Effect of partial wall motion on MHD mixed convection heat transfer undergoing in a porous cavity filled with Cu–water nanofluid with a centrally mounted heat source

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Abstract: The Magneto-hydrodynamics (MHD) convective heat transfer process in a partially driven cavity (PDC) containing a centrally mounted heat source is focused in the present study. The cavity is filled with Cu–water nanofluid saturated porous medium. Heated cavity fluid is cooled through the upper half of the two sidewalls. These cold sidewalls are allowed to move in upward direction at the same velocity. The horizontal walls and lower part of the sidewalls are insulated. The cavity subjected to an externally imposed uniform magnetic field. The flow structures and mixed convection heat transfer characteristics are analyzed numerically utilizing developed CFD code adopting FVM and SIMPLE algorithm. The thermo-fluid phenomena are analyzed under a range of governing parameters like Richardson number (Ri), Hartmann number (Ha), concentrations of nanoparticles (ϕ). The results reflect that the heat transfer characteristics of the PDC are greatly influenced by all of the above parameters. Heat transfer is enhanced significantly at the higher Ri and higher volume fraction.

Keywords: Partially driven cavity (PDC); Magneto-hydrodynamics (MHD); Cu-water nanofluid; Heat transfer enhancement.

1. Introduction
Because of enhanced thermal conductivity, the use of nanofluids as a working medium over the conventional working fluid is a recent technique for applying in various thermal systems. Furthermore, nanofluid flow through a porous substance can also be found in numerous applications [1,2]. Thermo-fluid flow behaviour of nanofluid flow through a porous medium becomes more complex, when it is subjected to an external magnetic field. Such a flow situation has can be found in diverse field like solar technology, material manufacturing technology, electromagnetic casting, oil and gas production process, cooling of electronic equipment, etc. When the natural convection alone is not sufficient enough for effective heat removal, mixed convection (shear induced flow due to wall translation) can efficiently improve the thermal behaviour. Many researchers have investigated the MHD mixed
convective heat transfer in classical lid-driven cavity geometry. A detailed account of review on the above subject topic can be found in refs. [3,4].

Many authors have investigated the different aspects of the thermal convection of fluid saturated porous substances in a lid-driven enclosure. Rashad et al. [5] examined the MHD thermal convection of Cu-water nanofluid filled U-shaped cavity with double lids motion and heated partially from the bottom. It is observed that, the increase in the nanoparticle volume fraction leads to the increase in the heat transfer rate. MHD mixed convection in a cavity with double lids motion and heating discretely from bottom wall was examined by Hussain et al. [6]. Using Ag–MgO/water hybrid nanofluid, recently Selimefendigil and Chamkha [7] investigated the MHD thermal convection in a partitioned (in a triangular shape) cavity packed with porous layers. Gibanov et al. [8] studied the mixed convective heat transfer in a lid driven cavity filled with of ferrofluid filled and porous layers under the inclined magnetic field. Utilizing, Cu-water nanofluid flow from external source in a bottom-heated porous cavity Biswas et al. [9] have reported enhanced heat transfer (~15%). They found that impinging flow velocity and concentration of nanoparticles controls the overall thermal performance. In other class of work mixed convective heat transfer in lid-driven porous cavity has been investigated in [10,11] and others.

Above mentioned literature survey signifies the importance of the Magneto-hydrodynamic convective heat transfer in a cavity with moving lid(s). In most of the cited work, translation of the whole wall(s) of the confined space/cavity has been considered. Whereas, the overall thermal performance of the cavity under the effect of partial wall(s) translation over the whole wall(s) movement is missing. There are very few studies on the effect of partially driven walls on the thermal performance. Mondal et al. [12] recently, studied the effect of partial wall motion on the MHD thermal performance of a corner heated porous cavity. It is observed that, the partial wall(s) motion can significantly improve the heat transfer, which is further affected by the direction and speed of the partial wall(s) motion. This motivates us to undertake the present work.

In view of the above mentioned literature survey, this work is formulated to examine the effect of partial wall on the mixed convective heat transfer of Cu–water nanofluid saturated porous cavity with a centrally mounted block heated isothermally in presence of external magnetic field. The results of the analysis are visualized using streamlines, and isotherms; global thermal performance is characterized by the average Nusselt number.

