The scalar induced gravitational waves are produced from primordial curvature perturbations in the second order of perturbations. We constrain the fractional energy density of scalar induced gravitational waves from gravitational waves observations. If there is no detection of the scalar induced gravitational waves, the fractional energy density of scalar induced gravitational waves is constrained by some upper limits. Depends on these upper limits, we can obtain the constraints on the power spectrum of the primordial curvature perturbations. For a power-law scalar power spectrum, the constraints from FAST project have a significant impact on the amplitude and spectral index, namely \( \ln(10^{10}A_s) = 3.024^{+0.015}_{-0.012} \) and \( n_s = 0.9468^{+0.0010}_{-0.0007} \) at 68% confidence level. We also consider the effects of LIGO, Virgo, LISA and IPTA detectors, while the constraints from CMB+BAO are totally within their upper limits of scalar induced gravitational waves.

I. INTRODUCTION

The cosmological perturbation theory has been developed so fast these decades owing to cosmological observations, such as the cosmic microwave background (CMB). The primordial perturbations affect not only temperature but also polarization by the scalar and tensor perturbations [1–4]. The polarization can be decomposed into E-mode and B-mode, while the B-mode component mainly comes from the tensor perturbation on very large scales and encodes the information of primordial gravitational waves. The upper limit on the tensor-to-scalar ratio is \( r_{0.05} < 0.038 \) at 95% confidence level from the combinations of Planck satellite [5], the BICEP/Keck Observations [6] and the Baryon Acoustic Oscillation (BAO) [7–9] which reflects the constraint on primordial gravitational waves.

Besides the primordial gravitational waves, the scalar induced gravitational waves are generated in the second order of perturbations. The curvature perturbations couple to the tensor perturbations at second-order which produce the scalar induced gravitational waves in the radiation dominated era. Although the scalar induced gravitational waves are suppressed by the square of curvature perturbations, but they can compare with primordial gravitational waves if the curvature perturbations are large enough. The enhancement of scalar induced gravitational waves can be realized in some models of inflation [10, 11] or scalar power spectrum [12–14].

The detection of scalar induced gravitational waves becomes important in cosmological perturbation theory. The gravitational waves detections provide the latest way to find scalar induced gravitational waves which include LIGO and Virgo detector [15], Laser Interferometer Space Antenna (LISA) detector [16] and two Pulsar timing array (PTA) projects [17], namely International Pulsar Timing Array (IPTA) [18] and Five-hundred-meter Aperture Spherical radio Telescope (FAST) [19, 20]. All of these detectors are sensitive to the fractional energy density which may contain information of scalar induced gravitational waves. If there is no detection of scalar induced gravitational waves, the fractional energy density of scalar induced gravitational waves is constrained by some upper limits. For LIGO and Virgo detector, the upper limit is \( \Omega_{GW} < 10^{-7} \) at frequency around 40 Hz. For LISA detector, the upper limit is \( \Omega_{GW} < 10^{-12} \) at frequency around 10^{-3} Hz. For IPTA detector, the upper limit is \( \Omega_{GW} < 10^{-16} \) at frequency 1.58 \( \times \) 10^{-9} Hz. For FAST detector, the upper limit is \( \Omega_{GW} < 10^{-19} \) at frequency 6.34 \( \times \) 10^{-10} Hz. Depends on these upper limits, we can obtain the constraints on the power spectrum of the primordial curvature perturbations. In this paper, we consider a power-law scalar power spectrum and constrain the power spectrum from the upper limits of scalar induced gravitational waves.
II. THE SCALAR INDUCED GRAVITATIONAL WAVES

