Spontaneous rotation in one-dimensional systems of cold atoms

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We theoretically study harmonically trapped one-dimensional Bose gases (e.g., $^7$Li, $^{23}$Na, $^{39}$K, $^{87}$Rb, etc.) with multibands occupied, focusing on effects of higher-energy bands. Combining the Ginzburg-Landau theory with the bosonization techniques, we predict that the repulsive interaction between higher-band bosons and the quantum fluctuation can induce the ground state with a finite angular momentum around the trapped axis. In this state, the $Z_2$ reflection symmetry (clockwise or anticlockwise rotations) is spontaneously broken.

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I. INTRODUCTION

For the last decade, cold-atom systems have been vigorously studied and experimental techniques of controlling them have been greatly developed. As a result, we can now tune various parameters of cold-atom systems, even including dimensionality. These developments have enabled us to encounter several phenomena: For instance, superfluid-insulator transitions on optical lattices, dynamics of Bose-Einstein condensed (BECs), BCS-BEC crossover in Fermi gases, etc. They are all difficult to be realized within traditional experiments using electronic systems in solids.

Among various new fields of cold-atom physics, in this paper, we focus on one-dimensional (1D) trapped Bose gases (e.g., $^7$Li, $^{23}$Na, $^{39}$K, $^{87}$Rb, etc.) with multibands occupied, focusing on effects of higher-energy bands. Combining the Ginzburg-Landau theory with the bosonization techniques, we predict that the repulsive interaction between higher-band bosons and the quantum fluctuation can induce the ground state with a finite angular momentum around the trapped axis. In this state, the $Z_2$ reflection symmetry (clockwise or anticlockwise rotations) is spontaneously broken.

Thanks to the recent studies of purely 1D atomic gases, their understanding has been greatly deepened. However, there still exist important issues in 1D confined systems: One of them is what happens if atoms occupy not only the lowest band but also higher ones. As yet, effects of higher bands have not been well discussed well. Since multibands may be regarded as an internal degrees of freedom such as electron spins, the system is expected to contain rich physical properties. In this paper, we will provide an answer to this problem. To this end, we consider a simple 3D single-component Bose gas system with a repulsive interaction and a 2D harmonic potential. As we will explain, due to the potential, each energy band takes an angular momentum $l$: the lowest band has $l = 0$, the second lowest ones have $l = \pm 1$ and are hence doubly degenerate, and so on. A positive (negative) $l$ corresponds to atoms rotating clockwise (anticlockwise) around the confinement axis. For simplicity, we will concentrate on the case where only the lowest band and the degenerate
second lowest ones (totally three bands) are occupied. In this situation, it is shown that at least if the 2D harmonic potential is sufficiently stronger than the interaction between atoms, the particle densities of the \( l = \pm 1 \) bands become imbalanced and the ground state spontaneously rotates with a finite angular momentum as shown in Fig. 1. We will explain in detail the mechanism of this spontaneous rotation phenomenon.

This paper is organized as follows. First, we define the 3D Hamiltonian of the Bose atoms confined by a 2D harmonic potential in Sec. II. Then, we reduce the 3D Hamiltonian to the 1D effective one in order to describe the low-energy physics. In Secs. III and IV, based on the reduced Hamiltonian, we analyze the low-energy physics of the trapped bosons. Employing the Ginzburg-Landau (GL) theory and the bosonization techniques, we show how certain, realistic conditions. The final Section V is devoted to the summary and the discussions for our results. In Appendix A, we summarize the symmetries of our Bose gas system, which play important roles in the analyses of Secs. III and IV.

II. BOSE GAS AND EFFECTIVE HAMILTONIAN

First, we introduce a 3D Bose gas in the presence of a 2D harmonic potential, whose Hamiltonian is defined as

\[
H = \int d\vec{r} \left[ \psi^\dagger \left( -\frac{\hbar^2 \nabla^2}{2m} - \mu + V(\vec{r}) \right) \psi + \frac{U}{2} \mathcal{D}^2 + \cdots \right],
\]

