A Heating Mechanism via Magnetic Pumping in the Intracluster Medium

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Abstract

Turbulence driven by active galactic nuclei activity, cluster mergers, and galaxy motion constitutes an attractive energy source for heating the intracluster medium (ICM). How this energy dissipates into the ICM plasma remains unclear, given its low collisionality and high magnetization (precluding viscous heating by Coulomb processes). Kunz et al. proposed a viable heating mechanism based on the anisotropy of the plasma pressure under ICM conditions. The present paper builds upon that work and shows that particles can be heated by large-scale turbulent fluctuations via magnetic pumping. We study how the anisotropy evolves under a range of forcing frequencies, what waves and instabilities are generated, and demonstrate that the particle distribution function acquires a high-energy tail. For this, we perform particle-in-cell simulations where we periodically vary the mean magnetic field \( B(t) \). When \( B(t) \) grows (dwindles), a pressure anisotropy \( P_{\perp} > P_{\parallel} (P_{\perp} < P_{\parallel}) \) builds up \( (P_{\perp} \text{ and } P_{\parallel} \text{ are, respectively, the pressures perpendicular and parallel to } B(t)) \). These pressure anisotropies excite mirror \( (P_{\perp} > P_{\parallel}) \) and oblique firehose \( (P_{\perp} > P_{\parallel}) \) instabilities, which trap and scatter the particles, limiting the anisotropy, and providing a channel to heat the plasma. The efficiency of this mechanism depends on the frequency of the large-scale turbulent fluctuations and the efficiency of the scattering the instabilities provide in their nonlinear stage. We provide a simplified analytical heating model that captures the phenomenology involved. Our results show that this process can be relevant in dissipating and distributing turbulent energy at kinetic scales in the ICM.

Unified Astronomy Thesaurus concepts: Intracluster medium (858); Plasma astrophysics (1261); Galaxy clusters (584); Galaxies (573); Magnetic fields (994); High energy astrophysics (739)

Supporting material: animation

1. Introduction

The intracluster medium (ICM) of galaxy clusters strongly affects the structure and evolution of the embedded galaxies and is of cosmological significance. It is therefore critical to understand the dynamics and energy balance of the ICM.

The observed X-ray emission from the ICM implies radiative cooling timescales shorter than their typical ages (e.g., Voigt & Fabian 2004). In the absence of a heat source that counteracts this cooling, significant mass inflow takes place (Fabian & Nulsen 1977), providing a continuous supply of cool gas that could fuel star formation. Further X-ray observational studies have inferred much lower inflow mass rates (Peterson et al. 2003) and star formation rates (Donahue et al. 2015) than predicted. This conundrum is well known as the cooling flow problem.

Among the various heating mechanisms that have been proposed to counteract the radiative cooling of the ICM (see Zweibel et al. 2018 for a review), active galactic nuclei (AGNs) feedback has been considered promising in terms of, for example, the amount of energy injected into the ICM and its self-regulating nature (Churazov et al. 2001; Reynolds et al. 2002; Omma et al. 2004; Karen Yang & Reynolds 2016). However, it is not well understood how the energy in AGN outflows is ultimately deposited and thermalized in the ICM.

Turbulent heating has been suggested as a channel of energy dissipation able to offset radiative cooling in the ICM (Zhuravleva et al. 2014), but the study of turbulent dissipation in the ICM presents several challenges both theoretically and observationally. The well-known presence of microgauss magnetic fields (Bonafede et al. 2010), its weakly collisional nature (i.e., the typical Coulomb collision time and mean free path are much longer than the ion Larmor period and Larmor radius, and in fact approach dynamical time and length scales), make the transport properties of the plasma, such as its thermal conduction and viscosity, anisotropic with respect to the direction of the local magnetic field and strongly dependent on the microphysics happening at kinetic scales. Additionally, the large ratio of thermal-to-magnetic pressure \( (\beta \equiv 8\pi P/\mathbf{B}^2 \sim 10–100, \text{ where } P \text{ is the isotropic thermal pressure and } \mathbf{B} \text{ the magnetic field strength}) \) put ICM plasmas in a regime that is not well studied experimentally or observed in space plasmas.

Microphysical processes driven by plasma pressure anisotropy appear to be particularly important. The compression, rarefaction, and shearing of collisionless, magnetized plasma generally produces pressure anisotropy with respect to the local magnetic field \( (P_{\perp,j} \neq P_{\parallel,j}) \), where \( P_{\perp,j} \text{ and } P_{\parallel,j} \text{ are, respectively, the pressures of the species } j \text{ perpendicular and parallel to the magnetic field}) \), as was first pointed out in the context of galaxy clusters by Schekochihin et al. (2005) and Schekochihin & Cowley (2006). These authors showed that even a small anisotropy can make the plasma easily unstable to microinstabilities such as mirror \( (P_{\perp,j} > P_{\parallel,j}) \) and oblique firehose \( (P_{\parallel,j} > P_{\perp,j}) \), the thresholds for which scale as \( \sim \beta^{-1} \).
A turbulent heating mechanism based on the anisotropy of the ICM plasma was proposed by Kunz et al. (2011). According to this mechanism, termed parallel viscous heating, the heating rate is determined by the level of pressure anisotropy, ΔP, in the system. These authors estimated ΔP by assuming it is pinned to marginal stability with respect to the mirror and firehose microinstabilities. In a similar approach, and with application to the solar wind, Lichko et al. (2017) considered heating and nonthermal particle energization under the assumption of a periodically compressing flux tube with a prescribed scattering frequency, in a process termed magnetic pumping.

The essence of this mechanism has also been described in previous works using different nomenclature. Kulsrud (1983) refers to it as “frictional heating by nonuniform velocities,” and Hollweg (1985) as “volumetric viscous heating rate.” We will follow Lichko et al. (2017) and refer to this mechanism as heating by magnetic pumping from now on. Magnetic pumping has the important property that it directly taps the energy of large-scale turbulence in weakly collisional plasmas. This is in notable contrast to the usual assumption made, for example, in hydrodynamic turbulence, that energy cascades from large scales to small in a loss-free manner, with dissipation becoming significant only at small scales.

In this work, we combine the approaches of Kunz et al. (2011) and Lichko et al. (2017) and present a heating mechanism acting via magnetic pumping in a periodically shearing, high-β plasma suitable for studying ICM conditions. The isotropizing effect necessary for the operation of magnetic pumping will be given by the scattering that mirror and firehose instabilities provide as they interact with the particles, and this will be self-consistently captured using 2.5 dimensional (i.e., electromagnetic fields and particles’ momenta have three components but only vary in two spatial dimensions), fully kinetic particle-in-cell (PIC) simulations. This mechanism allows the plasma to effectively retain energy coming from large-scale turbulent fluctuations after a complete pump cycle. One of the outcomes of our work is that, as predicted by Lichko et al. (2017), shorter cycles lead to greater energy retention, even when the scattering process is modeled self-consistently.

This paper is organized as follows. In Section 2 we present the analytical basis for plasma heating by magnetic pumping. In Section 3 we present our simulation method and setup. In Section 4 we quantify the amount of heating in our magnetic pumping configuration, show that our results are fairly independent of the numerical ion-to-electron mass ratio, and discuss their dependence on the rate at which B is driven. In Section 5 we describe how magnetic pumping operates in terms of the evolution of the pressure anisotropy along with the alternating excitation of mirror and firehose instabilities and discuss their dependence on different physical parameters. In Section 6 we provide a simplified heating model able to capture the essence of the magnetic pumping mechanism in presence of scattering by self-consistently generated kinetic instabilities. In Section 7 we summarize our results and present our conclusions.

2. Theoretical Basis

In this section, we develop the analytical basis for capturing the main features of the magnetic pumping mechanism, particularly how the presence of scattering becomes necessary to effectively heat the plasma.

Consider a uniform plasma subject to an external, incompressible forcing such as the shear velocity field considered in our numerical simulations (see Section 3). Consider first the case where there is no scattering mechanism. We can describe the evolution of the particle distribution function \( f = f(p_\perp, p_\parallel, t) \) by the drift kinetic equation (e.g., Kulsrud 1983, 2005)

\[
\frac{\partial f}{\partial t} + \frac{\partial p_\perp}{\partial p_\perp} \frac{\partial f}{\partial p_\perp} + \frac{\partial p_\parallel}{\partial t} \frac{\partial f}{\partial p_\parallel} = 0,
\]

where \( p_\perp \) and \( p_\parallel \) are the momenta perpendicular and parallel to the mean magnetic field, respectively. We will assume that the external motion drives the background sufficiently slowly that the particle magnetic moment \( \mathcal{M} \equiv p_\perp^2/B \) and longitudinal action \( \mathcal{J} \equiv \int \mathcal{F} \cdot dt \propto p_\parallel L \) are adiabatic invariants. For a pure shear motion, assuming conservation of particle number \( N = L\Delta n \) and magnetic flux \( \Phi = B \cdot dS \propto BA \), we can write

\[
\frac{dp_\perp}{dt} = \frac{p_\perp B}{2B}, \quad \frac{dp_\parallel}{dt} = -p_\parallel \frac{B}{B},
\]

which are valid for both nonrelativistic and relativistic particles. Substituting Equation (2) into Equation (1) gives

\[
\frac{\partial f}{\partial t} + \frac{B}{B} \left( p_\perp \frac{\partial f}{\partial p_\perp} - p_\parallel \frac{\partial f}{\partial p_\parallel} \right) = 0.
\]

In order to illustrate the presence of the heating rate by magnetic pumping and to obtain the evolution of the pressure anisotropy in the most transparent way, it is convenient to do a coordinate transformation from \( (p_\perp, p_\parallel) \) to \( (p, \mu) \), where \( p \equiv (p_\perp^2 + p_\parallel^2)^{1/2} \) and \( \mu \equiv p_\parallel/p \). The drift kinetic equation in the new coordinate system reads

\[
\frac{\partial f}{\partial t} - \frac{\dot{B}}{B} \left( P_\mu (\mu) \frac{\partial f}{\partial \mu} + 2(1 - \mu^2) \frac{\partial f}{\partial \mu} \right) = 0,
\]

where \( P_\mu (\mu) \equiv (3\mu^2 - 1)/2 \) is the Legendre polynomial of order 2. By multiplying Equation (4) by the particle energy, \( e \), and integrating over momentum space, we obtain

\[
\frac{dU}{dt} = -\frac{\dot{B}}{B} \Delta P = 0,
\]

where \( \Delta P \equiv P_\perp - P_\parallel \) and

\[
P_\perp = \int \frac{P_{\perp} \psi}{2} f d^3p, \quad P_\parallel = \int \frac{P_{\parallel} \psi}{2} f d^3p,
\]

which correspond to the pressures perpendicular and parallel to the magnetic field, respectively. The second term of the left-hand side in Equation (5) corresponds to the heating rate by magnetic pumping for a pure shear driving motion, and it is valid for both nonrelativistic and relativistic particles (Kulsrud 1983; Hollweg 1985). Note that magnetic pumping can either heat or cool the plasma, depending on the relative signs of \( \dot{B}/B \) and \( \Delta P \); heating will proceed whenever they have the same sign and cooling whenever they have opposite signs. We will make use of this feature in Section 4.
We can also obtain an evolution equation for $\Delta P$ by noting that
\begin{equation}
\Delta P = -\int p v p_z(\mu) f p^2 d\mu d\lambda.
\end{equation}

