Cross-Nucleation between Polymorphs: Quantitative Modeling of Kinetics and Morphology

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ABSTRACT: Cross-nucleation is defined as the nucleation of one polymorph on the surface of another polymorph of the same substance. Although the description of this particular form of heterogeneous nucleation is mainly phenomenological, recently dedicated quantitative studies are performed on several systems. In this work we propose a model framework that captures the phenomenon of cross-nucleation for a spherulitic seed-surface geometry, as well as the kinetic competition between the seed growth and the cross-nucleus formation, by the introduction of a tangential growth rate of the daughter polymorph. Regardless of the growth rate of the parent spherulite, this model describes the experimental data up to and including the final amount of cross-nuclei on its periphery, solely based on one parameter, the cross-nucleation rate. Furthermore, a strong temperature dependency of the kinetic competition between concomitantly growing α- and β-phase isotactic polypropylene is observed and related to the previously reported anomalous behavior of this cross-nucleating system.

INTRODUCTION

The phenomenon of cross-nucleation is a peculiar crystalization pathway, of interest for polymorphic substances. 1–8 In cross-nucleation, a polymorph has the ability to nucleate on the periphery of another polymorph of the same material, contrary to classical heterogeneous nucleation, where the seed-substrate is an alien substance. Generally the direction of cross-nucleation is indicated by defining the two polymorphs as “parent” and “daughter”. Provided that the crystal growth rate of the nucleating daughter is equal or larger than that of the parent, a nucleus can grow to a detectable size, despite the difference in thermal stability. 2,5,9 Various examples of both stable-on-metastable and metastable-on-stable can be found in the literature. 2,5,9 If the frequency of this special kind of heterogeneous nucleation is sufficiently high, the seed surface will eventually be overgrown by daughter-phase cross-nuclei.

Despite several experimental studies on organic systems 1,12–13 and macromolecules, 14–18 and numerical studies on spherical particles, 5,8 clathrate hydrates 19 and water, 20 the description of cross-nucleation is mainly phenomenological. In most cases, the cross-nucleation rate is experimentally determined by an oversimplification of the process, fitting a linear model to the data. 3,15,21 Commonly, the heterogeneous nucleation rate decreases with decreasing undercooling, 22,23 and most cross-nucleating systems tend to behave accordingly; α-on-γ polypivalolactone (PPVL), 21 α-on-δ D-mannitol, 10 and Form II-on-Form I isotactic polybutene (i-PBu) 17,18 all show a decreasing frequency of cross-nucleation when approaching the melting point. In our previous work, 24 we discussed the apparent contradictory case of isotactic polypropylene (i-PP), where the nucleation of the monoclinic α-phase on the pseudohexagonal β-phase occurs with increasing frequency for temperatures exceeding 140 °C. 14,24,25

It was argued by Yu et al. 10 that the growth rate of the parent polymorph affects the cross-nucleation kinetics. As epitaxial matching between cross-nucleating structures does not seem to be important for the phenomenon, 2,11,25 a hindering effect of the homopolymorphic secondary nucleation (i.e., growth of the parent phase) on the heteropolymorphic nucleation is suggested. On the basis of the radial growth rates of parent (Gp) and daughter (Gd), this hypothesis seems probable; the ratio of Gd/Gp in the temperature window where cross-nucleation is observed is close to 1 for the case of i-PP, 24 while it ranges from 2 to 1000 in other systems where cross-nucleation data are available. 10,26–28

In the present work, we propose a model framework which confirms that the competition between parent-phase growth and daughter-phase cross-nucleation is more pronounced in i-PP, as compared to other systems. Furthermore, by taking into account this growth competition, the observed cross-nucleation phenomenon is quantitatively described both from the morphological and kinetic point of view.

Received: February 15, 2018
Revised: May 8, 2018
Published: June 4, 2018
EXPERIMENTAL SECTION

Our modeling is based on several quantitative experimental studies reported in the literature. The growth rate data of PPVL is taken from Alfonso et al.26 i-PBu from Yamashita,27,28 D-mannitol from Yu et al.,3 and i-PP from our previous work.24 The cross-nucleation data is taken from Yu et al.3 for D-mannitol and from our previous work24 for i-PP.

On the morphological level, three ratios of $G_p/G_d$, are considered, as indicated in Figure 1. If the growth of the parent is larger than that of the daughter, no cross-nucleation will be observed due to the necessary kinetic constraint.6,5 Contrary, if the daughter polymorph grows considerably faster than the parent modification, hemispherical nuclei form on the periphery of the seed surface and are able to grow freely, since no hindering by the parent growth is present. The most complex yet interesting case is that where the growth rates are comparable; cross-nuclei grow in a “flower-like” shape on the growing crystal, see Figure 1.

