Study on Tunnelling Radiation in 4 Dimension Black Holes
Vector Particles

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Abstract. Recent studies show that the tunnelling radiation of vector particles has been studied successfully by WKB approximation and Hamilton-Jacobi method. In view of this, the main purpose of this paper is to study the Proca equation and the vector particles tunnelling radiation in a 4-dimensional black hole. Finally, the results here show that the temperature of the vector particle is the same as that of the Dirac particle.

Keywords: Vector particle tunnelling, Proca equation, 4-dimensional spacetime

1. Introduction
Since Hawking discovered black hole radiation [1], Hawking radiation research has attracted people's attention [2-14]. People have done a lot of research in this area and have obtained many significant research results. Later, many methods are proposed to further confirm Hawking radiation. In recent years, the semiclassical method of simulating Hawking radiation as tunnelling process has been widely used. In this semi classical method, Hawking radiation is regarded as a kind of tunnelling, and WKB approximation is used to calculate the imaginary part of the action of particles passing through the black hole horizon. When a particle passes through the horizon of a black hole, the solution of Hawking's temperature mainly depends on the imaginary part of its wave function. When Hawking radiation is interpreted as quantum tunnelling radiation, the action and tunnelling probability of the emitted particles can be expressed as $\Gamma \propto \exp(-2\text{Im}I)$.

In the first method, Parikh and Wilczek studied the penetrating behaviour of massless scalar particles [2] considering the background space-time of black hole changes, and obtained the radiation spectrum correction in spherically symmetric space-time. Their results show that the main correction of Hawking temperature is related to the energy of the emitted particles. Subsequently, the work is extended to a large number of charged scalar particles, and Hawking radiation from general black holes is studied [5-12]. For the case where the emergent particle is a vector particle, the motion equation of the particle is different from that of the massless particle. The trajectory of massless particles is zero geodesic equation, while the motion of massless particles satisfies Broglie wave, which is the phase velocity of outgoing particles. Another method is the Hamilton-Jacobi method used in [8-14].

In this paper, the WKB approximation and hamilton-Jacobi transformation are applied to the Proca equation, and the Hawking radiation of four dimensional GHS black hole vector particles is studied. We find that Hawking temperatures of vector particles are the same as those of other outgoing
particles. The main content of this paper is as follows: The first part summarizes the research status of Hawking radiation; The second part studies the tunnel radiation of vector particles in four-dimensional space-time. The third part gives our research results and conclusions.

In this paper, WKB approximation and Hamilton-Jacobi transformation are discussed to study the Hawking radiation of a four-dimensional GHS black hole vector particle using proca equation. We find that Hawking temperatures of vector particles are the same as those of other outgoing particles. The main content of this paper is as follows: The first part summarizes the research status of Hawking radiation; The second part studies the tunnel radiation of vector particles in four-dimensional space-time. The third part draws our conclusion.

2. Tunnelling of 4D Vector Particles Space-Time

This part will study on the 4D VECTOR PARTICLES space-time. To study following time and space:

\[ ds^2 = -Adt^2 + \frac{1}{B}dr^2 + C\theta^2 + D\phi^2 \]  

(D.1)

Determinants and inverse measures of this space-time are easy to calculate:

\[ D_{\mu \nu} = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \psi^{\nu}) \]

(D.2)

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\[ \psi_{\nu} = D_\nu \psi_{\mu} - D_\mu \psi_{\nu} = \partial_\nu \psi_{\mu} - \partial_\mu \psi_{\nu}, \psi_\nu = (\psi_0, \psi_1, \psi_2, \psi_3), \]

\[ D_\nu \psi_{\mu} \] are covariant derivatives and antisymmetric tensors respectively. Taking equation \[ D_\mu \psi_{\nu} = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \psi^{\nu}) \] into equation (2), we can obtain:

\[ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \psi^{\nu}) + \frac{m^2}{h^2} \psi^{\nu} = 0 \]  

(D.3)

In this time and space, the following relation is easily obtained:

\[ \psi^i = \psi_{\mu} \delta^{\mu i} = -\frac{1}{A} \psi_0, \psi^i = B \psi_1, \psi^3 = \frac{1}{C} \psi_2, \psi^0 = \frac{1}{B} \psi_3 \]  

(D.4)

Substituting equation (4) into equation (3) yields:

\[ \sqrt{\frac{B}{ACD}} \partial_\mu \left( -\sqrt{\frac{BCD}{A}} (\partial_\nu \psi_1 - \partial_\nu \psi_0) \right) + \partial_\nu \left( -\sqrt{\frac{D}{ABC}} (\partial_\mu \psi_2 - \partial_\mu \psi_0) \right) + \frac{m^2}{h^2} \left( \frac{1}{A} \psi_0 \right) = 0 \]

\[ \sqrt{\frac{B}{ACD}} \partial_\mu \left( -\sqrt{\frac{BCD}{A}} (\partial_\nu \psi_1 - \partial_\nu \psi_0) \right) + \partial_\nu \left( -\sqrt{\frac{D}{ABC}} (\partial_\mu \psi_2 - \partial_\mu \psi_0) \right) + \frac{m^2}{h^2} \left( \frac{1}{A} \psi_0 \right) = 0 \]
To understand equation (5), \( \psi_v \) is expressed in the following form using WKB approximation:

\[
\psi_v = (c_0, c_1, c_2, c_3) \exp \left( \frac{j}{\hbar} s(t, r, \theta, \phi) \right)
\]  

