Dynamic CT Reconstruction from Limited Views with Implicit Neural Representations and Parametric Motion Fields

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Abstract

Reconstructing dynamic, time-varying scenes with computed tomography (4D-CT) is a challenging and ill-posed problem common to industrial and medical settings. Existing 4D-CT reconstructions are designed for sparse sampling schemes that require fast CT scanners to capture multiple, rapid revolutions around the scene in order to generate high quality results. However, if the scene is moving too fast, then the sampling occurs along a limited view and is difficult to reconstruct due to spatiotemporal ambiguities. In this work, we design a reconstruction pipeline using implicit neural representations coupled with a novel parametric motion field warping to perform limited view 4D-CT reconstruction of rapidly deforming scenes. Importantly, we utilize a differentiable analysis-by-synthesis approach to compare with captured x-ray sinogram data in a self-supervised fashion. Thus, our resulting optimization method requires no training data to reconstruct the scene. We demonstrate that our proposed system robustly reconstructs scenes containing deformable and periodic motion and validate against state-of-the-art baselines. Further, we demonstrate an ability to reconstruct continuous spatiotemporal representations of our scenes and upsample them to arbitrary volumes and frame rates post-optimization. This research opens a new avenue for implicit neural representations in computed tomography reconstruction in general.

1. Introduction

Computed-tomography (CT) is a mature imaging technology with vital medical and industrial applications [19, 13, 8]. CT scanners capture x-ray data or sinograms by scanning a sequence of angles around an object. Reconstruction algorithms then estimate the scene from these measured sinograms. CT imaging in both 2D and 3D of static objects is a well-studied inverse problem with both theoretical and practical algorithms [20, 36, 1].

However, reconstruction of dynamic scenes (i.e. scene features changing over time), known as dynamic 4D-CT, is a severely ill-posed problem because the sinogram aggregates measurements over time which yields spatio-temporal ambiguities [12, 23, 50, 51, 29]. Analogous to motion blur, a static or quasi-static scene is only captured for a small angular range of the sinogram (motion blur analogy: short exposure), and this mapping is a function of the amount of scene motion relative to the CT scanner’s rotation speed. Traditional CT reconstruction algorithms have limited capability to address 4D-CT problems due to difficulty accounting for this motion. Yet solving these 4D-CT problems is critically important for a range of applications from clinical diagnosis to non-destructive evaluation for material characteristics and metrology.

4D-CT reconstruction techniques have been proposed in the literature to handle periodic motion [32, 23] as well as more general nonlinear, deformable motion [50, 51, 29]. While the latter methods achieve state-of-the-art for deformable motion, they typically assume slow motion relative to the scanner rotation speed. Specifically, these algorithms assume sparse measurements of the object that span the full angular range (0 – 360°) and typically require multiple revolutions around the sample to reconstruct images at each time step. This approach is called sparse view CT in the literature [30]. However, this setting is not always practical such as when the object motion is too fast for the scanner to make multiple revolutions. Instead, an alternative approach is to collect measurements over partial angular ranges in a single revolution – which is the more challenging limited-view reconstruction [42]. Even in the static case, the limited-view problem is challenging due to missing information often leading to significant artifacts [17].

Key Contributions: In this paper, we propose a novel,
training-data-free approach for 4D-CT reconstruction that works especially well in limited-view scenarios. Our method, illustrated in Figure 1, consists of an implicit neural representation (INR) [28] model that acts as the static scene prior coupled with a parametric motion field to estimate an evolving 3D object over time. The reconstruction is then synthesized into sinogram measurements using a differentiable Radon transform to simulate parallel-beam CT scanners. By minimizing the discrepancy between the synthesized and observed sinograms, we are able to optimize both the INR weights and motion parameters in a self-supervised, analysis-by-synthesis fashion to obtain accurate dynamic scene reconstructions without training data.

We leverage recent advances in INRs that use positional encoding to map input coordinates to volume density coefficients. In our experiments, we show that this approach is adept for inverting CT measurements and outperforms conventional methods in reconstruction quality and robustness to noise. Further, since our method learns a continuous representation of both volume and time, we are able to solve the reconstruction problem at low resolutions and then supersolve our scenes spatio-temporally after the fact.

