Grain shape effects in bed load sediment transport

Many planetary surfaces are fluid–regolith interfaces where flowing gases or liquids are in contact with granular sediment. On Earth, the resulting sediment transport down hillslopes, through rivers, along coasts and across deserts moves vast quantities of rock and other particulate material long distances. In bed load sediment transport, grains roll, hop and slide while generally remaining in contact with the sediment bed. This mechanism moves a landscape's coarsest grains, and it has an outsized role in shaping mountainous regions and building Earth's sedimentary record. Evidence of sediment transport on other worlds such as Mars and Titan can be used to infer past and present climates and to guide the search for habitable environments. Bed load transport also factors critically in numerous environmental and engineering contexts, including river delta formation, natural hazard mitigation and recovery, pollutant transport, infrastructure projects, and restoration of rivers and coasts. Because it is ultimately driven by the hydrologic cycle, bed load transport is sensitive to changes in climate, which complicates the already challenging task of predicting the behaviour of rivers.

Despite nearly a century of research on bed load transport, sediment flux predictions can be highly uncertain, with typical errors up to a factor of five or more. It has proven difficult to develop simple yet accurate models of bed load transport, which involves fully turbulent fluid flow interacting with a dense sediment slurry. Most widely used models are semi-empirical and predict sediment flux based on average flow and bed conditions using the form

\[ q^* = \alpha (\tau^* - \tau_c^*)^3, \]

where \( q^* \) is the nondimensional volumetric sediment flux per unit river width, \( \tau^* \), the Shields number, is the nondimensional shear stress on the bed, \( \tau_c^* \) is the critical value of the Shields number below which no transport occurs, and \( \alpha \) is a coefficient.

Natural variations in grain size, shape and density are possible sources of uncertainty in sediment transport predictions. Size and density effects on grain motion are well studied, and both appear in the standard nondimensionalization of the problem \( (q^*, \tau^* \text{ and } \tau_c^* \text{ in equation (1)}) \). Other effects have been implicated as sources of remaining uncertainty, including mixtures of different grain sizes and time-dependent bed structure. By contrast, the effects of grain shape have rarely been quantified, even though shape has long been hypothesized to influence sediment transport and is known to influence granular dynamics in general, and has been shown to modify properties relevant to sediment transport, such as the threshold of motion.

During bed load transport, the granular bed is sheared by the flow passing over it. Aspherical grains and rough surfaces generally increase the resistance to such shearing, and the tendency of aspherical grains to slide along the bed rather than roll further enhances frictional resistance. This argument is consistent with a compilation of bulk friction coefficients showing that less-spherical granular
materials generally have higher friction coefficients (Fig. 1c). The idea that aspherical grains are harder to transport is also in line with field and laboratory observations that isolated aspherical grains are transported more slowly than spheres under some conditions.22,23. 

Given these observations, it is tempting to assume that less-spherical grains always experience slower transport. However, fluid–granule interactions complicate the relationship between grain shape and sediment flux. Aspherical grains experience higher fluid drag force than spherical grains of the same volume, resulting in more-efficient fluid–granule momentum transfer under the same flow conditions. This occurs not only because irregular grain shapes generally impede flow around the grain, but also because grains in the flow tend to reorient such that their largest cross-sectional area is perpendicular to the flow.24. This argument is consistent with the observation that aspherical grains generally settle more slowly in still water than spherical grains (Fig. 1d). Enhanced fluid drag on aspherical grains counteracts the enhanced granular friction, partially obscuring the relation between grain shape and bed load transport parameters. Both of these competing effects generally get stronger as grain shape deviates from spherical (Extended Data Fig. 3), making it challenging to predict the net effect of grain shape on sediment transport.

**Shape-corrected bed load transport law**

We disentangle these competing effects by formulating a theory that accounts for grain shape effects on both fluid–granule and grain–granule interactions. Here shape is considered to include variability and irregularity in grain surface topography over a range of scales, from small-scale surface roughness through to meso-scale angularity all the way up to the scale of the grain (for example, sphere versus ellipsoid). We assume that the fluid drag forces driving grain motion can be described with an effective coefficient of drag \( C_D \) and that the resistance to bed load motion due to granular contacts can be described with a bulk friction coefficient \( \mu_b \). In the standard derivation of the Shields number, the coefficients of drag and friction are dropped, yet these are precisely the parameters that are sensitive to grain shape. We retain these terms, yielding a shape-corrected Shields number that includes a ratio of two dimensionless quantities that account for drag and friction (Methods).

The first quantity, \( C^* \), is the effective drag coefficient normalized by the drag coefficient of the volume-equivalent sphere (denoted with the subscript \( o \)): \( C^* = C_D/C_o = S_c D_{settle}/D_o \). The effective drag coefficient, \( C_D \), is obtained by multiplying the drag coefficient of grains settling in still water, \( C_{Dsettle} \), by the Corey shape factor, \( S_c \), to account for the fact that grains tumble during transport (Methods). The second quantity, \( \mu^* \), is the average bulk friction coefficient normalized by the bulk friction coefficient of spheres, both modified by the tangent of the bed angle \( \theta \) to account for a tilted bed: \( \mu^* = (\mu_s - \tan \theta)/(\mu_b - \tan \theta) \). Both \( C^* \) and \( \mu^* \) are equal to one for idealized spheres.

Introducing the shape-corrected Shields number \((C^*/\mu^*) \tau^*\) yields a shape-corrected bed load transport law,

\[
q^* = a_0 \left( \frac{C^*}{\mu^* \tau^* - r_{co}^*} \right)^{3/2},
\]

where the parameters \( a_0 \) and \( r_{co}^* \) are the transport coefficient and threshold of motion for idealized spheres and do not depend on grain shape. Equation (2) predicts that the sediment flux \( q^* \) for a given shape-corrected Shields number should be the same regardless of grain shape. It also summarizes the net effect of shape-influenced fluid drag and granular friction on sediment flux: if \( C^*/\mu^* > 1 \), the enhanced fluid drag on aspherical grains is more important than the enhanced granular friction, and aspherical grains have higher transport rates than spheres. If \( C^*/\mu^* < 1 \), the enhanced granular friction is more important, and aspherical grains have lower transport rates than spheres. This approach makes it possible to isolate the effects of bulk friction and fluid drag on sediment flux (equations (11), (12) and Methods).
Comparing equation (2) with the conventional bed load transport law, equation (1), reveals a second prediction of our theory: the conventional transport coefficient, α, and threshold of motion, τ∗, should depend on grain shape: α = a(C/μ)3/2 and τ∗ = τco(μ/C). Given measurements of bed load sediment flux over a range of bed shear stress for granular materials with different shapes, we can test these two key predictions of our theory: sediment flux, ω*, should follow a single trend described by the shape-corrected transport law, equation (2), for all grain shapes; and the conventional transport coefficient and threshold of motion should depend on grain shape. Both tests require measurements of C* and μ* for each granular material as well as estimates of αo and τ∗co, the transport coefficient and entrainment threshold for idealized spheres.