2. Mathematical modelling and numerical technique

The schematic diagram of the Cu-water filled porous cavity with a centrally mounted block (having width 0.2L and height 0.2L) heated isothermally (at a temperature $T_h$) under the influence of external magnetic field (of magnitude $B_o$) is depicted in Fig. 1. The domain is square cavity (2-D) of length $L$. Upper half portion (having dimension 0.5L) of the sidewalls are cold (at temperature $T_c < T_h$), which are moving (with a velocity $v_w = +1$) in the upward direction at the same speed. Cu-water nanofluid flow through the porous medium is assumed to be laminar, incompressible, Newtonian, and steady within the validity of Boussinesq limit. Joule heating, induced magnetic field, viscous dissipation effects are not considered. The non-dimensional governing equations in Cartesian coordinate system are transformed [10–12] as:

$$
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
$$

$$
\frac{1}{\epsilon^2} \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial X} + \nu_f \frac{\partial^2 U}{\partial X^2} + \frac{\nu_f}{\epsilon \text{Re}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{v_{nf}}{\nu_f} \frac{1}{\text{DaRe}} \frac{F_e(U^2 + V^2)}{\sqrt{\text{Da} \epsilon^{2/5}}} U
$$

$$
\frac{1}{\epsilon^2} \left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial Y} + \nu_f \frac{\partial^2 V}{\partial X^2} + \frac{\nu_f}{\epsilon \text{Re}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{v_{nf}}{\nu_f} \frac{1}{\text{DaRe}} \frac{F_e(U^2 + V^2)}{\sqrt{\text{Da} \epsilon^{2/5}}} V
$$

$$
-\frac{\rho_f}{\rho_{nf}} \frac{\sigma_{nf} \beta_{nf}}{\text{Re}} \frac{Ha^2}{V} + \frac{\rho_{nf} \beta_f}{\text{Re}} Ri \theta
$$

\( (\cdot)_{nf} \)
\[
\left( \frac{U \partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) = \frac{\alpha_{df}}{\alpha_f} \cdot \frac{1}{Re \cdot Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

where, \( Re = \frac{v \cdot L}{v_f}, \) \( Pr = \frac{v_f}{\alpha_f}, \) \( Da = \frac{K}{L^3} \), \( F_s = \frac{1.75}{\sqrt{150}} \), \( Ha = B_s \cdot L \sqrt{\sigma_f / \mu_f} \), \( Ri = \frac{Gr}{Re^{2/3}} \), \( Gr = \frac{8 \beta_f (T_h - T_c) L^3}{v_f^2} \)

are the Reynolds number, Prandtl number, Darcy number, Grashof number, Forchheimer coefficient, Hartmann number, Richardson number, and Grashof number respectively. The dimensionless equations (1)–(4) are transformed by the following factors:

\[
(X,Y) = (x,y)/L; (U,V) = (u,v)L/\alpha_f; \theta = (T - T_c)/(T_h - T_c); P = (p - p_c)L^2/\rho_f \alpha_f^2
\]

The thermophysical properties of the base liquid (water, showed by the subscript 'f') and Cu nanoparticles are taken from the ref. [9,13,14]. The properties of Cu-water nanofluid (designated by the subscript 'nf') are calculated using the volumetric concentration of nanoparticles (\( \phi \)) suspended in the base fluid. Following relations are utilized for obtaining the nanofluid density, specific heat and thermal expansion coefficient:

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_{Cu}; \quad (\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_{Cu}; \quad (\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_{Cu}
\]

The effective dynamic viscosity, thermal diffusivity \( \alpha_{df} \), thermal conductivity \((k_{nf})\) and effective electrical conductivity \( (\sigma_{nf}) \) of nanofluid [9] are given by

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^2}; \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}; \quad k_{nf} = k_f \left[ \frac{(k_f + 2k_f) - 2\phi(k_f - k_f)}{(k_f + 2k_f) + \phi(k_f - k_f)} \right] \quad \sigma_{nf} = \sigma_f \left[ 1 + \frac{3(\sigma_f - \sigma_{nf})}{2(\sigma_f - \sigma_{nf})} \right]
\]

**Figure 1.** Schematic diagram of the computational domain of the physical problem.

The dimensionless partial differential equations (1)–(4) are solved numerically using in-house CFD code adopting the FVM through ADI sweep, TDMA solver, using the SIMPLE algorithm [15]. The converged solution is obtained when continuity mass-defect becomes less than \( 10^{-10} \). The same code has been used in our earlier works and validated extensively under different problem of published literature [3,9,11,12,16,17]. In the present simulation, uniformly distributed 200×200 grid size is selected after conducting the grid independency study. With the grid size of 200×200, it is found the relative error in the estimation of average Nusselt number (with respect to immediate coarse grid) is less than 1%, which is within acceptable limit.