where \( \psi(\vec{r}) \) and \( \mathcal{D} = \psi^\dagger \psi \) are the annihilation field and the density operator of the bosons, respectively. Positive parameters \( m, \mu \) and \( U \) respectively mean the mass of bosons, the chemical potential and the repulsive coupling constant. The 3D s-wave scattering length \( a \) is related to \( U \) via \( U = 4\pi \hbar^2 a/m \). Three or more-body interactions are assumed to be quite small compared with \( U \). The 2D confinement potential \( V \) is a harmonic function: \( V(\vec{r}) = \frac{1}{2} m \omega_0^2 (y^2 + z^2) - \hbar \omega_0 \), where the axial direction is equal to the \( x \) axis (see Fig. 1). Here we note that the Hamiltonian (1) is invariant under the \( Z_2 \) reflection and the \( SO(2) \) rotation in the \( y-z \) plane \( [\psi(x, y, z) \rightarrow \psi(x, -y, z) \text{ and } \psi(\vec{r}) \rightarrow \psi(x, y \cos \Phi - z \sin \Phi, y \sin \Phi + z \cos \Phi)] \), where \( \Phi \in \mathbb{R} \).

Other symmetries of the model (1) are discussed in Appendix A.

In order to make the one dimensionality of the model (1) more visible, it is useful to expand \( \psi(\vec{r}) \) in terms of eigenstates of the 2D harmonic oscillator \( u_{n,l}(y, z) \) as follows:

\[
\psi(\vec{r}) = \sum_{n=0}^{\infty} \sum_{l} u_{n,l}(y, z) \phi_{n,l}(x),
\]

where \( l \) is the angular momentum in the \( y-z \) plane and can take \(-n, -n+2, \ldots, n-2, n\) for a given \( n \). The angular momentum is a good quantum number because of the SO(2) rotational symmetry of Eq. (1). The boson field \( \phi_{n,l} \) describes the physics of the \( x \) direction. Substituting Eq. (2) into Eq. (1), we obtain the single-particle energy dispersion \( \epsilon_{n,l}(k) = \frac{\pi^2}{2m} + \hbar \omega_0 n \) which is shown in Fig. 2.

It is noteworthy that all of the bands \( \epsilon_{n,l} \) with the same index \( n \) are degenerate, i.e., the \( (n+1) \)-fold degeneracy exists. For example, as we already mentioned, the second lowest bands \( \epsilon_{1, \pm 1} \) are doubly degenerate.

To discuss effects of multi bands, we assume that the bosons occupy the lower three bands \( \epsilon_{0,0}, \epsilon_{1, \pm 1} \) as in Fig. 2. To satisfy this situation, we impose the following conditions:

\[
h\omega_0 < \mu < 2h\omega_0, \tag{3}
\]
\[
a/\alpha_\perp < 1 \quad \text{(i.e., } h\omega_0 \gg U/a_\perp^3), \tag{4}
\]
\[
2h\omega_0 - \mu \gg U/a_\perp^3, \tag{5}
\]

where the confinement radius \( a_\perp = \sqrt{\hbar/m \omega_0} \) represents the width of the wave function \( u_{n,l}(y, z) \) around the origin \((y, z) = (0, 0)\). The first condition (3) ensures that the lower three bands have finite boson densities. The other ones (4) and (5) mean that the energy scale between neighboring bands is sufficiently larger than the strength of the interaction. These would allow us to neglect bosons in all of the higher bands \( \epsilon_{n>2,l} \) (i.e., bands \( \epsilon_{n>2,l} \) are nearly empty of bosons).

Under these assumptions, let us continue to study the model (1). Since the low-energy physics must be governed by the bosons on occupied bands with \( (n, l) = (0, 0), (1, \pm 1) \), the low-energy effective theory can be obtained by integrating out the degrees of freedom of the bosons on all the vacant bands. In order to carry out this integration, it is convenient to introduce the Euclidean action corresponding to the Hamiltonian (1) through the
path integral formalism as $S_{\text{tot}} = S_0 + S_1 + S_{\text{int}},$

$$S_0 = \int \! d\tau dx \sum_{n,l} \phi_{n,l}^* \left[ \partial_\tau - \frac{\hbar}{2m} \partial_x^2 - \mu + n\hbar \omega_0 \right] \phi_{n,l} + \frac{U}{8 \pi a_1} \left[ 2\rho_{0,0}^2 + \rho_{1,1}^2 + \rho_{-1,1}^2 + 4\rho_{0,0}(\rho_{1,1} + \rho_{-1,1}) 
+ 4\rho_{1,1}\rho_{-1,1} + 2(\phi_{0,0}^*\phi_{0,0}^*\phi_{1,1}\phi_{1,1} + \text{h.c.}) \right], \quad (6a)$$

