TRANSFORMATION OF THERMAL ENERGY IN ELECTRIC ENERGY IN AN INHOMOGENEOUS SUPERCONDUCTING RING

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Abstract

An inhomogeneous superconducting ring (hollow cylinder) placed in a magnetic field is considered. It is shown that a direct voltage appears on section with lowest critical temperature when it is switched periodically from the normal state in the superconducting state and backwards, if the magnetic flux contained within the ring is not divisible by the flux quantum. The superconducting transition can be first order in this case. In the vicinity of this transition, thermal fluctuations can induce the voltage in the ring with rather small sizes.

1 Introduction

Superconductivity is a macroscopic quantum phenomenon. One of the consequences of this is the periodical dependence of energy of a superconducting ring on a magnetic flux within this ring. This dependence is caused by a quantization of the velocity of the superconducting electrons \( v_s \). According to (See ref.[1])

\[
v_s = \frac{1}{m}(\hbar \frac{d\phi}{dr} - \frac{2e}{c}A) = \frac{2e}{mc} \left( \frac{\Phi_0}{2\pi} \frac{d\phi}{dr} - A \right)
\]

the velocity along the ring (or tube) circumference must have fixed values dependent on the magnetic flux because

\[
\int_l dl v_s = \frac{2e}{mc} (\Phi_0 n - \Phi)
\]

and \( n = \int_l dl (1/2\pi) d\phi/dr \) must be an integer number since the wave function \( \Psi = |\Psi| \exp(i\phi) \) of the superconducting electrons must be a simple function. Where \( \phi \) is the phase of the wave function; \( \Phi_0 = \pi \hbar c/e = 2.07 \times 10^{-7} G \ cm^2 \) is the flux quantum; \( A \) is the vector potential; \( m \) is the electron mass and \( e \) is the electron charge; \( l = 2\pi R \) is the ring circumference; \( R \) is the ring radius; \( \Phi = \int_l dA \) is the magnetic flux contained within the ring.

The energy of the superconductor increases with the superconducting electron velocity increasing. Therefore the \( |v_s| \) tends towards a minimum possible value. If \( \Phi/\Phi_0 \) is an integer number, the velocity is equal to zero. But \( v_s \) cannot be equal to zero if \( \Phi/\Phi_0 \) is not an integer number. Consequently, the energy of
the superconducting state of the ring depends in a periodic manner on the magnetic field value. The energy of the superconducting ring changes by two reasons: because the kinetic energy of the superconducting electrons \( n_s slmv^2/2 \) changes and because the energy of a magnetic field \( LI_s^2/2 \) induced by superconducting current \( I_s = s j_s = s2en_s \) changes. Here \( L \) is the inductance of the ring; \( n_s \) is the superconducting pair density; \( s \) is the area of cross-section of the ring wall.

The first is the cause of the Little-Parks effect \cite{2}. Little and Parks discovered that the critical temperature, \( T_c \), of a superconducting tube with narrow wall depends in a periodic way on a magnetic flux value within the tube. This effect has been explained by M.Tinkham \cite{3}. According to ref.\cite{2}, the critical temperature of the homogeneous ring (which we considered now) is shifted periodically in the magnetic field:

\[
T_c(\Phi) = T_c[1 - (\xi(0)/R)^2(n - \Phi/\Phi_0)^2]
\]

(3)
because the kinetic energy changes periodically with magnetic field. Here \( \xi(0) \) is the coherence length at \( T = 0 \). The value of \( (n - \Phi/\Phi_0) \) changes from -0.5 to 0.5. The \( T_c \) shift is visible if the tube radius is small enough (if \( R \approx \xi(0) \)).

The magnetic field energy \( F_L = LI_s^2/2 = Ls^22\pi^2v_s^2n_s^2 \) does not influence on the critical temperature value because it is proportional to \( n_s^2 \). A value of this energy depends on the temperature because the \( n_s \) value depends on the temperature. The \( n_s \) value change causes the superconducting current change and a voltage in consequence with electromagnetic induction law. Consequently the superconducting ring can be used for transformation the thermal energy into the electric energy if the velocity \( v_s \neq 0 \). A temperature change will induce a voltage in the ring if \( v_s \neq 0 \). In the present paper a most interesting case - a induction of direct voltage in an inhomogeneous ring - is considered.

