Precise Measurement of $B^0 - \bar{B}^0$ Mixing Parameters at the $\Upsilon(4S)$

CLEO Collaboration

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Abstract

We describe a measurement of $B^0 - \bar{B}^0$ mixing parameters exploiting a method of partial reconstruction of the decay chains $\bar{B} \rightarrow D^{*\pm} \pi^{\mp}$ and $\bar{B} \rightarrow D^{*\pm} \rho^{\mp}$. Using $9.6 \times 10^6 B\bar{B}$ pairs collected at the Cornell Electron Storage Ring, we find $\chi_d = 0.198 \pm 0.013 \pm 0.014$, $|y_d| < 0.41$ at 95% confidence level, and $|\Re(\epsilon_B)| < 0.034$ at 95% confidence level.
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The discovery of $B^0 - \overline{B^0}$ mixing in 1987 [1] signaled a large top quark mass and allows the anticipation of observable CP violating asymmetries in the $B^0$ meson in the near future. Well-known values of the parameters describing mixing will be necessary to extract precise values of CP violating parameters, as planned at asymmetric $B$ factories [2]. In addition, the size of $B^0 - \overline{B^0}$ mixing is characterized by the mass difference parameter $\Delta m_d$ and is proportional to the square of $|V_{td}^* V_{td}|$, the magnitude of one side of the “Unitarity Triangle” [3], which describes some of the mathematical constraints imposed by unitarity upon the elements of the CKM matrix [4]. Accurate measurements of $\Delta m$ for $B^0$ and $B_s^0$ mesons therefore provide an independent check on our understanding of CP violation in the Standard Model.

Mixing in the $B^0 - \overline{B^0}$ system may be described by the parameters $x_d, y_d, p, q$. The parameter $x_d = \Delta m_d/\Gamma_d$, where $\Delta m_d$ is the mass difference between the heavy and light eigenstates $B_H$ and $B_L$, $\Gamma_d$ is the average natural decay width. Similarly, $y_d$ is the normalized lifetime difference between the two eigenstates, and can be written as $\Delta \Gamma_d/2 \Gamma_d$. The parameters $p$ and $q$ describe the $B^0$ and $\overline{B^0}$ amplitudes, respectively, in the eigenstates $B_H$ and $B_L$. When the $\Upsilon(4S)$ is produced with symmetric energy electron positron collisions, the experimentally accessible quantity is $\chi_d$, the time-integrated probability for an initially produced $B^0$ or $\overline{B^0}$ meson to decay as its CP conjugate. It may be written in terms of $x_d$ and $y_d$:

$$\chi_d = \frac{\Gamma(B^0 \rightarrow \overline{B^0})}{\Gamma(B^0 \rightarrow B^0) + \Gamma(B^0 \rightarrow \overline{B^0})} \approx \frac{x_d^2 + y_d^2}{2(1 + x_d^2)}.$$ (1)

Under certain assumptions, a measurement of $\chi_d$ can be combined with direct determinations of $\Delta m_d$ and the $B$ meson lifetime in order to extract $y_d$. The ancillary variable, $\epsilon_B$, is analogous to the $K^0$-mixing parameter $\epsilon$, and is defined through the relation $p = (1 + \epsilon_B)/\sqrt{2(1 + |\epsilon_B|^2)}$. Limits on $\epsilon_B$ can be extracted by searching for a CP violating asymmetry in the events where mixing has occurred. The mixing parameters $y_d$ and $\epsilon_B$ are both expected to be of order $10^{-2}$ [3] with considerable uncertainty.

