Consistent approach to study gluon quasi-particles

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We discuss a novel approach to estimate the partition function in effective model frameworks when the effective potentials have multiple extrema, so that ascertaining a mean field becomes difficult. Using this approach we present a consistent model to study the thermodynamic properties of gluon quasi-particles as a function of temperature, both in the color confined and the color deconfined phases.

Keywords: Deconfinement, Polyakov loop, Center symmetry

I. INTRODUCTION

In the strong coupling regime the thermal physics of strongly interacting matter is best described by quantum chromodynamics (QCD) on space-time lattices [11]. Deconfinement of quarks and gluons and the chiral symmetry restoration at crossover temperatures $T_c \sim 150$ MeV is now well documented [12–14]. The deconfinement transition in a pure glue system is however found to be of first-order at temperatures $T_d \sim 270$ MeV [15–19]. The thermal average of the Polyakov loop in the fundamental representation $\hat{L}_F$ gives the static quark free energy, and is considered as the order parameter [16, 17]. Polyakov loop in the deconfined phase also breaks spontaneously the symmetry of the gluon action under $Z(3)$ twists on the gluon fields at the temporal boundary. Similarly, the Polyakov loop in the adjoint representation is related to the free energy of a static adjoint color source $\hat{L}_A$ [20–23]. But it is always invariant under the $Z(3)$ twists of the gluon fields at the physical boundary.

Significant efforts have been put in for building effective models for a spontaneous $Z(3)$ symmetry breaking, with $\hat{L}_F$ as order parameter, using Landau type of polynomial potentials [24–29]. Further models have been built [30–37] by introducing effective $\hat{L}_F$ fields in lieu of background temporal gluon fields in the chiral models like Nambu-Jona-Lasinio (NJL) model [38–43] or Chiral Sigma models [44–48]. These Polyakov loop enhanced chiral models give simple but insightful description of thermodynamics of strong interactions [49–82]. In these models the gluon pressure is obtained from the polynomial thermodynamic potential in $\text{Tr} \hat{L}_F$. But a more natural alternative seems to be in terms of gluon quasi-particles [83–87], including modifications due to the background $L_A$ [88–91]. Here $L_A$ is expected to induce statistical confinement of gluons in a similar way as $\hat{L}_F$ does for quarks in the Polyakov enhanced chiral models. But unfortunately the modified statistics result in a negative gluonic pressure below $T_d$. Various authors have argued for additional terms to preserve overall positivity. But the quasi-particle pressure itself still remains negative. This lacunae may have slowed down further progress in this direction.

Here we argue that the issue lies with the method of obtaining the statistics. Usually the saddle point approximation is employed to obtain the mean value of the Polyakov loop, which is then put back to obtain the thermodynamic potential. We propose a new prescription for obtaining the thermal averages and thermodynamic observables that can solve the issue and reliably predict various observables both below and above $T_d$.

II. FORMALISM

A. Standard approach

The Polyakov loop in the effective models is written in terms of the background temporal gluon field $A_0$, in the color diagonal form as,

$$\hat{L}_F \sim \exp[i(A_0^3 T_3 + A_8^8 T_8)/T].$$

(1)

Here $T_3$ and $T_8$ are the diagonal generators of SU(3). Accordingly, in terms of the class parameters $\theta_1$ and $\theta_2$ we have,

$$\hat{L}_F = \text{diag}(e^{i\theta_1}, e^{i\theta_2}, e^{-i(\theta_1 + \theta_2)}).$$

(2)
The normalized character and its complex conjugate are then given as,
\[ \Phi = \frac{1}{3} \text{Tr} \hat{L}_P; \quad \bar{\Phi} = \frac{1}{3} \text{Tr} \hat{L}_P. \] \tag{3}

In general for SU\((N_c)\) the group invariant Haar measure \(d\mu\) may be expressed in terms of the eigenvalues as,
\[ \int d\mu = \frac{1}{N_c!} \left( \prod_{i=1}^{N_c} \int_0^{2\pi} \frac{d\theta_i}{2\pi} \right) \delta \left( \sum_i \theta_i \right) \prod_{i<j} |e^{i\theta_i} - e^{i\theta_j}|^2 = 1. \] \tag{4}

