Quantum Gravity Evolution in the Hawking Radiation of a Rotating Regular Hayward Black Hole

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(Dated: January 11, 2022)

In this paper, we study two different phenomena (the Newman-Janis algorithm and the semiclassical Hamilton-Jacobi method) to analyze the Hawking temperature ($T_H$) for massive 4-dimensional regular Hayward BH with spin parameter. First of all, we compute the rotating regular Hayward black hole solution by taking the Newman-Janis algorithmic rule. We derive the $T_H$ for rotating regular Hayward BH with the help of surface gravity. We have also analyzed the effects of spin parameter $a$ and free parameter $l$ on $T_H$ with the help of graphs. Moreover, we investigate the quantum corrected Hawking temperature ($T_H'$) for rotating regular Hayward black hole. To do so, we utilize the Lagrangian filed equation in the background of GUP within the concept of WKB approximation and semiclassical Hamilton-Jacobi method. The $T_H'$ of rotating regular Hayward BH depends upon correction parameter $\beta$, BH mass $m$, spin parameter $a$, free parameter $l$ and BH radius $r_+$. We also study the graphical behavior of $T_H'$ versus event horizon $r_+$ for rotating regular Hayward BH and check the influences of quantum gravity parameter $\beta$, spin parameter $a$ and free parameter $l$ on the stability of corresponding black hole. Moreover, we study the significance’s of logarithmic entropy correction for regular rotating Hayward BH.

Keywords: Regular Hayward Black Hole; Newman-Janis algorithm; Lagrangian field equation; Tunneling phenomenon; Temperature analysis; Entropy correction

I. INTRODUCTION

The first idea of Black Hole (BH) was propounded by John Michell (1783). Black Holes are the most imperative discovery of the universe. Moreover, BH physics preserve the mystery of the present in the form of information paradox [1] but disregarding all profound investigations on BH physics the singularity at center is also an open problem until we introduce quantum gravity theory [2]. In order to solve the singularity issues many various solutions of BHs were introduced and these non-singular solutions are known as regular BHs. In recent years these models pulled a lot of attraction of researchers, particularly the solution of non-linear electrodynamics with theory of Einstein gravity. By using the non-linear electrodynamics many Bardeen like regular BHs were obtained to remove the singularities [3]. Furthermore, the evaporation and quantum corrections have also been discussed for regular BHs [4–6]. Hayward [7] obtained the same type of regular BH solution with well-defined asymptotic limits such as for $r \to \infty$ it becomes Schwarzschild and de-Sitter when $r \to 0$. Hallilsoy and his colleagues [8] have revisited the regular Hayward BH and constructed a thin shell wormhole by using the different equations of state. Molina and Villanueva [9] have derived the roots and studied the thermodynamics of Hayward BH. In order to study the thermodynamics of BHs many approaches have been introduced. Many authors have investigated the $T_H$ for various BHs by using the semiclassical tunneling method [10]–[28]. As a comparison between Hawking computation and tunneling strategy, it is not difficult to see that the Hawking technique is an immediate strategy however its complexity to speculation to any remaining space times is failed while the tunneling methods have been effectively applied to a wide scope of both the BH event and cosmological horizons. Lately, the generalized uncertainty principle (GUP) has been the subject of numerous curiously works and many research have been appeared in which the standard uncertainty principle
is generalized as microphysics framework. To investigate the quantum gravity effects on the tunneling strategy, it is fascinating to associate the tunneling analysis with a observable minimal length in the following expression

\[ \Delta p \Delta x \geq \frac{\hbar}{2} \left[ 1 + (\Delta p)^2 \beta \right], \]

(1)

Here \( \beta = \frac{\beta_0}{M_p} \) and \( M_p \) denotes the dimensionless parameter and Plank mass, respectively. The computation of the above equation depends upon the generalized standard commutation relation \([x_\mu, p_\nu] = i\hbar \delta_{\mu\nu}[1 + \beta p^2]\), where \( x_\mu \) and \( p_\nu \) stands for position and momentum operators, respectively. Ali and his colleagues [24] have investigated the gravity effects on Hawking radiations for charged black strings via Rastall gravity.

The modifications of the usual exchange relations is not unique. Many modifications of exchange relations are suggested to [25]-[27]. To get some information about the property of the gravity of quantum, these corrections are considered widely. Black holes are valuable objects for researching the impacts of gravity. Some fascinating outcomes and discoveries were acquired by the formation of the quantum of gravity impacts into BH theory via GUP [28]. The GUP effects have been analyzed to massive vector and scalar particles from warped DGP gravity BH [29]. Ling [30] has studied the tunneling approach for fermions particles from black lenses in 5D. As a result of their investigation they concluded that by using the tunneling approach one can calculate the correct values of \( T_H \) for rotating BHs.

