The Estimators of Vector Autoregressive Moving Average Model VARMA of Lower Order: VARMA (0,1), ARMA (1,0), VARMA (1,1), VARMA (1,2), VARMA (2,1), VARMA (2,2) with Forecasting

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Abstract. This research includes analyzing the relationship between two financial time series, which is a series of global monthly Silver price in dollar and global monthly gold price in dollar, a Vector Autoregressive Moving Average VARMA Model was used to analyze this relationship, for the period from January 2016 to December 2019, it is 48 monthly value, where the data has been transferred to get the Stationarity. Dieckey Fuller test for the Stationarity was conducted, lower order VARMA were estimated, the best order for the model was determined through a standard Akaike information AI, Akaike information corrected AICC, Hannan-Quinn criterion HQC, Schwarz Bayesian criterion SBC, Final prediction error criterion FPE, It turns out that the best model is VARMA(0,1), according to all standards, Some tests were conducted such as Portmanteau test, Jarque-Bera test, Autoregressive Conditional Heteroscedastic ARCH test to residuals for the selected model, with forecasting for the VARMA(0,1) model for the period from Jan 2020 to Jan 2021, It is 12 monthly value, It turns out that there is an increase in volatility for the two price forecasting series with increase in forecasting period. The simulation has been applied using three samples sizes, It turns out that the model is appropriate when the sample size (50), the result has been computed through the SAS Program.

1. Introduction
The vector autoregressive moving average VARMA models an important models, which is used to study interactions between variables, clarify the relationship between a variety of variables, whether internal and external variables or internal variables only, It is used in the currency markets and forecasting foreign exchange rates and in description of the behavior of economic variables, It is also used in the field of meteorology, VARMA model is a natural extension of the univariate autoregression moving average model, as analysis of multivariate time series has received wide attention from researchers, Tiao and Box (1981) It also has been studied widely by both Reinsel, 1993 Lutkepohl, 2005[1]. The aim of research is to estimate the VARMA model for two variable, that represents the first global silver price and second global gold price, and analyze their relationship and forecasting of these prices.
2. Materials and method [1] [2] [3]:

The VARMA (p,q) model of order (p,q) and for K variables, can be represented in the following formula:

\[ X_t = \sum_{i=1}^{p} \Phi_i X_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \Theta_j \varepsilon_{t-j} \]  

(1)

\( X_t = (x_{1t}, \ldots, x_{Kt})' \) denote an \((K x 1)\) vector of time series of variables.  
\( \Phi_i = (i = 1, 2, \ldots, p) \) denote an \((K x K)\) matrices of autoregression parameters.  
\( \Theta_j = (j = 1, 2, \ldots, q) \) denote an \((K x K)\) matrices of moving average parameters.  
\( \varepsilon_t \) denote an \((K x 1)\) vector of random error  
\( \Sigma = E(\varepsilon_t \varepsilon_t') \) positive definite and in order to achieve the stability in VARMA model and be appreciable, interpretation and invertible, the roots of the polynomial must be outside the unit circle, it is mean:

\|\Phi(B)\| \neq 0 \quad \text{and} \quad |\Theta(B)| \neq 0

Model (1) can be expressed as [4][5]:

\[ \Phi(B)X_t = \Theta(B)\varepsilon_t \]

\( B \) denote lagged operator  
\( B^i X_t = x_{t-i} \)

\[ \Phi(B) = I_k - \sum_{i=1}^{p} \Phi_i B^i \]

\[ \Theta(B) = I_k - \sum_{i=1}^{q} \Theta_i B^i \]

The forecasting for the model (1) and for \( h \) steps:

\[ X_{t+h/t} = \sum_{i=1}^{p} \Phi_i X_{t+h-i/t} - \sum_{j=1}^{q} \Theta_j \varepsilon_{t+h-j/t} \]  

(2)

2.1. Model order selection [2]

For the purpose of estimating VARMA model, it is requires determining the model order, a set of standards was adopted, such as AIC, AICC, HQC, SBC, FPE. since the formula of the AIC crietria is as follows:

\[ AIC = -2L + 2r \]

