Programming Constructs for Unstructured Data *

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1 Introduction

We investigate languages for querying and transforming unstructured data by which we mean languages than can be used without knowledge of the structure (schema) of the database. There are two reasons for wanting to do this. First, some data models have emerged in which the schema is either completely absent or only provides weak constraints on the data. Second, it is sometimes convenient, for the purposes of browsing, to query the database without reference to the schema. For example one may want to “grep” all character strings in the database, or one might want to find the information associated with a certain field name no matter where it occurs in the database.

The idea of using labeled trees for this purpose has been suggested by two data models. ACeDB (A C. elegans Database) [7] is a database system popular with biologists. It has a schema, but this only places very weak constraints on the database since any field in the deeply nested records that are common in ACeDB can be null. Recently Tsimmis [5] has been proposed as a data model for heterogeneous data integration. In Tsimmis there is no schema. The “type” is interpreted by the user from labels in the structure, which is quite flexible. In particular, a Tsimmis structure may be interpreted as a record or as a set. There is an analogy here with the dynamic type system of Lisp, whose one basic data structure, the s-expression, may be used to represent lists, trees, association lists, lambda terms, etc.

The approach we shall take is to extend structural recursion to labeled trees. This poses some interesting problems: first, it is no longer “flat” structural recursion, so that the usual syntactic forms and optimizations for collection types such as lists bags and sets may not be relevant. Second, we shall want to examine the possibility that the values we are manipulating may be cyclic. It is common in ACeDB, and generally in object-oriented databases, for objects to refer to each other, allowing the possibility of arbitrarily “deep” queries. Of course, such cyclic structures are usually constructed through the use of a reference/pointer type; however query languages are insensitive to these object identities and perform automatic dereferencing. We therefore want to understand what programs are well defined when we are allowed to make unbounded searches in the database.

This work is preliminary and serves only to describe languages that may be useful for unstructured data. While we believe that there are sound principles for justifying this choice of languages, they are at present mostly “articles of faith”. The paper is organized as follows. After specifying the data structure of interest, we first develop a

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variant of nested relational algebra which gives us the ability to construct queries to a fixed depth. Next we extend the idea of structural recursion to perform queries that can reach data at arbitrary depths in the tree. Finally we examine restrictions of this language that work on cyclic data. By a database “query” we usually understand a program that produces a relatively simple output from a complex input – the database. In what follows we are interested in producing data structures that may be as complex as the input. This is the problem of transforming databases, which is of paramount importance in heterogeneous database systems.

2 A labeled tree data type

As is common in this area we shall take a bibliographic database as a running example. The diagram in figure 1 shows an edge-labeled tree. At the top level we see four edges labeled doc indicating a set of documents. The first such document displayed is a tree with two distinct labels topic and book indicating a record.

The labels on the edges are drawn from some collection of basic types. For the sake of consistency with the systems mentioned above, we shall consider the type label to be the discriminated union of a number of basic types: character strings such as “Math”, “Wheels”; numbers such as 4, 3.1415; and symbols such as 'doc', 'article using the lisp notation for quoting (the quote mark is not shown on the symbol edges in figure 1). In general, symbols are used to mark internal edges, and other constants such as strings and numbers are used at the leaves, but this is not demanded by our model.

Having fixed a data type label, we can now define the type of a labeled tree to be a set of pairs, each consisting of a label and another tree. Using \( P_{\text{fin}}(S) \) for the finite subsets of \( S \), we can describe a labeled tree type by the recursive type equation

\[
\text{tree} = P_{\text{fin}}(\text{label} \times \text{tree})
\]

Before proceeding further we should remark that there are some differences between this type and the models used in Tsimmis and ACeDB. Tsimmis attaches values of base types such as \( \text{num} \) and \( \text{string} \) to the terminal nodes of the tree, and the edges are labeled only with symbols. Tsimmis also has object identities associated with the internal nodes. The transformation from Tsimmis is straightforward: we represent terminal nodes by terminal edges; and we may introduce object identities by simply adding a new object-identity base type. ACeDB, is much closer to our presentation in that numbers, strings etc. may be attached to non-terminal edges. It also allows one to build cyclic structures, which we shall discuss later. The transformation from ACeDB is obtained essentially
by transferring label information from the schema to the data; and a similar technique may be used to represent
other databases as trees.

