The nature of near-threshold XYZ states

Martin CLEVEN\textsuperscript{1}, Feng-Kun GUO\textsuperscript{2,3}, Christoph HANHART\textsuperscript{4}, Qian WANG\textsuperscript{4} and Qiang ZHAO \textsuperscript{1}

\textsuperscript{1}Institute of High Energy Physics and Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{2}Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany
\textsuperscript{3}State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, CAS, Beijing 100190, China
\textsuperscript{4}Institut für Kernphysik and Institute for Advanced Simulation, Forschungszentrum Jülich, D-52425 Jülich, Germany

E-mail: q.wang@fz-juelich.de

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We demonstrate that the recently observed X, Y, Z states cannot be purely from kinematic effect. Especially the narrow near-threshold structures in elastic channels call for nearby poles of the S-matrix which are qualified as states. We propose a way to distinguish cusp effects from genuine states and demonstrate that (not all of) the recently observed X, Y, Z states cannot be purely from kinematic effects. Especially, we show that the narrow near-threshold structures in elastic channels call for nearby poles of the S-matrix, since the normal kinematic cusp effect cannot produce that narrow structures in the elastic channels in contrast to genuine S-matrix poles. In addition, it is also discussed how spectra can be used to distinguish different scenarios proposed for the structure of those poles, such as hadro-quarkonia, tetraquarks and hadronic molecules. The basic tool employed is heavy quark spin symmetry.

KEYWORDS: Exotic mesons, Meson-meson interactions

1. Introduction

In the last decade many narrow structures were observed which do not fit into the well-established quark model in both charmonium and bottomonium sectors. Most of them are located close to nearby S-wave open flavor thresholds. For instance, in the charmonium sector X(3872) and Z\textsubscript{c}(3900) are close to the DD\textsuperscript{*} (here and in the following, one of the two open-flavor mesons contains a heavy quark and the other contains a heavy anti-quark) threshold and Z\textsubscript{c}(4020) is close to the D\textsuperscript{*}D\textsuperscript{*}, while in the bottomonium sector the Z\textsubscript{b}(10610) and Z\textsubscript{b}(10650) are close to BB\textsuperscript{*} and B\textsuperscript{*}B\textsuperscript{*}, respectively. Due to their proximity to the thresholds, various groups conclude that those structures are simply kinematical effects [1, 2] (and references therein) which occur near every S-wave threshold. In contrast to this there are many publications where the experimental signatures are interpreted as states with various suggestions for the underlying structure. The most prominent proposals are hadro-quarkonia [3, 4], hybrids [5–7], tetraquarks [8–14] and hadronic molecules [15–33].

This contribution consists of two parts. In the first part we demonstrate that although there is always a cusp at the opening of an S-wave threshold, it cannot produce a narrow pronounced structure in the elastic channel (the channel where the final state agrees to the channel that produces the cusp) from a perturbative rescattering [34, 35]. As a consequence of this, the narrow structures in elastic channels as observed in experiments necessarily call either for nonperturbative interactions between heavy mesons that lead to poles of the S-matrix or call for poles formed on the quark level. Having established that we need to talk about genuine states, in the second part of this contribution, we
employ heavy quark spin symmetry (HQSS) to distinguish three different dynamical models, i.e. hadro-quarkonia, tetraquarks and hadronic molecules based solely on their mass spectra [36].

2. Kinematic effect or S-matrix pole?

In this section, we analyse the existing data of $Z_c(3900)$ observed in $Y(4260) \rightarrow \pi DD^*$ to illustrate our argument about how to distinguish the presence of an S-matrix pole from a purely kinematic effect. However, it should be clear that the conclusion is more general and can be applied to all narrow structures near $S$-wave thresholds such as those $XYZ$ states mentioned above. To illustrate this point, we do not aim at field theoretical rigor but use a separable interaction for all vertices,

$$\mathcal{L}_I = g_Y \pi (DD^*)^\dagger Y^\mu + \frac{C}{2} (DD^*)^\dagger (DD^*) + \cdots,$$

and a Gaussian regulator

$$f_\Lambda(\vec p^2) = \exp\left(-2\vec p^2/\Lambda^2\right),$$

for each loop. Here $Y^\mu$, $D$, $D^*_f$ and $\pi$ are the fields for the $Y(4260)$, $D$, $D^*$ and $\pi$, respectively. The coupling $g_Y$ is the source term strength and $C$ is the $DD^*$ rescattering strength. It is the size of this parameter that will eventually allow us to decide whether the rescattering effect is perturbative or not. In the regulator, $\vec p$ is the three momentum of the $D$-meson in the center-of-mass (c.m.) frame of the $DD^*$ system and $\Lambda$ is the cut-off parameter. Therefore, the loop function reads

