Holographic Dark Energy in Brans-Dicke Theory

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In this paper, the holographic dark energy model is considered in Brans-Dicke theory where the holographic dark energy density \( \rho = \frac{3 c^2 M_{pl}^2}{L^2} \) is replaced with \( \rho_h = \frac{3 c^2 \Phi(t)}{L^2} \). Here \( \Phi(t) = \frac{1}{8 \pi G} \) is a time variable Newton constant. With this replacement, it is found that no accelerated expansion universe will be achieved when the Hubble horizon is taken as the role of IR cut-off. When the event horizon is adopted as the IR cut-off, an accelerated expansion universe is obtained. In this case, the equation of state of holographic dark energy \( w_h \) takes a modified form \( w_h = \frac{-1}{3} \left( 1 + \alpha + \frac{2}{c} \sqrt{\Omega_h} \right) \). In the limit \( \alpha \to 0 \), the 'standard' holographic dark energy is recovered. In the holographic dark energy dominated epoch, power-law and de Sitter time-space solutions are obtained.

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I. INTRODUCTION

The observation of the Supernovae of type Ia [1, 2] provides the evidence that the universe is undergoing accelerated expansion. Jointing the observations from Cosmic Background Radiation [3, 4] and SDSS [5, 6], one concludes that the universe at present is dominated by 70% exotic component, dubbed dark energy, which has negative pressure and push the universe to accelerated expansion. Of course, the accelerated expansion can attribute to the cosmological constant naturally. However, it suffers the so-called fine tuning and cosmic coincidence problem. To avoid these problem, dynamic dark energy models are considered, such as quintessence [7, 8, 9, 10], phantom [11], quintom [12] and holographic dark energy [13, 14] etc. To explain the accelerated expansion, modified gravity theories are explored too. For recent reviews, please see [13, 16, 17, 18, 19, 20]. Brans-Dicke theory [21] as a natural extension of Einstein’s general theory of relativity can pass the experimental tests from the solar system [22] and provide explanation to the accelerated expansion of the universe [25, 26, 27]. In Brans-Dicke theory, the gravitational constant is replaced with a inverse of time dependent scalar field, i.e. \( 8 \pi G = \frac{1}{\Phi(t)} \), which couples to gravity with a coupling parameter \( \omega \).

Recently, a model named holographic dark energy has been discussed extensively. The model is constructed by considering the holographic principle and some features of quantum gravity theory. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and has relations with the area of its boundary. By applying the principle to cosmology, one can obtain the upper bound of the entropy contained in the universe. For a system with size \( L \) and UV cut-off \( \Lambda \) without decaying into a black hole, it is required that the total energy in a region of size \( L \) should not exceed the mass of a black hole of the same size, thus \( L^3 \rho_\Lambda \leq LM_{pl}^{-2} \). The largest \( L \) allowed is the one saturating this inequality, thus \( \rho_\Lambda = \frac{3 c^2 M_{pl}^2}{L^2} \), where \( c \) is a numerical constant and \( M_{pl} \) is the reduced Planck Mass \( M_{pl}^2 = 8 \pi G \). It just means a duality between UV cut-off and IR cut-off. The UV cut-off is related to the vacuum energy, and IR cut-off is related to the large scale of the universe, for example Hubble horizon, event horizon or particle horizon as discussed by [13, 14]. In the paper [14], the author takes the future event horizon

\[
R_{eh}(a) = a \int_0^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{Ha'^2} \tag{1}
\]
as the IR cut-off \( L \). This horizon is the boundary of the volume a fixed observer may eventually observe. One is to formulate a theory regarding a fixed observer within this horizon. As pointed out in [14], it can reveal the dynamic nature of the vacuum energy and provide a solution to the fine tuning and cosmic coincidence problem. In this model,

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the value of parameter $c$ determines the property of holographic dark energy. When $c \geq 1$, $c = 1$ and $c \leq 1$, the holographic dark energy behaviors like quintessence, cosmological constant and phantom respectively.

