Primordial statistical anisotropy generated at the end of inflation

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Abstract. We present a new mechanism for generating primordial statistical anisotropy of curvature perturbations. We introduce a vector field which has a non-minimal kinetic term and couples with a waterfall field in a hybrid inflation model. In such a system, the vector field gives fluctuations of the end of inflation and hence induces a subcomponent of curvature perturbations. Since the vector has a preferred direction, the statistical anisotropy could appear in the fluctuations. We present the explicit formula for the statistical anisotropy in the primordial power spectrum and the bispectrum of curvature perturbations. Interestingly, there is the possibility that the statistical anisotropy does not appear in the power spectrum but does appear in the bispectrum. We also find that the statistical anisotropy provides the shape dependence to the bispectrum.

Keywords: cosmological perturbation theory, inflation, physics of the early universe

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1. Introduction

Current cosmological observations such as cosmic microwave background (CMB) observations tell us that our universe is likely to be homogeneous and isotropic. However, there is the possibility that the effect of rotational invariance violation has already been detected in the CMB temperature anisotropies, as an apparent alignment of the CMB multipoles on very large scales, the so-called ‘Axis of Evil’ [1]–[22]. If it is true, the statistical anisotropy might stem from the physics in the early universe. Hence, the possibility of statistical anisotropy deserves further investigation.

On the theoretical side, statistically homogeneous, isotropic and Gaussian density fluctuations had been supposed to be a robust prediction of the inflationary scenario. However, the possibility of statistical non-Gaussianity has been studied extensively. It turns out that it is not difficult to make a non-Gaussianity if we incorporate the multi-scalar fields into the system. In fact, there are many scalar fields in any realistic particle physics model. Hence, it is natural to test the non-Gaussianity seriously by making use of observational data. For the same reason, in this paper, we would like to argue that the statistical anisotropy should be checked observationally. The point is that there are many vector fields in addition to scalar fields in the particle physics model. As we will see, the vector field could violate the rotational invariance at the end of inflation if it couples with a waterfall field in a hybrid inflation model. That means the statistical anisotropy could be generated due to the ubiquitous vector fields. Of course, the curvature perturbations induced by the vector field should be subdominant in the total curvature perturbations in order to reconcile the scenario with the current data. Nevertheless, the effect of the vector field should not be overlooked under the current precision cosmology.

This paper is organized as follows. In section 2, we show a new mechanism generating the statistical anisotropy at the end of inflation. This model is based on [23]–[29], in which the authors consider the modified hybrid inflation adding a light scalar field.
other than inflaton in order to generate the large non-Gaussianity of primordial curvature perturbations. We use a massless vector field instead of a light scalar field. The vector field violates the rotational invariance and, as a consequence, generates the statistical anisotropy in the curvature perturbations. In section 3, we present the calculation of vector fluctuations. It allows us to explicitly show the effects of the violation of rotational invariance on the power spectrum and the bispectrum of primordial curvature perturbations. We also provide a simple example as an illustration. We find that the statistical anisotropy in the bispectrum could be more significant than that in the power spectrum. In section 4, we give a brief summary.

2. Basic idea

In this section, we give a basic idea of the new mechanism for generating the statistical anisotropy in primordial curvature perturbations. The explicit calculation will be given in the next section.

In the original hybrid inflation model, when the effective mass squared of a waterfall field becomes negative, i.e. the waterfall field becomes tachyonic, the tachyonic instability makes the waterfall field roll down toward the true vacuum state and the inflation suddenly ends. The critical value $\phi_e$, which is the value of the inflaton $\phi$ at the end of inflation, should be determined by the model parameter and hence $\phi_e$ does not fluctuate. However, if some other fields couple with the waterfall field, the situation would be different.

