Thermocapillary deformation in a locally heated layer of silicone oil

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Abstract. The processes of heat and mass transfer in systems with liquid-gas interface are of interest to a wide range of problems. Thermocapillary flows have an important role in such systems. Thermocapillary deformation of silicone oil layer was investigated using laser scanning confocal microscope Zeiss LSM 510 Meta. The numerical solution of the problem was obtained in the lubrication approximation theory for two-dimensional axisymmetric thermocapillary flow. The model takes into account the surface tension, viscosity, gravity and heat transfer in the substrate. Evaporation is neglected. The numerical algorithm for the joint solution of the energy equation and the evolution equation for the liquid layer thickness has been developed. Stationary solutions have been obtained by the establishment method. The dependences of the depth of thermocapillary deformation on the layer thickness were obtained for silicone oils of different viscosities. It was found that the value of the relative deformation decreases nonlinearly with increasing the initial layer thickness. There is a good qualitative agreement of numerical results and experimental data.

1. Introduction
The processes of heat and mass transfer in systems with liquid-gas interfacial surface are of interest to a wide range of fundamental and applied problems. Thermocapillary flows have an important role in such systems. Thermocapillary flows can be caused by even minor irregularities of interfacial surface temperature [1-3]. To a large extent thermocapillary effect appears in thin liquid layers with local heating. It is perspective to use thermocapillary effect in the locally heated thin liquid layers to determine the properties of the liquid and the layer thickness [4-5]. A method of measuring the liquid layer thickness using the laser-induced thermocapillary effect is proposed in [4]. A method for identifying liquids is described in [5]. Most of theoretical studies is considered that processes in thin liquid layers are modeled using an evolution equation for the layer thickness, resulting in the lubrication approximation (long-wave approximation) [6]. Velocity, temperature, pressure of the liquid, etc. are defined as a function of film thickness (the solution of this equation). This approach eliminates the complexity of the problem, caused by the presence of the free surface. However, the evolution equation is nonlinear and has a high order, and its solution is generally searched numerically. Steady state thermocapillary flows and deformations in locally heated horizontal liquid layer were numerically modelled in [7] for the axisymmetric case, the heat transfer in the substrate was not considered. In [8] unsteady solutions, with a local sharp changing in temperature of the surface layer, were obtained numerically for axisymmetric statement. In all these papers heat transfer...
in the substrate is not taken into account for calculating, so it is the reason that did not allow performing a quantitative comparison of the calculation results with the experimental data. The aim of this work is the experimental and theoretical study of thermocapillary deformations in the locally heated horizontal liquid layers with different properties using a confocal microscopy and numerical modeling based on the thin layer approximation.

2. Description of the experiment
Thermocapillary deformation of dimethylpolysiloxane layer (further PMS or silicone oil) was investigated using laser scanning confocal microscope Zeiss LSM 510 Meta. The main technical characteristics of the device: the length of the scanning beam wavelength - 405, 458, 477, 488, 514, 543, 633 nm (sources are 4 different types of lasers), scanning speed is up to 5 frames/s (at a resolution of 512 x 512 pixels), the maximum resolution frame is 2048 x 2048 pixels, the scan area is 1.4 x 1.4 mm. According to the results of mathematical processing of layered scans, the device creates a three-dimensional image of the sample with the elements of volume reconstruction, animation and quantitative analysis. This microscope allowed obtaining highly accurate data on thermocapillary deformation at the stage of steady flow. Measurements were conducted for five types of silicone oil: PMS-5, PMS-50, PMS-100, PMS-200 and PMS-400. The taken types of silicone oil cover the range of viscosity from 5 to 365 cP, other thermal properties are slightly varied. OPHIR NOVA meter with detector model 2A-SH (operating range from 60 mW to 2 W, precision ± 3%) was used to control the power of the laser beam, inducing thermocapillary flow.

