Constraining Dark Matter candidates
from structure formation

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Abstract

We show that collisional damping of adiabatic primordial fluctuations yields constraints on the possible range of mass and interaction rates of Dark Matter particles. Our analysis relies on a general classification of Dark Matter candidates, that we establish independently of any specific particle theory or model. From a relation between the collisional damping scale and the Dark Matter interaction rate, we find that Dark Matter candidates must have cross-sections at decoupling $\lesssim 10^{-33} \frac{m_{DM}}{M_{US}}$ cm$^2$ with photons and $\lesssim 10^{-37} \frac{m_{DM}}{M_{US}}$ cm$^2$ with neutrinos, to explain the observed primordial structures of $10^9 M_\odot$. These damping constraints are particularly relevant for Warm Dark Matter candidates. They also leave open less known regions of parameter space corresponding to particles having rather high interaction rates with other species than neutrinos and photons.

1 Introduction

Although Dark Matter appears as a necessary component of the Universe, its nature still remains a challenging question. While it has long been considered that Weakly-Interacting Massive Particles could provide a satisfactory solution to the Dark Matter puzzle, recent numerical computations led many authors to question this general belief by focusing,
for instance, on stronger interacting candidates [2] or by reviving Warm Dark Matter scenarios [3, 4]. Establishing, independently of any specific particle theory, what kind of Dark Matter particle mass and interaction rates are allowed or not thus appears to be useful. This can be achieved by requiring that collisional damping effects do not prevent the formation of the observed (galactic-size) primordial structures.

To this purpose, we estimate the Dark Matter collisional damping scale, taking into account the effects induced by all the species with which Dark Matter particles interact. In particular, we express this scale in terms of two contributions – shown to combine quadratically – referred to as self-damping and induced-damping. The former constrains Dark Matter properties while the latter constrains the interactions of Dark Matter with all other particles. By imposing that both self-damping and induced-damping scales be smaller than the length associated with the smallest primordial structure presently observed in the Universe, we obtain necessary conditions on the mass and interaction rates for any possible type of Dark Matter particles.

2 Collisional damping effects.

Let us consider a set of particle species $i$, including Dark Matter particles themselves, maintained in thermal equilibrium by collisional processes. For adiabatic fluctuations, the collisional damping scale ($l_{cd}$) accumulated until the Dark Matter decoupling, which occurs at a time $t_{dec(dm)}$, is given by

$$l_{cd}^2 = 2\pi^2 \int_0^{t_{dec(dm)}} \frac{\eta + \chi T}{\phi} \frac{n}{a^2(t)} dt,$$

with $\phi = \sum_i (\rho_i + p_i)$, \hspace{1cm} (1)

normalized so as to correspond to a mass scale $M_{cd} = 4\pi\rho l_{cd}^3/3$ over which all fluctuations are erased. The dissipative coefficients, namely the shear viscosity $\eta$ and the heat conduction $\chi T$, can be expressed in case of a mixture \hspace{1cm} (2)

as $\eta = \sum_i \frac{\rho_i v_i^2}{T_i}$ and $\chi T = \sum_i \frac{\rho_i v_i^2}{T_i} \frac{d \ln \rho_i}{d \ln T}$. Here, $\Gamma_i = \sum_j (\sigma v)_{ij} n_j$ denotes the interaction rate of the species $i$. The average cross-sections are, as usual in transport theory, weighted by the momentum transfer (shear viscosity) or energy transfer (heat conduction) associated with the interaction. For convenience, we will work with $\Gamma_i a^3$, the interaction rate calculated with comoving densities. The collisional scale $l_{cd}$ may finally be written as the quadratic sum

\hspace{1cm} (4)

We denote the cosmological scale-factor, normalized to unity at the present epoch, by $a(t)$, and the matter and radiation energy densities by $\rho_m$ and $\rho_r$, respectively. We will also use $g_*(T)$ as the effective number of interacting relativistic degrees of freedom and define the parameters $\kappa(T)$ and $\kappa_{dm}(T)$ describing the photon and Dark Matter temperature dependence on $a(t)$ as $T = \frac{T_0}{a(t)^{\kappa(T)}}$ and $T_{dm} = \frac{T_0}{a(t)^{\kappa_{dm}(T)}}$, respectively. Here, $T_0$ is the present photon temperature.

