Precision Supersymmetry Measurements at the $e^- e^-$ Collider

Hsin-Chia Cheng

Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, IL 60510

Abstract

Measurements of supersymmetric particle couplings provide important verification of supersymmetry. If some of the superpartners are at the multi-TeV scale, they will escape direct detection at planned future colliders. However, such particles induce nondecoupling corrections in processes involving the accessible superparticles through violations of the supersymmetric equivalence between gauge boson and gaugino couplings. These violations are analogous to the oblique corrections in the electroweak sector of the standard model, and can be parametrized in terms of super-oblique parameters. The $e^- e^-$ collision mode of a future linear collider is shown to be an excellent environment for such high precision measurements of these SUSY parameters, which will provide an important probe of superparticles beyond reachable energies.

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1 Introduction

If supersymmetry (SUSY) is relevant for the hierarchy problem, the supersymmetric partners of ordinary particles should have masses on the order of TeV scale. The discovery of supersymmetric particles at the present and future colliders is therefore promising. After the discovery, measurements of the superparticle properties such as their masses and couplings, will be the focus of studies. In particular, we need to check whether these new particles are indeed the superpartners of the Standard Model (SM) particles. This can be done by measuring the couplings of the superparticles, which is related to the couplings of the SM particles by supersymmetry. For example, SUSY implies the relations

\[ g_i = h_i, \]

where \( g_i \) are the SM gauge couplings, \( h_i \) are their SUSY analogues, the gaugino-fermion-sfermion couplings, and the subscript \( i = 1, 2, 3 \) refers to the U(1), SU(2), and SU(3) gauge groups, respectively. Unlike other relations, such as the unification of gaugino masses, these relations hold in all SUSY models and are true to all orders in the limit of unbroken SUSY. Therefore, they can serve as robust tests of SUSY. Such tests of SUSY were first considered by Feng et al in Ref. 1, in which the chargino production at a future \( e^+e^- \) linear collider is used. Tests with slepton production at an \( e^+e^- \) linear collider is then considered by Nojiri et al [2].

Because SUSY is broken, the relation (1) will receive radiative corrections due to SUSY breaking. Especially when some of the superpartners have large SUSY breaking masses, the deviations from Eq. (1) can be significant and may be used to probe SUSY breaking mass splittings. In fact, there are many models in which some number of the superpartners of ordinary matter and gaugino fields are very heavy and may be beyond the discovery range of the planned future colliders. These heavy particles decouple from most low energy processes. However, because of their large SUSY breaking masses, at the lower energy scale of the light superpartners, they induce deviations from the SUSY relations Eq. (1) through radiative corrections. The deviations
grow logarithmically as the heavy superpartner masses increase. Therefore, if the gaugino couplings can be measured to the precision sensitive to the typical deviation of (1) from such a heavy superpartner sector, not only can we test SUSY, but also probe the scale of the heavy superpartners. This can help to set the target energies of future colliders in searching for these particles. As we will see, the $e^-e^-$ collider is an excellent tool for the precision measurements of some of the superparticle couplings.

2 Super-oblique corrections

Before discussing the measurements at the $e^-e^-$ collider, we first discuss what kind of precision we would like to achieve, in order to be sensitive to the heavy superpartner scale. The models with some heavy superpartners can be roughly divided into two categories. In the first class of models, which we will refer to as “heavy QCD models,” the gluino and all the squarks are heavy. Examples of such models include the no-scale supergravity [3], models with gauge-mediated SUSY breaking [4], where strongly-interacting super-particles get large contributions to their masses, and models with a heavy gluino, which gives large contributions to the squark masses through renormalization group evolution. In the second class of models, “2–1 models,” the first two generation sfermions masses are heavy ($\mathcal{O}(10 \text{ TeV})$), while the third generation sfermions are at the weak scale [5]. Such models are motivated by the attempts to solve the SUSY flavor problem without the need for universality of sfermion masses or alignment, while avoiding the extreme fine-tuning problem by keeping the third generation sfermions which couple strongly to the Higgs sector at the weak scale.

The corrections to Eq. (1) are very similar to the oblique corrections of the standard model [6, 7]. In the standard model, nondegenerate SU(2) multiplets lead to different renormalizations of the propagators of the $W$ and $Z$ gauge bosons, inducing nondecoupling effects which grow with the mass splitting. Similarly, in SUSY theories, nondegenerate supermultiplets lead to inequivalent renormalizations of the propagators of gauge bosons and
gauginos, inducing nondecoupling effects that grow with the mass splitting. We will therefore refer to the latter effects as “super-oblique corrections” and parametrize them by “super-oblique parameters” \[ \tilde{U}_i \]. These corrections are particularly important, because they are universal in processes involving gauginos and enhanced by a number of factors.

