The generalised local model applied to Fibreglass

Sabrina Vantadori\textsuperscript{a}, Miguel Muñiz-Calvente\textsuperscript{b}, Daniela Scorza\textsuperscript{a}, Alfonso Fernández-Canteli\textsuperscript{b}, Adrián Álvarez Vázquez\textsuperscript{b}, Andrea Carpinteri\textsuperscript{a}

\textsuperscript{a}DICATeA, University of Parma, Parma, Italy
\textsuperscript{b}Department of Construction and Manufacturing Engineering, University of Oviedo, Gijón, Spain

Corresponding Author: Sabrina Vantadori sabrina.vantadori@unipr.it

ABSTRACT

In the present research work, the Generalised Local Model (GLM) is employed to characterize the failure of a commercial Fibreglass by determining the Primary Cumulative Distribution Function of Failure (PCDFF) through an experimental campaign and numerical computations. Four criteria used to define the failure condition are based on the following Generalised Parameters (GPs): (i) Maximum Principal Stress; (ii) von Mises Stress; (iii) Tsai-Hill GP; (iv) Hoffman GP.

For each GP adopted, the PCDFF is derived in two different ways: (a) by examining data obtained from tension tests only (GLM single application); (b) by collecting data, determined through tension tests and three-point bending tests, into a unique set (GLM joint application). Finally, a comparison between the obtained PCDFFs and the experimental failure results is presented.
KEYWORDS: failure assessment, fibreglass composite, Generalised Local Model, Weibull distribution

NOMENCLATURE

- $E$: elastic modulus
- $F_i, F_{ij}$: failure criteria parameters defined by the material strengths
- $FI$: Failure Index
- $GLM$: Generalised Local Model
- $GP$: Generalised Parameter
- $GP_{eq}$: equivalent Generalised Parameter
- $GP_H$: Hoffman Generalised Parameter
- $GP_{MS}$: Maximum Stress Generalised Parameter
- $GP_{TH}$: Tsai-Hill Generalised Parameter
- $GP_{VM}$: von Mises Stress Generalised Parameter
- $GPF$: Global Probability of Failure
- $i$: i-th specimen
- $j$: j-th finite element
- $k_i$: rank index
- $N$: total number of tested specimens
- $N_e$: total number of finite elements
- $P$: load at failure
- $PCDFF$: Primary Cumulative Distribution Function of Failure
- $P_{exp, fail}$: experimental cumulative failure probability
- $P_{fail}$: global cumulative failure probability
- $P_{fail, ij}$: local cumulative failure probability
- $S$: shear strength
- $SR$: Strength Ratio
- $S_{ref}$: reference size
- $S_{eq}$: equivalent size
- $SRVE$: Statistical Representative Volume Element
\( \beta \) Weibull shape parameter

\( \delta \) Weibull scale parameter

\( \Delta S_j \) size of \( j \)-th finite element

\( \varepsilon_f \) flexural strain

\( \varepsilon^u_T \) ultimate tensile strain

\( \lambda \) Weibull location parameter

\( \sigma_f \) flexural strength

\( \sigma^u_C \) compressive strength

\( \sigma^u_T \) tensile strength

1. INTRODUCTION

The application fields of composites are various, due to their high strength-to-weight and stiffness-to-weight ratios, good thermal properties, and potential for tailor-made design in mechanical, civil, marine and aerospace engineering. Their manufacture process (i.e. the way how fibres and matrix are combined and bounded together) is quite complex, which is reflected in numerous uncertainties related to design of composite structural components, due to the scatter of the mechanical, fracture and fatigue properties of the material. This aspect just distinguishes composites from other common structural materials [1].

Uncertainties in composite performance may be of three types [2,3]: aleatory uncertainty, epistemic uncertainty, and errors. The first type of uncertainty is related to the randomness of the system, i.e. due to fibres and matrix characteristics, their reciprocal interaction, manufacturing variations and so on. The
epistemic uncertainty is the consequence of the limited knowledge of the composite behaviour, and is related to the experimental procedures and modelling methods adopted (for instance, the assumptions regarding the material load-response). Finally, the third cause of uncertainty refers to errors, such as those due to instrumentation, regression method applied and so on, that some Authors include into the first category [2].

By focussing attention on the category of the aleatory uncertainty, different scales can be considered in designing composite structural components [2,4]:

(a) micro-scale, that is, at the level of constituents. The material modelling may be performed through stochastic simulations by employing a large number of random variables based on the fibre/matrix properties;

(b) macro-scale, that is, at the structural component level. The material global behaviour may be modelled by considering the randomness of the component properties (stiffness, strength, failure mode and so on);

(c) meso-scale, that is, at the ply level. Such level can be considered intermediate between micro- and macro-scale.

The choice of the scale depends on the type of problem analysed. It is often advantageous to adopt a multi-scale approach, by examining different modelling levels [5].

In such a context, a probabilistic (stochastic) approach is recommended to estimate the reliability of composite structural
components, since it may reduce the stochastic variability of the design data [4].

According to the uncertainty scale used, two classes of models are available in the literature [2]: Class A (models referring to micro-scale) and Class B (models referring to macro-scale). Moreover, an intermediate scale, named Class AB (related to meso-scale), has been recently proposed [2].

The models falling in Class A are focused on the constituents (fibre, matrix and fibre-matrix interface properties), whereas the material macroscopic behaviour is simulated through micro-mechanical models. Such a class is usually subdivided in two sub-classes: Class A1 and Class A2. The Class A1 models use probability distributions at level of constituents, and build up the uncertainties to macro-scale through stochastic mechanical analysers [6-9]. The Class A2 models are based on the concept of the Statistical Representative Volume Element (SRVE), which is assumed to be representative of the micro-structure [1,10-11].

