INVESTIGATION STUDY ON DETERMINATION OF FRACTURE STRAIN AND FRACTURE FORMING LIMIT CURVE USING DIFFERENT EXPERIMENTAL AND NUMERICAL METHODS

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Abstract. Forming Limit Curve (FLC) is a well-known tool for the evaluation of failure in sheet metal process. However, its experimental determination and evaluation are rather complex. From theoretical point of view, FLC describes initiation of the instability not fracture. During the last years Digital Image Correlation (DIC) techniques have been developed extensively. Throughout this paper, all the measurements were done using DIC and as it is reported in the literature, different approaches to capture necking and fracture phenomena using Cross Section Method (CSM), Time dependent Method (TDM) and Thinning Method (TM) were investigated. Each aforementioned method has some advantages and disadvantages. Moreover, a cruciform specimen was used in order to cover whole FLC in the range between uniaxial to equi-biaxial tension and as an alternative for Nakajima test. Based on above-mentioned uncertainty about the fracture strain, some advanced numerical failure models can describe necking and fracture phenomena accurately with consideration of anisotropic effects. It is noticeable that in this paper, dog-bone, notch and circular disk specimens are used to calibrate Johnson-Cook (J-C) fracture model. The results are discussed for mild steel DC01.

1. Introduction
A well-known method of material formability description is the Forming Limit Diagram (FLD). Forming Limit Curve (FLC) is a plot of major and minor principal strains in the plane of deformed shear where necking takes place. In sheet metal forming, failure mechanism is driven by necking and rupture. The transition between local necking and fracture is too fast and thereupon the failure terminates a forming operation. Failure prediction in sheet metal forming applications is mainly based on the experimentally measurement of FLC which was introduced by Keeler [1]. The concept still stands as the first safety criterion in deep drawing operations and is a conventional approach to assess sheet metal formability. Although this concept is very simple and well understood, its experimental determination is rather complex demanding a wide range of sheet metal forming tests. Therefore, in order to obtain FLC, Nakajima test is recommended vastly in the literature. Volk in [2] developed the so-called Time Dependent Method (TDM) based on the fact that the strain outside the localized region remains constant after the localization while the strain rate in the localization region increases sharply. Besides, Cross Section Method (CSM) is the strain distribution along a path in centre of specimen in different stages during an experiment and is approximated using quadratic function. The maximum value of major strain right before the fracture can represent the critical strain. CSM generally underestimates the formability in comparison with TDM. Fracture strain can be determined with two alternative techniques. The first one is most common in industry and is Digital Image Correlation (DIC) method, i.e. a contactless and material independent optical measuring technique which captures strains during the deformation. DIC can detect the strain path from beginning to fracture. During the experiment, the strain is increasing smoothly until the
necking, but from necking to fracture it shoots up and camera’s speed shutter is still constant without consideration of abrupt change in strain rate. Hence, this deficiency is attributed to DIC which can lead to underestimate the actual value of fracture strain. The second method is based on measurement of fracture strain using fractured thickness. Gorji in [3] used this method for Nakajima test in plane strain tension test:

\[
\begin{align*}
\varepsilon_{22} &= 0 \\
\varepsilon_{33} &= \ln \frac{t}{t_0}
\end{align*}
\]  

(1)

Where \(\varepsilon_{22}, \varepsilon_{33}, t\) and \(t_0\) are width, thickness strains, fractured and initial thicknesses, respectively. Afterwards, based on constant volume constraint condition \(\Delta \varepsilon_{11} = -\Delta \varepsilon_{33}\) (\(\Delta \varepsilon_{33}\) is the difference between thickness strains which were measured by thinning method and DIC technique), updated fracture strain is sum of fracture strain which was measured by DIC techniques and \(\Delta \varepsilon_{11}\), it can be expressed as follow:

\[
\varepsilon_{11}^f = \varepsilon_{11}^{DIC} + \Delta \varepsilon_{11}
\]  

(2)

Finally, the obtained value for \(\varepsilon_{11}^f\) is higher than fracture strain based DIC techniques. Another application of the measurement of fractured thickness is to estimate fracture strain which will be discussed in section 2.

