Gravitational redshift of galaxies in clusters from SDSS and BOSS

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The gravitational redshift effect allows one to directly probe the gravitational potential in clusters of galaxies. As such, it provides a fundamental test of general relativity (GR), and may help to constrain alternative theories of gravity. Following up on Wojtak, Hansen & Hjorth (2011), we present a new measurement. We take advantage of new data from the tenth data release of SDSS and BOSS, covering a range of redshift between 0.05 and 0.6. After selection, our dataset includes 60k galaxies, matched to 12k clusters, with an average cluster mass of $10^{14} M_\odot$. The analysis is focused on optimizing the selection method of clusters and of galaxies, taking into account possible systematic biases. We compare the light originating from the brightest cluster galaxies (BCGs), to that of galaxies at the outskirts of clusters. We find that BCGs have an average relative redshift of $-11 \pm 5$ km/s, with a standard deviation of $+7$ and $-5$ km/s. The result is consistent with the measurement of Wojtak et al., and is in good agreement with the predictions from GR. Considering the current systematic uncertainties, we can not distinguish between the baseline GR effect and the recently proposed kinematic modifications.

INTRODUCTION

The gravitational redshift (GRS) effect in clusters of galaxies is a feature in any metric theory of gravity. It is caused by the spatial variation of the gravitational potential; light traveling from deeper in the potential of a cluster is expected to be redshifted, compared to light originating from the outskirts of the cluster [1]. The GRS effect has the potential to constrain theories in which there are long-range non-gravitational forces acting on dark matter, modifying gravity on cluster scales [2]. The effect was first measured by Wojtak, Hansen & Hjorth (WHH) [3], a study which was subsequently repeated (with minor modifications) by Dominguez-Romero et al. [4].

WHH used 125k spectroscopic redshifts, taken from the seventh data release (DR7) [5] of the Sloan Digital Sky Survey (SDSS) [6], matched to 7.8k clusters from the GMBGC cluster catalog [7]. They used the brightest cluster galaxies (BCGs) as a proxy for the centers of clusters. They assumed that, in general, BCGs have relatively small velocity dispersions compared to other bound galaxies, and reside close to the bottom of the gravitational potential. WHH divided their galaxy sample into four bins, based on the transverse distance between cluster-galaxies and respective BCGs, $r_{ gc }$, extending up to 6 Mpc. In each bin, they calculated the velocity of galaxies in the rest-frame of the BCG,

$$v_{ gc } = c \frac{\bar{z}_{ gal } - \bar{z}_{ BCG }}{1 + \bar{z}_{ BCG }},$$

(1)

where $z_{ BCG }$ and $z_{ gal }$ respectively stand for the redshift of BCGs and of associated galaxies, and $c$ is the speed of light. On average, BCGs were found to have a redshift difference, $\Delta v_{ gc } \approx -10$ km/s, relative to the other galaxies.

WHH calculated the prediction for the signal from general relativity (GR), as well as from modified theories of gravity [8]. They first derived the GRS profile of a single cluster in the weak field limit,

$$\Delta_1(r_{ gc }) = \frac{2}{c} \int_{r_{ gc } }^{\infty} \left[ \Phi(r) - \Phi(0) \right] \frac{\rho(r) r}{\sqrt{r^2 - r_{ gc }^2}} dr,$$

(2)

where $\Phi$ is the gravitational potential, and $\rho$ and $\Sigma$ are respectively the 3D- and surface-density profiles of galaxies. They then convolved $\Delta_1$ with the distribution of cluster masses in their sample, estimated from the observed velocity dispersion profile, using stacked NFW models [9].

Subsequent works, notably those of Zhao et al. [10] and of Kaiser [11], modified the theoretical prediction; these took into account effects such as the so-called transverse Doppler shift and surface brightness modulation. The added corrections were found to be of the same order of magnitude as the GRS signal, some inducing redshifts and some blueshifts. Summed together, the prediction of Kaiser is of a relatively flat dependence of $\Delta v_{ gc }$ on $r_{ gc }$, with a mean value of $-9$ (GR only) or $-12$ km/s (GR and kinematic effects).

