Plasma Beams Free Vibration Investigation using the Boubaker Polynomials Expansion Scheme

K. Boubaker
Equipe de Physique des dispositifs a Semiconducteurs /ESSTT/, Faculte des Sciences de Tunis, Campus Universitaire, 2092 Tunis, Tunisia

Abstract: Problem statement: In the present study, plasma beams free vibration frequencies alteration has been investigated through an original protocol. Approach: The Boubaker Polynomials Expansion scheme BPES has been performed for deducing and ranging the dimensionless frequencies of the beam vibrations. Results: Natural frequencies of the plasma beam have been estimated for given parameters values. Conclusion: Yielded results have been compared and discussed. It was found that a good agreement as the values determined with experimental results determined by previous researchers.

Key words: Plasma beams, free vibration, natural frequency, mechanical vibrations, Metastability Exchange Optical Pumping (MEOP), avalanche ionization, generally laminated plasma

INTRODUCTION

In the two last decade, plasma beams have been given interesting electric and fire properties (Dohnalik et al., 2011; Vo et al., 2010; Ghanouchi et al., 2008; Chen 2003; Qindeel et al., 2007; Khare et al., 2004; Della and Shu, 2005; Kisa et al., 1998; Meirovitch, 1986). Sollier et al. (2011) studied plasma beams vibration patterns in terms of peak irradiance, pulse width and duration for energy pulses transmitted through the breakdown plasmas generated in fluids. Recorded data for 25 ns-1064 nm beam pulses outlined the roles of avalanche ionization and multiphoton ionization. Vo et al. (2010) studied the Metastability Exchange Optical Pumping (MEOP) performed at elevated 3He gas pressures using an annular beam and elaborated an accurate vibration model profiling. Many analytical methods of analysis have been used to study the vibration of plates, shells and beams (Khare et al., 2004; Della and Shu, 2005; Kisa et al., 1998; Meirovitch, 1986; Sollier et al., 2011; Osman and Hora, 2004; Osman et al., 2004; 2005).

In this study, a model on the vibration analysis of plasma beam has been developed and studied using a confirmed and tested scheme under some presumptions.

MATERIALS AND METHODS

According to the model presented in Fig. 1, The relationship between normal stress and bending moment, according to Bernoulli-Euler hypotheses, is given by Eq. 1:

\[ M = \frac{2h}{3\rho} \sum_{i=1}^{N} E_i (z_i - z_{i-1}) \]  

Fig. 1: Geometrical model of the studied beam

\[ \rho = \text{Beam curvature.} \]  
\[ H = \text{Beam radius.} \]  
\[ N = \text{The number of layers} \]  
\[ Z_k = k^{th} \text{ layer outer face distance to neutral plane} \]  
\[ E_x = \text{Elasticity modulus} \]  

Where:

\[ M = \frac{E_w I_{xx}}{\rho} = E_d I_{yy} \frac{d^2 W}{dx^2}, \]  
\[ E_d = \frac{8}{h} \sum_{i=1}^{N} (E_i) (z_i - z_{i-1}) \]  

Consequently, the relationship between the bending moment and the curvature can be written as Eq. 2 follows:
Where:

\( E_{ef} = \) Effective elasticity modulus.

\( I_{yy} = \) Beam cross-sectional inertia moment.

\( W = \) Lateral deflection \( x \)-dependent mode profile

Plasma beam flexural motion is described by the Eq. 3:

\[
4 \pi^2 \frac{E_{ef} I_{yy}}{\rho} \frac{\partial^2 w(x,t)}{\partial t^2} + \frac{\partial^2 w(x,t)}{\partial x^2} = 0
\]

With a trivial solution Eq. 4:

\[
w(x,t) = e^{\sigma_n t} w(x)
\]

where, \( \sigma_n \) is the frequency.

