The Photino Induced Distortion of the CMBR Blackbody Spectrum

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It is shown that photon–photino inter-conversions (if exist) may result in a detectable CMBR spectrum distortion which amplitude depends on photino properties, such as its mass. An upper bound on the distortion parameter determined from the recent COBE-FIRAS data, hence, sets a lower bound on the photino mass, $m_\gamma \gtrsim 300$ eV.

98.70.Vc, 98.80.Cq, 14.80.Ly

The elegant and generally accepted theory of supersymmetry in particle physics (for astrophysical implications, see, e.g., [1]) which unifies bosons and fermions remains neither confirmed nor rejected experimentally so far. This theory predicts the existence of new particles, the super-symmetric partners for all known particles, neither of which has been detected yet. In this Letter, we show that, under particular conditions, the photino $\tilde{\gamma}$ (the $\frac{1}{2}$-spin fermionic super-symmetric partner of the photon $\gamma$), if exists, may result in a measurable distortion of the blackbody Cosmic Microwave Background Radiation (CMBR) energy spectrum. The magnitude of the deviation from the Planckian spectrum is directly related to the value of the photino mass, $m_\gamma \neq 0$. We then use recent results [2] from the COsmic Background Explorer (COBE) mission to put some constraints on the value of $m_\gamma$. Recent evidence for the time variability of physical constants (e.g., the fine structure constant) [3] makes the measurement of the ‘photino induced’ CMBR spectrum distortion to be of great importance. Indeed, the CMBR provides information on physical laws existed in the early Universe (at redshifts $z \sim 10^3 - 10^8$) which cannot be obtained by means of other astronomical observations [4]. Thus, such an effect, if detected, would have profound implications for our understanding of fundamental physics.

Photons during the radiation dominated era ($10^2 \lesssim z \lesssim 10^8$) are in thermal and ionization equilibrium with matter. However, the process of photon energy equilibration throughout the spectrum is quite slow, because it proceeds via the (inverse) Compton scattering with electrons [3], which number density is small with respect to the number density of photons, $n_e/n_\gamma \sim 10^{-9}$. In fact, the rate of Comptonization of photons becomes comparable to the Universe expansion rate at redshift $z_c \sim 2 \times 10^5 \Omega_0^{-1/2} \sim 10^5$ (where $\Omega_0 = \rho_0/\rho_{crit}$ is the ratio of present density of matter in the Universe to the critical density). Thus, the characteristic time, $\tau_{\gamma e}$, required to establish the (spectral) equilibrium in the photon gas diverges as $z$ approaches $z_c$.

On the other hand, in the super-symmetry theory, the photon and photino are different “spin states” (with $s = 1$ and $s = \frac{1}{2}$, respectively) of a particle and have different interchange properties. The photon is known to be a massless particle, and the photino is believed to be massive. Thus, if a corresponding mixing angle $\theta$ of these two ‘states’ is not identically zero, one may expect the $\gamma \leftrightarrow \tilde{\gamma}$ oscillations to occur, with the probability $P^{osc}_{\gamma \rightarrow \tilde{\gamma}}(t) = \frac{1}{2} \sin^2 2\theta [1 - \cos ((E_\tilde{\gamma} - E_\gamma)h^{-1}t)]$, where the energy difference is $E_\tilde{\gamma} - E_\gamma \sim m_\gamma c^2$. Such a simple picture, however, is not correct, because the oscillations of a single particle are forbidden due to angular momentum and R-parity conservation. Thus, the $\gamma \leftrightarrow \tilde{\gamma}$ inter-conversions are allowed to occur only in reactions (collisions) with other particles. (We postpone the discussion of probable reaction channels intill the end of the Letter.) Consequently, the average rate $\nu_{\gamma\tilde{\gamma}}$ of $\gamma \leftrightarrow \tilde{\gamma}$ inter-conversions is, thus, the average collision frequency, $\langle \sigma v \rangle n$, times the probability of a $\gamma \leftrightarrow \tilde{\gamma}$ oscillation during collision time, $P^{osc}_{\gamma \rightarrow \tilde{\gamma}}(\Delta t_{coll})$, (note, $P^{osc}_{\gamma \rightarrow \tilde{\gamma}} = P^{osc}_{\tilde{\gamma} \rightarrow \gamma}$, by symmetry). It should be emphasized that $\gamma \leftrightarrow \tilde{\gamma}$ inter-conversions are incoherent in this regime, i.e., the quantum coherence of photon-photino oscillations is completely lost at times greater than $\nu_{\gamma\tilde{\gamma}}^{-1}$ due to classical, intrinsically chaotic motion of interacting particles. Photinos are, hence, in (at least, marginal) equilibrium with photons. The equilibrium fraction of $\tilde{\gamma}$’s in a system is proportional to the forward–to–reverse transition rate ratio and, hence, is independent of the mixing angle. The rate with which equilibrium is established is, however, affected by $\theta$. We now introduce the total characteristic time of a $\gamma \leftrightarrow \tilde{\gamma}$ transition as follows, $\tau_{\gamma\tilde{\gamma}} \simeq \tau_{\gamma\rightarrow\tilde{\gamma}} + \tau_{\tilde{\gamma}\rightarrow\gamma}$. The numbers of photons and photinos in the system, thus, fluctuate on the time-scale $\tau_{\gamma\tilde{\gamma}}$, while their total number is preserved.

