Multi-Threshold Image Segmentation Based on Improved Grey Wolf Optimization Algorithm

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Abstract. Threshold selection is the key to image threshold segmentation. The threshold determines the good or bad of the image segmentation results. As the number of thresholds increases, the computational process of image segmentation becomes more and more complicated. In order to select a better threshold for image segmentation, an improved Grey Wolf Optimization (IGWO) algorithm is proposed in this paper. The good point set method is applied to the GWO to generate the initial population, the weight is introduced in the position update of gray wolf hunting process, and the algorithm is used to solve the global optimization problem with the Kapur partition function as the objective function. The IGWO algorithm has good global convergence and computational robustness, can effectively avoid falling into local optimum, and is especially suitable for solving high-dimensional and multi-peak complex function problems, and can be well integrated into the image segmentation process. The results of theoretical analysis and simulation experiments show that compared with the image segmentation results of particle swarm optimization (PSO), the algorithm has better segmentation effect when multiple segmentation thresholds are selected. The segmentation efficiency, the threshold range obtained by optimization is more stable, and the segmentation quality is higher.

1. Introduction
The use of thresholds for image segmentation is one of the most commonly used methods in today's scientific research. So far, many methods for selecting thresholds and algorithms have been proposed. For example, Ostu et al. [1] have studied the largest inter-class variance method; Kapur et al. [2] have developed the maximum entropy threshold segmentation method; Dunn et al. [3] The threshold selection method for homogenization error is studied; Sahoo et al. [4] have proposed a threshold segmentation method based on Renyi entropy. Among them, Kapur et al.’s maximum entropy threshold selection method does not need to have a wealth of prior knowledge, but it can also achieve satisfactory results in segmentation of non-ideal bimodal histograms. However, it also has drawbacks, and when multiple thresholds are selected, the amount of calculation is greatly increased. This is a problem of optimization. The key to the problem is how to determine the optimal threshold quickly and accurately. This is an NP-hard problem.
With the rise of intelligent algorithms, scholars applied intelligent optimization algorithms to image segmentation and they achieved steady progress. The genetic algorithm (GA) inspired by biological evolution [5] has been widely used in threshold selection tasks in image segmentation; Li Kangshun et al. have improved the genetic algorithm (IGA) [6] and applied it to image segmentation. This segmentation method can adaptively exchange genetic control parameters based on individual fitness and population dispersion, which can accelerate the convergence speed; Evolutionary algorithms derived from group foraging behavior such as particle swarm optimization (PSO) [7] and the artificial bee colony algorithm (ABC) [8] is also in a frequently used state in the field of image segmentation. They choose the Kapur’s entropy function as the objective function for calculating the fitness value to search for the threshold for obtaining the optimal segmentation effect; Seyedali et al. [9] proposed the Grey Wolf algorithm (GWO) in the light of the leadership level and hunting mechanism of the gray wolf group in nature. The convergence of algorithm is better than the particle swarm optimization algorithm (PSO) and gravitation search algorithm (GSA).

As a new type of swarm intelligence optimization method, the grey wolf optimization algorithm has received extensive attention since its introduction, but it has the disadvantages of slow convergence speed and easy to fall into local optimum. In order to overcome its shortcomings, this paper proposes a weight-based improved grey wolf optimization algorithm for multi-level threshold image segmentation. In the improved algorithm, the good point set method is first applied to the GWO algorithm to generate and discrete the initial position of population [10], then the weight is added during the hunting position process, and finally the segmentation experiments are performed on several standard images. The results show that compared with the PSO intelligent optimization algorithm, the improved gray wolf optimization algorithm has stronger optimization performance and robustness, and the image segmentation performance is better.

2. Grey Wolf Optimization

2.1. Original GWO

The original GWO algorithm is an intelligent optimization algorithm based on simulating the gray hierarchy of the gray wolf and simulating its efficient collective hunting behavior. The gray wolves are divided into four categories according to their social status from high to low: \( \alpha \), \( \beta \), \( \delta \), \( \omega \).

