Alternative signatures of the quintuplet fermions at the LHC and future linear colliders

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Abstract

Large fermionic multiplets appear in different extensions of the Standard Model (SM), which are essential to predict small neutrino masses, relic abundance of the dark matter (DM) and the measured value of muon anomalous magnetic moment (muon (g-2)). Models containing quintuplet of fermions (Σ), along with other scalar multiplets, can address recent anomalies in the flavor sector while satisfying the constraints from the electroweak physics. In standard scenarios, the exotic fermions couple with the SM particles directly and there exists a strong limit on their masses from collider experiments such as the Large Hadron Collider (LHC). In this paper, we choose a particular scenario where the quintuplet fermions are heavier than the scalars, which is naturally motivated from the muon (g-2) data. A unique nature of these models is that they predict non-standard signatures at the colliders as the quintuplet fermions decay via the scalars once produced at the colliders. We study these non-standard interactions and provide alternative search strategies for these exotic fermions at the LHC and future linear colliders (such as $e^+e^-$ colliders). We also discuss their exclusion and discovery limits. For the doubly charged quintuplet fermion ($\Sigma^{\pm\pm}$), discovery is possible with 5\(\sigma\) significance at integrated luminosity of 3000 fb\(^{-1}\) at 14 TeV LHC if \(M_{\Sigma} \leq 980\) GeV. For the singly charged quintuplet fermion ($\Sigma^{\pm}$), the discovery is challenging at the LHC but there might be a possibility of 5 \(\sigma\) discovery with 1000 fb\(^{-1}\) luminosity at $e^+e^-$ collider for $M_{\Sigma} \leq 700$ GeV.

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1 Introduction

The Standard Model of particle physics has been observed with great precision at the experiments. The last missing piece, the Higgs boson, was discovered by ATLAS [1] and CMS [2] at the Large Hadron Collider (LHC) in 2012. However, it has some shortcomings – it is not possible to account for the small neutrino masses or the existence of dark matter in the SM, for instance. It is also difficult to explain the current measurement of muon (g-2) [3] and the outcomes of flavor experiments [4] within the SM framework.

The small neutrino masses can be achieved by introducing exotic particles at a high scale via the effective dimension-five Weinberg operator at tree level vis-a-vis the seesaw mechanism. These extra particles correspond to a heavy fermion singlet, a scalar triplet and a fermion triplet in type I, II and III seesaw mechanisms, respectively [5–12]. However, there are other models, where the exotic particle content involves one or more larger multiplets of scalars and fermions together. These models [13–22] not only explain the small neutrino masses but also provide an explanation for the muon (g-2) data, flavor anomalies while also predicting a suitable dark matter (DM) candidate in some cases. Such models also predict exotic signatures at the colliders.

A good example of such models are cascade seesaw-like scenarios [13–16], where the neutrino mass is generated via a higher dimensional \((5 + 4n)\) operator, where \(n = 1\) is the minimal scenario with three generations of Majorana quintuplets \(\Sigma_R\), with hypercharge \(Y = 0\) and a scalar quadruplet \(\Phi\), with hypercharge \(Y = -1\). Another example is the left-right symmetric (LRS) framework [17, 18] with an \(SU(2)_R\) quintuplet where the gauge group is extended to \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\). In these models, the presence of a right-handed neutrino in the particle spectrum is essentially governed by the gauge structure and hence, naturally explains the origin of light neutrino masses. Further, the neutral component of the quintuplet can be a DM candidate [19,20]. In models such as \(R\nu\)MDM [16,21], purely radiative neutrino masses are generated while also providing a viable DM candidate [22]. Models with both exotic fermions and charged scalars can also be motivated from the little Higgs scenarios [23], where the global symmetry is \(SU(6)/Sp(6)\). These models successfully explain the flavor anomalies [24] and the signatures can be studied at the colliders. Multicharged fermions also appear in models of warped extra dimensions and some other models as shown in Ref: [25–32].

In this paper, we are motivated by a model with one quartet and one septet scalar fields and quintuplet Majorana fermions (3 copies) [33]. The interactions between the SM \(SU(2)_L\) lepton doublets and these large multiplets induce neutrino masses which are suppressed by small VEV’s of the quartet and/or the septet and also by the inverse of the quintuplet fermion mass (\(\sim\) TeV). As a result, the scale of the neutrino Yukawa coupling can be reached to less than \(O(1)\) [33,34]. Further, this type of model is safe from any quantum anomaly, given the zero \(U(1)_Y\) charge of these quintuplet fermions. There is no contribution to \(SU(2)\) gauge anomaly as well.

The contents of the quintuplet are doubly charged fermions, singly charged fermions and a neutral fermion [33]. Signatures of quintuplet fermions at LHC have been studied in Ref: [35, 36]. These fermions are also good candidates for exotic particle search in the future collider experiments [37, 38]. In most of the phenomenological studies involving the quintuplet fermions, they decay directly into the SM particles. Even in scenarios as in Ref: [16], where interactions between the quintuplet fermions and the scalar multiplets are allowed, the fermions can not decay into the scalars as the scalars are slightly heavier than the fermions. However, in our scenario [34], a small mass difference between the
scalar multiplets and the fermionic quintuplet is naturally implied from muon \((g - 2)\) and in such a way that the quintuplet fermions are heavier than the scalars.

In this paper, we study the signatures of the singly and the doubly charged quintuplet fermions \((\Sigma_R)\) at the hadron collider \((pp\) collider) and linear colliders such as \(e^+e^-\) colliders, with each having its own advantages. Although the LHC has a much higher energy reach, the \(e^+e^-\) colliders provide a cleaner environment for distinguishing the signal from the background \([39]\) and are more suitable for precision measurements \([40]\). With many \(e^+e^-\) colliders, such as FCC-ee, ILC and CLIC in development stage, it would be interesting to study the discovery potential of the exotic quintuplet fermions at these colliders \(^3\).

