Expedited Pose Estimation Algorithm Involving Perturbance Affine Term Based on Projection Vector for Space Target

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\textbf{ABSTRACT} The present study primarily discusses the perturbance error indeterminacy that is caused by anisotropic and correlated non-identical gray distribution of feature points in vision measurement for space target pose parameters. On that basis, an expedited algorithm that involves perturbance affine term based on the novel statistical objective function is proposed. By invoking the inverse covariance matrix to model a novel data space, the pose estimation algorithm based on projection vector is capable of reducing the effect of different levels of disturbance error on the measured results, as well as effectively avoiding the poor or non-convergence attributed to data degradation. Furthermore, the repeated calculation is avoided by coupling each iteration, which significantly simplifies the computation. As a consequence, the calculation complexity of each iteration decreases from $O(n)$ to $O(1)$, and the expediting process is implemented significantly. Lastly, as revealed from the experimental results, the calculation efficiency is improved by 3.3 times, and the maximum measured error of the space target attitude is less than 0.1$^\circ$. Compared with the conventional methods, the proposed algorithm exhibits the effectively promoted speed-ability, precision and indeterminacy attenuation performance, suggesting that the proposed approach should have promising practical applications in deep-space target capture.

\textbf{INDEX TERMS} Vision measurement, pose estimation, statistical objective function, EIPAT algorithm.

\section{I. INTRODUCTION}

Pose estimation is the problem to solve the target position and attitude under required conditions, i.e., the known internal parameters of the camera, the set spatial point coordinates, as well as the corresponding planar image point coordinates [1], [2]. Overall, pose estimation is termed as the multi-point perspective problem, which has been prevalently employed in photogrammetry, robotics and industrial inspection [3]–[5].

To boost the accuracy of the target pose and reduce the effect of interference factors on the estimated results, numerous classical methods (e.g., Gaussian-Newton, Levenberg-Marquardt or other nonlinear optimization) have been presented successively [6], [7]. In the mentioned inchoate conventional methods, however, when a feasible initial value cannot be presented, more iterations are required, and even local convergence or convergence failure may occur. References [8], [9], proposed the nonlinear iterative algorithm with a scale coefficient, which exhibits high calculation accuracy and good robustness and meets the orthogonality constraint. However, this method should still meet high initial value requirements, and it remains low in practical applicability. The further extended algorithm in [10] repeatedly calculates the pose of the scaled orthographic projection camera linearly; it adopts the strategy of calculating the pose and depth-related scale factors respectively, whereas the precision of the model should be enhanced as impacted by the limitations of the model. Later, Hartley and Kahl used the traversing search space and the branch constraint to develop the global optimal solution of infinite norm; the calculation is significantly huge. These existing classic methods are the suboptimal solutions in a specific limited domain for the complex requirements of measurement scenarios.
Over the past decades, theories and applications of vision-based pose estimation have been significantly advancing. Some scholars adopted the normalized objective function to improve the accuracy and robustness of the estimated results. Thus, the impact of imaging error can be reduced to a certain extent, as suggested by the example [12]–[14] and the references therein. To satisfy the demands of motion control parameter input, the results achieved based on the visual feedback scheme of relative quaternion estimation [15] and coupled relative translation [16] are referenced for attitude control and orbit tracking. To enrich the available feature information, as highlighted in [17], a precise method for pose estimation by exploiting the object geometric structure is significantly effective, employing the corners of the solar panel as features. Moreover, [18], [19] suggested that the information-rich edge contour features lead to the advantages of small calculation, stable extraction, and strong anti-noise ability. Recently, [20] and [21] proposed an advanced approach based on optimal control to estimate the robustness of the pose of rigid objects in light field 3D imaging, as well as a navigation parameters method exhibiting high accuracy and strong stability using different cooperation markers and target shapes. Moreover, the constraint relationships between stereo multiple cameras are used to optimize the process of parameters measurement. Reference [22] demonstrated a novel and effective technique to acquire camera pose information with decoupling the distortion coefficients and intrinsic parameters, which achieved the more stable and reliable results. Some studies further verified the use of restrictive conditions, [23]–[25], where the flexible structure was built to decrease the effect of significant vibration and feature noise. For the global convergence and autonomous estimation, by an optimized method, the global solutions are obtained for the camera position and orientation based on a set of image pixel measurements associated with a scene of reference points of known three-dimensional positions, as illustrated in [26]. Literature [27] presented a novel method based on the contour features, which was extended by adopting projected features; with this method, the estimation limitations caused by the lack of features on free form objects could be addressed. Furthermore, [28] introduced an adaptive thresholding and a median-based Otsu method to determine the optimal threshold for ball grid array inspection in surface mounting machines.

Overall, a wide range of nonlinear optimization techniques are employed to minimize the objective function to determine the pose parameters precisely. However, the conventional objective function is commonly composed of the re-projection error of the feature points, and assuming that the disturbance noise exhibits an isotropical, independent and uniform distribution [14], [28], [29]. Under this assumption, the determined pose parameters are the maximum likelihood optimal results (as suggested by [30], [31] and the references therein). However, in practical applications, this is rarely the case complying with the ideal assumption [32]–[34]. On the whole, disturbance noise in feature points exhibits anisotropic and non-independent distribution. Nevertheless, it is often the case of significant directivity. In this scenario, the results achieved can inevitably exhibit the indeterminacy of the disturbance error, and the magnitude of the uncertainty may be not completely consistent. If the perturbation error of feature points is not distinguished, large deviations are likely to be caused between the estimated results and the actual value. It is noteworthy that the running time of the pose estimation algorithm considering perturbation error indeterminacy of noise will increase certainly, which should be accelerated to improve the operating efficiency. Thus, some rapid estimation algorithms employed exert a higher execution effect; recently, some fast vision measurement systems have been extensively investigated [35], [36], even though the search path planning may cause the complexity to increase. However, the mentioned strategies are not suitable for accelerating the pose estimation algorithm under the gray distribution mode. Thus, it is practically important to further study and build a rapid technique based on the minimization objective function with the indeterminacy of imaging feature points considered, with which the proposed algorithm can be suitable for practical applications. Note that the expedited pose estimation algorithm will be more challenging and sophisticated when the error indeterminacy is introduced in the process. For the pose estimation algorithm that involves the perturbation affine term based on the projection vector, it should be highlighted that no insightful investigations have been reported thus far on the scenario of expediting solution process.

