Improved Community Detection using Deep Embeddings from Multilayer Graphs

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Abstract

Community detection is a challenging, yet crucial, problem while mining large-scale graph structured data. Most existing approaches solve this problem by mapping nodes into a vector space and performing unsupervised learning with the resulting embeddings. In cases where multiple types of connectivity patterns exist for the set of nodes, commonly modeled as multilayer graphs, new strategies are required to model the inter-layer dependencies in order to perform effective inferencing. In this paper, we focus on learning embeddings for each node of a multilayer graph through neural modeling techniques, such that the complex dependencies can be concisely encoded into low-dimensional representations. Referred to as multilayer graph embeddings, these representations can be utilized for discovering community structure in a scalable fashion, even with a large number of layers. Furthermore, in order to ensure that the semantics that persist over a longer range in the network are well modeled, we propose to refine the multilayer embeddings via a proxy clustering loss and a graph modularity measure. Using real-world datasets, we demonstrate that this algorithm generates scalable and robust representations, and outperforms existing multilayer community detection approaches.

Introduction

Community Detection in Multilayer Graphs: Graphs are natural data structures to represent relational data, and hence modeling and inferencing with graph structured data have become central to a wide-range of applications, such as social network analysis (Eagle and Pentland 2006), recommendation systems (Rao et al. 2015), neurological modeling (Fornito, Zalesky, and Breakspear 2013) etc. Though some of these applications require supervised or semi-supervised learning formulations, mining large networks to identify cohesive clusters of densely-connected nodes is a highly prevalent idea in the graph mining literature (Blondel et al. 2008, Kim and Lee 2015). Referred to as community detection, this unsupervised learning problem is most commonly addressed by mapping nodes into a vector space and performing clustering using the resulting embeddings (Dong et al. 2012).

Constructing Node Embeddings: At their core, node embedding approaches attempt to identify low-rank representations that can best represent the network topology. Despite their broad applicability, several of these approaches produce linear embeddings for nodes, naturally motivating the use of deep neural networks to potentially produce more expressive, non-linear embeddings. Consequently, stacked graph auto-encoder style solutions have been proposed...

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(Yang et al. 2016), that directly transform the objective measure (e.g., modularity matrix) into an undercomplete representation through a reconstruction cost. In addition to producing non-linear mappings, deep learning approaches enable the use of robust reconstruction losses in lieu of a simple $\ell_2$ measure (Thiagarajan et al. 2016), and supports the inclusion of additional prior constraints on community structure (Yang et al. 2016). A known limitation of node embedding techniques has been their scalability (e.g., Eigen value decomposition) with large-scale graphs, and this issue persists even with graph autoencoders. In order to combat this limitation, recent approaches, such as DeepWalk (Perozzi, Al-Rfou, and Skiena 2014) and Node2Vec (Grover and Leskovec 2016), have resorted to a distributional hypothesis, popularly adopted in language modeling (Harris 1954), where co-occurrence of two nodes in short random walks implies a strong notion of semantic similarity. As a result, by extending highly scalable neural embedding techniques such as Word2Vec (Mikolov et al. 2013) to the construction of node embeddings, one can obtain state-of-the-art results in community detection with single-layer graphs.

**Proposed Work:** In this paper, we develop a novel scalable technique for obtaining deep node embeddings from multilayer graphs. We show that a naive extension of DeepWalk to the multilayer case, that performs independent random walks on each of the layers, can be worse than even simple baselines, thus emphasizing the need to explicitly model dependencies across the different layers. Consequently, we propose to parameterize virtual edges to allow information flow between layers. Furthermore, the premise of using short random walks to infer the underlying semantic structure relies on the assumption that the networks are highly sparse and the node co-occurrences follow a power law. However, by allowing inter-layer edges, that assumption can be violated in cases where the semantics can persist over even longer walks. We address this challenge by including a refinement stage, where the multilayer embeddings are fine-tuned to produce more cohesive communities. In particular, we use entropy based proxy clustering cost and modularity based refinement. We show that the proposed approach is highly effective for as many as 37 layers and it outperforms existing approaches for multilayer community detection.