Laboratory flume experiments
We conducted a series of flume experiments with five granular materials of similar size and density but different shapes (Fig. 2; Methods). Four of the materials were artificial glass (silica) grains with distinct shapes and the fifth was a naturally sourced river gravel (predominantly silica). Each material had a uniform or nearly uniform grain size. In each experiment, we supplied a constant water discharge and sediment feed at the upstream end of a narrow (2–3 grain diameters wide), inclined flume and made measurements after the system had reached an equilibrium in which sediment outflux matched sediment influx. We used a narrow flume to aid in visualizing flow and the transport process. Each experiment was recorded with a high-speed camera. Videos of representative experiments are provided in Supplementary Information.

Supplementary Videos 1, 2, 8, 9 show low- and high-intensity transport of spheres and natural gravel (Fig. 2a,d), whereas Supplementary Videos 3–7 show moderate-intensity transport of all five materials.

All five granular materials exhibit the scaling between dimensionless sediment flux and Shields number predicted by equation (1) (Fig. 3a). However, the dimensionless sediment flux for a given Shields number varies among the materials: it is at least a factor of 2.5 higher for spheres than for natural gravel, with an even larger difference close to the threshold of motion. This demonstrates that grain shape has an important influence on bed load transport, as has long been suspected based on tracking of individual grains undergoing sediment transport22,23. At the same time, some materials with very different shapes—such as rectangular prisms and rounded glass chips—plot close to one another (Fig. 3a), indicating that the net effect of grain shape on sediment flux is difficult to predict based on the appearance of grains.

We calculated C* and μ* for each material based on grain properties measured independently of the flume experiments. To obtain C*, we measured settling velocities and Corey shape factors for many individual grains (Methods), and to obtain μ*, we measured the angle of repose for each material (Methods). Using these experimental measurements, we first visualize separately the effects of shape-dependent bulk friction and fluid drag on sediment flux. Plotting sediment flux as a function of the drag-corrected Shields number, C*/μ, reveals the full effect of shape-dependent bulk friction on bed load transport. As described by equation (11) (Fig. 3b). More-aspherical granular materials are substantially more resistant to transport for a given C*/μ.

Conversely, plotting sediment flux as a function of the friction-corrected Shields number, τ*/μ, reveals the full effect of shape-dependent fluid drag, with aspherical grains more easily transported for a given τ*/μ owing to stronger fluid–grain coupling and momentum transfer, as described by equation (12) (Fig. 3c). Comparison of Fig. 3a–c also demonstrates that, although bed load transport of aspherical grains is generally inhibited by higher bulk friction (Fig. 3b) and enhanced by higher fluid drag (Fig. 3c), these two effects do not simply cancel each other out (C*/μ ≠ 1). Measurements of C* and μ* for six additional granular materials further illustrate the varying outcomes of this competition between drag and friction (0.84 ≤ C*/μ ≤ 1.52) (Extended Data Figs. 3, 4).

Next, we test the prediction that the conventional transport coefficient and threshold of motion are functions of grain shape. Rearranging the predicted functional forms of a and τ to separate C* and μ* yields a(C/μ)3/2 and τco(μ/C). By fitting equation (11) to the data in Fig. 3b, we extract the drag-corrected coefficient of transport, a(C/μ)3/2 and threshold of motion, C*/τ*, for each granular material. The drag-corrected transport coefficient scales with μ3/2 (Fig. 3d), as predicted, implying that frictionally stronger granular materials have lower bed load transport rates. The drag-corrected threshold of motion scales linearly with μ* (Fig. 3e), as predicted, implying that frictionally stronger granular materials require proportionally higher shear stress to initiate sediment transport. The latter observation is consistent with previous predictions for the threshold of motion22 as well as the derivation of our grain shape theory (Methods), which implies that

\[
τ∗ = (μ*/C*)τco = \frac{\mu∗ - \tanθ}{C_o}.
\]

Alternatively, we can perform this test using friction-corrected quantities and the corresponding expressions a(C/μ)3/2 = a(C/μ)3/2 and τco/μ* = τco/C*. Fitting equation (12) to the data in Fig. 3c, we extract the friction-corrected coefficient of transport and threshold of motion. Both friction-corrected quantities exhibit the predicted dependence on the drag parameter C*: a(C/μ)3/2 = C3/2 and τco/μ* = 1/C*, implying that grain shapes with stronger fluid coupling have higher bed load transport rates and require less shear stress to initiate sediment transport (Fig. 3f,g). In addition to supporting the proposed grain shape theory,
these results demonstrate that fluid and frictional effects during both entrainment and subsequent transport must be accounted for to reconcile transport rates of different grain shapes.

Finally, we compare the shape-corrected transport law (equation (2)) with the flume experiments. Given that \( q^* \) and \( \tau^* \) are measured from flume experiments, \( C^* \) is determined from independent settling velocity measurements, and \( \mu \) is determined from independent angle of repose measurements, this provides a stringent test of the theory. Plotting nondimensional sediment flux as a function of the shape-corrected Shields number (\( C^*/\mu \))\( \tau^* \), which accounts for the effects of grain shape on both fluid drag and granular friction, confirms that the five granular materials in our experiments collapse to a single trend. Moreover, this trend is well described by equation (2) with values of \( \alpha_h = \alpha(\mu^*/C^*\tau^*) = 11.991 \pm 0.455 \) and \( \tau_{\text{sh}}^* = \tau_{\text{sh}}(C^*/\mu^*) = 0.051 \pm 0.001 \), calculated by taking the mean and standard error across the five materials.

