Simulating Wess-Zumino Supersymmetry Model in Optical Lattices

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We study a cold atom-molecule mixture in two-dimensional optical lattices. We show that by fine-tuning the atomic and molecular interactions, Wess-Zumino supersymmetry (SUSY) model in 2+1-dimensions emerges in the low-energy limit and can be simulated in such mixtures. At zero temperature, SUSY is not spontaneously broken, which implies identical relativistic dispersions of the atom and its superpartner, bosonic diatom molecule. This defining signature of SUSY can be probed by single-particle spectroscopies. Thermal breaking of SUSY at a finite temperature is accompanied by a thermal Goldstone fermion, i.e., phonino excitation. This and other signatures of broken SUSY can also be probed experimentally.

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Introduction. – Wess and Zumino proposed the first space-time supersymmetry (SUSY) model (WZ-SUSY model) 36 years ago \cite{Wess1974}. Since then SUSY has become a fundamental ingredient of theories beyond the standard model in high-energy physics \cite{Ellis1981}. However, none of super partners of the known elementary particles have been found thus far; it remains to be seen if they can be detected in the energy range of Large Hadron Collider.

On a different front, nonrelativistic SUSY (a Bose-Fermi symmetry unrelated to space-time symmetry) has attracted considerable recent interest in the cold atom community, as it can be realized by using Bose-Fermi atom (molecule) mixtures which are loaded in optical lattices. Examples include attempts to simulate the nonrelativistic limit of superstring by trapping fermionic atoms in the core of vortices in a Bose-Einstein condensate \cite{Zhang2008}; study of the SUSY effect in an exactly solvable one-dimensional Bose-Fermi mixture with Bethe ansatz \cite{Jochi2008}; and SUSY models for nonrelativistic particles in various dimensions \cite{Berges2008,Chubykalo2008}. In Ref. \cite{Zhang2008}, we studied perhaps the simplest cold atom SUSY model and discussed detecting the Goldstino-like mode due to SUSY breaking by measuring a single fermion spectral function. In a further work \cite{Zhang2008}, we developed a SUSY response theory to photoassociation in a cold fermionic atom system. Although these studies are interesting and some results may be general for many SUSY systems \cite{Berges2008}, SUSY in these nonrelativistic systems is very different from the relativistic (or space-time) SUSY in high-energy physics.

In this Letter, we propose a way to simulate the simplest relativistic SUSY model, the WZ-SUSY model \cite{Wess1974}. We show that it can emerge in the low-energy limit of a cold atom-molecule mixture in properly chosen two-dimensional lattices. The first requirement is the existence of Dirac points in the Brillouin zone. Recently, such models based on a honeycomb lattice or graphene-like structure have been proposed \cite{Liu2010}. In these models, two Dirac points $K$ and $K'$ are related to each other by $K' = -K$, as required by time-reversal symmetry. This means This means that two fermionic atoms which form a usual BCS pair or a diatom molecule belong to two different Dirac points. To simulate the WZ-SUSY model, however, one needs a Klein-Gordon field as the Dirac fermions superpartner which corresponds to a diatom molecule made by two Dirac fermions from the same Dirac point. Such molecules carry a $2K \neq 0$ momentum and are energetically unfavorable as a result. In a recent work, Lee attempted to avoid this difficulty by introducing frustrated hopping for the molecules, such that the boson dispersion has minima at $\pm 2K$ instead of zero \cite{Lee2010}. It is found that the massless WZ-SUSY model emerges at the boson’s superfluid-insulator critical point.

In this work, we show that the WZ-SUSY model can emerge not only at the critical point. We use a lattice model studied recently by Liu et al. \cite{Liu2010} instead. This is a square lattice model in which the Dirac points at $K = (0, 0)$ and $K' = (0, \pi)$ are their own negatives, as $(0, -\pi) \equiv (0, \pi)$. This means a diatom molecule made of two atoms from the same Dirac points has zero-momentum for $2K = (0, 0)$ and $2K' = (0, 2\pi) \equiv (0, 0)$. With this setup, we can simulate the WZ-SUSY model more straightforwardly, after appropriate interactions are introduced and fine-tuned.