Models falling in Class B enable the global behaviour of composites to be investigated by examining the randomness of the material properties through a set of nominally identical experimental tests performed according to international standards, such as the ASTM, BS EN ISO and EN ISO standards [2,9,12-17]. Then, to describe the above random properties, such models resort to probability distribution functions such as Normal, Lognormal, Gamma or Weibull distributions [12,14,15,18-20].
A wide range of composite mechanical properties was analysed through probabilistic models. For instance, Sriramula and Chryssanthopoulos [17] developed a probabilistic model for a Glass-fibre reinforced polymer, based on experimental studies and Normal, Lognormal and Weibull probability distributions, in order to quantify the variability of material strength and stiffness. Lekou and Philippidis [18] proposed a probabilistic model based on experimental tests performed on a unidirectional Glass/Polyester composite and different probability distribution laws, to evaluate the variability of material strength, stiffness and thermal expansion coefficients. Nader and co-workers [14,21] presented a stochastic model based on tensile and compression tests performed on a Eglass/vinylester composite, by using Normal and Lognormal probability distributions to estimate material strength and stiffness.

Note that the majority of the probability distribution models proposed in the literature for composites implements two-parameter Weibull distributions to describe the composite random properties [2]. Alqam and co-workers compared three- and two-parameter Weibull distributions of material strength and stiffness, by using 26 data sets of fibre reinforced polymeric composites [22]. The Authors observed that both distributions produced similar results, and highlighted that the two-parameter distribution was more computationally efficient, whereas the three-parameter one was more robust and provided a better fitting of the data.
Further, the Global Probability of Failure (GPF) can be estimated by using a probabilistic approach, and different models are available in the literature. Some of them are based on the Weibull distribution law combined with the “weakest link principle” \([15,23-24]\), and other models take full advantage from the Daniels theory \([25, 26]\) or propose a modified weakest link method \([27,28]\). Moreover, in the case of multiaxial loading, failure criteria need to be implemented \([29-31]\).

In such a context, the Generalised Local Model (GLM) \([15,32-33]\) allows us to determine the GPF of a composite structural component by exploiting the weakest-link principle together with a formulation based on the three-parameter Weibull distribution. Such a model, starting from a set of experimental data, leads to estimate the Primary Cumulative Distribution Function of Failure (PCDFF) related to a given material failure type.

In the present work, the GLM is employed to analyse the GPF of a commercial Fibreglass. The novelty of this paper is that the most suitable failure Generalised Parameter (GP) to determine the GPF of a commercial Fibreglass is identified among four alternatives: Maximum Principal Stress, von Mises Stress, Tsai-Hill GP, and Hoffmann GP.

2. THE GENERALISED LOCAL MODEL

2.1 Formulation

The flowchart describing the Generalised Local Model \([15,32-33]\) is shown in Figure 1. The GLM allows us to determine the PCDFF,
which is a function of a Generalised Parameter characterising the failure of the structural component being examined.

**Figure 1.**

The GLM is formulated by assuming that the probability of material failure, $P_{\text{fail}}$, follows the three-parameter Weibull distribution given by \[15,34]\:

$$P_{\text{fail}} = 1 - \exp \left[ -\left( \frac{GP - \lambda}{\delta} \right)^\beta \right]$$  \hspace{1cm} (1)

where $\beta$, $\delta$ and $\lambda$ are three Weibull parameters (shape, scale, and location), and $GP$ is the above-mentioned Generalised Parameter.

More precisely, by starting from the data obtained from an experimental campaign, the $GP_i$ value is determined for the $i$-th specimen according to the adopted failure criterion (with $i=1,...,N$, $N$ being the total number of the tested specimens). After all the $GP_i$ values are sorted in increasing order and a rank index $k_i$ is associated to each one, being $k_i=i$, the experimental failure probability $P_{\text{exp fail},i}$ is computed by using the Bernard’s expression \[35]\:

$$P_{\text{exp fail},i} = \frac{k_i - 0.3}{N + 0.4}$$  \hspace{1cm} (2)

It is worth remarking that such a probability is related to the structural component experimentally examined and depends on both geometrical size and shape of such a component, loading conditions, material properties, and adopted failure criterion.
Therefore, the incipient failure condition of each experimental test is simulated through the finite element method by using the nominal values of both specimen sizes and material mechanical properties. Then, the $GP$ value is registered for the $j$-th finite element of the $i$-th discretised model ($GP_{ij}$), where $j=1,...,N_e$ identifies the $j$-th finite element and $N_e$ is the total number of finite elements in the numerical discretisation. This can be done automatically by a postprocessing finite element tool such as Abaqus2Matlab [36].

Then, the local cumulative failure probability for the $j$-th finite element of the model ($p_{fail,ij}$) may be expressed by an enhanced version of the Weibull distribution:

$$p_{fail,ij} = 1 - \exp \left( \frac{-\Delta S_{ij}}{S_{ref}} \left( \frac{GP_{ij} - \lambda}{\delta} \right)^\beta \right)$$

(3)

where $S_{ref}$ is an arbitrary reference size (for instance, an area or a volume), $\Delta S_{ij}$ represents the size (area or volume) of the $j$-th finite element, and initial values are assumed for $\beta, \delta, \lambda$.

