The Rise and Fall of Bioenergy.*

Michael Hoel†

January 24, 2017
Preliminary and incomplete

Abstract

The production and use of bioenergy may have a negative impact on the climate. However, if it has a less negative impact than carbon (fossil) energy, it may be socially optimal to have a significant increase in the use of bioenergy towards a future decarbonized economy. The paper demonstrates that due to the difference in the way the climate is affected by the two types of energy, the path of bioenergy towards a long-run steady state may be non-monotonic: It may be socially optimal to first have an increase in the use of bioenergy, and later a reduction towards the long-run value. The optimal taxes and subsidies are also derived, both for the first-best case and for the case of a constraint on the carbon tax or the energy price.

Keywords: bioenergy, renewable energy, climate policy, carbon tax, second best, subsidies.

JEL classification: Q42, Q48, Q54, Q58

*While carrying out this research I have been associated with CREE -Oslo Centre for Research on Environmentally friendly Energy. CREE is supported by the Research Council of Norway.

†Department of Economics, University of Oslo, P.O. Box 1095, Blindern, N-0317 Oslo, Norway. Email: mihoel@econ.uio.no
1 Introduction

In many countries there are various forms of direct and indirect subsidies to biofuels and other types of bioenergy. Such policies are often justified by an argument that bioenergy is "climate neutral", and that the production and use of bioenergy gives a reduction in the use of fossil energy. There are two problems with these arguments. First, recent contributions have questioned whether the production and use of bioenergy is climate neutral in the sense that it has no climate impact. Obvious sources of greenhouse gas emissions from bioenergy include the use of fertilizer when growing bioenergy crops (Crutzen et al, 2008), and the use of fossil energy in the harvesting and processing of bioenergy (Macedo et al, 2008). Land use change can lead to additional greenhouse gas emissions if the area of arable land is increased to accommodate increasing use of bioenergy. An alternative to converting grazing land or forest land into land for growing suitable crops for bioenergy production is to use the harvests from standing forests to produce bioenergy. However, the carbon stored in the forest is highest when there is little or no harvest from the forest. Hence, increasing the harvest from a forest in order to produce more bioenergy may conflict with the direct benefit of the forest as a sink of carbon.

Even if the production and use of bioenergy had no climate impact, the argument for subsidizing it is questionable. It is widely recognized among economists that a price on carbon emissions, through a carbon tax or a price on tradeable emission permits, is the most important policy instrument to reduce such emissions. Standard economic reasoning also implies that in the absence of other market failures, an appropriately set carbon price is the only instrument needed to achieve an efficient climate policy. If bioenergy had no direct climate impact, it should therefore be neither taxed nor subsidized. A possible reason for subsidizing bioenergy could be that tax on fossil energy is "too low", i.e. lower than the Pigovian rate (equal to the marginal environmental cost of carbon emissions).
To study these issues, this paper considers a simple model where fossil fuels (henceforth called carbon energy) and bioenergy are perfect substitutes. The model is presented in section 2, and has the following important features. Unlike carbon energy, bioenergy is climate neutral in the sense that it is possible to have a constant positive use of bioenergy without this giving any change over time in the carbon concentration in the atmosphere. This implies that in a long-run steady state, we will (under conditions specified in section 3) have positive bioenergy production and zero carbon energy production.

Although bioenergy is climate neutral in the sense given above, it has a negative climate impact in the sense that for any given time path of carbon energy, the carbon in the atmosphere is higher the higher is the level of the time path (constant or varying over time) of the production of bioenergy. Hence, both carbon energy and bioenergy have a negative climate impact, but the dynamics of these impacts differ. In section 3 the first-best optimum is derived, and it is shown that carbon energy production is zero in the long run, while bioenergy production is positive and constant. Due to the difference in how the climate is affected by the two types of energy, the time path of bioenergy production towards its steady-state level may be non-monotonic. Particular attention is given to the case in which marginal climate costs have an inverse L property: zero (or small) for carbon contents in the atmosphere below some threshold, and rapidly rising after the threshold is reached. In this case bioenergy production will first rise, and later decline towards its steady-state level.

