General holographic superconductor models with backreactions

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Abstract

We study general models of holographic superconductors away from the probe limit. We find that the backreaction of the spacetime can bring richer physics in the phase transition. Moreover we observe that the ratio $\omega g/T_c$ changes with the strength of the backreaction and is not a universal constant.

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I. INTRODUCTION

The anti-de Sitter/conformal field theories (AdS/CFT) correspondence\cite{1-3} states that a \(d\)-dimensional strongly coupled conformal field theory on the boundary is equivalent to a \((d+1)\)-dimensional weakly coupled dual gravitational description in the bulk. This remarkable finding provides a novel method to study the strongly coupled system at finite density and builds a useful connection between the condensed matter and the gravitational physics (for reviews, see Refs.\cite{4-6}). It has been shown that the spontaneous \(U(1)\) symmetry breaking by bulk black holes can be used to construct gravitational duals of the transition from normal state to superconducting state in the boundary theory\cite{7}. In \cite{8} the (2+1)-dimensional superconductor which consists of a system with a black hole and a charged scalar field in 3+1 dimensions was introduced in the probe limit where the backreaction of the matter fields on the metric is small and can be neglected. It is interesting to note that the properties of a (2 + 1)-dimensional superconductor can indeed be reproduced in this simple model. Motivated by the application of the Mermin-Wagner theorem to the holographic superconductors, there have been investigations on the effects of the curvature corrections on the (3 + 1)-dimensional superconductor and higher dimensional ones. It was shown that the Gauss-Bonnet coupling affects the condensation of the scalar field\cite{9-12} and the vector field\cite{13}, and the higher curvature correction makes the condensation harder to form. Further the curvature correction term will cause the unstable ratio \(\omega_g/T_c\)\cite{9,10}. The gravity models with the property of the so-called holographic superconductor have attracted considerable interest for their potential applications to the condensed matter physics, see for example\cite{14-25}.

A lot of studies in the holographic superconductor disclosed a second order phase transition in strongly interacting systems using the AdS/CFT correspondence. It was observed that a fairly wide class of phase transitions can be allowed if one generalizes the basic holographic superconductor model in which the spontaneous breaking of a global \(U(1)\) symmetry occurs via the St"uckelberg mechanism\cite{26}. This framework allows tuning the order of the phase transition which can accommodate the first order phase transition to occur, and for the second order phase transition it allows tuning the values of critical exponents\cite{27}. An interesting extension was done in\cite{28} by constructing general models for holographic superconductivity. It was found that except some universal model independent features, some important aspects of the quantum critical behavior strongly depend on the choice of couplings, such as the order of the phase transition and critical exponents of second-order phase transition. In addition to the numerical investigation, analytical understanding on
the phase transition of holographic superconductor was also provided in [29]. Rich phenomena in the phase transition were also found for the holographic superconductors in Einstein-Gauss-Bonnet gravity where the Gauss-Bonnet coupling can play the role in determining the order of phase transition and critical exponents in the second-order phase transition [30].

Most of available studies on the holographic superconductors are limited in the probe approximation, although they can give most qualitative results on the holographic superconductivity, the study on the effect of backreaction is called for. Recently there have been a lot of interest to study the holographic superconductor away from the probe limit and take the backreaction of the spacetime into account [28, 31–40]. Considering the backreaction, it was found that even the uncharged scalar field can form a condensate in the \((2 + 1)\)-dimensional holographic superconductor model [31]. In the p-wave superfluids system, it was argued that the order of the phase transition depends on the backreaction, i.e., the phase transition that leads to the formation of vector hair changes from the second order to the first order when the gravitational coupling is large enough [35]. However this result was not observed in studying the backreaction of \((3+1)\)-dimensional holographic superconductor in Einstein Gauss-Bonnet gravity [36].

It would be of great interest to further explore the holographic superconductivity with backreactions. In this work we will consider the effect of the spacetime backreaction on the sufficiently general gravity dual describing a system of a U(1) gauge field and the scalar field coupled via a generalized Stückelberg Lagrangian. We will examine the effects of the backreaction on the condensation of the scalar hair and conductivity. Furthermore we will investigate the phase transition when taking the backreaction into account. We will also generalize the discussion to the Einstein-Gauss-Bonnet gravity by considering the combined effects of the generalized Stückelberg mechanism and the backreaction.