The boundary conditions in the dimensionless form are:

- at the left and right walls - lower half \( (X = 0; 1; 0 \leq Y < 0.5) \), \( U = V = 0, \partial \theta / \partial X = 0 \) and for the upper half moving cold walls \( (X = 0; 1; 0.5 \leq Y \leq 1) \), \( U = 0, V = +1, \theta = 0 \).
- for the protruded heater \( (X = 0.4, 0.4 \leq Y \leq 0.6, X = 0.6, 0.4 \leq Y \leq 0.6) \) and \( Y = 0.4, 0.4 \leq X \leq 0.6, Y = 0.6, 0.4 \leq X \leq 0.6 \), \( U = V = 0, \theta = 1 \).
at the bottom adiabatic wall, \((Y = 0, 0 \leq X \leq 1)\),
\[ U = V = 0, \partial \theta / \partial Y = 0. \]

at the top adiabatic wall, \((Y = 1, 0 \leq X \leq 1)\),
\[ U = V = 0, \partial \theta / \partial Y = 0. \]

### 3. Results and discussion

In this work, MHD convective heat transfer of Cu-water nanofluid saturated porous cavity containing centrally mounted heated block is examined numerically. Upper half of the cold sidewalls are moving in the upward direction (such a configuration is termed as PDC). The analysis is carried out for the range of parameters such as Richardson number \((\text{Ri} = 0.1, 1, 10, 100)\), Hartmann number \((\text{Ha} = 0, 10, 30, 50, 70, 100)\), nanofluid volume fraction \((\phi = 0, 0.005, 0.01, 0.02, 0.03, 0.04, 0.05)\). The sliding speed of the cold walls is taken as fixed value through the Reynolds number \((\text{Re} = 200)\). The porosity \((\varepsilon = 0.8)\) and Darcy number \((\text{Da} = 10^{-3})\) are also taken as fixed. The results are visualized through streamlines, isotherms, and average Nusselt number \((\text{Nu})\).

#### 3.1. Effect of Richardson number (Ri)

On defining the operating flow regime, the choice of the Richardson number \((\text{Ri})\) signifies an imperative role. The impact of varying \(\text{Ri}\) values \((\text{Ri} = 0.1, 1, 10, 100)\) on the streamlines (top panel), and isotherms (bottom row) contours are presented in Fig. 2 for the fixed values of \(\text{Ha} = 30, \phi = 0.01\). As heat is released by the centrally mounted heated block surrounding fluid gets heated and moves upward (due to density difference) and then release heat through the moving cold sidewalls and then goes downward. Thus, the cavity is occupied by the two counter-rotating circulating cells. However, for the forced convection regime \((\text{Ri} \leq 0.1)\), wall shear induced flow dominates the buoyancy effect. As a result clockwise cell \((\text{CW})\) appears in the left part and anti-clockwise cell appears in the right part of the cavity. At \(\text{Ri} = 1\) (mixed convection regime), in addition two circulating cells (as seen with \(\text{Ri} = 0.1\)), there also appears two more counter rotating cells (due to weaker buoyancy effect) in the lower part of the cavity. When, \(\text{Ri}\) increases to 10 and 100, buoyancy force dominates the shear force.

| Ri = 0.1 | Ri = 1 | Ri = 10 | Ri = 100 |
|----------|--------|---------|----------|
| ![Streamlines](image1) | ![Streamlines](image2) | ![Streamlines](image3) | ![Streamlines](image4) |
| ![Isotherms](image5) | ![Isotherms](image6) | ![Isotherms](image7) | ![Isotherms](image8) |

**Figure 2.** Effect of Ri on the contours of streamlines (top row), and isotherms (bottom row) for \(\text{Ha} = 30, \phi = 0.01\). Isotherms and streamline contour intervals are 0.1 and 0.001, respectively.

This results in shrinkage of shear-induced flow (towards the moving cold sidewalls) and enlargement of the buoyancy-induced flow (from lower portion to the entire cavity). With the increase in Ri value,
Convection becomes stronger due to increase in the strength of the fluid circulation (as denoted by the maximum streamfunction). At Ri = 100, circulating cells stretched horizontally. The effect of different Ri values on the temperature distribution can be realised from the corresponding isotherm contours. For Ri ≤ 1, upper region of the cavity remain colder, whereas the bottom region of the enclosure is hotter due to distribution of high temperature lines about the middle horizontal portion (due to dominating shear-driven flow). However, for Ri ≥ 10, buoyancy-driven flow governs the convection mechanism; thus high temperature lines remains in the upper part of the enclosure. Due to stronger buoyancy effect, heat transfer rate increases markedly as indicated by the average Nu value.

3.2. Effect of Hartmann number (Ha)

The effect of externally applied horizontal magnetic field (defined by the term Hartmann number Ha) on the contours of streamlines (top row) and isotherms (bottom row) are shown in Fig. 3 for the different value of Ha = 0, 10, 50, 100 keeping other parameters fixed at Ri = 10, φ = 0.01. For easy of the understanding, the case of Ha = 0 (absence of magnetic field) is analyzed first. The thermo-fluid flow structure shows a symmetric distribution with respect to the vertical mid-plane. Now, strengthening the imposed magnetic field by increasing the Ha value = 10, 50 or 100, pattern wise overall flow structure are similar with the structure at Ha = 0. However, the buoyancy-induced circulating cells become weaker (as can be seen from the maximum streamfunction value). This leads to weaker heat transfer and this reduction is maximum at higher Ha = 100. Overall, the heat transfer reduction rate is about 0.37–21.23% compared to no-magnetic field, Ha = 0.

