Decoherence of Dirac-Quantumness for Open Particles in a Dilatonic Black Hole

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The quantumness of Dirac particles for quantized fields in a dilatonic black hole is estimated by means of quantum channel. A general Bloch vector representation of quantum channel in black hole spacetimes beyond single mode approximation is developed. The nonclassicality of Dirac particles can be measured by the minimization of quantum coherence over all orthonormal basis sets. The quantumness of the channel decreases as the dilaton field increases. The interplay between the external reservoir noise and dilaton black hole on the dynamical behavior of quantum coherence and steerability is investigated in the Pauli basis. The external environment is modeled by a random telegraph noise channel. The monotonous decay of quantum nonlocality occurs in the weak coupling case. The degradation and revival of quantum nonlocality are observed in the strong coupling condition. It is found that quantum fluctuation effects of the external reservoir can protect quantum coherence and steerability from the information loss of the black hole.

1. Introduction

The observation of gravitational waves and confirmation of black holes\(^1\) have received much attention. The quantum property of field modes near the event horizon plays a prominent role in the study of black hole thermodynamics.\(^2\) The relativistic effect on quantum dynamics in acceleration frames\(^3\) and vacuum fluctuation in black hole spacetimes\(^4\) have been studied in the field of relativistic quantum information.\(^5\) As is well known, Unruh acceleration effects\(^6,7\) and gravitational fields\(^6,7,8\) lead to information loss of particles or antiparticles in a relativistic framework. Recent studies have demonstrated that the geometric inflation and shadows have impacts on the nonclassicality of quasi-bound states of black holes.\(^10\) From the viewpoint of quantum nonlocality,\(^11\) quantumness of scalar and Dirac fields in the black holes can be quantified by quantum entanglement\(^12\) and quantum correlation.\(^13\) The relativistic metrology based on quantum Fisher information was also applied to the analysis of vacuum structures in curved spacetimes.\(^14,15\) The class of dilatonic black holes has become an interesting focus in astrophysical black holes.\(^16-20\) The superradiant instability and scattering properties of dilatonic black holes are studied in refs. \(^21, 22\). In the low energy limit of string theory, the static spherical charged dilatonic black hole solution was provided by Gibbons and Maeda,\(^23\) independently by Garfinkle, Horowitz, and Strominger (GMGHS). The study of dilatonic black holes contributes to a deeper understanding of quantum gravity.\(^25\) It is of a great interest to explore quantumness of dilatonic black holes in the combination of quantum information and general relativity.

On the other hand, quantum fluctuations in some curved spacetimes can be investigated in the context of open quantum system approach.\(^26\) As one feasible method, quantum channel can be used to characterize dynamics of quantum systems interacting with their reservoirs.\(^27,28\) The quantum resource theory\(^29\) provides an efficient way to measure quantumness of channels.\(^10\) The quantum channel can be represented as the evolution of Bloch states for quantized fields in relativistic spacetimes. It is assumed that the two observers can detect local field modes. After both of them share a quantum correlated initial state, one stays stationary at an asymptotically flat region and experiences quantum dissipation from an external reservoir, while the other moves with uniform acceleration and hovers near the event horizon of the dilatonic black hole. In the background of black holes, we expect to find a unified quantum channel which discovers the connection between gravitation and decoherence.

The paper is organized as follows. In the next section, the quantized fields of Dirac particles and antiparticles are discussed beyond single mode approximation. We suggest the generalized representation of quantum channel in the form of Bloch vectors. The available estimation of quantumness of the channel is obtained by means of quantum coherence. The relativistic effects from the dilaton field and Dirac mode approximation are considered. In Section 3, the interplay between the external environment and dilatonic black hole spacetime is analyzed in terms of quantum coherence and quantum steering. We put forward a unified method of quantum decoherence in the Pauli basis. We will discuss our conclusion in the last section.
2. Quantum Channel in the Dilatonic Black Hole

In accordance with string theory, the dilatonic black holes are formed by gravitational systems coupled to Maxwell and dilaton fields. The action of the 4D dilaton theory is written as[13,24]

$$S = \int d^4x \sqrt{-g} \left[ R - 2\phi + \frac{\partial \phi}{\phi} \right]^2$$