The quantum corrections for charged particles via tunneling method for modified Reissner BH have been investigated [31]. Gecim and Sucu [32] have studied the GUP effects on Hawking radiations in the background of (2 + 1)-dimensional Warped-AdS_3.

Cimdiker and his colleagues [33] have discussed the 4-dimensional BH in symmergent gravity and showed that the horizon radius, Hawking temperature, Bekenstein-Hawking entropy, photon deflection angle and shadow angular radius are sensitive investigations of the symmergent gravity of the fundamental quantum field theory. Övgün [34] has studied the solution of an exact confining charged BH to the scalar-tensor representation of regularized 4-dimensional Einstein-Gauss-Bonnet gravity and investigated the BH thermodynamics and physical properties for the corresponding BH i.e., Hawking temperature, specific heat, quasinormal modes and BH shadow. Okay and Övgün [35] have investigated the effects of nonlinear electrodynamics on non-rotating BHs, parametrized by magnetic charge and the field coupling parameter. They also discussed the Hawking temperature and heat capacity for this BH solution. Sakalli and Övgün [36]-[38] have studied the Hawking radiations phenomenon by using quantum tunneling method for spin-1 particles from Non-stationary Metrics, Rindler modified Schwarzschild BH as well as Lorentzian Wormholes in 3 + 1 dimensions.

The BH entropy was suggested by Hawking paper that the BH horizon area never decreases [39], as well as the evolution of this solution into the four laws of BH mechanics [40].

The main purpose of this paper is to check the quantum gravity effects for regular rotating Hayward BH and to discuss the stability condition of BH in the presence/absence of the quantum of gravity effects with the help of graphical analysis. This article is established in the following manner: In Sec. 2, we analyze a metric for regular rotating Hayward BH by considering the Newman-Jannis algorithmic rule and also compute the \( T_H \) for derived BH metric. Section 3 depicts the graphical behavior of \( T_H \) versus event horizon \( r_+ \) and describes the stability of corresponding BH. Section 4 investigates the \( T_H^* \) for regular rotating Hyward BH. Section 5 premises the influences of quantum gravity parameter \( \beta \), spin parameter \( a \) and free parameter \( l \) on regular rotating Hayward BH. with graphs. At last, Sec. 6 discusses the main results and conclusions.

II. REGULAR HAYWARD BLACK HOLE WITH ROTATION PARAMETER

The line element of spherically symmetric static Hayward non-singular BH enclosed in [8] is given as

\[ ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2d\theta^2 + r^2 \sin^2 \theta d\phi^2, \]

(2)

where \( F(r) = \left( 1 - \frac{2m^2}{r^2 - 2ml^2} \right) \), \( m \) and \( l \) represents the two free parameters.
By applying the Newman-Janis algorithm, we derive the regular Hayward BH metric with rotation parameter for the rotating case. So, through the transformation of \((t, r, \theta, \phi)\) to \((u, r, \theta, \phi)\) coordinates, we obtain

\[
d u = dt - \frac{dr}{F}.
\]  
(3)

After using the above transformation the metric (2) can be expressed as follows

\[
d s^2 = -F(r)du^2 - 2dudr + r^2d\theta^2 + r^2 \sin^2\theta d\phi^2.
\]  
(4)

In the null tetrad framework the metric can be written as

\[
g^{\mu\nu} = -l^\mu n^\nu - m^\mu \tilde{m}^\nu + m^\nu \tilde{m}^\mu.
\]  
(5)

The corresponding components are given as

\[
l^\mu = \delta^\mu_r, \quad n^\nu = \delta^\nu_r - \frac{1}{2}F \delta^\mu_r,
\]

\[
m^\mu = \frac{1}{\sqrt{2r}} \delta^\mu_\theta + \frac{i}{\sqrt{2r} \sin \theta} \delta^\mu_\phi,
\]

\[
\tilde{m}^\mu = \frac{1}{\sqrt{2r}} \delta^\mu_\theta - \frac{i}{\sqrt{2r} \sin \theta} \delta^\mu_\phi.
\]

For any point in the BH space-time the null vectors of the null tetrad satisfy the relations \(l^\mu l^\nu = n^\mu n^\nu = m^\mu m^\nu = \tilde{m}^\mu \tilde{m}^\nu = m^\mu \tilde{m}^\nu = m^\nu \tilde{m}^\mu = 0\) and \(l^\mu n^\nu = -m^\mu \tilde{m}^\nu = 1\) in the \((u, r)\) plane the coordinates transformation are \(u \to u\, (\text{real part}) - ia \cos \theta\, (\text{imaginary part}), r \to r\, (\text{real part}) + ia \cos \theta\, (\text{imaginary part})\), then we perform the transformation \(F(r) \to \tilde{F}(a, r, \theta)\) and \(\Sigma^2 = a^2 \cos^2 \theta + r^2\). The vectors in the \((u, r)\) space become

