1. Introduction

The ordinary (charge) Hall effect describes the accumulation of electric charge along the edges of a current-carrying surface in response to an applied magnetic field. A signature of this effect is the emergence of a non-vanishing transverse linear charge conductivity, \( \sigma_{xy} \) \cite{1}. In analogy with the charge Hall effect, the spin Hall effect (SHE) is the spin accumulation at the sample boundaries due to extrinsic \cite{2–4} or intrinsic \cite{5–7} spin–orbit coupling (SOC). The SHE was first predicted theoretically \cite{2, 3, 5} and later observed experimentally in GaAs quantum wells \cite{7, 8}. In recent years, monolayer transition metal dichalcogenides (TMDs) have been suggested as suitable materials for observing and investigating the SHE, because strong spin-valley coupling enhances the SHE lifetime and hence, eases the experimental observation \cite{9, 10}. Monolayer TMDs offer large direct bandgaps \cite{11}, broken crystal inversion symmetry, strong excitonic effects \cite{12–14}, and huge intrinsic SOC \cite{9, 15}, all of which make their optical and electronic properties unique \cite{16}. In addition, due to the intrinsic coupling of valley and spin degrees of freedom in TMDs, a valley Hall effect coexists simultaneously with the SHE \cite{9, 10}. This leads to the valuable possibility of manipulating the valley degree of freedom using the spin in valleytronics \cite{17, 18}.

In the absence of a magnetic field, the linear charge Hall effect induced by a time-dependent field in intrinsic monolayer TMDs vanishes due to the time-reversal symmetry (TRS), regardless of the frequency \cite{19}. In contrast, the linear SHE, characterized by \( \sigma_{xy}^{\text{SHE}} \) \cite{20}, can be non-zero in a specific frequency range, yet with a vanishing static limit (dc) \cite{20, 21}. Nonetheless, a finite dc SHE can be obtained in monolayer TMDs with various approaches, e.g. by introducing a uniaxial strain to break \( C_3 \) crystal symmetry \cite{22}, or by electron/hole doping \cite{9}. Recently, interest has focused on photo-induced Hall effects due to second-order (quadratic) nonlinear processes in non-centrosymmetric materials such as TMDs without breaking TRS \cite{19, 22–28}. Polarization-dependent charge Hall currents stemming from circular photo-galvanic and photon drag effects have been predicted and observed experimentally in TMDs \cite{24–26}. In addition, quadratic charge Hall currents can be induced in 1T' TMDs that emerge from a non-zero Berry curvature dipole.
due to reduced symmetry [23, 27, 28]. This effect, however, is absent in the usual 1H phase. Apart from being the natural synthesized form of monolayer TMDs, the 1H phase has been widely used for device applications such as transistors [29] and photodetectors [30, 31]. The alternative phases of monolayer TMDs are either not sufficiently stable [32, 33], e.g. the metallic 1T phase, or require strict chemical and thermal conditions [32], e.g. the topological 1T phase. So far, no mechanism for generating dc spin Hall currents in intrinsic 1H monolayer TMDs has been reported.

In this letter, we identify a quadratic photo-induced spin Hall mechanism leading to a static spin accumulation for monolayer TMDs in the common 1H phase. We estimate the magnitude of the nonlinear spin current, and show that it is sufficiently large to induce a measurable spin polarization. The induced spin polarization can be detected via spatially resolved Kerr rotation using a weak optical probe [8] or advanced coherent techniques [34–36]. In figure 1(a), we depict the Hall geometry to explore the proposed effect, where a normally incident, linearly polarized laser beam generates both dc charge and spin Hall currents. Under intense irradiation, a field $E$ oscillating at frequency $\omega$ induces a spin (or charge) dc current at frequency $0 = \omega + (-\omega)$ via optical rectification (OR), and simultaneously a fast-oscillating spin (or charge) current at frequency $2\omega = \omega + \omega$ via second-harmonic generation (SHG). The OR and SHG currents are proportional to $E^2$ and $E^2$, respectively, with frequency-dependent proportionality factors equal to the spin (charge) quadratic optical conductivity tensors $\sigma^{(2)}(\omega)$ and $\epsilon^{(2)}(\omega)$, respectively.

