PHASE CROSS-CORRELATION OF THE WMAP ILC MAP AND FOREGROUNDS

Pavel D. Naselsky1,2, Andrey G. Doroshkevich1, Oleg V. Verkhodanov1,4, naselsky@tac.dk, dorr@tac.dk, oleg@tac.dk

Submitted to The Astrophysical Journal Letters

ABSTRACT

We present a circular cross-correlation tests for the phases of the Internal Linear Combination Map (ILC) and WMAP’s foregrounds for all K–W frequency bands at the range of multipoles $\ell \leq 50$. We have found significant deviations from the expected Poissonian statistics for the ILC and the foregrounds phases. Our analysis shows that the low multipole range of the ILC power spectrum contains some of the foregrounds residues.

Subject headings: cosmology: cosmic microwave background — cosmology: observations — methods: data analysis

1. INTRODUCTION

The recently-published Wilkinson Microwave Anisotropy Probe (WMAP) data sets (see Bennett et al. 2003 a-c, Hinshaw et al. 2003 a-b)) strongly promote the development of the high sensitive statistics for testing the properties of the derived foregrounds and foreground cleaned maps (Komatsu et al. 2003), Tegmark, de Oliveira-Costa and Hamilton (2003), Chiang et al. (2003), Dineen and Coles (2003), Park (2003), Gaztanaga et al. (2003), Colley and Gott (2003), Naselsky et al. (2003)). The WMAP team produces the Internal Linear Combination (ILC) map and maps for the foregrounds (synchrotron, free-free, dust emission) for each K–W bands which are the basis for our analysis. Homogeneous and isotropic CMB Gaussian random fields, as a result of the simplest inflation paradigm, is a crucial test for the power spectrum estimation. We would like to point out that the detected non-Gaussianity of the foregrounds it would reflect directly contamination of the foregrounds at different levels.

In this Letter we present the result of a statistical test of the coupling the ILC map and the foreground taking from the WMAP data. The test is based on the circular cross-correlations analysis of the ILC and foreground maps. Our method complementing the mentioned above ones, exploits a natural assumption that phases of the “true” CMB signal should not correlate with the phases of the foregrounds. This allows us to detect significant ILC—“W band foregrounds” cross-correlations. These peculiarities determine the accuracy of the power spectrum estimation for the ILC map and the statistical properties of the ILC signal. We would like to point out that the detected non-Gaussianity of the ILC map (see for example, Eriksen et al. 2003, Vielva et al. 2003, Coles et al. 2003) is most likely related with foreground contaminations (see also Chiang et al. 2003) produced most likely by the Galactic dust emission.

2. ILC AND FOREGROUNDS PHASE CORRELATIONS

The basic idea of the phase analysis of the CMB maps was proposed and discussed in Chiang and Coles (2000), Naselsky, Novikov and Silk (2002), Chiang et al. (2002), Chiang et al. (2003), Naselsky et al. (2003), Matsubara (2003).

The CMB signal from can be expressed as a sum over spherical harmonics:

$$\Delta T(\theta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \varphi),$$

where $a_{\ell m}$ are the coefficients of expansion, $|a_{\ell m}|$ is the modulus and $\Psi_{\ell m}$ is the phase of each $a_{\ell m}$ harmonic. Homogeneous and isotropic CMB Gaussian random fields (GRFs), as a result of the simplest inflation paradigm, possess Fourier modes whose real and imaginary parts are independently distributed. The statistical properties of GRF are completely specified by its angular power spectrum $C_{\ell}$,

$$C(\ell, \ell', m, m') = \langle a_{\ell m} a_{\ell' m'}^* \rangle = \langle |a_{\ell m}||a_{\ell' m'}| \exp(i(\Psi_{\ell m} - \Psi_{\ell' m'})) \rangle = C(\ell) \delta_{\ell \ell'} \delta_{mm'}.$$  

where $\langle \rangle$ means averaging over the modulus and phase distribution functions. In Fig.1 we plot the cross-correlation of phases between the ILC map and the common foregrounds defined as a sum over synchrotron, free-free and dust emission maps for each frequency band.

