One-loop corrections to the processes $e^+e^- \rightarrow \gamma, Z_0 \rightarrow J/\psi \eta_c$ and $e^+e^- \rightarrow Z_0 \rightarrow J/\psi J/\psi$

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The cross section values of $J/\psi \eta_c$ and $J/\psi J/\psi$ production in $e^+e^-$ annihilation are estimated within one-loop approximation near $Z_0$ pole, as well as at higher energies. Both intermediate bosons, $\gamma$ and $Z_0$, are taken into account. It is shown that at $Z_0$ mass the NLO contribution increases the cross-section values by 3.5 times.

1. INTRODUCTION

An associative production $J/\psi \eta_c$ production in $e^+e^-$ annihilation was studied experimentally in details by Belle and Babar Collaborations at interaction energy about 10.6 GeV [1, 2]. It turned out that the cross-section value predicted within LO order approximation was more than 5 times lower than measured one. This gap launched an intensive study of different corrections and production mechanisms.

Two sources of corrections were mainly discussed. First source is the internal motion of quarks inside quarkonium (see the work [3], which initiated the discussion in this direction, as well as works [4–15]). The second source of enhancement is QCD loop corrections. One-loop corrections were estimated in [16, 17]. Today they are known for $B$-factories energies up to two loop accuracy [18]. Also it is worth to mention papers, where both types of corrections are taken into account [19, 20]. Evaluating the results of theoretical studies in the discussed area, we tend to believe that both mechanisms are needed to describe the data.
Study of charmonia production in the future is encouraged by two big projects: ILC and FCC. Both of them propose $e^+e^-$ collisions at energies of order of $Z$-boson’s mass: energy range announced for FCC is $\sqrt{s} = 90 \div 400$ GeV and $\sqrt{s} = 250$ GeV is proposed for ILC. Also it is worth to note, the $Z_0$-boson decays to two charmonia may be of some interest for experiments at the LHC. The $J/\psi \eta_c$ and $J/\psi J/\psi$ pair production near $Z_0$-boson pole were researched within LO in the papers [21, 22]. Recently in the work [23] $Z_0$-boson decays to two charmonia were studied in the framework of lightcone formalism, which allowed to take into account internal motion of quark inside charmonium. Complementing these works, we take into account another type of corrections to this decay, namely, one-loop corrections and also calculate the $J/\psi \eta_c$ and $J/\psi J/\psi$ pair production in $e^+e^-$ annihilation up to one-loop accuracy accounting both intermediate $Z_0$-boson and photon:

$$\begin{cases} 
  e^+e^- \xrightarrow{\gamma^*,Z_0^*} J/\psi, \eta_c, \\
  e^+e^- \xrightarrow{Z_0^*} J/\psi J/\psi.
\end{cases}$$

Discussing $J/\psi J/\psi$ production in $e^+e^-$ annihilation we should mention the theoretical study of this final state production via double photon exchange [24, 25] and so far unsuccessful attempts to find such a process at $B$-factories [1, 2, 26]. However, in this paper we restrict ourselves by studying $e^+e^-$ annihilation into one boson only.

In this paper we consider double charmonia production in close relation to the $B_c$ pair production in $e^+e^-$-annihilation, which was studied in our previous work [27] (see also [28], where the $B_c$ pair production was studied in $\gamma\gamma$-fusion).

2. THE METHOD

The discussed production of the charmonium pair through single boson exchange is affected by several selection rules. First of all neither photon nor $Z$-boson can decay to $\eta_c \eta_c$-pair because the pair of $\eta_c$ mesons in final state must simultaneously have the total angular momentum 1 and the symmetric wave function, which is impossible. Also $J/\psi J/\psi$ pair can not be produced by single photon exchange due to charge parity conservation. Equally to photon the vector part of $Z$-boson vertex can not contribute to $J/\psi J/\psi$ production. Also due to the charge parity conservation the axial part of $Z$-boson vertex does not contribute to $J/\psi \eta_c$ amplitude.
These selection rules were explicitly reproduced by our calculations and therefore provide the additional checks of our work.