Akike information corrected AICC:

\[ AICC = -2L + 2rT/(T - r - 1) \]

Hannan-Quinn criterion HQC:

\[ HQC = -2L + 2r \log(\log(T)) \]

Schwarz Bayesian criterion SBC:

\[ SBC = -2L + r \log(T) \]

Final prediction error criterion (FPE):

\[ FPE = \left( \frac{T + r_b}{T - r_b} \right)^k |\hat{\Sigma}| \]

\( L \): Log Likelihood  
\( R \): total number of parameters in the model  
\( K \): number of dependent variables  
\( T \): number of observations  
\( \hat{\Sigma} \): is the maximum likelihood estimate of the covariance matrix.
2.2 Multivariate Portmanteau test [6][7]

The portmanteau test is used to detect the autocorrelation and cross correlation in the residuals, the test statistics Lju-Box is defined as:

\[ Q_m = T^2 \sum_{l=1}^{m} (T-l)^{-1} \text{tr} \{ \hat{\rho}_e(l) \hat{\rho}_e(0)^{-1} \hat{\rho}_e(-l) \hat{\rho}_e(0)^{-1} \} \]

\[ m : \text{the maximum lag to be tested} \]
\[ \hat{\rho}_e(l) : \text{the cross correlation matrix of residuals} \]

The test statistics distributed as an approximate the distribution of \( Q_m \sim \chi^2 (K^2 m) \) under the null hypothesis, the null hypothesis states there is no autocorrelation in the residuals.

3. Application

The research data represent two time series, the first represent global monthly silver prices and the second represents global monthly gold prices for the period from January - 2016 until December - 2019 and is thus 48 monthly value of the prices, obtained from website: www.Kitco .com/script/hist charts/yearly graphs.plx

3.1 View and Discussion of the results

Due to the nature of prices and their fluctuations over time, Figure (1. (a) and (b)) shows series of silver and gold prices before conducting conversion.[1]

![Figure (1. (a) and (b))] (The original silver price and gold price series before conversion)

After taking the logarithm and the first difference of the two series to get the stationary, the Dickey-Fuller test performed to test the stationarity of the two series, table (1), it was found that the absolute value of the test statistics of the two series is greater than the tabular value, this means rejecting the Null hypothesis of having a unit root and that the two series is stationary.

| Table (1). Dickey-Fuller unit root tests |
|-----------------------------------------|
| Variable      | Type       | Rho   | Pr < Rho | Tau   | Pr < Tau |
|---------------|------------|-------|----------|-------|----------|
| Lsilverprice  | Zero Mean  | -28.25| <.0001   | -3.76 | 0.0003   |
|               | Single Mean| -28.47| 0.0004   | -3.71 | 0.0068   |
|               | Trend      | -28.55| 0.0031   | -3.67 | 0.0352   |
| Lgoldprice    | Zero Mean  | -30.88| <.0001   | -3.90 | 0.0002   |
|               | Single Mean| -31.39| 0.0004   | -3.80 | 0.0054   |
|               | Trend      | -31.42| 0.0012   | -3.75 | 0.0287   |
Table (2) shows the model order selection criterion for estimated VARMA models, Where the VARMA(0,1) model has less value and is the best according to AIC, AICC, HQC, SBC and the FPE criterion.

### Table (2). Information criterion for the models

| Information criterion | AICC     | HQC     | AIC     | SBC     | FPE       |
|-----------------------|----------|---------|---------|---------|-----------|
| VARMA(0,1)            | -703.235 | -702.069| -708.235| -691.777| 3.398E-8  |
| VARMA(1,0)            | -692.431 | -691.266| -697.431| -680.973| 3.081E-8  |
| VARMA(1,1)            | -692.081 | -694.551| -703.456| -679.684| 2.55E-8   |
| VARMA(1,2)            | -678.952 | -690.169| -701.619| -670.905| 2.639E-8  |
| VARMA(2,1)            | -665.248 | -676.465| -687.914| -657.200| 2.554E-8  |
| VARMA(2,2)            | -648.023 | -674.053| -688.197| -650.257| 2.085E-8  |

After selecting the best model, table (3) shows parameter estimates for VARMA(0,1) model, In which the model parameters are significant, except for MA1_1_1, It is not significant, as p value = 0.2429 greater than the significance level 0.05.

