Toward Communication-Learning Trade-Off for Federated Learning at the Network Edge

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Abstract—In this letter, we study a wireless federated learning (FL) system where network pruning is applied to local users with limited resources. Although pruning is beneficial to reduce FL latency, it also deteriorates learning performance due to the information loss. Thus, a trade-off problem between communication and learning is raised. To address this challenge, we quantify the effects of network pruning and packet error on the learning performance by deriving the convergence rate of FL with a non-convex loss function. Then, closed-form solutions for pruning control and bandwidth allocation are proposed to minimize the weighted sum of FL latency and FL performance. Finally, numerical results demonstrate that i) our proposed solution can outperform benchmarks in terms of cost reduction and accuracy guarantee, and ii) a higher pruning rate would bring less communication overhead but also worsen FL accuracy, which is consistent with our theoretical analysis.

Index Terms—Federated learning, network pruning, convergence analysis, bandwidth allocation.

I. INTRODUCTION

The increasing popularity of mobile devices and the rapid development of smart applications have led to a significant increase in the amount of user data [1]. To make full use of these distributed datasets while protecting users' privacy, federated learning (FL) has grabbed the limelight. However, there are still some challenges when deploying FL in wireless networks. On the one hand, the increasing number of model parameters induced by deep neural network (DNN) causes higher training and communication latency. On the other hand, the packet error over unreliable wireless channels affects the accuracy of global aggregation.

Recently, many existing works have focused on communication efficient FL [1]–[6]. To reduce both FL latency and energy consumption, Dinh et. al in [1] derived closed-form solutions for resource allocation. Yang et. al in [2] proposed an iterative algorithm to minimize the total energy. Likewise, Luo et. al in [3] optimized the number of selected users to minimize the total cost. Focusing on the long-term performance, Xu et. al in [4] developed a joint client selection and bandwidth allocation algorithm. Furthermore, the effect of the packet error on FL convergence was derived in [5]. Considering an edge computing-based FL system, Ren et. al in [6] minimized the weighted sum of communication and learning cost. However, in the above literature, the strong convexity assumption limits the applications of their schemes in DNN with non-convex loss function.

With the increase of model complexity, the training latency of FL becomes critical. To make large-size models compatible to resource-limited devices, model compression has drawn great attention [7]. Specifically, Li et. al in [8] proposed a flexible compression scheme to help to reduce communication latency. To further reduce training latency, adaptive network pruning was adopted in [9] and [10]. Jiang et. al in [9] proposed a pruning-based FL scheme by adjusting the model size to reduce FL latency. Furthermore, the effect of local pruning on FL convergence was analyzed in [10]. Although the network pruning can reduce the model size of DNN, the resulting information loss also leads to the deterioration of learning behavior.

Overall, through optimizing resource allocation, the works in [1]–[6] improved FL efficiency, but required clients with sufficient computing and storage capacity. Although the model compression methods in [8]–[10] can alleviate these issues, the effects of packet error on FL convergence are ignored. Motivated by this background, we are committed to improve the communication-learning trade-off in an adaptive network pruning supported FL system. The main contributions of this work include:

1) We derive the convergence upper bound of pruned FL, which reveals that both a higher pruning and packet error rate will worsen the convergence rate. Besides, users with more local training samples have greater impact on the achievable upper bound.
2) We formulate a non-convex problem to strike a balance between the communication and learning performance. Through decoupling problem, we provide closed-form solutions for network pruning rates and obtain the optimal bandwidth allocation with bisection method.
3) Numerical simulations are conducted to validate the effectiveness of the proposed schemes. Experimental results show that our solutions can obtain better identification accuracy and lower total cost than benchmarks.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a FL system with network pruning, where there are one base station (BS) and I user equipments (UEs), indexed by \( I = \{1, 2, \ldots, I\} \). Let \( D_i \) denote the local dataset of UE \( i \), where \( K_{i_{\text{max}}} = |D_i| \) denotes the number of samples owned by UE \( i \). At the \( s \)-th round, the BS first broadcasts the latest global model \( W_0 \) to all UEs. Upon reception, each UE prunes it to obtain a pruned model \( W_i^p \) and starts its training with local dataset. Then, all UEs uploads their gradients \( \nabla F(W_i^p) \) (\( \forall i \in I \)) to the BS for global aggregation and model update.