A ring (tube) with the narrow wall (the wall thickness \( w \ll R, \lambda \)) is considered. In this case \( \Phi = BS \approx H.S \), because the magnetic field induced by the superconducting current in the ring is small. Here \( H \) is the magnetic field induced by an external magnet, \( S = \pi R^2 \) is the ring area and \( \lambda \) is the penetration depth of the magnetic field. We consider a ring whose critical temperature varies along the circumference \( l = 2\pi R \), but is constant along the height \( h \). In such a ring, the magnetic flux shifts the critical temperature of a section with a lowest \( T_c \) value only. When the superconducting state is closed in the ring, the current of the superconducting electrons must appear as a consequence of the relation (2) if \( \Phi/\Phi_0 \) is not an integer number. Therefore the lowest \( T_c \) value will be shifted periodically in the magnetic field as well as \( T_c \) of the homogeneous ring.
2 Inhomogeneous superconducting ring as a thermal-electric machine of direct current

It is obvious that the current value must be constant along the circumference, because the capacitance is small. The value of the superconductor current must be constant if the current of the normal electrons is absent. Therefore the velocity of the superconducting electrons can not be a constant value along the circumference of an inhomogeneous ring if the superconducting pair density is not constant. (I consider the ring with identical areas along the circumference.)

Let us consider a ring consisting of two sections \( l_a \) and \( l_b \) (\( l_a + l_b = l = 2\pi R \)) with different values of the critical temperature \( T_{ca} > T_{cb} \). According to the relation (2) the superconducting current along the ring circumference, \( I_s \), must appear below \( T_{cb} \) if \( \Phi / \Phi_0 \) is not an integer number. Then

\[
I_s = I_{sa} = s_a j_{sa} = s_b j_{sb} = s_b 2e n_{sb} v_{sb}
\]

if the normal current is absent. Here \( n_{sa} \) and \( n_{sb} \) are the densities of the superconducting pair in the sections \( l_a \) and \( l_b \); \( v_{sa} \) and \( v_{sb} \) are the velocities of the superconducting pairs in the sections \( l_a \) and \( l_b \) and \( s_a \) and \( s_b \) are the areas of wall section of \( l_a \) and \( l_b \), \( s = s_a = s_b = wh \). \( \int dl v_s = v_{sa} l_a + v_{sb} l_b \). Therefore according to (2) and (4)

\[
v_{sa} = \frac{2e}{mc} \frac{n_{sb}}{(l_a n_{sb} + l_b n_{sa})} (\Phi_{o} n - \Phi); \quad v_{sb} = \frac{2e}{mc} \frac{n_{sa}}{(l_a n_{sb} + l_b n_{sa})} (\Phi_{o} n - \Phi)
\]

(5)

\[
I_s = \frac{4e^2}{mc} \frac{n_{sa} n_{sb}}{(l_a n_{sb} + l_b n_{sa})} (\Phi_{o} n - \Phi)
\]

(5a)

According to (5a) a change of the superconducting pair density induces a change of the superconducting current if \( \Phi_{o} n - \Phi \neq 0 \). The change of the superconducting current induces the change of the magnetic flux \( \Phi = H \pi R^2 + L(I_s + I_n) \) and, as a consequence, induces the voltage and the current of the normal electrons (the normal current, \( I_n \)). The total current \( I = I_s + I_n \) must be equal in both sections, because the capacitance is small. But \( I_{sa} \) can be no equal to \( I_{sb} \). Then \( I_{na} \neq I_{nb} \) and consequently, the potential difference \( dU/dt \) exists along the ring circumference. The electric field along the ring circumference \( E(r) \) is equal to

\[
E(r) = \frac{dU}{dl} - \frac{1}{l} \frac{d\Phi}{dt} = -\frac{dU}{dl} - \frac{L}{l} \frac{dI}{dt} = \frac{\rho_n}{s} I_n
\]

(6)

where \( \rho_n \) is the normal resistivity.

Because the normal current exists the relations (5) becomes no valid. The velocity \( v_{sa} \) and the current \( I_{ca} \) can be not equal zero even at \( n_{sb} = 0 \). The
current decreases during the decay time of the normal current \( L/R_{nb} \). \( R_{bn} = \rho_{bn}l_b/s \) is the resistance of the section \( l_b \) in the normal state.