$$G_\Lambda(E) = \int \frac{d^3q}{(2\pi)^3} \frac{f_\Lambda(\vec q^2)}{E - m_1 - m_2 - \vec q^2/(2\mu)} = \frac{\mu \Lambda}{(2\pi)^{3/2}} + \frac{\mu k}{2\pi} e^{-2k^2/\Lambda^2} \left[ \text{erfi}\left(\frac{\sqrt{2}k}{\Lambda}\right) - i \right]$$

where $k = \sqrt{2\mu(E - m_1 - m_2)}$, and $\text{erfi}(z) = 2/(\sqrt{\pi}) \int_0^z e^{t^2} dt$.

For the kinematic explanations of these $XYZ$ states [1, 2], it was claimed that the sum of tree-level (Fig.1 (a)) and one-loop (Fig.1 (b)) diagram can describe the invariant mass distribution quantitatively. In other words, the claim is that the rescattering process is perturbative and one does not need to add also the higher order loops. We confirm the result of the calculation by employing the tree-level diagram plus the one-loop diagram to fit the new data from BESIII [37] as shown by the red solid curve in Fig. 2 (a). The fitted parameters are

$$g_Y = 40.01 \text{ GeV}^{-3/2}, \quad C = 187.37 \text{ GeV}^{-2}, \quad \Lambda = 0.257 \text{ GeV}.$$
Fig. 2. The $DD^*$ invariant mass distribution in $Y(4260) \to \pi DD^*$. The data are from Ref. [37] and the green dotted, red solid, pink dashed and brown long dashed curves are the contributions from the tree level, full one-loop, full two-loop and the sum of all the loops, respectively. The dot-dashed line shows the full one-loop result with the strength of the rescattering requested to be small enough to justify a perturbative treatment as described in the text. Figure (a) and Figure (b) show the results with the full one-loop and the sum of all the loops fitted to the experimental data, respectively. The events below the $DD^*$ threshold are from the energy resolution that is not included in the theoretical calculations.

(Fig. 1 (a) + (b) + (c)) as dashed curve in Fig. 2 (a): The deviation of this curve from the full one-loop contribution is much larger than what is allowed in a perturbative scheme. In addition: a summation of the bubble leads to a bound state with a 9 MeV binding energy. This clearly illustrates that restricting oneself to a single rescattering is not self-consistent.

Turning the argument around, we can see that the perturbative requirement, namely that the two-loop contribution is at most half of the one-loop contribution, cannot produce the pronounced near-threshold structure as shown by the dot-dashed line in Fig. 2 (a). The crucial point of this analysis is that in the elastic channel the contribution from the source term (Fig. 1 (a)), as shown by the dotted curve in Fig. 2 (a), can be disentangled from the $DD^*$ rescattering process Fig. 1 (b). The former one is fixed by the $DD^*$ invariant mass distribution above $\sim 3.94$ GeV and the latter one is used to explain the near-threshold structure.

As discussed above, since the narrow near-threshold structure means a nonperturbative rescattering process, one needs to sum all the bubble loops to infinite order. Doing this to fit the data we find for the parameters

$$g_Y = 33.85 \text{ GeV}^{-3/2}, \quad C = 20.09 \text{ GeV}^{-2}, \quad \Lambda = 0.89 \text{ GeV}. \quad (5)$$

The fit results are shown in Fig. 2 (b). With the above fitted parameters, we find a pole below the $DD^*$ threshold with the binding energy 0.59 MeV. This demonstrates that a narrow pronounced near-threshold structure in the elastic channel requires a pole in the $S$-matrix and cannot be produced from kinematic effects alone.

Our reasoning is in contrast to that of Ref. [2], which is to our knowledge the only work so far that claims a kinematic origin for the near-threshold structures and also looks at the elastic channel. It should be stressed that in this work, while the structures in the inelastic channels are explained as cusps, those in the elastic channels come solely from the form factors employed. In this way our argument presented above is evaded. However, from our point of view the problem of this explanation is that it appears unnatural to explain both the narrow structure as well as the higher energy tail in the elastic channels as form factors because this calls for the simultaneous presence of drastically different length scales in the production vertex. We therefore do not regard the mechanism of Ref. [2]
as a plausible explanation for the near threshold structures observed. Thus, for the rest of these proceedings we regard it as established that the \( XYZ \) structures are states and discuss a certain proposal, how their internal structure could be disentangled experimentally.

