Testable Deviations from Exact Tribimaximal Mixing

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Abstract

A simple relation \( U_{PMNS} = V_{CKM}^{\dagger}U_{TB} \) between the lepton and quark mixing matrices
\((U_{PMNS} \text{ and } V_{CKM})\) is speculated under an ansatz that \( U_{PMNS} \) becomes an exact tribi-
maximal mixing \( U_{TB} \) in a limit \( V_{CKM} = 1 \). By using the observed CKM mixing parameters,
possible values of neutrino oscillation parameters are estimated: \( \sin^2 \theta_{13} = 0.024 - 0.028, \)
\( \sin^2 2\theta_{23} = 0.94 - 0.95 \) and \( \tan^2 \theta_{12} = 0.24 - 1.00 \) depending on phase conventions of \( U_{TB} \).
Those values are testable soon by precision measurements in neutrino oscillation experiments.

1 Introduction

Recently, there has been considerable interest in the magnitude of the neutrino mixing angle
\( \theta_{13} (\nu_e \leftrightarrow \nu_\tau \text{ mixing angle}) \), because it is a key value not only for checking neutrino mass matrix
models, but also for searching \( CP \)-violation effects in the lepton sector. (For a review of models
for \( \theta_{13} \), see, for example, Ref.[1].) Recent observed neutrino oscillation data are in favor of the
so-called “tribimaximal mixing” [2] which predicts \( \theta_{13} = 0, \tan^2 \theta_{12} = 1/2 \) and \( \sin^2 2\theta_{23} = 1 \),
since the present data yield the values \( \tan^2 \theta_{12} = 0.47_{-0.05}^{+0.06} \) [3] and \( \sin^2 2\theta_{23} = 1.00_{-0.13}^{+0.00} \) [4].
If the angle \( \theta_{13} \) is exactly zero or negligibly small, the observation of the \( CP \)-violation effects
in the lepton sector will be hopeless even in future, as far as neutrino oscillation experiments
are concerned. On the other hand, recently, Fogli \textit{et al.} [5] have reported a sizable value
\( \sin^2 \theta_{13} = 0.016 \pm 0.010 \) \((1\sigma)\) from a global analysis of neutrino oscillation data.

The tribimaximal lepton mixing is given by the form

\[
U_{TB}^0 = \begin{pmatrix}
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]

Such a form with beautiful coefficients seems to be understood from a discrete symmetry of
flavors [2]. In contrast to the lepton mixing matrix (Pontecorvo-Maki-Nakagawa-Sakata mixing
matrix [6]) \( U_{PMNS} \), the observed Cabibbo-Kobayashi-Maskawa [7] (CKM) quark mixing matrix
\( V_{CKM} \) seems to have no beautiful form with Clebsch-Gordan-like coefficients, and \( V_{CKM} \), rather,
looks like nearly \( V_{CKM} \simeq 1 \). It is unlikely that a theory which exactly leads to the tribimaximal
mixing (1) simultaneously gives the CKM mixing matrix with small and complicated mixing
values. Therefore, it is interesting to consider a specific case that a theory of flavor symmetry
gives $V_{CKM} = 1$ in the limit of $U_{PMNS} = U_{TB}$. We consider that the observed form of the CKM matrix $V_{CKM}$ is due to some additional effects (e.g. symmetry breaking effects for the flavor symmetry). If this is true, then, the observed lepton mixing $U_{PMNS}$ will also deviate from the exact tribimaximal mixing $U_{PMNS} = U_{TB}$ by additional effects which gives the deviation from $V_{CKM} = 1$. (Also see, e.g., Ref.[8] for a possible deviation of $U_{PMNS}$ from a bimaximal mixing (not tribimaximal mixing) related to $V_{CKM}$.)

Recently, Datta [9] has investigated possible flavor changing neutral current processes using the same assumption that $V_{CKM} = 1$ and $U_{PMNS} = U_{TB}$ in a flavor symmetry limit. By using a specific mass matrix model, he has discussed realistic mixings $V_{CKM}$ and $U_{PMNS}$ caused by a small breaking of the flavor symmetry. Also, Plentinger and Rodejohann [10] have investigated possible deviations from tribimaximal mixing by assuming a special form of the neutrino mass matrix. Furthermore, there are many works which discuss specific mass matrix models from the point of the so-called “quark-lepton-complementarity” [11]. In this paper, however, we start only from putting a simple ansatz stated later (in Eqs.(9) and (10)), without referring to any mass matrix model explicitly.