The differences between the gauge couplings \( g_i \) and the gaugino couplings \( h_i \) come from differences in wavefunction renormalizations, and hence are most analogous to the oblique parameter \( U \). We therefore define

\[
\tilde{U}_i \equiv h_i / g_i - 1 ,
\]

where the subscript \( i \) denotes the gauge group. For the two categories of models discussed above, we find

\[
\tilde{U}_1 \approx 0.35\% \ (0.29\%) \times \ln R \quad \text{(3)}
\]

\[
\tilde{U}_2 \approx 0.71\% \ (0.80\%) \times \ln R \quad \text{(4)}
\]

\[
\tilde{U}_3 \approx 2.5\% \times \ln R \quad \text{(5)}
\]

for 2–1 models (heavy QCD models), where \( R = M/m \) is the ratio of heavy and light superpartner mass scales, and can be \( O(10) \) or even \( O(100) \) (for 2–1 models). These parameters can also receive contributions from possible exotic supermultiplets. For example, contributions from vector-like (messenger) sectors have also been calculated, and were found that they can be significant only for highly split supermultiplets with masses \( \lesssim O(100\text{TeV}) \).

From Eqs. (3)–(5) we see that the corrections due to heavy superpartners can be a few percent, and are larger for stronger couplings. In Ref. 1, from chargino production at a linear \( e^+ e^- \) collider, the SU(2) gaugino coupling \( h_2 \) is found to be able to be measured to \( +30\% -15\% \) for a point in the gaugino region. While it can serve as a test of SUSY, it is not accurate enough to probe the super-oblique corrections. However, if we can also measure the sneutrino mass to 1\%, the uncertainties can be reduced to 2–3\%, starting to be sensitive to the heavy sector. In Ref. 2, the slepton production at the linear \( e^+ e^- \) collider was studied, and they found that \( h_1 \) can be measured to \( \sim 1\% \), which is also about the size of the correction from a heavy sector.
However, if we really want to constrain the heavy superpartner scale, higher precision is required. As we will see in the following, the $e^-e^-$ option of a linear collider may give the most precise determination of the super-oblique corrections.

### 3 Measurements at the $e^-e^-$ collider

The selectrons can be pair produced at an $e^-e^-$ collider through the $t$-channel neutralino exchange (Fig. 1). Here we will focus on the U(1) couplings and consider the $\tilde{e}_R$ pair production. There are several advantages with selectron production at an $e^-e^-$ collider. At an $e^-e^-$ collider, selectrons are produced only through $t$-channel neutralino exchange. The cross section for $\tilde{e}_R$ production is thus directly proportional to $h_1^4$. The coupling $h_1$ can be extracted directly from the total cross section, which is usually larger than at the $e^+e^-$ collider if the Bino mass $M_1$ is not too small. The backgrounds to selectron pair production at $e^-e^-$ colliders are very small. Most of the major backgrounds present in the $e^+e^-$ mode are absent; e.g., $W$ pair and chargino pair production are forbidden by total lepton number conservation. The remaining background $e^-\nu W^-$ can be suppressed by polarizing both $e^-$ beams right-handed. In addition, because a majorana mass insertion in the neutralino propagator is needed to flip the chirality, the dependence of the total cross section on $m_{\tilde{\chi}_1^0}$ is different at $e^-e^-$ colliders from at $e^+e^-$ colliders. This can be exploited to reduce theoretical systematic errors arising...
Figure 2: Contours of constant $\sigma_R = \sigma(e_R^- e_R^- \rightarrow \tilde{e}_R^- \tilde{e}_R^-)$ in fb in the $(m_{\tilde{e}_R}, M_1)$ plane for $\sqrt{s} = 500$ GeV.

from uncertainties in the $\tilde{e}_R$ and $\tilde{\chi}_1^0$ masses. We will come to this point in more detail later.

Now let us discuss what kind of precision can be achieved at the $e^- e^-$ collider. As we will see, the error in the total cross section could be well below 1%, which means the uncertainty in $h_1$ below 0.25%. Such a measurement could constrain the heavy superpartner scale to within a factor of 3 or even better.