Motivated by the mentioned observations, the present study aims to achieve accurate and reliable target pose estimated results from the coordinate data of the feature points with different degrees of perturbation error indeterminacy. The major contribution of the present study is that the versatile objective function is newly set with the perturbation affine term by analyzing feature point imaging gray distribution in depth. Furthermore, the IPAT (Involving Perturbance Affine Term, IPAT) algorithm for solving pose parameters based on the novel objective function is accelerated, upon which the EIPAT ( Expedited IPAT, EIPAT) algorithm is derived. The repeated calculation is eliminated by coupling each iteration, which significantly simplifies the computation. As a result, the calculation complexity of each iteration is reduced from \( O(n) \) to \( O(1) \) and manifestly expedited.

The rest of the present study is organized as follows. In Section 2, the novel objective function with the perturbation affine term is established by analyzing the inverse covariance matrix of the feature point region gray level. In Section 3, the expedited pose estimation algorithm involving the perturbation affine term is presented and assessed. In Section 4, the numerical and practical experiments are elucidated to verify the feasibility and effectiveness of the proposed technique. In Section 5, the conclusion is drawn.

II. RELATED WORKS
In practice, the corner points, circular marker points and other specific geometric shapes on the imaging plane of the target
exhibit a diverse gray distribution, which is anisotropic and non-independent. Thus, the unavoidable perturbation error of the reference point coordinate extraction is caused. In this section, the statistical objective function is set by involving perturbation affine term based on the collinearity error of projection vector.

A. PROBLEM FORMULATION

It is noteworthy that the measured value of the perturbation error is inconsistent with high directivity. If the target pose is evaluated directly without considering the perturbation error indeterminacy, the measured results may be far from the true value.

To qualitatively denote the inconsistent size of the perturbation error indeterminacy, the inverse covariance matrix referenced from [29], [37] is adopted for modelling. Hence, it is expressed as

$$Q^{-1} = \sum_{(u,v)\in \Omega} \omega(u,v,1) \begin{bmatrix} \nabla g_u \nabla g_u & \nabla g_v \nabla g_u & 0 \\ \nabla g_u \nabla g_v & \nabla g_v \nabla g_v & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(1)

For concise notation, where the covariance matrix $Q$ denotes the inconsistent size of perturbation error indeterminacy of the feature points; $\nabla g_u$ and $\nabla g_v$, respectively represent the gradient along $u$ and $v$ direction; $\Omega$ is the gray area and its center coincide exactly with the corresponding image point; $\omega$ is the sum of the pixel gray level in the elliptical or circle region $\Omega$. Before further proceeding, three diverse forms of imaging point perturbation error indeterminacy are illustrated in Figure 1.

In the mentioned model, $Q^{-1}$ denotes a region of indeterminate ellipse with the center at $p_i = (u_i, v_i)^T$. The size of the ellipse’s long axis $a$ and short axis $b$ respectively represent the size of the indeterminacy of the imaging point perturbation error. The angle of the major axes $a$ along the $u$ axis and that of the minor axes $b$ along the $v$ axis refer to the directions of perturbation error indeterminacy, respectively.

In Figure 1, the first case $a/b = 1$ indicates that the perturbation error indeterminacy is isotropic, and it is not related in the direction of $u$ and $v$ axis. Thus, the indeterminacy in both directions is equal and relatively small. Case II $a/b > 1$ or $a/b < 1$ indicates that the indeterminacy of perturbation error exhibits directionality, and yet it is not correlated in $u$ and $v$ axis. The last case is $a/b > 1$ or $a/b < 1$, whereas it is correlated in $u$ and $v$ axis. In such scenario, the covariance matrix of perturbation error indeterminacy is expressed as $Q$ with the $3 \times 3$ dimensions.

It is assumed that the objective function for pose estimation is built in the conventional manner and the indeterminacy of elliptical region illustrated in Figure 1, (I), the globally optimal pose solution can be yielded by iterative optimization. However, in such case of (II) and (III), a novel objective function should be built by considering the indeterminacy of perturbation error, rather than roughly adopting a conventional objective function.

B. NOVEL OBJECTIVE FUNCTION ESTABLISHED WITH THE PERTURBANCE AFFINE TERM

Based on the mentioned analysis of the effects of inconsistent distribution of imaging feature points, this section uses the covariance matrix of perturbation error indeterminacy to build the affine transformation matrix. To be specific, the perturbation error indeterminacy of the feature points is incorporated into pose estimation. As a result, the optimization objective function can be built with the perturbation affine term.

The covariance matrix of two-dimensional gray distribution $Q$ refers to a symmetric positive semidefinite matrix. It is therefore deduced that $Q = U \Sigma V^T$ by singular value decomposition of $Q$, where $\Sigma = \text{diag}(\sigma_{\min}^2, \sigma_{\max}^2, 1)$. $V$ denotes a real orthogonal rotation matrix with $3 \times 3$ dimensions; $\sigma_{\min}$, $\sigma_{\max}$ are the standard deviation of the indeterminacy along the maximum and minimum directions, respectively. The inverse matrix of the covariance matrix $Q$ is defined as:

$$Q^{-1} = U \text{diag} \left(1/\sigma_{\min}^2, 1/\sigma_{\max}^2, 1\right) V^T$$

(2)

It is obviously suggested that the perturbation error indeterminacy is inversely proportional to the weight. In other words, the larger the indeterminacy, the less the weighted contribution to objective function will be. Subsequently, the affine matrix $W$ can be obtained by a covariance matrix.

$$W = \Sigma^{-1/2} V^T$$

(3)
In Eq. (3), affine matrix $W$ can transform the coordinates of re-projection points into the covariance matrix $Q$ data space. It is assumed that the coordinates of re-projection points as $\hat{p}_i$, then the Euclidean coordinates $\hat{p}_i'$ after transforming by affine matrix transformation can be expressed as:

$$\hat{p}_i' = \left(\hat{u}', \hat{v}', 1\right)^T = W\hat{p}_i^T = W(\hat{u}, \hat{v}, 1)^T$$  \hspace{1cm} (4)

Eq. (4) expresses that perturbation error indeterminacy is added in the data space of covariance matrix after transforming by affine term $W$. Subsequently, the inconsistent perturbation error is invoked in the image space objective function, which is written as:

$$e = \sum_{i=1}^{N} \left\| p_i^T - W\hat{p}_i^T \right\|^2$$  \hspace{1cm} (5)

where $p_i$ denotes the coordinate in planar image coordinate system. The aim of this study is to achieve the maximal consistency between the imaging points and the re-projection points by continuous iterations, as well as to find the optimal solution for objective function with the affine item.

