Membranes and gauged supergravity*

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ABSTRACT

In 1980, Antonio Aurilia, Hermann Nicolai and I constructed an $N = 8$ supergravity with a positive exponential potential for one of the 70 scalar fields by adapting the dimensional reduction of 11D supergravity to allow for a non-zero 4-form field-strength in 4D. This model, now viewed as a particular gauged maximal supergravity, had little influence at the time because it has no maximally-symmetric vacuum. However, as shown here, it does have a domain-wall solution, which lifts to the M2-brane solution of $D = 11$ supergravity. A similar construction for other M-branes is also explored.

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1 Introduction

Early in 1980, Antonio Aurilia came to my office at CERN to tell me about his version, with Christodoulou and Legovini, of the “bag model” of hadrons; their bag was the space enclosed by a membrane coupled to a 3-form gauge potential such that the 4-form field strength was zero outside the bag but a non-zero constant inside it [1,2].

That was my introduction to the relativistic membrane, but my interest at the time was in the antisymmetric tensor fields that arise in supergravity theories, and I was intrigued by the idea that a 3-form potential could be physically relevant even though it has no propagating modes. While listening to Antonio, it occurred to me that a constant 4-form field strength would be a cosmological constant in a gravitational context. We soon worked out the details, in which the cosmological constant emerges as an integration constant in the solution of the field equation for the 3-form potential

\[ F \]

and we then had the idea of applying the result to 11D supergravity [5]; the 3-form gauge potential of that theory implies, in the context of dimensional reduction, a 3-form gauge potential for 4D \( N = 8 \) supergravity. Cremmer and Julia had recently found the full \( N = 8 \) supergravity action in this way [6], but they had set to zero the 4D 4-form field strength. Working with Herman Nicolai, we found a more general \( N = 8 \) supergravity action with an exponential scalar potential for one of the 70 scalar fields [7]. The cosmological constant had effectively been traded for the expectation value of a scalar field; this was interesting but the absence of any maximally symmetric 4D vacuum was disappointing.

The idea that a 3-form potential could replace a cosmological constant soon attracted attention. An example that is noteworthy here, because it has elements in common with the Aurilia-Chistodoulou-Legovini bag model, is the 1987 work of Brown and Teitelboim [9] in which it is shown that the cosmological constant, interpreted as the magnitude \( F \) of a 4-form field strength, could be dynamically reduced by nucleation of membrane bubbles if \( F \) is lower inside the bubble than outside. In contrast, our new \( N = 8 \) supergravity had very little impact, presumably because it was not related to other ideas of the time, and thus did not appear to be part of some bigger picture.

The bigger picture began to emerge a few months later, when Freund and Rubin showed that 11D supergravity has an \( AdS_4 \times S^7 \) solution with the 4-form field strength proportional to the volume form of \( adS_4 \) [10]. It was soon conjectured, and later proved, that the associated 4D theory is a gauged maximal supergravity with an \( SO(8) \) gauge group, which was constructed by de Wit and Nicolai in 1982 [11]. These developments were reviewed in 1986 by Duff, Nilsson and Pope [12]; by then it was understood that the de Wit-Nicolai theory is but one of many gauged maximal supergravity theories. A classification was achieved in relatively recent times; this was reviewed in 2008 by

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1 As was independently discovered around the same time by Duff and van Nieuwenhuizen [3], and (in the context of a superspace formulation of \( N = 1 \) supergravity) by Ogievetsky and Sokatchev [4].

2 We addressed this issue briefly in a conference report [8], where it was observed that a positive potential is what one might expect from a spontaneous partial breaking of the local supersymmetry.
Samtleben [13], who points out that the modified $N = 8$ supergravity of [7] is included. In other words, the $N = 8$ supergravity found in 1980 by Antonio, Hermann and myself was a gauged maximal supergravity theory avant la lettre.

The aim of this article is to show, in a different way, how this first gauged maximal supergravity fits into the bigger picture of M-theory. Although it has no Minkowski vacuum, it does have a membrane/domain-wall ‘vacuum’ solution that preserves half the supersymmetry. Considering that the starting point of my collaboration with Antonio was his idea that a relativistic membrane is a source for a 3-form gauge potential, it now seems surprising that we did not immediately look for a domain-wall solution of our new $N = 8$ supergravity theory. Of course, there was then no understanding of how a membrane coupling to the 3-form of 11D supergravity could be compatible with local supersymmetry; the 11D supermembrane lay seven years in the future [16]. However, if we had looked for, and found, the half-supersymmetric 4D domain wall solution, it would surely have been obvious that this must lift to a membrane solution of 11D supergravity. In fact, as will be shown here, it lifts to the the membrane solution found by Duff and Stelle in 1990 [17].

