OPTICAL SOLITONS TO THE FRACTIONAL PERTURBED NLSE IN NANO-FIBERS

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ABSTRACT. In this paper, we study the space-time fractional perturbed nonlinear Schrödinger equation under the Kerr law nonlinearity by using the extended sinh-Gordon equation expansion method. The perturbed nonlinear Schrödinger equation is a nonlinear model which arises in nano-fibers. Some family of optical solitons and singular periodic wave solutions are successfully revealed. The parametric conditions for the existence of valid solitons are stated. Under the choice of suitable values of the parameters, the 3-dimensional and 2-dimensional graphs to some of the reported solutions are plotted.

1. Introduction. The nonlinear Schrödinger’s equations (NLSEs) with their wide range of applications are the most commonly use nonlinear models in the field nonlinear science [32, 24]. Because of their remarkable stability properties, optical solitons are now the most fascinating research field of nonlinear wave propagation in optical fibres [59, 20, 43, 9, 14, 15, 49]. The theory of optical solitons is one of the interesting topics for the investigation of soliton propagation through nonlinear optical fibers [56]. These optical solitons are present in the nonlinear models that arise in the various fields of nonlinear science such as nano-fibers, optical fibers, quantum electronics, optoelectronics and photonics [33, 5, 34, 47].

In some past decades, various integration schemes have been used to investigate different kind of NLSEs, such as the semi-inverse variational principle [17, 16], the trial solution approach [35], the ansatz approach [46], the extended $G'/G$-expansion scheme [29], the improved tan($F(\Phi(\xi))/2$) expansion method [40] and several others [28, 50, 10, 38, 21, 45, 6, 22, 55, 11, 41, 27, 4, 57, 48, 23, 12, 25, 31, 26, 13, 58, 18].

For the past two decades, the field of fractional calculus is becoming one of the interesting area of research due to its wider range of applications in the several fields of nonlinear science [2]. There are several definitions of the fractional

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derivative that have been submitted to the literature [44, 1]. Recently, a new simple

definition of the fractional derivative known as conformable fractional derivative

has been developed by Khalil et al. [39]. There are several studies that have been

conducted in this context [54, 7, 36, 37, 52, 3, 8, 53, 30].

In this study, family of optical solitons to the conformable space-time fractional

derived nonlinear Schrödinger equation in nano-fibers [42] will be obtained by

using the extended sinh-Gordon equation expansion method (ShGEEM) [51].

The conformable space-time fractional perturbed nonlinear Schrödinger equation

[42] is given by

\[ iD_0^\alpha \psi + aD_0^\beta \psi + bF(|\psi|^2)\psi - i\sigma D_x^\beta (|\psi|^2\psi) - i\lambda D_x^\beta (|\psi|^2n)\psi = 0, \]  \tag{1}

where \( x, t > 0, \ 0 < \alpha, \beta \leq 1, \psi(x, t) \) is a complex valued wave function of \( x \); the non-dimen-

dional distance along the fiber and \( t \); the temporal variable in dimensionless form. The

nonzero real numbers \( a \) and \( b \) are the coefficients of group velocity dispersion and nonlinear term, respectively. The perturbation terms \( \sigma, \lambda \) and \( \gamma \) are the coefficients of the inter-modal dispersion, self-steepening term and nonlinear dispersion, respectively. The parameter \( n \) represents the full nonlinearity [42], \( F \) is a real-valued algebraic function which must have the smoothness of the function \( F(|\Psi|^2)\Psi : \mathbb{C} \to \mathbb{C} \). When the complex plane \( \mathbb{C} \) is considered as two-dimensional linear space \( \mathbb{R}^2 \), the function \( F(|\Psi|^2)\Psi \) is \( k \) times continuously differentiable [42].