Subsequently, the affine transformation is equivalently drawn into the minimum objective function of space colinear error. It is assumed that $\hat{p}_i^w$ is the reference point coordinate in the world coordinate system, and $\hat{P}_i^c$ denotes the reference point coordinate in the camera coordinate system. First, the mentioned reference points are processed by zero averaging as follows.

$$\left\{ \begin{array}{l}
\hat{P}_i^w = P_i^w - \bar{P}^w \\
\hat{P}_i^c = P_i^c - \bar{P}^c
\end{array} \right.$$  \hspace{1cm} (6)

where $\bar{P}^w = \frac{1}{N}\sum_{i=1}^{N} P_i^w$ and $\bar{P}^c = \frac{1}{N}\sum_{i=1}^{N} P_i^c$ represent the center of gravity of corresponding reference points.

Given the coordinate transformation relationship, $\hat{P}_i^c = R\hat{P}_i^w + t; R$ and $t$ respectively denote the rotation matrix and translation vector between the world coordinate system and the camera coordinate system.

Before further proceeding, $\bar{P}_{i|\perp}$ is assumed to be the projection of spatial point $\bar{P}_i^c$ on the camera line of sight. Under $\bar{P}_i^c$, the projection vector of the reference points along the line of sight, it is defined that $\hat{P}_{i|\perp} = \bar{P}_i^c \hat{v}_i$. Thus, the spatial feature points are finally mapped onto the image plane based on camera internal parameters, which is associated with the perturbation error indeterminacy caused by anisotropy of gray distribution. Now, the objective function with the perturbation affine term in camera reference coordinate system is defined as:

$$\Xi(R,t) = \sum_{i=1}^{N} \left\| \hat{P}_i^c - W\hat{P}_{i|\perp} \right\| = \sum_{i=1}^{N} \left\| (I - W\bar{v}_i^T\bar{v}_i) \hat{P}_i^c \right\|$$  \hspace{1cm} (7)

where $I$ denotes an identity matrix. Combined with $\hat{P}_{i|\perp} = R\hat{P}_i^w + t$, the novel objective function of colinear error in the object space can be set under the world coordinate system. As suggested from (7),

$$\Xi(R,t) = \sum_{i=1}^{N} \left\| (I - W\bar{v}_i^T\bar{v}_i) \left(R\hat{P}_i^w + t\right) \right\|^2$$  \hspace{1cm} (8)

The objective function complies with the collinearity error of all feature points in the unified coordinate system, and it invokes quantized inconsistent perturbation error; thus, a more statistically significant and meaningful novel objective function is set. Subsequently, the rotation and translation parameters of the target relative to the camera coordinate system can be determined by solving Eq. (8).

### III. Expedited Pose Estimation Algorithm Involving Perturbance Affine Term

Though the IPAT algorithm derived in the previous section exhibits certain advantages in the motion parameter measurement, the calculation burden increases since the perturbation error indeterminacy is invoked. Accordingly, in this section, the IPAT algorithm is expedited by simplifying the computation, so the operating efficiency of the pose parameters solution tends to be risen. On that basis, the EIPAT algorithm is proposed, and its performance is assessed.

#### A. Expediting Strategy of Algorithm

To achieve the ultimate goal, it is noticed that the set objective function considering the perturbation error indeterminacy should be updated in each iteration, and it is a function of $R$ and $t$. However, if there is a linear solution to $t$ in the case where $R$ is known, with the objective function as only a function of $R$, an expedited criterion for IPAT algorithm is derived in the following theorem.

**Theorem 1:** Since IPAT algorithm is derived by the set objective function (8), it is assumed that there exists a set $\{\hat{P}_i^w\}, i = 1, 2, \ldots, n$, $\exists k \in [1, n]$ that ensures $\hat{P}_i^w \neq \hat{P}_k^w$, and subsequently, the rotation matrix $R$ and translation vector $t$ can be obtained by EIPAT algorithm via vectorization and Kronecker product operation as follows:

$$\Xi(R,t) = vec(R)^T \Phi \, vec(R)$$  \hspace{1cm} (9)

$$t(R) = J \, vec(R)$$  \hspace{1cm} (10)

$$\varphi_k = \Upsilon \, vec \left( \hat{R}^{(k)} \right)$$  \hspace{1cm} (11)

where

$$\Phi \triangleq \left( \sum_{i=1}^{n} \hat{P}_i^w \otimes I + J^T \right) \left( I - W\bar{v}_i^T\bar{v}_i \right) \left( \hat{P}_i^w \otimes I + J^T \right)^T,$$

$$J \triangleq \frac{1}{N} \left( I - \frac{1}{N} \sum_{i=1}^{N} W\bar{v}_i^T\bar{v}_i \right)^{-1} \sum_{i=1}^{N} \left( \hat{P}_i^w \otimes W\bar{v}_i^T\bar{v}_i \right).$$

$$\Upsilon = \sum_{i=1}^{n} \left( \hat{P}_i^w \otimes \left( \hat{P}_i^w \right)^T \otimes W\bar{v}_i^T\bar{v}_i \right) + \sum_{i=1}^{n} \left( \hat{P}_i^w \otimes W\bar{v}_i^T\bar{v}_i \right) \left( J \right).$$

Vectorized matrix $\varphi_k$ is adopted to update the rotation matrix $R$. The matrix $\Upsilon$ can be solved before the iteration starts, and it is substantially considered a constant matrix.
Proof: According to vectorization vec(·) and Kronecker product, from the novel mode objective function (8), it yields:

\[
\Xi (R,t) = \sum_{i=1}^{N} \left\| (I - W_i \tilde{v}_i^T \tilde{v}_i) \left( R \tilde{p}_w^i + t \right) \right\|^2 \\
= \sum_{i=1}^{N} \left\| (I - W_i \tilde{v}_i^T \tilde{v}_i) \left( \left( \tilde{p}_w^i \right)^T \otimes I + J \right) \right\| vec(R) \right\|^2
\]

(12)

where, \( J \triangleq \frac{1}{N} \left( I - \frac{1}{N} \sum_{i=1}^{N} W_i \tilde{v}_i^T \tilde{v}_i \right) \cdot \sum_{i=1}^{N} \left( \left( \tilde{p}_w^i \right)^T \otimes W_i \tilde{v}_i^T \tilde{v}_i \right) \).