$$S_1 = \int \! d\tau dx \sum_{n=2}^\infty \sum_{l} \phi_{n,l}^* \left[ \partial_\tau - \frac{\hbar}{2m} \partial_x^2 - \mu + n\hbar \omega_0 \right] \phi_{n,l}, \quad (6b)$$

$$S_{\text{int}} = \frac{U}{2} \int \! d\tau dx \sum_{\{n\}} \sum_{\{l\}} P_{t_1,t_2,t_3,t_4}^{n_1,n_2,n_3,n_4} \times \phi_{n_1,t_1}^* \phi_{n_2,t_2}^* \phi_{n_3,t_3} \phi_{n_4,t_4}. \quad (6c)$$

where $\tau$ is the imaginary time, and $\rho_{n,l} = \phi_{n,l}^* \phi_{n,l}$ is a boson density of a band $(n,l)$. The prime of the summations in $S_{\text{int}}$ sums over all the possible sets of $(n_1,l_1)\ (i = 1, \ldots, 4)$ except for all of the interaction terms of $S_0$. Equations (6a) and (6b) are, respectively, the part including only the bosons of the occupied bands and the kinetic part of massive bosons of all the empty bands. The remaining part $S_{\text{int}}$ contains all the interaction terms including empty-band boson fields. The coupling constants $P_{t_1,t_2,t_3,t_4}^{n_1,n_2,n_3,n_4}$ in $S_{\text{int}}$ are expressed as

$$P_{t_1,t_2,t_3,t_4}^{n_1,n_2,n_3,n_4} = \int \! dy dz u_{n_1,t_1}^* u_{n_2,t_2}^* u_{n_3,t_3} u_{n_4,t_4}. \quad (7)$$

The integration of the empty-band bosons in $S_{\text{tot}}$ can be performed by making use of the representations [25-27] and the cumulant expansion. The resulting action is written as $S_{1D} = S_0 + (S_{\text{int}}) - \frac{1}{2}(S_{\text{int}}^2 - (S_{\text{int}})^2) + \cdots$, where $(\ldots) \cdot \cdot \cdot$ stands for the expectation value with respect to $S_1$. For the action $S_{1D}$, effects of higher-energy bands are taken into account through virtual scattering processes. In this calculation process, the Matsubara Green’s function $G_n(x,\tau,\sigma) = -\langle \bar{T}_\tau \phi_n(x,\tau) \phi_{n,-}\rangle(0,0)$ of the massive bosons with $\epsilon_n > 1$ may be approximated as $\lambda_n^{-1} \Theta_\alpha(\tau) \Theta_\alpha(\lambda_n - |\tau|)$, where $\alpha = \sqrt{2\pi \hbar/m\Delta_n}$ is the thermal de Broglie wave length, $\Delta_n = n\hbar \omega_0 - \mu$ is the gap of bosons, and $\Theta_\alpha$ is the Heaviside’s step function.

The resulting effective Hamiltonian corresponding to the action $S_{1D}$ is given as

$$H_{\text{1D}} = \int dx \ H_0 + H_{\text{int}}, \quad (8a)$$

$$H_0 = \frac{\hbar^2}{2m} \sum_{\alpha} \partial_x \phi_{\alpha}^* \partial_x \phi_{\alpha} - \mu_0 \rho_0 - \mu_1(\rho_1 + \rho_{-1}). \quad (8b)$$

$$H_{\text{int}} = \frac{U}{8 \pi a_1} \left[ 2\Gamma_0 \rho_0^2 + \Gamma_1 (\rho_1^2 + \rho_{-1}^2) + 4\Gamma_0(\rho_1 + \rho_{-1}) + 4\Gamma_{\pm 1} (\phi_{0,0}^*\phi_{0,0}^*\phi_{1,1}\phi_{1,1} + \text{h.c.}) \right]. \quad (8c)$$

where $\alpha = 0, \pm 1$ correspond to band indices $(n,l) = (0,0), (1, \pm 1)$, respectively. The Hamiltonian $H_0$ is the free boson part and $\mu_0 = \mu > 0$ and $\mu_1 = \mu - \hbar \omega_0 > 0$ are the chemical potentials. The first and second terms in $H_{\text{int}}$ are the two-body interactions in the same band. The third and fourth are those between the different bands. The final term is the tunneling between the lowest band and the doubly degenerate bands. The coupling constants $\Gamma_\alpha$ are modified from unity (the bare value in $S_0$) due to vacuum-band effects.

