BMS Symmetry via AdS/CFT

David A. Lowe

Department of Physics, Brown University, Providence, RI, 02912, USA

David M. Ramirez

Brown Theoretical Physics Center, Providence, RI, 02912, USA

Abstract

With a view to understanding extended-BMS symmetries in the framework of the $AdS_4/CFT_3$ correspondence, asymptotically AdS geometries are constructed with null impulsive shockwaves involving a discontinuity in superrotation parameters. The holographic dual is proposed to be a two-dimensional Euclidean defect conformal field localized on a particular timeslice in a three-dimensional conformal field theory on de Sitter spacetime. The defect conformal field theory generates a natural action of the Virasoro algebra. The large radius of curvature limit $\ell \to \infty$ yields spacetimes with nontrivial extended-BMS charges.
I. INTRODUCTION

The extended-BMS symmetry algebra \([1–7]\) appears to provide an infinite set of conserved quantities in asymptotically flat spacetime geometries and it is of great interest to understand to what extent these symmetries constrain the local dynamics and how these classical quantities generalize to the quantum level. For example, it has been suggested in \([8–10]\) that such quantities give rise to an infinite amount of quantum hair on black holes, which would be of profound importance for the information problem.

The goal of the present work is to study these quantities as a limit of asymptotically anti-de Sitter spacetimes where one may gain additional insight via the anti-de Sitter/conformal field theory correspondence. Some initial works in this direction are \([11, 12]\). It has been shown that the BMS algebra generalizes to a Lie algebroid in asymptotically anti-de Sitter spacetime \([13, 14]\). One concludes from this that in general the asymptotically anti-de Sitter geometries of interest involve nontrivial deformations of the boundary metric and lead to scenarios which are not well-understood from the holographic perspective.

In the present work we take a somewhat different approach to building a holographic description of asymptotically anti-de Sitter geometries with the analog of extended-BMS charges. We begin by considering null spherical impulsive shock waves where the metric across the shock undergoes a superrotation. The shock reflects off the boundary of anti-de Sitter spacetime, preserving the standard Dirichlet boundary conditions on the metric. This induces a nontrivial boundary stress energy tensor at the point of intersection, which is matched with the expectation value of the stress energy in the holographic dual. This suggests the shock should be identified with a two-dimensional Euclidean defect conformal field theory within a three-dimensional Lorentzian CFT on the boundary. In this setup, the defect CFT allows a natural action of Virasoro charges, in addition to the usual action associated with the global conformal symmetry of the three-dimensional CFT $SO(3, 2)$. The central charge of the algebra is non-zero, and is read off from the solution.

One may then take a large radius of curvature limit $\ell \to \infty$ of this setup to recover a holographic description of gravity in asymptotically flat spacetime. The limit is taken so that the defect CFT lives at spacelike infinity, once conformally compactified. In this limit the $SO(3, 2)$ contracts to $ISO(3, 1)$ \([13, 14]\). The defect CFT induces a nontrivial BMS charge when the asymptotically flat limit is taken. The full symmetry group includes the
generators realized on the defect CFT combined with actions on the fields of the three-
dimensional CFT. The more general supertranslation generators of the BMS algebra are not
so far manifest in this construction.

The shock solution is based on an exact solution which introduces an analytic diffeomor-
phism on the sphere [15]. These solutions were also considered in the context of superro-
rotations in [16]. We promote this to an asymptotically anti-de Sitter solution. The solution
has a mild Dirac delta function curvature singularity in affine null coordinates across the
shock, but is otherwise an exact solution of the vacuum Einstein equations with negative
cosmological constant.

Much is known about the general structure of conformal field theories, and there will be a
discrete infinity of quasi-primary operators with positive conformal weights $\Delta$. In the limit $\ell$
to infinity, the spectrum of the asymptotically flat theory will inherit this discrete structure.
Moreover we obtain a holographic realization of asymptotically flat spacetimes via a limit
of a three-dimensional CFT with a two-dimensional defect. The HKLL construction [17–19]
should extend straightforwardly to this setup and bulk fields receive contributions both from
smeared primaries in the three-dimensional CFT as well as operators in the two-dimensional
defect CFT, which alone carry the extended-BMS charges, though we leave the details of
this to future work.

