Double–lepton polarizations in $B \to \ell^+\ell^−\gamma$ decay

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Abstract

Double–lepton polarization asymmetries in the $B \to \ell^+\ell^−\gamma$ decay are calculated using the most general, model independent form of the effective Hamiltonian including all possible forms of the interaction. The dependencies of the asymmetries on new Wilson coefficients are investigated. The detectability the averaged double–lepton polarization asymmetries at LHC is also discussed.

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1 Introduction

Rare radiative leptonic $B_{s(d)} \rightarrow \ell^+\ell^-\gamma$ decays are induced by the flavor-changing neutral current transitions $b \rightarrow s(d)$. In the standard model (SM) such processes are described by the penguin and box diagrams and have branching ratios $10^{-10} - 10^{-15}$ (see for example [1]). These rare decays can not be observed at the running machines such as Tevatron, BaBar and Belle, but the $B_{s(d)} \rightarrow \mu^+\mu^-$ and $B_{s(d)} \rightarrow \mu^+\mu^-\gamma$ decays are already studied [2]. Moreover, the transition form factors and many experimental observables such as, the branching ratio, photon energy, dilepton mass spectra and charge asymmetries are calculated for the $B_{s(d)} \rightarrow \ell^+\ell^-\gamma$ decays in [3-9]. At the same time $B_{s(d)} \rightarrow \ell^+\ell^-\gamma$ decays might be sensitive to the new physics beyond the SM. New physics effects in these decays can appear in two different ways: either through the new operators in the effective Hamiltonian which are absent in the SM, or through new contributions to the Wilson coefficients existing in the SM. One efficient way for precise determination of the SM parameters and looking for new physics beyond the SM is studying the lepton polarization effects. It has been pointed out in [10] that some of the single lepton polarization asymmetries might be too small to be observed and might not provide sufficient number of observables in checking the structure of the effective Hamiltonian. In need of more observables, in [10], the maximum number of independent observables have been constructed by considering the situation where both lepton polarizations are simultaneously measured.

In the present work, we analyze the possibility of searching for new physics in the $B \rightarrow \ell^+\ell^-\gamma$ decay by studying the double–lepton polarization asymmetries, using the most general, model independent form of the effective Hamiltonian including all possible interactions.

The work is organized as follows. In section 2, the matrix element for the $B_s \rightarrow \ell^+\ell^-\gamma$ is obtained, using the general, model independent form of the effective Hamiltonian. In section 3, we calculate the double–lepton polarization asymmetries. Section for is devoted to the numerical analysis, discussions and conclusions.

2 Matrix element for the $B_q \rightarrow \ell^+\ell^-\gamma$ decay

In this section we derive the matrix element for the $B \rightarrow \ell^+\ell^-\gamma$ using the general, model independent form of the effective Hamiltonian. The matrix element for the process $B \rightarrow \ell^+\ell^-\gamma$ can be obtained from that of the purely leptonic $B \rightarrow \ell^+\ell^-$ decay. At inclusive level the process $B \rightarrow \ell^+\ell^-$ is described by $b \rightarrow q\ell^+\ell^-$ transition. The effective $b \rightarrow q\ell^+\ell^-$ transition can be written in terms of twelve model independent four–Fermi interactions in the following form [11]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{tb}^* \left\{ C_{SL} \bar{q}i\sigma_{\mu\nu} \frac{q^\nu}{q^2} \bar{L} b \ell\gamma^\mu\ell + C_{BR} \bar{q}i\sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \bar{\ell}\gamma^\mu\ell ight. $n+ C_{LL}^t q_L \gamma_\mu b_L \bar{\ell}_R \gamma^\mu \ell_L + C_{LR}^t q_L \gamma_\mu b_L \bar{\ell}_R \gamma^\mu \ell_R + C_{RL}^t q_R \gamma_\mu b_R \bar{\ell}_L \gamma^\mu \ell_L 
\left. + C_{RR}^t q_R \gamma_\mu b_R \bar{\ell}_L \gamma^\mu \ell_R + C_{LL} q_L \gamma_\mu b_L \bar{\ell}_L \gamma^\mu \ell_R + C_{LR} q_L \gamma_\mu b_L \bar{\ell}_R \gamma^\mu \ell_L 
\right. + C_{RL} q_R \gamma_\mu b_R \bar{\ell}_L \gamma^\mu \ell_L + C_{RR} q_R \gamma_\mu b_R \bar{\ell}_R \gamma^\mu \ell_L + C_{T} q_\sigma \mu \nu \bar{b} \ell \sigma_{\mu\nu} \ell + iC_{TE} e^{\mu\nu\alpha\beta} \bar{q} \sigma_{\mu\nu} b \ell \sigma_{\alpha\beta} \ell \right\}, \quad (1)$$
where \( C_X \) are the coefficients of the four–Fermi interactions and
\[
L = \frac{1 - \gamma_5}{2}, \quad R = \frac{1 + \gamma_5}{2}.
\]
The terms with coefficients \( C_{SL} \) and \( C_{BR} \) which describe penguin contributions correspond to \(-2m_sC_7^{eff}\) and \(-2m_bC_7^{eff}\) in the SM, respectively. The next four terms in this expression are the vector interactions. The interaction terms containing \( C_{LL}^{tot} \) and \( C_{LR}^{tot} \) in the SM have the form \( C_9^{eff} - C_{10} \) and \( C_9^{eff} + C_{10} \), respectively. Inspired by this \( C_{LL}^{tot} \) and \( C_{LR}^{tot} \) will be written as
\[
C_{LL}^{tot} = C_9^{eff} - C_{10} + C_{LL},
\]
\[
C_{LR}^{tot} = C_9^{eff} + C_{10} + C_{LR},
\]
where \( C_{LL} \) and \( C_{LR} \) describe contributions from new physics. The terms with coefficients \( C_{LRLR}, C_{RLLR}, C_{LRLR} \) and \( C_{RLRL} \) describe the scalar type interactions. The last two terms in Eq. (1) with the coefficients \( C_T \) and \( C_{TE} \) describe the tensor type interactions.

Having presented the general form of the effective Hamiltonian the next problem is the calculation of the matrix element of the \( B \to \ell^+\ell^-\gamma \) decay. This matrix element can be written as the sum of the structure–dependent and inner Bremsstrahlung parts
\[
\mathcal{M} = \mathcal{M}_{SD} + \mathcal{M}_{IB}.
\]
The matrix element for the structure–dependent part \( \mathcal{M}_{SD} \) can be obtained by sandwiching the effective Hamiltonian between initial \( B \) and final photon states, i.e., \( \langle \gamma | H_{eff} | B \rangle \). It follows from Eq. (1) that in order to calculate \( \mathcal{M}_{SD} \), the following matrix elements are needed
\[
\langle \gamma | \bar{s}\gamma_\mu(1 \mp \gamma_5)b | B \rangle,
\]
\[
\langle \gamma | \bar{s}\sigma_{\mu\nu}b | B \rangle,
\]
\[
\langle \gamma | \bar{s}(1 \mp \gamma_5)b | B \rangle.
\]
The first two of the matrix elements in Eq. (3) are defined as [3, 12, 13]
\[
\langle \gamma(k) | \bar{q}\gamma_\mu(1 \mp \gamma_5)b | B(p_B) \rangle = \frac{e}{m_B^2} \left\{ \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu}q^\lambda k^\sigma g(q^2) \pm i \left[ \varepsilon^{*\nu}(kq) - (\varepsilon^* q)k^\mu \right] f(q^2) \right\},
\]
\[
\langle \gamma(k) | \bar{q}\sigma_{\mu\nu}b | B(p_B) \rangle = \frac{e}{m_B^2} \epsilon_{\mu\nu\lambda\sigma} \left[ G\varepsilon^* k^\sigma + H\varepsilon^* q^\sigma + N(\varepsilon^* q)q^\lambda k^\sigma \right],
\]
respectively, where \( \varepsilon^* \) and \( k \) are the four vector polarization and momentum of the photon, respectively, \( q \) is the momentum transfer, \( p_B \) is the momentum of the \( B \) meson and \( g(q^2), f(q^2), G(q^2), H(q^2) \) and \( N(q^2) \) are the \( B \to \gamma \) transition form factors. The matrix element \( \langle \gamma(k) | \bar{s}\sigma_{\mu\nu}\gamma_5b | B(p_B) \rangle \) can be obtained from Eq. (5) using the identity
\[
\sigma_{\mu\nu} = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} \gamma_5.
\]
The matrix elements \( \langle \gamma(k) | \bar{s}(1 + \gamma_5)b | B(p_B) \rangle \) and \( \langle \bar{s}_i \sigma_{\mu\nu} q'' | B \rangle \) can be calculated by contracting both sides of the Eqs. (4) and (5) with \( q^\mu \) and \( q'' \), respectively. We get then
\[
\langle \gamma(k) | \bar{s}(1 + \gamma_5)b | B(p_B) \rangle = 0 , \tag{6}
\]
\[
\langle \bar{s}_i \sigma_{\mu\nu} q'' | B \rangle = \frac{e}{m_B^2} i \epsilon_{\mu\nu\alpha\beta} q'' q^\alpha k^\beta G . \tag{7}
\]

