THEORETICAL ANALYSIS OF ELECTROSTATIC ENERGY HARVESTER CONFIGURED AS BENNET’S DOUBLER BASED ON Q-V CYCLES

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Abstract – This paper presents theoretical analysis of a MEMS electrostatic energy harvester configured as the Bennet’s doubler. Steady-state operation of the doubler circuit can be approximated by a right-angled trapezoid Q-V cycle. A similarity between voltage doubler and resistive-based charge-pump circuit is highlighted. By taking electromechanical coupling into account, the analytical solution of the saturation voltage is the first time derived, providing a greater comprehension of the system performance and multi-parameter effects. The theoretical approach is verified by results of circuit simulation for two cases of mathematically idealized diode and of Schottky diode. Development of the doubler/multiplier circuits that can further increase the saturation voltage is investigated.

1. INTRODUCTION

Wireless sensor nodes (WSNs) are emerging as one of the most commonly used monitoring and sensing systems [1, 2]. Currently, most WSNs are powered by batteries. Energy harvesting from vibration becomes a potential alternative to obtain electrical energy for WSNs, especially in some circumstances where batteries may not be feasible. For the vibration energy harvesters, there are three common transduction mechanisms which includes piezoelectric, electromagnetic and electrostatic [3–5]. In this paper, we focus on the electrostatic energy harvesting system.

One of the problems associated with the electrostatic energy harvesters is the implementation of power management circuits. As an example, a conversion circuit consisted of a voltage source, a variable capacitor and two switches was presented in [6, 7]. Although energy transduction through this circuit is possible, the regime where the output voltage saturated was not discussed. Several solutions based on energy-renewal technique for extracting electrical energy were presented. For instance, Yen et al. proposed a configuration of single variable-capacitance harvester, combining an asynchronous charge-pump with an inductive fly-back circuit to recharge the scavenging capacitor [8]. Mitcheson et al. developed a buck-boost topology with bi-directional switches for rectifying and increasing the AC voltage obtained from a transducer [9]. These circuit topologies face the trade-off between power consumption of control unit and harvester efficiency.

The Bennet’s doubler was early introduced in 1787 by the Reverend Bennet and Kaye [10]. The device is used for the continuous doubling of an initial small charge through a sequence of operations with three plates. Based on this approach, de Queiroz proposed a promising variation of such a voltage doubler for macro-scale vibration energy harvesters composed by variable capacitors and diodes [11–13]. In order to adapt the concept to micro-scale electrostatic generators, several researches have been developed and investigated [14–17], including attempts to increase the charging current for a reservoir capacitor or to optimize the harvested power. In a recent work by Galayko [18], operation of the doubler configuration with a single variable capacitor was thoroughly analyzed in the electrical domain. The shape of Q-V diagram obtained from simulation is very close to be
rectangular. However, operation of a transducer configuration with two time-varying capacitors and the dependence of the saturated voltage on dynamic characteristics of the mechanical domain has not explored yet.

Since the saturation phenomenon was observed in experiments [16], the effect of the electromechanical coupling on it is of interest to study. This paper further presents a theoretical analysis of the Bennet’s doubler based on the Q-V cycle. A complete model of an anti-phase overlap-varying transducers electrically configured as a voltage doubler is investigated. Numerical results for both ideal- and non-ideal diodes are obtained by means of a SPICE simulator, which are used to support the analytical solutions. For further increase of the saturated voltage across the storage capacitor, alternative topologies are introduced and analyzed.

2. Steady state operation with mathematically idealized diodes

![Figure 1. Overlap-varying energy harvesters employing the Bennet’s doubler circuit.](image)

**Figure 1.** Overlap-varying energy harvesters employing the Bennet’s doubler circuit.

2.1. Theoretical analysis. The overlap-varying energy harvesters can be utilized in a charge-doubling circuit-configuration as shown in Figure 1. The proof mass is suspended by four folded-beam linear springs. The maximum displacement $X_{\text{max}}$ is defined by the mechanical end-stops. Two anti-phase variable capacitors $C_{1/2}(x) = C_0(1 \mp \frac{x}{x_0})$ are connected to three diodes $D_1$, $D_2$, $D_3$ and the storage capacitor $C_s$. Here $C_0$, $x_0$ and $x$ are the nominal capacitance, the nominal overlap and the proof mass displacement respectively. Operation of the doubler circuit does not require any control unit or switches but an initial bias voltage $V_0$.

