Electron hydrodynamic instabilities stimulated by asymmetry

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Abstract. Direct current in confined two-dimensional (2d) electron systems can become unstable with respect to the excitation of plasmons. Numerous experiments and simulations hint that structural asymmetry somehow promotes plasmon generation, but a constitutive relation between asymmetry and instability has been missing. We provide such relation in the present paper and show that bounded perfect 2d electron fluids in asymmetric structures are unstable under arbitrarily weak drive currents. To this end, we develop a perturbation theory for hydrodynamic plasmons and evaluate corrections to their eigenfrequency induced by carrier drift, scattering, and viscosity. We show that plasmon gain continuously increases with degree of plasmon mode asymmetry until it surrenders to viscous dissipation that also benefits from asymmetry.

1. General overview
In high-mobility semiconductors, the electrons at room temperature demonstrate a fluid-like (hydrodynamic) behaviour [1]. Several remarkable manifestations of electron hydrodynamics include negative resistance due to viscous flow [2], higher-than-ballistic conduction of constrictions [3], and strong violations of Wiedemann-Franz law [4]. An intriguing yet weakly explored area of electron hydrodynamics is related to turbulence, i.e. instabilities of electron drift. A strong effort in this field was triggered by the statement that HD electron flow in a confined gated two-dimensional electron system (2DES) is unstable with respect to the self-excitation of plasmons under certain boundary conditions, now known as Dyakonov-Shur (DS) instability [5]. An observable outcome of instability in confined 2DES is the electromagnetic emission due to radiative plasmon decay [6].

The boundary conditions providing instability in original work of DS assumed zero alternating current at the drain, which is hardly fulfilled in real transistors. A successful demonstration of instability-induced THz emission in an ordinary HEMT [7] in spite of boundary condition problem stimulated dozens of instability proposals in related systems, including ungated [8] and partly gated transistors [9], Corbino discs [10], and edges of 2DES [11]. Both experiments [12, 13] and theories [9, 14] for numerous plasmonic structures hinted that it is structural asymmetry that drives the instability, but not a specific boundary condition. However, the link between asymmetry and instability remained elusive so far, contrary to the well-established symmetry constraint for the inverse process, photodetection.

In this paper, we provide a constitutive relation between asymmetry of confined 2DES-based plasmonic resonators and stability of their plasmon modes. To this end, we develop an analogue
of quantum-mechanical perturbation theory for hydrodynamic equations in confined 2DES and express the growth/decay rates of plasmons through their field distributions in the absence of drift and dissipation. We show that the necessary condition for instability is the asymmetry of plasmon field: the mode gain turns to zero for even and odd field distributions, but is generally nonzero for modes without certain parity. The asymmetry in confined 2DES can be induced by inequivalence of source and drain contacts (which is the case of famous DS instability), asymmetry of gating environment, non-uniform doping, or all of them. The developed theory can be used for design of plasmonic oscillators with high gain and/or low threshold current.

2. Perturbation theory for electron hydrodynamics

Within this section, we shortly discuss the main points of our theory and its possible applications. A more detailed analysis can be found in [15].

Rewriting linearized hydrodynamic equations of a 2d charged fluid in an operator representation we managed to construct an analogue of the quantum-mechanical perturbation theory for an arbitrary transistor structure. It leads to the following expression for the plasmon eigenfrequency correction \( \delta \Omega \):

\[
\delta \Omega_{\lambda} = i \frac{1}{\Pi} \left\{ j_0 [K(L) - K(0)] - Q_{\text{loss}} \right\},
\]

where \( K(x) = m u_{\lambda}(x)^2/2 \) is the local kinetic energy in a plasmon mode, \( u_{\lambda} \) is the velocity variation, \( m \) – electron effective mass, \( \lambda \) enumerates plasmonic modes,

\[
\Pi = e^2 \int_0^L dx dx' n_{\lambda}(x) G(x, x') n_{\lambda}(x')
\]

is the potential energy of interacting charge density fluctuations, \( n_{\lambda} \) stands for electron density variation, \( G \) is the non-local Green’s function of the electrode system, the integration is performed over the transistor channel of length \( L \) and

\[
Q_{\text{loss}} = \frac{1}{2} \int_0^L dx \left\{ \frac{e^2 n_0}{m \Omega^2 \tau_p} E_{\lambda}^2 - u_{\lambda} \partial_x [\eta \partial_x u_{\lambda}] \right\}
\]

is the energy loss due to viscous friction and momentum non-conserving scattering, where \( e \) denotes the elementary charge, \( n_0(x) \) – electron density profile in the channel, \( \Omega \) – plasmon eigenfrequency, \( \tau_p \) – momentum scattering rate, \( E_{\lambda} \) – electric field in the channel, \( \eta \) – dynamic viscosity of the electron fluid.

