Theory Perspective: SCES ’05 Vienna

P. Coleman
Center for Materials Theory, Rutgers University, Piscataway, NJ 08855, U.S.A.

Abstract

Over the past decades, research into strongly correlated electron materials that has consistently outperformed our wildest expectations, with new discoveries and radically new theoretical insights. SCES ’05 reaffirmed this vitality. Highlight areas included parity violating superconductivity, a new, two dimensional Helium-3 “Kondo lattice” and the discovery of quantum critical ferroelectricity. I review these and many other developments, with an emphasis on open questions and prospects for the future.
I. EXTRAORDINARY FIELD

Strongly Correlated Electron Systems, 2005 (SCES05) was held in Vienna. This classical city, with its legendary coffee houses (many just outside the conference venue), proved a superb setting to review and discuss the latest discoveries in this active area of materials and condensed matter physics research.

SCES05 is a direct descendant of the first International Conference on Valence Fluctuations, held in Rochester New York in 1977[1]. The consistency with which the discovery of new materials and new concepts has outstripped expectations is simply extraordinary. In 1977, the concept that localized f-electrons could form heavy bands was treated with considerable skepticism and the possibility of electronically mediated superconductivity in dense local moment systems was regarded as highly radical! Our meeting in Vienna celebrated 30 years since these seminal discoveries, yet it was also a forward looking conference, with a host of incredible new discoveries. I came away from the meeting feeling like a kid, with a huge sense of excitement about the prospects for the future.

Strongly correlated electron physics is a field never far from applications, and strong links between research and the development of rare earth magnets, thermo-cooling, cryogenics, multiferroics and applied superconductivity continue today. We often forget however the equally important role of this field of physics as frontier science in the “middle ground” of our quantum universe. The domain of experiments reported at this conference spans a temperature range from room temperature to millikelvin, and within this playground, we have the opportunity to explore the general principles that govern collective behavior in matter, often using discoveries made on one scale, to understand the physics on another. For example, discoveries made in actinide and rare earth “heavy electron physics” have had broad influence on our understanding of magnetism and high temperature superconductivity in more complex oxide materials, yet they have also influenced research into quantum dots and mesoscopics. But the discoveries and ideas of strongly correlated electron physics also enjoy an influence that stretches far further along the energy axis, on one side out to the physics of the early universe, neutron stars, quark gluon plasmas and anisotropic color superconductivity, and on the other side, down to the low energy physics of Helium-3 and the tiny energy scales of atom traps (Fig. 1). Today, experimental discovery and theoretical ideas developed through strongly correlated electron systems continue to enjoy a strong
mutual influence with these kindred fields of research.

FIG. 1: Strongly correlated behavior of matter and its relationship to the research presented at this meeting (starred). Although the range of energy where research is carried out spans from roughly $10^4 K$ to $10^{-3} K$, ideas and research on strongly correlated electron systems in the lab have a broad connection with physics on far lower, and far greater energy scales.
II. NEW QUANTUM PHASES OF MATTER: \( URu_2Si_2, CePt_3Sn \) AND BILAYER \( ^3He \)

One of the themes of this field, is the quest to discover new quantum phases of matter and realize them in practical materials. Along the way, we need to learn the guiding principles that help us to navigate amongst the vast space of the periodic table. There were many new discoveries of new phases and materials reported at this meeting, and I’d like to highlight three of them.

- \( URu_2Si_2 \), where a “nexus” of new phases appear to cluster around a field-induced quantum critical point\cite{2, 3, 4, 5, 6}.
- \( CePt_3Si \) a broken parity superconductor, containing a coherent admixture of singlet and triplet Cooper pairs\cite{7, 8, 9, 10},
- Bilayer \( ^3He \) on graphite, which enters the scene as a new class of two dimensional heavy fermion fluid\cite{11, 12}.

A key observation of experimentalists, is that new phases of matter tend to accumulate in the vicinity of quantum critical points. The heavy electron superconductor \( URu_2Si_2 \) provides a fascinating realization of this phenomenon. Kim, Harrison et al.\cite{3} report on a plethora of new phases that develop in \( URu_2Si_2 \) upon application of high magnetic fields (Fig. 2). They have discovered that the high field specific heat of this material (doped with rhodium to suppress some of the order around the critical point) closely resembles that observed near a metamagnetic phase transition in \( CeRu_2Si_2 \). Metamagnetism involves a sudden cross-over magnetization from unpolarized to fully polarized Fermi liquid. Studies of the quasi two dimensional material \( Sr_3Ru_2O_7 \) suggest that metamagnetism takes place in materials that lie close to the critical end-point of a line of first order “ferromagnetic” phase transitions\cite{13}. A new analysis of Harrison et al. provides further support for the idea that quantum critical end points can act as nucleating point for new quantum phases.

I was also excited by new progress towards identifying the the “hidden order” (HO) that develops in this \( URu_2Si_2 \) below 17K\cite{14}. It is this order which is persists up to the high fields where nexus of new phases begin. Three new pieces of insight about the HO were announced at Vienna. It is now increasingly clear that the HO state is distinct from a large moment antiferromagnetic phase that phase-coexists with the hidden order under pressure.
FIG. 2: Nexus of critical points in $URu_2Si_2$ determined from a combination of magneto-caloric and resistivity measurements[2]. Fig. courtesy of N. Harrison. Color coding is determined by the resistivity. The extrapolation of $T^*$ to zero (red circle) is thought to be a quantum critical end point associated with a metamagnetic transition. Phase (I) is the “hidden order” phase of $URu_2Si_2$. Phase (IV) is a fully polarized Fermi liquid.

