Self-amplified spontaneous gamma-ray emission

A.A. Zadernovsky
Moscow State Technical University MIREA, Department of Physics,
78 Vernadsky Ave., 119454 Moscow, RUSSIA
e-mail: zadernovsky@mirea.ru

Abstract. We discuss a possibility for obtaining the self-amplified spontaneous emission of gamma radiation in an active medium with hidden population inversion of nuclear states. Hidden inversion appears in cooled nuclear ensembles due to assistance of the nuclear recoil in emission and absorption of gamma quanta. Establishing a hidden inversion allows an amplification of gamma radiation to occur in a particular frequency range without excess of the number of excited nuclei over the number of unexcited nuclei. We formulate a set of requirements to the pumping radiation and to the amplifying nuclear medium crucial to the development of gamma-ray lasing.

1. Introduction
In optical lasers, photon flux is amplified due to stimulated emission of radiation (this notion was first introduced by A. Einstein in 1916). Amplification is realized in an active medium with population inversion of quantum levels, if an increase in the number of photons due to stimulated emission exceeds the loss of the photons. The question arises as to whether this approach can be applied to create a gamma-ray laser on radiative nuclear transitions owing to the universal character of the Einstein emission laws.

For the photon flux propagating in an active medium, the gain coefficient is represented as

$$g = \sigma_s (n_n - n_i) S - \chi n,$$

where \(\sigma_s\) and \(\chi\) are the cross sections for the stimulated emission of photons and for the nonresonant photon losses, \(n_n\) and \(n_i\) are the concentrations of emitters in the excited state (at the upper quantum level) and in the unexcited state (at the lower quantum level), \(n\) is the total concentration of emitters, \(S=(2J_z+1)/(2J_i+1)\) is the degeneracy factor of quantum transition \((J_z\) and \(J_i\) are the angular momenta of the upper and lower quantum levels, respectively).

At the center of emission line with wavelength \(\lambda_0\), the cross section for stimulated gamma radiation is

$$\sigma_s = \frac{\lambda_0^2}{2\pi} \frac{\Delta \omega_{rad}}{\Delta \omega_{tot}} = \frac{\lambda_0^2}{2\pi} \frac{1}{\Delta \omega_{tot} \tau (1+\alpha)},$$

where \(\Delta \omega_{tot}\) is the total width of the emission line, which takes into account all types of excessive homogeneous and inhomogeneous broadening, \(\Delta \omega_{rad}\) - radiative width of the quantum transition (which is reciprocal value of the radiative lifetime \(\tau_{rad}\) in excited state), \(\tau\) is the lifetime in excited
state, \( \alpha \) is the transition quantum yield (for example, the internal electron conversion coefficient of the radiative nuclear transition).

If we pass from optics to gamma-rays, the wavelength \( \lambda_0 \) decreases by 4±5 orders of magnitudes, the ratio of the radiative width to the total width of emission line, \( \beta = \Delta \omega_{\text{rad}}/\Delta \omega_{\text{tot}} \), also decreases by 4±5 orders of magnitudes (since the total width of emission line \( \Delta \omega_{\text{tot}} \), being proportional to the nuclear transition energy, increases by 4±5 orders of magnitudes), so that the cross section for stimulated emission \( \sigma_{\text{st}} \) decreases at least by 12÷15 orders of magnitudes. However, an amplification condition, being proportional to the cross section for non-resonant photon losses \( \chi \), is obviously satisfied and the pump power could be decreased by 4÷5 orders of magnitudes (since the total width of emission line \( \Delta \omega_{\text{tot}} \) is close to its minimum value equal to unity. Thus, the product \( \Delta \omega_{\text{tot}} \tau \) is close to its minimum value equal to unity. Thus, the amplification condition \( \Delta \omega_{\text{tot}} \tau < 10^3 \) is obviously satisfied and the pump power could be decreased by using the Mössbauer nuclei with relatively long lifetimes in the excited states.

For this purpose, L. Rivlin [8, 9] first proposed to separate (employing, for example, radiochemical methods) and collect long-lived nuclear isomers (the lifetimes of isomeric excited states range for various isomers from several fractions of a second to hundreds of years) and then introduce these isomers into a crystal lattice for the creation of an active medium with population inversion of nuclear states or to grow active crystals containing these excited isomeric nuclei.

