Determination of HQET parameter $\lambda_1$ from Inclusive Semileptonic $B$ Meson Decay Spectrum

K. K. Jeong $^a$ and C. S. Kim $^{a,b}$

$^a$ Department of Physics, Yonsei University, Seoul 120-749, Korea

$^b$ School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea

Abstract

We estimate the heavy quark effective theory parameter $\lambda_1$ from inclusive semileptonic $B$-meson decay spectrum. By using recent CLEO double lepton tagging data of $B \rightarrow Xe\nu$, which shows the lepton momentum as low as 0.6 GeV, we extracted $\lambda_1 \sim -0.58$ GeV$^2$. We also derived $\bar{\Lambda} \sim 0.46$ GeV and $|V_{cb}| = 0.041 \pm 0.002$. 

$^1$kkjeong@theory.yonsei.ac.kr

$^2$kim@cskim.yonsei.ac.kr, http://phya.yonsei.ac.kr/~cskim/
1 Introduction

As is well known, the heavy quark symmetry breaking parameters $\lambda_1$ and $\lambda_2$ can affect the shape of $B$ meson semileptonic decay spectrum substantially. While it is easy to obtain the value of $\lambda_2$, the hyperfine splitting term, from mass difference between $B$ and $B^*$ mesons, it is very difficult to determine the value of parameter $\lambda_1$, which corresponds to the kinetic energy of heavy quark inside a heavy meson. So finding precise value of $\lambda_1$ is very important in understanding of heavy meson decay.

The CLEO collaboration measured the lepton spectrum in the inclusive $B \to X \ell \bar{\nu}$ decay both by one lepton tagging [1], and by double lepton tagging [2]. In single lepton tagging data, leptons from secondary charm decay ($b \to c \to s \ell \nu$) dominate the low lepton energy region. These secondary leptons have typically lower energy than the primary ones, because they are from $c$ quark decay. To obtain the $B \to X \ell \nu$ lepton spectrum in the low $E_\ell$ region from the single lepton tagging data, these secondary leptons must be separated by fitting the spectrum with some assumptions and models.

In Ref. [3], the parameter values of $\Lambda$ and $\lambda_1$ were estimated (with fixed value of $\lambda_2 = 0.12$ (GeV$^2$)$^{-1}$) by using lepton energy distribution of $E_\ell > 1.5$ GeV from CLEO data [4] of semileptonic decay $B \to X \ell \bar{\nu}$ with single lepton tagging. The advantage of using single lepton tagging data is small statistical error, though we cannot use low lepton energy part of the data ($E_\ell < 1.5$ GeV).

In Ref. [2], CLEO collaboration separated $B \to X \ell \bar{\nu}$ from cascade decays of $b \to c \to s \ell \nu$. They selected events with tagging leptons of momentum greater than 1.4 GeV, which are predominantly from semileptonic decay of one of the two $B$ mesons in an $\Upsilon(4S)$ decay. When a tag was found, they searched for an accompanying electron with minimum momentum 0.6 GeV. The main sources of these electrons are, (a) the secondary lepton from the same $B$, (b) the primary lepton from the other $B$ and (c) the secondary lepton from the other $B$. Lepton from (c) has the same charge as the tag lepton while leptons from (a) and (b) have opposite charge to the tag lepton. And leptons from (a) and (b) have different kinematic signatures so that their contributions are easy to separate. In the $\Upsilon(4S)$ decay, the $B$ and the $\overline{B}$ are produced nearly at rest. Hence there is little correlation between the directions of a tag lepton and an accompanying electron if they are from different $B$ mesons. If they are from the same $B$, there is a tendency for the tagged lepton and the electron to be back-to-back. They analyzed the data with double lepton tagging and separated the primary leptons from secondary leptons without model dependence.

In this paper by using the double lepton tagging data, we made a minimum $\chi^2$ analysis to determine value of the parameter $\lambda_1$. There is one difficulty in $\chi^2$ fitting for the data, as is well known. Since non-perturbative correction up to $1/m_\ell^2$ cannot predict the correct shape of lepton distribution near the end point, we have to exclude the high energy data.
points of the distribution. Choosing $E_{QCD}$, the maximum lepton energy that one can trust the shape of $1/m_b$ expansion, is very important in this fitting. Following Ref. [3], we choose $E_{QCD} = 2.0 \text{ GeV}$. The double tagged data has larger statistical error than the single tagged one, but we can use low energy lepton data model-independently. Therefore, this work can complement the work of Ref. [3].

