Nonlinear models for describing lettuce growth in autumn-winter

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ABSTRACT: The objectives of this study were to fit the Gompertz and Logistic models for the fresh and dry matter of leaves and the fresh and dry matter of shoots of three lettuce cultivars and indicate the best model to describe their growth in autumn-winter. The lettuce cultivars Gloriosa, Pira Verde, and Stella were evaluated in the autumn-winter of 2016 and 2017, in soilless in a protected environment. After transplantation, the fresh and dry matter of leaves and shoots were weighed every seven days. These dependent variables were fit using the accumulated thermal sum. The parameters of the Gompertz and Logistic models were estimated, the assumptions of the models were verified, the indicators of fit quality and critical points were calculated and the parametric and intrinsic curvature measures quantified. The Logistic and Gompertz models presented a satisfactory adjustment for the fresh and dry matter of leaves and the fresh and dry matter of shoots, for the lettuce cultivars Gloriosa, Pira Verde and Stella, in autumn-winter. The Logistic model best describes the growth of the lettuce cultivars.

Key words: dry matter, fresh matter, Gompertz, Lactuca sativa L., Logistic.

INTRODUCTION

Lettuce (Lactuca sativa L.) is a temperate leafy green vegetable (SALA & COSTA, 2012). Its leaves are consumed as raw salads, soups, and creams, and is a source of dietary fibers, vitamins, and minerals (NTSOANE et al., 2016). It is the main leafy green vegetable sold and consumed in Brazil, mainly because of its ease of production and acquisition.

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RESUMO: Os objetivos deste trabalho foram ajustar os modelos Gompertz e Logístico para as massas de matéria fresca e seca de folhas, e as massas de matéria fresca e seca de parte aérea de três cultivares de alface e indicar o modelo que melhor descreve o crescimento no outono-inverno. As cultivares de alface Gloriosa, Pira Verde e Stella, foram avaliadas no outono-inverno de 2016 e outono-inverno de 2017, em cultivo sem solo em ambiente protegido. Após o transplante, a cada sete dias, foram pesadas as massas de matéria fresca e seca de folhas e e as massas de matéria fresca e seca de parte aérea. Essas variáveis dependentes foram ajustadas em função da soma térmica acumulada. Foram estimados os parâmetros dos modelos Gompertz e Logístico, verificados os pressupostos dos modelos, calculados os indicadores de qualidade do ajuste e os pontos críticos e quantificadas as medidas de curvatura intrínseca e de parametrização. Os modelos Logístico e Gompertz apresentam ajuste satisfatório para as massas de matéria fresca e seca de folhas e para as massas de matéria fresca e seca de parte aérea, para as cultivares de alface Gloriosa, Pira Verde e Stella, no outono-inverno. O modelo Logístico é o que melhor descreve o crescimento das cultivares de alface.

Palavras-chave: Gompertz, Lactuca sativa L., Logístico, massa de matéria fresca, massa de matéria seca.
season to the growth of the crop is winter, because in this period temperatures vary between -3 to 18 °C (ALVARES et al., 2013).

One way to characterize plant growth is via modeling (STRECK et al., 2008). Adjusting growth models to plant species helps in the evaluation of plant response to environmental conditions, as well as understanding its growth pattern (LYRA et al., 2003). Growth models using the accumulated thermal sum allow to make inferences on precocity, velocity and stabilization of the plant growth through the interpretation of parameters and critical points of the adjusted model curve (MISCHAN & PINHO, 2014).

The accumulated thermal sum is a biological time measure in plants, being more accurate than days in the civil calendar or days after sowing/transplant (GILMORE & ROGERS, 1958; MCMASTER & SMIKA, 1988). The use of accumulated thermal sum as elapsed time of the crop cycle assumes a linear relation between growth or plant development and temperature (BONHOMME, 2000). However, this would not be realistic from the biological point of view, since the plant growth in response to the thermal accumulation is nonlinear. Therefore, nonlinear models are more often used to describe the growth of plants, generally, faster in its initial phase, then decreasing its speed and, finally, tending to a stability in the adult phase (PAINE et al., 2012; MISCHAN & PINHO, 2014).

