NEW NONLINEAR CHAOS-DYNAMICAL ANALYSIS OF ATMOSPHERIC RADON $^{222}$Rn CONCENTRATION TIME SERIES FROM BETA PARTICLES ACTIVITY DATA OF RADON MONITORS

The work is devoted to the development of the theoretical foundations of new universal complex chaos-dynamical approach to analysis and prediction of the atmospheric radon $^{222}$Rn concentration changing from beta particles activity data of radon monitors (with pair of the Geiger-Müller counters). The approach presented consistently includes a number of new or improved methods of analysis (correlation integral, fractal analysis, algorithms of average mutual information, false nearest neighbors, Lyapunov exponents, surrogate data, non-linear prediction schemes, spectral methods, etc.) to solve problems quantitatively complete modeling and analysis of temporal evolution of the atmospheric radon $^{222}$Rn concentration. There are firstly received data on topological and dynamical invariants for the time series of the $^{222}$Rn concentration, discovered a deterministic chaos phenomenon using detailed data of measurements of the radon concentrations at SMEAR II station of the Finnish Meteorological Institute.

1. Introduction

At present time one of the extremely important and too complex areas of elements, systems and devices physics and sensor electronics is study of regular and chaotic dynamics dynamics of non-linear processes in the different classes of quantum, quantum-generating systems and devices and quantum (atomic-molecular systems in an external electromagnetic field) [1-20]. It is worth to mention fulfilled by our group numerous studying of dynamics of the different quantum systems in external electromagnetic field, which has the features of the random, stochastic kind and its realization does not require the specific conditions.

The importance of studying a phenomenon of stochasticity or quantum chaos in dynamical systems is provided by a whole number of technical applications, including a necessity of understanding chaotic features in a work of different electronic devices and systems. New field of investigations of the quantum and other systems has been provided by a great progress in a development of a chaos theory methods [1-12]. In previous our papers [5-20] we have given a review of new methods and algorithms to analysis of different systems of quantum physics, electronics and photonics and used the nonlinear method of chaos theory and the recurrence spectra formalism to study quantum stochastic futures and chaotic elements in dynamics of atomic, molecular, nuclear systems in an free state and an external electromagnetic field, atmospheric and even environmental systems [21-71]. There were discovered non-trivial manifestations of a chaos phenomenon.

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II station of the Finnish Meteorological Institute (see details in Refs. [72-75]).

2. Universal chaos-dynamical approach in analysis of chaotic dynamics of the radon concentration time series

As many blocks of the presented approach have been developed earlier and are needed only to be reformulated regarding the problem studied in this paper, here we are limited only by the key moments following to Refs. [5-20]. Let us formally consider scalar measurements \( s(n) = s(t_n + n\Delta t) = s(n) \), where \( t_n \) is the start time, \( \Delta t \) is the time step, and \( n \) the number of the measurements. Further it is necessary to reconstruct phase space using as well as possible information contained in the \( s(n) \). Such a reconstruction results in a certain set of \( d \)-dimensional vectors \( y(n) \) replacing the scalar measurements. Packard et al. introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables \( s(n + \tau) \), where \( \tau \) is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector in \( d \) dimensions,

\[
y(n) = [s(n), s(n + \tau), s(n + 2\tau), \ldots, s(n + (d-1)\tau)],
\]

the required coordinates are provided. In a nonlinear system, the \( s(n + j\tau) \) are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension \( d \) is called the embedding dimension. Any time lag will be acceptable is not terribly useful for extracting physics from data. If \( \tau \) is chosen too small, then the coordinates \( s(n + j\tau) \) and \( s(n + (j + 1)\tau) \) are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if \( \tau \) is too large, then \( s(n + j\tau) \) and \( s(n + (j + 1)\tau) \) are completely independent of each other in a statistical sense. Also, if \( \tau \) is too small or too large, then the correlation dimension of attractor can be underestimated or overestimated respectively [3]. It is therefore necessary to choose some intermediate (and more appropriate) position between above cases. First approach is to compute the linear autocorrelation function

\[
C_\tau(\delta) = \frac{1}{N} \sum_{n=1}^{N} [s(m + \delta) - \bar{s}][s(m) - \bar{s}]
\]

\[
\bar{s} = \frac{1}{N} \sum_{n=1}^{N} s(m)
\]

and to look for that time lag where \( C_\tau(\delta) \) first passes through zero. This gives a good hint of choice for \( \tau \) at that \( s(n + j\tau) \) and \( s(n + (j + 1)\tau) \) are linearly independent. However, a linear independence of two variables does not mean that these variables are nonlinearly independent since a nonlinear relationship can differs from linear one. It is therefore preferably to utilize approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows. Let there are two systems, A and B, with measurements \( a_i \) and \( b_k \). The amount one learns in bits about a measurement of \( a_i \) from measurement of \( b_k \) is given by arguments of information theory [2,8,9]