2 Theoretical details

Following the heavy quark effective theory (HQET) [6], the mass of a pseudoscalar or a vector meson $M$ containing a heavy quark $Q$ can be expanded as

$$m_M = m_Q + \bar{\Lambda} - \frac{\lambda_1 + d_M \lambda_2}{2m_Q} + \cdots,$$

where $d_M = 3, -1$ for pseudoscalar and vector mesons, respectively, and

$$\lambda_1 = \frac{1}{2m_M} \langle M(v) | \bar{h}_v (iD)^2 h_v | M(v) \rangle,$$

$$\lambda_2 = \frac{1}{2d_M m_M} \langle M(v) | \bar{h}_v \frac{g}{2} \sigma^{\mu\nu} G_{\mu\nu} h_v | M(v) \rangle,$$

where $h_v$ is the heavy quark field in the HQET with velocity $v$. $\lambda_1$ parametrizes the mass shift due to the kinetic energy of heavy quark inside the meson, and $\lambda_2$ is related to the effect of chromomagnetic interaction between heavy quark and light degrees of freedom. In case of a $B$ meson, we can estimate the value of $\lambda_2$ quite accurately from the mass difference between $B$ and $B^*$ mesons.

$$m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b},$$

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b},$$

and approximately

$$\frac{1}{4}(m_{B^*}^2 - m_B^2) = \lambda_2 + O\left(\frac{1}{m_b}\right) \approx 0.12 \text{ GeV}^2.$$

Within HQET the lepton spectrum of the semileptonic decays of a $b$-flavored hadron ($H_b \rightarrow X_q \ell \nu$) is calculated in the Ref. [3, 7], and the result is

$$\frac{d\Gamma}{dx} = \Gamma_0 \theta(1 - x - \epsilon^2) 2x^2 \left[ (3 - 2x) - 3\epsilon^2 - \frac{3\epsilon^4}{(1-x)^2} + \frac{(3-x)^6}{(1-x)^3} \right]$$

$$+ G_b \left\{ \frac{6 + 5x}{3} - \frac{(6 - 4x)\epsilon^2}{(1-x)^2} + \frac{(3x - 6)\epsilon^4}{(1-x)^3} + \frac{5(6 - 4x + x^2)\epsilon^6}{3(1-x)^4} \right\}$$

$$+ K_b \left\{ -\frac{5x}{3} + \frac{(2x^2 - 5x)\epsilon^4}{(1-x)^4} + \frac{2(x^3 - 5x^2 + 10x)\epsilon^6}{3(1-x)^5} \right\},$$

(7)
where
\[ \Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{qb}|^2, ~ \epsilon = \frac{m_q}{m_b}, ~ x = 2E_\ell/m_b, \] (8)
\[ K_b = -\lambda_1/m_b^2, ~ G_b = 3\lambda_2/m_b^2, \] (9)

with \( m_q \) denoting the mass of the quark \( q = u, c \) in the final state, and \( V_{qb} \) is CKM matrix element \( [8] \). The terms in the second line and third line of Eq. (7) correspond to non-perturbative corrections (NP) to leading order Born approximation of the first line.

Perturbative corrections of the electron spectrum from \( b \)-decay were calculated in various references \([9, 10]\). The analytic form of order \( \alpha_s \) correction is given \([9]\) as
\[
\left( \frac{d\Gamma}{dx} \right)_{\alpha_s} = \frac{-2\alpha_s}{3\pi} \Gamma_0 \int_0^{y_m} dy \frac{12}{(1 - \xi y)^2 + \gamma^2} F_1(x, y, \epsilon^2),
\] (10)

with
\[ F_1(x, y, \epsilon^2) = H_1(x, y) + H_2(x, y, \epsilon^2) - H_2(x, y, z_m), \] (11)

where
\[ H_1(x, y) = 2(x - y)(xM - x + y)H_B(x, y) + \frac{\bar{Y}_p}{2p_\lambda}\left\{ (x - 4 + 5x) + y(4 - 6x - 5x^2) + y^2(1 + 10x) - 5y^3 \right. \] 
\[ \left. + \epsilon^2[1 - 2x + 5x^2 + y(5 - 16x) + 11y^2] + \epsilon^4(-2 + 6x - 7y) + \epsilon^6 \right\} \] 
\[ + \ln \epsilon[x(-1 + 2x) + y(1 - 4x) + 2y^2 + \epsilon^2(1 + x - y) - \epsilon^4] \] (12)

