Non-iterative Symmetric Two-Dimensional Linear Discriminant Analysis

Kohei INOUE†a, Kenji HARA†, and Kiichi URAHAMA†, Members

Linear discriminant analysis (LDA) is one of the well-known schemes for feature extraction and dimensionality reduction of labeled data. Recently, two-dimensional LDA (2DLDA) for matrices such as images has been formulated into symmetric 2DLDA (S2DLDA), which is solved by an iterative algorithm. In this paper, we propose a non-iterative S2DLDA and experimentally show that the proposed method achieves comparable classification accuracy with the conventional S2DLDA, while the proposed method is computationally more efficient than the conventional S2DLDA.

key words: symmetric two-dimensional linear discriminant analysis, face recognition, dimensionality reduction

1. Introduction

Linear discriminant analysis (LDA) is one of the well-known schemes for feature extraction and dimensionality reduction of labeled data. Recently, LDA, which was originally formulated for discriminating labeled vectors [1], has been extended to two-dimensional LDA (2DLDA) for discriminating labeled matrices such as images. Yang et al. [2] proposed a 2DLDA which performs the uncorrelated image matrix-based LDA (IMLDA) proposed by Yang et al. [3] twice: the first and the second times are in horizontal and vertical directions, respectively. Ye et al. [4] also proposed another 2DLDA which attempts to optimize two transformation matrices $L$ and $R$ for reducing dimensions of rows and columns, respectively. In Ye’s 2DLDA [4], $L$ and $R$ are optimized with different objective functions by an alternating algorithm. However, the optimization with $L$ may not be optimal for the objective function for $R$, and vice versa. Ye et al. [4] demonstrated that the accuracy curves for face recognition are stable with respect to the number of iterations, which led them to adopt a single-iteration renewal procedure. We proposed a non-iterative 2DLDA [5] which is a combination of Yang’s 2DLDA [2] and Ye’s one [4]. Luo et al. [6] pointed out the ambiguity problem on the objective function for 2DLDA, i.e., there are several choices for the objective function for 2DLDA, and derived a reasonable objective function for 2DLDA called symmetric 2DLDA (S2DLDA). However, S2DLDA is no longer reduced to any generalized eigenvalue problems. Therefore, they used a gradient ascent approach. However, their algorithm does not necessarily increase the objective function value monotonically as demonstrated in their paper [6], and therefore it is difficult to determine the number of iterations of the renewal procedure.

This work is motivated by the above observation of the state-of-the-art 2DLDA techniques. That is, Luo’s solution to the ambiguity problem in 2DLDA, i.e., S2DLDA, caused other problems that S2DLDA is no longer reduced to any generalized eigenvalue problems and therefore Luo et al. [6] adopted a gradient ascent approach which has the difficulty in determining the optimal number of iterations. If we can reformulate S2DLDA in a way that the reformulation is reduced to a generalized eigenvalue problem, then we will overcome the above problems.

In this paper, we propose a non-iterative S2DLDA which can be reduced to a generalized eigenvalue problem and therefore has an analytical solution. Experimental results show that the proposed method achieves comparable classification accuracy with S2DLDA [6] and is computationally efficient in the training stage.

The rest of this paper is organized as follows: Section 2 summarizes S2DLDA [4]. Section 3 proposes a non-iterative S2DLDA. Section 4 shows experimental results. Finally, Sect. 5 concludes this paper.

2. Symmetric 2DLDA

Let $A_i \in \mathbb{R}^{nxc}$ for $i = 1, \ldots, m$ be $m$ images categorized into $n$ classes $\Pi_1, \ldots, \Pi_c$. Let $M_j = \frac{1}{n_j} \sum_{i \in \Pi_j} A_i$ be the mean of the $j$th class for $j = 1, \ldots, n$ where $n_j = |\Pi_j|$ is the number of elements in $\Pi_j$, and let $M = \frac{1}{r} \sum_{j=1}^{n} \sum_{A_i \in \Pi_j} A_i$ be the global mean. In 2DLDA, the dimensions of $A_i$ are reduced by

$$B_i = L^T A_i R \in \mathbb{R}^{\tilde{n} \times \tilde{c}},$$

where $L \in \mathbb{R}^{nxr}$ for $\tilde{r} \leq r$ and $R \in \mathbb{R}^{cx\tilde{c}}$ for $\tilde{c} \leq c$. In symmetric 2DLDA (S2DLDA), $L$ and $R$ are obtained by solving the following maximization problem:

$$\max_{L,R} \text{tr} \left( (R^T S^L w R)^{-1} R^T S^L L \right) + \text{tr} \left( (L^T S^R w L)^{-1} L^T S^R R \right),$$

