Existence of black holes in Friedmann-Robertson-Walker universe dominated by dark energy

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We study the existence of black holes in a homogeneous and isotropic expanding Friedmann-Robertson-Walker (FRW) universe dominated by dark energy. We show that black holes can exist in such a universe by considering some specific McVittie solutions. Although these solutions violate all three energy conditions, the FRW background does satisfy the weak energy condition.

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I. INTRODUCTION

Astronomical observations indicate that about 2/3 of total energy of the universe is to be attributed to a fluid with equation of state $w < -1/3$, which drives the acceleration of universe, usually called dark energy [1]. Up to now, numerous cosmological models with dark energy have been proposed. Another fundamental physical issue is how dark energy affects the formation and evolution of black holes. Babichev et al. [2] considered the accretion of a relativistic perfect fluid onto black holes and showed that, if the expansion of the universe is dominated by phantom energy, black holes will decrease their mass due to phantom energy accretion, and tend to zero at the time of the big rip. Harada et al. [3] concluded that there is no self-similar black hole solution in a universe with a stiff fluid or scalar field or quintessence. Even some asserted that black holes might not exist in our real world, because the large negative pressure might prevent black holes from forming [4].

On the other hand, Cai and Wang [5] investigated the black hole formation from collapsing dust in the background of dark energy, and showed that the dark energy itself never collapses to form black holes, but when both the dark energy and the dust are present, black holes can be formed, due to the condensation of the dust. Similar results were obtained in [6]. Of course, cases of the most interesting are black holes in the background of our real universe. In 1933, McVittie [7] found the exact solution of Einstein’s equations for a perfect fluid, which in general describes a Schwarzschild black hole being embedded in a Friedmann-Robertson-Walker (FRW) universe. The solution was extended to the cases of charged black holes [8] and arbitrary dimensions [9], and the global structure of the solution was also investigated in some detail [10, 11]. Recall that dark energy usually violates the strong energy condition [12].

In this paper, we are going to re-examine McVittie’s solutions and show that black holes can exist even in the background of a dark energy dominated expanding universe. Specifically, in the next section, we review McVittie’s solutions and consider three special cases.

In Sec. III, we give a proof for the existence of black holes in such backgrounds, while in Sec. IV we study the evolution of the apparent horizons of the asymptotic Schwarzschild, singular, and Schwarzschild-de Sitter models, respectively. We conclude the paper with some comments in Sec. V.

II. MCVITTIE’S SOLUTIONS WITH DARK ENERGY

A. McVittie’s Solutions

McVittie’s solutions can be written in the form [7, 11]

$$ds^2 = - \left( \frac{1 - \frac{M}{2N}}{1 + \frac{M}{2N}} \right)^2 dt^2 + e^\beta \left( 1 + \frac{M}{2N} \right)^4 (dr^2 + r^2 d\Omega^2),$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2$, and

$$\beta = \beta(t), \quad M = m e^{-\frac{t}{2}},$$

with $m$ being the mass parameter. The functions $h(r)$ and $N(r)$ depend on the choice of the constant $k$, and are given, respectively, by

$$h(r) = \begin{cases} \sinh r, & k = -1, \\ r, & k = 0, \\ \sin r, & k = +1, \end{cases}$$

$$N(r) = \begin{cases} 2 \sinh \frac{r}{2}, & k = -1, \\ r, & k = 0, \\ 2 \sin \frac{r}{2}, & k = +1. \end{cases}$$

In 1994, Nolan [10, 11] studied the problem in some details, and gave the criteria for a point mass embedded in an open FRW universe. He showed that McVittie’s solution in the case $k = 0$ satisfies the criteria, but does not in the case $k = 1$. Therefore, we consider only the case $k = 0$ in present paper. In this case, the metric may be rewritten as

$$ds^2 = - \left( \frac{1 - \frac{m}{2u}}{1 + \frac{m}{2u}} \right)^2 dt^2 + e^\beta \left( 1 + \frac{m}{2u} \right)^4 (du^2 + v^2 d\Omega^2),$$
where
\[ u \equiv re^{\frac{a}{2}}. \] (2.5)

