Dodgeball—Can a Satellite Avoid Being Hit by Debris?

J. I. Katz
Department of Physics and McDonnell Center for the Space Sciences
Washington University, St. Louis, Mo. 63130

October 8, 2021

Abstract

Can a satellite dodge a collision with untracked orbiting debris? Can a satellite dodge collision with a tracked object, making only the avoidance manoeuvres actually required to avoid collision, despite the uncertainties of predicted conjunctions? Satellite-borne radar may distinguish actual collision threats from the much greater number of near misses because an object on a collision course has constant bearing, which may be determined by interferometric detection of the radar return. A large constellation of such radars may enable the determination of the ephemerides of all cm-sized debris in LEO.

1 Introduction

Large constellations of satellites are planned to be launched in the coming years, many to provide world-wide internet access, with cumulative totals of 100,000 or more. As the number of satellites and orbital debris increases, so does the threat collisions with other orbiting objects. Collisions not only damage or destroy the colliding objects, but also create multiple pieces of debris, increasing future hazards.

How may a satellite may manoeuvre to avoid an imminent collision? Predicted conjunctions, approaches closer than the orbital uncertainties risk collision, are comparatively frequent because these uncertainties are significant.
Avoiding all conjunctions would require a significant expenditure of manoeuvring fuel, growing as the number of orbiting objects grows. Is it possible to manoeuvre in the last-minute (or second) to dodge the rare objects that would actually collide with a satellite, while ignoring the large majority of those with predicted conjunctions that are not actually on a collision course? Is it possible to dodge collision with objects too small to be tracked from the ground, and only detected when they approach a satellite?

Approaching objects may be detected by radar. If detected at sufficient range and determined to be on a collision course, collision may be dodged by a displacement transverse to the relative velocity vector. The tiny fraction of nearby objects actually on collision courses have constant bearing, that may be measured interferometrically by comparing the phases of radar returns received by outriggers a short distance from the protected satellite. The combination of radar and aperture synthesis to determine if a scatterer has constant bearing provides a powerful method of distinguishing actual collision threats from harmless close approaches.

\section{Detection Ranges}

There are two distinct classes of threats:

1. Debris too small to be tracked individually;
2. Bodies that are tracked individually, but whose predicted conjunctions are uncertain enough that nearly all would be near-misses, whose avoidance would burden a satellite’s manoeuvring resources.

\subsection{Untracked Debris}

Untracked debris can arrive from almost any direction, as viewed from the satellite we wish to protect. For prograde orbits there will be little from the direction of motion, but, because of the broad distribution of orbital inclinations of threat objects, that information does not greatly limit the approach direction of threats.

Consider a satellite equipped with a radar that radiates isotropically. This cannot be achieved with a single dipole, whose antenna pattern has nulls, but can be reasonably approximated with two or three mutually orthogonal dipoles. The approximation of isotropic radiation provides an estimate of
what is possible. If the radar emits an energy $E$, in one or many pulses or
in a modulated continuous transmission, a dipole receiver with effective area
$\lambda^2/2\pi$ will receive, from an isotropic scatterer with total radar-scattering
cross-section $\sigma$ at range $R$, the returned energy

$$E_{\text{ret}} = \frac{\sigma \lambda^2}{(4\pi R^2)^2} \frac{E}{2\pi} \approx 9 \times 10^{-19} \frac{\sigma}{10^2 \text{cm}^2} \left(\frac{1 \text{ km}}{R}\right)^4 \left(\frac{\lambda}{30 \text{ cm}}\right)^2 E, \quad (1)$$

A high gain antenna would, of course, receive more energy, but this is pre-
cluded by the requirement of isotropic sensitivity—the arrival direction of an
untracked threat is unknown. We have scaled $\lambda$ to 30 cm (1 GHz) because
objects smaller than 10 cm would have $\sigma \ll 10^2 \text{cm}^2$ for waves of this wave-
length, while larger objects are likely to be tracked. Impact with a 10 cm
object, such as would have a radar cross-section $\mathcal{O}(10^2) \text{cm}^2$, is likely to be
catastrophic and must be avoided.

