Do consistent \( F(R) \) models mimic General Relativity plus \( \Lambda \)?

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Modified gravity models are subject to a number of consistency requirements which restrict the form that the function \( F(R) \) can take. We study a particular class of \( F(R) \) functions which satisfy various constraints that have been found in the literature. These models have a late time accelerating epoch, and an acceptable matter era. We calculate the Friedmann equation for our models, and show that in order to satisfy the constraints we impose, they must mimic General Relativity plus \( \Lambda \) throughout the cosmic history, with exponentially suppressed corrections. We also find that the free parameters in our model must be fine tuned to obtain an acceptable late time accelerating phase. We discuss the generality of this conclusion.

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I. INTRODUCTION

In recent years, experimental cosmology has provided strong evidence that the Universe is currently undergoing a phase of accelerated expansion. The physical phenomena responsible for this expansion has proved elusive, and there are numerous theories that attempt to explain the current epoch of the Universe. Typically, the observed acceleration is attributed to a new energy component, dark energy, which dominates at late times. Numerous models exist of this form, the simplest of which include introducing a cosmological constant \( \Lambda \), or alternatively a scalar field slowly rolling down a potential.

Although dark energy is undoubtedly the most popular explanation of the current epoch of the Universe, it is not the only way to obtain late time acceleration. In this paper we consider a class of models where gravity is modified at large scales, in such a way that the late-time expansion of the Universe could arise naturally from the new gravitational field equations. These modified gravity models have been considered by numerous authors, see for example [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

Modifying gravity in a manner consistent with experimental data has proved difficult. General Relativity (GR) is a very robust and well tested theory, and it has been found that even slight modifications often lead to instabilities, such as the propagation of ghosts [21], or in otherwise stable matter sources [3, 2]. When introducing additional terms into the gravitational action, we must be careful to respect the success of GR in both the low and high curvature regimes, to ensure that any potentially new model agrees with known observational tests of gravity.

In the literature, there are numerous simple conditions that exist which restrict the form that a modified gravity model may take. In this paper, we combine all of these conditions, and in doing so find that the modified gravity function \( F(R) \) is well constrained, even before considering stringent experimental tests. The philosophy adopted throughout is that before attempting to reconcile any potential modified gravity model with experimental data, that modified gravity model must first satisfy these consistency requirements.

We will take a trial \( F(R) \) model which satisfies all of the existing constraints. We find that our model exhibits a late-time accelerating solution, in the absence of a cosmological constant. We then consider the cosmology of our model, and obtain a modified Friedmann equation. We show that to satisfy all constraints, and to obtain an acceptable Newtonian limit for small \( R \), our model must act like GR with highly suppressed corrections throughout its cosmological history. In addition, we find that although our model has no true cosmological constant, we must still fine tune a particular combination of the free parameters in \( F(R) \) to obtain an acceptably small curvature in the current accelerating epoch. We conjecture that our results are generic features of models which satisfy the consistency requirements.

II. RELEVANT FIELD EQUATIONS IN THE JORDAN AND EINSTEIN FRAMES

Before we consider a specific \( F(R) \) model, we first briefly review the field equations in modified gravity theories. Throughout this paper, we will study the action

\[
S = \frac{M^2}{2} \int \sqrt{-g} d^4xF(R) + S_m,
\]
where $F(R)$ is some function of the Ricci scalar $R$, $M$ is the mass scale of our model, and $S_m$ the matter action. The field equations obtained by varying the action (1) with respect to the metric are

$$F'(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F'(R) + g_{\mu\nu} \Box F'(R) = \frac{1}{M^2} T_{\mu\nu},$$

(2)

where $T_{\mu\nu}$ is the energy momentum tensor of any matter present. Taking the trace of (2) gives the more useful equation

$$Q(R) + 3 \Box F'(R) = \frac{T}{M^2},$$

(3)

where the function $Q(R)$ is given by

$$Q(R) = R F'(R) - 2 F(R).$$

Vacuum solutions to the field equations satisfy $R = R_0 = \text{const}$ when $T = 0$, and hence $Q(R_0) = 0$.