Clearly, when \( m = 0 \) the McVittie solutions reduce to the FRW one with its expansion factor given by
\[ a(t) = e^{\beta(t)/2}. \] (2.6)

Making the transformation
\[ R = u(1 + \frac{m}{2u})^2, \] (2.7)
or inversely,
\[ u = \frac{1}{2} \left[ (R - m) - \epsilon \sqrt{R(R - 2m)} \right], \] (2.8)
we find that
\[ e^\beta \left(1 + \frac{m}{2u}\right)^4 \, dr^2 = \frac{1}{4} \beta^2 R^2 dt^2 \]
\[ + \epsilon \beta R \left(1 - \frac{2m}{R}\right)^{-\frac{3}{2}} \, dt \, dR \]
\[ + \left(1 - \frac{2m}{R}\right)^{-1} \, dR^2, \] (2.9)
\[ \frac{m - 2u}{m + 2u} = \epsilon \sqrt{1 - \frac{2m}{R}}, \] (2.10)
where \( \epsilon = \pm 1 \). From the above expressions, we can see that \( \epsilon \) must be chosen such that,
\[ \epsilon = \begin{cases} +1, & u \in (0, \frac{m}{2}), \\ -1, & u \in (\frac{m}{2}, \infty), \end{cases} \] (2.11)
for \( m \neq 0 \); and
\[ \epsilon = -1, \] (2.12)
for \( m = 0 \). Note that the radius \( R \) varies over the interval \( R \in (2m, \infty) \). As shown below, the spacetime usually is singular at \( R = 2m \).

Substituting Eqs. (2.9) and (2.10) into Eq. (2.4), we obtain
\[ ds^2 = -\left(1 - \frac{2m}{R} - \frac{1}{4} \beta^2 R^2\right) \, dt^2 \]
\[ + \epsilon \beta R \left(1 - \frac{2m}{R}\right)^{-\frac{3}{2}} \, dt \, dR \]
\[ + \left(1 - \frac{2m}{R}\right)^{-1} \, dR^2 + R^2 \, d\Omega^2, \] (2.13)
where the dot denotes the derivative with respect to \( t \).

From Einstein’s field equations one can obtain the corresponding energy density and isotropic pressure,
\[ 8\pi \rho = \frac{3}{4} \beta^2, \quad 8\pi p = -\frac{3}{4} \beta^2 + \epsilon \beta \left(1 - \frac{2m}{R}\right)^{-\frac{3}{2}}. \] (2.14)

Obviously, \( R = 2m \) is an intrinsic spacetime singularity for \( \beta \neq 0 \).

From Eq. (2.14) we can see that in the present case the energy density is always non-negative. Then, the weak energy condition reduces to
\[ \rho + p \geq 0. \] (2.15)

Equation (2.14) shows that the condition is satisfied when \( \epsilon \beta \geq 0 \).

B. Special Solutions of \( m = 0 \)

The energy density and pressure of the FRW background are given by setting \( m = 0 \) in Eq. (2.14), which read
\[ 8\pi \rho_0 = \frac{3}{4} \beta^2, \quad 8\pi p_0 = -\frac{3}{4} \beta^2 - \beta. \] (2.16)

In writing the above expressions, following Eq. (2.12) we had set \( \epsilon = -1 \). As already mentioned in the introduction, the dark energy satisfies the condition \( \rho_0 + 3p_0 < 0 \). Setting
\[ \frac{1}{2} \beta^2 + \beta \equiv F^2(t), \] (2.17)
one can show that the background is always filled with dark energy, where \( F(t) \) is a real and otherwise arbitrary function. For simplicity, we choose \( F(t) \) such that
\[ F^2(t) = c\beta^2 + \delta, \] (2.18)
where \( c \) and \( \delta \) are positive constants. Then, Eq. (2.17) has the first integral,
\[ \beta^2 = \alpha + \gamma e^{-2\frac{\delta}{c}}, \] (2.19)
where
\[ \alpha = b\delta, \quad b = \frac{2}{1 - 2c}, \] (2.20)
and \( \gamma \) is a real constant. To solve Eq. (2.19), it is found convenient to distinguish the three cases: \( \alpha = 0, \alpha < 0 \) and \( \alpha > 0 \).

