Polarizations in decays $B_{u,d} \rightarrow VV$ and possible implications for R-parity violating supersymmetry

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February 28, 2018

Abstract

Recently BABAR and Belle have measured anomalous large transverse polarizations in some pure penguin $B \rightarrow VV$ decays, which might be inconsistent with the Standard Model expectations. We try to explore its implications for R-parity violating (RPV) supersymmetry. The QCD factorization approach is employed for the hadronic dynamics of B decays. We find that it is possible to have parameter spaces solving the anomaly. Furthermore, we have derived stringent bounds on relevant RPV couplings from the experimental data, which is useful for further studies of RPV phenomena.

PACS Numbers: 12.60.Jv, 12.15.Mm, 12.38.Bx, 13.25.Hw

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1 Introduction

The study of exclusive nonleptonic weak decays of B mesons provides not only good opportunities to test the standard model (SM) but also powerful means to probe different new physics scenarios beyond the SM. Some important results from BABAR and Belle collaboration on the B decays to two light vector mesons $B \rightarrow V V$ [1, 2, 3, 4, 5, 6, 7, 8, 9] have caught considerable theoretical interests and have been extensively studied very recently [10, 11, 12, 13, 14, 15, 16].

In $B \rightarrow V V$ decay modes, both longitudinal and transverse polarization states are possible. It has been known to us for many years that longitudinal polarization fraction is dominant in the SM when both the final vector mesons are light [17]. In the heavy quark limit, dynamics for the situation is analyzed clearly by Kagan [18] and Grossman [11]. Recent BABAR and Belle measurements of decay modes $B \rightarrow \rho^+ \rho^-$, $\rho^0 \rho^+$ and $\rho^0 K^*$ have confirmed this prediction as the final states are dominated by the longitudinal polarization. However, for the observed $B \rightarrow \phi K^*$ and $\rho^+ K^{*0}$ decays, the transverse polarization fractions are measured to be anomalous as large as 50%, which has posed a challenge to theoretical explanation.

Recent studies have provided some possible resolutions to the anomaly. Kagan [14] has shown it could be solved in the SM by increasing the nonfactorizable contributions of annihilation diagrams, which are formally power suppressed and depend on some poorly known nonperturbative QCD parameters. Later on, using a different factorization approach, Li and Mishima [19] have found that the annihilation contribution is not sufficient to lower the longitudinal fraction in $B \rightarrow \phi K^*$ down to $\sim 50\%$, although it could help to alleviate the anomaly. It is also shown that the anomaly might be due to large charming penguin contributions and final-state-interactions (FSI) by Colangelo et al. [12] and Ladisa et al. [15]. However, Cheng et al. [16] have found the FSI effects not able to fully account for the anomaly. Hou and Nagashima [20] have given a model where the transverse $\phi$ descends from the transverse gluon from $b \rightarrow sg^*$. However, it should be noted that $\phi$ must couple to at least three gluons to neutralize color and conserve relevant quantum number. In this paper, we will explore the opportunity whether the RPV supersymmetry (SUSY) could provide a solution to the polarization anomaly and what kinds of constraints could be derived.

The possible appearance of RPV couplings [21], which will result in lepton and baryon number conservation, has gained full attention in searching for SUSY [22, 23]. The rich
phenomenology implied by RPV couplings in B decays have been extensively discussed in \cite{24, 25, 26, 27, 28}. In this paper, we extend these studies to decays $B \to VV$. We use the QCD factorization (QCDF) framework \cite{29} for hadronic dynamics. We show that the polarization anomaly could be solved in the presence of RPV couplings, due to the appearance of $\bar{q}(V \pm A)q\bar{b}(V+A)s$ interactions. Compared with the matrix element of $\langle K^*|\bar{b}(V-A)s|B\rangle$, the axial parts in $\langle K^*|\bar{b}(V+A)s|B\rangle$ have opposite sign, hence induce polarization phenomena different from the SM. Moreover, using the recent experimental data for branching ratios and longitudinal-polarization fractions, we obtain limits on the relevant RPV couplings.

This paper is organized as follows: In Sec. II, we calculate the $B \to VV$ decay amplitudes which contain the SM part and RPV effects using the QCDF approach. In Sec. III, we list the theoretical input parameters used in our analysis. Section IV is devoted to the numerical results and discussions, we also display the allowed regions of the parameter space that satisfy all experimental data. Finally, our summary is presented in Sec. V.

2 Amplitudes for $B \to VV$ decays in QCD factorization approach

Approaches for calculating amplitudes of B charmless nonleptonic decays always invoke factorization frameworks. Once factorization framework chosen, one can start from the $\Delta B = 1$ effective Hamiltonian of the underlying full electroweak (EW) theory at the renormalization scale $\mu \sim m_b$, which can be obtained from the full EW theory by integrating out heavy particles with mass larger than $m_b$ using the renormalization group equation. The QCDF \cite{29} developed by Beneke, Buchalla, Neubert and Sachrajda is a powerful approach, which will be employed in this paper. Details of QCDF could be found in papers \cite{29}.

2.1 The decay amplitudes in the SM

In the SM, the low energy effective Hamiltonian for the $\Delta B = 1$ transition has the form \cite{30}

$$H_{eff}^{SM} = \frac{G_f}{\sqrt{2}}[v_u(C_1O_1^u + C_2O_2^u) + v_c(C_1O_1^c + C_2O_2^c) - \epsilon_s(\sum_{i=3}^{10} C_iO_i + C_{7}\gamma O_{7}\gamma + C_{8g}O_{8g})] + H.c., \quad (1)$$
with the effective operators given by

\[
\begin{align*}
O_I^q & = (\bar{b}_\alpha u_\alpha)_{V^-A}(\bar{u}_\beta q_\beta)_{V^-A}, \\
O_1^q & = (\bar{b}_\alpha c_\alpha)_{V^-A}(\bar{c}_\beta q_\beta)_{V^-A}, \\
O_2^{(5)} & = (\bar{b}_\alpha q_\alpha)_{V^-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V^-A(V+A)}, \\
O_3^{(5)} & = (\bar{b}_\alpha q_\alpha)_{V^-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V^-A(V-A)}, \\
O_4^{(6)} & = (\bar{b}_\alpha q_\alpha)_{V^-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V^-A(V+A)}, \\
O_7^{(9)} & = \frac{3}{2} (\bar{b}_\alpha q_\alpha)_{V^-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V^-A(V-A)}, \\
O_{7\gamma} & = \frac{\epsilon}{8\pi^2} m_b \bar{b}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) q_\alpha F_{\mu\nu}, \\
O_{8g} & = \frac{g}{8\pi^2} m_b \bar{b}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T^{a}_\alpha q_\beta g_a^{\mu\nu}.
\end{align*}
\]

(2)

Where \( v_p = V^*_pb V_pq \) are the Cabibbo-Kobayashi-Maskawa (CKM) factors, \( C_i(\mu) \) are the effective Wilson coefficients and could be found in Ref. [30]; \( \alpha, \beta \) are the SU(3) color indices, \( \langle \bar{q}_1 q_2 \rangle_{V^\pm A} \equiv \bar{q}_1 \gamma^\mu (1 \pm \gamma_5) q_2, q = d, s \) and \( q' = u, d, s, c, b \).

In order to calculate the decay amplitudes and branching ratios for \( B \to VV \) decays, we need the hadronic matrix element of the local four fermion operators

\[
\langle V_1(\lambda_1)V_2(\lambda_2)| (\bar{q}_2 q_3)_{V^\pm A}(\bar{b} q_1)_{V^-A}|B \rangle,
\]

(3)

where \( \lambda_1, \lambda_2 \) are the helicities of the final-state vector mesons \( V_1 \) and \( V_2 \) with four-momentum \( p_1 \) and \( p_2 \), respectively. In the rest frame of \( B \) system, since the \( B \) meson has spin zero, two vectors have the same helicity therefore three polarization states are possible, one longitudinal (L) and two transverse, corresponding to helicities \( \lambda = 0 \) and \( \lambda = \pm \) (here \( \lambda_1 = \lambda_2 = \lambda \)). We use \( X(BV_1, V_2) \) to denote the factorizable amplitude for different chiral currents with the vector meson \( V_2 \) being factorized out. Under the naive factorization (NF) approach, we can express it as

\[
X(BV_1, V_2) \equiv \langle V_2|(\bar{q}_2 q_3) (1 - a \gamma_5) q_3) |0\rangle \langle V_1|(\bar{b} q_1) (1 - \gamma_5) q_1) |B \rangle,
\]

(4)

where \( a = +1 \) or \( -1 \) corresponding to \( (\bar{q}_2 q_3)_{V^-A} \) or \( (\bar{q}_2 q_3)_{V^+A} \) current, respectively.

