A general framework of covid-19 model with control strategies: the application of large-scale social restrictions, self-quarantine, and immune system improvement in Surabaya, Indonesia

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Abstract. A covid-19 is a virus disease of worldwide importance which spread not only in a small country such as Mongolia but also in a large country like the United State. It is caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) and well known as droplet-borne disease. The droplets containing coronavirus are spread through the air within a meter and deposited on the mucous membranes of susceptible person. The aim of this study was to produce a general framework model of corona virus disease with the application of large-scale social restriction, self-quarantine and immune system improvement as control strategies to reduce the spread of the disease. This study was conducted in Surabaya, Indonesia with a total of 299 positive patients infected with covid-19 until the fourth week of April, 2020. The transmission routes of the disease had been constructed based on the real problem. Therefore, human population have been divided into some sub-population based on the characteristics of disease symptoms. The spread of covid-19 have been formulated mathematically by using compartment model. Stability analysis had been done for a free disease equilibrium and an endemic equilibrium. Moreover, numerical simulation were also provided to quantify the disease dynamics.

Keyword: Covid-19, Mathematical Model, optimum control

1. Introduction
In early 2020, the world community was frightened by a virus that resulted in the death of more than one hundred thousand people worldwide. The virus was spread evenly in almost all countries in the world both small countries such as Mongolia and large countries like the United States. WHO stated that in the last week of April 2020, there were 152,551 people declared dead due to coronavirus. While, the number of corona-positive patients worldwide had reached 2,241,778 people [1]. The number of these patients continued to increase everyday in each country.

In the early discovery of coronavirus around the 1930s, the virus infected many respiratory tracts in animals such as chickens, while no cases of coronavirus infections were found in humans until around the 1960s [2], [3], [4]. The first case of coronavirus that infected humans is well-known as human coronavirus 229E (HCoV-229E) and human coronavirus OC43 (HCoV-OC43). Regard to some researchers, covid-19 was a disease caused by novel coronavirus named as severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). This virus was believed to originate from animals and had genetic similarities to the coronavirus found in bats [5], [6], [7]. All infections caused by coronavirus in humans attack the respiratory tract in all ages (children, adults, and elderly).

The transmission route of the virus infects humans through droplets from someone who has been infected earlier. Furthermore, it enter the mucous membrane of a healthy person and replicates...
itself in that person in the respiratory section. The virus takes approximately 14 days to be able to infect someone until the person can then infect others [8], [9]. Meanwhile, until the end of April 2020, there was no vaccine or drug treatment could be consumed by corona-positive patients.

Someone who had been infected by the coronavirus had to work very hard to improve his immune system. It is because the immune system was the only one believed can removed the virus from human body. Meanwhile, people who had not been infected by the virus, it was endeavored to maintain their health by behaving in a healthy lifestyle such as using masks in public places, washing hands with soap, using hand sanitizer, and spraying disinfectants into several places. Some countries in the world also made efforts to lock down the area to prevent the spread of the disease.

Surabaya as one of the major cities in Indonesia was participating in prevention by carrying out large-scale social restrictions, self-quarantine, and immune system improvement. It was carried out by the local government as one of the control strategy in reducing the spread of covid-19. The people of Surabaya were asked to remain at home for approximately 14 days from 28th of April 2020, to perform self-quarantine after returning from areas which were the epicenter of covid-19, and to improve immune system so then the patients of covid-19 could recover soon. In Indonesia, covid-19 patients were divided into 3 groups, namely people in monitoring (ODP), patients under supervision (PDP), and positive confirmed patient (Positive). This study was conducted to construct a general framework of the covid-19 model using large-scale social restriction, self-quarantine, and immune system improvement as a control of the disease spread.

2. Mathematical Model
In this study a mathematical model of covid-19 was built by considering large-scale social restrictions, self-quarantine, and immune system improvement as a control strategy to reduce its spread. It was built from a compartment model that perfectly matched the characteristics of the disease, such as: 1) the virus requires an average of 14 days of incubation to infect a healthy person, 2) An infected person who had a good immune system take approximately 20 days to recover, 3) An infected person without immune system will result in death, 4) a recovered person can be re-infected. Regard to this fact, the appropriate model for the spread of covid-19 is the SEIRS model. Meanwhile, the use of large-scale social restrictions was done to limit the movement of healthy people to stay at home so that it reduced the possibility of interacting with people who have been infected, self-quarantine was done to ensure if people who leave the house do not carry the virus or transmit it to others, and immune system improvement was done as an effort to fight the virus that was already in the human body.

Figure 1 shows the compartment model for the covid-19 transmission route in the human population. The controls used in the model were $u_1$ as large-scale social restrictions, $u_2$ as self-quarantine, and $u_3$ as immune system improvement.

