ORIGIN OF A BOTTOM-HEAVY STELLAR INITIAL MASS FUNCTION IN ELLIPTICAL GALAXIES
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ABSTRACT
We investigate the origin of a bottom-heavy stellar initial mass function (IMF) recently observed in elliptical galaxies by using chemical evolution models with a non-universal IMF. We adopt the variable Kroupa IMF with the three slopes ($\alpha_1$, $\alpha_2$, and $\alpha_3$) dependent on metallicities ([Fe/H]) and densities ($\rho_g$) of star-forming gas clouds and thereby search for the best IMF model that can reproduce (1) the observed steep IMF slope ($\alpha_2 \sim 3$, i.e., bottom-heavy) for low stellar masses ($m \leq 1 M_\odot$) and (2) the correlation of $\alpha_2$ with chemical properties of elliptical galaxies in a self-consistent manner. We find that if the IMF slope $\alpha_2$ depends on both [Fe/H] and $\rho_g$, then elliptical galaxies with higher [Mg/Fe] can have steeper $\alpha_2$ ($\sim 3$) in our models. We also find that the observed positive correlation of stellar mass-to-light ratios ($M/L$) with [Mg/Fe] in elliptical galaxies can be quantitatively reproduced in our models with $\alpha_2 \propto \beta [\text{Fe/H}] + \gamma \log \rho_g$, where $\beta \approx 0.5$ and $\gamma \approx 2$. We discuss whether the IMF slopes for low-mass ($\alpha_2$) and high-mass stars ($\alpha_3$) need to vary independently from each other to explain a number of IMF-related observational results self-consistently. We also briefly discuss why $\alpha_2$ depends differently on [Fe/H] in dwarf and giant elliptical galaxies.

Key words: galaxies: abundances – galaxies: elliptical and lenticular, cD – galaxies: evolution – galaxies: stellar content

Online-only material: color figures

1. INTRODUCTION
The stellar initial mass function (IMF) is a principal parameter for formation and evolution of star clusters and galaxies. Therefore, it has long been discussed observationally and theoretically whether and how the IMF could vary with physical conditions of star-forming clouds in galaxies (e.g., Larson 1998; Chabrier 2003; Elmegreen 2009; Bastian et al. 2010; Kroupa et al. 2013). One of the important recent observational discoveries regarding the possible IMF variation in galaxies is that the IMF in massive elliptical galaxies could be bottom-heavy for low-mass stars (e.g., van Dokkum & Conroy 2010; Conroy & van Dokkum 2012, hereafter CV12; Smith et al. 2012; Spiniello et al. 2012; Ferreras et al. 2013). Recent observational studies of early-type galaxies have also revealed a possible correlation between the IMF slope and galaxy properties such as velocity dispersions and chemical abundances (e.g., Cenarro et al. 2003; Cappellari et al. 2012; CV12; Ferreras et al. 2012). It is, however, theoretically unclear what physical mechanisms are behind the observed correlations between the IMF and physical properties of early-type galaxies.

CV12 have investigated the spectral absorption lines in early-type galaxies in order to provide strong constraints on IMFs of the galaxies by using their updated population synthesis model. They have found that the IMF for low-mass stars becomes increasingly bottom-heavy with increasing velocity dispersions ($\sigma$) and [Mg/Fe] in the 38 early-type galaxies. Although they have found no strong correlations of the IMF with total metallicity ([Z/H]), it could be possible that there exists a weak/marginal IMF-metallicity correlation. These results lead the authors to suggest that total metallicity is not a key factor that determines the IMF slope for low-mass stars. CV12 have also derived the particular three-part (Kroupa) IMF that can best match the observed spectral indices and thereby inferred $M/L$ (i.e., they did not directly measure $M/L$). CV12 have shown a strong correlation of K-band mass-to-light ratios ($M_{K}/L_{K}$) normalized to the Milky Way (MW) value ($M/L_{K}^{\text{MW}}$) with [Mg/Fe] and briefly discussed the origin of the correlation. The origin of the observed $M/L_{K}^{\text{MW}}$–[Mg/Fe] correlation has not been extensively investigated by theoretical studies of elliptical galaxy formation.

Narayanan & Davé (2012) have adopted a broken power-law IMF with fixed slopes yet variable break mass depending on star formation rates (SFRs) and thereby investigated the IMF evolution of elliptical galaxies in cosmological simulations. They have found that $M/L_{K}$ can be larger for more massive elliptical galaxies with higher central velocity dispersions ($\sigma$). Although the simulated $M/L_{K}^{\text{MW}}$–$\sigma$ correlation derived by Narayanan & Davé (2012) is consistent qualitatively with the observed one by CV12, the simulated slope is too shallow to be consistent quantitatively with the observed one. Furthermore, their IMF model, with a fixed slope and no dependence of the slope on [Fe/H], appears to be inconsistent with recent observational results, which have shown different IMF slopes in different galaxies and a dependence of the IMF slope on [Fe/H] (e.g., Geha et al. 2013). This apparent inconsistency suggests that we need to search for a better IMF model that can explain both (1) the observed bottom-heavy IMF of elliptical galaxies and (2) the observed dependences of IMF slopes on physical properties (e.g., [Fe/H] and [Mg/Fe]) of galaxies. Recently, Weidner et al. (2013) have pointed out that a time-independent bottom-heavy IMF cannot explain the observed metallicities of elliptical galaxies and suggested a two-stage formation scenario.

Marks et al. (2012, hereafter M12) have recently proposed a variable Kroupa IMF model with the three IMF slopes, $\alpha_1$ (for 0.08 $\leq m/M_\odot < 0.5$), $\alpha_2$ (0.5 $\leq m/M_\odot < 1$), and $\alpha_3$ (1 $\leq m/M_\odot$). In their model, $\alpha_1$ and $\alpha_2$ depend on [Fe/H], whereas $\alpha_3$ depends on [Fe/H] and gas densities ($\rho_g$) of star-forming gas clouds. Their model is promising, firstly because their model is derived from a detailed comparison between theoretical and observational results of globular cluster properties, and secondly because the model can naturally explain recent observational
results on the positive correlation of \( \alpha_2 \) with [Fe/H] in galaxies with a wide range of velocity dispersions and metallicities (Geha et al. 2013). Furthermore, recent numerical simulations with the variable Kroupa IMF model by M12 have shown that the observed correlation between SFR densities and the slope of the high-mass end of the IMF (\( \alpha_1 \)) can be naturally reproduced (Bekki & Meurer 2013).

However, it is clear that the proposed IMF with a variable slope \( \alpha_2 \) by M12 (i.e., \( \alpha_2 = 2.3 + 0.5[\text{Fe/H}] \)) cannot simply explain the observed strong \( \alpha_2 - [\text{Mg/Fe}] \) correlation yet no/little correlation between \( \alpha \) and metallicities (Z and [Fe/H]) in elliptical galaxies (\( \alpha_1 \) is assumed to be \( \alpha_2 - 1 \) in the present study so that \( \alpha_2 \) can be a sole key parameter for the IMF of low-mass stars in galaxies). Furthermore, observational support for the proposed dependence of \( \alpha_2 \) on [Fe/H] is significantly weaker in comparison with \( \alpha_3 \) in M12. These facts imply that the variable IMF model by M12 needs to be modified significantly by considering possible dependences of \( \alpha_2 \) on other physical properties of star-forming gas clouds, such as gas densities \( \rho_g \), temperature \( T_\text{e} \), and pressure \( P_\text{e} \). Thus, it is particularly important for theoretical studies to investigate how \( \alpha_2 \) in the variable Kroupa IMF needs to depend on physical properties of star-forming clouds so that the observed bottom-heavy IMF and its correlation with galaxy properties can be self-consistently explained.