For convenience of later discussions, we define the tribimaximal mixing by a form

$$U_{TB} = P_L^T U_{TB}^0 P_R,$$  

where

$$P_L = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}),$$

$$P_R = \text{diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3}),$$

by including freedom of the phase convention, although the tribimaximal mixing is conventionally expressed by the form (1). The purpose of the present paper is to speculate a possible form of the lepton mixing matrix $U_{PMNS}$ under the ansatz $V_{CKM} = 1 \leftrightarrow U_{PMNS} = U_{TB}$. We show, as stated later, that a natural realization of this ansatz leads to a simple relation

$$U_{PMNS} = V_{CKM}^T U_{TB}.$$  

By using the observed CKM mixing parameters, we estimate values of the neutrino oscillation parameters $\sin^2 \theta_{13}$, $\tan^2 \theta_{12}$ and $\sin^2 2\theta_{23}$, which are defined by

$$\sin^2 \theta_{13} \equiv |(U_{PMNS})_{13}|^2,$$

$$\tan^2 \theta_{12} \equiv |(U_{PMNS})_{12}/(U_{PMNS})_{11}|^2,$$

$$\sin^2 2\theta_{23} \equiv 4|(U_{PMNS})_{23}|^2|(U_{PMNS})_{33}|^2.$$  

First, let us give conventions of the mass matrices: the quark and charged lepton mass matrices $M_f$ ($f = u, d, e$) are defined by the mass terms $\bar{f}_L M_f f_R$, so that those are diagonalized as

$$U_{fL}^† M_f U_{fR} = D_f \equiv \text{diag}(m_{f1}, m_{f2}, m_{f3}),$$

and the neutrino (Majorana) mass matrix $M_\nu$ is defined by $\nu_L M_\nu \nu_L^*$, so that it is diagonalized as

$$U_{\nu L}^† M_\nu U_{\nu L} = D_\nu \equiv \text{diag}(m_{\nu1}, m_{\nu2}, m_{\nu3}).$$
Therefore, the quark and lepton mixing matrices, $V_{CKM}$ and $U_{PMNS}$, are given by

$$V_{CKM} = U_{uL}^\dagger U_{dL}, \quad U_{PMNS} = U_{eL}^\dagger U_{\nu L},$$

respectively. Hereafter, we refer to a flavor basis on which the mass matrix $M_f$ is diagonal (i.e. $D_f$) as “$f$-basis”. For example, in the $u$-basis, up-quark, down-quark, charged-lepton and neutrino mass matrices are given by $D_u = U_{uL}^\dagger M_u U_{uR}$, $D_d = U_{dL}^\dagger M_d U_{dR}$, $D_e = U_{eL}^\dagger M_e U_{eR}$ and $D_{\nu} = U_{\nu L}^\dagger M_{\nu} U_{\nu L}^*$, respectively.

2 Ansatz and speculation

Let us mention an ansatz which leads to the relation (4). We put the following ansatz:

In the limit of $U_{dL} \rightarrow 1$, the matrix $U_{eL}$ also becomes a unit matrix $1$, while the matrix $U_{\nu L}$ becomes the exact tribimaximal mixing

$$U_{TB}^\dagger M_{\nu}^u U_{TB}^* = D_{\nu}.$$  

(9)

Here, we have supposed that, in a symmetry limit, i.e. when an origin which causes $V_{CKM} \neq 1$ is switched off, the physical mass matrices $M_f$ become the diagonal forms $D_f$, while the neutrino mass matrix $M_{\nu}$ becomes a specific form $M_{\nu}^u$ defined by (9):

$$(M_u, M_d; M_e, M_{\nu}) \rightarrow (D_u, D_d; D_e, U_{TB}^\dagger D_{\nu} U_{TB}^*) .$$

(10)

In other words, we consider that a common origin in the down sector causes $D_d \rightarrow M_d$ and $D_e \rightarrow M_e$, and a common origin in the up sector causes $D_u \rightarrow M_u$ and $U_{TB}^\dagger D_{\nu} U_{TB}^* \rightarrow M_{\nu}$. Of course, this transformation (10) can not be realized by a flavor-basis transformation, because $M_f$ and $D_f$ are connected by Eqs.(6) and (7). It is well-known that physics at a low-energy is unchanged under any flavor-basis transformation.

