Agterberg and Dodgson Reply

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A reply to the Comment by Mineev and Champel.

In their Comment \cite{1}, Mineev and Champel have argued that our results are incorrect because of the simultaneous neglect of both non-linear and non-local terms in our theory. We disagree with this statement and point out that while Ref. \cite{1} argue that these terms should be included, they do not give a convincing physical argument that these terms are important at low fields. In the following, we explicitly show that non-linear and non-local terms can be safely ignored near $H_{c1}$ where our results are valid. Furthermore, our results agree with the assertion of Ref. \cite{2} that there is no true $A$ to $B$ phase transition in the vortex phase. In Ref. \cite{2}, we refer to the diverging correlation length that describes the $A$ to $B$ transition in the \textit{Meissner} phase.

To justify our claims, we give the missing steps between Eq. 2 and Eq. 3 of Ref. \cite{2}. The expression used for $\Psi_-$ in the $A$ phase was

\begin{equation}
\langle \xi_{A,-}^{-2} - \mathbf{D}^2 \rangle \Psi_-(\mathbf{r}) = \frac{\tilde{\kappa}}{\kappa} (D_x D_y + D_y D_x) \Psi_+(\mathbf{r})
\end{equation}

where $D_i = \partial_i + i 2\pi A_i/\Phi_0$. Eq. 3 in Ref. \cite{1} is found by setting the $-\mathbf{D}^2$ operator to zero. This operator cuts off the divergence that Ref. \cite{1} point out in their Eq. 3. We work with $\tilde{\kappa}/\kappa \ll 1$ and within a London approach. Taking $\Psi_+(\mathbf{r}) = |\Psi_+| e^{i\phi}$, $\Psi_- = e^{i\phi} \Psi_-$, and defining the superfluid velocity as $\mathbf{v} = \nabla \phi + 2\pi \mathbf{A}/\Phi_0$ we get

\begin{equation}
\langle \xi_{A,-}^{-2} + v^2 - 2i \mathbf{v} \cdot \nabla \Psi_-(\mathbf{r}) = |\Psi_+| \frac{\tilde{\kappa}}{\kappa} i \partial_x v_y + i \partial_y v_x - 2v_x v_y \rangle.
\end{equation}

This last equation shows that $\tilde{\Psi}_-$ will be proportional to $v$ (for small $v$). Consequently, for large vortex separations, the non-linear term in $v$ on the right hand side and the terms with $v$ on the left hand side of Eq. 2 can be ignored ($v \propto e^{-r/\lambda}/\sqrt{\pi \lambda}$ and the derivatives give a factor $1/\lambda \gg v$ for large $r$). After Fourier transforming and using the Maxwell relation $\mathbf{v} = \frac{2\pi \lambda^2}{\Phi_0} \nabla \times \mathbf{B}$, Eq. 2 becomes

\begin{equation}
\tilde{\Psi}_-(\mathbf{q}) = |\Psi_+| \frac{\tilde{\kappa}}{\kappa} \frac{q_x^2 - q_y^2}{\xi_{A,-}^2 + \mathbf{q}^2} \frac{2\pi \lambda^2}{\Phi_0} B(\mathbf{q}).
\end{equation}

Using this expression in the free energy and minimizing with respect to $B(\mathbf{q})$ gives the novel London equation that forms the basis of our results

\begin{equation}
\left[ 1 + \lambda^2 \mathbf{q}^2 + \frac{\lambda^2 \tilde{\kappa}^2 (q_x^2 - q_y^2)^2}{\kappa^2 \xi_{A,-}^2 + \mathbf{q}^2} \right] B(\mathbf{q}) = 0.
\end{equation}

This gives a 4-fold symmetry to the structure of a flux line out to the distance $\xi_{A,-}$, which diverges at the $A \rightarrow B$ transition temperature.

Using Eq. 3 $|\Psi_-(\mathbf{r})| \lesssim \frac{\tilde{\kappa}}{\kappa} |\Psi_+| \frac{2\pi \lambda^2 B(\mathbf{r})}{\Phi_0}$ (take $q_x^2 - q_y^2 \rightarrow \mathbf{q}^2$ in Eq. 3). This, with $\beta_1 |\psi_+|^2/\kappa = 1/\xi_{A,+}^2$ and $2\pi \lambda^2 B(\mathbf{r})/\Phi_0 = \lambda^2/d^2$, we find:

\begin{equation}
\frac{\langle \beta_1 |\Psi_-(\mathbf{r})|^4 \rangle}{\langle \kappa |\mathbf{D} \Psi_-(\mathbf{r})|^2 \rangle} \lesssim \frac{\tilde{\kappa}^2 \lambda^4}{\kappa^2 \xi_{A,+}^2}.
\end{equation}

where $d$ is the distance between vortices. Ref. \cite{1} uses this result as their Eq. 5. $H_{c1}$ marks a second order phase transition between the Meissner and the vortex phase. At this transition $B = 0$, which implies that as $H \rightarrow H_{c1}$, $d \rightarrow \infty$. Consequently, sufficiently near $H_{c1}$, the non-linear term can be safely neglected. Mineev and Champel argue that $d$ is cutoff when $d \approx \lambda/\sqrt{\ln(\lambda/\xi_{A,+})}$. Presumably, this accounts for the very small range of $H$ over which $B$ goes from $H_{c1}$ to zero (or $d$ goes from $\lambda$ to $\infty$). Nevertheless, this range is experimentally accessible, and furthermore, for flat plate-like samples the demagnetization factors force the applied field $H_{app} \propto B$, thus making the large $d$ limit
even more accessible. It is also argued that when Eq. 6 in Ref. [1] is not satisfied, then non-local corrections must be included (these are terms that are $O(q^4)$ or larger in Eq. 4). To counter this, we point out that Eq. 4 implies new physics at low fields (for small $q$ and large $d$) while non-local terms give new physics at high fields (for large $q$ and small $d$). In fact, we find non-local terms are not important when $\tilde{\kappa}/\kappa > \frac{\xi}{\lambda + \frac{\lambda}{d}}$. Consequently, non-local terms also become negligible in the large $d$ limit.

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[1] V.P. Mineev and T. Champel, preceding comment.
[2] D.F. Agterberg and M.J.W. Dodgson, Phys. Rev. Lett. 89, 017004 (2002).