The Improved Electromagnetic Equations and Superconductivity

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In this paper the macroscopic condition for dissipationless current in conductors is investigated and a set of improved electromagnetic equations are proposed. With these equations we show that the dissipation is particularly relative to free volume charges in conductors that originate from self-Hall-effect. The steadily normal current coexists with free volume charges and the steadily superconducting current doesn’t. Zero Hall coefficient is a candidate condition for nonexistence of free volume charges in conductors and a possible mechanism is exciton conduction or electron–hole pairing.

Keywords: improved electromagnetic equation; superconductivity; dissipationless current; macroscopic condition; free volume charge.

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1. Introduction

It is known that Maxwell equations are the foundation of classical electromagnetic theory and have become the important part of other physical phenomenological theories for example the theory of superconductivity [1, 2] though there were various generalizations [3-6]. These equations, however, are incapable of formulating the dissipation of current in conductors since they are independent of the Ohm’s law

\[ \mathbf{J} = \sigma \mathbf{E}, \]

which is closely related to the Joule law for current dissipation, where \( \mathbf{J} \), \( \mathbf{E} \) and \( \sigma \) are electric current density, electric field strength and conductivity, respectively. On the other hand, the unobservable current dissipation in steadily superconducting state has brought a big trouble to the Joule law and the Ohm's law. A model for this dissipationless current is taking superconductor as an idea conductor without Ohm resistance, or with \( \sigma = \infty \) in Eq. (1) [1]. But the infinite conductivity is found to violate the experiment of Meissner effect [1]. Indeed, to overcome this difficulty, one can turn to London [7] or Ginzburg-Landau [8] theory, which avoids the use of Eq. (1). But, the problem of dissipation has been evaded ably by the use of a two-fluid model [9, 1].

The macroscopic condition for dissipationless current in steadily superconducting state is still an open issue. In this paper, instead of Maxwell equations we propose a set of improved equations for the electromagnetic phenomena in isotropic media. These equations, from which the London equations can be derived under certain conditions, involve Maxwell equations and are compatible with Ginzburg-Landau ones. With these equations and the simple model of an infinitely long straight circular wire carrying a steady current, we obtain the relation between normal current and electric field and show that the dissipation is particularly relative to free volume charges that originate from the self-Hall-effect. It naturally leads to such a deduction: different from the normal state, the steadily superconducting state is that without the existence of free volume charges when
having a current. This nonexistence of free volume charges is found to be an explanation to the absence of Hall effect in a superconducting state [10-12]. A number of experiments show that the Hall coefficients of materials tend to zero when the materials go into steadily superconducting state [13, 14, 15]. And we find that zero Hall coefficient is a candidate condition for nonexistence of free volume charges in conductors with current. These would provide another angle of view for superconducting mechanism [16-21].

2. Theory

The derivation of our equations will be discussed later. Here we show them directly, which are of the forms

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2) \]
\[ \nabla \cdot \mathbf{B} = 0, \quad (3) \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (4) \]
\[ \nabla \cdot \mathbf{D} = \rho, \quad (5) \]
\[ \frac{\partial (\alpha \mathbf{J})}{\partial t} + \nabla \left( \frac{c^2 \alpha \rho}{\mu \varepsilon} \right) = -\mathbf{E}, \quad (6) \]
\[ \nabla \times (\alpha \mathbf{J}) = \mathbf{B}, \quad (7) \]

where \( \mathbf{D}, \mathbf{B}, \mathbf{H} \) and \( \rho \) are electric displacement, magnetic induction, magnetic field strength and free electric charge density, respectively; \( c, \mu, \varepsilon \) are the light speed in vacuum, the relative permeability and dielectric constant, respectively; \( \alpha = \alpha(r, t) \), a parameter depending on space coordinates and time in general. Equations (2)-(5) are just Maxwell equations and Eqs. (6) and (7) are new ones as the improvement of electromagnetic equations, which are from these four equations [see Eqs. (47) and (48)]

\[ \varphi - \frac{\partial \phi}{\partial t} = \frac{c^2 \alpha}{\mu \varepsilon} \rho, \quad (8) \]
\[ \mathbf{A} + \nabla \phi = \alpha \mathbf{J}, \quad (9) \]
\[ \mathbf{B} = \nabla \times (\mathbf{A} + \nabla \phi) = \nabla \times \mathbf{A}, \quad (10) \]
\[ \mathbf{E} = -\frac{\partial (\mathbf{A} + \nabla \phi)}{\partial t} \mathbf{t} - \nabla (\varphi - \frac{\partial \phi}{\partial t}) = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi, \quad (11) \]

where \( \mathbf{A} \) and \( \varphi \) are vector potential and scalar potential, respectively; \( \phi \) is a scalar function depending on space coordinates and time.