A. Communication Latency Model

The communication latency is defined as the time consumed by completing one communication round of pruned FL, which is composed of four parts as follows:

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1) Model Broadcasting Latency: The achievable downlink transmission rate of UE $i$ is

$$R_{dl}^i = B \log_2 \left( 1 + p^d h^d_{li} / B N_0 \right), \quad (1)$$

where $B$ is the entire bandwidth at the BS, $p^d$ is the transmit power, $h^d_{li}$ is the downlink channel gain from the BS to UE $i$, and $N_0$ denotes the noise power spectral density. The broadcast latency depends on the UE with the worst channel condition, which can be denoted by $t^d = \max_i \left\{ D_M / R_{dl}^i \right\}$, where $D_M$ is the data size of the global model.

2) Training Latency: Upon receiving the global model, each UE prunes it to get a local model. Let $p_i = D_p / D_M$, denote the pruning rate at UE $i$, where $D_p$ is the data size pruned by UE $i$. Using federated stochastic gradient descent (FedSGD), the training latency for UE $i$ is specified as

$$t^c_i = (1 - p_i) K_i d^c / f_i, \quad (2)$$

where $d^c$ denotes the CPU cycles required to compute one sample at the BS [10]. $K_i$ out of $K_i^{max}$ is the number of samples used by UE $i$ for local training and $f_i$ is the available CPU cycles per second at UE $i$. Compared with the training latency $t^c_i$, the time consumed to prune the global model is short, thus neglected here.

3) Gradient Uploading Latency: In this letter, the frequency domain multiple access (FDMA) is adopted for local gradient uploading. The achievable uplink transmission rate at UE $i$ is given by

$$R_{ul}^i = B_i \log_2 \left( 1 + p_i h^u_{li} / B_i N_0 \right), \quad (3)$$

where $B_i$ is the bandwidth allocated to UE $i$, $p_i$ is the maximum transmit power at UE $i$, and $h^u_{li}$ is the uplink channel gain from UE $i$ to the BS. Due to the similarity in the data size of gradient and model, the gradient uploading latency at UE $i$ can be written as $t^u_i = (1 - p_i) D_M / R_{ul}^i$.

4) Global Aggregation Latency: For simplicity, the global aggregation latency is given by a constant $t^a$. To sum up, the FL latency for completing one communication round is given by

$$t = \max_{i \in \mathcal{I}} \left\{ t^d + t^c_i + t^u_i + t^a \right\}. \quad (4)$$

**B. Federated Learning Model**

To characterize the effect of packet error on the convergence rate of pruned FL, we first give the packet error rate at UE $i$ as

$$q_i = 1 - \exp(-m_0 B_i N_0 / p_i h^d_{li}), \quad \text{where } m_0 \text{ is a waterfall threshold} [11].$$

We assume that each local gradient is uploaded as a single packet without retransmissions scheme. When the received local gradient contains errors, the BS will not aggregate it [5]. Thus, the global gradient at the $s$-th round is

$$g_s = \frac{\sum_{i=1}^I K_i \nabla F(\bar{W}_s^i) C(\bar{W}_s^i)}{\sum_{i=1}^I K_i C(\bar{W}_s^i)}, \quad (5)$$

where $C(\cdot)$ is the indicator of packet error, expressed as

$$C(\bar{W}_s^i) = \begin{cases} 1, & \text{with probability } 1 - q_i, \\ 0, & \text{with probability } q_i. \end{cases} \quad (6)$$

With the obtained $g_s$, the global model is updated by $W_{s+1} = W_s - \eta g_s$, where $\eta$ is the learning rate.