**Implications for sediment transport**

Our experiments demonstrate that grain shape can have a substantial effect on bed load sediment entrainment and transport (Fig. 3a). Close to the threshold of entrainment (\( \tau^* \approx 0.05 \)), a condition typical of bed load transport in gravel-bed rivers\(^{38} \), sediment flux varies by a factor of five or more across the different grain shapes. This magnitude of variability in transport rate for the same Shields number is comparable to that observed in compilations of flume data\(^{4,7,8} \), suggesting that grain shape effects may underlie some of this scatter.

Although abrasion during sediment transport can lead to convergence in shape\(^{39} \), natural sediment grains take on various shapes, owing to the varied mechanical properties of their source rocks and varied transport conditions\(^{40} \)(Extended Data Fig. 3). The magnitude of the grain shape effect in our experiments does not necessarily translate directly to natural scenarios, because the five granular materials do not precisely mimic the range of natural grain shapes. Still, some of the most common naturally occurring shapes are present in our experimental materials. These include platy grains derived from bedded or foliated rocks (rounded chips), grains with faceted mineral surfaces (faceted ellipsoids), blocky grains formed by intersecting fracture planes (rectangular prisms), well-rounded grains (spheres), and partially rounded grains (natural gravel).

Isolated aspherical grains in laboratory and field settings are observed to move downstream more slowly (including periods in motion and at rest) than spherical grains in the same flow\(^{22,23} \). This is generally consistent with the five experimental materials used here (Fig. 3a). For natural gravel in particular, enhanced fluid drag due to shape effects is not as important as enhanced granular friction, leading to slower transport. We suspect that the reduced influence of drag is due to the surface properties of the natural gravel, which could be the product of a feedback in which abrasion drives grains towards lower-drag shapes that are less likely to be entrained. Measurements of \( C^* \) and \( \mu^* \) for additional natural gravels yield \( C^*/\mu^* \leq 1 \), suggesting that this is also the case for other gravels (Extended Data Fig. 3).

However, some aspherical grain shapes may enhance bed load transport. Measurements of \( C^* \) and \( \mu^* \) for shell fragments and slick
but angular tempered glass chips yield \( C'\mu > 1 \) (Extended Data Fig. 3), indicating that enhanced fluid drag outweighs enhanced friction in these materials. These examples illustrate the difficulty of predicting how grain shape will affect bed load transport based on qualitative characterizations of grain shape. Our approach offers a quantitative means of predicting the net effect of shape on grain entrainment and transport.

In summary, we find that differences in the efficiency of bed load transport and the threshold of motion among different grain shapes can be reconciled by modifying the Shields number to account for enhanced fluid drag (via \( C' \)) and granular friction (via \( \mu' \)) in more-aspherical grains. These competing effects of grain shape can be characterized with familiar, easily measured quantities: the coefficients of drag and bulk friction. Although this approach does not capture all the possible effects of grain shape\(^4\), the ability to account for grain shape in sediment transport calculations is a major improvement over the usual practice of ignoring it. The difference between spheres and natural gravel is especially important because spheres are a favored tool in theoretical and experimental studies of sediment transport. Our results imply that inferences based on spheres may not translate directly to natural sediment, but our theory provides a framework for making such comparisons.

Scenarios with heterogeneous grain shapes commonly arise in evolving landscapes, such as coastal environments with grains sourced from a mix of mineral and biological materials, or streambeds with mixed rock types that weather differently. Better predictions of bed load flux (Fig. 3h) in such scenarios will aid interpretations of the sedimentary record and predictions of landscape evolution. A better understanding of the effect of grain shape on bulk transport dynamics also has broader applications ranging from river restoration\(^1\), coastal engineering\(^41\), and volcanic hazard\(^42\), to industrial processes involving granular materials, such as drilling\(^3\), energy supply\(^44\), and, more broadly, agriculture\(^45\). Our approach is simple enough to apply to grain shapes observed elsewhere in the Solar System\(^6\) and could help improve reconstructions of past and present climate on worlds such as Mars and Titan.

### Online content

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41586-022-05564-6.

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Methods

Grain shape theory for bed load transport

Shape-corrected Shields number. We derive a shape-corrected Shields number by retaining terms describing fluid drag and bulk grain friction in the derivation used to obtain the conventional Shields number. The Shields number compares the magnitudes of forces driving and resisting grain motion. As illustrated in the free-body diagram in Extended Data Fig. 1, a spherical grain with diameter \( d \), and density \( \rho \), resting on a bed inclined at an angle \( \theta \) from the horizontal and immersed in a moving fluid with density \( \rho_f \) experiences a gravitational force, \( F_g \), a buoyant force, \( F_b \), a fluid drag force, \( F_d \), a lift force, \( F_L \), a bed contact force, \( F_c \), and a frictional force, \( F_r \). At the threshold of grain motion, the slope-parallel driving and resisting forces are equal in magnitude,

\[
F_d + (F_g - F_b) \sin \theta = F_t = \mu_s \left((F_g - F_b) \cos \theta - F_c\right).
\]

where \( \mu_s \) is the coefficient of bulk static friction. The force magnitudes are

\[
F_0 = \frac{1}{2} \rho \left(C_D \mu^* \tau^* \right)^{3/2} \pi d^2 A
\]

\[
F_0 = \frac{1}{2} \rho C_I \mu^* \tau^* \left(f^* \right) A
\]

\[
F_0 = \rho g V
\]

\[
F_0 = \frac{\rho g V}{6}
\]

where \( C_0 \) is the drag coefficient, \( C_I \) is the lift coefficient, \( V \) is grain volume, \( A \) is grain cross-sectional area, \( g \) is gravitational acceleration, \( \tau^* = \frac{\tau}{\rho d^2} \), with \( \tau \) the shear stress due to the flow, \( f^* = \frac{u'}{u^*} \) is a velocity profile function, with \( u^* \) the square of the flow velocity, and \( \mu^* \) denotes averaging over A. We assume that \( f^* \) takes on an approximately constant value in most grain entrainment scenarios. All uses of \( \tau \) here and throughout the text refer to the stress at the grain bed. For a spherical grain, \( A = \pi d^2/4 \) and \( V = \pi d^3/6 \). Substituting equations (5a–5d) into equation (4) and rearranging to obtain a ratio of terms involving drag and lift to terms involving gravity and friction yields a definition of a ‘complete’ Shields number at the threshold of motion,

\[
\tau^*_{\text{complete}} = 1 - \frac{3}{4} \left( f^* \right) \left( \frac{C_0 + \mu_s C_I}{\rho g V} \right) \left( \frac{(\rho_g - \rho) g d_o \cos \theta (\mu_s - \tan \theta) \right). \tag{6}
\]