According to the uncertainty about the fracture strain value, an accurate simulation of a real-world experiment can provide better understanding about necking and fracture phenomena. The quality of simulation is basically dependent on knowledge of the material behavior, geometry, loading and boundary conditions.

Numerous researches have been conducted on ductile fracture of common metals such as steel and aluminum alloys over last decades. Earlier efforts have been made to investigate the effect of stress triaxiality on ductile fracture associated with nucleation, growth and coalescence of micro-voids. Their experiments and theoretical modelling have confirmed that fracture strain decreases monotonically as stress triaxiality increases [4-6]. Gurson in [7] proposed constitutive laws involving ductile fracture considering yield criteria under the framework of compressible plasticity for porous materials, i.e. softening is originated by micro-void nucleation, growth and coalescence. The model was modified by Tvergaard and Needleman [8] and as reported in the literature is called GTN.

Present paper aims to make a comparison between coupled and uncoupled ductile damage models. The former models are complex and computationally expensive. They require the characterization of more material constants. Moreover aspects such as convergence and localization due to softening should be considered. The latter models regarding to the aforementioned difficulties, ignore material degradation and damage, therefore failure modelling occurs without loss of strength and stiffness. In this case, damage variable is uncoupled and does not necessarily have representative volume element and fracture can happen without any modification in the material constitutive law. In such models, the failure is controlled by an external variable, when it attains a critical value, then material undergoes an abrupt decrease in the local resistance of the critical area. Many uncoupled models are reported in the literature and the pioneer one is proposed by Freudenthal [9], who postulated that failure occurs when the total work attains a critical value. In this work, dog-bone, notch and circular disk are used in order to calibrate J-C fracture model as an uncoupled ductile damage model for mild steel DC01. Besides, J-C failure model is incorporated with Hill’s 48 anisotropy yield criterion. On the other hand, calibrated GTN model [10] is utilized as a coupled damage model which was explained in the literature vastly and calibrated for mild steel DC01.

The cruciform was designed as an alternative for the Nakajima test, e.g. in Nakajima test many specimens with the variety of widths must be tested till whole FLC can be covered. Another deficiency is the friction between punch and sheet can alter the fracture location as well as fracture strain value. Furthermore, based on special design of the cruciform, necking can be induced in the central area and then the displacement ratios of the machines can control the strain path in the center without any friction. Moreover, the calibrated ductile damage model with anisotropy plasticity Hill
1948 is utilized for the cruciform specimen. Finally, FLC and Fracture Forming Limit Curve (FFLC) are obtained for mild steel DC01.

2. Uniaxial tensile test

The Lankford ratios and hardening behavior of mild steel DC01 are determined from uniaxial tensile tests on samples cut from the sheets. Three tensile specimens are extracted from each of rolling, transverse and 45° direction. As can be seen, figure 1 (a) indicates the geometry of the dog-bone specimen with thickness equal to 1.5 mm and extractions along the aforementioned directions. All the experiments are performed at an axial strain rate of about $10^{-3}/s$. During each experiment, the in-plane displacement field is monitored using planar DIC. In particular, evolutions of axial and width strains are captured using virtual extensometers of about 20 mm and 9 mm initial lengths for the respective directions. Afterwards, true width and thickness strains (assuming incompressibility condition) are computed, the Lankford ratios can be determined from the average slopes:

$$r = \frac{d\varepsilon^p_w}{d\varepsilon^p_{th}}$$

(3)

Figure 1. (a) geometry of dog-bone specimen, (b) extraction of the samples in rolling, transverse and 45° directions

Figure 2 (a) and (b) represent the true stress-logarithmic strain and average of true plastic width and thickness strains for mild steel DC01, respectively. The Lankford ratios are summarized in table 1:

Table 1. Measured values of the Lankford ratios for mild steel DC01

|       | $r_0$ | $r_{45}$ | $r_{90}$ |
|-------|-------|----------|----------|
|       | 1.66  | 1.11     | 1.57     |

3. Modelling of anisotropic plasticity

The aim of numerical simulation is to represent the real-world experiment as accurately as possible. For this sake, in this paper Hill’s 1948 is utilized to represent the plastic behavior of mild steel DC01 sheets. The Hill 1948 can be expressed in terms of equivalent stress in the case of plane stress ($\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$) as follow:

$$F\sigma_{22}^2 + G\sigma_{12}^2 + H(\sigma_{11} - \sigma_{22})^2 + 2N\sigma_{12}^2 = \bar{\sigma}^2$$

(4)

$F$, $G$, $H$ and $N$ are coefficients which can be calculated in terms of Lankford ratios.
Figure 2. (a) stress-strain curve, (b) transverse plastic strain versus through-thickness plastic strain

As it is mentioned, there is an uncertainty about the value of fracture strain. Beese in [11] proposed that equivalent fracture strain can be obtained from direct measurement of fracture strains at which the average value of Lankford ratio is \( r = 1.44 \). Therefore, the equivalent plastic strain can be expressed as follow:

\[
\varepsilon = (r + 1) \sqrt{\frac{1}{2r + 1} \left( \varepsilon_{11}^2 + \frac{2r}{1 + r} \varepsilon_{12}^2 + \varepsilon_{22}^2 \right)}
\]

(5)

Fracture strains can be calculated directly as follows:

\[
\varepsilon_{11} = \ln \frac{t}{t_0}; \quad \varepsilon_{22} = \ln \frac{w}{w_0}; \quad \varepsilon_{33} = \ln \frac{t}{t_0}
\]

(6)

Where \( t_0, w_0 \), and \( t_0 \) are initial length, width and thickness respectively. According to Thinning Method (TM) fractured specimen is cut by wire cut system and then the specimen is grinded and polished with sandpaper to ensure good visibility of the cracked cross-section. Finally, microscope pictures were used to determine the fracture thickness by measuring the number of pixels. Figure 3 illustrates the fractured thickness which is measured by microscope. The average value of equivalent fracture strain for six experiments is about 1.06. On other hand, the average value of the equivalent fracture strains which are measured by DIC is 1.02. This fact implies that thining method and surface method are in a close agreement for material in which Lankford ratios are greater than 1.

4. Johnson-Cook failure model

The criterion proposed by Johnson-Cook [6] postulates that fracture occurs when the accumulated plastic strain, calculated as a function of three independent variables including stress triaxiality, strain rate and temperature.

\[
\varepsilon^p = \omega_p(\eta)\omega_e(\dot{\varepsilon}^p)\omega_\theta(T) \leq \varepsilon_f^p
\]

(7)

Where \( \omega_p, \omega_e \), and \( \omega_\theta \) are the function of stress triaxiality \( \eta \), plastic strain rate \( \dot{\varepsilon}^p \) and temperature \( T \) respectively. In the present work, only the first term \( \omega_p(\eta) \) is taken into account and can be expressed as follow:

\[
\omega_p = D_1 + D_2 \exp(D_3 \eta)
\]

(8)
$D_1$, $D_2$, and $D_3$ are the material parameters which need to be calibrated. When $\bar{\varepsilon}^p$ reaches its critical value $\bar{\varepsilon}_{f}^p$, failure will happen.