Validation: Acquiring 4D-CT data is challenging and one of the primary bottlenecks for research in this area. While this is partly due to the expense and logistics of accessing CT scanners and data, it is also because acquiring ground truth in the 4D case is exceptionally challenging. While there are examples of real CT data [50], these datasets are specific to certain scanners and specialized, sparse sampling schemes. To address this lack of data and highlight our method’s ability to resolve 4D scenes from limited angles, we introduce a synthetic dataset for parallel beam CT. This dataset is generated with an accurate physics simulator for material deformation and used to benchmark ours and competing state-of-the-art methods. We also evaluate our algorithms performance on publicly available thoracic CT reconstructions where we resimulate x-ray measurements. In all cases, we find our method outperforms competitive baselines.

2. Related Work

3D-CT Sparse and Limited View Reconstruction: Traditional CT reconstruction is a mature imaging problem with applications in security, industrial and healthcare. For sparse-view 3D-CT, common techniques include the algebraic reconstruction technique (ART) and the filtered backprojection algorithm (FBP) [2, 20]. Model-based approaches have been proposed for the limited angle case [17, 48]. More recently, deep-learning based approaches utilize training data [14, 21, 18, 52, 48, 3, 24] to estimate scenes from sparse-view or limited angles. For a more comprehensive characterization on limited-angle tomography, we refer the reader to [10]. In our paper, we are interested in the 4D-CT problem, particularly for limited view sampling.

4D-CT for Periodic Motion: When recovering periodic motion, such as the breathing phases of clinical patients, several methods [23, 32, 47] gate measurements into phase/amplitude cycles to help reconstruct 3D image volumes. This gating limits the number of angular measurements per phase and can induce motion artifacts due to phase error [40]. Similar to our approach, these methods sometimes include parametric motion models for more general non-periodic motion (e.g., heart + breathing motion) [40, 37, 39]. These motion models perform image reg-
I. Introduction

The problem of 4D-CT reconstruction involves the recovery of a 3D scene from a sequence of 2D measurements over time. This is a challenging task due to the ill-posed nature of the inverse problem. The reconstruction process often requires the use of specialized sampling schemes or phase information, which can be expensive and difficult to obtain. In this section, we formulate our forward imaging model and discuss the key assumptions our algorithm makes for reconstructing scenes from CT angular projections. We also discuss our algorithmic pipeline to recover 4D scenes and under limited view sampling.

II. Implicit Neural Representations (INR)

Recent advances in neural networks have led to the development of Implicit Neural Representations (INRs), which can learn to represent complex 3D shapes and scenes. INRs have been used in a variety of applications, including texture completion, non-rigid registration, and light-field synthesis.

III. 4D-CT Forward Imaging Model

The primary task of this paper is to reconstruct 3D scenes in time from CT measurements (i.e., sinograms). In this section, we formulate our forward imaging model and discuss the key assumptions our algorithm makes for reconstructing scenes from CT angular projections. We also discuss our algorithmic pipeline to recover 4D scenes (i.e., 3D volume in time) from CT sinograms.

Mathematically, in a parallel-beam CT configuration, the three-dimensional Radon transform models the CT measurement of a dynamic scene at a particular viewing angle as follows:

$$p_\theta(r, z) = \int \int \sigma(x, y, z, t) \delta(x \cos(\theta(t)) + y \sin(\theta(t)) - r) dx dy,$$

where $\sigma(x, y, z, t)$ is the scene’s linear attenuation coefficient (LAC) at coordinates $(x, y, z)$ and time $t$, $\theta(t)$ is the view angle at time $t$, and $\delta(\cdot)$ is the Dirac delta function. The result of this transform, $p_\theta(r, z)$, is the summation of the scene’s LAC at view angle $\theta$, detector pixel $r$, and height $z$. In other words, $\{p_\theta(r, z) | (r, z) \in \mathbb{R}^2\}$ is the 2D projection of the 3D volume $\sigma(x, y, z, t)$ at the view angle $\theta$ and time $t$. We assume the scene’s LAC remains fixed for single projection, such that $||\sigma(x, y, z, t + dt) - \sigma(x, y, z, t)||_1 = 0$ where $dt$ is the exposure time of the CT scanner.

3.1. Modeling Assumptions:

Here we discuss the main assumptions made by our synthetic dataset and reconstruction algorithm.

- **Parallel-Beam Geometry:** Our first key assumption is that we assume a parallel-beam geometry, the geometry usually used by synchrotron CT scanners. Synchrotron scanners have vital applications for medical and industrial applications [31, 22, 15], and we expect our algorithm to be useful in these domains. Our reconstruction geometry would need to be modified to reconstruct CT measurements from cone-beam or helical measurements, typically used in medical imaging applications. The required modification is replacing our Radon transform with a differentiable volume ray tracer, for example those shown in [28, 33]. We leave this modification of our algorithm to future work.