II. \((2 + 1)\)-DIMENSIONAL SUPERCONDUCTING MODELS WITH BACKREACTION

We study the formation of scalar hair on the background of AdS black hole in \((3+1)\)-dimensions. The generalized action containing a U(1) gauge field and the scalar field coupled via a generalized Stückelberg Lagrangian reads

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G_4} (R - 2\Lambda) + \mathcal{L}_{\text{matter}} \right],
\] (1)
where $G_4$ is the 4-dimensional Newton constant and $\Lambda = -3/L^2$ is the cosmological constant. $\mathcal{L}_{\text{matter}}$ is the generalized Stückelberg Lagrangian \cite{26}

$$\mathcal{L}_{\text{matter}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_{\mu}\psi\partial^\mu\psi - \frac{1}{2}m^2\psi^2 - \frac{1}{2}\mathfrak{F}(\psi)(\partial_{\mu}p - A_{\mu})(\partial^{\mu}p - A^\mu),$$

where $\mathfrak{F}(\psi)$ is a general function of $\psi$. Here we consider the simple form of $\mathfrak{F}(\psi)$

$$\mathfrak{F}(\psi) = \psi^2 + c_4\psi^4,$$

which has been discussed in \cite{41,42}. Our study can be easily extended to a more general form $\mathfrak{F}(\psi) = \psi^2 + c_\lambda\psi^\lambda + c_4\psi^4$ with the model parameters $c_\lambda$, $c_4$ and $\lambda \in [3, 4]$ just as discussed in \cite{27,30}, which will not alter the qualitative result. Considering the gauge symmetry

$$A_\mu \rightarrow A_\mu + \partial_\mu\bar{\Lambda}, \quad p \rightarrow p + \bar{\Lambda},$$

we can fix the gauge $p = 0$ by using the gauge freedom.

We are interested in including the backreaction so we use the ansatz of the geometry of the 4-dimensional AdS black hole with the form

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2),$$

whose Hawking temperature, which will be interpreted as the temperature of the CFT, reads

$$T_H = \frac{g'(r_+)}{4\pi}e^{-\chi(r_+)/2}.$$  

$r_+$ is the black hole horizon defined by $g(r_+) = 0$. Choosing the electromagnetic field and the scalar field as

$$A = \phi(r)dt, \quad \psi = \psi(r),$$

we can obtain the equations of motion

$$\chi' + \gamma \left[ \frac{r}{2}\psi'^2 + \frac{r}{2g^2}e^{\chi^2} \mathfrak{F}(\psi) \right] = 0,$$

$$g' - \left(\frac{3r}{L^2} - \frac{g}{r}\right) + \gamma r g \left[ \frac{1}{4}\psi'^2 + \frac{1}{4g^2}e^{\chi^2} + \frac{m^2}{4}\psi^2 + \frac{1}{4g^2}e^{\chi^2} \frac{\mathfrak{F}(\psi)}{g} \right] = 0,$$

$$\phi'' + \left(\frac{2}{r} + \frac{\chi'}{2}\right)\phi' - \frac{\mathfrak{F}(\psi)}{g} = 0,$$

$$\psi'' + \left(\frac{2}{r} - \frac{\chi'}{2} + \frac{g}{g}\right)\psi' - \frac{m^2}{g}\psi + \frac{1}{2g^2}e^{\chi^2} \frac{\mathfrak{F}(\psi)}{g} = 0,$$

where the parameter $\gamma = 16\pi G_4$. In the probe limit where $\gamma \rightarrow 0$, \cite{5} goes back to the (2+1)-dimensional holographic superconductor model studied in \cite{27}. For nonzero $\gamma$ we take the backreaction of the spacetime into account.
The analytic solutions to Eq. (5) for $\psi(r) = 0$ are the asymptotically AdS Reissner-Nordström black holes

