Study of energy spectrum of $^{116}$Sn and $^{116}$Te nuclei by using surface delta and modified surface delta interactions

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Abstract. In this paper, surface delta and modified surface delta interactions have been used by applying the nuclear shell model to calculate values of excitation energies for isotopes of equal mass number containing two nucleons outside the closed core $^{114}$Sn, these nuclei are; The isotope (Tin) $^{116}$Sn contains two neutrons within the model space ($3s_{1/2}, 2d_{3/2}, 1h_{11/2}$) and the other isotope is $^{116}$Te (Tellurium) contains two protons within the model space ($1g_{7/2}, 2d_{5/2}, 3s_{1/2}, 2d_{3/2}, 1h_{11/2}$). The current results of the above isotopes are based on using the Fortran 95 program code to program the above two interactions equations to calculate the values of single particles energies, matrix elements in the state of mixing configurations of orbits for and through which it is possible to calculate the eigenvalues, which are necessary to calculate the excited energy of all isotopes used in this study. By comparing the current theoretical results with the practical results, the expected theoretical results appeared to be in good agreement with the practical values, as well as confirmed and determined of the total angular momentum and parities of some unconfirmed and undetermined practical values, in addition to obtaining a new theoretical energy levels were determined of unspecified practical energies and momentum and this increases the theoretical knowledge of the studied isotopes relative to the energy levels.

Keywords: Nuclear shell model, interactions, excitation energy

1. Introduction

The models of atomic shell succeeded in using several concepts and well to explain the behaviour of electrons in an atom, so nuclear physicists tried to apply these concepts to explain the nuclear structure through the nuclear shell model. The nuclear shell model treats nucleons as occupying various orbits of a single particle orbits that are occupied and bound according to the principle of Pauli exclusion, which states that two or more identical fermions cannot occupy the same quantum state within the quantum system and at the same time. Therefore, this principle supported the image of the shell model, where nucleons become more bounded and restricted in their orbitals. The particles of a single particle are successively rotated when nucleons are added to the closed nucleus. Magic numbers are interpreted as the closure of the main shell so that the nuclei of magic numbers are more stable [1-3]. Mayer and Jensen separately found that appropriate potentials efforts such as harmonic oscillator should include the spin-orbit interaction ($\mathbf{l} \cdot \mathbf{s}$) as the orbital and spin interaction result in individual energy levels being divided. In such a way that the magic numbers 2, 8, 20, 28, 50, 82 and 126 have large gaps in the energy spacing [4,5]. The quantum energy levels of a single nucleon can be calculated by solving the Schrodinger equation for the particle represented by a wave function ($\Psi$) moving in the field potential average due to the movement of all other nucleons in the nuclei as in the following equation [6,7]:

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \Psi(r) = E \Psi(r) \quad (1)$$

Where $m$ is the mass of the nucleon, $V$ is the average field potential, $E$ is the eigenvalue of energy, and $r$ is the nucleon distance from the centre of the potential.
2. Theory

The shell model can describe the nucleus as an inert core (the closest closed shell to the Fermi surface) as well as a description of nuclear properties created by excitations and interactions by valences nucleons. The shell model calculations have began with the assumption that one or more of these magic nuclei are inactive and have the closed shell configuration and the model space that follows is the space that determines the nuclear properties of the nuclei used in the study [7]. Among the other assumptions of the shell model is that there is a residual interaction (resulting between two valence nucleons), and it is defined as the collision force between the nucleons that occurs due to the perturbation in the Hamiltonian operation, which is equal to the sum of two fundamental parts and is given according to the following relationship [7-9].

\[
\hat{H} = H_D + \hat{V}
\]  
(2)

Where \(H_D\) is given by the following mathematical formula:

\[
H_D = \sum_{i=1}^{A} (\frac{\hbar^2}{2m} \nabla^2 + U_i)
\]  
(3)

As for \(\hat{V}\) can take the following form:

\[
\hat{V} = \sum_{\ell \neq k} V_{\ell k} (r_\ell - r_k) - \sum_{i=1}^{A} U_i (r_i)
\]  
(4)