The global cumulative failure probability, $P_{fail,i}$, of each numerical model is computed by applying the weakest link principle:

$$P_{fail,i} = 1 - \prod_{j=1}^{N_e} (1 - p_{fail,ij})$$

(4)

From the above-computed probability, an equivalent size, $S_{eq,i}$, may be determined for the $i$-th model by using the following equation:
where $GP_{i,\text{max}}$ is the maximum value of the GP evaluated for the $i$-th model. In such a way, the PCDF obtained at the end of the GLM procedure is independent of the size of the finite elements used in the discretisation.

By employing the above equivalent size (instead of the real one) together with the maximum value of the GP, the global cumulative failure probability may be determined by means of the enhanced version of the Weibull distribution, through an expression formally similar to Eq.\,(3) for the local cumulative failure probability:

$$P_{\text{fail},i} = 1 - \exp \left[ -\frac{S_{\text{eq},i} \left( \frac{GP_{i,\text{max}} - \lambda}{\delta} \right)^{\beta}}{S_{\text{ref}}} \right]$$

The global probability just obtained is equal to that computed by means of Eqs (3) and (4).

Up to this step of the procedure, $P_{\text{fail},i}$ from Eq. (6) is not yet the actual global probability of the $i$-th specimen since the values of $\beta, \delta$ and $\lambda$ have been only preliminarily estimated. Improved values of the Weibull parameters are achieved by equaling the above global cumulative failure probability ($P_{\text{fail},i}$, see Eq.(6)) to the experimental failure probability ($P_{\text{exp \, fail},i}$, see Eq.(2)) for each value of $i$, in order to write a system of $N$ equations. By solving such a system and fitting the three Weibull parameters, new values of $\beta, \delta$ and $\lambda$ can be determined and, as can be noted by the
The improved values are used to deduce new global probabilities (Eq. (4)) and new equivalent sizes (Eq. (5)). Such a loop is repeated until the Weibull parameters converge to constant final values. A detailed description of the parameter estimation process may be found in Refs. [32,33].

Finally, the PCDFF can be evaluated by using Eq. (1) and last values of the three Weibull parameters.

2.2 Single and joint application of GLM

The GLM described in Section 2.1 can be applied in two alternative ways:

(i) The first one, named GLM single application, consists in applying such a model to a homogeneous set of experimental data obtained from the same type of experimental test, such as tension test or three-point bending (3PB) test. The procedure described by the flowchart in Figure 1 is essentially the GLM single application.

(ii) The second one, named GLM joint application, consists in applying such a model to a heterogeneous set of experimental data, obtained from two or more types of experimental tests (such as tension tests together with 3PB tests). Since the loading conditions are different, an equivalent Generalised Parameter, $GP_{eq,i}$, has to be defined [15,33]:

$$GP_{eq,i} = \delta \left[ -\log \left( \frac{k_i - 0.3}{N + 0.4} \right)^{1/\beta} \right] + \lambda$$

(7)
and the global cumulative failure probability can be expressed as follows:

\[ P_{\text{fail},i} = 1 - \exp \left[ - \frac{S_{eq,i}}{S_{\text{ref}}} \left( \frac{GP_{eq,i} - \lambda}{\delta} \right)^\beta \right] \]  

(8)

In both cases above (i and ii), the PCDF obtained can be applied to estimate the probability of material failure for any other test type.

3. GENERALISED PARAMETERS EXAMINED

The Generalised Parameter GP characterises the failure behaviour of the structural component being examined, and hence it is linked to the failure criterion used. Four GPs are here considered, related to four failure criteria: Maximum Principal Stress [38], von Mises Stress [38], Tsai-Hill GP [39], and Hoffman GP [40].

3.1 GP related to the Maximum Stress Failure Criterion

According to the maximum stress theory, failure occurs when the maximum principal stress exceeds the ultimate material strength. By assuming the material to be macroscopically homogeneous, the condition of failure is given by [38]:

\[ \sigma_1 \geq \sigma_T^u \quad \text{or} \quad |\sigma_1| \geq \sigma_C^u \]  

(9)

where \( \sigma_1 \) is the maximum principal stress, \( \sigma_T^u \) is the material tensile strength, and \( \sigma_C^u \) is the material compressive strength.
Consequently, the Generalised Parameter, denoted $GP_{MS}$, based on the Maximum Stress Failure Criterion can be defined as follows:

$$GP_{MS} = \sigma_1$$  \hspace{2cm} (10)

### 3.2 GP related to the von Mises Stress Failure Criterion

The von Mises Stress Failure Criterion, also known as maximum distortion energy criterion, states that failure occurs when the distortion energy, which is the second deviatoric stress invariant, reaches a critical value [38]. Then the von Mises equivalent stress $\sigma_{eq}$ is defined, and the condition of failure is expressed as follows:

$$\sigma_{eq} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_x\sigma_z - \sigma_y\sigma_z + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)} \geq \sigma_T^u$$  \hspace{2cm} (11)

where $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}$ and $\tau_{yz}$ are the components of the stress tensor.

Consequently, the Generalised Parameter, denoted $GR_{VM}$, based on the von Mises Stress Failure Criterion can be defined as follows:

$$GR_{VM} = \sigma_{eq}$$  \hspace{2cm} (12)

### 3.3 GP related to the Tsai-Hill Failure Criterion

Such a failure criterion was proposed for orthotropic material [39,41]. By defining a Failure Index $FI$, the condition of failure can be written in tensor notation:

$$FI = F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1 \quad \text{with} \quad i, j = 1,\ldots,6$$  \hspace{2cm} (13)
where $\sigma_i$ and $\sigma_j$ are components of the stress tensor. The parameters $F_i$ and $F_{ij}$ are related to material strengths as is described in Table 1, where $\sigma_{T1}^u$, $\sigma_{T2}^u$ and $\sigma_{T3}^u$ are the tensile strengths along the principal directions 1, 2 and 3, respectively, $\sigma_{C1}^u$, $\sigma_{C2}^u$ and $\sigma_{C3}^u$ are the compressive strengths, and $S_{12}^u$, $S_{13}^u$ and $S_{23}^u$ are the shear strengths.

