Power loss prediction in asymmetric spur gear considering gear tooth dynamic load

Benny Thomas¹, K Sankaranarayanasamy², S Ramachandra¹ and Suresh Kumar SP¹
¹ Gas Turbine Research Establishment, DRDO, Bangalore, India-560093
² National Institute of Technology, Tiruchirapalli, India- 620015

Email ID: bennythomas@gtre.drdo.in

Abstract. In many gear drives, one side of the flank is subjected to relatively higher load for a longer duration than the other side. Asymmetric spur gears with drive side pressure angle higher than the coast side reflects this functional difference. Conventional design criteria and procedures followed for symmetric spur gears are suitably modified and applied to predict the gear tooth bending, contact stress and power loss in asymmetric spur gears. Quasi-static gear tooth load and empirical friction coefficient formulae were applied in the past to predict the sliding power loss in asymmetric spur gears. In the present work, Finite element method is used to determine the time varying mesh stiffness of the normal contact ratio asymmetric spur gear tooth. Computed gear tooth stiffness is used to predict the dynamic load at two different speeds under non-extended contact condition. Sliding power loss during the course of meshing is analytically calculated under quasi static and dynamic load conditions. Study demonstrates the difference in sliding power loss computed based on friction formulae and empirical friction coefficient formulae under static and dynamic load.

1. Introduction
Asymmetric gears are those in which drive flanks are designed with a higher pressure angle than the coast flank to enhance drive side load carrying capacity. Helicopter main drive gears, turboprop gearbox and gear pumps are some of the applications that demand unidirectional or relatively higher load on one side of the gear flank. Reduction in bending and contact stress, longer life and independent design of coast and drive side profile that helps in managing the gear tooth stiffness are some of the advantages of asymmetric gears. Relatively higher bearing load, non-standard gear tooth parameters for optimal performance and the need for special tool for manufacture are the disadvantages of asymmetric gears.

Performance of gear drives are mainly assessed based on life, dynamic behaviour and power loss. Conventional practice is to compute the gear tooth bending stress, contact stress and power loss under quasi static load. Research work to predict the power loss in transmission systems is given weightage with an intention to maximise the efficiency of the system and thereby reduce fuel burn and associated environmental effects. In geared systems sliding power loss is the major component that contributes to gear mesh loss and hence sliding power loss in asymmetric gears is the focus of present study.
Bernard et al [1] compared the empirical laws of friction with experimental law of friction and presented the variation in friction along the path of contact. Sekar et al [2] predicted the quasi static load in non-standard spur gears using Finite element method (FEM) and analytically computed the power loss. Enhancement in efficiency and load carrying capacity by using non-standard gears was demonstrated in their work. Yenti et al [3] found sliding power loss predicted based on friction coefficient formula proposed by ISO TC 60 to be closer to experimental results. Load and speed dependent losses were analysed by Heingartner et al [4] and found that sliding power losses increase with load. Hai Xu [5] introduced a new expression to determine friction coefficient based on elasto-hydrodynamic model for a specific grade of gear oil and validated it experimentally in his research work. Detail design procedures for asymmetric spur gears and direct gear design approach are presented by Kapelevich [6]. Thomas et al [7] introduced a novel approach to analytically predict the gear tooth bending stress in normal contact ratio asymmetric spur gears.

Researchers have predicted the power loss in asymmetric gears based on standard empirical friction laws considering quasi-static load. Successful performance of many geared systems designed based on conventional approach have proven the adequacy of the procedures. However, these methodologies do not consider the gear tooth dynamic behaviour and its effect on the life and performance of gears. This drawback in the general procedures gives space for possible performance enhancement based on the inferences from more accurate stress and dynamic behaviour predictions. Present work is focused on a mixed FE-M and analytical method to compute the sliding power loss in normal contact ratio (NCR) asymmetric spur gear under quasi static and dynamic load. Two different coefficient of friction models are used in this comparative study.