Eq. (12) can be further transformed into:

\[
\Xi (R,t) = vec(R)^T \left( \sum_{i=1}^{n} \left( \left( \tilde{p}_w^i \otimes I + J^T \right) \left( I - W_i \tilde{v}_i^T \tilde{v}_i \right) \left( \left( \tilde{p}_w^i \right)^T \otimes I + J^T \right) \right) \right) vec(R)
\]

(13)

where,

\[
\Phi \triangleq \left( \sum_{i=1}^{n} \left( \left( \tilde{p}_w^i \otimes I + J^T \right) \left( I - W_i \tilde{v}_i^T \tilde{v}_i \right) \left( \left( \tilde{p}_w^i \right)^T \otimes I + J^T \right) \right) \right).
\]

\( \hat{p}_w^i \) is the reference point coordinate by means of zero averaging and it can be calculated before iterating. It is noteworthy that \( \Phi \) remains constant. As a result, the computational complexity of the novel objective function decreases to \( O(1) \) according to Eq. (13).

Except for the output of the last iteration, the solution of \( t \) is only a temporary value during other iterations. Accordingly, the solution of \( t \) can be avoided by integration of the iteration process. According to Eq. (8), it holds that

\[
t(R) = \frac{1}{N} \left( I - \frac{1}{N} \sum_{i=1}^{N} W_i \tilde{v}_i^T \tilde{v}_i \right) \cdot \sum_{i=1}^{N} \left( I - W_i \tilde{v}_i^T \tilde{v}_i \right) R \tilde{p}_w^i
\]

(14)

With the Kronecker product expansion properties, Eq. (14) is rewritten as:

\[
t(R) = \frac{1}{N} \left( I - \frac{1}{N} \sum_{i=1}^{N} W_i \tilde{v}_i^T \tilde{v}_i \right)^{-1} \cdot \sum_{i=1}^{N} \left( \left( \tilde{p}_w^i \right)^T \otimes \left( W_i \tilde{v}_i^T \tilde{v}_i - I \right) \right) vec(R)
\]

(15)

Since all the reference points are centered and translated before starting the iteration, the sum of \( \left\{ \tilde{p}_w^i \right\} \) is 0. Thus, the identity matrix \( I \) can be eliminated, and the accelerated expression of \( t \) is yielded as:

\[
t(R) = J vec(R)
\]

(16)

Then, the projection points along the line of sight are transformed, and after introducing the perturbation affine term, formula (17) is satisfied

\[
\left( \hat{p}_w^i \right)^{(k)} = W_i \tilde{v}_i^T \tilde{v}_i \left( R^{(k)} \hat{p}_w^i + t^{(k)} \right)
\]

(17)

To adopt the absolute orientation to perform the optimal solution iteration, the matrix \( \Psi_k \) should be calculated in each iteration:

\[
\Psi^{(k)} = \sum_{i=1}^{n} \left( \left( \hat{p}_w^i \right)^{(k)} \right)^T \otimes \left( \hat{p}_w^i \right)^{(k)} - \sum_{i=1}^{n} \left( \left( \hat{p}_w^i \right)^{(k)} \right)^T \otimes \left( \hat{p}_w^i \right)^{(k)}
\]

(18)

where \( \left( \hat{p}_w^i \right)^{(k)} \) denotes the average of all projection points at the \( k - th \) iteration. By setting \( \phi_k = vec(\psi^{(k)}) \), combined with (18), it yields:

\[
\phi^{(k)} = \sum_{i=1}^{n} \left( \left( \hat{p}_w^i \right)^{(k)} \otimes \left( \tilde{v}_i \right)^T \right) vec(R^{(k)})
\]

(19)

Thus, \( \psi^{(k)} \) is determined and \( \phi^{(k)} = UDV^T \) is acquired by singular value decomposition, which allows the iterative algorithm to update \( vec(\psi^{(k+1)}) \). In (19) the matrix \( \Upsilon \) is expressed as:

\[
\Upsilon = \sum_{i=1}^{n} \left( \left( \hat{p}_w^i \right)^{(k)} \otimes \left( \tilde{v}_i \right)^T \right) vec(R^{(k)})
\]

(20)

Since \( \hat{p}_w^i \otimes \left( \tilde{v}_i \right)^T \) can be rewritten as \( \hat{p}_w^i \otimes \left( \tilde{v}_i \right)^T \), \( \Upsilon \) is rewritten as:

\[
\Upsilon = \sum_{i=1}^{n} \left( \hat{p}_w^i \otimes \left( \tilde{v}_i \right)^T \right) vec(R^{(k)})
\]

(21)

Eq. (21) suggests that the matrix \( \Upsilon \) is a constant matrix and can be solved before iterating to expedite the IPAT algorithm.

Then, the output of the EIPAT algorithm can be expressed. In the iteration process, \( \Upsilon \) matrix should be stored only. Besides, the iterative calculation is performed on \( R \), and it is outputted at the end of the iteration. Since all the reference points are zero-averaged at the beginning, vector \( t \) should be translated at the end. With Eq. (10), the final iteration result of \( t \) can be obtained by \( t_{out} \leftarrow t_{out} - R_{out} \hat{p}_w^i \). It can be
found from Theorem 1, the repeated calculation is avoided by coupling each iteration, which significantly simplifies the computation.