The total cross section $\sigma_R = \sigma(e_R^- e_R^- \rightarrow \tilde{e}_R^- \tilde{e}_R^-)$ at a 500 GeV $e^- e^-$ collider is shown in Fig. 2. For a wide range of the selectron mass and Bino mass $m_{\tilde{\chi}_1^0}$ (not too small), the cross section is quite large and is on the order of $\sim 2000$ fb. Assuming one year running at luminosity $\mathcal{L} \sim 20$ fb$^{-1}$/yr, we expect $\sim 40000$ events, yielding a statistical

$^1$We assume that we are in the gaugino region, $\tilde{\chi}_1^0 \approx \tilde{B}$, $m_{\tilde{\chi}_1^0} \approx M_1$. Neutralino mixings will be discussed later.
uncertainty of $\sim 0.5\%$ for the cross section. This can be further improved by longer runs or a larger luminosity. The major background is $e^-\nu W^-$ when followed by $W^- \to e^-\nu_e$, which results from $e^-_L$ contamination in the $\bar{e}_R$ polarized beams. If both beams are 90% right-polarized, i.e., if only 10% of the electrons in each beam are left-handed, the background is reduced to 12 fb [9]. In principle these backgrounds are calculable and can be subtracted, so the induced uncertainty in $\sigma_R$ should be negligible.

There are also theoretical systematic errors coming from uncertainties in the $\tilde{e}_R$ and $\tilde{\chi}^0_1$ masses. The $\tilde{e}_R$ and $\tilde{\chi}^0_1$ masses are constrained by electron energy distribution endpoints of the electrons from $\tilde{e}_R$ decays. The resulting allowed masses are positively correlated and lie in an ellipse-like region in the $m_{\tilde{e}_R} - m_{\tilde{\chi}^0_1}$ plane [10]. For example, for $m_{\tilde{e}_R} = 150$ GeV, $m_{\tilde{\chi}^0_1} = 100$ GeV, the typical allowed regions from a year’s worth of data are given by the ellipses in Fig. 3. On the other hand, the cross section increases as $M_1$ increases (in the regions in which we are interested) because the chirality-flipping mass insertion is needed, but decreases as $m_{\tilde{e}_R}$ increases. As a result, the contours of constant cross section run nearly parallel to the major axes of the ellipses, resulting in a very small error in the total cross section from the uncertainties of $m_{\tilde{e}_R}$ and $m_{\tilde{\chi}^0_1}$, about only 0.3% in this example. In constrast, at the $e^+e^-$ collider, the cross section decreases as either $m_{\tilde{e}_R}$ or $M_1$ increases. The contours of constant cross section will then run roughly perpendicular to the major axes of the ellipses, resulting in larger theoretical systematic errors.

Up to now we have assumed that the lightest neutralino is pure Bino. This is only true in the limit of $|\mu| \to \infty$. For finite $\mu$, neutralino mixings will appear at the $\tilde{e}_R - \tilde{\nu}_R - \tilde{\chi}^0_1$ vertices, so the cross section will also depend on other SUSY parameters in the neutralino mass matrix, $M_2$, $\mu$, and $\tan \beta$, in addition to $M_1$. The $M_2$ dependence is very small because $\tilde{B}$ and $\tilde{W}^3$ only mix indirectly, and hence can be neglected. The $\mu$ and $\tan \beta$ dependences are shown in Fig. 4. The variation in the cross section is less than 1% for $|\mu| \gtrsim 500 \sim 600$ GeV, but can be up to 2–4% for smaller $|\mu|$. Therefore, some information about $\mu$ and $\tan \beta$ is required to calculate $\sigma_R$ at high pre-
Figure 3: The allowed regions, “uncertainty ellipses,” of the \((m_{\tilde{e}_R}, m_{\tilde{\chi}_1^0})\) plane, determined by measurements of the end points of final state electron energy distributions with uncertainties \(\Delta E = 0.3\) GeV and 0.5 GeV. The underlying central values are \((m_{\tilde{e}_R}, m_{\tilde{\chi}_1^0}) = (150\) GeV, 100 GeV), and \(\sqrt{s} = 500\) GeV. We also superimpose contours (in percent) of the fractional variation of \(\sigma_R\) with respect to its value at the underlying parameters.
Figure 4: The fractional variation in $\sigma_R$ (in percent) in the ($\mu$, $\tan \beta$) plane, with respect to the $\mu \to \infty$ limit, for ($m_{\tilde{e}_R}, m_{\tilde{\chi}_1^0}$) = (150 GeV, 100 GeV), with $\sqrt{s} = 500$ GeV. $M_2$ is assumed to be $2M_1$, and for each point in the plane, their values are fixed by $m_{\tilde{\chi}_1^0}$. 
cision. Such information may be obtained from other processes in different colliders, for example, $\tilde{\chi}^0_1\tilde{\chi}^0_3$ production or chargino production in $e^+e^-$ collisions. Energies of $\sqrt{s} \sim 1$ TeV, if available, will therefore allow either a determination of $\mu$ or a sufficiently high lower bound on $\mu$ for us to obtain a precise prediction of $\sigma_R$ so that $h_1$ can be extracted with small uncertainties. Extra mixings at the $e_R - \tilde{e}_R - \tilde{\chi}^0_i$ vertices may exist if lepton flavor is not conserved, i.e., the lepton and slepton mass matrices are not diagonalized in the same basis. This may also cause some uncertainties. However, lepton flavor violation also can be studied at the same time. For instance, Ref. 11 shows that a mixing angle between the first and second generations of order $\sin \theta_{12} \sim 0.02$ will be probed at the $5\sigma$ level in $e^-e^-$ collisions. With such a precision, the induced uncertainty in $h_1$ is negligible.