Production of double heavy bound states is effectively described by NRQCD factorization. The factorization formalism is designed to factor out the perturbative degrees of freedom and therefore separate the production mechanism into hard and soft subprocesses. The physics reasoning lies in separation of short distance and long distance interactions. If we stand by scales hierarchy $m_c >> m_c v$, where $v$ — velocity of $c$-quark in charmonium then short distance interaction corresponds to perturbative production of $c\bar{c}$-pairs and long distance interaction can describe the bound state dynamics separately.

In this study we neglect the internal motion of quark in quarkonium and put equal the velocities of $c$- and $\bar{c}$- quarks before the projection onto the bound state $\Psi_{c\bar{c}}$. Four final $c$-quarks are fixed two by two and form either two vectors ($P_1$ and $P_2$) or vector and pseudoscalar ($P$ and $Q$ correspondingly):

$$
\begin{align*}
J/\psi & \begin{cases} p_c = P/2 \\
p_c = P/2 \end{cases} \\
\eta_c & \begin{cases} p_c = Q/2 \\
p_c = Q/2 \end{cases}
\end{align*}
$$

The bound states momenta $P, Q$ are kept on mass shells with equal masses: $P^2 = Q^2 = m^2$.

It should be clarified that we describe bound states in colour singlets only. We apply the projection technique to form the bound states. At the lowest order by $v$ spin sums $\sum \lambda_1(p_c)\bar{u}_{\lambda_2}(p_c)\langle(\frac{1}{2}, \lambda_1), (\frac{1}{2}, \lambda_2)|S, s_z\rangle$ over $\lambda_1, \lambda_2$ can be expressed in terms of projector operators:

$$
\Pi_{J/\psi}(P, m) = \frac{P - m}{2\sqrt{m}} \gamma^\mu \otimes \frac{1}{\sqrt{3}}, \quad \Pi_{\eta_c}(Q, m) = \frac{Q - m}{2\sqrt{m}} \gamma^5 \otimes \frac{1}{\sqrt{3}}
$$

(2)

where $m = 2m_c$ — mass of charmonium. These operators are placed in the vertices of bound states and close the fermion lines into traces. Figure 1 shows the diagrams for $e^+e^- \rightarrow J/\psi \eta_c$ process with the corresponding projectors. At next-to-leading order each of projectors can merge quark-antiquark pair either of one fermion line or from two different fermion lines.

The factorized matrix elements have the following form:

$$
\mathcal{M}[J/\psi + \eta_c] = \frac{1}{4\pi} R_{J/\psi}(0) R_{\eta_c}(0) \cdot \mathcal{A}_{J/\psi \eta_c}(P, Q),
$$

(3)

$$
\mathcal{M}[J/\psi + J/\psi] = \frac{1}{4\pi} R^2_{J/\psi}(0) \cdot \mathcal{A}_{J/\psi J/\psi}(P_1, P_2),
$$

(4)
where $A_{J/\psi \eta_c} = A^\mu (P, Q) \varepsilon_\mu (P)$, $A_{J/\psi J/\psi} = A^{\mu \nu} (P_1, P_2) \varepsilon_\mu (P_1) \varepsilon_\nu (P_2)$ and $R_{J/\psi, \eta_c}(0)$ are radial wave functions in origin.

Figure 1: Projection technique for $e^+e^- \to J/\psi \eta_c$ process: (a) at leading order; (b) and (c) at next-to-leading order. In accordance with (2) $P$ denotes $J/\psi$ vertex and $Q$ denotes $\eta_c$ vertex.

