The normal-state properties of underdoped high-$T_c$ cuprates are dominated by a gap in the density of states known as the pseudogap (PG)\cite{1,2}. Its origin is unknown. Unlike a superconducting gap which closes at $T_c$, angle-resolved photoemission spectroscopy (ARPES) and tunneling in underdoped cuprates have long suggested that the PG fills as $(1 - T/T^*)$ and disappears abruptly at $T^*$\cite{3,4}. If so then the lost low-energy spectral weight would be restored at $T^*$ with important thermodynamic consequences, as we shall see. The gap energy, $E_g$, and $T^*$ decrease with increasing doping, falling to zero at a critical doping $p = p_{crit} \approx 0.19$ holes per Cu\cite{2}. Generally $T^*$ is much greater than $T_c$ but when $p$ exceeds $\approx 0.16$ then $T^*$ falls below $T_c$\cite{5}.

Recent ARPES studies\cite{6,7} suggest that for $T < T^*$ the PG covers only part of the Fermi surface (FS) near the $(\pi,0)$ zone boundary. This leaves ungapped arcs on the FS ("Fermi arcs") which grow with increasing $T$. The arcs extend to an angle $\theta_0$ given by $\theta_0 = \frac{\pi}{4}(1 - T/T^*)$, where $\theta_0$ is measured from $(\pi,0)$. The gap is thus nodal at $T = 0$ and, with increasing $T$, retreats towards $(\pi,0)$ where it closes abruptly at $T^*$. Assuming that the PG continues to exist as the underlying normal state below $T_c$, the $T$-dependent restoration of the pristine FS would have important testable consequences, as follows:

(i) if the PG retreats abruptly to $(\pi,0)$ at $T^*$ the specific heat coefficient, $\gamma$, will exhibit an anomaly. The area under the anomaly is exactly equal to the restored entropy. We calculate this below for several Fermi arc scenarios and show this is not observed in the experimental data;

(ii) if the Fermi arcs grow as $T$ rises then the spectral weight taken up by the PG falls. So the superfluid density would first increase, then fall as $T$ approaches $T_c$, contrary to the observed monotonic decrease\cite{5};

(iii) and if the PG closes at $T^*$ then in the doping range $0.16 < p < 0.19$, where $T^* \leq T_c$, the PG should be open at $T = 0$ but closed when $T = T_c$. Again this is not observed, as we show in Fig. 1. Here $T_c(p)$ and $T^*(p) = E_g/2k_B$ are plotted for Bi$_2$Sr$_2$CaCu$_2$O$_8+\delta$ (Bi-2212). Panel (a) shows two properties at $T_c$: the jump in $\Delta \gamma$ and the electronic entropy $S(T_c)$. Panel (b) shows two ground state $T = 0$ properties: the superfluid density $\lambda_0^2$ and $x_{crit}$, the critical density of Zn required to suppress $T_c$. The data is from Tallon et al.\cite{9}. The two vertical dashed lines indicate where $T^* = T_c$ and $p_{crit} = 0.19$ where the PG closes.

FIG. 1: Thermodynamic data for Bi-2212 (a) two properties at $T_c$: the jump in specific heat coefficient $\Delta \gamma$ and the electronic entropy $S(T_c)$ in mJ/g.at.K; and (b) two properties at $T = 0$: the superfluid density $\lambda_0^2$ and $x_{crit}$ the critical Zn concentration giving $T_c = 0$. Vertical dashed lines indicate where $T^* = T_c$ and $p_{crit} = 0.19$ where the PG closes.
up on the Fermi arcs thereby obscuring the details of how the PG further evolves with \( T \), and in particular whether the PG itself becomes nodal at \( T = 0 \). Raman scattering allows the PG and SC gaps to be separately probed via the antinodal \((B_{1g})\) and nodal \((B_{2g})\) Raman response\(^{10, 11, 12}\). We calculate these for Bi-2212 using an ARPES derived energy-momentum dispersion. By comparing with Raman data we are able to confirm that the PG does not evolve significantly below \( T_c \).

