SO(10)-like Superstring Standard Model with R-parity from the Heterotic String

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Abstract. We construct a supersymmetric standard model in the context of the \( \mathbb{Z}_{12-I} \) orbifold compactification of the \( E_8 \times E'_8 \) heterotic string theory. The gauge group is \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \times [SO(10) \times U(1)]' \) with \( \sin^2 \theta_W = 3/8 \). We obtain three families of SO(10) spinor-like chiral matter states, and Higgs doublets. All other extra states are exactly vector-like under the standard model gauge symmetry. There are numerous standard model singlets, many of which get VEVs such that only the standard model gauge symmetry survives and desired Yukawa couplings can be generated at lower energies. In particular, all vector-like exotic states achieve superheavy masses and the R-parity can be preserved.

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1 Introduction

The standard model (SM) has been extremely successful in explaining most particle physics phenomena of the scale larger than \( 10^{-16} \) cm except gravity phenomena. String theory is a promising candidate for a fundamental theory, including quantum gravity as well as gauge interactions in the framework. However, the two approaches in theoretical physics, i.e. the bottom-up and top-down approaches do not meet each other. It would be an important task to connect the SM and string theory in particle physics.

The minimal supersymmetric standard model (MSSM) is one of the most promising candidates beyond the SM. It would be an effective theory appearing at an intermediate stage from the SM toward string theory. Thus, the features of the MSSM, which are listed below, are expected to play an important role of the guide for a realistic string model construction.

- Most of all, the gauge couplings in the MSSM, \( g_3, g_2, \sqrt{\frac{5}{3}} g_Y \) inferred from the RG evolutions are unified at \( 10^{16} \) GeV energy scale. Thus, the \( \sin^2 \theta_W \) (which is defined as \( g_2^2/[g_2^2+g_Y^2] \)) is \( \frac{3}{8} \) at \( 10^{16} \) GeV. It seems to imply the presence of a unified interaction described by a grand unified theory (GUT) at the \( 10^{16} \) scale. Particularly, the normalizing factor of the hypercharge, \( \frac{\sqrt{\frac{5}{3}}}{\sqrt{3}} \) could not be easily understood, were it not for GUTs such as SU(5) and SO(10) theories. It remains undetermined within SM or MSSM, even if all the gauge and gravity anomalies are considered.

- One family of chiral matter in the MSSM, which are 16 chiral superfields \( \{ Q, d^c, u^c, L, e^c, \nu^c \} \), are successfully embedded in the multiplets of GUTs. In SU(5), for instance, they are embedded in the tensor (10), (anti-)fundamental (\( \overline{5} \)), and singlet (1) representations, and in SO(10) embedded in a single spinor representation (16). Namely, \( 16 = (\pm \pm \pm \pm \pm) \) splits into \( \overline{5} + 10 + 1_5 \) under SU(5) \( \times U(1)_X \), and

\[
\begin{align*}
\overline{5} & \rightarrow \{(\pm - ; - - ) \quad d^c , \\
& \quad ( - - ; + + ) \quad L , \\
10 & \rightarrow \{(+ + ; + - ) \quad u^c , \\
& \quad (+ - ; + + ) \quad Q , \\
1_5 & \rightarrow \quad ( + + ; + + ) \quad \nu^c , 
\end{align*}
\]

where the underlined entries allow permutations.

- More matter fields, if exist, composing SU(5) multiplets leave intact the gauge coupling unification. Thus, the value of \( \sin^2 \theta_W \) at the electroweak scale (\( \approx 0.23 \)) could be a naturally predicted one in such GUTs.

The above features appearing in the MSSM seem to support the presence of GUT. The other aspects in the MSSM, however, make us doubtful of GUT.

- Unlike in chiral matter sector, the MSSM Higgs doublets are not well-embedded in GUT multiplets unless unwanted triplet partners \( \overline{3}, 3 \) are accompanied with them: \( \{ H_d, \overline{3} \} \subset \overline{5} \), \( \{ H_u, 3 \} \subset 5 \) in
SU(5), and \( \mathbf{3} \), \( \mathbf{5} \) are embedded in the vector representation \( \mathbf{10} \) in SO(10), i.e. \( \mathbf{10} = (\pm 1, 0, 0, 0) = \mathbf{5}_2 + \mathbf{5}_{-2} \) under SU(5) \( \times U(1)_X \), and

\[
\begin{align*}
\mathbf{5}_2 &= \begin{cases} 
(0,0,0;1,0) & H_d, \\
(1,0,0;0,0) & \mathbf{3},
\end{cases} \\
\mathbf{5}_{-2} &= \begin{cases} 
(0,0,0;-1,0) & H_u, \\
(-1,0,0;0,0) & \mathbf{3}.
\end{cases}
\end{align*}
\]

(2)

If the triplets \( \mathbf{3} \), \( \mathbf{3} \) remained light, the gauge coupling unification in the MSSM would be destroyed. However, making the triplets superheavy while keeping Higgs doublets light is indeed non-trivial. The doublet-triplet splitting is known to be a notorious problem in GUT.

