Josephson effect and spin-triplet pairing correlations in SF$_1$F$_2$S junctions

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We study theoretically the Josephson effect and pairing correlations in planar SF$_1$F$_2$S junctions that consist of conventional superconductors (S) connected by two metallic monodomain ferromagnets (F$_1$ and F$_2$) with transparent interfaces. We obtain both spin-singlet and -triplet pair amplitudes and the Josephson current-phase relations for arbitrary orientation of the magnetizations using the self-consistent solutions of Eilenberger equations in the clean limit and for a moderate disorder in ferromagnets. We find that the long-range spin-triplet correlations cannot prevail in symmetric junctions with equal ferromagnetic layers. Surprisingly, the long-range spin-triplet correlations give the dominant second harmonic in the Josephson current-phase relation of highly asymmetric SF$_1$F$_2$S junctions. The effect is robust against moderate disorder and variations in the layers thickness and exchange energy of ferromagnets.

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I. INTRODUCTION

Spin-triplet superconducting correlations induced in heterostructures comprised of superconductors with the usual spin-singlet pairing and inhomogeneous ferromagnets have attracted considerable attention recently. Triplet pairing which is odd in frequency was envisaged a long ago in an attempt to describe the A phase of superfluid $^3$He. Even though it was found that the pairing in superfluid $^3$He is odd in space (p-wave) rather than in time, it is predicted that even in space (s-wave) and odd in time (odd in frequency) pairing does occur in certain superconductor (S) – ferromagnet (F) structures with inhomogeneous magnetization. As a result, superconducting correlations can have a long-range propagation from SF interfaces, with penetration lengths up to $1\mu$m and a nonvanishing Josephson supercurrent through very strong ferromagnets.

The first evidence of long-range S–F proximity effect came from the experiments on long wires made of Ho conical ferromagnet and a fully-spin-polarized CrO$_2$ halfmetallic ferromagnet in which Josephson supercurrent was measured. These experimental findings were subsequently verified using different substrates and superconducting contacts. Very recently, the effect has also been observed in single-crystal ferromagnetic Co nanowires, a Heusler alloy Cu$_2$MnAl and synthetic antiferromagnets with no net magnetization where thin PdNi or Ho layers are combined with Co layers.

In SFS Josephson junctions with homogeneous or spiral magnetization, the projection of the total spin of a pair to the direction of magnetization is conserved and only spin-singlet $f_s$ and triplet $f_{t0}$ correlations with zero spin projection occur. These correlations penetrate into the ferromagnet over a short distance determined by the exchange energy. For inhomogeneous magnetization, odd-frequency triplet correlations $f_{t1}$ with nonzero ($\pm 1$) total spin projection are present as well. These correlations, not suppressed by the exchange interaction, are long-ranged and have a dramatic impact on transport properties and the Josephson effect.

Dominant influence of long-range triplet correlations on the Josephson current can be realized in SFS junctions with magnetically active interfaces, narrow domain walls between S and thick F layers with misaligned magnetizations, or superconductors with spin orbit interaction. In general, fully developed triplet proximity effect can be realized only after inserting a singlet-to-triplet “converter,” a thin (weak) ferromagnetic layer sandwiched between a superconductor and a thick (strong) ferromagnet, acting as a “filter” which suppresses the short-range correlations.

The simplest superconductor-ferromagnet heterostructures with inhomogeneous magnetization are F$_1$SF$_2$ and SF$_1$F$_2$S junctions with monodomain ferromagnetic layers having noncollinear in-plane magnetizations. These structures have been studied using Bogoliubov-de Gennes equation and within the quasiclassical approximation in diffusive and clean limits using Usadel and Elienberger equations, respectively. It has been expected that the critical supercurrent in SF$_1$F$_2$S junctions has a nonmonotonic dependence on angle between magnetizations due to the long-ranged spin-triplet correlations. However, in symmetric junctions a monotonic dependence has been found both in the clean and dirty limits, except for nonmonotonicity caused by $0 - \pi$ transitions.