Whereas: \(H_D\) is the diagonal Hamiltonian that describes the movement of independent nucleons of each other in the same field potential average (Hamiltonian factor without perturbation) and contains the individual particle energies of nucleons, \(\hat{V}\) is the residual interaction between the two particles [10]. The type of residual interaction can be defined as the SDI interaction used in the present study and the idea of the surface delta interaction as is based on a Pauli principle of exclusion, the interaction between nucleon is at the highest peak at the nuclear surface, the interaction of the delta has a short range and is represented by the free (N-N) interaction, and it has some interesting properties, which makes it a completely realistic interaction [10]. Where the interaction of surface delta proved its success in several previous studies, including; Researcher Plastino et al. discussed in 1966 the energy levels of the isotopes of Ni, Pb, Sn and the shell nuclei \(N = 82\) using surface delta interaction and the results appeared good and consistent with scientific results [11]. Researcher Chuu et al. Also discussed in 1979 the calculation of the energy spectrum of the \(^{90}Y, ^{91}Zr, ^{92}Nb, ^{93}Mo\) nuclei by applying the nuclear shell model and using a surface delta interaction where there was good agreement between scientific and theoretical results [12]. In addition, both researchers, ROKadhim and NASalman in 2018 discussed the effects of major polarization on some even-even levels by applying the shell model and using the modified surface delta interaction of the \(^{36}Ar, ^{32}S, ^{28}Si\) cores in the model space \(1d_{3/2} 1d_{5/2} 2s_{1/2}\) [13], the above studies of some researchers approved the success of the nuclear shell model in describing several nuclear properties in an acceptable manner and for several regions in the periodic table, the surface delta interaction has some assumptions as follows [10,14]:

1. The interaction takes place between nucleons only at the nuclear surface.
2. The interaction force of two particles is the force of the delta.
3. The probability that the particle is in the nuclear surface does not depend on the orbit of the shell model in which the particle is moving.

When considering the previous assumptions, the surface delta interaction (SDI) can be expressed in the following form [5]:

\[
V^{SDI} (\vec{r}_1, \vec{r}_2) = -4\pi g_T \delta (\vec{r}_1 - \vec{r}_2) \delta (r_1 - R_0)
\]  
(5)

Whereas: \((R_0)\) is the nuclear radius, \((g_T)\) and represents the interaction force of the surface delta interaction (SDI) which depends on the quantum number of isospin \((T)\) and the factor \(4\pi\) is randomly provided arbitrarily to avoid a factor similar to it in an expression matrix component of two particles for surface delta interaction.
The matrix element of two particles located outside the closed core can be found by using the surface delta interaction according to the following relationship [14,15]:

\[
<j_a|b|V^{SDI}(1,2)|j_c|d> = -(−1)^{n_a+n_b+n_c+n_d}\frac{G_T}{2(2J+1)} \times \\
\sqrt{\frac{(2J_a+1)(2J_b+1)(2J_c+1)(2J_d+1)}{(1+\delta_{J_aJ})(1+\delta_{J_cJ_d})}} \times \left[ \begin{array}{c} \{1 + (-1)^{J}\} <j_a\frac{1}{2}\frac{1}{2}|J1J1> + (-1)^{I_b+I_a+I_b-J_d} \times \\
\{1 - (-1)^{J+I_c+I_d}\} <j_b\frac{1}{2}\frac{1}{2}|J0J0> <j_c\frac{1}{2}\frac{1}{2}|J1J1> + (-1)^{I_b+I_a+I_b-J_d} \times \\
\{1 - (-1)^{J+I_c+I_d}\} <j_b\frac{1}{2}\frac{1}{2}|J0J0> <j_c\frac{1}{2}\frac{1}{2}|J1J1> \end{array} \right] 
\]

Whereas \(<j_a\frac{1}{2}\frac{1}{2}|J0J0> , <j_c\frac{1}{2}\frac{1}{2}|J1J1> > <j_a\frac{1}{2}\frac{1}{2}|J0J0> \) are Clebesh Gordan coefficients, \(J_a,J_c,J_b,J_d\) represent the single particle orbits, \(J\) is the total angular momentum for coupling of single particle orbits and \(G_T\) has the energy dimensions that include interaction force \((g_T)\) and radial integral value. According to the following mathematical formula [14]:

\[
G_T = g_T R_{ab}^a R_{cd}^c 
\]

