DC SQUID as a sensitive detector of dark matter

Bartłomiej Kiczek$^1$ Marek Rogatko$^2$ Karol I. Wysokinski$^3$

Institute of Physics
Maria Curie-Skłodowska University
20-031 Lublin, pl. Marii Curie-Skłodowskiej 1, Poland

ABSTRACT: The gauge-gravity duality has been applied to examine the properties of holographic superconducting quantum device (SQUID), composed of two S-N-S Josephson junctions, influenced by dark matter sector modeled by the auxiliary $U(1)$-gauge field coupled to the ordinary Maxwell one. The dark matter sector is known to affect the properties of superconductor and is expected to enter the current-phase relation. While the single junctions between two superconductors turn out to be very weakly sensitive to the presence of dark matter sector, the results obtained for the SQUID are encouraging. We find that the analyzed properties of the device may be used in the future experiments aimed at the detection of the dark sector of our Universe.

KEYWORDS: Gauge-gravity correspondence, Holography and condensed matter physics (AdS/CMT), Black Holes

$^1$bkiczek@kft.umcs.lublin.pl
$^2$rogat@kft.umcs.lublin.pl, marek.rogatko@poczta.umcs.lublin.pl
$^3$karol.wysokinski@poczta.umcs.lublin.pl
1 Introduction

The quest for the dark matter in the Universe has been one of the most important topics of the current research in cosmology and physics [1]. Contemporary astronomical observations of galaxies and primordial radiation endorse the fact that our Universe is made mostly (over 23 percent of its mass) of non-luminous dark matter. Several new types of fundamental particles have been claimed as candidates for dark matter sector. They are expected to interact with nuclei in suitable detector materials on Earth. It was claimed only by the DAMA collaboration [2, 3] that they observed modulation in the rate of interaction events which might be the trace of dark matter sector. Several groups want to reproduce the DAMA results but in vain [4]. This situation triggers the discussion concerning the composition, interaction with ordinary matter, the self-interaction and the possible ways to discriminate between various models of dark matter [5].

It was argued that due to the growing sense of ‘crisis’ in the dark matter particle community, arising from the absence of evidences for the most popular candidates for dark matter particles like WIMPs, axions and sterile neutrinos, diversifying the experimental effort should be paid attention to. These efforts ought to accomplish upcoming astronomical surveys, gravitational wave observatories which can provide us some complimentary information about the dark matter sector. They constitute our best hope for making progress in this direction. Unconventional experiments and techniques are also being looked for. One of the prospect directions is related to clever usage of molecular or condensed matter systems including superconductors or superconducting devices. Recent proposals authorize search for bosonic dark matter via absorption in superconductors [6], using superfluid helium [7] or optical phonons in polar materials [8] to detect light dark matter. More exotic proposal are based on the observations of color centers production in crystals [9], or the usage of bulk three-dimensional Dirac semi-metals [10] and other topological semiconducting compounds [11], as well as, multilayered optical devices [12], for the detection of possible candidates for dark matter sector.
The theoretical approach we have been used in our researches relies on the AdS/CFT correspondence [13, 14], which has been found to provide strong coupling description [15, 16] of many condensed matter models. In particular this gauge-gravity duality has been shown to describe $U(1)$ and $SU(2)$ symmetry breaking superconducting transitions. The applications of the holography to study various models of superconductivity have been recently reviewed in [17].

Our previous AdS/CFT studies [18]-[28] on holographic superconductors and superfluids reveal the influence of dark matter sector on various properties of them, e.g., superconducting transition temperature, Ginzburg-Landau ratio, vortices and condensation flow, viscosity bound for anisotropic superfluids and interacting currents in holographic Dirac fluid. These findings may in principle help to design future condensed matter experiments oriented on detection of dark matter. However, more sensitive probes are still searched for. One of them is proposed in the current paper, i.e., examination of the properties of DC SQUID device built of two Josephson junctions and influenced by hidden sector.

The Josephson effect [29] is the well known quantum phenomenon observed in devices consisting of two superconductors separated by thin layer of normal metal, insulator or a superconductor with much lower transition temperature. Such junctions, known as Josephson junctions, even without external bias sustain the direct current $I$ which magnitude is related to the phase difference $\phi = \phi_L - \phi_R$

$$I = I_{\text{max}} \sin \phi,$$

(1.1)

where $\phi_L$ and $\phi_R$ are the phases of the superconducting order parameter on the left and right hand side of the junction. This relation is at the heart of many devices able to detect tiny magnetic fields [30]. Recently the so-called second order Josephson effect has been proposed to detect axionic dark matter [31, 32].

The holographic model of Josephson junction was also paid attention to. In [33] a gravitational dual of S-N-S junction was constructed and the calculations on the gravity side reproduced the standard relation between the current across the junction and the phase difference of the considered condensate. In the probe limit, Maxwell field coupled to complex scalar one, was examined in the background of AdS-Schwarzschild black brane.

The holographic model of a Josephson junction array was considered in [34], where a model was proposed in the background of multi-supergravity theories on products of distinct asymptotically AdS spacetimes coupled by mixed boundary conditions. Among all it was found that the Cooper-pair condensates were described by a discretized Schrödinger-like equation. In a continuum limit the relation in question became a generalized Gross-Pitaevski equation, known from the long-wavelength description of superfluids.