### Table (3). Model parameter estimates for VARMA(0,1)

| Equation     | Parameter | Estimate | Standard Error | t Value | Pr > | Variable |
|--------------|-----------|----------|----------------|---------|-------|----------|
| Lsilverprice | CONST1    | 0.00055  | 0.00318        | 0.17    | 0.8637| 1        |
|              | MA1_1_1   | 0.27793  | 0.23496        | 1.18    | 0.2429| e1(t-1)  |
|              | MA1_1_2   | -0.71553 | 0.36923        | -1.94   | 0.0588| e2(t-1)  |
| Lgoldprice   | CONST2    | 0.00061  | 0.00276        | 0.22    | 0.8273| 1        |
|              | MA1_2_1   | 0.32833  | 0.16303        | 2.01    | 0.0499| e1(t-1)  |
|              | MA1_2_2   | -0.79116 | 0.30818        | -2.57   | 0.0136| e2(t-1)  |

Table (4). Shows the moving average coefficient estimates for VARMA(0,1) model. e1,e2 represent the parameters of the MA of the model.

| Lag | Variable | e1    | e2    |
|-----|----------|-------|-------|
| 1   | Lsilverprice | 0.27793 | -0.71553 |
|     | Lgoldprice  | 0.32833 | -0.79116 |

### Table (5). Covariances of innovations for VARMA(0,1).

| Variable | Lsilverprice | Lgoldprice |
|----------|--------------|------------|
| Lsilverprice | 0.00036 | 0.00018 |
| Lgoldprice  | 0.00018 | 0.00017 |
VARMA(0,1) model can be written in matrix form:

\[
\begin{pmatrix}
X_{1t}
X_{2t}
\end{pmatrix}
= (\begin{pmatrix}
\text{lsilver price}_t
\text{lngold price}_t
\end{pmatrix} = \epsilon_t - (0.27793
-0.71553
0.32833
-0.79116
\epsilon_{1t-1}
\epsilon_{2t-1}
\end{pmatrix}
\]

\[
\epsilon_t \sim N (0, \begin{pmatrix}
0.000036 & 0.00018 \\
0.00018 & 0.00017
\end{pmatrix})
\]

Portmanteau test was performed on the residuals for VARMA(0,1) model, table(6) shows the degrees of freedom DF and the values of the test statistic in which it is not significant for all lagged where Pr > ChiSq represents the p value and there values are greater than the significance level 0.05 for all lagged. In this case, the null hypothesis, which states that there is no autocorrelation in the residuals, is accepted. That is, the residuals are random.

**Table (6).** Portmanteau test for cross correlations of residuals for VARMA(0,1)

| Up To Lag | DF | Chi-Square | Pr > ChiSq |
|-----------|----|------------|------------|
| 2         | 4  | 5.39       | 0.2492     |
| 3         | 8  | 8.51       | 0.3849     |
| 4         | 12 | 13.60      | 0.3269     |
| 5         | 16 | 20.60      | 0.1946     |
| 6         | 20 | 22.67      | 0.3052     |
| 7         | 24 | 31.33      | 0.1446     |
| 8         | 28 | 33.78      | 0.2083     |
| 9         | 32 | 40.47      | 0.1446     |
| 10        | 36 | 44.37      | 0.1596     |
| 11        | 40 | 45.73      | 0.2464     |
| 12        | 44 | 52.88      | 0.1687     |

Table(7) shows the ARCH test, in which test statistic values are insignificant, the null hypothesis is accepted, which states that the residuals homoscedasticity and Jarque-Bera test was performed, shows the values of the test statistic of the residuals series for silver price, in which it is not significant, this means accepting the null hypothesis and that the residuals for silver price are normally distributed figure(2), but the test statistic of residuals series for gold price is significant, this means rejecting the null hypothesis and that the residuals for gold price are not normally distributed figure(3), as for test statistic of Durbin Watson, the value of test statistic is specified between (0.4). If the value of the statistic equals 2, this means that the residuals are independent.[8]