At this meeting, Sato et al.[4] showed measurements of the susceptibility under pressure which indicate that heavy fermion superconductivity is unique to the hidden order phase, and does not develop in the large moment antiferromagnet. Oscar Bernal et al. presented[5] NMR measurements on single crystal samples of this material that confirm a growth of tiny local fields inside the hidden order phase that had previously been seen in powder experiments[15]. Remarkably, these local fields - about 4-8 Gauss, do not change in a 14.5 Tesla field. This field-insensitivity adds support to the idea that these tiny fields derive from staggered orbital currents rather than from spin dipoles. In a separate development, Zhitormsky and Mineev[6] described how, in their recent work on the band-theory of $URu_2Si_2$ they have been able to newly identify nested regions of the Fermi surface, which are strong candidates as regions for the development of the hidden order. Zhitormsky and Mineev propose a novel kind of spin order, with a highly renormalized $g-$ hidden order. But whether the hidden order is spin, or orbital order, the identification of this nested Fermi surface as the origin of the Fermi surface gapping that takes place at $17K$ is an important step forward in this twenty year old mystery.
FIG. 3: Sketch of the temperature dependence of $1/T_1 T$ (normalized with respect to its value at $T_c$) for the parity violating heavy electron superconductor, $CePt_3Si$ after [16].

One of the new developments in superconductivity concerns $CePt_3Si$ [7, 8, 9, 10]. This new heavy fermion superconductor is part of a growing trend of condensed matter interest in materials that break parity inversion symmetry. $CePt_3Si$ has a non-centrosymmetric crystal structure, so that unlike most superconductors, the parity of the Cooper pairs is not a good quantum number [8]. The pair correlation function in a superconductor can be written

$$F_{\alpha\beta}(x) = \langle \psi_\alpha(x)\psi_\beta(0) \rangle$$

(1)

where

$$F_{\alpha\beta}(x) = \left[ i\sigma_2(F_s(x) + \vec{\sigma} \cdot \vec{F}_p(x)) \right]_{\alpha\beta}$$

(2)

is expanded in terms of a singlet, even-parity pair function $F_s(x) = F_s(-x)$ and a triplet, odd-parity pair function $\vec{F}_p(x) = -\vec{F}_p(-x)$. When parity is a good quantum number, either $F_s$ or $F_p$ are zero. In $CePt_3Si$, the absence of parity conservation implies that both singlet and triplet pairs coherently coexist. Experimental and theoretical work on this material has progressed at an impressive rate in the two years since its discovery. Mamoru Yogi presented new NMR data [9], which provides a striking confirmation of this co-existence of triplet and singlet pairs, exhibiting a small coherence peak characteristic of s-wave superconductivity,
slowly crossing over to a \( T^3 \) dependence at low temperatures that is characteristic of line
nodes around the Fermi surface (Fig. 3). Agterburg, Frigerie and collaborators have developed a phenomenological theory of this superconducting state which can account for
the NMR data. In their model, parity violation enters as a spin-dependent hopping.

The microscopic theory of heavy electron superconductors, is still a wide open question,
and \( CePt_3Si \) joins a long list of bad-actor superconductors, including \( UBe_{13}, CeCoIn_5 \) and
\( PuCoGa_5 \) for which there is almost no microscopic understanding. We do know that the
entropy associated with the Fermi liquid and the superconductor derives from f-spin entropy.
Heavy electron quasiparticles are really composite combinations of f-spins and conduction
electrons. The inclusion of this “Kondo physics” into heavy electron superconductivity is
a largely unexplored theoretical challenge. One of the paradoxes that needs to be explored
concerns the strong Coulomb repulsion between the f-electrons which we expect to severely
suppress the s-wave pairing component of the f-electrons, so that

\[
\langle f_{k\uparrow}^\dagger f_{-k\downarrow}^\dagger \rangle \sim 0
\]

In a Kondo lattice, this suppression is total, because of the local \( SU(2) \) gauge symmetry
associated with the f-spins. From this perspective, it is difficult to understand how
the pair-condensation entropy can be associated with s-wave pairing and f-electrons at the
same time, and this is an interesting paradox to motivate future work.

One of the most surprising discoveries announced at SCES05, is the discovery of heavy-
fermion behavior in bilayer \(^3\)He films on graphite. In the bulk, \(^3\)He is the historic
paradigm for Fermi liquid behavior and anisotropic superfluidity. Helium films adsorbed
on graphite form two dimensional quantum fluids, in which the Helium atoms move
coherently across the periodic triangular potential of the graphite surfaces. These are very
clean, model strongly correlated fermi systems, with no complications from crystal fields or
the separate electronic and nuclear contributions to the specific heat.

