Employment of the Limit-States Method and the Safety Theory in Geotechnical Design

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Abstract: In this paper, a methodology is proposed to apply the Safety Theory in the obtaining of safety factor systems required in the geotechnical design of the foundations using Limit-States Method. The entire mathematical formulation that facilitates the application of the safety theory as well as the integral methodology for the safety factor systems to be used is defined. In addition, a practical example is developed where it is allowed to value the security introduced in the design.

Keywords: Limit-States Method, Safety Factors, Foundation, Safety Theory

1. Introduction

The application of the limit-state method in geotechnical design is current research issue. Geotechnical design using the LSM has been mainly developed by more advanced countries in this field, such as Russia, Denmark, United States and Canada (Becker,1996), (Meyerhof,1970 and 1995). In Cuba, the study of the LSM started out in the mid eighties, and continues up to the present. Its main objective is to establish general methodology for geotechnical design using the limit-state method in the Cuban conditions. The paper present the main results obtained in this field of research in Cuba, like the introduction that has been done on the limit state method in engineering practice.

A general approach for the application of the limit-state method in the Cuban conditions is presented, bringing forward the entire mathematical procedure that supports the limit state method as well as its application to the main problems related to geotechnical design. The safety factor systems used in these designs are determined with the application of the probabilistic methods, safety theory, which are considered worldwide to be the most updated and rigorous from a scientific point of view. The entire mathematical formulation that facilitates the application of the safety theory as well as the integral methodology for the safety factor systems to be used is defined.

Finally, some of the main applications of there results during the last few years are shown. Thus, the economic and technical advantages of the proposed procedures are clearly demonstrated through the quantification of savings.

2. The General Methodology for the Application of the Limit State Method in Geotechnical Designing in Cuba

The general mathematical formulation for the introduction of the limit state method in geotechnical design in Cuba will be defined, which once later allow to establish, with a unique approach, the particular formulations for each one of the different geotechnical designs that are analyzed. As it is known, in the application of the limit-state method, the following limit states designs are defined:

1st limit state - The resistance limit state or stability limit state, known as the ultimate limit state

2nd limit state - The service limit state, which in the case of geotechnical design is converted into the deformation limit state

The design equation for the 1st limit state which will be employed is that which uses a system of three factors of partial safety, such as shown in formula (1).

\[ Y_1^* \leq \frac{Y_2^*}{\gamma_S} \]  

where: \( Y_1^* \) - operating loads function, with their calculation values.

\( Y_2^* \) - bearing loads function, determined with the
calculated values of the physical – mechanical soil properties. $\gamma_s$ - additional safety factor.

Procedure for the application of the factor of safety system

Once the general equation to be used for the application of the limit state method in geotechnical design is defined, the procedure for the introduction of the partial safety factors for the determination of the $Y^{*}_1$ and $Y^{*}_2$ function must be established.

In the case of the operating loads, in the 1st limit state summarized in Figure 1, the calculation values of the loads are determined according to:

$$Y^{*}_i = Y_{ik} \cdot \gamma_i$$  \hspace{1cm} (2)

$$Y^{*}_1 = \sum_{i=1}^{n} Y_{ik} \cdot \gamma_i$$  \hspace{1cm} (3)

where: $Y^{*}_i$ - calculation value of load $i$. $Y_{ik}$ - characteristic value of load $i$. $Y_i$ - mean value of load $i$. $\gamma_i$ - safety factor of load $i$.

