FRFD MIMO Systems: Precoded V-BLAST with Limited Feedback Versus Non-orthogonal STBC MIMO

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Abstract—Full-rate (FR) and full-diversity (FD) are attractive features in MIMO systems. We refer to systems which achieve both FR and FD simultaneously as FRFD systems. Non-orthogonal STBCs can achieve FRFD without feedback, but their ML decoding complexities are high. V-BLAST without precoding achieves FR but not FD. FRFD can be achieved in V-BLAST through precoding given full channel state information at the transmitter (CSIT). However, with limited feedback precoding, V-BLAST achieves FD, but with some rate loss. Our contribution in this paper is two-fold: i) we propose a limited feedback (LFB) precoding scheme which achieves FRFD in $2 \times 2$, $3 \times 3$ and $4 \times 4$ V-BLAST systems (we refer to this scheme as FRFD-VBLAST-LFB scheme), and ii) comparing the performances of the FRFD-VBLAST-LFB scheme and non-orthogonal STBCs without feedback (e.g., Golden code, perfect codes) under ML decoding, we show that in a $2 \times 2$ MIMO system with 4-QAM/16-QAM, FRFD-VBLAST-LFB scheme outperforms the Golden code by about 0.6 dB; in $3 \times 3$ and $4 \times 4$ MIMO systems, the performance of FRFD-VBLAST-LFB scheme is comparable to the performance of perfect codes. The FRFD-VBLAST-LFB scheme is attractive because 1) ML decoding becomes less complex compared to that of non-orthogonal STBCs, 2) the number of feedback bits required to achieve the above performance is small, 3) in slow-fading, it is adequate to send feedback bits only occasionally, and 4) in most practical wireless systems feedback channel is often available (e.g., for adaptive modulation, rate/power control).

I. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques can provide the benefits of spatial diversity and multiplexing gain [1]-[3]. Spatial multiplexing (V-BLAST) using $N_t$ transmit antennas achieves the full-rate of $N_t$ symbols per channel use. However, full transmit diversity of order $N_t$ is not achieved in V-BLAST. Orthogonal space-time block codes (e.g., $2 \times 2$ Alamouti code) achieve full transmit diversity, but suffer from rate loss for increased number of antennas [2]. Achieving both full-rate (FR) and full-diversity (FD) simultaneously is a desired goal in MIMO communications. We refer to MIMO systems that simultaneously achieve both FR and FD as FRFD MIMO systems.

One way to achieve FRFD in MIMO systems is to employ non-orthogonal STBCs [4]-[8], which offer the full-rate of $N_t$ symbols per channel use by having $N_t^2$ symbols in one $N_t \times N_t$ STBC matrix, and full-diversity under ML decoding. The $2 \times 2$ Golden code with 4 symbols in one STBC matrix is a well known non-orthogonal STBC for 2 transmit antennas [5]. A drawback with FRFD-achieving non-orthogonal STBCs is their high decoding complexities, because ML decoding of these STBCs involves joint decoding of $N_t^2$ symbols. ML decoding in V-BLAST, on the other hand, involves joint decoding of only $N_t$ symbols. The inability of V-BLAST to achieve transmit diversity can be overcome through the use of precoding at the transmitter [9]-[12]. Precoding based on knowledge of full channel state information at the transmitter (CSIT) and first-second-order statistics of the channel have been studied widely [9]-[12]. With full CSIT precoding, FRFD can be achieved in V-BLAST [12]. However, in limited feedback precoding schemes in V-BLAST, FD is achieved, but with some loss in rate [13]-[15]. For e.g., the precoding scheme in [13] is based on Grassmannian subspace packing, which does not allow simultaneous transmission of more than $N_t-1$ streams (i.e., achievable rate is $\leq N_t - 1$ symbols per channel use). In a $2 \times 2$ MIMO system, this means a rate loss of 50%. The same is true with any other Grassmannian subspace packing based scheme or transmit antenna selection based scheme [15]. Our contribution in this paper is two fold:

- First, we present a limited feedback (LFB) based precoding scheme for V-BLAST which achieves FRFD in small systems like $2 \times 2$, $3 \times 3$, and $4 \times 4$ MIMO. Since the proposed scheme is not based on subspace packing or antenna selection, there is no loss in rate. The proposed scheme involves the design of a codebook having a finite number $(N)$ of unitary precoding matrices, which are generated from a unitary matrix $(U_\theta)$ parametrized by a single angular parameter, $\theta \in \{ \frac{2\pi n}{N}, n = 0, \cdots, N-1 \}$. The receiver chooses the precoding matrix which maximizes the minimum distance with ML decoding, and sends the corresponding index to the transmitter. We refer to the proposed scheme as FRFD-VBLAST-LFB scheme.