Multiplying Equation (4) by $-p v p_z(\mu)$, integrating over momentum space and assuming that the anisotropy remains small so the term $\partial f / \partial \mu$ can be dropped, we get
\begin{equation}
\frac{d\Delta P}{dt} = -\frac{2}{5} \frac{\dot{B}}{B} \int \left(4 p v + p^2 \frac{d\nu}{dp}\right) f p^2 d\mu = 0.
\end{equation}

Using $\nu / dp = 1 / m \gamma^2$, the nonrelativistic and ultrarelativistic limits of Equation (8) are, respectively,
\begin{equation}
\frac{d\Delta P_{\text{NR}}}{dt} = -\frac{2}{5} \frac{\dot{B}}{B} U = 0,
\end{equation}
\begin{equation}
\frac{d\Delta P_{\text{R}}}{dt} = -\frac{4}{5} \frac{\dot{B}}{B} U = 0.
\end{equation}

We can then write, in general,
\begin{equation}
\frac{d\Delta P}{dt} = -\frac{\dot{B}}{B} U = 0,
\end{equation}
where $4/5 < \alpha < 2$. Note that while Equation (11) rests on the assumption of small anisotropy, Equation (5) is exact.

Using Equations (5) and (11), and changing the independent variable from $t$ to $\ln B$, we can see that there is a conserved quantity throughout the evolution:
\begin{equation}
\nu = \left(\frac{dU}{d\ln B}\right)^2 - \alpha U^2.
\end{equation}

Equation (12) implies that, no matter how intricate its time evolution, whenever $B$ returns to its initial value, so does $U$. This way, we have shown that, in absence of a scattering mechanism, the evolution of $U$ is completely adiabatic irrespective of the evolution of $B$.

The situation changes if we now allow the presence of scattering. To illustrate this, consider the addition of pitch-angle scattering, represented by a Lorentz operator so that Equation (4) takes the form
\begin{equation}
\frac{\partial f}{\partial t} = -\frac{\dot{B}}{B} \left(p v p_z(\mu) \frac{\partial f}{\partial p} + \frac{3 \nu}{2} (1 - \mu^2) \frac{\partial f}{\partial \mu}\right) = \frac{\partial}{\partial \mu} \left(\nu (1 - \mu^2) \frac{\partial f}{\partial \mu}\right).
\end{equation}

where $\nu$ is the scattering rate which, for simplicity, we assume is independent of $\mu$. The source of scattering can be either Coulomb collisions, interactions with a spectrum of electromagnetic fluctuations originating from self-generated microinstabilities, or just the high-frequency component of some preimposed turbulence.

While pitch-angle scattering does not heat the plasma in and of itself, it does have an indirect effect through the evolution of the pressure anisotropy. Assuming small anisotropy as before, Equation (11) is modified to
\begin{equation}
\frac{d\Delta P}{dt} = \alpha \frac{\dot{B}}{B} U - 3 \nu \Delta P,
\end{equation}
with the $\nu$ term resulting from breaking the adiabaticity of the evolution. The effect of this is most easily seen if we consider small-amplitude, periodic changes in $B$, e.g., $\dot{B}/B = \epsilon \sin \omega t$, where $\epsilon \ll 1$, and solve Equation (14) to first order in $\epsilon$. Whereas for $\nu = 0$, $\Delta P$ and $\dot{B}/B$ are out of phase by $\pi/2$ and yield no net heating when integrated over one magnetic cycle period, collisions introduce a phase shift in $\Delta P$ which is maximized for $\omega = \nu/3$, irreversibly heating the plasma (see Lichko et al. 2017, where $\nu$ is treated as a free parameter). A more general model of this effect is presented in Section 6.

In what follows, we will study how magnetic pumping can effectively heat the plasma when the magnetic field varies periodically in time and there is a scattering source provided self-consistently by kinetic microinstabilities such as mirror and firehose, in conditions similar to what can be encountered in the ICM (Borovsky 1986; Schekochihin et al. 2005; Lyutikov 2007).

3. Simulation Setup

We use the fully kinetic, relativistic particle-in-cell (PIC) code TRISTAN-MP (Bruneman 1993; Spitkovsky 2005) to simulate a shearing plasma made of singly charged ions and electrons (Riquelme et al. 2012). The plasma has a homogeneous magnetic field that initially points along the $x$-axis, $B=B_0 \hat{x}$. A periodic shear velocity field is imposed in our two-dimensional (2D) domain (via shearing box coordinates; see Riquelme et al. 2012), such that initially $v=-s \hat{x}$ (red arrows in Figure 1), where $x$ is the distance along the $x$-axis and $s$ is the shear rate. The velocity field then periodically changes its sign after a period of time $sB_0$, causing periodic reversals in the shear motion, as shown in Figure 1. This configuration allows us to capture the action of large-scale turbulence locally, and it was first presented by Melville et al. (2016). By magnetic flux conservation, the background magnetic field will vary accordingly, in both direction and magnitude. Initially, the shear amplifies the magnetic field $B$ in one direction during one shear time, $\tau_s$, such that its $y$-component evolves as $d B_y / dt = -s B_0$, while $dB_z / dt = dB_x / dt = 0$ (see Figure 1(a) and (b)). After one shear time $\tau_s$, the shear velocity changes its sign and causes the magnetic field to decrease during another shear time $\tau_s$, such that now $d B_y / dt = s B_0$, while $dB_z / dt = dB_x / dt = 0$, until it reaches its initial magnitude and direction (see Figures 1(c), (d), and (e)). This way, in an interval equal to $2\tau_s$, a full cycle is completed, with $B$ having a maximum increase of $\sqrt{2}$. Subsequently, a similar, “mirroring” cycle occurs, but in this case $dB_y / dt = s B_0$ initially, and then switches back to $dB_y / dt = -s B_0$. This way, in an interval equal to $4\tau_s$, two cycles involving opposite shear motions in the $y$-direction are completed. The simulations cover several cycles, which is essential to assess the efficiency of the magnetic pumping mechanism in the secular heating of the plasma.
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Figure 1. The simulation domain in our PIC simulations at different times. At $t=0$ (panel (a)), the shear motion is applied (red arrows) and the domain follows the shearing flow of the plasma. Magnetic flux conservation changes the magnitude and orientation of the background magnetic field $B$, being amplified between $0 < t - s < 1$ (panel (b)). At $t = s = 1$ (panel (c)), the shear is reversed and the domain follows the shear back to the initial configuration. By magnetic flux conservation, the strength of the background magnetic field decreases in this phase until it reaches its initial value (panel (d)). At $t = s = 2$ the domain is straight again and the shear motion continues and produces, again, an amplification of the magnetic field strength. The motion between $0 < t - s < 2$ constitutes one pump cycle, and it repeats until the end of the simulation.

In this configuration and in the absence of scattering, the particle magnetic moment $\mu_j = P_{ij}^2/(2m_jB)$ and longitudinal action $J_j = \int p_{ij}^2 dl$ are adiabatic invariants where $s \ll \omega_{cj}$, where $\omega_{cj} = eB_i m_c$ is the cyclotron frequency of particles of species $j$ and $e$ is the magnitude of the charge of the electron. This adiabatic invariance drives different pressure anisotropies ($P_{ji} > P_{ij}$ and $P_{ji} < P_{ij}$) depending on the variation of the magnetic field strength during the cycles, allowing—once the anisotropy exceeds a marginal stability threshold—the rapid excitation of kinetic instabilities that limit the anisotropy growth.

Given the parameter space here explored, when $P_{ji} > P_{ij}$, the mirror instability is excited (Chandrasekhar et al. 1958; Rudakov & Sagdeev 1961; Hasegawa 1969; Southwood & Kivelson 1993; Pokhotelov et al. 2002, 2004). Mirror modes are nonpropagating, with wavevectors highly oblique with respect to the mean magnetic field $B$ and maximum growth rate at $k_i R_{Li,i} \sim 1$, where $R_{Li,i}$ is the Larmor radius of the ions. It also presents Landau resonances with ions with very low parallel velocities with respect to $B$.

On the other hand, when $P_{ij} < P_{ji}$, the oblique firehose instability is excited (Yoon et al. 1993; Hellinger & Matsumoto 2000; Hellinger & Trávníček 2008). These modes are also nonpropagating, with wavevectors oblique with respect to $B$ with values of $kR_{Li,i}$ slightly smaller than $\sim 1$. This instability is also resonant with ions through cyclotron resonances. In our simulations we self-consistently excite both mirror and oblique firehose instabilities throughout the entire evolution.

The physical parameters of the plasma are the initial temperature of the ions and electrons ($T_i$ and $T_e$), the initial ratio between the ion pressure $P_{i,i}^{init}$ and the magnetic pressure $\omega_{ci,i}^{init} = 8\pi P_{i,i}^{init}/B_0^2$, the mass ratio between ions and electrons $m_i/m_e$, and the ion “magnetization,” defined as the ratio between the initial ion-cyclotron frequency, $\omega_{ci,i}^{init}$, and the shear rate, $s$, where $\omega_{ci,i}^{init} = eB_0/m_c$, with $B_0$ the initial magnetic field strength.

All the simulations reported here start with $\beta_i^{init} = 20$ and $k_BT_i/m_c e^2 = 0.1$. Ions and electrons are initialized with Maxwell–Boltzmann distributions (the relativistic generalization of Maxwell–Boltzmann) with $T_i = T_e$. A range of values for the mass ratio $m_i/m_e$ is used in our simulations and, given the current computational capabilities, using the realistic mass ratio $m_i/m_e \approx 1836$ becomes prohibitively expensive. The consequences of this constraint will be considered carefully, and our findings indicate that the mass ratio does not play a significant role in the main results here reported. A range of values for the magnetization parameter $\omega_{ci,i}^{init}/s$ is also used and the dependency of the results on this parameter will be discussed in the context of the turbulence expected in astrophysical environments such as the ICM. We note that the initial temperature of the electrons in our simulations make them more relativistic than ions, and this limits the applicability of our results in the context of the ICM.