II. CALCULATIONS

The goal is to gain insight into the flat spacetime limit of holography by considering
the limit where the radius of curvature $\ell \to \infty$ from the Anti-de Sitter side, where the
holographic dual is thought to be a well-defined conformal field theory. Some discussion of
the viewpoint has appeared in [11, 12].

We will be interested in a set of spherical impulsive gravitational wave solutions as stud-
ied in [15, 20]. These solutions were also considered in the context of superrotations in
[16]. These solutions are most naturally written in terms of Kruskal-like coordinates. A
generalization of the imploding-exploding solution studied in [15] to asymptotically anti-de
Sitter spacetime is [21]

$$ds^2 = \frac{\ell^2}{(1 + uv)^2} \left( \frac{4}{1 + z \bar{z}} |(u - v)dz + 2 [u F(z)\theta(u) + v G(z)(1 - \theta(v))] (1 + z \bar{z})^2 d\bar{z}|^2 - 4dudv \right)$$
where $\theta(x)$ is the Heaviside step function and $F(z)$, $G(z)$ are Schwarzian derivatives of holomorphic functions $f(z), g(z)$ of the form

$$F(z) = \frac{f'''(z)}{f'(z)} - \frac{3}{2} \left( \frac{f''(z)}{f'(z)} \right)^2,$$  \hspace{1cm} (1)$$

and similarly for $G(z)$. The holomorphic functions $f(z), g(z)$ parametrize superrotations. This is most easily seen by noting that across the shock (say at $u = 0$) the metric may be obtained by performing the coordinate transformation of the ordinary flat spacetime metric by

$$z \rightarrow f(z), \hspace{1cm} v \rightarrow \frac{1 + |f(z)|^2}{1 + |z|^2} \frac{v}{|f'(z)|}.$$  \hspace{1cm} (2)$$

For our purposes it will be convenient to transform by a 1/4 period in the global time
coordinate which amounts to a change of coordinates from \([15]\) of
\[ u \rightarrow \frac{u+1}{u-1}, \quad v \rightarrow \frac{v+1}{v-1}. \]

We also include the reflections of the waves off the boundary of AdS to yield a solution of the form
\[
ds^2 = \frac{\ell^2}{(1+uv)^2} \left( \frac{4}{(1+z\bar{z})^2} |(u-v)dz + (-u+1)(1-v)\theta(v-1)G + (u+1)(1-v)\theta(u+1)\bar{F} (1+z\bar{z})^2 d\bar{z}|^2 - 4dudv \right). \quad (3)
\]

The coordinates \((u,v)\) range from \(-\infty\) to \(\infty\) with \(v > u\), and with the timelike boundary \(uv < -1\) and the spacelike coordinate patch boundary \(uv < 1\). The figure shows a sequence of such solutions glued together along the spacelike boundaries of the coordinate patches. These impulsive gravitational waves are solutions of the vacuum Einstein equations with Dirac delta function curvature singularities along the null lines \(u = -1\) and \(v = 1\). We allow for stress energy localized on the boundary, which will be computed below, to enforce the standard Dirichlet boundary conditions at \(uv = -1\). In this case we set \(F = G\).

To become more familiar with these solutions, let us for the moment consider the case \(F = 0\) and map to embedding coordinates where (for simplicity we define our coordinates to be dimensionless, then restore units by multiplying the metric by \(\ell^2\) as above)
\[
-T^2 - X^2 + R^2 = -1.
\]

Then we can identify
\[
T - R = u(1+X), \quad T + R = v(1+X).
\]

The metric associated with the 2-sphere is
\[
dS^2 = R^2d\Omega^2 = \frac{4R^2dzd\bar{z}}{(1+z\bar{z})^2} \quad (4)
\]
where
\[
z = e^{i\phi} \tan \frac{\theta}{2}, \quad \bar{z} = e^{-i\phi} \tan \frac{\theta}{2}.
\]
Combining these we obtain the metric \(3\) in the unshaded region.

