Lunar Laser Ranging, Gravitomagnetism and Frame-Dragging

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Abstract

During the past century Einstein’s theory of General Relativity gave rise to an experimental triumph, however, there are still aspects of this theory to be measured or more accurately tested. One of the main challenges in experimental gravitation, together with the direct detection of gravitational waves, is today the accurate measurement of the gravitomagnetic field generated by the angular momentum of a body. Here, after a description of frame-dragging and gravitomagnetism and of the main experiments to detect these relativistic phenomena, we show that the fundamental tests of General Relativity performed by Lunar Laser Ranging do not, however, include a measurement of the intrinsic gravitomagnetic field generated by the angular momentum of a body.

Dedicated to John Archibald Wheeler, a master of physics of the XX century and father of the renaissance of General Relativity
1 Introduction

A number of experiments have been proposed and performed to accurately measure the gravitomagnetic field generated by the angular momentum of a body and frame-dragging \([1, 2, 3]\), from the complex space experiment Gravity Probe B, launched by NASA in 2004 after more than 40 years of preparation \([4]\), to the observations of the LAGEOS and LAGEOS 2 satellites \([5, 6]\) and from the LARES satellite, to be launched in 2009 by ASI (Italian Space Agency) \([6]\) using the new launching vehicle VEGA of ESA (European Space Agency), to Lunar Laser Ranging \([7]\), binary pulsars \([8]\) and other astrophysical observations \([9, 10]\), including a number of other space experiments currently proposed to international space agencies.

In Einstein’s gravitational theory the local inertial frames have a key role \([11, 12, 3]\). The strong equivalence principle, at the foundations of General Relativity, states that the gravitational field is locally ‘unobservable’ in the freely falling frames and thus, in these local inertial frames, all the laws of physics are the laws of Special Relativity. The local inertial frames are determined, influenced and dragged by the distribution and flow of mass-energy in the Universe; the axes of these non-rotating, local, inertial frames are determined by torque-free test-gyroscopes that are dragged by the motion and rotation of nearby matter, for this reason this phenomenon is called dragging of inertial frames or frame-dragging \([3, 1]\).

In General Relativity, a torque-free spinning gyroscope defines an axis non-rotating relative to the local inertial frames; the orbital plane of a test particle is also a kind of gyroscope. The frame-dragging effect on the orbit of a satellite, due to the angular momentum vector \(\vec{J}\) of a central body, is known as Lense-Thirring effect: \(\vec{\Omega}_{L-T} = \frac{2GJ}{c^3a(1-e^2)^{3/2}}\), where \(\vec{\Omega}_{L-T}\) is the rate of change of the longitude of the nodal line of the satellite, that is the intersection of its orbital plane with the equatorial plane of the central body, i.e., it represents the rate of change of the orbital angular momentum vector, \(a\) is the semi-major axis of the orbiting test-particle, \(e\) its orbital eccentricity, \(G\) the gravitational constant and \(c\) the speed of light. The frame-dragging by the Earth spin has been measured using the LAGEOS satellites with an accuracy of the order of 10 percent \([5, 6]\), might be detected by further Gravity Probe B data analysis \([4]\) and will be measured with improved accuracy by the LARES satellite.
2 Lunar Laser Ranging, gravitomagnetism and geodetic precession

In General Relativity there is another type of frame-dragging effect and precession of a gyroscope known as geodetic precession or de Sitter effect [3]. If a gyroscope is at rest with respect to a non-rotating mass, it does not experience any drag. However, if the gyroscope starts to move with respect to the non-rotating mass it acquires a precession that will again disappear when the gyroscope will stop relative to the non-rotating mass. The geodetic precession, due to the velocity $\vec{v}$ of a test-gyroscope, is:

$$\vec{\Omega}_{\text{geodetic}} = \frac{3GM}{2c^2r^3}\vec{x} \times \vec{v},$$

where $M$ is the mass of the central body and $\vec{x}$ and $r$ are position vector and radial distance of the gyroscope from the central mass.

A basic difference between frame-dragging by spin and geodetic precession is that in the case of the former (the Lense-Thirring effect) the frame-dragging effect is due to the additional spacetime curvature produced by the rotation of a mass, whereas in the case of the latter (the de Sitter effect) the frame-dragging effect is due to the motion of a test-gyroscope on a static background and its motion produces no spacetime curvature, (see below and section 6.11 of ref. [3]; for a discussion on frame-dragging and geodetic precession see refs [17, 18, 1]).

