The Fates of Merging Supermassive Black Holes and a Proposal for a New Class of X-Ray Sources

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ABSTRACT

We perform N-body simulations on some of the most massive galaxies extracted from a cosmological simulation of hierarchical structure formation with total masses in the range \(10^{12} M_\odot < M_{\text{tot}} < 3 \times 10^{13} M_\odot\) from \(4 \geq z \geq 0\). After galactic mergers, we track the dynamical evolution of the infalling black holes (BHs) around their host’s central BHs. From 11 different simulations, we find that, of the 86 infalling BHs with masses \(> 10^4 M_\odot\), 36 merge with their host’s central BH, 13 are ejected from their host galaxy, and 37 are still orbiting at \(z = 0\). Across all galaxies, 33 BHs are kicked to a higher orbit after close interactions with the central BH binary or multiple, after which only one of them merged with their hosts. These orbiting BHs should be detectable by their anomalous (not Low Mass X-ray Binary) spectra. The X-ray luminosities of the orbiting massive BHs at \(z = 0\) are in the range \(10^{28} - 10^{43}\) erg s\(^{-1}\), with a currently undetectable median value of \(10^{33}\) erg s\(^{-1}\). However, the most luminous \(\sim 5\%\) should be detectable by existing X-ray facilities.

Key words: Galaxy: kinematics and dynamics

1 INTRODUCTION

Simulations of structure formation in a ΛCDM universe show us how the massive galaxies we see today were formed through a myriad of minor and major mergers between (proto) galaxies. As most galaxies are believed to contain central massive black holes even at high redshifts, each of these mergers set the stage for a potential encounter between massive black holes. Here we investigate what happened to those black holes over a timeframe of billions of years.

What are possible outcomes for massive black holes during galaxy mergers? There are three possibilities: they merge or form a binary with the central massive black hole of their new host galaxy, they get ejected through three-body interactions with an existing central black-hole binary, or they remain in orbit in the merged galaxy. Combinations of these options are also possible with, e.g., the (spin-induced) kick at merger leading to an ejection or extended orbit of the merged black hole. We will argue in this paper that a very common outcome – especially for lower-mass black holes – is for the injected BHs to remain as orbiting X-ray sources in their host galaxies, waiting to be identified by current observational techniques.

But how common are encounters of massive black holes? For massive galaxies, we know that minor mergers are frequent from the observed evolution of the size and mass of these systems (van Dokkum & Brammer 2010; van Dokkum et al. 2008; Matharu et al. 2019). However, if we assume that all galaxy mergers lead to SMBH mergers we end up overpredicting the gravitational wave background, as constrained by observations from pulsar timing arrays (PTAs, see Sesana et al. 2008, 2009; Sesana 2013; McWilliams et al. 2014; Kulier et al. 2015; Taylor et al. 2016; Sesana et al. 2018; Inayoshi et al. 2018b,c; Middleton et al. 2018). In fact, dynamical studies of black hole encounters have shown that most encounters between two supermassive black holes will result in a stalling binary, unable to get close enough for gravitational wave emission to drive the merger (Begelman et al. 1980; Milosavljević & Merritt 2003).

However, we do not often see SMBH binaries at the centers of massive ellipticals (but see Charisi et al. 2016). So, what does happen? Ryu et al. (2018) indicate that dynamical interactions among multiple orbiting black holes, which will eject a non-negligible fraction of the mass, may solve this problem. The present paper also addresses this purported dynamical solution, focusing attention on the large fraction of lower-mass black holes that remain to be detected as they orbit in massive galaxies. We argue that, as a result of few-body interactions among the resident and infalling black holes, some are ejected, some remain in extended orbits, and some (the few most massive ones) do in fact merge. The merger rates we find do not exceed the PTA limits, and some of the orbiting BHs that our simulations predict to exist may actually be detectable via their ability to accrete gas and emit radiation.

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For this study, we look at the merger history of 11 exemplary galaxies across the galaxy mass spectrum extracted from a cosmological simulation of hierarchical structure formation. We investigate how, after merging with incoming galaxies, SMBHs sink into the cores of the hosts and interact with the resident black hole. We show that gravitational interactions of multiple SMBHs are most probable in high-mass galaxies with total mass $10^{12} M_\odot < M < 3 \times 10^{13} M_\odot$. Galaxies with lower masses have too few mergers with SMBH hosting galaxies. Galaxies with higher masses are more extended, making dynamical friction processes less efficient and hence failing to drive SMBHs into the host galaxy core. Kulkarni & Loeb (2012) and Ryu et al. (2018) performed similar N-body simulations of merging galaxies and black holes, although their halos were from dark matter-only merger trees from the Millennium Simulation (Springel et al. 2005), which have very different merger histories compared to the hydrodynamical simulations in this study. Additionally, most of the halo masses in the Kulkarni & Loeb (2012) simulations were of mass $M_{\text{halo}} > 10^{14} M_\odot$.