By introducing the expression (4) in Eq. 3, the following time-independent non-variable characteristic value problem is deduced Eq. 5:

\[
4 \pi^2 \frac{d^2 W(x)}{dx^2} - f_n^2 W(x) = 0
\]

where, \( f_n \) is the dimensionless frequency of the beam vibrations given by Eq. 6:

\[
f_n^2 = \frac{\sigma_n^2 \rho}{E_{ef} I_{yy}}
\]

Along with the boundary conditions, as per Eq. 7 Shu (2000):

\[
\begin{align*}
W(x)|_{x=0} &= 0 \\
\frac{dW(x)}{dx}|_{x=0} &= 0 \\
\frac{d^2W(x)}{dx^2}|_{x=L} &= 0
\end{align*}
\]

The solution of the system referring to Eq. 4 and 6 is obtained using the Boubaker Polynomials Expansion Scheme BPES (Awujoyogbe and Boubaker, 2009; Labiadh and Boubaker, 2007; Slama et al., 2008; 2009; Hossein et al., 2009; Fridjine and Amlouk, 2009; Belhadj et al., 2009a; 2009b), through establishing the expression Eq. 8:

\[
W(x) = \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k B_{4k}(x \times \frac{r_k}{L})
\]

where, \( B_{4k} \) are the 4\( k \)-order Boubaker polynomials, \( x \in [0, L] \) is the normalized distance, \( r_k \) are \( B_{4k} \) minimal positive roots, \( N_0 \) is a prefixed integer and \( \lambda_k \) are unknown pondering real coefficients.

Consequently, it comes for Eq. 5 that Eq. 9:

\[
\sum_{k=1}^{N_0} \lambda_k B_{4k}(x \times \frac{r_k}{L}) = 0
\]

The related boundary conditions expressed through Eq. 6. The Boubaker Polynomials Expansion Scheme (BPES) protocol ensures their validity regardless main equation features. In fact, thanks to Boubaker polynomials first derivatives properties are Eq. 10 and 11:

\[
\begin{align*}
\sum_{k=1}^{N_0} dB_{4k}(x) |_{x=0} &= -2N_0 \neq 0 \\
\sum_{k=1}^{N_0} dB_{4k}(x) |_{x=r_k} &= \sum_{q=1}^{N} H_q
\end{align*}
\]

And:

\[
\begin{align*}
\sum_{k=1}^{N_0} dB_{4k}(x) |_{x=0} &= 0 \\
\sum_{k=1}^{N_0} dB_{4k}(x) |_{x=r_k} &= \sum_{q=1}^{N} H_q
\end{align*}
\]

with: \( H_q = B_{4q}(r_q) \frac{4r_q \left[ 2 - r_q \right] \sum_{k=1}^{N_0} B_{4k}(r_k)}{B_{4q+1}(r_q)} \)

Boundary conditions are inherently verified Eq. 12:

\[
\begin{align*}
\frac{dW(x)}{dx} |_{x=0} &= \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k dB_{4k}(x) |_{x=0} = 0 \\
\frac{dW(x)}{dx} |_{x=L} &= \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k dB_{4k}(x) |_{x=L} = 0 \\
\sum_{k=1}^{N_0} \lambda_k H_q &= 0
\end{align*}
\]

The BPES solution is obtained by integrating, for a given value of \( N_0 \), the whole expression given by Eq. 8.
Table 1: Bream frequencies estimated using the PBES

| Mode No. | Frequency       | Error (%) |
|----------|-----------------|-----------|
| 1        | 49.85           | 2.3       |
| 2        | 363.53          | 2.8       |
| 3        | 998.93          | 2.4       |
| 4        | 1090.45         | 1.9       |
| 5        | 1210.55         | 2.2       |
| 6        | 1305.92         | 2.1       |
| 7        | 1403.15         | 2.0       |
| 8        | 1500.49         | 2.1       |

along the interval \([0, L]\), determining the set of coefficients where \(\tilde{\lambda}_k\) that minimizes the absolute difference Eq. 13:

\[
\Phi = \frac{1}{2N_0} \left| \sum_{k=1}^{N_0} \tilde{\lambda}_k \times \alpha_k - \zeta \sum_{k=1}^{N_0} \tilde{\lambda}_k \times \beta_k \right|
\]

with:

\[
\alpha_k = \int_0^L \frac{1}{L} \int_r^L dB_{nk}(x \times \frac{r}{L}) dx
\]

\[
\beta_k = \int_0^L B_{nk}(x \times \frac{r}{L}) dx
\]

And finally deducing, for increasing values of \(N_0\), the corresponding frequency using Eq. 6.

