Numerical investigation of a deep cavity with an overhanging lip considering aeroacoustic feedback mechanism

S. Schoder\textsuperscript{1,*}, I. Lazarov\textsuperscript{1}, M. Kaltenbacher\textsuperscript{1}

Abstract

In modern transport systems, passengers’ comfort is greatly influenced by flow-induced noise. In this study we investigate a generic deep cavity with an overhanging lip, mimicking a door gap in a vehicle, that is overflowed by air at two different free stream velocities, 26.8 m/s and 50 m/s. The turbulent boundary layer and the acoustic waves interact with the cavity’s geometry and form a strong feedback mechanism. In the present work, we focus on the details of the compressible turbulent flow structures and their variations concerning the velocity, the boundary layer as well as the domain dimensionality for a later acoustic simulation within a hybrid aeroacoustic workflow. Furthermore, we verify the feasibility of reducing the acoustic computational domain from 3D to 2D for this application by conducting a coherence study of acoustically active flow structures in the spanwise direction. The role of the three-dimensional Taylor-Görtler vortices from the recirculation regarding the vortex formation and the vortex-edge interaction was also evaluated. Remarkably for the lower approaching velocity (26.8 mm), we found a special vortex-edge interaction, namely an alternating sequence of complete clipping and a subsequent partial escape. Lastly, we assigned previous unknown peaks in the pressure spectrum to their corresponding mechanisms.

Keywords: Compressible flow, Deep cavity, Aeroacoustic feedback, Shear layer

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1. Introduction

In modern transport systems, passengers’ comfort is greatly influenced by flow-induced noise. A cavity with a lip represents a generic model of a vehicle door gap, involving an acoustic feedback mechanism on the underlying flow field. Due to complex interactions between acoustic, vortex, and entropy modes (see \cite{1}), various sound classifications of cavity problems have been introduced over the years. In the late 1970s, Rockwell and Naudascher \cite{2} proposed a classification into three mechanisms: fluid-dynamic, fluid-resonant, and fluid-elastic. Fluid-dynamic modes, known as Rossiter modes, are related to an aerodynamic feedback mechanism, which energetically feeds self-sustaining oscillations. On the other hand, fluid-resonant modes arise from acoustic resonance, e.g. Helmholtz resonance. This paper considers these two mechanisms and neglects fluid-elastic interaction, which represents a fluid-structure coupling resulting from oscillations of the cavity walls. If a fluid-dynamic and a fluid-resonant mode coincide, they can...
combine in the so-called lock-on state. In case of deep rectangular cavities \((D > L)\) at low Mach numbers \(M < 0.18\), it is likely that one of the first two Rossiter modes lock with the cavity’s depth mode \(3\).

We investigate a generic deep cavity with an overhanging lip that is overflowed by air at two different free stream velocities, namely \(26.8 \text{ m/s}\) and \(50 \text{ m/s}\). The turbulent boundary layer and the acoustic waves interact with the cavity’s geometry and form a strong feedback mechanism. Figure 1(a) illustrates the geometry and the problem definition that were initially introduced and experimentally studied within the *Third Computational Aeroacoustics (CAA) Workshop on Benchmark Problems* by NASA \(4\) and Henderson \(5\), respectively. Motivated by previous numerical studies (e.g. \(6\) \(7\)), we altered the CAA benchmark geometry from the prescribed cavity width \(W = 150 \text{ mm}\) to \(W = 15.9 \text{ mm}\). According to Ahuja and Mendoza \(7\), this change has an insignificant influence on the emitted sound, as long as the cavity width \(W\) exceeds the cavity mouth length \(L_M\). Investigations on the coherence of acoustically active structures confirm this reduction. Consequently, we lowered the number of finite volume cells and thus the computational burden \(8\). While the flow velocities have been well documented in the experimental study \(5\), the documentation of the boundary layer thicknesses were contradictory. As proposed by Farkas and Paal \(6\), we use a boundary layer thickness of \(10 \text{ mm}\) at the cavity’s leading edge (reference simulation). Furthermore, we vary the boundary layer thickness from \(6 \sim 14 \text{ mm}\) in our parametric study. In the experimental study, the fluid-dynamic pressure, including the sound pressure, is recorded inside the cavity.

