Interaction mechanism of the anti-slide pile and sliding mass based on the soil arching effect

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Abstract: Anti-slide pile has been one of the most used preventive measures for the landslide treatment in the recent years. The pile spacing has a significant impact on the arching effect which develops due to the relative compressibility of soil relative to the anti-slide piles. To reveal the interaction mechanism of pile and soil, the soil stress behind the anti-slide piles was studied based on the stress analysis of a semi-infinite plate. The influencing factors, such as time, pile spacing, and pile number were considered for the soil arching effect determination. A new method for determining the maximum pile spacing was presented, based on the Mohr-Coulomb failure criterion and the ultimate equilibrium method. The stress nephogram behind the anti-slide piles was obtained, and different distributions of soil arches, including the hyperbolic arch, extended shoulder arch, inverted bell arch, and circular arch were observed to be influenced by both the pile width and spacing. For the same pile width, the additional stress of soil decreases with the increase in the pile spacing. However, for the same pile spacing, a less apparent soil arching effect was observed when the pile width was smaller. This work can provide a simple solution for determining the soil stress and maximum pile spacing for the anti-slide pile design.

1. Introduction
Anti-slide pile is one of the most efficient preventive measures for improving the stability of both excavated and natural slopes. The pile mainly bears the horizontal forces of landslide thrust and soil resistance. This kind of pile is buried in a stable stratum, and the interaction between the piles, sliding mass, and bedrock is used to resist sliding. Compared with other kinds of anti-slide structures, the anti-slide pile has many advantages; it has a large anti-slide resistance and good supporting effect; the pile has a small section and little disturbance to the stability of the sliding mass; pile groups can be...
constructed at the same time with more working surfaces, thereby reducing the interference, thus it is more convenient for excavation and construction.

In the recent decades, researchers have done numerous investigations on the mechanism of pile-soil interaction. Poulos (1968, 1980) applied elastic theory to the study of pile-soil interaction and provided coefficient charts for the engineering design. Mylonakis and Gazetas (1999) proposed a method based on the Winkler model, to analyze the dynamic response and internal forces caused by the pressure load of both the single pile and pile groups in homogeneous, non-homogeneous, and layered soil. Küçükaral and et. al. (2003) adopted an improved Özdemir's non-linear model for the coupled equations of the piles and pile groups at the nodes of interaction. Levy et al. (2007) described the Winkler soil-pile interaction model, which systematically allows coupling between the two vertical directions by using the local yield surfaces of the pile. They provided a series of parametric studies of pile lengths, cross sections, applied loads, and changes in direction from previous loads. Wei et al. (2010) assumed that the landslide thrust was a triangular distribution and deduced the stress analytical solution of the soil behind the two piles with the elastic mechanics. Li et al. (2013) took the rectangular distribution of landslide thrust as an example; the analytic solution of the stress at an arbitrary point of the soil behind three piles and the maximum pile spacing were derived. Jegatheeswaran and Muthukkumaran (2016) compared the lateral load bearing capacity of the piles in the horizontal foundation and different slopes with a finite element software, which decreased as the slope increased. Zou et al. (2019) proposed the analytical method of a single pile in two-layered non-homogeneous soil under axial and torsional combined loads. Isbuga (2020) modified the governing differential equation of pile deflection, considering the pile-soil-pile interaction, and solved each pile analytically. The research proposed the coupling equation of pile-soil interaction and the soil stress behind the pile under a certain pile number, fixed distributed load, and the soil self-weight. However, these studies did not consider the thrust on the pile body as a function of the depth of the burial and the effect of the number of piles on the soil behind the pile. In addition, the size, shape, and position of the resultant force of the thrust on the cantilever section of the pile, are functions of time but the pile-soil interaction under consideration of time has rarely been investigated.