The matrix element \( \langle \bar{s}_i \sigma_{\mu\nu} q''(1 + \gamma_5)b | B \rangle \) can be written in terms of the form factors that are calculated in framework of the QCD sum rules [3] and light front model [4] as follows
\[
\langle \bar{s}_i \sigma_{\mu\nu} q''(1 + \gamma_5)b | B \rangle = \frac{e}{m_B^2} \left\{ \epsilon_{\mu\nu\alpha\beta} q'' q^\alpha k^\beta g_1(q^2) + i \left[ \epsilon^*_\mu(qk) - (\epsilon^*q)k_\mu \right] f_1(q^2) \right\} . \tag{8}
\]

It should be noted that these form factors were calculated in framework of the light–front model in [13]. Eqs. (5), (7) and (8) allow us to express \( G, H \) and \( N \) in terms of the form factors \( g_1 \) and \( f_1 \). Using Eqs. (4)–(8), \( \mathcal{M}_{SD} \) can be expressed as
\[
\mathcal{M}_{SD} = \frac{\alpha G_F}{4\sqrt{2\pi}} V_{tb} V_{ts}^* \frac{e}{m_B} \left\{ \frac{\bar{e} \gamma^\mu (1 - \gamma_5) \ell [A_1 \epsilon_{\mu\nu\alpha\beta} \epsilon^*_{\nu\tau} q^\tau k^\beta + i A_2 (\epsilon^*_\mu(qk) - (\epsilon^*q)k_\mu)]}{\bar{e} \gamma^\mu (1 + \gamma_5) \ell [B_1 \epsilon_{\mu\nu\alpha\beta} \epsilon^*_{\nu\tau} q^\tau k^\beta + i B_2 (\epsilon^*_\mu(qk) - (\epsilon^*q)k_\mu)]} + i \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma_{\mu\nu} \ell [G \epsilon^*_{\nu\tau} k^\tau + H \epsilon^*_{\nu\tau} q^\tau N(\epsilon^*q)q^\mu k^\beta] + i \bar{e} \sigma_{\mu\nu} \ell \left[ G_1 (\epsilon^*_{\mu\nu} k^\mu - \epsilon^*_{\nu\tau} q^\tau) + H_1 (\epsilon^*_{\mu\nu} q^\nu - \epsilon^*_{\nu\tau} q^\tau) + N_1 (\epsilon^*q)(q^\mu k^\nu - q^\nu k^\mu) \right] \right\} , \tag{9}
\]

where
\[
A_1 = \frac{1}{q^2} (C_{BR} + C_{SL}) g_1 + \left( C_{LL}^{\text{tot}} + C_{RL} \right) g , \quad A_2 = \frac{1}{q^2} (C_{BR} - C_{SL}) f_1 + \left( C_{LL}^{\text{tot}} - C_{RL} \right) f ,
\]
\[
B_1 = \frac{1}{q^2} (C_{BR} + C_{SL}) g_1 + \left( C_{LR}^{\text{tot}} + C_{RR} \right) g , \quad B_2 = \frac{1}{q^2} (C_{BR} - C_{SL}) f_1 + \left( C_{LR}^{\text{tot}} - C_{RR} \right) f ,
\]
\[
G = 4CT g_1 , \quad N = -4CT \frac{1}{q^2} (f_1 + g_1) , \quad H = N(qk) , \quad G_1 = -8CT g_1 , \quad N_1 = 8CT \frac{1}{q^2} (f_1 + g_1) , \quad H_1 = N_1(qk) . \tag{10}
\]

For the inner Bremsstrahlung part we get
\[
\mathcal{M}_{IB} = \frac{\alpha G_F}{4\sqrt{2\pi}} V_{tb} V_{ts}^* e f_B i \left\{ F_1 \ell \left[ \frac{\bar{B} B}{2p_1 k} - \frac{\bar{B} B}{2p_2 k} \right] \gamma_5 \ell \right. \]
\[
+ F_1 \ell \left[ \frac{\bar{B} B}{2p_1 k} + \frac{\bar{B} B}{2p_2 k} \right] + 2m_\ell \left( \frac{1}{2p_1 k} + \frac{1}{2p_2 k} \right) \bar{B} B \ell \right\} , \tag{11}
\]

3
where we have used

$$
\langle 0 | \bar{s} \gamma_\mu \gamma_5 b | B \rangle = -i f_B p_{B\mu} ,
$$

$$
\langle 0 | \bar{s} \sigma_{\mu \nu} (1 + \gamma_5) b | B \rangle = 0 ,
$$

and conservation of the vector current. The functions $F$ and $F_1$ are defined as follows

$$
F = 2 m_\ell \left( C_{LR}^{tot} - C_{LL}^{tot} + C_{RL} - C_{RR} \right) + \frac{m_B^2}{m_b} \left( C_{LRLR} - C_{RLLR} - C_{LRRL} + C_{RLRL} \right) ,
$$

$$
F_1 = \frac{m_B^2}{m_b} \left( C_{LRLR} - C_{RLLR} + C_{LRRL} - C_{RLRL} \right) .
$$

(12)

3 Double–lepton polarization asymmetries in $B_q \rightarrow \ell^+ \ell^- \gamma$ decay

In the present section we calculate the double–lepton polarization asymmetries, i.e., when polarizations of both leptons are taken into account. In order to calculate the double lepton polarization asymmetries we define the following orthogonal unit vectors $s^\pm_i$ in the rest frame of $\ell^\pm$

$$
s_L^- = (0, e_L^-) = \left( 0, \frac{\vec{p}_- \times e_L^-}{|\vec{p}_- \times e_L^-|} \right) ,
$$

$$
s_N^- = (0, e_N^-) = \left( 0, \frac{\vec{p}_- \times \vec{p}_+}{|\vec{p}_- \times \vec{p}_+|} \right) ,
$$

$$
s_T^- = (0, e_T^-) = \left( 0, \frac{\vec{p}_- \times \vec{p}_+}{|\vec{p}_- \times \vec{p}_+|} \right) ,
$$

$$
s_L^+ = (0, e_L^+) = \left( 0, \frac{\vec{p}_+ \times e_L^-}{|\vec{p}_+ \times e_L^-|} \right) ,
$$

$$
s_N^+ = (0, e_N^+) = \left( 0, \frac{\vec{p}_- \times \vec{p}_+}{|\vec{p}_- \times \vec{p}_+|} \right) ,
$$

$$
s_T^+ = (0, e_T^+) = \left( 0, \frac{\vec{p}_- \times \vec{p}_+}{|\vec{p}_- \times \vec{p}_+|} \right) ,
$$

(13)

where $\vec{p}_\pm$ and $\vec{k}$ are the three–momenta of the leptons $\ell^\pm$ and photon in the center of mass frame (CM) of $\ell^- \ell^+$ system, respectively. Transformation of unit vectors from the rest frame of the leptons to CM frame of leptons can be accomplished by the Lorentz boost. Boosting of the longitudinal unit vectors $s_{L}^{\pm \mu}$ yields

$$
(s_{L}^{\pm \mu})_{CM} = \left( \frac{\vec{p}_\pm}{m_\ell} , \frac{E_\ell \vec{p}_\pm}{m_\ell |\vec{p}_\pm|} \right) ,
$$

(14)

where $\vec{p}_+ = - \vec{p}_-$, $E_\ell$ is the energy of the lepton in the CM frame and $m_\ell$ is its mass. The unit vectors $s_N^{\pm \mu}$, $s_T^{\pm \mu}$ are unchanged under Lorentz transformation.

