Deflection of Thin-Walled Panels Loaded in Shear with Different Types of End Stiffeners

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Abstract, Numerical analysis for the load/deflection behavior of thin-web plate girders have been studied in this paper using a finite element program. An analytical approach presented in literature was discussed, and hence, it was used to calculate the load/deflection values in pre-buckling and post buckling stages. The validated finite element model is used to discuss the meaningful of the analytical approach. The effect of different parameters on load-deflection curves is proposed. Stiffener thickness (tₜ), end post type (no end post (NEP), rigid end post (REP), and non-rigid end post (NREP)) and end distance (e) are the main parameters considered in this paper. The analytical approach estimates well the load/deflection behaviors of REP and NREP plate girders with (a/hₑ < 2.2). On the other hand, the analytical approach can’t estimate either for the shear strength or the deflection of NEP plate girders.

KEYWORDS
Plate girders; Shear behaviour; In-plan deflection; End stiffeners; Finite element model.

1. INTRODUCTION
Plate girder is a structural element which contains steel plates assembled together by welded or bolted to one another. Rolled sections account for structures with relatively low loads and limited spans. However, when it comes to relatively heavy loads with large spans, as bridge girders with small to medium spans, designed plate girders are among the most widely used sections [1, 2]. Optimizing the plate girder dimensions presents deep thin web, that thin web tends to buckle. Hence transverse intermediate stiffeners are used to restrain the web and increase the buckling strength. At shear force, the thin web panels buckle. Therefore the shear behavior of thin web panel is one of the important design aspects of plate girder [3, 4].

A thin web panel subjected to shear was studied by several researchers [3-14]. The shear resistance of transversely stiffened plate girders was affected mainly by three factors [15]; 1) the buckling shear resistance, 2) the post-buckling shear resistance, and 3) flanges contribution to the shear resistance. The buckling shear resistance was studied and formulated by Porter et al. [16], hence Lee and Yoo [4] propose design equations for the determination of ultimate shear strength of the web panels. In the meanwhile, Ajam and Marsh express the elastic shear buckling strength based on the Tresca yield
criterion [17]. For all studied models, the shear buckling coefficient is the key parameter for the shear strength. As a simplification, and as an accurate approximation, all panel edges are assumed to be simply supported, although Galambos has presented the shear buckling coefficient for panel with fixed rotational restraints at the top and bottom [18].

Wilson in 1886 was one of the first to discuss the post-buckling behaviour [19]. In a major advance in 1950, Basler and Thurlimann [20] presented extensive studies on the post-buckling behaviour of plate girder web panels. They assumed that the transverse stiffeners doing as anchors and the yield was far from flanges. Then later Basler [21, 22] presented different approach on the post-buckling behaviour of thin web panel of a plate girder. In 1970s, Rockey and his co-workers predicted the ultimate strength of webs subjected to shear [16, 23].

As noted by many researchers, increasing flange dimensions develops larger shear strength, which means flanges contribute the shear strength by the frame action. However, Höglund [24] calculated the shear contribution from the frame action.

A numerical study of nonlinear large deflection behaviour of plate girder under shear was presented by Alinia et al. [7]. It was observed that plastic hinges induced by shear in the end panel only. These hinged are induced by the shear deformation near the supports. The researchers state that the end-posts are important elements in the behaviour and design of plate girder under shear.

A number of experimental and analytical studies have been carried out to establish a mechanical model to represent the out-of-plane and the in-plane deformation characteristics of web panel under shear [25, 26]. However the proposed model by Qian and Tan [25] does not take the effect of end-post into account, there is still a need for general analysis and design methodology to combine the effect of end-post on the out-of-plane and the in-plane deformation characteristics of web panels under shear.

Hence, this paper focuses on the shear behaviour of the thin web panel of plate girders. The principal aim is to improve the current presented model of the structural behaviour for the thin panel under shear formed by Qian and Tan [25]. The key parameters that affect this panel in the current investigation are the stiffeners thickness \( t_w \), the type of end-post, the web aspect ratio \( a/h_w \) and the end-post distance \( e \).

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The scope of this study can be summarized as:
1. Discuss the current literature model for calculating the in-plane deflection of web panels loaded in shear.
2. Propose and validate a detailed finite element 3D model used to address the behaviour of full-scale plate girders.
3. Investigate factors affecting the in-plane deflection, and compare between the finite element results and current literature model.

2. LITERATURE APPROACH FOR IN-PLANE DEFLECTION

Qian and Tan [25] presented a mechanical approach to calculate the in-plane deflection of web panel loaded in shear. Qian had used Cardiff theory to calculate the shear strength in his analytical approach. In Qian approach the overall deflection is divided into two stages, which are deflection of pre-buckling stage \( \delta_{cr} \) and deflection of post-buckling stage \( \delta_b \). Such that the total deflection for the panel loaded in shear \( \delta_{pb} \) could be calculated as follow:

\[
\delta_{pb} = \delta_{cr} + \delta_b
\]

