EFFICIENT SPARSE SECURE AGGREGATION FOR FEDERATED LEARNING

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ABSTRACT

Federated Learning enables one to jointly train a machine learning model across distributed clients holding sensitive datasets. In real-world settings, this approach is hindered by expensive communication and privacy concerns. Both of these challenges have already been addressed individually, resulting in competing optimisations. In this article, we tackle them simultaneously for one of the first times. More precisely, we adapt compression-based federated techniques to additive secret sharing, leading to an efficient secure aggregation protocol, with an adaptable security level. We prove its privacy against malicious adversaries and its correctness in the semi-honest setting. Experiments on deep convolutional networks demonstrate that our secure protocol achieves high accuracy with low communication costs. Compared to prior works on secure aggregation, our protocol has a lower communication and computation costs for a similar accuracy.

1 INTRODUCTION

Machine learning (ML) requires the collection of large volumes of data in order to train robust predictive models. In some healthcare applications, e.g. for rare diseases, this data collection necessarily involves data stemming from different locations. However, due to data sensitivity, it may be forbidden or extremely difficult to centrally collect them. Federated Learning (FL) introduced in (Shokri & Shmatikov, 2015; McMahan et al., 2016) is an approach to train an ML model that benefits from multiple datasets while keeping training data in place. FL algorithms typically iterate training rounds, during which model updates are usually aggregated after local training steps. While the approach is promising, two major challenges hinder the large-scale adoption of FL techniques in real-world use-cases: privacy concerns and expensive communication (Li et al., 2020).

Although leaving data at its source is a significant improvement for data privacy, sharing intermediate model updates indirectly leaks sensitive information (Bhowmick et al., 2018; Carlini et al., 2018; Melis et al., 2019). A standard protection technique is secure aggregation (Bonawitz et al., 2017; Dong et al., 2020). With this approach, only the aggregated models are revealed, while the local intermediate models are kept private, which reduces the effectiveness of attacks. Secure aggregation is often based on either Secure Multi-party Computation (SMC) protocols (Evans et al., 2018) or Homomorphic Encryption (HE) (Acar et al., 2017). The main bottleneck of these techniques is the additional computation and communication costs, which amplify the burden of communication costs in FL.

Due to its distributed nature, communication costs are significant in FL. In order to mitigate communication overhead, multiple neural network update compression techniques have been introduced in the last few years (Strom, 2015; Sattler et al., 2018; Tang et al., 2019). These techniques demonstrate that only the most significant bits of model updates’ components are required to obtain a good predictive model and very few of these components carry significant information. It permits one to greatly reduce the communication costs without compromising the final model accuracy. Unfortunately, these techniques are not directly compatible with efficient secure aggregation protocols, which limits their practical impact.

In this paper, instead of addressing each challenge separately, which leads to competing optimisations, we investigate them simultaneously, for one of the first times. Our main contribution is to introduce a set of new efficient protocols for secure aggregation in collaborative FL (see Sec. 3), which permits to achieve a good tradeoff between privacy and communication costs according to the security level required in each practical use case. These protocols combine neural network update compression techniques (Strom, 2015; Sattler et al., 2018; Tang et al., 2019) with additive secret sharing (Cramer et al., 2015). They are provably private against malicious adversaries when at most all servers except one collude (see Appendix E), have a low computation cost by reducing the secure aggregation to the secure evaluation of only additions (see

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We also introduce some notations related to the federated system. Let $C$ denote the number of clients jointly training a neural network, which consists of $N$ trainable parameters. Thus, the length of the vector of updates is equal to $N$. The clients jointly train this network during $R$ federated rounds. In each federated round, each client performs $E$ local update steps before aggregation.

2.2 Federated Learning

2.2.1 Collaborative or Cross-Silo Setting

Many current FL works such as (Bonawitz et al., 2019; Lim et al., 2019) study massive-scale use cases where millions of different devices participate to the training. In contrast, in this manuscript, we focus on collaborative FL applied to medical use cases. In this context, a small number of clients (generally less than 10) takes part in the federated training. Clients are already able to obtain useful trained models from only their own local dataset (which is generally moderately large), but they would like to augment their performance through collaboration. Unlike the massive-scale use-case, all clients are able to participate in each round with robust connectivity to the system.

2.2.2 Federated Averaging (McMahan et al., 2016)

In the studied FL setting, the goal is to obtain a single trained model from some datasets held by $C$ clients, $D_i \forall i \in [1, C]$, without moving these datasets to a single location. The most commonly used strategy in FL is Federated Averaging (FedAvg) (McMahan et al., 2016). In this algorithm, within each such federated round, indexed by $r \in [1, R]$, each client $i$ is provided with the current global model state $\theta^{(r)}$, performs $E$ local optimization steps $\text{Opt}_i$ to minimize its local loss $\mathcal{L}_i$, and evaluates its local update $U^{(r)}_i$:

$$\theta_i^{(0)} \leftarrow \theta^{(r)} \quad \text{(Local Initial State),} \quad (1a)$$

$$\theta_i^{(E)} \leftarrow \text{Opt}_i(\theta_i^{(0)}, D_i) \quad \text{(Local Training),} \quad (1b)$$

$$U^{(r)}_i \leftarrow \theta_i^{(E)} - \theta_i^{(0)} \quad \text{(Local Update) (1c)}$$

Then the local updates are transmitted to a central aggregator which maintains and updates the global model state,

$$\theta^{(r+1)} \leftarrow \theta^{(r)} + \sum_{i=1}^{C} \frac{|D_i|}{D} U^{(r)}_i, \quad \text{(Aggregate & Update) (2)}$$

where $D = \sum_{i=1}^{C} |D_i|$ is the total number of data samples across all datasets. Tens to thousands federated rounds are computed in sequence until the global model state converges or meets some desired performance metric.

When multiple local update steps are used ($E > 1$), the convergence of FedAvg is not guaranteed even in the convex setting due to data heterogeneity (Sahu et al., 2018; Karimireddy et al., 2019a). The trade-off for FedAvg, then, is to select a large enough $E$ so as to reduce the number of federated rounds required (and therefore communication overhead) to reach a desired performance metric, but not so large that the algorithm fails to converge.

2.2.3 Communication-Efficient FL

Over the past few years, many model update compression techniques such as (Seide et al., 2014b; Strom, 2015; Sattler et al., 2018; Tang et al., 2019) have shown that it is possible to greatly reduce the communication costs of distributed training while retaining the predictive accuracy of the final model. In (Seide et al., 2014b), gradients are encoded using only the direction (sign) of each coefficient. This technique is further enhanced in (Strom, 2015) where the authors demonstrate that very few of these components carry significant information, and thus many can be discarded, resulting in a so-called ternary coding of the model update (Wen et al., 2017). Both of these approaches make use of an error compensation (EC) scheme to stabilize training. EC has been studied analytically in (Karimireddy et al., 2019b; Stich et al., 2018), where it was proven that this technique achieves the same theoretical convergence guarantees as standard training. Recent works such as (Sattler et al., 2018; Tang et al., 2019) have showcased the utility of such model update compression schemes in practical federated settings.

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Sec. 3.2), and reduce its communication cost thanks to update compression techniques as demonstrated in Sec. 3.3. Our experiments on deep convolutional neural networks in MNIST and CIFAR-10 prove that our protocols obtain similar accuracy than non-secure uncompressed FL training with a lower communication cost (see Sec. 5). Compared to secure aggregation techniques (Bonawitz et al., 2017; Dong et al., 2020), our protocols have a lower computation and communication costs for a similar accuracy.
2.3 Additive Secret Sharing

Secure Multiparty Computation (SMC) is a cryptography subfield which studies techniques allowing several parties to jointly evaluate a function over their inputs while keeping these inputs private. Many protocols exist in this field, such as Yao garbled circuit (Yao, 1986) and secret sharing (Rivest et al., 1979). These protocols have different trade-offs between the computation and communication costs.

