Axion Production from Landau Quantization in the Strong Magnetic Field of Magnetars

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Abstract

We utilize an exact quantum calculation to explore axion emission from electrons and protons in the presence of the strong magnetic field of magnetars. The axion is emitted via transitions between the Landau levels generated by the strong magnetic field. The luminosity of axions emitted by protons is shown to be much larger than that of electrons and becomes stronger with increasing matter density. Cooling by axion emission is shown to be much larger than neutrino cooling by the Urca processes. Consequently, axion emission in the crust may significantly contribute to the cooling of magnetars. In the high-density core, however, it may cause heating of the magnetar.

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The axion is a hypothetical pseudoscalar particle. It is a pseudo-Goldstone boson associated with the Peccei-Quinn symmetry \( \text{PQ} \) and has been introduced as a solution to the strong CP-violation problem \( \text{SCP} \). The physics related to the axion has been discussed in many papers, e.g. \([4,7]\).

In particular, axion phenomenology in astrophysical environments has been extensively explored in Refs. \([8–13]\). Axions are candidates for the cold dark matter of the universe because they have non-zero mass and their interactions with normal matter should be small. In view of the lack of detections in recent WIMP searches, the study of axion production or detection is well motivated and axions become a compelling candidate for cold dark matter \([14–17]\). Axion dark matter can couple to two photons that can subsequently be observed \([18]\). However, various astronomical phenomena and laboratory experimental data \([4,17,19]\) have only placed upper limits on the axion mass and decay constants. Specifically, for hadronic axions the mass and couplings are expected to be proportional to each other.

Axions produced in a hot astrophysical plasma can transport energy out of stars or even reheat the interior plasma if they have a small mean free path. The strength of the axion coupling with normal matter and radiation is bounded by the condition that stellar evolution lifetimes and/or energy loss rates should not conflict with observation. Such arguments can also be applied to the physics of supernova explosions, where the dominant energy loss processes are thought to be the emission of neutrinos and anti-neutrinos along with axions via the mechanism of nucleon bremsstrahlung \([20–23]\).

Axions may be efficiently produced in the interiors of stars and act as an additional sink of energy. Therefore, they can alter the energetics of some processes, for example, type-II supernova explosions. Several authors have noted that the emission of axions \((a)\) via the nucleon \((N)\) bremsstrahlung process \(N + N \rightarrow N + N + a\) may drain too much energy from type-II supernovae, making them inconsistent with the observed kinetic energy of such events \([20–22,24,25]\).

In Refs. \([26,27]\) the thermal evolution of a cooling neutron star was studied by including axion emission in addition to neutrino energy losses. An upper limit on the axion mass of \(m_a < 0.06 – 0.3\ eV\) was deduced. Axion cooling is an interesting possibility for the cooling mechanism of the neutron stars \([26,29,34]\). In their pioneering study, Umeda et al. \([27]\) considered the axion radiation produced via the bremsstrahlung in \(NN\) collisions in bulk nuclear matter. Axion emission from a meson condensate \([33]\) was also studied.
Cosmological constraints may also provide upper and lower limits on the mass of the axion \[36\]. Nevertheless, there still remains a large region of the parameter space to be searched. One of the most well developed and sensitive experiments is the Sikivie haloscope \[37, 38\]. This approach exploits the inverse Primakoff effect whereby a magnetic field provides a source of virtual photons in order to induce axion-to-photon conversion via a two-photon coupling. The generated real photon frequency is then determined by the axion mass. This signal can be resonantly enhanced by a cavity structure and resolved above the thermal noise of the measurement system. It has been proposed \[37–39\] that in a haloscope with an axial DC magnetic field the expected power due to axion-to-photon conversion can be detected.