We write the factorized matrix elements in term of the decay constant and form factors \([32]\)

\[
\begin{align*}
\langle V(p, \varepsilon^*) | \bar{q} \gamma_\mu q' | 0 \rangle & = f_V m_V \varepsilon^*_\mu, \\
\langle V(p, \varepsilon^*) | \bar{b} \gamma_\mu q | B(p_B) \rangle & = \frac{2 V_{BV} (q^2)}{m_B + m_V} \epsilon^{\mu\nu\alpha\beta} \varepsilon^*_\nu p_B T^{a}_{\alpha\beta} \varepsilon^*_\mu, \\
\langle V(p, \varepsilon^*) | \bar{b} \gamma_5 q | B(p_B) \rangle & = i \left[ \varepsilon^*_\mu (m_B + m_V) A_{1BV}^{V} (q^2) - (p_B + p)_\mu (\varepsilon^* \cdot p_B) A_{2BV}^{V} (q^2) \right] \\
& - i q_\mu (\varepsilon^* \cdot p_B) \frac{2 m_V}{q^2} [A_{3BV}^{V} (q^2) - A_{5BV}^{V} (q^2)],
\end{align*}
\]

(7)
where $p_B(m_B)$ is the four-momentum (mass) of the $B$ meson, $m_{V_1}(\varepsilon^*_1)$ and $m_{V_2}(\varepsilon^*_2)$ are the masses (polarization vectors) of the vector mesons $V_1$ and $V_2$, respectively, $q = p_B - p$ is the transferred momentum, and the form factors obey the following exact relations [33]

$$A_3(0) = A_0(0),$$

$$A_3^{BV}(q^2) = \frac{m_B + m_V}{2m_V}A_1^{BV}(q^2) - \frac{m_B - m_V}{2m_V}A_2^{BV}(q^2).$$

(8)

One has the factorizable $B \to V_1V_2$ amplitude

$$X^{(BV_1V_2)} = -if_{V_2}m_{V_2}[(\varepsilon^*_1 \cdot \varepsilon^*_2)(m_B + m_{V_1})A_1^{BV_1}(m_{V_2}^2)$$

$$- (\varepsilon^*_1 \cdot p_B)(\varepsilon^*_2 \cdot p_B)\frac{2A_2^{BV_1}(m_{V_2}^2)}{m_B + m_{V_1}} + i\epsilon_{\mu\nu\alpha\beta}\varepsilon^*_2\varepsilon^*_1 p_\beta p_\alpha \frac{2V^{BV_1}(q^2)}{m_B + m_{V_1}}].$$

(9)

We assume the $V_1$ ($V_2$) meson flying in the minus(plus) $z$-direction carrying the momentum $p_1$ ($p_2$), Using the sign convention $^{0123} = -1$, we get

$$X^{(BV_1V_2)} = \begin{cases} \frac{if_{V_2}m_{V_2}}{2m_{V_1}}[(m_B^2 - m_{V_1}^2 - m_{V_2}^2)(m_B + m_{V_1})A_1^{BV_1}(m_{V_2}^2) - \frac{4m_B^2p_B^2}{m_B + m_{V_1}}A_2^{BV_1}(m_{V_2}^2)] = h_0, \\ if_{V_2}m_{V_2}[(m_B + m_{V_1})A_1^{BV_1}(m_{V_2}^2) + \frac{2m_Bp_B}{m_B + m_{V_1}}V^{BV_1}(m_{V_2}^2)] = h_{\pm}, \end{cases}$$

(10)

where $h_0$ for $\lambda = 0$ and $h_{\pm}$ for $\lambda = \pm$.

The QCDF approach [29] allows us to compute the nonfactorizable corrections to the hadronic matrix elements $\langle V_1V_2|O_i|B \rangle$ in the heavy quark limit. The nonfactorizable corrections can be normalized to the factorizable amplitudes, so that they enter the effective parameters $a_i$ as $\alpha_s$ corrections. They are calculated from the vertex corrections, hard spectator interactions, and QCD penguin-type contributions.

In QCDF approach, light-cone distribution amplitudes (LCDAs) play an essential role. The LCDAs of the light vector meson are given in [33]. In general, the light-cone projection operator for vector mesons in momentum space can be divide into two parts

$$\mathcal{M}^V = \mathcal{M}_\parallel^V + \mathcal{M}_\perp^V.$$  

(11)

The longitudinal projector and the transverse projector are given by [33, 34]

$$\mathcal{M}_\parallel^V = \frac{f_Vm_V(\varepsilon^* \cdot n_+)}{4} P_\parallel \Phi^V(u) + \frac{f_V^*m_V(\varepsilon^* \cdot n_+)}{4} \frac{\partial}{\partial E} \left[ -\frac{i}{2} \sigma_{\mu\nu} n_+^{\nu} H_\parallel^{(l)V}(u) \right]$$

$$-iE \int_0^u dv (\phi_\parallel^V(v) - H_\parallel^{(l)V}(v)) \sigma_{\mu\nu} \frac{\partial}{\partial k_+^{\nu}} + \frac{h_\parallel^{(s)V}(u)}{2} \right|_{k=up},$$

(12)
Here we suppose the vector meson moving in the $n_-$ direction, $n_\pm = (1, 0, 0, \pm 1)$ is the light-cone vectors, $u$ is the light-cone momentum fraction of the quark in the vector meson, $f_V$ and $f_V^{\perp}$ are vector and tensor decay constants, respectively, and $E$ is the energy of the vector meson in the $B$ rest system. In the main body of the paper we neglect power-suppressed higher-twist effects, i.e. we identify the meson momentum $p' \equiv En_-$ and set $\varepsilon^\ast \cdot n_- = 0$.

In the heavy quark limit, the light-cone projector for $B$ meson can be expressed as

$$\mathcal{M} = -\frac{if_{BM_B}}{4}[\varepsilon^\ast \gamma_5 \Phi^B_1(\vec{x}) + \Phi^B_2(\vec{x})],$$

with $v = (1, 0, 0, 0)$ and the normalization conditions

$$\int_0^1 d\xi \Phi^B_1(\xi) = 1, \quad \int_0^1 d\xi \Phi^B_2(\xi) = 0,$$

where $\xi$ is the momentum fraction of the spectator quark in the $B$ meson.

The coefficients of the flavor operators $a_i^\lambda$ which contain next-to-leading order coefficient and $O(\alpha_s)$ hard scattering corrections can be written as follows:

$$a_1^\lambda = C_1 + \frac{C_2}{N_C} + \frac{\alpha_s C_F}{4\pi N_C} C_2 [f_1^\lambda(1) + f_{11}^\lambda(1)],$$

$$a_2^\lambda = C_2 + \frac{C_3}{N_C} + \frac{\alpha_s C_F}{4\pi N_C} C_1 [f_1^\lambda(1) + f_{11}^\lambda(1)],$$

$$a_3^\lambda = C_3 + \frac{C_4}{N_C} + \frac{\alpha_s C_F}{4\pi N_C} C_4 [f_1^\lambda(1) + f_{11}^\lambda(1)],$$

$$a_4^\lambda = C_4 + \frac{C_5}{N_C} + \frac{\alpha_s C_F}{4\pi N_C} C_3 [f_1^\lambda(1) + f_{11}^\lambda(1)] + \frac{\alpha_s C_F}{4\pi N_C} \left\{ -C_1 \frac{v_u}{v_t} G^\lambda(s_u) + \frac{v_c}{v_t} G^\lambda(s_c) \right\}$$

$$+ C_3 \left[ G^\lambda(s_q) + G^\lambda(s_b) \right] + (C_4 + C_6) \sum_{q'=u}^b \left[ G^\lambda(s_{q'}) - \frac{2}{3} \right] + \frac{3}{2} C_9 \left[ e_q G^\lambda(s_q) + e_b G^\lambda(s_b) \right]$$

$$+ \frac{3}{2} (C_8 + C_{10}) \sum_{q' = u}^b e_{q'} \left[ G^\lambda(s_{q'}) - \frac{2}{3} \right] + C_8 G^\lambda_g,$$

$$a_5^\lambda = C_5 + \frac{C_6}{N_C} - \frac{\alpha_s C_F}{4\pi N_C} C_6 [f_1^\lambda(-1) + f_{11}^\lambda(-1)],$$

$$a_6^\lambda = C_6 + \frac{C_5}{N_C},$$

$$a_7^\lambda = C_7 + \frac{C_8}{N_C} - \frac{\alpha_s C_F}{4\pi N_C} C_8 \left[ f_1^\lambda(-1) + f_{11}^\lambda(-1) \right] - \frac{\alpha_s}{9\pi} N_C C^\lambda_\varepsilon,$$
\[ a_8^\lambda = C_8 + \frac{C_F}{N_C}, \]
\[ a_9^\lambda = C_9 + \frac{C_F}{N_C} + \frac{\alpha_s}{4\pi} C_9 \left[ f_1^\lambda(1) + f_{II}^\lambda(1) \right] - \frac{\alpha_e}{9\pi} N_C C_e^\lambda, \]
\[ a_{10}^\lambda = C_{10} + \frac{\alpha_s}{4\pi} C_F \left[ f_1^\lambda(1) + f_{II}^\lambda(1) \right] - \frac{\alpha_e}{9\pi} C_e^\lambda, \]

where \( C_F = (N_C^2 - 1)/(2N_C) \), \( s_i = m_i^2/m_b^2 \), and \( N_C = 3 \) is the number of colors. The superscript \( \lambda \) denotes the polarization of the vector meson.

