First $O(\alpha_s^3)$ Heavy Flavor Contributions to Deeply Inelastic Scattering

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We report on first NNLO results for the heavy flavor Wilson coefficients in deep–inelastic scattering. These Wilson coefficients factorize in the asymptotic limit $Q^2 \gg m^2$ into the massless Wilson coefficients and the universal heavy quark operator matrix elements, in this providing all non–power corrections in $m^2/Q^2$. At present, the heavy quark operator matrix elements are known to NLO. After calculating all necessary 2–loop $O(\varepsilon)$-terms in the unpolarized case, needed for the renormalization at 3–loops, we present first 3–loop results for fixed Mellin moments, considering terms proportional to the color factor $T_F$.

1 Introduction

In the region of small $Bjorken–x$, the unpolarized deep–inelastic structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$ obtain large $c\bar{c}$–contributions of the order of 20-40 %. Therefore, precision extractions of parton distribution functions and the measurement of $\Lambda_{QCD}$ require to extend the existing NLO corrections to these contributions to the $O(\alpha_s^3)$ terms, as reached in the massless case. Complete NLO corrections were calculated semi–analytically in $x$–space, [1]. For $Q^2 \gtrsim 10 m_c^2$, $F_{2c\bar{c}}(x,Q^2)$ is very well described by its asymptotic expression in the limit $Q^2 \gg m_c^2$, [2]. In this kinematic range, one can calculate the heavy flavor Wilson coefficients, the perturbative part of the structure functions $F_{2c\bar{c}}(x,Q^2)$ and $F_{Lc\bar{c}}(x,Q^2)$, analytically, which has been done for $F_{2c\bar{c}}(x, Q^2)$ to 2–loop order in [2, 3] and for $F_{Lc\bar{c}}(x, Q^2)$ to 3–loop order in [4]. First steps towards an asymptotic 3–loop calculation for $F_{2c\bar{c}}(x,Q^2)$ have been made by the present authors by calculating the $O(\varepsilon)$–terms of the 2–loop heavy quark operator matrix elements (OMEs), [5], contributing to the 3–loop heavy flavor Wilson coefficients via renormalization. In the present paper, we report on progress towards a full 3–loop calculation of moments of the heavy flavor Wilson coefficients. As part of the calculation, the $T_F^2$–terms of the even moments $N = 2...10$ of the NNLO non-singlet (NS) and pure-singlet (PS) anomalous dimensions given in [6], are confirmed in an independent calculation.

2 Heavy flavor Wilson coefficients in the asymptotic limit

As outlined in Ref. [2], the heavy flavor Wilson coefficients∗ in the limit $Q^2 \gg m^2$ are obtained as a convolution of the light flavor Wilson coefficients with the corresponding massive operator matrix elements of the flavor decomposed quarkonic and gluonic operators between massless parton states. Here we consider twist–2 operators. The light Wilson coefficients are known up to three loops [7] and carry all the process dependence, whereas

∗This paper was supported in part by SFB-TR-9: Computergestützte Theoretische Teilchenphysik, and Studienstiftung des Deutschen Volkes.

∗We consider extrinsic charm production only.
the OMEs are universal, process-independent objects. Their logarithmic contributions in $m^2/\mu^2$, as well as all pole terms in $1/\varepsilon$, are completely determined by renormalization. The latter provide checks on the calculation. Here, the single pole terms contain the respective contributions of the 3-loop anomalous dimensions.

3 Renormalization

We use dimensional regularization in $D = 4 + \varepsilon$ dimensions and the $\overline{\text{MS}}$-scheme, if not stated otherwise, and work in Feynman–gauge. For mass renormalization we use the on–shell scheme [8], whereas charge renormalization is done using the $\overline{\text{MS}}$ scheme. The remaining divergences originate from UV and collinear singularities, where the former are renormalized via the operator $Z$–factors and the latter are removed via mass factorization through the transition functions $\Gamma$. We denote the completely unrenormalized OMEs by a double–hat, $\hat{A}$, and those for which mass and coupling renormalization have already been performed, by a single hat. Operator renormalization and mass factorization then proceeds via $A = Z^{-1}\hat{A}\Gamma^{-1}$. Note that in the singlet case, this equation should be read as a matrix equation. The $Z$–factors are related to the anomalous dimensions of the twist–2 operators by $\gamma = \mu\partial/\partial\mu \ln Z(\mu)/\ln \mu$, which allows to express the $Z$–factors in terms of the anomalous dimensions up to an arbitrary order in the strong coupling constant $a_s := \alpha_s/(4\pi)$ (cf. [5] up to $O(a_s^3)$). Note that since we are dealing with diagrams containing at least one heavy quark, mass factorization is applied to those parts of the diagrams containing massless lines only. Finally, the $\text{PS}$ and $\text{NS}$ terms are related by $Z^{\text{PS}}_qq + Z^{\text{NS}}_qq = Z_{qq}$. From these relations, one can infer that for operator renormalization and mass factorization at $O(a_s^3)$, the anomalous dimensions up to NNLO, [6], together with the 1–loop heavy quark OMEs up to $O(\varepsilon)$ and the 2–loop heavy quark OMEs up to $O(\varepsilon^2)$ are needed. The last two quantities enter since they multiply $Z$– and $\Gamma$–factors containing poles in $\varepsilon$. This has been worked out in some detail in Ref. [5]. As a last remark, note that we consider charm quark contributions here, while for heavier quarks decoupling, [9], has to be applied.

