A microwave chip-based beam splitter for low-energy guided electrons

Jakob Hammer,* Sebastian Thomas, Philipp Weber, and Peter Hommelhoff
Department für Physik, Friedrich-Alexander-Universität Erlangen-Nürnberg, Staudtstr. 1, 91058 Erlangen, Germany
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We demonstrate the splitting of a low-energy electron beam by means of a microwave pseudopotential formed above a planar chip substrate. Beam splitting arises from smoothly transforming the transverse guiding potential for an electron beam from a single-well harmonic confinement into a double-well, thereby generating two separated output beams with 5 mm lateral spacing. Efficient beam splitting is observed for electron kinetic energies up to 3 eV, in excellent agreement with particle tracking simulations. Furthermore, we present a beam splitter potential that is numerically optimized towards coherent and adiabatic splitting of guided electron wave packets. Prospects for electron-based quantum matter-wave optics applications are discussed.

A plethora of electron interferometry experiments [10] was triggered by the invention of the electrostatic biprism in 1955 [11]. It is a relatively rugged transverse beam splitting element and also serves as a workhorse in modern commercial electron microscopes that employ holographic techniques [12, 13].

In this letter we show the concept and the experimental demonstration of a new beam splitter for slow electrons with energies in the electron-volt range, based on microstructured complex microwave fields. It may be of general interest to low-energy electron-based experiments and devices [14]. In particular, a novel quantum electron microscopy concept is emerging, aiming at the non-destructive imaging of biological samples [15, 16]. As a key feature such a quantum electron microscope exploits multiple splittings of a quantum particle’s wavefunction to consecutively probe a sample without inducing damage, with the beam splitter demonstrated here as potentially central element. However, we expect more experiments and applications to arise with this novel beam splitter device as it provides robust beam splitting.

Slow electrons with kinetic energies below 10 eV can be guided in the oscillating electric field of a linear quadrupole guide [17, 18], also known as linear Paul trap [19]. In order to achieve stable confinement of the electrons in the guide, the oscillating drive frequency needs to lie in the gigahertz range. The resulting tight transverse confinement is described by a microwave pseudopotential and enables the precise control of the transverse motion of guided electrons. Accordingly, the on-chip splitting of a guided electron beam can be achieved using a microstructured pseudopotential that gradually transforms from a single-well harmonic confinement into a double-well potential.

The implementation of this electron beam splitter, based on a planar, microfabricated guiding chip, is rendered possible due to the recent progress in microwave transmission line technology and micromachining of the chip electrodes with complex shapes. Based on a similar technology, surface-electrode ion traps have been developed comprising Y-, X- and T-junctions [20–25].

Electrons can be confined in the minimum of an alternating electric potential \( \phi(\vec{r}, t) = \phi_{RF}(\vec{r}) \cos(\Omega t) \) if their motion is slow compared to the drive frequency \( \Omega \). Effectively, a time-averaged pseudopotential is generated given by \( \Psi = Q^2/(4M\Omega^2) \cdot |\nabla \phi_{RF}(\vec{r})|^2 \) [26]. Here \( Q \) and \( M \) are the electron charge and mass. For the guiding of electrons we use a two-dimensional electric quadrupole potential \( \phi_{RF} \), which provides a transverse harmonic pseudopotential along the guide axis. The dynamics of an electron within the pseudopotential are then governed by an oscillatory macromotion with a frequency \( \omega = (q/\sqrt{8}) \Omega \) and a potential depth \( U = qQ/8 \cdot V_0 \). Here \( q \) denotes the stability parameter, where stable confinement of an electron requires \( 0 < q < 0.9 \) [26], and \( V_0 \) is the voltage amplitude applied to the electrodes.

Fig. 1(a) shows the chip electrode design of the planar beam splitter [27]. By means of a tapered central electrode the electric field above the guiding chip can be transformed from quadrupole to hexapole symmetry along the horizontal y-direction. This is indicated in Fig. 1(a) by the electric field line plots in the vertical \( xz \)-plane. An ideal two-dimensional intersection can be constructed from a hexapole electric field [28, 29]. The hexapole symmetry of the electric field leads to the creation of a double-well pseudopotential with two minima,
In order to achieve a smooth transition from quadrupole to hexapole electric fields along the y-axis we have used the Surface Pattern package [30–32] to numerically optimize the shape of the chip electrodes. Electrons are injected into the pseudopotential at the substrate edge at a height of 450 μm above the substrate and are guided along the potential minimum path towards the beam splitter junction. Due to the hexapole symmetry an additional, unwanted potential minimum path converges towards the path of guided electrons at the junction and weakens the confinement along the vertical z-direction. The numerical optimization of the electrode layout minimizes vertical pseudopotential gradients along the path of the electrons while keeping the confinement as constant as possible during the splitting process. This way unwanted excitations of the electron trajectories during the splitting process are reduced.