To achieve the first-best optimum, bioenergy must have a non-negative tax. In the limiting case of zero marginal climate costs for low levels of carbon in the atmosphere the tax will be zero initially, but will eventually become positive.

As mentioned above, the tax on carbon energy could for some reason be constrained to be lower than its optimal value. The implications of such a constraint for bioenergy production and bioenergy policies are studies in
section 4. In particular, it is shown that in this case it may be optimal to subsidize bioenergy for all or some of the time. For the inverse L climate cost function, it is optimal to subsidize bioenergy as long as carbon in the atmosphere is below the threshold. Moreover, during this phase the subsidy is increasing over time. However, once the threshold is reached the subsidy will start to decline. In the long run the tax on bioenergy may be negative (i.e. a subsidy) or positive, depending on parameters of the model.

Section 5 concludes.

2 The climate effects of carbon energy and bioenergy

The climate is affected by the amount of carbon in the atmosphere, and burning of fossil fuels (i.e. the use of carbon energy) adds carbon to the atmosphere. As we shall see below, this is true also for bioenergy, although the mechanism is somewhat different. For both types of energy, the model I use gives a very crude and simple description of how the atmospheric carbon is affected. The following quotation from Millner (2015) might help to justify the somewhat drastic simplifications I use: "Like many economic models, our is a fable in which reality is pushed to absurdly simplistic extremes in order to cleanly illustrate a mechanism that may help explain real world policy outcomes."

2.1 Atmospheric carbon from carbon energy

Using carbon energy gives an immediate release of carbon to the atmosphere. Over time, some of this carbon is transferred to the ocean and other sinks. However, a significant portion (about 25% according to e.g. Archer, 2005) remains in the atmosphere for ever (or at least for thousands of years). In this paper I simply assume that all the carbon released remains in the atmosphere.

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for ever. This simplification is of no importance for the long-run steady state, but the details of the dynamics toward the steady state would be slightly different if I had taken the partial depreciation into account.

In my model, \( x(t) \) measures the yearly production of carbon energy, and \( S_x(t) \) measures the stock of carbon in the atmosphere caused by the production of carbon energy. The relationship between the two is simply 
\[ \dot{S}_x(t) = x(t). \]

2.2 Atmospheric carbon from bioenergy

There are two possible sources of climate effects of producing and using bioenergy. The first of these is the emissions related to the production and transportation of bioenergy. Even if these are positive, one must be careful about what conclusions one draws from this. There are emissions related to practically all economic activity. It is only if the emissions related to production and transportation of bioenergy are larger per unit of general resources used (labour, capital etc.) than the emissions related to these resources being used elsewhere in the economy that one can conclude that increased use of bioenergy contributes to increased climate costs.

The second source of climate effects from producing and using bioenergy has to do with land use. If increased production of bioenergy uses land that otherwise would be barren and unproductive, there is no negative impact from the production of bioenergy on the climate. The same might be true if the land used for bioenergy production was previously used for food production. However, reduced land for food production may cause indirect land use changes: The increased value of land used for food production may lead to forest clearing to convert land for food production. This forest clearing will have the immediate effect of releasing carbon to the atmosphere, and thus increase climate costs. The same negative effect will of course be present if the increased production of bioenergy implies a direct land use change via forest land being converted directly to agricultural land for bioenergy crops.
Emissions from direct land-use change are treated by Fargione et al. (2008). Their study shows that converting different types of virgin land to crop land may give high initial emissions. Bioenergy crops are however often grown on existing agricultural land. Indirect land use change then refers to changes in land use that occurs through changes in the market prices for food and land. Both Searchinger et al. (2008) and Lapola et al. (2010) analyze indirect land-use change, and show that the effect upon emissions may be of great significance.

In my model I treat direct land-use change and indirect land-use change together. In particular, I assume that each unit of bioenergy production requires \( \ell \) units of land, and that each unit of land converted to bioenergy production will reduce the carbon sequestered on this land by an amount \( \sigma \). Producing \( y \) units of bioenergy hence increases the amount of carbon in the atmosphere by \( \sigma \ell \equiv \beta \) units.