This cavity problem has been numerically investigated in 2D (see \(9\) \(10\) \(11\) \(12\) \(13\) \(14\)) during the third and the fourth CAA Workshop on Benchmark Problems \(4\) \(15\). Zhang et al. \(13\) summarized the conducted numerical studies concerning Henderson’s \(5\) experimental results. The authors showed that despite the different nature of the flow fields (incompressible or compressible) and the used numerical approaches (URANS and DNS), the workshop participants have succeeded in qualitatively reproducing the experimental data. In most numerical studies the arising frequencies were underestimated and the pressure levels overestimated compared to the experiments. Zhang et al. \(13\) relate these frequency deviations to a thinner boundary layer that is used in most simulations. Besides, Lin et al. \(12\) assume that the pressure level deviations are a consequence of the incompletely resolved turbulent structures and the used

![Cavity geometry and computational domain.](image-url)
turbulence models. The authors pointed out that discrete tones are related to vortex detachment, whereas broadband sound components are related to turbulence. In general, throughout both CAA workshops it was found that the numerical results are strongly dependent on various parameters, such as the flow velocity, boundary layer thickness, grid refinement (especially in the vicinity of the cavity orifice), time step size, as well as dimensions (2D/3D) and size of the computational domain. Unwanted acoustic reflections at the domain boundaries impose an additional increase in both tone frequency and pressure level and lead to erroneous results (see [10, 11]). Therefore, a characteristic acoustic radiation boundary should be imposed on the acoustic free field boundaries and too small computational domains should be avoided. According to Kurbatskii and Tam [10], at least one wavelength should be used as a rule of thumb for this purpose.

Over the years, further 2D studies reinvestigated this cavity [16, 17, 18]. Carrying out a 2D compressible URANS study, Ashcroft et al. [16] have shown that the oscillation frequencies of the Rossiter mode and the pressure levels are proportional to the flow velocity. Using a hybrid method, the authors have quantified the radiation directivity patterns of this cavity as monopole in the far field. In addition, an inverse relationship between the boundary layer thickness and the semi-empirical constant $\kappa_v$ of the Rossiter’s formula

$$f_{R_n} = \frac{U}{L_M} \frac{n - \alpha}{M + \kappa_v^{1/4}}, \quad n \in \mathbb{N}^+, \quad (1)$$

which represents the ratio between the vortex convection speed and the flow velocity, has been observed. In Equ. (1), $f_{R_n}$ denotes the $n^{th}$ Rossiter mode, $U$ the free stream velocity, $L_M$ the length of the cavity mouth, $M$ the Mach number, and $\alpha$ the time delay between the moment of the vortex impinging on the trailing edge and the emission of the acoustic waves.

The latest publication of Farkas and Paal [6] studies the previous findings by suitable simulations and model variations. For these purposes, the authors have investigated the influence of various turbulence models (both in 2D and 3D domains) and flow parameters. Despite the relatively low Mach numbers of approximately 0.077 and 0.144 (where compressible effects are negligible compared to vortical effects), the authors have found that the compressible and incompressible fields differ significantly. This behavior is in agreement with the numerical studies from Wang et al. [18]. While in case of a compressible fluid the flow oscillates in the $1^{st}$ Rossiter mode (corresponds to one vortex in the cavity mouth), the incompressible fluid oscillates in the $2^{nd}$ Rossiter mode. Within a short parametric study Farkas and Paal [6] showed that a change in the fluid viscosity causes no significant influence on the shear layer oscillation frequency (similar to [7]). Furthermore, the authors had their biggest difficulties in dealing with acoustic reflections from the boundaries of the computational domain. Thus, it is highly recommended to use suitable non-reflecting boundary conditions at the free field boundaries.

In contrast to [6], the present paper focuses on the details of the compressible turbulent flow structures and their variations concerning the velocity, the boundary layer as well as the domain dimensionality for a later acoustic simulation within a hybrid aeroacoustic workflow. Since incompressible flow simulations lead to insufficient results, we exclusively use a compressible fluid model to gain a profound understanding of the cavity. The compressible flow equations are solved using finite volume methods as provided by
ANSYS Fluent 18.0 [19]. Large turbulent scales are resolved by a DES turbulence model on a 3D domain and in the far-field, we treat the acoustic component with acoustically absorbing boundaries based on the radiation characteristics. Furthermore, the grid study quantifies the most appropriate domain size and grid density for the numerical simulation. Overall, this work presents a robust setup for the flow simulation that may be used in a hybrid aeroacoustic method. As already mentioned, we verify the feasibility of reducing the acoustic computational domain from 3D to 2D for this application by conducting a coherence study of acoustically active flow structures in the spanwise direction. Thus we aim to quantify the influence of the 3D flow effects that originate from the recirculation flow and are attenuated by the cavity’s sidewalls. To our best knowledge, three-dimensional effects, such as Taylor-Görtler vortices have not been presented before, and so the flow field and its acoustically active structures are investigated in the spanwise direction. Remarkably for the lower approaching velocity (26.8 mm), we found a special vortex-edge interaction, namely an alternating sequence of complete clipping and a subsequent partial escape, which is in agreement with the experimental results of Rockwell and Knisely [20] for cavities without an overhanging lip.

