A Novel MS-MeMBer Filter for Extended Targets Tracking

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ABSTRACT

Conventional multi-sensor multi-target multi-Bernoulli (MS-MeMBer) filters are based on the assumption that each target produces at most one measurement per time step. However, this assumption is not always reasonable in practice as an extended target can generate multiple measurements per step due to the recent improvement in the sensor resolution. In this case, a potential estimation bias may occur in the current MS-MeMBer filters. Therefore, a novel extended target MS-MeMBer filter and its Gaussian inverse Wishart mixture implementation are given in this paper. Specifically, we modify the update process of the MS-MeMBer filter by assuming that the generation of extended target measurements follows an approximate Poisson-Body model. Simulation results validate that the proposed filter can effectively estimate the shape and position of the extended target.

INDEX TERMS

Multi-sensor multi-target multi-Bernoulli filter, extended target, approximate Poisson-body, Gaussian inverse Wishart.

I. INTRODUCTION

The random finite set (RFS) [1] has received much attention in the multi-target filtering domain due to its superiority in avoiding complicated data association steps [2]–[12]. Under the RFS framework, the target state estimation is transformed into a set-valued estimation problem. For single-sensor scenarios, the probability hypothesis density (PHD), the cardinalized PHD (CPHD), and the multi-Bernoulli filter were proposed since the optimal multiple target tracking is intractable [2]–[5]. The significant difference between the CPHD/PHD filter and the multi-Bernoulli filter is that the PHD and CPHD filters are moment-based approximations to the multi-target Bayes filter, while the multi-Bernoulli filter approximates the multi-target posterior as a multi-Bernoulli distribution. To obtain the target trajectories, B. T. Vo and B. N. Vo introduced a generalized labeled multi-Bernoulli (GLMB) filter, which can generate target track labels [6]. Subsequently, compared to [6], a more efficient labeled multi-Bernoulli (LMB) filter was designed in [7] by only allowing the propagation of a single set of track labels.

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For two-sensor scenarios, Mahler [8] proposed an accurate PHD filter based on binary segmentations. To process measurements from an arbitrary number of sensors, a parallel-combination approximate multi-sensor (PCAM) CPHD/PHD filter was presented in [9] and then an exact multi-sensor CPHD/PHD (MS-CPHD/PHD) filter was given in [10]. In [11], Saucan introduced a multi-sensor multi-target multi-Bernoulli (MS-MeMBer) filter via a Gaussian mixture (GM) implementation. Similarly, a generalization of the GLMB filter for the multi-sensor case, named multi-sensor GLMB (MS-GLMB) filter, was proposed in [12].

The filters above obey the “standard” observation model, that is, one target generates at most one measurement per time step, and each measurement originates from at most one target. However, in the high-resolution sensor system, one target may occupy multiple resolution cells and thus can produce multiple measurements. Such targets are known as extended targets [13], [14]. For the extended target tracking, Gilholm et al. proposed an approximate Poisson-Body (APB) model [15], in which the measurement is assumed to follow a multi-dimensional Poisson process. Based on the APB model, an extended target PHD
A novel MS-MeMBer filter for extended targets tracking is proposed in [16], and a GM implementation of it was given in [17]. Extensions of the ET-PHD filter for handling the shape estimation by using the random matrix model (RMM) [18]–[21] were given in [22], [23], and the resulting filters are the Gaussian inverse Wishart PHD (GIW-PHD) filter and the gamma Gaussian inverse Wishart PHD (GGIW-PHD) filter. In [22], the target kinematical state was defined by a Gaussian distribution, while the target extension was assumed to follow an inverse Wishart distribution. As an improvement on the GIW-PHD filter, the GGIW-CPHD (GGIW-PHD) filter can also obtain the estimation of target measurement rates. Besides, other filters based on the RFS, which include the gamma Gaussian inverse Wishart CPHD (GGIW-CPHD) filter, the extended target MeMBer filter, and the GGIW-GLMB/LMB filter, have also been introduced for the extended target case [24]–[27]. Extended models such as the random hypersurface model [28] and the star-convex [29] extended target case [24]–[27]. Extended models such as the random hypersurface model [28] and the star-convex [29] extended target case [24]–[27]. Extended models such as the random hypersurface model [28] and the star-convex [29] extended target case [24]–[27].

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The rest of this paper is organized as follows. In Section II, we present an overview of the APB model, the multi-Bernoulli RFS, and the MS-MeMBer filter. Our proposed ET-MS-MeMBer filter is derived in Section III. The GIW mixture implementation is provided in Section IV. Simulation experiments are presented in Section V. Conclusions are given in Section VI.

II. BACKGROUND

A. THE APB MODEL FOR EXTENDED TARGETS

In the scenario of tracking extended targets, a target may generate multiple measurements. Assume at time $k$, the measurement $z$ generated by the target with state $x$ on sensor $i$ obeys the spatial distribution $f_i(z) = h_{i,k}(z|x)$, and the number of extended measurements is Poisson-distributed with mean $\gamma_i(x)$. Then the likelihood function of $x$ can be denoted by

$$ f_{i,k}(Z|x) = e^{-\gamma_i(x)}\gamma_i(x)|Z| \prod_{z \in Z} f_i(z), \quad (1) $$

where $Z$ is a collection of all measurements generated by target $x$ on sensor $i$ at time $k$, and $|Z|$ is the cardinality of set $Z$.