![Figure 2. Equivalent circuit for mechanical domain and Bennet’s doubler configuration.](image)

**Figure 2.** Equivalent circuit for mechanical domain and Bennet’s doubler configuration.
Table 1. Model parameters

| Parameters              | Value       |
|-------------------------|-------------|
| Proof mass, m           | 1.022 mg    |
| Spring stiffness, k     | 3.595 N/m   |
| Thin-film air damping, b| 3.478e-5 Ns/m |
| Nominal overlap, x₀     | 80 µm       |
| Nominal capacitance, C₀ | 15 pF       |
| Parasitic capacitance, Cₚ| 7.5 pF      |
| Storage capacitance, Cₛ | 10 nF       |
| Contact stiffness, kᵢₘ | 3.361 MN/m  |
| Impact damping, bᵢₘ     | 0.435 Ns/m  |
| Maximum displacement, Xₘₐₓ | 80 µm       |

Figure 2 shows a complete lumped-model of the doubler configuration including equivalent circuit for the mechanical subsystem, where m - proof mass, b - mechanical damping, k - total spring stiffness, F - an external force, Fₑ - the electrostatic force and Cₚ - the parasitic capacitance of each transducer. The contact force Fᵢₘ is simply modeled as a spring-damper system Fᵢₘ = kᵢₘδ + bᵢₘδ for |x| ≥ Xₘₐₓ [19], where δ = |x| − Xₘₐₓ is relative displacement between the proof mass and the end-stops, kᵢₘ is the impact stiffness and bᵢₘ is the impact damping. For a sufficient voltage V₀ and an adequate input acceleration amplitude A, the voltage accumulated on the storage capacitor Cₛ initially increases. The vibration frequency is chosen f = f₀ = 1/2π√(k/m). Figure 3 shows that after certain cycles of transient regime, the steady state is achieved. The electrical energy is no longer harvested and the output voltage Vₜₐₜₜₜ is then maintained constant at Vₛ (i.e., saturation voltage).

The proof mass displacement amplitude X₀ changes in complicated manner: X₀ first reaches the maximum value X₀ ≈ Xₘₐₓ (i.e., which is limited by the mechanical end-stops), then decreases and kept fixed at X₀ ≈ Xₘₐₓ in saturation regime. For convenience, we define the rate of voltage...
evolution $v^*$ as a ratio of the maximum output voltage in two subsequent period

$$v^* = \frac{\max(V_{\text{out}}|_{T_i+1})}{\max(V_{\text{out}}|_{T_i})}.$$  

(2.1)

As shown in Figure 3, $v^*$ is modified over cycles under the variation of $X_0$ as follows. $v^*$ is small at the beginning and gradually increases, meanwhile $X_0 \approx X_{\text{max}}$. After reaching the maximum, $v^*$ decreases with reduction of $X_0$ and finally becomes one at steady state. Ultimately higher voltages through the conversion phase induce more effective electrical damping represented by electrostatic force in the transducers, causing a decrease of the proof mass displacement. As a consequence, the transducer capacitance ratio is reduced to $\eta = (C_{\text{max}} + C_p)/(C_{\text{min}} + C_p) \approx 1.72$, which is no more satisfied the condition of the doubler circuit operation $\eta_{\text{cr}} = 2$. Therefore, $V_{\text{out}}$ is saturated at a certain value. Detail of dynamic analyses and the model parameters (i.e., listed in Table 1) are referred to [20]. In this paper, the effect of the electrostatic force on $V_s$ is the major objective of investigation.

Figure 4 shows waveforms of the proof mass displacement, the voltages $V_1$, $V_2$ across $C_1$, $C_2$ and the currents $I_{D1}$, $I_{D2}$, $I_{D3}$ through three mathematically idealized diodes respectively. Operation of the doubler circuit at steady state can be divided into a sequence of four stages from $t_0$ to $t_4$. Based on the dynamic simulations, we observe that the relation of $Q_1$ and $V_1$ at steady state can be approximated by a right-angled trapezoid Q-V cycle diagram and the time interval between $\Delta t_{21} = t_2 - t_1$ and $\Delta t_{43} = t_4 - t_3$ are very small, as depicted in Figure 5.
Figure 5. Approximated Q-V diagram of variable capacitor $C_1(x)$ at steady state with mathematically ideal diodes.