Cases studied so far include multilepton and multijet signatures of the quintuplet fermions, both at the LHC \([16, 35]\) and the linear colliders \([37, 38, 41]\). As already stated, the quintuplet fermions in the scenario considered here decay into the SM particles via the scalars which leads to final states containing a large number of leptons and jets. Such signals for quintuplet fermions have not been studied before. Even though it is difficult to reconstruct the masses of the fermion quintuplets or the scalars, given these many particle final states, we show that by carefully choosing the final states from amongst the many possibilities, it is indeed possible to reconstruct both the masses. The masses of the exotic fermions, such as vectorlike quarks and leptons, are constrained to be more than \(\sim 1\) TeV \([42, 43]\) and \(\geq 740\) GeV \([44]\) respectively. However, for this model, we choose to explore masses much below 1 TeV as well as masses larger than 1 TeV, considering the nonstandard decay modes of the quintuplet fermions. For the singly charged scalar, the direct search limit from LEP is 80 GeV \([45]\). However, the limit on the production cross-section of the singly charged scalar as a function of its mass is given in Ref: \([46]\). Multilepton states where the doubly charged scalar has decayed into two same sign leptons are already studied in Ref: \([30, 47, 48]\). The lower mass limit ranges from about 230-870 GeV. However, these studies are based on several assumptions which do not apply to our case. In the analysis done here, the mass of the scalars is chosen to be well below the mass of the quintuplet fermions to facilitate the decay of the quintuplet fermion via the scalars.

The rest of the paper is arranged as follows: The model is described in section 2. Collider signatures at the LHC and linear collider are described in sections 3 and 4, respectively. We discuss the results and conclude in section 5.

### 2 Model

We consider a simple scenario based on the models proposed in Ref: \([33, 34]\). In the model of Ref: \([33]\), the interaction among the SM \(SU(2)_L\) lepton doublets and scalar multiplets, quartet \((\phi_4, Y = 1/2)\) and septet \((\phi_7, Y = 1)\), can induce neutrino masses, while preserving the \(\rho\) parameter. In order to achieve that it is also necessary to introduce a quintuplet Majorana fermion \(\Sigma_R\), with \(Y = 0\). The neutrino masses are suppressed by the small VEVs of the quartet or septet and an inverse of the quintuplet fermion mass, which explains the smallness of the neutrino mass while also relaxing the Yukawa hierarchies. In order to make the generation of neutrino mass more natural, additional quintet scalar field \((\phi_5, Y = 0)\) with a non-zero VEV can be introduced \([34]\). As a result, tree level neutrino mass is forbidden and the quartet scalar is an inert scalar while neutrino mass is generated at one-loop level. The model is further constrained from lepton flavor violation (LFV) and the muon anomalous magnetic moment \((\Delta a_{\mu})\).

\(^3\)This model also offers interesting signatures for the charged scalars which we do not address here.
In these models, the interesting point to note is that the quintuplet fermion $\Sigma_R$ does not decay to standard model particles directly. Instead, it happens via its interaction with the charged scalars. Our objective is to study this particular scenario. Hence, we choose a simplistic scenario with quintuplet fermion, $\Sigma_R$ ($Y = 0$) and the quartet scalar, $\phi_4$ ($Y = 1/2$), along with their components.

$$\Phi_4 = (\varphi^{++}, \varphi^+_2, \varphi^0, \varphi^-_1)^T,$$

$$\Sigma_R = [\Sigma_1^{++}, \Sigma_1^+, \Sigma_0^0, \Sigma_2^+, \Sigma_2^{++}]_R^T,$$  \hspace{1cm} (1)

($\Sigma_1^{\pm}$, $\Sigma_1^{\pm\pm}$) and ($\Sigma_2^{\pm}$, $\Sigma_2^{\pm\pm}$) are combined to make singly and doubly charged Dirac fermions, which we denote as $\Sigma^\pm$ and $\Sigma^{\pm\pm}$ respectively while $\Sigma_0^0$ remain a neutral Majorana fermion. The masses of each component are given by $M_{\Sigma}$ at the tree level where mixing between the SM leptons is negligibly small. The Yukawa interaction can be written as,

$$-\mathcal{L}_Y = (y_{\nu_L})_{ij} \bar{L}_{L_i} H e_{R_j} + (y_{\nu_R})_{ij} [\bar{L}_{L_i} \Phi_4 \Sigma_{R_j}] + (M_{R_i})_{ij} [\bar{\Sigma}^c_i \Sigma_{R_j}] + \text{h.c.},$$ \hspace{1cm} (2)

The components of the quintuplet fermion can decay into quartet scalars and SM leptons via the Yukawa interaction given by the second term of the above equation as,

$$-\mathcal{L}_{yuk} \supset (y_{\nu_L})_{ij} [\bar{L}_{L_i} \Phi_4 \Sigma_{R_j}] + \text{h.c.}$$

$$= (y_{\nu_L})_{ij} \left[ \bar{\nu}_{L_i} \left( \frac{1}{\sqrt{2}} \Sigma_0^{R_j} \varphi^0_1 + \frac{\sqrt{3}}{2} \Sigma_1^{R_j} \varphi^-_1 + \frac{1}{2} \Sigma_2^{R_j} \varphi^-_2 + \Sigma_2^{++ \pm R_j} \varphi^{--} \right) + \bar{\ell}_{L_i} \left( \frac{1}{\sqrt{2}} \Sigma_0^{R_j} \varphi^-_1 + \frac{1}{2} \Sigma_1^{R_j} \varphi^{--} + \frac{\sqrt{3}}{2} \Sigma_2^{R_j} \varphi^{0s} + \Sigma_2^{+- R_j} \varphi^{+s} \right) \right] + \text{h.c.}$$ \hspace{1cm} (3)

The coupling $y_{\nu_L}$, as given in Eq. 3, can be constrained from observables such as $\nu$-mass, $\Delta a_\mu$ and flavor observables. Following the benchmark points in [33,34], this coupling varies between (0.001–1), depending on the scalar particle content of the model. As we are studying a more general scenario, for the phenomenological purpose, we choose a very conservative limit of 0.1 for $y_{\nu_L}$.

The components of the quintuplet can be produced via gauge interactions given by,

$$\Sigma_R^\gamma \mu i D_\mu \Sigma_R \supset \Sigma^{++} \gamma \mu (2 e A_\mu + 2 g_{cW} Z_\mu) \Sigma^{++} + \Sigma^+ \gamma \mu (c A_\mu + g_{cW} Z_\mu) \Sigma^+$$

$$- \sqrt{2} g \Sigma^0 \gamma \mu W^+ \Sigma^+ - \sqrt{3} g \Sigma^+ \gamma \mu W^+ \Sigma^0 - \frac{\sqrt{5} g}{\sqrt{2}} \Sigma^+ \gamma \mu W^0 \Sigma^+$$

$$- \sqrt{2} g \Sigma^0 \gamma \mu W^- \Sigma^+ - \sqrt{3} g \Sigma^+ \gamma \mu W^- \Sigma^0 - \frac{\sqrt{5} g}{\sqrt{2}} \Sigma^0 \gamma \mu W^- \Sigma^+, \hspace{1cm} (4)$$

where $s_W(c_W) = \sin \theta_W(\cos \theta_W)$ with the Weinberg angle $\theta_W$.