The probability generation functional (PGFL) of $f_{i,k}(Z|x)$ is

$$ G_k[g|x] = e^{p_i(x)\gamma_i(x)-\gamma(x)}, \quad (2) $$

where $p_i(x) = \int g(z)h_{i,k}(z|x)dz$.

B. THE MULTI-BERNOULLI RFS

The Bernoulli RFS $X_B$ can be represented by the probability of existence $r$ and the density $p(x)$, then the distribution function is

$$ \pi(X_B) = \begin{cases} 1 - r, & X_B = \emptyset \\ r \cdot p(x), & X_B = \{x\}, \end{cases} \quad (3) $$

which can be further simplified to $\pi(X_B) = (r \cdot p)$.

The PGFL of $\pi(X_B)$ is

$$ G[h] = 1 - r + r \cdot (p \cdot h), \quad (4) $$

where $(p \cdot h) = \int p(x)h(x)dx$.

The PHD of $\pi(X_B)$ is

$$ D(x) = \frac{\delta G[h]}{\delta x} \bigg|_{h=1} = r \cdot p(x). \quad (5) $$

The multi-Bernoulli RFS $X_{MB}$ is a union of a fixed number of independent Bernoulli RFSs. Given an RFS set $X_{MB} = \{x_1, \ldots, x_n\}$, the distribution function is

$$ \pi([x_1, \ldots, x_n]) = \prod_{j=1}^{M} \left(1 - r^{(j)}\right) \times \sum_{1 \leq i_1 \neq \ldots \neq i_n \leq M} \prod_{j=1}^{n} \frac{r^{(j)}p^{(j)}(x_j)}{1 - r^{(j)}}, \quad (6) $$

where $M$ is the number of Bernoulli components. In this paper, the abbreviation $\pi(X_{MB}) = \{(r^{(j)}, p^{(j)})\}_{j=1}^{M}$ is adopted for the multi-Bernoulli distribution function. The PGFL of $\pi(X_{MB})$ is

$$ G[h] = \prod_{j=1}^{M} \left(1 - r^{(j)} + r^{(j)} \cdot p^{(j)}(h)\right). \quad (7) $$
The PHD of \( \pi(X_{MB}) \) is
\[
D(x) = \frac{\delta G[h]}{\delta x} \bigg|_{h=1} = \sum_{j=1}^{M} r^{(j)} \cdot p^{(j)}(x),
\]
(8)

C. THE MS-MEMBER FILTER

The prediction step of the MS-MeMBer filter is the same as that of the multi-Bernoulli filters [4], [5]. Suppose that at time \( k - 1 \), the posterior multi-target density is a multi-Bernoulli with
\[
\pi_{k-1|k-1} = \bigg\{ (r^{(i)}_{S;k|k-1}, p^{(i)}_{S;k|k-1}) \bigg\}_{i=1}^{M_{k-1}},
\]
and we make the following assumptions:
- The target can survive to the next step with a probability of \( \rho_{Sv} \);
- At time \( k \), the target is born with a probability of \( r_{bh,k} \) and a spatial density of \( p_{bh,k} \).

Then, the predicted density is also a multi-Bernoulli with
\[
\pi_{k|k-1} = \bigg\{ (r^{(j)}_{S;k|k-1}, p^{(j)}_{S;k|k-1}) \bigg\}_{j=1}^{M_{k-1}+M_{bh,k}},
\]
(9)

where \( M_{bh,k} \) is the number of newborn targets. The existence and the density of the survival target are given as follows
\[
r^{(j)}_{S;k|k-1} = r^{(j)}_{k-1|k-1} \cdot p^{(j)}_{k-1|k-1, \rho_{Sv}},
\]
(11)
\[
p^{(j)}_{S;k|k-1}(x) = \frac{f_{k-1|k-1}(x), \rho_{Sv} p^{(j)}_{k-1|k-1}}{r^{(j)}_{k-1|k-1, \rho_{Sv}}},
\]
(12)

where \( f_{k-1|k-1}(x) \) denotes the transfer function.

Assume that the predicted multi-target density is a multi-Bernoulli with
\[
\pi_{k|k-1} = \bigg\{ (r^{(j)}_{k|k-1}, p^{(j)}_{k|k-1}) \bigg\}_{j=1}^{M_{k|k-1}}.
\]
(13)

Then the updated density can be approximated by a multi-Bernoulli with
\[
\hat{\pi}_{k|k} = \bigcup_{P \in Q} \bigcup_{j=1}^{M_{k|k-1}} \bigg\{ (r^{(j)}_{P}, p^{(j)}_{P}) \bigg\},
\]
(14)

where each of the updated Bernoulli components \( (r^{(j)}_{P}, p^{(j)}_{P}) \) is given as follows
\[
r^{(j)}_{P} = \begin{cases} \alpha P \cdot \frac{r^{(j)}_{k-1|k-1} \cdot p^{(j)}_{k-1|k-1, \eta}}{1 - r^{(j)}_{k-1|k-1} + r^{(j)}_{k-1|k-1} (p^{(j)}_{k-1|k-1, \eta})}, & \text{if } W_{1:j} = \emptyset \ \text{and } W_{1:j} \neq \emptyset, \\ \alpha P, & \end{cases}
\]
(15)

The ET-MS-MeMBer filter is the same as that of the MS-MeMBer filter. Therefore, only the update step of the ET-MS-MeMBer filter is given in this section.