Let us consider two limiting cases. First, the spectral equilibration time is much smaller than the typical $\gamma \leftrightarrow \tilde{\gamma}$ fluctuation time, $\tau_{eq} \ll \tau_{\gamma\tilde{\gamma}}$. This is a general case, because super-symmetric particles are believed to be weakly interacting (otherwise they would be detected). In this case, the number of photons in the system fluctuates adiabatically slowly and CMBR photons have enough time to re-arrange throughout the spectrum. Hence, the Planckian (or Bose-Einstein [3]) spectrum is maintained. Second, if the spectral thermalization process is (externally) inhibited, e.g., by the Universe expansion, the opposite case may realize. Then

$$\tau_{\gamma\tilde{\gamma}} \ll \tau_{eq}, \quad \tau_{\gamma\tilde{\gamma}} \lesssim \tau_{\text{expan}} \sim (3H)^{-1},$$

(1a) (1b)
(where $H$ is the instantaneous Hubble constant) and
\[ \gamma \leftrightarrow \tilde{\gamma} \] transitions are faster than the photon transit time though the spectrum. Thus, different, ‘mean-field’ equilibrium, which is, in general, a weighted sum over all realizations, is created; rather than the ‘instantaneous’ Planckian equilibrium. [Note, inequality (11) ensures that $\gamma \leftrightarrow \tilde{\gamma}$ inter-conversions are fast enough, compared to the expansion rate]. The partition function and occupation numbers for a gas of particles with fluctuating quantum statistics were calculated elsewhere [6].

Since the CMBR spectrum is very close to Planckian with temperature $T_0 \approx 2.7$ K, we look for a small deviation only. Then, two free parameters, $p_f$ and $p_b$ (the boson and fermion probabilities, see Ref. [8] for details), become $p_b = 1 - p_f$ and $p_f \ll 1$. Thus, we may expand the occupation number $n$ in terms of $p_f$. From Eqs. (11,12) of Ref. [5], we straightforwardly obtain the effective bosonic and fermionic contributions ($n_b + n_f = n$):
\[ n_{\gamma} \equiv n_b = \frac{1}{e^{x}} \left( 1 + \sqrt{p_f} \sqrt{\frac{e^x}{e^{x} - 1}} \right) , \]  \hspace{1cm} (2a)
\[ n_{\tilde{\gamma}} \equiv n_f = \sqrt{p_f} \frac{1}{e^{x}(e^{x} - 1)} , \]  \hspace{1cm} (2b)
where $x = \epsilon/kT$ is the dimensionless energy. Thus, the deviation of the photon distribution from the Planckian one is
\[ \delta n_{\gamma} = n_{\gamma} - n_P = \sqrt{p_f} \sqrt{\frac{e^x}{(e^x - 1)^3}} . \]  \hspace{1cm} (3)

The parameter $p_f$ is related to the photino mass, $m_{\tilde{\gamma}}$, as follows. Let us notice that condition (11) states that photinos are in equilibrium with photons: $\partial_B n_\gamma = P_0 \to f n_\gamma - P_f \to 0 n_\tilde{\gamma} = 0$. The probability to change a ‘state’ from boson to fermi, $P_0 \to f$, is complimentary to that to stay in a bose ‘state’, $P_b$. Hence $P_b \to f = 1 - P_b \equiv p_f$, and we obtain:
\[ p_f \approx \frac{P_f}{p_b} = \frac{n_\tilde{\gamma}}{n_\gamma} . \]  \hspace{1cm} (4)

