Absorption of partial waves by three-branes

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We study the absorption of a class of fields in the geometry produced by extremal three-branes. We consider fields that do not mix with the ten-dimensional graviton. For these fields we solve the wave equations and find the absorption probabilities for all partial waves at leading order in the energy. We note that in some of these cases one needs an ‘intermediate’ region which interpolates between flat Minkowski space at infinity and the AdS geometry near the branes.

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1. Introduction

There has been renewed interest in the study of the background produced by 3-branes, due to their conjectured connection to large N Yang-Mills theories in four spacetime dimensions \([1][2][3][4][5]\). Close to the branes, the geometry reduces to an anti de-Sitter (AdS) metric times an \(S^5\), with a self-dual five form field strength producing the required curvature. It is conjectured that 10 dimensional IIB supergravity on this compactification is dual to large N Yang-Mills on the four dimensional boundary of the 5-dimensional AdS space.

Away from the immediate vicinity of the branes, the metric is not of the AdS form, and in fact far from the branes the spacetime approaches the Minkowski vacuum. The following process is of interest in this complete geometry. One imagines a low energy quantum incident on the branes, starting at spatial infinity \((r \to \infty)\). Some part of this wave tunnels through to the ‘horizon region’ close to the branes \(r \to 0\), where it then propagates freely inwards towards the branes, and this part of the wave may be considered as absorbed by the branes. The remainder of the wave is scattered back to \(r \to \infty\). One can compute the absorption probability for each spherical partial wave around the branes, or convert this to a cross section for absorption of different fields by the branes.

The above calculation is closely related to the connection between supergravity and Yang-Mills. In this connection one needs to find the coupling between supergravity modes and operators in the Yang-Mills, such that the former act as sources for the operators in the latter. Where would this set of couplings come from? Some constraint may arise from superconformal symmetry \([6]\), but this is not likely to be enough by itself. In earlier work it had been conjectured that the absorption of supergravity modes by the three branes could be obtained from coupling the branes to their background through a Born-Infeld type action. Such a coupling gives exact agreement for the three-charge black hole \([7]\), and for some modes of the dilaton in the case of 3-branes \([8][9][10]\).

In carrying out the classical calculation for absorption one notes that there are three regions of spacetime that one encounters in general. The region of small \(r\) is the AdS region, but this may not connect directly to the free propagation on flat space at \(r \to \infty\). The calculation of the absorption probability in some cases requires matching the solutions for small \(r\) and for large \(r\) by using an ‘intermediate region’. We say that this region is ‘needed’ in those cases where the solutions in the ‘outer’ and ‘inner’ (AdS) regions do not match on to each other as functions of \(r\).
In this paper we carry out this calculation for a class of massless fields of 10-dimensional supergravity propagating in the background produced by 3-branes. We consider fields that do not mix with the perturbations of the 10-dimensional gravitational field. For the fields that we study we consider all partial wave components, and construct solutions to the wave equation at leading order in energy. In all cases but one the fields separate out from each other, and for these cases we calculate the absorption probability for the incident quanta.

It is hoped that this analysis will shed light on the connection between supergravity and Yang-Mills, and also on how one might make a microscopic model to study absorption by the black 3-brane as well as by other black branes and black holes. This issue has been studied recently in [11][12].

2. Basic relations

2.1. Field equations and separation of variables

We will follow for the most part the notations in [13]. In this paper the authors had studied linearised perturbations of type IIB supergravity about $AdS_5 \times S^5$. While our geometry has this form only very close to the 3-branes, we have spherical symmetry at all $r$ and the eigenvalues of the Hodge-de Rham operator on $S^5$ can be taken from this reference. The computation of these eigenvalues is carried out in detail in [14].

The field equations are

$$R_{\hat{\mu}\hat{\nu}} = -\frac{1}{6} F_{\hat{\mu}\hat{\rho}\hat{\sigma}\hat{\tau}\hat{\kappa}} F_{\hat{\nu}}^{\hat{\rho}\hat{\sigma}\hat{\tau}\hat{\kappa}}$$  \hspace{1cm} (2.1)

$$F_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}'} = \frac{1}{5!} \epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}\hat{\rho}'\hat{\sigma}'\hat{\tau}'} F_{\hat{\rho}'\hat{\sigma}'\hat{\tau}'}$$  \hspace{1cm} (2.2)

$$D\hat{\mu} \partial_{[\hat{\mu} A_{\hat{\nu}\hat{\rho}]}} = -\frac{2i}{3} \hat{F}_{\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}\hat{\kappa}} D^{\hat{\sigma}} A^{\hat{\tau}\hat{\kappa}}$$  \hspace{1cm} (2.3)

$$D\hat{\mu} \partial_{\hat{\mu}} B = 0$$  \hspace{1cm} (2.4)

In terms of the fields of 10 dimensional supergravity,

$$A_{\hat{\mu}\hat{\nu}} = B^{NS}_{\hat{\mu}\hat{\nu}} + i B^{RR}_{\hat{\mu}\hat{\nu}}$$  \hspace{1cm} (2.5)

The field $B$ is a complex scalar describing the dilaton and the RR scalar. We study linearized perturbations around the background given by a collection of coincident 3-branes. The perturbations can be separated into two classes: those that involve or couple to the
fluctuations of the 10 dimensional metric, and those that decouple from these metric perturbations. We will study only the latter perturbations in this paper. We expand the fields around their background values. In the following the background value of the fields is denoted by a dot above the symbol giving the field. The coordinates on the $S^5$ surrounding the branes are called $y^\alpha$. The spatial coordinates long the brane are called $x^a$, $a = 1, 2, 3$. The coordinates $t, r, x^a$ are collectively denoted by $x^\mu$. (Here $r$ is a radial coordinate; the branes are located at $r = 0$.) These coordinates $x^\mu$ describe the $AdS_5$ geometry in the region close to the 3-branes. The 10-dimensional coordinates carry a ‘hat’. The $\epsilon$ symbol is a tensor with
\[ \epsilon_{12r03\alpha\beta\gamma\delta\epsilon} = \sqrt{-g} \] (2.6)
with $\alpha\beta\gamma\delta\epsilon$ giving a frame with positive orientation on $S^5$.