For our secure aggregation, we choose to only rely on Additive Secret Sharing (Cramer et al., 2015), owing to its low computation costs. Its communication cost is proportional to the number of multiplications to evaluate. To obtain an efficient secure aggregation, we adapt our aggregation such that only additions need to be securely evaluated.

In this section, we review the Additive Secret Sharing protocol in Sec. 2.3.1, as well as its communication and computation costs in Sec. 2.3.2, and justify its security aspects in Sec. 2.3.3.

2.3.1 Additive Secret Sharing (Cramer et al., 2015)

Alg. 1 presents in details how to securely evaluate the sum of integers held by $C$ clients, with additive secret sharing, thanks to the help of $S$ servers ($S \geq 2$). In this protocol, each client possesses a vector $X_i \in \mathbb{Z}_m^n$, and desires to know the sum of all client vectors, $X = \sum_{i=1}^{C} X_i \mod m$, but no client wants to reveal its own vector to other parties. During the evaluation, inputs, intermediate results and outputs are held by $S$ servers in the form of secret shares.

**Algorithm 1 SecureSum Protocol**

**Input:** Each client has a vector $X_i \in \mathbb{Z}_m^n$

**Locally on each Client** $i \in [1, C]$

// Create & Transmit Shares
R_{i,j} \in \mathbb{Z}_m^n \ \forall j \in [1, S - 1]$

$R_{i,s} \leftarrow X_i - \sum_{j=1}^{S} R_{i,j} \mod m$

Transmit $R_{i,j}$ to Server $j, \forall j \in [1, S]$

**Locally on each Server** $j \in [1, S]$

// Aggregate received shares
$R_j \leftarrow \sum_{i=1}^{C} R_{i,j} \mod m$

Broadcast $R_j$ to all clients

**Locally on each Client** $i \in [1, C]$

// Aggregate received shares
$Z \leftarrow \sum_{j=1}^{S} R_j \mod m$

Clients: $Z = \sum_{i=1}^{C} X_i \mod m$

Servers: $\emptyset$

**Output:** The SecureSum protocol does not require that the parties playing the role of the server are mutually exclusive from the client parties. No matter the architecture, the proposed protocol will only reveal to the clients the result of the secure function evaluation, with the caveat that if a client is also a server, then this party will observe the result by virtue of being a client itself.

2.3.2 Communication and Computation Costs

Compared to other SMC protocols such as Yao garbled circuit (Yao, 1986) and GMW (Goldreich et al., 1987), no expensive cryptographic operations are used in the SecureSum protocol. All parties have only to evaluate additions modulo $m$. Clients have an additional burden to produce $S - 1$ random vectors in $\mathbb{Z}_m^n$, but this additional computational cost is also minimal. Thus, the computation cost of the SecureSum protocol is very low.

About the communication cost, each client sends to each server a single vector in $\mathbb{Z}_m^n$ of $n \cdot \lceil \log_2 m \rceil$ bits, and then, each server sends back to each client one vector in $\mathbb{Z}_m^n$ of $n \cdot \lceil \log_2 m \rceil$ bits. Thus, the total communication cost required to execute this protocol is

$$2 \cdot S \cdot C \cdot n \cdot \lceil \log_2 m \rceil \text{ bits.}$$

2.3.3 Security Definitions

The security of an SMC protocol depends on the assumptions that are made on adversaries. In this paper, we only take into account semi-honest or malicious, static adversaries. A Semi-honest adversary follows the protocol but tries to infer as much information as possible from the observed messages. On the contrary, a Malicious adversary can use any kind of strategy to learn information, including sending fallacious messages. A Static adversary selects the parties to corrupt at the beginning of the protocol execution. This set of corrupted parties is fixed for the whole protocol execution.

An SMC protocol is determined to be secure if it is both correct and private. A protocol is correct if the output of the protocol is correct for the desired secure operation. A protocol is private if each party will learn their output and nothing else, except what they can infer from their own input and their own output.

**Theorem 1.** The SecureSum protocol (see Alg. 1) is correct against semi-honest adversaries and private against a malicious adversary controlling at most $S - 1$ servers and any number of clients.

**Proof.** It is well known that the SecureSum protocol is correct and private against semi-honest adversaries (Cramer et al., 2015). We demonstrate in Appendix E that this protocol is private against a malicious adversary controlling at most $S - 1$ servers and any number of clients.

It means that a malicious adversary cannot learn any in-
formation about the input data of honest parties, but it can modify messages such that the output is incorrect. In collaborative FL use cases, data owners benefit from the trained global model and therefore do not need to reduce their own performance. Therefore, semi-honest correctness is sufficient for our use case. In addition, all participants have an incentive to try to obtain information about data held by other participants. Hence, our protocol must be private against malicious adversaries.

2.4 Fixed-Point Representation

While ML models are very often parameterized by floating-point values, SMC operations are applied in a finite set (e.g. \( \mathbb{Z}_m \)). Thus, a fixed-point representation is required to convert these real values into the finite set. In our secure aggregation protocol (see Sec. 3.2), we would like to securely add only non-negative real values \( x \in \mathbb{R}^+ \). Thus, we adapted the fixed-point representation presented in (Cock et al., 2020) to only non-negative real values, \( Q : \mathbb{R}^+ \to \mathbb{Z}_{2^\lambda} : Q(x) = [2^\alpha \cdot x] \mod 2^\lambda \). (4)

In practice, all values \( x \) (inputs, intermediate results and outputs) belong to a finite interval. The integer \( \alpha \) is selected according to the wanted precision. To avoid overflow issues, the integer \( \alpha \) is set according to the largest value resulting from any step in the secure computation.

3 Proposed Method

In this paper, we focus on collaborative FL, a context often encountered in medical applications of federated techniques. The aim of this work is to present a secure FL technique which answers to the following four design constraints:

i) Perform the model update aggregation in a secure way. Nothing will be revealed to the aggregation servers. Only the clients will learn the aggregated updates, and thus whatever information can be inferred from these updates in conjunction with their local data and model updates. Specifically, our protocol must be:

a) Private in malicious settings. Clients and servers may deviate from the defined protocol to obtain information about other clients data.

b) Correct in semi-honest setting. Participants have an incentive to behave correctly in order to obtain a trained model with better performance than their local model.

ii) No compromise on accuracy. Our method must obtain a similar accuracy than non-secure uncompressed FedAvg.

iii) Computation Efficient. Our method must not use expensive cryptographic operations.

iv) Communication Efficient. Our method must have a smaller communication cost than non-secure uncompressed FedAvg and state of the art secure aggregation (Bonawitz et al., 2017; Dong et al., 2020).

In this section, we will first describe a communication-efficient FL algorithm which is amenable to efficient secure aggregation in Sec. 3.1. Then, in Sec. 3.2, we will present our efficient secure aggregation protocol, whose the main block is the SecureSum protocol (see Alg. 1). Our proposed method is summarized in Alg. 2.