\[
I_{AB}(a_i, b_k) = \log_2 \left( \frac{P_{AB}(a_i, b_k)}{P_A(a_i)P_B(b_k)} \right),
\]

where the probability of observing a out of the set of all A is \( P_A(a_i) \), and the probability of finding b in a measurement B is \( P_B(b_k) \), and the joint probability of the measurement of a and b is \( P_{AB}(a_i, b_k) \). The mutual information \( I \) of two measurements \( a_i \) and \( b_k \) is symmetric and non-negative, and equals to zero if only the systems are independent. The average mutual information between any value \( a_i \) from system A and \( b_k \) from B is the average over all possible measurements of \( I_{AB}(a_i, b_k) \),

\[
I_{AB}(a_i, b_k) = \log_2 \left( \frac{P_{AB}(a_i, b_k)}{P_A(a_i)P_B(b_k)} \right)
\]

To place this definition to a context of observations from a certain physical system, let us think of the sets of measurements \( s(n) \) as the A and of the measurements a time lag \( \tau \) later, \( s(n + \tau) \), as
B set. The average mutual information between observations at \( n \) and \( n + \tau \) is then

\[
I_{\mathbf{A}}(\tau) = \sum_{a \in \mathbf{A}} P_{\mathbf{A}}(a, b) I_{\mathbf{A}}(a, b) \tag{5}
\]

Now we have to decide what property of \( I(\tau) \) we should select, in order to establish which among the various values of \( \tau \) we should use in making the data vectors \( y(n) \). One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for non-linearity. The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process resulting in the vibrations amplitude level variations can be concluded.

The goal of the embedding dimension determination is to reconstruct a Euclidean space \( \mathbb{R}^d \) large enough so that the set of points \( \mathbf{d}_A \) can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding dimension, \( d_E \), must be greater, or at least equal, than a dimension of attractor, \( d_A \), i.e. \( d_E \geq d_A \). However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [1-20]. First, many of computations for extracting interesting properties from the data require searches and other operations in \( \mathbb{R}^d \) whose computational cost rises exponentially with \( d \). Second, but more significant from the physical point of view, in the presence of noise or other high dimensional contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal. In too large an embedding space one is unnecessarily spending time working around aspects of a bad representation of the observations which are solely filled with noise. It is therefore necessary to determine the dimension \( d_A \). There are several standard approaches to reconstruct the attractor dimension (see, e.g., [1-9]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, \( C(r) \), to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia is the most commonly used approach. According to this algorithm, the correlation integral is

\[
C(r) = \lim_{N \to \infty} \frac{2}{N(n-1)} \sum_{(i,j) \neq (i,j)} H(r - |y_i - y_j|) \tag{6}
\]

where \( H \) is the Heaviside step function with \( H(u) = 1 \) for \( u > 0 \) and \( H(u) = 0 \) for \( u \leq 0 \), \( r \) is the radius of sphere centered on \( y_i \) or \( y_j \), and \( N \) is the number of data measurements. If the time series is characterized by an attractor, then the integral \( C(r) \) is related to the radius \( r \) given by

\[
d = \lim_{r \to \infty} \frac{\log C(r)}{\log r} \tag{7}
\]

where \( d \) is correlation exponent that can be determined as the slope of line in the coordinates \( \log C(r) \) versus \( \log r \) by a least-squares fit of a straight line over a certain range of \( r \), called the scaling region.

If the correlation exponent attains saturation with an increase in the embedding dimension, then the system is generally considered to exhibit chaotic dynamics. The saturation value of the correlation exponent is defined as the correlation dimension \( (d_c) \) of the attractor. The method of surrogate data [1,8,9] is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data.

Often, a significant difference in the estimates of the correlation exponents, between the original and surrogate data sets, can be observed. In the case of the original data, a saturation of the correlation exponent is observed after a certain embedding dimension value (i.e., 6), whereas the correlation exponents computed for the surrogate data sets continue increasing with the increasing embedding dimension. It is worth consider another method for determining \( d_E \) that comes from asking the basic question addressed in the embedding theorem: when has one eliminated false crossing of the orbit with itself which arose by
virtue of having projected the attractor into a too low dimensional space? By examining this question in dimension one, then dimension two, etc. until there are no incorrect or false neighbours remaining, one should be able to establish, from geometrical consideration alone, a value for the necessary embedding dimension. Advanced version is presented in Refs. [8,9].

