A Distributed Online Algorithm for Promoting Energy Sharing Between EV Charging Stations

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Abstract—In recent years, electric vehicle (EV) charging stations have experienced an increasing supply-demand mismatch due to their fluctuating renewable generation and unpredictable charging demands. To reduce their operating costs, this paper proposes a distributed online algorithm to promote energy sharing between charging stations. We begin with the offline and centralized version of the EV charging stations operation problem, whose objective is to minimize the long-term time-average total cost. Then, we develop an online implementation approach based on the Lyapunov optimization framework. Although the proposed online algorithm runs in a prediction-free manner, we prove that by properly choosing the parameters, the time-coupling constraints remain satisfied. We also provide a theoretical bound for the optimality gap between the offline and online optimizers. Furthermore, an improved alternating direction method of multipliers (ADMM) algorithm with an iteration truncation is proposed to enable distributed computation. The proposed algorithm can protect privacy while being suitable for online implementation. Case studies validate the effectiveness of the theoretical results. Performance comparisons are carried out to demonstrate the advantages of the proposed method.

Index Terms—Electric vehicle, charging station, energy sharing, Lyapunov optimization, renewable energy.

NOMENCLATURE

Acronyms

ADMM Alternating direction method of multipliers.
CS Charging station.
EV Electric vehicle.
MPC Model predictive control.
PV Photovoltaic.
SOC State of charge.

Symbols

\( \alpha_i \) Unsatisfied charging demand coefficient.
\( \beta_{d,i} \) Ratio to restrict the demand shedding.
\( s(t) \) System state vector.
\( u(t) \) Control decision variable vector.
\( \delta \) Accuracy decision vector.
\( \Delta(\Theta(t)) \) One time slot Lyapunov drift.
\( \eta_c, \eta_d \) Charging/Discharging efficiency.
\( I_i, i/j \) Set and index of charging stations.
\( T, t \) Set and index of time slots.
\( V, v \) Set and index of EVs.
\( \rho \) Penalty parameter.
\( \Theta(t) \) Concatenated vector of virtual queues.
\( \theta_i(t) \) Perturbation parameter of battery energy queue in charging station \( i \).
\( \varepsilon_{i,j} \) Auxiliary variable.
\( B_i(t), H_i(t) \) Virtual queues for battery energy and charging demand shedding in station \( i \).
\( C(t) \) Total operation cost.
\( \hat{e}_g, e_g \) Electricity buying/selling price from/to the grid.
\( c_{b,i} \) Degradation coefficient of battery in charging station \( i \).
\( C_g,i \) Charging station \( i \)'s energy trading cost with the grid.
\( C_{sh,i} \) Energy sharing cost of station \( i \).
\( c_{sh} \) Energy sharing price.
\( d_{i,j} \) Dual variable.
\( E_{b,i}^{\min}, E_{b,i}^{\max} \) Minimum/Maximum value of \( E_{b,i}(t) \).
\( E_{b,i}(t) \) Battery storage energy in time slot \( t \).
\( e_{i,j}(t), \hat{e}_{i,j}(t) \) Energy that charging station \( i \) shares with charging station \( j \) in time slot \( t \), and final shared energy after update.
\( k_i \) Preset iteration threshold.
\( L(\Theta(t)) \) Lyapunov function.
\( p_{g,i}(t), p_{g,i}^d(t) \) Energy bought/sold from/to the grid by charging station \( i \).
\( p_{b,i}(t), p_{b,i}^d(t) \) Battery storage discharging/charging energy of charging station \( i \).
\( p_{d,i}^{\max}, p_{c,i}^{\max} \) Maximum value of \( p_{d,i}(t)/p_{c,i}(t) \).
\( p_{d,i}^{\min}, p_{c,i}^{\min} \) Minimum/Maximum value of \( p_{d,i}(t)/p_{c,i}(t) \).
\( p_{d,i}(t) \) Dispatched charging demand of charging station \( i \) in time slot \( t \).
\( p_{i,v}(t) \) EV charging energy in time slot \( t \).
\( p_{pv,i}(t) \) PV generation of charging station \( i \).
\( r_{i,v}(t) \) Demand shedding ratio of charging station \( i \) in time slot \( t \).
\( soc_{i,y}(t) \) EV initial SOC in time slot \( t \).
\( soc_{i,v}^{\text{ini}}, soc_{i,v}^{\text{req}} \) EV initial SOC and required SOC.
I. INTRODUCTION

Driven by the increasing number of electric vehicles (EVs), massive EV charging stations need to be built to meet the growing charging demand [1]. The charging stations can equip with renewable generation (e.g., photovoltaic (PV)) and battery energy storage to reduce the operating cost [2]. The flexible EV charging demands and renewable generation allow charging stations to play an important role in the future energy systems. Meanwhile, charging station changes to be an energy prosumer from a pure consumer. The power supply-demand mismatch caused by intermittent renewable generation and stochastic EV charging load poses new challenges to its operation [3]. It is crucial to develop an effective method to mitigate the mismatch of supply and demand and improve its self-consumption. Considering that different EV charging stations have distinct supply and demand patterns, energy sharing between them to make use of their complementary features has been considered as a promising solution [4].

Recently, there have been extensive studies on energy sharing in the literature. An energy trading strategy was developed for interconnected microgrids in [5], [6]. The microgrid with surplus renewable energy can trade with others in need of energy to gain mutual benefits. An energy sharing framework designed for an energy building cluster was studied in [7] considering the building thermal dynamics. A leader-follower game was used to model the interaction between an energy sharing provider and prosumers in a peer-to-peer (P2P) market [8]. The energy sharing model was extended to multi-energy systems in [9]. Generalized demand function based energy sharing schemes were developed for node-level prosumers [10] and networked prosumers [11], respectively, which were proved to have nearly social optimal economic efficiency. To encourage energy sharing between various prosumers, Nash bargaining based model [12] and mid-market ratio mechanism [13] were explored to allocate the payments among participants. The above studies adopt deterministic approaches without uncertainties. Some work considering uncertainties has been carried out. For example, stochastic programming was used to treat the uncertain PV generation, load, and electricity price in the day-ahead energy sharing decision-making stage [14]. A two-stage robust model for energy sharing was developed [15]. However, the above work usually operated in an offline manner, used predetermined time-of-use prices, and assumed complete information of the future realization of uncertainties. In practice, those data might be unavailable or inaccurate, and thus, the obtained solution might fail to adapt to the changes in real-time.

Various online optimization based methods have been used for real-time implementation. A common practice is the greedy algorithm which is shortsighted because it directly decomposes the time accumulated problem into each single time period ignoring the time-coupling constraints [16]. Another method is the model predictive control (MPC). An online MPC based optimal charging strategy was proposed for multiple charging stations to minimize the utility cost [17]. However, MPC still relies on a short-term forecast and the result is affected by the forecast accuracy [18]. Moreover, the rolling optimization can be computational expensive. An alternative real-time scheme is based on the Lyapunov optimization that takes the long-term benefit into account while requiring no prior knowledge of uncertainty [19]. Its decision-making only depends on the current state which is more flexible and practical. The Lyapunov optimization based real-time scheme has been widely applied in many fields such as online network resource allocation [20] and energy management in data center [21] and microgrid [22]. There are several studies related to energy sharing. For example, Lyapunov optimization was used to improve PV consumption of a cluster of nanogrids [23]. The energy trading between an end-user and the grid was optimally scheduled to maximize the profit via Lyapunov optimization [24]. However, the above studies were based on the centralized scheme with concerns about privacy and communication burdens. Distributed optimization methods, such as alternating direction method of multipliers (ADMM) [25], can help tackle these concerns. A real-time ADMM based algorithm was proposed to address the power balancing problem in a renewable-integrated power grid [26]. Reference [27] applied ADMM to operate a shared energy storage system for multiple users in real-time. An ADMM based online algorithm was developed to perform distributed energy management for multiple data centers [28]. However, the ADMM method may require a large number of iterations to converge. This may not be a big issue for the operation with a relatively long time scale. However, with the increasing penetration of renewable energy, the real-time operation runs in a smaller time resolution to cope with the volatility. Therefore, less and deterministic iterations are preferred, otherwise, the ADMM may not have enough time to converge. Motivated by the above discussions, a research gap can be found: an EV charging station energy sharing framework with online optimization to adapt to uncertainties, distributed implementation to protect privacy, and reduced iterations for fast execution has not been well explored yet.

In this paper, we propose a distributed online optimization for energy sharing between EV charging stations. Our main contributions are three-fold:

1) We propose an online implementation approach to manage the energy sharing between EV charging stations. First, an offline optimization that minimizes the long-term time-average total cost is formulated. To fit into the Lyapunov optimization framework, two virtual queues are introduced to turn the inter-temporal battery energy and charging demand dynamics into mean rate stable conditions. Then, an augmented objective function with a drift-plus-penalty term is built, based on which the online algorithm is derived. We prove that if the parameters are chosen within a particular range, the battery energy constraint is satisfied even if it is not explicitly considered. A theoretical bound for the optimality gap between the offline and online optimums is given. Compared with the offline approaches [7], [14] and the

\[ \text{Minimum/Maximum value of } \text{SOE}_{i,v}(t). \]

\[ t_{i,v}^{d}, t_{i,v}^{d} \]

Weight parameters.
In Fig. 1, an example is a university with a few neighboring community with a group of EV charging stations, as shown in [1160]. IEEE TRANSACTIONS ON SMART GRID, VOL. 14, NO. 2, MARCH 2023

The considered system is smaller than a node. Many other studies have focused on solving distribution network energy demand at a specific node of a city-sized distribution network. In this paper, we consider a residential area or an energy system of a distribution network. The total load of a university accounts only for a small fraction of the total demand at a specific node of a city-sized distribution network. Therefore, it is justifiable to neglect network constraints since the considered system is smaller than a node. Many other studies on charging station management, such as [5] and [7], also made a similar assumption and ignored network constraints. We consider a discrete time $t \in \mathcal{T} = \{1, \ldots, T\}$, and define $\mathcal{I}$ as the set of charging stations, each of which is indexed by $i \in \mathcal{I}$. For each charging station, its demand can be supplied by its own PV generation, battery storage, main grid, or the shared energy from neighboring charging stations.

