Quantum Dots with Disorder and Interactions: A Solvable Large-$g$ Limit

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We show that problem of interacting electrons in a quantum dot with chaotic boundary conditions is solvable in the $g \to \infty$ limit, where $g$ is the dimensionless conductance of the dot. The critical point of the $g = \infty$ theory (whose location and exponent are known exactly) that separates strong and weak-coupling phases also controls a wider fan-shaped region in the coupling-$1/g$ plane, just as a quantum critical point controls the fan in at $T > 0$. The weak-coupling phase is governed by the Universal Hamiltonian and the strong-coupling phase is a disordered version of the Pomeranchuk transition in a clean Fermi liquid. Predictions are made in the various regimes for the Coulomb Blockade peak spacing distributions and Fock-space delocalization (reflected in the quasiparticle width and ground state wavefunction).

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The union of interactions and disorder in electronic systems poses a nasty problem: Techniques that work when only one or the other is present fail when they coexist. Here we discuss a class of problems involving quantum dots (QD’s) where the evil twins can be tamed, thanks to a small parameter $1/g$, $g$ being the dimensionless conductance of an open QD. The problem is interesting for experiments in the Coulomb Blockade (CB) regime$^{1-4}$.

We consider $d = 2$ dots characterized by $E_F$, the Fermi energy, $E_T$, the Thouless energy (which is the amount by which the uncertainty principle broadens the electronic energy levels in the time it takes to traverse the dot), and $\Delta$, the average single particle level spacing, with $E_F \gg E_T >> \Delta$. $E_T$ also measures the band around the Fermi energy wherein the energy levels and wave functions ($g$ in number) may be described statistically$^5$ by Random Matrix Theory (RMT)$^6,7$. The randomness here is due to the chaotic boundary conditions, with the mean free path $l$ equal to the sample size $L$, and $E_T \approx E_c \approx \hbar v_F/L$, where $E_c$ is the level width due to scattering.

Our hamiltonian is:

$$H = \sum_\alpha \psi_\alpha^\dagger \psi_\alpha \epsilon_\alpha + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\gamma \psi_\delta$$

(1)

where $\epsilon_\alpha$ are single-particle levels that obey RMT statistics, have a mean spacing $\Delta$ and range from $-g\Delta/2$ to $g\Delta/2$. In the following we will suppress spin for simplicity, pointing out how its restoration modifies various results.

Choices for $V_{\alpha\beta\gamma\delta}$ range in previous work from all of them being independent gaussian variables$^8$ to the Universal Hamiltonian$^9-11$, wherein $V_{\alpha\beta\gamma\delta} = u_0 \Delta$, (all others zero). The sole parameter $u_0$ clearly couples to $Q^2$, $Q$ being the total charge. For the spinful case a term coupling to total $S^2$ is also included.

We employ the choice made by Murthy and Mathur$^{12}$ (MM) who appeal to the Renormalization Group (RG) approach developed by one of us$^{13}$, where one integrates out modes that lie outside a narrow band centered on the Fermi energy to expose the low-energy physics. In a clean system the RG leads to Landau’s Fermi Liquid (FL) parameters$^{14}$ as fixed-point couplings$^{13}$. With disorder one expects this RG to work till we come down to $E_T \approx E_F$, still leading to FL couplings since $E_T << E_F$. At this point disorder is included exactly by switching from the $k$ basis to the disorder basis:

$$V_{\alpha\beta\gamma\delta} = \frac{\Delta}{4}$$

$$\sum_{kk'} u(\theta - \theta') \left[ \phi_{\alpha}(k') \phi_{\beta}(k) - \phi_{\alpha}(k) \phi_{\beta}(k') \right] \ast (\alpha\beta \rightarrow \delta\gamma)$$

where $\theta$ and $\theta'$ are the angles of the momentum vectors $k$ and $k'$ and $V_{\alpha\beta\gamma\delta}$ is just the transcription of the Fermi liquid interaction $V_{FL} = \frac{E_F}{g} \sum_{kk'} u(\theta - \theta') n_k n_{k'}$. We will Fourier resolve the interaction as $u(\theta - \theta') = u_0 + \sum_{m=1}^{\infty} u_m \cos [m(\theta - \theta')]$. The constant $u_0$ controls $V_{FL}$ with nonzero average $\langle V_{\alpha\beta\gamma\delta} \rangle = \frac{\Delta}{g} \sum_{kk'} u(\theta, \theta') = u_0 \Delta$.

