Prospects of Detecting Baryon and Quark Superfluidity from Cooling Neutron Stars

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Baryon and quark superfluidity in the cooling of neutron stars are investigated. Observations could constrain combinations of the neutron or \(\Lambda\)–hyperon pairing gaps and the star’s mass. However, in a hybrid star with a mixed phase of hadrons and quarks, quark gaps larger than a few tenths of an MeV render quark matter virtually invisible for cooling. If the quark gap is smaller, quark superfluidity could be important, but its effects will be nearly impossible to distinguish from those of other baryonic constituents.

Pairing is unavoidable in a degenerate Fermi liquid if there is an attractive interaction in any channel. The resulting superfluidity, and in the case of charged particles, superconductivity, in neutron star interiors has a major effect on the star’s thermal evolution through suppressions of neutrino (\(\nu\)) emission processes and specific heats. Neutron (\(n\)), proton (\(p\)) and \(\Lambda\)-hyperon superfluidity in the \(^1S_0\) channel and \(n\) superfluidity in the \(^3P_2\) channel have been shown to occur with gaps of a few MeV or less. However, the density ranges in which gaps occur remain uncertain. At large baryon densities for which perturbative QCD applies, pairing gaps for like quarks have been estimated to be a few MeV. However, the pairing gaps of unlike quarks (\(u\), \(s\), and \(d\)) have been suggested to be several tens to hundreds of MeV through non-perturbative studies and kindling interest in quark superfluidity and superconductivity and their effects on neutron stars.

The cooling of a young (age \(< 10^5\) yr) neutron star is mainly governed by \(\nu\)-emission processes and the specific heat. Due to the extremely high thermal conductivity of electrons, a neutron star becomes nearly isothermal within a time \(t_w \approx 1–100\) years after its birth, depending upon the thickness of the crust. After this time its thermal evolution is controlled by energy balance:

\[
\frac{dE_{th}}{dt} = C_V \frac{dT}{dt} = -L_\gamma - L_\nu + H ,
\]

where \(E_{th}\) is the total thermal energy and \(C_V\) is the specific heat. \(L_\gamma\) and \(L_\nu\) are the total luminosities of photons from the hot surface and \(\nu\)S from the interior, respectively. Possible internal heating sources, due, for example, to the decay of the magnetic field or friction from differential rotation, are included in \(H\). Our cooling simulations were performed by solving the heat transport and hydrostatic equations including general relativistic effects (see \(\text{[3]}\)). The surface’s effective temperature \(T_e\) is much lower than the internal temperature \(T\) because of a strong temperature gradient in the envelope. Above the envelope lies the atmosphere where the emerging flux is shaped into the observed spectrum from which \(T_e\) can be deduced. As a rule of thumb \(T_e/10^6\) K \(\approx \sqrt{T/10^8}\) K, but modifications due to magnetic fields and chemical composition may occur.

The simplest possible \(\nu\) emitting processes are the direct Urca processes \(f_1 + \ell \to f_2 + \nu \chi, f_2 \to f_1 + \ell + \nu\ell\), where \(f_1\) and \(f_2\) are either baryons or quarks and \(\ell\) is either an electron or a muon. These processes can occur whenever momentum conservation is satisfied among \(f_1, f_2, \ell\) (within minutes of birth, the \(\nu\) chemical potential vanishes). If the unsuppressed direct Urca process for any component occurs, a neutron star will rapidly cool because of enhanced emission: the star’s interior temperature \(T\) will drop below \(10^9\) K in minutes and reach \(10^7\) K in about a hundred years. \(T_e\) will hence drop to less than 300,000 K after the crustal diffusion time \(t_w\). This is the so-called rapid cooling paradigm. If no direct Urca processes are allowed, or they are all suppressed, cooling instead proceeds through the significantly less rapid modified Urca process in which an additional fermion enables momentum conservation. This situation could occur if no hyperons are present, or the nuclear symmetry energy has a weak density dependence. The \(\nu\) emission rates for the nucleon, hyperon, and quark Urca and modified Urca processes can be found in \(\text{[3]}\).