1. \( \alpha = 0 \)

In this case, we have \( \delta = 0 \), and \( b \) can be written as
\[ b = \frac{4}{3(1 + w)}. \] (2.21)

Then, the equation of state reads
\[ p_0 = w\rho_0, \] (2.22)
which tells us that the FRW background is linear. It can easily be shown that the solution is given by

\[
\dot{\beta} = \frac{b}{t}, \quad \ddot{\beta} = -\frac{b}{t^2}.
\] (2.23)

Although Eq. (2.23) has the same forms as that of [10], the value of \( \beta \) is different. For dark energy, the region \( w \in (-\frac{\alpha}{3}, -1) \) corresponds to the region \( b \in (2, \infty) \), while the region \( w < -1 \) corresponds to \( b < 0 \). Obviously, for \( \epsilon = -1 \) the weak energy condition is satisfied when \( w \in (-\frac{\alpha}{3}, -1) \); and for \( \epsilon = 1 \) it is satisfied when \( w < -1 \).

2. \( \alpha < 0 \)

In this case, we find that \( b < 0 \) and \( c > \frac{1}{2} \). The solution is given by

\[
\dot{\beta} = \pm \sqrt{-\alpha} \cot \left( \frac{\sqrt{-\alpha}}{b} t \right), \quad \ddot{\beta} = \pm \frac{\alpha}{b} \csc^2 \left( \frac{\sqrt{-\alpha}}{b} t \right).
\] (2.24)

From Eqs. (2.24), it is clear that, when \( \epsilon = 1 \), the weak energy condition is satisfied for \( \dot{\beta} = \sqrt{-\alpha} \cot \left( \frac{\sqrt{-\alpha}}{b} t \right) \); and when \( \epsilon = -1 \), the weak energy condition is satisfied for \( \dot{\beta} = -\sqrt{-\alpha} \cot \left( \frac{\sqrt{-\alpha}}{b} t \right) \).

3. \( \alpha > 0 \)

When \( \alpha > 0 \), we have \( b \in (2, \infty) \) and \( c \in (0, \frac{1}{2}) \). The solution then is given by

\[
\dot{\beta} = \pm \sqrt{\alpha} \coth \left( \frac{\sqrt{\alpha}}{b} t \right), \quad \ddot{\beta} = \mp \frac{\alpha}{b} \left[ \sinh \left( \frac{\sqrt{\alpha}}{b} t \right) \right]^2.
\] (2.25)

Eq. (2.25) shows that, when \( \epsilon = 1 \) the weak energy condition holds for \( \dot{\beta} = \sqrt{\alpha} \coth \left( \frac{\sqrt{\alpha}}{b} t \right) \); and when \( \epsilon = -1 \) the weak energy condition holds for \( \dot{\beta} = \sqrt{\alpha} \coth \left( \frac{\sqrt{\alpha}}{b} t \right) \).

Note that all previous discussions of the weak energy condition were based on the cases where the interaction of matter and dark energy exist, that is \( m \neq 0 \). From Eq. (2.16) we can see that, for a dark energy dominated background, the weak energy condition is satisfied if and only if \( \dot{\beta} < 0 \).

**III. EXISTENCE OF BLACK HOLES**

In the previous section, we have derived three special solutions in a homogeneous and isotropic FRW universe with dark energy. The corresponding function \( \beta(t) \) is given, respectively, by Eqs. (2.22), (2.24) and (2.23), let us now show that, when \( m \neq 0 \), these solutions represent black holes in the background of a FRW universe filled with dark energy.