Equation (6) can alternatively be derived by defining \( \tau^*_{\text{complete}} \) as the ratio of driving to resisting forces and assuming flow conditions close to the threshold of motion (\( \tau^*_{\text{complete}} = 1 \)). We find that this is also a good approximation for \( \tau^*_{\text{complete}} > 1 \).

Making the common assumption that the lift term is negligible (\( \mu_s C_I = C_0 \)) simplifies this expression to

\[
\tau^*_{\text{complete}} = \frac{1}{\cos \theta} \left( \frac{C_0}{\rho g V} \right) \left( \frac{(\rho_g - \rho) g d_o \cos \theta (\mu_s - \tan \theta) \right). \tag{7}
\]

Ignoring the terms involving the drag coefficient, the bulk friction coefficient, and the bed slope yields the conventional definition of the Shields number,

\[
\tau^*_{\text{conventional}} = \frac{\tau}{(\rho_g - \rho) g d_o}. \tag{8}
\]

We account for grain shape effects by instead retaining the terms involving \( C_0, \mu_s, \) and \( \theta \) in equation (7). We additionally multiply by the normalizing factor \( \cos \theta (\mu_s - \tan \theta) \), yielding the shape-corrected Shields number:

\[
\tau^* = \frac{C_0 \mu_s - \tan \theta}{C_0 \mu_s - \tan \theta} \left( \frac{\tau}{(\rho_g - \rho) g d_o} \right) \left( \frac{C_0}{\rho g V} \right) \left( \frac{(\rho_g - \rho) g d_o \cos \theta (\mu_s - \tan \theta) \right. \tag{9}
\]

where the dimensionless quantities \( C^* \) and \( \mu^* \), representing shape-dependent fluid drag and bulk friction, are as defined in the main text. The normalizing factor, and the resulting definitions of \( C^* \) and \( \mu^* \), have the effect that the shape-corrected sediment transport model, equation (2), reduces to the conventional model, equation (1), for spheres. In addition, normalizing by \( C_0 \) ensures that \( C^* \) is not a function of grain size, whereas \( C_0 \) is a function of the particle Reynolds number \( \text{Re}_p = \frac{\rho_d d \nu}{\mu} \), and therefore implicitly a function of grain size for a given flow. We assume that \( C_0 = 0 \) for small \( \gamma \), and set \( \mu_s = \tan (24^\circ) \), consistent with spherical particles.

Interestingly, the dependence on bulk friction in equation (9) is consistent with a two-phase continuum model of sediment transport, which shows that the nondimensional sediment flux, \( q^* \), scales with \( \tau^*_{\text{conventional}}/(\mu_s (1 - \tan \theta)) \), where \( \gamma = 2 \) (ref. 30).

To incorporate the shape-corrected Shields number into a sediment transport model, we multiply the Shields number in equation (1) by \( (C^* / \mu^*) (\mu_s / C^* ) = 1 \). Moving the \( (\mu_s / C^* ) \) term into the coefficient of transport gives

\[
q^* = \frac{\alpha \mu^* \tau^*}{(\mu_s / C^*)^{3/2}} \frac{C^*}{(\mu_s / C^*)^{3/2}} \frac{\tau^*_{\text{complete}}}{\tau^*}_{\text{conventional}} \frac{1}{\tau^*_{\text{complete}}}. \tag{10}
\]

where the nondimensional volumetric sediment flux per unit width is defined as \( q^* = q / \left( (\rho_g / \rho - 1) d_o \right) \), \( q \) is the dimensional volumetric sediment flux per unit width, and the first term in parentheses is the shape-corrected Shields number (equation 9). Defining \( a_r = a(\mu_s / C^* ) \) and \( \tau^*_{\text{complete}} = \tau^*_{\text{complete}} (\mu_s / C^* ) \) leads to the shape-corrected transport law in equation (2).

We can also rewrite equation (10) in terms of a drag-corrected Shields number, \( \tau^* r^* \), which corrects for the effect of grain shape on fluid drag forces only,

\[
q^* = \frac{\alpha \mu^* \tau^*}{\tau^*_{\text{complete}} (\mu^* + \tau^*)} \frac{\tau^*_{\text{complete}} (\mu^* + \tau^*)}{\tau^*_{\text{complete}} (\mu^* + \tau^*)} \frac{1}{\tau^*_{\text{complete}}} \tag{11}
\]

or in terms of a friction-corrected Shields number, \( \tau^* / \mu^* \), correcting for the effect of shape on granular friction only,

\[
q^* = \frac{\alpha \mu^* \tau^*}{\tau^*_{\text{complete}} (\mu^* + \tau^*)} \frac{\tau^*_{\text{complete}} (\mu^* + \tau^*)}{\tau^*_{\text{complete}} (\mu^* + \tau^*)} \frac{1}{\tau^*_{\text{complete}}} \tag{12}
\]

Equation (11) isolates the frictional effects of grain shape, in the sense that variations in sediment flux \( q^* \) for a given value of \( C^* \tau^* \) should reflect only differences in bulk grain friction. Accordingly, the modified transport coefficient and threshold of motion in equation (11) are functions of \( \mu^* \) only: using the relation for \( a \) and \( \tau^*_{\text{complete}} \) above, \( a r^* C^* = a C^* r^* \) and \( C^* \tau^* = C^* \tau^* \). Equation (12) isolates the fluid drag effects of grain shape, and therefore the modified transport coefficient and threshold of motion in equation (12) are functions of \( C^* \) only: using the relations for \( a \) and \( \tau^*_{\text{complete}} \) above, \( a C^* r^* = a C^* r^* \) and \( \tau^* = \tau^* \).