**Mathematical Preliminaries**

**Definitions:** A single-layer undirected, unweighted graph is represented by $G = (V, E)$, where $V$ denotes the set of nodes with cardinality $|V| = N$, and $E$ denotes the set of edges. The goal of embedding techniques is to generate latent representations, $X \in \mathbb{R}^{N \times d}$, where $d$ is the desired number of latent dimensions. A multilayer graph is represented using a set of $L$ inter-dependent graphs $G^{(l)} = (V^{(l)}, E^{(l)})$, for $l = 1, \ldots, L$, where there exists a node mapping between every pair of layers to indicate which vertices in one graph correspond to vertices in the other.

**Deep Embeddings for Network Analysis:** The scalability challenge of factorization techniques has motivated the use of deep learning methods to obtain node embeddings. The earliest work to report results on this direction was the DeepWalk algorithm by Perozzi et al. (Perozzi, Al-Rfou, and Skiena 2014). Interestingly, it draws analogy between node sequences generated by short random walks on graphs and sentences in a document corpus. Given this formulation, the authors utilize popular language modeling tools to obtain latent representations for the nodes (Mikolov et al. 2013).

Let us consider a simple metric walk $W_t$ in step $t$, which is rooted at the vertex $v$. The transition probability between the nodes $v_i$ and $v_j$ can be expressed as

$$P(W_{t+1} = v_j | W_t = v_i) = h(||x_i - x_j||_2 / \sigma),$$

where $||x_i - x_j||_2$ indicates the similarity metric between the two vertices in the latent space to be recovered and $h$ is a linking function that connects the vertex similarity to the actual co-occurrence probability. With appropriate choice of the walk length, the true metric can be recovered accurately from the co-occurrence statistics constructed using random walks. Furthermore, the authors note that the frequency in which vertices appear in the short random walks follows a power-law distribution, similar to words in natural language.

Given a length-$S$ sequence of words, $(w_0, w_1, \ldots, w_{S-1})$, where $w_s$ denotes a word in the vocabulary, neural word embeddings attempt to obtain vector spaces that can recover the likelihood of observing a word given its context, i.e., $P(w_s | w_0, w_1, \ldots, w_{s-1})$ over all sequences. Extending this idea to the case of graphs, a random walk on the nodes, starting from node $v$, produces the sequence analogous to sentences in language data.

**Modularity based Community Detection:** A popular measure used in community detection algorithms is the modularity function $Q$ (Newman 2006), defined as the difference between the number of edges within cohesive communities and the expected number of edges over all pairs of nodes in a graph. Formally, this can be defined as follows:

$$Q = \frac{1}{2r} \sum_{i,j} \left( e_{ij} - \frac{n_in_j}{2r} \right) \delta(c_i, c_j)$$

where $e_{ij}$ denotes the connection between nodes $v_i$ and $v_j$, $n_i, n_j$ denote the degrees for node $v_i$ and $v_j$ respectively, $r = \frac{1}{2} \sum n_i$ is the total number of edges and $\delta(\cdot)$ is the Kronecker delta function which equals one only when the community memberships for nodes $v_i$ and $v_j$, namely $c_i$ and $c_j$, are the same. The Louvain method (Blondel et al. 2008) is a popular technique that employs an agglomerative optimization algorithm to search for the partition that maximizes $Q$. This algorithm is comprised of two stages that are iterated until a local optimum is reached: In the first step, each vertex $v_i$ is assigned to its own distinct community $C_i$. Then for each vertex $v_i$, its neighbors are sequentially added to $C_i$ if the inclusion results in a positive change in the modularity.