Note: We will call a field a p-form if it has $p$ indices of the kind $\mu$. This follows the language that is appropriate to studying the $AdS_5 \times S^5$ geometry in the region close to the branes. \footnote{In earlier work with 3-branes sometimes a different language was used. There one regarded the three space directions along the brane as ‘internal’ coordinates, and then the remaining 7-dimensional spacetime has coordinates $t, r$ and the five coordinates of type $\alpha$ on the $S^5$. In that language a p-form would have $p$ indices from these seven possibilities.}

The linearised perturbations around the background can be expanded as [13]
\[ A_{\mu\nu\alpha\beta} = \dot{A}_{\mu\nu\alpha\beta} + a_{\mu\nu\alpha\beta} \] (2.7)
\[ a_{\mu\nu\alpha\beta} = \sum_{I_{10}} b_{I_{10}}^{\mu\nu}(x) Y^{I_{10}}(y) \] (2.8)
\[ A_{\mu\nu} = \sum_{I_1} a_{I_1}^{\mu\nu}(x) Y^{I_1}(y) \] (2.9)
\[ A_{\mu\alpha} = \sum_{I_5} [a_{I_5}^{\mu}(x) Y^{I_5}_\alpha(y) + a_{I_5}^{\alpha} D_\alpha Y^{I_1}(y)] \] (2.10)
\[ A_{\alpha\beta} = \sum_{I_{10}} [a_{I_{10}}^{\alpha}(x) Y^{I_{10}}_{\alpha\beta}(y) + a_{I_5}^{\alpha} D_{[\alpha} Y^{I_5}_{\beta]}(y)] \] (2.11)
\[ B = \sum_{I_1} B_{I_1}^{1}(x) Y^{I_1}(y) \] (2.12)
Here $Y^{I_1}$ are the spherical harmonics appropriate to scalar functions. Similarly $Y^{I_5}_{\alpha}$ are the harmonics with one vector index along the $S^5$, and $Y^{I_{10}}_{[\alpha\beta]}$ are the harmonics for an antisymmetric tensor on the $S^5$.

The part $a^{I_5}$ in (2.11) and the part $a^{I_1}_{\mu}$ in (2.10) can be set to zero after a gauge transformation. The gauge transformation gives

$$\delta A_{\alpha\beta} = \partial_\alpha \Lambda_\beta - \partial_\beta \Lambda_\alpha, \quad \delta A_{\mu\alpha} = \partial_\mu \Lambda_\alpha - \partial_\alpha \Lambda_\mu$$

(2.13)

and we set

$$\Lambda_\beta = -\frac{1}{2} a^{I_5} Y^{I_5}_\beta (y), \quad \Lambda_\mu = a^{I_1}_{\mu} Y^{I_1}_\mu (y)$$

(2.14)

The background has the metric

$$ds^2 = H^{-\frac{1}{2}} [-dt^2 + dx^a dx^a] + H^\frac{1}{2} [dr^2 + r^2 d\Omega^2_5]$$

(2.15)

where

$$H = 1 + \frac{R^4}{r^4}$$

(2.16)

The background also has a 5-form field strength of the 4-form antisymmetric tensor potential:

$$F_{1203} = H^{-2} R^4 r^{-5}$$

(2.17)

We have a corresponding field strength for all five indices of $F$ on $S^5$, given through (2.2).

We will need to compute the following quantity

$$(n + 1) D^\alpha [D_\alpha B_{\beta_1...\beta_n}]$$

$$= (n + 1) g_{\beta_1\beta_1'} \cdots g_{\beta_n\beta_n'} \frac{1}{\sqrt{-g}} \partial_\alpha [\sqrt{-g} \partial_\beta_1 B_{\beta_1'...\beta_n'} g^{\alpha\alpha'} g^{\beta_1'\beta_1'} \cdots g^{\beta_n'\beta_n'}]$$

(2.18)

$$\equiv (\Delta_y B)_{\beta_1...\beta_n}$$

$$\Delta_y Y^k_{[\alpha\beta]} = -\frac{1}{r^2} (k + 2)^2 Y^k_{[\alpha\beta]}, \quad k = 1, 2, \ldots$$

(2.19)

$$\Delta_y Y^k_{\alpha} = -\frac{1}{r^2} (k + 1)(k + 3) Y^k_{\alpha}, \quad k = 1, 2, \ldots$$

(2.20)

$$\Delta_y Y^k = -\frac{1}{r^2} k(k + 4) Y^k, \quad k = 0, 1, \ldots$$

(2.21)

Here $\tilde{r}$ is the proper radius of the sphere on which the operator $\Delta_y$ acts:

$$\tilde{r} = r H^{\frac{1}{2}}$$

(2.22)

The functions $Y^k_{[\alpha\beta]}$ can be split into two sets under the action of

$$* D Y^k_{[\alpha\beta]} \equiv \epsilon_{\alpha\beta\gamma\delta\epsilon} \partial_\gamma Y^k_{[\delta\epsilon]} = \pm 2i (k + 2) Y^k_{[\alpha\beta]}$$

(2.23)

where $\epsilon_{\alpha\beta\gamma\delta\epsilon}$ is the volume form on the unit 5-sphere.
2.2. Some mathematical relations

We consider perturbations of the form

\[ \phi(t, r, y) = \phi(r) e^{-i\omega t} Y(y) \] (2.24)

We will construct the solutions for \( \phi(r) \) by dividing the radial coordinate \( r \) into three regions:

(a) The outer region, given by \( r > r_1 \) for some choice of \( r_1 \) satisfying \( \frac{r_1}{R} \gg 1 \) but also \( \omega r_1 \ll 1 \). [We are working to leading order in \( \omega \), so we can always assume that \( \omega \) is as small as we wish.] In the ‘outer part’ of this outer region, we will have \( \omega r \gg 1 \), while in the ‘inner part’ of this outer region we will have \( \omega r \ll 1 \).

(b) The intermediate region, given by \( r_1 > r > r_2 \), where \( \frac{r_2}{R} \ll 1 \), but \( v(r_2) \ll 1 \) (2.25)

\[ v \equiv \frac{\omega R^2}{r} \] (2.26)

In this region we can ignore \( \omega \) in the equation.