We then turn to the second issue as to how the PG evolves near \( T^* \). Does it really close at \( T^* \) as widely believed? We use the same dispersion to calculate the specific heat and show that the Fermi arc model leads to a large anomaly in \( \gamma(T) \) at \( T^* \) that is not observed. We go on to suggest these issues may be resolved by incorporating quasiparticle (QP) lifetime broadening.

We employ the six parameter tight-binding Bi-2212 dispersion, \( \epsilon(k) \), reported by Norman \textit{et al.}\(^{13}\) and assume a rigid single-band approximation. Inclusion of band splitting\(^{14}\) will not significantly affect the following results. We take the Fermi level to be 34meV above the \( (\pi, 0) \) van Hove singularity (vHS) near optimal doping\(^{13}\) and 96meV above the vHS in an underdoped sample with \( p = 0.11\).\(^{15}\) We interpolate between for intermediate doping levels. The imaginary part of the unscreened non-resonant Raman response at \( T = 0 \) is given by\(^{16}\)

\[
\chi_0''(q = 0, \omega) = \int d\varepsilon \frac{\omega - 2E(k)}{(2\pi)^2} |\Delta(k)|^2 |\gamma(k)|^2
\]

where the integral is over occupied states below \( E_F \), \( \Delta(k) = \frac{1}{2} \Delta_0 (\cos k_x - \cos k_y) \) is the \( d \)-wave SC gap function and \( E(k) = \sqrt{\epsilon(k)^2 + |\Delta(k)|^2} \). In the \( B_{1g} \) scattering symmetry \( \gamma(k)^{B_{1g}} = \gamma B_{1g} (\cos k_x - \cos k_y) \), giving a dominant response from the antinodal sections of the FS. For \( B_{2g} \), \( \gamma(k)^{B_{2g}} = \gamma B_{2g} \sin k_x \sin k_y \) and the response is mainly nodal. The magnitude of the SC gap, \( \Delta_0 \), is taken from the weak-coupling result \( 2\Delta_0 = 4.28 k_B T_c \) and \( T_c \) is given by the empirical relation\(^{17}\)

\[
\frac{T_c}{T_{c,\text{max}}} = 1 - 82.6 (p - 0.16)^2
\]

We adopt a PG of the form

\[
E_g = \begin{cases} 
E_{g,\text{max}} \cos \left( \frac{2\pi \theta}{\theta_0} \right) & (\theta < \theta_0) \\
E_{g,\text{max}} \cos \left( \frac{2\pi (\theta - \pi/2)}{\theta_0} \right) & (\theta > \frac{\pi}{2} - \theta_0) \\
0 & \text{otherwise}
\end{cases}
\]

where \( 0 \leq \theta \leq \pi/2 \). We initially assume

\[
\theta_0 = \frac{\pi}{4} \left( 1 - \frac{T}{T^*} \right) \quad (T < T^*)
\]

and \( T^* = E_{g,\text{max}}/k_B \). This form of the PG is illustrated in Fig. 2(c) and (d). Eqn. 3 models the linear \( T \)-dependence of the Fermi arc length inferred from ARPES\(^{18}\). At \( T = 0 \), \( \theta_0 = \pi/4 \) and the PG is fully nodal. As \( T \) rises, \( \theta_0 \) decreases resulting in a ‘filling-in’ of the PG and the growth of the Fermi arcs. The PG is a non-states-conserving gap\(^{18}\) i.e. unlike the SC gap there is no pile up of states outside the gap. This is implemented by removing states with \( \epsilon(k) < E_g \) from the integration in Eqn. 1.

The doping dependence of \( E_g \) is obtained from the reported leading-edge ARPES gap at 100K\(^{2}\).