### Algorithm 2 SMC-friendly Compressed Update FL

| Input: Number of rounds \( R \), number of local updates \( E \), initial global model state \( \theta^{(0)} \), local datasets \( D_i \), desired compression factor \( \rho \).
| Initialization (Error accumulator and model states) \( \delta_i^{(0)} \leftarrow 0 \), \( \theta_i^{(0)} \leftarrow \theta^{(0)} \), \( \forall i \in [1, C] \):
| for \( r = 0, 1, ..., R - 1 \) do
| Locally on each Client \( i \in [1, C] \):
| \( U_i \leftarrow \text{Opt}_E \{ \theta_i^{(r)}(D_i) - \theta_i^{(r)} \} \) Eq. (1)
| \( (\alpha_i, D_i) \leftarrow \text{TopBinary}(U_i + \delta_i^{(r)}) \) Eq. (6)
| \( \delta_i^{(r+1)} \leftarrow \delta_i^{(r)} + U_i - \alpha_i \cdot D_i \)
| Via SMC between clients and servers \( D \leftarrow \text{SecureSum}(D_1, ..., D_C) \) Sec. 3.2.2, 3.2.3
| \( \alpha \leftarrow \text{SecureSum}(\alpha_1, ..., \alpha_C) \) Sec. 3.2.4
| Locally on each Client \( i \in [1, C] \):
| \( \theta_i^{(r+1)} \leftarrow \theta_i^{(r)} + \frac{1}{C} \alpha \cdot D \) Eq. (2) with Eq. (8)
| end for

3.1 Communication-Efficient FL

3.1.1 TopBinary Coding

Our method is based on the compression of the updates sent from clients to the aggregation server with an error compensation to ensure convergence (Strom, 2015; Karimireddy et al., 2019b; Sattler et al., 2018). For the compression method, we used a combination of the Top-\( k \) sparsification (Stich et al., 2018) as well as 1-bit quantization (Bernstein et al., 2018) which we note here as TopBinary coding. The Top-\( k \) sparsification jointly compresses a vector \( X \) by retaining only the \( k \) components with largest magnitude. More precisely, let \( X \in \mathbb{R}^N \), then

\[
\forall i \in [1, N], \text{Top}-k(X)[\pi_i] = \begin{cases} X[\pi_i], & \text{if } \pi_i \leq k, \\ 0, & \text{otherwise.} \end{cases}
\] (5)

where \( \pi \) is a sorting permutation of \( [1, N] \) such that \( \forall i \in [1, N - 1], |X[\pi_i]| \geq |X[\pi_{i+1}]| \). In order to compare same
relative compression rate between models of varying architectures, we will introduce the term \( \rho \triangleq k/N \), which is interpreted as a desired model update sparsity level.

To further reduce communication costs, we also employ one-bit quantization to the \( k \) significant model update coefficients (Sattler et al., 2018; Strom, 2015). This scalar quantization maps each non-zero element of a vector \( X \) to the binary set \{ \(-\alpha, \alpha\)\} according to their signs. More precisely, we define our TopBinary coder as

\[
\text{TopBinary}(X;k) = \alpha \cdot \text{sgn} \circ \text{Top-k}(X),
\]

where \( \alpha \triangleq \frac{\|X\|_2}{\|\text{Top-k}(X)\|_2} = \frac{1}{\sqrt{k}} \|X\|_2 \) is a scaling factor used to preserve the Euclidean norm of \( X \). We note that the implementation of the proposed secure protocol does not depend tightly on this construction of \( \alpha \), and other definitions (Seide et al., 2014a; Sattler et al., 2018) could be chosen, as well.

The TopBinary coded vector can be split into two components: the scalar factor \( \alpha \) and the vector of signs \( D = \text{sgn} \circ \text{Top-k}(X) \). Furthermore, when \( \rho \) is small, the ternary-valued \( D \) can instead be represented by two \( k \)-length vectors, namely, the list of non-zero indices, \( V = [\pi_1, \ldots, \pi_k] \), and the signs of the coefficients at those non-zero locations, \( D = [D[\pi_1], \ldots, D[\pi_k]] \).

### 3.1.2 Separate Aggregation

Let \( \alpha_i \cdot D_i \) the updates of the client \( i \) obtained with the TopBinary encoding. Usually a direct aggregation (DirectAgg) is performed and the aggregated updates are defined as

\[
\frac{1}{C} \sum_{i=1}^{C} \alpha_i D_i \quad \text{(DirectAgg)}.
\]

Unfortunately, the secure evaluation of this DirectAgg requires the secure evaluation of multiplications, which is costly compared to the secure evaluation of additions. To have an efficient protocol, we would like to securely evaluate only additions. Thus, we suggest an alternative to aggregate compressed model updates called separate aggregation (SepAgg). This approach separately aggregates the scaling factors \( \alpha_i \) and the vector of signs \( D_i \), as follows:

\[
U = \frac{1}{C^2} \left( \sum_{i=1}^{C} \alpha_i \right) \left( \sum_{i=1}^{C} D_i \right) \quad \text{(SepAgg)}.
\]

We contrast these two approaches in Appendix B, where we demonstrate that when the norm of each original vector \( \|X_i\|_2 \) is close to the average norm \( \|X\|_2 \), then the SepAgg is an adequate estimation of the DirectAgg.

With SepAgg, the convergence of the federated training is not proved. However, in our experiments, federated training with this SepAgg provides similar trained model predictive performance than FedAvg training with DirectAgg. The main advantage of this SepAgg is to be SMC-friendly without compromising accuracy.

### 3.2 Compressed Secure Aggregation Protocol

In this section, we explain how the SecureSum algorithm (Alg. 1) is used as a building block to construct a secure protocol for our sparse aggregation protocol described in Sec. 3.1. The problem is the following: at the aggregation step, each client \( i \) has a vector \( X_i = \alpha_i D_i \in \mathbb{R}^N \) and one would like to securely evaluate \( U = \text{SepAgg}(X_1, \ldots, X_C) \). Our proposed secure aggregation protocol can be described in three steps:

\[
V \leftarrow \bigcup_{i=1}^{C} V_i = \bigcup_{i=1}^{C} \{ k \in [1, N] \mid D_i[k] \neq 0 \} \quad \text{(Union of Indices)}
\]

\[
D \leftarrow \sum_{i=1}^{C} (D_i)_{|V} \quad \text{(Sum of Signs)}
\]

\[
\alpha \leftarrow Q^{-1} \left( \sum_{i=1}^{C} Q(\alpha_i) \right) \quad \text{(Sum of Factors)}
\]

Finally, each client can locally evaluate \( U = \frac{1}{\tilde{\rho}} \cdot \alpha \cdot D \). During this protocol, each client learns \( V, D, \alpha \) and \( U \) and the servers learn nothing.

In the next sections, we will detail why the intermediate steps of this protocol are secure (Sec. 3.2.1), then we will provide a number of different approaches for the secure union (Sec. 3.2.2), the protocol for a secure sum of sign vectors (Sec. 3.2.3), and finally the protocol for the secure sum of scalar factors (Sec. 3.2.4).

#### 3.2.1 Security of Intermediate Outputs

At the end of the secure aggregation round, each client obtains the separate aggregated updates \( U \) (see Eq. 8). In this section, we will prove that from \( U \), each client can easily infer the intermediate outputs \( V, D \) and \( \alpha \). Thus, revealing the clients these intermediate outputs over the course of the protocol does not provide any additional information than what they could learn simply from their knowledge of their own inputs and result of the secure protocol at its completion. Hence, the proposed stepwise approach is secure as long as its individual steps are secure.