The voltage, the normal current, and superconducting current change periodically in time if the \( n_{sb} \) value changes periodically. But in addition a direct potential difference \( U_b \) can appear if the \( l_b \) section is switched from the normal state in the superconducting state and backwards i.e. some times \( t_n n_{sb} = 0 \) and some times \( t_s n_{sb} \neq 0 \). Let us consider two limit cases: \( t_n \ll L/R_{nb} \) and \( t_n \gg L/R_{nb} \). \( t_s \gg L/R_{nb} \) and \( l_a \gg l_b \) in the both cases.

In the first case the total current \( I \) is approximately constant in time and because \( t_s \gg L/R_{bn} \)

\[
I \approx s2en_{sa} < v_{sa} > \approx s \frac{4e^2}{mc} \frac{n_{sa} < n_{sb} >}{l_b n_{sa} + l_a < n_{sb} >} (\Phi_0 n - \Phi) \quad (7)
\]

Here \( < n_{sb} > \) is a average value of \( n_{sb} \). The average value of the resistivity of the \( l_b \) section, \( \rho_b \approx \rho_{bn} t_n/(t_s + t_n) \). Consequently,

\[
U_b = R_b I \approx \frac{l_b < n_{sb} >}{l_b n_{sa} + l_a < n_{sb} >} \frac{(\Phi_0 n - \Phi)}{\lambda_L^2} \rho_b \quad (8)
\]

where \( \lambda_L = (mc/4e^2 n_{sa})^{1/2} \) is the London penetration depth. According to (8) \( U_b \neq 0 \) at \( < n_{sb} > \neq 0 \) and \( \rho_b \neq 0 \). Consequently, the direct potential difference can be observed if we change the temperature inside the region of the resistive transition of the section \( l_b \) (where \( 0 < \rho_b < \rho_{bn} \)).

In the second case

\[
U_b \approx \frac{s < n_{sb} >}{l_b n_{sa} + l_a < n_{sb} >} \frac{(\Phi_0 n - \Phi)}{\lambda_L^2} L f \quad (9)
\]

where \( f \) is the frequency of the switching from the normal into the superconducting state.

Thus, the inhomogeneous superconducting ring can be used as a thermal-electric machine of direct current. The power of this machine is small. It decreases with the increasing of the ring radius. Let us to evaluate the maximum power at \( l_b n_{sa} < l_a < n_{sb} > \). This condition means that the temperature of \( l_b \) changes enough strongly. The power will be maximum at \( t_n \simeq t_s \simeq L/R_{bn} \). The frequency of the switching \( f = R_{bn}/2L \) in this case. The power of the ring can not exceed

\[
W = IU_b = \frac{s l_b \rho_{bn} \Phi_0^2}{2l_b \lambda_L^4} (n - \Phi/\Phi_0)^2 < \frac{s l_b \rho_{bn} \Phi_0^2}{8l_b \lambda_L^4} \quad (10)
\]

For example, at \( s = 0.1 \mu m, l_b = 0.1 \mu m, l_a = 1 \mu m, \lambda_L = 0.1 \mu m, \rho_{bn} = 100 \mu \Omega cm \) the power is smaller than \( W = 10^{-4}Vt \). The power can be increased by the ring height \( h \) increasing.

Above we considered the case when the changes of the temperature is enough strong. In the opposite case, when the temperature change is small, \( l_b n_{sa} \gg \)
\[ l_a < n_{sb} > \] and therefore the current and the voltage are proportional to the \[ < n_{sb} > / n_{sa} \] value. Because \[ n_{sb} \] must be equal zero some times, this means that the voltage is proportional to the amplitude of the temperature change. Consequently, the inhomogeneous ring is a classical thermal machine with a maximum efficiency in the Carno cycle \[ 4 \].

At a small \[ l_b/l_a \] value and an enough big \[ |(n - \Phi/\Phi_0)| \] value the superconducting transition of \[ l_b \] is first order. In this case in order to switch the \[ l_b \] section from the normal state in the superconducting state and backwards the temperature must be change on a finite value, because the hysteresis of the superconducting transition exists.