The shape-corrected transport law reveals how the conventional critical Shields number, \( \tau^*_{\text{c}} \), depends on the bulk friction coefficient, \( \mu_s \). Using the definitions of \( \mu^* \) and \( C^* \), the expression \( \tau^*_{\text{c}} = (\mu^* / C^*) \tau^*_{\text{c}} \) can be written as

\[
\tau^*_{\text{c}} = \frac{C_0}{C_0} \frac{\mu_s - \tan \theta}{\mu_s - \tan \theta} \frac{\tau}{(\rho_g - \rho) g d_o} \left( \frac{C_0}{\rho g V} \right) \left( \frac{(\rho_g - \rho) g d_o \cos \theta (\mu_s - \tan \theta) \right. \tag{13}
\]
This can be rearranged to give

$$
\tau = \frac{C_{D_v}}{\tau_0} \left( \frac{\mu - \tan \theta}{C_D} \right) \times \frac{\mu - \tan \theta}{C_D}.
$$

(14)

The proportionality holds because the first term is a constant for the same mean bed angle.

**Modified drag coefficient.** We measured grain drag coefficients, $C_{D_{settler}}$ by observing the grains settling in still water. Grains tend to settle in still water with their largest projected area perpendicular to the flow $^{32,36}$, which is the orientation with the largest drag force. Such a settling orientation is different from grain orientation during bed load transport, where grains tumble, presenting all faces to the flow. The measured drag coefficient for grain settling is therefore relevant to, but larger than, the effective drag coefficient during bed load transport.

We estimated the drag coefficient of a tumbling grain by correcting the measured still-water-settling drag coefficients for the effect of settling orientation while retaining the influence of other aspects of grain shape such as angularity. To illustrate the basis for our approach, we compare previously published drag coefficients for grains classified using two different measures of grain shape: gross grain shape, as measured by the Corey shape factor, $S_c$, and grain angularity. The Corey shape factor (defined in the main text) describes gross grain shape, which determines how projected area varies with orientation; $S_c$ is 1 for spheres and closer to zero for flatter grains. By contrast, although angularity influences the flow around a grain and therefore the drag coefficient, it does not substantially affect how projected area varies with orientation. Extended Data Fig. 2a shows trends of normalized drag coefficient versus $S_c$ for a compilation of grains $^{32}$ grouped qualitatively into three angularity categories (well-rounded, sub-angular and very angular). Each grey line in Extended Data Fig. 2a represents grains of the same angularity. For grains with a given angularity, the decrease in the drag coefficient as grains approach spherical ($S_c = 1$) scales approximately as $1/S_c$ (red dashed line).

We assume that the effect on drag of gross grain shape, as measured by the Corey shape factor, is equivalent to the effect of settling orientation, because gross grain shape controls the projected area of a grain in a given orientation. Therefore, to calculate an effective drag coefficient that reflects increases in drag due to grain angularity and roughness but corrects for the effect of settling orientation, we divide the measured settling drag coefficient by a factor of $1/S_c$, which removes the trend shown by the red dashed line in Extended Data Fig. 2a. This yields an effective ‘orientation-free’ drag coefficient $C_{D_v} = S_c C_{D_{settler}}$. The result is that, for example, two smooth, well-rounded grains with equal mass and volume but different gross grain shapes (for example, a sphere versus an oblate ellipse) have the same $C_{D_v}$ (Extended Data Fig. 2b). For rougher or more-angular grains $C_{D_v}$ is greater than the drag coefficient of a volume-equivalent sphere.

**Flume experiments.** We chose five different materials for the flume experiments (Fig. 2) with similar densities and sizes but different shapes. Each material had a unimodal or nearly unimodal grain size distribution. Extended Data Table I lists their key properties. We carried out the experiments in the narrow flume facility in the River Dynamics Laboratory at Simon Fraser University. The experimental set-up (Extended Data Fig. 5) consisted of a flume 4 m long, 45 cm tall and ~1 cm wide (slightly larger than two grain diameters) tilted 3° from horizontal. The narrow flume geometry was chosen to aid grain visualization. Water was recirculated at a fixed discharge of 0.91 s$^{-1}$ with a pump, creating a flow with a mean velocity of approximately 1 m s$^{-1}$. The mean water depth was 10 cm, and the mean hydraulic radius was 0.5 cm. This corresponds to a Reynolds number of ~5,000 and a Froude number close to 1. We fed grains into the flume with a grain hopper, making the sediment flux a fixed input parameter in each experiment. The base of the flume was a fixed bed of grains of different sizes.

In each experiment, we selected a constant sediment feed rate, which in turn caused the sediment bed to aggrade or degrade until a steady-state slope was reached, with sediment outflux at the downstream end of the flume equal to the sediment feed rate at the upstream end. This allowed us to create a different bed shear stress in each experiment by selecting a different sediment feed rate. The nondimensional transport rates sampled by our experiments span a factor of $>300$. These rates correspond to a sediment flux of ~10–20 grains per minute at the lowest transport rates, close to the threshold of grain motion, and a sediment flux of >5,000 grains per minute at the highest transport rates. Because we allowed the bed to aggrade or degrade until reaching a steady state, the steady-state bed slope in each experiment was typically larger or smaller than that of the flume base, and ranged from 1.8° to 7.4° with a mean of 3.5° across all experiments. Although the slope varied between experiments, each experiment formed a constant slope over the length of the flume. The minimum grain bed thickness was 5 cm (10 grain diameters).

Once at steady state, the experimental observations commenced. Observations consisted of measuring the mean bed and water slopes in the middle 2.5 m of the flume and measuring the mass flux of grains into a sediment trap at the end of the flume. The sediment transport was filmed with a high-speed camera in this middle section, and nine Supplementary Videos have been provided for different sediment transport rates and granular materials. To measure mass flux, the sediment trap was allowed to fill for a set period of time ranging from 30 to 900 s, depending on the mass flux. We then dried and weighed the collected grains and calculated the mass flux as the measured mass divided by the accumulation period. The reported mass flux for each experiment is the mean of at least three, and on average ten, individual mass flux measurements. We then divided the mass flux by the channel width and grain density to convert it to a volume flux per unit width.

