High frequency dielectric function of metals taking into account Umklapp processes

M E Veysman\textsuperscript{1}, G Röpke\textsuperscript{2,3} and H Reinholz\textsuperscript{2,4}

\textsuperscript{1} Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya 13
Bldg 2, Moscow 125412, Russia
\textsuperscript{2} University of Rostock, Universitätstr. 3, Rostock 18051, Germany
\textsuperscript{3} National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe Shosse 31, Moscow 115409, Russia
\textsuperscript{4} University of Western Australia, Crawley WA 6009, Australia

E-mail: bme@ihed.ras.ru

Abstract. An analytical model for the high-frequency dielectric function of metals accounting for electron–phonon interactions and Umklapp processes is derived using a quantum statistical approach within a linear response theory. Different limiting cases for radiation frequencies and electron temperatures, which are either low or high in comparison to the Fermi energy, are studied. New asymptotic formulas for high radiation frequencies or high electron temperatures are obtained. The application for the description of optical properties in warm dense aluminum plasma is discussed.

1. Introduction

Warm dense matter (WDM) is created in experiments by irradiating of materials with powerful charged particle beams or laser pulses [1–8]. The electronic subsystem can reach temperatures \( T_e \) much above the ionic ones \( T_i \). Optical properties of WDM can be described by the dynamic dielectric function (DF) for the appropriate frequencies. Collective electron–phonon interactions should be included in the case of \( T_i \lesssim T_{\text{melt}} \), where \( T_{\text{melt}} \) is the melting temperature.

A wide-range DF model for WDM for such a case was proposed previously [9]. According to this model, the real part of the effective collision frequency of electrons with heavy particles or excitations \( \nu_{\text{eff}} \) should decrease with increasing electron temperature \( T_e \) if assuming a fixed ion temperature \( T_i \). On the other hand, experiments (see, e.g., [10]) show, that the absorption coefficient \( A \) of different materials irradiated by laser pulses of intensity \( I_L \) can either be increasing or decreasing with \( I_L \) depending on whether \( I_L \) is low or high, respectively, in comparison to \( I_* \sim 10^{14} \) W/cm\(^2\). Assuming that the laser intensity is directly determining the electron temperature, we expect for the absorption coefficient the same behaviour in dependence on \( T_e \) as on the laser intensity. This can be extended further to an assumption, that the effective collision frequency has a similar dependence on \( T_e \). One possibility to explain this will be given below in section 2.

This contradiction with our previous results [9] is because of the fact that only the normal processes, which conserve the quasi-momentum of electrons, were accounted for in [9], while it is known [11–16], that Umklapp processes, in which the quasi-momentum of electrons is conserved...
only when accounting for the wave vector of the reverse lattice, can make a contribution to the conductivity.

The goal of the present paper is to derive a full expression for the complex effective collision frequency of electrons [9, 17], and hence for the DF, accounting for both normal and Umklapp processes. Then we will study the respective limiting cases for high and low frequencies as well as high and low electron temperatures. For this, the ions are assumed to be at a fixed temperature \( T_i \lesssim T_{\text{melt}} \), when one can assume conservation of long-range order of atoms and the existence of forward and reverse lattices. We use the same approach as in [17] and [9], but with an additional Hubbard term [18] in the Hamiltonian accounting for Umklapp processes during electron–electron interaction. This actually accounts for the contribution to the DF due to electrons scattering on Bragg planes. A similar approach was used earlier in [19] to calculate the contribution of Umklapp processes to the conductivity in the case of relatively low laser frequencies.

It is also worth mentioning, that unlike classical kinetic approaches [20], the used quantum statistical (QS) and linear response theory (LRT) method provides a consistent description for the real and the imaginary part of the complex effective electron–ion collision frequency and gives rise to expressions fulfilling the Kramers–Kronig relations [17].

### 2. Basic equations

Using QS and LRT approach, see [9, 17], we express the DF \( \varepsilon \) in the long-wavelength limit (i.e. neglecting spatial dispersions) in terms of a complex effective \( \nu(\omega) \) via the generalized Drude formula [17, 21, 22]

\[
\varepsilon(\omega) = 1 - \frac{\omega_{\text{pl}}^2}{\omega[\omega + i\nu(\omega)]},
\]

(1)

where \( \omega_{\text{pl}} = \sqrt{4\pi ne^2/m} \), \( n, e, m \) are the electrons plasma frequency, particle density, charge and mass, respectively.