Extending this definition to the case of multilayer graphs provides the multislice modularity defined in Mucha et al. (2010). In addition to measuring the intra-layer modularity in each layer, multislice modularity takes into account the coupling between layers through an inter-layer modularity function. A crucial advantage of this definition is that existing modularity optimization algorithms can be directly utilized to solve the maximization of the multislice modularity, thus enabling community detection in multilayer graphs.
Formally, this can be defined as:

\[
Q_{\text{multi}} = \frac{1}{2\mu} \sum_{i,j,l,m} \left[ \left( e_{ij}^{(l)} - \gamma_l n_i^{(l)} n_j^{(l)} \right) \frac{\delta(i, m)}{2l} + \delta(i, j) \sigma_j^{(l,m)} \right] \delta(c_i^{(l)}, c_j^{(m)}),
\]

where \( \mu \) is the normalization parameter for layer \( l \), \( \gamma_l \) is the resolution parameter for layer \( l \), \( \sigma_j^{(l,m)} \) is the coupling parameter between a pair of corresponding nodes in different layers, i.e., \( v_j^{(l)} \) and \( v_j^{(m)} \), and the other parameters are direct extensions from \( Q \) to each layer \( l \). As observed in eq. (3), the intra-layer modularity term is similar to eq. (2), whereas the inter-layer component measures the strength between node \( v_j \) and all its corresponding vertices in other layers.

Proposed Approach

Following the notation in the previous section, an \( L \)-layer graph is denoted by the set \( \{ G_i^{(l)} \}_{l=1,\ldots,L} \). Note that we expect different layers to contain common nodes, but do not require \( Y^{(l)} \) to be the same. The complete set of nodes in the multilayer graph is denoted as \( V = \bigcup_{l=1}^{L} V^{(l)} \), where \( \bigcup \) refers to concatenation. Under this setting, we formulate the graph community detection problem as follows: Given \( \{ G_i^{(l)} \} \), obtain a partition \( \{ P_k \}_{k=1,\ldots,K} \) of \( V \), such that a node \( v_i^{(l)} \in V \) is assigned to a single community \( C_k \). A key distinction between our formulation and existing multilayer graph fusion strategies is that we do not require the node counterparts in different layers, e.g., \( v_1^{(1)} \) and \( v_2^{(2)} \), to be assigned to the same community. This formulation is flexible and readily applicable to several real-world problems, wherein a node (e.g., a person) can have significantly different roles depending on the community he/she belongs to.

As illustrated in Figure 1, our two-stage approach takes as input a multilayer graph and infers \( d \)-dimensional latent representations of each of the nodes in \( V \). In the first stage, we model inter-layer dependencies by introducing edges between a node and its counterparts if they share the local neighborhood structure. More specifically, we allow the information sharing across layers by constructing a supra adjacency matrix encoding multilayer transitions (Kuncheva and Montana 2015). Subsequently, we infer multilayer latent representations using neural graph embedding techniques. Note that, this produces separate embeddings for each of the nodes in the superset \( V \). In the second stage, the representations are refined based on a proxy clustering cost defined in (Xie, Girshick, and Farhadi 2016), and the multi-slice modularity criterion (eq. 3). While the representations obtained after the first stage capture neighborhood information in the multilayer graph, varying levels of sparsity and uncertainties in each of the layers can lead to violation of the underlying assumptions made in deep graph embedding techniques. Consequently, the refinement strategy, driven entirely from a community detection standpoint, can produce more reliable representations. As the final step, the output representations are clustered using \( k \)-means algorithm to infer the community assignment of each node.

Stage 1: Deep Multilayer Graph Embeddings

This stage (the first two blocks in Figure 1) aims to exploit the dependencies across different layers by creating inter-layer graph connections. Such dependencies can then be encoded into the latent representations through deep graph embedding techniques such as DeepWalk. Since the quality of the random walks is crucial to obtaining effective embeddings, it is important to enable the walker to traverse across layers. For example, let us consider two common scenarios that can occur with multilayer graphs: (a) a community is shared by all or a subset of the layers, and (b) a community is discovered only in a specific layer and it is not apparent in other layers. This motivates the design of an embedding technique that respects the following constraints: (i) For a shared community, the set of random walks should encompass nodes located in different layers where the community can be discovered, thus emulating information flow across layers; and (ii) For a layer-specific community, the walks should be comprised of nodes within the same layer so that the latent embeddings will not be confused by communities from other layers. In order to achieve this, we introduce inter-layer edges based on the similarities between local neighborhood structure. For a pair of nodes \( v_i^{(l)} \) and \( v_j^{(m)} \) from the layers \( l \) and \( m \) respectively, we introduce inter-layer edges as follows:

\[
e_{ij}^{(l,m)} = \begin{cases} 0, & \text{if } i \neq j, \\ \frac{|\mathcal{N}_i^{(l)} \cap \mathcal{N}_j^{(m)}|}{|\mathcal{N}_i^{(l)} \cup \mathcal{N}_j^{(m)}|}, & \text{if } i = j \end{cases}
\]

where \( e_{ij}^{(l,m)} \) denotes the edge weight between the nodes \( v_i^{(l)} \) and \( v_j^{(m)} \), and \( \mathcal{N}_i^{(l)}, \mathcal{N}_j^{(m)} \) are the neighborhoods of \( v_i^{(l)} \) and \( v_j^{(m)} \) respectively. Note that, the edge weight is computed as the Jaccard coefficient of the agreement in the corresponding neighborhood lists between the two layers. In other words, this process ensures that each node in a layer is
Multilayer Node Embeddings
Initialized using reconstruction loss

Refined Embeddings
Fine-tune parameters based on proxy clustering cost and modularity measure

Stage 2: Representation Refinement

As discussed earlier, the success of deep graph embedding techniques relies on the assumption that graphs are sparse and short random walks reveal all necessary semantics for constructing node embeddings. However, because of the introduction of inter-layer edges, and the varying levels of inherent sparsity in each of the layers, this assumption can be violated in cases where the semantic relationships persist over longer walks. In order to address this challenge and to more reliably recover the community structure, we propose to refine the learned embeddings. More specifically, we employ a proxy clustering cost similar to the one proposed in [Xie, Girshick, and Farhadi 2016], along with modularity based reassignment to refine the embeddings. As illustrated in Figure 2(b), we employ a neural architecture $H$ to refine the multilayer graph embeddings from Stage 1 and the parameters of the network are adjusted to improve the quality of the discovered communities.

Architecture: We utilize a fully connected network with overall dimensions of $d – 128 – 256 – d$ to refine the representations. Sigmoid activation is applied in the hidden layers and linear activation is used in the output layer. Note that, the primary objective of the refinement stage is to retain the meaningful topological structure contained in multilayer graph embeddings and to only fine-tune the embeddings to reduce uncertainties in the community assignments. Since the representations learned in stage 1 already contain significant topological information as abstracted from the raw graph data, a reasonably shallow neural network is sufficient for the refinement process. As shown in Figure 2(b), the inputs to the network are the $d$-dimensional embeddings $X$ and the goal is to produce more refined embeddings $\tilde{X}$ (also $d$-dimensional), that can potentially lead to improved community detection. To this end, we initialize the network to act similar to an autoencoder, which minimizes a mean squared error reconstruction objective.

Proxy Clustering Loss: Since the network was initialized with a reconstruction loss, $X$ mimics the structure in $\tilde{X}$. However, it is not straightforward to measure uncertainties in the community assignment from the latent representations, and actually use them to optimize the parameters of $H$. Hence, we resort to a proxy clustering objective similar to the one proposed in [Xie, Girshick, and Farhadi 2016], where the core idea is to refine the embeddings such that the cluster assignment distribution has a low entropy. In other words, the embeddings must be expressive enough to ensure that the nodes can be assigned to one of the communities with high confidence. More specifically, this is carried out by first calculating the cluster assignment probabilities using the embeddings $\tilde{X}$ and then adjusting parameters of $H$ to reduce the entropy for each of the nodes.