\[
l^\mu = \delta^\mu_r, \quad n^\nu = \delta^\nu_r - \frac{1}{2} \tilde{F} \delta^\mu_r,
\]

\[
m^\mu = \frac{1}{\sqrt{2r}} \left( \delta^\mu_\theta + i a \sin \theta \left( \delta^\mu_r - \delta^\mu_\phi \right) + \frac{i}{\sin \theta} \delta^\mu_\phi \right),
\]

\[
\tilde{m}^\mu = \frac{1}{\sqrt{2r}} \left( \delta^\mu_\theta - i a \sin \theta \left( \delta^\mu_r - \delta^\mu_\phi \right) - \frac{i}{\sin \theta} \delta^\mu_\phi \right).
\]

From the definition of null tetrad the metric tensor \(g^{\mu\nu}\) in the \((u, r, \theta, \phi)\) coordinates are given by

\[
g^{uu} = \frac{a^2 \sin^2 \theta}{\Sigma^2}, \quad g^{ur} = g^{ru} = -1 - \frac{a^2 \sin^2 \theta}{\Sigma^2}, \quad g^{rr} = \tilde{F} + \frac{a^2 \sin^2 \theta}{\Sigma^2}, \quad g^{\theta\theta} = \frac{1}{\Sigma^2},
\]

\[
g^{\phi\phi} = \frac{1}{\Sigma^2 \sin^2 \theta}, \quad g^{u\phi} = g^{\phi u} = -\frac{a}{\Sigma}, \quad g^{r\phi} = g^{\phi r} = 0.
\]

The new line element in the tetrad transformation can be specified by

\[
d s^2 = -\tilde{F}(r)du^2 - 2a \sin^2 \theta \left( 1 - \tilde{F} \right) dud\phi - 2dudr + 2a \sin^2 \theta d\theta d\phi + \Sigma^2 d\theta^2 + \sin^2 \theta \left( \Sigma^2 - a^2 \left( \tilde{F} - 2 \right) \sin^2 \theta \right) d\phi^2.
\]  
(6)

Finally, we perform the transformation from coordinates \((u, r, \theta, \phi)\) to \((t, r, \theta, \phi)\) as

\[
d u = dt + \lambda(r)dr, \quad d\phi = d\phi + Z(r)dr,
\]  
(7)

where

\[
\lambda(r) = \frac{a^2 + r^2}{r^2 \tilde{F} + a^2}, \quad Z(r) = -\frac{a}{a^2 + r^2 \tilde{F}}, \quad \tilde{F}(r, \theta) = \frac{r^2 \tilde{F}(r) + a^2 \cos^2 \theta}{\Sigma^2}.
\]

The regular Hayward BH metric with new spin (rotation) parameter in \((u, r, \theta, \phi)\) can be derived in the following form

\[
d s^2 = -\left( 1 - \frac{2mr^4/r^3 + 2ml^2}{\Sigma^2} \right) dt^2 - 2a \left( \frac{2mr^4/r^3 + 2ml^2}{\Sigma^2} \right) \sin^2 \theta dt d\phi + \frac{\Sigma^2}{\Delta} dr^2 + \Sigma^2 d\theta^2
\]

\[+ \sin^2 \theta \left[ \Sigma^2 - a^2 \left( \frac{2mr^4/r^3 + 2ml^2}{\Sigma^2} \right) \sin^2 \theta \right] d\phi^2.
\]  
(8)
where 
\[ \Delta = r^2 + a^2 - \frac{2mr^4}{r^3 + 2ml^2}. \]

Now, we discuss the \( T_H \) from regular Hayward BH for rotating case with the help of surface gravity by using the given formula

\[ T_H = \frac{\kappa}{2\pi}, \quad \kappa = \frac{\Delta'_+}{2(r_+^2 + a^2)}, \quad (9) \]

The \( T_H \) from regular Hayward BH for rotating case can be derived as

\[ T_H = \frac{2r_+ \left( 2ml^2 + r_+^3 \right)^2 + 6mr_+^5 - 8mr_+^3 \left( 2ml^2 + r_+^3 \right)}{4\pi \left( 2ml^2 + r_+^3 \right)^2 \left( r_+^2 + a^2 \right)}. \quad (10) \]

The above \( T_H \) depend upon the mass \( m \) of BH, free parameter \( l \) and rotation parameter \( a \).

**III. GRAPHICAL ANALYSIS OF HAWKING TEMPERATURE \( T_H \) FOR REGULAR ROTATING HAYWARD BH**

This part investigate the behavior of \( T_H \) versus event horizon \( r_+ \) with the help of plots for regular rotating Hayward BH. We study the effects of arbitrary parameter \( l \) as well as spin parameter \( a \) for \( T_H \) of regular rotating Hayward BH. We analyze the behavior of \( T_H \) by choosing the fixed value of mass \( m = 1 \). We also investigate the stability of regular rotating Hayward BH under the influence of different parameters.