The effect of the pairing gaps on the emissivities and specific heats for massive baryons are investigated in \(\text{[4]}\) and are here generalized to the case of quarks. The principal effects are severe suppressions of both the emissivity and specific heat when \(T \ll \Delta\), where \(\Delta\) is the pairing gap. In a system in which several superfluid species exist the most relevant gap for these suppressions is the smallest one. The specific heat suppression is never complete, however, because leptons remain unpaired. Below the critical temperature \(T_c\), pairs may recombine, resulting in the emission of \(\nu\nu\) pairs with a rate that exceeds the modified Urca rate below \(10^{10}\) K; these processes are included in our calculations.

The baryon and quark pairing gaps we adopt are shown in Fig. 1. Note that gaps are functions of Fermi momenta \(p_F(i), i\) denoting the species) which translates into a density dependence. For \(p_F(n,p) \lesssim 200 – 300\) MeV/c,
nucleons pair in the $^1S_0$ state, but these momenta correspond to densities too low for enhanced $\nu$ emission involving nucleons to occur. At higher $p_F$’s, baryons pair in higher partial waves. The $n$ $^3P_2$ gap has been calculated for the Argonne V18, CD-Bonn and Nijmegen I & II interactions [3]. This gap is crucial since it extends to large $p_F(n)$ and can reasonably be expected to occur at the centers of neutron stars. For $p_F(n) > 350$ MeV/c, gaps are largely uncertain because of both experimental and theoretical uncertainties [3]. The curves [a], [b] and [c] in Fig. 3 reflect the range of uncertainty. The $p$ $^3P_2$ gap is too small to be of interest. Gaps for the $^1S_0$ pairing of $\Lambda$, taken from [4] and shown as dotted curves, are highly relevant since $\Lambda$s participate in direct Urca emission as soon as they appear [12]. Experimental information beyond the $^1S_0$ channel for $\Lambda$ is not available. $\Delta$s for $\Sigma$–hyperons remain largely unexplored. The quark ($q$) gaps are taken to be Gaussians centered at $p_F(q) = 400$ MeV/c with widths of 200 MeV/c and heights of 100 MeV [model D], 10 MeV [C], 1 MeV [B] and 0.1 MeV [A], respectively. The reason for considering quark gaps much smaller than suggested in [3] is associated with the multicomponent nature of charge-neutral, beta-equilibrated, neutron star matter as will become clear shortly.

We consider four generic compositions: charge-neutral, beta equilibrated, nucleons matter containing nucleons only ($np$), nucleons with quark matter ($npQ$), nucleons and hyperons ($npH$), and nucleons, hyperons and quarks ($npHQ$). In the cases involving quarks, a mixed phase of baryons and quarks is constructed by satisfying Gibbs’ phase rules for mechanical, chemical and thermal equilibrium [16]. The phase of pure quark matter exists only for very large baryon densities, and rarely occurs in our neutron star models. Baryonic matter is calculated using a field-theoretic model at the mean field level [17]; quark matter is calculated using either a bag-like model or the Nambu-Jona-Lasinio quark model [18]. The equation of state (EOS) is little affected by the pairing phenomenon, since the energy density gained is negligible compared to the ground state energy densities without pairing.

Additional particles, such as quarks or hyperons, have the effect of softening the EOS and increasing the central densities of stars relative to the $np$ case. For the $npQ$ model studied, a mixed phase appears at the density $n_B = 0.48$ fm$^{-3}$. Although the volume fraction of quarks is initially zero, the quarks themselves have a significant $p_F(q)$ when the phase appears. The $p_F$’s of the three quark flavors become the same at extremely high density, but for the densities of interest they are different due to the presence of negatively charged leptons. In particular, $p_F(s)$ is much smaller than $p_F(u)$ and $p_F(d)$ due to the larger $s$-quark mass. Use of the Nambu–Jona-Lasinio model, in which quarks acquire density-dependent masses resembling those of constituent quarks, exaggerates the reduction of $p_F(s)$. This has dramatic consequences since the pairing phenomenon operates at its maximum strength when the Fermi momenta are exactly equal; even small asymmetries cause pairing gaps to be severely reduced [3] [16]. In addition, one may also expect $p$-wave superfluidity, to date unexplored, which may yield gaps smaller than that for the $s$-wave. We therefore investigate pairing gaps that are much smaller than those reported for the case of $s$-wave superfluidity and equal quark $p_F$’s.

