On the dynamic nature of charge quantization

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Abstract

It is commonly observed that objects in a gravitational field experience a rate of acceleration that is independent of their mass and that, as a result, all massive objects with the same initial conditions follow the same trajectory. It is not generally recognized, however, that charged particles in an electric field experience an acceleration which is inversely proportional to their mass. This dynamical behavior is an interesting clue to the fundamental nature of the electric force, equally as important as the more familiar behavior of falling bodies, and seems to be the true significance of the observed fact that different charged particles have the same magnitude of charge $e$.

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1 Introduction

It is curious that different charged fundamental particles have the same magnitude of electric charge $e$, even though in general they have different masses. Electrons and protons, for example, have masses that differ by nearly a factor of 2000, yet they have the same magnitude of charge. This fact is widely recognized, but the implications of this for the dynamics of charged particles is not. A comparison between the effects of the gravitational force and the electric force on the acceleration of different particles sheds some interesting light on this issue.

2 Acceleration due to gravity

Galileo was perhaps the first to observe that all bodies fall at the same rate. In other words, they all experience the same rate of acceleration in a gravitational field,
independent of their masses. This is explained quantitatively by Newton’s second
law of motion
\[ F = ma, \tag{1} \]
and his universal law of gravitation,
\[ F_{\text{grav}} = G\frac{Mm}{r^2}. \tag{2} \]
When the gravitational force in Eq. (2) is substituted into Eq. (1), the mass \( m \) of
the falling object appears on both sides of the equation,
\[ ma = G\frac{Mm}{r^2}, \tag{3} \]
and cancels, giving the result that gravitational acceleration is independent of the
mass \( m \) of the falling body,
\[ a_{\text{grav}} = G\frac{M}{r^2}. \tag{4} \]
In the original context of Galileo’s observation, namely falling objects near the
Earth’s surface, the right side of Eq. (4) is simply equal to \( g = 9.8 \text{ m/s}^2 \) and we get
the familiar result that all objects fall with the same acceleration
\[ a_{\text{grav}} = g \quad \text{(independent of mass \( m \)).} \tag{5} \]
But behind this seemingly simple cancelation of the mass in Eq. (3) lies an unresolved
mystery. The two masses \( m \) that appear in Eq. (3) represent conceptually different
quantities. The mass that appears in equation (1) is the object’s inertial mass, a
measure of it’s resistance to change of motion when it is subject to any force, not
necessarily gravity. The mass that appears in Eq. (2) is the object’s gravitational
mass, a measure of its participation in the gravitational force, whether or not it is
being accelerated. Eq. (3) should more correctly read
\[ m_i a = G\frac{Mm_g}{r^2}, \tag{6} \]
where subscripts \( i \) and \( g \) have been added to distinguish between inertial mass and
gravitational mass respectively. Physicists and philosophers alike have repeatedly
pointed out that there is no fundamental reason why these two masses should be
equal and the mystery behind this equality is usually hidden from view by the
common practice of using the same symbol \( m \) to represent both quantities. That
these two masses are proportional to each other is an empirical observation that
was first confirmed by Newton in his experiments with pendulums and subsequently
confirmed with increasing precision by a number of other physicists. It
is, however, the dynamical consequence of this equivalence of inertial and gravitational masses, namely that the acceleration of a massive object in a gravitational field is independent of its mass, which offers a clue to the nature of the gravitational interaction.

This clue was, in fact, the starting point for Einstein’s development of his general theory of relativity. The realization that the motion of bodies in a gravitational field is independent of the nature of the bodies suggested to Einstein that the properties of the gravitational field could be attributed to the structure of spacetime itself. The success of general relativity lies in a mathematical tour de force that tells us how much spacetime must curve in order to reproduce the observed equivalence of gravitational and inertial mass. This is represented mathematically by Einstein’s field equation,

\[ R_{ik} - \frac{1}{2} g_{ik} R = -k T_{ik}. \]  

A detailed explanation of this equation, which can be found in any text on general relativity, need not concern us here. The only aspect of it that is important for the present discussion is that the left side describes how much spacetime curves and the right side describes the distribution of matter which produces this curvature. The equations of motion for objects moving in this curved spacetime then follow from the constraint that they move along the “straightest possible path” (a geodesic) through this curved spacetime.

Einstein’s success at “geometrizing gravity” led to numerous attempts to extend this approach to other forces. But while this geometrization works for the gravitational force, it can not be extended to other forces, such as the electromagnetic force, for which the motion of bodies does depend on the nature of the bodies. Furthermore, general relativity does not explain how (or why) matter causes space to curve, it only tells us how much space curves. In this sense, at least, the theory is incomplete. Einstein, who spent the last three decades of his life searching for a unified field theory that would describe both gravity and electromagnetism in terms of a pure field equation, was himself never fully satisfied with the presence of matter (the right side of Eq. (7)) separate from the field (the left side of Eq. (7)) in his field equation. Writing in 1949, over three decades after he published his theory of general relativity, Einstein wrote, referring to Eq. (7),