- In a simple SU(5) or SO(10), the masses of \( d \)-type quarks and charged leptons should be the same, because \( d \)-type quark Yukawa couplings, \( Q_i H_d e_j^c \), and those of charged leptons, \( L_i H_u e_j^c \), are unified in such GUTs: They are included in \( \mathbf{10, 5, 5} \), in SU(5), and in \( \mathbf{16, 16, 10} \) in SO(10). However, the resulting mass relation \( m_d = m_t \) is not realistic at all except for the bottom and tau masses. It is difficult to avoid the relation in GUT. In many GUT models, some vector-like pairs of heavy fields and additional adjoint Higgs are introduced to construct realistic Yukawa textures.

So far we have surveyed various features of the MSSM. Some aspects in the MSSM might imply embedding of it in a GUT, but other aspects seem to require that the MSSM should be rigorously just SM-like.

Let us define the “SO(10)-like MSSM,” which reflects the features discussed above. The SO(10)-like MSSM has the following properties:

- The gauge group is just the SM gauge group, SU(3)_c \( \times SU(2)_L \times U(1)_Y \).
- But the \( \sin^2 \theta_W \) is \( \frac{1}{2} \). It is a GUT property.
- Three families of chiral matter fields are contained in the three SO(10) spinor representations.
- The two Higgs doublets are contained in the SO(10) vector representation but the triplets are decoupled due to asymmetry between doublets and triplets.
- The \( d \)-type quark and charged lepton Yukawa couplings are not summarized in a single Yukawa couplings, avoiding \( m_d = m_t \).
- For stability of the proton and LSP, the R-parity (or matter parity) is necessary.

We will suggest a model based on the heterotic string theory, realizing the SO(10)-like MSSM. The heterotic string theory provides a good framework of unification, and its structure is rich enough to accommodate the MSSM. The heterotic string theory is defined in 10D space-time, and its gauge group is \( E_6 \times E_6 \). In order to reduce the number of space dimensions and also break gauge and supersymmetry (SUSY), we will employ the orbifold compactification. The orbifold compactification is relatively simple, and so easy to discuss resulting low energy physics. Moreover, in orbifold compactification, conformal field theory is still valid, and so provides useful tools for analysis.

2 The Model

We employ the \( Z_{12-I} \) orbifold compactification, which is specified with the twist vector, \( \phi = (\frac{5}{12}, \frac{1}{12}, \frac{1}{12}) \). It is associated with the boundary conditions of the strings in the compact 6D space. This twist vector leads to \( N = 1 \) SUSY in the 4D space-time. For the boundary conditions in the gauge coordinates, we take the following form of a shift vector \( V \) and a Wilson line \( a_3 \):

\[
V = (\frac{1}{2}, \frac{1}{12}, 0; \frac{5}{12}, \frac{1}{12}, 0) \equiv (\frac{1}{2}, 0; 0, 0)^c, \\
a_3 = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}; \frac{2}{3}, \frac{2}{3}; 0, 0)^c(0, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 0)^c.
\]

They satisfy all the conditions required for modular invariance. \( 12(V^2 - \phi^2) = 12, 12a_3^2 = 48, 12V \cdot a_3 = 12 \). Since \( \frac{1}{2} \)'s are aligned in the first five components in \( V \), the visible gauge symmetry is expected to be broken into SU(5). Due to the asymmetry between the first three \( (\frac{5}{12}) \) and the next two entries \( (\frac{1}{12}) \) in \( a_3 \), SU(5) would be further broken to SU(3) \( \times SU(2) \). From the above forms of \( V \) and \( a_3 \), SO(10)'s \( (\frac{1}{2}, 0; 0, 0)^c \) is expected in the hidden sector. We will clearly see them later.