A holographic model of S-I-S junction constructed by examining a complex scalar field coupled to Maxwell one, in the background of four-dimensional AdS soliton was examined in [35]. The dependence of the maximal current on the dimension of the condensate operator and the width of the junction were presented. On the other hand, a holographic configuration including a chiral time-reversal breaking superconductors in $(2+1)$-dimensions was discussed in [36], where the ansatz for $p_x + ip_y$ superconductors [37] was implemented. Such
kind of superconductor is believed to support topologically protected gapless Majorana-Weyl edge modes. On the other hand, the model with two coupled vector fields, was also implemented in a generalization of p-wave superconductivity, for the holographic model of ferromagnetic superconductivity [38].

The construction of a Josephson junction in non-relativistic case with a Lifshitz geometry as the dual gravity was discussed in [39], where the effect of the Lifshitz scaling was elaborated. Among all the standard sinusoidal relation between the current and phase difference was revealed for various values of scaling. The relation featuring exponential decreasing between condensate operator and the width of the weak link, as well as, the relation connected the critical current with the width were found.

Holographic s-wave and p-wave Josephson junctions with backreaction were studied in [40]. It turns out that the critical temperature of the considered junctions decreases with the increase of backreaction, while the tunneling current and condensation also decrease with the growth of backreaction. However, the relation between current and phase difference remains in the form of a sine-function. On the other hand, the problems of holographic models of hybrid and coexisting both types of junctions, i.e., s- and p-wave were analyzed in [41].

The studies of S-N-S junction in massive gravity theory unveil that the graviton mass parameter make it difficult for the normal metal-superconductor phase transition to occur [42]. Moreover the mass graviton parameter increase will cause the decrease of the maximal tunneling current.

In recent years there has been also an interest in building holographic model of superconducting quantum interference device (SQUID). In the model in question one considers Einstein-Maxwell gravity with complex scalar field in (3+1)-dimensional AdS Schwarzschild black brane background. One of the spatial dimensions is compactified into a circle and with the properly chosen profiles of the chemical potential constitute the main building blocks of the holographic SQUID [43, 44].

SQUID is known to be the device very sensitive to small changes of the magnetic field. It consists of two Josephson junctions embedded into the superconducting ring as shown in figure 1. There are two kinds of this device, i.e., the direct current and the radio-frequency SQUIDS. Each branch of the SQUID, in figure 1, contains one Josephson junction. In general the current phase relation reads

\[ I = I_{\text{max}} \sin \left( \Delta \phi + \frac{2eV}{h} t \right), \quad (1.2) \]

where \( V \) is the voltage drop across the junction, \( \Delta \phi \) the already introduced phase difference between two superconductors. Equation (1.2) shows that the current of the biased junction oscillates with frequency \( \frac{2eV}{h} \). In the following we shall consider only direct current (dc) SQUID with \( V = 0 \).

The presence of the the magnetic flux induces the screening currents in the superconductor or otherwise changes the phase relations in both branches. By the gauge invariance principle one finds that the flux \( \Phi \) piercing the SQUID loop modifies the relation between
Figure 1: The schematic view of the superconducting quantum interference device consisting of two Josephson junctions (dark-blue) embedded into the superconducting loop (yellow-bright). The current $I$ flowing across the system is split into $I_A$ in the upper and $I_B$ in the lower branch of the device. The presence of the magnetic flux $\Phi$ induces the interference in the system. For the holographic setup the compactified direction $\xi$ is also shown.

The phases of two junctions A and B and one gets [45]

$$\Phi = 2\pi\phi_0(\Delta\phi_A - \Delta\phi_B).$$

(1.3)

The total current $I$ entering the device from the left splits into $I_A$ and $I_B$. At the right hand side of the device both $I_A$ and $I_B$ fulfilling the relation (1.1) add to give again the current $I$. One thus gets (for $I_A = I_B$ and with the phase differences $\Delta\phi_A$ and $\Delta\phi_B$ in the respective branches)

$$I = I_A + I_B = 2I_{\text{max}} \sin\left(\frac{\Delta\phi_A + \Delta\phi_B}{2}\right) \cos\left(\frac{\Delta\phi_A - \Delta\phi_B}{2}\right).$$

(1.4)

As was shown in [46] the phase difference is related to the total flux $\phi_{\text{total}}$, which for the negligible inductance of the loop reduces to the external flux $\Phi$ as in the equation (1.3). With the symmetric distribution of the phase modifications

$$\Delta\phi_A = \Delta\phi + \frac{\pi\Phi}{\phi_0}$$

(1.5)

$$\Delta\phi_B = \Delta\phi - \frac{\pi\Phi}{\phi_0}$$

(1.6)

one gets

$$I = 2I_{\text{max}} \cos\left(\frac{\pi\Phi}{\phi_0}\right) \sin\left(\Delta\phi_B + \frac{\pi\Phi}{\phi_0}\right) = 2I_{\text{max}} \cos\left(\frac{\pi\Phi}{\phi_0}\right) \sin(\Delta\phi),$$

(1.7)

where $\phi_0 \equiv \hbar c/2e \approx 2.07 \cdot 10^{-7} Gs \cdot cm^2$ is the quantum flux. It is the small value of the quantum flux which enables measurements of very tiny changes of the magnetic fields [30].
From our point of view, the above relations can be interpreted in the slightly different manner. Suppose, that the phase difference, $\Delta \phi_A$ or $\Delta \phi_B$, in one of the branches of the considered device has changed as a result of external factors. In the model under inspection we understand this phenomenon as a dark matter particle appearing in one of the Josephson junctions being the part of the SQUID. This means that the current phase relation changes in the same way as the effective magnetic flux would appear. The large sensitivity of the device, rises the hope of detecting such transition of the dark matter particle, under the assumption of the validity of the coupling between hidden and visible sectors. In this paper we shall suppose that a single dark matter particle appears in one SQUID branch at a time, leaving aside the issue of possible fluctuations of the SQUID signal due to consecutive events. This problem will be studied in future publications.