In this letter, we report new measurements of the $B^0 - \overline{B^0}$ mixing parameters measured at the $\Upsilon(4S)$ resonance. We attempt to determine the beauty quantum number (or flavor) at decay for both of the $B$ mesons produced in the $\Upsilon(4S)$ decay using a novel method subject to systematic uncertainties very different from previous measurements [1], [5]. When the decay flavors of the two $B$ mesons in the event coincide, it indicates that the second $B$ has undergone mixing in the interval between the decays of the two $B$ mesons. The flavor of one $B$ meson at decay is tagged by a high-momentum lepton originating from the decay chain $B \rightarrow X \ell \nu$. The flavor at decay of the remaining $B$ meson is determined through partial reconstruction of the decay chain $\overline{B^0} \rightarrow D^{**} h_W^-$ (charge conjugate modes implied), where $h_W^-$ refers either to a $\pi^-$ or $\rho^-$. The electric charge of the $h_W^-$ identifies (tags) the value of the $B$ flavor at the time of its decay. (We assume the double Cabibbo suppressed decay $\overline{B^0} \rightarrow D^{*-} h_W^+$ is negligible.) By employing the hadronic flavor tag for one $B^0$ in the event, this method sacrifices statistical accuracy relative to methods where a semileptonic decay is used to tag the flavor of both $B$ mesons in the event. However, the systematic error due to the uncertainty in the charged to neutral $B$ meson production ratio at the $\Upsilon(4S)$
that dominates measurements of $\chi_d$ at the $\Upsilon(4S)$ using dileptons is substantially reduced [8]. As a result this method results in a significant improvement in precision over previous measurements of $\chi_d$ at the $\Upsilon(4S)$.

Four charge combinations of hadrons and leptons are possible: $h_W^+\ell^+$, $h_W^+\ell^-$, $h_W^-\ell^-$, and $h_W^-\ell^+$. In the absence of backgrounds or mistags, these correspond to the four flavor combinations $B^0\bar{B}^0$, $\bar{B}^0B^0$, $B^0\bar{B}^0$, and $\bar{B}^0B^0$, respectively. Then,

$$\chi_d = \frac{h_W^+\ell^- + h_W^-\ell^+}{h_W^+\ell^+ + h_W^-\ell^- + h_W^-\ell^+ + h_W^+\ell^-}.$$  

In practice, the raw counts recorded in each of the combinations must be corrected for processes that incorrectly tag the $B$ decay flavor (mistags). Mistags may be due either to leptons not arising from the primary decay $B \to X\ell\nu$ or hadrons not arising from the hypothesized decay chain $\bar{B}^0 \to D^{*+}h_W^-$. Backgrounds (non-$B^0$ events), which can contribute either to the denominator or numerator, must be subtracted.

The data were recorded at the Cornell Electron Storage Ring (CESR) with two configurations of the CLEO detector called CLEO II [7] and CLEO II.V. In the CLEO II.V configuration, the innermost wire chamber was replaced with a precision three-layer silicon vertex detector (SVX) [8]. The results presented here are based upon an integrated luminosity of 9.1 fb$^{-1}$ of $e^+e^-$ data taken at the $\Upsilon(4S)$ energy and 4.4 fb$^{-1}$ taken an average of 60 MeV below $B\bar{B}$ threshold. The Monte Carlo simulation of the CLEO detector was based upon GEANT [9] and simulated events were processed in the same manner as the data.

The method of partial reconstruction used in this Letter has been described in detail elsewhere [8]. By observing the $h_W$ and the soft pion from the decay $D^{*+} \to D^0\pi_s^+$, we deduce the kinematics of the decay chain $\bar{B}^0 \to D^{*+}h_W^-$ without reconstruction of the $D^0$. We denote the charged pion from the $h_W$ as $\pi_f$.

