Combined effect of Piezo-viscous dependency and non-Newtonian couple stresses in Annular Plates Squeeze-Film characteristics

Hanumagowda B N\textsuperscript{1}, Savitramma G\textsuperscript{2}, Salma A\textsuperscript{1} and Noorjahan\textsuperscript{1,3}

\textsuperscript{1}REVA University, Bangalore, India  
\textsuperscript{2}Navodaya Institute of Technology, Raichur, India.  
\textsuperscript{3}Huda National College, Bangalore, India. 
\textsuperscript{}corresponding author: hanumagowda123@rediffmail.com

Abstract: In this article, the theoretical analysis of the combined study of non-Newtonian couple stresses with piezo-viscous dependency for annular plates squeeze film bearings have been carried out, with help of stokes micro continuum theory along with the exponential variation of viscosity with pressure. An approximate analytical solution is found using a small perturbation method. The solution for pressure and load capacity with distinct values of viscosity-pressure parameter are calculated and compared with iso-viscous couple stress and Newtonian lubricants and the results reveals that the effect of couple stresses and pressure-dependent viscosity variation enhances the load-carrying capacity and lengthens the squeeze film time.

1. Introduction

Squeeze-film behavior between two approaching surfaces is significant in many engineering applications, such as rolling gears, damping films, bearings, piston engines and lubricating joints. This process is also noticed when faces of disc clutches is approached in lubricated condition. During mechanism action such as clipping, squeezing and so on, the temperature of lubricants’ changes along with density and viscosity through which the performance of bearings behavior also varies. Many authors have discussed squeeze film action of bearing with Newtonian lubricants such as Cameron \cite{1}; Hamrock \cite{2}; Gupta and Gupta \cite{3}, Murti \cite{4}, Bujurke et al. \cite{5}, Lin \cite{6}, Naduvinamani and Siddanagouda \cite{7}. The rapid growth in the modern engineering highlights the use of mechanism for squeeze-film of non-Newtonian lubricants. The feature of non-Newtonian lubricants are polymer-thickened oils, base oils and synovial bio-fluids mixed with additives of long chain. Various non-Newtonian fluid models have been used to analyse the performance of lubrication in different problems like Lin et al. \cite{8} calculated a modified lubrication equation for hydromagnetic non-Newtonian cylindrical squeeze films results were compared with non-conducting Newtonian lubricant through which it was seen that for circular squeeze films the hydromagnetic non-Newtonian lubricants performance was more advanced. Sharma et al. \cite{9} analysed the Effect in the behaviour of non-Newtonian lubricant and flexibility of bearing on the performance of slot-entry journal bearing. Nabhani, et al. \cite{10} deliberate the combined effects of viscous shear and non-Newtonian lubricants behaviour on porous squeeze film. Cheng-Hsing Hsu et al. \cite{11} found that the characteristics of Squeeze
film in conical bearings operating with Rabinowitsch fluid model having non-Newtonian lubricants. According to research by Barus [12] and Bartz and Ether [13], the dependency of viscosity pressure has taken a form of

\[ \mu = \mu_0 e^{\alpha p} \]  

(1)

where viscosity is taken as \( \mu \), pressure as \( p \), pressure-dependent viscosity co-efficient as \( \alpha \) and the viscosity at ambient pressure as \( \mu_0 \) and taking temperature to be constant. The relation studied above indicates the lubricant viscosity which raises exponentially and it changes the foreseen squeeze film performance of bearings.

As per earlier research study, the viscosity-pressure dependence is more significant in analysing the phenomena of high pressure in lubrication. In modern years, authors are more interested to describe the dependency of viscosity-pressure through which phenomenon of lubrication could be studied. Lin [14, 15] analyses the dependency of viscosity-pressure squeezing system on sphere plate and wide parallel plate along with couple stresses for non-Newtonian lubricant and from the obtained result there is increase in squeeze film pressure and load carrying capacity due piezo-viscous effects. Study on Piezo-viscous dependency on squeeze film for circular plates was examined by Singh [16]. It is found that, due to piezo-viscous effect, there is gradually increase in pressure, load capacity and response time. The combined study of surface roughness and Viscous-Pressure dependency of parallel circular plates was studied by Naduvinamani, et al. [17] and they found that the effects of couple stresses increases film time, and load capacity for both types of roughness patterns. Hanumagowda [18] deliberate the combined effect of couple stress on squeeze-film lubrication and pressure-dependent viscosity between circular step plates. He analyses that the non-dimensional values such as pressure, film time and load capacity, increases when compared with the case of iso-viscous lubricant.