The purpose of this paper is to investigate how \( \alpha_2 \) should depend on physical properties of star-forming gas clouds within galaxies so that the observed positive correlation between \( \alpha_2 \) and [Mg/Fe] in elliptical galaxies can be quantitatively reproduced. We adopt one-zone chemical evolution models of elliptical galaxy formation with a more generalized version of the variable Kroupa IMF in M12 and thereby search for the best IMF model that can explain the observed \( \alpha_2 - [\text{Mg/Fe}] \) correlation. We compare the spectroscopically inferred K-band \( M/L \) (normalized to the MW value) with the simulated one in order to derive the best functional form of \( \alpha_2 (= f(\text{[Fe/H]}, \rho_g)) \). In the present study, we consider that observational results on the \( \alpha_2 - [\text{Mg/Fe}] \) (or \( M/L - [\text{Mg/Fe}] \)) correlation by CV12, which do not show strong correlations between \( \alpha_2 \) and metallicities, can be used for determining the best variable Kroupa IMF model. However, Cenarro et al. (2003) reported a correlation between the IMF slope and metallicities ([Fe/H]) in elliptical galaxies. If we use the results by Cenarro et al. (2003) as a constraint on the functional form of \( \alpha_2 \), then the best IMF model would be quite different from the one that we can determine by using the \( \alpha_2 - [\text{Mg/Fe}] \) correlation as a constraint. Therefore, it should be noted that the choice of the best variable IMF model can depend on which observational results are used as a constraint on the functional form of the IMF.

The layout of this paper is as follows. In Section 2, we describe our new one-zone chemical evolution models with a variable IMF model. In Section 3, we present the results of the time evolution of \( \alpha_2 \), [Fe/H], and [Mg/Fe] for models with different parameters. In this section, we show the best variable Kroupa IMF model that can reproduce observations by CV12. In Section 4, we discuss IMF-related observational results that appear to be inconsistent with a bottom-heavy IMF in elliptical galaxies. The conclusions of the present study are given in Section 5. We mainly focus on correlations between IMF slopes and chemical properties of galaxies and accordingly do not discuss recent observational results on the mass-to-light ratios of early-type galaxies that suggest a non-universal IMF (e.g., Treu et al. 2010) in the present study.

2. THE MODEL

2.1. Outline

2.1.1. Comparing the Spectroscopically Implied \( M/L \) with the Modeled Ones

The main purpose of this study is to compare the observed and modeled \( M/L - [\text{Mg/Fe}] \) relations. First of all, it should be noted that CV12 derived \( M/L \) of elliptical galaxies from the gravity-sensitive spectral features by adopting a variable Kroupa IMF (with fixed \( \alpha_1 \) yet variable \( \alpha_2 \) and \( \alpha_3 \)). Therefore, the \( M/L \) in CV12 is not a direct measurement of \( M/L \) and can depend on the modeling of IMF. We however compare the observationally inferred \( M/L \) by CV12 with the modeled ones in order to derive physical meanings of the observed \( M/L - [\text{Mg/Fe}] \) relation. Also, as described later, a number of assumptions are made in deriving \( M/L \) of elliptical galaxies from chemical evolution models. Thus, there are some limitations both in inferring \( M/L \) from the observed spectral features in CV12 and in modeling \( M/L \) in the present study.

2.1.2. Two Approximations in \( M/L \) Modeling

We need to derive both [Mg/Fe] and \( M/L \) by using the present one-zone chemical evolution model and a publicly available stellar population synthesis code. Although it is straightforward to calculate [Mg/Fe] in the present one-zone model, we need to take the following steps to estimate \( M/L \). First, we derive \( \alpha_2 \) (and \( \alpha_1 = \alpha_2 - 1 \)) of a galaxy by using the adopted variable Kroupa IMF that depends on gas densities and [Fe/H] in one-zone chemical evolution models. Then we estimate the mean \( M/L \) of old stellar populations of the galaxy from the derived \( \alpha_2 \) (that is integrated over all time steps). It would be ideal that we adopt a fully self-consistent one-zone model with chemical yields depending on \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) at each time step and use a stellar population synthesis code to calculate \( M/L \) of old stellar populations with different Kroupa IMF slopes. However, most of the stellar population synthesis codes are for a fixed IMF (but see Conroy et al. 2009 for a new model with a variable Kroupa IMF), and we have not yet developed one-zone chemical evolution models with variable Kroupa IMFs. Therefore, we have to adopt the following two approximations as a compromise.

One is that chemical yields do not depend on \( \alpha_2 \) (\( \alpha_1 \)) in the present one-zone models in which \( \alpha_2 \) is assumed to vary with time. This approximation can be justified as follows. Chemical yields from supernovae and asymptotic giant branch (AGB) stars depend much more strongly on \( \alpha_1 \) than on \( \alpha_2 \) and \( \alpha_3 \). Therefore, the inclusion of time-varying \( \alpha_1 \) and \( \alpha_2 \) in one-zone chemical evolution models would not change significantly the present results. In order to demonstrate this point, we have investigated [Mg/Fe]–[Fe/H] relations of elliptical galaxies with different \( \alpha_1 \) and \( \alpha_2 \) (but fixed \( \alpha_3 = 2.3 \), like the Salpeter IMF) in the variable Kroupa IMF, and the results and their discussion are shown in Appendix A. Clearly, the final [Mg/Fe] does not depend strongly on \( \alpha_1 \) and \( \alpha_2 \) (but it depends more strongly on star formation timescales of galaxies). Thus, the adopted approximation can be regarded as good enough to discuss the final [Mg/Fe] of galaxies.

The other is that \( M/L \) predicted for a single-power-law IMF rather than \( M/L \) for a (variable/fixed) Kroupa IMF is used for estimating \( M/L \) of a galaxy. The adoption of this approximation means that the present model is not self-consistent (i.e., using a stellar population synthesis code based on a
variable single-power-law IMF for chemical evolution models with a variable Kroupa IMF. However, this is the best that we can do, because most of the publicly available stellar population synthesis codes are for a fixed IMF and we have used them so far. In order to demonstrate whether this approximation is good enough to discuss $M/L$, we have investigated possible IMF $M/L$ difference in single-power-law and variable Kroupa IMFs with different $\alpha_2$, and the results are shown in Appendix B.

It is clear in Appendix B that (1) the absolute values of $M/L$ can be slightly different between single-power-law ($\alpha_1 = \alpha_2$) and variable Kroupa ($\alpha_1 = \alpha_2 - 1$) IMFs for a given $\alpha_2$ ($= \alpha_3$) and (2) the $M/L$ difference does not depend strongly on $\alpha_2$. These mean that single-power-law IMFs are highly likely to overestimate $M/L$ by a similar amount in comparison with the variable Kroupa IMFs (with the same $\alpha_3$) for a wide range of $\alpha_2$ ($= \alpha_3$). These accordingly demonstrate that the slope of the $M/L$–[Mg/Fe] relation modeled in the present study (in which a stellar population synthesis code for a variable single-power-law IMF is adopted) can be very close to the true one that is derived self-consistently by using a stellar population synthesis code for a variable Kroupa IMF. Therefore, a comparison between the observed and modeled slopes can be regarded as reasonable. It should be noted, however, that $\alpha_1$ and $\alpha_2$ in the IMF adopted by CV12 can vary independently (0 $\leq$ $\alpha_1$, $\alpha_2$ $\leq$ 3), and therefore their IMF is different from ours, in which $\alpha_1 = \alpha_2 = 1$.