Table 1.

By assuming a macroscopically homogeneous and isotropic material, the parameters in Table 1 can be rewritten as follows:

\[
F_{11} = F_{22} = F_{33} = \frac{1}{\left(\sigma_{T}^u\right)^2}
\]

(14a)

\[
F_{44} = F_{55} = F_{66} = \frac{1}{\left(S^u\right)^2}
\]

(14b)

\[
F_{12} = F_{13} = F_{23} = -\frac{1}{2} \frac{1}{\left(\sigma_{T}^u\right)^2}
\]

(14c)

being $\sigma_{T1}^u = \sigma_{T2}^u = \sigma_{T3}^u = \sigma_{T}^u$ and $S_{12}^u = S_{13}^u = S_{23}^u = S^u$.

In order to define the GP based on the Tsai-Hill failure criterion, it is convenient to define a Strength Ratio, $SR$, also named safety factor, computed as the positive root of the following equation [42]:

\[
F_i\sigma_i(SR) + F_{ij}\sigma_i\sigma_j(SR)^2 - 1 = 0
\]

(15)
Such a root represents the safety factor for the material tensile strength \([42]\), and it results to be equal to:

\[
SR = \frac{1}{\sqrt{FI}}
\]  

(16)

Note that the coefficients in Eq. (15) are listed in \textbf{Table 1}.

Finally, the Generalised Parameter based on the Tsai-Hill Failure Criterion, \(GP_{TH}\), can be obtained by dividing the material tensile strength for the above safety factor, \(SR\):

\[
GP_{TH} = \frac{\sigma_T^\mu}{SR} = \sigma_T^\mu \cdot \sqrt{FI}
\]  

(17)

By combining Eq. (17) with Eqs (13) and (14), it results:

\[
GP_{TH} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x \sigma_y - \sigma_x \sigma_z - \sigma_y \sigma_z + a \cdot \left( \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2 \right)}
\]  

(18)

with \(a = \frac{S^\mu}{\sigma_T^\mu}\).

\textbf{3.4 GP related to the Hoffman Failure Criterion}

Such a failure criterion was proposed by Hoffman \([40]\) in order to take into account the different material strengths exhibited under tension and compression. Hoffman considered also the linear terms, \(F_i\), in Eq. (13) adopting the parameters reported in \textbf{Table 1}.

By assuming the material to be isotropic and macroscopically homogeneous, the parameters in \textbf{Table 1} can be re-written as follows:
\[ F_1 = F_2 = F_3 = \frac{1}{\sigma_T^u} \cdot \frac{1}{\sigma_C^u} \]  
(19a)

\[ F_{11} = F_{22} = F_{33} = \frac{1}{\sigma_T^u \cdot \sigma_C^u} \]  
(19b)

\[ F_{44} = F_{55} = F_{66} = \frac{1}{(S^u)^2} \]  
(19c)

\[ F_{12} = F_{13} = F_{23} = -\frac{1}{2} \frac{1}{\sigma_T^u \cdot \sigma_C^u} \]  
(19d)

The Strength Ratio can be determined by solving Eq. (15), and the Generalised Parameter, \( GP_H \), is defined as follows [42]:

\[ GP_H = \frac{\sigma_T^u}{SR} \quad \text{if} \quad SR > 0 \]  
(20a)

\[ |GP_H| = \frac{\sigma_C^u}{|SR|} \quad \text{if} \quad SR < 0 \]  
(20b)

4. GLM APPLIED TO A SHORT FIBRE-REINFORCED COMPOSITE

4.1 Experimental campaign

The failure behaviour of a short fibre-reinforced composite has been experimentally examined. The specimens, tested in the experimental campaign described in Ref. [15] have been extracted from commercial Fibreglass panels, with a fibre volume content equal to about 30\%, commonly used in the construction of caravans, commercial vehicles and tanks for storage of water, food and chemical substances.

The experimental campaign has consisted of:

1) tension tests on 33 specimens;
(2) three-point bending (3PB) tests on 38 specimens;

(3) four-point bending (4PB) tests on 34 specimens.

All tests have been performed according to the ASTM standards [43-45], by employing the universal testing machine Instron 8862 available at the “Testing Laboratory of Materials and Structures” of the University of Parma (Figure 2).

Figure 2.

Tests have been carried out under displacement control, with a head displacement rate computed according to the above-mentioned ASTM standards, by measuring the load through a load cell. Moreover, the Digital Image Correlation (DIC) technique has been applied in order to extract the 2D displacement field for each tested specimen.

The tensile and flexural strengths, $\sigma^u_T$ and $\sigma_f$, respectively, and the elastic modulus $E$ have been computed as a function of the final load $P$, according to the proper ASTM standard, whereas the corresponding strains, $\varepsilon^u_T$ and $\varepsilon_f$, have been evaluated by taking into account the displacement field through DIC analyses. Such values are summarised in Table 2 in terms of mean values and standard deviations for each test configuration. Further details may be found in Ref. [15].