3. Different scenarios for the \( XYZ \) states

Having established that the \( XYZ \) states require a near-threshold pole structure in the elastic channel we now discuss how to distinguish different models for these poles such as hadro-quarkonia, tetraquarks and hadronic molecules [36].

3.1 Hadro-quarkonia

The hadro-quarkonium picture was proposed by Voloshin [3, 4] based on the fact that several exotic candidates mainly decay into a heavy quarkonium plus light hadrons. Examples are the \( Y(4260) \) discovered in \( J/\psi \pi \pi \) channel, the \( Z_c(4430) \) discovered in \( \psi' \pi \) channel, and the \( Y(4360) \) and \( Y(4660) \) observed in \( \psi' \pi \pi \) channel. The basic idea is that the \( XYZ \) states contain a compact heavy-quarkonium core surrounded by a light-meson cloud. Based on the recent measurement that the cross sections for \( J/\psi \pi^+ \pi^- \) and \( h_c \pi^+ \pi^- \) at 4.26 GeV and 4.36 GeV in \( e^+e^- \) collisions are of similar size, Li and Voloshin include HQSS breaking to describe the \( Y(4260) \) and \( Y(4360) \) as a mixture of two hadro-charmonia [38]:

\[
Y(4260) = \cos \theta \psi_3 - \sin \theta \psi_1, \quad Y(4360) = \sin \theta \psi_3 + \cos \theta \psi_1,
\]

where \( \psi_1 \sim (1^+)_c \bar{c} \otimes (0^-)_{q\bar{q}} \) and \( \psi_3 \sim (1^-)_c \bar{c} \otimes (0^+)_h \) are the wave functions of the \( J^{PC} = 1^{--} \) hadro-charmonia with a \( 1^{+-} \) and \( 1^{--} \) \( c\bar{c} \) core charmonium, respectively. Since the leading order interaction between the heavy core and the light meson cloud is not dependent on the spin of the heavy core, spin partners for the heavy hadro-quarkonia can be identified by replacing the core with the corresponding spin partner [36].

Accordingly, the spin partners of \( Y(4260) \) and \( Y(4360) \) are found by replacing the \( \psi' \) in the core of \( \psi_3 \) by \( \eta'_c \) and replacing the \( h_c \) in the \( \psi_1 \) state by any of the three \( \chi_{cJ} \) states. The relative mixing amplitude can be obtained by constructing an \( CP \)-odd operator with the HQSS breaking [36]

\[
O_{\text{mixing}} = \frac{1}{4} (\chi^{\dagger} \cdot \sigma J') + \text{h.c.} = \vec{h}_c' \cdot \vec{\psi}' + \sqrt{3} \chi^{\dagger}_{c0} \eta'_c + \text{h.c.}.
\]

Hence, the mixing amplitude of the pseudoscalar sector is larger by a factor \( \sqrt{3} \) than that in the vector sector. The resulting masses of these hadro-charmonia are shown in Fig. 3 as an illustration. We notice
that the interpretation of $Y(4260)$ and $Y(4360)$ as mixed hadron-charmonia implies the existence of two states with $J^{PC} = 0^{-+}$.

The search for these partners will provide more insights into the nature of the two $Y$ states. For the two pseudoscalar states, i.e. $\eta_c(4140)$ and $\eta_c(4320)$, the large mixing amplitude leads to masses that allow both of them to decay into $\eta_c^{(')} \pi \pi$ and $\chi_{c0} \pi \pi$. Because of their quantum numbers the states cannot be produced directly in $e^+e^-$ collisions. An alternative way for searching for them is in $B$ meson decays, e.g. $B^\pm \rightarrow K^\pm \eta_c^{(')} \pi^+ \pi^-$ as suggested in Ref. [39] for the search of the spin partner of the $Y(4660)$. Another way to search for these two pseudoscalars is the radiative decays of $Y(4260)$ and $Y(4360)$ via their $\psi'$ component. Since the branching ratio of $\psi' \rightarrow \gamma \chi_{c0}$ is two orders of magnitude larger than that for $\psi' \rightarrow \gamma \eta_c^{(')}$, one could expect to observe these two states in the $e^+e^- \rightarrow \gamma \chi_{c0} \eta$ process at center-of-mass energies around the masses of the $Y(4260)$ and the $Y(4360)$. The states $\eta_c(4310)$ and $\eta_c(4350)$ can be searched for in analogous processes in the decays of $Y(4360)$ with the $\chi_{c0}$ in the final state replaced by the $\chi_{c1}$ and $\chi_{c2}$, respectively. The search for the above states can be performed at BESIII or a future high-luminosity super tau-charm factory.