- Second, we present a BER performance comparison between the two FRFD-achieving schemes, namely, the proposed FRFD-VBLAST-LFB scheme and non-orthogonal STBC MIMO using Golden/perfect codes under ML decoding. Our simulation results show that in a $2 \times 2$ MIMO system with 4-QAM/16-QAM, the proposed FRFD-VBLAST-LFB scheme outperforms the Golden code by about 0.6 dB. In $3 \times 3$ and $4 \times 4$ MIMO, the performance of FRFD-VBLAST-LFB scheme is comparable to the performance of perfect codes.

The proposed FRFD-VBLAST-LFB scheme is attractive because 1) ML decoding becomes less complex compared to that of non-orthogonal STBCs, 2) the number of feedback bits required to achieve the above performance is small, 3) in slow-fading channels it is adequate to send feedback bits only occasionally, and 4) in most practical wireless systems feedback channel is often available (e.g., for adaptive modulation, rate/power control).

The rest of this paper is organized as follows. In Section II we present the system model. The proposed limited feedback precoding scheme is presented in Section III. BER performance of the proposed scheme along with a performance comparison with Golden/perfect codes are presented.
Consider a precoded V-BLAST system with $N_t$ antennas at the transmitter and $N_r$ antennas at the receiver. Let $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denote the channel gain matrix, whose entries are i.i.d $CN(0,1)$. Perfect knowledge of $\mathbf{H}$ is assumed at the receiver but not at the transmitter. Let $\mathcal{F} = \{\mathbf{F}_0, \mathbf{F}_1, \cdots, \mathbf{F}_{N-1}\}$ denote the precoder codebook of size $N$, where the $\mathbf{F}_n$’s, $n = 0, 1, \cdots, N - 1$, are $N_t \times N_t$ unitary precoding matrices. This codebook is known to both transmitter and receiver. For a given channel $\mathbf{H}$, the receiver chooses the precoding matrix from $\mathcal{F}$ that maximizes the minimum distance with ML decoding, and sends the corresponding index to the transmitter. Let $B = \lceil \log_2 N \rceil$ denote the number of feedback bits needed to represent this index. Given this index, $k$, the transmitter uses the corresponding precoding matrix, denoted by $\mathbf{F} = \mathbf{F}_k$. Let $\mathbf{x} \in \mathbb{A}^{N_t}$ denote the complex data symbol vector at the transmitter, where $\mathbb{A}$ is the modulation alphabet. The transmitted signal vector, $\mathbf{u} \in \mathbb{C}^{N_t}$ is given by $\mathbf{u} = \mathbf{F}\mathbf{x}$. The received signal vector, $\mathbf{y} \in \mathbb{C}^{N_r}$ at the receiver is given by

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n},$$

(1)

where $n \in \mathbb{C}^{N_r}$ is the noise vector whose entries are i.i.d $CN(0, \sigma^2)$, where $\sigma^2$ is the average energy of the transmitted symbols, and $\gamma$ is the average received SNR per receive antenna.

III. PROPOSED LFB PRECODING SCHEME

For a non-precoded system (i.e., for $\mathbf{F} = \mathbf{I}_{N_t}$), the ML decision is given by

$$\hat{x} = \arg\min_{\mathbf{x} \in \mathbb{A}^{N_t}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2.$$  (2)

The probability of error in the decision depends on the minimum distance, $d_{\text{min}}$, which is given by

$$d_{\text{min}} = \min_{\mathbf{x}_j, \mathbf{x}_k \in \mathbb{A}^{N_t}, \mathbf{x}_j \neq \mathbf{x}_k} \|\mathbf{H}(\mathbf{x}_j - \mathbf{x}_k)\|_2^2.$$  (3)

It is known that precoding at the transmitter improves $d_{\text{min}}$ [3]. We illustrate this point using the following example and Fig. 1. Assume a $2 \times 2$ system with $\mathbf{H} = \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$ and PAM modulation. As can be seen from Fig. 1, $d_{\text{min}}(\mathbf{H}) = d_1 = 1.414$. Now, consider unitary precoding with $\mathbf{F} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$. The new effective channel matrix is given by $\mathbf{H}' = \mathbf{H}\mathbf{F}$. From Fig. 1 it can be seen that $d_{\text{min}}(\mathbf{H}') = d_2 = 2.875 > d_{\text{min}}(\mathbf{H})$.