One sees that Maxwell equations (2) and (3) can be derived from Eqs. (6) and (7) [this suggests in one aspect that Eqs. (6) and (7) are valid in physics]. So, the independent ones in equations (2)-(7) are only (4)-(7). Obviously, they are gauge invariant. It should be noticed that, when \( \mathbf{J} = 0 \) but \( \mathbf{B} \neq 0 \), or when both \( \mathbf{J} \) and \( \rho \) are zero but \( \mathbf{E} \neq 0 \), we can not find a parameter \( \alpha \) for Eqs. (6) and (7). In other words, we can not find a parameter \( k \), which make Eq. (43) held (see below). In these two cases, corresponding to the situation of order parameter \( \psi = 0 \) in Ginzburg-Landau theory, we have no Eqs. (6) and (7) and should come back to Maxwell equations (2)-(5). It is seen that the condition of \( \rho = 0, \mathbf{E} = 0 \) and \( \mathbf{J} \neq 0 \) makes Eqs. (6) and (7) held.
By letting $\alpha = -m/(ne^2) = constant$ and $\mu_e = e_r = 1$, Eqs. (6) and (7) become the London ones [7], where $m$ and $e$ are the electron mass and elementary charge, respectively; and $n_s$ the number density of superconducting electrons. Note in Ref. [7], electric charge density $\rho$ is nonzero in general, for example in a transiently superconducting state. The improved electromagnetic equations are compatible with Ginzburg-Landau equations. On scanning the second Ginzburg-Landau equation $J = i\hbar e/2m: (\psi^* \nabla \psi - \psi \nabla \psi^*) - e^2 / m \cdot \psi^* A$ [8] and letting $\psi(r) = \sqrt{n_s(r)} \exp(i\phi/\hbar)$, we obtain Eq. (9) and then Eq. (7) immediately, with $\alpha = -m/(n_s(r)e^2)$, where the number density of superconducting electrons is now not a constant. In this case, the first Ginzburg-Landau equation becomes a choice of our equations for determining parameter $\alpha$ in steady state. It should be noticed that, in steady state, Ginzburg-Landau equations are the sufficient conditions but not the necessary ones for Eqs. (9) and (7). In other words, Eqs. (7) and (9) are the necessary conditions for the second Ginzburg-Landau equation and need not be restricted by it. Equations (6) and (7) are of the same derivation. And the validity of Ginzburg-Landau equations suggests in another aspect that Eqs. (6) and (7) are valid in physics.

We now turn to discuss the dissipation of current in conductors then come back to the discussion of application of our equations to the superconductivity again. As mentioned above, Maxwell equations are independent of the Ohm's law and incapable of formulating the dissipation of current in conductors. We show here, however, the relation between electric current and electric field can be deduced from Eqs. (4)-(7) and Lorenz force formula under the condition of an infinitely long straight circular nonmagnetic wire carrying a steady current $J$. It is of the form

\[ J = neE_z / D \]  

[see Eq. (A26) in Appendix A], being from these two relations [see Eq. (A24) and Eq. (A25)]:

\[ E_z = -E_z B_y / D, \]  

\[ E_z = -J B_y / ne. \]  

Equation (12) is from Eqs. (4)-(7) and Eq. (13) from Lorenz force formula with a assumption that carriers are electrons, where $E_z$ is the electric field along the wire and $E_z$ is the Hall electric field along the radius of wire; $J = |J|$ and $B_y = |B|$; $n$ is the number density of free electrons and $D$ is a constant determined by experiment, to be the reciprocal of the mobility of free electrons, $\mu_e$.

The derivations of Hall electric field and the relation between electric current density and electric field $E_z$ suggest in the third aspect that Eqs. (6) and (7) are valid in physics. People always attribute the current dissipation to the Ohm resistance. It is certainly true since Joule heat density is $J^2 / \sigma$, which tells us that the Joule heat density is inversely proportional to the conductivity or directly proportional to the resistivity. We show here, however, behind the Ohm resistance, there
exists another particular factor for the dissipation of current. It is the free volume charges. Our formula for the heat density of current in a nonmagnetic conductor is of the form

$$Q = JE_z = -\frac{Dc^2 \rho}{\varepsilon_r}.$$  \hfill (14)

This is from improved electromagnetic equations [see Eq. (A17)]. In fact, with the Ohm's law, a similar formula can also be deduced by assuming a constant electric current density and considering the self-Hall-effect in the wire (see Appendix B). But here, we have no condition sufficiently in hand to ensure the number density of electrons therefore the electric current density being a constant, owing to the self-Hall-effect. It should be emphasized that the condition for Eq. (14) is that there exists a steady current \( J \). The free volume charges is generated by current than added from outside and \( \rho = -\varepsilon_r J^2 / (Dc^2 \sigma) \) when Ohm's law holds. Moreover, the generation of negative free volume charges at one place of circuit will result in the same magnitude of positive charges at another place of circuit due to the conservation of charges.