2. Gear tooth stiffness
Asymmetric gear with parameters as given in Table 1 is selected in the present work to investigate the influence of dynamic load on the sliding power loss. Figure 1a shows the model of three teeth of a 40 teeth 20°/31° drive to coast side pressure angle asymmetric spur gear used to generate a 2D meshed model to determine the gear tooth stiffness. Element size and the aspect ratio relations [8] is maintained in the selection of element size when a concentrated load is used to represent the gear tooth contact to determine the deflection within the interval of definition.

| Number of tooth \( z \) & 40  
| Gear ratio \( u \) & 1  
| Module \( m \) (mm) & 2  
| Tool dedendum coefficient \( h_{dc} \) & 1  
| Tool addendum coefficient \( h_{ac} \) & 1.25  
| Face width \( b_{fw} \) (mm) & 25  
| Tool tip radius \( TR_{dc} \) (mm) & 0.1  
| Coast side pressure angle \( \alpha_c \) (degree) & 20°  
| Drive side pressure angle \( \alpha_d \) (degree) & 31°  
| Backlash & 0  
| Young’s Modulus (GPa) & 210  
| Mass (kg) & 1  
| Applied normal tooth load (N) & 2916  |

![Table 1. Gear tooth parameters](image)

Deflection along the line of action corresponding to the point of load application is used to calculate the gear tooth stiffness \( k_q \) (N/m) at that point as per equation (1). Single tooth and double tooth time varying mesh stiffness (TVMS) is calculated based on equation (2)

\[
k_{qp,qB} = \frac{F_n}{\delta_{qp,qB}} \tag{1}
\]
where \( \delta_{q,p} \) is the pinion and gear tooth deflection (m) respectively along the line of action at \( q^{th} \) point due to the application of normal tooth load \( F_{n}(N) \) at that point.

Mesh stiffness of the \( j^{th} \) mesh pair at the \( q^{th} \) point of contact \( (k_{aj}) \) is given as

\[
k_{aj} = \frac{k_{pq}k_{pq}}{k_{pq} + k_{pq}}
\]  

Double tooth mesh stiffness is the direct summation of mesh stiffness of the two pairs of contact. Figure 1b shows the stiffness of the gear pair with parameters as per Table 1 during the course of meshing. Mesh stiffness calculated as per equation (2) is also presented in the figure.

3. Dynamic tooth load

Lumped parameter model is used in this study to analytically calculate the dynamic gear tooth load due to gear tooth stiffness. Rigid wheels representing the gear wheels are connected by spring elements with time varying mesh stiffness \( (k_{1} \text{ and } k_{2}) \) in N/m; and damper of damping coefficient \( c_{m} \) (N·m/sec). Y axis of the rectangular coordinate system in this model is chosen along the line of action.

\[
I_{bg} \ddot{\theta}_{g} + R_{bdg} F_{ST} = R_{bgd} (F_{DY1} + F_{DY1}) \pm \rho g_{l} \mu_{l} F_{DY1} \pm \rho g_{l} \mu_{l} F_{DY1}
\]

\[
I_{pg} \ddot{\theta}_{p} + R_{bdp} (F_{DI1} + F_{DI1}) = R_{bgp} F_{ST} \pm \rho \mu_{l} F_{DY1} \pm \rho \mu_{l} F_{DY1}
\]

\[
F_{ST} = \frac{T_{g}}{R_{bdg}} = \frac{T_{p}}{R_{bdp}}
\]

where \( I_{f} \) is the mass moment of inertia (Kg·m\(^2\)), \( \ddot{\theta} \) is the angular acceleration, \( \rho \) is the instantaneous radius (m) at the point of contact, \( R_{bd} \) is the drive side base circle diameter (m), \( \mu \) is the coefficient of friction between gear tooth surface in mesh, \( F_{DY} \) is the dynamic load (N), \( F_{ST} \) is the static gear tooth load (N) and I, II denotes gear mesh pair I and II respectively.