The relevant gauge interactions associated with $\phi_4$ can be obtained from the following kinetic term

$$|D_\mu \Phi_4|^2 \supset \frac{\sqrt{3}}{2} v_4 W^\pm W^\pm \varphi^{++} + \frac{g^2 v_4}{c_W} \left[ \frac{2}{s_W} \frac{Z_\mu W^{+\mu} \varphi^-_2 + \sqrt{3} (2 - s_W^2) Z_\mu W^{+\mu} \varphi^-_1 + \text{c.c.}}{2} \right].$$ \hspace{1cm} (5)

\footnote{Even in the models with more than one scalar multiplet, the components mix during mass diagonalization and charged and neutral scalar mass eigenstates are obtained.}

\footnote{The components of $\phi_4$ can also be produced at the collider via the gauge interaction, as shown in Ref. [33].}
Figure 1: Feynman diagrams for the production of doubly-charged fermions ($\Sigma^{\pm\pm}$) at $pp$-collider.

Here, we have neglected the mixings among the individual components of the multiplets and we choose the components of the quintuplet fermion as well as scalar multiplets to be degenerate, which we denote as $M_\Sigma$ and $M_\phi$ respectively [33]. We also consider $M_\Sigma > M_\phi$, which is naturally implied form the muon anomalous magnetic moment measurement, as shown in [34]. For the complete Lagrangian we refer to Ref: [33].

Further, considering the interactions in Eq. 3, $\Sigma^\pm$ and $\Sigma^{\pm\pm}$ can decay via the following modes,

\[
\begin{align*}
\Sigma^+ &\rightarrow \phi_2^+ \nu(\bar{\nu}) \\
\Sigma^± &\rightarrow \phi^± l^± \\
\Sigma^± &\rightarrow \phi^0 l^± \\
\Sigma^{±±} &\rightarrow \phi^{±±} \nu(\bar{\nu}) \\
\Sigma^{±±} &\rightarrow \phi^{±±} l^±
\end{align*}
\] (6)

The branching ratios are assumed to be same in each decay mode of $\Sigma^\pm$ and $\Sigma^{±±}$. Interactions in Eq. 5 allow the decays of the charged scalars into SM gauge bosons $\nu$,$\bar{z}$,

\[
\begin{align*}
\phi_2^± (\phi_1^±) &\rightarrow W^± Z \\
\phi^{±±} &\rightarrow W^± W^± \\
\phi^0 &\rightarrow W^+ W^-
\end{align*}
\] (8)

In the following sections, we perform a collider study of the $\Sigma^\pm$ and $\Sigma^{±±}$ when they decay via the charged scalars \(^6\). Even though there are many possible production modes, [35], we study the pair production of $\Sigma^\pm$ and $\Sigma^{±±}$. This is because, it will be easier to reconstruct the masses of the quintuplets if they are produced in pairs. As can be seen from the decay modes, the final states will have a rich collection of leptons and jets. The phenomenology of these alternative signatures is what we study next.

3 Phenomenology at the $pp$ Collider

In this section, we discuss the collider physics of the quintuplet fermions at the LHC. They can be produced in $pp$ collisions through $s$ and $t$ channel processes via $Z/\gamma$ and $\Sigma^\pm$ (or $\Sigma^{±±}$), respectively. The cross-sections for the production of the singly charged fermions, $pp \rightarrow \Sigma^± \Sigma^-$, are smaller in comparison to those for the doubly charged fermions, $pp \rightarrow \Sigma^{±±} \Sigma^{±±}$, as shown in Ref: [33]. Moreover, we have included the photon-photon fusion process, hence the matrix element squared of pair productions are enhanced by a factor of $(Q)^4$, where $Q$ is the charge of the fermion. We found the signal to background

\(^6\) We denote $\phi_2^± = \phi^\pm$ in the following sections.
ratio to be small for the singly charged fermions. Hence, we choose to study the pair production of only doubly charged fermions at the LHC and we show the Feynman diagrams in Fig: 1. In Fig: 2, we have shown the cross-section for the pair-production, \( pp \rightarrow \Sigma^{++}\Sigma^{--} \), for different values of \( \sqrt{s} \) at LHC where \( p = q, \bar{q}, \gamma \). For comparison, cross-section for pair-production of the singly charged fermion at \( \sqrt{s} = 14 \) TeV is also shown by the dotted curve in blue.

The inclusion of the photon PDF increases the signal cross-section significantly. Moreover, inclusion of photon PDF is important for the consistency of the calculation as the other PDF’s are determined up to NNLO in QCD. We would like to note that, in view of the above, NNPDF [49,50], MRST [51] and CTEQ [52] already include photon PDF in their definitions. In order to compute the cross-sections and generate events at the LHC, we incorporate the model Lagrangian in FeynRules (v2.3.13) [53,54]. Using FeynRules, we generate the model file for MadGraph5_aMC@NLO (v2.2.1) [55]. For the cross-sections, we use the NNPDF23LO1 parton distributions [56] with the factorization and renormalization scales at the central \( m_T^2 \) scale after \( k_T \)-clustering of the event.

![Figure 2](image_url)

**Figure 2:** Signal cross-section for the process, \( pp \rightarrow \Sigma^{++}\Sigma^{--} \), where \( p = q, \bar{q}, \gamma \) as a function of \( M_\Sigma \) at different \( \sqrt{s} \) is shown by the solid lines. The dotted line represents the process, \( pp \rightarrow \Sigma^{+}\Sigma^{-} \) at 14 TeV.