The experimental scheme is shown in figure 1a, where 1 microscope lens, 2 scanning beam microscope (wavelength \(\lambda = 488\) nm), 3 glass cuvette at the bottom of which a layer of (the cuvette radius \(R_c = 18\) mm), 4 colored lacquer (figure 1b) based on nitrocellulose, 5 layer of silicone oil, 6 ray semiconductor laser (wavelength 650 nm, power \(P = 16.5 \pm 0.5\) mW). The lacquer coating was transparent to the beam of scanning microscope, thus, absorbed about 99% of the semiconductor laser. In this way the bottom layer was induced by the heat source (heater), which was characterized by the parameter \(P_H = P / S_H\), where \(S_H\) - heating area. In the experiment, radius of the spot that is inducing laser on the substrate was \(R_h = 90\) \(\mu\)m (figure 1c), hence \(P_H = 0.62\) W/mm².

Figure 1. (a) The experimental scheme; (b) Photograph of a glass cuvette with a blue colored bottom; (c) The spot of laser which is inducing a localized heating of the silicone oil layer.

In the preparatory stage of the experiment it was formed a uniform layer of liquid in a cuvette, using a high-precision dispenser. The thickness of not deformed layer \(h_0\) was controlled by the 3D image from the microscope and, if necessary, altered by the addition/subtraction liquid. Thermocapillary deformation depth \(\Delta h = h_0 - h_c\) (hereinafter \(h_c\) is residual layer thickness in the center of the thermocapillary deformation) was determined by two complementary layer 3D images when the heater is off and on, figure 2. Moreover, from the moment when the laser heating is on to starting scanning, enduring a time of about 100 seconds required for the output thermocapillary flow.
on the stationary regime. If the duration of the scanning cycle is about 2 minutes, then because of microvibrations, interfacial surface of the liquid-gas gets a few blurred, but with a thin dark line through the middle. This ensured the reproducibility of the thickness measurement with pixel precision 3D image, i.e. about 3.5µm.

![Image](image_url)

**Figure 2.** Thermocapillary deformation of PMS-5 (non-deformed layer thickness $h_0 = 228$ µm)

### 3. Numerical modelling

The problem of thermocapillary deformation of the locally heated horizontal layer of silicone oil has been solved using the lubrication approximation theory for two-dimensional axisymmetric statement. The model takes into account important parameters such as capillary pressure, viscosity and gravity. Evaporation is neglected. Initially the liquid layer has flat surface and uniform temperature. The substrate is locally heated. Deformations of the liquid surface are determined by the properties of the liquid, substrate and heater. Stationary solutions have been obtained by the establishment method.

Dynamics of thin films is well described by the evolution equation, which has been obtained using the lubrication approximation theory [6,8,9]. The evolution equation in cylindrical coordinates for the axial symmetrical case has the following form:

$$h + \frac{1}{r} \frac{\partial}{\partial r} \left( r \left[ \frac{h^3}{3 \mu} \frac{\partial}{\partial r} \left( \rho g h + \sigma H \right) + \frac{h^2}{2 \mu} \sigma_T \frac{\partial T}{\partial r} \right] \right) = 0,$$

where $h$ is the thickness of the liquid layer, $\mu$ is dynamic viscosity, $\rho$ is density, $g$ is gravitational acceleration, $\sigma$ is surface tension, $\sigma_T$ is linear coefficient, which determines the dependence of the surface tension on temperature, $T$ is temperature, $H = h_r \sqrt{1 + h_r^2} + h \sqrt{1 + h_r^2}$ is double mean curvature of the liquid surface. Equation (1) is a nonlinear differential equation of the first order in time and the fourth order in spatial variables relative to unknown function $h(t,r)$.

Boundary conditions for equation (2) have a clear physical meaning:

$$h_r (t,0) = 0$$ the condition of the axial symmetry is given in the center of the cuvette,

$$h_r (t,R) = 0$$ the contact angle $\pi/2$ is given on the border of the cuvette,

wetting angle value does not matter in this problem

$$q(t,0) = 0$$ flow rate is equal to 0 in the center,

$$q(t,R) = 0$$ the condition of impermeability of liquid through the walls.

Here $q = \frac{h^3}{3 \mu} \frac{\partial}{\partial r} \left( \rho g h + \sigma H \right) + \frac{h^2}{2 \mu} \sigma_T \frac{\partial T}{\partial r}$ is the liquid flow rate along the substrate.