John Saunders described how his group had succeeded in measuring the properties
of bilayer \(^3\)He on graphite. In previous work, they established that monolayer \(^3\)He films
form a Fermi liquid which undergoes a Mott transition into a triangular lattice solid at a
critical filling factor \( n_{He} \sim 5nm^{-2} \). Using a slightly modified substrate, the same group
has now succeeded in adding more \(^3\)He atoms to form a bilayer quantum fluid. The new
bilayer system remains a Fermi liquid up to a much higher critical coverage around \( 9.9nm^{-2} \),
FIG. 4: (a) Schematic of bilayer $^3$He, showing almost localized lower layer of spins and delocalized upper “conduction sea”. Valence fluctuations between the two layers melt the lower layer to produce a two dimensional heavy fermion fluid. (b) Sketched evolution of $C_v/T$ as a function of filling, showing peak temperature $T_0$, and exponential decay $e^{-\Delta/T}$ that are used to define the characteristic energy scales. Inset: collapse of characteristic scales to zero at a quantum critical point where $n = 9.2\text{nm}^{-3}$.

where it undergoes a quantum phase transition into a magnetic state. Saunders notes that bilayer $^3$He may closely resemble a two-dimensional heavy fermion system, in which the lower layer of atoms is an almost localized spin system and the upper layer behaves as a conduction band. Zero point “valence fluctuations” between the layers may play an important role, causing spin exchange between the upper and lower fluid that closely resembles the physics of a two dimensional Kondo lattice (4. (a)). The observed specific heat curves show a maximum $T_0$ and appears to indicate the presence of an as-yet unexplained gap $\Delta$ in the excitation spectrum. As the filling of the second layer is increased, the inverse mass of the Fermi liquid appears to collapse linearly to zero as the critical coverage is approached, while $T_0$ vanishes with a power law. Mysteriously, the gap appears to drop to zero at a lower
value of the filling around $n_c \sim 9.2nm^{-2}$ (Fig. 4 (b)). This is an exciting new discovery that should be a great encouragement to the field, prompting new theoretical work and tending to support the idea that the physics of quantum criticality seen in electronic heavy fermion systems may in fact, be universal to a much broader class of fermionic matter.

III. THEORETICAL!

One of the dreams of numerical approaches, is to develop tools that on the one hand, can provide us with new insights into simple models but on the other hand, can guide us in the discovery of materials with new quantum properties. There is a strong tradition of creative numerical algorithms in the field of strongly correlated electrons, one that stretches back to Ken Wilson’s pioneering numerical renormalization group solution to the Kondo model[21]. This tradition continues today.

A modern descendant of Wilson’s numerical renormalization group approach is the density matrix renormalization group (DMRG)[22]. For one-dimensional models, this has become a state-of-the-art tool. An example of beautiful work in this direction, is provided by the DMRG study of the quarter filled, one dimensional Kondo lattice, by Hotta and Shibata[23]. Earlier DMRG work on this topic had proposed that at a quarter filling, the Kondo lattice becomes unstable to an insulating state, with with long-range staggered dimer order

$$\langle \vec{S}_j \cdot \vec{S}_{j+1} \rangle \sim \Psi \epsilon^{i\pi j}.$$  \hspace{1cm} (4)

Hotta and Shimizu have carefully studied the size dependence of dimer correlations using DMRG, triangulating their results by comparing even and odd-numbered chains. They conclude that the dimer order has power-law correlations, but no long range order. This work illustrates how, despite great advances, very careful scaling studies continue to be absolutely vital before drawing hard and fast conclusions about the thermodynamic limit.

Continued progress is also being made in the application of direct diagonalization approaches. One of the unsolved pieces of physics that challenges the theoretical community is the Doniach quantum phase transition that is believed to separate local moment magnetism, and heavy electron paramagnetism in the metallic Kondo lattice[24]. Zerec et al presented a finite temperature “Lanczos” diagonalization study of the two dimensional Kondo lattice[25], in which the temperature dependence of the specific heat shows a two-peak structure that
may signal the approach to the magnetic quantum critical point. I have the sense that these approaches will be amongst the first to provide us with the first detailed phase diagram of the two dimensional Kondo lattice.

One area of immense interest, is the application of dynamical mean field theory (DMFT) to strongly correlated systems. Dynamical mean field theory\cite{26} regards strongly correlated electron systems as an effective impurity, or cluster of atoms, embedded in a self-consistently determined environment. This approach is in many ways, the 21st century descendant of density-functional theory, but now, the Free energy is now a functional of the local, or cluster approximated Green’s functions. The challenge becomes to efficiently compute the functional of the local or cluster Green’s functions.

Many interesting papers were presented which marry the dynamical mean field theory with local, or cluster “solvers” provided by a variety of other methods. For example, Koller and Hewson\cite{27} showed how the combination of DMFT, with the use of numerical renormalization group to provide the impurity solver, could provide new insight into strong polaronic renormalization of electron masses in a Holstein-Hubbard model.

In an innovative piece of work, Hanke, Aichorn, Dahnken, Arrigoni and Potthoff\cite{28} demonstrated how they can extract the functional dependence of the free energy on cluster Greens functions from a Monte Carlo study of clusters. The function is then used to compute the translationally invariant lattice Green’s functions.

Dieter Vollhardt\cite{29} showed how the older density functional theory can be combined with the DMFT, and illustrated the tremendous potential of these hybrid methods for the study of metal-insulator transitions in vanadium oxide systems, notably $\text{SrVO}_3$. Vollhardt showed how these methods were able to track both the incoherent local background, and the dispersive component of the electron spectral functions, which contained “kinks” in the dispersion, corresponding to mass-renormalization effects. Frontier research of this sort is, I believe, slowly providing the framework for ab-initio many body studies of the future.

