Simple Energy Dissipation Model of Riveted Lap Joint in Aircraft Structure

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This investigation was conducted to propose an estimation method for energy dissipation in riveted lap joints. In this study, we made a simple energy dissipation model for riveted lap joint by replacing the joint part with a material that draws a hysteresis loop. This simple model does not require detailed contact analysis. Therefore, the calculation cost can be reduced. Hysteresis loops were obtained from cyclic loading tests. Coefficients were obtained by fitting the hysteresis loop assuming that the shape of the hysteresis loop is symmetric. The coefficients were applied to FEM as material properties. Then, load/displacement curves were obtained from FEM, and energy dissipation was calculated from the hysteresis loop. It was shown that the simple model in this study can reproduce the energy dissipation process with an error of less than approximately 13% in the range of the experiment.

Key Words: Structural Damping, Energy Dissipation, Rivet, Aircraft Structure

Nomenclature

$D$: energy dissipation
$f$: applied load
$u$: displacement
$C$: coefficient of load/displacement curve
$A$: cross section area of skin
$L$: gage length of extensometer
$\varepsilon$: strain
$\sigma$: stress
$\alpha$: area of slip region per unit load
$q$: frictional force per unit length
$K_{\text{skin}}$: stiffness of skin
$K_{\text{ST}}$: stiffness of stick region

Subscripts

max: maximum
min: minimum
amp: amplitude
1: loading process
2: unloading process
3: reloading process

1. Introduction

Vibration characteristics and flutter characteristics of aircraft structures are important evaluation items for confirming the safety of airplanes. Structural damping properties are used to evaluate these characteristics, but there is no method to estimate structural damping properties in the design phase. Therefore, it is necessary to estimate structural damping. In this study, we focused on the riveted lap joint because it is used most often in aircraft structures. The damping property of the riveted lap joint is characterized as energy dissipation per load cycle.

Metherell and Diller1) investigated energy dissipation in a shear lap joint subjected to axial loading. Panovko et al.2) made corrections to the coefficient of energy dissipation per cycle for a rod press-fit into a bushing under axial loading. Goodman3) studied the joint interfacial slip model described by Coulomb friction. Machida et al.4) made a new analytical model that can express energy dissipation in riveted lap joints. In this model, it was assumed that the relative slip area increases in proportion to applied load. Ito et al.5) measured energy dissipation of 3-row 3-column riveted lap joint using cyclic loading test. These studies showed that energy dissipation is proportional to the cube of applied load amplitude. From these results, energy dissipation can be estimated using Eq. (1).

$$D = \gamma f_{\text{amp}}^3$$  (1)

However, when the structural damping property is applied to finite element analysis, it is necessary to develop a method to reproduce the load/displacement curve of the energy dissipation process.

The process of energy dissipation can be reproduced by making a detailed FE-model of a riveted lap joint.6) This method is suitable for obtaining the information of a faying surface. However, this method requires a large calculation cost because the model requires detailed contact analysis. Thus, to reduce the calculation cost of FE-analysis, a simple model that can reproduce the load/displacement curve in the process of energy dissipation is required. Therefore, the purpose of this study is to reproduce the process of energy dissipation using a simple FE-model. In this study, experimental results were applied to FE-model based on Eq. (2).
\[ u_i = C_{i1}f^2 + C_{i2}f + C_{i3} \quad (i = 1-3) \] (2)

Equation (2) means that displacement can be expressed by a quadratic expression of the load. The experimental results were fitted based on Eq. (2). We confirmed that the experimental results can be fitted using Eq. (2) and applied the results to FE-model.

2. Experimental Procedure

2.1. Specimens

In this study, the type of specimen was 3-row 3-column riveted lap joint. The appearance of the specimen is shown in Fig. 1. This riveted lap joint specimen consisted of 2024-T3 clad sheet skins with a 1.27 mm thickness and NAS-1097 countersunk rivets. A rivet squeeze load of 13.25 kN was applied when the rivets and skins were assembled. Doubler suppressed the secondary bending during the chucking process.