### 3.1 Signal

Once pair-produced at the LHC, the decays of the doubly charged fermions produces the following states,

\[
\begin{align*}
\Sigma^{++} \rightarrow \phi^{++}\nu \rightarrow (W^+W^+\nu); & \quad \Sigma^{--} \rightarrow \phi^{-}\bar{\nu} \rightarrow (W^-W^-)\bar{\nu} \\
\Sigma^{++} \rightarrow \phi^{++}\nu \rightarrow (W^+W^+\nu); & \quad \Sigma^{--} \rightarrow \phi^{-}\bar{\nu} \rightarrow (W^-Z)\bar{l}^- \\
\Sigma^{++} \rightarrow \phi^{++}\bar{\nu} \rightarrow (W^+Z)\bar{l}^+; & \quad \Sigma^{--} \rightarrow \phi^{-}\bar{l}^- \rightarrow (W^-Z)l^- 
\end{align*}
\]

with conjugate processes included in each case. The branching ratio in each case is assumed to be the same, as discussed in the previous section. This gives rise to final states comprising of a number of leptons, jets, and missing energy, resulting in various multilepton, multijet and mixed states. After carefully analyzing each of them on the basis of performance over SM backgrounds and mass reconstruction of the doubly charged fermions, we decide upon two channels:

- **Channel I:** \( \geq 4\ell \) channel with two pairs of Same Sign (SS) leptons \((l^+l^+), (l^-l^-) + \text{MET} \), where both the pairs are oppositely charged,
• Channel II: $\geq 3\ell + 2$ jets channel with at least one pair of SS lepton ($l^+l^+$) + One isolated lepton ($l^\mp$) + MET,

Here, $\ell = e, \mu, \tau$. We also check the efficiency for the the channel $4\ell + 1/2$ jets, but the efficiency turns out to be less compared to channels I and II. For the $4\ell + 3/4$ jets channel, the signal and background cross-sections, both are significantly low. Even though it is possible to obtain a better significance in this channel, the number of signal events to be observed at 3000 fb$^{-1}$ integrated luminosity is less than 10. Hence, we do not pursue this channel. Also, in purely leptonic channel, the leptons in the final states are either coming from the decay of $\Sigma^{\pm \pm}$ or from $W/Z$. Hence, the transverse mass of the leptons can be reconstructed, but clear mass reconstruction of the quintuplet mass is difficult as there are different sources of MET. However, $\geq 4\ell$ channel with the additional criteria as in channel I, predicts a good $S/B$ ratio, and hence we study it. The requirement of at least 4 leptons as well as the two SS lepton pairs in channel I makes it much cleaner compared to other channels, even though we do not put any restriction on the number of jets. We do not impose any jet or b-jet veto in channel-I as this will result in lesser number of signal events. Also, we do not go beyond the 4-lepton requirement because the signal cross-section $\times$ BR falls off due to smaller branching ratio of $W$ and $Z$ into leptons. In channel II, we include both leptons and jets in the final states and we show that a clear mass reconstruction of $\phi^\pm$ or $\phi^\pm \pm^\pm$ and $\Sigma^{\pm \pm}$ is possible.

We choose five benchmark points (BP) in our study viz BP1: $(M_\phi = 200 \text{ TeV}, M_\Sigma = 300 \text{ TeV})$ BP2: $(M_\phi = 500 \text{ TeV}, M_\Sigma = 600 \text{ TeV})$, BP3:$(M_\phi = 700 \text{ TeV}, M_\Sigma = 800 \text{ TeV})$, BP3:$(M_\phi = 900 \text{ TeV}, M_\Sigma = 1000 \text{ TeV})$, BP4:$(M_\phi = 1000 \text{ TeV}, M_\Sigma = 1200 \text{ TeV})$. We do not study the signal for $M_\Sigma$ larger than 1.2 TeV due to two reasons. Firstly, the cross-section is small at a larger mass of $\Sigma$. Secondly, when $M_\Sigma > 1.2$ TeV, the decay products of $W$ and $Z$ bosons become colimated and the probability of observing them as a fatjet is larger and the analysis process will be very different. The fatjet scenario will also be ideal for 27 TeV com energy.

3.2 Backgrounds

The main backgrounds for channels I and II come from inclusive diboson production, $VV+$jets, where $V = W, Z$. There will also be contributions from triboson ($VVV+$jets), $HV+$jets, $t\bar{t}V+$jets. The contributions from the $t\bar{t}VV+$ jets and 4-top backgrounds are found to be negligible. For channel II, additionally, we get comparatively less contribution from $t\bar{t}+$ jets and $V+$ jets and for channel I, they are negligible. In [57] and the references therein, the cross-section of these channels has been discussed in detail.

3.3 Collider Analysis

As a potential signature, we prefer the same sign (SS) lepton pairs over the opposite sign (OS) leptons, due to the abundance of the former in the signal. This is because of the decay of the quintuplets via the charged scalars, as shown earlier. On the other hand, the SM backgrounds involving one or more than one $Z$, are more likely to involve OS pair of leptons. Hence the signatures involving the SS pair of leptons suffer from less SM background. The signal and background are optimised over a set of selections, which we list in Table 1.

(I) $\geq 4\ell$ channel with ($l^+l^+$) and ($l^\pm l^\mp$) pair+MET: In the multilepton searches performed for exotic particles (vector like leptons, charged scalars etc.), only one OS or SS pairs are identified in most of the cases, the only exception being the searches for multicharged scalars [47]. This is largely due

\footnote{As they have the same mass.}
to the standard decay modes of the BSM particles. Whereas, in channel I, we have the requirement for two SS lepton pair, with pairwise opposite charge. In channel I, the quintuplet fermions decay to $Z$ and/or $W$ via the charged scalars, and most of the final state leptons come from the decay of the $Z/W$ bosons. Hence, we do not apply any $Z/W$ veto. We arrange the leptons ($\ell_i$) in the descending order of $p_T$. In this channel, the SS leptons appear directly from the direct decay of $\Sigma^{\pm\pm}$ and from the decay of $W/Z$ decay in the same decay chain. We plot the lepton’s $p_T$ and the invariant mass distribution of the SS lepton pairs for 2 BP’s (BP2 and BP5) in Fig: 3. The solid line represents the invariant mass distribution if at least one SS pair is present, and the dotted line represents the same for the additional SS pair, where these two pairs have opposite charge. Based on the two body mass distribution, we have imposed the selection $M(\ell^\pm, \ell^\pm) > 100$GeV.