(1)

where \(\phi\) represents the dilaton and \(F_{\mu\nu}\) is the Maxwell field tensor. The coupling parameter between the dilaton and Maxwell field is given by the parameter \(a\). The original work in ref. [24] has presented the general solutions with different values of \(a\). The general metric for a dilaton black hole is written as

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\left(1 - \frac{Q^2}{r^2}\right)d\Omega^2$$

(2)

The parameters \(M\) and \(D\) are the mass of the black hole and dilaton field, respectively. For a charged dilatonic black hole, \(D = \frac{Q^2}{M}\) where \(Q\) denotes the charge. The term \(e^{-2\phi} = e^{-2\phi_0(1 - \frac{2M}{r})}\) when the dilaton value at spacelike infinity \(\phi_0 = 0\), the case implies an asymptotic flat manifold. We choose the spherically symmetric black hole with \(a = 1\) in order to obtain the vacuum structure for Dirac particles. According to the results of ref. [18, 31], the spherically symmetric line element under the circumstance of \(D < M\) is given by

$$ds^2 = -\left(\frac{2M}{r}\right)dt^2 + \left(\frac{2M}{r}\right)^{-1} dr^2 + r^2\left(1 - \frac{Q^2}{r^2}\right)d\Omega^2.$$  

Though the metric cannot represent all properties of different dilatonic black holes, it is asymptotically flat and only in this case the initial state can be prepared at the asymptotically flat regions. The natural units \(\hbar = G = c = k_B = 1\) are used. The event horizon of GMGH black hole is located at \(r_h = 2M\). The area of the sphere goes to zero when \(r = r_h = 2D\) and the surface is singular. The quantized Dirac fields in the dilatonic black hole satisfy the Dirac equation which is given by

$$\gamma^\mu \gamma^\nu \gamma^\rho \epsilon^\alpha_{\mu \nu \rho} = 0$$

(3)

\(\gamma^\mu\) is the Dirac matrix, \(\epsilon^\alpha_{\mu \nu \rho}\) corresponds to the inverse of the tetrad \(e^\alpha_{\mu \nu \rho}\) and \(\Gamma^\mu\) denotes the spin connection coefficient. By solving the Dirac equation, we can obtain the positive frequency outgoing solutions outside region and inside region of the event horizon, \(\Psi_{\pm,\pm} = \mathbf{R} e^{i\omega t\gamma^0}\) where \(\omega = \sqrt{\omega^2 - \epsilon^2}\), \(\epsilon^\alpha_{\mu \nu \rho}\) represents the two regions, \(k\) denotes a variable that labels the field mode, \(\mathbf{R}\) is a 4-component Dirac spinor and \(\omega\) is a monochromatic frequency of the Dirac field. The parameter \(\omega^0 = t - 2(M - D)\ln(r - 2M)/(2M - 2D)\) is the retarded time.

In terms of the complete orthogonal basis \(\Psi_{\pm,\pm}\), the Dirac field can be expanded as

$$\Psi = \sum_{m=0}^{\infty} \int d\omega (\hat{a}_{\omega}^\dagger \Psi_{m}^+ + \hat{b}_{\omega}^\dagger \Psi_{m}^-)$$

where \(\hat{a}_{\omega}\) and \(\hat{b}_{\omega}\) correspond to the fermion annihilation and antifermion creation operators acting on the state of the \(\pm\) region. Making the analytic continuations for \(\Psi_{\pm,\pm}\), we can quantize Dirac fields as the combination of Kruskal modes according to the suggestion of Domour–Ruffini.[31] The quantization of Dirac fields beyond single mode approximation[31] can be taken into account. Using the Bogoliubov transformations between the creation and annihilation operators in two kinds of coordinates, the operators for mode \(k\) are expressed as [14,35]

$$\hat{a}_{R,k} = \cos \sigma R_{\omega}^{\text{out}} - \sin \sigma R_{\omega}^{\text{in}},$$

$$\hat{b}_{R,k} = \cos \sigma R_{\omega}^{\text{in}} - \sin \sigma R_{\omega}^{\text{out}}$$