(c) The inner region, given by \( r < r_2 \). The region \( r \to 0 \) corresponds to \( v \to \infty \). A wave propagating to \( v \to \infty \) is considered to be absorbed into the branes. The boundary condition we impose in the computation of cross sections is that there be no wave propagating from \( v = \infty \) towards smaller \( v \).

In the outer region the wave equations turn out to take the form

\[ \phi_{,rr} + A_1 r^{-1} \phi_{,r} + \left[ \omega^2 - \frac{B_1}{r^2} \right] \phi = 0 \] (2.27)

The solution is

\[ \phi = C_1 \ r^{\frac{1}{2}[1-A_1]} \frac{J_{\sqrt{B_1+\frac{1}{2}(A_1-1)^2}}(\omega r)}{\sqrt{B_1+\frac{1}{2}(A_1-1)^2}} + C_2 \ r^{\frac{1}{2}[1-A_1]} \frac{N_{\sqrt{B_1+\frac{1}{2}(A_1-1)^2}}(\omega r)}{\sqrt{B_1+\frac{1}{2}(A_1-1)^2}} \] (2.28)

In the inner region the equations take the form

\[ \phi_{,rr} + A_2 r^{-1} \phi_{,r} + \frac{\omega^2 R^4}{r^4} \phi - B_1 r^{-2} \phi = 0 \] (2.29)

which is equivalent to

\[ \phi_{,vv} + (2 - A_2) v^{-1} \phi_{,v} + \left[ 1 - \frac{B_2}{v^2} \right] \phi = 0 \] (2.30)
The solutions are

$$\phi = C_5 v^{\frac{1}{2}(A_2 - 1)} J \sqrt{B_2 + \frac{1}{4}(1 - A_2)^2} (v) + C_6 v^{\frac{1}{2}(A_2 - 1)} N \sqrt{B_2 + \frac{1}{4}(1 - A_2)^2} (v)$$  \hspace{1cm} (2.31)

For $v \to \infty$ we have

$$J_k(v) + iN_k(v) \sim e^{iv}, \quad J_k(v) - iN_k(v) \sim e^{-iv}$$  \hspace{1cm} (2.32)

Thus to compute the absorption probability we will take in (2.31) a solution with

$$C_5 = 1, \quad C_6 = i$$  \hspace{1cm} (2.33)

In the outer region at $r \to \infty$, we have

$$J_k(\omega r) + iN_k(\omega r) \sim e^{i\omega r}, \quad J_k(\omega r) - iN_k(\omega r) \sim e^{-i\omega r}$$  \hspace{1cm} (2.34)

Thus from a solution of the form (2.28) we find for the probability of absorption of the spherical wave

$$P = 1 - \left| \frac{C_1 - iC_2}{C_1 + iC_2} \right|^2$$  \hspace{1cm} (2.35)

2.3. Scattering matrices

We will write out the solution for each field in each of the three regions mentioned above. In may cases it is not essential to introduce the intermediate region to solve the equation, but we introduce this region in all the cases since one of our goals is to compare the behavior in this region of the different fields. We also wish to see the relation between the power law solutions that arise here with the dimensions of operators in the Yang-Mills or Born-Infeld descriptions of the 3-branes.

If one wants to compute the absorption probabilities, one can simplify the calculation by using a comparison of fluxes at $r \to \infty$ and $r \to 0$ \cite{15}. In this case one does not need to compute both the solutions to the equation in each region, if one wants the result at leading order in the energy. We choose to instead to compute both solutions in each region, since from this we can extract the result to the following more general question. We can imagine the wave to be incident either from $r \to \infty$ or from $r \to 0$. The final waveform will have components that travel in each of these two directions. Thus there is a $2 \times 2$ scattering matrix describing the scattering/tunneling through the intermediate
region. We do not write this matrix explicitly, but it can be read off from the solutions in each case.

3. Absorption of the dilaton-axion

The equation (2.4) with the ansatz (2.24) becomes

$$r^{-5} \partial_r [r^5 B, r] + \omega^2 H B - k(k+4)r^{-2}B = 0$$ (3.1)

3.1. Outer region

Here the equation is

$$B_{,rr} + 5B_{,r} + \left[ \omega^2 \frac{k(k+4)}{r^2} \right] B = 0$$ (3.2)

The solution is

$$B = C_1 r^{-2} J_{(k+2)}(\omega r) + C_2 r^{-2} N_{(k+2)}(\omega r)$$ (3.3)

3.2. Intermediate region

Dropping the $\omega^2$ term, we have

$$B_{,rr} + 5r^{-1}B_{,r} - k(k+4)r^{-2}B = 0$$ (3.4)

The solution is

$$B = C_3 r^k + C_4 r^{-k-4}$$ (3.5)

Thus in this case we see that the equation does not have a nontrivial intermediate region.

3.3. Inner region

Here the equation reduces to

$$B_{,rr} + 5r^{-1}B_{,r} + \frac{\omega^2 R^4}{r^4} B - k(k+4)r^{-2}B = 0$$ (3.6)

The solutions are

$$B = C_5 v^2 J_{(k+2)}(v) + C_6 v^2 N_{(k+2)}(v)$$ (3.7)

4. Scalar from the antisymmetric two form

The field equation arises from (2.3), with the free indices taken to lie along the $S^5$:

$$r^{-1} \partial_r [H^{-1} r a_{\alpha, r}] + \omega^2 a_{\alpha \beta} - (k+2)^2 r^{-2} H^{-1} a_{\alpha \beta} \mp 4 H^{-2} R^4 r^{-6}(k+2)a_{\alpha \beta} = 0$$ (4.1)

Here the sign $\pm$ corresponds to the sign in the spherical harmonic $Y_{(\alpha \beta)}^{k, \pm}$ (eq. 2.23).
4.1. Outer region

The equation is

\[ a_{\alpha\beta,rr} + r^{-1}a_{\alpha\beta,r} + \left[ \omega^2 - \frac{(k + 2)^2}{r^2} \right] a_{\alpha\beta} = 0 \]  

(4.2)

The solutions are

\[ a_{\alpha\beta} = C_1 J_{(k+2)}(\omega r) + C_2 N_{(k+2)}(\omega r) \]  

(4.3)

4.2. Intermediate region

The equation is

\[ \phi_{,rr} + \frac{1}{r} \left[ \frac{r^4 + 5R^4}{r^4 + R^4} \right] \phi_{,r} - \frac{(k + 2)}{r^2} \left[ \frac{(k + 2)R^4 + (k + 2 + 4)R^4}{r^4 + R^4} \right] \phi = 0 \]  

(4.4)

The solution is

\[ a_{\alpha\beta} = C_3 r^{\pm(k+2)} + C_4 r^{\mp(k+2)} [1 + \frac{(k + 2)}{(k + 2 \pm 2) r^4}] \]  

(4.5)

Thus in this case we have a nontrivial intermediate region.