According to [13, 14, 15], black holes are defined by the existence of future outer apparent horizons. To study apparent horizons, we introduce two null coordinates \( \xi^+ \) and \( \xi^- \) via the relations

\[
d\xi^+ = f \left\{ \left( 1 - \frac{2m}{R} \right)^{\frac{1}{2}} dt + \frac{1}{2} \epsilon \beta R dt \right\},
\]

\[
d\xi^- = g \left\{ \left( 1 - \frac{2m}{R} \right)^{\frac{1}{2}} dt - \frac{1}{2} \epsilon \beta R dt \right\},
\]

where \( f \) and \( g \) are functions of \( t \) and \( R \) only, and satisfy the integrability conditions,

\[
\frac{\partial^2 \xi^\pm}{\partial t \partial R} = \frac{\partial^2 \xi^\pm}{\partial R \partial t}.
\] (3.2)

Without loss of generality, we assume that they are positive,

\[
f > 0, \quad g > 0.
\] (3.3)

Then, it can be shown that both \( \xi^+ \) and \( \xi^- \) are future-pointing, and along the lines of constant \( \xi^- \) the radial coordinate \( R \) increase towards the future, while along the lines of constant \( \xi^+ \) the coordinate \( R \) decreases towards the future. In terms of \( \xi^\pm \), the metric (2.13) can be written as

\[
ds^2 = -2e^{2\sigma(-\xi^-)} d\xi^+ d\xi^- + R^2(\xi^+, \xi^-) d\Omega^2,
\] (3.4)

where

\[
\sigma(\xi^+, \xi^-) \equiv -\frac{1}{2} \ln(2fg).
\] (3.5)

Introducing the two null vectors \( \xi_{(\pm)} \equiv \frac{\partial}{\partial \xi_{(\pm)}} \), which are future-pointing, we find that \( \xi_{(\pm)} \) define two null geodesic congruences,

\[
\xi^\mu_{(\pm)} \xi^\nu_{(\pm)} = 0,
\] (3.6)

and the expansions of these congruences are given by

\[
\theta_+ \equiv \xi_{(+\mu)} g^{\mu\nu} = \frac{1}{Rf} \left[ \left( 1 - \frac{2m}{R} \right)^{\frac{1}{2}} - \frac{1}{2} \epsilon \beta R \right],
\] (3.7)

\[
\theta_- \equiv \xi_{(-\mu)} g^{\mu\nu} = -\frac{1}{Rg} \left[ \left( 1 - \frac{2m}{R} \right)^{\frac{1}{2}} + \frac{1}{2} \epsilon \beta R \right].
\] (3.8)

Following [13, 14, 15], we define that a two sphere, \( S \), of constant \( t \) and \( R \), is said to be trapped if \( \theta_+ \theta_- > 0 \), untrapped if \( \theta_+ \theta_- < 0 \), and marginal if \( \theta_+ \theta_- = 0 \).
Assuming that on the marginally trapped surfaces $S$ we have $\theta_{+}\big|_{S} = 0$, then an apparent horizon is the closure $\overline{\Sigma}$ of a three-surface $\Sigma$ foliated by the trapped surfaces $S$ on which $\theta_{-}\big|_{\Sigma} \neq 0$. It is said outer, degenerate, or inner, according to whether $\mathcal{L}_{-}\theta_{+}\big|_{\Sigma} < 0$, $\mathcal{L}_{-}\theta_{+}\big|_{\Sigma} = 0$, or $\mathcal{L}_{-}\theta_{+}\big|_{\Sigma} > 0$ , where $\mathcal{L}_{-} = \mathcal{L}_{\xi(-)}$ denotes the Lie derivative along $\xi(-)$. In addition, if $\theta_{-}\big|_{\Sigma} < 0$ then the apparent horizon is said future, and if $\theta_{-}\big|_{\Sigma} > 0$ it is said past.

Black holes are usually defined by the existence of future outer apparent horizons [13, 14, 15]. However, in a definition given by Tipler [10] the degenerate case was also included [13].

From Eqs. (3.7) and (3.8), we find that on the apparent horizons we have,

$$\left(1 - \frac{2m}{R} - \frac{1}{4} \beta^{2} R^{2}\right)_{AH} = 0. \quad (3.9)$$

Thus, so long as

$$\beta^{2} < \frac{4}{27m^{2}}, \quad (3.10)$$

Eq. (3.9) has two solutions, $R = R_{H}$ and $R = R_{C}$, where

$$R_{H} < 3m < R_{C}. \quad (3.11)$$

It can also be shown that

$$\mathcal{L}_{-}\theta_{+}\big|_{AH} = \frac{1}{4fg} \left\{ \frac{R^{2}}{4} \left(1 - \frac{3m}{R}\right) \right. \right.$$

$$- \epsilon \beta \left(1 - \frac{2m}{R}\right)^{\frac{1}{2}} \left(\frac{R^{2}}{4}\right) \right. \left.^{-\frac{1}{2}} \right\}. \quad (3.12)$$