The DF \( \varepsilon \) and the effective collision frequency \( \nu \) are closely related to the absorption coefficient \( A \) of a matter. Particularly, in the case of a step-like profile of \( \varepsilon \), the absorption coefficient can be expressed as [23]

\[
A = 4 \text{Re}\{\zeta\}/|1 + \zeta|^2, \quad \zeta = 1/\sqrt{\varepsilon},
\]

(2)

where the expression for impedance \( \zeta \) in (2) is written with assumption of weak spacial dispersion. Equation (1) for \( \varepsilon \) can be rewritten as

\[
\varepsilon = 1 - \tilde{N}[1 - i\tilde{\nu}], \quad \tilde{N} = \frac{(\omega_{\text{pl}}/\omega)^2(1 - \nu''/\omega)}{(1 - \nu''/\omega)^2 + (\nu'/\omega)^2}, \quad \tilde{\nu} = \frac{\nu'/\omega}{1 - \nu''/\omega},
\]

(3)

where \( \nu' \) and \( \nu'' \) are the real and imaginary parts of the effective collision frequency \( \nu \), respectively. In the case \( \tilde{N} \gg 1 \), which is typical, e.g., for metals or dense overcritical plasmas, one can derive from (2) and (3), that

\[
A \approx C \frac{4b}{1 + b^2}, \quad b = \frac{\sqrt{1 + \tilde{\nu}^2} - 1}{\tilde{\nu}}, \quad C = \sqrt{\frac{\tilde{N}\tilde{\nu}}{2b}}.
\]

(4)

From (4) it follows, that

For \( \tilde{\nu} \gg 1 \)

\[
A \approx \frac{2\sqrt{2} |\nu'/\omega|^2/\tilde{\nu}}{\omega_{\text{pl}}/\omega} \approx 2\sqrt{2} \frac{\nu'/\omega}{\omega_{\text{pl}}};
\]

(5)

For \( \tilde{\nu} \ll 1 \)

\[
A \approx \frac{2\tilde{\nu}}{\omega_{\text{pl}}/\omega} \approx 2 \frac{\nu'}{\omega_{\text{pl}}},
\]

(6)
where the second expressions on the r.h. side of (5) and (6) are derived under the assumption \( \nu'' \ll \omega \).

From (5) and (6) one can infer, that for the considered case \( N \gg 1 \) and \( \nu'' \ll \omega \) the dependence of the absorption \( A \) on the electron temperature \( T_e \) resembles the dependence of the real part of the effective collision frequency, \( \nu' \), on \( T_e \), with exception for the case, when the electron density \( n \) is also dependent on \( T_e \) due to ionization and (or) expansion of a plasma.

For simplicity, we consider below a single-moment approach. In this case, the effective collision frequency is expressed via the dimensionless correlation function \( C_{11} \) of the first moment of the density operator \( \hat{n}_p \) as

\[
\nu(\omega) = \omega_{\text{au}} C_{11}(\omega), \quad C_{11}(\omega) = \frac{\langle \hat{P}_1; \hat{P}_1 \rangle_{\omega + i\eta}}{m n T_e \omega_{\text{au}}}, \quad \hat{P}_1 = \sum_p \hbar p \hat{n}_p, \quad (7)
\]

where \( \omega_{\text{au}} \) is the atomic unit of frequency, \( \eta \) is an infinitesimal small positive value, \( \hbar \omega_{\text{au}} = E_H = m e^2 / h^2 \approx 27.2 \text{ eV} \) is the Hartree energy, \( h \) is the Planck constant, and \( \hat{n}_p = \hat{a}_p^+ \hat{a}_p \), where \( \hat{a}_p^+ \) and \( \hat{a}_p \) are creation and annihilation operators of electrons with the wave vector \( p \). The correlation function of operators \( \hat{A}, \hat{B} \) is defined as Laplace transform of the Kubo scalar product:

\[
\langle \hat{A}; \hat{B} \rangle_z = \int_0^\infty dt e^{zt} \langle \hat{A}(t); \hat{B} \rangle, \quad \langle \hat{A}(t); \hat{B} \rangle = \int_0^1 \! d\tau \, \text{Tr} \left\{ \hat{A}(t - i\beta h\tau) \hat{B}^+ \hat{\rho}_0 \right\}, \quad (8)
\]

where \( \hat{A}(t) = e^{iHt/\hbar} \hat{A} e^{-iHt} \) is the Heisenberg representation of an operator, \( \hat{\rho}_0 \) is the equilibrium statistical operator,

\[
\hat{\rho}_0 = Z^{-1} \exp[-(\hat{H} - \mu \hat{n})/T_e], \quad Z = \text{Tr} \{ e^{-(\hat{H} - \mu \hat{n})/T_e} \},
\]

with \( \hat{n} = \sum_p \hat{n}_p \), the partition function \( Z \), the chemical potential \( \mu \) and the Hamiltonian \( \hat{H} \).

