A novel time delay estimation algorithm of acoustic pyrometry for furnace

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Abstract

Acoustic pyrometry is a non-contact measurement technology for monitoring furnace combustion reaction, diagnosing energy loss due to incomplete combustion and ensuring safe production. The accuracy of time of flight (TOF) estimation of an acoustic pyrometry directly affects the authenticity of furnace temperature measurement. In this paper presented is a novel TOF (i.e. time delay) estimation algorithm based on digital lock-in filtering (DLF) algorithm. In this research, the time-frequency relationship between the first harmonic of the acoustic signal and the moment of characteristic frequency applied is established through the digital lock-in and low-pass filtering techniques. The accurate estimation of TOF is obtained by extracting and comparing the temporal relationship of the characteristic frequency occurrence between received and source acoustic signals. The computational error analysis indicates that the accuracy of the proposed algorithm is better than that of the classical generalized cross-correlation (GCC) algorithm, and the computational effort is significantly reduced to half of that the GCC can offer. It can be confirmed that with this method, the temperature measurement in furnaces can be improved in terms of computational effort and accuracy, which are vital parameters in furnace combustion control. It provides a new idea of time delay estimation with the utilization of acoustic pyrometry for furnace.

Keywords: digital lock-in, time delay estimation, time of flight, acoustic pyrometry, furnace temperature

1. Introduction

In coal-fired power plants, the furnace temperature distribution is an essential factor, not only to safety, but also for monitoring combustion reaction[1, 2], improving combustion efficiency[1], and reducing pollution[3]. With the development of technologies in recent years, the non-contact temperature measurement in industrial processes is increasingly utilized. Two leading technologies used in coal-fired boilers for temperature distribution measurement are the acoustic [4, 5] and laser pyrometers[6]. According to the acoustic temperature measurement theory[7], the error of time of flight (TOF) measurement can directly affect the reconstruction accuracy of the temperature field. Therefore, the delay estimation of TOF is crucial for application of the technology.

The acoustic TOF is typically estimated using the direct cross-correlation method (DCC)[8], which determines the time delay by identifying the peak position of the cross-correlation function according to the similarity of two acoustic signals. Based on DCC, many extensions have also been developed, such as the generalized cross-correlation (GCC) algorithm [9], the Hilbert transform-based algorithm, the weighted cross-correlation [9, 10] and the phase...
correction cross-correlation [11]. By using the cross-correlation algorithm, one can achieve low measurement error even in a noisy signal background. However, such an algorithm requires a wide signal bandwidth. Theoretically, narrower the signal bandwidth is, lower the measurement error can be achieved.

In the zero-crossing method [12, 13], the received signals are obtained for a given detection threshold in the time domain, the TOF is estimated by calculating the difference between the reception time of two received signals. This method has the advantages of high calculation speed because of reduced number of sampling points, so that less computational effort is required. However, a high signal-to-noise ratio (SNR) is a pre-request to this method. The accuracy of TOF estimation depends heavily on SNR. The MAGNITUDE-squared coherence (MSC) function[14] is widely used in signal detection and time delay estimation[15]. It is a normalized cross-spectral density function and measures the strength of association and relative linearity between two stationary stochastic processes on a scale from zero to one. However, a significant limitation encountered in application of the MSC estimate is that only in some particular conditions, for example when coherence is zero or one, it is possible to provide the closed-form expression for the confidence interval. The phase spectrum estimation algorithm [16, 17] is another popular method, which uses correlation function and power spectral density (PSD) as the pair of mutual Fourier transforms. The TOF estimation is obtained by detecting the slope of the phase spectrum of the cross power frequency spectrum. For this method, the accuracy of TOF estimation is constrained by the limited sampling period and finite observation time. Long sampling period can significantly increase the computational complexity. The adaptive TOF estimation algorithm [18] has also been used, which does not require prior knowledge of the signal and noise and the algorithm can be realized through automatic adjustment of the adaptive filter parameters based on the statistical characteristics of the signal. Yet its convergence speed and steady-state misalignment are dependent upon the step size factor. Small step size can be used to improve the TOF estimation accuracy, though it can lead to a heavy computational load. The Least mean-square time delay estimate (LMSTDE) has also been mentioned by some authors [19, 20], in which, an adaptive FIR filter is used to model the time difference, and the filter weights are interpolated to obtain the delay time. In order to solve the problem of noise input due to limited filter length, many adjustments and deformations for LMSTDE have been made [21-23], however it is still not possible to obtain accurate TOF with a small number of filter taps in low SNR conditions [18]. Another Fourier transform based algorithm is known as ‘high-order statistical delay estimation algorithm’ [24-26]. This high-order spectral method of extended multidimensional Fourier transform [27] can be used to suppress stationary and correlated Gaussian white noise and extract the signal amplitude and phase information. However, to achieve a given accuracy, there are strict requirements for sufficient sampling length and high resolution of A/D converter.