We calculated bed shear stress using the depth–slope approximation for steady, uniform flow, $\tau = \rho g S$, where $R$ is hydraulic radius. We compared these shear stress estimates to shear stresses estimated by fitting the law of the wall to velocity profiles measured using particle image velocimetry (Extended Data Fig. 9). The two different approaches yield shear stress estimates that are proportional to one another but differ by a factor of approximately 3.3. This demonstrates that $\tau = \rho g S$ provides a good relative estimate but probably underestimates the true bed shear stress. We also performed a wall correction$^{33,34}$ to refine the shear stress estimates obtained from the depth–slope approximation. The wall-corrected estimates, although noisy, also suggest that the shear stress calculated using $\tau = \rho g S$ underestimates the true bed shear stress (Extended Data Fig. 9). Taking the mean wall correction factor for glass spheres and natural gravel of 2.41, we estimated the bed shear stress as $\tau = 2.41 \rho g S$. These wall-corrected shear stress estimates yield critical Shields numbers of $\approx 0.05$, in line with standard values$^{33}$.

Owing to the narrow width of the flume, we would ideally use the bed slope rather than the water surface slope for $S$. However, the water surface slope could be measured more reliably than the bed slope owing to persistent grain motion along the bed. Noting a consistent offset of approximately 1% grade between the bed slope and the water surface slope, we estimated the bed slope in each experiment by subtracting 1% grade from the measured water surface slope.

**Grain characterization**

**Grain density measurements.** We measured grain density by massing a sample of 15–40 grains in an empty 10 ml vial and again after the vial had been filled with water to the 10 ml mark. The difference between the mass of the vial when empty and when filled with water was used to calculate the volume of the grain sample. The procedure was repeated three times for each grain type.
Grain shape measurements. We characterized grain shape with three mutually perpendicular lengths \((a \geq b \geq c)\) (ref. 55) using the minimum bounding box method\(^6\) applied to between 10 and 1,611 grains for each material. Extended Data Fig. 6 shows distributions of grain shape measurements for the materials with variable grain shape. The spheres can be characterized by a single diameter (mean and standard deviation 4.9 ± 0.05 mm), as there was no variation in shape between grains. The faceted ellipsoids, which also had no variation in shape between grains, are circular in a cross-section perpendicular to their shortest axis, giving dimensions of \(a = b = 6.0 ± 0.1\) mm, \(c = 5.0 ± 0.1\) mm. The rectangular prisms are somewhat regular, with a square-cross-section perpendicular to the longest axis that has consistent dimensions from grain to grain. The variation in grain shape lies nearly entirely in the length of the longest axis \((a = 4.4 ± 0.77\) mm, \(b = 3.4 ± 0.27\) mm, \(c = 3.4 ± 0.2\) mm).

The rounded chips consist of smooth, rounded glass pieces with random shapes, and tend to have one dimension that is substantially shorter than the other two \((a = 8.7 ± 2.19\) mm, \(b = 5.7 ± 1.02\) mm, \(c = 3.5 ± 1.00\) mm). The natural river gravel was sourced from a beach near Vancouver, Canada, and sieved to a narrow grain size distribution \((4.0 \leq b \leq 5.6\) mm). We scanned approximately 1,600 grains with a microCT scanner, providing high-resolution shape data. We then measured the lengths \(a, b, c\) directly from the scanned shapes \((a = 5.8 ± 1.07\) mm, \(b = 4.4 ± 0.50\) mm, \(c = 3.3 ± 0.42\) mm). The faceted ellipsoids, rounded chips and rectangular prisms have a hole through the middle of each grain, which we measured and accounted for in the grain density calculation by assuming the hole is water-filled during sediment transport.

Drag coefficient measurements. The drag coefficient of a settling grain, \(C_{\text{dsett}}\), is calculated from the average settling velocity, \(u_s\), for each granular material using the relation\(^{57}\)

\[
C_{\text{dsett}} = \frac{4 (\rho / \rho_i - 1) g d_s^2}{u_s^2},
\]

(15)

which assumes that drag force balances the submerged weight of a grain settling in still water at terminal velocity. The settling velocity of a grain was measured by releasing it just beneath the surface of a 30-cm-deep tank of still, room-temperature water and filming its descent with a high-speed camera. Velocity was measured once the grain had stopped accelerating. This was repeated for 20–50 grains for each grain type, allowing for the characterization of the mean and standard deviation of the settling velocity. Extended Data Fig. 7 shows distributions of measured settling velocities.

Calculated drag coefficient of volume-equivalent spheres. We calculated the drag coefficient for a volume-equivalent sphere, \(C_{\text{ov}}\), using the estimated volume-equivalent sphere settling velocity and equation (15). The volume-equivalent sphere settling velocity, \(u_{s,ov}\), is estimated using the empirical equation\(^{12}\)

\[
\log W_s = -3.81564 + 1.94593 \log D_s - 0.09016 (\log D_s)^2
- 0.00855 (\log D_s)^3 + 0.00075 (\log D_s)^4
\]

(16)

where \(W_s = u_{s,ov} \rho_i / (g (\rho / \rho_i - 1) v_g D_s) = (\rho / \rho_i - 1) g d_s^2 / v^2\); and \(v\) is the kinematic viscosity of water. The parameters in equation (16) were found using an updated nonlinear least-squares fit to the data from ref. 12, and are slightly different than those provided in ref. 12. The difference is small, amounting to at most a few per cent difference in the estimated settling velocity.

Coefficient of static friction measurements. We estimated the coefficient of static friction from the angle of repose, \(\phi\), of the different granular materials using \(\mu_s = \tan \phi\) (ref. 58). We measured the internal angle of repose of each grain type using the fixed funnel method\(^{59}\). For each grain type we poured grains slowly from a height of a few centimetres onto an elevated disk with a diameter of 12 cm (approximately 24 grain diameters) and a rim height of 1 cm (approximately 2 grain diameters). We continued to pour grains until a conical pile grew to the diameter of the elevated disc and small avalanches began to occur, indicating that a steady-state slope had been achieved. Using OpenCV\(^{60}\) image processing software, we extracted the silhouette of the pile and fit a line to the silhouette on either side, excluding the parts of the pile close to the base or close to the peak (Extended Data Fig. 8). We repeated this procedure three to six times for each granular material. A few piles had distinctly convex or concave sides and were excluded from the analysis. The average slope of the fitted lines for each material was taken as the angle of repose \((6 \leq \phi \leq 12)\) (Extended Data Table 1).

