Massive particle pair production and oscillation in Friedman Universe: its consequence on inflation

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Abstract. We study the Friedman equation for the time-varying cosmological term and Hubble function $H$, and quantised field equation for massive modes $M \gg H$. Classical slow components $O(H^{-1})$ and quantum fast components $O(M^{-1})$ are coupled. Numerically solving these equations, we show the production of particle-antiparticle pairs and the oscillation of pair density and pressure. Their quantum-time averages effectively modify the classical equation. We present resultant impacts and consequences on inflation. The obtained relation of spectral index and tensor-to-scalar ratio agrees with recent observations.
1 Introduction

The gravitational particle creation in Friedman Universe expansion is an important theoretical issue [1–4] that has been intensively studied for decades [5–9]. Based on adiabaticity and non-back-reaction approximation for a slowly time-varying Hubble function $H(t)$, one adopted the semi-classical WKB approaches to calculating the particle production rate, which is exponentially suppressed $e^{-M/H}$ for massive particles $M \gg H$. However, the non-adiabatic back-reactions of particle creations on the Hubble function can be large and have to be taken into account. The non-adiabatic back-reactions of massive particle productions have a quantum time scale $\mathcal{O}(1/M)$ that is much smaller than classical Universe evolution time scale $\mathcal{O}(1/H)$. To properly include the back-reaction of particle production on Universe evolution, one should separate fast components $\mathcal{O}(1/M)$ from slow components $\mathcal{O}(1/H)$ in the Friedman equation. Many efforts [10–24] have been made to study non-adiabatic back-reaction and understand massive particle productions without exponential suppression. It is important for reheating, possibly accounting for massive dark matter and total entropy of the present Universe [25–45].

In this letter, we adopt the Chung-Kolb-Long approach [21] to describe the decomposition: scale factor $a = a_{\text{slow}} + a_{\text{fast}}$, Hubble function $H = H_{\text{slow}} + H_{\text{fast}}$, matter density $\rho = \rho_{\text{slow}} + \rho_{\text{fast}}$ and pressure $p = p_{\text{slow}} + p_{\text{fast}}$. The fast components vary much faster in time, but their amplitudes are much smaller than the slow components. The Friedman equations for a flat Universe read

$$H^2 = \frac{8\pi G}{3} \rho; \quad \dot{H} = -\frac{8\pi G}{2}(\rho + p),$$

(1.1)

$\rho \equiv \rho_{M} + \rho_{\Lambda}$, $p \equiv p_{M} + p_{\Lambda}$, and $p_{\Lambda} = -\rho_{\Lambda}$. The second equation is a generalised conservation law for time-varying cosmological term (dark energy) $\rho_{\Lambda}(t) \equiv \ddot{\Lambda}/8\pi G$ [46]. According to the order of small ratio $\lambda$ of fast and slow components, the Friedman equations (1.1) are decomposed into two sets. The slow components $\mathcal{O}(\lambda^0)$ obey the
same equations as usual Friedman equations

\begin{align}
H^2_{\text{slow}} &= \frac{8\pi G}{3}(\rho_M^{\text{slow}} + \rho_\Lambda^{\text{slow}}); \\
\dot{H}_{\text{slow}} &= -\frac{8\pi G}{2}(\rho_M^{\text{slow}} + \rho_\Lambda^{\text{slow}}),
\end{align}

where \( H_{\text{slow}} = \dot{a}_{\text{slow}}/a_{\text{slow}} \). The faster components \( O(\lambda^1) \) obey,

\begin{align}
H_{\text{fast}} &= \frac{8\pi G}{2 \times 3H_{\text{slow}}}(\rho_M^{\text{fast}} + \rho_\Lambda^{\text{fast}}); \\
\dot{H}_{\text{fast}} &= -\frac{8\pi G}{2}(\rho_M^{\text{fast}} + p_M^{\text{fast}}),
\end{align}

where slow components are approximated as constants in time, \( H_{\text{fast}} = \dot{a}_{\text{fast}}/a_{\text{slow}} \) and \( p_\Lambda^{\text{slow,fast}} = -\rho_\Lambda^{\text{slow,fast}} \) for the cosmological term.

We adopt the Park and Fulling approach [10] to describe the fast components of matter density \( \rho_{\text{fast}}^{M} \) and pressure \( p_{\text{fast}}^{M} \), that are attributed to the non-adiabatic production of particle and antiparticle pairs in fast variation \( H_{\text{fast}} = \dot{a}_{\text{fast}}/a_{\text{slow}} \). As a result, we find quantum pair production and oscillation effectively modified the Friedman equation (1.2) and study its consequence on inflation comparing with observations.