Let us consider a system constructed by two scalar fields, say, an inflaton $\phi$ and a waterfall field $\chi$, and a vector field $A_\mu (\mu = 0, 1, 2, 3)$ which couples with the waterfall field. The action can be written as

$$S = \frac{1}{2} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi \partial_\nu \phi + \partial_\mu \chi \partial_\nu \chi) + V(\phi, \chi, A_\mu) \right]$$

$$- \frac{1}{4} \int d^4x \sqrt{-g} g^{\mu\nu} g^{\rho\sigma} f^2(\phi) F_{\mu\nu} F_{\rho\sigma}, \quad (1)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the vector field, $V(\phi, \chi, A_\mu)$ is the potential of fields and an arbitrary function $f(\phi)$ represents gauge coupling. We have to choose $V(\phi, \chi, A_\mu)$ so as not to violate the gauge invariance. For the background universe to be isotropic, we need the condition that the energy of the inflaton should be dominant in the total energy of the universe during the inflation era. Under this condition, we have a small expectation value of the vector field. Here, we do not specify the form of the potential but we assume that the vector field is massless and has a small expectation value compared with the inflaton. Hence, we neglect the terms which come from the coupling with the vector field in the background equations of motion for the scalar fields and we treat the vector field perturbatively.

2.1. $\delta N$ formalism

First, we briefly review the $\delta N$ formalism [30]–[33] (or separate universe approach [34]), which is well known as the most powerful tool to calculate the evolution of curvature perturbations on super-horizon scales. Based on the $\delta N$ formalism, the curvature perturbations on super-horizon scales, where spatial gradients and anisotropic shear are
small, can be written by
\[ \zeta(t_F) \simeq \delta N(t_F, t_*), \tag{2} \]
where \( \delta N(t_F, t_*) \) represents the perturbation of the e-folding number \( N \) measured in a homogeneous background universe between the initial flat hypersurface at \( t = t_* \) and the final uniform energy density hypersurface at \( t = t_F \). More precisely, the e-folding number is given by \( N = \int_{t_*}^{t_F} H \, dt \), with the Hubble parameter \( H \). When we take the final time \( t_F \) to be a certain time after the complete convergence of the background trajectories in phase space, \( \zeta \) becomes constant. Let the initial time \( t_* \) be some time during the slow-roll inflation driven by the inflaton. Soon after the time that the scale of interest crosses the horizon, we can rewrite equation (2) as a Taylor expansion up to second order
\[ \zeta(t_F) = N_{I*} \delta \phi_1 + \frac{1}{2} N_{IJ*} \delta \phi_1 \delta \phi_2, \tag{3} \]
where \( N_{I*} \equiv \partial N(t_F, t_*)/\partial \phi^I(t_*) \) and \( N_{IJ*} \equiv \partial^2 N(t_F, t_*)/\partial \phi^I(t_*) \partial \phi^J(t_*) \). Here, we consider that the background trajectories are parameterized by the scalar field \( \phi^I \). Hence, the perturbations of the scalar field \( \delta \phi^I \) give rise to the difference between the evolutions of the adjacent background homogeneous trajectories.

### 2.2. Curvature perturbation generated at the end of inflation

As we mentioned in section 2.1, the curvature perturbations on super-horizon scales are given by the perturbations of e-folding number \( \delta N \). In the standard single scalar inflation or hybrid inflation, the inflation ends when the inflaton \( \phi \) reaches a critical value \( \phi_e \). On the other hand, in the multi-component inflation, the critical value \( \phi_e \) may fluctuate due to a light field other than the inflaton \( \phi \). Hence, in such a situation, there is a possibility of generating the curvature perturbations through the fluctuation of \( \phi_e \). Based on the models introduced by references [23, 24], in this paper, we introduce a massless vector field \( A^\mu \) as another light field. Using the \( \delta N \) formula, the curvature perturbation generated at the end of inflation \((t = t_e)\) can be expressed as
\[ \zeta_{\text{end}} = \frac{\partial N}{\partial \phi_e} \delta \phi_e + \frac{1}{2} \frac{\partial^2 N}{\partial \phi_e^2} \delta \phi_e^2 \]
\[ = \frac{\partial N}{\partial \phi_e} \frac{d \phi_e(A)}{d A^i} \delta A^i + \frac{1}{2} \left[ \frac{\partial N}{\partial \phi_e} \frac{d^2 \phi_e(A)}{d A^i d A^j} + \frac{\partial^2 N}{\partial \phi_e^2} \frac{d \phi_e(A)}{d A^i} \frac{d \phi_e(A)}{d A^j} \right] \delta A^i \delta A^j, \tag{4} \]
where we set \( A^0 = 0 \). That is, \( \phi_e \) can fluctuate due to fluctuations of the vector field \( A^i \). Let us take the hypersurface at the end of inflation \( t = t_e \) to be that of a uniform energy density. Then, the total curvature perturbations at the end of inflation \( t = t_e \) is given by
\[ \zeta(t_e) = \zeta_{\text{inf}} + \zeta_{\text{end}}, \tag{5} \]
where
\[ \zeta_{\text{inf}} = \frac{\partial N}{\partial \phi_*} \delta \phi_* + \frac{1}{2} \frac{\partial^2 N}{\partial \phi_*^2} \delta \phi_*^2. \tag{6} \]

