Modeling of the Opened Shell Forced Vibrations with a Small Associated Mass, with Hinged Operation by the Pade’ Approximation Method

A Yu Dobryshkin¹, O E Sysoev², Nyein Sitt Naing³

¹Department of Civil Engineering and Architecture, Komsomolsk-na- Amur State University, 27, Lenin Ave., Komsomolsk-na-Amur 681013, Russia
²Faculty of Cadastre and Civil Engineering, Komsomolsk-na-Amur State University, 27, Lenin Ave., Komsomolsk-na-Amur 681013, Russia
³Faculty of Cadastre and Civil Engineering. Komsomolsk-na-Amur State University, 27, Lenin Ave.,Komsomolsk-na-Amur. 681013, Russia

E-mail: wwwartem21@gmail.com, fks@knastu.ru, nyeinsisnaing51@gmail.com

Abstract. In many constructions of buildings and structures, aircraft, etc. open hinged shells of small curvature are used. During operation, such structures are subjected to cyclic loads that cause shells to oscillate. The presence of an attached mass - the outdoor units of air conditioners, roof fans, balconies, fuel tanks, etc. -also has a significant effect on vibrations. The mechanisms of oscillations of such structures have not been fully studied, and therefore emergencies arise due to the resonance phenomenon caused by forced oscillations. In the laboratory of building structures of Komsomolsk-on-Amur State University, a test bench was made and open shells tested to determine the numerical frequency characteristics of the vibrations of open shells. The boundary conditions of which are articulated on the elastic material at the edges. The article describes the obtained new mathematical models based on the results of experimental studies. Experimental verification confirmed the correctness of using a refined mathematical model of the mechanism of oscillations of an open shell with a small added mass.

1. Introduction
The widespread use of shells in construction and engineering is due to ease of manufacture and high efficiency, in terms of strength, and the optimal geometric shape of the shells. The occurrence of forced vibrations from intense dynamic loads during the operation of such structures does not limit the operation of open shells lying on an elastic base. Phenomena of this kind are often accompanied by attached mass to the shell. This circumstance has not been fully studied, and the exploitation of open shells continues. Therefore, sometimes destructions occur, despite large reserves of strength (Kz = 1.4 for snow load, Kz = 1.1 for wind load) interest in analyzing the dynamics of thin open shells is very high. [1-4]

2. Relevance
At present, there is no exact description of the mechanism of oscillations of open cylindrical shells carrying the added mass. When designing structures in accordance with the regulatory requirements of
modern joint ventures, technical regulations and other governing documents, accidents cannot be avoided, since the calculations and planning have not planned measures that prevent structural vibrations resulting from the presence of added mass.

To study the effect of the added mass in the laboratory of building structures of the KnASU, an experiment was set up, the goal of which is to determine the frequency characteristics of the oscillations of the open shell carrying the added mass. The prototype, located in the test bench (Figure 1), had a small attached mass, and was subjected to a short-term load to excite forced oscillations.

As a result of the study of forced oscillations of a thin rectangular in terms of an open shell, based on an elastic base on both sides, the equations of shell oscillations reflecting the dependence of the added mass on the numerical characteristics of the natural frequencies of the shell oscillations are obtained. Oscillations with moderate amplitudes of forced oscillations were decomposed according to equations [5, 6].

\[
D_1 \dddot{W}_{xxx} + 2D_3 \dddot{W}_{xxy} + D_2 \dddot{W}_{yyy} + \rho \dot{W} + C \ddot{W} = 0
\]

The discrete nonlinear model of oscillations of a thin shell, clamped at the edges, obtained in the course of research, was studied using the method of many scales.

The sample is an open shell, rectangular in plan, of galvanized steel St0. Geometric characteristics of the object: \(L = 890\) mm, \(B = 370\) mm, \(H = 0,4\) mm. The scheme of the experiment is shown in Figure 1.

The sample consists of steel St0. The shell model is fixed in a steel stand. This stand has the shape of a table made of equal-angle corners; a ceramic coating was made at the places where the shell touched, especially for this experiment. The boundary conditions are as close as possible to the real ones [7-10]. The attached mass is the accelerometer BC110, located on the sample according to Figure 1. The accelerometer BC110 measures the oscillation frequency with maximum accuracy.

The accelerometer BC110 transmits readings to a signal amplifier, which, amplifying a signal, transmits further to an analog-to-digital converter, then to a personal computer. The eddy current probe ZET 701, located as close as possible to the shell, but does not touch it, transmits readings on the Z-lab software, where, coupled with the vibration data, some parameters are influenced by others, in real time. Z-lab software allows you to display and record vibrations in real time. The heating...
element creates a temperature gradient. The block diagram of the experimental setup for the experiment is shown in Fig. 2.