For a receiver with noise temperature $T$ and a signal with matched pulse
width $\tau$ and bandwidth $\Delta \nu = 1/\tau$, the signal to noise ratio is

$$\frac{S}{N} = \frac{E_{\text{ret}}}{k_B T} \approx 28 \frac{E}{1 \text{ J}} \left(\frac{\sigma}{10^2 \text{ cm}^2}\right) \frac{30 \text{ K}}{T} \left(\frac{3 \text{ km}}{R}\right)^4 \left(\frac{\lambda}{30 \text{ cm}}\right)^2, \quad (2)$$

where $k_B$ is Boltzmann’s constant. Eq. (2) may be inverted to give the de-
tectability range

$$R_{\text{detect}} = \left(\frac{E \sigma}{k_B T (S/N)}\right)^{1/4} \frac{\lambda^{1/2}}{2^{5/4} \pi^{3/4}}$$

$$= 3.9 \left(\frac{E}{1 \text{ J}} \frac{\sigma}{10^2 \text{ cm}^2} \frac{10}{S/N} \frac{30 \text{ K}}{T}\right)^{1/4} \left(\frac{\lambda}{30 \text{ cm}}\right)^{1/2} \text{km}. \quad (3)$$

$R_{\text{detect}}$ is determined by the assumed pulse energy $E$, the receiver noise
temperature $T$, the cross-section $\sigma$ and the radar wavelength $\lambda$. $T$ is set by
the state of the electronic art, but the other parameters are design choices:
$\lambda$ and $\sigma$ set by the design goal ($\lambda \lesssim \sqrt{2\pi \sigma}$, where the scatterer is described
by an equivalent sphere of radius $a$, $\sigma \approx 2\pi a^2$ and $\lambda \lesssim 2\pi a$ is required for
efficient scattering; [1, 2]) and $E$ by the power available and the radar pulse
repetition frequency (PRF; itself set by the minimum acceptable $R_{\text{detect}}$).

Determination of whether an object is on a collision course requires
detection of two radar returns (Sec. 3). For a nominal relative velocity
$V_0 = 10 \text{ km/s}$, appropriate to LEO, and $R_{\text{detect}} \approx 3 \text{ km}$ (Eq. 3) PRF =

3
1 km/$V_0 = 10$/s and the mean radar power $P = E \times \text{PRF} = 10$ W, probably about the maximum acceptable for an add-on system to proliferated internet satellites.

A collision warning might be issued at a range of $R_{\text{warn}} = 2$ km, providing a time $\delta t = R_{\text{warn}}/V_0 = 0.2$ s for evasion. To displace the satellite by $\delta x = 1$ m (a nominal satellite size) in that time would require an acceleration $2\delta x/(\delta t)^2 \approx 5g$, likely infeasible. Because $R_{\text{detect}}$ and $\delta t$ scale $\propto (E\sigma)^{1/4}$, the acceleration scales $\propto (E\sigma)^{-1/2}$. This insensitivity means that it is not possible to dodge untracked objects with $\sigma = \mathcal{O}(10^2 \text{ cm}^2)$. Objects with larger radar cross-sections will be tracked in the foreseeable future [3, 4].

The prospect of dodging in GEO is more favorable, although the smaller density of objects there reduces the risk of collision and may make dodging unnecessary. In GEO $V_0 \approx 1$ km/s because the orbital velocities are less and the range of inclinations is less. This multiplies $\delta t$ tenfold and reduces the required acceleration to $\approx 0.05g$, which is likely to be feasible, even if produced by a point thruster.

### 2.2 Tracked Objects

The problem is easier if the threat objects are large enough that they are individually tracked and their ephemerides are known. These are not known accurately enough to predict conjunctions to within the dimensions of satellites, so avoidance maneuvers are made whenever there is any risk of collision, even though it be very small. The system proposed here could be used to render nearly all of these maneuvers unnecessary, provided a last-chance maneuver is possible in the rare cases when it is necessary. This would reduce the expenditure of propellant on maneuver by a large factor, as well as reducing any disruption of the satellite’s mission by maneuver.