Our aim in this paper is to model the Josephson junctions and the SQUID by using the gauge/gravity duality and the appropriate background of black brane or black soliton. We also consider the dark matter sector and find its signature in the performance of the studied devices. We use the dark matter model described by the auxiliary $U(1)$-gauge field coupled to the Maxwell one, being subject to the action

$$S_{dm+EM} = \int \sqrt{-g} \, d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\alpha}{4} F_{\mu\nu} B^{\mu\nu} \right),$$

(1.8)

where $F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$ stands for the ordinary Maxwell strength tensor, while the second $U(1)$-gauge field $B_{\mu\nu}$ is given by $B_{\mu\nu} = 2\nabla_{[\mu} B_{\nu]}$. $\alpha$ is a coupling constant between two gauge fields. The compatibility with the current observations authorizes its order to $10^{-3}$.

The justifications of such kind of models may be obtained from the top-down perspective [47], starting from the string/M-theory. It is very important from the AdS/CFT correspondence point of view, because string/M-theory constitutes a fully quantum description and this guarantees that any phenomena envisaged by the top-down reduction are physical.

The second term in the above action is connected with some hidden sector [47], while the interaction of the visible and hidden sectors are described by the so-called kinetic mixing term, for the first time introduced in [48], in order to describe the existence and subsequent integrating out of heavy bi-fundamental fields charged under the $U(1)$-gauge groups. The term in question is characteristic for theories having in addition to some visible gauge group an additional one, in the hidden sector. The hidden sector describes states in the low-energy effective theory, uncharged under the the Standard Model gauge symmetry groups. On the contrary they are charged under their own groups.

It is gravity which enables interaction between those two sectors [49, 50]. It turns out that the realistic embeddings of the Standard Model in $E8 \times E8$ string theory, as well as, in type I, IIA, or IIB open string theory with branes, require the existence of the hidden sectors for the consistency and supersymmetry breaking [51, 52]. On the other hand, in string phenomenology [51] the dimensionless kinetic mixing term parameter can be produced at an arbitrary high energy scale and it does not deteriorate from any kind of mass suppression from the messenger introducing it. This fact is of a great importance.
from the experimental point of view, due to the fact that its measurement can provide some interesting features of high energy physics beyond the range of the contemporary colliders.

Moreover the mixing term of two gauge sectors are typical for states for open string theories, where both $U(1)$-gauge groups are advocated by D-branes that are separated in extra dimensions. It happens in supersymmetric Type I, Type IIA, Type IIB models and results in the existence of massive open strings which stretch between two D-branes [53].

The organization of the paper is as follows. In the next section we pay attention to the holographic model of DC SQUID composed of two Josephson junctions in Einstein-Maxwell gravity with complex scalar field affected by the dark matter sector mimicked by the auxiliary $U(1)$-gauge field interacting with the ordinary Maxwell one. One considers the model of the junction, where a central part constitutes a normal metal (S-N-S). We also mentioned the case with an insulator (S-I-S). Section 3 is devoted to the numerical solutions of the adequate equations of motion, for the homogeneous case and the case of a dark matter beam. We propose an 'experiment' which helps to distinguish the presence of dark matter sector. In section 4 we concluded our investigations.

2 The holographic model and equations of motion

In this section we present the examined holographic model and the resulting equations of motion. We shall examine the holographic DC SQUID, composed of the two S-N-S Josephson junctions. The holographic model of the 2+1 dimensional system in question relies on the 3+1 dimensional gravitation background of AdS static Schwarzschild black brane spacetime.

The gravitational action in (3+1) dimensions is taken in the form
\[
S_g = \int \sqrt{-g} \, d^4x \left( R - 2\Lambda \right),
\]  
where $\Lambda = 6/L^2$ stands for the negative cosmological constant, while $L$ is the radius of the AdS spacetime. In what follows we set $L = 1$.

We shall examine the Abelian-Higgs sector coupled to the auxiliary $U(1)$-gauge field being the representative of the hidden sector coupled to the visible one by the kinetic mixing term, with a coupling $\alpha$
\[
S_{dm+EM+H} = \int \sqrt{-g} \, d^4x \left( - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - [\nabla_\mu \psi - iq A_\mu \psi]^\dagger [\nabla^\mu \psi - iq A^\mu \psi] - m^2 |\psi|^2 - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\alpha}{4} F_{\mu\nu} B^{\mu\nu} \right).
\]  
In many analysis of the hidden sector $\alpha$ is taken as a constant. In our considerations we shall elaborate the problem of the response of a superconducting junction/ SQUID to the appearance of dark matter particle. The nontrivial response of the SQUID is expected to appear only if one of its junctions is affected by a dark matter particle. Due to these circumstances we are forced to consider $\alpha$ as spacelike coordinate function.

As it was already mentioned, to mimic the passage of the dark matter particle via one of the junctions of the SQUID we assume that the coupling $\alpha$ depends on space-like...
coordinates. This is taken into account in the derivation of the underlying equations of motion. Later on we specify the particular dependence $\alpha(\xi)$ used to solve the model.

