Opportunistic Maintenance Optimization for Transmission Bays Based on Multistate Markov Process

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Abstract. Power equipment usually goes through multiple states during the transition from normal state to the failure state. In order to consider equipment multistate property, an opportunistic maintenance optimization model based on multistate Markov process is proposed considering economic dependence within the transmission bay. This model describes equipment state transition process using the multistate Markov process. And then, opportunistic maintenance strategy between dependent power equipment in a transmission bay is studied. The transmission bay availability is derived considering opportunistic maintenance and scheduled maintenance. Finally, system operation cost is quantified. System operation cost is minimized to optimize equipment maintenance strategies. Numerical studies demonstrate validity of the proposed model.

1. Introduction

With the expansion of modern power system, the safety and reliability of power equipment is becoming more and more important [1-2]. Nowadays, the development of condition-based maintenance technology has effectively improved the reliability and economy of power system [3]. In order to describe power equipment aging, multistate Markov model and semi-Markov model are widely used in power system maintenance scheduling [4-9]. For example, reference [4] establishes a multistate Markov model of power equipment and optimizes the condition-based maintenance thresholds. Reference [5] formulates equipment stochastic deterioration process as a maintenance-dependent continuous-time Markov model. Pareto-based multi-objective evolutionary algorithm is used to reduce the overall cost and improving system reliability. Reference [6] proposes a Fuzzy Markov model for aging power equipment to integrate uncertain parameters in Markov analysis. Reference [7] develops a Partially Observable Semi Markov Decision Process (POSMDP) for optimizing maintenance decisions. As the condition of the asset is not fully observable, this method can model asset deterioration in a more realistic way. Non-periodic inspection models are introduced in references [8-9] using Markov process in which the inspection rates will be accelerated with the increased deterioration of power equipment.

Most of the research mentioned above ignore economic dependence between dependent power equipment. In recent years, opportunistic maintenance has been proposed to benefit from the economic dependence. To consider equipment aging and the economic dependence, this paper proposes an opportunistic maintenance optimization model for transmission bays based on multistate Markov process. Demonstration of the transmission bay is shown in Figure 1. The transmission bay consists of...
equipment B₁, CT₁, T₁, CT₂, and B₂. In the transmission bay, the outage of any equipment will result in the outage of the whole bay. Thus, opportunistic maintenance can save the system outage cost by combining the maintenance of dependent equipment together.

Figure 1. Demonstration of the transmission bay

The remainder of this paper is organized as follows. In Section 2, multistate Markov model of power equipment is proposed. In Section 3, the availability function of the transmission bay under opportunistic maintenance is derived. System operation cost is quantified in Section 4. In Section 5, the formulation of the optimization model is presented. In Section 6, case studies are given to demonstrate the feasibility of the model. Section 7 gives the conclusion.

2. Multistate Markov model of power equipment

As shown in Figure 2, state 0 represents as-good-as-new state. State 1 and state 2 represent the deterioration states. State 3 represents the failure state. \( \lambda_{k,0} \), \( \lambda_{k,1} \) and \( \lambda_{k,2} \) are the deterioration rates of equipment \( k \). \( \mu_k \) is the repair rate.

Figure 2. Equipment state transition diagram

The state equation of the Markov model in Figure 2 is given by

\[
\frac{dp_k(t)}{dt} = p_{k}(t) \begin{bmatrix}
-\lambda_{k,0} & \lambda_{k,0} & \lambda_{k,1} & \lambda_{k,2} \\
\mu_k & -\lambda_{k,1} & \lambda_{k,1} & -\lambda_{k,2} \\
 & \mu_k & -\lambda_{k,2} & \mu_k \\
\end{bmatrix}
\]

In (1), \( s_k \) represents the initial state of equipment \( k \). \( p_{k,i}(t) \) represents the probability of state \( i \) given the initial state \( s_k \).

3. Availability function of the transmission bay under opportunistic maintenance

Let \( t_{s,m} \) represent the starting time of scheduled maintenance of transmission bay \( \pi \). \( d_k \) is the duration of scheduled maintenance of equipment \( k \). For a transmission bay composed of \( n \) equipment, there are \( n+1 \) cases to be considered to derive transmission bay availability.