The hadronic $B^0$ decay may be described by three angles. The angle formed, in the $B$ ($D$) rest frame, between the $D^*(D^0)$ flight direction and the direction of the lab frame, is called $\theta_B^*(\theta_D^*)$. A larger value of $\cos\theta_B^*(\cos\theta_D^*)$ corresponds to a higher momentum of the $h_W(\pi_s)$. The third angle, $\phi$, is the angle between the plane of the $\bar{B}^0 \to D^{*+}h_W^-$ decay and the plane that contains the $h_W^-$ and the $\pi_s^+$, as shown in Fig. [1]. All three angles have distinctive distributions for signal and background. The $\cos\theta_D^*$ distribution is constant for signal because the $B$ meson is a scalar particle. The distribution of $\cos\theta_D^*$ shows the 100% polarization in the $D^{*+}\pi^-$ mode from conservation of angular momentum, and shows the 87% polarization that has been measured in the $D^{*+}\rho^-$ mode. The distribution of $\cos\phi$ is a combination of the $\pi_s$ and $h_W$ momenta and the angle between them. For most signal events it reconstructs inside the physical region $-1 < \cos\phi < 1$. For non-signal events as well as signal events with imperfect measurements of the pion momenta, $\cos\phi$ can be calculated but may fail to have a physical value.

In order to select hadronic $D^{*+}h_W^-$ decay candidates for the analysis, both the $\pi_s$ and $\pi_f$ candidate tracks must be well-reconstructed and consistent with originating at the $e^+e^-$ interaction point, and must not be identified as a lepton. We reconstruct $\rho^\pm$ candidates from $\pi_f^\pm\pi^0$ combinations, where the $\pi^0$ is formed from a pair of photon candidates.

The $h_W$ momentum is required to fall in the kinematically allowed range for $\bar{B}^0 \to D^{*+}h_W^-$ decays, assuming $E_{B^0} = E_{beam}$. We require the momentum of the $\pi_s$ to be below 300 MeV/$c$. 

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The $\pi_s$ and $\pi_f$ are required to have opposite electrical charges. We require $|\cos \phi| < 7$ and $\cos \theta_{hW} \pi_s < -0.8$, where $\theta_{hW} \pi_s$ is the angle between the $h_W$ and $\pi_s$. In 8% of selected events, there is more than one combination of charged tracks that satisfy these criteria. In this case, we select the one that is reconstructed as $D^* \pi$ rather than $D^* \rho$. If more than one combination still remains, we choose the one for which the value of $\cos \phi$ is nearest 0.6 (the peak of the signal distribution.) The resulting bias in the $\cos \phi$ shape has a negligible effect upon our measurement of the mixing parameters. The events satisfying these criteria are used to determine the sample composition. For the events we use to measure yields (and therefore the mixing parameters), we use more restrictive criteria, requiring $\cos \theta_{hW} \pi_s < -0.95$, $|\cos \phi| < 2$, and $|\cos \theta_{D^*}| < 1$.

Lepton candidates are selected by requiring that the track is well-reconstructed, consistent with originating from the $e^+e^-$ interaction point, and well identified as either an electron or a muon. We require that the momentum of the lepton candidate is greater than 1.4 GeV/c and we use the angle between the lepton and the $\pi_s$ to suppress semileptonic decays from the unconstructed $D^0$. We veto leptons from the decays $J/\psi \rightarrow \ell^+\ell^-$. If the event has been partially reconstructed as $D^{*+}\pi^-$, we require that the lepton form a large angle with the thrust axis of the remainder of the event in order to suppress $e^+e^- \rightarrow q\bar{q}$ backgrounds, where $q = \{u, d, s, c\}$. If more than one lepton candidate in an event satisfies the criteria, we select the highest momentum candidate.

