TIDAL IMPRINTS OF A DARK SUB-HALO ON THE OUTSKIRTS OF THE MILKY WAY. II.
PERTURBER AZIMUTH

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ABSTRACT
We extend our analysis of the observed disturbances on the outskirts of the H I disk of the Milky Way. We employ the additional constraints of the phase of the modes of the observed H I image and asymmetry in the radial velocity to derive the azimuth of the perturber inferred to be responsible for the disturbances in the H I disk. Specifically, we carry out a modal analysis of the phase of the disturbances in the H I image and in high-resolution smoothed particle hydrodynamics (SPH) simulations of a Milky Way like galaxy tidally interacting with dark perturbers, the relative offset of which we utilize to derive the perturber azimuth. Under the assumption that the asymmetry in the radial velocity is due to the perturber, we derive the best fit to the radial velocity at \( l = 0, 180 \), and use this constraint to also derive the perturber azimuth. To make a direct connection with observations, we express our results in Sun-centered coordinates, predicting that the perturber responsible for the observed disturbances is between \(-50 \leq l \leq -10\). We explicitly show that the phase of the disturbances in the outskirts of simulated galaxies, our primary metric for the azimuth determination, is relatively insensitive to the equation of state (for the range of gas fractions of local spirals). Our calculations here represent our continuing efforts to develop the “tidal analysis” method of Chakrabarti & Blitz (CB09). CB09 employed SPH simulations to examine tidal interactions between perturbing dark sub-halos and the Milky Way. They found that the amplitudes of the Fourier modes of the observed planar disturbances are best fit by a perturbing dark sub-halo with mass \( \sim \text{one-hundredth that of the Milky Way} \) and a pericentric approach distance of \( \sim 5-10 \) kpc. The overarching goal of this work is to attempt to outline an alternate procedure to optical studies for characterizing and potentially discovering dwarf galaxies—whereby one can approximately infer the azimuthal location of a perturber, its mass, and pericentric distance (CB09) from analysis of its tidal gravitational imprints on the H I disk of the primary galaxy.

Key words: galaxies: dwarf – galaxies: evolution – galaxies: individual (Milky Way) – methods: numerical

Online-only material: color figures

1. INTRODUCTION

The detection and characterization of the ubiquitous dark matter in the universe is an outstanding problem in modern astrophysics. The overabundance of dark sub-halos (or dim dwarf galaxies) in theoretical simulations of the Milky Way (Klypin et al. 1999; Moore et al. 1999; Kravtsov et al. 2004) relative to observations of Local Group dwarfs has come to be known as the missing satellites problem. Recent discoveries of dark matter dominated Milky Way satellites (Willman et al. 2005; Belokurov et al. 2006; Walsh et al. 2007) have prompted much theoretical work on dwarf galaxies, mainly focused on the formation of dwarf galaxies (Governato et al. 2010, and references therein). These recent works have addressed a number of important problems, including the formation of bulgeless dwarfs (e.g., Governato et al. 2010). Nonetheless, the so-called missing satellites problem is still wanting a solution, and an approach that focuses on the dynamical interactions between dwarf galaxies and the primary galaxy would be extremely valuable toward this end.

The excess of sub-structure in simulations relative to observations of dwarf galaxies prompted us to ask—can dark dwarf galaxies be characterized from disturbances, left by tidal imprints of their passage, in the cold gas on the outskirts of galaxies? Thus, we were motivated to analyze observed perturbations (Levine et al. 2006a) on the outskirts of the gas disk of the Milky Way in an earlier paper (Chakrabarti & Blitz 2009, henceforth CB09), with the hypothesis that these disturbances arise from an external perturber that tidally interacted with the Milky Way. By quantitative comparison with a large set of high-resolution smoothed particle hydrodynamics (SPH) simulations of a Milky Way like galaxy tidally interacting with dark sub-halos, CB09 found that the best fit to the Fourier amplitudes of the planar disturbances occurred for sub-halos with mass ratio one-hundredth that of the Milky Way and a distance of closest approach \( \sim 5-10 \) kpc. A fundamental realization of CB09 was that the nonlinear response of the primary galaxy along the orbit of the satellite allows one to break the degeneracy between the mass and distance cubed in the tidal force—yielding a dynamical measure of the masses of satellites from analysis of the Fourier modes of the gas surface density. In CB09, we did not calculate the azimuthal location of this perturber. In this paper, we further develop the “tidal analysis” method of CB09 to outline our inference of the azimuthal location of perturbers from their tidal effects on gas disks. Specifically, we employ the additional constraints of the phase variation of the modes of the surface density in both the simulations and in the observations, as well as asymmetries in the radial velocity, to derive the perturber azimuth. To make a direct connection with observations, we express our results in Sun-centered coordinates.