and
\[ H_2(x, y, z) = \frac{f_1 + zf_2}{8(1 - y)p_\lambda(z)^2} + \frac{Y_p(z)(f_3 + zf_4)}{8[p_\lambda(z)]^3} + \frac{Y_p(z)(f_5 + zf_6)}{4p_\lambda(z)} \] 
\[ + \frac{1}{4} \ln(z)f_7 + \frac{\epsilon^2f_8}{2(1 - y)z} + [Li_2(w_+(z)) + Li_2(w_-(z))]f_9 - yz + 4yp_\lambda(z)Y_p(z), \] (13)

with
\[ H_B(x, y) = 1 - \ln(1 - x) - \ln(1 - y/x) - 2(\bar{p}_0/\bar{p}_\lambda Y_p - 1) \ln[(1 - x)(1 - y/x) - \epsilon^2] \] 
\[ + \frac{\bar{p}_0}{\bar{p}_\lambda}[Li_2\left(1 - \frac{\bar{p}_-\bar{w}_-}{x\bar{p}_+}\right) - Li_2\left(1 - \frac{\bar{w}_-}{\bar{w}_+}\right) - Li_2\left(1 - \frac{\bar{p}_-}{\bar{p}_+}\right) + Li_2\left(1 - \frac{1 - x}{\bar{p}_+}\right) \] 
\[ + Li_2\left(1 - \frac{x - y}{x\bar{p}_+}\right) - Li_2\left(1 - \frac{1 - x}{\bar{p}_-}\right) - Li_2\left(1 - \frac{x - y}{x\bar{p}_-}\right) \] 
\[ + 2\bar{Y}_p(\bar{w}_+ + 2\ln \epsilon) + \ln \epsilon \ln \left(\frac{\bar{w}_+ - x}{x - \bar{w}_-}\right) \] (14)

And
\[ f_1 = (1 - y)^3[5x^2 + y(5 - 2x)] - 4\epsilon^2(1 - y)[x^2 + y(1 - 5x + 2x^2) + y^2(2 - x)] \]
\[ + \epsilon^4[-x^2 + y(-1 + 6x - 3x^2) + y^2(-3 + 2x)], \]
\[ f_2 = -(1 - y)[5x^2 + y(5 + 18x + 3x^2) + y^2(3 - 2x)] + 4\epsilon^2(1 - y)[x^2 + y(1 - x)] + \epsilon^4[x^2 + y(1 - 2x)], \]
\[ f_3 = (1 - y)^2[-5x^2 + y(-5 - 8x + x^2) + y^2] + 2\epsilon^2(1 - y)[2x^2 + y(2 - 6x + x^2) + y^2] + \epsilon^4[x^2 + y(1 - 4x + x^2) + y^2], \]
\[ f_4 = 5x^2 + y(5 + 28x + 12x^2) + y^2(12 + 4x - x^2) - y^3 + 2\epsilon^2[4 - 10x - 4x^2 + y(-8 + 18x) - 4y^2] + \epsilon^4(1 - 4x + y), \]
\[ f_5 = -5 + 10x + y(5 + 24x + 8x^2) + y^2(5 - 18x) + 3y^2 + 2\epsilon^2[4 - 10x - 4x^2 + y(-8 + 18x) - 4y^2] + \epsilon^4(1 - 4x + y), \]
\[ f_6 = 5 + 10x - 4x^2 + y(14 + 10x) - 3y^2 - 2\epsilon^2(2 + 3x - 2y) - \epsilon^4, \]
\[ f_7 = -5 + 4x - 4x^2 + 6yx - y^2 + \epsilon^2[4(1 + x) - 10y] + \epsilon^4, \]
\[ f_8 = x(1 - x) + y(-1 + x + x^2) - 2y^2x + y^3 + \epsilon^2(1 - x)(x - y) \]
\[ f_9 = x + y(1 - 2x) + y^2 + \epsilon^2(x - y). \]

All the parameters and the kinematic variables in the above expressions are listed in Appendix.