The full 3-dimensional boundary at infinity corresponds to the surface \(uv = -1\). The induced metric is then conformal to three-dimensional de Sitter spacetime with metric
\[
ds^2 = \frac{4}{(1+z\bar{z})^2} \left| \left( u + \frac{1}{u} \right) dz \right|^2 - \frac{4}{u^2}du^2. \quad (5)
\]
Thus the shock at $u = -1$ meets the boundary at infinity on a spacelike 2-sphere with metric conformal to the standard round metric. It is interesting to note a similar $dS_3$ geometry appears at spacelike infinity in the construction of the symmetry generators in asymptotically flat spacetime [22, 25].

It will be helpful to re-express the Kruskal coordinates as Fefferman-Graham coordinates with a 3-dimensional de Sitter boundary metric. To do this let us define $t$ and $\rho$

$$t = \log\left(\frac{-u}{v}\right) = \log\left(\frac{-T + R}{T - R}\right), \quad \frac{4\rho}{(1 + \rho)^2} = 1 + uv = 1 + \frac{(T + R)(T - R)}{(1 + X)^2}$$

or equivalently

$$T = \frac{1 - \rho^2}{2\rho} \sinh \frac{t}{2}, \quad R = \frac{1 - \rho^2}{2\rho} \cosh \frac{t}{2}, \quad X = \frac{1 + \rho^2}{2\rho}$$

then the anti-de Sitter metric becomes

$$ds^2 = \ell^2 \frac{d\rho^2 + \frac{1}{16} (1 - \rho^2)^2 \left( (e^{t/2} + e^{-t/2})^2 d\Omega^2 - dt^2 \right)}{\rho^2}.$$

This matches the induced metric on the boundary (5) at $\rho \to 0$ with the change of variables $u = -e^{-t/2}$.

### III. HOLOGRAPHIC STRESS TENSOR

In this section we study the holographic mapping of the metric near the boundary of AdS to the stress energy tensor of the CFT. Following the procedure of [26] the $g^{(3)}_{ab}$ component of the metric in Fefferman-Graham coordinates

$$ds^2 = \ell^2 d\rho^2 + \left( g^{(0)}_{ab} + \rho^2 g^{(2)}_{ab} + \rho^3 g^{(3)}_{ab} + \cdots \right) dx^a dx^b$$

is identified with the expectation value of the CFT stress tensor. Here $x^a$ are the transverse coordinates, and $g^{(0)}_{ab}$ is the boundary metric. In this expansion $g^{(2)}_{ab}$ is determined in terms of $g^{(0)}_{ab}$, but $g^{(3)}_{ab}$ is independent data.

In the present situation the metric involves step functions, and leads to a curvature tensor that must be treated as a generalized function. This takes us beyond the original considerations of [26], however the main part of the derivation will carry over. To proceed, we continue to match $g^{(0)}_{ab}$ with the unperturbed metric [5]. One approach would be to
construct the Brown-York tensor [27] with the usual counter-terms at $\rho = \epsilon$. However this
surface cuts the shock, and potentially introduces corner-terms that will show up in the
boundary stress energy as $\epsilon \to 0$. See also [28] for an earlier review of junction conditions.
Instead it is more straightforward to carry over the derivation of [26] with the understanding
that the metric components become generalized functions in the time-direction.

In detail, we place a cutoff at $\rho = \epsilon$ in Fefferman-Graham coordinates and study the
metric in the interior of the gray-shaded wedge in [1]. The null shell $u = -1$ becomes the
surface $t = \log v$, $\rho = (1 - \sqrt{v})^2/(1 - v)$ (with $v \in (0, 1)$) while the shell at $v = 1$ becomes
the surface $t = -\log (-u)$, $\rho = (1 - \sqrt{-u})^2/(1 + u)$ with $u \in (-1, 0)$. Outside the wedge, the
holographic stress tensor [26], vanishes as $\epsilon \to 0$.

