Exact solutions of Lifshitz black hole coupled with nonlinear electrodynamics

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Abstract. In the paper, we find an exact black hole solution in Lifshitz space time coupled with nonlinear electrodynamics and investigate their thermodynamic quantities connected with the black hole in term of horizon radius. This black hole solution interpolates with the Lifshitz black hole in the absence of magnetic monopole charge and Bardeen black hole when $z = 0$.

1. Introduction

The strongly coupled theories in the presence of boundaries in the anti-de Sitter (AdS) spacetimes is study by the AdS/CFT correspondence which is proposed by the Maldacena [1, 2, 3]. This theory establish the relation between the strongly coupled theories and weakly coupled theories on the boundary and it is also known as gauge-gravity duality. It provides to study the nonrelativistic condensed matter theories using the holographic models corresponding to the Lifshitz-like [4] and Schrodinger-like [6] gravitation background respectively. In this paper, we are interested in Lifshitz-like geometry, which posses the scaling symmetry. The Lifshitz-like geometry is give as

$$x \rightarrow \lambda x, \quad t = \lambda^z t.$$ (1)

where $x$, $t$ and $x$ are spatial coordinate, temporal coordinate and dynamical critical exponent respectively with $z \geq 1$.

$$ds^2 = -r^{2z} dt^2 + \frac{dr^2}{r^2} + r^2 d\Omega_2^2.$$ (2)

This is the line element of Lifshitz space-time and invariant under the scale $r \rightarrow \lambda^{-1} r$. In the limit $z = 1$, it reduces to the AdSmetric in Poincare coordinates.

We find the exact solutions of Lifshitz Black Hole coupled with nonlinear electrodynamics. This solution is regular black hole in Lifshitz space-time. The first regular black hole is given by Bardeen [7, 8] based on Sakharov and Gliner Proposal [9] and exact solution is obtained after 30 years by AynonBeato and Gracia [10]. There are many other regular black hole solution[11, 14, 13, 12, 18, 15, 16, 17, 19, 21, 20] which is baesd on Bardeen proposal. The generlization of the regular black hole in EGB gravity [22, 26, 23, 24, 25], 4D EGB gravity [27, 28, 29], massive gravity [30], and also give the rotating counterpart by using the Newman Janis algorithm [31, 32] and more rotating black hole ware proposed [35, 33, 34, 36, 37]. In this paper, we have proposed first regular black hole in Lifshitz space-time which is based on the Bardeen proposal.

The paper is arranged as follow: The solutions of Lifshitz Black Hole coupled with nonlinear electrodynamics are studied in the Sec. II and their thermodynamics in Sec. III. The results and conclusion are given in Sec. IV.

2. Bardeen-Lifshitz black hole

Let us consider the action of the Einstein-Maxwell-Dilaton (EMD) system with two gauge fields axionic fields, cosmological constant in presence of nonlinear electrodynamics. The gravity action is given by,
\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( R - 2\Lambda + \alpha L_{GB} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} \sum_{i=1}^{2} e^{\lambda_i \phi} F_i^2 - 4e^{\phi \lambda_3} \mathcal{L}_F \right)
\]  

(3)

where \(F_1\) and \(F_2\) are two \(U(1)\) gauge fields, the role of first gauge field is to break Lorentz invariance and introduce Lifshitz-like geometry while second gauge field introduced to support the spherical horizon topology. The \(L_F\) is the lagrangian density of Bardeen source \([21, 19]\), given by:

\[
\mathcal{L}_F = \frac{3}{2s g^2} \left( \frac{\sqrt{2g^2F}}{1 + \sqrt{2g^2F}} \right)
\]

(4)

where \(g\) is the magnetic monopole charge, \(s\) is nonlinear parameter, which is connected to the mass \(M\) and magnetic monopole charge \(g\) by \(s = g/2M\), and \(L_F\) is the function of \(F = F_{\mu\nu}F^{\mu\nu}\) and \(F_{\mu\nu} = (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu})\) is the electromagnetic field tensor. The Einstein field equation for the above action is given by,

\[
G_{\mu\nu} = \frac{1}{2} \sum_{i=1}^{2} \frac{\lambda_i e^{\lambda_i \phi} F_i^2}{g} - \Lambda g_{\mu\nu} + \frac{1}{2} e^{\lambda_3 \phi} \left( \frac{\partial L_m}{\partial F} F_{\mu\nu} - \frac{1}{4} g_{\mu\nu} L_m \right)
\]