2. The conformable fractional derivative. Here, some basic facts about the conformable fractional derivative [39].

Definition 2.1. Let \( g : (0, \infty) \to \mathbb{R} \), then the conformable fraction derivative of \( g \)
of order \( \alpha \) is defined as

\[ T_\alpha (g)(t) = \lim_{\epsilon \to 0} \frac{g(t + \epsilon t^{1-\alpha}) - g(t)}{\epsilon}, \quad t > 0, \ 0 < \alpha \leq 1. \]  \tag{2}

We give some basic properties of the conformable fractional derivative [39] below

1. \( T_\alpha (bg + ch) = bT_\alpha (g) + cT_\alpha (h), \ b, c \in \mathbb{R}, \)

2. \( T_\alpha (t^\lambda) = \lambda t^{\lambda-\alpha}, \ \lambda \in \mathbb{R}, \)

3. \( T_\alpha (gh) = gT_\alpha (h) + hT_\alpha (g), \)

4. \( T_\alpha (\frac{g}{h}) = \frac{hT_\alpha (g) - gT_\alpha (h)}{h^2}, \)

5. if \( g \) is differentiable, then \( T_\alpha (g)(t) = t^{1-\alpha} \frac{dg}{dt}. \)

Theorem 2.2. Let \( g, h : (0, \infty) \to \mathbb{R} \) be differentiable and also \( \alpha \) differentiable

functions, then the following rule holds:

\[ T_\alpha (g \circ h)(t) = t^{1-\alpha} h'(t)g'(h(t)). \]  \tag{3}

3. Applications. In this section, we present the application of the extended ShGEEM to the fractional perturbed nonlinear Schrödinger equation [42].

It is known that Eq. (1) is integrable when \( n = 1 \) and in Kerr law media \( F(\psi) = \psi \)

[42].

Considering \( n = 1 \) under the Kerr law nonlinearity, Eq. (1) becomes

\[ iD_0^\alpha \psi + aD_x^\beta \psi + b|\psi|^2 \psi - i\sigma D_x^\beta (|\psi|^2 \psi) - i\lambda D_x^\beta (|\psi|^2n)\psi = 0. \]  \tag{4}

Thus, the Kerr law nonlinearity arises when a light wave in an optical fiber faces nonlinear responses from non-harmonic motion of electrons bound in molecules, caused by an external electric field [19, 60, 61].
Consider the wave transformation
\[ \psi = \Psi(\xi)e^{i\phi}, \quad \phi = -k^\beta x + p t^\alpha + \Omega, \quad \xi = \frac{x^\beta}{\beta} - \nu t^\alpha. \] (5)
Substituting Eq. (5) into Eq. (4), gives the following NODE:
\[ a\Psi'' - (p + ak^2 + k\sigma)\Psi + (b - \lambda k)\Psi^3 = 0 \] (6)
from the real part, and the relations
\[ \nu = -(2ak + \sigma) \quad \text{and} \quad \lambda = -\frac{2}{3}\gamma \] from the imaginary part.
By the extended ShGEEM, the solutions of any given nonlinear partial differential equation are assumed to be of the forms [51]
\[ \Psi(\theta) = \sum_{j=1}^{m} [b_j \sinh(\theta) + a_j \cosh(\theta)]^j + a_0, \] (7)
\[ \Psi(\xi) = \sum_{j=1}^{m} [\pm ib_j \sech(\xi) \pm a_j \tanh(\xi)]^j + a_0, \] (8)
\[ \Psi(\xi) = \sum_{j=1}^{m} [\pm bj \csch(\xi) \pm a_j \coth(\xi)]^j + a_0, \] (9)
\[ \Psi(\xi) = \sum_{j=1}^{m} [\pm b_j \sec(\xi) + a_j \tan(\xi)]^j + a_0, \] (10)
\[ \Psi(\xi) = \sum_{j=1}^{m} [\pm b_j \csc(\xi) - a_j \cot(\xi)]^j + a_0, \] (11)
where \( \theta' = \sinh(\theta) \) or \( \theta' = \cosh(\theta) \) and \( i = \sqrt{-1} \) [51].
Taking the balance between \( \Psi'' \) and \( \Psi^3 \) in Eq. (6), gives \( m = 1 \).
When \( m = 1 \), Eqs. (7), (8), (9), (10) and (11) take the forms
\[ \Psi(\theta) = b_1 \sinh(\theta) + a_1 \cosh(\theta) + a_0, \] (12)
\[ \Psi(\xi) = \pm ib_1 \sech(\xi) \pm a_1 \tanh(\xi) + a_0, \] (13)
\[ \Psi(\xi) = \pm b_1 \csch(\xi) \pm a_1 \coth(\xi) + a_0, \] (14)
\[ \Psi(\xi) = \pm b_1 \sec(\xi) + a_1 \tan(\xi) + a_0 \] (15)
and
\[ \Psi(\xi) = \pm b_1 \csc(\xi) - a_1 \cot(\xi) + a_0, \] (16)
respectively.
Inserting Eq. (12)) and its second derivative along with \( \theta' = \sinh(\theta) \) or \( \theta' = \cosh(\theta) \) into Eq. (6), gives a polynomial in degrees of hyperbolic functions. We sum the coefficients of the hyperbolic functions of the same degree and equate each summation to zero, this gives a class of algebraic equations. The set of algebraic equations is then simplified to get the values of the parameters involved. For each case, putting the obtained values of the parameters into Eqs. (13)-(15), gives the solutions to Eq. (4).
Case-1. When
\[ a_0 = 0, \quad a_1 = -\sqrt{\frac{a}{2k\lambda - 2\nu}}, \quad b_1 = a_1, \quad p = -\frac{1}{2}a(1 + 2k^2 - \sigma k), \]
we get
\[ \psi_1(x,t) = \pm \sqrt{\frac{a}{2k\lambda - 2b}} \left( i \sech \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] \right) e^{\left( -k \frac{x}{\beta} - \left( \frac{1}{2} a(1+2k^2) + \sigma k \right) \frac{t}{\alpha} + \Omega \right)} \]  
\[ + \tanh \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] e^{\left( -k \frac{x}{\beta} - \left( \frac{1}{2} a(1+2k^2) + \sigma k \right) \frac{t}{\alpha} + \Omega \right)} \]  
(17)