In Eq. (15), \( f_1^\lambda(\pm 1) \) contain the contributions from the vertex corrections, and given by

\[ f_1^0(a) = -12 \ln \frac{\mu}{m_b} - 18 + 6(1 - a) + \int_0^1 du \Phi^{\perp}(u) \left( 3 \frac{1 - 2u}{1 - u} \ln u - 3i\pi \right), \]
\[ f_1^\pm(a) = -12 \ln \frac{\mu}{m_b} - 18 + 6(1 - a) + \int_0^1 du \left( \frac{g^{(v)}(u) + \frac{ag^{(a)}(u)}{4}}{\xi} \right) \left( 3 \frac{1 - 2u}{1 - u} \ln u - 3i\pi \right). \]

For hard spectator scattering contributions, we use the notation that \( V_1 \) is the recoiled meson and \( V_2 \) is the emitted meson, explicit calculations for \( f_{II}^\lambda \) yield

\[ f_{II}^0(a) = \frac{4\pi^2}{N_C} \frac{if_B f_{V_1} f_{V_2}}{h_0} \int_0^1 d\xi \frac{\Phi^{B}(\xi)}{\xi} \int_0^1 dv \frac{\Phi^{V_2}(v)}{\bar{v}} \int_0^1 du \frac{\Phi^{V_2}(u)}{u}, \]
\[ f_{II}^\pm(a) = \frac{4\pi^2}{N_C} \frac{2if_B f_{V_1} f_{V_2} m_{V_2}}{m_B h_\perp (1 + 1)} \int_0^1 d\xi \frac{\Phi^{B}(\xi)}{\xi} \int_0^1 dv \frac{\Phi^{V_2}(v)}{\bar{v}^2} \times \int_0^1 du \left( g^{(v)}(u) - \frac{ag^{(a)}(u)}{4} \right) + \frac{4\pi^2}{N_C} \frac{2if_B f_{V_1} f_{V_2} m_{V_1} m_{V_2}}{m_B h_\perp} \int_0^1 d\xi \frac{\Phi^{B}(\xi)}{\xi} \times \int_0^1 dv du \left( g^{(v)}(v) + \frac{ag^{(a)}(v)}{4} \right) \left( g^{(v)}(u) + \frac{ag^{(a)}(u)}{4} \right) \frac{u + \bar{v}}{u\bar{v}}, \]

with \( \bar{v} = 1 - v \). In Eq. (17), when the asymptotical form for the vector meson LCDAs adopted, there will be infrared divergences in \( f_{II}^\pm \). As in [33, 36], we introduce a cutoff of order \( \Lambda_{QCD}/m_b \) and take \( \Lambda_{QCD} = 0.5 \) GeV as our default value.

The contributions of the QCD penguin-type diagrams can be described by the functions

\[ G^0(s) = \frac{2}{3} - \frac{4}{3} \ln \frac{\mu}{m_b} + 4 \int_0^1 du \Phi^{V_2}(u) g(u, s), \]
\[ G^\pm(s) = \frac{2}{3} - \frac{2}{3} \ln \frac{\mu}{m_b} + 2 \int_0^1 du \left( g^{(v)}(u) + \frac{ag^{(a)}(u)}{4} \right) g(u, s), \]

with the function \( g(u, s) \) defined as

\[ g(u, s) = \int_0^1 dx \, x \bar{x} \ln (s - x\bar{x}u - i\epsilon). \]
We have included the leading electro-weak penguin-type diagrams induced by the operators $O_1$ and $O_2$.

$$C_\ell^\lambda = \left[ \frac{v_u}{v_t} G^\lambda(s_u) + \frac{v_e}{v_t} G^\lambda(s_e) \right] \left( C_2 + \frac{C_1}{N_C} \right). \tag{21}$$

To calculate the coefficients $a_i$ in Eq. (15), we have also taken into account the contributions of the dipole operator $O_{8g}$, which are described by the functions

$$G_0^g = -\int_0^1 du \frac{2\Phi \Gamma_{V2}(u)}{1-u},$$

$$G_+^g = -\int_0^1 du \left( g_{(v)2}(u) + \frac{g_{(a)2}(u)}{4} \right) \frac{1}{1-u},$$

$$G_-^g = \int_0^1 \frac{du}{u} \left[ -\bar{u} g_{(v)2}(u) + \frac{\bar{u} g_{(a)2}(u)}{4} + \int_0^u dv \left( \Phi \Gamma_{V2}(v) - g_{(v)2}(v) \right) + \frac{g_{(a)2}(u)}{4} \right], \tag{22}$$

here we consider the higher-twist effects $k^\mu = uE n^\mu + k_+^\mu + \vec{k}_2^\perp n_+^\mu$ in the projector of Eq. (12).

The $G_-^g = 0$ in Eq. (22) if we consider the Wandzura-Wilczek-type relations, but we get $G_+^g \neq 0$ which is different from Ref. [10, 36].

With the coefficients in Eq. (15), we can obtain the decay amplitudes of the SM part $A^{SM}$.

$B \to VV$ decay amplitudes are given in Appendix A.

### 2.2 R-parity violating SUSY effects in the decays

Flavor changing neutral current (FCNC) is forbidden at tree level in the SM. FCNC processes could happen at one loop level, but are suppressed by Glashow-Iliopoulos-Maiani (GIM) mechanism. New physics effects could be comparable to the SM strength, so that penguin dominated rare B decays are sensitive to new physics. The RPV SUSY is an interesting scenario and its possible effects in B rare nonleptonic decays deserve our studies.

The R-parity symmetry was first introduced by Farrar and Fayet, which is assumed to forbid gauge invariant lepton and baryon number violating operators. The R-parity of a particle field is given by $R_p = (-1)^{L+2S+3B}$, where L and B are lepton and baryon numbers, and S is the spin. However, there is no deep theoretical motivation for imposing R-parity. The presence of RPV could give very rich phenomenology. Of course, it will get constraints from its phenomenology. The status of RPV SUSY and constraints on its parameters could be found in the recent reviews.
In the most general superpotential of the minimal supersymmetric Standard Model (MSSM), the RPV superpotential is given by \( W_{\text{RPV}} \) is the superpotential of the minimal supersymmetric Standard Model (MSSM), the RPV superpotential is given by \( W_{\text{RPV}} \) is the superpotential of the minimal supersymmetric Standard Model (MSSM),

\[
W_{\text{RPV}} = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c + \frac{1}{2} \lambda''_{i[jk]} \hat{U}_i \hat{D}_j^c \hat{D}_k^c,
\]

(23)

where \( \hat{L} \) and \( \hat{Q} \) are the SU(2)-doublet lepton and quark superfields and \( \hat{E}_c^c, \hat{U}^c \) are the singlet superfields, while \( i, j, \) and \( k \) are generation indices and \( c \) denotes a charge conjugate field.

The \( \lambda \) and \( \lambda' \) couplings in Eq. (23) break the lepton number, while the \( \lambda'' \) couplings break the baryon number conservation. There are 27 \( \lambda' \)-type couplings and nine each of the \( \lambda \) and \( \lambda'' \) couplings as \( \lambda_{ijk} \) is antisymmetric in \( i \) and \( j \), and \( \lambda''_{i[jk]} \) is antisymmetric in \( j \) and \( k \). The antisymmetry of the B-violating couplings, \( \lambda''_{i[jk]} \) in the last two indices implies that there are no operators generating the \( \bar{b} \to \bar{s}s \bar{s} \) and \( \bar{b} \to \bar{d}d \bar{d} \) transition.