One sees that, besides the Ohm resistance (note that in the case of carriers being the electrons, \( D = 1/ \mu_e \)), the existence of free volume charges is indispensable for the dissipation. The heat density is directly proportional free volume charge density in the wire. Equation (14) makes us think of the superconductivity. The steadily superconducting current means no dissipation. The absence of dissipation requires \( E_z = 0 \) when \( J \neq 0 \) (see Appendix A). It leads to

$$E_r = 0 \quad \text{if} \quad E_r = -E_z B_\theta / D \quad \text{[Eq. (A24)]}$$

immediately. And then \( \rho = 0 \)

(since \( \nabla \cdot (\varepsilon_0 \nabla E) = \rho \)), which is consistent with Eq. (14). Inversely, with the condition of \( \rho \neq 0 \), by Eq. (14) we know that there has a dissipation with current in this long straight wire. So, \( \rho = 0 \) would be the necessary condition for steadily superconducting state with \( J \neq 0 \). Note that the premise of traditional Joule heat formula \( J^2 / \sigma \) is that the Ohm's law \( J = \sigma E \) holds. And when \( \rho = 0 \), \( E_z = E_r = 0 \), the bridge connecting electric current and electric field is cut off (see Appendix A) and we have no the Joule heat formula \( J^2 / \sigma \).

The above discussion is based on the special relation Eq. (14), which is from the condition of an infinitely long straight circular nonmagnetic wire carrying a steady current. We now show directly from Eqs. (4)-(7) that the nonexistence of free volume charges is the sufficient condition of steadily superconducting state with \( J \neq 0 \). For a steady state, \( \partial (\sigma \mathbf{J}) / \partial t = 0 \) therefore

$$\nabla (\varepsilon_0 \partial \rho / \mu_e) = -\mathbf{E} \quad \text{by Eq. (6)}; \quad \text{If} \quad \rho = 0 \quad \text{and} \quad \mathbf{J} \neq 0 \quad \text{, then} \quad \mathbf{E} = 0 \quad \text{and} \quad \mathbf{J} \cdot \mathbf{E} = 0 \quad \text{, without dissipation of current, being a superconducting state [see an example described by the solution of Eqs.(A27) and (A28); note that if there have only Maxwell equations, we cannot reach this conclusion]. In fact, it is just the free volume charges who build up the bridge relating electric field and electric current in steady state, as shown in Eqs. (4)-(7). The absence of free volume}$$
charges will break down this relation and then we can have $E=0$ but $J \neq 0$. So it is not surprising that the Ohm law is invalid in the steadily superconducting state.

It is interesting to look into the relationship between the existence of free volume charges and Hall effect. One sees from the Appendix A that, free volume charges are symbiotic with the electric field $E_r$ due to self-Hall-effect in an infinitely long straight circular nonmagnetic wire carrying a steady current $J$. They are connected by Eqs. (12) and (14), from which we obtain $E_r = c^2 \rho B_\phi / (\varepsilon, J)$. This relation tells that, when $\rho = 0$, $E_r = 0$ and vice versa. Namely, the Hall electric field together with free volume charges disappears in the wire carrying steady current. On the other hand, since the early work of Onnes and Hof [10], it has been generally supposed that there is no Hall effect in a superconducting state. Lewis made another attempt to test the effect, but with a negative result [11]. After some of theoretical analysis based on the existing theories, Lewis concluded "that an explanation of the absence of a Hall effect in a superconductor will require an extension of present theory in a direction not easily foreseen" [12]. Equations (4)-(7) with condition $\rho = 0$ can give an explanation to this phenomenon: for a steady state of $\rho = 0$ and $J \neq 0$. As discussed above, it should be a superconducting state and $E = 0$. Indeed, this $E$ involves the Hall field. So there is no Hall effect in the superconductor. So far to the best of our knowledge, the Hall effect of superconductors has only been observed in the mixed states and intermediate states [13, 22-25] when the dissipative processes of currents take place [22]. This also suggests that the nonexistence of free volume charges would be necessary for the steadily superconducting state with $J \neq 0$. The above discussions lead us to such a picture: the steadily normal current coexists with free volume charges and the steadily superconducting current doesn’t. It should be noticed that the condition $\rho = 0$ then $E = 0$ for superconductivity is only in the steady state. For the transiently superconducting state, we have no this condition. According to the above discussion, in a long straight circular wire carrying a steady current, when Hall field $E_r = 0$, it has $\rho = 0$. And according to Eq. (12), coefficient $1/D = 0$ is the sufficient condition for $E_r = 0$. This suggests that Zero Hall coefficient with a material is a candidate condition for steadily superconducting state. In fact, there are a number of experiments [13, 14, 15] showing that the Hall coefficients of materials tend to zero when the materials go into steadily superconducting state. The model of exciton conduction or electron–hole pairing is that worthy of note [26-28], for which the Hall effect of electrons is expected to be counteracted by that of holes. In fact, in the mentioned experiments [13, 14, 15], near the transition temperature and under a suitable magnetic field, the Hall coefficient shows clearly an oscillation around the zero point, which would be the evidence of that there exist a competition between hole and electron conduction. A possible test material of electron–hole pairing is a semiconductor with two types of carriers, electrons and holes, with high concentration and at a proper ratio.