![Figure 1: Hawking temperature \( T_H \) via horizon \( r_+ \).](attachment:image.png)

**Fig. 1**: states the behavior of \( T_H \) for different values of \( l \) and for fixed values of spin parameter \( a = 1 \) and \( a = 5 \), respectively. The left hand plot shows the \( T_H \) after getting a maximum height eventually goes down and obtain an asymptotically flat case till \( r_+ \rightarrow \infty \). This case ensures the stable form of BH. We can also test that the \( T_H \) increases with the increasing value of \( l \).

The right hand plot shows the presentation of \( T_H \) for changing values of \( l \) as well as fixed value of spin parameter \( a = 5 \). One can observe the \( T_H \) decreases with the increasing values of \( r_+ \). This behavior with positive high \( T_H \) shows the stability of BH.
Fig. 2: Hawking temperature $T_H$ via horizon $r_+$. 

Fig. 2: gives the behavior of $T_H$ for varying values of spin parameter $a$ and fixed values of arbitrary constant $l = 100$ and $l = 500$, respectively. The left hand plot shows the presentation of $T_H$ for fixed value of $l = 100$. By the emission of Hawking radiation, the $T_H$ reaches a maximum height at the final stage of the evaporation and afterward suddenly drops to zero so that a stable remnant is showed up.

The right hand plot depicts the behavior of $T_H$ for different values of $a$ and fixed value of $l = 500$. The increasing values of $r_+$ shows a decreasing behavior of $T_H$ after a maximum height which shows the physical and stable behavior of BH. The $T_H$ decreases when we increase the values of $a$.

IV. QUANTUM CORRECTED TEMPERATURE FOR REGULAR ROTATING HAYWARD BH

The metric Eq. (8) can be re-written as

$$ds^2 = -Adt^2 + Bdr^2 + Cd\theta^2 + Ddy^2 + Edtdy,$$

(11)

here

$$A = \left(1 - \frac{2mr^2}{r^2 + 2ml^2} \right), \quad B = \frac{\Sigma^2}{\Delta}, \quad C = \Sigma^2, \quad D = \sin^2 \theta \left[ \Sigma^2 - a^2 \left( \frac{2mr^2}{\Sigma^2} + 2ml^2 \right) \sin^2 \theta \right],$$

$$E = 2a \left( \frac{2mr^2}{r^2 + 2ml^2} \right) \sin^2 \theta.$$

Lagrangian field equation with quantum gravity parameter is given by

$$\partial_\mu \left( \sqrt{-g} \phi^{\nu\mu} \right) + \sqrt{-g} \frac{m^2}{\hbar^2} \phi^{\nu} + \hbar^2 \beta \partial_\nu \partial_\mu \partial_0 \left( \sqrt{-g} g^{00} \phi^{0\nu} \right) - \hbar^2 \beta \partial_\nu \partial_0 \partial_\mu \left( \sqrt{-g} g^{\nu\mu} \phi^{00} \right) = 0,$$

(12)

where $\phi^{\nu\mu}$ shows the anti-symmetric tensor, $g$ denotes the coefficient matrix determinant and $m$ represents the mass of radiated particle. The $\phi^{\nu\mu}$ tensor is calculated by the given formula

$$\phi_{\nu\mu} = \left(1 - \hbar^2 \beta \partial_\nu \partial_\mu \right) \partial_\nu \phi_{\mu} - \left(1 - \hbar^2 \beta \partial_\mu \partial_\nu \right) \partial_\mu \phi_{\nu},$$

(13)

where $\beta$ is a parameter of quantum gravity.

The non-zero elements of $\phi^{\nu\mu}$ are given as

$$\phi^{0} = \frac{-D\phi_0 + E\phi_3}{E^2 + AD}, \quad \phi^{1} = \frac{1}{B}\phi_1, \quad \phi^{2} = \frac{1}{C}\phi_2, \quad \phi^{3} = \frac{E\phi_0 + A\phi_3}{E^2 + AD}, \quad \phi^{01} = \frac{-D\phi_0 + E\phi_3}{B(E^2 + AD)}, \quad \phi^{12} = \frac{1}{BC}\phi_{12},$$

$$\phi^{02} = \frac{-D\phi_{02}}{C(E^2 + AD)}, \quad \phi^{03} = \frac{(A^2 - AD)\phi_{03}}{(E^2 + AD)^2}, \quad \phi^{13} = \frac{1}{BAD + E^2}\phi_{13}, \quad \phi^{23} = \frac{E\phi_{02} + A\phi_{23}}{C(E^2 + AD)}.$$
The WKB approximation is given by

$$
\phi_v = c_v \exp \left( \frac{i}{\hbar} S_0(t, r, \theta, y) + \Sigma \hbar^2 \Theta_n(t, r, \theta, y) \right).
$$