and
\[ \psi_2(x,t) = \pm \sqrt{\frac{a}{2k\lambda - 2b}} \left( \coth \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] \right) e^{\left( -k \frac{x}{\beta} - \left( \frac{1}{2} a(1+2k^2) + \sigma k \right) \frac{t}{\alpha} + \Omega \right)} \]  
\[ + \csch \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] e^{\left( -k \frac{x}{\beta} - \left( \frac{1}{2} a(1+2k^2) + \sigma k \right) \frac{t}{\alpha} + \Omega \right)} \]  
(18)

where \( a(k\lambda - b) > 0 \) for valid solitons.

**Case-2.** When \( a_0 = 0, a_1 = -\sqrt{\frac{2a}{k\lambda - b}} \), \( b_1 = 0 \), \( p = -(2+k^2) - \sigma k \), we get
\[ \psi_3(x,t) = \pm \sqrt{\frac{2a}{k\lambda - b}} \tanh \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] e^{\left( -k \frac{x}{\beta} - (a(2+k^2) + \sigma k) \frac{t}{\alpha} + \Omega \right)} \]  
(19)

and
\[ \psi_4(x,t) = \pm \sqrt{\frac{2a}{k\lambda - b}} \coth \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] e^{\left( -k \frac{x}{\beta} - (a(2+k^2) + \sigma k) \frac{t}{\alpha} + \Omega \right)} \]  
(20)

where \( a(k\lambda - b) > 0 \) for valid solitons.

**Case-3.** When \( a_0 = 0, a_1 = 0, b_1 = -\sqrt{\frac{2a}{k\lambda - b}} \), \( p = a(1-k^2) - \sigma k \), we get
\[ \psi_5(x,t) = \pm \sqrt{-\frac{2a}{(k\lambda - b)}} \sech \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] e^{\left( -k \frac{x}{\beta} + (a(1-k^2) - \sigma k) \frac{t}{\alpha} + \Omega \right)} \]  
(21)

and
\[ \psi_6(x,t) = \pm \sqrt{-\frac{2a}{(k\lambda - b)}} \csch \left[ \frac{x}{\beta} + (2ak + \sigma) \frac{t}{\alpha} \right] e^{\left( -k \frac{x}{\beta} + (a(1-k^2) - \sigma k) \frac{t}{\alpha} + \Omega \right)} \]  
(22)

the valid solitons exist for both \( a(k\lambda - b) < 0 \) and \( a(k\lambda - b) > 0 \).