In addition to the above field equations in the Jordan frame, we will also want to consider the potential of the scalar field in the Einstein frame. To transform to the Einstein frame, we first write (1) as

$$S = \frac{M^2}{2} \int \sqrt{-g} d^4x \left[ F'(\phi) (R - \phi) + F(\phi) \right] + \int \sqrt{-g} d^4x L_m,$$

(4)

where $L_m$ is the matter Lagrange density. The actions (4) and (1) are equivalent if $F''(R) \neq 0$. We then use the conformal transform $\tilde{g}_{\mu\nu} = F'(\phi) g_{\mu\nu}$, which gives

$$R = F'(\phi) \left( \tilde{R} + 3 \Box \ln F' - \frac{3}{2(F')^2} \tilde{\nabla}_\alpha F' \tilde{\nabla}^\alpha F' \right),$$

(5)

where $R$ and $\tilde{R}$ are the Ricci scalars associated with the metrics $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively. Using this in (1) gives the following Einstein frame action

$$S = \int \sqrt{-\tilde{g}} d^4x \left[ \frac{M^2}{2} \tilde{R} - \frac{3M^2}{4(F')^2} \tilde{\nabla}_\alpha F' \tilde{\nabla}^\alpha F' - V(F'(\phi)) \right] + \int \sqrt{-\tilde{g}} d^4x \frac{1}{(F')^2} \tilde{L}_m,$$

(6)

where the Einstein frame potential is given by

$$V(\phi) = \frac{M^2}{2} \left( \frac{\phi F'(\phi) - F(\phi)}{F'(\phi)^2} \right).$$

(7)

Finally, by making the field redefinition $F'(\phi) = \exp(\sigma \sqrt{2}/\sqrt{3M^2})$, we obtain

$$S = \int \sqrt{-\tilde{g}} d^4x \left[ \frac{M^2}{2} \tilde{R} - \frac{1}{2} (\tilde{\nabla} \sigma)^2 - V(\sigma) + \frac{1}{(F')^2} \tilde{L}_m \right],$$

(8)

with potential

$$V(\sigma) = \frac{M^2}{2} \left[ \phi(\sigma) e^{\sigma \sqrt{2}/\sqrt{3M^2}} - F(\phi(\sigma)) \right] e^{-2\sigma \sqrt{2}/\sqrt{3M^2}}.$$

(9)

In many instances, it is preferable to study modified gravity in the Einstein frame, since the field equations are much simpler. Additionally, the potential (9) is useful since its critical points correspond to the vacuum solutions $Q(R_0) = 0$ in the Jordan frame. We stress that the two frames are globally equivalent only if $F''(R) \neq 0$. 


III. REVIEW OF CONSTRAINTS ON $F(R)$ MODELS

Here we briefly review the constraints in the literature imposed on the form of the function $F(R)$. We begin by demanding $F'(R) > 0$ and $F'' 
eq 0$ for all $R$. The first of these is to ensure stability; it has been shown \cite{19} that if $F'(R) < 0$ for any $R$ then the model will possess ghost instabilities in this region. The second condition $F''(R) 
eq 0$ is imposed since it has been found that models which have $F''(R) < 0$ will generically possess instabilities, when we consider the metric variational approach of our model \cite{20}. We note that the possibility of $F''(R)$ being zero for some $R$ has not been completely discounted in the literature. Nevertheless, we will impose $F'' 
eq 0$ for all $R$.

The next constraint that we impose on $F(R)$ is that in the limit $R \to \infty$, we have $F(R)/R \to 1$. Writing $F(R) = R + \epsilon(R)$, this implies that $\epsilon/R \to 0$ as $R \to \infty$. This in turn implies that $F'(R) \to 1$ as $R \to \infty$, from which we can deduce that $\epsilon(R) \to \text{const}$ as $R \to \infty$. This condition is imposed to ensure that the modified gravity model acts suitably like ΛCDM in the large $R$ regime, where gravity is well constrained by PPN constraints and the CMB.