Figure 3. Schematic of fractured thickness

In this paper, dog-bone, notch and circular disk specimens are chosen for the calibration of J-C model as an uncoupled fracture model. These specimens represent uniaxial, plane strain and equi-biaxial tension loading conditions, respectively. For the numerical simulations, GTN and J-C models were implemented in commercial code ABAQUS software. Moreover, in this paper, the calibration was done only for J-C model by inverse analysis through the comparison between numerical and experimental force-displacement curves. Experimental displacements were captured by virtual extensometer ARAMIS and the same coordinates used for the numerical simulation. Specimens were mounted to machine from holes and the same boundary condition was imposed for numerical simulation. Moreover, circular disk was secured with a 200 mm diameter die by a flange with 12 bolts. The radius of hemispherical punch was 47.5 mm with velocity of 1 mm/s. In addition, 4 sheets of teflon with grease between each two layers were used between the disk and the punch to reduce friction. In practice, necking and subsequent fracture occur slightly off-center due to friction. Figure 4 depicts the specimens geometries and their force-displacement responses. Table 2 indicates values of J-C fracture model parameters in equation (8):

|   | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|
|   | 0.320 | 0.201 | -0.110 |

5. GTN Failure Model

The yield function of the GTN’s model, which assumes isotropic hardening and isotropic damage, can be expressed by equation (9):

$$\phi(\sigma, k, f) = J_2(s) - \frac{1}{3} \left( 1 + q_3f^* - 2q_1f^* \cosh\left( \frac{q_23p}{2H} \right) \right) H^2$$

$$f^* = \begin{cases} f & f < f_c \\ f_c + \left( \frac{1}{q_1} - f_c \right) \left( \frac{f - f_c}{f_f - f_c} \right) & f \geq f_c \end{cases}$$

(9)

Where $k$, $f$, $J_2(s)$, $H$, $f^*$, $f_0$, $S_N$, $\epsilon_N$, $f_N$, $f_c$ and $f_f$ are thermodynamical force, volume void fraction, second invariant of the deviatoric stress tensor, hardening rule, effective porosity, initial volume void fraction, standard deviation, mean strain, volume void fraction at nucleation, critical volume void fraction and volume void fraction at fracture, respectively. Original GTN damage model is only for isotropic material and in this work is used just for cruciform specimen (section 6).

In addition, Tuninetti et al. in [10] calibrate GTN model and parameters for mild steel DC01 is summarized in table 3:
Table 3. GTN parameters for mild steel DC01

| \(f_0\) | \(S_N\) | \(\varepsilon_N\) | \(f_N\) | \(f_c\) | \(f_f\) |
|---------|---------|-----------------|---------|---------|---------|
| 0.0008  | 0.0008  | 0.42            | 0.0025  | 0.0055  | 0.135   |

Moreover, in this paper, for all numerical simulations, one quarter of the geometries are modelled using the same mesh size of 0.8 \(\text{mm}\) (in the necking and fracture zone) with 3D elements with 8 nodes (in ABAQUS C3D8 elements).

![Figure 4](image1.png)

Figure 4. (a) Notch specimen (b) Circular disk, (c) and (d) numerical and experimental reaction force-displacement curves for notch and circular disk specimens, respectively.

6. Cruciform specimen

FLC in sheet metal forming can be covered traditionally by Nakajima experiment. In Nakajima experiment, the friction between sheet and punch can alter the results. Therefore, many samples with variety of widths has to be tested to cover whole FLC in range between uniaxial to equi-biaxial. As an alternative, cruciform specimen can cover FLC in the aforementioned range based on a special design at which necking is induced to the central zone of the specimen. Furthermore, displacement ratios of machines can control the strain path in the central zone. Finally, in comparison with Nakajima test, cruciform specimen can provide FLC without friction.

![Figure 5](image2.png)

Figure 5 (a) and (b) represent the geometry of the cruciform specimen in front and cross-section views, respectively. As can be seen in figure 5 (b), in the central zone where necking occurs, the thickness is 0.3 \(\text{mm}\). However, it is not possible to construct the same groove on behind the specimen because of manufacturing limitation. Figure 6 illustrates the contour of equivalent plastic strain of the
cruciform specimen. As it is shown, the necking is induced to the central zone due to thickness reduction, then for different loading conditions, in order to plot strain paths (dash lines) just major and minor strains are needed. For experiment, strain path can be obtained just by point analysis in DIC software and for numerical using integration points in the central element. It is noticeable time dependent method and second derivation of thickness strain method [12] are utilized to predict the onset of necking for experimental and numerical strain paths, respectively. The latter is based on peaks in first (thinning rate) and second (thinning acceleration) derivations which are rigorously localized in time. In figure 6, GTN FLC, J-C FLC and J-C FFLC (solid lines) represent forming limit curve and fracture forming limit forming curve which are obtained from GTN and J-C failure models. As it is mentioned, GTN is a coupled damage model with assumption of isotropic material while J-C model is uncoupled and in this work is incorporated with Hill’s 48 yield criterion.