**Stage I:**
At $t = t_0$, $x(t_0) = -X_s$ and $V_1(t_0) \approx V_2(t_0) \approx V_s$, where $X_s$ is the maximum displacement at steady state. From $t_0$ to $t_1$, all three diodes $D_1$, $D_2$ and $D_3$ are blocking as the condition $V_{C_2} < V_0 < V_{C_1} < V_{C_2} + V_0$ is satisfied. The charges on the two transducers are

$$q_1(t_0) = V_s \left[ C_p + C_0(1 + \frac{X_s}{x_0}) \right],$$

$$q_2(t_0) = V_s \left[ C_p + C_0(1 - \frac{X_s}{x_0}) \right].$$

In the first stage, $q_1$ and $q_2$ are constants, $V_1$ and $V_2$ are given

$$V_1 \bigg|_{t \in [t_0, t_1]} = \frac{q_1}{C_1} = \frac{V_s \left[ C_p + C_0(1 + \frac{X_s}{x_0}) \right]}{C_p + C_0(1 + \frac{x_s}{x_0})},$$

$$V_2 \bigg|_{t \in [t_0, t_1]} = \frac{q_2}{C_2} = \frac{V_s \left[ C_p + C_0(1 - \frac{X_s}{x_0}) \right]}{C_p + C_0(1 + \frac{x_s}{x_0})}.$$  

**Stage II:**
At $t = t_1$, $V_1(t_1) \approx V_2(t_1) + V_s$ and diode $D_3$ starts to conduct. Since the time interval between $t_1$ and $t_2$ is very small (i.e., see Figure 5), the proof mass displacement at $t_1$ can be approximated as $x(t_1) \approx x(t_2) = X_s$, then

$$\frac{1 + \frac{C_0}{C_p} \left( 1 + \frac{X_s}{x_0} \right)}{1 + \frac{C_0}{C_p} \left( 1 - \frac{X_s}{x_0} \right)} = 1 + \frac{1 + \frac{C_0}{C_p} \left( 1 - \frac{X_s}{x_0} \right)}{1 + \frac{C_0}{C_p} \left( 1 + \frac{X_s}{x_0} \right)}.$$

The solution is given as

$$X_s = 3 \left( \frac{\sqrt{5}}{2} - 1 \right) x_0.$$

The peak values of voltages across $C_1$ and $C_2$ are

$$V_I = V_1(t_1) = V_1(t_2) = V_s \frac{\sqrt{5} + 1}{2},$$

$$V_{II} = V_2(t_1) = V_2(t_2) = V_s \frac{\sqrt{5} - 1}{2}.$$

In this stage, charges $\Delta Q_s$ and $\Delta Q$ are pumped from $C_1$ into $C_s$ and $C_2$ respectively. At steady state, $V_s$ is considered unchanged, thus $\Delta Q_s$ is neglected.

**Stage III:**
Therefore,

\begin{equation}
q_1\big|_{t\in[t_2,t_3]} = q_1(t_2) = V_s \left[C_p + C_0\left(1 + \frac{X_s}{x_0}\right)\right] - \Delta Q,
\end{equation}

\begin{equation}
q_2\big|_{t\in[t_2,t_3]} = q_2(t_2) = V_s \left[C_p + C_0\left(1 - \frac{X_s}{x_0}\right)\right] + \Delta Q.
\end{equation}

At \(t_3\), \(x(t_3) = x_3\), \(V_1(t_3) = V_s\) (2.12) and \(D_2\) starts to conduct transferring amount of charge \(\Delta Q^*\) from \(C_s\) into \(C_1\). Similarly, since \(V_s\) is treated as constant, \(\Delta Q^*\) is thus negligible. The relation (2.12) now can be written as

\begin{equation}
V_1(t_3) = \frac{q_1(t_3)}{C_1(t_3)} = \frac{V_s \left[C_p + C_0\left(1 + \frac{X_s}{x_0}\right)\right] - \Delta Q}{C_p + C_0\left(1 - \frac{X_s}{x_0}\right)} = V_s.
\end{equation}

Due to the small interval time between \(t_3\) and \(t_4\), \(x_3 \approx x(t_4) = -X_s\), resulting in \(\Delta Q \approx 0\). In other words, the charge transferred from \(C_1\) into \(C_2\) is insignificant.