The ansatz (9) states that the mixing matrix $U_{\nu L}$ in the neutrino sector, which is defined by $U_{\nu L}^\dagger M_{\nu} U_{\nu L}^* = D_{\nu}$, is given by

$$U_{\nu L} = U_{uL} U_{TB},$$

(11)

because $D_{\nu} = U_{TB}^\dagger M_{\nu}^u U_{TB}^* = U_{TB}^\dagger (U_{uL}^\dagger M_u U_{uL}) U_{TB}^*$. Therefore, the observed lepton mixing matrix $U_{PMNS}$ is given by

$$U_{PMNS} = U_{eL}^\dagger U_{\nu L} = U_{eL}^\dagger U_{uL} U_{TB} = U_{ed} V_{CKM}^t U_{TB},$$

(12)

where $U_{ed}$ is a flavor-basis transformation matrix defined by

$$U_{ed} = U_{eL}^\dagger U_{dL}.$$  

(13)

(The relation (12) is also derived by using relations $U_{eL}^u = U_{TB}$ and $U_{uL}^e = U_{uL}^\dagger U_{\nu L}$ in the $u$-basis as $U_{PMNS} = U_{eL}^\dagger U_{\nu L} U_{TB} = U_{ed} V_{CKM}^t U_{TB}$.) According to this notation, the CKM mixing matrix $V_{CKM}$ is expressed as $V_{CKM} = U_{ud}$. Since $U_{ed} = U_{ed} U_{ud} = U_{ue} V_{CKM}$, if we consider $U_{ue} = 1$, we obtain $U_{ed} = V_{CKM}$, so that we will obtain $U_{PMNS} = U_{TB}$ from the relation (11). However, such a case $U_{ue} = 1$ is unlikely under our ansatz $U_{ed} \rightarrow 1$ in the limit of $U_{dL} \rightarrow 1$. Generally speaking, $U_{ue}$ can vary from $U_{ue} = 1$ to $U_{ue} = V_{CKM}$, so that $U_{ed}$ varies.
from $U_{ed} = V_{CKM}$ to $U_{ed} = 1$ and Eq.(12) varies from $U_{PMNS} = U_{TB}$ to $U_{PMNS} = V_{CKM}^\dagger U_{TB}$. (Here, we have considered that $U_{ae}$ does, at least, not take a large mixing more than $V_{CKM}$ and a rotation to an opposite direction, $V_{CKM}^\dagger$.) Therefore, we can consider that the relation (4) describes a maximal deviation of $U_{PMNS}$ from $U_{TB}$. In spite of such a general consideration, we think that the case $U_{ed} = 1$ (or highly $U_{ed} \simeq 1$) is a most natural realization of our ansatz (10), because it means $U_{eL} \rightarrow 1$ in the limit $U_{dL} \rightarrow 1$. Therefore, in this paper, we adopt the case $U_{ed} = 1$, and investigate possible numerical values of the neutrino oscillation parameters $\sin^2 \theta_{13}$, $\tan^2 \theta_{12}$ and $\sin^2 2\theta_{23}$ under the relation (4).