The experiment is aimed at studying the plate's natural oscillations at rest and identifying the dependencies of forced and natural oscillations on the influence of the added mass or system of masses. Forced oscillations, going into natural oscillations, were set by means of a blow with a test hammer AUV03 [11-14]. There was also a contactless sensor measuring plate oscillations, not shown in the diagram. This sensor is a verifier, used to check and reject erroneous data readings of the accelerometer BC110.

3. Theoretical part
Considering the mathematical model of the natural vibrations of an orthotropic rectangular curved plate\((-\frac{a}{2} \leq \bar{x} \leq \frac{a}{2}; -\frac{b}{2} \leq \bar{y} \leq \frac{b}{2}\) with free edges laying on the elastic base of Winkler-Fuss.

The differential equation describing the oscillations of an orthotropic plate is as follows:

\[
D_1 W_{xxxx} + 2D_3 W_{xxyy} + D_2 W_{yyyy} + \rho W_t + CW = 0,
\]

where \(D_1 = \frac{E_1 h^3}{12(1-\nu_1\nu_2)}\), \(D_2 = \frac{E_2 h^3}{12(1-\nu_1\nu_2)}\), \(D_3 = D_1 v_2 + \frac{G h^3}{6}\), C - spread coefficient; \(D_1, D_2\) - cylindrical stiffness in the direction of the axes \(\bar{x}\) and \(\bar{y}\); \(D_3\) - torsional stiffness; \(E_1, E_2\) - modulus of elasticity in the direction of the axes \(\bar{x}\) and \(\bar{y}\); \(G\) - shear modulus; \(\nu_1, \nu_2\) - Poisson's ratios in directions \(\bar{x}\) and \(\bar{y}\).

After the separation of temporal and spatial variables \((W=W(x,y)*T(t))\) and dimensionless, the differential equation will take the following form:

\[
W_{xxxx} + 2\alpha_3 W_{xxyy} + \alpha_2 W_{yyyy} - \lambda W = 0
\]

here \(\alpha_3 = \frac{D_3}{D_1}; \alpha_2 = \frac{D_2}{D_1}; \lambda_1 = \frac{b^2(1-\nu_2^2)}{D_1}\).

Add to the differential equation the boundary conditions:

\[
W_{xxx} + (2 - \nu_1) W_{yy} = 0,
\]

\[
(1 - \varepsilon)W_x \pm k \varepsilon (W_{xx} + \nu_1 W_{yy}) = 0 \text{ at } x = \pm 0.5k;
\]

\[
W_{yyy} + (2 - \nu_2) W_{xx} = 0,
\]

\[
(1 - \varepsilon)W_y \pm \varepsilon (W_{yy} + \nu_2 W_{xx}) = 0 \text{ at } y = \pm 0.5;
\]

We represent the eigenvalue and the own form in the form of a recursive formulation of perturbation theory (RPT). Substituting these series into a differential equation and boundary conditions and splitting in powers of \(\varepsilon\), we obtain a recurrent sequence of boundary value problems:
\[
W_{0xxx} + 2 \alpha_3 W_{0xyy} + \alpha_2 W_{0yyyy} - \lambda_0 W_0 = 0
\]
\[
W_{0x} = 0, W_{0xxx} = 0 \text{ at } x = \pm 0.5k
\]
\[
W_{0y} = 0, W_{0yyyy} = 0 \text{ at } y = \pm 0.5
\]
\[
W_{jxxx} + 2 \alpha_3 W_{jxyy} + \alpha_2 W_{jyyyy} - \lambda_0 W_j = \sum_{i=1}^{j} \lambda_i W_{j-i}
\]
\[
W_{jxx} + (2 - \nu_1)W_{jyy} = 0
\]
\[
W_{jx} = \varepsilon k \sum_{i=0}^{j} (W_{ixx} + \nu_1 W_{iyy}) \text{ at } x = \pm 0.5k
\]
\[
W_{jyy} + (2 - \nu_2)W_{jxy} = 0
\]
\[
W_{jy} = \varepsilon \sum_{i=0}^{j} (W_{iyy} + \nu_2 W_{ixx}) \text{ at } y = \pm 0.5
\]