2.2. Measurement method for load/displacement curve and energy dissipation

Thirty-cycle sine load was applied to specimens using fatigue tester (INSTRON, 88R 8502), and displacement was measured using an extensometer (INSTRON, 2620-602) with a gauge length of 140 mm. Load/displacement curves of the last 10 cycles were obtained, and energy dissipation was calculated by averaging the area of the 10-cycle hysteresis loop. The specimen used for the experiment is shown in Fig. 2. The conditions for the loading tests are listed in Table 1.

3. Finite Element Analysis

In this study, ABAQUS-StandardTM ver. 6.13 (Dassault Systems S.A.) was used for finite element analysis. To consider the problem simply, a one-dimensional truss element was used. The appearance of the model is shown in Fig. 3. The length of the model in the longitudinal direction was 500 mm, and the energy dissipation property was applied to the center of the model as the user-defined material. The length of the material that was applied to the user-defined material was 140 mm. This length was the same as the gauge length. The load/displacement curve obtained from the experiment was used as the material property (stress and strain relations). This material property was defined using “UMAT” user subroutine. The material properties of A2024-T3 were applied to other areas. The mechanical properties of A2024-T3 are listed in Table 2. The load conditions were 1) loading to \( f_{\text{max}} \), 2) unloading to \( f_{\text{min}} \), and 3) reloading to \( f_{\text{max}} \).

The user-defined material was defined as shown in Fig. 4 using UMAT. In order to reproduce the process of energy dissipation, \( \sigma_1 \) at loading process, \( \sigma_2 \) at unloading process, and \( \sigma_3 \) at reloading process were defined as shown in Fig. 4. In this code, if the strain increment (DSTRAIN) is greater than zero and stress (STRESS) is greater than or equal to \( \sigma_1(\varepsilon) \), \( \sigma_1(\varepsilon) \) is given as a material property (stress and strain relations). If the strain increment is greater than zero and stress is smaller than \( \sigma_2(\varepsilon) \), \( \sigma_2(\varepsilon) \) is given as a material property. In the other case (strain increment is smaller than zero), \( \sigma_3(\varepsilon) \) is given as a material property. Then, the gradient of \( \sigma(\varepsilon) \) (DDSDDE) is calculated, and the stress increment is calculated by multiplying the strain increment and gradient of \( \sigma(\varepsilon) \).
4. Results and Discussion

4.1. Load/displacement curve and energy dissipation

The load/displacement curve obtained from the experiment is shown in Fig. 5. The energy dissipation calculated from the hysteresis loop is shown in Fig. 6. The relationship between energy dissipation and applied load magnitude was obtained using Eq. (3).

$$ D = 3.73 \times 10^{-5} \ (f_{\text{max}} - f_{\text{min}})^3 $$  \hspace{1cm} (3)

4.2. Simple model and coefficients obtained from experiment

The displacement is given as a quadratic expression.\(^1\)\(^4\)

$$ u_1 = C_{11} f^2 + C_{12} f + C_{13} \quad (4) $$
$$ u_2 = C_{21} f^2 + C_{22} f + C_{23} \quad (5) $$
$$ u_3 = C_{31} f^2 + C_{32} f + C_{33} \quad (6) $$

In this study, it is assumed that the shapes of the curves are symmetric (i.e., absolute values of $C_{11}$, $C_{21}$, and $C_{31}$ are the same).

$$ C_{11} = -C_{21} = C_{31} \quad (7) $$

In addition, the intercepts of $u_1(f)$ and $u_2(f)$ are assumed to be zero.

$$ C_{13} = C_{23} = 0 \quad (8) $$

From $u_1(f_{\text{max}}) = u_2(f_{\text{max}})$, $C_{22}$ is expressed as follow.

