Neutrino Radiative Decay
in an External Electromagnetic Field
via Vector Leptoquark

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Abstract

The massive neutrino radiative decay $\nu_i \to \nu_j \gamma \ (i \neq j)$ is investigated within the Minimal Quark-Lepton Symmetry of the Pati-Salam type based on the $SU(4)_V \times SU(2)_L \times G_R$ group in an external constant crossed field ($\vec{E} \perp \vec{H}$, $E = H$). The matrix element $\Delta M^{(X)}$ and the decay probability $W^{(X)}$ due to the leptoquark $X$ contribution are analysed. The effect of significant enhancement of the neutrino decay by the crossed field takes place. It is shown that the leptoquark contribution to the decay probability can dominate over the $W$-boson one in the case of strong suppression of the lepton mixing in the framework of the SM.

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1 Introduction

The Standard Model (SM) successfully describes all the available experimental data obtained at currently accessible energies. Although significant progress has been achieved in experimental studies of electroweak processes, some problems that are important for the theoretical description of elementary-particle physics remain unresolved. The phenomenon of fermion mixing in charged weak currents (whose mechanism has yet to be understood) is among the most intriguing problems of this kind. Mixing in the quark sector is described by the Cabbibo-Kobayashi-Maskawa (3\times3) unitary matrix $V_{ij}$. It is known to a rather high accuracy \[1\], and the information about mixing parameters is continually refined. It is natural to expect that a similar mixing phenomenon occurs in the lepton sector of the SM as well, provided that the neutrino mass spectrum is nondegenerate. It may lead to such interesting phenomena as neutrino oscillations \[2\], rare decays with lepton number violation of the types $\mu \to e\gamma \[3\]$, $\mu \to e\gamma\gamma \[4\]$, $\nu_i \to \nu_j\gamma \[3\]$, $\nu_i \to \nu_j\gamma\gamma \[5\]$, and other processes "forbidden" by the law of conservation of lepton number. Notice that both oscillations and above-mentioned radiative decays are continuously searched for in experiments \[6, 7\].

It is necessary to stress that the probabilities of these decays are strongly suppressed in the SM due to the well-known GIM cancellation by the factor

$$\left(\frac{m_l}{m_W}\right)^4 \ll 1,$$

(1.1)

The small values of lepton masses with respect to the $W$-boson mass lead to the conclusion that processes with such low probabilities can hardly be observed under laboratory conditions. It is worth noting that an additional type of suppression due to small mixing angles can appear. The experimental searches for neutrino oscillations, the main source of information on the lepton mixing angles, show that the mixing angles are most likely to be small. All attempts to find a possible manifestation of lepton mixing have given up to now negative results.

On the other hand, the continuing interest in neutrinos involving processes is due to that the study of their properties may lead to new physics beyond the SM \[7\]. It is conceivable that new symmetries will successively manifest themselves in experiments with increasing energy of colliding particles and that
$SU(2) \times U(1)$-symmetry of the SM is merely the first level in the as-yet-unknown hierarchy of symmetries. In this connection, it is interesting to find the next (in energy) level in this hierarchy. This brings up the question of what symmetry is restored after the electroweak one and of what mass scale follows $M_W$. There is a possibility of extending the SM in such a way that the rare radiative lepton decays:

1) take place in the absence of lepton mixing in the SM,
2) are not subjected to the above-mentioned strong suppressions.

We have in mind the Minimal Quark-Lepton Symmetry of the Pati-Salam type based on the $SU(4)_V \times SU(2)_L \times G_R$ gauge group, in which the lepton number is interpreted as the fourth color [8]. Let us assume that the right-hand symmetry $G_R$ is restored on a considerably higher mass scale than $SU(4)_V$; therefore, we do not specify the subgroup $G_R$. Fractionally charged color $X$ bosons – leptoquarks – which are responsible for the quark-lepton transitions, are the most exotic objects in this model. It was shown in [9] that a new type of mixing – mixing in quark-lepton currents – must be considered in this approach. The resulting additional freedom in the choice of parameters makes it possible to remove the lower bound on the vector-leptoquark mass $M_X$ obtained from rare decays of $\pi$ and $K$ mesons [1]. The only constraint that does not depend on mixing arises from the cosmological estimate of the width of the decay $\pi^0 \rightarrow \nu \bar{\nu}$ [10]: $M_X > 18$ eV.