Having obtained necessary expressions, we now define the double–polarization asymmetries as follows [10]:

4
\[ P_{ij}(q^2) = \left( \frac{d\Gamma(s_i^-, s_j^+)}{dq^2} + \frac{d\Gamma(-s_i^-, s_j^+)}{dq^2} - \frac{d\Gamma(s_i^-, -s_j^+)}{dq^2} - \frac{d\Gamma(-s_i^-, -s_j^+)}{dq^2} \right) , \] (15)

where, the first subindex \( i \) represents lepton and the second one antilepton. Using this definition of \( P_{ij} \), and after lengthy calculations, for the nine double–lepton polarization asymmetries we get

\[ P_{LL} = \frac{1}{\Delta} \left\{ -\frac{4}{m_\ell} m_B \hat{s}^2 (1 - \hat{s})(1 - v^2) \text{Im}[(A_2^* + B_2^*)H_1] \right. \]

\[ + 16\hat{s}(1 - \hat{s}) \text{Re}[(\hat{v}^2 G^* H - (1 - 2\hat{v}^2)G_1^* H_1)] \]

\[ + 16\hat{s}^2 [\text{Re}[(1 - 2\hat{v}^2)|H_1|^2] \]

\[ + \frac{1}{2} f_B^2 m_B \hat{s} \left[ (1 - \hat{s})^2 (I_4 + I_3) + [2\hat{s} + (1 + \hat{s}^2)v^2]I_3 + [2\hat{s} - (1 + \hat{s}^2)v^2]I_6 \right] |F|^2 \]

\[ + \frac{1}{2} f_B^2 m_B \hat{s} \left[ - (1 - \hat{s})^2 I_4 + v^2[1 + \hat{s}(2 - \hat{s} + 2\hat{v}^2)]I_3 \right] \]

\[ - (1 - \hat{s})^2 (I_4 + 2\hat{s}v(1 - v^2)I_6) + v^2[1 - \hat{s}(2 + \hat{s} - 2\hat{v}^2)]I_6 \right] |F_1|^2 \]

\[ - 4 f_B^2 m_B^2 \hat{s} v[(1 - \hat{s})vI_8 + (1 + \hat{s})I_9] \text{Re}[F^* H] \]

\[ - 4 f_B^2 m_B^2 \hat{s} v[(1 - \hat{s})vI_8 - (1 - \hat{s} - 2\hat{v}^2)I_6] \text{Im}[F_1^* H_1] \]

\[ + f_B^2 m_B^2 \hat{s} \left[ 8(1 - \hat{s} + 2\hat{v}^2) + m_B^2(1 - \hat{s})(4 + v^2 - 4\hat{s} + 3\hat{s}v^2 + 2\hat{s}v^4)I_8 \right] \]

\[ + m_B^2(1 - \hat{s})(3 - 3\hat{s} - 2\hat{v}^2 + 4\hat{s}v^2)I_9 \right] \text{Im}[F_1^* N_1] \]

\[ + f_B \left[ 8(1 - \hat{s} - 2\hat{v}^2) + m_B^2(1 - \hat{s})(4 + v^2 - 4\hat{s} + 3\hat{s}v^2 - 2\hat{s}v^4)I_8 \right] \]

\[ - m_B^2(1 - \hat{s})(1 - \hat{s} + 4\hat{v}^2 - 2\hat{s}v^2)I_9 \right] \text{Im}[F_1^* G_1] \]

\[ + f_B \left[ - 8(1 + \hat{s}) + m_B^2(1 - \hat{s})(4 - v^2 - 4\hat{s} + 3\hat{s}v^2)I_8 + m_B^2(1 - \hat{s})(1 + \hat{s} - 4\hat{v}^2)I_9 \right] \text{Re}[F^* G] \]

\[ + f_B m_B^2 \hat{s} \left[ - 8(1 + \hat{s}) + m_B^2(1 - \hat{s})(3 - \hat{s})v^2I_8 + m_B^2(1 - \hat{s}^2)I_9 \right] \text{Re}[F^* N] \]

\[ + \frac{1}{4 m_\ell} f_B m_B \left[ 4\hat{s}(1 + v^2 - \hat{s} + 3\hat{s}v^2) + m_B^2 \hat{s}(1 - \hat{s})(4 - 4\hat{s} - 3\hat{s}v^2 + 7\hat{s}v^2 + v^4 - \hat{s}v^4)I_8 \right] \]

\[ - m_B^2 \hat{s}(1 - \hat{s})(1 - \hat{s} + v^2 + 7\hat{s}v^2 - 4\hat{s}v^4)I_9 \right] \text{Re}[(A_2^* + B_2^*)F_1] \]

\[ - \frac{1}{2} m_B m_B \hat{s} \left[ 8(1 + \hat{s})v^2 + m_B^2(1 - \hat{s})(2 - 2\hat{s} - 2\hat{v}^2 + 2\hat{s}v^2 + v^4 + \hat{s}v^4)I_8 \right] \]

\[ - m_B^2(1 - \hat{s}^2)v^2I_9 \right] \text{Re}[(A_1^* + B_1^*)F] \]

\[ - \frac{8}{3}(1 - \hat{s})^2(1 - 3\hat{s}v^2) \left( |G|^2 + |G_1|^2 \right) \]

\[ + \frac{2}{3 m_\ell} m_B \hat{s}(1 - \hat{s})^2(1 - v^2) \left[ m_B \hat{s} \text{Im}[(A_2^* + B_2^*)N_1] - 2 \text{Im}[(A_2^* + B_2^*)G_1] \right] \]