The numerical parameters in our runs are the number of macroparticles per cell ($N_{ppc}$), the electron skin depth in terms of grid point spacing ($c/\sqrt{\omega_{pe}^2 + \omega_{pi}^2}/\Delta x$, where $\omega_{pe}^2 = 4\pi n_e e^2/m_e$ is the square of the electron plasma frequency and $n_e$ is the electron number density), and the box size in terms of the initial ion Larmor radius ($L/R_{Li,i}^{init}$, where $R_{Li,i}^{init} = v_{th,i}/\omega_{ci,i}^{init}$ and $v_{th,i} = k_BT_i/m_i$). Table 1 lists the names and parameters of our main simulations. We use $c/\sqrt{2}\omega_{pe}^2 + \omega_{pi}^2/\Delta x = 3.5$ in all simulations listed in Table 1. A series of simulations were performed to ensure that numerical convergence was reached in every case in terms of $N_{ppc}$, $c/\omega_{pe}/\Delta x$, and $L/R_{Li,i}^{init}$. Those simulations are not shown in Table 1.

In our simulations, both ions and electrons exhibit an excess energy gain due to numerical heating related to the inherent presence of electric field fluctuations generated by the discreteness in the number of particles in PIC simulations. We studied how this numerical heating behaves in the

| Runs | $\beta_i^{init}$ | $m_i/m_e$ | $\omega_{ci,i}^{init}/s$ | $N_{ppc}$ | $L/R_{Li,i}^{init}$ |
|------|----------------|---------|----------------|---------|----------------|
| Zb20m2w200 | 20 | 2 | 200 | 800 | 63 |
| Zb20m2w800 | 20 | 2 | 800 | 800 | 75 |
| Zb20m2w1600 | 20 | 2 | 1600 | 520 | 75 |
| Zb20m8w200 | 20 | 8 | 200 | 40 | 55 |
| Zb20m8w800 | 20 | 8 | 800 | 40 | 55 |
| Zb20m8w1600 | 20 | 8 | 1600 | 40 | 55 |
| Zb20m32w800 | 20 | 32 | 800 | 40 | 51 |
| Zb20m8w800 | 2 | 8 | 800 | 400 | 68 |
| Ekh20m2w200 | 20 | 2 | 200 | 1600 | 63 |
| Ekh20m2w800 | 20 | 2 | 800 | 800 | 63 |
| Ekh20m8w800 | 20 | 8 | 800 | 40 | 55 |

Notes. $\beta_i^{init} = 8\pi P_{i,i}^{init}/B_0^2$ is the initial ion plasma $\beta$, $m_i/m_e$ is the ion-to-electron mass ratio, $\omega_{ci,i}^{init}/s$ is the magnetization, where $\omega_{ci,i}^{init}$ is the initial ion gyrofrequency and $s$ is the shear frequency. $N_{ppc}$ is the number of particles per cell, and $L/R_{Li,i}^{init}$ is the size of the box in units of the initial ion Larmor radius. The reference simulation Zb20m2w800 is highlighted in bold and our characterization of numerical heating is based on the E simulations (last three rows).
Appendix by performing additional simulations designed to isolate the effect of numerical heating. We then subtracted it from the ion and electron internal energy, \( U_i(t) \) and \( U_e(t) \), here presented. This allowed us to reduce the discrepancy in energy gain to \( \sim 0.1\% \) for ions and \( \sim 12\% \) for electrons (see Figure 2). While subtracting the numerical heating rate does not completely nullify the effect of numerical heating, because the heating rate by magnetic pumping is probably not exactly the same as if numerical heating were not present, we hope it does indicate the size of this numerical effect.

4. Results

In this section we present our results of the periodic shear simulations described in Section 3. We show that heating by magnetic pumping (see Equation (5)) can effectively be retained after one pump cycle, with an efficiency that depends on the evolution of the mirror and firehose instabilities in each phase of the cycle. We also discuss how this depends on physical parameters like the mass ratio \( m_i/m_e \) and magnetization \( \omega_{ci}^{\text{init}}/s \).

4.1. Heating by Magnetic Pumping

Figure 2 shows the evolution of the internal energy gain \( \Delta U_j \equiv U_j(t) - U_j(0) \) after being corrected for numerical heating in a dashed black line and of the integrated magnetic pumping heating rate in solid green line (see Equation (5)). For comparison, the integrated heating by magnetic pumping considering a pure double-adiabatic CGL evolution for the pressure anisotropy \( \Delta P_j \) (i.e., without scattering) is shown in solid gray lines. The units on this plot and all subsequent heating plots are such that the ordinate represents the temperature increment in units of initial temperature.

Figure 2. The effective ion and electron heating by magnetic pumping as a function of time for our reference simulation Zb20m2w800 (see Table 1). All quantities are normalized by the initial pressure \( P_j(0) \) \((j = i, e)\). The ion (upper panel) and electron (lower panel) energy gains \( \Delta U_j = U_j(t) - U_j(0) \) are shown in dashed black lines after being corrected by numerical heating (see the Appendix). The heating by magnetic pumping integrated over time is shown in solid green lines (see Equation (5)). For comparison, the integrated heating by magnetic pumping considering a pure double-adiabatic CGL evolution for the pressure anisotropy \( \Delta P_j \) (i.e., without scattering) is shown in solid gray lines. The units on this plot and all subsequent heating plots are such that the ordinate represents the temperature increment in units of initial temperature.
pumping is smaller than the expected heating in the absence of scattering. This, however, is followed by a much shorter period of cooling \((1 < t \cdot s < 1.3)\), and then followed by another period of heating \((1.3 < t \cdot s < 2)\). The smaller heating obtained in the first cycle is then compensated with a much shorter period of cooling that allows most of the heat to be retained at the end of the cycle. This behavior is directly related to the evolution of the pressure anisotropy and \(\Delta B/\Delta t\) during the pump cycle. This evolution is determined by the excitation of mirror and firehose instabilities, which are the agents that provide the pitch-angle scattering in our simulations, as we will show in Section 5. Note that although the heating by magnetic pumping accounts for the ion energy gain almost entirely, there is a small additional amount of electron heating. This might be because the wave–particle interactions are not completely elastic (see discussion in Section 5.2) or because numerical heating acts differently on the electrons. However, we have not pinpointed the cause.

4.2. Mass Ratio and Magnetization Dependence

Figure 3(a) shows the evolution of the ion energy gain \(\Delta U_i = U_i(t) - U_i(0)\) in dashed lines after being corrected by numerical heating (see the Appendix) and the evolution of the integrated heating rate by magnetic pumping in solid lines for runs Zb20m2w800, Zb20m8w800, and Zb20m32w800, with the same magnetization \(\omega_{ci}^\text{int}/s = 800\) and mass ratios of \(m_i/m_e = 2\), \(m_i/m_e = 8\), and \(m_i/m_e = 32\), respectively. We can see that there is a net heating in all three cases, the energy gain is well tracked by the magnetic pumping heating, and it does not exhibit a significant dependence on mass ratio. A similar behavior is exhibited in the case of electrons in Figure 3(b), where a net heating is also obtained and the mass ratio does not play a significant role in their final energy gain, either, although the difference is larger than in the case of ions.

Analogously, Figures 3(c) and (d) show the evolution of the ion and electron energy gain \(\Delta U_j = U_j(t) - U_j(0)\) \((j = i, e)\) in dashed lines after being corrected by numerical heating and the integrated heating rate by magnetic pumping in solid lines for runs Zb20m2w200, Zb20m8w200, and Zb20m32w1600, with the same mass ratio \(m_i/m_e = 2\) and magnetizations \(\omega_{ci}^\text{int}/s = 200\), \(\omega_{ci}^\text{int}/s = 800\), and \(\omega_{ci}^\text{int}/s = 1600\), respectively. We can see that in all cases a net heating is obtained after a pump cycle, and in this case the heating rate decreases for larger magnetizations. This decrease in the heating rate is directly related to the evolution of the pressure anisotropy and the excitation of mirror and firehose instabilities, which depend on the magnetization parameter, as we will show in Section 5.

Before proceeding further, we calculate the heating by magnetic pumping predicted by the model of Kunz et al. 2011; (see also Lyutikov 2007; Sharma et al. 2007).\(^5\) In this model, \(\Delta P\) is maintained at its marginally stable values by the firehose and mirror instabilities,

\[
\Delta P = \frac{\xi_\pm}{\beta} P, \tag{15}
\]

where the \(\xi_\pm\) are constants of order unity and have the same sign as \(\Delta B/\Delta t\), with the plus and minus signs referring to increasing and decreasing \(B\), respectively. Equation (5) can then be written in the form

\[
\frac{dU}{dt} = \frac{\xi_\pm}{16\pi} \frac{dB^2}{dt}. \tag{16}
\]

For our magnetic field model, \(B^2\) doubles during the shearing phase of the cycle (i.e., \(0 < t \cdot s < 1\)) and returns to its original
Figure 4. (a) The evolution of the total pressure anisotropy $\Delta P \equiv \Delta P_i + \Delta P_e$, for the reference simulation Zb20m2w800 in Table 1 (green line). The dashed green line shows the corresponding double-adiabatic prediction (CGL; Chew et al. 1956). As a reference, the solid red and blue lines show approximate linear thresholds for the excitation of the mirror and firehose instabilities, respectively, where $\beta_\perp = \beta_{i,\perp} + \beta_{e,\perp}$ and $\beta_\parallel = \beta_{i,\parallel} + \beta_{e,\parallel}$ (Hasegawa 1969; Hellinger & Trávníček 2008). (b) Volume-averaged magnetic energy in $\delta B$ along different axes and as a function of time for run Zb20m2w800. Here $\delta B_\parallel$ (blue line) is the component parallel to $\langle B \rangle$, $\delta B_{\perp,xy}$ (red line) and $\delta B_\perp$ (green line) are, respectively, the components perpendicular to $\langle B \rangle$ in the plane and perpendicular to the plane of the simulation.

5. Anisotropy Evolution and Instability Excitation

Here we describe the evolution of the pressure anisotropy, $\Delta P$, in our simulations. After a brief linear phase consistent with Chew–Goldberger–Low (CGL) double-adiabatic scaling (Chew et al. 1956), the nonlinear evolution of $\Delta P$ is determined by the alternating excitation of the mirror and firehose instabilities during a pump cycle, limiting the anisotropy growth and providing the necessary scattering for the operation of magnetic pumping. It is this interplay which sets the heating efficiency in a pump cycle.