(4)

where \(\cos \sigma = \left(\varepsilon - e^{2\phi_0(1 - \frac{2M}{r})}\right)^{1/2}\) and sin \(\sigma = \left(\varepsilon - e^{2\phi_0(1 - \frac{2M}{r})}\right)^{1/2}\). \(\hat{a}_{R,k}\) denotes the annihilation operator for the right (R) or left (L) mode. According to the results of [24,36], the Hawking temperature for Equation (2) is \(T_h = \frac{1}{4\pi M}\) in the general Reissner–Nordström metric. However, we mainly consider the special case of spherically symmetric metric which has been investigated by some relevant references of ref. [9, 18]. This metric has the similar vacuum structure of Schwarzschild spacetime black hole. Using the method of ref. [18, 31], the statistical temperature \(T = \frac{1}{4\pi M}\) can be obtained. The quantum states of quantized Dirac field in the antisymmetric fermionic Fock space should be denoted by double-lined Dirac notation \(\langle\langle\rangle\rangle\) rather than the single-lined notations.[37] Assuming that \(\langle\langle\rangle\rangle\) is annihilated by the operator \(\hat{a}_{R,L,k}\), the Kruskal vacuum for mode \(k\) is defined as \(\langle\langle\rangle\rangle = \|\langle\langle\rangle\rangle\|_R, \|\langle\langle\rangle\rangle\|_L, \|\langle\langle\rangle\rangle\|_c\) = \(\langle\langle\rangle\rangle\)

\(\langle\langle\rangle\rangle\) is a three-dimensional vector, and the superscripts \(\langle\langle\rangle\rangle\) represent the particle and antiparticle. Because of the Pauli exclusion principle, only the first excited state for particle mode is obtained by

$$\|\langle\langle\rangle\rangle\|^+_K = [q_L|\hat{a}_{R,k}^\dagger \otimes I_L|\langle\langle\rangle\rangle\rangle]$$

(5)

with \(|q_L|^2 = |q_L|^2 = 1\). The creation operator \(\hat{a}_{R,k}^\dagger\) implies the creation of a fermion in the exterior vacuum and an antifermion in the interior vacuum of the black hole. The spontaneous creation of particles and antiparticles results in Hawking radiation with the total probability \(\|\langle\langle\rangle\rangle\|^+_K + \|\langle\langle\rangle\rangle\|^+_L = 1\). When the parameter \(|q_L|^2 = 1\), all the fermions propagate toward the outside of event horizon and all the antifermions move to the interior region. Similarly, the case of \(|q_L|^2 = 1\) represents that all the antifermions radiate toward the exterior region.

Since the exterior region is causally disconnected from the interior region in the dilatonic black hole, we can obtain the physical accessible outside states by tracing over the states of the interior region. The whole process of information loss can be characterized by a quantum channel. In the open quantum system approach, the dynamical map for the system of interest can be obtained by tracing over all the degrees of freedom of the other environment. With respect to the black hole spacetime, the outside states of all particles or antiparticles can be considered as the open system of interest. Thus, the quantum channel for Dirac particles in the outside region of the event horizon can be given by

$$\varepsilon^{\text{out}}(0)_{K}\langle\langle\rangle\rangle = q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle$$

$$\varepsilon^{\text{out}}(1)_{K}\langle\langle\rangle\rangle = q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle$$

$$\varepsilon^{\text{out}}(1)_{K}\langle\langle\rangle\rangle = q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle$$

$$\varepsilon^{\text{out}}(1)_{K}\langle\langle\rangle\rangle = q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle + q_L^2|\langle\langle\rangle\rangle\rangle$$