In each of the three cases described above, a tangential growth rate of the daughter is defined as the rate at which the “intersection” of parent and daughter phase moves parallel to the parent’s surface, indicated with $G_{d,tan}$ in Figure 1. From a modeling perspective, this tangential growth rate is the key parameter for an accurate determination of the parent overgrowth.

MODELING

The starting point is the generic equation, eq 1, where the time derivative of specific, i.e., per unit area, number of nuclei, $n$, is defined as the cross-nucleation rate $J$. Being strongly dependent on the crystallization temperature, the cross-nucleation rate can decrease with decreasing undercooling, in agreement with classical nucleation theory22,23 or increase, as is observed for the case of i-PP.14,24,25 The temperature dependence of the cross-nucleation rate $J$ is elaborately discussed in our previous work.24

\[ \dot{n} = J(T) \]  

Experimentally the study of cross-nucleation is limited to disk-like parent superstructures to ensure a proper detectability of every cross-nucleus. Hence, for a growing disk with initial seed surface area $A_0$, the time evolution of the undisturbed surface is given by

\[ A_p = A_0 + 2\pi h \int_{t_0}^{t} G_p(u) \, du \]

\[ = 2\pi h R_0 + 2\pi h \int_{t_0}^{t} G_p(u) \, du \]  

Figure 1. Schematic representation of the three regimes determined by the ratio of parent and daughter radial growth rates.

where $R_0$ denotes the radius of the parent seed at time zero ($t_0$) and $h$ is the sample thickness. The undisturbed area of the parent overgrown by the daughter phase depends on the momentary number of nuclei and the tangential growth rate ($G_{d,tan}$) of the daughter according to

\[ A_d = \int_{t_0}^{t} \dot{n}(t') \left( 2h \int_{t'}^{t} G_{d,tan}(u) \, du \right) \, dt' \]  

The time evolution of the experimentally observed, i.e., real number of nuclei, $\dot{n}_p$, is given by

\[ \dot{n}_p = (A_p - A_d) \frac{\dot{n}_p}{(A_p - A_d)} \]

This differential equation can be solved by calculating the macroscopic tangential growth rate as a function of time. In this case $G_{d,tan}$ is calculated from the two intersection points of two circular objects, one being the parent, and one representing the growing nucleus, nucleated at time $t'$, as indicated in Figure 2a.

The growth rate parallel to the parent surface is then given by

\[ G_{d,tan} = \frac{d}{dt} \left[ \frac{1}{2} \arccos \left( \frac{\vec{S}_1 \cdot \vec{S}_2}{||\vec{S}_1|| ||\vec{S}_2||} \right) \right] (R_0 + G_p t) \]

\[ = \frac{d}{dt} \left[ \frac{1}{2} \arccos \left( \frac{\vec{S}_1 \cdot \vec{S}_2}{||\vec{S}_1|| ||\vec{S}_2||} \right) \right] \]

where $\vec{S}_1$ and $\vec{S}_2$ denote the intersection points of parent and daughter polymorph. This tangential growth rate is a complex function of the sizes of parent and daughter, and could be solved numerically. However, a simplified, time-independent solution to eq 5 is given when the seed surface is considered flat, see Figure 2b:

\[ G_{d,tan,flat} = \frac{\sqrt{G_p^2 - c_p^2}}{2} \]

The tangential growth rate of the daughter polymorph is in this case solely determined by the individual radial growth rates of the parent and daughter polymorph. This approximation is valid in the regime where the size of the cross-nucleating phase is small compared to the radius of the parent spherulite.

Numerical determination of the tangential growth rate as a function of time shows, eq 5, for the systems considered, a nearly linear time-dependency, especially for the higher temperatures. In Figure 3 the tangential growth rate of the daughter polymorph of i-PP is shown as a function of time. By the introduction of a mean tangential growth rate $G_{d,tan,mean}$ eq 3 can, to a good approximation, be rewritten as

\[ A_d = 2h \bar{G}_{d,tan,mean} \int_{t_0}^{t} \dot{n}(t') \, dt' \]

\[ = 2h \bar{G}_{d,tan,mean} \dot{n}_p t \]  