Elasto-hydrodynamic lubrication (EHL) based coefficient of friction derived by Xu et al [5] is given by

\[
\mu = e^{f(SR, P_{h}, \nu_{0}, S)} P_{h} b_{1} |SR| b_{3} v_{0} b_{5} v_{0} b_{6} R_{R} b_{6}
\]

where \( f(SR, P_{h}, \nu_{0}, S) = b_{1} + b_{4} |SR| P_{h} \log_{10}(v_{0}) + b_{5} e^{-|SR| P_{h} \log_{10}(v_{0})} + b_{6} e^{S} \)
\[ b_j = -8.916465, 1.03303, 1.036077, -0.354068, 2.812084, -0.100601, 0.752755, -0.390958, 0.620305 \] for \( j = 1 \) to 9 respectively. SR is the slide to roll ratio, \( V_e \) is the entraining velocity (m/sec), \( P_h \) is the maximum Hertzian pressure (GPa), S is the surface finish (microns), \( \nu_0 \) is the dynamic viscosity in centipoise, \( R_R \) is the relative radius of curvature (m).

Mass moment of inertia of the gear and pinion wheels can be expressed as

\[
I_g = M_g (R_{bdg})^2 \\
I_p = M_p (R_{bdp})^2
\]

where \( M \) is the mass (kg)

Substituting the expression for mass moment of inertia from (7) and (8) and coefficient of friction \( \mu \) from (6) in equation (3) and (4), we get

\[
\ddot{x}_d + 2\omega \zeta \dot{x}_d + \omega^2 x_d = \omega^2 x_s
\]

where \( \ddot{x}_d \) is the relative acceleration along the line of action (m/sec\(^2\)), \( \dot{x}_d \) is the relative velocity along the line of action (m/sec), \( x_d \) is the relative displacement along the line of action (m), \( x_s \) is the static transmission error (m) and \( \omega \) is the frequency (Hz).

Equation (9) is numerically solved using fourth order Runge-Kutta Method. In the numerical solution damping ratio (\( \zeta \)) of 0.17 and kinematic viscosity (\( v \)) of 44.5cSt at 60°C is assumed. Length of action is divided into sufficiently large number of divisions, typically 500, to accurately capture the transition phase from double tooth contact to single tooth contact and vice versa. Iterative solution for \( \dot{x}_d \) and \( x_d \) is continued till a predefined accuracy is reached. Dynamic gear tooth load can be approximated as the product of the gear mesh stiffness and \( x_d \). Figure 2 (a) shows the gear tooth load under quasi static and dynamic condition. Oscillation seen in dynamic gear tooth load is the response of the gear of definite mass, mesh stiffness and damping to sudden load application during transmission.

Dynamic factor is the ratio of the maximum dynamic gear tooth load to the quasi-static load during the course of meshing. Figure 2(b) shows the variation in dynamic factor due to the influence of varying mesh stiffness and damping.

\[ \text{Figure 2. Quasi static load, dynamic tooth load at a given speed and dynamic factor for specified speed range for a 40 teeth asymmetric spur gear with parameters as per Table 1 (a) Quasi static load and dynamic tooth load computed under non-extended tooth contact at 2000 rpm (b) Variation in dynamic factor with speed} \]

4. Sliding power loss
Total gear tooth mesh loss is the sum of rolling and sliding friction loss. Present investigation is on the power loss due to sliding as the sliding friction gear mesh loss is relatively higher than rolling power loss. Magnitude of the instantaneous sliding friction power loss at a given point of contact between
Two gears in mesh is proportional to the coefficient of friction, normal tooth load shared by the gear pair and the sliding velocity.

Two models for predicting the coefficient of friction, first by Dowson and Higginson and second by Hai Xu [5] et al, is used in the present study to calculate the gear tooth sliding power loss under quasi-static and dynamic gear tooth load.

Coefficient of friction proposed by Dowson and Higginson is given by

$$\mu = 18.175 \nu^{-0.15} \left| \frac{V_e}{\nu_s} \right|^{-0.15} \nu_s^{-0.5} (R_r)^{-0.5}$$

(10)

where $\nu$ is the kinematic viscosity in centi-stokes, $V_e/\nu_s$ is the entraining to slide velocity, $\nu_s$ is the sliding velocity (mm/sec), $R_r$ is the relative radius of curvature at the contact point (mm). Variation in coefficient of friction, along the course of action defined with respect to roll angle, computed using the two models for two different speeds is depicted in Figure 3(a) and (b). Maximum value of $\mu$ predicted using equation (10) is limited to 0.2. Both the models show drop in coefficient of friction with increase in speed. However, the two models widely differ primarily in the single tooth contact zone.

