NUMERICAL SIMULATION OF EXCITATION AND PROPAGATION OF HELIOSEISMIC MHD WAVES: EFFECTS OF INCLINED MAGNETIC FIELD

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Abstract

Investigation of propagation, conversion, and scattering of MHD waves in the Sun is very important for understanding the mechanisms of observed oscillations and waves in sunspots and active regions. We have developed a three-dimensional linear MHD numerical model to investigate the influence of the magnetic field on excitation and properties of the MHD waves. The results show that surface gravity waves (f-modes) are affected by the background magnetic field more than acoustic-type waves (p-modes). Comparison of our simulations with the time–distance helioseismology results from Solar and Heliospheric Observatory/MDI shows that the amplitude of travel time variations with azimuth around sunspots caused by the inclined magnetic field does not exceed 25% of the observed amplitude even for strong fields of 1400–1900 G. This can be an indication that other effects (e.g., background flows and nonuniform distribution of the magnetic field) can contribute to the observed azimuthal travel time variations. The azimuthal travel time variations caused by the wave interaction with the magnetic field are similar for simulated and observed travel times for strong fields of 1400–1900 G if Doppler velocities are taken at the height of 300 km above the photosphere where the plasma parameter \( \beta \ll 1 \). For the photospheric level the travel times are systematically smaller by approximately 0.12 minutes than for the height of 300 km above the photosphere for all studied ranges of the magnetic field strength and inclination angles. Numerical MHD wave modeling and new data from the HMI instrument of the Solar Dynamics Observatory will substantially advance our knowledge of the wave interaction with strong magnetic fields on the Sun and improve the local helioseismology diagnostics.

Key words: Sun: oscillations – sunspots

1. INTRODUCTION

Local helioseismology has provided important results about the structures and dynamics of the solar plasma below the visible surface of the Sun, associated with sunspots and active regions (e.g., Duvall et al. 1996; Kosovichev 1996; Kosovichev et al. 2000; Zhao et al. 2001; Haber et al. 2000; Komm et al. 2008). The helioseismic inferences help us to understand the complicated processes of the origin of solar magnetic structures, formation and evolution of sunspots and active regions. These studies are based on measurements and inversions of variations of acoustic travel times and oscillation frequencies in the areas occupied by the magnetic field and around them.

The influence of inclined magnetic field on solar atmospheric oscillations has been extensively studied (e.g., Cally 2006; Schunker & Cally 2006; Schunker et al. 2008). There are several factors that may cause the observed variations of the oscillation properties, and it is very important to painstakingly investigate their effects to improve the reliability of the helioseismic inference (Bogdan 2000). Such studies are carried out both observationally by doing various experiments with the data analysis procedure, e.g., by masking the regions of strong field, doing “double-skip” experiments, etc. (e.g., Zhao & Kosovichev 2006), and theoretically by simulating wave propagation in various conditions of the solar convection zone and calculating how these conditions affect the helioseismic observables, such as the oscillation power spectrum and acoustic travel times (Georgobiani et al. 2007; Zhao et al. 2007). We emphasize that for correct interpretation of helioseismic results the theoretical modeling must include calculations of the actual observables, taking into account all important aspects of the data measurement procedure, such as data filtering and averaging, and geometrical factors (Nigam et al. 2007). If the actual helioseismic measurement procedure is not modeled, this may lead to incorrect conclusions about the role of various factors in the helioseismic results.

In general, the main factors causing variations in helioseismic travel times in solar magnetic regions can be divided into two types: direct and indirect. Direct effects are due to the additional magnetic restoring force, which changes the wave speed and may transform acoustic waves into different types of MHD waves. Indirect effects are due to the changes in the convective and thermodynamic properties in magnetic regions. These include depth-dependent variations of temperature and density, large-scale flows, and changes in wave-source distribution and strength. Both direct and indirect effects may be present in the observed travel time and frequency variations and cannot be easily disentangled by data analyses causing confusions and misinterpretations. Thus, it is very important to investigate the various aspects by numerically modeling the individual factors separately.

In particular, we have investigated the effects of the suppressed excitation of acoustic waves in sunspot regions, where strong magnetic field inhibits convective motions, which are the primary source of solar waves (Parchevsky & Kosovichev 2007a). The results showed that the suppression of acoustic sources may explain most of the observed deficit of acoustic power in sunspot regions (Parchevsky & Kosovichev 2007b), and also cause systematic shifts in the travel-time measurements. However, these shifts are significantly smaller than the observed variations of the travel times and also have the opposite sign (Parchevsky et al. 2008), and thus cannot affect the basic conclusions on the sunspot sound-speed structure, contrary to previous suggestions (e.g., Rajaguru et al. 2006).