The present status of the mass and coupling constant are well summarized and tabulated in Ref. \[40\]. Lower limits exist for the coupling constant, \(g_{a\gamma\gamma}\), in the Lagrangian,

\[
\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \phi_A ,
\]

where \(\phi_A\) is the axion field and \(F_{\mu\nu}\) is the electro-magnetic field strength tensor. Currently, from Helioscopes, \(|g_{a\gamma\gamma}| < 6.6 \times 10^{-11} \text{ GeV}^{-1}\) (95 % CL) for a mass range of, \(10^{-10} \text{ eV} < m_a < 1 \text{ eV}\). In addition, the analysis of gamma-rays from SN1987A \[28\] has led to the constraint that \(|g_{a\gamma\gamma}| \lesssim 5.3 \times 10^{-12} \text{ GeV}^{-1}\) and \(m_a < 4.4 \times 10^{-10} \text{ eV}\).

Axion couplings for fermions, \(g_{aNN}\) and \(g_{aee}\), in the Lagrangian

\[
\mathcal{L}_{aff} = -i g_{aff} \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi \phi_a
\]

are constrained to be \(\alpha_{aee} = g_{aee}^2 / 4\pi < 1.5 \times 10^{-26}\) and \(g_{aNN} = (3.8 \pm 3) \times 10^{-10}\) based upon many experiments and observations \[40\].

On the other hand, magnetic fields in neutron stars are much stronger than those in laboratory experiments. Hence, axion emission may play a vital role in the interpretation of many observed phenomena. In particular, magnetars, which are associated with super-strong magnetic fields, \[41, 42\] have many exotic features that distinguish them from normal the neutron stars. Thus, phenomena associated with magnetars can give information about the physical processes associated with strong magnetic fields.

It has been noted \[43\] that the characteristic magnetar spin down periods \((P/2\dot{P})\) (where \(P\) is the spin period) appear to be systematically overestimated compared to the ages of
the associated supernova remnants. Soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are believed to be to magnetars \[44\]. Magnetars emit energetic photons. Furthermore, the surface temperature of the magnetars is \( T \approx 280 - 720 \text{ eV} \). This is larger than that of normal neutron star which typically have a surface temperature of \( T \approx 10 - 150 \text{ eV} \) for similar ages \[45\]. Thus, the associated strong magnetic fields may have significant effects on these objects, and there must be a mechanism to convert the magnetic energy into thermal and radiant energies.

In this work we calculate the axion emission due to electrons and protons in the Landau quantization of the strong magnetic field. This mechanism is different from the previously considered bremsstrahlung or Primakoff mechanisms for axion production. Such axion emission from electrons has been previously calculated classically and quantum mechanically \[46, 47\]. However, the emission from protons was not taken into account. Here we show that the axion luminosity expected from the protons inside a magnetar is much larger than that due to electrons and it is high enough to be considered in the neutron star cooling (or reheating) process. In particular, contributions from the anomalous magnetic moment (AMM) of the protons becomes significant, as has been discussed in the pion production by the magnetic field \[48, 49\].

We assume a uniform magnetic field along the \( z \)-direction, \( \mathbf{B} = (0, 0, B) \), and take the electro-magnetic vector potential \( A^\mu \) to be \( A = (0, 0, xB, 0) \) at the position \( \mathbf{r} \equiv (x, y, z) \). The relativistic wave function \( \psi \) is obtained from the following Dirac equation:

\[
\left[ \gamma_\mu \cdot \left( i \partial^\mu - \zeta e A^\mu - U_0 \delta^\mu_0 \right) - M + U_s - \frac{e \kappa}{2M} \sigma_{\mu\nu} \left( \partial^\mu A^\nu - \partial^\nu A^\mu \right) \right] \psi_a(x) = 0, \tag{3}
\]

where \( \kappa \) is the AMM, \( e \) is the elementary charge and \( \zeta = \pm 1 \) is the sign of the particle charge. \( U_s \) and \( U_0 \) are the scalar field and time component of the vector field, respectively.