The rest of this paper is organized as follows: In Sec. 2 we present the simulation setup and the grid convergence study. Section 3 discusses parametric variations of the free stream velocity, the boundary layer thickness, and the effect of a reduced timestep size. Afterward, the 3D structures and the domain reduction for acoustic simulations are illustrated. The results of all simulations are then discussed in Sec. 4 and discrete pressure peaks are labeled by a source mechanism. Finally, Sec. 5 concludes our findings.

2. Simulation setup

We consider the compressible fluid dynamics equations using air, modeled as an ideal gas at ambient conditions ($p = 101325 \text{ Pa}$ and $T = 300 \text{ K}$). The partial differential equations are solved by a pressure-based solver and second-order spatial and temporal schemes as provided by ANSYS Fluent 18.0 [19]. The geometry of the investigated cavity is sketched in Fig. 1(a) and Fig. 1(b) depicts a side-view of the computational domain as well as the boundary conditions. All walls, including the cavity’s spanwise side walls, are modeled as perfectly smooth, non-penetrating, no-slip walls. The rest of the spanwise domain boundaries are periodic (see Fig. 1(b)). At the top and the outlet, a pressure outlet combined with a non-reflecting boundary condition for the compressible waves is introduced. These non-reflecting boundaries are based on the characteristics of the Euler equation. At the inlet, we prescribe the boundary layer profile accounting for the boundary layer thickness at the cavity and again a non-reflecting boundary condition. The boundary layer profiles are obtained by an auxiliary stationary flat plate simulation.

Although a structured grid could be easily generated for this simple geometry, we used a hybrid multi-block grid to reduce the number of finite volume cells. Each block is connected by identical discretizations at the boundaries, whereas different discretizations are used inside the blocks. Smooth and conform grid coarsening connects the different discretization densities inside these blocks. In this sense, the grid gradually becomes coarser with increasing distance from the cavity mouth. Figure 2 shows the finite volume grid that is designed for the flow velocity of 50 m/s and the SBES (Stress-Blended Eddy Simulation)
turbulence model, that is, a DES (Detached Eddy Simulation) type turbulence model. This grid consists of approximately 11.3 million cells and is denoted as the fine grid in this paper. The maximum length of the cell edges inside the cavity volume is approximate 0.1 mm, whereas the maximum cell length outside of the cavity is 8 mm. Especially in the vicinity of the cavity’s mouth and at the walls, free and wall-bounded shear layers need to be properly resolved. With the presented setup, a $y^+$ value of 0.97 at the leading edge of the cavity can be achieved. To do so, 46 wedge cells with a cell height of the first cells of 0.014 mm and a growth ratio of 1.1 are used. This simulation setup considers the expected parameter variations throughout the parametric study in the next section. Additionally to velocity scale based grid preparation, a grid convergence study verifies our simulation setup and determines the grid dependency of the results. The discretizations used for the grid convergence study are obtained by doubling the initial cell volumes $\Delta V$ of the fine grid. In contrast to URANS (Unsteady Reynolds-Averaged Navier-Stokes) turbulence models, where only the 1st Rossiter mode and its higher harmonics are captured, the SBES model resolves more turbulent structures and we assess the broad-banded pressure spectrum at the microphone position in our grid study. The instantaneous pressure fields (see Fig. 3) and the corresponding pressure spectra (see Fig. 4) reveal that the flow oscillates in the 1st Rossiter mode. Compared to the coarse (G-C) and the middle grids (G-M), the fine grid (G-F) resolves more turbulent flow structures. Hence we focus the discussion on the fine grid and chose it as a reference case for the following parametric study. Qualitatively, the three pressure spectra computed on different grids resolve similar structures (see Fig. 4). Besides the dominant 1st Rossiter mode (1671 Hz) and its higher harmonics (3341 Hz and 5012 Hz), further acoustic resonant modes are visible. The amplitude of the 1st Rossiter mode (134.3 dB) is well reproduced compared to the experimental data of Henderson [5] (134 dB). In agreement with previous numerical studies, the arising frequency is underestimated with a relative deviation of approximately 8.39 %. This discrepancy could be explained by the differences in the boundary layer thickness used in our simulation and those presented in the measurements. As shown in our parametric study, the peak frequencies and pressure levels are inversely dependent on the thickness of the approaching boundary layer above the leading edge of the cavity. Furthermore, the peak around 3552 Hz does not appear in Henderson’s [5] discussion.

Figure 2: 2D (left) and 3D (right) view of the fine computational grid consisting of approximately 11.3 million cells.
According to our simulation, this is the 1st harmonic of the 1st Rossiter mode. Although the literature concerning this cavity problem uses the term sound pressure level for describing the pressure spectra, the correct term is pressure level, since the pressure signal obtained at the microphone position (see Fig. 1(a)) includes the overall pressure (not just the acoustic part).