In Fig: 4, we plot the sum of lepton $p_T$ ($S_T(\ell)$) and missing transverse energy (MET) distributions for the signal and the total background. Note that, a substantial amount of MET is present in the signal, as well as in the SM background. Hence, we refrain from putting any cut on MET in order to get most of the signal events. Even though the signal has a very high $S_T(\ell)$ compared to the background, the set of selections optimise for $S_T(\ell) > 400$ GeV for the whole signal region under consideration. For example, if we focus on the region $M_2 > 1$ TeV only, $S_T(\ell) > 600$ GeV gives a much better $S/B$ ratio. But we choose to use only one value for the $S_T(\ell)$ selection for our BP points. Based on the plots of the kinematic variables, we optimize the selections at the given values in Table 1. We found that the cut on $S_T(\ell)$ is sufficient to suppress the background in channel I. Moreover, the leptons with heighest $p_T$ will have a large separation compared to the other leptons, as they are form the separate decay chains of the quintuplet in most of the cases. Thus we impose a selection on these leptons by requiring $\Delta R(\ell_0, \ell_1) > 1.5$. In Table 2, we summarize the effect of the selections in channel I with ($l^+l^+$) pair + MET. As there is no jet veto imposed, the majority of the backgrounds come from the diboson + jets events. Initially, this background cross section is comparably very high but after we impose the selection $S_2$, the background reduces further.

**Channel I**

| Selections | \( Channel \ I \) | \( Channel \ II \) |
|------------|----------------------|---------------------|
| S0         | \( N(\ell) \geq 4 \) | \( N(\ell) \geq 3 + N(j) \geq 2 \) |
| S1         | \( p_T(\ell) > 30 \text{ GeV} \) | \( p_T(\ell) > 30 \text{ GeV} \) |
|            | \( |\eta(\ell)| < 2.5 \) | \( |\eta(\ell)| < 2.5 \) |
|            | \( \Delta R_{\ell i/j} > 0.3 \) | \( \Delta R_{\ell i/j} > 0.3 \) |
| S2         | \( S_T(\ell) > 400 \text{ GeV} \) | \( S_T(\ell) > 200 \text{ GeV} \) |
|            | \( M(\ell^+, \ell^+), M(\ell^-, \ell^-) > 100 \text{ GeV} \) | \( M(\ell^\pm, \ell^\pm) > 100 \text{ GeV} \) |
|            | \( \Delta R(\ell_0, \ell_1) > 1.5 \) | \( \Delta R(\ell, \ell) > 1.5 \) |
| S3         | \( 60 \text{ GeV} < M(j, j) < 120 \text{ GeV} \) | \( (M_\phi - 100) < M(j, j, \ell) < (M_\phi + 100) \) |

Table 1: Selections S1, S2 and S3 for channels I and II.

**Channel with \( l^\pm l^\pm \) pair + \( l^\pm \) + \( \geq 2 \) jets channel:** In this channel, we require the presence of at least three leptons as well as two or more jets. In Fig: 5, we show the $p_T$ distributions of the jets, and also the sum of $p_T$ for the jets ($H_T(j)$). The $p_T$ distribution of the leptons are mostly the same as channel I but as the number of leptons in channel II is less than channel I, the $S_T(\ell)$ distribution peaks at a lower value compared to channel I. In channel II, we identify at least one SS
Figure 3: (Top) Distributions of $p_T(l)$ and (Bottom) same sign lepton pair invariant mass distributions for $M_\Sigma = 600$ GeV and 1200 GeV (BP2 and BP5) respectively in channel I at 14 TeV LHC. The solid and dashed lines correspond to the first and second pair of SS lepton pairs.

Figure 4: (left) Transverse missing energy (MET) and (right) sum of lepton $p_T$ ($S_T(\ell)$) distribution for $M_\Sigma = 600$ GeV and 1200 GeV, and total background (shadowed region) in channel I.
Table 2: (Left) The variation of the cross-section (fb) for each of the BPs, as the selections are imposed at 14 TeV LHC in channel I. (Right) The same is shown for the background.

| $M_{\Sigma}$ | $S_1$ (fb) | $S_2$ (fb) |
|-------------|------------|------------|
| BP1, 300 GeV | 2.335      | 1.093      |
| BP2, 600 GeV | 0.598      | 0.219      |
| BP3, 800 GeV | 0.196      | 0.093      |
| BP4, 1000 GeV | 0.063      | 0.032      |
| BP5, 1200 GeV | 0.024      | 0.013      |

| Major Backgrounds | $S_1$ (fb) | $S_2$(fb) |
|-------------------|------------|-----------|
| Di-Boson+jets     | 20.42      | 0.55      |
| $t\bar{t}V$       | 0.25       | 0.07      |
| Triboson          | 0.082      | 0.022     |
| $HV$+jets         | 0.048      | 0.021     |
| Total             | 20.80      | 0.66      |

Figure 5: (Top) Distributions of jet $p_T$ and (Bottom) sum of lepton $p_T$ ($S_T(\ell)$) (left) and sum of jet $p_T$ ($H_T(j)$) (right), for $M_{\Sigma} = 600$ GeV and 1200 GeV, respectively, for channel II at 14 TeV LHC. The shadowed region corresponds to the total background.

lepton pair in a manner as stated in channel I. The selections in channel II are summerized in Table 1. Additionally, we find that a cut on the minimum value of $H_T(j)$ is useful to minimise the background.

The main objectives in channel II are to construct the three and four body invariant mass distribution, $M(\ell\ell j)$ and $M(\ell\ell jj)$, for the reconstruction of $M_{\phi}$ and $M_{\Sigma}$, respectively. Note that, this is only possible when we consider the decay of the quintuplet via the singly charged scalar. Even though it is theoretically possible to reconstruct the mass of the quintuplet from $M(\ell jj)$ also, it is harder to select the exact jets for the distribution. Hence, we consider the case when $W$ and $Z$ decay through leptonic mode and hadronic mode respectively. At first, we select two SS leptons in such a way that they must come from the same decay chain. One lepton is coming from the decay of the quintuplet and the another is from the $W$. We demand $\Delta R(\ell\ell) > 1.5$ for these two leptons. Then we select two jets coming from the the decay of the $Z$ boson, by requiring $60 < M(jj) < 120$ GeV. The three body mass distribution $M(\ell\ell j)$ and the four body mass distribution $M(\ell\ell jj)$ reconstruct the masses of $\phi$ and $\Sigma^{\pm}$, which is shown in Fig: 6. We select the final events with $S_3$, where the events are required
Figure 6: (left) Three body invariant mass $M(\ell\ell jj)$ and (right) four body invariant mass $M(\ell\ell jj)$ for $M_\phi = 500, 700, 900 \text{ GeV}$ and $M_\Sigma = 600, 800, 1000 \text{ GeV}$, respectively, for channel II at 14 TeV LHC. The shadowed region corresponds to the total background.

To satisfy the three body invariant mass in the window of $M_\phi \pm 100 \text{ GeV}$. The signal and background cross-section after the cuts are shown in Tab 3. Clearly, the selection after $S_3$ gives a better signal to background ratio.