The goal of this paper is to model the excitation and propagation of helioseismic waves (both f- and p-modes) in the presence of high magnetic field.
of inclined magnetic field and investigate the importance of the inclined field in the time–distance helioseismology measurements by carefully modeling the measurement procedure and comparing with the observational results, obtained by Zhao & Kosovichev (2006). The issue of the influence of the inclined magnetic field was raised by Schunker et al. (2005), who found that the phase shift of the signal in the penumbra of a sunspot, measured by the acoustic holography technique varies with sunspot position on the disk. They attributed this to the variations of the angle between the inclined magnetic field of the penumbra and the line of sight (LOS). They suggested that the variations of the phase shift may affect the differences of the sound-speed distribution below sunspots, inferred by time–distance helioseismology. However, Zhao & Kosovichev (2006) repeated the analysis of the same sunspot using the time–distance technique, and found substantially smaller variations with the position on the disk, and no significant effect on the wave–speed profile. They also found that the variations due to the inclination angle exist only for the wave measurements using the Doppler-shift signal, and that the variations are absent when the travel times are measured from the simultaneous intensity observations from Solar and Heliospheric Observatory (SOHO)/MDI. This result indicates that the observed variations with the inclination angle of the Doppler-shift measurements are likely to be related to changes of the ratio between the vertical and horizontal components of the displacement vector of the solar oscillations in the penumbra, and not the wave transformation or other effects, which could affect the modal structure of the oscillations. The solar oscillation theory predicts that the ratio between the vertical and horizontal components mostly depends on the surface boundary conditions (e.g., Unno et al. 1989). In the sunspot umbra, the boundary conditions may change due to the inclined magnetic field or/and near surface flows, the Evershed effect, which is observed directly in the Doppler-shift data and shows a significant center-to-limb variation.

In this paper, we present the results of numerical modeling of the inclined magnetic field on the time–distance helioseismology measurements by isolating this effect in a simple magnetic configuration, and show that only 25% of azimuthal travel time variations measured by Zhao & Kosovichev (2006) can be explained by a direct influence of the inclined magnetic field on acoustic waves. In Section 2, we present the governing equation and describe the numerical method of three-dimensional modeling of helioseismic MHD waves. In Section 3, we present the code verification results comparing the numerical results with analytical solutions for simple cases. In Section 4, we present the simulation results of the wave propagation in regions with inclined magnetic field, calculation of the center-to-limb variations of the time–distance helioseismology measurements, and comparison with the observational results.

2. NUMERICAL MODEL

2.1. Governing Equations and Numerical Scheme

The propagation of MHD waves inside the Sun is described by the following system of linearized equations:

\[
\begin{align*}
\frac{\partial \rho'}{\partial t} + \nabla \cdot \mathbf{m}' &= 0, \\
\frac{\partial \mathbf{m}'}{\partial t} + \nabla \rho' - \frac{1}{4\pi} \left( (\nabla \times \mathbf{B}') \times \mathbf{B}_0 + (\nabla \times \mathbf{B}_0) \times \mathbf{B}' \right) &= \rho' \mathbf{g}_0 + \mathbf{S},
\end{align*}
\]

where \( \mathbf{m}' = \rho_0 \mathbf{v}' \) is the momentum perturbation, \( \mathbf{v}' \), \( \rho' \), \( p' \), and \( \mathbf{B}' \) are the velocity, density, pressure, and magnetic field perturbations respectively, and \( \mathbf{S} \) is the wave-source function. The quantities with subscript 0, such as gravity \( \mathbf{g}_0 \), sound speed \( c_0 \), and Brünt–Väisälä frequency \( \mathcal{N}_0 \) correspond to the background model, and \( \mathbf{B}_0 \) is the background magnetic field satisfying the usual hydrostatic equilibrium equation. The spatial and temporal behavior of the wave source is modeled by function \( f(x, y, z, t) \equiv AH(r)F(t) \):

\[
H(x, y, z) = \begin{cases} 
\left( \frac{1 - \frac{r^2}{R_{src}^2}} \right)^2 & \text{if } r \leq R_{src} \\
0 & \text{if } r > R_{src}
\end{cases}
\]