For this purpose, L. Rivlin [8, 9] first proposed to separate (employing, for example, radiochemical methods) and collect long-lived nuclear isomers (the lifetimes of isomeric excited states range for various isomers from several fractions of a second to hundreds of years) and then introduce these isomers into a crystal lattice for the creation of an active medium with population inversion of nuclear states or to grow active crystals containing these excited isomeric nuclei.

However, Il'inskii and Khokhlov [17] have found that the Mössbauer condition \( \Delta \omega_{\text{tot}} \tau \approx 1 \) can be realized only for the nuclear states with the lifetime \( \tau \) less than 10 \( \mu \)s and, thus, the total width of the Mössbauer line of the gamma transition can approach the natural width only for such relatively short-lived nuclei. This is due to the fact that, at longer lifetimes the natural emission line becomes so...
narrow that even insignificant distortions of the crystal lattice and/or the chemical and quadrupole shifts of the nuclear levels lead to an inevitable inhomogeneous broadening of the gamma emission line. In addition, the reason for the surplus inhomogeneous broadening that is inherent to even an ideal crystal is the magnetic dipole interaction with neighboring nuclei, since spins of the excited and unexcited nuclei are different and coordinates of the emitting nuclei are random. Owing to all these effects, the width of gamma emission line $\Delta \omega_{tot}$ at $\tau > 10 \mu s$ becomes independent of the lifetime $\tau$ and can be estimated as $10^5$ s$^{-1}$. The amplification condition $\Delta \omega_{tot} \tau < 10^3$ now leads to the upper-limit estimate $\tau < 10$ ms for the lifetime of excited nuclei.

Unfortunately, the above estimate does not allow rely neither on the growth of an active crystal consisting of such short-lived excited nuclei nor on direct pumping of nuclei in a crystal matrix for the realization of population inversion, since the flux densities needed for the pumping of such short-lived nuclear states are still too high and lead to the violation of the conditions at which the Mössbauer effect is observed or even to the destruction of the crystal. In spite of numerous and extremely refined proposals to improve the pumping process (see, for example, [18, 19]) as well as to narrow the emission line of long-lived isomers down to the natural value (see, for example, [17, 20]), the development of the standard Mössbauer scheme is almost stopped at present because of its impasse character. All these circumstances were discussed in details in the comprehensive reviews [21, 22] and they are often referred to as an unsolvable “gamma-ray laser dilemma”.

The primary reason for the harsh demand on pumping in Mössbauer scheme is the resonant nuclear absorption, which leads to the population inversion requirement that at least a half of total number of nuclei of an active medium should be excited. Another approach that seems very promising as an alternative to the Mössbauer scheme does not require the real population inversion but only a hidden population inversion of nuclear states [23]. This approach utilizes entirely new possibilities revealing themselves only in radiative processes with hard photons in cooled ensembles of free nuclei. One of them is the possibility to use the nuclear recoil in the emission or absorption of gamma-photons to establish a hidden inversion of state populations. The hidden inversion may ensure a gain of stimulated gamma radiation in a particular frequency range without an excess of the number of excited nuclei over the number of unexcited ones. Since a total population inversion is not required for the gamma-ray lasing, this may result in a considerable reduction in the threshold flux of x-ray pumping radiation. Simultaneously, a sufficiently high the linewidth ratio $\beta$ and the gain coefficient for amplified gamma-photon flux may be attained through deep cooling of atomic ensembles. The cooling may be accomplished by present-day or upcoming techniques [24] based on the light pressure of optical lasers. Together with some other attractive features [25] (the opportunity to use the so-called ‘two-level’ scheme for x-ray pumping of a cooled nuclear ensemble, the possibility of anisotropic amplification of a unidirectional gamma-photon flux without any reflecting mirrors, etc.) the concept of recoil-assisted gamma-ray amplification in deeply-cooled ensembles of free nuclei appears to be rather promising.