In general, a coal-fired boiler furnace has a large cross-section and there are severe noises, which poses serious challenges to the accurate estimation of acoustic TOF for measurement apparatus. With the in-depth study of the delay estimation algorithm [28], the GCC delay estimation algorithm is by far almost the best method in terms of accuracy at low SNR. But the problem of its large computational volume has been a cause of criticism as well. Therefore, an acoustic TOF estimation algorithm that can suppress noise interference while ensure a low computational effort and high accuracy is the key to true furnace temperature distribution mapping.

A TOF estimation algorithm based on a digital lock-in filter (DLF) has been developed by the authors of this paper. With this method, the relationship between the first harmonic and the sweeping moment of time for each frequency component is established via digital lock-in filtering of acoustic time-domain signal. The acoustic TOF is then obtained by finding the mean difference of the moment of time between the received signal and the acoustic source. The accuracy of the DLF algorithm was verified through experiments by comparing it with the classical GCC algorithm. The experimental results under multiple working conditions at different noise levels indicate that the algorithm has strong robustness, better accuracy and precision.

2. Principle of Acoustic Pyrometry and Time Delay Estimation

2.1 Principle of Acoustic Pyrometry

The acoustic pyrometry works based on the thermodynamic gas equation and the acoustic waveform equation. The temperature information of the gaseous medium is obtained according to the acoustic TOF and the sensor positions [7, 29]. The measuring principle of a single acoustic path pyrometry in a furnace is shown in Figure 1, where the loudspeaker and the microphone are mounted in two separate acoustic waveguides with sealed rears. The waveguides are perpendicular to the mean flow direction, so that the direction of soundwave is at 90° (orthogonal) with the direction of flue gas movement.

As is well known, the sound speed \( c \) in a gaseous flue medium at a temperature \( T \) is given by:

\[
\frac{c}{c} = \frac{L}{\tau} = \sqrt{\frac{\gamma R}{m T}}
\]
where $c$ is the sound speed in the given medium, $L$ is the distance between the microphone and loudspeaker in Figure 1, $r$ is the TOF, $\gamma$ is the isentropic exponent of the medium (flue gas), $R$ is the universal gas constant of an ideal gas, $m$ is the molar mass, and $T$ is the gas temperature.

From equation (1), it is can be seen that if $L$ is a known, for a given type of flue gas, $T$ can be found by measuring $r$. Obviously, an accurate measurement of acoustic TOF is the premise and guarantee for acoustic temperature measurement.

### 2.2 Time delay estimation

The TOF estimation algorithm established based on digital lock-in filter (DLF) links the two signals in the time and frequency domains according to their instantaneous frequencies. This algorithm can provide an accurate TOF estimation by using the mathematical expectation of characteristic frequency TOF. Different to other algorithms, the bandwidth of the acoustic source does not constraint the application of this algorithm. The feasibility of DLF algorithm will be validated under low SNR environment by comparing it with the GCC algorithm in the following sections.

#### 2.2.1 Digital Lock-in Filter Algorithm

It is well known that the acoustic frequency is the number of times that sound wave completes periodic vibrations per unit time. A loudspeaker driven by different source signals [7, 30-32] through power amplifier can produce different acoustic emission signals. In this research, the sinusoidal linear frequency-sweeping signal [10, 32] is used as the acoustic source, which is a sine signal with continuous frequency change, also known as the dominant variable bandwidth signal[33]. The angular frequency $\omega_{\text{linear}}$ of this sweeping waveform is expressed as:

$$\omega_{\text{linear}}(t) = 2\pi \left( \frac{B}{r_s} t + f_0 \right) \quad (0 \leq t \leq r_s)$$

where $f_0$ is the starting frequency, $B$ is the bandwidth in hertz, and $r_s$ is the pulse width. The waveform of this acoustic source is shown in Figure 2, the following three diagrams in Figure 2 are enlargements of the dashed sections. As the pulse width $r_s$ elapses from 0 to 0.2 s, the frequency of the acoustic source signal linearly increases from $f_0$ to $B+f_0$, i.e. from 4 kHz to 8 kHz.

