Steady Flow Profile Analysis of Ciliwung River Using Standard Step Method Simultaneous Procedures

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Abstract. The aim of this paper is to examine steady flow profiles over an inclined channel. Different steady flow profiles may appear depending on the type of flow, i.e. subcritical or supercritical. Here, the steady Saint-Venant model is adopted, its solution can be obtained through the application of the numerical integration method (trapezoid). When the water levels upstream and downstream of the channel are known, water level along the channel can be obtained as the solution of a non-linear system of equations, which is then solved by the Newton iteration. This method is known as the standard step method of simultaneous procedure. Furthermore, simulation is done using the measurement data at the downstream part of Ciliwung River. Field measurement data were collected in terms of the water level along the river, from TB Simatupang to PA. Manggarai. The result of numerical simulation is shown a good agreement with field measurement data.

1. Introduction

The river is a large and longitudinal flow of water that flows continuously from upstream to downstream [1, 2]. The shape of the water flow is called the flow profile. In reality various steady stream profiles can be formed, depending on the type of flow, for example subcritical or supercritical. Indeed the interesting case of this research is to find the profile of water height on inclined river, see Figure 1 for instance.

In this paper, the water level at the ends of the channel is known. However, the water discharge at the ends of the channel is unknown. Using numerical method, the water level and discharge along the river will be calculated. The mathematical equation that will be used is derived from the Saint-Venant equation for water flow in an unsteady state which is written as follows:

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q, \tag{1}
\]

\[
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\alpha \cdot Q^2}{A} \right) + g \cdot A \frac{\partial h}{\partial x} = -g \cdot A \cdot S_f. \tag{2}
\]
Figure 1: Sketch of a river with inclined bottom. The notations: $h_u$ is upstream height, $h_d$ is downstream height, $H$ is the average of water depth, $Z$ is the height of topography and $L$ is the long of river.

where, $t$ is the time, $x$ is the longitudinal distance, $h$ is the water level, $q$ is the lateral inflow, $\alpha$ is the momentum correction factor, $S_f$ is the friction slope caused by channel resistance.

Several ways are used to describe the flow profile under certain conditions, one of which uses numerical methods. To get a water level between the end of the channel and flow discharge, a numerical scheme to solve the boundary value problem is needed. Because the discharge value is unknown, the boundary value problem that will be solved as a non-linear equation system. The water level between the end of the channel and the flow discharge is the solution of the non-linear equation system that is solved by standard step method simultaneous procedures. The standard step method simultaneous procedures used because this method can solved the problem in any situation, the detail explanation can be found in [3]. Flow profiles that have been obtained will then be analyzed for the slope, location, and type.

Here, the numerical simulation will be carried out to illustrate the steady flow profile on the Ciliwung river in Jakarta. The Ciliwung River data used for the simulation was obtained directly from BBWS Ciliwung Cisadane Jakarta. Finally the organization of this paper is given as follow, in Section 2, open channel flow model will be given. Section 3, the numerical scheme which is called simultaneous solution procedure is elaborated. The sketch of flow profile for simulation is briefly explained in Section 4. Moreover, the discussion of results and conclusion are given in Section 5 and 6 respectively.

2. Open channel flow

The open channel Flow is a channel with surface is free, a cross section of the current is taken perpendicular to the flow velocity vector [4, 5, 3]. Open channel are divided into two parts, prismatic and non-prismatic. Figure 2 (left) shows that the cross section and the bed slope in prismatic is fixed. All artificial channels such as sewers are included in the prismatic channels. Non-prismatic channels (Figure 2 (right)) have varying cross section and bed slope, all natural channels are included in this channel.
2.1. Saint-Venant equation

Saint-Venant equation is derived from the Navier-Stokes equation, which describes the motion of fluids [6, 7, 8]. Mostly, Saint-Venant equation is used to model shallow water equation which has general characteristic, where the vertical dimension is much smaller than the typical horizontal scale. Here, Saint-Venant equation consists of two equations (1) and (2). The formula of 1-D Saint-Venant equation system for open channel flow in steady condition can be written as:

\[
\frac{dQ}{dx} = q,
\]

\[
\frac{dE}{dx} = \frac{d}{dx} \left( h + \alpha \cdot \frac{Q^2}{2g \cdot A^2} \right) = -S_f - \alpha \cdot \frac{Q}{g \cdot A^2} q.
\]

where \( E \) is the total head energy, \( g \) is the gravitational acceleration, \( Q \) is the discharge rate of the flow and \( S_f \) is the friction slope caused by channel resistance.

