The Pion–Nucleon Interaction as an Effective Field Theory*

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Abstract

After a brief survey of effective field theories, the linear $\sigma$ model is discussed as a prototype of an effective field theory of the standard model below the chiral–symmetry–breaking scale. Although it can serve as a toy model for the pion–nucleon interaction, the linear $\sigma$ model is not a realistic alternative to chiral perturbation theory. The heavy–baryon approach to chiral perturbation theory allows for a systematic low–energy expansion of Green functions and amplitudes with baryons. The chiral–invariant renormalization of the effective field theory for the pion–nucleon system to $O(p^3)$ in the chiral expansion is reviewed.

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1 Effective Field Theories

Effective field theories (EFT) are the quantum field theoretical implementation of the quantum ladder. As the energy increases and smaller distances are probed, new degrees of freedom become relevant that must be included in the theory. At the same time, other fields may lose their status of fundamental fields as the corresponding states are recognized as bound states of the new degrees of freedom. An EFT is characterized by a set of “asymptotic” fields and an energy scale \( \Lambda \) beyond which it must be replaced by a more “fundamental” theory. The development of modern physics can be viewed as a sequence of EFT culminating in the standard model of the fundamental interactions. We have every reason to believe that we have not arrived at the final stage of this development yet. All we know for sure, however, is that the associated energy scale \( \Lambda \) of the standard model is bigger than \( O(100 \text{ GeV}) \).

Conversely, as the energy is lowered, some degrees of freedom are frozen out and disappear from the accessible spectrum of states. To model the EFT at low energies, we rely especially on the symmetries of the “fundamental” underlying theory, in addition to the usual axioms of quantum field theory embodied in a corresponding effective Lagrangian. An important feature of an EFT is that its Lagrangian must contain all terms allowed by the symmetries of the fundamental theory for the given set of fields \[1\]. This completeness guarantees that the EFT is indeed the low–energy limit of the fundamental theory.

Two types of EFT can be distinguished \[2\].

A. Decoupling EFT

For energies below the scale \( \Lambda \), all heavy (with respect to \( \Lambda \)) degrees of freedom are integrated out leaving only the light degrees of freedom in the effective theory. No light particles are generated in the transition from the fundamental to the effective level. The effective Lagrangian has the general form

\[ L_{\text{eff}} = L_{d \leq 4} + \sum_{d>4} \frac{1}{\Lambda^{d-4}} \sum_{i_d} g_{i_d} O_{i_d} \] \hspace{1cm} (1.1)

where \( L_{d \leq 4} \) contains the potentially renormalizable terms with operator dimension \( d \leq 4 \), the \( g_{i_d} \) are dimensionless coupling constants expected to be of \( O(1) \), and the \( O_{i_d} \) are monomials in the light fields with operator dimension \( d \). At energies much below \( \Lambda \), corrections due to the non–renormalizable parts \( (d > 4) \) are suppressed by powers of \( E/\Lambda \). In such cases, \( L_{d \leq 4} \) can be regarded as the “fundamental” Lagrangian at low energies.

Examples of decoupling EFT:

i. QED for \( E \ll m_e \)

For energies much smaller than the electron mass, the electrons are integrated out to yield the Euler–Heisenberg Lagrangian for light-by-light scattering \[3\].

ii. Weak interactions for \( E \ll M_W \)

At low energies, the weak interactions reduce to the Fermi theory with \( d = 6 \).

iii. The standard model for \( E \ll 1 \text{ TeV} \)

There are many candidates for an underlying theory at smaller distances (composite
Higgs, SUSY, grand unification, superstrings, ...). With the exception of the Higgs sector, the standard model does not provide any clues for the scale $\Lambda$. There is no experimental evidence for terms in the effective Lagrangian with $d > 4$.

**B. Non–decoupling EFT**

The transition from the fundamental to the effective level occurs through a phase transition via the spontaneous breakdown of a symmetry generating light ($M \ll \Lambda$) pseudo–Goldstone bosons. Since a spontaneously broken symmetry relates processes with different numbers of Goldstone bosons, the distinction between renormalizable ($d \leq 4$) and non–renormalizable ($d > 4$) parts in the effective Lagrangian like in (1.1) becomes meaningless. The effective Lagrangian of a non–decoupling EFT is intrinsically non–renormalizable. Nevertheless, such Lagrangians define perfectly consistent quantum field theories [1, 4, 5, 6]. Instead of the operator dimension as in (1.1), the number of derivatives of the fields distinguishes successive terms in the Lagrangian.