In the zero approximation:
\[
\lambda_0 = \pi^4 \left[ \frac{m^4}{k^4} + 2 \alpha_3 \frac{n^2 m^2}{k^2} + \alpha_2 n^4 \right];
\]
\[
W_0 = X_0 Y_0 = \left\{ \sin \frac{\pi n y}{k}; \sin \frac{\pi m x}{k}; \right\}
\]
\[
\cos \frac{\pi n y}{k}; \cos \frac{\pi m x}{k}; \right\}
\]

The construction of further approximations will be considered in detail for centrally symmetric waveforms. In this case, \( n, m = 3, 5, 7, \ldots \). The boundary problem of the first approximation is as follows:
\[
W_{1xxx} + 2 \alpha_3 W_{1xyy} + \alpha_2 W_{1yyyy} - \lambda_0 W_1 = \lambda_2 \sin \pi n y \sin \frac{\pi m}{k} x;
\]
\[
W_{1xxx} + (2 - \nu_1)W_{1yy} = 0
\]
\[
W_{1x} = \pm k^2 \pi^2 \left( \frac{n^2}{k^2} + \nu_1 n^2 \right) (-1)^{n-1} \sin \pi n y \text{ at } x = \pm 0.5k;
\]
\[
W_{1yy} + (2 - \nu_2)W_{1xy} = 0
\]
\[
W_{1y} = \pm \pi^2 \left( \frac{n^2}{k^2} + \nu_2 \frac{m}{k} \right) (-1)^{n-1} \sin \frac{\pi m}{k} x \text{ at } y = \pm 0.5;
\]

The solution of the boundary value problem is sought as:
\[
W_1 = X_1(x) \sin \pi n y + Y_1(y) \sin \frac{\pi m}{k} x;
\]
\[
\lambda_1 = \lambda_{1x} + \lambda_{1y}
\]

Substituting expressions into a differential equation and boundary conditions, we have two one-dimensional problems:
\[
X_1^{(4)}(x) + 2\alpha_3 \pi^2 n^2 X_1''(x) - \pi^4 \left[ \frac{m^4}{k^4} + 2\alpha_3 \frac{n^2 m^2}{k^2} \right] X_1(x) = \lambda_{1x} \sin \frac{\pi m}{k} x;
\]
\[
X_1(x) = \pm k n^2 \left( \frac{m^2}{k^2} + \nu_1 n^2 \right) (-1)^{n-1}, \text{ at } x = \pm 0.5k;
\]
\[
X_1^{(4)}(x) - \pi^2 n^2 (2 - \nu_1)X_1'(x) = 0
\]
\[
Y_1^{(4)}(y) + 2\alpha_3 \pi^2 \frac{m^2}{k^2} Y_1''(y) - \pi^4 \left[ 2\alpha_3 \frac{n^2}{k^2} + \frac{m}{k} \right] Y_1(y) = \lambda_{1y} \sin \pi n y;
\]
\[
Y_1(y) = \pm \pi^2 \left( \frac{n^2}{k^2} + \nu_2 \frac{m}{k} \right) (-1)^{n-1}, \text{ at } y = \pm 0.5
\]
\[
Y_1'(y) - \pi^2 \frac{m}{k} (2 - \nu_2) Y_1'(y) = 0.
\]
After constructing the conditions for solvability, we give the final expressions:

\[ \lambda_{1x} = 4\pi^4 \left( \frac{m^2}{k^2} + \nu_1 n^2 \right) \left[ n^2 (2 - \nu_1 - 2\alpha_3) - \frac{m^2}{k^2} \right] \]
\[ \lambda_{1y} = 4\pi^4 \left( n^2 + \frac{m^2}{k^2} \right) \left[ \frac{m^2}{k^2} (2 - \nu_2 - 2\alpha_3) - n^2 \right] \]

Now we find the first amendment to our own form \( W_1 \):

\[ W_1 = \frac{\pi \left( \frac{m^2}{k^2} + \nu_1 n^2 \right)}{\left( \frac{m^2}{k^2} + \alpha_3 n^2 \right)} \left[ k \left( \frac{(-1)^{m-1}}{\pi} \right) \frac{(n^2 (2 - \nu_1) + \frac{m^2}{k^2})}{2\beta_2 \left( \frac{\sin \beta_2 k}{\sin \beta_2 \pi} \right)} \right] \left( \frac{\sin \pi n y}{\cos \pi n y} \right) \]

\[ - (-1)^m \frac{k}{n} \left( \frac{n^2 (2 - \nu_2 - 2\alpha_3)}{k^2} - \frac{m^2}{k^2} \right) \left( \frac{\cos \frac{\pi m}{k} x}{\sin \frac{\pi m}{k} x} \right) \]