From Figure 1 the expression that is used to determine the partial safety factor of the operating loads can be obtained, according to:

$$\gamma_i = \frac{Y^{*}_i}{Y_i}$$  \hspace{1cm} (4)

For the calibration of the safety factor system of to be used by the limit state method, it is necessary to establish procedures for determining the global safety factor $K$ that is introduced in the limit state method with the use of partial factors. The relationship between these partial and global factors can be observed in figure 3 and can be determined using equation (6).

$$K = \gamma_{f} \cdot \gamma_{g} \cdot \gamma_{s}$$  \hspace{1cm} (6)

In order to carry out the comparison between the global factors introduced by the limit state method and the global safety factor method, it is convenient to define the value of the global safety factor, measured from the characteristic values of the operating loads to the mean values of the bearing loads, $K_{km}$, which is determined according to:

$$K_{km} = \gamma_{fc} \cdot \gamma_{g} \cdot \gamma_{s}$$  \hspace{1cm} (7)

3. Formulation of the Limit State Method in Diverse Geotechnical Designs in Cuba

Taking as a basis the general methodologies previously defined, for the application of the limit state method and the safety theory in geotechnical design, diverse formulations of
specific designs have been done (Alvarez, 1998), (González-Cueto 1997 y 2000), (Ibañez, 2001), (Oliva, 1999), (Quevedo, 1987, 1988, and 1989), such as foundations, retaining walls, slopes and piles as a result of these investigations we rely on methodologies established with a unique approach. This fact has made it possible to place Cuba among the countries with more development in this field.

- Foundations.
  
  1st Limit state:
  \[
  N' \leq b' d' \left[ q'_s - q'_w + \frac{q'_v}{\gamma'} \right] \quad (8)
  \]

- Retaining walls.

  Overturning:
  \[
  \sum M'_{\text{Fuerzas desestabilizadas}} \leq \sum M'_{\text{Estabilizadas}} \quad (9)
  \]

  Sliding:
  \[
  \sum F'_{\text{Fuerzas actantes}} \leq \sum F'_{\text{Fuerzas resistentes}} \quad (10)
  \]

- Slopes.

  \[
  \sum M'_{\text{Fuerzas actantes}} \leq \sum M'_{\text{fuerzas resistentes}} \quad (11)
  \]

- Piles.

  Checking the vertical load:
  \[
  N'_{pi} \leq \frac{Q'_{pi}}{\gamma'} \quad (12)
  \]

  Checking the horizontal load:
  \[
  H'_{pi} \leq \frac{Q'_{pi}}{\gamma'} \quad (13)
  \]

4. General Methodology for the Application of the Safety Theory in Geotechnical Design in Cuba

From the different approaches analyzed for the application of the probabilistic methods, known by some authors as the safety theory, a design equation that defines the comparison between the levels of safety obtained in the design \( H_{\text{design}} \) and the level of safety required \( H_{\text{required}} \) would be considered as the starting point.

\[
H_{\text{design}} \geq H_{\text{required}} \quad (14)
\]

General mathematical formulation for the application of the safety theory

Once the basic equation for the application of the safety theory has been defined, it is necessary to develop the completely mathematical foundation on which its practical use is based, its fundamental expressions being the following:

\[
Y = Y_1 - Y_2 \quad (15)
\]

\[
\sigma^2 = \sigma_{Y1}^2 + \sigma_{Y2}^2 \quad (16)
\]

Where: \( \sigma_Y \) - deviation of the resulting function \( Y \)

\( \sigma_{Y1} \) - deviation of the function \( Y_1 \)

\( \sigma_{Y2} \) - deviation of the function \( Y_2 \)

It is possible to demonstrate that the level of safety is determined based on evaluating the integral of Laplace’s function \( \phi_\delta \) between \(-\beta\) and \(+\infty\), as shown:

\[
H = \phi_\delta [-\beta, +\infty] \quad (17)
\]

By considering a normal distribution for the function \( Y \) and its symmetry properties, and knowing that the evaluated distribution between 0 and \(+\infty\) is equal to 0.5, it is possible to define the level of safety as:

\[
H = 0.5 + \phi_\delta [\beta] \quad (18)
\]

\[
\phi_\delta [\beta] = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{\beta^2}{2\sigma^2}} \quad (19)
\]

By doing mathematical transformations with the previous equations and using the concept of additional safety factor \( k \), the equation, which relates the level of safety \( H \) with the global safety factor \( k \), can be obtained:

\[
H = 0.5 + \phi_\delta \left[ \frac{k - 1}{\sqrt{V_{Y1}^2 + k^2 V_{Y2}^2}} \right] \quad (20)
\]

To this end, equation (20) will be the basic equation for the application of the safety theory. Thus, once required security level value \( H_{\text{req}} \) is defined, it is very easy to determine the optimum safety factor, which should be used in the design, see figure 4.