Thus, the present model is not fully self-consistent in terms of the derivation of $M/L$ from the outputs of one-zone chemical evolution models owing to the adoption of the above two approximations. However, as long as we discuss the slope of the observed $M/L$–[Mg/Fe] relation, the present model greatly helps us to extract some important physical meanings of the observed $M/L$–[Mg/Fe] relation. We will be able to more properly estimate $M/L$ in our future studies by adopting the latest stellar population synthesis code for any combination of $\alpha_1$, $\alpha_2$, and $\alpha_3$ of a variable Kroupa IMF (e.g., Conroy et al. 2009). We consider that the main conclusion of this paper (i.e., $\alpha_2$ should be proportional to $-0.5[Fe/H] + 2 \log \rho_g$) will not change significantly in our future better models, because the conclusion is derived from a comparison between the observed and modeled slopes of the $M/L$–[Mg/Fe] relation (not from a comparison between the observed and simulated absolute values of $M/L$ themselves).

### 2.2. The Variable Kroupa IMF

We consider that the three slopes in the variable Kroupa IMF can vary according to the physical conditions of star-forming regions in the present study (this variable Kroupa IMF is illustrated in Figure 1). In the original M12's IMF, the low-mass end of the variable Kroupa IMF ($\alpha_1$ for 0.08 $\leq$ $m/M_\odot$ < 0.5) depends solely on [Fe/H] as follows:

$$\alpha_1 = 1.3 + 0.5 \times [Fe/H]. \quad (1)$$

The value of $\alpha_2$ for 0.5 $\leq$ $m/M_\odot$ < 1 is also determined solely by [Fe/H]:

$$\alpha_2 = 2.3 + 0.5 \times [Fe/H]. \quad (2)$$

The high-mass end of the variable Kroupa IMF $\alpha_3$ for 1 $\leq$ $m/M_\odot$ $\leq$ 100 is described as follows:

$$\alpha_3 = 0.0572 \times [Fe/H] - 0.4072 + \log \left( \frac{\rho_3}{10^6 M_\odot pc^{-3}} \right) + 1.9283, \quad (3)$$

where $\rho_3$ is the density of a rather high-density gaseous core where star formation can occur. This equation holds for $x_{th} \geq -0.87$, where $x_{th} = -0.1405[Fe/H] + \log(\rho_3/10^6 M_\odot pc^{-3})$, and $\alpha_3 = 2.3$ for $x_{th} < -0.87$ (M12). Although the two coefficients in Equation (3) are precisely described, they are simply the best-fit parameter values of their IMF that can explain observations (i.e., observations cannot determine the coefficients with such a high precision). In the present study, unlike in M12, it is assumed that $\alpha_3$ does not vary with densities and [Fe/H].

As briefly discussed in Section 1, the observed correlation between the IMF slope and chemical abundances in elliptical galaxies (CV12) cannot be simply explained by the above Equation (2). We therefore adopt the following more generalized version of the variable IMF for $\alpha_2$:

$$\alpha_2 = \alpha_{2,s} + \beta \times [Fe/H] + \gamma \times \log \rho_g, \quad (4)$$

where $\alpha_{2,s}$ is the value for the solar neighborhood and $\rho_g$ is the gas density of a star-forming gas cloud. This $\rho_g$ is the mean density of a star-forming cloud and thus different from the density of a molecular core ($\rho_{cl}$) in Equation (3). This functional form ($\alpha_2 = f([Fe/H], \rho_g)$) needs to ensure that $\alpha_2 \sim 2.3$ at the
solar neighborhood. Therefore, the above equation is modified as follows:

\[ \alpha_2 = 2.3 + \beta \times [\text{Fe}/\text{H}] + \gamma \times \log \frac{\rho_g + \rho_{\text{bh}}}{\rho_s + \rho_{\text{bh}}}, \tag{5} \]

where \( \rho_s \) is the typical gas density for star-forming gas clouds at the solar neighborhood and \( \rho_{\text{bh}} \) is introduced so that \( \alpha_2 \) cannot be too small for low-density star-forming regions. This \( \rho_{\text{bh}} \) can correspond to a threshold gas density beyond which star formation can occur. In the present study, \( \alpha_1 \) is assumed to be \( \alpha_2 - 1 \) throughout this paper, though in reality \( \alpha_1 \) could in principle vary independently from \( \alpha_2 \). We consider that gas density is a more fundamental parameter for \( \alpha_2 \) than SFR, because SFR can depend not only on gas density but also on other gas properties (e.g., molecular content and dynamical timescale). Also, the adopted relation between the IMF slope and gas density for \( \alpha_2 \) is more consistent with that for \( \alpha_1 \), in which gas density rather than SFR is a key parameter.

Our one-zone chemical evolution models (later described) can output \([\text{Fe}/\text{H}]\) and \(\rho_g\) so that we can investigate the time evolution of \(\alpha_2\) for a given \(\beta\) and \(\gamma\) by using Equation (5). Although we have investigated the models with \(\beta = 0\) and 0.5, we show the results of the models with \(\beta = 0.5\). This is firstly because \(\beta = 0.5\) is consistent with recent observations on the dependence of \(\alpha_2\) on \([\text{Fe}/\text{H}]\) (Geha et al. 2013), and secondly because the models with \(\beta = 0\) cannot explain the observed \(M/L - [\text{Mg}/\text{Fe}]\) relation. We investigate models with different \(\gamma\) (=0,0.5,1.0,1.5,2.0, and 2.5) and thereby try to find models that can reproduce the observed \(M/L_k - [\text{Mg}/\text{Fe}]\) correlation (which corresponds to an \(\alpha_2 - [\text{Mg}/\text{Fe}]\) correlation) in a quantitative manner. In the present study, we use chemical evolution models just for the purpose of finding the best variable Kroupa IMF with a certain value of \(\gamma\). Accordingly, in computing the predicted \([\text{Mg}/\text{Fe}]\), we adopt a standard (yet simple) Salpeter IMF model (\(\alpha_1 = \alpha_2 = \alpha_3 = 2.35\)) in which the IMF slope is fixed during chemical evolution of a galaxy. A justification of adopting such a model is given in Appendix A.

### 2.3. Chemical Evolution

Elliptical galaxies are assumed to form with initial massive starbursts at high redshifts, as often assumed in previous chemical evolution models (e.g., Arimoto & Yoshii 1987; Matteucci et al. 1998; Pipino & Matteucci 2004). The duration of the initial starbursts is assumed to be different in different models so that the final elliptical galaxies can have different \([\text{Fe}/\text{H}]\) and \([\text{Mg}/\text{Fe}]\) in the present study. We do not discuss other important aspects of elliptical galaxy formation, such as the origin of the color–magnitude or mass–metallicity relations among elliptical galaxies with different masses and luminosities. We adopt one-zone chemical evolution models that are essentially the same as those adopted in our previous studies on the chemical evolution of the Large Magellanic Cloud, LMC (Bekki & Tsujimoto 2012, hereafter BT12). Accordingly, we briefly describe the adopted models in the present study.