Sliding velocity during the mesh cycle calculated for 2000rpm and 7300rpm is shown in Figure 4(a) and (b). As seen from the figure, sliding velocity is zero at pitch point and at all other points sliding velocity increases with increase in speed.

Instantaneous sliding power loss $P_s$ at the $q^{th}$ point of contact is given by

$$ (P_s)_q = \mu_q (\nu_s)_q F_q$$

(11)

Sliding power loss during the course of meshing due to quasi static and dynamic load for two different speeds are shown in Figure 5(a) and (b). As mentioned earlier, sliding velocity at the pitch point is zero and hence the sliding power loss at this point is zero.

**Figure 3.** Variation in coefficient of friction along the course of action for gear with parameters as per Table 1 computed as per equation (6) and (10) (a) At 2000 rpm (b) at 7300 rpm

**Figure 4.** Variation in sliding velocity during the mesh cycle for gear with parameters as per Table 1 (a) At 2000 rpm (b) at 7300 rpm
Figure 5. Variation in sliding power loss during meshing computed for the gear as per Table 1 under quasi static and dynamic load (a) At 2000 rpm (b) at 7300rpm

5. Conclusion
Difference in coefficient of friction, computed based on two models, along the course of meshing of a NCR asymmetric spur gear is relatively higher in the single tooth contact zone. Both the models predict lower friction coefficient at higher speed and EHL model predicts relatively lower value of friction coefficient. Mesh sliding power losses computed based on EHL model is relatively lower than that based on empirical formula. Difference in power loss computed under quasi static and dynamic load is found to be relatively higher as the speed approaches resonance speed.

References
[1] Mushirabwoba Bernard, Lahcen Belfals, Najji Brahim, Lasri Abdelilah, A Comparative Study of Friction Laws Used in Spur Gear Power Losses Estimation, Contemporary Engineering Sciences, Vol. 9, 2016, no. 6, 279 – 288
[2] R Prabhu Sekar, V Edwin Geo and Leenus Jesu Martin, A mixed finite element and analytical method to predict load, mechanical power loss and improved efficiency in non-standard spur gear drives, Proc IMechE Part C: Journal of Mechanical Engineering Science February 2017, Volume: 231 issue: 11, page(s): 1408-1424, DOI: 10.1177/1350650117697594
[3] Chakrit Yenti, Surin Phongsupasamit, and Chanat Ratanasumawong, Analytical and Experimental Investigation of Parameters Affecting Sliding Loss in a Spur Gear Pair, Engineering Journal, Volume 17 Issue 1,pp.79-94, January 2013, DOI:10.4186/ej.2013.17.1.79
[4] Heingartner and David Determining power losses in the helical gear mesh; Case study, Proceedings of DETC’03 ASME 2003 Design Engineering Technical Conferences and Computers and Information in Engineering Conference Chicago, September 2-6, 2003 Paper No. DETC2003/PTG-48118, pp. 965-970; 6 pages, DOI:10.1115/DETC2003/PTG-48118
[5] Hai Xu, Development of a generalized mechanical efficiency prediction methodology for gear pairs, Dissertation, The Ohio State University, 2005
[6] Alexander L. Kapelevich, Direct gear design, CRC Press, Taylor & Francis Group, International Standard Book Number-13: 978-1-4398-7619-0
[7] Benny Thomas, K Sankaranarayanasamy, S Ramachandra, SP Suresh Kumar. Search method applied for gear tooth bending stress prediction in normal contact ratio asymmetric spur gears, Proc IMechE Part C: Journal of Mechanical Engineering Science, January 2018, Volume: 232 issue: 24, page(s): 4647-4663, DOI:10.1177/0954406217753235
[8] John J. Coy, Charles Hu-Chih Chao, A method of selecting grid size to account for Hertz deformation in Finite Element analysis of spur gears, NASA-TM-82623, September, 1981