The research interest in WZ-SUSY models has been renewed recently \cite{Lee2010}. No spontaneous breaking of the SUSY implies there are equal poles in the single-particle spectral functions of both the Dirac field and the Klein-Gordon field. A further calculation showed that these single-particle spectral functions are not renormalized from their free particle ones \cite{Lee2010}. This is the identifier of the SUSY and may be detected by the established techniques of the single-particle spectroscopies \cite{Fukuhara2010}. It is known that a thermal bath always breaks the SUSY \cite{Lee2010} and this thermal breaking of the SUSY is accompanied by a thermal Goldstone fermion, phonino \cite{Lee2010}; thus studying this model at finite temperature sheds light on physics of SUSY breaking.

There are many studies of SUSY in space-time lat-
tice models\textsuperscript{[13].} The significant difference between the the present work and those lattice SUSY models is that while the latter are supersymmetric on the lattices, we study the emergence of SUSY from a microscopic space lattice (but continuous time) model with no SUSY to begin with.

**Free Fermion Lattice Model and Continuum Limit.** We briefly recall the lattice model proposed in Ref. \textsuperscript{[10].} Consider a single-component fermionic atom gas loaded in a square lattice. The potential minimum in the sublattice $A$ is higher than that in the sublattice $B$. Two states with the energy difference $2M$, the $s$-orbital at the $A$-sites and the $p$-orbital at the $B$-sites, form a pseudospin-$1/2$ subspace. The sublattices are anisotropic with $1$, $2$, $3$, and $4$, the next nearest neighbor sites (Fig. 1(b)). The hopping amplitudes into account. The corresponding hopping amplitudes are $t_{AB} \delta_{k} \equiv t_{AB} \theta_{k}$ and $h_{2} = -M - t_{0} \cos(k_{x} a) - 2t \sin(k_{x} a) \sin(k_{y} a)$ with $t_{0} = t_{A1} - t_{B1} + t_{A2} - t_{B2}$ and $t = (t_{A1} - t_{B1} + t_{B2} - t_{A2})/2$. When $M = \pm t_{0} = 0$, there is a unique gapless Dirac point: either $K = (0, 0)$ or $K' = (0, \pi)$. We choose $\theta_{0} = \pi/2$ and define the 'speed of light' $v_{s} = 2t_{AB} a$. In the continuum limit and near the Dirac points, $p_{s}(K + \delta k) \sim 2t_{AB} a \delta k = v_{s} q_{x}$ and $p_{s}(K' + \delta k) \sim -2t_{AB} a \delta k = -v_{s} q_{x}$; $p_{s}(K + \delta k) \sim 2t_{AB} a \delta k \equiv v_{s} q_{x}$ and $p_{s}(K' + \delta k) \sim -2t_{AB} a \delta k \equiv v_{s} q_{x}$; and $h_{2}(K + \delta k) = -M - t_{0} = m_{0}$ and $h_{2}(K' + \delta k) = -M - t_{0} = m_{\pi}$. Thus, for the Dirac fields $\xi(x)$ near $K$ and $\zeta(x)$ near $K'$, the effective Hamiltonian reads

$$H(k) = p_{x}(k) \sigma_{x} + p_{y}(k) \sigma_{y} + h_{z}(k) \sigma_{z},$$

where

$$p_{x} = 2t_{AB} \sin(\theta_{0} \cos(k_{y} a)), \quad p_{y} = 2t_{AB} (\sin(k_{x} a) + \cos(\theta_{0} \sin(k_{y} a))) \text{ and } h_{z} = -M - t_{0} \cos(k_{x} a) \cos(k_{y} a) - 2t \sin(k_{x} a) \sin(k_{y} a) \text{ with } t_{0} = t_{A1} - t_{B1} + t_{A2} - t_{B2} \text{ and } t = (t_{A1} - t_{B1} + t_{B2} - t_{A2})/2.$$

We denote $\xi^{(0)}(x)$ the Dirac fermions in the open channel and $\xi^{(c)}(x)$ the Dirac fermion in the closed channel.

