NOTE

Local SAR compression algorithm with improved compression, speed, and flexibility

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Purpose: Local specific absorption rate (SAR) compression algorithms are essential for enabling online SAR monitoring in parallel transmission. A better compression resulting in a lower number of virtual observation points improves speed of SAR calculation for online supervision and pulse design.

Method: An iterative expansion of an existing algorithm presented by Lee et al is proposed in this work. The original algorithm is used within a loop, making use of the virtual observation points from the previous iteration as the starting subvolume, while decreasing the overestimation with each iteration. This algorithm is evaluated on the SAR matrices of three different simulated arrays.

Result: The number of virtual observation points is approximately halved with the new algorithm, while at the same time the compression time is reduced with speed-up factors of up to 2.5.

Conclusion: The new algorithm improves the original algorithm in terms of compression rate and speed.

KEYWORDS
local SAR, MRI, SAR, VOP compression, VOPs

1 INTRODUCTION

In today’s clinical MRI systems, single-channel and dual-channel transmit systems are still standard. In contrast, ultrahigh-field MRI systems often use multichannel parallel transmit RF systems to cope with the inhomogeneity introduced by the short wavelength of the RF fields.1,2 These systems generally offer more excitation flexibility at field strengths not limited to ultrahigh field3 by using arbitrary amplitudes and phases on the different transmit channels. Examples of techniques using parallel transmit systems are RF shimming,6,7 KT-points,8 2D spokes,9 3D tailored RF pulses,10 transmit SENSE,11,12 and TIAMO (time-interleaved acquisition of modes).13

Although parallel transmit systems allow alteration of the H-field (RF magnetic field), and therefore the \( B_1^+ \) field, distribution during transmission for improved excitation, the electric field is of course also altered, which changes the distribution of RF power absorption in the body tissue.
as well. In contrast, the distribution of RF power absorption in the body in single-channel systems can only be changed quantitatively, not qualitatively. To prevent damage to the subject's tissues, regulatory guidelines recommend constraints on specific absorption rate (SAR) averaged globally over the whole body (or, when appropriate, the region of the body exposed to RF fields) as well as averaged locally over any 10 g of tissue.

To ensure safe application of RF fields in MRI systems, knowledge about their distribution inside the human body is necessary. This information is taken from numerical simulations that use models of the arrays together with heterogeneous body models to calculate the fields inside the tissue. These simulations then provide maps of the SAR that can be used to determine global and 10 g-averaged SAR. In single-channel systems, this information can be used to define a maximum permissible input power for the respective transmit coil, which can be used for SAR prediction and SAR supervision. This is done by calculating the maximum local SAR for the applied field as well as the global and partial-body SAR. In multichannel systems, the situation is more complicated, as the SAR distribution varies with the varying phases and amplitudes of the transmit channels. In this case, a single scalar factor is not enough to relate the input signal to the SAR, and information about the variation of SAR with varying amplitudes and phases has to be retained. This information can be used for pulse design with SAR constraints as well as for online SAR supervision, but the number of voxels from a simulation with an anatomical body model is very high and can reach orders of $10^6$, resulting in high computational cost when calculating the local, global, and partial-body SAR.

To reduce the time for the SAR calculation, it is sensible to compress the data beforehand. For this purpose, the concept of virtual observation points (VOPs) was introduced by Eichfelder et al. The general idea is to trade the number of voxels against an overestimation of the actual local SAR by clustering the SAR matrices around a VOP that is a SAR matrix with added overestimation. This approach reduces the calculation effort from calculating the SAR over all voxels to calculating the SAR over a reduced number of VOPs, whereby the number of VOPs can be several orders of magnitude fewer.

Although the original clustering algorithm of Eichfelder and Gebhardt's original algorithm in terms of compression, it still leaves room for improvement due to the way the matrices are sorted and checked for dominance by the VOPs found previously.

We propose an algorithm that expands on Lee et al's algorithm to achieve better compression, higher speed, and increased flexibility, although there is still no proof that our approach is optimal.

