First numerical study of Neutrino-Dark Matter Mixed Damping

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Abstract. Mixed Damping is a physical effect that occurs when two fluids have interactions with each other but the particles in one fluid are already free-steaming. As such, it is a cross between collisional damping and free-streaming and has never been studied numerically. Mixed damping is particularly relevant in the context of dark matter-neutrino weak interactions and therefore should not be neglected. Here, we provide an accurate, self-consistent, description of the dark matter-neutrino interactions, which accounts for the different damping regimes. This work is critical to characterise the dark matter microphysics and will be extremely important if measurements of the matter power spectrum at small-scales indicate a departure from the ΛCDM predictions.
1 Introduction

Dark matter (DM) is a required ingredient in our universe to explain e.g. the galactic rotation curves, gravitational lensing, Cosmic Microwave Background (CMB) measurements and the growth of matter perturbations. Large-scale-structures observations by galaxy surveys and other measurements seem to be perfectly consistent with a Cold Dark Matter (CDM) component, that is, with a pressureless, non-interacting fluid. However, small-scale discrepancies indicate that the properties of dark matter could be more complex than the CDM hypothesis.

In this regard, dark matter interactions with Standard Model particles can change observations at small scales [1–25]. Dark matter-neutrino scattering in particular could affect both the distribution of matter in the universe and the structures in our cosmic neighbourhood as they may damp dark matter fluctuations on relatively large scales. Mixed damping [1, 4, 5] refers to the physical damping phenomenon in which dark matter is kinetically coupled to another species which itself is free streaming. It is particularly relevant in dark matter-neutrino interacting scenarios, the main focus of this study, but it might also apply to other dark matter interactions with a dark radiation component. The mixed damping effect is a cross between collisional damping and free-streaming; dark matter perturbations are damped because the dark matter follows the free-streaming neutrinos. Because mixed damping occurs when the neutrinos have kinetically decoupled, one can summarise the condition for mixed damping by

\[ \Gamma_{\text{DM} - \nu} > H > (\Gamma_{\nu} \equiv \Gamma_{\nu - e} + \Gamma_{\nu - \text{DM}}), \]  

(1.1)

where the collision rates between dark matter and neutrinos are given by

\[ \Gamma_{\nu - \text{DM}} = n_{\text{DM}} \sigma_{\nu \text{DM}}, \]  

(1.2a)

\[ \Gamma_{\text{DM} - \nu} = \frac{4\rho_{\nu}}{3\rho_{\text{DM}}} \Gamma_{\nu - \text{DM}}. \]  

(1.2b)
In addition $\Gamma_{\nu-e} = \sigma_{\nu-e} n_e$. To progress further, one needs to specify the dark matter number density. The observed dark matter relic density suggests that, if the dark matter was produced thermally, it has either annihilated or decayed since then \(^1\). In a thermal annihilating dark matter scenario, where the dark matter and anti dark matter number densities are supposed to be exactly the same, the observed relic density imposes a condition on the ratio of the dark matter mass to the freeze-out temperature (for a given annihilation cross section), which can be used to determine whether one needs to account for the evolution of the dark matter number density while neutrinos are decoupling [5]. In the following, we will disregard such a scenario and assume that dark matter has already annihilated when the neutrino fluid decouples. As a result the bounds that we quote in the following should be used with precaution when dark matter is lighter than $m_{DM} \lesssim O(\text{MeV})$.

In the past, several constraints on dark matter-neutrino interactions have been derived from a plethora of cosmological observations. Forecasts for next-generation of CMB and large scale structure surveys are also available in the literature, see e.g. [12]. Constraints are given in terms of a single parameter, namely

$$u_{\nu\text{DM}} = \frac{\sigma_{\nu\text{DM}}}{\sigma_{\text{Th}}} \left( \frac{m_{\text{DM}}}{100 \text{ GeV}} \right)^{-1},$$

(1.3)

and associate values can be found in Refs. [12, 18, 26]. The dark matter-neutrino scattering cross-section may be a power-law of the neutrino temperature, i.e. $\sigma_{\nu\text{DM}} \propto T_{\nu\text{DM}}^{n_{\nu\text{DM}}}$, and therefore can be expressed as

$$u_{\nu\text{DM}} = u_{\nu\text{DM},0} \times a^{-n_{\nu\text{DM}}},$$

(1.4)

with $u_{\nu\text{DM},0}$ a constant in which the temperature difference between neutrinos and photons can be absorbed and where we implicitly assumed that $a_0 = 1$. The most recent analyses provide $u_{\nu\text{DM}} \leq (4.5-9.0) \times 10^{-5}$ for $n_{\nu\text{DM}} = 0$ and $u_{\nu\text{DM},0} \leq (3.0-5.4) \times 10^{-14}$ for $n_{\nu\text{DM}} = 2$. Mixed damping is an important effect for these values and should have been included in the estimates of the angular and matter power spectra. In the past, studies have focused on the scenario where the neutrino decoupling was set by $\Gamma_{\nu} \sim \Gamma_{\nu-DM} \approx H$ and $\Gamma_{\text{SM}} < \Gamma_{\nu}$. However, there is a regime where the neutrino decoupling is set by $\Gamma_{\nu} \sim \text{max}(\Gamma_{\nu-DM}, \Gamma_{\nu-e}) \approx H$ and $\Gamma_{\text{DM}} > \Gamma_{\nu}$ and for which a systematic description of the damping effect is lacking.

In this manuscript, we review and extend the Boltzmann equations to propose a self-consistent formulation accounting for the mixed damping effect. We also test and improve the usual ultra-relativistic fluid approximation (UFA). In the presence of dark matter-neutrino interactions this approximation may (incorrectly) neglect high multipole moments in the Boltzmann hierarchy. Even though our findings show that these effects are not critical for the scales probed by current experiments, a coherent description of the dark matter-neutrino coupled system will become important as soon as high precision future cosmological probes become sensitive to smaller scales and improve the present precision on the attainable ones. Furthermore, a careful assessment of systematic effects in the theoretical predictions is mandatory to obtain meaningful constraints.