![Graphs showing 3 and 4 body invariant mass distributions with shaded background regions.]

| Major Backgrounds | $S_1$ | $S_2$ | $S_3$ (BP1) | $S_3$ (BP2) | $S_3$ (BP3) | $S_3$ (BP4) | $S_3$ (BP5) |
|-------------------|-------|-------|-------------|-------------|-------------|-------------|-------------|
| Di-Boson+jets     | 45.05 | 14.00 | 0.085       | $3 \times 10^{-4}$ | $6 \times 10^{-5}$ | $1 \times 10^{-5}$ |
| $tTV$             | 10.42 | 0.53  | 0.056       | $1 \times 10^{-3}$ | $2 \times 10^{-5}$ | $< 10^{-5}$ |
| Triboson          | 0.336 | 0.013 | 0.004       | $< 10^{-3}$ | $< 10^{-4}$ | $< 10^{-5}$ | $< 10^{-6}$ |
| HV+jets           | 1.2   | 0.012 | 0.003       | $< 10^{-3}$ | $< 10^{-4}$ | $< 10^{-5}$ | $< 10^{-6}$ |
| **Total**         | 61.06 | 14.57 | 0.148       | 0.0065      | 0.0005      | $7 \times 10^{-5}$ | $1 \times 10^{-5}$ |

Table 3: (Top) The variation of the signal cross-section (fb) for each of the BPs, as the selections are imposed at 14 TeV LHC. (Bottom) The same for the various backgrounds.

**Result** The significance for the discovery can be described as (see Ref: [58–60]),

$$Z_{dis} = \sqrt{2 \left( (s + b) \ln \left( \frac{(s + b) (b + \Delta_b^2)}{b^2 + (s + b) \Delta_b^2} \right) - \frac{b^2}{\Delta_b^2} \ln \left( 1 + \frac{\Delta_b^2 s}{b (b + \Delta_b^2)} \right) \right)^{1/2}}$$

(10)

Where $s$ and $b$ are number of signal and background events respectively, and $\Delta_b$ is the uncertainty in the measurement of the background. If $\Delta_b = 0$,

$$Z_{dis} = \sqrt{2 ((s + b) \ln (1 + s/b) - s)}$$
If $b$ is large,

$$Z_{\text{dis}} = s/\sqrt{b}$$

Thus, if $b$ is small, $s/\sqrt{b}$ overestimates the significance. We use $Z_{\text{dis}} > 5$ which corresponding to $p < 2.86 \times 10^{-7}$ for different values of $\Delta_b$. Similarly, the significance for exclusion is,

$$Z_{\text{exc}} = 2\left\{ s - b \ln\left(\frac{b + s + x}{2b}\right) - \frac{b^2}{2\Delta_b^2} \ln\left(\frac{b - s + x}{2b}\right) \right\}^{1/2}$$

$$x = \sqrt{(s + b)^2 - 4sb\Delta_b^2/(b + \Delta_b^2)}$$

(11)

If $\Delta_b = 0$,

$$Z_{\text{exc}} = \sqrt{2(s - b \ln(1 + s/b))}$$

For 95% confidence level (CL) exclusion ($p = 0.05$), we use $Z_{\text{exc}} > 1.645$ for different $\Delta_b$.

We calculate the significance using the formula in Eq. 10, in order to account for the uncertainty in the background, as the background is small in both the channels. The integrated luminosity for discovery and exclusion as a function of the mass of the doubly charged fermion ($M_{\Sigma}$) is shown in Fig: 7. The prediction in channel I is sensitive to the uncertainty in the background, which we have considered to be $\sigma_B = 0, 0.25 \times b, 0.5 \times b$. Channel II is not sensitive to $\sigma_B$ as the signal and background cross-sections, both, are small, as given in Table 3. We have found that channel I and II have a good discovery potential for masses up to 850 GeV and 1025 GeV respectively at 3000 fb$^{-1}$ luminosity with $\sigma_B=0$. In channel I, more than 3000 fb$^{-1}$ luminosity is required for discovery of $M_{\Sigma} > 850$ GeV with nonzero $\sigma_B$.

Masses up to 1.05 TeV and 1.2 TeV can be excluded with 95% CL (corresponds to $Z$ value =1.645) at 3000 fb$^{-1}$ luminosity, with no background uncertainty. In channel I, with integrates luminosity of 3000 fb$^{-1}$, the exclusion limit is 1 TeV and 920 GeV, for $\sigma_B=0.25$ and 0.5 respectively. Hence, we find that channel I and II have a good prospect for both exclusion and discovery of the doubly charged fermions in HL-LHC, with 3000 fb$^{-1}$, with the added advantage of mass reconstruction for the doubly charged fermion and the charged scalar in channel II.

4 Phenomenology at the $e^+e^-$ Collider

We have shown in Sec: 3 that the pair production cross-sections of the singly charged fermions ($\Sigma^\pm$) are smaller compared to the doubly charged fermions ($\Sigma^{\pm\pm}$) at p-p collision, where both are components of a fermionic quintuplet. The small cross-section makes it difficult to observe singly charged fermions at 14 TeV LHC when we look for the alternative signatures in our model. Even increasing the center of mass energy further up to 27 TeV does not solve the issue. These singly and doubly charged fermions can also be produced in linear colliders, such as the $e^+e^-$ collider, which in turn generate multiple leptons and jets in the final state. Even though it is possible to observe alternative signatures for both singly and doubly charged fermions at the $e^+e^-$ colliders, we restrict ourself to the case of the singly charged fermion. The production of the doubly charged fermions lead to more leptons and jets in the final state than the singly charged fermions. Here, we choose to study the final states once the singly charged fermions are produced in pair at $e^+e^-$ collider. The analysis for the doubly charged fermions
will be similar to this. Moreover, at LHC, being a pp collider, the multijet signals are complicated to study due to the heavy QCD backgrounds. But the $e^+e^-$ collider offers a much clean environment. Hence, the SM background for the signal involving multiple jets are remarkably small compared to pp collider.

4.1 Signal

The singly charged fermions($\Sigma^\pm$), can be produced in pairs at the $e^+e^-$-collider via the gauge couplings as described in the previous section. The Feynman diagrams for the pair production are shown in Fig: 8. In general, the process proceeds through s-channel via $\gamma$ and $Z$ boson exchange. But, in this particular model, there is an extra contribution coming from the t-channel diagram via the doubly charged scalar. The cross-section due to the t-channel diagram is large compared to the other diagram. However, the contribution in the total cross-section is not so large due to destructive interference between the s- ans the t-channel diagrams. The effect of the polarization of the electron and positron beam has been discussed in detail in Ref: [61], and we have followed the exact same polarization of the $e^+$ and $e^-$ beam which leads to maximum -60% left right asymmetry ($A_{LR}$).