III. THE EXTENDED TARGET MS-MEMBER FILTER

In this section, we present the proposed ET-MS-MeMBer filter. Since no measurement information is used in the prediction process, the prediction step of the ET-MS-MeMBer filter is the same as that of the MS-MeMBer filter. Therefore, only the update step of the ET-MS-MeMBer filter is given in this section.

A. MULTI-SENSOR MEASUREMENT PARTITIONING

In the update process of the ET-MS-MeMBer filter, all the partitioning forms of the multi-sensor measurements need to be obtained. Measurement partitioning aims to divide a measurement set into a finite number of mutually disjoint subsets. The principle is to put homologous measurements into the same subset. Suppose that at time \( k \), the measurement set generated by all sensors is \( Z_{1:k} = \{ Z_{1}, \ldots, Z_{S} \} \), where each \( Z_{i} \) contains all measurements from the \( i \)-th sensor. According to the number of the predicted Bernoulli components \( M_{k|k-1} \), the set \( Z_{1:k} \) can be repartitioned as \( P = Z_{1:k} = \{ W_{1,1}, W_{1,2}, \ldots, W_{M_{k|k-1}} \} \), where \( W_{0,1} \) is the clutter set from all sensors.

As the assumption that “one target produces at most one measurement” is invalid in the extended target case, we make the following definitions:
- Let \( W_{1:j} = w_{1}^{(j)} \oplus \cdots \oplus w_{S}^{(j)} \), where \( \oplus \) is the disjoint union, and \( W_{1:j} \) contains all clutter measurements from the \( i \)-th sensor.
- Let \( W_{j} = w_{1}^{(j)} \oplus \cdots \oplus w_{j}^{(j)} \), where \( j > 0 \) and \( j \) is the set of all measurements generated by target \( j \) on the \( i \)-th sensor.
- Let \( Z_{j} = w_{1}^{(j)} \oplus \cdots \oplus w_{M_{k|k-1}}^{(j)} \). The element \( w_{j} \) is allowed to be empty.

\[
p^{(j)}_P(x) = \begin{cases} \frac{p^{(j)}_P(x)f(W_{1:j}^{(j)}|x)}{\int p^{(j)}_P(x)f(W_{1:j}^{(j)}|x)dx}, & W_{1:j} = \emptyset, \\
\frac{\int p^{(j)}_P(x)f(W_{1:j}^{(j)}|x)dx}{\int p^{(j)}_P(x)f(W_{1:j}^{(j)}|x)dx}, & W_{1:j} \neq \emptyset,
\end{cases}
\]
(16)

\[
\alpha_P = \sum_{U \in Q} K_U \sum_{j=1}^{M_{k|k-1}} q^j_U W_{1:j} \bigg|_{W_{1:j}^{(j)} | x}.
\]
(17)

\[
q^j_U | w_{1:j} = \begin{cases} 1 - r^{(j)}_{k-1|k-1} + r^{(j)}_{k-1|k-1} p^{(j)}_P(x)f(W_{1:j}^{(j)}|x)dx,
\end{cases}
\]
(18)
Based on the definitions above, we can group different measurements from the same sensor together. Define the mapping function as
\[
T_{w_{i,s}} = \{j|w'_j \in W_{i,s}\},
\]
where \(w'_i \rightarrow i\) shows that the subset \(w'_j\) is generated by target \(j\) from the \(i\)-th sensor.

**B. ET-MS-MEMBER UPDATE**

Based on the APB model, the updating equations for extended targets are introduced as follows. Detailed derivations are given in the Appendix.

**Proposition 1:** In the extended target scenario, the undetected probability of the target with state \(x\) is
\[
\eta(x) = \prod_{i=1}^{s} \left(1 - p_{i,d}(x) + p_{i,d}(x)e^{-\gamma_i(x)}\right),
\]
where \(p_{i,d}(x)\) is the detection probability of the target by sensor \(i\), and \(\gamma_i(x)\) is the mean of the extended measurements.

**Proposition 2:** At time \(k\), the multi-sensor likelihood for the extended target is
\[
f(W_i^{j}|x) = \prod_{i \in T_{W_{i,s}}} \Pr\left(w'_j|\gamma_i(x)\right) \cdot \frac{p_{i,d}(x)L_i(w'_j|x)}{k_i(w'_j)} \times \prod_{i \notin T_{W_{i,s}}} \left(1 - p_{i,d}(x) + p_{i,d}(x)e^{-\gamma_i(x)}\right),
\]
where
\[
\Pr(n|\lambda) = \frac{\lambda^ne^{-\lambda}}{n!},
\]
\[
k_i(w'_j) = \prod_{z \in w'_j} c_{i,k}(z),
\]
\[
L_i(w'_j|x) = \prod_{z \in w'_j} h_{i,k}(z|x).
\]

\(\Pr(n|\lambda)\) is the Poisson probability distribution with mean \(\lambda\), \(c_{i,k}(z)\) is the spatial distribution of clutter at sensor \(i\), and \(L_i(w'_j|x)\) is the pseudo-likelihood of the extended target with state \(x\).