\hspace{1cm} (5)

Bulk viscosity is expected to be negligible. On the other hand, Dark Matter particle diffusion would add a further contribution to the self-damping similar to the one due to shear viscosity. We ignore it since it should not provide any different constraint.
of specific contributions, namely self-damping ($l_{sd}$) and induced-damping ($l_{id}$), where the index $i$ is relative to each species $i \neq dm$, so that

$$l_{sd}^2 = l_{sd}^2 + \sum_{i \neq dm} l_{id}^2 ,$$

(2)

with

$$l_{sd}^2 = \frac{2\pi^2}{3} \int_0^{t_{dec(dm)}} \frac{\rho_{dm}}{\rho} \frac{v_{dm}^2}{a^2 \Gamma_{dm}} (1 + \Theta_{dm}) \, dt ,$$

(3)

$$l_{id}^2 = \frac{2\pi^2}{3} \int_0^{t_{dec(dm-i)}} \frac{\rho_i}{\rho} \frac{v_i^2}{a^2 \Gamma_i} (1 + \Theta_i) \, dt .$$

(4)

In these expressions, $\Theta_x = \frac{\rho_x^2}{\rho} \frac{d \ln \rho_x}{d \ln T_x} \big|_{x=dm,i}$ is associated with the contribution of thermal conduction to the damping.

For an acceptable Dark Matter candidate, each of these scales must be smaller than the length $l_{struct}$ associated with the smallest primordial object presently observed. The latter is normalized to 100 kpc, corresponding to an object of approximately $10^9 M_\odot$ which could be a small galaxy or a Lyα cloud.

A systematic classification of all different Dark Matter particles may be achieved by considering the epoch at which these particles become non-relativistic (scale-factor $a_{nr}$), and the epoch at which they thermally decouple (scale-factor $a_{dec}$). These two specific scale-factors may then be compared to a third scale-factor $a_{eq} = \frac{\rho_e(T_0)}{\rho_m(T_0)}$. The latter is relevant even in the cases where Dark Matter particles would still be relativistic at $a_{eq}$, i.e. when this scale-factor no longer corresponds to the standard matter-radiation equality. The ordering of these three scale-factors defines six regions, shown in Fig. 1, corresponding to six general classes of Dark Matter particles, labelled from I to VI. As a matter of illustration, neutrinos of a few eV (or gravitinos of $\sim$ hundred of eV to keV), for instance, would belong to region I ($a_{dec} < a_{nr} < a_{eq}$), heavy supersymmetric particles to region II ($a_{nr} < a_{dec} < a_{eq}$) and baryon-like particles to region III ($a_{nr} < a_{eq} < a_{dec}$). The three other regions IV to VI, namely $a_{dec} < a_{eq} < a_{nr}$, $a_{eq} < a_{dec} < a_{nr}$, and $a_{eq} < a_{nr} < a_{dec}$, for which $a_{eq} < a_{nr}$, all correspond to light Dark Matter particles having masses less than a few eV. A complete calculation of damping scales for each of the six regions will be done in details in a follow-up paper [8]. Here, we only present the main results arising from this procedure.

### 3 Constraints from self-damping and free-streaming.

In all relevant cases, the accumulated collisional damping length turns out to be dominated by late epochs. So, equation (3) may simply be written as

$$l_{sd} \sim \pi \left[ \frac{\rho_{dm}}{\rho} \frac{H}{\Gamma_{dm}} (1 + \Theta_{dm}) \right]^{1/2} \frac{v_{dm}(t)}{a(t)} \bigg|_{t_{dec(dm)}} .$$

