Using topological characteristics to evaluate complex network models can be misleading

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Abstract

Graphical models are frequently used to represent topological structures of various complex networks. Current criteria to assess different models of a network mainly rely on how close a model matches the network in terms of topological characteristics. Typical topological metrics are clustering coefficient, distance distribution, the largest eigenvalue of the adjacency matrix, and the gap between the first and the second largest eigenvalues, which are widely used to evaluate and compare different models of a network. In this paper, we show that evaluating complex network models based on the current topological metrics can be quite misleading. Taking several models of the AS-level Internet as examples, we show that although a model seems to be good to describe the Internet in terms of the aforementioned topological characteristics, it is far from being realistic to represent the real Internet in performances such as robustness in resisting intentional attacks and traffic load distributions. We further show that it is not useful to assess network models by examining some topological characteristics such as clustering coefficient and distance distribution, if robustness of the Internet against random node removals is the only concern. Our findings shed new lights on how to reasonably evaluate different models of a network, not only the Internet but also other types of complex networks.

Keywords: Complex network model, Internet, Robustness, Traffic load distribution

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1. Introduction

The structural properties of a complex network are typically very complicated, due to the large size and intrinsic interconnection patterns of the network. To better understand the structure of a complex network and to analyze its dynamical behavior, it is almost always necessary to construct a simplified abstract model that can well reserve the most fundamental structural characteristics of the network.

Graphs, consisting of nodes with links connecting among them in some form, have been widely used to model and analyze the structures and dynamical behaviors of various complex networks [1]. Representative examples include the following. For social networks, Saban et al. [2] proposed a growth network model to represent the bilateral investment treaties (BIT) network; Kitsak et al. [3] proposed a scale-free model to describe the business firm network; Vieira et al. [4] investigated the sexual transmission of HIV within a population based on a small-world network model; Yang et al. [5] studied the spreading scheme of viral marketing based on a network model. For biological networks, Nicolau and Schoenauer [6] introduced a network model to reproduce some statistical measurements of the gene regulatory network; Nacher and Araki [7] suggested an evolutionary model to rebuild the degree distribution of the ncRNA-protein interaction network; Sneppen et al. [8] presented a simplified model to understand the large-scale regulatory networks; Ponten et al. [9] examined the relationship between structural and functional connectivity on the basis of the EEG neural mass model. For communication and transportation networks, Boas et al. [10] developed a modified geographical model to discuss worldwide highway networks; Wang and Loguinov [11] derived a wealth-based Internet model to study the AS-level Internet topology. Along this line of research from the graph-theoretic approach, many other types of examples can be easily given.

Once a model is constructed to describe a network, it immediately needs to be evaluated to see if it is “good” to represent the network and, moreover, if it is “better” than other existing models designed for the same network.

At present, researchers mainly rely on topological characteristics of a network to do such modeling, verification and comparison. A commonly adopted approach within the network science community is to consider a model to be “good” for a network if it can reproduce some basic topological characteris-
tics of that network. And, further, this model is considered to be superior to the others if it can better match the network in terms of these topological characteristics. In a word, topological characteristics are the litmus for testing network models today. Specifically, some well-studied topological characteristics such as node degree distribution, clustering coefficient, distance distribution, and spectrum of the adjacency matrix, are widely used to validate a model for a network or to evaluate and compare different models of the network. For instance, Nacher and Araki [7] used degree distribution to evaluate their proposed model for a ncRNA-protein interaction network. Wang and Loguinov [11] asserted that their proposed wealth-based Internet model is better than other existing models because it can better capture the clustering coefficient and the distance distribution of the AS-level Internet network using a set of real data. Toivonen et al. [12] utilized degree distribution, clustering coefficient, and community structure to compare different models of friendship and email networks.

A more careful examination of the modeling issue reveals that the performance of some specific functions of a network is more important than its topological characteristics, since a network (e.g., the Internet) is designed or formed for certain intended functioning and tasks, unless the latter truly determines the former.

