Primordial Fluctuations in the Warm inflation scenario with a more realistic coarse-grained field

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I study a semiclassical approach to warm inflation scenario introduced in previous works. In this work, I define the fluctuations for the matter field by means of a new coarse-grained field with a suppression factor $G$. This field describes the matter fluctuations on the now observable scale of the universe. The power spectrum for the fluctuations of the matter field is analyzed in both, de Sitter and power-law expansions for the universe. The constraint for the spectral index gives a constraint for the mass of the matter field in the de Sitter expansion and a constraint for the friction parameter in the power-law expansion for the universe.

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I. INTRODUCTION

The inflationary universe scenario asserts that, at some very early time, the universe went through a de Sitter phase expansion with scale factor $a(t)$ growing as $a \sim e^{H_o t}$. Inflation is needed because it solves the horizon, flatness, and monopole problems of the very early universe and also provides a mechanism for the creation of primordial density fluctuations. Quantum fluctuations and thermal fluctuations of matter fields play a prominent role in inflationary cosmology. They lead to density perturbations that would be responsible for the origin of structures in the universe. Structure formation scenarios, in particular, can receive important restrictions based on the measured $\delta T_r / < T_r > = 1.1 \times 10^{-5}$. According to the standard inflation model, the formation of large-scale structure in the universe has its origin in the growth of primordial inhomogeneities in the matter distribution.

The standard slow-roll inflation model separates expansion and reheating into two distinguished time periods. It is first assumed that exponential expansion from inflation places the universe in a supercooled phase. Subsequently thereafter the universe is reheated. Two outcomes arise from such a scenario. First, the required density perturbations in this cold universe are left to be created by the quantum fluctuations of the inflaton. Second, the temperature cliff after expansion requires a temporally localized mechanism that rapidly distributes sufficient vacuum energy for reheating.

The warm inflation scenario takes into account separately, the matter and radiation energy fluctuations. In this scenario the fluctuations of the matter field lead to perturbations for matter and radiation energy densities, which are responsible for the fluctuations of temperature. The field $\varphi$ interacts with the particles which are in a thermal bath with a mean temperature smaller than the GUT critical temperature, $< T_r > < T_{GUT} \sim 10^{15}$ GeV. This scenario was introduced by A. Berera, and developed in other work. In the Berera's formalism the temperature of the universe is constant during inflation and the temporal evolution for the amplitude of the fluctuations of temperature are not considered.

In the warm inflation era, the kinetic component of energy density $\rho_{\text{kinetic}}$ must be small with respect to the vacuum energy, which is given by the potential $V(\varphi)$

$$\rho(\varphi) \sim \rho_m \sim V(\varphi) \gg \rho_{\text{kinetic}}.$$
\[ \rho_{\text{kinetic}} = \rho_r(\varphi) + \frac{1}{2} \dot{\varphi}^2, \]

and the radiation energy density is

\[ \rho_r(\varphi) = \frac{\tau(\varphi)}{8H(\varphi)} \dot{\varphi}^2. \]

Here, \( \varphi \) is a scalar field of matter, \( \tau(\varphi) \) is a friction parameter due to the interaction of the matter field \( \varphi \) with other fields of the thermal bath with temperature \( T_r \). Furthermore, the dot denotes the derivative with respect to the time. The conventional treatment of scalar field dynamics assumes that it is pure vacuum energy dominated. The various kinematic outcomes are a result of specially chosen Lagrangians. In most cases the Lagrangian is unmotivated from particle phenomenology. Clear exceptions are the Coleman - Weinberg potential with an untuned coupling constant, which is motivated by grand unified theories and supersymmetric potentials [9]. Making an extension to the warm inflation picture, the behavior of the scale factor can also be altered for any given potential when the radiation energy is present.

In an alternative formalism of warm inflation [10] I demonstrated that, for a power - law inflation model, the amplitude for both, thermal fluctuations and mean temperature, decrease with time for a sufficiently rapid expansion of the scale factor of the universe. Thus, at the end of inflation the spectrum of the coarse - grained matter field can be calculated [5,6]. This is the most significant difference with the Berera’s formalism.

In this formalism a semiclassical expansion of the inflaton field was proposed [11]

\[ \varphi(\vec{x}, t) = \phi_c(t) + \phi(\vec{x}, t). \]

Here the expectation value of the field operator \( \varphi \), in some unknown state \( |E> \), is given by the classical field \( \phi_c(t) \) and the expectation value for the quantum fluctuations \( \phi(\vec{x}, t) \) is zero

\[ <E|\varphi(\vec{x}, t)|E> = \phi_c(t); \quad <E|\phi(\vec{x}, t)|E> = 0. \]

The field \( \phi_c \) is responsible for the expansion of the universe, while the quantum fluctuations \( \phi(\vec{x}, t) \) takes into account the local fluctuations of the matter field with respect to \( \phi_c \).

