity.

The applied voltage, and therefore the applied electrostatic force, is in the form of [30]:

\[ V(t) = V_{AC} \cos(\Omega_d t + \Delta \phi \text{ sgn}(\cos(\Omega_m t)) + \xi(t)) \]  

Here \( V_{AC} \) is the amplitude of the applied voltage, while \( \Omega_d \) denotes the resonant driving frequency. A phase modulation is added; it displays a square waveform described by the sign function \( \text{sgn} \), at frequency \( \Omega_m \) and a phase deviation of \( \Delta \phi \). Gaussian phase noise \( \xi(t) \) of zero-mean and standard deviation \( \xi_{RMS} \) (bandwidth \( B_\phi = 10 \text{ kHz} \)) such that \( \Omega_m \ll B_\phi \) is also applied on the nonlinear dynamic system. Under quasi-resonant forcing of the mechanical fundamental mode, a hysteresis behavior becomes prominent for \( V_{AC} > 5 \text{ V} \) and two stable fixed points co-exist in the bidimensional phase space of the oscillator (Fig. -bottom left and right). In the following, \( V_{AC} \) is set to 9 V in order to be deeply in the bistable regime; the driving frequency is set inside the hysteresis region at \( \Omega/2\pi = 2.824 \text{ MHz} \) in order to get equal probability of residence in each state (See Supplementary Information) and the system is systematically initially prepared in its upper state.

In the bistability regime, jumps between the two stable states can be induced by applying a slow modulation (\( \Omega_m \ll \Omega_d \)) with a sufficiently high phase deviation, phase noise strength or both. These jumps are investigated by tracing the amplitude and phase evolution of the fundamental mode with time and are also pictured in the X-Y phase plane. In the case of pure phase modulation, the system can transit or not from one state to the other depending on the values of \( \Omega_m \) and \( \Delta \phi \). Beyond the cut-off frequency \( \Omega_m/2\pi = 1 \text{ kHz} \), which is directly linked to the oscillator’s line width of 0.9 kHz [19], the output signal is not synchronized with the input signal, in amplitude or phase. For \( \Omega_m/2\pi = 500 \text{ Hz} \), every jumps in the input signal translate into a jump in the output signal for \( \Delta \phi > 1.83 \text{ rad} \) (see Supplementary Information). Similarly, in the case of pure noise-induced switching, the system starts to transit between the two 2D states, in amplitude and phase as noise strength increases. The occupancy between these two states becomes equiprobable for values of \( \xi_{RMS} \) close to 0.52 rad in our device. Such noise-induced transitions can also be quantified by the Kramers rate \( T_K = 1/\tau_K \) which is the inverse of the average time required to cross over the barrier [23] and reaches a value close to 100 Hz (see Supplementary Information). Contrary to amplitude noise which amounts to additive noise, phase noise acts here as a multiplicative noise. This feature is revealed through the non-constant dependence of the phase difference \( \Delta \theta \) between the two equilibria for increasing noise strengths (see Fig. 1) and is highlighted by the fourth term in the right hand side of Eq. 4. At weak phase noise (\( \xi_{RMS} < 0.4 \text{ rad} \)), uncertainties on the phase difference are large because the probability of residence in the lower state is weak (< 5%) and thus this state gets difficult to observe. Conversely, at strong phase noise (\( \xi_{RMS} > 0.6 \text{ rad} \)), the probability of residence of the upper state reduces, and this state is hardly observable.

The stochastic synchronization between the external noise and the weak coherent signal that occurs in stochastic resonance, takes place when the average waiting time between two noise-induced interwell transitions (\( T_K \)) is comparable to half the period of the periodic signal (\( T_R = 2\pi/\Omega_m \)). In order to match this time-scale condition, modulation frequency \( \Omega_m \) in phase is set at 50 Hz. The deviation \( \Delta \phi \) is also set to 0.09 rad (< 1.56 rad, the hysteresis width (Fig. )), a far too weak value to let the system switch periodically from one state to the other (see Fig. 3 upper line). When increasing the noise...
spectral power amplification is then given by the ratio of the crete Fourier Transform (DFT) of the time traces. The hysteresis cycle and the system drops to its lower state. Further increasing the noise distorts the transitions get stochastically synchronized with the modulation signal. For a given noise strength, occasional transitions occur, weakly locked to the modulation signal. For \( \xi_{RMS} = 0.52 \text{rad} \) and \( \Omega/2\pi = 2.824 \text{MHz} \).

To gain more insight into the observed dynamics, we compare our results to theoretical and numerical predictions of a stochastic amplitude equation. Fits of the experimental results are obtained by modeling the nanoelectromechanical oscillator by a simple forced stochastic Duffing oscillator whose dynamics can be described, in the limit of small injection and dissipation of energy, by:

\[
\dot{x} = -x - \epsilon \mu \dot{x} - \alpha x^3 + \epsilon^{3/2} F \cos((1 + \epsilon \sigma)t + \epsilon \phi_m(t) + \epsilon \sqrt{\eta_0} \Delta \phi) \tag{2}
\]

where \( x(t) \) accounts for the displacement of the membrane and \( \epsilon \) is a small control parameter (\( \epsilon \ll 1 \)). This parameter is introduced to properly balance the scaling between the dissipation and injection of energy in the system, and also control the frequency detuning. The natural frequency has been rescaled to one (\( \omega_0 = 1 \)), \( \mu \ll 1 \) is the damping coefficient that accounts for dissipation of energy, \( \alpha \) accounts for the nonlinear stiffness of the spring, which is positive (negative) for soft (hard) spring and \( F \) the strength of the driving. The near-resonant drive has an angular frequency of \( \omega_d = 1 + \sigma \), where \( \sigma \ll 1 \) stands for the detuning between the drive and the natural resonant frequency. The system is also subject to a slow phase modulation \( \phi_m(t) \) (\( \dot{\phi}_m \ll \omega_0 \phi_m \)) and a phase noise term in the form of a Wiener process \( \Delta \phi \) with Gaussian noise strength \( \eta_0 \). In the conservative limit and for small displacements, the system exhibits harmonic motion with a small arbitrary amplitude \( D \) such that \( x(t) = \Re[D e^{i \phi}] \). When considering the nonlinear terms, dissipation and forcing, the displacement of the membrane response can be approximated by