The equations of motion for the case when $\alpha = \alpha(x_m)$ are provided by

$$\left(\nabla_\mu - i A_\mu\right)\left(\nabla^\mu - i A^\mu\right)\psi - m^2 \psi = 0,$$

$$\nabla_\mu F^{\mu\nu} + \frac{1}{2} \nabla_\mu \left(\alpha B^{\mu\nu}\right) - i \left[\psi^\dagger \left(\nabla^\nu - i A^\nu\right)\psi - \psi \left(\nabla^\nu + i A^\nu\right)\psi^\dagger\right] = 0,$$

$$\nabla_\mu B^{\mu\nu} + \frac{1}{2} \nabla_\mu \left(\alpha F^{\mu\nu}\right) = 0.$$  \hspace{1cm} (2.3) \hspace{1cm} (2.4) \hspace{1cm} (2.5)

Consequently, using the last two equations one has that

$$\tilde{\alpha} \nabla_\mu F^{\mu\nu} + \frac{1}{2} \nabla_\mu \left(\alpha B^{\mu\nu} - \frac{1}{2} \alpha F^{\mu\nu}\right) - i \left[\psi^\dagger \left(\nabla^\nu - i A^\nu\right)\psi - \psi \left(\nabla^\nu + i A^\nu\right)\psi^\dagger\right] = 0,$$  \hspace{1cm} (2.6)

where we set $\tilde{\alpha} = 1 - \alpha^2/4$.  

As a gravitational background we shall consider the four-dimensional AdS-Schwarzschild black brane line element, given by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 \left(d\xi^2 + dx^2\right),$$  \hspace{1cm} (2.7)

where $f(r) = r^2 - r_0^3/r$ and $r_0$ is the horizon radius of the black brane, while the direction $\xi$ is compactified with the periodicity $-\pi R \leq \xi \leq \pi R$, where $R$ is the radius of the $\xi$-loop.

For further analysis it is convenient to assume that $r_0 = 1$ and $\pi R = 10$. According to the claim in [33], the coherence length for the considered model is estimated to be about $\sim 1.20$. We argue that the length of the SQUID loop should be much larger than the coherence length, in order to decrease the influence of the proximity effect and consequently the mixing of the phases. Fulfilling this condition, we obtain a homogeneous superconducting phase at the both ends of the $\xi$-direction. It results in a well defined phase difference for the junction under inspection. On the other hand, one wants to establish our weak link of the junction to be narrow enough, that its critical current receives a sufficiently high value. This is important from the point of view of the numerics, i.e., the precision of the solutions. Having all the above arguments in mind, we establish $R = 10/\pi$ as a reasonable choice.

In what follows one chooses the following components of the fields in the underlying theory:

$$\psi = |\psi| \ e^{i\phi}, \quad A_\mu = (A_t, \ A_r, \ A_\xi, \ 0), \quad B_\mu = (B_t, \ B_r, \ B_\xi, \ 0),$$  \hspace{1cm} (2.8)

where we assume that all the field components and the phase are real functions dependent on $(r, \ \xi)$-coordinates. Moreover, one defines the gauge invariant quantities $M_\mu = A_\mu - \partial_\mu \phi$. 

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Consequently, the equations of motion yield

$$\partial^2_r |\psi| + \frac{1}{r^2 f} \partial^2_\xi |\psi| + \left(\frac{2}{r} + \frac{\partial_r f}{f}\right) \partial_r |\psi| + \left[\frac{1}{f^2} M^2_t - \frac{1}{r^2 f} M_\xi - M^2_r\right] (2.9)$$

$$- \frac{m^2}{f} |\psi| = 0,$n

$$\partial_r M_r + \frac{2}{r^2 f} \partial_\xi M_\xi + \frac{2}{|\psi|} \left(M_r \partial_r |\psi| + \frac{M_\xi}{r^2 f} \partial_\xi |\psi|\right) + \left(\frac{2}{r} + \frac{\partial_r f}{f}\right) M_r = 0. (2.10)$$

$$\partial [\partial^2_t M_t - \frac{2}{r} \partial_\xi M_\xi] - 2 r^2 M_r |\psi|^2 + \frac{1}{2} \partial_\xi \left(\partial_\xi B_r - \partial_\xi B_\xi\right) (2.11)$$

$$\partial [\partial^2_M - \partial_\xi \partial_\xi M] - 2 r^2 M_r |\psi|^2 + \frac{1}{2} \partial_\xi \left(\partial_\xi B_r - \partial_\xi B_\xi\right) = 0,$$n

$$\partial [\partial^2_t M_t - \partial_\xi \partial_\xi M] + \frac{f}{2} \left(\partial_\xi M_\xi - \partial_\xi M_r\right) - \frac{2}{f} M_\xi |\psi|^2 (2.12)$$

$$+ \frac{1}{2} \partial_\xi \left[\frac{f}{2} \left(\partial_\xi B_r - \partial_\xi B_\xi\right) - \frac{\alpha f}{2 r^2} \left(\partial_\xi M_\xi - \partial_\xi M_r\right)\right] = 0. (2.13)$$

The components of the relation (2.5) imply

$$\frac{2}{r} \partial_r B_t + \partial^2_r B_t + \frac{1}{r^2 f} \partial^2_\xi B_t + \frac{1}{2} \partial_\xi \alpha \partial_\xi M_t + \frac{1}{2} \partial_\xi \alpha \frac{1}{r^2 f} \partial_\xi M_t (2.14)$$

$$+ \frac{\alpha}{2} \left(\frac{2}{r} \partial_r M_t + \partial^2_r M_t + \frac{1}{r^2 f} \partial^2_\xi M_t\right) = 0,$$n

$$\partial^2_r B_\xi - \partial_\xi \partial_\xi B_\xi + \frac{\partial_r f}{f} \left(\partial_\xi B_\xi - \partial_\xi B_r\right) + \frac{1}{2} \partial_\xi \alpha \left(\partial_\xi M_\xi - \partial_\xi M_r\right) (2.15)$$

$$+ \frac{\alpha}{2} \left[\partial^2_r M_\xi - \partial_\xi \partial_\xi M_\xi + \frac{\partial_r f}{f} \left(\partial_\xi M_\xi - \partial_\xi M_r\right)\right] = 0,$$n

$$\partial_\xi \left(\partial_\xi B_r - \partial_\xi B_\xi\right) + \frac{1}{2} \partial_\xi \alpha \left(\partial_\xi M_r - \partial_\xi M_\xi\right) + \frac{\alpha}{2} \partial_\xi \left(\partial_\xi M_r - \partial_\xi M_\xi\right) = 0. (2.16)$$