**Proposition 3:** In the extended target scenario, the coefficient \(K_P\) can be expressed as
\[
K_P = \left[\prod_{i=1}^{s} C_i^{(n)}(x)ight] \times \left[\prod_{j=1}^{M} \prod_{i \in T_{W_{i,s}}} \left(|w'_j|!\right)\right],
\]
where \(C_i^{(n)}(x)\) is the \(n\)-th-order derivative of function \(C_i(x) = e^{\lambda_i x} - \lambda_i^n\), and \(\lambda_i\) is the clutter intensity at sensor \(i\).

The other formulas for the ET-MS-MeMBer filter are the same as those for the MS-MeMBer filter.

**IV. THE GIW MIXTURE IMPLEMENTATION**

A closed-form solution to the ET-PHD filter has been established for GIW models in [22], where the kinematic state of the target is represented by a Gaussian distribution, and the target extension is described by an inverse Wishart distribution. In subsection IV-A, we present the basic form of the GIW model. The numerical implementation of the ET-MS-MeMBer filter is given in subsection IV-B and IV-C.

**A. THE BASIC MODEL**

Assume that at time \(k-1\), the target state \(x_{k-1}\) is composed of the kinematic state \(\tilde{x}_{k-1}\) and the target extension \(X_{k-1}\), i.e., \(x_{k-1} = [\tilde{x}_{k-1}, X_{k-1}]\). Then the motion model of the target can be expressed as:
\[
\tilde{x}_k = (F_{k|k-1} \otimes I_d) \tilde{x}_{k-1} + w_k,
\]
\[
w_k \sim N(0, Q_{k|k-1} \otimes X_k),
\]
where
\[
F_{k|k-1} = \begin{bmatrix} T_s & 1/T_s^2 \\ 0 & 1/T_s \end{bmatrix},
\]
\[
Q_{k|k-1} = \sigma^2 \begin{bmatrix} 1 - e^{-2\sigma^2} \end{bmatrix} \text{diag}([1, 0, 0, 1]).
\]
\(\otimes\) is the Kronecker product, and \(I_d\) is a \(d\)-dimensional identity matrix. \(N(\cdot; m, P)\) is a Gaussian distribution with mean \(m\) and variance \(P\). \(T_s\) is the sampling period. \(\sigma\) is the scalar acceleration standard deviation and \(\theta\) is the maneuver correlation time.

Assume that at time \(k\), the target generates \(n_k\) measurements, i.e., \(Z_k = \{z_k^{(j)}\}_{j=1}^{n_k}\). Then the observation model can be given by:
\[
z_k^{(j)} = (H_k \otimes I_d) \tilde{x}_k + e_k^{(j)},
\]
where
\(H_k = [100]\), and \(e_k^{(j)}\) is the white Gaussian noise with covariance determined by the target extension.

**B. THE NUMERICAL IMPLEMENTATION**

Assume that each Bernoulli density in (9) has a GIW mixture form:
\[
P_{k|k-1}^{(i)}(x) = \sum_{n=1}^{J_{n,k-1}} w_n^{(i)}(x) \mathcal{N}(\tilde{x}; m_n^{(i)}, P_n^{(i)} \otimes X),
\]
\[
\mathcal{IW}(X; v_{n,k-1}^{(i)}, V_{n,k-1}^{(i)}).
\]
where \(J_{n,k-1}\) is the number of the mixture, \(w_n^{(i)}\) is the weight of the \(n\)-th GIW component, and \(\mathcal{IW}(X; v_{n,k-1}^{(i)}, V_{n,k-1}^{(i)})\) is an inverse Wishart distribution with degrees of freedom \(v_{n,k-1}^{(i)}\) and inverse scale matrix \(V_{n,k-1}^{(i)}\).
Given the probability density of newborn targets:

$$p^{(i)}_{b,k}(x) = \sum_{n=1}^{j^{(i)}_{k-1}} w^{(i)}_{n,k} N(\tilde{x}; m^{(i)}_{n,k}, \Sigma^{(i)}_{n,k}) \cdot \mathcal{J}W(X; v^{(i)}_{n,k-1}, V^{(i)}_{n,k-1}),$$

Likewise, the predicted Bernoulli density has a GIM mixture form, and the legacy components become:

$$r^{(i)}_{S,k|k-1} = r^{(i)}_{k-1|k-1} - \rho v,$$

$$p^{(i)}_{S,k|k-1}(x) = \sum_{n=1}^{j^{(i)}_{k-1}} w^{(i)}_{n,k-1} N(\tilde{x}^{'}; m^{(i)}_{n,k-1}, \Sigma^{(i)}_{n,k-1}) \cdot \mathcal{J}W(X; v^{(i)}_{n,k-1}, V^{(i)}_{n,k-1}),$$

where

$$m^{(i)}_{n,k-1} = (F^{(i)}_{k-1} \otimes I_d) m^{(i)}_{n,k-1},$$

$$p^{(i)}_{n,k|k-1} = F^{(i)}_{k-1} p^{(i)}_{n,k-1} F^{T}_{k-1} + Q_{k|k-1},$$

$$v^{(i)}_{n,k-1} = v^{(i)}_{n,k-1} - d - 1,$$

In the predicted degrees of freedom (37), \( \tau \) is a temporal decay constant.