The geodetic precession has been measured on the Moon’s orbit by LLR with accuracy of the order of 0.6 percent [13, 7, 14], by Gravity Probe B with approximately 1 percent accuracy [4] and has been detected on binary pulsars [15, 8].

Lunar Laser Ranging (LLR) is a basic tool for testing fundamental physics and General Relativity. By short laser pulses, the range from an emitting laser on Earth and a retro-reflector on the Moon is today measured with an accuracy of the order of a centimeter, corresponding to a fractional error in the distance of approximately $2.6 \times 10^{-11}$. In addition to the important applications of LLR for the study of the dynamics of the Earth-Moon system and of the Moon internal structure, in fundamental physics LLR has provided accurate tests of the strong and weak equivalence principle, accurate measurements of the PPN (Parametrized Post Newtonian) parameters testing General Relativity [16], experimental limits on conceivable time variations of the gravitational constant $G$ and accurate tests of the geodetic precession [7, 14].

Recently, a number of authors have debated whether the gravitomagnetic
interaction and frame-dragging by spin have also been accurately measured on the Moon orbit by Lunar Laser Ranging \cite{19,20,21,22}. This is a recent chapter of a long debate on the meaning of frame-dragging and gravitomagnetism \cite{17,18,23,24,19,20,21,22,25,3}; a basic issue treated in \cite{19,20,21} is whether the effect detected by LLR is a frame-dependent effect or not.

In order to answer to this question, we propose here a distinction between gravitomagnetic effects generated by the translational motion of the frame of reference where they are observed, e.g., by the motion of a test-gyroscope with respect to a central mass (not necessarily rotating), and those generated by the rotation of a mass or by the motion of two masses (not test-particles) with respect to each other, without any necessary motion of the frame of reference where they are observed. The geodetic precession is a translational effect due to the motion of the 'Earth-Moon gyroscope’ in the static field of the Sun. The Lense-Thirring effect measured by the LAGEOS satellites, that might also be detected by further Gravity Probe B data analysis and by LARES, is due to the rotation of a mass, i.e., by the rotation of the Earth mass. In the following we show that the gravitomagnetic effect discussed in \cite{19} is just a translational effect that is substantially equivalent to the Moon’s geodetic precession. For this purpose, a rather illuminating formal analogy of General Relativity with electrodynamics is briefly described in the next section.

3 Gravitomagnetism and Electromagnetism

Whereas in electrodynamics an electric charge generates an electric field and a current of electric charge produces a magnetic field, in Newtonian gravitational theory the mass of a body generates a gravitational field but a current of mass, for example the rotation of a body, does not produce any additional gravitational field. On the other hand, Einstein’s gravitational theory predicts that a current of mass generates an additional gravitomagnetic field that exerts a force on surrounding bodies and changes the spacetime structure by generating additional curvature.

In General Relativity, the gravitomagnetic field due to the angular momentum $\vec{J}$ of a central body is, in the weak-field and slow-motion approximation:
\[ \vec{H} = \vec{\nabla} \times \vec{h} \cong 2 \ G \left[ \frac{\vec{J} - 3(\vec{J} \cdot \hat{x}) \hat{x}}{c^3 r^3} \right] \]  

where \( r \) is the radial distance from the central body, \( \hat{x} \) is the position unit-vector and \( \vec{h} \) is the so-called 'gravitomagnetic vector potential' (equal to the non-diagonal, space and time, part of the metric). The gravitomagnetic field generates frame-dragging of a gyroscope in a way formally similar to the magnetic field producing a change of orientation of a magnetic needle (magnetic dipole). Indeed, in General Relativity, a current of mass in a loop, that is a gyroscope, has a behavior formally similar to that of a magnetic dipole in electrodynamics which is made of an electric current in a loop (see Fig. 1). The precession \( \vec{\Omega}_S \) of the spin axis of a test-gyroscope by the angular momentum \( \vec{J} \) of a central body is:  
\[ \vec{\Omega}_S = 3 \left( G \vec{J} \cdot \hat{x} \right) \hat{x} - G \vec{J} \hat{c}^2 r^2, \]  
where \( \hat{x} \) is the position unit-vector of the test-gyroscope and \( r \) its radial distance from the central body.