The temperature of the liquid layer and cuvette is determined by the energy equation in cylindrical coordinates:
\[
\rho C_p \left( \frac{\partial T}{\partial t} + \frac{1}{r} \frac{\partial (r T)}{\partial r} + V \frac{\partial T}{\partial z} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} + Q,
\]

where, \( \rho \) is density, \( \lambda \) is coefficient of thermal conductivity; \( C_p \) is specific heat of the medium (solid or liquid). \( Q = Q(t, r, z) = \begin{cases} \text{const} > 0, & \text{in the heating area} \\ 0, & \text{outside the heating area} \end{cases} \) is bulk density of the heat sources. Heating area is a thin round layer in the centre of the cuvette bottom with radius \( R_h \), where the laser beam emission is absorbed. Boundary conditions are defined as follows:

\[
\left. \frac{\partial T}{\partial r} \right|_{r=0} = 0, \quad \left. \frac{\partial T}{\partial r} \right|_{r=R_h} = 0, \quad \lambda \frac{\partial T}{\partial n} \bigg|_{w} = \alpha_w (T_w - T_a), \quad \lambda \frac{\partial T}{\partial n} \bigg|_{S} = \alpha (T_s - T_a),
\]

axial symmetry condition, adiabatic side wall, the convective heat transfer coefficient is specified on the bottom wall of the cuvette and on the liquid surface. Here index \( W \) indicates to condition on the bottom wall, index \( S \) indicates to the liquid surface, \( T_a \) is a temperature of the ambient.

Initially the liquid layer has flat surface, temperature of the liquid surface and cuvette is uniform:

\[
h_{r=0} (r, z) = h_0, \quad T_{r=0} (r, z) = T_0 = \text{const}
\]

4. Calculating method

Splitting into physical processes, such as thermal conduction and flow of the liquid, is used for calculations. The grid in the space variables in the liquid and solid phases is uniform:

\[
h_i = r_i - dr_i, \quad i = 0, \ldots, N.
\]

The time step \( \tau \) is also uniform. The value of the liquid layer thickness in the time \( t_k \) and \( i \) node:

\[
h_i^k = h(r_i, t_k).
\]

The evolution equation of the liquid layer thickness (1) is approximated at grid’s nodes with finite volume method [10]. A discrete analogue of eq. (1) is written for each volume:

\[
\frac{h_i^{k+1} - h_i^k}{\tau} + \frac{1}{r_i} \frac{q_{i+1/2}^j - q_{i-1/2}^j}{dr} = 0
\]

Newton’s method is used to solve system of nonlinear algebraic equations (6). Numerical linearization is used to calculate the Jacobians [9]. The scheme has the second-order approximation for the spatial coordinates and the first-order in time.

When the temperature in the liquid is calculated, deformations of the surface are not taken into account. Since the laser beam locally heats the bottom of the cuvette, it is considered that the source of the heat is concentrated only in one top layer of nodes of the cuvette. The method of fractional steps is used to solve the difference analog of the energy equation (3), [11]:

\[
\begin{align*}
T_{y_{i+1/2}}^{n+1/2} - T_{y_i}^n &= \frac{\lambda}{2 \rho C_p} \left( \frac{1}{2r} \frac{dr}{d^3} T_{i+1/2, j}^{n+1/2} - 2T_{y_{i+1/2}}^{n+1/2} + \frac{1}{2r} \frac{dr}{d^3} T_{j, i+1/2}^{n+1/2} \right) + \frac{Q_y}{2 \rho C_p} \quad \text{at } i < N R_h / R, \quad j = M/2 + 1 \\
T_{y_{i+1/2}}^{n+1/2} - T_{y_{i-1/2}}^n &= \frac{\lambda}{2 \rho C_p} \left( \frac{1}{2r} \frac{dr}{d^3} T_{i+1/2, j}^{n+1/2} - 2T_{y_{i+1/2}}^{n+1/2} + \frac{1}{2r} \frac{dr}{d^3} T_{j, i+1/2}^{n+1/2} \right) + \frac{Q_y}{2 \rho C_p} \quad \text{at } i > 1/2, \quad j = M/2 + 1 \\
T_{y_{i}}^{n+1/2} - T_{y_i}^n &= \frac{\lambda}{2 \rho C_p} \left( \frac{1}{2r} \frac{dr}{d^3} T_{i, j+1/2}^{n+1/2} - 2T_{y_{i}}^{n+1/2} + \frac{1}{2r} \frac{dr}{d^3} T_{i, j+1/2}^{n+1/2} \right) + \frac{Q_y}{2 \rho C_p} \quad \text{at } i = 1/2, \quad j = M/2 + 1 \\
T_{y_{i}}^{n+1/2} - T_{y_{i}}^n &= \frac{\lambda}{2 \rho C_p} \left( \frac{1}{2r} \frac{dr}{d^3} T_{i, j+1/2}^{n+1/2} - 2T_{y_{i}}^{n+1/2} + \frac{1}{2r} \frac{dr}{d^3} T_{i, j+1/2}^{n+1/2} \right) + \frac{Q_y}{2 \rho C_p} \quad \text{at } i = 1/2, \quad j = M/2 + 1
\end{align*}
\]