The general structure of effective Lagrangians with spontaneously broken symmetries is largely independent of the specific physical realization. This is exemplified by two examples in particle physics.

a. The standard model without Higgs bosons

Even if there is no explicit Higgs boson, the gauge symmetry $SU(2) \times U(1)$ can be spontaneously broken to $U(1)_{em}$ (heavy Higgs scenario). As a manifestation of the universality of Goldstone boson interactions, the scattering of longitudinal gauge vector bosons is in first approximation analogous to $\pi\pi$ scattering.

b. The standard model for $E < 1$ GeV

At low energies, the relevant degrees of freedom of the standard model are not quarks and gluons, but the pseudoscalar mesons and other hadrons. The pseudoscalar mesons play a special role as the pseudo–Goldstone bosons of spontaneously broken chiral symmetry. The standard model in the hadronic sector at low energies is described by a non–decoupling EFT called chiral perturbation theory (CHPT).

2 From the Linear $\sigma$ Model to CHPT

The linear $\sigma$ model [7] is a seeming counterexample to the classification of Sect. [1: it is a renormalizable quantum field theory describing the spontaneous breaking of chiral symmetry. It is instructive to rewrite it in the form of a non–decoupling EFT to demonstrate the price of renormalizability: although it has the right symmetries by construction, the linear $\sigma$ model is not general enough to describe the real world [7]. It is instructive as a toy model, but it should not be mistaken for the EFT of QCD at low energies.

We rewrite the $\sigma$ model Lagrangian for the pion–nucleon system

$$
\mathcal{L}_\sigma = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - \frac{\lambda}{4} \left( \sigma^2 + \vec{\pi}^2 - v^2 \right)^2 + \bar{\psi} i \gamma_5 \partial \psi - g \bar{\psi} \left( \sigma + i \vec{\tau} \vec{\pi} \gamma_5 \right) \psi
$$

$$
\psi = \begin{pmatrix} p \\ n \end{pmatrix}
$$
in the form

\[ \mathcal{L}_\sigma = \frac{1}{4} (\partial_\mu \Sigma \partial^\mu \Sigma) - \frac{\lambda}{16} \left( \langle \Sigma^\dagger \Sigma \rangle - 2v^2 \right)^2 + \bar{\psi}_L i \partial_\mu \Sigma \psi_L + \bar{\psi}_R i \partial_\mu \Sigma \psi_R - g \bar{\psi}_R \Sigma \psi_L - g \bar{\psi}_L \Sigma^\dagger \psi_R \]  

(2.2)

\[ \Sigma = \sigma 1 - i \vec{\pi} \]  

(2.3)

to exhibit the chiral symmetry \( G = SU(2)_L \times SU(2)_R \):

\[ \psi_A \xrightarrow{G} g_A \psi_A \]  

(2.4)

For \( v^2 > 0 \), the chiral symmetry is spontaneously broken and the “physical” fields are the massive field \( \hat{\sigma} = \sigma - v \) and the Goldstone bosons \( \vec{\pi} \). However, the Lagrangian with its non–derivative couplings for the \( \vec{\pi} \) seems to be at variance with the Goldstone theorem predicting a vanishing amplitude whenever the momentum of a Goldstone boson goes to zero.

In order to make the Goldstone theorem manifest in the Lagrangian, we perform a field transformation from the original fields \( \psi, \sigma, \vec{\pi} \) to a new set \( \Psi, S, \vec{\varphi} \) through a polar decomposition of the matrix field \( \Sigma \):

\[ \Sigma = (v + S)U(\varphi) \]  

(2.3)

Under a chiral transformation,

\[ u(\varphi) \xrightarrow{G} g_R u(\varphi) h(g, \varphi)^{-1} = h(g, \varphi) u(\varphi) g_L^{-1}, \]  

(2.4)

defines a non–linear realization of \( G \) via the compensator field \( h(g, \varphi) \) \[8\]. Consequently,

\[ U \rightarrow g_R U g_L^{-1}, \]  

(2.5)

In the new fields, the \( \sigma \)–model Lagrangian (2.1) takes the form

\[ \mathcal{L} = \frac{v^2}{4} (1 + \frac{S}{v})^2 \langle u_\mu u^\mu \rangle \]

\[ + \bar{\Psi} i \nabla \Psi + \frac{1}{2} \bar{\Psi} \not{\partial} \not{\gamma}_5 \Psi - g(v + S) \bar{\Psi} \Psi + \ldots \]  

(2.6)

with a covariant derivative \( \nabla = \partial + \Gamma \) and

\[ u_\mu(\varphi) = i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) \]

\[ \Gamma_\mu(\varphi) = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) . \]  

(2.7)

The kinetic term and the self–couplings of the scalar field \( S \) are omitted in (2.6).