II. FORMALISM

The Lagrangian that describes the warm inflation scenario is

\[ L(\varphi, \varphi, \mu) = -\sqrt{-g} \left[ R - \frac{1}{2} g^{\mu\nu} \varphi,_{\mu} \varphi,_{\nu} + V(\varphi) \right] + L_{\text{int}}, \] (1)

where \( R \) is the scalar curvature, \( g^{\mu\nu} \) the metric tensor and \( g \) is the metric. The Lagrangian \( L_{\text{int}} \) takes into account the interaction of the field \( \varphi \) with other fields of the thermal bath with mean temperature \( <T_r> < T_{GUT} \sim 10^{15} \) GeV. This lower temperature condition implies that magnetic monopole suppression works effectively. As was recently showed, certain string inspired models solve all the cosmological puzzles and have a motivation from string theory [12,13]. All particlelike matter which existed before inflation would have been dispersed by inflation.

We consider the metric

\[ ds^2 = -dt^2 + a^2(t)d\vec{x}^2. \] (2)

The eq. (2) represents a flat Friedmann - Robertson - Walker (FRW) metric for a globally flat, isotropic and homogeneous universe. The Hubble parameter \( H[\varphi(\vec{x}, t)] \) can be written as an expansion around \( H_c(\phi_c) \)

\[ H[\varphi(\vec{x}, t)] = H_c(\phi_c) + \sum_{n=1}^{\infty} \frac{1}{n!} H^{(n)}(\phi_c) \phi^n(\vec{x}, t), \] (3)

where \( H^{(n)}(\phi_c) = \frac{d^n H(\varphi)}{d\varphi^n}|_{\phi_c} \equiv H^{(n)}(\phi_c) \). The equation of motion for the operator \( \varphi \) is

\[ \ddot{\varphi} - \frac{1}{a^2} \nabla^2 \varphi + [3H(\varphi) + \tau(\varphi)] \dot{\varphi} + V'(\varphi) = 0. \] (4)
The semiclassical Friedmann equation for a globally flat universe is given by

$$\langle E | H^2(\varphi) | E \rangle = \frac{8\pi}{3M_p^2} \langle E | \rho_m + \rho_r | E \rangle,$$  \hspace{1cm} (5)

where $M_p$ is the Planckian mass. Here $\rho_m$ and $\rho_r$ are

$$\rho_m(\varphi) = \frac{\dot{\varphi}^2}{2} + \frac{1}{a^2} \left( \nabla \varphi \right)^2 + V(\varphi),$$  \hspace{1cm} (6)

$$\rho_r(\varphi) = \frac{\tau(\varphi)}{8H(\varphi)} \dot{\varphi}^2.$$  \hspace{1cm} (7)

The scalar potential $V(\varphi)$ also can be written as an expansion around $V(\phi_c)$

$$V(\varphi) = V(\phi_c) + \sum_{n=1}^{\infty} \frac{1}{n!} V^{(n)}(\phi_c) \phi^n(\vec{x}, t),$$  \hspace{1cm} (8)

where $V^{(n)}(\phi_c) \equiv \frac{d^n V(\varphi)}{d\varphi^n} |_{\phi_c}$. Here the prime denotes the derivative with respect to the field $\varphi$. Furthermore $V'(\varphi)$ can be expanded as

$$V'(\varphi) = V'(\phi_c) + \sum_{n=2}^{\infty} \frac{1}{n!} V^{(n)}(\phi_c) \phi^{n-1}(\vec{x}, t).$$  \hspace{1cm} (9)

In the following I will consider $H(\varphi) \equiv H_c(\phi_c)$ and $V(\varphi)$ and $V'(\varphi)$ as first ordered expansions in $\phi$.