Furthermore, we derived compartment model into an ordinary differential equation, as follow.
\[ \frac{dS}{dt} = \Lambda - (1 - u_1)\beta S_0 \frac{I}{N} - (1 - u_1)\alpha S_0 - \mu S_0 + \theta R \]

\[ \frac{dS_1}{dt} = (1 - u_1)\alpha S_0 - (1 - u_2)\beta S_1 \frac{I}{N} - \mu S_1 \]

\[ \frac{dE}{dt} = (1 - u_3)\beta S_0 \frac{I}{N} + (1 - u_2)\beta S_1 \frac{I}{N} - (1 - u_3)\gamma E - u_3\rho E - \mu E \]

\[ \frac{dI}{dt} = (1 - u_3)\gamma E - \mu I - \delta I - \rho I \]

\[ \frac{dR}{dt} = u_3\rho E + \rho I - \mu R - \theta R \]

with \( S_0(0), S_1(0), E(0), I(0), R(0) \) given, where the human population was divided into 5 sub-populations in the following classes.

\( S_0 \): Susceptible

\( S_1 \): Susceptible who has returned from the epicenter area of covid-10 (ODP)

\( E \): Exposed

\( I \): Infected

\( R \): Recovered

By setting the right-hand side of the equations equal to zero, two equilibrium can be obtained, as follows.

a. Disease-Free Equilibrium

\[ DFE := \left( S_0 = -\frac{\Lambda}{\alpha u_1 - \alpha - \mu}, S_1 = \frac{(u_1 - 1)\Lambda\alpha}{(\alpha u_1 - \alpha - \mu)\mu}, E = 0, I = 0, R = 0 \right) \]

b. Endemic Equilibrium

\[ EE := (S_0 = S_0^*, S_1 = S_1^*, E = E^*, I = I^*, R = R^*) \]

It was not easy to determine the endemic equilibrium analytically due to the complexity of the equation, however it was still possible to show the endemic equilibrium numerically. As a routine stability analysis, if the basic reproduction number \( R_0 < 1 \) then disease-free equilibrium will be stable. Otherwise, endemic equilibrium will be stable.

Furthermore, we wanted to minimize our objective function \( [10] \), as follows

\[ J(u_1, u_2, u_3) = \int_0^{t_1} \left[ E(t) + I(t) + \frac{B_1}{2} u_1^2(t) + \frac{B_2}{2} u_2^2(t) + \frac{B_3}{2} u_3^2(t) \right] dt \]

where we want to minimize the exposed and infected population while also kept the cost of the control low. Our assumption that the costs of the control were nonlinear and took quadratic from there. The constant \( B_1, B_2 \) and \( B_3 \) were balancing cost factors due to size and importance of the three parts of the objective functional. We were looking to find an optimal control \( u_1^*, u_2^*, u_3^* \), and \( u_3^* \) such that

\[ J(u_1^*, u_2^*, u_3^*) = \min J(u_1, u_2, u_3) \]

\[ \Omega = \{(u_1, u_2, u_3) \in L^1(0, t_1) | a_i \leq u_i \leq b_i, i = 1, 2, 3 \} \] and \( a_i, b_i, i = 1, 2, 3 \) were positive constant. The following assumptions, \( \Lambda = \mu N \) and \( \delta = 0 \), were used to analyse our model therefore the total population \( N \) was constant.

Furthermore, Pontryagin’s Maximum Principle converts the model into a problem of minimizing Hamiltonian function with respect to all controls, as follows.

\[ H = z_0 \left[ E + I + \frac{B_1}{2} u_1^2 + \frac{B_2}{2} u_2^2 + \frac{B_3}{2} u_3^2 + z_1 \left[ \Lambda - (1 - u_1)\beta S_0 \frac{I}{N} - (1 - u_1)\alpha S_0 - \mu S_0 + \theta R \right] \right. \]

\[ + z_2 \left[ (1 - u_1)\alpha S_0 - (1 - u_2)\beta S_1 \frac{I}{N} - \mu S_1 \right] \]

\[ + z_3 \left[ (1 - u_1)\beta S_0 \frac{I}{N} + (1 - u_2)\beta S_1 \frac{I}{N} - (1 - u_3)\gamma E - u_3\rho E - \mu E \right] \]

\[ + z_4 \left[ (1 - u_3)\gamma E - \mu I - \delta I - \rho I \right] + z_5 [u_3\rho E + \rho I - \mu R - \theta R] \]

There are several conditions that must be met to determine optimal control, such as.
a. Hamiltonian condition
\[ \dot{z}_0 = - \frac{\partial H}{\partial x_0} = 0 \quad \text{WLOG } z_0 = 1 \]
\[ \dot{z}_1 = - \frac{\partial H}{\partial S_0} = z_1 \left[ \left( 1 - u_1 \right) \beta \frac{I}{N} + \left( 1 - u_1 \right) \alpha + \mu \right] + z_2 \left[ \left( 1 - u_1 \right) \alpha \right] + z_3 \left[ \left( 1 - u_1 \right) \beta \frac{I}{N} \right] \]
\[ \dot{z}_2 = - \frac{\partial H}{\partial S_1} = z_2 \left[ -\left( 1 - u_2 \right) \beta \frac{I}{N} + \mu \right] + z_3 \left[ -\left( 1 - u_2 \right) \beta \frac{I}{N} \right] \]
\[ \dot{z}_3 = - \frac{\partial H}{\partial E} = -1 + z_3 \left[ 1 - u_3 \right] \gamma + u_3 \rho + \mu] + z_4 \left[ -\left( 1 - u_3 \right) \gamma \right] + z_5 \left[ -u_3 \rho \right] \]
\[ \dot{z}_4 = - \frac{\partial H}{\partial I} = -1 + z_1 \left[ \left( 1 - u_1 \right) \beta \frac{S_0}{N} \right] + z_2 \left[ \left( 1 - u_2 \right) \beta \frac{S_1}{N} \right] + z_3 \left[ -\left( 1 - u_1 \right) \beta \frac{S_0}{N} - \left( 1 - u_2 \right) \beta \frac{S_1}{N} \right] + z_4 \left[ \mu + \delta + \rho \right] + z_5 \left[ -\rho \right] \]
\[ \dot{z}_5 = - \frac{\partial H}{\partial R} = z_1 \left[ -\theta \right] + z_5 \left[ \theta \right] \]