Uncertainty analysis. Unless stated otherwise, all error bars show the standard error of the mean, estimated as the standard deviation divided by the square root of the number of observations, and propagated using Gaussian error propagation when necessary. In some cases the error or uncertainty arises from limitations in measurement techniques, and in some cases there is variability in measured quantities owing to true variability between different grains in the same granular material. For example, the Corey shape factor varies among grains from a single granular material owing to differences in their shapes.

Data availability

The experimental flume data and measurements of grain properties and code used to support the conclusions and generate the figures in the main text and extended data items are available in an online repository. Flume data: https://doi.org/10.7910/DVN/GBC52U; grain properties: https://doi.org/10.7910/DVN/31KT36; main text figures: https://doi.org/10.7910/DVN/SPYJF; and extended data items: https://doi.org/10.7910/DVN/NQ33OD. Source data are provided with this paper.

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**Additional information**

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Extended Data Fig. 1 | Free body diagram of a single grain on a bed with inclination $\theta$. Force vectors shown correspond to forces in equations (4) and (5). The various forces are a gravitational force, $F_g$, a buoyant force, $F_b$, a fluid drag force, $F_D$, a lift force, $F_L$, a bed contact force, $F_c$, and a frictional force, $F_f$. 
Extended Data Fig. 2 | Effects of different aspects of grain shape on fluid drag coefficient. The measured drag coefficient of a grain settling in still water, $C_{D_{\text{settle}}}$, relative to the calculated drag coefficient of the volume-equivalent sphere, $C_o$, as a function of the Corey shape factor, a measure of gross grain shape. Coloured points show the materials used in our flume experiments and the additional materials in Extended Data Fig. 3a–e. Grey lines are fits to a large compilation of single-grain settling experiments that have been sorted by grain angularity, a measure of small-scale grain shape. Sketches in key show idealized grains of different angularity. Red dashed line is the trend $1/S_f$ for comparison. Error bars show one standard error of the mean.
Extended Data Fig. 3 | Additional granular materials and their shape properties. a, Tempered glass chips. b, Shell fragments. c–f, Natural gravels 2–5, respectively, with differences in gross grain shape, angularity and surface properties. All materials have similar average sizes and densities to each other and to the five granular materials used in the flume experiments (Extended Data Table 1). g, Normalized coefficient of friction, $\mu^*$, as a function of the Corey shape factor for the five granular materials used in the flume experiments and the six additional granular materials shown in a–f. h, Normalized coefficient of drag, $C^*$, as a function of Corey shape factor for the same 11 granular materials as in g and h. The parameters $C^*$ and $\mu^*$ tend to vary similarly for many of the measured materials, such that their ratio is close to one. However, several materials have ratios of $C^*/\mu^*$ distinctly different from one. This highlights the difficulty of guessing the net effect of grain shape on sediment transport based on qualitative inspection of grains. Error bars show one standard error of the mean.
Extended Data Fig. 4 | Competing effects of grain shape on bed load sediment transport. Same as Fig. 1, but including the six granular materials in Extended Data Fig. 3. a, Comparison of bulk coefficient of static friction with a measure of grain circularity, $S_c = 4\pi A/P^2$, where $A$ is the projected grain area and $P$ is the projected perimeter (values closer to 1 indicate more-circular grains), for a compilation of observations 47–49 and the materials measured here. b, Comparison of the still-water-settling drag coefficient, $C_{Dsettle}$, normalized by the drag coefficient for a sphere of the same volume (Methods) with another measure of grain shape, the Corey shape factor, $S_f = \frac{c}{\sqrt{ab}}$, where $a$, $b$, and $c$ are the long, intermediate, and short axes of a grain (values closer to 1 indicate more-spherical grains), for a compilation of observations 32 and the materials measured here. The coefficients of both friction and drag decrease with increasingly spherical grains. Error bars show one standard error of the mean.
Extended Data Fig. 5 | Schematic diagram of laboratory flume. Measurements of bed and water surface slope were made in the middle 2.5 m of the flume, where there were no visible entry or exit effects on grain motion. The flume is inclined 3°, but the sediment bed can develop a slope that is either steeper or less steep than the flume.
Extended Data Fig. 6 | Shape distributions of granular materials with variable grain shapes. a–c, Histograms of the three axes (a, b, and c) used to characterize grain shape. d–f, Corresponding histograms of the Corey shape factor. n is the sample size for each grain type.
Extended Data Fig. 7 | Distributions of settling velocities for the grain types used in flume experiments. $n$ is the sample size for each grain type.
Extended Data Fig. 8 | Measurement of the angle of repose of experimental materials. a, Spheres. b, Faceted ellipsoids. c, Rounded chips. d, Painted natural gravel. Painted gravel was used in the experiments to aid automated grain identification. e, Rectangular prisms. Blue and red lines are the right and left edges of the pile silhouette extracted with image analysis. Yellow lines are least-squares fits to these edges used to estimate the angle of repose. Vertical red line at the centre of each image is a plumb line used to determine the direction of gravity.
Extended Data Fig. 9 | Comparison of boundary shear stress estimates from different methods. For a subset of the flume experiments with spheres, flow velocity was measured using laser particle image velocimetry (PIV). a, Profiles of fluid velocity in the downstream direction as a function of distance above the grain bed (blue dotted lines), offset on the x-axis for visual clarity, are fit with the law of the wall (black lines), $u = (u^*\kappa)\ln(30z/d_0)$, where $\kappa = 0.4$ is the von Karman constant, $d_0$ is the grain diameter, and $u^* = \sqrt{\tau/\rho}$ is the shear velocity, which yields an estimate of the shear stress. The law of the wall is fit to the part of each profile between 20% and 80% of the maximum velocity (solid blue lines). b, Plot of the nondimensional bed shear stress estimated from $\tau = \rho g R S$ against the nondimensional shear stress estimated from the Law of the Wall (blue points) and the shear stress calculated by applying a wall correction factor to the original estimates of $\tau = \rho g R S$ (green points) for flume experiments with glass spheres. c, Same as b, but for the flume experiments with natural gravel, and without PIV-derived shear stress. Dashed lines are least-squares fits. The wall-corrected shear stress estimates for spheres and natural gravel are within error of each other and of the PIV-derived estimates. The average wall correction factor for the two grain types is $(2.7 + 2.1)/2 = 2.41$. Error bars show best estimate of uncertainty in shear stress estimates. For PIV-derived estimates this is the uncertainty of the log-linear fits in a; for the other estimates, it is the propagated standard error of the mean.
### Extended Data Table 1 | Grain properties