**A. Dynamics of the classical field**

The classical equation of motion for the field $\phi_c$ is

$$\ddot{\phi}_c + [3H_c(\phi_c) + \tau_c(\phi_c)] \dot{\phi}_c + V'(\phi_c) = 0,$$  \hspace{1cm} (10)

where $V'(\phi_c) \equiv \frac{dV(\varphi)}{d\varphi} |_{\phi_c}$ and $\tau(\varphi) \equiv \tau_c(\phi_c)$. Here, the laplacian term does not appears, since $\phi_c$ only depends on time. The evolution of $\phi_c$ and $H_c(\phi_c)$ are

$$\dot{\phi}_c = -\frac{M^2}{4\pi} H'_c \left( 1 + \frac{\tau_c}{3H_c} \right)^{-1},$$  \hspace{1cm} (11)

$$\dot{H}_c = H'_c \dot{\phi}_c = -\frac{M^2}{4\pi} (H'_c)^2 \left( 1 + \frac{\tau_c}{3H_c} \right)^{-1},$$  \hspace{1cm} (12)

where $H_c$ decreases with time due to the fact that $\dot{H}_c(t) < 0$. Furthermore, from eq. (5), the classical Friedmann equation is

$$H_c^2(\phi_c) = \frac{4\pi}{3M_p^2} \left[ \left( 1 + \frac{\tau_c}{4H_c} \right) \dot{\phi}_c^2 + 2V(\phi_c) \right].$$  \hspace{1cm} (13)

Replacing (11) in eq. (13), one obtains the classical scalar potential

$$V(\phi_c) = \frac{3M_p^2}{8\pi} \left[ H_c^2(\phi_c) - \frac{M_p^2}{12\pi} (H'_c)^2 \left( 1 + \frac{\tau_c}{4H_c} \right) \left( 1 + \frac{\tau_c}{3H_c} \right)^{-2} \right].$$  \hspace{1cm} (14)

This expression relates the potential with the Hubble and friction parameters.

On the other hand, the radiation energy density is

$$\rho_r(\phi_c) = \frac{\tau_c}{8H_c} \left( \frac{M_p^2}{4\pi} \right)^2 (H'_c)^2 \left( 1 + \frac{\tau_c}{3H_c} \right)^{-2}. $$  \hspace{1cm} (15)

From eq. (15) one obtains the mean temperature of the thermal bath, in which there are particles that would be dispersed during inflation

$$< T_r > \propto (\rho_r(\phi_c))^{1/4}.$$  \hspace{1cm} (16)

The eq. (16) gives the temperature of the background, but does not describes the local fluctuations $\delta T_r/ < T_r >$. 

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B. Dynamics of the quantum perturbations

I consider the quantum fluctuations of the matter field for an universe with a FRW metric \([3,4]\). I will consider the Hubble and friction parameters as spatially homogeneous \([i.e., \, H(\varphi) \equiv H_c(\phi_c) \text{ and } \tau(\varphi) \equiv \tau_c(\phi_c)]\). The equation of motion for the quantum fluctuations is

\[
\ddot{\phi} - \frac{1}{a^2} \nabla^2 \phi + [3H_c + \tau_c] \dot{\phi} + V''(\phi_c) \phi = 0, \tag{17}
\]

which can be simplified with the map \(\chi = e^{3/2 \int (H_c + \tau_c/3) dt} \phi\)

\[
\ddot{\chi} - a^{-2} \nabla^2 \chi - \frac{k_o^2}{a^2} \chi = 0. \tag{18}
\]

Here, \(k_o(t)\) is the wavenumber that separates the infrared sector \((k < k_o)\) and the short-wavelength sector \((k > k_o)\)

\[
k_o^2(t) = a^2(t) \left[ \frac{9}{4} \left( H_c(t) + \frac{\tau_c(t)}{3} \right)^2 - V''(t) + \frac{3}{2} \left( \dot{H}_c(t) + \frac{\dot{\tau}_c(t)}{3} \right) \right]. \tag{19}
\]

Here, the parameters \(H_c(\phi_c)\) and \(\tau_c(\phi_c)\) are evaluated in \(t\), due to the fact that \(\phi_c \equiv \phi_c(t)\). The infrared sector describes the macrophysics of the universe \(i.e.,\) describes the universe in a cosmological scale, while the short-wavelength sector takes into account the microphysics of it. The eq. (17) is a Klein-Gordon equation with a time dependent parameter of mass \(\mu(t) = \frac{k_o}{a}\).

We can write the redefined field \(\chi\) as a Fourier expansion — in terms of the modes \(\xi_k e^{i\vec{k}.\vec{x}}\) — in the \(k\)-space

\[
\chi(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[ a_k e^{i\vec{k}.\vec{x}} \xi_k(t) + H.c \right], \tag{20}
\]

where \(\xi_k(t)\) are the time dependent modes with wavenumber \(k\). The annihilation and creation operators \(a_k\) and \(a_k^\dagger\), satisfy the following commutation relations

\[
\left[ a_k, a_{k'}^\dagger \right] = \delta^{(3)}(k - k'); \quad [a_k, a_{k'}] = \left[ a_k^\dagger, a_{k'}^\dagger \right] = 0. \tag{21}
\]

Furthermore, the operators \(\chi\) and \(\dot{\chi}\) satisfy

\[
[\chi(\vec{x}, t), \dot{\chi}(\vec{x}, t)] = i\delta^{(3)}(\vec{x} - \vec{x}'). \tag{22}
\]

The interpretation for eq. (22) is just that \(\chi\) and \(\dot{\chi}\) are canonically conjugate variable if \(\vec{x}\) and \(\vec{x}'\) are within the same smeared point inside the light cone. Otherwise, \(\chi\) and \(\dot{\chi}\) can be measured independently.