\[ + \frac{\pi \left( n^2 + \nu_2 \frac{m^2}{k^2} \right)}{\left( n^2 + \alpha_3 \frac{m^2}{k^2} \right)} \left[ \frac{(-1)^{n-1}}{\pi} \frac{(n^2 (2 - \nu_2) n^2)}{2\beta_1 \left( \frac{\sin \beta_1 k}{\sin \beta_1 \pi} \right)} \right] \left( \frac{\sin \pi n y}{\cos \pi n y} \right) \]

\[ - (-1)^n \frac{1}{n} \left( \frac{m^2}{k^2} (2 - \nu_2 - 2\alpha_3) - n^2 \right) \left( \frac{\cos \frac{\pi m}{k} x}{\sin \frac{\pi m}{k} x} \right) \]

Considering the second approximation, we determine:
\[ \lambda_{2x} = 4\pi^4 \left[ n^2(2 - \nu_1 - 2\alpha_3) - \frac{m^2}{k^2} \right] \left\{ \frac{m^2}{k^2} + \nu_1 n^2 \right\} \]

\[ - \left\{ \frac{m^2}{k^2} + \nu_1 n^2 \right\} \frac{\pi k}{2} n^2(2 - \nu_1 + \frac{m^2}{k^2}) cth\left(\frac{m^2}{k^2}\right) \beta_2 - 2 \left( n^2(2 - \nu_1 - 2\alpha_3) - \frac{m^2}{k^2} \right) \]

\[ - 2\nu_1 \left( \frac{n^2 + \nu_2 m^2}{n^2 + \alpha_3 \frac{m^2}{k^2}} \right) \times \left( \frac{m^2}{k^2}(2 - \nu_2 - 2\alpha_3) - n^2 \right) - \frac{m^2}{k^2} \left( n^2 + \alpha_3 \frac{m^2}{k^2} \right) \]

\[ \times \left\{ \frac{\pi n^2}{2} \left( 2 - \nu_1 + \frac{m^2}{k^2} \right) + \frac{1}{2n^2} \left( \frac{m^2}{k^2}(2 - \nu_2 - 2\alpha_3) - n^2 \right) \right\} \]

\[ \times \left( \frac{m^2}{k^2} + \nu_1 n^2 \right) \left[ \frac{\pi k}{2} n^2(2 - \nu_1 + \frac{m^2}{k^2}) cth\left(\frac{m^2}{k^2}\right) \beta_2 - 2 \left( n^2(2 - \nu_1 - 2\alpha_3) - \frac{m^2}{k^2} \right) \right] \]

\[ \times \left( \frac{m^2}{k^2} + \nu_1 n^2 \right) \left\{ \frac{\pi k}{2} n^2(2 - \nu_1 + \frac{m^2}{k^2}) cth\left(\frac{m^2}{k^2}\right) \beta_2 - 2 \left( n^2(2 - \nu_1 - 2\alpha_3) - \frac{m^2}{k^2} \right) \right\} \}

\[ \left( \lambda_{2y}, \frac{m^2}{k^2}; n^2, \nu_1; \nu_2; \beta_1 \right) \rightarrow \left( \lambda_{2x}, \frac{m^2}{k^2}; n^2, \nu_1; \nu_2; \beta_2 \right). \]

In the isotropic case \( D_1 = D_2 = D_3 = D; \nu_1 = \nu_2 = \nu \) the expressions for \( \lambda \) and \( W \) take the form:

\[ \lambda = \pi^4 \left( n^2 + \frac{m^2}{k^2} \right)^2 - 4\pi^4 (1 + \nu) \left( n^2 + \frac{m^2}{k^2} \right) \nu + (\lambda_{2x} + \lambda_{2y}) \nu^2 + \cdots; \]
\[ W = \left( \sin \pi ny \sin \frac{\pi m}{k} x \right) \left( \cos \pi ny \cos \frac{\pi m}{k} x \right) \]

\[ + \frac{\pi}{\left( n^2 + \frac{m^2}{k^2} \right)} \left\{ k \left( \frac{(-1)^{m-1}}{-1} \right) \left( n^2 (2 - \nu) + \frac{m^2}{k^2} \right) \right\} \frac{2\beta_2 \left( \frac{\text{sh} \beta_2 k}{\text{ch} \beta_2 k} \right)}{\left( \text{sh} \frac{\beta_2 k}{2} \right)} \times \frac{\text{ch} \pi \beta_2 x}{\text{ch} \pi \beta_2 x} \]