To intuitively express the implementation process of the proposed EIPAT algorithm, the flow chart is given in the follow Figure 2.

B. ALGORITHM EVALUATION
The objective function built with the perturbation affine term in the covariance data space differs from that built in the original data space. Since in the original data space, the perturbation error of feature points exhibits totally different directional indeterminacy, and the anisotropy perturbation exhibits non-independent and inconsistent distribution. Thus, the novel objective function built in the present study has statistical significance compared with the conventional objective function.

Moreover, \( R \) and \( t \) need to be calculated once in each iteration of the conventional process. In fact, it is essential that \( R \) requires to be updated. Because every time \( R \) is updated, the

\[
\mathbf{E}(R^{(k+1)}, t^{(k+1)}) = \sum_{i=1}^{n} \left\| \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k+1)} \right\|^2
\]

\[
= \sum_{i=1}^{n} \left\| \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k+1)} \right\|^2 + \sum_{i=1}^{n} \left( \tilde{\mathbf{v}}_i^{(k+1)} - \tilde{\mathbf{v}}_i^{(k)} \right)^T \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i W_i^T
\]

\[
\times \left( 2 \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k)} \right)
\]

\[
= \sum_{i=1}^{n} \left\| \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k+1)} \right\|^2 + \sum_{i=1}^{n} \left( \tilde{\mathbf{v}}_i^{(k+1)} - \tilde{\mathbf{v}}_i^{(k)} \right)^T \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i W_i^T \left( 2 \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i \left( \tilde{\mathbf{v}}_i^{(k+1)} + \tilde{\mathbf{v}}_i^{(k)} \right) \right)
\]

\[
(22)
\]

\[
\sum_{i=1}^{n} \left( W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k)} \right)^2 = - \sum_{i=1}^{n} \left\| W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k+1)} - W_i \tilde{\mathbf{v}}_i^{T} \tilde{\mathbf{v}}_i^{(k)} \right\|^2
\]

\[
(23)
\]
optimal \( t \) can be calculated linearly. As a result, the solution of \( t \) is only the procedure value during the whole iterations, during which the updating of \( t \) is eliminated. Otherwise, the original algorithm should recalculate the projection points during each iteration, but the calculation of these projection points just updates \( R \) by using the absolute orientation. Accordingly, its computation is omitted during the period of iteration.

Based on the mentioned analysis, though the computational complexity of \( J \), \( \Phi \) and \( \Upsilon \) is \( O(n) \), as long as the iteration starts, \( J \) is calculated first, and then the \( \Phi \) and \( \Upsilon \) matrices are calculated. Next, only the \( \Phi \) and \( \Upsilon \) matrices are adopted to update the rotation matrix \( R \) in the iteration process and the objective function is accelerated by (13). Now the computational complexity of each iteration is descended to \( O(1) \). This proposed strategy can significantly reduce the amount of calculation in the iterative process. After the iteration is accomplished, the translation vector \( t \) can be obtained by (10), which effectively achieves the acceleration for the high-precision IPAT algorithm.

Next the convergence of the EIPAT algorithm is analyzed. First, Eq. (22), as shown at the bottom of the previous page, is satisfied from (8):

Moreover, it obviously holds that (23), shown at the bottom of the previous page.

According to the solution principle of \( R \), it holds that

\[
\sum_{i=1}^{n} \left\| \hat{\mathbf{p}}_i^{(k+1)} - W_i \mathbf{v}_i^T \hat{\mathbf{v}}_i \hat{\mathbf{p}}_i^{(k)} \right\|^2 \leq \sum_{i=1}^{n} \left\| \hat{\mathbf{p}}_i^{(k)} - W_i \mathbf{v}_i^T \mathbf{v}_i \hat{\mathbf{p}}_i^{(k)} \right\|^2 = \Xi \left( R^{(k)} \right) \tag{24}
\]

At this point, it yields:

\[
\Xi \left( R^{(k+1)}, t^{(k+1)} \right) \leq \Xi \left( R^{(k)}, t^{(k)} \right) - \sum_{i=1}^{n} \left\| W_i \mathbf{v}_i^T \hat{\mathbf{v}}_i \hat{\mathbf{p}}_i^{(k+1)} - W_i \mathbf{v}_i^T \hat{\mathbf{v}}_i \hat{\mathbf{p}}_i^{(k)} \right\|^2 \tag{25}
\]

If all the reference points are not consistent with each other, i.e., \( \sum_{i=1}^{n} \left\| W_i \mathbf{v}_i^T \hat{\mathbf{v}}_i \hat{\mathbf{p}}_i^{(k+1)} - W_i \mathbf{v}_i^T \hat{\mathbf{v}}_i \hat{\mathbf{p}}_i^{(k)} \right\|^2 \neq 0 \), the following formula is yielded:

\[
\Xi \left( R^{(k+1)}, t^{(k+1)} \right) < \Xi \left( R^{(k)}, t^{(k)} \right) \tag{26}
\]

It is therefore suggested that the novel objective function is strictly decreasing. More precisely, it can be demonstrated that the EIPAT algorithm for position and attitude calculation by involving the affine term of non-uniform perturbation error is global convergent.

**IV. EXPERIMENTAL RESULTS AND ANALYSIS**

In this section, both numerical and practical experiments are illustrated to assess the comprehensive performance of the proposed EIPAT algorithm, in which the statistical tests and practical trials are implemented.

**A. NUMERICAL EXPERIMENTS**

First, the numerical experiments are performed on a computer with 2.11GHz CPU and 4.0G RAM. The synthetic data is presented as follows: image size is set to 2048pixel \( \times \) 2048pixel; the spatial three-dimensional coordinates of the reference points are randomly generated in the range of \( [-2, 2] \times [-2, 2] \times [1, 3] \) (unit: m).

The randomly generated reference translation vector is assumed as \( t_{true} \), and the reference rotation matrix is expressed as \( R_{true} \). The absolute error of \( R_{true} \) and its estimated value \( \hat{R} \) is:

\[
e_{\text{rel}} = \max_{k=1}^{3} \left[ \frac{\text{arc} \cos \left( \dot{r}_{true} \cdot \hat{r}^k \right)}{\pi} \right] \times 180/\pi \tag{27}
\]

where \( \dot{r}_{true} \) and \( \hat{r}^k \) respectively denote the \( k \)-th column of \( R_{true} \) and \( R \). \( \dot{r} \) represents the inner product of these two vectors.