Replacing eq. (20) in eq. (22) one obtains the following condition for the time dependent modes \(\xi_k\)

\[
\xi_k \dot{\xi}_k^* - \dot{\xi}_k \xi_k^* = i. \tag{23}
\]

When the time dependent modes are real one obtains

\[
\xi_k \dot{\xi}_k^* - \dot{\xi}_k \xi_k^* = \xi_k \dot{\xi}_k - \dot{\xi}_k \xi_k = 0, \tag{24}
\]

and the operators \(\chi\) and \(\dot{\chi}\) commute \([5,6]\). Thus, the field \(\chi\) becomes classical when all the modes \(\xi_k(t)\) in eq. (20) are real. Note that \(k_o\) depends on time and continuously new time dependent modes \(\xi_k\) enters in the infrared sector \((k \ll k_o)\) from the short-wavelength sector \((k \gg k_o)\). We are interested in studying the sector of the universe where the redefined quantum fluctuations \(\chi\) become classical. Of course, this condition imposes restrictions over the vacuum \([5,6]\) and thus the asymptotic vacuum must be real in the infrared sector. However, the modes \(\xi_k\) — for \(k > k_o(t)\) — are complex, and during inflation \(\chi\) and \(\dot{\chi}\) does not commute.
C. The data COBE coarse-grained field

In this work I define a coarse-grained field which is obtained from the experimental data \[\chi_{C\text{cg}} = \frac{1}{(2\pi)^{3/2}} \int d^3k \; G(k,t) \left[ a_k e^{i \vec{k} \cdot \vec{x}} \xi_k(t) + H.c. \right], \tag{25}\]

where the suppression factor is

\[G(k,t) = \sqrt{\frac{1}{1 + \left( \frac{k_o(t)}{k} \right)^N}}, \tag{26}\]

with \(N = m - n\). Causality places a strict constraint on the suppression index: \(N \geq 4 - n\). A suppression factor like eq. (26) (with \(n \sim 1\)) also has been found in a model with cosmic strings plus cold or hot dark matter \[\text{[15][16]}\].

Furthermore, the squared fluctuations for the COBE coarse-grained field is \[\text{[17]}\]

\[\langle E \left| \chi_{C\text{cg}}^2 \right| E \rangle = \int_0^\infty \frac{dk}{k} P_{\chi_{C\text{cg}}}(k) \tag{27}\]

\[= \frac{1}{2\pi^2} \int_0^{k_o} dk \; k^2 |\xi_k(t)|^2 G^2(k,t), \tag{28}\]

where the power spectrum \(P_{\chi_{C\text{cg}}}(k)\) when the horizon exit is assumed as \[\text{[16]}\]

\[P_{\chi_{C\text{cg}}}(k) = A(t_\ast) \left( \frac{k}{k_o(t_\ast)} \right)^n f(k). \tag{29}\]

The power spectrum \(P_{\chi_{C\text{cg}}}\) contains four parameters: the amplitude \(A\), the spectral index \(n\), the suppression wavenumber \(k_o\), and the suppression index \(m\). Furthermore, \(t_\ast\) denotes the time when the horizon entry, for which \(k_o(t_\ast) \approx \pi H_o\) in comoving scale. Replacing eq. (25) in (18), and neglecting the terms with \(- \frac{1}{a(t)} \nabla^2 \chi_{C\text{cg}}\), since they are very small with respect to the another terms, one obtains the quantum stochastic equation for the COBE coarse-grained field

\[\ddot{\chi}_{C\text{cg}} = \left( \frac{k_o(t)}{a(t)} \right)^2 \chi_{C\text{cg}} = \frac{N}{k_o(t)} \left[ \xi_1(\vec{x},t) + \xi_2(\vec{x},t) \right]. \tag{30}\]

Here the noises \(\xi_1\) and \(\xi_2\) are given by

\[\xi_1(\vec{x},t) = -\frac{k_o N}{(2\pi)^{3/2}} \int d^3k \; k^{-N} \; G^3(k,t) \left[ a_k e^{i \vec{k} \cdot \vec{x}} \xi_k(t) + H.c. \right], \tag{31}\]

\[\xi_2(\vec{x},t) = \frac{k_o^{N-1}}{4(2\pi)^{3/2}} \int d^3k \; k^{-N} \; G^2(k,t) \left[ \frac{k_o}{k} \right]^N \left( 3k_o^2 - 2k_o \bar{k}_o \right) + 2k_o^2 (1 - N) - 2k_o \bar{k}_o \right] \times \left[ a_k e^{i \vec{k} \cdot \vec{x}} \xi_k(t) + H.c. \right]. \tag{32}\]

The function \(G(k,t)\) determines the stochastic properties and spectrum of the noises \(\xi_1\) and \(\xi_2\).