![Figure 4](image.png)

In order to be able to use equation (20), it is necessary to
establish the procedure to obtain the factors for variation of
the functions \( Y_1 \) and \( Y_2 \), which in turn consist of various
random variables.

5. Application of the Safety Theory in the
Calibration of the Safety Factors
System to be Employed in the Design
of Foundations

In order to show the results obtained in the application of
the safety theory in geotechnical design in Cuba, the
procedure followed in the case of foundations in soils that are
predominantly cohesive will be displayed in detail. This is
the first research work on the subject that is done in the
country (Quevedo, 1987) and it is the basis for further
generalization for the solution of similar geotechnical
problems.

For the application of the safety theory, equation (8) has
been simplified, considering that the load eccentricity exists
in only one plane of symmetry, therefore \( l' = l \), and that
the additional safety factor \( \gamma_s \) affects the gross load capacity \( q_{br}^* \).

Based on this, the following design equation is obtained:

\[
N' \leq b \cdot 1 \left[ \frac{q^*_{br}}{\gamma_s} \right] \tag{21}
\]

Next, the general methodology developed for the
application of the safety theory will be described.

1) Definition of the general parameters to be considered in
the analysis

1.1) Statistical characterization of all the random
parameters considered

In the case of geotechnical design the random parameters
considered are operating loads and physical–mechanical soil
properties, coefficients of variation of the internal friction
angle \( \phi \), of the cohesion \( c \), of the specific weight \( \gamma \), of the
dead load \( \gamma_{cm} \), of the live load \( \gamma_{vs} \), and of the extreme wind
load \( \gamma_{wind} \).

1.2) Design of the theoretical experiment

The variables analyzed for the application of the safety
theory are cohesion \( c \), internal friction angle \( \phi \), specific
weight \( \gamma \), load eccentricity \( e \), inclination of the load \( \delta \) and the
depth of the foundation \( d \).

1.3) Definition of the required safety level

In the case of geotechnical design via the 1st limit state, the
\( H_{required} \) to be used in the application of the probabilistic
methods is:

\[ H_{required} = 0.98. \]

II) Determination of the global safety design factor \( K_{design} \)
and the optimum safety factor \( K_{optimum} \).

II.1) Development of the mathematical apparatus

The expressions to determine the partial factors of
operating loads \( \gamma_f \) and of bearing loads \( \gamma_g \) introduced in the
design are:

\[
\gamma_f = \frac{N^*}{(b - 2e*) \ell} \tag{22} \]

\[
\gamma_g = \frac{q^*_{br}}{q_{br}} \tag{23} \]

In this way all the expressions used to determine de \( Y_1 \) \( Y_2 \),
are defined, whilst the global safety design factor \( K_{design} \) is
obtained from the following:

\[
K_{design} = \gamma_f \cdot \gamma_g \cdot \gamma_s \tag{24} \]

In order to determine the factor of optimum safety that
must be introduced in the design \( K_{optimum} \) it is necessary to
initially define the functions \( Y_1 \) and \( Y_2 \).

\[
Y_1 = N \tag{25} \]

\[
Y_2 = Q_{br} = b \cdot 1 \left[ q_{br}^* \right] \tag{26} \]

Thus, obtaining the equation that relates the level of safety
\( H \) to the factor of global safety \( K \), in the case of the design of
foundations using the load capacity is defined by:

\[
H = 0.5 + \phi \left[ \frac{k - 1}{\sqrt{V_N + V_{Qbr} \cdot k^2}} \right] \tag{27} \]