The soil arching effect caused by pile-soil interaction is an important factor in the mechanism of the analysis of slope reinforcement. The soil arching phenomenon, as first found by Terzaghi, is based on the trap door test (Terzaghi, 1943). Handy (1985) thought that the soil arching could be described as a trajectory of minor principal stress close to the catenary. Under the action of the lateral force, the soil arching effect is an important factor in the analysis and design of the slope reinforcement mechanism. After that, many scholars have spent a lot of effort in the investigation of the soil arching effect. Chen and Martin (2002, 2005) believed that the formation and shape of the arching region are related to the pile arrangement, pile shape, and interface roughness. Shelke and Patra (2008) analysis considered the influence of pile-soil parameters, such as the pile length, the pile diameter, the soil internal friction angle, and the pile-soil friction angle on the soil arching effect. Shelke and Mishra (2010) studied the pile embedding length, pile diameter, bending angle, pile surface characteristics, the group pile structure, the group pile spacing, and the influence of the characteristics on the ultimate tensile bearing capacity of the group. Ashour and Ardalan (2012) studied the type of pile and soil, position of the pile into the slope, depth of the failure surface of the pile, diameter of the pile, and the
distance between the piles, which affected the force transmitted from the pile down to the stable soil. Zhu and Gong (2014) used the particle flow code to show that the pile spacing and cushion thickness have an important effect on the soil arching. Deka (2016) studied the influence of the pile length and the pile diameter on the lateral bearing capacity of group piles with the increase of the pile spacing. He et al. (2018) used the orthogonal method to study the influence of uniform load, the pile spacing, and interface strength parameters on the soil arching under the condition of a row of piles. According to the Mohr-Coulomb strength theory and limit equilibrium theories, Chen et al. (2020) derived a new pile spacing calculation equation, which was verified by practical engineering. Liu et al. (2021) showed through examples that factors, such as the sliding force, pile width, cohesion, and soil internal friction angle influence the arching conditions of the inclined pile-soil arch. The research above studied the influence of the soil arching formation, the pile arrangement, the soil parameters, uniform load, the pile diameter, and the pile spacing on the soil arching. Evidently, the existing research lacks a unified understanding of the mechanical analysis of the soil arching. This will affect the analysis of the soil arching characteristics and critical pile spacing. In this paper, according to the stressing feature of the piles, the analytical equation for the stress of the sliding mass behind the piles, is established to explore the influence of the pile spacing, pile width, pile number, and different vertical distances behind the pile on the soil arching effect.

The main purpose of this paper is to study the soil arching effect caused by pile-soil interaction, to establish the theoretical model of the soil arching, and to obtain the distribution of stress nephogram, the influencing factors of the soil arching, and the control equation of maximum spacing. The rest of the paper is organized as follows. In Section 2, based on elastic mechanics, the time of thrust and depth of burial are fully considered. The distributed load on the soil behind the pile is equivalent to the stress effect of the concentrated force on an arbitrary point on the horizontal plane within a certain length. Then, according to the principle of stress superposition, \( \sigma_x, \sigma_y \), and \( \tau_{xy} \) are obtained for the point. In Section 3, the stress component nephogram of \( \sigma_x, \sigma_y \), and \( \tau_{xy} \) and the \( \sigma_z \) distribution diagram of different vertical distances and pile numbers are analyzed; in addition, using the analytical equation of the sliding mass stress behind the piles, the effects of the pile spacing, pile width, and pile number on the soil arching effect between the piles, are discussed. In Section 4, according to the Mohr-Coulomb strength criterion in soil mechanics, the maximum pile spacing is obtained by using the stress analytical solution in Section 2. Finally, some conclusions and further work are reflected in Section 5.

2. Elastic stress model of sliding mass behind the piles

2.1. Mechanical model of sliding mass behind the piles
To study the interaction model between the piles and sliding mass, people generally simplify the analysis of the soil arching effect into a plane problem (Zhao and Zhai 2014; Fan et al. 2015; Liang et al. 2020). Here a unit thick layer of sliding mass at a certain depth was taken for the mechanical analysis of elastic system, as shown in Fig.1(a). In this study, the soil layer of unit thick layer at depth \( z \) below the surface of the sliding mass behind the pile is selected as the analysis object, as shown in Fig.1(b). The emphasis is to analyze the stress analytical solution at point P on the plane behind the piles.
Fig. 1. Simplified plane of the stress soil arching analysis model between the piles.