In electromagnetism, in a frame where a test-particle with electric charge is at rest we only observe an electric field \( E^k \) but no magnetic field, however, in a frame that is moving relative to the charge we also measure a magnetic field \( B^k \). In General Relativity, in a similar way, in a frame where a non-rotating mass is at rest, the components of the gravitomagnetic vector potential \( h^k \) are zero. Nevertheless, if we consider an observer moving relative to the mass, in a local frame moving with the observer the components of the gravitomagnetic vector potential \( h^k \) are non-zero but can of course be annulled by a Lorentz transformation back to the original frame. Indeed, in a frame where a non-rotating mass \( M \) is at rest, the only components of the Schwarzschild metric \( g_{\alpha\beta} \) different from zero (written in standard Schwarzschild coordinates) are: \( g_{00} = -g_{rr}^{-1} = -\left( 1 - \frac{2GM}{c^2 r} \right) \), \( g_{0\theta} = r^2 \) and \( g_{\phi\phi} = r^2 \sin^2 \theta \) and the three non-diagonal components of the metric \( g_{0k} \), i.e., the components of the 'gravitomagnetic vector potential' \( h^k \), are zero. Nevertheless, if we perform a local Lorentz transformation with velocity \( v^k \) relative to the mass \( M \), the components of the gravitomagnetic vector potential \( g_{0k} \) are non-zero in the new frame. The orbital effects of this gravitomagnetic vector potential, arising from motion of the Earth-Moon system relative to the Sun mass, have been observed by LLR since the first measurements of the geodetic precession of the Moon orbit, i.e., of the Earth-Moon 'gyroscope' moving around the Sun. On the other hand, the angular momentum \( \vec{J} \) of a body generates a gravitomagnetic field and produces spacetime cur-
vature that cannot be eliminated by a simple change of frame of reference or by a coordinate transformation. This gravitomagnetic field generates the Lense-Thirring effect on the orbit of the LAGEOS satellites.

In order to distinguish between 'intrinsic' gravitomagnetic effects (the Lense-Thirring effect) and 'translational' ones (the geodetic precession), we have proposed to use spacetime curvature invariants. Here, below, we show that the phenomenon discussed in [19, 21] is a translational gravitomagnetic effect. In general, one cannot derive intrinsic gravitomagnetic effects from translational ones unless making additional theoretical hypotheses, such as the linear superposition of the translational gravitomagnetic effects; for example, the magnetic field generated by the intrinsic magnetic moment (Bohr magneton) is an intrinsic phenomenon due to the intrinsic spin of a particle that cannot be explained and derived as a translational effect by any Lorentz and frame transformation.

In electromagnetism, in order to characterize the electromagnetic field, using the electromagnetic field Lorentz-tensor $F_{\alpha\beta}$ we can build the scalar Lorentz-invariant $\star F \cdot F \equiv \frac{1}{4} F_{\alpha\beta} \star F^{\alpha\beta} = \vec{E} \cdot \vec{B}$, where $\star F^{\alpha\beta}$ is the dual of $F^{\alpha\beta}$, defined as: $\star F^{\alpha\beta} = \frac{1}{2} \varepsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$ and $\varepsilon^{\alpha\beta\mu\nu}$ is the Levi-Civita pseudotensor (that is equal to $+\sqrt{-g}$, i.e., plus the square root of minus the determinant, $g$, of the metric, if the indices are even permutations of (0,1,2,3), $-\sqrt{-g}$ for odd permutations of (0,1,2,3) and 0 if any indices are repeated). $\star F \cdot F$ is an invariant for Lorentz transformations (precisely a pseudo-invariant under coordinate reflections), i.e., is either null or not in every inertial frame. For example, in the rest frame of a test-particle with charge $q$ we have an electric field only and no magnetic field, and this invariant is zero, therefore even in a frames moving relative to $q$, where both $\vec{B} \neq \vec{0}$ and $\vec{E} \neq \vec{0}$, this invariant is zero. However, in a frame where a charge $q$ and a magnetic dipole $\vec{m}$ are at rest, we have in general $\star F \cdot F \neq 0$ and therefore this invariant is non-zero in any other inertial frame.

