Primordial Nucleosynthesis: The Predicted and Observed Abundances and Their Consequences

Gary Steigman∗†

Departments of Physics and Astronomy, Center for Cosmology and Astro-Particle Physics
The Ohio State University, Columbus, OH, USA
E-mail: steigman@mps.ohio-state.edu

For a brief time in its early evolution the Universe was a cosmic nuclear reactor. The expansion and cooling of the Universe limited this epoch to the first few minutes, allowing time for the synthesis in astrophysically interesting abundances of only the lightest nuclides (D, 3He, 4He, 7Li). For big bang nucleosynthesis (BBN) in the standard models of cosmology and particle physics (SBBN), the SBBN-predicted abundances depend on only one adjustable parameter, the baryon density parameter (the ratio by number of baryons (nucleons) to photons). The predicted and observed abundances of the relic light elements are reviewed, testing the internal consistency of primordial nucleosynthesis. The consistency of BBN is also explored by comparing the values of the cosmological parameters inferred from primordial nucleosynthesis for the standard model and for models with non-standard early Universe expansion rates with those determined from studies of the cosmic background radiation, which provides a snapshot of the Universe some 400 thousand years after BBN ended.

11th Symposium on Nuclei in the Cosmos, NIC XI
July 19-23, 2010
Heidelberg, Germany

∗Speaker.
†A footnote may follow.
1. Introduction

During its early evolution the Universe passed through a very brief epoch during which its high temperature and particle density allowed for nuclear reactions among nucleons building the lightest nuclides (D, $^3$He, $^4$He, $^7$Li) in astrophysically interesting abundances. As a result the comparison between the predicted and observed primordial abundances of the light nuclides provides a unique window on the early evolution of the Universe and a key probe of the standard models of cosmology and particle physics. Much later in the evolution of the Universe, after electrons combine with protons and alphas at “recombination”, the cosmic background photons are free to propagate, forming the cosmic microwave background (CMB) radiation, the black body spectrum of radiation observed today at $T_0 = 2.725$ K. Primordial, or big bang nucleosynthesis (BBN) and the CMB probe particle physics and cosmology at two very widely separated epochs in the evolution of the Universe: BBN when the Universe was only $\sim 20$ minutes old and the CMB some 400 thousand years later. Comparisons of the predictions and observations of BBN and the CMB provide tests of the standard models of cosmology and particle physics and offer constraints on new physics. For a recent review of these tests and constraints and for further references, see [1].

The standard model of cosmology uses General Relativity to predict the evolution of an expanding Universe filled with radiation, including three flavors of light neutrinos, and matter (baryons along with non-baryonic dark matter). Neither non-baryonic dark matter, dark energy (or a cosmological constant), or spatial curvature are relevant for our discussion here since they play no role in the physics and evolution of the early Universe. BBN in the standard model (SBBN) predicts the primordial abundances of the four light nuclides as a function of only one adjustable parameter, the “baryon abundance parameter”, the ratio by number of baryons (nucleons) to CMB photons, $\eta_B \equiv n_B/n_\gamma \equiv 10^{10} \eta_{10}$. $\eta_B$ provides a measure of the baryon asymmetry of the Universe whose value is a goal of but, at present, not a prediction of models of particle physics. A large class of models of cosmology and/or particle physics which go beyond the standard models predict a non-standard expansion rate during the early, radiation-dominated (RD) Universe. The Hubble parameter, $H$, measures the universal expansion rate; for the early RD evolution the age of the Universe, $t$, and $H$ are related by $Ht = 1/2$. A non-standard expansion rate may be parameterized by the “expansion rate parameter”, $S \equiv H'/H$, where $S = 1$ for the standard model. Historically, a non-standard particle content of the early Universe has been considered as the source of $S \neq 1$ [3]. In this case, $S$ may be related to the “equivalent number of additional neutrinos”, the “extra” energy density normalized to the energy density contributed by one flavor of standard model neutrinos: $\Delta N_\nu \equiv (\rho_\nu - \rho) / \rho_\nu \ (N_\nu = 3 + \Delta N_\nu)$. $S$ and $\Delta N_\nu$ provide equivalent parameterizations of non-standard expansion rates. Prior to $e^\pm$ annihilation, $S^2 = 1 + 7\Delta N_\nu/43 = 1 + 0.163\Delta N_\nu$, while post-$e^\pm$ annihilation, $S^2 = 1 + 0.134\Delta N_\nu$ (see, e.g., [3]). However, keep in mind that $\Delta N_\nu \neq 0$ need not be the result of extra neutrino flavors but could be due to other extensions of the standard models. In particular, $\Delta N_\nu$ need not be an integer and need not be positive. For example, if the gravitational constant were different in the early Universe (at BBN and/or at recombination) from its value today, $G_{\text{BBN}}/G_0 = 1 + 0.163\Delta N_\nu$ and $G_{\text{REC}}/G_0 = 1 + 0.134\Delta N_\nu$. While SBBN only depends on $\eta_B$, BBN depends on $S$ (or $\Delta N_\nu$) as well.

