Measurement-based noiseless linear amplification for quantum communication

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Entanglement distillation is an indispensable ingredient in extended quantum communication networks. Distillation protocols are necessarily non-deterministic and require advanced experimental techniques such as noiseless amplification. Recently, it was shown that the benefits of noiseless amplification could be extracted by performing a post-selective filtering of the measurement record to improve the performance of quantum key distribution. We apply this protocol to entanglement degraded by transmission loss of up to the equivalent of 100 km of optical fibre. We measure an effective entangled resource stronger than that achievable by even a maximally entangled resource passively transmitted through the same channel. We also provide a proof-of-principle demonstration of secret key extraction from an otherwise insecure regime. The measurement-based noiseless linear amplifier offers two advantages over its physical counterpart: ease of implementation and near-optimal probability of success. It should provide an effective and versatile tool for a broad class of entanglement-based quantum communication protocols.

The impossibility of determining all properties of a system, as exemplified by Heisenberg's uncertainty principle1, is a well-known signature of quantum mechanics. It results in phase and amplitude fluctuations in vacuum, enables applications such as quantum key distribution (QKD), and is at the heart of fundamental research into the no-cloning theorem2, quantum limited metrology3 and the unavoidable addition of noise during amplification4–6. This last constraint means that even an ideal quantum amplifier cannot be used for entanglement distillation7–9, which is a critical step in the creation of large-scale quantum information networks8–10.

Distillation protocols, originally conceived for discrete variables6–7, initially proved more elusive in the continuous-variable (CV) regime. The most experimentally feasible and theoretically well-studied class of CV states and operations are the Gaussian states and the operations that preserve their Gaussianity11. Protocols that distill Gaussian states have been discovered12–14 that involve an initial non-Gaussian operation that increases the entanglement, followed by a 'Gaussification' step that iteratively drives the output towards a Gaussian state. More recently, noiseless linear amplification has been identified as a simpler method of distilling Gaussian entanglement13–15.

The noiseless linear amplifier (NLA) avoids the unavoidable noise penalty by moving to a non-deterministic protocol. This ingenious concept and a linear optics implementation have been proposed13,16,17 and experimentally realized for the amplification of coherent states18–21, manipulation of qubits22–24, and the concentration of phase information25. All these were extremely challenging experiments, with only ref. 18 demonstrating entanglement distillation and none directly showing an increase in Einstein–Podolsky–Rosen (EPR) correlations26. Moreover, the success probability of these experiments was substantially worse than the maximum set by theoretical bounds. In the context of QKD, refs 27 and 28 proposed the possibility of implementing a non-deterministic measurement-based NLA (MB-NLA) to improve performance. This represents a significant advantage as the difficulty of sophisticated physical operations can be moved from a hardware implementation, where one must suffer penalties related to source and detector efficiencies, to a software implementation where we are limited primarily by quantum theory and the statistics of our sample. Here, we apply this protocol to EPR entanglement and observe improvements in the measured correlations consistent with distillation of the entanglement. We emphasize that this method is only equivalent to entanglement distillation for certain applications; specifically, it is only in situations where the desired distillation operation immediately precedes the measurement of the target mode that the two are indistinguishable.

We first derive some general conditions on the limits to implementing arbitrary quantum operations on an ensemble by conditionally filtering the measurement results. Using this method we experimentally implement an MB-NLA protocol, achieving significant distillation with a much improved probability of success. Furthermore, we illustrate the critical benefit of distillation in combating decoherence by considering the distribution of EPR entanglement through a lossy channel. We measure an output level of entanglement that exceeds the maximum achievable without distillation, even if one could use a perfect initial entangled state.

In any quantum information application the final result is always some classical measurement record, drawn from a set of possible outcomes \( k \) and described by some probability distribution \( p(k) \). In an application where the proposed distillation would take place immediately prior to measurement, for example in QKD, one could imagine emulating the operation on an ensemble via post-selective measurements. We first consider the process of emulating arbitrary operations via conditioning on measurements in a general setting before describing the results of refs 27 and 28, in which an explicit procedure applicable to the NLA was proposed. Our analysis will allow us to clarify some of the previous work as well as showing that the \( g^k \) operator key to the operation of the NLA is particularly well suited to emulation via post-selection.