$$g = \frac{r^2}{L^2} - \frac{2M}{r} + \frac{\gamma\rho^2}{4r^2}, \quad \phi = \rho \left( \frac{1}{r^+} - \frac{1}{r} \right), \quad \chi = 0,$$

where $M$ and $\rho$ are the integration constants that can be interpreted as the mass and the charge density of the solution, respectively. When $\gamma = 0$, the metric coefficient $g$ goes back to the Schwarzschild AdS black hole. In order to get the solutions with nonzero $\psi(r)$, we have to count on the numerical method which has been explained in detail in [28, 31, 32]. The equations of motion (8) can be solved numerically by doing integration from the horizon out to the infinity. At the asymptotic AdS boundary ($r \to \infty$), the scalar and Maxwell fields behave like

$$
\psi = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}, \quad \phi = \mu - \frac{\rho}{r},
$$

with

$$
\lambda_{\pm} = \frac{1}{2}(3 \pm \sqrt{9 + 4m^2}),
$$

where $\mu$ and $\rho$ are interpreted as the chemical potential and charge density in the dual field theory, respectively. Notice that both of the falloffs are normalizable for $\psi$, so one can impose boundary condition that either $\psi_+$ or $\psi_-$ vanishes [8, 31]. For simplicity, we will take $\psi_- = 0$ and the scalar condensate is now described by the operator $\langle \mathcal{O}_+ \rangle = \psi_+$. In this work we will discuss the condensate $\langle \mathcal{O}_+ \rangle$ for fixed charge density. Moreover, we will consider the values of $m^2$ which must satisfy the Breitenlohner-Freedman (BF) bound $m^2 \geq -(d-1)^2/4$ [43] for the dimensionality of the spacetime $d = 4$ in the following analysis.
A. The effects on the scalar condensation and phase transition

FIG. 1: (color online) The metric function $\chi(r)$, the matter functions $\psi(r)$ and $\phi(r)$, the effective mass $m^2_{\text{eff}}$ for different values of the backreacting parameter $\gamma$, i.e., $\gamma = 0$ (red), 0.1 (green), 0.3 (blue) and 0.5 (black) with the fixed value of $\psi_+ = 3.0$.

In our computation we fix $r_+ = 1$. In order to show the effect of the backreaction, we choose different values of $\gamma$ in presenting our numerical results. In Fig. 1 we give the typical solutions with different values of the backreaction $\gamma = 0$ (red), 0.1 (green), 0.3 (blue) and 0.5 (black) for the metric function $\chi(r)$, the scalar field $\psi(r)$, the Maxwell field $\phi(r)$ and the effective mass of the scalar field described by

$$m^2_{\text{eff}} = m^2 + g^{tt}A_t^2 = - \frac{2}{L^2} - \frac{\phi^2}{ge^{-\chi(r)}}.$$  \hspace{1cm} (12)

For clarity, we set $\psi_+ = 3.0$, $c_4 = 0$ and $m^2L^2 = -2$ in our calculation. The solutions for other values of $\psi_+$, $c_4$ and the scalar mass $m$ are qualitatively similar. Considering the backreaction, $\chi(r)$ in the metric functions becomes nonzero. With the increase of the backreaction, we see that $\chi$ deviates more from zero near the black hole horizon, this indicates that with stronger backreaction the black hole deviates more from the usual Schwarzschild AdS black hole as we observed in the probe limit. For the Maxwell field, $\phi(r) = 0$ at the horizon, and we observed that it increases slower near the horizon for the stronger backreaction. This
shows that stronger backreaction will hinder the growth of the Maxwell field near the horizon. Although the scalar field $\psi$ will increase with the backreaction near the horizon, the coupling between the Maxwell field and the scalar field will be reduced when the backreaction is enhanced, which leads the effective mass to develop more shallow and narrow wells out of the horizon. The negative effective mass is the crucial effect to cause the formation of the scalar hair and the more negative effective mass will make it easier for the scalar hair to form \([7]\). The dependence of the effective mass on the backreaction shows that with the stronger backreaction the scalar condensate will develop harder.

$$m^2/L^2 = \text{Eq} \pm \text{Min}$$

![Diagram](image.png)

**FIG. 2:** (Color online) The critical temperature $T_c$ as a function of the backreaction $\gamma$ for different scalar mass $m$. The three dashed lines from top to bottom correspond to increasing mass, i.e., $m^2L^2 = -2$ (black), $m^2L^2 = -1$ (blue) and $m^2L^2 = 0$ (red), respectively.

