The fate of Schwarzschild-de Sitter Black Holes in $f(R)$ gravity

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The semiclassical effects of antievaporating black holes can be discussed in the framework of $f(R)$ gravity. In particular, the Bousso-Hawking-Nojiri-Odintsov antievaporation instability of degenerate Schwarzschild-de Sitter black holes (the so-called Nariai space-time) leads to a dynamical increasing of black hole horizon in $f(R)$ gravity. This phenomenon causes the following transition: emitting marginally trapped surfaces become space-like surfaces before the effective Bekenstein-Hawking emission time. As a consequence, Bousso-Hawking thermal radiation cannot be emitted in an antievaporating Nariai black hole. Possible implications in cosmology and black hole physics are also discussed.

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1. INTRODUCTION

In [1], it is discussed the antievaporation effect for degenerate Schwarzschild-de Sitter black holes in the context of $f(R)$ gravity. The authors demonstrated that the black hole radius increases because of extra gravitational contribution from the extended gravitational action. In [2], the generalization to charged black holes is studied, showing the same effect. An analogous phenomena was previously studied by Bousso and Hawking [3]. Antievaporation is also discussed in the contest of bigravity in [4].

In this paper, we take into account implications of such a phenomena in a quantum semiclassical regime. One could expect that also in this case Bousso-Hawking-Nojiri-Odintsov antievaporation and Bekenstein-Hawking evaporation would be both present in $f(R)$ black holes. However, we shall demonstrate that Bekenstein-Hawking evaporation is completely suppressed by Bousso-Hawking-Nojiri-Odintsov antievaporation in the context of Nariai space-time (Nariai black holes), for a large class of $f(R)$ gravity models. This result may appear surprising. However, let us remark that the derivation of Bekenstein-Hawking radiation is based on a non-dynamical background metric. In other words, they consider black holes in the limit of infinite mass, heat capacity and static event horizon. As a consequence, their argument cannot be rigorously applied in a dynamical case. We are going to show that the Bousso-Hawking-Nojiri-Odintsov antievaporation introduces an extra focusing term into the Raychaudhuri equation. This implies that an emitting marginally trapped surface transits from a time-like to a space-like surface in a short range of time. In other words, a time-like emitting horizon surface is almost suddenly trapped into the black hole space-like interior. However, the Bekenstein-Hawking radiation cannot be emitted from space-like surface. This result is well known in literature: the original Bekenstein and Hawking argument, using Bogolubov coefficients [5, 6], tunnel effect calculations [7], or eikonal approximations [8] lead to this conclusion. As a consequence, the Bekenstein-Hawking radiation is suppressed in an antievaporating space-time. As a shown in [9–11], a pair creation in a dynamical space-time violates the energy conservation, i.e the (quantum) stress-energy tensor conservation. We will show below how similar argument can be applied for $f(R)$ gravity black holes.

The paper is organized as follows: in Section 2, we briefly review the main results on Bousso-Hawking-Nojiri-Odintsov antievaporation; Section 3 is devoted to the mechanism of Bekenstein-Hawking radiation suppression starting from the path integral formalism, developed in Section 3.1. In Section 3.2, our main argument is discussed. In Section 4, we draw conclusions and give outlooks on possible cosmological implications.