We can draw the following conclusions:

i. The Goldstone theorem is now manifest at the Lagrangian level: the Goldstone bosons \( \vec{\varphi} \) contained in the matrix fields \( u_\mu(\varphi), \Gamma_\mu(\varphi) \) have derivative couplings only.
ii. By construction, S–matrix elements are unchanged under the field transformation (2.3), but the Green functions are very different. For instance, in the pseudoscalar meson sector the field $S$ does not contribute at all at lowest order, $O(p^2)$, whereas $\sigma$ exchange is essential to repair the damage done by the non–derivative couplings of the $\bar{\pi}$. The linear $\sigma$ model ascribes an importance to the field $\bar{\sigma}$ which does not match the relevance of the corresponding physical state. At $O(p^4)$ in the meson sector, the chiral Lagrangian is known to be dominated by meson–resonance exchange [4, 9]. However, the scalar resonances are much less important than the vector and axial–vector mesons [1].

iii. The manifest renormalizability of the Lagrangian (2.1) has been traded for the manifest chiral structure of (2.6). Of course, the Lagrangian (2.6) is still renormalizable, but this renormalizability has its price. It requires specific relations between various couplings that have nothing to do with chiral symmetry and, which is worse, are not in agreement with experiment. For instance, the model contains the Goldberger–Treiman relation [10] in the form ($m$ is the nucleon mass)

$$m = g v \equiv g_{\pi NN} F_\pi .$$

(2.8)

Thus, instead of the physical value $g_A = 1.26$ for the axial–vector coupling constant $g_A$ the model has $g_A = 1$ (compare with the CHPT Lagrangian (3.3) below). As already emphasized, the problems with the linear $\sigma$ model are even more severe in the meson sector. In the form of (2.1) or (2.6), the linear $\sigma$ model is a toy model, but not a realistic EFT of QCD in the pion–nucleon sector.

Of course, nobody uses the original $\sigma$ model (2.1) nowadays for actual phenomenological analysis. By introducing additional terms in the Lagrangian, one may reconcile the model with experimental data for the price of abandoning renormalizability. Compared with the alternative general approach of CHPT described in the next section, such a procedure has several conceptual drawbacks that tend to obscure the relation to the underlying “fundamental” theory of QCD. To make the point, let me consider an example of such an approach inspired by the linear $\sigma$ model.

In the model of Goudsmit et al. [11], the relevant interaction terms (among others, including vector mesons) are given by the Lagrangians

$$\mathcal{L}^\text{int}_{\pi N} = - \frac{g_{\pi NN}}{1 + x} \bar{\psi} \gamma_5 \bar{\pi} \left( i x \bar{\pi} + \frac{1}{2m_N} \partial \bar{\pi} \right) \psi,$$

$$\mathcal{L}^\text{int}_\sigma = - g_{\pi \sigma} M_\pi \bar{\pi} \pi \sigma - g_{\sigma NN} \bar{\psi} \sigma \psi .$$

(2.9)

In the last paper of Ref. [11], the authors have performed a fit of their model to the $\pi N$ phase shifts at low energies. As explicitly stated in their paper, chiral symmetry is nowhere implemented. One interesting consequence of their analysis is that the effective scalar coupling $G_\sigma$ defined as

$$G_\sigma = \frac{g_{\pi \sigma} g_{\sigma NN}}{M_\sigma^2}$$

and with a corresponding vector coupling $G_\rho$. Two extreme cases are listed in Table [1]. Although the authors deplore that the parameters $x$, $G_\sigma$ and $G_\rho$ cannot be well determined
Table 1: Parameters of the model of Ref. [11] extracted from a fit to $\pi N$ phase shifts. The quality of fit is the same for the two sets of parameters.

| $x$  | $G_\sigma$ (GeV$^{-2}$) | $G_\rho^V$ (GeV$^{-2}$) |
|------|-------------------------|-------------------------|
| 0.2  | 43                      | 30                      |
| 0    | 23                      | 60                      |

at present, the content of Table 1 is actually a field transformation in action. For $x = 0.2$, the scalar parameter $G_\sigma$ must be big in order to repair the chiral–symmetry violation by the pseudoscalar pion–nucleon coupling. For a pure pseudo–vector coupling ($x = 0$), $G_\sigma$ is significantly smaller while the vector exchange becomes more important. The quality of fit is the same in both cases as one would expect for identical $S$–matrix elements.