The production cross-sections are computed in MadGraph5_aMC@NLO (v2.6.5) with the normalisation and factorisation scales set at $m_Z$ and shown in Fig: 9. For further study, we choose the following benchmark points: $M_\Sigma = 200$ GeV at $\sqrt{s} = 500$ GeV, $M_\Sigma = 300, 400$ GeV at $\sqrt{s} = 1000$ GeV, and $M_\Sigma = 500, 600, 700$ GeV for $\sqrt{s} = 1500$ GeV.

Figure 7: The integrated luminosity for discovery (left) and exclusion (right) as a function of the mass of the doubly charged fermion at 14 TeV LHC. The solid, dashed and dotted lines correspond to 0%, 25% and 50% uncertainty in the total background, respectively.

Figure 8: Feynman diagrams for the production of singly charged quintuplet fermion at the $e^+e^-$ collider.
The decays of the singly charged fermions lead to the following final states involving W/Z bosons,

![Graphs showing cross-sections](image)

Figure 9: Pair production cross-sections for the singly charged fermions of different masses as a function of center of mass energy, at $e^+e^-$ collider.

governed by the equations in Sec: 2. Among all final states, the leptonic final states or final states of leptons+jets suffer from lower effective cross-section due to small branching ratio of W/Z into leptons. The signals with multiple jets have the advantage over multilepton states as the branching ratio of W/Z is more into jets than leptons. The final states involving multiple jets can have a maximum of 6 jets, coming from the decays of W/Z. Here, we show a detailed analysis of two final states:

- Channel(A): One lepton ($\ell^\pm$) + 4 jets.
- Channel(B): Two opposite sign lepton pair ($\ell^+\ell^-$) + 4 jets.

These type of signals in $e^+e^-$ collider have a great chance for discovery due to smaller background, which is also shown in [64].

4.2 Backgrounds

The major backgrounds for the channels under study get contribution from di-boson ($WW, ZZ$), $t\bar{t}$, $tV$, Triboson ($VVV=ZZZ, ZVV$) and $HZ$ production. The variation of these major backgrounds with $\sqrt{s}$ is already shown in [62]. Along with multileptons, as the channels under investigation include multiple jets, we demand inclusive cross-section of these backgrounds by producing at least two jets in association, such as, di-boson+ 2 jets production($VVjj$), $t\bar{t}$+ 2 jets and $HZ$+2 jets. The contribution from the $\ell\ell$+ 2 jets, 4-jets and 4-top production are found to be small. We include these backgrounds in “others” category. Among all the backgrounds, the cross-section of $ZZjj$ is found to be larger.

4.3 Collider Analysis

In order to generate events, we use MadGraph5_aMC@NLO (v2.2.1) [55], where the showering and hadronization are done in a similar way as mentioned before in the LHC part. In FastJet, the jets are reconstructed with distance parameter $R = 0.4$ using anti-$K_t$ algorithm. In Delphes, we use the Delphes ILD card [65] for detector simulation. The signal and background events are required to pass through selections on different kinematic distributions, as given in Table 4. At first, we select events with basic cuts, $A1$. Later, while selecting the single lepton or the oppositely charged lepton pair, we make sure that it is well isolated from the jets coming from the decays of W/Z by requiring a moderate isolation cut in $A2$. We have also imposed a cut on $M(\ell^+, \ell^-)$ in channel (B) to reduce the background further.
The signal and background cross-sections after the cuts are shown in Table 5 and Table 6, respectively. We found the background to be small enough to give a very good signal to background ratio \(S/B\), after the initial cuts \(A_1\) for channel (A). For channel (B), in order to improve the \(S/B\) ratio, we have imposed further cuts in \(A_2\) on selected opposite sign lepton pair \((\ell^+\ell^-)\), as shown in Table 4. The requirement of exactly two leptons with opposite sign in channel (B) makes the cross section smaller than channel (A). The largest background contribution comes from \(t\bar{t}+jets\) due to the large cross-section. We further check that the additional cuts on kinematic variables such as \(H_T\), \(S_T\) or \(MET\) would reduce the signal efficiency effectively, hence we did not impose them. Even though the signal cross section in channel (B) is less, we find that it is an excellent channel to reconstruct the invariant

### Table 4: Selections \(A_1\) and \(A_2\) for channel (A) and channel (B).

| Selections | Channel (A) | Channel (B) |
|------------|-------------|-------------|
| \(A_0\)    | \(N(\ell) \geq 1 + N(j) \geq 4\) | \(N(\ell) \geq 2 + N(j) \geq 4\) |
| \(A_1\)    | \(p_T(\ell) > 10\) GeV, \(|\eta(\ell)| < 2.5\), \(\Delta R(\ell, \ell/j) > 0.4\), \(p_T(j) > 20\) GeV, \(|\eta(j)| < 5.0\), \(\Delta R_{jj} > 0.4\) | \(p_T(\ell) > 10\) GeV, \(|\eta(\ell)| < 2.5\), \(\Delta R(\ell, \ell) > 0.4\), \(p_T(j) > 20\) GeV, \(|\eta(j)| < 5.0\), \(\Delta R_{jj} > 0.4\) |
| \(A_2\)    | \(\Delta R(\ell, j) > 1.5\), \(M(\ell^+, \ell^-) > 100\) GeV | \(\Delta R(\ell, j) > 1.5\), \(M(\ell^+, \ell^-) > 100\) GeV |

### Table 5: Cross-sections for the signal \(e^+e^- \rightarrow \Sigma^+\Sigma^-\) before and after the selections. Channel (A) corresponds to \(l^+ + 4\) jets and Channel (B) corresponds to \(l^\pm l^\pm + 4\) jets.