The goal of this work is to attempt to delineate a procedure by which one characterizes dark perturbers, in particular to infer the azimuthal location of perturbers from analysis of the observed disturbances on the gas disks of galaxies. If
successful, this method may allow for dark (or effectively dark, i.e., with large mass-to-light ratios) sub-halos to be discovered in infrared surveys, even if they are not very deep. To date, dwarf galaxies have been discovered in the optical (e.g., Ibata et al. 1994). Cold dark matter (CDM) sub-structure in galaxies can alternately be studied via gravitational lensing (Vegetti et al. 2010), a method that like the tidal analysis method of CB09 does not rely on the stellar light distribution, but is subject to uncertainties in the projected mass distribution. With the advent of upcoming radio surveys by the Square Kilometer Array (SKA) and ALMA, we wish to develop further our alternate approach of characterizing CDM sub-structure by analysis of observed perturbations in the gas surface density, as the new generation of radio telescopes will be able to image extended H i disks out to higher redshift, promising an independent method of understanding the redshift evolution of CDM sub-structure. The general theme of learning about the distribution of dark matter from its visible effects is explored in a recent contribution by Peña-Rubia et al. (2010)—they employ $N$-body simulations to find that the tidal disruption of dwarf galaxies is highly sensitive to the inner density profile of the dark matter halo of the Milky Way. Here, we carry out a similar analysis, but look at the gravitational effects of dark sub-halos on the cold responsive gas disk of the primary galaxy. Our hypothesis here is a simple one, namely, that the disturbances are due to an external perturber. We examine this assumption in detail in a future work where we examine the effects of multiple perturbers and asymmetries in the dark matter halo. We discuss the possibility of known dwarf galaxies (such as the LMC, the Sagittarius dwarf, and Canis Major) in producing the observed disturbances in the H i disk in Section 4. Specifically, we discuss the detailed work of Peña-Rubia et al. (2005), which is in a similar spirit to the analysis carried out here and is designed to explain the production of the Monoceros tidal stream. This paper is organized as follows: in Section 2, we present our results and analysis with a brief synopsis of methods used; in Section 3 we discuss our results, future work, and caveats; and we conclude in Section 4.

2. RESULTS

The simulations and methodology have been previously presented in CB09. We briefly review them here. We used the GADGET-2 simulation code (Springel 2005) to examine tidal interactions between perturbers and the Milky Way disk in CB09 for a set of $\sim$50 simulations to find that the amplitudes of the low-order ($m = 1$–4) Fourier modes of the H i data are best fit by a 1:100 perturber with a pericentric approach distance of $R_{\text{peri}} \sim 5$ kpc, at a current radial location $\sim 80$ kpc. A representative set of simulations and parameters surveyed is summarized in Table 1 and was earlier described in CB09. We use the same nomenclature here, i.e., $f_{\text{gas}}$ refers to the gas fraction of the disk, EQS refers to the equation of state (with 0 being isothermal and 0.25 referring to the fiducial multiphase model). By plotting the variances of the $m = 1$ mode relative to the variance of modes 1–4 (a variance versus variance plot necessarily situates the best-fit simulations close to the origin), we showed that variations in initial conditions of the simulated Milky Way disk, such as gas fraction and equation of state, do not produce effects that are statistically significant. (This is true as long as the gas fraction is varied to within the range of typical local spirals.) We showed explicitly that varying orbital inclination for parabolic orbits led to small changes on the variance versus variance plot. Significant changes on this variance versus variance plot were primarily driven by changes in gross parameters, such as the mass of the perturber and distance of closest approach.

We also showed that it is possible to break the degeneracy between the mass and distance cubed in the tidal force under certain conditions (the first term in the approximation to the tidal force is proportional to $M/R^3$ when the pericenter distance is large compared to the galaxy size). This can be done if the time it takes the satellite to traverse the distance from $R_0(M_\ast)$ (the minimum distance at which a satellite of mass $M_\ast$ starts to significantly affect a parcel of the primary galaxy) to the pericentric approach distance $R_{\text{peri}}$ is greater than the time it takes for the gas to shock. If so, then disturbances will first form on the scale of $R_0(M_\ast)$ (at which point the pull of the satellite starts to overwhelm the local gravity in the primary galaxy), resulting in a visibly different response for two satellites of differing mass that exert an equivalent tidal force at their respective pericentric approach distances (Figure 3 in CB09). This is therefore a dynamical method of determining the masses of satellites, from analysis of the observed H i map, which does not require knowledge of the stellar velocity dispersion. A review of masses of Milky Way satellites from the observed stellar velocity dispersion can be found in the work by Walker et al. (2009), and in the subsequent work of Wolf et al. (2010). A central result of these papers is that the mass–velocity anisotropy degeneracy can be broken at the half-light radius of a pressure-supported system embedded in a dark matter halo, which has been shown analytically earlier by Peña-Rubia et al. (2005).