From \( U \), the client can infer the union of the list of indices via \( \tilde{V} = \{ k \in [1, N] \mid U[k] \neq 0 \} \), where \( \tilde{V} \subset V = \bigcup_{i=1}^{C} \{ k \in [1, N] \mid D_i[k] \neq 0 \} \) and \( V \setminus \tilde{V} = \{ k \in V \mid \sum_{i=1}^{C} D_i[k] = 0 \} \). Then, the client can deduce the sum of the scalar factors via \( \tilde{\alpha} = \frac{C^2}{\tilde{\rho}} \cdot \min_{k \in \tilde{V}} |U[k]| \), where \( \tilde{\alpha} = \alpha \) if at least one index of the sum of the signs
For an analysis of the average amount of adversary knowledge of support locations can be indicative of training features, the effectiveness of many white-box privacy attacks on distributed ML systems has only been demonstrated for exact, real-valued model updates, and not for model update support. In the case of computer-vision models, such a security perimeter may be acceptable to the system designer.

PartialSecUnion In this protocol, each client will create two Boolean vectors $B_i$ representing their list of indices $V_i$ such that $\forall k \in [1, N]$, 

$$B_i[k] = \begin{cases} 1, & \text{if } k \in V_i, \\ 0, & \text{otherwise} \end{cases}. \quad (9)$$

Using this representation, the clients can securely calculate the union of indices through the use of Alg. 1 by evaluating the sum of their Boolean vectors $B_i$ in $\mathbb{Z}^N_{C+1}$. At the end of the secure sum evaluation, each client will obtain $B = \sum_{i=1}^C B_i \mod (C + 1)$, from which they may easily infer the union of indices $V$ by selecting the positions of non-zero values in $B$ and the servers learn nothing. In addition, $B$ reveals to the clients how many clients have non-zero values at a given index, however it does not reveal which clients.

SecUnion Here, each client will create a vector $A_i$ representing its list of indices $V_i$ such that $\forall k \in [1, N]$, 

$$A_i[k] = \begin{cases} R_{i,k}, & \text{if } k \in V_i \text{ where } R_{i,k} \in \mathbb{Z}_2, \\ 0, & \text{otherwise} \end{cases}. \quad (10)$$

Subsequently, the clients will securely evaluate the sum of the vectors $A_i$ in $\mathbb{Z}_2^N$ using Alg. 1. At the end of the secure evaluation, each client will obtain $A = \sum_{i=1}^C A_i \mod 2^q$ from which each client can easily infer the union of indices by selecting the positions of non-zero values.

When using the SecUnion protocol, the resulting union of indices may have some false negatives. Specifically, when more than two clients have the same index, the sum of random values $R_{i,k}$ will be equal to 0 with probability $1/2^q$, causing $k$ to be omitted from $V$ when it should be present. Increasing $q$ can reduce the occurrence of such false negatives, but at the cost of increased communication cost. We refer to Appendix C for an analysis of the average amount of false negatives. We note that, in experimentation (see Tab. 3), the training with our SecUnion protocol is observed empirically to be quite robust to such dropped support.

PlaintextUnion Here, the clients will only learn the union of indices, however the first server will learn the list of indices of each client. In this protocol, each client sends to the first server the Boolean vector $B_i$ representing their list of indices. The first server evaluates the union of indices by OR-ing the received Boolean vectors and transmitting back the resulting Boolean vector to each client.

While such a protocol is not advisable for every setting, especially for NLP tasks or recommender systems, where the knowledge of support locations may be indicative of training features, the effectiveness of many white-box privacy attacks on distributed ML systems has only been demonstrated for exact, real-valued model updates, and not for model update support. In the case of computer-vision models, such a security perimeter may be acceptable to the system designer.

NoUnion If the compression rate is very low, the split of the sum of signs into a union of indices and a sum on restricted signs vectors is not beneficial. In this case, it is more efficient to perform the secure aggregation in two steps: i) the secure sum of signs vectors $D_i$ directly, as $V = [1, N]$, and ii) the secure sum of scalar factors.

3.2.3 Secure Sum of Signs

In the second step of our compressed secure aggregation protocol, each client restricts its dense vector of ternary values $D_i$ to the support resulting from the secure union of indices $V$, and then together they securely evaluate the sum of these binary vectors of signs, $(D_i)_V$. More precisely, the parties would like to securely evaluate the sum of $C$ vec-

| Protocol                  | Communication Cost (bits) | Clients Learn (besides $V$) | Servers Learn             |
|---------------------------|---------------------------|-----------------------------|----------------------------|
| PlaintextUnion            | $2 \cdot C \cdot N$       | Nothing                     | Clients’ list of indices $V_i$ |
| PartialSecUnion           | $2 \cdot S \cdot C \cdot N \cdot \lceil\log_2(C+1)\rceil$ | How many clients have selected each index | Nothing |
| SecUnion                  | $2 \cdot S \cdot C \cdot N \cdot q$ | If they alone selected each index | Nothing |

Table 1. Comparison of secure union protocols for the proposed secure aggregation.
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| Protocol                                      | Sec. Agg. | Communication Cost per Federated Round (bits) |
|-----------------------------------------------|-----------|-----------------------------------------------|
| Uncompressed FedAvg                          | ✗         | $2 \cdot C \cdot 32 \cdot N$                  |
| FedAvg w/ TopBinary & DirectAgg              | ✗         | $C \cdot (32 \cdot (1 + |V|) + 2 \cdot N + [\rho \cdot N])$ |
| FedAvg w/ TopBinary & SepAgg                 | ✗         | $C \cdot (2 \cdot 32 + 2 \cdot N + [\rho \cdot N] + |V| \cdot [\log_2(2C + 1)])$ |
| Our method w/ NoUnion                        | ✓         | $(2 \cdot S \cdot C \cdot N \cdot [\log_2(2C + 1)]) + (2 \cdot S \cdot C \cdot 32)$ |
| Our method w/ SecUnion                       | ✓         | $(2 \cdot S \cdot C \cdot N \cdot q) + (2 \cdot S \cdot C \cdot |V| \cdot [\log_2(2C + 1)]) + (2 \cdot S \cdot C \cdot 32)$ |
| Our method w/ PartialSecUnion                | ✓         | $(2 \cdot S \cdot C \cdot N \cdot [\log_2(C + 1)]) + (2 \cdot S \cdot C \cdot |V| \cdot [\log_2(2C + 1)]) + (2 \cdot S \cdot C \cdot 32)$ |
| Our method w/ PlaintextUnion                 | ✓         | $C \cdot (2^C \cdot a_K + (5C - 4)a_S + 2N[\log_2 T])$ |
| (Bonawitz et al., 2017)                       | ✓         | $2SCN[\log_2(2C + 1)]$                       |
| (Dong et al., 2020) with TSS                  | ✓         | $4CN(\text{prec} + \text{pad})$               |
| (Dong et al., 2020) with HE                   | ✓         |                                                |

Table 2: Communication costs per federated round for the different protocols where $|V|$ is the size of the union of indices, for (Bonawitz et al., 2017), $a_K = a_S = 256$ are the number of bits in a public key and in an encrypted share, all operations are evaluated in $\mathbb{Z}_T$ with $T = 2^{32}$, for (Dong et al., 2020), prec = 24 and pad = 8 are HE parameters used to avoid overflows.

3.3 Communication Costs

In this section, we estimate the total communication cost per federated round of FedAvg without secure aggregation and of our proposed method with secure aggregation, as summarized in Table 2. We choose a 32-bit floating-point representation for all “unquantized” real values.

3.3.1 Non-secure FedAvg

For non-secure uncompressed FedAvg, each client sends its updates to one server (32 · $N$ bits) and then the server sends the aggregated updates to each client (32 · $N$ bits). Thus, the total communication cost for one aggregation is equal to $2 \cdot C \cdot 32 \cdot N$ bits.