2.2. Normal and critical depth

Based on the governing equation (4), the total energy \( E \) is function of water depth \( H \) where \( Q \) is given and \( A \) depends on \( H \). This means the specific energy \( E_s \) is the relative energy on the bottom channel also function of \( H \). The formula shown by the following equations:

\[
E_s = H + \frac{\alpha \cdot Q^2}{2g \cdot A^2}.
\]

The specific energy profile as a function of \( H \) when discharge \( Q \) is given, can be shown in Figure 3. The curves in Figure 3 show that for the certain specific energy \( E_s \), there are two possible of depths, called alternate depth. The condition when \( E_s \) minimum is known as the critical condition. In this case there is only one depth denoted as \( H_c \) that corresponds to energy \( E \). Since \( H_c \) is an extreme minimum point, \( dE_s/dH \) at \( H_c \) should be equal to zero. This yields a formula presented in (7).

\[
\frac{dE_s}{dH} = 0,
\]

\[
\frac{Q^2}{g} = \frac{A^3}{B}.
\]

Here, the value of \( H_c \) is obtained by finding the root of (7). On other side, normal depth \( H_n \) is a depth in uniform flow. Uniform flow is an open channel flow where the flow velocity and depth do not change along a certain length of the channel [5]. This mean normal depth \( H_n \) can be obtained when the channel resistance balances out the weight force, hence the resultant force is zero and there is no acceleration.
Since the depth and the acceleration not changed, the equation (4) can be written as:

$$\frac{dZ}{dx} = -s = -S_f,$$

where $Z$ is the datum and $s$ is the slope of the channel.

It can be seen that (8) involves the channel resistance. There are several equations which can be used to model the channel resistance, with each equation uses different approaches and various variables. For the model in this paper, Manning equation [9, 10] is used. The formula of Manning equation is given as,

$$S_f = \frac{n_m^2 \cdot Q^2}{R^{1/3} \cdot A^2},$$

(9)

where $n_m$ is Manning roughness coefficient and $R$ is hydraulics radius. Substituting (9) into (8), then formula for $H_n$ is given as

$$Q = \frac{1}{n_m} R^{2/3} \cdot s^{1/2} \cdot A(H).$$

(10)

Here, $H_n$ is the root of (10).

3. Numerical scheme

3.1. Numerical integration for Saint-Venant equation

As explained in the previous section, the governing equation of the model is (4). However, (4) is a differential equation. Thus, to obtain the solution, integration steep is needed.

Suppose the computational domain $[0, L]$ is discretized into $N$ partitions. Thus there will be $N + 1$ grids with the interval of $\Delta x$ unit length. Assuming $q = 0$, integration of (4) can be written as:

$$\int_{E_i}^{E_{i+1}} dE = - \int_{x_i}^{x_{i+1}} S_f \, dx,$$

(11)

where $E_{i+1}$ and $E_i$ are the total energy head at point $x_{i+1}$ and $x_i$. The integral of $S_f$ with respect to $x$ from $x_i$ to $x_{i+1}$ can be approximated numerically with various methods [11, 12]. Here, the
approximation is done with trapezoidal rule. Applying the trapezoidal rule towards (11) and further by elaborating the total energy head and the Manning formula, after rearrangement, we obtain:

\[
\left( Z_{i+1} + H_{i+1} + \frac{Q^2}{2g \cdot A_{i+1}^2} \right) = \left( Z_i + H_i + \frac{Q^2}{2g \cdot A_i^2} \right) - \frac{\Delta x}{2} \left( \frac{n_m^2 \cdot Q^2}{R_i^{4/3} \cdot A_i^2} + \frac{n_m^2 \cdot Q^2}{R_{i+1}^{4/3} \cdot A_{i+1}^2} \right),
\]

(12)

in further discussion, equation (12) will be solved using the simultaneous procedure.