**Two-channel Model.** We take $m_{0} = 0$ and $m_{\pi} \neq 0$, and integrate out $\zeta$ in the low-energy limit where only the states with their energy lower than $\min\{m_{\pi}, E_{A}\}$ are relevant. We now extend our Hamiltonian to a two-channel model, i.e., the lowest two hyperfine atom states with two-atom scattering states in open channel and the two-atom bound state (Feshbach molecule) in closed channel. We denote $\xi^{(o)}(x)$ the Dirac fermions in the open channel and $\xi^{(c)}(x)$ the Dirac fermion in the closed channel. Analogous to the many body theory of the atom-molecule coherence in Ref. \textsuperscript{[18]}, the effective Lagrangian describing this two-channel Dirac fermion model is given by

$$\mathcal{L} = -\xi^{(o)\dagger} \sigma^{\mu} \partial_{\mu} \xi^{(o)} - \xi^{(c)\dagger} \sigma^{\mu} \partial_{\mu} \xi^{(c)} + U(c) \xi^{(o)}(1) \xi^{(c)}(2) \xi^{(c)}(3) + U^{(c)} \xi^{(o)}(1) \xi^{(c)}(2) \xi^{(c)}(3) + \text{h.c.},$$

where $\sigma^{\mu} = (1, \sigma_{x}, \sigma_{y})$ and $\partial_{\mu} = (\partial_{x}, v_{s} \nabla)$. $U^{(c)}$ and $U^{(co)}$ are the interaction between closed channel fermions and the interchannel interaction, respectively. We have neglected the background interaction in open channel. By introducing the pairing field $\Delta(r, t)$ for $\xi^{(c)}$ via a Hubbard-Stratonovich transformation and integrating out $\xi^{(c)}$, the resulting Lagrangian is given by

$$\mathcal{L}(\xi^{(o)}, \Delta) = -\frac{1}{2} \xi^{(o)\dagger} \sigma^{\mu} \partial_{\mu} \xi^{(o)} - \frac{|\Delta|^{2}}{U^{(c)}} + \text{Tr} \ln G^{(c)-1}.$$  

FIG. 1: (a) Josephson tunneling between the atom-molecule mixture (lower lattice plane) and the dimolecule Bose-Einstein condensate nearby (upper plane). The orange dots are molecules in the mixture and red dots are dimolecules. Fermionic atoms in the lattice sites. (b) The square lattice where 1, 2, 3, 4 denote the next nearest neighbor sites.
The inverse of the propagator $G^{(-)}$ of $\xi^{(c)}$ is given by

$$G^{(-)} = \left( \begin{array}{cc} 0 & i\sigma^{\mu} \partial_{\mu} \\
-i\sigma^{\mu} \partial_{\mu} & 0 \end{array} \right) + \left( \begin{array}{cc} 0 & 0 \\
0 & 0 \end{array} \right) \Xi \Xi^\dagger,$$

where $\Xi = \Delta + U^{(c)} \xi_1 \xi_2$. Expanding the Lagrangian in powers of $\Delta$ and its gradients yields

$$\mathcal{L}[\phi] = -\frac{1}{2} \partial_{\mu} \phi^{\dagger} \partial^{\mu} \phi - \frac{1}{2} \varepsilon_{m} |\phi|^{2} - \frac{\lambda}{8} |\phi|^{4} + O(|\phi|^{6}),$$