II. MATHEMATICAL FORMULATION

A. System Overview

In this paper, we consider a residential area or an energy community with a group of EV charging stations, as shown in Fig. 1. An example is a university with a few neighboring buildings, each of which has a charging station. The total load of a university accounts only for a small fraction of the total demand at a specific node of a city-sized distribution network. Therefore, it is justifiable to neglect network constraints since the considered system is smaller than a node. Many other studies on charging station management, such as [5] and [7], also made a similar assumption and ignored network constraints. We consider a discrete time $t \in \mathcal{T} = \{1, \ldots, T\}$, and define $\mathcal{I}$ as the set of charging stations, each of which is indexed by $i \in \mathcal{I}$. For each charging station, its demand can be supplied by its own PV generation, battery storage, main grid, or the shared energy from neighboring charging stations.

B. Modelling of Charging Station

For each charging station, its modeling can be divided into two types: 1) Modeling of external energy flows, including energy trading with the main grid and energy sharing with neighboring stations; 2) Modeling of internal energy flows, including PV power generation, battery energy storage, and EV charging demand.

1) Energy Trading With the Main Grid: In addition to utilizing the energy from local PV, the charging station can purchase energy directly from the grid. When local PV generation exceeds the charging demand, the charging station can sell the surplus energy to the grid. We denote $p^{b}_{g,i}(t)$ and $p^{s}_{g,i}(t)$ as the purchasing and selling energy from/to the grid by charging station $i \in \mathcal{I}$ in time slot $t$. Both $p^{b}_{g,i}(t)$ and $p^{s}_{g,i}(t)$ should satisfy physical limits:

$$p^{b}_{g,i}(t) \geq 0, \quad p^{s}_{g,i}(t) \geq 0, \quad (1)$$

The energy trading cost of charging station $i \in \mathcal{I}$ in time slot $t$ incurred by trading with the main grid is:

$$C_{g,i}(t) = \left( p^{b}_{g,i}(t) c^{b}_{g}(t) - p^{s}_{g,i}(t) c^{s}_{g}(t) \right), \quad (2)$$

where $c^{b}_{g}(t)$ and $c^{s}_{g}(t)$ are the unit electricity purchase and sale prices, respectively; and $c^{b,\min}_{g} \leq c^{b}_{g}(t) \leq c^{b,\max}_{g}$, $c^{s,\min}_{g} \leq c^{s}_{g}(t) \leq c^{s,\max}_{g}$. In fact, electricity prices $c^{b}_{g}(t)$ and $c^{s}_{g}(t)$ are uncertain and only known in time slot $t$. Further, they should meet the following requirement

$$c^{s}_{g}(t) < c^{b}_{g}(t), \quad (3)$$

which can ensure that charging stations will not arbitrage by buying from and selling back to the grid at the same interval.

2) Energy Sharing With Other Charging Stations: The renewable power supply and charging demand of adjacent charging stations may differ greatly due to the different charging patterns and facility capacities. Therefore, charging station $i$ can share energy with its interconnected neighbor $j \in \mathcal{I}\setminus i$. Through energy sharing, charging stations with surplus energy can transfer part of the energy to other charging stations that lack energy. This can enhance the energy efficiency of the entire system and reduce the amount of energy bought/sold from/to the main grid.

Let $e_{i,j}(t)$ be the amount of energy that charging station $i$ shares with charging station $j$ in time slot $t$. $e_{i,j}(t) > 0$ means charging station $i$ purchases energy from $j$. Conversely, if $e_{i,j}(t) < 0$, it means that charging station $i$ sells energy to $j$. The energy sharing between charging stations should meet the following coupling constraint:

$$e_{i,j}(t) + e_{j,i}(t) = 0, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{I}, \forall j \in \mathcal{I}\setminus i. \quad (4)$$

In addition, $e_{i,j}(t)$ should be within the allowable range with minimum $e^{\min}_{i,j}$ and maximum $e^{\max}_{i,j}$:

$$e^{\min}_{i,j} \leq e_{i,j}(t) \leq e^{\max}_{i,j}, \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{I}, \forall j \in \mathcal{I}\setminus i. \quad (5)$$

To be fair, we have $e^{\min}_{i,j} = e^{\min}_{j,i}$ and $e^{\max}_{i,j} = e^{\max}_{j,i}$. The loss of energy during transmission is ignored as in [5]. To encourage energy sharing among charging stations, we assume that the trading price $c_{sh}$ for sharing energy is lower than the electricity prices $c^{b}_{g}(t)$ and $c^{s}_{g}(t)$.
purchasing price from the grid \( (e^p_g) \) and greater than the selling price \( (e^s_g) \), i.e.,
\[
e^s_g(t) < e^d(t) < e^p_g(t). \tag{6}
\]
Otherwise, charging stations have no incentive to participate in energy sharing. The cost of station \( i \) incurred by energy sharing in time slot \( t \) is calculated by
\[
C_{sh,i}(t) = \sum_{j \in \mathcal{I}} e_{i,j}(t)c_{sh}(t). \tag{7}
\]
Budget balance is achieved for the overall charging stations system, i.e., \( \sum_i C_{sh,i}(t) = 0 \), which is due to (4).

3) **Local PV Power Generation:** Compared with other renewable energy sources such as wind power, PV generation is noiseless, and PV panels can be easily deployed on the rooftop of charging stations. We let \( p_{pv,i}(t) \) denote the generated PV energy of charging station \( i \in \mathcal{I} \) in time slot \( t \). PV power generation is random and highly depends on the current solar irradiance. We assume no prior knowledge of \( p_{pv,i} \) nor its statistics. In addition, unlike conventional generators, PV power generation has a nearly zero cost.

4) **Battery Energy Storage:** Let \( p_{b,i}^d(t) \) and \( p_{b,i}^c(t) \) denote the battery storage discharging and charging energy of charging station \( i \in \mathcal{I} \) in time slot \( t \). \( p_{b,i}^d(t) \) and \( p_{b,i}^c(t) \) should meet the following physical constraints:
\[
0 < p_{b,i}^d(t) \leq p_{b,i}^{d,\text{max}},
\]
\[
0 < p_{b,i}^c(t) \leq p_{b,i}^{c,\text{max}},
\]
where \( p_{b,i}^{d,\text{max}} \) and \( p_{b,i}^{c,\text{max}} \) are the maximum discharging and charging energy of battery, respectively.

The battery energy dynamics \( E_{b,i}(t) \) can be represented by:
\[
E_{b,i}(t+1) = E_{b,i}(t) - \frac{1}{\eta_d} p_{b,i}^d(t) + \eta_c p_{b,i}^c(t), \tag{10}
\]
where \( \eta_d \) and \( \eta_c \) are the discharging and charging efficiency. The battery energy should be always within its allowable range to avoid over-discharging or over-charging:
\[
E_{b,i}^{\text{min}} \leq E_{b,i}(t) \leq E_{b,i}^{\text{max}}, \tag{11}
\]
where \( E_{b,i}^{\text{min}} \), \( E_{b,i}^{\text{max}} \) are the minimal/maximal level.

We assume that the battery capacity is large enough to accommodate two consecutive charges or discharges, i.e., \( A1: 2 \max [p_{b,i}^{d,\text{max}}/\eta_d, p_{b,i}^{c,\text{max}}/\eta_c] \leq E_{b,i}^{\text{max}} - E_{b,i}^{\text{min}}, \forall i \in \mathcal{I} \).

Both frequent charging and discharging can cause battery degradation. Here, the battery degradation cost is considered:
\[
C_{b,i}(t) = c_{b,i}(p_{b,i}^d(t) + p_{b,i}^c(t)), \tag{12}
\]
where \( c_{b,i} \) is the coefficient to measure the degradation caused by charging and discharging.

**Remark:** With continuous charging and discharging, the battery may undergo aging/degradation, such as a capacity decrease after a long period (e.g., hundreds of cycles) [29]. Thus, in a long-term operation scenario, the influence of battery aging should be considered for battery safety, and the battery capacity value should be calibrated after a certain period. Model-based (e.g., electrochemical mechanism model or equivalent electrical circuit model) and data-driven based (e.g., artificial neural network, support vector machine) battery aging methods can be used [30]. In this paper, we focus on real-time market operation and the time period studied (days to a week) will not be too long, so the battery capacity can be considered as unchanged.

5) **EV Charging Demand:** EVs are flexible loads, whose charging demand can be partially shed in response to the power supply conditions. For charging station \( i \in \mathcal{I} \), its charging demand \( p_{d,i}(t) \) in time slot \( t \) satisfies:
\[
p_{d,i}^{\text{min}}(t) \leq p_{d,i}(t) \leq p_{d,i}^{\text{max}}(t), \tag{13}
\]
where \( p_{d,i}^{\text{min}}(t) \) is the minimum charging energy in time slot \( t \) required by EVs that cannot be shed, and \( p_{d,i}^{\text{max}}(t) \) is the maximum charging energy preferred by EVs.

To protect privacy of EV owners, we assume that the charging station operator does not know the detailed information of each EV but only knows their available aggregate power flexibility region \([p_{d,i}^{\text{min}}(t), p_{d,i}^{\text{max}}(t)], \forall t \) [31]. The individual EV charging behaviors are implicitly reflected in the parameters \([p_{d,i}^{\text{min}}(t), p_{d,i}^{\text{max}}(t)], \forall t \), whose determination method can be found in Appendix A.