**III. CONVERGENCE AND PROBLEM FORMULATION**

**A. Convergence Analysis**

To facilitate analysis, we make the following assumptions.

**Assumption 1:** The loss function $F(\cdot)$ is $\beta$-smooth [10]:

$$F(V) \leq F(W) + (V - W, \nabla F(W)) + \frac{\beta}{2} \| F(V) - F(W) \|^2. \quad (7)$$

**Assumption 2:** The loss $f_{ik}(\bar{W}^i)$ calculated on the $k$-th sample with the local model $W^i$ satisfies:

$$\left\| \nabla f_{ik} (\bar{W}^i) \right\|^2 \leq \xi_1 + \xi_2 \| \nabla F(W) \|^2, \quad (8)$$

where $\xi_1$ and $\xi_2$ are two non-negative constants [5].

**Assumption 3:** The model weight is bounded by a non-negative constant $D$ [10], that is,

$$\mathbb{E} \left[ \| W \|^2 \right] \leq D^2. \quad (9)$$

When Assumptions 1–3 hold, the average $l_2$-norm of global gradient is used by us to evaluate the expected convergence rate of pruned FL, which is given in the following theorem.

**Theorem 1:** Let $d = 1 - 8\xi_2$. The expected convergence rate of pruned FL after $S$ communication rounds is given by

$$\mathbb{E} \left[ \| W_s \|^2 \right] \leq \frac{F(W_0) - F(W^*)}{d (S + 1) / 2\beta} + \frac{2\xi_1}{dK^2} \sum_{i=1}^I K_i \overline{p}_i + \frac{2\beta^2 D^2}{dK^2} \sum_{i=1}^I K_i^2 \overline{p}_i, \quad (10)$$

where $\overline{p}_i$ and $\overline{p}_i$ are the average pruning rate and the average packet error rate at UE $i$ during $S + 1$ rounds, respectively.

**Proof** Please see Appendix A.

Based on Theorem 1, we can find that the average $l_2$-norm of global gradient is bounded by the sum of three terms. The first term is affected by the gap between the initial model and the optimal model, which converges to zero as $S$ goes to infinity. The second term reflects the effect of average packet error rate. A higher average packet error rate $\overline{q}_i$ will result in a larger upper bound of the average $l_2$-norm of global gradient. This result is consistent with the actual situation, since a higher packet error rate means a higher probability that the local gradient may not be applied to the global aggregation,
which reduces the learning performance. The third term is related to the average local pruning rate \( \bar{\rho}_i \). A higher pruning rate causes a greater deviation between the pruned model and global model, thus increases the upper bound.

Since the effects of packet error and network pruning cannot be mitigated by only increasing communication rounds \( S \), we focus on reducing them through optimization.

It can be found from (10) that optimizing the average \( L_2 \)-norm of global gradient is equivalent to optimize the one-round convergence upper bound, which is defined by

\[
g = \psi + m \sum_{i=1}^{t} K_i (q_i + K_i \rho_i),
\]

where \( m = \max\{\delta_{\ell}/dK, 2\beta^2ID^2/dK^2\} \) and \( \psi = 2\beta(F(W_0) - F(W^*)) / d(S + 1) \).

### B. Problem Formulation

Both FL latency and learning performance are important to the practical implementation of pruned FL at the network edge, thus it is desirable to minimize the FL latency in (4) while reducing the upper bound in (11). However, it is difficult to minimize two metrics at the same time. For example, increasing the local pruning rates is helpful to reduce FL latency, but it inevitably degrades the convergence rate. Therefore, to strike a trade-off between the communication and learning performance, we formulated a weighted sum optimization problem, which is given by

\[
\min_{\rho, B} \quad (1 - \lambda) t + \lambda g
\]

\[
\text{s.t.} \quad 0 \leq \rho_i \leq \rho_i^{\max}, \quad \forall i,
\]

\[
\sum_{i=1}^{t} B_i \leq B,
\]

\[
B_i \geq 0, \quad \forall i,
\]

where \( \lambda \in [0, 1] \) is a weight to balance two metrics, \( \rho = [\rho_1, \rho_2, \ldots, \rho_t]^T \) and \( B = [B_1, B_2, \ldots, B_t]^T \) denote the network pruning and bandwidth allocation vectors, respectively.