We investigate the time evolution of the gas mass fraction \((f_g(t))\), the SFR \((\psi(t))\), and the abundance of the ith heavy element \((Z_i(t))\) for a given accretion rate \((A(t))\), IMF, and ejection rate of interstellar medium (ISM) \((u(t))\). The basic equations for the adopted one-zone chemical evolution models are described as follows:

\[ \frac{df_g}{dt} = -\alpha_{\text{lock}} \psi(t) + A(t) - u(t), \tag{6} \]

\[ \frac{d(Z_i f_g)}{dt} = -\alpha_{\text{lock}} Z_i(t) \psi(t) + Z_{A,i}(t) A(t) + y_{\text{II},i} \psi(t) \]

\[ + y_{\text{Ia},i} \int_0^t \psi(t - t_b) g(t_b) dt_b + \int_0^t \nu_{\text{agb},i}(m_{\text{agb}}) \psi(t - t_{\text{agb}}) h(t_{\text{agb}}) dt_{\text{agb}} - W_i(t), \tag{7} \]

where \(\alpha_{\text{lock}}\) is the mass fraction locked up in dead stellar remnants and long-lived stars; \(y_{\text{II},i}\), \(y_{\text{Ia},i}\), and \(\nu_{\text{agb},i}\) are the chemical yields for the ith element from Type II supernovae (SNe II), from SNe Ia, and from AGB stars, respectively; \(Z_{A,i}\) is the abundance of heavy elements contained in the infalling gas, and \(W_i\) is the wind rate for each element. The quantities \(t_b\) and \(t_{\text{agb}}\) represent the time delay between star formation and SN Ia explosion and that between star formation and the onset of AGB phase, respectively. The terms \(g(t_b)\) and \(h(t_{\text{agb}})\) are the distribution functions of SNe Ia and AGB stars, respectively. The term \(t_{\text{agb}}\) controls how much AGB ejecta can be returned into the ISM per unit mass for a given time in Equation (7). The total gas masses ejected from AGB stars depend on the original masses of the AGB stars (e.g., Weidemann 2000). Therefore, this term \(t_{\text{agb}}\) depends on the adopted IMF and the age–mass relation of the stars. We adopt the same models for \(g(t_b)\) and \(h(t_{\text{agb}})\) as those used in BT12. The wind and ejection rates \((u(t))\) and \((W(t))\), respectively, are set to be 0 in all models of the present study. Thus, Equation (6) describes the time evolution of the gas due to star formation and gas accretion. Equation (7) describes the time evolution of the chemical abundances due to chemical enrichment by SNe and AGB stars.

The SFR \((\psi(t))\) is assumed to be proportional to the gas fraction with a constant star formation coefficient and thus is described as follows:

\[ \psi(t) = C_{\text{sf}} f_g(t). \tag{8} \]

This \(C_{\text{sf}}\) given in dimensionless units can control the strength of a starburst in each model, and its value is assumed to be different between different models. The star formation is assumed to be truncated at \(t_{\text{run}}\), after which elliptical galaxies can evolve passively without star formation. For the accretion rate, we adopt the formula in which \(A(t) = C_a \exp\left(-t/t_f\right)\) and \(t_f\) is a free parameter controlling the timescale of the gas accretion. The normalization factor \(C_a\) is determined such that the total gas mass accreted onto an elliptical galaxy can be 1 for a given \(t_s\) and \(t_{\text{run}}\).

Although we investigated models with different \(t_s\), we show the models with \(t_s = 0.5, 1, 2, 3, \) and 4 Gyr. The initial \([\text{Fe}/\text{H}]\) of the infalling gas is set to be \(-2\), and we assume an SN II like enhanced \([\alpha/\text{Fe}]\) ratio (e.g., \([\text{Mg}/\text{Fe}]\approx0.4\)) for the gas.

We adopt the nucleosynthesis yields of SNe II and Ia from Tsujimoto et al. (1995) to deduce \(y_{\text{II},i}\) and \(y_{\text{Ia},i}\). We adopt a fixed Salpeter IMF, \(dN/dm = \xi(m) \propto m^{-2.35}\), where \(\alpha\) is the IMF slope fixed at 2.35 for the calculation of chemical yields. The fraction of the stars that eventually produce SNe Ia for 3–8 \(M_\odot\) has not been observationally determined and thus is regarded as a free parameter, \(f_b\). Although we investigate models with \(f_b = 0.05, 0.1, \) and 0.15, we describe the models with \(f_b = 0.05\). The present chemical evolution models of elliptical galaxies with \(f_b = 0.05\) can show the mean \([\text{Mg}/\text{Fe}]\) as low as \(-0.1\) only after \(\approx 6\) Gyr. If we adopt \(f_b = 0.15\) (as Pipino & Matteucci 2004 did), then the mean \([\text{Mg}/\text{Fe}]\) can more rapidly become as low as 0.1. We consider that using 20 representative models with \(f_b = 0.05\) is enough to find the...
best γ for successful reproduction of observations regarding the IMF variation in elliptical galaxies. The parameter values of the 20 representative models (M1–M20) and four additional ones (M21–M24) with the variable Kroupa IMF models for Appendix A are summarized in Table 1.

| Model | t_a | t_trunc/t_b | C_α | IMF Type |
|-------|-----|------------|-----|----------|
| M1    | 0.5 | 2          | 0.4 | Salpeter |
| M2    | 1.0 | 2          | 0.4 | Salpeter |
| M3    | 2.0 | 2          | 0.4 | Salpeter |
| M4    | 3.0 | 2          | 0.4 | Salpeter |
| M5    | 4.0 | 2          | 0.4 | Salpeter |
| M6    | 0.5 | 1          | 0.4 | Salpeter |
| M7    | 1.0 | 1          | 0.4 | Salpeter |
| M8    | 2.0 | 1          | 0.4 | Salpeter |
| M9    | 3.0 | 1          | 0.4 | Salpeter |
| M10   | 4.0 | 2          | 0.4 | Salpeter |
| M11   | 0.5 | 2          | 0.2 | Salpeter |
| M12   | 1.0 | 2          | 0.2 | Salpeter |
| M13   | 2.0 | 2          | 0.2 | Salpeter |
| M14   | 3.0 | 2          | 0.2 | Salpeter |
| M15   | 4.0 | 2          | 0.2 | Salpeter |
| M16   | 0.5 | 2          | 0.8 | Salpeter |
| M17   | 1.0 | 2          | 0.8 | Salpeter |
| M18   | 2.0 | 2          | 0.8 | Salpeter |
| M19   | 0.5 | 2          | 0.8 | Salpeter |
| M20   | 0.5 | 2          | 0.8 | Salpeter |

Notes.

* The gas accretion timescale in units of Gyr.
* The ratio of the SF truncation timescale (t_trunc) to the gas accretion timescale t_a.
* The dimensionless parameter that controls SF rates. The larger C_α is in a galaxy of a model, the higher the SF rate is.
* For the single-power-law Salpeter IMF (M1–M20), α = 2.35 (i.e., α_2 = α_3 = 2.35) is adopted for all models. For the Kroupa IMF (M21–M24), different α_1 and α_2 are adopted, but α_3 is fixed at 2.3.

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2.4. Derivation of α_2 and M/L

In order to estimate α_2 by using Equations (5)–(8), the (typical) gas density of a star-forming cloud in a forming elliptical galaxy at each time step is calculated as follows:

$$\rho_g(t) = C_{den} F_g(t),$$

where C_{den} is a normalization factor for ρ_g and F_g(t) is the ratio of the total gas mass at a time t to the total mass of gas and stars at t = t_{trunc}. This F_g is calculated from Equations (6) and (7). In the present study, ρ_g and ρ_{gas} are set to be 1 and 0.5, respectively. Therefore, C_{den} should be 10–20 to ensure that an MW-like galaxy with F_g = 0.05–0.1 can have α_2 = 2.3 in Equation (5). Although we investigate the models with C_{den} = 5, 10, and 20, we show the results of the models with C_{den} = 20 (i.e., higher density of gas clouds). This is because the models with C_{den} = 5 and 10 do not show high α_2 as observed. Such models with lower C_{den} would be reasonable for disk galaxies with low mean mass densities.