The question remains, however, whether destructive impact of x-ray pumping radiation on the active medium of cooled free nuclei brings to violation the conditions for hidden population inversion and breakdown the development of gamma-ray lasing process. We discuss in this paper some destructive features of x-ray pumping and formulate a set of requirements to the pumping radiation and to the amplifying nuclear medium crucial to the development of recoil-assisted gamma-ray lasing. We discuss, as well, the efficiency of pumping with modern sources of x-ray radiation.

2. Self-amplified spontaneous gamma-ray emission
Since in gamma region of wavelengths no mirrors exist to reflect the radiation effectively, the development of a gamma-ray laser source is likely to be based on self-amplified spontaneous emission principle. Spontaneous gamma radiation, propagating along the axis of cylindrically shaped nuclear active medium with hidden population inversion of nuclear states (which are assumed below to be the first excited and the stable ground ones), undergoes an amplification with the gain coefficient (1) with $n_1=0$. 

In fact, it is well known that the centers of emission and absorption lines in ensemble of free nuclei are shifted relative to each other due to recoil momentum acquired by the nuclei in the radiative processes. At low temperatures, $k_B T < E_{\text{reco}}$ (where $E_{\text{reco}} = (E_0)^2 / 2MC^2$ gives the recoil energy of a nucleus with mass $M$ and $k_B$ is the Boltzmann constant), sufficient for a radical suppression in an overlap between the emission and absorption lines, a hidden population inversion is established for a gamma-ray propagating oppositely to the direction of pumping x-ray [23, 25]. Since in this case the emitted photons cannot be resonantly absorbed by unexcited nuclei (but they are still capable of stimulating emission of the excited nuclei), the population difference merely equal to the concentration of excited nuclei and, accordingly, the gain coefficient (1) does not involve the concentration $n_1$ of unexcited nuclei.

![Figure 1. Amplification of spontaneous gamma radiation, propagating along the axis of cylindrically shaped nuclear ensemble with hidden population inversion](image)

If we assume that the pump pulse duration exceeds the time for single pass of gamma-radiation, $L/c$, along the axis of a cylindrical nuclear medium with length $L$ (see figure 1) or, alternatively, the pump is arranged in such a way that the traveling wave of nuclear excitation occurs in the direction of expected gamma-ray, then the output flux of self-amplified spontaneous emission into a solid angle $\Delta \Omega = \pi (R/L)^2$ (where $R$ is the radius of the cylinder) could be collected from all over the length $L$. The output density of amplified photon flux integrated over all frequencies of emitted gamma quanta is

$$\Phi = \frac{\omega}{\pi} \int_0^n S_{\omega}(\omega) \left[ \int_0^L e^{-(L-z)} dz \right] d\omega,$$  \hspace{1cm} (3)

where $S_{\omega}(\omega) = (n_2/\tau_{\omega}) f(\omega - \omega_0)$ is the spectral density of the spontaneous gamma emission rate into unit volume. With approximation for the shape-factor of emission line $f(\omega - \omega_0)$ by a constant value of $(\Delta \omega_{\text{tot}})^{-1}$ within the spectral bandwidth $\Delta \omega_{\text{tot}}$ around the central frequency $\omega_0$, the amplified
photon flux density $\Phi$ (3) can be estimated, as compared to out-of-axis flux density of spontaneous gamma emission, $\Phi_0$, by a factor of

$$\frac{\Phi}{\Phi_0} = \frac{G - 1}{\ln G},$$  \hspace{1cm} (4)

where $G = \exp(gL)$ is the single-pass gain of stimulated gamma photons over the length $L$.

Truly exponential amplification of spontaneous gamma radiation occurs if $gL \geq 1$, the condition which is assumed to be a starting point to determine in the next sections a set of requirements crucial to the recoil assisted gamma-ray lasing process.