For the events selected by these criteria, the $B$ decay modes contributing to the $D^{*+}h_{W}^-$ candidates can be divided into five categories: signal ($D^{*+}\pi^-$ and $D^{*+}\rho^-$), other two-body and semi-leptonic $\bar{B}^0$ decays (such as $\bar{B}^0 \rightarrow D^+\rho^-$ and $\bar{B}^0 \rightarrow D^+\ell^-\nu$), two-body and semi-leptonic $B^{\pm}$ decays (such as $B^- \rightarrow D^{*0}\pi^-$ and $B^- \rightarrow D^{*0}\ell^-\nu$), random combinatoric backgrounds, and events of the type $e^+e^- \rightarrow q\bar{q}$ (continuum). For the distributions of two-body $B^0$ and $B^+$ decays in $\cos\theta_{D^*}^*$ and $\cos \phi$, we rely on the simulation. We include 10 two-body and semi-leptonic decay modes of the $B^0$ in the definition of two-body $B^0$ decays, and 12 in the definition of the two-body $B^+$ decays [12]. These decays are well-measured [4] and are reliably modeled by the simulation. Combinations of $h_W$ and $\pi_s$ that
satisfy the analysis requirements, yet originate from neither the signal decays nor the twobody decays, are considered random combinatoric backgrounds. To model these, we use a synthetic distribution in $\cos \theta_{D^*}$ and $\cos \phi$, generated by combining track pairs drawn at random from the observed spectrum of all $B\bar{B}$ track momenta in data. The cosine of the angle between them, $\cos \theta_{hW \pi_s}$, is distributed uniformly. The simulation predicts that the distribution generated with this procedure provides an excellent approximation to the distributions of these decays, and indicates that 40% of the combinatoric background comes from $B^+B^−$ events. Distributions from continuum $q\bar{q}$ production are directly measured in data taken below the $\Upsilon(4S)$ resonance.

In order to determine the composition of our event sample, we divide the sample into two subsets based on the value of $\cos \theta_{hW \pi_s}$. Events for which $−0.9 < \cos \theta_{hW \pi_s} < −0.8$ are the sideband sample; events for which $\cos \theta_{hW \pi_s} < −0.95$ make up the signal sample. We perform a binned two-dimensional maximum likelihood fit [11] simultaneously to the $\cos \phi$ and $\cos \theta_{D^*}$ distributions of the signal sample and the sideband sample. The fit determines the normalization of the first four categories of events; that of the continuum is fixed by the relative luminosity of the data taken at and below the $\Upsilon(4S)$.

The results of the fit are shown in Table I. The two projections of the fits in the mixing sample (more restrictive selection criteria) are shown in Fig. 2. We find that the fit has a Baker-Cousins $\chi^2$ of 129.7 for 151 degrees of freedom. From simulations we find that this corresponds to a confidence level of 91%.

![Fig. 2](image.png)

**FIG. 2.** Projections of the fit to the signal sample onto (a) the $\cos \phi$ axis and (b) the $\cos \theta_{D^*}$ axis. In each plot, the points are the data and the histograms are the best-fit shapes from the simulation.

We also show in Fig. 3 that the distributions of the subset of events in Fig. 2 that contribute to the numerator of Eqn. 2 ($h^{±}l^{±}$ events), are well described by the fit.

The partially reconstructed hadronic tag may incorrectly identify the flavor of the decaying $B$ meson. The dominant source of mistagged events is $B^0$ candidates formed from random combinations of tracks in $B^+B^−$ or $B^0\overline{B}^0$ events. We determine that the mistag
FIG. 3. Projections of the fit to the sample of mixed events onto (a) the cos $\phi$ axis and (b) the cos $\theta_{D^*}$ axis. In each plot, the points are the data and the histograms are the best-fit shapes from the simulation.

TABLE I. The composition of the mixing sample, as determined by the fit. For comparison, the continuum subtracted data sample size is 1865 events. The mistag fraction uncertainties are statistical only.

| Event Type         | Fit          | Mistag Fraction |
|--------------------|--------------|-----------------|
| Signal             | 1241±52      | 0.0006±0.0006   |
| Two-body $B^0$     | 262±60       | 0.020±0.005     |
| Two-body $B^\pm$   | 172±45       | 0.050±0.010     |
| Combinatoric       | 192±21       | 0.21±0.12       |
| Total              | 1867±92      | 0.031±0.012     |

rate of combinatoric events is (21±12)% (where the uncertainty is statistical only), using a separate sample of fully-reconstructed $B \to D^*\ell\nu$ decays in the data. We combine the composition as determined by the fit (Table I) with the mistag rates for each individual component to determine the hadronic mistag rate, also shown in Table I. We calculate a total hadronic mistag rate of (3.1±1.2)%. The uncertainty includes the statistical uncertainties from the fit and in the random combinatoric mistag rate.

