Simplified Matrix Focusing Imaging Algorithm for Ultrasonic Nondestructive Testing

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Simplified Matrix Focusing Imaging Algorithm for Ultrasonic Nondestructive Testing

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Abstract: Full matrix focusing method has been proved with the advantages of good signal-to-noise ratio and high-resolution image. However, it is still suffering from the problems of the time-consuming data acquisition and processing. In order to solve the problem, two kind simplified matrix focusing methods are provided in this paper. One is a triangular matrix focusing algorithm based on the principle of reciprocity for the multi-channel ultrasonic system. The other is a trapezoidal matrix focusing algorithm based on the energy weight of the different channel to the imaging area. In order to prove the validity of the provided algorithms, both side-drilled hole and oblique crack defects are used for the imaging comparation. The results show that the imaging quality of the triangular matrix focusing algorithm is basically consistent to that of the full matrix focusing method, and the imaging quality of the trapezoidal matrix focusing algorithm can be slightly reduced as the amount of multi-channel data decreases. In addition, both data acquisition and computational efficiency using the provided algorithms have been improved significantly.

Keywords: Ultrasonic phased array • Full matrix focusing • Triangular matrix focusing • Trapezoidal matrix focusing

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1 Introduction

In recent years, ultrasonic phased array technology has been gradually applied in the field of industrial nondestructive testing [1-4]. Phased array imaging algorithms have been widely studied by domestic and foreign scholar [5-8]. Holmes and Drinkwater proposed a full-matrix focus imaging algorithm [5], it can more clearly characterize the geometric features of internal defects with the help of the multi-channel technology. And the image quality is significantly better than the traditional phased array imaging algorithm with steering and focusing time delays. Wu Bin, et al. extend the total focusing imaging method to the multi-mode case with both direct and reflection waves [9]. P Zheng used a total focusing imaging method to quantitatively simulate the cracks [10]. However, most commercial phased array instruments currently do not support the full focus imaging method. The main reason is that the large amount of data lead to an inefficient transmission and calculation. In order to improve the efficiency of full-focus imaging, H R Jin introduced a wavenumber algorithm for the multi-layered medium imaging [11]. S Bannouf proposed an algorithm based on point spread function to enhance the efficiency [12]. H w Hu used sparse matrix for two-layer medium to simplify the total focusing imaging algorithm [13].

Two kind simplified matrix focusing methods are provided in this paper. One is a triangular matrix focusing algorithm based on the reciprocity principle of the transmitter and receiver channel. The other is a trapezoidal matrix focusing algorithm based on the energy weight of the different channel to the imaging area. These image algorithms corresponding to different data regions, thus we named as full, triangular and trapezoidal matrix focusing method, respectively. In the second part of this paper, the data acquisition and imaging algorithm of full matrix focusing method is introduced, then the simplified imaging methods of the triangular and trapezoidal matrix are provided. In the third part, side-drilled holes and oblique crack defects are used to compare the imaging quality and computational efficiency according to three methods.

2 Focus Imaging Algorithm

2.1 Full Matrix Focused Data Acquisition

Before focus imaging, these A-scan data must be acquired and arranged in a matrix form. Assuming the linear array has \( n \) elements, the first element is excited and all
elements are received at the same time, named as \( A_{11}, A_{12}, \ldots, A_{nn} \) and placed in the first row, then next element is excited in turn until all the \( n \times n \) A-scan data are acquired as shown in (1).

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & \cdots & A_{1n} \\
A_{21} & A_{22} & A_{23} & \cdots & A_{2n} \\
A_{31} & A_{32} & A_{33} & \cdots & A_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_{n1} & A_{n2} & A_{n3} & \cdots & A_{nn}
\end{bmatrix}
\]  

(1)

2.2 Full Matrix Focusing Imaging Algorithm

Imaging algorithm according to the acquired full matrix data as shown in Fig.1. Firstly, the imaging region below the line array is discretized into a many focus points in alignment. Taking point \( P \) as an example, the distances between the point \( P \) and the center of two array elements \( i, j \) are calculated, and the corresponding time delay \( t_0 \) can be determined, the amplitude \( A_{ij}(t_0) \) in the the signal \( A_{ij} \) can be acquired. Similarly, these corresponding amplitudes of all these A-scan signals can be determined and summated, finally the digital amplitude of the focus point \( P \) \( I_p(x, z) \) is obtained according to (2)

\[
I_p(x, z) = \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij}(t_0)
\]

(2)

\[
t_0 = \left( \sqrt{(x - x_i)^2 + z^2} + \sqrt{(x - x_j)^2 + z^2} \right) / c
\]

(3)

Where \( c \) is the sound velocity in the material.

![Schematic diagram of the full matrix focus imaging algorithm](image)

Figure 1  Schematic diagram of the full matrix focus imaging algorithm

2.3 Triangular Matrix Focusing Imaging Algorithm

The full-matrix focus imaging algorithm needs all A-scan data acquisition and superposition, so it is a time-consuming method. Considering the reciprocity principle of multi-channel linear time invariant system, that means, a good consistency can be maintained before and after the transmit and receive channels are interchanged. Thus, \( A_{ij} \) (the \( i \) transmission channel and the \( j \) reception channel) and \( A_{ji} \) (the \( j \) transmission channel and the \( i \) reception channel) should have a good consistency. Therefore, the full matrix data shown in (1) should be a symmetric matrix. If the symmetric matrix is right, that will be enough for only the upper triangular matrix signal used for imaging calculation, as shown in (4), here we call it as the triangular matrix focusing imaging algorithm, as shown in (5).

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & \cdots & A_{1n} \\
A_{21} & A_{22} & A_{23} & \cdots & A_{2n} \\
A_{31} & A_{32} & A_{33} & \cdots & A_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_{n1} & A_{n2} & A_{n3} & \cdots & A_{nn}
\end{bmatrix}
\]

(4)

\[
I_p(x, z) = \sum_{i=1}^{N} \sum_{j=i}^{N} A_{ij}(t_0)
\]

(5)

According to the symmetry of the full matrix data, the data acquisition and imaging are reduced from the amount of \( n \times n \) to \( n \times (n+1)/2 \). Then both the amount of data transfer, storage and calculation will be reduced nearly one half.

2.4 Trapezoidal Matrix Focusing Imaging Algorithm

Furthermore, in order to improve the computational efficiency, the energy weight of the different channel to the imaging area can also be taken into account. Here we view the problem from the transducer aspect. As the distance of the transmitter to the receiver element increases, that means the transmitter is far away from the receiver, the A-scan signal received will weaken gradually. Therefore, we can reserve those data near the diagonal of the triangular matrix, ignore those data far from the diagonal. Thus, the trapezoidal matrix focusing imaging algorithm can be determined as shown in (6) and (7)

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & \cdots & A_{1n} \\
A_{21} & A_{22} & A_{23} & \cdots & A_{2n} \\
A_{31} & A_{32} & A_{33} & \cdots & A_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_{n1} & A_{n2} & A_{n3} & \cdots & A_{nn}
\end{bmatrix}
\]

(6)

\[
I_p(x, z) = \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij}(t_0)
\]

(7)
The RF A-scan signals are used in our image algorithm so that the waveform average denoising can work well. After obtaining the superposition amplitude of each focus point, the image data needs to be normalized for the image display. Due to the image color bar with 256 colors is often used in the ultrasonic NDT field, the superposition amplitude data is normalized from -127 to 128 using (8) and (9).

If $I_p(x, z) > 0$

$$I_p(x, z) = \frac{I_p(x, z)}{I(x, z)_{\text{max}}} \times 128 \quad (8)$$

If $I_p(x, z) \leq 0$

$$I_p(x, z) = -\frac{I_p(x, z)}{I(x, z)_{\text{min}}} \times 127 \quad (9)$$

3 Experimental verification

A series of experimental data corresponding to symmetric side-drilled holes and asymmetric oblique cracks are used to compare the difference in imaging quality and calculation efficiency. The data acquisition and imaging experiments is based on an AOS 64 × 64 ultrasonic phased array system and the self-developed imaging software. The sampling frequency of each channel is 100MHz. A linear array probe with 5 MHz center frequency and 0.6 mm element center distance is used in our testing. Due to the large amount of data collected by the full matrix focus method, we used only the 32 channels and elements in the data acquisition.

Symmetric side-driller holes with 1.5mm diameter are fabricated in an aluminum alloy specimen, as shown in Fig. 2(a). Three asymmetric cracks with oblique angle of 30°, 45° and 60° are fabricated at the bottom of the low carbon steel, as shown in Fig. 2(b).

3.1 Comparison of Signal Consistency

First, considering the reciprocity principle of the acoustic signal after the transmitter and receiver channels are interchanged. The full matrix acquisition data contain all kinds of signals. Some A-scan signals are selected from twin random transmitter and receiver channels, here Fig.3(a) shows the signal of the symmetric side-drilled holes and Fig.3(b) (c) (d) show the signal of asymmetric oblique cracks. It can be seen from these plots that the $A_{521}$ signal keep good agreement with $A_{215}$ signal, almost the same one in both signal phase and amplitude for the flaw reflection wave part, and slightly different amplitude for the near field cross talk part. This kind regular rule can also be found in any pair of $A_{ij}$ and $A_{ji}$ channels. Therefore, the full matrix data shown in (1) meet the condition of symmetric matrix.

Figure 2 The Experiment Specimens
3.2 Comparison of Imaging Quality
Comparison of different image methods, named full-matrix, triangular and trapezoidal matrix focus imaging algorithm, are shown in Fig.4-7. Fig.4 shows the symmetric defects and Fig.5-7 show the asymmetric cracks with oblique angle of 30°, 45° and 60°, respectively. It is difficult to identify the difference of the four algorithms in those imaging results with naked eyes. The holes and crack tip signals with different oblique angles can be found clear using these methods. And as the crack oblique angle increases, the absent bottom reflection signals become larger from Fig.5 to Fig.7. These patterns can be found in a good agreement for all of three methods.

![Figure 4](image1)  
(a) Full matrix  
(b) Triangular matrix  
(c) Trapezoidal matrix (k=16)  
(d) Trapezoidal matrix (k=12)

**Figure 4** Comparison of different image methods for side-drilled holes

![Figure 5](image2)  
(a) Full matrix  
(b) Triangular matrix  
(c) Trapezoidal matrix (k=16)  
(d) Trapezoidal matrix (k=12)

**Figure 5** Comparison of different image methods for 30° crack
In order to analyze detailed behaviors of three methods, four typical A-scan signals of side-drilled holes and oblique cracks are shown in Fig.8. As can be seen in these figures, the synthesized A-scan signals obtained by the three methods also have good consistency in both amplitude and phase. Poor signal-to-noise ratio (SNR) can be found in those near surface part, also the effectiveness of this simplification can be proved by a good agreement among three methods.

In order to further quantify the difference of imaging results, the signal-to-noise ratio of each defect echo signal are calculated separately. The peak value of the signal is regarded as $V_s$, and these data outside the main signal area are regarded as noise signal $V_n$. Then the signal-to-noise ratio can be calculated according to the signal peak and the average amplitude of noise as shown in (10).

$$SNR = 20 \times \log_{10} \left( \frac{V_s}{V_n} \right)$$

These SNR of three crack-like defects and side-driller holes defects are shown in Fig.9. Obviously, the SNR of...
the triangular matrix focusing algorithm is almost the same as that of the full matrix focusing algorithm. And slightly different can be found in that of the trapezoidal matrix focusing algorithm. As the decreasing of the number of A-scan data (from k=16 to 12), the SNR of the trapezoidal matrix becomes worse.

FIGURE 9 Comparison of signal-to-noise ratios of four imaging methods for crack and holes

By contrast, the computational efficiencies of three methods are figured out as shown in Fig.10. Here a B-scan image with $115 \times 150$ pixels is used to compare the imaging time for three methods. Obviously, the full matrix focusing is a kind of time-consuming method. The calculation time of the triangular matrix focusing can be cut in half to that of full matrix focusing. The trapezoidal matrix focusing spends shorter time as decreasing the number of A-scan data (from k=16 to 12). The computation efficiency can be improved greatly with the help of both triangular and trapezoidal matrix algorithms.

FIGURE 10 Time comparison of four imaging methods

4 Conclusion
(1) Based on the reciprocity principle of the transmitter and receiver channel, the triangular matrix focusing algorithm is provided to simplify the full matrix
focusing method. Furthermore, the trapezoidal matrix focusing algorithm is proposed based on the energy weight of the different channel to the imaging area.

(2) The experimental results of data acquisition and imaging for oblique cracks and side-drilled holes show that three imaging methods keep great agreement in image and synthesized A-scan signal. The SNR of the triangular matrix does not change much compared with full matrix focusing algorithm. With the amount of A-scan data deceasing, the SNR of the trapezoidal matrix are slightly lower.

(3) In terms of computation efficiency, both triangular and trapezoidal matrix algorithms can ensure good imaging quality with better computation efficiency.

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Declarations

Availability of data and materials
All data generated or analyzed during this study are included in this published article.

Competing interests
The authors declare that they have no competing interests.

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Authors’ contributions
ZHAO Xin-Yu analyzed and interpreted the principles and rules of simple matrix focusing imaging, and was a major contributor in writing the manuscript. MA Ze-Min analyzed the data regarding the full matrix focusing imaging and the simple matrix focusing imaging, and calculated SNR of different imaging algorithms. ZHANG Jia-Ying disposed data and compared the simple matrix focusing imaging with full matrix focus imaging, and strengthened and optimized the manuscript.

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