In our model charged particles are protons and electrons. The mean-fields are taken to be zero for electrons, while for protons they are given by relativistic mean-field (RMF) theory \[50\]. The single particle energy is then written as

\[
E(n, p_z, s) = \sqrt{p_z^2 + (\sqrt{2eBn + M^*} - se\kappa B/M)^2 + U_0} \tag{4}
\]

with \( M^* = M - U_s \), where \( n \) is the Landau number, \( p_z \) is a \( z \)-component of momentum, and \( s = \pm 1 \) is the spin. The vector-field \( U_0 \) plays the role of shifting the single particle energy.
and does not contribute to the result of the calculation. Hence, we can omit the vector field in what follows.

We obtain the differential decay width of the proton from the pseudo-vector coupling for the axion-proton (electron) interaction,

$$\frac{d^3\Gamma}{dq^3} = \frac{g_a^2}{8\pi^2 e_a} \sum_{n_f, s_f} \frac{\delta(E_f + e_a - E_i)}{4E_iE_f} W_{if} f(E_i) [1 - f(E_f)] , \quad (5)$$

with

$$W_{if} = \text{Tr} \left\{ \rho_M(n_i, s_i, p_z) O_A \rho_M(n_f, s_f, p_z - q_z) O_A^\dagger \right\} , \quad (6)$$

where $e_a$ is the energy of the emitted axion, $q \equiv (q_x, q_y, q_z)$ is the axion momentum, $g_a$ is the pseudo-scalar axion coupling constant, and

$$\rho_M = \left[ E\gamma_0 + \sqrt{2eBn\gamma^2 - p_z\gamma^3} + M^* + (eB\kappa/M)\Sigma_z \right]$$

$$\times \left[ 1 + \frac{s}{\sqrt{2eBn + M^*2}} (eB\kappa/M + p_z\gamma_5\gamma_0 + E\gamma_5\gamma_3) \right] , \quad (7)$$

while

$$O_A = \gamma_5 \left[ \mathcal{M}(n_i, n_f) \frac{1 + \zeta \Sigma_z}{2} + \mathcal{M}(n_i - 1, n_f - 1) \frac{1 - \zeta \Sigma_z}{2} \right] . \quad (8)$$

In the above equation, the harmonic oscillator (HO) overlap function $\mathcal{M}(n_1, n_2)$ is defined as

$$\mathcal{M}(n_1, n_2) = \int_{-\infty}^{\infty} dx h_{n_1}(x - \frac{q_T}{2\sqrt{eB}}) h_{n_2}(x + \frac{q_T}{2\sqrt{eB}}) , \quad (9)$$

where $q_T = \sqrt{q_x^2 + q_y^2}$, and $h_n(x)$ is the HO wave function with quantum number $n$.

The mass and coupling constants of the axion are still ambiguous. The axion mass is much smaller than the energy difference between different Landau levels in the present work, and its value does not affect the final results. In this work we choose the axion-nucleon coupling to be $g_aNN = 6 \times 10^{-12}$ and the axion-electron coupling to be $g_aee = 9 \times 10^{-15}$, which are $10^{-2}$ below the maximum value deduced in Ref. \[26\]. These parameters are chosen to impose the condition that the axion emission be negligible compared to the neutrino emission in normal neutron stars.

Furthermore, we use the parameter-sets in Ref. \[51\] for the equation of state (EOS) of the neutron-star matter, which we take to be comprised of neutrons, protons and electrons. In this work we take the temperature to be very low, $T \ll 1$ MeV, and use the mean-fields at zero temperature.
In Fig. 1 we show the temperature dependence of the axion luminosity per nucleon at $B = 10^{15}$G for baryon densities of: (a) $\rho_B = 0.1\rho_0$; (b) $\rho_B = 0.5\rho_0$; (c) $\rho_B = \rho_0$; and (d) $\rho_B = 2\rho_0$. The solid, dot-dashed and long-dashed lines represent the contributions from protons with the AMM, without the AMM, and that of electrons, respectively. For comparison, we also exhibit the neutrino luminosities from the modified Urca (MU) process [52] (dashed lines) and those from the direct Urca (DU) process [53] (dotted lines). (Note that the contribution from the AMM is omitted in the DU process.)