Assume that each predicted Bernoulli density in (13) has a GIM mixture form:

$$p^{(i)}_{k|k-1}(x) = \sum_{n=1}^{j^{(i)}_{k-1}} w^{(i)}_{n,k-1} N(\tilde{x}; m^{(i)}_{n,k-1}, \Sigma^{(i)}_{n,k-1}) \cdot \mathcal{J}W(X; v^{(i)}_{n,k-1}, V^{(i)}_{n,k-1}).$$

Set the detection probability and the extended measurement rate as constants, i.e., \( p_{d,d}(x) = p_{d,d}, \gamma(x) = \gamma \). Then the updated Bernoulli density is calculated by

$$r^{(i)}_{\text{P}} = \begin{cases} 0 & \alpha \neq 0, \\ r^{(i)}_{k|k-1} \prod_{i=1}^{s} (1 - p_{i,d} + p_{i,d}e^{-\gamma}), \\ 1 - r^{(i)}_{k|k-1} + r^{(i)}_{k|k-1} \prod_{i=1}^{s} (1 - p_{i,d} + p_{i,d}e^{-\gamma}), \\ \alpha \neq 0, \end{cases}$$

$$p^{(i)}_{\text{P}}(x) = \begin{cases} 0 & \alpha \neq 0, \\ \sum_{n=1}^{j^{(i)}_{k-1}} w^{(i)}_{n,k-1} N(\tilde{x}; m^{(i)}_{n,k-1}, \Sigma^{(i)}_{n,k-1}) \cdot \mathcal{J}W(X; v^{(i)}_{n,k-1}, V^{(i)}_{n,k-1}), \\ \sum_{n=1}^{j^{(i)}_{k-1}} w^{(i)}_{n,k} N(\tilde{x}; m^{(i)}_{n,k}, \Sigma^{(i)}_{n,k}) \cdot \mathcal{J}W(X; v^{(i)}_{n,k}, V^{(i)}_{n,k}), \end{cases}$$

where

$$w^{(i)}_{n,k} = \left( \prod_{i=1}^{s} \operatorname{Pr}(|w_i^j|; \gamma) \right) \cdot \prod_{i=1}^{s} \left( 1 - p_{i,d} + p_{i,d}e^{-\gamma} \right),$$
C. A GREEDY PARTITIONING MECHANISM FOR EXTENDED TARGETS

The exact solution for the ET-MS-MeMBer filter is computationally infeasible since all partitioning hypotheses are needed in the update step. Hence, in this section, we propose an efficient approximate solution based on the greedy mechanism.

Firstly, all measurements from the same source are formed as a subset. For each measurement set $Z_i$, define $w$ as a subset of $Z_i$, and $w$ contains extended measurements generated by the same target. Define $W(Z_i)$ as the collection of all subsets. For example, if the measurement set $Z_i = \{z_{i1}^{(1)}, z_{i2}^{(2)}, z_{i3}^{(3)}\}$, then $W(Z_i) = \{(z_{i1}^{(1)}), (z_{i2}^{(2)}), (z_{i3}^{(3)}), (z_{i1}^{(1)}, z_{i2}^{(2)}), (z_{i1}^{(1)}, z_{i3}^{(3)}), (z_{i2}^{(2)}, z_{i3}^{(3)}), (z_{i1}^{(1)}, z_{i2}^{(2)}, z_{i3}^{(3)})\}$. Note that $Z_i$ also includes clutter measurements. And we define the subset $w$ containing only one element to be a clutter subset. Thus, in this section, only the subset that is more likely generated by the target is obtained. By using Algorithm 1, we can obtain all subsets that satisfy the above conditions.

Algorithm 1 Greedy Elements Selection

1: function Greedy_elements_selection($Z_i, D_U, D_L$),
   where $Z_i$ contains all the measurements from sensor $i$.
2: Part = Distance_Partitioning($Z_i, D_U, D_L$)
3: cell = []
4: for $j = 1$ : length (Part)
5: Part($j$) = Subpartition (Part($j$))
6: Part($j$) = Cut_length (Part($j$), 1)
7: cell = (cell, Part($j$))
8: end for
9: \{w^1_i, \ldots, w^{n_i}_i\} = Find_unique (cell)
10: return \{w^1_i, \ldots, w^{n_i}_i\}
11: end function

In Algorithm 1, the inputs are the measurement sets $Z_i$, the upper threshold $D_U$, and the lower threshold $D_L$. According to the distance partition method in [17], partitions of $Z_i$ are obtained (line 2 in Algorithm 1). The idea of the distance partition method is to group the adjacent measurements into the same subset. We can get different partitions based on different maximum partition distances $d_{\text{max}} \in [D_L, D_U]$. As the measurements generated by adjacent targets can also be allocated to the same subset, the subpartition algorithm [17] is applied (line 5 in Algorithm 1). In line 6, we delete the subset containing only one measurement. In the end, identical subsets are discarded to keep each subset unique.