(14)

After putting all the given values in Eq. (12), we get the set of field equation in this form

$$
\frac{D}{(E^2 + AD)B} \left[ c_1(\partial_0 S_0)(\partial_1 S_0) + \beta c_1(\partial_0 S_0)^3(\partial_1 S_0) - c_0(\partial_1 S_0)^2 - \beta c_0(\partial_1 S_0)^4 \right] - \frac{E}{B(E^2 + AD)} \left[ c_3(\partial_1 S_0)^2 + \beta c_3(\partial_1 S_0)^4 - c_1(\partial_1 S_0)(\partial_3 S_0) - \beta c_1(\partial_1 S_0)(\partial_3 S_0)^2 \right] + \frac{D}{C(E^2 + AD)} \left[ c_2(\partial_0 S_0)(\partial_2 S_0) + \beta c_2(\partial_0 S_0)(\partial_2 S_0)^3(\partial_3 S_0) - c_0(\partial_2 S_0)^2 - \beta c_0(\partial_2 S_0)^4 \right] - m^2 \frac{Dc_0 - Ec_3}{(E^2 + AD)} = 0,
$$

(15)

$$
- \frac{D}{B(E^2 + AD)} \left[ c_2(\partial_0 S_0)^2 + \beta c_2(\partial_0 S_0)^4 - c_0(\partial_0 S_0)(\partial_2 S_0) - \beta c_0(\partial_0 S_0)(\partial_2 S_0)^3 \right] + \frac{E}{B(E^2 + AD)} \left[ c_3(\partial_0 S_0)(\partial_1 S_0) + \beta c_3(\partial_0 S_0)(\partial_1 S_0)^3 - c_1(\partial_1 S_0)^2 - \beta c_1(\partial_1 S_0)^4 \right] - \frac{m^2 c_1}{B} = 0,
$$

(16)

$$
+ \frac{D}{C(AD + E^2)} \left[ c_0(\partial_0 S_0)^3(\partial_2 S_0) - \beta c_0(\partial_0 S_0)(\partial_2 S_0)^3 \right] - \frac{E}{C(E^2 + AD)} \left[ c_2(\partial_0 S_0)(\partial_3 S_0) + \beta c_2(\partial_0 S_0)(\partial_3 S_0)^3(\partial_3 S_0) - c_0(\partial_3 S_0)^2 - \beta c_2(\partial_3 S_0)^4 \right] - \frac{m^2 c_2}{C} = 0,
$$

(17)

$$
+ \frac{(AD) - A^2}{(E^2 + AD)^2} \left[ c_3(\partial_0 S_0)^2 + \beta c_3(\partial_0 S_0)^4 - c_0(\partial_0 S_0)(\partial_3 S_0) - \beta c_0(\partial_0 S_0)(\partial_3 S_0)^3 \right] - \frac{D}{C(E^2 + AD)} \left[ c_3(\partial_1 S_0)^2 + \beta c_3(\partial_1 S_0)^4 - c_1(\partial_1 S_0)(\partial_3 S_0) - \beta c_1(\partial_1 S_0)(\partial_3 S_0)^3 \right] - \frac{E}{C(E^2 + AD)} \left[ c_2(\partial_0 S_0)(\partial_2 S_0) + \beta c_2(\partial_0 S_0)(\partial_2 S_0)^3(\partial_3 S_0) - c_0(\partial_2 S_0)^2 - \beta c_2(\partial_2 S_0)^4 \right] - \frac{m^2 (Ec_0 - Ac_3)}{(E^2 + AD)} = 0.
$$

(18)

Using separation of variables technique, we can choose

$$
S_0 = -(E - j\omega)t + W(r) + f y + v(\theta),
$$

(20)

where $E$ stands for the energy of particle, $f$ shows the particles angular momentum.

By using Eq. (20) into set of Eqs. (15-18), we obtain a $4 \times 4$ matrix equation as follows

$$
K(c_0, c_1, c_2, c_3)^T = 0.
$$
So, the elements of the above matrix equation are defined as

\[ K_{00} = -\frac{D}{B(E^2 + AD)} \left[ W_1^2 + \beta W_1^4 \right] - \frac{D}{C(E^2 + AD)} \left[ j^2 + \beta j^4 \right] - \frac{AD}{(E^2 + AD)^2} \left[ v_1^2 + \beta v_1^4 \right] - \frac{m^2 D}{(E^2 + AD)}, \]

\[ K_{01} = -\frac{D}{B(E^2 + AD)} \left[ (E - j\omega) + \beta(E - j\omega)^3 \right] + \frac{E}{B(E^2 + AD)} \left[ v_1 + \beta v_1^3 \right], \]