5.1. Pressure Anisotropy and Mirror/Firehose Evolution

We can see the evolution of the pressure anisotropy and the development of mirror and firehose instabilities in Figure 4. Figure 4(a) shows the total pressure anisotropy, $\Delta P \equiv \Delta P_i + \Delta P_e$, normalized by the total parallel pressure, $\langle P_\parallel \rangle \equiv P_{\parallel,i} + P_{\parallel,e}$, as a function of time as a green solid line, and Figure 4(b) shows the time evolution of the magnetic energy in $\delta B$ along different directions for run Zb20m2w800 in Table 1 ($\delta B \equiv B - \langle B \rangle$, where $\langle \rangle$ denotes a volume average over the entire simulation box). The decomposition of $\delta B$ is done in terms of the direction parallel to $\langle B \rangle$ ($\delta B_\parallel$, blue line), perpendicular to $\langle B \rangle$ in the plane of the simulation ($\delta B_{\perp,xy}$, red line) and perpendicular to $\langle B \rangle$ and out of the simulation plane ($\delta B_\perp$, green line). Initially, the particle distribution is isotropic. The initial amplification of the magnetic field drives $\Delta P > 0$ and an early evolution consistent with double-adiabatic scalings (dashed green line). The mirror instability’s hydromagnetic threshold $1/\beta_\perp$, where $\beta_\perp = \beta_{i,\perp} + \beta_{e,\perp}$ (Hasegawa 1969), is shown as a solid blue line in Figure 4(a).\footnote{Even when the fastest growing mirror modes in our simulations have $kr_i \sim 1$, where $r_L$ is the Larmor radius of the ions (see Figure 7), we do not see important differences when comparing the evolution of $\Delta P$ with more accurate kinetic instability thresholds such as the ones provided by Pokhotelov et al. (2004); the anisotropy $\Delta P$ always largely surpasses the threshold for the excitation of the mirror instability and it does not stay at marginal stability afterwards.}

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The evolution of the parallel magnetic moment breaking are shown in Figure 5. On the other hand, the trapped particles can propagate through the modes. We will refer to these populations as trapped and passing, respectively, following Kunz et al. (2014). The breaking of the particles’ magnetic moments occurs first for the passing particles, right at the end of the linear growth of mirror modes. On the other hand, the trapped particles keep conserving their magnetic moment through the linear and secular phase of mirror growth, and it is only at the end of the secular phase that the trapped particles break their magnetic moment and start to become passing. These two stages of magnetic moment breaking are shown in Figure 5(a) for run Zb20m2w800, where we can see that the averaged magnetic moment of passing ions (solid red line) is broken first at $t \cdot s \sim 0.5$, consistent with the end of the linear growth of mirror modes (see $\delta B_2^2$ in Figure 4(b)). On their part, the trapped ions maintain their averaged magnetic moment approximately constant until $t \cdot s \sim 0.75$, also consistent with the end of the secular growth of mirror modes (from $t \cdot s \sim 0.75$ to $t \cdot s \sim 1$; see $\delta B_2^2$ in Figure 4(b)). We have defined the trapped and passing populations of particles according to the behavior of their parallel momentum: During the interval of time $\Delta t_{\text{trapp}}$, defined from the end of the linear growth of mirror modes ($t \cdot s \sim 0.5$) until the shear motion reverses its direction ($t \cdot s = 1$), a particle is considered trapped if the median of its parallel momentum over $\Delta t_{\text{trapp}}$ is smaller than its standard deviation over $\Delta t_{\text{trapp}}$. Correspondingly, a particle is considered passing if the median of its parallel momentum over $\Delta t_{\text{trapp}}$ is greater or equal than its standard deviation over $\Delta t_{\text{trapp}}$. In Figure 5(b) we show an example of the evolution of the parallel momentum of a trapped (solid blue line) and a passing particle (solid red line) for run Zb20m2w800, defined according to this criterion. We can see the characteristic oscillatory behavior of the parallel momentum for the trapped particle, whereas the passing particle streams freely along magnetic field lines.

This way, the growth of $\Delta P$ is limited and regulated throughout the initial half-cycle by mirror modes. It is important to note that, after surpassing the mirror instability threshold, $\Delta P$ does not reach a marginally stable state $\sim 1/\beta_i$, but it saturates at a larger value, $\Delta P/P_i \sim 0.15$, similar to that reported by Melville et al. (2016). We will come back to this point in Section 5.2, where we compare different magnetization values. Note that the relative amplitudes of all components of $\delta B$ are quite large, suggesting that linear wave physics alone does not describe these fluctuations.

The excitation of the mirror instability is also consistent with the dominance of $\delta B_2^2$ during the initial half-cycle, especially from $0.5 \lesssim t \cdot s \lesssim 0.6$. Two explanations may be proposed for the origin of this feature in $\delta B_2^2$. On the one hand, it is consistent with the appearance of the non-coplanar component of mirror modes when the fastest growing mode has $k R_L \sim 1$ in the kinetic regime (Pokhotelov et al. 2004; see Figures 7(a) and (b)). On the other hand, the structure of the modes in $\delta B_2^2$ and their parallel propagation (see Figures 6(c) and 7(b)) also resemble ion-cyclotron modes, although they are expected to be subdominant for large plasma $\beta$, based on linear theory and simulations (Riquelme et al. 2015). The disentanglement of the true nature of these fluctuations has been reported to be a difficult task (Pokhotelov et al. 2004). As the overall dynamics is still dominated by mirror modes at this stage of the pump cycle, the proper identification of the modes seen in $\delta B_2^2$ is inessential to this work. However, it is interesting to note that after its initial decay at $t \cdot s \approx 0.6$, $\delta B_2^2$ grows again and begins oscillating; $\delta B_2^2 \perp B_z$ begins to oscillate, too. We will see that these behaviors are consistent with the appearance of short-lived, parallel propagating whistler modes localized in the regions of low magnetic field associated with mirror modes, commonly known as “lion roars” (Baumjohann et al. 1999; Breuillard et al. 2018), which will be investigated more completely in future work (F. Ley et al. 2023, in preparation).

At $t \cdot s = 1$, the shear is reversed and the magnetic field starts to dwindle, driving a decrease in $\Delta P$. Since $\dot{B}$ becomes negative.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{(a) The evolution of the volume-averaged magnetic moments $\langle \mu_i \rangle = \langle p_{\|,i}^2 / B \rangle$ of trapped (solid blue line) and passing (solid red line) ions during the first pump cycle for run Zb20m2w800. The average was done over 812 trapped ions and 436 passing ions. (b) The evolution of the normalized parallel momentum for a trapped ion (solid blue line) and a passing ion (solid red line) during the first pump cycle for run Zb20m2w800. The dashed black line at $t \cdot s = 1$ marks the time when the shear motion reverses its direction. (c) The evolution of the parallel (solid green line) and perpendicular (solid black line) kinetic energy of a passing ion during the first pump cycle for run Zb20m2w200.}
\end{figure}
while $\Delta P$ is positive, the plasma undergoes cooling, but, as shown in Figures 2 and 3, the duration of this phase is short and the cooling is slight. Because mirror instability has kept $\Delta P$ at a lower value than that predicted by CGL, the decrease of $\Delta P$ does not end at a complete isotropic state at the end of the cycle but grows further to $\Delta P < 0$, driving the system anisotropic again (but with opposite sign), and setting the conditions for the excitation of the oblique firehose instability. The approximate threshold 

$$-1.4/(\beta_{\|} + 0.11)$$

for this instability is shown as a solid red line in Figure 4(a), where $\beta_{\|} \equiv \beta_i + \beta_e$ (Hellinger & Trávníček 2008). Indeed, at $t \cdot s \approx 1.5$, we can see in Figure 4(b) that $\delta B^z$ is now the dominant component, consistent with firehose modes (Hellinger & Matsumoto 2000).

The pitch-angle scattering that these modes provide quickly stops $\Delta P$ from growing more negative, bringing it closer to isotropy until the end of the second half of the pump cycle ($t \cdot s = 2$). In this case $\Delta P$ is able to reach values much closer to marginal stability and is well tracked by the oblique firehose instability threshold. The presence of firehose modes is also reflected in the evolution of the particle magnetic moment shown in Figure 5(b).

Figure 6. Upper row: parallel and perpendicular components of the magnetic fluctuation $\delta B$ with respect to $\langle B \rangle$ for run Zb20m2w800 at time $t \cdot s = 0.6$; left panel shows the in-plane perpendicular component $\delta B_{\perp}$, middle panel shows the parallel component $\delta B_{\parallel}$, and right panel shows the out-of-plane perpendicular component $\delta B_z$. The black arrows show the direction of the mean magnetic field $\langle B \rangle$ at $t \cdot s = 0.6$. Bottom row: same as upper row but at time $t \cdot s = 1.5$. An animated version of this figure, running for one pump cycle, is available in the online journal.

(An animation of this figure is available.)

The scattering that the particles are subjected to by waves is such that they exchange energy between parallel and perpendicular components in each interaction; this can be clearly seen in Figure 5(c), where the perpendicular and parallel (with respect to $B$) kinetic energies are shown for a passing particle for run Zb20m2w200, during the first pump cycle. Before the onset of the mirror instability ($0 < t \cdot s \lesssim 0.6$), we can see the continuous increase of perpendicular kinetic energy in accordance with the conservation of magnetic moment. Then, once the mirror modes are developed ($t \cdot s \sim 0.6$), we can see a continuous exchange of energy between the two components of kinetic energy, from perpendicular to parallel components in the mirror-dominated stage of the pump cycle ($0.6 < t \cdot s < 1$) and then from parallel to perpendicular in the firehose-dominated stage of the pump cycle ($1 < t \cdot s < 2$). This process

Note that the trapped/passing distinction makes sense only for the first half of the pump cycle when mirror modes dominate the scattering. When firehose modes are the main source of scattering in the second half of the pump cycle, the distinction between passing and trapped particles is not relevant anymore, as trapped particles become passing.

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7 Note that the trapped/passing distinction makes sense only for the first half of the pump cycle when mirror modes dominate the scattering. When firehose modes are the main source of scattering in the second half of the pump cycle, the distinction between passing and trapped particles is not relevant anymore, as trapped particles become passing.
is essential for retaining part of the heating by magnetic pumping after a pump cycle (see Section 4.1). The next cycles present mainly the same phenomenology.

This way, Figure 4(a) shows how the pressure anisotropy undergoes periodic stages of growth, decrease and negative growth, always bounded between values determined by the efficiency of the isotropizing processes provided by mirror and firehose modes (and not necessarily constrained by marginal stability). As we will show in Section 5.2, this efficiency will depend on the magnetization parameter, $\omega^{\text{init}} / s$.

The overall alternating excitation of mirror and firehose instabilities in the parameter space studied here is qualitatively consistent with the results by Melville et al. (2016) for both mirror-to-firehose and firehose-to-mirror regimes. In particular, we see that mirror and firehose modes decay in a relatively short time after the shear changes direction and stops driving the corresponding anisotropy (although mirror modes decay much more slowly than firehose modes). In this sense, no memory of preceding wave excitations is retained after one pump cycle. This is consistent with Melville et al. (2016) in the regime where $s \beta < \omega_{ci}$ relevant to this work.

The presence of mirror and firehose modes at different stages of the pump cycle can be seen in Figure 6, where we show two snapshots of $\delta B$ in the same three different directions used in Figure 4(b), and black arrows representing the direction of $\langle B \rangle$. Because $\langle B \rangle$ is almost the same in the two cases, the clear differences in the character of the fluctuations can only be attributed to the anisotropy of the plasma.