We assume that all fields decay into a radiation component through the sudden reheating just after the end of inflation. Under this assumption, the evolution of the universe
can be described by one parameter, namely, radiation energy density, after the end of inflation. Hence, the curvature perturbations on a uniform energy density hypersurface become constant after the end of inflation and the iso-curvature mode does not persist until the present. At the leading order, the power spectrum of curvature perturbations is given by

\[ \langle \zeta_k \zeta_{k_2} \rangle = P_\zeta(k_1) \delta^{(3)}(k_1 + k_2) = N^2 e \frac{d^2 \phi_e (A)}{dA^2} \frac{d^2 \phi_e (A)}{dA^2} \langle \delta A_{*k_1}^i \delta A_{*k_2}^j \rangle, \]

where \( N_e = \partial N/\partial \phi_e \), \( N_r = \partial N/\partial \phi_r \) and \( \langle \delta A_{*k_1}^i \delta A_{*k_2}^j \rangle \) is the power spectrum of the perturbation of the vector field, which we will calculate in the next section. Here, we neglect the cross-terms such as \( \langle \delta \phi_{el} \delta \phi_{ek} \rangle \) and assume that \( \delta \phi_r \) has pure Gaussian distributions. To the leading order, the bispectrum is also given by

\[ \langle \zeta_k \zeta_{k_2} \zeta_{k_3} \rangle \equiv (2\pi)^{-3/2} B_\zeta(k_1, k_2, k_3) \delta^{(3)}(k_1 + k_2 + k_3) = (2\pi)^{-3/2} N^2 e N_{\text{ee}} [P_\phi(k_1) P_\phi(k_2) + 2 \text{ perms}] \delta^{(3)}(k_1 + k_2 + k_3) \]

\[ + N^2 e \frac{d^2 \phi_e (A)}{dA^2} \frac{d^2 \phi_e (A)}{dA^2} \langle \delta A_{*k_1}^i \delta A_{*k_2}^j \delta A_{*k_3}^l \rangle \]

\[ + N^2 e \frac{d^2 \phi_e (A)}{dA^2} \frac{d^2 \phi_e (A)}{dA^2} \left( \frac{1}{N_e^2 dA^2} + \frac{N_e d^2 \phi_e (A) d^2 \phi_e (A)}{N_e^2 dA^2} \right) \]

\[ \times \left[ \langle \delta A_{*k_1}^i \delta A_{*k_2}^j \delta A_{*k_3}^l \rangle + 2 \text{ perms} \right], \]

where \( N_{\text{ee}} = \partial^2 N/\partial \phi_e \partial \phi_e \), \( N_r = \partial^2 N/\partial \phi_r \partial \phi_e \) and \( * \) denotes the convolution.

From these expressions, we find that one can obtain the curvature perturbations which have the direction dependence due to the vector field perturbations.

3. Primordial statistical anisotropy

In this section, we first quantize the vector field with a non-minimal kinetic term and obtain the power spectrum of the vector fluctuations. Next, we derive the explicit formula for the power spectrum and the bispectrum of curvature perturbations. Then, we present a simple example as an illustration.