From Eq. (23), we can obtain the following four fermion effective Hamiltonian due to the exchanging of the sleptons:

\[
H_{2u-2d}^{4f} = \sum_i \frac{\lambda'_{ijk} \lambda''_{ik}}{2m^2_{UL,i}} \eta^{-8/\beta_0} (\bar{d}_m \gamma^\mu P_R d_i) (\bar{u}_k \gamma^\mu P_L u_j) s,
\]

\[
H_{4d}^{4f} = \sum_i \frac{\lambda_{ijk} \lambda''_{ik}}{2m^2_{UL,i}} \eta^{-4/\beta_0} (\bar{d}_m \gamma^\mu P_R d_i) (\bar{d}_k \gamma^\mu P_L d_j) s.
\]

(24)

The four fermion Hamiltonian due to the exchange of the squarks can be written as

\[
H_{2u-2d}^{4f} = \sum_n \frac{\lambda''_{mik} \lambda''_{mnj}}{4m^2_{\tilde{d}_a,n}} \eta^{-4/\beta_0} \left\{ \left[ (\bar{u}_i \gamma^\mu P_R u_j) (\bar{d}_k \gamma^\mu P_R d_i) - (\bar{u}_i \gamma^\mu P_R u_j) (\bar{d}_k \gamma^\mu P_R d_i) s \right] - \left[ (\bar{d}_k \gamma^\mu P_R u_j) (\bar{u}_i \gamma^\mu P_R d_i) - (\bar{d}_k \gamma^\mu P_R u_j) (\bar{u}_i \gamma^\mu P_R d_i) s \right] \right\},
\]

\[
H_{4d}^{4f} = \sum_n \frac{\lambda''_{mik} \lambda''_{mnj}}{4m^2_{\tilde{d}_a,n}} \eta^{-4/\beta_0} \left\{ \left[ (\bar{d}_i \gamma^\mu P_R d_j) (\bar{d}_k \gamma^\mu P_R d_i) - (\bar{d}_i \gamma^\mu P_R d_j) (\bar{d}_k \gamma^\mu P_R d_i) s \right] - \left[ (\bar{d}_k \gamma^\mu P_R d_j) (\bar{d}_i \gamma^\mu P_R d_i) - (\bar{d}_k \gamma^\mu P_R d_j) (\bar{d}_i \gamma^\mu P_R d_i) s \right] \right\}.
\]

(25)

Where \( P_L = \frac{1-\gamma_5}{2}, P_R = \frac{1+\gamma_5}{2}, \eta = \frac{\alpha_s(m_{\tilde{q}})}{\alpha_s(m_t)} \) and \( \beta_0 = 11 - \frac{2}{3} n_f \). The subscript for the currents \((j_{\mu})_{1,8}\) represent the current in the color singlet and octet, respectively. The coefficients \( \eta^{-4/\beta_0} \) and \( \eta^{-8/\beta_0} \) are due to the running from the sfermion mass scale \( m_{\tilde{f}} \) (100 GeV assumed) down to the \( m_s \) scale. Since it is always assumed in phenomenology for numerical display that only one sfermion contributes one time, we neglect the mixing between the operators when we use the renormalization group equation (RGE) to run \( H^{4f} \) down to low scale. The \( H^{4f} \) for the relevant decay modes are written down in Appendix B.
Compared with the operators in the $\mathcal{H}_{\text{eff}}^{SM}$, there are new operators $(\bar{q}_2 q_3)_{V^\pm A}(\bar{b} q_1)_{V^\pm A}$ in the $\mathcal{H}^R$. We will use the (’) denote the $(\bar{q}_2 q_3)_{V^\pm A}(\bar{b} q_1)_{V^\pm A}$ current contribution. In the NF approach, the factorizable amplitude can be expressed as

$$X'(BV_1, V_2) = \langle V_2 |(\bar{q}_2 \gamma_\mu(1-\alpha \gamma_5)q_3) |0 \rangle \langle V_1 |(\bar{b} \gamma^\mu(1+\gamma_5)q_1) |B \rangle.$$ \hspace{1cm} (26)

Taking the $V_1$ ($V_2$) meson flying in the minus (plus) z-direction and using the sign convention $\epsilon^{0123} = -1$, we have

$$X'(BV_1, V_2) = \begin{cases} 
-\frac{if_{V_2}}{2m_{V_1}} \left[ (m_B^2 - m_{V_1}^2 - m_{V_2}^2)(m_B + m_{V_1})A_1^{BV_1}(m_{V_2}^2) \right. \\
\left. - \frac{2m_B m_{V_1}}{m_B + m_{V_1}} A_2^{BV_1}(m_{V_2}^2) \right] \equiv h'_0, \\
- if_{V_2} m_{V_2} \left[ (m_B + m_{V_1})A_1^{BV_1}(m_{V_2}^2) \pm \frac{2m_B m_{V_1}}{m_B + m_{V_1}} V^{BV_1}(m_{V_2}^2) \right] \equiv h'_\pm.
\end{cases} \hspace{1cm} (27)

Applying the QCDF approach and supposing $V_1$ ($V_2$) is the recoiled (emitted) meson, we obtain the vertex corrections $f_{I^0}^a(a)$ and hard spectator scattering corrections $f_{II}^\pm(a)$ as follows:

$$f_{I^0}^a(a) = 12 \ln \frac{\mu}{m_b} + 18 - 6(1 + a) - \int_0^1 du \Phi_{V_2}^{(v)}(u) \left( 3 \frac{1-2u}{1-u} \ln u - 3i\pi \right),$$

$$f_{I^\pm}^a(a) = 12 \ln \frac{\mu}{m_b} + 18 - 6(1 + a) - \int_1^0 du \left( g_{V_2}^{(v)}(u) \pm \frac{a g_{V_2}^{(a)}(u)}{4} \right) \left( 3 \frac{1-2u}{1-u} \ln u - 3i\pi \right),$$

$$f_{II}^{0}(a) = \frac{4\pi^2}{N_C} \frac{i f_B f_{V_2} m_{V_2}}{h'_0} \int_0^1 d\xi \frac{\Phi_{I^0}^B(\xi)}{\xi} \int_0^1 dv \Phi_{II}^V(v) \int_0^1 du \frac{\Phi_{V_2}^V(u)}{u},$$

$$f_{II}^{\pm}(a) = -\frac{4\pi^2}{N_C} \frac{2i f_B f_{V_2} m_{V_2}}{m_B h'_\pm} (1 \pm 1) \int_0^1 d\xi \frac{\Phi_{II}^B(\xi)}{\xi} \int_0^1 dv \frac{\Phi_{I^0}^V(v)}{v^2} \times \int_0^1 du \left( g_{V_2}^{(v)}(u) \pm \frac{a g_{V_2}^{(a)}(u)}{4} \right) + \frac{4\pi^2}{N_C} \frac{2i f_B f_{V_2} m_{V_2} m_{V_1}}{m_B h'_\pm} \int_0^1 d\xi \frac{\Phi_{II}^B(\xi)}{\xi} \times \int_0^1 dv du \left( g_{V_2}^{(v)}(v) \mp \frac{a g_{V_2}^{(a)}(v)}{4} \right) \left( g_{V_2}^{(v)}(u) \pm \frac{a g_{V_2}^{(a)}(u)}{4} \right) \frac{u + \bar{v}}{uv^2}. \hspace{1cm} (28)$$

Since we are considering the leading effects of RPV, we need only evaluate the vertex corrections and the hard-spectator scattering. The R-parity violating contribution to the decay amplitudes $\mathcal{A}^R$ can be found in Appendix C.

### 2.3 The polarized fraction and branching ratio

With the QCDF approach, we can get the total decay amplitude

$$\mathcal{A}_\lambda = \mathcal{A}_\lambda^{SM} + \mathcal{A}_\lambda^R. \hspace{1cm} (29)$$
The expressions for the SM amplitude $\mathcal{A}_{\lambda}^{SM}$ and the RPV amplitude $\mathcal{A}_{\lambda}^{R}$ are presented in Appendices A and C. From the amplitude in Eq. (29), the branching ratio reads

$$Br(B \to VV) = \frac{\tau_B |p_c|}{8\pi m_B^2} (|A_0|^2 + |A_+|^2 + |A_-|^2),$$

(30)

where $\tau_B$ is the lifetime of B meson, $p_c$ is the center of mass momentum, and given by

$$|p_c| = \frac{1}{2m_B} \sqrt{[m_B^2 - (m_{V_1} + m_{V_2})^2][m_B^2 - (m_{V_1} - m_{V_2})^2]}.$$  

(31)

In order to compare the size of helicity amplitudes, we express the longitudinal polarization fraction

$$\frac{\Gamma_L}{\Gamma} = \frac{|A_0|^2}{|A_0|^2 + |A_+|^2 + |A_-|^2}.$$  

(32)

The ratios of $\Gamma_L/\Gamma$ measure the relative strength of longitudinally polarizations in the decay.

### 3 Input Parameters

#### A. Wilson coefficients

To proceed we use the next-to-leading Wilson coefficients calculated in the naive dimensional regularization (NDR) scheme and at $m_b$ scale [30]:

- $C_1 = 1.082$, $C_2 = -0.185$, $C_3 = 0.014$, $C_4 = -0.035$, $C_5 = 0.009$, $C_6 = -0.041$, $C_7/\alpha_e = -0.002$, $C_8/\alpha_e = 0.054$, $C_9/\alpha_e = -1.292$, $C_{10}/\alpha_e = 0.263$, $C_{8g} = -0.143$.

#### B. The CKM matrix element

As for the CKM matrix elements, we will use the Wolfenstein parametrization [12]:

$$\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.$$  

(33)

We shall use the values [13]

- $|V_{cb}| = 0.0413$, $\lambda = 0.22$, $\bar{\rho} = 0.20$, $\bar{\eta} = 0.33$,

where $\bar{\rho} = \rho(1 - \frac{\lambda^2}{2})$ and $\bar{\eta} = \eta(1 - \frac{\lambda^2}{2})$. 