\[
F(t) = (1 - 2\tau^2)e^{-\tau^2},
\]

where \( R_{src} \) is the source radius,

\[
r = \sqrt{(x - x_{src})^2 + (y - y_{src})^2 + (z - z_{src})^2}
\]

is the distance from the source center, \( \tau \) is given by equation

\[
\tau = \frac{\omega_0(t - t_0)}{2}, \quad t_0 \leq t \leq t_0 + \frac{4\pi}{\omega},
\]

where \( \omega_0 \) is the central source frequency and \( t_0 \) is the moment of the source initiation. This source model provides the wave spectrum, which closely resembles the solar spectrum. It has a peak near the central frequency \( \omega_0 \) and spreads over a broad frequency interval. The source spectrum is

\[
|\tilde{F}(\omega)| \equiv \left| \int_{-\infty}^{\infty} F(t)e^{-i\omega t} dt \right| = 4\sqrt{\pi} \frac{\omega^2}{\omega_0^2} e^{-\frac{\omega^2}{\omega_0^2}}.
\]

The source spectrum for the central frequency of \( \omega_0 = 3.5 \) mHz is shown in Figure 1. In our simulations we used sources of two types: vertical force \( \mathbf{S} = (0, 0, f' \mathbf{z}) \) and pressure perturbation \( \mathbf{S} = \nabla f \). A superposition of such randomly distributed sources describes very well the observed solar oscillation spectrum (Parchevsky & Kosovichev 2007a).

For numerical solution of Equation (1) a semidiscrete finite difference scheme of high order was used. At the top and bottom boundaries nonreflective boundary conditions based on the perfectly matched layer (PML) technique were set. We used the standard solar model \( \mathbb{S} \) (Christensen-Dalsgaard et al. 1996) with a smoothly joined model of the chromosphere of Vernazza et al. (1976), as the background model. In this paper, we consider the uniform background magnetic field, which does not modify the hydrostatic pressure balance of the background model. The background model was modified near the photosphere to make it convectively stable. Details of numerical realization of the code and the background model can be found in Parchevsky & Kosovichev (2007a).

2.2. Code Verification for Different Types of MHD Waves

To verify the code and estimate the total error of the method we compare numerical results with one-dimensional analytical
solution. We assume that the background model is uniform and all quantities \(U = (\rho', u', v', w', B_x', B_y', p')^T\) depend on time \(t\) and one spatial coordinate \(x\) only. The one-dimensional linearized MHD equations in the infinite interval \(-\infty < x < \infty\) have a solution in the form of plane waves

\[
U = \hat{U} \exp[i(kx - \omega t)].
\]

There are three types of waves: Alfvén, slow, and fast MHD waves with phase velocities \(\pm V_A, \pm V_S, \pm V_F\), respectively, and amplitudes

\[
\hat{U}_{\pm A} = (0, 0, \pm V_A, 0, -B_0 \cos \theta, 0, 0)^T
\]

\[
\hat{U}_{\pm S} = \left(\rho_0, \pm V_S, 0, \pm \frac{c_A^2 V_S \sin \theta \cos \theta}{V_A^2 - V_S^2}, 0, -\frac{B_0 V_S^2 \sin \theta}{V_A^2 - V_S^2}, c_A^2 \rho_0\right)^T.
\]

\[
\hat{U}_{\pm F} = \left(\rho_0, \pm V_F, 0, \pm \frac{c_A^2 V_F \sin \theta \cos \theta}{V_A^2 - V_F^2}, 0, -\frac{B_0 V_F^2 \sin \theta}{V_A^2 - V_F^2}, c_A^2 \rho_0\right)^T.
\]

The phase velocities are

\[
V_A = c_A |\cos \theta|,
\]

\[
V_S = \frac{1}{\sqrt{2}} \sqrt{c_A^2 + c_A^2 + \sqrt{c_A^2 + c_A^2 - 2c_A^2 c_A^2 \cos 2\theta}},
\]

\[
V_F = \frac{1}{\sqrt{2}} \sqrt{c_A^2 + c_A^2 + \sqrt{c_A^2 + c_A^2 - 2c_A^2 c_A^2 \cos 2\theta}},
\]

where \(\rho_0, c_A,\) and \(B_0\) are respectively the density, sound speed, and magnetic field in the background state, \(c_A = B_0/\sqrt{4\pi \rho_0}\) is the Alfvén speed, \(\theta\) is the angle between the wave vector and background magnetic field \(B_y^{(0)} = B_0 \cos \theta, B_z^{(0)} = 0, B_0 = B_0 \sin \theta\). The sign “+” corresponds to the waves propagating in the positive direction of the \(x\)-axis and the sign “−” corresponds to the waves propagating in the opposite direction.