3.2. Simultaneous solution procedure
Simultaneous solution procedure can be seen in [5] in detail. The goal is to obtain the solution of (12) numerically. Assume the channel length of \( L \) is divided into \( N \) points, with length \( \Delta x_i (i = 1, 2, 3, ...) \). Here, implementing (12) for all cells depicted in Figure 4, will yield \( N - 1 \) algebraic equations with \( N + 1 \) \( H_i, i = 1, ..., N \) and \( Q \). Two others equations will be obtained from evaluating (12) for the first cell and the last cell, respectively; they reads:

\[
H_1 + Z_1 + \frac{\alpha Q^2}{2 \cdot g \cdot A^2} - h_u = 0,
\]

(13)

\[
H_N + Z_N + \frac{\alpha Q^2}{2 \cdot g \cdot A^2} - h_d = 0,
\]

(14)

where as \( h_u, h_d \) denote upstream, downstream water levels, respectively.

For each cells, the equation is given as follow

\[
\frac{\Delta x}{2} \left( \frac{n_3^2 \cdot Q^2}{R(\text{H}_{N+1})^{4/3} A(\text{H}_{N+1})^2} + \frac{n_3^2 \cdot Q^2}{R(\text{H}_N)^{4/3} A(\text{H}_N)^2} \right) = Z_N - Z_{N+1}
\]

\[
\frac{\Delta x}{2} \left( \frac{n_3^2 \cdot Q^2}{R(\text{H}_{N+1})^{4/3} A(\text{H}_{N+1})^2} + \frac{n_3^2 \cdot Q^2}{R(\text{H}_N)^{4/3} A(\text{H}_N)^2} \right) = Z_N - Z_{N+1}
\]
Since $Q$ value is unknown, the equation (15) is formulated into a non-linear condition. Further, the equation is formed into a linear equation below:

$$AX = B \tag{16}$$

where,

- $A$, coefficient matrix with $[N+1 \times N+1]$ dimension. The value of matrix $A$ can be found from (15).
- $B$, vector of known variable.
- $X$, vector of unknown variable ($H_1, H_2 \cdots H_N, Q$)

Moreover, structure of 16 is described in figure 5.

![Figure 5: Structure of matrix $A$, vector $X$ and vector $B$.](image)

### 4. Sketch of flow profile

The water surface profile along the channel describe how the flow depth changes longitudinally [5, 3, 10]. Profiles are be classified based on the relationship between the actual water depth ($H$), the normal depth ($H_n$) and depth critical ($H_c$).

Generally, the basic slope of the channel is classified into five category, namely:

- **Mild slope** (M) when $H_n > H_c$.
- **Steep slope** (S) when $H_n < H_c$.
- **Critical slope** (C) when $H_n = H_c$.
- **Adverse** (A) if $s < 0$ (negative slope).
- **Horizontal** (H) if $s = 0$ (zero slope).

Figure 6 shows flow profile of each category created in different area called zone. Qualitative observations of flow profiles can be done by simplifying the equation (3) and (4). So that the equation is obtained:

$$\frac{dH}{dx} = \frac{s - S_f}{1 - F_r^2} \tag{17}$$
where, \( F_r \) is the Froude number.

By determining the numerator and denominator of equation (17), then sketches of the flow profile can be determined without calculation. In [5], the first step is to make some general statements, namely:

- water depth (\( H \)) will increase with distance when the value of \( \frac{dH}{dx} \) is positive, and
- water depth (\( H \)) will decrease with distance when the value of \( \frac{dH}{dx} \) is negative.

In [5], it explained that \( S_f = S_w = s \) if the value of \( H = H_n \). Therefore, it is obtained from the Manning equation, for a certain \( Q \),

\[
S_f > s \quad \text{if} \quad H < H_n,
\]

and

\[
S_f < s \quad \text{if} \quad H > H_n.
\]

The relationship above is used to determine the numerator. Whereas to determine the denominator, the following relationship is used:

\[
H > H_c \quad \text{if} \quad F_r^2 < 1,
\]

and

\[
H < H_c \quad \text{if} \quad F_r^2 > 1.
\]

When \( H \rightarrow H_n \), then \( S_f \rightarrow s \), equation (17) requires \( \frac{dh}{dx} \rightarrow 0 \). This means that the water level \( H \) goes to asymptotic \( H_n \). When \( H \rightarrow H_c \), then \( F_r^2 \rightarrow 1 \), and the equation (17) indicates \( \frac{dh}{dx} \rightarrow \infty \). This means that the water level of \( H \) goes to \( H_c \) vertically.
5. Results and discussion