The charging demand shedding will cause discomfort for EVs, which is measured by
\[
C_{l,i}(t) = \alpha_i(p_{d,i}^{\text{min}}(t) - p_{d,i}(t))^2, \tag{14}
\]
where \( \alpha_i \) is a positive coefficient to indicate the sensitivity of charging station \( i \) to the unsatisfied charging demand \( p_{d,i}^{\text{max}}(t) - p_{d,i}(t) \). In addition, to prevent excessive charging load shedding, an upper bound is imposed on the time-average charging load shedding rate:
\[
r_{d,i}(t) = \frac{p_{d,i}^{\text{max}}(t) - p_{d,i}(t)}{p_{d,i}^{\text{max}}(t) - p_{d,i}^{\text{min}}(t)}, \tag{15}
\]
\[
\lim_{T \to \infty} \frac{1}{T} \mathbb{E} \left[ r_{d,i}(t) \right] \leq \beta_{d,i}, \tag{16}
\]
where \( \beta_{d,i} \) is a positive constant, responsible for controlling the quality of charging services.

**Remark:** The shedding of EV charging energy may affect the quality of service for EV owners. To take that into account, Reference [32] proposed a two-stage sizing and packetized energy scheduling for charging stations considering the quality of services. The proposed model in Appendix A can also take into account EV users’ willingness to be shed and charging priorities.

For example, if an EV user is less willing to be shed, it can report a tighter capacity constraint \([p_{d,i}^{\text{min}}(t), p_{d,i}^{\text{max}}(t)], \forall t \). In the extreme case when the EV user is unwilling to be shed in time slot \( t \), it can just let \( p_{d,i}^{\text{min}}(t) = p_{d,i}^{\text{max}}(t) \). Moreover, charging priorities can be incorporated by adding a term \( \sum_{v \in \mathcal{I}_o} \gamma_v (\text{soc}_{i,v}(T^f) - \text{soc}_{i,v}^{\text{req}})^2 \) to objective function of the aggregate flexibility problem (A.2). An EV with a higher priority is assigned a larger \( \gamma_v \). In addition, a larger sensitivity parameter \( \alpha_i \) in (14) and a lower target shedding ratio \( \beta_{d,i} \) in (16) can also lead to a lower charging demand shedding and a higher quality of service.

Each charging station \( i \in \mathcal{I} \) should maintain energy balancing in each time slot \( t \), which is described as
\[
p_{d,i}(t) = p_{b,i}^d(t) - p_{b,i}^c(t) + \sum_{j \in \mathcal{I}} e_{i,j}(t)
+ p_{pv,i}(t) + p_{b,i}^{d}(t) - p_{b,i}^{c}(t). \tag{17}
\]
C. Offline Optimization of the Charging Stations System

Our goal is to minimize the long-term operational cost of the overall charging stations system, which can be achieved by co-
optimizing the charging and discharging of battery storage, the energy exchanges with the grid and adjacent stations, and the supply to charging demand over the scheduling time horizon.

Define the system state vector \( s(t) \) in time slot \( t \) as a collection of PV generations, charging demands, and electricity prices, i.e.,
\[
\begin{align*}
    s(t) &= (s_1(t), s_2(t), \ldots, s_i(t)), \quad \forall i \in I, \\
    s_i(t) &= (p_{pv,i}(t), p_{d,min}^i(t), p_{d,max}^i(t), c_b(t)).
\end{align*}
\]  

The system state is uncertain and we have no prior knowledge about their statistics.

Define the control decision variable vector \( u(t) \) as
\[
\begin{align*}
    u(t) &= (u_1(t), u_2(t), \ldots, u_i(t)), \quad \forall i \in I, \\
    u_i(t) &= (p_{pv,i}(t), p_{d,min}^i(t), e_g(t), p_{d,max}^d(t), p_{d,j}(t), p_{d,j}(t)).
\end{align*}
\]

The total operational cost at time \( t \) includes the energy trading cost with grid, energy sharing cost with neighboring stations, battery degradation cost, and load shedding cost:
\[
C(t) = \sum_{i \in I} C_i(t) = \sum_{i \in I} (C_{g,i}(t) + C_{sh,i}(t) + C_{b,i}(t) + C_{l,i}(t)).
\]

Remark: The investment cost to have energy sharing between charging stations is not reflected in (22). This is because that we are not considering the long-term planning problem of EV charging stations in which the investment decisions are made, but a real-time operation problem. In the real-time operation, the investment decisions have been fixed and the investment cost is a sunk cost (can be viewed as a constant in the objective function). Neglecting it will not impact the optimal solution of the real-time operation problem. Therefore, we did not model the investment cost explicitly. Moreover, the proposed model does not require that every charging station has both energy storage systems and renewable energy sources. For the case without an energy storage system, we can set battery maximum charging/discharging power \( p_{b,i}^{d,max}, p_{b,i}^{c,max} \), \( \forall i \) to zero. For the case without a renewable generator, we can set the PV power generation \( p_{pv,i}(t), \forall i \) to zero.

The offline optimization of the charging stations system is formulated as the following stochastic optimization that minimizes the time-average total operational cost:
\[
P1 : \min_{u(t)} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[C(t)],
\]
\[
\text{s.t. } (1), (4), (5), (8) - (11), (13), (16), (17),
\]
where the expectation symbol \( \mathbb{E}[\cdot] \) in the objective function is with regard to the random system state and corresponding control decisions.

The above offline optimization cannot be solved directly for two reasons: 1) It requires complete information of the future. However, in practice, the real-time electricity price, PV generation, and charging load are uncertain. 2) It runs in a centralized manner which requires private information of each charging station.

For the first issue, online algorithms are desired. The main difficulty that hinders online implementation is the time-coupling constraints for battery energy dynamics (10). Common online algorithms, such as MPC and greedy algorithm, either require prior knowledge of uncertainties or simply ignore time-coupling constraints. To tackle the above challenges, this paper adopts the Lyapunov optimization framework. Though Lyapunov optimization has been widely used for queueing problems, the time-coupling constraints of battery storage make it hard to be applied in energy system problems. In Section III, we deal with this difficulty by introducing virtual queues to reformulate the problem. For the second issue, we develop an improved ADMM based distributed algorithm with iteration reduction in Section IV to protect privacy and reduce computational burdens. This enables fast execution of the algorithm and fits for real-time implementation.

III. ONLINE ALGORITHM

In this section, we turn the offline problem \( P1 \) into an online tractable form and prove that battery energy bound constraint (11) is always satisfied though not explicitly considered in the online problem; the optimality gap with \( P1 \) is also provided.

A. Problem Modification

Lyapunov optimization can solve a stochastic optimization problem with time-average constraints. Therefore, to solve \( P1 \), the time-coupling constraint (10) needs to be converted into a time-average one. First, both sides of (10) are summed over \( t \in \{1, \ldots, T \} \) and then divided by \( T \) yielding
\[
\frac{1}{T} \sum_{t=1}^{T} p_{b,i}(t) = -\frac{E_{b,i}(T+1)}{T} - \frac{E_{b,i}(1)}{T},
\]
\[
p_{b,i}(t) = -\frac{1}{\eta_d} p_{b,i}^d(t) + \eta_c p_{b,i}^c(t).
\]

We then take expectations on both sides of (24) and take limits over \( T \) to infinity yielding
\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[p_{b,i}(t)] = \lim_{T \to \infty} \mathbb{E} \left[ \frac{E_{b,i}(T+1)}{T} \right] - \lim_{T \to \infty} \mathbb{E} \left[ \frac{E_{b,i}(1)}{T} \right],
\]
\[
\text{Due to the constraint (11), both } E_{b,i}(T+1) \text{ and } E_{b,i}(1) \text{ are finite. Thus, the right hand side of (26) equals to zero, i.e.,}
\]
\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[p_{b,i}(t)] = 0.
\]

Now the problem \( P1 \) turns to be
\[
P2 : \min_{u(t)} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[C(t)],
\]
\[
\text{s.t. } (1), (4), (5), (8) - (10), (13), (16), (17), (27).
\]
In fact, constraint (27) is a relaxed version of constraints (10)-(11). Thus, any feasible solution to P1 is also a feasible solution to P2, i.e., P2 is less constrained than P1. The above relaxation step is critical which enables us to employ the Lyapunov optimization framework.

B. Lyapunov Optimization Based Method

Now the Lyapunov optimization framework can be used, and the key steps are briefly described below: 1) Construct virtual queues to transform the time-average constraints into queue stability constraints; 2) Define the Lyapunov function to obtain the Lyapunov drift and the drift-plus-penalty; 3) Minimize the upper bound of the drift-plus-penalty term.

1) Construct Virtual Queues: Lyapunov optimization aims to transform time-average constraints into queue stability constraints. We define two virtual queues to deal with time-average constraints (16) and (27) in problem P2.

Battery Energy Queue: The virtual battery energy queue $B_i(t)$ is defined as follows:

$$B_i(t) = E_i(t) - \theta_i(t),$$

where $\theta_i(t)$ is a perturbation parameter designed to ensure the feasibility of constraint (11), which will be explained later. The dynamics of virtual battery energy queue is obtained as

$$B_i(t + 1) = B_i(t) + p_{b,i}(t).$$

Comparing (30) with (10), it can be observed that $B_i(t)$ is actually a shifted version of $E_{b,i}(t)$ of battery storage. But different from $E_{b,i}(t)$, the virtual energy queue $B_i(t)$ could be negative because of the perturbation parameter $\theta_i(t)$. This shift can ensure that the constraint (11) is met. In addition, due to (27), it can be easily derived that the virtual battery energy queue $B_i(t)$ is also mean rate stable, i.e.,

$$\lim_{t \to \infty} \frac{\mathbb{E}[B_i(t)]}{t} = 0.$$  

(31)

It means that the queue does not grow linearly over time.