With this bound, we have $F'(R) \to 1$ as $R \to \infty$. However, in ref. \cite{21} it was found that for a given $F(R)$ theory to be stable to perturbations in the large $R$ regime, we must have $F'' > 0$ as $R \to \infty$. This means that $F'(R)$ must asymptote to unity from below, since $F''(R) > 0$ implies that it must be an increasing function. Next, we can use the fact that we have imposed $F''(R) \neq 0$ for all $R$. This means that there can be no turning points in $F'(R)$, implying that if $F'(R)$ asymptotes to unity from below, then it must remain less than unity for all $R$. However, we have also imposed $F'(R) > 0$. From this we conclude that $F'(R)$ must be an increasing function of $R$, and must satisfy $0 < F'(R) < 1$.

Another set of constraints on $F(R)$ are discussed in ref. \cite{22}, where the cosmology of $F(R)$ models was considered. In order for a theory to be a viable alternative to GR, it must exhibit an acceptable cosmological history; that is it must have suitable radiation, matter and late time accelerating eras. It was found that in order for a particular $F(R)$ to have a valid matter era, $F(R)$ must satisfy $m(r) \approx +0$ and $\frac{dF}{dr} > -1$ at $r = -1$, where $m(r)$ and $r$ are given by $m = RF''/F'$ and $r = -RF'/F$. In addition, to obtain a late time accelerating era such as we are currently in, the function $m$ must satisfy either: (a) $m = -r - 1$, ($\sqrt{3} - 1)/2 < m \leq 1$ and $dm/dr < -1$ or (b) $0 \leq m < 1$ at $r = -2$. If these are not satisfied for any $R$, then the model will not have an acceptable cosmological history.

IV. A TRIAL $F(R)$ MODEL

In this section, we give an example $F(R)$ function which satisfies all of the conditions in the previous section. Given that $F'(R)$ must be an increasing function, and lie in the range $0 < F'(R) < 1$, an obvious choice is

$$F'(R) = \frac{1}{2} [1 + \tanh(aR - b)],$$

(10)

where $a > 0$ and $b$ are free parameters. This function has the appropriate behaviour $F'(R) \to 1$ as $R \to \infty$ and $F' \to 0$ as $R \to -\infty$. Integrating this $F'(R)$ gives us

$$F(R) = \frac{R}{2} + \frac{1}{2a} \log \left[ \cosh(aR - b) \right] + A,$$

(11)

where $A$ is an integration constant. We are looking for $F(R)$ models that exhibit late time accelerating solutions in the absence of a cosmological constant. Therefore to specify $A$, we impose that $F(0) = 0$. Hence we find

$$A = -(1/2a) \ln \cosh(c)$$

and

$$F(R) = \frac{1}{2} R + \frac{1}{2a} \log \left[ \cosh(aR - \tanh(b) \sinh(aR)) \right].$$

(12)

It is this $F(R)$ that will be studied for the remainder of this paper.

We will begin by considering some of the general properties of our model. First, we look for vacuum solutions to the field equations, that is solutions to $Q(R_{0}) = 0$. The behaviour of $Q(R)$ is exhibited in Fig. \ref{fig:1}(a). We have found that $Q(R)$ for our $F(R)$ generically possesses three zeros, for any $a$ and $b \gtrsim 1.2$. This means that our $F(R)$ model can have three vacuum states, which are Minkowski space ($R = 0$), and two de-Sitter vacua, one of which is stable.

