Holographic description of glueballs in a deformed AdS-dilaton background

Frédéric Jugeau

INFN - Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy (frederic.jugeau@ba.infn.it)

Abstract. We investigate the mass spectra of scalar and vector glueballs in the so-called bottom-up approach of the AdS/QCD correspondence. The holographic model of QCD includes a static dilaton background field. We study the constraints on the masses coming from perturbing the dilaton field and the geometry of the bulk.

Keywords: AdS/QCD correspondence, glueball spectroscopy, holographic constraints.

INTRODUCTION

A breakthrough in the attempt to understand strongly coupled Yang-Mills theories came with the AdS/CFT correspondence, proposed by Maldacena [1], that conjectures a connection between the large $N$ limit of a maximally $\mathcal{N} = 4$ superconformal $SU(N)$ gauge theory defined in $d$ dimensions and the supergravity limit of a superstring/M-theory living on a $d + 1$ anti-de Sitter (AdS) space times a compact manifold [2, 3]. However, the application of this conjecture to a theory such as QCD is not straightforward, being QCD neither supersymmetric nor conformal so that its gravity dual theory remains unknown. Witten proposed a procedure to extend Maldacena’s duality to such gauge theories [4]: the conformal invariance is broken by compactification, while supersymmetry is broken by appropriate boundary conditions on the compactified dimensions. The AdS geometry of the dual theory is then deformed into an AdS-Black-Hole geometry where the horizon plays the role of an IR brane.

Adopting this (so-called up-down) approach, analyses of glueball spectroscopy in 3 and 4 dimensions have been carried out, obtaining, for instance, that the scalar glueball with $J^{PC} = 0^{++}$ corresponds, in the supergravity side, to the massless dilaton field propagating in the 10 dimension black-hole geometry [5]. Then, the glueball mass is computable by solving the dilaton wave equation and gives results in reasonable agreement with the available lattice data [6].

However, instead of trying to warp the original 11-dimensional $AdS_7 \times S^4$ geometry in order to obtain a 4-dimension gauge theory with similarities with QCD, one could adopt a different strategy investigating what features the dual theory should have in order to reproduce known QCD properties. In this (so-called bottom-up) approach, or AdS/QCD correspondence, one attempts to construct a 5-dimensional holographic model able to...
reproduce the main features of QCD. As pioneered by Polchinski and Strassler [7], it turned out to be possible to modify the AdS/CFT duality, aimed at describing a confining gauge theory, by considering a truncated $AdS_5$ holographic space-time on the 4-dimensional boundary of which QCD is defined. In this so-called IR hard wall approximation, the typical size of this $AdS_5$ slice stands for an IR cutoff associated to the QCD mass gap. The IR hard wall model has been widely used in order to investigate, for instance, light hadron spectrum and form factors [8]. Another holographic model of QCD has been proposed, which consists in inserting a static dilaton field in the $AdS_5$ space-time. This particular background allows one to recover the Regge behaviour believed to be satisfied by mesons [9], at odds with what happens with the IR hard wall model (which rather appears to be dual to a bag model of QCD) or starting from a general string theory and attempting to deform it [10].

Even if somehow cumbersome shortcomings subsist when constructing holographic models of QCD, namely, for instance, the stringy corrections $O(1/N)$, the role of the remaining compact manifold $S^5$ or the accurate range of the holographic coordinate that is effectively dual to the QCD energy scale, there is the hope that they do not spoil the main features of these dual models.

THE 5d HOLOGRAPHIC MODEL DUAL TO QCD

Following [9], we consider a five dimensional conformally flat spacetime (the bulk) described by the metric:

$$g_{MN} = e^{2A(z)} \eta_{MN}, \quad ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2),$$

(1)

where $X^M = (x^\mu, z)$ and $\eta_{MN} = \text{diag}(-1, 1, 1, 1, 1)$. $x^\mu$ ($\mu = 0, \ldots, 3$) represent the usual space-time (the boundary) coordinates and $z$ is the fifth holographic coordinate running from zero to infinity. The metric function $A(z)$ satisfies the condition

$$A(z) \to \ln \left( \frac{R}{z} \right),$$

(2)

to reproduce the $AdS_5$ metric close to the UV brane $z \to 0$. In the following, we will take the simplest choice compatible with the constraints displayed in [9]: $A(z) = -\ln z$. Besides, we consider a background dilaton field $\phi(z) = c^2 z^2$ which only depends on the holographic coordinate $z$ and vanishes at the UV brane. The large $z$ dependence of the dilaton is chosen to reproduce the Regge behaviour of the low-lying mesons, and all the masses will be given with respect to the scale parameter $c$. Moreover, the introduction of this background dilaton allows one to avoid ambiguities in the choice of the bulk field boundary conditions at the IR wall.