Secondly, we apply the two-step partition method [10], [11] to obtain several partitioning hypotheses with higher weights. In the first step, the best associations for each Bernoulli component are obtained. As shown in Figure 1, the collection \{w^1_i, \ldots, w^{n_i}_i\} calculated by Algorithm 1 is associated with the given $j$-th Bernoulli component ($A^j_k, a^j_k$). At most $W_{\text{max}}$ subsets are retained in each recursion. Meanwhile, the sensors are processed sequentially.

After all the sensors are processed, the second step (as shown in Figure 2) is applied to select top $P_{\text{max}}$ partitions with the highest weight at most. Meanwhile, the Bernoulli components are processed sequentially. Note that there is no overlap between each measurement set $W_{1:s}$ in the same partition.

V. SIMULATION EXPERIMENTS AND ANALYSIS

In this section, simulation experiments are conducted to verify the superiority of the proposed ET-MS-MeMBer filter. A comparison of the ET-MS-MeMBer filter and the IC-GGIW-CPHD filter is also given in this section.

A. PARAMETER SETTINGS

Consider a two-dimensional surveillance area of the size $[-800, 800]m \times [-800, 800]m$, with three sensors and up to four targets observed in clutter. The detection probability of each sensor $p_{i,d} = 0.6$, and the clutter intensity $\kappa_k = 5$.

Suppose that at time $k$, the target extension $X_{k}^{(i)}$ is determined by

$$X_{k}^{(i)} = R_{k}^{(i)} \text{diag}(\lambda^2 \alpha^2) (R_{k}^{(i)})^T,$$

where $R_{k}^{(i)}$ is a rotation matrix that ensures the extension’s major axis being aligned with the target’s direction of motion. $\lambda$ and $\alpha$ are the length of the major and minor axes,
respectively. Here we assume that the number of measurements generated by the extended target is related to the target size:

\[
y^{(i)}_k = \left[ 2\sqrt{A_i}a_i + 0.5 \right]. \tag{57}
\]

The parameters of the target are set as follows:

- \( A_1 = A_2 = A_3 = A_4 = 5; a_1 = a_2 = a_3 = a_4 = 20; \)
- \( m_1 = [-500, -500, 30, 30, 0, 0], t^{(i)}_b = 5, t^{(i)}_d = 40; \)
- \( m_2 = [500, -500, -30, 30, 0, 0], t^{(2)}_b = 20, t^{(2)}_d = 55; \)
- \( m_3 = [0, 700, -35, -35, 0, 0], t^{(3)}_b = 35, t^{(3)}_d = 60; \)
- \( m_4 = [0, 700, 35, -35, 0, 0], t^{(4)}_b = 45, t^{(4)}_d = 70; \)

where \( m_i = [x_i, y_i, \dot{x}_i, \dot{y}_i, \ddot{x}_i, \ddot{y}_i] \) contains the position, velocity and acceleration of the target. The duration time is 100 s. \( t^{(i)}_b \) and \( t^{(i)}_d \) are the target birth and death time, respectively. The target trajectories are shown in Figure 3. Figure 4 presents the measurements of sensor 1.

The parameters of the motion model are \( T_s = 1s, \theta = 1s, \sigma = 0.1, \) and \( \tau = 5s. \) Set the newborn model according to the initial states of the target: \( M_{b,k} = 3, r^{(1)}_{b,k} = r^{(2)}_{b,k} = r^{(3)}_{b,k} = 0.1, m^{(1)}_{b,k} = [-500, -500, 0, 0, 0, 0], m^{(2)}_{b,k} = [500, -500, 0, 0, 0, 0], m^{(3)}_{b,k} = [0, 700, 0, 0, 0, 0], P^{(1)}_{b,k} = P^{(2)}_{b,k} = P^{(3)}_{b,k} = \diag([100^2, 50^2, 50^2]), v^{(1)}_{b,k} = v^{(2)}_{b,k} = v^{(3)}_{b,k} = 7, V^{(1)}_{b,k} = V^{(2)}_{b,k} = V^{(3)}_{b,k} = \diag([11]), \)

In Algorithm 1, we set \( D_U = 60, \) and \( D_L = 20. \) In the two-step measurement partition method, we set \( W_{\text{max}} = 4 \) and \( P_{\text{max}} = 4, \) i.e., the maximum number of subsets and hypotheses are both 4.

For a fair comparison, we adopt the same pruning threshold for both the ET-MS-MeMBer and the IC-GGIW-CPHD filter, and the threshold \( T_{\text{cut}} = 10^{-5}. \) The number of mixed components is no more than 100. In the IC-GGIW-CPHD filter, the merge thresholds of Gamma, Gaussian, and inverse Wishart components are set as \( U_Y = 10, U_G = 50, \) and \( U_W = 50, \) respectively.