For a steady state with $\rho = 0$ and $J \neq 0$, according to the above discussion, it is a superconducting state. What we should consider in Eqs. (4)-(7) are only two equations $\nabla \times (\alpha \mathbf{J}) = \mathbf{B}$ and $\nabla \times \mathbf{H} = \mathbf{J}$ with a parameter $\alpha$. In general, the parameter cannot be determined by classical electrodynamics and other branches in physics and is an interesting issue to be explored. The
simplest case is $\alpha = \text{constant}$, for example $\alpha = -m/\hbar c^2$, as that found by London brothers, however is only an approximation. In general, $\alpha$ is a function of space coordinates. As mentioned above, Ginzburg-Landau equations [8] are one of choices for determining this parameter. An example of approximate solution of Ginzburg-Landau equations for a homogeneous type II superconductor just below the upper critical field is [29]

$$\psi(x, y) = \sum_{n=-\infty}^{\infty} c_n \exp(inky) \exp(-1/2\kappa^2 (x - nk / \kappa^2)^2)$$

with the periodicity condition $c_{n+N} = c_n$, where $N$ is a positive integer; $\kappa$ is a parameter relative to critical magnetic field; $k$ and $c_n$ are the parameters determined by minimum free energy. In this case,

$$\alpha = \alpha_0 / |\psi(x, y)|^2 = \alpha_0 / |\sum_{n=-\infty}^{\infty} c_n \exp(inky) \exp(-1/2\kappa^2 (x - nk / \kappa^2)^2)|^2,$$

where $\alpha_0$ is a constant. The parameter $\alpha$ determined by London formula or Ginzburg-Landau equations tells the local relation between $J$ and $A$. According to Eq. (9), however, the form of parameter $\alpha$ can be more general. For example, it can be a nonlinear function of vector potential $A$, or further, its response can be nonlocal. So the relation between $J$ and $A$ can be nonlinear and nonlocal in general.

We now begin to derive the Eqs. (2)-(9) basing on the charge conservation law and differential geometry, with consulting Maxwell equations. Here we consider an isotropic medium, in which the electromagnetic field has no singularity, i.e., the space-time manifold related to the medium is a simply connected one. In this space-time manifold, the charge conservation law

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$$

(16)
can be expressed as

$$d\omega_3 = 0,$$

(17)
where $d$ is the exterior differential operator [30, 31] and $\omega_3 = R_{123} dx^{123} + R_{012} dx^{012} + R_{013} dx^{013} + R_{023} dx^{023}$ is a 3-form; $dx^{123} = dx^1 \wedge dx^2 \wedge dx^3$, $dx^{012} = dx^0 \wedge dx^1 \wedge dx^2$, $dx^{013} = dx^0 \wedge dx^1 \wedge dx^3$ and $dx^{023} = dx^0 \wedge dx^2 \wedge dx^3$ ($x^0 = ct$); and in a rectangular coordinate system, $x^1 = x$, $x^2 = y$, $x^3 = z$; $R_{123} = c \rho$, $R_{012} = -J_x$, $R_{013} = J_y$, $R_{023} = -J_z$. By Eq. (17) the charge conservation law is now becomes

$$R_{123,0} - R_{012,3} + R_{013,2} - R_{023,1} = 0,$$

(18)
where $R_{123,0} = \partial R_{123} / \partial x^0$, $R_{012,3} = \partial R_{012} / \partial x^3$, $R_{013,2} = \partial R_{013} / \partial x^2$, $R_{023,1} = \partial R_{023} / \partial x^1$. From Eq. (17) we know that $\omega_3$ is a closed form. By Poincaré lemma [30] we know that $\omega_3$ should also be an exact form, which means that there exists a non-closed 2-form $\omega_2 = G_{01} dx^{01} + G_{02} dx^{02} + G_{03} dx^{03} + G_{12} dx^{12} + G_{13} dx^{13} + G_{23} dx^{23}$ making
\[ \omega_3 = d \omega_2 \] 

or

\[ R_{123} = G_{12,3} - G_{13,2} + G_{23,1} \]  

\[ R_{012} = G_{01,2} - G_{02,1} + G_{21,0} \]  

\[ R_{013} = G_{01,3} - G_{03,1} + G_{13,0} \]  

\[ R_{023} = G_{02,3} - G_{03,2} + G_{23,0} \] 