(6)
where the partial trace over all the states of the interior region and the antiparticle states of the outside is carried out. For single particle state, the density matrix of the state $\rho$ can be expressed by the Bloch vector $\mathbf{v}$, that is, $\rho = \frac{1}{2} (I + \sum_{\sigma=1}^{3} v_{\sigma} \sigma_{\sigma})$ where $v_{\sigma} = \text{Tr}(\rho \sigma_{\sigma})$. ($\mu = 1, 2, 3$) and $\sigma_{\sigma} (\mu = 1, 2, 3)$ denote the three Pauli operators. In the pauli basis of $\{\sigma_{1}, \sigma_{2}, \mu (\mu = 1, 2, 3)\}$, the dynamical process from an arbitrary initial particle vector $X = (1, v^{(0)})$ to the evolved one $Y = (1, v)$ is written as

$$Y = \hat{\Phi}_{\text{out}}^{+} X = \left( \begin{array}{c} 1 \\ \Gamma_{\text{out}}^{+} \end{array} \right) \left( \begin{array}{c} 0 \\ v^{(0)} \end{array} \right)$$

(7)

where $0^{\text{T}} = (0, 0, 0)$ and the superscript T denotes the transpose. The quantum channel $\epsilon_{\text{out}}^{+}(\cdot)$ is equivalent to the action operator $\Phi$ which is composed of the mapping vector and mapping matrix.

Here, $\lambda_{\mu}(\mu = 1, 2, 3) = \text{Tr}[\Psi_{\mu}^{\text{T}} \rho_{\text{in}}]$. The elements of the matrix $\Gamma$ are obtained by $\Gamma_{\nu, \mu}(\nu, \mu = 1, 2, 3) = \text{Tr}[\Psi_{\mu}^{\text{T}} \rho_{\text{in}} \sigma_{\nu}^{\text{T}}]$. By using Equation (6), we obtain the channel matrix and vector as

$$\Gamma_{\text{out}}^{+} = \begin{pmatrix} \text{Re}(q_{\theta}) \cdot c & -\text{Im}(q_{\theta}) \cdot c & 0 \\ \text{Im}(q_{\theta}) \cdot c & \text{Re}(q_{\theta}) \cdot c & 0 \\ 0 & 0 & |q_{\theta}|^{2} \cdot c^{2} \end{pmatrix}$$

$$\lambda_{\text{out}}^{+} = \begin{pmatrix} 0 \\ 0 \\ (1 - |q_{\theta}|^{2}) c^{2} - s^{2} \end{pmatrix}$$

(8)

We consider a real parameter, that is, $\text{Re}(q_{\theta}) = q_{\theta} \in [0, 1]$ and $\text{Im}(q_{\theta}) = 0$ in this case. The evolved particle state is expressed as $\rho = \Gamma_{\text{out}}^{+} \cdot v^{(0)} + \lambda_{\text{out}}^{+}$ in the form of Bloch vectors. It is known that a geometric picture of an arbitrary pure state is a symmetric sphere which corresponds to a Bloch vector $v^{(0)} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)^{T}$, $\theta \in [0, \pi], \phi \in [0, 2\pi]$ without information loss. Under the condition of the small values of $q_{\theta}$, the Bloch sphere for Dirac particle states shrinks drastically. The geometrical contraction can verify the fact about the loss of quantum information induced by quantum fluctuations near the event horizon of the black hole.

Moreover, we try to quantify the quantumness of the channel which describes the thermal radiation of particles near the event horizon of the dilatonic black hole. The squared $l_{i}$ norm can be used to measure quantum coherence of a particle state. The quantity of quantum coherence is given by $C(\rho) = \sum_{i,j} |a_{ij}|$ where $a_{ij}$ is the element of the state density matrix. When a particle state is subjected to the action from quantum channel, the decay of quantum coherence depends both on the channel and incoherent basis. The nonunitary evolution of states addresses the question of how much quantumness a channel can preserve. By averaging on quantum coherence of all states and minimizing over all orthonormal basis sets,\(^{(10)}\) the measure of nonclassicality of the quantum channel $\epsilon_{\text{out}}^{+}$ can be obtained as

$$Q(\epsilon_{\text{out}}^{+}) = \min_{\mathbf{m}^{\text{T}}} \left[ \text{Tr}(M) - n^{T} M n \right]$$

(9)

where $M = \frac{1}{2} (\Gamma_{\text{out}}^{+} M^{(3)} + 5 \lambda_{\text{out}}^{+} \cdot \lambda_{\text{out}}^{+} \cdot \lambda_{\text{out}}^{+})$ is determined by the mapping vector and matrix. The minimization is achieved when $\mathbf{n}$ is chosen to be an eigenvector of the matrix $M$ with the largest eigenvalue. Therefore, the quantumness of the channel is calculated by $Q(\epsilon_{\text{out}}^{+}) = \text{Tr}(M) - n^{T} M n$.

