Upside down Magic, Bimagic, Palindromic Squares and Pythagoras Theorem on a Palindromic Day - 11.02.2011

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Abstract

In this short note we have produced different kinds of upside down magic squares based on a palindromic day 11.02.2011. In this day appear only the algorisms 0, 1 and 2. Some of the magic squares are bimagic and some are palindromic. Magic sums of the magic squares of order 3×3, 4×4 and 5×5 satisfies the Pythagoras theorem. Three different kinds of bimagic squares of order 9×9 are also produced. The bimagic square of order 9×9 with 8 digits is palindromic numbers. We have given bimagic squares of order 16×16 and 25×25, where the magic sum S1 in both the cases is same. In order to make these magic squares upside down, i.e., 180° degree rotation, we have used the numbers in the digital form. All these magic square are only with three digits, 0, 1 and 2 appearing in the day 11.02.2011.

1 Introduction

It interesting to observe that the day 11.02.2011 is palindromic and has only three digits 0, 1 and 2. A similar kind of palindromic day shall also appear next year 21.02.2012 having the same three digits. In this paper our interest is to produce upside down magic squares, bimagic squares and palindromic magic squares using only these three algorisms, 0, 1 and 2. A similar kind of study can be seen in another author’s work on the day October 10, 2010 [5]. Using these three digits we have [8] made equivalence with classical magic squares, the one is ”Lo-Shu” magic square of order 3×3 and another is ”Khajurao” magic squares of order 4×4.

Before we proceed, here below are some basic definitions:

(i) A **magic square** is a collection of numbers put as a square matrix, where the sum of element of each row, sum of element of each column and sum of each element of two principal diagonals have the same sum. For simplicity, let us write it as S1.

(ii) **Bimagic square** is a magic square where the sum of square of each element of rows, columns and two principal diagonals are the same. For simplicity, let us write it as S2.
(iii) **Upside down**, i.e., if we rotate it to 180° degree it remains the same.

(iv) **Mirror looking**, i.e., if we put it in front of mirror or see from the other side of the glass, or see on the other side of the paper, it always remains the magic square.

(v) **Universal magic squares**, i.e., magic squares having the property of upside down and mirror looking are considered *universal magic squares*.

In this short note we have produced different kinds of upside down magic squares using only the algorithms 0, 1 and 2. Some of the magic squares are bimagic and some are palindromic. Magic sum of the magic squares of order 3×3, 4×4 and 5×5 satisfies the Pythagoras theorem. Bimagic squares of order 9x9 are produced with 4, 6 and 8 digits. The bimagic square of order 9×9 with 8 digits is palindromic while with 6 digits is a combination of palindromic numbers. We have given bimagic squares of order 16×16 and 25×25, where the magic sum S1 is the same. In order to make these magic squares upside down we have used the numbers in the digital form.

All these magic squares are only with three digits, 0, 1 and 2. In order to do so, we have used the numbers in the digital form:

\[0, 1 \text{ and } 2\]

These digits generally appear in watches, elevators, etc. We observe that the above three digits are rotatable to 180°, and remains the same. These also be considered as universal, because in the mirror 2 becomes 5, while 0 and 1 remains the same. In these situations the magic sums are different. This we leave to reader to verify.

### 2 Upside Down Magic Squares and the Pythagoras Theorem

In this section we shall present magic squares of order 3×3, 4×4 and 5×5 having only the three digits 0, 1 and 2 in the digital form. Interesting the magic sum S1, in this case satisfies the Pythagoras theorem.

- **Magic squares of order 3×3**

Here below are two magic squares of order 3×3 with \(S_{1_{3\times3}} := 33\) and \(S_{1_{4\times3}} := 3333\) respectively. The first one is with two digits combinations while the second one is with four digits combinations:
We observe from the second magic square that it is **palindromic**. In order to have upside down we have considered 110, 220 as 0110, 0220 to be symmetry in the result.

- **Magic squares of order 4×4**

    Here below is a magic square of order 4x4 with \( S_{1,4} := 4444 \)

    \[
    \begin{array}{cccc}
    1212 & 2222 & 0101 & 2112 \\
    2002 & 0000 & 0220 & 0110 \\
    0110 & 2222 & 1111 & 2222 \\
    1212 & 1010 & 2112 & 0220 \\
    \end{array}
    \]

- **Magic squares of order 5×5**

    Here below is a magic square of order 5×5 with \( S_{1,5} := 5555 \)

    \[
    \begin{array}{cccc}
    0000 & 0220 & 1102 & 2022 & 2211 \\
    2002 & 2222 & 0014 & 0200 & 1120 \\
    0211 & 1100 & 2020 & 2202 & 0022 \\
    2220 & 0002 & 0222 & 1111 & 2000 \\
    1122 & 2011 & 2200 & 0020 & 0202 \\
    \end{array}
    \]

    The above magic square is pan diagonal.
2.1 Pythagoras Theorem

From the above magic squares of orders 3×3, 4×4 and 5×5 with four digits, we have the following result:

\[(S_{13×3})^2 + (S_{14×4})^2 = (S_{15×5})^2,\]

i.e.,

\[3333^2 + 4444^2 = 5555^2\]

i.e.,

\[11108889 + 19749136 = 30858025.\]

This means that if we consider square of any line, or column or principle diagonal, from one of the above magic squares we shall always have the same value, for example,

\[(1221 + 1111 + 1001)^2 + (1012 + 2101 + 1210 + 0121)^2\]

\[= (2002 + 2222 + 0011 + 0200 + 1120)^2\]

The above result gives us an upside down Pythagoric equation when we use the digital letters:

\[0221\times1110\times0221\times1110\times0000+201+120+0200+210+120+020\]

\[= (2002+2222+0011+0200+1120)\times(2002+2222+0011+0200+1120)\]

180° degrees rotation:

\[0221\times1110\times0221\times1110\times0000+201+120+0200+210+120+020\]

\[= (2002+2222+0011+0200+1120)\times(2002+2222+0011+0200+1120)\]

We observe from the second equation that the numbers are different, but the sum is same.

