Brane Gas Inflation

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We consider the brane gas picture of the early universe. At later stages, when there are no winding modes and the background is free to expand, we show that a moving 3-brane, which we identify with our universe, can inflate even though it is radiation-dominated. The crucial ingredients for successful inflation are the coupling to the dilaton and the equation of state of the bulk. If we suppose the brane initially forms in a collision of higher-dimensional branes, then the spectrum of primordial density fluctuations naturally has a thermal origin.

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In recent work, Alexander et al. [1] proposed a dynamical origin of the non-compact spatial dimensions of the universe. In their picture, the universe starts as a hot, dense gas of the fundamental states of string theory, namely D-branes. Considering 11-dimensional M-theory compactified on $S^1$, they show that winding modes will allow only some spatial dimensions to grow large; a result which generalizes that of Brandenberger and Vafa [2]. The relevant equations of motion (see also [3]) emphasize the importance of the dilaton to this proposal.

Here we consider what might be the late-time behavior of such a universe. We will suppose that $d$ spatial dimensions have become large and that all degrees of freedom that have been able to interact have annihilated with one another. In particular, we assume that there are no more winding modes so that the universe is free to expand. In a similar set-up, Park et al. [4] envisaged the universe as a gas of Dp-branes in the context of Brans-Dicke theory. As for our model, we will be more precise shortly as to the dominant contribution to the energy density of the universe, but for now we imagine that are a number of 3-branes in this universe. Furthermore, we will suppose what we think of as our universe is, in fact, one of these branes. Our proposal, therefore, is similar in spirit to that of mirage cosmology [3].

In this paper, we show that a 3-brane, moving in this background, can inflate. This is true even though the brane, assumed formed in a collision, is radiation-dominated. Consequently, primordial density fluctuations are seen to be thermal in origin. The parameter space of inflationary solutions is spanned by the coupling to the dilaton and the bulk barotropic index. The set-up has elements in common with those of Alexander [1] and Burgess et al. [5]. However, in our case inflation is not due to brane—anti-brane interaction and we require the 3-brane to be moving rather quickly.

In our scenario, the time evolution of our universe is governed by the matter on the brane and its dynamics in the expanding background. However, we do not consider the self-gravity of the brane, this being still an open problem. Solving for the brane dynamics then is analogous to determining planetary motion in that we assume a background and do not take back-reaction into account. In particular, we do not have the usual brane world junction conditions which are, in any case, difficult to apply to objects with codimension greater than one. We start by considering the background.

If we assume the background is flat and roughly homogeneous and isotropic in the $d$ spatial dimensions, then the metric can be written as

$$g_{\mu\nu} = \text{diag}(1, -a^2(t), \ldots, -a^2(t)),$$

where $\mu = 0 \ldots d$ and we will let $x^\mu$ label the coordinates, with $t \equiv x^0$. Following [1], we take the low-energy effective action of the bulk to be the dilaton-gravity action in $D = d + 1$ dimensions:

$$S_B = \int d^Dx\sqrt{-g}\left\{ e^{-2\phi} \frac{2\kappa_D^2}{2\kappa_D^2} [R + 4(\nabla \phi)^2] + \mathcal{L}_B \right\},$$

where $\phi$ is the dilaton, $\mathcal{L}_B$ describes the matter in the bulk, $\kappa_D$ is related to the $D$-dimensional Newton’s constant in the usual way and we have supposed there is no bulk cosmological constant.

Taking the bulk matter to be of perfect fluid form, the equations of motion for the background are

$$\frac{1}{2}d(d-1)H^2 = e^{2\phi}\kappa_D^2 \rho - 2\phi(\dot{\phi} - dH)$$

$$\frac{d}{dt}(d-1)\dot{H} + 2dH = e^{2\phi}\kappa_D^2 \rho + 2(\ddot{\phi} - H\dot{\phi})$$

$$-2\phi(\dot{\phi} - dH)$$

$$dH + \frac{1}{2}d(d+1)H^2 = 2\phi - 2\phi(\dot{\phi} - dH),$$

where $\rho$ is the density and $p$ the pressure of the fluid, and $H = \dot{a}/a$. The case of pure Einstein gravity is recovered if we let $\phi = 0$ and drop [1].

