Modes of nanosatellite aerodynamic oscillations in atmosphere

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Abstract. The paper is devoted to the results of investigating the dependencies of nanosatellite aerodynamic oscillations frequency on attack angle at different altitudes up to 70 km are defined. The oscillations bandwidths are determined with respect to the geometric parameters for a nanosatellite with 10 kg mass and 6000 kg/m³ average density. The model allows estimating the bandwidth aerodynamic oscillations in the suborbital nanosatellite trajectory based on the given geometry and mass-dimensional parameters.

1. Introduction

The paper investigates the aerodynamic oscillations bandwidth of an axisymmetric high-fineness-ratio configuration body with an aerodynamic skirt in a hypersonic flow. This objective is relevant in the context of the nanosatellite electromagnetic launch [1]. This research is concerned with suborbital motion at altitudes up to 70 km. The nanosatellite aerodynamic configuration is shown in Fig. 1.

![Figure 1. Nanosatellite aerodynamic configuration.](image)

The nanosatellite aerodynamic coefficients in hypersonic flow in the body-fixed coordinate system are estimated. The aerodynamic coefficients are calculated using the Newtonian flow theory [2]. The corresponding formulas for the coefficients of the axial force, normal force and pitching moment are:

\[
C_R = \frac{2}{S_{mid}} \int_0^l d\cdot r \cdot \tan \beta \int_0^{\gamma_{stag}} d\gamma \cdot \overline{p} ;
\]

\[
C_N = \frac{2}{S_{mid}} \int_0^l d\cdot r \int_0^{\gamma_{stag}} d\gamma \cdot \overline{p} \cdot \cos \gamma ;
\]

\[
m_z = \frac{2}{l \cdot S_{mid}} \int_0^l d\cdot r \cdot x \int_0^{\gamma_{stag}} d\gamma \cdot \overline{p} \cdot \cos \gamma ;
\]

(1)
where  
\( r \) is the local radius of a rotation body;
\( l \) is the nanosatellite length;
\( \bar{p} = 2(\sin \beta \cdot \cos \alpha - \sin \alpha \cdot \cos \beta \cdot \cos \gamma)^2 \) \hspace{1cm} \text{(2)}

is the surface pressure coefficient;
\( \beta \) is a local angle of the streamlined surface to the nanosatellite axis;
\( \alpha \) is an angle of attack;
\( \gamma \) is an integrating parameter equal to the angle between the attack angle plane and the meridian plane. The range \( \gamma > \gamma_{\text{Shadow}} \) corresponds to the aerodynamic shadow area, it is marked by shading in Fig. 1a.

The external moment in relation to the center of mass is calculated using the formula below [2]:
\[
M_z = \left( m_z - C_N \cdot \frac{X_{CM}}{l} \right) \cdot l \cdot S_{\text{mid}} \cdot q,
\]

where \( X_{CM} \) is the center of mass position relative to nanosatellite forebody;
\( q = \frac{\rho \cdot V^2}{2} \) is dynamic pressure.

We assume the speed of nanosatellite to be \( V = 8000 \text{ m/s} \), and the atmosphere parameters correspond to those at 30 km altitude. In accordance with [3], \( \rho = 1.84 \cdot 10^{-2} \text{ kg/m}^3 \), so the dynamic pressure is \( q = 588 \text{ kPa} \).

The following nanosatellite parameters are used in the work: head cone aspect ratio is assumed to be constant \( X_1 = 4 \text{ d}_{\text{mid}} \); the length of cylindrical part is \( X_{\text{cil}} = 20 \text{ d}_{\text{mid}} \); the skirt length is \( X_s = 2.2 \text{ d}_{\text{mid}} \). The skirt diameter \( D_s \), varies from 1.3 \text{ d}_{\text{mid}} to 2.2 \text{ d}_{\text{mid}} \); The midsection diameter \( d_{\text{mid}} \) must enable 10 kg satellite mass, with the uniform distribution of material of 6000 kg/m³ density.

The dependencies of the aerodynamic moment on the attack angle \( M(\alpha) \) for varying skirt diameters are shown in Fig. 2. The corresponding behavior patterns are numbered:
- \( D_s = 1.85 \text{ d}_{\text{mid}} \) – curve I;
- \( D_s = 1.5 \text{ d}_{\text{mid}} \) – curve II;
- \( D_s = 1.3 \text{ d}_{\text{mid}} \) – unnumbered curve;
- \( D_s = 2.2 \text{ d}_{\text{mid}} \) – curve III.

\[ \text{Figure 2. Aerodynamic moment as a function of the attack angle for varying skirt diameters} \]

The curves clearly show the influence of the aerodynamic shadow on the nanosatellite flight dynamics: the break in the curves corresponds to the critical angle of attack \( \alpha_c \), where the aerodynamic shadow from the cylindrical part reaches the edge of the skirt.
It is evident that the dependence of the aerodynamic moment on the angle of attack can assume three characteristic patterns. Worth noting is the geometry benefit that enables Pattern I of the M(α) dependence: When the moment is constant, in order to apply the control pulse, only the direction of the deviation from the zero angle of attack must be known – consequently, the control system problem is simplified.

Let us investigate the possible bandwidths of nanosatellite aerodynamic oscillations for different geometric configurations.

2. Behavior pattern I
The dependence may be approximated by a piecewise linear function:

$$M_\varepsilon = \begin{cases} \lambda \cdot \alpha, & \alpha < \alpha_*; \\ \lambda \cdot \alpha_* > \alpha > \alpha_*; \end{cases}$$

(4)

where $\lambda = \frac{\partial M_\varepsilon}{\partial \alpha}$ is a derivative of the aerodynamic moment at small angles of attack.

