A Configurable Protocol for Quantum Entanglement Distribution to End Nodes

Leonardo Bacciottini  
Dept. of Information Engineering  
Universities of Florence and Pisa  
Florence and Pisa, Italy  
leonardo.bacciottini@phd.unipi.it

Luciano Lenzini  
Dept. of Information Eng.  
University of Pisa  
Pisa, Italy  
lenzini44@gmail.com

Enzo Mingozzi  
Dept. of Information Eng.  
University of Pisa  
Pisa, Italy  
enzo.mingozzi@unipi.it

Giuseppe Anastasi  
Dept. of Information Eng.  
University of Pisa  
Pisa, Italy  
giuseppe.anastasi@unipi.it

Abstract—The primary task of a quantum repeater network is to deliver entanglement among end nodes. Most of existing entanglement distribution protocols do not consider purification, which is thus delegated to an upper layer. This is a major drawback since, once an end-to-end entangled connection (or a portion thereof) is established it cannot be purified if its fidelity (F) does not fall within an interval bounded by Fmin (greater than 0.5) and Fmax (less than 1). In this paper, we propose the Ranked Entanglement Distribution Protocol (REDiP), a connection-oriented protocol that overcomes the above drawback. This result was achieved by including in our protocol two mechanisms for carrying out jointly purification and entanglement swapping. We use simulations to investigate the impact of these mechanisms on the performance of a repeater network, in terms of throughput and fidelity. Moreover, we show how REDiP can easily be configured to implement custom entanglement swapping and purification strategies, including (but not restricted to) those adopted in two recent works.

Index Terms—Quantum Repeater Networks, Entanglement Distribution, Quantum Network Protocol, Entanglement Swapping, Entanglement Purification

I. INTRODUCTION

In the last few years there have been different proposals for a quantum Internet protocol stack [1], but a common trait of all near-term quantum network architectures is entanglement distribution through the Purify-and-Swap scheme [2], which glues many entanglement connections between adjacent quantum repeaters into an end-to-end entangled pair. This will probably change in the future when more advanced quantum hardware will enable support for quantum error correcting codes [3]. This work focuses on the former approach, as it can be achieved with near-term hardware.

To be completed, entanglement swapping also requires the transmission of two bits of classical information to let one of the recipients know in which Bell state the qubit pair has landed. This requirement introduces a new problem, that is determining the order in which this classical synchronization between repeaters takes place. Such decision is very important because it defines – and is defined by - the order of performing entanglement swapping among nodes, as shown in Fig. 1. We call this choice Entanglement Swapping Strategy (ESS). In this paper we focus on three different ESSs: (i) Consecutive ESS (Fig. 1a), where a node performs entanglement swapping only after receiving the classical message from the previous node on the chain, and then sends a message to the next node. (ii) Nested ESS (Fig. 1b), where the swapping order is recursively determined by extracting the nodes of the chain in odd positions - with positions starting from zero - until only the end nodes remain. (iii) Parallel ESS (Fig. 1c), where all intermediate nodes perform entanglement swapping concurrently and transmit the result directly to one of the end nodes. If we include entanglement purification in this discussion, we define the Purification and Entanglement Swapping Strategy (PESS) as the policy that determines both the order of entanglement swapping other than when and where (i.e., at what node) purification rounds are to be carried out.

To quote only two proposals on several, in 2020 Kozlowski, Dahlberg and Wehner [4] defined a connection-oriented protocol for entanglement distribution that exploits Parallel ESS to counter the effect of low memory coherence times. Moreover, in 2022 Li, Xue, Wei, and Yu [5] designed another connection-oriented protocol with a detailed description of the connection establishment phase, involving a resource allocation mechanism based on quantum memory slots partitioning. Their protocol uses Consecutive ESS to have a strict control over the timing of the actions performed by each node. The limit of these previous works is that these protocols are tightly bonded to a specific ESS and do not support entanglement purification as an integrated mechanism. This leads to a lack of flexibility that is proved in section III. Of course, it is always possible to have a recursive protocol stack where several connections and purification protocols are installed one on top of the other to implement an arbitrary PESS, as done in [6] and suggested in [4], but this inevitably introduces an overhead and complicates network configuration and resource allocation.

In this paper we present the Ranked Entanglement Distribution Protocol (REDiP). Our protocol overcomes the aforementioned limits by letting users configure each single connection to implement the ESS that better suits their needs. We point out that REDiP is not stuck to the above strategies only. It also allows the implementation of any user-defined ESS. Moreover, the protocol empowers ESSs by enabling them to cooperate with entanglement purification to adopt an arbitrary PESS. Our results show that this configurability is essential to satisfy user requirements in a quantum network subject to quantum errors and link conditions that may change over time and space.
Finally, section IV draws some final remarks and future work.

This paper is structured as follows: section II describes REDiP design, then section III shows the results of a simulation campaign that evaluates the performances of different PESSs implemented on REDiP under varying scenarios. Finally, section IV draws some final remarks and future work.

II. PROTOCOL DESIGN

REDiP is a connection-oriented protocol. Therefore, a connection on a path between the two end nodes must be created before distributing the entanglement. This connection conveys classical bits and is provided by a conventional infrastructure (for example, the classical Internet). To avoid ambiguities with entangled connections, we will refer to this preliminary connection as tunnel. Since the tunnel establishment is not the main focus of this paper, we refer to the work done in [4] and [5] as to how this phase can be carried out with a one-way or two-way handshake respectively. This approach is borrowed from classical Internet protocols like the Resource Reservation Protocol (RSVP).

On the other hand, taking as reference the quantum protocol stack introduced in [7], REDiP is placed at the Network layer. The underlying Link layer has the task of generating heralded entangled Bell pairs on physical links that are consumed by Network layer protocols to distribute end-to-end entanglement. We will simply refer to upper layer protocols as Users of REDiP. Before delving deeper into the analysis of the protocol, we define some reference variables that will be used across this section:

- $N$ is the number of nodes composing the path.
- $L$ is the vector of node identifiers, in the order they are traversed. $L_i$ indicates the $i$th node on the path, $i \in \{0, \ldots, N-1\}$.
- $R$ is the vector of node ranks, where each element $R_i$ is an integer indicating the rank of node $L_i$ (see section II-A).
- $P$ is the purification vector, where each element $P_r$ is an integer indicating how many purification rounds must be performed by nodes whose rank is $r$, $r \in \{0, \ldots, \max\{R_i \in R\}\}$ (see section II-A).
- $K$ is the number of end-to-end pairs that a given tunnel must deliver to the destination end nodes.

Finally, we assume that the quantum network architecture where REDiP is used has an addressing scheme where each entangled qubit pair has an identifier shared at least among the nodes holding the two ends of the pair. For the sake of clarity, we call an entangled qubit pair between adjacent nodes an entangled link or just a link. We also use the term entangled segment or just segment when the entangled pair resides on non-adjacent nodes. Finally, we call end-to-end entangled connection or just connection an entangled segment that resides on the end nodes. Clearly the two qubits of an entangled pair embody the endpoints of a link, segment, or connection.

A. Opening and Configuring the Tunnel

REDiP becomes active at an end node $A$ when a user, typically an upper layer protocol, submits a NEW_TUN message containing the four $(L, R, P, K)$ variables. As shown in Fig. 2, this triggers the RSVP-like tunnel establishment where resources are reserved and the tuple $(L, R, P, K)$ is shared among all nodes along the path. Furthermore, as in [4], we assume that the Link layer protocol continuously generates entangled links once the tunnel is open. For any given node, from now on we will refer to the directions towards the rightmost and leftmost nodes on the path $L$ as upstream and downstream respectively.

Entanglement swapping is carried out in consecutive steps. The rank of a node, specified inside the vector $R$, indicates the step at which that node must swap. By design, nodes whose rank is zero will swap entangled links as soon as they can, whereas nodes whose rank is $r > 0$ are allowed to swap an entangled segment only after a (classical) signal from an $(r-1)$-ranked node notifies that the segment is ready. A constraint for REDiP to work properly is that end nodes of the connection must share the same maximum rank $r_{max}$, so that they are notified only when the end-to-end connection is ready. For what concerns purification, the user can specify how many purification rounds should be performed by nodes belonging to each specific rank. For example, a vector $P = [2, 0, 1]$ means that 0-ranked nodes purify entangled links twice before swapping, 1-ranked nodes do not need purification, whereas 2-ranked nodes (end nodes in this example) purify again once before delivering the end-to-end connection to the user. REDiP allows the negotiation of any purification algorithm supported by the nodes’ hardware during the tunnel establishment phase as part of the resource allocation process. Once established, the tunnel remains open until $K$ end-to-end connections have been delivered and consumed by the user.
B. Generating Entangled Connections

REDiP uses the allocated tunnel to generate entangled connections. The control operations of this phase are carried out by three types of classical message called (i) SWAP_UPDATE, used to notify nodes about the result of an entanglement swapping, (ii) PURIF_SOLICIT and (iii) PURIF_RESPONSE used during the entanglement purification procedure. We will gradually go through their definition and usage as the description moves forward.

When the tunnel has been established, each intermediate node $L_i$ processes the parameters $(L, R, P, K)$ and computes the following variables:

- $L_i$ Swapping Destinations ($SD^i$), a two components vector where its components $SD^i_0$ and $SD^i_1$ are picked as the downstream and upstream nodes closest to $L_i$ whose rank is higher than $R_i$.
- $L_i$ Swapping Neighbors ($SN^i$), a two components vector where its components $SN^i_0$ and $SN^i_1$ are picked as the downstream and upstream nodes closest to $L_i$, whose rank is equal or higher than $R_i$.

These variables are used to determine whether an intermediate node $L_i$ should generate or not a SWAP_UPDATE message after the execution of the entanglement swapping. Taking the Nested ESS (Fig. 1b) as an example, all nodes must generate both an upstream and a downstream SWAP_UPDATE after they swap and send them to their swapping destinations. On the contrary, in the Parallel ESS (Fig. 1c) only the second and the penultimate nodes on the path generate an upstream and downstream SWAP_UPDATE respectively. All other nodes limit to update and forward incoming updates to their swapping neighbors. To implement this behavior, the node $L_i$ generates one or two SWAP_UPDATEs only if $SD^i$ and $SN^i$ partially or totally coincide. We provide a more specific description in section II-B1.

The generation of entangled connections begins when all nodes have computed their swapping neighbors and destinations. As shown in Fig. 3, during this phase, an endpoint held by an $r$-ranked node of a connection can be in one of the following possible logical states:

- WAIT is the initial state for endpoints signaled by the lower layer when an entangled qubit pair is generated.
- PURIF is entered if $r = 0$, otherwise it is entered when a SWAP_UPDATE message arrives from a node whose rank is $r - 1$. An endpoint stays in this state if the link, segment, or connection must be purified.

1) Swap Updates: SWAP_UPDATE messages share some similarities with the TRACK message from [4], in the sense that they serve the purpose of keeping track of what endpoints have been swapped and the final Bell state of the new segment. The difference lies in the fact that TRACK messages are always forwarded by all nodes on the path towards an end node, whereas SWAP_UPDATES are forwarded by swapping neighbors and delivered to a swapping destination which, as highlighted before, may or may not be an end node.

Fig. 3 shows the behavior of an $r$-ranked node $L_i$ when its swapping neighbors and destinations do not coincide. We can see that $L_i$ receives an upstream SWAP_UPDATE from $SN^i_0$ (time $T_0$), but it has not swapped its two local endpoints yet, so the message cannot be immediately forwarded. As soon as $L_i$ has an upstream endpoint in ELIGIBLE state, it is swapped with the endpoint specified by the received message (time $T_1$). The final Bell state measurement is collected and saved in a temporary record as it is done in [4], and the two swapped endpoints transition to the RELEASE state. At this point $L_i$ can add the stored measurement result to the upstream SWAP_UPDATE, which is forwarded to $SN^i_1$ (time $T_2$). When $L_i$ receives the SWAP_UPDATE from $SN^i_1$, it immediately updates and forwards downstream the message (time $T_3$). When both SWAP_UPDATES are delivered to the corresponding swapping destination, the two endpoints of the new segment between $SD^i_0$ and $SD^i_1$ transition to the state PURIF.
ancillas, depending on the purification algorithm. The policy used for this choice also depends on the chosen purification algorithm. For example, a recurrence purification pattern like Deutsch algorithm [8] requires that ancillas must have the same fidelity as the purified link, whereas an entanglement pumping pattern like the one proposed in [2] allows the exploitation of ancillas with a lower fidelity than the purified link. The Initiator node then executes the purification algorithm and sends a PURIF_SOLICIT message containing the identifier of the link to purify, ancillas and their purification measurement outcomes. The Solicited node receives the message and applies itself the purification algorithm. If the measurement outcome matches with the one in the PURIF_SOLICIT message, then the purification was successful. In any case, the Solicited node replies with a PURIF_RESPONSE message, containing the purified link identifier and ancillas (piggyback approach), and the purification outcome (OK or FAIL). If the purification was successful, the purification counter \( i \) is increased by one, otherwise both endpoints are released back to the Link layer. Ancillas are always released to the Link layer right after their measurement. We show an example in Fig. 5 where the purification requires only one ancilla (pair B). This is the case for popular purification patterns like Deutsch algorithm [8]. The procedure is repeated until the purification counter \( i \) reaches the value \( P_r \), where \( r \) is the rank of the Initiator node. To determine the actor roles there is a simple rule: the lowest ranked node is the Initiator. The tie-breaking rule is left up to REDiP implementation; one could simply set the downstream node to always be the Initiator when the ranks are equal.

C. Implementing Different Strategies

In this section we show how REDiP configurability allows to implement the Parallel ESS, Consecutive ESS, and Nested ESS. For each one of these strategies, we provide a way to compute the rank vector \( R \) given the number of nodes \( N \).
1) Parallel strategy: If we set the ranks as $R = [1, 0, 0, \ldots, 0, 0, 1]$, we reproduce the Parallel ESS used in [4]. However, this REDiP configuration performs slightly better because SWAP UPDATEs are generated by the nodes adjacent to the end node. TRACK messages from [4] are instead generated by the end nodes themselves. This saves one transmission hop for each SWAP UPDATE, which translates in less idle time spent by qubits inside of the end nodes quantum memories.

2) Consecutive strategy: The rank vector $R = [N - 2, 0, 1, 2, \ldots, N - 3, N - 2]$ reproduces the Consecutive ESS from [5], where entanglement swapping is sequentially carried out by intermediate nodes along the path in the upstream direction. According to this REDiP version, it is not up to the downstream end node to trigger entanglement swapping on the successive repeater, which is instead the approach used in [5]. This again saves one transmission hop for each SWAP UPDATE, leading to the same improvement seen for the Parallel ESS.

3) Nested strategy: The Nested ESS is implemented by a rank vector $R$ where each element $R_i$ is defined as follows:

$$R_i = \max_{r \in \{0, 1, \ldots, k\}} \{r | i \mod 2^r = 0\},$$

where $N = 2^k + 1$ for some $k > 0$. This last condition is an assumption for the applicability of this strategy.

III. SIMULATION ANALYSIS

We implemented a REDiP simulator to estimate the impact of several PESSs on the protocol performances in terms of throughput (pairs/s) and fidelity. The simulator was implemented as a Python package on top of Netsquid [9], a simulation engine for quantum networks. Below we define the assumptions of the simulation campaign and then we show some representative scenarios. From the simulation outcomes, it is clear that the assignments of REDiP ranks and purification rounds have a significant impact on the entanglement distribution performance.

A. Assumptions

In our simulation scenarios we assume all repeaters and links have the same hardware specifications. In particular, we modeled an implementation of the Midpoint Source (MS for short) Link layer protocol defined in [10], and we set its parameters according to the optimistic parameter set used in [11]. The rationale behind this choice is that [11] employs the most advanced emerging technology for multi-mode quantum memories.

To model quantum errors, we introduced three sources of noise: (i) An initial depolarization probability $p_0^d$ applied on both endpoints of a newly generated link, so that its initial fidelity is $F_0 < 1$. (ii) A quantum memory exponential dephasing rate $r_d$, which simulates the decoherence of a qubit state $\rho$ stored inside a quantum memory, so that after $\Delta t$ time the new state $\rho'$ of the qubit can be written as

$$\rho' = f(\rho, \Delta t) = e^{-r_d\Delta t}\rho + (1 - e^{-r_d\Delta t})Z\rho Z.$$  (2)
IV. CONCLUSION

In this paper we presented REDiP, a quantum data plane protocol for entanglement distribution that integrates both entanglement swapping and purification to provide a configurable service to the upper layer. A preliminary performance evaluation of REDiP proved that its configurability is essential to meet user requirements and mitigate the effect of quantum errors. We left as further study the optimization problem to assign ranks and purification rounds of a REDiP tunnel, given a set of user requirements and network conditions.

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