For SU\((3)\) the corresponding Haar measure is given by,
\[ \frac{1}{3!} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{d\theta_1 d\theta_2 d\theta_3}{2\pi^3} \text{Det}_{VdM}[\theta_1, \theta_2] = 1, \] \tag{5}

where the Vandermonde determinant \(\text{Det}_{VdM}\) is given as \(\text{Det}_{VdM} = 64 \sin^2 \left( \frac{\theta_1 - \theta_2}{2} \right) \sin^2 \left( \frac{2\theta_1 + \theta_2}{2} \right) \sin^2 \left( \frac{\theta_1 + 2\theta_2}{2} \right) = 27 \left[ 1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2 \right]. \) \tag{6}

Correspondingly, in both polynomial and quasi-particle potentials, a Vandermonde term can be included. Thus the \(n^{th}\) order polynomial potential becomes,
\[ \Omega'_{\text{poly}} = \left[ \Omega_{\text{poly}} \left( \alpha_1 = \cdots = \alpha_r \right), \Phi, \bar{\Phi} \right] + \kappa \text{Det}_{VdM} T^4, \] \tag{7}

where \(\alpha_i, \kappa\) are model parameters. The quasi-particle potential becomes,
\[ \Omega'_{\text{qp}} = \Omega_{\text{qp}} + \kappa \text{Det}_{VdM} T^4, \] \tag{8}

where,
\[ \Omega_{\text{qp}} = 2T \int \frac{d^3 p}{(2\pi)^3} \ln \det \left( 1 - \hat{L}_A e^{-\frac{i\bar{\Phi}}{T}} \right) = 2T \int \frac{d^3 p}{(2\pi)^3} \ln \left( 1 + \sum_{n=1}^{8} a_n e^{-\frac{n\bar{\Phi}}{T}} \right). \] \tag{9}

The coefficients \(a_n\) for \(n = 1 \cdots 8\), are,
\[ a_8 = 1; \quad a_1 = a_7 = 1 - 9\Phi \bar{\Phi} \]
\[ a_2 = a_6 = 1 - 27\Phi \bar{\Phi} + 27(\Phi^3 + \bar{\Phi}^3) \]
\[ a_3 = a_5 = 2[-1 + 9\Phi \bar{\Phi} - 27(\Phi^3 + \bar{\Phi}^3) + 81(\Phi \bar{\Phi})^2], \] \tag{10}

The adjoint Polyakov loop is given as,
\[ \hat{L}_A = \text{diag} \left( 1, 1, e^{i(\theta_1 - \theta_2)}, e^{-i(\theta_1 - \theta_2)}, e^{i(2\theta_1 + \theta_2)}, e^{-i(2\theta_1 + \theta_2)}, e^{i(\theta_1 + 2\theta_2)}, e^{-i(\theta_1 + 2\theta_2)} \right), \] \tag{11}

with the corresponding normalized character
\[ \Phi_A = \frac{1}{N_c^2 - 1} \text{Tr} \hat{L}_A = \frac{1}{8} \left( 9\Phi \bar{\Phi} - 1 \right). \] \tag{12}

Given \(\Omega\) from either Eq. \(7\) or Eq. \(8\) one can solve for
\[ \frac{\partial \Omega}{\partial \Phi} = 0; \quad \frac{\partial \Omega}{\partial \bar{\Phi}} = 0, \] \tag{13}

obtaining the saddle point estimate for the mean fields \(\Phi_m\) and \(\bar{\Phi}_m\), and the mean thermodynamic potential \(\Omega = \Omega(\Phi_m, \bar{\Phi}_m)\).

In the quasi-particle picture this mean field approach gives satisfactory results for \(T > T_d\) \(\text{[88–91]}\). Below \(T_d\) \(\Phi_m = \bar{\Phi}_m = 0\), and the thermodynamic potential becomes,
\[ \Omega_{\text{qp}}(\Phi_m, \bar{\Phi}_m \to 0) = 2T \int \frac{d^3 p}{(2\pi)^3} \left[ \ln \left( 1 - e^{-\frac{3(1+i)\Phi}{T}} \right)^2 \right. \]
\[ + \left. \ln \left( 1 - e^{-\frac{3(1-i)\bar{\Phi}}{T}} \right) \right]. \tag{14}

The last two terms are positive, resulting in an overall temperature dependent negative pressure for \(T < T_d\).