2. Theoretical framework

A minimal model for the TMD band structure is obtained with the massive Dirac Hamiltonian in the basis of $|d_{x^\pm}\rangle$ and $(|d_{y}\rangle + |d_{xy}\rangle)/\sqrt{2}$ orbitals of the metal atom, $\hat{H}_0 = \hbar v_F (\kappa_x \hat{x} + \tau \kappa_y \hat{y}) + \Delta d_{x^\pm}/2$ [9]. Here, $v_F$, $\Delta$, and $\tau = \pm 1$ are the Fermi velocity, bandgap, and valley index, respectively, $\kappa_{xy}$ are the normalized (to lattice constant $a_0$) wavevector components measured with respect to $K$ or $K'$, $\hat{x}$, $\hat{y}$, and $\hat{z}$ denote the Pauli matrices. The SOC can be introduced in this model by replacing $\Delta$ with $\Delta + \tau \kappa_\tau$, where $\Delta$ denotes the spin splitting and $\tau = \pm 1$ is the spin index [9]. Note that this way of introducing SOC splits the conduction and valence bands equally. In reality, the SOC mainly lifts the valence band degeneracy, i.e. the conduction band splitting is much smaller than the valence band splitting. For instance, in monolayer MoS$_2$, the valence and conduction band splittings are 150 and 3 meV, respectively [39]. Despite the presence of SOC, the coupling between bands with different spins is typically weak for TMDs [39]. Therefore, the optical response can be obtained simply by summing the contributions from the two spins. Hence, for undoped samples, where the Fermi level resides in the bandgap, one is allowed to shift the spin-up and -down bands with respect to each other, since only the energy differences for each spin matters [40]. In contrast, the different spin splittings are of importance for the optical spectra of electron/hole-doped samples (where the Fermi level crosses the bands) [9, 41].

The massive Dirac Hamiltonian, which is linear in wavevector $k$, has proven useful to characterize the electronic properties of TMDs [9, 10, 42–45]. However, it fails to account for any even-order nonlinear response (in the dipole approximation) due to the presence of full rotation symmetry, $C_{\infty v}$. By including terms up to second order in $k$, threefold rotation symmetry, $C_3$, is recovered and trigonal warping (TW) of isoenergy contours is captured [39]. Hence, the resulting TW Hamiltonian is written as
\[ \hat{H}_{\text{TW}} = \hat{H}_0 + i\hbar v_{\text{f}} \zeta \begin{pmatrix} 0 & (\kappa_x + i\tau \kappa_y) \nu \kappa_x \end{pmatrix} \],

where \( \zeta \) specifies the degree of trigonal warping \([39]\). This Hamiltonian not only reproduces the band structure more accurately, but also leads to non-vanishing even-order responses. The typical band structure of monolayer TMDs near the K/K' valleys is shown in figure 1(c).

Excitons are known to significantly influence the optical response of monolayer TMDs \([12–14, 45, 46]\) due to the reduced screening and enhanced confinement of electrons and holes \([47, 48]\). The excitonic energies and wave functions can be determined by solving the Bethe–Salpeter equation \(\text{(BSE)}\) with an appropriate electron–hole interaction kernel \([49, 50]\). In two-dimensional \(2\text{D}\) materials, the electron–hole interaction is accurately captured by the Rytova–Keldysh potential \([51, 52]\). Despite the similarities with 2D Hydrogen atoms, the exciton energy spectrum deviates considerably from the hydrogen-like one \([12, 14, 53, 54]\), which is a signature of Bloch band geometry \([43, 55]\) and non-local screening \([12, 14]\). Note that we neglect the exchange interaction in the BSE Kernel for our calculations, since it mainly shifts the excitonic energies slightly (less than five percent \([56]\)). We are interested in excitation at normal incidence (perpendicular to the sample), and hence only in-plane excitons with zero center of mass momenta are considered here. The in-plane excitonic states are labeled in analogy to the 2D Hydrogen atom \((s, p, \text{etc.})\) and whether they are formed by the upper or lower valence band \((A \text{ and } B)\) as shown schematically in figure 1(d).