In the GWO algorithm, the hunt behavior performed by \( \alpha \), \( \beta \) and \( \delta \), the first three are followed by \( \omega \) to track the cofferdam of prey, and finally the predation task is completed. When the GWO algorithm is used to solve the continuous function optimization problem, it is assumed that the number of gray wolves in the gray wolf population is \( N \) and the search space dimension is \( d \). The position of the i-th gray wolf in the d-dimensional space can be expressed as \( X_i = (x_{i1}, x_{i2}, \cdots, x_{id}) \), the current optimal individual in the population recorded as \( \alpha \), and the corresponding individuals ranked second and third in the fitness value are recorded as \( \beta \) and \( \delta \), and the remaining individuals are recorded as \( \omega \). The position of the prey corresponds to the global optimal solution of the optimization problem. The optimization process of the GWO algorithm is: Generate a group of gray wolves in the search space randomly. During the evolution process, the position of the prey (global optimal solution) is evaluated and positioned by \( \alpha \), \( \beta \), \( \delta \) and the rest of the group calculates the self and the standard. The distance between the prey and the complete approach to the prey, encirclement, attack, etc., and finally capture the prey.

The three definitions in the algorithm are given below.

Definition 1 The distance between the grey wolf and the prey. During the predation process, the grey wolf first needs to surround the prey. In the GWO algorithm, it need to determine the distance between the individual and the prey:

\[
\overline{D} = \| \overline{C} \cdot \overline{X}_p(t) - \overline{X}(t) \| \quad (1)
\]
Where: $X_p(t)$ indicates the position of the prey at the t-th generation; $X(t)$ indicates the position of the gray wolf individual at the t-th generation; the constant $C$ is the oscillating factor, which is determined by

$$
C = 2r_1
$$

Where: $r_1$ is a random number between $[0, 2]$.

Definition 2 The Location Update of Gray Wolf:

$$
\bar{X}(t+1) = \bar{X}_p(t) - \bar{A}D
$$

$$
\bar{A} = 2\alpha \cdot r_2 - \alpha
$$

Where: $A$ is the convergence factor; $r_2$ is a random number of $[0, 1]$; $\alpha$ is linearly decreasing from 2 to 0 as the number of iterations increases.

Definition 3 Prey position location.

When the gray wolf judges the position of the prey, it will be led by the head wolf $\alpha$ to pursue the pursuit. In the wolves, the $\alpha$, $\beta$, $\delta$ are closest to the prey, and the position of the three can be used to determine the position of the prey. The mathematical description of the individual's tracking prey orientation in the wolves is as follows:

$$
\bar{D}_\alpha = |\bar{C}_1 \cdot \bar{X}_\alpha(t) - \bar{X}(t)|, \bar{D}_\beta = |\bar{C}_2 \cdot \bar{X}_\beta(t) - \bar{X}(t)|, \bar{D}_\delta = |\bar{C}_3 \cdot \bar{X}_\delta(t) - \bar{X}(t)|
$$

$$
\bar{X}_1 = \bar{X}_\alpha - \bar{A}_1 \cdot \bar{D}_\alpha, \bar{X}_2 = \bar{X}_\beta - \bar{A}_2 \cdot \bar{D}_\beta, \bar{X}_3 = \bar{X}_\delta - \bar{A}_3 \bar{D}_\delta
$$

$$
\bar{X}_p(t+1) = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{3}
$$

2.2. Improvement of Weighted Grey Wolf Algorithm

The scale of the wolves is assumed to be N. In the initialization phase of the improved discrete gray wolf algorithm, N feasible solutions (the position of the wolf is the threshold). Before the iteration, the initial population of the basic GWO algorithm is randomly generated, and it is difficult to guarantee the diversity of the initial population, which affects the search efficiency of the algorithm to a certain extent. The good point set is an effective and capable point selection. Compared with the random method, the point collection method can be more evenly distributed in the search space [11]. At present, the good point set method has been successfully applied in the group intelligent optimization algorithms such as GA, PSO, DE, and ABC. Therefore, this paper applies the good point set method to the GWO algorithm to generate the initial population $(1, 2, \ldots, N)$. Since the image threshold is a set of discrete quantities, all integers, it is done by rounding to zero. The specific principle of using the good point set to generate the initial population is detailed in the literature [12].