Figure 2(a) and (b) show the nodal \((B_{2g})\) and antinodal \((B_{1g})\) Raman response for six dopings spanning the range 0.12 to 0.19. We consider two scenarios:
(i) Firstly, we have assumed that the length of the Fermi arc becomes fixed at the onset of superconductivity, implemented by setting $T = T_c$ in Eqn. 3. Fig. 2(a) shows the nodal ($B_{2g}$) response. Leaving aside the anomalous electronic Raman continuum above the pair-breaking gap, the calculations closely resemble the recent results of Le Tacon et al.\cite{11} which are reproduced in Fig. 3. The PG peak maximum in the $B_{1g}$ response shifts monotonically to higher energies with decreasing doping. Simultaneously the intensity of this peak rapidly reduces with underdoping. In contrast, the SC peak maximum in the $B_{2g}$ response is found to shift to lower energies in the underdoped regime. The magnitude of the $B_{2g}$ peak persists relatively undiminished down to the lowest doping levels. Also reproduced is the increased linear slope of the response at very low doping.

(ii) Secondly, we show in Fig. 4 the Raman response in the alternative case where we have assumed that the Fermi arcs continue to collapse below $T_c$. This is done setting $T = 0$ in Eqn. 3 resulting in a fully nodal PG. In this case the $B_{2g}$ peak shifts monotonically to higher energies with decreasing doping and the intensity reduces rapidly. This behavior is not observed experimentally. We cannot say that the Fermi arc freezes exactly at $T_c$ but our analysis indicates that it remains finite at $T=0$ and evolves only weakly below $T_c$.

We turn now to the question as to how the PG evolves above $T_c$ as $T \rightarrow T^\ast$. It is widely assumed that the PG closes at $T^\ast$, as indeed Eqn. 3 suggests, thus exposing a pristine FS. In fact, Kanigel et al.\cite{7} suggest that a discrete jump in $\theta_0$ occurs at $T^\ast$ so that the PG retreats to the flat sections of the FS near $(\pi,0)$ from whence it abruptly disappears at $T = T^\ast$. This model is given by

$$\theta_0 = \begin{cases} \frac{\pi}{4} \left( 1 - 0.68 \frac{T}{T^\ast} \right) & (T < T^\ast) \\ 0 & (T \geq T^\ast) \end{cases}$$

(4)

and is illustrated in Fig. 2(d). The closure of the PG, whether according to Eqn. 3 or Eqn. 4 will restore the entropy to the bare-band value. This, quite generally, will result in a $\gamma$ anomaly, the area of which equals the restored entropy. The lower is $T^\ast$ the greater is the anomaly. We evaluate this here.

Using the method described previously\cite{8}, together with the above tight-binding dispersion, we have computed $\gamma(T)$ for three cases with $p \approx 0.138$. In Fig. 5 we compare these with experimental data for Bi-2212. The three cases are shown in Fig. 2(d). They are (i) the linear behavior described by Eqn. 3, (ii) the sudden jump in $\theta_0$ inferred by Kanigel et al. and described by Eqn. 4 and (iii) the smooth evolution of the gap given by Eqn. 5:

$$\theta_0 = \frac{\pi}{4} \left( 1 - \tanh \left( \frac{T}{T^\ast} \right) \right)$$

(5)

As can be seen, cases (i) and (ii) lead to substantial anomalies in $\gamma$ which are clearly not found experimentally. The experimental data, shown by the data points, are from a previous study on the specific heat of Bi-2212\cite{18} with the closest corresponding doping state. The smooth evolution of the PG given by Eqn. 5 satisfactorily describes the data and generally discounts the possibility that the PG closes abruptly at $T^\ast$.

We have so far shown that (i) below $T_c$, the Fermi arcs do not continue to shrink notably; and (ii) above $T_c$ the Fermi arcs cannot spread out to abruptly form a connected pristine FS at $T^\ast$. (iii) Both results are confirmed by the data in Fig. 4 in the interesting case where $T^\ast$ lies below $T_c$. These problems seriously prejudice the current picture of Fermi arcs but could be resolved as follows by invoking a $T$-dependent scattering rate.

Firstly, one is led to this view by the observations of Norman et al.\cite{19}. They modelled the QP peak using a QP self energy with a scattering rate or inverse lifetime $\Gamma_0$ which, in underdoped samples, grows with $T$.