For non-secure FedAvg with TopBinary compression, each client sends to one server its factor $\alpha$ (32 bits), its list of non zero indices $V$ ($N$ bits) and the signs of the coefficients at those non-zero locations $(D)_V ([\rho \cdot N]$ bits). Let $|V|$ the size of the union of indices $V$. With DirectAgg, the server sends to each client the aggregated updates composed of the list of the non-zero indices ($N$ bits) and the coefficients at those non-zero locations (32 · $|V|$ bits). For SepAgg, the server sends to each client the aggregated updates composed of the aggregated factor (32 bits) and the aggregated vector of directions ($N + |V| \cdot [\log_2(2C + 1)]$ bits).

3.3.2 Our method with secure aggregation

We can easily evaluate the total communication cost in bits of each sub-protocol of our secure aggregation from Eq. (3):

$$2 \cdot S \cdot C \cdot N \cdot [\log_2(C + 1)]$$ (PartialSecUnion)
$$2 \cdot S \cdot C \cdot N \cdot q$$ (SecUnion)
$$2 \cdot C \cdot N$$ (PlaintextUnion)
$$2 \cdot S \cdot C \cdot |V| \cdot [\log_2(2C + 1)]$$ (Sum of Signs)
2 \cdot S \cdot C \cdot 32 \quad \text{(Sum of Factors)}

where $|V|$ is the size of the union of indices $V$.

4 RELATED WORK

To date, proposed secure aggregation schemes in FL have relied on both SMC and HE approaches. Secure aggregation methods based on HE (Truex et al., 2019; Zhang et al., 2020) require a single round of communication, but also require computationally expensive cryptographic operations. On the contrary, secure aggregation methods based on SMC (Bonawitz et al., 2017; Xu et al., 2019; Bell et al., 2020) have a lower computational cost, but a higher communication cost.

A secure aggregation protocol based on Threshold Secret Sharing (TSS) is introduced in (Bonawitz et al., 2017). Contrary to our proposed method (see Section 3), this protocol requires the use of a single aggregation server, this server learns the aggregated models and this protocol is resilient against clients dropout. With respect to computational requirements, this protocol relies on key agreements, asymmetric cryptography, and the use of cryptographically-secure pseudo-random number generators (PRNG). These cryptographic operations are computationally expensive and increase the computational burden for each participating party. To be resilient against dropouts, in the protocol of (Bonawitz et al., 2017), public keys and encrypted shares are routed from one client to another through the central coordination server, incurring extra communication costs. More precisely, for one round, the total communication cost is

$$C \cdot (2C \cdot a_K + (5C - 4)a_S + 2N \lceil \log_2 T \rceil) \text{ bits,}$$  \hspace{1cm} (13)

where $a_K = a_S = 256$ are the number of bits respectively in a public key and in an encrypted share and all operations are evaluated in $\mathbb{Z}_T$ with $T = 2^{32}$. We refer to Appendix A for more details about this communication cost. The main drawbacks of this approach are its high computation and communication costs and the fact that aggregated updates are revealed to the aggregator server.

This line of work has been extended by (Bell et al., 2020). In (Bonawitz et al., 2017), each client speaks to all clients through the central server to share public keys and encrypted shares. In (Bell et al., 2020), they prove that each client is only required to share public keys and encrypted shares with part of the clients. That permits to reduce the communication and computation costs while keeping the privacy guarantees. For instance, if they have $10^8$ clients, each client is only required to speak to $150$ clients. Unfortunately, this improvement is not applicable to collaborative setting where we have less than $10$ clients (see Theorem 3.10 in (Bell et al., 2020)). In collaborative setting, each client is required to speak with all clients.

To the best of our knowledge, only one proposal combines neural network update compression with secure aggregation (Dong et al., 2020). In this work, the authors make use of TernGrad (Wen et al., 2017) for model update compression and subsequently suggest two methods to perform the secure aggregation: one based on HE and the second based on TSS. A vector compressed with TernGrad consists of the multiplication of a scalar factor by a ternary vector with values in $\{-1, 0, 1\}$. In their secure aggregation, the authors propose to securely evaluate the sum of the ternary vectors and subsequently each client updates their local model from this aggregated ternary vector and according to their own local factor. With this aggregation, contrary to traditional FL methods, each client obtains a different intermediate model at the beginning of each federated round and at the end of training. It is not explicit how these different models should be used or aggregated in one model. Contrary to this approach, we focus on the federated training of a single model across decentralized clients. With respect to computation cost, the authors propose two different approaches to perform the secure aggregation, each of which requires its own flavor of costly cryptographic procedures. For the version based on TSS, the implementation requires the evaluation of some polynomial interpolations, while their version based on Paillier requires some expensive homomorphic encryption and decryption operations. For communication costs, the authors do not take into account the sparsity of the ternary vector and thus, their communication cost is not optimal with respect to the significant information contained in the model update. More precisely, the total communication cost in bits per round for their secure aggregation is equal to

$$2SCN[\log_2(2C + 1)] \quad \text{(for TSS-based Secure Agg.)}$$

$$CN(\text{prec} + \text{pad}) \quad \text{(for HE-based Secure Agg.)}$$

where $\text{prec} = 24$ and $\text{pad} = 8$ are HE parameters used to avoid overflows.

In our proposed method, we adapt a compressed FL learning strategy to be more amenable to secure computation in order to obtain a secure aggregation which has a lower communication and computation cost than reported in (Bonawitz et al., 2017; Dong et al., 2020). Table 2 summarizes the communication costs of the secure aggregations from (Bonawitz et al., 2017; Dong et al., 2020).

5 EXPERIMENTS

In this section, we perform experiments on $C = 5$ clients and $S = 2$ servers, leading to a moderately sized collaborative federated system, representative of e.g. healthcare FL applications.


| Strategy                          | Sec. Agg. | Parameters       | Metrics          |
|----------------------------------|-----------|------------------|------------------|
|                                  |           | E    R   ρ E | [V] Comp. cost | Comm.       |
| Uncompressed FedAvg              | X         | 100  15 - - | + 35.31MB      |
| FedAvg w/ TopBinary & DirectAgg  | X         | 100  17 0.10 - | + 7.23MB       |
| FedAvg w/ TopBinary & SepAgg     | X         | 100  17 0.10 18 253 | + 2.05MB       |
| (Bonawitz et al., 2017)          | ✓         | 100  15 - - | ++ 35.37MB      |
| (Dong et al., 2020) w/ TSS       | ✓         | 100  17 - - | ++ 10.00MB      |
| (Dong et al., 2020) w/ HE        | ✓         | 100  17 - - | ++ 80.03MB      |
| Our method w/ NoUnion            | ✓         | 100  17 0.10 - | + 10.01MB      |
| Our method w/ SecUnion (q=1)     | ✓         | 100  22 0.10 14 344 | + 6.25MB      |
| Our method w/ SecUnion (q=5)     | ✓         | 100  17 0.10 18 037 | + 15.43MB      |
| Our method w/ PartialSecUnion    | ✓         | 100  17 0.10 18 253 | + 10.46MB      |
| Our method w/ PlaintextUnion     | ✓         | 100  17 0.10 18 253 | + 4.21MB       |