4.3. Inner region

The equation here is

\[ a_{\alpha\beta,rr} + 5r^{-1}a_{\alpha\beta,r} + \left[ \omega^2 R^4 r^{-4} - [(k + 2)^2 \pm 4(k + 2)] r^{-2} \right] a_{\alpha\beta} = 0 \]  

(4.6)

The solutions are

\[ a_{\alpha\beta} = C_5 v^2 J_{(k+2\pm2)}(v) + C_6 v^2 N_{(k+2\pm2)}(v) \]  

(4.7)

5. Vector from the two-form field

The wave equation in this case arises from (2.3) with one free index on the $S^5$ and one free index $\rho$ in the remaining 5 directions. For $\rho = 0$ we get

\[ \frac{1}{r^3} \partial_r \left[ r^3 (\partial_r a_0 + i\omega a_r) \right] - \frac{(k + 1)(k + 3)}{r^2} a_0 = 0 \]  

(5.1)

For $\rho = r$ we get

\[ \partial_r a_0 = \frac{1}{i\omega} \left[ \omega^2 - \frac{(k + 1)(k + 3)}{r^2 H} \right] a_r \]  

(5.2)
For \( \rho = 1 \) we get
\[
\frac{1}{r^3} \partial_r(r^3 \partial_r a_1) + \left[ \omega^2 H - \frac{(k + 1)(k + 3)}{r^2} \right] a_1 = 0
\] (5.3)

[The equations for \( \rho = 2, 3 \) are similar to the equation for \( \rho = 1 \) so we do not need to write them.]

We find that \( a_0 \) and \( a_r \) can be algebraically determined from each other. For \( a_r \) we have the equation
\[
\frac{1}{r} \partial_r \left( \frac{r}{H} a_r \right) + i \omega a_0 = 0
\] (5.4)

Then we find the following equation for \( a_r \):
\[
\partial_r \left[ \frac{1}{r} \partial_r \left( \frac{r}{H} a_r \right) \right] + \left[ \omega^2 - \frac{(k + 1)(k + 3)}{r^2 H} \right] a_r = 0
\] (5.5)

and for \( a_0 \) we have the equation
\[
\partial_r \left[ r^3 \partial_r a_0 \left( \omega^2 r^2 H - (k + 1)(k + 3) \right)^{-1} \right] + a_0 = 0
\] (5.6)

5.1. Outer region

The equations are
\[
a_{1,rr} + 3 \frac{a_{1,r}}{r} + \left[ \omega^2 - \frac{(k + 1)(k + 3)}{r^2} \right] a_1 = 0
\] (5.7)
\[
a_{r,rr} + r^{-1} a_{r,r} + \left[ \omega^2 - \frac{(k + 1)(k + 3)}{r^2} \right] a_r = 0
\] (5.8)

The solutions are
\[
a_1 = \tilde{C}_1 r^{-1} J_{k+2}(\omega r) + \tilde{C}_2 r^{-1} N_{k+2}(\omega r)
\] (5.9)
\[
a_r = C_1 J_{(k+2)}(\omega r) + C_2 N_{(k+2)}(\omega r)
\] (5.10)

5.2. Intermediate region

The equations are
\[
\frac{1}{r^3} \partial_r (r^3 \partial_r a_1) - \frac{(k + 1)(k + 3)}{r^2} a_1 = 0
\] (5.11)
\[
\partial_r \left[ \frac{1}{r} \partial_r \left( a_r \frac{r}{H} \right) \right] - \frac{(k + 1)(k + 3)}{r^2} \left( a_r \frac{r}{H} \right) = 0
\] (5.12)
The solutions are

\[ a_1 = \tilde{C}_3 r^{k+1} + \tilde{C}_4 r^{-k-3} \]  
\[ a_0 = [C_3 r^{k+1} + C_4 r^{-k-3}] \]

(5.13) (5.14)

We find that \( a_r \) vanishes if \( \omega \) is set to zero, but it is still helpful to write it down since one can match either \( a_r \) or \( a_0 \):

\[ a_r = -(i\omega)\left[C_3(k + 3)^{-1}H r^{k+2} - C_4(k + 1)^{-1}H r^{-(k+2)}\right] \]

(5.15)

5.3. Inner region

The equations are

\[
\frac{1}{r^3} \partial_r (r^3 \partial_r a_1) + \left[\frac{\omega^2 R^2}{r^4} - \frac{(k + 1)(k + 3)}{r^2}\right] a_1 = 0
\]

(5.16)

\[
a_{r,rr} + 9r^{-1} a_{r,r} + \left[\frac{R^4 \omega^2}{r^4} - \frac{(k + 1)(k + 3)}{r^2}\right] a_r = 0
\]

(5.17)

The solutions are

\[ a_1 = \tilde{C}_5 v J_{(k+2)}(v) + \tilde{C}_6 v N_{(k+2)}(v) \]
\[ a_r = C_5 v^4 J_{(k+2)}(v) + C_6 v^4 N_{(k+2)}(v) \]
\[ a_0 = -i\omega^2 R^2 [C_5 (v^2 J'_{(k+2)}(v) - v J_{(k+2)}(v)) + C_6 (v^2 N'_{(k+2)}(v) - v N_{(k+2)}(v))] \]

(5.18) (5.19) (5.20)