3 First order superconducting phase transition

According to \((5)\) the \[ v_{sb} \] value decreases with the \[ n_{sb} \] value increasing. Therefore the dependence of the energy of the superconducting state on the \[ n_{sb} \] value can have a maximum in some temperature region at \[ T \simeq T_{cb}(\Phi) \]. The presence of such a maximum means that the superconducting transition is a first order phase transition.

The existence of the maximum and the width of the temperature region where the maximum exists depends on the \[ n - \Phi/\Phi_0 \] value and on the ring parameters: \[ l_a, l_b, w, h \] and \[ T_{ca}/T_{cb} \]. These dependencies can be reduced to two parameters, \[ B_f \] and \[ L_I \], which are introduced below. It is obvious that the maximum can exist at only \[ n - \Phi/\Phi_0 \neq 0 \]. Therefore only this case is considered below.

The Ginsburg-Landau free energy of the ring can be written as

\[ F_{GL} = s[l_a((\alpha_a + \frac{mv^2_{sa}}{2})n_{sa} + \frac{\beta_a}{2}n^2_{sa})] + l_b((\alpha_b + \frac{mv^2_{sb}}{2})n_{sb} + \frac{\beta_b}{2}n^2_{sb})] + \frac{LI^2}{2} \quad (11) \]

Here \[ \alpha_a = \alpha_{a0}(T/T_{ca} - 1) \], \[ \beta_a \], \[ \alpha_b = \alpha_{b0}(T/T_{cb} - 1) \] and \[ \beta_b \] are the coefficients of the Ginsburg-Landau theory. We do not consider the energy connected with the density gradient of the superconducting pair. It can be shown that this does not influence essentially the results obtained below.

The Ginsburg-Landau free energy \((11)\) consists of \[ F_{GL,la} \] (the energy of the section \( l_a \)), \[ F_{GL,lb} \] (the energy of the section \( l_a \)) and \[ F_L \] (the energy of the magnetic field induced by the superconducting current):

\[ F_{GL} = F_{GL,la} + F_{GL,lb} + F_L \quad (12) \]

Substituting the relation \((4)\) for the superconducting current and the relation \((5)\) for the velocity of the superconducting electrons into the relation \((11)\), we obtain

\[ F_{GL,la} = sl_a(\alpha_a(\Phi,n_{sa},n_{sb})n_{sa} + \frac{\beta_a}{2}n^2_{sa}) \quad (12a) \]
\[ F_{GL, l_b} = s l_b (\alpha_b (\Phi, n_{sa}, n_{sb}) n_{sb} + \frac{\beta_b}{2} n_{sb}^2) \]  \hspace{1cm} (12b)\\

\[ F_L = \frac{2 L s^2 e^2 (\Phi_0 n - \Phi)^2 n_{sa}^2 n_{sb}^2}{mc (l_a n_{sb} + l_b n_{sa})^2} \]  \hspace{1cm} (12c)\\

Here

\[ \alpha_a (\Phi, n_{sa}, n_{sb}) = \alpha_{ao} \frac{T}{T_{ca}} - 1 + (2 \pi \xi_a (0)) ^2 \frac{(n - \Phi/\Phi_0)^2 n_{sb}^2}{(l_a n_{sb} + l_b n_{sa})^2} \]\\

\[ \alpha_b (\Phi, n_{sa}, n_{sb}) = \alpha_{bo} \frac{T}{T_{cb}} - 1 + (2 \pi \xi_b (0)) ^2 \frac{(n - \Phi/\Phi_0)^2 n_{sa}^2}{(l_a n_{sb} + l_b n_{sa})^2} \]\\

\[ \xi_a (0) = (h^2 / 2 m \alpha_a)^{1/2}; \xi_b (0) = (h^2 / 2 m \alpha_b)^{1/2} \] are the coherence lengths at \( T = 0 \).