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Consider an arbitrary quantum map applied to an incoming state \( \rho \) that can be written using the Kraus decomposition as\(^2\):

\[
\mathcal{E}(\rho) = \sum_{i} E_i \rho E_i^\dagger
\]  

(1)

Here, \( \{E_i\} \) are the Kraus operators. Note that this decomposition is valid for any completely positive operator, including those that do not preserve the trace, as is the case with many useful conditional operations in quantum optics, including photon addition and subtraction and the NLA. In the latter case the Kraus operators fail to satisfy the usual relation \( \sum_i E_i E_i^\dagger = I \), with the extra information needed to restore conservation of probability being the success probability, \( P \), of the conditional process\(^3\).

If this map is immediately followed by a positive-operator valued measure (POVM) described by operators \( \{\pi_k\} \) the corresponding probability distribution is given by

\[
p(k) = \text{tr}\left\{ \bar{\pi}_k \sum_i E_i \rho E_i^\dagger \right\} = \text{tr}\{ \bar{\pi}_k \rho \}
\]

(2)

where \( \bar{\pi}_k = \sum_i E_i^\dagger \pi_k E_i = \tilde{\mathcal{E}}(\pi_k) \) are a new set of POVM elements obtained by applying the mapping \( \tilde{\mathcal{E}} \) to the desired output POVM set. Equation (2) tells us that we may obtain the statistics of a probability distribution is given by \( \tilde{\mathcal{E}}(\rho) \) by conditioning upon measurements made with a transformed set \( \{\bar{\pi}_k\} \) on the original input state.

Although the above procedure is quite general, it is not arbitrary in that it does not allow the reconstruction of any desired POVM set in combination with any desired operation. If one wishes to implement arbitrary operations \( E \), one is restricted to certain final POVM sets \( \{\pi_k\} \) and vice versa. Intuitively, we expect that, to correctly reconstruct the statistics of an arbitrary POVM upon an arbitrary state, it is necessary to obtain maximum information about that state, that is, to make measurements capable of complete tomographic reconstruction. This requirement can be derived by considering equation (2) and noting that the POVM set with which one must actually measure \( \{\tilde{\mathcal{E}}(\pi_k)\} \) is not necessarily physical for arbitrary \( E \) and \( \{\pi_k\} \). The unphysicality occurs because some of the operations that we wish to emulate are not themselves physical. For example, the NLA itself, as will be discussed later, is trace increasing (see Supplementary Section I). If one demands access to arbitrary operations, then a sufficient condition on \( \{\tilde{\mathcal{E}}(\pi_k)\} \) would be that it maps to physical output states and is capable of uniquely determining an arbitrary completely positive map. This is precisely the same condition required of a POVM set for it to be classified as informationally complete (IC)\(^31,32\).

Conversely, if one is only able to experimentally realize a certain POVM set, then one is limited in the range of operations that can be faithfully implemented. Further details regarding the status of IC-POVMs as a sufficient condition for implementation via post-selection in combination with non-deterministic operations are provided in Supplementary Section I.

Noiseless amplification is commonly defined as the ability to increase the amplitude of an unknown coherent state without any noise penalty, effecting the transformation \( |\alpha\rangle \rightarrow |g\alpha\rangle \), with \( g > 1 \). By considering the annihilation and creation operators describing a bosonic mode it becomes clear that such a transformation would violate the canonical commutation relations \([\hat{a}, \hat{a}^\dagger] = 1\). Consistency with quantum mechanics can be restored if one instead implements a non-deterministic version of this transformation \( |\alpha\rangle \rightarrow P(|\alpha\rangle \langle |\alpha\rangle | (1 - P)|0\rangle \langle 0| \) which succeeds with probability \( P \). Provided the success is heralded, one may enjoy the benefits of entirely noiseless amplification at least some fraction of the time.