BBN at $\sim 3$ minutes and the CMB (recombination) some $\sim 400$ thousand years later provide complementary probes of the physics and early evolution of the Universe. This review of BBN ad-
addresses the following questions. Do the BBN-predicted abundances agree with the observationally-inferred primordial abundances (i.e., is BBN internally consistent)? Do the values of $\eta_B$ inferred from BBN and the CMB agree? Do the values of $\Delta N_{\nu}$ derived from BBN and the CMB agree and, are $\Delta N_{\nu}(\text{BBN}) = \Delta N_{\nu}(\text{CMB}) = 0$?

2. SBBN-Predicted And Observationally-Inferred Primordial Abundances

The observations of value in inferring the primordial abundances are diverse, from the oldest, most metal-poor stars in the Galaxy ($^7\text{Li}$), to H II regions in the Galaxy ($^3\text{He}$) and extragalactic H II regions ($^4\text{He}$), to cool, neutral (H I) gas in the Lyman-alpha forest (D). To a greater or lesser extent, the material in these regions have experienced some post-BBN nuclear processing, potentially modifying the original relic abundances. The post-BBN evolution of deuterium is simple and monotonic: as gas is cycled through stars D is burned to $^3\text{He}$ and beyond \[4\]. As a result, the deuterium abundance observed anywhere, at any time in the evolution of the Universe provides a lower bound to the primordial value and, the observed deuterium abundance should reveal a plateau at the primordial value in regions at high redshift and/or low metallicity where minimal stellar processing has occurred. In contrast, the post-BBN evolution of $^3\text{He}$ is considerably more complicated.

While some of the $^3\text{He}$ incorporated into stars (along with any D which is burned to $^3\text{He}$) survives in the cooler, outer regions, most is burned away in the hotter interiors and, newly synthesized $^3\text{He}$ is produced by incomplete hydrogen burning in some stars. The net effect of these competitive processes depends on detailed models of stellar structure and evolution, along with models of galactic chemical evolution. If there is net production of $^3\text{He}$ in the course of chemical evolution, the $^3\text{He}$ abundance should correlate with the "metal" (i.e., CNO...) abundance which, in turn, is correlated with location in the Galaxy; for net destruction an anticorrelation would be expected. Since stars burn hydrogen to helium in the course of their evolution, the post-BBN evolution of $^4\text{He}$ is simple and monotonic (similar to that of deuterium). The observationally-inferred abundance of $^4\text{He}$ should correlate with metallicity (i.e., CNO...), extrapolating to the primordial value at zero metallicity. Finally, like deuterium, most lithium is destroyed when gas is cycled through stars. However, some stars appear to be net producers of $^7\text{Li}$ and collisions between cosmic ray and interstellar nuclei also produce post-BBN lithium. All these effects must be kept in mind when using the observations to infer the primordial abundances.