At present the investigation of electroweak processes in strong external fields are of special interest. In this case, the method of exact solutions of relativistic wave equations in external electromagnetic fields is quite effective and allows one to go beyond the perturbation theory and predict new phenomena. In particular, an external field can induce novel lepton transitions of the types $\nu_i \rightarrow \nu_j \ (i \neq j)$ [11], $\mu \rightarrow e$ [12], forbidden by energy-momentum conservation in vacuum, and influence substantially the neutrino processes with flavour violation (one-loop as minimum) [13, 14].

In the present work we study the massive neutrino radiative decay $\nu_i \rightarrow \nu_j \gamma \ (i \neq j)$ within the Minimal Quark-Lepton Symmetry of the Pati-Salam type based on the $SU(4)_V \times SU(2)_L \times G_R$ group in an external constant crossed field ($\vec{E} \perp \vec{H}$, $\mathcal{E} = H$).
2 The amplitude of $\nu_i \rightarrow \nu_j \gamma$ in the external crossed field

2.1 The quark-lepton interaction Lagrangian

The Lagrangian describing the interaction of the $up$–type fermion with the leptoquark has the form:

$$\mathcal{L}_X = \frac{g_s(M_X)}{\sqrt{2}} [\mathcal{U}_{iq}(\bar{\nu}_i \gamma_\alpha q^c)X^c_\alpha + h.c.] ,$$

(2.1)

where $c$ is the $SU(3)$ color index, the indices $i$ and $q$ correspond to the $up$–fermions: the neutrino states $\nu_i$ and quarks $q = u, c, t$ are the eigenstates of the mass matrix. The constant $g_s(M_X)$ can be expressed in terms of the strong coupling constant $\alpha_s$ at the leptoquark mass scale $M_X$, $g^2_s(M_X)/4\pi = \alpha_s(M_X)$. Our knowledge of the properties of the unitary matrix $\mathcal{U}$ is restricted to the fact that the product of $\mathcal{U}$ and a similar matrix $\mathcal{D}$ in the interaction Lagrangian of $down$–fermions with leptoquarks equals the standard Cabbibo-Kobayashi-Maskawa matrix: $V = \mathcal{U}^+ \mathcal{D}$.

If the momentum transferred $k^2 \ll M^2_X$, then the Lagrangian (2.1) leads to an effective four-fermion interaction of the quark-lepton vector currents. By using the Fiertz transformation, it can be presented as the interaction of the scalar, pseudoscalar, vector and axialvector lepton and quark currents:

$$\mathcal{L}_{eff} = \frac{2\pi \alpha_s(M_X)}{M^2_X} \mathcal{U}_{iq} \mathcal{U}^*_{jq} \left\{ (\bar{\nu}_j \gamma_\alpha \nu_i)(\bar{q}_q q) - (\bar{\nu}_j \gamma_5 \nu_i)(\bar{q} \gamma_5 q) \right. \right.$$

$$\left. - \frac{1}{2}(\bar{\nu}_j \gamma_\alpha \nu_i)(\bar{q} \gamma_5 q) - \frac{1}{2}(\bar{\nu}_j \gamma_\alpha \gamma_5 \nu_i)(\bar{q} \gamma_\alpha \gamma_5 q) \right\} .$$

(2.2)

In constructing the effective Lagrangian of lepton-quark interactions, we must take QCD corrections into account, which can be estimated according to well-known procedures [13]. In our case QCD corrections are reduces to the enhancement factor for the scalar and pseudoscalar couplings [16]. However, as we shall see later the scalar and pseudoscalar currents give small contribution to the ultrarelativistic neutrino decay with respect to the axial current. By this means taking into account QCD corrections isn’t essential in this case.
2.2 The amplitude of \( \nu_i \to \nu_j \gamma \) decay

The amplitude of the neutrino radiative decay in the one-loop approximation is described by the Feynman diagram, represented in Fig.1, where double lines imply the influence of the external field in the propagators of intermediate fermions \( (f = l, q; l = e, \mu, \tau, q = u, c, t) \). The field-induced contribution to the matrix element \( \Delta M \) is:

\[
\Delta M = \Delta M^{(W)} + \Delta M^{(X)},
\]

(2.3)

where \( \Delta M^{(W)} \) and \( \Delta M^{(X)} \) are the contributions from \( W \)-boson and leptoquark \( X \), respectively. The purpose of this paper is to investigate the leptoquark contribution to this process as the \( W \)-boson one was investigated in detail in the uniform magnetic field \([13]\) and in the monochromatic wave field \([14]\). The crossed field limit of the \( W \)-boson contribution to the amplitude \( \Delta M^{(W)} \) can be obtained, for example, from the matrix element \( \Delta S (i \neq j) \) \([14]\) in the regular way when the wave frequency \( \omega \) tends to zero with fixed field strengths.