(1)

where tr denotes the matrix trace, $S^R_w$ and $S^L_w$ are the within-class scatter matrices defined by

$$S^R_w = \sum_{j=1}^{n} \sum_{A_i \in \Pi_j} (A_i - M_j) R R^T (A_i - M_j)^T,$$
respectively, and \( S \) and \( S_b \) are the between-class scatter matrices defined by

\[
S_b = \sum_{j=1}^{n} n_j (M_j - M)^T (M_j - M),
\]

\[
S = \sum_{j=1}^{n} n_j (M_j - M)(M_j - M)^T,
\]

respectively. We also define the column-column within-class and between-class scatter matrices [5] as

\[
S^L_w = \sum_{j=1}^{n} \sum_{i \in \Pi_j} (A_i - M_j)^T LL^T (A_i - M_j),
\]

\[
S^R_b = \sum_{j=1}^{n} n_j (M_j - M)^R (M_j - M)^T,
\]

\[
S^L_b = \sum_{j=1}^{n} n_j (M_j - M)^L (M_j - M),
\]

The proposed algorithm.

| Algorithm: Non-iterative symmetric 2DLDA |
|------------------------------------------|
| 1. Compute \( S^T \), \( S^R \), \( S^L \), and \( S^c \). |
| 2. Compute \( S_w \) and \( S_b \). |
| 3. Compute the first \( K \) principal eigenvalues \( \lambda_1 \geq \cdots \geq \lambda_K \) of \( \tilde{S}_w^{-1} \tilde{S}_b \) and the corresponding eigenvectors \( \tilde{u}_1, \ldots, \tilde{u}_K \). |
| 4. Initialize \( L = {}^T [1] \) and \( R = [1] \). |
| 5. For \( k = 1, \ldots, K \), do the following: |
| 5-1. Let \( x = [\tilde{u}_{k,1}, \ldots, \tilde{u}_{k,J}]^T \) and \( y = [\tilde{u}_{k,1} \cdots, \tilde{u}_{k,K}]^T \), where \( \tilde{u}_{k,j} \) is the \( j \)-th element of \( \tilde{u}_k \). |
| 5-2. If \( ||x|| \geq ||y|| \), then renew \( L \leftarrow [L, x] \) or else \( R \leftarrow [R, y] \). |
| 6. Reduce the dimension of \( A_i \) by \( \tilde{B}_i = \tilde{L}^T \tilde{A}_i \tilde{R} \). |

We define the row-row within-class and between-class scatter matrix

\[
S^c_i = \sum_{j=1}^{n} n_j (M_j - M)^T (M_j - M),
\]

respectively. Then we form a combined within-class scatter matrix

\[
\tilde{S}_w = \begin{bmatrix} S^r_w & 0 \\ 0 & S^c_w \end{bmatrix},
\]

and a combined between-class scatter matrix

\[
\tilde{S}_b = \begin{bmatrix} S^r_b & 0 \\ 0 & S^c_b \end{bmatrix}.
\]

Now we formulate a non-iterative S2DLDA as follows:

\[
\max_{\tilde{U}} \text{tr} \left( \tilde{U}^T \tilde{S}_w \tilde{U} \right)^{-1} \tilde{U}^T \tilde{S}_b \tilde{U} \right). \tag{2}
\]

Since \( S^r_w \), \( S^r_b \), \( S^c_w \), and \( S^c_b \) do not contain \( L \) and \( R \) and therefore \( \tilde{S}_w \) and \( \tilde{S}_b \) do not contain \( L \) and \( R \) either, the solution to (2) is given by the principal eigenvectors of \( \tilde{S}_w^{-1} \tilde{S}_b \) as in the conventional LDA [1]. Therefore, we can avoid using a gradient ascent approach which has the difficulty in determining the number of iterations.

The procedure of the non-iterative 2S2DLDA is summarized in Table 1. Since the projection matrices \( L \) and \( R \) are intermingled in \( \tilde{U} \), we separate them in steps 5-1 and 5-2 in Table 1 and express them as \( \tilde{L} \) and \( \tilde{R} \). Since \( \tilde{S}_w \) and \( \tilde{S}_b \) are block diagonal, \( \tilde{S}_w^{-1} \tilde{S}_b \) is also block diagonal. Therefore, the \( k \)-th eigenvector \( \tilde{u}_k \) of \( \tilde{S}_w^{-1} \tilde{S}_b \) has the form \( \tilde{u}_k = [\tilde{u}_{k,1}, \ldots, \tilde{u}_{k,J}, 0, \ldots, 0]^T = [x^T, 0, \ldots, 0]^T \) or \( \tilde{u}_k = [0, \ldots, 0, \tilde{u}_{k,J+1}, \ldots, \tilde{u}_{k,K}]^T = [0, \ldots, 0, y^T]^T \). In step 5-2, we select a nonzero subvector \( x \) or \( y \) by comparing their Euclidean norm.