To see these vacuum states more explicitly, we move to the Einstein frame. A plot of the potential $V(\sigma)$ for our $F(R)$ is given in Figs. \ref{fig:2}(a), \ref{fig:2}(b). We can clearly see that the potential has two minima, one at $V = 0$ (which corresponds to the Minkowski vacuum) and the other at some $V = V_0 > 0$. This second minimum corresponds to a
FIG. 1: (a) is a plot of the function $Q(R)$. This function has three zeros, which correspond to three vacuum states of our model. (b) are the functions $m$ (solid), $r$ (dotted) and $dm/dr$ (dashed), plotted parametrically as functions of $R$. We can clearly see that there exists an $R$ such that $r = -2$ and $m < 1$, suggesting a late-time accelerating phase. In addition, we have $m \sim 0$ as $r \to -1$, which is the condition required for an acceptable matter era \[22\]. We have set $a = 2$, $b = 1.5$.

FIG. 2: (a) The Einstein frame potential. Note the presence of two minima; one at $V = 0$, and one at some $V = V_0 > 0$. It is this minima (shown in (b)) that corresponds to a stable de Sitter solution in the Jordan frame. (b) is $V(\sigma)$ in the small $\sigma$ regime. We have set $a = 2$, $b = 1.5$.

stable de Sitter vacuum solution in the Jordan frame, and hence our model has a stable accelerating solution, in the absence of a cosmological constant. This is exactly the behaviour we seek. Finally, the third vacuum in the Jordan frame corresponds to the maxima in the potential \[2(a)\] and is unstable.

Following \[22\], we also briefly consider the functions $m(r)$, $r$ and $dm/dr$ for our model, as given in section \[11\]. We plot these functions parametrically as functions of $R$ in Fig. \[1(b)\]. We see that there is a region in which $m \approx 0$ and $dm/dr > -1$ for $r \approx -1$, which corresponds to a matter era. As $R \to \infty$, we have $r \to -1$ and $m \to 0$, suggesting that the condition for a matter epoch is satisfied asymptotically. Further, as $R$ decreases, there is a region where $r = -2$, at which $0 < m < 1$. This region corresponds to the late-time de-Sitter phase. Hence our model connects the points $(-1, 0)$ and $(-2, a < 1)$ in the $(r, m)$ plane, as is required \[31\] for it to have an acceptable cosmic history.
V. NEWTONIAN LIMIT

We now consider the Newtonian limit of our model. In doing so, we will be able to constrain the free parameters $a$ and $b$. The constant $a$ will be fixed by the following two conditions; we require the de-Sitter vacuum solution to be $R = R_0 \approx 12H_0^2$, and as we will see shortly, we require $F''(R_0) \ll F'(R_0)$. Using these conditions, the constant $b$ will not be fixed, but we will find that it must be large.

To see how $a$ and $b$ are determined, we consider the Newtonian limit of modified gravity theories. The full field equations for the action (1) are

$$F'(R)R_{\mu \nu} - \frac{1}{2} F(R)g_{\mu \nu} - \nabla_\mu \nabla_\nu F'(R) + g_{\mu \nu} \Box F'(R) = \frac{1}{M^2} T_{\mu \nu}. \quad (13)$$

We would like to perform a weak-field expansion of these equations around the de-Sitter minima of our model. In doing so, we hope to obtain an acceptable Newtonian limit. An analysis has been performed previously in Ref. [28], and expanding the field equations about a symmetric de-Sitter vacuum gives

$$F'(R_0)\delta R_{\mu \nu} - \frac{1}{4} F''(R_0) R_0 g_{\mu \nu} \delta R - \frac{1}{2} F(R_0) \left( \delta g_{\mu \nu} - \frac{1}{2} g_{\mu \nu} \delta g \right) - F''(R_0) \left( \nabla_\mu \nabla_\nu R + \frac{1}{2} g_{\mu \nu} \Box R \right) = \frac{1}{M^2} \left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right), \quad (14)$$

where

$$\delta R_{\mu \nu} = \frac{1}{2} \left( \nabla_\mu \nabla_\nu \delta g_{\alpha \beta} + \nabla_\nu \nabla_\mu \delta g_{\alpha \beta} - \frac{1}{2} R_0 \delta g_{\mu \nu} - \frac{1}{12} R_0 g_{\mu \nu} \delta g - \frac{1}{2} \Box \delta g_{\mu \nu} - \frac{1}{2} \nabla_\mu \nabla_\nu \delta g. \quad (15)$$