For testing the numerical simulations we use our three-dimensional code with initial conditions depending only on the \(x\)-variable. The calculations are carried out in the Cartesian geometry in the domain of \(15.46 \times 15.46 \times 3.05\) Mm with the numbers of grid points: \(N_x = 104, N_y = 71, N_z = 1\). All boundary conditions are chosen periodic simulating an infinite spatial domain. Wavevector \(k = 6\pi/(N_x + 1)\Delta x\) is chosen in a way to match the periodic boundary conditions. In dimensionless variables \(\tilde{\rho} = \rho/\bar{\rho}, \tilde{\rho} = p/\bar{\rho} \tilde{c}_A^2, \tilde{v} = \tilde{v}/\tilde{c}_A,\) and \(\tilde{B}^2 = \bar{B}^2/4\pi \bar{\rho} \tilde{c}_A^2\) parameters of the background model are

\[
\tilde{\rho} = 1, \quad \tilde{c}_A = 1, \quad \tilde{B} = 0.5, \quad c_A = 0.5, \quad \theta = \pi/4.
\]

The amplitude of the Alfvén wave (in chosen dimensionless variables) traveling in the positive direction of the \(x\)-axis is

\[
\hat{U}_{\alpha A} = (0, 0, -1, 0, 0, 0)^T.
\]

The slow and fast MHD waves traveling in the same direction these amplitudes are

\[
\hat{U}_{\alpha S} = (1, 0.33108, 0, 2.6894, 0, -2.5184)^T \quad \text{and} \quad \hat{U}_{\alpha F} = (1, 1.0679, 0, -0.13146, 0, 0.39708, 1)^T,\]

respectively. The cyclic frequencies of these waves are \(\omega_A = k V_A, \omega_S = k V_S,\) and \(\omega_F = k V_F,\) respectively. The results of our numerical and analytical solutions for the moment of time \(t = 20\) minutes for all three waves are shown in Figure 2. Panels (a), (b), and (c) represent the results for the Alfvén, slow, and fast MHD waves, respectively. Only the variables \(v\) and \(B_x\) are nonzero in the Alfvén wave. They are shown by the solid and dashed curves in panel (a), respectively. The exact solution

Figure 1. Spectrum of the source localized near the central frequency \(\nu = 3.5\) mHz and spread out on the interval of approximately 2–6 mHz (full width at half-maximum (FWHM)). The source depends on time as Ricker’s wavelet and has finite lifetime.

Figure 2. Comparison of analytic (circles) and numerical (markers) solutions for plain Alfvén (a), slow MHD (b), and fast MHD (c) waves, respectively. For Alfvén wave only \(v\) and \(B_x\) (marked by the solid and dashed curves, respectively) are nonzero. The circles mark an analytical solution for \(v\). The solid, dashed, dashed, and dotted curves in panels (b) and (c) correspond to \(\rho, u, w,\) and \(B_t,\) respectively. The analytical solution for \(w\) is shown by the circles.
for $\nu$ is shown by circles. In the slow and fast MHD waves, variables $\rho$, $u$, $w$, $B_z$, and $p$ are all nonzero. The dimensionless pressure coincides in amplitude and phase with the density and is not shown in Figure 2. Variables $\rho$, $u$, $w$, and $B_z$ are shown in panels (b) and (c) by the solid, dash-dotted, dashed, and dotted curves, respectively. The exact analytical solution for $w$ is shown by circles. The relative accuracy of the numerical solution is $|U_{\text{num}}(x, t) - U_{\text{exact}}(x, t)|/U_{\text{exact}} < 1.5 \times 10^{-4}$, where $U_{\text{num}}(x, t)$ and $U_{\text{exact}}(x, t)$ are the numerical and exact solutions, respectively, and $U_{\text{exact}}$ is the amplitude of the exact solution. We see that our numerical solutions reproduce the amplitudes, phases, and velocities of the Alfvén, slow, and fast MHD waves very well.