All couplings have nonzero fluctuations given by

$$\langle V_{\alpha\beta\gamma\delta}^2 \rangle - \langle V_{\alpha\beta\gamma\delta} \rangle^2 = \frac{\Delta^2}{4g^2} \sum_{m=1}^{\infty} u_m^2.$$  (2)

Couplings with zero average are discarded in the Universal Hamiltonian$^{9-11}$. Can these couplings be relevant in the RG sense and dominate low-energy physics, despite their small size in the original Hamiltonian? To answer this, MM$^{12}$ resorted to further fermionic RG$^{15}$ within $E_T$. Their one-loop $\beta$-function yielded a critical point at $u_m^* = -1/\ln 2$ for $m > 0$. (Upon including spin the instabilities can occur in the charge or spin channels, the critical coupling being $-1/2\ln 2$ for each channel.) For $u_m > -1/\ln 2$, the flow led to the Universal Hamiltonian, while for $u_m < -1/\ln 2$ there was a runaway to strong coupling. The two phases seemed clearly related to two types of behavior in level spacings and the nature of quasiparticles. However, uncertainty surrounding the nature of the strong-coupling phase and even its very existence, predicated as it was on a one-loop calculation and a fixed point of order unity, impeded further progress.
We have verified that the theory is solvable in the limit $g \to \infty$, with $1/g$ playing the role of $1/N$ in large-$N$ theories. The key idea is illustrated by proving that the four point function $G_{\alpha\beta,\gamma\delta}$ is just a sum of repeated particle-hole bubble diagrams. Recall that in theories with interaction $V = \lambda/N(\sum \psi_i^\dagger \psi_i)(\sum \psi_j^\dagger \psi_j)$, only such iterated bubbles survive, since only they have a free sum over $N$ for each extra loop. The situation here is similar. Consider the one-loop diagrams in our theory. Each vertex carries a sum over two momenta ($k$ and $k'$ in Fig. 1a for the bare vertex), and each propagator is diagonal in the disorder eigenvalue index, but not the $k$ index. In Fig. 1b, the internal states involve a sum over terms of the form $\phi^*_\mu(k_1)\phi^*_\mu(k_2)\phi^*_\mu(k_3)\phi^*_\mu(k_4)$ Replacing the sum (over $\mu, \nu, l, m$) by the ensemble averages using

$$\langle \phi^*_\mu(k_1)\phi^*_\mu(k_2)\phi^*_\mu(k_3)\phi^*_\mu(k_4) \rangle = \frac{\delta^{12}\delta_{34}}{g^2} + O(1/g^3)$$

(and neglecting fluctuations down by $1/g$) we find that $m = 1$ and that there is a free sum over $g$ values of either one of them. (An exception occurs when either vertex involves $u_0$ which does not flow.) The reader may check that other possible one-loop particle-hole and particle-particle diagrams are down by $1/g$ because one or more of the external $k$ labels creep into the diagram. The same logic holds for higher orders. Consequently, MM’s one-loop $\beta$-function and critical exponent $\nu = 1/\beta'(u^*) = 1$ are exact and the phase transition is real. We now ask what it corresponds to.

Let us consider just one $u_m < 0$ and factorize:

$$\exp\left[\frac{\Delta}{2}\sum_{kk'} |u_m| \cos(m\theta - m\theta') \right] =$$

$$\int d\sigma \exp\left[-\frac{\sigma^2}{2|u_m|\Delta} - \bar{\psi}_\alpha \sigma \cdot M_{\alpha\beta} \psi_\beta \right]$$

where $\sigma = (\sigma^1, \sigma^2)$ has two components, as does $M$:

$$(M^1, M^2) = \sum_k \phi_\alpha^*(k) \phi_\beta(k) (\cos m\theta, \sin m\theta)$$

Integrating out the fermions we get an effective action

$$S = -\frac{\sigma^2}{2|u_m|\Delta} + Tr \ln [(i\omega - \varepsilon_\alpha)I - \sigma \cdot M]$$

where $I$ is the unit matrix. The quadratic part of the action is

$$S_0 = -\int d\tau \left[\frac{\dot{\sigma}^2}{4g^2\Delta^2} + \frac{\sigma^2}{2\Delta} \left[\frac{1}{|u_m|} - \ln 2\right] \right]$$

upon using the disorder-averaged result

$$\left\langle \sum_{\alpha\beta} N_\beta - N_\alpha \right\rangle = \frac{2}{\Delta^2} \int_0^{\frac{\Delta}{\varepsilon_1 + \varepsilon_2}} d\varepsilon_1 d\varepsilon_2 = \frac{2g^2 \ln 2}{\Delta}$$

where $N_\alpha$ is the Fermi occupation factor of level $\alpha$. The $\sigma^2$ term is valid for frequencies well below $\Delta$. (We shall not be interested in faster motion of the collective mode.)