For the calculation of the correlation function \( C_{11} \), the following Hamiltonian is used:

\[
\hat{H} = \sum_{k,i,\sigma} E_{k,i} \hat{a}_{k,i,\sigma}^+ \hat{a}_{k,i,\sigma} + \sum_{q,\lambda} \hbar \omega_{q,\lambda} \hat{b}_{q,\lambda}^+ \hat{b}_{q,\lambda} + \sum_{q,\lambda,\sigma} g_k(q, i, i', \lambda) \hat{a}_{k+i,\sigma}^+ \hat{a}_{k+i',\sigma}^+ \hat{b}_{q,\lambda}^+ \hat{b}_{-q,\lambda} + \frac{U}{2n} \sum_{k,k',q,g,i,\sigma} \hat{a}_{k+q-g,i,\sigma}^+ \hat{a}_{k+q-g,i'-\sigma}^+ \hat{a}_{k'-i,\sigma} \hat{a}_{k'-i,\sigma}, \quad (9)
\]

where the first two terms on the r.h. side of (9) represent electron and phonon kinetic energies, respectively. The third term represents the electron–phonon interaction in the Fröhlich form [24]. The fourth term represents the electron–electron interaction with account for Umklapp processes in the Hubbard [18] form, where \( g \) is the wave vector of the reverse lattice. \( i, i' \) are electron band numbers, \( \lambda \) is the phonon mode number, \( \sigma \) is the spin quantum number, \( \hat{a}_{k,i,\sigma}^+ \) and \( \hat{b}_{q,\lambda}^+ \) are creation and annihilation operators of electrons and phonons, respectively, and \( E_{k,i} = \hbar^2 k^2 / (2m m_i) + E_{0,i} \)

and \( \omega_{q,\lambda} \) are the energy of electrons in \( i \)-th zone and the frequency of phonons in \( \lambda \)-th mode, respectively, \( m_i \) is the ratio of the electrons effective mass in the \( i \)-th band to the free electron mass. \( E_{0,i} \) is the energy of the bottom of the \( i \)-th band (for the conduction band it can be assumed to be zero) and \( g_k(q, i, i', \lambda) \) is the coefficient of the electron–phonon interaction. \( U \) describes the Coulomb interaction of electrons with opposite spin orientations.

Below we consider the case of electrons interacting with a single phonon mode containing longitudinal optical (LO) phonons with a frequency independent of the wave vector [24]. LO phonons have been considered for simplicity. One can expect, that the somewhat more difficult consideration of acoustical phonons gives similar results for ion temperatures greater than the
Debye one. This consideration can be done similarly as it was done, e.g., in [25] for the treatment of acoustical modes for the case of direct-current conductivity.

Also we disregard interband transitions between different bands \( n, i \) and consider only the contribution due to free–free transitions of electrons with effective mass \( m_i = m_e \) in the conduction band. Besides that, we disregard cross terms from contributions of different type of interactions (normal and Umklapp processes) in the 1st moment of the density operator. In this way, and using the same procedure as described in [9], we obtain in the 1st Born approximation:

\[
\mathbf{c}_{11} = \mathbf{c}_{e-ph} + \mathbf{c}_U,
\]

where

\[
\mathbf{c}_{e-ph} = \frac{i e^3}{2 \pi^2 \epsilon T} \frac{w_{LO}}{\omega_{LO}} \frac{m_e^2 C_{eph}}{\omega_a \omega_u} \int_0^\infty y dy \int_{-\infty}^{\infty} dx \left( \frac{1}{w - x + w_{LO} + i \eta} + \frac{1}{w + x - w_{LO} + i \eta} \right)
\]