3.1. The spectrum of vector perturbations

Here, we give the spectrum of vector perturbations, following reference [35], which appear in the expression of the spectrum of curvature perturbations (7) and (8).

Following reference [35], we adopt the Coulomb gauge where \( \delta A_0 (x^\mu) = 0 \) and \( \partial_j A^j (x^\mu) = 0 \). Then, we obtain the evolution equation for the linear perturbation as

\[ A''_i - \frac{f''}{f} A_i - a^2 \partial_j \partial^j A_i = 0, \]

where \( A_i \equiv f \delta A_i \), a prime represents the derivative with respect to a conformal time, \( \tau = \int dt/a \), and we neglected the potential term. Here, we assumed the gauge invariance
for the vector field. Hence, there should be a derivative coupling between the vector field and the waterfall field. However, this term is not relevant to our analysis.

Now, we can expand the vector field in terms of annihilation and creation operators, $\hat{a}_\lambda^\dagger$ and $\hat{a}_\lambda$, as

$$A_i(\tau, x^i) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\lambda = 1}^2 \epsilon_{i\lambda}(k) \left[ v_k(\tau) \hat{a}_\lambda e^{ik\cdot x} + v_k^*(\tau) \hat{a}_\lambda^\dagger e^{-ik\cdot x} \right],$$

(10)

where $k$ is the comoving wavenumber and the transverse polarization vectors $\epsilon_{i\lambda}$ defined by

$$\sum_{\lambda = 1}^2 \epsilon_{i\lambda}(k) \epsilon_{j\lambda}(k) = \delta_{ij} - \frac{k_i k_j}{k^2},$$

(11)

are introduced for a consistent quantization in the Coulomb gauge. With the change of the variable $\psi_k = a \hat{a}_k$, the equation of motion for the mode function $\psi_k$ is given by

$$\psi_k'' + \left( k^2 - \frac{f''}{f} \right) \psi_k = 0.$$  

(12)

Using this variable, we obtain the power spectrum of vector perturbations as

$$\langle \delta A_i(k) \delta A_j(k') \rangle = \left| \frac{\psi_k}{a^2 f} \right|^2 \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \delta^{(3)}(k + k'),$$

(13)

where $k_i = \delta_{ij} k^j$. In the short-wavelength limit $k^2 \to \infty$, from equation (12), we see one can take the Bunch–Davies vacuum

$$\psi_k \to \frac{1}{\sqrt{2k}} e^{-ik\tau}.$$  

(14)

Let us consider the case in which the vector perturbations $\delta A_i$ stay constant in time on super-horizon scales. From equation (13), we have to obtain $|\psi_k| \propto a(\tau) f(\tau)$ in order to achieve such a situation. In the slow-roll limit ($\epsilon = -\dot{H}/H^2 \to 0$), we have

$$-\tau = \frac{1}{aH} (1 + O(\epsilon)).$$

(15)

Hence, we assume that $d\tau = da/(a^2 H)$ and assuming that the form of the gauge coupling function $f \propto a^\alpha$, we have

$$\psi_k'' + \left( k^2 - \frac{\alpha(\alpha + 1)}{\tau^2} \right) \psi_k = 0.$$  

(16)

With the Bunch–Davies vacuum (14), this equation can be solved easily as

$$\psi_k = \left( \frac{\pi}{4k} \right)^{1/2} \exp \left[ i(\alpha + 1) \frac{\pi}{2} \right] (-k\tau)^{1/2} H_{\alpha + 1/2}^{(1)}(-k\tau),$$

(17)

where $H_{\nu}^{(1)}(x)$ is the Hankel function of the first kind. If we take $\alpha = -1$, in the long-wavelength limit $-k\tau \to 0$, we obtain the asymptotic solution for $\psi_k$ as

$$\psi_k(\alpha = -1) \to \frac{-i k \tau}{\sqrt{2k}} (1 + O((-k^2)^2)) + \frac{1}{\sqrt{2k}} (1 + O((-k^2)^2)).$$