Study of piezo-viscous dependency and Couple stress on squeeze film performance for parallel stepped plate was examined by Naduvinamani et al. [19] recently and through the result obtained we found that due to couple stresses and pressure-dependent viscosity, variation increases the load capacity and lengthen film time. However, Combined study of piezo-viscous dependency and non-Newtonian couple stress between annular plates squeeze-film characteristics was not yet analysed so far. Hence an attempt is made in the present analysis.

2. Formulation Of The Problem

A geometry of squeeze film lubrication between annular plates approaching each other with \( V(= -dh/dt) \) a normal velocity is seen in Figure 1, assuming that the body couple and force is absent, the basic equation of motion are given by

\[ \frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{\partial v}{\partial y} = 0 \]  

(2)

\[ \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial r} \]  

(3)

\[ \frac{\partial p}{\partial y} = 0 \]  

(4)

where velocity components \( u \) and \( v \) are in the directions of \((r,y)\) coordinates, pressure \( (p) \), dynamic viscosity \(( \mu \) ), and material constant \(( \eta \) ) is responsible for the couple stress fluids.

The boundary conditions are

When \( y=h \), for upper surface, we have

\[ u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0 \]  

(5a)
When $y=0$, for lower surface, we have

\[ u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0 \]  \hspace{1cm} (6a)

\[ v = 0 \] \hspace{1cm} (6b)

By using the boundary conditions (5a, 6a), and equation (1), the solution of equation (3) is given in the form

\[ u = \frac{1}{2\mu} \frac{\partial p}{\partial r} \left[ y^2 - hy + 2l^2 \left( 1 - \frac{Cosh(\frac{2y-h}{2l})}{Cosh(\frac{h}{2l})} \right) \right] \] \hspace{1cm} (7)

Where $l = \left( \frac{\eta}{\mu_0} \right)^{1/2}$ couple stress parameter.

Substituting the value ‘$u$’ from (7) in the continuity equation (2), and using the boundary conditions (5b, 6b), integrating across the film thickness, the equation (2) is given as

\[ \frac{\partial}{\partial r} \left[ A(h,m,\alpha,p) r \frac{\partial p}{\partial r} \right] = 12r \mu_0 \frac{dh}{dt} \] \hspace{1cm} (8)

Where, $A(h,m,\alpha,p) = h^3 e^{-\alpha p} - 12m^2 h e^{-2\alpha p} + 24m^3 e^{-2.5\alpha p} \tanh \left( \frac{he^{\alpha p/2}}{2m} \right)$

Using the non-dimensional parameters in equation (8),

\[ r^* = \frac{r}{b}, a^* = \frac{a}{b}, C = \frac{m}{h_0}, h^* = \frac{h}{h_0}, p^* = \frac{ph_0}{\mu_0 b^2 (-dh/dt)}, G = \frac{\alpha \mu_0 b^2}{h_0^3} \frac{(-dh/dt)}{h_0^3} \]