The structure of the paper is as follows. Section 2 describes the conditions and the parameter space in terms of the scale factor and of the scattering strength for which mixed

\(^1\)There may be other mechanism in the thermal scenario that could explain why the dark matter number density nowadays is so small. Alternatively dark matter may have been produced non-thermally.
damping will take place. In Sec. 3, we introduce the Boltzmann equations and the formalism that needs to be implemented to accurately describe dark matter-neutrino interactions in a self-consistent and robust way. We also review the ultra-relativistic fluid approximation when “beyond the Standard Model physics” in the neutrino sector is present. Section 4 contains a deep, quantitative study of the evolution of the perturbations in the presence of mixed damping. We draw our conclusions in Sec. 5.

2 Mixed damping regime and parameter space

As mentioned in the introduction, mixed damping occurs when neutrinos decouple from dark matter before the reverse process sets in. The two interaction rates $\Gamma_{\text{DM}-\nu}$ and $\Gamma_{\nu-\text{DM}}$ are not expected to decouple simultaneously because of the differences between the neutrino and dark matter number densities (c.f. Eq. (1.2)). In the Standard Model, neutrinos kinetically decouple from the thermal bath when they decouple from the electrons. However, in models where neutrinos interact with dark matter, the neutrino interaction rate is given by $\Gamma_{\nu} \equiv \Gamma_{\nu-\text{DM}} + \Gamma_{\nu-e}$. The neutrino kinetic decoupling is thus determined by their last interactions, which might be with either the electrons (if $\Gamma_{\nu} \sim \Gamma_{\nu-e} \simeq H$) or dark matter (if $\Gamma_{\nu} \sim \Gamma_{\nu-\text{DM}} \simeq H$). Each option corresponds to a different dark matter decoupling epoch and, consequently, to a different damping scale.

By requesting the existence of an epoch where $\Gamma_{\text{DM}-\nu} > H > \Gamma_{\nu}$, we ensure that dark matter stays coupled to neutrinos after the neutrinos have kinetically decoupled and thereby experiences a phase of mixed damping. Therefore, the minimum scattering rate for which mixed damping can occur is set by the simultaneous decoupling of dark matter from neutrinos and of neutrinos from electrons. This condition implies $\nu_{\text{min}} > 2.4 \times 10^{-14}$, $1.4 \times 10^{-33}$ and $7.4 \times 10^{-53}$ for $n_{\nu_{\text{DM}}} = 0, 2$ and 4 respectively. The scale factor at decoupling in this configuration is $a_{\text{min}} = a (T_{\gamma} = 1 \text{ MeV}) = 2.35 \times 10^{-10}$ and the corresponding ”mixed damping scale” is rather small, lying beyond the current experimental reach: $r_{\text{min}} = \frac{1}{aH} (T_{\gamma} = 1 \text{ MeV}) = 0.11 \text{ kpc}$, or $M_{\text{min}} = 0.2 M_{\odot}$.

Equation (1.1) also imposes an upper limit on the scales which can be affected by the mixed damping. Its value depends on the thermal history of the neutrino and, in particular, whether their last interactions are with electrons or with dark matter. In general, the mixed damping effect is larger when the neutrino last scattering is set by its interactions with dark matter, once neutrinos have decoupled from the electrons. In this case, the magnitude of the effect is determined by the requirement $\Gamma_{\text{DM}-\nu} > \Gamma_{\nu-\text{DM}}$, with $T_{\text{dec}(\nu-\text{DM})} < T_{\text{dec}(\nu-e)}$. In heavy thermal dark matter scenarios (where $m_{\text{DM}} \gg O(\text{MeV}))$, this condition is automatically satisfied since the dark matter is expected to have annihilated and frozen-out before nucleosynthesis. Therefore, in these scenarios, the magnitude of the effect and the upper limit is set by the DM-$\nu$ cross section. We provide a quantitative estimate of the respective scales in the following.

2.1 Neutrino decoupling by weak interactions

The mixed damping effect is the sole process responsible for erasing dark matter perturbations larger than $r_{\text{min}}$ when the neutrino decoupling is set by its Standard Model interactions, i.e. $\Gamma_{\text{DM}-\nu} > \Gamma_{\nu-e} > \Gamma_{\nu-\text{DM}}$ (assuming a monotonous thermal evolution of both dark matter and neutrinos in the early Universe). In this case, the magnitude of the effect is determined by the dark matter decoupling only ($\Gamma_{\text{DM}-\nu} \simeq H$); the later the decoupling, the bigger the effect. However, we cannot make $\Gamma_{\text{DM}-\nu}$ arbitrarily large. Indeed, the relation given by
Eq. (1.2b) together with the condition $\Gamma_{\nu-DM} < \Gamma_{\nu-e}$ implies that the maximal decoupling time is given by $\Gamma_{\nu-DM} \simeq \Gamma_{\nu-e}$. The dark matter decoupling condition $\Gamma_{DM-\nu} \simeq H$ can be therefore recast as

$$\frac{4\rho_\nu}{3\rho_{DM}} < \frac{\Gamma_{\nu-e}}{\Gamma_{\nu-DM}} \simeq H,$$

which translates into a maximal damping scale $r_{wdec}$, which corresponds to 79 kpc ($M_{wdec} = 8 \times 10^7 M_\odot$), 2.6 kpc ($M_{wdec} = 3 \times 10^3 M_\odot$) and 0.8 kpc ($M_{wdec} = 1 \times 10^2 M_\odot$) for $n_{\nu DM} = 0, 2$ and 4 respectively.

The above condition on the respective interaction rates can only be fulfilled if $u_{\nu DM,0}^{\text{min}} < u_{\nu DM,0}^{\text{wdec}}$, with $u_{\nu DM,0}^{\text{wdec}} = 1.3 \times 10^{-8}, 7.4 \times 10^{-28}$ and $4.1 \times 10^{-47}$ for $n_{\nu DM} = 0, 2$ and 4 respectively. For these limiting parameters we depict the evolution of the interaction rates in Fig. 1, where the left edge of the green shaded region indicates the time of neutrino Standard Model decoupling.