$$ C_{22} = 2 C_{11} f_{\text{max}} + C_{12} \quad (9) $$

From $u_3(f_{\text{max}}) = u_3(f_{\text{min}})$ and $u_2(f_{\text{min}}) = u_3(f_{\text{min}})$, $C_{32}$ and $C_{33}$ are expressed as follows.

$$ C_{32} = -2 C_{11} f_{\text{min}} + C_{12} \quad (10) $$
$$ C_{33} = 2 C_{11} f_{\text{max}} f_{\text{min}} \quad (11) $$

Equations (4)–(11) mean that the relations between displacement and applied load in each process can be expressed by $C_{11}$, $C_{12}$, $f_{\text{max}}$, and $f_{\text{min}}$.

Transforming Eqs. (4)–(6) into the stress/strain relationship, Eqs. (12)–(14) are obtained.

$$ \varepsilon_1 = \frac{C_{11} A^2}{L} \sigma^2 + \frac{C_{12} A}{L} \sigma $$  \hspace{1cm} (12)
$$ \varepsilon_2 = -\frac{C_{11} A^2}{L} \sigma^2 + \frac{(2 C_{11} f_{\text{max}} + C_{12}) A}{L} \sigma $$  \hspace{1cm} (13)
$$ \varepsilon_3 = \frac{C_{11} A^2}{L} \sigma^2 + \frac{(-2 C_{11} f_{\text{min}} + C_{12}) A}{L} \sigma + \frac{(2 C_{11} f_{\text{max}} f_{\text{min}})}{L} $$  \hspace{1cm} (14)
In this study, the cross sectional area was 76.2 mm² and gauge length was 140 mm. The physical model of lap joint described by Coulomb friction showed that the parameters $C_{ij}$ are expressed as functions of skin stiffness, frictional force, and so on.1,3 But it is very difficult to measure contact pressure and frictional force on a faying surface experimentally. Therefore, in this study, $C_{11}$ and $C_{12}$ were determined from energy dissipation and displacement amplitude obtained from the experiment. Energy dissipation is equal to the area of the hysteresis loop. Therefore, energy dissipation is expressed as follows, using $C_{11}$.

$$D = \int_{f_{max}}^{f_{min}} u_2 df - \int_{f_{min}}^{f_{max}} u_3 df$$

$$= \frac{1}{3} C_{11}(f_{max} - f_{min})^3$$ \hspace{1cm} (15)

From Eq. (5), the displacement amplitude is expressed as follows.

$$u_{amp} = \frac{C_{21}(f_{max} - f_{min})^2 + C_{22}(f_{max} - f_{min})}{2}$$ \hspace{1cm} (16)

Thus, $C_{12}$ can be determined from Eqs. (7), (9), and (16).

$$C_{12} = \frac{2u_{amp} - C_{11}(f_{max} - f_{min})^2}{f_{max} - f_{min}}$$ \hspace{1cm} (17)

$C_{11}$ and $C_{12}$ are calculated to be $1.12 \times 10^{-2}$ mm·kN⁻² and $2.38 \times 10^{-2}$ mm·kN⁻¹ from Eqs. (3) and (16), and Eq. (17), respectively.

### 4.3. Validity of the simple model

In this section, we confirm the validity of the assumption that the shape of the hysteresis loop is symmetric and the intercepts of $u_1(f)$ and $u_2(f)$ are zero. The physical model of the lap joint, which assumed that the relative slip area increases proportional to the applied load, showed that $C_{21}$, $C_{22}$, $C_{31}$, and $C_{32}$ can be expressed as follows.4)

$$C_{21} = \frac{-\alpha_2^2 q_2}{K_{skin}}$$ \hspace{1cm} (18)

$$C_{22} = \frac{2}{K_{skin}} (\alpha + \alpha f_{max}) + \frac{1}{K_{ST}} (1 - \alpha_2 q_2)(l - 2\alpha_2 f_{max})$$ \hspace{1cm} (19)

$$C_{31} = \frac{\alpha_3^2 q_3}{K_{skin}}$$ \hspace{1cm} (20)

$$C_{32} = \frac{2}{K_{skin}} [\alpha - \alpha_3^2 q_3(f_{max} + f_{min}) + \alpha_3 f_{max}]$$

$$+ \frac{1}{K_{ST}} (1 - \alpha_3 q_3)(l - 2\alpha_3 f_{max})$$ \hspace{1cm} (21)