11
C. Masses and lifetime

For quark masses, which appear in the penguin loop corrections with regard to the functions $G^\lambda(s)$, we take

$$m_u = m_d = m_s = 0, \quad m_c = 1.47 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}.$$  

To compute the branching ratio, the lifetime of $B$ meson and masses of meson are also taken from [43]

$$\tau_{B_u} = 1.671 \text{ ps}, \quad m_{B_u} = 5279 \text{ MeV}, \quad \tau_{B_d} = 1.536 \text{ ps}, \quad m_{B_d} = 5279 \text{ MeV},$$

$$m_{K^{*\pm}} = 892 \text{ MeV}, \quad m_\phi = 1019 \text{ MeV}, \quad m_{K^{*0}} = 896 \text{ MeV}, \quad m_\rho = 776 \text{ MeV}.$$  

D. The LCDAs of the vector meson

For the LCDAs of the vector meson, we use the asymptotic form [33]

$$\Phi^V_\parallel(x) = \Phi^V(x) - g_\perp^V = 6x(1-x),$$

$$g_\perp^V = \frac{3}{4}[1 + (2x - 1)^2].$$  

As for the two $B$ meson wave functions in Eq. (34), we consider only the $\Phi^B_1(\xi)$ contribution to the nonfactorizable corrections as done in the literature [29, 44], since $\Phi^B_2(\xi)$ is power suppressed. We adopt the moments of the $\Phi^B_1(\xi)$ defined in Ref. [29, 44] for our numerical evaluation:

$$\int_0^1 d\xi \frac{\Phi^B(\xi)}{\xi} = \frac{m_B}{\lambda_B},$$  

with $\lambda_B = 0.46 \text{ GeV}$ [45]. The quantity $\lambda_B$ parameterizes our ignorance about the $B$ meson distribution amplitudes and thus brings considerable theoretical uncertainty.

E. The decay constant and form factors

The decay constant and form factors are nonperturbative parameters. They are available from the experimental data or estimated with theories, such as lattice calculations, QCD sum rules, etc. For the decay constant, we take the latest light-cone QCD sum rule results (LCSR) [32] in our calculations:

$$f_{K^*} = 217 \text{ MeV}, \quad f_\rho = 205 \text{ MeV}, \quad f_\phi = 231 \text{ MeV}$$

$$f_{K^*_c} = 156 \text{ MeV}, \quad f_{\rho^*} = 147 \text{ MeV}, \quad f_{\phi^*} = 183 \text{ MeV},$$
and \( f_{B_{u(d)}} = 161 \text{ MeV} \). For the form factors involving the \( B \rightarrow K^* \) and \( B \rightarrow \rho \) transition, we adopt the results given by \(^{32}\)

\[
A_1^{B_{u(d)}K^*}(0) = 0.292, \quad A_2^{B_{u(d)}K^*}(0) = 0.259, \quad V^{B_{u(d)}K^*}(0) = 0.411, \\
A_1^{B_{u(d)}\rho}(0) = 0.242, \quad A_2^{B_{u(d)}\rho}(0) = 0.221, \quad V^{B_{u(d)}\rho}(0) = 0.323.
\]

4 Numerical results and Analysis

In this section we will give our estimations in the SM and compare with the relevant experimental data, then we show the RPV effects to the branching ratios and longitudinal polarization fractions. With the RPV effects we can do two things. First, it is possible to have small longitudinal polarization fractions for the pure penguin processes completely different from the SM predictions. Second, one can obtain more stringent bounds on the product combinations of RPV couplings if the measured values of the decay modes are consistent with the SM predictions.

In the numerical analysis, we assume that only one sfermion contributes one time with a mass of 100 GeV. So for other values of the sfermion masses, the bounds on the couplings in this paper can be easily obtained by scaling them by factor \( \tilde{f}^2 \equiv \left( \frac{m_{\tilde{f}}}{100 \text{ GeV}} \right)^2 \). We consider all possible sfermion contributions when we obtain the bounds on the couplings.

4.1 \( B^0 \rightarrow \phi K^{*0} \) and \( B^+ \rightarrow \phi K^{*+} \)

\( B^0 \rightarrow \phi K^{*0} \) and \( B^+ \rightarrow \phi K^{*+} \) decays are induced by the underlying quark transition \( \bar{b} \rightarrow \bar{s}s\bar{s} \) at one loop level. In the SM, these decays are called pure penguin processes. However, they could happen at tree level in SUSY models with RPV. Our estimations in the SM and recent data from the BABAR and Belle collaborations are presented in the Table I and II.

From Table I and II, we can see that the predictions from the two factorization framework are very similar. It is also noted that the latest LCSR \(^{32}\) results for factors are smaller than their pervious values \(^{46}\). With the form factors, we find that the SM predictions for the branching ratios are smaller that the BABAR \(^1\) \(^7\) and Belle \(^5\) \(^8\) measurements, and the
Table I: Branching ratios of $B \to \phi K^*$ decay modes.

| Mode            | BABAR($\times 10^6$) | Belle($\times 10^6$) | Average($\times 10^6$) | QCDF in SM($\times 10^6$) |
|-----------------|-----------------------|----------------------|-------------------------|---------------------------|
| $B_u^+ \to \phi K^{*+}$ | $12.7^{+2.2}_{-2.0} + 1.1$ | $6.7^{+2.1+0.7}_{-1.9-1.0}$ | $9.5 \pm 1.7$ | $4.60$ |
| $B_d^0 \to \phi K^{*0}$    | $9.2 \pm 0.9 \pm 0.5$  | $10.0^{+1.6+0.7}_{-1.5-0.8}$ | $9.4 \pm 0.9$ | $4.21$ |

Table II: The longitudinal polarization fractions of $B \to \phi K^*$ decay modes.

| Mode            | BABAR     | Belle     | Average    | QCDF in SM |
|-----------------|-----------|-----------|------------|-------------|
| $B_u^+ \to \phi K^{*+}$ | $0.46 \pm 0.12 \pm 0.03$ | $0.51 \pm 0.08 \pm 0.03$ | $0.49 \pm 0.07$ | $0.861$ |
| $B_d^0 \to \phi K^{*0}$    | $0.52 \pm 0.05 \pm 0.02$  | $0.45 \pm 0.05 \pm 0.02$ | $0.49 \pm 0.04$ | $0.861$ |

Longitudinal polarizations are as large as $\sim 90\%$ in contrast to $\sim 50\%$ measured by BABAR and Belle.

Now we turn to RPV effects which may give a possible solution to the polarization anomaly. Using the formula derived in previous sections, we can calculate RPV effects. The RPV effects to the branching ratios and longitudinal polarization fractions of the $B \to \phi K^*$ decays are displayed by curves in the Fig.1. We find that the $\lambda^\prime_{i22}\lambda^{*}_{i32}$ term could not enhance transverse polarization because the corresponding current has a structure $\bar{s}\gamma_\mu (1 + \gamma_5)s\bar{b}\gamma^\mu (1 - \gamma_5)s$. Its matrix element is the same as the SM one, since $\langle \phi | \bar{s}\gamma_\mu \gamma_5 s | 0 \rangle = 0$. However, the $\lambda^\prime_{i23}\lambda^{*}_{i22}$ term could provide a solution to the polarization anomaly because its current has the structure $\bar{s}\gamma_\mu (1 - \gamma_5)s\bar{b}\gamma^\mu (1 + \gamma_5)s$. The right handed ($\bar{b}s$) current will flip the signs of the axial parts of the matrix $\langle K^* | \bar{b}\gamma^\mu (1 - \gamma_5)s | B \rangle$ appearing in the SM contributions. We also find that $|\lambda^\prime_{i23}\lambda^{*}_{i22}| < 3.0 \times 10^{-3}\tilde{\nu}_{Li}^2$ by the branching ratios; however, there is a much stronger bound from the polarization ratios.

Combining both the branching ratios and the polarization ratios which we get by QCDF (the same as follow), the resolution is obtained in a very narrow parameter interval $|\lambda^\prime_{i23}\lambda^{*}_{i22}| \in [1.5 \times 10^{-3}\tilde{\nu}_{Li}^2, 2.1 \times 10^{-3}\tilde{\nu}_{Li}^2]$. Fortunately, the parameter space is not ruled out yet, the existing upper limit is $|\lambda^\prime_{i23}\lambda^{*}_{i22}| < 2.3 \times 10^{-3}$ [22, 24]. We note that similar strength of the RPV couplings is also needed in the recent studies to solve the CP problem in $B \to \phi K$ [47] and the $\eta t$ puzzle in $B \to \eta t K$ [20].
Figure 1: The branching ratios and the longitudinal polarization fractions for $B \to \phi K^*$ as functions of the RPV couplings $\lambda' \lambda''$. The solid curves represent our theoretical results by QCDF. The dash lines are the predictions by NF for comparisons. The horizontal solid lines are the experimental data and the SM predictions with QCDF as labelled respectively. The horizontal dot lines represent the $1\sigma$ error-bar of the measurements. (The same in Fig. 2 and 3).