As usual, the conservation equation, here $\dot{\rho} + dH(\rho + p) = 0$, follows from the field equations or it can be derived from $\nabla_{\nu}T^{\mu\nu} = 0$. Thus, if we augment the system [1]-[6] with an equation of state $p = w\rho$, we find that
\[ \rho \sim a^{-(1+w)}. \]  

This suggests the following ansatz for the dilaton which is indeed the general solution at low curvature scales \[3\]:

\[ e^{2\phi} \sim a^n, \]  

where \( n \) is a constant. The equations of motion are satisfied only if

\[ n = d - \frac{1}{w} \quad \text{and} \quad a \sim t^{\frac{2}{w+n}} = t^{\frac{2w}{1+w}}. \]  

Thus, the background expands for all \( w > 0 \) but it is not hard to see that bulk inflation is not possible for any choice of \( w \) and \( d \). This is in contradiction with models of dilaton-driven inflation because we have assumed the background has evolved into a low curvature regime. It should also be pointed out that the case of \( n = 0 \) is not pathological. Even though \( n \to -\infty \), the density and scale factor simply become constant while \( e^{2\phi} \sim t^{-2} \).

However, we will have no use for \( w < 0 \) solutions since, as we will see, the bulk and the brane expand or contract together. Having said that, \( w < 0 \) is allowed in pure Einstein gravity; there \( n = 0 \) and \( w \) and \( d \) are unconstrained.

Bulk inflation occurs for \(|w| + 1 < 2/d\). Having determined the evolution of the bulk, we now turn to the question of the action for the brane. We are led by the usual Dirac-Born-Infeld action but will make some simplifications. We will suppose that the brane is not charged under any fields living in the bulk and, as in \[4\], we will assume that usual antisymmetric tensor field \( B_{\mu\nu} \) vanishes. Additionally, we adopt a phenomenological approach to enable us to put ordinary forms of matter on the brane. We suppose the brane action is:

\[ S_b = \int d^4\sigma \sqrt{-\gamma} \mathcal{L} = \int d^4\sigma \sqrt{-\gamma} \{ e^{-\phi} \lambda + \xi e^{-m\phi} \mathcal{L}_b \}, \]  

(9)

where \( \sigma^i \) are the world-volume coordinates of the brane \( i = 0, \ldots, 3 \), \( \gamma_{ij} \) the induced metric, \( \lambda \) the tension of the brane, and \( m \) and \( \xi \) are dimensionless constants which determine the coupling of the dilaton to the brane matter given by \( \mathcal{L}_b \). Note that this action allows us to consider the usual gauge fields that might live on the brane (by expanding the usual square-root term in powers of the string tension) but that it does not include the effects of the brane self-gravity.

The induced metric on the 3-brane follows from its embedding in the bulk, i.e. \( x^i = X^i(\sigma) \). We choose the static gauge \( \sigma^i = x^i \) and suppose an embedding of the form

\[ X^i = x^i, \quad X^A = X^A(t), \]  

(10)

where \( A = 4 \ldots d \). Then the induced metric on the brane is

\[ \gamma_{ij} \equiv g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^i} \frac{\partial X^n}{\partial \sigma^j} = \text{diag}(1 - V^2, -a^2, -a^2, -a^2), \]  

(11)

where \( V^2 \equiv -\dot{X}^A \dot{X}_A \geq 0 \). Note that the scale factor on the brane is the same as that in the bulk. However, because of the motion of the brane in the transverse directions, the brane time \( \tau \), defined via

\[ d\tau = dt\sqrt{1 - V^2}, \]  

(12)

is not the same as the bulk time. Accordingly, a brane-bound observer will see a quite different evolution of the scale factor, with the discrepancy becoming more pronounced as \( V^2 \to 1 \).

Thus, we will have brane inflation when

\[ \alpha - \beta < 0, \]  

and scale factor simply become constant while \( e^{2\phi} \sim t^{-2} \). However, we will have no use for \( w < 0 \) solutions since, as we will see, the bulk and the brane expand or contract together. Having said that, \( w < 0 \) is allowed in pure Einstein gravity; there \( n = 0 \) and \( w \) and \( d \) are unconstrained.

Bulk inflation occurs for \(|w| + 1 < 2/d\).