An important feature of the studied challenge is absence of corrections for real gluon radiation (since we hold two colour-singlet final states). Thereby complete analysis of QCD corrections involves interference between LO and NLO amplitudes as well as interference between intermediate $\gamma$ and $Z$. The full square of amplitude is of order of $O(\alpha^2 \alpha_s^3)$ and comprises the following seven terms:

$$
|\mathcal{A}|^2 = |\mathcal{A}_{LO}^\gamma|^2 + |\mathcal{A}_{LO}^Z|^2 + 2 \text{Re} \left( \mathcal{A}_{LO}^\gamma \mathcal{A}_{LO}^{\gamma*} \right) + \\
+ 2 \text{Re} \left( \mathcal{A}_{LO}^Z \mathcal{A}_{NLO}^{\gamma*} \right) + 2 \text{Re} \left( \mathcal{A}_{Z}^{LO} \mathcal{A}_{Z}^{NLO*} \right) + 2 \text{Re} \left( \mathcal{A}_{\gamma}^{LO} \mathcal{A}_{\gamma}^{NLO*} \right) + \ldots \quad (5)
$$

The renormalization procedure is organized by building counter-terms from the leading order diagrams. The renormalization constants are listed in (7)–(9). "On shell" scheme is fixed for mass and spinors renormalization and $\overline{\text{MS}}$ scheme is fixed for coupling constant. At next-to-leading order we start the automatic calculation of $\tilde{\mathcal{A}}^{NLO}$ with physical spinors, masses and coupling constant. Final $c$-quarks are kept on mass shells: $p_c^2 = m_c^2$. The isolated singularities are further cancelled with singular parts of $\mathcal{A}^{CT}$ so that $\mathcal{A}^{NLO} = \tilde{\mathcal{A}}^{NLO} + \mathcal{A}^{CT}$ remains a finite expression for renormalized amplitude:

$$
\mathcal{A}^{CT} = Z_2^2 A^{LO} \bigg|_{m \to m_{\text{ren}}, g_s \to g_s}.
$$

### Notes
- The notation $A = A^\mu (P, Q) \varepsilon_\mu (P)$ suggests that $A$ is a polarization vector in the $e^+e^-$ collision. $\varepsilon_\mu (P)$ is a polarization tensor.
- $R_{J/\psi, \eta_c}(0)$ refers to radial wave functions specifically for $J/\psi$ and $\eta_c$.
- The figures (a), (b), and (c) illustrate the projection technique for the $e^+e^- \to J/\psi \eta_c$ process.
- The renormalization constants $Z_2$ are a function of the running coupling constant $\alpha_s$ and must be evaluated at the appropriate scale.
- The on-shell scheme implies that only physical states are considered, excluding virtual states.
- The $c$-quark mass $m_c$ is assumed to be fixed and not renormalized.
\[
Z_{m}^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_\epsilon \left[ \frac{3}{\epsilon_{UV}} + 4 \right] + O(\alpha_s^2), \tag{7}
\]
\[
Z_{2}^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_\epsilon \left[ \frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 4 \right] + O(\alpha_s^2), \tag{8}
\]
\[
Z_{g}^{MS} = 1 - \frac{\beta_0 \alpha_s}{2} \frac{1}{4\pi} \left[ \frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi) \right] + O(\alpha_s^2). \tag{9}
\]

3. CALCULATION DETAILS

Working with Feynman diagrams one can factorize the annihilation process and firstly consider the \(Z\)-boson’s decay into two \(c\bar{c}\) bound states (see Figure 2) as well as the same decay of virtual photon and then square the matrix element along with electric current. There are 4 diagrams at leading-order and 86 diagrams with one loop for \(Z^*\) decay and the same number of diagrams for \(\gamma^*\) decay. The diagrams and the corresponding analytic expressions are generated with FeynArts-package [29] in Mathematica.