3. RESULTS AND DISCUSSION

3.1. MHD Waves Generated by a Single Source in the Uniform Background Magnetic Field

In this section, we present our results of numerical simulation of excitation and propagation of MHD waves generated by a single source of vertical force with central frequency $\nu = 3.5$ mHz in a rectangular region of size $15.5 \times 15.5 \times 12.5$ Mm$^3$ (104 $\times$ 104 $\times$ 70 nodes). The question of the depth of the acoustic sources in the Sun is still open. We chose shallow (100 km deep) sources, as suggested by Nigam & Kosovichev (1999) and Kumar & Basu (2000). The horizontal grid is uniform with $\Delta x = \Delta y = 150$ km. The vertical grid is nonuniform. The grid step $\Delta z$ varies from 50 km near the photosphere to 600 km near the bottom of the computational domain. Time step $\Delta t = 0.5$ s was chosen to satisfy the Courant stability condition. Vector $B_0 = (B_0 \sin \gamma, 0, B_0 \cos \gamma)^T$, $B_0 = 625$ G of the uniform inclined background magnetic field lies in the XZ-plane and has an inclination angle of $\gamma = 45^\circ$ with respect to the top boundary normal. Horizontal layer with $\beta = 1$ has the height of 200 km above the photosphere. Plasma parameter $\beta$ represents the ratio of the gas pressure to the magnetic pressure.

The simulation results are shown in Figure 3. The left panels represent the horizontal cuts of the computational domain at the level of the photosphere (shown in the right panels by the white horizontal line near the top boundary). The right panels represent the vertical slices of the domain through the source position. The vertical maps of $\rho \omega / \nu$ and $B_1$ (right panels (b) and (c), respectively) reveal a strong wave (marked as A in the right panel (b)), which propagates downward along the background-inclined magnetic field lines (the waves propagating upward are absorbed by the top PML). The wave source generates a mixture of the Alfvén, slow, and fast MHD waves. The fast MHD wave (marked as F) has speed much greater than the wave A and does not make contribution to its amplitude. We do not see a noticeable amplitude of wave A in density perturbations (right panel (a)).

The concentric waves in the left panels represent a mixture of fast MHD waves (analogous to $p$-modes in the absence of the magnetic field) and the surface magnetogravity waves (analogous to $f$-modes). Since the wave speed depends on the angle between the vector of the background magnetic field and the wavevector, the wave fronts are anisotropic.

Due to the different dispersion relations the magnetogravity and fast MHD waves are easily separated on time–distance diagram (see Figure 4). To make the wave structure more clear we presented time–distance diagram for simulations in bigger region (216 Mm $\times$ 216 Mm in the horizontal direction). The ridges formed by the waves reflected from the photosphere are clearly seen. The lowest ridge corresponds to the direct wave (first bounce). Higher ridges correspond to the reflected waves (second, third, and higher bounces, respectively). The magnetogravity wave analogous to the $f$-mode in the absence of the magnetic field is marked by the arrows. It has characteristic “zebra” structure due to the difference between phase and group velocities of the $f$-mode. The depth of the region, the background model, and magnetic field are the same as in Figure 3. The solid black curve represents a theoretical time–distance curve for $p$-modes and standard solar model in the absence of the magnetic field, calculated for the quiet Sun in the ray approximation (Kosovichev & Duvall 1997).

3.2. Phase and Group Travel Time Variations Along the Wave Front

To study the travel time variations we performed simulations in a rectangular box of size $48 \times 48 \times 12.5$ Mm$^3$ (320 $\times$ 320 $\times$ 70 nodes) for different values $B_0$ and inclination angles $\gamma$ of the uniform background magnetic field: (625 G, 70$^\circ$), (1400 G, 45$^\circ$), and (1900 G, 30$^\circ$). The plasma parameter $\beta$ for these values of $B_0$ is shown in Figure 5. The heights of the horizontal layers with $\beta = 1$ relative to the photosphere for these cases are 200 km, -4.8 km, and -100 km, respectively (the negative values mean that the corresponding layers are below the photosphere). In these simulations, we analyzed the wave behavior at two different levels: at the photospheric level and at the horizontal layer 300 km above the photosphere. The grid step size, source type, and depth were chosen the same as in Section 3.1. Total simulation time equals 6 hr of solar time.