In the conformal Newtonian gauge, the metric about the Friedmann-Robert-Walker background is taken as
\[
ds^2 = a^2 \left\{- (1 + 2\Phi) d\eta^2 + (1 - 2\Phi) \delta_{ij} + \frac{h_{ij}}{2} \right\} dx^i dx^j, \tag{1}
\]
where \(a(\eta)\) is the scale factor, \(\eta\) is the conformal time, \(\Phi\) is the scalar perturbation and \(h_{ij}\) is the gravitational waves perturbation. In the Fourier space, the tensor perturbation \(h_{ij}\) is
\[
h_{ij}(\eta, x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left( e^{+}_{ij}(k) h^+_k(\eta) + e^{-}_{ij}(k) h^-_k(\eta) \right) e^{ik \cdot x}, \tag{2}
\]
where the plus and cross polarization tensors are
\[
e^{+}_{ij}(k) = \frac{1}{\sqrt{2}} (e_i(k) e_j(k) - \epsilon_i(k) \epsilon_j(k)), \quad e^{-}_{ij}(k) = \frac{1}{\sqrt{2}} (e_i(k) \epsilon_j(k) + \epsilon_i(k) e_j(k)), \tag{3}
\]
the normalized vectors \(e_i(k)\) and \(\epsilon_i(k)\) are orthogonal to each other and to \(k\). The gravitational waves satisfy following wave equations
\[
h''_k(\eta) + 2H h'_k(\eta) + k^2 h_k(\eta) = 4S_k(\eta), \tag{4}
\]
where the prime denotes derivative with respect to conformal time and \(H = a'/a = aH\) is the conformal Hubble parameter. The source term is given by
\[
S_k = \int \frac{d^3q}{(2\pi)^{3/2}} e_{ij}(k) q_i q_j \left( 2\Phi q \Phi_{k-q} + \frac{4}{3(1+\omega)} \left( H^{-1} \Phi' + \Phi_q \right) \left( H^{-1} \Phi'_{k-q} + \Phi_{k-q} \right) \right). \tag{5}
\]
The power spectrum of scalar induced gravitational waves is defined as
\[
\langle h_k(\eta) h_{k'}(\eta) \rangle = \frac{2\pi^2}{k^3} \delta^{(3)}(k + k') \mathcal{P}_h(\eta, k), \tag{6}
\]
and the fractional energy density is
\[
\Omega_{GW}(\eta, k) = \frac{1}{24} \left( \frac{k}{aH} \right)^2 \mathcal{P}_h(\eta, k). \tag{7}
\]
After calculation, the power spectrum of scalar induced gravitational waves takes the form
\[
\mathcal{P}_h(\eta, k) = 4 \int_0^{\infty} dv \int_{|1-v|}^{1+v} du f^2(v, u, x) \mathcal{P}_\xi(ku) \mathcal{P}_\zeta(ku), \tag{8}
\]
where \(\mathcal{P}_\zeta(k)\) is the power spectrum of the primordial curvature perturbations. The function \(f(v, u, x)\) is defined as
\[
f(v, u, x) = I(v, u, x) \frac{4v^2 - (1 + v^2 - u^2)^2}{4vu}, \tag{9}
\]
where function \(I(v, u, x)\) comes from the source term.

For a power-law scalar power spectrum
\[
\mathcal{P}_\xi(k) = A_s \left( \frac{k}{k_s} \right)^{n_s-1}, \tag{10}
\]
the power spectrum of scalar induced gravitational waves is given by \([12]\)
\[
\mathcal{P}_h(\eta, k) = \frac{24Q(n_s)}{(kn)^2} A_s^2 \left( \frac{k}{k_s} \right)^{2(n_s-1)}, \tag{11}
\]
where $A_s$ is the amplitude of power-law spectrum, $n_s$ is the scalar spectral index, $k_*$ is the pivot scale and $Q(n_s)$ is coefficient. The fractional energy density becomes

$$
\Omega_{GW}(\eta, k) = Q(n_s)A_s^2 \left( \frac{k}{k_*} \right)^{2(n_s-1)}.
$$

The gravitational waves detections show the sensitivity curves of frequency and $\Omega_{GW}$ in the detectible ranges which can be used to find scalar induced gravitational waves. If there is no detection of scalar induced gravitational waves, the fractional energy density of scalar induced gravitational waves is constrained by some upper limits. Combine these upper limits with Eq. (12), we can obtain the constraints on the power spectrum of the primordial curvature perturbations.

