Nonlinear dynamics enabled systems design and control

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Abstract. There is a growing interest towards design of high-performance structures and devices by seeking ways to exploit advantageously different nonlinearities at different scales rather than constraining operations to avoid nonlinear phenomena. Tools of robust nonlinear modeling and analysis are shown to be turned into design tools for achieving high levels of vibration control authority and synthesis of engineered systems and materials. A brief overview of methods and results on active resonance cancellation and passive nonlinear hysteretic vibration absorbers is illustrated. Recent results on the diffused hysteresis exhibited at the nano-microscale in nanocomposites due to the powerful nonlinear stick-slip mechanism exhibited by carbon nanotubes dispersed in a hosting matrix are discussed. The optimization of the main microstructural parameters is shown to lead to unprecedented levels of damping capacity in next-generation nanostructured materials tailored for wide-band vibrational energy dissipation.

1. Introduction

The field of passive and active nonlinear vibration absorbers has been widely investigated in the last century. Passive autoparametric vibration absorbers (VAs) exploit nonlinear coupling between the main system and the absorber [1] thus overcoming the drawbacks associated with the increase of the system degrees of freedom when the VA is linearly coupled to the main system. Cartmell and Lawson [2] enhanced the performance of the autoparametric VA by using a computerized closed-loop controller. The autoparametric VA is often represented by a pendulum in applications such as for control of the parametric resonance in cantilever beams or suppression of the 1/3-order subharmonic resonance in magnetically levitated (MAGLEV) bodies [3].

Control architectures with passive autoparametric VAs capable of coping with certain nonlinear resonances sometimes turn out to be ineffective for other dynamic instabilities. For example, in the context of a parametrically excited MAGLEV body with an attached autoparametric pendulum, the autoparametric coupling can excite chaotic motions in the main system [4]. In such instances, nonlinear active resonance rejection methods may be necessary. An open-loop control strategy designed via a perturbation procedure turned the pendulum into an active VA achieving a full rejection of the parametric resonance [4].

Different nonlinear mechanisms (e.g., saturation [5], quenching, sub/superharmonic resonances, autoparametric resonances, 0:1 resonances) can be leveraged to produce effective actuator actions for resonance rejection in nonlinear lumped- and distributed-parameter systems. These control laws can be designed to be effective also when excitations and actuations are
noncollocated for which classical linear control techniques break down. The nonlinear resonance rejection strategies can be shaped by perturbation techniques [6]. Noncollocated disturbances causing primary and parametric resonance oscillations were addressed via nonlinear actuator actions in a pendulum-type crane architecture [7], in a MAGLEV body [4], in parametrically excited hinged-hinged shallow arches [8], and in simply supported beams subject to parametric-resonance pulsating thrusts [9].

As an alternative to nonlinear active control, passive control achieved by multiple passive viscoelastic VAs linearly coupled with the main system has been extensively investigated. Notwithstanding their desirable fault-tolerance characteristics, viscoelastic (VE) absorbers, often realized as a parallel arrangement of linear springs and dashpots, are rather sensitive to mistuning problems due to heating of the viscoelastic component or due to slight changes of the mass/stiffness distribution. Hysteretic (HYS) absorbers were proposed by Lacarbonara and Vestrioni [10, 11] to overcome the limitations of VE absorbers. The rheological hysteretic component can be a device or an assembly of material components with nonlinear (memory-dependent) constitutive behaviors [12] for which unloading occurs on distinct curves different from the so-called virgin loading curve (e.g., a series arrangement of a spring and a friction element, short steel wire ropes, shape memory wires [13, 14]). These nonlinear VAs are proved to be effective for reducing by orders of magnitude the effects of sustained primary resonances in structures such as beams/bridges [15] or for control of self-excited mechanisms such as flutter in aircraft wings [16] and suspension bridges [17, 18].

For applications requiring wide-band dissipation of vibrational energy and noise, the next-generation damping devices will make use of carbon nanotube-based multi-phase composites which exhibit high damping capacities. This is due to the interfacial friction between the nanofillers (CNTs) and the polymer matrix. The combination of extremely large surface area, weak interfacial bonding with the polymer, together with low mass density of the CNTs implies that frictional sliding of nanotubes with the matrix cause significant dissipation of energy with a minimal weight penalty. Recent experimental studies have shown that the damping capacity of polymer nanocomposites was enhanced by over 200% by incorporating a small amount of CNTs [19]. The mechanism behind this enhancement can be explained by shear lag theory and stick-slip theory. A micromechanics-based formulation is proposed [20, 21] to describe the space-wise distributed stick-slip phenomenology and quantify the loss factor and damping ratio. The approach allows to investigate the optimal combination of microstructural parameters to achieve unprecedented increments of damping by over 1000% as in epoxy and PEEK nanocomposites.