2.1 Deuterium

Given its simple post-BBN evolution and the sensitivity of the BBN-predicted abundance to the baryon abundance parameter \((D/H)_{\text{DP}} \propto \eta_B^{-1.6}\), so that a ~10% determination of $(D/H)_{\text{P}}$ yields a ~6% measurement of $\eta_B$, deuterium is the baryometer of choice. There is, unfortunately, a very small set of D abundances inferred from the spectra of only seven, high-redshift, low-metallicity, QSO Absorption Line Systems [5] (the observations require high resolution spectra on the world’s largest telescopes). Even more problematic, the seven data points exhibit an unexpectedly large dispersion (given the quoted errors). These variations in D abundances show no obvious correlation with redshift or metallicity. From their data Pettini et al. [5] derive $\log y_{\text{DP}} = 5 + \log(D/H)_{\text{P}} = 0.45 \pm 0.03$, where the error has been inflated from the error in the mean to account for the large
Figure 1: The Izotov & Thuan 2010 (IT10) [8] helium and oxygen abundances. The solid line is the IT10 best fit for a linear $Y$ versus $O/H$ relation; see the text.

dispersion. In the context of SBBN this estimate of the primordial D abundance corresponds to a baryon abundance of $\eta_{10}(\text{SBBN}) = 5.80 \pm 0.27$.

2.2 Helium-3

For the baryon abundance parameter inferred from SBBN using the Pettini et al. deuterium abundance, the SBBN-predicted $^3\text{He}$ abundance is $y_{3P}(\text{SBBN}) \equiv 10^5 (^3\text{He}/\text{H})_P = 1.08 \pm 0.04$. Aside from the solar system [3], $^3\text{He}$ is observed in Galactic H II regions via the spin-flip transition (the analog of the 21 cm line in neutral hydrogen) in singly-ionized $^3\text{He}$. The most complete data set is that of Bania, Rood & Balser [7]. The gas in these Galactic H II regions has been processed through several generations of stars so, not surprisingly, the $^3\text{He}$ abundances reveal a large variation, likely due to post-BBN production/destruction. However, these $^3\text{He}$ abundances show no clear correlation with either metallicity (e.g., O/H) or location in the Galaxy (see, e.g., Figure 9 and the discussion in Steigman 2007 [1]). The lowest $^3\text{He}$ abundances, $y_3 \geq 1.1 \pm 0.2$, which are adopted by Bania, Rood & Balser as an estimate of (or, a lower bound to) the primordial abundance, are in excellent agreement with the SBBN-predicted abundance, providing support for the internal consistency of SBBN (two predicted abundances, one free parameter). However, the lack of a correlation between the $^3\text{He}$ abundances and metallicity or location in the Galaxy is puzzling.

2.3 Helium-4

As the second most abundant nuclide in the Universe $^4\text{He}$ is observed in many astrophysical environments. The most useful observations for BBN are those of the hydrogen and helium recom-
2.4 Lithium-7

For the D-determined baryon density parameter the SBBN-predicted lithium abundance is $[\text{Li}]_{\text{SBBN}} \equiv 12 + \log(\text{Li}/\text{H})_{\text{SBBN}} = 2.65 \pm 0.06$. In Figure 2 are shown the lithium abundances derived from observations of the oldest, most metal-poor stars in the Galaxy [10, 11, 12, 13]. These stars should provide a sample of the “lithium plateau”, the nearly primordial abundance in systems which have experienced very little stellar processing. It is difficult to identify a lithium plateau.
from the data shown in Figure 2 and it is clear that none of the observationally-inferred lithium abundances is even close to the SBBN-predicted value. The difference is $\sim 0.5 - 0.6$ dex or, a factor of 3 – 4. Lithium is a problem. However, since the target stars have had $\gtrsim 10$ Gyr to modify their surface abundances by mixing with the lithium-depleted interior material, it is unclear if the problem is one for cosmology or particle physics or, for stellar astrophysics.