\[ K_{02} = -\frac{D}{C(E^2 + AD)} \left[ (E - j\omega) + \beta(E - j\omega)^3 \right], \]

\[ K_{03} = -\frac{E}{B(E^2 + AD)} \left[ W_1^2 + \beta W_1^4 \right] - \frac{AD}{C(E^2 + AD)} \left[ (E - j\omega) + \beta(E - j\omega)^3 \right] + \frac{m^2 E}{(E^2 + AD)^2}, \]

\[ K_{10} = -\frac{D}{B(E^2 + AD)} \left[ (E - j\omega)W_1 + \beta(E - j\omega)W_1^3 \right] - \frac{m^2}{B}, \quad K_{21} = \frac{1}{BC} \left[ j + \beta j^3 \right] W_1, \]

\[ K_{11} = -\frac{D}{B(E^2 + AD)} \left[ (E - j\omega)^2 + \beta(E - j\omega)^4 \right] + \frac{E}{B(E^2 + AD)} \left[ v_1 + \beta v_1^3 \right] (E - j\omega) - \frac{1}{BC} \left[ j^2 + \beta j^4 \right] \]

\[ - \frac{1}{B(E^2 + AD)} \left[ v_1 + \beta v_1^3 \right] - \frac{m^2}{B}, \quad K_{12} = \frac{1}{BC} \left[ W_1 + \beta W_1^3 \right] J, \]

\[ K_{13} = -\frac{E}{B(E^2 + AD)} \left[ W_1 + \beta W_1^4 \right] (E - j\omega) + \frac{1}{B(E^2 + AD)^2} \left[ W_1 + \beta W_1^3 \right] v_1, \]

\[ K_{20} = \frac{D}{C(E^2 + AD)} \left[ (E - j\omega)J + \beta(E - j\omega)j^3 \right] + \frac{E}{C(E^2 + AD)} \left[ (E - j\omega) + \beta(E - j\omega)^3 v_1 \right], \]

\[ K_{22} = \frac{D}{C(E^2 + AD)} \left[ (E - j\omega)^2 + \beta(E - j\omega)^4 \right] - \frac{1}{BC} + \frac{E}{C(E^2 + AD)} \left[ (E - j\omega) + \beta(E - j\omega)^3 \right] - \frac{m^2}{C}, \]

\[ K_{23} = \frac{A}{C(E^2 + AD)} \left[ j^2 + \beta j^4 \right] v_1, \quad K_{31} = \frac{1}{B(E^2 + AD)} \left[ v_1 + \beta v_1^3 \right] W_1, \]

\[ K_{32} = \frac{E}{C(E^2 + AD)} \left[ J + \beta j^3 \right] (E - j\omega) + \frac{A}{C(E^2 + AD)} \left[ v_1 + \beta v_1^3 \right] J, \]

\[ K_{33} = \frac{(AD - A^2)}{(E^2 + AD)} \left[ (E - j\omega)^2 + \beta(E - j\omega)^4 \right] - \frac{1}{B(E^2 + AD)} \left[ W_1^2 + \beta W_1^4 \right] - \frac{A}{C(E^2 + AD)} \left[ j^2 + \beta j^4 \right] \]

\[ - \frac{m^2 A}{(E^2 + AD)}, \]

where \( J = \partial_S S_0, \ W_1 = \partial_{\tau} S_0 \) and \( v_1 = \partial_{\theta} S_0 \). In order to get a significant imaginary solution, we put the determinant of matrix \( |K| = 0 \), so

\[ \text{Im}W^\pm = \pm \int \sqrt{\frac{(E - j\omega)^2 + Z_1 \left[ 1 + \beta Z_2 \right]}{(E^2 + AD)}} \, dr, \]

\[ = \pm i\pi \frac{(E - j\omega) + \left[ 1 + \beta Z_2 \right]}{2\kappa(r_+)} \text{,} \quad \text{(21)} \]

here

\[ Z_1 = \frac{BE(E - j\omega) v_1}{(E^2 + AD)} + \frac{ABv_1^2}{(E^2 + AD)} - Bm^2, \]

\[ Z_2 = \frac{BD(E - j\omega)^2}{(E^2 + AD)} + \frac{BE(E - j\omega)^3}{C(E^2 + AD)} - \frac{ABv_1^4}{(E^2 + AD)} - W_1^4. \]
The charged particles tunneling probability is defined as

\[ \Gamma = \frac{\Gamma_{\text{emission}}}{\Gamma_{\text{absorption}}} = \exp \left[ -2\pi \frac{(E - j\omega)}{\kappa(r_+)} \right] \left[ 1 + \beta \Xi \right]. \]  

(22)

here

\[ \kappa(r_+) = \frac{2r_+ \left( 2ml^2 + r_+^3 \right) + 6mr_+^5 - 8mr_+^3 \left( 2ml^2 + r_+^3 \right)}{2 \left( 2ml^2 + r_+^3 \right)^2 \left( r_+^2 + a^2 \right)}. \]  