Replacing eq. (20) in eq. (18), one obtains the equation of motion for the modes \(\xi_k(t)\)

\[\dot{\xi}_k(t) + \omega_k^2 \xi_k(t) = 0, \tag{33}\]

where \(\omega_k(t) = a^{-1} [k^2 - \mu_k^2(t)]^{1/2}\) is the oscillation frequency of the modes. In a de Sitter expansion \[\text{[18]}\] these frequencies are constant. Most generally, the frequencies depends on time. Note that for \(k < k_o\) the squared frequencies \(\omega_k^2\) are positives and the solutions of (33) real, but it does not occur for \(k > k_o\) where the solutions of (33) become complex.
D. Classicality conditions for the COBE coarse-grained field

As in previous works [5,6,10,11,18], we are interested in studying the classicality conditions for the quantum stochastic equation (40). Observe that all the modes $\xi_k(t)$ on the infrared sector are real. If we write the modes as a complex function with components $u_k(t)$ and $v_k(t)$

$$\xi_k(t) = u_k(t) + iv_k(t),$$

the condition for that the modes to be real is

$$\alpha_k(t) = \left| \frac{v_k(t)}{u_k(t)} \right| \ll 1. \quad (35)$$

The condition for the COBE coarse-grained field $\chi_{Ccg}$ to be classical during inflation becomes [5,6]

$$\frac{1}{M(t)} \sum_{k \geq k_o(t)} \alpha_k(t) \ll 1, \quad (36)$$

where $M(t)$ is the time dependent number of degrees of freedom in the infrared (large-wavelength) sector. When all the modes of the infrared sector are real (i.e., when $\alpha_{k \geq k_o} \ll 1$).

E. Heisenberg representation for the COBE coarse-grained field

During inflation the field $\chi_{Ccg}$ obeys the quantum stochastic equation

$$\ddot{\chi}_{Ccg} - \left( \frac{k_o(t)}{a(t)} \right)^2 \chi_{Ccg} = \frac{N}{k_o(t)} [\xi_{c1}(\vec{x},t) + \xi_{c2}(\vec{x},t)], \quad (37)$$

which can be written as

$$\ddot{\chi}_{Ccg} - \left( \frac{k_o(t)}{a(t)} \right)^2 \chi_{Ccg} + \xi_{c}(\vec{x},t) = 0, \quad (38)$$

where $\xi_{c}(\vec{x},t) = -\frac{N}{k_o(t)} [\xi_{c1}(\vec{x},t) + \xi_{c2}(\vec{x},t)]$. The new noises $\xi_{c1}$ and $\xi_{c2}$ are

$$\xi_{c1}(\vec{x},t) = -\frac{k_o N}{2(2\pi)^{3/2}} \int d^3k \frac{k^{-N}}{k_o} G^3(k,t) \left[ a_k e^{i\vec{k} \cdot \vec{x}} \dot{\xi}_k(t) + a_k^* e^{-i\vec{k} \cdot \vec{x}} \xi_k(t) \right], \quad (39)$$

$$\xi_{c2}(\vec{x},t) = \frac{k_o^{-1}}{4(2\pi)^{3/2}} \int d^3k \frac{k^{-N}}{k_o} G^5(k,t) \left[ \left( \frac{k_o}{k} \right)^N \left( 3k_o^2/k - 2k_o \dot{k}_o + 2k_o^2(1 - N) - 2k_o \ddot{k}_o \right) \right]$$

$$\times \left[ a_k e^{i\vec{k} \cdot \vec{x}} \xi_k(t) + a_k^* e^{-i\vec{k} \cdot \vec{x}} \xi_k(t) \right]. \quad (40)$$

These noises become from the increasing number of degrees of freedom of the infrared sector, as a consequence of the modes which enter in the infrared sector from the short-wavelength sector. In general, $\xi_{c1}$ is a colored noise, while $\xi_{c2}$ gives non-local dissipation. Under special circumstances (i.e., for $N \to \infty$), the noise $\xi_{c1}$ is nearly delta correlated and describes a nearly white and gaussian noise. Furthermore, in this case $\xi_{c2}$ generates local dissipation.

