Avoiding the dark energy coincidence problem with a cosmic vector

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Abstract. We show that vector theories on cosmological scales are excellent candidates for dark energy. We consider two different examples, both are theories with no dimensional parameters nor potential terms, with natural initial conditions in the early universe and the same number of free parameters as ΛCDM. The first one exhibits scaling behaviour during radiation and a strong phantom phase today, ending in a "big-freeze" singularity. This model provides the best fit to date for the SNIa Gold dataset. The second theory we consider is standard electromagnetism. We show that a temporal electromagnetic field on cosmological scales generates an effective cosmological constant and that primordial electromagnetic quantum fluctuations produced during electroweak scale inflation could naturally explain, not only the presence of this field, but also the measured value of the dark energy density. The theory is compatible with all the local gravity tests, and is free from classical or quantum instabilities. Thus, not only the true nature of dark energy could be established without resorting to new physics, but also the value of the cosmological constant would find a natural explanation in the context of standard inflationary cosmology.

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INTRODUCTION

The fact that today matter and dark energy have comparable contributions to the energy density, $\rho_\Lambda \sim \rho_M \sim (2 \times 10^{-3} \text{ eV})^4$ in natural units, poses one of the most important problems for models of dark energy. Thus, if dark energy is a cosmological constant, its energy density would remain constant throughout the history of the universe, whereas those of the rest of components (matter and radiation) grow as we go back in time. Then the question arises as to whether it is a coincidence (or not) that they have comparable values today when they have differed by many orders of magnitude in the past. Notice also that if $\Lambda$ is a fundamental constant of nature, its scale (around $10^{-3}$ eV) is more than 30 orders of magnitude smaller than the natural scale of gravitation, $G = M_P^{-2}$ with $M_P \sim 10^{19}$ GeV. On the other hand, alternative models in which dark energy is a dynamical component rather than a cosmological constant also require the introduction of unnatural scales in their Lagrangians or initial conditions in order to account for the present phase of accelerated expansion. Such models are usually based on new physics, either in the form of new cosmological fields or modifications of Einstein’s gravity [1,2,3,4,5] and they are generically plagued by additional problems such as classical or quantum instabilities, or inconsistencies with local gravity constraints.

Therefore, we would like to find a description for dark energy without dimensional scales (apart from Newton’s constant $G$), with the same number of free parameters as ΛCDM, with natural initial conditions, with good fits to observations and no consistency
problems. In this work we will show that vector theories can do the job. With that purpose, we present two examples of such theories which have been recently proposed \[6, 7\] (for other vector models see references in those works).

**SCALING VECTOR DARK ENERGY**

The action in this case reads \[6\]:

\[
S = \int d^4x \sqrt{-g} \left( -\frac{R}{16\pi G} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\nabla_\mu A^\mu)^2 + R_{\mu\nu} A^\mu A^\nu \right)
\] (1)

Notice that the theory contains no free parameters, the only dimensional scale being the Newton’s constant. The numerical factor in front of the vector kinetic terms can be fixed by the field normalization. Also notice that the "mass" term \(R_{\mu\nu} A^\mu A^\nu\) can be written as a combination of derivative terms as \(\nabla_\mu A^\mu \nabla_\nu A^\nu - \nabla_\mu A^\nu \nabla_\nu A^\mu\) and therefore the theory contains no potential terms. This action resembles that of Maxwell electromagnetism in the Feynman gauge with a mass term.

The classical equations of motion derived from the action in (1) are the Einstein’s and vector field equations:

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G (T_{\mu\nu} + T^A_{\mu\nu})
\] (2)

\[
\Box A_\mu + R_{\mu\nu} A^\nu = 0
\] (3)

where \(T_{\mu\nu}\) is the conserved energy-momentum tensor for matter and radiation and \(T^A_{\mu\nu}\) is the energy-momentum tensor coming from the vector field. For the simplest isotropic and homogeneous flat cosmologies, we assume that the spatial components of the vector field vanish, so that \(A_\mu = (A_0(t), 0, 0, 0)\) and that the space-time geometry will be given by:

\[
ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j,
\] (4)

For this metric (3) reads:

\[
\ddot{A}_0 + 3H \dot{A}_0 - 3 \left[ 2H^2 + \dot{H} \right] A_0 = 0
\] (5)