Figure 5. (a) geometry of the cruciform specimen (b) Variations of thickness along the specimen

7. Conclusion and future research

The present work was a review study of numerical and experimental methods about determination of fracture strain and forming limit curve. As it was reviewed, the calculation of the exact value of equivalent fracture strain is not possible. Therefore, based on this uncertainty about the exact value, advanced numerical failure models can describe the fracture phenomenon and improve this procedure. In this work GTN and J-C failure models are used as couple and uncouple damage models, respectively. As it is explained above, coupled damage models are complex and computationally expensive, with more parameters for calibration in comparison with uncoupled model. Besides, original GTN is suitable only for isotropic material. According to table 1, mild steel DC01 represents anisotropic behavior, so it is necessary to take into account anisotropy effects. Furthermore, in this work Hill’s 48 is utilized in combination with J-C model in order to describe the real behavior of mild steel DC01 as accurately as possible. The results in figure 6 demonstrate the high performance of J-C model. Throughout this paper, the strain increment ratio were \(-0.5, 0\) and \(1\) which are corresponding to the uniaxial, plane strain and equi-biaxial tension loading conditions, respectively. For the future research, authors would like to use different strain increment ratios for right side of FLD and subsequently, use more complicated failure models.
Figure 6. Strain paths and obtained FLCs based on TDM and second derivation method

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References
[1] Keeler S and Backofen W 1963, Plastic instability and fracture in sheets stretched over rigid punches, J. Transaction of American society for metals, 56, pp.25-48
[2] Volk W and Hora P 2010, New algorithm for a robust user-independent evaluation of beginning instability for the experimental FLC determination, Int. J. Mater. Form 4, pp.339-46
[3] Gorji M 2015, Instability and fracture models to optimize the metal forming and bending Crack behaviour of Al-alloy composite, PhD thesis, ETH Zurich, pp.53-61
[4] Garrison J and Moody N 1987 Ductile fracture, J. Phys. Chem. Solids, 48, pp.1035-74
[5] Hancock J and Mackenzie A 1976, On the mechanisms of ductile failure in high-strength steels subjected to multi-axial stress-states, J. Mech.Phys. Solids, 24, pp.147-60
[6] Johnson G and Cook W 1985, Fracture characteristics of metals subjected to various strains, strain rates, temperatures and pressures, Eng. Fract. Mech 21,pp.31-48
[7] Gurson A 1977, Continuum theory of ductile rupture by void nucleation and growth, J. Eng. Mater. Trans, ASME Ser. H 99, pp.2-15
[8] Tvergaard V and Needleman A 1984, Analysis of the cup-cone fracture in a round tensile bar, Acta Metall. Mater, 32, pp.157-69
[9] Freudenthal A 1950, The inelastic behaviour of engineering materials and structures. John Wiley& Sons, New York
[10] Tuninetti V, Yuan S, Gilles G, Guzman C.F, Habraken A.M and Duchene L 2016, Modelling the ductile fracture and the plastic anisotropy of DC01 steel at room temperature and low strain rates, J. Physics: conference series, 734.
[11] Beeese AM 2008, Plasticity and phase transformation of stainless steel 301LN sheets, MS thesis, Massachusetts Institute of Technology
[12] Petek A, Pepelnjak T Kuzman K 2005, An improved method for determining a forming limit diagram in the digital environment, J. Mech. Eng. 51, pp.330-45