We assign the peak at 2151 Hz to the expected Helmholtz resonance frequency

$$f_H = \frac{c}{2\pi} \sqrt{\frac{\pi R'^2}{V(D_M + \frac{\pi R'^2}{2})}} = 2149 \text{ Hz},$$

(calculated with the speed of sound $c$ (347.411 m/s), the total cavity volume $V$, the depth of the cavity mouth $D_M$, and the equivalent hydraulic radius $R' = \sqrt{A/\pi}$, where $A$ stands for the area of the cavity orifice. This resonant peak is comparable to the one from Henderson [5] (2016 Hz) and Loh et al. [14] (2062 Hz). Henderson [5] has no explanation for the peak around 2861 Hz. We assume this acoustic

![Figure 3: Slice in the middle plane (at $z = 0 \text{ mm}$) of the instantaneous pressure field for G-C (left), the G-M (middle) and G-F (right).](image)

![Figure 4: Pressure level spectra for the convergence study compared to the experimental data of Henderson [5].](image)
resonance is a transversal cavity duct mode in the depth direction

\[ f_{Cy} = \frac{c}{4(D + D_M)} = 3102 \text{ Hz} \]  

(3)

modulated by the cavity orifice, where \( c \) denotes the speed of sound and \( (D + D_M) \) the total cavity depth. However, the other lowest longitudinal cavity resonances are outside the range of investigation at about 11 kHz. The peak at 1190 Hz in the pressure spectrum of the simulation using the fine grid may be a result of recirculation or vortex pairing. According to Loh et al. [14], this peak could be a subharmonic of the Helmholtz resonance. Nevertheless, further studies are needed to classify the origin of this peak.

Additionally to the physical peaks, we detected an artificial computational domain resonance at 480 Hz of 88.2 dB in the pressure spectrum of the simulation on the fine grid. This non-physical resonance arises due to not fully absorbing boundary conditions and acoustic wavelength coincidences with the computational domain size. For the coarse and medium grid density, this peak is masked by the turbulent fluctuations at the measurement location.

To conclude the grid convergence study, the most appropriate computational grid is the one with the fine discretization. In the following section, we investigate the influence of various flow velocities \( U \), boundary layer thicknesses \( \delta \), and time-step sizes \( \Delta t \) for the fine grid.

3. Parametric study

Table 1 summarizes the performed CFD simulations during the grid and the parametric studies. Based on the the fine grid (G-F), parameter changes are highlighted by bold symbols. In total, we performed nine CFD simulations with different parameter combinations and computational grids.

| Simulation code | Grid  | \( \delta \) (mm) | \( U \) (m/s) | \( \Delta t \) (\( \mu \)s) | Steps | \( h_{CPU} \) (h) |
|-----------------|-------|------------------|--------------|------------------|-------|---------------|
| Grid study      |       |                  |              |                  |       |               |
| G-C coarse      |       | 9.68             | 50           | 20               | 7500  | 1282          |
| G-M middle      |       | 9.68             | 50           | 20               | 7500  | 2808          |
| G-F fine        |       | 9.68             | 50           | 20               | 7500  | 6247          |
| Parameter study |       |                  |              |                  |       |               |
| P-D06 fine      |       | 6.06             | 50           | 20               | 7500  | 6366          |
| P-D08 fine      |       | 8.06             | 50           | 20               | 7500  | 5652          |
| P-D12 fine      |       | 11.96            | 50           | 20               | 7500  | 5440          |
| P-D14 fine      |       | 14.03            | 50           | 20               | 7500  | 6000          |
| P-U26 fine      |       | 9.7              | 26.8         | 20               | 10000 | 6814          |
| P-T06 fine      |       | 9.68             | 50           | 1                | 114000| 49532         |

Table 1: CFD simulations during the parametric and the grid convergence study. The specified processor hours \( h_{CPU} \approx N_{CPU} \cdot t_{run} \) are estimated by the number of processors \( N_{CPU} \) and the simulation run time \( t_{run} \). Note that the step numbers presented here are overall time steps. For data evaluation we used the last time span of approximately 1 s, which corresponds to 5000 steps in case of \( \Delta t = 20 \mu \text{s} \) and 100000 steps for \( \Delta t = 1 \mu \text{s} \).
3.1. Boundary layer thickness

Figure 5 shows the pressure level of the 1st Rossiter mode and its peak frequency as a function of the boundary layer thickness $\delta$. Both quantities exhibit an inverse proportional monotonic decrease for the boundary layer thickness $\delta$ (e.g. see [6]). This behavior meets the expectations that a thinner boundary shear layer potentially excites stronger oscillations and increases the pressure level. Familiar with the fundamentals of Rossiter’s formula, the increasing frequency is a consequence of a higher convective speed of disturbances inside thinner shear layers. A comparison of the result of Rossiter’s formula to the flow resonance suggests that our analytically used convection speed ($\kappa_v = 0.43$) is the appropriate one for a boundary layer thickness of $\delta = 8.06\,\text{mm}$. Furthermore, in literature, both phenomenons are described by the mass reduction of a thinner shear layer leading to higher-frequency oscillations.