Table 2.
4.2 Single GLM application

The GLM is applied to the test results presented in the above Sub-Section. The PCDFF is determined by considering the specimens subjected to tension tests (single GLM application) and alternatively employing one of the four GPs discussed. The numerical analyses are performed by means of the commercial software Strauss 7® [46].

In order to verify which one of the four adopted Generalised Parameters most suitably describes the failure behaviour of the tested material, the Weibull parameters related to the PCDFF for tension (listed in Table 3) are used to evaluate the PCDFF for both three- and four-point bending.

Table 3.

For each loading configuration, the probability of failure (computed through Eq.(6)) and the experimental failure probability values (computed through Eq.(2)) are plotted in Figure 3, by adopting the Maximum Stress Failure Criterion (i.e. $GP_{MS}$ as the GP). It can be remarked that numerical results represented by the curves in continuous line are in a satisfactory agreement with the experimental data represented by symbols, since more than 50% of the experimental data falls within the ±5% scatter band (dashed lines) and all data are within the ±10% scatter band (dash-dot lines) for all the loading conditions (see also the last three columns in Table 3).
For each loading configuration, the numerical probability of failure and the experimental failure probability values are plotted in Figure 4 by using the von Mises Stress Failure Criterion (i.e. $GP_{VM}$ as the GP). In the case of tension and three-point bending conditions, all the experimental data fall within the $\pm10\%$ scatter band, whereas $94\%$ of data falls inside the $\pm15\%$ scatter band for four-point bending condition (see Table 3).

For each loading configuration in Figure 5, the numerical probability of failure and the experimental failure probability values are plotted by using the Tsai-Hill Failure Criterion (i.e. $GP_{TH}$ as the GP). In the case of tension and three-point bending conditions, all the experimental data fall within the $\pm10\%$ scatter band, whereas $85\%$ of data falls inside the $\pm15\%$ scatter band for four-point bending condition (see Table 3).

For each loading configuration, the numerical probability of failure and the experimental failure probability values are plotted in Figure 6 by using the Hoffman Failure Criterion (i.e. $GP_{H}$ as the GP). In the case of tension and three-point bending conditions, all the experimental data fall within the $\pm10\%$ scatter
band, whereas only 73% of data falls inside the ±15% scatter band for four-point bending condition (see Table 3).

Figure 6.

4.3 Joint GLM application

The GLM is here applied by considering the results obtained from the specimens subjected to both tension and three-point bending as a unique set of data, as is described in Section 2.2. The Weibull parameters, computed for each Generalised Parameter, are listed in Table 3.

For each loading condition, the numerical probability of failure (computed through Eq. (8)) and the experimental failure probability values (computed by using Eq. (2)) are plotted in Figure 7 by using the von Mises failure criterion (i.e. $G_{VM}$ as the GP). As can be noted, the PCDFs (continuous curves) are in a quite satisfactory agreement with the experimental probability values (represented by symbols) since more than 50% of the experimental data falls within the ±5% scatter band (dashed lines) for all the loading combinations. Moreover, all data are in the ±10% scatter band (dash-dot lines) for tension and three-point bending conditions, whereas 91% of data falls inside the ±15% scatter band for four-point bending condition (see Table 3).

Figure 7.

For each loading condition, the numerical probability of failure and the experimental failure probability values are plotted in
Figure 8 by using the Tsai-Hill failure criterion (i.e. $GP_{TH}$ as the GP). In the case of tension and three-point bending conditions, all the experimental data fall within the $\pm 5\%$ scatter band, whereas 91% of data falls inside the $\pm 15\%$ scatter band for four-point bending condition (see Table 3).

Figure 8.

5. CONCLUSIONS

In the present paper, the Generalised Local Model (GLM) has been applied in order to analyse the failure behaviour of a short fibre-reinforced material represented by a commercial Fibreglass composite. The novelty of the paper is that the most suitable failure Generalised Parameter (GP) to determine the GPF of a commercial Fibreglass is identified among four possible Generalised Parameters related to the corresponding failure criteria. For each GP adopted, the Primary Cumulative Distribution Function of material Failure (PCDFF) has been derived through both a GLM single application and a GLM joint application.

The main conclusions of this research work are here summarised:

(a) The PCDFFs based on the Maximum Stress Generalised Parameter provide the best results, particularly in the case of single application for which all the experimental failure probability values fall within the $\pm 10\%$ scatter band for all loading conditions. Moreover, by considering such a parameter, the
percentage of experimental data lying inside the ±5% scatter band is higher than 50%, for all the examined combinations.

(b) The Generalised Parameter related to the Hoffman criterion provides the worst results for the four-point bending condition, by analysing both single and joint GPL applications.

(c) In general, the PCDFFs obtained through both single and joint applications are conservative and in a quite satisfactory agreement with the experimental data.

In conclusion, the present paper has proved that the GLM is a promising tool to assess the cumulative failure probability of structural components made of short-fibre reinforced materials, although more complex load configurations need to be processed in order to devise a robust procedure suitable for practical component design.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support provided by the Italian Ministry for University and Technological and Scientific Research (MIUR), Research Grant PRIN 2015 No. 2015JW9NJT on “Advanced mechanical modeling of new materials and structures for the solution of 2020 Horizon challenges”.
REFERENCES

[1] Naskar S, Mukhopadhyay T, Sriramula S, Adhikari S. Stochastic natural frequency analysis of damaged thin-walled laminated composite beams with uncertainty in micromechanical properties. Composite Structures 2017; 160: 312-334.