After using the above all formulae, the electron distribution in semileptonic decay of \( B \) meson can be written as
\[
\frac{d\Gamma_{\text{theory}}}{dE_\ell} = \left( \frac{d\Gamma}{dE_\ell} \right)_{\text{Born}} + \left( \frac{d\Gamma}{dE_\ell} \right)_{NP} + \left( \frac{d\Gamma}{dE_\ell} \right)_{\alpha_s},
\]
where \( \left( \frac{d\Gamma}{dE_\ell} \right)_{\text{Born}} \) is leading order Born approximation, \( \left( \frac{d\Gamma}{dE_\ell} \right)_{NP} \) is non-perturbative correction using the HQET and \( \left( \frac{d\Gamma}{dE_\ell} \right)_{\alpha_s} \) the perturbative \( \alpha_s \) correction. We define the CKM-matrix independent decay rate
\[
\gamma_q = \frac{\Gamma_{\text{theory}}(B \to X_q\ell\nu)}{|V_{q\ell}|^2},
\]
and then, semileptonic decay rate \( \Gamma_{SL} \) can be written as
\[
\Gamma_{SL} = \gamma_c |V_{cb}|^2 + \gamma_u |V_{ub}|^2.
\]
Since \( |V_{ub}|^2 \ll |V_{cb}|^2 \), we can neglect \( b \to u \) decay. Integrating over \( E_\ell \) of Eq. (24), we obtain
\[
\Gamma_{SL} = \gamma_c |V_{cb}|^2 = \Gamma_0 \left[ z_0 \left\{ 1 - \frac{2\alpha_s(m_b)}{3\pi}g(\epsilon) \right\} + \frac{1}{2}z_0(G_b - K_b) - 2z_1G_b \right],
\]
where \( z_0 \) and \( z_1 \) are defined as
\[
z_0 = 1 - 8\epsilon^2 + 8\epsilon^6 - \epsilon^8 - 24\epsilon^4 \ln \epsilon, \quad z_1 = (1 - \epsilon^2)^4,
\]
and \( g(\epsilon) \) is a complicated function of \( \epsilon \), which can be approximated \[12\] to
\[
g(\epsilon) = \left( \pi^2 - \frac{31}{4} \right) (1 - \epsilon)^2 + \frac{3}{2} .
\] (30)

To obtain the mass ratio \( \epsilon = m_c/m_b \), we use the relation
\[
m_b - m_c = (m_B - m_D) - \frac{1}{2}(\lambda_1 + 3\lambda_2) \left( \frac{1}{m_c} - \frac{1}{m_b} \right)
\approx (m_B - m_D) \pm (\sim 1\%) \approx m_B - m_D = 3.41 \text{ GeV}.
\] (31)

We note that if we use instead the other relations, e.g.
\[
m_b - m_c = (m_{B^*} - m_{D^*}) - \frac{1}{2}(\lambda_1 - \lambda_2) \left( \frac{1}{m_c} - \frac{1}{m_b} \right)
\approx m_{B^*} - m_{D^*} = 3.32 \text{ GeV},
\] (32)
or
\[
m_b - m_c = (\overline{m}_B - \overline{m}_D) - \frac{\lambda_1}{2} \left( \frac{1}{m_c} - \frac{1}{m_b} \right)
\approx \overline{m}_B - \overline{m}_D = 3.35 \text{ GeV},
\] (33)

where \( \overline{m}_B = \frac{1}{4}(m_B + 3m_{B^*}) \) and \( \overline{m}_D = \frac{1}{4}(m_D + 3m_{D^*}) \),
the values of correction would become as large as \( \sim +(2 \sim 4\%) \) depending on \( \lambda_1 \).

For \( b \to c\ell\nu \), Figs. 1(a-b) illustrate the dependencies of various corrections on \( m_b \) and \( \lambda_1 \). All figures in Fig. 1 are with \( |V_{cb}| = 0.04 \). The value of \( m_b \) determines mainly the overall size of decay width, while other parameters determine the shape of the distribution. As can be seen from Figure 1(a), we find that the dependence of \( \Gamma_{SL} \) on \( m_b \) is rather weak on the contrary to the naive estimation of \( \Gamma_{SL} \propto m_b^5 \), and is very sensitive to the quark mass difference \( (m_b - m_c) \) \[11\]. Note also that the shapes of Born approximation and perturbative correction are almost insensitive to the value of \( m_b \), while non-perturbative correction is quite sensitive to both \( m_c/m_b \) and \( \lambda_1 \).