## 2 Theory

Although Lee et al’s algorithm clearly outperforms Eichfelder and Gebhardt's original algorithm in terms of compression, it still leaves room for improvement due to the way the matrices are sorted and checked for dominance by the VOPs found previously.

We can illustrate this with a simple example with the full set of matrices $V_{full}$ containing three matrices in a two-channel setup:

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; M_2 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.1 \end{pmatrix}; M_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0.5 \end{pmatrix}.$$

These matrices are already ordered according to their respective highest eigenvalue in descending order. For the overestimation, we define a diagonal matrix with all of the diagonal elements equal to 5% of the highest eigenvalue:

$$D_{0.05} = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.05 \end{pmatrix}. $$

Lee et al’s algorithm initially includes $M_1$ as the first element in the subset $V_{sub}$. In the next step, it takes $M_2$ and checks whether it is dominated by $M_1 + D_{0.05}$ (meaning $M_1 + D_{0.05} - M_2$ is positive semidefinite). Because it is not, $M_2$ is included as the next element in $V_{sub}$. In the last step, the algorithm checks whether $M_3$ is dominated by the set of now two VOPs. Because there are no real, positive coefficients $c_{w,j}$ with $\sum_{w \in V_{sub}} c_{w,v} = 1$ to fulfill,

$$M_3 \leq c_{1,3}M_1 + c_{2,3}M_2 + D_{0.05},$$

where $M_3$ is also included in the subset. In the end, all three matrices have been included in the set of VOPs. However, there are solutions to fulfill:

$$M_2 \leq c_{1,2}M_1 + c_{3,2}M_3 + D_{0.05}.$$

For example,

$$\begin{pmatrix} 0.6 & 0 \\ 0 & 0.1 \end{pmatrix} \leq 0.6 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + 0.4 \begin{pmatrix} 0 & 0 \\ 0 & 0.5 \end{pmatrix} + \begin{pmatrix} 0.05 & 0 \\ 0 & 0.05 \end{pmatrix}.$$
Expressed differently: \( P = 0.6M_1 + 0.4M_3 - M_2 + D_{0.05} \) is positive semidefinite.

This implies that VOPs that are included in the set later can make VOPs included earlier redundant. This deficiency of the algorithm cannot be mitigated by simply choosing another norm (e.g., the Frobenius norm).

We propose an enhanced algorithm that can reduce the redundant VOPs by iteratively reducing the overestimation. An overview of the algorithm is shown in Figure 1. In its core, the algorithm uses Lee et al’s algorithm together with a speed enhancement proposed by Kuehne et al.\(^{21}\) It starts by sorting all matrices by their respective highest eigenvalue in decreasing order. The matrix with the highest eigenvalue is used as the first matrix in the subset \( V_{\text{sub}} \). A matrix \( D \) for the overestimation term is chosen that can, for example, be derived from global SAR,\(^{20}\) local SAR,\(^{22}\) or simply be a diagonal matrix.

Then Lee et al’s algorithm is run with Kuehne’s expansion. Instead of finishing the VOP calculation after completing a run of Lee et al’s algorithm, the resulting subset \( V_{\text{sub}} \) containing the matrices identified as VOPs (but without overestimation) is used at the start of a rerun of the algorithm with the full set of matrices, but with a lower overestimation than in the previous run. After each run of Lee et al’s algorithm, the resulting set of VOPs can be saved and represents a valid solution for the compression with its respective overestimation. This makes the algorithm flexible in that a break condition can be defined, such as a maximum number of VOPs, and the algorithm iterates until this condition is reached.