In constraint (12b), \( \rho_i^{\max} \) denotes the maximum pruning rate at UE \( i \), which depends on the acceptable information loss. Since the total bandwidth is limited, we have constraint (12c). Constraint (12d) restricts the bandwidth allocation to UEs. Due to the close coupling of the optimization variables in (12a), problem (12) is a non-convex optimization problem, which is hard to solve directly. In the following, by introducing an auxiliary variable to transform (12) equivalently, we provide closed-form solutions for UEs’ local pruning rates and bandwidth allocation.

### IV. PROPOSED SOLUTION

Before solving problem (12), we first rearrange (12a) as

\[
F(\rho, B) = (1 - \lambda) \max_i \{t_i^p + t_i^n\} + v + \lambda (g - \psi),
\]

where \( v = (1 - \lambda)(t_i^p + t_i^n) + \lambda \psi \) is independent with all optimization variables. By introducing an auxiliary variable \( \bar{\rho} \), problem (12) can be equivalently transformed to

\[
\min_{\rho, B, i} \quad (1 - \lambda) \bar{\rho} + \lambda m \sum_{i=1}^{t} K_i (q_i + K_i \rho_i)
\]

\[
\text{s.t.} \quad t_i^p + t_i^n \leq \bar{\rho}, \quad \forall i,
\]

\[
(12b) \quad -(12d),
\]

where (14a) is our considered total cost, the weighted sum of FL latency and learning cost. To solve this problem, we decouple it into two sub-problems and derive the corresponding closed-form solutions.

#### A. Optimization of Pruning Rates

Given the bandwidth allocation scheme \( B \), problem (14) can be rewritten as

\[
\min_{\rho, \bar{\rho}} \quad (1 - \lambda) \bar{\rho} + \lambda m \sum_{i=1}^{t} K_i^2 \rho_i
\]

\[
\text{s.t.} \quad (12b) \quad \text{and (14b)},
\]

which is a linear programming problem of \( \rho \) and \( \bar{\rho} \). From (15a), it is always efficient to utilize the minimal pruning rates, which can be derived from (14b) as

\[
\rho_i^{\min}(\bar{\rho}) = \max \left\{ 1 - \frac{D_M / R_i^n + K_i d^c / f_i}{0} \right\}, \quad \forall i.
\]

Substituting (16) into problem (15) yields:

\[
\min \quad (1 - \lambda) \bar{\rho} + \lambda m \sum_{i=1}^{t} K_i \rho_i^{\min}(\bar{\rho})
\]

\[
\text{s.t.} \quad \bar{\rho}_{\min} \leq \bar{\rho} \leq \bar{\rho}_{\max},
\]

where \( \bar{\rho}_{\min} = \max_{i \in I} \{ (D_M / R_i^n + K_i d^c / f_i) (1 - \rho_i^{\max}) \} \) and \( \bar{\rho}_{\max} = \max_{i \in I} \{ D_M / R_i^n + K_i d^c / f_i \} \). The objective function (17a) is a piece-wise linear function, where the required FL latency with no pruning \( t_i^{np} \) is derived from (14b) as

\[
\rho_i^{np} \geq t_i^{np} \geq \ldots \geq t_i^{np}.
\]

Further, we introduce an auxiliary variable \( i^* \). If \( t_i^{np} \geq i^* \), then set \( i^* = i \). Otherwise we search for \( i \) that satisfies \( t_i^{np} \leq i^* \). This to end, the closed-form optimal solution \( i^* \) for the sub-problem (17) is given in the following proposition.