(4)

where $\phi \propto \Delta / U^{(c)}$ is the Feshbach molecular field with the detuning energy $\varepsilon_{m}$ and the interacting strength $\lambda \propto (U^{(c)})^{2}$. We have $v_{b} = v_{s}$ in the weak coupling limit (i.e., $U^{(c,co)}$ much smaller all other energy scales in the system including $m_{s}$ and $E_{q}$) due to (emergent) Lorentz invariance. Lattice effects (which break Lorentz invariance) give rise to nonuniversal corrections to $v_{b}$ and $v_{s}$; thus tuning of one parameter (e.g., molecule dispersion through an additional lattice potential seen by the molecule only) is needed to ensure $v_{b} = v_{s}$ to maintain Lorentz invariance in the low-energy limit. Also included in $\mathcal{L}$ is the Yukawa coupling between $\phi$ and $\xi^{(c)}$, i.e.,

$$\mathcal{L}_{\phi,\xi} = -\frac{g}{2}(\phi \xi^{(c)}_{1} \xi_{1} + \phi^{\dagger} \xi^{(c)}_{2} \xi_{2}) \quad \text{with} \quad g \propto -2U^{(c)}.$$

WZ-SUSY Model: Massless. – For simplicity, we drop the superscript of $\xi^{(c)}$ hereafter. By combining $\mathcal{L}$, $\mathcal{L}_{\phi,\xi}$ and the Yukawa coupling together, the effective Lagrangian after neglecting $O(|\phi|^{6})$ is given by

$$\mathcal{L}(\xi, \phi) = -\frac{1}{2} \partial_{\mu} \phi^{\dagger} \partial^{\mu} \phi - \frac{1}{2} \varepsilon_{m} |\phi|^{2} - i\xi^{(c)} \sigma^{\mu} \partial_{\mu} \xi - \frac{\lambda}{8} |\phi|^{4} - \frac{g}{2}(\phi \xi^{(c)}_{2} \xi_{1} + \phi^{\dagger} \xi^{(c)}_{1} \xi_{2}).$$

(5)

Tuning $\varepsilon_{m} = 0$ by varying $U^{(c)}$, and further tuning pair-pair (or molecule-molecule) interaction by varying $U^{(c,co)}$ so that the coupling constant $\lambda = g^{2}$, the effective Lagrangian $\mathcal{L}(\xi, \phi)$ is exactly the massless WZ-SUSY model with the SUSY under the WZ-SUSY transformations $\delta \phi = e^{i\sigma^{\mu} \xi} \partial_{\mu} \phi$ and $\delta \xi = \sigma^{\mu} \sigma^{\nu} \partial_{\mu} \phi^{\dagger} - \frac{g}{2} \phi^{2} \xi$ where $\epsilon$ is a constant two-component spinor parameter.

WZ-SUSY Model: Massive. – To have a massive WZ-SUSY model, we need to introduce an external source. This can be realized by putting a Bose-Einstein condensate of dimolecules nearby, which is made of pairs of molecules (or 4-atom molecules) (see Fig. 1(a)). Through Josephson tunneling with an amplitude $\kappa$, the dimolecule condensate exchanges pairs of molecules with the mixture. The effective Lagrangian reads

$$\mathcal{L}(\xi, \phi, \Psi) = -\frac{1}{2} \partial_{\mu} \phi^{\dagger} \partial^{\mu} \phi - i\xi^{(c)} \sigma^{\mu} \partial_{\mu} \xi - \frac{g}{2} |\phi|^{4} - \frac{g}{2}(\phi \xi^{(c)}_{2} \xi_{1} + \phi^{\dagger} \xi^{(c)}_{1} \xi_{2}) + \kappa(\Psi^{\dagger} \phi^{2} + \Psi \phi^{2} \xi^{2}),$$

