Light hadron masses with an $O(a^2)$ improved NNN action

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Meson and baryon masses in the light (u,d and s) sector are calculated using tadpole-improved gauge field and fermion actions. These are corrected to order $O(a^2)$ on the classical level using next-nearest-neighbour terms. The results, obtained at lattice spacings of 0.4 and 0.27fm, are compared to Wilson action calculations. Improved actions hold great promise for moving lattice field theory closer to phenomenology.

Additional terms in the action that are nonleading in powers of the lattice spacing $a$ can be used to reduce discretization errors. These can be significantly suppressed further by incorporating tadpole factors $[1]$ with the nonleading terms in the action.

It is desirable to push improvement to the next-to-leading order in $a$. Alford et al proposed the so-called D234 action $[2]$. This action is corrected to $O(a^2)$ and tadpole improved. It is built by adding clover $[3]$ and next-nearest-neighbour terms to the Wilson action.

A simpler next-nearest-neighbour (NNN) action was considered some time ago $[4]$. Tree-level $O(a^2)$ improvement is achieved by employing 2-link terms for the fermionic part and 6-link rectangles for the gauge field. We here use this NNN action with tadpole improvement to compute light hadron masses. Calculations with this action were also reported by Bock $[5]$ and by Boriçi and de Forcrand $[6]$.

The $SU(3)$ gauge field part of the action is

$$S_G(U) = \beta \left[ \sum_{pl}(1 - \frac{1}{3}\text{Re} \text{Tr} U_{pl}) + C_{rt} \sum_{rt}(1 - \frac{1}{3}\text{Re} \text{Tr} U_{rt}) \right]. \tag{1}$$

The Wilson action is recovered by replacing the coefficient $4/3$ by $1$ and dropping NNN terms.

Calculations were carried out in quenched approximation for the NNN and the Wilson action at matching physical lattice spacings $a_{st}$ determined from the string tension $[7]$. Two set of simulations were done at 0.4 and 0.27fm. The lattice parameters are shown in Tab. 1.

Quark propagators were calculated for a range of $\kappa$ values using a stabilized biconjugate gradient algorithm. Periodic boundary conditions were imposed on the quark fields in spatial directions.
but in the time direction a Dirichlet, or fixed, boundary condition was used. The source position was two time steps in from the boundary in all simulations.

Meson and baryon correlators, constructed from standard local interpolating fields, were obtained for both local and smeared sinks with local sources. Gaussian smearing was used.

Masses were calculated using an analysis procedure motivated by Bhattacharya et al. This involves a combination of effective mass functions from local-local and local-smeared correlators. Except for the baryon spin-3/2 channel, it was found that compatible masses could be obtained using time averages starting 2 or 3 time steps away from the source.

For each simulation the masses were extrapolated as a function of pion mass to the chiral limit $M_\pi = 0$. The choice of extrapolation function was either $M = M_0 + cM_\pi^2$ or $M = M_0 + cM_\pi^2 + dM_\pi^3$ depending on whether the coefficient $d$ could be determined to be nonzero within the statistical errors. The errors in masses and in mass ratios were estimated using a bootstrap procedure with 500 samples.

In the strange quark sector we have fixed $\kappa_s$ such that $K^*/K$ equals the experimentally observed 1.8. This was done by linear interpolation (or extrapolation) using the two $\kappa$ values from our set nearest to $\kappa_s$, see Tab. 1.

Mass ratios extrapolated to the chiral limit are given in Tab. 2. The ratio $M_N/M_\rho$ sets an overall scale of baryon masses relative to meson masses. This seems to be the quantity most effected by discretization errors. By 0.27fm the improved action results are fairly close to experiment and are compatible with Wilson action calculations done at small lattice spacing.

An interesting way of looking at the relative quark mass dependence of pseudoscalar and vec-
tor meson masses is to examine \( M_V^2 - M_P^2 \). Empirically, this quantity is approximately constant (\( \approx 0.5 \)), a fact which has been linked to chiral symmetry [12]. Therefore, its behaviour in lattice QCD may shed light on the role of chiral symmetry breaking Wilson-like terms in the action [13]. In Fig. 1 we plot the squared mass difference in the limit of zero quark mass (\( M_\pi = 0 \)). At comparable lattice spacings the NNN results are much closer to \( M_V^2 - M_P^2 = \text{const} \) than the Wilson results.

In Fig. 2 we have compiled some results for the ratio of the lattice spacing extracted from the string tension to the lattice spacing determined by the \( \rho \)-meson mass [9,11,14–16]. The improved action and Wilson action results show the same qualitative behaviour only shifted in lattice spacing by about a factor of 3. A remarkable feature seen in Fig. 2 is that different improved fermion actions exhibit a high degree of universality even in the non-scaling region.

If simulations done with improved actions on coarse lattices are to be useful the results should extrapolate smoothly to the continuum limit. The ratio of nucleon to \( \rho \)-meson mass has been calculated a number of times. A sample of Wilson action results [9,11,17] and results for tadpole improved actions [15,16] are presented in Fig. 3. It is encouraging that our improved action values at 0.27 and 0.4fm are compatible with the Wilson action results at smaller lattice spacing.

In this work we have seen that, compared to the Wilson action, the tadpole improved NNN action operates much closer to the continuum limit. Not surprisingly, the closeness of the mass spectrum to phenomenology, see Fig. 4, confirms this conclusion.

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Figure 3. Ratio $M_N/M_\rho$ versus $a_\rho$ for improved actions (open symbols) and the Wilson action (solid symbols).

Figure 4. Level scheme of hadron masses from the NNN action at $\beta = 6.8$ and experimental values.

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