## 2 Quantum pair production and oscillation

In the Friedman Universe, a quantised matter field in a volume \( V \) reads

\[ \Phi(x, t) = \sum_n A_n Y_n(x) \psi_n(t), \]

\[ \int_V Y_n(x) Y_{n'}^\dagger(x) h^{1/2} d^3 x = \delta_{nn'}. \]

The \( A_n \) and \( A_n^\dagger \) are time-independent annihilation and creation operators satisfying the commutation relation \( [A_n^\dagger, A_n] = \delta_{n,n'} \). The time-separate equation for \( \psi_n(t) \) is

\[ \partial_t^2 \psi_n(t) + \omega_n(t)^2 \psi_n(t) = 0, \quad \omega_n(t)^2 = \frac{n(n+1)}{a^2(t)} + M^2, \]

and \( \psi_n(t) \partial_t \psi_n^*(t) + \psi_n^*(t) \partial_t \psi_n(t) = i/\alpha^3(t) \). Expressing

\[ \psi_n(t) = \frac{1}{(2V \omega_n)^{1/2}} \left( \alpha_n^*(t) e^{-i \int \omega_n dt} + \beta_n^*(t) e^{i \int \omega_n dt} \right) \]

in terms of \( \alpha_n(t) \) and \( \beta_n(t) \), Equation (2.2) becomes

\[ \partial_t \alpha_n(t) = C_n e^{-2i \int \omega_n dt} \beta_n(t); \]
\[ \partial_t \beta_n(t) = C_n e^{2i \int \omega_n dt} \alpha_n(t), \]

and \( |\alpha_n|^2 - |\beta_n|^2 = 1 \), where \( C_n \equiv 3H\omega_n^{-2}[n(n+1)/3a^2 + M^2/2] \). In an adiabatic process for slowly time-varying \( H_{\text{slow}} \), the particle state \( \alpha_n(0) = 1 \) and \( \beta_n(0) = 0 \).
evolve to $|\alpha_n(t)| \lesssim 1$ and $|\beta_n(t)| \neq 0$. Positive and negative frequency modes get mixed, leading to particle productions of probability $|\beta_n(t)|^2 \propto e^{-M/H_{\text{fast}}}$.

We focus on studying particle production in non-adiabatic processes for rapidly time-varying $H_{\text{fast}}$, $\alpha_n$ and $\beta_n$. Park and Fulling introduced \[10\] the transformation for the lowest lying massive mode $n = 0$,

$$A_0 = \gamma^* B + \delta B^\dagger, \quad B = \delta A_0^\dagger - \gamma A_0,$$

\[(2.5)\]

$[B, B^\dagger] = 1$, and two mixing constants obeying $|\gamma|^2 - |\delta|^2 = 1$. For a given $A_n$ and its Fock space, the state $|N_{\text{pair}}\rangle$ is defined by the conditions $A_n \neq 0 \Rightarrow |N_{\text{pair}}\rangle = 0$ and

$$B^\dagger B |N_{\text{pair}}\rangle = N_{\text{pair}} |N_{\text{pair}}\rangle.$$

\[(2.6)\]

The $B^\dagger$ and $B$ are time-independent creation and annihilation operators of the pair of mixed positive frequency $A_0$ particle and negative frequency $A_0^\dagger$ antiparticle. The state $|N_{\text{pair}}\rangle$ contains $N_{\text{pair}} = 1, 2, 3, \ldots$ pairs, and $|N_{\text{pair}} = 0\rangle$ is the ground state of non-adiabatic interacting system of fast varying $H_{\text{fast}}$ and massive pair production and annihilation \[1\]. It is a coherent superposition of states of particle and anti-particle pairs. In this state $|N_{\text{pair}}\rangle$, the quantum pressure and density of quantised coherent pair field were obtained

$$p^\text{fast}_M = -\frac{M(2N_{\text{pair}} + 1)}{2\pi^2 V} \left\{ \Re[\gamma^* \delta(|\alpha|^2 + |\beta|^2)] + (2|\delta|^2 + 1)\Re(\alpha^* \beta e^{2\Lambda t}) \right\},$$

\[(2.7)\]

and

$$\rho^\text{fast}_M = \frac{M(2N_{\text{pair}} + 1)}{\pi^2 V} \left\{ \Re[\gamma^* \delta^* \alpha \beta] + (|\delta|^2 + 1/2)(|\beta|^2 + 1/2) \right\},$$

\[(2.8)\]

where $\omega_{n=0} = M$, $\alpha_{n=0} = \alpha$ and $\beta_{n=0} = \beta$. The $p^\text{fast}_M$ is positive definite and $\rho^\text{fast}_M$ is negative.