When one row of the piles is inserted in the landslide, the slope reaches a stable state. The thrust force $q_i$ acting on the back of the piles is the reaction on the sliding mass exerted by the piles, where $i$ represents the number $i$ of the pile from the left or right side of the centerline, which is denoted as the x-axis in Fig.1. Considering the symmetry of the analysis model, the pile on the far-left side of the center line is named as “pile $i$,” and the pile on the far-right side of the center line is named as “pile $i^*$,” $b$ is the pile width, and $L$ is the pile spacing.

The x-axis in Fig.1 represents the horizontal position of the piles, and the cross-section when $x=0$ represents the symmetrical center of the piles. As mentioned above, the y-axis represents the center
line of one row of the piles, indicating the direction opposite to the main sliding direction of the landslide.

Generalize the sliding mass behind the pile to an elastic mechanics model, the assumptions are as follows: soil behind the piles is homogeneous; reaction force of the piles acts on the semi-infinite soil completely; the stress of the soil layer per unit thickness is limited to the horizontal direction; it is assumed that the horizontal displacement of the piles is zero; the piles reaction has no effect on self-weight.

2.2. Stress analytic model of the sliding mass behind the piles

Xu (2006) combined the semi-inverse solution method with the Saint-Venant principle and solved in detail the $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ with the force concentrated on the boundary of the semi-infinite plane. In Fig.1(b), the action of the piles on the sliding mass can be assumed as pressure. Here, we give pressure on the boundary of the semi-infinite plane. Then, the stress distribution function of the sliding mass behind the piles in different times, different space positions considering the number of the piles, is derived by using the elastic mechanics.

According to the mechanism of the thrust load causing landslides, the effect of the cantilever section of the pile $i$ on the soil behind the pile is $q_i$, and the analysis model is shown in Fig.2.

\[ dF = q_i d\zeta \]

Fig.2. Distributed load analysis model for the semi-infinite plane boundary.

Xu (2006) proposed the $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ formula for the semi-infinite plane under the force on the boundary:

\[
\begin{align*}
\sigma_x &= \frac{2}{\rho} D \cos \varphi = -\frac{2F}{\pi \rho} \cos \varphi \\
\sigma_y &= 0 \\
\tau_{xy} &= 0
\end{align*}
\]

where $D$ is the parameter, $\rho$ is the polar radius, $\varphi$ is the polar angle, and $F$ is the force.
By means of material mechanics oblique section stress formula, the stress components represented by the polar coordinates are transformed into the Cartesian coordinates:

\[
\begin{align*}
\sigma_x &= \sigma_\rho \cos^2 \varphi - \frac{2F}{\pi \rho} \cos^3 \varphi \\
\sigma_y &= \sigma_\rho \sin^2 \varphi - \frac{2F}{\pi \rho} \sin^2 \varphi \cos \varphi \\
\tau_{xy} &= \sigma_\rho \sin \varphi \cos \varphi - \frac{2F}{\pi \rho} \sin \varphi \cos^2 \varphi
\end{align*}
\]  

(2)

By transforming formula (2) into the Cartesian coordinates, the analytical solutions of stress at the point in the semi-infinite plane is expressed as follows:

\[
\begin{align*}
\sigma_x &= -\frac{2F}{\pi} \frac{x^3}{(x^2 + y^2)^3} \\
\sigma_y &= -\frac{2F}{\pi} \frac{xy^2}{(x^2 + y^2)^3} \\
\tau_{xy} &= -\frac{2F}{\pi} \frac{x^2y}{(x^2 + y^2)^3}
\end{align*}
\]  

(3)

where \( x \) is the vertical distance from \( F \) to the point, and \( y \) is the horizontal distance from \( F \) to the point.