As was shown in refs 13 and 17, just such a transformation is performed by the operator \( g^\dagger \hat{n} g \) where \( \hat{n} = \hat{a}^\dagger \hat{a} \) is the number operator. Furthermore, acted upon one arm of a two-mode Gaussian EPR state written in the number basis as

\[
|\chi, \chi\rangle = \sqrt{1 - \chi^2} \sum_n \chi^n |n\rangle |n\rangle
\]

(3)

where \( \chi \in [0,1] \) characterizes the entanglement, we find the operation results in \( g^\dagger |\chi\rangle |\chi\rangle \rightarrow P_{|\chi\rangle \langle \chi|} g^\dagger |\chi\rangle |\chi\rangle + (1 - P) |0\rangle |0\rangle \). In other words the entanglement is probabilistically increased. Applying the NLA to an EPR state that has been distributed through a lossy channel (Fig. 1a) results in an output with a greater degree of initial entanglement that appears to have suffered less loss\(^13\). We note that although \( g^\dagger \) appears Gaussian in the sense of being quadratic in annihilation/creation operators, it is in fact non-unitary and unbounded. These properties are also the reason that such an operation falls beyond the purview of the no-go theorem\(^33,35\), which states that Gaussian entanglement cannot be distilled via purely Gaussian operations. In fact, an exact implementation of \( g^\dagger \) would necessitate a success probability of zero. However, when considering a given set of input states one may explicitly construct physical operations that have arbitrarily high fidelity with \( g^\dagger \) while succeeding with a finite probability. The most intuitive version of this method, proposed in ref. 13 and utilized in subsequent experiments, is to use a generalized quantum scissors scheme\(^36\) and truncate in the photon-number basis, faithfully amplifying low-energy input states that have negligible higher-order terms. However, these truncated experiments are by no means trivial, with all demonstrations limited to the single-photon case except for ref. 23 in which two stages were achieved. It would therefore be extremely valuable to devise an easier method of implementing the distillation, albeit for a more restricted set of applications. Here, we implement a measurement-based version of this protocol (Fig. 1b) where the

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**Figure 1** | Schematic of equivalent methods of entanglement distillation with physical and measurement-based noiseless linear amplifiers. a, b, Two-mode EPR entanglement is represented by two orthogonally juxtaposed squeezed states. One arm of the EPR entanglement is transmitted through a lossy channel before being noiselessly amplified. In the physical implementation (a) a quantum scissor set-up is used to implement the probabilistic amplification before the final measurement. In the measurement-based implementation (b) a post-selective filter is used to keep a remaining fraction of data.
original state is first measured using heterodyne detection on Bob’s side. A sub-ensemble is then post-selected according to a filter function defined by the desired NLA gain.

The MB-NLA is also in compliance with the no-go theorem of refs 33 and 34 in that we do not distill free propagating Gaussian entanglement. Nevertheless, it is remarkable that, under certain circumstances, we achieve useful results utilizing only hardware from the experimentally friendly Gaussian toolbox. We emphasize that, with respect to the post-selected ensemble, the protocol works shot by shot.

The exact filter function corresponding to $g^h$ can be derived following ref. 27 by considering the coherent state projection, or the Q function, on an arbitrary input state $\rho$, $Q_{\rho}(\alpha) = (\alpha|\rho|\alpha)/\pi$. Recalling that the action of the NLA on a coherent state is given by\textsuperscript{13}

$$g^h(\alpha) = e^{\frac{1}{2}g^2 - \frac{1}{2}|\alpha|^2}(|\alpha\rangle)$$

we can write down the Q function of the amplified state $\rho'$ as

$$Q_{\rho'}(\alpha) = (\alpha|\rho' g^h|\alpha)$$

$$= e^{\frac{1}{2}g^2 - \frac{1}{2}|\alpha|^2}(|\alpha\beta\rangle)$$

$$= e^{\frac{1}{2}g^2 - \frac{1}{2}|\alpha|^2}(|\alpha\rangle)$$

where $\beta = g\alpha$. This equation allows us to determine the particular probabilistic filter and rescaling we must apply to the original heterodyne data in order to obtain the same output as a heterodyne measurement applied to the same input state after noiseless amplification with gain $g$. Clearly, for $g > 1$, the filter defined above does not qualify as a sensible weighting probability as it is always greater than 1. Thus, we must renormalize to some cutoff, thereby implementing an approximation to the ideal operation. This is analogous to the fact that although the success probability for $g^h$ has to be zero, one can experimentally achieve a good approximation of an ideal NLA with finite probability. In the measurement-based picture, this corresponds to implementing an approximate operation while keeping a finite fraction of the data after post-selection. In both cases, however, the approximation can be made arbitrarily close to perfect while retaining a finite success probability.