By the way, we are also interested in whether those values are dependent on the phase parameters $\alpha_i$ and $\gamma_i$ defined in Eq.(3). The relation (12) is invariant under the rephasing $U_{fL} \rightarrow U_{fLP_f}$ $(f = u, d, e)$ because of $V_{CKM} \rightarrow P_u^* V_{CKM} P_d$, $U_{PMNS} \rightarrow P_e^* U_{PMNS}$, $U_{ed} \rightarrow P_e^* U_{ed} P_d$ and $U_{TB} \rightarrow P_n^* U_{TB}$ under the rephasing (note that $U_{eL}$ does not have such a freedom of rephasing). Therefore, the phase matrices $P_L$ and $P_R$ originate in the mass matrix $M^{(u)}_\nu$ as shown in Eq.(9). Then, Eq.(9) can be rewritten as

$$
(U_{TB}^0)^T M^{(u)}_\nu U_{TB}^0 = D_\nu P_R^2,
$$

where

$$
\tilde{M}^{(u)}_\nu = P_L M^{(u)}_\nu P_L.
$$

Since the matrix $U_{TB}^0$ is orthogonal, the mass matrix $\tilde{M}^{(u)}_\nu$ has to be real. In other words, the phase matrix $P_L$ is determined from the form $M^{(u)}_\nu$ so that $\tilde{M}^{(u)}_\nu$ is real. On the other hand, the phase matrix $P_R$ is fixed so that $D_\nu P_R^2$ is real. Then, we find that the numerical results for $|\langle U_{PMNS}\rangle_{ij}|$ are independent of the phases $\gamma_i$ in $P_R$, because $U_{PMNS}$ is expressed by $U_{PMNS} = U^{P_R=1}_{PMNS} P_R$, so that the quantities $|\langle U_{PMNS}\rangle_{ij}| = |\langle U^{P_R=1}_{PMNS}\rangle_{ij} e^{i\gamma_j}|$ are independent of the phase parameters $\gamma_j$. The results are only dependent on the phase parameters $\alpha_i$ in $P_L$.

Hereafter, for simplicity, we put $P_R = 1$.

Let us show that the neutrino oscillation parameters $\sin^2 \theta_{13}$ and $\sin^2 2\theta_{23}$ are only dependent on a relative phase parameter $\alpha \equiv \alpha_3 - \alpha_2$. Since $|\langle U_{PMNS}\rangle_{i3}|$ is expressed as

$$
|\langle U_{PMNS}\rangle_{i3}| = \sum_k (V_{CKM})^\dagger_{ik} e^{-i\alpha_k} (U_{TB}^0)_{kj} = \frac{1}{\sqrt{2}} \left[ -(V_{CKM})^\dagger_{2k} e^{-i\alpha_2} + (V_{CKM})^\dagger_{3k} e^{-i\alpha_3} \right],
$$

the values $|\langle U_{PMNS}\rangle_{i3}|$ are dependent only on the parameter $\alpha$. We illustrate the behaviors of $\sin^2 \theta_{13}$ and $\sin^2 2\theta_{23}$ versus $\alpha$ in Fig.1 and Fig.2, respectively. Here, for numerical evaluation, we have used the Wolfenstein parameterization [12] of $V_{CKM}$ and the best-fit values [13] $\lambda = 0.2272$, $A = 0.818$, $\rho = 0.221$ and $\eta = 0.340$. We find that the values $\sin^2 \theta_{13}$ and $\sin^2 2\theta_{23}$ are almost insensitive to the value $\alpha$, and those take $\sin^2 \theta_{13} = 0.024 - 0.028$ and $\sin^2 2\theta_{23} = 0.94 - 0.95$. Those values are consistent with the present experimental data. As shown in Fig.1, if we take the result $\sin^2 \theta_{13} = 0.016 \pm 0.010$ (1$\sigma$) obtained from a global analysis of neutrino oscillation data by Fogli et al. [5], we can obtain allowed bounds for $\alpha$. The sizable value $\sin^2 \theta_{13}$ is within a reach of forthcoming neutrino experiments planning by Double Chooz, Daya Bay, RENO, OPERA, and so on. The value $\sin^2 2\theta_{23} = 0.94 - 0.95$ is consistent with the present observed
value \[4\] \(\sin^2 2\theta_{23} = 1.00_{-0.13},\) and the predicted value will also be testable soon by precision measurements in solar and reactor neutrino experiments.