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Let us review now some basic aspects of antievaporation in $f(R)$ gravity. Details of this derivation can be found in [1]. Let us consider the $f(R)$ gravity action
\[ I = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R) + S_m \] written in units $G_N = c = 1$. The field equations are
\[ f'(R)R_{\mu\nu} - \frac{1}{2} f(R)g_{\mu\nu} - [\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box] f'(R) = 8\pi T_{\mu\nu}, \] where $T_{\mu\nu}$ is stress-energy tensor of matter. For $T_{\mu\nu} = 0$, assuming the Ricci tensor covariantly constant and proportional to $g_{\mu\nu}$, the field equations reduced to
\[ f(R) - \frac{1}{2} R f'(R) = 0. \] The Nariai space-time is a solution of Eq.(3), having the following form:
\[ ds^2 = \frac{1}{M^2} \left[ \frac{1}{\cos h^2 x} (dx^2 - dt^2) + d\Omega^2 \right], \] where $M$ is a mass scale, and the $d\Omega^2$ is the solid angle on a 2-sphere $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. The Ricci scalar of Nariai space-time is a constant being $R = 4M^2$. Dynamical aspects are achieved by perturbing the Nariai space-time. Let us assume the general expression for the metric
\[ ds^2 = e^{2\rho(x,t)} (dx^2 - dt^2) + e^{-2\phi(x,t)} d\Omega^2 \] where
\[ \rho = -\ln(M \cosh x) + \delta \rho, \quad \phi = \ln M + \delta \phi. \] Substituting these expressions into the field equations, a set of equations in $\delta \rho, \delta \phi$ is obtained. The horizon radius has the form
\[ r_H = \frac{1}{M} e^{-\phi_0 \cosh^2 \beta t}, \] with the parameterization
\[ \delta \phi = \phi_0 \cosh \omega t \cosh^2 x = \phi_0 \cosh^2 \beta t, \] and $\omega, \beta$ real parameters related each other by the field equations and the horizon definition $g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi = 0$. If $\phi_0 < 0$, $r_H$ increases, i.e. antievaporation phenomena happen.\(^1\)

According to [1], let us consider a class of $f(R)$ models
\[ f(R) = \frac{R}{2k^2} + f_2 R^2 + f_0 M^{4-2n} R^n. \] In this case, antievaporation occurs for $n > 2$ and $\zeta > -n/2$ & $\zeta < \frac{32-10n}{6}$ and for $n < 2$ and $\zeta < -\frac{n}{2}$ & $\zeta > \frac{32-10n}{6}$; where
\[ \zeta = \frac{f_2}{n-1} [2(n-2)n^2]^{\frac{1}{2-n}} \frac{1}{f_0} \frac{1}{M^{4-n}}. \] As a consequence, antievaporation is happening in a large space of parameters. In other gravity theories, extra polynomial terms in the Newtonian potential can lead to the destabilizations of geodetics, as shown in [49].

\(^1\) Another branch of solution, corresponding to $\omega, \beta$ complex parameters, leads to more exotic solutions: an infinite number of evaporating and antievaporating horizons is conjectured. This case will not be discussed in this paper.
3. THE SUPPRESSION OF BEKENSTEIN-HAWKING RADIATION

Let us discuss now the suppression of Bekenstein-Hawking radiation in $f(R)$-gravity-Nariai black holes considering theoretical approaches where such a suppression comes out\(^2\).

3.1. The Path integral approach

In general, the path integral over all Euclidean metrics and matter fields $\phi_i, \psi_j, A^\mu_k, ..$ is

$$Z_E = \int Dg D\phi_i D\psi_j DA^\mu_k e^{-I[g, \phi_i, \psi_j, A^\mu_k, ...]}$$  \hspace{1cm} (11)

where $g$ is the Euclidean metric tensor. Adopting a semiclassical limit of General Relativity, the leading terms in the action are \(^{12}\)

$$I_E = -\int_{\Sigma} \sqrt{g} d^4x \left( L_m + \frac{1}{16\pi} R \right) + \frac{1}{8\pi} \int_{\partial\Sigma} \sqrt{h} d^3x (K - K^0)$$  \hspace{1cm} (12)

where $L_m$ is the matter Lagrangian

$$L_m = \frac{Y^{ii'}}{2} g_{\mu\nu} \partial\phi^{i\mu} \partial\phi^{i'}{}^{\nu} + ...$$  \hspace{1cm} (13)