The filter function, or acceptance probability, of the $g^h$ modified post-selection filter with a finite cutoff is given by

$$P(\alpha) = \begin{cases} e^{\frac{1}{2}g^2 - \frac{1}{2}|\alpha|^2}(\pi g - 1), & \alpha < \alpha_C \\ 1, & \alpha \geq \alpha_C \end{cases}$$

where $\alpha = \frac{1}{2g}(x + ip)$ is the coherent state projection for each heterodyne measurement. Examples of the function $P(\alpha)$, and the resultant histograms and probability distributions for various gains, are shown in Fig. 3a. As the fidelity of the truncated filter with the ideal $g^h$ operation is input state-dependent, we choose a finite cutoff $\alpha_C$ that optimizes post-selection rates while preserving high fidelity with the ideal filter. This also ensures that output distributions remain statistically close to a normal distribution, allowing us to only consider the second-order moments of the measured distribution to characterize the correlations (see Supplementary Section II).
Figure 3 | Results of the measurement-based NLA implemented at the receiver (Bob) station. a, Bob’s experimental data: (i) acceptance probability function of the post-selective filters used to obtained (ii) the resulting measurement histograms, and (iii) the final normalized probability distributions. Gain \( g \) is increased by selecting a filter function with increasingly lower acceptance probability. As the gain is increased, the variance of Bob’s final measurement distribution increases. This corresponds to stronger two-mode EPR entanglement. b, EPR criterion as a function of post-selection success probability for direct (\( \mathcal{I}_{A|B}^\text{direct} \), red) and reverse (\( \mathcal{I}_{B|A}^\text{reverse} \), green) inferences for input states with initial EPR entanglement strengths of \( 0.484 \pm 0.001 \) and \( 0.492 \pm 0.001 \), respectively. Data points are the post-selected ensemble average of ten experimental runs. Solid lines show the theoretical distillation of an ideal implementation of \( g \) given the same input state. Shading represents a 2\( \sigma \) confidence interval on the variance of the implemented filter. Inset: effect of distillation on the inseparability criterion, \( \mathcal{I}_{A|B} \), with the same data set. c, Effect of EPR entanglement distillation as a function of probability of success for different losses (0%, 25% and 50%) for i, direct (\( \mathcal{E}_{A|B}^\text{direct} \)); ii, reverse (\( \mathcal{E}_{B|A}^\text{reverse} \)).

For each input state we implement a cutoff of between four and five standard deviations of Bob’s measured state. While the energy of Bob’s input state defines the theoretical maximum gain of our amplifier[^15], the size of the initial data ensemble effectively places a stronger restriction on the maximum gain that can be applied while retaining reasonable statistical precision.

Let us consider that Alice and Bob share a symmetric two-mode EPR state, with no transmission loss between either of their measurement stations. In this scenario, an MB-NLA on Bob’s side is indistinguishable from an implementation on Alice’s side, and the observed distillation should be symmetric for both parties. A two-mode EPR state demonstrates EPR-type correlations if it demonstrates a Reid EPR criterion of \( \mathcal{I} < 1 \), which is a condition on the product of the conditional quadrature variances[^37]. As such, it is an inherently directional quantity and we borrow terminology from QKD, and refer to Bob’s ability to infer Alice’s state as the direct inference \( \mathcal{E}_{A|B} = V_{x_A|x_B} V_{p_A|p_B} \), and the converse as the reverse inference \( \mathcal{E}_{B|A} = V_{x_B|x_A} V_{p_B|p_A} \). Either \( \mathcal{E}_{B|A} < 1 \) or \( \mathcal{E}_{A|B} < 1 \) is a sufficient but not necessary condition for entanglement. We also examine the symmetric inseparability criterion of Duan et al.[^38], denoted \( \mathcal{I}_{A|B} \). For Gaussian states \( \mathcal{I}_{A|B} < 1 \) is both a necessary and sufficient condition for entanglement.

