A major challenge in the domain of hadronic physics is the understanding of states not having the standard $qq$ and $qqq$ structures existing in the traditional Constituent Quark Model (CQM). The prediction of an antidecuplet of exotic particles (pentaquarks) within the framework of the Chiral Soliton Model ($\chi$SM) [1] spawned major experimental efforts worldwide. Recently, this interest in pentaquarks has been renewed with the claim of a charmed pentaquark by the LHCb Collaboration [2]. It is an open question whether this newly-discovered state may have its partners at lower masses.

Beginning with the first pentaquark announcement, from LEPS collaboration [3], there were many reports confirming the observation of the lightest member of the proposed antidecuplet, the $\Theta^+(1538)$ baryon [4]. In 2004, the Particle Data Group quoted it as an established $3^-$ star particle [5]. Somewhat later, however, most of these results were announced to be statistical fluctuations [4]. Nonetheless, three groups LEPS [6], DIANA [7], and SVD-2 [8], still insist on this finding. In 2012, a part of the CLAS Collaboration reported a new high-statistics signal which could be associated with the $\Theta^+$ [9]. More recently, however, an experiment at J-PARC has found no evidence for this particle [10].

In 2004, a modified SAID PWA of $\pi N$ scattering data allowed for two $P_{11}$ candidates for the second member of the antidecuplet, the non-charmed pentaquark, with masses near 1.68 and 1.73 GeV [11]. To be compatible with the data existing at that time, these candidate states were required to be very narrow and have a small branching to $\pi N$. In this context, the observation of a narrow enhancement at $W \sim 1.68$ GeV in $\eta$ photoproduction on the neutron (the so-called ”neutron anomaly”) appeared to be an important piece of the puzzle. The effect was first observed at GRAAL [12] and then confirmed by the LNS [13], CBELSA/TAPS [14, 15] and A2@MAMI [16] Collaborations. This structure was not seen in the previous measurements of $\eta$ photoproduction on the proton [17]. Recent precise measurements of the cross section for this reaction, at A2@MAMI-C, have revealed a narrow dip at this same energy [18]. A narrow resonance-like structure at $W \sim 1.685$ GeV was also observed in the $\gamma p \to \eta p$ beam asymmetry data from GRAAL [19]. A narrow peak at this energy was found in Compton scattering on the neutron $\gamma n \to \gamma n$ while neither peak was seen in the $\gamma n \to \pi^0 n$ cross section [21].

This whole assembly of experimental findings has generated a number of explanations. In line with the pentaquark hypothesis, these may signal a nucleon resonance with unusual properties: a mass $M \sim 1.68$ GeV, a narrow ($\Gamma \lesssim 25$ MeV) width, a strong photo-excitation on the neutron, and a suppressed decay to $\pi N$ final state [11, 22-27]. The properties of this putative resonance coincide surprisingly well with those expected for the second member of the antidecuplet, the non-strange $P_{11}$ pentaquark [26, 27]. However, contradictory explanations also exist, with several groups explaining the bump in the $\gamma n \to \eta n$ cross section in terms of i) the interference of well-known and broader resonances [28] or ii) the sub-threshold $K\Lambda$ and $K\Sigma$ production (cusp effect) [29]. Therefore it is of interest to reexamine this problem using elastic $\pi N$ scattering data.

Much of our knowledge of the baryon resonances was obtained by through the analysis of $\pi N$ scattering. In general, theory predicts only weak couplings of pentaquark states to the elastic $\pi N$ channel. Therefore, experimental data should be of very high precision. On the other hand the analysis of such data would have some ad-
vantages: i) the structure of $\pi N$ amplitude is essentially simpler than that of photoproduction; ii) the $\pi N$ partial waves are quite well known from phase shift analysis; iii) there is isospin symmetry in the $\pi N$ system.

In the years from 2005 to 2013, the EPECUR Collaboration measured $\pi^+ p \rightarrow \pi^+ p$ elastic scattering over an energy range of $p_{lab} = 800 - 1300$ MeV/c and for angles $\theta_{cm}$ from 40 to 120 degrees [33]. In total, about 10000 new data points have been obtained. These data have been produced with a momentum resolution of $\sim 1$ MeV and with $\sim 1\%$ statistical errors.