Thus, the pre-buckling deflection can be calculated as:

\[
\delta_{cr} = 2 \tau_{cr} a (1+v)/E
\]

in which:
- \( a \) is the panel length;
- \( v \) is the Poisson’s ratio;
- \( E \) is the elastic modulus;
- \( \tau_{cr} \) is the shear buckling stress of the web panel \( = K \frac{E}{12(1-v^2)} \left( \frac{t_w}{h_w} \right)^2 \);
- \( t_w \) and \( h_w \) are the web panel thickness and height respectively.
$K$ is the shear buckling coefficient in case of web panel with transversal stiffeners only and equals to
\[
\begin{cases} 
5.34 + 4/(a/h_w)^2 & a/h_w \geq 1.0 \\
4 + 5.34/(a/h_w)^2 & a/h_w < 1.0 
\end{cases}
\]
In the meanwhile, the post-buckling deflection was derived for the shear case as follows:
\[
\delta_b = \frac{(a_t^2 c)^2 c h_w t_w}{E (V_{ult} - 4 M_{pf})}
\]
where:
- $c$ is the distance assumed between the plastic hinge and the end bearing stiffener
- $M_{pf}$ is the plastic moment capacity of flanges ($= 0.25 f_y A_f t_f$)
- $\sigma_t^2$ and $V_{ult}$ are the tensile yield stress of web panel under shear and the post-buckling shear force component, respectively, which can be calculated as:
\[
\sigma_t^2 = -\frac{3}{2} \tau_{cr} \sin 2\theta + \sqrt{f_{yw}^2 + \tau_{cr}^2 [1.5 \sin 2\theta]^2 - 3}
\]
\[
V_{ult} = \sigma_t^2 h_w t_w \sin^2 \theta \left(\cot \theta - \frac{a}{h_w}\right) + 4 \sigma_t^2 t_w \sin \theta \frac{M_{pf}}{\sigma_t^2 t_w}
\]
Thus, $\theta$ is the inclination of the tensile field, the angle $\theta$ may be calculated by iteration to give the maximum value of $V_{ult}$ or can be approximated as $\theta = 2/3 \tan^{-1}(h_w/a)$ as given by Rockey et al. [24].

However, equation 5 is subject to the following control [27]:
\[
\frac{M_{pf}}{h_w^2 f_{yw}} < 0.125 \left(\frac{h_w}{a}\right)^2 \left\{-\frac{3}{2} \tau_{cr} + \sqrt{1 - 0.25 \left(\frac{\sqrt{3} \tau_{cr}}{f_{yw}}\right)^2}\right\}
\]
If the left-hand side is greater than the right-hand side stated in equation 6, the value of $V_{ult}$ is given by:
\[
\frac{V_{ult}}{\tau_{cr} t_w h_w} = 4\sqrt{3} \left(\frac{h_w}{a}\right) \frac{M_{pf}}{h_w^2 f_{yw} t_w} + \left\{-\frac{3\sqrt{3} \tau_{cr}}{4} \frac{M_{pf}}{f_{yw}} + \sqrt{0.75 - \frac{3}{16}\left(\frac{\sqrt{3} \tau_{cr}}{f_{yw}}\right)^2}\right\}
\]

3. **FINITE ELEMENT MODEL**

A detailed finite element (FE) 3D model used to address the behaviour of transversely stiffened, full-scale plate girders was developed and verified. Nonlinear behaviour of the thin web panel was presented. The verified FE model is used to promote the current literature equations for in-plane deflection and study additional factors affecting the behaviour of thin web panels under shear force.

3.1 **Model description**

Transversely stiffened plate girders were modelled using reduced-integrated shell element S4R. The element S4R, from non-linear finite element analysis of the ABAQUS platform element library [28], has four nodes each with six degrees of freedom. The full-scale 3D model includes both geometric and material nonlinearity. The mesh convergence study verified that the element with size 30 mm x 30 mm gives reasonable accuracy.

Girders with no end post (NEP), with rigid end post (REP), and with non-rigid end post (NREP) were included. Plated girder dimensions notations and various end supports were introduced, according to EN 1993-1-5 [29], as shown in figure 1.
3.2 Boundary conditions and load procedures

The boundary conditions and load procedures follow what is indicated in [7], to assure quite small bending moment and constant shear force for the end panel. However, two-point loads were applied at the third points, and simply supported boundary conditions were applied to the end section. Both load and boundary conditions are presented in figure 2.

3.3 Material properties

Bilinear elastic-plastic curve with linear strain hardening was used to simulate the behavior of the steel material for web, flanges and stiffeners. The slope of the linear elastic part of the curve, which mean...
Young's modulus, was used as $E = 200$ GPa, and Poisson’s ratio was taken $\nu = 0.3$. The slope of the hardening part was taken as 2 GPa. The used steel material is S355, which has ultimate stress 510 MPa and yield stress 355 MPa.

3.4 Model validation
To determine the accuracy of the FE model, the model was used to simulate the behaviour of specimens exhibited in the experimental tests of Zhu and Zhao [26]. As well as, the numerical model studied by Alinia et al. [7]. Table 1 presents the comparison between the available results in the literature and the FE model results. The mean value for the ratio of the available ultimate load in literature ($P_u$) to the FE ultimate load ($P_{u,FE}$) is 0.97 with standard deviation of 0.05.

The failure modes of plate girders in [26] were compared with the failure of FE model, as shown in figure 3, which indicates a good agreement. In the meanwhile, the failure modes stated by [7] for studied plate girders were also confirmed by the FE study, as shown in figure 4. Finally, the comparisons between load-deflection curves for the literature plate girders and the FE load deflection curves are presented. Where the FE model accurately predicts the load-deflection curves, as can be seen in figure 5.