FIG. 4: Calculated Raman response at $T = 0$ with a fully nodal PG for (a) nodal $B_{2g}$ and (b) antinodal $B_{1g}$ symmetry. The intensity scale is the same for all plots.

FIG. 5: Specific heat coefficient calculated for $\theta_0$ given by Eqns. (3), (4) or (5) and compared with the experimental data for Bi-2212\cite{18} where every 20th data point is shown.
FIG. 6: Simulated symmetrized ARPES quasiparticle EDCs ranging from the antinode to the node (θ = 0 to 45°). The broadening is fixed at 0.45E_g(0). The crossover from double to single peaks (bold curve) presents an apparent, though false, closing of the PG and recovery of Fermi arcs. Inset (a) true normalized gap and apparent gap for two broadenings (= 0.45 × and 0.6 × E_g(0)). (b) curve: apparent Fermi arc length (FAL) assuming broadening ∝ T/T*; data: from ref. [7]. And equals the gap magnitude precisely at T* (see Fig. 2(b) of ref. [19]). This would lead to a smearing out of the gap at E_F resulting in a single peak in the dispersion at E_F that looks like a recovered pristine FS. But of course the gap is still there, and interestingly is reported to be T-independent, just as we concluded above.

In Fig. 6 we have simulated the symmetrized QP EDCs as the sum of two Lorentzians. We use a fixed broadening of 0.45E_g(0), corresponding to a particular temperature below T*. The k-dependent gap value, E_g(θ), is assumed to be d-wave-like with E_g(θ) = E_g(0) cos(2θ). Fifteen angles ranging from θ = 0 (the antinode) to θ = 45° (the node) are shown and the data qualitatively reproduces the reported experimental data [7, 19]. These EDCs reveal a crossover from double to single peaks, with the flat bold curve lying at the boundary. Within the Fermi arc model this would be interpreted as a closure of the PG at θ = 30° with a pristine Fermi arc extending from θ = 30° to 45°. But, as shown in inset (a), the true gap does not close until the node at θ = 45°. The apparent gap, found by reading off the peak positions, is also plotted in inset (a) and this falls to zero at θ = 30°. We also show the apparent gap for a larger broadening (= 0.6E_g(0)) which would correspond to a higher temperature, closer to T*. Here the apparent gap closes at 23°.

Inset (b) in Fig. 6 shows the apparent (though fictitious) Fermi arc length obtained from these simulated EDCs when the broadening Γ = Γ*(T/T*)*, where Γ* is the critical value that “closes” the gap at the antinode. The arc length exhibits a rapid change at T* just like the data of Kanigel et al. [7] which is also plotted. This rapid change arises from the flat part of the d-wave gap when θ → 0 and does not signal an abrupt recovery of the FS.

Thus QP lifetime broadening with a d-wave gap accounts for all apparent Fermi arc features, including the abrupt jump in arc length reflected in Eqn. 4. We expect therefore that the T-variation of the PG is not in Fermi arcs but in the scattering rate. Both the PG and the SC gap parameters should be replaced by complex terms of the form E_g(θ) = \frac{E_g(θ)}{1-iΓ/ε(θ)} and Δ(θ) = \frac{Δ(θ)}{1-iΓ/ε(θ)}.

This naturally leads to a “U-shaped” gap [20], and instead of frozen Fermi arcs below T_c the Raman data would then insist on a frozen scattering rate.

In summary, we have used an ε(k) dispersion and a model for the normal-state PG, both based on ARPES results, to show that the shrinking Fermi arc picture is inconsistent with Raman data below T_c and thermodynamic data near T*. Only by freezing the length of the Fermi arcs at or near T_c do we find that the calculations mimic the experimental data. This implies that the PG does not evolve greatly below T_c. We have further shown that closure of the PG at T* and the associated recovery of a pristine FS would lead to a large specific heat anomaly that is not observed. We thus question the concept of Fermi arcs and suggest that they are probably an artifact of a T-dependent lifetime scattering rate.

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