(5)
The coefficient $\Theta_{dm}$ is negligible, except in region V where it is of order $f_{\text{dm}}$, so that thermal conduction can be neglected in the computation of the self-damping scale in most of the cases. The ratio $\rho_{dm}/\rho$ may be very small in regions I and IV (if Dark Matter has been in contact with particles of the thermal bath) while in regions II and V already, but especially in regions III and VI, it gets close to unity. The ratio $\frac{H}{\Gamma_{dm}}$ is equal to unity when taken at $t_{\text{dec}(dm)}$. The self-damping scale (5) is then seen to be smaller, or comparable (regions II and III), to the free-streaming scale which reads

$$l_{fs} = \pi \frac{v_{dm}(t) t}{a(t)} \left|_{\max(t_{\text{dec}(dm)}-t_0)} \right.$$  \hspace{1cm} (6)

As a result, although self-damping does not in general significantly modify the limits obtained from free-streaming, it appears that in many cases the former does actually erase a large part of the scale-fluctuations. In addition, in the special cases for which the non-linear collapse would occur before the Dark Matter decoupling (which corresponds to the upper parts of regions III and VI, respectively denoted by regions III’ and VI’), only collisional damping constraints are left since free-streaming no longer acts on Dark Matter primordial fluctuations. In these regions, the self-damping effect – which has to be estimated at the non-linear collapse epoch and no longer at the Dark Matter decoupling time – is nevertheless greatly reduced since, for large interaction rates, the factor $\left(\frac{H}{\Gamma_{dm}}\right)_{\text{collapse}}$ may be significantly smaller than unity.

4 Constraints from induced-damping.

The largest damping effects appear, from eq. (4), to be induced by particles which are both relativistic and late decoupling. This leads to consider neutrinos as well as photons as the primary source of induced-damping, as it was already the case when baryons were thought to be the only matter component of the Universe [10, 11, 12]. The constraints we obtain turn out to correspond to an epoch where the Universe is radiation dominated so that the collisional damping of interest to our purpose is only due to shear viscosity.

In the standard scheme, neutrinos are expected to decouple at a temperature of $\sim 1$ MeV. If Dark Matter decouples from neutrinos at this epoch, or earlier, we find that the neutrino induced-damping scale is $l_{\nu, d} \lesssim 100$ pc, which is of reduced cosmological interest. This nevertheless provides constraints on the Dark Matter parameters in case one is led to require the formation of primordial structures of less than $\sim 1\text{M}_\odot$. If, on the other hand, Dark Matter decouples from neutrinos at a temperature $T < 1$ MeV, an additional source of damping is expected. In regions I to III, where neutrinos are much more numerous than Dark Matter particles, the Dark Matter-neutrino interaction rate ($\Gamma_{dm-\nu}$) may be larger than the neutrino-Dark Matter rate ($\Gamma_{\nu-\text{dm}}$). In this case, Dark Matter can remain coupled to freely-propagating neutrinos which, in turn, induce “collisional” damping effects! From a rough estimate of the new corresponding damping
scale, we find that Dark Matter candidates must satisfy
\[< \sigma v >_{\nu-dm} \lesssim 1 \times 10^{-27} \text{cm}^3/\text{s} \, \frac{g_s^{1/2}(T)}{\kappa^2(T)} \, \frac{m_{dm}}{1 \text{MeV}} \, \left(\frac{l_{\text{struct}}}{100 \text{ kpc}}\right)^2, \tag{7}\]
at the Dark Matter-neutrino decoupling time, corresponding to cross-sections smaller or of the order of \(10^{-37} \frac{m_{dm}}{1 \text{MeV}} \text{cm}^2\). This bound, valid for any type of Dark Matter particles, is of potential interest to constrain Dark Matter properties.

The photon induced-damping scale is given by \(l_{\gamma d} \simeq 2.2 \text{ Mpc} \, \frac{\kappa(T)}{g_{\ast}(T)} \left[\frac{a(t)}{10^{-4}}\right]^{3/2}\), taken at Dark Matter-photon decoupling time. So, as a rule of thumb, the decoupling must occur somewhat before the epoch of the standard matter-radiation equality to avoid prohibitive damping effects. More specifically, this implies
\[< \sigma v >_{\gamma-dm} \lesssim 7 \times 10^{-24} \text{cm}^3/\text{s} \, \frac{m_{dm}}{1 \text{MeV}} \, \left[\frac{g_\ast(T)}{\kappa^4(T)}\right]^{1/2} \left(\frac{l_{\text{struct}}}{100 \text{ kpc}}\right)^{1/2}, \tag{8}\]
that is cross-sections smaller or of the order of \(10^{-33} \frac{m_{dm}}{1 \text{MeV}} \text{cm}^2\).