Table 1. Comparison between available ultimate load in the literature ($P_u, L$) and the FE one ($P_{u,FE}$).

| Details of experimental tests of Zhu and Zhao [26] | Details of the simulated plate girder studied by Alinia et al. [7] |
|--------------------------------------------------|---------------------------------------------------------------|
| Plate girder | $h_w$ [mm] | $t_w$ [mm] | $b_f$ [mm] | $t_f$ [mm] | $L$ [mm] | $a$ [mm] | $f_{yw}$ [MPa] | $E$ [GPa] | $P_u, L$ [kN] | $P_{u,FE}$ [kN] | $P_u, L / P_{u,FE}$ |
| G1 | 600 | 8 | 180 | 16 | 2700 | 900 | 280 | 208 | 1200 | 1291 | 0.93 |
| G2 | 600 | 8 | 180 | 16 | 3600 | 1200 | 280 | 208 | 985 | 1037 | 0.95 |
| G(A) | 1000 | 4 | 300 | 9 | 4000 | 1000 | 345 | 210 | 800 | 773 | 1.03 |
| G(B) | 1000 | 3.33 | 300 | 7.5 | 4000 | 1000 | 345 | 210 | N/A | 551 | N/A |
| G(C) | 1000 | 3.33 | 300 | 3.33 | 4000 | 1000 | 345 | 210 | N/A | 312 | N/A |
| Mean* |  |  |  |  |  |  |  |  |  |  | 0.97 |
| Standard deviation |  |  |  |  |  |  |  |  |  |  | 0.053 |

* The ultimate load $P_u, L$ for the plate girder G(B) and G(C) are unavailable, hence they were excluded from the mean and the standard deviation

Figure 3. Lateral torsional buckling of tested specimen G2: a) Experimental results [26], b) F.E. results
Figure 5. Comparison between literature girders and FE load-deflection curves.

4. **NUMERICAL STUDY**
4.1 Assumptions and parameters

In this section, the analytical and numerical load/deflection behaviour of thin-walled panel loaded in shear is investigated. Based on the main target of that section for studying the shear failure, the span length of the modelled plate girders was taken as 3 times the panel width (3a), so the bending effect on the studied panel was minimized. Regarding the geometrical limitation of thin-walled panel, h_w/t_w ratio was taken as 200 for all studied panels. Twenty (a/h_w) ratios were investigated, such that (a/h_w) ratio was varied from 3.5 to 0.8. The initial imperfections values (w_o) were introduced for the model with scale factors (h_w/200), which considers a greater value for the bow of the web.

The current study contains four hundred full scale thin-web plate girders, as shown in table 2, covering three main parameters; stiffener thickness (t_s), end-post type (REP, NREP and NEP) and end distance (e).
Table 2. Details of studied specimens.

| Parameter                     | No. of specimens | \( h_w \) (mm) | \( w/ h_w \) | \( t_w \) \( \times \) \( t_w \) (mm) | \( h_w/t_w \) | \( b_f \) \( (L) \) (mm) | \( t_f \) (mm) | End stiffeners type                  | Initial imperfection \( w_o \) (mm) | Effect of stiffener thickness \( t_s \) |
|-------------------------------|------------------|----------------|--------------|-----------------------------|----------------|----------------|----------------|--------------------------------------|-------------------------------------|-------------------------------------|
| Effect of stiffener thickness \( t_s \) | 160              | 1000           | 3.5, 3.4, 3.3, 3.2, 2.4, 2.2, 2.0, 1.8, 1.7, 1.6, 1.5, 1.4, 1.3, 1.2, 1.0, 0.8 | 5              | 200             | 3a             | 300           | 25                    | Rigid end post (REP), Non-rigid end post (NREP) | \( h_w/200 \)                      | 5                                    |
| Effect of end-post            | 60               | 1000           | 3.5, 3.4, 3.3, 3.2, 2.4, 2.2, 2.0, 1.8, 1.7, 1.6, 1.5, 1.4, 1.3, 1.2, 1.0, 0.8 | 5              | 200             | 3a             | 300           | 25                    | No-end post (NEP), Rigid end post (REP), Non-rigid end post (NREP) | \( h_w/200 \)                      | 10                                   |
| Effect of end-distance \( e \) | 180              | 1000           | 3.5, 3.4, 3.3, 3.2, 2.4, 2.2, 2.0, 1.8, 1.7, 1.6, 1.5, 1.4, 1.3, 1.2, 1.0, 0.8 | 5              | 200             | 3a             | 300           | 25                    | \( e = 100 \text{ mm} \) for NEP, REP, NREP | \( h_w/200 \)                      | 10                                   |
|                              |                  |                |              |                             |                |                |                |                                      | \( e = 150 \text{ mm} \) for NEP, REP, NREP |                                      |                                     |
|                              |                  |                |              |                             |                |                |                |                                      | \( e = 300 \text{ mm} \) for NEP, REP, NREP |                                      |                                     |
4.2 Results discussion

The specimens listed in table 2 were solved numerically using (ABAQUS), hence load-deflection curves for these specimens were determined numerically and analytically. Investigating the effect of different parameters on the finite element load-deflection curves, then conforming these results to the theoretical approach is the main goal of this study.