2. Passive versus active nonlinear vibration absorbers

An autoparametric VA in the form of a pendulum – in a 1:2 frequency ratio with the main system – was proved [3] to prevent the occurrence of the 1/3-order subharmonic resonance in a MAGLEV body such as the one shown in Fig. 1. The autoparametric VA cannot suppress, however, the principal parametric resonance since large chaotic motions of the main system and VA were found in experiments [4]. This evidence motivated the development of nonlinear control methods aimed at direct rejection of the parametric resonance without using the autoparametric energy transfer.

A simplified model of MAGLEV body [3] is considered in Fig. 1 with the origin \( O \) placed at the pendulum pivot point on the body of mass \( m_1 \) in its equilibrium under the repulsive magnetic forces in which the gap is denoted by \( z_{b0} \). Let the vertical motion of the body and the pendulum angle be denoted by \( z \) and \( \theta \), respectively. The pendulum of length \( r \) and tip mass \( m_2 \) is actuated by the torque \( \tau(t) \). The base magnet is subject to the prescribed vertical displacement \( z_{b0} = z_{b01} \cos \Omega t \).
µ force, only if it is of the same order of the parametric resonance term, \(2\) with frequency equal to \(1\). This reaction force can counteract the parametric resonance force pendulum oscillations will exhibit the frequency \(\nu/\omega\) transfer of energy when with the pendulum \(V_A\).

The nondimensional equations of motion expanded in Taylor series up to cubic terms are
\[
\ddot{z} + (1 - 2\alpha_2 \epsilon \cos \nu t) z = -\mu z \ddot{z} + \epsilon \cos \nu t - \alpha_2 z^2 - \alpha_3 z^3 + \mu \dot{\theta}^2 + \mu \nu \omega_0^2 \dot{\theta}^2
\]
\[
\ddot{\theta} + \left( \frac{\dot{z}}{r} \right) \theta = -\mu \dot{\theta} + \omega_0^2 b \cos(\nu_r t + \phi) + \frac{1}{\mu} \nu \omega_0^2 \dot{\theta}^2
\]
where \(\mu := m_2/(m_1 + m_2)\) is the mass ratio; \(\epsilon := z_{d1}/z_0\) is the nondimensional excitation amplitude; \((\mu_z, \mu_\theta)\) are the damping coefficients; \(\alpha_2\) and \(\alpha_3\) are the coefficients of \(z^2\) and \(z^3\), respectively, in the expansion of the magnetic force; and the nondimensional control torque is \(\tau = b \cos(\nu_r + \phi)\). Time and lengths in Eqs. (1) and (2) have been rescaled by \(1/\omega_z\) and \(z_0\), respectively, where \(\omega_z\) is the natural frequency of the body (when the pendulum is locked).

The term proportional to \(\theta \ddot{z}\) in Eq. (2) causes the (principal autoparametric resonance) transfer of energy when \(\omega_\theta \approx 1/2\). The pendulum torque is a direct excitation term. Setting its frequency to one-half the external excitation frequency \(\nu \approx 2\) (i.e., \(\nu_r = \nu/2\)), the resultant pendulum oscillations will exhibit the frequency \(\nu/2\) and, by means of part of the vertical reaction force, \(\mu \dot{\theta}^2\), at the pendulum pivot point, the pendulum will impart to the body a resonant input with frequency equal to \(1\). This reaction force can counteract the parametric resonance force only if it is of the same order of the parametric resonance term, \(2\alpha_2 \epsilon \nu \cos \nu t\).
An asymptotic expansion of the dynamic response can be pursued using the method of multiple scales [4]. The nearness of the principal parametric resonance is expressed introducing the detuning parameter $\sigma$ such that $\nu = 2 + \epsilon \sigma$ and $\nu_r = \nu/2 = 1 + \epsilon \sigma/2$. By omitting the full details (see [4]), the slow modulation of the complex-valued amplitude $A_z(t) = B_z(t) \exp(i \epsilon \sigma t/2)$ of the MAGLEV motion is expressed at the onset of the parametric resonance by

$$
\dot{B}_z = -\frac{i}{2} \hat{\mu}_z B_z - i \left( \frac{z}{2} + P_{z1} e^{i 2 \phi} \right) \dot{B}_z - i \left( \frac{z}{2} + P_{z2} \dot{\phi}^2 \right) B_z
$$