In Figure 6(b) ($t \cdot s = 0.6$), we can see that $\delta B_\parallel$ has the largest amplitude, and the structure of oblique mirror modes can also be observed. This is consistent with the expectation for mirror modes having their components mainly in the plane $(k, B)$ (Pokhotelov et al. 2004). On the other hand, in Figure 6(f) ($t \cdot s = 1.5$), we can see that now $\delta B_\perp$ is the largest component, exhibiting the presence of oblique firehose modes, consistent with the dominance of $\delta B^2$ at this time (see Figure 4(b)).

The oblique nature of mirror modes is also evident from Figure 7(a). The spatial Fourier transform (FT) of $\delta B_\parallel$ is shown at $t \cdot s = 0.5$, with the solid and dashed black lines showing the direction along and perpendicular to $\langle B \rangle$ at $t \cdot s = 0.5$, respectively. The modes span angles $\theta_k = \cos^{-1}(k \cdot \langle B \rangle / kB)$ between $\sim 45^\circ$ and $\sim 90^\circ$, although most of the power is concentrated in a range $60^\circ \lesssim \theta_k \lesssim 80^\circ$.

We can better characterize the two different stages seen in the evolution of $\delta B^2$ during $0 \lesssim t \cdot s \lesssim 0.6$ and $0.6 \lesssim t \cdot s \lesssim 1.2$ in Figures 7(b) and (c). We can see the modes excited at $t \cdot s = 0.5$ in $\delta B_\parallel$ propagating quasi-parallel to $\langle B \rangle$, and most of the power concentrates around wavelengths $\sim 9 R^\text{init}_{ci} L^\text{init}_{ci}$ in Figure 7(b). Additionally, between $0.6 \lesssim t \cdot s \lesssim 1.2$, the peak power oscillates around the parallel direction in a narrow cone, so the waves always propagate nearly parallel to $\langle B \rangle$, with most of the power concentrating at wavelengths $\sim 15 R^\text{init}_{ci} L^\text{init}_{ci}$ (for $m_i/m_e = 2$), as shown in Figure 7(c). The oblique character of firehose modes can also be seen in Figure 7(d), where the spatial FT of $\delta B_\perp$ is shown at $t \cdot s = 1.5$. In this case, most of the power is concentrated in modes with angles in the range $20^\circ \lesssim \theta_k \lesssim 65^\circ$.

Finally, in Figure 8 we show the FT in time averaged over the simulation box of the three components of $\delta B$, namely $FT_t(\delta B_\parallel)$, $FT_t(\delta B_\perp)$, and $FT_t(\delta B_\perp)$, for run Zb20m2w800 in four time intervals. During $0 < t \cdot s < 0.5$ (Figure 8(a)), we can see that most of the power is concentrated at low frequencies, especially in $\delta B_\parallel$, consistent with the nature of mirror modes (Pokhotelov et al. 2002, 2004). During $0.5 < t \cdot s < 1$ (Figure 8(b)), the power at low frequency still dominates, consistent with the dominance of mirror modes in this interval, but also the transverse components $\delta B_{L,xy}$ and $\delta B_{L,z}$ exhibit a subdominant, broad resonant feature at $\omega \sim 0.2\omega^{\text{init}}_{ci} \sim 0.1\omega^{\text{init}}_{ci}$ (for $m_i/m_e = 2$), whereas $\delta B_{c,z}$ and $\delta B_{L,x}$ exhibit narrower resonances around $\omega \sim \omega^{\text{init}}_{ci}$. The broad resonant feature at $\omega \sim 0.1\omega^{\text{init}}_{ci}$ in both perpendicular components of $\delta B$ is consistent with the expected frequency of whistler lion roars (Baumjohann et al. 1999). The nature of the narrower resonances at $\omega \sim \omega^{\text{init}}_{ci}$ in the $\delta B_{L,x}$ is less clear, but could be related to the resonant nature of mirror instability via Landau resonances. During $1.2 < t \cdot s < 1.5$ (Figure 8(c)), we can see that the broad resonance at $\omega \sim 0.1\omega^{\text{init}}_{ci}$ is not present anymore, but the narrow resonant peaks still appear, although with less power. During $1.5 < t \cdot s < 2$ (Figure 8(d)), we can see that the power in $\delta B_{L,x}$ dominates in power and resides at low frequencies, consistent with the nature of oblique firehose modes (Hellinger & Matsumoto 2000). There are also narrow peaks at $\omega \sim \omega^{\text{init}}_{ci}$, which is consistent with oblique firehose instability being resonant via cyclotron resonances (Hellinger & Trávníček 2008).

5.2. Mass Ratio and Magnetization Dependence

In this section, we explore the dependency of the heating on the mass ratio of ions to electrons, $m_i/m_e$, and the magnetization parameter, $\omega^{\text{init}} / s$.

Figure 9(a) shows the evolution of the total magnetic fluctuations $\delta |B|/(t^2)$ in one cycle ($0 < t \cdot s < 2$) for simulations with $\omega^{\text{init}} / s = 200, 800$, and $1600$ (runs Zb20m2w200, Zb20m2w800, and Zb20m2w1600 in Table 1). In all three simulations the particles have the same physical parameters other than the magnetization: $m_i/m_e = 2$, $k_B T_i = k_B T_e = 0.1 m_e c^2$, $\beta^{\text{init}} = 20$. We can see that overall the evolution of the background behavior: exciting mirror modes in the first half-cycle and firehose modes in the second half-cycle, and mirror modes are excited at relatively earlier times for increasing magnetizations.8 Mirror fluctuations saturate around the same value for all runs, whereas for firehose modes it decreases with increasing magnetizations.

Figure 9(b) shows the pressure anisotropy evolution for the same set of simulations as in Figure 9(a). The overall evolution is similar in all cases: both mirror and firehose instabilities are excited and have time to limit the anisotropy growth. The main difference is reflected in the value that the anisotropy can reach before starting to get regulated by the instabilities. We will refer to this point as the overshoot in $\Delta P$. This can be seen more clearly at the beginning of each cycle (e.g., $t \cdot s \approx 0.5$), where the overshoot that $\Delta P / P_{\parallel}$ undergoes is largest for $\omega^{\text{init}} / s = 200$ and decreases for larger values of magnetization. This also happens when the shear motion is reversed, right before the firehose modes act and regulate the pressure anisotropy (e.g., $t \cdot s \approx 1.3$). This effect has been shown and discussed in previous works for both hybrid and fully kinetic PIC simulations in the context of a continuous shear motion (Kunz et al. 2014; Riquelme et al. 2015, 2018). This difference

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8 Note that, in our simulations, the larger the magnetization, the smaller the numerical value of the shear frequency, so the shear occurs more slowly with respect to the initial ion-cyclotron frequency, therefore providing more time for the instabilities to develop. This translates to the apparent earlier excitation of the instabilities for larger magnetizations in Figure 9(a).
in the overshoot peak value implies that, for lower magnetizations, mirror and firehose modes are less efficient in pitch-angle scattering the particles, taking longer to grow (in units of s⁻¹), and this allows ΔP/P₁ to keep growing to larger values until it effectively saturates due to scattering (Riquelme et al. 2015).

For completeness, in Figures 9(c) and (d) we show the same plots as in Figures 3(c) and (d), respectively, i.e., the ion and electron energy gain ΔUᵢ, ΔUₑ (dashed lines) after being corrected by numerical heating (see the Appendix) and the integrated magnetic pumping heating rate as a function of time (solid lines; see Equation (5)) for the same runs as in

Figure 7. Fourier transform (FT) in space of two different components of δB at four different times for run Zb20m2w800. The wavenumbers k₁ and k₂ are normalized to the initial ion Larmor radius Rₐinit. In all panels, the solid and dashed black lines represent, respectively, the direction along (B) and perpendicular to (B) at the corresponding time. (a) Magnitude of the FT of δB₁ at t·s = 0.5. (b) Magnitude of the FT of δB₂ at t·s = 0.5. (c) Magnitude of the FT of δB₁ at t·s = 0.9. (d) Magnitude of the FT of δB₂ at t·s = 1.5.

Figure 8. Volume-averaged power spectrum in time of δB along its three components, δB₁ (green line), δB₂ (red line), and δB₃ (blue line), as a function of angular frequency for run Zb20m2w800 in a time interval 0 < t·s < 0.5 (panel (a)), 0.5 < t·s < 1 (panel (b)), 1.2 < t·s < 1.5 (panel (c)), and 1.5 < t·s < 2 (panel (d)). The frequencies are normalized by the initial cyclotron frequency ωₐinit of the ions.

Figure 9. Comparison between different magnetizations ωₐinit/s. (a) The evolution of the magnetic field fluctuations δB² for runs Zb20m2w200, Zb20m2w800, and Zb20m2w1600, all with the same mass ratio mᵢ/mₑ = 2 and magnetizations of ωₐinit/s = 200 (blue line), ωₐinit/s = 800 (orange line), and ωₐinit/s = 1600 (green line). (b) Evolution of the total pressure anisotropy (ΔPᵢ + ΔPₑ)/(Pᵢ,0 + Pₑ,0) for the same runs as in panel (a). The dashed lines show the approximate linear thresholds for the excitation of mirror (dashed blue line) and firehose (dashed red line) instabilities, and the dashed-dotted gray line shows the expected double-adiabatic evolution of the pressure anisotropy. (c) Same as Figure 3(c), the integrated ion heating by magnetic pumping for the same runs as in panel (a) as shown as solid lines (see Equation (5)) for magnetizations ωₐinit/s = 200 (blue line), ωₐinit/s = 800 (red line), and ωₐinit/s = 1600 (green line). The dashed lines show the ion energy gain ΔUᵢ after being corrected for numerical heating. (d) Same as Figure 3(d), the same quantities as in panel (c) but for electrons.
Figure 9(a). We can see that in all cases a net heating is obtained in each pump cycle, with the difference that the heating rate decreases for larger magnetizations. The reason for the decrease in the heating rate for larger magnetizations is directly related to the decrease of the peak value of $\Delta P/P_\parallel$ during the overshoot and also to the anisotropy decrease in the secular mirror modes regime. Indeed, the heating provided by the pressure anisotropy during this phase is retained after the pump cycle and, looking at Equation (5), it will be larger for larger values of the anisotropy. This sets a direct dependence of the efficiency of the heating by magnetic pumping on the overshoot and secular values that $\Delta P/P_\parallel$ produces during a pump cycle for different values of $\omega_{ci}/s$. This way, higher shear frequencies (i.e., smaller $\omega_{ci}/s$) will produce less heating, whereas low shear frequencies (large $\omega_{ci}/s$) will produce less heating. In a turbulent medium like the ICM, where the plasma can be driven by a spectrum of turbulent eddies with different frequencies, the contribution of all these frequencies can be relevant in the total amount of heating the plasma can get via magnetic pumping.