The results and the primary parameters are presented and discussed herein, where $P_{u,FE}$ is the ultimate load carrying capacity calculated numerically using the FE model, which is the maximum load stated, and was typically followed by a drop in the load value. $\delta_{pb,FE}$ is the maximum FE vertical deflection under loading points for the post buckling stage, which is followed by failure stage (an increase in the deflection with nearly constant load).

In the meantime, the shear deflection $\delta_{pb,a}$ calculated by the analytical approach were expressed in equation 1, and the ultimate shear force $P_{u,a}$ is calculated such that;

$$P_{u,a} = \tau_{cr}(A_w) + V_{ult}$$

4.3 Effect of stiffeners thickness

To investigate the effect of stiffeners thickness, 160 specimens were studied as listed in table 2. Specimens with REP and NREP were examined. The specimens had the same cross section dimensions, but different stiffeners thickness (5 mm, 10 mm, 15 mm, and 20 mm). Twenty ratios of ($a/h_w$) were studied separately and combined, based on the aspect ratio of the web panels to evaluate the elastic shear buckling strength and post buckling strength, [4].

Table 3 & 4 present the comparison between the analytical results of the presented approach and the FE results for girders with stiffeners thickness 5 and 20, respectively. When the thickness of stiffeners were 5 mm, the mean values of $P_{u,a}/P_{u,FE}$ for REP and NREP are 0.96 and 1.00, with standard deviations of 0.17 and 0.19, respectively. Then the mean values $P_{u,a}/P_{u,FE}$ are 0.9 and 0.95 with standard deviations 0.11 and 0.15, for REP and NREP girders respectively, with stiffeners thickness 20 mm. Which indicates a good agreement with the analytical results.

For girders with thin stiffener ($t_s = 5$ mm), the mean values of $\delta_{pb,a}/\delta_{pb,FE}$ for REP and NREP are 0.80 and 0.82, with standard deviations of 0.11 and 0.12, respectively. Hence for girders with thick stiffeners ($t_s = 20$ mm), the mean values of $\delta_{pb,a}/\delta_{pb,FE}$ for REP and NREP are 0.82 and 0.87 respectively, with standard deviations of 0.10, for both. Deflection values indicate slight discrepancies between the FE and analytical approach. The authors believe that the presented approach underestimate the deflection values, since it only considers the shear deflection. But the FE model were subjected to shear and bending effects. Furthermore, results confirmed the small impact of the stiffener thickness on both shear strength and deflection of thin girders.

Figures 6, 7 and 8 present comparisons between the analytical and the FE load-deflection curves for different specimens, where it can be seen that the analytical approach accurately predicts the load-deflection curves for the studied specimens, especially for girders with panel aspect ratio ranged from $a/h_w = 1.7$ to $a/h_w = 2.2$, hence the performance has some discrepancy for $a/h_w > 2.2$. That was probably as discussed, such that for $a/h_w > 2.2$, the effect of bending was significant as the panel and the girder length increased. Although these slight deviations between analytical predictions and numerical results, the given approach still exhibits acceptable accuracy compared with these FE especially for $a/h_w < 2.2$. In the meantime, the stiffeners thickness has negligible effect on the load-deflection behavior.
Table 3. Analytical and FE loads/deflections of the studied specimens with \((t_s = 5 \text{ mm})\) values.

| \(h_w, t_w \text{ (mm)}\) | \(a/ h_w\) | Analytical \(\delta_{ba}\) | FE (REP) \(\delta_{b,FE}\) | FE (NREP) \(\delta_{b,FE}\) | Ratio \(P_{u,a}/P_{u,FE}\) | Ratio \(\delta_{ba}/\delta_{b,FE}\) |
|---------------------------|----------|------------------|------------------|------------------|------------------|------------------|
| 1000 x 5                  | 3.5      | 3500             | 701              | 19.14            | 954              | 23.20            | 0.73             |
|                           | 3.4      | 3400             | 714              | 18.12            | 950              | 21.41            | 0.75             |
|                           | 3.3      | 3300             | 727              | 17.12            | 952              | 19.84            | 0.76             |
|                           | 3.2      | 3200             | 742              | 16.15            | 961              | 22.40            | 0.77             |
|                           | 3.1      | 3100             | 757              | 15.21            | 969              | 20.64            | 0.78             |
|                           | 3.0      | 3000             | 773              | 14.30            | 976              | 18.36            | 0.79             |
|                           | 2.8      | 2800             | 808              | 12.57            | 987              | 15.37            | 0.82             |
|                           | 2.6      | 2600             | 848              | 10.95            | 994              | 12.53            | 0.85             |
|                           | 2.4      | 2400             | 894              | 9.46             | 1005             | 12.73            | 0.89             |
|                           | 2.2      | 2200             | 947              | 8.08             | 1005             | 10.91            | 0.94             |
|                           | 2.0      | 2000             | 1009             | 6.83             | 1027             | 8.29             | 0.98             |
|                           | 1.8      | 1800             | 1081             | 5.69             | 1083             | 6.73             | 0.98             |
|                           | 1.7      | 1700             | 1123             | 5.16             | 1096             | 6.02             | 1.00             |
|                           | 1.6      | 1600             | 1168             | 4.67             | 1095             | 5.07             | 1.07             |
|                           | 1.5      | 1500             | 1217             | 4.20             | 1092             | 4.45             | 1.11             |
|                           | 1.4      | 1400             | 1272             | 3.77             | 1135             | 3.98             | 1.12             |
|                           | 1.3      | 1300             | 1332             | 3.36             | 1164             | 3.46             | 1.14             |
|                           | 1.2      | 1200             | 1399             | 2.99             | 1193             | 4.42             | 1.17             |
|                           | 1.0      | 1000             | 1556             | 2.33             | 1282             | 3.33             | 1.21             |
|                           | 0.8      | 800              | 1769             | 1.82             | 1408             | 3.26             | 1.26             |
| Mean                     |          |                  |                  |                  |                 |                  | 0.96             |
| Standard deviation        |          |                  |                  |                  |                 |                  | 0.17             |
Table 4. Analytical and FE loads/deflections of the studied specimens with \((t_s = 20 \text{ mm})\) values.