The purpose of this study is to revise the measurement of WHH. The main modifications are as follows:

- we reprocessed, higher-quality spectra, derived from the tenth data release (DR10) [12] of the SDSS, as well as measurements taken with the Baryon Oscillation Spectroscopic Survey (BOSS) [13]. Our initial sample, equivalent in composition to the WHH dataset, included 426k galaxies, matched to 31k clusters. This constituted a fourfold increase in the number of analyzed spectra, and an extension of the maximum redshift from 0.4 to 0.6;
the large dataset allowed us to compare alternative selection methods, to remove overlapping cluster-configurations and to address ambiguities in cluster-galaxy association;

- we analyzed the galaxy-velocities in $r_{gc}$-bins of variable length, accounting for the intrinsic sizes of clusters; we used a sliding window to probe the dependence of the GRS signal on $r_{gc}$;

- we explored multiple possible sources of systematic bias, which were not explicitly addressed by WHH; we incorporated the uncertainties on individual redshifts into the analysis, and validated the fitting procedure of the $v_{gc}$-distribution using a model-independent method;

- finally, we compared our results with the above-mentioned, improved theoretical predictions.

In the next sections we describe the analysis in detail, following up with our results and conclusions.

**METHODOLOGY**

**Dataset**

The spectroscopic redshifts of galaxies included in DR10 were measured using the SDSS and the BOSS spectrographs; they may be divided into four target subsamples, the SDSS main and LRG galaxy samples, and the BOSS LOWZ and CMASS samples, having redshift completeness generally higher than 98% [14].

We associated the DR10 data with galaxy clusters, using the catalog of Wen, Han & Liu (WHL) [15]. The WHL sample includes $\sim 130k$ clusters, detected using a friends-of-friends algorithm, based on SDSS and BOSS photometric data. The virial radius of a cluster is commonly approximated by $r_{200}$, the radius within which the mean density of a cluster is 200 times that of the critical density of the universe [16]. The latter is additionally used to define $m_{200}$, the cluster mass within $r_{200}$. The WHL catalog is nearly complete for clusters with masses, $m_{200} > 2 \cdot 10^{14} M_{\odot}$, and redshifts, $z < 0.5$, and is $\sim 75\%$ complete for $m_{200} > 0.6 \cdot 10^{14} M_{\odot}$ and $z < 0.42$. We estimated the mass of clusters in our sample using the scaling relation between mass and optical richness for WHL clusters. Our selected cluster sample was found to have an average cluster mass, $m_{200} = 1.3 \cdot 10^{14} M_{\odot}$, with values ranging between 1 and $3 \cdot 10^{14} M_{\odot}$. This is comparable to the mean value of cluster masses in the WHH dataset (accounting for the conversion from the quoted value, to our convention of using $m_{200}$).

In the initial stage of the analysis, the spectroscopic redshifts were subjected to various quality cuts, ensuring e.g., that the uncertainty on the redshift is below $10^{-4}$, and that the confidence in the likelihood-fit of the redshift is high. We then matched galaxy spectra to BCG positions, keeping only those clusters for which the BCG had a corresponding spectrum. Additionally, each cluster had to contain at least one galaxy within transverse distance, $r_{gc} < 6$ Mpc, and velocity, $|v_{gc}| < 4,000$ km/s. Conversion from angular to physical distances was performed using a flat ΛCDM cosmology, with $\Omega_m = 0.307$ and the Hubble constant, $H_0 = 67.8$ km s$^{-1}$ Mpc$^{-1}$ [17].

The initial selection left us with 31k clusters and 426k associated galaxies, which we denote as our baseline sample. Following the selection procedure discussed in the next section, we were left with 60,626 galaxies and 12,600 clusters, having $r_{gc} \lesssim 3$ Mpc; an additional 25k galaxies and 5k clusters were used for systematic checks. The respective redshift distribution of galaxies, occupying the range 0.05 to 0.6, is shown in Fig. 1(a).

**Fitting procedure**

WHH employed a Markov chain Monte Carlo (MCMC) to fit the velocity distribution to the model,

$$f(v_{gc}) = p_{cl} \cdot f_{Gauss}(v_{gc}) + (1 - p_{cl}) \cdot f_{Lin}(v_{gc}) , \quad (3)$$

where $f_{Gauss}$ is a convolution of two Gaussian distributions, having a common mean value, $\Delta v_{gc}$, and $f_{Lin}$ is a linear function. The quasi-Gaussian contribution is a phenomenological model, representing galaxies bound to clusters. It accounts for the intrinsic non-Gaussianity of velocity distributions of individual clusters, and for the variation in cluster masses in the sample. The linear part of the model represents a uniform background of interlopers (line-of-sight galaxies which are not gravitationally bound to the cluster). The fraction of bound galaxies, $p_{cl}$, is a free parameter of the MCMC which is marginalized over, as are the two coefficients of $f_{Lin}$, the width of the two Gaussian functions, and the relative normalization of the two.