Interestingly, a jump in the pressure level of roughly 3 dB occurs for the simulation with $\delta = 8.06\,\text{mm}$. Henderson [5] addressed this switching phenomena as a random process. In contrast to the monotonous character of the pressure level, previous 2D URANS or viscous flow simulations have shown non-monotonous (see [10, 16]) or even constant behavior (see [13]). According to the aforementioned literature, these deviations can be explained by the turbulence models in 2D, combined with the viscous flow, the stability of the main flow, and the boundary conditions. The authors explained their findings partly by the stability characteristics of the main flow profile. Concerning the present 3D study, we assume that the dimensionality of the computational flow domain allows vortex pairing as well as recirculation and consequently a change in the shear layer dynamics. Furthermore, Zhang et al. [13] showed for 2D simulations that different boundary conditions on the top boundary change the pressure level amplitude. For the pressure outlet boundary condition, the pressure level amplitude remained constant for a varying boundary layer thickness. In contrast to this, the symmetry boundary condition leads to a pressure level drop with increasing boundary layer thickness. In this case, the pressure outlet condition with a characteristic boundary for far-field radiation is the appropriate choice. In addition to the thickness, the shape of the prescribed boundary layer differs widely throughout the studies. Initially, the benchmark case was proposed with a one-seventh power law for the boundary layer. Since this definition does not represent reality, we focused our study on a developed turbulent boundary layer on a plate.

![Figure 5: Pressure level (left) of the 1st Rossiter peak and its corresponding frequency $f$ (right) as a function of the boundary layer thickness $\delta$.](image-url)
3.2. Flow velocity

Figure 6 shows the influence of the velocity variation on the pressure level fluctuations. Similar to [6], our study deviates from Henderson’s experiments [5] at a first glance. We found that a partial vortex-edge interaction causes a subharmonic peak at 800 Hz. Farkas and Paal [6] accounted for their discrepancy to the low approach velocity, at which the driving mechanisms of the fluid-dynamic and fluid-resonant modes are more competitive than in the case of higher flow velocities. During our investigation, the number of discrete peaks decreases for lower flow velocities, which we attribute to lower turbulent kinetic energy inside the flow.

Henderson [5] addresses the pressure level peak at 1168 Hz to a fluid-dynamic mode that analytically corresponds to the 1st Rossiter mode. Observing the pressure field (see Fig. 7), we indicate that the flow does not oscillate in the 1st but in the 2nd Rossiter mode, which correlates to [6]. We assume that the change in the Rossiter mode was misinterpreted by Henderson’s experiment through the occurrence of the subharmonic peak. The already mentioned vortex edge interaction and the different boundary layer

![Figure 6: Pressure level spectra for the flow speed variation of the parametric study compared to the experimental data of Henderson [5].](image)

![Figure 7: Slice in the middle plane (at $z = 0$ mm) of the instantaneous static pressure $p_{\text{stat}}$ field for P-U26. Scaling of the $p_{\text{stat}}$ values between 101300 Pa (blue) and 101350 Pa (red).](image)

thickness explains the difference in the frequency and the amplitudes. Concluding from the higher flow speeds, doubling the boundary layer thickness can reduce the resonance peaks by almost 10 dB, as long as the flow structures remain unchanged. In our case, the air trapped inside the cavity oscillates with
two expansion and two compression phases per period (see [17]). Our findings raise the question if the indicated peaks from Henderson [5] may be reassigned to a different source mechanism. To clarify these findings, a further study focusing on the lower flow velocity (26.8 m/s) should be conducted.

A profound coherence study in the spanwise direction indicated that the recirculating flow inside the cavity plays an important role for both the P-U26 and the G-F cases (see Tab. 1). While the flow structures around the shear layer were mainly two-dimensional, three-dimensional effects (Taylor-Görtler vortices) inside the cavity mouth participate in the main vortex formation inside the shear layer. Figure 8 shows that small-scale vortices shed from the lower edge of the cavity lip, driven by the three-dimensional recirculating flow, and interact with the shear layer instability by pushing in the vertical direction. In this manner, an alternating sequence of complete clipping and a subsequent partial escape vortex-edge interaction (see [20]) can be observed (see Fig. 9). This means that only every second vortex hits the trailing edge of the cavity while the other vortex partially escapes the cavity.