The spectrum of the environment background is usually extracted prior to other signal processing in DLF to guarantee a superior SNR. The source signal frequency band is chosen according to that of background noise to avoid the frequency band overlap as much as possible. The acoustic source signal is amplified and emitted through the loudspeaker and propagated in the flue for a particular time before it is received by the microphone. As shown in Figure 6, although the acoustic waves are subject to attenuation and distortion by the soot particles in a gas medium [34-37], the frequency information of the acoustic signal is retained. Compared to the acoustic source, only the phase of the received signal has been shifted, and the amount of phase-shift depends on the distance between the loudspeaker and microphone. When the phase difference between the source and the received signal is found, the TOF of the acoustic on the propagation path can be determined.

From equation (2), it can be seen that within the pulse width $r_s$, there is a linear relationship between the frequency and time. There is a single independent frequency at a given moment of time. In an acoustic measurement system, the received acoustic signals are nonstationary signals. Constrained by the characteristics of the Fourier transform, the traditional frequency-domain analysis method cannot establish a precise relationship between signal representations in the time and the frequency domains. In contrast, the instantaneous frequency can better represent the local characteristics of the nonstationary signal in different periods.

It is assumed that $x_1(n)$ is sampled before the loudspeaker, propagated in the flue and received by the microphone as signal $x_2(n)$. These two signals can be expressed by the following set of equations[32]:

$$\begin{align*}
x_1(n) &= s(n) + \mu_1(n) \\
x_2(n) &= \xi s(n + D) + \mu_2(n)
\end{align*}$$

where $s(n)$ is the acoustic source signal. $\mu_1(n)$ and $\mu_2(n)$ are the noise-contaminated random components in the received signals. $\xi$ is the attenuation coefficient of the acoustic signal.
$D$ is the TOF between the two signals. $s(n)$, $\mu_1(n)$, and $\mu_2(n)$ are assumed uncorrelated.

The amplitude and phase of each sinusoidal frequency component in a signal can be obtained by phase-sensitive demodulation [38]. For a frequency-sweeping signal composed of multiple frequency components, the TOF of the propagation path can be found using the mean transit time of all frequency components. Since $x_l(n)$ is sampled in front of the loudspeaker, the SNR is very high, and for naming convenience, we call $x_l(n)$ the source signal. A brief flowchart in Figure 3 describes how the time-frequency relationship between the source signal $x_l(n)$ and received signal $x_r(n)$ is obtained. Both the source signal and the received signal are processed with a digital lock-in and a low-pass filter to extract the amplitude and phase of a signal operating at a known frequency[39], which is likely buried in a noisy background. The first harmonic $R_f$ can be extracted from the source and received signals using a digital lock-in processing technique [40-42]. For $R_f$, its demodulated components $X$ and $Y$, expressed as $X_i(t)$ and $Y_i(t)$ are obtained through a multiplier and a low-pass filter, where the source and received signals are multiplied with the two sets of orthogonal reference signals ($\cos(2\pi fm_0t)$ and $\sin(2\pi fm_0t)$)[43], and the resultant signals are filtered.

Therefore, the modulus of the first harmonic $R_f$ can be calculated using equation (4).

$R_f(t) = \sqrt{X_i^2(t) + Y_i^2(t)}$  

Through the peak detection of the first harmonic, the moment $f_{m_0}$ corresponding to the frequency $f_m$ can be found, and the relationship between the time and frequency of the acoustic source and the received signal can then be established. The TOF $(\tau)$ is computed using the average of TOF at the characteristic frequencies over the pass-band $B$:

$$\tau = \frac{1}{B} \sum_{f_m} B \left( t_{\text{received},f_m} - t_{\text{source},f_m} \right)$$

where $t_{\text{received},f_m}$ and $t_{\text{source},f_m}$ are the moment of time of the received and source signals at the frequency of $f_m$ respectively.

Since it is constant for the signal source by the loudspeaker every time, and its time-frequency information is known. Therefore, during on-line monitoring, only digital lock-in processing is needed for each received signal $x_r(n)$ in Figure 3, and then its TOF $(\tau)$ can be obtained by comparing with the known time-frequency relationship of $x_l(n)$. This makes the amount of calculation each time depend only on the length of the received signal. Moreover, by analysing and extracting the first harmonic $R_f$ from the source and received signals through digital lock-in processing, it is possible to avoid both the complex mathematical operations of Fourier expansion and convolution of the cross-correlation algorithm discussed in section 2.2.2. Therefore, this method is very effective in reducing the computational burden.