3 Upside down bimagic squares of order 9x9

Here below are three bimagic squares of order 9x9. The first one is with four digits combinations, the second one is with six digits combinations and the third one is with eight digits combinations.

- Bimagic squares of order 9x9 with four digits
Also we have sum of each block of order 3x3 is 9999 and the squares of sum of each term in each block of order 3x3 is also 17169495. We observe that the above bimagic square still has the day’s number 11.02.2011 in two parts 1102 and 2011.

• Bimagic squares of order 9×9 with six digits

Also we have sum of each block of order 3x3 is 999999 and square of sum of each term in each block of order 3x3 is also 172916950695. Here we observe that each number is a composition of three digit palindromic numbers.

• Bimagic squares of order 9x9 with eight digits
Also we have sum of each block of order $3 \times 3$ is $99999999$ and square of sum of each term in each block of order $3 \times 3$ is $1717172174949490$. Here the numbers are palindromic including the day’s number: 11.02.2011.

4 Upside down bimagic squares of order $16 \times 16$ and $25 \times 25$ with same magic sum

Here below are bimagic squares of order $16 \times 16$ and $25 \times 25$. Both these magic square have the same magic sum, $S_{16 \times 16} = S_{25 \times 25} = 222222220$.

- Bimagic square of order $16 \times 16$

\[
\begin{array}{cccccccccccccccc}
0000000 & 0123210 & 0211201 & 0123210 & 0211201 & 0123210 & 0211201 & 0123210 & 0211201 & 0123210 & 0211201 & 0123210 & 0211201 & 0123210 & 0211201 & 0123210 \\
1201201 & 1101011 & 1201201 & 20122021 & 2122212 & 2122212 & 0021202 & 0101010 & 0220202 & 1000001 & 1201201 & 1101011 & 1201201 & 20122021 & 2122212 & 2122212 \\
2021202 & 2110122 & 2021202 & 0021202 & 0101010 & 0220202 & 1000001 & 1201201 & 1101011 & 1201201 & 20122021 & 2122212 & 2122212 & 0021202 & 0101010 & 0220202 \\
0220202 & 0111010 & 0100001 & 1201201 & 10022021 & 1201201 & 2122212 & 2012202 & 1101011 & 1201201 & 20122021 & 2122212 & 2122212 & 0021202 & 0101010 & 0220202 \\
1201201 & 0200020 & 0022200 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 \\
0211202 & 1200221 & 1101101 & 2112212 & 2202222 & 2202222 & 0020200 & 0121210 & 0222221 & 1122221 & 1010010 & 1101101 & 1200221 & 1101101 & 2112212 & 2202222 \\
2021202 & 2100102 & 2012010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 \\
2102202 & 2211222 & 2100102 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 & 0212010 & 0120210 \\
0000001 & 1201201 & 1101011 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 & 1201201 \\
\end{array}
\]

\[
S_{9 \times 9} := 99999999 \\
S_{2 \times 9} := 1717172174949490
\]
\[ S_{16 \times 16} := 222222220 \]
\[ S_{2 \times 16} := 4097520801469040. \]

Here each block of order 4\times4 is a magic square of sum \( S_{4 \times 4} := 55555555 \). Also we observe that the numbers are not palindromic, but still include the day’s number: 11.02.2011.

- Pan diagonal bimagic square of order 25\times25

\begin{tabular}{|cc|cc|cc|}
\hline
222222220 & 222222220 & 222222220 & 222222220 \\
222222220 & 222222220 & 222222220 & 222222220 \\
\hline
\end{tabular}

**Part 1:** \( 11_{25 \times 15} \)

**Part 2:** \( 12_{25 \times 10} \)
\begin{align*}
S_{125 \times 25} & := 222222220 \\
S_{225 \times 25} & := 3169428014410330
\end{align*}

Here each block of order $5 \times 5$ is a magic square of sum $S_{5 \times 5} := 44444444$. We observe that the numbers are not palindromic, but still include the day’s number: \textbf{11.02.2011}.

\section{Final Comments}

In this short paper we have brought magic squares of different kinds using only the digits 0, 1 and 2 appearing in a palindromic day - 11.02.2011. Another palindromic day having the same digits shall also appear next year - 21.02.2012. The magic squares obtained are upside down, i.e., when we make a rotation of $180^\circ$ degrees they still remain the magic squares. This happens using the letters in the digital. Some of the magic squares are palindromic. Bimagic squares are of order $9 \times 9$, $16 \times 16$ and $25 \times 25$. The numbers 9, 16 and 25 remember us a Pythagoras theorem, i.e., $3^2 + 4^2 = 5^2$. We have produced magic squares of orders $3 \times 3$, $4 \times 4$ and $5 \times 5$ using only three digits 0, 1 and 2 and the sum $S_1$ satisfies the Pythagoras theorem, i.e., $(S_{3 \times 3})^2 + (S_{4 \times 4})^2 = (S_{5 \times 5})^2$. Interestingly, the sum $S_1$ in case of bimagic squares of order $16 \times 16$ and $25 \times 25$ is the same and can also be made upside down. Most of the magic squares have the palindromic number 11.02.2011. The digits 0, 1 and 2 also appears in many others days during the years 2010, 2011 and
In another work [8], we have brought equivalent versions of two classical magic squares of order $3 \times 3$ and $4 \times 4$ using only these three digits 0, 1 and 2. For more studies on magic squares see the references below.

**References**

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