In this paper, simulations were carried out in two different trapezoidal sides slopes and areas. The first area is Kalibata bridge - P.A. Manggarai, we called domain 1 and the second area is TB. Simatupang - Kalibata bridge, we called domain 2. The first sides slope \(MS_1\) is calculated by linear regression from the original topographic data. Moreover, second side slope \(MS_2\) is obtained by calculating the slope of a straight line drawn from the bottom surface to the surface of the original topographic data. This calculation is done with the gradient \(\cot(\Theta)\). Topographic data from each simulation area, is shown in Table 1.

Table 1: The topographic of each channels.

| Topographic                        | Channels                              |
|-----------------------------------|---------------------------------------|
| Channel length \((L)\)            | Kalibata bridge - P.A. Manggarai       |
|                                   | TB. Simatupang - Kalibata bridge      |
| Bottom width \((b)\)              | 10620 m                               |
|                                   | 1150 m                                |
| First trapezoid sides slope \((MS_1)\) | 14 m                                 |
|                                   | 14 m                                  |
| Second trapezoid sides slope \((MS_2)\) | 4.45                                 |
|                                   | 4.45                                  |
| River slope \((s)\)               | 2.86                                  |
|                                   | 2.86                                  |
| Manning coefficient \((n_M)\)      | 0.030                                 |
|                                   | 0.030                                 |
| Water level at upstream \((h_u)\) | 14.88                                 |
|                                   | 22.2                                  |
| Water level at downstream \((h_d)\) | 8.82                                 |
|                                   | 21.78                                 |
| Bottom height at upstream \((Z_u)\) | 9.20 m                             |
|                                   | 15.3 m                                |
| Bottom height at downstream \((Z_d)\) | 3.66 m                             |
|                                   | 14.65 m                               |

The discrete domain used is \(\Omega [0, L] \ m\), partition width \(\Delta x = 106.2\) and \(\Delta x = 11.5\) are used for domain 1 and domain 2 respectively, the base height of the channel is calculated by the equation \(Z = -sx + Z_u\), the initial guess for \(Q = 100 \ m^3/s\), the initial guess for \(H\) is written with the equation \((h_u - Z(1) + h_d - Z(N))/2\).

5.1. Simulation result in domain 1

The results of simulation in Kalbiata bridge - P.A. Manggarai are shown in Table 2. In Table 2, the results are obtained using two different slopes \((MS_1\) and \(MS_2\). Here, the value of \(MS_1\) = 4.45 and \(MS_2\) = 2.86 are obtained from approximation of observation data (see Table 1). Using slope from linear regression \((MS_1\), high value of discharge is obtained, 342.07 \(m^3/s\). Meanwhile using approximation of straight line \(MS_2\), 272.28 \(m^3/s\) of discharge is gained.

Table 2: Simulation result in Kalbiata bridge - P.A. Manggarai.

| Variables                  | Trapezoid side slopes |
|----------------------------|-----------------------|
| Discharge \((Q)\)          | 342.07 \(m^3/s\)     |
| Froude number \((F_r)\)    | 0.0165                |
| Friction slope \((S_f)\)   | 0.00057               |
| Normal depth \((H_n)\)     | 5.45                  |
| Critical depth \((H_c)\)   | 2.90                  |
| Friction slope \((S_f)\)   | 0.00057               |
| Normal depth \((H_n)\)     | 5.44                  |
| Critical depth \((H_c)\)   | 2.78                  |

Sketch of flow profile in entire of domain is shown in Figure 7. From Figure 7, we can see the water depth of \(H\) for each trapezoid side slopes (A and B) have a small difference. Depth
of $H$ at a trapezoidal slope of $MS_1$ is higher than the trapezoidal slope of $MS_2$. The side slope is mild, because condition $H_n > H_c$ is observed.

Moreover form Figure 7, since $S < s$ and $F_r^2 < 1$, then analysis of flow profile with (17) is obtained:

$$\frac{dH}{dx} = s - S - F_r^2 = + = +.$$ (18)

This shows that this simulation resulting the flow profile formed in zone 2 ($M_2$) (see Figure 6 for more detail).