$K$ is the trace of the curvature induced on the boundary $\partial\Sigma$ of the region $\Sigma$ considered, $h$ is the metric induced on the boundary $\partial\Sigma$, and $K^0$ is the trace of the induced curvature embedded in flat space. The last term is a contribution from the boundary. We consider infinitesimal perturbations of matter and metric as $\phi = \phi_0 + \delta\phi$, $A = A_0^0 + \delta A$, (...) and $g = g_0 + \delta g$, so that

$$I[\phi, A, ..., g] = I[\phi_0, A_0, ...g_0] + I_2[\delta\phi, \delta A, ...\delta g] + \text{higher orders},$$  \hspace{1cm} (14)

$$I_2[\delta\phi, \delta A, ..., \delta g] = I_2[\delta\phi, \delta A, ...] + I_2[\delta g].$$  \hspace{1cm} (15)

$$\log Z = -I[\phi_0, A^0, ..., g_0] + \log \int D\delta\phi D\delta A (...) D\delta g e^{-I_2[\delta\phi, \delta A, ...]}$$  \hspace{1cm} (16)

In an Euclidean Schwarzschild solution, the metric has a time dimension compactified on a circle $S^1$, with periodicity $i\beta$, and

$$\beta = T^{-1} = 8\pi M$$

Here $T, M$ are the Bekenstein-Hawking temperature and mass respectively. The Euclidean Schwarzschild metric has the form

$$ds^2_E = \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right) dr^2 + r^2 d\Omega^2$$  \hspace{1cm} (17)

A convenient change of coordinates

$$x = 4M \sqrt{1 - \frac{2M}{r}}$$

leads to

$$ds^2_E = \left(\frac{x}{4M}\right)^2 + \left(\frac{r^2}{4M^2}\right)^2 dx^2 + r^2 d\Omega^2$$  \hspace{1cm} (18)

\(^2\) Our derivation is inspired to the papers \(^9\)\(^11\).
Eq. (18) has no more a singularity in $r = 2M$. The boundary $\partial \Sigma$ is $S^1 \times S^2$ where $S^2$ has a a conveniently fixed radius $r_0$. The path integral becomes a partition function of a (canonical) ensemble, with an Euclidean time related to the temperature $T = \beta^{-1}$. The leading contribution to the path integral is

$$Z_{ES} = e^{-\frac{S}{\beta}}$$

(19)

Contributions to this term are only coming from surface terms in the gravitational action, i.e. bulk geometry not contributes to Eq. (19).

The average energy (or internal energy) is

$$\langle E \rangle = -\frac{d}{d\beta}(\log Z) = \frac{\beta}{8\pi}$$

(20)

On the other hand, the free energy $F$ is related to $Z$ as

$$F = -T \log Z$$

(21)

Finally the entropy is

$$S = \beta(\log Z - \frac{d}{d\beta}(\log Z)) = \frac{\beta^2}{16\pi} = \frac{1}{4} A.$$ \hspace{1cm} (22)

As a consequence, Bekenstein-Hawking radiation can be related to the partition function as follows:

$$S = \beta(\log Z - \frac{d}{d\beta}(\log Z)) = \frac{\beta^2}{16\pi} = \frac{1}{4} A.$$ \hspace{1cm} (23)

We can reformulate the Euclidean approach in $f(R)$ gravity. The action, in semiclassical regime, is now

$$I = -\frac{1}{16\pi} \int d^4x \sqrt{-g} f(R) - \frac{1}{8\pi} \int d^4x \sqrt{h} f'(R)(K - K_0)$$

(24)

Let us assume a generic spherical symmetric static solution for $f(R)$-gravity with an Euclidean periodic time $\tau \rightarrow \tau + \beta$ where $\beta = 8\pi M$,

$$ds_E^2 = J(r)dr^2 + J(r)^{-1}d\tau^2 + r^2d\Omega^2.$$ \hspace{1cm} (25)

As in General Relativity, the leading contribution is zero from the bulk geometry. On the other hand, the boundary term has a non-zero contribution. One can evaluate the boundary integral considering suitable surface $\partial \Sigma$. In this case the obvious choice is a $S^2 \times S^4$ surface with with radius $r$ of $S^2$. We obtain

$$\int_{\partial \Sigma} d^3x \sqrt{h} f'(R)(K - K_0) = f'(R_0) \int_{\partial \Sigma} d^3x \sqrt{h}(K - K_0) = 8\pi \beta r - 12\pi \beta M - 8\pi \beta r \sqrt{1 - \frac{r_S}{r}},$$