To see the effect of the backreaction on the scalar condensation more directly, we plot the behavior of the critical temperature $T_c$ with the change of the backreaction for different scalar mass $m$ in Fig. 2. It is clear that for the same scalar mass $m$, the critical temperature $T_c$ drops if $\gamma$ increases, which shows that the backreaction makes the scalar condensation harder. Fitting the numerical data, we have

$$T_c \approx 0.118 \cdot \exp(-1.21 \cdot \gamma) \sqrt{\rho}, \quad \text{for} \quad m^2L^2 = -2,$$

$$T_c \approx 0.0974 \cdot \exp(-3.33 \cdot \gamma) \sqrt{\rho}, \quad \text{for} \quad m^2L^2 = -1,$$

$$T_c \approx 0.0882 \cdot \exp(-5.31 \cdot \gamma) \sqrt{\rho}, \quad \text{for} \quad m^2L^2 = 0,$$

(13)

which shows the influence of the backreaction on the critical temperature $T_c$. The exponential dependence of $T_c$ on $\gamma$ is the same as that disclosed in the 3 + 1 dimensions \([36]\). For the fixed $\gamma$, $T_c$ decreases when $m^2$ becomes less negative, which shows that the larger mass of the scalar field can make the scalar hair harder to form \([14]\).
FIG. 3: (Color online) The condensate $\langle O_+ \rangle$ as a function of temperature with fixed values $c_4$ for different values of $\gamma$, which shows that a different value of $\gamma$ can separate the first- and second-order behavior. The five lines in each panel from right to left correspond to increasing $\gamma$, i.e., 0 (red), 0.1 (green), 0.2 (blue), 0.3 (purple) and 0.4 (black). For clarity the dashed line in these panels corresponds to the case of the critical value $\gamma_c$ which can separate the first- and second-order behavior for different $\mathcal{F}(\psi)$.

It is of great interest to see the influence of the backreaction on the phase transition. In the generalized St"uckelberg mechanism, broader descriptions of phase transitions were provided. Here we will discuss whether the backreaction can play the role in the description of the phase transition. In Fig. 3 we exhibit the condensate of $\langle O_+ \rangle$ for selected values of the parameter $c_4$ in $\mathcal{F}(\psi)$ and the backreaction $\gamma$. Keeping $c_4 = 0$, we find that the phase transition is always of the second order. The condensate approaches zero as $\langle O_+ \rangle \propto (T_c - T)^{1/2}$ no matter how big is the backreaction we consider. This shows that when we take $\mathcal{F}(\psi) = \psi^2$ the backreaction will not change the mean field result allowing the second order phase transition with the critical exponent $\beta = 1/2$ as predicted in the probe limit [8]. When the $\psi^4$ term appears in $\mathcal{F}(\psi)$ with the strength $c_4 \geq 1.0$, the condensate $\langle O_+ \rangle$ does not drop to zero continuously at the critical temperature and this behavior does not alter for choosing different values of $\gamma$. This phenomenon also appears when we consider the condensate $\langle O_- \rangle$ but with different $c_4$ range. In the probe limit, this phenomenon was attributed to the change of the phase transition from the second order to the first order [27]. Here we find that the backreaction cannot influence the result when the strength of the $c_4 \geq 1$. When $0 < c_4 < 1$, we see
that the phenomenon appearing in the condensate to exhibit the change of the second order phase transition to the first order emerges when the backreaction is strong enough. This observation supports the argument in the study of the holographic p-wave superfluids that the backreaction plays the role in the phase transition \[35\]. For selected values of $c_4$ within the range of $0 < c_4 < 1$, we get the critical value of $\gamma$ to allow the change of the order of the phase transition, i.e., $\gamma_c = 0.2$ for $c_4 = 0.5$, $\gamma_c = 0.1$ for $c_4 = 0.7$ and $\gamma_c = 0$ for $c_4 = 1$. It shows that $\gamma_c$ becomes smaller when $c_4$ is bigger. Thus, when the strength of the $\psi^4$ term is not strong enough in the $\mathcal{F}(\psi)$, the backreaction will combine with the strength $c_4$ to tune the order of the phase transition. With the backreaction of the spacetime, we see richer physics in the phase transition.