3. Active nuclear medium

The request for $gL \geq 1$ brings to the requirement to the relative concentration of excited nuclei

$$\frac{n_2}{n} \geq \frac{2\pi (\chi / \lambda_0^2 + 1/\lambda_0^2 nL)}{\Delta \omega_{nd}/\Delta \omega_{tot}}.$$  \hspace{1cm} (5)

Taking into account that the concentration of excited nuclei $n_2$ is obviously less than the total concentration of nuclei $n$, we obtain from (5) the following requirement to the linewidth ratio

$$\frac{\Delta \omega_{nd}}{\Delta \omega_{tot}} \geq 2\pi \left( \frac{\chi}{\lambda_0^2} + \frac{1}{\lambda_0^2 nL} \right),$$  \hspace{1cm} (6)

the condition which is, actually, to be considered as a demand on cooling degree of nuclear ensemble, since

$$\frac{\Delta \omega_{nd}}{\Delta \omega_{tot}} = \frac{1}{1 + \alpha} \sqrt{\frac{T_{ct}}{T}} \hspace{1cm} \text{at} \hspace{0.2cm} T \geq T_{ct},$$  \hspace{1cm} (7)

where $T_{ct}$ is the critical temperature of nuclear ensemble at which the total width of emission line (which is basically dominated by the Doppler width) is reduced to the decay width of an upper nuclear level $\Delta \omega_2 = \Delta \omega_{tot} (1 + \alpha)$.

Even in deeply cooled nuclear ensembles the emission linewidth $\Delta \omega_{tot}$ exceeds its minimum value equal to the decay width of the upper nuclear state $\Delta \omega_2$, which generally contains the contributions both from the radiative nuclear decay and the decay due to internal electron conversion. From the requirement (6) the relation $\Delta \omega_{tot} \gg \Delta \omega_2$ gives for the dimensionless parameter of active medium, $\varepsilon = \lambda_0^2 nL$, involved into relations (5) and (6), the following restriction

$$\varepsilon \geq \frac{2\pi (1 + \alpha)}{1 - 2\pi (1 + \alpha) \chi/\lambda_0^2}.$$  \hspace{1cm} (8)

For a numerical example we assume that the nuclear transition has an energy of $E_0 = 10$ keV. The total losses of the gamma radiation with the corresponding wavelength of $\lambda_0 = 1.24$ Å, are mainly due to the photoelectric effect with atomic cross section within the range of $\chi = (10^{21} + 10^{20})$ cm$^2$ (for instance, $\chi = 1.14 \times 10^{21}$ cm$^2$ for Al, $1.57 \times 10^{20}$ cm$^2$ for Fe, $4.32 \times 10^{20}$ cm$^2$ for Pb, $7.90 \times 10^{21}$ cm$^2$ for Rb, $5.55 \times 10^{21}$ cm$^2$ for Se, $2.82 \times 10^{20}$ cm$^2$ for W and etc. [1]). For nuclear transitions with internal electron conversion coefficient $\alpha = 10$ we obtain the estimate $2\pi (1 + \alpha) \chi/\lambda_0^2 = 4.5 \times (10^4 + 10^5) \ll 1$, which leads from (8) to the requirement $\varepsilon \geq \varepsilon_{min} = 2\pi (1 + \alpha) \approx 70$. Accordingly, if the dimensionless parameter $\varepsilon = 200$, then gamma-ray lasing medium might have the total concentrations of nuclei of $n = 1.3 \times 10^{16}$ cm$^{-3}$ for the lengths of amplification $L = 1$ m or $n = 2.6 \times 10^{15}$ cm$^{-3}$ for $L = 5$ m.
Figure 2 shows the dependence of the relative concentration of excited nuclei \( n_2/n \) against the relative reciprocal temperature \( T_{cr}/T \) of an active medium with taken \( \varepsilon = \lambda_0^2 nL \) as a dimensionless parameter of a family of curves. Gamma-ray lasing occurs for a set of nuclear medium parameters falling into a shaded domain in figure 2 with the boundaries putted by the restrictions (5), (6) and (8). In the case of \( \varepsilon = \varepsilon_{min} = 2\pi(1+\alpha) \) this domain consists of only a single point, then spreading gradually together with growth of \( \varepsilon \) above the minimum value.

![Figure 2](image_url)

**Figure 2.** Dependence of the relative concentration of excited nuclei \( n_2/n \) against the relative reciprocal temperature \( T_{cr}/T \) of active medium with \( \varepsilon = \lambda_0^2 nL \) taken as a dimensionless parameter of a family of curves

One can see from the figure 2 that the opportunity to use for gamma-ray lasing the lowest (for a given \( \varepsilon \)) concentration of excited nuclei,

\[
\frac{n_2}{n} = \frac{\varepsilon_{min}}{\varepsilon} = \frac{2\pi(1+\alpha)}{\varepsilon}, \tag{9}
\]

is conditioned by the need of deep cooling of the atomic ensemble down to the critical temperature \( T_{cr} \).