Inside the wedge, Fefferman-Graham coordinates correspond to

$$u = -e^{-t/2} \frac{1 - \rho}{1 + \rho}, \quad v = e^{t/2} \frac{1 - \rho}{1 + \rho}$$

where the metric is

$$ds^2 = \frac{\ell^2}{\rho^2} \left( d\rho^2 - \frac{1}{16} \left( \rho^2 - 1 \right)^2 dt^2 + \frac{1}{(1 + z\bar{z})^2} \left( 1 - \rho^2 \right) \cosh \left( \frac{t}{2} \right) d\bar{z} - (1 + z\bar{z})^2 \left( 1 + \rho^2 + (\rho^2 - 1) \cosh \left( \frac{t}{2} \right) \right) d\bar{z} \bar{F}(\bar{z}) \right)^2.$$  

(6)

The mapping between the CFT stress energy tensor and the Fefferman-Graham expansion
is [26]

$$\left\langle T_{ab}^{\text{CFT}} \right\rangle = \frac{3}{16\pi G_N} g_{ab}^{(3)}.$$  

(7)

To be able to apply (7) we need to take into account the step functions which localize the
solution (6) to the interior of the shaded wedge, while outside we have the solution with
$\bar{F} = 0$. To do this we Taylor expand the step functions near $\rho = 0$ that modulate the $\bar{F}$
terms, and the $F\bar{F}$ terms

$$\theta(t + 4 \tanh^{-1} \rho) - \theta(t - 4 \tanh^{-1} \rho) = \left( \theta(t + 4 \tanh^{-1} \rho) - \theta(t - 4 \tanh^{-1} \rho) \right)^2 = 8\rho \delta(t) + \frac{64}{3} \rho^3 \delta'(t) + \frac{8}{3} \rho^3 \delta(t) + O(\rho^4).$$

This $\rho$ dependence induces a $g_{ab}^{(3)}$ term in the metric, and (7) leads to

$$\left\langle T_{ab}^{\text{CFT}} \right\rangle = \frac{3}{16\pi G_N} g_{ab}^{(3)} = -\frac{2\ell^2}{\pi G_N} \begin{pmatrix} 0 & 0 & 0 \\ 0 & F(\bar{z}) & 0 \\ 0 & 0 & \bar{F}(\bar{z}) \end{pmatrix} \delta(t).$$  

(8)
The lower order components $g_{ab}^{(0)}$ and $g_{ab}^{(2)}$ simply match the vacuum solution with $F(z) = 0$.

This expectation value is exactly what one expects for the vacuum expectation value of a two-dimensional defect CFT inserted at $t = 0$ after undergoing a holomorphic coordinate transformation $z' = f(z)$

$$\langle T_{zz}^{2d,CFT} \rangle = -\frac{c}{12} \{f(z), z\}$$

where $\{f(z), z\} = F(z)$ is the Schwarzian derivative \cite{1}. Recall this is indeed the coordinate transformation on the boundary \cite{2}. We therefore tentatively identify the central charge of the 2d defect CFT with

$$c = \frac{24\ell^2}{\pi G_N}.$$ 

Clearly the flat space limit will then correspond to a large central charge limit if $G_N$ is held fixed.

The one-point function \cite{9} can be evaluated for a general analytic diffeomorphism that maps the $t = 0$ slice to itself. One may therefore use the one-point function to iteratively generate insertions of the boundary stress tensor $T_{zz}^{2d}$ on the $t = 0$ slice. Therefore we expect to get a general set of $n$-point functions of the 2d stress tensor that are constrained by the Virasoro algebra.