In this paper, first, taking models of the AS-level Internet topology as the underlying test-bed, we show that the current approach of using purely the topological characteristics to evaluate different models may be misleading in model selection. Specifically, we show that different existing Internet models can have little difference in resisting random removals although they are very different in major topological metrics, namely clustering coefficient, distance distribution, the largest eigenvalue of the adjacency matrix, and the gap between the first and the second largest eigenvalues. As a result, if the robustness of the Internet against random removals is the main concern, then Internet model should not be assessed based only on such model topological characteristics.

In this paper, furthermore, we show that some models that can closely match the Internet in terms of the aforementioned topological metrics can be very unsuitable to use for investigating the robustness of of the Internet in resisting intentional attacks and traffic load distribution. As a result, such criteria for Internet modeling seem to be misleading in selecting good models for describing the real Internet, at least at the AS level.

The rest of the paper is organized as follows. Section 2 provides some
background and preliminaries on network models and their topological metrics. Section 3 compares some basic topological characteristics and Section 4 compares robustness against random failures and intentional attacks, and data traffic performance of the Internet, all by simulations. Section 5 briefly concludes the present investigation.

2. Network models and their topological metrics

For the AS-level Internet, since the first observation by Faloutsos et al. [13], several power-law models, such as the BA [14], EBA [15], Fitness [16], GLP [17], HOT [18], PFP [19], MLW [20], and WIT [11] models, have been proposed to describe the Internet topology, despite the fact that many of them were not intended for the Internet. In order to evaluate and compare these models against the real Internet, many basic topological characteristics have been examined and discussed, including the following:

Clustering coefficient—it is defined to measure how close the neighbors of a node are interconnected, popularly known as the probability of two friends of a person being friends themselves in a social network. It is an important characteristic to the robustness performance resisting removals of nodes-links, and even to routing algorithms in computer networks since a node with higher clustering coefficient generally means higher path diversity of the node.

Distance distribution—it is to measure the probability that a randomly selected pair of nodes are separated by a pre-designated distance. As a global topology characteristic, it plays a vital role in many Internet applications, such as routing and resisting virus spreading.

The first largest eigenvalue and the gap between the first and the second largest eigenvalues of the adjacency matrix—eigenvalues of the adjacency matrix of a network represent another global characteristic of the network. Particularly, the first largest eigenvalue of the network adjacency matrix and the gap between the first and the second largest eigenvalues are very important because the former is key to the network robustness on removals of nodes-links and the latter is closely related to the maximum traffic throughput of the network. Here, the adjacent matrix, \( \{a_{i,j}\}_{N \times N} \), is defined by setting \( a_{i,j} \) to be 1 if a pair of nodes \( i \) and \( j \) is connected, and 0, otherwise.

These three basic topological characteristics have been frequently used to evaluate newly proposed models for the Internet. For example, Wang
and Loguinov [21] compared the wealth-based evolution model with the BA model, the generalized linear preferential model [22], and the HOT model [18], by examining whether they can reproduce the average clustering coefficient and average distance characteristics of the real Internet topology. They also used the dynamical behaviors of average clustering coefficient, average distance, and the second smallest nonzero eigenvalue of the normalized Laplacian matrix, to compare different models [11]. Bu and Towsley [17] argued that their proposed model is better than the others by evaluating the degree of resemblance to the Internet in terms of power-law exponent, average clustering coefficient and average distance.

On the other hand, in the computer networking community two usually concerned and widely studied issues are the following:

Robustness in resisting random failures and intentional attacks—On the Internet, events such as equipment failures, power lost, traffic overload, and distributed DoS attacks, occur frequently. Such incidents are expected to have little effect on the effective operation of the entire network, namely the Internet should be robust against them.

Traffic load distribution—In Internet data traffic engineering, the traffic load distribution pattern of the Internet is very important because it can be used to measure the potential traffic on nodes-links and potential congestion points in the Internet.

By taking all the aforementioned network metrics and concerned issues into consideration, the objective of this paper is to answer the following question: for a “good” Internet model that “closely” matches the real Internet in terms of the three key topological characteristics mentioned above, is it also “good” to the Internet in capturing the robustness of the network against random failures and intentional attacks and in reproducing the Internet traffic load distribution pattern?

To address this question, the familiar BA, EBA, Fitness, and MLW models are used below, because they can be precisely formulated and programmed, to investigate the AS-level Internet topology constructed based on the daily data collected by UCLA [23] on 15 May 2005.

