ULTRA-RELATIVISTIC GRAZING COLLISIONS OF BLACK HOLES

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We study gravitational wave emission, zoom-whirl behavior and the resulting spin of the remnant black hole in highly boosted collisions of equal-mass, non spinning black-hole binaries with generic impact parameter.

Keywords: Black Holes, Numerical Relativity

Introduction: Ultra-relativistic black hole (BH) scattering simulations are of high interest for a variety of areas in contemporary physics. These include attempts to resolve the hierarchy problem by adding extra dimensions, the resulting possibility to produce BHs in particle colliders and ultra high-energy cosmic ray interactions with the atmosphere, the AdS/CFT correspondence, as well as fundamental aspects of black-hole dynamics. Here we summarize results obtained for highly boosted collisions of BH binaries with generic impact parameter, see also Refs. 10,11.

Simulations: We have performed our numerical simulations using the LEAN code, which employs the moving puncture method. The code is based on the Cactus toolkit, uses CARPET mesh-refinement, a spectral solver for BH initial data and Thornburg’s AHFinderDirect. For more details see 12. We set up a coordinate system such that the BHs start on the x-axis separated by a coordinate distance d and with radial (tangential) momentum \( P_x \) \( (P_y) \). The impact parameter is \( b \equiv L/P = P_y d/P \), where \( P \) is the linear momentum of either hole and \( L \) the initial angular momentum. Our set of simulations consists of three sequences, characterized by \( d \) and the Lorentz boost \( \gamma \equiv (1 - v^2)^{-1/2} \): (1) \( \gamma = 1.520 \) \( (v = 0.753) \) and \( d/M = 174.1 \); (2) \( \gamma = 1.520 \) \( (v = 0.753) \) and \( d/M = 62.4 \); (3) \( \gamma = 2.933 \) \( (v = 0.940) \) and \( d/M = 23.1 \), where \( M \) is the total BH mass. Along each sequence we increase the impact parameter starting from the head-on limit \( b = 0 \).

Results: For all sequences we identify three distinct regimes in the \( b \) parameter
space: (i) immediate mergers, (ii) non-prompt mergers and (iii) the scattering regime where no common apparent horizon forms. These regimes are separated by two special values of $b$, the threshold of immediate merger $b^*$ and the scattering threshold $b_{\text{scat}}$. Roughly speaking, for $b < b^*$, the holes merge within the first encounter, whereas for $b^* < b < b_{\text{scat}}$ they do not, but radiate enough energy to enter a bound state that eventually results in merger. In the left panel of Fig. 1 we illustrate these regimes by plotting the trajectories of one binary member, respectively, for three simulations of sequence 1 with $b/M = 3.34, 3.39$ and $3.40$. Analysis of the entire sequences 1 and 2 shows that merger occurs only for $b/M < 3.4$, consistent with the estimate $b_{\text{crit}}/M \sim (2.5 \pm 0.5)/v$ of Shibata et al.\cite{Shibata11}. For sequence 3, however, we obtain $2.3 \lesssim b_{\text{scat}}/M \lesssim 2.4$, indicating that Ref.\cite{Shibata11} might overestimate $b_{\text{scat}}$ for large $\gamma$. We emphasize in this context the excellent agreement of our findings with the point-particle approximation\cite{point-particle} a cross section estimate from high-energy scattering off a Kerr BH with $j \simeq 0.98$ gives $b_{\text{scat}}/M \simeq 2.36$.

The notion of a threshold of immediate merger arises in the context of the geodesic limit. The argument is that this threshold should generically be accompanied by behavior akin to zoom-whirl orbits in the geodesic limit\cite{zoom-whirl}. The number of “whirls” exhibits exponential dependence on the impact parameter according to $n = C - \Gamma \ln |b - b^*|$, where $C$ is a constant and $\Gamma$ is inversely proportional to the Lyapunov instability exponent of the limiting spherical orbit; see Ref.\cite{zoom-whirl} for details.

We have analyzed sequence 1 by estimating the number of orbits $n$ in the whirl phase using (a) the puncture trajectories ($n_p$) and (b) the gravitational wave flux ($n_{GW}$); cf. Fig. 1 in\cite{point-particle}. These two estimates are shown in the center and right panel of Fig. 1 and indicate that the above mentioned relationship between $n$ and $b$ is valid with a slope $\Gamma \sim 0.2$ to within 50%.

The threshold of immediate merger $b^*$ appears to also play a special role when we consider GW emission and final spin in cases where merger occurs: both quantities show maxima for $b \sim b^*$. For $v \sim 0.75$ the radiated energy increases from $\sim 2.2\%$ of the total energy of the system for $b = 0$ to $\gtrsim 23\%$ near $b^*$. Even for

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Left panel: Puncture trajectories for an immediate merger ($b = 3.34 \, M < b^*$), a non-prompt merger ($b^* < b = 3.39 \, M < b_{\text{scat}}$) and a scattering orbit ($b = 3.40 \, M > b_{\text{scat}}$). Center and right panel: Estimated number of orbits $n_p$ and $n_{GW}$ as functions of distance from $b^*$ for immediate and non-prompt merger cases in sequence 1.}
\end{figure}
this comparatively small boost we thus exceed the maximum of $14 \pm 4\%$ reported for ultra relativistic head-on collisions\textsuperscript{10}. For sequence 3 with $v \sim 0.94$ and $b \sim b^*$ the radiated energy is $35 \pm 5\%$. Extrapolation to $v = 1$ indicates enormous luminosities $\gtrsim 0.1$ corresponding in physical units to $\sim 3.6 \times 10^{58}$ erg s$^{-1}$. Similarly we observe a maximum in the final spin of the post-merger hole near $b^*$. For sequence 2 and $2.7 \lesssim b/M \lesssim b^*/M$ we obtain a dimensionless spin parameter $j_{\text{fin}} > 0.9$. For example, $b = 3.04 M$ results in $j_{\text{fin}} = 0.96 \pm 0.03$, close to extremal and larger than any values reported so far in the literature\textsuperscript{24,25}.

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