What can one learn from an analysis of this type? The suggested greater generality of the model compared to a pure pseudo–vector pion–nucleon coupling is spurious. In particular, there is nothing in the $\pi N$ phase shifts that would argue against using a manifestly chirally symmetric framework like CHPT with $x = 0$. At the same time, the scalar field is reduced to a role in agreement with the status of the corresponding $I = 0$ s–wave meson resonance $f(1000)$ in the 1994 edition of the Review of Particle Properties [12]: an inconspicuous, highly elastic and very broad ($\Gamma \simeq 700$ MeV) $\pi\pi$ resonance. There is nothing special about the $\sigma$ meson [13]!

3 Heavy mass expansion

CHPT for the $\pi N$ system starts from the most general chiral–invariant effective Lagrangian [4, 14]

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{meson}} + \mathcal{L}_{\pi N} \]  
\[ \mathcal{L}_{\text{meson}} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots \]  
\[ \mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle \]  
\[ \mathcal{L}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \ldots \]  
\[ \mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}(i \nabla - m + \frac{gA}{2} \gamma_5) \Psi \]  

\[ u_\mu = i \{ u^\dagger (\partial_\mu - ir_\mu) u - u (\partial_\mu - il_\mu) u^\dagger \} , \quad \chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u \]  
\[ \nabla = \partial + \Gamma , \quad \Gamma_\mu = \frac{1}{2} \{ u^\dagger (\partial_\mu - ir_\mu) u + u (\partial_\mu - il_\mu) u^\dagger \} . \]

The fields $u_\mu$, $\Gamma_\mu$ are as in (2.7), except that they now contain also the external gauge fields (like photon and $W$ boson)

\[ r_\mu = v_\mu + a_\mu , \quad l_\mu = v_\mu - a_\mu . \]

The scalar field $\chi$ includes the quark masses responsible for the explicit chiral–symmetry breaking:

\[ \chi = 2B_0 \text{ diag}(m_u, m_d, m_s) + \ldots \]
At lowest order in the derivative expansion, the effective chiral Lagrangian for the pion–nucleon system contains four parameters, the nucleon mass $m$, the axial coupling constant $g_A$ and the mesonic parameters $F$ and $B_0$ related to the pion decay constant and the quark condensate, respectively:

\[
F_\pi = F[1 + O(m_{\text{quark}})] = 93.2 \text{MeV}
\]

\[
\langle 0 | \bar{u}u | 0 \rangle = -F^2 B_0 [1 + O(m_{\text{quark}})]
\]

\[
M^2_{\pi^+} = B_0 (m_u + m_d) [1 + O(m_{\text{quark}})].
\]  

Comparing with (2.6), we realize that the $\sigma$ model has indeed the correct chiral structure, but with the axial coupling constant $g_A = 1$.

The effective Lagrangian (3.1) is the starting point for a systematic low–energy expansion of Green functions and amplitudes. To satisfy unitarity and analyticity, it is essential to consider the chiral Lagrangian not only at tree level, but to take it seriously as an EFT by including loops. There is however a difference between the purely mesonic and the pion–nucleon sector [14]: in contrast to mesonic amplitudes, the loop expansion and the derivative expansion of amplitudes do not coincide in the presence of baryons. The reason is very simple: unlike the pseudoscalar meson masses, the nucleon mass $m$ does not vanish in the chiral limit. Therefore, the nucleon four–momentum can never be soft. The size of the nucleon mass suggests a simultaneous expansion in

\[
\frac{p}{4\pi F} \quad \text{and} \quad \frac{p}{m},
\]

but there is an essential difference between the two denominators ($p$ stands for a generic meson four–momentum or nucleon three–momentum): $F$ appears only in vertices of the effective Lagrangian (3.1), while the nucleon mass enters via the nucleon propagator. To put these two quantities on the same footing, one has to find a way to move the nucleon mass from the propagator to the vertices of some effective Lagrangian.