This paper is organized as follows: in Section 2 we describe the cosmological simulations from which we use the merger history to set up our idealized numerical simulations. We present the few-body integration code, AR-CHAIN that we used for our simulations of SMBH dynamics, and the modifications we made to this code in order to deal with a host galaxy's gravitational potential. In Section 3, we show the results of our 11 exemplary simulations of galaxies growing with time and acquiring new SMBHs. We analyze how the SMBHs spiral into the core of their new host galaxies due to dynamical friction, and how interactions with the host black hole and other orbiting BHs leads to near-ejections or mergers. We then estimate the X-ray luminosities expected of the orbiting black holes. The final Section 5 contains a discussion of the results and our conclusions.

2 METHODS
2.1 Overview of simulation

The computational challenge of this investigation is a wide range of time-scales. Galactic evolution and galactic orbits take hundreds of millions to billions of years, while orbital times of BH binaries may require time steps of years for accurate integration. We overcome this challenge by simplifying the model where possible, and by focussing on the main drivers of BH dynamics: the galactic background potential, dynamical friction, and strong BH-BH encounters.

Our simulations focus on elliptical galaxies with central SMBHs. The galaxies were given a dark matter background potential using the Stone-Ostriker profile (Stone & Ostriker 2015), which is a three-parameter potential-density pair, whose quantities such as density, potential, and binding energy can be written in closed form, having a profile similar to that of a truncated isothermal sphere. The galaxies were evolved from $4 \geq z \geq 0$. Orbiting black holes were periodically introduced into the "host" galaxy following mergers and their dynamical interaction with the background potential and central SMBH were followed, as described further below.

For the numerical simulations presented here, we used a modified version of the algorithmic chain integrator AR-CHAIN developed by Mikkola & Merritt (2006). It uses algorithmic chain regularization for high-precision integration of few-body dynamics, and is capable of handling velocity-dependent forces efficiently. It includes relativistic post-Newtonian terms up to order 2.5 (Mikkola & Merritt 2008).

2.2 Merger tree Data

The merger tree data we used as the input to our simulations is the result of previous work by Cen (2011a,b, 2012b,a, 2013) (known as the LAOZI simulations Cen 2014), as well as Lackner et al. (2012), and Kulier et al. (2015). A brief synopsis of the work that led to our input data is described below.

Cen (2011a) studied the "cosmic downsizing" effect (e.g., Cowie et al. 1996) using high-resolution large-scale hydrodynamic galaxy formation simulations. His work produced physical parameters for galaxies such as position, velocity, total mass, stellar mass, gas mass, mean formation time, mean stellar metallicity, mean gas metallicity, star formation rate, luminosities, etc.

Lackner et al. (2012), using Cen (2011a)'s work as a basis, created galactic merger trees that compared the properties of in-situ and accreted stellar mass of galaxies at redshift snapshots between $4 \geq z \geq 0$.

Kulier et al. (2015) furthered the merger tree data produced by Lackner et al. (2012) by estimating the evolution of the SMBH population. They assume growth of SMBHs to be proportional to that of the bulge stellar mass of the galaxies. They further assume a constant relation between bulge and total stellar mass at all redshifts, although some observations suggest that the bulge mass fraction for a fixed stellar mass should increase with decreasing redshift (Somerville et al. 2012; Guo et al. 2013; Avila-Reese et al. 2014), with the result that their SMBHs grow faster at high redshift than would be expected in reality.

The primary data we extracted from the above work was organized as follows:

- Galactic properties - Each galaxy was distinguished by an id number, with properties such as stellar mass, dark matter (DM) mass, central black hole id number, and orbiting black hole id numbers, recorded at 37 redshift slices between $4 \geq z \geq 0$.
- Black Hole properties - Each black hole was also distinguished by an id number, with properties such as seed mass, accreted mass, its host galaxy id number, and time after $z = 4$ at which the black hole entered its host galaxy, if it was not the central black hole.

The merger tree data from Lackner et al. (2012) and Kulier et al. (2015) provided 1,830 galaxies in total. However, not all the galaxies were suitable for our simulations. We placed further requirements as follows:

- The galaxies had to exist through the entire simulation ($4 \geq z \geq 0$). If they merged with other galaxies, they had to have been the "surviving" galaxy at each merger.
- They had to have accumulated orbiting black holes by $z = 0$.

The above criteria narrowed the field of eligible galaxies down to 51. In considering time and resources, we chose the 11 galaxies that had the largest number black holes to run the simulations. The black holes in these galaxies accounted for nearly 84% of all orbiting black holes across all the galaxies, so it represented a reasonable balance between completeness and efficiency.