\[
\times \frac{(e^{i x} - 1)^{-1} - (e^{4 \omega_{LO}} - 1)^{-1}}{x - \alpha \omega_{LO}} \ln \left[ \frac{1 + \exp[\varepsilon_{\mu} - (y - x/y)^2]}{1 + \exp[\varepsilon_{\mu} - (y + x/y)^2]} \right],
\]

is the contribution due to electron–phonon interactions, derived earlier in [9], where \( C_{eph} \) is a constant of the order of unity, \( \alpha = T_e / T_i \); \( w = h \omega/(4 T_e) \), \( \omega_a = h \omega_{au}/T_e \), \( w_{LO} = h \omega_{LO}/(4 T_e) \) with the frequency of the longitudinal optical phonons \( \omega_{LO} \) and \( \eta \) is an infinitesimal small value. The dimensionless chemical potential

\[
\varepsilon_{\mu} = \mu / T_e = X_{1/2} \left( 2 e_F^{3/2} / 3 \right),
\]

is expressed through the reverse function \( X_{1/2}(x) \) of the Fermi integral \( I_{1/2}(x) \), where

\[
\varepsilon_F = E_F / T_e, \quad E_F = \hbar^2 k_F^2 / (2m), \quad k_F = (3 \pi^2 n)^{1/3}.
\]

The contribution of Umklapp processes to the correlation function is given by the second term in (10),

\[
\mathbf{c}_U = \frac{9 i m_e U^2 T_e^2}{4 E_F^2 E_H} \sum_g g^2 J_\Omega(g) J_E(W, \varepsilon_B, \varepsilon_{\mu}),
\]

\[
J_\Omega(g) = \frac{1}{(4 \pi)^4} \iiint d\Omega d\Omega' d\Omega_1 d\Omega_1' \delta \left( \vec{k}_1 + \vec{k}_1' - \vec{k} - \vec{g} \right),
\]

\[
J_E(W, \varepsilon_B, \varepsilon_{\mu}) = \iiint \frac{dx_1 dx'_1 dx x x'}{x_1 + x'_1 - x - x' + W + i \eta} \frac{e^{x_1 + x'_1} - e^{x + x'}}{x_1 + x'_1 - x - x'}.
\]

In (14), the vectors \( \vec{g}, \vec{k}_1, \vec{k}_1', \vec{k}, \vec{k}' \) are taken dimensionless, as ratio to the absolute value \( k_F \). The integration in (14) is performed on solid angles for each of the respective wavevectors \( \vec{k}_1, \vec{k}_1', \vec{k}, \vec{k}' \). Therefore, the integral (14) is dependent only on \( \vec{g} \). Furthermore \( W = \hbar \omega / T_e = 4w; \varepsilon_B = E_B / T_e, E_B \) is the energy of electrons on the surface of the 1st Brillouin zone boundary. In (15), \( n_x = 1/[1 + e^\varepsilon] \) is the Bose distribution function.

3. Real part of correlation function and its asymptotic expansions

In equations (11) and (13), the integral over \( dx \) can be performed applying the Sokhotski–Plemelj formula

\[
\int_{-\infty}^{\infty} \frac{f(x)}{x + i \eta} dx = -i \pi f(0) + \text{v.p.} \int_{-\infty}^{\infty} \frac{f(x)}{x} dx,
\]
where v.p. denotes the principal value of the integral. In this case, real and imaginary part of the correlation function can easily be derived. The real parts are found as

$$\mathcal{C}_{e-ph} = \frac{\epsilon^3}{2|\hbar|^3/2} \sum_{\sigma = \pm 1} \frac{(e^{i|\omega + \sigma \omega|} - 1) - (e^{i\omega_{LO} \alpha} - 1)}{\omega_{LO}(\alpha - 1) - \sigma \omega} \times \int_{-\epsilon}^{\epsilon} y dy \ln \left[ 1 + \exp \left( \frac{\epsilon_{\mu} - \left| y - \omega_{LO} + \sigma \omega \right|}{y} \right)^2 \right]$$

and

$$\mathcal{E}_U = \frac{9}{4} C_UG^2 m_e u \frac{U^2 t^2}{E_F E_H} J_E^2,$$

$$J_E = \frac{4}{W} \int_{-2\epsilon}^{2\epsilon} dt \left[ \frac{1}{e^{e - W} - 1} - \frac{1}{e^\epsilon - 1} \right] \ln \left[ \frac{e^{i/2} + e^{-B/2}}{e^{i/2} - 1} \right] \ln \left[ \frac{e^{i/2 - W/2} + e^{-B/2}}{e^{i/2 - W/2} - 1} \right],$$

where $\epsilon = \Delta_E/T_e$, where $\Delta_E$ is the energetic distance between Fermi surface and Brillouin zone boundary, $B = \epsilon_{\mu} + \epsilon_D$.