Calculating lower-order cumulants carefully, one can find that, for example, massive bosons on $\epsilon_{2,1}$ varies coupling constants as $\Gamma_\alpha \approx 1 + \sum_{n=1}^\infty c_{\alpha,n}(a/a_\perp)^n$ ($c_{\alpha,n}$ are nonuniversal constants). This result strongly supports that the cumulant expansion is reliable and works out under the conditions (3)-(5) and $\Gamma_\alpha = 1$ is semi-quantitatively correct. The cumulant expansion generates other three- or more-body interactions, but they are also negligible compared with the two-body ones in $H_{\text{int}}$.

### III. ANALYSIS I

In the preceding section, we have obtained the 1D effective Hamiltonian [5]. Here, analyzing it, let us investigate the low-energy properties of the Bose gas under the conditions (3)-(5). First, to evaluate the mean density profile of each band, we introduce the following GL potential from $H_{\text{int}},$

$$F_{\text{GL}} = -\mu_0 \rho_0 - \mu_1 \rho_1 + \frac{U}{8 \pi a_1^2} \left[ 2\Gamma_0 \rho_0^2 + \Gamma_1 (\rho_1^2 + \rho_{-1}^2) + \Gamma_0 \rho_1 + \Gamma_{\pm 1} \right]$$

$$+ 4\Gamma_0(\rho_1 + \rho_{-1}) + 2\Gamma_{\pm 1} \rho_0 \rho_{\pm 1} \sqrt{\rho_0^2 - \rho_{\pm 1}^2} \cos(\theta_0 - \theta_{\pm 1}), \quad (9)$$

where $\theta_\alpha = (\Gamma_1 \pm 2\Gamma_{\pm 1})/2$ and we have defined $\phi_\alpha = \rho_{\alpha}^{1/2} e^{i\theta_\alpha}$, $\rho_{\alpha} = \rho_1 \pm 1$ and $\theta_\alpha = (\theta_1 \pm \theta_{-1})/2$. The quantity $\rho_0$ is the total density in the second lowest bands, while $\rho_1$ stands for the angular momentum density in the same bands [the total angular momentum is $L = \sum_{n,l} \int dx \ \rho_{n,l}(x)$]. Therefore, a solution with $\rho_0 \neq 0$, if possible, means the emergence of the spontaneous rotation and the breakdown of the $Z_2$ reflection symmetry in the trapped plane. Each term in $F_{\text{GL}}$ has the following physical meanings: (i) Positive $\mu_0, \Gamma_0$ induce finite densities $\rho_{\pm 1} \neq 0$ and the relation $\mu_0 \gg \mu_1$ must yield $\rho_0 \gg \rho_{\pm 1}$, (ii) $\Gamma_0(> 0)$ tend to decrease $\rho_{\pm 1}$, while $\Gamma_0(< 0)$ promotes the growth of $\rho_{\alpha}$. (iii) $\Gamma_1(> 0)$ favors the decrease of $\rho_{\alpha}$, and (iv) the tunneling $\Gamma_{\pm 1}(> 0)$ inversely enhances $\rho_{\pm 1}$ and decreases $\rho_0$. Recalling the condition (3), we can guess that bosons on the second lowest bands interact only with the lowest-band bosons with a kinetic energy $e_{\alpha,l}(k) \sim \hbar \omega_0$, and hardly influence the bosons with lower energies. It is hence expected that $\Gamma_0$ and $\Gamma_{\pm 1}$ are overestimated in $F_{\text{GL}}$. Here, we simply regard both the density amplitude $\rho_\alpha$ and the phase $\theta_\alpha$ as $c$ numbers, although for real quantum systems, we cannot fix them simultaneously. This approximation would also overestimate the effects of $\Gamma_{\pm 1}$. 
The GL equations $\partial \mathcal{F}_{\text{GL}} / \partial \rho_a = \partial \mathcal{F}_{\text{GL}} / \partial \theta_a = 0$ are solved as