**Table (7).** Univariate model white noise diagnostics for VARMA(0,1)

| Variable       | Durbin Watson | Normality | ARCH       |
|----------------|---------------|-----------|------------|
|                |               | Chi-Square| Pr > ChiSq | F Value | Pr > F |
| Lsilverprice   | 1.91210       | 5.86      | 0.0534     | 1.21    | 0.2774 |
| Lgoldprice     | 1.95309       | 6.77      | 0.0338     | 0.03    | 0.8668 |
VARMA(0,1) model was forecasting, table (8) shows forecasting values for silver prices from the period Jan 2020 to Jan 2021, as it start to increase from the first to the third period, then it decreases from the fourth to the fifth period, then it increases from the sixth to the ninth period, then it decreases from the tenth to the last period, us for gold prices we note similarity in fluctuation of forecasting values for silver prices, this indicates that there is volatility in the two price forecasting series.

As for standard error values, it start to increase from the first to the last period in ascending order for silver and gold prices, this indicates that there is an increase in volatility for the two price forecasting series with increase in forecasting period, this means, the variance increasing if the model is used to forecasting for a long time. Figure (11), (14) shows the forecasting series for silver and gold prices, respectively.

Table (8). Forecasts for VARMA(0,1)

| Variable  | Obs | Time    | Forecast | Standard Error | 95% Confidence Limits |
|-----------|-----|---------|----------|----------------|-----------------------|
| Lsilverprice | 49  | JAN2020 | 1.24506  | 0.02695        | 1.19223, 1.29789       |
|           | 50  | FEB2020 | 1.25104  | 0.04020        | 1.17224, 1.32984       |
|           | 51  | MAR2020 | 1.23749  | 0.05006        | 1.13937, 1.33561       |
|           | 52  | APR2020 | 1.22952  | 0.05828        | 1.11530, 1.34374       |
|           | 53  | MAY2020 | 1.21732  | 0.06547        | 1.08901, 1.34564       |
|           | 54  | JUN2020 | 1.22821  | 0.07194        | 1.08721, 1.36922       |
|           | 55  | JUL2020 | 1.24936  | 0.07788        | 1.09671, 1.40201       |
|           | 56  | AUG2020 | 1.28617  | 0.08340        | 1.12271, 1.44963       |
|           | 57  | SEP2020 | 1.31157  | 0.08857        | 1.13797, 1.48517       |
|           | 58  | OCT2020 | 1.29833  | 0.09346        | 1.11515, 1.48151       |
|           | 59  | NOV2020 | 1.28722  | 0.09811        | 1.09494, 1.47951       |
|           | 60  | DEC2020 | 1.28557  | 0.10254        | 1.08459, 1.48655       |
| Lgoldprice| 49  | JAN2020 | 3.13341  | 0.01525        | 3.10352, 3.16329       |
|           | 50  | FEB2020 | 3.14282  | 0.02913        | 3.08573, 3.19992       |
|           | 51  | MAR2020 | 3.13647  | 0.03827        | 3.06146, 3.21148       |
|           | 52  | APR2020 | 3.12481  | 0.04561        | 3.03541, 3.21421       |
|           | 53  | MAY2020 | 3.13078  | 0.05193        | 3.02900, 3.23256       |
|           | 54  | JUN2020 | 3.15546  | 0.05756        | 3.04265, 3.26827       |
|           | 55  | JUL2020 | 3.17236  | 0.06268        | 3.04951, 3.29521       |
|           | 56  | AUG2020 | 3.19797  | 0.06742        | 3.06584, 3.33010       |
|           | 57  | SEP2020 | 3.20158  | 0.07184        | 3.06078, 3.34238       |
|           | 58  | OCT2020 | 3.19681  | 0.07601        | 3.04784, 3.34578       |
|           | 59  | NOV2020 | 3.18955  | 0.07996        | 3.03284, 3.34626       |
|           | 60  | DEC2020 | 3.16086  | 0.08372        | 2.99677, 3.32495       |
Figure (2). Residuals for the $L_{\text{silver}}$ Price Series