The lepton in the event may also mistag the flavor of the decaying $B$ meson for several reasons. Leptons may arise from the secondary decay chain $B \to DX, D \to X\ell\nu$. The magnitude of this source is well-constrained by measurements of the $B \to DX$ spectrum [14] and the known form factors governing $D$ semileptonic decays [13]. We also correct for leptons from $q\bar{q}$ events, $B \to \psi X, \psi \to \ell^+\ell^-$ events [8], misidentified hadrons [12], leptons from $D_s^+$ [14] and other upper vertex ($b \to \tau$) production [17], in-flight decays, $\pi^0$ Dalitz decays, $\gamma$ conversions, and $\delta$ rays [12]. Altogether we find that (3.6±0.5)% of electrons and (3.8±0.5)% of muons incorrectly tag the $B$ decay flavor. The uncertainties are the total systematic uncertainties obtained by adding in quadrature the uncertainties associated with

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TABLE II. Data event yields, sequentially subjected to the corrections, and the value of $\chi_d$ computer from these yields. The uncertainties are statistical only.

| Correction                | $h^\pm \ell^\pm$ | $h^\pm \ell^\mp$ | Corrected $\chi$ |
|---------------------------|-------------------|-------------------|------------------|
| None (raw yield)          | 458±21            | 1524±39           | 0.231±0.010      |
| Continuum                 | 401±23            | 1464±40           | 0.215±0.011      |
| Electron mistags          | 377±23            | 1487±31           | 0.202±0.011      |
| Muon mistags              | 359±24            | 1505±42           | 0.193±0.012      |
| Hadron mistags            | 321±25            | 1543±43           | 0.172±0.012      |
| $B^+B^-$ Background       | 321±25            | 1294±38           | 0.198±0.013      |

TABLE III. Summary of systematic uncertainties for $\chi_d$.

| Source                                | Uncertainty |
|---------------------------------------|-------------|
| Hadronic Mistag Fraction              | 0.009       |
| $B^\pm$ Background                    | 0.007       |
| Two-Body Distributions                | 0.006       |
| Mixed:Unmixed Efficiency Difference   | 0.004       |
| Lepton Mistag Fraction                | 0.003       |
| Mixed:Unmixed Mistag Rate Difference  | 0.003       |
| Signal Shape                          | 0.002       |
| Combinatoric Shape                    | 0.002       |
| Total                                 | 0.014       |

The yields for the mixing sample (more restrictive selection criteria) in the possible charge combinations, and subsequent corrections, are summarized in Table II. We correct the continuum subtracted raw yields of Table II for the mistag levels that we have determined for the leptonic and hadronic tags, then subtract $B^+B^-$ background, which contributes to the denominator even when the beauty quantum number is correctly reconstructed. The total charged $B$ background is (13.3±2.5)%, which is the sum of the two-body $B^+B^-$ decays and the 40% of the combinatoric background that is attributed to $B^+B^-$. The fully-corrected result is $\chi_d = 0.198 ± 0.013 ± 0.014$. The systematic uncertainties are listed in Table III.