(26)

where $r_S = 2M$ and $R_0$ is the scalar curvature of the classical black hole background. In the limit of $r \rightarrow \infty$, the resulting action, partition function and entropy are

$$I = f'(R_0) \beta^2, \quad Z_E = e^{-f'(R_0) \beta^2}, \quad S = 16\pi f'(R_0) \frac{A}{4}.$$ \hspace{1cm} (27)

See also [13]. This result seems in antithesis with our statements in the introduction: Eqs. (27) leads to a Bekenstein-Hawking-like radiation. In fact, as mentioned, a Nariai solution is nothing else but a Schwarzschild-de Sitter one with $J(r) = 1 - J(r)_{\text{Schwarzschild}} - \frac{M}{r} r^2$, with a black hole radius $r \simeq H^{-1}$ (limit of black hole mass $M \rightarrow \frac{1}{2} \Lambda^{-1/2}$), with mass scale $M = \Lambda$. However, the result (27) is based on a strong assumption on the metric (25): the gravitational action does not give dynamical evolution. For example, in Nariai solution obtained by Nojiri and Odintsov in $f(R)$-gravity, $J(r, t)$ is also a function of time: the mass parameter is a function of time $r_S(t)$. As a consequence, the result got in this section has to be considered with caution: Eq. (27) can be applied if and only if one has a spherically symmetric stationary and static solution in $f(R)$ gravity. This means that because the Birkhoff theorem cannot be, in general, applied to $f(R)$ gravity (27), non-static cases can be considered.

Let us also comment that the same entropy in (27) can be obtained by the Wald entropy charge integral. The Wald entropy is

$$S_W = -2\pi \int_{S^2} d^2x \sqrt{-h^{(2)}} \left( \frac{\delta L}{\delta R_{\mu\nu\rho\sigma}} \right)_{S^2} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} = - \frac{A}{4G_{eff}}$$

(28)
where $\hat{\epsilon}$ is the antisymmetric binormal vector to the surface $S^2$ and

$$(2\pi G_{eff})^{-1} = -\left(\frac{\delta L}{\delta R_{\mu\rho\sigma}}\right)_{S^2} \hat{\epsilon}_{\mu\nu} \hat{\epsilon}_{\rho\sigma}$$

leading to $G_{eff} = G/f'(R_0)$ [14].

However, again, this result can be applied if and only if the spherically symmetric solution is static. As discussed above, this is not the case of Nariai black holes in $f(R)$ gravity. This point deserves a further discussion.

The Euclidean path integral approach is supposed to be an Euclidean black hole inside an ideal box in thermal equilibrium with it. Furthermore, the thermodynamical limit can be applied only for systems in equilibrium, so that an approach based on statistical mechanics can be reasonably considered. However a dynamical space-time inside a box is, in general, an out-of-equilibrium system and this fact could lead to misleading results. In the next section, we will show a simple argument leading to the conclusion that black hole evaporation is suppressed by the increasing of the Nariai horizon in $f(R)$ gravity. As a consequence, a thermal equilibrium in an external ideal box at $T_{BH}$ will never be reached by a dynamical Nariai black hole.

3.2. No particles emission

Let us consider a Bekenstein-Hawking pair in a dynamical horizon. These are created nearby black hole horizon and they become real in the external gravitational background. Now, one of this pair can pass the horizon as a quantum tunnel effect, with a certain rate $\Gamma$. However, the horizon is replacing outward the previous radius because of antievaporation effect. As a consequence, the Bekenstein-Hawking pair will be trapped in the black hole interior, in a space-like surface $A_{space-like}$. From, such a space-like surface, the tunneling effect of a particle is impossible. As a consequence, the only way to escape is if $\Gamma < \Delta t$, where $\Delta t$ is the minimal effective time scale (from an external observer in a rest frame) from a $A_{time-like}$ to $A_{space-like}$ transition - from a surface on the black hole horizon $A_{time-like}$ to a surface inside the black hole horizon $A_{space-like}$. However, $\Delta t$ can also be infinitesimal of the order of $\lambda$, where $\lambda$ is the effective separation scale between the Bekenstein-Hawking pair. In fact, defining $\Delta r$ as the radius increasing with $\Delta t$, it is sufficient $\Delta r > \lambda$ in order to "eat" the Bekenstein-Hawking pair in the space-like interior. But, for black holes with a radius $r_S >> l_P$, the tunneling time is expected to be $\Gamma_{bh}^{-1} = \Delta t$. As a consequence, a realistic Bekenstein-Hawking emission is impossible for non-Planckian black holes. The same argument can be iteratively applied during all the evolution time and the external horizon. In [8–11], it was rigorously proven that the Bekenstein-Hawking radiation cannot be emitted from a space-like surface by tunneling approach [7], eikonal approach [8] and Hawking’s original derivation by Bogolubov coefficients [6].