A. Codebook Design

Let $\mathcal{U}_{N_t}(\theta) = \{\text{all } N_t \times N_t \text{ unitary matrices parametrized by a single angular variable, } \theta\}$. For e.g., for $N_t = 2$, $\mathcal{U}_{2}(\theta)$ is the set of all possible matrices of the form

$$\begin{bmatrix} A_1(\theta) & A_2(\theta) \\ A_3(\theta) & A_4(\theta) \end{bmatrix},$$

where $A_i(\theta)$ is a real or complex scalar function of only $\theta$, and

i) $A_1(\theta)A_2(\theta) + A_3(\theta)A_4(\theta) = 0$,

ii) $|A_1(\theta)|^2 + |A_3(\theta)|^2 = |A_2(\theta)|^2 + |A_4(\theta)|^2 = 1, \forall \theta \in (0, 2\pi)$.

In words, $\mu(\zeta_N(\mathcal{U}_{N_t}(\theta)))$ is the expected ratio of the maximum squared $d_{\text{min}}$ with precoding to that without precoding. Then, the optimal finite precoding codebook with $B = \log_2 N$ feedback bits is $\zeta_N(\mathcal{U}_{N_t}^{\text{opt}}(\theta))$, where $\mathcal{U}_{N_t}^{\text{opt}}(\theta)$ is given by

$$\mathcal{U}_{N_t}^{\text{opt}}(\theta) = \arg\max_{\mathcal{U}_{N_t}(\theta), \theta} \mu(\zeta_N(\mathcal{U}_{N_t}(\theta))).$$  (6)

Obtaining an exact solution for $\mathcal{U}_{N_t}^{\text{opt}}(\theta)$ analytically is difficult. In the absence of a solution to the above problem, we tried out several $\mathcal{U}_{N_t}(\theta)$ matrices for small values of $N_t$ (e.g., $N_t = 2, 3, 4$), which are of interest in practical MIMO systems, and found that the following designs for $N_t = 2, 3, 4$ work very well in the proposed scheme:

$$\mathcal{U}_{2}\theta(\cdot) = \begin{bmatrix} e^{j\theta} & 1 \\ 1 & e^{-j\theta} \end{bmatrix},$$

(7)

1Our computer simulations show that these designs for $N_t = 2, 3, 4$ achieve very good BER performance (as we will see in Sec. IV).
\[ U_{N_t=3}(\theta) = \frac{1}{3} \begin{bmatrix} 2e^{j\theta} & -2 & e^{j\theta} \\ e^{j\frac{2\pi}{3}} & 2e^{-j\frac{\pi}{3}} & e^{j\frac{2\pi}{3}} \\ 2 & e^{-j\theta} & -2 \end{bmatrix}, \] (8)

\[ U_{N_t=2m+1}(\theta) = \frac{1}{\sqrt{2}} \begin{bmatrix} U_{2m}(\theta) & I_{2m} \\ -I_{2m} & U_{2m}(\theta) \end{bmatrix}. \] (9)

### B. Precoding Matrix Selection

At the receiver, given the knowledge of \( H \), we define \( d_{\text{min}}(H, i) \), \( i = 0, 1, \ldots, N - 1 \), as

\[ d_{\text{min}}(H, i) \triangleq \min_{\mathbf{x}_j, \mathbf{x}_k \in \Lambda^N, \mathbf{x}_j \neq \mathbf{x}_k} \| \mathbf{H} \mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi}{m}} (\mathbf{x}_j - \mathbf{x}_k) \|_2. \] (10)

The receiver sends to the transmitter the index \( p \), given by

\[ p = \arg\max_i \{ d_{\text{min}}(H, i) \}, \] (11)

using \( B \) bits of feedback. Hence, the optimum precoding matrix chosen is given by

\[ F_p = U_{N_t}(\theta)|_{\theta=\frac{2\pi}{m}}. \] (12)

### C. Receiver Processing: Feedback Computation and Signal Detection

Signal detection is performed using the sphere decoding algorithm [16]. In the following, we present the computation of \( d_{\text{min}}(H, i) \) in (10) for \( i = 0, 1, \ldots, N - 1 \).