\begin{align}
\tilde{\rho}_0 &= \frac{4\pi a^2}{U} \Gamma s \mu_0 - (2\Gamma_0 - 2\Gamma_{\perp,1}) \mu_1, \\
\tilde{\rho}_s &= \frac{4\pi a^2}{U} - (2\Gamma_0 - \Gamma_{\perp,1}) \mu_0 + 2\Gamma_0 \mu_1, \\
\tilde{\rho}_a &= 0, \\
\tilde{\theta}_0 - \tilde{\theta}_s &= \pi/2.
\end{align}

Furthermore, computing $\partial^2 \mathcal{F}_{\text{GL}} / \partial \rho_a \partial \rho_b$, we find that the above solution is stable and minimize $\mathcal{F}_{\text{GL}}$. Because of $\tilde{\rho}_a = 0$, the GL argument shows no symmetry breaking. If we straightforwardly adopt the approximation $\Gamma_{\alpha} = 1$, $\tilde{\rho}_\alpha$ becomes negative under the condition (3). This is unphysical and attributed to large $\Gamma_{01}$ and $\Gamma_{0 \perp 1}$, as expected. To recover a physical solution $\tilde{\rho}_0 > \tilde{\rho}_s \neq 0$, we should replace $\Gamma_0$ and $\Gamma_{\perp,1}$ with small values, $\tilde{\Gamma}_0$ and $\tilde{\Gamma}_{\perp,1}$, respectively. For example, when we set $\tilde{\Gamma}_0 = \tilde{\Gamma}_{\perp,1} = a/a_{\perp} \ll 1$, the GL equations offer a reasonable solution $\tilde{\rho}_s \approx \frac{8\pi a^2}{3\mu_1}(\mu_1 - \frac{a}{2\pi} \mu_0)$.

Let us next take into account the quantum fluctuation around the physical GL solution. To this end, the bosonization [9, 13, 14] is powerful and unbiased. It makes the density and field operators transform as

\begin{align}
\rho_\alpha &\approx \left( \tilde{\rho}_\alpha + \frac{1}{\pi} \frac{\partial_x \varphi_\alpha}{a} \right) \sum_{n=-\infty}^{\infty} e^{i2n(\varphi_\alpha - \pi \rho_{\alpha x})}, \\
\phi_\alpha &\approx \sqrt{\tilde{\rho}_\alpha + \frac{1}{\pi} \frac{\partial_x \varphi_\alpha}{a}} \sum_{n=-\infty}^{\infty} e^{i2n(\varphi_\alpha - \pi \rho_{\alpha x})} e^{-i(\tilde{\theta}_\alpha + \theta_\alpha)},
\end{align}

where the phase fields $\theta_\alpha$ and $\varphi_{\alpha'}$ ($\alpha, \alpha' = 0, \pm 1$) represent the quantum fluctuation and obey $[\theta_\alpha(x), \partial_x \varphi_{\alpha'}(x')] = [\varphi_\alpha(x), \partial_x \theta_{\alpha'}(x')] = i\pi \delta_{\alpha \alpha'} \delta(x - x')$. Here we define $\varphi_{\pm, \alpha} = \varphi_{\alpha} \pm \varphi_{\alpha + 1}$. Substituting the formula (11) into Eq. (8), we obtain the phase-field Hamiltonian, which is invariant under the symmetry operations (A1) and (A2). Remarkably, due to $\Gamma_{\alpha} < 0$, the coefficient of $(\partial_x \varphi_\alpha)^2$ becomes negative in the Hamiltonian. This means that the $(\varphi_\alpha, \theta_\alpha)$ sector has an instability against the fluctuation of $\rho_\alpha$. It might be restored by sufficiently large higher-order differential terms such as $(\partial_x \varphi_\alpha)^4$ [15], but the emergence of those terms would not be expected under the condition (3). We thus conclude that the quantum fluctuation violates the GL solution and induces a ground state with a finite angular momentum $\tilde{\rho}_\alpha \neq 0$. It is known that similar scenarios of symmetry breakings can occur in a few systems. [16, 17, 18]