(18)
Hence, we achieve $\langle \delta A_i \delta A_j \rangle \sim \text{constant in time on super-horizon scales} \ (k/aH \to 0)$. Thus, we have the vector power spectrum

$$
\langle \delta A_{i+k} \delta A_{j+k'} \rangle = \frac{H_s^2}{2k^3f_s^2} \left( \delta^{ij} - \frac{k_i k_j}{k^2} \right) \delta^{(3)}(k+k')
$$

$$
= P_{\phi}(k)f_s^{-2} \left( \delta^{ij} - \frac{k_i k_j}{k^2} \right) \delta^{(3)}(k+k'),
$$

where we defined $P_{\phi}(k) = H_s^2/2k^3$. Strictly speaking, in order to evaluate the bispectrum we need to calculate $\langle \delta A_{i+k_1} \delta A_{j+k_2} \delta A_{\ell+k_3} \rangle$. However, as we saw in this subsection, initially the vector perturbation in Coulomb gauge can be considered as the Gaussian perturbation as well as the perturbation of a scalar field in the standard slow-roll inflation model. In the standard slow-roll inflation model, as is well known, the three-point correlation function of the perturbation of the scalar field is suppressed by the slow-roll parameters [36], that is, we can consider that the perturbations of the scalar field at the horizon crossing time $t = t_*$ almost obey Gaussian statistics. In this model, the vector field can be considered to be slowly rolling, so the deviation from Gaussian statistics of the vector perturbations $\delta A_i$ at $t = t_*$ would be too small to be detected in future experiments, e.g. Planck [37,38]. Hence, we assume, here, that the non-Gaussian feature of the vector field perturbation at $t = t_*$ is negligible.

3.2. Rotation violating power spectrum and bispectrum

Now, we are in a position to write down the formula for the power spectrum and the bispectrum of curvature perturbations.

Using equation (19), we can rewrite the expression for the power spectrum of curvature perturbations in the modified hybrid inflation model (7) as

$$
\langle \zeta_{k_1} \zeta_{k_2} \rangle = P_\zeta(k_1)\delta^{(3)}(k_1 + k_2) = \left[ N_s^2 + \left( \frac{N_{e}}{f_s} \right)^2 q_i q_j \left( \delta^{ij} - \frac{k_i^2 k_j^2}{k^2} \right) \right] P_{\phi}(k_1)\delta^{(3)}(k_1 + k_2),
$$

where we have defined $q_i \equiv d\phi_e/dA^i$, $q_{ij} \equiv d^2\phi_e/dA^i dA^j$. In order to compare this expression with that of [18], $P_\zeta$ can be rewritten as

$$
P_\zeta(k) = P^{\text{iso}}_\zeta(k) \left[ 1 + g_\beta (\hat{q} \cdot \hat{k})^2 \right],
$$

where we used the notation $\hat{q} = q/|q|$, $\hat{k} = k/|k|$ and the isotropic part

$$
P^{\text{iso}}_\zeta(k) = \frac{P_{\phi}(k)}{2\epsilon_s} (1 + \beta),
$$

is separated. The coefficient of the anisotropic part is

$$
g_\beta = \frac{\beta}{1+\beta}, \quad \beta \equiv \left( \frac{N_{e}}{N_s f_s} \right)^2 |q|^2,
$$

where we defined $N_s = 1/\sqrt{2\epsilon_s}$. Here, $\epsilon \equiv V_{\phi}^2/2V^2$ is a slow-roll parameter.
The bispectrum (8) also becomes

\[ B_\zeta(k_1, k_2, k_3) = \left\{ \left[ N_s^2 N_{ss} + \left( \frac{N_e}{f_\zeta} \right)^4 q_1 q_2 \left( \frac{1}{N_e} q_1 t_2 + \frac{N_{et}}{N_e^2} q_1 q_2 \right) \left( \delta t_1 - \frac{k_1^2 k_1^2}{k_2^2} \right) \right. \right. \]
\[ \times \left( \delta t_2 - \frac{k_2^2 k_2^2}{k_2^2} \right) \right\} P_\phi(k_1) P_\phi(k_2) + 2 \text{ perms}, \]