In General Relativity, the gravitomagnetic 'vector' potential $h^k$ can be zero or not depending on the frame where it is calculated. Nevertheless, the curvature of a manifold is a coordinate independent quantity [11, 12, 3]. Therefore, in order to test for intrinsic gravitomagnetic effects, i.e., independent of the coordinate system (and not eliminable with a coordinate transformation) we have to use the Riemann curvature tensor $R_{\alpha\beta\mu\nu}$ and the spacetime invariants built with it [25, 3]. Given a metric $g_{\alpha\beta}$ in some coordinate system (with or without the so-called 'magnetic' components $g_{0k}$), in a way
similar to electromagnetism, using the Riemann curvature tensor $R_{\alpha\beta\mu\nu}$ we can build the spacetime curvature invariant $*R \cdot R \equiv *R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu}$, where $*R^{\alpha\beta\mu\nu} \equiv \frac{1}{2} \varepsilon^{\alpha\beta\sigma\rho} R_{\sigma\rho \mu\nu}$ is the dual of $R_{\alpha\beta\mu\nu}$. Here below and in [3] the exact explicit expression of the Riemann curvature invariant $*R \cdot R$ is given for some spacetime solutions of the Einstein field equation. For example, in the case of the Kerr metric generated by the angular momentum $J$ and the mass $M$ of a rotating body, this invariant (precisely a pseudo-invariant under coordinate reflections) is equal to [30]:

$$\frac{1}{2} \varepsilon^{\alpha\beta\sigma\rho} R_{\sigma\rho \mu\nu} R_{\alpha\beta\mu\nu} = 1536 \frac{J \cdot M}{r^5} \cos \theta \left( r^5 \rho^{-6} - r^3 \rho^{-5} + \frac{3}{16} r^3 \rho^{-4} \right)$$

where $\rho = \left( r^2 + \left( \frac{J}{\sqrt{M^2}} \right)^2 \cos^2 \theta \right)$; this expression of $*R \cdot R$ is then different from zero if and only if $J \neq 0$, e.g., it is zero in the case of the Schwarzschild metric generated by the mass only of a non-rotating body (with $J = 0$). In the case of Earth with angular momentum $J\oplus$, the invariant $*R \cdot R$ is at the lowest order: $*R \cdot R \simeq 288 \frac{G^2 J \cdot M\oplus}{c^5 \rho^7} \cos \theta + ...$, where $\theta$ is the colatitude, thus the Lense-Thirring effect on the LAGEOS satellites is an intrinsic gravitomagnetic effect [25, 3] that cannot be eliminated by a change of frame of reference. However, in the next section we show that the effect discussed in [29, 19], accurately measured by Lunar Laser Ranging, is just a 'translational' gravitomagnetic effect which depends on the frame of reference used in the analysis; the invariant $*R \cdot R$ is indeed null on the ecliptic plane (apart from the intrinsic gravitomagnetic terms due to $J\oplus$ and $J\odot$) and the gravitomagnetic term discussed in [29, 19], when analyzed in a different frame, is substantially equivalent to the geodetic precession.

4 Lunar Laser Ranging and gravitomagnetic effects

In [29, 19] is analyzed a gravitomagnetic perturbation of the Moon orbit consisting in a change of the Earth Moon distance of about 5 meters with monthly and semi-monthly periods. This variation of the Earth Moon distance is, in the Moon’s geodesic equation of motion, due to the gravitomagnetic acceleration [29]:

$$\ddot{a}_I = \frac{4}{c^2} \sum_{j \neq I} \ddot{v}_I \times (\ddot{v}_J \times \ddot{G}_{IJ}),$$

(3)
where the index $I$ indicates the Moon and $J$ the Sun and Earth, $\vec{v}_I$ and $\vec{v}_J$ are their velocities, $\vec{a}_{IJ} = \frac{GM_J}{r_{IJ}} \vec{r}_{IJ}$ is the standard Newtonian acceleration vector, $\vec{r}_{IJ}$ the position vector from body $I$ to body $J$ and $\vec{r}_{IJ} = |\vec{r}_{IJ}|$.

In a frame of reference comoving with the Sun, we find the gravitomagnetic acceleration (2) of the Moon:

$$\frac{4}{c^2} \vec{v}_{M}^{(S)} \times (\vec{v}_E^{(S)} \times \vec{G}_M^{(S)})$$

i.e., the term analyzed in [29, 19], where the upper letter within parenthesis indicates if the corresponding quantity is measured with respect to the Sun: $(S)$, or to Earth: $(E)$, i.e., $\vec{v}_M^{(E)}$ is the velocity of the Moon with respect to Earth and $\vec{v}_E^{(S)}$ is the velocity of Earth with respect to the Sun. In the Moon’s equation of motion there is another gravitomagnetic term due to the velocity of the Earth-Moon system around the Sun:

$$2 \vec{\Omega}_{geodetic} \times \vec{v}_M^{(E)}$$

where $\vec{\Omega}_{geodetic} = \frac{3GM_\odot}{2c^2R^3} \vec{R} \times \vec{v}_E^{(S)}$ is the geodetic precession of a gyroscope comoving with Earth and $R$ is the radial distance of Earth from the Sun.