To illustrate the algorithm, we apply it to the previous example. For overestimation, we add a diagonal matrix \( D_{0.1} \) with all of the diagonal elements equal to 10% of the highest eigenvalue:

\[
D_{0.1} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}.
\]

(Figure 1) Schematic of the proposed virtual observation point (VOP) algorithm. In its core, it uses Lee et al’s algorithm\(^{20}\) with a speed enhancement proposed by Kuehne et al\(^{21}\)
If we now run Lee et al’s algorithm on the matrices, $M_2$ is dominated by $M_1 + D_{0.1}$ and only $M_3$ is now included in the subset already containing $M_1$. If we now run Lee et al’s algorithm again with $D_{0.05}$ for overestimation, but starting with the subset from the previous run with $D_{0.1}$ for overestimation, we find that $M_2$ is dominated by the already assembled subset ($M_1$ and $M_3$), and therefore not part of the VOPs.

3 | METHODS

3.1 | Coil arrays

To compare the enhanced algorithm to Lee et al’s original algorithm, the SAR matrices of three different arrays made from micro strip lines with meanders and operating at the proton resonance frequency of 7 T were used. The first array was a local eight-channel array placed directly on the body, while the second and third arrays were remotely positioned behind the bore liner in a 2 × 4 channel and 1 × 16 channel configuration, respectively (Figure 2). All simulations were performed in CST Microwave Studio 2017 (CST, Darmstadt, Germany) and considered the MR environment (ie, patient table, bore liner, gradient coil, and cryostat) as well as the coil housing for the local array.

The local array was tuned to resonance and matched to 50 Ω using a capacitor network in a co-simulation. The remote arrays were each ideally tuned and decoupled by applying a decoupling matrix consisting of lossless inductors and capacitors that interconnects the transmit elements. No decoupling matrix was applied to the local array.

A heterogeneous body model (male, 174 cm, 70 kg, tissue resolution 2 × 2 × 2 mm³) in head-first supine position with the liver-kidney region longitudinally centered in the coils was used for all arrays. The simulation domain was discretized with approximately 65 million mesh cells. Matrices for 10 g–averaged local SAR were calculated for all three setups, resulting in 7.6 million matrices for the eight-channel local array, 6.5 million matrices for the 2 × 4 channel remote array, and 6.2 million for the 1 × 16 channel remote array. Different numbers of mesh cells resulted from different meshing due to the different coil geometries.

3.2 | Algorithm implementation and calculations

The algorithms were implemented in MATLAB (The MathWorks, Natick, MA) with a high degree of vectorization and other optimizations to speed up the calculations. The software developed for this paper is openly available at sourceforge.net (https://sourceforge.net/projects/enhanced-sar-compression/). All compressions were performed on a PC with two 6-core Xeon Gold 6128 processors (Intel, Santa Clara, CA) with 192 GB of 2666-MHz RAM.

The enhanced algorithm is compared with the exact same implementation of Lee et al’s algorithm, as used in the core loop of the enhanced algorithm to minimize the influence of the implementation on timing differences between the algorithms. In both cases, the overestimation is defined by a diagonal matrix with all diagonal elements equal to a fraction of the worst-case local SAR.

The coefficients $c_{wv}$ were calculated with an iterative algorithm ([I + 1] evolution strategy) $P = \sum_{w \in V_{sub}} c_{wv} S_{v, 10g} + D - S_{v, 10g}$, with $S_{v, 10g}$ being the SAR$_{10g}$ matrix currently under test. The resulting coefficients were then used to find other matrices that are dominated.

The reduction of the overestimation was done by multiplying the overestimation of the preceding iteration by a factor $R$.

4 | RESULTS

Figure 3 shows the comparison between Lee et al’s algorithm and the enhanced algorithm for all three arrays. The
FIGURE 3  Comparison of the compression achieved with the original (Lee et al) and the enhanced algorithm for all three arrays. The left column shows the results in terms of number of VOPs; the right column shows the respective calculation time. Please note that the duration given for the calculation time of the enhanced algorithm includes the time to calculate the results for the respective higher overestimation factors, as they are intermediate results. For comparison, for the lowest overestimation, Eichfelder’s algorithm concluded with 10 788 VOPs after 362 minutes (eight-channel local array), 6270 VOPs after 527 minutes (eight-channel remote array), and 2802 VOPs after 441 minutes (16-channel remote array). Results obtained with Eichfelder’s algorithm are not provided in the plots, to increase readability.
The left column shows the resulting number of VOPs for different overestimations given in percent of the worst-case local SAR, whereas the right column shows the time necessary to calculate the results. For the two 8-channel arrays, the calculations with the enhanced algorithm were started at 5% overestimation, whereas for the 16-channel array it was started at 20% overestimation. Two different reduction factors $R$ were used.