| \(b_w \times t_w \) (mm) | \(a/ b_w\) | \(b_h \times t_h \) (mm) | \(a\) | \(t_s\) | Analytical Eq. | FE (REP) | FE (NREP) | Ratio (REP) | Ratio (NREP) | \(P_{u, a}/ P_{u, FE}\) | \(\delta_{p, a}/ \delta_{p, FE}\) | \(P_{u, a}/ P_{u, FE}\) | \(\delta_{p, a}/ \delta_{p, FE}\) |
|------------------------|-------------|--------------------------|-----|-----|--------------|----------|------------|--------------|--------------|----------------|----------------|----------------|--------------|
| 1000 x 5               | 300 x 25    | 20                       | 3.5 | 3500 | 701          | 19.14    | 946        | 23.00        | 946          | 22.86          | 0.74          | 0.83          | 0.74          | 0.84          |
|                        |             |                          | 3.4 | 3400 | 714          | 18.12    | 945        | 21.22        | 951          | 21.05          | 0.76          | 0.85          | 0.75          | 0.86          |
|                        |             |                          | 3.3 | 3300 | 727          | 17.12    | 966        | 19.66        | 947          | 19.66          | 0.75          | 0.87          | 0.77          | 0.87          |
|                        |             |                          | 3.2 | 3200 | 742          | 16.15    | 970        | 24.20        | 958          | 18.19          | 0.76          | 0.67          | 0.77          | 0.89          |
|                        |             |                          | 3.1 | 3100 | 757          | 15.21    | 978        | 22.36        | 962          | 22.42          | 0.77          | 0.68          | 0.79          | 0.68          |
|                        |             |                          | 3.0 | 3000 | 773          | 14.30    | 985        | 20.46        | 966          | 20.46          | 0.79          | 0.70          | 0.80          | 0.70          |
|                        |             |                          | 2.8 | 2800 | 808          | 12.57    | 994        | 15.10        | 975          | 12.95          | 0.81          | 0.83          | 0.83          | 0.97          |
|                        |             |                          | 2.6 | 2600 | 848          | 10.95    | 1002       | 12.91        | 985          | 11.52          | 0.85          | 0.85          | 0.86          | 0.95          |
|                        |             |                          | 2.4 | 2400 | 894          | 9.46     | 1016       | 11.70        | 995          | 12.64          | 0.88          | 0.81          | 0.90          | 0.75          |
|                        |             |                          | 2.2 | 2200 | 947          | 8.08     | 1023       | 10.45        | 1018         | 10.25          | 0.93          | 0.77          | 0.93          | 0.79          |
|                        |             |                          | 2.0 | 2000 | 1009         | 6.83     | 1054       | 8.26         | 1026         | 7.62           | 0.96          | 0.83          | 0.99          | 0.90          |
|                        |             |                          | 1.8 | 1800 | 1081         | 5.69     | 1119       | 6.65         | 1074         | 6.73           | 0.97          | 0.86          | 1.01          | 0.85          |
|                        |             |                          | 1.7 | 1700 | 1123         | 5.16     | 1115       | 5.66         | 1077         | 5.70           | 1.01          | 0.91          | 1.04          | 0.91          |
|                        |             |                          | 1.6 | 1600 | 1168         | 4.67     | 1151       | 5.00         | 1097         | 5.02           | 1.01          | 0.93          | 1.06          | 0.93          |
|                        |             |                          | 1.5 | 1500 | 1217         | 4.20     | 1200       | 4.45         | 1124         | 4.47           | 1.01          | 0.94          | 1.08          | 0.94          |
|                        |             |                          | 1.4 | 1400 | 1272         | 3.77     | 1251       | 3.92         | 1153         | 3.92           | 1.02          | 0.96          | 1.10          | 0.96          |
|                        |             |                          | 1.3 | 1300 | 1332         | 3.36     | 1311       | 3.40         | 1185         | 3.41           | 1.02          | 0.99          | 1.12          | 0.99          |
|                        |             |                          | 1.2 | 1200 | 1399         | 2.99     | 1387       | 4.32         | 1229         | 2.94           | 1.01          | 0.69          | 1.14          | 1.02          |
|                        |             |                          | 1.0 | 1000 | 1556         | 2.33     | 1577       | 3.12         | 1351         | 3.13           | 0.99          | 0.75          | 1.15          | 0.74          |
|                        |             |                          | 0.8 | 800  | 1769         | 1.82     | 1833       | 2.81         | 1497         | 2.17           | 0.97          | 0.65          | 1.18          | 0.84          |

| Mean                  | 0.90        | 0.82        | 0.95        | 0.87        |
| Standard deviation    | 0.11        | 0.10        | 0.15        | 0.10        |
Figure 6. Comparison between analytical equation and FE load-deflection curves for REP and NREP plate girders with ($a/h_w = 1.0$).