The higher the temperature of an atomic ensemble, the greater concentrations of excited nuclei are required and at the highest (for a given \( \varepsilon \)) temperature, \( T_{max} = T_{cr} (\varepsilon/\varepsilon_{min})^2 \), still suitable for development of the recoil assisted gamma-ray lasing, all the available nuclei of the atomic ensemble must be pumped into the excited state.

The critical temperature, being measured in \( \mu K \), can be estimated by \( T_{cr} = 0.41A(E_0T_{1/2})^{-2} \), where \( A \) is the nuclear mass number, the transition energy \( E_0 \) is measured in keV and the half-life in the nuclear excited state, \( T_{1/2} = \tau \ln 2 \), is in ns. For an ensemble of nuclei with \( A=100 \) and \( E_0=10 \) keV the estimate gives \( T_{cr} = 0.41 \mu K \) for \( T_{1/2} = 1 \) ns and \( T_{cr} = 41 \mu K \) for \( T_{1/2} = 0.1 \) ns.

4. X-ray pumping radiation

In attempting to propose any pump scheme, it should be noted that the pumping must interact with a previously-prepared cooled atomic ensemble. This is necessitated by the lifetime of the nuclear excited state, which lies in the nanosecond and even sub-nanosecond range, thereby precluding the reverse
sequence of pumping and cooling operations. Therefore, the pumping radiation must not damage the initial monokinetic quality of the cooled atomic ensemble. This imposes some restrictions on the non-destructive pumping x-ray radiation.

There are a number of perturbations introducing by the radiative pumping into an active medium. Among them are: (1) an additional broadening of the initial velocity distribution in the group of excited nuclei caused by the broadband radiative pumping; (2) an additional broadening of the initial velocity distribution in the group of excited nuclei caused by an angular divergence of the pumping radiation; (3) the inter-atomic collisions destroying the monokinetic quality of the group of excited nuclei; and (4) the processes that are not directly relative to pumping of nuclei: coherent and incoherent x-ray scattering, absorption of x-rays by atomic electron shell, photoelectric x-ray absorption.

First of all, it is important to note that the pumping with a beam of resonant monochromatic and highly collimated x-ray photons preserves initial monokinetic quality of atomic medium, but through the recoil effect only adds a new narrow line into the velocity distribution. This line corresponds to a group of excited nuclei moving with the nuclear recoil velocity

\[ \Delta v_{\text{rec}}^{(1)} = \frac{\hbar}{M} \Delta \omega_{\text{pump}}. \]  

and, accordingly, to an additional broadening which can be neglected, however, since the corresponding linewidth

\[ \Delta \omega_{\text{pump}}^{(1)} = \frac{\omega_0}{c} \Delta v_{\text{rec}}^{(1)} = \frac{E_0}{M c^2} \Delta \omega_{\text{D}} \]  

is apparently far less than the initial Doppler linewidth \( \Delta \omega_{\text{D}} \).

Angular divergence of the pumping x-ray radiation also introduces an additional perturbation of the monokineticity into the atomic medium. Providing that the x-ray is confined to a narrow cone with the plane angle \( \Delta \theta \) centered close to the axis of cylindrically shaped atomic medium (see figure 1), we obtain the following estimates for an additional dispersion of the longitudinal velocities of pumped nuclei

\[ \Delta v_{\text{rec}}^{(2)} = \frac{E_0}{2M c} \left( \frac{\Delta \theta}{2} \right)^2 \]  

and for the corresponding additional width

\[ \Delta \omega_{\text{pump}}^{(2)} = \frac{\omega_0}{c} \Delta v_{\text{rec}}^{(2)} = \frac{E_0 \hbar}{M c^2} \left( \frac{\Delta \theta}{2} \right)^2. \]  