The number densities of photons and photinos are calculated integrating Eqs. (3) over energies. Photons are relativistic ($\epsilon = pc$) and the degeneracy $g_\gamma = 2$ (due to two polarizations); photinos are assumed non-relativistic ($m_{\tilde{\gamma}} c^2 \gg kT_\gamma$) and $g_{\tilde{\gamma}} = (2s + 1) = 2$. Finally, we obtain:
\[ \sqrt{p_f} \approx \frac{1}{\zeta(3)} \sqrt{\frac{\pi}{8}} \left( \frac{m_{\tilde{\gamma}} c^2}{kT_c} \right)^{3/2} e^{-m_{\tilde{\gamma}} c^2/kT_c} , \]  \hspace{1cm} (5)
where $\zeta$ is the Riemann $\zeta$-function, $\zeta(3) \approx 1.2$ and $T_c$ is the temperature at redshift $z_c$, $T_c \approx T_0 z_c \approx 2.7 \times 10^5$ K, to satisfy condition (13).

The CMBR spectrum has been measured with great accuracy by the FIRAS (Far-InfraRed Absolute Spectrophotometer) on board of the COBE satellite. For our analysis, we use the CMBR residuals (after subtraction of the Planck blackbody and galactic emission spectra) of both monopole and dipole components of Ref. [8] (referred to as the ‘96-monopole’ and ‘96-dipole’ data sets, respectively) and, for comparison, some older data of Ref. [9] (‘94-monopole’ data set) and Ref. [9] (‘94-dipole’ data set). Note, if a dipole component is associated with the Earth motion only (with respect to the CMBR), both monopole and dipole spectra should be identical. In general, a dipole spectrum is less sensitive to systematic errors (e.g., absolute calibration), but statistical errors may be higher, because a dipole component is of much smaller amplitude than monopole.

Since deviations from the Planck spectrum are small, a linear fit can be performed:
\[ I_{\text{residual}}(\nu) = \Delta T \frac{\partial B_0}{\partial T} + \sum_{\text{models}(\alpha)} \alpha \frac{\partial S_\alpha(\nu)}{\partial \alpha} . \]  \hspace{1cm} (6)
Here $B_0$ is the Planck blackbody spectrum, $S_\alpha$ represents a spectral model, and $\alpha$ is a distortion parameter which quantify the deviation from $B_0(T_0)$. It is important to have the first term since $\partial B_0/\partial T$ correlates with other models $\partial S_\alpha/\partial \alpha$. We consider three spectral models. First, the ‘photino induced’ distortion (referred to as the $\sqrt{p_f}$-model):
\[ \frac{\partial S_\sqrt{p_f}}{\partial \sqrt{p_f}} = 2 \hbar c^2 \nu^3 \sqrt{\frac{\epsilon_{\nu_0}}{(\epsilon_{\nu_0} - 1)^3}} , \]  \hspace{1cm} (7)
which readily follows from Eq. (3), where $x_0 = hcv/kT_0$ and $\nu$ is measured in cm$^{-1}$. The distortion parameter is $\sqrt{p_f}$. Second, the Bose-Einstein quasi-equilibrium photino distribution [5] with dimensionless chemical potential $\mu$ ($\mu$-model). Third, the Comptonized spectrum characterized by the Kompaneets [14] parameter $y$ ($y$-model). (For details regarding the last two models, see, e.g., Ref. [6].) Note that all three distortion parameters, $\sqrt{p_f}$, $\mu$, and $y$, must be positive by physical meaning. The normalized spectrum distortions and the effect of a temperature shift $\partial B_0/\partial T$ are shown in Fig. 4. Note very strong similarity of the $\sqrt{p_f}$- and $\mu$-models, which precludes their simultaneous fit. However, preference to one of the models may be given, due to their opposite contributions to the CMBR spectrum.