**IV. ANALYSIS II**

In order to examine and to enhance the validity of the above prediction of $\tilde{\rho}_0 \neq 0$, let us reconsider the effective theory [8] using another approximation. As we already mentioned, $\Gamma_0$ and $\Gamma_{\perp,1}$ terms are expected to be overestimated in Eq. (9). Therefore, at first we daringly drop the tunneling term with $\Gamma_{\perp,1}$ and replace $\Gamma_{01}$ with a small effective value $\tilde{\Gamma}_{01}$. For this simplified case, the GL potential is written as

$$
\tilde{\mathcal{F}}_{\text{GL}} = -\mu_0 \rho_0 - \mu_1 \rho_s + \frac{U}{8\pi a_{\perp}^2} \left[ 2\Gamma_0 \rho_0^2 + 2\Gamma_0 \rho_s^2 + \Gamma_0 \rho_a^2 + 4\tilde{\Gamma}_{01} \rho_0 \rho_s \right].
$$

The GL equations $\partial \tilde{\mathcal{F}}_{\text{GL}} / \partial \rho_a = 0$ lead to $\rho_0 = \frac{2\pi a^2}{U} \Gamma_{01} \mu_0 - 2\tilde{\Gamma}_{01} \mu_1$, $\rho_s = \frac{4\pi a^2}{U} \Gamma_{01} \mu_1 - \Gamma_{01} \mu_0$ and $\tilde{\rho}_a = 0$. Namely, the GL solution again suggests no angular momentum. However, considering the stability of the solution, we see that the density profile $(\tilde{\rho}_0, \tilde{\rho}_s, \tilde{\rho}_a)$ corresponds to the saddle point for $\tilde{\mathcal{F}}_{\text{GL}}$, and it is minimized at $\rho_0 \to \pm \infty$ which is unphysical. The reason for this instability is that the interband interaction $\Gamma_{\pm,1}$ is stronger than intraband one $\Gamma_1$. To recover a physically proper solution, we can add the following phenomenological term $\frac{U}{8\pi a_{\perp}^2} \Gamma_{01} (\tilde{\Gamma}_{01} > 0)$ to $\tilde{\mathcal{F}}_{\text{GL}}$. It is interpreted that this term originates from the higher-order corrections of the cumulant expansion or many-body interactions neglected in the original Hamiltonian (1). This modification offers a stable, physical solution

$$
\tilde{\rho}_a = \pm \sqrt{\frac{2\Gamma_0}{\Gamma_{01}}}.
$$

From the consideration above, we see that the GL approach with neglecting the tunneling term naturally leads to a finite angular momentum.

As in the previous analysis in Sec. III, let us bosonize the Hamiltonian (8) based on the GL solution (13). At this stage, we recover the tunneling term. The resulting phase-field Hamiltonian is represented as

$$
\mathcal{H}_{1D} \approx \int dx \sum_{\alpha = 0, \pm 1} \frac{\nu_{\alpha}}{2\pi} \left\{ K_{\alpha} (\partial_x \theta_\alpha)^2 + K_{-1} (\partial_x \varphi_\alpha)^2 \right\} \\
+ \tilde{g}_{\alpha} \partial_x \theta_\alpha \partial_x \theta_\alpha + \tilde{g}_0 \partial_x \varphi_0 \partial_x \varphi_0 \\
+ \tilde{g} \cos[2n(\theta_0 - \theta_s)] (n \in \mathbb{Z})\right. \\
+ & \tilde{g}_h \cos[2n(\theta_0 - \theta_s)] + \cdots.
$$

Coefficients of $(\partial_x \theta_\alpha)^2$, $(\partial_x \theta_\alpha)^2$ and $(\partial_x \varphi_\alpha)^2$ are proportional to $\tilde{\rho}_0$, $\tilde{\rho}_s$ and $\tilde{\rho}_a$, respectively, while $\tilde{g}_{\alpha} \propto \tilde{\rho}_a$.