The introduction of hyperons does not change these generic trends. In the case $npH$, the appearance of hyperons changes the lepton and nucleon $p_F$’s similarly to the appearance of quarks although with less magnitude. While the appearance of quarks is delayed by the existence of hyperons, at high densities the $p_F$’s of nucleons and quarks remain similar to those of the $npQ$ case. The existence of either hyperons or quarks, however, does allow the possibility of additional direct Urca processes involving themselves as well as those involving nucleons by decreasing $p_F(n) - p_F(p)$. For the $npQ$ and $npHQ$ models studied, the maximum masses are $\approx 1.5M_\odot$, the central baryon densities are $\approx 1.35$ fm$^{-3}$, and the volume fractions of quarks at the center are $\approx 0.4$.

Cooling simulations of stars without hyperons and with hyperons are compared, in Figs. 2 and 3, respectively, to available observations of thermal emissions from isolated neutron stars. Sources for the observational data can be found in [2]. However, at the present time, the inferred temperatures must be considered as upper limits because the total flux is contaminated, and in some cases dominated, by the pulsar’s magnetospheric emission and/or the emission of a surrounding synchrotron nebula. Furthermore, the neutron star surface may be reheated by...
magnetospheric high energy photons and particles; late-time accretion for non-pulsing neutron stars is also possible. Other uncertainties arise in the temperature estimates due to the unknown chemical composition and magnetic field strength in the surface layers, and in the age, which is based upon the observed spin-down time. In these figures, the bolder the data symbol the better the data.

![Figure 2](image_url)

**FIG. 2.** Cooling of 1.4 M⊙ stars with np matter (continuous curves) and npQ matter (dashed and dotted curves). The curves labelled as [a], [b], and [c] correspond to n 3P2 gaps as in Fig. 1: [z] corresponds to zero n gap. Models labelled [A], [B], [C] and [D] correspond to quark gaps as in Fig. 1: [Z] corresponds to zero quark gap.

The np case is considered in Fig. 2, in which solid lines indicate the temperature evolution of a 1.4 M⊙ star for quarkless matter: case [z] is for no nucleon pairing at all, and cases [a], [b] and [c] correspond to increasing values for the neutron 3P2 gap, according to Fig. 1. The field-theoretical model employed for the nucleon interactions allows the direct nucleon Urca process, which dominates the cooling. The unimpeded direct Urca process carries the temperature to values well below the inferred data. Pairing suppresses the cooling for T < Tc, where T is the interior temperature, so Tc increases with increasing Δ. If the direct Urca process is not allowed, the range of predicted temperatures is relatively narrow due to the low emissivity of the modified Urca process. We show an example of such cooling (curve [Mc]) using the n 3P2 gap [c] for a 1.4 M⊙ with an EOS [20] for which the direct Urca cooling is not allowed.