In the model, carbon energy and bioenergy are measured in the same units, with 1 unit being equal to the amount of carbon energy that releases 1 unit of carbon to the atmosphere. Hence the denomination of \( y \) is tons of carbon per year. The denomination of \( \ell \) is land units per \( y \), and the denomination of \( \sigma \) is tons of carbon per land unit. Hence, the denomination of \( \beta \) is tons of carbon per \( y \), i.e. tons of carbon/(tons of carbon per year), i.e. "years". Using this approach, Greaker et al. (2016) show that estimates in the literature of \( \ell \) and \( \sigma \) suggest that \( \beta \) typically will be in the range of about 9-24 years.

In the reasoning above it has been implicitly assumed that bioenergy is based on food crops. An alternative to converting grazing land or forest land into land for growing suitable crops for bioenergy production is to use the harvests from standing forests to produce bioenergy. However, wood-based bioenergy from standing forests is not unproblematic from a climatic point of view. The carbon stored in the forest is highest when there is little or no harvest from the forest. Hence, increasing the harvest from a forest in order
to produce more bioenergy may conflict with the direct benefit of the forest as a sink of carbon.

Wood-based bioenergy may take many forms, including e.g. raw firewood, processed charcoals, and pellets. The possibility of producing liquid bioenergy from cellulosic biomass may also be a promising alternative to using food crops, see e.g. Hill et al. (2006). To the extent that bioenergy is produced from residuals that otherwise would gradually rot and release carbon, the climate impact of this type of bioenergy is limited. If increased production of bioenergy implies increased harvesting from the forest, the time path of the carbon in the forest biomass will be shifted downwards, giving an unambiguously negative climate effect (i.e. increased climate costs). This is illustrated in figure 1, where $g(F)$ is a standard biological growth function showing the growth of the forest as a function of its volume $F$ (measured in tons of carbon). With harvesting $h$ the net growth is hence $g(F) - h$. If yearly harvesting increases from a constant value $h^*$ to a new constant value $h^{**}$, the forest volume will eventually decline from $F^* = g^{-1}(h^*)$ to $F^{**} = g^{-1}(h^{**})$. Assume that one unit increased bioenergy production implies $\delta$ units increased harvest, implying that one unit permanent increase of bioenergy production will give a permanent reduction of the volume of the forest equal to $-\delta g'(h)$. Moreover, assume that a fraction $\xi$ of the carbon released from the forest is permanently added to the carbon in the atmosphere. Then one unit permanent increase of bioenergy production will give a permanent increase in the carbon in the atmosphere equal to $-\xi \delta g'(h) = \beta$. Also in this case, $\beta$ is measured in years, and in this case it will depend strongly on how intensively the forest initially is harvested. If the initial harvest $h$ is close to the maximal sustainable yield value (i.e. close to max $g(F)$), $g'$ will be small and hence $(g')^{-1}$ and $\beta$ will be large.

In my model, $y(t)$ measures the yearly production of bioenergy, and $S_y(t)$ measures the stock of carbon in the atmosphere caused by the production of carbon energy. The relationship between the two is simply $S_y(t) = \beta y(t)$. 

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Although such a relationship might be reasonable in the long run, it is of course quite a drastic simplification in the short run. From the reasoning above it is clear that a change in $y(t)$ from one constant level $y^1$ to a new constant level $y^2$ will not immediately change $S_y(t)$ from $\beta y^1$ to $\beta y^2$. In particular, the reduction in $S_y$ caused by a reduction in $y$ may take considerably longer time than my simple model suggests.