**Case-4.** When
\[ a_0 = 0, a_1 = \sqrt{\frac{a(-2ab + \lambda \left( -\sigma + \sqrt{\sigma^2 - 2a(a + 2p)} \right)}{2a(2b^2 + \lambda^2) + 4\lambda(p\lambda + b\sigma)}}} \]  
\[ b_1 = a_1, \]
where $a$ and $\sigma$ are the same as in Case-5.

When $a(2a(2b^2 + \lambda^2) + 4\lambda(p\lambda + b\sigma) > 0$ for valid solitons.

**Case-5.** When

$$a_0 = 0, a_1 = -2 \sqrt{\frac{a^3}{2a^2 b + a\lambda \sigma - \sqrt{a^2 \lambda^2(\sigma^2 - 4a(2a + p))}}, b_1 = 0,$$

$$k = \frac{1}{2a^2 \lambda} \left( -a \lambda \sigma + \sqrt{a^2 \lambda^2(\sigma^2 - 4a(2a + p))} \right),$$

we get

$$\psi_9(x, t) = \pm 2 \sqrt{\frac{a^3}{2a^2 b + a\lambda \sigma - \sqrt{a^2 \lambda^2(\sigma^2 - 4a(2a + p))}}}
\times \tanh \left[ \frac{x^\beta}{\beta} + (2ak + \sigma) \frac{t^\alpha}{\alpha} \right] e^{i\left(-k x^\beta + \sigma x^\alpha + \Omega\right)}$$

(25)

and

$$\psi_{10}(x, t) = \pm 2 \sqrt{\frac{a^3}{2a^2 b + a\lambda \sigma - \sqrt{a^2 \lambda^2(\sigma^2 - 4a(2a + p))}}}
\times \coth \left[ \frac{x^\beta}{\beta} + (2ak + \sigma) \frac{t^\alpha}{\alpha} \right] e^{i\left(-k x^\beta + \sigma x^\alpha + \Omega\right)},$$

(26)

where $a^3(2a^2 b + a\lambda \sigma - \sqrt{a^2 \lambda^2(\sigma^2 - 4a(2a + p))}) < 0$ and $\sigma^2 - 4a(2a + p) > 0$ for valid solitons.

**Case-6.** When

$$a_0 = 0, a_1 = 0, b_1 = -2 \sqrt{\frac{a^3}{\sqrt{a^2 \lambda^2(4a(a - p) + \sigma^2)} - a(2ab + \sigma)}}.$$
where the valid wave solutions exist for both

\[
\psi_{930} = \left( -a\lambda\sigma + \sqrt{a^2\lambda^2(4a(a-p) + \sigma^2)} \right),
\]

we get

\[
\psi_{11}(x, t) = \pm 2 \sqrt{\frac{a^3}{\sqrt{a^2\lambda^2(4a(a-p) + \sigma^2)} - a(2ab + \sigma)}}
\times \text{sech} \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] e^{-i\left( -k\frac{x\beta}{\alpha} + \frac{t^\alpha}{\alpha} + \Omega \right)}
\] (27)

and

\[
\psi_{12}(x, t) = \pm 2 \sqrt{\frac{a^3}{\sqrt{a^2\lambda^2(4a(a-p) + \sigma^2)} - a(2ab + \sigma)}}
\times \text{csch} \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] e^{-i\left( -k\frac{x\beta}{\alpha} + \frac{t^\alpha}{\alpha} + \Omega \right)},
\] (28)

where the valid solitons exist for both \(a^3\left( \sqrt{a^2\lambda^2(4a(a-p) + \sigma^2)} - a(2ab + \sigma) \right) > 0\)
and \(a^3\left( \sqrt{a^2\lambda^2(4a(a-p) + \sigma^2)} - a(2ab + \sigma) \right) < 0\), and for \(4a(a-p) + \sigma^2 > 0\).