Assuming that the universe has gone through radiation and matter phases in which the contribution from dark energy was negligible, we can easily solve this equation in those periods. In that case, the above equation has a growing and a decaying solution:

\[
A_0(t) = A_0^+ t^{\alpha_+} + A_0^- t^{\alpha_-}
\] (6)

with \(A_0^\pm\) constants of integration and \(\alpha_\pm = -(1 \pm 1)/4\) in the radiation era, and \(\alpha_\pm = (-3 \pm \sqrt{33})/6\) in the matter era. On the other hand, the (00) component of Einstein’s equations reads:

\[
H^2 = \frac{8\pi G}{3} \left[ \sum_{\alpha=\text{M}, \text{R}} \rho_\alpha + \rho_A \right]
\] (7)
where the vector energy density is given by:

$$\rho_A = \frac{3}{2} H^2 A_0^2 + 3 H A_0 \dot{A}_0 - \frac{1}{2} \dot{A}_0^2 \tag{8}$$

Using the growing mode solution from (6), we obtain $\rho_A = \rho_{A0} a^\kappa$ with $\kappa = -4$ in the radiation era and $\kappa = (\sqrt{33} - 9)/2 \simeq -1.63$ in the matter era. Thus, the energy density of the vector field starts scaling as radiation at early times, so that $\rho_A / \rho_R = $ const. However, when the universe enters its matter era, $\rho_A$ starts growing relative to $\rho_M$ eventually overcoming it at some point, in which the dark energy vector field would become the dominant component (see Fig. 1). Notice that since $A_0$ is essentially constant during radiation era, solutions do not depend on the precise initial time at which we specify it. Thus, once the present value of the Hubble parameter $H_0$ and the constant $A_0$ during radiation (which fixes the total matter density $\Omega_M$) are specified, the model is completely determined, i.e. this model contains the same number of parameters as $\Lambda$CDM, which is the minimum number of parameters of a cosmological model with dark energy. As seen from Fig.1 the evolution of the universe ends at a finite time $t_{\text{end}}$ where $a \rightarrow a_{\text{end}}$ with $a_{\text{end}}$ finite, $A_0(t_{\text{end}}) = M_P/(4\sqrt{\pi})$, $\rho_{DE} \rightarrow \infty$ and $p_{DE} \rightarrow -\infty$. This corresponds to a Type III (big-freeze) singularity according to the classification in [8].

We can also calculate the effective equation of state for dark energy as:

$$w_{DE} = \frac{p_A}{\rho_A} = \frac{-3 \left( \frac{5}{2} H^2 + \frac{4}{3} \dot{H} \right) A_0^2 + 3 H A_0 \dot{A}_0 - \frac{3}{2} \dot{A}_0^2}{\frac{3}{2} H^2 A_0^2 + 3 H A_0 \dot{A}_0 - \frac{1}{2} \dot{A}_0^2 \tag{9}}$$

Again, using the approximate solutions in (6), we obtain: $w_{DE} = 1/3$ in the radiation era and $w_{DE} \simeq -0.457$ in the matter era. As shown in Fig. 1, the equation of state can cross the so called phantom divide, so that we can have $w_{DE}(z = 0) < -1$.

In order to confront the predictions of the model with observations of high-redshift supernovae type Ia, we have carried out a $\chi^2$ statistical analysis for two supernovae datasets, namely, the Gold set [9], containing 157 points with $z < 1.7$, and the more recent SNLS data set [10], comprising 115 supernovae but with lower redshifts ($z < 1$).
Table 1: Best fit parameters with 1σ intervals for the vector model (VCDM) and the cosmological constant model (ΛCDM) for the Gold (157 SNe) and SNLS (115 SNe) data sets. \(w_0\) denotes the present equation of state of dark energy. \(A_0\) is the constant value of the vector field component during radiation. \(z_T\) is the deceleration-acceleration transition redshift. \(t_0\) is the age of the universe in units of the present Hubble time. \(t_{\text{end}}\) is the duration of the universe in the same units.

In Table 1 we show the results for the best fit together with its corresponding 1σ intervals for the two data sets. We also show for comparison the results for a standard ΛCDM model. We see that the vector model (VCDM) fits the data considerably better than ΛCDM (in more than 2σ) in the Gold set, whereas the situation is reversed in the SNLS set. This is just a reflection of the well-known 2σ tension [11] between the two data sets. Compared with ΛCDM, we see that VCDM favors a younger universe (in \(H_0^{-1}\) units) with larger matter density. In addition, the deceleration-acceleration transition takes place at a lower redshift in the VCDM case. The present value of the equation of state with \(w_0 = -3.53^{+0.46}_{-0.57}\) which clearly excludes the cosmological constant value \(-1\). Future surveys [12] are expected to be able to measure \(w_0\) at the few percent level and therefore could discriminate between the two models.

We have also compared with other parametrizations for the dark energy equation of state [13]. Since our one-parameter fit has a reduced chi-squared: \(\chi^2/d.o.f = 1.108\),
VCDM provides the best fit to date for the Gold data set. We see that unlike the cosmological constant case, throughout radiation era $\rho_{DE}/\rho_R \sim 10^{-6}$ in our case. Moreover the scale of the vector field $A_0 = 3.71 \times 10^{-4} M_P$ in that era is relatively close to the Planck scale and could arise naturally in the early universe without the need of introducing extremely small parameters (for instance in an inflationary epoch), thus avoiding the coincidence problem.