4 Fixed values of $N$ at three loops

The diagrams to calculate are of the 3–loop self energy type with the external particle massless and on–shell. Furthermore, they contain two inner scales: the mass of the heavy quark, and the Mellin–variable $N$ of the operator insertions emerging in light–cone expansion. The necessary diagrams are generated using QGRAF [10]. They are genuinely given as tensor integrals due to the operators contracted with the light–cone vector $\Delta$, $\Delta^2 = 0$. The idea is, to first undo this contraction and to construct a projector, which, applied to the tensor integrals, provides the results for the diagrams for a chosen specific (even) Mellin $N$. So far, we implemented the projector for the first 5 contributing Mellin moments, $N = 2, ..., 10$, where the color factors are calculated using [11]. A generalization to higher moments is straightforward, however, one quickly runs into computing time problems. The diagrams are then translated into a form, which is suitable for the program MATAD [12], which does the expansion in $\varepsilon$ for the corresponding massive three–loop tadpole–type diagrams. We have implemented these steps into a FORM–program, cf. [13], and tested it against various two–loop results and found agreement.

The first 3–loop objects being investigated are the terms $\propto T_2^3$ of the OMEs $A_{qq}^{\text{NS}}$ and $A_{qq}^{\text{DIS 2008}}$. 

DIS 2008
Table 1: Mellin moments 2 and 10 for the $T_F^2$ terms of the $O(\varepsilon^0)$ terms of the unrenormalized OME $\hat{A}_q^{(3),\text{NS}}$ as obtained from MATAD.

$A_{Qq}^{\text{PS}}$. Note that in the NS–case, the ± and ν terms do not differ. All diagrams contain two inner quark loops, where the quark to which the operator insertion couples is heavy and the other one may be heavy or light. The latter two cases can be distinguished by a factor $n_f$, denoting the number of light flavors, in the result.

5 Results for $A_{Qq}^{(3),\text{PS}}$ and $A_{Qq}^{(3),\text{NS}}$

From the general structure of the renormalization prescription, we obtain the pole structure of the completely unrenormalized PS and NS OMEs. We have calculated both OMEs using MATAD for $N = 2, ..., 10$ and all pole terms agree with the predicted pole structure. Table 1 shows as an example the constant terms in $\varepsilon$ we have obtained for the momenta $N = 2$ and $N = 10$ in the non-singlet case. Note, that in Table 1 there is an overall factor $C_F T_F^2$, which we do not show explicitly, see [14] for more details.

Additionally, from the pole-structure of the moments of the PS and NS OMEs, one can deduce the corresponding moments for the 3–loop anomalous dimension $\gamma_{\text{PS}}^{(2)}|_{T_F^2}$ and $\gamma_{\text{NS}}^{(2)}|_{T_F^2}$, [14]. We have extracted these anomalous dimensions and checked that they agree with the results from Refs. [6]b.

As an example for a renormalized result, consider the second moment in the PS case, for which we obtain

$$
A_{Qq}^{(3),\text{PS}} \bigg|_{N=2,T_F^2} = C_F T_F^2 \left\{ -\frac{128}{81} \ln^3 \left( \frac{m^2}{\mu^2} \right) - \frac{32}{27} \ln^2 \left( \frac{m^2}{\mu^2} \right) - \frac{5344}{243} \ln \left( \frac{m^2}{\mu^2} \right) + \frac{53144}{2187} \right. \\
- \frac{3584}{81} \ln \left( \frac{m^2}{\mu^2} \right) + n_f \left( \frac{128}{81} \ln^3 \left( \frac{m^2}{\mu^2} \right) + \frac{32}{27} \ln^2 \left( \frac{m^2}{\mu^2} \right) - \frac{5104}{243} \ln \left( \frac{m^2}{\mu^2} \right) - \frac{34312}{2187} + \frac{1024}{81} \right) \right\}.
$$

We observe for all moments in the PS and NS renormalized case that the terms proportional to $\zeta_2$ disappear after renormalization, since the corresponding terms in the light flavor Wilson coefficients do not contain even $\zeta$-values. This is a general observation made in many $D = 4$ calculations.

bHere, one has to make the replacement $n_f \rightarrow n_f(2T_F)$, with $T_F = 1/2$, and multiply with 2, to account for the different convention for the $Z$–factors we adopted.
6 Conclusions and outlook

We installed a program chain to calculate first 3–loop diagrams, contributing to the unpolarized heavy quark OMEs to $O(\alpha_s^3)$, using MATAD. As a first step, we derived moments of the terms $T_F^2$ of the heavy quark OMEs $\hat{A}^{(3),NS}_{Qq,Q}$ and $\hat{A}^{(3),PS}_{Qq}$, for which we found agreement with the general pole structure expected from renormalization. This provides us with a good check on the method we apply for our calculation. In the near future, we expect first complete results on the different OMEs contributing.

Acknowledgments

We would like to thank M. Steinhauser and J. Vermaseren for useful discussions and M. Steinhauser for a FORM 3.0 compatible form of the code MATAD.

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