According to the analytical solutions of stress at the point in the semi-infinite plane (3), the stress caused by the force \( dF \) of the small segment on the point \( P \) is expressed as follows:

\[
\begin{align*}
d\sigma_x &= -\frac{2q_i \, d\xi}{\pi} \frac{x^3}{(x^2 + (y - \xi)^2)^3} \\
d\sigma_y &= -\frac{2q_i \, d\xi}{\pi} \frac{xy^2}{(x^2 + (y - \xi)^2)^3} \\
d\tau_{xy} &= -\frac{2q_i \, d\xi}{\pi} \frac{x^2y}{(x^2 + (y - \xi)^2)^3}
\end{align*}
\]  

(4)

where \( q_i \) is horizontal stress, and \( \xi \) is the distance from \( dF \) to the origin.

For the integral of the pile width (\( AB \) section), the pressure can cause stress at the point \( P \) is expressed as follows:

\[
\begin{align*}
\sigma_x &= -\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{q \, x^3 \, d\xi}{(x^2 + (y - \xi)^2)^3} \\
\sigma_y &= -\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{q \, xy^2 \, d\xi}{(x^2 + (y - \xi)^2)^3} \\
\tau_{xy} &= -\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{q \, x^2y \, d\xi}{(x^2 + (y - \xi)^2)^3}
\end{align*}
\]  

(5)
\[
\begin{align*}
\sigma_x &= \frac{q}{\pi} \left[ \arctan \frac{y + b}{x} - \arctan \frac{y - a}{x} + \frac{x(y + b)}{x^2 + (y + b)^2} - \frac{x(y - a)}{x^2 + (y - a)^2} \right] \\
\sigma_y &= \frac{q}{\pi} \left[ \arctan \frac{y + b}{x} - \arctan \frac{y - a}{x} + \frac{x(y + b)}{x^2 + (y + b)^2} + \frac{x(y - a)}{x^2 + (y - a)^2} \right] \\
\tau_{xy} &= \frac{q}{\pi} \left[ \frac{x^2}{x^2 + (y + b)^2} - \frac{x^2}{x^2 + (y - a)^2} \right] 
\end{align*}
\]

According to the plane model in Fig.1, assuming that the total number of piles is \( n \), the coordinate transformation of formula (6) can be obtained:

\[
\begin{align*}
\sigma_x' &= \frac{q}{\pi} \left[ \arctan \frac{2(x + m) + (3 - 2i)L + b}{2y} - \arctan \frac{2(x + m) + (3 - 2i)L - b}{2y} + k_i \right] \\
\sigma_y' &= \frac{q}{\pi} \left[ \arctan \frac{2(x + m) + (3 - 2i)L + b}{2y} - \arctan \frac{2(x + m) + (3 - 2i)L - b}{2y} - k_i \right] \\
\tau_{xy}' &= \frac{q}{\pi} \left[ \frac{-16y'b(2(x + m) + (3 - 2i)L)}{(2(x + m) + (3 - 2i)L)^2 + 4y^2 - b^2} + 16y'b^3 \right] \\
k_i &= \frac{4y^3b^2(2(x + m) + (3 - 2i)L)^2 - 4y^2b^2}{(2(x + m) + (3 - 2i)L)^2 + 4y^2 - b^2} + 16y'b^3 \\
m &= \frac{1}{2} \left( bn + (L - b)(n - 1) \right) - \frac{1}{2} (L + b) 
\end{align*}
\]

The stress effects of all the piles on the point \( P \) are superimposed:

\[
\begin{align*}
\bar{\sigma}_x &= \sum_{i=1}^{n} \frac{q_i}{\pi} \left[ \arctan \frac{2(x + m) + (3 - 2i)L + b}{2y} - \arctan \frac{2(x + m) + (3 - 2i)L - b}{2y} + k_i \right] \\
\bar{\sigma}_y &= \sum_{i=1}^{n} \frac{q_i}{\pi} \left[ \arctan \frac{2(x + m) + (3 - 2i)L + b}{2y} - \arctan \frac{2(x + m) + (3 - 2i)L - b}{2y} - k_i \right] \\
\bar{\tau}_{xy} &= \sum_{i=1}^{n} \frac{q_i}{\pi} \left[ \frac{-16y'b(2(x + m) + (3 - 2i)L)}{(2(x + m) + (3 - 2i)L)^2 + 4y^2 - b^2} + 16y'b^3 \right] \\
k_i &= \frac{4y^3b^2(2(x + m) + (3 - 2i)L)^2 - 4y^2b^2}{(2(x + m) + (3 - 2i)L)^2 + 4y^2 - b^2} + 16y'b^3 \\
m &= \frac{1}{2} \left( bn + (L - b)(n - 1) \right) - \frac{1}{2} (L + b) 
\end{align*}
\]