What we aim to do here and earlier in CB09 is to present a complementary means of deriving satellite masses, from analysis of observed disturbances in the gas disk of the primary galaxy.

The fact that one can break the degeneracy between the mass of the satellite and the pericentric distance when one is not in the impulse approximation, coupled with the relative lack of sensitivity on the initial conditions of the primary galaxy and orbital inclination—these two taken together are what allow us to characterize dark perturbers from their tidal imprints on gas disks. In a forthcoming paper, we perform a linear analysis of the problem and develop scaling relations that allow us to infer the satellite mass from the Fourier amplitudes directly (Chang & Chakrabarti 2011).

Our calculations track the locations of the particles used to simulate the Milky Way, as well as the perturber. Therefore, we were able to determine the current radial location of the perturber at the best-fit time snapshots in CB09. The notion of a best-fit time here derives from the fact that the disturbances in the gaseous component damp out on the order of a dynamical time. Our metric of comparison to the data were the azimuthally integrated radially varying Fourier modes—which yielded the mass of the perturber and distance of closest approach.

We did not make use of all the information present in the images of the data and simulations in our analysis in CB09—an image is composed of the Fourier amplitudes as well as the

| Sim   | $f_{\text{gas}}$ | $V_{\text{vir}}$ (km s$^{-1}$) | EQS | $R_{\text{peri}}$ (h$^{-1}$ kpc) | Inclination |
|-------|------------------|-------------------------------|-----|-------------------------------|-------------|
| 100E0R5 | 0.2              | 160                           | 0   | 5                             | h           |
| 100R5   | 0.2              | 160                           | 0.25| 5                             | h           |
| 100v4R5 | 0.2              | 165                           | 0.25| 5                             | h           |
| 100v0.1R5 | 0.1         | 165                           | 0.25| 5                             | h           |
| 100E0R5md | 0.2            | 160                           | 0   | 5                             | md          |
phases of the modes. Here we determine the azimuthal location of the perturber in Sun-centered (or galactocentric) coordinates as follows: we compute the phase of individual modes of the simulations and the data for the sub-set of simulations (and at the best-fit times for a given simulation) that give the best fit to the amplitudes of the Fourier modes as determined by CB09; choosing this sub-set guarantees that we consider those class of simulations with perturber masses and pericentric approach distances that reasonably fit the amplitudes of the observed disturbances. For projected gas surface density denoted $\Sigma(r, \phi)$, we calculate the phase of individual modes “$m$” by taking the FFT:

$$\phi(r, m) = \arctan \frac{\text{Imag} \text{FFT} \Sigma(r, \phi)}{\text{ReFFT} \Sigma(r, \phi)}. \tag{1}$$

The phase of the modes contains information on the shape of the spiral planform, i.e., tightly wrapped spirals produced by self-excited spiral structure inside of the Inner and Outer Lindblad Resonances will have a sharp gradient in the phase, while open spirals as produced by tidal interactions will have a flatter profile (Shu 1984). This type of Fourier analysis is similar to observational analyses by Considere & Athanassoula (1982) who fit logarithmic spirals to study the distribution of H II regions in images of external galaxies (for more detailed recent examples see Garcia-Gomez et al. 2004; Block et al. 2009). Since the simulations are not performed in the observational frame, there is no reason why—even if the phase variation of the simulations ($\phi^{\text{sim}}(r, m)$) and data ($\phi^{\text{data}}(r, m)$) are similar in shape—the relative offset between $\phi^{\text{sim}}(r, m)$ and $\phi^{\text{data}}(r, m)$ will be zero. We make use of this relative offset between $\phi^{\text{sim}}(r, m)$ and $\phi^{\text{data}}(r, m)$ to translate our determination of the perturber azimuth in the simulation frame to the observational frame. This procedure is similar to visual matching of (dominant) features in the data and simulations. We then use standard conversions to express our results in the Sun-centered frame.