The $\pi^- p \rightarrow \pi^- p$ data revealed two narrow structures, at $W \sim 1.686$ and at $W \sim 1.720$ GeV, which were not seen in $\pi^+ p$ scattering [30]. This clearly shows that the observed structures appear in the isospin $I=1/2$ sector only. It is interesting to note that a structure at $W \sim 1.720$ GeV was also recently found in Compton scattering off the proton [31] and $\eta$-photoproduction off the neutron [32].

In Ref. [30], a preliminary analysis of the data from Ref. [33] was presented, with the finding that these structures could be described by two narrow (width $\sim 25$ MeV) $S_{11}$ and $P_{11}$ resonances. In this paper, an analysis of the full EPECUR database [33] is presented. Here we attempt to explain observed structures in terms of both couplings to inelastic channels and resonance contributions. For that purpose, we employ a K-matrix approach based on the effective Lagrangians described in Refs. [24, 35], and applied to both $\pi N$ scattering and photoproduction in Ref. [36].

It is assumed that the K-matrix, as a solution to equations yielding the scattering amplitude, can be described in terms of a sum of the tree-level Feynman diagrams with vertices obtained from an effective Lagrangian. The model includes four-star PDG [57] resonances in the $s$- and $u$-channels and $\sigma, \rho, a_0$ and $K^*$ exchange in the $t$ channel. To describe the high energy tail in $\pi^+ p$ data, the three-star $P33(1900)$ resonance was also included. Two new isospin-1/2 resonances were added, as well, to reproduce observed structures in the $\pi^- p$ data, as we describe below. In total, the 5-channel analysis took into account elastic, $2\pi$ (effective), $\eta n$, $K\Lambda$, and $K\Sigma$ production.

As the main goal of this work was to explore the nature of narrow structures in $\pi^- p$ elastic scattering, a detailed description of inelastic channels was not attempted. This reduced the number of free parameters, resonance masses and couplings, used in the fits. The employed database included the EPECUR data, the total cross-sections for $\pi^- p \rightarrow \eta n$ [38], and data for the differential cross sections of $\pi^- p \rightarrow K\Lambda$ and $\pi^- p \rightarrow KS$ [39]. To achieve the consistency with the data on elastic $\pi N$ scattering, at the energies below the EPECUR data, single-energy solutions from the $XP15$ [33] partial wave analysis (PWA) were added to the data base.

The $XP15$ solution was the result of a SAID PWA analysis which included the EPECUR data. This solution provided a rather good description of the whole data set getting a $\chi^2 \sim 3$ per point. However, a description of the abovementioned sharp structures was absent. This is clear from Figs. 2 in which the dotted lines correspond to the $XP15$ solution. The results of our calculations without any narrow resonances are shown in this figure by the solid lines. It should be noted that the $XP15$ parameterization included the inelastic channels $\pi\Delta, \rho N,$ and $\eta N,$ but no $K\Lambda$ or $K\Sigma$ channels.
dependence seen in the $\pi^- p$ differential cross section close to the $\pi^- p \to K\Sigma$ threshold at the angles $\sim 90$ degrees. Such an effect is not seen in the $\pi^+ p$ data.

A qualitative explanation of this phenomenon is evident from Fig. 4 in which the energy dependence of the total $\pi p \to K\Sigma$ cross section for different charged states is shown. One can see that the $\pi^- p \to K^0\Sigma^0$ and $\pi^- p \to K^+\Sigma^-$ plots vary rapidly near the threshold $W \sim 1690$ GeV, while the energy dependence of the $\pi^+ p \to K^+\Sigma^+$ reaction is more smooth, and therefore does not generate sharp structures in the $\pi^+ p$ scattering data. Our results for the $\pi^- p \to K\Sigma$ total cross section are shown in Fig. 3 by solid lines. The present calculation reproduces these data quite well.

As a next step, two resonances were added in an attempt to improve the fit quality around 90$^{\circ}$. Here, the overall $\chi^2$ per datum is not a good parameter to estimate the quality of the fit, as the structure is evident in only $\sim 200$ data points among 5000 in total. Thus, the overall $\chi^2$ would be overwhelmed by the quality of fit to the background behavior. To compare the different fits with additional resonances, $\chi^2$ in the restricted energy interval of $p_{lab} = 980 - 1140$ GeV/c was calculated. While different quantum numbers for the added resonances were tested, only S11 for the first and P11 for the second gave a reasonable $\chi^2$. The inclusion of these resonances lead to a significant improvement of $\chi^2 \sim 1.5$ as compared with $\chi^2 \sim 2.6$ for the background. The results are shown in Fig. 4 and Table 1.