6. Antisymmetric tensor from the 4-form

In (2.2) we take two of the free indices to be of type \( \alpha\beta \) and the rest to be of type \( \mu \). We note further that the spherical harmonics \( Y_{[\alpha\beta]}^{I_{10}} \) are independent of \( \partial_{[\alpha} Y_{\beta]}^{I_{15}} \). Then we find the equation

\[ 3\partial_{[\mu} a_{\nu\rho]}_{\alpha\beta} = \frac{1}{4} \epsilon_{\mu\nu\rho\sigma} \epsilon^{\gamma\delta\epsilon} \partial_\gamma a_{\delta\epsilon\sigma} \]

(6.1)

For \( \mu, \nu, \rho = 0, 1, 2 \) we get

\[ b_{3r} = \pm \frac{\omega r H}{k + 2} b_{12} \]

(6.2)

For \( \mu, \nu, \rho = r, 0, 3 \) we get

\[ b_{03,r} - i\omega b_{3r} = \pm i r^{-1}(k + 2) b_{12} \]

(6.3)
For $\mu, \nu, \rho = r, 1, 2$ we get
\[ b_{12,r} = \mp ir^{-1}(k + 2)b_{03} \quad (6.4) \]

Eliminating $b_{03}$ and $b_{3r}$ we get
\[ r^{-1}\partial_r[rb_{12,r}] + [\omega^2 H - \frac{(k + 2)^2}{r^2}]b_{12} = 0 \quad (6.5) \]

6.1. Outer region

The equation is
\[ b_{12,rr} + r^{-1}b_{12,r} + [\omega^2 - \frac{(k + 2)^2}{r^2}]b_{12} = 0 \quad (6.6) \]

The solutions are
\[ b_{12} = C_1 J_{k+2}(\omega r) + C_2 N_{k+2}(\omega r) \quad (6.7) \]

6.2. Intermediate region

The equation is
\[ b_{12,rr} + r^{-1}b_{12,r} - \frac{(k + 2)^2}{r^2}b_{12} = 0 \quad (6.8) \]

The solutions are
\[ b_{12} = C_3 r^{k+2} + C_4 r^{-(k+2)} \quad (6.9) \]

Thus the field equation does not have a nontrivial intermediate region.

6.3. Inner region

The equations are
\[ b_{12,rr} + r^{-1}b_{12,r} + \left[\frac{\omega^2 R^4}{r^4} - \frac{(k + 2)^2}{r^2}\right]b_{12} = 0 \quad (6.10) \]

The solution is
\[ b_{12} = C_5 J_{k+2}(v) + C_6 N_{k+2}(v) \quad (6.11) \]

7. Two form from the antisymmetric tensor

The wave equation follows from (2.3)
\[ 3\frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} \partial_\nu a_{\mu_1 \mu_2} g^{\mu' \mu} g^{\nu' \nu_1} g^{\mu_2' \mu_2}] + \frac{1}{\sqrt{-g}} \partial_\alpha [\sqrt{-g} \partial_\nu a_{\mu_1 \mu_2} g^{\alpha' \alpha} g^{\nu' \nu_1} g^{\mu_1' \mu_1} g^{\mu_2' \mu_2}] + 2iF^{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5} \partial_{\mu_3} a_{\mu_4 \mu_5} = 0 \quad (7.1) \]
Setting the free indices $\mu_1 \mu_2$ to $r, 3$ we get

$$a_{r3}[\omega^2 H_{\mu_1}^\mu - \frac{k(k+4)}{H_{\mu_1}^\mu r^2}] = \omega[i a_{03,r} H_{\mu_1}^\mu - \frac{4R^4}{H_{\mu_1}^\mu r^5} a_{12}] = 0 \quad (7.2)$$

Setting $\mu_1 \mu_2$ to $1, 2$ we get

$$\omega^2 H_{\mu_1}^\mu a_{12} + \frac{1}{r^5} r^5 [r^5 H a_{12,r}] - 4 i a_{03,r} \frac{R^4}{r^5} - a_{r3} - \frac{k(k+4)H}{r^2} a_{12} = 0 \quad (7.3)$$

Setting $\mu_1 \mu_2$ to $0, 3$ we get

$$- \frac{1}{r^5} \partial_r [a_{03,r} H r^5] - \frac{i \omega}{r^5} \partial_r [H r^5 a_{r3}] + \frac{k(k+4)}{r^2} H a_{03} - \frac{4 i R^4}{r^5} a_{12,r} = 0 \quad (7.4)$$

From the first and last equations we find

$$\frac{1}{r^3} \partial_r [r^3 a_{r3}] + i \omega H a_{03} = 0 \quad (7.5)$$

### 7.1. Outer region

In this region $(7.2)$ becomes

$$\frac{1}{r^3} \partial_r [r^3 a_{r3}] + i \omega a_{03} = 0 \quad (7.6)$$

From this we get

$$\partial_r \left[ \frac{1}{r^3} \partial_r [r^3 a_{r3}] \right] + i \omega a_{03,r} = 0 \quad (7.7)$$

$(7.2)$ gives

$$a_{r3}[\omega^2 - \frac{k(k+4)}{r^2}] = \omega[i a_{03,r}] \quad (7.8)$$

Substituting $(7.7)$ into $(7.8)$ we get

$$a_{r3,rr} + 3r^{-1} a_{r3,r} + [\omega^2 - \frac{(k+4)+3}{r^2}] a_{r3} = 0 \quad (7.9)$$

The solution is

$$a_{r3} = C_1 r^{-1} J_{(k+2)}(\omega r) + C_2 r^{-1} N_{(k+2)}(\omega r) \quad (7.10)$$

We can then get $a_{03}$ from $(7.6)$ and we find

$$a_{03} = \frac{i}{\omega}[a_{r3,r} + 3r^{-1} a_{r3}]$$

$$= \frac{i}{\omega} C_1 [r^{-1} \partial_r J_{(k+2)}(\omega r) + 2r^{-2} J_{(k+2)}(\omega r)] + \frac{i}{\omega} C_2 [r^{-1} \partial_r N_{(k+2)}(\omega r) + 2r^{-2} N_{(k+2)}(\omega r)] \quad (7.11)$$