Figure 7. Comparison between analytical equation and FE load-deflection curves for REP and NREP plate girders with ($a/h_w = 1.7$).
4.4 Effect of end-post
Sixty plate girders having the same cross-sections and the same spans but with different end-post profiles were studied. Table 5 presents a comparison between analytical and FE load/deflection results for the studied plate girders. Since the analytical approach of Qian and Tan [25] neglected the effect of end-post, the analytical load/deflection results did not confirm any difference with different end-posts. Unlike the analytical results, the FE results highlight the effect of end-post rigidity on the load-deflection behaviour. The results show that providing a REP increases the ultimate shear strength ($P_{u,FE}$) of the girder, if compared with NREP. In the meantime, changing end-post from REP to NREP has insignificant effect on maximum vertical deflection ($\delta_{Pb,FE}$), in the most studied cases. It also shows that, ($P_{u,FE}$) of the same plate girder considerably decrease by providing NEP. However NPE profile has irregular effect on ($\delta_{Pb,FE}$).

The column charts presented in figure 9 shows the ratio between the analytical approach and finite element results for REP, NREP and NEP plate girders with $a/h_w = 0.8, 1, 1.4, 1.7, 2, 2.2$ and $3$. The analytical approach estimates well the shear strength of REP and NREP plate girders, hence it can’t estimate the shear strength or deflection for NEP plate girders. In the meantime, the presented analytical approach gives underestimates deflection results for both REP and NREP plate girders.

4.5 Effect of end-distance ($e$)
The current sub-section studies the effect of end-distance ($e$) in the load-deflection curves, note that the cross-section dimensions of the studied plate girders remained constant. However, $a/h_w$ ratio was varied from 0.8 to 3.5, as listed in tables 5, 6 and 7 for ($e$) equals 100, 200 and 300 mm, respectively.

Figure 10 presents the load-deflection curves of plate girders ($a/h_w = 2$) with different end-post profiles. Accordingly, the end distance ($e$) are 100 mm, 150 mm and 300 mm. The FE results show that increasing the end-distance ($e$) has no effect on the initial stiffness of the plate girders. However, it slightly increases the ultimate shear strength of the girders. That was expected, since increasing the end-distance influence the rigidity of end-post and hence the position of plastic hinges on flanges at failure stage. Hence, that explains the insignificant change of the maximum deflection at the post buckling stage for the studied girder. In other word, the end-distance becomes effective only in the failure stage. Figure 10 also states the misestimating of the analytical approach for the NEP girders. The same conclusions can be seen clearly form tables 5, 6 and 7, increasing the end-distance has insignificant effect on the deflection of the studied girders. Given that, for ($e$) equal 100, 150 and 300
mm, the mean values of $\delta_{pu,a}/\delta_{pu,FE}$ are 0.84, 0.85 and 0.85 respectively for REP girders. And equals 0.86, 0.87 and 0.86 for NREP girders.
Figure 9. Comparison between the analytical and FE results (loads/deflections) with different end-post profiles.
Table 5. Comparison between analytical and FE loads/deflections results for specimens (ts = 10 mm, e = 100 mm).

| Cross section | Analytical Eq. | FE (REX) | FE (NEP) | Ratio (REX/NEP) | Ratio (NRP/NEP) |
|---------------|---------------|----------|----------|-----------------|----------------|
|               | P       | δ       | P       | δ       | P       | δ       | P       | δ       | \( P_u \) | \( u \) |
| 3.5           | 701     | 19.14   | 315     | 8.96    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 3.4           | 714     | 18.12   | 304     | 8.91    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 3.3           | 727     | 17.12   | 302     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 3.2           | 742     | 16.15   | 300     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 3.1           | 757     | 15.21   | 298     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 3.0           | 773     | 14.30   | 296     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.9           | 808     | 13.47   | 295     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.8           | 848     | 12.67   | 294     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.7           | 894     | 11.90   | 292     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.6           | 947     | 11.17   | 291     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.5           | 1009    | 10.52   | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.4           | 1061    | 9.90    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.3           | 1123    | 9.34    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.2           | 1185    | 8.83    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.1           | 1247    | 8.37    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 2.0           | 1310    | 7.95    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.9           | 1373    | 7.57    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.8           | 1436    | 7.24    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.7           | 1500    | 6.94    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.6           | 1563    | 6.67    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.5           | 1627    | 6.44    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.4           | 1691    | 6.25    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.3           | 1755    | 6.09    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.2           | 1820    | 5.94    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.1           | 1884    | 5.80    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |
| 1.0           | 1949    | 5.67    | 290     | 8.90    | 1.05   | 0.94   | 0.98   | 0.90   | 1.00   | 0.99 | 0.94 |

Mean: \( \bar{P} = 0.94 \) \( \bar{u} = 0.94 \) \( \bar{\sigma} = 0.94 \)
Table 6. Comparison between analytical and FE loads/deflections results for specimens (\(ts = 10\) mm, \(e = 150\) mm).