We focus our analysis of the phase variation on the $m = 1$ and $m = 3$ modes. We do this first because, as we discuss in a longer forthcoming paper, the $m = 1$ and $m = 3$ modes are particularly affected by perturbers with mass ratios $\sim 1:100$ with $R_{\text{per}} \sim 5–10$ kpc. Second, the Fourier analysis, particularly the analysis of the phase of the modes, is affected by the wedges cut out of the H I data (see Levine et al. 2006b for the details of the size of the wedges). The H I map cannot be reliably determined close to $l \sim 0$, 180 because the velocities along the line of sight are too small with respect to random motions to establish reliable distances. Therefore, the final map has wedges cut out of the data that are symmetric in galactic longitude but asymmetric in the galactocentric $\phi$ coordinate. We have taken here the original map and rendered the wedges symmetric in $\phi$, as depicted in Figure 1(a) for the raw H I data and in Figure 1(b) for a representative simulation (100R5). It can be easily demonstrated that symmetric wedges affect the even modes primarily (as the wedges themselves contain even modes) using simple functions such as a constant function $f(r, \phi) = \text{constant}$. Thus, for an image with wedges cut out that are symmetric in $\phi$, the phase of the odd modes is more reliable than the even modes. The 100R5 simulation was a case that fit the amplitudes of the Fourier modes of the data well; the images of 100R5 simulation shown in Figure 1(b) correspond to the best-fit time snapshot. The images of the odd modes are shown in Figures 2 for the raw H I data and simulation.

Figures 3(a) and (b) depict the phase variation of the $m = 1$ and $m = 3$ modes from a representative simulation (100R5), as compared to the phases of the raw H I data. Of particular note is the steep variation of $\phi_{1}(r)$ in the inner regions ($r \lesssim 15$ kpc) where the dominant mode of spiral structure arises from the gas being forced by the stellar spiral arms inside of the Outer Lindblad Resonance (Chakrabarti et al. 2003; Chakrabarti 2009) for both the simulations and the data. In contrast, $\phi_{1}(r)$ is relatively flat in the outer regions (for $r \gtrsim 15$ kpc), a signature of a tidal interaction (Shu 1984), where the wavelength of the disturbance increases outward (similar to taking a bedsheets and flapping it). In other words, tidal interactions produce open spiral planforms, while self-excited spiral structure is more tightly wrapped. The higher order modes have “$m$” branches at a given radius, while being similar to the $m = 1$ mode in having a flat variation of the phase in the outer regions of the galaxy. To determine the translation between the simulation frame and the observational frame, we simply determine the relative offset from the phases, i.e., $\delta \phi = \phi^{\text{sim}}(r) - \phi^{\text{data}}(r)$. For the $m = 3$ mode, we have a priori three branches to select from to determine the offset; we select the branch of $m = 3$ that is in magnitude closest to the $m = 1$ branch for both the simulations and the data, in order to determine the offset. (In detail, we select the branch of $m = 3$ by subtracting each branch of $\phi(r, m = 3)$ from $\phi(r, m = 1)$ and choosing the branch that gives the smallest median (computed between $19$ kpc $< r < 23$ kpc) absolute difference.) This leads to, for instance, a median value of $\delta \phi = -1.9$ for $19$ kpc $< r < 23$ kpc.
Figure 2. Images of the odd modes in raw H \(\text{I}\) data (a) \(m = 1\), (b) \(m = 3\), (c) Images of the odd modes in representative simulation 100R5 for \(m = 1\), and (d) \(m = 3\). The box extends from \(-30\) kpc to 30 kpc.

Figure 3. Comparison of phase of (a) \(m = 1\) and (b) \(m = 3\) modes for 100R5 relative to raw H \(\text{I}\) data. See the text for description of how the azimuth of the perturber is determined from the relative offset between the phase of the simulations and data.

(A color version of this figure is available in the online journal.)

from the \(m = 1\) mode comparison, and \(\delta\phi = -0.63\) from the \(m = 3\) mode comparison, when selecting the branch of \(m = 3\) closest to \(m = 1\) for both the simulations and data. To determine the azimuth of the perturber, we employ the phase information from the images with symmetric wedges (as in Figure 1) as the information in the odd modes should be unaffected by symmetric wedges. Figures 4(a) and (b) display the phase of the odd modes in the simulations and data with symmetric wedges, which is slightly different from that shown in Figures 3, leading to \(\delta\phi\) values derived from the \(m = 1\) and \(m = 3\) modes that are somewhat more comparable.