The beam splitter substrate consists of a 0.76 mm thick microwave compatible Rogers RO4350B laminate coated with a 20 μm layer of gold-plated copper. The electrodes are defined by chemical etching of 50 μm wide gaps along the electrode contours into the metal layer. The microwave signal is delivered to the signal electrodes (drawn in red in Fig. 1) by a coplanar waveguide structure on the backside of the chip (not shown), which is interconnected to the top side by laser-machined, plated through-holes with a diameter of 20 μm. The experiments are performed with a drive frequency of \( \Omega = 2\pi \cdot 990 \text{MHz} \) and an injected microwave power of 7.8 W. Taking into account the impedance mismatch of the beam splitter electrode layout, with a characteristic impedance of 15 Ohm, and the feeding coplanar waveguide structure, 55% of the microwave power is transmitted to the signal electrodes [33]. This results in a microwave voltage amplitude of \( V_0 \approx 16 \text{ V} \) on the electrodes [34].

A thermionic electron source provides an electron beam with kinetic energies ranging from 1 to 10 eV and beam currents on the order of several ten femtoamperes. The beam is collimated using two apertures resulting in a full opening angle of 14 mrad and a spot diameter of about 100 μm at the guide entrance. Electrons that leave the guiding chip at \( y = 40 \text{ mm} \) are detected on a microchannel plate (MCP) electron detector after traveling 10 mm in free space. Images of the phosphor screen behind the MCP are recorded by a CCD camera.

Fig. 2(a) shows the detector signal recorded for an electron kinetic energy of 1.5 eV and the microwave parameters given above. We clearly observe an electron signal with two symmetrically split up components, with a separation of 5 mm. Additionally a faint signal of lost electrons is detected in between the two guided components. The fraction of guided electrons comprises 80% of the detected signal.

In order to fully understand the observed features, we perform classical particle tracking simulations taking into account the oscillating electric field of the beam splitter chip according to the experimental parameters. The simulations gather 1000 particle trajectories that are released at \( y = 0 \text{ mm} \) and initially form a circular beam with a diameter of 100 μm. Fig. 2(b) shows the simulated electron signal, which is in excellent agreement with the experimentally observed beam splitter signal. The color scale illustrates the initial lateral displacement of the electrons along the x-axis. Due to the planar symmetry of the beam splitter potential the electron trajectories obey the same symmetry when being released along
the negative or positive x-axis. Apparently, electrons released closest to the symmetry axis of the beam splitter potential [blue dots in Fig. 2(b)] are preferentially lost.

This can be understood by considering the extreme case of an electron traveling perfectly on the symmetry axis. Such a classical trajectory does not encounter any potential curvature and therefore no deflecting force along the x-axis. As a result, this trajectory cannot follow the pseudopotential minimum paths of the separating double-well and is only deflected away perpendicular to the substrate.

For this reason electrons that propagate closest to the symmetry axis may become lost from the beam splitter potential. Using quantum mechanical simulations we will show later that lossless, adiabatic splitting of an electron beam can be achieved by means of an optimized beam splitter potential.

To further characterize the dependence of the beam splitting process on the initial position of the electron beam we simulate electron trajectories for a centered and a displaced electron source. Fig. 3(a) shows a top view of the simulated electron trajectories in the xy-plane. Clearly, the electrons perform oscillations after injection into the guiding potential with a spatial period of 14 mm corresponding to a trap frequency of $\omega = 2\pi \times 50$ MHz at an electron kinetic energy of 1.5 eV. For a centered electron beam the trajectories (drawn in red) become symmetrically separated in the region from $y = 20$ mm to 25 mm. Electrons that are released at a positive (negative) x-position end up in the output beam at positive (negative) x-values. In contrast, for an electron source displaced about 125 $\mu$m along the positive or negative x-direction all trajectories of the beam (drawn in green and blue) end up in the same output port. This is because the initial lateral displacement of the trajectories sets the potential energy of the transverse electron oscillation. For the displaced beam the potential energy of the electron oscillation is larger and electrons may cross the potential barrier in the splitting region once more compared to the centered beam.

Implicitly, in these classical particle trajectory simulations the initial conditions fully determine the path of the electrons within the beam splitter potential.

The same behavior is found experimentally when the electron source is displaced in the x-direction [see Fig. 3(b) and (c)], where all electrons end up in a single output beam. When the electron gun is centered we obtain a symmetric splitting, as shown in Fig. 3(d).

Finally, we have varied the electron kinetic energy from 1 eV up to 4 eV [Figs. 3(d)- 3(f)]. From 1 eV to 3 eV, we find that the signal of lost electrons becomes larger with energy. This is because with increasing electron energy the transverse curvature of the beam splitter potential becomes insufficient to significantly deflect the electrons.
in the lateral $x$-direction. Accordingly, the electron trajectories cannot follow the separating paths of the potential minimum and are lost from the potential. As a consequence, for energies above $4$ eV we observe no splitting anymore and all electrons are detected around $x = 0$ mm in Fig. 3(f).