Apparently, the long-range Josephson effect is not feasible in the junctions with only two F layers, except in highly asymmetric SF$_1$F$_2$S junctions at low temperatures, as we will show here. In this case, the long-range spin-triplet effect manifests itself as a second harmonic ($I_2 \gg I_1$) in the spectral decomposition of the Josephson current-phase relation, $I(\phi) = I_1 \sin(\phi) + I_2 \sin(2\phi) + \cdots$. The ground state in Josephson junctions with ferromagnet can be 0 or $\pi$ state. The energy of the junction is proportional to $\int_0^\pi I(\phi')d\phi'$, hence a second harmonic leads to degenerate ground states at $\phi = 0$ and $\phi = \pi$. Small contribution of the first harmonic lifts the degeneracy which results in coexistence of stable and metastable 0 and $\pi$ states.

In this article, we study the Josephson effect and influ-
ence of odd-frequency spin-triplet superconducting correlations in clean and moderately disordered SF$_1$F$_2$S junctions with transparent interfaces. Magnetic interlayer is composed of two monodomain ferromagnets with arbitrary orientation of in-plane magnetizations. We calculate pair amplitudes and the Josephson current from the self-consistent solutions of the Eilenberger equations. Pair amplitudes $f_s$ and $f_{20}$ are short-ranged, while $f_{11}$ is long-ranged.

We show that the influence of misalignment of magnetizations on the Josephson current in symmetric SFFS junctions both in the ballistic and diffusive regimes is given in Sec. IV.

II. MODEL AND FORMALISM

We consider a simple model of an SF$_1$F$_2$S heterojunction consisting of two conventional (s-wave, spin-singlet pairing) superconductors (S) and two uniform monodomain ferromagnetic layers (F$_1$ and F$_2$) of thickness $d_1$ and $d_2$, with angle $\alpha = \alpha_2 - \alpha_1$ between their in-plane magnetizations (see Fig. 1). Interfaces between layers are fully transparent and magnetically inactive.

Superconductivity is described in the framework of the Eilenberger quasiclassical theory. Fermagnetism is modeled by the Stoner model, using an exchange energy shift $2\hbar$ between the spin subbands. Disorder is characterized by the electron mean free path $l = v_F\tau$, where $\tau$ is the average time between scattering on impurities, and $v_F$ is the Fermi velocity assumed to be the same everywhere.

Both the clean and moderately diffusive ferromagnetic layers are considered. In the clean limit, the mean free path $l$ is larger than the two characteristic lengths: the ferromagnetic exchange length $\xi_F = \hbar v_F/\alpha$, and the superconducting coherence length $\xi_S = \hbar v_F/\pi\Delta_0$, where $\Delta_0$ is the bulk superconducting pair potential. For moderate disorder $\xi_F < l < \xi_S$.

In this model, the Eilenberger Green functions $g_{\sigma\sigma'}(x, v_x, \omega_n)$, $g_{\sigma\sigma'}^1(x, v_x, \omega_n)$, $f_{\sigma\sigma'}(x, v_x, \omega_n)$, and $f_{\sigma\sigma'}^1(x, v_x, \omega_n)$ depend on the Cooper pair center-of-mass coordinate $x$ along the junction axis, angle $\theta$ of the quasiclassical trajectories with respect to the $x$ axis, projection $v_x = v_F\cos\theta$ of the Fermi velocity vector, and on the Matsubara frequencies $\omega_n = \pi k_B T(2n + 1)$, $n = 0, \pm 1, \ldots$. Spin indices are $\sigma = \uparrow, \downarrow$.

The Eilenberger equation in particle-hole $\otimes$ spin space can be written in the compact form

$$\hbar v_x \partial_x \hat{g} + \left[ \omega_n \hat{\tau}_3 \otimes \hat{1} - i \hat{V} - \Delta + \hbar \langle \hat{g} \rangle/2\tau, \hat{g} \right] = 0, \quad (1)$$

with normalization condition $\langle \hat{g} \rangle^2 = \hat{1}$. We indicate by $\cdots$ and $\cdot \cdot \cdot$ $2 \times 2$ and $4 \times 4$ matrices, respectively. The brackets $\langle \cdot \cdot \cdots \rangle$ denote angular averaging over the Fermi surface (integration over $\theta$), and $[\cdot, \cdot]$ denotes a commutator. The quasiclassical Green functions

$$\hat{g} = \begin{bmatrix} g_{\uparrow\uparrow} & g_{\uparrow\downarrow} & f_{\uparrow\uparrow} & f_{\uparrow\downarrow} \\ g_{\downarrow\uparrow} & g_{\downarrow\downarrow} & f_{\downarrow\uparrow} & f_{\downarrow\downarrow} \\ -f_{\downarrow\uparrow}^\dagger & -f_{\uparrow\downarrow}^\dagger & -g_{\uparrow\uparrow}^\dagger & -g_{\uparrow\downarrow}^\dagger \\ -f_{\downarrow\uparrow}^\dagger & -f_{\uparrow\downarrow}^\dagger & -g_{\downarrow\uparrow}^\dagger & -g_{\downarrow\downarrow}^\dagger \end{bmatrix} \quad (2)$$