Charging Demand Shedding Queue: Similarly, we define a virtual queue $H_i(t)$ to deal with the time-average constraint in (16). It evolves as follows:

$$H_i(t + 1) = \max\{H_i(t) - \beta_{d,i}, 0\} + r_{d,i}(t).$$

(32)

Let $H_i(0) = 0$. The virtual queue $H_i(t)$ accumulates the portion of unsatisfied charging load ratio at each time slot $t$. Because of constraint (16), $H_i(t)$ is also mean rate stable,

$$\lim_{t \to \infty} \frac{\mathbb{E}[H_i(t)]}{t} = 0.$$  

(33)

Intuitively, to keep the virtual queue $H_i(t)$ stable, the arrival rate in time slot $t$, i.e., the shedding ratio $r_{d,i}(t)$, should not exceed the threshold $\beta_{d,i}$.

2) Obtain Lyapunov Function and Drift-Plus-Penalty: We define $\Theta(t) = (B(t), H(t))$ as the concatenated vector of virtual queues, where

$$B(t) = (B_1(t), \ldots, B_I(t)),$$

$$H(t) = (H_1(t), \ldots, H_I(t)).$$

(34)

(35)

A Lyapunov function is then defined as follows

$$L(\Theta(t)) = \frac{1}{2} \sum_{i \in I} B_i(t)^2 + \frac{1}{2} \sum_{i \in I} H_i(t)^2,$$  

(36)

where $w$ is the weight. $L(\Theta(t))$ can be considered as a measure of the queue size. A smaller value of $L(\Theta(t))$ is preferred to push virtual queues $B_i(t)$ and $H_i(t)$ to be less congested. Continually, the conditional one time slot Lyapunov drift is defined as follows:

$$\Delta(\Theta(t)) = \mathbb{E}[L(\Theta(t + 1)) - L(\Theta(t))|\Theta(t)],$$

(37)

where the expectation is taken with respect to the random $\Theta(t)$.

The Lyapunov drift is a measure of the expectation of the queue size growth given the current state $\Theta(t)$. Intuitively, by minimizing the Lyapunov drift, virtual queues are expected to be stabilized. However, only minimizing the Lyapunov drift may lead to a high total operational cost. Therefore, following the drift-plus-penalty method, we add the expected total operational cost (22) in one time slot to (37). The drift-plus-penalty term is obtained as follows

$$\Delta(\Theta(t)) + V\mathbb{E}[C(t)|\Theta(t)],$$

(38)

where $V$ is a weight parameter that controls the trade-off between virtual queues stability and operational cost minimization. We will show later in Proposition 1 how to choose the value of this parameter.

3) Minimizing the Upper Bound: Problem (38) is still time-coupled due to the definition of $\Delta(\Theta(t))$. To adapt to an online implementation, instead of directly minimizing the drift-plus-penalty term, we minimize the upper bound to obtain the control decision. To be specific, we first derive the one time slot Lyapunov drift expression as follows:

$$L(\Theta(t + 1)) - L(\Theta(t)) = \frac{1}{2} \sum_{i \in I} \left[ B_i(t + 1)^2 - B_i(t)^2 \right]$$

$$+ w \left[ H_i(t + 1)^2 - H_i(t)^2 \right].$$

(39)

Based on the queue update equations (30) and (32), we have

$$\frac{1}{2} \left[ B_i(t + 1)^2 - B_i(t)^2 \right]$$

$$\leq B_i(t)p_{b,i}(t) + \frac{1}{2} \max\left\{ \eta_i p_{b,i}^{\text{max}} \left( \frac{1}{\eta_d p_{d,i}^{\text{max}}} \right)^2, \left( \frac{1}{\eta_d p_{d,i}^{\text{max}}} \right)^2 \right\},$$

(40)

$$\frac{1}{2} \left[ H_i(t + 1)^2 - H_i(t)^2 \right]$$

$$\leq H_i(t)(r_{d,i}(t) - \beta_{d,i}) + \frac{1}{2} \left( 1 + \beta_{d,i}^2 \right).$$

(41)

Continuously, we substitute inequalities (40) and (41) into drift-plus-penalty term yielding

$$\Delta(\Theta(t)) + V\mathbb{E}[C(t)|\Theta(t)] \leq A + \sum_{i \in I} B_i(t)\mathbb{E}[p_{b,i}(t)|\Theta(t)]$$

$$+ w \sum_{i \in I} H_i(t)\mathbb{E}[r_{d,i}(t) - \beta_{d,i}|\Theta(t)]$$

$$+ V\mathbb{E}[C(t)|\Theta(t)],$$

(42)
where $A = \frac{1}{2} \sum_{i \in \mathcal{I}} \max\{(\eta_d p_{b,i}^{c,\max})^2, (\eta_d p_{d,b,i}^{d,\max})^2\} + \frac{1}{2} w \sum_{i \in \mathcal{I}} (1 + \beta_i^2)$ is a constant.

Recalling that the main principle of the Lyapunov optimization based method is to minimize the upper bound, we obtain the following real-time optimization problem

$$
\text{P3} : \min \sum_{i \in \mathcal{I}} B_i(t)p_{b,i}(t) + w \sum_{i \in \mathcal{I}} H_i(t)r_{d,i}(t) + VC(t) \\
\text{s.t. (1), (4), (5), (8) – (9), (13), (17),} 
$$

where $B_i(t)$ and $H_i(t)$ are firstly updated based on (30) and (32) before solving P3 in each time slot. In each time slot, given the current system state $s(t)$ and virtual queue state $\Theta(t)$, the proposed method determines the control decision $u(t)$ by solving problem P3. The term $wH_i(t)\beta_d,i$ is ignored since it is a constant in the problem. Hereafter, the original offline optimization problem P1 has been decoupled into simple real-time (online) problems, which can be executed at each time slot without requiring a high-complex solver and a prior knowledge of uncertain states.

### C. Feasibility Guarantee and Performance Analysis

Comparing constraints of P1 with those of P3, it can be observed that constraint (11) is not considered in P3. We may be concerned about whether the solution generated by P3 is feasible for P1. In fact, by carefully designing the perturbation parameter $\theta_i(t)$, this bound constraint (11) of battery energy state can be guaranteed, which is stated in the following proposition.

**Proposition 1:** When Assumption A1 holds, if we let

$$
\theta_i(t) = E_{b,i}^{\min} + \frac{1}{\eta_d} p_{b,i}^{d,\max} + \frac{V}{\eta_c} (c_{b,i}^{c,\max} + c_{b,i}), \forall t, 
$$

where

$$
0 \leq V \leq V_{\max}^{\min} = \min_{i \in \mathcal{I}} \frac{E_{b,i}^{\min} - E_{b,i}^{\max} - \eta_c p_{b,i}^{c,\max} - \eta_d p_{d,b,i}^{d,\max}}{\eta_c (c_{b,i}^{c,\max} + c_{b,i} (\eta_d + \frac{1}{\eta_c})}, 
$$

the sequence of optimal solutions obtained by the online problem P3 satisfies constraint (11).

The proof of Proposition 1 can be found in Appendix B. Moreover, another important issue we care about is: what’s the gap between the optimal solutions of online problem P3 and offline problem P1? This is addressed below.

**Proposition 2:** Denote the achieved long-term time-average cost objective value of P1 and P3 as $C^*$ and $\tilde{C}$, respectively. We have

$$
0 \leq \tilde{C} - C^* \leq \frac{1}{V} A, 
$$

where $A$ is a constant mentioned in (42).

The proof of Proposition 2 can be found in Appendix C. The optimality gap is affected by the control parameter $V$. A bigger $V$ value can decrease the optimality gap but it increases the size of virtual queues. In contrast, a smaller $V$ value makes queues more stable but leads to a larger optimality gap.

### IV. DISTRIBUTED IMPLEMENTATION

Due to the need to protect privacy and reduce computational burden, we propose an ADMM based algorithm with iteration truncation to solve the energy sharing problem.

#### A. Distributed Optimization Formulation

First, auxiliary variable vectors $\epsilon_i, \forall i \in \mathcal{I}$ are introduced, and each $\epsilon_i = \{\epsilon_{i,j}(t), \forall t \in T, \forall j \in \mathcal{I}_i\}$. Then the constraint (4) is replaced by

$$
\epsilon_{i,j}(t) = \epsilon_{i,j}(t), \forall t \in T, \forall i \in \mathcal{I}, \forall j \in \mathcal{I}_i. 
$$

The corresponding augmented Lagrangian function of problem P3 is written as follows:

$$
L(u_i(t)) = \sum_{i \in \mathcal{I}} [B_i(t)p_{b,i}(t) + wH_i(t)r_{d,i}(t) + VC_i(u_i(t)) \\
+ \sum_{j \in \mathcal{I}_i} \rho \left(\epsilon_{i,j}(t) - \epsilon_{i,j}(t) + \frac{d_{i,j}(t)}{\rho} \right)^2], 
$$

where $\rho$ is a positive penalty parameter of the augmented term, and $d = [d_{i,j}(t), \forall t \in T, \forall j \in \mathcal{I}_i]$ is the vector of associated dual variables of the equality constraint (47).

Let $U_i$ denote the constraint set of decision variable $u_i(t)$ derived from (43). After the above transformation, the problem can be solved in an iterative process. In the first step, each charging station $i$ updates its own variables $u_{i,k+1}(t)$ by solving the local optimization problem:

$$
\min_{u_i(t)} B_i(t)p_{b,i}(t) + wH_i(t)r_{d,i}(t) + VC_i(u_i(t)) \\
+ \sum_{j \in \mathcal{I}_i} \rho \left(\epsilon_{i,j}(t) - \epsilon_{i,j}(t) + \frac{d_{i,j}(t)}{\rho} \right)^2, 
$$

where $k$ is the iteration number. After obtaining the $\epsilon_{i,k+1}$ of each charging station, the second step is to update the auxiliary variables $\epsilon_{i,k+1}, \forall i$ by solving the optimization problem:

$$
\min_{\epsilon_{i,j}(t)} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}_i} \rho \left(\epsilon_{i,j}(t) - \epsilon_{i,j}(t) + \frac{d_{i,j}(t)}{\rho} \right)^2, 
$$

s.t. (48), given $\rho, \epsilon_{i,j}(t), d_{i,j}(t)$.