The expression for the \( \Delta M^{(W)} \) corresponds to Fig.1 with \( f = l \) and can be presented in the form:

\[
\Delta M^{(W)} = \frac{eG_F}{4\sqrt{2}\pi^2} \sum_{l=e,\mu,\tau} K_{il} K_{jl}^* \\
x \left\{ e(\bar{f}f^*) \left( \frac{k F F j}{k F F k} \right) J_1(\eta_l) + \frac{e}{8m_l^2} (F \bar{f}^*) (k j) J_{2,1}(\eta_l) \right\}
\]

(2.4)
\[ j_{\mu} = \bar{\nu}_j(p') \gamma_{\mu}(1 + \gamma_5) \nu_i(p), \quad f_{\alpha\beta}^* = k_\alpha \epsilon_\beta^* - k_\beta \epsilon_\alpha^*; \]

where \( p, p', k \) and \( \epsilon \) are the 4-vectors of the momenta of the initial and the final neutrinos and the photon, and the photon polarization, respectively; \( m_l \) is the mass of the intermediate charged lepton \( l \); \( K_{il} \) is the unitary lepton mixing matrix of Kobayashi-Maskawa type; \( F_{\mu\nu} \) and \( \tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}/2 \) are the tensor and the dual tensor of the constant crossed field; \( e > 0 \) is the elementary charge; \( G_F \) is the Fermi constant. In equation (2.4) \( J_{i,\sigma}(\eta_f) \) \((i = 1, 2, 3, \sigma = \pm 1, f = l, q)\) are the integrals of the Hardy-Stokes function \( f(\eta_f) \):

\[
\begin{align*}
J_{1}(\eta_f) &= \int_0^1 dt \eta_f f(\eta_f), \\
J_{2,\sigma}(\eta_f) &= \int_0^1 dt (1 + \sigma t^2) \eta_f f(\eta_f), \\
J_{3,\sigma}(\eta_f) &= \int_0^1 dt (1 - t^2) (3 + \sigma t) \eta_f^2 \frac{df(\eta_f)}{d\eta_f},
\end{align*}
\]

\( f(\eta_f) = i \int_0^\infty du \exp \left[ -i (\eta_f u + \eta_f^3/3) \right], \)

\( \eta_f \equiv \left\{ \frac{16m_f^6}{e_f^2(kF F k)} \cdot \frac{1}{(1 - t^2)^2} \right\}^{1/3}. \)

The functions \( J_{i,\sigma}(\eta_f) \) (2.5) are presented in the general form, as the amplitude \( \Delta M^{(X)} \) we shall analyze below can be written in terms of the integrals \( J_{i,\sigma}(\eta_q) \).

Let us discuss more detail the field-induced leptoquark contribution \( \Delta M^{(X)} \) to the amplitude \( \Delta M \). In the second order of the perturbation theory, the expression for the matrix element \( \Delta M^{(X)} \) of the decay corresponding to Fig.1 with \( f = q \) has the form:

\[
\Delta M^{(X)} = -6i \pi \cdot \frac{e_q \alpha_s(M_X)}{M_X^2} \sum_{q=u,c,t} U_{iq} U_{jq}^* e^*_\beta(k) \int d^4 X e^{-iXk} \\
\times \left\{ (\bar{\nu}_j \nu_i) \text{Sp} [S(-X)S(X)\gamma_\beta] \\
- (\bar{\nu}_j \gamma_5 \nu_i) \text{Sp} [S(-X)\gamma_5 S(X)\gamma_\beta] \right\}.
\]
\[
- \frac{1}{2} (\bar{\nu}_j \gamma_\alpha \nu_i) S_p [S(-X) \gamma_\alpha S(X) \gamma_\beta] \\
- \frac{1}{2} (\bar{\nu}_j \gamma_\alpha \gamma_5 \nu_i) S_p [S(-X) \gamma_\alpha \gamma_5 S(-X) \gamma_\beta] \}
\]

where \( e_q = eQ_q \), \( Q_q \) is a relative quark charge in the loop.