Mathematical models must be able to reproduce the plants behavior as closest as possible to the real. The adjustment of nonlinear models have been applied to describe the growth of Allium sativum L. (PUIATTI et al., 2013; REIS et al., 2014) and production of the Cucurbita pepo and Capsicum annuum (LÚCIO et al., 2015), cherry tomatoes (LÚCIO et al., 2016) and strawberry (DIEL et al., 2018). According to TERRA et al., 2010), models allow condensing information from a series of data, taken over time, into a small set of biologically interpretable parameters.

It has been shown that for lettuce, the models Gompertz, Logistic, and Expolinear fit well the cultivars Grand Rapids, Regina, and Great Lakes, in a hydroponic system during the summer (LYRA et al., 2013). Growth models using the accumulated thermal sum allow to make inferences on precocity, velocity and stabilization of the plant growth through the interpretation of parameters and critical points of the adjusted model curve (MISCHAN & PINHO, 2014).

Two experiments were carried out with lettuce cultivars, one in the autumn-winter of 2016 (experiment 1) and other in the autumn-winter of 2017 (experiment 2). The plants were grown using a closed soilles system, in a protected environment of umbrella type, with 115 m² (5×23 m) environment covered with 150 μm anti-UV polyethylene. The location is between coordinates 29º42'S, 53º49'W and 95 m altitude. According to Köppen’s classification, the climate of the region is humid subtropical Cfa, with hot summers and no defined dry season (ALVARES et al., 2013).

The lettuce cultivars evaluated were: Gloriosa (iceberg - light green leaves, crisp, consistent, prominent ribs, compact head), Pira Verde (crisp green - consistent and loose leaves that do not form head), and Stella (butterhead - delicate and smooth leaves with loosely formed head). These cultivars were recommended by the seed companies for autumn-winter conditions. The seedlings were produced in the floating system in 200-cell expanded polystyrene trays filled with commercial Plantmax® substrate. Transplanting was carried out when the plants developed four to five leaves, on 30/Jun/2016 (experiment 1) and 04/Jun/2017 (experiment 2).

Plants were grown in six benches of corrugated fiber cement sheets, 3.66 m long, 1.10 m wide, 6 mm thick, with six troughs of 5 cm in depth. The troughs were covered with clear 100-μm-thick plastic film and filled with washed gravel number two. The benches were raised (0.85 m) on fixed masonry blocks at the two end portions, with slope of 2%. This slope allowed the nutrient solution to return to the 500 L plastic storage tank. The solution was pumped by a low-power submersible motor pump (with a timer) to a PVC pipe (25 mm diameter). From this pipe derived four drip hoses with pots placed under the drippers at a distance of 30 cm between the plants in the row, to a density of 11.11 m² plants. Each bench consisted of four rows, totaling 44 three-liter volume pots (11 pots per row), filled with washed sieved coarse sand, with 0 dS m⁻³ electrical conductivity.

The nutrient solution consisted of the following macronutrient composition (in mmol L⁻¹): 10.36 NO₃⁻; 1.0 H₂PO₄⁻; 3.36 NH₄⁺; 1.0 SO₄²⁻; 4.0 de K⁺; 2.0 Ca²⁺; 1.0 Mg²⁺; and micronutrients (mg L⁻¹): 1.0 Fe; 0.50 Mn; 0.22 Zn; 0.26 B; 0.06 Cu, and 0.03 de Mo, for lettuce, with 1.33 dS m⁻³ electrical conductivity (EC) and pH 5.5 to 6.5. The EC and pH were monitored throughout the
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The assumptions of normality, homogeneity, and independence of errors were met in both the Gompertz and Logistic models for fresh and dry matter of leaves and fresh and dry matter of stems.

RESULTS AND DISCUSSION

The assumptions of normality, homogeneity, and independence of errors were met in both the Gompertz and Logistic models for fresh and dry matter of leaves and fresh and dry matter of stems.