b. Transversality condition
\[ S_0(t_1), S_1(t_1), E(t_1), I(t_1), R(t_1) \text{ arbitrary} \]
Target were in \( R^5 \) then for \( n = 6 \), choose \( k = 0 \)
\[ z^1 = 0 \]
\[ z_1(t_1) = z_2(t_1) = z_3(t_1) = z_4(t_1) = z_5(t_1) = 0 \]
The following characterization holds
The control \( u_1 \) optimum when \( \frac{\partial H}{\partial u_1} = 0 \)
\[ \frac{\partial H}{\partial u_1} = 0 = B_1 u_1 + z_1 \left[ \beta S_0 \frac{I}{N} + \alpha S_0 \right] + z_2 \left[ -\alpha S_0 \right] + z_3 \left[ -\beta S_0 \frac{I}{N} \right] \]
\[ u^*_1 = \min(\max(a_1, u_1), b_1) \]
The control \( u_2 \) optimum when \( \frac{\partial H}{\partial u_2} = 0 \)
\[ \frac{\partial H}{\partial u_2} = 0 = B_2 u_2 + z_2 \left[ \beta S_1 \frac{I}{N} + \beta S_1 \frac{I}{N} \right] + z_3 \left[ -\beta S_1 \frac{I}{N} \right] \]
\[ u^*_2 = \min(\max(a_2, u_2), b_1) \]
The control \( u_3 \) optimum when \( \frac{\partial H}{\partial u_3} = 0 \)
\[ \frac{\partial H}{\partial u_3} = 0 = B_3 u_3 + z_3 \left[ \gamma E - \rho E \right] + z_4 \left[ -\gamma E \right] + z_5 \left[ \rho E \right] \]
\[ u^*_3 = \min(\max(a_3, u_3), b_1) \]

3. Results and Discussion
As we wanted to study the effect of optimal control strategy on coronavirus transmission, the simulation model was divided into two parts. First part was a model without control and the second part was a model that considered large-scale social restrictions, self-quarantine, and immune system improvement. Furthermore, numerical calculations were performed to determine the controls that
correspond to the results of analytical calculations using parameters which were consistent with the characteristic of covid-19.

| Description                          | Parameter | Value |
|--------------------------------------|-----------|-------|
| Large-Scale Social Restriction       | $u_1$     | 0.95  |
| Self Quarantine                      | $u_2$     | 0.19  |
| Immune System Improvement            | $u_3$     | 0.71  |

Table 1. Parameters value

The optimum control parameters used in this study are presented in table 1. The forward fourth-order Runge Kutta method was used to find the solution of the state equation. Meanwhile, the backward forth-order Runge Kutta was used to find the solution of the co-state equation.

In Fig 2 (i), due to no controls involved in the system, the number of susceptible person went down from about 6,000 people to 800. Where as, the number of positive confirmed patients (Positive) and the number of patients under supervision (PDP) were increase significantly above 1,000 and 2000 respectively. It also influenced the number of recovered patients. In the other hand, fig 2 (ii) which some control strategies were included in the model, the number of susceptible person increased significantly. Meanwhile, the number of patients under supervision (PDP) were less than 2,000. The number of recovered people also increased compare to the number of recovered people in the system without any controls.

The difference between two model also can be seen in the figure 3. The total number of patients covid-19 in Surabaya decreased not only in number but also in time. Fig 3 (i), model without control, show that the number of patients increased dramatically since beginning. Where as in fig 3 (ii), model with control strategy, the number of patients decreased in the beginning and then increased significantly but still less than 3,000 people.
Figure 3. (i) Numerical simulation for total number of covid-19 cases without optimal control and (ii) Numerical simulation for total number of covid-19 cases with optimal control

4. Conclusion
The application of large-scale social restrictions, self-quarantine, and immune system improvement influenced the spread of the disease significantly. The control strategies which have been applied to the model can reduce not only the number of patient but also the period of the spread. It also can save the number of susceptible person. However, regard to the simulation model, it can be concluded that the coronavirus will remain to infect human people for long period. The characteristic of Covid-19 tend to be similar to other diseases caused by viruses such as influenza, measles, etc.

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