Noticing the argument [24] that the degree distribution of the AS-level Internet is not a power-law but a Weibull distribution or something else, we plot Figure 1 here for verification, to show the cumulative degree distributions of the UCLA data collected on 15 May from 2004 to 2010. During this period of a total of six years, these cumulative degree distributions of real Internet
data turned out to be very similar to each other and they all look like power-law (though not exactly), albeit the Internet size has increased dramatically in the six years. For this reason, some traditional Internet models, such as random graph [25], Tiers and Transit-Stub models [22], are excluded from our comparisons below. Meanwhile, previous observations have shown [26] that the three characteristics, namely average degree, degree distribution, and joint degree distribution, are key to reproduce an Internet-like topology. Therefore, the models considered here, namely the BA, EBA, Fitness, and MLW models, will apply the same set of values of these three characteristics, whenever possible (see [20] for more details in performing such simulations).

It should be remarked that our emphasis here is not to claim which model is the best one to represent the Internet or optimize a model to best fit the Internet topology, although this comparison will be made from time to time, but rather to demonstrate that topological metrics should not be used as criteria for modeling the Internet. In other words, the concern is the performance especially the robustness of the model versus the real Internet, therefore it is often not necessary to tune the model parameters to best fit the snapshots of the Internet topology data in simulations.

3. Comparison of basic topological characteristics

The parameter values of some basic topological metrics obtained from our extensive simulations, such as the network size (number of nodes), power-law exponent, assortativity coefficient, average clustering coefficient, average distance, and the largest eigenvalue of the adjacency matrix, are summarized in Table I.

It can be observed from Table I that the MLW and EBA models are closer to the Internet in terms of average clustering coefficient, average distance, and the largest eigenvalue. Clearly, the MLW and EBA models are better than the BA and Fitness models if models are compared by these topological characteristics.

Figure 2 shows the relationship between the clustering coefficient and the node degree $k$ for the Internet and all models studied. It can be seen that high-degree nodes of the Internet have lower clustering coefficients while low-degree nodes have higher clustering coefficients, which is consistent with the observations [27] that the core is loosely connected and the structure is clearly hierarchical in the Internet. It can also be observed from Figure
Table 1: Values of topological parameters for the Internet and the network models. \( N \) is the number of nodes, \( \gamma \) is the power-law exponent, \( r \) is the assortativity coefficient, \( \bar{C} \) is the average clustering coefficient, \( \bar{d} \) is the average distance between nodes, and \( \lambda \) is the largest nonzero eigenvalue of the adjacency matrix.

|          | Internet | BA  | EBA | Fitness | MLW |
|----------|----------|-----|-----|---------|-----|
| \( N \)  | 21999    | 21999| 21999| 21999   | 21999|
| \( \gamma \) | 2.18    | 3.0 | 2.69 | 2.45   | 2.36 |
| \( r \)   | -0.18    | -0.02| 0.02 | -0.11   | 0.03 |
| \( \bar{C} \) | 0.46    | 0.003| 0.01 | 0.01   | 0.24 |
| \( \bar{d} \) | 3.49    | 4.14 | 3.49 | 3.71   | 3.45 |
| \( \lambda \) | 141.12  | 27.82| 62.83| 39.16   | 111.87|

2 that the clustering coefficient of the MLW model is closer to that of the Internet as compared to the other models.

Figure 3 displays the distance distributions of the Internet and the models. It can be seen that the BA and Fitness models have a Poisson-like distance distribution, with a peak around a certain distance \( d_0 \) and decaying exponentially when distance \( d \) is far away from \( d_0 \). Clearly, the MLW and EBA models are better than the BA and Fitness models in capturing the characteristic of distance distribution of the Internet.

Figure 4 depicts the first and second largest eigenvalues of the adjacency matrix of the Internet and the models. For the Internet, the first largest eigenvalue is quite large and there is a big gap between the first and the second largest eigenvalues. It can be observed that the first largest eigenvalue and the gap between the first and the second largest eigenvalues are both bigger in the MLW and EBA models, but they are smaller in the BA and Fitness models. Clearly, if only the first largest eigenvalue is concerned, the MLW and EBA models are better than the BA and Fitness models. Furthermore, in evaluating both the first largest eigenvalue and the gap between the first and the second largest eigenvalues, the same can be concluded.