The coefficient of determination \( R^2 = 1 - \frac{S_{res}^2}{S_{tot}^2} \) was used to assess the quality of fit of the models, and the best fit was considered the model with the coefficient closest to 1 or 100%. The Akaike Information Criterion (AIC = \( \ln(n\sigma^2) + 2(p + 1)/n \)) in which the lower its value the better the model (that is, the more suitable the model is to describe the study), and the Residual Standard Deviation \( RSD = \sqrt{\frac{S_{res}^2}{n-3}} \), define the best fit of the model with values closer to zero. The intrinsic curvature measures (ICM) and curvature measures of the parameter effect (PE) were quantified using the geometric concept of curvature (BATES & WATTS, 1998). The selection of the best model to describe plant growth, is based on the one that provides the lowest values of intrinsic or parametric curvature measures. Were calculated according to the equations described in MISCHAN & PINHO (2014), the inflection point (IP) to Gompertz \( IP_x = \frac{b}{c} \) and to Logistic \( IP_x = \frac{-b}{c} \) and to Logistic \( IP_y = \frac{a}{e} \), the maximum acceleration point (MAP) to Gompertz \( MAP_x = \frac{b - in\left(\frac{a+3}{c}\right)}{c} \) and to Logistic \( MAP_x = \frac{-1 - b - ln(2 + \sqrt{3})}{c} \) and \( MAP_y = \frac{a}{3 + \sqrt{3}} \) the maximum deceleration point (MDP) to Gompertz \( MDP_x = \frac{b - in\left(\frac{-a}{c}\right)}{c} \) and \( MDP_y = a \left(\frac{-3 - \sqrt{3}}{c}\right) \) to Logistic

Intenerces about plant growth were made from these critical points. The calculations were performed using the Microsoft Office Excel® applications and the R software, with the nls function (R DEVELOPMENT CORE TEAM, 2018).

The assumptions of normality, homogeneity, and independence of errors were met in both the Gompertz and Logistic models for fresh and dry matter of leaves and fresh and dry matter of stems.

The estimates of the parameters \((a, b, c)\) for each response were compared between the experiments for each cultivar, and between the cultivars in each experiment, by overlapping confidence intervals (CI) of the parameter estimates in each model. For this purpose, we calculated the lower and upper limits of the 95% confidence interval.

The assumptions of normality, homogeneity, and independence of errors were met in both the Gompertz and Logistic models for fresh and dry matter of leaves and fresh and dry matter of stems.

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shoots of lettuce cultivars in both experiments, as the Shapiro-Wilk, Durbin-Watson, and Breusch-Pagan tests had p-values equal to or greater than 0.05. Similar results were reported by RIBEIRO et al. (2018), in which the assumptions were taken to nonlinear models.

The estimates of \(a\) are the asymptotic values, that is, in the case of lettuce, represent the maximum mass achieved. For all the characters of the cultivars, the \(a\) values for the Gompertz model were higher than for the Logistic model (Tables 1 and 2). The estimation of \(b\), in theory, provides a concept of the ratio between the initial values and the missing amount to reach the asymptote. The estimate of parameter \(c\), represents the growth speed, which was higher in the Logistic model (Tables 1 and 2).

The estimates of the parameters \((a, b, \text{ and } c)\) of each character for the Gompertz and Logistic models were compared between each experiment (Tables 1 and 2) and between the each cultivars (Tables 3), by the criterion of overlapping confidence intervals. This comparison criterion was used by WHEELERN et al. (2006) and by BEM et al. (2018), to verify if the growth curves have differed according to the treatments.

To clarify the comparison by the criterion of overlapping 95% confidence intervals (CI), the FLM of cv. Pira Verde will be used as an example to compare the estimate of the parameter \(a\) of the Logistic model between experiments 1 and 2 (Table 2). The following results were reported: the estimate of parameter \(a\) (354.7561) in experiment 1 is within the confidence interval of the estimate of parameter \(a\) in experiment 2 (329.5156 to 385.3911). Also, the estimate of parameter \(a\) (357.4533) in experiment 2 is within the confidence interval of the estimate of parameter \(a\) of experiment 1 (225.9895 to 483.5227). Therefore, the estimates of the parameter \(a\) are not different between the experiments. When at least one of the estimates is within the CI of the other, it can be concluded that the effect is not significant. If the two parameter estimates are outside the CI of the other, it can be concluded that the effect is significant.