**Proposition 1:** The optimal solution \( i^* \) can be expressed as

\[
i^* = \begin{cases} \bar{\rho}_{\min}, & 1 - \lambda - \lambda m \sum_{i=1}^{t} D_M / R_i^n + K_i d^c / f_i \geq 0, \\ i^*, & \text{otherwise}, \end{cases}
\]

where \( t_i^{np} \) is the inflection point of function (17a).

With calculated \( i^* \), we can calculate the optimal local pruning rate \( \rho_i^{\bar{\rho}} \) of UE \( i \) according to (16).

#### B. Optimization of Bandwidth Allocation

Given \( \rho \) and \( \bar{\rho} \), problem (14) can be simplified as

\[
\min_{B} \quad \lambda m \sum_{i=1}^{t} K_i q_i
\]

\[
\text{s.t.} \quad (12c), (12d) \quad \text{and (14b)},
\]

We temporarily remove the constraint (12c) and use it to verify the feasibility of the solution to the simplified problem latter. Without constraint (12c), problem (19b) can be decoupled to \( I \) independent sub-problems, each related to one UE. The bandwidth allocation subproblem for UE \( i \) is given by

\[
\min_{B_i} \quad m \lambda K_i q_i
\]

\[
\text{s.t.} \quad R_i^n \geq \frac{(1 - \rho_i)D_M}{t - (1 - \rho_i)K_i d^c / f_i},
\]

\[
B_i \geq 0,
\]

where (20b) is transformed from (14b). Rewrite \( q_i \) and \( R_i^n \) into the form of functions, as \( q_i(B_i) \) and \( R_i^n(B_i) \), whose important properties are shown in the following lemma.

**Lemma 1:** Both \( q_i(B_i) \) and \( R_i^n(B_i) \) are monotonically increasing functions of variable \( B_i \).
Algorithm 1: Overall Algorithm for Solving Problem (14)

1: Initialize $\lambda$, $K_i$, $D_M$, $d^e$, $m$, $f_i$.
2: Set the iteration number $j = 1$.
3: repeat
4: Given $B^*(j)$, compute $\hat{r}^*(j)$ using (18).
5: With obtained $\hat{r}^*(j)$, compute $\rho^*(j) = \{\rho^*_i(j)\}$ using (16).
6: Given $\rho^*(j)$, solve (21) with bisection method to obtain $B^{(j+1)} = \{B^{*}_i(j+1)\}$.
7: $j = j + 1$.
8: until the termination criterion is reached.
9: Output the convergent solution $\{\rho^*, B^*, \hat{r}^*\}$.

Proof: This is obtained by computing their first order derivation. The detailed proof is omitted. □

Based on Lemma 1, the optimal bandwidth allocation for UE $i$ equals to the minimum bandwidth under all constraints of problem (20), which satisfies

$$R_i^u(B_i^*) = \frac{(1 - \rho_i)D_M}{\sum_i (1 - \rho_i_K_i d^e / f_i)}.$$  \hspace{1cm} (21)

With the monotonicity of $R_i^u(B_i^*)$, $B_i^*$ can be obtained by bisection method and its optimality is given in the following lemma.

Lemma 2: Since $\sum_{i=1}^{K} B_i^* \leq B$ always holds, $B^*$ is the optimal solution of problem (19).

Proof: We denote the bandwidth allocation in the $n$-th iteration as $B^{(n)} = \{B_1^{(n)}, B_2^{(n)}, \ldots, B_i^{(n)}\}$. In the $(n+1)$-th iteration, since $B_i^{(n+1)}$ is the minimal feasible bandwidth under all constraints of (20), we have $B_i^{(n+1)} \leq B_i^{(n)}$, $\forall i$. Thus, we can get $\sum_{i=1}^{K} B_i^{(n+1)} \leq \sum_{i=1}^{K} B_i^{(n)} \leq B$. □

Therefore, our algorithm is summarized in Algorithm 1. The complexity of one iteration is $O(I\log_2(1/\delta))$, where $\delta$ is the convergence accuracy of bisection method.