| Granular materials used in flume experiments | Additional granular materials |
|-------------------------------------------|-------------------------------|
| Spheres | Faceted ellipsoids | Rounded chips | Natural gravel | Rectangular prisms | Tempered glass chips | Shell fragments | Natural gravel 2 | Natural gravel 3 | Natural gravel 4 | Natural gravel 5 |
| **Фigure** | 2a | 2b | 2c | 2d | 2e | ED 3a | ED 3b | ED 3c | ED 3d | ED 3e | ED 3f |
| _ρ _ (kg/m³) | 2558 ± 128 | 2412 ± 148 | 2349 ± 59 | 2471 ± 122 | 2392 ± 280 | 2487 ± 2 | 2792 ± 3 | 2293 ± 2 | 2753 ± 3 | 2513 ± 3 | 2434 ± 3 |
| _a_ (mm) | 4.9 ± 0.05 | 6.0 ± 0.18 | 8.7 ± 0.40 | 5.8 ± 0.03 | 4.4 ± 0.24 | 8.5 ± 0.18 | 10.5 ± 0.18 | 8.2 ± 0.14 | 9.7 ± 0.21 | 8.7 ± 0.12 | 8.2 ± 0.18 |
| _b_ (mm) | 4.9 ± 0.05 | 6.0 ± 0.18 | 5.7 ± 0.19 | 4.4 ± 0.01 | 3.4 ± 0.09 | 6.1 ± 0.04 | 7.7 ± 0.14 | 6.3 ± 0.07 | 6.8 ± 0.08 | 6.4 ± 0.09 | 5.9 ± 0.08 |
| _c_ (mm) | 4.9 ± 0.05 | 5.0 ± 0.10 | 3.6 ± 0.18 | 3.3 ± 0.01 | 3.4 ± 0.06 | 5.1 ± 0.09 | 2.4 ± 0.11 | 4.8 ± 0.06 | 3.8 ± 0.07 | 4.1 ± 0.09 | 3.5 ± 0.10 |
| _d_ (mm) | 4.9 ± 0.08 | 5.0 ± 0.11 | 5.7 ± 0.48 | 4.1 ± 0.41 | 3.9 ± 0.18 | 6.4 ± 0.11 | 5.4 ± 0.15 | 5.8 ± 0.14 | 5.2 ± 0.17 | 5.5 ± 0.15 | 5.2 ± 0.16 |
| _w_ (mm/s) | 386 ± 2.3 | 360 ± 4.8 | 286 ± 7.4 | 285 ± 8.4 | 263 ± 3.8 | 299 ± 4.2 | 218 ± 5.5 | 322 ± 4.3 | 278 ± 4.4 | 380 ± 4.2 | 262 ± 5.8 |
| _w_ (mm/s) | 499 | 490 | 500 | 436 | 413 | 556 | 560 | 490 | 539 | 517 | 490 |
| _Re_ | 4900 | 6000 | 8733 | 5775 | 4400 | 8534 | 10546 | 8243 | 9670 | 8726 | 8194 |
| _C_0 | 0.43 | 0.61 | 0.64 | 0.67 | 0.93 | 1.01 | 0.72 | 0.74 | 0.61 | 0.72 | 0.74 |
| _M_ | 1.00 ± 0.01 | 0.83 ± 0.02 | 0.51 ± 0.02 | 0.68 ± 0.00 | 0.88 ± 0.02 | 0.71 ± 0.02 | 0.27 ± 0.01 | 0.66 ± 0.01 | 0.50 ± 0.01 | 0.54 ± 0.01 | 0.51 ± 0.02 |
| _C_ | 0.43 | 0.50 | 0.33 | 0.45 | 0.41 | 0.81 | 0.72 | 0.19 | 0.42 | 0.38 | 0.36 |
| _C_ | 0.41 | 0.41 | 0.41 | 0.42 | 0.43 | 0.40 | 0.41 | 0.41 | 0.41 | 0.41 | 0.41 |
| _G_ | 0.14 | 0.09 | 0.08 | 0.16 | 0.07 | 0.84 | 0.08 | 1.04 | 0.04 | 1.52 | 0.06 |
| _G_ | 1.02 ± 0.02 | 1.05 ± 0.08 | 1.01 ± 0.08 | 0.84 ± 0.08 | 1.04 ± 0.04 | 1.52 ± 0.06 | 1.09 ± 0.09 | 1.03 ± 0.04 | 1.02 ± 0.06 | 0.87 ± 0.05 | 1.03 ± 0.07 |

Measured grain density _ρ_ measured grain dimensions _a_, _b_, and _c_, estimated volume equivalent sphere diameter _d_, measured mean settling velocity _w_, calculated settling velocity of the volume equivalent sphere _D_ = S_f/Dsettle, the particle Reynolds number _Re_ = 4Dv/7dν associated with the settling experiments (where _D_ is the mean grain diameter and _ν_ is the kinematic viscosity of water), the mean coefficient of drag _C_0, the calculated settling velocity _D_ settle, the calculated drag coefficient of the volume equivalent sphere _C_0, the normalized drag coefficient _C*_, the mean measured angle of repose of the granular material _φ_ and the associated coefficient of static friction _μ_*, the normalized friction coefficient _C_0, and the ratio of the two, _C*_ / _μ_* All uncertainties are one standard error of the mean.