3 The optimal use of bioenergy

3.1 The model

Let $x$ and $y$ denote the production and use of carbon energy and bioenergy, respectively. I assume they are perfect substitutes so the gross benefit of using these two types of energy is $u(x + y)$, which is strictly increasing and concave. In some of the reasoning I assume that $u'(0)$ is finite, although this is not a crucial assumption. Production costs for carbon energy and bioenergy are given by the strictly convex cost functions $c(x)$ and $b(y)$, respectively.\(^1\)

To insure an interior solution in the absence of climate costs I assume $c'(0) = b'(0) < u'(0)$, although the equation $c'(0) = b'(0)$ is is not important for the main results. Climate costs are given by an increasing and convex function of the total amount of carbon in the assumed in the atmosphere, $D(S)$. Unless otherwise stated, I assume that this function is strictly convex.

The objective function in the social optimization problem is

$$W = \int_0^\infty e^{-rt} [u(x(t) + y(t)) - c(x(t)) - b(y(t)) - D(S(t))] \, dt$$

(1)

where $r$ is the discount rate. In addition to the non-negativity constraints

\(^1\)I disregard a resource constraint on the production of carbon energy. An obvious interpretation is that the resource reserves are so large that they due to climate considerations do not imply a binding constraint on the production of carbon energy.
on $x$ and $y$, we have the following constraints (explained in section 2):

\begin{align}
S(t) & = S_x(t) + S_y(t) \quad (2) \\
\dot{S}_x(t) & = x(t) \quad (3) \\
S_y(t) & = \beta y(t) \quad (4)
\end{align}

Notice that with this formulation we have implicitly chosen units of fossil fuel and carbon in the atmosphere in the same units, e.g. tons of carbon, while the units of bioenergy are such that one unit of bioenergy is equally useful to the user as one unit of fossil energy.

The current value Hamiltonian corresponding to the maximization of $W$ subject to (2)-(4) is (ignoring the time references)

\begin{equation}
H = u(x + y) - c(x, y) - D(S_x + \beta y) + (-\lambda)x \quad (5)
\end{equation}

and the conditions for the optimum are

\begin{align}
u'(x + y) - c'(x) - \lambda & \leq 0 \quad [= 0 \text{ for } x > 0] \quad (6) \\
v'(x + y) - b'(y) - \beta D'(S) & \leq 0 \quad [= 0 \text{ for } y > 0] \quad (7) \\
\dot{\lambda} & = r\lambda - D'(S) \quad (8) \\
\lim_{t \to \infty} e^{-rt} \lambda(t) & = 0 \quad (9)
\end{align}

The costate variable $\lambda(t)$ may be interpreted as the optimal carbon tax (i.e. tax on fossil energy) in a market economy. From (8) and (9) we can derive

\begin{equation}
\lambda(t) = \int_0^\infty e^{-r\tau} D'(S(t + \tau))d\tau \quad (10)
\end{equation}

This well-known formula for the optimal carbon tax, or social cost of carbon, has a straightforward interpretation: The optimal carbon tax at time $t$ is equal to the discounted value of the marginal climate cost caused at all future dates from one unit of carbon use at time $t$. 
The term $\beta D'$ may be interpreted as the optimal tax on bioenergy. Clearly this is always positive (or at least non-negative, but zero for any $t$ such that $D'(S(t)) = 0$). Without more assumptions about the function $D(S)$ and the parameter $\beta$ it is not possible to say much about the relative sizes or relative growth rates of the two energy taxes ($\lambda$ and $\beta D'$), at least outside of steady state. After discussing the steady-state in the next sub-section, I therefore consider in more detail four special cases in the subsequent subsections.

\subsection{Steady state}

Assuming $D'' > 0$, there are two possible types of steady states. In the first there is positive production of bioenergy, i.e. $y^* > 0$. This steady state follows directly from the equations in the previous subsection. For $\dot{\lambda} = \dot{\delta}_x = 0$ we have

\begin{align*}
x^* &= 0 \quad (11) \\
u'(y^*) &= c'(0) + \lambda^* \quad (12) \\
u'(y^*) &= b'(y^*) + \beta D'(S_x^* + \beta y^*) \quad (13) \\
\lambda^* &= \frac{1}{r} D'(S_x^* + \beta y^*) \quad (14)
\end{align*}

In the long run, the optimal carbon tax is constant and exactly so high that fossil energy production is zero. This level of the carbon tax is achieved by cumulative carbon energy production giving so much carbon in the atmosphere that discounted future damages are equal to this tax rate. Bioenergy will be produced at the level that is privately optimal given a tax equal to $\beta D'$. The ratio between this tax and the carbon tax hence $\beta r$.