### 3 Results and discussions

To compare CLEO data with theoretical calculation, we use minimum \( \chi^2 \) method with
\[
\chi^2 = \sum_{E_i, E_i < E_{QCD}} \frac{[\mathcal{B}(E_i) - F_{\text{theory}}(E_i; \epsilon, \lambda_1)]^2}{\sigma(E_i)^2}
\] (34)

where \( \mathcal{B}(E_i) \) and \( \sigma(E_i) \) are experimental data of differential branching ratio and error at lepton energy \( E_i \), and \( F_{\text{theory}}(E_i; \epsilon, \lambda_1) \) is theoretical prediction at \( E_i \) as a function of parameters \( \epsilon \equiv m_c/m_b \) and \( \lambda_1 \). We normalized the decay distribution to have branching ratio of 10.49\% as in Ref. \[3\], \( \mathcal{B}(B \to X\ell\nu) = (10.49 \pm 0.17 \pm 0.43)\% \), i.e.,
\[
F_{\text{theory}}(E_i; \epsilon \equiv m_c/m_b, \lambda_1) = \frac{0.1049}{\Gamma_{SL}} \left[ \frac{d\Gamma_{\text{theory}}}{dE_\ell} \right]_{E_\ell = E_i}.
\] (35)
We note that, because of exact cancellation between $\Gamma_{SL}$ and $\Gamma_{\text{theory}}$, $F^{\text{theory}}$ is independent of $\Gamma_0$, and therefore independent of $|V_{cp}|$ and $m_b^5$. $F^{\text{theory}}$ is only indirectly dependent on $m_b$ through the definition of $x$ in Eq. (8). Following Ref. [11], we use the $b$ quark mass, $m_b = 4.8 \pm 0.1$ GeV, which has been derived from QCD sum rule analysis of the $\Upsilon$ system [13]. For $\alpha_s$, we use $\alpha_s = 0.22$, as in Ref. [3].

We here comment on determination of $E_{QCD}$, which is crucial for this analysis. As we can see in the Figs. 1(b) and 2, non-perturbative corrections are significant only in large electron energy region (i.e. $E_\ell > 1.5$ GeV). Therefore, we have to include as many data points up to $E_{QCD}$, in which theory can give correct shape of lepton energy distribution. Otherwise, we cannot fully see the effect of non-perturbative correction which determines the value of $\lambda_1$. However, if we include the data points over $E_\ell > E_{QCD}$, the result will be meaningless because the shape of the lepton energy spectrum is not reliable above $E_{QCD}$ region. The numerical value of $E_{QCD}$ can be estimated from the value of $m_b$ [3]: For $b \to u$ decay with $m_u = 0$, $E_{QCD} \approx 0.9 \cdot m_b/2 \sim 2.15$ GeV. For $b \to c$ decay, smaller smearing range is required near end point, and

$$E_{QCD} \approx 0.9 \cdot (m_b^2 - m_c^2)/2m_b \sim 2.0 \text{ GeV.} \quad (36)$$

Since we are dealing only with lepton energies less than 2.15 GeV, we neglect $b \to u$ decay and set $0.6 < E_\ell < 2.0$ GeV.

We tabulated the results in Table 1. Since we fixed $(m_b - m_c)$, changing the value of $m_b$ means changing mass ratio $\epsilon \equiv m_c/m_b$ together, and this mass ratio affects the results. The values of $\overline{\Lambda}$ are determined from the mass relation Eq. (1). All values of $\lambda_1$ in Table 1 are much larger than the value in Ref. [3] which is $\lambda_1 = -0.19 \pm 0.10$ GeV$^2$, or the values in [14, 15] which are $\sim -0.1$ GeV$^2$, but consistent with [16, 17, 18] which are in the range $-0.4 \sim -0.7$ GeV$^2$.

The values of $\lambda_1$ show significant dependencies on the input value of $m_b$, but still each value is consistent within 1$\sigma$ error range. As explained before, this large sensitivity comes from mass ratio $m_c/m_b$. Indeed, for $m_b = 4.8$ GeV, if we change the value of $(m_b - m_c)$ to $3.35$ GeV, i.e. $m_c^2/m_b^2 = 0.091$, then $\lambda_1 = -0.69 \pm 0.22$ GeV$^2$. Changing the value of $m_b$ with fixed $m_c^2/m_b^2 = 0.084$, we obtain $\lambda_1 = -0.55 \pm 0.18$ GeV$^2$ for $m_b = 4.7$ GeV.