Figure (3). Residuals for the $L_{\text{gold}}$ Price Series
Figure (4). Residuals Autocorrelation for the Lsilver price Series

Figure (5). Residuals Histogram and Normality Q-Q for the Lsilver price Series
Figure (6). Residuals Autocorrelation for the L.goldprice Series

Figures (4),(6) shows the residuals autocorrelation function (ACF) , the plot is shown in the upper left side , the partial autocorrelation function (PACF) and the inverse autocorrelation function (IACF) are plotted . a plot of individual autocorrelations is shown in the lower right side , for the L.silver price and L.gold price respectively

Figure (7). Residuals Histogram and Normality Q-Q for the L.goldprice Series

Figure(5) , (7) shows the distribution of the residuals plots for the fit of the normal distribution for the series of L.silver price and L.gold price respectively .
Figures (8), (9) shows that the actual data and the predicted are closed to each other for the series of Lsilver price and Lgold price respectively.
Figure (10). Actual and predicted for L.silver price series

Figure (11). Predicted and forecasts for L.silver price series
Figure (12). Forecasts for Lsilverprice series

Figure (13). Actual and predicted for Lgoldprice series
4. Simulation

4.1 View and discussion of the simulation results

Values for the variables $y_1, y_2$ were generated and three sizes for samples (50, 100, 250). VARMA(0,1) model was estimated for these two variables. Table (9), (10), (11) shows the results of the final estimated for the three samples are as follows:

| Equation | Parameter   | Estimate | Standard Error | t Value | Pr > | Variable       |
|----------|-------------|----------|----------------|---------|-------|----------------|
|          | y1          |          |                |         |       |                |
|          | MA1_1_1     | -0.21659 | 0.13170        | -1.64   | 0.1065| e1(t-1)        |
|          | MA1_1_2     | 0.17761  | 0.15338        | 1.16    | 0.2525| e2(t-1)        |
|          | y2          |          |                |         |       |                |
|          | MA1_2_1     | -0.27985 | 0.12893        | -2.17   | 0.0348| e1(t-1)        |
|          | MA1_2_2     | -0.23690 | 0.14061        | -1.68   | 0.0984| e2(t-1)        |
Table (10). Model parameter estimates n=100

| Equation | Parameter | Estimate  | Standard Error | t Value | Pr > | Variable |
|----------|-----------|-----------|----------------|---------|-------|----------|
|          | y1        | MA1_1_1   | -0.38276       | 0.09070 | -4.22 | 0.0001  | e1(t-1) |
|          |           | MA1_1_2   | 0.24446        | 0.12489 | 1.96  | 0.0531  | e2(t-1) |
|          | y2        | MA1_2_1   | -0.28672       | 0.10937 | -2.62 | 0.0101  | e1(t-1) |
|          |           | MA1_2_2   | -0.09705       | 0.12257 | -0.79 | 0.4304  | e2(t-1) |

Table (11). Model parameter estimates n=250

| Equation | Parameter | Estimate  | Standard Error | t Value | Pr > | Variable |
|----------|-----------|-----------|----------------|---------|-------|----------|
|          | y1        | MA1_1_1   | -0.44923       | 0.06271 | -7.16 | 0.0001  | e1(t-1) |
|          |           | MA1_1_2   | 0.19390        | 0.08069 | 2.40  | 0.0170  | e2(t-1) |
|          | y2        | MA1_2_1   | -0.23669       | 0.07272 | -3.25 | 0.0013  | e1(t-1) |
|          |           | MA1_2_2   | -0.14081       | 0.08207 | -1.72 | 0.0875  | e2(t-1) |

Portmanteau test was performed on the residuals for VARMA(0,1) model, table(12), when sample size (n=50) the values of the test statistic in which it is not significant for all lagged, in this case, the null hypothesis, which states that there is no autocorrelation in the residuals, is accepted. but when (n=100 , n=250), the test statistic is significant for all lagged, that is, rejecting the null hypothesis table (13), (14) respectively.