The effective Hamiltonian associated with eq. (38) is

$$H_{eff}(\chi_{Ccg},t) = \frac{1}{2} P_{Ccg}^2 + \frac{1}{2} \mu^2(t) \chi_{Ccg}^2 + \xi_c \chi_{Ccg}, \quad (41)$$

where $P_{Ccg} \equiv \dot{\chi}_{Ccg}$ and $\mu^2(t) = \frac{k_o^2}{a^2}$. Observe that $\xi_c$ plays the role of an external classical stochastic force in the effective Hamiltonian (1). Thus, one can write the Schrödinger equation

$$i \frac{\partial}{\partial t} \psi(\chi_{Ccg},t) = \frac{1}{2} \frac{\partial^2}{\partial \chi_{Ccg}^2} \psi(\chi_{Ccg},t) + \left[ \frac{1}{2} \mu^2(t) \chi_{Ccg}^2 + \xi_c \chi_{Ccg} \right] \psi(\chi_{Ccg},t). \quad (42)$$
Here, $\psi(\chi_{Ccg}, t)$ is the wave function that characterize the fluctuations of the matter field in the observable universe. Since the time dependence of $\mu(t)$, the Hamiltonian is non-conservative, also in the case in which one would neglects the stochastic force. The only case where $\mu$ does not present a time dependence is a de Sitter expansion of the universe. In this case $\mu$ is constant \(^{13}\) and eq. \(^{(11)}\) represents a harmonic oscillator with a stochastic external force $\xi_c$. In this case we have a forced linear harmonic oscillator and the solution is a coherent state with the displacement due to the action of the external force \(^{13}\).

The effective Hamiltonian \(^{(11)}\) represents an oscillator that experiences both, the squeezing, due to the time dependent frequency, and the external force, due to the inflow of the short modes in the infrared sector.

The probability to find the universe with a given $\chi_{Ccg}$ in a given time $t$ is

$$P(\chi_{Ccg}, t) = \psi(\chi_{Ccg}, t)\psi^*(\chi_{Ccg}, t),$$

where the asterisk denotes the complex conjugate. The problem with the Heisenberg’s representation is that is very complicated to find solutions for $\psi(\chi_{Ccg}, t)$. However, it gives a clear conceptual notion of the physical problem under consideration.

### III. EXAMPLES

#### A. de Sitter expansion: supercooled scenario

I consider a de Sitter expansion for which $H_o(\phi_c) \equiv H_o$ = constant. In this case $H' = 0$ and thus, from eq. \(^{(13)}\) one obtains a supercooled expansion for the universe \(^{13}\) ($\rho_c = 0$). This means that the dissipation parameter is zero ($\tau_c = 0$) during the expansion. From eq. \(^{(14)}\), one obtains a constant potential

$$V_o = \frac{3M_p^2}{8\pi} H_o^2. \hspace{1cm} (44)$$

For a massive inflaton with mass $m$ the square parameter $\mu^2(t) = \frac{k^2(t)}{\nu^2}$ (where $a \propto e^{H_o t}$ ) also becomes time independent

$$\mu^2 = \frac{9}{4} H_o^2 - m^2. \hspace{1cm} (45)$$

The equation for the time dependent modes $\xi_k(t)$ is

$$\ddot{\xi}_k(t) + \left[ k^2 e^{-2H_o t} - \frac{9}{4} H_o^2 + m^2 \right] \xi_k(t) = 0. \hspace{1cm} (46)$$

The general solution for $\xi_k$ is

$$\xi_k(t) = A_1 \mathcal{H}^{(1)}_{\nu} \left[ \frac{k}{H_o} e^{-H_o t} \right] + A_2 \mathcal{H}^{(2)}_{\nu} \left[ \frac{k}{H_o} e^{-H_o t} \right], \hspace{1cm} (47)$$

where $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H_o^2}} < 3/2$. Here $\mathcal{H}^{(1)}_{\nu}$ and $\mathcal{H}^{(2)}_{\nu}$ are the first and second species Hankel functions. For sufficiently large $t$, i.e., for $t \gg H_o^{-1}$ one obtains the expression for $\mathcal{H}^{(1)}_{\nu}$ and $\mathcal{H}^{(2)}_{\nu}$

$$\mathcal{H}^{(1,2)}_{\nu} \left[ \frac{k}{H_o} e^{-H_o t} \right] \approx \frac{1}{\Gamma(1+\nu)} \left( \frac{k}{2H_o} e^{-H_o t} \right)^{\nu} \pm \frac{i}{\pi} \Gamma(\nu) \left( \frac{k}{2H_o} e^{-H_o t} \right)^{-\nu}. \hspace{1cm} (48)$$