[2] Sriramula S, Chryssanthopoulos MK. Quantification of uncertainty modelling in stochastic analysis of FRP composites. Composites Part A: Applied Science and Manufacturing 2009; 40(11): 1673-1684.

[3] Agarwal H, Renaud JE, Preston EL, Padmanabhan D. Uncertainty quantification using evidence theory in multidisciplinary design optimization. Reliability Engineering and System Safety 2004; 85(1-3): 281-294.

[4] Ngah MF, Young A. Application of the spectral stochastic finite element method for performance prediction of composite structures. Composite Structures 2007; 78(3): 447-456.

[5] Wisnom MR. Size effects in the testing of fibre-composite materials. Composites Science and Technology 1999; 59(13): 1937-1957.

[6] Shiao MC, Chamis CC. Probabilistic evaluation of fuselage-type composite structures. Probabilistic Engineering Mechanics 1999; 14(1-2): 179-187.

[7] Chamis CC, Probabilistic simulation of multi-scale composite behavior. Theoretical and Applied Fracture Mechanics 2004; 41(1-3): 51-61.

[8] Mustafa G, Crawford C, Suleman A. Fatigue life prediction of laminated composites using a multi-scale M-LaF and Bayesian inference. Composite Structures 2016; 151: 149-161.

[9] Shaw A, Sriramula S, Gosling PD, Chryssanthopoulos MK. A critical reliability evaluation of fibre reinforced composite materials based on probabilistic micro and macro-mechanical analysis. Composites Part B: Engineering 2010; 41(6): 446-453.

[10] Pindera MJ, Khatam H, Drago AS, Bansal Y. Micromechanics of spatially uniform heterogeneous media: A critical review and emerging approaches. Composites Part B: Engineering 2009; 40(5): 349-378.

[11] Akmar ABI, Kramer O, Rabczuk T. Probabilistic multi-scale optimization of hybrid laminated composites. Composite Structures 2018; 184: 1111-1125.

[12] Jeong HK, Shenoi RA. Probabilistic strength analysis of rectangular FRP plates using Monte Carlo simulation. Computers and Structures 2000; 76(1): 219-235.

[13] Carbillet S, Richard F, Boubakar L. Reliability indicator for layered composites with strongly non-linear behavior. Composites Science and Technology 2009; 69(1): 81-87.

[14] Nader JW, Dagher HJ, Lopez-Anido R, El Chiti F, Fayad GN, Thomson L. Probabilistic finite element analysis of modified ASTM D3039 tension test for marine grade polymer matrix composites. Journal of Reinforced Plastics and Composites 2008; 27(6): 583-597.

[15] Carpinteri A, Fernández-Canteli A, Fortese G, Muñiz-Calvente M, Ronchey C, Scorza D, Vantadori S. Probabilistic failure
assessment of Fibreglass composites. Composite Structures 2017; 160: 1163-1170.

[16] Philippidis TP, Lekou DJ, Aggelis DG. Mechanical property distribution of CFRP filament wound composites. Composite Structures 1999; 45(1): 41-50.

[17] Sriramula S, Chryssanthopoulos MK. Probabilistic models for spatially varying mechanical properties of in-service GFRP cladding panels. Journal of Composites for Construction 2009; 13(2): 159-167.

[18] Lekou DJ, Philippidis TP. Mechanical property variability in FRP laminates and its effect on failure prediction. Composites Part B: Engineering 2008; 39(7-8): 1247-1256.

[19] Swolfs Y, Verpoest I, Gorbatikh L. A review of input data and modelling assumptions in longitudinal strength models for unidirectional fibre-reinforced composites. Composite Structures 2016; 150: 153-172.

[20] Bansal M, Singh IV, Mishra BK, Sharma K, Khan IA. A two-scale stochastic framework for predicting failure strength probability of heterogeneous materials. Composite Structures 2017; 179: 294-325.

[21] Nader JW, Dagher HJ, El Chiti F, Lopez-Anido R. Probabilistic finite element analysis of ASTM D6641 compression test for marine grade polymer matrix composites. Journal of Reinforced Plastics and Composites 2009; 28(8): 897-911.

[22] Algam M, Bennett RM, Zureick AH. Three-parameter vs. two-parameter Weibull distribution for pultruded composite material properties. Composite Structures 2002; 58(4): 497-503.

[23] Barbero E, Fernández-Sáez J, Navarro C. Statistical analysis of the mechanical properties of composite materials. Composites Part B: Engineering 2000; 31(5): 375-381.

[24] Lan C, Wu J, Bai N, Qiang D, Li H. Size effect on tensile strength of parallel CFRP wire stay cable. Composite Structures 2017; 181: 96-111.

[25] Sentler L. Reliability of high performance fibre composites. Reliability Engineering and System Safety 1997; 56(3): 249-256.

[26] Paramonov Y, Cimanis V, Varickis S, Kleinhofs M. Modeling the Residual Strength of a Fibrous Composite Using the Residual Daniels Function. Mechanics of Composite Materials 2016; 52(4): 497-506.

[27] Wu WF, Cheng HC, Kang CK. Random field formulation of composite laminates. Composite Structures 2000; 49(1): 87-93.

[28] Paramonov Y, Andersons J. Analysis of the fiber length dependence of its strength by using the weakest-link approach 1. A family of weakest-link distribution functions. Mechanics of Composite Materials 2008; 44(5): 479-486.

[29] Wu W, Cheng H, Kang C. Random field formulation of composite laminates. Composite Structures 2000; 49(1): 87-93.