By solving (51), we can get the closed-form solution of $\epsilon_{i,k+1}, \forall i \in \mathcal{I}$ as follows:

$$
\epsilon_{i,j}^{k+1}(t) = \frac{1}{2} \left( \epsilon_{i,j}^{k+1}(t) - \epsilon_{i,j}^{k+1}(t) + \frac{1}{\rho} (d_{i,j}(t) - d_{i,j}(t)) \right), 
$$

$$
\epsilon_{j,i}^{k+1}(t) = \frac{1}{2} \left( \epsilon_{j,i}^{k+1}(t) - \epsilon_{j,i}^{k+1}(t) + \frac{1}{\rho} (d_{j,i}(t) - d_{j,i}(t)) \right). 
$$

Based on the updated $\epsilon_{i,k+1}(t)$ and $\epsilon_{i,k+1}(t)$, the dual variable is updated in the third step by

$$
d_{i,j}^{k+1}(t) = d_{i,j}^{k+1}(t) + \rho \left(\epsilon_{i,j}^{k+1}(t) - \epsilon_{i,j}^{k+1}(t) \right). 
$$
The algorithm convergence criterion is set as:

$$r = \|d^{k+1} - d^k\| \leq \delta,$$

(54)

where $r$ is the error and $\delta$ is the accuracy tolerance.

B. Efficiency Improvement for Real-Time Implementation

It may take a large number of iterations for the aforementioned algorithm to converge. However, in real-time applications with a small time resolution, this algorithm may not have enough time to converge. As shown in Fig. 2, the required iteration number is uncertain and depends on the situation in each time slot. In the following, an iteration truncation method is developed to improve the efficiency of real-time energy sharing. It stops the algorithm when it exceeds a preset iteration threshold $k_s$.

Since the shared energy between a pair of charging stations $i$ and $j$ may not be balanced at the stop iteration $k_s$, an energy balancing mechanism is proposed. For the sake of secure operation, the shared energy quantities are determined by the minor one between $i$ and $j$. For charging station $i$, if $|e^{k}_{i,j}|$ is greater than $|e^{k}_{j,i}|$, then charging station $i$ will decrease the shared energy $e^{k}_{i,j}$ according to the following update rule where $\hat{e}^{k}_{i,j}$ is the new shared energy after update,

$$\hat{e}^{k}_{i,j} = -e^{k}_{j,i}, \forall i \in \mathcal{I}, \forall j \in \mathcal{I} \setminus i,$$

(55)

and charging station $j$ keeps no change,

$$\hat{e}^{k}_{j,i} = e^{k}_{j,i}, \forall i \in \mathcal{I}, \forall j \in \mathcal{I} \setminus i.$$

(56)

Since $e^{min}_{i,j} = e^{min}_{j,i}$ and $e^{max}_{i,j} = e^{max}_{j,i}$, both $\hat{e}^{k}_{i,j}$ and $\hat{e}^{k}_{j,i}$ are feasible. So far, the shared energy balance condition is guaranteed. However, this shared energy update inevitably affect the system energy balance (17) obtained at iteration $k_s$ for charging station $i$. The caused imbalance should be offset by adjusting dispatchable resources (grid or battery) of the charging station. Here, we choose to adjust the purchasing/selling energy from/to the grid as follows

$$\begin{cases}
p^b_{g,i} = p^b_{g,i} + (e^{k}_{i,j} + \hat{e}^{k}_{i,j}) & \text{if } e^{k}_{i,j} \geq 0 \\
p^b_{g,i} = p^b_{g,i} - (e^{k}_{i,j} + \hat{e}^{k}_{i,j}) & \text{if } e^{k}_{i,j} < 0.
\end{cases}$$

(57)

On the contrary, if $|e^{k}_{j,i}|$ is less than $|e^{k}_{i,j}|$, then charging station $i$ will maintain unchanged, and charging station $j$ will update its shared energy to $\hat{e}^{k}_{i,j}$ according to the similar update rule (55) and adjust the grid power like (57).

Algorithm 1 ADMM With Iteration Truncation

1: Set iteration index $k = 0$, convergence error tolerance $\delta > 0$, penalty parameter $\rho > 0$.
2: Initialize dual variables $d^k = 0$, auxiliary variable $\{e^k_i = 0, i \in \mathcal{I}\}$.
3: repeat
4: for Each charging station $i \in \mathcal{I}$ do
5: Formulate the energy management subproblem for each charging station as (50) and solves it with $e^k_i$ and $d^k_i$.
6: Update $e^k_{i}$. \quad end for
7: Update $e^{k+1}_i$ with $e^{k+1}_i$ and $d^{k+1}_i$ via (52).
8: Update dual variables $d^{k+1}_i$ with the new $e^{k+1}_i$ and $e^{k+1}_i$.
9: Set $k = k + 1$.
10: until convergence criterion (54) is satisfied or $k$ reaches the threshold $k_s$.
11: if $k_s$ is reached then
12: Update the shared energy according to (55) and (56).
13: Adjust grid output based on (57) to offset the imbalance caused by energy sharing update.
14: end if
15: Update the battery energy state of each charging station.
16: Update virtual energy queues (30) and load shedding queues (32) of each charging station.

A complete description of the proposed distributed algorithm is shown in Algorithm 1 that will be executed in each time slot. The data exchanged among charging stations only include their shared energy while other private data (e.g., charging load, battery storage SOC) are well protected.

V. SIMULATION RESULTS AND DISCUSSION

In this section, we evaluate the effectiveness of the proposed method and compare it with the state-of-the-art approaches. The proposed methodology does not depend on the size of the charging stations, energy storage, and renewable energy sources. In the simulation, different charging station settings are adopted, where the EV charging load requirement, PV power generation, and battery storage capacity are different. The simulation results validate the effectiveness of the proposed methodology under different sizes. All the simulations are carried out by MATLAB and run on an Intel Core i7-8550U CPU at 1.80 GHz computer with a 16 GB RAM.

A. Simulation Setup

We first consider three charging stations (CS1, CS2, CS3) that interconnect with each other. All of them are equipped with PV panels and battery storage, but their capacities are different. The battery related parameters are $E_{b,1} = 100$ kWh, $E_{b,2} = 200$ kWh, $E_{b,3} = 200$ kWh, $p_{b,1}^{c,max} = p_{b,2}^{d,max} = 10$ kW, $p_{b,3}^{c,max} = p_{b,3}^{d,max} = 20$ kW, $p_{b,1}^{c} = p_{b,2}^{d} = 20$ kW, $\eta_d = \eta_c = 0.95$, $c_{b,1} = c_{b,2} = c_{b,3} = 0.01$. The used data of real-time electricity
price [33], PV power profile [34], and EV charging load [33] are obtained from the real world to reflect the strong uncertainty. They are shown in Fig. 3. Specifically, we consider an entire period of 7 days (i.e., one week) with 10-minute per time slot, namely 1008 time slots in total. The three charging stations have different charging demand patterns. In particular, charging station 2 is with moderate charging demand while charging stations 1 and 3 are with heavy charging demands.

Charging station 2 has sufficient PV energy over the load while charging stations 1 and 3 are with heavy charging demands. The feed-in tariff $c^f$ is set to be 0.01 $/kWh at any time slot. Referring to the commonly used mid-market rate mechanism, the internal energy sharing price $c^g$ is chosen as an intermediate value between $c^f$ and $c^b$, satisfying (6).

B. Simulation Results

1) Performance Comparison: To show the advantage of the proposed distributed online algorithm, four widely used baselines in the literature are employed.

- The first baseline (B1) individually operates each charging station and no energy sharing is allowed. Each station only minimizes the operational cost in the current time slot. Additionally, when the electricity price is below a threshold (0.1 USD), the battery storage will be charged at its maximum charging power.
- The second baseline (B2) uses the same greedy algorithm as B1, but energy sharing is incorporated.
- The third baseline (B3) uses a MPC-based algorithm, which can look ahead and minimize the cost incurred over the prediction time window. Only the first step of the obtained control sequence will be applied. We assume B3 has accurate predictions over the future 6 time slots.
- The fourth baseline (B4) solves the offline optimization $P_1$, assuming complete information of the future. Though not realistic, it provides a theoretical benchmark to verify the performance of other methods.

Fig. 4 shows the accumulated operational costs over time under different methods. The B1 algorithm has the worst performance and the highest cost. The cost of B2 is lower than that of B1 thanks to energy sharing. With precise predictions for PV generation, charging loads, and real-time energy prices in the future 6 time slots, B3 outperforms B1 and B2. However, the forecast-and-rolling procedure inevitably increases the computational complexity. In contrast, our proposed algorithm performs better than B3 as time goes on and requires no future prediction, which is more practical. In addition, the gap between the proposed algorithm and the greedy baseline B2 gradually widens over time. This implies that the proposed algorithm has a long-term vision to achieve a better overall performance in the end. The offline optimization B4 has the best performance, which however is usually impossible in practice. Overall, the proposed algorithm can achieve nearly offline optimal performance and is easy to implement.

Table I summarizes the costs under different methods. The total cost of each charging station comprises the energy trading cost with the grid, battery cost, shedding cost, and sharing cost. Compared with B1, the proposed algorithm significantly reduces the total cost of three stations from 2981.75 USD to 2739.6 USD, with a larger drop of 8.12%, while the reductions brought by B2 and B3 are merely 1.51% and 6.74%, respectively. The result of the proposed algorithm is the closest to that of offline optimization B4.