I will begin with an exposition of the construction of [7], which I will refer to as the “ANT construction”, after its authors. In contrast to the detailed exposition there, no attempt will be made here to include all supergravity fields, including fermions. Instead, I will start with a simplified model of gravity in a spacetime of general dimension $D = d + n$, coupled to a $d$-form field strength $F_d$. This suffices for an explanation of the basic idea, which yields (in this simplified but also generalized context) a $d$-dimensional dilaton-gravity model; for $(d, n) = (4, 7)$ it is a consistent truncation of the modified $N = 8$ supergravity theory found in [7]. For certain other values of $(d, n)$ a similar construction may apply for other M-theory branes, as will be discussed.

## 2 The ANT construction

Our starting point will be the following Lagrangian density

$$\mathcal{L}_D = \sqrt{-g^{(D)}} \left\{ R^{(D)} - \frac{1}{2} |F_d|^2 \right\},$$

(2.1)

where $|F_d|^2$ is defined such that, in local coordinates for which the $D$-metric is $g_{MN}$ and $F_d$ has components $F_{M_1 \ldots M_d}$,

$$|F_d|^2 = \frac{1}{d!} g^{M_1 N_1} \cdots g^{M_d N_d} F_{M_1 \ldots M_d} F_{N_1 \ldots N_d}.$$

(2.2)

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3To my knowledge, this idea originated in Antonio’s 1977 paper with Legovini [14]; it is, of course, a straightforward generalization of the Kalb-Ramond coupling of strings to a 2-form potential [15].
We will be interested in an \(n\)-dimensional reduction to a \(d\)-dimensional spacetime \(\mathcal{M}_d\) (so \(D = d + n\)) with a reduction/truncation ansatz for which

\[
\begin{align*}
    ds^2_D &= e^{-2\alpha\phi} ds^2(\mathcal{M}_d) + e^{2[(d-2)/n]\alpha^2} ds^2(\mathbb{E}^n) \\
    F_d &= \text{vol}(\mathcal{M}_d) f ,
\end{align*}
\]

where \(\text{vol}(\mathcal{M}_d)\) is the volume \(d\)-form on \(\mathcal{M}_d\), and \(\alpha\) is an arbitrary constant. Although both \(\phi\) and \(f\) are scalar fields on \(\mathcal{M}_d\) that are constant on the Euclidean \(n\)-space \(\mathbb{E}^n\), the scalar \(f\) is constrained to be the Hodge dual of a \(d\)-form field strength on \(\mathcal{M}_d\).

The metric ansatz has been chosen such that \(ds^2(\mathcal{M}_d)\) is the Einstein conformal frame metric. A choice of the constant \(\alpha\) is equivalent to a normalization for \(\phi\), and if we choose

\[
\alpha = \sqrt{\frac{n}{2(d-2)(D-2)}},
\]

then the Lagrangian density for the \(d\)-dimensional theory is

\[
\mathcal{L}_d = \sqrt{-g} \left\{ R - \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} e^{2(d-1)\alpha\phi} f^2 \right\}.
\]

For \((d, n) = (4, 7)\) this is a consistent truncation of the full \(N = 8\) supergravity found by dimensional reduction of 11D supergravity. If the scalar \(f\) were unconstrained, its equation of motion would be \(f = 0\); we could then back substitute to trivially eliminate \(f\) from the action, which would then be, for \((d, n) = (4, 7)\), a consistent truncation of the standard Cremmer-Julia \(N = 8\) supergravity.

However, \(f\) is not unconstrained; the unconstrained variable is the \((d-1)\)-form potential for (the Hodge dual of) \(f\) and its field equation has the general solution

\[
f = \mu e^{-2(d-1)\alpha\phi},
\]

for arbitrary mass parameter \(\mu\). For \(\mu \neq 0\) it is not legitimate to substitute for \(f\) into the Lagrangian density of (2.5) because the variation of the action is not zero for this solution of the field equations. This can be remedied by first adding to the Lagrangian density a \(\mu f\) term, which is a total derivative because of the constraint on \(f\). It is now legitimate to substitute for \(f\), and this yields the new ‘dual’ Lagrangian density

\[
\mathcal{L}_d = \sqrt{-g} \left\{ R - \frac{1}{2} (\partial \phi)^2 - 2\Lambda e^{-a\phi} \right\},
\]

where

\[
a = 2(d-1)\alpha, \quad \Lambda = (\mu/2)^2.
\]

Notice that the sign of the potential term is opposite to what one would find by the illegitimate substitution for \(f\) in (2.5). This sign change may be checked by verifying that the field equations of (2.5) are equivalent, for \(f\) given by (2.6), to the field equations of (2.7).
Our conventions are now those of [18] except that here is \(-\Lambda\) there; defined here, \(\Lambda\) becomes the usual cosmological constant (negative for anti-de Sitter) when \(a = 0\). Let us record here that, as a consequence of (2.4),

\[
a^2 = \frac{2n(d-1)^2}{(d-2)(D-2)}.
\]