One sees that Eq. (20) is just Eq. (5) and Eqs. (21)-(23) are just Eq. (4), which involve the relations.

\[ \mathbf{H} = (G_{01}, G_{02}, G_{03}), \quad \mathbf{D} = (G_{23}, -G_{13}, G_{12}). \]  

In fact, in our 4-dimension space-time manifold, besides \( \omega_2 \) it permits that there exists a closed 2-form \( \tau_2 = \int_{0}^{1} \omega_2 \), which makes the following equation held:

\[ \omega_3 = d(\omega_2 + \tau_2). \]  

Of course we have

\[ F_{12,3} - F_{13,2} + F_{23,1} = 0 \]  

\[ F_{01,2} - F_{02,1} + F_{12,0} = 0 \]  

\[ F_{01,3} - F_{03,1} + F_{13,0} = 0 \]  

\[ F_{02,3} - F_{03,2} + F_{23,0} = 0 \] 

According to Poincaré lemma [30], \( \tau_2 \) should be an exact form, which means that there exists a non-closed 1-form \( \omega_1 = A_0 dx^0 + A_1 dx^1 + A_2 dx^2 + A_3 dx^3 \) making

\[ \tau_2 = d \omega_1 \]  

It immediately leads to

\[ F_{01} = A_{1,0} - A_{0,1}, \quad F_{02} = A_{2,0} - A_{0,2}, \quad F_{03} = A_{3,0} - A_{0,3}, \quad F_{12} = A_{1,2} - A_{2,1}, \quad F_{13} = A_{1,3} - A_{1,3}, \quad F_{23} = A_{3,2} - A_{3,3}. \]  

For an arbitrary closed 1-form \( \tau_1 = C_0 dx^0 + C_1 dx^1 + C_2 dx^2 + C_3 dx^3 \), which is generally permitted to exist, we have also
\[ \tau_2 = d(\omega_1 + \tau_1) \]  

(32)

and

\[ C_{1,0} - C_{0,1} = 0, \quad C_{2,0} - C_{0,2} = 0, \quad C_{3,0} - C_{0,3} = 0 \]
\[ C_{2,1} - C_{1,2} = 0, \quad C_{3,2} - C_{2,3} = 0. \]  

(33)

Again, according to Poincaré lemma [30], \( \tau_1 \) should be an exact form, which means that there exists a non-closed 0-form \( \phi \) making

\[ \tau_1 = d\phi. \]  

(34)

Namely,

\[ C_0 = \phi_0, \quad C_1 = \phi_1, \quad C_2 = \phi_2, \quad C_3 = \phi_3 \]  

(35)

Up to this point, we have not discussed the physics of \( \tau_2, \omega_1, \tau_1 \) and \( \phi \) yet. To endow these forms with physics, we now build the relations between \( \tau_2 \) and \( \omega_2 \) as well as that between \( \omega_3 \) and \( (\omega_1 + \tau_1) \) with the aid of Hodge \( \ast \) operation [31], an algebraic operation stock, which maps an \( r \)-form onto a (\( n-r \))-form (\( r<n \)) and depends on the metric of space-time manifold considered. In our case, \( n=4 \), so the Hodge \( \ast \) operation can map a 2-form onto a 2-form as well as map a 3-form onto a 1-form. To perform the Hodge \( \ast \) operation we need to introduce a metric for our space-time manifold. For convenience and without losing the generality we consider the following metric for an isotropic medium:

\[ g^{\lambda\nu} = \begin{cases} 
-\mu, & \lambda = \nu = 0 \\
1, & \lambda = \nu = 1,2,3 \\
0, & \text{others}
\end{cases} \]  

(36)

With the metric we can map the non-closed 2-form \( \omega_2 \) onto a 2-form \( \beta_2 \) (\( d\beta_2 = 0 \)), i.e., \( \ast \omega_2 = \beta_2 \). The \( \beta_2 \) would not be just equal to \( \tau_2 \). But it is possible to introduce a parameter \( f \) depending on space coordinates and time, which makes

\[ f(\ast \omega_2) = \tau_2. \]  

(37)