**Case-7.** When

\[ a_0 = 0, \ a_1 = -\sqrt{\frac{a}{2(k\lambda - b)}}, \ b_1 = a_1, \ p = \frac{1}{2}(a - 2ak^2 - 2k\sigma) \]

we get

\[
\psi_{13}(x, t) = \pm \sqrt{\frac{a}{2(k\lambda - b)}} \left( \text{sec} \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] \right.
\]
\[ + \tan \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] e^{i\left( -k\frac{x\beta}{\alpha} + \frac{t^\alpha}{\alpha} + \Omega \right)} \] (29)

and

\[
\psi_{14}(x, t) = \pm \sqrt{\frac{a}{2(k\lambda - b)}} \left( \text{cot} \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] \right.
\]
\[ + \csc \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] e^{i\left( -k\frac{x\beta}{\alpha} + \frac{t^\alpha}{\alpha} + \Omega \right)}, \] (30)

where the valid wave solutions exist for both \(a(k\lambda - b) > 0\) and \(a(k\lambda - b) < 0\).

**Case-8.** When

\[ a_0 = 0, \ a_1 = \sqrt{\frac{a(-2ab + \lambda(-\sigma + \sqrt{2a(a-2p) + \sigma^2})}{a(4b^2 - 2\lambda^2) + 4\lambda(p\lambda + b\sigma)}}, \ b_1 = -a_1, \]

\[
k = -\frac{1}{2\alpha} \left( \sigma + \sqrt{2a(a-2p) + \sigma^2} \right), \]

we get

\[
\psi_{15}(x, t) = \pm \sqrt{\frac{a(-2ab + \lambda(-\sigma + \sqrt{2a(a-2p) + \sigma^2})}{a(4b^2 - 2\lambda^2) + 4\lambda(p\lambda + b\sigma)}} \left( \text{sec} \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] \right.
\]
\[ + \tan \left[ \frac{x\beta}{\alpha} + (2ak + \sigma)\frac{t^\alpha}{\alpha} \right] e^{i\left( -k\frac{x\beta}{\alpha} + \frac{t^\alpha}{\alpha} + \Omega \right)} \] (31)
and

$$\psi_{16}(x,t) = \pm \sqrt{\frac{a(-2ab + \lambda(-\sigma + \sqrt{2a(a-2p)+\sigma^2})}{a(4b^2 - 2\lambda^2) + 4\lambda(p\lambda + b\sigma)}} \left( \cot\left(\frac{x^\beta}{\beta} + (2ak + \sigma)^\frac{t^\alpha}{\alpha}\right) \right)$$

$$+ \csc\left(\frac{x^\beta}{\beta} + (2ak + \sigma)^\frac{t^\alpha}{\alpha}\right) e^{i\left(-k\frac{x^\beta}{\beta} + p\frac{t^\alpha}{\alpha} + \Omega\right)},$$

(32)

where the valid wave solutions exist for $2a(a-2p)+\sigma^2 > 0$.

**Remarks.** Solutions (17) and (23) are combined dark-bright optical solitons. Solutions (19) and (25) are dark optical solitons. Solutions (21) and (27) are bright optical solitons. Solutions (18) and (24) are combined singular solitons. Solutions (20), (22), (26) and (28) are singular solitons, and finally, solutions (29), (30), (31) and (32) are singular periodic wave solutions.

**Figure 1.** The 3D and 2D surfaces of Eq. (19) at $\alpha = \beta = 0.7$.

**Figure 2.** The 3D and 2D surfaces of Eq. (19) at $\alpha = \beta = 0.8$. 
4. Conclusions. This paper revealed the dark, bright, singular, compound dark-bright, compound singular optical solitons and singular periodic wave solutions to the fractional perturbed nonlinear Schrödinger equation in nano-fibers by using the extended sinh-Gordon equation expansion approach. The Kerr law nonlinearity is considered. We presented the 2-dimensional and 3-dimensional graphics to some of the obtained solutions with the suitable values of $\alpha, \beta \in (0, 1)$. The reported results
Figure 6. The 3D and 2D surfaces of Eq. (22) at $\alpha = \beta = 0.8$.

Figure 7. The 3D and 2D surfaces of Eq. (29) at $\alpha = \beta = 0.7$.

Figure 8. The 3D and 2D surfaces of Eq. (29) at $\alpha = \beta = 0.8$.

provide a lot of encouragements for future studies. The extended sinh-Gordon equation expansion approach gives family of optical solitons that may be useful in explaining the physical meaning of several complex nonlinear models that arise in the various fields of nonlinear science.
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