II. FRIEDMANN EQUATION IN BRANS-DICKE THEORY

In this paper, we generalized the holographic dark energy model to that in the framework of Brans-Dicke theory which has already been considered by many authors [28, 29, 30, 31, 32]. Then, it takes the general form

$$\rho_h = 3c^2 \Phi(t) L^{-2},$$

where $\Phi(t) = \frac{1}{8\pi G}$ is a reverse of time variable Newton constant. In a spatially flat FRW cosmology filled dark matter and holographic dark energy, the gravitational equations can be written as

$$3 \Phi \left[ H^2 + \frac{\dot{\Phi}}{\Phi} - \frac{\omega \dot{\Phi}^2}{6 \Phi^2} \right] = \rho_m + \rho_h,$$  (3)

$$2 \ddot{a} + H^2 + \frac{\omega \dot{\Phi}^2}{2 \Phi^2} + 2H \frac{\dot{\Phi}}{\Phi} + \frac{\ddot{\Phi}}{\Phi} = -p_h,$$  (4)

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, $\rho_m$ is dark matter energy density, $\rho_h$ is the holographic dark energy density and $p_h$ is the pressure of holographic dark energy. The scalar field evolution equation is

$$\ddot{\Phi} + 3H \frac{\dot{\Phi}}{\Phi} = \frac{\rho_m + \rho_h - 3p_h}{2\omega + 3}.$$  (5)

Considering the dark matter energy conservation equation

$$\dot{\rho}_m + 3H \rho_m = 0,$$  (6)

and jointing it with Eq. (3), Eq. (4) and Eq. (5), one obtains the holographic dark energy conservation equation

$$\dot{\rho}_h + 3H (\rho_h + p_h) = 0.$$  (7)

Here, we have considered non-interacting cases. The Friedmann equation (3) is

$$H^2 = \frac{\rho_m + \rho_h}{3\Phi} - H \frac{\dot{\Phi}}{\Phi} + \frac{\omega \dot{\Phi}^2}{6 \Phi^2}.$$  (8)

With the assumption $\Phi/\Phi_0 = (a/a_0)^\alpha$, the Eq. (8) is rewritten as

$$H^2 = \frac{2}{(6 + 6\alpha - \omega \alpha^2)\Phi} (\rho_m + \rho_h).$$  (9)

It is easy to find out that, in the limit case $\alpha \to 0$, the standard cosmology is recovered. To make the Friedmann equation (9) to have physical meanings, i.e. to make $(6 + 6\alpha - \omega \alpha^2) > 0$, one has the following constraints on the values of $\alpha$

$$\frac{3 - \sqrt{9 + 6\omega}}{\omega} < \alpha < \frac{3 + \sqrt{9 + 6\omega}}{\omega}, \quad \omega > 0,$$

$$\alpha < \frac{3 - \sqrt{9 + 6\omega}}{\omega} \quad \text{or} \quad \alpha > \frac{3 + \sqrt{9 + 6\omega}}{\omega}, \quad -3/2 \leq \omega < 0,$$

$$\Re, \quad \omega < -3/2.$$  (10)

However, the solar system experiments predict the value of $\omega$ is $|\omega| > 40000$ [22]. However, the value of parameter $\omega = -3/2$ is a boundary of ghost [23]. So, in this paper, when considering these constraints, the second line of Eq. (10) will be omitted and $\omega > 40000$ will be consider in this paper. In fact, authors [24] have used the cosmic observations to constrain the parameter $\omega$. In [24], the authors found that $\omega$ can be smaller than 40000 in cosmological scale.
III. HUBBLE HORIZON AS IR CUT-OFF

At first, we consider the Hubble horizon as the IR cut-off, i.e. \( L = H^{-1} \). Then, the holographic dark energy is rewritten as

\[
\rho_h = 3c^2 \Phi H^2. \tag{11}
\]

Inserting Eq. (11) into Eq. (9), one has

\[
H^2 = H_0^2 \frac{\Omega_{m0}}{1 - \Omega_{h0}} \left( \frac{a_0}{a} \right)^{3+\alpha}, \tag{12}
\]

where \( \Omega_{m0} = \frac{2}{(6+6\alpha-\omega\alpha^2)} \frac{\rho_{m0}}{\Phi_0 H_0^2} \) and \( \tilde{\Omega}_{h0} = \frac{6c^2}{6+6\alpha-\omega\alpha^2} \). Then, the equation (12) has the solution

\[
a(t) = \left[ \frac{3 + \alpha}{2} \frac{\Omega_{m0} H_0^2}{1 - \tilde{\Omega}_{h0}} \right]^{\frac{3+\alpha}{2}} a_0 t^{\frac{2}{3+\alpha}} \tag{13}.\]