We begin by estimating the cluster assignments for all multilayer nodes using $k$-means clustering on the embeddings $\tilde{X}$. Subsequently, we calculate the likelihood for a
node to be assigned to each of the communities as follows:

\[ q_{ik} = \frac{1}{\sum_{k'=1}^{K} \left( 1 + ||x_i - \mu_k||^2 / \alpha \right)^{-\frac{\alpha+1}{2}}} \]

where \( \mu_k \) is the cluster centroid for community \( C_k \) and the parameter \( \alpha \) is set to 1. Note that the graph layer index \( l \) for each node is omitted for clarity. However, the calculation is performed for every node on the multilayer graph, i.e. \( v_i^{(l)} \).

Given these assignment probabilities, a natural strategy is to iteratively refine the clustering assignment by learning from the regions of high confidence. Interestingly, this is conceptually similar to discriminative clustering algorithms (Ye, Zhao, and Wu 2008) that iteratively perform clustering (e.g. k-Means) and supervised dimensionality reduction (e.g. linear discriminant analysis) based on current cluster labels to identify the discriminant function for maximal linear separability. In order to perform iterative, discriminative clustering with deep architectures, we adopt an strategy that defines an auxiliary distribution, which is a reduced-entropy variant of the likelihoods \( q \). Here, \( f_k = \sum_k q_{ik} \) are the soft cluster frequencies. Note, by constructing this auxiliary distribution, we identify a target distribution for the deep architecture to match, such that each node is assigned to one of the clusters with high certainty. Consequently, the network parameters are adjusted by minimizing the KL-divergence loss between the true distribution \( Q \) and the target \( P \) for each of the nodes:

\[ L = KL(Q||P) = \sum_i \sum_k q_{ik} \log \left( \frac{q_{ik}}{p_{ik}} \right). \]

The optimization is carried out using Stochastic Gradient Descent (SGD). After this step, the algorithm updates \( q_{ik} \) again according to eq. (5) and repeats this discriminative clustering procedure until convergence, in terms of cluster assignment.

**Modularity-Driven Refinement:** Though the proxy clustering cost can be highly effective in producing reliable community assignments, it relies heavily on the quality of the initial clustering, i.e., the high confidence predictions should be correct. In cases where this assumption is violated, we can incorporate heuristic measures extracted from the graph structure to provide additional information during the refinement procedure. In this paper, we propose to utilize multislice modularity while refining the node embeddings. Specifically, inspired by the extremal optimization algorithm for modularity maximization (Duch and Arenas 2005), during each iteration of the refinement process, we move samples which have low modularity contribution to other communities, such that the modularity gain is maximized. The soft assignment probabilities are subsequently re-calculated to reflect the new assignment.

### Algorithm 1: Proposed two-stage algorithm for multi-layer community detection.

We first rewrite the multislice modularity in eq. (3) in order to define the contribution from each graph node. Note that this procedure essentially extends the “fitness score” definition in (Duch and Arenas 2005) to multilayer graphs:

\[
Q_{\text{multi}} = \frac{1}{2\mu} \left[ \sum_l \sum_i \sum_{j \in C_i^{(l)}} \left( e_{ij}^{(l)} - \gamma^{(l)} \frac{n_{ij}^{(l)} n_{j}^{(l)}}{r_l} \right)^2 + \sum_l \sum_i \sum_{m \neq l} \sum_{j \in C_i^{(l)}} \sigma_i^{(l,m)} \right] \\
= \frac{1}{2\mu} \left[ \sum_l \sum_i \left( n_{C_i} - \gamma n_{ij}^{(l)} \frac{r_{C_i}}{r_l} + \sum_i \sigma n_{C_i} \right) \right] \\
= \frac{1}{2\mu} \sum_l \sum_i \left( n_{C_i} - \gamma n_{ij}^{(l)} \frac{C_i^{(l)}}{r_l} + \sigma n_{C_i} \right)
\]

In the derivation we assume \( \gamma, \sigma_i^{(l,m)} \) are constants for all nodes. For simplicity, we use \( n_{C_i} \) to denote the number of edges in layer \( l \) that connect node \( v_i^{(l)} \) to other nodes in community \( C_i^{(l)} \). On the other hand, \( n_{C_i} \) denotes the number of counterpart nodes of \( v_i^{(l)} \) which also belong to community \( C_i^{(l)} \).