The simulation results for $B_0 = 625$ G and $\gamma = 70^\circ$ are shown in Figure 6. The top row presents $k-\nu$ diagrams, the bottom one shows corresponded horizontal snapshots of the $z$-component of momentum $\rho \omega w'$ at the level of the photosphere for the moment of time $t = 30$ minutes. The usual technique of fitting of the cross-covariance function by Gabor’s wavelet was used to calculate the travel times. This technique was developed for $p$-modes (Kosovichev & Duvall 1997). The source of the vertical ($z$-component) of force generates a strong gravity wave (the lowest ridge of the $k-\nu$ diagram). This wave has a different dispersion relation from the $p$-modes and has to be filtered out. For each field strength we adjusted the $f$-mode filter for better separation of the modes. The results of separation of $p$- and $f$-modes are shown in panels (b) and (c), respectively. The maps of $\rho \omega w'$ for the $p$- and $f$-modes (bottom row) were obtained by taking the inverse Fourier transform of the corresponding filtered three-dimensional spectra.

The background magnetic field causes asymmetry of the wave amplitude along the wave front and changes the shape of the wave front itself. This is clearly seen in Figure 7. The left panel represents the same picture as the bottom-left panel in Figure 6 plotted in polar coordinates with the origin at the source location. As was shown before, the $f$- and $p$-modes are spatially separated after approximately 30 minutes due to the different propagation speed. Hence, the wave front marked by the white sinusoidal curve belongs to the $f$-modes while the white vertical line marks the wave front which belongs to the $p$-modes. Variations of the $f$-mode wave front shape are about 0.7 Mm, while the wave front of the $p$-mode remains almost circular. The solid and dashed curves in the right panel present respectively the amplitude variations of the $f$- and $p$-modes (marked in the left panel by the white curves) along the wave front. We see that the wave amplitude variations along the wave front are about 10 times bigger for the $f$-modes than for the $p$-modes.
Before we proceed to the calculation of the travel times we can briefly summarize the contribution of the background magnetic field to the variations of the wave amplitude. The uniform inclined magnetic field causes asymmetry of the amplitude along the wave front (the stronger field the bigger asymmetry). For the density variations in the $p$-modes this asymmetry is more noticeable for the waves reflected from the surface (second and higher bounces on the Figure 4), which can be an evidence that the inclined magnetic field changes the reflection properties of the surface. The wave amplitude variations along the wave front are approximately 10 times stronger for the $f$-modes than for the $p$-modes, and the shape of the wave front is distorted more strongly for the $f$-modes as well.

As a result of the anisotropy of the wave properties, the wave travel times measured from the LOS component of the displacement velocity depend on the direction of the wave propagation and also on the viewing angle. We have investigated this effect for the case of the uniform inclined magnetic field. The choice of the coordinate system and geometry is shown in Figure 8. The horizontal $XY$-plane coincides with the photosphere. The origin of the Cartesian coordinate system is placed at the point $O$ above the wave source (the source itself is at the depth of 100 km below the photosphere). The uniform inclined background magnetic field lies in the $XZ$-plane ($B_y = 0$) and has an angle $\gamma$ with the normal to the photosphere. The location of the point of observations $P_{\text{obs}}$ is defined by the distance $\Delta$ from the wave source and the azimuthal angle $\alpha$. The LOS direction is defined by two angles: angle $\theta$ between the LOS direction and the local normal, and azimuthal angle $\psi$ between $OX$-axis and the projection of the LOS on the local horizontal plane. First, we build three-dimensional $k-\nu$ diagrams for each velocity component $u$, $v$, and $w$, filter out the $f$-modes as shown in Figure 6, and calculate the Cartesian components of velocity perturbation $u_p$, $v_p$, and $w_p$ for the $p$-modes by taking the
inverse Fourier transform of the corresponding spectra. Then, we calculate cross covariance

\[ C(r_1, r_2, \tau) = \frac{1}{T} \int_0^T v_{\text{LOS}}^{(p)}(r_1, t)v_{\text{LOS}}^{(p)}(r_2, t + \tau) dt \]  

(10)
of LOS velocities

\[ v_{\text{LOS}}^{(p)} = u_p \cos \psi \sin \theta + v_p \sin \psi \sin \theta + w_p \cos \theta \]  

(11)
in the origin of the system of coordinates and in the observational point \( P_{\text{obs}} \) (\( \Delta = 7.9 \) Mm). Then we fit the cross-covariance function with Gabor’s wavelet

\[ G(\Delta, \tau) = A \cos(\omega_0(\tau - \tau_p)) \exp \left[ -\frac{\delta \omega^2}{4} (\tau - \tau_g)^2 \right], \]  

(12)
where \( \Delta = |r_1 - r_2| \) is the distance between points where LOS velocities \( v_{\text{LOS}}^{(p)}(r_1, t) \) and \( v_{\text{LOS}}^{(p)}(r_2, t) \) are measured, \( A \) is the amplitude, \( \omega_0 \) is the central frequency, \( \tau_p \) and \( \tau_g \) are the phase and group travel times, respectively, and \( \delta \omega \) is the bandwidth. Parameters \( A, \omega_0, \tau_p, \tau_g, \) and \( \delta \omega \) are free and have to be determined from the fitting procedure. Repeating this procedure for different observational points with the same \( \Delta \) but different azimuthal angle we obtain \( \tau_p \) and \( \tau_g \) as functions of \( \alpha \).