Figure 2. Schematic representation of (a) the vectors $\vec{S}_1$ and $\vec{S}_2$ used in eq 5 to determine the overgrown area of parent spherulite, and (b) a simplification the tangential growth rate in the case that a flat seed surface is assumed, eq 6.
Substitution of eq 2 and 7 into eq 4, makes that the phenomenon of cross-nucleation is defined by the differential equation:

\[ \dot{n}_t + (an_t - b)t = c \]  

with:

\[ a = 2\pi h G_{d,tan} \]
\[ b = 2\pi h G_p \]
\[ c = 2\pi h R_0 \]

which can be solved analytically under the condition that \( n \geq 0 \). The physical interpretation of this constraint implies the number of cross-nuclei can only grow for the time that there is area available; a situation where \( A_d > A_p \) is not possible. In the time where cross-nucleation occurs, the exact solution to eq 8 is given by

\[ n(t) = \frac{b}{a} \left[ 1 - \exp \left( -\frac{at^2}{2} \right) \right] + \frac{c\sqrt{\pi/2}}{\sqrt{a}} \exp \left( -\frac{at^2}{2} \right) \operatorname{erfi} \left( \frac{\sqrt{a}t}{\sqrt{2}} \right) \]  

(9)

where the first term corresponds to the homogeneous solution that arises when the initial seed area equals zero. The second term, the particular solution, accounts for the consumption of seed surface by daughter overgrowth.

The key parameter in this model framework is \( G_{d,tan} \), the tangential growth of the daughter. In the next section we show that by computing this parameter, we can quantitatively describe the number density of cross-nuclei in time for materials with widely different polymorphic growth and nucleation kinetics.

### RESULTS AND DISCUSSION

Experimental studies on cross-nucleation in different systems \(^3,10,17,18,21,24\) revealed the concomitant growth of the two growing modifications on the macroscopic level. The angle of the phase boundary, i.e., the angle between parent and daughter polymorph at the intersection point, significantly changes with temperature in the case of i-PP, whereas for PPVL (and also for i-PBu\(^17\) and D-mannitol\(^3\)) this angle is almost temperature independent, as is shown in Figure 4. For i-PP, Figure 4a, only slightly above the crossover temperature of the radial growth rates, the angle (defined as \( \theta \)) is small, because the ratio of the growth rates is close to 1. Upon a slight temperature increase, the difference in growth rate between the crystal phases increases, and as a result \( \theta \) increases. In PPVL on the other hand, the angle remains constant over a temperature range of 20 °C; see Figure 4b. A nearly constant angle for PPVL indicates a temperature-independent ratio between the \( \alpha \) and \( \gamma \) modification growth rates, while in i-PP the angle increases with temperature because of the increasing ratio of the growth rates \( G_{d}/G_p \).

By applying the flat surface approximation of a growing seed surface competing with a cross-nucleus, this morphological effect of the polymorph’s growth rates is numerically calculated and shown in Figure 5. For each of the materials considered,
the coordinates of one of the intersection points is plotted for a unit time in a Cartesian coordinate system for various temperatures. The origin of the axes is the cross-nucleation point. The insets show the radial growth rates of the parent and daughter, together with the temperature range in which cross-nucleation is observed (gray area).

Contrary to other materials, i-PP displays cross-nucleation in a temperature window where the growth rates are very comparable. Reconsidering eq 6, it can be seen that when the divergence of the individual growth rates of the polymorphs is more important than their absolute decrease with temperature, the tangential growth rate of the daughter may increase with decreasing undercooling. This is the case for i-PP at temperatures just above the crossover temperature of the radial growth rates (140 °C), see Figure 5a. Upon further

Figure 5. Location of the parent/daughter intersection point as indicated in Figure 1 in a unit time for (a) isotactic polypropylene, (b) isotactic polybutene, (c) polypivalolactone, and (d) D-mannitol at different temperatures. The individual growth rates of parent and daughter are shown in the insets, together with the temperature range where cross-nucleation is observed (gray area).

Figure 6. Ratio of the tangential growth rate and the radial growth rate as a function of temperature.

Figure 7. Comparison between the cross-nucleation model, based on the average tangential growth rate of α-phase i-PP, and the literature data.
increasing the temperature, the tangential growth rate of the \( \alpha \)-daughters goes through a maximum.

For i-PBu, where the growth rate of the parent is really negligible as compared to the daughter, the latter grows freely along the (flat) seed surface, at a rate of \( G_{\alpha,tan} = G_{\beta,tan} \). See Figure 5b.