### 3.2 Tetraquarks

Among the different tetraquark scenarios we here focus on the compact diquark-antidiquark states suggested by Maiani et al. [8] for simplicity. It is a straightforward extension of the quark model. Since the four-quark system is bound by the effective gluon exchanges, the isoscalar tetraquark is mostly degenerate with the isovector tetraquark similar to the $\rho-\omega$ degeneracy in the light meson sector.

In this model, the mass of a tetraquark is given by [8, 36]

$$M = M_{00} + B_c \frac{L(L+1)}{2} + a[L(L+1) + S(S+1) - J(J+1)] + \kappa_{cq} [s(s+1) + \bar{s}(ar{s}+1) - 3],$$

with $s$ and $\bar{s}$ the total spin of diquark and antidiquark system, $S$ the total spin, $L$ the relative orbital angular momentum between diquark and antidiquark system, $J$ the total angular momentum. The parameters $B_c$, $a$ and $\kappa_{cq}$ are defined such that they are positive to fit to the masses of selected states. As a result, the mass of the tetraquarks increases with increasing $L$ and $S$, but decreases for growing $J$, which is a rather unusual feature for composite systems.

As discussed above, the most striking feature is a very rich spectroscopy emerging in the tetraquark model, as shown in Fig. 4. In what follows, we will limit the discussion of the implications of the model to $S$-wave and $P$-wave tetraquark states only [36]. Since the isospin singlet and triplet are almost degenerate, there should be in total 24 $S$-wave tetraquark states even without considering radial excitations, which are expected to be about 400 MeV heavier than the corresponding ground states [8]. Among the ground states, the authors of Ref. [8] identified the $X(3872)$ as the isoscalar...
1++ ground state and \( Z_c(3900) \) and \( Z_c(4020) \) as the iso-triplet 1++ ground states, respectively. One may also assign \( X(3915) \) and \( X(3940) \) as 0++ and 2++, respectively [8], although there are sizeable deviations between the predicted and the empirical values of the masses, cf. Fig. 4. Therefore, at least 15 more \( S \)-wave tetraquarks still need to be observed or identified.

For the \( P \)-wave tetraquarks, there are four iso-singlet 1-- states without radial excitations. Three of them are identified as the \( Y(4008), Y(4260) \) and \( Y(4630) \), and the other one was identified as one of two structures, called \( Y(4220) \) and \( Y(4290) \) observed in \( e^+e^- \rightarrow h_\pi^+\pi^- \) [8]. The states \( Y(4360) \) and \( Y(4660) \) were assigned to be the radial excitations of the \( Y(4008) \) and \( Y(4260) \) [8]. Thus, besides the first radial excitations, only 6 of 112 (28 if considering only the iso-singlet ones) \( P \)-wave tetraquark states have candidates so far. As shown in Fig. 4 (b), there are two exotic quantum numbers, i.e. 0-- and 1--, and two 0++ states which might mix with each other. Since the mass decreases with the increasing \( J \), a rather light charmonium(-like) state with \( J = 3 \) is the particular feature of the discussed tetraquark picture [8].

3.3 Hadronic molecules

A hadronic molecule is an extended object which is composed of two or more narrow hadrons via their nonperturbative interactions. Because of their relatively narrow widths the ground state hadrons have a long enough life time to form a bound state. Since some of the XYZ states are close to \( S \)-wave thresholds and strongly couple to the corresponding continuum states, an interpretation as hadronic molecules appears quite natural. In this contribution, we focus on the interaction of the members of two narrow charmed meson doublets with one of them contain a \( c \) quark. Those are characterized by the quantum numbers of their light degrees of freedom, namely \( s^D = \frac{1}{2} \) for the doublet \((D, D^*)\) and \( s^D = \frac{3}{2} \) for the doublet \((D_1(2420), D_2(2460))\). In particular, we discuss the \( \frac{1}{2} + \frac{1}{2} \) and \( \frac{1}{2} + \frac{3}{2} \) hadronic molecules. There are two kinds of interaction between them: the long-ranged one-pion exchange and short-ranged interactions. The latter need to be fixed by a fit to experimental data. Since there are not enough data available, for now we mostly restrict the discussion to qualitative statements based on the pion exchange potential. As a result, instead of a prediction for the spectrum we show the potentially relevant thresholds for the \( \frac{1}{2} + \frac{1}{2} \) molecules and \( \frac{1}{2} + \frac{3}{2} \) molecules (cf. Fig.5). However, this already allows us to highlight some striking features of hadronic molecules which can be used to distinguish them from the hadro-charmonium and tetraquark scenarios.