**RESULTS**

Plasma beam frequencies estimated using the boubaker polynomials expansion scheme PBES are gathered in Table 1.

**DISCUSSION**

The natural frequency changes as a direct result of the change in the stacking sequence causes resonance if the changed frequency becomes closer to the working frequency. This feature is in good agreements with the results published by Aly et al. (2010); Alsultanny (2006); Mardani (2008) and Hashemi et al. (2007).

With relation to the deviations of the numerical results in relation to those recorded in the related literature, some possible measurement errors can be pointed out such as measurement noise or non-uniformity in the beam properties (voids, variations in radius, non uniform outer shape ..). Such factors are not taken into account during the numerical analysis, since the model considers a strictly homogeneous beam, what rarely occurs in practice.

**CONCLUSION**

This study presents a protocol for the calculation of natural frequency of the plasma beam. Calculations performed by means of Boubaker Polynomials Expansion Scheme (PBES) resulted in coherent similar results. Changes in the stacking sequence as well as damping effects, which can have a large influence on the path of the beam, are the subject of coming investigations.

**REFERENCES**

Alsultanny, Y.A., 2006. Laser beam analysis using image processing. J. Comput. Sci., 2: 109-113. DOI: 10.3844/jcssp.2006.109.113

Aly, M.F., I.G.M. Goda and G.A. Hassan, 2010. Experimental investigation of the dynamic characteristics of laminated composite beams. Int. J. Mech. Mechatronics, 10: 59-68.

Awojoyogbe, O.B. and K. Boubaker, 2009. A solution to Bloch NMR flow equations for the analysis of hemodynamic functions of blood flow system using m-Boubaker polynomials. Curr. Applied Phys., 9: 278-283. DOI: 10.1016/j.cap.2008.01.019

Belhadj, A., J. Bessrour, M. Bouhafs and L. Barrallier, 2009. Experimental and theoretical cooling velocity profile inside laser welded metals using keyhole approximation and Boubaker polynomials expansion, J. Thermal Ann. Calorim., 97: 911-915. DOI: 10.1007/s10973-009-0094-4

Belhadj, A., O.F. Onyango and N. Rozibaeva, 2009. Boubaker polynomials expansion scheme-related heat transfer investigation inside keyhole model. J. Thermophy. Heat Transfer, 23: 639-640

Chen, C.N., 2003. Buckling equilibrium equations of arbitrarily loaded nonprismatic composite beams and the DQEM buckling analysis using EDQ. Applied Math. Modell., 27: 27-46. DOI: 10.1016/S0307-904X(02)00084-7

Della, C.N. and D. Shu, 2005. Free vibration analysis of composite beams with overlapping delaminations. Eur. J. Mech. A/Solids 24: 491-503. DOI: 10.1016/j.euromechsol.2005.01.007

Dohnalik, T., A. Nikiel, T. Palas, M. Suchanek and G.Collier et al., 2011. Optimization of the pumping laser beam spatial profile in the MEOP experiment performed at elevated 3He pressures. Eur. Phys. J. Applied Phys., 54: 20802-20809. DOI: 10.1051/epjap/2011100493

Fridjine, S. and M. Amlouk, 2009. A new parameter: An ABACUS for optimizig functional materials using the Boubaker polynomials expansion scheme. Modern Phys. Lett., B 23: 2179-2182.