The coefficients of variation of the vertical operating
load \( \gamma_N \) and the load capacity \( \gamma_{Qbr} \) are determined
accordingly:

\[
V_N = \sigma_N N \tag{28} \]

\[
V_{Qbr} = \sigma_{Qbr} Q_{br} \tag{29} \]

where: \( \sigma_N \) - deviation of the vertical operating load.
\( \sigma_{Qbr} \) - deviation of the load capacity in the base of the
foundation

In order to determine \( \sigma_{Qbr} \), \( Q_{br} \) the method of
linearization of the function and the general theorem of
deviation [13], known by as the method of development in
Taylor’s series [3] is applied to the expression for \( Q_{br} \), thus
obtaining:
values of $K$ from: $K$ always greater than the optimum, $K$ therefore the designs are irrational. Factors of safety greater than those required are obtained and application of the partial factors in the limit state method, indicates that, using the proposed procedure for the achievement of the required level of safety $H_{\text{required}}$ which indicates that, using the proposed procedure for the application of the partial factors in the limit state method, factors of safety greater than those required are obtained and therefore the designs are irrational.

III) Calibration of the factors of safety

III.1) Analysis of the influence of the main variables in the values of $K_{\text{design}}$ and $K_{\text{optimum}}$

In this research, the analysis of the influence of all the random variables considered in the values of $K_{\text{design}}$ and $K_{\text{optimum}}$ is done.

In figure 5 the results from the analysis for the influence of the coefficient of variation $v_{\text{tg}}$ is shown, but integrated in that same figure is the variation of $K_{\text{design}}$ and $K_{\text{optimum}}$ for all the combinations analyzed, which allows the comparative assessment of the laws of variation of both parameters in relation to the variable of study.

Furthermore an assessment of the influence of the rest of the variables, the variation coefficient of cohesion $v_{c}$, the eccentricity $e$ and the angle of inclination of the load $\delta$, on the values of $K_{\text{design}}$ and $K_{\text{optimum}}$ was done.

![Figure 5. Relationship of $K_{\text{optimum}}$ and the $K_{\text{design}}$ with $v_{\text{tg}}$.](image)

| Variation coefficients | Variation interval | $K_{\text{design}}$ | $K_{\text{optimum}}$ |
|------------------------|--------------------|----------------------|----------------------|
| $v_{\text{tg}} = 0.07$ | 1.48 – 1.76        | 1.89 – 2.65          | 1.3 – 1.6            |
| $v_{\text{tg}} = 0.138$ | 1.42 – 1.68        | 2.34 – 4.38          | 1.5 – 2.1            |
| $v_{\text{tg}} = 0.20$ | 1.37 – 1.63        | 2.72 – 6.16          | 1.9 – 2.6            |
| $v_{\text{tg}} = 0.26$ | 1.37 – 1.63        | 2.72 – 6.16          | 1.9 – 2.6            |
| $v_{\text{tg}} = 0.336$ | 1.37 – 1.63        | 2.72 – 6.16          | 1.9 – 2.6            |

III.2) Determination of the worst combinations with regard to the relationship $K_{\text{design}}$ and $K_{\text{optimum}}$

The worst combinations obtained are:

- Internal friction angle $\phi = 5^\circ$
- Load eccentricity $e = b/3$
- Load inclination $\delta = 10^\circ$

III.3) Calibration of the safety factor system to be used in the limit state method

This is a genuine engineering process, and by using all the available information from the previous analyses a new safety factor system is established, which will be used for the design of foundations via the limit state method, in such a way that it is guaranteed for all the possible cases of safety required, obtaining for those most critical the fulfillment of $K_{\text{design}} = K_{\text{optimum}}$ and therefore $H_{\text{design}} = H_{\text{required}}$.