Because the sinusoidal linear frequency-sweeping signal contains multiple frequency components, there exists intermodulation distortion in the output signal of the loudspeaker. Coupled with the harmonic distortion in the acoustic transmission, such intermodulation distortion can cause a nonlinear relationship between the instantaneous frequency and the time in the received signal. In this research, this problem is overcome by using the DLF technology, with which the time-frequency relationship can be fitted linearly. The least-squares regression method is the most commonly used linear fitting solution[44, 45]. This method is featured with maximizing the fitting accuracy by minimizing the sum of squared errors so that an optimized relationship can be identified.

Assume that the linear equation of the time-frequency relationship is:

$$f_i = \alpha + \beta t_i + \varepsilon$$

where $\alpha$ and $\beta$ are the intercept and slope respectively, and $\varepsilon$ are the residuals between the actual and the fitting frequency values.

The residual sum of squared error $\Delta f (\alpha, \beta)$ can be determined as follows:

$$\Delta f (\alpha, \beta) = \sum_{i=1}^{n} (f_i - \alpha - \beta t_i)^2$$

where $f_i$ is the actual value. $\alpha + \beta t_i$ is the targeting value. By making the partial derivatives of the residual sum $\Delta f(\alpha, \beta)$ with respect to $\alpha$ and $\beta$ to zeros, the optimized $\alpha$ and $\beta$ can be identified to achieve the minimum $\Delta f(\alpha, \beta)$.

2.2.2 Generalized Cross-correlation Algorithm

The generalized cross-correlation algorithm is the most widely used TOF estimation method in acoustic measurement technology, which is usually further divided into DCC [8, 46] and GCC [9, 47]. In the GCC algorithm, signal is usually weighted in a way to reduce the sensitivity
to noise[9]. The TOF (τ) in equation(1) can be obtained using the GCC algorithm. A brief flowchart in Figure 4 describes its principle.

The cross-correlation between the source signal x1(n) and the received signal x2(n) is related to the power spectral density (PSD) function by the well-known Fourier transform relationship[9]:

\[ G_{x_1x_2} (f) = F_x x_1(n) F^* x_2(n) \]  \hspace{1cm} (8)

\[ R_{x_1x_2} (\tau) = \int_{-\infty}^{\infty} \psi(f) G_{x_1x_2} (f) \exp(j2\pi f \tau) df \]  \hspace{1cm} (9)

where \( F \) stands for Fourier transform. \( R_{x_1x_2} (\tau) \) is the GCC coefficient. \( G_{x_1x_2} (f) \) is the cross PSD function. \( \psi(f) \) denotes the general frequency weighting function.

It can be seen from (9) that TOF(τ) can be obtained through peak detection of the GCC coefficient function \( R_{x_1x_2} (\tau) \).

The principle of the GCC algorithm in Figure 4 shows that, during on-line monitoring, each TOF (τi) calculation requires the participation of the known source signal \( x_1(n) \). The Fourier transform and Fourier inverse transform are performed by \( x_1(n) \) with the \( k \)-th received signal \( x_k(n) \). Moreover, the data size of the cross-correlation coefficient is twice as large as that of the source signal, which will undoubtedly increase the system burden.

In comparison with GCC, the amount of data involved in the computation of the DLF algorithm is halved, and only digital lock-in processing is required for each received signal. Moreover, in the DLF, complex mathematical calculations such as Fourier transform and inverse transform are not required, which leads to a significant reduction in the computational effort for time delay estimation.

3. Experimental setup and result analysis

In this research, the feasibility of the DLF algorithm was investigated experimentally for acoustic waves at different distances and under different SNR conditions. The distance and noise intensity were controlled with reference to the actual operating conditions of utility boilers [29, 48]. The maximum length is set as 10 m and the maximum TOF is controlled within 30 ms. The GCC algorithm was operated synchronously in parallel with DLF for comparison purpose. In order to avoid interference from other environmental sources in the experiments, the waveguides were used to ensure that the acoustic wave propagates inside the pipe only. Since the length of the waveguide and the temperature were known, the theoretical acoustic TOF can be obtained as a reference as shown in Figure 5, the experimental system mainly consists of an acoustic waveguide, two loudspeakers, a microphone receiver. A data acquisition card, a power amplifier and an industrial computer were also included, but not shown in this figure. The loudspeakers \( S_0 \) and \( S_1 \) were used to emit acoustic signals and the Gaussian white noise with adjustable energy levels respectively, to create different SNR conditions.