While deriving (17) and (18) from (13), the substitution $x = (t + r)/2$, $x' = (t - r)/2$, $x_1 = (t + r_1)/2$, $x'_1 = (t - r_1)/2$ was conducted, integrals over $r$ and $r_1$ were calculated explicitly and the $\delta$-function was accounted for while integrating over $t_1$. Expression (14) is approximated, taking the term

$$\sum_\gamma g^2 J_\Omega(g) \approx C_U g^2 N_g.$$  

$C_U$ is a constant of the order of 1, which can be found, e.g., from optical measurements, like in [26–28]. $N_g$ is the number of different wavevectors of the reverse lattice in the first Brillouin zone, which coincides with the number of nearest neighbors in the reverse lattice for the point $g = 0$. $g^2$ is the average value of $g^2$ in the first Brillouin zone.

The asymptotic expansions for the real part of the full correlation function $\mathcal{C}_{11}$ according to (10) are determined by respective asymptotic expansions for $\mathcal{C}_{e-ph}$ and $\mathcal{E}_U$. Asymptotic expressions for $\mathcal{C}_{e-ph}$ were found in [9]. Here we present them for the most important case of small phonon frequencies, $h \omega_{LO} \ll T_e$, $h \omega_{LO} \ll T_1$. Then for high laser frequencies, $h \omega \gg T_e$, one has

$$\mathcal{C}_{e-ph} \approx \frac{m^2 C_{\text{eph}}}{6 \pi^3/2} \frac{T_{\text{ion}}}{\sqrt{E_H h \omega}}.$$

For low laser frequencies, $\omega \leq \omega_{LO}$, one has

$$\mathcal{C}_{e-ph} \approx \frac{m^2 C_{\text{eph}}}{2 \pi^3/2} \frac{T_{\text{ion}}}{\sqrt{E_H T_e}} \left( E_F/T_e \right)^{-3/2} \ln(1 + e^{\mu/T_e}) \times \left\{ \begin{array}{ll} 1, & \omega \ll \omega_{LO}, \\
9/4, & \omega = \omega_{LO}, \end{array} \right.$$  

For the high- and low-temperature case the upper line of the last equation can be written as

$$\mathcal{C}_{e-ph} = \frac{m^2 C_{\text{eph}}}{2 \pi^3/2} \frac{T_{\text{ion}}}{\sqrt{E_H E_F}}, \quad T_e \ll E_F,$$

$$\mathcal{C}_{e-ph} = \frac{m^2 C_{\text{eph}}}{2 \pi^3/2} \frac{T_{\text{ion}}}{\sqrt{E_H E_F}}, \quad T_e \gg E_F.$$
The asymptotic expressions for $c'_1$, are now derived. First, we consider the low-temperature asymptote, when $E_F/T_e \gg 1$, $\mu \approx E_F$, $\Delta_E/T_e \gg 1$. For this case, one has from (18):

$$J'_E = \frac{2\pi^2}{3} \left[ 1 + \frac{\hbar^2\omega^2}{4\pi^2 T_e^2} \right], \quad c'_U = \frac{3\pi^2}{2} C_U N_g g^2 m_* \frac{U^2}{E_F^4 E_H^4} \left[ T_e^2 + \frac{\hbar^2\omega^2}{4\pi^2} \right]$$  

(23)

for $\hbar\omega \ll E_F$, and

$$J'_E = \frac{E_F + \Delta_E}{\hbar\omega} \left[ \frac{\pi^2}{3} + \frac{2\Delta_E^2}{T_e^2} \right], \quad c'_U = \frac{3\pi^2}{4} C_U N_g g^2 m_* \frac{U^2 (E_F + \Delta_E)}{E_F^4 E_H \hbar\omega} \left[ T_e^2 + \frac{6}{\pi^2} \Delta_E^2 \right]$$  

(24)

for $\hbar\omega \ll E_F, |\mu|$, and

$$J'_E = \frac{2 \Delta_E^3}{\hbar\omega T_e^2}, \quad c'_U = \frac{9}{2} C_U N_g g^2 m_* \frac{U^2 (1 + \Delta_E/E_F)^3}{\hbar\omega E_H}$$  

(25)

for $\hbar\omega \gg E_F, |\mu|$. Equation (23) gives the same asymptotic dependence on electron temperature and laser frequency, as already known from the literature [11–16]. Contrary to that, equations (24)–(26) give new, quite different asymptotes. Particularly, equations (24) and (26) demonstrate, that for high laser frequencies the correlation function is reversely proportional to the laser frequency. Equation (25) demonstrates reverse proportionality to forth power of $T_e$ for low laser frequency and high electron temperature $T_e$.