![Figure 2: Sample diagrams for \(\gamma^*, Z^* \rightarrow J/\psi \eta_c (J/\psi J/\psi)\) at next-to-leading-order.](image-url)

The computation strategy is based on the following toolchain: \textbf{FeynArts} \to \textbf{FeynCalc} [30] (FeynCalcFormLink [31], TIDL) \to \textbf{Apart} [32] \to \textbf{FIRE} [33] \to \textbf{X}-package [34]. The amplitudes are moved from \textbf{FeynArts} to \textbf{FeynCalc} which provides algebraic calculations with Dirac and colour matrices. Taking traces is established in both \textbf{FeynCalc} and \textbf{FORM}. However we don’t evaluate \textbf{FORM} code directly rather work with package \textbf{FeynCalFormLink} which embegges \textbf{FORM}. 


into Mathematica and allows one to save time. The Passarino-Veltman reduction is conducted with TIDL library which also appears in FeynCalc. After this reduction we keep only scalar expressions by loop momentum $k$. $\textit{Apart}$ function does the extra simplification by partial fractioning for IR-divergent integrals. At last FIRE package provides the complete reduction to master integrals. It implements several strategies for IBP reduction mostly based on the Laporta algorithm \cite{35}. The master integrals are evaluated by substitution of their analytical expressions with the help of X-package. It is worth mentioning that all the calculations are analytical. The masses and other parameters are set at last step to obtain the numerical values.

The regularization technique is performed in conventional dimensional regularization (CDR) scheme where all the momenta live in $D$ dimensions: loop momentum as well as external momenta. Since Dirac matrices are also $D$-dimensional the problem of $\gamma^5$ interpretation arises as it is badly determined in $D$ dimensions. The so-called naive interpretation is imposed for this challenge: $\gamma^5$ anticommutes with all other matrices and therefore disappears in traces with an even number of $\gamma^5$. For traces with an odd number of $\gamma^5$ the following expression is applied for the left one:

$$
\gamma^5 = -\frac{i}{24} \varepsilon_{\alpha\beta\sigma\rho} \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\rho,
$$

where $\varepsilon_{\alpha\beta\sigma\rho}$ is either 4- or $D$-dimensional. It is checked that calculation result does not depend on $\varepsilon_{\alpha\beta\sigma\rho}$ dimension in \textcolor{red}{(10)}. The choice of $\varepsilon_{\alpha\beta\sigma\rho}$ dimension slightly affects the traces evaluation process but it has no effect on the renormalized amplitudes.

The set of evaluated diagrams contains ones with triangle loops (see diagram 4 in the Figure \textcolor{red}{2}). These diagrams do not contribute to any of examined processes because of $P$-parity violation and assume the verification test for calculations. The contribution from diagrams with two distinct traces projection (see diagram 3 in the Figure \textcolor{red}{2}) is relevant only for $J/\psi \eta_c$ production. However it is pretty small (about 3% of total amplitude). We treat bubble loops (see diagram 8 in the Figure \textcolor{red}{2}) massive for $t$, $b$- and $c$-quarks while masses of light quarks $u, d, s$ are neglected. One of the additional checks is that we explicitly obtain zero for prohibited process of $\eta_c \eta_c$ production at both LO and NLO levels.

It is worth mentioning that usage of FIRE reduction raises terms $\sim \frac{1}{D-4}$ in amplitudes. Therefore one should carefully execute the processing of master integrals — the terms $\mathcal{O}(\varepsilon)$ in masters asymptotics might contribute to the finite part of amplitude. However in this study
the amplitude terms $\sim \frac{1}{D-4}$ cancel each other (unlike the study \[27\]). After FIRE we treat only one-, two- and three-point integrals $A_0, B_0, C_0$. The infinite part coming from divergent masters $A_0, B_0$ carries poles $O(1/\varepsilon)$ only. Working with automatic tools we do not distinguish $\varepsilon_{IR}$ and $\varepsilon_{UV}$ rather handle them together: $\varepsilon_{IR} = \varepsilon_{UV} = \varepsilon$.

| Table I: Parameters involved in calculations. |
|------------------------------------------------|
| $m_c = 1.5 \text{ GeV}$                  |
| $m_b = 4.5 \text{ GeV}$                  |
| $m_t = 172.8 \text{ GeV}$               |
| $M_Z = 91.2 \text{ GeV}$                |
| $R_{J/\psi}^2 = 1.1 \text{ GeV}^2$     |
| $R_{\eta_c}^2 = 1.1 \text{ GeV}^2$     |
| $\Gamma_Z = 2.5 \text{ GeV}$            |
| $\sin^2 \theta_w = 0.23$               |