IV. NEW PHENOMENOLOGY

A vital middle ground for research in our field, is the development of new, phenomenological approaches. Condensed matter physics is driven largely by experimental discovery. Phenomenology is not only a stepping stone to deeper theoretical insight, it is also the key
to a closer interplay between theory and experiment.

Two interesting pieces of phenomenology caught my eye at SCES05. Achim Rosch\textsuperscript{30} discussed “Grüneisen” parameter $\Gamma_T$ can be used to characterize pressure or field-tuned quantum phase transitions. The Grüneisen parameter is the ratio of the thermal expansivity $\beta = V^{-1}\partial V/\partial T$ to the specific heat $C_P$

$$\Gamma_T = \frac{\beta}{C_P}. \quad (5)$$

Near a quantum critical point, the characteristic scale of the Fermi liquid $T_0(x) \sim x^{\nu z}$, collapses to zero as power of the separation $x$ from the critical point. Simple scaling analyses show that the collapse of this scale to zero leads to a divergence of the temperature dependent Grüneisen parameter at a quantum critical point, given by $\Gamma_T \sim 1/T^{1/\nu z}$\textsuperscript{31}. Rosch showed how degree of divergence of the Gruneissen parameter is an important method to delineate between “local” and “spin density wave” magnetic quantum critical points. Moreover, the Grüneissen parameter should should change sign at a quantum critical point, making it an extremely useful tool for hunting for quantum phase transitions.

One of the most useful unifying trends in strongly correlated electron systems is the Kadowaki Woods relation\textsuperscript{32}

$$\frac{A}{\gamma^2} = W \sim 1 \times 10^{-5}\mu\Omega\text{cm} \left(\frac{K\text{mol}}{m\text{J}}\right)^2 \quad (6)$$

between the quadratic “$A$” coefficient of the resistivity ($\rho = \rho_0 + AT^2$) and the specific heat coefficient $\gamma = C_V/T$. The constancy of this quantity over a wide range of strongly correlated materials is a reflection of the local character of the quasi-particle interactions. Tsujii et al.\textsuperscript{33} have made the observation that several $Yb$ compounds with a large orbital degeneracy, develop a consistently lower Kadowaki Woods ratio. In a lovely piece of phenomenology, in which they analyze the dependence of the Fermi liquid properties on spin degeneracy $N$, Tsujii et al find that both $A$ and $\gamma$ scale as $N(N - 1)^2$\textsuperscript{34},

$$A \propto \left(\frac{N(N - 1)/2}{T_0^2}\right), \quad \gamma \sim \left(\frac{N(N - 1)/2}{T_0}\right), \quad (7)$$

where $T_0$ is the characteristic scale of the Fermi liquid, so that the Kadowaki Woods relation becomes

$$\frac{A}{\gamma^2} \times \frac{N(N - 1)}{2} = W \sim 1 \times 10^{-5}\mu\Omega\text{cm} \left(\frac{K\text{mol}}{m\text{J}}\right)^2 \quad (8)$$
This degeneracy dependence accounts for the low KW ratio in Yb compounds, and when taken into account, brings them into line into align with the broad majority of lower spin degeneracy materials.

There were so many fascinating discoveries I encountered in the meeting that seem prime fodder for future phenomenological work. Here are three phenomena that may fall into this category:

- Gegenwart pointed out a fascinating property of the Fermi liquid ground-state of YbRh$_2$Si$_2$, as it is field tuned towards the quantum critical point $^{35, 36}$. Although the specific heat coefficient $\gamma = C_V/T$, $\chi$ and the quadratic coefficient of the resistivity all diverge, it is the modified Kadowaki Woods ratio $A/\chi^2$ rather than $A/\gamma^2$ that remains constant. What simple model for the momentum-dependent interactions in a Fermi liquid model can account for this scaling?

- At the metal to Kondo-insulator phase transition observed by Slabarski and Spalek $^{37, 38}$ in tin doped CeRhSb, the conductivity $\sigma$ scales with the magnetic susceptibility $\sigma/\chi = \text{cons.}$ Why?

- In the heavy electron superconductor CeCoIn$_5$, a Fulde-Ferrell-Larkin-Ovchinikov (FFLO) phase, (in which the superconducting order parameter is spatially modulated) is observed at low temperatures, near the Pauli-limited upper critical field $^{39, 40, 41, 42}$. As the field is rotated away from the c-axis, a sequential heirachy of FFLO phases develop $^{39, 41}$, manifested by a sequence of plateau’s in the magnetization. This physics does not fit into the existing understanding of FFLO phases, and it may involve the stabilization of vortex structures through the development of antiferromagnetism in the vortex cores. Traditionally, FFLO phases are treated using non-uniform solutions of the Boguilubov de Gennes equation $^{43}$, but the interplay with magnetism makes such an approach far too complicated for these systems. Would it be possible, I wonder, to develop a Landau Ginzburg theory for FFLO phases that can take antiferromagnetic vortex cores into account?
V. CHARGE AND QUANTUM CRITICALITY.

Quantum criticality was a very central topic at this meeting. Zachary Fisk has likened quantum critical points to a kind of black-hole - an essential singularity located at absolute zero in the material phase diagram\[44, 45\]. We’re fascinated by quantum criticality in part because

- as we saw in the case of $URu_2Si_2$, materials quantum critical points provide a kind of nucleation point for the formation of new phases of quantum matter\[2, 13\].