The following cosmological parameters were used in our simulations, consistent with both Lackner et al. (2012) and Kulier et al. (2015): $\Omega_M = 0.28$, $\Omega_b = 0.046$, $\Omega_{\Lambda} = 0.72$, $\sigma_8 = 0.82$, $H_0 = 100h^{-1}$Mpc$^{-1} = 70$km s$^{-1}$Mpc$^{-1}$, and $n = 0.96$.

2.2.1 Stellar Mass Adjustment

As we will explain further in Section 2.3.1, in our simulations orbiting black holes are inserted into the host galaxy at the effective...
radius, $R_e$, which is partially dependent on stellar mass, $M_*$ (see Equation 3). For every doubling of the stellar mass, $R_e$ increases by a factor of 1.66. Additionally, the velocity dispersion at the effective radius, $\sigma(R_e)$, which is used to set the parameter $\eta_0$ (see Equation 1) of our dark matter profile, is also dependent on $M_*$, with a factor of 1.18. These variations have direct effects on the ability of the orbiting black holes to merge with the central black hole, and thus can affect our merger statistics. Therefore, it is important to estimate our stellar masses with reasonable accuracy.

In cosmological simulations, it is common to overproduce stellar mass, with a general over-efficiency in the range of 2-4 times those measured in observations (Katz et al. 1996; Guo et al. 2010; Oser et al. 2010). One cited reason for this overproduction is a lack of modeling for AGN feedback, which may be responsible for solving the phenomenon called the “cooling flow paradox” (Fabian et al. 2001) that suppresses star formation.

Lackner et al. (2012), which is the source of our stellar masses, noted that the efficiency of star formation in their simulations, defined as $f_{\text{bary}} = M_*/M_{\text{DM}}(\Omega_{\text{DM}}/\Omega_0)$ (where $M_{\text{DM}}$ is the mass of the dark matter halo), was approximately 0.6. Compared with the expected range of $0.10 \leq f_{\text{bary}} \leq 0.15$ that they referenced from Leauthaud et al. (2012), their stellar masses were a factor of roughly 4 times greater.

Given the overestimate of stellar mass in the merger tree data, we rescaled our stellar masses in order to match the observational and abundance matching data from Kravtsov et al. (2018). In their study, they estimated star formation efficiency and the stellar mass-halo mass relation in massive haloes. They obtained their data from two sources: a study of nine nearby clusters ($z < 0.1$) using Chandra X-ray observations (Vikhlinin et al. 2009), and one of 12 clusters from Gonzalez et al. (2013).

Figure 1 is a partial, approximate, replication of Figure 11 from Kravtsov et al. (2018) (abundance matching line). We rescaled our stellar masses at $z = 0$ by using a lognormal distribution. For a given galaxy’s dark matter mass, the mean of the distribution was taken to be the point on the abundance matching line at that dark matter mass. The standard deviation of that mean was taken arbitrarily to be 0.3.$\mu$.

Table 2.2.1 shows the original and rescaled stellar masses.

### 2.3 Simulation Physics

In this section, we describe the underlying physics of our simulations, including choice of galaxy background potential, BH diffusion through the galaxy, and gravitational wave kicks as a result of black hole mergers.

#### 2.3.1 Galaxy background potential

To model the dark matter halo distribution in our simulations, we used the Stone-Ostriker profile (Stone & Ostriker 2015), which is a three-parameter potential-density pair. Quantities such as density, potential, and binding energy can be written in closed form, which speeds up computations. It is essentially an analytic form of a finite, cored isothermal mass distribution:

$$\rho(r) = \frac{\rho_c}{(1 + r^2/r_c^2)^{(1 + \gamma)/2}}. \quad (1)$$

Here $\rho_c$ is the central density, $r_c$ is the core radius, and $r_h$ is the outer halo radius.

The central density, $\rho_c$, can be found from inverting Equation 5 in Stone & Ostriker (2015) for the galaxy’s total mass:

$$M_{\text{DM}} = \frac{2\pi^2 r_c^3 \rho_c}{r_h + r_c}. \quad (2)$$

In order to parameterize $r_c$ and $r_h$, we can utilize the relationship between the distribution of light and stellar velocity dispersion at the effective radius, and the shape of the dark matter profile. The effective radius $R_e$ and velocity dispersion at the effective radius

![Figure 1. Approximation of Figure 11 from Kravtsov et al. (2018). $f_{\text{bary}}$ is the universal baryon fraction as a function of total halo mass. Purple dots shown are examples of our rescaled stellar masses. We rescaled our stellar masses at $z = 0$ by using a lognormal distribution. For a given galaxy’s dark matter mass, the mean of the distribution was taken to be the point on the abundance matching line at that dark matter mass. The standard deviation of that mean was taken arbitrarily to be 0.3.](image)
\( \sigma(R_c) \), respectively, can be obtained from (Nipoti et al. 2009; Oser et al. 2010; McWilliams et al. 2014):

\[
R_c = 2.5 \text{kpc} \left( \frac{M_*}{10^{11} M_\odot} \right)^{0.73} (1 + z)^{-0.98},
\]