We construct a 5d model that can be considered as a cut-off $AdS$ space: a smooth cut-off in the IR replaces the hard-wall IR cutoff that would be obtained by allowing the holographic variable $z$ to vary from zero to a maximum value $z_m \simeq \frac{1}{\Lambda_{QCD}}$. To investigate

---

2 In the following, we put the AdS radius $R$ to unity.
the mass spectra of the QCD scalar and vector glueballs, we consider the two lowest
dimension operators with the corresponding quantum numbers and defined in the field
theory living on the 4d boundary [11]:

\[
\begin{align*}
O_S &= Tr(F^2), \\
O_V &= Tr(F(DF)F),
\end{align*}
\]

(with \(D\) the covariant derivative) having conformal dimension \(\Delta = 4\) and \(\Delta = 7\) respectively. The operator corresponding to the vector glueball satisfies the Landau-
Pomeranchuk-Yang selection rule [12]. According AdS/CFT correspondence, the con-
formal dimension of a \((p\text{-form})\) operator on the boundary is related to the \((\text{AdS mass})^2\) of his dual field in the bulk as follows [2, 3]:

\[
(\text{AdS mass})^2 = (\Delta - p)(\Delta + p - 4).
\]

In the following, we assume that the mass \(m_5^2\) of the bulk fields is given by this expres-
sion.

A 5d massless scalar field \(X(x,z)\) can be constructed as the correspondent of \(Tr(F^2)\),
described by the action in the gravitational background:

\[
S = \frac{1}{2} \int d^5x \sqrt{-g} e^{-\phi(z)} g^{MN} (\partial_M X)(\partial_N X),
\]

with \(g = det(g_{MN})\). Scalar glueballs are identified as the normalizable modes of \(X\) satisfying the equations of motion obtained from (5), corresponding to a finite action.

For the spin 1 glueball, we introduce a 1-form \(A_M\) described by the action:

\[
S = -\frac{1}{2} \int d^5x \sqrt{-g} e^{-\phi(z)} \left[ \frac{1}{2} g^{MN} g^{ST} F_{MS} F_{NT} + m_5^2 g^{ST} A_S A_T \right],
\]

with \(F_{MS} = \partial_M A_S - \partial_S A_M\) and \(m_5^2 = 24\), and study its normalizable modes. Notice that the action (6), with a different value of \(m_5^2\), describes \textit{a priori} fields that are dual to other operators in QCD, namely those describing hybrid mesons with spin one, which is an explicit example of different QCD operators having similar bulk fields as holographic correspondents.

**SCALAR AND VECTOR GLUEBALL SPECTROSCOPY**

The field equations of motion obtained from the actions (5)-(6) can be reduced in the form of a one dimensional Schrödinger equation in the variable \(z\):

\[
-\psi'' + V(z) \psi = -q^2 \psi,
\]

involving the function \(\psi(z)\) obtained applying a Bogoliubov transformation \(\psi(z) = e^{-B(z)/2} \tilde{Q}(q,z)\) to the Fourier transform \(\tilde{Q}\) of the field \(Q \ (Q = X,A_M)\) with respect to the boundary variables \(x^\mu\). The function \(B(z)\) is a combination of the dilaton and the
metric function: \( B(z) = \phi(z) - aA(z) \), with the parameter \( a \) given by: \( a = 3 \) and \( a = 1 \) in cases of \( \chi \) and \( AM \) fields, respectively. The condition \( q^2 = -m^2 \) identifies the mass of the normalizable modes of the two fields.