Finally, many of the above considerations apply also to left-handed selectrons. If kinematically accessible, their production cross section $\sigma_L$ at $e^-e^-$ colliders may also be used to precisely measure gaugino couplings, since the $\tilde{e}_L\tilde{e}_L$ pair production cross section receives contributions from both $t$-channel $\tilde{B}$ and $\tilde{W}^3$ exchange, and hence depends on both $h_1$ and $h_2$. For equivalent mass selectrons, $\sigma_L$ is generally even larger than $\sigma_R$. Note also that $\tilde{e}_L$ and $\tilde{e}_R$ production may be separated either by beam polarization, or, if the selectrons are sufficiently nondegenerate, by kinematics [10] or by running below the higher production thresholds. If the $\tilde{\chi}^\pm_1$ and $\tilde{\chi}^0_2$ decay channels are not open, the only decay is $\tilde{e}_L^- \rightarrow e^-\tilde{\chi}^0_1$ and we will have a large clean sample of events for precision studies. However, in general, the decay patterns may complicate the analysis. The cross section also depends strongly on $m_{\tilde{\chi}^0_2}$ (in the gaugino region), which could be measured either directly from $\tilde{\chi}^0_2\tilde{\chi}^0_2$ production in $e^+e^-$ collisions, or indirectly by measuring $M_1$, $M_2$, $\mu$ and $\tan \beta$ from chargino and $\tilde{\chi}^0_1$ properties. In the end, a measurement of $\sigma_L$ bounds a certain combination of $h_1$ and $h_2$. Under the assumption that the heavy sparticles are fairly degenerate, the deviations $\tilde{U}_1$ and $\tilde{U}_2$ are related and determined by the same heavy scale $M$, and so $\sigma_L$ also provides a probe of the heavy scale $M$, which, in fact, is generically more sensitive, since $\tilde{U}_2 > \tilde{U}_1$ in most models. Of course, in the event that both $\tilde{e}_R$ and $\tilde{e}_L$ are studied, both
$\bar{U}_1$ and $\bar{U}_2$ may be determined, and we may check that their implications for the heavy scale $M$ are consistent or find evidence for nondegeneracies in the heavy sector.

4 Conclusions

In summary, we have seen that the $e^-e^-$ collider may provide the most precise determination of the superparticle couplings. Of course, it is important that the experimental systematic errors from uncertainties in various collider parameters, including the beam energy, luminosity, and so on, have to be controlled in order to obtain the high precision measurements. From this workshop, it seems that required precision of these collider parameters should be able to be achieved. The implications of such precision measurements of super-oblique parameters depend on the scenario realized in nature. At the first step, it provides a stringent test of supersymmetry. If some number of superpartners are not yet discovered, bounds on the super-oblique parameters may lead to bounds on the mass scale of the heavy particles. If, on the other hand, all superpartners of the standard model particles are found, the consistency of all super-oblique parameters with the predicted values will be an important check of the supersymmetric model with minimal field content. If instead deviations are found, such measurements will provide exciting evidence for new exotic sectors with highly split multiplets not far from the weak scale [6]. These insights could also provide a target for future superparticle searches, and could play an important role in evaluating future proposals for colliders with even higher energies.

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