(3)

\[
\sigma(R_c) = 190 \text{km s}^{-1} \left( \frac{M_*}{10^{11} M_\odot} \right)^{0.2} (1 + z)^{0.47}.
\]

(4)

We also give \( R_c \) an initial value of 100 pc, which is reasonable for a cored massive system. We can then equate the value for \( \sigma(R_c) \) obtained from Equation 4 to the analytic expressions for \( \sigma \) in the Stone-Ostriker profile (cf. Equations 9 and A1-A4 in Stone & Ostriker 2015). The only unknown is \( r_h \), which we can solve for using a simple recursive Newton method. Whether \( \sigma_{\text{near}} \) or \( \sigma_{\text{far}} \) is used from Stone & Ostriker (2015) is determined by whether \( R_c \) is less than or greater than \( \sqrt{5} r_h \).

At each timestep in the code, \( r_h \) is updated according to the stellar mass and Equations 3 and 4. The core radius, \( r_c \), is recalculated only if an orbiting black hole enters within \( r_c \). The work done by diffusion, as described in Section 2.3.2 is calculated, the total potential energy and dynamical friction are updated, and \( r_c \) is solved for from Equation 8 in Stone & Ostriker (2015) based on conservation of energy.

### 2.3.2 Phase-space diffusion

Much of this subsection was borrowed from Stone et al. (2017), Section E1, which made use of the same modified version of AR-CHAIN.

Weak encounters with background stars and dark-matter particles will let the SMBHs diffuse through phase space while they are orbiting within the gravitational potential of the galaxy. The change can be expressed in terms of the Schwarzschild radius of a BH by \( \Delta \hat{v} \) per unit time. We can split this change into a component along the direction of motion of the SMBH, and one perpendicular to that. Following Binney & Tremaine (2008), the diffusion coefficients can be expressed as

\[
D[\Delta v_{||}] = \frac{4\pi G^2 \rho(r) M_{BH} \ln \Lambda}{\sigma^2} f(\chi),
\]

(5)

\[
D[\Delta v_{\perp}^2] = \frac{4\pi G^2 \rho(r) M_{BH} \ln \Lambda}{\sigma^2} \left( f(\chi) - \frac{f(\chi)}{\chi} \right),
\]

(6)

\[
D[\Delta v_{\perp}^2] = \frac{4\pi G^2 \rho(r) M_{BH} \ln \Lambda}{\sigma^2} \left( f(\chi) - 2\chi \frac{\exp(-\chi^2)}{\sqrt{\pi}} \right),
\]

(7)

where \( \Delta v_{||} \equiv \Delta \hat{v} \cdot \hat{v} / v \) is the velocity change in direction of motion, and \( \Delta v_{\perp} \equiv \Delta \hat{v} \cdot \hat{v} / v \) is the velocity change perpendicular to the direction of motion. Here, \( M_{BH} \) is the mass of the black hole, and \( \chi = \frac{v}{\sqrt{\sigma(r)}} \). The function \( f(\chi) \) is given by

\[
f(\chi) \equiv \frac{1}{2\chi^2} \left( \text{erf}(\chi) - 2\text{erfc}(\chi) \right).
\]

(8)

We approximate the factor \( \Lambda \) in the Coulomb logarithm as

\[
\Lambda \equiv \left( \frac{M_{DM}}{M_{BH}} \right) \left( \frac{r}{r_h} \right).
\]

(9)

We can identify Equation 5 as the dynamical friction term. The second term introduces a variance of the friction term, and even allows the BHs to be accelerated when the velocity of a BH is sufficiently small. The third term introduces a change in velocity perpendicular to the direction of motion of the BH. It is a randomly oriented vector, and hence causes the BHs to depart from smooth orbits due to the small random velocity kicks. The last two terms will establish that the BHs are ultimately in energy equipartition with the background stars. The velocity changes \( \Delta v_{||} \) and \( \Delta v_{\perp} \) per unit time \( \Delta t \) can be computed with the above equations. Both changes are normally distributed, where the mean, \( \mu \), and the variance, \( \Sigma \), of the distributions are given by

\[
\mu_{||} = D[\Delta v_{||}] \Delta t,
\]

(10)

\[
\Sigma_{||} = D[\Delta v_{||}^2] \Delta t,
\]

(11)

\[
\mu_{\perp} = 0,
\]

(12)

\[
\Sigma_{\perp} = D[\Delta v_{\perp}^2] \Delta t.
\]

(13)

We compute the diffusion coefficients for each black hole at each time step, and modify its velocity on a Monte Carlo basis. For each time step we draw a random orientation before adding the perpendicular velocity change to the respective BH. Hence, the BH’s modified velocity, \( v_{\perp} \), is computed using

\[
\vec{v}_{\perp} = \vec{v}_0 + \Delta v_{||} \hat{v}_\parallel + \Delta v_{\perp} \hat{v}_\perp,
\]