### 2.2 Neutrino decoupling set by dark matter interactions

Mixed damping also plays an important role if the neutrino kinetic decoupling is determined by dark matter-neutrino interactions. This is more relevant for larger modes, which are easier to access observationally. To illustrate the situation, first consider a small mode, which enters the horizon during a period where $\Gamma_{DM-\nu} > \Gamma_{\nu-DM} > H$. This mode will be initially subject to collisional damping and, upon neutrino decoupling, will experience a transition to the mixed damping regime. A larger mode, on the other hand, enters the horizon later and thus can be subject to mixed damping only. We shall discuss this situation in detail in Sec. 4. For now it is important to note that a transition from the collisional to the mixed damping regime can only occur if the ratio of densities in Eq. (1.2b) is larger than unity by the time dark matter decouples from the neutrino fluid. The corresponding upper limit on the decoupling time is

$$\frac{4\rho_\nu}{3\rho_{DM}} \bigg|_{a=a_{\text{max}}} = 1 \quad \iff \quad a_{\text{max}} = 1.9 \times 10^{-4} \left(\frac{\Omega_{DM} h^2}{0.1186}\right)^{-1} \left(\frac{\Omega_b h^2}{0.0223}\right)^{-1}. \quad (2.1)$$

The largest scales that will be affected by mixed damping have therefore to enter the horizon before $a_{\text{max}}$. The comoving Hubble radius at $a_{\text{max}}$ is given by $r_{\text{max}} = \frac{1}{a_{\text{max}} H(a_{\text{max}})^2}$ and the mass enclosed in a spherical volume of size $r_{\text{max}}$ is $M_{\text{max}} = 6 \times 10^9 M_\odot$. Finally, the criterion of neutrinos decoupling from the dark matter before $a_{\text{max}}$ translates into a maximum value of the scattering rate, which reads

$$u_{\nu DM,0}^{\text{max}} = 1.97 \times 10^{-2} \times a_{\nu DM,0}^{\text{max}} \times \left(\frac{0.1186}{\Omega_{DM} h^2}\right)^2 \sqrt{1.0 + 0.066 \left(\frac{\Omega_b h^2}{0.0223}\right) \left(\frac{0.1186}{\Omega_{DM} h^2}\right)}. \quad (2.2)$$

We find $u_{\nu DM,0}^{\text{max}} \simeq 1.98 \times 10^{-2}, 7.12 \times 10^{-10}$ and $2.5 \times 10^{-17}$, for $n_{\nu DM} = 0, 2$ and 4 respectively. Note that these values fall within the current observational limits [12, 16], indicating the importance of mixed damping while deriving bounds on scenarios with dark matter-neutrino interactions.

The evolution of scattering rates in this limiting case is depicted in Fig. 1 along with the Hubble rate. The right edge of the green region indicates $a_{\text{max}}$. Irrespective of the value

\[\text{Numerically, } r_{\text{max}} = \frac{1}{a_{\text{max}} H(a_{\text{max}})^2} = \frac{71.0 \text{ Mpc} \left(\frac{\Omega_{DM} h^2}{0.1186}\right)}{\sqrt{1.0 + 0.009 \left(\frac{\Omega_b h^2}{0.0223}\right) \left(\frac{0.1186}{\Omega_{DM} h^2}\right)}}.\]
chosen for $u_{\nu,0}$ and $n_{\nu,0}$, we always have $\Gamma_{\nu-DM}(a_{\text{max}}) = \Gamma_{DM-\nu}(a_{\text{max}})$ on this edge. Mixed damping occurs whenever $u_{\nu,0}^{\text{min}} < u_{\nu,0} < u_{\nu,0}^{\text{max}}$, or, in terms of Fig. 1, whenever the neutrino and dark matter decoupling times fall within the green region. Depending on the size of a mode it might either be subject to mixed damping solely or to a period of collisional damping followed by mixed damping.

### 3 Boltzmann Equations

In the Newtonian gauge,\footnote{This is a typical gauge choice for gravitational waves, where the gauge condition is $\epsilon_{ij} \nabla^i \nabla^j \phi = 0$.}

$$ds^2 = a^2(\tau) \left[ -(1 + 2\psi) d\tau^2 + (1 - 2\psi) dx_i dx^i \right], \quad (3.1)$$

where $\phi$ and $\psi$ are the metric perturbations, the scattering processes between dark matter and massless neutrinos are governed, for the dark matter fluid, by its evolution equations,

$$\dot{\delta}_{\text{DM}} = -\theta_{\text{DM}} + 3\dot{\phi}, \quad (3.2a)$$

$$\dot{\theta}_{\text{DM}} = k^2 \psi - \mathcal{H}\theta_{\text{DM}} - S\dot{\kappa}_{\nu,\text{DM}} (\theta_{\text{DM}} - \theta_{\nu}). \quad (3.2b)$$
Here $S = 4\rho_\nu/(3\rho_{DM})$, and the scattering rate is $\dot{\kappa}_{\nu,DM} = an_{DM}\sigma_{\nu,DM}$. For massless neutrinos the modified Boltzmann hierarchy is [27]

\[
\dot{\delta}_\nu = \frac{4}{3}\theta_\nu + 4\dot{\phi},
\]
\[
\dot{\theta}_\nu = k^2 \left( \frac{\delta_\nu}{4} - \sigma_\nu \right) + k^2 \psi - \dot{\kappa}_{\nu,DM} (\theta_\nu - \theta_{DM}),
\]
\[
2\dot{\sigma}_\nu = \frac{8}{15}\theta_\nu - \frac{3}{5}kF_{\nu,3} - \alpha_2 \dot{\kappa}_{\nu,DM}\sigma_\nu,
\]
\[
\dot{F}_{\nu,l} = \frac{k}{2l+1} \left[ F_{\nu,l-1} - (l+1)F_{\nu,l+1} \right] - \alpha_l \dot{\kappa}_{\nu,DM}F_{\nu,l},
\]
\[
\dot{F}_{\nu,l_{\text{max}}} = k \left[ F_{\nu,l_{\text{max}}-1} - \frac{l_{\text{max}}+1}{k\tau}F_{\nu,l_{\text{max}}} \right] - \alpha_l \dot{\kappa}_{\nu,DM}F_{\nu,l_{\text{max}}}.\tag{3.3e}
\]