PPVL and d-mannitol show a comparable behavior, except for the temperature coefficient of the growth rates. From Figure 5c,d it can be deduced that the competition between the growth of seed and cross-nucleus, and thus the initial angle at which a cross-nucleus grows, is practically temperature independent, or at least far less temperature dependent as in \( \alpha \)-on/\( \beta \)-cross-nucleation in i-PP.

An effective way of describing this competition between the concomitantly growing polymorphs is the ratio of the tangential and radial growth rate of the daughter polymorph, \( G_{\alpha,tan} / G_{\alpha} \). This ratio is shown in Figure 6 as a function of temperature for the various systems. For i-PBu, PPVL and d-mannitol, \( G_{\alpha,tan} / G_{\alpha} \) is close to 1, and more importantly, constant, implying a temperature-independent competition. On the other hand, i-PP displays a steep increase from 0 to 1 in the temperature range where cross-nucleation is observed, as a result of the previously discussed maximum in the tangential growth rate of the \( \alpha \)-daughters, see Figure 5a.

Among the investigated systems, i-PP is the only one which displays an anomalous temperature dependence of cross-nucleation kinetics, i.e., the \( \alpha \)-on/\( \beta \)-nucleation rate increases with decreasing undercooling. In our previous work,\(^{24}\) we rationalized this observation by proposing a temperature-dependent probability of growing a cross-nucleus to a detectable size. This probability is lower when the competition between seed and daughter-phase growth is more important. We note that, since \( G_{\alpha,tan} / G_{\alpha} \) quantifies this growth competition, it might reasonably be related to such probability. However, a direct link between this geometrically derived parameter \( G_{\alpha,tan} / G_{\alpha} \) and the anomalous temperature dependence of cross-nucleation reported for i-PP, cannot be established at this stage. In fact, this would require detailed hypotheses on the molecular mechanism of nucleation between polymorphs and is out of the scope of the present work.

The use of an average tangential growth rate to compute the consumption of seed surface, eq 7, is validated by comparing the exact solution given in eq 9 with the available experimental data on cross-nucleation.

In Figure 7 the average number of i-PP \( \alpha \)-phase cross-nuclei on a given \( \beta \)-phase seed (\( R_0 = 100 \mu m \) and \( h = 30 \mu m \)) is shown as a function of time. On the basis of the cross-nucleation rates reported in our previous work,\(^{24}\) a remarkable agreement between the model and experimental data is obtained, in the whole temperature range where the \( \alpha \)- and \( \beta \)-phase grow concomitantly. In particular, when the experimental data is collected for sufficiently long times, the saturation of the parent surface with cross-nuclei is accurately predicted on the basis of the (bulk) radial growth rates of the individual phases.

A further demonstration of the applicability of the model is shown in Figure 8 for pure d-mannitol and a d-mannitol/polyvinylpyrrolidone mixture.\(^3\) In both these systems the crystallization is not seeded, and the cross-nucleation rate decreases with undercooling, contrary to the case of i-PP. Irrespective of the temperature dependence of the cross-nucleation rate, the parent overgrowth in the late stages of the process is accurately captured. The initial slope of the exact solution, being the cross-nucleation rate \( J_{n,\alpha} \), corresponds well with the values determined from the linearized approach used in the original experimental works.\(^{3,24}\)

Since this model framework can accurately predict the saturation value of cross-nuclei on a parent polymorph of given dimensions, it can be applied to derive the kinetic of cross-nucleation indirectly from the final morphology: by measuring the saturation density of cross-nuclei on a seed surface, the cross-nucleation rate can be determined implicitly when the radial growth rates are known.

**CONCLUSION**

For various systems, the kinetic competition between two concomitantly growing polymorphs is modeled based on radial growth rates taken from the literature. A combination of these growth rates and the seed geometry allows one to calculate the time-evolution of daughter-overgrown area.

Irrespective of the temperature dependence of the cross-nucleation rate, the proposed model can be used to predict the final number of cross-nuclei on a parent spherulite of given dimensions. The implementation of this parent overgrowth allows for a quantitative description of the average number of nuclei in time, solely based on one independent parameter, i.e., the cross-nucleation rate. When this strategy is inverted, the
cross-nucleation rate can be determined simply from the saturation density of daughter-phase nuclei on the parent seed. Furthermore, the importance of growth competition between the polymorphs is quantified by the ratio of the daughter growth rates in the direction parallel and perpendicular to the seed surface. Contrary to the other systems, this ratio varies strongly with temperature for i-PP, for which an inverse temperature dependence of the cross-nucleation rate is also observed.

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**Notes**

The authors declare no competing financial interest.

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