According to the relevant literature, the relationship between the thrust and the buried depth displayed a parabolic distribution (Jiao et al. 2013; Liu et al. 2018). From the thrust distribution curve drawn by Tang et al. (2014), the shape of the parabolic curve is adjusted with time. Considering the time factor for the evolution of landslides, we can get:

\[
q_i(t,z) = a_i(t)z^2 + b_i(t)z + c_i(t) 
\]  

where \( z \) is the buried depth from the pile top; \( a_i, b_i, c_i \) are the coefficients of the fitting equation, and \( t \) is the evolutionary time of the landslide.

The soil behind the piles under the action of self-weight stress, the stress at \( P \) at depth \( z \) is as
follows:

\[
\begin{align*}
\sigma_z &= \gamma \cdot z \\
\sigma_x &= \sigma_y = K_0 \gamma \cdot z
\end{align*}
\]  

(10)

where \( \gamma \) is the natural weight of the sliding mass and \( K_0 \) is the coefficient of the lateral pressure of the soil. \( K_0 \) is the physical quantity describing the state of the stress.

The total stress at point \( P \) is composed of the self-weight stress and the subsidiary stress exerted by the piles. The stress at point \( P \) is obtained according to the superposition of the self-weight stress and the subsidiary stress and calculated as follows:

\[
\begin{align*}
\bar{\sigma}_i &= \sum_{i=1}^{n} q_i \left[ \frac{-16y^2b(2(x+m)+(3-2i)L)}{(2(x+m)+(3-2i)L)^2+4y^2-b^2} \right] + 16y^2b^2 \\
\bar{\sigma}_y &= \gamma \cdot z \\
\bar{\tau}_{xy} &= \sum_{i=1}^{n} q_i \left[ \frac{4yb[(2(x+m)+(3-2i)L)^2-4y^2-b^2]}{(2(x+m)+(3-2i)L)^2+4y^2-b^2} \right] + 16y^2b^2 \\
k_i &= \frac{1}{2} \left[ \frac{1}{bn+(L-b)(n-1)} \right] - \frac{1}{2} \frac{1}{L+b} \\
m &= \frac{1}{2}
\end{align*}
\]  

(11)

3. Spatial distribution of the soil stress behind the piles

3.1. Stress component of the soil stress behind the piles

According to the findings by Li et al. (2013), the distribution of soil stress under different pile numbers at a certain time is analyzed. The specific calculation parameters are: the pile spacing \( L=6.0 \) m, pile width \( b=2.0 \) m, pushing force \( F=800 \) kN/m, cantilever section length \( H=8.0 \) m. It is assumed that the distribution of the landslide thrust is rectangular. Therefore, the actual load acting on the pile is expressed as follows:

\[
q = \frac{FL}{Hb}
\]  

(12)

According to the formula (8), the stress of the three piles can be calculated \( \sigma_x, \sigma_y, \) and \( \tau_{xy} \). The stress can be expressed in the form of a nephogram, as shown in Fig.3(a), Fig.3(b) and Fig.3(c).
Fig.3. Stress nephogram of the soil stress behind the three piles. (a) $\sigma_x$; (b) $\sigma_y$; (c) $\tau_{xy}$.

It can be seen from the $\sigma_x$ nephogram that the $\sigma_x$ exerted by the piles on the soil is relatively small. Within 1 m vertical distance directly behind the piles, $\sigma_x$ is relatively large, and the soil adjacent to the piles is close to 300 kPa. The $\sigma_x$ between the piles first increases and then decreases with the increase in the vertical distance. The non-zero part of the soil $\sigma_{xx}$ is in the “crescent” shape, connected with the non-zero area of $\sigma_{xx}$ behind the piles to form a continuous arch shape.