2) Feasibility Analysis: We have proved that by appropriately designing the parameter $\theta(t)$, the battery energy bound constraint (11) can be met even though it is not explicitly considered in $P_3$. Here, simulation results are given to demonstrate the feasibility of the proposed algorithm. Fig. 5 shows the energy evolution of battery storage over the time in each charging station. As seen in the figure, all battery energy states are within the allowable range without exceeding the upper and lower bounds, which justifies Proposition 1. Meanwhile, the battery energy curve drops during the spikes of real-time electricity price. It means that the battery discharges its stored energy when the price is very high and vice versa. In addition, batteries remain at a medium energy level without large deviations, which allows it to quickly release or absorb energy when needed. This is very helpful when working in future energy systems full of uncertainties.

Similarly, we replace the charging demand shedding ratio constraint (16) by a virtual queue. To examine its effectiveness, the time-average load shedding ratio over time is shown in Fig. 6. At the beginning, the ratios are higher than the individual requirement of charging service quality. Under the control of the proposed algorithm, they rapidly drop and meet the required time-average constraint as the time goes on.

Note that the above results are obtained under the weight parameter $V = V_{\max}$. Recall that parameter $V$ controls the
trade-off between stabilizing virtual queues and minimizing the total cost in the objective function (43). Here we investigate the impact of $V$ by varying its value in the allowable range. The results are shown in Fig. 7. The final time-average load shedding ratio and the total cost are both nonlinear function in $V$. A larger $V$ would bring the time-average load shedding ratio closer to the requirement boundary, i.e., the queue turns to be more unstable, while the cost becomes lower.

The long-term average constraint (16) will impact the energy satisfaction of EV charging demand. We investigate this by comparing different target values $\beta_{d,i}$. The original target values are $\beta_{d,1} = 0.4$, $\beta_{d,2} = 0.5$, $\beta_{d,3} = 0.6$. With decreasing target values, the time-average load shedding ratio also gradually reduces, but still, the target is satisfied, as shown in Fig. 8. A lower target value $\beta_{d,i}$ means a lower charging demand shedding level, i.e., EV can obtain more charging energy. For example, if the target values $\beta_{d,i} = 0.2$, $\forall i$ and the EV charging power is 3.3kWh per hour, then an EV can receive approximately $3.3 \times (1 - 0.2) \times 8 = 21.12$ kWh during 8 hours over night.

For the cost parameter $\alpha_i$, its original value is 0.01. When we increase the value of $\alpha_i$ from 0.01 to 0.04, the time-average load shedding ratio gradually decreases, and is farther away from the target value, as shown in Fig. 9. A bigger $\alpha_i$ means the charging load is less willing be to shed, and results in a smaller time-average load shedding ratio, which is reasonable.

3) Energy Sharing Analysis: The complementary nature of energy supply and demand between different charging stations makes energy sharing possible. Fig. 10(a) shows the energy

|      | Grid cost | Battery cost | Shedding cost | Sharing cost | Cost of each charging station | Total cost | Reduction |
|------|-----------|--------------|--------------|--------------|-----------------------------|-----------|-----------|
| B1   | CS1       | 1224.96      | 14.96        | 94.50        | 0                           | 1334.41   |           |
|      | CS2       | 457.00       | 26.64        | 40.25        | 0                           | 523.96    | 2981.75   |
|      | CS3       | 910.93       | 29.91        | 182.54       | 0                           | 1123.38   | -         |
| B2   | CS1       | 1010.08      | 14.96        | 94.50        | 193.84                      | 1313.37   |           |
|      | CS2       | 750.04       | 29.66        | 45.67        | -314.1                      | 511.27    | 2936.59   | 1.51%     |
|      | CS3       | 781.35       | 29.91        | 180.41       | 120.27                      | 1111.95   |           |
| B3   | CS1       | 976.30       | 6.47         | 94.50        | 206.27                      | 1283.53   |           |
|      | CS2       | 741.73       | 9.05         | 47.63        | -357.3                      | 441.11    | 2780.74   | 6.74%     |
|      | CS3       | 708.86       | 10.72        | 185.49       | 151.03                      | 1056.10   |           |
|      | CS1       | 533.3        | 13.63        | 94.5         | 568.73                      | 1230.16   |           |
|      | CS2       | 643.79       | 27.22        | 56.39        | -376.13                     | 351.27    | 2632.48   | 11.71%    |
|      | CS3       | 1030.94      | 27.22        | 185.49       | -192.6                      | 1051.04   |           |
| B4   | CS1       | 588.09       | 6.76         | 106.32       | 545.41                      | 1246.78   |           |
|      | CS2       | 954.40       | 14.11        | 62.83        | -611.02                     | 420.32    | 2739.60   | 8.12%     |
|      | CS3       | 805.59       | 14.74        | 186.54       | 65.62                       | 1072.50   |           |

Fig. 5. Battery storage energy over time.

Fig. 6. Time-average charging load shedding ratios.
sharing result of three charging stations in each time slot under the proposed algorithm. Generally, CS1 and CS3 import energy from CS2, since CS2 has the most PV energy generation and the least charging load. Fig. 10(b) shows the energy sharing solution under B2 algorithm. Comparing the two figures, we can find that the proposed algorithm can greatly enhance the energy sharing between charging stations.

In addition, as seen in Table I, compared with B1 (the no energy sharing case), B2 reduces the total cost by 1.51% with the help of energy sharing. However, the improvement in social welfare is marginal. The proposed algorithm further considers the long-term benefit through the Lyapunov optimization, achieving a significant cost reduction (8.12%) without violating time-coupling constraints. Moreover, the proposed algorithm can reduce the cost of individual charging station compared with B1 and B2. Therefore, the charging stations have the incentive to participate in energy sharing.

We further investigate the impacts of internal energy sharing price and energy sharing limits on the cost of each charging station and the overall cost. The charging stations $i$ and $j$ have the same energy sharing price $c_{sh}^{i}(t)$ and $c_{sh}^{j}(t) + c_{sh}^{j}(t) = 0$, thus, budget balance is always achieved for the overall charging station system: $\sum_{i} C_{sh,i} = \sum_{j} \sum_{i \neq j, i} c_{ij}(t) c_{sh}(t) = 0$. Therefore, the sharing price $c_{sh}$ can affect the cost of an individual charging station but will not affect the overall cost of the whole system. Since the energy sharing price $c_{sh}$ is between $c_{sh}^{i}$ and $c_{sh}^{j}$, it can be represented as $c_{sh} = (1 - r_{sh}) c_{sh}^{i} + r_{sh} c_{sh}^{j}$, where $r_{sh} > 0$ is the ratio. We let $r_{sh}$ equals to 0.2, 0.4, 0.6, 0.8, respectively. Fig. 11 shows the cost of individual charging stations under different energy sharing prices. As the energy sharing price increases, the cost of charging station 1 (CS1) increases while the cost of CS2 decreases. This is because CS2 has a lot of surplus energy and can earn more through higher energy sharing prices, while CS1 has to pay more for buying shared energy. The cost of CS3 also decreases slightly from 1085.9 USD at $r_{sh} = 0.2$ to 1068.9 USD at $r_{sh} = 0.8$. The total cost of the three charging stations at different energy sharing prices remains the same, validating the budget balance conclusion.

The amount of shared energy $e_{i,j}$ is restricted by the physical energy sharing limits $e_{i,j}^{min}$ and $e_{i,j}^{max}$. With a tighter energy sharing limit, the charging station has less flexibility in sharing energy with other charging stations and it has to turn to other energy sources such as the main grid, the battery, etc. This will further influence the overall cost. We investigate the impact of energy sharing limit by changing its value from 0 to 200 kW. As shown in Fig. 12, with a growing energy sharing limit, the overall cost first decreases significantly and then remains stable. This is because when the energy sharing limit is large enough, it is no longer the binding constraint and thus will not further influence the cost.

4) Convergence and Efficiency Analysis: Computational efficiency is a critical issue in real-time application. Fig. 2 has shown the iteration number required to reach convergence without iteration truncation. The corresponding error is given in Fig. 13, which quickly drops at the beginning but then slowly converges. This allows us to stop the algorithm at a smaller number of iterations with little impact on the performance of the algorithm, which is the iteration truncation method in Section IV-B. The selection of the threshold $k_{s}$ involves a trade-off between the computational time and optimality. A large $k_{s}$ will result in a computing time that is even longer than the market’s time resolution. Thus, the algorithm may fail to give the energy sharing results at the end of each period. On the contrary, a small $k_{s}$ may lead to sub-optimal results. The threshold $k_{s}$ can be determined according
to statistics analysis. We extract the distribution of the iterations to reach convergence over the 1008 scenarios in Fig. 2. We can find that most iterations are less than 4, which also satisfy the real-time operating requirement. Therefore, we set the $k^*$ to 4 in this paper. We compare the performance of the conventional ADMM algorithm with the proposed algorithm in terms of total cost, computing time and average iteration number. As shown in Table II, the proposed algorithm achieves a higher computational efficiency ($-26\%$) and fewer iterations at slight cost increase ($+0.12\%$).

5) Scalability: In the following, we further investigate the scalability of the proposed algorithm. The computational time over the entire time horizon (1008 time slots) and the total cost are used as criteria. The results under different number of charging stations are shown in Fig. 14. As the number of charging stations increases, the proposed algorithm takes much less time than the conventional ADMM with little impact on the total cost. For the extreme scenario with 100 charging stations, the proposed algorithm takes 692 s in total while the conventional ADMM needs 1574 s. The proposed algorithm is more scalable.