In summary, the MLW is the best choice among the studied models to fit the Internet topology, if the models are evaluated by their topological characteristics such as the average clustering coefficient, average distance, clustering coefficient distribution, distance distribution, the first largest eigenvalue, and the gap between the first and the second largest eigenvalues.
4. Comparison of performances in robustness and data traffic

The robustness of a network against attacks and failures can be studied by discussing $S_f$, the size of the largest connected component after a fraction of nodes, $f$, in the network were randomly or intentionally removed from the original network $S_0$. Clearly, the ratio $S_f/S_0$ measures the capability of the network regarding, after the $f$ portion of nodes have been randomly or intentionally removed, how many nodes remain functioning in communicating with each other.

Figure 5 shows a comparison of robustness resisting random removals of nodes. One can observe that all models have little difference. However, as discussed above, all these models are very different in their topological characteristics, namely clustering coefficient, distance distribution, the first largest eigenvalue, and the gap between the first and the second largest eigenvalues. As a conclusion, if the resistance ability of the Internet against random removals is the only concerned, then a good model of the Internet should not assessed by topological metrics.

One can also observe that the BA and Fitness models have different power-law exponents but they behave similarly in resisting random removals. Therefore, concerning the robustness of the Internet against random removals, it is not necessary to require a model to be able to exactly reproduce the value of the power-law exponent of the real Internet. Even the simplest toy BA model can roughly reflect the Internet’s robustness against random removals. This also shows that the so-called “robust yet fragile” property of the Internet [28], or its BA model, does not essentially depend on the power-law distribution of its topology.

Figure 6 compares the robustness of resisting intentional attacks. Here, as usual, intentional attacks mean that nodes are removed one after another following the decreasing order of the node degrees. It can be observed from the figure that again the MLW model is the best while the EBA model is the worst in reflecting the Internet’s robustness in resisting intentional attacks. However, both the MLW and EBA models are better than the BA and Fitness models in terms of reproducing the average clustering coefficient and average distance. Therefore, if Internet models are evaluated based on the average clustering coefficient and average distance, as did in [17], then the “best” or a “better” model so selected will be truly misleading. Note also that the EBA model is better than the Fitness model in reproducing the Internet’s topological characteristics, including the clustering coefficient, dis-
tance distribution, the first largest eigenvalue, and the gap between the first and the second largest eigenvalues. However, the Fitness models is closer to the Internet than the EBA model in matching the robustness against intentional attacks. Thus, one way or another, if these models are evaluated based on topological characteristics, then the results will be misunderstanding.

Next, to investigate the traffic load distribution, it is natural to study \( T(r) \), the ratio of the traffic load of the first \( r \) largest nodes over the total traffic load of the whole network. Here, it is assumed that a data packet is sent from node \( i \) to \( j \), for every possible pair of nodes \((i, j)\). For simplicity, we do not take into account the time delay of data transmission at nodes and links, and adopt the Open-Shortest-Path-First (OSPF) routing protocol to transmit data packets. Thus, the traffic load of a node is defined as the total number of packets passing through it when all pairs of nodes send and receive one packet between them.

Figure 7 shows the traffic load distribution of the Internet and the models. It can be seen that the traffic load distribution of the Internet is quite heterogeneous: a small fraction of the first largest nodes occupy most traffic load of the network, while a large number of low-degree nodes occupy only a small portion of the total traffic. Compared to the BA and Fitness models, the MLW and EBA models significantly underestimate the heterogeneity of the traffic load distribution of the Internet. Again, a “better” model determined by using the average clustering coefficient and average distance or by using the clustering coefficient, distance distribution, the first largest eigenvalue, and the gap between the first and the second largest eigenvalues, can be misunderstanding too — A model that is closer to the Internet in topological characteristics can be very bad in reflecting the traffic load distribution, or vice versa.