The equation for $a_{12}$ decouples from the other fields in the outer region:

$$\frac{1}{r^5} \partial_r [r^5 a_{12,r}] + \omega^2 a_{12} - \frac{k(k+4)}{r^2} a_{12} = 0 \quad (7.12)$$

The solution is

$$a_{12} = \tilde{C}_1 r^{-2} J_{(k+2)}(\omega r) + \tilde{C}_2 r^{-2} N_{(k+2)}(\omega r) \quad (7.13)$$
7.2. Intermediate region

We get from (7.3) and (7.12)

\[
\frac{1}{r^5} \partial_r [r^5 H a_{12},r] - \frac{k(k+4)H}{r^2} a_{12} - 4ia_{03},r \frac{R^4}{r^5} = 0
\]  

(7.14)

\[
\frac{1}{r^5} \partial_r [a_{03},r H r^5] - \frac{k(k+4)}{r^2} H a_{03} + \frac{4iR^4}{r^5} a_{12},r = 0
\]  

(7.15)

Define

\[
\phi_\pm \equiv a_{12} \mp i a_{03}
\]  

(7.16)

Then we get

\[
\frac{1}{r^5} \partial_r [r^5 H \phi_+,r] - \frac{k(k+4)H}{r^2} \phi_\pm + \frac{4R^4}{r^5} \phi_\pm,r = 0
\]  

(7.17)

The solutions are

\[
\phi_- = C_3^{-} r^{-k} [r^4 + R^4]^{-1} + C_4^{-} r^{k+4} [r^4 + R^4]^{-1}
\]  

(7.18)

\[
\phi_+ = C_3^{+} r^k + C_4^{+} r^{-(k+4)}
\]  

(7.19)

7.3. Inner region

Here we use the factorized form of the equations. In this region the equation can be written as

\[
\left[ \frac{2k}{R} + i \ast D \right] \left[ \frac{2(k+4)}{R} - i \ast D \right] a_{\mu \nu} = 0
\]  

(7.20)

\[
(\ast Da)_{\mu \nu} = -\epsilon_{\mu \nu} \epsilon_{\mu_1 \mu_2 \mu_3} \partial_{\mu_1} a_{\mu_2 \mu_3}
\]  

(7.21)

where

\[
\epsilon_{1203} = \frac{r^3}{R^3}
\]  

(7.22)

sets the sign of the volume form in the $AdS_5$.

Assume that the second factor vanishes; the analysis is similar when the first factor vanishes. Then we get

\[
\left[ \frac{2(k+4)}{R} - i \ast D \right] a_{\mu \nu} = 0
\]  

(7.23)

which reads

\[
\frac{2(k+4)}{R} a_{\mu \nu} + i\epsilon_{\mu \nu} \epsilon_{\mu_1 \mu_2 \mu_3} \partial_{\mu_1} a_{\mu_2 \mu_3} = 0
\]  

(7.24)
The $r3$ component is
\[
\frac{(k + 4)}{R}r_{3} + \frac{\omega R^3}{r^3}a_{12} = 0
\] (7.25)

The $12$ component is
\[
\frac{(k + 4)}{R}a_{12} + \frac{\omega r}{R}a_{r3} - \frac{i r}{R}a_{03,r} = 0
\] (7.26)

The $03$ component is
\[
\frac{(k + 4)}{R}a_{03} + i r a_{12,r} = 0
\] (7.27)

From the $03$ equation we find $a_{03}$ and substitute it into the $12$ equation. Then we get
\[
\frac{1}{r} \partial_r [ra_{12,r}] - \frac{(k + 4)^2}{r^2}a_{12} + \frac{\omega^2 R^4}{r^4}a_{12} = 0
\] (7.28)

The solution is
\[
a_{12} = C_5 J_{(k+4)}(v) + C_6 N_{(k+4)}(v)
\] (7.29)

From this we then get
\[
a_{r3} = -\frac{\omega R^4}{r^3(k+4)}[C_5 J_{(k+4)}(v) + C_6 N_{(k+4)}(v)]
\] (7.30)

\[
a_{03} = -\frac{ir}{(k+4)}[C_5 \partial_r J_{(k+4)}(v) + C_6 \partial_r N_{(k+4)}(v)]
\] (7.31)

If in (7.20) we assume that the first factor vanishes (The two factors commute in the inner region) we get the solutions
\[
a_{12} = C_5 J_{(k)}(v) + C_6 N_{(k)}(v)
\] (7.32)

\[
a_{r3} = \frac{\omega R^4}{r^3k}[C_5 J_{(k)}(v) + C_6 N_{(k)}(v)]
\] (7.33)

\[
a_{03} = \frac{ir}{k}[C_5 \partial_r J_{(k)}(v) + C_6 \partial_r N_{(k)}(v)]
\] (7.34)

7.4. The case $k = 0$.

The case with $k = 0$ is special because in this case we have a gauge freedom given by a function $\Lambda_\mu(x)$ which was not fixed by the choices (2.13), (2.14) [13]. This case was studied in [16]. To make the solutions (7.18), (7.19) in the intermediate region agree with the corresponding solutions given in this reference, we have to note that this gauge freedom allows us to add arbitrary constants to $a_{12}$ and $a_{03}$. This freedom removes two of the four constants that appeared in the solution for general $k$, and corresponds to the fact that only one propagating degree of freedom exists for $k = 0$ in contrast to two degrees of freedom for higher $k$. Thus we can solve the dynamical equations for $a_{12}$ and then note that for $k = 0$ we can find $a_{03}$ algebraically from the equations of motion.
8. Absorption probabilities, structure of solutions

First we consider the fields for which the solution in the intermediate region is a pure power law. In these cases we can ignore the intermediate region in the calculation of scattering at leading order in $\omega$, since the solution form in the outer part of the inner region directly joins up with the solution in the inner part of the outer region. The fields in this category are

(a) The dilaton-axion scalar $B$
(b) The two form field in the $x$ space arising from the 4-form field $A_{\mu \nu \alpha \beta}$
(c) The vector $A_{\mu}$ in the $x$ space arising from the two form field $A_{\mu \alpha}$. Here we have to study the transverse components $a_{a}, a = 1, 2, 3$ and the longitudinal component described by $a_{r}, a_{0}$; both have trivial intermediate region solutions.