The relative error between \( t_{true} \) and its estimated value \( \hat{t} \) is defined as \( e_{\text{rel trans}} (\%) = \left| t_{true} - \hat{t} \right|/\left| t_{true} \right| \times 100\% \). It is effectively known from Section 2 and Section 3 that the only difference between EIPAT and IPAT algorithms is that the IPAT has not been accelerated. In the numerical experiments, a comparison is drawn of the direct linear transformation (DLT) [38] method, the regular objective function (ROF) [39] based method and the scalar-weighted objective function (SWOF) [11] from the EIPAT method. To conduct an unbiased assessment of the performance test results achieved by the mentioned algorithms, the initial values of all pose estimation methods are calculated by weak perspective model [40], and the position and orientation parameters are determined by the Levenberg-Marquardt algorithm. Moreover, different performance tests are performed under the identical conditions (e.g., the number of feature points, noise level, and perturbation error indeterminacy).

Furthermore, to make the experimental results more statistically significant, all the trials in this section are implemented multiple times for each parameter condition, and then the average value is taken as the experimental result. The numerical statistical test considers four situations, i.e., 1) effects of inconsistent perturbation error indeterminacy, 2) efficiency assessment, 3) performance of noise resistance; 4) accuracy and convergence analysis.

1) RELATIONSHIP BETWEEN POSE ESTIMATION ACCURACY AND INCONSISTENT PERTURBATION ERROR INDETERMINACY

To evaluate the robustness of the mentioned five different methods under different levels of perturbation error indeterminacy, the ellipticity of the uniformly varying error indeterminacy gradually rises from 1 to 30, where \( \sqrt{\sigma_{\text{min}}} = 0.01 \) remains constant, and \( \sqrt{\sigma_{\text{max}}} \) increases from 0.01 to 0.30. Moreover, the direction of the ellipse varies stochastically. Accordingly, the simulation codes run 100 times in a loop and takes the average value as the test results; as a result, the methods can be statistically tested, and their ability to attenuate perturbation error indeterminacy can be fully demonstrated.
Figure 3 noticeably indicates that the accuracy of the ROF method and the SWOF method is equivalent whilst the DLT method exhibits the largest error. The precision of EIPAT and IPAT algorithm is distinctly higher than that of the other three methods. As the ellipticity increases, the error of the ROF, SWOF, and DLT methods grows rapidly. In contrast, the error of EIPAT and IPAT algorithms rises marginally, even in the case of large indeterminacy \( \sqrt{\sigma_{\text{max}}/\sigma_{\text{min}}} = 30 \) with the slight error. The primary reason for this is that the three reference methods do not consider the indeterminacy of perturbation error. Moreover, the SWOF method only considers the uncertainty of the gray point isotropy. Though the direction of the indeterminate ellipse changes randomly in the range \([0^\circ, 360^\circ]\), the EIPAT algorithm allows for the non-uniform perturbation error caused by gray direction of feature points.

When under the condition of case \( a/b = 1 \), the perturbation error indeterminacy is not related in the direction of \( u \) and \( v \) axis; the covariance matrix can be yielded by \( Q^{-1} = U \text{diag} \left( \frac{1}{\sigma_2}, \frac{1}{\sigma_2}, 1 \right) V^T \). Thus, the indeterminacy in both directions is equal and relatively small, which is a rare and conventional form for pose estimation. However, while in the case \( a/b > 1 \) or \( a/b < 1 \), the covariance matrix of perturbation error indeterminacy requires to be obtained by Eq. (2).

Then the perturbation error indeterminacy can be added in the data space of covariance matrix after transforming by affine term \( W \), upon which from (7) the pose parameters can be solved rapidly with the aid of Theorem 1. Figure 3 intuitively displays the superiority of EIPAT algorithm that adapts to different degrees of elliptical indeterminacy.

2) EFFICIENCY ASSESSMENT

Now the runtime of EIPAT algorithm is analyzed by comparing with other listed methods. Verification is performed by the time taken for the calculation of each algorithm under different number of feature points. The number of control points is increased from 5 to 100, and 20 independent trials are performed to record the time consumed, as shown in Figure 4.

Figure 4 suggests that with the increase in the number of control points, the computation time of all five algorithms rises steadily. Undoubtedly, the IPAT method takes the most time, and the operation efficiency is the minimum; thus, results are achieved for the error indeterminacy caused by the imaging gray distribution that is considered in the iterative process to improve the accuracy of parameter estimated results. Otherwise, the time consumed of IPAT algorithm is slightly longer than the other methods, since the running time is still on the same order of magnitude. Nonetheless the running time is slightly longer, the advantages shown by IPAT algorithm in other aspects cannot be ignored. In other words, it can be acceptable for high precision and robustness. The operation time of the DLT method is much faster than the ROF method as well as the SWOF method, but the accuracy of the linear solution for DLT reaches the bottom even with plenty of feature points.

It is noteworthy that the EIPAT method has distinct advantages over the IPAT, ROF and SWOF methods. With the number of control points increasing to 100, the time cost of the EIPAT algorithm is 14.90ms whilst the IPAT algorithm takes by 50.17ms that is 3.3 times as EIPAT. The reason is that in the EIPAT algorithm, just \( R_1, R_2, \ldots, R_l \) require to be updated, where \( l \) denotes the number of iterations.
When matrix $J$ is calculated first, matrices $\Phi$ and $\Upsilon$ can be computed. Next, only the $\Phi$ and $\Upsilon$ matrices are adopted to update the rotation parameters $R_1, R_2, \ldots, R_l$, demonstrating that the computational complexity of each iteration is descended from $O(n)$ to $O(1)$. The optimal $l$ can be calculated linearly at the end of the acceleration process. In contrast, by the other four algorithms, $R_1, R_2, \ldots, R_l$ and $t_1, t_2, \ldots, t_l$ should be overall updated. Nevertheless, no acceleration strategy is formulated when solving the external parameter matrices. Figure 4 obviously suggests that the EIPAT algorithm is significantly expedited by simplifying the computation.