The Lyapunov’s exponents (LE) are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local LE. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of LE is considered a measure of the effect of perturbing the initial conditions of a dynamical system. In fact, if one manages to derive the whole spectrum of the LE, other invariants of the system, i.e. Kolmogorov entropy (KE) and attractor’s dimension can be found. The inverse of the KE is equal to an average predictability. Estimate of dimension of the attractor is provided by the Kaplan and Yorke conjecture:

\[
\sum_{j} \lambda_j = a + \frac{\sum_{j} \lambda_j}{|\lambda_{j+1}|}, \tag{8}
\]

where \( j \) is such that \( \sum_{a=1}^{j} \lambda_a > 0 \) and \( \sum_{a=1}^{j} \lambda_a < 0 \), and the LE \( \lambda_j \) are taken in descending order. There are a few approaches to computing the LE. One of them computes the whole spectrum and is based on the Jacobi matrix of system. In the case where only observations are given and the system function is unknown, the matrix has to be estimated from the data. In this case, all the suggested methods approximate the matrix by fitting a local map to a sufficient number of nearby points. To calculate the spectrum of the LE from the amplitude level data, one could determine the time delay \( \tau \) and embed the data in the four-dimensional space. In this point it is very important to determine the Kaplan-Yorke dimension and compare it with the correlation dimension, defined by the Grassberger-Procaccia algorithm. The estimations of the KE and average predictability can further show a limit, up to which the amplitude level data can be on average predicted. Other details can be found in Refs. [5-20].

### 3. Data on chaotic elements in time series of the radon concentration and conclusion

The concentration of atmospheric radon \(^{222}\text{Rn}\) was determined by measuring the activity of beta particles in atmospheric aerosol using radon monitors. Measurements of the radon concentrations at SMEAR II station (61 ° 51’N, 24 ° 17’E, 181 m above sea level; near the Hyytiälä, Southern Finland) was done by group of experts of the Finnish Meteorological Institute (FMI) and was actually integrated into the system long-term measurements (see details in Ref.[74] and [75-77] too). The continuous measurement was performed during 2000-2006 on the seventh heights (from 4.2 m to 127 m). Technologically for the detection of beta particles there are used the device with a pair of the Geiger-Müller counters, arranged in the lead corymbs. Registration of the beta particles was cumulatively carried in 10-minutes intervals. Effectiveness of a detection was 0.96% and 4.3% for beta radiation from \(^{214}\text{Pb}\) and \(^{214}\text{Bi}\) respectively. Estimate of the 1-σ statistical counting \( \pm 20\% \) for stable concentrations of \(^{222}\text{Rn}\) (1 Bq/m\(^3\)). The mean-daily values of atmospheric \(^{222}\text{Rn}\) concentrations were in the range from <0.1 to 11 Bq/m\(^3\). In fact, the lower limit of this range was limited by a hardware detection limit of the radon monitors. The corresponding data meet the log-normal distribution with a geometric mean of 2.5 Bq/m\(^3\) (a standard geometric deviation of 1.7 Bq/m\(^3\)). The average geometric value for the daily average radon concentrations was amounted to 2.3 to 2.6 Bq× m\(^3\) per year. In general during 2000-2006 as hourly, as daily values of a parameter, which corresponds to the radon concentration, were ranged from about 1 to 5 Bq/m\(^3\). In Figure 1 there is presented the typical time dependent curve of the radon concentration , received on the base of measurements at SMEAR II station (61 ° 51’N, 24 ° 17’E, 181 m above sea level; near the Hyytiälä, Southern Finland) (see [74]).
Below in Table 1 we list the results of computing different dynamical and topological invariants and parameters (Time delay $\tau$, correlation dimension ($d_2$), embedding space dimension ($d_E$), Lyapunov exponent ($\lambda_i$), Kolmogorov entropy ($K_{ent}$), Kaplan-York dimension ($d_L$), the predictability limit ($Pr_{max}$) and chaos indicator ($K_{ch}$)) for radon concentration time series (2001).

Table 1. Time delay $\tau$, correlation dimension ($d_2$), embedding space dimension ($d_E$), Lyapunov exponent ($\lambda_i$), Kolmogorov entropy ($K_{ent}$), Kaplan-York dimension ($d_L$), the predictability limit ($Pr_{max}$) and chaos indicator ($K_{ch}$) for radon concentration time series (2001)

| Year | $\tau$ | $d_2$ | $d_E$ |
|------|--------|-------|-------|
| 2001 | 12     | 5.48  | 6     |

| Year | $\lambda_i$ | $\lambda_2$ | $K_{ent}$ |
|------|--------------|--------------|-----------|
| 2001 | 0.0182       | 0.0058       | 0.024     |

| Year | $d_L$ | $Pr_{max}$ | $K$  |
|------|-------|------------|------|
| 2001 | 5.36  | 42         | 0.80 |

The resulting Kaplan-York dimension is very close to the correlation dimension, which is determined by the algorithm by Grassberger and Procaccia; Moreover, the Kaplan-York dimension is smaller than the dimension of attachment, which confirms the correctness of the choice of the latter. Therefore, using the new uniform chaos-dynamical approach we have carried out modeling and analysis of temporal evolution of the atmospheric radon $^{222}\text{Rn}$ concentration, firstly received data on topological and dynamical invariants for the time series of the $^{222}\text{Rn}$ concentration and discovered a deterministic chaos phenomenon. The results are of great theoretical and practical interest as for the dynamical systems and chaos theories for applied scientific applications such as nuclear physics, photoelectronics, atmospheric and environmental (environmental radioactivity) sciences etc.