(5)

The equations of motion of the matter field \(s\) are obtained as,

\[
\nabla^2 \phi = \frac{1}{4} \sum_{i=1}^{2} \lambda_i e^{\lambda_i \phi} F_i^2
\]

(6)

\[
0 = \nabla_{\mu} \left( e^{\lambda_1 \phi} F_{\mu\nu} \right)
\]

(7)

\[
0 = \nabla_{\mu} \left( e^{\lambda_3 \phi} \partial_{\mu} \frac{\partial L_m}{\partial F} F_{\mu\nu} \right)
\]

(8)

To find the regular black hole solution in the Lifshitz space time, the line element of the Lifshitz-like metric is given by

\[
ds^2 = -\frac{r^{2x}}{12} f(r) dt^2 + \frac{r^2 dr^2}{r^2 f(r)} + r^2 d\Omega^2_2
\]

(9)

with

\[
q_i = r^{-x+3} e^{\lambda_i \phi} (A_i)_{t}, \quad F_{0\phi} = g \sin^2 \Theta.
\]

(10)

considering \(q_i\) as the charges of two electromagnetic fields. Here the role of \(q_1\) is the used to introduce Lifshitz-scaling and \(q_2\) is to support spherical horizon whereas \(g\) is magnetic monopole charge.

and \(\phi = \gamma \log r\).

Using the gravity solution, the parameters of given model are related by, \([38]\)

\[
\gamma = 2 \sqrt{z - 1}, \quad \lambda_1 = -\frac{4}{\gamma}, \quad \lambda_2 = \frac{2(z - 1)}{\gamma}, \quad \lambda_3 = -\frac{\gamma}{2}
\]

(11)

\[
q_1 = \sqrt{2(z - 1)(z + 2)}, \quad q_2 = 2 \sqrt{\frac{z - 1}{z}}
\]

(12)
with \( \Lambda = \frac{(z+1)(z+2)}{2l^2} \).

Now, we are able to obtain the black hole solution by solving the Eq. (5) with matter field Eq. (6), (7), (8). The regular black hole in Lifshitz space-time is

\[
f(r) = 1 - \frac{l^2}{z^2r^2} + \frac{2M r^{1-z}}{(r^2 + g^2)^{3/2}} \tag{13}
\]

This is a solution Bardeen Lifshitz black hole which is characterized by the mass \((M)\), magnetic charge \((g)\), AdS length \((l)\) and dynamical critical exponent \(z\). The solution reduces to the Lifshitz black hole in the absence of magnetic monopole charge.

3. Thermodynamics

Now, we study the thermodynamics of the Bardeen Lifshitz black hole in term of horizon radius \(r_+\).

The mass of the black hole is obtained by \(f(r) = 0\). The mass of the Bardeen Lifshitz black hole is

\[
M_+ = \frac{(r_+^2 + g^2)^{3/2}}{2r_+^{1-z}} \left( \frac{r^2}{z^2 r_+^2} - 1 \right) \tag{14}
\]

This is the mass of the Bardeen Lifshitz black hole and reduces to the mass of Lifshitz black hole when the magnetic monopole charge swithed off. The Hawking temperature of black hole is calculated by the following expression which is

\[
T_+ = \frac{r_+^{z+1} f'(r_+)}{4\pi} \tag{15}
\]

Substituting the value of \(f(r)\) in Eq. (15) the temperature of the Bardeen Lifshitz black hole becomes

\[
T_+ = \frac{1}{4\pi} \frac{r_+^{z-4} \left( g^2 (3-z) l^2 - r_+^2 z^2 (1-z) - r_+^2 (l^2 - r_+^2 (z+2)) \right)}{(r_+^2 + g^2)^{3/2}} \tag{16}
\]

So we shall consider the range \(1 \leq z < 2\). This is the temperature of the Bardeen Lifshitz black hole. It reduces to the temperature of the Bardeen black hole when \(z = 0\) \([19]\) and Lifshitz black hole in the absence of magnetic monopole charge.

4. Results and Conclusion

In this paper, we find an exact solution of regular Lifshitz black hole in the presence of Bardeen source and the black hole is known as Bardeen Lifshitz black hole. This black hole solutions interpolate with the Lifshitz black hole in the limit of magnetic monopole charge \(g = 0\) and Bardeen black hole when \(z = 0\). In addition, we also study the thermodynamic quantities associated with the black hole in term of horizon radius.

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