Low energy field spectrum in a model is determined by (1) the massless conditions and (2) the GSO projection. The massless conditions for the left and right movers on the orbifold \( Z_{12-I} \) are

\[
\begin{align*}
\text{left movers:} & \quad \frac{(P + kV_f)^2}{2} + \sum_i N_i^L \phi_i - c_k = 0, \\
\text{right movers:} & \quad \frac{(s + k\phi)^2}{2} + \sum_i N_i^R \phi_i - c_k = 0,
\end{align*}
\]

where \( k = 0, 1, 2, \cdots, 11, V_f = (V + m_f a_3) \) with \( m_f = 0, +1, -1 \), and \( i \) runs over \( \{1, 2, 3, 1, 2, 3\} \). Here \( \phi_i \equiv k\phi_j \mod Z \) such that \( 0 < \phi_i \leq 1 \), and \( \phi_j \equiv -k\phi_j \mod Z \) such that \( 0 < \phi_j \leq 1 \). \( N_i^L \) and \( N_i^R \) indicate oscillating numbers for the left and right movers. \( P \) and \( s \equiv (s_0, s) \) are the \( E_8 \times E_8' \) and SO(8) weight vectors, respectively. The values of \( c_k, c_k \) are found in Refs. [2, 3].

The multiplicity for a given massless state is calculated with the GSO projector in the \( Z_{12-I} \) orbifold,

\[
\mathcal{P}_k(f) = \frac{1}{12 \cdot 3} \sum_{l=0}^{11} \tilde{\chi}(\theta^k, \theta^l) e^{2\pi il\phi_k},
\]

where \( f = \{f_0, f_+, f_-\} \) denotes twist sectors associated with \( kV_f = kV, k(V + a_3), k(V - a_3) \). The phase \( \Theta_k \) is given by

\[
\Theta_k = \sum_i (N_i^L - N_i^R) \phi_i + (P + \frac{1}{2}V_f) V_f - (s + \frac{1}{2}\phi) \phi,
\]

where \( \phi_j = \phi_j \) and \( \phi_j = -\phi_j \). Here, \( \chi(\theta^k, \theta^l) \) is the degeneracy factor summarized in Ref. [3]. Note that \( \mathcal{P}_k(f_0) = \mathcal{P}_k(f_+) = \mathcal{P}_k(f_-) \) for \( k = 0, 3, 6, 9 \).

In addition, the left moving states should satisfy

\[
P \cdot a_3 = 0 \mod Z \quad \text{in the } U, T_3, T_6, T_9 \text{ sectors.} 
\]
2.1 Gauge symmetry and Weak mixing angle

The untwisted sector \((k = 0)\) contains the gauge and chiral multiplets as well as the gravity multiplet. The gauge group and gauge quantum numbers are determined from the massless left mover states. Since \(\hat{s} \cdot \phi = 0\) for the gauge and gaugino fields, only gauge multiplets satisfying \(P \cdot V = \text{integer}\) and \(P \cdot a_3 = \text{integer}\) survives the projection with \(\Theta_0 = 0 \mod \text{integer}\). The root vectors of \(E_8 \times E_8\) satisfying \(P \cdot V = \text{integer}\) and \(P \cdot a_3 = \text{integer}\) in this model are only

\[
(1, -1, 0; 0, 0, 0^3)(0^8)', \quad (0, 0, 0; 1, -1, 0^8)(0^8)',
\]

\[
(0^8)(0^8; \pm 1, \pm 1, 0, 0, 0').
\]

They are the root vectors of \(E_8 \times E_8\) satisfying the first five components only.

\[
\begin{align*}
(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}; \frac{1}{3}, \frac{1}{3}, \frac{1}{3}; 0^3)(0^8)', \\
(0, 0, 0; 1, -1, 0^8)(0^8)', \\
(0^8)(0^8; \pm 1, \pm 1, 0, 0, 0').
\end{align*}
\] (3)

They are the root vectors of \(SU(3)\), \(SU(2)\), and \(SO(10)\). Since orbifold compactification preserves the rank of \(E_8 \times E_8\), the gauge group of the model is

\[
SU(3)_c \times SU(2)_L \times U(1)_{Y} \times U(1)^4 \times [SO(10) \times U(1)]^3.
\] (4)

In this model, \(U(1)_Y\) is defined using only the first five components again:

\[
Y = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}; \frac{1}{3}, \frac{1}{3}, \frac{1}{3}; 0^3)(0^8)',
\]

which is orthogonal to all non-Abelian root vectors in Eq. (3). Thus, the SM quantum numbers of states are read only from the first five components of \(P + kV_f\):

\[
P + kV_f = (SM; x, x, x)(x, x, x, x, x, x, x, x, x, x, x, x)'.
\]

We will discuss later how the other visible gauge symmetries except the SM gauge symmetry in Eq. (4) can be broken.