- quantum criticality influences a broad swath of the material phase diagram, leading, to the formation of metals with very unusual transport properties - variously refereed to as “strange”, or “non-Fermi liquid” metals\[46\].

- of the possibility of new universality classes of critical behavior that lie outside those previously observed at classical critical points.

SCES ’05 introduced a number of fascinating new factors into quantum criticality. There were two new examples of quantum criticality that involve charge, rather than spin order:

1. Quantum critical ferro-electricity\[47\].

2. Critical valence fluctuations and their possible relation with unconventional superconductivity.

In a post-deadline poster, Stephen Rowley et al.\[47\] described the measurement of a divergence of the dielectric constant in strontium titanate that follows a $1/T^2$ behavior, rising up to values in excess of $\chi = 30,000$ in the approach to a ferro-electric quantum critical point (Fig.3). Strontium titanate may offer the possibility of a text-book quantum phase transition, because there are no complicating effects of electron damping in insulators. In this system, the temporal quantum fluctuations of the electric polarization should count as one additional dimension, so the quantum critical behavior in strontium titanate should be equivalent to an Ising phase transition in four-dimensions. This is the marginal, or “upper critical” dimension for the Ising model. Rowley et al point out that the $1/T^2$ divergence may be understood in terms of Gaussian, quantum fluctuations in the dipole moments that exist beyond four dimensions, but that there may be important marginal logarithmic corrections associated with the critical dimensionality.
FIG. 5: Temperature dependence of the dielectric constant of $SrTiO_3$ after Stephen Rowley et al.\cite{47}, showing (a) cross-over from $1/2$ Curie behavior at high temperature to $1/T^2$ divergence at low temperature (b) small low temperature maximum that believed to be associated with a first order transition.

Kazumasa Miyake\cite{48} advanced a fascinating proposal that the superconducting phase diagram (Fig. 6) of $CeCu_2(Ge, Si)_2$ exhibits a maximum in the transition temperature in close vicinity to a valence-changing critical point\cite{49}. Finite temperature critical end-points associated with first order valence changing phase transitions, such as the $\alpha - \gamma$ transition in elemental cerium, been known for decades. Miyake argues that superconductivity may develop around the region where the critical end point is suppressed to zero, to become a quantum critical point.

To these new developments, I should also add the recent discovery of a charge Kondo effect in Thallium doped $PbTe$ by a group at Stanford\cite{50,51}, where charge $2e$ fluctuations of the thallium ions give rise to a charge scattering analog of the Kondo effect. This is the charge $2e$ companion to the phenomena discussed by Miyake. These new developments make me believe that we have underestimated the importance of slow charge fluctuations at quantum critical points, and that charge will play an increasingly important role in our consideration of quantum criticality in the years ahead.
FIG. 6: Schematic phase diagram for $CeCu_2(Ge, Si)_2$ after [48] illustrating possible valence fluctuation critical point beneath the superconducting dome at high pressures.

There are substantial challenges that still face us, in trying to understand magnetic quantum critical points. The idea of a phase transition between the heavy electron paramagnet and the local moment antiferromagnet dates back to Doniach’s original proposal [24] at the Rochester conference, that heavy electrons are dense Kondo lattice systems. Despite the vintage of this idea, we still do not have a theory for how zero-point spin fluctuations melt antiferromagnetic order in a metal. Indeed, our understanding of such quantum phase transitions is far better evolved for insulating antiferromagnetic order, than it is for metallic antiferromagnets. At this meeting, Qimiao Si [52] emphasized how many of the properties of quantum critical heavy electron systems can not be understood in terms of a Hertz and Moriya spin fluctuation picture of quantum phase transitions [53, 54]. Si, Ingersent, Smith and Rabello [55] have proposed that the bad-actor heavy electron quantum critical points are quasi-two dimensional spin fluids in which the important critical spin fluctuations are critically correlated in time. Local quantum criticality is certainly the most mature theory of heavy electron quantum criticality that we have at the current time. Many other ideas have also been advanced as contenders. One of the ideas floating around the community, is that the composite electrons which form the heavy electron fluid may break into spinons and holons at the quantum critical point. This has been an idea explored by Catherine Pepin in her recent work [56]. The idea of deconfined spinons also features in the ideas of “deconfined criticality” advanced by Senthil et al. [57]. None of these approaches is yet able
to provide a clear idea of how to connect the a state of three dimensional magnetic order to
the three-dimensional heavy electron paramagnet.

My own belief is that progress on quantum criticality may benefit by partially refocusing
our effort towards developing a mean-field theory to connect local moment magnetism with
the heavy fermion metal. Just as Landau and Weiss mean theory of ferromagnetism provided
the key insights into the soft modes of classical criticality, I suspect that a proper mean-field
theory linking the Kondo lattice to local moment magnetism will provide key insight into
the unusual zero modes responsible for the quasi-linear resistivity $\rho \sim T^{1+\eta}$ and the unusual
logarithmic temperature dependence of the specific heat coefficient $C_v/T \sim (T_0/T) \ln(T_0/T)$
seen in heavy electron criticality\[46].