We now describe constructors for the type tree. Trees are sets, so we have \( \phi \) for the empty set and \( e_1 \cup e_2 \) to
construct the union of sets \( e_1 \) and \( e_2 \). In addition we have the expression \( \{ a \mapsto e \} \) to describe a singleton set
consisting of a tree formed by attaching the edge labeled with \( a \) to the root of the tree \( e \). The types of these
constructors are as follows:

\[
\begin{align*}
\phi & : \text{tree} \\
\{ a \mapsto e \} & : \text{label} \times \text{tree} \rightarrow \text{tree} \\
\_ \cup \_ & : \text{tree} \times \text{tree} \rightarrow \text{tree}
\end{align*}
\]

We shall also make use of the following abbreviations for constructing trees: \( \{ a_1 \mapsto e_1, a_2 \mapsto e_2, \ldots, a_n \mapsto e_n \} \) for
\( \{ a_1 \mapsto e_1 \} \cup \{ a_2 \mapsto e_2 \} \cup \ldots \cup \{ a_n \mapsto e_n \} \). Also \( a \mapsto \phi \), appearing within \( \{ \ldots \} \) may be abbreviated to \( a \). Thus \( \{ 1, 2, 3 \} \)
is an abbreviation for the “flat” tree \( \{ 1 \mapsto \phi, 2 \mapsto \phi, 3 \mapsto \phi \} \)

As a more elaborate example, the tree depicted in figure 1 can be built with the following syntax:

\[
\begin{align*}
bib = \{ 'doc' \mapsto \{ 'topic' \mapsto \{ "Genetics" \}, \\
\quad 'book' \mapsto \{ 'title' \mapsto \{ "Cartoon Guide to Genetics" \}, \\
\quad \quad 'authors' \mapsto \{ "Gonick", "Wheels" \} \} \\
\quad 'doc' \mapsto \{ 'topic' \mapsto \{ "Math" \}, \\
\quad 'book' \mapsto \{ 'title' \mapsto \{ "Cartoon Guide to Statistics" \}, \\
\quad \quad 'authors' \mapsto \{ "Gonick" \} \} \\
\quad 'doc' \mapsto \{ 'topic' \mapsto \{ "Genetics", "Database" \}, \\
\quad 'article' \mapsto \{ 'title' \mapsto \{ "FlyBase-the Drosophila database" \}, \\
\quad \quad 'authors' \mapsto \{ "The FlyBase Consortium" \} \}
\end{align*}
\]

3 Nested relational algebra on trees

The previous section gave a syntax for constructing trees. We now extend this to the syntax of a programming
language for trees. To our types \text{label} and \text{tree} we add a boolean type \text{bool} with the usual boolean connectives\( ^1 \).

We also add

- An equality test \( a = b \) on labels. Equality is of type \text{label} \times \text{label} \rightarrow \text{bool}.
- An emptiness test \( \text{null}(t) \) on trees. \( \text{null} : \text{tree} \rightarrow \text{bool} \)
- A conditional \( \text{if } b \text{ then } e_1 \text{ else } e_2 \) in which \( b \) is a boolean expression and \( e_1, e_2 \) denote trees.

Since trees are necessarily sets, we first consider structural recursion on sets as the basic programming paradigm,
and following [3] use the restricted form of structural recursion given by functions \( h \) of the form

\[
\begin{align*}
h(\phi) &= \phi \\
h(\{ a \mapsto t \}) &= f(a, t) \\
h(t_1 \cup t_2) &= h(t_1) \cup h(t_2)
\end{align*}
\]

\( ^1 \)The introduction of a boolean type is inessential. We can simulate \text{false} with the empty set and \text{true} with some non-empty set. See [3] for details.
In this, the meaning of the function $h$ of type $\text{tree} \rightarrow \text{tree}$ is determined by the function $f : \text{label} \times \text{tree} \rightarrow \text{tree}$. Note that this is a mathematical definition, which suggests an implementation. The syntax we will use for $h(S)$ when the function $f$ is defined by $f(a,t) = e$ is $\text{ext}(\lambda(a,t).e)(S)$.