(6)

where $\Psi$ is the external dimolecular field. There is a global U(1) symmetry (called R-symmetry) under $\xi \rightarrow e^{i\theta} \xi, \phi \rightarrow e^{2i\theta} \phi$ and $\Psi \rightarrow e^{i\theta} \Psi$, if $\Psi$ slowly varies in space-time, it is also SUSY invariant under $\delta \phi = e^{i\sigma^{\mu} \xi} \partial_{\mu} \phi$ and $\delta \xi = \sigma^{\mu} \sigma^{\nu} \partial_{\mu} \phi^{\dagger} - \frac{g}{2} \phi^{2} \xi$ and by taking $\Psi$ to be its condensed order parameter $\langle \Psi \rangle = (\Psi^\dagger) = m^{2}/8\epsilon$. The R-symmetry is broken and reduced to a discrete $\mathbb{Z}_{2}$ symmetry with $\xi \rightarrow i\xi$ and $\phi \rightarrow -\phi$, and the on-shell WZ Lagrangian appears (up to an additive constant)

$$\mathcal{L}(\xi, \phi, m) = -\frac{1}{2} \partial_{\mu} \phi^{\dagger} \partial^{\mu} \phi - i\xi \sigma^{\mu} \partial_{\mu} \xi - \frac{g^{2}}{8}(\phi^{2} - m^{2}/g^{2} )(\phi^{2} - m^{2}/g^{2}) \Xi 0$$

(7)

The SUSY is exact by replacing $\Psi$ with $\langle \Psi \rangle$ in the SUSY transformations. The $\mathbb{Z}_{2}$ symmetry is always spontaneously broken in one of the degenerate ground states with $\phi = \phi \pm m/g$. The SUSY Lagrangian with spontaneous breaking of $\mathbb{Z}_{2}$ becomes

$$\mathcal{L} = -\frac{1}{2} \partial_{\mu} \phi^{\dagger} \partial^{\mu} \phi - i\xi \sigma^{\mu} \partial_{\mu} \xi - \frac{1}{2} m(\xi_{1}^{\dagger} \xi_{1} + \xi_{2}^{\dagger} \xi_{2}) - \frac{g^{2}}{8} |\phi|^{4} - \frac{m^{2}}{4} |\phi|^{2} + \frac{2m\phi^{\dagger} \phi}{8}(\phi^{2} + \phi^{\dagger} \phi) - \frac{g}{2}(\phi \xi_{2}^{\dagger} \xi_{1} + \phi^{\dagger} \xi_{1} \xi_{2})$$

(8)

This is the 2+1-dimensional reduction of the original WZ-SUSY model in 3+1 dimensions. The SUSY transformation generated by $Q$ for a field $O$ reads $\delta O = -i\xi \sigma^{\mu} \partial_{\mu} \phi + i\phi \sigma^{\mu} \partial_{\mu} \phi^{\dagger}$. We focus on the on-shell model with the $\mathbb{Z}_{2}$ symmetry spontaneously broken, where the on-shell supercurrent $J_{\mu}^{\phi}$ is given by $J_{\mu}^{\phi} = i\sigma^{\mu} \sigma^{\nu} \partial_{\mu} \phi^{\dagger} + i\phi \sigma^{\mu} \partial_{\mu} \phi^{\dagger}$.

Nonrenormalization. – In this simplest WZ-SUSY model, single-particle Green’s functions are not renormalized due to (unbroken) SUSY. For example, the renormalization to Klein-Gordon field’s propagator in a one-loop self-energy calculation is given by $g^{2} - m^{2} \rightarrow g^{2} - m^{2}(q)$ with $m(q) = m + g_{R}(A)_{0} + O(g^{2}_{R}(A)_{0}^{2})$ where $A = Re \phi$. For 2+1 dimensions, due to the nonzero anomalous critical exponents, the coupling constant may be renormalized to $g_{R}$. However, $(A)_{0} \propto \langle \{ Q, \xi \} \rangle = 0$ because the SUSY is not spontaneously broken. The mass of the Dirac field is also not renormalized as required by SUSY. Therefore, the single-particle Green’s functions, both of the Dirac and Klein-Gordon fields, are not renormalized from their free version. The spectral functions of the Green’s functions can
be measured by the single-particle spectroscopic technique which has been developed recently [13]. The non-renormalization of the Green’s functions implies sharp peaks in their spectral functions, with identical relativistic dispersions for the atoms and molecules. Experimentally, this would be the hallmark of achieving SUSY.