The results show in all cases that after very few iterations the enhanced algorithm approximately halves the number of VOPs compared with the original algorithm. The smaller reduction factor $R$ performs slightly better.

The effect on calculation time in the right column of Figure 3 shows a strong dependence on the array model. Although the smaller reduction step (higher $R$ value) takes approximately twice the calculation time as Lee et al’s algorithm for the eight-channel local array, it is faster for the $2 \times 4$ channel remote array. The larger reduction step (lower $R$ value) proved to be faster in all cases. It should be noted that the duration given for the calculation time of the enhanced algorithm includes the calculation of all previous intermediate results.

Figure 4 shows scatter plots for local SAR results of the compressed data versus the results of the uncompressed data for the eight-channel local array for one million random excitation vectors. The results for the original algorithm (Figure 4A) and the proposed algorithm (Figure 4B) look very similar, apart from a barely noticeable difference: In the results of the enhanced algorithm, the points are spread a little more in the gap between the line of identity and the line for maximum absolute overestimation. The mean absolute overestimation in Figure 4A is 99.19% of the maximum overestimation, whereas in Figure 4B it is 97.86%. In all evaluated scenarios and maximum overestimations (apart from the starting point of course), the mean overestimation was found to be slightly lower for the enhanced algorithm, but only by less than 2 percentage points. No underestimation occurred in any scenario.

5 | DISCUSSION

The results show that the proposed algorithm is capable of approximately halving the number of VOPs for a certain overestimation. When the reduction factor $R$ is chosen carefully, the enhanced algorithm is also faster than the original algorithm. The reason for this is two-fold: First, a lower number of VOPs makes the optimization in Lee et al’s algorithm faster when trying to find the coefficients to dominate a certain matrix; second, the search for other dominated matrices in the step introduced by Kuehne et al finds more matrices that are dominated during the first runs of Lee et al’s algorithm, making successive runs faster.
The algorithm was started with different overestimation factors for the first iteration step because of the difference in complexity among the three models. As shown in Figure 3, the 16-channel array produced well over 100 VOPs for approximately 5% overestimation, whereas the compressions for the other models resulted in about 20. It is recommended to start with a low number of VOPs for the first run, to not accumulate too many redundant VOPs in this run.

Although only a simple diagonal matrix was used to define the overestimation in this work for simplicity reasons, the presented algorithm is compatible with methods that are more sophisticated and can further reduce the number of VOPs when considering a fixed relative overestimation.22 Because the new method is independent of the type of overestimation term used, any benefits of different overestimation terms should be cumulative. Furthermore, individual overestimation matrices might further enhance the compression and should be the subject of future investigations.

The fact that the proposed algorithm outputs a complete set of VOPs after every iteration makes it more convenient to use. If there is, for example, a limit to the number of VOPs that an online supervision system can handle, the calculation can be started with a high overestimation factor and run until the maximum number is reached. Although this was not investigated in the paper, it is possible to change the reduction factor when approaching the maximum number of VOPs, to achieve a result with a number of VOPs very close to the allowed maximum.

6 | CONCLUSIONS

The presented algorithm is an expansion to the algorithm presented by Lee et al. It significantly reduces the number of VOPs for a given overestimation, while at the same time being potentially faster. Furthermore, the algorithm has more flexibility, because it provides intermediate results that are valid VOP sets with an overestimation decreasing with each iteration step.

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DATA AVAILABILITY STATEMENT

The code that supports the findings of this study is openly available in source forge at https://sourceforge.net/projects/enhanced-sar-compression/.

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