| \(a/b\) | Cross section | \(P_{uv}\) | \(\delta_{ph,uv}\) | \(P_{u,FE}\) | \(\delta_{ph,FE}\) | \(P_{u,FE}\) | \(\delta_{ph,FE}\) | \(P_{u,FE}\) | \(\delta_{ph,FE}\) | \(P_{u,FE}/P_{u,FE}\) | \(\delta_{ph,FE}/\delta_{ph,FE}\) | \(P_{u,FE}/P_{u,FE}\) | \(\delta_{ph,FE}/\delta_{ph,FE}\) |
|---------|---------------|----------|-----------------|-------------|-----------------|-------------|-----------------|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 3.5     | 701           | 19.14    | 952             | 23.06       | 952             | 23.11       | 234             | 10.40       | 0.74            | 0.83            | 0.74            | 0.83            | 3.00            | 1.84            |
| 3.4     | 714           | 18.12    | 944             | 21.29       | 947             | 21.28       | 234             | 9.94        | 0.76            | 0.85            | 0.75            | 0.85            | 3.05            | 1.82            |
| 3.3     | 727           | 17.12    | 967             | 19.68       | 956             | 19.59       | 234             | 9.30        | 0.75            | 0.87            | 0.76            | 0.87            | 3.11            | 1.84            |
| 3.2     | 742           | 16.15    | 972             | 18.18       | 960             | 18.17       | 234             | 8.91        | 0.76            | 0.89            | 0.77            | 0.89            | 3.17            | 1.81            |
| 3.1     | 757           | 15.21    | 977             | 16.78       | 965             | 18.17       | 234             | 8.84        | 0.77            | 0.91            | 0.78            | 0.84            | 3.24            | 1.72            |
| 3.0     | 773           | 14.30    | 983             | 20.64       | 967             | 18.37       | 234             | 8.48        | 0.79            | 0.69            | 0.80            | 0.78            | 3.30            | 1.69            |
| 2.8     | 808           | 12.57    | 994             | 15.21       | 978             | 12.86       | 232             | 6.96        | 0.81            | 0.83            | 0.83            | 0.98            | 3.48            | 1.81            |
| 2.6     | 848           | 10.95    | 1004            | 11.29       | 986             | 11.46       | 235             | 5.39        | 0.84            | 0.97            | 0.86            | 0.96            | 3.61            | 2.03            |
| 2.4     | 894           | 9.46     | 1030            | 12.66       | 998             | 12.72       | 234             | 6.57        | 0.87            | 0.75            | 0.90            | 0.74            | 3.82            | 1.44            |
| 2.2     | 947           | 8.08     | 1060            | 10.25       | 1023            | 10.31       | 235             | 6.34        | 0.89            | 0.79            | 0.93            | 0.78            | 4.03            | 1.27            |
| 2.0     | 1009          | 6.83     | 1049            | 8.29        | 1050            | 8.32        | 234             | 6.00        | 0.96            | 0.82            | 0.96            | 0.82            | 4.31            | 1.14            |
| 1.8     | 1081          | 5.69     | 1144            | 6.68        | 1104            | 6.73        | 233             | 5.66        | 0.97            | 0.85            | 1.00            | 0.85            | 4.64            | 1.01            |
| 1.7     | 1123          | 5.16     | 1114            | 5.84        | 1077            | 6.00        | 234             | 9.95        | 1.01            | 0.88            | 1.04            | 0.86            | 4.80            | 0.52            |
| 1.6     | 1168          | 4.67     | 1129            | 5.00        | 1066            | 4.04        | 234             | 10.40       | 1.03            | 0.93            | 1.10            | 1.16            | 4.99            | 0.45            |
| 1.5     | 1217          | 4.20     | 1157            | 4.48        | 1116            | 4.48        | 227             | 3.75        | 1.05            | 0.94            | 1.09            | 0.94            | 5.36            | 1.12            |
| 1.4     | 1272          | 3.77     | 1205            | 3.98        | 1138            | 4.00        | 230             | 3.78        | 1.06            | 0.95            | 1.12            | 0.94            | 5.53            | 1.00            |
| 1.3     | 1332          | 3.36     | 1259            | 3.47        | 1167            | 3.49        | 236             | 3.91        | 1.06            | 0.97            | 1.14            | 0.96            | 5.64            | 0.86            |
| 1.2     | 1399          | 2.99     | 1326            | 3.00        | 1202            | 3.02        | 243             | 3.84        | 1.06            | 1.00            | 1.16            | 0.99            | 5.76            | 0.78            |
| 1.0     | 1556          | 2.33     | 1498            | 3.29        | 1297            | 3.31        | 236             | 3.72        | 1.04            | 0.71            | 1.20            | 0.70            | 6.59            | 0.63            |
| 0.8     | 1769          | 1.82     | 1723            | 3.17        | 1443            | 2.43        | 251             | 2.28        | 1.03            | 0.57            | 1.23            | 0.75            | 7.05            | 0.80            |