In order to obtain a systematic description between the experimental data and the predictions of the surface delta interaction for the excited energy levels, some limits have been added to the surface delta interaction which represents a linear mixing of the Wickher and Heisenberg limits which are considered the modification for surface delta interaction that is called (MSDI) and is given according to the following equation [10, 15]:

\[
V_{mix}^{SDI}(1,2) = -4\pi g_T (\delta(\Omega_s)\delta(r_i-R)\delta(r_j-R) + B (r_1 - r_2) + C). 
\]

\(A, B, C\) represent the interaction force and is determined from the experimental spectrum and given according to the following relationships:

\[
A_o = A = B = \frac{2S}{A} MeV \quad \ldots \ldots 
\]

\(B (r_1 - r_2) + C \quad : \quad T = 1 \ldots \ldots \quad (10)\)

While the matrix interactions of the two particles using MSDI have been calculated by assuming that the residual effect of the residual interaction \(V_{ij}\) is the surface delta interaction (SDI) which is given by the following equation [15]:

\[
<j_a|b|V_{MSDI}(1,2)|j_b> = \frac{1}{2}A_T(−1)^{n_a+n_b+n_c+n_d} \times \\
\sqrt{\frac{(2J_a+1)(2J_b+1)(2J_c+1)(2J_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}} \times \left( -1 \right)^{I_b+I_a+I_b-J_d} <j_a\frac{1}{2}\frac{1}{2}|J1J1> + \left< j_b\frac{1}{2}\frac{1}{2}|J0J0> \right> \left< j_c\frac{1}{2}\frac{1}{2}|J1J1> \right> + \left< j_d\frac{1}{2}\frac{1}{2}|J1J1> \right> \left< j_d\frac{1}{2}\frac{1}{2}|J1J1> \right> 
\]

Mixed states of mixed eigenvalues are given by linear sets of unperturbation wave functions [16]:

\[
\Psi_{S,T} = \sum_{k=1}^{n} a_k (S,T) 
\]

Where \(S\) is the number of mixing configurations and takes values \((S = 1, 2, \ldots, n)\), the \(a_k\) parameters satisfy the condition:

\[
\sum_{k=1}^{n} |a_k|^2 = 1 
\]

Through the elements of the Hamiltonian matrix of mixing configuration, the eigenvalues of energy have been calculated in this present study [17-18]:

\[
\langle H \rangle_{11} = \epsilon_{j_1} + \epsilon_{j_2} + \langle j_1, j_2|V_{1,2}|j_1, j_2 \rangle \quad (14) 
\]

\[
\langle H \rangle_{21} = \epsilon_{j_1} + \epsilon_{j_2} + \langle j_1, j_2|V_{1,2}|j_2, j_1 \rangle \quad (15) 
\]

\[
\langle H \rangle_{12} = \langle H \rangle_{21} = \langle j_1, j_2|V_{1,2}|j_1, j_2 \rangle \quad (16) 
\]
In the equations (16,14) in the energy levels of the pure configuration, are calculated only, and for the three equations as a whole (16,15,14) from which the energy levels of the mixed configuration, are calculated.

3. Results and discussion

The nuclear shell model of \(^{116}\text{Sn}\) and \(^{116}\text{Te}\) isotopes containing two neutrons and two protons have been applied, respectively, to calculate excited energies. In this calculations, a different model space is represented by the orbits \((3s_{1/2}, 2d_{3/2}, 1h_{11/2})\) and \((1g_{7/2}, 2d_{5/2}, 3s_{1/2}, 2d_{3/2}, 1h_{11/2})\) for the nuclei \(^{116}\text{Sn}\) and \(^{116}\text{Te}\) respectively. The current results of the aforementioned nuclei on the use of the Fortran 95 program code to program the equations of surface delta and the modified surface delta interactions in the case of mixed orbital configuration of the isotopes above. Currently, the accounts for the nuclei used in the study will be discussed as follow:

3.1 \(^{116}\text{Sn}\) isotope

3.1.1 Calculations of the Surface Delta and Modified Surface Interactions

After calculating the binding energies values for the single particle nucleus represented by \(^{115}\text{Sn}\) and the binding energy of close core nucleus \((^{114}\text{Sn})\), the individual particles energies have been calculated in the mixed order of orbits \((3s_{1/2}, 2d_{3/2}, 1h_{11/2})\). These energy values are shown in Table (1). The values of the total angular momentum and the parity of the mentioned mixed configuration are listed in Table (2). The elements of the matrix have been calculated using the surface delta and modified surface delta interactions according to the equations (6 and 11), by choosing the values of the interaction strengths \(A_T\) of the surface delta interaction represented by the value \((A_T=0.3445\text{MeV})\) and the coefficients values for the interaction strengths of the modified surface delta interaction represented by the values of \(\{A'=0.2030\text{MeV}, B=0.2723\text{MeV}, C=0.3838\text{MeV}\}\)