For all these fields the absorption probability (2.35) is

$$P = 4\pi^2 (\frac{\omega R}{2})^{4k+8} [\Gamma(k+2)\Gamma(k+3)]^{-2} \quad (8.1)$$

Now we consider the scalar in the $x$ space arising from the 2-form field $A_{\alpha \beta}$. The spherical harmonics for this case split into two classes denoted by $Y^{k, \pm}_{[\alpha \beta]}$. The absorption probabilities for these two classes are given by

$$P_+ = 4\pi^2 (\frac{\omega R}{2})^{4k+12} [\Gamma(k+3)\Gamma(k+4)]^{-2} \quad (8.2)$$

$$P_- = 4\pi^2 (\frac{\omega R}{2})^{4k+4} [\Gamma(k+1)\Gamma(k+2)]^{-2} \quad (8.3)$$

For the 2-form field in the $x$ space arising from $A_{\mu \nu}$ we have seen that the longitudinal and transverse components do not decouple, except for $k = 0$ when the longitudinal component is not dynamical. It appears to not be helpful to try to define cross sections in this case, so we present instead the relation between the solutions at $r \to 0$ and the solutions at $r \to \infty$.

In the outer region we have found in (7.13) (7.10) (7.11)

$$a_{12} = \tilde{C}_1 r^{-2} J_{(k+2)}(\omega r) + \tilde{C}_2 r^{-2} N_{(k+2)}(\omega r) \quad (8.4)$$
$$a_{r3} = C_1 r^{-1} J_{(k+2)}(\omega r) + C_2 r^{-1} N_{(k+2)}(\omega r) \quad (8.5)$$
$$a_{03} = \frac{i}{\omega} [a_{r3,r} + 3r^{-1} a_{r3}] \quad (8.6)$$

In Table 1 we present the result of matching the solutions across the intermediate region. The first column gives the choice of solution at $r \to 0$. The other columns give the solution at $r \to \infty$ in terms of the coefficients in (8.5), (8.6).

In the first two cases the solution in the intermediate region is a pure power law, as it gives $\phi_+$ (eq. (7.19) ) in this region. In the last two cases the solution is not trivial in the intermediate region, since it gives $\phi_-$ (eq. (7.18) )
Table 1: Scattering matrix for the two form from the antisymmetric tensor

| $a_{12}$ | $\tilde{C}_1$ | $\tilde{C}_2$ | $C_1$ | $C_2$ |
|----------|--------------|--------------|-------|-------|
| $J_{(k+4)}(v)$ | 0 | $-\frac{\pi R^2}{\Gamma(k+2)\Gamma(k+6)} \left( \frac{\omega R}{k} \right)^{2k+6}$ | 0 | $-\frac{\omega}{k} \tilde{C}_2$ |
| $N_{(k)}(v)$ | $-\frac{R^2 \Gamma(k)\Gamma(k+3)}{\pi} \left( \frac{\omega R}{k} \right)^{-(2k+2)}$ | 0 | $\frac{\omega}{k+4} \tilde{C}_1$ | 0 |
| $N_{(k+4)}(v)$ | $-\frac{R^2 \Gamma(k+4)\Gamma(k+5)}{\pi} \left( \frac{\omega R}{2} \right)^{-(2k+6)}$ | 0 | $-\frac{\omega}{k+4} \tilde{C}_1$ | 0 |
| $J_{(k)}(v)$ | 0 | $-\frac{\pi R^2}{\Gamma(k+1)\Gamma(k+2)} \left( \frac{\omega R}{k} \right)^{2k+2}$ | 0 | $\frac{\omega}{k} \tilde{C}_2$ |

9. Analysis of solutions

Let us comment on the nature of the calculation and its results. We note that in the outer and the inner regions the waveform attains the form of a freely traveling wave. The intermediate region acts as a barrier that the solution has to tunnel through, and at small $\omega$ the tunneling is small. We can imagine having the wave incident from either side of this barrier. If we throw in a particle from infinity and have it absorbed by the brane, then the particle is incident from the outer region, and the boundary condition on the solution is that there be no outgoing wave at $r \to 0$ in the inner region. Perhaps for emission of a quantum from the branes we should take the particle to be incident from the inner region.

The results on the absorption probabilities are summarized in Table 2. The first column gives the the field and its expansion in spherical harmonics. The second column gives the range of the index for the harmonics. The third column lists the cases that occur in the supergravity multiplet of the 5-dimensional AdS space. (These fields are labelled by $\odot$ in [13], and we have used the same notation here.) The fourth column gives the absorption probabilities. For the antisymmetric tensor with no indices along the $S^5$ the components mix in a way that does not make it helpful to define an absorption probability. One can choose combinations of fields incident on the branes such that the reflected fields are a multiple of the combination that was incident. In this case one can define an absorption probability for the incident combination, but it turns out that in this basis the two power laws of $\omega$ that arise during the calculation mix with each other, so the diagonalization obscures the $\omega$ dependence of the solution.

For the case $k = 0$ however we have seen that only one degree of freedom propagates, and the absorption probability can be computed for this mode. The result from [16] can
be written as
\[ P_\text{=} = \frac{4\pi^2}{[\Gamma(k+3)\Gamma(k+4)]^2} \left( \frac{\omega R}{2} \right)^{4k+12} \]  
(9.1)

which agrees with the general pattern of results in Table 1.

In the case of the scalar field \( B \) the above computation was carried out for all partial waves in \( \mathbb{S} \), and our result for this agrees with the results presented there.

The probability for passage from one side of the barrier to the other is the same, whichever side the particle is incident from, as would be expected on general grounds. In the intermediate region (the barrier) we set \( \omega = 0 \). There are two solutions to a second order wave equation, and these solutions are power laws in \( r \) in the inner part of the intermediate region, and also in the outer part of the intermediate region. The powers are not the same in these two parts of the intermediate region in general, since the solutions need not be a simple power of \( r \) throughout the region. Let the powers in the inner part of the intermediate region be
\[ r^{a+b}, \quad r^{-a+b} \]  
(9.2)

Let the powers in the outer part of the intermediate region be
\[ r^{c+d}, \quad r^{-c+d} \]  
(9.3)

Here we have assumed that \( a \geq 0, \ c \geq 0 \). Then the absorption probability has the dependence
\[ P \sim \omega^{2a+2c} \]  
(9.4)

The parameters \( a \) and \( c \) are the indices of the Bessel function \( J \) in the inner and outer regions respectively.