V. SIMULATION RESULTS

We examine our theoretical results in a pruned FL system with $I = 5$ users. Each UE’s CPU cycle is set as 5 GHz and other parameters are shown in Table I. The shallow neural network ($\eta = 10^{-3}$) and a DNN ($\eta = 10^{-4}$) are trained on the MNIST and Fashion-MNIST dataset, respectively.\(^1\) Cross-entropy is adopted as the loss function.

Along with the exhaustive search with exponential complexity, the following schemes are also considered as benchmarks.

- Greedy bandwidth allocation (GBA): Bandwidth allocation is proportional to the reciprocal of channel gain.
- Fixed pruning rate (FPR): The pruning rates for users are preset as constants, e.g., $\rho_i \in \{0, 0.35, 0.7\}, \forall i$.
- Ideal FL: All local models are not pruned. Meanwhile, the packet error rates are assumed to be zero.

Fig. 2 presents the impact of the maximum transmit power on the total cost. The total cost decreases as the maximum transmit power at UEs grows. That is because higher transmit power is helpful to reduce FL latency and learning cost. When UEs’ maximum transmit power is low, higher pruning rates help to reduce the total cost. With the increased maximum transmit power, the importance of FL latency relative to learning cost reduces, so reducing pruning rates brings benefits. From this figure, we observe that the proposed solution outperforms baselines such as GBA and FPR algorithms, and is close to the exhaustive search. Then we investigate the impact of the data size of global model in Fig. 3. In the case of low data size, the proposed solution, GBA algorithm and FPR algorithm with $\rho_i = 0$ show similar performance, because the total bandwidth is sufficient to transmit the low data size local gradient even with no pruning. As data size increases, the proposed solution is always close to exhaustive search and enjoys a significant performance gain than others. Finally, the impact of $\lambda$ is studied in Fig. 4. As $\lambda$ grows, the system is more concentrated on the minimization of learning cost. To this end, FL latency increases while learning cost decreases.

Fig. 5 shows the test accuracy of training a shallow neural network on MNIST dataset. The results are averaged over twenty simulations. Without considering pruning and packet error, the accuracy of the ideal FL is usually the highest. The FPR algorithm with $\rho_i = 0$ shows a slightly lower convergence rate than that of the ideal FL algorithm because of the packet error. Note that the test accuracy of our proposed solution is usually slightly lower than that of ideal FL and FPR algorithm with $\rho_i = 0$, because it prunes the local models to reduce FL latency. Due to the high pruning rates, the accuracy achieved

\(^1\)The shallow neural network has one hidden layer of 60 neurons. The DNN has two hidden layers of 60 and 20 neurons, respectively.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $p_i(V_i)$ | 23 dBm | $\rho_i^\text{max}(V_i)$ | 0.7 |
| $D_M$ | 1.6 Mbit/s | $B$ | 15 MHz |
| $m_0$ | 0.023 dB | $\lambda$ | 0.0004 |
| $N_0$ | -174 dBm/Hz | $d^e$ | 0.168 GHz |

$K_i \in \{30, 40, 50\}, V_i \in \mathcal{Z}$, local SGD step 1
by the FPR algorithm with $\rho_1 = 0.7$ is the lowest. In Fig. 6, we present the test accuracy of training a DNN on Fashion-MNIST dataset. We notice that the proposed algorithm is close to the ideal FL, while the achievable performance of FPR algorithm with $\rho_1 = 0.7$ is extremely lower than others. Besides, a smaller $\lambda$ leads to lower accuracy of our algorithm.

VI. CONCLUSION

In this letter, a network pruning supported wireless FL system was studied. We first theoretically analyzed the effect of the packet error and local pruning rates on the FL convergence upper bound. By capturing the trade-off between communication and learning, the closed-form solutions were derived to solve the formulated non-convex problem. Numerical results demonstrated that our proposed scheme can reduce the total cost while maintaining the learning performance.