Travel time variations along the wave front for different \( B_0 \) and different LOS angles are shown in Figure 9. Panels (a), (b), and (c) correspond to \( B_0 \) equal to 625 G, 1400 G, and 1900 G with the inclination angles of 70°, 45°, and 30°, respectively. The solid, dashed, and dash-dotted curves correspond to \( \psi = \{0°, 90°, 180°\} \), respectively. The angle between the LOS and the local normal is \( \theta = 20.5° \). The mean phase travel times show variations of about 0.5 minutes along the wave front, and the variation amplitude is a little smaller for strong (1400–1900 G) magnetic fields than for the weak 625 G field. For weak magnetic fields, changes of \( \psi \) change the shape of the curve, but not its average value. Strong magnetic fields cause anisotropy, and the mean value of the travel times changes with angle \( \psi \). Thus, averaging along the wave front for strong magnetic fields gives variations of the observed mean travel times of about 0.1–0.3 minutes with angle \( \psi \) (see detailed discussion in the next section).

### 3.3. Azimuthal Dependence of Phase Travel Times in Sunspots: Comparison with Observations

Zhao & Kosovichev (2006) observed different behavior of azimuthal dependence of phase travel times obtained from the SOHO/MDI Doppler shift data (Scherrer et al. 1995) in sunspots depending on their position on the disk. As far as we want to reproduce similar conditions (and geometry) in our simulations we give a detailed description of their algorithm of the phase travel time calculation. We will apply the same technique to our simulated data.

The sunspot is located at latitude \( \phi \) and longitude \( \lambda \) on the eastern part of the disk as shown in Figure 10. The sunspot meridian plane is shown in gray. Orth \( i \) of the global system of coordinate with the origin in the center of the Sun is aimed at the center of the visible solar disk, orth \( k \) points to the north, and orth \( j \) points to the west. The local system of coordinates with the origin in the center of the sunspot is chosen in such a way that \( e_x \) coincides with the local normal to the surface, \( e_y \) is directed to the west, and \( e_z \) is directed along the meridian and forms a right vector triplet with \( e_x \) and \( e_y \).

Observational point \( P_{\text{obs}} \) is chosen inside the sunspot penumbra. Azimuthal \( A \) is counted counterclockwise from local west direction \( e_z \). Two signals are calculated: the LOS velocity at \( P_{\text{obs}} \), and the LOS velocity averaged along the annulus (between inner and outer radii of a ring with average radius \( \Delta = 8 \) Mm with the origin in the observational point). Cross covariance of these two signals is fit with Gabor’s wavelet. Fitting procedure gives us mean phase and group travel times. We assume, that the annulus is small enough that sunspot magnetic field \( B_0 \) inside the annulus can be considered as uniform. Hence the problem is reduced to simulations with the uniform inclined magnetic field shown in Figure 8. To compare results of the simulations with the observations we have to find from what direction we have to look at our simulation domain to match the observations. In other words, we have to find a relation between azimuthal angle \( A \) of the sunspot center and \( \psi \). Angle \( \psi \) here has the same meaning as in Figure 8. This is the angle between the projection of the background magnetic field on the local horizon (XY-plane) and projection of the LOS on the same plane. The unit vector in LOS direction in the sunspot local system of coordinates is given by equation

\[ i = \sin \lambda \ e_x - \sin \phi \cos \lambda \ e_y + \cos \phi \cos \lambda \ e_z. \]  

(13)
The projection of \( i \) on the local horizontal plane (defined by the vectors \( e_x \) and \( e_y \)) has coordinates \( i_p = (- \sin \lambda, - \sin \phi \cos \lambda, 0) \) (nonunit vector). Angle \( \psi \) is given
The systematic shift of the calculated mean travel times with respect to the observations is caused by temperature effects (the sound speed is smaller inside sunspots than in the quiet Sun), which were not included in these simulations. The travel times systematically decrease when the strength of the magnetic field increases because the speed of the fast MHD wave increases with the magnetic field. For studying the inclined field effect we are

by equation

$$\psi = -A - \angle \mathbf{p} \mathbf{e}_x = -A - \arccos \left( \frac{-\sin \lambda}{\sqrt{\sin^2 \lambda + \sin^2 \phi \cos^2 \lambda}} \right).$$