Using $p^\text{fast}_M$ \[(2.7)\] and $\rho^\text{fast}_M$ \[(2.8)\], we search for a solution of fast component Friedman equation \[(1.3)\] and quantum fluctuating mode equations \[(2.4)\] in the period $[-t, t]$ of the microscopic time $t \sim H_{\text{fast}}^{-1}$, which is around the macroscopic time $t_{\text{slow}} \sim H_{\text{slow}}^{-1}$, when the slow components $\rho_{\text{slow}}$, $H_{\text{slow}}$, $\rho_{M,\Lambda}^\text{slow}$ and $p_M^\text{slow}$ are valued, following the Friedman equations \[(1.2)\]. The integrals $\int_t \omega_n dt$ are over the microscopic time $t$ characterised by the Compton time scale $1/M$. Its lower limit is $t = 0$ by setting $t_{\text{slow}} = 0$ as a reference time, when $\alpha_{\text{fast}}(0) = 0$,

$$H_{\text{fast}}(0) = \dot{H}_{\text{fast}}(0) = 0; \quad \alpha(0) = 1, \quad \beta(0) = 0.$$  

The real value $\gamma^* \delta$ condition leads to the time symmetry: $\alpha^\text{fast}(t) = \alpha^\text{fast}(-t)$, $\alpha(t) = \alpha^*(-t)$ and $\beta(t) = \beta^*(-t)$. When $t \leftrightarrow -t$, positive and negative frequency modes

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\[1\] Analogously, we discussed the back and forth processes of massive fermion and antifermion pairs production and annihilation in spacetime $S \leftrightarrow F + F$ in Refs. [24, 45].

\[2\] The renormalisation of $n \neq 0$ contributions was discussed, e.g., using $\zeta(x)$-function regularisation.
interchange. In Ref. [10], $a_{\text{slow}} = 0$, $H_{\text{slow}} = 0$ (i.e., $H = H_{\text{fast}}$, $a = a_{\text{fast}}$) and a small spherical volume $V \sim (H_{\text{fast}})^3$ at the cosmic origin were adopted for studying the avoidance of cosmic singularity for the $\rho_\Lambda = 0$ and curved Universe. Whereas, to study the pair production of massive particles and antiparticles $M \gg H_{\text{slow}}$, we use $a_{\text{slow}} \neq 0$, $H_{\text{slow}} \neq 0$ and the thin spherical layer of the co-moving width $a \lambda_M \approx a_{\text{slow}} \lambda_M$ and the radius $(H a)^{-1} \approx (H_{\text{slow}} a_{\text{slow}})^{-1}$. The layer physical width $\lambda_M$ is a few thousand times of the Compton length $1/M$ and volume $V \sim \lambda_M H_{\text{slow}}^{-2}$, in which massive matter fields are quantised.

Figure 1 shows results for $C_0 = (3/2) H_{\text{fast}}$ and verified condition $|\alpha|^2 - |\beta|^2 = 1$. In the quantum period of microscopic time $t$, the negative quantum pressure $p_{\text{fast}} > 0$ and back-reaction effects lead to the quantum pair oscillation characterised by the frequency $\omega = M$ of massive quantised pair fields. The positive quantum pair density $\rho_{\text{fast}} > 0$ indicates particle creations without $e^{-M/H}$ suppression. It is consistent with increasing Bogoliubov coefficient $|\beta(t)|^2$ that mixes positive and negative energy modes. The sum $\rho_{\text{fast}} + p_{\text{fast}} > 0$ is positive definite, leading to the decreasing $H_{\text{fast}}(t)$ (1.3). As a consequence, for time $t > 0$, the fast components $H_{\text{fast}}$ and $p_{\text{fast}}$ decrease in time, in order for pair production. Whereas for time $t < 0$, $H_{\text{fast}}$ and $\rho_{\text{fast}}$ increases, due to pair annihilation. The small $a_{\text{fast}}(t)$ varies around $a_{\text{slow}}$ at $t_{\text{slow}} \equiv 0$. This phenomenon is dynamically analogous to the plasma oscillation of electron-positron pair production in an external electric filed $E$ [47] and pair production rate is not exponentially suppressed by $e^{-M^2/E}$ [48]. The coherent plasma state of electron-positron pairs is analogous to the coherent pair state $|N_{\text{pair}}\rangle$ (2.6).

3 Modified Friedman equation and inflation

Averaged $\langle \cdots \rangle$ over microscopic time period, non-vanishing fast quantum components have effective contributions to slow classical components, therefore impact on the Friedman equations (1.2) evolving in macroscopic time. In order to study these impacts, us-
ing quantum pressure (2.7), density (2.8) and positive average $\langle \rho_{M}^{fast} + p_{M}^{fast} \rangle \propto M^{2} H_{slow}^{2}$ in Fig. 1, we model effective mass and number densities of massive pair plasma as

$$\rho_{M}^{H} \equiv 2 \chi m^{2} H_{slow}^{2}, \quad n_{M}^{H} \equiv \chi m H_{slow}^{2}; \quad m^{2} \equiv \sum_{f} g_{d}^{f} M_{f}^{2},$$