$4.2 \quad B_u^+ \to \rho K^*$ decays

$B_u^+ \to \rho K^*$ decays are due to $\bar{b} \to \bar{u}u\bar{s}$ or $\bar{b} \to \bar{d}d\bar{s}$ transitions at the quark level. The $B_u^+ \to \rho^0 K^{*0}$ decay is a pure penguin process in the SM. Its longitudinal polarization fraction were measured to be unexpected low $\sim 0.5$ very recently by Belle [9], which is inconsistent with the SM prediction. While the $B_u^+ \to \rho^0 K^{*+}$ has both tree and penguin amplitude, experimental measurements by BABAR [11] have shown the decay dominated by longitudinal polarization, which is consistent with the SM predictions.

For comparison, our estimations in the SM and recent data from experiments are presented in the Table III and IV.
Table III: Branching ratios of $B \to \rho K^*$ decay modes.

| Mode            | $B_{ABAR} \times 10^6$ | Belle$(\times 10^6)$ | QCDF in SM$(\times 10^6)$ |
|-----------------|------------------------|----------------------|---------------------------|
| $B_u^+ \to \rho^0 K^{*+}$ | $10.6^{+3.0}_{-2.6} \pm 2.4$ | $\cdots$ | 2.58                       |
| $B_u^+ \to \rho^+ K^{*0}$ | $6.6 \pm 2.2 \pm 0.8$ | $\cdots$ | 3.79                       |

Table IV: The longitudinal polarization fractions of $B \to \rho K^*$ decay modes.

| Mode            | $B_{ABAR}$ | Belle | QCDF in SM |
|-----------------|------------|-------|------------|
| $B_u^+ \to \rho^0 K^{*+}$ | $0.96^{+0.04}_{-0.15} \pm 0.04$ | $\cdots$ | 0.906       |
| $B_u^+ \to \rho^+ K^{*0}$ | $0.50 \pm 0.19^{+0.05}_{-0.07}$ | $\cdots$ | 0.905       |

From Table III, we can see the SM predictions for the branching ratios are smaller than the measurement by $B_{ABAR}$ and Belle. As shown in Table IV, for the longitudinal polarization fraction in the tree dominant decay $B^+ \to \rho^0 K^{*+}$, the SM prediction agrees with the $B_{ABAR}$ measurement very well. However, for the pure penguin process $B^+ \to \rho^+ K^{*0}$, the SM predicts dominant longitudinal polarization, which is in contrast to Belle measurement.

In the $B_u^+ \to \rho^0 K^{*+}$ decay, since the quark content of $\rho^0$ is $(u\bar{u} - d\bar{d})/\sqrt{2}$, the decay could be induced by superpartners of both up-type and down-type fermions. For example, $\bar{b} \to d\bar{s}$ could be induced by sneutrino, while $\bar{b} \to u\bar{u}s$ could be induced by slepton with the same $\lambda_1^{\prime} \lambda_2^{n*}$ product. We take $\lambda_1^{\prime} \lambda_2^{n*}$ and $\lambda_3^{\prime} \lambda_2^{n*}$ contribute to $B^+ \to u\bar{u}s$ and $\bar{b} \to d\bar{s}$ at the same time, so the effects of $\lambda_1^{\prime} \lambda_2^{n*}$ will be eliminated if taking $m_{\tilde{\nu}_i} = m_{\tilde{\nu}_i}$.

Our results for the RPV contributions to $B_u^+ \to \rho K^*$ are summarized in Fig.2. We can get a lot of information from this figure. We find that the longitudinal polarization in $B_u^+ \to \rho^0 K^{*+}$ is sensitive to $|\lambda_1^{\prime}\lambda_1^{n*}|$ and $|\lambda_2^{\prime}\lambda_2^{n*}|$ arising from exchanging $\bar{d}$ and $\bar{u}$, between the quark currents $\bar{u}\gamma_\mu(1 + \gamma_5)u \otimes \bar{b}\gamma_\mu(1 + \gamma_5)s$ and $d\gamma_\mu(1 + \gamma_5)d \otimes \bar{b}\gamma_\mu(1 + \gamma_5)s$, respectively, and insensitive to the effect from exchanging $\bar{\nu}_i$ between the quark currents $d\gamma_\mu(1 + \gamma_5)d \otimes \bar{b}\gamma_\mu(1 + \gamma_5)s$. However, the pure penguin process $B_u^+ \to \rho^+ K^{*0}$ is sensitive to the RPV couplings $|\lambda_3^{\prime}\lambda_2^{n*}|$ and $|\lambda_3^{\prime}\lambda_1^{n*}|$ which associate with operators $d\gamma_\mu(1 + \gamma_5)s \otimes \bar{b}\gamma_\mu(1 + \gamma_5)d$ and $d\gamma_\mu(1 - \gamma_5)s \otimes \bar{b}\gamma_\mu(1 + \gamma_5)d$, respectively.
Figure 2: The branching ratios and longitudinal polarization fractions for $B \to \rho K^*$ as functions of the RPV couplings $\lambda' \lambda''$ and $\lambda''' \lambda'''$. 

From Fig.2, the polarization problem in $B_u^+ \to \rho^0 K^{*+}$ could be solved by RPV effects. However, combining the constraints from polarization fraction and branching ratios of $B_u^+ \to \rho K^*$, we obtain very narrow parameters spaces for relevant coupling constants. It implies that these couplings could be pinned down or ruled out by refined measurements in the very near future at BABAR and Belle. Our constraints are listed in Table V.

For comparison, we have also listed existing bounds on these quadric coupling constant products. We can see that the product $|\lambda''_{31}\lambda''_{121}|$ is severely constrained by double nucleon decay [23]. Our bounds on $|\lambda'_{11}\lambda''_{132}|$ is weaker than that by $\Delta m_K$ [48]. These couplings also contribute to $B \to K \pi$ decays. In an interesting study [27], constraints on these couplings have been derived from the experiment measurement of branching ratios of $B \to K \pi$. As shown in Table V, our bounds are lower than theirs [27]. However, in view of new data from BABAR and Belle, it would be interesting to investigate whether the RPV couplings could solve the
Therefore, these decays could give strong constraints on relevant RPV couplings. The RPV contributions are presented in Fig.3. From the figure, we can know that longitudinal polarization fractions for $B \rightarrow \rho \rho$ are not sensitive to the RPV couplings since they are tree dominated in the SM. By the decay $B_u^+ \rightarrow \rho^0 \rho^+$, we get the bounds on $|\lambda'_{i11}\lambda''_{i31}|$:

$$|\lambda'_{i11}\lambda''_{i31}| \in \begin{cases} [6.3 \times 10^{-3} \rho^2_{Li}, 8.6 \times 10^{-3} \rho^2_{Li}] & \text{for } \lambda_{i11}\lambda''_{i31}>0 \\ [8.6 \times 10^{-4} \rho^2_{Li}, 2.5 \times 10^{-3} \rho^2_{Li}] & \text{for } \lambda_{i11}\lambda''_{i31}<0 \end{cases}$$

by $B_u^+ \rightarrow \rho^0 \rho^+$. (36)

The existing bounds on the products by $B \rightarrow \pi \pi$ decays are $|\lambda'_{i11}\lambda''_{i31}| < 1.6 \times 10^{-2}$ [27]. The parameter spaces constrained by $B_u^+ \rightarrow \rho^0 \rho^+$ are very narrow, which could be closed easily be future refined measurements if the tight bounds from branching ratio and polarization ratio do not overlap. This could not be done by the $B \rightarrow PP, PV$ decays. We see again the rich phenomena of $B \rightarrow VV$ decays and its power for bounding new physics.

