Scaling properties of flow harmonics of deuterons and protons in Au+Au reactions at 1.23 AGeV

Paula Hillmann$^{1,2,3,4}$, Jan Steinheimer$^2$, Tom Reichert$^1$, Vincent Gaebel$^1$, Marcus Bleicher$^{1,2,3,4}$, Sukanya Sombun$^5$, Christoph Herold$^5$, Ayut Limphirat$^5$

$^1$ Institut für Theoretische Physik, Goethe Universität Frankfurt, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany
$^2$ Frankfurt Institute for Advanced Studies, Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany
$^3$ GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstr. 1, 64291 Darmstadt, Germany
$^4$ John von Neumann-Institut für Computing, Forschungszentrum Jülich, 52425 Jülich, Germany
$^5$ School of Physics and Center of Excellence in High Energy Physics & Astrophysics, Suranaree University of Technology, Nakhon Ratchasima 30000, Thailand

Abstract. We explore the scaling properties of the elliptic and quadrangular flow of deuterons and protons in Au+Au reactions at a beam energy of 1.23 AGeV within the UrQMD approach. These investigations are of great interest for the HADES experiment at GSI that has recently studied the flow of light nuclei. In the present studies, deuterons are formed via phase-space coalescence. Our results agree well with the experimental data, if for this low energy a hard equation of state is considered. We observe that protons and deuterons show direct scaling with the mass number $A$. We also see scaling of higher order flow harmonics, e.g. $v_4 \sim v_2^2$.

1. Introduction
Heavy-ion collisions in modern accelerators offer the opportunity to study hot and dense nuclear matter in the lab as it was present in the early universe and nowadays is expected to be found in neutron stars or other compact stellar objects or even neutron star mergers. While in the case of low temperatures and high densities the nuclear equation of state (EoS) dominates the research interest, at high temperatures and low baryon chemical potential the exploration of the Quark-Gluon-Plasma is the main goal. For both areas the collective flow is a promising observable to study the collectivity of the evolving system due to its sensitivity to initial pressure gradients [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The flow components $v_n$ are given by the Fourier-transform of the momentum distribution,

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_{RP})] \right).$$  

To calculate the flow components one has to average over all particles in events within a given centrality class [2]:

$$v_n(p_T, y) = \langle \cos[n\varphi] \rangle.$$
### Parameters used in the UrQMD Skyrme potential for a hard equation of state [24].

| Parameters | Value |
|------------|-------|
| $\alpha$ [MeV] | -124 |
| $\beta$ [MeV] | 71 |
| $\gamma$ | 2.00 |

Table 1.

In transport simulations the reaction-plane is known by the initial geometry of the system (i.e. $\Psi_{RP} = 0$ in the calculations, assuming that fluctuations can be neglected) and therefore one only has to calculate the azimuthal angle $\varphi$.

The formation of nuclear clusters is currently a wide discussed topic and can have a huge impact on the physics happening at low energies. The HADES experiment has acquired a substantial amount of data on gold-gold collisions at 1.23 AGeV and is therefore able to measure higher order flow components of light nuclei with high precision [12, 13, 14, 15]. The main goal of this paper is to study the scaling properties of deuteron and proton elliptic and quadrangular flow at the HADES energy regime using the UrQMD transport model with deuteron formation via coalescence [16, 17, 18, 19].

### The UrQMD model

For the study of the collective flow we apply the Ultra relativistic Quantum Molecular Dynamics (UrQMD) transport model. The model includes many processes such as binary elastic and inelastic scattering of hadrons, resonance excitations and decays as well as string dynamics and strangeness exchange reactions [17, 18, 19]. The model uses a geometrical interpretation of the nuclear cross section which is taken from experimental data [20], if possible. Less known processes are described by effective model calculations. The cascade version of the transport model describes the mean particle production well [6, 26]. At the HADES beam energy of 1.23 AGeV potentials need to be included in the simulation to describe the dynamics of the hadrons well [21, 22].

For the current studies a hard EoS is applied [23]. The interaction with the nuclear mean-field is given by a Skyrme-type potential $V_{Sk}$ [24]:

$$V_{Sk} = \alpha \cdot \left( \frac{\rho_{int}}{\rho_0} \right) + \beta \cdot \left( \frac{\rho_{int}}{\rho_0} \right)^\gamma. \quad (3)$$

The parameters $\alpha$, $\beta$ and $\gamma$ describe the stiffness of the EoS. In the calculations we do not include a momentum dependent or isospin dependent potential which were discussed in the literature [21, 22, 25]. We use the same set-up which was successful to describe the collective flow of protons previously [23].