(3.1)

and classically kinematic pressure $p_{M}^{H} = \omega_{M}^{H} \rho_{M}^{H}$. The $\omega_{M}^{H} \approx 0$ for $m \gg H_{slow}$ and its upper limit is $1/3$. The introduced mass parameter $m$ represents possible particle masses $M_{f}$, degeneracies $g_{d}$ and the mixing coefficient $\delta$ (2.5). While the width parameter $\chi \ll 1$ characterises the width $\lambda_{M} \sim (\chi m)^{-1}$ of thin layer on horizon surface area $4\pi H_{slow}^{-2}$, where massive pair productions dominantly occur $^{3}$. Massive pair plasma energy and number densities (3.1) follow “area” law $\propto H^{2}$, rather than “volume” law $\propto H^{3}$. It is in accordance with the spirit of holographic principle [49–51].

Including effective pair plasma contribution (3.1), the Friedman equations (1.2) is modified as

$$H^{2} = \frac{8\pi G}{3}(\rho_{M} + \rho_{\Lambda} + \rho_{M}^{H}),$$

$$\dot{H} = -\frac{8\pi G}{2}(\rho_{M} + p_{M} + \rho_{M}^{H} + p_{M}^{H}),$$

(3.2)

henceforth sub- and super-scripts “slow” are dropped, $\rho_{M}$ and $p_{M} = \omega_{M} \rho_{M}$ are normal matter density and pressure. Up to macroscopic time $H^{-1}$, we estimate the total number of particles produced inside the Hubble sphere $N \approx n_{M}^{H} H^{-3}/2$ and mean pair production rate w.r.t. macroscopic time

$$\Gamma_{M} \approx \frac{dN}{2\pi dt} \approx \frac{\chi m}{4\pi} \epsilon, \quad \epsilon \equiv -\frac{\dot{H}}{H^{2}}.$$  

(3.3)

This is not a theoretical derivation, but modeling parametrized by $\chi$ and Universe evolution rate $\epsilon < 1$. Here we neglect the back-reactions of slow time-varying components $H, \rho_{\Lambda, M}$ and $p_{\Lambda, M}$ on fast components $H_{fast}, \rho_{M}^{fast}$ and $p_{M}^{fast}$. Equations (3.2) and (3.3) are basic equations to describe inflation, effective mass $m$ and width $\chi$ parameters are fixed by observations.

In inflation, normal matter $\rho_{M} \approx 0$ and $p_{M} \approx 0$ are negligible. Equations (3.2) are governed by cosmological term $\rho_{\Lambda}$ and induced massive pair plasma density $\rho_{M}^{H}$ (3.1) and pressure $p_{M}^{H} \approx 0$. We analytically obtain the inflationary solution of slowly decreasing $H$

$$H \approx H_{*}(a/a_{*})^{-\epsilon}, \quad \epsilon \approx \chi (m/m_{pl})^{2} \ll 1,$$

(3.4)

where $a_{*}$ and $H_{*}$ are the characteristic inflation scale corresponding to the interested quantum modes of pivot scale $k_{*}$ crossed the horizon ($c_{s} k_{*} = H_{*} a_{*}$) for CMB observations. The scalar, tensor power spectra and their ratio read [52]

$$\Delta_{R}^{2} = \frac{1}{8\pi^{2}} \frac{H_{*}^{2}}{m_{pl}^{2} c_{s}}, \quad \Delta_{h}^{2} = \frac{2}{\pi^{2}} \frac{H_{*}^{2}}{m_{pl}^{2}}; \quad r \equiv \frac{\Delta_{h}^{2}}{\Delta_{R}^{2}} = 16 \epsilon c_{s},$$

(3.5)

$^{3}$We approximately obtained $\chi \approx 1.85 \times 10^{-3}$ for massive fermion pair productions in a slowing varying $H_{slow}(t) \ll M$ [24].

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where the time-dependent background sound velocity \( c_s < 1 \), and the spectra index \( n_s \approx 1 - 2\epsilon \) at the leading order of scale-invariance deviations. Based on two CMB observational values at the pivot scale \( k_* = 0.05 \, \text{(Mpc)}^{-1} \) [53]: (i) the spectral index \( n_s \approx 0.965 \), from Eq. (3.4) we obtain
\[
\epsilon \approx \chi (m_*/m_{pl})^2 \lesssim (1 - n_*)/2 \approx 0.0175; \tag{3.6}
\]
(ii) the scalar amplitude \( A_s = \Delta^2 (k_*) \approx 2.1 \times 10^{-9} \), Equation (3.5) gives
\[
H_* = 3.15 \times 10^{-5} (r/0.1)^{1/2} m_{pl}. \tag{3.7}
\]
As a result, the energy-density ratio of pair plasma and cosmological term densities is
\[
\rho_H M^3 \rho_{\Lambda} \bigg|_{H_*} \approx 2 \chi (m_*/m_{pl})^2 \frac{2}{3} \chi \left( \frac{m_*/m_{pl}}{m_*} \right)^2 \approx 1.17 \times 10^{-2}, \tag{3.8}
\]
and \( H_*^2 \approx \rho_H^*/(3m_{pl}^2) \).