Choosing $A_1 = 0$, and from the relation $\xi_k \dot{\xi_k} - \dot{\xi_k} \xi_k = i$, one obtains $A_2 = \frac{i}{2} \sqrt{\frac{\pi \Gamma(1+\nu)}{\nu H_o \Gamma(\nu) \Gamma(\nu + 1)}}$, and thus

$$\xi_k(t) \approx \frac{i}{2} \sqrt{\frac{\pi}{\nu H_o \Gamma(\nu) \Gamma(\nu + 1)}} \left[ \frac{k}{2H_o} e^{-H_o t} \right]^{\nu} + \frac{1}{2} \sqrt{\frac{\pi \Gamma(\nu + 1) \Gamma(\nu)}{\nu H_o \Gamma(\nu + 1)}} \left[ \frac{k}{2H_o} e^{-H_o t} \right]^{-\nu}. \hspace{1cm} (49)$$

For very large $t$, one obtains the asymptotic modes

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\[ \xi_k(t) \big|_{t \gg 1} \approx \frac{1}{2} \sqrt{\frac{\Gamma(\nu)\Gamma(\nu + 1)}{\nu H_0^2}} \left[ \frac{k}{2H_0} e^{-H_0 t} \right]^{-\nu}. \]  

(50)

Thus, the squared fluctuations on the observable scale, for very large \( t \), are

\[ \langle E | \delta^2_{\phi_{\text{Cyc}}} | E \rangle \approx \frac{2^{2\nu-3}}{\pi^3} \left[ \frac{\Gamma(\nu + 1)\Gamma(\nu)}{\nu} \right] H_o^{2\nu-1} e^{2(\nu-3/2)H_o t} \int_0^{k_p} dk \, k^{2(1-\nu)} G^2(k), \]

(51)

where \( \phi_{\text{Cyc}} = e^{-3H_0 t} \chi_{\text{Cyc}} \) and \( k_p \) is the wavenumber for the Planckian wavelength. Comparing (51) with (27) and (29), one obtains the following relation for the spectral index

\[ n - 1 = 2(1 - \nu). \]

(52)

The standard choice \( n = 1 \) was first advocated by Harrison [20] and Zel’dovich [21] on the ground that it is scale invariant at the epoch of horizon entry. The constraint \(|n - 1| < 0.3\)

(53)

gives the following values for the mass of the inflaton field

\[ 1.36 H_0 < m < 1.46 H_0. \]

(54)

Thus, the parameter \( \nu \) only can take the values

\[ 0.351 < \nu < 0.65. \]

(55)

The power spectrum for the COBE coarse-grained field \( \phi_{\text{Cyc}} \), is

\[ P_{\phi_{\text{Cyc}}}(k) \propto k^{2(1-\nu)} G^2(k). \]

(56)

Furthermore, the amplitude \( A(t) \) decreases with time during the inflation era

\[ A(t) = \frac{2^{2\nu-3}}{\pi^3} \left[ \frac{\Gamma(\nu + 1)\Gamma(\nu)}{\nu} \right] H_o^{2\nu-1} e^{2(\nu-3/2)H_o t}, \]

(57)

for \( m \neq 0 \). After the horizon entry (i.e., for \( t > t_* \)) the amplitude \( A(t_*) \) becomes

\[ A(t_*) = \frac{2^{2\nu-3}}{\pi^3} \left[ \frac{\Gamma(\nu + 1)\Gamma(\nu)}{\nu} \right] H_o^{2\nu-1} e^{2(\nu-3/2)H_o t_*}. \]

(58)

Due to \( |\delta_k|^2 = P_{\phi_{\text{Cyc}}}(k) \) [17], the spectral density is \( |\delta_k| = k^{1-\nu} G(k) \). From the condition (55), one obtains a positive exponent for \( k \) in the spectral density. To obtain a negative exponent in the spectral density \( |\delta_k| \), the mass of the inflaton field must be very small

\[ m < \frac{\sqrt{5}}{2} H_0. \]

(59)

**B. Power-law inflation**

In this example I consider a power-law expansion for the universe. In this case the scale factor and the Hubble parameter are respectively \( a(t) \propto (t/t_0)^p \), and \( H_*(t) = p/t \). The temporal evolution of the classical field is \( e^{-\phi(t)/m} = (H_0/p)t \). Furthermore, the temporal evolution for the radiation energy density is [see eq. (13)]

\[ \rho_r(t) = \left( \frac{M_P^2}{4\pi} \right)^2 \frac{p^2}{m^2} \left( 1 + \frac{\tau_c t}{3p} \right)^{-2} t^{-2}. \]