1 Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
2 Rostov State University, Space Research Department, Zorge,5, 344091, Russia
3 Special Astrophysical Observatory, Nizhnij Arkhyz, Karachaj-Cherkesia, 369167, Russia
4 see http://lambda.gsfc.nasa.gov
5 Naselsky et al. 2003 have shown that for any linear separations of the CMB signal and foregrounds the phases of derived CMB signal should be correlated with the foreground phases at different level depending on separation technique.
6 Below we will use the term foreground for such a combinations of each component.
As one can see from Fig. 1 the distribution of the ILC map phases and corresponding foregrounds seems to be non-correlated one. However, in order to test such a distribution, for these phase diagram we will apply circular statistics (Fisher 1993) to investigate the possible cross-correlation quantitatively.

3. Correlations of the ILC and Foregrounds Phases

Naselsky et al. (2003) have shown that for linear methods of the foreground separation from the CMB signal we can expect to find some cross-correlations between the phases of the cleaned signal, $\phi_s$, and the foregrounds, $\psi_f$, which can be used to characterize the degree of separation achieved. To do this, here we will use the circular correlation coefficients which reflect directly the $2\pi$-phase periodicity.

For the K–W frequency band the maps were taking from the WMAP web-site and all the phases are obtained by the spherical harmonic decomposition using the HEALPix (Górski et al. 1999) and the GLESP (Doroshkevich et al. 2003) codes. We consider the ranges of multipoles, $2 \leq \ell \leq \ell_{\text{max}} = 50$ for which the WMAP own foreground are presented in the WMAP web site. In addition to the WMAP foregrounds for each band we produce the five maps as a difference between signal (S) in the band and the ILC signal: $F = S - \text{ILC}$. We will call the F-map as the derived foreground. Let $\psi$ and $\phi$ be the foreground and ILC phases, respectively, for given value of $\ell$ and all the corresponding values of $m$. Following Fisher (1993) we define the statistics

$$x_m = \cos(\psi_m), \quad y_m = \sin(\psi_m), \quad \nu_m = \cos(\phi_m), \quad \mu_m = \sin(\phi_m),$$

$$M_p = \frac{1}{\ell_{\text{max}}} \sum_{m}^{\ell_{\text{max}}} \exp(ip(\phi_m - \langle \phi \rangle)), \quad \langle \phi \rangle = \tan^{-1}(\frac{\sum_{m} y_m}{\sum_{m} x_m}), \quad \langle \psi \rangle = \tan^{-1}(\frac{\sum_{m} y_m}{\sum_{m} x_m}),$$

$$M_{1p} = \frac{1}{\ell_{\text{max}}} \sum_{m}^{\ell_{\text{max}}} \exp(ip(\psi_m - \langle \psi \rangle)), \quad R_{sf}(\ell) = \ell^{-1} \sum_{m=1}^{\ell_{\text{max}}} \cos(\phi_m - \psi_m),$$

$$r_{sf} = (\ell_{\text{max}} - \ell + 1)^{-1} \sum_{\ell} R_{sf}(\ell)$$

where $M_p$ and $M_{1p}$ are the $p$-th trigonometric moments of the phase samples, $\langle \phi \rangle$ and $\langle \psi \rangle$ are the corresponding mean directions of the samples, $R_{sf}(\ell)$ is the circular cross-correlation coefficient in each mode $\ell$ and $r_{sf}$ is the mean circular cross-correlation coefficient for all phases. For $m = 0$ and for all $\ell$ phases $\phi(\ell, 0) = \psi(\ell, 0) = 0$ and here we neglect them.

For the K–W bands the circular coefficients, $R_{sf}(\ell)$ are plotted in Fig. 2 for the ILC and its own foreground, and for the ILC and the derived foreground.