The inflation slows down and eventually ends at \( a = a_{end} \) and \( H = H_{end} \),
\[
H_{end} = H_* \exp (-\epsilon N_{end}), \tag{3.9}
\]
where \( N_{end} = \ln (a_{end}/a_*) \) is the e-folding numbers from the inflation scale \( H_* \) to the inflation ending scale \( H_{end} \). It can be preliminarily determined by the inflationary rate being smaller than the mean pair-production rate namely
\[
H_{end} < \Gamma_M = (\chi m_*/4\pi) \epsilon. \tag{3.10}
\]
However, this inequality provides the upper bound on \( H_{end} \), whose value should be calculated by studying the dynamical transition from inflation to reheating. Using Eqs. (3.9) and (3.10), we give the upper limit on the tensor-to-scalar ratio \( r \) in terms of the e-folding numbers \( N_{end} \),
\[
r < 1.01 \times 10^8 \left( \frac{\Gamma_M}{m_{pl}} \right)^2 e^{2\epsilon N_{end}}
= 7.97 \times 10^4 \chi (1 - n_*)^3 e^{(1 - n_*) N_{end}}, \tag{3.11}
\]
where \( \epsilon = \chi (m_*/m_{pl})^2 \approx (1 - n_*)/2 \) (3.6) is used. Non-vanishing \( \chi \) implies \( r \neq 0 \). In Fig. 2, we plot the upper limit (3.11) compared with data.

From Eq. (3.9), the inflation ending scale \( H_{end} \) is given by
\[
H_{end} \approx H_* e^{-(1 - n_*) N_{end}/2} \approx (0.42, 0.35) H_*; \tag{3.12}
\]
for \( N_{end} = (50, 60) \) and \( r = (0.02, 0.028) \). It shows small \( H \)-variation
\[
H_{end}^2 = \frac{\rho_{H_{end}}^*}{3m_{pl}^2} \gtrsim \frac{\rho_M^*}{3m_{pl}^2}; \quad \frac{\rho_{H_{end}}}{\rho_{\Lambda}} \ll 1, \tag{3.13}
\]
Figure 2. On the Figure 5 of Ref. [54], the upper limit (3.11) is plotted for $\chi = 10^{-3}$. The red zone bound by $N_{\text{end}} = 50, 60$ curves agrees with the blue constraint zone. The model name abbreviation was QFC (quantum field cosmology) [55] or $\tilde{\Lambda}$CDM (time-varying $\tilde{\Lambda}$) [56].

and $\rho_{\Lambda}^{\text{end}} \approx 3m_{\text{pl}}^2 H_{\text{end}}^2$. Equations (3.8) and (3.13) imply the time-varying $\Lambda(t) \propto H^2$ “area law” in inflation.

It is worthwhile to mention pre-inflation, where all slow components set zero. The Friedman equations (1.1) become

$$H_{\text{fast}}^2 = \frac{8\pi G}{3}(\rho_{\Lambda}^{\text{fast}} + \rho_{M}^{\text{fast}}), \quad \dot{H}_{\text{fast}} = -\frac{8\pi G}{2}(\rho_{M}^{\text{fast}} + p_{M}^{\text{fast}}),$$  \hspace{1cm} (3.14)

with a spherical volume $V \sim H_{\text{fast}}^{-3}$ in $\rho_{\Lambda}^{\text{fast}}$ (2.7) and $p_{M}^{\text{fast}}$ (2.8). The initial values are (2.9), but $H_{\text{fast}}^2(0) \approx (m_{\text{pl}}^{-2}/3)\rho_{\Lambda}^{\text{fast}}(0) \neq 0$ and $a_{\text{fast}}(0) \neq 0$, due to nontrivial cosmological term $\rho_{\Lambda}^{\text{fast}}(0) = \Lambda(0)/(8\pi G)$ \textsuperscript{4}. Numerically integrating Eqs. (3.14) and (2.4), we show that quantum pair production and oscillation do not decrease the scale factor $a_{\text{fast}}(t)$, which instead exponentially increases, leading to inflation. It shows that the weak energy condition of $\rho_{\Lambda}^{\text{fast}} > 0$ and $\rho_{M}^{\text{fast}} + p_{M}^{\text{fast}} > 0$ is satisfied, but the strong energy condition $\rho_{\Lambda}^{\text{fast}} + 3p_{M}^{\text{fast}} > 0$ is violated, for details see Fig. 4 in Supplemental Material.