(24)

where we assumed that \( \delta A_\zeta \) is Gaussian. Now, we define the nonlinear parameter \( f_{NL} \) as the bispectrum normalized by the isotropic part of the power spectrum \( P_{\zeta\text{iso}}(k) \):

\[ \frac{6}{5} f_{NL}(k_1, k_2, k_3) \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_{\zeta\text{iso}}(k_1) P_{\zeta\text{iso}}(k_2) + 2 \text{ perms}} = \left[ \frac{F(k_1, k_2, k_3)}{\sum_i k_i^3} + 2 \text{ perms} \right], \]

(25)

where we assumed the scale-invariant power spectrum and defined

\[ F(k_1, k_2) \equiv (1 + \beta) - \frac{2 N_s}{N_s^2} + \frac{g_3^2 N_{et}}{N_s^2} \left[ 1 - (\hat{q} \cdot \hat{k}_1)^2 \right] \left[ 1 - (\hat{q} \cdot \hat{k}_2)^2 \right] \]
\[ + \frac{g_3^2 q_1 t_2}{N_s |q|^2} \left[ \delta t_1 - (\hat{q} \cdot \hat{k}_1) \hat{k}_1^3 \right] \left[ \delta t_2 - (\hat{q} \cdot \hat{k}_2) \hat{k}_2^3 \right]. \]

(26)

Strictly speaking, we should have defined the nonlinear parameter by using the full power spectrum instead of the isotropic one. However, the resultant expression is too complicated to display here. When the anisotropic part in the power spectrum is small, the above definition gives a good approximation to the true nonlinear parameter. Indeed, we will discuss this interesting possibility later. Otherwise, we should go back to the original formula for the bispectrum.

Under the slow-roll approximation, we obtain \( N_s / N_s^2 \sim N_{et} / N_s^2 = O(\epsilon) \). Hence, neglecting the first and second terms in the right-hand side of the above equation, we can further simplify the expression (26) as

\[ F(k_1, k_2) \approx \frac{g_3^2 q_1 t_2}{N_s |q|^2} \left[ \delta t_1 - (\hat{q} \cdot \hat{k}_1) \hat{k}_1^3 \right] \left[ \delta t_2 - (\hat{q} \cdot \hat{k}_2) \hat{k}_2^3 \right], \]

(27)

where \( \delta t_1 t_2 = q_1 t_1 t_2 / |q|^2 \). We can decompose the nonlinear parameter into the isotropic part and the anisotropic part as

\[ \frac{6}{5} f_{NL}(k_1, k_2, k_3) = \frac{6}{5} f_{NL}^{\text{iso}}(1 + f_{\text{ani}}(k_1, k_2, k_3)), \]

(28)

where the isotropic part is

\[ \frac{6}{5} f_{NL}^{\text{iso}} = \sqrt{2 \epsilon} g_3^2 \hat{q} i j \hat{q}^j i \]

(29)

and the anisotropic part is deduced as

\[ f_{\text{ani}}(k_1, k_2, k_3) = \frac{g_3^2}{k_1^3 k_2^3 k_3^3} \left\{ \left[ -\hat{q}^i \left( \hat{k}_i^j (\hat{q} \cdot \hat{k}_1) + \hat{k}_i^j (\hat{q} \cdot \hat{k}_2) \right) + \hat{k}_i^j \hat{k}_j^k (\hat{q} \cdot \hat{k}_1) (\hat{q} \cdot \hat{k}_2) \right] \right\} \times \left[ \sum_i k_i^3 \right] + 2 \text{ perms}. \]

(30)

Here, we used the relation \( N_s = 1 / \sqrt{2 \epsilon} \). Taking a look at the above formula, we notice that the statistical anisotropy gives the shape to the bispectrum even for the local model.
3.3. Simple example