To have an accelerated expansion, the condition \( \frac{2}{3+\alpha} > 1 \) is required, i.e. \( \alpha < -1 \). By combining Eq. (10) and taking the solar system constraint into account, one can only take the value of \( \omega < -40000 \) to make an accelerated expansion of the universe in Brans-Dicke theory when the Hubble horizon is taken as the IR cut-off. However, if one considers the current value of \( \Phi_0 = 1/8\pi G \) and lets \( \Omega_{m0} = \frac{2}{(6+6\alpha-\omega\alpha^2)} \frac{\rho_{m0}}{\Phi_0 H_0^2} = \frac{8\pi G \rho_{m0}}{3H_0^2} \), the relations \( \omega_0 = 6 \) and \( \tilde{\Omega}_{h0} = c^2 \) will be derived. Then, combining with solar system constraint \( |\omega| > 40000 \) and avoidance of ghost, one obtains

\[
0 < \alpha < -\frac{3}{20000}. \tag{14}\]

So, under this stronger condition, no accelerated expansion universe will be achieved in Brans-Dicke theory without interactions, when the Hubble horizon is taken as the IR cut-off.

IV. EVENT HORIZON AS IR CUT-OFF

Now, as done in [14], the event horizon \( R_{eh} \) is taken as the IR cut-off. Then, the holographic dark energy is

\[
\rho_h = \frac{3c^2 \Phi}{R_{eh}}. \tag{15}\]

And, the Friedmann Eq. (9) is rewritten as

\[
H^2 = H_0^2 \Omega_{m0} \left( \frac{a_0}{a} \right)^{(3+\alpha)} + \Omega_h H^2 = H_0^2 \Omega_{m0} a^{-3(3+\alpha)} + \Omega_h H^2, \tag{16}\]

where \( \Omega_h = \frac{2}{6+6\alpha-\omega\alpha} \frac{\rho_{h0}}{\Phi_0 H_0^2} = \tilde{\Omega}_{h0} \frac{1}{H_0 R_{eh}} \). For convenience, the scale factor \( a \) has been normalized to \( a_0 = 1 \). Jointing Eq. (15) and Eq. (11), one has

\[
\int_a^\infty \frac{d\ln a'}{Ha'} = \frac{1}{aH} \sqrt{\tilde{\Omega}_{h0} \over \Omega_h}. \tag{17}\]

From Eq. (16), one obtains

\[
\frac{1}{Ha} = \sqrt{a^{(1+\alpha)} (1 - \Omega_h)} \frac{1}{H_0 \sqrt{\Omega_{m0}}}. \tag{18}\]

Inserting the above equation into Eq. (17), one has

\[
\int_x^\infty e^{(1+\alpha)x'} \sqrt{1 - \Omega_h} dx' = e^{\frac{(1+\alpha)x}{2}} \sqrt{\tilde{\Omega}_{h0}} \sqrt{\frac{1}{\Omega_h}} - 1, \tag{19}\]
where \( x = \ln a \). Taking derivative with respect to \( x = \ln a \) from both sides of the above equation, one has the differential equation of \( \Omega_h \)

\[
\Omega_h' = \Omega_h \left( 1 - \Omega_h \right) \left( 1 + \alpha + \frac{2}{\sqrt{\Omega_{h0}}} \sqrt{\Omega_h} \right),
\]

(20)

where \('\) denotes the derivative with respect to \( x = \ln a \). This equation describes the evolution of dimensionless energy density of dark energy. It can be solved exactly,

\[
\ln \Omega_h - 2 \ln \left( 1 + \alpha + 2 \frac{\sqrt{\Omega_{h0}}}{\sqrt{\Omega_{h0}}} \right) - \frac{(1 + \alpha)}{2} \sqrt{\Omega_{h0}} \left[ \ln(1 - \sqrt{\Omega_h}) - \ln(1 + \sqrt{\Omega_h}) \right] + \frac{(1 + \alpha)^2}{4} \Omega_{h0} \ln(1 - \Omega_h) = - (1 + \alpha) \left( \frac{1 + \alpha}{4} \Omega_{h0} - 1 \right) \ln a + C_0,
\]

(21)

where \( C_0 \) is an integration constant which can be obtained by setting \( a_0 = 1 \)

\[
C_0 = \ln \Omega_{h0} - 2 \ln \left( 1 + \alpha + 2 \frac{\sqrt{\Omega_{h0}}}{\sqrt{\Omega_{h0}}} \right) - \frac{(1 + \alpha)}{2} \sqrt{\Omega_{h0}} \left[ \ln(1 - \sqrt{\Omega_h}) - \ln(1 + \sqrt{\Omega_h}) \right] + \frac{(1 + \alpha)^2}{4} \Omega_{h0} \ln(1 - \Omega_h) - \ln \Omega_{h0}.
\]