| Variation coefficient | Maximum values of $\gamma_{\text{tg}}$ or $\gamma_{c}$ |
|------------------------|-----------------------------------------------|
| $v_{\text{tg}} = 0.20$ | $\gamma_{\text{tg}}\text{max} = 1.20$       |
| $v_{\text{tg}} = 0.26$ | $\gamma_{\text{tg}}\text{max} = 1.40$       |
| $v_{\text{tg}} > 0.20$ | $\gamma_{\text{tg}}\text{max} = 1.25$       |
| $v_{\text{tg}} > 0.26$ | $\gamma_{\text{tg}}\text{max} = 1.45$       |

For foundation design in cohesive soils, via bearing
capacity and applying the limit state method, the results from the calibration is summarized in table # 2.

III.4). Obtainment of new values of $K_{\text{design}}$ with the calibrated factors

With the new safety factor system that has been proposed, all the values of $K_{\text{design}}$ are determined.

### Table #3. Variation of the coefficients $\gamma_f$, $\gamma_g$, $K_{\text{design}}$, $K_{\text{optimum}}$ obtained.

| Variation coefficient | Variation interval | $\gamma_f$ | $\gamma_g$ | $K_{\text{design}}$ | $K_{\text{optimum}}$ |
|-----------------------|--------------------|------------|------------|---------------------|----------------------|
| $v_{\text{opt}} \leq 0.20$ | 1.42 – 1.76 | 1.21 – 1.92 | 1.71 – 3.38 | 1.3 – 2.1 |
| $v_{\text{opt}} > 0.20$ | 1.37 – 1.63 | 1.44 – 2.17 | 1.98 – 3.54 | 1.9 – 2.6 |

An analysis similar to that which was carried out in III.1 was done with the new factors of safety, thus figure 6 shows how these factors vary with respect to those obtained previously without the limitation of the coefficients.

III.5) Assessment of the results and formulation of conclusions

If we analyze comparatively figures 5 and 6, it is possible to see that once the values of factors $\gamma_{\text{opt}}$ and $\gamma_g$ are limited, the $K_{\text{design}}$ reduces, mainly maximum values, while the $K_{\text{optimum}}$ hardly changes, thus obtaining a zone between the $K_{\text{optimum}}$ and $K_{\text{design,min}}$ curves, where the critical combinations for which the factors have been calibrated are sure to be located.

### Table #4. The design results.

| Foundation | I. Proposed LSM | II. Previous LSM | % savings I-II | III. GSFM | % savings I-III |
|------------|----------------|-----------------|---------------|-----------|----------------|
| Internal   | B(m) | $K_{\text{min}}$ | $K_{\text{max}}$ | B(m) | $K_{\text{min}}$ | $K_{\text{max}}$ | B(m) | $K_{\text{min}}$ | $K_{\text{max}}$ |
| External   | B(m) | $K_{\text{min}}$ | $K_{\text{max}}$ | B(m) | $K_{\text{min}}$ | $K_{\text{max}}$ | B(m) | $K_{\text{min}}$ | $K_{\text{max}}$ |