According to the mean field approximation the transition into the superconducting state of the section \( l_b \) occurs at \( \alpha_b (\Phi, n_{sa}, n_{sb}) = 0 \). Because \( n_{sa} \neq 0 \) at \( T = T_{cb} \) the position of the superconducting transition of the \( l_b \) section depends on the magnetic flux value:

\[ T_{cb} (\Phi) = T_{cb} [1 - (2 \pi \xi_b (0)) ^2 \frac{(n - \Phi/\Phi_0)^2 n_{sa}^2}{(l_a n_{sb} + l_b n_{sa})^2}] \]  \hspace{1cm} (12d)\\

At \( l_a = 0 \) the relation (12d) coincides with the relation (3) for a homogeneous ring. A similar result ought be expected at \( l_b \gg l_a \). But at \( l_b \ll l_a \) the \( T_{cb} (\Phi) \) value depends strongly on the \( n_{sa} \) value. At \( n_{sa} = 0 \) \( T_{cb} (\Phi) = T_{cb} [1 - (2 \pi \xi_b (0)) ^2 \frac{(n - \Phi/\Phi_0)^2}{l_a n_{sb}^2}] \) whereas at \( l_a n_{sb} \gg l_b n_{sa} \) \( T_{cb} (\Phi) = T_{cb} [1 - (2 \pi \xi_b (0)) ^2 \frac{(n - \Phi/\Phi_0)^2}{l_a n_{sb}^2}] \). Consequently a hysteresis of the superconducting transition ought be expected in a ring for \( l_b \ll l_a \).

To estimate the dependence of the hysteresis value on the ring parameters, we transform the relation (12) using the relations for the thermodynamic critical field \( H_c = \Phi_0 / 2 \sqrt{2 / \pi} \lambda L \xi; \alpha^2 / 2 \beta = H_c^2 / 8 \pi \) and for the London penetration depth \( \lambda_L = (cm/4 e^2 n_s)^{1/2} \). We consider a ring with \( l_a \gg \xi_a (T) = \xi_a (0) (1 - T/T_{ca})^{0.5} \). \( n_{sa} \simeq -\alpha_a / \beta_a \) in this case.

\[ F_{GL} = F_{GLa} + F_{l_b} (T + \frac{1}{(n_{sb}^2 + 1)^2} + n_{sb}^2 (B + \frac{1}{(n_{sb}^2 + 1)^2} (2 + L_I))) \]  \hspace{1cm} (13)\\

Here \( n_{sb}^l = l_a n_{sb} / l_b n_{sa} \):

\[ F_{GLa} = -s l_a \frac{H_c^2}{8 \pi} (\frac{(2 \pi \xi_a (T))^2}{l_a^2} (\frac{(n - \Phi/\Phi_0)^4}{(n_{sb}^l + 1)^4})) \]

Because \( l_a \gg 2 \pi \xi_a \), \( F_{GLa} \simeq -s l_a H_c^2 / 8 \pi \).

\[ F = \frac{s \xi_a (T) H_c^2}{2} + \frac{2 \pi \xi_a (T)^2}{l_a} (n - \Phi/\Phi_0)^2 \]
be observed in the ring with $B_f$ (if $\beta_f$ depends on the $0_k$).

The hysteresis will be observed if the maximum transformation of thermal fluctuation energy

$$\tau = \left( \frac{T}{T_{cb}} - 1 \right) (n - \Phi/\Phi_0)^{-2} \frac{l_b^2}{(2\pi\xi_b(0))^2}$$

$$B_f = 0.5 \frac{\beta_b}{\beta_a} \frac{l_b}{l_a} \frac{l_b^2}{(2\pi\xi_b(0))^2} (n - \Phi/\Phi_0)^{-2}$$

$$L_I = 4\pi \frac{s}{\lambda_{L_a}^2} \frac{L}{l_a}$$

For $h > R$, $L = k4\pi R^2/h$ where $k = 1$ at $h \gg R$. Consequently, $L_I = 4\pi(l/l_a)(l w/\lambda_{L_a}^2(T))$ in this case. At $h, w \ll R, L \approx 4\ln(2R/w)$, therefore $L_I = 16\pi(l/l_a)(s/\lambda_{L_a}^2(T))\ln(2R/w)$ in this case.