(14)

\[
\Delta v_{||} = N(\mu_{||},\Sigma_{||}),
\]

(15)

\[
\Delta v_{\perp} = N(\mu_{\perp},\Sigma_{\perp}),
\]

(16)

The change of energy, \( dE_{BH} \), of the orbiting black hole due to phase-space diffusion is given back to the galactic background potential, with \( dE = -dE_{BH} \). As a consequence of this energy transfer, inspiralling black holes will cause an expansion of the galactic mass distribution. For this purpose we calculate the change in potential energy, \( dW \), of the host galaxy using

\[
E = T + W = \frac{1}{2} W_r,
\]

(17)

\[
dW = -2dE_{BH}.
\]

(18)

where we made use of the virial theorem \( 2T + W = 0 \). With this change in potential energy we can calculate a new radius for the galactic background potential at each integration step. For simplicity, we assumed that the relatively low mass of the infalling black holes compared to the core mass of the host galaxies would only affect the size of the core radius, and hence ignored the feedback effect when the black holes were outside of the core radius.

### 2.3.3 Gravitational wave recoils and escape

The code AR-CHAIN includes post-Newtonian terms up to order 2.5. The SMBHs can therefore merge via gravitational wave emission. We include gravitational wave recoils following the prescription outlined in Kulier et al. (2015), which is based on the fitting formula by Lousto et al. (2012). We assume that a merger will be inevitable when the separation between two SMBHs becomes smaller than four times the Schwarzschild radius of the more massive black hole. At the moment of the merger, we also assume that the spin vectors of the two SMBHs are randomly aligned.

A result of the anisotropic emission of gravitational waves is that the merged binary often experiences a linear kick in the opposite direction of the emission of the GW due to conservation of linear momentum. This kick can result in velocities of the merged binary from approximately 200 \( \text{km s}^{-1} \) to \( \text{500 km s}^{-1} \) (see González et al. 2007b,a; Campa et al. 2007; Lousto & Zlochower 2011). In our simulations, the binaries never stray more than a few tens of parsecs from the nucleus of the host galaxies.

Black holes can also eject each other via strong three-body
interactions. We remove SMBHs from the simulations once they move beyond $r_h$, assuming that it will take them more than a Hubble time to find their way back into the center of the host galaxy, or that they have achieved escape velocity.

2.4 Simulation setup

For each galaxy in the simulations, we only used black holes that would theoretically have merged with the center of the host galaxy within a dynamical friction time, $t_{\text{fric}}$, of less than 100 times a Hubble time, with $t_{\text{fric}}$ defined from Binney & Tremaine (2008) as:

$$t_{\text{fric}} = \frac{19}{6} \left( \frac{R_e}{5 \text{kpc}} \right)^2 \frac{\sigma(R_e)}{200 \text{km s}^{-1}} \frac{10^8 M_\odot}{M_{\text{BH}}} \text{Gyr},$$

(19)

The orbiting black holes were injected into their host galaxy at redshift $z$, at a distance from the galactic center of $R_e$ (Equation 3) following galaxy mergers. Their initial velocity was arbitrarily chosen to be the circular velocity at that radius, $v_c = \sqrt{GM(R_e)/R_e}$, with orientation, $v_x$, $v_y$, and $v_z$, randomly chosen.

3 SIMULATION RESULTS

Here we review the primary results of our simulations, focusing on the various interactions of the orbiting black holes with their host SMBHs. In the next section, we will investigate the luminosity characteristics of the orbiting black holes that did not merge at $z = 0$.

Figure 2 shows the distribution of merged, ejected and orbiting black holes (stacked) across all galaxy simulations as a function of the mass of the infalling black holes.

Of the 86 SMBHs used for the simulations, 36 black holes merged with their respective host black hole within a Hubble time, 37 remained orbiting at the end of the simulations, and 13 were ejected from their galaxies as a result of sufficiently large kicks due to three-body interactions with the host and other black holes.

Given the log distribution of masses, one can see the general trend of larger mass BHs ($\geq 10^7 M_\odot$) tending to merge with their respective host BH, while lower mass BHs continue to wander at larger radius. It is also evident from the plot that all but two of the black holes that had not been kicked by $z = 0$ have mostly approached and entered within the galactic core radius, $r_c$ (which never exceeded 1 kpc in any of the galaxies).

3.1 Mergers

The tendency of larger mass black holes to merge with their host can perhaps be more clearly seen by looking at the merger mass ratio between the incoming black hole and the host. Figure 3 displays the fraction of galaxy mergers that result in black hole mergers versus the merger mass ratio, defined as $m_2/m_1$, where $m_2$ is the orbiting black hole and $m_1$ is the host black hole. The plot monotonically increases with increasing mass ratio, reaching nearly unity for mass ratio of $10^{-1}$, meaning nearly all galaxy mergers with a black hole mass ratio greater than $10^{-1}$ resulted in the orbiting black hole merging with the host.