| Cohesive soils. ($c \neq 0 ; \phi \leq 25^\circ$) |

| Applications | Limit state/ Level of safety | Factors $\gamma_{\text{opt}}$ or $\gamma_{\text{design}}$ | $V \leq 0.26$ | $V > 0.26$ | $v_{\text{opt}} \leq 0.20$ | $V_{\text{opt}} > 0.20$ |
|--------------|-----------------------------|---------------------------------|-----------------|-----------------|---------------------|-----------------|
| Foundations. Slopes. | 1st Limit state/H=0.98 | $\gamma_{\text{opt}}=1.40$ | $\gamma_{\text{design}}=1.45$ | $\gamma_{\text{opt}}=1.40$ | $\gamma_{\text{design}}=1.45$ | $\gamma_{\text{opt}}=1.40$ | $\gamma_{\text{design}}=1.45$ |
| Retaining walls. | 2nd Limit state/H=0.85 | $\gamma_{\text{opt}}=1.30$ | $\gamma_{\text{design}}=1.35$ | $\gamma_{\text{opt}}=1.30$ | $\gamma_{\text{design}}=1.35$ | $\gamma_{\text{opt}}=1.30$ | $\gamma_{\text{design}}=1.35$ |
| Piles. | 3rd Limit state/H=0.85 | $\gamma_{\text{opt}}=1.20$ | $\gamma_{\text{design}}=1.25$ | $\gamma_{\text{opt}}=1.20$ | $\gamma_{\text{design}}=1.25$ | $\gamma_{\text{opt}}=1.20$ | $\gamma_{\text{design}}=1.25$ |

From the results obtained a number of conclusions that confirm several of the hypotheses considered in the present task can be drawn, the main ones being the following:

In interior foundations, which work at axial loading with eccentricity zero, using the proposed limit state method the factors of safety are required to be lower than those for the exterior foundations which have eccentric loads.

The rationality of the proposed limit state method in relation to the global safety factor method which has been used frequently the engineering practice is demonstrated, obtaining savings in terms of direct cost of the foundation ranging from 23 – 38%, depending on the type of foundation and the type of soil.

Safety Factors System for geotechnical design using the limit state method in Cuba

Following the methodology developed for the application of the probabilistic methods with the objective of obtaining the safety factors system to be used in geotechnical design using the limit state method, a group of research work have been carried out in Cuba, (Alvarez, 1998), (González-Cueto.
1997 y 2000), (Ibañez, 2001), (Oliva, 1999), (Quevedo, 1987, 1988, 1989 and 2000), which included the study of the following problems:
- Foundations
- Retaining walls
- Slopes
- Piles

Table # 5 shows the unified safety factors system, for cohesive soils, likewise a table was also prepared for frictional soils to be used in geotechnical design via the limit state method.

6. Conclusions

1. Based on the international experience and the results from research, a proposed of a general procedure for the introduction of the limit state method in geotechnical design in Cuba is made. This proposal is characterized by the use of a system of three factors of partial safety, one related to the acting loads, the other to the physical–mechanical soil characteristics and the third to additional safety, from which the general mathematical apparatus which supports its rational application in any geotechnical design problem is defined.

2. The general mathematical apparatus is established for the application of the safety theory and probabilistic methods, thus proposing a solution for various problems that are not addressed in the international literature, such as the analysis of functions with correlated random variables. For the first time the coincidence of approaches followed by the two main schools the western and the Russian, on the subject is demonstrated when it comes to applying probabilistic methods. The mathematical apparatus created can be used for any geotechnical problem.

3. A general methodology is proposed for the application of the probabilistic methods in geotechnical design, which is characterized by the creative combination of the mathematical precision of this method with the engineering solutions, in such a way that the results obtained can be introduced easily in practice.

4. Based on this research and on the assessment of the international experience, it is possible to summarize the statistic characterization of the main random variables that take part in geotechnical design, the operating loads and the physical-mechanical soil properties, which facilitate the application of the probabilistic methods in any geotechnical problem to be tackled.

5. Based on the results obtained from the application of the safety theory in the determination of the factor of safety system of various geotechnical problems, foundations, walls, slopes and piles, the practical effectiveness of the general proposed methodology is demonstrated, stating furthermore general factors of safety system, distinctive for cohesive and frictional soils, applicable to the geotechnical design of foundations using both limit states of any of the problems under study.

6. Using the results obtained from the application of the general methodology for the geotechnical design of foundations, its technical-economical effectiveness is demonstrated, the influence of each one of the new aspects introduced in the design thus quantified obtaining savings ranging from 15 ~ 40 % for direct cost of the foundation, depending on the procedure used in the design with which it is compared and the peculiarities of the soils and the operating loads of the structure under study.

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