Here \( Q_y = P / \pi R_h^2 dz \) at \( i < N R_h / R \), \( j = M/2 + 1 \) and \( Q_y = 0 \) for others \( i, j \).
Boundary and initial conditions are set in the finite differences equations:

\[ j = 1, \ldots, M, \quad T(j, 0) = T(j, 2) \text{ symmetry, } T(j, N + 1) = T(j, N - 1) \text{ adiabatic right wall.} \]  

\[ i = 1, \ldots, N, \quad T(0, i) = T(2, i) - \frac{\alpha}{\lambda} (T(1, i) - T_a) 2dz, \text{ heat transfer coefficient on the liquid surface} \]  

\[ T(M + 1, i) = T(M - 1, i) - \frac{\alpha}{\lambda_w} (T(M, i) - T_w) 2dz, \text{ heat transfer coefficient on bottom wall} \]  

Initial conditions: \( T_{ij}^0 = 0, \quad i = 0, \ldots, N, \quad j = 1, \ldots, M - 1 \).

The resulting system of linear algebraic equations is solved at each step with the tridiagonal matrix algorithm. Calculations are performed sequentially. The time step for the evolution equation is done after the time step for the energy equation.

The mathematical model accounted such defining parameters as the geometry of the problem, parameters of the liquid, properties of the substrate and the heater materials, heating methods. The calculation results have shown that all these parameters have a significant impact on the distribution of the heat and deformations of the liquid surface.

5. Analysis of the experimental results and calculations

Dependences on the silicone oil thickness of the thermocapillary deformation’s depth have been measured experimentally and calculated by the model (figure 3). Numerical calculations were made for two marks of silicone oil: PMS-5 and PMS-50. The thinner the layer, the greater the difference \( \Delta h \), when other things being equal. In thin layers thermocapillary deformation depth reaches 30% or more of the initial thickness, but quickly decreases with increasing \( h_0 \) (see figure 3). When the layer thickness is \( h > 400\mu m \) values of deformations are practically the same, because viscosity variations for thick layers have less effect on the value of deformation. There is a good qualitative agreement between the calculating results and experimental data. The differences in the calculating results for PMS-5 and PMS-50 are explained by differences in the coefficients of surface tension and coefficients of dependence of the surface tension on temperature for these liquids.

![Figure 3](image-url)  

**Figure 3.** Thermocapillary deformation depth dependence on the thickness of the layer of silicone oil. \( h_0 \) – thickness of non-deformed layer, \( \Delta h = h_0 - h_c \) – thermocapillary deformation depth, \( h_c \) - layer thickness over the center of the heater at the stage of steady thermocapillary flow.
6. Conclusion
There have been measured and numerically calculated deformations in locally-heated horizontal layers of silicone oils of different types and thickness. It has been found that the value of the relative deformation of the layer decreases nonlinearly with increasing the layer thickness, when other conditions being equal. The results of modeling using the thin layer approximation are predicting well the main features of the experimental dependences of the thermocapillary deformations on the layer thickness for silicone oils of different marks. So the calculation results are in good qualitative and quantitative agreement with experimental data. The experimental results can be used to test a wide class of computer programs that simulate the processes of heat and mass transfer in multiphase systems with liquid-gas interfaces.

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