[30] Frangopol DM, Recek S. Reliability of fiber-reinforced composite laminate plates. Probabilistic Engineering Mechanics 2003; 18(2):119-137.

[31] Karsh PK, Mukhopadhyay T, Dey S. Spatial vulnerability analysis for the first ply failure strength of composite laminates
including effect of delamination. Composite Structures 2018; 184: 554-567.

[32] Muñiz-Calvente M, PhD. Thesis: Generalized Local Model, University of Oviedo. 2017.

[33] Muniz-Calvente M, Ramos A, Shlyannikov V, Lamela MJ, Fernández-Canteli A. Hazard maps and global probability as a way to transfer standard fracture results to reliable design of real components. Engineering Failure Analysis 2016. 69:135-146.

[34] Weibull W. The phenomenon of rupture in solids. Ing. Vetenskaps Akad. Handlinger, 1939

[35] Bernard A, Bos-Levenbach EC. The Plotting of Observations on Probability-paper. Stichting Mathematisch Centrum Statistische Afdeling., 1955

[36] Papazafeiropoulos G, Muñiz-Calvente M, Martínez-Pañeda E. Abaqus2Matlab: A suitable tool for finite element post-processing. Advances in Engineering Software 2017. 105: 9-16.

[37] Muniz-Calvente M, Ramos A, Pelayo F, Lamela MJ, Fernández-Canteli A. Statistical joint evaluation of fracture results from distinct experimental programs: An application to annealed glass. Theoretical and Applied Fracture Mechanics 2016. 85:149-157.

[38] Dowling N. Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture, and Fatigue. Fourth Edition. Pearson Ed. Edinburgh, UK, 2013.

[39] Tsai SW. Strength characteristics of composite materials, NASA CR-224, 1945.

[40] Hoffman O. The brittle strength of orthotropic materials. J. Composite Materials 1967; 1: 200-206.

[41] Hill R. The mathematical Theory of Plasticity, Oxford University Press, London, 1950

[42] Daniel IM. Composite materials: Testing and design. 6th Conference. ASTM STP 787, 1982.

[43] ASTM D 6272-10. Standard Test Method for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials by Four-Point Bending. West Conshohocken, PA: ASTM International, 2010.

[44] ASTM D 3039/D 3039M-14. Standard Test Method for Tensile Properties of Polymer Matrix Composite Materials. West Conshohocken, PA: ASTM International, 2014.

[45] ASTM D 790-15e2. Standard Test Methods for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials. West Conshohocken, PA: ASTM International, 2015.

[46] Straus7 (2004), Theoretical manual: theoretical background for Straus7 finite element analysis systems, Sydney, G+D Computing.
Submitted to COMPOSITE STRUCTURES

Special Issue Dedicated to the
Memory of the Founding Editor, Professor Ian Marshall
February 2018

The generalised local model applied to Fibreglass

Sabrina Vantadori\textsuperscript{a}, Miguel Muñiz-Calvente\textsuperscript{b}, Daniela Scorza\textsuperscript{a}, Alfonso Fernández-Canteli\textsuperscript{b}, Adrián Álvarez Vázquez\textsuperscript{b}, Andrea Carpinteri\textsuperscript{a}

\textsuperscript{a}DICATeA, University of Parma, Parma, Italy
\textsuperscript{b}Department of Construction and Manufacturing Engineering, University of Oviedo, Gijón, Spain

Corresponding Author: Sabrina Vantadori Sabrina.vantadori@unipr.it
Figure 1. Flowchart of the Generalised Local Model.
Table 1: Parameters related to the Tsai-Hill criterion [39] and the Hoffman criterion [40].

| PARAMETER | Tsai-Hill [39] | Hoffman [40] |
|-----------|----------------|--------------|
|           | $1/\text{MPa}^2$ | $1/\text{MPa}^2$ |
| $F_1$     | 0              | $\frac{1}{\sigma_{T_1}^u} - \frac{1}{\sigma_{C_1}^u}$ |
| $F_2$     | 0              | $\frac{1}{\sigma_{T_2}^u} - \frac{1}{\sigma_{C_2}^u}$ |
| $F_3$     | 0              | $\frac{1}{\sigma_{T_3}^u} - \frac{1}{\sigma_{C_3}^u}$ |
| $F_{11}$  | $\frac{1}{(\sigma_{T_1}^u)^2}$ | $\frac{1}{\sigma_{T_1}^u \cdot \sigma_{C_1}^u}$ |
| $F_{22}$  | $\frac{1}{(\sigma_{T_2}^u)^2}$ | $\frac{1}{\sigma_{T_2}^u \cdot \sigma_{C_2}^u}$ |
| $F_{33}$  | $\frac{1}{(\sigma_{T_3}^u)^2}$ | $\frac{1}{\sigma_{T_3}^u \cdot \sigma_{C_3}^u}$ |
| $F_{44}$  | $\frac{1}{(S_{23}^u)^2}$ | $\frac{1}{(S_{23}^u)^2}$ |
| $F_{55}$  | $\frac{1}{(S_{13}^u)^2}$ | $\frac{1}{(S_{13}^u)^2}$ |
| $F_{66}$  | $\frac{1}{(S_{12}^u)^2}$ | $\frac{1}{(S_{12}^u)^2}$ |
| $F_{12}$  | $-\frac{1}{2} \left[ \frac{1}{(\sigma_{T_1}^u)^2} + \frac{1}{(\sigma_{T_2}^u)^2} - \frac{1}{(\sigma_{T_3}^u)^2} \right]$ | $-\frac{1}{2} \left[ \frac{1}{\sigma_{T_1}^u \cdot \sigma_{C_1}^u} + \frac{1}{\sigma_{T_2}^u \cdot \sigma_{C_2}^u} - \frac{1}{\sigma_{T_3}^u \cdot \sigma_{C_3}^u} \right]$ |
| $F_{13}$  | $-\frac{1}{2} \left[ \frac{1}{(\sigma_{T_3}^u)^2} + \frac{1}{(\sigma_{T_1}^u)^2} - \frac{1}{(\sigma_{T_2}^u)^2} \right]$ | $-\frac{1}{2} \left[ \frac{1}{\sigma_{T_3}^u \cdot \sigma_{C_3}^u} + \frac{1}{\sigma_{T_1}^u \cdot \sigma_{C_1}^u} - \frac{1}{\sigma_{T_2}^u \cdot \sigma_{C_2}^u} \right]$ |
| $F_{23}$  | $-\frac{1}{2} \left[ \frac{1}{(\sigma_{T_2}^u)^2} + \frac{1}{(\sigma_{T_3}^u)^2} - \frac{1}{(\sigma_{T_1}^u)^2} \right]$ | $-\frac{1}{2} \left[ \frac{1}{\sigma_{T_2}^u \cdot \sigma_{C_2}^u} + \frac{1}{\sigma_{T_3}^u \cdot \sigma_{C_3}^u} - \frac{1}{\sigma_{T_1}^u \cdot \sigma_{C_1}^u} \right]$ |
**Figure 2.** Experimental testing set-up for three point bending tests.