\[ + \frac{4}{3 m_\ell} m_B \hat{s}(1 - \hat{s})^2(1 - v^2) \text{Re}[(A_1^* + B_1^*)G] \]
\[
\begin{align*}
\mathcal{P}_{LN} & = \frac{1}{\Delta} \left\{ 2 f_{BM}^3 \hat{m}_\ell \hat{s}(1 - \hat{s}) v^2 \text{Im}[F_1 F^*](I_2 + I_4) \\
& \quad + f_{BM}^3 \hat{s}(1 - \hat{s}) v^2 \text{Im}[A_1^*(F_1 + F) + B_1^*(F_1 - F)]I_7 \\
& \quad + 8 f_{BM}^2 \hat{m}_\ell \hat{s}(1 - \hat{s}) v^2 \text{Im}[F_1^*G]I_7 \\
& \quad + \pi m_B \hat{s}(1 - \hat{s}) v^2 \text{Re}[(A_1^* + A_2^* + B_1^* + B_2^*)G_1] \\
& \quad - \pi m_B \hat{s}(1 - \hat{s}) v^2 \text{Im}[(A_1^* - A_2^* - B_1^* - B_2^*)G] \\
& \quad + 2 \pi m_B \hat{s}(1 - \hat{s}) v^2 \text{Re}[(A_1^* + B_1^*)H_1] - \text{Im}[(A_1^* - B_1^*)H] \\
& \quad - 4 \pi f_{BM}^2 \hat{m}_\ell \hat{s}(1 - \hat{s})(1 - \sqrt{1 - v^2}) \text{Im}[(A_2^* - B_2^*)F_1 + (A_2^* + B_2^*)F] \right\},
\end{align*}
\] 
\[
\mathcal{P}_{NL} = \frac{1}{\Delta} \left\{ 2 f_{BM}^3 \hat{m}_\ell \hat{s}(1 - \hat{s}) v^2 \text{Im}[F_1 F^*](I_2 + I_4) \\
& \quad + f_{BM}^3 \hat{s}(1 - \hat{s}) v^2 \text{Im}[A_1^*(F_1 - F) + B_1^*(F_1 + F)]I_7 \\
& \quad + 8 f_{BM}^2 \hat{m}_\ell \hat{s}(1 - \hat{s}) v^2 \text{Im}[F_1^*G]I_7 \\
& \quad + \pi m_B \hat{s}(1 - \hat{s}) v^2 \text{Re}[(A_1^* + A_2^* + B_1^* - B_2^*)G] \\
& \quad + \pi m_B \hat{s}(1 - \hat{s}) v^2 \text{Im}[(A_1^* + A_2^* - B_1^* + B_2^*)G] \\
& \quad + 2 \pi m_B \hat{s}(1 - \hat{s}) v^2 \text{Re}[(A_1^* + B_1^*)H_1] + \text{Im}[(A_1^* - B_1^*)H] \\
& \quad + 4 \pi f_{BM} \hat{m}_\ell \hat{s}(1 - \hat{s})(1 - \sqrt{1 - v^2}) \text{Im}[(A_2^* - B_2^*)F_1 - (A_2^* + B_2^*)F] \right\},
\] 
\[
\mathcal{P}_{LT} = \frac{1}{\Delta} \left\{ - \frac{1}{\sqrt{\hat{s}}} f_{BM}^3 \hat{m}_\ell \hat{s}(1 - \hat{s}) v \left[ (1 - \hat{s}) |F_1|^2 + (1 + \hat{s}) |F|^2 \right] (I_2 + I_4) \\
& \quad + 8 f_{BM}^2 \hat{m}_\ell \hat{s}(1 - \hat{s}) v \left( \text{Im}[F_1^*H_1] + \text{Re}[F^*H] \right)I_7 \\
& \quad + f_{BM}^3 \hat{s}(1 - \hat{s}) v |F_1| \text{Re}[(A_1^* + B_2^*)F_1]I_7 \\
& \quad + \frac{4}{\hat{v}} f_{BM} \hat{m}_\ell \hat{s}(1 - \hat{s}) (1 - \sqrt{1 - v^2}) \text{Re}[(A_2^* - B_2^*)F] \\
& \quad + \frac{4}{\hat{v}} f_{BM} \hat{m}_\ell \hat{s}(1 - \hat{s})(1 - \sqrt{1 - v^2}) \text{Re}[(A_1^* - B_1^*)F_1] \\
& \quad - \frac{4}{\sqrt{\hat{s}}} \hat{m}_\ell (1 - \hat{s}) v \left( |G_1|^2 + |G|^2 \right) \\
& \quad - 8 \pi \hat{m}_\ell \hat{s}(1 - \hat{s}) v \text{Re}[G_1^*H_1 + G^*H] \right\}.
\]
\[ P_{TL} = \frac{1}{\Delta} \left\{ \frac{1}{\sqrt{s}} f_B m_B^2 \hat{m}_\ell (1 - \hat{s}) v [(1 - \hat{s}) |F_1|^2 + (1 + \hat{s}) |F|^2] (\mathcal{I}_2 + \mathcal{I}_4) \right. \\
+ 8 f_B m_B^2 \hat{m}_\ell \sqrt{s} (1 - \hat{s}) v \left[ \text{Im}[F^*_1 H_1] + \text{Re}[F^* H] \right] \mathcal{I}_7 \\
+ f_B m_B^3 \sqrt{s} (1 - \hat{s})^2 v \text{Re}[(A^*_1 + B^*_1) F_1] \mathcal{I}_7 \\
- 4 v f_B m_B \sqrt{s} (1 - \hat{s}) (1 - \sqrt{1 - v^2}) \text{Re}[(A^*_1 - B^*_1) F] \\
+ 4 v f_B m_B \sqrt{s} [1 - \hat{s} (1 - 2v^2)] (1 - \sqrt{1 - v^2}) \text{Re}[(A^*_1 - B^*_1) F_1] \\
- 4 v \hat{m}_\ell (1 - \hat{s})^2 v \left[ |G_1|^2 + |G_1|^2 \right] \\
- 8 \hat{m}_\ell \sqrt{s} (1 - \hat{s}) v \text{Re}[G^*_1 H_1 + G^* H] \\
+ \pi m_B \sqrt{s} (1 - \hat{s})^2 v \text{Im}[(A^*_1 + A^*_2 - B^*_1 + B^*_2) G_1] \\
+ \pi m_B \sqrt{s} (1 - \hat{s})^2 v [\text{Re}[(A^*_1 + A^*_2 + B^*_1 - B^*_2) G] - 2 m_B \hat{m}_\ell \text{Re}[A^*_1 A_2 - B^*_1 B_2]] \\
- 2 \pi m_B \sqrt{s} (1 - \hat{s}) v [\text{Im}[(A^*_1 - B^*_1) H_1] - \text{Re}[(A^*_1 + B^*_1) H]] \\
- 4 v f_B m_B \sqrt{s} (1 + \hat{s}) (1 - \sqrt{1 - v^2}) \text{Re}[(A^*_1 + B^*_1) F] \\
+ 32 v \hat{m}_\ell (1 - \hat{s}) (1 - \sqrt{1 - v^2}) \text{Im}[F^*_1 G_1] \\
+ 32 v \hat{m}_\ell (1 - \sqrt{1 - v^2}) \text{Re}[F^* G] \right\}, \tag{19} \]

\[ P_{NT} = \frac{1}{\Delta} \left\{ - 16 \hat{s} (1 - \hat{s}) v \text{Re}[G^* H_1 + G^*_1 H] \\
- 16 m_B \hat{m}_\ell \hat{s} (1 - \hat{s}) v \text{Im}[(A^*_2 + B^*_2) H] \\
+ \frac{16}{v^2} f_B \hat{s} \left[ 2v - (1 - v^2) \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Re}[F^* G_1] \\
+ 2 f_B m_B^3 \hat{m}_\ell (1 - \hat{s})^2 v \text{Im}[A^*_1 (F_1 - F) + B^*_1 (F_1 + F)] (\mathcal{I}_8 - \mathcal{I}_9) \\
- 32 v^2 \text{Im}[H^*_1 H] \\
- f_B m_B^3 \text{Im}[F^*_1 F] \left\{ v [3 + v^2 - \hat{s}(1 - v^2)] \mathcal{I}_3 - (1 - v^2) [(1 - \hat{s}) \mathcal{I}_5 + (1 + \hat{s}) \mathcal{I}_6] \right\} \\
- 4 f_B m_B^3 (1 - \hat{s}) [\hat{s} - 2 \hat{m}_\ell^2 (1 + \hat{s})] v \text{Im}[F^*_1 N \mathcal{I}_8] \right\}. \tag{20} \]
\[ + 16 f_B \hat{s} \left[ 2v - (1 - v^2) \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Im}[F_1^* G] \]

\[ - 2 f_B m_\ell^2 \hat{\tilde{m}}_{\ell}(1 - \hat{s})^2 v \text{Im}[(A_2^* + B_2^*) F](\mathcal{I}_8 - \mathcal{I}_9) \]

\[ + 4 f_B m_\ell^2 \hat{s} v \text{Re}[F^* H_1] \left[ (1 - \hat{s}) \mathcal{I}_8 + (1 + \hat{s}) \mathcal{I}_9 \right] \]

\[ + 4 f_B m_\ell^2 \hat{s} v \text{Im}[F_1^* H] \left[ (1 - \hat{s}) v^2 \mathcal{I}_8 + (1 - \hat{s} + 2 \hat{s} v^2) \mathcal{I}_9 \right] \]

\[ + \frac{16}{v^2} f_B m_\ell^2 \hat{s} \left[ 2v - \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Re}[F^* N_1] \]

\[ - \frac{8}{v^2} f_B m_\ell^2 \hat{s} \left[ 2v - (1 - v^2) \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Im}[(A_2^* - B_2^*) F_1] \]

\[ - \frac{16}{3} m_B \hat{\tilde{m}}_{\ell}(1 - \hat{s})^2 v \text{Re}[(A_1^* + A_2^* + B_1^* - B_2^*) G_1 + m_\ell^2 \hat{s} (A_1^* - B_2^*) N_1] \]

\[ - \frac{8}{3} m_B (1 - \hat{s})^2 v \text{Im}[2 \hat{\tilde{m}}_{\ell}(A_1^* + A_2^* + B_1^* - B_2^*) G - m_{\ell} \hat{s} (A_1^* B_1 + A_2^* B_2)] \]

\[ + \frac{8}{3} m_\ell^2 \hat{s} (1 - \hat{s})^2 v \left( \text{Re}[G^* N_1 + G_1^* N + m_\ell^2 \hat{s} N_1^* N] + m_B \hat{\tilde{m}}_{\ell} \text{Im}[(A_1^* + B_2^*) N] \right), \quad (21) \]

\[ P_{TN} = \frac{1}{\Delta} \left\{ - 16 (1 - \hat{s}) v \text{Re}[G^* H_1 + G_1^* H] \right\} \]

\[ - 16 m_B \hat{\tilde{m}}_{\ell}(1 - \hat{s}) v \text{Im}[(A_2^* + B_2^*) H] \]

\[ + \frac{16}{v^2} f_B \hat{s} \left[ 2v - (1 - v^2) \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Re}[F^* G_1] \]