However, employing the Rytova–Keldysh potential lifts the degeneracy between \(s\) and \(p\) states, while the lifted degeneracy between \(p^+\) and \(p^-\) states stems from the nonzero Berry curvature \([55]\). In addition, the TRS relates these excitonic states at K and K' valleys. For instance, the lowest in-plane exciton, \(A_{1\alpha}\), is formed by the spin-up electron–hole pair at the K valley (with angular momentum of \(+1\)) or spin-down electron–hole pair at the K' valley (angular momentum of \(-1\) \([40]\). Note that the labeling of excitons by their angular momenta requires special attention due to the freedom of the Bloch phase \([40, 57]\).

The nonlinear conductivity tensors can be determined by solving perturbatively the master equation for the density matrix, \(\hat{\rho}(t)\), \(i\hbar \partial \hat{\rho}/\partial t = [\hat{H}_i, \hat{\rho}] + i\hat{\mathcal{L}}(\hat{\rho})\), where \(\hat{H}_i\) and \(\mathcal{L}(\hat{\rho})\) are the total Hamiltonian and Lindblad superoperator, respectively \([58–61]\). The total Hamiltonian consists of an unperturbed part (free electron plus the electron–hole interaction) and a light-matter interaction part. In the independent particle approximation \((\text{IPA})\), the electron–hole part is ignored, whereas it is treated in the mean-field approximation when excitonic effects are included. Moreover, the interaction Hamiltonian in the dipole approximation reads \(\hat{H}_d(t) = \mathbf{r} \cdot \mathbf{E}(t)\), where \(\mathbf{r}\) and \(\mathbf{E}(t)\) are the position operator and time-dependent electric field, respectively. Finally, the Lindblad superoperator is evaluated in the context of the relaxation-time approximation using phenomenological broadening parameters. We use two different relaxation rates: \(\Gamma_\nu\), for the coherences \((\rho_{\nu \kappa}, \rho_{\nu \kappa})\) and \(\Gamma_\Pi\) for the band populations \((\rho_{\nu \kappa}, \rho_{\nu \kappa})\). Despite its simplicity, this approximation captures accurately the dynamics of the system \([61, 62]\).

The quadratic spin current density, \(\mathbf{J}^{(2S)}\), is evaluated as the trace of the second-order density matrix, \(\mathbf{J}^{(2S)} = \text{Tr}[\rho^{(2)} \mathbf{J}]\), where the spin current den-
sity operator for spin moment polarized along the z-direction (perpendicular to the TMD plane) is given by \( \hat{f} = -e(\hat{z}\hat{\psi} + \hat{\psi}^\dagger) / 2A \) [63]. Here, \( \hat{\psi} \) and \( \hat{A} \) are the velocity operator and crystal area, respectively, and \( \hat{z}_s \) is the \( z \) component of the spin operator with the eigenvalues \( s = \pm 1 \). Hereafter, we consider excitation by a monochromatic beam, i.e. \( \mathcal{E}(t) = \mathcal{E}_0 \exp(-i\omega t) + \text{c.c.} \), which generates a dc spin current density, \( \mathbf{J}_n^{(25)} \), due to the optical rectification process. Hence, the quadratic spin conductivity tensor is defined using \( \mathbf{J}_n^{(25)} = 2 \sum_{\alpha\beta} \sigma_{\alpha\beta}^{(25)} \mathbf{E}_\alpha \mathbf{E}_\beta \), where \( \eta, \alpha, \beta = x, y \) denote the Cartesian coordinates (\( \mathbf{e}_\eta \) is the unit vector along the \( \eta \)-direction) using the TW Hamiltonian. We can distinguish the contributions of spin-up/down electron at the K/K' valleys, which are denoted by \( \sigma_{\alpha\beta}^{(25)} \). Upon determining \( \sigma_{\alpha\beta}^{(25)} \), the total spin conductivity reads \( \sigma_{\alpha\beta}^{(25)} = \sum_{\eta=\tau}\sigma_{\eta\beta}^{(25)} \) [21].