The right side is a formal condensation of all things whose comprehension in the sense of a field theory is still problematic. Not for a moment, of course, did I doubt that this formulation was merely a makeshift in order to give the general principle of relativity a preliminary closed expression. For it was essentially not anything more than a theory of the gravitational field, which was somewhat artificially isolated from a total field of as yet unknown structure.
While it is tempting to accept the curvature of spacetime as the “explanation” for the equivalence of gravitational and inertial mass, it must be recognized that general relativity does not answer related questions such as “why do massive objects cause space to curve in just the right way to preserve this equivalence of masses?” and “why do the other forces behave differently from gravity?” Until a theory is advanced which unifies the gravitational and electromagnetic forces and answers these questions, it is perhaps best to withhold judgment on whether general relativity gives us the final word on the equivalence of masses. Especially when comparing this situation with that presented by the electric force in the next section, it may yet be fruitful to continue to ponder the question “why is the acceleration of a massive object in a gravitational field independent of its mass?”

3 Acceleration due to the electric force

The situation with the electric force does not suffer from the confusion between the two masses because the electric force is proportional to electric charge rather than to gravitational mass, as is evident in Coulomb’s Law,

\[ F_{\text{elec}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}. \]  

(8)

The details of the source charge(s) are commonly absorbed into the electric field \( E \),

\[ F = qE. \]  

(9)

This has the advantage that it makes it easier to focus on the effects of the electric interaction on the charge \( q \) without the distraction of the details of the sources that produce the electric field. In an electric field \( E \), different charged particles do not accelerate at the same rate as they do in a gravitational field, but if we consider two different charged fundamental particles in the same electric field, we observe an effect which is equally interesting. Let particle 1 have mass \( m_1 \) and charge \( q_1 = e \), and particle 2 have mass \( m_2 \) and charge \( q_2 = e \), where we explicitly use the fact that different charged fundamental particles have the same unit of electric charge \( e \). The particles could be, for example, a \( \mu^- \) lepton (charge \(-e\)) and a \( \pi^+ \) meson (charge \(+e\)). Applying Newton’s second law to both of these particles (ignoring the minus sign which determines the direction of the acceleration),

\[ m_1 a_1 = eE, \quad m_2 a_2 = eE, \]  

(10)

it immediately follows that

\[ m_1 a_1 = m_2 a_2. \]  

(11)
There is, of course, a long list of fundamental particles with different masses but the same magnitude of charge $e$, and what Eq. (11) is saying is that in an electric field they all experience an *acceleration that is inversely proportional to their mass*,

$$a_{\text{electric}} \propto \frac{1}{m} \quad (12)$$

This dynamic behavior of charged particles in an electric field seems to be the real significance of the observation that different fundamental charged particles have the same magnitude of charge $e$. Nature seems to be giving us a clue here which sheds a different light on the quantization of electric charge. Instead of asking why different fundamental particles have the same magnitude of charge $e$, we should be asking “why is the acceleration of a charged particle in an electric field inversely proportional to its mass?”

4 Conclusion

In the gravitational interaction, it is the fact that the acceleration of all bodies is independent of their mass that allows the construction of a theory of gravity in terms of the curvature of spacetime and independent of the properties of the objects. In contrast, the electric interaction causes a different acceleration for different objects and a similar geometric theory of the electric force following the pattern of general relativity is not possible. This, I believe, is the ultimate reason that the attempts of Einstein and others to develop a unified theory of gravity and electromagnetism based on the model of general relativity have failed.

A brief survey of more recent advances in theoretical physics shows that we are no closer to understanding this difference in the dynamics of particles in gravitational and electric fields. The current theory of elementary particles, the standard model, does not address this issue directly. It assumes the masses and charges of the elementary particles as inputs to the theory but cannot account for the observed mass spectrum and sheds no light on why different charged particles have the same magnitude of charge. String theory offers an enticing suggestion that different masses correspond to different resonances of a fundamental string, but the mathematical formalism is not yet developed enough to allow the calculation of the mass spectrum from fundamental principles, and like the standard model also offers no clue as to why different elementary particles have the same charge. And finally, active research into quantum gravity seems directed more toward understanding the properties of spacetime itself than toward the behavior of the objects that populate it.

Nature is offering us clues to a deeper understanding of the fundamental particles and their interactions, and we would do well to pay attention to them. The
two facts that (1) gravitational acceleration is independent of mass and (2) electric acceleration is inversely proportional to mass are clues that should be given equal importance. Add to these the equally unexplained mystery of why gravity is always attractive but the electric force can be both attractive and repulsive, and we get a sense of the limits of our understanding of these fundamental forces. Any theory which attempts to unify gravity and electromagnetism must account for these differences in behavior. Existing theory has not been able to resolve this issue. We need a new perspective, a new way of thinking about this issue. Instead of thinking of charge quantization as a particle property, we should be thinking of it as a consequence of the dynamics of particle interactions. Instead of asking “why do different charged particles have the same magnitude of charge $e$?” we should be asking “why do different charged particles experience accelerations that are inversely proportional to their masses?”

References

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