2.1. Phase Variation: Secular versus Tidal Response; Dependence on Initial Conditions

We wish to underscore the visual signature of a tidal interaction that is present in Figures 3 and 4. We do this by comparison of the phase of the odd modes of the 100R5 interacting model to that of an isolated galaxy simulation. Figure 5(a) depicts the phase of the \(m = 1\) mode from an isolated galaxy simulation overplotted with the \(m = 1\) mode of the 100R5 interacting model, and Figure 5(b) shows the same for the \(m = 3\) mode. As is clear, the \(m = 1\) mode of the isolated galaxy simulation does not display the continuous flat profile as evinced by the 100R5 tidally interacting model; it discontinuously goes to zero in the outer regions. The phase of the \(m = 3\) mode for the isolated galaxy simulation declines outward, in contrast to that of the tidally interacting system shown in green, which is flat in the
outer regions. It is the case, as is expected, that the inner regions \((r \lesssim 15 \text{ kpc})\) do display a gradient in \(\phi_1(r)\) as the gas in the inner regions responds to the forcing by the stellar spiral arms. It is unlikely that cosmologically grown asymmetrical dark matter halos would reproduce both the amplitudes and phase of the Fourier modes in the absence of external perturbers, a point that we return to in Section 3.

Second, we wish to highlight the relatively insensitive dependence on initial conditions for variants of the 100R5 simulations at the best-fit time to the Fourier amplitudes (where by variants, we mean variations in the initial conditions of the simulated Milky Way disk, with the parameters varied denoted in Table 1 of CB09). Figure 6 depicts the phase variation of the \(m = 1\) mode of several of the 100R5 variants (for the nomenclature, see Table 1) at the best-fit time. As is clear, the phase of the 100R5 variants at the best-fit time is quite similar in the outskirts of the galaxy \((R \gtrsim 15 \text{ kpc})\). Figure 7 shows the images of the odd modes and further illustrate the similarity of the phase of two of the 100R5 variants \((100E0R5 \text{ and } 100v4R5)\) at the best-fit time. There are differences in the images of the modes for the isothermal 100E0R5 and the multiphase 100R5 simulation (shown earlier in Figure 2) and the 100v4R5 simulation in the inner regions of the galaxy. This is due to higher energy injection from supernovae in the multiphase 100R5 model thereby rendering it more stable, with the isothermal 100E0R5 model displaying a greater density contrast as it is more susceptible to fragmentation without the energy injection from supernovae. All of the images are however remarkably similar in the outskirts. The reason this is so is that in the Springel & Hernquist (2003) multiphase interstellar medium (ISM) model, the energy injection is proportional to the star formation rate. In the outskirts of the galaxy, there is very little star formation, and as such little difference arises due to variations in the equation of state of the gas. Due to the relative lack of star formation in the outskirts of galaxies (i.e., relative to the inner regions; although see Thilker et al. 2005 for observations of UV emission that indicate a low level of star formation in the outskirts of galaxies), the study of gas dynamics in the outskirts of galaxies is comparatively simpler as it is less dependent on processes such as star formation and feedback that are difficult to model in detail and at present generally prescribed in a sub-grid manner.

2.2. Radial Velocity Constraint: \(v_\pi(R)\)

Levine et al. (2006b) define a quantity \(v_\pi(R)\) that represents the radial velocity at \(l = 180\), and allows one to correct for the asymmetry between the surface densities at Galactic longitudes on either side of \(l = 0\) and \(l = 180\) (Henderson et al. 1982). In essence, the quantity \(v_\pi(R)\) represents the magnitude of the elliptical corrections to circular motions. Assumption of pure circular motions produces a surface density map that is discontinuous on either side of \(l = 0\) and \(l = 180\). Levine et al. (2006b) derive the functional form of \(v_\pi(R)\) by requiring continuity of the surface density in these regions. One may assume that this asymmetry in the radial velocity arises from a perturber forcing the gas in the outskirts. If so, \(v_\pi(R)\) potentially gives us another estimate of the azimuth of the perturber, in addition to that already given by the relative offset in the phases of the surface density maps of the simulations and the data.
In Figures 8, we compare the radial velocity for a representative simulation (100R5) at the time snapshot that gives the best fit to the amplitudes of the Fourier modes of the raw HI data. Specifically, Figure 8(a) shows the comparison of $v_\pi(R)$ (in asterisks) and $v_\text{sim}^r$ (crosses) at the best-fit azimuth. The general behavior of the two curves is in agreement, although the simulation curve overestimates the radial velocity by ~2 in the outer regions, relative to $v_\pi(R)$ in the outer regions. It is important to note that the functional form of $v_\pi(R)$ as given in Levine et al. (2006b) is itself an average and has some intrinsic scatter. As such, disagreement at the ~2 level may not be significant. It is difficult to refine this comparison further due to lack of the full velocity field. We address this problem in a future paper for M83 where we compare both the HI map and velocity map with the analogous simulation quantities.