5. Conclusions

Several comparable network models of the AS-level Internet have been investigated, analyzed and compared, in terms of their topological characteristics such as clustering coefficient, distance distribution, and the first largest eigenvalue as well as the gap between the first and the second largest eigenvalues of the adjacency matrix. It reveals that that a model that are better than the others in matching the topological characteristics of the Internet may actually be worst in representing some critical performances and behaviors such as the robustness against random or intentional attacks, and
traffic load distribution, of the Internet. The conclusion is therefore that evaluating complex network models based on current topological metrics can be misleading, at least in the scenario of the AS-level Internet. Our findings may shed new lights on realistic modeling of more general complex networks.

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References

[1] R. Albert, A.L. Barabási, Rev. Mod. Phys. 74 (2002) 47.
[2] D. Saban, F. Bonomo, N. E. Stier-Moses, Phys. A 389 (2010) 3661.
[3] M. Kitsak, et al., Phys. Rev. E 81 (2010) 036117.
[4] I. T. Vieira, R. C. H. Cheng, P. R. Harper, V. Senna, Ann. Oper. Res. 178 (2010) 173.
[5] J. Yang, C. Yao, W. Ma, G. Chen, Phys. A 389 (2010) 859.
[6] M. Nicolau, M. Schoenauer, BioSystems 98 (2009) 137.
[7] J. C. Nacher, N. Araki, BioSystems 101 (2010) 10.
[8] K. Sneppen, S. Krishna, S. Semsey, Annu. Rev. Biophys. 39 (2010) 43.
[9] S. C. Ponten, A. Daffertshofer, A. Hillebrand, C. J. Stam, NeuroImage 52 (2010) 985.
[10] P. R. V. Boas, F. A. Rodrigues, L. F. Costa, Phys. Lett. A 374 (2009) 22.
[11] X. Wang, D. Loguinov, IEEE Trans. Networking, 18 (2010) 257.
[12] R. Toivonen, et al., Social networks, 31 (2009) 240.
[13] M. Faloutsos, P. Faloutsos, C. Faloutsos, Comp. Comm. Rev. 29 (1999) 251.
[14] A.L. Barabási, R. Albert, Science 286 (1999) 509.
[15] R. Albert, A. L. Barabási, Phys. Rev. Lett. 85 (2000) 5234.
[16] C. Bianconi, A. L. Barabási, Europhys. Lett. 54 (2001) 436.
[17] T. Bu, D. Towsley, Proceedings of IEEE InfoCom., IEEE, New York, 2002, pp. 638-647.
[18] J. M. Carlson, J. Doyle, Phys. Rev. E 60 (1999) 1412.
[19] S. Zhou, Phys. Rev. E 74 (2006) 016124.
[20] Z. P. Fan, G. R. Chen, Y. N. Zhang, Phys. Lett. A 373 (2009) 1601.
[21] X. Wang, D. Loguinov, Proceedings of IEEE InfoCom 2006, IEEE, Barcelona, 2006, pp. 1-11.
[22] M. B. Doar, IEEE GLOBECOM, IEEE/ACM, London, 1996, pp. 86-93.
[23] B. Zhang, R. Liu, D. Massey, L. Zhang, ACM SIGCOMM Computer Communication Review 35 (2005) 53.
[24] H. Haddadi, et al., IEEE Comm. Surveys Tutorials, 10 (2008) 48.
[25] B. M. Waxman, IEEE J. Selec. Areas in communications 6 (1988) 1617.
[26] P. Mahadevan, D. Krioukov, K. Fall, A. Vahdat, Proceedings of ACM SIGCOMM, ACM, Pisa, 2006, pp. 135-146.
[27] G. Q. Zhang, Q.-F. Yang, S. Q. Cheng T. Zhou, New J. Phys. 10 (2008) 123027.
[28] R. Albert, H. Jeong, A.-L. Barabási, Nature 406(2000) 387.
Figure 1: Cumulative degree distribution of the AS-level Internet on May 15 from 2004 to 2010.

Figure 2: Comparison of clustering coefficient.
Figure 3: Comparison of distance distribution.

Figure 4: Comparison of the first and second largest eigenvalues of adjacent matrix.
Figure 5: Comparison of robustness in resisting random removals.

Figure 6: Comparison of robustness in resisting intentional attacks.
Figure 7: Comparison of traffic load distribution.