First, we see that the axion luminosity varies slowly when $T \gtrsim 10$ keV, while it changes rapidly in the low temperature region. It is well known that the low temperature expansion leads to a power law temperature dependence of the the emission luminosity, i.e., $L = cT^a$.

In the semi-classical approach [54], the axion luminosity from an electron was shown to
be proportional to \( T^a \) with \( a = 13/3 \approx 4.3 \). In our results the electron contributions can be fitted with \( a = 3.6 − 3.8 \) in the high temperature region; these values are similar to those obtained in the semi-classical approach. However, one should also consider realistic low magnetar temperatures \( T \lesssim 1 \text{ keV} \). In this case, the temperature dependence of the luminosity is more complicated. In particular, to satisfy the power law, one requires that the particle energies be continuous. In a strong magnetic field, however, the transverse momentum is discontinuous.

The energy of the emitted axion, \( \varepsilon_a \), for a charged particle transition is obtained as

\[
\varepsilon_a = E(n_i, p_z, s_i) - E(n_f, p_z - q_z, s_f)
\]

\[
= \sqrt{2eBn_i + p_z^2 + M^*^2} - \sqrt{2eB(n_i - \Delta n_{if}) + (p_z - q_z)^2 + M^*^2} - \frac{eB \kappa}{M} \Delta s_{if}
\]

\[
\approx \frac{eB}{\sqrt{2n_i eB + M^*^2}} \Delta n_{if} + \frac{p_z q_z}{\sqrt{2n_i eB + M^*^2}} - \frac{eB \kappa}{M} \Delta s_{if},
\]

(10)

where \( \Delta n_{if} = n_i - n_f, \Delta s_{if} = (s_i - s_f)/2 \), and \( n_{i,f} \gg \Delta n_{if} \) is assumed.

As the initial Landau number increases, the decay width for PS particle emission becomes larger \[49\], and the state at \( p_z \approx q_z \approx 0 \) gives the largest contribution. Furthermore, the energy of emitted particles at the largest decay strength is proportional to the mass of the produced particle \[49\]. The axion mass is negligibly small, and the largest contribution comes from \( \Delta n_{if} = 1 \).

In the low temperature region, the initial and final states are near the Fermi surface and \( p_z \approx q_z \approx 0 \), so that the energy interval of the dominant transition is given by

\[
\varepsilon_a \approx \Delta E = \frac{eB}{E_F^*} - \frac{eB \kappa}{M} \Delta s_{if}.
\]

(11)

with \( E_F^* = E_F - U_0 \), where \( E_F \) is the Fermi energy.

The luminosities are proportional to the Fermi distribution of the initial state and the Pauli-blocking factor of the final state, \( f(E_i)[1 - f(E_f)] \). In the low temperature expansion, it is assumed that energies of the initial and final states populate the region with \( E_F - T \lesssim E_{i,f} \lesssim E_F + T \) because the factor \( f(E_i)[1 - f(E_f)] \) becomes very small except in this region.

When \( T \lesssim \Delta E \approx eB/E_F^* \), neither the initial nor the final states reside in the above region. Hence, the luminosities rapidly decrease at low temperature as the magnetic field becomes weaker.

When \( B = 10^{15} \text{G}, \sqrt{eB} = 2.43 \text{ MeV}, eB/E_F^* = 6.6 \text{ keV} \) at \( \rho_B = 0.1 \rho_0 \), while \( eB/E_F^* = 9.4 \text{ keV} \) at \( \rho_B = \rho_0 \) for protons, and \( eB/E_F^* = 43 \text{ keV} \) at \( \rho_B = 0.1 \rho_0 \). For electrons
\[ eB/E_{F}^{*} = 6.7 \text{ keV at } \rho_B = \rho_0. \] As can be seen in Fig. 1, indeed, the change of the axion luminosities becomes more abrupt for \( T \lesssim eB/E_{F}^{*}. \)

The energy step is much larger for protons than electrons because the proton mass is much larger than the electron mass, and the proton axion luminosity becomes the dominant source.