(2.9)

3 Domain walls and M-branes

The domain-wall solutions of the field equations that follow from the Lagrangian density of (2.7) take the form [18, 19]

\[
ds_d^2 = H^{\frac{4}{(d-2)}} ds^2(\text{Mink}_{d-1}) + H^{\frac{4(d-1)}{(d-2)}} dy^2
\]

\[
e^\phi = H^{\frac{2n}{d-2}},
\]

(3.1)

where \(H\) is linear in \(y\) with \(dH = \mp \sqrt{\Lambda \Delta} dy\), and

\[
\Delta \equiv a^2 - \frac{2(d-1)}{(d-2)}.
\]

(3.2)

From (2.9) we see that, for us,

\[
\Delta = \frac{2(n-1)(d-1)}{d+n-2}.
\]

(3.3)

Our aim now is to lift these domain wall solutions to solutions of the \(D\)-dimensional theory from which we started, using the ansatz (2.3). The spacetime \(\mathcal{M}_d\) appearing in this ansatz is now the above domain-wall spacetime, and

\[
\text{vol}(\mathcal{M}_d) = H^{\frac{4(d-1)}{(d-2)}} \text{vol}(\text{Mink}_{d-1}) \wedge dy.
\]

(3.4)

Let us also record here that for the domain-wall solution, the function \(H(y)\) is such that

\[
dH^{-1} = \pm \sqrt{\Delta} H^{-2} dy.
\]

(3.5)

On substituting the domain-wall metric and dilaton configurations of (3.1) into the ansatz (2.3), one finds that

\[
ds_D^2 = H^{-\frac{2}{d-1}} ds^2(\text{Mink}_{d-1}) + H^{-\frac{2}{d-1}} ds^2(\mathbb{E}^{n+1})
\]

\[
F_d = \pm \frac{2}{\sqrt{\Delta}} \text{vol}(\text{Mink}_{d-1}) \wedge dH^{-1}.
\]

(3.6)

We have been assuming that \(H\) is a function only of \(y\), but this is now a special case of a more general \((d-1)\)-brane solution of the the \(D\)-dimensional theory from which we started; in general \(H\) is a harmonic function on the \((n+1)\)-dimensional transverse Euclidean space. A simple choice, with \(SO(n+1)\) symmetry, is

\[
H = 1 + \left(\frac{r_0}{r}\right)^{n-1},
\]

(3.7)
for constant $r_0$. Near the singularity of this function at $r = 0$, the $D$-metric takes the asymptotic form

$$H \sim e^{-(n-1)\rho/r_0} ds^2(Mink_{d-1}) + d\rho^2 + r_0^2 d\Omega_n \quad [\rho = r_0 \ln(r/r_0)], \quad (3.8)$$

where $d\Omega_n$ is the $SO(n+1)$-invariant metric on $S^n$. This is an AdS$_d \times S^n$ metric, which implies that $r = 0$ is a Killing horizon of the full $D$-metric, near which the solution asymptotes to AdS$_d \times S^n$.

The configuration (3.6) is a solution of a $D$-dimensional supergravity in those cases of table 1 which is adapted from a similar table in [20]. The first column gives the type of solution. The second column gives the spacetime dimension $D$ and the next two columns gives the values of $d$ and $n$. The last column gives the value of $\Delta$ according to the formula (3.3). Notice that entries with $d \neq n$ occur in ‘dual’ pairs for which the values of $d$ and $n$ are interchanged; these have the same value of $\Delta$, as is manifest from the formula (3.3).

| Solution type               | $D$ | $d$ | $n$ | $\Delta$ |
|----------------------------|-----|-----|-----|---------|
| M2                         | 11  | 4   | 7   | 4       |
| M5                         | 11  | 7   | 4   | 4       |
| D3                         | 10  | 5   | 5   | 4       |
| Self-dual black string     | 6   | 3   | 3   | 2       |
| Magnetic black string      | 5   | 3   | 2   | 4/3     |
| Tangerlini black hole      | 5   | 2   | 3   | 4/3     |
| Reissner-Nordström black hole | 4 | 2   | 2   | 1       |

Table 1: Supergravity solutions with an ‘AdS$\times$S’ near-horizon geometry.

For $(d, n) = (4, 7)$, in which case $\Delta = 4$, we recover the half-supersymmetric Duff-Stelle membrane solution of 11D supergravity [17] as a lift to $D = 11$ of a domain-wall solution of the modified $N = 8$ supergravity theory of [7].