The non-dimensional Reynolds equations (9) is observed to be highly non linear.
\[
\frac{\partial}{\partial r} \left\{ f^*(h^*, C, G, p^*) \right\} r^* \frac{\partial p^*}{\partial r} = -12r^*
\]
(9)
where
\[
f^*(h^*, C, G, p^*) = e^{-Gp^*} h^{*3} - 12C^2 e^{-2Gp^*} h^{*5} + 24C^3 e^{-2.5Gp^*} \tanh(e^{0.5Gp^*} h^*/2C)
\]
To obtain analytical solution of first order, viscosity parameter is taken as 0 ≤ G ≪ 1 the film pressure is acquired by using
\[
p^* = p^*_0 + Gp^*_1
\]
(10)
by introducing film pressure in Reynold’s type equation, the following equations are obtained
\[
\frac{\partial}{\partial r} \left\{ r^* \frac{dp^*_0}{dr^*} \right\} = -12r^*
\]
(11)
\[
\frac{d}{dr^*} \left\{ r^* \frac{dp^*_1}{dr^*} \right\} = -f^*_0(h^*, C) \frac{d}{dr^*} \left\{ p^*_0 r^* \frac{dp^*_0}{dr^*} \right\}
\]
(12)
where
\[
f^*_0(h^*, C) = h^{*3} - 12C^2 h^{*5} + 24C^3 \tanh(h^*/2C)
\]
\[
f^*_1(h^*, C) = -h^{*3} + 6C^2 h^{*5} \left( 4 + \text{sech}^2(h^*/2C) \right) - 60C^3 \tanh(h^*/2C)
\]
By using the boundary conditions
\[
\frac{dp^*_0}{dr^*} = 0 \text{ at } a^* = 0
\]
(13)
\[
p^*_0 = p^*_1 = 0 \text{ at } a^* = 1
\]
(14)
Solving the two equations (10) and (11), the non-dimensional pressure is developed in the film region as
\[
p^* = \frac{3(a^{*2} - 1) \left[ \log r^* - \frac{r^{*2} - 1}{2} \right]}{f^*_0(h^*, C) \left[ \log a^* - \frac{a^{*2} - 1}{2} \right]} - G \left[ \frac{3(a^{*2} + 1)}{2f^*_0(h^*, C)} \right] \left[ \frac{\left( \log r^* \right) (r^{*2} - 1)}{\left( \log a^* \right) (a^{*2} - 1)} \right]^2
\]
(15)
The load-carrying capacity \( W \) is given in the form as
\[
W = 2\pi \int_a^b prdr
\]
(16)
The non-dimensional load-carrying capacity \( W^* \) is obtained as
\[
W^* = \frac{3\pi(a^{*2} - 1)^2}{2f^*_0(h^*, C)} \left[ \frac{1}{\log a^*} - \frac{(a^{*2} + 1)}{(a^{*2} - 1)} \right] - G \frac{9\pi(a^{*2} + 1)}{4f^*_0(h^*, C)} \left[ \frac{3(a^{*2} + 1)}{2\log a^*} - \frac{2(a^{*4} + a^{*2})}{3(a^{*2} - 1)} \right]
\]
(17)
Where
\[
W^* = \frac{Wh^3}{\mu_b b^3 \left( -dh/dt \right)}
\]
The non-dimensional squeezing time is given by
\[ T' = \int \left[ \frac{3\pi(a^2 - 1)^2}{2f_0(h', C)} \left\{ \frac{1}{\log a} \left( \frac{(a^2 - 1)}{(a^2 - 1)} \right) - G\frac{9\pi(a^2 - 1)^2 f_1(h', C)}{4f_0(h', C)} \left\{ \frac{2(a^2 - 1)}{\log a} \left( \frac{(a^2 - 1)}{(a^2 - 1)} \right) - \frac{2(a^4 + a^2 + 1)}{3(a^2 - 1)} \right\} \right\} dh' \]  

Where

\[ T^* = \frac{Wh_0^2}{\mu_0b^4} \]  

3. Results and Discussions

The combined study of couple stress with viscosity pressure dependency between annular plates using Stokes couple stress fluid model for lubricants is carried out in present analysis to discuss the squeeze film performance. The dimensional pressure, load capacity and film time are plotted for distinct parametric values of couple stress \( C \) and PDV \( G \). To discuss the characteristics of squeeze film following range is considered

\[ C = 0, 0.1, 0.2, 0.3, 0.4, 0.5 \text{ and } G = 0, 0.02, 0.04, 0.06, 0.08, 0.1. \]

3.1 Squeeze Film Pressure

The graph of non-dimensional pressure \( P' \) along \( r' \) for various values of couple stress parameter \( C \) with \( h' = 0.6 \), \( G = 0.04 \), \( a' = 0.3 \) as fixed is depicted in the fig.2. On Comparing the iso-viscous Newtonian case \( (C=0) \) the pressure gradually increases for increasing of \( C \) along with \( r' \). Thus piezo-viscous analysis is significantly more compared to iso-viscous analysis for \( C \). Figure-3 represents graph of \( P' \) along \( r' \) for distinct values of \( G \) with \( h' = 0.6 \), \( a' = 0.3 \) fixed and it is found that pressure significantly raises along with \( G \).