The numerical calculations show that the $F_{GL}(n_{sb})$ dependence (13) has a maximum at small enough values of $B_f$ and $L_I$ in some region of the $\tau$ values. The width of the $\tau$ region with the $F_{GL}(n_{sb})$ maximum depends on the $B_f$ value first of all. At $L_I \approx 2$ the maximum exists at $B_f < 0.4$. For example at $B_f = 0.2$ and $L_I \ll 2$ the maximum takes place at $\tau \approx 1.02 \gamma_k < \tau < 0.89$. This means that the transition into the superconducting state of the section $l_b$ occurs at $\tau \approx 1.02$, (that is at $T_{cs} = T_{cb}(1 - 1.02(n - \Phi/\Phi_0)^2(2\pi\xi_b(0)/l_b)^2)$ and the transition in the normal state occurs at $\tau \approx -0.89$, (that is at $T_{cn} = T_{cb}(1 - 0.89(n - \Phi/\Phi_0)^2(2\pi\xi_b(0)/l_b)^2)$) if thermal fluctuations are not taken into account.

The inequality $L_I \ll 2$ is valid for a tube (when $h > R$) with $2\pi lw \ll \lambda_{L_a}^2(T)$ and for a ring (when $h < R$) with $8\pi hw \ll \lambda_{L_a}^2(T)$. The hysteresis value increases with decreasing $B_f$ value and decreases with increasing $B_f$ value. The $B_f$ value is proportional to $(n - \Phi/\Phi_0)^{-2}$. Consequently, the hysteresis value depends on the magnetic field value. Because the hysteresis is absent at $B_f > 0.4$, it can be observed in the regions of the magnetic field values, where $\Phi/\Phi_0$ differs essentially from an integer number. The width of these regions depends on the $0.5(\beta_b/\beta_a)l_b^2/(2\pi\xi_b(0))^2l_a$ value (see above the relation for $B_f$). Since $(n - \Phi/\Phi_0)^2 < 0.25$ and $\beta_b \simeq \beta_a$ in the real case, the hysteresis can be observed in the ring with $l_b^2 < 0.2(2\pi\xi_b(0))^2l_a$). For example in the ring with $l_b = 2\pi\xi_b(0)$ and $l_a = 10l_b$, the hysteresis can be observed at $|n - \Phi/\Phi_0| > 0.35$ (if $\beta_a = \beta_b$). At $|n - \Phi/\Phi_0| = 0.5$ $B_f = 0.2$ and the hysteresis is equal to $T_{cn} - T_{cs} \approx 0.03T_{cb}$ in this ring.

4 Transformation of thermal fluctuation energy into electric energy

The hysteresis of the superconducting transition can be observed if the maximum is high enough. The maximum height is determined by a parameter $F$, which is introduced below. The hysteresis will be observed if the maximum height is much greater than the energy of the thermal fluctuation, $k_BT$. In the
opposite case the thermal fluctuation switches the $l_b$ section from the normal state into the superconducting one and backwards at $T \approx T_{cb}(\Phi)$.

Above we have used the mean field approximation which is valid when the thermal fluctuation is small. In our case the mean field approximation is valid if the height of the $F_{GL} - F_{GLa}$ maximum, $F_{GL,max}$, is much greater than $k_B T$. This height depends on the $F$, $\tau$, $B_f$ and $L_f$ values: $F_{GL,max} = FH(\tau, B_f, L_f)$. The $F$ parameter is determined above. The $H(\tau, B_f, L_f)$ dependence can be calculated numerically from the relation (8). To estimate the validity of the mean field approximation we ought to know the maximum value of the $H(\tau)$ dependence: $H_{max}(B_f, L_f)$. We can use the mean field approximation if $FH_{max}(B_f, L_f) \gg k_B T$. This is possible if the height of the ring is large enough, namely

$$h \gg \xi_a(0) \frac{1}{\pi H_{max}(B_f, L_f)} \frac{l_a}{w} \frac{G_1^{1/2}}{T_{ca}/T_{cb} - 1} \left( n - \frac{\Phi}{\Phi_0} \right)^{-2}$$