Fits (6) have been performed with the help of MATHEMATICA 3.01 standard package for Linear Regression analysis, which uses the weighted least square $[\sum_{\nu} \left( \Delta_\nu / \sigma_\nu \right)^2 / \sum_{\nu} (1/\sigma_\nu)^2]$ method, where $\Delta_\nu$ is the difference of the model and the data at frequency $\nu$, and $\sigma_\nu$ is the uncertainty. Test fits of the $y$-model and the $\mu$-model separately to the both 96- and 94-monopole spectra yield values of $y$ and $\mu$ consistent with those of Refs. [8],[9]. In the dipole fits, $y$ and $\mu$ tend to negative (unphysical) values, so do their simultaneous fits, i.e., the $(\mu + y)$-model, for both monopole and dipole components. All
the fits, which include the both \( \mu \)- and \( \sqrt{p_f} \)-models, yield large statistical errors resulting from very strong correlation of these models, so they have been rejected. Thus, there are only two statistically reliable models, namely the \( \sqrt{p_f} \)-model and the \( \sqrt{p_f} + y \)-model. The results with \( 1\sigma \) uncertainties (95\% confidence level) are presented in Table I. There is an obvious inconsistency between the 96-monopole and 96-dipole fits for both models. Since the results for the 96-dipole and 94-monopole spectra calculated in the \( \sqrt{p_f} \)-model are also inconsistent with those in the \( \sqrt{p_f} + y \)-model (even within 3\( \sigma \) uncertainty), we consider them as statistically unreliable. What causes such anomalously large correlations of residuals (especially in the 96-dipole data set) is not known. This issue requires further investigation. (One of possible ‘candidates’ is residual galactic contamination, which is strongly coupled to the \( y \)-model.) Other fits, i.e., for the 94-monopole and 94-dipole data, with both models yield results which are consistent with each other and include the ‘null’ result (\( \sqrt{p_f} = y = 0 \)) within \( \sim 1.4\sigma \) uncertainty. Weak trend toward positive values may, however, be discerned. Using the values with smaller statistical uncertainties, we set the following conservative upper limit on the parameter \( \sqrt{p_f} \):

\[
\sqrt{p_f} \lesssim 10^{-4} . \tag{8}
\]

From Eq. (3), a lower bound on the photino mass is

\[
m_{\tilde{\gamma}} \gtrsim 300 \text{ eV}/c^2 , \tag{9}
\]

which is much lower than generally accepted: \( m_{\tilde{\gamma}} \sim 0.1 - 10 \text{ GeV}/c^2 \). Note, however, that the ‘null’ result cannot yield absolutely reliable lower bound on \( m_{\tilde{\gamma}} \), because such a result can also be due to breaking of condition (11), which accounts for \( \gamma \leftrightarrow \tilde{\gamma} \) kinetics.

Now we check the consistency of our result with conditions (1). In fact, we should check (11) only, because (14) is satisfied at \( z \sim z_c \) by definition. Since binary collisions are most frequent, the leading processes are: (i) the binary photino collisions, \( \gamma \gamma \rightarrow \tilde{\gamma} \tilde{\gamma} \); (ii) the R-parity ‘exchange’, \( \gamma \tilde{x} \rightarrow \tilde{\gamma} x \), where \( x \) is any particle (possibly an electron, \( e \), as the most abundant) and ‘tilde’ denotes its super-symmetric partner; and (iii) the decay-type processes, \( \tilde{\gamma} \rightarrow \tilde{x}_i \), \( \tilde{x}_i \) may be an axino, \( \tilde{a} \) (the super-symmetric partner of the axion), or another hypothetical particle. We estimate cross sections of these processes and show that only the last reaction cannot be completely ruled out. Of course, the reaction channels that could influence the \( \gamma \rightarrow \tilde{\gamma} \) transitions are beyond the scope of this Letter, we restrict ourselves considering the transition processes mentioned above. Since the results below are illustrative, we assume, for simplicity, that \( \sin^2 \theta \sim 1 \) and \( (E_\gamma - E_\nu) \sim m_{\tilde{\gamma}} c^2 > \hbar \Delta t_{\text{coll}}^{-1} \), so that the fast oscillating cosine term vanishes upon time and ensemble averaging. However, the mixing angle and oscillation time contributions may be trivially recovered when needed.

First, let us consider \( \gamma \gamma \rightarrow \tilde{\gamma} \tilde{\gamma} \) process. In equilibrium, according to (11), \( \langle \sigma v \rangle_{\gamma \gamma} n_\gamma^2 = \langle \sigma v \rangle_{\tilde{\gamma} \tilde{\gamma}} n_{\tilde{\gamma}}^2 \), where \( \sigma \) is the effective cross-section of a process. The characteristic times are \( \tau_{\gamma \gamma} \sim \langle \sigma v \rangle_{\gamma \gamma} n_\gamma \), and \( \tau_{\tilde{\gamma} \tilde{\gamma}} \sim \langle \sigma v \rangle_{\tilde{\gamma} \tilde{\gamma}} n_{\tilde{\gamma}} \), and \( 3H \) may be replaced [due to condition (11)] with the Compton scattering inverse rate \( \tau_{\gamma \gamma}^{-1} \sim \langle \sigma_{\gamma\gamma} v_{\gamma\gamma} \rangle n_\gamma \), etc. Since complete transitions may be trivially recovered when needed.