In Ref. \(\text{[88]}\) the authors proposed a hybrid approach including glue-balls implemented as dilaton fields, resulting in an overall positive pressure below \(T_d\). In Ref. \(\text{[89]}\), an additional pure matrix interaction term was used along with \(\Omega_{\text{qp}}\) and a Weiss mean field analysis was done. In a related Weiss averaging procedure \(\text{[56]}\), a model parametrization was invoked similar to the polynomial potential. In Ref. \(\text{[91]}\), a Bag term was introduced along with \(\Omega_{\text{qp}}\) and a thermodynamic consistency analysis was done. Truly each of these additional terms may have significant physical inputs. But the negativity of gluon quasi-particle pressure below \(T_d\) remains unaddressed.

### B. Alternate approach

Here we propose to use a matrix model for the background Polyakov loop. We begin with the corresponding partition function given as,
\[ Z_{PL} = \int D\theta_1 D\theta_2 \exp \left[ -\frac{1}{T} \int d^3 x \Omega_{\text{qp}}[\theta_1(x), \theta_2(x)] \right] \]
\[ = \int \prod_{x} \frac{1}{24\pi^2} d\theta_1(x) d\theta_2(x) \text{Det}_{VdM} \]
\[ \times \exp \left[ -\frac{1}{T} \int d^3 x \Omega_{\text{qp}}[\theta_1(x), \theta_2(x)] \right]. \] \tag{15}

where \(\Omega_{\text{qp}}\) is given in Eq. \(9\). Additional terms as in Refs. \(\text{[88–91]}\) could be introduced but are not relevant for the physics discussed here. As we shall see that in our approach, even this simple \(Z_{PL}\) is sufficient to describe the pure gauge lattice field theory data quite satisfactorily.

The Polyakov loop is an oscillating function of \(\theta_1\) and \(\theta_2\), and so will be the thermodynamic potential. Hence the configurations away from the saddle point may have a significant measure. In fact below \(T_d\), where \(\langle \Phi \rangle = 0\), configurations with \(|\Phi| \sim 1/3\) is most preferred \(\text{[90]}\). Here instead, we compute the partition function \(Z_{PL}\), by numerically integrating over all the finite periodic interval...
of the \( \theta_1 \) and \( \theta_2 \) fields. The difficulty is with taking the \( V \to \infty \) limit, and this is why the saddle point analysis is the usual choice. However a simplification arises by noting that the effective action contains no derivatives of the \( \theta_1 \) and \( \theta_2 \) fields. Therefore the configuration space can be split up into \( N \to \infty \) independent and equivalent points, such that the partition function becomes,

\[
Z_{PL} = z_{PL}^N
\]

where,

\[
z_{PL} = \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \text{Det}_{V_{dM}} \exp \left[ -\frac{v}{\beta_1} O_{gpp}[\theta_1, \theta_2] \right]
\]

and \( v \) is a parameter with the dimension of volume. This is the only free parameter and may be suitably related with \( T_d \), the only physical scale in the finite temperature SU(3) pure gauge field theory. We assume \( v = (\beta_1/T_d)^3 \), where \( \beta_1 \) is some constant. We further scale out the momentum variable as \( |\vec{p}| = |\vec{p}|/T \), whereby the partition function may be expressed in terms of the scaled temperature \( T/T_d \) as,

\[
z = \int \frac{1}{24\pi^2} d\theta_1 d\theta_2 \text{Det}_{V_{dM}} e^{\exp \left[ -\frac{2}{\beta_1} \frac{T}{T_d} \frac{1}{(2\pi)^3} \ln \left( 1 + \sum_{n=1}^{N} a_n e^{-n|\vec{p}|} \right) \right]}
\]