In order to study the model stability we have considered the evolution of metric and vector field perturbations. Thus, we obtain the dispersion relation and the propagation speed of scalar, vector and tensor modes. For all of them we obtain $v = (1 - 16\pi G A_0^2)^{-1/2}$ which is real throughout the universe evolution, since the value $A_0^2 = (16\pi G)^{-1}$ exactly corresponds to that at the final singularity. Therefore the model does not exhibit exponential instabilities. As shown in [14], the fact that the propagation speed is faster than $c$ does not necessarily implies inconsistencies with causality. We have also considered the evolution of scalar perturbations in the vector field generated by scalar metric perturbations during matter and radiation eras, and, again, we do not find exponentially growing modes.

In order to determine whether such bounds conflict with the model predictions or not, we should know the predicted value of the field at solar system scales, which in principle does not need to agree with the cosmological value. Indeed, $A_0^2$ will be determined by the mechanism that generated this field in the early universe characterized by its primordial spectrum of perturbations, and the subsequent evolution in the formation of the galaxy and solar system. Another potential difficulty arising generically in vector-tensor models is the presence of negative energy modes for perturbations on sub-Hubble scales. They are known to lead to instabilities at the quantum level, but not necessarily at the classical level as we have shown previously. Work is in progress in order to determine which are the necessary conditions to avoid the presence of such states in this model.

**IS THE NATURE OF DARK ENERGY ELECTROMAGNETIC?**

In the previous section, we have seen that a generic vector field whose action resembles that of electromagnetism with a mass term could be a good candidate for dark energy, but what about standard electromagnetism? In a very recent work [7], it has been shown using the covariant (Gupta-Bleuler) formalism that it is indeed possible to explain cosmic acceleration from the standard Maxwell’s theory.
We start by writing the standard electromagnetic action including a gauge-fixing term in the presence of gravity:

\[
S = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\lambda}{2} (\nabla_{\mu} A^{\mu})^2 \right]
\] (10)

The gauge-fixing term is required in order to define a consistent quantum theory for the electromagnetic field [16], and we will see that it plays a fundamental role on large scales. Still, this action preserves a residual gauge symmetry \( A_\mu \rightarrow A_\mu + \partial_\mu \phi \) with \( \Box \phi = 0 \). Electromagnetic equations derived from this action can be written as:

\[
\nabla_\nu F^{\mu\nu} + \lambda \nabla^\mu \nabla_\nu A_\nu = 0
\] (11)

Notice that since we will be using the covariant Gupta-Bleuler formalism, we do not a priori impose the Lorentz condition.

We shall first focus on the simplest case of a homogeneous electromagnetic field \( A_\mu = (A_0(t), \vec{A}(t)) \) in a flat Robertson-Walker background. In this space-time, equations (11) read:

\[
\ddot{A}_0 + 3H\dot{A}_0 + 3\dot{H}A_0 = 0
\]

\[
\ddot{\vec{A}} + H\dot{\vec{A}} = 0
\] (12)

We can solve (12) during the radiation and matter dominated epochs when the Hubble parameter is given by \( H = p/t \) with \( p = 1/2 \) for radiation and \( p = 2/3 \) for matter. In such a case the solutions for (12) are:

\[
A_0(t) = A_0^+ t + A_0^- t^{-3p}
\] (13)

\[
\vec{A}(t) = \vec{A}_+ t^{1-p} + \vec{A}^-
\] (14)

where \( A_0^\pm \) and \( \vec{A}^\pm \) are constants of integration. Hence, the growing mode of the temporal component does not depend on the epoch being always proportional to the cosmic time \( t \), whereas the growing mode of the spatial component evolves as \( t^{1/2} \) during radiation and as \( t^{1/3} \) during matter, i.e. at late times the temporal component will dominate over the spatial ones.

The energy densities of the temporal and spatial components read:

\[
\rho_{A_0} = \lambda \left( \frac{9}{2} H^2 A_0^2 + 3HA_0 \dot{A}_0 + \frac{1}{2} A_0^2 \right)
\] (15)

\[
\rho_{\vec{A}} = \frac{1}{2a^2} (\dot{\vec{A}})^2
\] (16)