There is one exception to this lack of predictive power: as discussed in [29], to leading order the potentials for 1++ and 2++ are identical, which means an iso-singlet hadronic molecule with quantum number 2++ near the \( D^*D^* \) threshold would have a large probability to exist if the \( X(3872) \) is assigned as an iso-singlet \( DD^* \) molecule.

As is well-known the one-pion exchange potential has different signs in the iso-singlet and iso-triplet channels. Hence it is natural that an iso-triplet state does not exist if the \( D \) meson pair forms an iso-singlet and vice versa. This means we do not expect any iso-triplet state with quantum numbers 1++ or 2++. Due to the same reason, once \( Z_c(3900) \) and \( Z_c(4020) \) are assigned as isovector \( DD^* \) and \( D^*D^* \) hadronic molecules, one would not expect to find isoscalar hadronic molecules with quantum number 1++ at the similar mass region. For the 0++ channel the absence of the one-pion exchange in the \( DD \) diagonal potential prevents us from speculating about the existence of molecular structures with these quantum numbers.

For the \( \frac{1}{2} + \frac{3}{2} \) system, because there are two kinds of one-pion exchange contributions, i.e. \( t \)-channel and \( u \)-channel exchange diagrams. They depend on different products of unknown coupling constants, and thus no strong conclusion can be drawn regarding the existence or non-existence of these states. Yet, one may still have reasonable speculations without dynamical analysis. For instance, since hadronic molecules exist near the \( S \)-wave thresholds, the lightest \( \frac{1}{2} + \frac{3}{2} \) molecular state is expected be close to the \( D_1D \) threshold (cf. Fig. 5), and have quantum numbers of either 1-- or 1++. If a 0-- molecule exists, it should be around the \( D_1D^* \) threshold which is similar to that in the
The two-body thresholds in the charmonium mass range potentially related to the formation of hadronic molecules with fixed quantum numbers. The left and right panel are for \( \frac{1}{2} + \frac{1}{2} \) and \( \frac{1}{2} + \frac{3}{2} \) hadronic molecules, respectively.

tetraquark picture and very different from that in the hadro-charmonium picture where the \( 0^{--} \) state is expected to be the lightest one. Another way to distinguish between molecular and tetraquark scenarios is the location of the \( J = 3 \) state. It will be the lightest state in the tetraquark picture, but close to the \( D_2D^* \) threshold (cf. Fig. 5) in the molecular picture.

4. Summary and Outlook

In this contribution, we demonstrate that a narrow pronounced peak in the elastic channel cannot be produced by purely kinematic effects. A consistent treatment of these narrow structures necessarily calls for poles in the \( S \)-matrix which correspond to physical states. To further explore the nature of these XYZ states, namely whether they are hadro-charmonia, tetraquarks or hadronic molecules, we study their spectra in those different scenarios using mainly HQSS and identify their distinctive features. The prominent components of those puzzling states need to be identified by further experiments and detailed analysis, which will deepen our understanding of QCD in the nonperturbative regime.