Complementary to Figure 9, Figure 10(a) shows the evolution of the total magnetic fluctuations $|\delta B|^2$ in one cycle ($0 < t \cdot s < 2$) now for simulations with $m_i/m_e = 2, 8$, and 32 (runs Zb20m2w800, Zb20m8w800, and Zb20m32w800 in Table 1). The three simulations share the same physical parameters: $\omega_{ci} = 800$, $k_BT_i = k_BT_e = 0.1m_e c^2$, $\beta_i^{init} = 20$. We can see a similar behavior in all three cases: the mirror modes saturate at the same level in the first half of the pump cycle, whereas the firehose modes saturates at a slightly higher level for increasing mass ratio.

Figure 10(b) shows the pressure anisotropy evolution for the same set of simulations as in Figure 10(a). In this case, the evolution is very similar for all three simulations; in the first half of the cycle ($0 < t \cdot s < 1$) and after a brief double-adiabatic (CGL) evolution, they reach a similar overshoot level (as they all have the same magnetization) and start to be regulated by the interaction with mirror modes. In the second half of the cycle ($1 < t \cdot s < 2$) the anisotropies start to increase in absolute value and quickly start to be regulated by firehose modes, after reaching an overshoot that is also very similar in all three mass ratio cases.

For completeness, in Figures 10(c) and (d) we show the same plots as in Figures 3(a) and (b). As discussed in Section 4, we can see that in all three cases we obtain a net heating after one pump cycle, and its efficiency is very similar in all three simulations. This is consistent with the evolution of the pressure anisotropy as seen in Figure 10(b) in terms of the overshoot level in the first and second halves of the pump cycle. The same happens for electrons: they are also heated after one pump cycle, and the net heating at the end of one cycle is slightly lower for larger mass ratios. In the case of electrons, however, given the limited range in mass ratio, we do not fully capture the electron-scale instabilities that could be present in a realistic scenario and which could also participate in the pitch-angle scattering of electrons.

Thus, this analysis shows that the action of heating by magnetic pumping on the ions is fairly independent of $m_i/m_e$, and its efficiency depends on $\omega_{ci}/s$, where for larger magnetizations the retained heating becomes smaller.

Figure 11 summarizes our findings. The energy gain of ions (blue symbols) and electrons (red symbols) after one pump cycle are shown as a function of magnetization for different mass ratios. We obtain scaling relations $\Delta U_i \propto (\omega_{ci}/s)^{\alpha_i}$, with $\alpha_i = -0.29 \pm 0.02$ for ions, and $\Delta U_e \propto (\omega_{ci}/s)^{\alpha_e}$, with $\alpha_e = -0.19 \pm 0.02$ for electrons. These relations can be used to estimate the heating rate of a turbulent spectrum of different frequencies (or shear rates $s$). It is worth noting that, for sufficiently high magnetizations, the energy gain is expected to become independent of magnetization and is given by the values of the anisotropy at marginal stability determined by mirror and firehose instabilities. According to Equation (17), the energy gain $\Delta U/P_\parallel$ obtained in this case is $3\%$ per cycle. To illustrate this point, we plot this energy gain in Figure 11 as horizontal dotted blue and red lines for ions and electrons, respectively, at the expected value of magnetization $\omega_{ci}/s$ according to the scaling relations above.9 These levels provide a lower limit to the scaling relations we have derived.

The scaling relations that we have obtained show shallower dependencies with magnetization than what can be obtained by the anisotropy overshoot scalings from, for example, Kunz et al. (2014), specifically for the mirror instability case (as it is the case in which most of the heating is obtained in our simulations). Using Equation (5) at the end of the first cycle and $B/B \sim s$, with $dU_i/dt \sim s\Delta U \sim s\Delta P$, the mirror anisotropy overshoot scaling $\max(\Delta P/P) \propto s^{1/2}$ (Kunz et al. 2014) then predicts $\Delta U \propto s^{1/2}$. Interestingly, in our simulations we have measured an anisotropy overshoot scaling of $\max(\Delta P/P) \propto s^{0.48-0.51}$ (not shown), very similar to the one reported in Kunz et al. (2014). Consequently, the heating we obtained in one cycle is not only dependent on the anisotropy overshoot, and we argue that the disagreement between our scaling and the predicted $\Delta U \sim s^{1/2}$ scaling, is related to additional heating occurring in the “plateau” stage that the anisotropy reaches immediately after the overshoot. This plateau stage keeps the anisotropy at a higher level than what is given by the marginal stability threshold of $1/\beta_\perp$, therefore contributing to gain more heat (see Figure 9(b), from $t \cdot s \sim 0.5$ to $t \cdot s \sim 1$), at least for the magnetization range here explored. For higher magnetizations or higher initial plasma $\beta$, this plateau might not be as high and could be closer to the $1/\beta_\perp$ threshold. This shows that the nonlinear evolution of the mirror instability is also important for the efficiency of magnetic pumping.

5.3. Plasma $\beta_i^{init}$ Dependence

We also explored how the heating by magnetic pumping acts for the case of a lower plasma beta, such as the solar wind, where it could coexist with other mechanisms of thermal heating and/or nonthermal acceleration mediated by micro-instabilities, such as resonant wave–particle interaction with ion-cyclotron waves or whistlers (Riquelme et al. 2017; Ley et al. 2019; Cerri et al. 2021; Riquelme et al. 2022).

Figure 12 shows the results of a periodic shear simulation with the same fixed physical parameters as the previous simulations except for the initial ion plasma beta: $\beta_i^{init} = 2$ (run Zb2m8w800 in Table 1). Figure 12(a) shows that, for a fixed value of magnetization ($\omega_{ci}/s = 800$), mirror modes take longer to be excited, reaching saturation around $t \cdot s \approx 1$ (in contrast to $t \cdot s \sim 0.4$ in the $\beta_i^{init} = 20$ case), and consequently they do not have time to reach a nonlinear stage before the shear reverses. In the second half of the cycle ($1 < t \cdot s < 2$) we can see $\delta B_i^2$ slowly decaying and $\delta B_e^2$ staying at very low amplitudes after a quick decay, exhibiting a slight rise toward $t \cdot s = 2$. This means that firehose modes are not being excited

9 In a real setup, however, one could still expect some overshoot in the anisotropy at these values of magnetization.
sufficiently rapidly before the shear reverses, so they never gain sufficient energy to effectively scatter particles. This behavior is consistent with the $\beta^{-1}$ dependence of the pressure anisotropy instability thresholds of both instabilities, which are higher in this case. This is explicitly shown in Figure 12(b), where we can see that the approximate threshold for mirror instability is surpassed at a later time than for higher beta ($t \cdot s \approx 0.5$). Analogously, we can see that in the second half of the cycle ($1 < t \cdot s < 2$) the anisotropy can barely meet the approximate threshold for the excitation of firehose modes, consistent with the evolution of $\delta B^2$ in Figure 12(a).

The initial excitation of mirror modes in the first half of the pump cycle and the absence of firehose modes in the second half translates into an inefficient scattering process in one pump cycle, therefore we would not expect the evolution of the energy density to be very different from pure CGL evolution. Figure 12(c) shows the ion energy gain after being corrected by numerical heating and the integrated magnetic pumping heating rate as a function of time. We see that, indeed, during the first half of the pump cycle ($0 < t \cdot s < 1$), mirror modes start to interact with ions, and this somewhat extends to the second half of the cycle ($1 < t \cdot s < 2$). This allows retention of a small fraction of the energy injected by the shear motion. However, the absence of scattering provided by firehose modes does not allow the heating to continue growing, and the ion internal energy stays relatively constant.

In the second cycle, the evolution differs from the previous one and the cycles are no longer reproducible, unlike the high-$\beta$ case. The subsequent heating around $t \cdot s \approx 3$ is completely reversible as the scattering becomes inefficient, and there is no additional energy gain. We see that the efficiency of the heating by magnetic pumping is much lower for $\beta_{\text{init}}^i = 2$. This is directly related to the inefficiency of the scattering processes during a pump cycle, and is set by the slower excitation of mirror and firehose instabilities with respect to the length of the cycle.
6. Pitch-angle Scattering Model

A self-consistent treatment of the interaction of mirror and firehose modes with particles in a pump cycle would require solving Equation (13) coupled with equations for the scattering frequency, $\nu$, assuming, for example, a quasi-linear regime for each instability. This procedure turns out to be rather difficult, as the instabilities do not closely follow a quasi-linear evolution (Hellinger & Trávníček 2008; Hellinger et al. 2009; Rincón et al. 2015), but reach nonlinear amplitudes in our simulations. Alternatively, we consider a simplified model similar to the bounded anisotropy model from Hellinger & Trávníček (2008), where we directly evolve the scattering frequencies representing the interaction with mirror and firehose modes coupled to the evolution of the pressure anisotropy and plasma beta.

Assuming for simplicity one particle species and a nonrelativistic plasma, we can obtain the nonrelativistic evolution equation for $\Delta P$ by multiplying Equation (13) by $-\nu P_\beta(\mu)$ and integrating over momentum space. In order to perform this integration, we assume $\nu$ independent of $\mu$ in Equation (13) and we expand the distribution function in Legendre polynomials $f(t, p, \mu) = \sum f(t, p) P_n(\mu)$, where $P_n(\mu)$ is the Legendre polynomial of order $n$. For our purposes here, it is sufficient to expand $f$ up to $n = 2$, $f = f_0 P_0 + f_2 P_2$, where $f_0$ is the isotropic part of the distribution function and $f_2$ naturally couples to $\Delta P$. This leads to

$$\frac{d\Delta P}{dt} = \frac{3P - \Delta P dB}{B^2 dt} - 3\nu \Delta P.$$  \hspace{1cm} (18)

We then characterize the scattering via waves excited by positive and negative pressure anisotropies, so $\nu = \nu_+ + \nu_-$. Waves that are not excited are assumed to be damped, and both damping and excitation processes are linear. This translates to the following pair of equations:

$$\frac{d\nu_+}{dt} = \gamma_+ \nu_+ \left( \frac{\Delta P}{P} - \frac{\xi_+}{\beta} \right) + \nu_b,$$ \hspace{1cm} (19)

$$\frac{d\nu_-}{dt} = -\gamma_- \nu_- \left( \frac{\Delta P}{P} - \frac{\xi_-}{\beta} \right) + \nu_b,$$ \hspace{1cm} (20)

where $\beta = 8\pi P/B^2$, $\gamma_+$, $\gamma_-$ are the wave growth rates, $\xi_+$, $\xi_-$ are the constants of order unity related to the linear instability thresholds introduced in Equation (15), and $\nu_b$ is a constant numerical parameter that we include to set the evolution on the right path without the final result being sensitive to its value, and is taken to be small. Given the forms of Equations (19) and (20), we can redefine Equation (18) to an equation for $A \equiv \Delta P/P$:

$$\frac{dA}{dt} = -3\nu A + \frac{3 - A - \frac{\xi_+}{3} dB}{B} \frac{dB}{dt}. \hspace{1cm} (21)$$

Finally, using $U = 3P/2$ along with Equation (5) we obtain an evolution equation for $\beta$:

$$\frac{d\beta}{dt} = -2\beta \left( 1 - \frac{A}{3} \right) \frac{dB}{B} \frac{dB}{dt}. \hspace{1cm} (22)$$

It is important to note that for larger magnetization values, the instabilities would have more time to develop (relative to the shear frequency $s$) and to interact with the particles. The heating could therefore be relatively more efficient, although the anisotropy thresholds would be the same for any magnetization value for a given initial beta, setting a more stringent limiting factor for the overall efficiency.