There is a reasonable level of agreement between $v_\text{sim}^r$ and $v_\pi(R)$ at the best-fit angle, which when translated to Sun-centered coordinates agrees with the azimuth calculated from the phase offset to within ~15° for the variants of the 100R5 simulations (variations in initial conditions, orbits, etc., represent the variants in 100R5 as discussed in CB09). Figure 8(b) shows that the square of $v_\text{sim}^r$ deviates from the square of $v_\pi(R)$ significantly at all angles except at the best-fit angle. This suggests that the asymmetry in the radial velocity can be effectively analyzed in this way to find the best-fit angle for the perturber, which corresponds to the $\phi$ value at which $|v_r^2 - v_\pi(R)^2|/v_\pi(R)^2$ is a minimum. We also note that the simulations of isolated galaxies differ from tidally interacting galaxies not only in the phase variation in the outskirts of the galaxy (with tidally interacting galaxies displaying open spirals or a radially flat variation of the phase) but also in that the isolated galaxy simulations do not show as good of an agreement with $v_\pi(R)$. Specifically, if we take the quantity $M \equiv |v_r^2 - v_\pi(R)^2|/v_\pi(R)^2$ as our metric of comparison, then variants of 100R5 have a minimum in $M$ of order unity, while isolated galaxies have minima in $M \gtrsim 10$, indicating that the 100R5 tidally interacting systems give a better fit overall to the observed asymmetry in the radial velocity.

We now describe the coordinate transformation performed to calculate the azimuth of the perturber in Sun-centered coordinates. We take the Sun–galactic center distance $R_0 = 8.5$ kpc, and the radial location of the perturber at the best-fit time as determined by CB09 to be $R \sim 80$ kpc. From the analysis of CB09, we also know the perturber azimuth in simulation

![Figure 6](image_url)

**Figure 6.** Comparison of phase of $m = 1$ mode for variants of 100R5. Note that the phases of the variants of 100R5 are broadly similar.

(A color version of this figure is available in the online journal.)

![Figure 7](image_url)

**Figure 7.** Images of odd modes for the 100E0R5 simulation done with the isothermal equation of state, (a) $m = 1$ and (b) $m = 3$. These images of the phase of the odd modes of the isothermal 100E0R5 simulation should be compared to Figure 2, which was performed with the SH03 multiphase ISM (see the text for details). Both of these simulations have very similar phase of the disturbances in the outskirts although having different equation of state, (c) images of odd modes for the 100v4R5 simulation for $m = 1$ and (d) $m = 3$. (A color version of this figure is available in the online journal.)
coordinates in addition to the current radial location of the perturber. We denote the azimuth of the sub-halo in simulation coordinates as $\phi_{\text{sat,avg}}$ which is an average value over the 100R5 variants (with a range of a degree) equal to 307° in center-of-mass coordinates. The angle of the perturber in galactocentric coordinates as determined by the relative offset in the phase is given by $\phi_{\text{perturber}} = \phi_{\text{sat,avg}} + \phi_{\text{data}}$, where the $\phi_{\text{data}}$s are the phases of modes “m” in the simulations and data. With these quantities and simple geometry, we then find that the longitude “l” is given by

$$l = a \sin \left( \frac{R}{r'} \sin(90° - \phi_{\text{perturber}}) \right)$$

(2)

$$r' = \left( R_0^2 + R^2 - 2 R R_0 \cos(90° - \phi_{\text{perturber}}) \right)^{1/2},$$

(3)

which gives $-50 \leq l \leq -20$ for the best-fit cases. The derived azimuth values for the perturber are shown in Figure 9 overlaid on a map of the Milky Way. The scatter in the azimuth values is due to the variation in $\phi_m(r)$ for simulations where initial conditions are being varied, as shown in Figure 6. The phase of the $m = 1$ mode is broadly similar in different simulations where the initial conditions are varied (the variants of 100R5). However, there are local differences as a function of radius, which lead to the scatter in the azimuth, as the azimuth is derived from the relative offset of the phase in the simulations and the data.

In a separate paper (Chakrabarti et al. 2011), we demonstrate the validity of this method for determining the azimuthal loci of satellites from the phase by applying it to galaxies with known optical companions (that have come close to the galaxy in the recent past; see Section 3 for a discussion of why the known Milky Way satellites are unlikely to have produced the observed disturbances in the H1 disk). Specifically, we showed that deriving the azimuth from the relative offset of the phase enables us to correctly identify the location of the satellites for M51 and NGC 1512. These galaxies have satellites that span a large range in perturber to primary galaxy mass ratio (1:3–1:100). The application of this method to galaxies with known optical companions and its demonstrated validity lends credence to its use here as well.