(1) Case 0: no failure occurs during the interval \([0, t_{s,m}]\).
In this case, scheduled maintenance will be performed as planned. The transmission bay will be repaired to as-good-as-new state. Transmission bay availability can be written as

\[
B_{x,\text{con}i}(t) = \begin{cases} 
\prod_{k=1}^{n} \left[1 - \tilde{P}_{k,i}^{n}(t)\right], & t < t_{x,m} \\
0, & t_{x,m} \leq t < t_{x,m} + d_{x,m} \\
\prod_{k=1}^{n} \left[1 - \tilde{P}_{k,i}(t_{x,m})\right] A_{0,0}(t - t_{x,m} - d_{x,m}), & t \geq t_{x,m} + d_{x,m}
\end{cases}
\]  \hspace{1cm} (2)

Where \( \tilde{P}_{k,i}(t) \) is the reliability function of transmission bay. \( d_{x,m} \) is the maintenance duration of the transmission bay. \( \tilde{P}_{k,i}(t) \) can be computed as

\[
\begin{align*}
\tilde{P}_{k}(t) &= \left[\tilde{P}_{k,0}^{n}(t), \tilde{P}_{k,1}^{n}(t), \tilde{P}_{k,2}^{n}(t), \tilde{P}_{k,3}^{n}(t)\right] \\
\frac{d\tilde{P}_{k}(t)}{dt} &= \tilde{P}_{k}(t) \\
\tilde{P}_{k,0}(0) &= 1 \\
\tilde{P}_{k,3}(t) &= \tilde{P}_{k,0}^{n}(t) + \tilde{P}_{k,1}^{n}(t) + \tilde{P}_{k,2}^{n}(t)
\end{align*}
\]  \hspace{1cm} (4)

\( d_{x,m} \) can be computed as

\[
d_{x,m} = \max(d_1, d_2, \ldots, d_n)
\]  \hspace{1cm} (5)

(2) Case \( i = 1, \ldots, n \): the \( i \)th equipment fails during the interval \([0, t_{x,m}]\).

In the case \( i \), the \( i \)th equipment will fail before time \( t_{x,m} \). Then, the availability of the transmission bay can be computed as

\[
B_{x,\text{con}i}(t) = \sum_{m=0}^{n} B_{x,\text{con}i}(t)
\]  \hspace{1cm} (6)

Where \( B_{x,\text{con}i}(t) \) represents the availability function of transmission bay in state \( \omega \). Let \([\tau_k, t_{x,m}]\) represent the allowed maintenance period of equipment \( k \). \( \tau_k \) is the starting time of opportunistic maintenance. That is if the \( i \)th equipment fails during the time interval \([\tau_k, t_{x,m}]\), then opportunistic maintenance will be performed on equipment \( k \). For simplicity, we sort the starting time of opportunistic maintenance according to the size of order. And we can get

\[
0 \leq \tau_{o(1)} \leq \cdots \leq \tau_{o(n-1)} < t_{x,m}
\]  \hspace{1cm} (7)

Where \( \tau_{o(i)} \) represent the starting time of \( i \)th equipment. Specially, let \( \tau_{o(0)} \) represent \( \tau_i \).

\( B_{x,\text{con}i}(t) \) can be computed as follows.

1. If \( 0 \leq t < \tau_l \), then \( B_{x,\text{con}i}(t) \) is written as

\[
B_{x,\text{con}i}(t) = \int_{0}^{t} A_{0,0}(u) \prod_{j=1}^{n} A_{0,i_{j}}(t-u) d\tilde{G}_{o(0)}(u) d\tilde{Q}_{o(0)}(u)
\]  \hspace{1cm} (8)

\[
\tilde{G}_{o(0)}(u) = 1 - e^{-\lambda_{o(0)} u}, \quad \tilde{Q}_{o(0)}(u) = \prod_{i=1}^{n} P_{o(0),i_{i}}(u) d\tilde{P}_{o(0),i}(u)
\]  \hspace{1cm} (9)

Where \( \tilde{G}_{o(i)} \) represents the state of equipment \( o(i) \) corresponding to state \( \omega \).