Furthermore, one can see that there are shoulders in the density dependence of the luminosity for protons with the AMM included at \( T \sim 1 \text{ keV when } \rho_B = 0.5\rho_0 \) and at \( T \sim 2 \text{ keV when } \rho_B = \rho_0. \) The transition of \( s_i = -1 = -s_f \) is dominant in the higher temperature region while the transition \( s_i = +1 = -s_f \) becomes dominant in the lower temperature region. The spin non-flip transition seldom contributes to the emission of PS particles [48, 49]. The roles of the two contributions reverse at the temperature of the shoulders. In addition, this reversal occurs at \( T \sim 3 \text{ keV when } \rho_B = 0.1\rho_0 \) though the shoulder is not very evident.

When \( \rho_B = \rho_0 \) and \( B = 10^{15} \text{ G}, eB\kappa/M = 7.43 \text{ keV.} \) In the transition of \( s_i = -1 = -s_f, \) the AMM interaction for the initial state is repulsive, while at the final state attractive. The additional energy contributes to the transition. When the temperature is high enough, this positive additional energy causes the luminosity to increase. When the temperature is very low, however, the positive additional energy makes the energy interval \( \Delta E \) larger than the temperature. This suppresses the luminosity.

In Fig. 2 we show the density dependence of the total axion luminosity for \( B = 10^{15} \text{ G (a) and } B = 10^{14} \text{ G (b).} \) The solid lines show the results at \( T = 0.7 \text{ keV, 2 keV and 5 keV from below to above.} \) For comparison, we plot the neutrino luminosities in the DU process (dotted line) and those in the MU processes (dashed lines) in the right panel (b), which are independent of the magnetic field strength.

The luminosity at \( T = 0.7 \text{ keV first increases and then decreases with some fluctuations as the baryon density increases. All other results increase monotonously, but they become more or less saturated at higher densities.} \)

As argued before, the luminosity is mainly determined by the factor \( f(E_i)[1 - f(E_f)]. \) The \( z \)-component of the momentum is not changed much for the PS-particle emission [49], and \( E_i \) and \( E_f \) can be thought of as having discrete energy levels so that the density dependence of the factor \( f(E_i)[1 - f(E_f)] \) does not smoothly for strong magnetic fields and very low temperatures.
FIG. 2. (Color online) Axion luminosity versus baryon density at temperatures $T = 0.7 \text{ keV}$, $T = 2 \text{ keV}$ and $T = 5 \text{ keV}$ (from bottom to top) when $B = 10^{15} \text{ G}$ (a) and $B = 10^{14} \text{ G}$ (b). The dashed and dotted lines in the right panel (b) indicate the results of the MU and DU processes.

In addition, the axion luminosities are much larger than that of neutrinos in the MU process in the present calculation even when we take the coupling constant to be $10^{-2}$ of the upper limit in Ref. [26]. So, the axion luminosity can be expected to give an important contribution to magnetar cooling.

Furthermore, we notice that the results at $T = 0.7$ and $1 \text{ keV}$ are smaller at $B = 10^{15} \text{ G}$ than those for $B = 10^{14} \text{ G}$. This is counter intuitive: the luminosity becomes larger as the magnetic field increases. When $B = 10^{14} \text{ G}$, $eB/E_F^* = 0.04 \text{ keV}$ and $e\kappa_p/M = 4.8 \text{ keV}$ for protons, and the discretization of energy levels does not contribute to the final results. Indeed, the results for $T = 5 \text{ keV}$ are larger at $B = 10^{15} \text{ G}$ than that at $B = 10^{14} \text{ G}$.

One can attempt to determine the upper limit of the axion coupling constant from the calculation results. One usually expects the axion luminosity to not exceed the (anti-)neutrino luminosity in neutron star cooling. As discussed above, axions produced in a low density region contribute to the neutron star cooling, which is dominantly caused by the MU process. Then, we use $4.0 \times 10^{-25} \text{ keV}$, which is the anti-neutrino luminosity for the MU process per nucleon at $T = 0.7 \text{ keV}$ and $\rho_B = 0.1\rho_0$, as a baseline value.