4 Discussions

Since dark matter dominates over normal matter today, we suppose that major massive pairs produced in pre-inflation and inflation should be dark-matter particles. In addition to quantum pair oscillating modes, pair plasma oscillation appears, when the pair density $\rho_{M}^{H}$ is large enough. The acoustic wave of the density perturbation $\delta \rho_{M}^{H}/\rho_{M}^{H}$ is formed and described by the sound velocity $c_{s}^{M} = (\partial H_{M}^{H}/\partial p_{M}^{H})^{1/2} = (\omega_{M}^{H})^{1/2}$. The quantum pair oscillating modes and density perturbations exited and reentered the

\textsuperscript{4}The quantum-gravity origin of $\Lambda(0)$ is not an issue here. $\Lambda(0)$ possibly represents the correlation length (characteristic scale) of quantum gravity field theory, $\Lambda(0) \sim \xi^{-2} = m_{\text{pl}}^2/a_{\text{fast}}^2(0)$ [57–59], analogously to $\Lambda_{\text{QCD}}$ of the quantum chromodynamics field theory.
horizon, which should imprint on both CMB and matter density power spectra. Using Eqs. (3.2) and $\epsilon$ (3.3), we recast the scalar spectrum (3.5) as

$$\Delta^2_R(k) \approx \frac{1}{12\pi^2} \frac{\rho_\Lambda}{c_s^2 m^2_{pl}(1 + \omega_H^M)}.$$  \hspace{1cm} (4.1)

From pre-inflation $H > H^*$ to inflation $H \approx H^*$, $\rho_\Lambda$ and $c_s$ are almost constants, and the variation $\omega_H^M$ is $1/3$ at most. Therefore the scalar spectrum $\Delta^2_R(k)$ (4.1) decreases $3/4$, as the scalar spectrum goes to the large distance scale of CMB observations, exploring high-energy scale of horizon crossing. This probably explains the large-scale anomaly of the low amplitude of CMB power spectrum at low-$\ell$ multipole, namely the CMB power spectrum drops $3/4$ at $\ell = 2$. These discussions are preliminary and further studies are required.

References

[1] L. Parker, Particle creation in expanding universes, Phys. Rev. Lett. 21 (1968) 562.

[2] L. Parker, Quantized fields and particle creation in expanding universes. ii, Phys. Rev. D 3 (1971) 346.

[3] L. Parker, Quantized fields and particle creation in expanding universes. i, Phys. Rev. 183 (1969) 1057.

[4] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, Cambridge Monographs on Mathematical Physics. Cambridge Univ. Press, Cambridge, UK, 2, 1984, 10.1017/CBO9780511622632.

[5] E. Mottola, Particle creation in de sitter space, Phys. Rev. D 31 (1985) 754.

[6] S. Habib, C. Molina-Paris and E. Mottola, Energy momentum tensor of particles created in an expanding universe, Phys. Rev. D 61 (2000) 024010 [gr-qc/9906120].

[7] P. R. Anderson and E. Mottola, Instability of global de sitter space to particle creation, Phys. Rev. D 89 (2014) 104038 [1310.0030].

[8] P. R. Anderson and E. Mottola, Quantum vacuum instability of “eternal” de sitter space, Phys. Rev. D 89 (2014) 104039 [1310.1963].

[9] A. Landete, J. Navarro-Salas and F. Torrenti, Adiabatic regularization and particle creation for spin one-half fields, Phys. Rev. D 89 (2014) 044030 [1311.4958].

[10] L. Parker and S. A. Fulling, Quantized matter fields and the avoidance of singularities in general relativity, Phys. Rev. D 7 (1973) 2357.

[11] L. H. Ford, Gravitational particle creation and inflation, Phys. Rev. D 35 (1987) 2955.

[12] E. W. Kolb, A. D. Linde and A. Riotto, Gut baryogenesis after preheating, Phys. Rev. Lett. 77 (1996) 4290 [hep-ph/9606260].
[13] B. R. Greene, T. Prokopec and T. G. Roos, *Inflaton decay and heavy particle production with negative coupling*, Phys. Rev. D 56 (1997) 6484 [hep-ph/9705357].

[14] E. W. Kolb, A. Riotto and I. I. Tkachev, *Gut baryogenesis after preheating: Numerical study of the production and decay of x bosons*, Phys. Lett. B 423 (1998) 348 [hep-ph/9801306].

[15] D. J. H. Chung, E. W. Kolb and A. Riotto, *Superheavy dark matter*, Phys. Rev. D 59 (1998) 023501 [hep-ph/9802238].

[16] D. J. H. Chung, P. Crotty, E. W. Kolb and A. Riotto, *On the gravitational production of superheavy dark matter*, Phys. Rev. D 64 (2001) 043503 [hep-ph/0104100].