The pumping is non-destructive if the additional width \( \Delta \omega_{\text{pump}}^{(2)} \) is much less than the initial Doppler linewidth \( \Delta \omega_{\text{D}} \). This leads to the following requirement to the angular divergence of pumping radiation

\[ \Delta \theta << 4 \left( \frac{k_B T \ln 2}{E_{\text{rec}}} \right)^{1/4}. \]  

Numerical estimate gives \( \Delta \theta << 0.1 \) rad for the active medium of nuclei with mass number \( A=100 \), the nuclear transition energy \( E_0 = 10 \) keV and the temperature \( T=10 \) \( \mu \)K.
As concern the inter-atomic collisions, in spite of an appreciable value for the nuclear recoil velocity, which is estimated to be 32 m/s for a nucleus with \( A=100 \) and \( E_0=10 \) keV, the path covered by a pumped nucleus during its lifetime in excited state, \( \tau = 1 \) ns, is less than the mean free path of atoms for the concentrations up to \( 10^{16} \) cm\(^{-3} \). Taking into account that the velocity of thermal atomic motion at \( T=10 \) \( \mu \)K is only 5 cm/s to be by three orders of magnitude less than the nuclear recoil velocity, one can conclude that the inter-atomic collisions cannot destroy the monokinetic quality of the considered group of excited nuclei. This group creates a hidden population inversion of nuclear states for the gamma-ray propagating opposite to the pumping x-ray.

Among other processes introducing perturbations into the nuclear active medium, the photoelectric absorption of pumping radiation is likely to be the most destructive one. The recoil momentum acquired by a free excited nucleus in the process of atomic ionization knocks it out of the participants of the gamma-ray lasing process. Therefore, the pumping radiation can be considered as non-destructive if the rate of creation of excited nuclei exceeds the rate of devastation of their concentration due to the photoelectric effect, that is

\[
\left( S \lambda_0^2 / 4 \right) \left( n / \tau_\omega \right) P(\omega_b) > n_2 \chi \Delta \omega_{\text{pump}} P(\omega_b),
\]

where \( P(\omega_b) \) is the spectral photon flux density of pumping radiation on the nuclear transition frequency, \( \Delta \omega_{\text{pump}} \) is the frequency bandwidth of pumping radiation (it is assumed \( \Delta \omega_{\text{pump}} \gg \Delta \omega_\omega \)), \( S = (2J_z+1)/(2J_1+1) \) is the spin factor of the nuclear transition, \( \tau_\omega \) is the radiative lifetime in excited state. The condition (15) is consistent with the need to have concentrations of excited nuclei above the critical value (9) only for the x-ray pumping radiation confined to the following relative spectral bandwidth

\[
\frac{\Delta \omega_{\text{pump}}}{\omega_b} < \frac{S \lambda_0^2 \chi}{[4 \pi (1+\alpha)]^2 \chi \tau}.
\]

In the case \( \varepsilon = 200 \), \( \lambda_0 = 1.24 \) Å (\( E_0=10 \) keV), \( \alpha=10 \), \( S=1 \), \( \tau = 0.1 \) ns and \( \chi = 10^{-21} \) cm\(^2\) we obtain for the upper-limit estimate of the degree of monochromaticity of pumping radiation the value of \( 10^{-5} \).

At last we estimate intensity of pumping radiation. The growth of the concentration of excited nuclei can be describe as

\[
n_2 \approx n S \left( P/P_{\text{sat}} \right) \left( 1 - e^{-i\tau} \right)
\]

where \( P_{\text{sat}} = 4(1+\alpha)/\lambda_0^2 \) is the saturation spectral photon flux density.

The critical concentration (9) can be exceeded only with the help of intense pumping radiation of a high enough spectral photon flux density

\[
P(\omega_b) \geq P_{\text{ct}} = S(\varepsilon_{\text{min}}/\varepsilon) P_{\text{sat}}
\]

The spectral photon flux density of pumping radiation \( P(\omega_b) \) in (18) is expressed in cm\(^2\), whereas the same value expressed in more relevant units of photons/(cm\(^2\) s keV) is given by \( P(\omega_b)/h \) with Planck constant \( h = 6.58 \times 10^{-19} \) keV s. Numerical estimate gives \( P_{\text{ct}}/h = 10^{15} \) photons/(cm\(^2\) s keV). The critical spectral photon flux density of pumping radiation approaches the peak fluxes that could be supplied with the upcoming x-ray free electron lasers (HASYLAB/DASY, Hamburg, Germany).