After observing the flow structures in Fig. 9, we have found that the vertical shear layer oscillations and the vortex shedding occur at a frequency of 1600 Hz. The aforementioned complete clipping interaction takes place for every second vortex with a frequency of 800 Hz (see Fig. 9), present as a subharmonic peak in the P-U26 spectrum (see Fig. 6). All remaining peaks (2411 Hz, 3211 Hz, 4012 Hz and 4812 Hz) are higher harmonics of the shear layer oscillation. An interesting point is that the 113.6 dB peak at 1601 Hz is as strong (in terms of pressure level) as the peak at 800 Hz. We conclude that both mechanism, vortex
shedding, and the interaction with the trailing edge, are energetically important effects.

3.3. Time step size

As depicted in Fig. 10, both the peak frequencies and the pressure level remain nearly unchanged for different time-step sizes. However, the high-frequency components are better resolved with a reduced time step size. If the flow simulation is designed for a hybrid aeroacoustic workflow, this effect on the frequency resolution must be taken into account. Nevertheless, the main flow features, up to 3500 Hz, are captured well by both simulations, which justifies the use of the coarser time step size for lower frequencies. For a higher time resolution, we resolve additional scales between approximately 800 Hz and 1100 Hz. These scales arise due to deviations in the vortex-edge interaction, similar to the partial clipping in full escape at the low free stream velocity. According to the coherence study, incoherent structures occur at the trailing edge compared to the reference simulation. Furthermore, the subharmonic peak at around 1250 Hz, which is connected to the 3D effects driven by recirculation and vortex pairing, is more pronounced as a consequence.

Figure 10: Pressure level spectra for the time step size variation of the parametric study compared to the experimental data of Henderson [5].

3.4. Coherence Study

In this section, we verify the feasibility of reducing the acoustic computational domain from 3D to 2D for this application by conducting a coherence study of acoustically active flow structures in the spanwise direction. A well known aeroacoustic analogy is the inhomogenous wave equation of Lighthill [21, 22]

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \right) \left[ c^2 (\rho - \rho_0) \right] = \nabla \cdot \nabla \cdot \mathbf{L},
\]

with the speed of sound \( c \), the density \( \rho \) of the real fluid, the density \( \rho_0 \) of an ideal linear acoustic fluid, and the Lighthill stress tensor \( \mathbf{L} \). Due to the low Mach number \( M^2 \ll 1 \) and after neglecting thermal
and dissipative viscous effects we approximate the Lighthill stress tensor as

$$\mathbf{L} \approx \rho \mathbf{uu}. \quad (5)$$

Using Equ. (5), we performed a comprehensive coherence study for \( q_a = \nabla \cdot \nabla \cdot \mathbf{L} \), which allowed us to investigate the source field properties in the spanwise dimension. For these purposes, we used 54 equidistant acoustic source term probes \( q_{a,i} \) in the spanwise direction in the region near the cavity mouth, where the dominant sources occur (see Fig. 11). After defining a reference probe \( q_{a,\text{ref}} \) at the middle of the cavity’s span, the coherence

$$\gamma_i^2(f) = \frac{|G_{q_{a,\text{ref}}q_{a,i}}(f)|^2}{G_{q_{a,\text{ref}}q_{a,\text{ref}}}(f) \cdot G_{q_{a,i}q_{a,i}}(f)}, \quad 0 \leq \gamma_i^2(f) \leq 1 \quad (6)$$

was calculated with regard to all other 53 probes. In Equ. (6) \( G_{q_{a,\text{ref}}q_{a,i}}(f) \) denotes the cross spectral density between reference probe and probe \( i \), whereas \( G_{q_{a,\text{ref}}q_{a,\text{ref}}}(f) \) and \( G_{q_{a,i}q_{a,i}}(f) \) denote the power spectral densities of both probes, \( q_{a,\text{ref}} \) and \( q_{a,i} \).[23]

Figure 11: Segments of the coherence study’s test region.

The results of the coherence study for G-F and P-T06 are summarized in Tab. 2. To distinguish the coherence over the free shear layer growth, we subdivided the region of interest into four segments, denoted with A, B, C, and D as displayed in Fig. 11. It should be noted that the coherence results are presented for a frequency of 1671 Hz, which was the dominant frequency. Inside the cavity neck, 3D effects are dominant, whereas in plane 6 and 11 the Lighthill sources cohere and behave two-dimensional. In the case of P-T06, we observe 3D effects in plane 11 of segment D that results from the impingement process, which occurs (in contrast to G-F) due to the temporary finer resolved vortex structures.

According to Larchevêque [24], the use of side walls generates a bifurcated flow that often leads to a switch in the dominant Rossiter mode and non-linear energy transfer. Nevertheless, our coherence study showed that in our case the cavity sidewalls pose minor influence on the flow structures in terms of 3D effects (see Tab. 2).