are related to the corresponding Gor’kav-Nambu Green functions $\hat{G} = -i(T(\Psi \Psi^\dagger))$ integrated over energy,

$$\hat{g} = \frac{i}{\pi} \hat{\tau}_3 \otimes \hat{1} \int d\omega \hat{G}, \quad (3)$$
where \( \Psi = (\psi_1, \psi_2, \psi_1^\dagger, \psi_2^\dagger)^T \) and \( \varepsilon_k = \hbar^2 k^2 / 2m - \mu \). The matrix \( \hat{V} \) is given by

\[
\hat{V} = \mathbb{1} \otimes \text{Re} \left[ \mathbf{h}(x) \cdot \hat{\mathbf{\sigma}} \right] + i \hat{\tau}_3 \otimes \text{Im} \left[ \mathbf{h}(x) \cdot \hat{\mathbf{\sigma}} \right],
\]

(4)

where the components \( \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z \) of the vector \( \hat{\mathbf{\sigma}} \), and \( \hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3 \) are the Pauli matrices in the spin and the particle-hole space, respectively. The in-plane \((y-z)\) magnetizations of the neighboring F layers are not collinear in general, and form angles \( \alpha_1 \) and \( \alpha_2 \) with respect to the \( z \)-axis in the left (F1) and the right (F2) ferromagnets. The exchange field in ferromagnetic layers is \( \mathbf{h}(x) = h_1(0, \sin \alpha_1, \cos \alpha_1) \) and \( h_2(0, \sin \alpha_2, \cos \alpha_2) \).

We assume the superconductors are identical, with

\[
\Delta = \begin{bmatrix} 0 & \hat{\sigma}_2 \Delta \tau \times \hat{\sigma}_x \tau \end{bmatrix}
\]

(5)

for \( x < -d_1 \) and \( x > d_2 \). The self-consistency condition for the pair potential \( \Delta = \Delta(x) \) is given by

\[
\Delta = -i \lambda 2 \pi N(0) k_B T \sum_{\omega_n} \langle f_{\uparrow \downarrow} \rangle,
\]

(6)

where \( \lambda \) is the coupling constant, \( N(0) = m k_F^2 / 2 \pi^2 \hbar^2 \) is the density of states per spin projection at the Fermi level \( E_F = \hbar^2 k_F^2 / 2m \), and \( k_F = m v_F / \hbar \) is the Fermi wave number. In F layers \( \Delta = 0 \).

The supercurrent is obtained from the normal Green function through the following expression

\[
I(\phi) = \pi e N(0) S k_B T \sum_{\omega_n} \sum_{\sigma} \langle v_2 \text{Im} g_{\sigma \sigma} \rangle,
\]

(7)

where \( \phi \) is the macroscopic phase difference across the junction, and \( S \) is the area of the junction. In examples, the current is normalized to the resistance \( R_N = 2 \pi^2 \hbar / e^2 k_F^2 \).

Pair amplitudes, singlet \( f_s \), and triplet \( f_{00} \) and \( f_{11} \), with 0 and \( \pm 1 \) projections of the total spin of a pair, are defined in terms of anomalous Green functions

\[
f_s(x, t) = -i \pi N(0) k_B T \sum_{\omega_n} \langle f_{\uparrow \downarrow} - f_{\downarrow \uparrow} \rangle e^{-i \omega_n t},
\]

(8)

\[
f_{00}(x, t) = -i \pi N(0) k_B T \sum_{\omega_n} \langle f_{\uparrow \downarrow} + f_{\downarrow \uparrow} \rangle e^{-i \omega_n t},
\]

(9)

\[
f_{11}(x, t) = -i \pi N(0) k_B T \sum_{\omega_n} \langle f_{\uparrow \uparrow} + f_{\downarrow \downarrow} \rangle e^{-i \omega_n t}.
\]

(10)