We can rewrite the system model equation (1) for the precoded system, when precoded with the \( i \)th precoding matrix as

\[ y = H_i x + n, \] (13)

where \( H_i \triangleq H \mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi}{m}} \).

Using \( \mathcal{R}\{\cdot\} \) and \( \mathcal{I}\{\cdot\} \) to denote the real and imaginary parts of a complex argument, the above equation can be transformed into an equivalent real-valued model as

\[ \tilde{y} = \tilde{H}_i \tilde{x} + \tilde{n}, \] (14)

where \( \tilde{y} = [\mathcal{R}\{y^T\} \quad \mathcal{I}\{y^T\}]^T \), \( \tilde{x} = [\mathcal{R}\{x^T\} \quad \mathcal{I}\{x^T\}]^T \), \( \tilde{n} = [\mathcal{R}\{n^T\} \quad \mathcal{I}\{n^T\}]^T \), and

\[ \tilde{H}_i = \begin{bmatrix} \mathcal{R}\{H_i\} & -\mathcal{I}\{H_i\} \\ \mathcal{I}\{H_i\} & \mathcal{R}\{H_i\} \end{bmatrix}. \] (15)

Here, \( \tilde{y} \in \mathbb{R}^{2N_r \times 1}, \tilde{x} \in \mathbb{R}^{2N_r \times 1}, \tilde{n} \in \mathbb{R}^{2N_r \times 1} \) and \( \tilde{H}_i \in \mathbb{R}^{2N_r \times 2N_t} \). Also, \( \mathbb{S} \) is the real PAM constellation corresponding to \( \tilde{a}_k \). Henceforth, we shall work with the real-valued system in (14). With the new system model in (14), we can re-write (10) as

\[ d_{\text{min}}(H, i) \triangleq \min_{\mathbf{x}_j, \mathbf{x}_k \in \Lambda^N, \mathbf{x}_j \neq \mathbf{x}_k} \| \mathbf{H} \mathbf{U}_{N_t}(\theta)|_{\theta=\frac{2\pi}{m}} (\mathbf{x}_j - \mathbf{x}_k) \|_2 = \min_{\tilde{\mathbf{x}}_j, \tilde{\mathbf{x}}_k \in \mathbb{S}^N, \tilde{\mathbf{x}}_j \neq \tilde{\mathbf{x}}_k} \| \tilde{H}_i (\tilde{\mathbf{x}}_j - \tilde{\mathbf{x}}_k) \|_2 = \min_{\mathbf{z} \in \mathbb{D}^N, \mathbf{z} \neq \mathbf{0}} \| \tilde{H}_i \mathbf{z} \|_2, \] (16)

where \( \mathbb{D} \) is the difference constellation of \( \mathbb{S} \). For example, for square \( M \)-QAM modulation, \( \mathbb{S} \) is given by \( \{ \pm 1, \pm 3, \ldots, \pm \sqrt{M-1} \} \) and \( \mathbb{D} \) is \( \{ \pm 2, \pm 4, \ldots, \pm (\sqrt{M-1}) \} \).

### IV. Simulation Results

We evaluated the BER performance of the proposed limited feedback precoding scheme as a function of average received SNR per receive antenna, \( \gamma \), through simulations for \( N_t = N_r = 2, 3, 4 \). For comparison purposes, we also evaluate the performance of Golden code/perfect codes in MIMO systems with \( N_t = N_r = 2, 3, 4 \).

Figure 2 shows the simulation results for \( 2 \times 2 \) and \( 4 \times 4 \) V-BLAST without and with the proposed precoding, for 4-QAM and 16-QAM using sphere decoding. BER performance plots for different levels of quantization requiring different number of feedback bits \( B = 3 \) are shown.