(23)

The generalized \( T'_H \) of regular rotating Hayward BH can be computed by considering the Boltzmann factor \( \Gamma_B = \exp \left[ \frac{(E - j\omega)}{T'_H} \right] \) as follows

\[ T'_H = \frac{2r_+ \left( 2ml^2 + r_+^3 \right)^2 + 6mr_+^5 - 8mr_+^3 \left( 2ml^2 + r_+^3 \right)}{4\pi \left( 2ml^2 + r_+^3 \right)^2 \left( r_+^2 + a^2 \right)} \left[ 1 - \beta \Xi \right]. \]  

(24)

The \( T'_H \) of BH depends upon \( \beta, m, a \) and \( r_+ \). We also conclude that the \( T'_H \) of the charged vector particles is smaller than the \( T_H \). So, quantum corrections decelerate the increase in \( T_H \).

Equating with the \( T'_H \), the GUP-modified particle energy radiate in regular rotating Hayward BH [18, 42] is computed as

\[ E_{\text{GUP}} \geq E[1 - \beta \Xi] \]  

(25)

the \( E_{\text{GUP}} \) relates on arbitrary parameter \( \Xi \) and correction parameter \( \beta \). The \( E_{\text{GUP}} \) increases with the increasing values of correction parameter.

It is important to mention here that if we put \( (\beta = 0) \), we get the original Hawking temperature \( T_H \) of Eq. (10) for regular rotating Hayward BH. When \( l = 0, \beta = 0 \), we obtain the Hawking temperature of Kerr BH [43]. Moreover, for \( a = 0, l = 0, \beta = 0 \), we recover the temperature of Schwarzschild BH [44].

V. GRAPHICAL ANALYSIS OF CORRECTED TEMPERATURE \( T'_H \) FOR REGULAR ROTATING HAYWARD BH

This section provide the graphical analysis of \( T'_H \) via horizon \( r_+ \) for regular rotating Hayward BH. We study the effects of quantum correction parameter \( \beta \), spin parameter \( a \) and free parameter \( l \) on \( T'_H \). We analyze the behavior of \( T'_H \) for fixed value of mass \( m = 1 \) and varying values of different parameters.

Figure 3: Hawking temperature \( T'_H \) versus horizon \( r_+ \).
Fig. 3: states the behavior of $T'_H$ for fixed values of spin parameter $a$, free parameter $l$ and different values of correction parameter $\beta$ in the range $0 \leq r_+ \leq 15$.

The left hand side plot shows the behavior of $T'_H$ for fixed values of $a = 2.5$, $l = 60$ and varying values of gravity parameter $\beta$. The temperature for these values of $\beta$ satisfies the GUP relation with positive temperature and shows the stable condition of BH.

The right hand plot depicts the behavior of $T'_H$ for fixed values of $a = 1$, $l = 50$ for different values of correction parameter. One can observe that the $T'_H$ decreases as horizon increases and remnant left at maximum temperature with non-zero horizon. The $T'_H$ decreases with the increasing values of correction parameter.

![Graph of Hawking temperature $T'_H$ versus horizon $r_+$.](image)

Fig. 4: Hawking temperature $T'_H$ versus horizon $r_+$.

Fig. 4: describes the graphical presentation of $T'_H$ for fixed values of correction parameter and changing values of $a$ and $l$.

The left hand side plot gives the presentation of $T'_H$ for fixed value of $\beta = 0.5$, $l = 40$ and changing values of $a$. We can also observe that in the presence of $\beta$ influences the $T'_H$ also decreases with the increasing values of $a$. The presentation shows the stable condition of BH.

The right hand side plot gives the presentation of $T'_H$ for constant value of $\beta = 0.9$, $a = 10$ and changing values of $l$. We can see after a height the $T'_H$ decreases as $r_+$ increases and temperature increases with the increasing values of $l$ as compared to absence of quantum gravity effects case. This representation ensures the stability of BH in the range $0 \leq r_+ \leq 15$.

It has also worth to mention here that in the case of $\beta$ effects, we get the smaller $T'_H$. So, Fig. 3 & 4 shows that the $\beta$ effects decelerate the increase in $T'_H$.