3) PERFORMANCE OF NOISE RESISTANCE

To delve into the anti-noise performance of the proposed algorithm, the factors (e.g., camera intrinsic parameter error and image extraction error of feature points) are considered. On that basis, the Gaussian noise with mean 0 and variance from 0 to 6 is introduced at each projection point.

For the purpose of preventing accidental test results, all the tests are performed 100 times to obtain the error curves of the rotation matrix and the translation vector under different noise levels. The relevant results of attitudes and positions are presented in Figure 5.

Overall, Figure 5 shows that with the elevation of the image noise level, the pose estimation error of the five methods tends to grow, and the stability inevitably declines. The protocol for generating the input data used throughout this section is governed by the added growing Gaussian noise. As the noise increases further, the noise resistance performance of SWOF and ROF methods becomes increasingly worse, and the DLT is the worst, especially for the attitude angle measurement. These three methods for comparison only take the collinearity geometric constraints of the image plane or the object space into account, and even the joint constraint of the object points and the image points cannot resist the influence of the different gray distribution. By contrast, the EIPAT and IPAT algorithms exhibit the identical anti-noise performance, since both of them invoke the non-uniform perturbation error. With the aid of $I - W_t \tilde{v}_t \tilde{v}_t^T$ in the novel objective function (8), the noise at the feature point can be suppressed within a certain range during projection mapping; accordingly, the mentioned two algorithms can effectively suppress image noise.

Overall, as the mean square error level of the feature points continues to be elevated, the EIPAT algorithm exhibits better performance in anti-noise and remains stable. Especially when the noise level is larger, the advantage is more prominent.

4) ACCURACY AND CONVERGENCE ANALYSIS

In this part, the accuracy and rate of algorithm convergence is investigated. The pitch, roll and yaw angles of the target are selected as $[35^\circ, 10^\circ, 20^\circ]$, and the translation vector takes the different values in $[50, 70, -100]$ unit: mm). About 30 independent trials are conducted, and the convergence results achieved are presented in Figure 6.

In Figure 6, as the iterative convergence tends to be stable, the EIPAT algorithm can reach at the desired value in 10 iterations, the convergence rate of which is higher than that of the other methods. In contrast, the convergence accuracy of SWOF algorithm is slightly higher than that of ROF. Since the EIPAT algorithm complies with the projection vector and considers the minimized objective function set by the non-uniform perturbation error, the EIPAT algorithm exhibits the maximum accuracy. Moreover, this algorithm is proven to effectively avoid the poor or non-convergence from the achieved results. Furthermore, the DLT refers to a linear solution method without the demands of iteration, and yet the solution error is the maximum. Combined with the mentioned numerical experiment results and analysis, it is manifested that the EIPAT algorithm exhibits the superiority for running speed, estimation accuracy and numerical stability.

B. PRACTICAL EXPERIMENTS

To further verify the effectiveness of the proposed algorithm, the visual measurement system is built, as shown in Figure 7. The resolution of the 4M140MCX digital camera employed is taken as $2048 \times 2048$ pixels; the pixel size $du \times dv = 5 \times 5$ um$^2$; focal length $f = 35$ mm. The position of the camera can be flexibly tuned in accordance with different targets and...
tasks. The aircraft under test is simulated by a high-precision turntable to verify the effectiveness of the measurement algorithm. To be more convincing, the high-precision turntable adopted is extremely similar to the actual ground vehicle in structure and movement trajectory.

1) VERIFICATION OF FEATURE POINT REPROJECTION ACCURACY CONSIDERING GRAY DISTRIBUTION

First, the corner points on the sailboard of the satellite simulator in Figure 7 are identified and then extracted, and the non-uniformity of the gray distribution of the feature points is considered in the extraction.

The types of perturbation error indeterminacy of the target feature points are shown in Figure 8. The angle of the corner point imaging varies continuously in the target movement, so the difference is generated in the gray scale distribution of the same feature point under different frames. Besides, the inconsistent distribution is modeled by the approach in Section 2.

According to the empirical evidence based on large amounts of testing data, when the satellite simulator rotates relative to the camera coordinate system per 10°, the distribution of the grayscale pattern of feature points on the solar sailboard imaged on the area array CCD will change greatly. Correspondingly, the angle between \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) in Figure 8 tends to be different. In other words, the indeterminacy of perturbation error exhibits the changing directionality. Therefore, all subsequent pose estimation tests need to consider the angle of the major axes along the \( u \) axis and that of the minor axes along the \( v \) axis refer to the directions of perturbation error indeterminacy.

Also, the target should be placed at three different posture states. To draw the comparison of re-projection of the EIPAT and SWOF algorithms, the perturbation error indeterminacy is introduced to the corner points under the different positions.
The spatial distribution of the point cloud re-projection is illustrated in Figure 9. Figure 9(b) indicates that under the added perturbation error indeterminacy $\sqrt{\sigma_{\text{max}}/\sigma_{\text{min}}} = 20$, the distribution of spatial re-projection points with the SWOF algorithm is significantly disturbed. Hence, the re-projection error trends to increase. Otherwise, the EIPAT algorithm exhibits a more significant performance against the additive perturbation error indeterminacy (Figure 9(d)). The covariance matrix in (2) brings the inconsistent size of an ellipse region with the center at each feature point in the EIPAT algorithm which can be maintained steadily and consistent with the actual situation. Furthermore, in the case of Figure 9 (a) and (c), 12 discrete feature points are semi-randomly taken for testing, but these points diffusely distribute in various regions (e.g., body edge, center and regular area of the space target). The relevant re-projection results are listed in Table 1.
Table 1 obviously suggests that slight error exists in the image coordinates transformed by the affine term when the coordinates are re-projected. Thus, it is effectively verified that the perturbation error indeterminacy can be adequately diminished by the EIPAT algorithm by considering the gray distribution of the feature points. The accuracy of the proposed algorithm is accurately given by the re-projection data.