### III. CONSTRAINTS ON PRIMORDIAL CURVATURE PERTURBATIONS FROM THE SCALAR INDUCED GRAVITATIONAL WAVES

We use the publicly available codes Cosmomc [21] to constrain the scalar induced gravitational waves and the power spectrum of primordial curvature perturbations. In the standard $\Lambda$CDM model, the six parameters are the baryon density parameter $\Omega_b h^2$, the cold dark matter density $\Omega_c h^2$, the angular size of the horizon at the last scattering surface $\theta_{MC}$, the optical depth $\tau$, the scalar amplitude $A_s$ and the scalar spectral index $n_s$. Usually we introduce a new parameter, namely the tensor-to-scalar ratio $r$, to quantify the tensor amplitude $A_t$ compared to the scalar amplitude $A_s$ at the pivot scale:

$$
r \equiv \frac{A_t}{A_s}.
$$

We extend this model by adding the tensor-to-scalar ratio $r$ and constrain these seven parameters in the $\Lambda$CDM+$r$ model from the combinations of CMB+BAO and CMB+BAO+FAST, respectively. Our numerical results are given in the Table 1 and Fig. 1.

In the $\Lambda$CDM+$r$ model, the constraint on the amplitude of power-law spectrum is

$$
\ln \left( 10^{10} A_s \right) = 3.049 \pm 0.014,
$$
at 68% CL from CMB+BAO, and

$$
\ln \left( 10^{10} A_s \right) = 3.024^{+0.015}_{-0.012},
$$
at 68% CL from CMB+BAO+FAST. The constraint on the scalar spectral index is

$$
n_s = 0.9654 \pm 0.0037,
$$
at 68% CL from CMB+BAO, and

$$
n_s = 0.9468^{+0.0010}_{-0.0007},
$$
at 68% CL from CMB+BAO+FAST.

### TABLE I: The 68% limits on the cosmological parameters in the $\Lambda$CDM+$r$ model from the combinations of CMB+BAO and CMB+BAO+FAST, respectively.

| Parameter | CMB+BAO | CMB+BAO+FAST |
|-----------|---------|--------------|
| $\Omega_b h^2$ | $0.02241 \pm 0.00013$ | $0.02220 \pm 0.00012$ |
| $\Omega_c h^2$ | $0.11954 \pm 0.00091$ | $0.12235 \pm 0.00075$ |
| $100\theta_{MC}$ | $1.04099 \pm 0.00029$ | $1.04068^{+0.00029}_{-0.00028}$ |
| $\tau$ | $0.0567^{+0.0070}_{-0.0071}$ | $0.0415^{+0.0074}_{-0.0060}$ |
| $\ln \left( 10^{10} A_s \right)$ | $3.049 \pm 0.014$ | $3.024^{+0.015}_{-0.012}$ |
| $n_s$ | $0.9654 \pm 0.0037$ | $0.9468^{+0.0010}_{-0.0007}$ |
| $r_{0.05}$ (95% CL) | $< 0.038$ | $< 0.033$ |

1 CMB: Planck18+BK18=TT+TE+EE+lensing+BAO+ BK18
at 68% CL from CMB+BAO+FAST. The constraint on the tensor-to-scalar ratio is

\[ r < 0.038, \]  

(18)

at 95% CL from CMB+BAO, and

\[ r < 0.033, \]  

(19)

at 95% CL from CMB+BAO+FAST. We see that the constraints on the power spectrum of primordial curvature perturbations are significant affected by adding the upper limit of scalar induced gravitational waves from FAST project. If there is no detection of scalar induced gravitational waves from FAST project, the amplitude and the spectral index of power-law spectrum are much smaller than the constraints from CMB+BAO which are obvious in Fig. 1. We also consider the effects of LIGO, Virgo, LISA and IPTA detectors. While the constraints on the amplitude and the spectral index from CMB+BAO are totally within the upper limits of scalar induced gravitational waves from these detectors.
In this paper, we constrain the fractional energy density of scalar induced gravitational waves from gravitational waves observations. If there is no detection of the scalar induced gravitational waves, the fractional energy density of scalar induced gravitational waves is confined by some upper limits. Depends on these upper limits, we can obtain the constraints on the power spectrum of the primordial curvature perturbations. For a power-law scalar power spectrum, the constraints on the power spectrum are significant affected by adding the upper limit of scalar induced gravitational waves from FAST project. We also consider the effects of LIGO, Virgo, LISA and IPTA detectors, while the constraints from CMB+BAO are totally within their upper limits of scalar induced gravitational waves.

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