In the Gompertz model, the parameters \(b\) and \(c\) for FLM and FSM of cv. Gloriosa were not different between the experiments (Table 1). However, the parameter \(a\) differed in all the characters, with higher values of FLM and FSM in experiment 1, which indicates higher matter production in relation to experiment 2. Opposite behavior was observed for DLM and DSM. For cv. Pira Verde, the estimates were not different for the characters except for DLM in relation to parameter \(c\). These results indicated that, for this cultivar, there was no difference of the Gompertz models between the experiments. However, for cv. Stella, no differences were observed for FLM and FSM between the experiments. The DLM and DSM were not different asymptotically.

The Logistic model of cv. Gloriosa showed that DLM and DSM differed between experiments for parameters \(a\), \(b\), and \(c\) (Table 2). FLM and FSM showed differences only in the asymptotic values and were higher in the experiment 1 than in the experiment 2, which indicated that the plants had higher production of green matter in experiment 1. Characters differed of cultivar Pira Verde with respect to the parameters \(b\) and \(c\), and did not differ for parameter \(a\). Asymptotic values of cultivar Stella were not different between experiments for all the characters, the estimate of \(b\) was similar for FLM and FSM. However, all characters differed for growth rate.

These results suggested that the growth models had different behaviors between experiments 1 and 2. Similar results were reported for genotype tomato in two years, in the Cordillera and Ellen genotypes were more premature in 2015/2016 crop, and the Gaucho genotype was more premature in the 2016/2017 crop (SARI et al. 2019).

Comparing the cultivars in each experiment, we found that cvs. Gloriosa and Pira Verde, in the Gompertz model, experiment 1, showed no difference between the characters (Table 3). This means that the Gompertz model makes no difference between these cultivars. Conversely, Gompertz model differed for all the characters of cvs. Pira Verde and Stella, since at least one of the three parameters \((a, b, \text{ and } c)\) was significant. This same behavior was observed between cvs. Gloriosa and Stella. In experiment 2, the cultivars Gloriosa and Pira Verde were not different for FLM, while Pira Verde and Stella did not differ for DLM and DSM.

The estimates of the Logistic model parameters, for FSM of cvs. Pira Verde and Stella in experiment 1, for FSM of cvs. Gloriosa and Pira Verde in experiment 2, and for DLM and DSM of cvs. Pira Verde and Stella in experiment 2, were not different (Table 3). All other comparisons differed in at least one of the three parameters of the Logistic model. For the Gompertz and Logistic models there was a predominance of differences, which indicated the need of specific models per character and cultivar. Different models were also required in groups of garlic accesses (REIS et al., 2014).

Goodness-of-fit indicators are used to define the most suitable model. The Logistic and Gompertz models presented acceptable goodness-of-fit values (high R², low AIC and RSD) and close
Table 1 - Estimates of the parameters $a$, $b$, and $c$, lower limit (LL) and upper limit (UL) of the confidence interval (CI 95%) of the Gompertz model for the characters as a function of accumulated thermal sum (in °C) of lettuce cultivars (Gloriosa, Pira Verde, and Stella) in two experiments.