The discussion of the effect of term (2) on the Moon orbit and its interpretation to be equivalent to the intrinsic gravitomagnetic effects generated by the angular momentum of a central body [29], are based and have been carried out by writing the Moon’s equation of motion in a frame comoving with the Sun (whereas the orbit of the Moon is measured in a frame comoving with Earth, i.e., the ‘observable’ quantity is the round-trip travel time of laser pulses from Earth to Moon measured on Earth). To elucidate with a simple example that the interpretation of term (2) depends on the velocity of the frame of reference where the calculations are performed, let us for example consider a single mass only, e.g., $M = M_\odot$, i.e., the mass of Earth. In the weak-field and slow-motion approximation of General Relativity, the corrections to the Newtonian gravitational theory of order $\frac{v^2}{c^2} \sim \frac{GM}{c^2r} << 1$ are described by the so-called ‘post-Newtonian’ metric [16]. Let us then consider the post-Newtonian expression of the Schwarzschild metric generated by $M_\odot$. This post-Newtonian metric can simply be obtained by expanding the Schwarzschild metric at the lowest post-Newtonian order in $\frac{2GM}{c^2r}$. Let us then perform a local Lorentz transformation with velocity $v^k$, in the new local frame we have the non-zero gravitomagnetic metric components: $g_{0k} \sim \frac{(GM_\odot v_k)}{c^2r}$. In this new frame moving with velocity $v^k$ with respect to
using the geodesic equation of motion of a test-particle, e.g., the Moon, we then find the term \( \sim \frac{1}{c^2} \vec{v}_M \times \vec{v} \times \vec{G}_{M\oplus} \), i.e., the frame-dependent term (2) of the Moon’s equation of motion, that is different from zero in this moving frame and is, however, zero in a frame at rest relative to \( M_{\oplus} \). This example shows that the gravitomagnetic acceleration discussed in [29, 19], i.e., the term (2), is a frame-dependent effect: when we go back to the original frame, where the mass \( M_{\oplus} \) is at rest, this term of the Moon acceleration is zero.

Let us now consider the post-Newtonian metric generated by the masses of both Sun and Earth, the non-diagonal components, \( g_{0k} \), of this metric, i.e., the components of the gravitomagnetic ‘vector’ potential, are [16]:

\[
g_{0k} = \frac{7}{2} \frac{GM_{\oplus} v_{\oplus k}}{c^3 r_{\oplus}^3} + \frac{1}{2} \frac{GM_{\oplus} v_{\oplus k} (\vec{r}_{\oplus} \cdot \vec{v}_{\oplus})}{c^3 r_{\oplus}^3} + \frac{1}{2} \frac{GM_{\odot} v_{\odot k} (\vec{r}_{\odot} \cdot \vec{v}_{\odot})}{c^3 r_{\odot}^3} + \frac{1}{2} \frac{GM_{\odot} v_{\odot k} (\vec{r}_{\odot} \cdot \vec{v}_{\odot})}{c^3 r_{\odot}^3}.
\]

In a frame of reference comoving with the Sun, the components of the gravitomagnetic ‘vector’ potential are then: \( g_{0k} = \frac{7}{2} \frac{GM_{\oplus} v_{\oplus k}}{c^3 r_{\oplus}^3} + \frac{1}{2} \frac{GM_{\oplus} v_{\oplus k} (\vec{r}_{\oplus} \cdot \vec{v}_{\oplus})}{c^3 r_{\oplus}^3} \) and by using this gravitomagnetic potential in the Moon’s geodesic equation of motion, we find the acceleration (3), i.e., the gravitomagnetic acceleration (2) of the Moon written in a frame comoving with the Sun.