B. The effects on the conductivity

In order to investigate the influence of the backreaction on the conductivity, we consider the time-dependent perturbation with zero momentum $A_x = a_x(r)e^{-i\omega t}$ and $g_{tx} = f(r)e^{-i\omega t}$ which can get the equations of motion decoupled from other perturbations \[28, 31, 32\]

$$a_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2}\right)a_x' + \left[\frac{\omega^2}{g^2}e^\chi - \frac{\mathcal{F}(\psi)}{g}\right]a_x + \frac{\phi'}{g}e^\chi \left(f' - \frac{2f}{r}\right) = 0,$$  \hspace{1cm} (14)

$$f' - \frac{2f}{r} + \gamma \phi'a_x = 0.$$  \hspace{1cm} (15)

Substituting Eq. (15) into Eq. (14), we have the equation of motion for the perturbed Maxwell field

$$a_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2}\right)a_x' + \left[\left(\frac{\omega^2}{g^2} - \frac{\gamma \phi'^2}{g}\right)e^\chi - \frac{\mathcal{F}(\psi)}{g}\right]a_x = 0.$$  \hspace{1cm} (16)

Near the horizon, we solve the above equation by imposing the ingoing boundary condition

$$a_x(r) \propto g(r)^{-i\omega/(4\pi T_H)}.$$  \hspace{1cm} (17)

In the asymptotic AdS region, the asymptotic behavior of the perturbations can be expressed as

$$a_x = a_x^{(0)} + \frac{a_x^{(1)}}{r}, \quad f = r^2 f^{(0)} + \frac{f^{(1)}}{r}.$$  \hspace{1cm} (18)

Thus, we have the conductivity of the dual superconductor by using the AdS/CFT dictionary \[31\]

$$\sigma(\omega) = -\frac{\omega a_x^{(1)}}{\omega a_x^{(0)}}.$$  \hspace{1cm} (19)

For the general forms of the function $\mathcal{F}(\psi) = \psi^2 + c_4 \psi^4$, we can obtain the conductivity by solving the Maxwell equation numerically. In our computation we fix $m^2 L^2 = -2$. Other values of the scalar mass present the same qualitative results \[10, 14\].
In the probe limit, it was argued in [14] that there is a universal relation between the gap $\omega_g$ in the frequency dependent conductivity and the critical temperature $T_c$, $\omega_g/T_c \approx 8$, which is roughly two times bigger than the BCS value 3.5 indicating that the holographic superconductors are strongly coupled. However this so-called universal relation was challenged when the higher curvature corrections are taken into account [9, 10]. When taking the backreaction of the spacetime into account, we find that the claimed universal relation can be modified even without the high curvature correction. In Fig. 4 we plot the frequency dependent conductivity obtained by solving the Maxwell equation numerically for $c_4 = 0$ and 0.1 with different strength of the backreaction, i.e., $\gamma = 0$, 0.1 and 0.2 respectively at temperature $T/T_c \simeq 0.2$. It clearly shows that for fixed $c_4$, the gap frequency $\omega_g$ becomes larger when the backreaction is stronger. The deviation from $\omega_g/T_c \approx 8$ becomes bigger with the increase of $\gamma$. This behavior is consistent with the observation in five dimensional Einstein-Gauss-Bonnet gravity [37]. Here we show that the backreaction also changes the ratio $\omega_g/T_c$ in lower dimensional backgrounds without higher curvature corrections. For the selected $\gamma$, the ratio $\omega_g/T_c$ also depends on the model parameter $c_4$, which agrees with the result obtained for general holographic superconductor models with Gauss-Bonnet corrections in the probe limit [30].