The angular coefficients $\alpha_l$, which appear in the interaction terms of the higher-order multipoles, are generally of $O(1)$. Nevertheless, their precise numerical value is set by the dependence of the matrix element for the scattering process $|M_{\nu,DM}|^2$ on the cosine of the angle between the incoming and the outgoing neutrino $\mu$. Previous works on dark matter-neutrino scattering have adopted different choices for $\alpha_l$ ($\alpha_2 = 2$ and $\alpha_1 = 1$ for $l \geq 3$ [12], or $\alpha_2 = 9/5$ and $\alpha_1 = 1$ for $l \geq 3$ [18]). Here, we follow Ref. [28], which provides a self-consistent formalism to compute the higher-order multipole coefficients,

\[
\alpha_l = \frac{\int dp \; p^4 \left( \frac{\partial f_\nu}{\partial p} \right) \left[ A_0(p) - A_l(p) \right]}{\int dp \; p^4 \left( \frac{\partial f_\nu}{\partial p} \right) \left[ A_0(p) - A_1(p) \right]},
\]

where $f_\nu$ denotes the Fermi-Dirac equilibrium neutrino distribution and

\[
A_l(p) = \int_{-1}^1 d\mu \; P_l(\mu) \left( \frac{1}{\eta_{DM}\eta_\nu} |M_{\nu,DM}|^2 \right) \bigg|_{t=2p^2(\mu-1)}^{s=m_{DM}^2+2m_{DM}p}.
\]

In the expressions above, $P_l$ are the Legendre Polynomials, $p$ is the momentum of the incoming neutrino, which we assume not to change during the scattering process, $t$ and $s$ are Mandelstam variables and $\eta_{DM}$ and $\eta_\nu$ the internal degrees of freedom of the dark matter particle and neutrinos respectively. Within this formalism, the scattering rate can be expressed as

\[
\dot{\kappa}_{\nu,DM} = \frac{a \rho_{DM}}{128\pi^3 m_{DM}^2 \rho_\nu} \eta_\nu \int_0^\infty dp \left( \frac{\partial f_\nu}{\partial p} \right) p^4 \left[ A_0(p) - A_1(p) \right].\tag{3.6}
\]

We make use of the classification of dark matter-neutrino interaction scenarios introduced in [19] to give an overview of the (properly) derived values for $\dot{\kappa}_{\nu,DM}$ and $\alpha_l$ in Tab. 2 in Appendix A. To account for all possible configurations of the interaction term, we have modified the publicly available Boltzmann solver code CLASS$^3$ (version v2.7) introducing three new parameters. Namely, $w_{\nu,DM}$ represents the coupling strength parameter (see Eq. (1.3)), $n_{\nu,DM}$ governs the temperature dependence of the cross-section (see Eq. (1.4)) and $\alpha_l$ refers to the higher-order multipole coefficients appearing in Eq. (3.3). If not stated otherwise, we choose $\alpha_l = 3/2$ for $l \geq 2$ and $n_{\nu,DM} = 2$, motivated by the most-common values (see Tab. 2 in Appendix A).

$^3$Our modified CLASS version is publicly available and can be downloaded from https://gitlab.dur.scotgrid.ac.uk/dm-interactions/class_v2.7_udm.git
3.1 The Ultra-relativistic Fluid Approximation

The evolution of neutrino perturbations is described by an infinite hierarchy of moment equations (c.f. Eqs. (3.3)), that must be truncated at some maximum multipole $l_{\text{max}}$. At early times $l_{\text{max}}$ typically is of $O(10)$. Once neutrino perturbations are well inside the horizon, the numerical Boltzmann solver code CLASS uses the ultra-relativistic fluid approximation (UFA) [29], which truncates the multipole hierarchy after $l = 2$ at a time $k\tau = k\tau_{UFA}$. The advantage is twofold. First, the computational costs to describe the late time evolution of neutrino perturbations are reduced. Second, to avoid unphysical reflections, which are caused by the inevitable truncation of the a priori infinite Boltzmann hierarchy, for a mode of wavenumber $k$ at some time $\tau$ one has to choose the maximum multipole moment $l_{\text{max}} \geq k\tau$. Truncating the Boltzmann hierarchy in a consistent way as earlier as possible then allows to choose a smaller value for $l_{\text{max}}$ during the early evolution and hence also benefits the computational costs prior to the UFA.

The UFA truncation scheme in general differs from the ordinary truncation scheme for Boltzmann equations proposed in Ref. [27]. Its implementation in CLASS reads

$$
\dot{\sigma}_{ur} = -\frac{3}{\tau}\sigma_{ur} + \frac{2}{3}\left(\theta_{ur} - 6\dot{\phi}\right),
$$

where the subscript $ur$ refers to any ultra-relativistic species. Previous studies dealing with dark matter-neutrino interactions have tried to generalise the expression above as

$$
\dot{\sigma}_{ur} = -\frac{3}{\tau}\sigma_{ur} + \frac{2}{3}\left(\theta_{ur} - 6\dot{\phi}\right) - \bar{\kappa}_{\nu\text{DM}}\sigma_{ur}.
$$

Instead of this economical approach, we follow here a different avenue. We start by choosing $k\tau_{UFA}$ large enough to make sure that neutrinos have decoupled from dark matter when the UFA starts. Then, we evolve the neutrino perturbations accordingly to Eqs. (3.3) while the coupling to dark matter is still active, and accordingly to Eq. (3.7) afterwards. In parallel, we ensure that $l_{\text{max}}$ is large enough to avoid unphysical reflections while the Boltzmann hierarchy is evolved. We perform an optimisation procedure for the parameters $k\tau_{UFA}$ and $l_{\text{max}}$ to check that the final result, i.e. the matter power spectrum, is stable and the computational errors are minimised against variations in these two parameters. In the following we shall compare our treatment to the usual, default UFA assumptions, bearing in mind that our method avoids the application of the UFA regime when neutrinos are not free streaming particles.

In the mixed-damping regime neutrinos have to decouple from dark matter before $a_{\text{max}}$, i.e. when the universe is still dominated by radiation. Therefore, the scale-factor at dark matter-neutrino decoupling $a_{\nu,\text{dec}}$ can be approximated as

$$
a_{\nu,\text{dec}}^{n_{\nu,\text{DM}}+1} = \frac{3M_P^2\Omega_{DM} u_{\nu,\text{DM}} \sigma_{\text{TH}} H_0}{8\pi \sqrt{10} \times 100 \text{ GeV}} = 1.19 \times 10^{-2} \times u_{\nu,\text{DM},0} \times \left(\frac{\Omega_{DM}h^2}{0.1186}\right),
$$

where $\Omega_{DM}h^2$ is the current dark matter energy density, and we have introduced the factor $\epsilon_{UFA}$ by requiring that the UFA truncation is not switched on before $\Gamma_{\nu-\text{DM}} \leq \epsilon_{UFA}H$. With