In the presented calculations the strong coupling constant is taken with two loops accuracy:

$$\alpha_S(Q) = \frac{4\pi}{\beta_0 L} \left( 1 - \frac{\beta_1 \ln L}{\beta_0^2 L} \right),$$  \hspace{1cm} (11)

where $L = \ln (Q^2/\Lambda^2)$; $\beta_0 = 11 - \frac{2}{3} N_f$ and $\beta_1 = 102 - \frac{38}{3} N_f$ with $N_f = 6$; the reference value is $\alpha_S(M_Z) = 0.1185$. We have chosen the same scale for renormalization and coupling scales: $Q = \mu_R = \mu$. The fine structure constant is fixed in Thomson limit $\alpha = 1/137$. Numerical values for all the rest parameters are shown in Table II.

4. RESULTS

We present the analytic expressions for leading order cross sections in Equations (12) to (15) and highlight the contributions from $\gamma$ annihilation, $Z$ annihilation and interference between $\gamma$ and $Z$. At next-to-leading order expressions are quite large so it is more reasonable to introduce the reference values (see Table II).

$$\sigma_{J/\psi \eta_c} = \frac{131072\pi \alpha^2 \alpha_s^2 R_{J/\psi}^2 R_{\eta_c}^2 (1 - 4m^2/s)^{3/2} (1 + a_{\gamma Z} + a_Z)}{243 s^4},$$  \hspace{1cm} (12)

$$\sigma_{J/\psi J/\psi} = \frac{32\pi \alpha^2 \alpha_s^2 R_{J/\psi}^4 (1 - 4m^2/s)^{5/2} \left( \csc^4 \theta_w - 4 \csc^2 \theta_w + 8 \right) \sec^4 \theta_w}{27 s^2 \left( (M_Z^2 - s)^2 + \Gamma^2 M_Z^2 \right)},$$  \hspace{1cm} (13)

where:

$$a_{\gamma Z} = \frac{\tan^2 \theta_w \left( 3 \csc^4 \theta_w - 20 \csc^2 \theta_w + 32 \right)}{16} \frac{s \left( s - M_Z^2 \right)}{(M_Z^2 - s)^2 + \Gamma^2 M_Z^2},$$  \hspace{1cm} (14)

$$a_Z = \frac{\tan^4 \theta_w \left( \csc^4 \theta_w - 4 \csc^2 \theta_w + 8 \right) \left( 8 - 3 \csc^2 \theta_w \right)^2}{512} \frac{s^2}{(M_Z^2 - s)^2 + \Gamma^2 M_Z^2}.$$  \hspace{1cm} (15)
Table II: Cross section values at NLO for different collision energies and renormalization scales.

| µ = √<i>s</i> | √<i>s</i> = 0.25MZ | √<i>s</i> = 0.5MZ | √<i>s</i> = MZ | √<i>s</i> = 2MZ |
|---------------|-------------------|-----------------|---------------|-----------------|
| σ<sub>J/ψ η_c</sub>, fb | 3.01 · 10⁻² | 1.21 · 10⁻⁴ | 2.66 · 10⁻⁵ | 1.98 · 10⁻⁹ |
| σ<sub>J/ψ J/ψ</sub>, fb | 4.27 · 10⁻⁶ | 4.68 · 10⁻⁷ | 2.25 · 10⁻⁵ | 1.18 · 10⁻¹⁰ |
| µ = 10 GeV | σ<sub>J/ψ η_c</sub>, fb | 4.03 · 10⁻² | 2.10 · 10⁻⁴ | 6.05 · 10⁻⁵ | 5.84 · 10⁻⁹ |
| | σ<sub>J/ψ J/ψ</sub>, fb | 5.68 · 10⁻⁶ | 8.14 · 10⁻⁷ | 5.16 · 10⁻⁵ | 3.56 · 10⁻¹⁰ |

