Reliability Assessment and Optimization of Double Random Vibration Systems based on PDEM

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Abstract. The dynamic responses analysis method of structure with random parameters under stochastic seismic loads is proposed. The generalized probability density evolution equation for compound stochastic process is solved by the finite difference method. Then, for the first passage criterion, the dynamic reliability is analysed by establishing the one-dimensional integral in safe domain and the absorbing boundary condition. A new optimization method is coming up combining improved genetic algorithm and PDEM. Taking the parameters optimization of a three-story floor-shear frame structure with TMD under random seismic loads as an example, the correctness of the proposed method is verified by comparing interlayer displacement response and dynamic reliability.

1. Introduction

The tuned mass damper (TMD) absorbs the vibration energy by using the whip lash effect of small mass spring system attached to the permanent structure. Obviously, the reasonable selection of TMD parameters has a great influence on the effect of shock absorption. In practical engineering, random excitation such as earthquake always controls the performance of structure with uncertain parameters. Therefore, it is necessary to consider the randomness of seismic loads and structural parameters when optimizing the parameters of TMD system.

In addition, the methods of dynamic reliability analysis of engineering structure based on the two failure laws are proposed, namely, the first passage criterion and accumulated damage evaluation criterion [1]. In the analysis of dynamic reliability for the first passage criterion, the methods including leapfrog process theory, diffusion process theory and PDEM are developed. The leapfrog process analysis method [2] is widely applied at present, but it is usually difficult to grasp the probability information of structure response comprehensively. The diffusion process theory method based on Fokker-Planck-Kolmogorov (FPK) equation [3] has not been widely used because the solution of FPK equation is very difficult. In recent years, J Li and J B Chen have developed reliability analysis method of the first passage criterion based on the absorbing boundary condition [4] and PDEM [5], which considering the influence of double random factors. Dynamic reliability solved is successfully applied to the solving general multi-degree-of-freedom structural system.

In this paper, by considering the randomness of external loads and structure parameters, establishes the PDEE of double random nonlinear response characteristics and solves the system reliability in safe domain. The mass optimization design method is developed by establishing optimization model of a multi-degree-of-freedom floor-shear frame structure with TMD based on improving genetic algorithm.
The successful application of the reliability analysis and optimization design of double random vibration system with TMD is realized.

2. The Probability Density Evolution Method
Generally, the motion equation of n-dimensional double random vibration systems can be expressed as

\[ M(\Theta)\ddot{X} + C(\Theta)\dot{X} + G(\Theta, X) = \Gamma F(\Theta, t) \]  \hspace{1cm} (1)

in which \( \dot{X}, \ddot{X}, X \) is n-dimensional acceleration, velocity and displacement vector, respectively; \( M(\Theta), C(\Theta) \) is \( (n \times n) \) mass and damping matrix; \( G(\Theta, X) \) is the n-dimensional linear or nonlinear restoring force vector; \( \Gamma \) is \( (n \times r) \) excitation influence matrix; \( F(\Theta, t) \) is r-dimensional excitation vector; \( \Theta = (\Theta_1, \Theta_2, \ldots, \Theta_s) \) is the random vector including structural parameters and the external excitation process, where \( s \) represents the number of all random variables in a double random system.

The physical quantity of interest is recorded as \( Z_\Theta = (Z_1(t), Z_2(t), \ldots, Z_m(t)) \). For well-posed dynamical system (the physical solution exists, unique and continuously depends on the basic parameters), there must be

\[ Z = H_\Theta(\Theta, t), \dot{Z} = h_\Theta(\Theta, t) \]  \hspace{1cm} (2)

in which \( H_\Theta = (H_{z_1}, H_{z_2}, \ldots, H_{z_m})^T, h_\Theta = (h_{z_1}, h_{z_2}, \ldots, h_{z_m}) \). Obviously, formula (2) is the solution of equation (1) if the quantity in question is displacement.

The joint probability density function \( p_{z_\Theta}(z, \Theta, t) \) of a conservative stochastic system \( (Z, \Theta) \). According to the random event description of probability conservation principal, there is

\[ \frac{D}{Dt} \int_{\Omega_1 \times \Omega_3} p_{z_\Theta}(z, \Theta, t) dz d\Theta = 0 \]  \hspace{1cm} (3)

where \( \frac{D}{Dt} \) is the total derivative; \( \Omega_1 \times \Omega_3 \) is the soluble region for any original space at time \( t \). A series of mathematical derivations of equation (3) can obtain

\[ \frac{\partial p_{z_\Theta}(z, \Theta, t)}{\partial t} + \sum_{i=1}^{m} \dot{Z}(\Theta, t) \frac{\partial p_{z_\Theta}(z, \Theta, t)}{\partial z_i} = 0 \]  \hspace{1cm} (4)

In particular, the equation (4) can be simplified to a first-order partial differential equation when only one physical quantity is considered.

\[ \frac{\partial p_{z_\Theta}(z, \Theta, t)}{\partial t} + \dot{Z}(\Theta, t) \frac{\partial p_{z_\Theta}(z, \Theta, t)}{\partial z} = 0 \]  \hspace{1cm} (5)

Equation (4) and (5) are the generalized probability density evolution equation (PDEE). Obviously, the dimension of equation (4) depends only on the number of physical quantities examined, and is independent of the degree-of-freedom (DOF) of the original physical system.