3. DISCUSSION

Our methodology to characterize dark sub-halos here is a simple one—in essence, we found in CB09 that the mass of the perturber was given by the amplitude of the Fourier modes, and disturbances were excited on scales of order $R_0(M_*)$ (the minimum distance at which a satellite of mass $M_*$ starts to affect a parcel of the primary galaxy so as to induce radial motions of order the sound speed). This leads to therefore disturbances being produced on larger scales for more massive satellites (see Figure 3 in CB09) as their $R_0$ is larger—producing a visually different response for two satellites of differing mass that exert the same tidal force at their respective pericentric approach distances. The nonlinear response of gas in the primary galaxy along the course of the orbit of the perturber (from $R_0(M_*)$ to $R_{\text{peri}}$) allowed us to break the degeneracy between the mass of the perturber and distance of closest approach. If $R_{\text{peri}}/R_0 \gg 1$ (where $R_0$ is the scale length of the galaxy) then the condition stated in CB09 cannot be realized, i.e., the time it takes the satellite to traverse from $R_0$ to $R_{\text{peri}}$ becomes infinitesimally small as $R_0$ approaches $R_{\text{peri}}$ in this limit (for non-major mergers). Similarly, for smaller and smaller masses, $R_0(M_*)$ approaches $R_{\text{peri}}$ and the degeneracy cannot be broken. However, it is the regime where the pericentric distance is of order or a few times greater than the scale length wherein a significant tidal force is exerted—it is in this regime that CB09 found the $M, R^3$ degeneracy can be broken.

We have utilized three separate and independent constraints to characterize dark perturbers in this paper: (1) the radial variation of the Fourier modes (to determine the mass and pericentric distance—these are results we utilize from CB09), (2) the phase of the Fourier modes (to determine the azimuth of the perturber from the relative offset in phase between the simulations and data), and (3) the asymmetry in the radial velocity, which yields another measure of the azimuth of the perturber in addition to that derived from the relative offset of the phase. We also find that interactions of 1:100 satellites with $R_{\text{peri}} \sim 5–10\text{kpc}$ preserve the observed constraint of the thin stellar disk of the Milky Way, in agreement with prior studies (Purcell et al. 2009; Kazantzidis et al. 2008). The physical reason for this is that the sub-halos that we consider are not point masses—they have some physical extent and circular velocity, which mediates the amount of energy transfer to the stellar disk. Specifically, $\sim 1:100$ sub-halos have a circular velocity (at $R_{\text{peri}}$) that is comparable to the velocity dispersion of the galactic stellar disk. For extended bodies the maximum velocity that can be imparted is the escape velocity—as such, a kick from such sub-halos can at most produce an order unity effect as the velocity imparted will be stabilized by a comparable stabilizing force due to the intrinsic velocity dispersion of the stellar disk. Finally, it is not unreasonable to posit that these disturbances arise from a satellite close to the galactic plane—dust obscuration close
to the plane of the galaxy, to within $\sim 5^\circ$ of the galactic plane, renders optical identification of dwarf galaxies difficult (using dust extinction from Schlegel et al. 1998). Finally, Stewart (2009) find that the stellar to dark matter content of galaxies is lower at higher redshifts. Thus, if this dwarf galaxy formed at $z \sim 1$ and fell into the Milky Way halo recently, its intrinsic light-to-mass ratio may be lower than dwarf galaxies formed more recently.

We comment on the possibility of known dwarf galaxies producing these disturbances. We noted in CB09 that the LMC is too far away to raise sufficient tides on the galactic disk, and that it is more difficult to rule out the Sagittarius dwarf (Sgr) as the dominant architect of these disturbances. Cited masses for Sgr range from $\sim 10^8 M_\odot$ to $\sim 10^{11} M_\odot$ (Jiang & Binney 2000) although more recent work (Niederste-Ostholt et al. 2010) that analyzes the stellar debris from SDSS and 2MASS cites a mass of $\sim 10^{10} M_\odot$. If the more recent numbers are considered, then the crucial quantity is the last pericentric approach, i.e., not the current one at 24 kpc from the Sun (Ibata et al. 1997) but the previous approach as the disturbances manifest about a dynamical time after pericentric approach. As noted also in CB09, earlier calculations (e.g., Johnston et al. 1999) find pericentric approach at the prior passage to be $\sim 40$ kpc. More recent cosmological simulations that treat the collisionless component of the Milky Way (Purcell et al. 2011) find that with a fairly massive Sgr ($\sim 10^{10} M_\odot$), the percenter approach is closer ($R_{\text{peri}} \sim 20$ kpc), but the odd modes excited are too low at the present time compared to the observations to indicate that Sgr is the culprit that produced these disturbances. It is possible that inclusion of a cold (and thereby more responsive) component may enhance the response (however such a cold component will also be dissipative and therefore the disturbances will not last for more than a dynamical time), and we explore that possibility in future studies.