Figure 5 shows the mass distribution (stacked) of kicked and not-kicked black holes, including those that merged and those that did not merge. The vast majority of larger SMBHs ($\geq 10^7 M_\odot$) were not kicked to higher orbits nor ejected from their galaxies, which at least partly explains why most of the merged SMBHs are in the greater mass range. In contrast, most of the SMBHs below $10^7 M_\odot$ are kicked (with a few of them being ejected). In our simulations, no black holes that are kicked to higher orbits ever returned to merge with their central host SMBH.

Figure 6 is a realization of the final radii of the remaining wandering black holes at $z = 0$. There is a slight inverse relationship between black hole mass and final radius (solid line), but with a wide dispersion. It is also evident from the plot that all but two of the black holes that had not been kicked by $z = 0$ have mostly approached and entered within the galactic core radius, $r_c$ (which never exceeded 1 kpc in any of the galaxies).

4 SIGNATURES OF ORBITING BLACK HOLES

The large fraction of orbiting black holes found in our simulations motivates a back-of-the-envelope calculation to see if orbiting black holes produce detectable signatures. In the following section, we...
Figure 3. Fraction of BHs that merge with the central BH as a function of the BH mass ratio. Each point represents the fraction of galaxy mergers that led to BH mergers for each BH merger ratio bin. Note that a "bin" at each point is comprised of the mass ratio at that point up to, but not including, the next point on the x-axis.

Figure 4. Time required for BHs to merge with host SMBH after galaxy merger. Form for fitted line is $-0.379M_{BH} + 5.766$ in log-log space.

Figure 5. Histogram of kicked and not-kicked SMBHs at $z=0$. Colored arrows along upper x-axis represent median masses of respective sets. Histograms are stacked.

Figure 6. Final radii of remaining orbital black holes at $z=0$. Kicked black holes generally had a low likelihood of finding their way back to the center by $z=0$. There is a slight inverse relationship between BH mass and final distance from center (solid line).

4.1 Galaxy Gas X-Ray Thermal Luminosity

First, we construct realistic X-ray thermal luminosities for our host galaxies. We compute the X-ray thermal luminosity of the hot gas in these galaxies, adapting Equation 26 from Choi et al. (2012). In lieu of summing over discrete particles, we use the beta model of the gas as defined in our equation 23 and integrate:

$$L_x = \frac{1.2 \times 10^{-24}}{(\mu m_p)^2} \left( \frac{kT_x}{1\text{keV}} \right)^{1/2} \rho_0^2 4\pi \int_0^\infty \frac{r^2}{(1+r^2/r_c^2)^3} dr \text{ erg s}^{-1}. \quad (20)$$

The integral evaluates to $\pi r_c^5/16. m_p$ is the mass of a proton and $\mu$ is the mean molecular weight. To calculate $\mu$, we use metallicity $Z = 2Z_\odot = 0.0268$, mass fraction of Helium $Y = Y_\odot = 0.2485$, and mass fraction of Hydrogen $X = 0.7247$. This results in $\mu = 0.606$. 

estimate the X-ray thermal luminosities due to gas accreted by these BHs while orbiting through their host galaxies.
The resulting X-ray thermal luminosities are shown in Figures 7 and compared against a sample from Babyk et al. (2018).

4.2 Black Hole X-Ray Thermal Luminosity

4.2.1 Assumptions and Method Employed

Since we model the black holes in this work as point particles for purposes of the simulations, we do not assume any particular structure to them. Therefore, we adopt the approach and assumptions from Inayoshi et al. 2018a and Inayoshi et al. 2019 in estimating the X-ray luminosities of the remaining black holes left wandering in our subject galaxies. Their approach assumes rotating accretion flows along with the simple linear fits (in log-log space) and resulting gas properties used for our galaxies:

$$\rho_0 = 10^{0.6\log([\sigma_c/r_\text{c}]) - 25} \text{ g cm}^{-3},$$

$$T_x = 10^{-2.5\log([\sigma_c/r_\text{c}]) - 0.6} \text{ keV}.$$

The dispersion of the data around the fit for $\rho_0$ is at most 1.5-2.0 orders of magnitude. This leads to a similar variation in the gas density surrounding the black hole, in addition to the ultimate accretion rate of the black hole, due to both having a linear dependence on the central gas density (see Equations (23) and (25) below).

The dispersion of the X-ray temperature data around the fit is approximately 1-1.5 orders of magnitude. The gas sound speed (see Equation (24) below) has a $C_s\rho_0^{1/2}$ dependence on temperature. However, given the dependence of the accretion rate (see Equation (25) below) on gas sound speed of $M_{\text{acc}}\rho_0^{1/2}C_s^{-3}$, the accretion rate is dependent on temperature to the order of $T_x^{-3/2}$. This is another source of variation in our estimate of accretion rate.