It should be noted, that the above derivation is valid only under the assumption, that the ion temperature is less than $T_{\text{melt}}$, or, alternatively, the time period for which the ion temperature has been above $T_{\text{melt}}$ is less than the time between collision $\tau_m$. The latter is given by the interatomic distance $r_a = (4\pi n_{\text{at}}/3)^{1/3}$ divided by the sound velocity $v_s = \sqrt{ZT_e/m_{\text{at}}}$, where $n_{\text{at}}, m_{\text{at}}$ are concentration and mass of the heavy particles, respectively, i.e.

$$t < \tau_m = r_a/v_s = k_m A_{\text{at}}^{5/6} / (ZT_e/T_1)^{1/2} \tilde{\rho}^{-1/3}, \quad k_m \approx 7.5 \text{ fs}, \quad T_1 = 1 \text{ eV},$$  

(27)

where $A_{\text{at}}$ is the atomic number and $\rho$ is the mass density of matter. For the case of aluminum we find $\tau_0 \approx 11 \text{ fs}$ for $Z = 3$, $\rho = 2.71 \text{ g/cm}^3$ and $T_e = 20 \text{ eV}$.

4. Numerical calculations

Exemplary calculations of the absorption coefficient (2)\textsuperscript{1} for step-like aluminum plasma with solid density $\rho = 2.71 \text{ g/cm}^3$ and average ion charge $Z = 3$, irradiated by a laser of wavelength $\lambda = 0.4 \mu$m. For this, the DF (1) with the effective collision frequency, calculated from expressions (10), (16), (18), was determined. The effective mass $m_*$ was calculated according to the Huttner model [29], see also [30]. The value of $C_{\text{eph}} \approx 5.73$ in (11) was chosen in such a way to reproduce low-frequency cold reflectivity of aluminum [26, 30] for the considered laser wavelength. The value of $\omega_{\text{LO}}$ was determined by the position of the maximum of the phonon spectrum for aluminum [31] as $\omega_{\text{LO}} \approx 0.006 \text{ eV}$. Furthermore, the following parameters are $C_U = 1.5$, $U = 2 \text{ eV}$, $\Delta_E = 7.3 \text{ eV}$, $g^2 = 2$. The value $N_g = 8$ is chosen, assuming fcc lattice structure and bcc reverse lattice structure for aluminum.

Figure 1 shows the results for (a) the absorption coefficient $A$, equation (2), and (b) the real part of the effective collision frequency, calculated for two different fixed ion temperatures. The results are compared with a similar model as considered in the work of Veysman et al [17], where

\textsuperscript{1} Below, we disregard the imaginary part of the effective collision frequency $\nu(\omega)$ when calculating $A$ via (2).
Figure 1. (a) The absorption coefficient $A$ and (b) real part of the complex collision frequency of electrons $\nu'$ as functions of electron temperature $T_e$, for a solid-density aluminum plasma of $\rho = 2.71$ g/cm$^3$ and laser radiation of wavelength $\lambda = 0.4$ $\mu$m. Thick (online in green) and thin (online in blue) solid curves are calculations by the model (10), (16), (18) for different ion temperatures $T_i$. For more details see text. Curves with markers are according to the models cited in the text (see the legend).

For $T_e > T_m$ ($T_m \approx 11.6$ eV for solid-density aluminum), in accordance with the asymptotic behaviour (25), $\nu'$ is decreasing as $\nu' \sim T_e^{-4}$. This is much faster than the scaling $\nu' \sim T_e^{-3/2}$ for the case of electron–ion interaction in a high-temperature plasma [17]. For this also compare the solid curves and the curve with rectangular markers in figure 1(b). In real experimental situations, where ultra-short (femtosecond) intense energy fluxes act on matter, in principle the following situations depending on laser and matter parameters are both possible:

(i) $T_i < T_m$ or $T_i > T_m$, but $t < \tau_m$;
(ii) $T_i > T_m$, $t > \tau_m$. 