Since $c'(0) < b'(y^*)$ for $y^* > 0$, these equations can only be valid if $\beta D' < \lambda^*$, i.e. if $r\beta < 1$. This seems a reasonable assumption: If e.g. $\beta = 10$ (see section 2) and the interest rate is $r = 0.02$, we have $r\beta = 0.2$. With these numbers we therefore have a steady state with positive bioenergy production. Moreover, in this steady state bioenergy should be taxed, but at a much lower
rate than carbon energy.

In the Appendix I show that if the initial value of $S_x$ is sufficiently small, this steady state is reached asymptotically with monotonically rising values of $\lambda(t)$, $S_x(t)$, and $S(t)$.

If $r \beta > 1$ the production of both carbon energy and bioenergy will be zero in the steady state. Equations (11)-(14) remain valid, except that $y^* = 0$, and $=$ is replaced by $<$ in (13). Notice that even if $y^* = 0$, it may be optimal to have $y(t) > 0$ for some $t$. To see this, assume $y(t) = 0$ for all $t$. With positive production of carbon energy $\lambda(t)$ will be positive and rising for $D'' > 0$ (see (10)). However, it may well be the case that $D'$ is small or even zero for low values of $S_x$, implying that we may have $\beta D'(S(t)) < \lambda(t)$ for some $t$ even if $r \beta > 1$. But if this is the case it follows immediately from (6) and (7) that $x(t) > 0$ and $y(t) = 0$ cannot be optimal. This contradiction proves that even if $r \beta > 1$, it may be optimal to have positive production for some $t$, although this production must go to zero in the long run.

In the remainder of this paper I restrict myself to the case of $r \beta < 1$. I now give an analysis both of the steady state and the dynamics for four special cases.

### 3.3 A fixed price of carbon energy and bioenergy

Consider the limiting case of $u'' = 0$, so that $u'$ is constant. Without any climate costs the production of the two types of energy would be independent of each other, determined by $c'(x) = b'(y) = u'$. With climate costs the steady state is given as in the general case, with $\lambda^* = u'$ and $y^*$ and $S_x^*$ determined by (13) and (14). The dynamics are also straightforward: From (6) it is clear that the rising value of $\lambda(t)$ implies that $x(t)$ is declining. Moreover, since $S$ and $D'(S)$ are increasing, it follows from (7) that $y(t)$ is declining.
3.4 Inelastic demand for carbon energy and bioenergy

In this case the utility function $u$ is irrelevant, and we instead have $x + y = E$, where $E$ is exogenous. Using (6) and (7) we find

$$b'(y(t)) - c'(E - y(t)) = \lambda(t) - \beta D'(S(t))$$

The l.h.s. is a strictly increasing function of $y$. It is not obvious that the r.h.s is monotonically increasing over time. However, in the long run (asymptotically) we must have $x = 0$, $y = E$, and

$$b'(E) = c'(0) + \left( \frac{1}{r} - \beta \right) D'(S^*)$$

This equation determines $S^*$. Provided $S(0) < S^* - \beta E$, there will be a phase with $x > 0$ and $y < E$.

3.5 Linear climate costs

If the climate cost function is linear, $D'$ is independent of $S$. In this case the carbon tax is constant and given by $\lambda = D'/r$, and the optimal tax on bioenergy is also constant, equal to $\beta D'$. The ratio between the tax on bioenergy and fossil energy is $\beta r$, as in the steady-state situation for the general climate cost function.