With inspiration from heavy quark effective theory, Jenkins and Manohar [15] have reformulated baryon CHPT in precisely such a way as to transfer the nucleon mass from propagators to vertices. The method is called “heavy baryon CHPT” and it can be interpreted [16] as a clever choice of variables for performing the fermionic integration in the path integral representation of the generating functional of Green functions

\[
e^{iZ[j,\eta,\bar{\eta}]} = N \int [dud\Psi d\bar{\Psi}] \exp \left[ i \left( S_{\text{meson}} + S_{\pi N} + \int d^4x (\bar{\eta}\Psi + \bar{\Psi}\eta) \right) \right].
\]  

The action $S_{\text{meson}} + S_{\pi N}$ corresponds to the effective Lagrangian (3.1), the external fields $v_\mu$, $a_\mu$, $\chi$ are denoted collectively as $j$ and $\eta, \bar{\eta}$ are fermionic sources. Heavy baryon CHPT can be formulated in a manifestly Lorentz covariant way by defining velocity–dependent fields [16]

\[
N_v(x) = \exp[i m v \cdot x] P^+_v \Psi(x) \quad \text{(3.7)}
\]

\[
H_v(x) = \exp[i m v \cdot x] P^-_v \Psi(x)
\]

\[
P^\pm_v = \frac{1}{2}(1 \pm \gamma^0), \quad v^2 = 1.
\]
In the nucleon rest frame $v = (1, 0, 0, 0)$ and $\overline{N}_v, H_v$ correspond to the usual non–relativistic projections of a Dirac spinor into upper– and lower–component Pauli spinors. In general, we may call the $\overline{N}_v (H_v)$ the light (heavy) components of the nucleon field $\Psi$. In the functional integral (3.6), one first integrates out the heavy components $H_v$ and then expands the resulting non–local action in a power series in $1/m$. The resulting effective pion–nucleon Lagrangian contains only the light components $\overline{N}_v$ together with the pion fields:

$$L_{\pi N}(\overline{N}_v, \phi) = \overline{\Psi}(iv \cdot \nabla + g_{A} S \cdot u)\Psi + O(p^n), \quad n \geq 2$$

(3.8)

$$S^\mu = \frac{i}{2} \gamma_5 \sigma^{\mu \nu} v_\nu, \quad S \cdot v = 0, \quad S^2 = -\frac{3}{4} 1.$$  

(3.9)

The nucleon mass appears only in powers of $1/m$ in the higher–order terms in (3.8). The $\overline{N}_v$ propagator is

$$\frac{iP^+}{v \cdot k + i\varepsilon}$$

(3.9)

according to the Lagrangian (3.8), independent of the nucleon mass. Thus, the goal has been achieved to move the nucleon mass from the propagator to the vertices of an effective Lagrangian. Consequently, loop and derivative expansion coincide again as in the mesonic case.

There is a small price one has to pay for the systematic low–energy expansion in the presence of baryons. Any given order in the chiral expansion of the generating functional will in general not be independent of the time–like unit vector $v$, because a change in $v$ involves different chiral orders (reparametrization invariance [18]).

### 4 Renormalization

With the effective pion–nucleon Lagrangian of the last section, all Green functions and amplitudes with a single incoming and outgoing nucleon can be calculated in a systematic chiral expansion [14, 13, 20, 21]: nucleon form factors, $\pi N \rightarrow \pi \ldots \pi N, \gamma^* N \rightarrow \pi \ldots \pi N, W^* N \rightarrow \pi \ldots \pi N$.

Up to and including $O(p^2)$, only tree–level amplitudes contribute. At $O(p^3)$, loop diagrams of the type shown in Figs. 1, 2 must be taken into account. Those diagrams are in general divergent requiring regularization and renormalization. Since we have a non–decoupling EFT that is intrinsically non–renormalizable, the divergences must be cancelled by counterterms of $O(p^3)$. Those counterterms are part of the general chiral–invariant pion–nucleon Lagrangian (3.8).