Table (12). Portmanteau test for cross correlations of residuals n=50

| Up To Lag | DF | Chi-Square | Pr > ChiSq |
|-----------|----|------------|-----------|
| 2         | 4  | 6.14       | 0.1890    |
| 3         | 8  | 7.68       | 0.4657    |
| 4         | 12 | 12.59      | 0.3996    |
| 5         | 16 | 13.88      | 0.6079    |
| 6         | 20 | 21.13      | 0.3895    |
| 7         | 24 | 26.82      | 0.3128    |
| 8         | 28 | 36.79      | 0.1236    |
| 9         | 32 | 39.14      | 0.1798    |
| 10        | 36 | 45.50      | 0.1333    |
| 11        | 40 | 55.03      | 0.0572    |
| 12        | 44 | 56.67      | 0.0953    |

Table (13). Portmanteau test for cross correlations of residuals n=100

| Up To Lag | DF | Chi-Square | Pr > ChiSq |
|-----------|----|------------|-----------|
| 2         | 4  | 25.99      | <.0001    |
| 3         | 8  | 33.17      | <.0001    |
| 4         | 12 | 45.23      | <.0001    |
| 5         | 16 | 49.49      | <.0001    |
| 6         | 20 | 54.13      | <.0001    |
| 7         | 24 | 56.18      | 0.0002    |
| 8         | 28 | 62.03      | 0.0002    |
Table (14). Portmanteau test for cross correlations of residuals n=250

| Up To Lag | DF | Chi-Square | Pr > ChiSq |
|-----------|----|------------|-----------|
| 2         | 4  | 47.89      | <.0001    |
| 3         | 8  | 55.26      | <.0001    |
| 4         | 12 | 64.00      | <.0001    |
| 5         | 16 | 68.29      | <.0001    |
| 6         | 20 | 78.89      | <.0001    |
| 7         | 24 | 84.78      | <.0001    |
| 8         | 28 | 93.28      | <.0001    |
| 9         | 32 | 96.68      | <.0001    |
| 10        | 36 | 99.20      | <.0001    |
| 11        | 40 | 105.35     | <.0001    |
| 12        | 44 | 107.04     | <.0001    |

Table (15) shows ARCH test, in which test statistic values are insignificant, the null hypothesis is accepted, which states that the residues are homoscedasticity for three samples, and Jarque-Bera test was performed, shows the values of the test statistic of the residuals series for $y_1$ and $y_2$ when ($n=50$), are not significant, this means accepting the null hypothesis and that the residuals for $y_1$ and $y_2$ are normally distributed figure (16), (17).

when the sample size ($n=100$, $n=250$) the statistic values is not significant for the variable $y_1$, this means accepting the null hypothesis and that the residuals for $y_1$ are normally distributed figure(20),(22), but the test statistic for $y_2$ is significant when sample size ($n=100$, $n=250$), that is rejecting the null hypothesis and that the residuals for $y_2$ are not normally distributed figure(21),(23).

Table (15). Univariate model white noise diagnostics

| n   | Variable | Durbin Watson | Normality | ARCH |
|-----|----------|---------------|-----------|------|
|     |          |               |           | F Value | Pr > F |
| 50  | $y_1$    | 1.68048       | 2.16      | 0.3390 | 0.69   | 0.4097 |
|     | $y_2$    | 1.94483       | 5.26      | 0.0719 | 1.03   | 0.3156 |
| 100 | $y_1$    | 1.70027       | 5.41      | 0.0667 | 2.01   | 0.1598 |
|     | $y_2$    | 1.88901       | 8.27      | 0.0160 | 0.95   | 0.3318 |
| 250 | $y_1$    | 1.75009       | 4.04      | 0.1329 | 0.01   | 0.9380 |
|     | $y_2$    | 1.91155       | 8.79      | 0.0123 | 0.01   | 0.9426 |