The propagator of the quark \( S(X) \) in the crossed field in the proper time formalism \([17]\) can be represented in the following form:

\[
S(X) = -\frac{i}{16\pi^2} \int_0^\infty \frac{ds}{s^2} \left[ \frac{1}{2s} (X\gamma) + \frac{ie_q}{2} (X\bar{F}\gamma)\gamma_5 \right] \\
- \frac{se_q^2}{3} (XFF\gamma) + m_q - \frac{sm_q e_q}{2} (\gamma F\gamma) \right] \\
\times \exp \left( -i \left[ m_q^2 s + \frac{1}{4s} X^2 + \frac{se_q^2}{12} (XFFX) \right] \right),
\]

where \( X_\mu = (x - y)_\mu \); \( \gamma_\mu, \gamma_5 \) are Dirac \( \gamma \)-matrices (where \( \gamma^5 = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \)), \( m_q \) is the mass of the up-quark in the loop.

The integration of the amplitude (2.6) with respect to \( X \) can be reduced to the Gaussian integrals:

\[
I = \int d^4X \: e^{-i \left( \frac{1}{4} XGX + kX \right)} = -(4\pi)^2 (\det G)^{-1/2} e^{ikG^{-1}k},
\]

\[
I_\mu = \int d^4X \: X_\mu e^{-i \left( \frac{1}{4} XGX + kX \right)} = i \frac{\partial I}{\partial k^\mu},
\]

\[
I_{\mu\nu} = \int d^4X \: X_\mu X_\nu e^{-i \left( \frac{1}{4} XGX + kX \right)} = - \frac{\partial^2 I}{\partial k^\mu \partial k^\nu},
\]

\[
G_{\alpha\beta} = \frac{s + \tau}{s\tau} g_{\alpha\beta} + (s + \tau) \frac{e_f^2}{3} (FF)_{\alpha\beta},
\]

where \( s \) and \( \tau \) are the proper time variables. With (2.7) and (2.8) the invariant amplitude (2.6) can be presented in the form:

\[
\Delta M^{(X)} = 3Q_q^2 \frac{\alpha_s(M_X)}{M_X^2} \sum_q U_{iq} U^*_{jq}.
\]
\begin{align}
\times & \left\{ - \bar{\nu}_j \gamma_5 \nu_i \right. i(F^* f) 2m_q J_{2,1}(\eta_q) + (\bar{\nu}_j \gamma_5 \nu_i) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right. \\
+ & \left. \bar{\nu}_j \gamma_5 \nu_i \right) \left( \bar{F} f^* \right) \frac{m_q}{m_q} J_1(\eta_q) \right.
\end{align}

As can be readily checked, the amplitude (2.9) is evidently gauge invariant, as it is expressed in terms of the tensors of the external field \( F_{\mu\nu} \) and the photon field \( f_{\mu\nu} \). We note that this expression does not contain the suppression factor \( \sim m_q^2/M_X \ll 1 \) which is analogous to the well-known GIM suppression factor \( \sim m_q^2/M_W^2 \ll 1 \) of the decay amplitude \( \nu_i \rightarrow \nu_j \gamma \) in vacuum \([3]\). Below we analyze a physically more interesting case of the ultrarelativistic neutrino decay \( E_\nu \gg m_\nu \).

### 2.3 Ultrarelativistic neutrino \( (E_\nu \gg m_\nu) \)

Notice that in the ultrarelativistic limit the kinematics of the decay \( \nu_i(p) \rightarrow \nu_j(p') + \gamma(k) \) is such that the momentum 4-vectors of the initial neutrino \( p \) and the decay products \( p' \) and \( k \) are almost parallel to each other. As the analysis shows, the contribution due to the interaction in the axialvector currents dominates in the expression for amplitude (2.9) which can be simplified and reduced to the form:

\[ \Delta M^{(X)} \approx \frac{4 \alpha s (M_X)}{3 M_X^2} \left( \bar{\nu}_j \gamma_\alpha \nu_i \right) \left( \bar{F} f^* \right) \left( k FF \right)_\alpha \left( k FF k \right)_\alpha < J_1(\eta_q) > \]

\[ \approx \frac{8 \sqrt{2}}{3} \cdot \alpha s (M_X) \left( \epsilon^* \bar{F} p \right) \left( 1 - x + \frac{m_j^2}{m_i^2} (1 + x) \right)^{1/2} < J_1(\eta_q) >, \]

\[ < J_1(\eta_q) > = \sum_{q=u,c,t} U_{iq} U_{j\bar{q}}^* J_1(m_q). \]