Since black holes are defined by the existence of future outer apparent horizons, we must have $\theta_{+}\big|_{AH} = 0$, $\theta_{-}\big|_{AH} < 0$, and $\mathcal{L}_{-}\theta_{+}\big|_{AH} < 0$. From Eqs. (3.7), (3.8) and (3.12), it is clear that, if and only if

$$\epsilon \beta > 0, \quad (3.13)$$

and

$$\epsilon \beta > \frac{4}{R^{2}} \left(1 - \frac{3m}{R}\right) \left(1 - \frac{2m}{R}\right)^{\frac{1}{2}}, \quad (3.14)$$

black holes exist in the FRW universe. Note that these conditions not only apply to the dark energy dominated universe, but also to the matter-dominated universe. For $\epsilon = -1$, we find that, even there exist apparent horizons, they cannot be future outer apparent horizons in an expanding universe. This is consistent with the result obtained in [10].

Now let us show that, for $\epsilon = 1$, solutions satisfying these conditions indeed exist in an expanding universe $\beta > 0$. From Eq. (3.9), on the hypersurface $R = R_{H}$ we have

$$\dot{\beta} = \frac{2}{R_{H}} \left(1 - \frac{2m}{R_{H}}\right)^{\frac{1}{2}}. \quad (3.15)$$

Then, we find

$$\ddot{\beta} = - \frac{\dot{R}_{H}}{2 \left(1 - \frac{2m}{R_{H}}\right)} \left[ \frac{4}{R_{H}^{2}} \left(1 - \frac{3m}{R_{H}}\right) \left(1 - \frac{2m}{R_{H}}\right)^{\frac{1}{2}} \right]. \quad (3.16)$$

In all the three cases, $\alpha = 0$, $\alpha < 0$, and $\alpha > 0$, one can show that $\dot{R}_{H} < 0$. Then, considering Eq. (3.16) and the fact that $1 - \frac{3m}{R_{H}} < 0$, we find that the condition (3.14) requires

$$I_{H} = 2 \left(1 - \frac{2m}{R_{H}}\right) + \dot{R}_{H} > 0. \quad (3.17)$$

As shown in Fig. 1 and Fig. 2, solutions satisfying Eq. (3.17) indeed exist in the cases, $\alpha = 0$ and $\alpha < 0$. For the case $\alpha > 0$, Eq. (3.17) is obviously satisfied, since when $t \to \infty$, one has $R_{H} > 2m$ and $\dot{R}_{H} \to 0$. Therefore, we conclude that black holes exist even in dark energy dominated universe. It should be noted that $R_{C}$ may be interpreted as the location of the cosmological apparent horizon.
IV. EVOLUTION OF APPARENT HORIZONS

In this section, we consider the evolution of the apparent horizons for $\alpha = 0$, $\alpha < 0$ and $\alpha > 0$, respectively. All of these models have a big bang singularity. From Eq. (3.9), we find that the apparent horizons $R_H$ and $R_C$ are given by

$$R_H = \frac{4}{\sqrt{3} |\beta|} \cos \left( \frac{\Psi}{3} + \frac{\pi}{3} \right), \quad (4.1)$$

$$R_C = \frac{4}{\sqrt{3} |\beta|} \cos \left( \frac{\Psi}{3} - \frac{\pi}{3} \right), \quad (4.2)$$

where

$$\Psi = \arccos \left( \frac{3\sqrt{3}}{2} m |\beta| \right). \quad (4.3)$$

Since we are dealing with black holes in an expanding FRW universe filled with dark energy, in the following discussions we consider only the case $\dot{\beta} > 0$ and $\epsilon = 1$.

A. $\alpha = 0$

In this case, the function $\beta(t)$ is given by Eq. (2.23), from which we see that the spacetime is singular at $t = 0$, which may be interpreted as the big bang singularity [10].