| $m_b$  | $\epsilon^2 \equiv m_c^2/m_b^2$ | $\lambda_1$            | $\overline{\Lambda}$ |
|--------|---------------------------------|-------------------------|------------------------|
| 4.7 GeV| 0.075                           | $-(0.45 \pm 0.19)$ GeV$^2$ | 0.57 $\pm$ 0.018 GeV  |
| 4.8 GeV| 0.084                           | $-(0.58 \pm 0.23)$ GeV$^2$ | 0.46 $\pm$ 0.022 GeV  |
| 4.9 GeV| 0.092                           | $-(0.70 \pm 0.27)$ GeV$^2$ | 0.34 $\pm$ 0.026 GeV  |
and $\lambda_1 = -0.52 \pm 0.29$ GeV$^2$ for $m_b = 4.9$ GeV, which are very similar to the case with $m_b = 4.8$ GeV, as shown in Table 1.

Once we know the parameter values $m_b$, $\lambda_1$, we can extract $|V_{cb}|$ from the relation

$$|V_{cb}|^2 = \frac{\mathcal{B}(B \rightarrow X_c \ell \nu)}{\tau_{B\gamma_c}}.$$ \hspace{1cm} (37)

For $\tau_B$, we averaged $\tau_{B^\pm}$ and $\tau_{B^0}$ from Particle Data Book \[19\]

$$\begin{align*}
\tau_{B^\pm} &= (1.62 \pm 0.06) \times 10^{-12}\text{sec}, \\
\tau_{B^0} &= (1.56 \pm 0.06) \times 10^{-12}\text{sec}.
\end{align*}$$

This gives the value

$$|V_{cb}| = 0.041 \pm 0.002,$$ \hspace{1cm} (38)

where the error includes the errors from semileptonic branching ratio of CLEO data, $B$ meson lifetime, uncertainties from $\lambda_1$ and $b$ quark mass. This result is consistent with CLEO result with ISGW model which is $|V_{cb}| = 0.040 \pm 0.001 \pm 0.002$ \[2\], and with recent Particle Data Book result $|V_{cb}| = 0.0395 \pm 0.0017$ \[19\].

Figure 2 shows the best fit result of differential branching ratio compared with CLEO data as a function of charged lepton energy, with $m_b = 4.8$ GeV and $\lambda_1 = -0.58$ GeV$^2$. It shows the relative size and shape of the various corrections for $m_b = 4.8$ GeV. Non-perturbative term is about $\sim -4.5\%$ and perturbative term is $\sim -12\%$ from leading approximation. From these facts, it is clear that non-perturbative correction determines the shape and not much effect on total decay rate, while perturbative term has little effect on the shape but its contribution on the total decay rate is quite large. Fitting with data points between 1.0 GeV $< E_i < 2.0$ GeV, we obtain $\lambda_1 = -0.57 \pm 0.19$ GeV$^2$ and $\overline{\Lambda} \simeq 0.46$ for $m_b = 4.8$ GeV, which are almost the same with the results from 0.6 GeV $< E_i < 2.0$ GeV. Finally we note the dependence on $\alpha_s$ is very weak. Changing $\alpha_s$ to 0.35 we get $\lambda_1 \sim -0.54$ GeV$^2$ and $\overline{\Lambda} \simeq 0.46$ GeV for $m_b = 4.8$ GeV, which are almost same values in Table 1.

We finally note that recently CLEO collaboration measured \[20\] the first and the second moments of the hadronic mass-squared distribution in the inclusive decay $B \rightarrow X_c \ell \nu$ and also made a preliminary determination of the first and the second moments of the lepton energy distribution from the spectrum in Ref. \[3\]. Using those four moments, they obtained the values of $\overline{\Lambda}$ and $\lambda_1$, but there appeared to be inconsistencies in the results which suggests either experimental error or problems in the HQET. However, if we consider only the moments of the lepton energy distribution, the preliminary CLEO analysis \[20\] gives $\lambda_1 \sim -0.75 \pm 0.20$ GeV$^2$, which is rather in a good agreement with our results.
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A Appendix – Kinematic Variables

In reference [9], the kinematic variables are defined as:

- \( b, q, G, \ell, \nu \): four-momenta of the \( b \)-quark, lighter quark, gluon, lepton, neutrino.
- \( P = q + G, W = \ell + \nu \): four-momentum of the quark-gluon system and the virtual \( W \)
- \( \lambda_G \) stands for the scaled gluon mass (\( \lambda_G \equiv m_G/m_b \ll \epsilon \))