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76. Jean-Louis Pinault, Jean-Claude Baubron Signal processing of diurnal and semidiurnal variations in radon and atmospheric pressure’ A new tool for
The work is devoted to the development of the theoretical foundations of new universal complex chaos-dynamical approach to analysis and prediction of the atmospheric radon concentration changing from beta particles activity data of radon monitors (with pair of the Geiger-Müller counters). The approach presented consistently includes a number of new or improved methods of analysis (correlation integral, fractal analysis, algorithms of average mutual information, false nearest neighbors, Lyapunov exponents, surrogate data, non-linear prediction schemes, spectral methods, etc.) to solve problems quantitatively complete modeling and analysis of temporal evolution of the atmospheric radon concentration. There are firstly received data on topological and dynamical invariants for the time series of the radon concentration, discovered a deterministic chaos phenomenon using detailed data of measurements of the radon concentrations at SMEAR II station of the Finnish Meteorological Institute.

**Key words:** chaotic dynamics, time series of the radon concentration, universal complex chaos-dynamical approach, analysis and prediction
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НЕЛИНЕЙНЫЙ ХАОС-ДИНАМИЧЕСКИЙ АНАЛИЗ ВРЕМЕННЫХ РЯДОВ
КОНЦЕНТРАЦИЙ АТМОСФЕРНОГО РАДОНА 222Rn НА ОСНОВЕ ДАННЫХ
АКТИВНОСТИ БЕТА ЧАСТИЦ РАДОНОВЫХ МОНИТОРОВ

Резюме

Работа посвящена разработке теоретических основ нового универсального комплексного хаос-динамического подхода к анализу и прогнозированию временных изменений концентрации атмосферного радона 222Rn на основе данных активности бета-частиц радоновых мониторов (с парой счетчиков Гейгера-Мюллера). Подход последовательно включает в себя ряд новых или улучшенных методов анализа (метод корреляционного интеграла, фрактальный анализ, алгоритмы средней взаимной информации, ложных ближайших соседей, показателей Ляпунова, схемы нелинейного прогнозирования, спектральные методы и т.д.) для решения проблемы количественно полного моделирования и анализа временной эволюции концентрации атмосферной радона 222Rn. Впервые получены данные о топологических и динамических инвариантах для временных рядов концентрации 222Rn, открыт феномен детерминированного хаоса, используя подробные данные измерений концентраций радона на станции SMEAR II Финского метеорологического института.

Ключевые слова: Хаотическая динамика, временные ряды концентрации 222Rn, универсальный комплексный хаос-динамический подход, анализ и прогнозирование

УДК 541.13

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НЕЛИНЕЙНЫЙ ХАОС-ДИНАМИЧЕСКИЙ АНАЛИЗ ЧАСТОВЫХ СЕРИЙ
КОНЦЕНТРАЦИЙ АТМОСФЕРНОГО РАДОНА 222Rn НА ОСНОВЕ ДАННЫХ
АКТИВНОСТИ БЕТА ЧАСТИЦ РАДОНОВЫХ МОНИТОРОВ

Резюме

Работа присвячена разработке теоретических основ нового универсального комплексного хаос-динамического подхода до анализу и прогнозированию часовых змін концентрації атмосферного радону 222Rn на основі даних активності бета-частинон радонових моніторів (з парою лічильників Гейгера-Мюллера). Підхід послідовно включає в себе ряд нових або поліпшених методів аналізу (метод кореляційного інтеграла, фрактальний аналіз, алгоритми середньо взаємної інформації, помилкових найближчих сусідів, показників Ляпунова, схеми неелінійного прогнозування, спектральні методи і т.і.) для вирішення проблеми кількісно повного моделювання та аналізу часової еволюції концентрації атмосферної радону 222Rn. Вперше отримані дані про топологічні і динамічні інваріанти для часовых рядів концентрації 222Rn, відкрито феномен детермінованого хаосу, використовуючи детальні дані вимірювань концентрації радону на SMEAR II станції Фінського метеорологічного інституту.

Ключові слова: Хаотична динамика, часові ряди концентрації 222Rn, універсальний комплексний хаос-динамічний підхід, аналіз і прогнозування