- **Data Pre-Processing:** We assume CT measurements have undergone pre-processing to account for beam hardening and truncation. First, we consider the development of our algorithm for synchrotron systems which employ a monochromatic x-ray source and are therefore not affected by beam hardening. In cases where the effect is unavoidable, many methods exist for correcting beam hardened measurements and subsequently enabling high quality reconstructions [16, 4]. Truncation is not common in scientific and industrial imaging, but when present in measurements, methods exist for correcting it [49].

- **Limited View Sampling:** Previous 4D-CT methods assume either phase-based information or a sparse set of angular samples in order to reconstruct the scene. However, objects undergoing general deformations are not amenable to phase gating and sparse sampling schemes require scanner rotation speed to be fast relative to scene motion. Our method relaxes these assumptions — we do not require a specialized sampling scheme or phase information for reconstruction. Rather, we assume only that the scene is
4. System Architecture

As described earlier, limited view 4D CT is an ill-posed problem both due to scene motion as well as the incomplete data captured from dense angular sampling. Our key insight is to leverage implicit neural representations to jointly learn continuous functions of the scene volume and its evolution in time. In this section, we present our algorithmic pipeline consisting of three main parts: (1) an INR to estimate a template reconstruction of the static 3D volume LACs; (2) a parametric motion field that warps the template in time; and (3) a differentiable Radon transform to synthesize an estimate of the sinogram measurements. This pipeline is optimized jointly via analysis-by-synthesis with the ground truth sinogram measurements in a self-supervised fashion.

Template Estimation: In order to estimate a template of our measured volume’s LACs, we utilize an implicit neural representation architecture, as illustrated in the upper left portion of Figure 1. We denote the tunable parameters of the MLP as $\phi$. Specifically, we use a multi-layer perceptron (MLP), denoted by the function $\tilde{\sigma}$, that maps scene coordinates $(x, y, z)$ to a template reconstruction of the scene’s LACs $\tilde{\sigma}(x, y, z)$, such that $\tilde{\sigma}(x, y, z) \mapsto \tilde{\sigma}(x, y, z)$. Note that $\tilde{\sigma} \neq \sigma$, i.e., this template reconstruction does not equal the actual reconstruction of the scene’s LACs until we use the parametric motion field to warp the template to be consistent with the measurements given in the sinogram. In implementation, we input a grid of $(x, y, z)$ coordinates. Specifically, let $V \in \mathbb{R}^{3 \times 3}$ be a voxel representation of the scene, and the scene boundaries defined as $[-1, 1]^3$. For our experiments, we set $\beta = 80$ and sample our INR with $80^3$ linearly spaced coordinates at each iteration. Importantly, we perturb these coordinates randomly within their voxel so the INR is forced to learn a continuous representation of the scene.

Our INR architecture is inspired by [28] — we use 4 fully MLP layers with ReLU activations. We utilize Gaussian random Fourier feature (GRFF) [46] to randomly encode input coordinates with sinusoids of a random frequency. Formally, let $v = (x, y, z)$ be a coordinate from the input grid. Its GRFF is computed as $\gamma(v) = [\cos(2 \pi k \mathbf{B} v), \sin(2 \pi k \mathbf{B} v)]$, where $\cos$ and $\sin$ are performed element-wise; $\mathbf{B}$ is a vector randomly sampled from a Gaussian distribution $N(0, I)$, and $\kappa$ is the bandwidth factor which controls the sharpness of the output from the INR. Similarly to [46], we find that tuning the $\kappa$ parameter regularizes our reconstruction. As shown in the supplemental material, setting $\kappa$ too low prevents the INR from fitting high frequency content in the scene. Conversely, setting it too high causes the INR to fit spurious features in the measured sinogram resulting in poor reconstruction quality.