Eq. (7) is a one dimensional Schrödinger equation where \( V(z) \) plays the role of a potential. It reads as:

\[
V(z) = \frac{1}{4} (B'(z))^2 - \frac{1}{2} B''(z) + \frac{m_5^2}{z^2} = V_0(z) + \frac{m_5^2}{z^2} ,
\]

with

\[
V_0(z) = c_4 z^2 + a_2 + 2a_4 z^2 + c_2 (a - 1) .
\]

With this potential, eq. (7) can be analytically solved. Regular solutions at \( z \to 0 \) and \( z \to \infty \) correspond to the spectrum:

\[
m_n^2 = c^2 \left[ 4n + 1 + a + \sqrt{(a+1)^2 + 4m_5^2} \right] ,
\]

with \( n \) an integer (we identify it as a radial quantum number), while the corresponding eigenfunctions read as:

\[
\psi_n(z) = A_n e^{-c^2 z^2/2} (c_2)_{g(a,m_5^2)+1/2} _1F_1 (-n, g(a,m_5^2) + 1, c^2 z^2) ,
\]

with \(_1F_1\) the Kummer confluent hypergeometric function, \( A_n \) a normalization factor and \( g(a,m_5^2) = \sqrt{(a+1)^2 + 4m_5^2} \). From these relations, we obtain the spectrum of scalar and vector glueballs, respectively:

\[
m_n^2 = c^2 (4n + 8) , \quad m_n^2 = c^2 (4n + 12) .
\]

A few remarks are in order in respect to the results derived in [9]. First, both the spectra have the same dependence on the radial quantum number \( n \) as the mesons of spin \( S \): \( m_n^2 = c^2 (4n + 4S) \). This is a consequence of the large \( z \) behaviour chosen for the background dilaton. Second, both the lowest lying glueballs are heavier than the \( \rho \) mesons, the spectrum of which reads: \( m_n^2 = c^2 (4n + 4) \). Finally, the vector glueball turns out to be heavier than the scalar one.

More precisely, comparing our result to the computed \( \rho \) mass, we obtain for the lightest scalar \( (G_0) \) and vector \( (G_1) \) glueballs

\[
\frac{m_{G_0}^2}{m_{\rho}^2} = 2 ; \quad \frac{m_{G_1}^2}{m_{\rho}^2} = 3 ,
\]

which implies that these glueballs are expected to be lighter than as predicted by other QCD approaches [6]. The result \( m_{G_1}^2 - m_{G_0}^2 = m_{\rho}^2 \) predicts indeed a lightest vector glueball with mass below 2 GeV.

It is interesting to investigate how it is possible to modify the \( z \) dependence of the background dilaton field and of the metric function \( A \), and how the spectra change, an issue discussed in the following section.
There are other choices for the background dilaton $\phi$ and the metric function $A$ which allow us to reproduce the Regge behaviour of the low-lying mesons and to recover the $AdS_5$ metric close to the UV brane when $z \to 0$. As a matter of fact, it is possible to add to the background fields terms of the type $z^\alpha$ with $0 \leq \alpha < 2$. Considering the simplest case: $\alpha = 1$, this can be done in two different ways. Either we modify the dilaton field including a linear contribution which is subleading in the IR regime ($z \to \infty$) or we modify the metric function which now acquires a linear term subleading in the UV regime ($z \to 0$):

$$\begin{align*}
\phi(z) &= c^2 z^2 + \lambda cz \\
A(z) &= -\ln z
\end{align*}$$

(15)

with $\lambda$ a real dimensionless parameter. The two choices produce different results. Modifying the dilaton field, the potential (7) becomes:

$$V(z) = V_0(z) + \lambda V_1(z) + \frac{\lambda^2 c^2}{4} + \frac{m_5^2}{z^2} f(z, \lambda) \quad \text{with} \quad \begin{align*}
V_1(z) &= c (c^2 z + \frac{a}{z}) \\
f(z, \lambda) &= 1
\end{align*}$$

(16)

while modifying the metric in the IR, the potential term reads as:

$$V(z) = V_0(z) + \lambda \tilde{V}_1(z) + \frac{\lambda^2 c^2 a^2}{4} + \frac{m_5^2}{z^2} f(z, \lambda) \quad \text{with} \quad \begin{align*}
\tilde{V}_1(z) &= a c (c^2 z + \frac{a}{z}) \\
f(z, \lambda) &= e^{-2\lambda cz}
\end{align*}$$