An example of this form of definition is a selection function:

$$\text{sel}(b)(\phi) = \phi$$
$$\text{sel}(b)(\{a \Rightarrow t\}) = \text{if } a = b \text{ then } \{a \Rightarrow t\} \text{ else } \phi$$
$$\text{sel}(b)(t_1 \cup t_2) = \text{sel}(b)(t_1) \cup \text{sel}(b)(t_2)$$

In our syntax, $\text{sel}(b)(S)$ can be written as $\text{ext}(\lambda(a,t).\text{if } a = b \text{ then } \{a \Rightarrow t\} \text{ else } \phi)(S)$. Its effect is to discard from $S$ all edges that are not labeled with $b$, together with their subtrees. For example, $\text{sel}(1)(\{1 \Rightarrow \{10\}, 2 \Rightarrow \{20\}\}) = \{1 \Rightarrow \{10\}\}$.

Another useful function is projection, defined as $\text{proj}(b)(S) \overset{\text{def}}{=} \text{ext}(\lambda(a,t).\text{if } a = b \text{ then } t \text{ else } \phi)(S)$. This function takes the union of the trees at the ends of $b$ edges and discards the others. Note how it differs from $\text{sel}(b)$: $\text{proj}(1)(\{1 \Rightarrow \{10\}, 2 \Rightarrow \{20\}\}) = \{10\}$.

A flattening function defined as $\text{flat}(S) \overset{\text{def}}{=} \text{ext}(\lambda(a,t).t)(S)$ will also be useful. This function removes one level of edges out of the root and takes the union of the subtrees at their ends: $\text{flat}(\{1 \Rightarrow \{10\}, 2 \Rightarrow \{20\}\}) = \{10, 20\}$.

To summarize the language at this point, we assume we have an infinite collection of typed variables for labels (ranged over by $a$) and for trees (ranged over by $t$). In addition we have a set of constants of type $\text{label}$ as described above. An expression in the language is built up from the variables and constants with the following rules:

$$\begin{align*}
\phi &: \text{tree} \\
1 &: \text{label} \\
et &: \text{tree} \\
\text{null} &: \text{bool} \\
el_1 &: \text{label} \\
el_2 &: \text{label} \\
b &: \text{bool} \\
e_1 &: \text{tree} \\
e_2 &: \text{tree} \\
e' &: \text{tree} \\
\text{ext}(\lambda(a,t).e') &: \text{tree}
\end{align*}$$

We also assume the boolean constants and operations with the obvious typing rules together with other appropriate operations on labels. We shall call this language $\text{EXT}$.

**Nested Relational Algebra.** We now have a language equivalent in expressive power to the nested relational algebra, since it includes all the operations described in [3]. Note that although we have not introduced an explicit pairing operation into $\text{EXT}$, it can be simulated with the operators already available. If we fix two labels, $1, 2$, the pair $e_1, e_2$ can be expressed as $\{1 \Rightarrow e_1, 2 \Rightarrow e_2\}$ and the projection operations are $\text{proj}(1), \text{proj}(2)$. In particular, we may simulate a (flat) relational database by constructing for each tuple $(t_1, \ldots, t_n)$ in relation $R(A_1, \ldots, A_n)$ a tree $\{R \Rightarrow \{A_1 \Rightarrow \{t_1\}, \ldots, A_n \Rightarrow \{t_n\}\}\}$ and taking the union of all such trees.

To illustrate the types of queries and transformations that we can perform with $\text{EXT}$ we give some examples in the spirit of [5]. To simplify their presentation, we will use the following abbreviations: $e.a$ for $\text{proj}(a)(e)$, $e | a$ for $\text{sel}(a)(e)$, and $a$ in $e$ for $\neg \text{null}(\text{sel}(a)(e))$.