| \(\sqrt{s}\) (GeV) | \(M(\Sigma)\) (GeV) | \(\sigma\) (fb) | \(\sigma^A_{A_2}\) (fb) | \(\sigma^B_{A_2}\) (fb) |
|-------------------|---------------------|----------------|--------------------------|--------------------------|
| 500 GeV           | 200                 | 0.706          | 4.45                     | 0.049                     |
| 1 TeV             | 200                 | 0.218          | 15.70                    | 0.131                     |
|                   | 300                 | 0.209          | 14.63                    | 0.125                     |
|                   | 400                 | 0.175          | 13.50                    | 0.122                     |
| 1.5 TeV           | 500                 | 0.089          | 7.56                     | 0.107                     |
|                   | 600                 | 0.077          | 7.24                     | 0.1001                    |
|                   | 700                 | 0.05           | 4.95                     | 0.09                      |

### Table 6: Cross-sections for various backgrounds corresponding to channels A \((l^+ + 4jets)\) and B \((l^\pm l^\pm + 4jets)\) after the selections.

| Background         | \(\sqrt{s} = 500\) GeV | 1 TeV | 1.5 TeV | \(\sqrt{s} = 500\) GeV | 1 TeV | 1.5 TeV |
|--------------------|--------------------------|-------|---------|--------------------------|-------|---------|
| \(diboson + jets\) | 8.08                     | 3.04  | 1.63    | 0.0                       | 0.0   | 0.0     |
| \(t\bar{t} + jets\)| 82.5                     | 24.75 | 11.25   | 1.1                       | 0.33  | 0.15    |
| \(t\bar{t}V\)     | 1.121                    | 1.79  | 1.083   | 0.039                     | 0.0614| 0.037   |
| \(VVV\)           | 2.85                     | 5.0   | 3.67    | 0.0024                    | 0.035 | 0.0031  |
| \(HV + jets\)     | 2.85                     | 0.65  | 0.3     | 0.045                     | 0.0   | 0.0     |
| \(others\)        | –                        | –     | –       | 0.045                     | 0.0425| 0.035   |
| \(Total\)         | 97.4                     | 34.35 | 17.94   | 1.186                     | 0.437 | 0.225   |
Figure 10: Three body invariant mass $M(\ell j j)$ for $M_\Sigma = 400$ GeV (left) and $M_\Sigma = 600$ GeV (right) for channel (B) at 1 TeV and 1.5 TeV $e^+e^-$ collider respectively. The shadowed region represents the total background.

Figure 11: The integrated luminosity for discovery (Solid line) as a function of the mass of the singly charged fermion ($M_\Sigma$) is shown for channel (A) and (B) at 1 TeV (left) and 1.5 TeV (right) $e^+e^-$ collider. The integrated luminosity for exclusion (dotted line) is also shown for channel B.

mass of $M_\Sigma$ from the $\ell^+\ell^- j j$ distribution. We show the distribution for two cases, $M_\Sigma=400$ and 600 GeV at $\sqrt{s}=1$ and 1.5 TeV respectively in Fig:10.

**Result**  For the study in $e^+e^-$ collider, the background is larger compared to the LHC scenario. Hence, Eq:10 reduces to a simple form of $S/\sqrt{B}$. The integrated luminosity for discovery as a function of the mass of the singly charged fermion is shown in Fig:11. We find that, the discovery potential of channel (A) is much better than channel (B), i.e, the required integrated luminosity is less in channel (A), for $M_\Sigma \leq 450$ GeV and $M_\Sigma \leq 750$ GeV, at $\sqrt{s} = 1$TeV and 1.5 TeV respectively. With $\leq 20$ fb$^{-1}$ luminosity, it is possible to discover in the region of $M_\Sigma \leq 700$ GeV with 5$\sigma$ significance in channel (A). Whereas, the required luminosity for 5$\sigma$ discovery is $\leq 1000$ fb$^{-1}$, for the same in channel (B). For 95% exclusion limit, the entire mass region can be probed with luminosity less than 100 fb$^{-1}$ in channel (B). The required luminosity in channel (A) is very small for the same, hence we do not plot them.
5 Conclusion and Outlook

We have discussed the discovery potential of the singly and the doubly charged fermions, which are components of a quintuplet, at the LHC and future $e^+e^-$ colliders. Such a specific model, as we have considered, with quintuplet fermions and a scalar multiplet, predicts certain signatures which require alternate search strategies.

In the study of signatures at the LHC, we have discussed the possible multilepton and multi(lepton+jet) signatures of the doubly charged fermions, as they have larger cross-sections compared to that of the singly charged fermions. For the doubly charged quintuplet fermion ($\Sigma^{\pm\pm}$), $5\sigma$ discovery might be possible at integrated luminosity of 3000 fb$^{-1}$ at 14 TeV LHC if $M_{\Sigma} \leq 980$ GeV. The exclusion limit can be extended up to 1.2 TeV with the same parameters.

On the other hand, linear colliders, such as the $e^+e^-$ collider, offer a much cleaner environment to study the signatures associated with multiple jets. Thus, the signals have the advantage of a larger cross-section $\times$ BR, where the W/Z bosons decay into jets. We find that the singly charged fermion ($\Sigma^{\pm}$) shows a great discovery potential at the $e^+e^-$ collider, unlike the case of the LHC. There might be a possibility of $5\sigma$ discovery with 1000 fb$^{-1}$ luminosity at $e^+e^-$ collider for $M_{\Sigma} \leq 700$ GeV. Similar kind of final states also exist for the doubly charged quintuplet fermion but with more leptons and jets, making the analysis much more complicated. Thus, we will address it somewhere else. The cross-sections for the pair production of the doubly charged fermions at the $e^+e^-$ collider are shown in Fig: 12 (left and middle).

An $e^+e^-$ linear collider can also be operated as a $\gamma\gamma$ and an $e^-\gamma$ collider, as illustrated in Ref: [66,67]. The highly intense photons for the collision are obtained by Compton back-scattering laser photons on intense high-energy electron beams. Due to the coupling with photon, charged particles can be produced with a considerably high cross-section at these photon colliders. In the present model, the production of the singly charged quintuplet fermions is possible via $\gamma\gamma \rightarrow \Sigma^+\Sigma^-$, $e^-\gamma \rightarrow \Sigma^+\phi^{-}-$ and $e^-\gamma \rightarrow \Sigma^-\phi^0$ modes, along with the conjugate process in each case. The production cross-section for the singly-charged quintuplet fermion at $\gamma\gamma$ collider is shown in Fig: 12 (right) as a function of the center of mass energy. These production modes, alone or combined with $e^+e^-$ collision, show a great potential at future linear colliders. Over all, the nonstandard decay modes of the quintuplet fermions
offer different signals which require alternate search strategies and there might be an opportunity for discovery and/or exclusion at the HL-LHC and future linear colliders.

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