The final summation term in eq. (8) defines the contribution from \( v_i^{(l)} \) to the multislice modularity. In analogy to extremal optimization, we further calculate the fitness score by normalizing the intra- and inter-layer contribution terms...
Table 1: Details for all datasets used in the experiments.

| Dataset                  | $|\mathcal{V}|$ | $\sum_{l} |\mathcal{E}^{(l)}|$ | $L$ | Labeled? |
|--------------------------|-----------------|---------------------------|-----|----------|
| Leskovec-Ng Dataset      | 214             | 536                       | 4   | Yes      |
| Reinnovation             | 1728            | 8460                      | 12  | Yes      |
| Pierre Auger             | 965             | 7153                      | 16  | No       |
| Air Transportation       | 2034            | 3588                      | 37  | No       |

where

$$\lambda_{i}^{(l)} = \frac{n_{C_{l}}}{n_{i}^{(l)}} - \frac{r_{C_{l}}^{(l)}}{r_{i}} + \sigma \frac{n_{C_{l}}}{n_{i}^{(l,m)}}$$  \quad (9)

where $n_{i}^{(l,m)}$ refers to the total number of counterpart nodes of $\nu_{i}^{(l)}$.

Following the probabilistic selection process in [Duch and Arenas 2005, Boettcher 2001], nodes in $\mathcal{V}$ are first sorted in ascending order based on their fitness scores, and then sampled according to the probability:

$$p(s) \propto s^{-\tau}$$  \quad (10)

where $s$ is the rank of node after sorting and $\tau \sim 1 + \frac{1}{\ln(|\mathcal{V}|)}$.

Next, we move the sampled node $i'$ to community $k'$ which gives the largest gain of $Q_{\text{multi}}$. To reflect the updated assignment for the overall refinement process, we increase the community likelihoods $q_{i',k'}$ (eq. [5]) by a constant $c$ and re-normalize the probabilities. Note that the change of community structure requires $\lambda_{i}^{(l)}$ to be re-calculated before next node is sampled. This sample and update process is repeated for a pre-specified number of iterations. Subsequently, we carry out the remaining steps of the refinement procedure (calculation of eq. [6] and optimization of eq. [7]) as described in the previous section. The overall two-stage algorithm for obtaining robust embeddings is listed in Table 1.

### Experimental Results

We begin by describing the multilayer graph datasets that are used in our experiments. Subsequently, we report results obtained using our method in comparison to the baseline modularity-based and DeepWalk-based approaches.

### Datasets

We consider two labeled multilayer graph datasets: Leskovec-Ng collaboration dataset[^5] and Reinnovation dataset. Leskovec-Ng dataset consists of a temporal multilayer graph which represents the coauthorship between researchers over a 20-year span. Each layer contains a 5-year interval and as a result, there are 4 layers in total. The ground-truth label denotes whether a researcher is a member of Jure Leskovec’s or Andrews Ng’s group at Stanford University. On the other hand, the Reinnovation dataset characterizes the similarities between 144 countries according to attributes such as infrastructure, health, and primary education. Each attribute defines a layer where two countries are connected if they contain high similarity. The development level of each country is annotated as the node-level ground-truth label. Note that for both datasets, the corresponding nodes of all layers share the same label. Although this formulation is different from ours, it is automatically accommodated by our algorithm when the inter-layer weighting threshold $\ell$ is set to a low value.

In real-world social, transportation and biological multilayer graphs, the large number of vertices often lack ground-truth labeling. Hence, we consider two unlabeled multilayer graph datasets and utilize implicit metrics to evaluate the quality of the inferred community structure. The first one, Pierre Auger Observatory dataset[^5], contains the collaboration of 514 scientists on 16 research tasks at the Pierre Auger Observatory. Each task corresponds to a layer and there is an edge between the scientists if they co-authored a report. The second unlabeled dataset is Air Transportation multiplex[^5], where each layer models whether there is a direct flight between cities for a certain European airline. There are 450 cities and 37 airlines in total. We summarize the details of all datasets used in our experiments in Table 1.