(a) LeNet on MNIST (98% Acc. Target)

| Strategy                          | Sec. Agg. | Parameters       | Metrics          |
|----------------------------------|-----------|------------------|------------------|
|                                  |           | E    R   ρ E | [V] Comp. cost | Comm.       |
| Uncompressed FedAvg              | X         | 100  101 - - | + 6.61GB       |
| FedAvg w/ TopBinary & DirectAgg  | X         | 100  110 0.10 - | + 1.60GB       |
| FedAvg w/ TopBinary & SepAgg     | X         | 100  110 0.10 663 630 | + 0.41GB       |
| (Bonawitz et al., 2017)          | ✓         | 100  101 - - | ++ 6.61GB       |
| (Dong et al., 2020) w/ TSS       | ✓         | 100  110 - - | ++ 1.80GB       |
| (Dong et al., 2020) w/ HE        | ✓         | 100  110 - - | ++ 14.40GB      |
| Our method w/ NoUnion            | ✓         | 100  110 0.10 - | + 1.80GB       |
| Our method w/ SecUnion (q=1)     | ✓         | 100  114 0.10 574 599 | + 1.08GB       |
| Our method w/ SecUnion (q=5)     | ✓         | 100  110 0.10 657 164 | + 2.92GB       |
| Our method w/ PartialSecUnion    | ✓         | 100  110 0.10 663 630 | + 2.03GB       |
| Our method w/ PlaintextUnion     | ✓         | 100  110 0.10 663 630 | + 0.90GB       |

(b) AlexNet on CIFAR-10 (80% Acc. Target)

Table 3. Total communication cost to reach the targeted accuracy with FedAvg strategy and our approach on five clients.

5.1 Datasets and Model Architectures

To demonstrate the utility of our proposed approaches, we conduct a series of image classification experiments on MNIST (LeCun & Cortes, 2010) and CIFAR-10 (Krizhevsky et al.). For client data, we use i.i.d. dataset partitions to mimic clients holding data samples from the same data distribution. The MNIST dataset is composed of a 60k training image dataset and a 10k test image dataset. For MNIST experiments, the training dataset is randomly shuffled and then partitioned into a training dataset of 50k samples and a validation dataset of 10k samples. Then, the training dataset is partitioned into five sets, and each is given to one of the C = 5 clients, thus we have 10k samples per client. We similarly partition the 40k training dataset of CIFAR10.

For our MNIST experiments, we train a simple LeNet-5 network (Lecun et al., 1998) which consists of N = 61,706 trainable parameters. This architecture is detailed in Table 6 in Appendix D. For CIFAR-10, we train the same AlexNet (Krizhevsky et al., 2017) architecture as demonstrated in the original FedAvg work (McMahan et al., 2017), which consists of N = 1,756,426 trainable parameters. This architecture is detailed in Table 7 in Appendix D. We also utilize the same image preprocessing pipeline as was proposed in this work, e.g. random flipping, and color normalization.

The goal of our experiments is to demonstrate that with our proposed method, compared to FedAvg (McMahan et al., 2016) with no compression, we obtain a computation and communication efficient secure aggregation without compromising accuracy. We note that more modern neural architectures could be utilized, as well as larger datasets across more varied tasks. However, as our approach is agnostic to the underlying task, we utilize these simpler ML experiments for the sake of clarity of our demonstration.

1Such homogeneity cannot be expected in practice but data heterogeneity is out of the scope of the current work.
5.2 Hyperparameter Selection

To select appropriate local model training hyperparameters, we perform a grid search on the entire training dataset without FL (i.e., a single client), evaluating performance on the held-out validation dataset. For MNIST, we obtained an accuracy of 99.2\% with a learning rate of 0.01, a batch size of 64 and a momentum of 0.9. For CIFAR-10, we obtained an accuracy of 80.6\% with a learning rate of 0.10, a batch size of 64 and no momentum. Although this approach cannot be utilized in a practical FL use-case, we use it for our FL experiments and defer optimal hyperparameter tuning to future works. Indeed, we stress that thanks to the communication efficiency of the proposed approach, hyperparameter optimisation in FL could be eased by the proposed method.

5.3 Targeted accuracy

One of main constraints is to not compromise the prediction performance obtained using our proposed techniques in comparison to non-secure non-compressed FedAvg. The accuracy obtained with non-secure non-compressed FedAvg training on five clients with the selected hyperparameters from Sec 5.2 is equal to 99.0\% for MNIST (with $E = 100$ and 50 epochs) and 81.3\% for CIFAR-10 (with $E = 100$ and 150 epochs). From those performances, we selected a desired performance level to achieve: 98\% accuracy for MNIST and 80\% accuracy for CIFAR-10.

5.4 Results

We conduct several ML training experiments to define the number of federated rounds $R$ required to obtain the targeted accuracy (98\% for MNIST and 80\% for CIFAR-10) for FedAvg and our proposed methods. For experiments with TopBinary coding, we set $\rho = 0.1$ and we also report the mean size of the aggregated union of indices over the course of federated training, $|V|$. For the method presented in (Bonawitz et al., 2017), no update compression is used and thus, the number of federated round $R$ to reach the targeted accuracy is equal to that of uncompressed FedAvg. The method presented in (Dong et al., 2020) uses the TermGrad model-update compression technique, which is similar to our TopBinary coding. Thus, we assume that this technique with TermGrad compression requires a similar number of federated rounds to obtain the same level of accuracy as our proposed techniques. We recall that these approaches introduced in (Dong et al., 2020) cannot be used in practice because at the end of the FL training, each client obtains a different trained models and we do not know how to aggregate these models into one model without compromising accuracy.

To compare the computation cost, we compare the cost of the most expensive operations. For non-secure aggregations and our proposed methods, only additions (noted “++” in Table 3) are performed. (Dong et al., 2020) with TSS is based on polynomial interpolation (noted “++” in Table 3) which is more expensive than additions. (Dong et al., 2020) with HE and (Bonawitz et al., 2017) are based on the most expensive operations respectively Paillier HE and asymmetric encryption (noted “+++” in Table 3).

Finally, we analytically estimate the communication costs associated with these strategies (see Table 2), the results of which are detailed in Table 3. These experimental results showcase the low total communication cost of our proposed compressed secure aggregation approach compared to other techniques with or without secure aggregation. This communication cost can be further reduced with more fine tuning, as higher compression rates (i.e., $\rho < 0.02$) may be achievable for the same performance level (Sattler et al., 2018).

We emphasize that our SecUnion protocol is not correct, since some indices are missing in the union result. In this protocol, the parameter $q$ is used to manage this amount of false negatives: higher is $q$, smaller is the amount of false negatives. With $q = 1$, we observed that many selected gradients are not used during the secure aggregation due to the incorrectness of the SecUnion protocol. Among all our experiments, our method with SecUnion ($q = 1$) achieves the best trade-off between privacy and communication cost. In addition, this strategy has a lower communication cost than (Bonawitz et al., 2017; Dong et al., 2020).

6 Conclusion

In this article, we design a set of secure and efficient protocols for federated learning. These protocols are based on quantization to highly reduce the communication cost of the aggregation without compromising accuracy. We adapt quantized federated learning techniques to secure computation in order to be able to efficiently and securely evaluate the aggregation steps during a federated training. Thanks to these adaptations, our secure aggregation protocols solely rely on additions, without any expensive cryptographic operations, and thus have a very low computation cost. Since our secure aggregations are only based on additions, it can be easily implemented by non-cryptographic experts. We also prove that our protocols are private against malicious adversaries and correct against semi-honest adversaries. Finally, our experiments show their efficiency in real use cases. Compared to prior works, they have a lower computation cost, as no expensive operations are used, and a lower communication cost thanks to model update compression and lightweight SMC protocols.