After the feasible solution is initialized, first judge whether it is in $[lb, ub]$, if it is, then calculate its fitness value $Fitness$; otherwise, use the formula (8) to pull the search agent back into the search space, and then round to zero to calculate the fitness value.

$$
X_i = \left( (X_i \cdot (u+l)) + ub \cdot u + lb \cdot l \right)
$$
Among them, $u$ and $l$ are the upper and lower limits of the parameters respectively. $u = X_i > ub, l = X_i < lb$

The fitness value calculation formula (9), among the fitness values of all feasible solutions, select the optimal three feasible solution assignment values, and record all fitness values and feasible solutions.

$$
\text{Fitness}(x_i) = \frac{1}{1 + |f(x_i)|} \begin{cases} 0 & \text{if } f(x_i) \leq 0 \\ 1 & \text{if } f(x_i) > 0 \end{cases}
$$

(9)

Among them, $f(x_i)$ is obtained by the formula (12).

In order to make the algorithm approach the optimal solution more quickly, this paper will improve the role of the current optimal solution in location update by weighting.

$$
\bar{X}(t+1) = w_1 \cdot \bar{X}_1 + w_2 \cdot \bar{X}_2 + w_3 \cdot \bar{X}_3
$$

(10)

In equation (14), $w_1, w_2, w_3$ are the corresponding weights, which are calculated by formula (11).

$$
w_1 = \frac{f_1}{F}, w_2 = \frac{f_2}{F}, w_3 = \frac{f_3}{F}
$$

(11)

Among them: $f_1, f_2, f_3$ are the corresponding fitness value, which are calculated by the formula: $F = f_1 + f_2 + f_3$.

3. Kapur’s entropy

Kapur’s entropy is one of the early methods applied to single-threshold image segmentation (binary image segmentation), which has been extended to the field of multi-threshold segmentation by many scholars. This method is based on the entropy threshold conversion method, a more effective image segmentation technique combined with the probability distribution of image histograms. Entropy gets the maximum when the optimal threshold is correctly selected and assigned. The image can be divided into $k$ classes with $K-1$ thresholds. Therefore, the objective function of a multi-level threshold is defined as:

$$
f(TH) = \sum_{i=1}^{k} H_i^\lambda, \quad \lambda = \{1, 2, 3\} \quad \text{if RGB}\quad \lambda = \{1, 2\} \quad \text{if Gray}
$$

(12)

In the Formula, $TH = [th1, th2, \ldots, th_{k-1}]$ is a vector containing multiple thresholds, $i$ is a specific gray level, with a range of $[0, L-1]$. $\lambda$ is a parameter of depending on whether the image is a grayscale image or a RGB image.

$$
H_1^\lambda = \sum_{i=1}^{th1} \frac{P_{h_i}}{\alpha_i^\lambda} \ln \left( \frac{P_{h_i}}{\alpha_i^\lambda} \right), \quad H_2^\lambda = \sum_{i=th1+1}^{th2} \frac{P_{h_i}}{\alpha_i^\lambda} \ln \left( \frac{P_{h_i}}{\alpha_i^\lambda} \right), \ldots, H_k^\lambda = \sum_{i=th_{k-1}+1}^{L} \frac{P_{h_i}}{\alpha_i^\lambda} \ln \left( \frac{P_{h_i}}{\alpha_i^\lambda} \right)
$$

(13)

Among them, $h_i^\lambda$ (Histogram) is the pixel number of gray level $i$, $P_{h_i}$ is the probability distribution of histogram normalization. The probability values $\{\alpha_0^\lambda, \alpha_1^\lambda, \ldots, \alpha_{k-1}^\lambda\}$ generated by $k$ class are obtained from the following formula (14). Finally, we need to use (15) formula to divide the pixels into their respective classes, so as to achieve multi-threshold image segmentation.