J Li and J B Chen established a numerical analysis method of PDEE using finite difference method [6]. In order to fully consider the precision deviation caused by dissipation and dispersion in the calculation process, this paper uses the TVD scheme with flux limiter to solve the numerical [7].

3. Dynamic Reliability
According to the failure laws of structure, considering the response of structural interlayer displacement, the system reliability function of the first passage criterion is generally defined as

\[ R(t) = \Pr\{X(\tau) \in \Omega_1, 0 \leq \tau \leq t\} \]  \hspace{1cm} (6)
where $X(\tau)$ is the physical quantity that leads to structural failure; $\Omega_s$ is safe domain; $t$ is the structure life or time period that needs to be inspected.

Mandatory absorbing boundary conditions is

$$\bar{p}_{X_0}(x, \theta, t) = 0, x \notin \Omega_s$$  \hspace{1cm} (7)

The residual probability density function is obtained by solving the PDE

$$\bar{p}_{X}(x, t) = \int_{\Omega_s} \bar{p}_{X_0}(x, \theta, t) d\theta$$  \hspace{1cm} (8)

Then the dynamic reliability of structure is solved by equation (9).

$$R(t) = \int_{-\infty}^{\infty} \bar{p}_{X}(x, t) dx$$  \hspace{1cm} (9)

4. Improved Genetic Algorithm

Improved genetic algorithm [8] is a stochastic search method to simulate the biological evolution mechanism. By establishing the information exchange between individuals and the strategy of group search, the problem is converted to the optimal state. Because the improved genetic algorithm does not depend on the gradient information of the objective function, taking the structural displacement is the constraint condition for solving the optimal mass of TMD in the nonlinear analysis of the multi-degree-of-freedom double-random vibration system [9]. Inter layer displacement response at top of structure $f(x)$ is the objective function when using genetic algorithm to solve the optimal mass $m_d$ of TMD, the model can be defined as

$$\min f(x), x = x_1, x_2, \cdots, x_n$$  \hspace{1cm} (10)

where $x = x_1, x_2, \cdots, x_n$ is the mass that TMD may achieve; $\min f(x)$ is the optimal solution of objective function. In order to facilitate the application of improved genetic algorithm, the above constrain problem is transformed into unconstrained problem, which can be expressed as

$$\varphi_i(x) = f(x)(1 + PC)$$  \hspace{1cm} (11)

where $P$ is the effect of default on the objective function. $P=1$ represents a default, which means that the individual is more likely to be eliminated. When $i$ individual constraint is satisfied there is $C_i = 0$, otherwise $C_i = 0.2$.

Fitness function represents individual adaptation to population. During the iteration process, individual fitness of each generation population needs to be determined, and the individual fitness of each generation population is

$$\mu_j = 1 - \frac{\varphi_j(x)}{\varphi(x)_{\min} + \varphi(x)_{\max}}$$  \hspace{1cm} (12)

where $u_j$ is the fitness of the $j$ individuals in this population.

5. Numerical Example

In order to verify the correctness of study method in this paper, the solution of dynamic reliability and the parameter optimization of TMD of three-story floor-shear frame structure with random stiffness parameters under seismic excitation as an example. The mass of the structure from the bottom to the top is 2762 kg, 2760 kg and 2300 kg. The lateral stiffness of each layer is $1.243 \times 10^4 \text{kN/m}$, $0.961 \times 10^4 \text{kN/m}$ and $0.761 \times 10^4 \text{kN/m}$. In the rigidity parameter, all three random variables are subject to normal distribution, and the coefficient of variation is 0.2. In the case, the damping model
based on Rayleigh damping, and the damping matrix is determined by mass matrix and rigidity matrix, namely \( C = aM + bK \). Because \( K \) is a random matrix, the damping matrix is also a random matrix.

Basement input acceleration time history

\[
\ddot{x}_g = \xi_1 \ddot{x}_{g1} + \xi_2 \ddot{x}_{g2}
\]  

where \( \ddot{x}_{g1} \) and \( \ddot{x}_{g2} \) are the East-West and North-South components of the seismic acceleration record of the El Centro respectively, their peaks are all \( 2m/s^2 \). \( \xi_1 \) and \( \xi_2 \) are the random variables of identically independent and normal distribution, the means value is all 0.5, and the coefficient of variation is all 0.2.

In addition, the value ranges of the initial design variables of TMD are \( m_t \in (0.03, 10.52)t \), \( c_d = 0.9 \times c_{11} \), and \( k_x = 0 \), where \( c_{11} \) is the first data of the damping matrix in double random vibration systems with TMD.