With a linear climate cost function, production of fossil energy and bioenergy are constant over time. Provided fossil energy production is positive, this means that the stock of carbon in the atmosphere is continuously increasing. This is of course in stark contrast to the climate goals that were agreed upon in the Paris agreement. These goals are more in line with what we might call an "inverse L" cost function, with negligible costs for carbon concentrations (or temperature) below some threshold, and "infinitely" high costs above the threshold. Hence, we therfore consider this case below.
3.6 A binding threshold on carbon in the atmosphere

Assume now that \( D' = 0 \) for \( S < S \) and \( D' = \infty \) for \( S > S \). (Our results would not be changed much if we instead assumed a small and constant marginal cost for values of \( S \) below the threshold \( S \)). The interpretation of \( D' \) for this case is that it is an endogenous shadow price associated with the constraint \( S \leq S \).

Provided \( S_x(0) \) is small enough, it follows from the monotonicity result above that the solution will have two phases: First (before \( t_1 \) in Figure x) \( S(t) < S \) and rising, and then a phase with \( S(t) = S \).

Consider first the phase where \( S(t) < S \). In this phase the carbon tax \( \lambda(t) \) rises at the rate of interest, while the bioenergy tax \( \beta D' \) is zero. This gives a declining \( x(t) \), a rising \( y(t) \), and a declining \( x(t) + y(t) \), as illustrated in Figure 2.

When \( S(t) \) reaches its upper limit \( S \), the term \( D' \) becomes positive, reducing \( y(t) \). As long as \( x(t) \) is positive, \( y(t) \) must be declining, since \( S_x(t) + \beta y(t) \) is constant during this phase. In Figure 2 this phase lasts till \( t_2 \), after which we have the steady-state outcome \( (x, y) = (0, y^*) \). However, in reality this phase is reached only asymptotically (i.e. \( t_2 = \infty \)).

4 Second-best policy

Assume that the energy taxes are \( \theta_x(t) \) and \( \theta_y(t) \), so that an interior market equilibrium is given by

\[
\begin{align*}
  u'(x(t) + y(t)) &= c'(x(t)) + \theta_x(t) \quad (15) \\
  u'(x(t) + y(t)) &= b'(y(t)) + \theta_y(t) \quad (16)
\end{align*}
\]

As discussed previously, this equilibrium will coincide with the social optimum if the taxes \( \theta_x(t) \) and \( \theta_y(t) \) are equal to the \( \lambda(t) \) and \( \beta D'(S) \) derived in section 3. Assume now that, for whatever reason, \( \theta_x(t) \) is exogenously set at
a level satisfying
\[ \theta_x(t) < \int_0^{\infty} e^{-\tau t} D'(S(t + \tau))d\tau \] (17)

In other words, there is an exogenous carbon tax that is lower than the social cost of carbon.

For a given time path of \( y(t) \), (15) and (16) determine the time paths of \( x(t) \) and \( \theta_y(t) \). We write this relationship as \( x = x(y, \theta_x) \). Choosing \( y(t) \) to maximize our objective function (5) gives the following Hamiltonian (ignoring the time references):

\[ H = u(x(y, \theta_x) + y) - c(x(y, \theta_x)) - b(y) - D(S_x + \beta y) + (-\gamma)x(y, \theta_x) \] (18)

The conditions for an interior optimum are

\[ u'(x + y) = b'(y) + \beta D'(S) + \left[ (\gamma + c' - u') \frac{\partial x(y, \theta_x)}{\partial y} \right] \] (19)

\[ \gamma = r\gamma - D'(S) \] (20)

\[ \lim_{t \to \infty} e^{-rt}\gamma(t) = 0 \] (21)

The costate variable \( \gamma(t) \) may as in the first-best case be interpreted as the social cost of carbon. From (20) and (21) we can derive

\[ \gamma(t) = \int_0^{\infty} e^{-\tau t} D'(S(t + \tau))d\tau \] (22)

Define

\[ k = - \frac{\partial x(y, \theta_x)}{\partial y} \] (23)

\[ \phi(t) = \gamma(t) - \theta_x(t) \] (24)

and

\[ \alpha(t) = \frac{\phi(t)}{\gamma(t)} \] (25)

From (15) and (16) it follows that \( k = \frac{u''}{w' c'} \), implying that \( k \in (0, 1) \). The
variable $\phi(t)$ is a measure of the tax distortion, i.e. the distance from the actual exogenous carbon tax to the social cost of carbon. The variable $\alpha(t)$ is a similar measure, but now in relative terms.