\[ + 2 f_B m_\ell^2 \hat{\tilde{m}}_{\ell}(1 - \hat{s})^2 v \text{Im}[A_1^*(F_1 + F) + B_1^*(F_1 - F)](\mathcal{I}_8 - \mathcal{I}_9) \]

\[ - 32 \hat{s}^2 v \text{Im}[H_1^* H] \]

\[ - f_B^2 m_\ell^4 \hat{s} \text{Im}[F_1^* F]\left\{ v[3 + v^2 - \hat{s}(1 - v^2)] \mathcal{I}_3 + (1 - v^2)[(1 - \hat{s}) \mathcal{I}_5 + (1 + \hat{s}) \mathcal{I}_6] \right\} \]

\[ - 4 f_B m_\ell^4 (1 - \hat{s}) \left[ \hat{s} - 2 \hat{\tilde{m}}_{\ell} (1 + \hat{s}) \right] v \text{Im}[F_1^* N] \mathcal{I}_8 \]

\[ + 16 f_B \hat{s} \left[ 2v - (1 - v^2) \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Im}[F_1^* G] \]

\[ - 2 f_B m_\ell^2 \hat{\tilde{m}}_{\ell}(1 - \hat{s})^2 v \text{Im}[(A_2^* + B_2^*) F](\mathcal{I}_8 - \mathcal{I}_9) \]

\[ + 4 f_B m_\ell^2 \hat{s} v \text{Re}[F^* H_1] \left[ (1 - \hat{s}) \mathcal{I}_8 + (1 + \hat{s}) \mathcal{I}_9 \right] \]

\[ + 4 f_B m_\ell^2 \hat{s} v \text{Im}[F_1^* H] \left[ (1 - \hat{s}) v^2 \mathcal{I}_8 + (1 - \hat{s} + 2 \hat{s} v^2) \mathcal{I}_9 \right] \]

\[ + \frac{16}{v^2} f_B m_\ell^2 \hat{s} \left[ 2v - \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Re}[F^* N_1] \]

\[ + \frac{8}{v^2} f_B m_B \hat{\tilde{m}}_{\ell}(1 - \hat{s}) \left[ 2v - (1 - v^2) \ln \left( \frac{1 + v}{1 - v} \right) \right] \text{Im}[(A_2^* - B_2^*) F_1] \]

\[ - \frac{16}{3} m_B \hat{\tilde{m}}_{\ell}(1 - \hat{s})^2 v \text{Re}[(A_1^* - A_2^* + B_1^* + B_2^*) G_1 - m_\ell^2 \hat{s} (A_1^* - B_2^*) N_1] \]

\[ + \frac{8}{3} m_B (1 - \hat{s})^2 v \text{Im}[2 \hat{\tilde{m}}_{\ell}(A_1^* - A_2^* + B_1^* - B_2^*) G - m_{\ell} \hat{s} (A_1^* B_1 + A_2^* B_2)] \]

\[ + \frac{8}{3} m_\ell^2 \hat{s} (1 - \hat{s})^2 v \left( \text{Re}[G^* N_1 + G_1^* N + m_\ell^2 \hat{s} N_1^* N] + m_B \hat{\tilde{m}}_{\ell} \text{Im}[(A_2^* + B_2^*) N] \right), \quad (22) \]
\[P_{NN} = \frac{1}{\Delta} \left\{ -16m_B \hat{m}_\ell \hat{s}(1 - \hat{s}) \text{Im}[(A^*_2 + B^*_2)H_1] \\
+ 16\hat{s}v^2 \left[(1 - \hat{s})\text{Re}[G^* H] + \hat{s} |H|^2 \right] \\
- 16\hat{s}(1 - \hat{s})\text{Re}[G^*_1 H_1 - v^2 G^* H] \\
- 16\hat{s}^2 \left(|H_1|^2 - v^2 |H|^2 \right) \\
+ f_B^2 m_B^4 \hat{s} \left[(1 + v^2)\mathcal{I}_3 - (1 - v^2)\mathcal{I}_6 \right] |F|^2 \\
- f_B^2 m_B^4 \hat{s} v \left\{v^2 (1 - \hat{s} - v^2)\mathcal{I}_3 - (1 - v^2) \left[(1 - \hat{s})\mathcal{I}_5 + \hat{s}v\mathcal{I}_6 \right] \right\} |F_1|^2 \\
- 4f_B m_B^2 \hat{s}v^2 \text{Re}[F^* H] \left[(1 - \hat{s})\mathcal{I}_8 + (1 + \hat{s})\mathcal{I}_9 \right] \\
+ 4f_B m_B^2 \hat{s} \text{Im}[F^*_1 H_1] \left[(1 - \hat{s})v^2\mathcal{I}_8 + (1 - \hat{s} + 2\hat{s}v^2)\mathcal{I}_9 \right] \\
+ \frac{16}{v} f_B m_B \hat{m}_\ell \hat{s} \left[2v - (1 - v^2) \ln \left(\frac{1 + v}{1 - v} \right) \right] \text{Re}[(A^*_2 + B^*_2)F_1] \\
+ \frac{16}{v} f_B \hat{s} \left[2v - (1 - v^2) \ln \left(\frac{1 + v}{1 - v} \right) \right] \text{Im}[F^*_1 G_1 - \text{Re}[F^* G]] \\
+ \frac{8}{v} f_B m_B^2 \hat{s} \left[4v - (3 - \hat{s} - v^2 + \hat{s}v^2) \ln \left(\frac{1 + v}{1 - v} \right) \right] \text{Im}[F^*_1 N_1] \\
- \frac{16}{v} f_B m_B^2 \hat{s} \left[2v - \ln \left(\frac{1 + v}{1 - v} \right) \right] \text{Re}[F^* N] \\
+ \frac{4}{3} m_B^2 \hat{s}(1 - \hat{s})^2 v^2 \left(2\text{Re}[A^*_2 B_1 + A^*_2 B_2 - G^* N] - m_B^2 \hat{s} |N|^2 \right) \\
+ \frac{4}{3} m_B^2 \hat{s}(1 - \hat{s})^2 (3 - 2v^2) \left(2\text{Re}[G^* N_1] + m_B^2 \hat{s} |N_1|^2 \right) \right\}, \tag{23}
\]