\[\text{Note:} \quad \epsilon_{UFA} = 0.01 \quad \text{for} \quad k\tau_{UFA} = 30 \quad \text{and} \quad l_{\text{max}} = 17.\]
these definitions in mind, we find
\[
\tau_{\text{UFA}} = \frac{a_{\nu,\text{dec}}}{H_0 \sqrt{\Omega_r}} = \begin{cases} 
5.53 \times 10^3 \text{Mpc} \times (\frac{u_{\nu,\text{DM}}}{c_{\text{UFA}}}) \times \left(\frac{a_{\nu,\text{DM}}}{5.1186}\right)^\frac{1}{2} & \text{if } n_{\nu,\text{DM}} = 0 \\
10.6 \times 10^4 \text{Mpc} \times (\frac{u_{\nu,\text{DM}}}{c_{\text{UFA}}}) \times \left(\frac{a_{\nu,\text{DM}}}{5.1186}\right)^\frac{1}{2} & \text{if } n_{\nu,\text{DM}} = 2 \\
19.2 \times 10^4 \text{Mpc} \times (\frac{u_{\nu,\text{DM}}}{c_{\text{UFA}}}) \times \left(\frac{a_{\nu,\text{DM}}}{5.1186}\right)^\frac{1}{2} & \text{if } n_{\nu,\text{DM}} = 4
\end{cases}
\]
(3.10)
where we have approximated the conformal time \(\tau_{\text{UFA}}\) as
\[
\tau_{\text{UFA}} \simeq \left[a H\right]^{-1} \quad \text{at} \quad \Gamma_{\nu-\text{DM}} = \epsilon_{\text{UFA}} H.
\]
(3.11)
If the largest wavenumber we are interested in is \(k_{\text{max}}\) we have to choose \(l_{\text{max}} \geq k_{\text{max}} \tau_{\text{UFA}}\).

We choose two benchmark scenarios to investigate the impact of the UFA approach on the CMB temperature auto-correlation, E-mode polarisation auto-correlation and the temperature-E-mode cross-correlation spectra. Namely, the parameters we consider are \(n_{\nu,\text{DM}} = 0\) and \(u_{\nu,\text{DM}} = 4.5 \times 10^{-5}\) (upper limit from Ref. [18]) and \(n_{\nu,\text{DM}} = 2\) and \(u_{\nu,\text{DM},0} = 5.4 \times 10^{-14}\) (upper limit derived from Ref. [26]). The remaining six \(\Lambda\)CDM parameters are set to the mean values obtained in the Planck 2018 data release [30], and we consider three massless neutrinos with identical interactions to dark matter. For each scenario, we delay the UFA regime by increasing the value of \(k \tau_{\text{UFA}}\), both varying and keeping fixed the value of \(l_{\text{max}}\). We find that the effect of delaying the UFA regime on the CMB spectra is completely negligible for both scenarios.

However, the impact on the matter power spectrum is clearly noticeable, as we shall now illustrate, focusing exclusively on the \(n_{\nu,\text{DM}} = 0\) case. Figure 2 shows that, up to the first oscillation peak, the power spectra computed with CLASS UFA default settings, (Eq. (3.7), see dashed lines) and with a delayed UFA regime approach (solid lines) typically agree. However, at smaller scales, non-negligible discrepancies show up. Figure 3 compares the results obtained with a truncation according to Eq. (3.7) and Eq. (3.8) for the CLASS default UFA settings (dashed lines) and a delayed onset of the UFA regime (solid lines). Note that Ref. [26] uses the truncation scheme of Eq. (3.8) but with default CLASS parameters for the onset of the UFA. Both truncation schemes result in a very similar power spectra if the UFA regime is delayed sufficiently. This behaviour is the expected one since, in this case, the scattering term in Eq. (3.8) is small and should have no impact. On the other hand, increasing \(l_{\text{max}}\) while not delaying the onset of the UFA method leaves the power spectrum unchanged. Most importantly, however, the modified truncation scheme of Ref. [26] is not able to reproduce the result to which both codes converge for a delayed UFA regime for default UFA parameters. We therefore conclude that, to obtain precise predictions for the matter power spectrum on small scales, it is crucial to delay the onset of the UFA regime sufficiently long. Otherwise discrepancies will show up due to the fact that, with the default UFA settings, neutrinos are treated as free-streaming during epochs in which their coupling to dark matter is still active. Since in the UFA approach all multipoles beyond \(\sigma_{ur}\) are neglected, the effect on the matter power spectrum will be very severe if the UFA regime is not delayed. Eventually, neutrinos decouple from dark matter at a time where the reverse dark matter-neutrino coupling is still active (c.f. Sec. 2). Once this happens, the application of the UFA regime is well justified and hence the damping of the dark matter fluctuations at intermediate scales is predicted accurately irrespective of the UFA settings.

As the power law of the dark matter-neutrino interaction cross section steepens, i.e. \(n_{\nu,\text{DM}}\) in Eq. (1.4) increases, the duration of the mixed damping regime is shortened. This
Figure 2. The matter power spectrum computed with the default CLASS UFA settings (dashed lines) and with a delayed onset of the UF approximation ($k\tau = l_{\text{max}} = 200$, solid lines). Non-negligible differences appear beyond the first oscillation peak.

Figure 3. Comparison between the matter power spectrum obtained with the truncation scheme of Eq. (3.7) and Eq. (3.8) for default UFA settings (dashed lines) and a delayed onset of the UFA regime (solid lines).

trend is clearly visible in Fig. 1. Hence, the time interval during which default UFA setting would erroneously treat neutrinos as free streaming particles tends to be shorter for larger values of $n_{\nu DM}$. Nevertheless, we advocate to conduct a careful convergence study as the accuracy of the results will depend on the precise combination of $u_{\nu DM,0}$ and $n_{\nu DM}$ considered.

To estimate the impact of the UFA treatment on small-scale observables, we compute the number of Milky satellites $N_{\text{sat}}$ following Ref. [31]. Table 1 summarises the results in the $n_{\nu DM} = 0$ case for different values of the neutrino-dark matter interaction parameter

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6See also the works of Refs. [6, 11, 13] for devoted simulations within different possible interacting dark matter scenarios.
4. The effect of mixed damping on the evolution of perturbations

Having discussed all the relevant technical aspects required to obtain accurate predictions for the evolution of perturbations in the mixed damping scenario, we now turn to the evolution of individual modes and discuss how these are affected by mixed damping. We start by illustrating some relevant aspects of the evolution of perturbations, like neutrino free streaming, in the canonical $\Lambda$CDM case. Subsequently, a comprehensive examination of the mixed damping effect is presented.