As mentioned in Section 2.1, although we adopt a variable Kroupa IMF in our chemical evolution models, we cannot calculate M/L for the modeled galaxies in a fully self-consistent manner, because of the lack of stellar population synthesis codes for the variable Kroupa IMF with three different IMF slopes. Therefore, as a compromise, we use the code “MILES,” which is a new population synthesis code for a variable IMF with a single-power-law form and made publicly available by Vazdekis et al. (2010). MILES can output M/L for different metallicities and different IMF slopes (for a single-power-law IMF with a slope α). At each time step in a one-zone chemical evolution model, α_1 and α_2 can be derived by using Equation (5) and a relation of α_1 = α_2 − 1. Since MILES adopts a fixed single lower-law slope (i.e., α_1 = α_2 = α_3 = α), we have to use the derived α_2 as α in MILES and thereby estimate M/L by using the tabulated values of M/L in the single stellar population (SSP) of MILES. For example, if α_2 = 2.5 (thus α_1 = 1.5) at a time step of a model, then we use the tabulated M/L of α = 2.5 (i.e., α_1 = α_2 = α_3 = 2.5) for the age (and metallicity) of a stellar population formed at the time step in order to calculate the M/L. We estimate the mean M/L by using M/L of all stars formed in a model.

It should be noted that CV12 did not measure the masses of elliptical galaxies but instead compute the expected masses from the IMF required to fit the observed spectrum of the galaxies by using the observed absorption lines that are sensitive to stellar gravity. Furthermore, the MILES and an original SSP code are used in this study and CV12, respectively. Therefore, the M/L normalized by the MW value in CV12 is not exactly the same as M/L calculated in the present study. The simulated M/L is normalized by M/L for an SSP with a solar metallicity and an age of 12.6 Gyr so that the simulated M/L range can be similar to the observed one by CV12. This normalization is done just for convenience. We consider that a comparison between the present results and observational ones by CV12 enables us to derive the best variable Kroupa IMF model.

3. RESULTS

3.1. α_2 Evolution

Figure 2 shows the time evolution of [Mg/Fe] and galaxy evolution on the [Mg/Fe]–[Fe/H] plane for the five representative models with C_α = 0.4 and different t_a. Owing to the adoption of the prompt SN Ia model, [Mg/Fe] can rapidly decrease with time from the early evolutionary phase (t < 0.2 Gyr) for these models. The models with shorter t_a and thus stronger initial starburst can have larger final [Mg/Fe] and smaller [Fe/H] in these models. For the adopted f_b = 0.05 in these models, active star formation needs to continue at least ~6 Gyr so that galaxies can have the mean [Mg/Fe] (not the instantaneous one as shown in this figure) as small as 0.1. Although models with such a relatively long continuous star formation might not be reasonable for giant elliptical galaxies, observations by CV12 showed that some elliptical galaxies have [Mg/Fe] ~ 0.1. We therefore consider that these models (M4 and M5) can represent stellar populations of some elliptical galaxies with [Mg/Fe] ~ 0.1.

Figure 3 shows the time evolution of galaxies on the α_2–[Fe/H], α_2–[Fe/H], α_2–[Mg/Fe], and α_2–[Mg/Fe] planes for the five models with γ = 2.0. Here α_2 is an instantaneous value at a time t, whereas α_2 is the mean α_2 averaged for all stars formed by the time t. The values of [Fe/H] and [Mg/Fe] are those at a given time step (not the average over all stars formed to that time step). Since α_2 is proportional to 0.5[Fe/H] + 2 log ρ_g, α_2 can increase with time owing to (1) increasing [Fe/H] and (2) higher ρ_g in these models. After α_2...
Figure 2. Evolution of galaxies on the [Mg/Fe]−[Fe/H] plane (upper) and the time evolution of [Mg/Fe] for the five chemical evolution models, M1 with \( t_a = 0.5 \) Gyr (red, solid), M2 with \( t_a = 1.0 \) Gyr (blue, dotted), M3 with \( t_a = 2.0 \) Gyr (green, short-dashed), M4 with \( t_a = 3.0 \) Gyr (magenta, long-dashed), and M5 with \( t_a = 4.0 \) Gyr (cyan, dot-dashed). The star formation is assumed to be truncated at \( t = 2t_a \) in these models.

(A color version of this figure is available in the online journal.)

Figure 3. Evolution of \( \alpha_2 \) (upper two) and \( \alpha_{2,m} \) (lower two) as a function of \([\text{Fe/H}]\) (left) and [Mg/Fe] (right) for the five models, M1 (red, solid), M2 (blue, dotted), M3 (green, short-dashed), M4 (magenta, long-dashed), and M5 (green, dot-dashed). Here \( \alpha_2 \) is an instantaneous value at each time \( t \), whereas \( \alpha_{2,m} \) is the mean averaged for all stars formed by the time \( t \). The dotted lines indicate a canonical value of \( \alpha_2 \).

(A color version of this figure is available in the online journal.)

reaches its peak value at a certain [Fe/H], it starts to decrease owing to the lower \( \rho_g \). In this decreasing phase, \( \alpha_2 \) can decrease with decreasing [Mg/Fe]. The models with higher SF can show both higher final mean \( \alpha_2 (\alpha_{2,m}) \) and higher [Mg/Fe], which means that there should be a positive correlation between [Mg/Fe] and \( \alpha_2 \). Clearly, the models M1 and M2 show the final mean \( \alpha_2 \) significantly larger (i.e., steeper) than the canonical Salpeter IMF \((\alpha_2 = 2.3)\), which is consistent with observational results by CV12.

Figure 4 shows the locations of final elliptical galaxies on the \( M/L_K-[\text{Mg/Fe}] \) and \( \alpha_2-[\text{Mg/Fe}] \) planes in the 20 models with different \( C_{sf}, t_a, \) and \( t_{trun} \) for \( \gamma = 2.0 \). It is clear that galaxies with higher [Mg/Fe] can have higher \( M/L_K \) and larger \( \alpha_2 \) in these models. Owing to the adopted dependence of \( \alpha_2 \) both on [Fe/H] and on \( \rho_g \), there can be a dispersion in \( M/L_K \) and \( \alpha_2 \) for a given [Mg/Fe]. It should be stressed that the present models can reproduce not only the bottom-heavy IMF \((\alpha_2 \sim 3)\) at high [Mg/Fe] but also an IMF shallower than the Salpeter at low [Mg/Fe].

The original variable Kroupa IMF model for \( \alpha_2 \) (M12) depends only on [Fe/H], and therefore \( \alpha_2 \) can be only 2.4 at [Fe/H] = 0.2 in the model. This value of \( \alpha_2 \) is significantly smaller than the observed bottom-heavy IMF \((\alpha_2 \sim 3)\) for metal-rich giant elliptical galaxies, which means that the present variable IMF model has an advantage in reproducing the observed large \( \alpha_2 \) in giant elliptical galaxies.

3.2. Comparison with Observations

Figure 5 shows a comparison between the observed \( M/L_K-[\text{Mg/Fe}] \) correlation for elliptical galaxies (CV12)
and the predicted one for $\gamma = 2$. If $\alpha_2$ is assumed to be $0.5[\text{Fe/H}] + 2 \log \rho_e + 2.3$, then the predicted correlation appears to be very similar to the observed one both in the slope and in the dispersion in $M/L_K$. Although the ways to derive $M/L_K$ in observations and models are not exactly the same, this similarity would suggest that $\alpha_2$ depends on $\rho_e$ in star-forming gas clouds of elliptical galaxies at their formation. In the present models with $\alpha_2 \propto \beta[\text{Fe/H}] + \gamma \log \rho_e$, $\alpha_2$ and $M/L_K$ can be both larger in galaxies with higher $\rho_e$ for which star formation can proceed more rapidly so that $[\text{Mg/Fe}]$ can be higher for $\gamma > 0$. However, in order to reproduce the observed steep dependence of $M/L_K$ on $[\text{Mg/Fe}]$ (CV12), $\gamma$ needs to be as large as 2. These results imply that the observed slope in the $M/L_K-[\text{Mg/Fe}]$ correlation of elliptical galaxies can be used to give strong constraints on the IMF model for low-mass stars in forming galaxies.