3. Relativistic Decoherence with External Noise

The Dirac particles escaping from the outside region may experience external noisy environments. When one observer detects states of open Dirac particles, the effects from external noises need be considered. It is necessary to study the impacts from external noises on quantum nonlocality in the background of black hole spacetime. In a protocol, two observers can detect local field modes of Dirac particles. Both of them share an initial state of two Dirac particles. One observer stays stationary at an asymptotically flat region and the local Dirac particle $A$ suffers from an external reservoir. While the other detects the state of Dirac particle $B$ near the event horizon of the black hole. In terms of quantum channels, the two Dirac particles experience two independent channels of $\epsilon_{\text{in}}$ and $\epsilon_{\text{out}}$ respectively. In general, an arbitrary state for two Dirac particles $A$ and $B$ can be written as $\rho_{AB}^{(0)} = \frac{1}{2} \sum_{\mu, \nu = 0}^{3} \Theta_{\mu, \nu}^{(0)} (\sigma_{\mu} \otimes \sigma_{\nu})$. Here, the matrix element $\Theta_{\mu, \nu}^{(0)} = \text{Tr}(\rho_{AB}^{(0)} \sigma_{\mu} \otimes \sigma_{\nu})$ is obtained. In the pauli basis, we can obtain the equivalent form of the initial state by using

![Figure 1](https://www.ann-phys.org/)
the matrix

$$\Theta^{(0)} = \left( \frac{1}{d_A} \delta^{(0)}_{AB} \right)^T$$

(10)

where $\delta^{(0)}_{AB}$ denotes the initial Bloch vector of the particle A or B. The matrix $T^{(0)}$ implies the correlation information.\(^{[38,39]}\) Each quantum channel is determined by the mapping vector $\lambda$ and matrix $\Gamma$. In the Bloch vector representation, the transformation of quantum channel is given by $v = \Gamma \cdot v^{(0)} + \lambda$. In the context of open quantum systems, the evolved state is obtained by $\rho_{AB} = \epsilon \otimes \epsilon_B (\rho_{AB}^{(0)})$. We can expand $\rho_{AB}$ as

$$\rho_{AB} = \frac{1}{4} \sum_{\alpha, \beta = 0}^3 \Theta^{(0)}_{\alpha} \epsilon_A (\sigma_{\alpha, a}) \otimes \epsilon_B (\sigma_{\beta, b})$$

(11)

where the part of $\epsilon_A (\sigma_{\alpha, a})$, $\alpha = A, B$ is given by

$$\epsilon_A (\sigma_{\alpha, a}) = \frac{1}{4} \sum_{\mu, \nu = 0}^3 \Gamma_{\nu, \mu, a} \cdot \sigma_{\mu, a} \cdot \sigma_{\nu, a}$$

(12)

The matrix $\Theta$ for the evolved state is $\Theta = \left( \frac{1}{d_A} \delta^{(0)}_{AB} S \right)^T$ and the vectors satisfy that $v_a = \Gamma_a \cdot (\lambda + \lambda_a)$, $\alpha = A, B$. According to Equations (11) and (12), the correlation matrix $S$ for the evolved state is obtained by

$$S = \lambda \cdot \sigma^T \cdot (v_a - \lambda_a) \cdot \sigma^T + \Gamma_a \cdot T^{(0)} \cdot \Gamma_a^T$$

(13)

Here, $\epsilon_A = \epsilon^{\mu, \mu}$ is given by the quantum channel for Dirac particles from the outside region near the event horizon of the black hole.