The local gas density $\rho$ surrounding the black hole is obtained from a simple isothermal beta model (King 1962; Cavaliere & Fusco-Femiano 1976, 1978):

$$\rho = \frac{\rho_0}{(1 + r^2/r_c^2)^{1.5}} \text{ g cm}^{-3},$$

where $G$ is the gravitational constant, $M_{\text{tot}}$ is the total galaxy mass (stars + DM), and $r_h$ is the outer halo radius as defined in the Stone-Ostriker profile (cf. equation 1).

Figure 8 and 9 present the data from Babyk et al. (2018), along with the simple linear fits (in log-log space) and resulting gas properties used for our galaxies:

$$\rho_0 = 10^{0.6\log([\sigma_c/r_\text{c}]) - 25} \text{ g cm}^{-3},$$

$$T_x = 10^{-2.5\log([\sigma_c/r_\text{c}]) - 0.6} \text{ keV}.$$

The dispersion of the data around the fit for $\rho_0$ is at most 1.5-2.0 orders of magnitude. This leads to a similar variation in the gas density surrounding the black hole, in addition to the ultimate accretion rate of the black hole, due to both having a linear dependence on the central gas density (see Equations (23) and (25) below).

The local gas density $\rho$ surrounding the black hole is obtained from a simple isothermal beta model (King 1962; Cavaliere & Fusco-Femiano 1976, 1978):

$$\rho = \frac{\rho_0}{(1 + r^2/r_c^2)^{1.5}} \text{ g cm}^{-3},$$

where $\rho_0$ is as defined above, $r$ is the black hole’s distance from the center of the galaxy, and $r_c$ is the core radius. Since the orbiting black hole masses are $10^{-5} - 10^{-5}$ $M_{\text{tot}}$, we assume there is no change in $M_{\text{tot}}$ after black holes are ejected.

The sound speed of the gas, $C_s$, is derived from the ideal gas law:

$$C_s = \sqrt{1.67T_x \times 1.602 \times 10^{-16}/n_\text{p} \text{ km s}^{-1}},$$
where $n_p$ is the proton mass in kilograms. Since $T_x$ is in keV, the factor of $1.602 \times 10^{-16}$ is added to convert the temperature to Joules.

### 4.2.3 Black Hole Mass Accretion and Luminosity

At large distances, the black hole mass accretion rate is governed by the Bondi-Hoyle formula (Bondi & Hoyle 1944 and Bondi 1952):

$$\dot{M}_B = \frac{4\pi G^2 M_B^2 \rho}{(C_s^2 + v^2)^{3/2}} \text{g s}^{-1},$$

(25)

where $\rho$ is the density of the surrounding gas, $C_s$ is the sound speed, and $v$ is the velocity of the black hole.

However, due to convective (i.e., inefficient) accretion interior to the Bondi radius (see Inayoshi et al. 2018a and Inayoshi et al. 2019), the net accretion to the black hole is often substantially lower than the Bondi-Hoyle accretion. It can be inferred from the fitting relation given in equation 31 of Inayoshi et al. (2019):

$$\frac{\dot{M}_*}{\dot{M}_{Edd}} \approx 1.5 \times 10^{-6} T_7^{-4/5} \left( \frac{a}{0.01} \right)^{0.37} \left( \frac{\dot{m}_B}{10^{-3}} \right)^{3/5} \left( \frac{f_c}{0.1} \right)^{2/5},$$

(26a)

$$\dot{m}_B = \frac{\dot{M}_B}{\dot{M}_{Edd}},$$

(26b)

$$\dot{M}_{Edd} = \frac{L_{Edd}}{0.1 c^2} \text{g s}^{-1},$$

(26c)

$$L_{Edd} = 1.26 \times 10^{38} \left( \frac{M_{BH}}{M_{\odot}} \right) \text{erg s}^{-1}.$$  

(26d)

$\dot{M}_*$ is the net accretion to the black hole, $T_7$ is the temperature of the ambient gas in units of $10^7 K$, $a$ is the strength of viscosity (generally accepted to be 0.1; see Shakura & Sunyaev 1973; Balbus & Hawley 1991; Matsumoto & Tajima 1995; Stone et al. 1996; Balbus & Hawley 1998; Sano et al. 2004; Inayoshi et al. 2018a), and $f_c$ is the conductivity suppression factor (generally accepted to be 0.1; see Narayan & Medvedev 2001; Maron et al. 2004, and Inayoshi et al. 2019). The Eddington accretion rate is $\dot{M}_{Edd}$, $c$ is the speed of light, and $L_{Edd}$ is the Eddington luminosity.