Therefore, when $\tilde{\rho}_a$ is smaller enough than $\tilde{\rho}_0, \tilde{\rho}_s$, any instability of restoring $\tilde{\rho}_a = 0$ does not originate from the differential terms. In addition, the symmetry operations (A2) indicate that any vertex term with $\varphi_{\alpha}$ or $\theta_{\alpha}$ is forbidden in the Hamiltonian (13). Thus, we can say that $\tilde{\rho}_a \neq 0$ is not destroyed by the quantum fluctuation and it agrees with our previous approach in Sec. III. The symmetries also tell us that for all of the vertex operators, only $\cos[2n(\theta_0 - \theta_s)] (n \in \mathbb{Z})$ are allowed to appear in the phase-field Hamiltonian. The final term of Eq. (14) is the most relevant in the allowed vertex interactions. Generally, for 1D boson systems with repulsive short-range interactions, $\cos[2n(\theta_0 - \theta_s)] (n \in \mathbb{Z})$ is expected to dominate, and it induces a ground state with a finite angular momentum $\tilde{\rho}_a \neq 0$. It is known that similar scenarios of symmetry breakings can occur in a few systems. [16, 17, 18]
interactions, the TLL parameter $K_{0,s,a}$ are known to be larger than unity. [3] The vertex term $\cos(2\theta_0 - 2\theta_s)$ thus must be strongly relevant [14] and $\theta_0 - \theta_s$ is locked at $\pi/2$, which is consistent with our previous result [10b]. Introducing new fields $(\varphi_{s,\pm}, \theta_{s,\pm}) = (\varphi_s \pm \varphi_s, \theta_s \pm \theta_s)/\sqrt{2}$, we can interpret that the cosine term opens a gap in the $(\varphi_{s,\pm}, \theta_{s,\pm})$ sector. Following the method in Ref. [20], we can trace out the degrees of freedom of the gapped sector. After that, we finally obtain a two-component TLL Hamiltonian with the $(\varphi_s, +)$ and $(\varphi_s, -)$ sectors as the low-energy effective theory.

On the other hand, when $\rho_a$ is too large in Eq. (13), we should adopt $(\bar{\rho}_1, \bar{\rho}_1) = (\bar{\rho}_a, 0)$ or $(0, \bar{\rho}_a)$ as the proper GL solution instead of Eq. (13). For this case, neglecting all of the parts with the field $\phi_1$ in the Hamiltonian [8], we again derive a two-component TLL Hamiltonian.

Consequently, we conclude from the second analysis above that the ground state possesses a finite angular momentum $\rho_a \neq 0$ and the low-energy excitations are described by a two-component TLL. [21]

V. SUMMARY AND DISCUSSIONS

In this paper, we have studied the 1D harmonically trapped Bose gas [1] with bosons on multi-transverse modes. Starting from the 3D boson systems with 2D confinement potential, the 1D effective Hamiltonian is derived by integrating out the degrees of freedom of all the vacant bands. Applying two theoretical ways based on the GL approach and the bosonization to the 1D effective Hamiltonian [8], we have shown that when bosons are filled in the lower three bands under the conditions [3]-[5], a angular momentum becomes finite and the $Z_2$ reflection symmetry in the trapped plane is spontaneously broken. This phenomenon essentially originates from the larger repulsive interaction between the degenerate bands $(n, l) = (1, \pm 1)$ than the intraband repulsion. The second analysis in Sec. [14] predicts that the low-energy excitations on the rotating ground state are described by a two-component TLL. [21] It is known that when the bosons occupy only the lowest bands, the low-energy physics is governed by a one-component TLL. We thus can draw the phase diagram in Fig. 3.

The conjugate field for $\rho_a$ is not realistic in the model [1]. Any trigger of the rotating ground state therefore seems to be absent. However, the real trap potential must deviate from the harmonic type. Such a small deviation would help the system circulate clockwise or anticlockwise. Thermal fluctuations and large deformations of the trapped potential would generate kink structures between clockwise and anticlockwise rotating regimes. When Bose atoms carry an electric charge, the predicted rotation means spontaneous loop current and magnetic flux. If we suddenly change the form of the trap potential and then observe the real-space boson density profile, we can detect, in principle, whether the bosons rotate clockwise or anticlockwise.

APPENDIX A: SYMMETRY ARGUMENT

Here, we briefly summarized the symmetries of our Bose gas system [1]. They are often used in our analyses of Secs. [11] and [14].