Equation (14) can be inverted to find \( \partial \sqrt{-\gamma} \mathcal{L} / \partial X^A \) are constants of the brane motion. To be precise,

\[ c_A = \frac{\partial \sqrt{-\gamma} \mathcal{L}}{\partial X^A} = \frac{\partial \mathcal{L}_0}{\partial X^A} \delta_{00} = \sqrt{-\gamma} \dot{X}_A T^{00} \]  

(13)

are constant. It follows that

\[ c \equiv \sqrt{-e^A c_A} = \frac{a^3 V}{\sqrt{1 - V^2}} T^0_0 \]  

(14)

is a positive constant, where

\[ T^0_0 = e^{-\phi} \alpha + \xi e^{-m\phi} \rho_b \]  

(15)

and \( \rho_b \) is the energy density of the matter on the brane. Equation (14) can be inverted to find \( V \) which is then inserted into (12). We find

\[ d\tau = \frac{a^3 T^0_0}{\sqrt{c^2 + a^6 (T^0_0)^2}} dt. \]  

(16)

Before applying this equation to a definite scenario, a small mathematical digression will be helpful. It is not hard to show that if

\[ d\tau \sim a^\beta \, dt \quad \text{and} \quad a \sim t^{\frac{1}{n}}, \]  

(17)

then

\[ a \sim (\epsilon t)^{\frac{1}{n}} \]  

(18)

where \( \epsilon = \text{sgn}(\alpha + \beta) \).

Thus, we will have brane inflation when \(|\alpha + \beta| < 1\) and, in particular, exponential inflation when \(\alpha + \beta = 0\). We will use the variables \( \alpha \) and \( \beta \) in what follows.

We are now in a position to consider a particular physical situation. Our idea is that the 3-brane which is our universe came about as a result of a collision process, say 5-5 brane annihilation \[12\]. We make two assumptions about this collision. Firstly, that the resulting velocity
of the 3-brane in the transverse directions is, at least initially, relativistic. In other words
\[ c \gg a^3 T_0. \]  

Furthermore, we suppose a large amount of energy is deposited on the brane in the collision and that this dominates over the brane tension, i.e.
\[ \rho_b \gg \lambda \]  
or, to be slightly more accurate, \( \xi e^{-m\phi} \rho_b \gg e^{-\phi} \lambda. \)

The equation of state for brane matter most consistent with a collision event is \( \rho_b = \rho_b/3 \) which corresponds to radiation on the brane or the excitation of massless scalar modes. If, as is usual, we assume the matter on the brane remains confined to it, then energy conservation determines that
\[ \rho_b \sim a^{-4}. \]  

From (8) and substituting (19)-(21) into (10), we have that
\[ \alpha + \beta = \frac{1}{2} [d(1 + w) - n(1 + m)] - 1. \]  

The parameter space of solutions is quite constrained. On M-theoretic grounds we expect \( d \leq 10 \) and we need \( d \geq 4 \) for the background to be able even to contain a moving 3-brane. However, if our universe is the result of 5-5 brane annihilation, then we would require \( d > 5 \). Furthermore, if there are a number of 3-branes in the bulk, then we would prefer our universe to avoid them. Since two \( p \)-branes will interact in at most \( 2p+1 \) large spatial dimensions, it follows that we must have \( d > 7 \), \( d = 7 \) being marginal.

Next, we will limit ourselves to \( 0 < w \leq 1 \). The lower bound is required to have expansion at all and the upper bound is the usual case of stiff matter. Actually, we might expect \( w \) to be no greater than \( 1/d \) which would correspond to radiation in the bulk. In the case of Einstein gravity, we will consider \(-1 \leq w \leq 1\).

We now search for inflationary solutions. The case of minimal coupling of the dilaton to brane matter can be dealt with immediately. Putting \( m = 0 \) into (22), we find \( \alpha + \beta < 1 \) when \( dw^2 - 4w + 1 < 0 \), but this only has solutions for \( d < 4 \). Thus, inflation in this scenario will require non-minimal coupling to the dilaton.

The typical effect of the coupling to the dilaton is illustrated in fig. 1. The number of spatial dimensions has been set to seven and the shaded regions indicate the values of \( (w, m) \) which give rise to inflation on the brane. These regions grow as \( d \) gets smaller, and shrink for larger values of \( d \). Their behavior as \( w \rightarrow 1/d \), in other words \( n \rightarrow 0 \), is due to the dilaton becoming independent of the scale factor. The interesting result here is that it is possible to obtain brane inflation with ordinary matter both on the brane and in the bulk.