Scaling the separation between galaxies and associated BCGs by $r_{200}$ takes advantage of the self-similarity of clusters; we therefore used $r_{gc}$-bins defined in units of $r_{200}$. We fitted the $v_{gc}$-distribution in each bin using MultiNest, a Bayesian inference tool employing importance nested sampling [18]. For illustration, Fig. 1(b) shows fitted velocity distributions. The observed velocity dispersions are of the order of several hundred km/s. The trend is for the dispersion to increase with $r_{gc}$, and for the corresponding relative number of interlopers to grow. The fits for the various $r_{gc}$-bins are compatible with the data, scoring better than 99% in K-S tests.

The GRS signal, $\Delta v_{gc}$, is an order of magnitude smaller than typical velocity dispersions, and is therefore difficult to confirm visually. In order to validate that the MultiNest fitting procedure does not introduce biases in the signal, we computed the following, model-independent quantity:

$$S_{\pm}^v = \begin{cases} -I_+^v / I_-^v , & \text{if } I_+^v > I_-^v \\ I_+^v / I_-^v , & \text{if } I_-^v < I_+^v . \end{cases} \quad (4)$$
Here $I_\nu$ stand for the integrated $v_{gc}$-distributions of negative and of positive velocities, after correcting for outlier galaxies. Choosing $\nu = 1,000$ km/s, the initial integrals were derived from spline-fits within $-\nu < v_{gc} < 0$ (for $I_\nu^-$), or within $0 < v_{gc} < \nu$ (for $I_\nu^+$). We then subtracted from each integral the contribution of outliers, where the latter was estimated by a linear fit of the velocity distribution for $|v_{gc}| > \nu$. While the value of $S_\nu^\pm$ is not directly related to the GRS signal, one would expect the two to be correlated. Comparing the fitted values of $\Delta v_{gc}$ with the respective values of $S_\nu^\pm$, we got a Pearson coefficient of 0.97, indicating that the fitting process is indeed unbiased. Additionally, we wrote a simple Metropolis-Hastings MCMC and cross-checked the fit-results.

Sample composition and systematic tests

The baseline dataset may be utilized in various ways to perform the measurement. One of the main sources of ambiguity is that we are interested in galaxies which are several Mpc away from the corresponding BCGs. On average, the distance between close pairs of clusters in our dataset corresponds to $2.3 \cdot r_{200}$, where for the bulk of the cluster sample, $0.8 < r_{200} < 1.2$ Mpc. Many galaxies are therefore likely to be associated with multiple clusters, depending on their extent and separation.

We define a pair of overlapping clusters as having a transverse separation, $r_{cc} < 4 \cdot r_{200}$, and a velocity difference, $|v_{cc}| < 4,000$ km/s. We tested several galaxy selection schemes, with different restrictions on overlapping configurations.

One possible selection procedure is to exclude all overlapping cluster pairs from the analysis. Another option is to exclude all but one member from any configuration of overlapping clusters. Alternatively, we may choose not to take into account cluster overlaps at all. In this case a given galaxy may have more than one matched cluster. We then accept only those galaxies which have only one cluster association, effectively performing exclusive selection on galaxies instead of on clusters. In order to reduce the effect of double-counting, we assigned galaxy-weights that are inversely proportional to the number of matched clusters, in any of these scenarios.

The issue of overlapping configurations was not explicitly addressed by WHH, who randomly determined the membership of galaxies in cases of ambiguity. Therefore, as a cross-check of our selection procedure, we created a dataset comparable to that of WHH: using the GMBCG catalog with exclusive SDSS data, our measurement of $\Delta v_{gc}$ was consistent with that of WHH. However, we noted that the result was sensitive to small changes in the width of the $r_{gc}$-bins.