Rather than continually searching all nearby space for possible threat objects, requiring transmission of $\sim 10$ radar pulses per second, it would only be necessary to resolve the rare predicted conjunctions. More energetic radar pulses would be possible, extending the range of detection, because threatened conjunctions are rare and infrequent. The energy $E$ of a radar pulse would be limited only by capacitive energy storage, not by the mean radiated power.

Tracked objects approach a conjunction from a known direction. That permits use of high gain antennae for both transmission and reception, and detection at much greater ranges. As a nominal example, take a radar fre-
frequency of 22 GHz (\(\lambda = 1.36 \text{ cm}\)); at this frequency a dish 40 cm (30\(\lambda\)) in diameter has an area \(A = 1.3 \times 10^3 \text{ cm}^2\) and a gain \(G = 4\pi A/\lambda^2 \approx 40 \text{ dB}\) in the center of its \(2^\circ\) (0.03 radian, \(10^{-3}\) sterad) beam. It is compact and light enough for inclusion as an add-on system on a satellite, without requiring great precision of figure (\(\lambda/20 = 0.7 \text{ mm}\)). It can also be rapidly slewed, although with conjunctions predicted hours or days in advance this is unlikely to be necessary.

22 GHz is in the middle of an atmospheric water vapor absorption band, with typical (but dependent on location, weather and season) attenuation of 20 dB from the zenith to the ground. Attenuation minimizes any interference with other systems, partly by attenuating the the radar radiation and partly because atmospheric attenuation means that this band is little used. Interference might become an issue if a large number of communications satellites were equipped with collision avoidance systems, both for their own immediate protection and to avoid multiplication of secondary debris, although if radars are only turned on when conjunctions are predicted, infrequent events, this is not expected to be an issue.

An object tracked from the ground is likely to have a scattering cross-section \(\sigma \gtrsim 10^2 \text{ cm}^2\), and a satellite (rather than a debris object) is likely to have \(\sigma \sim 10^4 \text{ cm}^2\). Replacing the dipole effective area \(\lambda^2/2\pi\) by a dish of area \(A\), Eq. 3 becomes

\[
R_{\text{detect}} = \left( \frac{\sigma A G}{16\pi^2 (S/N) k_B T} \right)^{1/4}
\]

\[
\approx 60 \left( \frac{\sigma}{10^2 \text{ cm}^2} \frac{A}{10^3 \text{ cm}^2} \frac{E}{1 \text{ J}} \frac{10}{S/N} \frac{30 \text{ K}}{T} \frac{40 \text{ dB}}{G} \right)^{1/4} \text{ km.}
\]

If the factors in parentheses are unity, \(\delta t \approx 6\text{ s}\) and the acceleration required for a displacement \(\delta x = 1\text{ m}\) would be \(\approx 0.005g\), a quite modest value. Over such times and distances the gravitational acceleration must be taken into account, but this is not an obstacle in principle.

If the antenna is correctly pointed when the target is first within the range \(R_{\text{detect}}\) (Eq. 4), then comparison of the amplitudes detected by multiple antennae (as discussed in Sec. 3, the proposed system requires at least three antennae) can be used to refine the pointing and track the target.

A large conjunction uncertainty \(C \gg 1\text{ km}\) would imply that the 40 dB gain achievable with a 40 cm dish at 22 GHz would be useless—it would not be possible to point the beam with sufficient accuracy at the detection range.
Eq. 4 to illuminate the target. In this case deliberate defocusing would be necessary to ensure that the target would be within the beam. Then the best possible gain would be $G \approx 4\pi (R_{\text{detect}}/C)^2$ and Eq. 4 would become

$$R_{\text{detect}} = \sqrt{\frac{\sigma A E}{4\pi C^2 (S/N) k_B T}}$$

$$\approx 140 \sqrt{\frac{\sigma}{10^2 \text{cm}^2} \frac{A}{10^3 \text{cm}^2} \frac{E}{1 \text{ J}} \frac{10}{S/N} \frac{30 \text{ K}}{T} \left( \frac{1 \text{ km}}{C} \right)} \text{ km.}$$

(5)

3 Will it Collide?

Is an object a collision threat? Radar provides its range and closing speed, but more is required; most tracked objects that come within a conjunction uncertainty $C$ will not collide, nor will most untracked objects within the range of Eq. 3. As pilots and sailors know, an object on a collision course keeps a constant bearing as it approaches. A single receiver cannot determine bearing, but a minimal form of aperture synthesis can. The relative phases among three non-colinear receivers determine bearing in the two angles required to describe the direction to the scatterer. Absolute bearing is required, so any relative motion (such as rotation around the satellite, if they are deployed centrifugally) among the receivers must be compensated; in general, this motion will be known or determined from the phases of a distant source.