### 2.5 Summary for SBBN

The observationally-inferred primordial abundances of the light nuclides are subject to difficult to quantify systematic uncertainties along with the usual statistical errors. The fact that D, $^3$He, $^4$He, and $^7$Li are observed by different astronomical techniques in different targets ensures that their derived abundances are not affected by the same systematics. Within the errors, the SBBN-predicted and observationally-inferred primordial abundances of D, $^3$He, and $^4$He are in good agreement, providing support for the standard models of particle physics and cosmology ($e.g.$, $\Delta N_{\nu} = 0$). However, lithium is a problem. Since non-standard BBN offers a second free parameter, $S$ or $\Delta N_{\nu}$, it is of interest to see how the BBN abundances constrain the combination of $\eta_B$ and $\Delta N_{\nu}$ and, if this additional freedom can help to ameliorate the lithium problem.

### 3. Non-Standard BBN

For the more general BBN case the relic abundances depend on two adjustable parameters, the baryon density parameter $\eta_B$ and the expansion rate parameter $S$ (or, equivalently, $\Delta N_{\nu}$). In Figure 3 are shown the D and $^4$He isoabundance contours, $y_{DP}$ (dotted, blue) and $Y_P$ (solid, red) in the $S - \eta_{10}$ plane [14, 1]. The point with the error bars represents the observationally-inferred D and $^4$He abundances from §2. Since the $y_{DP}$ and $Y_P$ contours form a grid in the $S - \eta_{10}$ plane, the D and $^4$He abundances constrain $S$ (or, $\Delta N_{\nu}$) and $\eta_{10}$, as may be seen from Figure 3. The adopted D and $^4$He abundances predict $\eta_{10}(BBN) = 6.07 \pm 0.33$ and $\Delta N_{\nu} = 0.62 \pm 0.46$ ($= 0$ at $\sim 1.3\sigma$).

Also shown in Figure 3 (solid, green) are the $^7$Li isoabundance contours. The D and $^4$He constrained, BBN-predicted lithium abundance with both $\eta_B$ and $S$ ($\Delta N_{\nu}$) as free parameters yields $[\text{Li}]_{BBN} = 2.66 \pm 0.06$. Lithium is still a problem.

### 4. Comparison Between BBN And The CMB

The CMB offers a probe of the later, but still early, evolution of the Universe which complements that from BBN. In particular, the CMB temperature anisotropy spectrum is sensitive to the values at recombination of both the baryon abundance parameter and the expansion rate parameter (or, $\Delta N_{\nu}$).

From the 7-year WMAP data Komatsu et al [15] find $\eta_{10} = 6.190 \pm 0.145$. If it is assumed that $N_{\nu} = 3$ (SBBN), for which $\eta_{10}(\text{SBBN}) = 5.80 \pm 0.27$, then SBBN and the CMB agree to within $\sim 1.3\sigma$ on the value of the baryon density parameter at a few minutes and some 400 thousand years later. The comparison of the SBBN and CMB inferred values of $\eta_{10}$ are shown in the likelihood distributions of Figure 4. However, Komatsu et al find some evidence (at the $\sim 1.5\sigma$ level) in support of $\Delta N_{\nu} \neq 0$. For BBN ($\Delta N_{\nu} \neq 0$), $\eta_{10}(BBN) = 6.07 \pm 0.33$, which is in even better agreement with CMB result for $\eta_{10}$, as may be seen in Figure 5.
Figure 3: The BBN isoabundance contours in the $S$ versus $\eta_{10}$ plane \[14, 1\] for $\gamma_{DP} \equiv 10^5(D/H)$ (dashed, blue) and $Y$ (solid, red). $\gamma_{DP}$ decreases from left to right from 4.0 to 3.0 to 2.0 respectively. $Y$ increases from bottom to top, from 0.24 to 0.25 to 0.26 respectively. Also shown are the isoabundance contours (solid, green) for $[\text{Li}] \equiv 12 + \log(\text{Li/H})$. $[\text{Li}]$ increases from left to right from 2.6 to 2.7 to 2.8 respectively. The data point with error bars corresponds to the adopted D and $^4\text{He}$ abundances; see the text.