(22)

It is obvious that \( \Omega_h \) is a function of \( \alpha, c, \Omega_{h0} \) and \( \Omega_{\phi0} \) (or \( \omega \)). Then, the Friedmann equation (16) is rewritten as

\[
H^2 = H_0^2 \frac{\Omega_{m0} a^{-3(1+\alpha)}}{1 - \Omega_h} = H_0^2 \frac{\Omega_{m0}(1 + z)^{(3+\alpha)}}{1 - \Omega_h},
\]

(23)

where \( a_0/a = 1 + z \) is used in the second equal sign. From the conservation equation of the holographic dark energy (4), on has the equation of state (EoS) of holographic dark energy

\[
w_h = -1 - \frac{1}{3} \frac{d \ln \rho_h}{d \ln a} = -\frac{1}{3} \left( 1 + \alpha + \frac{2}{\sqrt{\Omega_{h0}}} \sqrt{\Omega_h} \right) = \frac{1}{3} \left( 1 + \alpha + \frac{2}{c} \sqrt{\Omega_h} \right),
\]

(24)

where \( w_h = p_h/\rho_h \). The formula \( \rho_h = \frac{\Omega_{h0}}{1 - \Omega_h} \rho_{m0} a^{-3} \) and the relation Eq. (20) is used in the second equal sign. The third equal sign is obtained by inserting the relation \( \Omega_{h0} = c^2 \). From the above equation, one finds the EoS of holographic dark energy is in the range of

\[
-\frac{1}{3} \left( 1 + \alpha + \frac{2}{c} \right) < w_h < -\frac{1}{3} (1 + \alpha),
\]

(25)

when one considers the holographic dark energy density ratio \( 0 \leq \Omega_h \leq 1 \). Also, by using the Eq. (4) and the assumption \( \Phi/\Phi_0 = (a/a_0)^\alpha \), one obtains the deceleration parameter as follows

\[
q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \alpha + \frac{\alpha}{8 + 2\alpha} + \frac{6w_h \Omega_h}{4 + \alpha}.
\]

(26)

It is clear that the ‘Standard’ holographic dark energy will be recovered in the limit \( \alpha \to 0 \). In Brans-Dicke theory case, the dynamic behavior of the holographic dark energy is determined by the parameters \( c \) and \( \alpha \). The holographic dark energy can be quintessence, phantom and quitom as that in the Standard case. The cosmic observational constraints on holographic dark energy have been discussed by many authors [33, 34, 35, 36, 37, 38, 39, 40]. The joint analysis of SN, CMB shift parameter and BAO datasets, see [40], gives the results of the parameters in 1\( \sigma \) range: \( c = 0.91^{+0.26}_{-0.18} \) and \( \Omega_{m0} = 0.29 \pm 0.03 \). In this paper, instead of giving any cosmic observational constraints to the holographic dark energy in Brans-Dicke theory, we are going to give some characteristic values of parameters to describe the possible properties and evolutions of this kind of dark energy. Obviously, with a positive value of \( \alpha \), the range of value the EoS of the holographic dark energy is enlarged. In Fig 1, the evolutions of EoS \( w_h(z) \) and dimensionless density parameter \( \Omega_h(z) \) of the holographic dark energy and the deceleration parameter \( q(z) \) with respect to the redshift \( z \) are plotted, where different parameter values \( \Omega_{h0} = 0.73, c = 0.31 (c = 0.91) \) and \( \alpha = 0.00005 \) are adopted. It is clear that an accelerated expansion of the universe is obtained as shown in the bottom panels of Fig. 1. Now, one will
FIG. 1: The evolutions of EoS $w_h(z)$, dimensionless density parameter $\Omega_h(z)$ of the holographic dark energy and the deceleration parameter $q(z)$ with respect to the redshift $z$ in Brans-Dicke theory, where the values $\Omega_{h0} = 0.73$, $c = 0.31 (c = 0.91)$ and $\alpha = 0.00005$ are adopted.