Due to the point-group symmetry \( D_{6h} \) of the honeycomb lattice, there can be only two independent tensor components obeying \( \sigma_{\alpha\alpha}^{(25)} = \sigma_{\beta\beta}^{(25)} = \sigma_{\beta\alpha}^{(25)} = \sigma_{\alpha\beta}^{(25)} \), where (\( \alpha, \beta \)) are \( (x,y) \) or \( (y,x) \) \[64\]. In addition,TRS relates the tensor components at the K and K' valleys. The tensor symmetries are summarized in equation (2), in which dots/triangles designate equal magnitudes and open/filled symbols indicate a relative sign difference,

\[
\sigma_{\alpha\beta}^{(25)} : \uparrow K \xleftarrow{\text{TRS}} \uparrow K', (2)
\]

Using these symmetry relations, it is straightforward to see that \( \sigma_{\alpha\alpha}^{(25)} = 0 \). Note that \( \sigma_{\alpha\beta}^{(25)} \) (\( \sigma_{\gamma\eta}^{(25)} \)) has identical (opposite) sign in the two valleys due to the TRS.

In the dipole approximation, the calculation of optical conductivities in periodic systems involves handling the ill-defined position operator, \( \hat{r} \), which is done by formally separating it into its interband (e) and intraband (i) parts, i.e. \( \hat{r} = \hat{r}^{(e)} + \hat{r}^{(i)} \) \[59, 60, 64-66\]. Furthermore, four different combinations of inter- and intraband terms, denoted by ec, ie, ei and ii, are obtained for the quadratic conductivity \[59, 64\]. At zero temperature, when the Fermi level resides in the middle of bandgap, the ei and ii terms vanish \[59\]. The purely interband (ee) and mixed intraband–interband (ie) terms are simply referred to as inter- and intraband contributions \[47\], i.e. \( \sigma_{ee}^{(25)} = \sigma_{ie}^{(25)} = \sigma_{ei}^{(25)} \). For a two-band system, the interband part, \( \sigma_{ie}^{(25)} \), originates from the band populations, \( \rho_{ie}^{(2)} \) and \( \rho_{ei}^{(2)} \), whereas the intraband term, \( \sigma_{ei}^{(25)} \), emerges from the coherences, \( \rho_{ie}^{(2)} \) and \( \rho_{ei}^{(2)} \).

In the absence of excitonic effects, we are able to determine \( \sigma_{\gamma\eta}^{(25)} \) and \( \sigma_{\alpha\alpha}^{(25)} \) analytically at zero temperature using the TW Hamiltonian

\[
\sigma_{\alpha\alpha}^{(25)} = iC_\alpha^{25} P \left[ \mathcal{G}_i(\hat{h}\omega_i, -\omega_i) - \mathcal{G}_i'(\hat{I}_\alpha, -\omega_i) \right],
\]

\[
\sigma_{\eta\eta}^{(25)} = \frac{\tau C_\eta^{25}}{2} P \left[ \mathcal{F}_\eta(\hat{h}\omega_\eta, -\omega_\eta) + \mathcal{F}_\eta'(\hat{h}\omega_\eta, -\omega_\eta + i\eta) \right],
\]

Here, \( C_\alpha^{25} = e^2 a_0/(4\pi\hbar) \), \( \omega_i \equiv \omega_\eta + \hat{I}_\alpha \), \( \mathcal{G}_i(a, b) \equiv \Delta_i \tan^{-1}(a/\Delta_i) / (ab) \), and \( P \) is defined as \( P[f(\omega)] \equiv f(\omega) + f(-\omega) \). The expressions for \( \mathcal{F}_\eta(a, b) \) and \( \mathcal{F}_\eta'(a, b) \) are provided in the supplementary material due to their complicated form. The subscript \( i \) in \( \mathcal{F} \) or \( \mathcal{G} \) implies that the corresponding term is of interband (intra-band) origin. Note that TRS leads to a vanishing ee contribution for \( \sigma_{ee}^{(25)} \), whereas both ec and ie terms contribute to \( \sigma_{ie}^{(25)} \). Including the excitonic effect, the conductivity should be calculated numerically and the inter/intraband contributions are given by

\[
\sigma_{\eta\eta}^{(25)} = C_0 \sum_{m,n} \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)} R_{m0}^{(0)}}{(i\hbar \omega_\eta + E_m - E_n)} \times \left[ \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)}}{(i\hbar \omega_\eta + E_m + E_n)} \right] - \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)}}{(i\hbar \omega_\eta + E_m)} \left[ \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)}}{(i\hbar \omega_\eta + E_m + E_n)} \right],
\]