The discovery of heavy fermion behavior in bilayer $^3He$ announced at this conference[11]
provides tremendous new impetus to theoretical studies of heavy fermion criticality. Here
we have an absolutely two dimensional heavy fermion system to which we may compare our
theories. Bilayer $^3He$, is in essence, a localized layer of Heisenberg spins, coupled vertically
via a Kondo coupling to an upper layer of mobile fermions. One of the challenges this poses,
is to connect the phase diagram of the two dimensional quantum antiferromagnet, with the
two dimensional Kondo lattice (Fig. 7).

There are two ways for zero point quantum fluctuations to melt the magnetic order in a
quantum antiferromagnet:

- by increasing the zero-point fluctuations of the spin (by reducing the size $S$ of the spin
  or increasing the magnetic frustration), or

- by coupling the spin fluid via a Kondo coupling to a two-dimensional fluid of fermions.

These two processes define two orthogonal axes of a phase diagram, as shown in Fig. 7.
Along the “frustration (x-) axis” that loosely defines insulating antiferromagnets, there is a
quantum phase transition from a spin liquid to an ordered two dimensional antiferromagnet.
This quantum phase transition has been extensively studied, and is the source of inspiration
for “deconfined quantum criticality” advanced by Senthil et al. as a theory for critical
behavior which is dominated by the formation of a fluid of deconfined spinons[57]. Along
the Kondo (y-) axis, there is the quantum phase transition from antiferromagnet to heavy
electron Fermi liquid. A working mean-field theory would show us how these two phase
transitions are connected. For highly frustrated, low spin lattices, we will also have to
understand how a gapped spin liquid can make a transition to the heavy Fermi liquid, and whether, as seems the case in both MnSi and UCoAl, there is an intermediate non-Fermi liquid ground-state separating the antiferromagnet from the fully developed Fermi liquid.

There are some interesting technical practical problems to be solved to actually realize this phase diagram in a controlled calculation. One important aspect to the problem, is finding a way to control the fluctuations to form a mean-field theory. In classical criticality, this kind of control was obtained by generalizing the three component magnetization to an $N$ component order parameter, the so called “spherical model”, where the $O(3)$ rotation group is replaced by the $O(N)$ rotation group. In heavy fermion physics, we have yet to find the appropriate large $N$ expansion to described the phases on both sides of the quantum critical point. For a long time, our field has followed the example of the particle physics, in trying to use the $SU(N)$ group for this purpose. The problem with $SU(N)$ group, is that singlets are baryons, and they require complexes of $N$ particles,

$$|\text{singlet SU (N)}\rangle = f_1^\dagger f_2^\dagger \ldots f_N^\dagger |0\rangle$$
But in condensed matter physics, the formation of singlets between pairs of particles - electrons in superconductors, spins in antiferromagnets, plays a very important role. An important group for our purposes, may well be the group $SP(2N)$, which preserves the special relationship between up and down electrons. In SP $(2N)$, the spin labels on the states are given by $(\uparrow, a)$ and $(\downarrow, a)$, where $a$ runs from 1 to $N$, and a pair operator still defines a singlet

$$|\text{singlet SP (2N)}\rangle = \sum_{a=1,N} f^{\dagger}_{\uparrow,a} f^{\dagger}_{\downarrow,a} |0\rangle$$

One of the nice properties of this group, is that it preserves a local $SU(2)$ particle-hole gauge symmetry, yet surprisingly, this symmetry has not yet been applied to the Kondo impurity, or Kondo lattice problems.

Another important technical problem, is how to tune $S$, as one does in insulating antiferromagnets, while maintaining a perfectly screened Kondo lattice. This part of the problem was fortunately solved some years ago by Parcollet and Georges, who pointed out that that one could consider a family of Kondo lattice models of different spin $S$, in which each spin is coupled to $2S$ screening channels. The “mean-field theory” that appears in this kind of Eliashberg or dynamical mean field theory. Recent results of Jerome Rech at Rutgers, and my collaborators Gergely Zarand and Olivier Parcollet indicate that it may be possible to exactly compute the mean-field phase diagram in the large $N$ limit using this scheme, using Schwinger bosons. Schwinger bosons are an effective method to describe low dimensional antiferromagnetism, and our most recent work confirms that they can also describe the fully screened Fermi liquid physics of the one and two impurity Kondo model. One of the exciting aspects of this kind of approach, is that the initial mathematical formulation involves a gauge theory of electrons interacting with spinons, which carry the local moment spin, and charged spinless fermions, or “holons” which mediate the Kondo interaction. Remarkably, when one solves the equations for the fully screened Kondo model, one finds that spinon and holons develop a gap in the Fermi liquid. Antiferromagnetism corresponds to the condensation of spinons, and when this happens, the gap for both holons and spinons must collapse suggesting a kind of critical behavior featuring a co-existence of critical spin and charge fluctuations (see Fig.7). Taken seriously, this appears to predict that a heavy electron Fermi liquid near to a quantum critical point may have new, low lying spinon and holon excitations. We hope to report on these results in future meetings.
VI. POSTSCRIPT.

I came to my first conference as a new Ph.D., twenty one years ago, in Cologne 1984, just as the ideas of heavy electron bands, and heavy electron superconductivity were becoming accepted phenomena. I was excited, had the sense that realistically, we’d probably had our fair quota of big discoveries. Had anyone tried to convince me that d-wave superconductivity at $135K$ superconductivity, quantum dots, routine experiments at $10^{-6}K$ in things called atom traps, and a completely new class of phase transition, would be part of the near future, I would certainly have dismissed it as wonderful science fiction! Yet all of this happened in the past twenty years. So perhaps you today - as a student are wondering, if now, finally, the big discoveries are past?