**Example 1:** Find the titles of all books on Genetics.

$$\text{ext}(\lambda(a,t).\text{if } a = \text{"doc"} \text{ and } \text{\"Genetics\" in t.\text{topic}} \text{ then t.\text{book}} |\text{title else } \phi)(\text{bib})$$

**Example 2:** Find the authors of all documents, regardless of the type of document.

$$\text{ext}(\lambda(a,t).\text{if } a = \text{\"doc\" then (flat t.) \text{\"authors else } \phi)(bib)$$}
Example 3: Find the title and topic of all books by Gonick and Wheels.

\[ \text{ext}(\lambda(a,t).\text{if } (a = 'doc') \text{ and } ('Gonick' in t.'book.'author) \text{ and } ('Wheels' in t.'book.'author) \text{ then } \{ 'book' \Rightarrow \{ t.'topic' \} \cup \{ t.'book.'title' \} \} \text{ else } \phi)(\text{bib}) \]

The last example does not return a subtree of the original tree, and illustrates how the result can restructure information. Such restructuring cannot be performed in OEM, the language of [5].

It should also be observed that the queries in these examples assume a particular structure on the trees, i.e., that the labels of interest appear at predetermined depths. In the next section, we will see how to specify queries which operate on trees in which labels can appear at arbitrary depths.

4 Structural recursion on trees

We now consider a form of structural recursion that one would naturally associate with trees.

\[
\begin{align*}
\text{h}(\phi) &= \phi \\
\text{h}(\{a\Rightarrow t\}) &= f(a, \text{h}(t)) \\
\text{h}(t_1 \cup t_2) &= \text{h}(t_1) \cup \text{h}(t_2)
\end{align*}
\]

The only difference between this and our previous form of structural recursion is that \( h \) acts recursively on the subtrees of a tree. As before we will use the syntax \( \text{text}(\lambda(a,r).e)(S) \) for \( h(S) \) when the function \( f \) is defined by \( f(a,r) = e \).

For example, to change each 'topic' label to a 'subject' label we may use the function \( \text{change\_lab} \) defined by

\[
\begin{align*}
\text{change\_lab}(\phi) &= \phi \\
\text{change\_lab}(\{a\Rightarrow t\}) &= \text{if } a = 'topic' \text{ then } \{ 'subject' \Rightarrow \text{change\_lab}(t) \} \text{ else } \{ a \Rightarrow \text{change\_lab}(t) \} \\
\text{change\_lab}(t_1 \cup t_2) &= \text{change\_lab}(t_1) \cup \text{change\_lab}(t_2)
\end{align*}
\]

This will change labels at any depth in the tree. It is expressed using \( \text{text}(\lambda) \) as

\[
\text{change\_lab}(S) \overset{\text{def}}{=} \text{text}(\lambda(a,r).\text{if } a = 'topic' \text{ then } \{ 'subject' \Rightarrow r \} \text{ else } \{ a \Rightarrow r \})(S)
\]

We may also write a selection function that operates over the whole tree. \( \text{tsel}(p)(S) \) selects only those edges in \( S \) that satisfy \( p \); the other edges are lost and their subtrees become inaccessible. It is defined by

\[
\text{tsel}(p)(S) \overset{\text{def}}{=} \text{text}(\lambda(a,r).\text{if } p(a) \text{ then } \{ a \Rightarrow r \} \text{ else } \phi)(S)
\]

Applying this to the \( \text{bib} \) structure with the predicate \( p(x) = \neg(x = 'topic') \) will result in the topic labels and the associated strings being removed from the tree.

We may also build a "flat" tree of all the edges in \( S \) with

\[
\text{flat\_trees}(S) \overset{\text{def}}{=} \text{text}(\lambda(a,r).\{ a \} \cup r)(S)
\]

Then \( \text{flat\_trees}(\text{bib}) \) results in \{ 'doc', 'topic', "Genetics", 'book' ... \}. With such a transformation and the use of the discriminating function for strings, we can easily find all the strings in the database.
A more interesting example is to find a tree containing the set of all paths from the root of the tree. We represent a path by a list, or “vertical” tree, so that the path consisting of the sequence of labels 'doc', 'book', 'title' is \( \{ \text{'doc}\Rightarrow \{ \text{'book}\Rightarrow \{ \text{'title} \} } \} \). We can obtain the set of all paths with

\[
\text{all\_paths}(S) \overset{\text{def}}{=} \text{text}(\lambda(a, r).\{ a \} \cup \text{cxt}(\lambda(a_1, r_1).\{ a_1 \Rightarrow r_1 \}))(S)
\]