It can be seen from the $\sigma_y$ nephogram behind the pile that the $\sigma_y$ exerted by the anti-slide pile on the soil has a greater effect than $\sigma_x$. There are obvious soil arching effects behind the piles. These results also show an agreement with those obtained by Li et al. (2013) and four forms of soil arching shape can be obtained, which are hyperbolic arch, extended shoulder arch, inverted bell arch, and circular arch. The $\sigma_y$ between the piles is relatively small, and in the range of 0–2 m vertical distance, the $\sigma_y$ is close to 0 kPa. The $\sigma_y$ behind the piles is greater than that between the piles.

It can be seen from the $\tau_{xy}$ nephogram that “stress bubbles” with equal absolute values and opposite signs are present at the corners of each pile. The maximum value of $\tau_{xy}$ is close to 100 kPa, concentrated near the corner of the piles. With the increase of the vertical distance, the shear stress decreases, and the $\tau_{xy}$ behind the piles is less than the shear stress of the soil on both sides.

Comparing Fig.3(a), (b), and (c), $\sigma_y$ is generally much larger than $\sigma_x$ and $\tau_{xy}$ with a significant
impact on the spatial distribution of the soil stress. Therefore, the influences on the soil stress by the pile number, distance behind piles, pile width and pile spacing were discussed based on $\sigma_y$ distributions in the following sections.

3.2. The influence of different factors on $\sigma_y$ distribution

3.2.1. The comparison of $\sigma_y$ distributions at different positions behind the piles.

Four locations behind the piles at 1 m, 3 m, 5 m, and 7 m, are selected for stress comparison, as shown in Fig.4. The soil stress behind the piles shows an axially symmetric distribution with the central axis of a single row of the piles as the axis of symmetry; the stress behind the pile is large, which reaches the maximum in the central location behind the pile, and the pile stress shows a downtrend. At 1 m and 3 m behind the piles, $\sigma_y$ values are 250 kPa and 125 kPa, respectively, and $\sigma_y$ values of the soil between piles are 0 kPa and 75 kPa. The values indicate that the closer we are to the pile body, the larger is the $\sigma_y$ of the soil behind the piles, the smaller the $\sigma_y$ of the soil between the piles. At 5 m and 7 m behind the pile, $\sigma_y$ value varied within a range of 75–100 kPa, showing no significant fluctuations, and a circular stress arching is formed.

![Fig.4. $\sigma_y$ distribution of the soil arching behind the piles with different vertical distances.](image)

3.2.2. The comparisons of $\sigma_y$ distributions considering the different number of the piles.

To compare the impact of the number of piles on $\sigma_y$ distribution, different cases with 1, 2, 3, 4, and 5 piles are discussed, consecutively. The distribution of 3 piles is shown in Fig.3(b), whereas Fig.5(a), Fig.5 (b), Fig.5(c), and Fig.5(d) show the distribution of 1, 2, 4, and 5 piles. Overall, $\sigma_y$ value is large closer to the pile, which decreases as the vertical distance increases, and $\sigma_y$ between the piles first increases and then decreases with the increase of the distance. Under the single pile condition, the soil behind the pile does not form stress arching. After the vertical distance is larger than 4 m, the subsidiary stress value is close to 0 kPa. The anti-slide pile has no obvious effect on the soil behind the pile. By comparing the stress nephograms of the two piles and the central two piles among those four piles, it was found that the subsidiary stress shape of the sliding mass behind the two piles changed significantly. Therefore, the outward two piles significantly influence the soil stress behind the two piles in the middle. With the increase in the number of anti-slide piles, the subsidiary stress applied by
the pile on the slide mass behind the pile, is increased.

Fig.5. $\sigma_y$ nephograms of the soil arching behind the piles. (a) one pile; (b) two piles; (c) four piles; (d) five piles.