Let us consider a simple example given by

\[ V(\phi, \chi, A^i) = \frac{\lambda}{4} (\chi^2 - v^2)^2 + \frac{1}{2} g^2 \phi^2 \chi^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} h^2 A^\mu A_\mu \chi^2, \]  

(31)

where \( \phi, \chi \) and \( A^i \) are the inflaton, the waterfall field and the light vector field, respectively. The coupling constants are denoted by \( \lambda, g, h \), the inflaton mass is given by \( m \) and the vacuum expectation value for \( \chi \) is represented by \( v \). The effective mass term of the vector can be accommodated in the system without violating gauge invariance. Indeed, the potential (31) looks like an Abelian Higgs model in unitary gauge. Namely, \( \chi \) could be the radial direction of a complex scalar field which is invariant under a local \( U(1) \) gauge transformation and \( \phi \) would be a gauge singlet. For this potential, we can realize the hybrid inflation and obtain the evolution of inflaton field, \( \phi \), as

\[ \phi \propto a^{-m^2/V_0}, \]  

(32)

where \( V_0 = \lambda v^4/4 \). Hence, when \( f(\phi) \propto \phi^{V_0/m^2} \), we have \( f \propto a^{-1} \). This behavior was assumed in the previous section 3.1 in order to achieve \( \delta A_i \sim \text{const} \) on super-horizon scales. The effective mass squared of the waterfall field \( \chi \) is given by

\[ m^2_\chi \equiv -\lambda v^2 + g^2 \phi^2 + h^2 A^i A_i, \]  

(33)

where we set \( A^0 = 0 \). At the end of inflation, the equality

\[ \lambda v^2 = g^2 \phi^2 + h^2 A^i A_i \]  

(34)

is satisfied. As is expected, the critical value, \( \phi_c \), depends on \( A^i \). Hence, we have

\[ \hat{q}^i = -\frac{A^i}{|A|}, \quad \hat{q}^{ij} = -\frac{1}{\phi_c} \left( \frac{g^2 \phi_c^2}{h^2 |A|^2} \delta^{ij} + \hat{q}^i \hat{q}^j \right), \]  

(35)

and

\[ \beta \simeq \frac{1}{f_*^2} \left( \frac{h^2 |A|}{g^2 \phi_c} \right)^2, \]  

(36)

where we used the approximation \( N_* \simeq N_e \).

Now, we can write down the power spectrum:

\[ P_\zeta(k) = P^{\text{iso}}_\zeta(k) \left[ 1 - \frac{(1/f_*^2)(h^2 |A|/(g^2 \phi_c))^2}{1 + (1/f_*^2)(h^2 |A|/(g^2 \phi_c))^2} (\hat{q} \cdot \hat{k})^2 \right]. \]  

(37)

It should be noted that the power spectrum is scale-invariant although there is a direction dependence. The magnitude of statistical anisotropy is determined by the parameter, \( g\beta \).

Let us move on to the bispectrum, which is more interesting. Substituting the expressions (35) and (36) into equations (29) and (30), we have the isotropic part of the nonlinear parameter:

\[ \frac{6}{5} f^{\text{iso}}_{NL} \simeq -\eta_6 g^2 \left( 1 + \frac{g^2 \phi_c^2}{h^2 |A|^2} \right), \]  

(38)
where \( \eta = V_{\phi \phi} / V \). The anisotropic part is

\[
\begin{align*}
\mathcal{f}_{\text{ani}}(k_1, k_2, k_3) &= -\left\{ \left[ (\hat{\mathbf{q}} \cdot \hat{\mathbf{k}}_1)^2 + (\hat{\mathbf{q}} \cdot \hat{\mathbf{k}}_2)^2 \right] \frac{k_3^3}{\sum_i k_i^3} + 2 \text{ perms} \right\} \\
&\quad + \left( 1 + \frac{g^2 \phi_c^2}{h^2 |A|^2} \right)^{-1} \left\{ \left[ (\hat{\mathbf{q}} \cdot \hat{\mathbf{k}}_1)^2 (\hat{\mathbf{q}} \cdot \hat{\mathbf{k}}_2)^2 + \frac{g^2 \phi_c^2}{h^2 |A|^2} (\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) (\hat{\mathbf{q}} \cdot \hat{\mathbf{k}}_1) \right] \frac{k_3^3}{\sum_i k_i^3} + 2 \text{ perms} \right\},
\end{align*}
\]