In order to obtain an acceptable Newtonian limit from these perturbative equations, we require that $F'(R_0) \sim O(1)$, $F(R_0) \sim O(R_0) \ll 1$ and $F''(R_0) \ll F'(R_0)x^2$, where $x$ is a macroscopic distance. If this is the case, then the perturbative equations (2) will be approximately given by

$$F'(R_0)\delta R_{\mu \nu} \approx \frac{1}{M^2} \left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right). \quad (16)$$

Equation (16) is simply the weak field expansion of General Relativity, from which the standard Newtonian limit is recovered. The only difference is the coefficient of the first term in (16), which is $F'(R_0) \sim O(1)$ as opposed to unity. Note that to obtain the correct limit we must have $F''(R_0) \ll F'(R_0)$; if this were not the case then fourth order derivative terms such as $\nabla_\mu \nabla_\nu \delta R$ would be present in (16), and it has been shown that these terms would lead to strong curvature at all length scales [31].

For our model (and generically in $F(R)$ theories), we have $F(R_0) \sim O(R_0) \ll F'(R_0)$ for $R \sim H_0^2$, which is the first condition to obtain an acceptable Newtonian limit. However, the second condition, $F''(R_0) \ll F'(R_0)x^2$, is not generically satisfied, and we must choose our free parameters to obtain the correct behaviour. From (10), we find $F''(R)$ to be

$$F''(R) = \frac{a}{2} \text{sech}^2(aR - b), \quad (17)$$

and hence to ensure that $F''(R_0)$ is very small, we require $aR_0 - b \gg 1$. We can use this condition to fix $a$ in terms of $R_0$ and $b$. To do so, we note that in the vicinity of the de-Sitter minima, $R \sim R_0$, we can expand $F(R)$ and $F'(R)$ for large $\alpha = aR - b$,

$$F(R) \approx R - \frac{\log(1 + e^{2b})}{2a} + \frac{1}{2a} e^{-2\alpha} + O(e^{-4\alpha}) \quad (18)$$

$$F'(R) \approx 1 - e^{-2\alpha} + O(e^{-4\alpha}) \quad (19)$$

$$F''(R) \approx 2ae^{-2\alpha} + O(e^{-4\alpha}) \quad (20)$$

and hence from the vacuum field equations $Q(R_0) = 0$, we can use our approximations (18) to give
from which we find $a \approx \frac{1}{12H_0^2} \log \left(1 + e^{2b}\right)$. This fixes the free parameter $a$ in terms of $R_0$ and the remaining free parameter $b$. Next, we can constrain $b$ from the condition $F''(R_0) \ll F'(R_0) \approx 1$. Using this, and expanding $F''(R)$ in powers of $e^{-2a}$, we find that $b$ must satisfy

$$\log(1 + e^{2b}) \cosh^2 b \ll 2R_0,$$

which is satisfied for large $b$, such that $8be^{-2b} \ll 2R_0$.

To review our conditions; we are working with a model with two free parameters, $a$ and $b$, and we have imposed two constraints on this model, which are that $R_0 = 12H_0^2 \ll 1$ and $F''(R_0) \ll 1$. These constraints force us to choose $a$ and $b$ such that $a \approx \frac{1}{12H_0^2} \log \left(1 + e^{2b}\right)$ and $8be^{-2b} \ll 2R_0$. Together, they imply that $b \gg 1$, and hence $a \approx \frac{2b}{R_0}$. We note that we have not been forced to specify the constant $b$, other than imposing the condition $b \gg 1$. This parameter controls how closely our model mimics GR; the larger we set $b$ the more suppressed our corrections $\exp(-2a)$ become.