The results shown in this section set a lower, magnetization-dependent limit in $\beta$ for the action of this heating mechanism, therefore for moderate values of $\beta$ the heating rate is an increasing function of $\beta$. 

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Figure 12. Results of run Zb2m8w800 for initial $\beta^{\text{init}} = 2$. (a) The three components of the volume-averaged magnetic energy in $\delta B$ as a function of time. Here $\delta B_r$ (blue line) is the component parallel to $\langle B \rangle$, $\delta B_{\perp r}$ (red line) and $\delta B_r$ (green line) are, respectively, the components perpendicular to $\langle B \rangle$ in the plane and perpendicular to the plane of the simulation. (b) Evolution of the total pressure anisotropy $(\Delta P_i + \Delta P_e)/(P_{i,i} + P_{i,e})$. The dashed lines show the approximate linear thresholds for the excitation of mirror (dashed blue line) and firehose (dashed red line) instabilities. (c) The integrated ion heating by magnetic pumping is shown as a solid green line and the ion energy gain $\Delta U_i$ after being corrected by numerical heating is shown as a dashed black line. The solid gray line shows the expected magnetic pumping heating rate if the pressure anisotropy evolved according to CGL double-adiabatic prediction.
Equations (19), (20), (21), and (22) constitute the basic equations for the model. In order to compare with the results of our simulations, we prescribe the evolution of the magnetic field to be the same as in our previous PIC runs, and set \( \gamma \pm \) by fitting an exponential growth rate to the linear stages of mirror and firehose fluctuations: \( \delta B^2_\pm (0.4 < t \cdot s < 0.52) \) and \( \delta B^2_\pm (1.22 < t \cdot s < 1.31) \), respectively (see Figure 4). Importantly, the growth rates in our simulations need not be constant; the rates may depend on \( \Delta P \) through the instability parameter \( (\Delta P/P - \xi/\beta) \). To account for this dependence, we fit a function of the form

\[
\delta B^2_{ci}(t) \propto \exp(\Gamma t),
\]

assuming constant \( \Gamma \pm \) over the fitted time interval. Then, in our model Equations (19)-(20), we take \( \gamma \pm \) equal to \( \Gamma \pm \) divided by the average of \( (\Delta P/P - \xi/\beta) \) over the same fitted time interval and measured directly from the simulations.

Additionally, choosing to normalize \( \nu \) by the initial cyclotron frequency, \( \tilde{\nu} = \nu/\omega_{ci,init}^{r\text{hose}} \), by the shear frequency, \( \gamma_\pm = \gamma_\pm/\nu_0 \), and time by the inverse shear frequency, \( \tilde{t} = t/\nu_0^{-1} \), the equations remain unaltered except for the equation for \( A \):

\[
\frac{dA}{dt} = -3 \left( \frac{\omega_{ci,init}^{r\text{hose}}}{\nu_0} \right) \tilde{\nu} A + \frac{3 - A - \frac{2}{3} A^2}{B} \frac{dB}{dt}.
\]  

(24)

It is worth noting that the magnetization parameter \( \omega_{ci,init}^{r\text{hose}}/s \) naturally appears in the first term of the right-hand side of Equation (24), causing the same effect that we see in our simulations: a higher magnetization leads to a smaller overshoot and a faster regulation of the pressure anisotropy.

In Figure 13 we show the results for one pump cycle. We set \( \omega_{ci,init}^{r\text{hose}}/s = 800 \), \( \gamma_+ = 152.18 \) s, \( \gamma_- = 5406.17 \) s, \( \nu_0 = 10^{-4} \) s, \( \xi_+ = 1 \) (i.e., the approximate mirror linear threshold), \( \xi_- = -1.4 \) (i.e., the approximate oblique firehose threshold), and the initial conditions \( \nu_{+,0} = \nu_{-,0} = 0 \), \( A_0 = 0 \), \( P_0 = 0.4 \), and \( \beta_0 = 40 \). In Figures 13(b) and 13(c) we also include the respective results from our PIC simulation, run Zb20m2w800 (black lines).

Additionally, in Figure 13(c) we include the results for magnetization \( \omega_{ci,init}^{r\text{hose}}/s = 1600 \) (solid green line), which we compare with the PIC run Zb20m2w1600 (green dashed line). For this case, we set \( \gamma_+ = 218.96 \) s, \( \gamma_- = 1967.160 \) s, and the rest of the parameters and initial conditions are the same as in the \( \omega_{ci,init}^{r\text{hose}}/s = 800 \) case.

We can see that our model is phenomenologically consistent with our PIC simulations: the anisotropy is effectively regulated by the scattering frequencies \( \nu \pm \) that are alternately excited, as shown in Figure 13(a). When the anisotropy \( A(t) \) surpasses the given mirror threshold \( \xi_+/\beta \) (solid green curve in Figure 13(b)), \( \nu_+ \) is excited, reaches a saturated value and subsequently decays when \( B(t) \) starts to decrease. Analogously, \( \nu_- \) is not excited until the second half of the cycle, just when \( A(t) \) surpasses the firehose threshold \( -\xi_-/\beta \), reaching a similar saturated value as \( \nu_+ \) and regulating the anisotropy rapidly. It is interesting to note that, as \( \nu_- \) is only excited when \( A(t) \) surpasses the firehose threshold and the decrease of \( B(t) \) continuously drives more anisotropy, this produces a series of \( \nu_- \) excitations whenever \( A(t) \) surpasses the firehose threshold (solid green curve in Figure 13(b)), generating an oscillatory behavior between stable and unstable regions. This is a distinctive feature of the oblique firehose instability related to its self-destructive properties (Hellinger & Trávníček 2008; Hellinger 2017).

We can also see the presence of anisotropy overshoots in both mirror (\( 0 < t \cdot s < 1, \ A > 0 \)) and firehose (\( 1 < t \cdot s < 2, \ A < 0 \)) stages in Figure 13(b) (solid green curve). The overshot of \( A(t) \) in the model during the mirror phase (\( t \cdot s \approx 0.5 \)) coincides very

![Figure 13. Results of our pitch-angle scattering model up to \( t \cdot s = 2 \).](image-url)
well with our PIC run (dashed purple line), whereas in the firehose phase the overshoot is larger. Furthermore, we can see that $A(t)$ can be effectively pinned to the approximate mirror threshold $\xi_m/\beta$ in the model, in contrast to the nonlinear evolution of the anisotropy in PIC runs, which saturates at a larger value above the approximate mirror threshold. This feature is not captured by the model since it assumes linear wave excitation only, and demands further investigation on the nonlinear evolution of mirror modes and their conditions for marginal stability, in order to provide a more precise instability threshold. The discrepancy between the PIC simulations and the model may be evidence for a wave physics effect, such as a nonlinear damping mechanism, such that a higher level of plasma anisotropy is required to maintain a steady state than predicted by linear theory.

The evolution of the energy gain $\Delta U$ is also consistent with our PIC runs, and heating is retained in each pump cycle, as evident in Figure 13(c) (solid black curve). In the first half of the cycle, we can see that the energy gain increases following a CGL evolution until the anisotropy $A(t)$ is limited by the action of $\nu_\perp$ ($t \cdot s = 0.5$), after which it continues to increase but at a lower rate, given that $A(t)$ is now pinned to the mirror threshold $\xi_m/\beta$ at a lower amplitude. The energy gain in our PIC runs evolves similarly, but the change in slope is less pronounced, as the anisotropy saturates at a value not very far from the overshoot (see dashed yellow curve in Figure 13(b)). This also produces a slight difference in the energy gained by $t \cdot s = 1$. In the second half of the cycle, the energy gain evolves very similarly to the PIC run; the difference both curves show at $t \cdot s = 1$ is somewhat compensated by a larger cooling produced in the PIC run, given the larger amplitude at which the anisotropy saturates (see Figure 13(b)). Finally, by the end of the cycle, the energy gain from the model reaches a very similar value to the PIC run.

7. Summary and Conclusions

In this work, we have studied how ions and electrons can be heated by magnetic pumping in a weakly collisional, high-$\beta$ plasma subject to a magnetic pumping driving configuration, where the scattering process is provided by mirror and oblique firehose fluctuations that grow from instabilities driven by pressure anisotropy $\Delta P = P_i - P_e$ in the plasma. By performing 2D PIC simulations of a periodically sheared domain we were able to self-consistently excite these instabilities during several pump cycles, allowing a detailed study of their linear and nonlinear stages and their respective roles in the heating mechanism. Our simulation parameters are given in Table 1. Because the plasma is not compressed, the heating is provided entirely by shear. However, heating by magnetic pumping has also been demonstrated in a compressing plasma at lower beta (Sironi & Narayan 2015; Ley et al. 2019), so we do not expect the restriction to pure shear to be a fundamental requirement.

When initially $\beta^{\text{init}} = 20$, the magnetization $\omega^{\text{init}}_{ci}/s = 800$, and $k_BT_i/m_ee^2 = 0.1$, we saw that the plasma can effectively retain about 40% of the energy transferred during half a shear cycle in a CGL evolution ($t \cdot s = 1$; solid gray curve in Figure 2). This corresponds to $2 \pm 3$ times the rate that holds for marginally stable anisotropy with no overshoot. The efficiency of the heating is not strongly dependent on $m_i/m_e$, but does show a dependence on magnetization $\omega^{\text{init}}_{ci}/s$; larger magnetization provides less heating per shear cycle. Physically, the dependence of the heating efficiency on magnetization is directly related to the evolution of the pressure anisotropy $\Delta P$ throughout the cycle, and consequently on the evolution of mirror and firehose instabilities. At the lowest magnetization we studied ($\omega^{\text{init}}_{ci}/s = 200$), $\Delta P/P$ overshoots the mirror instability threshold by more than 50%, but the overshoot decreases with increasing magnetization. It is expected, however, that for large magnetizations, even with a very small overshoot, the heating reaches a nonzero value given by the marginal stability values at which the anisotropy should pin to.

For the range of magnetizations we studied, the heating rate scales with shear frequency $s$ as approximately $s^{0.3}$ in the case of ions and $s^{0.2}$ in the case of electrons (Figure 11).

On the other hand, simulation at lower beta (Section 5.3) with higher mirror thresholds indicate that at too low a magnetization the instability will not have time to develop at all, implying that there is a particular value of $\beta$ and shear-amplitude-dependent magnetization at which the heating is maximized.