Among them:

\[
s(t, r, \theta, \phi) = s_0(t, r, \theta, \phi) + \hbar s_1(t, r, \theta, \phi) + \hbar^2 s_2(t, r, \theta, \phi) + \ldots \quad (7)
\]

Combine (6) and (7) into (5):

\[
\begin{bmatrix}
\frac{B}{A} \left[ C_1 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] + \frac{1}{AC} \left[ C_2 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] + \frac{1}{A} \partial_\theta \psi_v \left( \frac{\partial}{\partial \theta} \psi_v \right) + \frac{1}{A} \partial_\phi \psi_v \left( \frac{\partial}{\partial \phi} \psi_v \right) \\
- \frac{C}{D} \left[ C_3 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] + \frac{1}{A} \partial_\theta \psi_v \left( \frac{\partial}{\partial \theta} \psi_v \right) + \frac{1}{A} \partial_\phi \psi_v \left( \frac{\partial}{\partial \phi} \psi_v \right) \\
- \frac{1}{AB} \left[ C_2 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] - \frac{B}{C} \left[ C_3 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] \\
- \frac{1}{CD} \left[ C_2 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] + \frac{B}{D} \left[ C_3 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] \\
- \frac{1}{CD} \left[ C_2 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] + \frac{B}{D} \left[ C_3 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] \\
- \frac{1}{CD} \left[ C_2 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] + \frac{B}{D} \left[ C_3 (\frac{\partial}{\partial t} S_0) (\frac{\partial}{\partial r} S_0) - C_0 (\frac{\partial}{\partial r} S_0)^2 \right] \\
\end{bmatrix} = 0
\]

Considering the space-time characteristics of black holes, the following variables are used for separation:

\[
I = -\omega t + \theta(r) + \phi + J \phi \quad (9)
\]

Among them, \( \omega \) and \( J \) represent the energy and angular momentum of tunnelling particles respectively. After substituting equation (9) into equation (8), the following matrix equation can be obtained:

\[
\begin{pmatrix}
\Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} & C_0 \\
\Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & C_1 \\
\Lambda_{20} & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & C_2 \\
\Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & C_3
\end{pmatrix} = 0
\]

Among them:
The matrix of equation (10) is equal to zero, so when \( \text{Det} (\Lambda) = 0 \):

\[
AC \left[ J^2 + D \left( \frac{m^2}{2} + B \left( \varphi, \theta \right) \right) \right] + AD \left( \varphi, \theta \right)^2 - CD
\]  

In equation (12), the \( r^+ \) and \(-r^+\), so:

\[
\varphi, \theta = \pm \sqrt{\frac{-ACJ^2 - AC\omega^2 - AD\varphi^2 + C\omega^2}{ABCD}}
\]

\[
\varphi(r) = \pm \int dr \sqrt{-ACJ^2 - AC\omega^2 - AD\varphi^2 + C\omega^2}
\]  

Next, the tunnelling radiation of a four-dimensional GHS black hole will be considered. The measurement of GHS black holes can be expressed as:

\[
ds^2 = -Adt^2 + \frac{1}{B}dr^2 + C\Delta \theta^2 + D\Delta \phi^2
\]

\[
= -f(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2\sin^2 \theta d\phi^2
\]

Among them:

\[
f(r) = \left\{ 1 - \frac{2Me^{\Phi}}{r} \right\} \left( 1 - \frac{Q^2 e^{2\Phi}}{Mr^2} \right) = \frac{r - r_1}{r - r_2}
\]

\[
h(r) = \left\{ 1 - \frac{2Me^{\Phi}}{r} \right\} \left( 1 - \frac{Q^2 e^{2\Phi}}{Mr^2} \right) = \frac{r - r_1}{r^2}
\]

\[
r_1 = 2Me^{\Phi}, r_2 = \frac{Q^2 e^{2\Phi}}{M}, Q^2 < 2e^{-\Phi} M^2
\]

\[
\varphi(r) = \pm \int dr \sqrt{-ACJ^2 - AC\omega^2 - AD\varphi^2 + C\omega^2} = \pm i\pi r_1 \omega = \pm 2i\pi Me^{\Phi} \omega
\]

Where \(+(-)\) corresponds to an emergent solution (incident solution). Under the classical limit, the probability of particles entering the black hole should be unified, that is, the black hole can absorb all materials near the horizon \([14]\), then the absorption rate at the horizon of the black hole is \( P_{\text{absorption}} = 1 \). Therefore, the tunnelling rate of vector particles is:
\[ \Gamma = \frac{P_{\text{emission}}}{P_{\text{absorption}}} = \exp\left(-\frac{2}{\hbar} \Im \{ I_+ \right) = \exp\left(-\frac{2}{\hbar} \Im \{ \theta \} \right) = \exp\left(-\frac{8\pi M_0^0}{\hbar} \alpha \right) \]  \tag{17}

Use the following formula:

\[ \Gamma = \exp\left(\frac{E}{T}\right) \]  \tag{18}

the Hawking T of the black hole can be calculated as follows:

\[ T = \frac{1}{8\pi M_0^0} \]  \tag{19}

This temperature is Hawking temperature of GHS black hole, which is the same as Hawking temperature of Dirac particle.

3. Conclusion
In this paper, the Hawking radiation of the vector particles of the GHS black hole is studied. Using the application of WKB approximation and Hamilton-Jacobi transformation in Proca equation, the expected Hawking temperature of 4D VECTOR PARTICLES space-time is obtained. The results show that the Hawking T of vector is the same as that of Dirac particles and other particles. In the future, what we need to do is to further develop this method into other gravitational frameworks.

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