Then the eigenvalues of the energy have been extracted according to the equations (14-16). These values are important for calculating the energy levels values for all allowable angular momentum in relation to the ground state, which are considered as final values of the excited energy levels. Figure (1) shows a comparison between the excited energy levels of the \(^{116}\text{Sn}\) nucleus with the experimental available values [19]. Through this comparison it is clear that:

1- Theoretical energy levels \((2.1425\text{ MeV}; 2^+_1\text{ and } 2.4642\text{MeV}; 5^+_1)\) appeared for the modified surface delta interaction and \((1.9830\text{MeV}; 2^+_1\text{ and } 2.2858\text{MeV}; 5^+_1)\) for the surface delta interaction agreed well with the practical levels \((2.1123\text{MeV}; 2^-\text{ and, } 2.3659\text{MeV}; 5^-)\) respectively.

2- The total angular momentum has been confirmed and the parity of the practical level \((2.5457; 0^-)\) corresponds well with the theoretical level \((2.4389; 0^-)\) expected for the modified surface delta interaction and \((2.2325; 0^-)\) for the surface delta interaction.

3- It is predicted that there has been the compatibility of theoretical levels \((2.5135\text{ MeV}; 1^+_1\text{, } 2.7298\text{ MeV}; 6^+_1)\text{ and } (2.7264\text{ MeV}; 1^-_1\text{, } 2.9427\text{ MeV}; 6^-_1)\) for the surface delta and modified surface delta interactions levels respectively with the practical levels \((2.5855\text{MeV}, 1^-\text{, } 2.773\text{MeV}, 6^-)\).

4- Theoretical energy levels \((2.7819\text{ MeV}; 7^-_1)\text{ and } (2.9631\text{MeV}; 7^-_1)\) for the surface delta and modified surface delta interactions are well agreed with the practical level \((2.908\text{MeV}, 7^-)\).

5- It is expected a good compatibility with theoretical level \((2.8614\text{ MeV}; 2^+_1\text{ and } 3.0675\text{MeV}; 2^-)\)for the surface delta and modified surface delta interactions levels, respectively, with the practical level \((3.0886\text{MeV}; 2^-)\).

6- A well-predicted theoretical levels of \((3.1373\text{MeV}; 0^+_1\text{ and } 3.3492\text{MeV}; 0^-_1)\) has been shown a compatibility of the surface delta and the modified surface delta interactions respectively with the practical level \((3.2360,0^+)\).

7- It has been predicted the determination of the total angular momentum and parity of the practical level\((3^+_1, 4^+_1, 5^-_1)\text{ 3.257}\text{ MeV};\) that corresponds well to the theoretical levels \((3.1507\text{MeV}; 5^+_2)\text{ and } (3.1507\text{ MeV}; 5^-_2)\) for the surface delta and the modified surface delta interactions.
8- It is expected to confirm the total angular momentum and parity of the practical level (3.35 MeV; $5^+$) that correspond well with the theoretical levels (3.1735 MeV; $4^+$) and (3.3673 MeV; $4^+$) for the surface delta interaction and modified surface delta interactions respectively.

9- It has been predicted a good agree for practical levels with values of {3.4930 MeV; $8^+$, 3.5472 MeV; $10^+$, 3.4162 MeV; $2^+$, 3.427 MeV; $4^-$} with theoretical levels {3.2271 MeV; $4^-$, 3.3337 MeV; $8^+$, 3.3813 MeV; $10^+$, 3.2039 MeV; $2^+$} and the values {3.44 MeV; $4^+$, 3.4117 MeV; $2^+$, 3.5389 MeV; $8^+$, 3.5899 MeV; $10^+$} for both the surface delta and modified surface delta interactions levels, respectively.

10- The total angular momentum of the practical level (3.5136; $2^+$) that agrees well with the theoretical level (3.4741; $6^+$) of surface delta interaction and (3.2271; $6^+$) of the modified surface delta interaction has been confirmed.