To proceed further, we ought to choose the form of $\alpha(x_i)$. One of the possible representations of it is a Gaussian function form

$$\alpha(\xi) = \alpha_0 e^{-|\xi - \xi_0|^2 / \lambda^2}, (2.17)$$

where $\alpha_0$ is the peak value of the coupling and $\lambda$ its decay length (half of the Gaussian width). In principle $\alpha$ could depend on the $r$ coordinate, but we shall neglect this assuming the above is obtained for $r \to \infty$. The interpretation of the Gaussian as a wave packet representing the dark particle allows to think about $\lambda$ as the de Broglie wave length of the particle. The latter being inversely proportional to its mass. This is an important aspect of the calculations as the ratio of $\lambda$ to the SQUID characteristic length-scale $\pi R$, factor and
weak link shape of the Josephson junctions are affecting the SQUID’s response and thus impact the possibility of detecting dark matter particle.

In principle, there are two possible attitudes for modeling the local presence of the dark sector fields in this theory. We can either pick $\alpha$ to be constant and impose specific boundary conditions for dark matter fields or as it was mentioned before, promote the coupling with a spatial dependence and keep the fields in the homogeneous scenario. In this approach we selected the latter one, so we can choose the $x$-independent and source free solution of $B_\mu$ fields, which simply yields

$$B_\alpha \, dx^\alpha = \mu_D \left(1 - \frac{r_+}{r} \right) dt. \quad (2.18)$$

By putting this assumption into our system of differential equations we receive a significant simplification, with only four functions to be obtained numerically:

$$\partial_r^2 |\psi| + \frac{1}{r^2 f} \partial_\xi^2 |\psi| + \left(\frac{2}{r} + \frac{\partial_r f}{f}\right) \partial_r |\psi| + \left[\frac{1}{f^2} M_t^2 - \frac{1}{r^2 f^2} M_\xi^2 - M_r^2 \left(1 - \frac{r}{r_0} \right)\right] |\psi|^2 = 0, \quad (2.19)$$

$$\tilde{\alpha} \left[\partial_\xi^2 M_t + \frac{2}{r} \partial_r M_t + \frac{1}{r^2 f^2} \partial_\xi^2 M_t \right] - \frac{2}{f} M_t |\psi|^2 = 0, \quad (2.20)$$

$$\tilde{\alpha} \left[\partial_\xi^2 M_\xi - \partial_\xi \partial_r M_\xi \right] - 2 r^2 M_r |\psi|^2 - \frac{1}{2} \partial_\xi \alpha \frac{1}{r^2 f^2} \partial_\xi M_t = 0, \quad (2.21)$$

$$\tilde{\alpha} \left[\partial_\xi^2 M_\xi - \partial_\xi \partial_\xi M_\xi + \frac{\partial_r f}{f} \left(\partial_\xi M_\xi - \partial_\xi M_\xi \right) \right] - \frac{2}{f^2} M_\xi |\psi|^2 = 0. \quad (2.22)$$

The above forms of the equations envisage that comparing to the case when $\alpha = 0$, the modification is significant. Some terms are multiplied by $\tilde{\alpha}$ and moreover we have additional terms with $\alpha \partial_\xi \alpha$ factor. Numerical solutions of these equations will be presented in the subsequent part of the paper.

As we have remarked at the beginning of the section, the presented model describes SQUID with S-N-S type Josephson junctions. Nevertheless, for the completeness of the results, we shall mention the modeling of S-I-S Josephson junction. In the holographic model of the insulator the AdS soliton line element will play the crucial role. Namely, performing the double Wick rotation on the metric (2.7), we arrive at the AdS soliton line element [54]. It is provided by

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{f(r)} + r^2 dx^2 + f(r) d\chi^2, \quad (2.23)$$

with the same function $f(r)$ as before. However, the coordinate $\chi$ has the period $\beta = 4\pi l/3r_0$, in order to avoid the conical singularity at $r = r_0$.

As in the previous case we assume that the following components of the gauge field are non-zero $A_\mu = (A_t, A_r, A_\xi, 0)$, and that all the field components and the phase $\phi$ are real functions dependent on $(r, x)$-coordinates. The equations of motion are very similar
to the ones obtained in the previous case. In fact only the factor in front of $\frac{\partial}{\partial r} M_t$ instead of $\frac{1}{r}$ reads $\frac{\partial}{\partial r}$. The results will differ only quantitatively, therefore they will be not analyzed numerically.