To further explore the decoherence of Dirac particles, we study the dynamics of quantum coherence and quantum steering. The Werner state is chosen to be an initial one. In the Hilbert space spanned by $\{ (00), (01), (10), (11) \}$, $\rho_{AB} = 1 (0) I + \eta |\psi^-\rangle \langle \psi^- |$ where $|\psi^-\rangle = \sqrt{\frac{1}{2}} (|01\rangle - |10\rangle)$ and the parameter $\eta \in [0, 1]$. In this case, the evolved state has a real X shaped form. According to quantum resource theory, the measure of quantum coherence based on $l_1$ norm is obtained by

$$C(\rho_{AB}) = \frac{1}{2} \left( |S_{11} - S_{22}| + |S_{11} + S_{22}| \right)$$

(14)

where $S_{ij}$ are the elements of the correlation matrix for the evolved state. Recently, the inequality has been developed to judge whether a bipartite state is steerable when both of the observers are allowed to measure observations in their local sites.\(^{[40]}\) This inequality is composed of a finite sum of bilinear expectation values, which is given by $F_{\rho} (\eta, \mu) = \frac{1}{V^2} \sum_{i=1}^n \langle A_i \otimes B_i \rangle \leq 1$.

Here, $\lambda = u \cdot \sigma, B_i = \nu_i \cdot \sigma$ where $u_i \in \mathbb{R}^3$ are unit vectors and $\nu_i \in \mathbb{R}^3$ are orthonormal vectors and $\zeta = \{ u_1, \ldots, u_n, v_1, \ldots, v_n \}$ is the set of measurement directions. The CJWR inequality for $n = 2$ is used to quantify steerability $Q(\rho_{AB}) = \max_0 \left( \frac{2}{F_{\rho} (\eta, \mu)} - 1 \right)$ where $F_2 (\rho_{AB}) = \max_0 \left( F_{\rho} (\eta, \mu) \right)$ and $F_{\rho} (\eta, \mu) = \max_0 \left( F_{\rho} (\eta, \mu) \right)$. In this case, the steerability is obtained by

$$Q(\rho_{AB}) = \max_0 \left( \frac{\sqrt{\text{Tr}(S^T S) - \min_0 (S^T) - 1}}{\sqrt{2}} \right)$$

(15)

To clearly demonstrate the effects of external noises on the quantumness of Dirac particles, we consider a general open system model which is applicable to a qubit subjected to the fluctuating quantum fields with random phase noises. This kind of reservoir is modeled by the random telegraph noise channel.\(^{[41,42]}\) The noise is characterized by the autocorrelation function of $\langle \chi (\tau) \chi (\delta) \rangle = \delta^2 e^{-\gamma \tau}$ and used to mimic the quantum fluctuations outside a black hole. The parameter $\gamma$ denotes the stochastic variable and $\delta$ represents the coupling strength between the system and the reservoir. And $\gamma$ signifies the fluctuation rate of the noise. In the Kraus representation, the quantum channel for the random telegraph noise is given by $\epsilon (\rho) = \sum K_i \rho K^T_i$ where the Kraus operators are $K_i (\nu) = \sqrt{\frac{1 + \nu}{2}} (1 + \nu) - 1$ and $\nu = \gamma \tau$. In the Bloch representation, we can rewrite the random telegraph noise channel by using the mapping matrix and vector,

$$\Gamma_{RTN} = \text{Diag} [\epsilon (\nu), \epsilon (\nu), 1], \quad \lambda_{RTN} = 0$$

(16)

where the mapping matrix $\Gamma_{RTN}$ has the diagonal form. If the Dirac particle A suffers from the dissipation of random telegraph noise, the correlation matrix $S$ in the pauli basis is obtained by

$$S = \text{Diag} \left[ -\pi \eta (\nu) q_\mu \cos r, -\pi \eta (\nu) q_\mu \cos r, -\pi |q_\mu|^2 \cos^2 r \right]$$

(17)