We use the optimal subpattern assignment (OSPA) distance [31] as the metric for evaluating the filtering performance, in which the order \( c = 1, \) and the cutoff threshold \( p = 100. \) Given an estimated target state set \( \hat{Y} = \{g_1, \ldots, g_n\} \) and a truth target state set \( Y = \{y_1, \ldots, y_m\}(n \leq m). \) The OSPA distance can be defined as

\[
\tilde{d}_p^{(c)}(G, Y) = \left( \min_{\pi \in \Pi_m} \frac{1}{m} \sum_{i=1}^{n} d^{(c)}(g_{\pi(i)}, y_{\pi(i)})^p + c^p(m-n) \right)^{1/p}, \tag{58}
\]
where $\Pi_m$ is the set of permutations on $\{1, \ldots, m\}$.

$$d^{(c)}(g_i, y_{\pi_i}) = \min(d(g_i, y_{\pi_i}), c),$$

which denotes the distance between vector $g_i$ and $y_{\pi_i}$ cutting off at a threshold $c$.

B. SIMULATION RESULTS

The filtering results of the ET-MS-MeMBer filter are shown in Figure 5. It can be seen that the proposed filter can effectively estimate the shape and position of the extended target.

Figure 6 and Figure 7 compare the performance of the ET-MS-MeMBer filter and the MS-MeMBer filter over 100 Monte Carlo runs. It is clearly shown in Figure 6 that the MS-MeMBer filter has a significant bias in estimated cardinality, thus leading to a large OSPA distance in Figure 7. In contrast, the ET-MS-MeMBer filter can estimate the target cardinality more accurately and has a much smaller OSPA distance.

We also compare the performance of the ET-MS-MeMBer filter with the IC-GGIW-CPHD filter concerning different numbers of sensors. As before, 100 Monte Carlo runs are performed. Figure 8 and Figure 9 present the OSPA and cardinality estimation of the two filters with a varying number of sensors. The average OSPA and average running time
of the two filters are shown in Figure 10 and Figure 11, respectively. From Figure 9 and Figure 10, we can observe that the estimation accuracy of both filters improves as the number of sensors increases.

Meanwhile, compared with the IC-GGIW-CPHD filter, the ET-MS-MeMBer filter has a smaller average OSPA distance. From Figure 11, it can be seen that the average running time of the ET-MS-MeMBer filter is shorter than the IC-GGIW-CPHD filter. Therefore, we can conclude that the proposed ET-MS-MeMBer filter has a more satisfactory filtering performance with a lower computational cost.

In Figure 12 and Figure 13, we present the performance of the ET-MS-MeMBer filter and the IC-GGIW-CPHD filter with different clutter intensities. The monitoring area is observed by three sensors. We can see that with the increase of the clutter intensity, the average OSPA of the ET-MS-MeMBer filter rises very slowly, while in contrast, the average OSPA of the IC-GGIW-CPHD filter grows strikingly and there is a steep growth between $\lambda_i^\kappa = 5$ and $\lambda_i^\kappa = 65$. In the meantime, in Figure 13, the average running time of the two filters show a similar trend as it is in Figure 12. Thus we can see that our proposed ET-MS-MeMBer filter has a superior performance in both filtering and efficiency.

VI. CONCLUSION

In this paper, we propose a novel ET-MS-MeMBer filter for extended target tracking to handle the bias of the estimated cardinality that arises in the existent MS-MeMBer filter. In addition, a GIW mixture implementation is introduced for the proposed ET-MS-MeMBer filter. We demonstrate that the proposed ET-MS-MeMBer filter can effectively estimate the shape and position of the extended target. Furthermore, the performance of the ET-MS-MeMBer filter is compared with the IC-GGIW-CPHD filter under different numbers of sensors, and simulations verify that the ET-MS-MeMBer filter outperforms the IC-GGIW-CPHD filter. It is also shown that the increasing number of sensors can lead to a higher filtering accuracy at the expense of more computational burden.

APPENDIX

In this section, we prove Propositions 1-3 given in Section III.B. Let $G_{k|k}[u]$ denote the PGFL of the updated multi-Bernoulli density at time $k$. Consider the original multivariate function $F[g_{1:s}, u]$ and the derivative $\frac{\delta F[g_{1:s}, u]}{\delta Z_{1:s}}$ introduced in [11]

$$
F[g_{1:s}, u] = \prod_{i=1}^{s} C_i ((c_i, g_i)) \cdot \prod_{j=1}^{M} G_j^r [u \prod_{i=1}^{s} \phi_{g_i}(x)],
$$

(A.1)

$$
\frac{\delta F[g_{1:s}, u]}{\delta Z_{1:s}} = \sum_{P \in Q} \left[ \prod_{i=1}^{s} \frac{\delta C_i}{\delta W_{1:s}} \delta G^1 \cdots \delta G^M \right].
$$

(A.2)

$$
\frac{\delta F[g_{1:s}, u]}{\delta Z_{1:s}} = \sum_{P \in Q} \left[ \prod_{i=1}^{s} \frac{\delta C_i}{\delta W_{1:s}} \delta G^1 \cdots \delta G^M \right].
$$