where the relation between wavenumber vectors holds: \( \hat{\mathbf{k}}_1 + \hat{\mathbf{k}}_2 + \hat{\mathbf{k}}_3 = 0 \). From the above expression, we see that the amplitude of the non-Gaussianity \( f_{\text{NL}}^{\text{iso}} \) is dependent on the magnitude of statistical anisotropy, \( g_\beta \). For large \( \beta (\beta \gg 1) \), we can approximate \( g_\beta \approx 1 \), and then the statistical anisotropy appearing in the primordial power spectrum (22) becomes large. On the other hand, for small \( \beta (\beta \ll 1) \), we can approximate \( g_\beta \approx \beta \ll 1 \). Naively, in this limit, the statistical anisotropy is small and the nonlinear parameter \( f_{\text{NL}} \) is also too small to be detected in future experiments. However, equation (29) tells us that there may be a possibility of generating large non-Gaussianity even for the small statistical anisotropy in the power spectrum if we choose a small value for \( f_* \) and a much larger value for \( g^2 \phi_c^2 / (h^2 |A|^2) \) while keeping \( g_\beta \ll 1 \). From equations (28) and (39), we also find that the anisotropic part has the same order as the isotropic part in the bispectrum, in contrast to the power spectrum. Hence, it may be possible to detect the statistical anisotropy in the bispectrum with future experiments. It will give us information about a new physics in the early universe associated with the violation of the rotational invariance.

In this model, it is important to set the initial value of the vector field to be small. In general, the vector field \( A^\mu \) may not be trapped near the ground state. In such a case, however, the prediction contradicts the observations, that is, this case predicts the anisotropic universe. Hence, it is not our universe. Of course, there may be a more attractive scenario which does not resort to the anthropic argument.

### 4. Summary

We presented a new mechanism for generating a primordial statistical anisotropy of curvature perturbations. We have considered a vector field coupled with a waterfall field in a hybrid inflation model. We have shown that, in such a system, the statistical anisotropy could appear not only in the primordial power spectrum but also in the bispectrum of curvature perturbations. From the formula for the power spectrum and bispectrum of the curvature perturbations, we see the magnitude of the direction dependence is related to that of non-Gaussianity of the curvature perturbations. The degree of the anisotropy depends on the coupling parameter between the vector field and the waterfall field and also the coupling between the inflaton field and the waterfall field. Intriguingly, it turned out that there is the possibility that the statistical anisotropy does not appear in the power spectrum but does appear in the bispectrum. It is also interesting to observe that the shape of the bispectrum changes due to the statistical anisotropy. It is known that the shape of the bispectrum is different from model to model. The statistical anisotropy may have to be taken into account when the data is interpreted.
Recently, many authors discussed the statistical anisotropy of CMB temperature anisotropies, the so-called ‘Axis of Evil’. The resulting expressions for the spectrum of curvature perturbation (22), (29) and (30) can be compared with observations. If this statistical anisotropy were detected significantly, the mechanism presented in this paper would explain such effects without contradicting other current observations, just by tuning the coupling parameters.

In this paper, in order to obtain the spectrum of perturbations of the vector field we considered a system in which conformal invariance is violated based on [35] and assumed that $\alpha = -1$, which represents the dependence of the coupling function $f$ on a scale factor $a$. We think that other forms of function $f$ are also possible. It would be interesting to investigate those cases. We can also consider other systems where the conformal invariance is broken [39]–[43]. The model we presented in this paper is just a toy model. We need to investigate realistic models based on high energy physics which can accommodate our new mechanism. This would be an issue for future work.

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