Moreover, we compare the proposed algorithm with the B3 approach, i.e., MPC. Fig. 15 shows the average computational time at a single time slot. As seen, the proposed method is faster than B3. This is because the proposed method only needs to solve a single time step problem while the MPC needs to calculate the decisions over the prediction horizon which is relatively computation intensive. In addition, with a growing number of charging stations, the time required by the proposed method maintains around 0.02-0.03 s. For the case with 100 charging stations, it can be estimated that the total computational time over the 1008 time slots for the proposed algorithm would be $0.03 \times 100 \times 1008/3600 = 0.84$ hour, while B3 may take $1.51 \times 100 \times 1008/3600 = 42.28$ hour.

VI. CONCLUSION

This paper proposes a distributed online algorithm to promote energy sharing among EV charging stations. Compared with the existing online algorithms, long-term benefits are considered through the Lyapunov optimization technique where the time-coupling constraints are decoupled with the help of virtual queues. We provide guidance for selecting the parameters to ensure the satisfaction of battery-related constraint. We also prove theoretically that the optimality gap between the proposed online algorithm and its offline counterpart is inversely proportional to the weight coefficient used in the drift-plus-penalty term. To protect privacy of individual charging stations, an improved ADMM algorithm with iteration truncation is proposed. Simulation results demonstrate the effectiveness and scalability of the proposed algorithm, and we have the following findings:

1) The proposed online algorithm can achieve a nearly offline optimum, with a cost reduction of 6.7% and 1.5% compared to the greedy and MPC based algorithms, respectively.

2) Compared with the conventional ADMM algorithm, the computational time is reduced by 26% with little sacrifice in total cost ($+0.12\%$).

3) The participation level of energy sharing can be enhanced by the proposed algorithm.

Future research directions include: 1) a more realistic EV charging model considering the real charging characteristics of lithium batteries and the quality of charging services; 2) an improved algorithm with an adaptive tuning of the iteration threshold parameter for applications in various scenarios; 3) a more complicated energy sharing setting with distribution network resources.

APPENDIX A

DETERMINATION OF $[p_{d,i}^{\min}, p_{d,i}^{\max}]$, $\forall t$

To protect privacy of EV owners, we only provide the charging station operator with the aggregate power flexibility region $[p_{d,i}^{\min}(t), p_{d,i}^{\max}(t)]$, $\forall t$ of all EVs instead of their detailed information. The individual EV charging demand features are implicitly reflected in the parameters $[p_{d,i}^{\min}(t), p_{d,i}^{\max}(t)]$, $\forall t$, i.e.,
constraint (13). To be specific, the parameters can be generated through a rolling horizon as follows:

When an EV $v \in \mathcal{V}$ arrives at charging station $i$, it will report its information $(t_{i}^{d},v^{d},soc_{i}^{ini},soc_{i}^{req})$ to an EV management system, including the arrival time $t_{i}^{d}$, the departure time $t_{i}^{d}$, the initial SOC $soc_{i}^{ini}$, and the minimum required SOC $soc_{i}^{req}$ when leaving. The constraints for an individual EV $v \in \mathcal{V}$ are

\[
soc_{i,v}(t + 1) = soc_{i,v}(t) + \frac{p_{i,v}(t)}{E_{i,v,cap}}, \quad \forall t \neq T, \tag{A.1a}
\]
\[
soc_{i,v}^{min} \leq soc_{i,v}(t) \leq soc_{i,v}^{max}, \quad \forall t \tag{A.1b}
\]
\[
p_{i,v}^{min} \leq p_{i,v}(t) \leq p_{i,v}^{max}, \quad \forall t \tag{A.1c}
\]
\[
soc_{i,v}(t_{i}^{d}) = soc_{i}^{ini} \tag{A.1d}
\]
\[
soc_{i,v}(t_{i}^{d}) \geq soc_{i,v}^{req} \tag{A.1e}
\]

The aggregate power flexibility problem can be formulated as:

\[
\max_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}} (p_{d,v}^{max}(t) - p_{d,v}^{min}(t)), \tag{A.2a}
\]
\[
\text{s.t. } p_{d,v}^{max}(t) = \sum_{v \in \mathcal{V}} \tilde{p}_{i,v}(t), \quad \forall t \tag{A.2b}
\]
\[
p_{d,v}^{min}(t) = \sum_{v \in \mathcal{V}} \tilde{p}_{i,v}(t), \quad \forall t \tag{A.2c}
\]
\[
\tilde{p}_{i,v}(t), soc_{i,v}(t), \text{ satisfy (A.1)} \tag{A.2d}
\]
\[
\tilde{p}_{i,v}(t), soc_{i,v}(t), \text{ satisfy (A.1)} \tag{A.2e}
\]

To take into account the random arrival/departure time, we determine the parameters $[p_{d,v}^{min}(t), p_{d,v}^{max}(t)]$, $\forall v$ in the following way:

- In time slot $t = 1$, we have the information $(t_{i}^{d},v^{d},soc_{i}^{ini},soc_{i}^{req})$ of $n_1$ EVs. Then, we solve the problem (A.2) and get the optimal solution $[p_{d,v}^{min}(t), p_{d,v}^{max}(t)]$, $\forall v$. The parameters for the first time slot $[p_{d,v}^{min}(1), p_{d,v}^{max}(1)]$ are used for the charging station management problem P3 at $t = 1$.

- In time slot $t = 2$, $n_2$ EVs suddenly leave while $n_2'$ EVs suddenly arrive. Then, we update the information $(t_{i}^{d},v^{d},soc_{i}^{ini},soc_{i}^{req})$ for $n_1 + n_2 - n_2'$ EVs. For the EVs that suddenly leave, we set $p_{i,v}^{min} = p_{i,v}^{max} = 0$ for the following time slots and remove the constraint (A.1e). For the EVs that suddenly arrive, the related constraints (A.1) are added to the problem (A.2). Moreover, to keep the parameters consistent, we add the following constraints to (A.2):

\[
p_{d,v}^{min}(1) = p_{d,v}^{min}(1), p_{d,v}^{max}(1) = p_{d,v}^{max}(1). \tag{A.2a}
\]

Then we can solve (A.2) again and update the optimal solution $[p_{d,v}^{min}(t), p_{d,v}^{max}(t)]$, $\forall v$. The parameters for the second time slot $[p_{d,v}^{min}(2), p_{d,v}^{max}(2)]$ are used for the charging station management problem P3 at $t = 2$.

- In time slot $t = 3$, a similar procedure can be conducted.

...
In addition, since \( \frac{1}{\eta_d} p_{b,i}^{d,\max} + E_{b,i}^{\min} \leq E_{b,i}(t) \), we can obtain
\[
E_{b,i}(t + 1) \geq \frac{1}{\eta_d} p_{b,i}^{d,\max} + E_{b,i}^{\min} - \frac{1}{\eta_d} p_{b,i}^{d}(t) + \eta_c p_{b,i}^{e}(t) \\
\geq E_{b,i}^{\min}.
\]

Case 3: \( V(\frac{1}{\eta_c} p_{b,i}^{b,\max} - \eta_d p_{b,i}^{b,\min} + c_{b,i}(\eta_d + \frac{1}{\eta_c})) + E_{b,i}^{\min} + \frac{1}{\eta_d} p_{b,i}^{d,\max} < E_{b,i}(t) \leq E_{b,i}^{\max} \). Due to (45), we have \( V(\frac{1}{\eta_c} p_{b,i}^{b,\max} - \eta_d p_{b,i}^{b,\min} + c_{b,i}(\eta_d + \frac{1}{\eta_c})) + E_{b,i}^{\min} + \frac{1}{\eta_d} p_{b,i}^{d,\max} \leq E_{b,i}^{\max} - \eta_c p_{b,i}^{e} \). Similar to Case 1, we then derive the partial derivative of the objective function of P3 with respect to \( p_{b,i}^{d}(t) \), i.e.,
\[
\frac{\partial P3(t)}{\partial p_{b,i}^{d}(t)} = V \frac{\partial C(t)}{\partial p_{b,i}^{d}(t)} - B(t) \frac{1}{\eta_d} c_{b,i}^{b,\min} \\
- \left( E_{b,i}(t) - E_{b,i}^{\min} - \frac{1}{\eta_d} p_{b,i}^{d,\max} - V \left( c_{b,i}^{b,\max} + c_{b,i} \left( \eta_d + \frac{1}{\eta_c} \right) \right) \right) \frac{1}{\eta_d} \\
+ \left[ V \left( \frac{1}{\eta_c} p_{b,i}^{b,\max} - \eta_d p_{b,i}^{b,\min} + c_{b,i} \left( \eta_d + \frac{1}{\eta_c} \right) \right) \right] \\
+ E_{b,i}^{\min} + \frac{1}{\eta_d} p_{b,i}^{d,\max} - E_{b,i}(t) \frac{1}{\eta_d} < 0.
\]

Thus, the objective function is strictly decreasing with respect to \( p_{b,i}^{d}(t) \). Therefore, the optimal solution is \( p_{b,i}^{d}(t) = p_{b,i}^{d,\max} \).

Since charging and discharging cannot happen at the same time, we have \( p_{b,i}^{d}(t) = 0 \). According to (10), we have \( E_{b,i}(t + 1) = E_{b,i}(t) - \frac{1}{\eta_d} p_{b,i}^{d,\max} \) and hence
\[
E_{b,i}^{\min} \leq E_{b,i}(t + 1) \leq E_{b,i}^{\max} - \frac{1}{\eta_d} p_{b,i}^{d,\max} \leq E_{b,i}^{\max}.
\]

Therefore, we have proved that the hard constraint (11) still holds for all time slots.

**APPENDIX C**

**PROOF OF PROPOSITION 2**

Proposition 1 tells that the solution generated by P3 is also feasible for P1. Since P1 is the offline optimization that minimizes the long-term time-average cost, we have \( \hat{C} \geq C^* \).