![FIG. 1. The shaded regions indicate inflationary scenarios for a 3-brane moving in \( d = 7 \) spatial dimensions. The regions grow as \( d \) is decreased, and shrink as \( d \) is made larger. Parameter space is spanned by \( m \), which gives the coupling of the dilaton to brane matter, and \( w \), the ratio of the bulk pressure to the bulk density.](image-url)

A natural question to ask is how inflation ends in this scenario. It turns out that in cases of successful inflation (19) remains valid but the brane tension eventually becomes dominant, reversing (20). Now we have \( \alpha + \beta = d(1 + w)/2 - n + 3 \) and a simple analysis reveals it is not possible to have inflation for the range of parameters we are considering. However, this fact allows us to make a straightforward estimation of the number of \( e \)-folds of inflation, \( N \). If we suppose inflation ends when \( e^{-\phi} \lambda = \xi e^{-m\phi} \rho_b \), then
\[ N = \frac{\ln \xi + \ln (\frac{\phi_0}{\lambda}) - (m - 1) \phi_0}{\frac{1}{2} n(m-1) + 4}, \]  

where subscript 0 indicates the value of the quantity at the start of inflation and we are now letting \( \rho \) be the density of matter on the brane. The denominator is positive for inflationary solutions but it diverges to infinity as \( w \rightarrow 0 \). Since we can expect realistically \( \xi \sim 1 \) and \( \ln (\rho_0/\lambda) \lesssim 10 \), we see that the initial value of the dilaton will, in general, be crucial to ensuring the usual requirement for inflation that \( N \gtrsim 70 \).

For example, when \( w > 1/d \), \( n \) is positive so that the value of the dilaton when the 3-brane is formed can be arbitrarily large. Furthermore, as seen in fig. 1 we can have inflation for \( m < 1 \), thus the large initial value of the dilaton translates to a large value of \( N \). On the other hand, an interesting case arises when \( w \lesssim 1/d \). Then the size of \( \phi_0 \) can be irrelevant because \( m \) must large and negative for inflation to occur. However, we expect such values of \( m \) to be theoretically unlikely.

The fact that we do not have inflation when \( \lambda \) dominates may seem a little odd until we remember we have
neglected the self-gravity of the brane in our analysis. Without doubt, until this is included, details of the all-important transition from inflation to radiation domination will not be known and we will be unable to accurately determine the number of e-folds of inflation. However, although it is still an open problem how to address the issue of self-gravity, it seems plausible that at early brane times the assumptions \[14\] and \[24\] mean that the motion of the brane through its background is more important than its internal dynamics in determining the evolution of the brane. It is tempting, however, to suppose that when self-gravity effects become important, it may be that \(\lambda\), attenuated by its coupling to the dilaton, generates late-time acceleration on the brane.

The construction of inflationary solutions is quite different in the case of pure Einstein gravity. It follows when \(n = 0\) is substituted into \[14\] that brane inflation can only occur for \(w\) negative. However, there is an interesting region \(-1 + 2/d < w < -1 + 4/d\) where the brane inflates but the bulk does not. Once again, inflation ends when the brane tension starts to dominate the matter density on the brane. In this instance though, it will be difficult to have sufficient e-folds of inflation. This can be seen by putting \(n = \phi_0 = 0\) in \[23\]; we require \(\rho_1\) to be bigger than \(\lambda\) to a fantastic degree in order to have \(N\) large. Therefore, it appears that dilaton gravity is essential to the inflationary scenarios we have considered here.

One of the more compelling features of the usual models of inflation is that they naturally give rise to primordial density fluctuations. In the scenario presented here this is also the case. There are two contributions. The first are thermal fluctuations on the brane, coming from the collision, for which the spectrum will likely need to be quite red in order to fit with observations. The details will depend crucially on the manner of the formation of the 3-brane \[14\]. The second contribution comes from bulk fluctuations which are then induced on the brane. The need here will be for an account of the background evolution. And of course, in both cases, self-gravity will have to be included in order to trace the primordial spectrum through to the present day.

To summarize, we have proposed that our universe could be a 3-brane moving in a late-time, brane gas background. Of particular note is that this brane can, depending on the details of the coupling of brane matter to the dilaton and the nature of the bulk matter, successfully inflate. Furthermore, if the brane is a result of a collision, there appears to be a natural mechanism by which density fluctuations would arise on the brane.

There are a number of avenues worth exploring however. Firstly, one could include explicitly the gauge fields which live on the brane. Since the usual interpretation is that these fields reflect the existence of open strings ending on the brane, one is further led to consider brane-brane interactions. Perhaps the “near miss” of another 3-brane during the evolution of our universe is responsible for the varying of the fine structure constant? Secondly, one might suppose the 3-brane is charged under bulk Ramond-Ramond fields. On a more phenomenological level, one could consider different couplings to the dilaton and a varying bulk equation of state. Indeed, it is not to hard to see that brane inflation could begin or end because of a change in \(w\). Lastly, the most pressing need is to understand the effect of brane self-gravity. We have argued that this may not be important at early brane times, however it will be vital if one is to turn our proposal into a fully viable cosmological model. One requirement will be that the self-gravity is confined to the brane so as not to violate Newton’s Law.

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