Flavor symmetry breaking of the nucleon sea in the statistical approach

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Abstract. The flavor structure of the nucleon sea provides unique information to test the statistical parton distributions approach, which imposes strong relations between quark and antiquarks. These properties for unpolarized and helicity distributions have been verified up to now by recent data. We will present here some new updated results which are a real challenge, also for forthcoming accurate experimental results, mainly in the high Bjorken-\(x\) region.

Keywords: Statistical parton distributions, nucleon sea asymmetry
1. Introduction

The structure of the nucleon sea is an important topic which has been the subject of several relevant review papers \[1, 2\]. In spite of considerable progress made in our understanding, several aspects remain to be clarified, which is the goal of this paper. The properties of the light quarks \(u\) and \(d\), which are the main constituents of the nucleon, will be revisited in the quantum statistical parton distributions approach proposed more than one decade ago. It is well known that the \(u\)-quark dominates over the \(d\)-quark, but for antiquarks, \(SU(2)\) symmetry was assumed for a long time, namely the equality for the corresponding antiquarks, \(\bar{u} = \bar{d}\), leading to the Gottfried sum rule \[3\]. However the NMC Collaboration \[4\] found that this sum rule is violated, giving a strong indication that \(\bar{d} > \bar{u}\). In the statistical approach we impose relations between quarks and antiquarks and we treat simultaneously unpolarized distributions and helicity distributions, which strongly constrains the parameters, a unique situation in the literature. As we will see, this powerful tool allows us to understand, not only the flavor symmetry breaking of the light sea, but also the Bjorken-\(x\) behavior of all these distributions and to make challenging specific predictions for forthcoming experimental results, in particular in the high-\(x\) region.

2. Formalism

Let us now recall the main features of the statistical approach for building up the parton distributions function (PDFs). The fermion distributions are given by the sum of two terms, a quasi Fermi-Dirac function and a helicity independent diffractive contribution:

\[
qx^h(x, Q_0^2) = \frac{A_q X_{0q}^h x^{b_q}}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1},
\]

\[
\bar{q}x^h(x, Q_0^2) = \frac{\tilde{A}_q (X_{0\bar{q}}^{-h})^{-1} x^{\tilde{b}_q}}{\exp[(x + X_{0\bar{q}}^{-h})/\bar{x}] + 1} + \frac{A_q x^{b_q}}{\exp(x/\bar{x}) + 1},
\]

at the input energy scale \(Q_0^2 = 1\text{GeV}^2\). We note that the diffractive term is absent in the quark helicity distribution \(\Delta q\), in the quark valence contribution \(q - \bar{q}\) and in \(u - d\) if one assumes \(\tilde{A}_u = \tilde{A}_d\).

In Eqs. (1,2) the multiplicative factors \(X_{0q}^h\) and \((X_{0\bar{q}}^{-h})^{-1}\) in the numerators of the first terms of the \(q\)'s and \(\bar{q}\)'s distributions, was justified in our attempt to generate the transverse momentum dependence of the PDFs. The parameter \(\bar{x}\) plays the role of a universal temperature and \(X_{0q}^\pm\) are the two thermodynamical potentials of the quark \(q\), with helicity \(h = \pm\). They represent the fundamental parameters of the approach. Notice that following the chiral properties of QCD, we have \(X_{0q}^h = -X_{0\bar{q}}^{-h}\) in the exponentials.

For a given flavor \(q\) the corresponding quark and antiquark distributions involve the free parameters, \(X_{0q}^\pm, A_q, \tilde{A}_q, b_q, \tilde{b}_q\) and \(\bar{b}_q\), whose number is reduced to seven by the valence sum rule, \(\int (q(x) - \bar{q}(x))dx = N_q\), where \(N_q = 2, 1\) for \(u, d\), respectively.
From a fit of unpolarized and polarized experimental data we have obtained for the potentials the values [5]:

\[ X_u^+ = 0.475 \pm 0.001, \quad X_u^- = X_d^- = 0.307 \pm 0.001, \quad X_d^+ = 0.244 \pm 0.001. \]  

It turns out that two potentials have identical numerical values, so for light quarks we have found the following hierarchy between the different potential components

\[ X_u^+ > X_u^- = X_d^- > X_d^+. \]  

We notice that quark helicity PDFs increases with the potential value, while antiquarks helicity PDFs increases when the potential decreases.