Table 1. Represents the single nucleon energies of the $^{116}$Sn nucleus within the model space ($3s_{1/2}, 2d_{3/2}, 1h_{11/2}$)

| Orbits       | Nucleon energy (MeV) |
|--------------|-----------------------|
| $3s_{1/2}(n)$| - 7.54785             |
| $2d_{3/2}(n)$| - 7.05052             |
| $1h_{11/2}(n)$| - 6.8342             |

Table 2. Possible configurations for the $^{116}$Sn nucleus within the model space ($3s_{1/2}, 2d_{3/2}, 1h_{11/2}$).

| Total angular momentum ($J^g$) | configurations                  |
|-------------------------------|---------------------------------|
| $0^+,1^+$                     | $(3s_{1/2}(n))^2$               |
| $1^+,2^+$                     | $3s_{1/2}(n),2d_{3/2}(n)$       |
| $5^+,6^+$                     | $3s_{1/2}(n),1h_{11/2}(n)$     |
| $0^-,2^+$                     | $(2d_{3/2}(n))^2$               |
| $4^+,5^+,6^+,7^+$             | $2d_{3/2}(n),1h_{11/2}(n)$     |
| $0^+,2^+,4^+,6^+,8^+,10^+$    | $(1h_{11/2}(n))^2$             |
Figure 1. Exciting energy diagram for $^{116}$Sn nucleus for SDI and MSDI levels compared to experimental energies [19].
3.2. $^{116}$Te isotope

3.2.1 Calculations of the surface delta interaction (SDI)

Theoretical calculations of the $^{116}$Te nucleus have been performed to obtain the exciting energy levels by using equations (6 and 14-16). By calculating the binding energies of the single particle nucleus $^{115}$Sb relative to the closed core nucleus $^{114}$Sn, the individual particle energies of the nucleons have been obtained within the mixed configurations of the orbits (1g7/2, 2d5/2, 3s1/2, 2d3/2, 1h11/2) for the $^{116}$Te nucleus are listed in Table (3). In accordance with the possible configurations for the states of total angular momentum and parity listed in Table (4), the matrix elements of the two particles outside the closed core have been calculated according to equation (6) and depending on the values of the coefficients chosen for the interaction strength of the surface delta interaction that are represented by the values $\Delta T = 0.2030$ MeV, the values of the final excited energy levels have been calculated. Figure 2 shows a comparison of the energy levels of the $^{116}$Te nucleus with the experimental levels [20].

1- The excited energy levels have been appeared for the values{(1.3128;2+), (1.7601;6+), 1.6308;4+} per MeV unit slightly compatible with practical values levels ,{(1.746;4+), 1.2191;2+},(2.0022;6+) in MeV unit.

2- The total angular momentum is confirmed by the values {(6), (6), (5)} for the practical levels(2.5645,2.3398,2.9662) per MeV unit respectively, These levels appeared to correspond well with the expected theoretical levels {2.9819;5, 2.3637;2, 2.3236;4} in MeV unit.

3- A well matching has been appeared with the theoretical energy level (2.9302 MeV;7+1) with the practical level (3.0273 MeV;7).

4- The total angular momentum has been confirmed for the practical levels {3.2451; (7+) , 3.1901;(7)} per MeV Which has appeared in good conformity with the expected theoretical levels {3.2553;6+1,3.1191;2+1}.

5- A good matching has appeared for the expected theoretical level (3.1191;8) with the practical level (3.175;8).

6- We confirmation , the total angular momentum has been predicted and uniformity of the practical plane (3.7465;9) which has appeared well to be corresponded with the expected theoretical level (3.6611;12+).

7- The overall angular momentum of the practical plane has been confirmed (4.2282;11) with angular momentum and parity 6;5 which has appeared good matching with practical levels (4.1909).

8- Total angular momentum and practical parity have been confirmed{(11), (12), (12+), (13+), (13)} for the practical levels for the values {4.9952,4.9200, 4.5856, 4.4366, 4.3285 MeV} with theoretical angular momentum for levels {4.8554;6+1,4.8554;4+1, 4.6089;2+1,4.3828;10+1,4.3453;3- MeV} respectively.

9- A confirmation of total angular momentum and parity for practical levels have been predicted {(4.9200MeV;13), (5.4725MeV;14), 5.2363MeV;(14)} with the total theoretical angular momentum and parity of the levels {(5.2586;4+1, 5.2277;2+1, 4.8554;4+1 MeV} respectively.