### Table V: Bounds on the quadric coupling constant products. For comparison, the exiting bounds are listed in last column.

| Couplings | Bounds | Process | Previous bounds |
|-----------|--------|---------|-----------------|
| $|\lambda'_{i21}\lambda''_{i31}|$ | $\leq 5.1 \times 10^{-3} \rho_2^2$, $\leq 2.5 \times 10^{-3} \rho_2^2$ | $B_u^+ \rightarrow \rho^0 K^{*+}$ | $8.2 \times 10^{-4}$ [23] |
| $|\lambda''_{i31}\lambda'_{i21}|$ | $\leq 4.1 \times 10^{-3} \rho_2^2$, $[4.0 \times 10^{-3} \rho_2^2, 8.3 \times 10^{-3} \rho_2^2]$ and $\leq 1.1 \times 10^{-3} \rho_2^2$ | $B_u^+ \rightarrow \rho^0 K^{*+}$ | $1.0 \times 10^{-2}$ [27] |
| $|\lambda''_{i121}\lambda'_{i31}|$ | $\leq 1.8 \times 10^{-2} \rho_2^2$, $[1.6 \times 10^{-2} \rho_2^2, 3.8 \times 10^{-2} \rho_2^2]$ and $\leq 4.7 \times 10^{-3} \rho_2^2$ | $B_u^+ \rightarrow \rho^0 K^{*+}$ | $2 \times 10^{-8}$ [23] |
| $|\lambda''_{i231}\lambda'_{i11}|$ | $\leq 1.5 \times 10^{-3} \rho_2^2$, $[1.6 \times 10^{-3} \rho_2^2, 4.0 \times 10^{-3} \rho_2^2]$ and $\leq 1.2 \times 10^{-3} \rho_2^2$ | $B_u^+ \rightarrow \rho^+ K^{*0}$ | $2.1 \times 10^{-3}$ [27] |
| $|\lambda''_{i11}\lambda'_{i32}|$ | $\leq 1.4 \times 10^{-3} \rho_2^2$, $\leq 4.1 \times 10^{-3} \rho_2^2$ | $B_u^+ \rightarrow \rho^+ K^{*0}$ | $4.7 \times 10^{-4}$ [18] |
| $|\lambda''_{i31}\lambda'_{i21}|$ | $\leq 2.5 \times 10^{-3} \rho_2^2$, $\leq 6.3 \times 10^{-3} \rho_2^2$ | $B_u^+ \rightarrow \rho^+ K^{*0}$ | $1.0 \times 10^{-2}$ [27] |

**known πK puzzle** [49, 50].

### 4.3 $B \rightarrow \rho \rho$ decays

Recent measurements of tree dominated vector-vector charmless modes $B_u^+ \rightarrow \rho^0 \rho^+$ and $B_d^0 \rightarrow \rho^+ \rho^-$ by $BABAR$ [1, 4] and Belle [2] have shown that the decays are dominated by longitudinal polarizations, which are just as the SM expectations.

Our estimations in the SM and recent data from the $BABAR$ and Belle collaborations are presented in Table VI and VII. We find that the SM predictions for both branching ratios and longitudinal polarization fractions agree with the $BABAR$ and Belle measurements very well. Therefore, these decays could give strong constraints on relevant RPV couplings.

The RPV contributions are presented in Fig.3. From the figure, we can know that longitudinal polarization fractions for $B \rightarrow \rho \rho$ are not sensitive to the RPV couplings since they are tree dominated decay in the SM. By the decay $B_u^+ \rightarrow \rho^0 \rho^+$, we get the bounds on $|\lambda'_{i11}\lambda''_{i31}|$: 

$$|\lambda'_{i11}\lambda''_{i31}| \in \begin{cases} 6.3 \times 10^{-3} \rho_2^2, 8.6 \times 10^{-3} \rho_2^2 & \text{for } \lambda_{i11}\lambda''_{i31}>0 \\ 8.6 \times 10^{-4} \rho_2^2, 2.5 \times 10^{-3} \rho_2^2 & \text{for } \lambda_{i11}\lambda''_{i31}<0 \end{cases}$$

by $B_u^+ \rightarrow \rho^0 \rho^+$. (36)
Table VI: Branching ratios of $B \to \rho\rho$ decay modes.

| Mode          | BABAR ($\times 10^6$) | Belle ($\times 10^6$) | Average ($\times 10^6$) | QCDF in SM ($\times 10^6$) |
|---------------|------------------------|-----------------------|-------------------------|-----------------------------|
| $B_u^+ \to \rho^0\rho^+$ | $22.5^{+5.7}_{-5.4} \pm 5.8$ | $31.7 \pm 7.1^{+3.8}_{-0.7}$ | $26.2 \pm 6.2$ | $15.30$ |
| $B_d^0 \to \rho^+\rho^-$ | $30 \pm 4 \pm 5$ | ... | ... | $25.75$ |

Table VII: The longitudinal polarization fractions of $B \to \rho\rho$ decay modes.

| Mode          | BABAR | Belle | Average | QCDF in SM |
|---------------|-------|-------|---------|-------------|
| $B_u^+ \to \rho^0\rho^+$ | $0.97^{+0.03}_{-0.07} \pm 0.04$ | $0.948 \pm 0.106 \pm 0.021$ | $0.964 \pm 0.056$ | $0.941$ |
| $B_d^0 \to \rho^+\rho^-$ | $0.99 \pm 0.03^{+0.04}_{-0.03}$ | ... | ... | $0.935$ |

Figure 3: The branching ratios and longitudinal polarizations for $B \to \rho\rho$ as a function of the RPV couplings $\lambda'\lambda^{rs}$ and $\lambda''\lambda^{rs}$.

We note that RPV coupling $\lambda''^i \lambda''^{i*}$ is eliminated because the factorizable amplitudes $X^{(B_u^+\rho^0,\rho^+)} = X^{(B_d^+\rho^+,\rho^0)}$ and cancelled each other. The effect of $\lambda'^i \lambda'^{i*}$ also is eliminated.
for the same reason in $B_u^+ \rightarrow \rho^0 K^{*+}$.

The decay $B_d^0 \rightarrow \rho^+ \rho^-$ give constraint $\lambda''_{132} \lambda''_{112} \in [9.3 \times 10^{-4} \tilde{s}^2, 1.1 \times 10^{-3} \tilde{s}^2]$. However, this parameter space is already ruled out in previous studies [22, 23]. Again, we see the standard model works well for tree dominant B decays.

Through the numerical analysis in this section, we can conclude that the experimental value of the polarization anomaly in pure penguin decays can be solved with RPV effects; however, the solution parameter spaces are always very narrow.

5 Conclusions

Motivated by the polarization anomaly observed recently by BABAR and Belle, we have studied $B \rightarrow VV$ modes with QCD factorization approach both in the SM and in RPV SUSY theories. We have found a set of RPV couplings can give possible solution to the polarization anomaly. However, the windows of the RPV couplings intervals are found to be always very narrow. It implies that these couplings might be pinned down from the rich polarization phenomena in these decays. However, it also implies the window could be closed easily with refined measurements from BABAR and Belle in the near future.

Since the hadronic dynamics for B nonleptonic decays are generally tangled with nonperturbative dynamics, we need to known how to separate perturbative and nonperturbative dynamics; of course, we also need to know the value for the parameters of nonperturbative dynamics to give reliable predictions based on electroweak theories. At the present stage, QCDF is a working scheme. Generally, we can believe that QCDF calculations for polarization fractions could be more accurate than that for branching ratios, since many uncertainties could be cancelled in the fractions. Therefore the constraints from polarization measurements would be more well-founded than those from branching ratio measurements.

Comparing our prediction with the recent experimental data, we have obtained bounds on the relevant products of RPV couplings. We find that many bounds are stronger than the existing limits [22, 23], which may be useful for further studying the RPV SUSY.

In conclusion, we have shown that RPV SUSY could give possible solution to the polarization anomaly in pure penguin decays $B \rightarrow \phi K^*$ and $B^+ \rightarrow \rho^+ K^{*0}$ observed by BABAR and Belle.
Acknowledgments

We thank Alex Kagan for helpful communications and comments on this paper. The work is supported in part by National Science Foundation under contract No.10305003, Henan Provincial Foundation for Prominent Young Scientists under contract No.0312001700 and in part by the project sponsored by SRF for ROCS, SEM.

Appendix

A. The amplitudes in the SM

$A^\lambda_{\text{SM}}(B_u^+ \to \phi K^{*+}) = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} a^\lambda_3 + a^\lambda_4 + a^\lambda_5 - \frac{1}{2} (a^\lambda_7 + a^\lambda_9 + a^\lambda_{10}) X(B_u^+ K^{*+},\phi), \quad (37)$

$A^\lambda_{\text{SM}}(B_d^0 \to \phi K^{*0}) = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} a^\lambda_3 + a^\lambda_4 + a^\lambda_5 - \frac{1}{2} (a^\lambda_7 + a^\lambda_9 + a^\lambda_{10}) X(B_d^0 K^{*0},\phi), \quad (38)$

$A^\lambda_{\text{SM}}(B_u^+ \to \rho^0 K^{*+}) = \frac{G_F}{2} \left\{\left[V_{ub}^* V_{us} a^\lambda_2 - V_{tb}^* V_{ts} \frac{3}{2} (a^\lambda_7 + a^\lambda_9)\right] X(B_u^+ K^{*+},\rho^0) + \left[V_{ub}^* V_{us} a^\lambda_4 - V_{tb}^* V_{ts} (a^\lambda_7 + a^\lambda_9)\right] X(B_u^+ K^{*+},\rho^+ + \rho^0)\right\}, \quad (39)$

$A^\lambda_{\text{SM}}(B_u^+ \to \rho^+ K^{*0}) = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} a^\lambda_3 - a^\lambda_5 - \frac{1}{2} (a^\lambda_7 + a^\lambda_9 + a^\lambda_{10}) X(B_u^+ K^{*0},\rho^0), \quad (40)$

$A^\lambda_{\text{SM}}(B_u^+ \to \rho^0 \rho^+ + \rho^-) = \frac{G_F}{2} \left\{V_{ub}^* V_{ud} a^\lambda_1 - V_{tb}^* V_{td} (a^\lambda_7 + a^\lambda_9)\right\} X(B_u^+ \rho^0,\rho^- + \rho^+). \quad (41)$

$A^\lambda_{\text{SM}}(B_u^+ \to \rho^0 \rho^0 + \rho^-) = \frac{G_F}{\sqrt{2}} [V_{ub}^* V_{ud} a^\lambda_1 - V_{tb}^* V_{td} (a^\lambda_7 + a^\lambda_9)] X(B_u^+ \rho^0,\rho^- + \rho^+). \quad (42)$

where we neglected the annihilation matrix element contributions.