(4a)

\[
\sigma_{\eta\eta}^{(25)} = C_0 \frac{2T}{\Gamma_\eta} \sum_{n,m} \left[ \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)} R_{m0}^{(0)}}{(i\hbar \omega_\eta + E_m - E_n)} \times \left[ \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)}}{(i\hbar \omega_\eta + E_m + E_n)} \right] - \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)}}{(i\hbar \omega_\eta + E_m)} \left[ \frac{E_{mn} R_{0m}^{(0)} R_{0n}^{(0)}}{(i\hbar \omega_\eta + E_m + E_n)} \right] \right],
\]

(4b)
Fermi distribution only affects the optical response marginally in undoped monolayer TMDs, since the bandgap is much larger than $k_B T$. The bandgap shrinks approximately 5% if $T$ is raised from 4 K to 300 K \cite{67}, and thereby simply red-shifts the entire spectrum. To ease comparison, this shift is not included in the plots. In contrast, the interband broadening, $\Gamma_\text{e}$, varies significantly with temperature, and leads to strong modifications of the spectra \cite{68–70}.

The different temperature behaviors of $\Gamma_1$ and $\Gamma_2$ follow from their distinct origin. Physically speaking, $\Gamma_1$ originates mainly from carrier–carrier (electron or hole) scattering, and is closely related to the Drude response in doped TMDs \cite{71, 72}. In contrast, $\Gamma_2$ includes various phonon-assisted, impurity/defect-related and pure dephasing scattering mechanisms \cite{70}. While the disorder-related contributions are nearly independent of the temperature, the pure dephasing and phonon-assisted parts increase with $T$ due to enhanced carrier–phonon scattering \cite{68–70}.

Regarding the phonon-assisted contribution (where the exciton center-of-mass momentum is changed due to the interaction with a phonon), the intravalley exciton-phonon scattering has been shown to be the main mechanism in MoS$_2$ and MoSe$_2$, whereas the intervalley phonon scattering is dominating in WS$_2$ \cite{68}. For monolayer MoS$_2$, $\Gamma_1$ is measured to be approximately 25 meV \cite{70}, whereas $\Gamma_2$ is estimated to be 4, 10, 53 meV \cite{67} at $T = 4, 77, 300$ K, respectively.

Note that the long-range exchange scattering process has a negligible effect on the generated spin current here, since a linearly polarized light is employed for the excitation. In the long-range exchange scattering process, an exciton is annihilated in one valley and another is created in the time-reversal valley (the exciton center-of-mass momentum remains the same) \cite{73, 74}. This elastic process depends weakly on the temperature, and is highly important as a decoherence mechanism for circularly polarized excitations \cite{74}. However, here, due to the perfect balance between the population of the excitations at the two valleys for linearly polarized light, both processes of scattering from K to K’ or from K’ to K occur with the same rate (approximately no net exciton scattering between valleys).

Neglecting excitons for the moment (IPA limit), the spin OR conductivity of monolayer MoS$_2$ is shown in figure 2(a), where smaller values of $\Gamma_2$ at lower temperatures mean sharper spectral features. Including excitonic effects, the spin OR conductivity is illustrated in figure 2(b). In contrast to theIPA results, the nonlinear conductivities exhibit discrete resonances inside the bandgap as typically observed for excitonic optical responses \cite{47, 48}. Here, the first three resonances at 1.9, 2.05, 2.15 eV are labeled according to their characteristic wave function symmetry as $A_{1\text{tr}}$, $B_{1\text{tr}}$, and $A_{2\text{p}}$, respectively \cite{75}. In contrast to the linear optical response, where mainly $s$ excitons are bright \cite{57, 76}, both $s$ and $p$ excitons manifest themselves in the quadratic optical response \cite{53, 75}. Regarding the temperature-dependence, a significant difference from the IPA result is observed: increasing the temperature not only broadens the spectral features, but also inverts the sign of the conductivity. Hence, changing the temperature may reverse the direction of the spin current. This prominent and peculiar behavior is a purely excitonic effect that stems from the relative change of inter- and intraband contributions. At low $T$, the response is dominated by contributions that arise from coherence terms. By increasing the temperature, exciton-phonon scattering is enhanced significantly, which leads to larger change of band populations with respect to their equilibrium values. Thus, the relative weight of band population contributions is increased compared to coherence terms. We expect that the peculiar temperature behavior persists for TMD samples with low doping level. However, for high doping levels, the $ii$ and $ei$ terms may drastically change the quadratic spin conductivity. Moreover, doping will affect the screening and consequently soften the excitonic resonances. Last, a linear spin Hall effect in response to a static electric field may exist due to a non-vanishing Berry curvature integral \cite{9}.