**Table 2.** Experimental results in terms of mean values (m.v.) and standard deviations (s.d.) for each testing configuration examined.

| LOADING       | $P$ [N] | $\sigma_f$ [MPa] | $\varepsilon_f$ [-] | $E$ [GPa] |
|---------------|---------|------------------|---------------------|-----------|
|               | m.v.    | s.d.             | m.v.               | s.d.      | m.v.     | s.d.     |
| TENSION       | 6276.32 | 417.41           | 86.60               | 5.85      | 0.014    | 0.002    | 7.64     | 0.96     |
| 3P BENDING    | 63.23   | 6.33             | 145.90              | 9.81      | 0.034    | 0.003    | 6.16     | 0.41     |
| 4P BENDING    | 87.72   | 11.30            | 137.90              | 12.82     | 0.033    | 0.002    | 6.32     | 0.50     |
Table 3. Weibull parameters obtained from both single and joint application of GPA, and percentage of data falling in scatter bands equal to ± 5%, ± 10% and ± 15%, for the different GPs and loading conditions examined.

| GP  | PCDIFF | WEIBULL PARAMETERS | LOADING | % DATA IN SCATTER BAND |
|-----|--------|-------------------|---------|------------------------|
|     |        | \( \beta = 10.16 \) | TENSION | 96 100 100 |
| TENSION |        | \( \delta = 97.91 \) | 3PB     | 94 100 100 |
|        |        | \( \lambda = 34.28 \) | 4PB     | 51 100 100 |
| \( GP_{MS} \) | | | | |
| TENSION + 3PB | | \( \beta = 8.83 \) | TENSION | 100 100 100 |
|        |        | \( \delta = 87.15 \) | 3PB     | 94 100 100 |
|        |        | \( \lambda = 42.13 \) | 4PB     | 73 82 94 |
| \( GP_{VM} \) | | | | |
| TENSION + 3PB | | \( \beta = 10.25 \) | TENSION | 100 100 100 |
|        |        | \( \delta = 92.63 \) | 3PB     | 100 100 100 |
|        |        | \( \lambda = 34.28 \) | 4PB     | 52 73 91 |
| \( GP_{TH} \) | | | | |
| TENSION + 3PB | | \( \beta = 8.59 \) | TENSION | 96 100 100 |
|        |        | \( \delta = 102.33 \) | 3PB     | 87 100 100 |
|        |        | \( \lambda = 34.28 \) | 4PB     | 30 73 85 |
| \( GP_{H} \) | | | | |
| TENSION + 3PB | | \( \beta = 8.08 \) | TENSION | 65 96 100 |
|        |        | \( \delta = 107.10 \) | 3PB     | 26 100 100 |
|        |        | \( \lambda = 34.28 \) | 4PB     | - 7 88 |
Figure 3. Comparison between global probability of failure (Eq. (6)) computed by using $GP_{MS}$ and experimental failure probability values (Eq. (2)) for: (a) tension; (b) three-point bending; (c) four-point bending.
Figure 4. Comparison between global probability of failure (Eq. (6)) computed by using $GP_{VM}$ and experimental failure probability values (Eq. (2)) for: (a) tension; (b) three-point bending; (c) four-point bending.
Figure 5. Comparison between global probability of failure (Eq. (6)) computed by using $GP_{th}$ and experimental failure probability values (Eq. (2)) for: (a) tension; (b) three-point bending; (c) four-point bending.
Figure 6. Comparison between global probability of failure (Eq. (6)) computed by using $GP_H$ and experimental failure probability values (Eq. (2)) for: (a) tension; (b) three-point bending; (c) four-point bending.
Figure 7. Comparison between global probability of failure (Eq. (8)) computed by using \( GP_{VM} \) and experimental failure probability values (Eq. (2)) for: (a) tension; (b) three-point bending; (c) four-point bending.
Figure 8. Comparison between global probability of failure (Eq. (8)) computed by using $GP_{TH}$ and experimental failure probability values (Eq. (2)) for: (a) tension; (b) three-point bending; (c) four-point bending.