Here it is important to note that, as they are defined, the (momentum-transfer weighted) average cross-sections that we use take into account the efficiency of each reaction in changing the particle momentum [7] and cannot be assimilated to ordinary thermal averages. In this formulation, the \(dm - \nu (dm - \gamma)\) momentum-weighted thermal average cross-sections significantly differ from the \(\nu - dm (\gamma - dm)\) ones, by a factor \(\frac{v_{dm}^2}{c^2} |t_{dec}| \sim 3 T_{dec}/m_{dm}\), so that
\[< \sigma v >_{dm-i}/c \sim (3 T_{dec}/m_{dm}) < \sigma v >_{i-dm}/c, \tag{9}\]
where \(< \sigma v >_{i-dm}/c\), for relativistic particles \((i = \nu, \gamma)\) scattered by a heavy target \((dm)\), are close to the corresponding total cross-sections.

These constraints, relevant in the parts of regions I, II, III and possibly VI where the Dark Matter particles escape free-streaming and self-damping constraints, become evidently more stringent if primordial structures of less than \(10^9 M_\odot\) are required to form.

5 Dark Matter scenarios.

In addition to the damping requirements, Dark Matter particles must have an acceptable relic density. For instance, non-annihilating Dark Matter particles must decouple extremely early from relativistic species, to avoid overclosing the Universe. The latter requirement turns out to provide much more stringent constraints on the Dark Matter mass than the ones obtained from induced-damping when Dark Matter decouple from the relativistic species after inflation: only small masses \(\lesssim \text{keV}\) (or conceivably up to \(\sim \text{MeV}\) or even more, if the number of interacting relativistic degrees of freedom at decoupling
were very large) are compatible with the observed relic density. Taking into account the limit on the Dark Matter mass ($\gtrsim$ keV) obtained from the self-damping and free-streaming estimate, one can see that, in the best case, only particles having masses in the $\sim$ keV (up to $\sim$ MeV or so ...) range are allowed. On the other hand, if Dark Matter particles decouple before or during an epoch of inflation (which then must be tuned to dilute them by just the right amount), or if they can annihilate after their non-relativistic transition, any mass $\gtrsim$ keV is allowed.

In Fig. 1, we plot the limits arising from both self-damping and free-streaming requirements in the plane defined by the Dark Matter mass and interaction rate. This allows us to define the different scenarios to which Dark Matter particles belong. One still has to keep in mind that each of the allowed candidates on this figure has to satisfy, also, both relic density and induced-damping requirements, not graphically represented there.

We now discuss the various possible Dark Matter scenarios. Hot Dark Matter (HDM) usually refers to particles for which galactic-scale fluctuations are damped by free-streaming. Since self-damping and free-streaming are seen to behave in a similar way, as discussed in section 3, we suggest to call HDM particles those for which both self-damping and free-streaming effects prevent structure formation. Conversely, Cold Dark Matter (CDM) scenarios are defined as the ones for which collisional damping and free-streaming are negligible whereas Warm Dark Matter (WDM) scenarios are those for which the damping is just at the edge to allow the formation of galaxies.

**HDM.** With our definition, we recover usual candidates, relativistic at the moment of their decoupling (region I) and having a mass less than a few keV [13]. But we see that HDM extends further into regions II and III for particles with masses up to even a few MeV, despite their small velocity at decoupling. In addition, we see that HDM scenarios extend into region III', where Dark Matter remains thermally coupled a very long time and where only self-damping constraints are left.