From the perspective of general 3d CFT this behavior is surprising. We expect linear couplings between the 2d operators and the 3d operators, and at best the 3d theory will enjoy a local $SO(3,2)$ symmetry, not an infinite dimensional Virasoro symmetry. We therefore do not expect the 2d theory to have an exact Virasoro symmetry constraining its correlation functions. On the other hand we can only trust the gravity results (of which \cite{9} is an example) for $\ell^2 \gg G_N$ so we are necessarily working in a large central charge limit. It is possible then for an enhanced symmetry to emerge, broken by perturbative corrections in a $1/c$ expansion. One may view the 3d CFT correlators (including mixed correlators involving insertions at $t = 0$) to be determined by usual holographic map from gravity correlators to boundary correlators. It will be interesting to work out the details of this map to see whether there is a sense Virasoro symmetry constrains the correlators of general operators at $t = 0$, but we leave this for future work.

In summary, we conjecture the expectation value \cite{8} is most simply explained by the insertion of a boundary two-dimensional conformal field theory at the surface $t = 0$ within the three-dimensional $dS_3$ boundary. Recall that the shock wave solutions we are studying
are constructed by gluing together two patches with the transformation $z \to f(z)$ along the shock. The expectation values of the stress tensor on the plane vanish in the three-dimensional vacuum state ($f(z) = z$), and we are left with the Schwarzian term above when $f(z)$ is chosen more generally. This is certainly a highly non-trivial proposal and many more consistency checks should be made to ascertain its validity.

IV. ASYMPTOTICALLY FLAT LIMIT

To construct the asymptotically flat limit, we take the metric in the form (3) and define rescaled coordinates $\tilde{u} = \ell u$, $\tilde{\bar{u}} = \ell \bar{v}$. Taking the limit $\ell \to \infty$ with $\tilde{u}$, $\tilde{\bar{u}}$ fixed, we get

$$ds^2 = -4d\tilde{u}d\tilde{\bar{u}} + \frac{4}{(1 + z\bar{z})^2} \left| (\tilde{u} - \tilde{\bar{u}})dz + \tilde{F}(\tilde{z})d\tilde{z}(1 + z\bar{z})^2 \right|^2$$

where $\tilde{F} = \ell \bar{F}$. Next we transform to Bondi coordinates on $I^+$ defining

$$\tilde{\bar{u}} = \tilde{u} + \sqrt{\xi^2 + (1 + z\bar{z})^4} \left| \tilde{F}(\tilde{z}) \right|^2$$

and replacing $\tilde{u} \to u/2$ and $\tilde{F} \to F$ to give

$$ds^2 = -du^2 - \left( 2 - \frac{1}{\xi^2}(1 + |z|^2)^4 |F(z)|^2 \right) dud\xi - \frac{1}{\xi} \partial_z \left( (1 + |z|^2)^4 |F(z)|^2 \right) dzdu -$$

$$\frac{1}{\xi} \partial_z \left( (1 + |z|^2)^4 |F(z)|^2 \right) d\bar{z}du - 4\xi F(z)dz^2 - 4\xi \tilde{F}(\tilde{z})d\tilde{z}^2 +$$

$$\frac{\xi^2}{(1 + |z|^2)^2} dzd\bar{z} + 8(1 + |z|^2)^2 |F(z)|^2 dzd\bar{z} + \cdots$$

where $\cdots$ refers to higher order terms in the $1/\xi$ expansion. Reading off the Bondi parameters following the notation of [5] we find

$$M = 0, \quad N_{AB} = 0, \quad C_{AB} = \begin{pmatrix} -4F(z) & 0 \\ 0 & -4\tilde{F}(\tilde{z}) \end{pmatrix}, \quad D_{AB} = 0, \quad N_A = -\frac{3}{32} \partial_A C_{CD}C^{CD}$$

here the indices $A, B$ run over $z, \bar{z}$. Putting these together, we compute the integrable component of the superrotation charge [5] as

$$Q_Y = \frac{1}{16\pi G_N} \int d^2z \frac{2}{(1 + |z|^2)^2} Y^A \left( 2N_A + \frac{1}{16} \partial_A C_{CD}C^{CD} \right)$$

$$= \frac{1}{16\pi G_N} \int d^2z \frac{2}{(1 + |z|^2)^2} Y^A \left( -\frac{1}{8} \partial_A C_{CD}C^{CD} \right)$$
where $Y^A(z, \bar{z})$ is a vector field on the 2-sphere that can be chosen to compute the desired moment of the integrand. The superrotation charge is therefore in general non-zero at quadratic order in the perturbation.$^{[29]}$