\[P_{TT} = \frac{1}{\Delta} \left\{ 16m_B \hat{m}_\ell \hat{s}(1 - \hat{s}) \text{Im}[(A^*_2 + B^*_2)H_1] \\
- 16\hat{s}v^2 \left[(1 - \hat{s})\text{Re}[G^* H] + \hat{s} |H|^2 \right] \\
+ \frac{1}{2} f_B^2 m_B^4 \left\{- (1 - \hat{s})^2 (1 - v^2)\mathcal{I}_1 + [(1 - \hat{s})^2 - v^2 + 3\hat{s}v^2 (2 - \hat{s}) + 2\hat{s}^2 v^4] \mathcal{I}_3 \\
- (1 - v^2)(1 - \hat{s})^2 \mathcal{I}_4 - 2\hat{s}(1 - \hat{s}) v (1 - v^2)\mathcal{I}_5 + (1 - v^2) \left[(1 - \hat{s}) (2 - \hat{s} + 2\hat{s}v^2) \mathcal{I}_6 \right] \right\} |F_1|^2 \\
+ \frac{1}{2} f_B^2 m_B^4 \left\{- (1 - \hat{s})^2 (1 - v^2)\mathcal{I}_1 + [1 - v^2 - 4\hat{s} + \hat{s}^2 (1 - v^2)] \mathcal{I}_3 \\
- (1 - v^2)(1 - \hat{s})^2 \mathcal{I}_4 + (1 - v^2)(1 - \hat{s}^2)\mathcal{I}_6 \right\} |F|^2 \\
- 4f_B m_B^2 \hat{m}_\ell (1 - \hat{s})^2 \text{Re}[(A^*_1 + B^*_1)F] \mathcal{I}_8 - \mathcal{I}_9 \\
+ 4f_B m_B^2 \hat{s}v^2 \text{Re}[F^* H] \left[(1 - \hat{s})\mathcal{I}_8 + (1 + \hat{s})\mathcal{I}_9 \right] \\
- 4f_B m_B^2 \hat{s} \text{Im}[F^*_1 H_1] \left[(1 - \hat{s})v^2\mathcal{I}_8 + (1 - \hat{s} + 2\hat{s}v^2)\mathcal{I}_9 \right] \\
- f_B m_B \hat{m}_\ell (1 - \hat{s}) \text{Re}[(A^*_2 + B^*_2)F_1] \left[8 - m_B^2 (4 - v^2 - 4\hat{s} + 5\hat{s}v^2) \mathcal{I}_8 \\
+ m_B^2 (3 - 3\hat{s} + 4\hat{s}v^2) \mathcal{I}_9 \right] \\
+ f_B m_B^2 \hat{s} \text{Re}[F^* N] \left[8(1 + \hat{s}) - m_B^2 (1 - \hat{s}) (3 - \hat{s})v^2 \mathcal{I}_8 - m_B^2 (1 - \hat{s}^2)\mathcal{I}_9 \right] \\
- f_B m_B^2 \hat{s} \text{Im}[F^*_1 N_1] \left[8(1 - \hat{s} + 2\hat{s}v^2) - m_B^2 (1 - \hat{s}) (4 - v^2 - 4\hat{s} + 5\hat{s}v^2 - 2\hat{s}v^4) \mathcal{I}_8 \right] \right\}, \tag{24}
\]
\[ + m_B^2 (1 - \hat{s})(1 - \hat{s} - 2\hat{v}^2)I_9 \]
\[ - f_B \text{Im}[F^*_1 G_1] \left[ 8(1 - \hat{s} + 2\hat{v}^2) - m_B^2 (1 - \hat{s}) (4 - 5\hat{v}^2 - 4\hat{s} + 9\hat{v}^2 - 2\hat{v}^4)I_8 \right] \]
\[ + m_B^2 (1 - \hat{s})(3 - 3\hat{s} - 4\hat{v}^2 + 6\hat{v}^2)I_9 \]
\[ + f_B \text{Re}[F^*G] \left[ 8(1 + \hat{s}) + m_B^2 (1 - \hat{s}) (4 - 4\hat{s} - 3\hat{v}^2 + \hat{v}^2)I_8 \right] \]
\[ - m_B^2 (1 - \hat{s})(5 - 3\hat{s} - 4\hat{v}^2)I_9 \]
\[ + \frac{16}{3} m_B \hat{m}_\ell (1 - \hat{s})^2 \left[ 2\text{Im}[(A_2^* - B_2^*)G_1] - 2\text{Re}[(A_1^* + B_1^*)G] \right] \]
\[ + m_B \hat{m}_\ell \left( |A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2 \right) \]
\[ + \frac{64}{3\hat{s}} m_B^2 (1 - \hat{s})^2 \left( |G_1|^2 + |G|^2 \right) \]
\[ + \frac{8}{3} m_B^2 (1 - \hat{s})^2 \left( \hat{s} \text{Re}[A_1^* B_1 + A_2^* B_2] + m_B \hat{m}_\ell \text{Im}[(A_2^* + B_2^*)N_1] \right) \]
\[ + \frac{4}{3} m_B^4 \hat{s}^2 (1 - \hat{s})^2 \left[ (1 - 2\hat{v}^2) |N_1|^2 + \hat{v}^2 |N|^2 \right] \]
\[ + \frac{8}{3} m_B^2 \hat{s} (1 - \hat{s})^2 \text{Re}[(1 - 2\hat{v}^2)G_1^* N_1 + \hat{v}^2 G^* N] \right) , \tag{24} \]

where,

\[ \Delta = 16 m_B \hat{m}_\ell (1 - \hat{s})^2 \left( \text{Im}[(A_2^* + B_2^*)G_1] - \text{Re}[(A_1^* + B_1^*)G - m_B \hat{m}_\ell (A_1^* B_1 + A_2^* B_2)] \right) \]
\[ + 48 m_B \hat{m}_\ell \hat{s} (1 - \hat{s}) \text{Im}[(A_2^* + B_2^*)H_1] \]
\[ - 8 m_B^3 \hat{m}_\ell \hat{s} (1 - \hat{s})^2 \text{Im}[(A_2^* + B_2^*)N_1] \]
\[ + \frac{2}{3} (1 - \hat{s})^2 \left[ 4(3 - \hat{v}^2) \left( |G_1|^2 + |G|^2 \right) + m_B^2 \hat{s}(3 + \hat{v}^2) \left( |A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2 \right) \right] \]
\[ + 16 \hat{s} \hat{v}^2 \left[ (1 - \hat{s}) \text{Re}[G^* H] + \hat{s} |H_1|^2 \right] \]
\[ + 16 \hat{s} (3 - 2\hat{v}^2) \left[ (1 - \hat{s}) \text{Re}[G_1^* H_1] + \hat{s} |H_1|^2 \right] \]
\[ - \frac{4}{3} m_B^3 \hat{s} (1 - \hat{s})^2 (3 - 2\hat{v}^2) \left( 2 \text{Re}[G_1^* N_1] + m_B^2 \hat{s} |N_1|^2 \right) \]
\[ - \frac{4}{3} m_B^3 \hat{s} (1 - \hat{s})^2 \hat{v}^2 \left( 2 \text{Re}[G^* N] + m_B^2 \hat{s} |N|^2 \right) \]
\[ - \frac{1}{2} \frac{f_B^2 m_B^3 |F|^2}{f_B} \left\{ (1 - \hat{s})^2 v^2 (I_1 + I_4) - (1 + \hat{s}^2 + 2\hat{s} \hat{v}^2)I_3 - [1 - \hat{s}(4 - \hat{s} - 2\hat{v}^2)]I_6 \right\} \]
\[ + \frac{1}{2} \frac{f_B^2 m_B^3 |F|^2}{f_B} \left\{ - (1 - \hat{s})^2 v^2 (I_1 + I_4) + [1 - \hat{s}(2 - \hat{s} - 4\hat{v}^2 + 2\hat{v}^2 - 2\hat{v}^4)]I_3 \right\} \]
\[ - 2\hat{s}(1 - \hat{s}) v(1 - \hat{v}^2)I_3 + [1 - \hat{s}(2 - \hat{s} + 2\hat{v}^2 - 2\hat{v}^4)]I_6 \}
\[ - 4 f_B m_B^2 \hat{s} \text{Re}[F^* H] \left[ (1 - \hat{s}) v^2 I_8 + (1 + \hat{s})I_9 \right] \]
\[ - 4 f_B m_B^2 \hat{s} \text{Im}[F^*_1 H_1] \left[ (1 - \hat{s}) v^2 I_8 + (3 - 2\hat{v}^2 - 3\hat{s} + 4\hat{s} v^2)I_9 \right] \]
\[ + 2 f_B m_B \hat{m}_\ell \text{Re}[(A_1^* + B_1^*)F] \left[ 8(1 + \hat{s}) + m_B^2 (1 - \hat{s})^2 v^2 I_8 + m_B^2 (1 - \hat{s})(1 - 3\hat{s})I_9 \right] \]
\[ - f_B m_B \hat{m}_\ell (1 - \hat{s}) \text{Re}[(A_1^* + B_1^*)F_1] \left[ 8 + m_B^2 (1 - 5\hat{s}) v^2 I_8 + m_B^2 (3 - 3\hat{s} + 4\hat{s} v^2)I_9 \right] \]
\[ + f_B \text{Im}[F_1 G_1] \left[ -24(1 - \hat{s} + 2\hat{s}v^2) + m_B^2(1 - \hat{s})(1 + 3\hat{s} - 6\hat{s}v^2)v^2 I_8 \right. \\
+ m_B^2(1 - \hat{s})(1 - \hat{s} - 2\hat{s}v^2)I_9 \] \\
+ f_B \text{Re}[F^* G] \left[ -24(1 + \hat{s}) + m_B^2(1 - \hat{s})(1 + 3\hat{s})v^2 I_8 - m_B^2(1 - \hat{s})(1 - 7\hat{s} + 4\hat{s}v^2)I_9 \right. \\
+ f_B m_B^2 \hat{s} \text{Im}[F^* N_1] \left[ -8(1 - \hat{s} + 2\hat{s}v^2) + m_B^2(1 - \hat{s})(3 + \hat{s} - 2\hat{s}v^2)v^2 I_8 \right. \\
+ m_B^2(1 - \hat{s})(3 - 2v^2 - 3\hat{s} + 4\hat{s}v^2)I_9 \right. \\
+ f_B m_B^2 \hat{s} \text{Re}[F^* N] \left[ -8(1 + \hat{s}) + m_B^2(1 - \hat{s})(3 - \hat{s})v^2 I_8 + m_B^2(1 - \hat{s}^2)I_9 \right]. \tag{25} \]