\[
x(t) = e^{3/2} D(T = \epsilon t) e^{i [t + \epsilon (\sigma t + \phi_m(t)) + \sqrt{\eta_0} \Delta \phi]} + \frac{\alpha e^{9/2}}{8} D^3 e^{3 i [t + \epsilon (\sigma t + \phi_m(t)) + \sqrt{\eta_0} \Delta \phi]} + \text{c.c.} + o(\partial^3)
\]
phase modulation is sinusoidal with an amplitude $\Delta \phi$, that is,

$$\xi$$

$$d\phi = \xi$$

where $T$ accounts for the slow temporal scale ($\dot{D} \approx \epsilon D$ and $\frac{D}{\epsilon^2}$), and the symbol $\epsilon$ stands for complex conjugate. Introducing the above ansatz in Eq. (2) to order $\epsilon^{3/2}$ and using the rules of calculus in stochastic normal form theory [12] one finds the stochastic nonlinear amplitude equation:

$$\frac{dD}{dT} = -\left(\frac{\mu}{2} + i(\sigma + \frac{d\phi_m}{dT} + \sqrt{\eta_0} \xi)\right)D + \frac{3\epsilon}{8} |D|^2 D - \frac{iF}{2}$$

where $\xi = dW_{\phi}/dT$ is a zero-mean and delta-correlated white Gaussian noise term. Note that $\phi_m$ is a slow phase modulation, that is, $d\phi_m/dt = \epsilon d\phi_m/dT$. To derive the above model, we have considered ansatz (3) as a change of variable. Here, the Stratonovich prescription for noise has been adopted. Namely, the stochastic term can induce a non-zero drift, $\langle \xi(T) D(T) \rangle \neq 0$. Note that even though Eq. (2) would give rise to additive noise with time-dependent coefficients in a Fokker-Planck equation, the reduced equation (Eq. (4)) for the response amplitude of the oscillations satisfies a stochastic differential equation with multiplicative noise as a result of the stochastic normal-form derivation [12].

Stochastic numerical simulations of Eq. (4) are performed with the help of the XMDS2 package [14]. We use the semi-implicit numerical scheme which converges to the Stratonovich integral. The time step is kept fixed in the simulation and is chosen to be $dT = 0.1$. The slow phase modulation is sinusoidal with an amplitude $\Delta \phi = 5 \times 10^{-5}$ and an angular frequency $\Omega_m = 2\pi/2 \times 10^5$. The detuning is $\sigma = 1.77 \times 10^{-3}$. The model reproduces well the bistable response in amplitude and phase of our nano-electromechanical oscillator (see Fig. bottom), as well as the temporal evolution of the response in amplitude or phase, in the case of pure phase modulation, pure phase noise (see Supplementary Information) and stochastic resonance (see Fig. 3 and 4). Moreover, multiplicative noise shall translate into a shift of the operating point in the hysteresis and thus into an effective detuning in Eq. (4) which reduces to $\sigma_{eff} = \sigma + \eta_0/2$. Physically, this translates in a drift of the operating point for increased noise strengths, a signature of the multiplicative nature of the added noise, as observed in our experiment (see Fig. 2). The measured $\Delta \theta$ is slightly smaller in the experiment compared to theory presumably because of extra low-frequency noise sources which are not taken into account in the model.

Stochastic resonance amplification of the modulated signal is here limited by the relative orientation of the modulation and of the minimal energy path between the two basins of attraction, which is almost in a direct straight line (see Fig. 4). In the same frame, the added phase modulation shakes the upper state preferentially in the azimuthal direction. These two orientations being not parallel, higher amplification value can not be achieved in this configuration. This reveals the importance of the modulation format of the signal: optimal stochastic resonance would certainly require a mixed amplitude-phase format to follow the minimal energy path in the nanomechanical oscillator phase space. The distribution of the two states in the phase plane gets also distorted: The system switches between a symmetric branch (with a quasi-circular state in the phase portrait) to an asymmetric branch (with an elongated state in the polar plot). Such distortion is reminiscent to thermal noise squeezing observed e.g. in parametrically-driven oscillators [8, 30, 38, 40].

In conclusion, we have demonstrated phase stochastic resonance with phase noise in a bidimensional nonlinear
oscillator consisting of a nano-electromechanical device. The applied phase noise reveals to act as a multiplicative noise on the system which introduces an effective detuning that plays a crucial role in the residence probability asymmetry. The derived stochastic amplitude equation is a universal model that describes the evolution of the envelope of the oscillations near a nonlinear resonance and subjected simultaneously to phase noise and to a phase modulation. That is, it applies to any nonlinear oscillator with such forcing provided one makes use of a suitable nonlinear and periodic change of variables in the initial equations that describe the system. Our model applies to e.g. dispersive optical bistability that plays an important role in nonlinear optical science and can thus shed a new light on coherent processes involving phase fluctuations in these systems. Such stochastic resonance obtained by the assistance of phase noise may also enable various noise-aided applications, including signal transmission in particular involving phase fluctuations in these systems. Such anisotropy that plays a crucial role in the residence probability

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