The impact of the number of piles on $\sigma_y$ distribution is compared using the stress nephograms. We chose the changing law of stress with the number of piles at the vertical distance of 3 m behind the pile as an example, as shown in Fig.6. The subsidiary stress increases as the number of piles increases, exhibiting an exponential relationship; under single pile, double-pile, and three-pile conditions, the subsidiary stress at the same position varies greatly, exhibiting a rapidly growing trend. The subsidiary stress at the same position when there are five piles, remains constant, indicating that the subsidiary stress is affected when the number of piles approaches five. Notably, the subsidiary stress value of other piles at this point is small.
3.2.3. The comparisons of $\sigma_y$ distributions considering the different pile spacing.

Yang et al. (2007) studied the influence of pile width and spacing on the soil arching effect between piles by the centrifugal model test. It is considered that the pile width and spacing play an important role in the rationality of the pile design. In this section, based on the example data given in section 3, the influence of the pile spacing and width on the soil arching effect between the piles will be discussed by using the analytical equation of the stress of the sliding mass.

We discuss the effect of the pile spacing on the subsidiary stress when the pile width is constant ($b=2$ m). $L/b$ takes the values of 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9, 9.5, 10.0 ($L$ is the center-to-center spacing between the two adjacent piles). The changes in the subsidiary stress value between piles at the vertical distance of 2 m behind the pile, are analyzed. Based on the discussion regarding Fig.6 in the prior section, it is considered that in case of 6 piles or above in the calculation model, other piles have little effect on the subsidiary stress at a point behind the pile. In the view of this, we will merely discuss the relationship between the stress near the soil arching and the pile width ratio under the conditions of 2, 3, 4, and 5 piles.

As shown in Fig.7, the subsidiary stress of the soil at the same position between piles (2 m behind the pile) decreases with the increase of the pile spacing and the subsidiary stress approaches to 0 kPa; within the range of pile spacing 4.5 times the pile width, the subsidiary stress changes significantly; the smaller the pile spacing, the larger the subsidiary stress at the same position under different pile numbers; when $L/b=2.5$, the subsidiary stress under different pile numbers are equal, and the effect of the pile number on the subsidiary stress between piles, can be ignored.
Fig. 7. Relationship between the soil subsidiary stress and $L/b$ under variable pile spacing.

3.2.4. The comparisons of $\sigma_y$ distributions considering the different pile widths.

We discuss the effect of the pile width on the subsidiary stress when the former is a constant value ($b=1$ m, 2 m, 3 m, and 4 m) and there are four piles. $L/b$ takes the values of 1.0, 2.0, 3.0, 5.0, 6.0, 7.0, 8.0, 9.0, and 10.0.

It can be seen from Fig. 8 that the subsidiary stress of the soil at the same position between piles (1 m behind the pile) under the same $L/b$ decreases with the increase of the pile width; when the pile width is 4 m, $L/b=1$ (subsidiary stress is 99.92 kPa) and when the pile width is 2 m, $L/b=2$ (subsidiary stress is 33.97 kPa); the difference in the subsidiary stress under the two conditions is thrice as much, indicating that when the pile spacing is the same, the smaller the pile width, the more insignificant the stress soil arching effect behind the pile. The larger the pile width, the faster the subsidiary stress decreases between piles with $L/b$, indicating that with the larger pile width, the subsidiary stress changes in a smaller ratio range of pile spacing and width. For example, when the pile width is 4 m, after the pile spacing is 3 times greater than the pile width, the subsidiary stress approaches to 0 kPa. But when the pile width is 2 m, after the pile spacing is 5 times greater than the pile width, the subsidiary stress approaches to 0 kPa.
Fig. 8. Relationship between the soil subsidiary stress and $L/b$ under the variable pile width.