| Character (1) | Parameter | Estimates | CI 95% | Estimates | LL  | UL  |
|--------------|-----------|-----------|--------|-----------|-----|-----|
|              |           | Experiment 1 | | Experiment 2 | |
| Gloriosa     | $a^{(2)}$ | 929.8712 | 597.8730 | 1261.8694 | 455.6514 | 370.6886 | 540.6942 |
|              | $b^{(1)}$ | 2.3061 | 1.8969 | 2.7154 | 2.4371 | 2.0036 | 2.8707 |
|              | $c^{(1)}$ | 0.0076 | 0.0049 | 0.0102 | 0.0082 | 0.0061 | 0.0102 |
| Pira Verde   | $a^{(2)}$ | 16.7031 | 15.3232 | 18.0831 | 25.3207 | 17.9166 | 32.7249 |
|              | $b^{(1)}$ | 4.2032 | 2.9665 | 5.4398 | 2.3264 | 1.8240 | 2.8288 |
|              | $c^{(1)}$ | 0.0198 | 0.0138 | 0.0258 | 0.0073 | 0.0048 | 0.0098 |
| Stella       | $a^{(2)}$ | 984.0988 | 617.5294 | 1350.6682 | 535.0355 | 420.6123 | 649.4587 |
|              | $b^{(1)}$ | 2.2898 | 1.8898 | 2.6897 | 2.3731 | 1.9682 | 2.7780 |
|              | $c^{(1)}$ | 0.0074 | 0.0048 | 0.0100 | 0.0076 | 0.0056 | 0.0095 |
| DSM          | $a^{(2)}$ | 17.8364 | 16.2972 | 19.3755 | 30.4005 | 19.9823 | 40.8187 |
|              | $b^{(1)}$ | 4.0599 | 2.8834 | 5.2364 | 2.2844 | 1.8061 | 2.7627 |
|              | $c^{(1)}$ | 0.0189 | 0.0132 | 0.0246 | 0.0068 | 0.0043 | 0.0092 |

(1)FLM: fresh leaf matter, as g plant$^{-1}$; DLM: dry leaf matter, as g plant$^{-1}$; FSM: fresh shoot matter, as g plant$^{-1}$; and DSM: dry shoot matter, as g plant$^{-1}$.

(2)Comparison of the parameters estimates ($a$, $b$, and $c$) between the experiments: *Significant effect at 0.05 probability of error. **Non-significant.

The Gompertz and Logistic models satisfactorily described the growth curve of lettuce cultivars, with $R^2$ values equal to or higher than 0.913, except for cv. Stella, which showed lower Goodness-of-fit in experiment 2 ($0.769 \leq R^2 \leq 0.826$), of both

to each other (Tables 4 and 5). The $R^2$ indicator was used by LIRA et al. (2003) to study the growth curve of lettuce cultivars. However, it is recommended to use more than one fit quality indicator to increase the reliability of the model choice.
models. LYRA et al. (2003) adjusted growth models for dry leaf matter in lettuce cultivars in the summer in Viçosa, Minas Gerais, Brazil, and obtained results partially similar to the present study, with a coefficient of determination equal to or greater than 0.98.

Although, the models presented satisfactory Goodness-of-fit for FLM and FSM of cv. Gloriosa in experiment 1, the Gompertz model overestimated the parameter \( a \) with 929.8712 for FLM and 984.0988 for FSM (Table 1), that is, these estimates were higher