The deuterons for the flow calculations are formed via phase-space coalescence after kinetic freeze-out [16]. To calculate the clusters, one boosts into the local rest-frame of all possible proton-neutron pairs. If the relative coordinate distance $\Delta r$ and the relative momentum distance $\Delta p$ fulfill the conditions $\Delta r = |r_p - r_n| < \Delta r_{max} = 3.575$ fm and $\Delta p = |p_p - p_n| < \Delta p_{max} = 0.285$ GeV, a deuteron is formed with the probability of $3/8$ (probability of spin-isospin coupling). With this set of parameters one can successfully describe deuteron production for a large body of different systems and energies [16].

### Results

In the following we discuss the mass number scaling properties of elliptic and quadrangular flow of protons and deuterons as function of transverse momentum for semi-peripheral Au+Au reactions for the case of low temperatures and dense matter ($E_{lab} = 1.23$ AGeV).
At first we have a look on the transverse momentum spectra of the elliptic flow of protons and deuterons in comparison. Figures 1 and 2 show the elliptic flow of protons (left) and deuterons (right) as function of transverse momentum for various rapidity bins in Au+Au collisions at a fixed target beam energy of 1.23 AGeV. The lines denote the UrQMD calculations \((b = 6−9\text{ fm})\) and the symbols denote the preliminary HADES data. The calculations agree well with the experimental data. One can observe that the deuterons show a similar behaviour than the protons for the same rapidity windows.

Due to the effect that the deuteron carries the momentum of two nucleons and \(v_2\) being additive one can expect a mass number scaling of the elliptic flow of protons and deuterons as a direct consequence of coalescence.

Figure 3 shows the elliptic flow (left) as function of transverse momentum divided by the mass number \(A\) in Au+Au collisions at a fixed target beam energy of 1.23 AGeV. The solid lines show the UrQMD calculations of the proton flow and the dashed lines the deuteron flow \((b = 6−9\text{ fm})\) divided by the mass number \(A\). As expected above one can observe a direct scaling of the elliptic flow of protons and deuterons. This scaling was also confirmed by the HADES data.

Figure 4 shows the quadrangular flow \(v_4\) as function of \(p_T^2\) for protons and deuterons in Au+Au collisions at a beam energy of 1.23 AGeV. The solid line shows the UrQMD calculations of the proton flow and the dashed line the deuteron flow \((b=6-9\text{ fm})\). Even more surprising is that a mass number scaling can be also found for the quadrangular flow. This is a strong indicator that deuterons are formed via phase-space coalescence. Furthermore, a nonzero \(v_4\) with respect to the reaction plane indicates a connection between the initial stage and the expansion stage of the system.

4. Summary
We used the Ultra-relativistic Quantum Molecular Dynamics transport model including phase-space coalescence to calculate the proton and deuteron flow in semi-peripheral gold-gold collisions. The lines indicate the UrQMD calculations \((b = 6−9\text{ fm})\) and the symbols denote the preliminary HADES data.

**Figure 1.** [Color online] Elliptic flow of protons in Au+Au collisions as a function of transverse momentum and for various rapidity bins at a fixed-target beam energy of 1.23 AGeV. The lines indicate the UrQMD calculations \((b = 6−9\text{ fm})\) and the symbols denote the preliminary HADES data [13].

**Figure 2.** [Color online] Elliptic flow of deuterons in Au+Au collisions as a function of transverse momentum and for various rapidity bins at a fixed-target beam energy of 1.23 AGeV. The lines indicate the UrQMD calculations \((b = 6−9\text{ fm})\) and the symbols denote the preliminary HADES data [15].
The elliptic and quadrangular flow. Furthermore the spectra agree well with the HADES data and a mass number scaling is observable both for free protons (solid line) and deuterons (dashed line) in Au+Au collisions as a function of transverse momentum and for $|y| < 0.05$ scaled with the mass number $A$ at a fixed-target beam energy of 1.23 A GeV. The lines indicate the UrQMD calculations ($b = 6 - 9$ fm).

For $E_{lab}=1.23$ A GeV a hard Skyrme-type equation of state was used. The flow spectra agree well with the HADES data and a mass number scaling is observable both for the elliptic and quadrangular flow. Furthermore the $v_3 \neq 0$ with respect to the reaction plane indicates a connection between the expansion of the system and the initial stage at low energies.

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