10- New energy levels for total angular momentum and parity have been predicted {4.5;0+1, 3.2+1, 5;3+1, 3+1, 9;8+1, 8;0+1, 4+1, 6+1, 2+2, 3+4, 2;1+1, 2+1, 5+2+1, 6+2, 7+2, 0;1+1, 2+1, 2+1, 5;4+1} they are not limited to the energies and practical momentum so far.

3.2.2 Calculations of the modified surface delta interaction (MSDI)

The excited energies have been calculated in addition to determining the coefficients values for the interaction strength of the modified surface delta interaction{$\Delta T =0.2510$MeV, $B = 0.2723$ MeV, C= 0.3838 MeV} for the mixed configuration depending on the values of the matrix elements that have been calculated for this interaction from equation (11) and then calculating eigenvalues of energy through equations (14, 15 and 16). From them, the final energies representing the values of the excitation energies have been calculated (the energies values in relation to the ground state).
According to Figure (2), which shows a comparison between the theoretical values of the expected energy levels and the values of experimental energies [20], the following is shown:

1- Theoretical energy levels have been found for values {2.3987; 6\textsuperscript{+}, 2.1987; 4\textsuperscript{+}, 1.7188; 2\textsuperscript{+}} slightly compatible with practical levels of values {2.0022; 6\textsuperscript{+}, 1.8118; 4\textsuperscript{+}, 1.2191; 2\textsuperscript{+}}.

2- The total angular momentum of the practical state has been confirmed (2.9165; 4\textsuperscript{+}) compatible with theoretical level (2.5645; 6\textsuperscript{+}).

3- It is expected to determine the total angular momentum for practical levels {4.2282; (11)\textsuperscript{+}, 3.7465; (7), 3.1901; (9)} with total angular momentum and parity of theoretical levels, {3.3541; 9\textsuperscript{+}, 4.2119; 5\textsuperscript{+}, 3.7759; 8\textsuperscript{+}}.

4- The theoretical level has been appeared agreed (3.5263; 7\textsuperscript{+}) with the practical levels (3.0273; 7\textsuperscript{+}).

5- The total angular momentum and parity for practical levels {3.7645; (9), 4.5856; (12)\textsuperscript{+}, 4.4366; (12)\textsuperscript{+}, 4.3285; (11)} has been predicted respectively with by the overall angular momentum and parity of the expected theoretical levels {4.5528; 7\textsuperscript{+}, 4.533; 2\textsuperscript{+}, 4.3178; 1\textsuperscript{+}, 4.0566; 2\textsuperscript{+}}.

6- The total angular momentum for practical levels {5.1975; (14)\textsuperscript{+}, 4.92; (13)\textsuperscript{+}, 4.2282; (11)} has been confirmed with angular momentum and parity of the expected theoretical levels {4.8883; 3\textsuperscript{+}, 4.8476; 4\textsuperscript{+}, 4.2268; 5\textsuperscript{+}}.

7- The total angular momentum for practical levels {5.2363; (14)\textsuperscript{+}, 4.9952; (13)\textsuperscript{+}, 5.4726; (14)\textsuperscript{+}} has been respectively confirmed with levels {5.4406; 7\textsuperscript{+}, 5.2648; 2\textsuperscript{+}, 4.8883; 3\textsuperscript{+}}.

8- The overall and practical angular momentum and parity (15\textsuperscript{+}, 15\textsuperscript{+}, 16\textsuperscript{+}) have been found uncertain for the practical levels {6.106, 5.7758, 5.7215} so it is predicted that angular momentum and parity of the expected theoretical level {5.9114; 4\textsuperscript{+}, 5.8785; 2\textsuperscript{+}, 5.7635; 0\textsuperscript{+}}.