To estimate the magnitude of the nonlinear SHE and compare it with the typical linear spin Hall currents, we consider the experiments reported for bulk

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**Figure 2.** Spin OR spectra for monolayer MoS$_2$ at $T = 300$ K (green), 77 K (blue) and 4 K (red) with the IPA model (a) or with excitons (b), normalized to $\sigma_2 = 1 \times 10^{-15}$ S m V$^{-1}$. The employed parameters for MoS$_2$: $\zeta = 0.144, \alpha_i = 3.18$ Å, $\alpha_e = 44.3$ Å, $\Delta = 2.5$ eV, $\lambda = 75$ meV, $\nu_p = 6.32 \times 10^6$ m/s, $\Gamma_i = 25$ meV, and $\Gamma_e = 4/10/53$ meV at temperature $T = 4/77/300$ K (for more details, see supplementary material).
GaAs. In [8], the linear spin Hall current density in bulk GaAs due to n-doping is estimated to be 10 nA μm⁻² for an electric field of 20 mV μm⁻¹. To make a fair comparison, we first transform the bulk current density to a surface current density by multiplication with the sample thickness (400 nm), i.e. J_{exp} ∼ 4 nA/μm. In order to generate the same nonlinear spin Hall current in monolayer TMD at photon energies coinciding with the A or B excitons, an electric field of approximately E_0 ∼ 300 mV μm⁻¹ is required (since E_{yy} ≈ J_{exp}/2σ_{yy}^{(2S)} with σ_{yy}^{(2S)} ∼ 25 × 10⁻¹⁵ S m V⁻¹). This field strength is readily accessible and within the common range applied in nonlinear optics, e.g. see [77–79]. Since the linear spin current in [8] induces a measurable Kerr rotation, the nonlinear SHE in monolayer TMDs should be experimentally observable under similar experimental conditions.

We can also compare the magnitude of the induced nonlinear SHE to the linear SHE in monolayer TMDs. The linear dc spin current is given by J_{dc}(0) ∼ σ_{dc}^{(1)} E_{dc} where σ_{dc}^{(1)} and E_{dc} are the dc spin conductivity and electric field, respectively. The photo-induced nonlinear spin current is given by J_{exc}(2S) ∼ 2σ_{dc}^{(2S)} E_{dc}^2 with E_{dc} as the optical field strength. Roughly, σ_{dc}^{(2S)} ∼ ε^2/h, where σ_{dc}^{(2S)} ∼ A_{exc} e\Delta \hbar / (h \Delta), where A_{exc} shows the response enhancement factor due to excitons. Therefore, the ratio of the nonlinear to linear spin current is 2Y E_{dc}^2 / E_{dc} with Y ≡ σ_{dc}^{(2S)} / σ_{dc}^{(1)} ∼ 0.1 A_{exc} nm V⁻¹. In a typical gated device based on monolayer TMD, the dc electric field can be E_{dc} ∼ 1 V μm⁻¹ [80], hence, with an excitonic enhancement factor of A_{exc} = 50, the nonlinear SHE will be of the same magnitude as the linear SHE for an optical field strength of 10 μV μm⁻¹. This field strength is available with typical pulsed lasers and can be used to experimentally observe the nonlinear SHE in monolayer without excessively heating the sample [79,81].

4. Conclusions

In conclusion, we have shown that a nonlinear dc spin Hall current is induced by linearly polarized light in intrinsic 1H monolayer TMDs. The spin current, that stems from the large SOC, does not require any strain, external static electric or magnetic field. Similarly to the linear optical response, excitons significantly modify the quadratic optical response. We predict that the spin current direction can be inverted, by varying the temperature exclusively due to excitonic effects.

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