So far, we conclude that the gap ratio $\omega_g/T_c$ does not only change in the AdS spacetimes with higher curvature correction, it also alters in the presence of the backreaction and the $\psi^4$ term in the $\mathcal{F}(\psi)$. Thus there is no universal relation $\omega_g/T_c \approx 8$ for general holographic superconductors.
III. GAUSS-BONNET SUPERCONDUCTING MODELS WITH BACKREACTION

In the following we generalize the above discussion to the Gauss-Bonnet superconducting models. We extend the Lagrangian in Eq. (1) to (4 + 1)-dimensional Einstein-Gauss-Bonnet gravity

\[
S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R + \frac{12}{L^2} + \frac{\alpha}{2} \left( R_{\mu\nu\lambda\delta} R^{\mu\nu\lambda\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \right) + 16\pi G_5 L_{\text{matter}} \right],
\]

(20)

where \( G_5 \) is the 5-dimensional Newton constant, \( \alpha \) is the Gauss-Bonnet coupling constant and \( L_{\text{matter}} \) is the same generalized Stückelberg Lagrangian given in Eq. (2). The Gauss-Bonnet parameter \( \alpha \) has an upper bound called the Chern-Simons limit \( \alpha = L^2/4 \) which can guarantee a well-defined vacuum for the gravity theory \[44\], and a lower bound \( \alpha = -7L^2/36 \) determined by the causality \[45–50\].

Taking the Ansatz for the metric in the five-dimensional spacetime

\[
ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2 + dz^2),
\]

(21)

we can have the Hawking temperature as expressed in Eq. (6) and get the equations of motion

\[
\chi' + \frac{2}{3}\gamma \frac{r^2}{r^2 - 2\alpha g} \left( \frac{r}{2} \psi'^2 + \frac{r}{2g} e^\chi \phi^2 \tilde{g}(\psi) \right) = 0,
\]

\[
g' - \frac{r^2}{r^2 - 2\alpha g} \left( \frac{4\sqrt{r}}{L^2} - \frac{2g}{r} \right) - \frac{2}{3} \gamma g \left[ \frac{1}{4} \psi'^2 + \frac{1}{4g} e^\chi \phi'^2 + \frac{m^2}{4g^2} \psi'^2 + \frac{1}{4g^2} e^\chi \phi'^2 \tilde{g}(\psi) \right] = 0,
\]

\[
\phi'' + \left( \frac{3}{r} + \frac{\chi'}{2} \right) \phi' - \frac{\tilde{g}(\psi)}{g} \phi = 0,
\]

\[
\psi'' + \left( \frac{3}{r} - \frac{\chi'}{2} + \frac{g'}{g} \right) \psi' - \frac{m^2}{g^2} \psi + \frac{1}{2g^2} e^\chi \phi^2 \tilde{g}(\psi) = 0,
\]

(22)

where we have set the backreaction \( \gamma = 16\pi G_5 \). In the limit \( \gamma \to 0 \), (22) reduce to describe the general (3 + 1)-dimensional holographic superconductor model with Gauss-Bonnet corrections in the probe limit \[30\].

An exact solution to Eq. (22) is the charged Gauss-Bonnet black hole described by \[36\]

\[
g = \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 - \frac{4\alpha}{L^2} + \frac{4\alpha M}{r^3} - \frac{4\gamma \rho^2}{3r^6}} \right), \quad \phi = \rho \left( \frac{1}{r^2} - \frac{1}{r^2} \right), \quad \chi = \psi = 0.
\]

(23)

Obviously, it is not easy to find other analytic solutions to these nonlinear equations. So we have to count on the numerical computation. The boundary conditions at the asymptotic AdS boundary \( r \to \infty \) are

\[
\psi = \frac{\psi_-}{r^\lambda_-} + \frac{\psi_+}{r^\lambda_+}, \quad \phi = \mu - \frac{\rho}{r^2},
\]