In accordance with the results of Equations (14) and (15), the measures of quantum coherence and quantum steering are obtained by $C(\rho_{AB}) = \min_0 (\sqrt{2} n \eta (\nu) q_\mu \cos r)$ and $Q(\rho_{AB}) = \max_0 (\frac{\sqrt{2} n \eta (\nu) q_\mu \cos r}}{\sqrt{2} - 1})$. The interplay between the external noise and quantum fluctuations near the event horizon of the black hole is numerically analyzed. It is seen that the maximum of quantum nonlocality can be attainable in the case of $\eta = 1, q_\mu = 1$. The dynamical behavior of quantum coherence with the reservoir evolution time $\tau$ and dilato field $D$ is illustrated in Figure 2. In the condition of weak couplings $\left( \frac{\beta}{\gamma} \right)^2 < 1$, the values of quantum coherence are monotonically decreased with the increase of the reservoir parameter in Figure 2. The dilaton effect suppresses quantum coherence when the dilaton parameter is large. In the strong coupling of $\left( \frac{\beta}{\gamma} \right)^2 > 1$, Figure 3 demonstrates that the revivals of quantum coherence oscillate with time. The result shows that the quantum fluctuation from the external reservoir leads to the oscillation of quantum coherence. The similar behavior of quantum steerable can be shown in Figure 4. In the evolution process, the values of quantum steerability are diminished to zero in the weak coupling case. While
The death and revival of the steerability are presented in the strong coupling condition.

It is interesting to make a further study about the asymmetric properties of quantum steering with respect to different observers. The geometric features of quantum steering ellipsoids are investigated. The observing behaviors of quantum steering are different from those of quantum correlation and entanglement. If an arbitrary positive operator valued measure (POVM) is applied to the particle \(i\), the steered state of particle \(j\) is given by \(\rho_{ij}\) whose steering features are geometrically described by the ellipsoid center \(\bar{c}_{ij}\) and orientation matrix \(\bar{m}_{ij}\),

\[
\bar{c}_{ij} = \frac{\mathbf{v}_i - S^T \mathbf{v}_j}{1 - |\mathbf{v}_j|^2} \quad (i,j = A, B),
\]

\[
\bar{m}_{ij} = \frac{1}{1-|\mathbf{v}_j|^2}(S^T - \mathbf{v}_j \mathbf{v}_j^T)(I + \frac{\mathbf{v}_j \mathbf{v}_j^T}{1-|\mathbf{v}_j|^2})(S - \mathbf{v}_j \mathbf{v}_j^T).
\]

The orientation and semi-axes lengths of the ellipsoid are given by the eigenvectors and eigenvalues of the orientation matrix. In the case of \(\eta = 1, q_R = 1\), the Bloch vectors for Dirac particle \(A\) and \(B\) are expressed by \(\mathbf{v}_A = (0, 0, 0)^T, \mathbf{v}_B = (0, 0, -1)^T\). The correlation matrix \(S = \text{Diag}(-\kappa_c, -\kappa_c, -\kappa_c)\). It is easily seen that quantum steering ellipsoids with respect to different observers are demonstrated by the different centers and different orientation matrices.

These results demonstrate that the evolutions of quantum coherence and steerability are dependent on the interplay between the external noise and quantum fluctuations near the event horizon of the dilatonic black hole. The non-monotonic decoherence effects from the external environment can protect quantum nonlocality against the information loss from the relativistic effects of the dilatonic black hole.

4. Discussion

The effects of black hole’s dilaton and external reservoir fluctuation on the quantumness for Dirac particles are investigated in the unified quantum channels. The process of information loss from quantum fluctuation near the event horizon of the black hole is modeled by the quantum channel beyond single mode approximation. The geometric feature and quantity of quantumness for the channel are obtained. We find out that the quantumness for Dirac particles escaping from the outside region can be preserved to a high extent under the circumstance of \(q_R \to 1\) and small dilaton. In order to explore the decoherence of quantum nonlocality, we also put forward a general method by using the Pauli basis. The interplay between the external environmental noise and relativistic effects of the dilatonic black hole can be further studied. For an instance of external reservoir, the random telegraph noise channel is chosen. In the weak coupling condition, the monotonous degradation of quantum coherence and quantum steering with time can be observed when the dilaton field is increased. The oscillation revivals of quantum coherence and steerability are also presented when the strong coupling is
considered. From the viewpoint of relativistic quantum information, this kind of non-monotonous decoherence contributes to the protection of quantum resources from the information loss near the event horizon of the black hole.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Research data are not shared.

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