Motion Estimation: To map the estimated template LAC to the sinogram measurements, we introduce a parametric motion field to warp the template to different time values (i.e., $\tilde{\sigma} \mapsto \sigma(x, y, z, t_0), \sigma(x, y, z, t_1), \ldots, \sigma(x, y, z, t_N)$). Specifically, we define a tensor $C \in \mathbb{R}^{\beta^3 \times k \times 3}$. This tensor contains $k$ polynomial coefficients at each scene voxel in $\beta^3$ in the 3 spatial dimensions $(x, y, z)$. Next, we define $N$ time samples linearly spaced within $[0, 1]$ where $N$ is the number of angular measurements as $t_i = \{0, \ldots, N-1\}$. To warp a voxel to a specific time $t_i$, we compute the polynomial $W(C, t_i) = C_0 t_i^0 + C_1 t_i^1 + \ldots + C_K t_i^K$, where $W(C, t_i) \in \mathbb{R}^{\beta^3 \times 3}$ is the warp field of the scene at $t_i$, and $C_k$ is $C(t, k, :).$ We generate scene frames using a differentiable grid sampling function as introduced in [34]. $\text{warp.fn}(W(C, t_i), \tilde{\sigma}) = \sigma(x, y, z, t_i)$. Typically, we observe that polynomials of order $k = 5$ are sufficient for describing the deformable and periodic motion present in our empirical studies.

Hierarchical motion model: We observed that attempting to estimate the warp field at the full volume of $\beta^3$ results in poor motion reconstruction. To address this issue, we introduce a hierarchical coarse-to-fine procedure for estimating motion. Specifically, our initial motion field at a base resolution $\alpha$ such that $C_0 \in \mathbb{R}^{\alpha^3 \times k \times 3}$ where
We iteratively increase $\alpha$ throughout training (e.g., $2^3 \rightarrow 4^3 \rightarrow 8^3 \rightarrow 16^3 \rightarrow \ldots$) and use linear upsampling to progressively grow our warp field like $C_{\alpha+1} = U(C_{\alpha})$ where $U : \mathbb{R}^2 \rightarrow \mathbb{R}^{2^2}$. This strategy encourages our optimization to first recover simple motion and then iteratively recover more complex deformations.

Differentiable Radon Transform: After estimating a sequence of LAC volumes $\sigma(t_0), \ldots, \sigma(t_N)$ by applying our motion field $W$ to the LAC volume template $\hat{\sigma}$, we then project the LAC volumes through the CT forward imaging model to synthesize CT measurement using the 3D Radon transform. Thus we can compare our synthesized measurements to the ground truth CT measurements provided by the captured sinograms. To enforce a loss, we implement the 3D Radon transform in an autograd enabled package so that the intensity of each projected pixel is differentiable with respect to the viewing angle. We backpropagate derivatives through this operation and update the INR and motion field parameters for analysis-by-synthesis.

The weights of the INR and the coefficients of the parametric motion field are updated via gradient descent to minimize our loss function

$$\min_{\phi, C} \lambda_1 \| R_{\theta(t)}(\sigma(x, y, z, t) - R_{\theta(t)}(GT))_1 + \lambda_2 \text{TV}(C), \quad t \in [0, 1].$$

The first term in our loss function is an L1 loss between our synthesized and given sinogram measurements. Additionally, we regularize the motion field by penalizing the spatial variation of the coefficients. The weight of $\lambda_2$ governs the allowed spatial complexity of the motion field. Higher values create smoother warp fields but may underfit complex motion, and lower values allow fitting of complex motion but are more prone to noisy solutions.

Continuous Volume and Time Representation: Due to memory constraints, we sample our INR at resolution of $80^3$ points during optimization. However, similar to [28], we randomly perturb these points at each iteration, encouraging our INR to learn a continuous mapping from $(x, y, z)$ to scene LAC. This continuous mapping property is useful because it allows us to query our INR at an arbitrary resolution post-optimization. Similarly, we sample our motion field at random times $t$ during the optimization which encourages the polynomial coefficients to fit a continuous representation of the scene motion. We use this fact to upsample our scenes to arbitrary frame rates post-optimization. Due to the parametric representation of the motion field, we are constrained to simple trilinear interpolation for upsampling the field itself. We find that this works well in practice, but is something to be addressed in future work.

Using the upsampling functionality, we show that we can optimize our scene on a set of relatively low-resolution measurements (e.g. $80^3$) at 10 time frames, and then upsample the measurements to $256^3$ at 90 time frames ($256 \times 256 \times 256 \times 90$), making our method practically viable while also bypassing substantial GPU memory requirements. We demonstrate results of this fact in Section 6 and videos in the supplement.

5. Implementation

Datasets: We benchmark our algorithm and competing SOA methods on a dynamic 4D-CT dataset of object deformation that we created (D4DCT Dataset). This dataset represents a time-varying object deformation to demonstrate damage evolution due to mechanical stresses over time for the study of materials science and additive manufacturing. The deformation by the damage evolution provides crucial information about the performance and safety of the material of interest, more accurate, physics-based simulation is needed. To this end, we generated dataset using the material point method (MPM) [38] to accurately represent deformation of a type of aluminum under various loading conditions. We then simulated 4D sinogram data with the provided angular ranges of 180 and 720 degrees where the number of uniformly spaced projections is 90 and the detector row size is 80. The dimension of the ground truth volume is $80^3$ and the number of ground truth frames for algorithm evaluation is 10. We plan to open source this dataset to encourage reproducibility in 4D-CT research.