In order to check the dependence of the signal on the composition of our dataset, we ran the analysis on subsets of the data. For instance, we chose to only use SDSS data, or to only match LRG-BCGs to MAIN-galaxies. In another case, we matched BCGs to both SDSS and BOSS spectra, but only used SDSS data for associated galaxies. Most of these tests did not significantly affect the measurement. However, tests involving BOSS-BCGs associated with SDSS-galaxies revealed a systematic positive bias of a few km/s.
So as to understand this effect, we define the quantity, \( \delta m^{gc}_{r} = m^{*}_{r} - m^{c}_{r} \), where \( m^{*}_{r} \) and \( m^{c}_{r} \) respectively stand for the \( r \)-band magnitude of a BCG, and that of the brightest matched galaxy within one \( r_{200} \) of the BCG. Positive \( \delta m^{gc}_{r} \) values correspond to BCGs which are indeed found to be the brightest source within the area of a cluster. We observed on average, \( \delta m^{gc}_{r} = -0.3 \) for configurations in which the two surveys were mixed. The implication of this is that for this sub-sample, it is likely that BCGs were misidentified in the cluster catalog. As a result, selected BCGs were less likely to represent the bottom of the gravitational potential well of clusters, effectively suppressing the GRS signal. One should also keep in mind that the difference between SDSS and BOSS redshifts is almost an order of magnitude smaller than the uncertainties on the redshifts. It is therefore possible that the bias originates e.g., from changes made in the template-fitting procedure between data-releases.

Another important systematic is the treatment of clusters with high multiplicities (numbers of matched galaxies), which we denote by \( n_{gal} \). These configurations are subject to two types of bias. The first is due to the fact that \( r_{gc} \) depends on the redshifts of both a galaxy and the corresponding BCG; consequently, an error in the redshift of a given BCG affects all matched galaxy-redshifts in a correlated way. The second effect which we observed, was that the value of \( \delta m^{gc}_{r} \), while generally positive, tends to decrease as \( n_{gal} \) increases. We therefore concluded that clusters become more susceptible to misidentification of the BCG with growing multiplicities.

In order to mitigate these effects, we normalized the contribution of each cluster in the \( v_{gc} \)-distribution by the number of galaxies; we thus prevented the small number of high-\( n_{gal} \) clusters from dominating the result. The downside of this procedure is that the statistical significance of low-multiplicity clusters increases. We checked that this did not significantly affect the GRS signal, by changing the minimal threshold value of \( n_{gal} \). That is, we rejected clusters which have fewer than \( n_{gal}^{min} \) matched galaxies, taking \( 1 < n_{gal}^{min} < 7 \). (This may be compared to the WHH selection, for which \( n_{gal}^{min} = 5 \).

An additional possible source of bias is the uncertainty associated with individual spectroscopic redshifts, which we addressed in two ways. The first was to propagate the redshift uncertainties to the velocity distribution. We did this by generating random positive and negative shifts of \( v_{gc} \), distributed according to the corresponding uncertainties. The second method was to suppress redshifts with high uncertainties, which we did by weighting galaxies inversely proportional to the respective uncertainties. We found that neither test affected the results of the analysis.

**RESULTS**

We estimated \( \Delta v_{gc} \) using a sliding window for the transverse separation between galaxies and clusters. The sliding window nominally had a width of 0.5 \( r_{200} \), and a step size of 0.1 \( r_{200} \). For our primary selection, we elected to discard configurations in which SDSS- and BOSS-spectra were mixed together. This reduced the number of clusters and galaxies by 16 and 11\%, respectively. We also excluded all overlapping cluster pairs, further reducing these numbers by a respective 34 and 46\%. The final dataset was composed of 12,600 clusters and 60,626 matched galaxies. The measurement was restricted to transverse separation values below 2.5 \( r_{200} \).

The reason for this last condition may be inferred from Fig. 2(a). The figure shows the dependence on \( r_{gc} \) of the number of galaxies, \( n_{gal} \), of the number of associated clusters, \( n_{clst} \), and of the ratio, \( n_{gal}/n_{clst} \). One may observe that for \( r_{gc} < 1.3 \cdot r_{200} \), the multiplicities of matched clusters and galaxies decrease; this is in accordance with the expected trend for the surface density of galaxies in clusters (see e.g., figure 8 in [19]). However, for \( r_{gc} \geq 2 \cdot r_{200} \), both the multiplicities and the galaxy-to-cluster ratio increase. This comes about as galaxies at large \( r_{gc} \) have an increasingly higher probability of being associated with another cluster. Such configurations therefore tend to suppress the signal of the GRS, and should be rejected from the analysis.

The dependence of \( \Delta v_{gc} \) on \( r_{200} \) is presented in Fig. 2(b). We find that on average, \( \Delta v_{gc} = -11^{+7}_{-5} \) km/s for \( 1 < r_{gc}/r_{200} < 2.5 \), with uncertainties given as one standard deviation of the average signal. The top axis of the figure shows the median value and the width of the distribution of \( r_{gc} \)-values, corresponding to four of the bins. In physical scales, the measurement extends up to galaxy-cluster transverse separations of \( \sim 3 \) Mpc.