The Hamiltonian [11] is invariant under the following symmetry operations: The translation and the parity for the $x$-axis $[\psi(x, y, z) \rightarrow \psi(x + \delta, y, z) \ (\delta \in \mathbf{R})$ and $\psi(x, y, z) \rightarrow \psi(-x, y, z)]$, the $Z_2$ reflection and the SO(2) rotation in the $y$-$z$ plane $[\psi(x, y, z) \rightarrow \psi(x, -y, z)$ and $\psi(r) \rightarrow \psi(x, y \cos \Phi - z \sin \Phi, y \sin \Phi + z \cos \Phi) \ (\Phi \in \mathbf{R})]$, and the global U(1) gauge transformation $[\psi(x, y, z) \rightarrow e^{i\theta} \psi(x, y, z) \ (\Theta \in \mathbf{R})]$. For the bosonized phase-field Hamiltonians (e.g., Eq. (11)), it is important to note that all of the above symmetry operations can be translated to the bosonization language. [18] The translation and the parity operation for the $x$-direction, the $Z_2$ reflection and the SO(2) rotation in the $y$-$z$ plane, and the global U(1) gauge transformation are, respectively, expressed as

\[
\begin{align*}
[\varphi_a(x, \theta_a(x))] & \rightarrow [\varphi_a(x + \delta) + \pi \bar{\rho}_a \delta, \theta_a(x + \delta)], \\
[\varphi_a(x, \theta_a(x))] & \rightarrow [-\varphi_a(-x), \theta_a(-x)], \\
[\varphi_a(x, \theta_a(x), \bar{\rho}_a)] & \rightarrow [\varphi_a(x, -\alpha \Phi), \bar{\rho}_a], \\
[\varphi_a(x, \theta_a(x))] & \rightarrow [\varphi_a(x, \theta_a(x) - \Phi)], \\
[\varphi_a(x, \theta_a(x))] & \rightarrow [\varphi_a(x, \theta_a(x) + \theta)],
\end{align*}
\]

where $\alpha = 0, \pm 1$. Using another set of the phase fields $(\varphi_{s,a}, \theta_{s,a}) = (\varphi_s \pm \varphi_s, \theta_s \pm \theta_s)/\sqrt{2}$, we can further transform Eq. (A1) as

\[
\begin{align*}
[\varphi_a(x, \theta_a(x))] & \rightarrow [\varphi_a(x + \delta) + \pi \bar{\rho}_a \delta, \theta_a(x + \delta)], \\
[\varphi_a(x, \theta_a(x))] & \rightarrow [-\varphi_a(-x), \theta_a(-x)], \\
[\varphi_a(x, \theta_a(x), \bar{\rho}_a)] & \rightarrow [-\varphi_a(x, \theta_a(x) - \Phi)], \\
[\varphi_a(x, \theta_a(x))] & \rightarrow [\varphi_a(x, \theta_a(x) + \theta)], \\
[\varphi_a(x, \theta_a(x))] & \rightarrow [\varphi_a(x, \theta_a(x) - \Phi)].
\end{align*}
\]
for \((\varphi_a, \theta_a)\), and
\[
\begin{align*}
[\varphi_s(x), \theta_s(x)] &\rightarrow [\theta_s(x + \delta), \varphi_s(x + \delta) + \pi \rho_s \delta], \quad (A3a) \\
[\varphi_s(x), \theta_s(x)] &\rightarrow [-\varphi_s(-x), \theta_s(-x)], \quad (A3b) \\
[\varphi_s(x), \theta_s(x)] &\rightarrow [\varphi_s(x), \theta_s(x)], \quad (A3c) \\
[\varphi_s(x), \theta_s(x)] &\rightarrow [\varphi_s(x), \theta_s(x) + \Theta], \quad (A3d) \\
[\varphi_s(x), \theta_s(x)] &\rightarrow [\varphi_s(x), \theta_s(x + \delta) + \pi \rho_s \delta]. \quad (A3e)
\end{align*}
\]

for \((\varphi_s, \theta_s)\). Note that \(\tilde{\rho}_a\) changes the sign under the \(Z_2\) reflection in the \(y-z\) plane shown in Eq. (A2c).

For instance, applying these symmetries, we can strongly restrict possible operators in the bosonized Hamiltonian \((14)\).

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