The scaled masses and lepton energies

\[
\epsilon \equiv \frac{m_q}{m_b} = \left( \frac{q^2}{b^2} \right)^{\frac{3}{2}}, \quad x \equiv \frac{2 E_\ell}{m_b}, \quad y \equiv \frac{W^2}{b^2}, \quad z \equiv \frac{P^2}{b^2}, \quad \xi \equiv \frac{m_b^2}{M_W^2}, \quad \gamma \equiv \frac{\Gamma_W}{M_W}
\] (39)

vary in the region

\[
0 \leq x \leq x_M \equiv 1 - \epsilon^2 \quad (40)
\]

\[
0 \leq y \leq y_m \equiv x(x_M - x)/(1 - x) \quad (41)
\]

\[
(\epsilon + \lambda_G)^2 \leq z \leq z_m \equiv (1 - x)(1 - y/x). \quad (42)
\]

Frequently used kinematic variables which characterize the quark-gluon system are

\[
p_0(z) \equiv \frac{1}{2}(1 - y + z),
\]

\[
p_3(z) \equiv \frac{1}{2}[1 + y^2 + z^2 - 2(y + z + yz)]^{1/2};
\]

\[
p_\pm(z) \equiv p_0(z) \pm p_3(z),
\]

\[
Y_p(z) \equiv \frac{1}{2} \ln \frac{p_+(z)}{p_-(z)} = \ln \frac{p_+(z)}{\sqrt{z}}, \quad (43)
\]

and similarly for the virtual \( W \)

\[
w_0(z) \equiv \frac{1}{2}(1 + y - z),
\]

\[
w_3(z) \equiv \frac{1}{2}[1 + y^2 + z^2 - 2(y + z + yz)]^{1/2},
\]

\[
w_\pm(z) \equiv w_0(z) \pm w_3(z),
\]

\[
Y_w(z) \equiv \frac{1}{2} \ln \frac{w_+(z)}{w_-(z)} = \ln \frac{w_+(z)}{\sqrt{z}}. \quad (44)
\]

For \( G = 0 \), which implies \( z = \epsilon^2 \), the abbreviations

\[
\bar{p}_0 \equiv p_0(\epsilon^2), \quad \bar{p}_3 \equiv p_3(\epsilon^2), \quad \text{etc.,}
\]

\[
\bar{w}_0 \equiv w_0(\epsilon^2), \quad \bar{w}_3 \equiv w_3(\epsilon^2), \quad \text{etc.} \quad (45)
\]

will be useful. Polylogarithms are defined as real functions, and in particular

\[
\text{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln |1 - t|. \quad (46)
\]
Figure captions

Fig. 1 Contributions of each terms in lepton spectra of $b \rightarrow c\ell\nu$ decay. In all figures, $|V_{cb}| = 0.04$. (a) Born approximation and perturbative $\alpha_s$ correction with $m_b = 4.7$ GeV (solid line), $m_b = 4.8$ GeV (dashed line), $m_b = 4.9$ GeV (dotted line) and $\alpha_s = 0.22$. (b) Non-perturbative correction with $\lambda_1 = 0.3$ GeV$^2$ (solid line), $\lambda_1 = 0.4$ GeV$^2$ (dashed line), $\lambda_1 = 0.5$ GeV$^2$ (dotted line). $\lambda_2$ and $m_b$ are fixed with the values $\lambda_2 = 0.12$ GeV$^2$ and $m_b = 4.8$ GeV.

Fig. 2 Best fit result for $m_b = 4.8$ GeV with Born approximation (long dashed line), non-perturbative correction (short dashed line), perturbative correction (dotted line) and sum of the all (solid line). Dots with error bars represents CLEO data. Parameter values are $\lambda_1 = -0.58$ GeV$^2$, $\lambda_2 = 0.12$ GeV$^2$ and $\alpha_s = 0.22$. 
Figure 1: (b)

$\lambda_1 = -0.3 \text{ GeV}^2$
$\lambda_1 = -0.4 \text{ GeV}^2$
$\lambda_1 = -0.5 \text{ GeV}^2$

Figure 2:

- Born approx.
- $\alpha_s$ correction
- N.P. correction
- sum of the all terms
- CLEO data

$E_t$

$dB/dE_t$