Figure 6 shows a comparison between the observed $M/L_K-[\text{Mg/Fe}]$ correlation for elliptical galaxies (CV12) and the predicted one for $\gamma = 0$ (i.e., no dependence on $\rho_e$). Clearly, the predicted $M/L_K-[\text{Mg/Fe}]$ correlation is qualitatively inconsistent with the observed one, which means that $[\text{Fe/H}]$ is not a key parameter for the observed $M/L_K-[\text{Mg/Fe}]$ correlation. Given that galaxies with higher $[\text{Mg/Fe}]$ are likely to have lower $[\text{Fe/H}]$ in the present chemical evolution models, these results mean that there can be no/little correlation between $[\text{Fe/H}]$ and $M/L_K$ (and between $[\text{Fe/H}]$ and $\alpha_2$). It should be noted here that a positive $[\text{Fe/H}]-\alpha_2$ correlation is suggested by previous observations by Cenarro et al. (2003), though such a correlation was not found in CV12.

Figure 7 shows that the predicted $M/L_K-[\text{Mg/Fe}]$ correlation in the models with $\gamma = 1$ is significantly shallower than the observed one. If the dependence of $\alpha_2$ on $\rho_e$ is weaker, then neither the high $M/L_K (>1.5)$ nor the steep slope in the observed $M/L_K-[\text{Mg/Fe}]$ correlation can be quantitatively reproduced in the present models. These results confirm the importance of $\rho_e$ in controlling the IMF slope for low-mass stars in galaxies. It is confirmed that if $\gamma = 2.5$, then the final mean $\alpha_2$ in some models can be too large (3.5) to be consistent with observations. Therefore, $\gamma$ needs to be in a certain range for successful reproduction of observations. Given that $P_g$ depends on $\rho_e$ through a thermodynamic equation, the derived steep dependence of $\alpha_2$ on $\rho_e$ implies that $P_g$ could be also a key parameter for the IMF slope of low-mass stars.

4. DISCUSSION

4.1. Origin of the Observed Different Dependences of the IMF Slope $\alpha_2$ on $[\text{Fe/H}]$ between Dwarfs and Giant Elliptical Galaxies

Geha et al. (2013) have recently shown that the IMF slopes ($\alpha_2$) of dwarfs galaxies in the Local Group (e.g., LMC, Small Magellanic Cloud, and ultra-faint dwarfs) are steeper in more metal-rich systems. Although the total number of galaxies with known IMF slopes for low-mass stars is still very small, this
result appears to be consistent at least qualitatively with the proposed IMF dependence on $[\text{Fe}/\text{H}]$ by M12. This result, however, appears to be inconsistent with the result by CV12, who found no clear $[\text{Fe}/\text{H}]$ dependence of the IMF slope in giant elliptical galaxies. It should be noted here that the $[\text{Fe}/\text{H}]$ range of elliptical galaxies in CV12 is only 0.3 dex and thus corresponds to only 0.15 variation in $\alpha_2$ (for the derived relation of $\alpha_1 \propto 0.5 \times [\text{Fe}/\text{H}]$ in Geha et al. 2013). The current observational data would not enable us to clearly distinguish between such a small $\alpha_2$ variation and no $\alpha_2$ variation with $[\text{Fe}/\text{H}]$ in elliptical galaxies. The apparently inconsistent results by CV12 and Geha et al. (2013) could possibly mean that the IMF slope for low-mass stars does not depend simply on $[\text{Fe}/\text{H}]$. So a key question here is why dwarfs appear to show more clearly the dependence of $\alpha_2$ (or $\alpha_1$) on $[\text{Fe}/\text{H}]$.

As shown in Figure 3 of the present study, $\alpha_2$ depends almost linearly on $[\text{Fe}/\text{H}]$ in the early chemical evolution phases (i.e., $-2 \leq [\text{Fe}/\text{H}] \leq -0.5$) of forming elliptical galaxies, even though $\alpha_2$ is assumed to depend on both $[\text{Fe}/\text{H}]$ and $\rho_\star$. This result suggests that if star formation can be truncated by stellar winds from massive stars and SNe in the early chemical evolution phases ($[\text{Fe}/\text{H}] < -0.5$) for dwarfs, and if the truncation epochs are earlier for less massive dwarfs, then the dwarfs can show a correlation between $\alpha_2$ and $[\text{Fe}/\text{H}]$ (and a mass–metallicity relation). Thus, the truncation epoch of star formation is a key parameter for the final $\alpha_2$ for dwarfs.

Figure 3 also shows that $\alpha_2$ can decrease after $\alpha_2$ takes its peak values at higher metallicities ($[\text{Fe}/\text{H}] > -0.5$) owing to the dependences of $\alpha_2$ on $\rho_\star$ (i.e., lower $\alpha_2$ in lower densities in late gas-poor phases of galaxy formation). Therefore, $\alpha_2$ does not depend simply on $[\text{Fe}/\text{H}]$ for galaxies that can continue star formation beyond $[\text{Fe}/\text{H}] \sim -0.5$. Giant elliptical galaxies that formed with high star formation efficiencies and thus high metallicities accordingly do not show a strong dependence of $\alpha_2$ on $[\text{Fe}/\text{H}]$. The star formation timescale, which depends primarily on $\rho_\star$, can be a key parameter for these metal-rich elliptical galaxies. This is one of the possible explanations for the observed different dependences of $\alpha_2$ on $[\text{Fe}/\text{H}]$ between dwarfs and giant elliptical galaxies. We need to investigate whether $\alpha_2$ depends differently on $[\text{Fe}/\text{H}]$ between dwarfs and giant elliptical galaxies by using self-consistent chemodynamical simulations of galaxy formation with the same variable IMF model in our future study.

4.2. Bottom-heavy $\alpha_2$ yet Slightly Top-heavy $\alpha_3$?

Greggio & Renzini (2012, hereafter GR12) convincingly discussed whether top-heavy/bottom-heavy IMFs can explain a number of key observed properties of elliptical galaxies in a self-consistent manner. They clearly demonstrated that the canonical IMF ($\alpha = 2.3$) can better explain both (1) $B$-band mass-to-light ratios and their correlations with mean stellar ages observed in elliptical galaxies and (2) oxygen and silicon mass-to-light ratios ($M_{\text{O}}/L_B$ and $M_{\text{Si}}/L_B$, respectively) in clusters of galaxies. They furthermore showed that both bottom-heavy and top-heavy IMFs fail to explain these observations by using some idealized models of elliptical galaxy formation. Then, how can we explain these two observations, if elliptical galaxies really have a bottom-heavy IMF for low-mass stars? We provide a clue to this puzzling problem regarding the IMF of elliptical galaxies as follows.