where $i$ is the imaginary unit, the overbar indicates the complex conjugate and $P_{z1}$ is the effective controller coefficient. The term proportional to $\dot{B}_z$ is the effective parametric resonance force which has to exceed a threshold force for the onset of the parametric resonance. Therefore, by suitably tuning the control phase $\phi$ and gain $b$, the effective parametric resonance force can be demoted below the threshold force thus achieving full rejection of the resonance. To this end, the appropriate phase such to minimize the magnitude of the coefficient of $\dot{B}_z$ is $\phi = 0$ if $\text{sgn}(\alpha_2 P_{z1}) = -1$ or $\phi = \pm \pi/2$ if $\text{sgn}(\alpha_2 P_{z1}) = 1$ where $\text{sgn}$ denotes the signum function. The stability of the trivial solution is ensured (in which case the parametric resonance is fully rejected) if

$$
\left| \frac{1}{2} \alpha_2 \epsilon + 2 P_{z1} e^{i 2 \phi} b^2 \right| < \mu_z
$$

which yields in closed form the range of effective control gains shown in Fig. 3.

In the experimental setup shown in Fig. 2, the base ferrite magnet is excited by an electromagnetic shaker while the other magnet is fixed on the lower part of the body. The torque signal was fed to the rotary motor as a sinusoidal signal generated by the (PLL) phase-lock loop circuit (LM565CN) and a power supply. In addition, the phase difference between the body and the pendulum was locked on $\pi/2$ by the PLL device and the phase-shift circuit [4]. Figure 4 portrays the theoretical and experimental frequency-response curves of the uncontrolled and controlled system in which $a_1$ indicates the amplitude of the levitated body.

The same concepts were applied in theory and experiments to reject principal parametric resonances in structural systems such as shallow arches and beams. A hinged-hinged arch subject to a parametric resonance of the lowest skew-symmetric mode was controlled by a transverse force at the midspan tuned to be in a 1/2-order subharmonic resonance with the excited mode [8]. On the other hand, the principal parametric resonance of a simply supported beam subject to a harmonic end thrust was controlled by noncollocted piezoceramic actuators driven by nonlinear voltage control laws [9].

### 3. Passive Hysteretic Vibration Absorbers

Passive nonlinear absorbers may in general allow higher vibration attenuation in wider frequency bands due to nonlinear frequency self-tuning. The class of proposed hysteretic VAs comprises rheological elements that exhibit rate-independent hysteresis such as the softening/hardening hysteretic restoring forces possessed by short wire ropes (see Figs. 5a and 6), or the pinched hysteretic forces (see Fig. 5b), or the pseudoelastic forces of shape memory wires (see Fig. 5c).

A hysteretic vibration absorber (HYS VA) (see Fig. 6) was investigated in [10, 11]. The restoring force of the VA mass is provided by short wire ropes under flexure (see Figs. 6c,d). The hysteresis loops of the steel wire ropes, obtained by a Universal Testing Machine, exhibit softening or hardening hysteresis depending on whether the clamps of the wire ropes are laterally unconstrained or constrained when displaced in the vertical direction. The identified loops are represented by solid lines. The constitutive law of the model can be expressed as $N = c \dot{y} + k y + z$ where $y$ denotes the VA displacement and $z$ is the hysteretic part of the force governed by the Bouc-Wen differential flow law [12].
The nondimensional equations of motion of the main (spring-mass-damper) system endowed with the HYS VA are:

\[
(1 + \mu) \ddot{u} + \tilde{C} \dot{u} + u + \mu \ddot{y} = 0, \\
\mu \ddot{u} + \mu \ddot{y} + \tilde{c} \dot{y} + \tilde{k} y + \tilde{k}_3 y^3 + z = 0, \\
\dot{z} = [\tilde{k}_2 - |z|^n (\tilde{\gamma} + \tilde{\beta} sgn(z) sgn(\dot{y}))] \dot{y}
\]

where \( u \) and \( y \) are the nondimensional displacements of the main mass and VA mass; \( \mu := m/M \) is the mass ratio; \( (\tilde{k}, \tilde{k}_2, \tilde{k}_3, \beta, n) \) are the nondimensional constitutive parameters of the Bouc-Wen model with the hardening term.

The mass ratio is set to 2% and the nondimensional amplitude of the base excitation is varied from 0.1 to 1.0 to obtain the frequency-response curves in Fig. 7a. The continuation analysis performed on the 5-dimensional system (in state space) reveals the occurrence of Neimark-Sacher bifurcations in the region of the out-of-phase mode (denoted by points A, B and C, D in Fig. 7a). The bifurcation diagram in the frequency band between the Neimark-Sacher bifurcations shown in Fig. 7b exhibits a scenario of loss of stability to quasiperiodicity, phase-locking, and torus breakdown [12]. The relevance of this finding is that the emergence of these nonlinear attractors is highly beneficial to the HYS VA performance since the amplitude-modulated or low-frequency periodic responses possess lower amplitudes than those exhibited during the softening mode of operation of the VA for which the resonance of the out-of-phase mode gives rise to stable large-amplitude responses [11].