It is not necessary to determine the scatterer’s actual bearing, but only to determine if it is changing. If changing, it is not a collision threat. For this purpose, the receiving system must comprise a minimum of three non-colinear receivers. Radar returns from the receivers are compared coherently—this is a minimal synthesized aperture or interferometer, with two visibility functions.

Consider (Fig. 1) a satellite $S$ threatened by an object, perhaps debris, $D$. The satellite has two colinear outrigger radar receivers at 1 and 2, each a distance $r$ away. The threat object is at a bearing $\theta$ from the normal to the line connecting the outriggers and satellite. Radar signals emitted from the satellite are scattered by the threat object, with path lengths $R_1$ and $R_2$ to the receivers. The phase difference between signals at the two receivers

$$\Delta \phi_{12} = \frac{2\pi}{\lambda} (R_1 - R_2) \approx \frac{4\pi}{\lambda} r \sin \theta,$$  

(6)
Figure 1: Path difference between the returns of a radar signal emitted from a satellite S and received from a threat object D by two outrigger receivers 1 and 2. The time derivative of the phase difference (Eq. 7) is non-zero unless $\cos \theta = 0$, a possibility dealt with by providing a third receiver not colinear with the first two, or $\dot{\theta} = 0$, a collision course.

where $\lambda$ is the radar wavelength and $\theta$ the angle between the line from satellite to the threat and the line connecting the outriggers and the satellite. This result is accurate to first order in $r/R$.

If the threat object is not on a collision course then $\theta$ is changing, as is the phase difference:

$$\Delta \dot{\phi}_{12} = \frac{4\pi}{\lambda} r \dot{\theta} \cos \theta.$$  

(7)

$\Delta \dot{\phi}_{12}$ is nonzero if the object is not on a collision course. It is zero if either the object is on a collision course or if $\cos \theta = 0$. The latter case is dealt with by having a third receiver, not colinear with the first two. Then the third receiver and the satellite will not be colinear with either of the first two receivers (colinear geometry was illustrated only to ease visualization). The three receivers might all be coplanar with the satellite, perhaps on centrifugally extended booms or wires, equidistant and equally spaced in angle; this is not
necessary, but may be optimal.

Fig. 2 (for simplicity, coplanar geometry has been chosen for this illustration) shows the threat object’s trajectory if the miss distance is \( m \). This is related to the range \( R \) and the angles

\[
m = R \sin \left( \gamma + \theta - \frac{\pi}{2} \right),
\]

where \( \theta \) has the same meaning as in Fig. 1 and \( \gamma \) is the angle between the threat’s path and the line joining the receivers. Differentiating this equation with respect to time (\( m \) is constant) we find

\[
\dot{\theta} = \frac{1}{R} \frac{dR}{dt} \cot (\gamma + \theta),
\]

where \( dR/dt \approx V_0 \), the threat’s velocity in the frame of the satellite, to first order in \( m/R \ll 1 \). \( R \) and \( dR/dt \) are directly obtainable from the radar return.