2. If \( \tau_{o(1)} \leq t < \tau_{o(k+1)} \) \((k=1, \ldots, n-1)\), then \( B_{x,\text{con}i}(t) \) is written as

\[
\int_{\tau_{o(k)}}^{t} A_{0,0}(u) \prod_{j=1}^{n} A_{0,i_{j}}(t-u) d\tilde{G}_{o(0)}(u) d\tilde{Q}_{o(0)}(u)
\]  \hspace{1cm} (10)
\[
B_{\tau,s}(t) = \sum_{j=0}^{n} \int_{t_{e_{ij}}}^{t_{e_{ij}}+t_{d}} \int_{t_{e_{ij}}}^{t_{e_{ij}}+t_{d}} A_{d_{ij}}(t-u-y) \prod_{k} \left( A_{d_{ij}}(t-u) dG_{e_{ij}}(y) dQ_{e_{ij}}(u) \right) + \int_{t_{e_{ij}}}^{t_{e_{ij}}+t_{d}} A_{d_{ij}}(t-u-y) \prod_{k} \left( A_{d_{ij}}(t-u) dG_{e_{ij}}(y) dQ_{e_{ij}}(u) \right) \tag{11} \]

3. If \( t > t_{x,m} \), then we have

\[
B_{\tau,s}(t) = \sum_{j=0}^{n} \int_{t_{e_{ij}}}^{t_{e_{ij}}+t_{d}} \int_{t_{e_{ij}}}^{t_{e_{ij}}+t_{d}} A_{d_{ij}}(t-u-y) \prod_{k} \left( p_{\gamma}^m(t-u)p_{\gamma}^m(t-u) \prod_{k} \left( A_{d_{ij}}(t-u) dG_{e_{ij}}(y) dQ_{e_{ij}}(u) \right) \right) \tag{12} \]

\[
p_{\gamma}^m(t-u) = \prod_{h \in \text{set}} \left[ 1 - \tilde{p}_{(\gamma,3,t_{x,m} - u)} \right] \tag{13} \]

\[
p_{\gamma}^m(t-u) = \prod_{h \in \text{set}} \left[ A_{d_{ij}}(t-u-y) dP_{(\gamma,3,t_{x,m} - u)} \right] \tag{14} \]

\[
p_{\gamma}^m(t-u) = \prod_{h \in \text{set}} \left[ A_{d_{ij}}(t-t_{x,m} - d_{y}) \right] \tag{15} \]

Where \( Z_{j+1} \) is the maintenance scenario set of the equipment which is not under opportunistic maintenance strategy. \( \text{re} \) is the reliable equipment set in scenario \( \gamma \) and \( \text{ure} \) is the unreliable equipment set in scenario \( \gamma \).

Combining the \( n+1 \) cases mentioned above, transmission bay availability can be computed as

\[
B_s(t) = \prod_{r=0}^{n} B_{r,s}(t) \tag{16} \]

4. Formulation of system operation cost

Let \( T \) represent the planning maintenance horizon. System operation cost is computed as

\[
pr_s(t) = \prod_{s \in \text{BA}_s} B_s(t) \prod_{s \in \text{BU}_s} \left[ 1 - B_s(t) \right] \tag{17} \]

\[
\text{RI}_{\text{system}} = \sum_{j=1}^{s} \sum_{s \in \text{NS}_j(t)} \text{pr}_s(t) \text{sev}_s(t) \tag{18} \]

Where \( s \) represents system contingency; \( \text{NS}(t) \) is the system contingency set; \( \text{pr}_s(t) \) is the occurrence probability of contingency \( s \); \( \text{sev}_s(t) \) is the loss of load of contingency \( s \) in period \( t \); \( c_t \) is the unit loss of load; \( \text{BA}_s \) is the equipment set that is in the operation state in contingency \( s \); \( \text{BU}_s \) is the equipment set that is in the outage state in contingency \( s \).

5. Formulation of the maintenance optimization model

The proposed maintenance optimization model is formulated in (19)-(20) with the objective of minimizing the system operation cost.

\[
\min \text{RI}_{\text{system}} \tag{19} \]

subject to

\[
\begin{align*}
X_{\tau}(t) \in [0,1], & \quad \text{if } t_{x,m} \leq t \leq t_{x,m} \\
X_{\tau}(t) = 0, & \quad \text{otherwise}
\end{align*} \tag{20} \]

Where \( X_{\tau}(t) \) is a binary variable which is equal to 1 if opportunistic maintenance is performed on equipment \( k \) in period \( t \) and 0 otherwise.