Let us consider the situation from the energy conservation point of view. In stationary black holes, as in Schwarzschild ones in General Relativity, the black hole horizon is necessary a Killing bifurcation surface. In fact, one can define two Killing vector fields for the interior and the exterior of the black hole. In the exterior region, the Killing vector $\zeta^\mu$ is time-like, while in the interior is space-like. This aspect is crucially connected to particles energies: the energy of a particle is $E = -p_\mu \zeta^\mu$, where $p^\mu$ is the 4-momentum. As a consequence, energy is always $E > 0$ outside the horizon. While $E < 0$ inside the horizon. In the Killing horizon, a real particle creation is energetically possible. On the other hand, in the dynamical case, to define a conserved energy of a particle $E$ for a dynamical space-time is impossible, i.e it is impossible to define Killing vector fields in a dynamical space-time. As discussed above, the Bekenstein-Hawking particle-antiparticle pair will be displaced inside the horizon in a space-like region. The creation of a real particle from a space-like region is a violation of causality. In fact, it is an a-causal exchange of energy, i.e of classical information. In fact, a particle inside the horizon is inside a light-cone with a space-like axis.

As shown in [8], one can distinguish marginally outer trapped 3-surface emitting Hawking’s pair (timelike surface), from the outer non-emitting one (space-like). Let us consider the null or optical Raychaudhuri equation for null geodesic congruences:

$$\dot{\theta} = -\theta^2 - 2\hat{\sigma}_{ab} \sigma^{ab} + \hat{\omega}_{cd} \hat{\omega}_{cd} - R_{\mu\nu} k^\mu k^\nu,$$

where hat indicates that expansion, shear, twist and vorticity are defined for the transverse directions. Latin indexes are for spatial components. The Ricci tensor encodes the dynamical proprieties of gravitational field. For $f(R)$ gravity, it can be easily derived by an algebraic manipulation of field equations [2]. Let us also specify that $\dot{\theta} = \frac{d}{d\lambda} \theta$, where $\lambda$ is the affine parameter, while $k^a$ is $k^a = \frac{dx^a}{d\lambda}$, with $k^2 = 0$, and $\dot{\theta} = k^a \theta$ also defined as the relative variation of the cross sectional, is

$$\dot{\theta} = \frac{1}{A} \frac{dA}{d\lambda}.$$
From the above definition, one can define an emitting marginally outer 2-surface $A_{\text{time-like}}$ and the non-emitting inner 2-surface $A_{\text{space-like}}$. Let us call the divergence of the outgoing null geodesics $\hat{\theta}_+$ in a $S^2$-surface. With the increasing of the black hole gravitational field, $\hat{\theta}_+$ is decreasing (light is more bended). On the other hand, the divergence of ingoing null geodesics is $\hat{\theta}_- < 0$ everywhere, while $\hat{\theta}_+ > 0$ for $r > 2M$ in the Schwarzschild case. The marginally outer trapped 2-surface $A_{\text{space-like}}$ is rigorously defined as a space-like 2-sphere with

$$\hat{\theta}_+(A_{\text{space-like}}) = 0$$

As mentioned above, in a Schwarzschild black hole, the radius of the $S^2$-sphere $A_{\text{space-like}}$ is exactly equal to the Schwarzschild radius. As a consequence, $S^2$-spheres with smaller radii than $r_S = 2M$ will be trapped surfaces (TS) with $\theta(A_{\text{TS}}) < 0$.