These solutions with vanishing ADM momentum and non-vanishing $C_{AB}$ are noted in $^{[8]}$. For example, supertranslations can be used to induce non-vanishing $C_{AB}$ from the state where all Bondi parameters vanish. As in that case, there exist transformed Poincare generators that leave the solution invariant, as is clear from the construction of the solution as a diffeomorphism of flat spacetime $^{[15]}$. These solutions are related to the results of $^{[30]}$ where solutions with vanishing ADM momentum but non-vanishing ADM angular momentum are studied (though in the present work $F(z) = 0$ for any globally defined $SL(2, C)$ transformation, so we only expect to see superangular momenta).

We note a linearized version of the Schwarzian term appears in the work of $^{[31]}$ when $N_{AB}$ and $C_{AB}$ undergo a superrotation. In the present work the Bondi news $N_{AB}$ vanishes except for a Dirac delta function term localized on the shock. In the limit considered in this section the shock itself does not appear in the asymptotically flat region. However shocks that cross the asymptotically flat region can similarly be constructed, and are briefly commented on below.

V. DISCUSSION

In the present work, we have constructed a shock solution in asymptotically local anti-de Sitter spacetime and found the corresponding boundary stress tensor has an interpretation in terms of a two-dimensional Euclidean conformal field theory living on a timeslice within a three-dimensional conformal field theory. In the limit of asymptotically flat spacetime the shock gives a patch of spacetime with nontrivial superrotation charges. In a more general context, one may insert 2d CFT operators and build more general Virasoro charges along with the holomorphic coordinate transformation that appears in the shock solution. One may follow through the usual construction of perturbative bulk fields around this solution $^{[17, 18]}$ and there will be contributions from the 2d CFT and the more standard contributions from the 3d CFT. The contributions from operators in the 2d CFT will contain the leading order information about the BMS charges, while the 3d CFT will only have the usual global conformal charges $SO(3, 2)$ after explicitly breaking the symmetry with the defect. These
global charges contract to the 4d Poincare group in the flat space limit $ISO(3,1)$ \cite{13}.

One might also consider shocks crossing the asymptotically flat region that can give rise to other defect timeslices in the 3d CFT. It is possible there is some connection between this picture and the suggestion that strings pierce $I^+$ in the asymptotically flat limit, destroying asymptotic flatness in general \cite{32}. Conversely, one might rule out such additional shocks if one insists on a well-defined asymptotically flat limit. Note there do exist Penrose shock solutions involving more general BMS transformations \cite{16, 33} and it would be interesting to study these from the present perspective.

A primary motivation for the present work is to gain better insight into how the BMS symmetries generalize to the quantum case. The proposal of the present work is that the generalization of a class of spacetimes with nontrivial superrotation charges to asymptotically AdS spaces are dual to a conventional 3d CFT with a 2d CFT living at a particular timeslice. Such a system should have a self-contained quantum description. Another approach to accommodate BMS like symmetries in AdS is to allow for more general boundary conditions \cite{13, 14}, however this avenue appears to lead to boundary gravity theories that seem difficult to describe.

The idea of a defect CFT living at a particular timeslice is perhaps a rather novel idea, however in Euclidean signature this is a relatively familiar construction. In Lorentzian signature, one can view the defect as the coincident limit of a pair of local operator quenches. This coincident limit takes one out of the realm of the 3d CFT (or equivalently involves infinite energies in the 3d CFT) and requires the specification of the 2d CFT to make it well-defined. It will be very interesting to further study the details of this construction to better develop the structure of this 2d CFT and its coupling to the 3d CFT. Thus far, we have only been able to read off the central charge and see the Schwarzian vacuum energy emerge.
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