9- New theoretical energy levels for total angular momentum and parity are predicted {0\textsuperscript{2+}, 4\textsuperscript{2+}, 3\textsuperscript{1+}, 5\textsuperscript{1+}, 3\textsuperscript{1+}, 5\textsuperscript{2+}, 5\textsuperscript{2+}, 1\textsuperscript{2+}, 2\textsuperscript{2+}, 6\textsuperscript{2+}, 0\textsuperscript{4+}, 5\textsuperscript{5+}, 4\textsuperscript{2+}, 3\textsuperscript{3+}, 6\textsuperscript{2+}, 8\textsuperscript{2+}, 2\textsuperscript{1+}, 10\textsuperscript{2+}, 4\textsuperscript{3+}, 3\textsuperscript{4+}, 1\textsuperscript{5+}, 5\textsuperscript{4+}, 6\textsuperscript{1+}, 2\textsuperscript{2+}, 3\textsuperscript{1+}, 5\textsuperscript{1+}, 6\textsuperscript{2+}, 2\textsuperscript{2+}, 6\textsuperscript{1+}, 6\textsuperscript{1+}, 8\textsuperscript{1+}} and they are not a specified with practical energies and momentum yet.

Table 3. Represents the single nucleon energies of the $^{116}\text{Te}$ nucleus within the model space (2d\textsubscript{3/2}, 1h\textsubscript{11/2}, 2d\textsubscript{5/2}, 3s\textsubscript{1/2}, 1g\textsubscript{7/2}).

| Orbits | Nucleon energy (MeV) |
|--------|----------------------|
| 1g\textsubscript{7/2}(p) | − 2.50034 |
| 3s\textsubscript{1/2}(p) | − 2.45352 |
| 2d\textsubscript{3/2}(p) | − 2.15219 |
| 1h\textsubscript{11/2}(p) | 1.92371 |
| 2d\textsubscript{5/2}(p) | − 1.48772 |
Table 4. Possible configurations for the $^{116}\text{Te}$ nucleus within the model space $\left(1g_{7/2},2d_{5/2},3s_{1/2},2d_{3/2},1h_{11/2}\right)$.

| $(J^P)$ Total angular momentum | Configuration |
|-------------------------------|--------------|
| $0^+,2^+,4^+,6^+$ | $(1g_{7/2}(p))^2$ |
| $1^+,2^+,3^+,4^+,5^+,6^+$ | $1g_{7/2}(p),2d_{5/2}(p)$ |
| $3^+$ | $1g_{7/2}(p),3s_{1/2}$ |
| $2^+,3^+,4^+,5^+$ | $1g_{7/2}(p),2d_{3/2}(p)$ |
| $2^+,3^+,4^+,5^+,6^+,7^+,8^+,9^+$ | $1g_{7/2}(p),1h_{11/2}(p)$ |
| $0^+,2^+,4^+$ | $(2d_{5/2}(p))^2$ |
| $2^+,3^+$ | $2d_{5/2}(p),3s_{1/2}(p)$ |
| $1^+,2^+,3^+,4^+$ | $2d_{5/2}(p),2d_{3/2}(p)$ |
| $3^+,4^+,5^+,6^+,7^+,8^+$ | $2d_{5/2}(p),1h_{11/2}(p)$ |
| $0^+$ | $(3s_{1/2}(p))^2$ |
| $1^+,2^+$ | $3s_{1/2}(p),2d_{3/2}(p)$ |
| $5^+,6^+$ | $3s_{1/2}(p),1h_{11/2}(p)$ |
| $0^+,2^+$ | $(2d_{5/2}(p))^2$ |
| $4^+,5^+,6^+,7^+$ | $2d_{3/2}(p),1h_{11/2}(p)$ |
| $0^+,2^+,4^+,6^+,8^+,10^+$ | $(1h_{11/2}(p))^2$ |
Figure 2.a. exciting energy diagram for $^{116}$Te nucleus for SDI and MSDI levels compared to experimental energies [20]
Figure 2.b. Exciting energy diagram for $^{116}$Te nucleus for SDI and MSDI levels compared to experimental energies [20].
4- Conclusions :

By using surface delta and modified surface delta interactions of $^{116}$Sn and $^{116}$Te even-nuclei, for mixed configurations, results for the exciting energy levels have been obtained, as follow:

- The nuclear shell model has been successful in the valence region chosen for the two studied nuclei by the appearance of a well matched for the experimental energy levels with the expected theoretical levels for both even- nuclei in this study.
- It has also appeared that the modified surface delta interaction, its accuracy and spaces to the experimental levels are better than when using the surface delta interaction with confirmation and determination of the total angular momentum and the parity of a number of experimental energy levels in both of them.
- New values of exciting energies have been appeared compared to the experimental levels that have not been met so far, and this increases the nuclear theoretical knowledge of these isotopes relative to the expected energy levels.

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