Equation (1) is the main point of our theory. We readily observe that the origin of plasmon growth, \( \text{Im} \delta \Omega > 0 \), is the excess of kinetic energy entering the mode at source over the energy lost at the drain. The necessity for asymmetry to achieve wave growth now becomes apparent. Indeed, for zero-order functions with certain parity \( u_{\lambda}(L) = u_{\lambda}(0) \), and only the loss term retains in (1). For modes without parity, the compensation of ingoing and outgoing energy can appear only accidentally. Therefore, the plasmon modes of any asymmetric nanostructure with perfect electron fluid (\( \eta = 0, \tau_p^{-1} = 0 \)) are unstable with respect to arbitrary weak drive current. This property hallmarks the instabilities of confined hydrodynamic plasmons from Smith-Purcell [16] and beam [17] instabilities in extended systems. The latter typically develop above the threshold drift velocity order of plasmon phase velocity.

The need for asymmetry becomes even more apparent from Fig.1, top, where we show the dependencies of plasmon growth rate for a partly gated transistor structure (shown on the inset) at different gate lengths and density ratios. The orange line shows the growth rates of
Figure 1. Top: calculated instability growth rates for the third plasmon mode in a partly gated FET (shown in inset) with different gate lengths and carrier densities. The growth rates are normalized by $u_0/L$, where $u_0$ is the drift velocity at the drain (same for all curves), the density at the drain $n_0$ is also fixed. The instability benefits if the drift is directed from low- to high-density region and is especially pronounced if low-density region is ungated. A structure with uniform density (orange line) also supports instabilities due to the asymmetry of electrical environment. Bottom: spatial distribution of plasmon potential in partly gated FET for two modes: (1) with almost symmetric in-plane distribution of potential and zero gain (2) with highly asymmetric in-plane potential and high gain.
plasmon modes in FET with uniform carrier density and metal gate adjacent to drain side. It demonstrates an oscillatory behaviour with varying gate length with a relatively small maximum growth rate \( \sim u_0/L \), where \( u_0 \) is the drift velocity in the ungated region. Addition of extra density asymmetry, \( n_g \neq n_u \), results in much more pronounced instability (for \( n_g > n_u \)) or more pronounced current-induced plasmon decay (for \( n_g < n_u \)).

Discussion
With relation (1), it becomes possible to study the 2d plasmon instabilities without going into complicated simulations of turbulent flows [14]. All necessary ingredients to judge on the possibility of gain are the plasmon mode profiles \( \varphi_0(x) \) ("zero-order functions") that can be obtained with commercial electromagnetic simulators. Our perturbative expressions for plasmon eigen frequency allowed to reveal the competition between current-driven driven and viscosity both of which increase with the degree of asymmetry. Moreover, they allowed us to find an "optimal degree of asymmetry" for which the plasmon gain is highest or instability threshold current is the lowest. There exists a universal lower bound of Reynolds number for development of plasmon instability in 2DES with fixed voltage drop between source and drain, which equals \( R_{\text{min}} = 2\sqrt{3} \).

References
[1] Gurzhi R 1968 Physics-Uspekhi 11 255–270
[2] Bandurin D A, Torre I, Kumar R K, Ben Shalom M, Tomadin A, Principi A, Auton G H, Khestanova E, Novoselov K S, Grigorieva I V, Ponomarenko L A, Geim A K, Polini M 2016 Science 351 1055–1058
[3] Kumar R K, Bandurin D A, Pellegrino F M D, Cao Y, Principi A, Guo H, Auton G H, Ben Shalom M, Ponomarenko L A, Falkovich G, Watanabe K, Taniguchi T, Grigorieva I V, Levitov L S, Polini M and Geim A K 2017 Nat. Phys. 13 1182–1185
[4] Crossno J, Shi J K, Wang K, Liu X, Harzheim A, Lucas A, Sachdev S, Kim P, Taniguchi T, Watanabe K, Ohki T A, Fong K C 2016 Science 351 1058–1061
[5] Dyakonov M and Shur M 1993 Phys. Rev. Lett. 71, 2465
[6] Chaplik A 1985 Surf. Sci. Rep. 5 289
[7] Knap W, Lusakowski J, Parenty T, Bollaert S, Cappy A, Popov V and M. Shur 2004 Appl. Phys. Lett. 84 2331
[8] Dyakonov M and Shur M 2005 Appl. Phys. Lett. 87 111501
[9] Petrov A S, Svintsov D, Rudenko M, Ryzhii V and Shur M S 2016 Int. Jour. High Speed Electron. Syst. 25 1640015
[10] Sydoruk O, Symes R and Solymar L 2010 Appl. Phys. Lett. 97 263504
[11] Dyakonov M 2008 Semicond. 42 984
[12] El Fatimy A, Dyakonova N, Meziani Y, Otsuji T, Knap W, Vandenbrouck S, Madjour K, Theron D, Gaquiere C, Poisson M, Delage S, Prystawko P and Skierbiszewski C 2010 J. Appl. Phys. 107 024504
[13] Boubanga-Tombet S, Yadav D, Knap W, Popov V V and Otsuji T 2018 Plasmonic instabilities and terahertz waves amplification in graphene metamaterials arXiv preprint arXiv:1801.04518
[14] Koseki Y, Ryzhii V, Otsuji T, Popov V V and Satou A 2016 Phys. Rev. B 93 245408
[15] Petrov A S, Svintsov D 2018 arXiv preprint arXiv:1802.03994
[16] Mikhailov S 1998 Phys. Rev. B 58 1517
[17] Krasheninnikov M and Chaplik A 1980 Sov. Phys. JETP 79 555