The current algebra in the heterotic string theory fixes the normalization of \(Y\) such that \(U(1)_Y\) is embedded in it. Let us consider a properly normalized \(Y\):

\[
Y = x \times Y = x \times \left[\sqrt{\frac{2}{3}} \frac{g_3}{\sqrt{2}} - \frac{g_2}{\sqrt{2}}\right],
\]

where \(u\) indicates a normalization factor of the hypercharge \(Y\), and \(g_3\) and \(g_2\) are orthonormal bases: \(g_3 = \frac{1}{\sqrt{6}}(1, 1, 1; 0, 0, 0^3)(0^8)\)' and \(g_2 = \frac{1}{\sqrt{12}}(0, 0, 0; 1, 0, 1; 0^8)(0^8)'\). For \(Y\) to be embedded in the heterotic string theory, \(u\) should be fixed such that \(u^2(\frac{2}{3} + 1) = 1\) or \(u^2 = \frac{3}{2}\) [2]. This hypercharge normalization leads to the gauge coupling normalization \(g_1^2 = \frac{2}{3}g_2^2\), where \(g_1\) is unified at the string scale with the other non-Abelian gauge couplings such as the \(SU(2)_L\) gauge coupling \(g_2\). Thus, in this model the weak mixing angle at the string scale is given by

\[
\sin^2 \theta_W = \frac{1}{1 + (g_2^2/g_1^2)} = \frac{3}{8}.
\]

It reflects a GUT property that this model has.

However, there is a considerable discrepancy between the string and the GUT scales, \(A_{\text{string}}/A_{\text{GUT}} \sim 10\). It could be avoided by simply assuming the heterotic M-theory, in which \(A_{\text{string}}\) can be lowered to \(A_{\text{GUT}}\). As another possibility, one could assume the relatively large 6D space \(\sim 1/A_{\text{GUT}}\). Then the SM gauge group can be embedded in a simple group \(SU(8)\) at the GUT scale, which protects \(\sin^2 \theta_W = \frac{3}{8}\) up to the string scale.

### Table 1. Matter states from the \(U\) sector

| \(P \cdot V\) States \((P)\) | \(\chi\) | SM | \(\Gamma\) |
|---|---|---|---|
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(Q_3\) | +1 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(L_3\) | -3 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(d_3^\prime\) | +1 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(v_{\nu}^f\) | +1 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(u_{d}^f\) | -3 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(e_{d}^f\) | +5 |

\[
(0, 0, 0; 0, 0, 0; 0, 0, 0)(0^8)', \quad L \quad H_d \quad -2
\]

\[
(0, 0, 0; -1, 0, -1, 0, 0, 0)(0^8)', \quad L \quad H_u \quad +2
\]

\[
(0, 0, 0; 0, 1, 0, -1, 0, 0, 0)(0^8)', \quad L \quad 1_0 \quad 0
\]

### Table 2. Some matter states from the \(T^4_1\) sector

| States \((P + 4V)\) | \(\chi\) | \(P_4\) | SM | \(\Gamma\) |
|---|---|---|---|---|
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(2\) | \(2 \cdot d^c\) | +1 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(2\) | \(2 \cdot L\) | -3 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(2\) | \(2 \cdot e^c\) | +5 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(2\) | \(2 \cdot e^c\) | +1 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(2\) | \(2 \cdot e^c\) | -3 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(2\) | \(2 \cdot Q\) | +1 |
| \((-\cdots-;\cdots;0;0;0)(0^8)\)' | L | \(2\) | \(2 \cdot Q\) | +1 |

2.2 Chiral matter

The polarizations of matter states in the untwisted sector are in the directions of the internal 6D space, and so \(\hat{s} \cdot \phi\) for the states with the left handed chirality \((\chi = L)\) is \(\frac{\pi}{3}\), \(\frac{\pi}{3}\), or \(\frac{\pi}{3}\) mod \(Z\). For \(\Theta_0 = 0 \mod Z\), \(P \cdot V\) should be one of \(\{ \frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3} \}\) mod \(Z\). The \(E_8 \times E_8\) root vectors satisfying it and \(P \cdot a_3 = \text{integer}\) are listed in Table 1. Note that the first five entries in \(P\) of the states from the \(U_1\) and \(U_3\) sectors take exactly the form of \(SO(10)\) spinors 16 shown in Eq. (1). Thus, we get one family of the MSSM chiral matter from the untwisted sector. From the \(U_3\) sector, we have the MSSM Higgs doublets and a SM singlet. The Higgs doublets are pieces of an \(SO(10)\) vector shown in Eq. (2). However, the unwanted triplets are absent in the untwisted sector, because they can not satisfy the projection conditions.