4. Discussion

According to the third CAA workshop [4] two edge tone frequencies are expected between 0 Hz and 2000 Hz, whereas frequencies related to the longitudinal cavity modes occur between 2000 Hz and 4000 Hz.
Table 2: Coherence of the Lighthill source terms from G-F and P-T06 in the span wise direction for the 1st, 6th and 11th planes of the investigated region. Note that the coherence is plotted for a frequency of 1660 Hz.

Table 3 summarizes and compares the dominant peaks from the simulation cases G-F and P-T06 to the experimental data in [5]. We associate each peak with a source mechanism and discuss it accordingly. This simulation captures the 1st Rossiter mode and its higher harmonics (3341 Hz and 5012 Hz) at the microphone position. The amplitude of the 1st Rossiter mode (134.3 dB) is well reproduced compared to the experimental data of Henderson [5] (134 dB). Although the peak frequency is underestimated, this discrepancy could be explained by the differences in the boundary layer thickness used in our simulation and those presented in the measurements. As shown in the parametric study, a boundary layer thickness of about $\delta = 8.06$ mm matches the measured peak frequency. For a higher time resolution, we resolve additional scales between approximately 800 Hz and 1100 Hz. This is due to deviations in the vortex-edge interaction, similar to the partial clipping and full escape at the low free stream velocity.

We assign the peak at 2151 Hz to the expected Helmholtz resonance ($f_H = 2149$ Hz), close to the results of other studies [5] [11]. We explain the origin of the peak around (89 dB, 2861 Hz) by a transversal cavity mode in depth direction (analytically at $f_C = 3102$ Hz) modulated by the cavity orifice.

The peak at 1190 Hz in the pressure spectrum of the simulation may be a result of recirculation or vortex pairing. As already mentioned during the discussion of Fig. 8, a 3D vortex below the leading edge convects vertically, participates directly in the new vortex formation, and influences the shear layer growth.
instability. Nevertheless, further studies are needed to classify the origin of this peak.

| Mechanism | f (Hz) | PL (dB) |  | f (Hz) | PL (dB) |  |  |  |
|-----------|--------|---------|-----|--------|---------|-----|---|---|
| artificial domain resonance | 480 | 88.2 | 442 | 87.7 |  |  |  |  |
| shear layer-edge interaction | 1190 | 93.7 | 1250 | 98.9 |  |  |  |  |
| 1st Rossiter mode \(f_R\) | 1671 | 134.3 | 1702 | 136.1 |  |  |  |  |
| Helmholtz resonance \(f_H\) | 2152 | 102 | 2154 | 96.2 |  |  |  |  |
| transversal cavity mode \(f_{C_y}\) | 2861 | 89 | 2952 | 87.6 |  |  |  |  |
| 1st harmonic of \(f_R\) | 3341 | 106 | 3404 | 101.8 |  |  |  |  |
| 1st harmonic* of \(f_H\) | 3822 | 75.7 | 3875 | 89.4 |  |  |  |  |
| 2nd harmonic of \(f_R\) | 4543 | 72 | 4654 | 79.3 |  |  |  |  |
| 1st harmonic* of \(f_{C_y}\) | 5012 | 78 | 5106 | 93.9 |  |  |  |  |
| 1st harmonic* of \(f_{C_y}\) | 5502 | 58.3 | 5577 | 80.6 |  |  |  |  |

Table 3: Numerical pressure peak values at the high flow velocity (50 m/s) compared to the experimental data of Henderson [5]. Mechanisms denoted with □* are based on assumptions and further investigations are needed.

5. Conclusion

In this paper, we investigated a generic deep cavity with an overhanging lip that is overflowed by air at two different free stream velocities, namely 26.8 m/s and 50 m/s. This cavity geometry and the problem definition were initially introduced and experimentally studied within the Third Computational Aeroacoustics (CAA) Workshop on Benchmark Problems by NASA [4] and Henderson [5], respectively.

The turbulent boundary layer and the acoustic waves interact with the cavity’s geometry and form a strong feedback mechanism.

The present work focused on the details of the compressible turbulent flow structures and their variations concerning the velocity, the boundary layer as well as the domain dimensionality for a later acoustic simulation within a hybrid aeroacoustic workflow. For these purposes, we resolved the large turbulent scales by the DES-based SBES turbulence model on a 3D domain. In the far-field, we treated the acoustic component with acoustically absorbing boundaries based on the radiation characteristics and succeeded to reduce the reflections from the domain boundaries, which are proven to contaminate the results (see [10, 16, 6]). Furthermore, we quantified the most appropriate domain size and grid density for the numerical simulation within a grid study.