5.2. Simulation result in domain 2
The results of simulation at TB. Simatupang - Kalibata bridge are shown in Table 3. Here the discharge of slopes $MS_1$ and $MS_2$ is obtained $486.54 \text{ m}^3/\text{s}$ and $376.54 \text{ m}^3/\text{s}$ respectively. Similar in domain 1, here, the discharge from $MS_1$ is higher than discharge from $MS_2$.

| Variables       | Trapezoid side slopes | $MS_1$ | $MS_2$ |
|-----------------|-----------------------|--------|--------|
| Discharge ($Q$) | 486.54 m$^3$/s        | 376.54 m$^3$/s |
| Froude number ($F_r$) | 0.0115               | 0.0130 |
| Friction slope ($S_f$) | 0.000365            | 0.000365 |
| Normal depth ($H_n$) | 6.38 m               | 6.35 m |
| Critical depth ($H_c$) | 3.50 m             | 3.30 m |

Sketch of flow profile is shown in Figure 8. The results shown that, the side slope is mild. The analysis of flow profile with (17) is given as:

$$\frac{dH}{dx} = s - S - F_r^2 = + = +.$$ (19)

Since $H > H_n$, then flow profile formed in zone 1 ($M_1$), see Figure 6 for more detail.
5.3. Comparison of numerical result and observation data
The simulation results of the Ciliwung river flow profile in the Kalibata bridge area - P.A. Manggarai and the area between TB. Simatupang - Kalibata bridge with two different trapezoidal slopes has been shown in Figures 7 and 8.

Here, the comparison of numerical results with observation data can be seen in Figures 9 and 10. This comparison is given in order to measure the robustness of numerical method.

From Figure 9, comparisons of observation data and numerical result using slope $MS_1$ and $MS_2$ are shown in a good agreement. Even though in area 4000-6000 meters of channel, small difference height between observation data and numerical result still can be seen for both slopes.

Similarly in Figure 10, it can be seen that the discrepancy between observation data and numerical results is observed small. Numerical simulation is shown satisfying approximate the observation data for both different slopes.

In order to measure the closeness of numerical result and observation data, the error can be given. The error value of each simulations is calculated by Root Mean Square Error (RMSE). The results of RMSE can be found in Table 4.

In Table 4, RMSE for domain 1 using side slope $MS_1$ and $MS_2$ is obtained 0.2769 and 0.2894 respectively. Moreover, RMSE using side slope $MS_1$ and $MS_2$ in domain 2 is obtained 0.1184.
Figure 10: The comparison of numerical result with observation data at TB. Simatupang - Kalibata bridge. A. trapezoid slope $MS_1$ and B. trapezoid slope $MS_2$.

Table 4: Root Mean Square Error (RMSE) results for each simulations.

| Simulations area                          | Trapezoid side slopes |
|------------------------------------------|------------------------|
| Kalibata bridge - P.A. Manggarai         | $MS_1$ 0.2769, $MS_2$ 0.2894 |
| TB. Simatupang - Kalibata bridge        | $MS_1$ 0.1184, $MS_2$ 0.1245 |

and 0.1245 respectively. Here using $MS_1$, which is slope from linear regression is observed has smaller error than using rough straight line approximation ($MS_2$).

6. Conclusion

The simulations of steady flow profile in Ciliwung river has been successfully carried out. In the Kalibata bridge - P.A Manggarai (domain 1) using two different trapezoid side slopes, the result of flow profile is mild in zone 2 (M2). Meanwhile, in the area between the TB. Simatupang - Kalibata bridge (domain 2) flow profile is mild in zone 1 (M1). Numerical simulation results show small differences from the observation data. In domain 1, RMSE using slopes $MS_1$ and $MS_2$ is observed 0.2769 and 0.2894 respectively. In domain 2, using slope $MS_1$, RMSE is obtained 0.1184 and using slope $MS_2$, RMSE is gained 0.1245. From this numerical simulation, it can be said that numerical results from the standard step method simultaneous procedures for Ciliwung river flow profile simulation in the Kalibata bridge - P.A. Manggarai and TB. Simatupang - Kalibata bridges is accurate.

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