Behaviour with energy for cross sections with full annihilation account is performed in the Figures 3–6. The peak in the Figure 5 represents the natural width of Z-boson. In these plots the scale µ is chosen as µ = √<i>s</i>. Variation of the scale parameter µ provides the appropriate way to present the calculation uncertainties. Thereby we vary µ in the range √<i>s</i>/2 < µ < 2√<i>s</i> and present the shaded area Figures 9–12 duplicating the same plots with µ fixed.

It can be seen that NLO contribution significantly enhances the leading order values. The maximums for cross sections correspond to √<i>s</i> ≈ 7 GeV for J/ψ η<sub>c</sub> and to √<i>s</i> ≈ 9 GeV for J/ψ J/ψ. One can make sure that our results for J/ψ η<sub>c</sub> production at low energies reproduce ones of the earlier works [16, 17, 19, 20]. Particularly we obtain σ<sub>J/ψ η_c</sub> ≈ 15.5 fb at B-factories energy 10.6 GeV if setting the scale µ = m. At low energies virtual photon exchange plays the dominant role and therefore J/ψ J/ψ production through virtual Z<sub>0</sub>-boson decay is highly suppressed with respect to J/ψ η<sub>c</sub> case. This suppression is of order 10⁻⁶ at energies below 10 GeV and goes down as energy gets closer to M<sub>Z</sub>.

Cross sections for J/ψ J/ψ and J/ψ η<sub>c</sub> production are very close in magnitude near Z-boson’s pole. At energies close to M<sub>Z</sub> the annihilation dominantly goes through Z<sub>0</sub>-exchange and therefore the suppression factor caused by prohibition for photon exchange becomes not relevant. It is interesting to compare the ratio \( r = \sigma_{J/ψ η_c}/\sigma_{J/ψ J/ψ} \) at energy √<i>s</i> = M<sub>Z</sub> with the ratio \( R = \Gamma (Z \rightarrow J/ψ η_c)/\Gamma (Z \rightarrow J/ψ J/ψ) \) for decay widths. Our estimations are in a pretty good agreement with results of the paper [23], where R was calculated within LO accuracy:

\[
r_{LO} = 1.22, \quad r_{NLO} = 1.18, \quad R_{LO} = 1.20. \tag{16}
\]

It is interesting to note, that as it is seen from the comparison of the Figures 4 and 5 the cross sections for J/ψ J/ψ production at energies of B-factories and at energies near Z-pole are comparable in magnitude.
Figure 3: $\sigma_{LO}$ and $\sigma_{NLO}$ dependence on collision energy for $e^+e^- \rightarrow J/\psi \eta_c$.

Figure 4: $\sigma_{LO}$ and $\sigma_{NLO}$ dependence on collision energy for $e^+e^- \rightarrow J/\psi J/\psi$.

Figure 5: $\sigma_{LO}$ and $\sigma_{NLO}$ dependence on $\sqrt{s}$.

Figure 6: $\sigma_{NLO}/\sigma_{LO}$ ratio dependence on $\sqrt{s}$. 
The asymptotic behaviour of cross-sections with collision energy is established: the leading order cross-sections decrease with energy as $O(1/s^4)$ while at next-to-leading order decreasing is slower $O(\ln^2 s/s^4)$. Figure 6 demonstrates rising of the ratio $\sigma_{NLO}/\sigma_{LO}$ with energy. It seems that such behaviour of NLO corrections cannot be compensated by the scale choice at very high energies. As a result at energies of collision below $2M_Z$ the NLO contribution enhances cross-sections up to 5 times.