Ghanouchi, J., H. Labiad and K. Boubaker, 2008. An attempt to solve the heat transfrt equation in a model of pyrolysis spray using 4q-order m-Boubaker polynomials. Int. J. of Heat Tech., 26: 49-53.
Hashemi, S.H., R. Rahgozar and A.A. Maghsoudi, 2007. Finite element and experimental serviceability analysis of HSC beams strengthened with FRP sheets. Am. J. Applied Sci., 4: 725-735. DOI: 10.3844/ajassp.2007.725.735

Khare, R.K., T. Kant and A.K. Garg, 2004. Free vibration of composite and sandwich laminates with a higher-order facet shell element. Comp. Struct., 65: 405-418. DOI: 10.1016/j.compstruct.2003.12.003

Kisa, M., J. Brandon and M. Topcu, 1998. Free vibration analysis of cracked beams by a combination of finite elements and component mode synthesis methods. Comp. Struct., 67: 215-223. DOI: 10.1016/S0045-7949(98)00056-X

Labiadh, H. and K. Boubaker, 2007. A Sturm-Liouville shaped characteristic differential equation as a guide to establish a quasi-polynomial expression to the Boubaker polynomials. Diff. Eq. Cont. Proc., 2: 117-133.

Mardani, E., 2008. Analysis of a beam made of physical nonlinear material on nonlinear elastic foundation under a moving concentrated load. Am. J. Eng. Applied Sci., 1: 324-328. DOI: 10.3844/ajeassp.2008.324.328

Meirovitch, L., 1986. Elements of Vibration Analysis. 2nd Edn., McGraw-Hill, New York, ISBN: 0071002715, pp: 560.

Osman, F. and H. Hora, 2004. Suppression of instabilities and stochastic pulsation at laser-plasma interaction by beam smoothing. Am. J. Applied Sci., 1: 76-83. DOI: 10.3844/ajeassp.2004.76.83

Osman, F., P. Evans and H. Hora, 2004. An investigation of the nature properties of plasma. Am. J. Applied Sci., 2: 168-175. DOI: 10.3844/ajeassp.2004.168.175

Osman, F., P. Evans, P. Toups, H. Hora and S. Glovacz et al., 2005. Laser interaction and related plasma phenomena. Am. J. Applied Sci., 2: 403-409. DOI: 10.3844/ajeassp.2005.403.409

Shu, C., 2000. Differential Quadrature and its Application in Engineering. 1st Edn., Springer, New York, ISBN: 1852332093, pp: 340.

Slama, S., M. Bouhafs, K.B. and A. Ben Mahmoud, 2008. Boubaker polynomials solution to heat equation for monitoring a3 point evolution during resistance spot welding. Int. J. Heat Technol., 26: 141-146.

Slama, S., J. Bessrour, M. Bouhafs, K.B. Ben, 2009. Numerical distribution of temperature as a guide to investigation of melting point maximal front spatial evolution during resistance spot welding using boubaker polynomials. Heat Transf. Part A.: Appli., 55: 401-408. DOI: 10.1080/10407780902720783

Sollier, A., L. Berthe and R. Fabbro, 2001. Numerical modeling of the transmission of breakdown plasma generated in water during laser shock processing. Eur. Physical J. Applied Phys., 16: 131-139. DOI: 10.1051/epjap:2001202

Hossein, S.A., A.E. Tabatabaei, T. Zhao, O.B. Awojoyogbe and F.O. Moses, 2009. Cut-off cooling velocity profiling inside a keyhole model using the Boubaker polynomials expansion scheme. Int. J. Heat Mass Transfer., 45: 1247-1251. DOI: 10.1007/s00231-009-0493-x

Vo, T.P., J. Lee and K. Lee, 2010. On triply coupled vibrations of axially loaded thin-walled composite beams. Comp. Struct., 88: 144-153. DOI: 10.1016/j.compstruc.2009.08.015

Qindeel, R., N. Bidin and Y.M. Daud, 2007. IR laser plasma interaction with glass. Am. J. Applied Sci., 4: 1009-1015. DOI: 10.3844/ajeassp.2007.1009.1015