In this expression, \( r \) is bound recursively to all the paths of the subtree below the edge \( a \). The set of paths we want includes the single edge \( a \) together with the paths that are formed by tacking \( a \) onto the beginning of each of the paths in \( r \), which is done with an application of \( \text{cxt}(\_) \). The result of this query will be

\[
\{ \text{'doc},
\text{'doc}\Rightarrow \{ \text{'topic} \},
\text{'doc}\Rightarrow \{ \text{'topic}\Rightarrow \{ "Genetics" \} \},
\text{'doc}\Rightarrow \{ \text{'book} \},
\text{'doc}\Rightarrow \{ \text{'book}\Rightarrow \{ \text{'title} \} \},
\ldots
\}
\]

As a final example of the use of \( \text{text}(\_) \), consider the expression

\[
\text{blow\_up}(S) \overset{\text{def}}{=} \text{text}(\lambda(a, r).\{ a \Rightarrow r \} \cup r)(S)
\]

When \( S \) the linear tree \( \{ 1 \Rightarrow \{ 2 \Rightarrow \{ 3 \} \} \} \) \( \text{blow\_up}(S) \) produces \( \{ 1 \Rightarrow \{ 2 \Rightarrow \{ 3 \} \}, 2 \Rightarrow \{ 3 \}, 3 \} \). When \( S \) is a linear tree with \( n \) edges it produces a tree with \( 2^n - 1 \) edges. We shall see later that this apparent growth does not imply that we are performing transformations that are outside PTIME.

The preceding examples show that a number of queries that are expressed with “path variables” [5] can be computed in \( \text{EXT} \) together with \( \text{text}(\_) \). We will refer to this extended version of \( \text{EXT} \) as \( \text{TEXT} \). However there is one extremely useful query which presents some difficulty; it is that of computing the union of a tree with all of its subtrees. A refinement of this problem is to obtain all the edges that satisfy a certain property together with their subtrees. For example, we might want all the books in the \textit{bib} structure. In order to build such a tree we need a more general form of recursion

\[
\begin{align*}
h(\phi) &= \phi \\
h(\{ a \Rightarrow t \}) &= f(a, t, h(t)) \\
h(t_1 \cup t_2) &= h(t_1) \cup h(t_2)
\end{align*}
\]

in which the function \( f \) is now a three-place function taking the edge label \( a \), the input subtree \( t \), and the result of recursively computing \( h \) on that subtree. We use the form \( \text{text}'(\lambda(a, t, r).e)(S) \) for this form, where the function \( f \) is defined by \( f(a, t, r) = e \). The union of a tree \( S \) with all of its subtrees is now given by:

\[
\text{all\_trees}(S) \overset{\text{def}}{=} \text{text}'(\lambda(a, t, r).\{ a \Rightarrow t \} \cup r))(S)
\]

As a further example, to find all the books in the \textit{bib} database together with their subtrees we can write

\[
\text{text}'(\lambda(a, t, r).\{ \text{if } a = \text{'book then } \{ a \Rightarrow t \} \text{ else } \phi \} \cup r))(\textit{bib})
\]

Note that if we have one \textit{'book} tree below (or inside) another, this function will extract both of them.
\[ X_1 \cup X_2 \text{ where} \]
\[ X_1 = \{ \text{doc} \Rightarrow \{ \text{topic} \Rightarrow \{ "Genetics" \}, \text{book} \Rightarrow \{ \text{title} \Rightarrow \text{"Cartoon Guide to Genetics"}, \text{authors} \Rightarrow Y_1 \cup Y_2 \} \} \]
\[ X_2 = \{ \text{doc} \Rightarrow \{ \text{topic} \Rightarrow \{ "Math" \}, \text{book} \Rightarrow \{ \text{title} \Rightarrow \text{"Cartoon Guide to Statistics"}, \text{authors} \Rightarrow Y_1 \} \} \]
\[ Y_1 = \{ \text{"Gonick"}, \text{papers} \Rightarrow X_1 \cup X_2 \} \]
\[ Y_2 = \{ \text{"Wheels"}, \text{papers} \Rightarrow X_1 \} \]

Figure 2: A specification of a cyclic structure

It is possible to implement \( \text{text}(\_\_) \) in \( \text{TEXT} \), so that this form of structural recursion presents nothing new. However, the implementation involves the use of projection, which can cause problems when we consider operations on cyclic structures. The direct use \( \text{text}(\_\_) \) is more closely related to a form of structural recursion that we shall now examine.