Combining Eqs. 7 and 9, the rate of change of the phase difference between the two receivers

\[
\dot{\Delta \phi_{12}} = \frac{4\pi r}{\lambda} \frac{dR}{dt} \cos \theta \cot (\gamma + \theta) \approx \frac{r}{R} \frac{4\pi}{\lambda} m V_0 \cos \theta,
\]

because for a distant threat object \( m \ll R \) \( \gamma + \theta \approx \pi/2 \) and \( \cot (\gamma + \theta) \approx m/R \). Over a time interval \( \Delta t \) (between the returns of two radar pulses) the phase difference

\[
\Delta \phi_{12} \approx \frac{4\pi r}{\lambda} \frac{dR}{dt} \cos \theta \cot (\gamma + \theta) \Delta t \approx \frac{4\pi r}{\lambda} \frac{m}{R} V_0 \cos \theta \Delta t.
\]

Numerically, for a miss distance \( m \leq 3 \) m (corresponding to a likely impact on a nominal 3 m satellite), outriggers at \( r = 3 \) m, relative velocity \( V_0 = 10 \) km/s, range \( R = 3 \) km, \( \lambda = 1.36 \) cm (22 GHz), \( \cos \theta = \mathcal{O}(1) \) and \( \Delta t = 0.1 \) s

\[
\Delta \phi_{12} \sim \Delta \phi \Delta t \sim 10 \text{ rad}.
\]

This is readily detectable even with a modest signal to noise ratio. Threat objects with larger \( m \) that would miss, would have larger \( \Delta \phi_{12} \) and could be distinguished from actual collision risks.

Eq. 12 is the phase difference in the returns from an object that risks striking the satellite. If \( \Delta \phi_{12} \) is larger, then, for the assumed vulnerable
Figure 2: Encounter geometry, with threat object at range $R$ and miss distance $m$, simplified to the coplanar case. The horizontal line joins the receivers and the satellite, as in Fig. 1 and the threat object’s trajectory is $AB$. 
radius of $r_{sat} = 3 \text{ m}$, we know that the satellite will not be struck. Only $\mathcal{O}(r_{sat}/R_{detect})^2 \sim 10^{-6}$ of the objects entering a 3 km range of detectability really threaten collision. The remaining 99.9999% have $m > 3 \text{ m}$ and $\Delta \phi_{12} > 10$ radians and are excluded as threats by measurement of their $\Delta \phi_{12}$. For the very large $R_{detect}$ of Eq. 4, $R_{detect}$ is replaced by the conjunction uncertainty $C \gg 1 \text{ m}$, and the fraction that would actually collide is $\mathcal{O}(3 \text{ m}/C)^2 \ll 1$. The problem of objects with small $\cos \theta$ is solved by having three (or more) non-colinear outriggers; $\cos \theta$ will not be small for some pairs of outriggers.

4 Required $\Delta V$

To dodge a tracked object on a collision course detected at range $R_{detect}$ (Eq. 4) requires a velocity increment

$$\Delta V_{dodge} = \frac{r_{sat} V_0}{R_{detect}},$$

where the necessary displacement is $r_{sat}$, taken as the satellite’s (including Solar panels and other appendages) radius.

The alternative is to displace the satellite by the conjunction uncertainty $C$ each time an object is predicted to come within that distance. Transverse $\Delta V$ must produce the required displacement in the time $P_{orb}/2\pi$ because transverse perturbations about a circular orbit are simple harmonic motion with the orbital period. It is more effective to apply $\Delta V$ along the satellite orbit because there is no restoring force in that direction; a parallel velocity increment changes the orbital period and the displacement accumulates secularly over the warning time $t_{CA}$. A velocity increment

$$\Delta V_C = C/t_{CA}$$

is sufficient to displace the satellite from the conjunction uncertainty region; it need only be displaced in one dimension to avoid collision. Displacement along-track can suffice, even if the orbits remain intersecting, by changing the time at which the satellite passes through the intersection.

For each predicted conjunction that would actually lead to a collision there are about $(C/r_{sat})^2$ near misses, and the total of the velocity increments

$$\Delta V_{tot} = C/t_{CA} \left( \frac{C}{r_{sat}} \right)^2.$$
The ratio of these two $\Delta V$ is the comparative cost CC of dodging only those predicted conjunctions that would actually lead to collision:

$$CC = \frac{\Delta V_{dodge}}{\Delta V_{Ctot}} = \left(\frac{r_{sat}}{C}\right)^3 \frac{V_0 t_{CA}}{R_{detect}} \approx \begin{cases} 1 \times 10^{-3} & \text{LEO} \\ 3 \times 10^{-6} & \text{GEO} \end{cases}$$

where the numerical results assume $r_{sat} = 3 \text{m}$, $R_{detect} = 60 \text{km}$ (Eq. 4), $V_0 = 10 \text{km/s}$ in LEO and $V_0 = 1 \text{km/s}$ in GEO, $C = 1 \text{km}$ in LEO and $C = 5 \text{km}$ in GEO and $t_{CA} = 3 \text{days}$ in LEO and $t_{CA} = 10 \text{days}$ in GEO [5].