[17] D. J. H. Chung, E. W. Kolb, A. Riotto and I. I. Tkachev, *Probing planckian physics: Resonant production of particles during inflation and features in the primordial power spectrum*, Phys. Rev. D 62 (2000) 043508 [hep-ph/9910437].

[18] D. J. H. Chung, *Classical inflation field induced creation of superheavy dark matter*, Phys. Rev. D 67 (2003) 083514 [hep-ph/9809489].

[19] D. J. H. Chung, E. W. Kolb, A. Riotto and L. Senatore, *Isocurvature constraints on gravitationally produced superheavy dark matter*, Phys. Rev. D 72 (2005) 023511 [astro-ph/0411468].

[20] Y. Ema, R. Jinno, K. Mukaida and K. Nakayama, *Gravitational particle production in oscillating backgrounds and its cosmological implications*, Phys. Rev. D 94 (2016) 063517 [1604.08898].

[21] D. J. H. Chung, E. W. Kolb and A. J. Long, *Gravitational production of super-hubble-mass particles: an analytic approach*, JHEP 01 (2019) 189 [1812.00211].

[22] Y. Ema, K. Nakayama and Y. Tang, *Production of purely gravitational dark matter*, JHEP 09 (2018) 135 [1804.07471].

[23] L. Li, T. Nakama, C. M. Sou, Y. Wang and S. Zhou, *Gravitational production of superheavy dark matter and associated cosmological signatures*, JHEP 07 (2019) 067 [1903.08842].

[24] S.-S. Xue, *Cosmological λ driven inflation and produced massive particles*, 1910.03938.

[25] L. Kofman, A. D. Linde and A. A. Starobinsky, *Reheating after inflation*, Phys. Rev. Lett. 73 (1994) 3195 [hep-th/9405187].

[26] L. Kofman, A. D. Linde and A. A. Starobinsky, *Towards the theory of reheating after inflation*, Phys. Rev. D 56 (1997) 3258 [hep-ph/9704452].

[27] V. Kuzmin and I. Tkachev, *Ultrahigh-energy cosmic rays, superheavy long living particles, and matter creation after inflation*, JETP Lett. 68 (1998) 271 [hep-ph/9802304].
[28] V. Kuzmin and I. Tkachev, *Matter creation via vacuum fluctuations in the early universe and observed ultrahigh-energy cosmic ray events*, Phys. Rev. D 59 (1999) 123006 [hep-ph/9809547].

[29] E. W. Kolb, D. J. H. Chung and A. Riotto, *Wimpzillas*, AIP Conf. Proc. 484 (1999) 91 [hep-ph/9810361].

[30] G. Bertone, D. Hooper and J. Silk, *Particle dark matter: Evidence, candidates and constraints*, Phys. Rept. 405 (2005) 279 [hep-ph/0404175].

[31] B. A. Bassett, S. Tsujikawa and D. Wands, *Inflation dynamics and reheating*, Rev. Mod. Phys. 78 (2006) 537 [astro-ph/0507632].

[32] E. W. Kolb, A. A. Starobinsky and I. I. Tkachev, *Trans-planckian wimpzillas*, JCAP 07 (2007) 005 [hep-th/0702143].

[33] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine and A. Mazumdar, *Reheating in inflationary cosmology: Theory and applications*, Ann. Rev. Nucl. Part. Sci. 60 (2010) 27 [1001.2600].

[34] A. V. Frolov, *Nonlinear dynamics and primordial curvature perturbations from preheating*, Class. Quant. Grav. 27 (2010) 124006 [1004.3559].

[35] L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, *Freeze-in production of fimp dark matter*, JHEP 03 (2010) 080 [0911.1120].

[36] J. L. Feng, *Dark matter candidates from particle physics and methods of detection*, Ann. Rev. Astron. Astrophys. 48 (2010) 495 [1003.0904].

[37] M. A. Amin, M. P. Hertzberg, D. I. Kaiser and J. Karouby, *Nonperturbative dynamics of reheating after inflation: A review*, Int. J. Mod. Phys. D 24 (2014) 1530003 [1410.3808].

[38] Y. Ema, R. Jinno, K. Mukaida and K. Nakayama, *Gravitational effects on inflaton decay*, JCAP 05 (2015) 038 [1502.02475].

[39] M. Garny, M. Sandora and M. S. Sloth, *Planckian interacting massive particles as dark matter*, Phys. Rev. Lett. 116 (2016) 101302 [1511.03278].

[40] M. Garny, A. Palessandro, M. Sandora and M. S. Sloth, *Theory and phenomenology of planckian interacting massive particles as dark matter*, JCAP 02 (2018) 027 [1709.09688].