4. Discussion

Based on the analysis of the soil arching effect of the interaction of piles and soil, and further solving the problem of maximum pile spacing, when a point is in the plane stress state, the main stress can be solved by the following formula (Das, 2008):

$$
\begin{align*}
\sigma_{max} &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
\sigma_{min} &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}
\end{align*}
$$

(13)

Combining formulas (11) and (13), the main stress expression can be obtained as follows:

$$
\sigma_{max} = \sum_{i} q \left[ \frac{\text{arctan}}{\pi} \left\{ \frac{2(x + m) + (3 - 2i)L + b}{2y} - \frac{2(x + m) + (3 - 2i)L - b}{2y} \right\} + K_{xy} \cdot z \right] \\
+ \left\{ \sum_{i} q \cdot k \right\} \left( \sum_{i} q \right) + \left\{ \sum_{i} q \right\} \left( \sum_{i} q \right) \left( \frac{-16y^2 b(2(x + m) + (3 - 2i)L)}{(2(x + m) + (3 - 2i)L)^2 + 4y^2 - b^2} \right)^2 + 16y^2 b
$$

(14)

$$
\sigma_{min} = \sum_{i} q \left[ \frac{\text{arctan}}{\pi} \left\{ \frac{2(x + m) + (3 - 2i)L + b}{2y} - \frac{2(x + m) + (3 - 2i)L - b}{2y} \right\} + K_{xy} \cdot z \right] \\
- \left\{ \sum_{i} q \cdot k \right\} \left( \sum_{i} q \right) + \left\{ \sum_{i} q \right\} \left( \sum_{i} q \right) \left( \frac{-16y^2 b(2(x + m) + (3 - 2i)L)}{(2(x + m) + (3 - 2i)L)^2 + 4y^2 - b^2} \right)^2 + 16y^2 b
$$

(15)

If the hyperbolic stress soil arching crown (the vertical distance $L$ from center of soil between piles) is taken, the critical pile spacing in case of the soil failure reaches the maximum (Li et al. 2013), according to the Mohr-Coulomb strength criterion in soil mechanics, the limit equilibrium condition of the soil is as follows:
\[ \sigma_{\max} = \sigma_{\max} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) + 2c \cdot \tan(45^\circ + \frac{\phi}{2}) \] (16)

Or:

\[ \sigma_{\max} = \sigma_{\max} \cdot \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) - 2c \cdot \tan(45^\circ - \frac{\phi}{2}) \] (17)

In the design of piles, the cross-section of the pile \( b \) is known, and the pile spacing \( L \) is calculated. By substituting the coordinates of the hyperbolic stress arch crown into the formulas (11), (14), (15), and (16), the maximum pile spacing can be solved.

5. Conclusions

Based on the elasticity theory, the stress analysis model of the sliding mass behind the pile is established. The control equation of the stress distribution function and the maximum spacing of the sliding mass behind the piles at different times (evolution stage) and different spatial positions with different pile numbers, are obtained. Therefore, the soil arching effect of the sliding mass behind the pile with different pile numbers, spacing, and section size, is studied.

It can be seen from the nephogram of \( \sigma_{xx}, \sigma_{yy}, \tau_{xy} \) that the \( \sigma_y \) exerted by the pile on the soil has a great influence. There are obvious soil arching effects behind the piles, and four forms of the soil arching shape can be obtained, namely hyperbolic arch, extended shoulder arch, inverted bell arch, and circular arch.

The subsidiary stress behind the piles increases with the number of piles, and the relationship between them is exponential. When the number of piles reaches 5, the subsidiary stress at the same position tends to be constant. It shows that the subsidiary stress is greatly affected by the five adjacent piles, and the subsidiary stress of other piles is relatively small at this point.

Under the same pile width condition, the smaller the pile spacing, the greater the subsidiary stress is at the same position between piles under different pile numbers. In the range of 4.5 times of pile width, the subsidiary stress changes significantly. When \( L/b=2.5 \), the subsidiary stress is equal under different pile number conditions, and the influence of pile numbers on the subsidiary stress between piles is ignored.

Under the same pile spacing condition, the smaller the pile width, the less obvious is the soil arching effect behind the pile. The larger the pile width, the faster the stress value of the attachment between piles decreases with \( L/b \), indicating that when the pile width is larger, the subsidiary stress changes within the range of the smaller ratio of pile spacing to pile width.

Some conclusions are obtained from the interaction between the anti-slide piles and the sliding mass. It is expected to clarify the equations and conclusions by experiments and data in the subsequent research and verify their applicability.

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