**CDM.** The original scenario of massive weakly-interacting particles [14] refers to particles belonging to region I, heavy enough ($\gtrsim$ keV) to escape the free-streaming constraint [13]. Such particles, if they are non-annihilating, are CDM candidates only if their number density is adequately diluted by inflation, at least considering thermal relics only. (Indeed, if they would decouple from relativistic species after inflation, they would be WDM particles as indicated by their allowed range of mass.) Hence, these early-decoupling particles are required to have extremely low interaction rates with relativistic species and are not expected to suffer from induced-damping effects.

However, CDM scenarios also involve massive particles which annihilate and thermally decouple after their non-relativistic transition [1], still escaping the free-streaming constraint. Such Weakly-Interacting Massive Particles (WIMPs) are considered to be the most favored CDM candidates and may be illustrated by supersymmetric particles. These
particles are usually considered to be essentially collisionless (so that their expected place is a priori at the bottom of region II) although they must in any case have a minimal amount of interactions, at least for being able to annihilate. Weakly-interacting candidates may then suffer from induced-damping effects related to their interactions with neutrinos (eq. (7)) or photons (eq. (8)). An estimate of the corresponding scales is thus necessary to claim that a particle is a good Dark Matter candidate. This is especially relevant for weakly-interacting particles decoupling from neutrinos at a late time, which, if allowed, may actually be in the upper part of region II.

In addition, we see that other Dark Matter candidate possibilities exist. Provided their coupling with neutrinos and photons is moderate, CDM also includes particles having much larger cross-sections (region III), which may even be comparable to those of the baryonic matter. Even more strongly-interacting particles are allowed, for nearly any mass, in case they remain collisional up to the epoch of structure formation (region III').

We refer to this strongly-interacting Dark Matter as SDM. Such particles may escape the Tremaine-Gunn [15] bound and be even of quite low mass if bosonic. A potential problem, however, is the observed phase-space of the structures [16]: if clustering is hierarchical, Dark Matter particles cannot have been collisional during their gravitational collapse.

**WDM.** WDM candidates usually are non-annihilating but rather weakly-interacting particles (region I), barely escaping the free-streaming bound, with a mass not much above 1 keV. We find, in addition, that more strongly-interacting (but still non-annihilating) particles (regions II and III), with masses between the keV and MeV range, could also belong to this scenario. More generally, WDM candidates can be associated with any kind of particles as long as they are just at the limit of the region allowed by self-damping and free-streaming. This is also the case for particles suffering from induced-damping effects at this limit. Should these CDM-looking particles, having a mass above 1 MeV in the upper region II or in region III (Fig. 1), which experience moderate induced-damping, be considered as a new kind of WDM?

Altogether this analysis shows that the constraints arising from the damping of adiabatic primordial fluctuations are to be considered very seriously, whatever the Dark Matter candidate, in addition to the relic density requirements.

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Figure 1: The different Dark Matter scenarios (HDM, CDM, WDM and SDM) may be classified according to the particle mass (more precisely the product $m_{\text{dm}} \kappa_{\text{dm}} = 3 T_0 / a_{\text{nr}}$ where the scale-factor $a_{\text{nr}}$ characterizes the epoch at which Dark Matter particles become non-relativistic), as well as the Dark Matter interaction rate $\Gamma_{\text{dm}} a^3$. This rate is evaluated at the epoch of Dark Matter decoupling or at the onset of structure formation, whichever occurs first. The two resulting regimes are separated by the horizontal dotted line corresponding to $\Gamma_{\text{dm}} a^3 \simeq 7 \times 10^{-20}$ s$^{-1}$. The arrow on the right corresponds to the value of $\Gamma_{\text{dm}} a^3$ implied by the Spergel-Steinhardt [2] scenario. The dark hatched regions are excluded by collisional damping or free-streaming when we require fluctuations of scale above 100 kpc, corresponding to $10^9 M_\odot$, to survive. The light hatched regions are those excluded by the relic density requirement for particles which do not annihilate after becoming non-relativistic and do not decouple before or during inflation. The new additional constraints due to induced-damping are not represented here. We nevertheless indicate by the label “(WDM)” particles which, due to induced-damping, are only marginally allowed.