No! Absolutely not! Today, the reasons for optimism about research in strongly correlated electrons are at least as bright as twenty years ago, and if the past is any lesson for the future, major discoveries - both experimental and theoretical, will come that will continue to outstrip our expectation. I think think as experimentalists and theorists, especially the students in the audience - we can not be too ambitious with our research at this time. It is still an exciting time, and I wish you good spirits and good luck as you return to your home institutions.

Acknowledgement

This research was supported by the National Science Foundation, USA, grant DMR-0312495. I am indebted to John Saunders, for discussions about his bilayer $^3He$ data, and to Neil Harrison and Jaime Moreno for providing figure 2. at very short notice. Thanks to Stephen Julian for drawing the possible non-Fermi liquid phase of $UCoAl$ to my attention.

[1] Valence Instabilities and Narrow Band Phenomena, edited by R. Parks, (Plenum 1977).
[2] K. H. Kim, N. Harrison, M. Jaime, G. S. Boebinger and J. A. Mydosh, Phys. Rev. Lett. 91, 256401 (2003)
[3] N. Harrison, A. V. Silhanek, C. D. Batista, M. Jaime, A. Lacerda, H. Amitsuka, J. A. Mydosh, Th-FHO-5, p. 104.
[4] N. K. Sato, S. Uemura, G. Motoyama and T. Nishioka, Th-FHO-10, p 170.

[5] O. Bernal, M. E. Moroz, H. Murukawa, A. P. Reyes, P. L. Kuhns, D. E. MacLaughlin, H. G. Lukefahr, J. A. Mydosh, T. J. Gortenmulder and H. Amitsuka, Th-FHO-9, p 170.

[6] M. E. Zhitormsky and V. P. Mineev, Th-FHO-6, p. 105.

[7] E. Bauer et al, Physical Review Letters, 92, 027003/1-4, (2004).

[8] Daniël Agterburg, Raminder P. Kaur, Manfred Sigrist, Paolo Frigeri and Akihisa Koga, Th-SCO-6, 98.

[9] Mamoru Yogi, Hidekazu Mukuda, Yoshio Kitaoka, Shi Hashimoto, Takashi Yasuda, Rikio Setta, Tatsuma D. Matsuda, Yoshimori Haga, Yoshichika Onuki, Peter Rogl and Ernst Bauer, Th-SCO-7, p 99.

[10] P. A. Frigeri, M. Sigrist and D. Agterburg, Fr-NSC-35, p 272.

[11] M. Neumann, Andrew Casey, Jan Nyéki, Brian Cowan and John Saunders, "3He Films as Model Strongly Correlated Fermion Systems: observation of a magnetic quantum critical point", Fr-HF-5, (191).

[12] A. Casey, H. Patel, J. Nyéki, B. P. Cowan, and J. Saunders, Phys. Rev. Lett. 90, 115301/1-4 (2003).

[13] R. S. Perry, A. J. Schofield, A.J., M. Chiao, S. R. Julian, G. G. Lonzarich, S. Ikeda, Y. Maeno, A. J. Millis, and A. P. Mackenzie, Science 294, 329 (2001).

[14] V. Tripathi, P. Chandra and P. Coleman, J. Phys.: Condens. Matter 17, 5285 (2005).

[15] O.O. Bernal, C. Rodrigues, A. Martinez, H. G. Lukefahr, D. E. MacLaughlin, A. A. Menovsky, and J. A. Mydosh, Phys. Rev. Lett. 87, 153, (2001).

[16] M. Yogi, Y. Kitaoka, S. Hashimoto, T. Yasuda, R. Setta, T. D. Matsuda, Y. Haga, Y. Onuki, P. Rogl, and E. Bauer Phys. Rev. Lett. 93, 027003 (2004).

[17] I. Affleck, Z. Zou, T. Hsu and P. W. Anderson, “Local SU(2) symmetry of the Heisenberg model”, Phys. Rev. B 38 754-759 (1988).

[18] P. Coleman and N. Andrei, J. Phys. Cond. Matt C 1., 4057-4080 (1989).

[19] L. D. Landau, “Theory of Fermi Liquids”, Zh. Eksperim. i Teor. Fiz. 30, 1058 (1956); [English transl: Soviet Phys. [JETP 3, 920 (1956)].

[20] D. S. Greywall and P. A. Busch, Phys. Rev. Lett. 62, 1868-1871 (1989).

[21] K. G. Wilson, Rev. Mod. Phys. 47, 773, (1976).

[22] Steven R. White, Phys. Rev. Lett. 69, 2863 (1992).
[23] Chisa Hotta and Naokazu Shibata, Fr-HF-36, p 209.

[24] S. Doniach, Valence Instabilities and Narrow Band Phenomena, edited by R. Parks, 34, (Plenum 1977); S. Doniach, Physica B91, 231 (1977).

[25] Iviva Zerec, Burkhard Schmidt, Peter Thalmeier and Peter Fulde, Fr-HF-32, p 207.

[26] A. Georges, G. Kotliar, W. Krauth, and MJ Rozenberg, Rev. Mod. Phys. 68, 13 (1996).