B. The Hamiltonian for RPV

$H^R_{\text{eff}}(B_u^+ \to \phi K^{*+}) = \frac{\lambda^{L_3 L_2 L_1}}{2 m_d^2} \eta^{-8/30} (\bar{s} \gamma^\mu P_L s)(\bar{b} \gamma_\mu P_R s) + \frac{\lambda^{L_2 L_3 L_1}}{2 m_d^2} \eta^{-8/30} (\bar{s} \gamma^\mu P_R s)(\bar{b} \gamma_\mu P_L s),$

$H^R_{\text{eff}}(B_d^0 \to \phi K^{*0}) = \frac{\lambda^{L_3 L_2 L_1}}{2 m_d^2} \eta^{-8/30} (\bar{s} \gamma^\mu P_L s)(\bar{b} \gamma_\mu P_R s) + \frac{\lambda^{L_2 L_3 L_1}}{2 m_d^2} \eta^{-8/30} (\bar{s} \gamma^\mu P_R s)(\bar{b} \gamma_\mu P_L s),$

$H^R_{\text{eff}}(B_u^+ \to \rho^0 K^{*+}) = \frac{\lambda^{L_1 L_2 L_3}}{2\sqrt{2} m_d^2} \eta^{-4/30} \left\{[(\bar{u} \gamma^\mu P_R u) \bar{b} \gamma_\mu P_L s] - (\bar{u} \gamma^\mu P_R u) \bar{b} \gamma_\mu P_L s\right\}$

$- \frac{\lambda^{L_1 L_2 L_3}}{4\sqrt{2} m_u^2} \eta^{-4/30} [(\bar{d} \gamma^\mu P_R d) \bar{b} \gamma_\mu P_L s] - (\bar{d} \gamma^\mu P_R d) \bar{b} \gamma_\mu P_L s\right\}$
\[ \mathcal{H}_{\text{eff}}(B_u^+ \to \rho^+ K^{0*}) = \frac{\lambda'_{11} \lambda'_{22}}{8 N_C m_d^2} \eta^{-4/30} \left[ \left( \bar{u}_R^\mu P_R u_s \right) (\bar{b}_R^\mu P_R s) - \left( \bar{d}_R^\mu P_R d \right) (\bar{b}_R^\mu P_R s) \right] \eta^{-5/30} \]

\[ + \frac{\lambda'_{11} \lambda'_{22}}{8 N_C m_d^2} \eta^{-4/30} \left[ (\bar{d}_R^\mu P_R d) (\bar{b}_R^\mu P_R s) \right] \eta^{-5/30} \]

\[ \mathcal{H}_{\text{eff}}(B_d^0 \to \rho^0 K^{*0}) = \frac{\lambda'_{11} \lambda'_{22}}{8 N_C m_d^2} \eta^{-4/30} \left[ (\bar{d}_R^\mu P_R d) (\bar{b}_R^\mu P_R s) \right] \eta^{-5/30} \]

\[ C. \ The \ amplitudes \ for \ RPV \]

\[ A_\lambda^R(B_u^+ \to \phi K^{*+}) = \frac{\lambda'_{13} \lambda'_{12}}{8 N_C m_d^2} \eta^{-8/30} \left[ \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(1) + f_{II}^\lambda(1) \right) \right] X(B_u^+ K^{*+}, \phi) \]

\[ + \frac{\lambda'_{12} \lambda'_{13}}{8 N_C m_d^2} \eta^{-8/30} \left[ \frac{1}{N_C} - \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(-1) + f_{II}^\lambda(-1) \right) \right] X(B_u^+ K^{*+}, \phi) \]

\[ A_\lambda^R(B_d^0 \to \phi K^{*0}) = \frac{\lambda'_{13} \lambda'_{12}}{8 N_C m_d^2} \eta^{-8/30} \left[ \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(1) + f_{II}^\lambda(1) \right) \right] X(B_d^0 K^{*0}, \phi) \]

\[ + \frac{\lambda'_{12} \lambda'_{13}}{8 N_C m_d^2} \eta^{-8/30} \left[ \frac{1}{N_C} - \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(-1) + f_{II}^\lambda(-1) \right) \right] X(B_d^0 K^{*0}, \phi) \]

\[ A_\lambda^R(B_u^+ \to \rho^0 K^{*+}) = \frac{\lambda'_{13} \lambda'_{12}}{8 \sqrt{2} m_d^2} \eta^{-4/30} \left[ \frac{1}{N_C} - \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(-1) + f_{II}^\lambda(-1) \right) \right] X(B_u^+ K^{*+}, \rho^0) \]

\[ + \frac{\lambda'_{12} \lambda'_{13}}{8 \sqrt{2} m_d^2} \eta^{-4/30} \left[ \frac{1}{N_C} - \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(-1) + f_{II}^\lambda(-1) \right) \right] X(B_u^+ K^{*+}, \rho^0) \]

\[ + \frac{\eta^{-8/30}}{N_C \sqrt{2}} \left( \frac{\lambda'_{13} \lambda'_{12}}{8 m_d^2} - \frac{\lambda'_{12} \lambda'_{13}}{8 m_d^2} \right) \left[ 1 + \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(1) + f_{II}^\lambda(1) \right) \right] X(B_u^+ K^{*+}, \rho^0) \]

\[ - \frac{\lambda'_{12} \lambda'_{13}}{8 N_C \sqrt{2} m_d^2} \eta^{-8/30} \left[ \frac{1}{N_C} - \frac{\alpha_s}{4 \pi} \frac{C_F}{N_C} \left( f_I^\lambda(-1) + f_{II}^\lambda(-1) \right) \right] X(B_u^+ K^{*+}, \rho^0) , \]
\[ \mathcal{A}_\lambda^R (B_u^+ \to \rho^+ K^{*0}) = \frac{\lambda'''_{12} \lambda''_{11}}{16 m_{\xi_i}^2} \eta^{-4/30} \left[ 1 - \frac{1}{N_C} - \frac{\alpha_s C_F}{4 \pi N_C} \left( f_I^\lambda(-1) + f_H^\lambda(-1) \right) \right] X^R(B_u^+ \rho^+, K^{*0}) \]
\[ + \frac{\lambda''_{12} \lambda''_{11}}{8 N_C m_{\bar{V}_{L_i}}^2} \eta^{-8/30} \left[ 1 + \frac{\alpha_s C_F}{4 \pi N_C} \left( f_I^\lambda(1) + f_H^\lambda(1) \right) \right] X^R(B_u^+ \rho^+, K^{*0}) \]
\[ + \frac{\lambda''_{12} \lambda''_{11}}{8 N_C m_{\bar{V}_{L_i}}^2} \eta^{-8/30} \left[ 1 - \frac{1}{N_C} - \frac{\alpha_s C_F}{4 \pi N_C} \left( f_I^\lambda(-1) + f_H^\lambda(-1) \right) \right] X^R(B_u^+ \rho^+, K^{*0}) \]
\[ + \frac{\eta^{-8/30}}{N_C \sqrt{2} \left( \lambda_{13} \lambda_{11}^{*} \right)} \left( \frac{\lambda_{13} \lambda_{11}^{*}}{8 m_{\bar{V}_{L_i}}^2} - \frac{\lambda_{13} \lambda_{11}^{*}}{8 m_{\bar{V}_{L_i}}^2} \right) \left[ 1 + \frac{\alpha_s C_F}{4 \pi N_C} \left( f_I^\lambda(1) + f_H^\lambda(1) \right) \right] X^R(B_u^+ \rho^+, \rho^+) \]
\[ - \frac{\lambda''_{12} \lambda''_{11}}{8 N_C \sqrt{2} m_{\bar{V}_{L_i}}^2} \eta^{-8/30} \left[ 1 - \frac{1}{N_C} - \frac{\alpha_s C_F}{4 \pi N_C} \left( f_I^\lambda(-1) + f_H^\lambda(-1) \right) \right] X^R(B_u^+ \rho^-, \rho^+) \]
\[ A^R_\lambda (B_d^0 \to \rho^+ \rho^-) = -\frac{\lambda''_{12} \lambda''_{11}}{8 m_{\xi_i}^2} \eta^{-4/30} \left[ 1 - \frac{1}{N_C} - \frac{\alpha_s C_F}{4 \pi N_C} \left( f_I^\lambda(-1) + f_H^\lambda(-1) \right) \right] X^R(B_d^0 \rho^-, \rho^+) \]

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