In a detailed study of the Monoceros tidal (for more than a dynamical time), and we explore that possibility especially with the inclusion of gas. Such a study is beyond the scope of the present paper, and the presence of this possibility may well yield a better fit to one of these constraints. It is unlikely however, that all three of these constraints can be fit significantly better. We do consider in future papers the effects of deviations from spherical symmetry in the dark matter halo on the generation of these disturbances (see Law & Majewski 2010 for a description of the evolution of Sgr in a triaxial Milky Way halo with $N$-body simulations), as well as the effect of multiple perturbers, magnetic instabilities, and variations in the inner structure of perturbers (where the latter is likely to only have a second-order effect on our analysis as the perturbers we consider here are much less massive than the primary). We comment briefly on the consideration of a triaxial halo as it has received considerable attention in the literature. Generally, dark matter halos are observationally inferred to be rounder than what is predicted by collisionless cosmological CDM simulations, from gravitational lensing studies (Mandelbaum et al. 2006) and X-ray isophotal studies (Buote et al. 2002). This may be due to the condensation of baryonic components leading to rounder halos (Kazantzidis et al. 2004; Debattista et al. 2008), or possibly due to uncertainties in the observational determination of the triaxiality of halos as the observational inferences are based on the projected mass distribution. Therefore, while the investigation and determination of discriminants between secular effects, due to a triaxial dark matter halo, and effects from external perturbers are a topic worthy of serious investigation, it is not immediately clear that the prevalence of non-spherical halos found in collisionless cosmological simulations is reflected in reality. Nonetheless, a comprehensive understanding of the differences between secular effects with the inclusion of non-spherical halos and external perturbers would be very useful. However, it is difficult to produce stable initial conditions for multi-component disks with non-spherical halos, especially with the inclusion of gas. Such a study is beyond the scope of the present paper, and the presence of this possibility may be considered a caveat to the model presented here. We emphasize in closing that the distribution of subhalos in the vicinity of spiral galaxies is remarkably sensitive to the inner structure of the dark matter halo of the primary galaxy (and therefore the very nature of dark matter), as found in the work by Peñarrubia et al. (2010). Thus, our tidal analysis method, which allows us to infer the loci of satellites, may in the future allow for constraints to be set on the structure of the dark matter halo of the primary galaxy.

4. CONCLUSION

1. We have examined the phase variation of the low-order modes ($m = 1$–4) in both the simulations and the data with specific attention to the odd modes to find that the phase of the modes in the outer regions (for $r \gtrsim 15$ kpc) is relatively
flat as a function of radius, i.e., qualitatively of the form of a tidal interaction. This is true for the variants of the 100R5 simulations at the best-fit time (the time that best fits the Fourier amplitudes of the data as determined in CB09). The phase variation for both the simulations and the data shows a sharp gradient in the inner regions (as expected when gas is forced by the stellar spiral arms). To translate between the observed frame and the simulation frame, we find the relative offset between the phases of the modes, and use this offset to calculate the azimuthal location of the perturber, in both galactocentric and Sun-centered coordinates. We find that the perturber responsible for the disturbances in the H I disk of the Milky Way is at a longitude $-50 \lesssim l \lesssim -10$, assuming that the angles calculated from the modes are independent.

2. The phase variation of the simulated galaxies at the best-fit time is largely insensitive to the initial conditions. This lack of sensitivity to the initial conditions (specifically, the equation of state) for the phase of the modes in the outskirts of the simulated galaxy arises from the relative lack of star formation (i.e., relative to the inner regions) which serves to set the effective equation of state and energy injection from supernovae. Therefore, even though global simulations of galaxies presently treat star formation in a simple, prescriptive manner and the equation of state of the gas is an unknown parameter, one can reach conclusions about the evolution of gas on the outskirts of galaxies that are relatively independent of processes such as star formation and feedback that are difficult to model in detail.

3. We have compared the radial velocity in the simulations to find not only the angle at which $v_r$ gives the best fit to the asymmetry in radial velocity ($v_r(R)$) as observed by Levine et al. (2006b) but also the distribution of chi-squared’s as a function of angle. The angle at which $v_r$ most closely matches $v_r(R)$ is comparable to that derived from the phase comparison, to within $\sim 15^\circ$. Moreover, the distribution of chi-squared values of $v_r$ in the simulation as a function of angle increases sharply when moving away from the best-fit angle.

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