The above hierarchy implies the following hierarchy on the quark helicity distributions for any \( x, Q^2 \),

\[ xu_+(x, Q^2) > xu_-(x, Q^2) = xd_-(x, Q^2) > xd_+(x, Q^2) \]  

and also the obvious hierarchy for the antiquarks, namely

\[ xd_-(x, Q^2) > xd_+(x, Q^2) = x\bar{u}_+(x, Q^2) > x\bar{u}_-(x, Q^2). \]

3. Results

For illustration we show in Figs. [1,2] the resulting distributions at \( Q^2 = 54\text{GeV}^2 \), a particular value which will be explained later. It is important to note that these inequalities Eqs. (5,6) are preserved by the next-to-leading order QCD evolution, at least outside the diffractive region, namely for \( x > 0.1 \). We have also checked that the initial analytic form Eqs. (1,2), is almost preserved by the \( Q^2 \) evolution with some small changes of the parameters. This general pattern displayed in Figs. [1,2] does not change much for different \( Q^2 \) values. We also remark that the largest distribution is indeed \( xu_+(x, Q^2) \), which has a distinct maximum around \( x = 0.3 \), a relevant feature as we will see below. In our approach one can conclude that, \( u(x, Q^2) > d(x, Q^2) \) implies a flavor symmetry breaking of the light sea, i.e. \( \bar{d}(x, Q^2) > \bar{u}(x, Q^2) \), which is clearly seen in Fig. [2] A simple interpretation of this result is a consequence of the Pauli exclusion principle, based on the fact that the proton contains two \( u \)-quarks and only one \( d \)-quark.

We now turn to more significant outcomes concerning the helicity distributions which follow from Eqs. (5,6). First for the \( u \)-quark

\[ x\Delta u(x, Q^2) > 0 \quad x\Delta \bar{u}(x, Q^2) > 0. \]  

Similarly for the \( d \)-quark

\[ x\Delta d(x, Q^2) < 0 \quad x\Delta \bar{d}(x, Q^2) < 0. \]  

So once more, quarks and antiquarks are strongly related since opposite signs for the quark helicity distributions, imply opposite signs for the antiquark helicity distributions, (see Eqs. (1,2)), at variance with the simplifying flavor symmetry assumption \( x\Delta \bar{u}(x, Q^2) = x\Delta \bar{d}(x, Q^2) \).

Our predicted signs and magnitudes have been confirmed [5] by the measured single-helicity asymmetry \( A_L \) in the \( W^\pm \) production at BNL-RHIC from STAR [6]. For the
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Figure 1. The different helicity components of the light quark distributions \( x f(x, Q^2) \) (\( f = u_+, u_- = d_-, d_+ \)), versus \( x \), at \( Q^2 = 54 \text{GeV}^2 \), after NLO QCD evolution, from the initial scale \( Q_0^2 = 1 \text{GeV}^2 \).

Figure 2. The different helicity components of the light antiquark distributions \( x f(x, Q^2) \) (\( f = \bar{d}_-, \bar{d}_+ = \bar{u}_+, \bar{u}_- \)), versus \( x \), at \( Q^2 = 54 \text{GeV}^2 \), after NLO QCD evolution, from the initial scale \( Q_0^2 = 1 \text{GeV}^2 \).

The extraction of helicity distributions, this process is expected to be clearer than semi-inclusive DIS, because it does not involve fragmentation functions.

Another important earlier prediction concerns the Deep Inelastic Scattering (DIS) asymmetries, more precisely \( (\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2))/(q(x, Q^2) + \bar{q}(x, Q^2)) \) (\( q = u, d \)), shown in Fig. 3. Note that the JLab [7] data and the COMPASS [9] data are in agreement with these predictions, in particular in the high-\( x \) region where there is a great accuracy. Beyond \( x = 0.6 \), this is a new challenge for the JLab 12 GeV upgrade,
with an extremely high luminosity, will certainly reach a much better precision.

There are two more strong consequences of the equalities in Eqs. (5,6), which relate unpolarized and helicity distributions, namely for quarks

\[ xu(x, Q^2) - xd(x, Q^2) = x\Delta u(x, Q^2) - x\Delta d(x, Q^2) > 0, \]  

(9)

and similarly for antiquarks

\[ x\bar{d}(x, Q^2) - x\bar{u}(x, Q^2) = x\Delta \bar{u}(x, Q^2) - x\Delta \bar{d}(x, Q^2) > 0. \]  

(10)

These equalities mean that the flavor asymmetry of the light quark and antiquark distributions is the same for the corresponding helicity distributions, as noticed long time ago, by comparing the isovector contributions to the structure functions \( 2xg_1^{(p-n)}(x, Q^2) \) and \( F_2^{(p-n)}(x, Q^2) \), which are the differences on proton and neutron targets [11].

We have checked that using Eq. (9) it is possible to predict the helicity distributions from the unpolarized distributions, as displayed in Fig. 4. This difference, which is indeed positive, has a pronounced maximum around \( x = 0.3 \), reminiscent of the dominance of \( xu_+ (x, Q^2) \).

Similarly one can use Eq. (10), to predict the difference of the antiquark helicity distributions and the result is shown in Fig. 5.

Although compatible with zero it is slightly positive, but JLab 12GeV upgrade is expected to reach a much better accuracy [10].

Let us inspect the \( x \)-behavior of all these components \( xu_+ (x, Q^2), ..., x\bar{u}_- (x, Q^2) \), which are all monotonic decreasing functions of \( x \) at least for \( x > 0.2 \), outside the region.
Figure 4. The data on $x[\Delta u(x, Q^2) - \Delta d(x, Q^2)]$ from Refs. [7] (JLab) and [9] (COMPASS), compared to the statistical model prediction, using Eq. (9) with the corresponding error band.