4. Experimental Results

In this section, we evaluate the performance of the proposed non-iterative 2S2DLDA compared with the conventional S2DLDA [6]. We use the ORL face image database [7] in our experiments. Figure 1 shows some face images in this database, which contains the face images of 40 persons. There are 10 different images per person. We use 3 images per person for training, and the remaining 7 images for testing. The size of each image is \( 112 \times 92 \) pixels, i.e., \( r = 112 \) and \( c = 92 \). We set \( \tilde{r} = 10 \) and \( \tilde{c} = 6 \) by leave-one-out
cross-validation in non-iterative S2DLDA. Figure 2 shows the variation in the objective function value \( J \) for S2DLDA, where the horizontal axis denotes the number of iterations and the vertical axis denotes the value of \( J \). As demonstrated in [6], the value of \( J \) fluctuates with the increase in the number of iterations. That is, the gradient ascent approach by Luo et al. [6] does not guarantee that \( J \) increases monotonically. One of the reasons that \( J \) fluctuates in their approach is that \( L \) and \( R \) are normalized once every \( c = 3 \) iterations where \( c \) is a parameter used in their paper [6]. Figure 3 shows the CPU time for training in S2DLDA. The CPU time linearly increases with the number of iterations. We performed our experiments using MATLAB on a Pentium 4 CPU 3.40 GHz machine with 2.00 GB RAM. Figure 4 shows the recognition rate for S2DLDA, where we classified the test set with the nearest-neighbor rule [8]. The highest recognition rate 0.85 is obtained at the 7 iterations.

Next, we show the results for non-iterative S2DLDA. Figure 5 shows the maximum objective function value in (2), where the horizontal axis denotes the dimension of \( \tilde{B}_i = \tilde{L}_i^T \tilde{A}_i \tilde{R} \) or the number of the elements in \( \tilde{B}_i \). The CPU time for training in non-iterative S2DLDA is 0.218 seconds, which is shorter than that (0.469 seconds) for one iteration in S2DLDA. Figure 6 shows the recognition rate for non-iterative S2DLDA. The highest recognition rate 0.889 is obtained when the number of dimensions is \( \tilde{r} = 9 \) and \( \tilde{c} = 6 \). Thus, the proposed non-iterative S2DLDA achieves comparable recognition rate with the conventional S2DLDA, and the CPU time for training in non-iterative S2DLDA is shorter than that in S2DLDA.

Figure 7 shows a confusion matrix for the proposed method. The highest rate in the off-diagonal elements is 0.8 which is the (17, 36) element, i.e., the 17th person is likely to be confused with the 36th person. Figures 8 and 9 show the face images of the 36th person for training and that of the 17th person for testing, respectively.

We compared the performance of the proposed method with that of the conventional methods: no dimension reduction (baseline), PCA+LDA, 2DLDA [4], S2DLDA [6] and image shrinkage by a photo-retouch software (Adobe Photoshop CS). We excluded the conventional LDA from...
Table 2  Classification accuracy.

| Dataset  | baseline | PCA+LDA | 2DLDA | S2DLDA | photo-retouch | proposed |
|----------|----------|---------|-------|--------|---------------|----------|
| ORL      | 0.857    | 0.768   | 0.829 | 0.811  | 0.864         | 0.889    |
| IFD      | 0.778    | 0.750   | 0.534 | 0.773  | 0.824         | 0.818    |
| UMIST    | 0.556    | 0.569   | 0.272 | 0.484  | 0.600         | 0.663    |
| CHFD     | 0.654    | 0.520   | 0.109 | 0.640  | 0.548         | 0.815    |

5. Conclusion

In this paper, we proposed a non-iterative symmetric 2DLDA for supervised dimensionality reduction of matrices such as two-dimensional images. Experimental results show that the proposed method achieves comparable recognition rate with the conventional symmetric 2DLDA and is computationally efficient in the training stage of face recognition. We also compared the performance of the proposed method with that of no dimension reduction method (baseline), PCA+LDA, 2DLDA [4] and image shrinkage by a photo-retouch software on four benchmark datasets.

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