The attack angle dynamics is described by the equation:

$$J \cdot \ddot{\alpha} + M_\varepsilon = 0.$$  

(5)

where $J$ is the nanosatellite’s moment of inertia relative to the transverse axis through the center of mass. Taking into account (4), the dynamics equation is written down as:

$$\ddot{\alpha} + \frac{\tau}{2} \left( |\alpha_* - \alpha| - |\alpha_* + \alpha| \right) = 0.$$  

(6)

The oscillations period depends on the amplitude $\alpha_0$. When the amplitude is less than critical, harmonic oscillations occur; at large angles nonlinear oscillations take place:

$$T = \begin{cases} \frac{2\pi r_0}{\lambda}, & \alpha_0 < \alpha_*; \\ 4\tau_0 \left( \sqrt{1 - \mu^2} + \arcsin(\mu) \right), & \alpha_0 > \alpha_*; \end{cases}$$

(7)

where two parameters with the temporal dimension and their ratio are introduced:

$$\tau_0 = \sqrt{\frac{J}{\lambda}}; \quad \tau_1 = \frac{\alpha_*}{\dot{\alpha}(0)}; \quad \mu = \tau_1 \frac{1}{\tau_0}.$$  

(8)

Here $\dot{\alpha}(0)$ is amplitude angular velocity, $\alpha_*$ is critical angle.

For further analysis the results are conveniently presented in form of the reduced oscillation frequency $\nu = \frac{2\pi}{\tau_0}$ as a function of the reduced amplitude obtained by the integration of dynamic equations (4, 5):

$$\bar{\alpha}_0 = \frac{\alpha_0}{\alpha_*} = \frac{1 + \mu^4}{2\mu^2}.$$  

The corresponding graph is shown in Fig. 3a. The graphs in Fig. 3b allow estimating the bandwidth of nanosatellite aerodynamic oscillations in hypersonic flow with dynamic pressure $q = 588$ kPa, $D_s = 1.85$ $d_{\text{mid}}$. With the amplitude angles up to $12^\circ$ the nanosatellite undergoes oscillations in the bandwidth $\nu = 8 - 15$ Hz.
The limiting form of (4) is of interest:

$$M_z = M_z \cdot \text{Sign}(\alpha)$$  \hspace{1cm} (9)

The formal solution (5, 9) is a periodical temporal function, with each half-period having a parabola form. The frequency dependence on the amplitude is plotted as a dashed line in Fig. 3b and takes the following form:

$$\nu(\alpha_0) = \frac{1}{4} \sqrt{\frac{M_z}{2J \cdot \alpha_0}}$$  \hspace{1cm} (10)

Thus, determining the bandwidth boundaries does not involve the motion equations (5, 4). The upper limit is determined by a derivative of the aerodynamic moment at small angles of attack. The lower limit is provided by the asymptotics (10), and is supported by the assumption of the amplitude angle not exceeding $5^\circ$:

$$\bar{\nu} \in \left(1, \frac{1}{2}\right)$$  \hspace{1cm} (11)

3. **Behavior patterns II and III**

M($\alpha$) dependence is represented by a piecewise-linear approximation with angular coefficients in the following form:

$$M_z = \begin{cases} \lambda \cdot \alpha, & \alpha_0 < \alpha_*; \\ \lambda_+ \cdot \alpha + (\lambda - \lambda_+) \cdot \alpha_0, & \alpha_0 > \alpha_* \end{cases}$$  \hspace{1cm} (12)

$\lambda_+ > 0$ corresponds to Pattern III of angular momentum dependence on the angle; $\lambda_+ < 0$ corresponds to Pattern II. The lower limit of the dynamic systems bandwidth (5, 12) is determined by the angular coefficient $\lambda_+$, the upper limit is determined by $\lambda$. Fig. 4 a, b show the dependencies for $\lambda_+ > 0$ and the formal representation of the dependence for $\lambda_+ < 0$, respectively. Asymptotics is plotted as a dashed line.

Formal asymptotics:

1. when $\lambda_+ > 0$ is $\bar{\nu} \to \sqrt{\frac{\lambda_+}{\lambda}}$ for $\alpha_0 \to \infty$;

2. when $\lambda_+ < 0$ is $\bar{\nu} \to -\sqrt{\frac{\lambda_+}{\lambda}}$ for $\alpha_0 \to \infty$.

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**Figure 3a.** Reduced oscillation frequency as a function of reduced amplitude

**Figure 3b.** Frequency as a function of amplitude
We propose, by analogy with (11), an estimate of the frequency range:

\[ \bar{v} \in \left\{ \frac{1}{2} \left( 1 + \sqrt{\frac{\lambda}{\lambda'}} \right), 1 \right\} \text{ for } \lambda > 0; \]

\[ \bar{v} \in \left\{ \frac{1}{2} \left( 1 - \sqrt{\frac{\lambda}{\lambda'}} \right), 1 \right\} \text{ for } \lambda < 0. \]

The bandwidths for the given altitude and geometry parameters are obtained by scaling by a factor of \( \sqrt{\frac{J}{\lambda}} \) in \( y \) direction, and by a factor of \( \alpha \) in \( x \) direction.

4. Conclusions.
The dependencies of nanosatellite aerodynamic oscillations frequency on attack angle at different altitudes up to 70 km are defined. The oscillations bandwidths are determined with respect to the geometric parameters for a nanosatellite with 10 kg mass and 6000 kg/m\(^3\) average density. The model allows estimating the bandwidth aerodynamic oscillations in the suborbital nanosatellite trajectory based on the given geometry and mass-dimensional parameters [1].

References
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