As shown in Fig. 3, the two apparent horizons appear at the same moment $t = t_0 \equiv \frac{3\sqrt{3}}{2} mb$. As time increases, $R_H$ becomes smaller and smaller, while $R_C$ becomes larger and larger. When $t \to \infty$, we have $R_H \to 2m$ and $R_C \to \infty$. Therefore, this is the asymptotic Schwarzschild model.

B. $\alpha < 0$

In this model, our spacetime can be defined within $0 \leq t \leq -\frac{b}{\sqrt{1-\alpha}}$. The evolution of the apparent horizons with time are shown in Fig. 4, where $t = 0$ is a big bang singularity. The apparent horizons first develop at $t = t_0 = -\frac{b}{\sqrt{1-\alpha}} \cot^{-1}\left(\frac{2}{3m\sqrt{3\alpha}}\right)$ and immediately bifurcate. As time increases, $R_H$ decreases monotonically, while $R_C$ increases monotonically. When $t = t_s = \frac{b}{\sqrt{1-\alpha}} \coth^{-1}\left(\frac{2}{3m\sqrt{3\alpha}}\right)$, the apparent horizon $R_H$ coincides with the singularity at $R = 2m$, and $R_C$ expands to infinity. This is the asymptotically singular model. Note that, if our spacetime is defined within $-\frac{b}{\sqrt{1-\alpha}} \leq t \leq -\frac{b}{\sqrt{1-\alpha}}$, the evolution of the apparent horizons is similar to the time reversal of the previous case.

C. $\alpha > 0$

In this case, the function $\beta(t)$ is that of Eq. (2.25). As shown in Fig. 5, the apparent horizons first develop at $t = \frac{b}{\sqrt{1+\alpha}} \cot^{-1}\left(\frac{2}{3m\sqrt{3\alpha}}\right)$ and immediately bifurcate. As time increases, $R_H$ decreases, and $R_C$ increases. When $t \to \infty$, they tend asymptotically to two different constants. Consequently, the model is asymptotically Schwarzschild-de Sitter.

FIG. 3: The evolution of the apparent horizons for the asymptotic Schwarzschild model ($\alpha = 0$). The apparent horizons first develop at the moment $t = t_0 \equiv \frac{3\sqrt{3}}{2} mb$.

FIG. 4: The evolution of the apparent horizons for the asymptotically singular model ($\alpha < 0$). The apparent horizons develop at the moment $t = t_0 \equiv -\frac{b}{\sqrt{1-\alpha}} \cot^{-1}\left(\frac{2}{3m\sqrt{3\alpha}}\right)$. 

FIG. 5: The evolution of the apparent horizons for the asymptotically Schwarzschild-de Sitter model ($\alpha > 0$). The apparent horizons develop at the moment $t = \frac{b}{\sqrt{1+\alpha}} \cot^{-1}\left(\frac{2}{3m\sqrt{3\alpha}}\right)$ and immediately bifurcate.
FIG. 5: The evolution of the apparent horizons for the asymptotic Schwarzschild-de Sitter model ($\alpha > 0$). The apparent horizons develop at the moment $t = t_0 \equiv b \sqrt{\frac{\alpha}{m}} \coth^{-1} \left( \frac{2}{3m \sqrt{\alpha}} \right)$.

\[ \dot{\beta} > \frac{4}{R^2} \left( 1 - \frac{3m}{R} \right) \left( 1 - \frac{2m}{R} \right)^{\frac{1}{2}}, \]  

\[ (5.2) \]

are satisfied, black holes in a dark energy dominated expanding FRW background exist. It is interesting to note that the background of these solutions satisfies the weak energy condition. Therefore, the speculations that black holes do not exist in our universe due to the presence of dark energy is groundless.

To show the above claim explicitly, we have considered three special McWittie solutions, which are the asymptotically Schwarzschild, asymptotically singular, and asymptotically Schwarzschild-de Sitter solutions, given, respectively, by Eqs. (2.23)-(2.25) with $\dot{\beta} > 0$. For the asymptotic Schwarzschild solution, the FRW background is linear. For the asymptotically singular solution, the time of evolution of universe is finite. For the asymptotic Schwarzschild-de Sitter solution, the final form of the equation of state of the FRW background is $w = -1$.

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