VI. LOGARITHMIC ENTROPY CORRECTION FOR REGULAR ROTATING HAYWARD BH

In this section, we compute the entropy corrections for regular rotating Hayward BH. Banerjee and Majhi [45]-[47] have calculated the corrected temperature and entropy by considering the back-reaction effects with the help of null geodesic method. We investigate the entropy corrections for regular rotating Hayward BH by utilizing the generic formula for first order corrections to Bekenstein-Hawking formula [48]. We calculate the logarithmic entropy corrections with the help of corrected temperature $T'_H$ and standard entropy $S_o$ for regular rotating Hayward BH. The entropy corrections can be calculated by the given formula

$$S = S_o - \frac{1}{2} \ln \left| T^2_H, S_o \right| + \ldots .$$  

(26)

The standard entropy for regular rotating Hayward BH can be computed by the general formula

$$S_o = \frac{A_+}{4}.$$  

(27)
here

\[
A_+ = \int_0^{2\pi} \int_0^\pi \sqrt{g_{\theta\theta}} d\theta dy,
\]

\[
= 2\pi \left( r^2 + a^2 \right) \left( r^3 + 2l^2 m \right) \left( 2l^2 m + r^3 + mr^4 \right).
\]

(28)

So the standard term for entropy is given as

\[
S_o = \frac{\pi \left( r^2 + a^2 \right) \left( r^3 + 2l^2 m \right)}{2 \left( 2l^2 m + r^3 + mr^4 \right)}.
\]

(29)

After putting the values from Eq. (24) and (29) into Eq. (26), we calculate the corrected entropy in the following form

\[
S = \frac{\pi \left( r^2 + a^2 \right) \left( r^3 + 2l^2 m \right)}{2 \left( 2l^2 m + r^3 + mr^4 \right)} - \frac{1}{2} \ln \left[ \frac{2r_+ \left( 2ml^2 + r^3_+ \right) + 6mr_+^6 - 8mr_+^3 \left( 2ml^2 + r^3_+ \right) \left( 1 - \beta \Xi \right)}{32\pi \left( 2ml^2 + r^3_+ \right)^3 \left( r^2_+ + a^2 \right) \left( 2l^2 m + r^3 + mr^4 \right)} \right]^2 + \ldots
\]

(30)

The above equation gives the corrected entropy for regular rotating Hayward BH.

VII. CONCLUSIONS

In this paper, we have effectively applied two different phenomena (the Newman-Janis algorithm and the semiclassical Hamilton-Jacobi method) to analyze the \(T_H\) for massive 4-dimensional regular Hayward BH with spin parameter. By considering the Newman-Janis algorithm, the Hayward BH solution with spin parameter is computed. We have derived the temperature for regular rotating Hayward BH with the help of surface gravity. The temperature depends upon the BH mass \(M\), parameter \(a\), free parameter \(l\) and BH radius \(r_+\). We have also studied the effects of spin parameter \(a\) and free parameter \(l\) on Hawking temperature with the help of graphs. The temperature decreases with the increasing values of spin parameter \(a\) and increases with the increasing values of free parameter \(l\).

To study the Hawking temperature of massive vector particles, we have utilized the Lagrangian filed equation in the background of GUP within the concept of WKB approximation and semiclassical Hamilton-Jacobi method. By taking into account the Hamilton-Jacobi phenomenon and the complex path integration, we have calculated the tunneling probability of the regular rotating Hayward BH, which is dominated by the well known Boltzmann factor. Then, the temperature found from the leading tunneling probability is established to the \(T_H^l\) of the regular rotating Hayward BH. The \(T_H^l\) of regular rotating Hayward BH depends upon correction parameter \(\beta\), BH mass \(m\), spin parameter \(a\), free parameter \(l\) and BH radius \(r_+\). If \((\beta = 0)\), we analyzed the original \(T_H\) of Eq. (10) for regular rotating Hayward BH. When \(l = 0, \beta = 0\), we found the Hawking temperature of Kerr BH. Moreover, for \(a = 0, l = 0, \beta = 0\), we observed the temperature of Schwarzschild BH. Moreover, we have studied the graphical presentation of \(T_H^l\) via horizon \(r_+\) for regular rotating Hayward BH. We have investigated the effects of quantum gravity \(\beta\), spin parameter \(a\) as well as free parameter \(l\) on corrected temperature. We have concluded that, we observed an increasing behavior of temperature for the increasing values of \(l\) and decreasing behavior for increasing values of \(a\) and \(\beta\). The temperature at maximum height with non-zero horizon gives BH remnant. After maximum height the temperature eventually goes down and get an asymptotically flat condition till \(r_+ \rightarrow \infty\), which ensures the stable case of BH. Comparing with the \(T_H^l\), the GUP-modified particle energy radiate in regular rotating Hayward BH also computed. The particle energy \(E_{GUP}\) radiate in regular rotating Hayward BH depends on quantum gravity.
Finally, we study the entropy corrections for regular rotating Hayward BH by observing the generic formula for first order corrections to Bekenstein-Hawking formula \[ 48 \] and analyze the logarithmic entropy corrections with the help of \( T'_{ij} \) and standard entropy \( S_0 \) for regular rotating Hayward BH.

**ACKNOWLEDGMENTS**

We are very much grateful to the honorable referee and to the editor for the illuminating suggestions that have significantly improved our work in terms of research quality, and presentation.

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