| Character | Parameter | Estimates | CI 95% | Estimates | CI 95% |
|-----------|-----------|-----------|--------|-----------|--------|
|           |           | Experiment 1 |        | Experiment 2 |        |
|           |           | LL          | UL    | LL          | UL    |
| FLM       | \( a \) (1) | 644.5429 | 548.4506 | 740.6351 | 374.9478 | 336.2122 | 413.6834 |
| FLM       | \( b \)   | -5.1549 | -5.8057 | -4.5041 | -5.0146 | -5.7298 | -4.2994 |
| FLM       | \( c \)   | 0.0174 | 0.0140 | 0.0208 | 0.0159 | 0.0131 | 0.0187 |
| FSM       | \( a \)   | 16.1507 | 15.1791 | 17.1223 | 19.8721 | 16.8842 | 22.8600 |
| FSM       | \( b \)   | -6.9451 | -8.6093 | -5.2809 | -4.9363 | -5.7612 | -4.1114 |
| FSM       | \( c \)   | 0.0301 | 0.0224 | 0.0378 | 0.0149 | 0.0117 | 0.0182 |
| FSM       | \( a \)   | 671.9417 | 568.1495 | 775.7339 | 423.4933 | 375.6487 | 471.3380 |
| FSM       | \( b \)   | -5.1427 | -5.7829 | -4.5026 | -5.0110 | -5.6958 | -4.3261 |
| FSM       | \( c \)   | 0.0172 | 0.0139 | 0.0206 | 0.0154 | 0.0127 | 0.0181 |
| FSM       | \( a \)   | 17.1579 | 16.1129 | 18.2030 | 22.6678 | 18.9117 | 26.4239 |
| FSM       | \( b \)   | -6.7920 | -8.3663 | -5.2178 | -4.9890 | -5.7998 | -4.1783 |
| FSM       | \( c \)   | 0.0291 | 0.0219 | 0.0364 | 0.0146 | 0.0114 | 0.0179 |
| FSM       | \( a \)   | 354.7561 | 225.9895 | 483.5227 | 357.4533 | 329.5136 | 385.3911 |
| FSM       | \( b \)   | -6.9162 | -8.2985 | -5.5339 | -5.4368 | -6.3065 | -4.5671 |
| FSM       | \( c \)   | 0.0275 | 0.0189 | 0.0361 | 0.0185 | 0.0151 | 0.0219 |
| FSM       | \( a \)   | 12.9143 | 10.9345 | 14.8941 | 13.4350 | 12.0807 | 14.7892 |
| FSM       | \( b \)   | -8.1440 | -10.4198 | -5.8683 | -5.3736 | -6.7002 | -4.0469 |
| FSM       | \( c \)   | 0.0366 | 0.0249 | 0.0483 | 0.0194 | 0.0141 | 0.0248 |
| FSM       | \( a \)   | 371.9859 | 233.7199 | 510.2519 | 373.1430 | 343.2436 | 403.0423 |
| FSM       | \( b \)   | -6.9521 | -8.3426 | -5.5615 | -5.4148 | -6.2733 | -4.5563 |
| FSM       | \( c \)   | 0.0276 | 0.0189 | 0.0362 | 0.0183 | 0.0150 | 0.0216 |
| FSM       | \( a \)   | 13.5725 | 11.5050 | 15.6401 | 14.2652 | 12.7645 | 15.7659 |
| FSM       | \( b \)   | -8.2774 | -10.6061 | -5.9486 | -5.2873 | -6.5615 | -4.0313 |
| FSM       | \( c \)   | 0.0371 | 0.0252 | 0.0490 | 0.0189 | 0.0137 | 0.0240 |

(1) FLM: fresh leaf matter, as g plant. DLM: dry leaf matter, as g plant. FLM: fresh shoot matter, as g plant. DLM: dry shoot matter, as g plant.

(2) Comparison of the parameters estimates (\( a, b \) and \( c \)) between the experiments. *Significant effect at 0.05 probability of error. ns: Non-significant.
Intrinsic curvature measures (ICM) and curvature measures of the parameter effect (PE) help us choose the best model. We found that the Logistic model had lower ICM for most of the characters of the cultivars in the two experiments and the PE was always smaller than in the Gompertz model (Tables 4 and 5). The lower ICM and, especially, the lower PE indicate better suitability of the Logistic model, when compared to the Gompertz model.

Considering the five Goodness-of-fit indicators ($R^2$, AIC, RSD, ICM, and PE), we can infer that the Logistic model had suitable behavior regardless of cultivar, character, and experiment and is the best indicated to describe the growth of the lettuce cultivars. To exemplify the growth curve shape of the Logistic model for each character, with the respective critical points, we selected cv. Gloriosa of experiment 2 (Figure 1). The other growth curves can be constructed with the respective estimates of the parameters (Table 2).