Under the metric described by Eq. (36), we know that

\[ f(\ast \omega_2) = f \sqrt{-g}(G_{20}dx^0 - G_{10}dx^1 + G_{02}dx^3 + G_{01}dx^3 - g^{00}G_{00}dx^0 + g^{01}G_{01}dx^0 + g^{02}G_{02}dx^2 + g^{03}G_{03}dx^3), \]  

(38)

where \( \sqrt{-g} = \sqrt{\mu, \epsilon} \). On substituting Eq. (38) into Eq. (37) and using Eqs. (26)-(29), we obtain

\[ (aG_{01})_1 + (aG_{02})_2 + (aG_{03})_3 = 0, \]  

(39)

\[ (bG_{21})_2 + (bG_{13})_3 - (aG_{03})_0 = 0, \]  

(40)
\[(bG_{32})_3 - (bG_{12})_3 + (aG_{00})_0 = 0, \quad (41)\]
\[(bG_{13})_3 + (bG_{23})_2 + (aG_{00})_0 = 0, \quad (42)\]

where \( a = \mu, \varepsilon, \sqrt{\mu, \varepsilon, f} \) and \( b = \sqrt{\mu, \varepsilon, f} \). By noting Eq. (24), equations (39)-(42) can be interpreted as Maxwell equations (2) and (3) as long as let \( f = -\mu_0 / (\varepsilon, \sqrt{\mu, \varepsilon, f}) \), which shows the existence of relation (37) and shows the validity of differential geometry method used here.

Similarly, we can map the 3-form \( \omega_3 \) onto a 1-form \( \beta_3 \), i.e., \( *\omega_3 = \beta_3 \); and then introduce another parameter \( k \) depending on space coordinates and time, which makes
\[ k(*\omega_a) = \omega_a + \tau_a. \quad (43) \]

Under the metric mentioned above,
\[ k(*\omega_a) = k\sqrt{-g} \left( -R_{123}dx^0 + g^{00}R_{203}dx^1 - g^{00}R_{301}dx^2 + g^{00}R_{012}dx^3 \right). \quad (44) \]

On the other hand (see the expressions of \( \omega_a \) and \( \tau_a \) above),
\[ \omega_a + \tau_a = (A_0 + C_0)dx^0 + (A_1 + C_1)dx^1 + (A_2 + C_2)dx^2 + (A_3 + C_3)dx^3. \quad (45) \]

Therefore,
\[ A_0 + C_0 = -k\sqrt{\mu, \varepsilon}, R_{123}, \quad A_1 + C_1 = -k\mu, \varepsilon, \sqrt{\mu, \varepsilon}, R_{203}, \]
\[ A_2 + C_2 = k\mu, \varepsilon, \sqrt{\mu, \varepsilon}, R_{013}, \quad A_3 + C_3 = -k\mu, \varepsilon, \sqrt{\mu, \varepsilon}, R_{021} \quad (46) \]

Denoting \( \alpha = -k\mu, \varepsilon, \sqrt{\mu, \varepsilon} \) and \( A_0 = -\varphi / c \), then we obtain
\[ \varphi - \frac{\partial \varphi}{\partial t} = \frac{c^2 \alpha}{\mu, \varepsilon} \rho, \quad (47) \]
\[ \mathbf{A} + \nabla \varphi = \alpha \mathbf{J} \quad (48) \]

Here Eq. (35) has been used. Further, by Eqs. (24), (32) and (37)-(42), we reach Eqs. (6) and (7) ultimately. In fact, Eqs. (6) and (7) can be obtained directly by using Eqs. (32), (37) and (43). It should be noticed that in Eq. (25), closed 2-form \( \tau_2 \) is rather arbitrary. The introduction of Eqs. (37) and (43), however, has excluded those independent of the electromagnetic field. On the other hand, the above four relations (19), (32), (37) and (43) will lead to this relation
\[ d[k(*\omega_2)] = f * \omega_2 \quad (49) \]

which involves six differential equations that can, in principle, determine six parameters \( f, k \) and \( g^{\mu\nu} (\mu = 0, 1, 2, 3) \) if the 2-form \( \omega_2 \) (or the electromagnetic field) has been given.

3. Conclusion

In conclusion, we have proposed a set of improved electromagnetic equations of isotropic
media, with which and the simple model of an infinitely long straight circular wire carrying a steady current, we have shown that the dissipation of current is relative to free volume charges that originates from self-Hall-effect; and the nonexistence of free volume charges is the condition for steadily superconducting state with $J \neq 0$. Zero Hall coefficient with a material is a candidate condition for nonexistence of free volume charges and a possible mechanism is exciton conduction or electron–hole pairing [26-28]. A possible test material is a semiconductor with two types of carriers, electrons and holes, with high concentration and at a proper ratio. The relations between the nonexistence of free volume charges and the formation of Cooper pairs [16-21], would be an interesting issue in the future.