[41] E. W. Kolb and A. J. Long, *Superheavy dark matter through higgs portal operators*, Phys. Rev. D 96 (2017) 103540 [1708.04293].

[42] S. Hashiba and J. Yokoyama, *Gravitational reheating through conformally coupled superheavy scalar particles*, JCAP 01 (2019) 028 [1809.05410].

[43] S. Hashiba and J. Yokoyama, *Gravitational particle creation for dark matter and reheating*, Phys. Rev. D 99 (2019) 043008 [1812.10032].
[44] J. Haro, W. Yang and S. Pan, Reheating in quintessential inflation via gravitational production of heavy massive particles: A detailed analysis, *JCAP* **01** (2019) 023 [1811.07371].

[45] S.-S. Xue, Cosmological $\lambda$ converts to reheating energy and cold dark matter, 2006.15622.

[46] S.-S. Xue, How universe evolves with cosmological and gravitational constants, *Nucl. Phys. B* **897** (2015) 326 [1410.6152].

[47] Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper and E. Mottola, Pair production in a strong electric field, *Phys. Rev. Lett.* **67** (1991) 2427.

[48] R. Ruffini, G. Vereshchagin and S.-S. Xue, Electron-positron pairs in physics and astrophysics: from heavy nuclei to black holes, *Phys. Rept.* **487** (2010) 1 [0910.0974].

[49] G. ’t Hooft, Dimensional reduction in quantum gravity, *Conf. Proc. C* **930308** (1993) 284 [gr-qc/9310026].

[50] L. Susskind, The world as a hologram, *J. Math. Phys.* **36** (1995) 6377 [hep-th/9409089].

[51] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Effective field theory, black holes, and the cosmological constant, *Phys. Rev. Lett.* **82** (1999) 4971 [hep-th/9803132].

[52] D. Baumann and L. McAllister, *Inflation and String Theory*, Cambridge Monographs on Mathematical Physics. Cambridge University Press, 5, 2015, 10.1017/CBO9781316105733, [1404.2601].

[53] PLANCK collaboration, Planck 2018 results. vi. cosmological parameters, *Astron. Astrophys.* **641** (2020) A6 [1807.06209].

[54] BICEP, Keck collaboration, Improved constraints on primordial gravitational waves using planck, wmap, and bicep/keck observations through the 2018 observing season, *Phys. Rev. Lett.* **127** (2021) 151301 [2110.00483].

[55] D. Bégué, C. Stahl and S.-S. Xue, A model of interacting dark fluids tested with supernovae and baryon acoustic oscillations data, *Nucl. Phys. B* **940** (2019) 312 [1702.03185].

[56] L.-Y. Gao, Z.-W. Zhao, S.-S. Xue and X. Zhang, Relieving the $h\theta$ tension with a new interacting dark energy model, *JCAP* **07** (2021) 005 [2101.10714].

[57] S.-S. Xue, Detailed discussions and calculations of quantum regge calculus of einstein-cartan theory, *Phys. Rev. D* **82** (2010) 064039 [0912.2435].

[58] S.-S. Xue, The phase and critical point of quantum einstein-cartan gravity, *Phys. Lett. B* **711** (2012) 404 [1112.1323].

[59] S.-S. Xue, Quantum regge calculus of einstein-cartan theory, *Phys. Lett. B* **682** (2009) 300 [0902.3407].
# Supplemental Material: quantum pair oscillation details

In microscopic time, we plot the Bogoliubov coefficient $|\beta|^2$, the quantum pair density $\rho^\text{fast}_\Lambda$ and pressure $p^\text{fast}_\Lambda$, as well as the fast components of Hubble function $H^\text{fast}$, scale factor $a^\text{fast}$ and cosmological term $\rho^\text{fast}_\Lambda$.

![Graphs showing quantum pair oscillation details](image)

**Figure 3.** Corresponding to Fig. 1, the details of quantum pair oscillation in a tiny layer $\lambda_M H^2\text{slow}$ are shown. The oscillatory $H^\text{fast}$ and $\rho^\text{fast}_\Lambda$ structures are too small to see.
Figure 4. Quantum pair production and oscillation in a sphere $H_{\text{fast}}^{-3}$ are illustrated for pre-inflation, using $M = 10^{-1} m_{\text{pl}}$, $H_{\text{fast}}(0) = 10^{-2} m_{\text{pl}}$, $N_{\text{pair}} = 10^2$, and $a_{\text{fast}}(0) = 1$ at initial time $t = 0$. Oscillating Bogoliubov $|\beta|^2$ shows nontrivial particle productions. The time variation $H_{\text{fast}} \propto - (\rho_{\text{fast}} M + p_{\text{fast}} M)$. $H_{\text{fast}}$ and $\rho_{\text{fast}}$ decrease very slowly. The scale factor $a_{\text{fast}}$ increases exponentially.