From the 2d surfaces, one can construct a generalized definition for 3d surfaces. The dynamical horizon is a marginally outer trapped 3d surface. It is foliated by marginally trapped 2d surfaces. In particular, a dynamical horizon can be foliated by a chosen family of $S^2$ with $\theta(a)$ of a null normal vector $n_a$ vanishing while $\theta(n_a) < 0$, for each $S^2$. In particular, one can distinguish among an emitting marginally outer trapped 3d surface $A_{\text{time-like}}$ and a non-emitting one $A_{\text{time-like}}$ by their derivative of $\hat{\theta}_m$ with respect to an ingoing null tangent vector $n_a$.

$$\hat{\theta}_m(A_{\text{time-like}}) = 0, \quad \partial \hat{\theta}_m(A_{\text{time-like}})/\partial n^a > 0$$

and the non-emitting one is define as

$$\hat{\theta}_m(A_{\text{space-like}}) = 0, \quad \partial \hat{\theta}_m(A_{\text{space-like}})/\partial n^a < 0$$

Now, adopting these definitions, let us demonstrate that the antievaporation will displace the emitting marginally outer trapped 3d surface to a non-emitting space-like 3d surface. We can consider the Raychaudhuri equation associated to our problem. Let us suppose an initial condition $\theta(\lambda) > 0$ with $\lambda$ an initial value of the affine parameter $\lambda$. In the antievaporation phenomena, the null Raychaudhuri equation is bounded as

$$\frac{d\hat{\theta}}{d\lambda} < -R_{ab}k^a k^b$$

Let us consider such an equation for an infinitesimal $\Delta t$. We can expand the Schwarzschild radius in the Nariai space-time according to the definition $[7]$ for $f(R)$ gravity in the above O’Hanlon picture. It is

$$r_S = \frac{1}{M} e^{-\phi_0} - \frac{1}{M} \beta^2 e^{-\phi_0} \phi_0 t^2 + \frac{1}{6M} \beta^4 e^{-\phi_0} \phi_0 (-2 + 3\phi_0) t^4 + O(t^5)$$

where we are considering only the first 0th leading term. For $\lambda > \lambda$, it is $R_{ab}k^a k^b > C > 0$, where $C$ is a constant associated to the 0-th leading order of $R_{ab}k^a k^b$ with time. As a consequence, $\hat{\theta}$ is bounded as

$$\hat{\theta}(\lambda) < \hat{\theta}(\lambda) + C(\lambda - \lambda)$$

leading to $\hat{\theta}(\lambda) < 0$ for $\lambda > \lambda_1 + \hat{\theta}_1/C$, where $\lambda_1, \hat{\theta}_1$ are defined at a characteristic time $t_1$. Even for a small $\Delta t$, a constant 0th contribution coming from antievaporation will cause an extra effective focusing term in the Raychaudhuri equation. On the other hand, the dependence of the extra focusing term on time is exponentially growing. This formalizes the argument given above. As a consequence, an emitting marginally trapped 3d surface will exponentially evolve to a non-emitting marginally one. Bekenstein-Hawking emission is completely suppressed by this dynamical evolution because of space-like surface cannot emit thermal radiation. It is important to stress that solutions of Raychaudhuri equations are strictly related to the energy conditions. In $f(R)$ gravity, energy conditions, like the null energy condition, are generically not satisfied as shown in $[13, 16]$.