5 Cyclic structures

The languages \( \text{EXT} \) and \( \text{TEXT} \) operate on labeled trees. Surprisingly, \( \text{EXT} \) and an important fragment of \( \text{TEXT} \), which we call \( \text{VEXT} \), can be extended naturally from trees to cyclic structures. This is due to the fact that the queries in \( \text{EXT} \) and \( \text{VEXT} \) can be computed by independently processing each edge of a cyclic structure, without needing to chase every path in the structure.

Syntactically, we describe cyclic structures with the aid of variables and equations defining these variables. Semantically, cyclic structures are rooted, labeled graphs.

Consider the syntactic specification of a cyclic structure given in figure 2. It uses the variables \( X_1, X_2, Y_1, Y_2 \), and four equations defining them. We recover a tree for such a specification by textually substituting in \( X_1 \cup X_2 \) each variable with the right hand side of the equation defining it, and by repeating this process until all variables are eliminated, thus unfolding a labeled tree. E.g. consider the same example from figure 2, but with the definitions of \( Y_1, Y_2 \) changed to: \( Y_1 = \text{"Gonick"} \) and \( Y_2 = \text{"Wheels"} \). Then by unfolding we get a subtree of the tree of figure 1. In general the unfolded tree may be infinite. Note that we may have different syntactic specifications denoting the same tree: these should be regarded as equal. Also, exactly those (infinite) labeled trees are meanings of syntactic specifications of cyclic structures, which are rational, i.e. for which the set of subtrees is finite.

Formally, a syntactic specification of a cyclic structure is:
\[
e \text{ where} \]
\[
X_1 = e_1 \\
\ldots \\
X_k = e_k
\]

Here \( e, e_1, \ldots, e_k \) are labeled trees with markers \( X_1, \ldots, X_k \). More precisely, they are expressions built up using the three constructors \( \emptyset, \{ \Rightarrow \_, \} \), and \( \cup \), and which may have the variables \( X_1, \ldots, X_k \) on their leaves. The type
Figure 3: The graph associated to a cyclic structure (the label $\varepsilon$ stands for $\varepsilon$)

tree_{x_1, \ldots, x_k}$ of labeled trees with markers $X_1, \ldots, X_k$ is defined by:

$$tree_{x_1, \ldots, x_k} = \mathcal{P}_{fin}(label \times tree_{x_1, \ldots, x_k} \cup \{X_1, \ldots, X_k\})$$

We write $\{\text{name} \Rightarrow \text{“Joe Doe”}\} \cup X_1 \cup X_2$ instead of the official $\{\text{name} \Rightarrow \text{“Joe Doe”}, X_1, X_2\}$.

The semantics of cyclic structures is given by rooted, labeled graphs, $G = (V, E, r, l)$. Each such graph has a distinguished vertex $r \in V$ called the root, and the edges are labeled with elements from $label \cup \{\varepsilon\}$, i.e. $l : E \rightarrow label \cup \{\varepsilon\}$, where $\varepsilon$ is a special symbol not occurring in $label$. E.g. the cyclic structure of figure 2 will be interpreted as the graph given in figure 3. Notice how we use $\varepsilon$-edges to connect an occurrence of some variable $X_i$ with its definition.

It is on these graphs that we can now define equality. Namely we say that two graphs $G = (V, E, r, l)$, $G' = (V', E', r', l')$ are bisimilar iff there exists a binary relation $\sim \subseteq V \times V'$ s.t. (1) $r \sim r'$, and (2) if $v \sim v'$ then for any label $a \in label$, there exists a path $v \xrightarrow{a} \ldots \xrightarrow{a} w$ in $G$ iff there exists some path $v' \xrightarrow{a} \ldots \xrightarrow{a} w'$ and $w \sim w'$. We state here without proof that two graphs are bisimilar iff their (potentially infinite) unfolded labeled trees are equal.