The dashed box in Figure 5 indicates that the length of the acoustic waveguide was adjustable to create different theoretical delay time. The diameter of the acoustic waveguide used in the experiments was 45 mm. The microphone was 1/2-inch in diameter and non-directional with their sensitivity of 67.1 mv/pa, dynamic range of 20 dB to 136 dB, and frequency response range of 20 Hz to 20 kHz. The data acquisition card used was NI-USB6356, which contains eight simultaneous analog input channels, each channel having a sampling rate of 1.25 MS/s with a resolution of 16-bit.

3.1 Time Delay Estimation at Different Distances

The first part of experiments was conducted to verify the accuracy of the DLF algorithm under different distance conditions by adjusting the length of the acoustic waveguide. The temperature of the measurement environment was kept constant during the experiment. In these experiments, \( S_0 \) did not emit noise, and the SNR of the experimental environment was constant and above 10 dB. The acoustic waveguide was set at 8 different lengths of 0.5 m, 1.0 m, 2.0 m, 3.0 m, 4.0 m, 5.0 m, 7.0 m, and 10.0 m respectively. The laboratory environment temperature was controlled at 26.3 °C.

Figure 6 shows the time-domain waveforms and PSD functions of the received acoustic signals at waveguide length of 3 m. The red and black curves in Figure 6(a) are the...
source waveforms and filtered received signal with the normalized amplitudes. The enlarged portion of the dashed box in Figure 6(a) shows that both the sound source and the received signal are sinusoidal. Since the signal from the microphone was received synchronously when $S_0$ started emitting the waveform, the received signal has a certain time delay to the acoustic source signal due to waveform propagation in the medium for the given distance. This delay is TOF, which depends on the length of the acoustic waveguide. From the PSD functions of the two signals in Figure 6(b), it is clear that both the source and the received signals are linear frequency-sweeping signals. Compared to the frequency components outside the signal band, the power of both the source and the received signal is strong. The signal is received after propagation, and its PSD is slightly attenuated. According to the time-frequency correspondence, it can be seen that the range of power reduction in the frequency domain is roughly the same as the attenuation range of the time-domain waveform plot. However, since the amount of PSD attenuation in the effective frequency band was much higher than that of the frequency components outside the signal band, i.e. the frequency component of interest was relatively little attenuated, the TOF estimation was not affected.

Figure 6. Waveform and PSD of source and received signals with 3 m long waveguide

The time-frequency relationship with different TOFs was obtained by using the DLF algorithm to process the received signals at previously mentioned eight different waveguide lengths. Figure 7 shows the time-frequency relationship of the received signals for the waveguide of 2 m and 10 m. From this figure, it can be seen that when the received signals were directly processed using the DLF algorithm, nonlinear relationship occurred around the starting and ending points of the time-frequency curves. This is due to the intermodulation distortion generated during the loudspeaker emission and the harmonic distortion produced during the acoustic transmission. The time-frequency curves were fitted with the least-squares linear regression method proposed in 2.2.1, and the fitting results are shown in Figure 7. Based on the fitting curves, the time-frequency relationship can be regarded as linear over the frequency range from 4 kHz to 8 kHz.

Figure 7. Time-frequency relationship for 2 m and 10 m long acoustic waveguides

The time-frequency relationship between the acoustic source and the received signals was tested and obtained at the given eight conditions. From Figure 8, it is evident that the time-frequency lines are parallel for these different waveguide lengths. With the waveguide length increases, the acoustic signal remains constant in frequency and only shifts with time. Larger the waveguide length is, greater the amount of TOF becomes.

Figure 8. Time-frequency relationship between the acoustic source and eight received signals

In the experiments, the two algorithms analysed in sections 2.2.1 and 2.2.2 were used to solve the acoustic TOF for each of the eight conditions. The number of measurements $n$ at each distance is 2000. The standard deviation ($SD$) and the systematic error $\delta$ are given by:
\[ SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\tau(i) - \hat{\tau})^2} \]  
(10)

\[ \delta = \frac{1}{n} \sum_{i=1}^{n} |\tau(i) - \hat{\tau}(i)| \]  
(11)

where \( \tau(i) \), \( \bar{\tau} \) and \( \hat{\tau}(i) \) denote measured values, average values and reference values; \( n \) is the number of measurements.