In the following we will characterize singlet amplitudes by the zero-time \( f_s = f_s(x, 0) \). However, since zero-time triplet amplitudes identically vanish in agreement with the Pauli principle (with or without self-consistency)\( \hat{\mathbf{\sigma}} \), we characterize triplets by auxiliary functions using summation over negative frequencies only,

\[
f_{00}^{<} = -i \pi N(0) k_B T \sum_{\omega_n < 0} \langle f_{\uparrow \downarrow} + f_{\downarrow \uparrow} \rangle,
\]

(11)

\[
f_{11}^{<} = -i \pi N(0) k_B T \sum_{\omega_n < 0} \langle f_{\uparrow \uparrow} + f_{\downarrow \downarrow} \rangle.
\]

(12)

Note that in previous definitions of triplet pair amplitudes the total spin of a pair is projected on the \( z \)-axis. Physically it is more reasonable to take directions of magnetizations in F layers as the spin quantization axes. New triplet amplitudes can be introduced in the F1 layer \((-d_1 < x < 0)\) by simple rotation

\[
- i \hat{f}_{00} = \cos(\alpha_1)(-i \hat{f}_{00}) - \sin(\alpha_1)\hat{f}_{11},
\]

(13)

\[
\hat{f}_{11} = \sin(\alpha_1)(-i \hat{f}_{00}) + \cos(\alpha_1)\hat{f}_{11}.
\]

(14)

The same expressions with \( \alpha_1 \to \alpha_2 \) hold for F2 layer, \( 0 < x < d_2 \). Auxiliary functions \( f_{00}^{<} \) and \( f_{11}^{<} \) are related to \( f_{00}^{<} \) and \( f_{11}^{<} \) in the same way.

For \( \phi = 0 \), amplitudes \( f_{s} \) and \( f_{00}^{<} \) are real, while \( f_{11}^{<} \) is imaginary, and for \( \phi = \pi \) the opposite is true. For \( 0 < \phi < \pi \), all amplitudes are complex. In the examples, we normalize the amplitudes to the value of \( f_s \) in the bulk superconductors,

\[
f_{sb} = 2 \pi N(0) k_B T \sum_{\omega_n} \frac{\Delta_0}{\sqrt{\omega_n^2 + \Delta_0^2}}.
\]

(15)

Here, in the summation over \( \omega_n \), the high-frequency cutoff of \( 10 \Delta_0 \) is used. The temperature dependence of the bulk pair potential \( \Delta_0 \) is given by \( \Delta_0(T) = \Delta_0(0) \tanh \left( 1.74 \sqrt{T_c / T - 1} \right) \).

We consider only transparent interfaces and use continuity of the Green functions as the boundary conditions. For planar SF1F2S junctions (three dimensional case) Eq. (11) is solved iteratively together with the self-consistency condition, Eq. (4). The averaging over \( \theta \) is given by \( \langle ... \rangle = (1/2) f_0^e \langle ... \rangle d(\cos \theta) \).

Numerical computation is carried out using the collocation method. Iterations are performed until self-consistency is reached, starting from the stepwise approximation for the pair potential

\[
\Delta = \Delta_0 \left[ e^{-i \phi / 2} \Theta(-x - d_1) + e^{i \phi / 2} \Theta(x - d_2) \right],
\]

(16)

where \( \Theta(x) \) is the Heaviside step function. For finite electron mean free path in ferromagnets, the iterative procedure starts from the clean limit. We choose the appropriate boundary conditions in superconductors at the distance exceeding \( 2 \xi_S \) from the SF interfaces. These boundary conditions are given by eliminating the unknown constants from the analytical solutions in stepwise approximation. To reach self-consistency starting from the stepwise \( \Delta \), five to ten iterative steps were sufficient for results shown in this article.

III. RESULTS

We illustrate our results on SF1F2S planar junctions with relatively weak ferromagnets, \( \hbar / E_F \sim 0.1 \), and the ferromagnetic exchange length \( \xi_F \sim 20 k_F^{-1} \). Superconductors are characterized by the bulk pair potential at
zero temperature $\Delta_0(0)/E_F = 10^{-3}$, which corresponds to the superconducting coherence length $\xi_S(0) = 636k_F^{-1}$. We assume that all interfaces are fully transparent and the Fermi wave numbers in all metals are equal ($k_F^{-1} \sim 1\text{Å}$).

