TWO PHOTON WIDTH OF $\eta_c$

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We discuss the measured partial width of the pseudoscalar charmonium state, $\eta_c$, into two photons. Predictions from potential models are examined and compared with experimental values. Including radiative corrections, it is found that present measurements are compatible both with a QCD type potential and with a static Coulomb potential, with $\alpha_s$ evaluated at two loops. Results are also compared with those from $J/\psi$ data through the NRQCD model.

1 Introduction

In this note, we examine the theoretical predictions for the electromagnetic decay of the simplest and lowest lying of all the charmonium states, i.e. the pseudoscalar $\eta_c$. We shall compare the two photon decay width with the leptonic width of the $J/\psi$, which has been measured with higher precision and found to be 15% higher than in previous measurements. The most recently reported Particle Data Group average is given by

$$\Gamma_{\text{exp}}(\eta_c \rightarrow \gamma\gamma) = 7.4 \pm 1.4 \text{ keV}$$  \hspace{1cm} (1)

2 Relation to $J/\psi \rightarrow e^+e^-$ width

The two photon decay width of a pseudoscalar quark-antiquark bound state can be written as

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = 12e_q^4\alpha_{\text{QED}}^24\pi\frac{|\psi(0)|^2}{M^2}\left[1 + \frac{\alpha_s}{\pi}\left(\frac{\pi^2 - 20}{3}\right)\right] \approx \Gamma^P_B(1 - \alpha_s)$$  \hspace{1cm} (2)

where $\psi(0)$ is the wavefunction of the interquark potential evaluated at the origin. It is useful to compare eq. (2) with the expressions for the vector state $J/\psi$, i.e.

$$\Gamma(J/\psi \rightarrow ee) = 4e_q^2\alpha_{\text{QED}}^24\pi\frac{|\psi(0)|^2}{M^2}\left(1 - \frac{16\alpha_s}{3\pi}\right) \approx \Gamma^V_B(1 - 1.7\alpha_s).$$  \hspace{1cm} (3)

The expressions in eq. (2) and (3) can be used to estimate the radiative width of $\eta_c$ from the measured values of the leptonic decay width of $J/\psi$, if
one assumes that the $\psi(0)$ values for both the pseudoscalar and the vector state should be the same. This is true up to errors of $O(\alpha_s/m_c^2)$. From

$$\Gamma_{\exp}(J/\psi \to e^+e^-) = 5.26 \pm 0.37 \text{ keV}$$

(4)

expanding in $\alpha_s$ one has:

$$\Gamma(\eta_c \to \gamma\gamma) \approx \frac{4}{3} \left(1 - \frac{3.38\alpha_s}{\pi} + 1.96\frac{\alpha_s}{\pi} + O(\alpha_s^2)\right)$$

(5)

From the value $\alpha_s(M_Z) = 0.118 \pm 0.003$ the renormalization group evolution gives $\alpha_s(Q = 2m_c = 3.0 \text{ GeV}) = 0.25 \pm 0.01$. Combining the formulae (4) and (5) we obtain

$$\Gamma(\eta_c \to \gamma\gamma) \pm \Delta\Gamma(\eta_c \to \gamma\gamma) = 8.18 \pm 0.57 \pm 0.04 \text{ keV}$$

(6)

This estimate agrees within 1σ with the value given in formula (1).

3 Potential models predictions

We shall extract now the wave function at the origin from potential models. For the calculation of the wave function we have used four different potential models, like the Cornell type potential $V(r) = -\frac{k}{r} + \frac{r}{a}$ with parameters $a = 2.43$, $k = 0.52$, the Richardson potential $V_R(r) = -\frac{4}{3}\frac{12\pi}{33-2N_f} \int \frac{d^3q}{(2\pi)^3} q^2 \log(1+q^2/\Lambda^2)$ with $N_f = 3$, $\Lambda = 398 \text{ MeV}$, and the QCD inspired potential $V_J$ of Igi-Ono $V_J(r) = V_{AR}(r) + d r e^{-\gamma r} + a r$, $V_{AR}(r) = -\frac{4}{3}\frac{\alpha_s}{\pi} (r)$ with two different parameter sets, corresponding to $\Lambda_{\overline{MS}} = 0.5 \text{ GeV}$ and $\Lambda_{\overline{MS}} = 0.3 \text{ GeV}$ respectively. We also show the results from a Coulombic type potential with the QCD coupling $\alpha_s$ frozen to a value of $r$ corresponding to the Bohr radius of the quarkonium system, $r_B = 3/(2m_c\alpha_s)$ (see for instance [2]). The error sources in calculation are given by the choice of scale in radiative correction, the choice of various potential parameters and the fluctuations in results from different models. The $\Gamma(\eta_c \to \gamma\gamma)$ potential models prediction gives a range of values:

$$\Gamma(\eta_c \to \gamma\gamma) = 7.6 \pm 1.5 \text{ keV}$$

(7)

4 Octet Component model

We will present now another model which admits other components to the meson decay beyond the one from the colour singlet picture (Bodwin, Braaten...
NRQCD has been used to separate the short distance scale of annihilation from the nonperturbative contributions of long distance scale. This model has been successfully used to explain the larger than expected \( J/\psi \) production at the Tevatron. According to\(^1\), in the octet model for quarkonium the decay widths of charmonium states involve four unknown long distance coefficients which can be reduced to two by means of the vacuum saturation approximation:

\[
G_1 \equiv \langle J/\psi | O_1 (3S_1) | J/\psi \rangle = \langle \eta_c | O_1 (1S_0) | \eta_c \rangle \\
F_1 \equiv \langle J/\psi | P_1 (3S_1) | J/\psi \rangle = \langle \eta_c | P_1 (1S_0) | \eta_c \rangle,
\]

correct up to \( O(v^2) \), the velocity of the quarks inside the meson. We use the \( J/\psi \) experimental decay widths as input in order to determine the long distance coefficients \( G_1 \) and \( F_1 \). This result in turn is used to compute the \( \eta_c \) decay widths.

The BBL model gives the following decay width of the \( \eta_c \) meson:

\[
\Gamma(\eta_c \to \gamma\gamma) = 9.02 \pm 0.65 \pm 0.14 \text{ keV}
\]

This value agrees with experimental data within \( 1\sigma \).

## 5 Summary

We present in fig. 1 a set of predictions coming from different methods:

\[
\Gamma(\eta_c \to \gamma\gamma)
\]

![Figure 1. Potential Models results; BBL model with input from \( J/\psi \) decay data; Lattice evaluation of \( G_1 \) and \( F_1 \) factors; Singlet picture with \( G_1 \) obtained from \( J/\psi \to e^+e^- \) and \( J/\psi \to LH \) processes respectively. The vertical lines represent the PDG average value and its indetermination.](image)

results from potential models; BBL model with \( G_1 \) and \( F_1 \) extracted from the
$J/\psi$ decay data; lattice calculation of the long distance terms for the BBL model. Singlet picture: $G_1$ extracted from $J/\psi \rightarrow e^+e^-$ decay width, and singlet picture: $G_1$ extracted from $J/\psi \rightarrow LH$ decay width.

6 Conclusions

The $\Gamma(\eta_c \rightarrow \gamma\gamma)$ decay width prediction of the potential models considered gives the value $7.6 \pm 1.5$ keV which is consistent with the PDG average. The Coulombic model is in agreement with other models prediction. Predictions of the BBL model for the $\eta_c \rightarrow \gamma\gamma$ decay width is consistent with the experimental measures, for both the long distance terms $G_1$ and $F_1$ extracted from the $J/\psi$ experimental decay widths and the one evaluated from lattice calculations.

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