We experimentally prepare a two-mode EPR resource by interfering two identical-amplitude squeezed states on a 50:50 beamsplitter with their relative phase controlled to be in quadrature. The two arms of the resulting two-mode EPR state are then sent to two independent measurement stations that we identify with Alice and Bob (Fig. 2). We implement an MB-NLA on Bob’s subsystem, amounting to a heterodyne measurement of the amplitude and quadratures \( X \) and \( Y \), followed by the appropriate post-selection procedure. Alice implements a homodyne measurement of her subsystem, alternating between measurement of \( X \) and \( Y \). These measurements allow characterization of the covariance matrix of the two-mode EPR state and, thereafter, the corresponding EPR criterion[^39] \( \mathcal{E}_{B|A}^\text{direct} \), \( \mathcal{E}_{A|B}^\text{reverse} \) and inseparability criterion[^40] \( \mathcal{I}_{A|B} \).
Our initial entangled resource demonstrates an EPR criterion violation of $\mathcal{E}_{\text{EPR}} = 0.484 \pm 0.001$ and $\mathcal{E}_{\text{EPR}} = 0.492 \pm 0.001$ with an initial ensemble size of $8.3 \times 10^9$ data points. We then apply the post-selection function of equation (6) followed with a rescaling by $1/g$, emulating $g^k$. A linear increase in the amplifier gain sees an exponential reduction in the probability of success, but results in a more correlated subset of the measurement record, equivalent to a more entangled two-mode EPR state. Figure 3b demonstrates our improvement in the EPR criterion as a function of success probability. The declining probability of success as we apply increasingly larger gains to obtain stronger correlations is manifested in increased statistical uncertainty. The solid lines represent the theoretical inseparability of our input state, with an applied post-selection filter of a defined gain ($g = 1.0, 1.1, \ldots, 1.5$) as a function of channel transmission.

Figure 4 | Improvement in the inseparability criterion of the two-mode EPR state for a series of lossy channels. For each transmissivity, a series of post-selections corresponding to an NLA gain (specified by the legend) are applied. The boundary of the shaded area describes the theoretical inseparability of a perfect EPR state—ininitely squeezed—subject to the same channel transmissivity. Post-selection allows access to an entangled resource beyond that accessible with even a perfect initial resource. Solid lines represent the theoretical inseparability of our input state, with an applied post-selection filter of a defined gain ($g = 1.0, 1.1, \ldots, 1.5$) as a function of channel transmission.

Perhaps the most interesting regime for the performance of the MB-NLA occurs at very high loss. In Fig. 4 we plot the inseparability criterion of the two-mode EPR state as a function of the channel transmission encountered by Bob’s subsystem. For each channel, we consider distillation using the MB-NLA with the maximum loss of 99% equivalent to 100 km of optical fibre (assuming a loss of 0.02 dB per kilometre). The boundary of the shaded area describes the theoretical inseparability of a perfect EPR state in the limit of infinite squeezing, subject to the same channel transmissivity. We find that post-selection allows access to final EPR correlations that, without our protocol, are inaccessible, even considering a perfect initial EPR resource.

Finally, we turn to the application that sparked interest in this protocol, and investigate the performance of an entanglement-based CV-QKD protocol including an MB-NLA (see Supplementary Section III). We conduct a very cautious analysis in which all measured imperfections are attributed to the eavesdropper, such that our EPR source is interpreted as a pure EPR source followed by a decohering channel. In Fig. 5 we show that, starting from a situation in which key distribution is impossible, application of a sufficiently high gain restores security. Figure 5 only considers the effect on the key rate of the post-selected ensemble. When considering the overall key rate, the maximum gain is unlikely to be the optimal gain; rather, the optimal gain recovers a secure key while balancing post-selection rates. Further discussions and a calculation of the key rate are given in Supplementary Section III.

The primary significance of these results is twofold. First, we have experimentally demonstrated the equivalence of the MB-NLA to the implementation of a physical NLA for entanglement distillation when considering scenarios where amplification directly precedes measurement. This equivalence ensures that this technique has immediate applications for CV-QKD, where the advantage of an NLA has already been studied. Furthermore, we provided a generalized theoretical explanation of the conditions in which an arbitrary quantum operation could in principle be implemented on an ensemble via post-selective measurements. Second, this equivalence is of great practical importance, because the MB-NLA is significantly less demanding than the existing physical implementations of $g^k$, where inefficiencies in sources and measurement
restrict the physical NLA to very small input states.\textsuperscript{13,18,19,23} In contrast, the MB-NLA is more flexible in that it can be used on a wide variety of input states without experimental reconfiguration, and more scalable, in that by circumventing several experimental inefficiencies it achieves near-optimal success probability for an implementation of $g^*$ of arbitrary precision. Although there are clear restrictions on the scenarios where this MB-NLA can be substituted for its physical counterpart, when applicable, it is certainly advantageous to do so. The achievable entanglement distillation is now highly amenable by the amount of data collected. Here, for feasible sample sizes, we demonstrate distillation of correlations in excellent agreement with close to the theoretical ideal performance of $g^*$. For moderate-loss channels we have shown the recovery of EPR correlations from an entangled state, and for high-loss channels have demonstrated levels of entanglement that are impossible without a distillation process.