Here $G_1 = (k_B T_{ca}/\xi_a(0)^3 H_{ca}^2)^2$ is the Ginsburg number of a three-dimensional superconductor. We have used the relation for the $F$ parameter (see above). For conventional superconductors $G_1 = 10^{-11} - 10^{-5}$. $H_{max} \approx 10^{-2}$ for typical $B_f$ and $L_f$ values. For example in the ring with $B_f = 0.2$ and $L_f \ll 2$ the $H(\tau)$ dependence has a maximum $H_{max}(B_f = 0.2, L_f \ll 2) = 0.024$ at $\tau \approx -0.94$. Consequently, the value of $h$ cannot be very large. As an example for a ring with parameter value $B_f = 0.2$, $L_f \ll 2$, $l_a/w = 20$, $T_{ca}/T_{cb} - 1 = 0.2$, and fabricated from an extremely dirty superconductor with $G_1 = 10^{-5}$, the mean field approximation is valid at $h \gg 20 \xi_a(0)$ if $|n - \Phi/\Phi_0| \approx 0.5$.

If the mean field approximation is not valid, we must take into account the thermal fluctuations which decrease the value of the hysteresis. The probability of the transition from normal into superconducting state and that of the transition from superconducting into normal state are large when the maximum value of $F_{GL} - F_{GLa}$ is no much more than $k_B T$. Therefore the hysteresis cannot be observed at $FH_{max}(B_f, L_f) < k_B T$. This inequality can be valid for a ring made by lithography and etching methods from a thin superconducting film, where $h$ is the film thickness in such a ring.

As a consequence of the thermal fluctuations, the density $n_{ab}(r,t)$ changes with time. We can consider $n_{ab}(r,t)$ as a function of the time only if $h, w, l_b \simeq or < \xi_a(T)$. At $T \approx T_{cb}(\Phi)$ (at the resistive transition) $l_b$ is switched by the fluctuations from the normal state in the superconducting state and backwards i.e. some times ($\simeq t_n$) $n_{ab} = 0$ and some times ($\simeq t_s$) $n_{ab} \neq 0$. Consequently, according to the relations (8) and (9) the direct potential difference can appear in the region of the resistive transition of the section $l_b$. Thus, the energy of the thermal fluctuations can be transformed into the electric energy of direct current in the inhomogeneous ring at the temperature of the resistive transition of the section with the lowest critical temperature.

In order to evaluate the power of this transformation one ought take into
account that in the consequence of the thermal fluctuation the Ginsburg-Landau free energy $F_{GL}$ changes in time with amplitude $k_B T$. According to the relations (7), (9) and (12c) $U_b I/f \simeq F_L$. Because $F_L$ is a part of $F_{GL}$ (see (12)) the power can not exceed $k_B T f$. The maximum value of the switching frequency $f$ is determined by the characteristic relaxation time of the superconducting fluctuation $\tau_{GL}$:

$$f_{max} \simeq 1/\tau_{GL}.$$  

In the linear approximation region $\bar{\hbar}$

$$\tau_{GL} = \frac{\hbar}{8k_B(T - T_c)}$$  

(14)

The width of the resistive transition of the section $l_b$ can be estimated by the value $T_{cb}G_{ib}$. $G_{ib} = (k_B T/H_s^2(0)l_b s)^{1/2}$ is the Ginsburg number of the section $l_b$. Consequently the power value can not be larger than

$$W = \frac{8G_{ib}}{\hbar}(k_B T_{cb})^2$$  

and the $U_b$ value can not exceed

$$U_{b, max} = \left(\frac{8R_b G_{ib}}{\hbar}\right)^{1/2}k_B T_{cb}$$  

(16)

The $U_{b, max}$ value is large enough to be measured experimentally. Even at $T_c = 1 K$ and for real values $R_b = 10\Omega$ and $G_{ib} = 0.05$, the maximum voltage is equal to $U_{b, max} \simeq 3\mu V$. In a ring made of a high-Tc superconductor, $U_{b, max}$ can exceed 100$\mu V$. One ought to expect that the real $U_b$ value will be appreciably smaller than $U_{b, max}$. This voltage can be determined by the periodical dependence on the magnetic field value (see the relations (8) and (9)).

Transformation of thermal fluctuation energy into electric energy does not contradict the second thermodynamic law, because it is valid to within the thermal fluctuations $\bar{\hbar}$.

**ACKNOWLEDGMENTS**

I thank the National Scientific Council on "Superconductivity" of SSTP "AD-PCM" (Project 95040) and the International Association for the Promotion of Co-operation with Scientists from New Independent States (Project INTAS-96-0452) for financial support.

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