NLO study of exclusive production of quarkonium states is known to face problems in double logarithmic terms that specify the corrections magnitude at high energies. In this study we confirm the result of the previous researches [36, 37] for the process $e^+e^- \rightarrow J/\psi \eta_c$ and obtain the double logarithmic terms in the expansion at $\sqrt{s} >> m$:

$$A_{NLO}^A / A_{LO}^A \sim \alpha_s \left( c_3 \ln^2 s + c_2 \ln s + c_1 \ln \mu + c_0 \right).$$

Also we demonstrate for the first time the same behaviour both for the processes $e^+e^- \rightarrow Z_0 \rightarrow J/\psi \eta_c$ and $e^+e^- \rightarrow J/\psi J/\psi$. In this context pair charmonia production differs from pair $B_c$ production. As it is shown in paper [27] NLO QCD corrections to $e^+e^- \rightarrow B_c(\ast)B_c(\ast)$ are stable with energy increasing. This fact requires separate consideration, which will be carried out in the next study.

We provide the Figures 7 and 8 to demonstrate the importance of account for $Z_0$ exchange in $J/\psi \eta_c$ production. At $Z_0$ mass $\sigma(Z^* + \gamma^*)/\sigma(\gamma^*) \approx 60$. At energies away from $Z_0$ mass...
deviation of this ratio from one is related to the sign of $\gamma - Z$ interference member (see Figure 8). Roughly we can estimate that $\sigma(Z^* + \gamma^*)/\sigma(\gamma^*) > 1.1$ in the range $0.8M_Z < \sqrt{s} < 2M_Z$.

It is worth to mention that $P$-symmetry is not violated in the studied challenge. Although $P$-asymmetry could be induced by $V - A$ interference in $Z$–quarks coupling it has no place in the particular case due to selection rules. Charge parity conservation lets only one part of $V - A$ structure to act (either vector or axial vector). Both of them are not able to violate $P$-symmetry alone.

Besides the interest in studying double charmonia production in $e^+e^-$ collisions we can not fail to refer to searches for $Z \rightarrow J/\psi J/\psi$ decays at LHC. Now the study of $Z$ decays to double quarkonia states is motivated by the first CMS search \cite{38} where Higgs and $Z$ boson decays to $J/\psi$ and $\Upsilon$ pairs are examined in the four-muon final state. In this way our work complements predictions of \cite{23} showing that $\Gamma(Z \rightarrow J/\psi J/\psi)$ and $\Gamma(Z \rightarrow J/\psi \eta_c)$ should be approximately 3.5 times larger at next-to-leading order of NRQCD.

5. CONCLUSIONS

We calculate cross sections for associative $J/\psi \eta_c$ and $J/\psi J/\psi$ production in $e^+e^-$ annihilation at next-to-leading order of NRQCD. We consider single boson annihilation including $\gamma$ exchange, $Z$-boson exchange and $\gamma - Z$ interference. It is shown that at energies near $Z$-bosons mass as well as at higher energies annihilation with photon only becomes insufficient. QCD corrections $\mathcal{O}(\alpha_S)$ significantly enhance the leading-order cross sections. At energies below $2M_Z$ cross sections are enhanced up to 5 times. In $Z$-pole we obtain $\sigma_{NLO} \approx 3.5 \sigma_{LO}$. The same enhancement in 3.5 times applies to widths of decays $Z \rightarrow J/\psi J/\psi$ and $Z \rightarrow J/\psi \eta_c$.

The results performed in the paper might be relevant for future study of charmonia physics at ILC or FCC colliders. In addition they are directly related to searches for rare decays of $Z$ to double quarkonia states at LHC.

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Figure 9: $\sigma_{NLO}$ dependence on collision energy for $e^+e^- \rightarrow J/\psi \eta_c$ at different scales.

Figure 10: $\sigma_{NLO}$ dependence on collision energy for $e^+e^- \rightarrow J/\psi J/\psi$ at different scales.

Figure 11: $\sigma_{NLO}$ dependence on $\sqrt{s}$ for different scales.

Figure 12: $\sigma_{NLO}/\sigma_{LO}$ ratio dependence on $\sqrt{s}$ for different scales.
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