Now, we study the scalar field solution in holographic dark energy dominated epoch, under the assumptive solution $\Phi / \Phi_0 = (a/a_0)^\alpha$. In the holographic dark energy dominated epoch, the dimensionless energy density of holographic
dark energy is $\Omega_h \approx 1$, and the dark matter energy density can be neglected, i.e. $\rho_m \approx 0$. Also, in this epoch, the holographic dark energy density equation (15) can be rewritten as

$$\rho_h = \frac{3\alpha^2 \Phi}{R_{ch}^2} = 3H^2 \Phi. \quad (31)$$

So, the evolution equation (5) of the scalar field $\Phi$ is reduced to

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{\rho_h - 3\rho_h}{2\omega + 3} = \beta H^2 \Phi, \quad (32)$$

where $\beta = \frac{\alpha^2 + 2\alpha + 2/\omega}{4-\alpha}$ is a constant, and the second equal sign is obtained by considering the EoS (24) of the holographic dark energy and Eq. (31). Substituting the assumptive solution of $\Phi/\Phi_0 = (a/a_0)^\alpha$ into the above equation, one has

$$\alpha \eta \ddot{\eta} + (\alpha^2 + 2\alpha - \beta) \dot{\eta}^2 = 0, \quad (33)$$

where $\eta = a/a_0$ is used. This differential equation has the solutions of power law and exponential. The power law solution is

$$\eta = C_1 \left[ (\alpha^2 + 3\alpha - \beta) t - C_2 \alpha \right]^{(\alpha^2 + 3\alpha - \beta)}, \quad (34)$$

where $C_1$ and $C_2$ are integral constant which are determined by the initial condition $\eta_0 = 1$ and $\dot{\eta}_0 = \frac{\omega h}{a_0} = H_0$,

$$1 = C_1 \left[ (\alpha^2 + 3\alpha - \beta) t_0 - C_2 \alpha \right]^{(\alpha^2 + 3\alpha - \beta)}, \quad (35)$$

$$H_0 = C_1 \alpha \left[ (\alpha^2 + 3\alpha - \beta) t_0 - C_2 \alpha \right]^{(\alpha^2 + 3\alpha - \beta)^{-1}}, \quad (36)$$

here $t_0$ denotes the present time or the time when cold dark matter can be neglected. To obtain a power law accelerated expansion, one needs $\alpha/ (\alpha^2 + 3\alpha - \beta) > 1$, i.e. $0 < \alpha < \frac{-5 + \sqrt{1 + 8/c}}{2}$ and $c < 1/3$. When $\alpha^2 + 3\alpha - \beta = 0$ is respected, one has an exponential solution

$$\eta = C_1 \exp(\lambda t), \quad (37)$$

where $\lambda = H_0$ and $C_1 = \exp(-\lambda t_0)$. Here, the case of $\lambda < 0$ is omitted, for its no accelerated properties. This solution describes a de Sitter time-space, the result is consistent with conventional consciousness. But in this case ($c \leq 1/5$), $\alpha$ and $c$ has the algebraic relation

$$\alpha = \frac{-6 + \sqrt{8/c - 4}}{2}, \quad (38)$$

when $c \rightarrow 1/5$, one has $\alpha \rightarrow 0$, for example $c = 0.199998$, $\alpha = 0.000017$. This is not surprising, because we have assume the special solution $\Phi/\Phi_0 = (a/a_0)^\alpha$. So, in the limit of holographic dark energy dominated epoch, a de Sitter like time-space can be obtained.

VI. CONCLUSIONS

In this paper, the holographic dark energy model is explored in Brans-Dicke theory where the holographic dark energy density $\rho_h = 3c^2 M_p^2 L^{-2}$ is replaced with $\rho_h = 3c^2 \Phi(t) L^{-2}$. Here $\Phi(t) = \frac{1}{8\pi G}$ is a time variable Newton constant. With this replacement in Brans-Dicke theory, it is found that no accelerated expansion universe will be achieved when the Hubble horizon is taken as the role of IR cut-off. When the event horizon takes the role of IR cut-off, an accelerated expansion universe is obtained. In this case, the equation of state of holographic dark energy $\omega_h$ takes in a modified form $\omega_h = -\frac{1}{3} (1 + \alpha + \frac{\omega}{2} \sqrt{16\alpha})$. In the limit $\alpha \rightarrow 0$, the 'standard' holographic dark energy is recovered. In the Brans-Dicke theory case of holographic dark energy, the properties of the holographic dark energy is determined by the parameter $c$ and $\alpha$ together. These parameters would be obtained by confronting with cosmic observational data. In stead of doing that, some characteristic values of the parameters are given to describe the possible properties and evolutions of the holographic dark energy in Brans-Dicke case, see Fig. 1. With this special solution $\Phi/\Phi_0 = (a/a_0)^\alpha$, one find power law and de Sitter like time-space solutions in holographic dark energy dominated epoch.
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