In Eqs. \((16)-(25)\), \(\hat{s} = q^2/m_B^2\), \(v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}\) is the lepton velocity with \(\hat{m}_\ell = m_\ell/m_B\), and \(I_i\) represent the following integrals

\[ I_i = \int_{-1}^{+1} F_i(z)dz, \]

where

\[ F_1 = \frac{z^2}{(p_1 \cdot k)(p_2 \cdot k)}, \quad F_2 = \frac{z}{(p_1 \cdot k)(p_2 \cdot k)}, \quad F_3 = \frac{1}{(p_1 \cdot k)(p_2 \cdot k)}, \]

\[ F_4 = \frac{z^2}{(p_1 \cdot k)^2}, \quad F_5 = \frac{z}{(p_1 \cdot k)^2}, \quad F_6 = \frac{1}{(p_1 \cdot k)^2}, \]

\[ F_7 = \frac{z}{(p_2 \cdot k)^2}, \quad F_8 = \frac{z^2}{p_1 \cdot k}, \quad F_9 = \frac{1}{p_1 \cdot k}. \]

\section{4 Numerical analysis and discussion}

We now proceed by presenting our numerical analysis for all possible double–lepton polarizations. The values of the input parameters which have been used in the present work are: \(|V_{tb}V_{ts}^*| = 0.0385\), \(m_\mu = 0.106\ \text{GeV}\), \(m_\tau = 1.78\ \text{GeV}\), \(m_b = 4.8\ \text{GeV}\). For the SM values of the Wilson coefficients we have used \(C_7^{SM}(m_b) = -0.313\), \(C_9^{SM}(m_b) = 4.344\) and \(C_{10}^{SM}(m_b) = -4.669\). The magnitude of \(C_7^{SM}\) is quite well constrained from the \(b \to s\gamma\) transition, and hence it is well established. Therefore the values of \(C_{BR}\) and \(C_{SL}\) are fixed by the relations \(C_{BR} = -2m_b C_7^{eff}\) and \(C_{SL} = -2m_s C_7^{eff}\). It is well known that the Wilson coefficient \(C_9^{SM}\) receives also long distance contributions which have their origin in the real \(\bar{c}c\) intermediate states, i.e., \(J/\psi, \psi', \cdots [14]\). In the present work we restrict ourselves only to short distance contributions.

In performing the numerical analysis, as is obvious from the expressions of \(P_{ij}\) given in Eqs. \((16)-(24)\), we need to know the values of the new Wilson coefficients. During the numerical calculations, we will vary all new Wilson coefficients in the range \(-|C_7^{SM}| \leq C_X \leq |C_7^{SM}|\) and assume that all new Wilson coefficients are real. The experimental results on the branching ratio of the \(B \to K^*(K)\ell^+\ell^-\) decays \([15,16]\) and the bound on the branching ratio of \(B \to \mu^+\mu^- [17]\) suggest that this is the right order of magnitude for the Wilson coefficients describing the vector and scalar interaction coefficients. But
present experimental results on the branching ratio of the $B \to K^*\ell^+\ell^-$ and $B \to K\ell^+\ell^-$ decays impose stronger restrictions on some of the new Wilson coefficients. For example, $-2 \leq C_{LL} \leq 0, 0 \leq C_{RL} \leq 2.3, -1.5 \leq C_T \leq 1.5$ and $-3.3 \leq C_{TE} \leq 2.6$, and all of the remaining Wilson coefficients vary in the region $-|C^{SM}_{10}| \leq C_X \leq |C^{SM}_{10}|$.

In further numerical analysis, as can easily be seen from Eqs. (16)–(24), explicit forms of the form factors are needed, which are the main and most important parameters in the calculation of $P_{ij}$. These form factors are calculated in the framework of the QCD sum rules in [3, 12, 13] and their $q^2$ dependence can be represented, to a very good accuracy, in the following forms

$$g(q^2) = \frac{1 \text{ GeV}}{(1 - \frac{q^2}{(5.6 \text{ GeV})^2})^2}, \quad f(q^2) = \frac{0.8 \text{ GeV}}{(1 - \frac{q^2}{(6.5 \text{ GeV})^2})^2},$$

$$g_1(q^2) = \frac{3.74 \text{ GeV}^2}{(1 - \frac{q^2}{40.5 \text{ GeV}^2})^2}, \quad f_1(q^2) = \frac{0.68 \text{ GeV}^2}{(1 - \frac{q^2}{30 \text{ GeV}^2})^2},$$

which we will be using in the numerical calculations.

Numerical results are presented only for the $B_s \to \ell^+\ell^-\gamma$ decay. It is clear that in the SU(3) limit the difference between the decay rates is attributed to the CKM matrix elements only, i.e.,

$$\frac{\Gamma(B_d \to \ell^+\ell^-\gamma)}{\Gamma(B_s \to \ell^+\ell^-\gamma)} \simeq \frac{|V_{td}V_{ts}^*|^2}{|V_{ub}V_{ub}^*|^2} \simeq \frac{1}{20}.$$

It follows from the explicit expressions of the double-lepton polarization asymmetries that they depend on $q^2$ and the new Wilson coefficients. For this reason there may appear difficulties in studying the dependencies of the physical properties on both parameters at the same time. Hence, it is necessary to eliminate the dependence of $P_{ij}$ on one of these parameters. Here in the present work, we eliminate $q^2$ dependence of $P_{ij}$ by performing integration over $q^2$ in the kinematically allowed region. The averaging of $P_{ij}$ over $q^2$ is defined as

$$\langle P_{ij} \rangle = \frac{\int_{4m^2_B}^{m^2_B} P_{ij} \frac{dB}{dq^2} dq^2}{\int_{4m^2_B}^{m^2_B} \frac{dB}{dq^2} dq^2}$$

(26)

The reason why we study the dependence of $\langle P_{ij} \rangle$ on new Wilson coefficients is that in doing so we directly establish new physics beyond the SM, if the value of $\langle P_{ij} \rangle$ turns out to be different compared to that predicted by the SM.

In Figs. (1)–(8) we present the dependence of the averaged double-lepton polarization asymmetries on the new Wilson coefficients. From these figures we get the following results:

- $\langle P_{LL} \rangle$ exhibits very strong dependence on scalar, tensor interactions as well as on the vector interaction with coefficient $C_{RL}$. When scalar interaction coefficients vary in
the region $-4 \leq C_{\text{scalar}} \leq -0.8$ the value of $\langle P_{LL} \rangle$ is positive; in the region $-0.8 \leq C_{\text{scalar}} \leq 0.8$ it gets negative value and when it varies in the region $0.8 \leq C_{\text{scalar}} \leq 4$ it again gets positive values. We should remind that in the SM $\langle P_{LL} \rangle$ is negative and its magnitude is $|\langle P_{LL} \rangle| \approx 0.7$. Similar situation holds for tensor interaction with the coefficient $C_{TE}$, as can easily be seen in Fig. (1). Our analysis shows that $\langle P_{LT} \rangle$ depends more strongly on scalar and tensor interactions, as is the case for $\langle P_{LL} \rangle$. Therefore we can conclude that measurement of the sign and magnitude of $\langle P_{LL} \rangle$ and $\langle P_{LT} \rangle$ can give essential information about the existence of new physics beyond the SM.

- $\langle P_{TL} \rangle$ is very sensitive to the existence of the scalar interaction only. More essential than that, $\langle P_{TL} \rangle$ changes its sign when scalar interaction coefficients vary in the allowed region. Such behavior can serve as a good test for establishing new type of scalar interaction.

- $\langle P_{TT} \rangle \approx - \langle P_{NN} \rangle$, and both are quite sensitive to the existence of tensor and scalar interactions, and to the vector interaction with coefficient $C_{LR}$. In the presence of tensor, scalar and vector interactions the values of $\langle P_{TT} \rangle$ and $\langle P_{NN} \rangle$ can exceed the SM results 6, 5 and 2.5 times, respectively. Moreover, when scalar interaction coefficients $C_{RLLR}(C_{RLRL})$ is negative, then the value of $\langle P_{TT} \rangle$ is positive (negative). When $C_{RLLR}(C_{RLRL})$ is positive, $\langle P_{TT} \rangle$ becomes negative (positive). For the case when vector interaction coefficient $C_{LR}$ is negative, $\langle P_{TT} \rangle$ is also negative, while, when $C_{LR}$ changes sign and becomes positive, $\langle P_{TT} \rangle$ turns out to be positive as well. Therefore determination of the sign and magnitude of $\langle P_{NN} \rangle$ and $\langle P_{TT} \rangle$ can give unambiguous information about the existence of scalar and vector interactions. Departure of $\langle P_{NT} \rangle$ and $\langle P_{TN} \rangle$ from SM results is of not considerable importance, and hence we do not present their dependence on $C_X$.

