Google Distance Between Words

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Rudi Cilibrasi and Paul Vitányi have demonstrated that it is possible to extract the meaning of words from the world-wide web. To achieve this, they rely on the number of webpages that are found through a Google search containing a given word and they associate the page count to the probability that the word appears on a webpage. Thus, conditional probabilities allow them to correlate one word with another word’s meaning. Furthermore, they have developed a similarity distance function that gauges how closely related a pair of words is. We present a specific counterexample to the triangle inequality for this similarity distance function.

I. INTRODUCTION

When the Google search engine is used to search for word \( x \), Google displays the number of hits that word \( x \) has. The ratio of this number of hits to the total number of webpages indexed by Google represents the probability that word \( x \) appears on a webpage. Cilibrasi and Vitányi [CV] use this probability to extract the meaning of words from the world-wide-web. If word \( y \) has a higher conditional probability to appear on a webpage, given that word \( x \) also appears on the webpage, than it does by itself, then it can be concluded that words \( x \) and \( y \) are related. Moreover, higher conditional probabilities imply a closer relationship between the two words. Thus, word \( x \) provides some meaning to word \( y \) and vice versa.

Cilibrasi and Vitányi’s normalized Google distance (NGD) function measures how close word \( x \) is to word \( y \) on a zero to infinity scale. A distance of zero indicates that the two words are practically the same. Two independent words have a distance of one. A distance of infinity occurs for two words that never appear together.

Although Cilibrasi and Vitányi’s NGD function was sensibly derived from its basic axioms (which allow it to theoretically yield the values mentioned above), it does not account for the presence of multi-themed webpages. In other words, the NGD function does not account for webpages, such as dictionary sites and other long pages, which encompass many unrelated subjects. For this reason, it is necessary to renormalize the NGD formula to achieve the desired values.

II. NGD’S EXPECTATION VALUE

On the average, two random words should be independent of one another. Hence, two random words should have an NGD of one. To test this assumption, we randomly selected two sets of five words from the dictionary. In addition, we randomly chose one set of ten words from two different news articles (five words were taken from each article). We then proceeded to evaluate the NGD among the different word pairs in each set using Cilibrasi and Vitányi’s formula

\[
\text{NGD}(x, y) = \frac{\max \{\log f(x), \log f(y)\} - \log f(x, y)}{\log M - \min \{\log f(x), \log f(y)\}},
\]

(1)

where \( f(x) \) and \( f(y) \) are the number of hits of words \( x \) and \( y \), respectively, and \( M \) is the total number of webpages that Google indexes.

Our first set, which consisted of the words micrometeorite, transient, denature, pentameter, and reside, yielded an expectation value of 0.64 for the NGD with a standard deviation of 0.14. Our second set, which consisted of the words detrition, unity, interstice, abrupt, and reside, had an expectation value of 0.75 for the NGD with a standard deviation of 0.12. Our last set, which consisted of the words agency, diabtic, enforcement, federal, hormone, illegal, intelligence, measure, spread, and war, yielded an expectation value of 0.77 with a standard deviation of 0.15. Averaging the above values (weighing them correctly according to the number of words in each set), we obtain an expected value of 0.7325 for the NGD.

A different type of analysis brought us to a similar expectation value. We call this evaluation the triangle difference, (TD). The reason for this name is that we
evaluated the following difference:

\[ \text{TD} \equiv \text{NGD}(x, y) + \text{NGD}(y, z) - \text{NGD}(x, z). \]  

The reason to evaluate such difference stemmed from the possibility that the sum of two distances, between words x and y, and y and z, might be smaller than the distance between x and z. If such were the case, then it would be sensible to redefine the distance between two words such that it minimizes all possible NGD sums:

\[ \text{NGD}^*(x, z) \equiv \min \{ \text{NGD}(x, y) + \text{NGD}(y, z), \text{NGD}(x, z) \}, \]  

for all words y. The triangle difference, however, is only violated by extremely rare exceptions, so it is not necessary to perform such minimization. Nevertheless, we proceeded to evaluate the expected triangle difference for each of our sets. They were 0.69, 0.79, and 0.60, for the first, second, and third sets, respectively. Combining them, they yield an expected triangle difference of 0.67. This, of course, is close to the expected value of NGD, as the triangle difference for random words is

\[ E[\text{TD}] \equiv E[\text{NGD}(x, y)] + E[\text{NGD}(y, z)] - E[\text{NGD}(x, z)] \]

\[ E[\text{TD}] = 2E[\text{NGD}] - E[\text{NGD}] = E[\text{NGD}]. \]  

Therefore, the expectation value of the triangle difference should be equal to the expectation value of the NGD. A rough average of our expectation values obtained through each method is 0.7.

### III. NOTES REGARDING NGD

In an arduous effort to find a set of words that would violate the triangle difference, we obtained a set that illustrates a few interesting properties of NGD. The set consists of the words Rolling Stones, Beatles, and salmonflies and it is, among the many word sets that we attempted, the only one that violates the triangle difference. Our first evaluation of the pertinent distances was the following:

| Word Pair         | NGD  |
|-------------------|------|
| Rolling Stones, Beatles | 0.23 |
| Beatles, salmonflies | 0.81 |
| Rolling Stones, salmonflies | 1.06 |

Table 1. First evaluation of NGD values among the words Rolling Stones, Beatles, and salmonflies.

As can be observed, the NGD between Rolling Stones and salmonflies (1.06) is slightly higher than the addition of the Google distances between Rolling Stones and Beatles, and between Beatles and salmonflies (1.04). Therefore, our first observed property of NGD is that, even in the rare cases in which the triangle difference is violated, it is not by much. Furthermore, it is important to indicate that our example worked because of the high propensity that people have to misspell the word beetle as beatle, thus decreasing the distance between Beatles and salmonflies.

The second property that we observed, which deserves much attention, is the NGD dependence on the Google server to which a user connects. A second evaluation of the distances in question yielded the following result:

| Word Pair         | NGD  |
|-------------------|------|
| Rolling Stones, Beatles | 0.27 |
| Beatles, salmonflies | 0.82 |
| Rolling Stones, salmonflies | 1.14 |

Table 1. Second evaluation of NGD values among the words Rolling Stones, Beatles, and salmonflies.

This second set of distances was obtained by connecting to Google through a different internet service provider and it shows that Google distances are not stable values. In fact, from our example, they can vary by as much as 17% (for the Rolling Stones-Beetles word pair).

The last property that this set of words depicts is the plausibility of Google distances that are higher than unity. With an NGD of 1.14, the word pair Rolling Stones-salmonflies has the highest distance that we have encountered. Indeed, Google distances greater than one are very rare, and before this example, the only one we had encountered was between the words transient and pentameter (1.02). Moreover, these cases are so rare that even Cilibrasi and Vitányi’s conjecture about the words by and with having an NGD higher than one is false. The actual distance, per the Google server currently in use, is 0.19.

### IV. CONCLUSIONS

Although Google can be used to extract the meaning of words, it is important to modify Equation 1 in order to obtain the desired distance values between words. The expectation value of NGD, which is the distance between two random and therefore independent words, is 0.7. To achieve the desired value of unity between independent words, it is only necessary to recalibrate the NGD formula by dividing by 0.7:

\[ \text{NGD}^*(x, y) = \frac{\text{NGD}(x, y)}{0.7}. \]  

It is also important to remember that NGD values are not exact. They depend on the number of hits that each word has, which makes them unstable. Factors such as the Google server to which one connects and the number of websites connected to the world-wide-web can cause...
discrepancies as high as 17%, which our Rolling Stones-Beatles example showed.

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