Figure 5. The data on $x[\Delta \bar{u}(x, Q^2) - \Delta \bar{d}(x, Q^2)]$ from Refs. [8] (HERMES) and [9] (COMPASS), compared to the statistical model prediction, using Eq. (10) with the corresponding error band.

dominated by the diffractive contribution (see Figs. [12]). As already said, $xu_+(x, Q^2)$ is the largest of the quark components and similarly $xd_-(x, Q^2)$ is the largest of the antiquark components.

The ratio $xd(x, Q^2)/xu(x, Q^2)$ value is one at $x = 0$, because the diffractive contribution dominates and, due to the monotonic decreasing, it decreases for increasing $x$. This $x$-behavior is strongly related to the values of the potentials $X_{0q}^h$.

This falling $x$-behavior has been verified experimentally from the ratio of the DIS structure functions $F_2^d/F_2^p$ and from the charge asymmetry of the $W^\pm$ production in $\bar{p}p$ collisions [12].

Similarly if one considers the ratio $xd(x, Q^2)/x\bar{u}(x, Q^2)$, its value is one at $x = 0$, because
the diffractive contribution dominates and, due to the slightly larger value of $\bar{d}^{-}$ over $\bar{u}^{-}$, it increases for increasing $x$ (see Fig. 2).

By looking at the curves (See Figure 6), one sees similar behaviors. In both cases in the vicinity of $x = 0$ one has a sharp behavior due to the fact that the diffractive contribution dominates. In the high-$x$ region there is a flattening out above $x \simeq 0.6$ and it is remarkable to see that these ratios have almost no $Q^2$ dependence.

In the introduction we have recalled the first indication by the NMC Collaboration for a flavor asymmetry of the nucleon sea $\bar{d}(x) > \bar{u}(x)$. There is another way to probe this asymmetry, which is the ratio of the proton-induced Drell-Yan process $\sigma(pd)/2\sigma(pp)$ on a deuterium and an hydrogen targets. At forward rapidity region, the Drell-Yan cross section is dominated by the annihilation of a $u$-quark in the incident proton with the $\bar{u}$-antiquark in the target. Assuming charge conjugaison one can show that

$$\sigma(pd)/2\sigma(pp) \sim 1/2[1 + \bar{d}(x_2)/\bar{u}(x_2)],$$

where $x_2$ refers to the momentum fraction of antiquarks.

The major advantage of the Drell-Yan process is that it allows to determine the $x$ dependence of $\bar{d}/\bar{u}$. We show in Fig. 7 the E866 from Ref. 14 compared to our prediction. By assuming SU(3) symmetry it is possible to generate the PDFs for the baryon octet and to calculate the corresponding Drell-Yan cross section ratios. The results shown in Fig. 7 might be of interest for future hyperon beams at LHC in fixed-target mode, to study the sea structure of the hyperons. In the $\Lambda$ one expects no flavor symmetry breaking since it contains one $u$-quark and one $d$-quark and under SU(3) symmetry one has, $\bar{u}_\Lambda = \bar{d}_\Lambda = (\bar{u} + \bar{d})/2$. This is not the case for the $\Sigma^\pm$ and under SU(3) symmetry one expects $\bar{u}_\Sigma/\bar{s}_\Sigma = \bar{u}/\bar{d}$, but this prediction can be largely modified (see Ref. 15).

The extracted ratio $\bar{d}(x)/\bar{u}(x)$ for $Q^2 = 54$GeV$^2$ displayed in Fig. 8 shows a remarkable agreement with the statistical model prediction up to $x = 0.2$. 


Figure 6. The ratios $d(x, Q^2)/u(x, Q^2)$ (top) and $\bar{d}(x, Q^2)/\bar{u}(x, Q^2)$ (bottom) versus $x$ for $Q^2$ (GeV$^2$) = 1 solid, 10 dashed, 100 dashed-dotted.

Figure 7. The Drell-Yan cross section ratios of $\sigma(Bd)/2\sigma(Bp)$ versus $x_2$ (momentum fraction of the target partons) data from (E866) Ref. [14]. The curves are the statistical model predictions for different incoming beams $B = p, \Lambda, \Sigma^+, \Sigma^-$ at $p_{lab} = 800$GeV.
4. Conclusions

To conclude a monotonic increase of the ratio $x\bar{d}(x, Q^2)/x\bar{u}(x, Q^2)$ is predicted in our approach, as a consequence of strong relations between polarized quark distributions (see Eq. (10)). Very recently there was a serious indication from the preliminary results of the SeaQuest collaboration, that this ratio rises beyond $x = 0.2$, at variance with several other model predictions, as reported in Figs. 7 and 8 of Ref. [2]. This prediction is a real challenge for the statistical approach, whose strong predictive power will be confronted with several other forthcoming accurate data, mainly in the high-$x$ region, a region which remains poorly known.

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‡ It is interesting to recall that this trend was already detected in a primitive version of the statistical model [13] (see Fig. 11).
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