Inflection points, maximum acceleration and maximum deceleration are used to infer the crop growth, having as a base the general behavior to the cultivars Gloriosa, Pira Verde and Stella (Tables 4 and 5).
Table 4 - Coefficient of determination ($R^2$), Akaike information criterion (AIC), residual standard deviation (RSD), intrinsic curvature measures (ICM), curvature measures of the parameter effect (PE), inflection point (IP), maximum acceleration point (MAP), and maximum deceleration point (MDP) of the Gompertz model for characters ($^{(1)}$) as a function of the accumulated thermal sum (in °C) of lettuce cultivars (Gloriosa, Pira Verde, and Stella) in two experiments.

| Statistic | FLM | DLM | FSM | DSM | FLM | DLM | FSM | DSM |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| $R^2$     | 0.968 | 0.918 | 0.968 | 0.922 | 0.969 | 0.952 | 0.970 | 0.952 |
| AIC       | 7.283 | 1.309 | 7.296 | 1.371 | 6.334 | 0.786 | 6.453 | 0.953 |
| RSD       | 35.802 | 1.851 | 36.758 | 1.909 | 22.596 | 1.410 | 23.982 | 1.533 |
| ICM       | 0.165 | 0.128 | 0.164 | 0.134 | 0.147 | 0.191 | 0.146 | 0.192 |
| PE        | 6.975 | 0.787 | 7.448 | 0.851 | 2.575 | 4.373 | 3.111 | 5.477 |
| IP        | x 305.065 | 212.742 | 309.348 | 214.657 | 298.835 | 319.371 | 314.040 | 337.689 |
|           | y 342.080 | 6.145 | 362.030 | 6.562 | 167.625 | 9.315 | 196.829 | 11.184 |
| MAP       | x 177.750 | 164.029 | 179.324 | 163.771 | 180.824 | 187.248 | 186.681 | 195.418 |
|           | y 67.831 | 1.218 | 71.786 | 1.301 | 33.238 | 1.847 | 39.029 | 2.218 |
| MDP       | x 432.380 | 261.455 | 439.372 | 265.543 | 416.846 | 451.494 | 441.398 | 479.961 |
|           | y 634.654 | 11.400 | 671.665 | 12.174 | 310.990 | 17.282 | 365.171 | 20.749 |
| $R^2$     | 0.964 | 0.935 | 0.964 | 0.935 | 0.960 | 0.913 | 0.961 | 0.915 |
| AIC       | 5.769 | 5.335 | 5.846 | 5.442 | 6.665 | 0.998 | 6.727 | 1.066 |
| RSD       | 16.940 | 1.126 | 17.608 | 1.188 | 26.674 | 1.569 | 27.519 | 1.623 |
| ICM       | 0.324 | 0.229 | 0.327 | 0.207 | 0.141 | 0.213 | 0.143 | 0.220 |
| PE        | 87.261 | 5.582 | 92.807 | 5.552 | 1.627 | 1.904 | 1.708 | 2.043 |
| IP        | x 345.989 | 217.964 | 349.716 | 218.552 | 266.933 | 247.837 | 269.886 | 251.721 |
|           | y 454.300 | 5.943 | 490.928 | 6.266 | 145.992 | 5.286 | 153.548 | 5.662 |
| MAP       | x 211.724 | 164.335 | 214.251 | 165.325 | 175.678 | 167.763 | 176.310 | 167.631 |
|           | y 90.082 | 1.178 | 97.345 | 1.242 | 28.948 | 1.048 | 30.447 | 1.123 |
| MDP       | x 480.254 | 271.592 | 485.181 | 271.780 | 358.189 | 327.910 | 363.462 | 335.811 |
|           | y 842.853 | 11.026 | 910.808 | 11.625 | 270.855 | 9.807 | 284.873 | 10.505 |
| $R^2$     | 0.935 | 0.925 | 0.935 | 0.923 | 0.821 | 0.769 | 0.824 | 0.769 |
| AIC       | 6.474 | 0.359 | 6.552 | 0.489 | 8.090 | 1.997 | 8.185 | 2.151 |
| RSD       | 24.130 | 1.139 | 25.093 | 1.215 | 54.362 | 2.583 | 57.029 | 2.791 |
| ICM       | 0.057 | 0.065 | 0.060 | 0.067 | 0.360 | 0.311 | 0.340 | 0.312 |
| PE        | 4.705 | 1.388 | 5.087 | 1.506 | 2.182 | 2.860 | 2.371 | 3.232 |
| IP        | x 222.486 | 206.749 | 223.651 | 207.711 | 262.976 | 255.899 | 267.333 | 262.513 |
|           | y 122.108 | 4.225 | 128.759 | 4.505 | 111.380 | 4.659 | 121.122 | 5.198 |
| MAP       | x 175.906 | 175.329 | 176.094 | 175.661 | 199.373 | 185.208 | 199.646 | 186.506 |
|           | y 24.213 | 0.830 | 25.531 | 0.893 | 22.085 | 0.924 | 24.017 | 1.031 |
| MDP       | x 269.067 | 238.169 | 271.208 | 239.760 | 326.578 | 326.589 | 335.020 | 338.519 |
|           | y 226.545 | 7.839 | 238.885 | 8.359 | 206.642 | 8.644 | 224.714 | 9.643 |