The largest systematic uncertainty in $\chi_d$ is due to the uncertainty in the total hadronic mistag rate which in turn is dominated by the uncertainty in the mistag rate of combinatoric background events. The systematic uncertainty in the combinatoric background mistag rate is determined by comparing the mistag rates of energetic pions in data and simulated events, using samples in which the $B$ flavor has been tagged using the decay $D^*+\ell^-\bar{\nu}$. Smaller contributions to the total mistag rate come from the statistical uncertainty of the fit and uncertainty in the two-body mistag rates. We evaluate mistag rates for hadronic and leptonic tags independently, assuming no correlation. The difference in efficiencies and mistag rates for mixed and unmixed events are both found to be consistent with zero in large samples of simulated signal events, and the uncertainty in $\chi_d$ reflects the statistical uncertainty of the finite simulation samples. We assign a systematic uncertainty in $\chi_d$ due to the uncertainty in the mistag rate totaling 9%.
The uncertainty in the $B^+B^-$ background is dominated by uncertainty in the percentage of random combinations that are due to $B^+B^-$ decays. The assigned uncertainty allows the fraction of random combinations arising from $B^+B^-$ decays to vary uniformly from 0 to 100%.

The uncertainty due to the distributions in $\cos \theta_D$ and $\cos \phi$ of two-body decays is evaluated by repeating the analysis, varying in turn each two-body decay mode’s weight according to the experimental uncertainty in its branching fraction and the limited statistics of the simulation.

The uncertainty due to the shape of the signal distribution is assessed by examining the variation of $\chi_d$ as the data are refit with modified signal distributions. Modifications include variations in $D^*\rho$ polarization, overall tracking efficiency, solution multiplicity, and beam energy. The uncertainty due to the fitting distribution of combinatoric decays is evaluated by considering possible variations in the momentum spectrum of random tracks.

We consider three variations, corresponding to decreasing the average momentum of soft tracks, increasing the average momentum of high-momentum tracks and changing the shape of the low-momentum spectrum as if there were 50% more $D^*$’s than expected.

By comparing the yield of $B^0\bar{B}^0$ candidate events to the yield of $\bar{B}^0\bar{B}^0$ candidates, we can limit $\epsilon_B$ from this measurement. We compare $\chi_d$ in events with positively charged leptons to $\chi_d$ with negatively charged leptons, defining $\chi_d = h_W^+\ell^+/h_W^-\ell^-$, and $A_{CP} = (\chi_+ - \chi_-)/(\chi_+ + \chi_-)$, where $A_{CP} = 4\Re(\epsilon_B)$. Charge asymmetries in lepton identification cancel with this method. From studies of detection asymmetries for hadrons and from measurements of hadronic fake contributions, we find negligible systematic bias in the measurement and estimate a systematic uncertainty of 1.4% on $A_{CP}$. We determine $A_{CP} = 0.017 \pm 0.070 \pm 0.014$, corresponding to $\Re(\epsilon_B) = 0.004 \pm 0.018 \pm 0.003$, or $|\Re(\epsilon_B)| < 0.034$ at the 95% confidence level.

We are also able to provide a non-trivial limit on $y_d$ using Eqn.(1) under two assumptions. We assume that any possible $\Delta \Gamma_d$ has negligible impact upon the extraction of $\Delta m_d$ from the experimental results listed in [3], and we assume that indirect CP violation is not present ($|p/q| = 1$). We combine our measurement of $\chi_d$ with the values $\Delta m_d = 0.464 \pm 0.018\text{ps}^{-1}$ and $\tau_{B^0} = 1.56 \pm 0.04\text{ps}$ [4], to find $|y_d| < 0.41$ at 95% confidence level.

We have described a measurement of $B^0 - \bar{B}^0$ mixing parameters $\chi_d$ and $\epsilon_B$, and have combined our result with direct measurements of $\Delta m_d$ to extract limits on $y_d$. We exploit a method of partial reconstruction of the decay chains $B \to D^{*\pm}\pi^\mp$ and $B \to D^{*\pm}\rho^\mp$ subject to systematic uncertainties different from previous measurements. We note that this result is independent of previous CLEO mixing analyses which used leptons to tag the beauty quantum numbers at decay for both $B$ mesons in the event [5]. This measurement provides the first non-trivial limits on $y_d$.

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