Considering the voltage across the capacitor \(C_2\) at \(t_3\)

\begin{equation}
V_2(t_3) = \frac{q_2(t_3)}{C_2(t_3)} = \frac{V_s \left[C_p + C_0\left(1 - \frac{X_s}{x_0}\right)\right] + \Delta Q}{C_p + C_0\left(1 + \frac{X_s}{x_0}\right)} \approx V_s.
\end{equation}

Therefore, \(D_1\) also starts to conduct at \(t_3\) since the condition \(V_2 \approx V_s\) holds.

**Stage IV:**

From \(t_3\) to \(t_4\), \(D_1\) is conducting and \(\Delta Q\) is transferred from \(C_2\) into \(C_1\). The charge \(q_1\) is

\begin{equation}
q_1(t_4) = q_1(t_3) + \Delta Q = V_s \left[C_p + C_0\left(1 + \frac{X_s}{x_0}\right)\right].
\end{equation}

The condition \(q_1(t_4) = q_1(t_0)\) (2.16) is fulfilled, showing that the state of the doubler circuit at \(t_4\) is exactly the same as when \(t = t_0\), and a new cycle starts. This also proves that the right-angled trapezoid Q-V cycle diagram is capable of describing the operation of the doubler circuit.

### 2.2. Similarity of Bennet’s doubler and charge-pump circuit.

Among different circuit topologies for the interface electronics of MEMS capacitive energy harvesters [21, 22], the charge pump circuit early presented by Roundy et al. [23] is one of the most promising topologies. Another variation with inductive fly-back circuitry was developed by Yen et al. [8]. The simplest way to implement fly-back is to use a load resistance, originally reported in [24]. Such a fly-back configuration was thoroughly analyzed in [25].

Comparing the results shown in the literature with the one obtained in this paper, it is worth to note that the Q-V cycle for the charge pump circuit with resistive fly-back is very similar to that of Bennet’s doubler circuit. Both topologies can be approximated by trapezoidal conversion cycle. At the steady state of the idealized charge pump and the voltage doubler, the Q-V cycle is degenerated to a line (i.e., see Figure 5).

### 3. Approximation of the Saturation Voltage with Mathematically Ideal Diode

The electrostatic force \(F_e\) plays an important role in saturation of the output voltage and is thoroughly analyzed in this Section. \(F_e\) is modeled as

\begin{equation}
F_e = -\frac{\partial W_e}{\partial x} = -\frac{1}{2} \frac{\partial C_1(x)}{\partial x} V_1^2 - \frac{1}{2} \frac{\partial C_2(x)}{\partial x} V_2^2 = \frac{1}{2} \frac{C_0}{x_0} (V_1^2 - V_2^2)
\end{equation}

where \(W_e\) is the electrostatic energy of the transducers. \(V_1\) and \(V_2\) can be simplified as anti-phase sinusoidal signals for the sake of analysis although it is more complicated than that in reality. Based
on the dynamic simulations, we observe that the phase difference between the input acceleration and the voltage across $C_1$ is negligibly small and is ignored. The waveforms of $V_1$ and $V_2$ are then presented as

$$V_1 = \frac{V_i + V_s}{2} + \frac{V_i - V_s}{2} \sin(\omega t) = V_s \frac{3 + \sqrt{5}}{4} + V_s \frac{-1 + \sqrt{5}}{4} \sin(\omega t),$$

$$V_2 = \frac{V_{II} + V_s}{2} - \frac{V_s - V_{II}}{2} \sin(\omega t) = V_s \frac{1 + \sqrt{5}}{4} - V_s \frac{3 - \sqrt{5}}{4} \sin(\omega t)$$

yielding

$$V_1^2 - V_2^2 = \frac{2 + \sqrt{5}}{4} V_s^2 (1 + \sin(\omega t)) \left(1 + \frac{(\sqrt{5} - 2)^2}{2} \sin(\omega t)\right).$$

The coefficient $\frac{(\sqrt{5} - 2)^2}{2} \approx 0.028 \ll 1$ is negligible, the electrostatic force is then

$$F_e = \frac{2 + \sqrt{5}}{8} \frac{C_0}{x_0} V_s^2 (1 + \sin(\omega t)) = F_0 (1 + \sin(\omega t)).$$

where $F_0 = \frac{2 + \sqrt{5} C_0}{8} x_0 V_s^2$. The harmonic term of $F_e$ is in phase with the input acceleration.