**Thermal breaking of SUSY.** – By replacing $t$ by $i\tau$, the imaginary time, the Euclidean version of Lagrangian \[ \mathcal{L} \] describes WZ-SUSY model in finite temperature $T$. When $T \neq 0$, SUSY is always broken because $\langle \{ Q, Q^{\dagger} \sigma^a \} \rangle = \langle \{ \sigma^a P_{\mu} \} \rangle \neq 0$ with $P_{\mu}$ being the energy-momentum operator [14], due to the nonvanishing thermal energy. This SUSY thermal breaking is accompanied by a thermal Goldstone fermion (photonino) but not necessarily by a phonon because the Lorentz symmetry is also broken by $\langle P_0 \rangle_T \neq 0$ [12]. The photonino dispersion is given by $\omega_0 = \pm v_m |q|$ where the SUSY sound velocity $v_s = v_m / 3$ for $T \gg m$ and $v_s = T v_m / m$ for $T \ll m$.

To detect the photonino mode, one can consider the response to an external ‘fermionic’ field coupled to the supercurrent. The photonino is a pole of the supercurrent-supercurrent correlation function. This external ‘fermionic’ field can be a combination of an external photon with another hyperfine state of the fermionic atom which is decoupled to the mixture. We have studied this kind of SUSY response theory for a nonrelativistic SUSY mixture [3]. However, the difficulty in the present case is that the supercurrent is not so simple as that in the nonrelativistic theory, and thus the coupling between the external ‘fermionic’ field and the supercurrent is not that easy to be experimentally handled.

Replacing $\langle A \rangle_0$ by $\langle A \rangle_T$, the masses are thermally renormalized. The masses of $A$, $B$ ($\phi = A + iB$) and the spinor $\xi$ have been calculated in low temperature and high temperature limits [12]. Namely, in 2+1-dimensions up to one-loop, for $T \gg m$, one has $m_A = m$, $m_B^2 - m_B^2 \propto g^2 m_{\phi} \alpha \propto g^2 m_{\phi} \xi - m_A \propto g^2 m_{\phi} \alpha$, where $\alpha = 2 T / x_m \propto m / T$, for $T \gg m$, $m_B = m$, $m_A^2 = m^2 - 2 g^2 T$, $m_B^2 = m^2 + g^2 T$.

These unequal masses of these fields signal SUSY breaking, and can be probed quantitatively. In particular, we spectroscopy measurements to show double peaks in the molecule spectral function due to the unequal masses between $A$ and $B$ components, while the atom spectral function has a single peak with a mass of the Dirac field equal to neither $m_A$ nor $m_B$.

**Experimental Challenges.** – Optical lattices that trap cold atoms can be routinely set up in laboratory. The staggered Peierls phase originates from production of the artificial magnetic field [16]. As discussed earlier, one needs to tune three parameters to achieve SUSY capabilities. Perhaps the biggest challenge is finding the right fermionic atom, which needs to have a highly tunable interaction through a $p$-wave Feshbach resonance. It also needs to support a sufficiently stable dimolecule (or 4-atom bound) state, whose condensates provides the source term in Eq. \[ \langle \Omega \rangle \], which gives rise to equal particle masses. Experimentally, one needs to overcome the atom loss due to the heating of the atom gas caused by three- and four-body collisions. Without the last ingredient however, one can still realize the massless version of the WZ-SUSY model, Eq. \[ \langle \Omega \rangle \] (with $\epsilon_m$ tuned to zero), which already contains very rich SUSY physics. Despite these and other challenges, we believe simulating the WZ-SUSY model using cold atom-molecule mixtures is a worthwhile endeavor, as as generalizing the present model to a (3+1)-dimensional model is straightforward, and then it provides new opportunities to explore the real space-time SUSY physics.

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