\[ + (-1)^m \frac{k}{m} \left( n^2 + \frac{m^2}{k^2} \right) x \left( \text{cos} \frac{\pi m}{k} x \right) \left( \text{sin} \pi ny \right) \left( \text{cos} \pi ny \right) \]

\[ + \left( n^2 + \frac{m^2}{k^2} \right) \frac{\left( (-1)^{m-1} \left( \frac{-1}{(-1)^m} \right) \right) \left( \frac{\text{ch} \frac{\pi m}{k^2}}{\text{sh} \frac{\pi m}{k^2}} \right)}{\left( \text{ch} \frac{\pi m}{k^2} \right)} \times \left( \text{sh} \pi \beta_1 y \right) \left( \text{ch} \pi \beta_1 y \right) \]

\[ + (-1)^n \frac{1}{n} \left( n^2 + \frac{\nu^2}{k^2} \right) y \left( \text{cos} \pi ny \right) \left( \text{sin} \pi ny \right) \left( \text{cos} \frac{\pi m}{k} x \right) \left( \text{sin} \frac{\pi m}{k} x \right) e + \cdots, \]

\[ \lambda_{2x} = 8\pi^4 (1 + \nu) \left( vn^2 + \frac{m^2}{k^2} \right) \left[\frac{k}{2} \left( \frac{n^2 (2 - \nu) + \frac{m^2}{k^2}}{n^2 + \frac{m^2}{k^2}} \right) - \frac{m^2}{k^2} \left( \frac{n^2}{k^2} + \frac{m^2}{k^2} \right) \right] - 4\pi^4 \left( \frac{4m^2}{k^2} + vn^2 \right)^2 \]

\[ + 8\pi^4 \left( \frac{m^2}{k^2} + vn^2 \right) \left( \frac{n^2 + \frac{m^2}{k^2}}{k^2 + vn^2} \right) \left[\frac{k}{2} \left( \frac{n^2 (2 - \nu) + \frac{m^2}{k^2}}{n^2 + \frac{m^2}{k^2}} \right) \right] \times \frac{\pi \text{th} \beta_2 k}{2} \]

\[ + 2 \left( vn^2 + \frac{m^2}{k^2} \right) + 2\nu \left( n^2 + \frac{m^2}{k^2} \right) + \nu \left( n^2 + \frac{m^2}{k^2} \right) \]

\[ \times \left[ \frac{\left( \frac{\nu n^2}{k^2} (2 - \nu) + n^2 \right)}{k^2} \left( \frac{m^2}{k^2} (1 - 2\nu) + n^2 \right) + \left( \frac{\nu n^2 + \frac{m^2}{k^2}}{2n^2} \right) \left( \frac{m^2 + \nu^2}{k^2} \right) \right] \]

\[ \left( \lambda_{2y}; \frac{m^2}{k^2}; \beta_2 \right) \rightarrow \left( \lambda_{2x}; \frac{m^2}{k^2}; \beta_1 \right). \]

Results of experimental studies
1 – Data from recursive perturbation theory;  
2 - Data obtained using the Pade׳ approximation;  
3- Experimental data;  

Figure 3. The first proper number for a square in the plan of an open shell on an elastic base.

4. Conclusions

Let us compare the results of the first eigenvalue of oscillations of a free square isotropic plate with other methods for determining oscillations. The approximation of the Pade segment of the series has the form \( \lambda_1(\varepsilon) = \frac{a_0 + a_1 \varepsilon}{1 + b_1 \varepsilon} \). The first proper number of the task at \( \nu = 1/6 \), obtained using the Pade approximation, equally \( \lambda_1 = (1.100 \pi)^4 \), Bubnov – Galerkin method – \( \lambda_1 = (1.2295 \pi)^4 \), (error – 10.51%).

For \( \nu = 0.3 \) Pade approximation gives \( \lambda_1 = (1.1198 \pi)^4 \), Bubnov – Galerkin method – \( \lambda_1 = (1.1683 \pi)^4 \) (погрешность – 4.15%), Southwell method \( \lambda_1 = (1.1424 \pi)^4 \) (error – 1.14%). It should be noted that the methods of Bubnov-Galerkin and Southwell give an upper estimate of the eigenvalue. The zero approximation of the recursive formulation of the perturbation theory and the Padé approximation with \( \varepsilon = 1 \), respectively, give the upper and lower estimates for the eigenvalues of the problem.

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