![FIG. 3: Total cross section for $\pi p \to K\Sigma$. The data are from [39]. Solid lines are from the present work.](image)

Both resonances have the small widths and the small couplings to the elastic $\pi N$ channel. This is in agreement with the predicted properties of the non-strange pentaquark state, the second member of the antidecuplet.

Concerning the narrow resonance contributions, narrow structures are also seen in Compton scattering [20].

![FIG. 4: $\pi^- p$ elastic scattering with added resonances. Solid line gives the present calculation.](image)

### TABLE I: Resonance parameters.

|       | S11 (MeV) | P11 (MeV) |
|-------|-----------|-----------|
| Mass  | 1686      | 1724      |
| $T_{el}$ | 5.0       | 8.5       |
| $T_{\eta n}$ | 2.3       | 19.8      |
| $T_{2\pi}$ | 0.3       | 7.1       |
| $T_{KK}$ | 10.0      | 4.8       |
| $T_{K\Sigma}$ | 4.0       |
| $T_{tot}$ | 17.6      | 44.2      |

Having concentrated on structure in $\pi N$ scattering, it is important to see how the added resonances would appear in inelastic channels. In Fig. 5 the total cross section for $\pi^- p \to \eta n$ is presented.

One may see that the data are not in conflict with resonance contributions but also do not prove their existence. The dotted line in Fig. 5 gives the S-wave contribution to $\pi^- p \to \eta n$. As was shown in Ref. [25], a minimum of the S-wave contribution near $p_{lab} \sim 1050$ MeV/c could be explained through interference of the $S_{11}(1535)$ and $S_{11}(1650)$ resonances. Different signs for the coupling constants of the $\eta$ meson with these resonances was found, in agreement with Refs. [28]. But opposite to these works, the interference does not produce any sharp peak in the $\pi^- p \to \eta n$ reaction. Moreover, a very small (1%) branching ratio of the $S_{11}(1650)$ resonance to $\eta n$ is found in the present work.

Figs. 6 and 7 show the results for $\pi^- p \to K\Lambda$ differential cross sections and their energy dependence for $\cos(\theta_{cm})=0.65$. Again, the existing large experimental errors do not make a definite conclusion possible regarding the existence of the added narrow resonances.

We conclude that two narrow structures observed in elastic $\pi^- p$ scattering can be explained by a combination of threshold effects and two narrow resonances S11(1686) and P11(1720). These contributions we discuss separately below.
FIG. 5: Comparison of the measured and calculated $\pi^- p \rightarrow \eta n$ total cross sections. The solid line presents the calculations from this work. The dotted line indicates the S-wave contribution.

FIG. 6: $\pi^- p \rightarrow K\Lambda$ differential cross section. Solid line - present calculations.

and $\eta$ photoproduction off the neutron. What is the nature of these structures? The interference of well-known resonances suggests a delicate relation between incoming and outgoing vertexes. It is unlikely that this relation is valid for all three reactions, namely $\pi^- p$ scattering, Compton scattering and $\eta$ photoproduction. Further work is required before a definitive conclusion can be drawn concerning this possibility.

Another contribution is available via the cusp effect, i.e. the influence closed channels on incoming amplitude due to the analyticity condition. This element requires further development and remains a hypothesis which requires further detailed verification. It is worth noting that not all threshold effects result in sizable cusp effects. For instance, no clear structure is seen in $\pi^- p$ elastic scattering near the $K\Lambda$ threshold ($P_{lab}=900$ MeV/c).

Finally, in the energy region around 1686 MeV, possible electromagnetic effects must be taken into consideration. Indeed, just below the $K\Sigma$ threshold, a bound atomic-like state of $K^+\Sigma^-$ could be created. If it exists, this state could in fact be seen in the $\pi^- p$ and $\gamma n$ reactions only. The existence of these electromagnetic effects could be checked, for example, by measuring the cross-sections for two isospin-symmetric reactions $\pi^- p \rightarrow \eta n$ and $\pi^+ n \rightarrow \eta p$. Accordingly, the isospin symmetry cross-sections of reactions should be the same but the $K^+\Sigma^-$ system would exist for the first reaction only. No such effect exists for a narrow $P_{11}(1724)$, and we consider this resonance, which has the nucleon quantum numbers to be the best candidate for the non-strange member of an exotic antidecuplet. New high precision experimental data on $\pi^- p \rightarrow K\Lambda$ and $\pi^- p \rightarrow \eta n$ are needed to achieve a decisive conclusion.

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