Given the above, we estimate the bolometric luminosity of the black holes as:

$$L_{\text{bol}} = \epsilon \dot{M}_* c^2,$$

(27a)

$$\log \epsilon = \begin{cases} -1.0 - (0.0162/\dot{m})^2 & \text{for } 0.023 < \dot{m} < 0.023, \\ \sum_n a_n (\log \dot{m})^n & \text{for } 10^{-3} < \dot{m} < 0.023, \\ \sum_b b_n (\log \dot{m})^n & \text{for } 10^{-8} < \dot{m} < 10^{-4}, \end{cases}$$

(27b)

where $\dot{m} = \dot{M}_* / \dot{M}_{Edd}$, and $\epsilon$ is the radiative efficiency (cf. Equation 13 in Inayoshi et al. 2019). The fitted values are $a_0 = -0.807$, $a_1 = 0.27$, $a_2 = 0 (n \geq 2)$, $b_0 = -1.749$, $b_1 = -0.267$, $b_2 = -0.07492$, and $b_b = 0 (n \geq 3)$.

For sources having $\dot{M}_B / \dot{M}_{Edd} > 10^{-3}$, we take $L_x = 0.1 L_{\text{bol}}$, due to the higher net accretion onto the black hole as a result of cooling of the accretion disc and hence higher gas density. For those having $\dot{M}_B / \dot{M}_{Edd} < 10^{-5}$, we take $L_x = L_{\text{bol}}$.

### 4.2.4 Central Black Hole Influence

Within the influence radius of the central black hole, which is defined as the radius at which the total mass of the galaxy within that radius (stars + dark matter) equals the mass of the central black hole, the gas density is much higher than otherwise, and equation (23) cannot be used in the region where the BH Kepler potential dominates. We start with the equation for hydrostatic equilibrium, assuming isothermality,

$$C_s^2 \frac{d\rho}{dr} = -\frac{GM(r)}{r^2}.$$  

(28)

Inside the sphere of influence, $M(r)$ is constant, and we can integrate Equation (28) with respect to $r$, leading to

$$ln \left( \frac{\rho}{\rho_1} \right) = \frac{GM}{C_s^2} \left( \frac{r}{r_1} - 1 \right),$$

(29)

where $\rho$ is the gas density at the location of the wandering black hole, $\rho_1$ is the density at the radius where the central black hole mass equals the galaxy mass, $r$ is the distance of the wandering black hole from the center, and $r_1$ is the radius where the central black hole mass equals the galaxy mass.

Figure 10 shows the difference in luminosity when taking into account the influence of the central black hole for the 17 wandering black holes that were within the influence radius. In all cases, the final luminosity is higher than the original luminosity.

### 4.2.5 Final X-ray Luminosities of Wandering Black Holes, and Expected Detection Spectra

Figure 11 is an approximation to Figure 12 from Inayoshi et al. (2019), along with the final $L_{\text{bol}} / L_{Edd}$ ratio of the wandering black holes from this study, although extended below $10^{-6}$ $\dot{M}_B / \dot{M}_{Edd}$ for the very low accretion-rate sources. Indeed, such low luminosity sources are common, with some estimates of $L_{\text{bol}} / L_{Edd}$ as low as $10^{-10}$ (e.g., Yuan et al. 2003, Quataert 2004, Ho 2008, Ho 2009, Inayoshi et al. 2018a).

Figure 12 represents a histogram of the final X-ray luminosities
of the black holes, overlaid by a lognormal distribution, obtained with a mean $\mu = 1.0 \times 10^{33}$ erg s$^{-1}$ and standard deviation $\sigma = 4.5 \times 10^{3}$ erg s$^{-1}$.

The orbiting black holes should be detectable via their anomalous soft X-ray spectra. Sgr A*, the black hole at the center of the Milky Way Galaxy, has been observed to fit a power law spectrum with photon index $\Gamma \sim 2.7^{+1.2}_{-0.4}$ Baganoff et al. (2003). Conversely low-mass X-ray binaries (LMXRB) have been fit to a power law spectrum with $\Gamma \sim 1.5 – 2.0$ (Hailey et al. 2018), and we expect our orbiting sources to be more similar to Sgr A* than to normal LMXRBs.

5 CONCLUSIONS
We performed N-body simulations on 11 galaxies ranging in mass between $10^{12} \ M_\odot < M < 3 \times 10^{13} \ M_\odot$, from $4 \geq z \geq 0$. A total of 86 orbiting black holes were followed in our simulations, resulting in 42% mergers with their host galaxy’s resident black hole, while another 43% remained in orbit at $z = 0$. Those remaining orbiting black holes had X-ray luminosities in the range of $10^{28} - 10^{43}$ erg s$^{-1}$. The median luminosity of $10^{33}$ erg s$^{-1}$ would be undetectable with present instruments, although the top 5% most luminous from this set could be detectable. If detected, these orbiting massive black holes would constitute a new category of X-ray source.

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