[27] Winfried Koller, Alex Hewson, David Edwards and Dietrich Meyer Fr-HF-29, p205.

[28] Werner Hanke, Markus Aichorn, Christopher Dahnken, Enrigo Arrigoni and Michael Potthoff, W2-SCE1-2, p 11.

[29] D. Vollhardt, WE-SCE1-1, p10.

[30] Achim Rosch, W2 QC-3, p5.

[31] Lijun Zhu, Markus Garst, Achim Rosch and Qimiao Si, Phys. Rev. Lett. 91, 066404 (2003).

[32] K. Kadowaki and S. B. Woods, Solid State Commun. 58, 507 (1986).

[33] N. Tsujii, H. Kontani and K. Yoshimura Fr-HF-54, p218.

[34] N. Tsujii, H. Kontani and K. Yoshimura, Phys. Rev. Lett. 94, 057201 (2005).

[35] Frankziska Weickert, Julia Ferstl, Teodara Radu, Philipp Gegenwart, Christoph Geibel and Frank Steglich, We-QC-11, p 14.

[36] P. Gegenwart, J. Custers, Y. Tokiwa, C. Geibel, and F. Steglich Phys. Rev. Lett. 94, 076402 (2005).

[37] Andrzej Slebarski and Joze Spalek, We-NFL-7, p 41.

[38] A. Slebarski and J. Spalek, Phys. Rev. Lett. 95, 046402 (2005).

[39] H. A. Radovan, N. A. Fortune, T. P. Murphy, S. T. Hannahs, E. C. Palm, S. W. Tozer and D. Hall, Nature 425, 51-55 (2003).

[40] A. Bianchi, R. Movshovich, C. Capan, A. Lacerda, P. G. Pagliuso and J. L. Sarrao, Phys. Rev. Lett 91 ,187004/1-4, (2003).

[41] H. A. Radovan, Th-SCO-3, p 97.

[42] K. Kumagai, H. Kakuyanagi, M. Saito, K. Kumagai, S. Takashina, M. Nohara, H. Takagi, Y. Matsuda, Th-SCO-4, p97.

[43] See e.g. S. Matsuo, Y. Nagato and K. Nagai K., “Phase diagram of the Fulde-Ferrell-Larkin-Ovchinnikov state in a three-dimensional superconductor” Journal of the Physical Society of Japan, 67, 280, (1998).

[44] Piers Coleman and Andrew J. Schofield, Nature 433, 226-229 (2005).
[45] R. B. Laughlin, G. G. Lonzarich, P. Monthoux and D. Pines, “The quantum criticality conundrum” Advances in Physics, 50, 361-5 (2001).

[46] P. Coleman, P., C. Pépin, C., Q. Si and R. Ramazshvili., J. Phys.: Condens. Matter 13, R723-R738 (2001).

[47] Stephen E. Rowley, S.S. Saxena and G. G. Lonzarich, post-deadline poster.

[48] Kazumasha Miyake, Th-P-1, p 95.

[49] A. T. Holmes, D. Jaccard and K. Miyake, “Signatures of valence fluctuations in CeCu2Si2 under high pressure”, Phys. Rev. B 69, 024508 (2004).

[50] Y. Matsushita, H. Bluhm, T. H. Geballe, and I. R. Fisher, “Evidence for Charge Kondo Effect in Superconducting Tl-Doped PbTe”, Phys. Rev. Lett. 94, 157002/1-4 (2005).

[51] M. Dzero and J. Schmalian, “Superconductivity in Charge Kondo Systems”, Phys. Rev. Lett. 94, 157003/1-4 (2005)

[52] Qimiao Si, We-P-1, p 3.

[53] J. A. Hertz, Phys. Rev. B 14, 1165 (1976).

[54] T. Moriya and J. Kawabata, J. Phys. Soc. Japan 34, 639 (1973); J. Phys. Soc. Japan 35, 669 (1973).

[55] Q. Si, S. Rabello, K. Ingersent and J. L. Smith, Nature

[56] C. Pépin, Phys. Rev. Lett. 94, 066402 (2005)

[57] T. Senthil, Ashvin Vishwanath, Leon Balents, Subir Sachdev, M. P. A. Fisher, Science 303, 1490-4 (2004).

[58] N. Doiron-Leyraud, I. R. Walker, L. Taillefer, M. J. Steiner, S. R. Julian, G. G. Lonzarich, Nature 425, 595 (2003).

[59] A. J. Bograd, J. S. Alwood, D. J. Mixon, J. S. Kim and G. R. Stewart, www.phys.ufl.edu/REU/2002/reports/bograd.pdf unpublished.

[60] T. H. Berlin and M. Kac, “The Spherical Model of a Ferromagnet” Phys. Rev. 86, 821-835 (1952).

[61] K. G. Wilson, Phys. Rev. D 7, 2911 (1973).

[62] Subir Sachdev and N. Read, “Large N expansion for frustrated and doped quantum antiferromagnets”, International Journal of Modern Physics B 5, 219 (1991).

[63] O. Parcollet and A. Georges, PRL 79, 4665-8 (1997).

[64] J. Rech, P. Coleman, O. Parcollet and G. Zarand, Phys. Rev. Lett. in press (2005).
[65] P. Coleman, I. Paul and J. Rech, Phys. Rev. B 72, 094430 (2005).