Nevertheless, the interpretation of the nature and meaning of term (2) is simple in a frame comoving with Earth, where its measurable effect is equivalent to the geodetic precession. Indeed, in a geocentric frame of reference the gravitomagnetic ‘vector’ potential components are:

\[
g_{0k} = \frac{7}{2} \frac{GM_{\odot} v_{\odot k}}{c^3 r_{\odot}^3} + \frac{1}{2} \frac{GM_{\odot} v_{\odot k} (\vec{r}_{\odot} \cdot \vec{v}_{\odot})}{c^3 r_{\odot}^3}
\]

and by using this gravitomagnetic potential in the Moon’s geodesic equation of motion, we then find the Moon’s gravitomagnetic acceleration:

\[
\frac{4}{c^2} \vec{v}_M \times (\vec{v}_M \times \vec{G}_{M\odot}) \approx \frac{4}{c^2} \vec{v}_M \times [\vec{v}_M \times \vec{G}_{M\odot} \left( \vec{R} - \vec{r}_{M\oplus} \right)] = 0
\]

\[
= \frac{4}{c^2} \left[ GM_{\odot} \left( \vec{R} \times \vec{v}_M^{(S)} \right) \right] \times \vec{v}_M - \frac{4}{c^2} \vec{v}_M \times (\vec{v}_M \times \vec{G}_{M\odot} \vec{r}_{M\oplus})
\]

(6)

this acceleration of the Moon corresponds to the acceleration (2) but it is written in a frame comoving with Earth: the first term of the last expression of (6) is clearly equivalent to the geodetic precession (5), apart from a numerical factor, and the second term is today too small to be measured; of course, in order to describe the Moon orbit, one must also include all the other terms in the Moon’s equation of motion, however, here we have only been interested in analyzing the nature of the term (2) discussed in [29].
A second argument shows that the interpretation of the term (2) as an intrinsic gravitomagnetic effect is in fact frame-dependent. On the other hand, in the case of a spacetime geometry generated by both the angular momentum and the mass of a body (e.g., the Kerr metric), the gravitomagnetic effects generated by the angular momentum, for example that of Earth, are intrinsic to the spacetime geometry and cannot be eliminated by a coordinate transformation, indeed the angular momentum generates spacetime curvature, as manifestly displayed by the curvature invariants.

This can rigorously be shown by using the curvature invariant \( ^*R \cdot R \), described in section 3, formally similar to the electromagnetism invariant \( ^*F \cdot F = \vec{E} \cdot \vec{B} \). In the case of a point-mass metric generated by Earth and Sun (neglecting the angular momenta of Earth and Sun that produce effects that are presently unmeasurable on the Moon orbit) this invariant is: \( \sim \vec{G} \cdot \vec{H} \), where \( \vec{G} \) is the standard Newtonian electric-like field of Sun and Earth and \( \vec{H} \) is the magnetic-like gravitational field; for a test-particle moving with velocity \( \vec{v} \), similarly to electrodynamics, this magnetic-like gravitational field is \( \sim \vec{v} \times \vec{G} \) and then, for any motion on the ecliptic plane, the invariant \( ^*R \cdot R \) is zero. Indeed, its expression, as calculated \([30]\) in quasi-cartesian coordinates in a frame with origin at the Sun (\( x \) and \( y \), and \( z \) are respectively the coordinates on and off the ecliptic plane), is at the lowest order in \( G/c^2 \):

\[
^*R \cdot R \simeq 288 \frac{G^2 M_\oplus M_\odot}{c^5 r_M^4} r_\odot^4 \left( v_\odot^y y_\odot - v_\odot^x x_\odot \right) (\hat{x}_M \cdot \hat{r}_M) \tag{7}
\]

where \( \vec{x}_\odot \) and \( \vec{x}_M \) are the position vectors of Earth and Moon from the Sun and \( z_M \) is the distance of the Moon from the ecliptic plane; this expression is then: \( ^*R \cdot R = 0 \) on the ecliptic plane (even by considering that the Moon orbit is slightly inclined of 5 degrees on the ecliptic plane, its \( z \) component would only give a contribution to the change of its radial distance from Earth of less than 1 % of the total change discussed in \([29]\)).