Future works could investigate the interplay between the
proposed protocols and heterogeneity, which is another important challenge for FL. Other compression approaches could also be used to further reduce the communication cost. Finally, there still exist large communication differences between the different security profiles of the secure union techniques we propose. It would be of interest to find better trade-offs in the security perimeter of such protocols against communication requirements.

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A Practical Secure Aggregation (Bonawitz et al., 2017)

Communication Cost

The communication cost of the Practical Secure Aggregation protocol is presented in Section 7.1 of (Bonawitz et al., 2017). For each federated round, each client transmits to the server

\[ 2n \cdot a_K + (5n - 4) \cdot a_S + m \log_2 R \] bits,

where \( n \) is the number of clients, \( m \) is the number of trainable parameters, \( a_K = 256 \) is the number of bits in a public key, \( a_S = 256 \) is the number of bits in an encrypted share, and \( R \) is the field in which all operations are evaluated. And at the end of the federated round, the server transmits the aggregated model parameters to each client at a cost of \( m \log_2 R \) bits. Thus, the total communication cost for one federated round is equal to

\[ n \cdot (2n \cdot a_K + (5n - 4) \cdot a_S + 2m \log_2 R) \] bits.

Using our notation, the total communication cost for a single federated round is thus given as

\[ C \cdot (2C \cdot a_K + (5C - 4) \cdot a_S + 2N \log_2 R) \] bits,

where \( C \) is the number of clients, \( N \) is the number of trainable parameters and \( R \) is the field in which all operations are evaluated.

B Further details on TopBinary Aggregation

Let us recall that we want to operate on the aggregation of some real-valued vectors \( X_i \in \mathbb{R}^N \), of which we have \( C \) different realizations. From each of these vectors, we calculate one new vector \( D_i = \text{sgn} \circ \text{Top}-k(X_i) \) and one scalar value \( \alpha_i = \frac{||X_i||}{||D_i||} \), such that \( ||\alpha_i D_i|| = ||X_i|| \). Because of the \text{sgn} \circ \text{Top}-k operation, the result \text{sgn} vector belongs to the space \( D_i \in \{-1, 0, 1\}^N \). Now, since the value of \( k \) is fixed, we know, trivially, the number of non-zero entries: \( k \). And, since the non-zero entries are \( \pm 1 \), we know \( ||D_i|| = \sqrt{k} \) \( \forall i \), thus \( \alpha_i = \frac{1}{\sqrt{k}} ||X_i|| \).

Let us now define the different variants of the aggregation again,

\[ \tilde{D} = \frac{1}{C} \sum_{i=1}^C \alpha_i D_i = \frac{1}{C \sqrt{k}} \sum_{i=1}^C L_i D_i, \quad \text{(DirectAgg)} \]

\[ \hat{D} = \frac{1}{C^2} \sum_{i=1}^C \sum_{i=1}^C \alpha_i \cdot D_i = \frac{1}{C \sqrt{k}} \cdot \frac{1}{C} \sum_{i=1}^C L_i \cdot \sum_{i=1}^C D_i, \quad \text{(SepAgg)} \]

where \( L_i \hat{=} ||X_i||_2 \) is the Euclidean norm of the original vector \( X_i \). Our desire is to compute the value of \( \tilde{D} \), however, for the efficiency of SMC, we may only make use of the estimate \( \hat{D} \). Given this, how far off will we be, and what are the effects of using such an estimate?

Let \( \bar{L} \hat{=} \frac{1}{C} \sum_{i=1}^C L_i \), i.e. the average norm of all of the original vectors \( X_i \), then

\[ \bar{D} = \frac{\bar{L}}{\sqrt{k}} \cdot \frac{1}{C} \sum_{i=1}^C D_i = \frac{1}{C \sqrt{k}} \sum_{i=1}^C \bar{L} D_i, \]

where we can see that the average norm \( \bar{L} \) serves as an approximation for the individual norms \( L_i \). We now evaluate the MSE difference between the \text{SepAgg} and \text{DirectAgg} as

\[ ||\hat{D} - \bar{D}||^2 = \frac{1}{C^2} \sum_{j=1}^N \left( \sum_{i=1}^C ((L_i - \bar{L}) D_i[j]) \right)^2, \]

which shows us that if the expectation of the difference \( L_i - \bar{L} \) is small, we can expect that the \text{SepAgg} will be an adequate estimation of the direct aggregation.

C Further details on SecUnion protocol

For the analysis of the SecUnion protocol, let us assume that each client \( i \) has selected independently and uniformly at random their list of \( k \) indices \( V_i \subset \{1, N\} \), then, for each client the probability of selecting the index \( x \in \{1, N\} \) is equal to \( k/N \). Thus, the probability that exactly \( t \) clients have selected the index \( x \) is

\[ P(N, k, C, t) = \binom{C}{t} \left( \frac{k}{N} \right)^t \left( \frac{N-k}{N} \right)^{C-t} \]

C.1 Expected number of false negatives

When using the SecUnion protocol, false negatives occur only when at least 2 clients have selected the same index \( x \) and \( \sum_{i=1}^C A_i[x] \mod 2^q \) is equal to zero. The probability that at least 2 clients have selected the same index \( x \) is equal to \( 1 - P[N, k, C] = 1 - P[N, k, C, 0] - P[N, k, C, 1] \). When one index is selected by at least 2 clients, the probability to obtain a false negative for this index is equal to \( 1/2^q \). Thus, the expected number of false negatives in \( V \) is given by \( N \times P[t \geq 2 \mid N, k, C] \times \frac{1}{2^q} \). The Table 4 presents the average amount of false negatives in the SecUnion protocol for different parameters.

C.2 One client learns that he alone has selected a given index

Let us assume the client \( i \) observes that \( A_i[x] = R_{ix} \) for one index \( x \). The client \( i \) is not alone to have selected this
Table 4. Average amount of false negatives $E$ in the SecUnion protocol.

| $N$  | $\rho$ | $k = \lfloor N \cdot \rho \rfloor$ | $C$ | $q$ | $E$  |
|------|--------|----------------------------------|----|----|-----|
| 61706 | 0.1    | 6170 | 5 | 1 | 2513 |
| 61706 | 0.1    | 6170 | 5 | 5 | 157  |
| 61706 | 0.1    | 6170 | 5 | 10| 5   |
| 1756426 | 0.1 | 175642 | 5 | 1 | 71539 |
| 1756426 | 0.1 | 175642 | 5 | 5 | 4471 |
| 1756426 | 0.1 | 175642 | 5 | 10| 140  |

Table 5. If one client $i$ observes that $A[x] = R_{i,x}$, we evaluate the probability that this client alone has selected this index $x$.

| $N$ | $\rho$ | $k = \lfloor N \cdot \rho \rfloor$ | $C$ | $q$ | Probability |
|-----|--------|----------------------------------|----|----|-------------|
| 61706 | 0.1    | 6170 | 5 | 1 | 97.39% |
| 61706 | 0.1    | 6170 | 5 | 5 | 98.94% |
| 61706 | 0.1    | 6170 | 5 | 10| 99.99% |
| 1756426 | 0.1 | 175642 | 5 | 1 | 97.39% |
| 1756426 | 0.1 | 175642 | 5 | 5 | 99.84% |
| 1756426 | 0.1 | 175642 | 5 | 10| 99.99% |

Table 6. Architecture of the LeNet used for experiments on MNIST.

| Layer Type         | Filters | Kernel | Stride | Padding | Output Shape |
|--------------------|---------|--------|--------|---------|--------------|
| Conv2D+ReLU        | 6       | (5,5)  | (1,1)  | (2,2)   | 28 × 28 × 6  |
| Max Pooling        | (2,2)   |        |        |         | 14 × 14 × 6  |
| Conv2D+ReLU        | 16      | (5,5)  | (1,1)  | (2,2)   | 10 × 10 × 16 |
| Max Pooling        | (2,2)   |        |        |         | 5 × 5 × 16   |
| FC+ReLU            |         |        |        |         | 120          |
| FC+ReLU            |         |        |        |         | 84           |
| FC                 |         |        |        |         | 10           |

Table 7. Architecture of the AlexNet used for experiments on CIFAR-10. The LocalResponseNorm layers have the following parameters: size = 4, $\alpha = 0.001/9$, $\beta = 0.75$, $k = 1$.