Mean: 0.91 0.85 0.96 0.87 4.42 1.28
| δ  | Cross section | Analytical Eq | FE (REP) | FE (NREP) | FE (NEP) | Ratio (REP) | Ratio (NREP) | Ratio (NEP) |
|----|---------------|---------------|----------|-----------|----------|-------------|--------------|-------------|
| 3.5|               | P_{u,a}       | δ_{p,a}  | P_{u,FE}  | δ_{p,FE} | P_{u,FE}    | δ_{p,FE}     |            |
|    |               | 701           | 19.14    | 955       | 23.09    | 955         | 23.09        | 275         |
|    |               | 714           | 18.12    | 946       | 21.30    | 947         | 21.29        | 276         |
| 3.3|               | 727           | 17.12    | 970       | 19.68    | 965         | 19.66        | 276         |
|    |               | 742           | 16.15    | 975       | 18.18    | 972         | 18.17        | 275         |
| 3.1|               | 757           | 15.21    | 982       | 16.78    | 978         | 18.16        | 275         |
|    |               | 773           | 14.30    | 987       | 20.06    | 981         | 20.65        | 276         |
| 2.8|               | 808           | 12.57    | 998       | 19.43    | 992         | 12.86        | 276         |
|    |               | 848           | 10.95    | 1014      | 11.31    | 1001        | 11.37        | 275         |
| 2.4|               | 894           | 9.46     | 1044      | 12.68    | 1020        | 12.69        | 276         |
|    |               | 947           | 8.08     | 1075      | 10.28    | 1046        | 10.29        | 276         |
| 2.2|               | 1009          | 6.83     | 1062      | 8.30     | 1039        | 8.29         | 270         |
| 2.0|               | 1081          | 5.69     | 1133      | 6.69     | 1105        | 6.70         | 275         |
| 1.8|               | 1123          | 5.16     | 1155      | 5.85     | 1095        | 5.74         | 276         |
| 1.7|               | 1168          | 4.67     | 1191      | 5.01     | 1110        | 5.00         | 275         |
| 1.6|               | 1217          | 4.20     | 1240      | 4.50     | 1140        | 4.48         | 273         |
| 1.5|               | 1272          | 3.77     | 1299      | 4.00     | 1169        | 4.00         | 277         |
| 1.4|               | 1332          | 3.36     | 1366      | 3.50     | 1201        | 3.49         | 277         |
| 1.3|               | 1399          | 2.99     | 1449      | 3.03     | 1237        | 3.03         | 278         |
| 1.2|               | 1556          | 2.33     | 1647      | 3.36     | 1338        | 3.29         | 282         |
| 1.0|               | 1769          | 1.82     | 1893      | 2.48     | 1477        | 2.44         | 294         |

Table 7. Comparison between analytical and FE loads/deflections results for specimens (ts = 10 mm, e = 300 mm).
| Mean     | 0.88 | 0.86 | 0.94 | 0.86 | 3.76 |
|----------|------|------|------|------|------|
| Standard deviation | 0.10 | 0.10 | 0.16 | 0.09 | 0.60 |
CONCLUSIONS

The load/deflection of thin-walled panel loaded in shear were studied using numerical and analytical methods. The Qian and Tan [25] mechanical approach, to calculate the in-plane deflection web panel loaded in shear, has been applied to calculate pre- and post-buckling deflection. And hence the non-linear large deflection FE analysis of the full-scale plate girders were studied. The comparison between the results of two methods concluded that:

- The analytical approach presented by Qian and Tan [25] can be used to simulate the load/deflection behaviour of thin-walled panel loaded in shear for $a/h_w < 2.2$, with REP and NREP. Such that, it estimates well the shear strength of REP plate girders and slightly overestimates the shear strength for NREP plate girders.
- The presented analytical approach can’t be used for calculating the deflection of NEP plate girders. And more studied is needed to find an accurate approach for NEP panels.
- The stiffeners thicknesses have insignificant effect on the load-deflection behaviour.
- The type of end-post directly affects the ultimate shear strength ($P_u$). The results show that providing a REP increases the ultimate shear strength ($P_{u,FE}$) of the girder, if compared with NREP.
- Changing end-post from REP to NREP has insignificant effect on maximum vertical deflection ($\delta_{P_u,FE}$).
- The FE results show that increasing the end-distance ($e$) has no effect on the initial stiffness of the plate girders. However, it slightly increases the ultimate shear strength of the girders.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.
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**Nomenclature**

| Symbol | Description |
|--------|-------------|
| $\sigma_y$ | tensile yield stress of web panel under shear; |
| $a$ | panel length; |
| $a/ h_w$ | web panel aspect ratio; |
| $A_w$ | area of the web; |
| $b_f$ | width of the flange; |
| $c$ | distance assumed between the plastic hinge and the end stiffener; |
| $E$ | elastic modulus; |
| $f_{yf}$ | yield stress of the flange plate; |
| $f_{yw}$ | yield stress of the web plate; |
| $h_w$ | height of the web; |
| $h_w/ t_w$ | web slenderness ratio; |
| $K$ | shear buckling coefficient; |
| $L$ | span of the plate girder; |
| $M_{pf}$ | plastic moment capacity of flanges($= 0.25f_{yf}A_f t_f$); |
| $P_{u,a}$ | ultimate shear force |
| $t_f$ | thickness of the flange; |
| $t_s$ | thickness of the stiffeners; |
| $t_w$ | thickness of the web; |
| $V_{ult}$ | post-buckling shear force component; |
| $w_o$ | initial imperfection |
| $\delta_{pa}$ | deflection of web panel from pre-buckling to the post buckling stage; |
| $\delta_{pb}$ | deflection of web panel at the pre-buckling stage; |
| $\delta_{ps}$ | total deflection of web at the post buckling stage for a panel loaded in shear; |
| $\nu$ | Poisson’s ratio; |
| $\tau_{cr}$ | shear buckling stress of the web panel |
| $\theta$ | inclination of the tensile field; |