Using the definitions above, it follows from (15) that (19) may be rewritten as

$$u'(x + y) = b'(y) + \beta D'(S) - k\phi(t)$$

Using (16) the second-best optimal tax on bioenergy follows:

$$\theta_y(t) = \beta D'(S) - k\phi(t)$$  \hspace{1cm} (26)

or

$$\theta_y(t) = \beta D'(S) - k\alpha(t)\gamma(t)$$  \hspace{1cm} (27)

To interpret (26), it is useful to rewrite it as $\theta_y = \beta D'(S) - k\gamma + k\theta_x$. The first term in this expression, $\beta D'(S)$, measures the direct climate cost of producing and using bioenergy. The second term, $-k\gamma$, represent the reduced climate costs from carbon energy caused by the increased use of bioenergy. The final term, $k\theta_x$, represent a non-environmental indirect cost of increasing the use of $y$. By increasing the use of $y$ the use of carbon energy declines. This reduction gives a social cost (ignoring the climate effect, since this is taken care of by the second term), since the marginal benefit of using carbon energy exceeds the marginal cost of supplying it. This difference is due to the carbon tax, since the carbon tax is identical to the difference between the user and producer price.

4.1 Steady state

Without any assumptions about the exogenous carbon tax $\theta_x(t)$, we cannot know whether a steady state exists. Hence, we assume that $\theta_x(t)$ in the long run is equal to (or approaches asymptotically) a constant value $\theta^*_x$. If this is
the case and $D'' > 0$ there is a steady state given by

\[
x^* = 0
\]
\[
u'(y^*) = c'(0) + \theta_x^*
\]
\[
u'(y^*) = \nu'(y^*) + \theta_y^*
\]
\[
\gamma^* = \frac{1}{r}D^*(S_x^* + \beta y^*)
\]
\[
\theta_x^* = (1 - \alpha^*)\gamma^*
\]
\[
\theta_y^* = \beta D^*(S_x^* + \beta y^*) - k\alpha^*\gamma^*
\]

Since $c'(0) < \nu'(y^*)$ for $y^* > 0$, these equations can only be valid if $\theta_y^* < \theta_x^*$. From the equations above it is clear that this inequality is identical to $r\beta < 1 - (1 - k)\alpha^*$. The condition for $y^* > 0$ is thus stricter in the second-best case than in the first-best optimum, and is stricter the larger is the carbon tax distortion $\alpha^*$. In the remainder of this section I assume that the inequality $r\beta < 1 - (1 - k)\alpha^*$ holds. In other words, I only consider cases where bioenergy production is positive in a steady state.

From the equations above we see that the sign of $\theta_y^*$ is ambiguous. It is positive (i.e. we have a tax on bioenergy in the long run) if $r\beta \in (k\alpha^*, k\alpha^* + 1 - \alpha^*)$,\footnote{Notice that it can never be optimal to tax bioenergy if carbon energy is subsidized, i.e. if $\alpha^* > 1$.} and negative (i.e. a subsidy to bioenergy in the long run) if $r\beta < k\alpha^*$. In other words, with a small carbon tax distortion (small $\alpha^*$) bioenergy should be taxed in the long run (as in the first-best case), while bioenergy should be subsidized in the long run if the tax distortion is sufficiently high.

I assumed above that $\theta_x(t)$ in the long run was equal to (or approaches asymptotically) a constant value $\theta_x^*$. Without more assumptions on the time path of $\theta_x(t)$ is is of course not possible to say anything about the dynamics toward the steady state. In the next section I shall nevertheless briefly consider the dynamics of the second-best equilibrium for the case of an inverse damage function and a particular assumption about the time path of $\theta_x(t)$.\footnote{Notice that it can never be optimal to tax bioenergy if carbon energy is subsidized, i.e. if $\alpha^* > 1$.}
4.2 A binding threshold on carbon in the atmosphere and a constant parameter of tax distortion.