Before moving onto the cosmology of our model, we remark that the above weak-field expansion was performed around the de-Sitter vacuum $R = R_0$. It has been noted [27] that for local tests of gravity, the expansion should instead be around the local energy density of the solar system, which is much larger than $R_0$. In a subsequent paper we will perform a detailed analysis of the local tests of gravity for our model, but for now we simply expand around $R = R_0$. Performing a weak-field expansion around a solar system background will not significantly modify our conclusion; we are simply stating that in order to obtain an acceptable weak-field limit for small $R$, we must choose the constants $a$ and $b$ such that $aR - b \gg 1$ is satisfied.

**VI. COSMOLOGICAL EVOLUTION**

Having considered the Newtonian limit of our models, we now evaluate the Friedmann equation, and attempt to reconstruct the cosmic history of our models. We will find that this model mimics general relativity very closely throughout its cosmic history.

To show this, we first write down the equivalent Friedmann equation for a general $F(R)$ model, obtained from the $(0, 0)$ component of the field equations (2).

$$-3\frac{\ddot{a}}{a}F'(R) + \frac{F}{2} + 3H\partial_t[F'(R)] = \frac{\rho}{M^2}.$$  

(23)

With our highly non-trivial $F(R)$, this equation corresponds to a complex, non-linear, third-order differential equation in the scale factor $a$. However, we can use our approximations (18) to write (23) in a much simplified form. We first note that the approximations (18) are valid for the entire cosmic history of this model, because they are applicable for large $a = aR - b$. Since $R > R_0$ throughout the cosmic history, and since $a$ is at its smallest value at $R = R_0$, then given that we have constrained $a$ to be large at $R = R_0$ then it must be large for all $R > R_0$.

Using the approximations (18) in (23), we find the following modified Friedmann equation,

$$3H^2 \frac{b}{2a} + e^{-2a} \left(3\frac{\ddot{a}}{a} + \frac{1}{4a} + 6aH\partial_t R \right) + O \left(e^{-4a}\right) = \frac{\rho}{M^2},$$

(24)

where we have used the condition that $b$ must be large, and hence $\log(1 + \exp(2b)) \approx 2b$. The first two terms are what we would expect from standard General Relativity with a fine tuned cosmological constant $\Lambda = \frac{b}{2a} \approx R_0$. The remaining terms on the left hand side are corrections to the Friedmann equation, but they are highly suppressed by a factor of $e^{-2a} \ll 1$ for all $R > R_0$. It follows that our model will not significantly differ from standard GR. The exponentially suppressed corrections to general relativity that we have found are small in the current epoch, and decrease in significance as we track our model back through its cosmic history. Additionally, we note that we still have a fine tuning problem, in that we have two free parameters, and we have chosen them specifically such that the combination $\Lambda = \frac{b}{2a} \satisfies \Lambda \ll 1$. We stress however that the combination $b/2a$ in (24) is not a true cosmological constant; rather our model mimics standard general relativity with a small vacuum energy.
The fine-tuning found in our model is a generic feature of modified gravity theories. Since we demand that $F''(R) \sim 0$ for small $R$ to obtain an acceptable weak field limit, it follows that in the vicinity of the vacuum $R = R_0$, we must have $F''(R) \approx 0$, and hence $F(R) \approx cR + d$, where $c$ and $d$ are constants. This means that for vacuum $R \sim R_0$, we can expand $F(R)$ in the form $F(R) \approx cR + d$, where $c$ and $d$ will be some combination of constants in the model. This is exactly what we have done for our $F(R)$ model above; we have expanded $F(R)$ as $F(R) \approx R - \frac{b}{6}$ for $R \sim R_0$. It follows that to obtain an acceptable vacuum solution $R_0 = 12H_0^2 \ll 1$, we must fine-tune the parameters in the model.