5. Conclusions
We have formulated a set of requirements to the pumping radiation and to the amplifying nuclear medium crucial to the development of recoil assisted gamma-ray lasing. Performed analysis has revealed several most significant parameters to be fixed for the active nuclear medium and for the
x-ray pumping radiation in order to obtain a self-amplified spontaneous gamma-ray emission. Unfortunately, the required values are far beyond that of available with current technology. First of all it concerns the pumping radiation. We have found that in order to preserve the initial monokinetic quality of the cooled nuclear ensemble, a highly collimated x-ray photon beam with $\Delta \theta \ll 0.1 \text{ rad}$ should be employed for pumping. Additionally, in order to diminish the destructive impact of pumping radiation through the atomic photoelectric effect, the x-ray photon beam should be of a high degree of monochromaticity with $\Delta \omega_{\text{pump}} / \omega_0 \ll 10^{-5}$. We have estimated the critical spectral photon flux density of pumping radiation. It falls into a hard-to-reach region of photon fluxes that could be supplied only with the upcoming x-ray free electron lasers. Summarizing, we may conclude that the lack of sources producing x-ray free electron lasers is the primary impediment to realizing the gamma-ray lasing process in a cooled nuclear ensemble.

Acknowledgements
Author is grateful to L.A. Rivlin for fruitful discussions of the problem considered in this paper. This research was supported by the Ministry of Education and Science of the Russian Federation, the contract number 14.B37.21.0756.

References
[1] NIST Photon Cross Sections Database: http://www.nist.gov/pml/data/xcom/index.cfm
[2] Khokhlov R V 1972 JETP Lett. 15 414
[3] Il’inskii Yu A and Khokhlov R V 1974 Phys. Usp. 16 565
[4] Andreev A V, Volkov R V, Gordienko V M, Mikheev P M, Savel’ev A B, Tkalya E V, Chutko O V, Shashkov A A and Dykhne A M 1999 JETP Lett. 69 371
[5] Ledingham K W D et al 2000 Phys. Rev. Lett. 84 899
[6] Cowan T E et al 2000 Phys. Rev. Lett. 84 903
[7] Stedile F et al 2001 Phys. Rev. C 63 024320
[8] Rivlin L A 1961 USSR Inventor’s Certificate No. 621256 on Jan. 10
[9] Rivlin L A 1979 USSR Bull. Inventions 23 220
[10] Rivlin L A 1962 Vopr. Radioelektron. ser. 1 Elektron. 6 60
[11] Rivlin L A 1963 Vopr. Radioelektron., ser. 1 Elektron. 6 42
[12] Chirikov B V 1963 Sov. Phys. JETP 17 1355
[13] Vali V and Vali W 1963 Proc. IEEE 51 182
[14] Vali V and Vali W 1963 Proc. IEEE 51 1248
[15] Baldwin G C, Neissel J P, Terhune J, and Tonks L 1963 Trans. Am. Nucl. Soc. 6 178
[16] Baldwin G C, Neissel J P, Terhune J, and Tonks L 1963 Proc. IEEE 51 1247
[17] Il’inskii Yu A and Khokhlov R V 1974 Sov. Phys. JETP 38 809
[18] Goldansky V I and Kagan Yu M 1973 Sov. Phys. JETP 64 90
[19] Letokhov V S 1973 Sov. Phys. JETP 64 1555
[20] Andreev A V, Il’insky Yu A and Khokhlov R V 1975 Sov. Phys. JETP 40 819
[21] Baldwin G C and Solem J C 1997 Rev. Mod. Phys. 69 1085
[22] Rivlin L A 2007 Quantum Electron. 37 723
[23] Rivlin L A 1999 Quantum Electron. 6 467.
[24] Metcalf H J and van der Straten P 1999 Laser Cooling and Trapping (New York: Springer)
[25] Carroll J J, Karamian S A, Rivlin L A and Zadernovsky A A 2001 Hyperfine Interact. 135 3