For BBN and the observationally-inferred D and $^4\text{He}$ relic abundances, $\Delta N_\nu = 0.62 \pm 0.46$, which is consistent with $\Delta N_\nu = 0$ at $\sim 1.3\sigma$. From the imprint on the CMB at recombination Komatsu et al. [15] find $\Delta N_\nu = 1.30 \pm 0.87$ (see also [16]), which differs from $\Delta N_\nu = 0$ by $\sim 1.5\sigma$. The very good agreement between BBN and the CMB, $\Delta N_\nu(\text{CMB}) - \Delta N_\nu(\text{BBN}) = 0.68 \pm 0.98$, as well as their overlap with $\Delta N_\nu = 0$, is illustrated by the likelihood distributions in Figure 3.

5. Discussion

The predictions of SBBN ($\Delta N_\nu = 0$) are consistent, within the errors, with the observationally-inferred relic abundances of D, $^3\text{He}$, and $^4\text{He}$, as well as with the value of the baryon density parameter derived from the CMB. But, lithium is a problem since its SBBN predicted abundance differs from that derived from observations of very metal-poor halo and globular cluster stars by a factor of three or more. The problem could be in the stars (e.g., depletion or dilution of surface lithium during the lifetime of the oldest stars in the Galaxy) or in the cosmology (e.g., late-time or renewed BBN initiated by the decay of long-lived massive particles).

For non-standard BBN with $\Delta N_\nu \neq 0$ comparison between the predicted and observed abundances of D and $^4\text{He}$ constrains the allowed values of the baryon density parameter ($\eta_B$) and the expansion rate parameter ($S$) when the Universe was only a few minutes old. The BBN results
Figure 4: The likelihood distributions for $\eta_{10}$ inferred from SBBN (dashed, red) and the CMB (solid, blue).

Figure 5: The likelihood distributions for $\eta_{10}$ inferred from BBN (red) and the CMB (blue).
Figure 6: The likelihood distributions for $N_\nu \equiv 3 + \Delta N_\nu$ inferred from BBN (red) and the CMB (blue). The vertical line (magenta) is for the standard model at $N_\nu = 3$.

are consistent with the CMB inferred values of these parameters at recombination, some 400 thousand years later. But, even in this case lithium remains a problem. Lithium aside, the very good agreement between these independent probes of cosmology during widely separated epochs in the evolution of the Universe validates using the combined constraints from BBN and the CMB to probe non-standard models of cosmology and particle physics.

5.1 Comparing The Universe At BBN And At Recombination

During most epochs in the evolution of the Universe entropy is conserved. The number of CMB photons in a comoving volume provides a measure of the entropy in that comoving volume. A comparison of the number of photons in the comoving volume at BBN and at recombination provides a test of entropy conservation. Of course, it is necessary to define the size of the comoving volume, which can be accomplished by identifying the number of baryons (nucleons) in it, so that $N_\gamma = N_B/\eta_B$. Given the very good constraints on baryon non-conservation, the number of baryons in a comoving volume is (should be!) preserved from BBN to recombination, so that a comparison of $\eta_B$ derived from BBN and from the CMB constrains any entropy production in the intervening epochs: $N_\gamma(\text{REC})/N_\gamma(\text{SBBN}) = 0.94 \pm 0.05$ and $N_\gamma(\text{REC})/N_\gamma(\text{BBN}) = 0.98 \pm 0.06$.