Therefore, $\alpha$ and $q$ can be determined from Eqs. (18)–(21). $C_{21}$, $C_{22}$, $C_{31}$, $C_{32}$, $\alpha$, and $q$ of the simple model and experiment are shown in Table 3.

The value of $\alpha$ for the simple model agreed well with experimental values. The value of $q$ for the simple model was the same order as that for the experimental values. From these results, it was confirmed that the simple model can be represented by the stick-slip model as well as experimentally. Although the shapes of the hysteresis loops are slightly different from each other when the shapes of the loops are assumed to symmetrical, the result shows that the physical phenomenon of the simple model is almost the same as that of the experiment.

It must also be confirmed whether or not the simple model can reproduce the load/displacement curve and energy dissipation in all ranges of load. The energy dissipation and displacement amplitude of each load range are shown in Table 4. The results show that the simple model can reproduce energy dissipation and the hysteresis loop well in each load range. Especially, in higher load ranges, the difference in energy dissipation between the experiment and the simple model is very small (0.8% when $f_{max} = 6.360$ kN, 0.5% when $f_{max} = 5.088$ kN). On the other hand, in lower load ranges, there is a slight difference (9.2% when $f_{max} = 3.816$ kN). This difference was caused by secondary bending. Generally, energy dissipation increases by secondary bending, which reduces contact pressure and thus causes relative slip. In this study, experimental results were obtained under different amounts of secondary bending, which caused differences between experimental results and the simple model. The results at $f_{max} = 6.360$ kN and $5.088$ kN were more greatly affected by secondary bending than the result at $3.816$ kN. If the influence of secondary bending on energy dissipation is clarified, more accurate estimation can be obtained. But this does not mean that the simple model is not useful. When the load range of FE-analysis is lower, more accurate estimation can be obtained when $C_{11}$ and $C_{12}$ are obtained from experimental data of lower load ranges.

Displacement amplitude was also reproduced well by the simple model. The differences between the experiment and the simple model when $f_{max} = 6.360$ kN, $f_{max} = 5.088$ kN, and $f_{max} = 3.816$ kN were 0.1%, 1.5%, and 2.9%, respectively.

### 4.4. Results of finite element analysis

Finite element analysis was conducted using the coefficients, $C_{11}$ and $C_{12}$, obtained using the process explained in Section 4.2. Load/displacement curves obtained from

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**Table 3.** Comparison of simple model and experiment.

| $f_{max}$ kN | $D_{experiment}$ mm·kN⁻¹ | $D_{amp, experiment}$ mm·kN⁻¹ | $D_{simple model}$ mm·kN⁻¹ | $D_{amp, simple model}$ mm·kN⁻¹ |
|--------------|--------------------------|-------------------------------|----------------------------|-------------------------------|
| 6.360        | 7.04                     | 0.0701                        | 7.00                       | 0.0700                        |
| 5.088        | 3.56                     | 0.0556                        | 3.58                       | 0.0563                        |
| 3.816        | 1.37                     | 0.0414                        | 1.51                       | 0.0425                        |

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**Table 4.** Comparison of simple model and experiment.