Optimization the mass of TMD using improved genetic algorithm, where selection probability is 0.3, crossover probability is 0.2 and mutation probability is 0.2. The population sizes are 20, 30 and 40 respectively. The number of iterations is 40 times and the calculation results are shown in table 1. The analysis of table 1 shows that the structure of TMD has a good effect on reducing the displacement response when TMD mass is 8.79t, and the maximum displacement response after optimization is 49.93% than before optimization. And the calculation results tend to be stable with the increase of population size.

The displacement response of the top layer of structure is \( x_i \). Table 2 shows the reliability of absolute value of displacement response of top layer not exceed the given limit. It shows that there is great variability in the reliability of structural displacement when displacement limit value is small. However, TMD has a good effect on reducing the displacement response and increasing the reliability when displacement limit value is large.

| Population size | The optimal mass of TMD (t) | Optimization ratio |
|-----------------|-----------------------------|--------------------|
| 20              | 8.64                        | 48.94%             |
| 30              | 8.79                        | 49.93%             |
| 40              | 8.79                        | 49.88%             |

| Table 2. The top layer displacement historical reliability. |
|----------------------------------------------------------|
| \( x_i / m \) | Before optimization | 20 populations | 30 populations | 40 populations |
|----------------|---------------------|----------------|----------------|----------------|
| \([-0.02,0.02]\) | 0.6682              | 0.4689         | 0.4674         | 0.4925         |
| \([-0.04,0.04]\) | 0.8812              | 0.9023         | 0.9024         | 0.8902         |
| \([-0.06,0.06]\) | 0.9486              | 1.0000         | 1.0000         | 1.0000         |
| \([-0.08,0.08]\) | 1.0000              | 1.0000         | 1.0000         | 1.0000         |

Figure 1 shows the mean and standard deviation of top layer displacement of structure before and after optimization. As can be seen from figure (a), TMD has better control effect on structure displacement response when it is larger, while the control effect is not obvious when it is smaller. In general, TMD has a good effect on reducing the displacement response of structure, and the maximum displacement response after optimization is 49.93% than before optimization. Figure (b) shows that the discreteness of displacement response of TMD structure is smaller than that of uncontrolled structure when the randomness of structural parameters and external excitation is considered. Therefore, TMD can effectively reduce the discreteness of structural response.
Figure 1. The mean and standard deviation (std.D) of top layer displacement of structure before and after optimization.

Figure 2. The displacement PDF surface of the top floor.

Figure 3. The contour of the PDF surface of the top floor.

Figure 4. The PDF at certain time instants.

Figure 5. The CDF at certain time instants.

Combined with the displacement probability density function (PDF) surface of the top floor of structure in figure 2 and the corresponding contour of the PDF surface in figure 3, it can be seen that the PDF change with time, which reflect the result of the probability flowing in the state space. Obviously, the probability distribution of structural response is the irregular surface. Its evolution
process likes a continuous mountain range, and its contour of PDF surface liking water flowing in a river. So, this is a non-stationary random process and the different widths at different times.

Figure 4 shows that the PDF at certain time instants and figure 5 shows that the cumulated probability density function (CDF) at certain time instants. Where, CDF is the integral of PDF. For the response analysis of double random vibration systems, the PDF is not the normal distribution or other special distribution, even assuming that the randomness of structural parameters and external incentives is the normal distribution. Moreover, it is a multi-peak curve at many times.

6. Conclusions
In this paper, the effect of TMD on the response of the double random vibration system is considered, and then the improved genetic algorithm and PDEM are adopted to optimize the parameters of TMD. The conclusions are as follows:

(1) TMD can effectively reduce the maximum peak and discreteness of structural displacement response. In the case, the maximum peak of structural displacement is reduced by 49.93\% after optimization, which greatly improved the structural reliability. There is great variability in the reliability of structural displacement when displacement limit value is small. However, TMD has a good effect on reducing the displacement response and increasing the reliability when displacement limit value is large.

(2) The probability density distribution of the structural response is a multi-peak curve and varies with time. Therefore, it is necessary to describe the whole process with probability.

The ideas and methods presented in this paper provide a new way to solve the reliability of double random vibration system, and provide technical support for structural optimization design using TMD for shock absorption control, which has good engineering practicability.

References
[1] Zhang Z H and Yang W J 2012 Spatial Struct. 18 64-75.
[2] Kawano K and Venkataramana K 1999 Comput. Method. Appl. M. 168 255-72.
[3] Langley R S 1998 J. Sound. Vib. 123 213-27.
[4] Li J and Chen J B 2004 Soil. Dyn. Earthq. Eng. 2004 39-44.
[5] Yang J Y, Chen J B and Li J 2015 J. Southwest Jiaotong Univ. 50 1047-54.
[6] Chen J B and Li J 2008 Chinese Quart. Mech. 21-8.
[7] Li and Chen JB 2009 Stochastic Dyn. Struct. (John Wiley & Sons Pte Ltd).
[8] Greco A, Cannizzaro F and Pluchino A 2016 Eng. Struct. 143 (2017): 152-168.
[9] Pourzeynali S, Salimi S and Kalesar H E 2013 Sci. Iran. 20 207-21.