(24)

with

\[
\lambda_{\pm} = 2 \pm \sqrt{4 + m^2 L_{\text{eff}}^2},
\]

(25)
where \( L_{\text{eff}}^2 = 2\alpha/(1 - \sqrt{1 - 4\alpha/L^2}) \) is the effective asymptotic AdS scale [44]. We will concentrate on the scalar condensate \( \langle O_+ \rangle = \psi_+ \) and set \( \psi_- = 0 \). For concreteness, we will take the fixed mass of the scalar field \( m^2L^2 = -3 \). The alternative choice of the mass \( m^2L_{\text{eff}}^2 = -3 \) will not qualitatively change our results [9, 10].

![Image](image_url)

**FIG. 5:** (Color online) The condensate \( \langle O_+ \rangle \) as a function of temperature with fixed Gauss-Bonnet correction term \( \alpha \) and model parameter \( c_4 \) for different values of \( \gamma \), which shows that the critical value \( \gamma_c \) (the blue and dashed line in each panel) decreases as \( \alpha \) or \( c_4 \) increases. The five lines in each panel from right to left correspond to increasing \( \gamma \) for fixed \( \alpha \) and \( c_4 \).

**TABLE I:** The critical value \( \gamma_c \) which can separate the first- and second-order behavior for different Gauss-Bonnet correction term \( \alpha \) with fixed \( \bar{\delta}(\psi) = \psi^2 + c_4 \psi^4 \).

| \( \alpha \) | -0.1 | 0   | 0.1 | 0.2 | 0.25 |
|-------------|------|-----|-----|-----|------|
| \( c_4 = 0.1 \) | 0.35 | 0.24 | 0.16 | 0.09 | 0.05 |
| \( c_4 = 0.2 \) | 0.18 | 0.12 | 0.07 | 0.03 | 0.01 |

In the probe limit we have already observed that different values of Gauss-Bonnet correction term and model parameters can determine the order of phase transition [30]. Considering the backreaction, we obtain richer descriptions in the phase transition. When \( c_4 = 0 \), no matter what values of the Gauss-Bonnet factor and the backreaction we choose, \( \langle O_+ \rangle \) always drops to zero continuously at the critical temperature, which indicates that the phase transition is always of the second order. When \( c_4 \) deviates from zero, for chosen \( \alpha \), \( \langle O_+ \rangle \) can become multivalued near the critical temperature when the backreaction is strong enough, which indicates that the first order of the phase transition can happen. In Fig. 5, we exhibit the results of the condensate \( \langle O_+ \rangle \) for chosen values of \( c_4 \) and \( \alpha \) but variable strength of the backreaction \( \gamma \). The critical values of the backreaction to allow the condensate not dropping to zero continuously at the critical temperature are listed in Table I for different values of Gauss-Bonnet correction term \( \alpha \) with the selected...
$c_4 = 0.1$ and $0.2$. For bigger values of $\alpha$ or $c_4$, we observe that the critical $\gamma$ is smaller to accommodate the first order phase transition.

In the Gauss-Bonnet gravity, we conclude that the backreaction $\gamma$ together with the Gauss-Bonnet factor $\alpha$ and the model parameter $c_4$ plays the role in determining the order of the phase transition.

IV. CONCLUSIONS

In this work we have studied the general holographic superconductors away from the probe limit. We have considered the four-dimensional and five-dimensional Einstein and Einstein-Gauss-Bonnet gravity backgrounds. We observed that the backreaction can make the condensation harder to be formed. In addition to the model parameters in $\mathcal{F}$ and the Gauss-Bonnet factor, we found that the spacetime backreaction can also bring richer descriptions in the phase transition. When the curvature correction term or model parameter is larger, smaller backreaction can trigger the first order phase transition. This observation supports the finding in the holographic p-wave superfluids with backreactions in $3 + 1$ dimensions \[35\]. Extending the analysis to the conductivity, we further disclosed the fact that there is no universal relation for $\omega_g/T_c$ in the holographic superconductor when the backreaction is taken into account.

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