(60)

The potential is given by
From the condition $n = 1 = 2(1 - \nu)$ one obtains
\[ n - 1 = 2 \left[ 1 - \frac{1}{2(p-1)} \sqrt{1 + 4K^2} \right], \tag{70} \]
where $K^2$ is given by eq. (66). The constraint $|n - 1| < 0.3$ in eq. (70) gives the following conditions
\begin{align*}
K^2 &< \frac{\left[1.3(p-1)\right]^2 - 1}{4}, \tag{71} \\
K^2 &> \frac{\left[0.702(p-1)\right]^2 - 1}{4}. \tag{72}
\end{align*}
For example, taking $p = 4$, with the scale $m = 1$, one obtains the condition

$$7.1065 < \gamma < 7.1286,$$

(73)

which implies that

$$7.1065 H_c(t) < \tau_c < 7.1286 H_c(t).$$

(74)

When the horizon entry, the condition (74) becomes

$$7.1065 H_c(t_\ast) < \tau_c(t_\ast) < 7.1286 H_c(t_\ast).$$

(75)

### IV. CONCLUSIONS

In this thermal scenario the rapid cooling followed by rapid heating of the standard inflation is replaced by a smoothened dissipative mechanism. The classical field $\phi_c(t)$ generates the expansion of the scale factor of the universe. Furthermore the fluctuations of the matter field, with respect to the homogeneous field $\phi_c(t)$, are described by the field $\phi(\vec{x},t)$. In the framework of a more realistic treatment, the fluctuations on the sub-Hubble scale are described by the COBE coarse-grained field $\chi_{Ccg}$, which is defined by a suppression factor $G(k)$, which tends to zero for $k \to 0$. This field describes the observable universe after the horizon entry. The quantum to classical transition of the COBE coarse-grained field $\chi_{Ccg}$, is due to the complex to real transition of the modes $\xi_k$, during inflation. The infrared sector takes into account only the modes with wavelength much bigger than the size of the horizon $k^{-1} > k_o^{-1}(t)$. In particular, during inflation $\chi_{Ccg}(\vec{x},t)$ does not commutes with $\chi_{Ccg}(\vec{x}',t)$ for $|\vec{x} - \vec{x}'| < k_o^{-1}$. This is due to $\chi$ and $\chi'$ are canonically conjugate variables if are “measured” inside a causally connected region of the spacetime. Otherwise, $\chi$ and $\chi'$ can be “measured” independently. The now observable universe is composed by causally desconected domains during inflation.

During inflation, the number of degrees of freedom in the infrared sector $M(t)$ is constantly increasing, since short-wavelength modes cross the horizon from the short-wavelength sector. This effect is seen in the second order stochastic equation as a noise $\xi_c$ and produces quantum decoherence [22] and non-local dissipative effects in the infrared sector. The stochastic properties of $\xi_c$ are given by the suppression factor $G$. Furthermore, the noise $\xi_c$ can be seen as a stochastic force in the framework of the Heisenberg’s representation for the redefined quantum fluctuations $\chi_{Ccg}$. The effective Hamiltonian in the Heisenberg’s representation describes a damped oscillator with a variable $\mu(t)$. This oscillator experiences both, squeezing, due to the time dependent $\mu(t)$, and a stochastic external force, due to the inflow of the short-wavelength modes in the infrared sector.

Finally, two examples were considered. In the example for a de Sitter expansion of the universe, the radiation component for the energy density becomes zero. In this case the parameter of mass $\mu = \frac{4}{3}\pi$ does not depends on time. Furthermore, constraint $|n - 1| < 0.3$ for the spectrum index gives restrictions for the mass of the inflaton field $(1.35 \ H_o < m < 1.46 \ H_o)$ and the parameter $\nu (0.351 < \nu < 0.65)$. Here, the spectral density being given by $|\delta_k| = k^{1-\nu}G(k)$ and the amplitude $A(t)$ decreases with time as $A(t) \approx e^{2(\nu - 3/2)H_o t}$ for $m \neq 0$. When the horizon entry (for $t = t_\ast$), this amplitude becomes freeze with value $A(t_\ast)$. In the example for a power-law expansion for the universe, the constraint $|n - 1| < 0.3$ for the spectrum index gives restrictions for the friction parameter, which is now time dependent. For the particular case $\tau_c = p\gamma t^{-1}$ and $p = 4$, one obtains that $7.1065 H_c(t_\ast) < \tau_c(t_\ast) < 7.1286 H_c(t_\ast)$.

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