Similarly, the combined results of BBN and the CMB may be used to compare the value of the gravitational constant, $G$, at BBN and at recombination with each other and with the present value ($G_0$). Since $G_{\text{BBN}}/G_0 = 1 + 0.163\Delta N_\nu(\text{BBN})$ and $G_{\text{REC}}/G_0 = 1 + 0.134\Delta N_\nu(\text{REC})$ [3], the values of $\Delta N_\nu$ at BBN and at recombination constrain the early Universe strength of gravity, leading to: $G_{\text{REC}}/G_{\text{BBN}} = 1.07 \pm 0.13$, $G_{\text{BBN}}/G_0 = 1.10 \pm 0.07$ and, $G_{\text{REC}}/G_0 = 1.17 \pm 0.12$. 
As a last example of the value of combining the constraints from BBN and the CMB consider the effect of a massive particle which decays after BBN but before recombination. If the decay occurs too late for the relativistic decay products to be thermalized (producing extra CMB photons whose presence has already been constrained above), the energy they carry will, nevertheless, contribute to the total energy density when the Universe is radiation dominated. Since 
\[ \frac{\rho_R^b}{\rho_R} = 1 + 0.163 \Delta N_\nu (\text{BBN}) \]
and
\[ \frac{\rho_R^b}{\rho_R} = 1 + 0.134 \Delta N_\nu (\text{REC}) \]
\( \Delta N_\nu \) measures the presence of “extra” relativistic energy. Comparing \( \Delta N_\nu \) at BBN with its value at recombination it is seen that they agree,
\[ \frac{\rho_R^b}{\rho_R} = \frac{\rho_R^b}{\rho_R} \text{BBN} = \frac{\rho_R^b}{\rho_R} \text{REC} \text{ within } 0.5 \sigma \]

6. Conclusions

For \( N_\nu \approx 3 \) BBN is internally consistent (but lithium is a problem!) and is in agreement with the CMB. The very good agreement between BBN and the CMB permits their use in combination to constrain some examples of non-standard models of cosmology and particle physics. While celebrating the success of BBN it should be kept in mind that some challenges remain. For example, why is the dispersion among the observed deuterium abundances so large? Or, why don’t the \(^3\)He abundances observed in Galactic H II regions correlate with the oxygen abundances or with location in the Galaxy? And, for \(^4\)He, the second most abundant element in the Universe, how large are the systematic errors in its observationally-inferred primordial abundance and, are there observing strategies to reduce or eliminate at least some of them? This active research area of importance for cosmology and particle physics would benefit greatly from more data.

Acknowledgments

I am pleased to acknowledge informative discussions and correspondence with R. Cyburt, G. Ferland, Y. Izotov, A. Korn, and T. Prodanović. The research reported here is supported at The Ohio State University by a grant from the U. S. Department of Energy.

References

[1] G. Steigman, *Ann. Rev. Nucl. Part. Sci.*, 57 (2007) 463.
[2] G. Steigman, D. N. Schramm, J. E. Gunn, *Phys. Lett.*, B66 (1977) 202.
[3] V. Simha & G. Steigman, *JCAP*, 06 (2008) 016.
[4] R. J. Epstein, J. Lattimer, D. N. Schramm, *Nature*, 263 (1976) 198.
[5] M. Pettini, B. J. Zych, M. T. Murphy, A. Lewis, C. C. Steidel, *MNRAS*, 391 (2008) 1499.
[6] J. Geiss & J. G. Gloeckler, *Space Sci. Rev.*, 84 (1998) 239.
[7] T. M. Bania, R. T. Rood, D. S. Balser, *Nature*, 415 (2002) 54.
[8] Y. I. Izotov & T. X. Thuan, *ApJL*, 710 (2010) L67.
[9] E. Aver, K. A. Olive, E. D. Skillman, *JCAP*, 05 (2010) 003.
[10] A. M. Boesgaard, A. Stephens, & C. P. Deliyannis, *ApJ*, 633 (2005) 398.
[11] M. Asplund, *et al., ApJ*, 644 (2006) 229.
[12] W. Aoki, et al, ApJ, 698, (2009) 1803.
[13] K. Lind, et al, A&A, 503 (2009) 545.
[14] J. P. Kneller & G. Steigman, New J. Phys., 6 (2004) 117.
[15] E. Komatsu et al, ApJ submitted (2010) arXiv:1001.4538v2 [astro-ph.CO]
[16] G. Steigman, JCAP, 04 (2010) 029.