2) TEST OF VIRTUAL PYRAMID

In this section, re-projection transformation is conducted via the virtual pyramid formed by the selected feature points on the aircraft. Subsequently, compared with the actual pose of the measured target, the effect of pose estimation can be identified intuitively. If the pose estimated results are accurate, the re-projection of the virtual pyramid can show the actual pose of the target in the real scene. Provided that the pose estimated results are not the global optimal solutions, there will exist a deviation and distortion between the re-projection of the virtual cone and the true motion tendency. The test results are presented in Figure 10.

Likewise, the selected feature points are extracted in Figure 10(a). With the model built in the Part A, Section II, gradient $\nabla g_u$ and $\nabla g_v$ along $u$ and $v$ direction can be
determined. The calculated gray value of \( \Omega \) is 5616, 5447, 5320, 5506, 5684 in sequence. Note that the perturbation error indeterminacy is transferred to the minimum objective function to determine the estimated parameters, demonstrating that the results in Figure 10(c) are determined by the EIPAT algorithm. When the affine matrix \( W_i \) is calculated by Eq. (3), the minimum coordinates \( \| \hat{P}_c^i - W_i \hat{P}_c^i \| \) can be obtained. Combined with the novel objective function in (8), the term of \( I - W_i \vec{v}^T_i \vec{v}_i \) can restrict the changes of coordinates in a controllable range, which complies with the accuracy and stability verification results in the numerical simulation experiments. It is suggested that the virtual pyramid is capable of effectively reflecting the target attitude, and the positions of the virtual control points are consistent with the rotation direction of the measured target. In Figure 10(b), the perturbation error indeterminacy is not considered by the SWOF algorithm, and it is indicated that there exists the overestimation. As revealed from the comparison of the intermediate images of group (b) and (c) in Figure 10, the position direction of virtual pyramid vertices obtained by the EIPAT more significantly complies with the actual scene pose estimation than that acquired by the SWOF algorithm, which effectively suggests that the EIPAT algorithm is more suitable for the actual measurement circumstance.

3) ACTUAL ESTIMATION RESULTS OF TARGET DYNAMIC PARAMETERS

Subsequently, the target motion parameters are measured by the EIPAT algorithm, and the motion trajectory of the turntable space vehicle is a sine curve with the amplitudes rising from 0° to 35°. The motion attitude angle is decomposed in the yaw, pitch and roll directions; the dynamic measured results are sampled per 5°. To be more intuitive, the reconstructed target pose spatial pose is illustrated in Figure 11.

Figure 11 obviously indicates that the results achieved by EIPAT method are relatively stable, since the perturbation error indeterminacy is introduced into the objective function, by which the imaging uncertainty of the feature point can be well resisted in the process of target motion. In this experiment, the satellite simulator moves faster. Consequently, the imaging effect of the feature points is worse than that in a static state. Therefore, the testing results of three-component in Figure 11 (a), (b) and (c) are more challenging. It is satisfactory that the reconstructed flight trajectory and attitude of the satellite simulator are consistent with actual orbit. Apparently, no overestimation occurs, and the subsequent pose calculation is stabilized.

It is worth noting that the target placement positions should cover the measurement space as much as possible to ensure the accuracy and stability of the calibrated camera intrinsic parameters, which is the premise of the vision-based pose estimation. Additionally, a multi-constraint equation set is established to achieve the acquisition of the internal parameters [41]. Given the calibrated parameters of stereo cameras, Table 2 lists the rotation matrix \( R \) and translation vector \( t \) between the world coordinate system and the stereo camera coordinate system determined by the EIPAT algorithm.

Lastly, the dynamic error of measured attitude angles and positions of the target by EIPAT method are presented in Figure 12 and 13. Figure 12 suggests that each error of yaw, pitch, and roll angles is in the range of ±0.1°, and the angle error obviously exhibits the sinusoidal distribution. The roll angle accuracy is marginally worse than that of the pitch and
TABLE 2. Experimental parameters.

| Test space          | 4m×4m×2m             |
|---------------------|----------------------|
| Sampling frequency  | 100Hz                |
| Exposure time       | 30ms                 |

| Intrinsic parameters of the left camera |
|-----------------------------------------|
| $A_l = \begin{bmatrix} 3066.310 & 0 & 1026.008 \\ 0 & 3066.652 & 1026.354 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\mathbf{k}_l = [0.1465, 0.1398, -0.0002, 0.0002]^T$ |

| Intrinsic parameters of the right camera |
|------------------------------------------|
| $A_r = \begin{bmatrix} 3084.524 & 0 & 1022.600 \\ 0 & 3083.917 & 1023.027 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\mathbf{k}_r = [-0.1503, 0.1286, 0.0002, -0.0003]^T$ |

| System calibration error | 0.052nm |

FIGURE 12. Error of estimated attitude angles of the target.

FIGURE 13. Error of estimated positions of the target.

Moreover, the EIPAT algorithm obviously exhibits its limitations. The proposed method only applies to the measurement for cooperative targets instead of non-cooperative targets. For another limitation, the novel objective function adopted in EIPAT algorithm cannot be directly applied in the condition where the feature points cling to the curved surface. When adopting this method, the curvature of the surface should be considered since the feature points on curved surface should be mapped to the two-dimensional image plane. The desired curvature value should be set in the range of [0, 0.1].

V. CONCLUSION AND FUTURE WORK

The present study primarily aims to solve the problem of accuracy and speed of target pose estimation. Based on the analysis of the gray distribution of imaging feature points with a proper treatment of directional indeterminacy in the data, this study sets a novel minimizing objective function
with affine term containing perturbation error indeterminacy to avoid falling into local convergence and obtain a global optimal solution. Afterwards, the pose estimation algorithm based on the newly set objective function is accelerated. Thus, the strategy formulated in this study combines this meaningful objective function and powerful expediting strategy that reduces the calculation complexity of each iteration from $O(n)$ to $O(1)$. The experimental results are presented to verify the performance of the proposed method; it is therefore demonstrated that the EIPAT algorithm is more suitable for the actual measurement circumstance.

It is expected that one subsequent study is considered to explore the characteristics of imaging gray-scale distribution under the saturated and low-light level condition to derive a more accurate positional indeterminacy estimation formula that supports arbitrary pixel covariance. Another aspect refers to the widespread use to expedite the pose acquirement of the multi-camera system, as an attempt to enhance the real-time performance of full measured field.

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