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6 References

References

[1] Ciufolini I 2007 Nature 449 41-48

[2] Thorne K S, Price R H and Macdonald D A 1986 The Membrane Paradigm (Yale Univ. Press, NewHaven)

[3] Ciufolini I and Wheeler J A 1995 Gravitation and Inertia (Princeton Univ. Press, Princeton, New Jersey)

[4] GRAVITY PROBE-B update at: http://einstein.stanford.edu/

[5] Ciufolini I and Pavlis E C 2004 Nature 431 958-960

[6] Ciufolini I et al 2008 Proc. of the First International School of Astrophysical Relativity ”John Archibald Wheeler”: Frame-Dragging, Gravitational-Waves and Gravitational Tests (Erice, Italy, 2006) Ciufolini I and Matzner R eds. (Springer)

[7] Williams J G, Turyshev S G and Boggs D H 2004 Phys. Rev. Lett. 93 261101-1-4

[8] Stairs I H, Thorsett S E and Arzoumanian Z 2004 Phys. Rev. Lett. 93 141101-1-4

[9] Nordtvedt K 1988 Int. J. Theor. Phys. 27 1395-1404

[10] Cui W et al 1998 Astrophysical J. 492 L53-L58

[11] Misner C W, Thorne K S and Wheeler J A 1973 Gravitation (Freeman, San Francisco)

[12] Weinberg S 1972 Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity (Wiley, New York)

[13] Bertotti B, Ciufolini I and Bender P L 1987 Phys. Rev. Lett. 58 1062-5

[14] Williams J G, Newhall X X and Dickey J O 1996 Phys. Rev. D 53 6730-6739
[15] Weisberg J M and Taylor J H 2002 *Astrophysical J.* **576** 942-949

[16] Will C M 1993 *Theory and Experiment in Gravitational Physics* 2nd edn (Cambridge Univ. Press, Cambridge, UK)

[17] Ashby N and Shahid-Saless B 1990 *Phys. Rev. D* **42** 1118-22

[18] O’Connell R F 2005 *Class. Quantum Grav.* **22** 3815-16

[19] Murphy T W Jr, Nordtvedt K and Turyshev S G 2007 *Phys. Rev. Lett.* **98** 071102-1-4

[20] Kopeikin S M 2007 *Phys. Rev. Lett.* **98** 229001

[21] Murphy T W Jr, Nordtvedt K and Turyshev S G 2007 *Phys. Rev. Lett.* **98** 229002

[22] Ciufolini I 2007 arXiv:0704.3338v2 [gr-qc]; see also Pavlis E and Ciufolini I 2006 *Proc. of 15th International Laser Ranging Workshop (Camberra, Australia, October 16-20, 2006)*

[23] Barker B M and O’Connel R F 1979 *Gen. Rel. Grav.* 11, 149-175

[24] Khan A R and O’Connell R F 1976 *Nature* **261**, 480

[25] Ciufolini I 1994 *Class. Quantum Grav.* **11** A73-A81

[26] Schäfer G 2004 *J. Gen. Rel. and Grav.* **36** 2223-35

[27] Ciufolini I 1986 *Physical Review D* **34** 1014-1017

[28] Ciufolini I and Demianski M 1986 *Physical Review D* **34** 1018-1020

[29] Nordtvedt K 2005 *Proc. of the School “Gravitation: from the Hubble Length to the Planck length (Frascati, Italy, 2002) Ciufolini I et al eds.* (IOP)

[30] The curvature invariants have been calculated using MathTensor, a system for doing tensor analysis by computer by L Parker and S M Christensen
Fig. 1 In panel a, I show the gravitomagnetic field $\vec{H}$ generated by the spin $\vec{J}$ of a central body and the dragging of an inertial frame of reference whose axes are determined by test-gyroscopes. In panel b, I show how to get a measurement of the spacetime curvature and of the gravitomagnetic field by local measurements only. One first measures the relative accelerations, $\ddot{\delta}x^\alpha$, between a number of test-particles which follow spacetime geodesics then, using the geodesic deviation equation \[11, 12, 3\], one obtains the spacetime curvature, i.e., one determines all the components of the Riemann tensor, $R_{\alpha\beta\mu\nu}$. Finally, using the Riemann tensor components, one obtains the spacetime invariant discussed in the text that is a function of the angular momentum and, in general, of the mass-energy currents. In electromagnetism, using the Lorentz force equation, in order to measure the six independent components of the electromagnetic field tensor $F^{\alpha\beta}$, one needs to use at least two test-particles endowed with electric charge [11]. In General Relativity, in order to measure the twenty independent components of the spacetime curvature, i.e., of the Riemann tensor, the minimum number of test-particles to be used is six, however, in vacuum, in order to measure the ten independent components of the spacetime curvature, is sufficient to use four test-particles [27, 28].