| $f_{max}$ kN | $D_{experiment}$ $\times 10^3$ J | $D_{amp, experiment}$ $\times 10^3$ J | $D_{simple model}$ $\times 10^3$ J | $D_{amp, simple model}$ $\times 10^3$ J |
|--------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 6.360        | 7.04                          | 0.0701                        | 7.00                          | 0.0700                        |
| 5.088        | 3.56                          | 0.0556                        | 3.58                          | 0.0563                        |
| 3.816        | 1.37                          | 0.0414                        | 1.51                          | 0.0425                        |
The finite element analysis are shown in Figs. 8–10. The energy dissipation and displacement amplitude of each load range are listed in Table 5. The differences of energy dissipation between FE-analysis and the experiment are 4.2% when $f_{\text{max}} = 6.360$ kN, 5.3% when $f_{\text{max}} = 5.088$ kN, and 13.3% when $f_{\text{max}} = 3.816$ kN. The differences of displacement amplitude between FE-analysis and the experiment are 0.3% when $f_{\text{max}} = 6.360$ kN, 0.1% when $f_{\text{max}} = 5.088$ kN, and 0.4% when $f_{\text{max}} = 3.816$ kN. The difference in energy dissipations is larger than that reported in Table 4 because of the calculation method for stress increment in this study. In this study, stress increment was calculated by multiplying the strain increment and gradient of $\sigma(e)$. This means the stress increment was calculated linearly, although the stress and strain relationship is non-linear in reality. Therefore, the difference in energy dissipation became larger than that shown in Table 4. If a smaller load increment is set, the energy dissipation obtained from FE-analysis becomes closer to that of the experiment. Thus, the hysteresis loop was reproduced using the simple FE-model with an error of less than approximately 13%.

In this study, the energy dissipation process was simply expressed by fitting the experimental results and applying

![Fig. 8. Load/displacement curve obtained from FEM ($f_{\text{max}} = 6.360$ kN): (a) whole graph and (b) partially enlarged graph.](image)

![Fig. 9. Load/displacement curve obtained from FEM ($f_{\text{max}} = 5.088$ kN): (a) whole graph and (b) partially enlarged graph.](image)

![Fig. 10. Load/displacement curve obtained from FEM ($f_{\text{max}} = 3.816$ kN): (a) whole graph and (b) partially enlarged graph.](image)

| $f_{\text{max}}$ | $D_{\text{experiment}}$ | $u_{\text{amp, experiment}}$ | $D_{\text{FEM}}$ | $u_{\text{amp, FEM}}$ |
|------------------|------------------------|-----------------------------|-----------------|---------------------|
| 6.360            | 7.04                   | 0.0701                      | 7.35            | 0.0700              |
| 5.088            | 3.56                   | 0.0556                      | 3.76            | 0.0557              |
| 3.816            | 1.37                   | 0.0414                      | 1.58            | 0.0416              |
the results to FE-model. When a riveted specimen has different configurations from those used for this study, energy dissipation should be different from the study results. For example, the energy dissipation of a countersunk riveted specimen is larger than that of a universal riveted specimen. Reducing the squeezing force increases energy dissipation because this causes relative slip to occur more easily. Furthermore, a thicker skin shows smaller energy dissipation because of suppressing secondary bending. Reducing a row of rivets increases energy dissipation because the load transferred to each rivet increases. However, the simple modeling method proposed in this study can be applied to various types of rivets only after performing cyclic loading tests under a few different loading conditions.

On the other hand, if the riveting pattern is same as used for this study (n-row 3-column specimen), the results of this study can be used because it is geometrically the same, even if the number of rivets increases.

5. Conclusion

In this study, we made a simple energy dissipation model by replacing the joint part with the material that draws the hysteresis loop.

1. Load/displacement curves in loading, unloading, and reloading processes can be represented by $C_{11}$, $C_{12}$, $f_{\text{max}}$, and $f_{\text{min}}$ if the hysteresis loop is assumed symmetrical. These can be applied to FE-model as material properties.

2. The energy dissipation process can be reproduced simply using FE-analysis with the ABAQUS user subroutine “UMAT.” Because this method does not require contact analysis, the calculation cost can be reduced.

The model in this study can reproduce the energy dissipation process with an error of less than approximately 13% in the range of the experiment.

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