$$
\alpha_0^\lambda(th) = \sum_{i=1}^{th1} P_{h_i}, \alpha_1^\lambda(th) = \sum_{i=th1+1}^{th2} P_{h_i}, \ldots, \alpha_{k-1}^\lambda(th) = \sum_{i=th_{k-1}+1}^{L} P_{h_i}
$$

(14)
In a summary, the multi-threshold segmentation problem is formulated to maximize Kapur’s entropy. Its objective function, such as formula (12), will be optimized by using the improved grey wolf algorithm. Finally, we need to use the formula (15) to separate pixels into their classes. Threshold transformation is a process that divides pixels of an image into classes or classes according to L. This classification must select a threshold (th) or follow the following multilevel threshold conversion formula criteria:

\[ C_i \leftarrow p \text{ if } 0 \leq p < \text{th}_1, \ldots, C_i \leftarrow p \text{ if } \text{th}_k \leq p < \text{th}_{k+1}, \ldots, C_n \leftarrow p \text{ if } \text{th}_n \leq p < L - 1 \] (15)

Among them, L is the gray level, the value is 256, P is one of the pixels \((m \times n)\) in the gray image \(I_{Gr}\), it represents the level in the L gray level, \(C_i\) belongs to the class of the pixels \(p\); \(\text{th}_1, \text{th}_2, \ldots, \text{th}_k\) are the different threshold.

Multilevel threshold switching problem is the threshold of th choose the right recognition class. Kapur’s entropy is known as a method for determining these thresholds. The objective function proposed by this method must be maximized in order to find the best threshold. In this paper, we use the formula (16) to obtain the K dimension optimal threshold and maximize the Kapur’s entropy objective function.

\[ th = \arg \max(\sum_{j=0}^{k-1} \omega_j^i (\text{th})) \] (16)

4. Image segmentation steps of IGWO algorithm

Step1: Read the image for judgment, if it is RGB image, it will be separated into three channels \(I_r, I_g, I_b\); if it is gray image, it will be stored in \(I_{Gr}\).

Step2: Get the histogram of the image. For grayscale images, it will be remarked as \(H_{Gr}\); for RGB images, it will be remarked as \(H_r, H_g, H_b\) respectively.

Step3: Calculate the probability distribution and histogram of each gray value according to formula (3).

Step4: Initialize GWO’s population number N and parameter E, A, C and MaxIter.

Step5: Use good point set to initialize and discrete location of grey wolf population.

Step6: Calculate the fitness value of each search agent by using the Kapur’s entropy function as the fitness function.

Step7: According to the size of the fitness value, assign the position of the best search agent to the wolf \(\alpha\), assign the position of the second good search agent to the wolf \(\beta\), and assign the position of the third good search agent to the wolf \(\delta\).

Step8: Calculate \(A_1, A_2, A_3, C_1, C_2, C_3, D_1, D_2, D_3\) and \(X_1, X_2, X_3\) according to formula (5) ~ (6), respectively, and then update the location of the current search agent according to formula (10).

Step9: Increase the circular pointer Q by 1; if \(Q > \text{Max_iter}\) or satisfy the stopping condition of the algorithm, complete the iteration process and jump to step 10; otherwise, jump to step 6.

Step10: Take the position of the wolf with the best objective function value as the optimal segmentation threshold.

Step11: Output the optimal segmentation threshold and the image before and after segmentation.