### E Privacy Proof of SecureSum Algorithm

**Theorem 2.** The SecureSum Protocol is private against a malicious adversary controlling at most $S - 1$ servers and any number of clients.

The privacy proof of this theorem is mainly based on the following lemma.

**Lemma 1.** Let $b \in \mathbb{Z}_m$. If $r$ is a uniform random element in $\mathbb{Z}_m$, then $b + r$ is also a uniform random element in $\mathbb{Z}_m$ (even if $b$ is not a uniform random element in $\mathbb{Z}_m$).

We will now prove Theorem 2.

**Proof.** Let $A$ be an adversary controlling $p < S$ servers, $Server_{i_1}, \ldots, Server_{i_p}$, and $q \leq C$ clients, $Client_{i_{q+1}}, \ldots, Client_{i_s}$. In this proof, we will use the following notations for different sets of servers and clients, $\{J = \{j_1, \ldots, j_p\}\}$ and $\{I = \{i_1, \ldots, i_q\}\}$ and $\{\overline{J} = \{j \in [1,S] \text{ and } j \notin J\}\}$ and $\{\overline{I} = \{i \in [1,C] \text{ and } i \notin I\}\}$.

We will prove the privacy of our protocol in the ideal-real paradigm (Lindell, 2017). We will first describe the simulator $S_A$, who simulates the view of the adversary $A$ in the ideal world. Then, we will prove that the views of the adversary $A$ in the ideal world and in the real world are indistinguishable. We will split the proof in three cases, depending on the number of clients the adversary controls: I. no clients ($q = 0$), II. at least one and up to $C - 2$ clients ($1 \leq q \leq C - 2$), and III. all clients, or all clients except one ($C - 1 \leq q \leq C$).

**CASE 1:** $q = 0$

**Simulation.** $S_A$ picks $p \times C$ uniformly random vectors $R_{i,j}, \forall (i, j) \in [1, C] \times J$ into $\mathbb{Z}_m^n$ and sends them to the adversary $A$.

**Indistinguishability of Views.** We will now prove that the views of $A$ in the real world and in the ideal world are indistinguishable. In the real world, the adversary $A$ receives
from honest parties the following messages

\[ \forall (i, j) \in [1, C] \times J, R_{i,j}. \quad (17) \]

If \( S \not\in J \), then by construction, \( \forall (i, j) \in [1, C] \times J \), \( R_{i,j} \) are independently and uniformly random in \( \mathbb{Z}_m^n \) (see L.2 in Alg. 1). Thus, the views of the adversary \( A \) in the real world and in the ideal world are indistinguishable.

If \( S \in J \), since \( J \subseteq [1, S] \), then \( [1, S - 1] \cap J \) is not empty. Let \( j^* \) be one element of \( [1, S - 1] \cap J \), and with it rewrite \( R_{i,S} \) as

\[ R_{i,S} = \left( X_i - \sum_{j \in [1, S - 1] \setminus j^*} R_{i,j} \right) - R_{i,j^*} \mod m. \quad (18) \]

By construction, \( \forall i \in [1, C] \), the values \( R_{i,j^*} \) are uniformly random in \( \mathbb{Z}_m \) (see L.2 in Alg. 1). By applying Lemma 1, \( \forall i \in [1, C] \), the values \( R_{i,S} \) are uniformly random in \( \mathbb{Z}_m \).

This proves that all messages received by \( A \) are independently and uniformly random in \( \mathbb{Z}_m \), and thus, the views of the adversary \( A \) in the real world and in the ideal world are indistinguishable.

**CASE II: \( 1 \leq q \leq C - 2 \)**

**Simulation.** \( S_A \) picks \( (C - q) \times p + (S - p) \) uniformly random vectors into \( \mathbb{Z}_m^n \) noted \( \tilde{R}_{i,j}, \forall (i, j) \in \tilde{T} \times J \) and \( R_j, \forall j \in J \) and sends them to the adversary \( A \).

**Indistinguishability of Views.** In the real world, the adversary \( A \) receives from honest parties the following messages:

\[
\begin{cases}
R_{i,j}, & \forall (i, j) \in \tilde{T} \times J \\
R_j, & \forall j \in J
\end{cases}
\quad (19)
\]

With a similar proof to CASE I, we can demonstrate that \( \forall (i, j) \in \tilde{T} \times J \), the values \( R_{i,j} \) are independently and uniformly random in \( \mathbb{Z}_m^n \). Now, let \( i^* \) be an element in \( \tilde{T} \), where \( \tilde{T} \) is not empty because the adversary controlled at most \( (C - 2) \) clients. Then, \( \forall j \in J \), we can rewrite \( R_j \) as

\[ R_j = \left( \sum_{i \in [1, C] \setminus i^*} R_{i,j} \right) + R_{i^*,j} \mod m. \]

By construction, for all \( j \in J \), \( R_{i^*,j} \) are independently and uniformly random in \( \mathbb{Z}_m^n \) (see L.2 in Alg. 1). By applying Lemma 1, \( \forall j \in J \), the values \( R_j \) are uniformly random in \( \mathbb{Z}_m^n \).

This proves that all messages received by \( A \) are independently and uniformly random in \( \mathbb{Z}_m^n \), and thus, the views of the adversary \( A \) in the real world and in the ideal world are indistinguishable.

**CASE III: \( C - 1 \leq q \leq C \)**

Assume that the adversary controls all the clients \( (q = C) \). Then, the adversary knows all inputs and outputs. Thus, our protocol is private according to the privacy definition outlined in Sec. 2.3.

Now assume that the adversary controls all clients *except* the client \( i^* \). Then, the inputs of the parties controlled by the adversary are \( \{X_i\}_{i \in [1, C] \setminus i^*} \) and the outputs of the parties controlled by the adversary are \( X = \sum_{i \in [1, C] \setminus i^*} X_i \). The adversary can easily infer the input of the client \( i^* \) as \( X_{i^*} = X - \sum_{i \in [1, C] \setminus i^*} X_i \mod m \). Thus, the adversary cannot learn anything more, because they already know everything. Thus, our protocol is private according to the privacy definition in Sec. 2.3.

\[ \square \]

We just proved that the SecureSum protocol (Alg. 1) is private against a malicious adversary controlling at most \( (S - 1) \) servers and any number of clients. That means that a malicious adversary cannot learn more than what he can infer from the inputs and outputs of the parties they control. In practice, if the adversary controls all clients, the adversary already knows everything. If the adversary controls all clients *except one*, he can deduce the inputs of this honest client from the inputs and outputs of the clients that are under his control. If the adversary controls at most \( (C - 2) \) clients, the adversary can infer the sum of the inputs of the honest clients from the inputs and outputs of the clients that he controls, but not the individual inputs of the honest parties. In this manner, the security of the individual clients is preserved, however the sum of vectors of honest parties is not.