Figure 9(a) shows the acoustic TOF for each of the eight conditions calculated by the two algorithms. Since the TOFs obtained using the two algorithms were very close, the ranges of the left and right axes in Figure 9(a) are adjusted slightly inconsistently to allow the respective calculation results and error bars to be clearly expressed. Moreover, in order to make the comparison of error bars obvious in the figure, the standard deviations of the both methods are enlarged by 20 times. Figure 9(a) shows that the TOFs calculated using the two algorithms were almost linearly related to the measurement distance, i.e. waveguide length, and the SD values increases with distance. The maximum errors were 0.030 ms and 0.039 ms for DLF and GCC respectively. The TOF standard deviations for DLF algorithm at all 8 distances were lower than that of GCC algorithm, which demonstrated that the precision of TOF measurement using DLF algorithm is superior over that of GCC algorithm, showing good robustness of DLF method.

![Figure 9](image.png)

**Figure 9. Time delay estimation results of GCC and DLF algorithms**

Figure 9(b) shows the TOF systematic error \( \delta \) of the DLF and GCC algorithms. Similar to the variation, the \( \delta \) values of the two algorithms also increase with distance. The maximum errors of 0.034 ms and 0.055 ms were recorded for DLF and GCC respectively. The DLF algorithm has achieved smaller bias value at each given distance, which shows DLF has better trueness over GCC.

### 3.2 Time Delay Estimation under Different SNR conditions

Since the experiments with variable distances in Section 3.1 were conducted without noise interference, it was impossible to verify their accuracy and applicability in noisy environments. For the purposes of applications, several key practical factors should be considered, such as environmental interference, main frequency bands, and the power level of background noise\[32\]. For the flue gas in power plants, the main component of noise is Gaussian White Noise\[49\], and according to the research \[32\], it has been known that the main frequency band of the internal noise in the power plant furnace is below 3 kHz.

In the experiments described in this section, the loudspeaker \( S_0 \) was still used to emit acoustic source signal and Loudspeaker \( S_1 \) was to emit Gaussian white noise signals with adjustable intensity to set the SNR of the measurement environment. The noise energy was controlled by varying the intensity of the noise source and the gain of the power amplifier. In the experiments, the following SNR: 0.086 dB, -3.020 dB, -6.322 dB, -9.380 dB, -11.698 dB, -14.937 dB, and -16.167 dB were used. The number of measurements \( n \) under each SNR condition is 2000.

![Figure 10](image.png)

**Figure 10. Time delay estimation results under different SNR conditions**

Figure 10 shows the statistical results of TOF estimation for the two algorithms in different energy level of background noise with a fixed waveguide length of 2.03 m. The temperature of the measurement environment was kept constant during the experiments. With the decrease of SNR, the systematic error \( \delta \) of the both algorithms has an increasing trend as a whole, with slight fluctuation. It can be seen that although the difference of \( \delta \) between the two algorithms under each SNR condition were very small, the DLF algorithm has better results at every given SNR conditions. This indicates that although both have certain anti-noise capability, the DLF algorithm is more robust to interference. The SD of the DLF is smaller than that of the GCC for all seven SNR conditions with the maximum SD deviation occurred at the SNR of -6.322 dB. Neither the \( \delta \) nor the SD of both algorithms tended to increase significantly with the enhanced noise. Compared to the classical GCC
algorithm, the good measurement precision and robustness of DLF had been achieved.

4. Conclusion

In this paper, the TOF estimation algorithm using the digital lock-in filter (DLF) is proposed. The TOF estimation by using this method is obtained via harmonic peak detection after the signals are processed by digital lock-in and low-pass filtering. Theoretical and experimental results demonstrate that the accuracy (both bias and precision) of the proposed algorithm is better than that of the GCC, and the computational effort is halved compared to the GCC algorithm, which greatly reduce the computational burden of the system. The computational error analysis also indicates that the TOF measurement with DLF is robust even in noisy environments. The average δ of the DLF algorithm is 0.012 ms lower than that of the GCC algorithm, which means that the accuracy is improved by 4.8% with the DLF algorithm. The maximum SD values of the two algorithms are 0.125 ms and 0.119 ms, respectively, in an environment with SNR ranging from 0.086 dB to -16.167 dB, showing good accuracy and robustness over GCC. With further development, this novel method can provide an alternative, more accurate and robust temperature distribution measurement for acoustic pyrometry used in power plant furnaces.

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