Having discussed the model setup we shall describe appropriate boundary conditions for our model. Starting with AdS boundary where $r \to \infty$, the asymptotic of the fields in question are provided by

$$|\psi| = \frac{|\psi^{(1)}(\xi)|}{r^{\Delta_{(1)}}} + \frac{|\psi^{(2)}(\xi)|}{r^{\Delta_{(2)}}} + O(r^{-3}),$$

$$M_t = \mu(\xi) - \frac{\rho(\xi)}{r} + O(r^{-2}),$$

$$M_r = O(r^{-3}),$$

$$M_\xi = \nu(\xi) + \frac{J(\xi)}{r} + O(r^{-2}),$$

where

$$\Delta_{(1)} = \frac{1}{2} (3 - \sqrt{9 + 4m^2}), \quad \Delta_{(2)} = \frac{1}{2} (3 + \sqrt{9 + 4m^2}).$$

In the boundary field theory the quantities $\mu(\xi)$, $\rho(\xi)$, $\nu(\xi)$, and $J(\xi)$ are connected with the chemical potential, charge density, superfluid velocity and current, respectively. On the other hand, $|\psi^{(1)}(\xi)|$ and $|\psi^{(2)}(\xi)|$ may be considered as the source and the vacuum expectation value of the dual operator of the scalar field $|\psi(\xi)| = \langle O \rangle$. In the numerical calculation we set $|\psi^{(1)}(\xi)| = 0$ (as turning off the source), because one has that the U(1)-gauge symmetry be broken. At the black brane event horizon we demand that $g^{tt} M_t$ ought to be regular, but because of the fact that $g^{tt}$ is divergent on the horizon, $M_t$ should vanish at $r = 1$. The remaining fields should be finite at the event horizon and the simplified equation of motion place the boundary conditions therein. In our numerical calculations for simplicity we set $m^2 = -2$ which is above Breitenlohner-Freedman limit $m^2 \geq -\frac{9}{4}$, therefore $\Delta_{(1)} = 1$ and $\Delta_{(2)} = 2$.

3 Numerical results

This section will be devoted to the results of our numerical solutions of the equations of motion for the Josephson junction and DC SQUID device. At first we pay attention to the case of homogeneous holographic superconductor influenced by the dark matter sector. Then we proceed to analyze the model of SQUID with a beam of dark matter, i.e., we allow for $\xi$ dependence of $\alpha$ in the kinetic mixing term.

3.1 Homogeneous superconductor - warm up

To commence with, let us consider the properties of homogeneous holographic superconductor influenced by the dark matter sector, such as critical temperature, critical chemical potential and value of the condensation. For this calculation we straight out the $\xi$ dimension renaming it to $x$. Then by neglecting the spatial dependence of $x$-coordinate in the
equations of motion, we receive the simplified system of differential equations

\[
\begin{align*}
\partial_r^2 |\psi| + \left(\frac{2}{r} + \frac{\partial_r f}{f}\right) \partial_r |\psi| + \left[\frac{1}{f^2} M_t^2 - \frac{1}{r^2 f} M_x - \frac{m^2}{f}\right] |\psi| &= 0, \quad (3.1) \\
\partial_r^2 M_t + \frac{2}{r} \partial_r M_t - \frac{2}{\alpha f} M_t |\psi|^2 &= 0, \quad (3.2) \\
\partial_r^2 M_x + \frac{\partial_r f}{f} \partial_r M_x - \frac{2 M_x}{\alpha f} |\psi|^2 &= 0. \quad (3.3)
\end{align*}
\]

where all the above three functions, i.e., $|\psi|$, $M_t$, $M_x$, depend only on $r$.

These equations establish the boundary conditions for a homogeneous region of superconductor in $x$-dependent approach, but for the analysis of phase transitions we set the current to zero, which implies that $M_x = 0$. Now we can move towards numerical solution of our equations. In order to proceed further, we perform coordinate transformation $z = 1 - \frac{1}{r}$ in order to work on a $(0, 1)$ grid, where $z = 0$ is the black hole horizon and $z = 1$ is the holographic boundary. We solve the boundary value problem using the standard relaxation method for different values of temperature and chemical potential, to investigate the superconductor-normal phase transition. In figures 2 and 3 we plot the condensation operator $\langle O \rangle$ as a function of temperature and the chemical potential for the different values of dark matter $\alpha$-coupling constants.

One concludes that the bigger $T/T_c$ is the smaller value of $\langle O \rangle / T_c$ we obtain. In the case when we fix the value of $T/T_c$, then the bigger $\alpha$-coupling constant is taken into account the smaller $\langle O \rangle / T_c$ we get. Just, the condensation diminishes when dark matter coupling constant grows. On the other hand, the bigger $\mu$ one considers, the larger value of $\langle O \rangle$ we receive, for the fixed value of $\alpha$-coupling constant. In the case when $\alpha$ grows, we deduce that, the larger $\alpha$ is, the smaller value of the condensation operator one perceives.

In the both types of the analyzed phase transitions, the presence of dark matter does not shift the phase transition point (critical temperature, critical chemical potential), yet

![Figure 2](image-url)
it changes the value of condensation.

Figure 4 depicts the relationship between the condensation operator and dark matter coupling constant, for the fixed temperature and chemical potential. The differences are not too big, but they are visible and the lines are distinguishable. However there might be a problem with measuring such a deviation. Our idea to overcome this obstacle is presented in the next section – by using superconducting devices.

The calculations of this section served as a test of our numerical procedure. The procedure passed the test as the closer inspection of the equations of motion (3.1) - (3.3) shows that the parameter $\tilde{\alpha}$ can be eliminated by rescaling the $\tilde{\psi}$ field, as $\tilde{\psi} = \psi \sqrt{\tilde{\alpha}}$, with $\tilde{\psi}$ being independent on $\alpha$. This rescaling leads to the similar rescaling of the condensation parameter $\langle O \rangle$, leading to the solution with $\alpha$ independent value $\langle O \rangle_0$. The expected behavior $\langle O \rangle = \sqrt{\tilde{\alpha}} \langle O \rangle_0$ is clearly visible in figure 4 as are the concomitant changes of other parameters depending on $\langle O \rangle$.