We need two more families of the MSSM chiral matter. We find them from the \(T^4_1\) twist sector. The \(T^4_1\) sector is the sector constructed with the string states satisfying the boundary conditions, \(4 \times V\). The gauge quantum numbers \((P + 4V)\) of some massless states and the numbers of the corresponding states determined with \(P_4\) are listed in Table 2. We see again the \(SO(10)\) spinor’s structure from the first five entries in \(P + 4V\). They are identified as two copies of 16s. Hence, including one family of 16 from the untwisted sector, we have in total three families of \(16s\). Of course, there are more matter states in \(T^4_1\) and a lot of massless states including SM singlets also arise in other sectors. They all should be counted for modular invariance of the model. However, all the other matter except the
MSSM chiral fields found in the above turn out to be exactly vector-like under the SM gauge symmetry [1].

In Ref. [1], all gauge and gravity anomalies in this model have been checked out. Only one U(1)A symmetry defined with \( Q_A = (\pm 6^i; 1, 1, 1)(1, -1; 0^j) \) turns out to be anomalous. The anomaly could be cancelled via the Green-Schwarz mechanism. It is a general feature in the heterotic string theory.

### 2.3 Yukawa couplings and Desirable vacuum

In order to discuss Yukawa couplings and desired vacuum, we need to know the allowed superpotential in the string model. The superpotential are obtained from vertex operators obeying the orbifold conditions [2]. They are summarized as the following selection rules:

(a) Gauge invariance.
(b) the \( H \)-momentum, which is defined by \( R_j = (\tilde{s} + k\phi + \tilde{r}_-)_j - N_j^L + N_j^L \), should be preserved:
\[
\sum_z R_1(z) = 1 \text{ mod } 12, \quad \sum_z R_2(z) = 1 \text{ mod } 3, \quad \sum_z R_3(z) = 1 \text{ mod } 12,
\]

where \( z \) denotes the index of states participating in a vertex operator. The \( H \)-momentum conservation is a remnant of the Lorentz symmetry of the internal 6D space, and interpreted as a discrete R-symmetry in the 4D space-time.

(c) Since a discrete symmetry associated with \( Z_{12-t} \) has been introduced, the space group selection rules should be satisfied:
\[
\sum_z k(z) = 0 \text{ mod } 12, \quad \sum_z [km_f](z) = 0 \text{ mod } 3.
\]

In this model, the allowed renormalizable Yukawa couplings from the untwisted sector are
\[
U \text{ sector: } Q_3 H_u \nu^c_3, \quad L_3 H_d e^c_3, \quad L_3 H_u \nu^g_3.
\]

But the R-parity violating couplings are forbidden by the selection rules. The above allowed Yukawa couplings would be phenomenologically quite acceptable with large \( \tan \beta \) (\( \sim 50 \)) except for the missing bottom quark Yukawa coupling. The Yukawa couplings in the untwisted sector are from the 10D gauge couplings. The superpotential are obtained from untwisted sector are from the 10D gauge couplings.

Once the SM singlets develop VEVs of order \( \Lambda^2 \) and be decoupled from low energy physics, and all the needed SM Yukawa couplings are induced while the R-parity violating terms absent (or suppressed) [1].

In fact, many trials to make all exotics heavy by large \( \langle \Phi \rangle \) end up with R-parity breaking. In this model, however, it is possible to separate the SM singlets into two (or more) classes I and II such that the requirements listed above are fulfilled [1]. The SM singlets belonging to the class I develop VEVs \( \sim \Lambda_{string} \), by which all the exotics achieve superheavy masses and all the needed SM Yukawa couplings are induced with leaving intact the R-parity. On the other hand, the VEVs of the singlets in the class II are assumed to be zero (or small enough), which make it possible to define the exact (or effective) R-parity.

### 3 Conclusion

Thus, based on the heterotic string theory compactified on the \( Z_{12-t} \) orbifold, we have successfully realized the “SO(10)-like MSSM” defined in Introduction.

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\[ \sum_i \delta_{ii} = 0 \text{ mod } 12, \quad \sum_i [km_f](i) = 0 \text{ mod } 3. \]