Assume that the exogenous carbon tax develops in a manner that makes the variable $\alpha(t)$, which is a measure of the tax distortion, constant over time. As in the first-best optimum, the equilibrium will have two phases. First $S(t) < \bar{S}$, and then a phase with $S(t) = \bar{S}$.

Consider first the phase where $S(t) < \bar{S}$. In this phase the carbon tax $\alpha \gamma(t)$ rises at the rate of interest. Since $D'$ is zero in this phase, it follows from (27) that bioenergy is subsidized, and that the subsidy is increasing over time. This gives a declining $x(t)$ and a rising $y(t)$, as in the first-best case.

When $S(t)$ reaches its upper limit $\bar{S}$, $y(t)$ is declining for $x(t) > 0$, since $S_x(t) + \beta y(t)$ is constant during this phase. This is achieved through a gradual reduction in the subsidy to bioenergy, which might even eventually turn into a tax.

5 Concluding remarks

In recent years it has become clear that the production of bioenergy in most cases will have a negative direct climate impact. This suggests that contrary to what is the case in many countries, bioenergy ought to be taxed, and certainly not subsidized. The present paper has shown that the dynamics of the climate impact of bioenergy and carbon energy differ. This difference may make it optimal to have an initially rising, but later declining, production of bioenergy. Moreover, the optimal taxes of the two types of energy may have very different time paths.

If policy makers for some reason are not willing to tax carbon energy as high as the climate considerations require, it may be optimal to subsidize bioenergy. However, even if such a such a subsidy is (second-best) optimal
in the short run, it may be optimal to eventually reduce the subsidy and perhaps tax bioenergy in the long run.

The present study has completely ignored the possibility of CCS (Carbon Capture and Storage). An interesting extension would be to include the possibility of CCS, both in a first-best and second-best setting.

6 Appendix: The dynamics towards steady state.

For an interior solution it follows from (6) and (7) and the properties of the functions that

\[
\begin{align*}
x &= x(\lambda, S_x) \\
y &= y(\lambda, S_x)
\end{align*}
\] (34)

It follows that

\[
S = S_x + \beta y(\lambda, S_x) = S(\lambda, S_x)
\] (36)

To see that \(S\) is increasing in \(S_x\), assume the opposite. Then it would follow from (6) and (7) that an increase in \(S_x\) for given \(\lambda\) would make \(y\) higher or unchanged, and hence increase \(S\). This contradiction proves the last sigh in (36).

The phase diagram in Figure 3 follows from (3) and (8). From (3) and (34) it is clear that the line for \(x(\lambda, S_x) = 0\) is upward sloping, and that \(\dot{S}_x\) is zero above this line and positive below.

When \(\dot{\lambda} = 0\) it follows from (8) that

\[
\lambda = \frac{1}{r} D'(S_x + y(\lambda, S_x))
\]
When $x = 0$ it follows from (7) that $y$ is independent of $\lambda$, and since $S$ and $D'$ is higher the higher is $S_x$, the curve for $\dot{\lambda} = 0$ must be upward sloping above the line for $x(\lambda, S_x) = 0$. The slope below this line is not obvious, but is of no importance. In any case we must have $\dot{\lambda} = r\lambda - D'(S(\lambda, S_x)) > 0$ to the left of the curve giving $\dot{\lambda} = 0$.

If the initial value of $S_x$ is higher than the steady-state value $S^*_x$, the optimal solution is trivial: We should have $x = 0$ for all $t$, and $y$ constant and determined by (7) for all $t$.

If the initial value of $S_x$ is lower than the steady-state value $S^*_x$, as in Figure 3, the dynamics are as illustrated: $S_x(t)$ and $\lambda(t)$, and hence also $S(t)$, rise monotonically and asymptotically towards their steady-state values.

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