In Fig. 3 we show the magnetic field dependence of the maximum axion coupling at
$T = 0.7 \text{ keV}$ and $\rho_B = 0.1 \rho_0$. The dot-dashed, solid and dashed lines represent the upper limits to the axion-nucleon coupling constant $g_{aNN}$ with the maximum luminosity being $4.0 \times 10^{-23} \text{ keV/s}$, $4.0 \times 10^{-25} \text{ keV/s}$ and $4.0 \times 10^{-28} \text{ keV/s}$, respectively.

The shaded region exhibits the region $g_{aNN} \geq 3.8 \times 10^{-10} \text{ GeV}^{-1}$, which is the present upper limit. The upper limits of $g_{aNN}$ are much lower than this value.

Furthermore, one can see that $g_{aNN}$ obtains minimum at a minimum values at $B \approx 9 \times 10^{13} \text{G}$. This indicates that the luminosity is maximum at this strength of $B$.

In addition, the dotted line indicates the upper limit of the axion-electron coupling $g_{aee}$ when the maximum luminosity is $4.0 \times 10^{-25} \text{ keV}$. It is shown to increase in an oscillatory manner with increasing magnetic field. So, the strength of the magnetic field which gives the maximum luminosity is less than $B = 10^{13} \text{G}$. It was shown in Ref. 47 that the axion luminosity from electrons decreases with a similar oscillation manner when $B > m_e^2/e \approx 4.41 \times 10^{13} \text{G}$ at a temperature of $T \geq 5 \text{ keV}$ and an electron density of $\rho_e = 10^{-4} \text{ fm}^{-3} \approx 0.006 \rho_0$. We give a comment on the $B$ dependence of the axion luminosity.

As mentioned above, the discontinuity of the energy levels affects the results in regions of large magnetic field strength. Assuming this to be a generic behavior, the peak magnetic...
field is given by the following equation

\[ T \sim \frac{eB_{\text{max}}}{E_F^*}, \quad B_{\text{max}} \sim TE_F^*/e. \]

With \( T = 0.7 \text{ keV} \), we estimate that the magnetic field strength at the maximum is \( B_{\text{max}} \sim 8.4 \times 10^{13} \text{ G} \) for protons and \( B_{\text{max}} \sim 1.7 \times 10^{12} \text{ G} \) for electrons. This estimate for protons is close to the exact calculated results.

As the baryon density increases, the \( E_F^* \) of protons slightly decreases with \( B \) in the density region considered here. This is because \( B_{\text{max}} \) does not have a strong density dependence. On the other hand, since the \( E_F \) for electrons increases, \( B_{\text{max}} \) must become smaller as the density increases.

It is well known that the axion can couple with two photons, and that the axion and the real photon are mixed in a magnetic field and oscillate as \( a \rightarrow \gamma \rightarrow a \rightarrow \cdots \) [11, 55].

In the large magnetic field limit the wave length is given by \( \lambda \approx \frac{4\pi}{|g_{a\gamma\gamma}|B} \). When \( |g_{a\gamma\gamma}| \lesssim 5.3 \times 10^{-12} \text{ GeV}^{-1} [28], \lambda \gtrsim 120 \text{ fm}. \) Even if the coupling becomes smaller, the wavelength must be much smaller than the magnetar radius, \( \sim 10 \text{ km} \). Many of the photons are absorbed by charged particles, so that most of the axions are absorbed by the medium.

We can also ask if axion production could be more effective in normal neutron stars in addition to magnetars. In the weak magnetic fields of normal neutron stars, the energy intervals are very small and calculations become more involved. We defer this topic to a future publication. Nevertheless, since our results show that the axion luminosity is much larger than that due to neutrinos, in a future publication we plan to consider axion emission from normal neutron-stars in the relativistic quantum approach.