4.1 Evolution of perturbations in the $\Lambda$CDM scenario

Figure 4 depicts the evolution of density perturbations for two modes, which both enter the horizon prior to matter radiation equality ($a_{eq} \approx 3.6 \times 10^{-4}$). The evolution of the dark matter perturbations is the simplest: after entering the horizon, $\delta_{DM}$ grows as $\delta_{DM} \propto \log a$ while the universe is radiation dominated and as $\delta_{DM} \propto a$ once the universe has transitioned to matter domination. Upon horizon entry, massless neutrinos diffuse out of overdense regions, leading to a decrease in $\delta_{\nu}$. In the subsequent evolution, the competition between gravitational forces and the neutrino pressure leads to oscillations in $\delta_{\nu}$, apparent in Fig. 4 for both modes.

Photons and baryons are initially tightly coupled by Thomson scattering and oscillate together. Their perturbations experience collisional or Silk damping, the impact of which is most significant when the mode’s momentum is comparable to the scattering rate. A corresponding significant decrease in $\delta_{\nu}$ and $\delta_{\gamma}$ is clearly noticeable for the smaller mode ($k = 27$ Mpc$^{-1}$, lower panel of Fig. 4). The decrease is less pronounced for the larger mode, whose momentum becomes comparable to the Thomson scattering rate at a time

\[^7\text{The sudden termination of the oscillations in the evolution of the } k = 27 / \text{Mpc mode is not a physical effect. It is rather caused by the use of the radiation streaming approximation (RSA) in CLASS. Instead of resolving the oscillations, the numerical code uses an approximation scheme where the average amplitude of } \delta_{\nu} \text{ over many oscillations is the one which is evolved.}\]
when the photons are about to decouple from the baryons. Upon the recombination era ($a_{\text{rec}} \simeq 1.1 \times 10^{-4}$), baryons decouple from photons and fall into the potential wells created by dark matter implying a rapid increase in $\delta_b$. Photons, on the other hand, continue their free-streaming evolution.

4.2 The effects of mixed damping

Let us now explore how mixed damping modifies the evolution of the perturbations with respect to the $\Lambda$CDM scenario. The mixed damping effect is difficult to investigate for a number of reasons. Firstly, there is an important issue with numerical accuracy: even when the UFA is delayed until neutrinos have decoupled from dark matter, the fact that the perturbations are now damped makes the evolution of large modes subject to numerical instabilities. In the matter power spectrum, shown in Fig. 5, these instabilities translate into wiggles on small scales, clearly visible for the largest interaction strengths considered. In the evolution of a single mode these instabilities can lead to more drastic effects causing
e.g. the perturbations to grow faster. As previously explained, in order to obtain robust results we firstly conduct a convergence study for the UFA parameters $l_{\text{max}}$ and $k\tau$, see Sec. 3.1 for details. Once our results have converged to an acceptable level of accuracy, we verify that delaying the onset of the radiation streaming approximation does not affect our outputs. Finally, we optimise the remaining precision parameters to achieve a good compromise between numerical accuracy and computational speed.

The mixed damping-only regime is also difficult to access due to its narrow parameter space. A given mode has to be large enough to enter the horizon after neutrinos have decoupled, but small enough to evolve within the mixed-damping regime for some considerable time. On the other hand, a direct and quantitative comparison between the collisional and mixed damping effects is difficult to perform as a mode evolves in either one or the other regime. Therefore, we focus here exclusively on the qualitative description of the perturbation evolution in both the collisional and mixed damping regimes.

We show several examples for the evolution of a mode in the collisional and the mixed damping regime in Fig. 6. Namely we consider the following configurations:

- $u_{\nu \text{DM}} = 10^{-3}$ and $k = 10 \text{ Mpc}^{-1}$ (top left panel): this mode spends most of its evolution in the collisional damping regime and only switches to the mixed damping regime at late times.
- $u_{\nu \text{DM}} = 10^{-5}$ and $k = 10 \text{ Mpc}^{-1}$ (bottom left panel): this mode experiences a very brief period of mixed damping before dark matter decouples from neutrinos.
- $u_{\nu \text{DM}} = 10^{-5}$ and $u_{\nu \text{DM}} = 10^{-6}$ for $k = 60 \text{ Mpc}^{-1}$ (top and bottom right panels): both modes evolve in the mixed-damping regime for some substantial period of time.

When the mixed damping is the dominant damping mechanism, neutrino perturbations evolve very similarly to the $\Lambda$CDM case: upon horizon entry free streaming leads to a decrease of the perturbations’ magnitude followed by rapid oscillations. This behaviour was expected as the neutrino interactions have decoupled before the mode entered the horizon, and hence
Figure 6. The evolution of two small modes, which both experience considerable damping. Depending on the value of $u_{\nu DM}$, the relevant effect is mixed damping, collisional damping or a mixture of both following each other in time. Neutrino (dark matter) perturbations are shown in pastel (darker) colours. Arrows indicate the cosmic time when the neutrino/dark matter interaction rates match the Hubble expansion rate.

the effects of the additional neutrino scattering terms should be minimal. A small decrease in the maximum oscillation amplitudes is present. This is either caused by forces exercised by the dark matter interaction, by the damping of dark matter fluctuations and the associated shallowness of gravitational potentials, or by a combination of both. Note that this decrease leaves the average amplitude unaffected and hence is irrelevant once the RSA is turned on. When $k = 60$ Mpc, $u_{\nu DM} = 10^{-5}$, some additional offset between the $\Lambda$CDM and the coupled neutrino evolution is evident. It is most likely caused by the fact that in this scenario neutrinos decouple very close to horizon entry and the coupling leads to some
residual effects.

In the mixed damping cases, the dark matter perturbations initially follow the free-streaming evolution of the neutrinos closely. This is most noticeable for the mode in the top right panel of Fig. 6, which spends the longest time in the mixed damping regime. As the rate for dark matter interactions with neutrinos decreases, an out-of-phase evolution between the neutrino and the dark matter perturbations becomes noticeable. Finally, once \( \Gamma_{\text{DM-}\nu} \) has dropped below the Hubble rate, the dark matter evolution completely decouples from that of neutrinos. In the most extreme case (\( u_{\nu\text{DM}} = 10^{-5}, \ k = 10 \text{Mpc}^{-1} \)) the dark matter perturbation follows only one oscillation of the neutrino fluid. Without the neutrino’s pressure support, dark matter starts clustering and the perturbations grow as they would in the ΛCDM scenario, i.e. proportional to \( \ln a \) during radiation and proportional to \( a \) during matter domination. However, because this growth is delayed with respect to the ΛCDM evolution and starts from a smaller initial value, the overall power on all scales affected by mixed damping is reduced, as visible in Fig. 2.