(14)

We performed numerical simulations of propagation of MHD waves in the presence of the uniform inclined magnetic field for different inclination angles and strengths of the magnetic field. The goal of these simulations was to calculate contribution of the inclined magnetic field effect to variations of the mean travel times with the azimuthal angle. This is why we used horizontally uniform standard solar model as a background model. The same model was used to obtain Figure 3. The mean travel times obtained for the photospheric level (panel (a)) and the level of 300 km above the photosphere (panel (b)) are shown in Figure 11. The solid curve corresponds to the magnetic field $B_0 = 625$ G with inclination angle $\gamma = 70^\circ$. The dashed and dash-dotted curves are corresponded to cases $B_0 = 1400$ G, $\gamma = 45^\circ$ and $B_0 = 1900$ G, $\gamma = 30^\circ$, respectively. The systematic shift of the calculated mean travel times with respect to the observations is caused by temperature effects (the sound speed is smaller inside sunspots than in the quiet Sun), which were not included in these simulations. The travel times systematically decrease when the strength of the magnetic field increases because the speed of the fast MHD wave increases with the magnetic field. For studying the inclined field effect we are
mostly interested in relative variations of the mean travel times. We provide the absolute values to give a general impression about the magnitude of contributions to the mean travel times from the temperature effect and variations of the fast MHD wave speed with the magnetic field. A model of a sunspot with both temperature and magnetic field effects will be presented in a future publication.

The amplitude of the mean travel time variations for weak (625 G) magnetic field is 10 times smaller than the observed quantity for the height of 300 km and even smaller for the photospheric level. For stronger magnetic fields (1400–1900 G) the amplitude of travel time variations is about 25% of the
observed amplitude. Behavior of the travel times depends on the magnetic field strength and the level of observation of Doppler velocities. For the weak magnetic field (625 G) the travel time variations are opposite for simulations and observations for both levels. For the height of 300 km above the photosphere where plasma parameter $\beta \ll 1$ for both magnetic field strengths (1400 G and 1900 G) the observed and simulated travel times have similar variations with azimuth. For the photospheric level (zero height) and the field strength of 1900 G the simulated and observed travel times also have similar azimuthal variations, while for $B_0 = 1400$ G the variations of the simulated travel times are opposite in sign to the variations of the observed travel times.

The curves in Figure 11 were obtained for the annulus radius of 8 Mm and the sunspot located at heliospheric latitude $\theta = 19.5$ and longitude $\lambda = -6.5$. The angle between LOS and local normal to the photosphere at the center of the sunspot ($\theta$ angle) is $20.5$. The parameters are chosen to match corresponding angles for Figure 1(b) of Zhao & Kosovichev (2006).

**4. CONCLUSION**

Numerical three-dimensional simulations of excitation and propagation of magnetoacoustic-gravity waves in the solar interior and atmosphere show that the presence of inclined background magnetic field causes amplitude variations along the wave front. These variations are stronger for the $f$-modes than for the $p$-modes; the shape of the wave front is distorted more strongly for the $f$-modes as well. The interaction of the wave source with the background magnetic field generates a mixture of fast, slow, and Alfvén waves. The helioseismic travel times, obtained from cross covariance of the $p$-mode LOS velocities at the observation point and the source point, show variations of about 0.5 minutes along the wave front (the amplitude depends on inclination $\theta$ of the LOS). Due to the anisotropy, the travel time averaged along the wave front (as in the observational procedure) is not zero. Comparison of the variations of the mean travel times versus the azimuthal angle of the observing point shows that simulated and observed travel times are the same when Doppler velocities are taken at the level of 300 km above the photosphere (at the same height as the observed velocities are obtained). The travel time weakly depends on the height of observations. The amplitude of variations of the travel times obtained from simulations is about 25% of the observed amplitude even for strong fields of 1400–1900 G. It can be an indication that other effects (for example, background flows or nonuniform distribution of magnetic field) can contribute to the observed travel time variations. The developed three-dimensional MHD wave propagation code provides an important tool for further investigations of local helioseismology in regions with strong magnetic field.

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