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Appendix A

Here we consider an infinitely long straight circular wire carrying a steady current $J$. For convenience, we suppose that the wire made of single material is nonmagnetic, i.e., $\mu_r = 1$. In this case, Eqs. (4)-(7) become

$$\nabla (\frac{\alpha J}{\varepsilon_r}) = -\frac{\mathbf{E}}{c^2}, \quad \text{(A1)}$$

$$\nabla \times (\alpha \mathbf{J}) = \mathbf{B}, \quad \text{(A2)}$$

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad \text{(A3)}$$

$$\nabla \cdot (\varepsilon_0 \varepsilon_r \mathbf{E}) = \rho. \quad \text{(A4)}$$

Further, we choose a circular cylindrical coordinates with its z-axis along the direction of current $J$. Obviously, $J$ has only a component $J_z$. Denoting the component as $J$ and using $\nabla \cdot \mathbf{J} = 0$, we have

$$\frac{\partial J}{\partial z} = 0, \quad \text{(A5)}$$

which means that $J$ is independent of $z$. From Eqs. (A2) and (A3), we can deduce

$$\nabla (\nabla \cdot (\alpha \mathbf{J})) - \nabla^2 (\alpha \mathbf{J}) = \mu_0 \mathbf{J}, \quad \text{(A6)}$$

which is

$$\frac{\partial}{\partial r} \frac{\partial}{\partial z} (\alpha J) e_r + \frac{1}{r} \frac{\partial^2}{\partial \theta^2} (\alpha J) e_\theta + \frac{\partial^2}{\partial z^2} (\alpha J) e_z - \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial (\alpha J)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} (\alpha J) + \frac{\partial^2}{\partial z^2} (\alpha J) \right] e_z = \mu_0 J e_z,$$

where $e_r$, $e_\theta$, and $e_z$ are three unit vectors of circular cylindrical coordinates, respectively. This vector equation is equivalent to the following three scalar ones

$$\frac{\partial}{\partial r} \frac{\partial}{\partial z} (\alpha J) = 0, \quad \text{(A7)}$$

$$\frac{1}{r} \frac{\partial^2}{\partial \theta^2} (\alpha J) = 0, \quad \text{(A8)}$$

$$- \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial (\alpha J)}{\partial r} \right) - \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} (\alpha J) = \mu_0 J, \quad \text{(A9)}$$
Equations (A7) and (A8) give
\[ J = \frac{y(z)}{\partial z} = \frac{\partial \alpha}{\partial z}. \] (A10)

Let us consider Eq. (A1), which includes these three scalar equations
\[ \frac{\partial}{\partial r} \left( \frac{ap}{\varepsilon_r} \right) = -\frac{E_r}{c^2}, \] (A11)
\[ \frac{\partial}{\partial z} \left( \frac{ap}{\varepsilon_r} \right) = -\frac{E_z}{c^2}, \] (A12)
\[ \frac{\partial}{\partial \theta} \left( \frac{ap}{\varepsilon_r} \right) = -\frac{E_\theta}{c^2}. \] (A13)

It is helpful to consider the symmetry of our system before going forwards. In fact, the symmetry of our system requires \(E_r, E_z, \rho\) and \(\varepsilon_r\) to be independent of \(\theta\) and \(z\) besides \(E_\theta = 0\) and \(B_z = B_r = 0\). So we conclude by Eq. (A13) that \(\alpha\) is independent of \(\theta\), which leads to \(\frac{\partial (\alpha J)}{\partial \theta} = 0\). By assuming \(\rho \neq 0\) and with these conditions, Eq. (A12) becomes
\[ \frac{ap}{\varepsilon_r} = -\frac{E_z}{c^2} z + g(r) \quad \text{or} \quad \alpha = -\frac{\varepsilon_r E_z}{c^2 \rho} z + h(r), \] (A14)
where \(g(r)\) and \(h(r)\) are two arbitrary functions depending on \(r\). Using Eq. (A10) we obtain
\[ J = -\frac{y(z)c^2 \rho}{\varepsilon_r E_z}. \] (A15)

Function \(y(z)\) should be independent of \(z\) and is a constant since \(J, E_z, \rho\) and \(\varepsilon_r\) are all independent of \(z\). By denoting the constant as \(D\), Eq. (A15) can be rewritten as
\[ J = -\frac{Dc^2 \rho}{\varepsilon_r E_z}, \] (A16)
or
\[ Q = JE_z = -\frac{Dc^2 \rho}{\varepsilon_r}. \] (A17)