The evolution of neutrino perturbations is different in the collisional damping case, which becomes apparent considering the evolution of the \( k = 10 \text{Mpc}^{-1} \) mode for \( u_{\nu\text{DM}} = 10^{-3} \) (top left panel of Fig. 6). Although the immediate decrease upon horizon entry is less pronounced for the neutrino perturbations, in the subsequent evolution collisional damping leads to a steeper slope in the decrease of \( |\delta_\nu| \) than found for the free streaming evolution. Dark matter perturbations closely follow the neutrino evolution, much alike the mixed damping scenario. There is a brief period of mixed damping after neutrinos have decoupled. However, due to the proximity of decoupling times, it is hard to isolate these mixed damping effects. Eventually, dark matter decouples from neutrinos and starts clustering. As in the mixed damping case, the lower initial value and later initial time for this process to happen leads to an overall suppression in power, visible in the matter power spectrum.

5 Conclusions

Non-standard dark matter scenarios, which predict a smaller number of structures at small-scales with respect to ΛCDM, have become popular in the last decade, as they have the potential to alleviate the various problems that ΛCDM may face. This includes interacting dark matter scenarios with massless or very light Standard Model particles, such as photons or neutrinos. In these scenarios, dark matter fluctuations are usually erased because of the scattering of dark matter particles off radiation.

However, there exists an additional (and equally relevant) damping regime, dubbed mixed damping. Mixed damping is of particular relevance for dark matter-neutrino interaction scenarios. It takes place when dark matter remains coupled to neutrinos after that they started to free-stream. This regime was disregarded until now in numerical studies. Yet, all current limits constrain interaction rates, which automatically lead to a phase of mixed damping (whether it is mixed damping exclusively, or the combined effect of a collisional period and a mixed damping epoch).

Future cosmological observations will be sensitive to very small scales and therefore will test even smaller dark matter-neutrino interaction rates, exploring the mixed damping regime further. It is therefore of critical importance to robustly establish the parameter space for which mixed damping changes predictions in interacting dark matter scenarios and to accurately model the evolution of perturbations.
Here, we provide for the first time a comprehensive study of the mixed damping effect. We also provide a robust calculation of the higher-order multipole moments in the Boltzmann hierarchy, a key ingredient to properly describe the mixed damping regime, which was still missing in the literature. We present a self-consistent method to compute the higher-order coefficients, based on precise calculations of the matrix elements governing the dark matter-neutrino scattering processes. Furthermore, we also present an optimisation of the so-called ultra-relativistic fluid approximation for scenarios in which dark matter interacts with neutrinos. Our approach avoids unphysical reflections when the Boltzmann hierarchy is evolved. It also circumvents misleading treatments of the neutrino fluid, which usually lead to inaccurate features in the matter power spectrum, because neutrinos are treated as a free-streaming fluid while they remain coupled to the dark matter. The proper description of the mixed damping will be crucial for future cosmological surveys, which are expected to reach unprecedented precision at the smallest scales. We have customised numerical tools, and manage to achieve an excellent compromise between high accuracy and computational time.

Last but not least, the interest and the reach of our results are not limited to the mixed damping regime. Rather, the correct description of the higher-order multipole coefficients, together with a suitable treatment of the UFA regime, are basic and indispensable pieces to analyse dark matter interactions with light or massless degrees of freedom. The calculations carried out in this paper should be of broad interest for a large number of non-standard cosmological perturbation theory scenarios.

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References

[1] C. Boehm, P. Fayet and R. Schaeffer, Constraining dark matter candidates from structure formation, *Phys. Lett.* **B518** (2001) 8 [astro-ph/0012504].

[2] C. Boehm, A. Riazuelo, S. H. Hansen and R. Schaeffer, Interacting dark matter disguised as warm dark matter, *Phys. Rev.* **D66** (2002) 083505 [astro-ph/0112522].

[3] X.-l. Chen, S. Hannestad and R. J. Scherrer, Cosmic microwave background and large scale structure limits on the interaction between dark matter and baryons, *Phys. Rev.* **D65** (2002) 123515 [astro-ph/0202496].

[4] C. Boehm, H. Mathis, J. Devriendt and J. Silk, Non-linear evolution of suppressed dark matter primordial power spectra, *Mon. Not. Roy. Astron. Soc.* **360** (2005) 282 [astro-ph/0309652].

[5] C. Boehm and R. Schaeffer, Constraints on dark matter interactions from structure formation: Damping lengths, *Astron. Astrophys.* **438** (2005) 419 [astro-ph/0410591].
6. C. Boehm, J. A. Schewtschenko, R. J. Wilkinson, C. M. Baugh and S. Pascoli, *Using the Milky Way satellites to study interactions between cold dark matter and radiation*, Mon. Not. Roy. Astron. Soc. **445** (2014) L31 [1404.7012].

7. G. Mangano, A. Melchiorri, P. Serra, A. Cooray and M. Kamionkowski, *Cosmological bounds on dark matter-neutrino interactions*, Phys. Rev. **D74** (2006) 043517 [astro-ph/0606190].

8. P. Serra, F. Zalamea, A. Cooray, G. Mangano and A. Melchiorri, *Constraints on neutrino – dark matter interactions from cosmic microwave background and large scale structure data*, Phys. Rev. **D81** (2010) 043507 [0911.4411].

9. R. J. Wilkinson, J. Lesgourgues and C. Boehm, *Using the CMB angular power spectrum to study Dark Matter-photon interactions*, JCAP **1404** (2014) 026 [1309.7588].

10. A. Schneider, *Structure formation with suppressed small-scale perturbations*, Mon. Not. Roy. Astron. Soc. **451** (2015) 3117 [1412.2133].

11. J. A. Schewtschenko, C. M. Baugh, R. J. Wilkinson, C. Bœhm, S. Pascoli and T. Sawala, *Dark matter–radiation interactions: the structure of Milky Way satellite galaxies*, Mon. Not. Roy. Astron. Soc. **461** (2016) 2282 [1512.06774].