Detailed analysis is given for symmetric junctions with relatively thin ($d_1 = d_2 = 50k_F^{-1}$) and thick ($d_1 = d_2 = 500k_F^{-1}$) ferromagnetic layers, Figs. 2, 3 and for an highly asymmetric junction ($d_1 = 10k_F^{-1}$ and $d_2 = 990k_F^{-1}$), Figs. 4 and 5. In these examples $h_1 = h_2 = 0.1E_F$, $T/T_c = 0.1$, and both the clean limit ($l \to \infty$) and moderate disorder in ferromagnets ($l = 200k_F^{-1}$) are considered. The influences of temperature, thickness and exchange field variations are shown in Figs. 3 and 5 in the clean limit.

Short-ranged pair amplitudes $f_s$ and $f_{t_0}^{<}$ decay spatially from the FS interfaces in the same oscillatory manner. They decay algebraically with length $\hbar v_F/\hbar$ in the clean limit, and exponentially with the characteristic length $\sqrt{\hbar D/\hbar}$ in the dirty limit, where diffusion coefficient $D = v_F l/3$.

The long-ranged pair amplitude $f_{t_1}^{<}$ is not suppressed by the exchange field and penetrates the ferromagnet on the scale $\hbar v_F/k_BT(\sqrt{\hbar D/\hbar})$ in the clean (dirty) limit. In symmetric junctions ($d_1 = d_2$, $h_1 = h_2$, and $\alpha_1 = -\alpha/2$, $\alpha_2 = \alpha/2$) all pair amplitudes are practically the same in the clean limit ($l \to \infty$) and for moderate disordered ($l = 200k_F^{-1}$) in ferromagnets, which is illustrated in Figs. 2, 3 and 5. In symmetric junctions, $f_{t_1}^{<}$ does not prevail; it can be seen in Fig. 5 that $\alpha$-dependence of the Josephson critical current is monotonic.

Note that the spatial oscillations of $f_{t_1}^{<}$ in Figs. 2, 3 are due to our choice of $z$ axis as the spin quantization axis. After choosing the magnetization in F layers as the spin quantization axis, the rotated amplitude $f_{t_1}^{<}$ no longer oscillates but instead exhibits monotonic spatial variation with the jump at $x = 0$ (dash-dotted curve in Fig. 3). It can be seen that the amplitude $f_{t_1}^{<}$ decays from F1F2 interface. This is because $f_{t_1}^{<}$ in each F layer is generated at the interface itself by the projection $f_{t_1}^{<}\sin\alpha$.
from the neighboring layer. The triplet pair amplitudes penetrate into superconductors and monotonically decay over the same distance (the bulk superconducting coherence length \( \xi_S \)) as the singlet amplitude saturates to the bulk value (see Fig. 4).

For thick F layers (when short-ranged amplitude \( \tilde{f}_{10}^< \) is highly suppressed at the F1F2 interface) the generated long-ranged component \( f_{11}^< \) is very small. Therefore, influence of misalignment of magnetizations on the Josephson current in symmetric SF1F1S junctions cannot be attributed to the emergence of spin-triplet correlations: For thin ferromagnetic layers all amplitudes are equally large, while for thick ferromagnetic layers the long range triplet \( f_{11} \) is very small.

Fully developed long-range spin-triplet proximity effect emerges in asymmetric junctions with particularly thin and thick F layers (or weak and strong ferromagnets). The thin (weak) F layer acts as a “triplet-generator”, while the thick (strong) F layer is a “filter” which suppresses the short-ranged components. This is illustrated in Fig. 6 for thin F1 layer (\( d_1 = 10k_F^{-1} \)) and thick F2 layer (\( d_2 = 990k_F^{-1} \)) of equal strength \( h_1 = h_2 = 0.1E_F \), with orthogonal magnetizations, \( \alpha_1 = -\pi/2 \) and \( \alpha_2 = 0 \). Here, magnetization in the thick (F2) layer is taken along the \( z \)-axis and along the \( y \)-axis in the thin (F1) layer. Because of this choice, the \( x \)-dependence of \( f_{11}^< \) is monotonic in F2. The thin layer thickness \( d_1 \) is chosen to give maximum triplet current for a moderate disorder in ferromagnets \( l = 200k_F^{-1} \). This explains why \( f_{11}^< \) in Fig. 6 is larger for the finite electron mean-free path than in the clean limit.