Note, that in the work of Povarnitsyn et al [26] different expressions were used for the effective collision frequency in the low-temperature ($T_e < E_F$) case, ascending curve of round markers in figure 1(b), and in the high-temperature $T_e > E_F$ case, descending curve of round markers in figure 1(b).
Figure 2. (a) The absorption coefficient $(2)$, (b) the reflection coefficient $R = 1 - A$ and (c) the real part of the complex collision frequency of electrons $(10)$, each in dependence on electron temperature $T_e$. The same calculations as in figure 1, but for fixed ion temperature $T_i = 0.04$ eV and different laser photon energies $\hbar \omega$ (shown on the legend).

According to the above considerations, one can expect much lower values of $\nu'$ in the case (i) than in the case (ii) for temperatures $T_e > T_F$ considered here.

The dependence of the absorption coefficient $A$ on electron temperatures $T_e$ for the considered optical frequencies is similar to the dependence of $\nu'$ on $T_e$, in accordance with expressions $(5)$ and $(6)$. The maximum values of $A$, calculated by the model $(10)$, $(16)$, $(18)$ as derived above, is relatively close to that of the semi-empirical model [26] and is close to the values of $A$, calculated by the QS and LRT model of the work [17] for the case of electron–electron and electron–ion interactions in high-temperature plasmas for the same electron temperatures in the vicinity of the peak of $A(T_e)$, see figure 1(a).

It should be noted, that under conditions of figure 1, the contribution of Umklapp process $(18)$ exceeds the contribution of electron–phonon part $(16)$, so the influence from different ion temperatures is mainly due to the dependence of effective electron mass $m_e$ on ions temperatures, calculated by means of the Huttner model [29] (compare thin and thick solid curves in figure 1).
Figure 2 presents similar calculations, as in figure 1, but for fixed ion temperature and for different frequencies $\omega$ of laser radiation. For $\hbar\omega \lesssim E_F = 11.6$ eV for Al, the dependence of $\nu'(\omega)$ is increasing for low or moderate electron temperatures $T_e \lesssim E_F$, in accordance with (23). Contrary to that, the dependence $\nu'(\omega)$ is decreasing for $\hbar\omega \gg E_F$, in accordance with (24) and (26). Also it is important to note, that for $\hbar\omega > \hbar\omega_{pl} \approx 16$ eV for solid-density aluminum plasma this plasma becomes transparent to radiation. That leads to absorption coefficient of step-like semi-infinite plasma approaching to 1 (and reflection coefficient approaching to 0) with increasing of $\hbar\omega$ for $\hbar\omega > \hbar\omega_{pl}$, see figure 2(a, b).

5. Discussion
With the QS approach within LRT, we used a Fröhlich Hamiltonian describing electron–phonon interactions and a Hubbard Hamiltonian describing electron–electron interactions in metals. From this, we have derived new simple expressions for correlation functions and the DF for WDM in the case of arbitrary laser frequencies, arbitrary electron temperatures, and ion temperatures lower than melting temperature or above for times lower than the characteristic melting time only.

These expressions account for both electron–phonon interactions and the Umklapp process. Using these expressions, new asymptotic formulas for the case of high laser frequencies $\hbar\omega \gg E_F$ (24), (26) and the case of high electrons temperatures $T_e \gg E_F$ (25), (26) are obtained.

The derived model permits one to obtain dependence of absorption coefficient on electrons temperature with a peak in the vicinity of the Fermi temperature, without appealing to any kind of fitting.

Further progress of the theory requires a smooth link from the results obtained in this work for the case of relatively small ion temperatures $T_i < T_{melt}$ with results obtained in our previous work [17] for the case of $T_i > T_{melt}$. This will be the subject of our further activity.

6. Conclusions
The simple wide-range (over radiation frequencies and electron temperatures) expressions for the correlation functions, the complex effective collision frequency and the permittivity are obtained using QS approach, LRT, Fröhlich Hamiltonian for electron–phonon and Hubbard Hamiltonian for electron–electron interactions.

The derived model accounts for electron–phonon interactions and Umklapp process in metallic plasmas.

New asymptotic are found for the case of high laser frequencies or high electron temperatures.

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