The excitation and evolution of both mirror and firehose instabilities are essentially consistent with theoretical expectations in the kinetic regime, in both linear (Pokhotelov et al. 2004) and nonlinear stages (Rincon et al. 2015), and with previous studies of fully kinetic and hybrid simulations (Matteini et al. 2006; Hellinger & Trávníček 2008; Kunz et al. 2014; Hellinger & Trávníček 2015; Riquelme et al. 2015; Sironi & Narayan 2015; Melville et al. 2016; Riquelme et al. 2018). In the first half of the pump cycle ($0 < t \cdot s < 1$), we saw that mirror modes grow to relatively large amplitudes, have low frequencies, and $kR_L \sim 1$ initially. After reaching nonlinear amplitudes, we also saw the appearance of short-wavelength, parallel propagating modes in regions of low magnetic field with frequencies $\omega \sim 0.1\omega^{\text{crit}}_{ce}$ and $kR_L \sim 1$, consistent with the nature of whistler ion roars (Baumjohann et al. 1999; Breuillard et al. 2018). The effect of these modes on the global evolution and the nature of their interaction with the particles (e.g., resonant particle acceleration; Riquelme et al. 2017; Ley et al. 2019) is deferred to future investigations (F. Ley et al. 2023, in preparation).

During the nonlinear mirror stage, the pressure anisotropy is regulated and maintained at a relatively constant amplitude above the marginally stable value, at least for the range of magnetizations studied here. In the second half of the pump cycle ($1 < t \cdot s < 2$), we saw the rapid excitation of low-frequency, oblique firehose modes also with $kR_L \sim 1$ that quickly regulate the pressure anisotropy, maintaining it close to marginally stable values. Each subsequent cycle presents essentially the same evolution.

Even though the interaction of mirror and oblique firehose modes with the particles can become quite complex, especially in their nonlinear stages, we showed that a simplified model in which the effective pitch-angle scattering frequency only depends on the growth rates of mirror and firehose modes when the approximate instability thresholds are met (similar to the bounded anisotropy model from Hellinger & Trávníček 2008) accurately reproduces the heating rate seen in our simulations. However, the model fails to capture the incomplete relaxation of the pressure anisotropy to the linear instability threshold during the mirror-dominated phase. This is evidence for an unidentified, possibly nonlinear damping process for the mirror modes, such that supercritical pressure anisotropy is needed to maintain them.
Lichko et al. (2017) have shown that a magnetic pumping configuration similar to\textsuperscript{10} the one studied in this work can create nonthermal tails in the distribution after several pump cycles, and their growth can be enhanced in the presence of particle trapping (Lichko & Egedal 2020). Nonthermal tails appear on our simulations, as well, and will be explored in forthcoming work (F. Ley et al. 2023, in preparation).

In the context of the ICM of galaxy clusters, we can expect to have a fully developed turbulent cascade with a large range of frequencies evolving simultaneously, possibly fed at high frequencies by cosmic rays. If we associate our local shear frequency $s$ with the frequency of the local turbulent motion, we can think of several channels of magnetic pumping heating acting together, with different frequency-dependent efficiencies. Assuming that an anisotropy overshoot can be developed by any variation of the local magnetic field by an ensemble of large-scale turbulent eddies, our results could constitute the building blocks to construct a plausible channel for turbulent dissipation in the ICM (Arzamasskiy et al. 2022).

In this study, however, our results regarding the evolution of electrons are not complete, as our initial temperatures are too high to have a direct application to a nonrelativistic environment such as the ICM, especially for simulations with higher mass ratios. Electrons can also excite their own, electron-scale instabilities that can in turn interact with the electron population, contributing to the pitch-angle scattering, and they are not fully captured here.

We emphasize that the parameters considered here represent tiny scales. For example, at a magnetization of 800, if $B = 0.1 \, \mu G$, the time to complete a full pump cycle ($t \cdot s = 2$) is only $1.7 \times 10^6$ s. There is probably little turbulent power at such scales; otherwise, the heating rate could easily become too large. In future work, we plan to integrate our results with plausible turbulence models to produce a full theory of heating by magnetic pumping in the ICM.

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\textsuperscript{10} Lichko et al. (2017) used compression instead of shear, but other works have found little difference between these two types of pumping (Ley et al. 2019).

Appendix

Numerical Heating

The inherent discreteness of the macroparticles in PIC simulations introduces finite electric field fluctuations that can interact with the particles and affect the evolution of their physical properties. In our simulations, this effect introduces a numerical heating source in both ions and electrons that is comparable to their energy gain in a single pump cycle. In general, this numerical effect can be reduced by significantly increasing the number of particles per cell, $N_{\text{ppc}}$, quickly making the simulations prohibitively expensive given our computational resources, especially for larger mass ratios $m_i/m_e$ and magnetizations $\omega_{ci}/s$. In this section, we characterize this numerical heating in terms of its dependency on numerical and physical parameters and describe the method we applied to subtract it from the energy gain of each species.

A.1. Weak Shear Simulations

The characterization of the numerical heating was done by performing “weak shear” simulations, where the initial configuration is the same as the runs presented in previous sections (see Figure 1(a)) but now reversing the shear at a much earlier time, $\tau_s = 0.1 \, s^{-1}$ (see Section 3), so the variation in magnitude and direction of the magnetic field $B$ is so small that no significant pressure anisotropy is developed, no instability is excited, and therefore the system does not exhibit any of the dynamics presented in the previous sections but the underlying numerical heating. This way, we can isolate the effect that the numerical heating produces upon the system.

Figure 14 shows the evolution of several physical quantities in one weak shear simulation, in which the reversal time is $\tau_s = 0.1 \, s^{-1}$ (run Eb20m2w200 in Table 1). We can see in Figure 14(a) that the energy in all components of the magnetic field fluctuations $\delta B$ stays constant at a low level throughout the simulation, so we confirm that no instability is developed. Similarly, in Figure 14(b) we see that the anisotropy $\Delta P/P_\parallel$ exhibits periodic variations due to the shear reversals, but always consistent with a double-adiabatic evolution (dashed purple line). It also never surpasses the threshold for mirror instability, consistent with the evolution of the magnetic field fluctuations. We can see the evolution of the energy gain $\Delta U_j(t) = U_j(t) - U_j(0)$ ($j = i, e$) for ions and electrons in solid red lines in Figures 14(c) and (d), respectively, as well as the integrated magnetic pumping heating rate for each species, shown in solid green lines. Consistent with the double-adiabatic evolution of the pressure anisotropy, the expected energization of ions and electrons by magnetic pumping evolves as periodic phases of heating and cooling but ultimately leading to zero net heating. Notwithstanding the above, the ion and electron energy gains clearly exhibit a steady growth that, in the absence of any other physical heating source, reveals the action of numerical heating. In both Figures 14(c) and (d), the dashed black lines show the best linear fit to $\Delta U_j \approx a_j(t s) + b_j$. These best-fit relations are what we will be using to subtract the numerical heating from the particle energy gain in the rest of the simulations presented in this work.

A.2. Dependency on Physical and Numerical Parameters

We have shown that the numerical heating in our simulations grows linearly with time. Consequently, more heating will be
accumulated for longer simulations. This is the case for higher mass ratios \(m_i/m_e\) and higher magnetizations \(\omega_{ci}/s\).

Figure 15 shows a comparison between the evolution of the ion and electron energy gains between two different weak shear runs (Eb20m2w200 and Eb20m2w800 in Table 1) with \(m_i/m_e = 2\), \(\tau_s = 0.1 \text{ s}^{-1}\), and different magnetizations, \(\omega_{ci}^\text{init}/s = 200\) (red line) and \(\omega_{ci}^\text{init}/s = 800\) (green line). In this case, the time shown in the horizontal axis is not normalized by their respective shear frequency \(s\) but it shows the number of time steps in the simulation. In terms of normalized time, both runs Eb20m2w200 and Eb20m2w800 lasted until \(t \cdot s = 1\). With this choice, we can explicitly see that, even though the \(\omega_{ci}^\text{init}/s = 800\) run is \(\sim 4\) times longer, in both runs the numerical heating acts at a similar rate.

The numerical heating behaves similarly also for different mass ratios, as shown in Figure 16. In this case, we show the ion and electron energy gains for two weak shear runs (Eb20m2w800 and Eb20m8w800 in Table 1) now with \(\omega_{ci}^\text{init}/s = 800\), \(\tau_s = 0.1 \text{ s}^{-1}\), and different mass ratios, \(m_i/m_e = 2\) and \(m_i/m_e = 8\). We can see that both ions and electrons are numerically heated at a very similar rate, as well, indicating that numerical heating is comparable among simulations with different mass ratios.

A.3. Numerical Heating Subtraction

Having characterized the nature of numerical heating in our simulations, we now show how it can be subtracted from the total energy gain in our periodic shear simulations. Given the linear evolution of the ion and electron energy gain by pure numerical heating, for every periodic shear run we show in this work we performed a corresponding weak shear run with the same physical parameters and do a linear fit to the particle energy gain, similar to Figures 14(c) and (d). We then use the best-fit parameters thus obtained to subtract the numerical heating from the total energy gain in the periodic shear runs.

The results of this procedure are described in Figure 17 for ions (left panel) and electrons (right panel) for run Zh20m2w200. In both panels the dashed red line shows the total energy gain of the particles, including the contribution from numerical heating. The solid purple line shows the linear best fit to the energy gain by pure numerical heating from the corresponding weak shear run Eb20m2w200 (dashed black lines from Figures 14(c) and (d)). The numerical heating is then subtracted from the total particle energy gain using this linear fit, the result of which is shown as a solid green line. We can see that it exhibits a better agreement with the integrated magnetic pumping heating rate (dashed black lines).
Figure 15. Panels (a) and (b) show, respectively, the ion and electron energy gain by the action of numerical heating as a function of time steps (not normalized) in the simulation for two runs with the same $m_i/m_e = 2$ and magnetizations $\omega_{ci}/s = 200$ (Eb20m2w200, red line) and $\omega_{ci}/s = 800$ (Eb20m2w800, green line).

Figure 16. Panels (a) and (b) show, respectively, the ion and electron energy gain by the action of numerical heating as a function of time steps (not normalized) in the simulation for two runs with the same $\omega_{ci}/s = 800$ and mass ratios $m_i/m_e = 2$ (Eb20m2w800, red line) and $m_i/m_e = 8$ (Eb20m8w800, green line).

Figure 17. The ion and electron energy gain for run Zb20m2w200. Upper row: the dashed–dotted red line shows the evolution of the total ion energy gain including numerical heating. The solid purple line shows the linear best fit to the ion energy gain by pure numerical heating in a weak shear simulation (run Eb20m2w200; see Figures 14(c) and (d)). The solid green line shows the subtraction of the numerical heating from the total energy gain using the linear best fit. Finally, the dashed black line shows the integrated ion magnetic pumping heating rate. Bottom row: same as upper row but for electrons.
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