This is the Joule heat density. Substituting Eq. (A16) into Eq. (A9) and noting \(\frac{\partial (\alpha J)}{\partial \theta} = 0\), we obtain
\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \left( \alpha \frac{Dc^2 \rho}{\varepsilon_r E_z} \right) \right) = \mu_0 \frac{Dc^2 \rho}{\varepsilon_r E_z}. \] (A18)

Since \(E_z\) is independent of \(z\), we get immediately the result \(\frac{\partial E_z}{\partial r} = 0\) in the wire by equation \(\nabla \times \mathbf{E} = 0\), which leads to the simplified form of Eq. (A18)
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \left( \frac{\alpha \rho}{\varepsilon_r} \right) \right) = -\mu_0 \frac{\rho}{\varepsilon_r} .
\]  
(A19)

Substituting Eq. (A11) into (A19) we get
\[
\frac{1}{r} \frac{\partial}{\partial r} (r E_r) = \mu_0 \frac{\varepsilon_r^2 \rho}{\varepsilon_r} .
\]  
(A20)

And Eq. (A4) gives another equation
\[
\frac{1}{r} \frac{\partial}{\partial r} (r \varepsilon_r E_r) = \frac{\rho}{\varepsilon_0} .
\]  
(A21)

Therefore, we have \( \varepsilon_r = \text{constant} \) incidentally from equations (A20) and (A21). It is not surprising as what we considered is an isotropic medium.

By using Eq. (A16), Eq. (A20) can be rewritten as
\[
\frac{1}{r} \frac{\partial}{\partial r} (r E_r) = -\frac{\mu_e E_z}{D} J .
\]  
(A22)

And from Eq. (A3) we can deduce
\[
\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) = \mu_0 J .
\]  
(A23)

Here we have used the relation of \( \theta = B_\theta \). Comparing Eq. (A22) with Eq. (A23), we find that
\[
E_r = -\frac{E_z}{D} B_\theta . \tag{A24}
\]

On the other hand, by Lorenz force formula, we find immediately that the Hall electric field is of the form
\[
E_r \varepsilon_r = \mathbf{v} \times \mathbf{B} = -\frac{J}{ne} B_\theta \varepsilon_r , \tag{A25}
\]
for which we have assumed that the carriers are free electrons with a number density of \( n \). In fact, the electric field \( E_r \) in Eq. (A24) is, indeed, the same as that in Eq. (A25). So we have
\[
J = \frac{1}{D} neE_z . \tag{A26}
\]

This is the relation between electric current density and electric field \( E_z \), which can be interpreted as the Ohm's law as long as let \( 1/D = \mu_e \), the mobility of free electrons. In fact, the relation \( 1/D = \mu_e \) can be determined by experiment. Now, we know that it is just the free volume charges who maintain the electric field \( E_z \) relative to the current \( J \).

When \( \rho = 0 \), what we should consider are two equations from Eqs. (A7)-(A9):
\[
J \frac{\partial \alpha}{\partial z} = y(z) , \tag{A27}
\]
\[-\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial (\alpha J)}{\partial r} \right) = \mu_0 J. \quad (A28)\]

Here we consider the simplest case, \( \frac{\partial \alpha}{\partial z} = 0 \), i.e., \( \alpha = \text{constant} \), then we obtain immediately from Eqs. (A28) that \( J = J_0 I_0(r/\lambda)/I_0(R/\lambda) \), where \( R \) is the radius of the wire, \( I_0(x) \) is the 0-order modified Bessel function of the first kind; \( \lambda = \sqrt{\alpha/\mu_0} \), the penetration depth; and \( J_0 \) is the electric current density at \( r=R \).

### Appendix B

We still consider an infinitely long straight circular wire carrying a steady current \( J \) and assume that the wire made of single material is nonmagnetic. But here, we assume further the current \( J \) is a constant, i.e., \( J \) is independent of \( r \). Therefore, the magnetic induction in the wire is

\[ B_\theta = \frac{\mu_0}{2} Jr. \quad (B1) \]

Substituting Eq. (B1) into Eq. (A25), we obtain

\[ E_r = -\frac{\mu_0}{2ne} J^2 r. \quad (B2) \]

Further, on substituting Eq. (B2) into Eq. (A4), it yields

\[ -\frac{1}{ne} \mu_0 e_0 e_r J^2 = \rho. \quad (B3) \]

By the Ohm's law \( J = e_n e E_z \), we reach finally that

\[ JE_z = -\frac{e^2 \rho}{\mu_0 e_r}, \quad (B4) \]

which is very similar to Eq. (14). But here, free volume charge density is a constant. And in Eq. (14), the free volume charge density can vary with \( r \).

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