Notice that we need \( \lambda > 0 \) in order to have positive energy density for \( A_0 \). In fact, it is possible to show that imposing canonical normalization for the corresponding creation and annihilation operators we get \( \lambda = 1/3 \) [17]. Besides, when inserting the growing modes of the fields into these expressions we obtain that \( \rho_{A_0} = \rho_{A_0}^0, \rho_{\vec{A}} = \rho_{\vec{A}}^0 a^{-4} \) and \( \nabla_\mu A^{\mu} = \text{const.} \) Thus, the field behaves as a cosmological constant throughout the
evolution of the universe since its temporal component gives rise to a constant energy density whereas the energy density corresponding to $\vec{A}$ always decays as radiation. Moreover, this fact prevents the generation of a non-negligible anisotropy which could spoil the highly isotropic CMB radiation. Finally, when the universe is dominated by the electromagnetic field, both the Hubble parameter and $A_0$ become constant (one can straightforwardly check that this is a solution of the complete system of equations) so the energy density is also constant and the electromagnetic field behaves once again as a cosmological constant leading therefore to a future de Sitter universe. As the observed fraction of energy density associated to a cosmological constant today is $\Omega_\Lambda \simeq 0.7$, we obtain that the field value today must be $A_0(t_0) \simeq 0.3 M_P$.

The effects of the high electric conductivity $\sigma$ can be introduced using the magnetohydrodynamical approximation and including the current term $J_i = \sigma(\partial_0 A_i - \partial_i A_0)$ on the r.h.s. of Maxwell’s equations. Notice that because of the universe electric neutrality, conductivity does not affect the evolution of $A_0(t)$. The infinite conductivity limit simply eliminates the growing mode of $\vec{A}(t)$ in (14).

We still need to understand which are the appropriate initial conditions leading to the present value of $A_0$. In order to avoid the cosmic coincidence problem, such initial conditions should have been set in a natural way in the early universe. In a very interesting work [17], it was suggested that the present value of the dark energy density could be related to physics at the electroweak scale since $\rho_\Lambda \sim (M_{EW}^2 / M_P)^4$, where $M_{EW} \sim 10^3$ GeV. This relation offers a hint on the possible mechanism generating the initial amplitude of the electromagnetic fluctuations. Indeed, we see that if such amplitude is set by the size of the Hubble horizon at the electroweak era, i.e. $A_0(t_{EW})^2 \sim H_{EW}^2$, then the correct scale for the dark energy density is obtained. Thus, using the Friedmann equation, we find $H_{EW}^2 \sim M_{EW}^4 / M_P^2$, but according to (15), $\rho_{A_0} \sim H^2 A_0^2 \sim \text{const.}$, so that $\rho_{A_0} \sim H_{EW}^4 \sim (M_{EW}^2 / M_P)^4$ as commented before.

A possible implementation of this mechanism can take place during inflation. Notice that the typical scale of the dispersion of quantum field fluctuations on super-Hubble scales generated in an inflationary period is precisely set by the almost constant Hubble parameter during such period $H_I$, i.e. $\langle \vec{A}_0^2 \rangle \sim H_I^2$ [18]. The correct dark energy density can then be naturally obtained if initial conditions for the electromagnetic fluctuations are set during an inflationary epoch at the scale $M_I \sim M_{EW}$.

Despite the fact that the background evolution in the present case is the same as in $\Lambda$CDM, the evolution of metric perturbations could be different, thus offering an observational way of discriminating between the two models. In fact, the evolution of the scalar perturbation $\Phi_k$ with respect to the $\Lambda$CDM model gives rise to a possible discriminating contribution to the late-time integrated Sachs-Wolfe effect [19]. The propagation speeds of scalar, vector and tensor perturbations are found to be real and equal to the speed of light, so that the theory is classically stable. We have also checked that the theory does not contain ghosts and it is therefore stable at the quantum level.

On the other hand, using the explicit expressions in [15] for the vector-tensor theory of gravity corresponding to the action in (10), it is possible to see that all the parametrized post-Newtonian (PPN) parameters agree with those of General Relativity, i.e. the theory is compatible with all the local gravity constraints for any value of the homogeneous background vector field [20].
The presence of large scale electric fields generated by inhomogeneities in the $A_0$ field opens also the possibility for the generation of large scale currents which in turn could contribute to the presence of magnetic fields with large coherence scales. This could shed light on the problem of explaining the origin of cosmological magnetic fields. Work is in progress in this direction.

CONCLUSIONS

We have shown that vector theories offer a simple and accurate description of dark energy in which the coincidence problem could be easily avoided. In our first example, the scaling behaviour during radiation and the natural initial conditions for the vector field offer a neat way around the problem. Moreover, in our second example, the presence of a cosmological electromagnetic field generated during inflation provides a natural explanation for the cosmic acceleration. This result not only offers a solution to the problem of establishing the true nature of dark energy, but also explains the value of the cosmological constant without resorting to new physics. In this scenario the fact that matter and dark energy densities coincide today is just a consequence of inflation taking place at the electroweak scale. Present and forthcoming astrophysical and cosmological observations will be able to discriminate these proposals from the standard $\Lambda$CDM cosmology.

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