We summarize some information about the solutions in Table 3. In the first column we list the fields that we study. Here the symbols \( a_+, a_- \) correspond to the two kinds of spherical harmonics that we have for this field, \( Y^{k,+}_{[\alpha\beta]} \) and \( Y^{k,-}_{[\alpha\beta]} \) respectively. The second column gives the two powers of \( r \) for each field in the inner part of the intermediate region, eq. (9.2). The third column gives the powers of \( r \) in the outer part of the intermediate region, eq. (9.3).

The fourth column gives the dimension \( \Delta \) of the field in the Yang-Mills theory which corresponds to the mode of supergravity having the given power law behaviors in the inner region. This dimension is given by \( [8] [17] \)
\[ \tilde{m}^2 = (\Delta - p)(\Delta + p - 4) \]  
(9.5)
where $\tilde{m}^2$ is the eigenvalue of the Maxwell operator on the appropriate p-form in the 5-dimensional AdS geometry in the inner region. These $\tilde{m}^2$ values are also the same as those listed in [13]. Note that $\Delta$ is connected to the power $a + b$ of $r$ in the solution of the field equation in the outer part of the inner region [4]; this is the power that decays as we go towards smaller $r$ values. The relation is

$$\Delta = a + b - p + 4$$

(9.6)

Where $p$ is the rank of the form in the AdS space.

The last column gives the $\omega$ dependence of the absorption probability, which is given by (9.4). For the antisymmetric tensor with no indices along the $S^5$ the components mix and we do not get a statement analogous to (9.4). For the case $k = 0$, the absorption probability (9.1) agrees with the pattern of results in Table 2 for the $\Delta = 6$.

| Field          | $k$     | $\sigma$ | $\mathcal{P}$                                                                 |
|----------------|---------|----------|--------------------------------------------------------------------------------|
| $B = B^{I_1}Y_{I_1}$ | $k \geq 0$ | $k = 0$  | $\mathcal{P} = \frac{4 \pi^2}{\Gamma(k+1)\Gamma(k+3)^2} (\frac{\omega R}{2})^{4k+8}$ |
| $A_{\alpha\beta} = a^{I_{10,+}}_{[\alpha}Y_{I_{10},+}^{\beta]}$ | $k \geq 1$ | $k = 1$  | $\mathcal{P}_- = \frac{4 \pi^2}{\Gamma(k+3)\Gamma(k+4)^2} (\frac{\omega R}{2})^{4k+12}$ |
| $A_{\alpha\beta} = a^{I_{10,-}}_{[\alpha}Y_{I_{10},-}^{\beta]}$ | $k \geq 1$ | $k = 1$  | $\mathcal{P}_+ = \frac{4 \pi^2}{\Gamma(k+1)\Gamma(k+2)^2} (\frac{\omega R}{2})^{4k+4}$ |
| $A_{\mu\alpha} = a^{I_5}_{\mu}Y_{I_5}^{\alpha}$ | $k \geq 1$ |           | $\mathcal{P} = \frac{4 \pi^2}{\Gamma(k+1)\Gamma(k+3)^2} (\frac{\omega R}{2})^{4k+8}$ |
| $a_{\mu\nu\alpha\beta} = a^{I_{10,\pm}}_{\mu\nu}Y_{I_{10},\pm}^{\alpha\beta}$ | $k \geq 1$ |           | $\mathcal{P} = \frac{4 \pi^2}{\Gamma(k+1)\Gamma(k+3)^2} (\frac{\omega R}{2})^{4k+8}$ |
| $A_{\mu\nu} = a^{I_1}_{\mu\nu}Y_{I_1}$ | $k \geq 1$ | $k = 1$  | $\mathcal{P} = \frac{4 \pi^2}{\Gamma(k+1)\Gamma(k+3)^2} (\frac{\omega R}{2})^{4k+8}$ |
|                | $k \geq 0$ |           | $\mathcal{P} = \frac{4 \pi^2}{\Gamma(k+1)\Gamma(k+3)^2} (\frac{\omega R}{2})^{4k+8}$ |

Table 2: Fields and corresponding absorption probabilities.

10. Discussion

In the geometry produced by the 3-branes we have the asymptotic regions of large $r$ and small $r$, in each of which we have the maximal number of supersymmetries (32 real components). In the intermediate region the number of supersymmetries is only half that
Table 3: Power law behavior for the fields in the inner and outer regions, and the dependence of absorption probability on $\omega$.

(16 real). Consider any partial wave, and imagine that it is incident from the large $r$ region. In passing through the intermediate region there will be a change in the fields, such that a possibly different basis of components will emerge as the natural one in the $r \to 0$ region.

It would be interesting to seek a microscopic model of the 3-branes which reflects the action of the intermediate region. What appears to be passage into the inner region in the classical picture is expected to be an excitation of the quantum states on the branes in a microscopic description. Note that we have considered linear perturbations on the background, but in general the strength of the perturbation grows as the wave moves deeper into the inner region. (This phenomenon occurs also for perturbations on the three charge hole in 4+1 dimensions.)

Recently it has been shown that in the three charge black hole (in the dilute gas regime) the two point function of supergravity fields in the AdS region can be used to reproduce the agreement between the microscopic and classical absorption cross sections, for $s$-wave absorption \cite{18}. It is important in this calculation to use the correctly normalized two-point function for the field \cite{3} \cite{19}. It is not clear however how such a calculation would
be extended to higher partial waves. From Table 2 we observe that if \( \Delta = k + m \), then the power of \( \omega \) is \( 4k + 2m \). Thus the case \( k = 0 \) might not exhibit the full structure of the relation between the classical calculation and the microscopic theory. We hope to return to this issue in the future.

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