Practical implementations of HYS VAs deal with control of the base-excited primary resonance and random vibrations of beams and footbridges [15], flutter control of aircraft wings [16] and suspension bridges [18]. For suspension bridges, the indicial function approach and a linearized (condensed) model of suspension bridges were employed to study the onset of the flutter condition [17]. The VA parameters were tuned according to an optimization procedure...
Figure 7. (a) Frequency-response curves of the main mass (see Fig. 6) subject to harmonic base excitations. The dashed lines indicate unstable periodic responses. (b) Bifurcation diagram (Poincaré map) past the Neimark-Sacher bifurcations.

[18]. Multiple HYS VAs (with a mass ratio equal to 0.1 % for an overall mass of 27 tons) distributed on the two sides of the deck of the Runyang Suspension Bridge (span=1490 m) are tuned to suppress flutter of the symmetric torsional mode. The HYS VAs perform better than linear VE VAs since the oscillation amplitudes of the bridge and the HYS devices are an order of magnitude smaller than those achieved with the VE absorbers (see Fig. 8).

Figure 8. (a) Flutter mode shape, (b) vertical deflection and (c) torsional rotation of the bridge at the flutter wind speed. The uncontrolled response is compared with the response of the structure subject to VE and HYS VAs, here denoted by VE-TMD and H-TMD.

4. Stick-slip hysteresis in carbon nanotube-based composites

During a loading process, a nanocomposite starts elongating and shear stresses are generated between the CNT walls and the surrounding matrix due to the difference in their elastic properties. During the *sticking phase*, the load is transferred to the CNTs which start to elongate with the matrix, if they are well bonded. As the external load exceeds a threshold value that causes a partial debonding of CNTs from the matrix, the CNTs stop elongating together with the matrix and a further increase of the load can only result in further deformation of the matrix. Thus, the polymer starts to flow over the surface of CNTs and deformation energy is dissipated through the frictional slippage between the CNTs and the matrix: this is the *slipping phase*. The CNTs embedded in the hosting matrix are assumed to be aligned with one preferential
direction. The domain of the model is given by two sub-domains, \( B = B_M \cup B_C \), the matrix and the CNTs (see Figs. 9a,b). To study the mechanism of interfacial slippage between the CNTs and the matrix, a micromechanics-based approach was developed [20, 21] extending the concepts of Eshelby’s inclusion theory together with the MOri-Tanaka averaging, and introducing the plastic eigenstrain concept to describe via plastic shear strain increments the distributed slippage of the CNTs. For simplicity, a one-dimensional problem is considered by increasing the shear strain field so that the shear stress along the CNTs walls can be determined using the proposed stick-slip constitutive law and the inelastic shear strain evolutive law characterizing the micromechanics model (see Fig. 9c). Two thermosetting polymers are chosen for the matrix, epoxy resin and PEEK, which exhibit excellent strength and chemical resistance. The constitutive parameters of the adopted model are the CNT volume fraction \( \phi_C \), the CNT-matrix post-sticking shear stiffness expressed by the exponent \( M \) in the plastic flow law, the interfacial shear strength \( S_0 \), and the reference plastic strain velocity \( \dot{\gamma}^P \).

The calculated damping ratios shown in Figs. 9d,e are highly sensitive to the strain amplitude prescribed in the cyclic tests. A peak value is attained at a given strain amplitude depending on the microstructural parameters. The conducted optimization in terms of all microstructural parameters has led to damping ratios as high as 26 % while the baseline matrices are very lightly damped (well below 1 %) thus achieving increments of damping capacity exceeding 1000 %.

![Figure 9](image_url)

**Figure 9.** (a) Nanocomposite with CNTs inclusions, (b) shear stress on the CNTs walls, (c) stress-strain curve during stick-slip and associated works, (d) damping ratio \( \xi \) vs. shear strain amplitude for various \( S_0 \) and (e) for various \( \phi_C \) for the CNT/PEEK composite.

5. Conclusions

An overview of methods and results on nonlinear active resonance cancellation and passive nonlinear hysteretic vibration absorbers has been offered. Recent results on the diffused hysteresis exhibited at the nano-microscale in nanocomposites due to the nonlinear stick-slip
mechanism exhibited by carbon nanotubes dispersed in a polymeric hosting matrix have been documented showing the potential to reach unprecedented levels of damping capacity in next-generation nanostructured materials for wide-band vibrational energy dissipation and noise control.

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