(A.2)
Using (A.2), we get

$$
\frac{\delta G^j}{\delta W_{1:s}^j} \left[ u \prod_{l=1}^{s} \phi_{c_l}(x) \right] = \left\{ \begin{array}{ll}
1 - r^{(j)} + r^{(j)} \left( p^{(j)}; u \prod_{l=1}^{s} \phi_{c_l}(x) \right), & W_{1:s}^j = \emptyset \\
& \\
\int u(x)p^{(j)}(x) \prod_{l=1}^{s} \phi_{c_l}(x) \cdot \prod_{l \in T_{W_1:s}^j} \left( 1 - p_{i,l,d}(x) \right) \cdot \prod_{l \in T_{W_1:s}^j} h_{l,k}(z|x) \cdot \prod_{l \in T_{W_1:s}^j} \gamma_l(x)^{w^j_l} dx, & W_{1:s}^j \neq \emptyset.
\end{array} \right.
$$

(A.3)

Under the APB model, the function $\phi_{c_l}(x)$ can be expressed as

$$
\phi_{c_l}(x) = 1 - p_{i,l,d}(x) + p_{i,l,d}(x)e^{\gamma_l(x)p_{c_l}(x) - \gamma_l(x)}. \tag{A.4}
$$

Thus, the derivative term in (A.3) can be written as

$$
\frac{\delta}{\delta W_{1:s}^j} \left( \prod_{l=1}^{s} \phi_{c_l}(x) \right) = \prod_{l \notin T_{W_1:s}^j} \phi_{c_l}(x) \cdot \prod_{l \in T_{W_1:s}^j} \left( 1 - p_{i,l,d}(x) \right) \cdot \prod_{l \in T_{W_1:s}^j} h_{l,k}(z|x) \cdot \prod_{l \in T_{W_1:s}^{(j)}} \gamma_l(x)^{w^j_l}. \tag{A.5}
$$

Substituting (A.5) into (A.3), we get

$$
\frac{\delta G^j}{\delta W_{1:s}^j} \left[ u \prod_{l=1}^{s} \phi_{c_l}(x) \right] = \left\{ \begin{array}{ll}
1 - r^{(j)} + r^{(j)} \left( p^{(j)}; u \prod_{l=1}^{s} \phi_{c_l}(x) \right), & W_{1:s}^j = \emptyset \\
& \\
\int u(x)p^{(j)}(x) \prod_{l \notin T_{W_1:s}^j} \phi_{c_l}(x) \cdot \prod_{l \in T_{W_1:s}^j} \left( 1 - p_{i,l,d}(x) \right) \cdot \prod_{l \in T_{W_1:s}^j} h_{l,k}(z|x) \cdot \prod_{l \in T_{W_1:s}^{(j)}} \gamma_l(x)^{w^j_l} dx, & W_{1:s}^j \neq \emptyset.
\end{array} \right.
$$

(A.6)

Also, $\sum_{i=1}^{s} \frac{\delta G^j}{\delta W_i}$ follows

$$
\sum_{i=1}^{s} \frac{\delta G^j}{\delta W_i} = \prod_{i=1}^{s} \Gamma_i \cdot \prod_{i=1}^{s} C_i^{(W_i^j)} ((c_i, g_i))
$$

(A.7)

where $\Gamma_i = \prod_{z \in Z_i} c_i(z)$.

Define that $\varphi_{W_1:s}^{(j)} [u] = \frac{\delta G^j}{\delta W_{1:s}^j}$, and we can get

$$
\varphi_{W_1:s}^{(j)} [u] = \left\{ \begin{array}{ll}
1 - r^{(j)} + r^{(j)} \left( p^{(j)}; u \prod_{l=1}^{s} \phi_{c_l}(x) \right), & W_{1:s}^j = \emptyset \\
& \\
\int u(x)p^{(j)}(x) \prod_{l \notin T_{W_1:s}^j} \phi_{c_l}(x) \cdot \prod_{l \in T_{W_1:s}^j} \left( 1 - p_{i,l,d}(x) \right) \cdot \prod_{l \in T_{W_1:s}^j} h_{l,k}(z|x) \cdot \prod_{l \in T_{W_1:s}^{(j)}} \gamma_l(x)^{w^j_l} dx, & W_{1:s}^j \neq \emptyset.
\end{array} \right.
$$

(A.8)
Hence, according to (A.8), we obtain equations (20)-(24). Finally, (A.2) becomes
\[
\frac{\delta^2 F}{\delta Z_{1:s}}[0, \ldots, 0, u] = \left[ \prod_{i=1}^{s} \gamma_i \right] \cdot \sum_{P \in Q} \left[ \prod_{j=1}^{M} \frac{\phi_{w_j}^{(i)}[u]}{\phi_{w_j}^{(0)}} \right] \cdot \left[ \prod_{j=1}^{M} \frac{q_{w_j}^{(i)}[u]}{q_{w_j}^{(0)}} \right] 
\]
As the updated PGFL no longer has the multi-Bernoulli form in (7), we approximate it with a multi-Bernoulli component. Hence, we have established the coefficient (25) as required.

Therefore, we can obtain the updated multi-Bernoulli density according to (A.10).

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