3.3. Effect of nanoparticle concentration (φ)

The increase in Cu nanoparticle volume fraction φ (= 0, 0.005, 0.02, and 0.05) on the streamlines, isotherms contours and average Nu are illustrated in Fig. 4 for the fixed values of Ri = 10, Ha = 30. It is noted that, the inclusion of nanoparticles in the water leads to the increase in the strength of fluid circulation, thus improvement in the heat transfer rate. The reason behind this improvement is due to the raise in φ leads to the increase in the effective thermal conductivity the working fluid. Thus, improvement in the buoyancy effect, and associated heat transfer rate (as indicated by the average Nu value).
value). In general, the heat transfer enhancement rate is about 0.46–4.46% compared to base fluid (φ = 0).

| φ  | Nu  |
|----|-----|
| 0.0 | 9.239 |
| 0.005 | 9.281 |
| 0.02 | 9.407 |
| 0.05 | 9.651 |

Figure 4. Effect of φ on the streamlines (top row) and isotherms (bottom row) contours at Ri = 10, Ha = 30. Isotherms and streamline contour intervals are 0.1 and 0.001, respectively.

3.4. Heat transfer characteristics

The average Nusselt number (Nu) is used to calculate the heat transfer characteristics at the cold sidewalls as

\[
\text{Nu} = \frac{k_{\text{nf}}}{k_f} \left( \frac{1}{\beta} \int_{0}^{1} \left( \frac{\partial \theta}{\partial X} \right) dY \right) + \frac{k_{\text{nf}}}{k_f} \left( \frac{1}{\beta} \int_{0}^{1} \left( \frac{\partial \theta}{\partial Y} \right) dX \right)
\]

The distribution of average Nu with the variation of Ri are plotted in Fig. 5 (a and b) for different Ha and φ keeping other parameters at fixed values. In Fig. 5a, the average Nu shows a consistently decreasing trend with the increasing Ha (for φ = 0.01). This happens due to the presence of negative source term with Ha in the Y momentum equation (3), which counteracts the positive effect of the buoyancy effect. The decreasing trend is almost similar for all Ri values except Ri = 1 (as Nu values shows a little bit increasing trend). Furthermore, the Nu curve at Ri = 0.1 remains above that of Ri = 1. The reason behind this fact may be understood by observing the flow structure at Ri = 1 (as in Fig. 2). Here the shear force is counteracted by the buoyancy force. As a result the heat transfer process is affected severely. Thus, setting Ri plays a crucial role on the thermal performance of such PDC.

Furthermore, selection of the Ha value influences the heat transfer process.

The effect of volumetric concentration of Cu-water nanofluid under different Ri values are reflected in the Fig. 5b at Ha = 30. The Fig. 5b clearly reflects that, with the increase in φ average Nu increases for any Ri values. The heat transfer process is more influenced by the nanoparticles in the convection dominated mode (Ri = 100). Inclusion of nanoparticles leads to increase in the effective thermal conductivity of the working fluid; it leads to increase in the circulation strength. Thus more amount of heat is transferred from heat source to the heat sink. In general, the heat transfer enhancement rate varying the Cu nanoparticles volume fraction (φ = 0.005–0.05) is about 0.81–8.40%
(for Ri = 0.1), 1.04–11.12% (for Ri = 1), 0.61–5.94% (for Ri = 100) compared to the base fluid (\( \phi = 0 \)).

![Figure 5. Heat transfer characteristics with varying Ri, (a) Ha, (b) \( \phi \) keeping other parameters fixed.](image)

**4. Conclusions**

In this study, the effects of partial wall motion on mixed convection heat transfer in Cu-water nanofluid filled porous cavity with a centrally mounted heat source under the externally applied magnetic field is investigated numerically. The thermo-fluid phenomena in a PDC are explored under different parametric influences like Ri, Ha, and \( \phi \). The significant conclusions are:

a) Under the partial wall motion, selection of Ri is the key factor for the convective heat transfer process. The heat transfer characteristics is increases significantly, when Ri > 1 and it is maximum at Ri = 100.

b) Increase in the magnetic field strength, buoyancy effect counteracted by the magnetic force. This results in decrement in the heat transfer rate. Such decrement is more at higher Ha = 100.

c) Increase in the Cu nanoparticle volume fraction, the heat transfer rate increases ~ 1.04–11.12% compared to base fluid (\( \phi = 0 \)).

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