4. CONCLUSIONS AND OUTLOOKS

In this paper, Schwarzschild-de Sitter solutions of $f(R)$-gravity have been considered in view of studying their thermodynamical properties. If the Schwarzschild radius is comparable to the Hubble radius, we are dealing with Nariai black holes. In $f(R)$-gravity, Nariai solutions have been obtained by Nojiri and Odintsov $[1]$. They have shown that Nariai black holes are unstable: they antievaporate since their radii exponentially increase with time.
(measured by an external observer in a rest frame). This phenomenon is standard for a large class of $f(R)$ gravity models. We have shown that Bekenstein-Hawking radiation is turned off by antievaporation in $f(R)$ gravity. In case, the dynamical evolution of the space-time traps the emitting surfaces in the black hole space-like interior before the effective Bekenstein-Hawking emission time. In the limit of very slow black hole antievaporation process a part of Bekenstein-Hawking radiation can be emitted. However, the instability is exponentially growing, so that the Bekenstein-Hawking radiation could be emitted with a very slow antievaporation and only during a limited black hole story.

This result opens intriguing possibilities in cosmology. In fact, a Nariai black hole is a realistic solution for primordial black holes, where the Hubble radius is comparable to the primordial black hole radius. In this case, a black hole becomes a Nariai-like solution. $f(R)$-gravity has its most successful applications in cosmology, as a natural and elegant extension of General Relativity, providing a successful inflation mechanism and reconstructing an effective cosmological term; without the introduction of any extra inflaton field. $f(R)$-gravity seems also in good agreement with Planck data and large scale structure. As a consequence, to study its primordial or final black hole solutions is crucially important. In particular, Nariai primordial black holes present antievaporation instability which allow them to decay into one (or more than one) final stable vacuum state(s) that can be de-Sitter vacua. Bekenstein-Hawking radiation cannot reduce the black hole mass, but gravitational instantons can destabilize Nariai black holes, opening wormholes among the Euclidean Schwarzschild-de-Sitter space-time.

In principle, the transition probability can be evaluated. A gravitational instanton associated to such a transition has a geometrical action $I_S = \pi M^2_{Pl} R^2$, where $R = 2 M_{Pl}^2 = 2 G_N M$ is the radius, $M$ black hole mass, so that $I_S = 4 \pi M^2 / M_{Pl}^2$. Using a dilute instanton gas approximation, one can estimate a transition rate for volume unit as $\Gamma = A e^{-I_S}$, where $A \approx O(1). M^{-1} M_{Pl}^{-1}$. So, the decay rate has a form $\Gamma \approx O(1) M_{Pl}^{-2} M^{-1} \exp(-4 \pi M^2 / M_{Pl}^2)$. For a large black hole $M \approx M_{Pl}$, $\Gamma^{-1}$ is much higher than the whole universe time-life. However, for primordial black holes of mass $M \approx M_{Pl}$, one can estimate a decay rate that is only $\Gamma \approx 10^{-6}$. One can also consider multi-wormholes’ transitions from an Euclidean Nariai solution to multi white holes/de-Sitter vacua. In fact, one can ”glue” the Penrose diagram of a Nariai solution of radius $R$ with Penrose diagrams of $N$ white holes/de-Sitter of radius $R/N$. This fact could have intriguing connections with cyclic ekpyrotic cosmology: a Big Crunch/Big Bang transition can be viewed as a Nariai black hole decay into a white hole/de Sitter vacuum. However, this is a violation of the null energy condition at Planck scale, that, in general, is dynamically violated in $f(R)$ gravity. See for example [62] for a realization of a wormhole solution in $f(R)$-gravity, violating null energy conditions. $f(R)$-gravity can naturally be related to cyclic ekpyrotic cosmologies [34, 35, 36, 41], where a new universe emerges and inflates from the final Big Crunch [42]. Such cyclic Big Bounce is strongly motivated by quantum gravity approaches as string theory and loop quantum gravity. See for example [43, 46, 3]. Let us also note that in string theory, the presence of higher derivative terms, extending the Einstein-Hilbert action, can be generated by stringy instantons. For a review on stringy instantons see [47].

Stringy instantons have also other intriguing implications in particle physics as discussed in [48, 50, 59, 61]. Finally, if a classical Nariai black hole is reinterpreted as an ensemble of horizonless naked singularities, as recently suggested in [62, 63], one could relax conditions considered in the Bousso-Hawking-Nojiri-Odintsov analysis. This hypothesis deserves future investigations both from thermodynamical and cosmological points of view.

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