Table (16) shows forecasting values for two variables $y_1$ and $y_2$, in case ($n=50$), the standard error values start to increase from the first to the last period for $y_1$ and $y_2$, this indicates that there is an increase in volatility for the two forecasting series with increase in the forecasting period, this means the variance increasing if the model used to forecasting for a long time.
| Variable | Obs | Time     | Forecast | Standard Error | 95% Confidence Limits |
|----------|-----|----------|----------|----------------|-----------------------|
| y1       | 51  | JAN2020  | 0.20858  | 1.88477        | -3.48550  3.90266   |
|          | 52  | FEB2020  | -0.60353 | 1.95546        | -4.43617  3.22910   |
|          | 53  | MAR2020  | 0.16992  | 2.02369        | -3.79643  4.13628   |
|          | 54  | APR2020  | 0.20977  | 2.08969        | -3.88594  4.30548   |
|          | 55  | MAY2020  | -0.33789 | 2.15366        | -4.55900  3.88321   |
|          | 56  | JUN2020  | 0.61613  | 2.21579        | -3.72675  4.95900   |
|          | 57  | JUL2020  | 1.48562  | 2.27623        | -2.97571  5.94695   |
|          | 58  | AUG2020  | 2.39789  | 2.33510        | -2.17883  6.97461   |
|          | 59  | SEP2020  | 1.69098  | 2.39253        | -2.99829  6.38024   |
|          | 60  | OCT2020  | -1.02020 | 2.44860        | -5.81938  3.77897   |
|          | 61  | NOV2020  | -2.21046 | 2.50343        | -7.11708  2.69617   |
|          | 62  | DEC2020  | 0.88368  | 2.55707        | -4.12809  5.89545   |
| y2       | 51  | JAN2020  | 0.10325  | 1.68725        | -3.20369  3.41019   |
|          | 52  | FEB2020  | 0.08468  | 1.80144        | -3.44609  3.61544   |
|          | 53  | MAR2020  | -1.43616 | 1.90882        | -5.17738  2.30506   |
|          | 54  | APR2020  | -1.03498 | 2.01047        | -4.97543  2.90547   |
|          | 55  | MAY2020  | 1.26955  | 2.10722        | -2.86053  5.39963   |
|          | 56  | JUN2020  | 0.69661  | 2.19972        | -3.61477  5.00799   |
|          | 57  | JUL2020  | 0.07210  | 2.28849        | -4.41326  4.55745   |
|          | 58  | AUG2020  | 0.74788  | 2.37394        | -3.90495  5.40071   |
|          | 59  | SEP2020  | 1.50520  | 2.45641        | -3.30928  6.31968   |
|          | 60  | OCT2020  | 1.69884  | 2.53621        | -3.27204  6.66972   |
|          | 61  | NOV2020  | 0.73718  | 2.61357        | -4.38533  5.85968   |
|          | 62  | DEC2020  | -0.73596 | 2.68871        | -6.00573  4.53382   |
Figure (16). residuals for y1 series

Figure (17). residuals for y2 series

Figure (18). actual and predicted for y1 series (n=50)
Figure (19). actual and predicted for y2 series (n=50)

Figure (20). residuals for y1 series (n=100)

Figure (21). residuals for y2 series (n=100)
5. Conclusions

- The best model was chosen from the estimated models. Comparison was based on the criterion order selection AIC, AICC, HQC, SBC, FBE, the best model was VARMA(0,1).
- The Parameter Estimates for VARMA(0,1) model are significant, except for MA1_1_1, It is not significant.
- The residuals for VARMA(0,1) model are random, according to portmanteau and ARCH test.
- The residuals for silver price series are normally distributed but the residuals for gold price are not.
- The variance in the forecasting values for silver and gold prices increases with increase in the forecast period.
- In simulation, VARMA(0,1) were appropriate for two variable y1 and y2, when sample size (n=50).
- The variance in the forecasting values for the two variables y1 and y2 increases with increase in the forecast period, when sample size (n=50).

Figure (22). residuals for y1 series (n=250)

Figure (23). residuals for y2 series (n=250)
6. Proposal

It is suggested to estimate, Bayesian VAR model BVAR, VARMA with Exogenous Variables model VARMAX, VAR with Exogenous Variables model VARX.

7. References

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