From Fig. 2, it is observed that the proposed precoding scheme achieves significantly better diversity than the ‘no precoding’ scheme. In fact, a comparison of this precoded performance for \( 2 \times 2 \) V-BLAST with the performance of \( 2 \times 2 \) Golden code shows that both these curves run parallel illustrating that, just like Golden code, the proposed scheme also achieves full diversity. Another interesting observation in Fig. 2 is the effect of feedback bits on the BER performance. It can be seen that the BER with \( B = 4 \) and \( B = 8 \) are almost the same, showing that the performance in the proposed scheme remains robust even with a nominal quantization of using 4 bits.

Next, in Figs. 3 to 5, we compare the BER performances of the proposed precoding scheme and the Golden code/perfect codes in \( 2 \times 2 \) (Fig. 3), \( 3 \times 3 \) (Fig. 4), and \( 4 \times 4 \) (Fig. 5) systems, using sphere decoding. The channel is assumed to remain constant for \( N_t \) consecutive channel uses in V-BLAST in order to facilitate the comparison between V-BLAST and Golden code/perfect codes (which are assumed to have a quasi-static interval of \( N_t \) channel uses) under similar quasi-static channel conditions.

From Fig. 3, it can be seen that, for both 4-QAM and 16-QAM, the performance curves of the proposed scheme runs parallel to those of the Golden code, showing that the proposed scheme achieves the full diversity of 4. It is interesting to further observe that the proposed scheme exhibits some coding gain advantage compared to the Golden code. Particularly, the coding gain attained by the proposed scheme over Golden Code is about 0.7 dB at a BER of \( 10^{-3} \).
This is significant since precoding achieves this better performance with a lower decoding complexity (joint detection of 2 symbols in one channel use) than the Golden code (joint detection of 4 symbols in one STBC matrix).

Next, the BER comparison in Fig. 4 for $3 \times 3$ system shows that the proposed scheme achieves the same diversity as that of $3 \times 3$ perfect code, but is inferior to perfect code in terms of coding gain. This performance gap, however, is small (about 0.3 dB). In $4 \times 4$ system in Fig. 5 the performance gap in terms of coding gain is about 1 dB. This suggests that better precoding strategies for larger $N_t$ can be investigated to achieve close to perfect code performance. A likely approach can be to consider multiple parameter based precoder designs.

A key advantage of the proposed precoder approach compared to the non-orthogonal STBC approach is its lesser decoding complexity. This advantage is captured in the complexity comparison plots in Fig. 6 where the complexities in both the approaches using sphere decoding, in terms of number of real operations per decoded symbol, are plotted as a function of $N_t = N_r = 2, 3, 4$ at SNRs corresponding to a target BER of $10^{-2}$. It can be seen that the complexity in the proposed approach is much less because it needs to jointly detect only $N_t$ symbols, whereas in the non-orthogonal STBC approach joint detection is over $N_t^2$ symbols. We note that the complexity comparison in Fig. 6 does not include the complexity involved in the optimization to choose the optimum precoding matrix at the receiver. It is pointed out that the data decoding complexity dominates over the precoding selection complexity, as decoding is done on a per channel use basis whereas, in slow fading, the precoding selection computation need not be carried out so frequently.

V. CONCLUSIONS

We presented a simple, single angular parameter based codebook design for limited feedback precoding in V-BLAST.
The proposed precoding scheme achieves full-rate for any $N_t$ by design, whereas the achievability of full-diversity was established through BER simulations for $N_t = N_r = 2, 3, 4$ under ML decoding for 4-QAM/16-QAM. Our simulation results showed that in a $2 \times 2$ MIMO system, the proposed scheme outperformed the Golden code by about 0.6 dB. It performed comparable to perfect codes in $3 \times 3$ and $4 \times 4$ MIMO systems as well. The decoding complexity in the proposed scheme was shown to be much less compared to that of Golden/perfect codes. It is noted that the feedback channel is an additional resource required in precoding schemes. However, given that a feedback channel is often available in most practical wireless systems (e.g., for adaptive modulation, rate/power control, etc.), and that the feedback bandwidth required will be very less in slow fading, the proposed scheme can be quite attractive for its full-rate, full-diversity, and low-complexity attributes. Investigations related to applicability of the proposed approach to large antenna systems using multiple-parameter based precoder designs can be carried out further.

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