In this case, the argument \( \eta_q \) of the Hardy-Stokes function \( f(\eta_q) \) in the integral \( J_1(\eta_q) \) (see Eq (2.5)) assumes the form:

\[ \eta_q = 4 \left[ (1 + x)(1 - t^2) \left( 1 - \frac{m_j^2}{m_i^2} \right) \chi_q \right]^{-\frac{2}{3}}, \]
\[ \chi_q^2 = \frac{e_q^2 (p F F p)}{m_q^6}. \]

where \( x = \cos \theta \), \( \theta \) is the angle between vectors \( \vec{p} \) (the momentum of the decaying ultrarelativistic neutrino) and \( \vec{k}' \) (photon momentum in the rest system of the decaying neutrino \( \nu_i \)). \( \chi_q^2 \) is the dynamic parameter which corresponds to the up-quark with the charge \( e_q \) and the mass \( m_q \). Notice that, as \( F F = F \tilde{F} = 0 \) in the crossed field, the dynamic parameter is the single field invariant, by which the decay probability is expressed.

3 The neutrino decay probability

The decay probability \( W^{(X)} \) due to the leptoquark contribution can be expressed in terms of the integral of the squared amplitude over the variable \( x \):

\[ W^{(X)} \simeq \frac{1}{32 \pi E_\nu} \left( 1 - \frac{m_j^2}{m_i^2} \right) \int_{-1}^{+1} dx \left| \Delta M^{(X)} \right|^2 \]

\[ = \frac{4 \alpha^2}{9 \pi} \frac{\alpha_s^2(M_X)}{E_\nu M_X^4} (p F F p) \left( 1 - \frac{m_j^2}{m_i^2} \right) \]

\[ \times \int_{-1}^{+1} dx \left[ (1 - x) + \frac{m_j^4}{m_i^4} (1 + x) \right] \left| < J_1(\eta_q) > \right|^2. \tag{3.1} \]

Using the asymptotic expansions of the Hardy-Stokes functions both at large and small values of its argument

\[ J_1(\chi_q) = O(\chi_q^2), \quad \chi_q \ll 1, \]

\[ J_1(\chi_q) = -1 + O(\chi_q^{-2/3}), \quad \chi_q \gg 1, \]

and the unitarity property of mixing matrix \( U_{ij} \), we present here the expression (3.1) in the most reasonable case of the values of the dynamic parameter \( \chi_u \gg 1, \chi_{c,t} \ll 1 \):

\[ W^{(X)} \simeq \frac{8 \alpha^2}{9 \pi} \frac{\alpha_s^2(M_X)}{E_\nu M_X^4} (p F F p) \left( 1 - \frac{m_j^4}{m_i^4} \right) |U_{iu} U_{ju}^*|^2. \tag{3.2} \]
Here the strong hierarchy of the dynamic parameter $\chi_q$ ($\chi_u : \chi_c : \chi_t = m_u^{-3} : m_c^{-3} : m_t^{-3}$) was taken into account. Because the dynamic parameter $\chi_q \sim (E_\nu/m_q)(F/F_q)$ is proportional to the neutrino energy, we can see from (3.2) that, as the energy of the decaying neutrino increases, the decay probability increases linearly. It is worth noting, that consideration of the other limit values of the dynamic parameter $\chi_t(1 - m_j^2/m_i^2) \gg 1$ (although it isn’t a physically realistic case) shows that $W^{(X)}$ becomes finally a constant:

$$W^{(X)} \simeq 28.8 \left( \frac{\alpha}{\pi} \right)^{3/2} \frac{\alpha_s^2(M_X)}{E_\nu M_X^4} m_t^2 \sqrt{p F F_p} |U_{it} U_{jt}^*|^2,$$