The other curves in the figure illustrate the effects of quarks upon the cooling. The dotted curves [Z] are for vanishingly small quark gaps; the dashed curves ([A], [B], [C] and [D]) are for quark gaps as proposed in Fig. 1. For nonexistent ([z]) or small ([a]) nucleon gaps, the quark Urca process is irrelevant and the dependence on the existence or the size of the quark gaps is very small. However, for large nucleon gaps ([b] and [c]), the quark direct Urca process quickly dominates the cooling as the nucleon direct Urca process is quenched. It is clear that for quark gaps of order 1 MeV or greater ([B], [C] or [D]), the effect of quarks is again very small. There is at most a slight increase in the stars temperatures at ages between 10^4 to 10^5-6 years due to the reduction of p_F(n) and the consequent slightly larger gap (Fig. 2). Even if the quark gap is quite small ([A]), quarks have an effect only if the nucleon gap is very large ([b] or [c]), i.e., significantly larger than the quark gap: the nucleon direct Urca process is suppressed at high temperatures and the quark direct Urca process has a chance to contribute to the cooling. We find that the effects of changing the stellar mass M are similar to those produced by varying the baryon gap, so that only combinations of M and Δ might be constrained by observation.

The thermal evolution of stars containing hyperons has been discussed in [22,23], but we obtain qualitatively different results here. Hyperons open new direct Urca channels: Λ → p + e + ν_e and Λ + e → Σ^- + ν_e if Σ^-'s are present, with their inverse processes. Previous results showed that the cooling is naturally controlled by the smaller of the Λ or n gap. However, this is significantly modified when the Λ gap is more accurately treated. At the Λ appearance threshold, the gap must vanish since p_F(Λ) is vanishingly small. We find that a very thin layer, only a few meters thick, of unpaired or weakly bound neutrons can lead to a significant effect.

![Figure 3](image_url)

**FIG. 3.** Cooling of stars with npH (continuous lines) and npHQ matter (dotted lines) for various stellar masses (in M⊙). n 3P2 gaps are from case [c] while quark gaps, when present, are from model [C] of Fig. 1.
paired $\Lambda$’s is sufficient to control the cooling. This effect was overlooked in previous works perhaps because they lacked adequate zonal resolution.

In Fig. 2 we compare the evolution of stars of different masses made of either $npH$ or $npHQ$ matter. We find that all stars, except the most massive $npH$ ones, follow two distinctive trajectories depending on whether their central density is below or above the threshold for $\Lambda$ appearance ($= 0.54 \text{ fm}^{-3}$ in our model EOS, the threshold star mass being $1.28M_\odot$). In the case of $npH$ matter, stars with $M > 1.50M_\odot$ are dense enough so that the $\Lambda^1S_0$ gap vanishes and hence undergo fast cooling, while stars made of $npHQ$ matter do not attain such high densities. The temperatures of $npH$ stars with masses between 1.3 and 1.5 $M_\odot$ are below the ones obtained in the models of Fig. 2 with the same $n^3P_2$ gap [b], which confirms that the cooling is dominated by the very thin layer of unpaired $\Lambda$’s (the slopes of these cooling curves are typical of direct Urca processes). Only if the $n^3P_2$ gap $\lesssim 0.3$ MeV do the cooling curves fall below what is shown in Fig. 2. Notice, moreover, that in the mass range 1.3 – 1.48 $M_\odot$ the cooling curves are practically indistinguishable from those with unpaired quark matter shown in Fig. 2. In these models with $npH$ or $npHQ$ matter, there is almost no freedom to “fine-tune” the size of the gaps to attain a given $T_c$: stars with $\Lambda$’s will all follow the same cooling trajectory, determined by the existence of a layer of unpaired or weakly paired $\Lambda$’s, as long as the $n^3P_2$ gap is not smaller. It is, in some sense, the same result as in the case of $np$ and $npQ$ matter: the smallest gap controls the cooling and now the control depends on how fast the $\Lambda^1S_0$ gap increases with increasing $p_F(\Lambda)$.

Our results indicate that observations could constrain combinations of the smaller of the neutron and $\Lambda$–hyperon pairing gaps and the star’s mass. Deducing the sizes of quark gaps from observations of neutron star cooling will be extremely difficult. Large quark gaps render quark matter practically invisible, while vanishing quark gaps lead to cooling behaviors which are nearly indistinguishable from those of stars containing nucleons or hyperons. Moreover, it also appears that cooling observations by themselves will not provide definitive evidence for the existence of quark matter itself.

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