The scaled pressure can be expressed in terms of the scaled temperature as,

\[
p/T^4 = \left( \frac{T}{V} \ln Z_{PL} / T^4 \right) = \frac{1}{N_v N \ln z_{PL}/T^3}
\]

\[
= \ln z_{PL}/(\beta_1 T/T_d)^3 .
\]

III. RESULTS

A. Parameter Fitting

The variation of \( p/T^4 \) with \( T/T_d \) obtained from Eq. 19 is completely consistent throughout the range of temperatures (Fig. 1). We chose \( \beta_1 \sim 2 \). However for quantitative agreement with data from lattice SU(3) field theory, we consider a temperature dependent effective mass \( m_g(T) \) for the gluon quasi-particles. Effects of such constant mass was studied in [88], while a temperature dependent ansatz was used in [89], following the studies in [92]. We substitute \( |\vec{p}| = \sqrt{\vec{p}^2 + m_g(T)^2} \) in Eq. 18 where \( m_g(T) = m_g(T)/T \). The lattice data [93] for pressure was solved for the scaled masses from the pressure equation \( p/T^4|_{\text{model}} = p/T^4|_{\text{lattice}} \) to an accuracy of \( 10^{-6} \) or better. The momentum rescaling is undefined for \( T \to 0 \). Also the numerical uncertainties were insignificant only above \( T/T_d \sim 0.45 \). Therefore the partition function at a given \( T/T_d \) was normalized with the one at \( T/T_d = 0.45 \).

![FIG. 2: Scaled mass of gluon quasi-particles as function of scaled temperature.](image)

The extracted scaled masses \( \tilde{m}_g(T) = m_g(T)/T \) are shown in Fig. 2 as data points. The functional dependence of \( \tilde{m}_g(T) \) has an abrupt change close to \( T/T_d = 1 \), and may have a functional form,

\[
m_g(T)/T = \alpha + \beta/\ln(\gamma/T_d), \quad \text{for } T/T_d > 1 \quad (20)
\]

\[
= \zeta (T_d/T)^2, \quad \text{for } T/T_d < 1. \quad (21)
\]

With \( T_d \) arbitrary and assuming \( \beta = (2/T_d)^3 \), the functional fit is shown in Fig. 2. The parameters \( \alpha, \beta \) and \( \gamma \) are obtained by least square fit using the "gnuplot" software, and summarized in Table I.

| \( \alpha \) | \( \beta \) | \( \gamma \) | \( \zeta \) | \( \tilde{m}_g(\text{GeV}^{-3}) \) |
|---|---|---|---|---|
| 0.548 | 0.174 | 1.083 | 2.70 | (0.5T_d)^{-3} |
| \( \pm 0.006 \) | \( \pm 0.005 \) | \( \pm 0.004 \) | \( \pm 0.07 \) |

![TABLE I: Model parameters](image)

Given the mass parametrization a comparative plot of the scaled pressure obtained in our model vis-a-vis the lattice QCD data is shown in Fig. 3.
B. Thermodynamic Observables

Given $p/T^4$ as a function of $T/T_d$, other thermodynamic observables like entropy density ($s$), energy density ($\epsilon$), specific heat ($c_V$) and speed of sound ($v_s$) may be obtained as,

$$s/T^3 = \frac{1}{T^3} \frac{\partial p}{\partial T} = \frac{1}{(T/T_d)^3} \frac{\partial [(p/T^4)(T/T_d)^4]}{\partial (T/T_d)} ,$$

(22)

$$\epsilon/T^4 = p/T^4 - s/T^3 ,$$

(23)

$$c_V/T^3 = \frac{1}{T^3} \frac{\partial \epsilon}{\partial T} = \frac{1}{(T/T_d)^3} \frac{\partial [(\epsilon/T^4)(T/T_d)^4]}{\partial (T/T_d)} ,$$

(24)

$$v_s^2 = \frac{\partial p}{\partial \epsilon} = \frac{\partial p}{\partial T}/\frac{\partial \epsilon}{\partial T} = \frac{s/T^3}{c_V/T^3} .$$

(25)

The scaled interaction measure (Fig. 4a) is expected to capture the deviation of thermal system from that of a relativistic non-interacting gas of gluons. However for $T << T_d$ due to the heavy effective mass of the gluon quasi-particles as well as the confinement-like interactions both $p/T^4$ and $\epsilon/T^4$ are insignificant, and so is the interaction measure. With increasing $T/T_d$, both $m_g/T$ and the confinement effect decrease, thereby increasing the interaction measure. A turnover occurs for $T/T_d > 1$ inside the gluonic phase where the measure gradually decreases towards relativistic ideal gas limit.

A more direct observable for transition from the non-relativistic confined phase to the relativistic gluonic phase is the conformal measure ($\epsilon - 3p)/\epsilon$, which varies from 1 to 0 between the two phases respectively (Fig. 4b). This behavior follows the general trend of $m_g/T$. At $T/T_d = 1$ there is a sudden gap arising out of the sudden changes in $m_g/T$ and the deconfining effects.

For a conformal theory in $d$ dimensions, $\epsilon = d.p$ and $c_V/T^3 = (1 + d)\epsilon/T^4$. In Fig. 5a we show a direct comparison of scaled specific heat with $1/T^2$. The gap in the $\epsilon/T^4$ gives an estimate of the latent heat of transition. The scaled specific heat is both discontinuous and divergent right at $T/T_d = 1$. The agreement with lattice data could be ascertained only for $T/T_d > 1$ from the measurements reported in Ref. [1].

Finally we present the behavior of the squared speed of sound ($v_s^2$) which is supposed to be an important transport coefficient determining the hydrodynamic evolution in the heavy-ion collisions [94]. In the conformal limit $v_s^2 = p/\epsilon = 1/3$. A comparison of our estimation for the speed of sound along with the ratio $p/\epsilon$ and measurements on the lattice [1] is shown in Fig 5b. Note that $p/\epsilon$ is three times the additive inverse of the conformal measure. Reflection of such a variation is seen in the $v_s^2 - T$ curve. The softest equation of state is supposed to be at $T/T_d = 1$, where $v_s^2$ drops towards zero.

Thus our model results for various sensitive thermodynamic observables are in excellent numerical agreement with the lattice data. In fact the parametrizations obtained by fitting data from Ref. [93] (Fig. 4), makes excellent predictions for the data from Ref. [1] (Fig. 5).

C. Order Parameter

As discussed earlier, $\langle \Phi \rangle$ is expected to vanish in the $Z(3)$ symmetric confined phase. In the deconfined phase the system may be in any one of the spontaneously chosen ground states. For numerical implementation, choosing the ground state requires some biasing. For example with a source term for $\Phi$ towards one of the ground states, one can consider the sequential limits $V \to \infty$ and source
characteristics of a first order phase transition.

![FIG. 7: Thermal evolution of \(\langle \Phi_A \rangle\) and \(d\langle \Phi_A \rangle/dT\) (inset).](image)

Unlike \(\Phi\), \(\Phi_A\) is invariant under the Z(3) transformation. Therefore no complications arise in its evaluation. The temperature dependence of \(\langle \Phi_A \rangle\) is shown in Fig. 7. Though a discontinuity corresponding to the one present in \(m_g/T\) appears at \(T/T_d = 1\), the variation indicates a crossover rather than a phase transition. This is further confirmed from the temperature variation of the thermal derivative of \(\langle \Phi_A \rangle\) (inset of Fig. 7), having a gap at \(T/T_d = 1\) and an inflection point at some \(T/T_d > 1\). This can be attributed to the fact that within this current model framework, quasi-gluons have a finite mass at temperatures below \(T_d\). However a clear understanding of the relation between the fundamental and adjoint representations of the Polyakov loop is still to be investigated.

![FIG. 8: Casimir scaling as a function of \(T/T_d\) (see [23] for notations).](image)

Given that \(\Phi\) is renormalization dependent on the lattice, it may not agree with model results (as seen in Fig. 8). On the lattice the Polyakov loop in various representations have been found to follow Casimir scaling above \(T/T_d = 1\) [23]. We find \(\langle \Phi \rangle\) and \(\langle \Phi_A \rangle\) do follow the scaling reasonably well for \(T/T_d > 1\) (Fig. 8). Below \(T/T_d = 1\), the scaling breaks because \(\langle \Phi_A \rangle\) is non-zero. This is natural as the quasi-gluons have finite mass, and the situation resembles the crossover observed in \(\langle \Phi \rangle\) in the presence of low mass quarks in models as well as on...
where, \( \tilde{q} \) for simplicity we shall use \( \Omega \) function of temperature for various quark masses. Again \( \tilde{q} \) be 2. The full potential will be the sum of \( \Omega \) is number of quark flavors, which we shall consider to proceed sections, except that we now specify \( T \) but an artefact of the discontinuity of the variation of \( m_f \) with \( T \). This can be taken care of in a detailed analysis with dynamical quarks that will be explored elsewhere.

IV. PRELIMINARY EXPLORATION INCLUDING QUARKS

The results discussed in the previous sections for the pure glue model is expected to hold true in the presence of infinitely heavy quarks. For practical purposes it should hold true even for a system of quarks whose masses are much higher than the temperature scales. Here we make a preliminary discussion on the presence of heavy quarks in the present model. Neglecting effects of chiral physics altogether and incorporating the Polyakov loop modified quark quasiparticle contribution we have the additional potential,

\[
\Omega_{gqp} = 2N_fT \int \frac{d^3p}{(2\pi)^3} \left\{ \ln \left[ 3(\Phi + \Phi e^{-E_\pi})e^{-E_\pi} + 1 + e^{-3E_\pi} \right] + \ln \left[ 3(\bar{\Phi} + \Phi e^{-E_\pi})e^{-E_\pi} + 1 + e^{-3E_\pi} \right] \right\},
\]

where, \( E_\pi = \sqrt{\vec{p}^2 + m_q^2} \), \( m_q \) being the quark mass. \( N_f \) is number of quark flavors, which we shall consider to be 2. The full potential will be the sum of \( \Omega_{gqp} \) and \( \Omega_{gqp} \) (given in Eq. 4) and to be used in Eq. 15. In this preliminary study with quarks we assume all parameters of \( \Omega_{gqp} \) to remain unchanged from those obtained in the preceding sections, except that we now specify \( T_d = 270 \) MeV. We shall discuss the behavior of the \( \Phi_{m_f} \) as a function of temperature for various quark masses. Again for simplicity we shall use \( \Omega_{gqp} \) for various \( m_q \), some of which are smaller than \( T \).

FIG. 9: Comparison of \( \Phi_{m_f} \) with varying quark masses.

The effect of introducing the quarks can be seen from Fig. 9. For \( m_q = 3 \) GeV the results are identical with the pure gauge results. With reducing masses we find the corresponding deconfinement temperature, as well as the gap of \( \Phi_{m_f} \) to decrease. Subsequently between 1.65 Gev < \( m_q < 1.8 \) GeV the transition goes over to a crossover. These results are commensurate with the findings in the literature [99]. The variation of \( \Phi_{m_f} \) with \( T \) shows a dip at the \( T = T_d \) of the pure glue model, which is nothing but an artefact of the discontinuity of the variation of \( m_g \) with \( T \). This can be taken care of in a detailed analysis with dynamical quarks that will be explored elsewhere.

V. DISCUSSION

The gluon quasi-particle models are usually found to become inconsistent in the confined phase. We identified the problem to lie with the saddle point method. To overcome the problem we discussed a novel prescription of obtaining the thermodynamic observables by a pseudo path integral formalism. Essentially instead of considering only the saddle point solution for the field variable, all possible field variables are considered with their appropriate thermal weight functions. By implementing this approach we predicted a variety of sensitive thermodynamic quantities to a high level of accuracy. In addition, we observed that while the temperature variation of \( \Phi \) indicates a first order phase transition, that of \( \Phi_q \) is almost like a crossover. The latter seems natural as it is similar to thermal variation of \( \Phi \) when the quark masses are finite in chiral models. However the deeper connection between the different representations of the Polyakov loop in our model would need further investigation. A preliminary study including quarks with heavy masses give consistent results with existing literature.

We conclude that the quasi-particle model presented here is the most consistent one for studies of color deconfinement and gluon thermodynamics from cosmology to heavy-ion collision experiments.

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