### Performance Evaluation

For labeled datasets, we utilized the metrics Normalized Mutual Information (NMI), Adjusted Rand Index (ARI) and Fowlkes Mallows Index (FMI) (Boutemine and Bouguessa 2017) for evaluation. In order to quantify the results, we compare the proposed algorithm with the baselines of GenLouvain and DeepWalk based approaches. For the GenLouvain algorithm, we used the number of communities automatically detected based on the $\gamma$ and $\sigma$ parameters. We performed grid search on $\gamma \in \{0.5, 1, 1.5, 2\}$, $\sigma = \{0.5, 1, 1.5, 2\}$ and selected the best combination which yields the actual number of communities, and the highest metric scores. For the DeepWalk baseline, representations were obtained by performing random walks on each layer independently. We fixed the embedding dimension $d = 64$ and

[^5]: sites.google.com/site/pinyuchenpage/datasets
[^6]: deim.urv.cat/~manlio.dedomenico/data.php
[^7]: complex.unizar.es/~atnmultiplex/
window length $S = 5$. The number of short random walks and walk length have slightly larger effect on the learned representations and are tuned separately for each dataset. Finally, for our proposed approach, we fixed the constant to be added to the association probabilities, $c = 0.1$, in the modularity-driven refinement process, and tuned the inter-layer connectivity threshold $t$ in intervals of 0.2.

In order to investigate the effect of different algorithmic components to the performance of the proposed approach, we also present results obtained using the multilayer deep embeddings from Stage 1 and refinement with only the proxy clustering loss. The results are summarized in Table 2. The first observation is that by effectively utilizing the modularity measure, the baseline of GenLouvain is capable of achieving high performance on both datasets. On the other hand, the baseline of DeepWalk fails to generate meaningful representations, thus leading to inferior results. Interestingly, by simply adopting the proposed multilayer deep embeddings, the algorithm achieves similar or even better performance than GenLouvain. This observation demonstrates that significant dependencies exist between nodes on different layers and they should not be ignored during the representation learning process. Finally, the refinement steps contributes further in adjusting the embeddings, thus producing communities that agree the most with ground-truth labels.

Next, we consider the evaluation of our approach on unlabeled datasets, with the focus on comparison to GenLouvain, the state-of-the-art method for multilayer community detection. Due to the lack of knowledge about the community structure, the clustering results must be evaluated implicitly. To this end, we utilize modularity, coverage and conductance as the measures. In this evaluation, the number of communities is an important parameter and can largely affect the partitioning result. To provide a comprehensive comparison, we show the results for different number of communities estimated using GenLouvain, by varying $\gamma$ and $\sigma$ parameters. With each estimated number of communities, we performed $k$-means clustering on the learned representations. The performance comparisons on the two unlabeled datasets are visualized in Figure 3 and 4, respectively. We observe that the performance of GenLouvain is highly sensitive to the number of communities chosen. In many cases, the algorithm fails to discover meaningful communities. On the contrary, the performance of the proposed approach is consistently better and stable. This clearly evidences the need to construct multilayer embeddings by effectively exploring the dependencies between layers.

**Conclusions**

This paper presents a novel framework to extract node embeddings from multilayer graphs for community detection. While the locally adaptive inter-layer edge construction facilitates complex dependency modeling for DeepWalk, the refinement stage for the node representations improves the modeling of longer-range semantics. Experimental results demonstrate that the framework is scalable to the size of the multilayer graph and the learned embeddings can achieve improved community detection performance. In fu-
ture work, we plan to investigate incorporating the multislice modularity criterion directly into the learning of neural graph embeddings, such that community-specific representations can be discovered in a single pass.

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