$^{(1)}$ FLM: fresh leaf matter, as g plant$^{-1}$; DLM: dry leaf matter, as g plant$^{-1}$; FSM: fresh shoot matter, as g plant$^{-1}$; and DSM: dry shoot matter, as g plant$^{-1}$.

and 5). The maximum acceleration point occurred at the beginning of the curve, when the plants showed slow growth, which is related to smaller plants and still young leaves. For most cultivars, in both experiments, the inflection point (IP) coincided with the phase close to harvest point, with the appearance of senescent basal leaves, which in practice is one of the criteria to classify the produce commercially. In general, independent of the experiment, in the Gompertz model, the cultivars reached the IP with lower STa than in the Logistic model. Among the cultivars, Gloriosa required the greatest accumulation of thermal sum and had the highest dry and fresh matter compared with cvs. Stella and Pira Verde. The results showed that

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iceberg cultivars need higher thermal sum during the autumn-winter period, due to the process of formation of a compact commercial head (YURI et al., 2017). The characters fresh leaf matter and fresh shoot matter represent the edible part of the lettuce, that is, the part of major commercial interest. Thus, the fresh mass has greater relevance than the dry matter. Accordingly, the inflection points of the fresh leaf mass and aerial part can be used in a practical way, since the accumulated thermal sum (IPx) reflects the amount of mass accumulated (IPy) near the harvest point. Therefore, this information is useful for producers and researchers that work with this crop.
The results of this study showed that the Logistic nonlinear growth model and its critical points are relevant to help the selection of promising lettuce cultivars. The Logistic model was also used to describe the growth curve of dry matter of the aerial part, the bulb and the whole plant of the onion culture (PÔRTO et al., 2006), the production of genotype tomato (SARI et al., 2019) and to describe the production of strawberry cultivars from different seedling origins grown on organic substrates (DIEL et al., 2018).

The parameters of the Logistic model were estimated as a function of the relations between the productive characters and the accumulated thermal sum. The parameters (a, b and c) estimated in this study can be used for simulation and prediction of growth of cvs. Gloriosa, Pira Verde, and Stella in the autumn-winter period, for research or production. However, we recommend the use of the thermal sum from the crop site, because the lettuce crop is more influenced by temperature during its vegetative phase (LOPES et al., 2004). Thus, this prediction can be used, but the values obtained will be approximated to those reported in this study and should follow its growth curve pattern. In addition, because we found no studies focusing on growing these cultivars in autumn-winter, the models developed here can become a reference for further research.

CONCLUSION

The growth models developed show differences between the experiments (years) and among the cultivars. The Logistic and Gompertz growth models showed a satisfactory Goodness-of-fit for the fresh and dry matter of leaves and fresh and dry matter of shoots of the lettuce cultivars Gloriosa, Pira Verde, and Stella, in autumn and winter. The Logistic model best describes the growth of the lettuce cultivars.

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DECLARATION OF CONFLICT OF INTERESTS

The authors declare no conflict of interest. The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

AUTHORS’ CONTRIBUTIONS

The authors contributed equally to the manuscript.

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