(3.3)

It is of interest to compare the contributions to the decay probability due to the $W$-boson and the leptoquark $X$ for the following values of the dynamic parameters $\chi_u, \chi_e \gg 1, \chi_{c,t}, \chi_{\mu,\tau} \ll 1$:

$$\frac{W^{(X)}}{W^{(W)}} \simeq \frac{16}{9} \sin^4 \Theta_W \left( \frac{\alpha_s(M_X)}{\alpha} \right)^2 \left( \frac{M_W}{M_X} \right)^4 \left| \frac{\alpha}{\pi} \right| \frac{\alpha_s^2(M_X)}{E_\nu M_X^4} m_t^2 \sqrt{p F F_p} |U_{it} U_{jt}^*|^2$$

(3.4)

$$\simeq 10^{-12} \left( \frac{100 TeV}{M_X} \right)^4 \left| \frac{\alpha}{\pi} \right| \frac{\alpha_s^2(M_X)}{E_\nu M_X^4} m_t^2 \sqrt{p F F_p} |U_{it} U_{jt}^*|^2$$

It follows from (3.4) that if the mixing in the lepton lector of the SM is strongly suppressed the leptoquark contribution $W^{(X)}$ can dominate over $W^{(W)}$.

Comparing (3.2) with the decay probability $\nu_i \to \nu_j \gamma$ in vacuum [3]:

$$W_0 \simeq \frac{27\alpha}{32\pi} \frac{G_F^2}{192\pi^3} \frac{m_i^6}{E_\nu} \left( \frac{m_\tau}{m_W} \right)^4 \left( 1 + \frac{m_j^2}{m_i^2} \right) \left( 1 - \frac{m_j^2}{m_i^2} \right)^3 \left| K_{i\tau} K_{j\tau}^* \right|^2,$$

(3.5)

we can see the catalyzing influence of the external field on the neutrino radiative decay in the $SU(4)_V \times SU(2)_L \times G_R$ Model in case of the leptoquark contribution to the decay probability:

$$\frac{W^{(X)}}{W_0} \simeq \frac{211}{9} \sin^4 \Theta_W \left( \frac{\alpha_s(M_X)}{\alpha} \right)^2 \chi_u^2 \left( \frac{m_u}{m_i} \right)^6 \left( \frac{M_W}{m_\tau} \right)^4 \left( \frac{M_W}{M_X} \right)^4 \left| \frac{\alpha}{\pi} \right| \frac{\alpha_s^2(M_X)}{E_\nu M_X^4} m_t^2 \sqrt{p F F_p} |U_{it} U_{jt}^*|^2$$

(3.6)
Such a gigantic enhancement is that the external field removes the main suppression caused by the smallness of the neutrino mass.

4 Conclusion

The influence of the external crossed field on the massive neutrino radiative decay $\nu_i \to \nu_j \gamma$ ($i \neq j$) was studied in the framework of the Minimal Quark-Lepton Symmetry of the Pati-Salam type based on the $SU(4)_V \times SU(2)_L \times G_R$ group. The matrix element $\Delta M^{(X)}$ and the decay probability $W^{(X)}$ due to the leptoquark $X$ contribution were obtained.

1. The most substantial manifestation of the catalyzing influence of the external field is that the field removes the main suppression associated with the smallness of the mass of the decaying neutrino (in vacuum this suppression is $W_0 \sim (m^6_\nu/E_\nu)^6$ for decay in flight). The decay probabilities (3.2) and (3.3) do not depend of the specific neutrino masses, if only the threshold factor $(1 - m^4_j/m^4_i)$ is not close to zero.

2. In the expressions (3.2) and (3.3) the suppression factor $\sim m^2_\gamma/M^2_X \ll 1$ which is analogous to the well-known GIM suppression factor $\sim m^2_l/M^2_W \ll 1$ of the decay $\nu_i \to \nu_j \gamma$ in vacuum [3] is absent.

3. The leptoquark contribution to the decay probability can dominate over the $W$-boson one (see eq. (3.4)) in the case of the strong suppression of the lepton mixing in the framework of the SM:

$$|U_{iq}U_{jq}^*| > 10^6 \left( \frac{M_X}{100 TeV} \right)^4 |K_{ii}K_{jj}^*|,$$

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