The uncertainty bands were estimated from the variations in \( \Delta v_{gc} \) due to a series of systematic checks, combined with the uncertainty on the model-fit. Besides the tests described above, we took into account the effect of including the rejected clusters and galaxies. Additionally, we varied analysis parameters, such as those defining the overlap of clusters (\( r_{cc} \) and \( v_{gc} \)). Among the tests, the upper uncertainty bound was found to be dominated by configurations in which \( r_{cc} \) was decreased. Correspondingly, for larger cluster-cluster separations, \( \Delta v_{gc} \) became more negative. This corroborates our assumption, that the leading effect diluting the signal is mis association of galaxies with clusters.

In addition to the primary result, we present a measurement with an increased \( r_{gc} \)-bin width. The two results are consistent within uncertainties. We note that the set-up with the wider bin has a radial resolution which is slightly too low to describe the GRS effect at low values of \( r_{gc} \). On the other hand, for high \( r_{gc} \)-values, the increase in statistics in each bin seems to stabilize the result. Finally, we also include a measurement of \( \Delta v_{gc} \),
in which SDSS and BOSS spectra are used congruently. This change incurs a systematic shift of a few km/s, as discussed above.

Calculating the GR prediction for \( \Delta v_{\text{gc}} \) is beyond the scope of this study. However, the features of our dataset are similar to those of the WHH sample. We therefore refer to the corresponding estimate of Kaiser of \( \Delta v_{\text{gc}} \). Bins of \( r_{\text{gc}} \) are defined by a sliding window with a width of 0.5 \( \cdot r_{200} \), where each data-point is placed in the center-position of the corresponding bin.

(b): Dependence of the signal of the GRS, \( \Delta v_{\text{gc}} \), on \( r_{\text{gc}} \), where the width of the sliding window is denoted by \( w_{\text{sl}} \). The shaded areas around the two nominal results (circles and squares) correspond to the variations in the signal due to the systematic tests described in the text, combined with the uncertainty on the model-fit. On average, \( \Delta v_{\text{gc}} = -11^{\pm 7} \) km/s for \( 1 < r_{\text{gc}}/r_{200} < 2.5 \). The third dataset (triangles) includes configurations in which SDSS and BOSS redshifts are not separated, and the bold lines represent the GR predictions of Kaiser [11], with and without his added kinematic effects, as indicated. The top axis specifies the median values and the width of the distribution of \( r_{\text{gc}} \) (in Mpc) for four bins of width 0.5 \( \cdot r_{200} \), centered at (0.5, 1, 1.5 and 2) \( \cdot r_{200} \). (Color online.)

**SUMMARY**

The gravitational redshift effect allows one to directly probe the gravitational potential in clusters of galaxies. As such, it provides a fundamental test of GR.

Following up on the analysis of Wojtak, Hansen & Hjorth, we present a new measurement with a larger dataset. We use spectroscopic redshifts taken with SDSS and BOSS, and match them to the BCGs of clusters from the catalog of Wen, Han & Liu. The analysis is based on extracting the GRS signal from the distribution of the velocities of galaxies in the rest frame of corresponding BCGs. We focus on optimizing the selection procedure of clusters and of galaxies, and take into account multiple possible sources of systematic biases not considered by WHH.

Our nominal dataset includes 12,600 clusters and 60,626 associated galaxies, with an additional 5k clusters and 25k galaxies used for systematic checks. We restrict the transverse galaxy-cluster separations to values, \( r_{\text{gc}} < 2.5 \cdot r_{200} \), corresponding to \( \sim 3 \) Mpc, and use sliding windows with a width of either 0.5 or 1 \( \cdot r_{200} \) to derive the signal. We find an average redshift of \( -11 \) km/s with a standard deviation of +7 and \( -5 \) km/s for \( 1 < r_{\text{gc}}/r_{200} < 2.5 \). The result is consistent with the measurement of WHH, and in good agreement with the GR predictions. Considering the current systematic uncertainties, we can not distinguish between the baseline GR predictions.
effect and the recently proposed kinematic modifications.

With the advent of future spectroscopic surveys, such as Euclid and DESI [20], we will have access to larger, more homogeneous datasets. We expect that the new spectra will help to significantly reduce the systematic uncertainties on the measurement. This could be done, e.g., by increasing the number of usable isolated clusters, which may require specialized target selection. Additionally, new data will facilitate novel techniques of detecting the GRS signal, such as the cross-correlation method suggested in [21].

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