In summary, we have studied axion emission from neutron-star matter with the strong magnetic fields, \( B = 10^{15} \text{ G} \) and \( 10^{14} \text{ G} \) in the relativistic quantum approach. We calculated the axion luminosities due to the transitions of protons and electrons between two different Landau levels without invoking any any classical approximation.

The axion luminosities turn out to be much larger than that of neutrinos due to the MU process in the present calculation even when we take the coupling constant to be \( 10^{-2} \) of the upper limit. The axion couplings are not yet completely constrained, but our axion luminosity is about \( 10^4 \) times larger than the neutrino luminosity when \( B = 10^{15} \text{ G} \) and \( \rho_B = 0.1\rho_0 \). Therefore, the axion luminosity can be expected to make an important contribution to magnetar cooling. One more point to be noted is that magnetic fields of about \( 10^{14} \text{ G} \)
may be present, leading to a maximum in the axion luminosity at low temperatures.

Fully quantum calculations provide a higher yield for particle production than the semi-classical and/or the perturbative calculations for pions \[48, 49\] and axions. Hence it would be worthwhile to investigate the heating processes of magnetars \[57\] by calculating particle production from other mechanisms such as photons from synchrotron radiation in the quantum approach.

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[1] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).
[2] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).
[3] J.E. Kim, Phys. Rev. Lett. **43**, 103 (1979).
[4] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. **B104** (1981) 199.
[5] M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. **B166**, 493 (1980).
[6] J.E. Kim, Phys. Rep. **150**, 1 (1987).
[7] H.-Y. Cheng, Phys. Rep. **158**, 1 (1988).
[8] M.I. Vysotsky, Y. Zeldovich, M.Y. Khlopov and V.M. Chechketkin, Pisma Zh. Eksp. Teor. Fiz. 27 (1978) 533.
[9] M.S. Turner, Phys. Rep. **197** (1990) 67.
[10] G.G. Raffelt, Phys. Rep. **198** (1990) 1.
[11] G.G. Raffelt, Stars as Laboratories for Fundamental Physics: the Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles, The University of Chicago Press, U.S.A. (1996):
[12] G.G. Raffelt, Ann. Rev. Nucl. Part. Sci. 49 (1999) 163.
[13] G.G. Raffelt, Lect. Notes Phys. 741, 51: (2008).
[14] J. Preskill, M.B. Wise, and F. Wilczek, Phys. Lett. 120B, 127 (1983).
[15] J. Ipser and P. Sikivie, Phys. Rev. Lett. 50, 925 (1983).
[16] M. Dine and W. Fischler, Phys. Lett. 120B, 137 (1983).
[17] L.F. Abbott and P. Sikivie, Phys. Lett. 120B, 133 (1983).
[18] B.T. McAllister, S.R. Parker and M.E. Tobar, Phys. Rev. Lett. 116, 161804 (2016).
[19] L.B. Leinson, JCAP 1408, 031 (2014).
[20] R.P. Brinkmann and M.S. Turner, Phys. Rev. D38, 2338 (1988).
[21] A. Burrows, M.S. Turner and R.P. Brinkmann, Phys. Rev. D39 (1989) 1020.
[22] C. Hanhart, D.R. Phillips and S. Reddy, Phys. Lett. B499, 9 (2001).
[23] G.G. Raffelt and D. Seckel, Phys. Rev. D52, 1780 (1995).
[24] A. Burrows, M.S. Turner and R.P. Brinkmann, Phys. Rev. D42, 3297 (1990).
[25] H.-T. Janka, W. Keil, G.G. Raffelt and D. Seckel, Phys. Rev. Lett. 76, 2621 (1996).
[26] N. Iwamoto, Phys. Rev. Lett. 53, 1198 (1984).
[27] H. Umeda, N. Iwamoto, S. Tsuruta, L. Qin, and K. Nomoto, in Proceedings of the Neutron Stars and Pulsars: Thirty Years after the Discovery, edited by N. Shibazaki (Universal Academy Press, Tokyo, 1998), p. 213.
[28] A. Payez, C. Evoli, T. Fischer, M. Giannotti, A. Mirizzib and A. Ringwalda, JCAP 1502, 006 (2015).
[29] A. Sedrakin, Phys. Rev. 93, 065044 (2016).
[30] N. Iwamoto, Phys. Rev. D64, 043002 (2001).
[31] M. Nakagawa, Y. Kohyama, and N. Itoh, ApJ. 322, 291 (1987).
[32] M. Nakagawa, T. Adachi, Y. Kohyama, and N. Itoh, ApJ. 326, 241 (1988).
[33] F. Weber, Pulsars as Astrophysical Laboratories for Nuclear and Particle Physics, Studies in High Energy Physics, Cosmology and Gravitation (CRC Press, Boca Raton, 1999).
[34] A. Sedrakin, Prog. Part. Nucl. Phys. 58, 168 (2007).
[35] T. Muto, T. Tasumi and N. Iwamoto, Phys. Rev. D50, 6089 (1994).
[36] L.J. Rosenberg, Proc. Natl. Acad. Sci. U.S.A. 112, 12278 (2015).
[37] P. Sikivie, Phys. Rev. Lett. 51, 1415 (1983).
[38] P. Sikivie, Phys. Rev. D32, 2988 (1985).
[39] J. Hoskins, J. Hwang, C. Martin, P. Sikivie, N.S. Sullivan, D.B. Tanner, M. Hotz, L.J. Rosenberg, G. Rybka, A. Wagner, S.J. Asztalos, G. Carosi, C. Hagmann, D. Kinion, K. van Bibber, R. Bradley, and J. Clarke, Phys. Rev. D84, 121302 (2011).