Depicted in Figs. (4)–(7) are the dependence of $\langle P_{yi} \rangle$ on the new Wilson coefficients for the $B_s \to \tau^+\tau^-\gamma$ decay. Similar to the $B_s \to \mu^+\mu^-\gamma$ decay, we observe that several of the double-lepton polarization asymmetries are very sensitive to the existence of new physics. More precisely, we can briefly summarize the results as follows:

i) except $C_{LL}$, $\langle P_{LL} \rangle$ is very sensitive to the existence of all new Wilson coefficients.

ii) $\langle P_{LT} \rangle$ and $\langle P_{TL} \rangle$ exhibit strong dependence on all Wilson coefficients, except the vector interactions $C_{LL}$ and $C_{RR}$. These quantities show very strong dependence, especially, on all types of scalar interactions, vector interactions with coefficients $C_{LR}$ and $C_{RL}$, and tensor interaction with coefficient $C_T$.

iii) $\langle P_{TT} \rangle = - \langle P_{NN} \rangle$ are very sensitive to the existence of scalar interactions $C_{RLLR}$ and $C_{LRLL}$, when these coefficients are both positive. More important than that, $\langle P_{TT} \rangle = - \langle P_{NN} \rangle$ change their sign when scalar coefficients vary in the region $-3 > C_{\text{scalar}} > +3$.

iv) $\langle P_{NT} \rangle \approx \langle P_{TN} \rangle$ are both strongly dependent only on the tensor interaction with coefficient $C_{TE}$. For all other new Wilson coefficients the values of $\langle P_{NT} \rangle \approx \langle P_{TN} \rangle$ are very close to the SM prediction, i.e., to zero. Furthermore, when $C_{TE}$ is negative (positive), these quantities get positive (negative) values, with a considerable departure from the SM about 15%. Therefore, determination of the sign and and magnitude of $\langle P_{TN} \rangle$ and $\langle P_{NN} \rangle$ can give direct information solely about the existence of the tensor interaction.
v) Similar situation holds for the double–lepton polarization asymmetry \( \langle P_{LN} \rangle \) as well, i.e., \( \langle P_{LN} \rangle \) shows strong dependence only on the tensor interaction with coefficient \( C_{TE} \). When \( C_{TE} \) is negative (positive), \( \langle P_{LN} \rangle \) gets negative (positive) values. Hence, departure from the SM prediction (in SM \( \langle P_{LN} \rangle \approx 0 \)) can reach to 4%. Therefore analysis of \( \langle P_{LN} \rangle \) can serve as a good test for establishing the presence of tensor interactions. The values of \( \langle P_{NL} \rangle \) is negligibly small (maximum departure from the SM is being about 1.5%) and for this reason we do not present its dependence on \( C_X \).

At the end of this section, let us discuss the possibility of measurement of the lepton polarization asymmetries in experiments. Experimentally, to measure an asymmetry \( \langle P_{ij} \rangle \) of the decay with the branching ratio \( \mathcal{B} \) at \( n \sigma \) level, the required number of events (i.e., the number of \( B \bar{B} \) pair) are determined by the following expression

\[
\mathcal{N} = \frac{n^2}{\mathcal{B}s_1s_2\sigma_{ij}^2},
\]

where \( s_1 \) and \( s_2 \) are the efficiencies of the leptons. Efficiency of the \( \mu \) lepton is practically equal to one, and typical values of the efficiency of the \( \tau \) lepton ranges from 50% to 90% for the various decay modes [18].

From the expression for \( \mathcal{N} \) we see that, in order to obtain the double–lepton polarization asymmetries in \( B_s \to \ell^+\ell^-\gamma \) decays at \( 3\sigma \) level, the minimum number of required events are (for the efficiency of \( \tau \)-lepton we take 0.5):

- for the \( B_s \to \mu^+\mu^-\gamma \) decay
  \[
  \mathcal{N} = \begin{cases} 
  \sim 10^9 & \langle P_{LL} \rangle , \\
  \sim 7 \times 10^{10} & \langle P_{TT} \rangle , \\
  \sim 3 \times 10^{10} & \langle P_{NN} \rangle \approx \langle P_{LT} \rangle \approx \langle P_{TL} \rangle ,
  \end{cases}
  \]

  which yields that, for detecting \( \langle P_{LN} \rangle, \langle P_{NL} \rangle, \langle P_{NT} \rangle \) and \( \langle P_{TN} \rangle \), more than \( 10^{13} \) \( B \bar{B} \) pairs are required.

- for \( B_s \to \tau^+\tau^-\gamma \) decay
  \[
  \mathcal{N} = \begin{cases} 
  \sim 10^{10} & \langle P_{LL} \rangle, \langle P_{TT} \rangle, \langle P_{NN} \rangle , \\
  \sim 3 \times 10^{11} & \langle P_{LT} \rangle = \langle P_{TL} \rangle , \\
  > 10^{13} & \langle P_{LN} \rangle, \langle P_{NL} \rangle, \langle P_{TN} \rangle, \langle P_{NT} \rangle .
  \end{cases}
  \]

The number of \( B \bar{B} \) pairs that will be produced at LHC is around \( \sim 10^{12} \). As a result of a comparison of this number of \( B \bar{B} \) pairs with that of \( \mathcal{N} \), we conclude that \( \langle P_{LL} \rangle, \langle P_{TT} \rangle, \langle P_{NN} \rangle, \langle P_{TL} \rangle \) and \( \langle P_{LT} \rangle \) in \( B_s \to \mu^+\mu^-\gamma \) decay, and \( \langle P_{LL} \rangle, \langle P_{TT} \rangle \) and \( \langle P_{NN} \rangle \) in \( B_s \to \tau^+\tau^-\gamma \) decay can be detectable in future experiments at LHC. Note that for calculation of the branching ratio, we take its SM result, i.e., \( \mathcal{B}(B_s \to \mu^+\mu^-\gamma) \approx 1.3 \times 10^{8} \) and \( \mathcal{B}(B_s \to \tau^+\tau^-\gamma) \approx 6 \times 10^{8} \). In obtaining these values, minimal value for the photon energy is taken to be 50 MeV.

In conclusion, we calculate nine double–lepton polarization asymmetries using the most general, model independent form of the effective Hamiltonian including all possible form of interactions. The sensitivity of the averaged double–lepton polarization asymmetries to the new Wilson coefficients are studied. Finally we discuss the possibility of experimental measurement of these double–lepton polarization asymmetries at LHC.
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Figure captions

Fig. (1) The dependence of the averaged double–lepton polarization asymmetry $\langle P_{LL} \rangle$ on the new Wilson coefficients for the $B_s \rightarrow \mu^+\mu^-\gamma$ decay.

Fig. (2) The same as in Fig. (1), but for the averaged double–lepton polarization asymmetry $\langle P_{TL} \rangle$.

Fig. (3) The same as in Fig. (1), but for the averaged double–lepton polarization asymmetry $\langle P_{TT} \rangle$.

Fig. (4) The same as in Fig. (1), but for the $B_s \rightarrow \tau^+\tau^-\gamma$ decay.

Fig. (5) The same as in Fig. (4), but for the averaged double–lepton polarization asymmetry $\langle P_{LT} \rangle$.

Fig. (6) The same as in Fig. (4), but for the averaged double–lepton polarization asymmetry $\langle P_{TL} \rangle$.

Fig. (7) The same as in Fig. (4), but for the averaged double–lepton polarization asymmetry $\langle P_{TT} \rangle$. 
Figure 1:

\[ \langle P_{LL} \rangle (B \rightarrow \mu^- \mu^+ \gamma) \]

Figure 2:
Figure 3:

Figure 4:
\[ \langle P_{TL} | B \rightarrow T^+ \rangle \]

\[ \langle P_{TL} | B \rightarrow T^+ \rangle \]

Figure 5:

![Graph 1](image1)

Figure 6:

![Graph 2](image2)
Figure 7: