The Intracluster Gas Fraction in X–ray Clusters: Constraints on the Clustered Mass Density

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ABSTRACT

The mean intracluster gas fraction of X–ray clusters within their hydrostatic regions is derived from recent observational compilations of David, Jones & Forman and White & Fabian. At radii encompassing a mean density 500 times the critical value, the individual sample bi-weight means are moderately (2.4σ) discrepant; revising binding masses with a virial relation calibrated by numerical simulations removes the discrepancy and results in a combined sample mean and standard error $f_{\text{gas}}(r_{500}) = (0.060 \pm 0.003) \, h^{-3/2}$. For hierarchical clustering models with an extreme physical assumption to maximize cluster gas content, this value constrains the universal ratio of total, clustered to baryonic mass $\Omega_m/\Omega_b \leq 21.1\, h^{3/2}$, combining with the primordial nucleosynthesis upper limit on $\Omega_b$ results in $\Omega_m \, h^{1/2} < 0.60$. A less conservative, physically plausible approach based on low D/H inferences from quasar absorption spectra and accounting for baryons within cluster galaxies yields an estimate $\Omega_m \, h^{2/3} = 0.28 \pm 0.07$ with sources of systematic error involved in the derivation providing approximately 35\% uncertainty. Additional effects which could provide consistency with the Einstein–deSitter case $\Omega_m = 1$ are presented, and their observable signatures discussed.

Key words: cosmology: observations – cosmology: theory – cosmology: dark matter – galaxies: clusters

1 INTRODUCTION

Clusters of galaxies provide a number of interesting cosmological diagnostics. In particular, the relative amount of baryons and dark matter within their hydrostatic regions provides a measure of the cosmic mix of these components, i.e., a measure of the ratio of density parameters $\Omega_m/\Omega_b$ in the Friedmann–Lemaître world model.\footnote{In this paper, $\Omega_m$ refers to the contribution of all clustered matter (including baryons) to the stress–energy density.}

Employing this measure in practice requires accurate observational data along with estimates of possible systematic biases. Likely sources of bias include systematic errors in component mass estimates and deviation of the local cluster ratio of baryonic–to–total mass arising, for example, from the different dynamical histories of the two components. Within the context of hierarchical clustering scenarios, these effects have been calibrated by numerical simulations of cluster formation. The results, discussed in detail below, indicate that the magnitude of these effects are small — a few tens of percent or less — in observationally accessible regions of clusters. If our current description of cluster formation dynamics is physically accurate, then determination of the cosmic baryon fraction from X–ray cluster data is straightforward.

The baryonic component of the richest clusters is dominated by the X–ray emitting intracluster gas rather than the mass associated with the optical light of the cluster galaxies (Forman & Jones 1982; Sarazin 1986; Mushotzky 1994). Taking Coma as an example, White et al. (1993) estimate the ratio of gas to galaxy mass to be $M_{\text{gas}}/M_{\text{gal}} = (5.5 \pm 1.5) \, h^{-3/2}$ (with $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$). This number is typical of the values seen in larger samples for clusters comparable in temperature to Coma (David et al. 1990; Arnaud et al. 1992). The data also indicate a dependence of the ratio $M_{\text{gas}}/M_{\text{gal}}$ on cluster temperature, with...
poorer clusters and groups possessing less gas per massive
galaxy than their larger counterparts.

The gas mass thus provides not only a formal lower limit
to the total baryon mass in clusters, but a fairly accurate es-
timate of the baryon mass in rich clusters if the dark mat-
ter is assumed non-baryonic. Recent compilations of X-ray
cluster data by White & Fabian (1995) and David, Jones &
Forman (1995) provide gas and total mass estimates for sam-
ple of moderate size. In this paper, I use these data, along
with guidance from numerical simulations, to estimate the
sample mean mass fraction $f_{\text{gas}}$ within a characteristic
radius defining the hydrostatic boundary of clusters. Moti-
vations for the specific approach adopted here are provided
in §2 and the observational data are analysed in §3. Implied
constraints on the universal baryon fraction and the value of
$\Omega_{\text{m}}$ are provided in §4, including a thorough discussion of
systematic errors.

2 PHYSICAL EXPECTATIONS

The value of any individual cluster’s baryon-to-total mass
inferred from observations will, in general, differ from the
cosmic value. The difference can be partly intrinsic — re-
fl ecting a true baryon enhancement or deficit within the
cluster — and partly due to errors in estimates of the com-
ponent masses.

To the extent that the physical processes responsible
for a cluster’s structure are independent of its total mass,
any intrinsic bias in the baryon fraction can, to first approx-
imation, be expressed as a function of a scaled radial vari-
able, equivalent to the local density contrast. Introducing
notation used below, $\delta_c$ denotes the mean interior density
contrast with respect to the critical value $\delta_c \equiv \rho(< r) / \rho_c$
with $\rho_c \equiv 3 \, H_0^2 / 8\pi G$. Similarly, $r_{\delta_c}$ is the radius at which a
density contrast $\delta_c$ is attained.

2.1 Intrinsic component bias

Because the horizon mass scale at the baryogenesis epoch
is many orders of magnitude smaller than the typical clus-
ter mass (e.g., Kolb & Turner 1990), there is no causal
mechanism which can generate primordial fluctuations in
the baryon–to–total mass ratio on cluster scales. This argu-
ment holds as long as inflation precedes baryogenesis. Any
intrinsic bias must therefore be set by dynamical processes
operating differentially on the baryonic and dark matter.

Gravity alone is not a powerful segregating mechanism.
In the simplest example, consider clusters forming from
spherically symmetric, scale–free initial density perturba-
tions. Self–similar solutions to the dynamical equations (Fill-
more & Goldreich 1984; Bertschinger 1985; Chièze, Teyssier
& Alimi 1996) exhibit two characteristic regions — a nearly
hydrostatic, inner body surrounded by an outer, infalling
envelope which merges seamlessly into the Hubble flow at
large radii. The flow changes discontinuously at the bound-
ary, implying the development of a shock for the collisional
baryons or a caustic surface for the collisionless dark mat-
ter (or galaxies). The position and velocity of the shock for
a $\gamma = 5/3$ ideal gas is very close to that of the outermost
caucic surface for the dark matter; together they define a
unique radius (commonly referred to as the virial radius
$r_{\text{vir}}$) within which the mean enclosed density is $\sim 100$ times
the background value. Since all matter outside the virial sur-
face is infalling with the cosmic mix of components, then, by
continuity, the baryon mass fraction measured at the virial
radius must be unbiased $f_b(r_{\text{vir}}) \equiv \Omega_b / \Omega_m$.

The realistic case differs from the spherical one in sev-
eral respects. Clusters forming in a fully three–dimensional,
hierarchical fashion experience lumpy, asymmetric accretion
directed by connecting filaments (West, Dekel & Oemler
1987; Frenk et al. 1990; Evrard 1990a,b; Kang et al. 1994;
Navarro, Frenk & White 1995; Tormen 1996). In addition,
the ability of the intracluster gas to lose entropy via radia-
tive cooling or increase it via feedback from galactic winds
calls into question lessons learned assuming gravitationally
induced shocks are the only entropy changing mechanism.

These issues led White et al. (1993) to explore extreme
models for dynamical baryon enhancement. They examined
the evolution of infinitely dissipational (pressureless) gas
and dark matter using both a spherical model based on
Bertschinger’s (1985) solutions and three dimensional, gas
dynamic simulations. Following their notation, define $\Upsilon(\delta_c)$
as the ratio of enclosed baryon fraction within radius $r_{\delta_c}$
to the cosmic value

$$f_b(r_{\delta_c}) \equiv \Upsilon(\delta_c) \frac{\Omega_b}{\Omega_m}. \tag{1}$$

For the extreme case of a zero temperature gas, the White
et al. three dimensional simulations show baryon enhance-
ments smaller than the spherical case. For example, at
$\delta_c = 500$, the spherical model predicts $\Upsilon(500) = 1.6$ while
the average of twenty simulations is $\Upsilon(500) = 1.25$ and the
maximum simulation value is 1.5. Although the simulation results
strictly apply to the case of a standard CDM initial fluctu-
ation spectrum, results for other cosmologically reasonable
$\Omega_m = 1$ power spectra should not differ substantially, as the
sensitivity of cluster dynamical histories to spectral shape is
fairly mild (Lacey & Cole 1993; Crone, Evrard & Richstone
1994; Navarro, Frenk & White 1996).

When a realistic gas equation of state is used, com-
bined N-body and gas dynamic simulations generally show
the gas to be slightly more extended than the dark matter
(Evrard 1990a,b; Thomas & Couchman 1992; Cen & Ostriker
1993; Kang et al. 1994; Metzler & Evrard 1994; Navarro,
Frenk & White 1995; Lubin et al. 1996), though the evi-
dence is not universal (Anninos & Norman 1996). Energy
transfer between these components during major mergers is
a likely physical explanation (Navarro & White 1993; Pearce,
Thomas & Couchman 1994), though the origin may be un-
related to mergers (Chièze, Teyssier & Alimi 1996). The ex-
tended gas structure implies a weakly rising baryon fraction
with radius $f_{\text{gas}}(r) \sim r^\gamma$ with $\gamma \sim 0.1 – 0.2$ near the virial ra-
dius, and a modest, overall baryon diminution $\Upsilon(500) \sim 0.9$
(Frenk et al. 1996). The mild rise of gas fraction with radius
appears consistent with the observations discussed below.

2.2 Cluster mass estimation

The bias and variance in the mass estimates inferred from
observations have also been calibrated by numerical experi-
ments of cluster formation incorporating gravity and gas dy-
namics. A number of independent experiments over the years
(Evrard 1990a,b; Tsai, Katz & Bertschinger 1994; Metzler &
Evrard 1994; Navarro, Frenk & White 1995; Schindler 1996; Evrard, Metzler & Navarro 1996; Roettiger, Burns & Loken 1996) show that, when exercised judiciously, accurate binding mass estimates can be made with the standard, \( \beta \)-model approach (Cavaliere & Fusco-Fumiano 1976). “Judicious exercising” here means avoiding the cores of clusters where cooling flows and other complications occur (Tsai, Katz & Bertschinger 1994), avoiding clusters engaged in obvious major mergers (Roettiger et al. 1996), and avoiding extrapolating to very large radii where an equilibrium assumption is not justifiable.

From an analysis of gas velocity moments, Evrard et al. (1996, hereafter EMN) propose \( r_{500} \) as a conservative choice for the outer hydrostatic boundary of clusters. Using a sample of 56 cluster simulations evolved in different cosmological backgrounds, and with a subset including input of mass and energy from galactic winds, they show that, within this radius, the gas is very close to hydrostatic, with a mean, mass weighted, radial Mach number of only a few percent (see Tables 3 and 4 of EMN). Binding mass estimates based on the standard, \( \beta \)-model approach are nearly unbiased and have an intrinsic scatter of \( \approx 30\% \) at \( r_{500} \). This compares well with the 15\% \( \text{rms} \) deviation quoted by Schindler (1996) and the tens of percent variations seen in the experiments of Roettiger et al. (1996).

In addition, EMN find that the variance in the mass estimates can be considerably reduced by eliminating the \( \beta \) parameter entirely (i.e., ignoring the X-ray image) and deriving the binding mass directly from the global, emission weighted temperature \( T_X \). The resulting scaling relations are consistent with virial equilibrium expectations at a fixed density contrast \( T \sim GM/r \sim \delta \rho \rho_i r^2 \). Calibrating the relation with the 56 experiments at \( \delta_c = 500 \) yields

\[
r_{500}(T_X) = (1.24 \pm 0.09) \left( \frac{T_X}{10 \text{ keV}} \right)^{1/2} h^{-1} \text{Mpc},
\]

\[
M_{500}(T_X) = (1.11 \pm 0.16) \left( \frac{T_X}{10 \text{ keV}} \right)^{3/2} \times 10^{15} h^{-1} M_\odot.
\]

For the numerical sample, a standard deviation of only 15\% in mass estimates results from this relation.

Gas masses can be recovered with typically higher accuracy than the dark matter (Metzler & Evrard 1994; Roettiger et al. 1996), but this is based upon the assumption that the gas is smoothly distributed within intracluster space, not bound into filamentary or knotty clumps. There is no well defined physical model for such clumpiness, rather the motivation for such a model comes from a desire to minimize the gas fraction in clusters while still providing the observed emission measure for bremsstrahlung. Since the latter scales as \( \rho_{gas}^2 \) while the former as \( \rho_{gas} \), a large, local "clumping factor" \( C \equiv \rho_{gas}^2 / \rho_{gas} > 1/2 \) could reduce the amount of gas required to produce a fixed X-ray emission by a factor \( \sim C \).

There is some observational evidence arguing against such clumping. A very high signal-to-noise X-ray image of the Coma cluster shows no signs of fluctuations in the emission apart from that associated with individual galaxies (White, Briel & Henry 1993). In addition, measurements of the Sunyeav–Zel’dovich (SZ) decrement in clusters are consistent with expectations based on no significant clumping for reasonable values of the Hubble constant (Birkenshaw, Hughes & Arnaud 1991; Herbig et al. 1995; Carlstrom, Joy & Grego 1996). For example, again in Coma, Herbig et al. (1995) derive a Hubble constant \( H_0 = 71_{-12}^{+30} \text{ km s}^{-1} \text{ Mpc}^{-1} \) from OVRO observations of the SZ effect combined with the X-ray model of Hughes (1989) for the intracluster gas, which assumes no clumping. However, given the present uncertainty in these and other Hubble constant determinations (Kennicutt, Freedman & Mould 1995), there appears room for a systematic effect from clumping at the tens of percent level, and perhaps higher. This and other systematic effects are discussed in §4.3 below.

### 3 THE MEAN CLUSTER GAS FRACTION

In this section, I determine the mean gas fraction within \( r_{500} (T_X) \) — calibrated by the numerical experiments, eqn(2) — for the observational samples presented by White & Fabian (1995) and David, Jones & Forman (1995).

White & Fabian (1995; hereafter WF) present gas fractions for 19 clusters derived from archival *Einstein* IPC observations and temperatures compiled from the literature, many from David et al. (1993). They quote values for \( f_{gas} \) at both a fixed metric radius of 0.5 \( h^{-1} \) Mpc and the cluster X-ray radius \( r_X \) which is governed by the image quality. To evaluate gas fractions at \( r_{500} (T_X) \), I use a mild, power law extrapolation of the data quoted at \( r_X \)

\[
f_{gas}(r_{500}(T_X)) = f_{gas}(r_X) \left( \frac{r_{500}(T_X)}{r_X} \right)^\eta
\]

with \( \eta = 0.17 \). This value of \( \eta \) is derived from the numerical simulations of EMN, and is consistent with the rise of gas fraction with radius in both the WF and DJF samples. The mean values of \( \eta \) in the WF data, derived by comparing the baryon fractions at 0.5 \( h^{-1} \) Mpc and \( r_X \), is 0.13. This is biased somewhat low by the few clusters with short lever arm — removing 3 clusters with \( r_X < 0.6 h^{-1} \) Mpc yields a mean \( \eta = 0.15 \), with all but one value in the range 0.05—0.28.

The resultant gas fractions at \( r_{500} (T_X) \) for the WF sample are plotted against cluster temperature as filled circles in Figure 3. Error bars on the gas fraction are 90\% confidence limits and include contributions from the binding and gas mass errors. The errors quoted in Table 2 of WF include only the gas contribution. To estimate binding mass uncertainty, I assume a fractional error equal to that in cluster temperature. Because the latter is generally asymmetric, so also is the binding mass error. Temperature values and errors are taken from the deprojected values in Table 1 of WF, except for two cases of very hot clusters — A2142 and A2163 — which have unusually large allowed lower temperature ranges from the deprojection method. For these, I use the “reference” lower bound on temperature quoted by WF rather than the deprojected values.

The temperature errors are comparable in magnitude to those of the gas; average 1\( \sigma \) fractional errors are 8.5\% for the gas and 6.6/12.8\% for the upper/lower temperature uncertainties. Because the gas emissivity in the IPC band is fairly insensitive to temperature over most of the range spanned by the WF data, the derived gas masses are nearly independent of temperature. Errors in gas and total masses can then be assumed uncorrelated, implying the fractional
The intracluster gas fraction within $r_{500}$ is shown in Figure 1. The errors in the gas and total masses are due to the individual cluster asymmetric Gaussian contributions, as shown in the figure. Estimates of the samples' cumulative parent probability distributions, derived from the normalized sum of the individual cluster asymmetric Gaussian contributions, are shown by the solid (WF) and dotted (DJF) lines.

David et al. (1995; hereafter DFJ) provide in their Figure 5 the gas fraction as a function of overdensity $\delta$, directly. They present data on a range of systems, from elliptical galaxies to clusters; I use only the seven groups and clusters with $T_X > 1$ keV; namely, A539, A262, A2589, A2063, A1795, A85 and A2029. Errors in the binding masses for these systems are again taken to be proportional to the temperature errors listed in Table 1 of DFJ. Gas mass errors are not quoted in their Table, for these I assume a fixed 1% overestimate of the actual typical error, given the improved image quality of the ROSAT PSPC over the Einstein IPC.

The results, shown in panel (a) of Figure 2, are very nearly symmetric and Gaussian, the DJF sample less so because of the influence of A2063, an outlier with $f_{\text{gas}}(r_{500}) = 4.9 h^{-3/2}\%$ (see Figure 1). The bi-weight means and standard errors of the samples — $5.82 \pm 0.45 h^{-3/2}\%$ (WF) and $7.77 \pm 0.69 h^{-3/2}\%$ (DJF) — differ at the 2-$\sigma$ level, where the significance quoted is $|x_1 - x_2|/\sqrt{\sigma_1^2 + \sigma_2^2}$, appropriate for independent Gaussian distributions with means $x_1$ and $x_2$ and standard deviations $\sigma_1$ and $\sigma_2$. This is supported by the appearance of the distributions in Figure 2(a).

The direction of the discrepancy — lower temperature DJF groups having higher gas fractions than the richer clusters of the WF sample — is surprising, because adding the galaxy contribution to the baryon mass would serve to exaggerate the difference, not correct it. Adding the galaxies would impliesignificantly larger baryon fractions in poor groups compared to rich clusters within the virial radius, an outcome not anticipated in hierarchical clustering scenarios. In fact, the opposite is the more likely expectation. Feedback from galactic winds drives baryons preferentially out of low temperature systems (Yahil & Ostriker 1973; White 1991; Metzler & Evrard 1994). It is possible that the discrepancy arises from sample selection criteria, but the fact that
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neither sample is statistically well defined makes it difficult to address this issue. Qualitatively, it is difficult to understand why the generally lower quality IPC imaging should be biased in favor of lower gas fraction clusters.

A possibility addressed here is that modest biases in the binding masses are responsible for the discrepancy. Though both are based on an underlying assumption of equilibrium, WF and DJF use slightly different approaches for estimating binding masses. Only a small (∼25%) systematic effect is required to bring the two sample means into agreement. Analysis of the numerical sample of EMN indicated the virial scaling relation provided a more accurate — in the sense of minimizing variance — mass estimator than the standard, β-model method. If real galaxy clusters satisfy virial equilibrium to the degree established in the simulations, then application of eq’n (3) to the observations should produce similarly accurate mass estimates. This suggests a re-analysis of the data, employing revised binding mass estimates derived from eq’n (3) and the measured cluster temperatures $T_c$.

The result of repeating the bootstrap procedure with the revised binding masses is shown in Figure 2(b). Appropriate corrections have been made to maintain estimates at a density contrast $δ_c = 500$. The WF data shift by a small amount, indicating good agreement between the virial scaling relation and the original deprojection mass estimates, and the variance decreases slightly. The DJF data shift more substantially to lower gas fractions. The direction and magnitude of the shift can be traced directly to the mean value of $β$, which enters linearly in the original mass estimates. The seven groups and clusters of the DJF sample have $β = 0.63$, about 25% lower than the “magic” value of 0.79 inferred from inserting the radial scaling, eq’n (2), into the observations should produce similarly accurate mass estimates. This suggests a re-analysis of the numerical sample of EMN indicated the virial scaling relation provided a more accurate — in the sense of minimizing variance — mass estimator than the standard, β-model method. If real galaxy clusters satisfy virial equilibrium to the degree established in the simulations, then application of eq’n (3) to the observations should produce similarly accurate mass estimates. This suggests a re-analysis of the data, employing revised binding mass estimates derived from eq’n (3) and the measured cluster temperatures $T_c$.

The revised distributions provide consistent estimates of the mean cluster gas fraction between the two samples. Combining the data sets results in a highly statistically accurate estimate of the bi-weight mean gas fraction within $r_{500}$ for X-ray clusters

$$f_{gas}(r_{500}) = (0.060 \pm 0.003) \ h^{-3/2}$$

where the quoted error is 1σ. For comparison, the same limits derived from combining the original sample data is 0.063 ± 0.004. In aligning the two samples, the revised mass estimates provide a small reduction in the uncertainty while not significantly affecting the location of the mean. The analysis which follows employs the revised gas fraction value, but clearly similar numbers result if the original data are employed.

This analysis of the population mean should not be misinterpreted as presenting the value for the gas fraction within $r_{500}$ for all clusters. Simulations indicate intrinsic variations in the gas fraction at about the 15% level arise naturally from the different dynamical histories/states of a coeval population (White et al. 1993; Cen & Ostriker 1993; Kang et al. 1994). Galactic feedback can cause gas loss by subsonic winds after cluster formation or by hindering collapse itself through early preheating. Though nearly all the 26 clusters in Figure 2 have 90% confidence limits which overlap the limits in eq’n (3), there are clusters which have significantly less gas. An example is A576, which has a gas fraction lower by about a factor 2 than the mean (Mohr et al. 1996). Compact groups are extreme in this regard (Ponman et al. 1996; Pillis, Evrard & Bregman 1996). Loewenstein & Mushotzky (1996) present two cool clusters with different baryon fractions within their virial regions. Intrinsic variations in cluster gas content are thus both expected and observed, with the caveat that large amplitude variations go in only one direction, that of reducing gas content. There is currently no empirical evidence or theoretical justification supporting large baryon enhancements near the virial radius in clusters.

4 IMPLICATIONS FOR $\Omega_M$

The arguments presented in §2 indicate that the mean baryon fraction within $δ_c = 500$ is enhanced by at most a factor 1.25 by dynamical means during hierarchical clustering and is more likely slightly below the universal value. This leads to two avenues for using the mean intracluster gas fraction to constrain $\Omega_m$ which are considered here.

4.1 Upper limit approach

Because the contribution of baryon sources add linearly, the mean baryon content of clusters must exceed the mean gas content $f_b \geq f_{gas}$. The mean gas fraction can thus be used to place an upper limit on $\Omega_m/\Omega_b$ in eq’n (3)

$$\frac{\Omega_m}{\Omega_b} \leq \Upsilon(δ_c) \bar{f}_{gas}^{-1} (r_{500})$$

From the perspective of maximizing $\Omega_m$, it is appropriate to take the 5%-ile lower limit on $\bar{f}_{gas}$ along with the largest possible baryon enhancement. The simulations of White et al. yield an average enhancement $\Upsilon(500) = 1.25$, and it is reasonable to assume this is appropriate for the effect on the population mean. The upper bound on the ratio of total, clustered to baryonic mass density is then

$$\frac{\Omega_m}{\Omega_b} < 23.1 \ h^{3/2}$$

Comparison between the light elemental composition of the universe and primordial nucleosynthesis expectations places an upper limit on the mean baryon density. This can be combined with the above to express an upper limit on $\Omega_m$ directly. The exact value of the baryon density derived in this way continues to be a subject of current debate (Krauss & Kernan 1994; Copi, Schramm & Turner 1995; Steigman 1995; Sasselov & Goldwirth 1995; Hata et al. 1996; Tytler, Fan & Burles 1996; Rogers & Hogan 1996). However, a consensus view is that a firm upper limit $\Omega_b \leq 0.026 h^{-2}$ exists simply from the abundance of fragile deuterium in the hostile environment of the local solar neighborhood (Linsky et al. 1995). This value produces the constraint

$$\Omega_m \ h^{1/2} < 0.60$$

The Einstein–deSitter case $\Omega_m = 1$ requires a very low Hubble constant $h \leq 0.36$ (Bartlett et al. 1995).

4.2 Best estimate approach

This upper limit is generous in that it: (i) ignores the contribution of galaxies to the baryon fraction; (ii) assumes maximal, dynamical baryon enhancement through use of an unphysical gas equation of state and (iii) uses the largest realistic value of $\Omega_b$ combined with the smallest allowed value of $\Omega_m$. Loewenstein & Mushotzky (1996) present two cool clusters with different baryon fractions within their virial regions. Intrinsic variations in cluster gas content are thus both expected and observed, with the caveat that large amplitude variations go in only one direction, that of reducing gas content. There is currently no empirical evidence or theoretical justification supporting large baryon enhancements near the virial radius in clusters.
\[ f_{gas}(r_{500}) \]. An alternate perspective is to use “realistic” parameter values to provide a “best” estimate of \( \Omega_m \). For this, I will assume: (i) galaxies make a 20% contribution relative to the gas \( f_b(r_{500}) = (1 + 0.2 h^{3/2}) f_{gas}(r_{500}) \) as appropriate for Coma (White et al. 1993) and (ii) a realistic equation of state for the gas leads to a modest baryon diminution \( T(500) = 0.85 \). Utilizing the central value of \( f_{gas}(r_{500}) \) from eq’n (5) results in

\[
\frac{\Omega_m}{\Omega_b} = \frac{1.2 h^{3/2}}{1 + 0.2 h^{3/2}} \simeq (11.8 \pm 0.7) h^{4/3}
\]

which is a factor of two smaller than the generous upper limit in eq’n (6). The 1\( \sigma \) error quoted above is derived from the propagated statistical error of \( f_{gas} \). The odd-looking slope of \( 4/3 \) in the second expression is a power law approximation to the \( 1.2h^{3/2}/(1 + 0.2h^{3/2}) \) term; it is accurate to 2 percent over the range \( h \in [0.45, 1] \).

The final step to estimate \( \Omega_m \) requires a value for \( \Omega_b \). Recent inferences of the primordial deuterium abundance from quasar absorption line spectra produce two different preferred values, a low value \( \Omega_b h^2 = 6.2 \pm 0.8 \times 10^{-3} \) (Rugers & Hogan 1996) and a high value \( \Omega_b h^2 = 0.024 \pm 0.006 \) (Tytler, Fan & Burles 1996) derived from their respective high and low estimates for D/H. These values result in estimates and 1\( \sigma \) errors of

\[
\begin{align*}
\Omega_m h^{4/3} & = 0.07 \pm 0.01 & \text{high D/H.} \\
\Omega_m h^{4/3} & = 0.28 \pm 0.07 & \text{low D/H.}
\end{align*}
\]

The latter value should be preferred over the former for several reasons. First, a recent analysis of high resolution Keck spectra of Q0014+813 — the best “high D/H” candidate — by Tytler, Burles & Kirkman (1996) provides no support for a level of high deuterium absorption, and suggests hydrogen interlopers as a likely explanation for the data. In addition, the value \( \Omega_m \sim 0.3 \) is currently concordant with large-scale structure observations (e.g., Ostriker & Steinhardt 1995), while the low value \( \Omega_m < 0.1 \) deduced from the Rugers & Hogan D/H estimate is decidedly difficult in this regard.

### 4.3 Systematic effects

The modest statistical error in these estimates is deceptive, since there are systematic uncertainties in the steps involved in the derivation. In the “minimal” approach adopted above, there is perhaps a factor two uncertainty in the contribution of galaxies relative to gas, implying about a 20% error in the baryon fraction estimate. In addition, there is approximately 15% uncertainty in the appropriate value of \( T(500) \). Conservatively adding these contributions leads to an estimate of the overall systematic uncertainty of 35%. In addition, there are systematic effects in the nucleosynthesis determination of \( \Omega_b \) which would enlarge the quoted statistical error (Audouze, Olive & Truran 1997), but values \( \Omega_b h^2 > 0.024 \) are unlikely because of additional constraints from \(^4\)He, \(^6\)Li and \(^7\)Li abundances (e.g., Lemoine et al. 1997).

The actual error in this analysis could be larger if other systematic effects play a significant role. Current possibilities include the following.

(i) **Multi-phase intracluster gas** — One can imagine a multi-phase structure, with cooler gas perhaps entrained along loops of magnetic field surrounded by hotter, more dilute plasma. In order to be competitive with thermal pressure and create a significant mass estimate bias through clumping, the magnetic pressure must be close to the thermal pressure outside the loops and the mass filling factor within the loops should be large. No specific model for the origin and maintenance of such a configuration has been proposed. On energetic grounds alone, it is difficult to imagine galaxies supplying enough magnetic field, and observations of Faraday rotation in background sources indicate that any strong fields must have small coherence lengths (Kim et al. 1990; Kronberg 1994). Another energetic argument against significant magnetic pressure is the fact that the specific thermal energy of the intracluster gas is very nearly equal to the specific kinetic energy of the galaxies (Jones & Forman 1984; Lubin & Bahcall 1993), as expected if the thermal and kinetic pressures respectively support each component within the same potential well (the assumption underlying the \( \beta \)-model of Cavaliere & Fusco-Femiano 1976).

Spatially resolved X-ray spectroscopy, particularly of line emission in cooler (2 – 6 keV) clusters, will provide tests of such models, which are broadly similar to multi-phase cooling flow models (Sarazin 1996). Limited information is already available in broad–beam colors (Henriksen & White 1996). Data from ASCA, SAX, and soon XMM and AXAF, coupled with realistic, dynamical and thermodynamical modeling (Teyssey, Chièze & Alimi 1996) should place stringent constraints on such multiphase models.

In addition, since the electron pressure structure in the gas in a multi-phase model will differ from the standard, unclumped model, Sunyaev–Zel’dovich measurements can be used to provide independent constraints on clumping. Roettiger’s (1996) gas dynamic merger simulations show that errors in SZ Hubble constant determinations are small in the standard scenario, providing hopeful prospects of detecting a modest signal from clumpiness. Present data probably cannot rule out clumping factors as large as 50%, but factors \( \geq 2 \) seem unlikely.

(ii) **Mass estimate errors** — The simulations may be giving a misleadingly simple picture of mass estimate accuracy. The good agreement of independent codes employing different gas dynamic methods on the same problem points the finger at missing physics, rather than numerical inaccuracy, if a significant effect is to be found. Neither feedback from galactic winds (EMN) nor the inclusion of non-equilibrium thermodynamics in the intracluster plasma (Teyssey, Chièze & Alimi 1996) significantly alter the virial scaling relation, eq’n (6). The fact that this binding mass estimator is able to reconcile the modest WF and DjF sample difference can be taken as a measure of empirical support for the simulation results, but that could be a misleading interpretation. Independent measures of cluster mass, particularly weak gravitational lensing (Tyson, Valdes & Wenk 1990; Kaiser & Squires 1993) are needed to provide additional measures of mass estimate accuracy. Comparison of the two methods near the virial radius in a few clusters indicates consistency within modest (\( \sim 50\% \)) statistical error ranges (Squires et al. 1996).

Extending such studies to larger, statistical samples is imperative.

(iii) **Extrapolation errors** — The X–ray data of the WF sample do not extend to \( r_{500} \), making extrapolation necessary. The extrapolation in gas fraction is modest, about
$1 h^{-3/2} \%$ on average. As a further check on the employed procedure, the data values at $r_X$ can be used to predict values at $0.5 \, h^{-1} \, \text{Mpc}$, and the underlying distribution function constructed at that radius. The resultant $5, 50$ and $95\%$ confidence limits using the extrapolated gas fraction of $9.7, 13.8$, and $22.9\%$, respectively (quoted for $h=0.5$), agree well with the directly determined values of $10.0, 13.8$ and $22.3$ from WF (their Figure 6). Significant errors from extrapolation are thus very unlikely.

(iv) Hot dark matter — A sea of massive, light neutrinos would cluster differently from the cold dark matter typically assumed in the dynamical simulations; their high entropy would prevent them from clustering in small potential wells. Experiments using viable cold plus hot dark matter (CHDM) models show that this effect is negligible in rich clusters at radii close to $r_{500}$ (Kofman et al. 1996).

(v) Additional baryon contributions — Sources of baryons in clusters besides gas and galaxies exist, in the form of diffuse intracluster stars (Uson, Boughn, & Kuhn 1991; Theuns & Warren 1996) and MACHOS (Gates, Gyuk, & Turner 1995; Alcock et al. 1996). The latter have not been directly detected in intracluster environments, and they may be simply connected to the former. Regardless, extra baryons only drive the constraints on $\Omega_m$ to lower values. Their contribution probably does not exceed that of galactic starlight.

(vi) Initial baryon fraction inhomogeneities — An early universe model which generates pre-inflationary baryon–to–total mass fluctuations and manages to preserve them into the post-inflationary epoch would circumvent the causality argument mentioned in §2. Such a model would need to naturally couple high baryon overdensity to high mass overdensity in order to produce an overestimate of the cosmic baryon fraction in clusters. Such a correlation would also avoid producing baryon deficient clusters, a comforting situation since no large population of deep, “empty” potential wells exists (although Bonnet, Mellier & Fort (1994) have a candidate in the field of CL0024+1654). Consistency with $\Omega_m=1$ would presumably be a natural feature of such models.

The sources of systematic uncertainty above (except the last) added in quadrature allow room for perhaps $70\%$ additional upward error in $\Omega_m$. However, the magnitude of these effects — particularly of gas clumping for which there are no specific, dynamical models — are currently not well understood. Forcing all effects in the same direction could probably manage consistency with $\Omega_m=1$ and, in this case, the observational tests cited above should start uncovering the effects responsible in the near future.

5 SUMMARY

The gas fraction in clusters of galaxies provides information on the cosmic baryon mass fraction. Dynamical biases of clusters’ baryon content can be minimized by measuring masses near the virial radius, where gas dynamic effects show equilibrium is valid and segregation processes inefficient. At $r_{500}$, the radius where the mean interior density is a factor 500 times the critical value, the bi–weight mean gas fraction of the combined, revised WF and DJF X–ray cluster samples is $0.060 \pm 0.003 h^{-3/2}$. This value, when combined with our current understanding of cluster formation history and limits on $\Omega_m$ from primordial nucleosynthesis, strongly favors $\Omega_m h^{2/3} \sim 0.3$ and rules out the possibility of $\Omega_m=1$ with high statistical significance, unless the Hubble constant is very low $h<0.4$. These conclusions reinforce, at greater statistical significance, earlier work based on different methods and data than those used here (e.g., Henriksen & Mamon 1994; Steigman & Felten 1995; Lubin et al. 1996).

How serious is the case against $\Omega_m=1$? A die–hard Einstein–deSitter advocate could espouse all the elements of the analysis leading to the upper limit in eq’n (5) and claim only a small ($30\%$ for $h=0.7$) systematic effect is missing. However, this approach accounts for neither the hot, X–ray emitting gas nor the galaxies in clusters, and one might well be suspicious of an argument which ignores these two principal, observable components. The best estimate approach leaves one short by a factor $\gtrsim 3$. It is possible that this gap is plugged not by a single large effect, but by several effects acting in concert, a situation reminiscent of the so–called “$\beta$–discrepancy” in clusters (Evrard 1990b; Lubin & Bahcall 1995). There remains the possibility that a new element of X–ray cluster physics is simply missing from the present picture. Observational constraints on known sources of systematic error should be vigorously pursued, along with more sophisticated theoretical modeling of intracluster plasma dynamics and thermodynamics.

The arguments presented here are independent of the value of the cosmological constant. If one favors a spatially flat universe, as motivated by simple models of inflation, obtained through a non-zero cosmological constant $\Lambda$, then the limits on the clustered matter component predict values of the required vacuum energy density $\Omega_\Lambda \equiv \Lambda/3 \, H_0^2 = 1 - \Omega_m$. For example, for $h=0.7$, the upper limit on $\Omega_m$ requires $\Omega_\Lambda \gtrsim 0.28$ while the best estimate approach gives $\Omega_\Lambda = 0.64 \pm 0.09$. The latter is consistent with the limit $\Omega_\Lambda < 0.66$ derived from gravitational lensing arguments (Kochanek 1996) while marginally inconsistent with the $\Omega_\Lambda < 0.51$ inferred recently by Perlmutter et al. (1996) from the magnitude–redshift relation for Type-Ia supernovae.

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