APPENDIX: PROOF OF THEOREM 1

The update function of global model can be rewritten as $W_{s+1} = W_s - \eta (\nabla F(W_s) - g_s)$, where $o = \nabla F(W_s) - g_s$. Let $\eta = \frac{1}{I}$ and further take the expectation of both sides of assumption 1, we have:

$$\mathbb{E} [ F(W_{s+1}) ] \leq F(W_s) - \frac{\mathbb{E} [ ||\nabla F(W_s)||^2 ]}{2\beta} + \frac{\mathbb{E} [ ||o||^2 ]}{2\beta}.$$  (22)

Note that $K = \sum_{i=1}^{I} K_i$, we further derive $\mathbb{E} [ ||o||^2 ]$ as

$$\mathbb{E} [ ||o||^2 ] = \mathbb{E} [ ||\nabla F(W_s) - \nabla F(W_s') + \nabla F(W_s') - g_s||^2 ] \leq 2\mathbb{E} [ \frac{1}{K} \sum_{i=1}^{I} K_i \nabla F(W_s') - \nabla F(W_s) ||^2 ] + 2\mathbb{E} [ ||\epsilon||^2 ] \leq \sum_{i=1}^{I} \frac{K_i^2 \mathbb{E} [ ||\nabla F(W_s') - \nabla F(W_s)||^2 ]}{K^2/2I} + 2\mathbb{E} [ ||\epsilon||^2 ],$$  (23)

where $\epsilon = \nabla F(W_s') - g_s$. Similar to the proof in Appendix A of [5], we have

$$\mathbb{E} [ ||\epsilon||^2 ] \leq \frac{4}{K} \mathbb{E} [ \xi_1 + \xi_2 ||\nabla F(W_s)||^2 ] \sum_{i=1}^{I} K_i q_i.$$  (24)

With (23) and (24), the $l_2$-norm of gradients is bounded by

$$\mathbb{E} [ ||\nabla F(W_s)||^2 ] \leq \mathbb{E} [ F(W_s) ] - \mathbb{E} [ F(W_{s+1}) ]$$

$$+ \frac{4\xi_1}{\beta K} \sum_{i=1}^{I} K_i q_i + \frac{\beta I}{K} \sum_{i=1}^{I} K_i^2 \mathbb{E} [ ||W_s - \tilde{W}_s||^2 ],$$  (25)

where $A = \left( K - 8\xi_2 \sum_{i=1}^{I} K_i q_i \right) / 2\beta K$, which should be greater than zero to guarantee convergence. Because $2\beta \leq 1/A \leq 2\beta / (1 - 8\xi_2)$, we substitute the upper bound of $1/A$ into the right hand of (25):

$$\mathbb{E} [ ||\nabla F(W_s)||^2 ] \leq \frac{2\beta \mathbb{E} [ F(W_s) ] - \mathbb{E} [ F(W_{s+1}) ]}{d}$$

$$+ \frac{8\xi_1}{dK} \sum_{i=1}^{I} K_i q_i + \frac{2\beta^2 I D^2}{dK^2} \sum_{i=1}^{I} K_i^2 \rho_i,$$  (26)

where $d = 1 - 8\xi_2$. Sum up inequalities from $s = 0$ to $s = S$, the average $l_2$-norm of gradients is derived as

$$\frac{1}{S+1} \sum_{s=0}^{S} \mathbb{E} [ ||\nabla F(W_s)||^2 ] \leq \frac{F(W_0) - F(W^*)}{d (S+1) / 2\beta} + \frac{8\xi_1 \sum_{i=1}^{I} K_i q_i}{dK} + \frac{2\beta^2 I D^2 \sum_{i=1}^{I} K_i^2 \rho_i}{dK^2}.$$  (27)

where (b) stems from $\mathbb{E} [ ||W_s - \tilde{W}_s||^2 ] \leq \rho_i \mathbb{E} [ ||W_s||^2 ] \leq \rho_i D^2$ and the fact $F(W^*) \leq F(W_{s+1})$.

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