Since GR12 adopted a simple IMF model with a single-power-law slope in discussing the above IMF problems, a bottom-heavy IMF means bottom-heavy for both low-mass and
Figure 8. Mass fraction of high-mass stars with \( m \geq 8 M_\odot \) \( (f_{3N}, \text{upper}) \) and that of low-mass stars with \( m \leq 1 M_\odot \) \( (f_{3M}, \text{lower}) \) as a function of \( \alpha_2 \) for \( \alpha_3 = 1.9 \) (red, solid), 2.1 (blue, dotted), 2.3 (green, short-dashed), 2.5 (magenta, long-dashed), and 2.7 (cyan, dot-dashed). The canonical \( \alpha_2 \) (2.3) and \( f_{3N} \) and \( f_{3M} \) estimated for the canonical IMF with \( \alpha_2 = \alpha_3 = 2.3 \) are shown by dotted lines for comparison.

(A color version of this figure is available in the online journal.)

high-mass stars. On the other hand, \( \alpha_2 \) for low-mass stars and \( \alpha_3 \) for high-mass stars can vary independently from each other in a variable Kroupa IMF model so that a bottom-heavy IMF for low-mass stars does not necessarily mean bottom-heavy for high-mass stars. As a result of this, elliptical galaxies with bottom-heavy IMFs for low-mass stars can have high \( M_o/L_B \) and \( M_{Si}/L_B \) as the canonical IMF predicts, if \( \alpha_3 \) is only slightly top-heavy. Figure 8 shows the mass fraction of stars with \( m \geq 8 M_\odot \) \( (f_{3N}) \) as a function of \( \alpha_2 \) for five \( \alpha_3 \) values. This \( f_{3N} \) can be used as a more accurate measure for \( M_o/L_B \) and \( M_{Si}/L_B \), because oxygen and silicon abundances come largely from SNe II (GR12). In this simple model, \( f_{3N} \) is 0.21 for the canonical IMF with \( \alpha_2 = \alpha_3 = 2.3 \), and \( f_{3N} \) at a given \( \alpha_2 \) is lower for larger \( \alpha_3 \) (i.e., more bottom-heavy). However, \( f_{3N} \) can be as high as 0.21 for \( \alpha_3 = 2.1 \) even for \( \alpha_2 \sim 3 \). This implies that the observed bottom-heavy IMF for low-mass stars is not inconsistent with the observed \( M_o/L_B \) and \( M_{Si}/L_B \), as long as the IMF for high-mass stars is only slightly top-heavy.

Figure 8 also shows the mass fraction of low-mass stars with \( m \leq 1 M_\odot \) \( (f_{3M}) \) as a function of \( \alpha_2 \) for five \( \alpha_3 \) values. If a galaxy is dominated by low-mass stars (i.e., higher \( f_{3M} \)), then the galaxy can have rather high \( M_o/L_B \) (GR12). It is clear that the larger \( \alpha_2 \) is, the higher \( f_{3M} \) is (i.e., more dwarf-dominated), independent of \( \alpha_3 \). The model with a slightly top-heavy IMF for high-mass stars \( (\alpha = 1.9) \) can show \( f_{3M} \) at \( \alpha_3 = 3 \) as low as \( \sim 0.44 \) estimated for the canonical IMF with \( \alpha_2 = \alpha_3 = 2.3 \). This result means that even if the IMF for low-mass stars is bottom-heavy, \( M_o/L_B \) cannot be so high (i.e., cannot be dominated by low-mass dwarf stars), as long as the IMF for high-mass stars is slightly top-heavy. This result suggests that the observed range of \( M_o/L_B \) (2–14 shown in GR12) in elliptical galaxies is not inconsistent with the observed bottom-heavy IMF for low-mass stars. Thus, it is possible that future models of elliptical galaxy formation with a variable Kroupa IMF can explain a number of observational results regarding the IMF slopes in a fully self-consistent manner.

Although elliptical galaxies with bottom-heavy IMF for low-mass stars and slightly top-heavy IMF in high-mass stars might form, as the present study suggests, it is theoretically unclear why such a combination of bottom-heavy/top-heavy IMFs is possible in a single star-forming gas cloud. Elmegreen (2004) considered that different parts of the IMF can be independently determined and thereby demonstrated that the entire IMF can be constructed by using three log-normals, each of which has its own characteristic stellar mass. The shape of his multi-component IMF model depends basically on the amplitudes of the three log-normals: it would be possible in principle that a certain combination of the three amplitudes can yield an IMF with bottom-heavy \( \alpha_2 \) and top-heavy \( \alpha_3 \). If we understand how the basic nine parameters determining the amplitudes of the three log-normals depend on physical properties of star-forming clouds, such as [Fe/H], interstellar radiation fields, \( P_g \), and \( f_{SN} \), then we could better understand in what physical conditions a star-forming cloud an IMF with a bottom-heavy \( \alpha_2 \) and a top-heavy \( \alpha_3 \) is possible. Thus, extensive investigation on the dependences of the basic parameters of the IMF on physical properties of star-forming gas clouds will greatly advance our understanding of the origin of the IMF in elliptical galaxies.

5. CONCLUSIONS

We have investigated the origin of the observed bottom-heavy IMF in elliptical galaxies by using one-zone chemical evolution models. A principal assumption is that the Kroupa IMF slope \( \alpha_2 \) depends on both metallicities ([Fe/H]) and gas densities \( (\rho_g) \) of star-forming gas clouds in such a way that \( \alpha_2 \) is proportional to \( \beta [\text{Fe}/\text{H}] + \gamma \log \rho_g \) (\( \beta \) is fixed at 0.5). We have searched for the best parameter value of \( \gamma \) that can reproduce the observed \( M/L_K - [\text{Mg}/\text{Fe}] \) relation (corresponding to \( \alpha_2 - [\text{Mg}/\text{Fe}] \) relation) in elliptical galaxies. Although the present model for \( M/L \) has some limitations (e.g., using SSPs for a fixed IMF), we have found the following important results.

1. Our chemical evolution models with \( \alpha_2 = 2.3 + 0.5 \) [Fe/H] + 2 log \( \rho_g \) (i.e., \( \gamma = 2 \)) can reproduce the observed positive \( M/L_K - [\text{Mg}/\text{Fe}] \) correlation (i.e., higher \( M/L_K \) for higher [Mg/Fe]) in a quantitative manner. Furthermore, some models with \( \gamma = 2 \) can show larger \( \alpha_2 \) (\( \sim 3 \)), which is consistent with recent observations (e.g., CV12). However, our models with low \( \gamma (\sim 0.1) \) cannot reproduce the \( M/L_K - [\text{Mg}/\text{Fe}] \) correlation. These results suggest that the IMF slope for low-mass stars needs to depend more strongly on \( \rho_g \) than on [Fe/H].

2. The observed different dependences of \( \alpha_2 \) on [Fe/H] in dwarf and giant elliptical galaxies cannot be simply explained by a variable IMF model that depends only on [Fe/H] for low-mass stars. Instead, such differences suggest that \( \alpha_2 \) needs to depend on both [Fe/H] and \( \rho_g \). A key parameter for \( \alpha_2 \) is suggested to be the truncation epoch of star formation for dwarfs and the timescale of star formation for giant elliptical galaxies. We will attempt to understand why gas density can be a fundamental parameter for \( \alpha_2 \) in a future study.
3. The observed bottom-heavy IMF for low-mass stars ($\alpha_2$) in elliptical galaxies would not be a problem in explaining the observed $M/L_B$ and $M_\text{tot}/L_B$ in galaxy clusters, as long as the variable Kroupa IMF for high-mass stars (i.e., $\alpha_3$) is slightly more top-heavy. Both the observed bottom-heavy IMF and the cluster metal content (for which the Salpeter IMF is suggested to be required) could be self-consistently explained in a model in which $\alpha_2$ and $\alpha_3$ can vary independently from each other.