We also benchmark our algorithm on thoracic CT data [5]. This dataset contains volumetric reconstructions of the chest cavity at 10 breathing phases. There is motion present due to periodic functions of the diaphragm and heart. We project the 10 reconstructions from an $80^3$ volume into sinogram space with 90 uniform angular projection between 0 and 180 degrees to emulate the real sinogram data.

Comparisons: To our knowledge, we are the first method to propose solving 4D CT in the limited angle regime without the use of motion phase information. However, we benchmark against two baseline methods typically used for sparse angular views on our limited angle datasets: TIMBIR [29] and Warp and Project [50]. TIMBIR uses sparse angular views with interleaved sampling to recover 4D-CT reconstructions. Warp and Project jointly solve for motion and the object reconstruction from sparse angular views. We note that these methods are expected to perform poorly in our limited angle sampling regime as both are designed for sparse sampling in time. We also benchmark the datasets with the filtered back projection (FBP) method for static 3D CT. This method is not designed to account for motion and serves to illustrates the degrading effects motion has on reconstruction performance. We note that we utilize source code for both [29] and [50] given to us by the authors for running our experiments.

Algorithm Implementation Details: Our algorithm is implemented in PyTorch to run on two Titan X GPUs for 15 minutes per recovery of an $80^3$ LAC volume in time from a
Figure 3. (A) Reconstruction results of our method and competing baseline methods on two objects (Left: Object Alum#1. Right: Object Alum#2.) from our aluminum deformation dataset at the beginning ($T = 0.00$) and end ($T = 1.00$) of the deformation. On the left, two plates compress the center mass of the object, and on the right, the center of the object is squeezed by two bars. In each tile, we display 3D rendering of the object to the left, and on the right, a white 2D inlet containing an XY slice through the center of the object. The PSNR of each frame is shown in white at the upper right corner of each method’s tile. (B) Ablation of key motion field regularization components.

6. Experimental Results

Here we benchmark the performance of our algorithm against baselines on our D4DCT dataset and the thoracic dataset. While we show these results at a resolution of $80^3$ to compare with our baselines, we also demonstrate our ability to upsample our results to a more practical CT resolution of $256^3$. In addition, we show the ability to upsample our videos to arbitrary frame rates, but refer the reader to the supplemental videos for an example of this capability. Finally, we show ablations on our method and its sampling schemes at the end of this section.

D4DCT Dataset: As shown in Figure 3(A) and summarized in Table 1, our proposed method drastically outperforms competing methods in peak signal-to-noise ra-
warped through time, meaning our optimization leverages a
different scheme because we optimize a single reconstruction that is
consistent across time. This also allows our method to perform
significantly better in this sampling regime. We believe that
many algorithms expect a sparse set of samples in order to form
efficient initial estimates of the object at each time step; these
sampling schemes prevent these methods from constructing suf-
ficient detail. These artifacts exist because the limited view sam-
pling scheme of Figure 3(A) contain much more severe ar-
tifacts. These artifacts are caused by the limited view sam-
pling scheme and by the way the reconstruction is optimized for sparse angular sampling.

6.1. Ablation Studies

INR Reconstruction: Using an INR in our reconstruction pipeline allows us several key advantages. First, we
observe that our INR outperforms conventional reconstruction
methods on 3D scene reconstructions. In Figure 7, we asked an INR and the two conventional reconstruction
methods (SART and FBP), the 3D reconstruction technique
used by our 4D-CT baselines [50, 29], to reconstruct a 3D Shepp-Logan phantom. The INR is implemented as a MLP
and takes GRFF features of \((x,y)\) coordinates as input to predict the LAC at each coordinate. The scene is projected
to sinogram space with the Radon transform and compared to the given limited view measurements to enforce a loss
via gradient updates of the INR weights. We observed that the INR gave better reconstruction PSNR. We also tested
this performance in the presence of additive noise and still observe better performance under these conditions. We be-
lieve this performance gap extends to the 4D problem since our 4D reconstruction results drastically outperform bas-
elines methods that use SART and FBP.