This gives us an effective procedure for deciding whether two syntactic specifications are equal. Namely (1) convert the two specifications to rooted graphs, and (2) test whether the two graphs are bisimilar. Note that testing for bisimilarity is a PTIME problem. By contrast, testing for graph isomorphism is believed to be outside of PTIME. See [1] for a discussion of the relevance of bisimulation in query languages with object identities.

Now we will extend our languages to cyclic structures. First notice that all operations in $EXT$ can be extended straightforwardly to cyclic structures. This is obvious in the case of $\emptyset, \{\cdot \rightarrow \cdot\}$, and $\cup$. To apply $null$ of $ext(f)$ to some syntactic specification $t$, we first have to expose the topmost set in $t$. For this we convert $t$ into a rooted, labeled graph $G$, and then restructure into a graph $G'$ bisimilar to $G$, in which no $\varepsilon$-edge leaves the root (this is always possible). Next we convert $G'$ back to a syntactic specification $t'$, and on $t'$ we apply $null$, or $ext(f)$.

But $text$ cannot be extended to cyclic structures. Indeed, consider the query $all\_paths$, of section 4. On some

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*During the unfolding of a graph, the $\varepsilon$ edges will be removed.*
infinite rational tree as input, i.e. a cyclic structure, it will return as output an infinite non-rational tree, i.e. a non-cyclic structure.

Fortunately there exists a natural restriction of text pointed to us by Val Tannen [6], which allows us to define most of the queries of section 4, and which generalizes naturally to cyclic structures. We call this restriction vect.

To define vect, we start by discussing another primitive operation, substitution. Let $X$ be some variable. We add a new primitive to the ones already mentioned, namely the substitution $\text{subst}_X: \text{tree}_X \times \text{tree}_X \rightarrow \text{tree}_X$. $\text{subst}_X(s,t)$ will simply replace every occurrence of $X$ in $s$ with $t$. Formally:

$$\text{subst}_X(X,t) \overset{\text{def}}{=} t$$
$$\text{subst}_X(\{a \Rightarrow s\},t) \overset{\text{def}}{=} \{a \Rightarrow \text{subst}(s,t)\}$$
$$\text{subst}_X(t_1 \cup t_2,t) \overset{\text{def}}{=} \text{subst}_X(t_1,t) \cup \text{subst}_X(t_2,t)$$

Notice that on “lists”, i.e. trees in which each interior node has exactly one son, like $\{a \Rightarrow \{a_2 \Rightarrow \{a_3 \Rightarrow X\}\}\}$, $\text{subst}_X$ becomes append.

Finally we are ready to introduce the vect construct. Namely for any function $f_X: \text{labels} \rightarrow \text{tree}_X$, $h = \text{vect}_X(f_X)$ is defined by:

$$h(\emptyset) \overset{\text{def}}{=} \emptyset$$
$$h(\{a \Rightarrow t\}) \overset{\text{def}}{=} \text{subst}_X(f_X(a),h(t))$$
$$h(t_1 \cup t_2) \overset{\text{def}}{=} h(t_1) \cup h(t_2)$$

Again, notice that on “lists” vect$_X$ is simply ext on lists.

We now informally describe how vect$_X(f)$ acts on some syntactic specification of a cyclic structure $t = (e$ where $X_1 = e_1, \ldots, X_k = e_k)$. The syntactic specification $t' = \text{vect}_X(f_X)$ is obtained from $t$ by processing every subexpression of the form $\{a \Rightarrow e\}$ of $e, e_1, \ldots, e_k$ as follows: we first fetch a fresh variable $Y$, next replace the subexpression $\{a \Rightarrow e\}$ with $f_Y(a)$, and finally add the equation $Y = e'$. Intuitively there is a high degree of parallelism in the computation of vect$_X$, which can be visualized even better on rooted, labeled graphs. Namely here, vect$_X(f_X)(S)$ is computed by independently replacing each edge $v \overset{\varepsilon}{\Rightarrow} w$ with the tree $f_X(a)$ having $v$ as the root, and by drawing $\varepsilon$-edges from the leaves $X$ of $f_X(a)$ to $w$; the $\varepsilon$-edges in the original graph are left untouched.