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References

[1] E. S. Swanson, arXiv:1409.3291 [hep-ph].
[2] E. S. Swanson, arXiv:1504.07952 [hep-ph].
[3] M. B. Voloshin, Prog. Part. Nucl. Phys. 61, 455 (2008).
[4] S. Dubynskiy and M. B. Voloshin, Phys. Lett. B 666, 344 (2008).
[5] S.-L. Zhu, Phys. Lett. B 625, 212 (2005).
[6] E. Kou and O. Pene, Phys. Lett. B 631, 164 (2005).
[7] F. E. Close and P. R. Page, Phys. Lett. B 628, 215 (2005).
[8] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 89, 114010 (2014) [arXiv:1405.1551 [hep-ph]].
[9] L. Maiani, V. Riquer, R. Faccini, F. Piccinini, A. Pilloni and A. D. Polosa, Phys. Rev. D 87, no. 11, 111102 (2013) [arXiv:1303.6857 [hep-ph]].
[10] H. Högaasen, J. M. Richard and P. Sorba, Phys. Rev. D 73, 054013 (2006) [hep-ph/0511039].
[11] F. Buccella, H. Høgaasen, J. M. Richard and P. Sorba, Eur. Phys. J. C 49, 743 (2007) [hep-ph/0608001].
[12] T. Guo, L. Cao, M. Z. Zhou and H. Chen, arXiv:1106.2284 [hep-ph].
[13] H. Høgaasen, E. Kou, J. M. Richard and P. Sorba, Phys. Lett. B 732, 97 (2014) [arXiv:1309.2049 [hep-ph]].
[14] F. Stancu, J. Phys. G 37, 075017 (2010) [arXiv:0906.2485 [hep-ph]].
[15] N. A. Törnqvist, Phys. Lett. B 590, 209 (2004) [hep-ph/0402237].
[16] S. Fleming, M. Kusunoki, T. Mehen and U. van Kolck, Phys. Rev. D 76, 034006 (2007) [hep-ph/0703168].
[17] C. E. Thomas and F. E. Close, Phys. Rev. D 78, 034007 (2008) [arXiv:0805.3653 [hep-ph]].
[18] G.-J. Ding, Phys. Rev. D 79, 014001 (2009).
[19] I. W. Lee, A. Faessler, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D 80, 094005 (2009) [arXiv:0910.1009 [hep-ph]].
[20] Y. Dong, A. Faessler, T. Gutsche, S. Kovalenko and V. E. Lyubovitskij, Phys. Rev. D 79, 094013 (2009) [arXiv:0903.5416 [hep-ph]].
[21] E. Braaten and J. Stapleton, Phys. Rev. D 81, 014019 (2010) [arXiv:0907.3167 [hep-ph]].
[22] D. Gamermann and E. Oset, Phys. Rev. D 80, 014003 (2009) [arXiv:0905.0402 [hep-ph]].
[23] T. Mehen and R. Springer, Phys. Rev. D 83, 094009 (2011) [arXiv:1101.5175 [hep-ph]].
[24] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk and M. B. Voloshin, Phys. Rev. D 84, 054010 (2011) [arXiv:1105.4473 [hep-ph]].
[25] J. Nieves and M. P. Valderrama, Phys. Rev. D 84, 056015 (2011) [arXiv:1106.0600 [hep-ph]].
[26] J. Nieves and M. P. Valderrama, Phys. Rev. D 86, 056004 (2012) [arXiv:1204.2790 [hep-ph]].
[27] Q. Wang, C. Hanhart and Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013).
[28] Q. Wang, M. Cleven, F.-K. Guo, C. Hanhart, U.-G. Meißner, X.-G. Wu and Q. Zhao, Phys. Rev. D 89, 034001 (2014) [arXiv:1309.4303 [hep-ph]].
[29] F.-K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D 88, 054007 (2013) [arXiv:1303.6608 [hep-ph]].
[30] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang and Q. Zhao, Phys. Lett. B 725, 127 (2013) [arXiv:1306.3096 [hep-ph]].
[31] T. Mehen and J. Powell, Phys. Rev. D 88, 034017 (2013) [arXiv:1306.5459 [hep-ph]].
[32] J. He, X. Liu, Z. F. Sun and S. L. Zhu, Eur. Phys. J. C 73, 2635 (2013) [arXiv:1308.2999 [hep-ph]].
[33] X. H. Liu, L. Ma, L. P. Sun, X. Liu and S. L. Zhu, Phys. Rev. D 90, 074020 (2014) [arXiv:1407.3684 [hep-ph]].
[34] C. Schmid, Phys. Rev. 154, 1363 (1967).
[35] F. K. Guo, C. Hanhart, Q. Wang and Q. Zhao, Phys. Rev. D 91, no. 5, 051504 (2015) [arXiv:1411.5584 [hep-ph]].
[36] M. Cleven, F. K. Guo, C. Hanhart, Q. Wang and Q. Zhao, Phys. Rev. D 92, no. 1, 014005 (2015) [arXiv:1505.01771 [hep-ph]].
[37] M. Ablikim et al. [BESIII Collaboration], arXiv:1509.01398 [hep-ex].
[38] X. Li and M. B. Voloshin, Mod. Phys. Lett. A 29, 1450060 (2014) [arXiv:1309.1681 [hep-ph]].
[39] F.-K. Guo, C. Hanhart and U.-G. Meißner, Phys. Rev. Lett. 102, 242004 (2009) [arXiv:0904.3338 [hep-ph]].