6. Case study

The proposed model is verified by a typical substation depicted in Figure 3. The planning maintenance horizon is set to one year which is divided into 52 weeks. 4 transmission bays can be found in the substation: \{T1, G1\}, \{T2, G2\}, \{T3, G3\} and \{T4, G4\}. For simplicity, only equipment T1, T2, T3 and G4 will be undergo scheduled maintenance. These equipment are initially in state 2. Other equipment are
assumed to be in state 0. The peak load is 155 MW and the weekly load profile of the 52 weeks of load point Lp1 and Lp2 can be seen in reference [10-11]. $c_f$ is set to 1,053 $/\text{MWh}$.

Figure 3. Substation configuration

Table 1. Equipment maintenance parameters

| Equipment   | $\lambda_0$ (week$^{-1}$) | $\lambda_1$ (week$^{-1}$) | $\lambda_2$ (week$^{-1}$) | $\mu$ (week$^{-1}$) | $d$ (week) |
|-------------|-----------------------------|-----------------------------|-----------------------------|---------------------|-------------|
| Transformer | 0.00833                     | 0.00847                     | 0.01932                     | 0.35                | 2           |
| Generator   | 0.0083                      | 0.0225                      | 0.084                       | 0.2                 | 3           |
| Circuit breaker | 0.0042                    | 0.07                        | 0.042                       | 0.57                | --          |

Table 2. Maintenance time constraints

| Equipment | $\tau$ (week) |
|-----------|---------------|
| T_1       | 6             |
| T_2       | 5             |
| T_3       | 7             |
| G_4       | 2             |

To compare different opportunistic maintenance strategies, the following two strategies are analysed.

1) Strategy 1: Scheduled maintenance of transformer T_1 is in the 34th period and opportunistic maintenance is not considered.

2) Strategy 2: The same as Strategy 1 except that opportunistic maintenance is considered.

Table 3 compares the system operation cost of the two strategies.

Table 3. System operation cost

| Maintenance strategies | System operation cost ($) |
|------------------------|---------------------------|
| Strategy 1             | 2815.1                    |
| Strategy 2             | 2705.6                    |

It can be seen from Table 3, system operation cost in strategy 1 and strategy 2 are 2815.1$ and 2705.6$, respectively. A total cost of 3.9% can be saved by strategy 2. We can see that opportunistic maintenance have a great impact on system operation cost.

To further investigate the impact of opportunistic maintenance on system maintenance scheduling, the following two schemes are analyzed.

1) Scheme 1: equipment opportunistic maintenance and scheduled maintenance is optimized by minimizing the system operation cost.

2) Scheme 2: the same as Scheme 1 except that opportunistic maintenance is not considered.
Table 4. Maintenance schedules

| Equipment | Scheme 1 | Scheme 2 |
|-----------|----------|----------|
| T₁        | 36       | 15       |
| T₂        | 32       | 13       |
| T₃        | 11       | 7        |
| G₄        | 2        | 2        |
| System operation cost ($) | 2262.5 | 2380.2 |

Table 5. Opportunistic maintenance strategy in scheme 1

| Equipment | Opportunistic maintenance strategy |
|-----------|-----------------------------------|
| T₁        | Strategy 1                         |
| T₂        | Strategy 1                         |
| T₃        | Strategy 2                         |
| G₄        | Strategy 2                         |

It can be seen from Table 4 and Table 5, opportunistic maintenance will be only performed on equipment T₁ and T₂. The total cost under scheme 1 is 2262.5 $ which is 4.9% reduced, compared with scheme 2. Thus, opportunistic maintenance strategy should be optimized to benefit from economic dependence within the transmission bay.

7. Conclusion
This paper proposes an opportunistic maintenance optimization model for transmission bays to benefit from the economic dependence. To incorporate aging effect of power equipment, multistate Markov process is used. System operation cost is minimized to get the optimal maintenance strategies. Test results show the significance of the proposed model.

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