Previously, Plentinger and Rodejohann [10] have predicted possible deviations from tribimaximal mixing by assuming a specific form of the neutrino mass matrix and by assuming a CKM-like hierarchy of the mixing angles \((\theta_{12}^\prime = \lambda, \theta_{23} = A\lambda^2, \theta_{13}^\prime = B\lambda^3)\) in the charged lepton sector. Furthermore, they have assumed the quark-lepton-complementarity (QLC) [11], and put an ad hoc relation \(\theta_{12}^\prime = \theta_C\) (\(\theta_C\) is the Cabibbo mixing angle). Then, they have obtained a relation

\[
|(U_{PMNS})_{13}| \simeq \frac{1}{\sqrt{2}} |(V_{CKM})_{us}|.
\]

Their result (17) agrees with our result \(\sin^2 \theta_{13} = 0.024 - 0.028,\) because

\[
|U_{MNS}|_{13}^2 = \frac{1}{2} |(V_{C\Lambda M})_{cd} - (V_{CKM})_{id}e^{-i\alpha}|^2 \simeq \frac{1}{2} |(V_{CKM})_{us}|^2 \simeq 0.025,
\]

from Eq.(16).

On the other hand, for the value \(\tan^2 \theta_{12},\) there is no simple situation (one-parameter dependency). The values \((U_{PMNS})_{11}\) and \((U_{PMNS})_{12}\) are given by

\[
(U_{PMNS})_{11} = \frac{1}{\sqrt{6}} \left[2(V_{CKM})_{11}^* e^{-i\alpha_1} - (V_{CKM})_{21}^* e^{-i\alpha_2} - (V_{CKM})_{31}^* e^{-i\alpha_3}\right],
\]

\[
(U_{PMNS})_{12} = \frac{1}{\sqrt{3}} \left[(V_{CKM})_{11}^* e^{-i\alpha_1} + (V_{CKM})_{21}^* e^{-i\alpha_2} + (V_{CKM})_{31}^* e^{-i\alpha_3}\right],
\]

so that the values \(|(U_{PMNS})_{11}|\) and \(|(U_{PMNS})_{12}|\) depend not only on \(\beta \equiv \alpha_2 - \alpha_1\) but also on \(\alpha \equiv \alpha_3 - \alpha_2\). However, since the observed CKM matrix parameters show \(1 \gg |(V_{CKM})_{cd}|^2 \gg |(V_{CKM})_{id}|^2\), we can neglect the terms \((V_{CKM})_{i1}e^{-i\alpha_1}\) compared with \((V_{CKM})_{11}e^{-i\alpha_1}\) and \((V_{CKM})_{21}^* e^{-i\alpha_2}\), so that the value \(\tan^2 \theta_{12}\) approximately depends only on the parameter \(\beta\).

We illustrate the behavior of \(\tan^2 \theta_{12}\) versus \(\beta \equiv \alpha_2 - \alpha_1\) in Fig.3, in which we take typical values of \(\alpha\) such as \(\alpha = 0\) and \(\alpha = -2\pi/3\). We can see that \(\tan^2 \theta_{12}\) is, in fact, insensitive to the parameter \(\alpha\). In contrast to the cases of \(\sin^2 \theta_{13}\) and \(\sin^2 2\theta_{23}\), the value of \(\tan^2 \theta_{12}\) are highly sensitive to the parameter \(\beta\) as shown by

\[
|(U_{PMNS})_{12}| \simeq \frac{1}{\sqrt{3}} \left[1 - |(V_{CKM})_{us}| \cos \beta\right],
\]

from Eq.(20). The similar result has been obtained by Plentinger and Rodejohann [10]. The value of \(\tan^2 \theta_{12}\) takes from 0.24 to 1.00 according to the variation in \(\beta\). In order to fit the observed value \([3]\) \(\tan^2 \theta_{12} \approx 0.5,\) we must take \(\beta \approx \pm \pi/2\). This will put a constraint on scenarios which give a tribimaximal mixing.