To reconfirm the large-$g$ nature of the theory, one defines $\bar{\sigma} = \sigma/g$ and evaluates the $Tr \ln$, term by term, and finds that $S$ has a $g^2$ in front, which plays the role of $1/h$. If $1/g^2 = g'' = 0$, we are in the classical limit and spontaneous symmetry breaking is possible (even) for a single degree of freedom, while if $1/g^2 > 0$, this is impossible due to “quantum” fluctuations. If we include more $u_m$’s, the corresponding $\sigma$’s will make additive contributions to $S$. As soon as one of them breaks symmetry, the rest will not matter.

The phase diagram is shown in Figure 2. Let us first focus on $1/g = 0$. For $u_m > u^*_m$, $\langle \sigma \rangle = 0$. For $u_m < u^*_m$, $\langle \sigma \rangle \neq 0$ with a Mexican hat minimum located by balancing the quadratic term we have above with the rest of $S$. This leads to $\langle |\sigma| \rangle = C g \sqrt{|r|}$ where $C$ is a number of order unity and $r = u^{-1}_m - (u^*_m)^{-1}$. At $1/g = 0$ there are no fluctuations and $\sigma$ can sit anywhere on a circle.

What happens at $1/g > 0$ follows from its role as the pre-factor in the action of $\bar{\sigma}$: the critical point can be felt within a critical “fan” just as a $T = 0$ critical point can influence $T > 0$ physics. The fan is $V$-shaped since both $r$ and $1/g$ scale with exponent 1.

For $1/g > 0$ the symmetry of the Mexican hat valley is explicitly broken by sample-specific corrections to next order in $1/g$ yielding a unique minimum at some angle $\theta$. The stability of this minimum to “quantum” fluctuations (at nonzero $1/g$) is examined by reading off the hamiltonian for $\sigma$ from Eqn.(6) for the effective action. The angular part of $\sigma$ is governed by

$$H_\sigma = \frac{L^2}{g} + gV(\theta)$$

where $L$ is the angular momentum conjugate to $\theta$, and factors other than $g$ are suppressed. The radial part of $\langle \sigma \rangle \simeq g$ which appears in both kinetic and potential terms is represented by its average value of order $g$. The potential term dominates at large $g$ and ensures that the wavefunction is localized near the minimum of $V$. Unlike in the two phases, the critical point (and fan region) has very large fluctuations in $\sigma$: From the hamiltonian

$$H_\sigma = gP^2 + \frac{\sigma^4}{g^2}$$

we estimate the ground state wave function to have a width $O(g^{1/2})$ in $\sigma$. As the interaction increases we cross
over from the Universal Hamiltonian phase through the critical region to the symmetry-broken phase.

What does the order parameter correspond to? In a pure system when any Landau parameter $u_m < -2 \, ( \approx -1 \text{ with spin})$, the Fermi surface undergoes a Pomeranchuk shape transition. For example if $m = 2$ the shape is an oval, with the director being a Goldstone mode. The present transition is the disordered version of this, with small disorder terms making the Goldstone mode massive. (An anisotropic gap at the Fermi surface could also arise when considering instabilities in the spin channel). Note that the phenomenology described above can occur on top of a mesoscopic Stoner transition.

The phase diagram clarifies at least two issues. First, we can relate the fluctuations of $\sigma$ to “delocalization in Fock space”, which concerned the crossover of the quasiparticle width to a Breit-Wigner form as a function of its energy. Note that $\sigma$ acts as a background field for the fermions in the Hartree-Fock Hamiltonian

$$H_{HF} = \sum_{\alpha \beta} \psi^{\dagger}_{\alpha} \left[ \varepsilon_{\alpha} \delta_{\alpha \beta} + (\sigma) \cdot M_{\alpha \beta} \right] \psi_{\beta}. \quad (10)$$

Outside the critical fan $H_{HF}$ is controlled by one sharply peaked value of $<\sigma>$ (either zero or somewhere on the Mexican hat’s valley) and states are descendants of a single Slater determinant. In the fan $\sigma$ suffers large critical fluctuations. Thus we expect the ground state to go from being localized to delocalized and back to localized as we raise the coupling at fixed $g$. This re-entrance has been seen in numerics. We connect the quasiparticle width to its energy $\varepsilon$, by arguing that $\varepsilon$ plays the same role as $1/g$ or $T$, and that there will be a fan in the $u - \varepsilon$ plane. As the quasiparticle energy increases in either phase, one hits the crossover fan with its delocalized states.