### 3.1 Other cases

What about the other cases of the table? For $(d, n) = (7, 4)$, we recover the half-supersymmetric fivebrane solution of 11D supergravity, but in terms of a 7-form field strength for the 6-form ‘dual’ potential that is defined only on solutions of the 11D supergravity equations. This is probably only a technical difficulty since the ANT construction could be recast as a purely on-shell construction, but this will not be attempted here. The remaining $\Delta = 4$ case involves the self-dual 5-form field strength of IIB 10D supergravity, and we now face the complication that a non-zero 5-form on the 5-dimensional spacetime of the maximal 5D supergravity obtained by dimensional reduction must be accompanied by a non-zero 5-form field strength on the Euclidean...
5-space on which we reduce; again, some modification of the ANT construction may allow for this but this will not be investigated here either.

What about the $D < 10$ cases with $\Delta < 4$? The are various intersecting M-brane solutions of 11D supergravity for which the asymptotic geometry near the intersection is of ‘$\text{AdS} \times S \times E$’ form. The four geometries of this type obtainable in this way, with a particular realization in terms of intersecting M2-branes and M5-branes are given in table 2 which is adapted from a similar table in [21]. The symbol $\perp$ indicates either (i) an orthogonal intersection in which p-branes self-intersect on $(p-2)$-planes, which may then also intersect on $(p-4)$-planes, or (ii) the orthogonal intersection of an M2 with an M5 on a line; these intersections are among those that preserve some fraction of supersymmetry and the fraction preserved is given in the last column of the table.

| $M2 \perp M5$ | $\text{adS}_3 \times S^3 \times E^6$ | $1/4$ |
| $M2 \perp M2 \perp M2$ | $\text{adS}_2 \times S^4 \times E^6$ | $1/8$ |
| $M5 \perp M5 \perp M5$ | $\text{adS}_2 \times S^2 \times E^6$ | $1/8$ |
| $M2 \perp M2 \perp M5 \perp M5$ | $\text{adS}_2 \times S^2 \times E^7$ | $1/8$ |

Table 2: Intersecting M-branes with ‘$\text{AdS} \times S \times E$’ near-horizon geometries.

Consider the 6D self-dual black string solution of the minimal $(1,0)$ 6D supergravity, which is a consistent truncation of the maximal 6D supergravity obtained by dimensional reduction of 11D supergravity. An application of the ANT construction would, if successful, lead to a $d=3 \ N=4$ supergravity with a domain-wall vacuum that lifts to the 6D self-dual string. However, the 3-form field strength that supports this solution is self-dual, and this presents the same difficulty that we found for the D3 case. Next is the magnetic black string: a half-supersymmetric solution of minimal 5D supergravity, which is a consistent truncation of the maximal 5D supergravity obtained by dimensional reduction of 11D supergravity. Here we have the same issue that arose for the M5 case, the 3-form field strength that we need for an application of the ANT construction is defined only on-shell. These difficulties (which may be purely technical) do not arise for the remaining, extreme black hole, cases but for these the ANT construction would require us to dimensionally reduce to 2D, which is the dimension for which there is no Einstein-frame metric.

So, the ANT construction of [7] leading to a modified $N=8$ supergravity with a domain-wall ‘vacuum’ that lifts to the M2-brane of 11D supergravity does not immediately generalise to the other M-theory possibilities, although a modified construction may work in some of these other cases.

4 Comments

My interaction with Antonio Aurilia was brief but intense. Apart from our joint work with Hermann Nicolai, we also co-authored a paper with Antonio’s long-term (and...
long distance) collaborator Yasushi Takahashi (of Ward-Takahashi fame) on another application of a non-propagating 3-form gauge potential \cite{22}. I eventually met my co-author Takahashi at the Edmonton Summer Institute of 1987, although he was speaking at a parallel event on superconductivity \footnote{My talk at the other event was on branes, as was that of Kellogg Stelle with whom I co-wrote a conference proceedings with the rhetorical title Are 2 branes better than 1 \cite{23}.}. I introduced myself to him after his talk, mentioning our joint paper but he had no recollection of it!

I think Antonio and I met only once again after our 1980 collaboration at CERN, but his influence on me was very significant. I paid attention to his ideas about membranes at the same time that I was working with him on 11D supergravity, and I returned to wondering how to connect these topics after the Green-Schwarz superstring revolution of 1984. Eventually, in Trieste in January 1987, Eric Bergshoeff, Ergin Sezgin and I were able to solve the problem of how to couple 11D supergravity to a membrane \cite{16}. This set the course for much of my subsequent research, although branes only became mainstream in 1996. Before then, Antonio was a fellow heretic; see, for example, his 1993 paper with Spallucci \cite{24}. He would have been thrilled to know that our joint work on 11D supergravity would also turn out to be related to the topic so close to his heart – the relativistic membrane.

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