The fact that our model must necessarily mimic GR with suppressed corrections is a consequence of the constraints that we have imposed. Specifically, we have used the fact that to obtain an acceptable weak field expansion, we require $F''(R) \sim 0$ for small $R$. In addition, we have also assumed that in the large $R$ regime $F(R)/R \to 1$, which suggests that $F''(R) \sim 0$ as $R \to \infty$. Since we are considering models which have $F''(R) \neq 0$ for all $R$, then it follows that $F''(R)$ must be small throughout the cosmic history.

We expect that any $F(R)$ function that satisfies the consistency conditions that we have considered will possess a Friedmann equation of the form (24), that is standard GR plus $\Lambda$ with small correction terms. These correction terms will not necessarily be exponentially suppressed as they are for the $F(R)$ considered in this paper, but they must be suppressed throughout the expansion history of the Universe regardless of the particular form of $F(R)$. However, although these models will mimic GR, this does not imply that they are observationally identical to GR if we consider all aspects of gravity. For example, it might be possible to observe small corrections to GR by considering solar system tests of gravity or the binary pulsar. In a subsequent paper we hope to find distinct observational signatures of our model by analyzing local gravity constraints.

Finally, we observe that the conclusions that we have reached do not apply if we were to relax any of the constraints on $F(R)$, for example by allowing $F'' = 0$ for some $R$ in the past. Hence it might be possible to construct an acceptable model that does not obey all of the constraints at all times, and hence will not mimic GR throughout the entire cosmic history. In addition, it is not strictly necessary for an acceptable modified gravity model to obey the consistency conditions for all $R$, since they can only be probed for $R \gtrsim R_0$. We could have an acceptable model which has $F''(R) = 0$ for some $R < R_0$ in the future. However, there is a possibility of such models evolving to a final state containing instabilities.

VII. CONCLUSION

We have taken the consistency constraints that exist in the literature for modified gravity models, and used them to construct a particular modified gravity function $F(R)$. A study of the model has shown that in order to satisfy all constraints, it must act like GR with highly suppressed corrections throughout cosmic history. In addition, we find that even though we do not have a true cosmological constant in our setup, we must still fine-tune the parameters in our $F(R)$ function to obtain an acceptable late time accelerating epoch. Although we have come to this conclusion for our specific model, we expect our results to hold true for a large class of $F(R)$ models which satisfy all of the constraints that we have considered.

While we were completing this work we became aware of Refs. [31, 32], which come to similar conclusions. In Ref. [31], it was found that in order to satisfy local gravity constraints, the function $m(r)$ as defined in section [11] must satisfy $m(r) \ll 10^{-58}$ for $R \approx H_0^2$. This places a severe constraint upon $F''(R)$, which must be very small in the current epoch of the Universe. The model considered in this paper can satisfy the constraint $m(r) \ll 10^{-58}$ by making the free parameter $b$ sufficiently large. However, such a stringent constraint is likely to yield a model that is virtually indistinguishable to GR.

In Ref. [32], a modified gravity function of the form

$$\tilde{F}(R) = R - m^2 \frac{c_1 (\frac{R}{m^2})^n}{c_2 (\frac{R}{m^2})^n + 1}$$

was considered, where $m$, $n$, $c_1$ and $c_2$ are free parameters. The model considered in this paper shares many similarities with (25); the condition $\tilde{F}'(R) \approx 1$ was imposed in Ref. [32] both at $R \approx H_0^2$ and as $R \to \infty$. This implies that the function $F(R)$ can be expanded as

$$\tilde{F}(R) \approx R - \frac{c_1}{c_2} m^2 + \frac{c_1}{c_2} \frac{m^2}{R}$$

(26)
for small $\frac{m^2}{R} \ll 1$, which is valid throughout the cosmic history of the model. We have arrived at a very similar expansion for the $F(R)$ function considered in this paper, (18). The difference between the two models is that (26) contains power law corrections to GR, whereas (18) contains exponentially suppressed corrections.

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