3.2 Load Carrying Capacity

The non-dimensional load carrying capacity \( W' \) with \( h' \) for various values of couple stress parameter \( C \) with \( G = 0.04 \), \( a = 0.3 \) is depicted in the fig.4. It is found that when the values of couple stress parameter \( C \) increases the load carrying capacity increases. Figure-5 displays the non-dimensional load carrying capacity \( W' \) for various values of \( G \) fixed \( C = 0.3 \), \( a = 0.3 \). It is seen that when the values of parameter \( G \) increases the load carrying capacity increases. Further, figure.6 represents variation of non-dimensional load carrying capacity \( W' \) against \( a' \) for various values of \( G \) fixed \( C = 0.3 \), \( h' = 0.3 \) and observed that when the values of \( G \) increases the load carrying capacity increases. Thus from the all three graphs we can conclude that \( W' \) enhances along with \( C \), \( G \) and \( a' \).

3.3 Squeeze Film Time

The variation of squeeze film time \( T' \) with \( h' \) for distinct values of \( C \) with \( G = 0.04 \), \( a' = 0.3 \) is depicted in the fig7 and for distinct values of \( G \) with \( C = 0.3 \), \( a' = 0.3 \) is depicted in the fig 8. From both graphs we found that squeeze film enhances significantly for \( C \) and \( G \). The plot of squeeze film time \( T' \) along \( h' \) for distinct values of \( a' \) fixed \( G = 0.04 \), \( C = 0.3 \) is displayed in the fig.8 and found that when the value of \( a' \) increases the squeeze film time also increases.
Figure 2: Variation of non-dimensional pressure $P^*$ with $r^*$ for distinct values of $C$ with $h^* = 0.6, G = 0.04, a^* = 0.3$.

Figure 3: Variation of non-dimensional pressure $P^*$ with $r^*$ for distinct values of $G$ with $h^* = 0.6, C = 0.3, a^* = 0.3$.

Figure 4: Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for distinct values of $C$ with $G = 0.04, a^* = 0.3$.

Figure 5: Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for distinct values of $G$ with $C = 0.3, a^* = 0.3$.

Figure 6: Variation of non-dimensional Load carrying capacity $W^*$ with $a^*$ for distinct values of $G$ with $C = 0.3, h^* = 0.6$.

Figure 7: Variation of squeeze film time $T^*$ with $h^*$ for distinct values of $C$ with $G = 0.04, a^* = 0.3$.
4. Conclusion

Combined study of Piezo-viscous dependency and couple stresses for Annular Plates is carried out to discuss the non-Newtonian Squeeze-Film characteristics using Barus and Bartz [12] and Ether [13] for VPD and the fluid theory of Stoke’s couple stress. In the present paper result is theoretically analyzed as follows

1. The non-dimensional pressure and load capacity significantly enhance when there is an effect of Piezo-viscous dependency as well as the couple stresses.
2. The response time increases due to the effect of both the couple stresses and Piezo-viscous dependency.

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6. Nomenclature

$B_0$ applied magnetic field in the direction in the z-direction.

$h^*$ thickness of squeeze film.

$C$ parameter of couple stress $\left(\frac{g}{\mu}\right)^{\frac{1}{2}}$

$P^*$ squeeze film pressure for non-dimensional surface $\frac{-E\left(p \right)h_0^3}{\mu\left(\frac{dh}{dt}\right)\pi ab}$

$T^*$ squeeze film time for non dimensional surface is approach $\left(\frac{\mu}{\rho \pi R^2}\right)$

$V$ velocity of squeezing $\left(\frac{dh}{dt}\right)$

$W^*$ load carrying capacity for non-dimensional surface $\frac{-E\left(W \right)h_0^3}{\mu\left(\frac{dh}{dt}\right)\pi^2 a^2 b^2}$

$\eta$ material constant of couple stresses

$\mu$ Coefficient of viscosity

$\sigma$ Conductivity of the fluid

$R$ radius of the annular plate

$\mu_0$ atmospheric pressure with viscosity