3.2 Analysis of the holographic SQUID

To imitate the two insulating regions embedded into the superconducting ring we take the chemical potential as given in [43]

$$
\mu(\xi) = h - \sum_{i=1, 2} \frac{d_i}{d_i} \left[ \tanh \left( \frac{k_i (\xi - p_i + w_i)}{\pi} \right) - \tanh \left( \frac{k_i (\xi - p_i - w_i)}{\pi} \right) \right], \quad (3.4)
$$

where $i = 1, 2$ stand for two junctions in the SQUID ring, and $h$, $d_i$, $k_i$, $p_i$, and $w_i$ are related to the highest value of the chemical potential (inside the superconductor), depth, slope, position, and width of the junction $i$, respectively. The idea behind this choice is the following: from the previous studies we know that with and/or without dark matter it is
the value of the chemical potential which tunes the transition between insulator and metal and the temperature decrease induces metal - superconductor transition. To construct a junction it is thus enough to make the chemical potential in the normal part of the junction lower than its critical value for superconducting transition, while leaving it above this value in the superconducting parts of the junction. This is just embodied in the equation (3.4) and shown in figure 5, for the weak links located at $p_1 = 5$ and $p_2 = -5$ with $h = 5$ and identical other parameters $d_1 = d_2 = 0.7$ widths $w_1 = w_2 = 1.6$ and slopes $k_1 = k_2 = 7$.

In the discussion of the holographic SQUID we change $\Delta \phi_A$, $\Delta \phi_B$ for $\gamma_L$ and $\gamma_R$, defined in what follows. In our model we assume that the dark matter is present only in one of the junctions of SQUID. Such approach gives us a possibility to calculate the difference of the phases between junctions caused by the dark matter coupling. In the trivial case (without the presence of dark matter sector) the effective magnetic flux defined as $\Phi = \gamma_L - \gamma_R$ vanishes, therefore the critical current

$$J_c = \sqrt{J_L^2 + J_R^2 + 2J_LJ_R \cos(\Phi)},$$

becomes constant and equals to $J_L + J_R$. On the contrary, when dark sector particle is present in one of the junctions, we receive a "yes or no" type criteria for the dark matter occurrence.

In adverse to the previously obtained results [43], we do not impose the continuity conditions in the node points $\xi = 0$ and $\xi = \pm 10$. Moreover we argue that these points can be singular due to the presence of the additional source terms. Especially the supercurrent might not be well behaved when it inflows and outflows from our system. If we wish to
Figure 5: The schematic profile of the chemical potential in the superconducting quantum interference device consisting of two Josephson junctions (lower values of $\mu$) embedded into the superconducting loop with $\mu = 5$.

model the SQUID with two currents flowing parallel, we have to take $J \sim sgn(\xi)$, because current flows into the system at $\xi = 0$, then in one branch it flows to $\xi = -10$ and in the another to $\xi = 10$, against and with the $\xi$ axis, respectively.

As far as the boundary value problem is concerned, for the AdS/CFT boundary one has the asymptotic expansions, for the black hole horizon we require that $M_t = 0$, while the conditions for the remaining functions are given by the adequate equation of motion. On the other hand, for the $\xi$ boundary, in both cases, we impose that the functions $|\psi|$, $M_t$ and $M_x$ have to be even and $M_r$ is an odd function with respect to the central point of the junction.

In our numerical computations we implement the pseudo-spectral method with Chebyshev points, which constitutes an extremely efficient tool for smooth functions we deal with in the problem. For convenience we compactify the $r$-coordinate using following transformation $z = 1 - 1/r$, after which $z = 0$ corresponds to the black hole horizon and $z = 1$ to the AdS boundary. Also we squeeze $\xi$ dimension into $(-1, 1)$ range which is the domain of the Chebyshev polynomials. Generally pseudo-spectral methods require only few points in the spatial discretization to achieve satisfying results, therefore we use the grid with 37 points along $\xi$ direction and 20 points along $z$ axis. By expanding the functions into Chebyshev series we translate a differential problem stated by equations of motion into an algebraic one. Then we deal with a system of nonlinear algebraic equations by standard Newton-Raphson method. In the explained way we solve numerically our set of partial differential equations (2.19)-(2.22) for different values of $J_0$, varying from $-0.06$ to $0.06$. In figure 6 we plot the exemplary solutions of the underlying equations of motion for $J = 0.03$ and $\alpha_0 = 0.2$. We can see the imposed shape of the chemical potential at the boundary $z = 1$ of the $M_t$ function. The remaining functions react accordingly to the presence of the
normal phase. The shapes of the solution are similar to the ones obtained in [33, 35, 43, 44]. However it contains the traces of the influence of $\alpha$ coupling, but they are too small to be visible on such 3D plots. Nevertheless they do have the influence on some of the properties, which will be elaborated below.

After solving our differential equations we obtain the phase differences for the left and right Josephson junctions separately, using the following formula:

$$\gamma_L = -\int_{-10}^{0} (\nu(\xi) - \nu(0)) d\xi$$ \hspace{1cm} (3.6)

$$\gamma_R = -\int_{0}^{10} (\nu(\xi) - \nu(0)) d\xi$$ \hspace{1cm} (3.7)

Next, for each Josephson junction we fit in a sine relation between $J_{L/R}$ and $\gamma_{L/R}$, to obtain the value of the maximum current.