![Fig. 2.— The circular correlation coefficient between the cleaned signal and the foreground phases in V–W channels vs. the harmonic index, $\ell$. Thick solid line corresponds to the WMAP own foreground. Thin solid line corresponds to derived foreground. Shadow area represent 1σ error bars level taking from 200 random realizations.](http://lambda.gsfc.nasa.gov/product/map/m_products.cfm)
cross-correlation of the ILC phases and the derived foreground seems to be more significant. The same tendency follows from the estimation of the mean coefficients, \( r_{sf} \), for \( 2 \leq \ell \leq 50 \), listed in Table 1 for the two samples of the foregrounds. As is expected, for the first three channels these coefficients are small but they become significant for channels V and W.

### Table 1

Circular cross-correlation coefficients between phases of ILC and the WMAP own foregrounds (ILC\(^{(o)}\)) and the ILC and derived foregrounds (ILC\(^{(d)}\)).

| \( \ell \) | K | KA | Q | V | W |
|------|---|---|---|---|---|
| ILC\(^{(o)}\) | -0.026 | -0.031 | -0.030 | -0.033 | -0.033 |
| ILC\(^{(d)}\) | -0.017 | 0.002 | 0.022 | 0.112 | 0.262 |

To test the sensitivity of circular statistics, we used the so called “WMAP simulator”, in which we simulated the W band with all own WMAP foregrounds and the CMB signal with random phases instead of the ILC signal\(^9\). After that we repeat the circular phase analysis in the same way as it was done for the ILC map. Our results are shown in Fig. 3.

![Fig. 3.— \( R_{sf} \) for the “WMAP simulator”. Thick solid line corresponds to the simulated random CMB signal, cross-correlated with WMAP own foreground. For the CMB and derived foregrounds we have the same curve. The dashed line is the cross-correlation between the foreground, defined as the W band signal minus the simulated CMB signal (not the ILC !), and the simulated CMB signal.](image)

For the simulator, corresponding mean cross-correlation coefficients are \( r_{sf} = -0.036 \) for the CMB–Foreground correlations and \( r_{sf} = -0.194 \) for “wrong” foreground (actual signal minus simulated CMB), corresponds to the dashed line in Fig. 3. Thus, if the CMB map and foreground are extracted correctly, corresponding mean cross-correlation coefficient should be less than statistical error (\( \sim 5\% \)), while the wrong foreground and CMB should have significant positive or negative bias. For the ILC map it means that a positive bias for \( r_{sf} \) for the V and W bands (see Table 1) reflects directly the cross-correlation with the foreground phases.

### 4. Conclusion

The circular cross-correlation analysis for the ILC map and K–W foregrounds and shows that the ILC map has significant correlation with the derived foregrounds obtained by subtraction the ILC map from the V and W bands signals. Some of these correlations could be linked with non-Gaussianity of the ILC map detected by Park (2003), Eriksen et al. (2003), Coles et al. (2003), Vielva et al. (2003). For example, Vielva et al. (2003) has reported about characteristic scale of non-Gaussianity in order to \( 10^9 \), which corresponds to \( \ell \sim 10 \). Our analysis shows that the cross-correlation coefficients \( r_{sf} \) at \( \ell = 11 \) have the highest maxima for both V and W bands. At high multipole range \( \ell \sim 40 \) corresponding peculiarities are above 95\% confidential level. If we take into account that for \( \ell \sim 40 \) all \( m \sim 35 - 40 \) lies along Galactic plane in the map, we can conclude that they are related with the V and W bands signal at the same longitude. Some of the peculiarities need an additional investigation which would be published soon.

### Acknowledgments

This paper was supported by Danmarks Grundforskningsfond through its support for the establishment of the Theoretical Astrophysics Center. We thank Max Tegmark et al. for providing their processed maps and making them public with openness. We thank Igor Novikov and Lung-Yih Chiang for useful discussions. We also acknowledge the use of HEALPix \(^{10}\) package (Górski, Hivon & Wandelt 1999) to produce \( a_{tm} \) from WMAP maps the use of GLESP package of the TAC CMB collaboration.

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