Hence, the \( \tilde{\gamma} \tilde{\gamma} \) cross-section is too large compared to that expected from particle physics experiments. [Such a cross-section corresponds to the photino mass (using (11)) \( m_{\tilde{\gamma}} \lesssim 0.5 \text{ eV}/c^2 \), which contradicts to (9).] Thus, such a process is (probably) ruled out. Second, the same is true for \( \gamma \tilde{x} \rightarrow \tilde{\gamma} x \) process. Indeed, the cross-sections (estimated for electrons, \( x \equiv e \)) are

\[
\frac{\langle \sigma v \rangle_{\gamma \tilde{e}}}{\langle \sigma_{\gamma \tilde{e}} v_{\gamma \tilde{e}} \rangle} \sim \frac{n_{\tilde{\gamma}}}{n_\gamma} \sim 10^{-9} , \tag{10}
\]

which are also too large. Third, the situation is better for \( \gamma \rightarrow \tilde{\epsilon} \) process. Denoting \( \Gamma_{\tilde{\epsilon}} \) to be the \( \tilde{\epsilon} \)-decay rate, we obtain:

\[
\frac{\langle \sigma v \rangle_{\gamma \tilde{\epsilon}}}{\langle \sigma_{\gamma \tilde{\epsilon}} v_{\gamma \tilde{\epsilon}} \rangle} \sim \frac{n_{\tilde{\gamma}}}{n_\gamma} \sim 10^8 , \tag{11}
\]
\[ \frac{\Gamma_{\text{decay}}}{p_f} \gtrsim 3H_c \sim 10^{-2} \text{sec}^{-1}, \quad \frac{\langle \sigma v \rangle_{\tilde{\alpha} \gamma}}{\langle \sigma v \rangle_{\tilde{\gamma} \tilde{\gamma}}} \gtrsim \frac{n_e}{n_{\tilde{\alpha}}}, \quad (12) \]

where \( H_c = H(z_c) \). Since the axino is a possible non-baryonic dark matter candidate, the ratio \( n_e/n_{\tilde{\alpha}} \) may be quite small, too. Estimated from the \( \tilde{\gamma} \)-decay rate [12], the photino mass is \( m_{\tilde{\gamma}} \gtrsim 3 \text{ MeV/c}^2 \) (for the Peccei-Quinn symmetry breaking scale \( V_{PQ} = 10^9 \text{ GeV} \)). Since \( m_{\tilde{\gamma}} c^2 \gg kT_c \), fine-tuning \( m_{\tilde{\gamma}} \simeq m_{\tilde{\alpha}} \) is required; otherwise the inverse decay \( \tilde{\alpha} \gamma \rightarrow \tilde{\gamma} \) is strongly suppressed. Assuming \( m_{\tilde{\gamma}} \sim m_{\tilde{\alpha}} \sim 3 \text{ MeV/c}^2 \), we may estimate the cosmological abundance of axinos (for the \( \Omega_{\text{tot}} = 1 \) Universe) to be \( n_{\tilde{\alpha}}/n_e \sim \Omega_{\text{barion}}/m_{\tilde{\alpha}} \lesssim 10^5 \), which yields a reasonable \( \tilde{\alpha} \gamma \) cross-section.

It is important to note that the restrictions imposed by condition (1b) may, however, be significantly relaxed by admitting (i) multi-channel \( \gamma \leftrightarrow \tilde{\gamma} \) inter-conversions (e.g., \( \gamma \rightarrow \tilde{\gamma} \) and \( \tilde{\gamma} \rightarrow \gamma \) conversions may proceed through different processes), (ii) super-symmetry theories beyond the standard model, or (iii) theories with R-parity violation. Detailed study of these issues is, obviously, beyond the scope of this Letter.

To conclude, we have shown that, under appropriate conditions, the \( \gamma \leftrightarrow \tilde{\gamma} \) inter-conversions may result in a detectable distortion of the CMBR blackbody energy spectrum. Since the ‘bare’ \( \gamma \leftrightarrow \tilde{\gamma} \) process is forbidden by angular momentum and R-parity conservation, kinetics of super-symmetric particle–matter interactions plays an important role. Thus, the amplitude of the CMBR spectrum distortion is related to photino properties, e.g., its mass, \( m_{\tilde{\gamma}} \). Recent data from the COBE-mission are used to evaluate the distortion parameter \( \sqrt{p_f} \) and photino mass.

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