12. M. Escudero, O. Mena, A. C. Vincent, R. J. Wilkinson and C. Bœhm, *Exploring dark matter microphysics with galaxy surveys*, JCAP **1509** (2015) 034 [1505.06735].

13. J. A. Schewtschenko, R. J. Wilkinson, C. M. Baugh, C. Bœhm and S. Pascoli, *Dark matter–radiation interactions: the impact on dark matter haloes*, Mon. Not. Roy. Astron. Soc. **449** (2015) 3587 [1412.4905].

14. Y. Ali-Ha¨ ımoud, J. Chluba and M. Kamionkowski, *Constraints on Dark Matter Interactions with Standard Model Particles from Cosmic Microwave Background Spectral Distortions*, Phys. Rev. Lett. **115** (2015) 071304 [1506.04745].

15. R. Murgia, A. Merle, M. Viel, M. Totzauer and A. Schneider, "Non-cold” dark matter at small scales: a general approach, JCAP **1711** (2017) 046 [1704.07838].

16. R. Wilkinson, *Deciphering Dark Matter with Cosmological Observations*, Ph.D. thesis, Durham U., IPPP, 2016-06-24.

17. J. A. D. Diacoumis and Y. Y. Y. Wong, *Using CMB spectral distortions to distinguish between dark matter solutions to the small-scale crisis*, JCAP **1709** (2017) 011 [1707.07050].

18. E. Di Valentino, C. Boehm, E. Hivon and F. R. Bouchet, *Reducing the H0 and σ8 tensions with Dark Matter-neutrino interactions*, Phys. Rev. **D97** (2018) 043513 [1710.02559].

19. A. Olivares-Del Campo, C. Boehm, S. Palomares-Ruiz and S. Pascoli, *Dark matter-neutrino interactions through the lens of their cosmological implications*, Phys. Rev. **D97** (2018) 075039 [1711.05283].

20. J. A. D. Diacoumis and Y. Y. Wong, *Prior dependence of cosmological constraints on dark matter-radiation interactions*, 1811.11408.

21. M. Escudero, L. Lopez-Honorez, O. Mena, S. Palomares-Ruiz and P. Villanueva-Domingo, *A fresh look into the interacting dark matter scenario*, JCAP **1806** (2018) 007 [1803.08427].

22. S. Kumar, R. C. Nunes and S. K. Yadav, *Cosmological bounds on dark matter-photon coupling*, Phys. Rev. **D98** (2018) 043521 [1803.10229].

23. J. Stadler and C. Boehm, *Constraints on γ-CDM interactions matching the Planck data precision*, JCAP **1810** (2018) 009 [1802.06589].

24. J. Stadler and C. Boehm, *Is it Mixed dark matter or neutrino masses?*, 1807.10034.

25. L. Lopez-Honorez, O. Mena and P. Villanueva-Domingo, *Dark matter microphysics and 21 cm observations*, Phys. Rev. **D99** (2019) 023522 [1811.02716].
A Interaction formalism

We present in Tab. 2 the values of \( \dot{\kappa}_{\nu DM} \) and \( \alpha_l \) which allow for a proper calculation of high-order multipoles in the neutrino-dark matter interacting sector. We exploit the classification of dark matter-neutrino interaction scenarios provided in Ref. [19].
Table 2. Possible scenarios for dark matter interactions and the corresponding matrix element, scattering cross section and coefficients for the higher multipoles in the Boltzmann equations.

| Scenario                              | Matrix Element | Scattering Rate $\tilde{\kappa}_{\nu, \text{DM}}$ | Higher Order Coefficients $\alpha_l l \geq 2$ |
|---------------------------------------|----------------|-----------------------------------------------|-----------------------------------------------|
| Complex DM                           | $g^4 \left( \sum_{s \rightarrow 0, \ell} \frac{1}{\eta_0 \eta_{\text{DM}}} |\mathcal{M}_{s\ell}\rangle^2 \right)$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Dirac mediator $N_R$                   | $g^4 \left( 2s + t - 2m_{\text{DM}}^2 \right)^2 \left[ m_{\text{DM}}^4 - 2s m_{\text{DM}} + s(s + t) \right]$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Majorana mediator $N_L$               | $g^4 \left( 2s + t - 2m_{\text{DM}}^2 \right)^2 \left[ m_{\text{DM}}^4 - 2s m_{\text{DM}} + s(s + t) \right]$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Dirac DM scalar mediator $\phi$       | $g^4 \left( s + t - m_{\text{DM}}^2 \right)^2 / \left( s + t + m_{\text{DM}}^2 \right)^2$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Majorana DM scalar mediator $\phi$    | $g^4 \left( s + t - m_{\text{DM}}^2 \right)^2 / \left( s + t + m_{\text{DM}}^2 \right)^2$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Vector DM fermionic mediator $N_L$     | $g^4 \left( s + t - m_{\text{DM}}^2 \right)^2 / \left( s + m_{\phi}^2 - 2m_{\text{DM}}^2 \right)^2$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Scalar DM vector mediator $Z_{\mu}$   | $g^4 \left( m_{\text{DM}}^2 - 2s m_{\text{DM}}^2 + s(s + t) \right)$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Dirac DM vector mediator $Z_{\mu}$    | $g^4 \left( m_{\text{DM}}^2 - s \right)^2 + 4st + 2t^2 / \left( m_{\text{DM}}^2 - t^2 \right)^2$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |
| Majorana DM vector mediator $Z_{\mu}$ | $g^4 \left( 2m_{\text{DM}}^2 + 2s^2 + 2st + t^2 - 4m_{\text{DM}}^2(s + t) \right) / (t - m_{\text{DM}}^2)^2$ | $2 g^4 p^2_{\text{DM}} (\mu + 1) / (m_{\text{DM}}^2 - m_{\text{DM}}^2)$ | $\alpha_l = 3$ |

Notes:
- $\eta_0 \eta_{\text{DM}}$ are the coupling constants of the dark matter to the standard model.
- $s = m_{\text{DM}}^2 + 2p^2_{\text{DM}}$ represents the energy transfer in the scattering process.
- $t = 2p^2_{\text{DM}}(\mu - 1)$ is the momentum transfer.
- $\tilde{\kappa}_{\nu, \text{DM}}$ is the scattering rate for dark matter interactions.
- $\alpha_l$ are the coefficients for the higher multipoles in the Boltzmann equations.