Many avenues for further research remain. Beyond the aforementioned applications in CV-QKD, the NLA could find use in other quantum communication protocols including teleportation and remote state preparation. This would be of particular interest as it would enable us to extend these conditioning distillation techniques to improve the quality of a still propagating, albeit unentangled, quantum mode. Furthermore, our theory is sufficiently general to allow extensions to other conditional processes. For example, using precisely the same set-up described here it is also possible to implement the photon addition operation, which has been extensively studied.\textsuperscript{17,41–44} As well as targeting other operations one could also use this formalism to consider conditioning on different POVM sets, opening up many promising candidates for future applications.

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References

1. Heisenberg, W. Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik. Z. Phys. 43, 172–198 (1927).
2. Wootters, W. K. & Zurek, W. H. A single quantum cannot be cloned. Nature 299, 802–803 (1982).
3. Giovannetti, V., Lloyd, S. & Maccone, L. Advances in quantum metrology. Nature Photon. 5, 222–229 (2011).
4. Caves, C. M. Quantum limits on noise in linear amplifiers. Phys. Rev. D 26, 1817–1839 (1982).
5. Caves, C. M., Combes, J., Jiang, Z. & Pandey, S. Quantum limits on phase-preserving linear amplifiers. Phys. Rev. A 86, 063802 (2012).
6. Bennett, C. et al. Purification of noisy entanglement and faithful teleportation via noisy channels. Phys. Rev. Lett. 76, 722–725 (1996).
7. Horodecki, M., Horodecki, P. & Horodecki, R. Inseparable two-spin-1/2 density matrices can be distilled to a singlet form. Phys. Rev. Lett. 78, 574–577 (1997).
8. Browne, D., Eisert, J., Scheel, S. & Plenio, M. Driving non-Gaussian to Gaussian states with linear optics. Phys. Rev. A 67, 062320 (2003).
9. Duan, L. M., Lukin, M. D., Cirac, J. I. & Zoller, P. Long-distance quantum communication with atomic ensembles and linear optics. Nature 414, 413–418 (2001).
10. Kimble, H. J. The quantum internet. Nature 453, 1023–1030 (2008).
11. Weedbrook, C. et al. Gaussian quantum information. Rev. Mod. Phys. 84, 621–669 (2012).
12. Eisert, J., Browne, D., Scheel, S. & Plenio, M. Distillation of continuous-variable entanglement with optical means. Ann. Phys. 311, 431–458 (2004).
13. Ralph, T. C. & Lund, A. P. Nondeterministic noiseless linear amplification of quantum systems in Proceedings of the 9th International Conference on Quantum Communication Measurement and Computing (ed. Ivlevsky, A. I.) 155–160 (American Institute of Physics, 2009).
14. Ralph, T. C. Quantum error correction of continuous-variable states against gaussian noise. Phys. Rev. A 84, 022339 (2011).
15. Walk, N., Lund, A. P. & Ralph, T. C. Nondeterministic noiseless amplification via non-symplectic phase space transformations. New J. Phys. 15, 073014 (2013).
16. Marek, P. & Filip, R. Coherent-state phase concentration by quantum probabilistic amplification. Phys. Rev. A 81, 022302 (2010).
17. Fiurášek, J. Engineering quantum operations on traveling light beams by multiple photon addition and subtraction. Phys. Rev. A 80, 053822 (2009).
18. Xiang, G. Y., Ralph, T. C., Lund, A. P., Walk, N. & Pryde, G. J. Heralded noiseless linear amplification and distillation of entanglement. Nature Photon. 4, 316–319 (2010).
19. Ferreyrol, F. et al. Implementation of a nondeterministic optical noiseless amplifier. Phys. Rev. Lett. 104, 123603 (2010).