The observed dependence of the beam splitting on the kinetic energy indicates that for efficient beam splitting the longitudinal propagation along the beam splitter potential has to be slow compared to the transverse electron motion. More specifically, in order to achieve smooth and lossless splitting any variation in the pseudopotential landscape experienced by an electron has to be slow with respect to the time scale set by the transverse trap frequency. In the following we investigate the splitting of a quantum mechanical electron wave packet taking into account the discretized eigenstates of the microwave pseudopotential and the subsequent temporal quantum evolution of the electron wave packet.

We simulate the quantum mechanical ground state probability density over a length $L = 100$ µm in the range along the splitter electrodes where the splitting evolves. The splitting parameter $s$ is defined in the main text. (b) Ground state probability density for the optimized beam splitter potential, according to the optimization of the splitting process speed $ds/dt$, shown in (c). (d) State populations of the three lowest symmetric states during the optimized splitting process. More than 90% of the population end up in the ground state. Only symmetric states are considered as transitions occur only between states of the same parity.

verse confining potential. We refer to adiabatic splitting if more than 90% of the state population ends up in the ground state after the splitting process.

The adiabaticity of the splitting process can be increased by following an optimization routine [35]. The separation of the double-well along the $y$-axis is parameterized by a splitting parameter $s$. For $s = 0$ the potential is given by a single-well confinement, whereas $s = 1$ corresponds to a double-well with a lateral distance set to $13$ µm here. The splitting process (from $s = 0$ to $s = 1$) takes a time $T$, given by the kinetic energy of the electron and the length $L$ along $y$. The adiabaticity of the splitting process can be increased by optimizing the process speed $ds/dt$ such that excitations of the ground state wave packet are minimized. Following the optimization routine we obtain an improved splitting process speed $ds/dt$ [red line in Fig. 4(c)]. Fig. 4(b) shows the quantum ground state probability density that results from the adiabaticity optimization of the beam splitter potential. As expected, a smooth transition into the split-up paths by means of a small splitting angle $\alpha$ is required and obtained from the optimization.

We calculate the state population $|c_i(t)|^2$ for the three lowest symmetric quantum states ($i = 0, 2, 4$) by solving the time-dependent Schrödinger equation, assuming that initially only the ground state is populated. Fig. 4(d) shows the temporal evolution of the state populations during the splitting process for the optimized beam splitter and the microwave parameters given below. For a kinetic energy of $1$ eV we find transition probabilities into higher excited transverse states to be below 10% after the splitting process, even though during splitting the excited state population may be transiently higher.

It is instructive to specify the scaling behavior of the adiabatic splitting with respect to the electron energy, which can be regarded as a general result when fast and adiabatic splitting is required based on the optimization routine employed here. To increase the range of adiabatic splitting the transverse trap frequency $\omega$ can be raised by increasing the drive frequency $\Omega$ ($\omega \propto \Omega$, c.f. page 1). From numerical simulations we find empirically that the maximum attainable kinetic energy for adiabatic splitting scales according to $E_{\text{kin}}^{\text{max}} \approx a(U) \alpha^{-2} \arctan[0.255 \cdot \Omega/(2\pi \cdot 1\text{GHz})]$, where $a$ is a function of the potential depth $U$.

The adiabatic splitting of a $1$ eV electron beam is obtained for a fivefold increased trap frequency $\omega$ with $\Omega = 2\pi \cdot 5$ GHz [36]. A constant stability parameter $q$ is assured by increasing the voltage amplitude to $V_0 = 75$ V and scaling the section of the beam splitter potential that underlies the probability density simulation in Fig. 4 (b) in the longitudinal and transverse dimensions to a length $L = 40$ mm and a splitting angle $\alpha = 0.22$ mrad.

The adiabatic splitting of an electron wave packet by means of the so optimized splitting potential is of utmost interest as the transverse motional quantum state of the
guided electrons could be used as a carrier of quantum information in future guided matter-wave interferometry experiments. Non-destructive imaging of biological samples represents a potential application [15, 16]. Here quantum effects in the motion of electrons, which are confined by a structured guiding potential, can be employed to perform an interaction-free measurement protocol [37].

A prerequisite of such a quantum electron microscope is that it requires multiple adiabatic splittings of the guided electron wave packet.

Investigating the adiabatic splitting in phase-space we find that the discussed beam splitter leads to an amplitude division of the electron wave packet. Here the incident wave packet is divided into two separate copies with each copy occupying the identical volume in phase space as the original wave packet but with half the amplitude. As a result both copies can be recycled as equivalent parts in a quantum-enhanced microscopy scheme. In contrast, wavefront beam splitters, like the electrostatic biprism, cut the original phase space volume into two halves, where each individual part cannot be re-imaged onto the original phase space volume. To this end, the manipulation of electrons using the pseudopotential of a planar microwave chip augments the already available, rich electron toolkit and provides a novel electron optics platform heralding new quantum optics experiments with free electrons.

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* jakob.hammer@fau.de

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