In contrast to URANS turbulence models, where only the 1st Rossiter mode and its higher harmonics are captured, the SBES model resolved more turbulent structures and therefore acoustic resonant effects. During the grid and parameter studies, we assessed the broadband pressure spectrum at the evaluation position, where the peak frequencies and pressure levels are inversely dependent on the thickness of the approaching boundary layer above the leading edge of the cavity. Close to the results of other studies [5, 13], we assigned the peak at 2151 Hz to the expected Helmholtz resonance \((f_H = 2149 \text{ Hz})\). Although Henderson [5] had no explanation for the peak around 2861 Hz, we assumed this acoustic resonance to be a transversal cavity duct mode in the depth direction (analytically at \(f_{C_y} = 3102 \text{ Hz}\).
modulated by the complicated cavity orifice. Additionally to the physical peaks, we detected an artificial computational domain resonance at 480 Hz of 88.2 dB in the pressure spectrum of the G-F simulation. This non-physical resonance arises due to not fully absorbing boundary conditions but has a small influence on the computations. For the coarse and medium grid density, this peak is masked by the turbulent fluctuations at the measurement location.

Furthermore, we verified the feasibility of reducing the acoustic computational domain from 3D to 2D for this application by conducting a coherence study of acoustically active flow structures in the spanwise direction. This coherence study showed that in our case the cavity side walls have a minor influence on the acoustically active flow structures in terms of 3D effects. After visualizing relevant flow structures, we investigated and determined the vortex-edge interactions as previously observed in experiments (see Rockwell and Knisely [20]). Similar to Ashcroft et al. [16], the 3D Taylor-Görtler vortices glide along the lower edge of the cavity lip thus creating a vertical flow that is convected towards the shear layer (see Fig. 8). Remarkably for the lower approaching velocity (26.8 mm), we found a special vortex-edge interaction, namely an alternating sequence of complete clipping and a subsequent partial escape.

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References

References

[1] B. T. Chu, L. S. G. Kovasznay, Non-linear interactions in a viscous heat-conducting compressible gas.
[2] D. Rockwell, E. Naudasher, Review – self-sustaining oscillations of flow past cavities.
[3] L. F. East, Aerodynamically induced resonance in rectangular cavities.
[4] Third computational aeroacoustics (caa) workshop on benchmark problems – category 6.
[5] B. Henderson, Automobile noise involving feedback – sound generation by low speed cavity flows.
[6] B. Farkas, G. Paal, Numerical study on the flow over a simplified vehicle door gap – an old benchmark problem is revisited.
[7] K. K. Ahuja, J. Mendoza, Effects of cavity dimensions, boundary layer, and temperature on cavity noise with emphasis on benchmark data to validate computational aeroacoustic codes.
[8] I. Lazarov, Aeroacoustic simulation of a deep cavity with a lip, master thesis, Master’s thesis (Apr. 2018).
[9] Y. J. Moon, S. R. Koh, Y. Cho, J. M. Chung, Aeroacoustic computations of the unsteady flows over a rectangular cavity with lip.

[10] K. K. Kurbatskii, C. K. W. Tam, Direct numerical simulation of automobile cavity tones.

[11] G. B. Ashcroft, K. Takeda, X. Zhang, Computations of self-induced oscillatory flow in an automobile door cavity.

[12] W. H. Lin, R. H. Loh, Numerical solutions to the fourth and second computational aeroacoustics (caa) workshop benchmark problems.

[13] Z. Zhang, R. Barron, C.-F. An, Spectral analysis for air flow over a cavity.

[14] C. Y. Loh, P. C. E. Jorgenson, Computation of tone noises generated in viscous flows.

[15] Fourth computational aeroacoustics (caa) workshop on benchmark problems – category 5, problem 2.

[16] G. B. Ashcroft, K. Takeda, X. Zhang, A numerical investigation of the noise radiated by a turbulent flow over a cavity.

[17] S. R. Koh, Y. Cho, Y. J. Moon, Aeroacoustic computation of cavity flow in self-sustained oscillations.

[18] Z. K. Wang, G. Djambazov, C. H. Lai, K. Pericleous, Numerical simulation of flow-induced cavity noise in self-sustained oscillations.

[19] ANSYS Fluent Theory Guide (Release 15.0) (2013).

[20] D. Rockwell, C. Knisely, Vortex-edge interaction: Mechanisms for generating low frequency components.

[21] M. J. Lighthill, On sound generated aerodynamically I. General theory, Proceedings of the Royal Society of London 211 (1951) 564–587.

[22] M. J. Lighthill, On sound generated aerodynamically II. Turbulence as a source of sound, Proceedings of the Royal Society of London 222 (1953) 1–32.

[23] J. S. Bendat, A. G. Piersol, Engineering applications of correlation and spectral analysis.

[24] L. Larchevêque, P. Sagaut, O. Labbé, Large-eddy simulation of a subsonic flow including asymmetric three-dimensional effects.