Secondly, our INR allows us to upsample the scene to arbitrary resolutions in the post-optimization, as shown in
Figure 5. Impressively, our INR yields sharp results that resemble ground truth at the high resolution of \(256^3\) despite
being optimized on data at the low resolution of \(80^3\). Further, we observe that it qualitatively and quantitatively out-
performs naive trilinear upsampling methods. However, our
method is not perfect and fails to capture fine details like ar-
teries at the high resolution. This is possibly because these
subtle artifacts were too degraded at the optimization resolution \(80^3\) for the INR to recover this structure.

Parametric Motion Field Regularization: We regular-
ize our motion field both by choosing an appropriate order for
its polynomial equation and with the hierarchical motion
model. In Figure 3(B), we illustrate the importance of these
methods on the final reconstruction quality of our scenes.
Of these two methods, the coarse-to-fine ablation affected
the largest change on the reconstruction PSNR with a 6 dB performance improvement. This result is expected as the
motion field recovery is extremely ill-posed — simplifying its initial estimation ensures the motion field does not imme-
diately overfit to noisy solutions. For the warp field polyno-
mial, we observed that a parametric motion field with low
order polynomials underfits non-linear motion, and thus re-
quired order 3 or higher for satisfactory performance.

Effects of Angular Sampling on Performance: We
observe enhanced reconstruction performance of our tested
methods when we increase the angular range of our pro-
jections (i.e., make the samples more sparsely situated).
In Figure 6 we show the reconstruction results from our
method and TIMBIR [29] on data captured with 90 pro-
jective views. Please see supplemental materials for a video of these reconstructions.

Thoracic CT Data [5]: In Figure 4, we display our re-
construction results for 3 breathing phases of our thoracic
CT data. In the upper half of each transverse slice, the top
portion of the diaphragm is observed progressively rising and
occupying more of the scene at each breathing phase.
Our method recovers this motion and the overall geometry
of the thoracic cavity. We encourage the reader to view the
supplemental material videos to view the full reconstruc-
tions in time. Compared to the TIMBIR reconstruction,
we observe that our method preserves sharper details and
achieves a better estimate of the motion. We also bench-
marked the Warp and Project method on this data, but the
reconstruction quality was subpar as noted quantitatively in
Table 1. This may be due to the fact that its code implement-
ation is optimized for sparse angular sampling and not
robust to limited angular sampling.
Figure 5. We demonstrate an ability to effectively upsample our scenes to arbitrary resolutions by sampling our INR’s continuous representation of the scene. In this experiment, we use the thoracic data to $80^3$ as ground truth (first column), and performed scene reconstruction with our method (second column). In the third column, we show the ground truth thoracic data at its native resolution $256^3$. In the last two columns from left to right, we show the results of upsampling our $80^3$ reconstruction with a trilinear interpolation, and upsampling by querying our INR at an upsampled set of input $(x, y, z)$ coordinates.

Figure 6. More sparsely sampled projections results in better reconstruction performance for ours and SOA method TIMBIR [29]. Here we show the PSNR and visual difference of reconstructing with 90 uniform projections between 180 degrees (top row) and between 720 degrees (bottom row).

Figure 7. The INR model outperforms conventional CT reconstruction methods [20, 2] in static 3D reconstructions. We show transverse slices of the 3D Shepp-Logan for viewer reference.

ejections within 180 and 720 degrees. Our method resolves the object geometry and deformation in both cases, whereas TIMBIR only begins to capture the underlying geometry in the latter case.

7. Discussion

We demonstrate that our proposed algorithm outperforms SOA methods in reconstructing limited view 4D-CT measurements of deformable motion. These results have the potential to enable CT scanners to measure rapidly moving scenes with a fidelity that was previously unattainable. Generally, this research has the potential to enable more efficient CT scans in industrial and clinical settings. We plan to open source both our code and the D4DCT dataset for reproducible research (after paper acceptance).

We also address two limitations of our work. First, we only consider a parallel-beam scanning geometry. While this makes our method directly applicable to synchrotron scanners, our method needs to be modified to reconstruct cone-beam data. Several other works provide implementations of differentiable ray tracers capable of modeling this geometry [45, 11], but we leave this modification to future work. Second, we show promising results of upsampling our scenes post-optimization. However, the efficacy of this upsampling needs to be further explored and compared with running the optimization at the full resolution. Even so, we believe the upsampling results we show are a promising method for achieving super-resolution in memory-hungry regimes — a few very recent works also show impressive
supper-resolution results with INRs [43, 44]. We hope our work sparks interest in dynamic 4D-CT reconstructions that leverage INRs in the future.

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