20. Ferreyrol, F., Blandino, R., Barbieri, M., Tuale-Broui, R. & Grangier, P. Experimental realization of a nondeterministic optical noiseless amplifier. Phys. Rev. A 83, 063801 (2011).
21. Zavatta, A., Fiurášek, J. & Bellini, M. A high-fidelity noiseless amplifier for quantum light states. Nature Photon. 5, 52–60 (2010).
22. Osorio, C. I. et al. Heralded photon amplification for quantum communication. Phys. Rev. A 86, 023815 (2012).
23. Koksis, S., Xiang, G. Y., Ralph, T. C. & Pryde, G. J. Heralded noiseless amplification of a photon polarization qubit. Nature Phys. 9, 23–28 (2012).
24. Múñoz, M. et al. Noiseless loss suppression in quantum optical communication. Phys. Rev. Lett. 109, 180503 (2012).
25. Usuga, M. A. et al. Noise-powered probabilistic concentration of phase information. Nature Phys. 6, 767–771 (2010).
26. Reid, M. D. et al. Colloquium: the Einstein–Podolsky–Rosen paradox: from concepts to applications. Rev. Mod. Phys. 81, 1727–1751 (2009).
27. Fiurášek, J. & Cerf, N. Gaussian postselection and virtual noiseless amplification in continuous-variable quantum key distribution. Phys. Rev. A 86, 060302 (2012).
28. Walk, N., Ralph, T. C., Symul, T. & Lam, P. K. Security of continuous-variable quantum cryptography with Gaussian postselection. Phys. Rev. A 87, 020303 (2013).
29. Hellwig, K. & Kraus, K. Operations and measurements. II. Commun. Math. Phys. 16, 142–147 (1970).
30. Ferreyrol, F., Spagnolo, N., Blandino, R., Barbieri, M. & Tuale-Broui, R. Heralded processes on continuous-variable spaces as quantum maps. Phys. Rev. A 86, 062327 (2012).
31. Prugovečki, E. Information-theoretical aspects of quantum measurement. Int. J. Theor. Phys. 16, 321–331 (1977).
32. Busch, P. & Lahti, P. J. The determination of the past and the future of a physical system in quantum mechanics. Found. Phys. 19, 633–678 (1989).
33. Eisert, J., Scheel, S. & Plenio, M. Distilling Gaussian states with Gaussian operations is impossible. Phys. Rev. Lett. 89, 137903 (2002).
34. Fiurášek, J. Gaussian transformations and distillation of entangled gaussian states. Phys. Rev. Lett. 89, 137904 (2002).
35. Giedke, G. & Cirac, J. I. Characterization of Gaussian operations and distillation of Gaussian states. Phys. Rev. A 66, 032316 (2002).
36. Pegg, D., Phillips, L. & Barnett, S. Optical state truncation by projection synthesis. Phys. Rev. Lett. 81, 1604–1606 (1998).
37. Reid, M. D. Demonstration of the Einstein–Podolsky–Rosen paradox using nondegenerate parametric amplification. Phys. Rev. A 40, 913–923 (1989).
38. Duan, L. M., Giedke, G., Cirac, J. I. & Zoller, P. Inseparability criterion for continuous variable systems. Phys. Rev. Lett. 84, 2722–2725 (2000).
39. Reid, M. D. & Drummond, P. D. Quantum correlations of phase in nondegenerate parametric oscillation. Phys. Rev. Lett. 66, 2731–2733 (1998).
40. Blandino, R. et al. Improving the maximum transmission distance of continuous-variable quantum key distribution using a noiseless amplifier. Phys. Rev. A 86, 012327 (2012).
41. Lee, S.-Y., Ji, S.-W., Kim, H.-J. & Nha, H. Enhancing quantum entanglement for continuous variables by a coherent superposition of photon subtraction and addition. Phys. Rev. A 84, 012302 (2011).
42. Kim, H.-J., Lee, S.-Y., Ji, S.-W. & Nha, H. Quantum linear amplifier enhanced by photon subtraction and addition. Phys. Rev. A 85, 013839 (2012).
43. Barbieri, M. et al. Non-Gaussianity of quantum states: an experimental test on single-photon-added coherent states. Phys. Rev. A 82, 063833 (2010).
44. Zavatta, A., Viciani, S. & Bellini, M. Quantum-to-classical transition with single-photon-added coherent states of light. Science 306, 660–662 (2004).