Having obtained the values of the critical currents for each junction we can proceed to calculation of the dark matter particle induced effective magnetic flux $\Phi$, as defined previously. We wish to see how the critical current of SQUID, defined by the relation (3.5), changes with the presence of dark matter in the system, hence one elaborates the case when

Figure 6: Plots of numerical solutions of equations of motion (2.19)-(2.22) for one of SQUID’s Josephson junction with parameters $J = 0.03$ and $\alpha_0 = 0.2$. We can see the imposed chemical potential boundary condition for $M_t$ at the AdS boundary and the reaction of the remaining functions to it.
Figure 7: Relative change of critical current ratio from the relation (3.8) versus the effective magnetic flux, which appears to be non-zero in the presence of dark matter. The red curve represents solution when $\alpha_0 = 0.5$, while in the zoomed area for the blue one $\alpha_0 = 0.05$. In the case with $\alpha = 0$ such a dependence would not occur, resulting a trivial SQUID with two identical Josephson junctions and $\delta J \equiv 0$.

$\alpha \neq 0$. To proceed further let us define the critical current ratio, given by

$$\delta J = \frac{J_c(\Phi) - J_c(0)}{J_c(0)}.$$  \hspace{1cm} (3.8)

In figure 7 the critical current ratio as a function of the magnetic flux, for different values of $\alpha_0$, is depicted. In our model such behavior is only possible when dark matter particle appears in one of the junctions. Otherwise, if $J_c$ is independent of $\Phi$ the interference does not occur. This gives us the possibility to establish the criterion for detection of the discussed coupling which is correlated with the dark sector.

While the residuals of numerical solution of our equations (2.19)-(2.22) and the constraint relation (2.10) are relaxed to the values less than $10^{-12}$, the relation from the figure 7 carries a burden of inaccuracy of the least squares method. Namely the sine function is not a perfectly fitting one and the best result we can obtain for the norm of residuals is to the order of $10^{-6}$. It results that the uncertainty of the critical current is $\sim 10^{-4}$. Having this in mind, for little and yet realistic values of coupling our computed values of $J(\Phi)$ are below the level of accuracy, so we cannot state its direct value. Although when working on differences, like in our ratio given by the relation (3.8), these uncertainties are subtracted, so using this quantity seems sensible.
Figure 8: The critical current of dark matter influenced Josephson junction as a function of gaussian broadening $\lambda$ from the relation (2.17). It is clearly seen that there is a maximal sensitivity for some value of $\lambda$. It means that SQUID might be specially designed for a particle desired parameters. In this plot we used $\alpha_0 = 0.5$.

We also investigated the influence of increasing the Gaussian packet decay length $\lambda$ on the current-phase relation of one of the SQUID’s Josephson junctions. By implementing to our code various values of $\lambda$ (from 1 to 5), we have found out an interesting behavior which is depicted in figure 8. It envisages the critical Josephson current as a function of the wavelength of the decay length $\lambda$. This plot might be interpreted as the sensitivity of the SQUID on dark matter sector and it is a very interesting result from the experimental point of view. Namely, by adjusting the size of the normal/insulating part of the Josephson junctions one can tune up the system to react most sensitively on the desired mass (wavelength) of an investigated particle. The obtained value of the optimal $\lambda \approx 1.5$ is correlated, as it is expected to be, with the width $w$ of the normal region of the junction affected by the hidden sector particle. Having a possible range of masses of the searched object this property gives us a possibility to prepare an adequate detector for it. In the present approach we are assuming that the dark matter particle passes the center of the junction. In real experiment one expects the dark matter particle to move slightly off the center. In such situation the sensitivity can be roughly read of from the curve shown in figure 8, as the decrease/increase of $\lambda$ can mimic the distance of the wave packet from the center of the junction.
4 Discussion and conclusion

Using the holographic approach we have studied the possibility to detect dark matter particles by means of the SQUID. To this end we have assumed gravity background consisting of the four-dimensional AdS Schwarzschild brane. The holographic modeling of the device is achieved by use of the special form of the chemical potential mimicking the two insulating/normal regions embedded into superconducting ring. Moreover one assumes that dark matter sector is present only in one of the Josephson junctions of the SQUID under inspection.

This supposition enables us to calculate the difference of the phases between the junctions, caused by dark matter $\alpha$-coupling constant. We solve numerically equations of motion and receive values of $M_i$ and thus the phase, for the specific choice of the current $J$ and $\alpha$-coupling. This procedure leads to the critical current and the phase for each of the examined junctions and allows establishing the current phase relation which has a typical sinusoidal character.

The passage of the dark matter particle by one of the SQUID junctions modifies the phase difference in that junction and leads to the modification of the critical current. This shows up as the appearance of the effective, dark matter particle induced effective flux $\Phi$. The difference of the critical currents of the SQUID measured for $\alpha \neq 0$ and $\alpha = 0$, signals the dark matter particle. This gives us the criterion for the observation of the dark sector particles.

We have assumed that the presence in the junction of the dark matter particle can be described by the Gaussian type of dependence of the coupling $\alpha$ on the $\xi$ coordinate. It turns out that the increase of the Gaussian packet decay length $\lambda$ has an effect on the current-phase relation of one of the SQUID’s Josephson junctions. Moreover, the critical Josephson current is a function of the decay length $\lambda$. This fact supplies us the tool for changing the sensitivity of the SQUID in the dependence on the expected properties dark matter sector particles. It is a very interesting result from the experimental point of view, because by adjusting the size of the Josephson junctions one can tune up the system to react at the desired mass (wavelength) of the investigated hidden sector particle.

To conclude, we remark that the presented results have been obtained for the s-wave Josephson junction and SQUID composed of such junctions. They envisage some features which can be utilized in future experiments aimed at detecting dark matter sector particles. Of course, we are aware that examination of more complicated models like p-wave or $p_x + ip_y$ might lead to even better detection methods. We shall investigate these problems elsewhere.

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