Note that, from the relation (4), we can obtain a \(CP\) violating observable

\[
J_{CP}^\prime \simeq \frac{1}{6} |(V_{CKM})_{us}| \sin \beta,
\]

(22)
as well as in a model given in Ref.[10]. Therefore, if we require a maximal CP violation in the lepton sector, we obtain $\beta \simeq \pm \pi/2$ as pointed out in Ref.[10], which is compatible with the constraint from the observed value $\tan^2 \theta_{12} \simeq 0.5$.\[14]

3 Summary

In conclusion, under the ansatz “$U_{PMNS} \rightarrow U_{TB}$ in the limit of $V_{CKM} \rightarrow 1$”, we have speculated a simple relation $U_{PMNS} = V_{CKM}^{\dagger} U_{TB}$. We have not referred an explicit mechanism (model) which gives such a CKM mixing $V_{CKM} = 1$ in the limit of $U_{PMNS} = U_{TB}$. For example, a model [10] by Plentinger and Rodejohann is one of mass matrix models which explicitly realize our ansatz because they have put an ad hoc assumption $\sin \theta_{12} = \sin \theta_C$. A model [9] by Datta is also one of such models. However, such a model-building is not a purpose of the present paper.

We have started our investigation by admitting the relation $U_{PMNS} \rightarrow U_{TB}$ as $V_{CKM} \rightarrow 1$ as an ansatz. The relation $U_{PMNS} = V_{CKM}^{\dagger} U_{TB}$ is widely valid for all models which are consistent with our ansatz.

By using the observed CKM matrix parameters, we have estimated the lepton mixing parameters $\sin^2 \theta_{13}$, $\sin^2 2 \theta_{23}$ and $\tan^2 \theta_{12}$. The values of $\sin^2 2 \theta_{23}$ and $\sin^2 \theta_{13}$ are almost independent of the phase convention, and they take values $\sin^2 \theta_{13} = 0.024 - 0.028$ and $\sin^2 2 \theta_{23} = 0.94 - 0.95$. The sizable value of $\sin^2 \theta_{13}$ is within a reach of forthcoming neutrino experiments planning by Double Chooz, Daya Bay, RENO, OPERA, and so on. The value of $\sin^2 2 \theta_{23}$ is also testable soon by precision measurements in solar and reactor neutrino experiments. On the other hand, the value of $\tan^2 \theta_{12}$ has highly depended on the phase convention of the tribimaximal mixing, and the value has been in a range $0.24 < \tan^2 \theta_{12} < 1.00$. Note that the phase matrix $P_L$ cannot be absorbed into the rephasing of $V_{CKM}$, although it seems to be possible from the expression (4). Since the present observed value of $\tan^2 \theta_{12}$ is $\tan^2 \theta_{12} \simeq 0.5$, the phase parameter $\beta$ is constrained as $\beta \simeq \pm \pi/2$. This put a strong constraint on models which lead to the exact tribimaximal mixing (2). The requirement of a maximal CP violation in the lepton sector is interestingly related to the observed value $\tan^2 \theta_{12} \simeq 0.5$.

If the predicted values $\sin^2 \theta_{13} = 0.024 - 0.028$ and $\sin^2 2 \theta_{23} \simeq 0.94 - 0.95$ are denied by forthcoming neutrino oscillation experiments, it means a denial of the simple view that the lepton mixing $U_{PMNS}$ becomes the exact tribimaximal mixing $U_{TB}$ in the limit of $V_{CKM} \rightarrow 1$. We will be compelled to consider that the view stated above is oversimplified and the situation of quark and lepton flavor mixings is more complicated. The observed values of neutrino oscillation parameters will provide us a promising clue to a possible structure of $U_{ed}$, although we simply assumed $U_{ed} = 1$ in the expression (12). This will shortly become clear by forthcoming experiments.

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Fig. 1 Behavior of $\sin^2 \theta_{13}$ versus $\alpha = \alpha_3 - \alpha_2$. The horizontal dashed and dotted lines denote the analysis $\sin^2 \theta_{13} = 0.016 \pm 0.010 \ (1\sigma)$ by Fogli et al. [5].

Fig. 2 Behavior of $\sin^2 2\theta_{23}$ versus $\alpha = \alpha_3 - \alpha_2$. The predicted value is consistent with the observed data [4] $\sin^2 2\theta_{23} = 1.00 - 0.13$.

Fig. 3 Behavior of $\tan^2 \theta_{12}$ versus $\beta = \alpha_2 - \alpha_1$. The horizontal dashed and dotted lines denote the observed values [3] $\tan^2 \theta_{12} = 0.47^{+0.06}_{-0.05}$. 