We call VEXT the language obtained by extending EXT with the vect construct. Obviously the following relationships between languages holds:

$$\text{EXT} \subset \text{VEXT} \quad \subset \quad \text{TREES}$$

Here are some of the queries from section 4 expressed with vect$_X$:

$$\text{changeLab}(S) \overset{\text{def}}{=} \text{vect}_X(\lambda a.\text{if } a = '\text{topic} \text{ then } \{\text{subject} \Rightarrow X\} \text{ else } \{a \Rightarrow X\})(S)$$
Notice how sharing allows us to avoid combinatorial explosion of the data structure resulting from the function blow. E.g. on the input tree $S = \{a_1 \Rightarrow \{a_2 \Rightarrow \ldots \{a_n \Rightarrow \emptyset \} \ldots \}$, blow will return:

$\begin{align*}
X_0 & \text{ where} \\
X_0 & = \{a_1 \Rightarrow X_1 \} \cup X_1 \\
X_1 & = \{a_2 \Rightarrow X_2 \} \cup X_2 \\
& \quad \ldots \\
X_{n-1} & = \{a_{n-1} \Rightarrow X_n \} \cup X_n \\
X_n & = \{a_n \Rightarrow \emptyset \}
\end{align*}$

When we unfold this compact syntactic specification, we obtain a tree with $2^n$ nodes. In general, we have:

**Proposition 5.1** $\text{VEXT}$ is in PTIME.

Unlike text however, $\text{vext}_X$ does not seem to be able to express alltrees. However we can express alltrees with a slight generalization of $\text{vext}_X$, from one variable $X$ to an arbitrary number of variables. First we define $\text{subst}_{X_1,\ldots,X_n}(t,t_1,\ldots,t_n)$ to substitute simultaneously $X_1$ with $t_1,\ldots,X_n$ with $t_n$ in $t$. Then $\text{vext}_{X_1,\ldots,X_n}(f_1,\ldots,f_n)$ is a construct which allows us to define simultaneously $n$ functions $h_1,\ldots,h_n$ by iteration on the cyclic structure (we omit the subscripts $X_1,\ldots,X_n$):

$\begin{align*}
& h_1(\emptyset) \overset{\text{def}}{=} \emptyset \\
& h_1(\{a \Rightarrow s\}) \overset{\text{def}}{=} \text{subst}(f_1(a),h_1(s),\ldots,h_n(s)) \\
& h_1(t_1 \cup t_2) \overset{\text{def}}{=} h_1(t_1) \cup h_1(t_2) \\
& \quad \ldots \\
& h_n(\emptyset) \overset{\text{def}}{=} \emptyset \\
& h_n(\{a \Rightarrow s\}) \overset{\text{def}}{=} \text{subst}(f_n(a),h_1(s),\ldots,h_n(s)) \\
& h_n(t_1 \cup t_2) \overset{\text{def}}{=} h_n(t_1) \cup h_n(t_2)
\end{align*}$

Then we can compute allpath using $\text{vext}_{X_1,\ldots,X_n}$. More interestingly, for any regular expression on labels, $R$, we can write in $\text{VEXT}$ an expression $\text{project}_R(S)$ which, on a given tree $S$, returns the set of all subtrees which can be reached from the root of $S$ using a path in $R$. E.g. when $R = (a(bc)^*)^*$, then $\text{select}_R(S)$ will return the set of all subtrees in $S$ which can be reached from the root by a path of the form $abc\ldots babc\ldots bc \ldots abc \ldots bc$. We invite the reader to check that, for any regular expression $R$, $\text{select}_R(S)$ can be written using $\text{vext}_{X_1,\ldots,X_n}$. It suffices to take $n$ as the number of states in the deterministic automaton accepting $R$.

6 Conclusions

We believe that the forms of structural recursion on trees described in this paper offer good prospects for the development of powerful query languages for unstructured data. However to demonstrate this, considerable additional work is needed. First, we need to substantiate the claim that the graph/bisimulation model provides an effective semantics for the language developed in section 5. Second, there appear to be some optimization
techniques for these languages, but the generality of these is an open question. An interesting question is how schema information, when it is present, may be used in optimization as suggested in [2]. Third, there are good syntactic forms for languages similar to EXT[4] that resemble the syntax of popular database query languages. We hope that it will be possible to find as attractive ways of representing the more powerful constructs developed in this paper.

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