[40] C. Patrignani et al. (Particle Data Group), Chin. Phys. C40, 100001 (2016).

[41] For a review, G. Chanmugam, Annu. Rev. Astron. Astrophys. 30, 143 (1992).

[42] B. Paczyński, Acta. Astron. 41, 145 (1992).

[43] T. Nakano, K. Makishima, K. Nakazawa, H. Uchiyama and T. Enoto et, AIP Conf. Proc. 1427, 126 (2012).

[44] S. Mereghetti, Annu. Rev. Astron. Astrophys., 15, 225 (2008).

[45] A.D. Kaminker, A.Y. Potekhin, D.G. Yakovlev, and G. Chabrier, MNRAS 395, 2257 (2009), and references therein.

[46] A.V. Borisov and V.Yu. Grishina, JETP 79, 837 (1994).

[47] M. Kachelriess, C. Wilke, and G. Wunner, Phys. Rev. D56, 1313 (1997).

[48] T. Maruyama, M.-K. Cheoun, T. Kajino, Y. Kwon, G.J. Mathews, C.Y. Ryu, Phys. Rev. D 91, 123007 (2015).

[49] T. Maruyama, M.-K. Cheoun, T. Kajino, G.J. Mathews, Phys. Lett. B75, 125 (2016).

[50] B.D. Serot and J.D. Walecka, Int. J. Mod. Phys. E6, 515 (1997).

[51] T. Maruyama, J. Hidaka, T. Kajino, N. Yasutake, T. Kuroda, T. Takiwaki, M.K. Cheoun, C.Y. Ryu, G.J. Mathews, Phys. Rev. D90, 067302 (2014).

[52] O.V. Maxwell, ApJ 319, 691 (1987); D.G. Yakovlev, K.P. Levenfish A&A 297, 717 (1995).

[53] L.B. Leinson and A. Pérez, Phys. Lett. B518, 15 (2001).

[54] A.V. Borisov and V.Yu. Grishina, JETP 79, 837 (1994).

[55] G.G. Raffelt and L. Stodolsky, Phys. Rev. D37, 1237(1988).

[56] S.L. Cheng, C.G. Geng and W.-T. Ni, Phys. Rep. 52 (1995) 3132.

[57] A.M. Beloborodov and X. Li, ApJ, 833.