(17)

with $V_0(z)$ given in (9). Considering (15)-(17), one sees that the mass term is the main responsible of the difference between the scalar and vector cases when the geometry is perturbed, while its effect turns out to be the same when the background dilaton is modified. Eq. (7) with the new potentials (16) and (17) can be solved perturbatively and, for small values of the parameter $\lambda$, the spectra are modified:

$$m_n^2 = m_{n, (0)}^2 + \lambda m_{n, (1)}^2 .$$

(18)

The detailed analysis can be found in [11]. Different predictions at $O(\lambda)$ for the lowest-lying vector and scalar glueball mass difference are then obtained, modifying either the dilaton or the geometry:

$$m_{G_1}^2 - m_{G_0}^2 = c^2 \left(4 - \frac{3\sqrt{\pi}}{128}\lambda\right) \quad \text{(modifying the dilaton)},$$

(19)

$$m_{G_1}^2 - m_{G_0}^2 = c^2 \left(4 - \frac{1899\sqrt{\pi}}{128}\lambda\right) \quad \text{(modifying the metric function)}.$$

(20)

Therefore, the mass splitting between vector and scalar glueballs increases if $\lambda$ is negative, and the maximum effect is produced, for the same value of $\lambda$, when the metric function is perturbed. This can be considered as an indication on the type of constraints the background fields in the bulk must satisfy.
CONCLUSIONS

We have discussed how the QCD holographic model proposed in [9], with the hard IR wall replaced by a background dilaton field, allows one to predict the light glueball spectra [11]. Vector glueballs turn out to be heavier than the scalar ones, and the dependence of their masses on the radial quantum number is the same as obtained for $\rho$ and higher spin mesons. Combining the calculations of the glueball and $\rho$ masses in the same holographic model, the glueballs turn out to be lighter than predicted in other approaches [6].

We have also investigated how the masses change as a consequence of perturbing the dilaton in the UV or the bulk geometry in the IR, finding that constraints in the bottom-up approach can be found if information on the spectra from other approaches is considered. Such constraints should be taken into account in the attempt to construct the QCD gravitational dual.

ACKNOWLEDGMENTS

I am very indebted to my collaborators at the INFN sezione di Bari who make my stay so pleasant and fruitful. I am also grateful to the organizers who gave me the opportunity to present my research activities during this workshop. This work was supported in part by the EU Contract No. MRTN-CT-2006-035482, "FLAVIAnet".

REFERENCES

1. J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231 [Int. J. Theor. Phys. 38 (1999) 1113] (hep-th/9711200).
2. E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253.
3. S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B 428 (1998) 105 (hep-th/9802109).
4. E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys.2, 505, 1998 (hep-th/9803131).
5. C. Csaki, H. Ooguri, Y. Oz, J. Terning, Glueball mass spectrum from supergravity, JHEP 9901 (1999) 017 (hep-th/9806021).
6. C. J. Morningstar and M. J. Peardon, The glueball spectrum from an anisotropic lattice study, Phys. Rev. D 60 (1999) 034509 (hep-lat/9901004); B. Lucini and M. Teper, SU(N) gauge theories in four dimensions: Exploring the approach to infinity, JHEP 0106 (2001) 050 (hep-lat/0103027).
7. J. Polchinski, M. J. Strassler, Hard scattering and gauge/string duality, Phys. Rev. Lett. 88 (2002) 031601 (hep-th/0109174).
8. H. R. Grigoryan and A. V. Radyushkin, Form Factors and Wave Functions of Vector Mesons in Holographic QCD (hep-ph/0703069).
9. A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Linear confinement and AdS/QCD, Phys. Rev. D 74 (2006) 015005 (hep-ph/0602229).
10. E. Schreiber, Excited mesons and quantization of string endpoints (hep-th/0403226); M. Shifman, Highly excited hadrons in QCD and beyond, Frascati 2005, Quark-hadron duality and the transition to pQCD p.171-191 (hep-ph/0507246).
11. P. Colangelo, F. De Fazio, F. Jugeau and S. Nicotri, On the light glueball spectrum in a holographic description of QCD, Phys. Lett. B652 (2007) 73-78 (hep-ph/0703316).
12. V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Are All Hadrons Alike?, Nucl. Phys. B 191 (1981) 301.