4. In the variable Kroupa IMF, the mass fraction of low-mass stars with $m \leq 1 M_\odot$ ($f_{\text{LM}}$) depends on $\alpha_1$ for a given $\alpha_2$ such that $f_{\text{LM}}$ can be lower for smaller $\alpha_3$. Therefore, elliptical galaxies with $\alpha_2 \sim 3$ (i.e., bottom-heavy) can have $f_{\text{LM}}$ as low as 0.44 estimated for the canonical IMF with $\alpha_2 = \alpha_3 = 2.3$, if the IMF for high-mass stars is only slightly top-heavy ($\alpha_3 \sim 2$). This implies that the observed $B$-band mass-to-light ratios ($M/L_B$) in elliptical galaxies are not inconsistent with a bottom-heavy IMF for low-mass stars. A more detailed modeling with variable $\alpha_2$ and $\alpha_3$ is necessary to confirm that both the bottom-heavy $\alpha_2$ and $M/L_B$ in elliptical galaxies can be self-consistently explained by a variable IMF model.

The present study suggests that the Kroupa IMF slopes, $\alpha_2$ and $\alpha_3$, would need to vary independently from each other for more self-consistent explanations of different observational results regarding possible IMF variations in elliptical galaxies. Since the present study did not extensively discuss the physics behind these independently varying IMF slopes, in a future study we will investigate why and how the three IMF slopes depend on physical properties of star-forming gas clouds.

Although the present study has adopted somewhat idealized models for elliptical galaxy formation, the formation processes are significantly more complicated in recent hierarchical galaxy formation models (e.g., Naab 2013). Thus, in a future study we will investigate whether the variable Kroupa IMF model can really explain the observed bottom-heavy IMF of elliptical galaxies in a more sophisticated formation model of elliptical galaxies.

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**APPENDIX A**

[Mg/Fe]–[Fe/H] RELATIONS FOR DIFFERENT KROUPA IMFs

In order to demonstrate that final [Mg/Fe] in a one-zone chemical evolution model does not depend so strongly on $\alpha_1$ and $\alpha_2$ of the (fixed) Kroupa IMF, we have investigated the models with different $\alpha_1$ and $\alpha_2$ (yet a fixed $\alpha_3 = 2.3$). These models (M21–M24) have $t_\text{SN} = 0.5$ Gyr, $t_{\text{turn}}/t_\text{SN} = 2$, and $C_{\text{SE}} = 0.4$ so that they can be compared with the model M1 with the Salpeter IMF. Figure 9 shows that the final [Mg/Fe] is very similar between these models with different $\alpha_1$ and $\alpha_2$ and M1 with the Salpeter IMF, though final [Fe/H] depends on these IMF slopes. The present study adopted an approximation of a fixed IMF slope ($\alpha_2 = 2.35$, i.e., Salpeter IMF) in chemical evolution models, though $\alpha_1$ and $\alpha_2$ are assumed to vary with time at each time step in the models. The results in Figure 9 mean that if the time evolution of chemical yields due to varying $\alpha_1$ and $\alpha_2$ is included in one-zone models, the present results cannot change significantly. These results therefore demonstrate that the adopted approximation of a fixed IMF slope in one-zone models can be justified and thus good enough to discuss the final [Mg/Fe] of elliptical galaxy formation models in the present study.

**APPENDIX B**

POSSIBLE $M/L$ DIFFERENCES BETWEEN VARIABLE KROUPA AND SINGLE-POWELAW IMFs

In order to discuss how $M/L$ could be possibly different between variable Kroupa and single-power-law IMFs, we have investigated (1) the mass fraction ($f_{\text{SN}}$) of massive stars with stellar masses ($m$) equal to or larger than $8 M_\odot$ (i.e., those that explode as SNe II and leave stellar remnants) and the mass fraction ($f_{\text{LM}}$) of low-mass stars with $m < 1 M_\odot$. Since we cannot directly estimate $M/L$, we discuss the possible $M/L$ differences between variable Kroupa and single-power-law IMFs by using these $f_{\text{SN}}$ and $f_{\text{LM}}$. By definition, a variable Kroupa IMF has $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = \alpha_3$, whereas a single-power-law IMF has $\alpha_1 = \alpha_2 = \alpha_3$. A galaxy can have a larger number of stellar remnants for a more top-heavy IMF so that $f_{\text{SN}}$ (thus $M/L$) can be larger. A galaxy is more dominated by dwarf stars for a more bottom-heavy IMF so that $f_{\text{LM}}$ (thus $M/L$) can be larger.

Figure 10 shows that (1) $f_{\text{SN}}$ is slightly larger in variable Kroupa IMFs than in single-power-law IMFs, (2) $f_{\text{LM}}$ is larger in single-power-law IMFs than in variable Kroupa IMFs, and (3) $f_{\text{SN}}$ differences between the two IMF models are significantly smaller in comparison with $f_{\text{LM}}$ differences. For example, $f_{\text{SN}}$ ($f_{\text{LM}}$) is 0.21 (0.45) for a variable Kroupa IMF with $\alpha_2 = 2.3$ and 0.16 (0.58) for a single-power-law IMF with $\alpha_2 = 2.3$. These results therefore mean that $M/L$ should be systematically larger in single-power-law IMFs than in variable Kroupa IMFs for a given $\alpha_2$ ($=\alpha_3$). The bottom line in this figure is that $f_{\text{LM}}$ is always slightly larger in single-power-law IMFs than in the variable Kroupa IMFs and the $f_{\text{LM}}$ differences do not depend strongly on $\alpha_2$.

These results indicate that $M/L$ can be always slightly overestimated by a very similar amount in the present models with different $\alpha_2$ (in comparison with the true $M/L$ for variable Kroupa IMFs). These therefore demonstrate that the slope in the

Figure 9. Evolution of galaxies on the [Mg/Fe]–[Fe/H] plane for the Salpeter IMF model M1 with $\alpha = 2.35$ (red, solid) and four variable Kroupa IMF models, M21 with $\alpha_1 = 1.3$ and $\alpha_2 = 2.3$ (blue, dotted), M22 with $\alpha_1 = 2.3$ and $\alpha_2 = 2.3$ (green, short-dashed), M23 with $\alpha_1 = 2.3$ and $\alpha_2 = 3.3$ (magenta, long-dashed), and M24 with $\alpha_1 = 0.3$ and $\alpha_2 = 1.3$ (cyan, dot-dashed). For M21–M24, $\alpha_3$ is fixed at 2.3. The model parameters for star formation histories ($t_\text{turn}$ and $t_{\text{SN}}$) are exactly the same between these five models. (A color version of this figure is available in the online journal.)
Figure 10. Mass fraction of high-mass stars with \( m \geq 8 M_\odot \) (\( f_{SN} \), upper) and that of low-mass stars with \( m \leq 1 M_\odot \) (\( f_{LM} \), lower) as a function of \( \alpha_2 \) for a variable Kroupa IMF (red, solid) and a variable single-power-law IMF (blue, dotted). Here \( \alpha_2 = \alpha_3 \) is adopted.

(A color version of this figure is available in the online journal.)

The simulated \( M/L-[\text{Mg/Fe}] \) relation can be very close to the true value (estimated self-consistently by using a stellar population synthesis code for a variable Kroupa IMF). Accordingly, as long as we discuss the slope of the observed \( M/L-[\text{Mg/Fe}] \) relation (i.e., not the absolute value of \( M/L \) itself), the usage of the MILES code for variable single-power-law IMFs can be justified. However, the simulated absolute magnitudes of \( M/L \) can slightly deviate from the true ones in the present study so that a direct comparison between the observed and simulated absolute values of \( M/L \) cannot be valid.

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