The divergent part of the one–loop functional of $O(p^3)$ can be calculated in closed form [22] by using the heat kernel method (see Ref. [23] for a review) for the meson and nucleon propagators in the presence of external fields (the propagators appearing in Fig. 1). In this way, one can not only renormalize all single–nucleon Green functions once and for all, but one also obtains the so–called chiral logs for all of them. Although one should be wary of doing phenomenology with chiral logs only, they determine the scale dependence of the renormalized coupling constants of $O(p^3)$. The details of the calculation can be found in Ref. [22]. A non–trivial part of this calculation consists in finding a heat kernel representation.
Table 2: Some counterterms and their β functions as defined in Eqs. (4.2), (4.3). The complete list of such terms with non–zero β_i is given in [22].

| i | \( O_i \)                                                                                  | \( \beta_i \)                                      |
|---|-------------------------------------------------------------------------------------------|----------------------------------------------------|
| 1 | \[ i[u_\mu, v \cdot \nabla u^\mu] \]                                                        | \( g_A^4/8 \)                                      |
| 2 | \[ i[u_\mu, \nabla^\mu v \cdot u] \]                                                        | \( -(1 + 5g_A^2)/12 \)                           |
| 3 | \[ iv \cdot u, v \cdot \nabla v \cdot u \]                                                 | \( (4 - g_A^4)/8 \)                              |
| 4 | \( S \cdot u(u \cdot u) \)                                                                 | \( g_A(4 - g_A^4)/8 \)                           |

for the inverse of the differential operator

\[ iv \cdot \nabla + g_A S \cdot u \ . \] (4.1)

This is precisely the nucleon propagator in the presence of external fields appearing in the diagrams of Fig. 1.

The renormalization program at \( O(p^3) \) can be summarized in the following way, in complete analogy to the mesonic case at \( O(p^4) \) \[4, 5\]. By choosing a convenient regularization, the one–loop functional is decomposed into a divergent and a finite part. This decomposition introduces an arbitrary scale parameter \( \mu \): although the total one–loop functional is independent of \( \mu \), the two parts are not. The divergent part is then cancelled by a corresponding piece in the general effective Lagrangian of \( O(p^3) \),

\[
L^{(3)}_{\pi N}(x) = \frac{1}{(4\pi F)^2} \sum_i B_i N_v(x) O_i(x) N_v(x) ,
\] (4.2)

through the decomposition

\[
B_i = B_i^r(\mu) + (4\pi)^2 \beta_i \Lambda(\mu) \] (4.3)

of the dimensionless coupling constants \( B_i \). The quantity \( \Lambda(\mu) \) is divergent and the coefficients \( \beta_i \) are chosen such that the divergent part of (4.2) cancels the divergent piece of the one–loop functional. The complete generating functional of \( O(p^3) \) then consists of the finite one–loop functional and the tree–level functional due to (4.2), with the couplings \( B_i \) replaced by the renormalized coupling constants \( B_i^r(\mu) \). The complete functional of \( O(p^3) \) is finite and independent of the scale \( \mu \) by construction. In Table 4, some of the operators \( O_i \) are listed together with their coefficients \( \beta_i \).

The situation for the pion–nucleon system to \( O(p^3) \) is now comparable to the mesonic sector at \( O(p^4) \) \[4, 5\]. It remains to extract the low–energy constants \( B_i^r(\mu) \), the analogues of the mesonic constants \( L_i^r(\mu) \) \[3\], as well as the scale–independent constants of \( O(p^2) \) from pion–nucleon data \[24\]. Another important task is to try to understand the actual values of these parameters, in particular to investigate systematically the effect of meson and baryon resonances.

5 Conclusions

CHPT is the effective field theory of the standard model in the hadronic sector at low energies. It is a “non–renormalizable”, yet fully consistent quantum field theory giving rise
to a systematic low–energy expansion of amplitudes. The relevant scale for this expansion is $4 \pi F_\pi$, which is of the same order of magnitude as the nucleon mass $m$. The heavy mass expansion for the meson–baryon part of the effective Lagrangian allows for a simultaneous expansion in inverse powers of $4 \pi F_\pi$ and $m$. The renormalization has been fully implemented in a manifestly chiral–invariant way to $O(p^4)$ in the meson and to $O(p^3)$ in the pion–nucleon sector.

Among the future developments in the meson–baryon system are the renormalization at $O(p^4)$, which again involves only one–loop diagrams, inclusion of higher baryon states, extension to chiral $SU(3)$ and applications for the non–leptonic weak interactions of baryons.

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Figure Captions

**Fig. 1:** Irreducible one–loop diagrams. The full (dashed) lines denote the nucleon (meson) propagators. The double lines indicate that the propagators (as well as the vertices) have the full tree–level structure attached to them as functionals of the external fields.

**Fig. 2:** Feynman diagrams for $\pi\pi$ photo–(electro–)production off nucleons as explicit examples for the diagrams of Fig. 1.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9407240v1