The current-phase relation for an highly asymmetric SF1F2S junction (the same parameters as in Fig. 4) is shown in Fig. 7. The critical current for orthogonal magnetizations is an order of magnitude larger than for collinear magnetizations even though the first harmonic is absent. The second harmonic is dominant like at 0-

FIG. 5: The current-phase relation \( I(\phi) \) for a symmetric SF1F2S junction with \( d_1 = d_2 = 500k_F^{-1} \), \( h_1 = h_2 = 0.1E_F \), \( l = 200k_F^{-1} \), \( T/T_c = 0.1 \), and for three values of the relative angle between magnetizations: \( \alpha = 0 \) (solid curve), \( \pi/2 \) (dashed curve), and \( \pi \) (dash-dotted curve).

FIG. 6: Spatial dependence of singlet and triplet pair amplitudes \( f_s, f_{10}^< \), and \( f_{11}^< \), normalized to the bulk singlet amplitude \( f_{sb} \). We consider an asymmetric SF1F2S junction (\( d_1 = 10k_F^{-1} \) and \( d_2 = 990k_F^{-1} \)) at low temperature, \( T/T_c = 0.1 \), with the electron mean free path in ferromagnets \( l = 200k_F^{-1} \) (thick curves) and in the clean limit \( l \rightarrow \infty \) (thin curves). The phase difference is \( \phi = 0 \). Magnetization in the thin layer is along \( y \)-axis, \( \alpha_1 = -\pi/2 \), and along \( z \)-axis in the thick layer, \( \alpha_2 = 0 \). Here, \( h_1 = h_2 = 0.1E_F \) and \( \Delta_0(0)/E_F = 10^{-3} \).

FIG. 7: The current-phase relation \( I(\phi) \) for an asymmetric SF1F2S junction with \( d_1 = 10k_F^{-1} \) and \( d_2 = 990k_F^{-1} \), \( h_1 = h_2 = 0.1E_F \), \( l = 200k_F^{-1} \), \( T/T_c = 0.1 \), and for three values of the relative angle between magnetizations: \( \alpha = 0 \) (solid curve), \( \pi/2 \) (dashed curve), and \( \pi \) (dash-dotted curve).
dominant second harmonic and the resulting ground state degeneracy of the Josephson junction (like at 0-π transition, \(T_c\)) is experimentally accessible in asymmetric SFS junctions at 0-π transition,\(^{47,48}\) where the second harmonic is dominant, the Josephson current drops to zero. However, this is not the case in the clean SFS junctions where the Josephson current is smaller but finite at the 0-π transition.\(^{46}\)

IV. CONCLUSION

We have studied the Josephson effect in SF$_1$F$_2$S planar junctions made of conventional superconductors and two monodomain ferromagnetic layers with arbitrary thickness, strength, and angle between in-plane magnetizations. We carried out a detailed analysis of spin-singlet and -triplet pairing correlations and the Josephson current in the clean limit and for moderate disorder in ferromagnets by solving self-consistently the Eilenberger equations. While the spin-singlet and -triplet correlations with zero spin projection are short range, the triplet correlations with a nonzero spin projection are long range and may have a dramatic impact on transport properties and the Josephson effect.

In symmetric SFFS junctions with ferromagnetic layers of equal strength, the long-range spin-triplet correlations have no substantial impact on the Josephson current both in the ballistic and moderately diffusive regimes: For thin (weak) ferromagnetic layers all amplitudes are equally large, while for thick (strong) layers the long-range triplet amplitude is very small. This explains the previous results of Refs. 22–23.

We have found that fully developed long-range spin-triplet proximity effect occurs in highly asymmetric SF$_1$F$_2$S junctions at low temperatures and manifests itself as a dominant second harmonic in the Josephson current-phase relation. In contrast to the temperature-induced 0-π transition in which the first harmonic is restored away from the transition temperature, the magnitude of the second harmonic due to long-range spin-triplet correlations increases as the temperature is lowered. The triplet-induced second harmonic is robust against moderate disorder and variations in the layers thickness and exchange energy of ferromagnets.

Dominant second harmonic and the resulting ground state degeneracy of the Josephson junction (like at 0-π transitions) is experimentally accessible in asymmetric SF$_1$F$_2$S junctions with small interface roughness at low
temperatures. In addition, the half-periodicity of $I(\phi)$ in the considered junctions can be used for quantum interferometers (SQUIDs) which operate with two times smaller flux quantum and can exhibit the superposition of macroscopically distinct quantum states even in the absence of an external magnetic field. This has a potential application in the field of quantum computing.

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