If the conjunction uncertainty is anisotropic (larger uncertainties in the along-orbit direction might be expected because errors in the orbital period produce errors in true anomaly that accumulate in time, just as does the displacement resulting from along-orbit $\Delta V$), $C$ is a tensor. It is likely to have a much greater principal axis along the threat’s orbit.

The actual ratios of $\Delta V$ costs are even smaller than those of Eq. 16. When the satellite’s and threat’s orbits intersect at an angle $\phi \ll 1$ the length of the conjunction uncertainty volume in the direction parallel to the orbit in Eq. 14 is $C \csc \phi$. If the orbits have nearly the same speed (as for conjunctions between nearly circular orbits), the $V_0$ in Eq. 13 is multiplied by $\sin \phi$. For these near-parallel conjunctions $\Delta V_{dodge}/\Delta V_C$ is multiplied by $\sin^2 \phi$. For such near-parallel conjunctions it may be necessary to use transverse $\Delta V$, in which case $t_{CA}$ is replaced by $P_{orb}/2\pi$ in Eq. 14. The effect is to reduce CC further.

Unless $C$ is much reduced, last-minute (or last-second) dodging demands much less propulsion than avoiding all predicted conjunctions because only a tiny fraction of predicted conjunctions actually need to be dodged.

The extreme ratios of Eq. 16 are only applicable if threat objects are numerous enough, and the satellite is in orbit long enough, that the probability of a collision that must be dodged is $\geq O(1)$, so that $\geq O(C/r_{sat})^2$ predicted conjunctions that would actually be near misses must be avoided. In the present environment that is not so; rather, the probability that any collisions would occur, in the absence of avoidance, is very small and the number of predicted conjunctions is $\ll (C/r_{sat})^2$. CC remains a figure of merit of a Dodgeball system, but has a different significance.
5 Survey of Small Debris

The short wavelength radar system described in Sec. 2.2 can provide an in situ measure of the density of debris too small to track individually from the ground, but large enough to pose a significant threat to satellites. 22 GHz waves have a scattering cross-section \( \sigma \approx \frac{2 \pi a^2}{\lambda^2 / 2 \pi} \approx 0.3 \text{ cm}^2 \) for objects of equivalent radius \( a \approx \frac{\lambda}{2 \pi} \approx 2 \text{ mm} \) \cite{1, 2}. In lower LEO these may be detectable by powerful ground-based radars, but in higher orbits they can only be detected in situ, either by radar or by observation of impact damage.

5.1 Survey Rate

Eq. 4 provides an estimate of the maximum detection range. The beam has a solid angle \( \Omega \approx 4 \pi / G \text{ sterad} \). The volume surveyed in one pulse

\[
V \approx \frac{4 \pi}{3G} R_{\text{detect}}^3 \approx \frac{4 \pi}{3} \left( \frac{\sigma A}{16 \pi^2 (S/N) k_B T} \right)^{3/4} G^{-1/4}
\]

\[
\approx 3 \left( \frac{\sigma}{1 \text{ cm}^2} \left( \frac{A}{10^3 \text{ cm}^2} \right) \frac{E}{10 \text{ J}} \frac{30 \text{ K}}{T} \right)^{3/4} \left( \frac{10^4}{G} \right)^{1/4} \text{ km}^3.
\]

For the nominal parameters (in parentheses) of Eq. 17, a single pulse would detect scatterers with density > 1 km\(^{-3}\). In survey mode (slewing the beam between pulses by an angle \( \geq \Omega^{1/2} \) to avoid overlap, although objects with a relative speed of \( V_0 \) move through the beam in a few tenths of a second) gives a volume sweep rate

\[
SW = PRF \times V.
\]

With PRF = 10/s and the nominal parameters of Eq. 17, \( SW \approx 30 \text{ km}^3 / \text{s} \).