Denote \( \hat{p}_{b,i}(t) \), \( \hat{r}_{d,i}(t) \) and \( \hat{C}(t) \) as the optimal results based on the optimal solution of P3 in time slot \( t \). Denote \( p_{b,i}^{*}(t) \), \( r_{d,i}^{*}(t) \) and \( C^*(t) \) as the optimal results of P1 in time slot \( t \). According to (42), we have
\[
\Delta(\Theta(t)) + V E[\hat{C}(t) | \Theta(t)] \\
\leq A + \sum_{i \in I} B_i(t) E[p_{b,i}^{*}(t) | \Theta(t)] \\
+ \sum_{i \in I} H_i(t) E[r_{d,i}(t) - \beta_i | \Theta(t)] + V E[\hat{C}(t) | \Theta(t)] \\
\leq A + \sum_{i \in I} B_i(t) E[p_{b,i}^{*}(t) | \Theta(t)] \\
+ \sum_{i \in I} H_i(t) E[r_{d,i}(t) - \beta_i | \Theta(t)] + V E[C^*(t) | \Theta(t)] \\
= A + \sum_{i \in I} B_i(t) E[p_{b,i}^{*}(t)] \\
+ \sum_{i \in I} H_i(t) E[r_{d,i}^{*}(t) - \beta_i] + V E[C^*(t)].
\]

Since the system state \( s(t) \) is i.i.d., \( p_{b,i}^{*}(t) \) and \( r_{d,i}(t) \) are also i.i.d. stochastic process. Then, according to the strong law of large numbers, we obtain
\[
E[L(\Theta(t + 1)) - L(\Theta(t)) | \Theta(t)] + V E[\hat{C}(t) | \Theta(t)] \\
\leq A + \sum_{i \in I} B_i(t) \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[p_{b,i}^{*}(t)] \\
+ \sum_{i \in I} H_i(t) \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[r_{d,i}^{*}(t) - \beta_i] + V E[C^*(t)].
\]

By taking expectation of the above inequality, we have
\[
E[L(\Theta(t + 1)) - L(\Theta(t)) | \Theta(t)] + V E[\hat{C}(t)] \\
\leq A + \sum_{i \in I} B_i(t) \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[p_{b,i}^{*}(t)] \\
+ \sum_{i \in I} H_i(t) \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[r_{d,i}^{*}(t) - \beta_i] + V E[C^*(t)] \\
\leq A + V E[C^*(t)].
\]

By summing the above inequality over time slots \( t \in \{1, 2, \ldots, T\} \), we have
\[
\sum_{t=1}^{T} V E[\hat{C}(t)] \\
\leq AT + V \sum_{t=1}^{T} E[C^*(t)] - E[L(\Theta(T + 1)) + E[L(\Theta(1))].
\]

Since \( L(\Theta(T + 1)) \) and \( L(\Theta(1)) \) are finite, we divide both sides of the above inequalities by \( VT \) and take limits as \( T \to \infty \) yielding
\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[\hat{C}(t)] \leq A \left( V + \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[C^*(t)] \right).
\]

So far, We have finished the proof.

**REFERENCES**

[1] Q. Yang, S. Sun, S. Deng, Q. Zhao, and M. Zhou, “Optimal sizing of PEV fast charging stations with Markovian demand characterization,” *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 4457–4466, Jul. 2019.

[2] Q. Yan, B. Zhang, and M. Kezunovic, “Optimized operational cost reduction for an EV charging station integrated with battery energy storage and PV generation,” *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 2096–2106, Mar. 2019.

[3] S. Wang, Z. Y. Dong, F. Lao, K. Meng, and Y. Zhang, “Stochastic collaborative planning of electric vehicle charging stations and power distribution system,” *IEEE Trans. Ind. Informat.*, vol. 14, no. 1, pp. 321–331, Jan. 2018.

[4] W. Tushar, T. K. Saha, C. Yuen, D. Smith, and H. V. Poor, “Peer-to-peer trading in electricity networks: An overview,” *IEEE Trans. Smart Grid*, vol. 11, no. 4, pp. 3185–3200, Jul. 2020.

[5] H. Wang and J. Huang, “Incentivizing energy trading for interconnected microgrids,” *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 2647–2657, Jul. 2018.

[6] H. Kim, J. Lee, S. Bahrami, and V. W. S. Wong, “Direct energy trading of microgrids in distribution energy market,” *IEEE Trans. Power Syst.*, vol. 35, no. 1, pp. 639–651, Jan. 2020.

[7] S. Cui, Y. Wang, and J. Xiao, “Peer-to-peer energy sharing among smart energy buildings by distributed transaction,” *IEEE Trans. Smart Grid*, vol. 10, no. 6, pp. 6491–6501, Nov. 2019.
[8] L. Chen, N. Liu, C. Li, and I. Wang, “Peer-to-peer energy sharing with social attributes: A stochastic leader–follower game approach,” IEEE Trans. Ind. Informat., vol. 17, no. 4, pp. 2545–2556, Apr. 2021.

[9] R. Jing, M. N. Xie, F. X. Wang, and L. X. Chen, “Fair P2P energy trading between residential and commercial multi-energy systems enabling integrated demand-side management,” Appl. Energy, vol. 262, Mar. 2020, Art. no. 115551.

[10] Y. Chen, C. Zhao, S. H. Low, and S. Mei, “Approaching prosumer social optimum via energy sharing with proof of convergence,” IEEE Trans. Smart Grid, vol. 12, no. 3, pp. 2484–2495, May 2021.

[11] Y. Chen, C. Zhao, S. H. Low, and A. Wierman, “An energy sharing mechanism considering network constraints and market power limitation,” IEEE Trans. Smart Grid, early access, Aug. 16, 2022, doi: 10.1109/TSG.2022.3198721.

[12] S. Fan, Q. Ai, and L. Piao, “Bargaining-based cooperative energy trading for distribution company and demand response,” Appl. Energy, vol. 226, pp. 469–482, Sep. 2018.

[13] W. Tushar, T. K. Saha, C. Yuen, P. Liddell, R. Bean, and H. V. Poor, “Two-stage stochastic and load scheduling algorithm for the end-user in smart grid,” IEEE Trans. Smart Grid, vol. 8, no. 1, pp. 258–269, Jan. 2020.

[14] N. Liu, M. Cheng, X. Yu, J. Zhong, and J. Lei, “Energy-sharing provider for PV Prosumer clusters: A hybrid approach using stochastic programming and Stackelberg game,” IEEE Trans. Ind. Electron., vol. 65, no. 8, pp. 6740–6750, Aug. 2018.

[15] S. Cui, Y. Wang, J. Xiao, and N. Liu, “A two-stage robust energy sharing management for prosumer microgrid,” IEEE Trans. Ind. Informat., vol. 15, no. 5, pp. 2741–2752, May 2019.

[16] Z. Guo, P. Pinson, Q. Wu, S. Chen, Q. Yang, and Z. Zhang, “An asynchronous online negotiation mechanism for real-time peer-to-peer electricity markets,” IEEE Trans. Power Syst., vol. 37, no. 3, pp. 1868–1880, May 2022.

[17] Y. Zheng, Y. Song, D. J. Hill, and K. Meng, “Online distributed MPC-based optimal scheduling for EV charging stations in distribution systems,” IEEE Trans. Ind. Informat., vol. 15, no. 2, pp. 638–649, Feb. 2019.

[18] E. Stai, C. Wang, and J.-Y. Le Boudec, “Online battery storage management via Lyapunov optimization in active distribution grids,” IEEE Trans. Control Syst. Technol., vol. 29, no. 2, pp. 672–690, Mar. 2021.

[19] S. Fan, G. He, X. Zhou, and M. Cui, “Online optimization for networked distributed energy resources with time-coupling constraints,” IEEE Trans. Smart Grid, vol. 12, no. 1, pp. 251–267, Jan. 2021.

[20] T. Chen, A. Mokhtari, X. Wang, A. Ribeiro, and G. B. Giannakis, “Stochastic averaging for constrained optimization with application to online resource allocation,” IEEE Trans. Signal Process., vol. 65, no. 12, pp. 3078–3093, Jun. 2017.

[21] L. Yu, T. Jiang, and Y. Zou, “Distributed real-time energy management in data center microgrids,” IEEE Trans. Smart Grid, vol. 9, no. 4, pp. 3748–3762, Jul. 2018.

[22] W. Shi, N. Li, C. Chu, and R. Gadh, “Real-time energy management in microgrids,” IEEE Trans. Smart Grid, vol. 8, no. 1, pp. 228–238, Jan. 2017.

[23] N. Liu et al., “Online energy sharing for nanogrid clusters: A Lyapunov optimization approach,” IEEE Trans. Smart Grid, vol. 9, no. 5, pp. 4624–4636, Sep. 2018.

[24] D. Liu, J. Xiao, J. Liu, X. Yuan, and S. Zhang, “Dynamic energy trading and load scheduling algorithm for the end-user in smart grid,” IEEE Access, vol. 8, pp. 189632–189645, 2020.

[25] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, “Distributed optimization and statistical learning via the alternating direction method of multipliers,” Found. Trends Mach. Learn., vol. 3, no. 1, pp. 1–122, 2011.

[26] S. Sun, M. Dong, and B. Liang, “Distributed real-time power balancing in renewable-integrated power grids with storage and flexible loads,” IEEE Trans. Smart Grid, vol. 7, no. 5, pp. 2337–2349, Sep. 2016.

[27] W. Zhong, K. Xie, Y. Liu, C. Yang, S. Xie, and Y. Zhang, “Online control and near-optimal algorithm for distributed energy storage sharing in smart grid,” IEEE Trans. Smart Grid, vol. 11, no. 3, pp. 2552–2562, May 2020.