Second, we can understand the dependence of the CB peak spacing distribution on $u_{m}$22,23. $H_{HF}$ (Eqn.(10)) is a sum of two random Hamiltonians whose widths can be added in quadrature to yield an effective level spacing

$$\Delta' = \sqrt{\Delta^2 + 6 <\sigma>^2 / g^2} \quad (11)$$

In the weak-coupling phase $<\sigma> \approx 0$ and there is no memory of $u_m$. In the broken symmetric phase we have $<\sigma> = C g \Delta \sqrt{r}$, and the effective level spacing increases with $r$, and thus $u_m$. There is a crossover between these two regimes in the critical fan.

Let us now briefly analyze the relevant experiments. The Sivan et al and Patel et al2 experiments are done on gated GaAs 2DEG samples, for which $r_s \approx 1 - 1.2$ in the bulk. Using the area of the sample and the fact that these dots are in the ballistic limit we can find both $\Delta$ and $E_T$, and hence find $g \approx 7 - 14$. Sivan et al2 find that the CB peak spacing is about 5 times broader than that predicted by the Universal Hamiltonian, which describes the weak coupling region of our phase diagram. However, Patel et al2 find it to be in accord with this prediction, after accounting for “experimental noise” which is determined by measuring the magnetic field asymmetry of the CB spacings (which indicates motion of the dopant atoms, or some other scrambling of the single-particle potential). Thus, the Patel et al data2 seem to lie in the weak-coupling region. Accounting for experimental noise could also put the Sivan et al data in the weak-coupling region, consistent with the equality of parameters in the two experiments.

The experiments by Simmel et al and Abusch-Magder et al2 are performed on Si quantum dots. Including valley degeneracy one finds $r_s \approx 2.2$, and $g \approx 18$. While one cannot directly relate $r_s$ to $u_m$, one expects that some $u_m$ might become more negative as $r_s$ increases. (For $r_s \geq 2$ local charge density correlations develop in the dot24, similar to the classical limit25. While a Fermi surface distortion is not a charge density wave, it enhances the susceptibility for one, and could thus be a precursor). Indeed, two signatures of the critical fan are found in this experiment3. The CB peak-spacing distribution is found to be 7-8 times wider than expected from the Universal Hamiltonian (assuming spin degeneracy). The most striking feature of the data is that the width of the CB peaks does not vanish as $T \to 0$. This is just what is expected for a system was located in the critical fan: The ground and low-lying excited states are “Fock-space delocalized”, and single-particle states are broad even at low energy.

There are many problems one can attack using our approach. The $d = 2$ disordered, interacting bulk system, which shows experimental26 and theoretical27 indications of undergoing a metal-insulator transition, with a Stonertype instability that occurs for arbitrary negative triplet $u_{0\alpha}28$, can be modeled as a $d = 2$ lattice of quantum dots. In the strong-coupling phase, each dot will have a slow collective degree of freedom, which could lead to phase-breaking and lack of coherent backscattering of quasiparticles at very low energies. Persistent current measurements in systems with a flux, which have resisted theoretical explanation29, may succumb to our treatment, as may disordered gapless superconductors exhibiting a novel metal-insulator transition30, since the eigenvalue and eigenvector statistics of their grains are expected to be governed by one of four newly discovered RMT universality classes31. Effects of finite $T$ on the CB peak spacing32 in the strong-coupling phase constitute a natural extension of our work.
In summary, by identifying a small parameter $1/g$, we have managed to control the problem of electrons subject to interactions and disorder in ballistic quantum dots. At $1/g = 0$ the (exact) saddle-point result shows that order parameter $\sigma$ acquires a nonzero average past a critical coupling, and the correlation length based on the one-loop $\beta$ the disordered version of the Pomeranchuk transition of a control fan in which $\rho$ shows that order parameter $\sigma$ describes the effective space delocalization is just $\sigma$ delocalization. The nonzero $\langle \sigma \rangle \approx O(g)$ in the strong-coupling phase is seen to explain the significant dependence of the CB peak spacing on the interaction, just as $\langle \sigma \rangle = 0$ in the weak-coupling regime makes the distribution insensitive to interaction. Finally, note that the increase of the effective level spacing leads to an enhancement of the persistent current.

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