In a year a single radar would sweep a volume of \( 10^9 \text{ km}^3 \) and detect objects with a spatial density \( \gtrsim 10^{-9} \text{ km}^{-3} \). The minimum detected cross-section \( \sigma = 1 \text{ cm}^2 \) corresponds to a radius \( a \approx 0.4 \text{ cm} \). Smaller particles would be detected only if closer, but their density is expected to be higher, compensating for the smaller \( V \) and \( SW \); their detection is inefficient if \( a \ll \lambda / 2 \pi \) because then \( \sigma \ll \pi a^2 \), which effectively limits the survey to objects with \( a \gtrsim \lambda / 2 \pi \).

5.2 Particle Properties

Combining the measured \( E_{\text{ret}} \) and the range \( R \) measured by the delay of the radar return, Eq. 11 permits inference of the scattering cross-section \( \sigma \), and
hence of the size of the scatterer \( a \approx \sqrt{\sigma / 2\pi} \).

Correlation of the results of \textit{in situ} radar observations with impact damage can provide additional diagnostic information. Radar cross-sections are largely determined by the greatest linear dimension of an object, while damage is determined by its kinetic energy (in the frame of the damaged satellite), and hence by its mass.

### 5.3 Ephemerides of Small Debris

The range to the scatterer would be determined accurately (to < 1 cm) but its location in the two cross-beam dimensions only to \( R_{\text{detect}}/G^{1/2} \approx 0.6 \text{ km} \) (for the nominal parameters). With radars operating from a large number of satellites it might be possible, using the range measurements and accurate satellite coordinates determined from GPS to construct accurate ephemerides of even these cm-size scatterers. With radars on \( 10^5 \) satellites in future constellations, the entire \( 3 \times 10^{11} \text{ km}^3 \) of LEO space (altitudes 400–1400 km) could be swept in about a day. If the PRF is increased, with a corresponding decrease in \( E \) so that the mean radiated power is held constant, \( SW \) increases as \( \text{PRF}^{1/4} \).

Eq. 17 shows that survey with a broad angle emitter (like a dipole) with \( G \approx 1 \) would be ten times faster than for the assumed \( G = 40 \text{ dB} \). This would require an additional transmitter and receiver, distinct from the high gain system used to predict collision threats, but would reduce the survey time to a few hours.

In principle, six range measurements from satellites of known location are sufficient to determine a scatterer’s Newtonian orbit, although this is more complex for the non-Keplerian orbits of the Earth’s distorted gravitational field. Radar provides not only a range measurement, but, if the pulse is many cycles long, also a Doppler measurement of radial velocity with respect to the satellite. Unfolding orbits from such data, requiring association of different detections of the same scatterer separated in time when a very large number of scatterers are observed, is computationally challenging, but may be possible in principle.
6 Conclusion

It is not possible to dodge small untracked objects with radar cross-sections $\sigma \lesssim 10^2 \text{cm}^2$ with which conjunctions are not predictable and that may approach from any direction. It is possible, with feasible maneuvers, to dodge larger tracked objects with $\sigma \gtrsim 10^2 \text{cm}^2$ that have predictable conjunctions.

Search radars on contemplated constellations of $10^5$ satellites may enable the determination of the ephemerides of all cm-sized debris in LEO space. This task would be formidable, including computationally, but is not precluded.

7 Acknowledgment

I thank P. Dimotakis, D. Finkbeiner, J. Goodman, K. Pister, C. Stubbs and J. Tonry for useful discussions.

References

[1] Born, M. B. and Wolf, E. *Principles of Optics* 3rd ed. (Pergamon, Oxford 1965).

[2] van de Hulst, H. C. *Light Scattering by Small Particles* (Dover, New York 1981).

[3] http://www.peterson.spaceforce.mil accessed August 16, 2021.

[4] http://www.leolabs.space accessed August 16, 2021.

[5] *Spaceflight Safety Handbook for Satellite Operators* Version 1.5 August 2020 http://www.space-track.org