5. Experiment

5.1. Experimental parameter setting

In order to analyze the algorithm, two standard gray images are used to test the DGWO algorithm. IGWO and PSO algorithm are compared. To avoid the randomness of the results, appropriate statistical measures are used to compare the effectiveness of these algorithms. The stopping criteria for each
In order to verify the stability, the standard deviation (STD) is calculated according to formula (17) at the end of each test.

\[
STD = \sqrt{\frac{\sum_{i=1}^{\text{MaxIter}} (\theta - \epsilon)^2}{\text{MaxIter}}}
\]  

(17)

In addition, peak signal-to-noise ratio (PSNR) compares the similarity between the segmented image and the reference image based on the mean square error (MSE) of each pixel. The higher the PNSR value, the better the segmentation result and the optimized performance.

\[
PSNR = 20 \log_{10} \left( \frac{255}{\text{MSE}} \right), (dB)
\]  

(18)

\[
\text{MSE} = \frac{\sum_{i=1}^{m-1} \sum_{j=1}^{n-1} [I(i, j) - K(i, j)]^2}{m \times n}
\]  

(19)

Among them, m and N represent the size of the image, MSE represents the mean square error of each pixel of the original image and the segmented image, and I and K represent the original image and the segmented image respectively.

5.2. Experimental results and analysis

![Figure 1. Hunter image, corresponding gray histogram and results segmented by IGWO algorithm](image)

![Figure 2. Baboon image, corresponding gray histogram and results segmented by IGWO algorithm](image)
Table 1. Maximum entropy, corresponding threshold combination and running time of two images

| Image | n/Iteration | Optimal thresholds | Max-e | Exh (s) | PSO (s) | IGWO (s) |
|-------|-------------|---------------------|-------|--------|--------|---------|
| Hunter | 3/70        | 86,129,179          | 23.866| 14.045 | 0.378  | 0.194   |
|       | 4/104       | 76,114,149,183      | 25.523| 690.455| 0.612  | 0.294   |
|       | 5/142       | 66,99,124,154,1     | 30.562| 28833.423| 0.853 | 0.422   |
| Baboon | 3/70        | 52,100,146          | 14.676| 12.832 | 0.382  | 0.185   |
|       | 4/104       | 44,79,113,152       | 17.276| 610.677| 0.602  | 0.281   |
|       | 5/142       | 43,74,104,134,1     | 19.883| 28789.245| 0.809 | 0.402   |

By comparison, it can be seen that the IGWO running time is significantly shortened compared to the exhaustive algorithm. For example, the 5 threshold segmentation of the Hunter image, the exhaustive algorithm requires 28833.423s, and the IGWO algorithm only takes 0.5002s. And as the number of thresholds increases, the effect becomes more apparent. It shows that the application of intelligent optimization algorithm can overcome the shortcomings of using traditional algorithms to solve the multi-threshold segmentation problem.

Table 2. Multi-threshold search results for two images

| Image | k | PSNR | PSO | MEAN | PSNR | IGWO |
|-------|---|------|-----|------|------|------|
|       |   |      | STD | MEAN | STD | MEAN |
| Hunter | 3 | 17.122 | 0.072 | 22.878 | 20.396 | 0.043 | 23.074 |
|       | 4 | 18.342 | 0.085 | 25.102 | 21.762 | 0.052 | 25.318 |
|       | 5 | 19.547 | 0.087 | 30.124 | 22.899 | 0.087 | 30.252 |
| Baboon | 3 | 15.008 | 0.082 | 21.088 | 18.632 | 0.035 | 21.107 |
|       | 4 | 17.574 | 0.085 | 24.375 | 20.480 | 0.055 | 24.927 |
|       | 5 | 20.224 | 0.087 | 30.994 | 22.070 | 0.063 | 30.436 |

According to the data in the table, the peak signal-to-noise ratio (PNSR) of IGWO algorithm is generally higher than that of PSO algorithm, which indicates that its segmentation effect is better. Moreover, with the increase of threshold series, the higher the PSNR is, the better the image segmentation effect is.

6. Conclusion
A Gray Wolf Optimization Algorithm based on weight improvement is proposed in this paper. The algorithm is applied to the field of multi threshold image segmentation, so as to achieve multi threshold image segmentation. The experimental results show that the image segmentation method based on this algorithm not only achieves better image segmentation quality, but also has obvious advantages in running time and convergence compared with PSO method.

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