Figure 6 shows the comparison between the input acceleration and the electrostatic force, at the same time duration as Figure 4. These simulation results along with expression of $F_e$ in (3.5) confirm that our assumption is reasonable.

The differential equation of the spring-mass-damping system, which is set in continuous oscillation by a sinusoidal force acting on the mass, is

$$m \ddot{x} + b \dot{x} + kx = mA \sin(\omega t) - F_e.$$

The steady-state solution of (3.6) is $x = -\ddot{x} + x_h$, where $\ddot{x} = \frac{F_0}{k}$ is the offset displacement and the harmonic term is [26]

$$x_h = X_0 \sin(\omega t + \varphi).$$

where $X_0 = \frac{(mA - F_0)/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (\frac{b}{m})^2 \omega^2}}$. Since $\omega = \omega_0 = \sqrt{\frac{k}{m}}$ and the proof mass displacement barely reaches its constraint, the peak value of $x_h$ is $X_0 = X_s = \frac{mA - F_0}{b \omega_0}$. The ratio $\ddot{x}$ obtained from simulations is less than 2.1% for all $A \in [1, 2]$ g, therefore $\ddot{x}$ is assumed negligible. By considering

![Figure 6](image-url)
amplitudes of the harmonic term and ignoring phase differences, the saturation voltage is

\[ V_s = \frac{8}{2 + \sqrt{5}} \left( \frac{mA - \frac{3}{2} (\sqrt{5} - 2)x_0 b\omega_0}{c_2 x_0} \right). \]  

Although performance of the harvesting system using mathematically ideal diode is analyzed, the power loss due to diode imperfections such as leakage current and junction capacitance is still an open room for investigation. This issue will be explored in the next section.

4. Operation of the Bennet’s Doubler with Schottky diode

4.1. Approximated Q-V Cycle at steady state. In the same manner of the Dragunov’s work [14,27], the Schottky diode 1N6263 is used to assess effect of diode losses on the harvesting system performance, where the magnitude of reverse current is comparable with the charging current through the storage capacitor, and the zero bias junction capacitance is in the range of transducer nominal capacitance.

Figure 7 shows waveforms of the proof mass displacement and the voltages \( V_1 \) and \( V_2 \) across \( C_1 \) and \( C_2 \) respectively. Similarly, operation of the doubler circuit at steady state can be divided into
a sequence of four stages, which is more clearly than considerations of mathematically idealized diode (i.e., the time interval between stages is significant). In general, the relation of \( Q_1 \) (\( Q_2 \)) and \( V_1 \) (\( V_2 \)) at steady state can be approximated by a right-angled trapezoid Q-V cycle diagram in Figure 8. Charges transferred from or into \( C_s \) are neglected since the output voltage is unchanged at steady state. Differently from previous section, the proof mass displacements at \( t_1 \) and \( t_3 \) are still unknown.

**Stage I:**
The same as previous analysis, the charges on the two generators and variations of \( V_1 \) and \( V_2 \) from \( t_0 \) to \( t_1 \) are presented by equations (2.2), (2.3), (2.4) and (2.5).

**Stage II:**
At \( t = t_1 \), \( x(t_1) = x_1 \), \( V_1(t_1) = V_2(t_1) + V_s \) and diode \( D_3 \) starts to conduct, this yields

\[
\frac{1 + \frac{C_p}{C_p} \left( 1 + \frac{X_3}{x_0} \right)}{1 + \frac{C_p}{C_p} \left( 1 - \frac{x_1}{x_0} \right)} = 1 + \frac{1 + \frac{C_0}{C_p} \left( 1 - \frac{X_s}{x_0} \right)}{1 + \frac{C_0}{C_p} \left( 1 + \frac{x_1}{x_0} \right)}.
\]

From \( t_1 \) to \( t_2 \), charge \( \Delta Q \) is pumped from \( C_1 \) into \( C_2 \).

**Stage III:**
From \( t_2 \) to \( t_3 \), all diodes are blocked, \( q_1 \) and \( q_2 \) are constants that are described by (2.10) and (2.11). At \( t = t_3 \), \( D_2 \) starts to conduct due to \( V_1(t_3) = V_s \) (4.2). This condition is expressed by (2.13), which results in

\[
\Delta Q = V_s C_0 \left( \frac{X_s + x_3}{x_0} \right).
\]

The voltage across \( C_2 \) at \( t_3 \) is

\[
V_2(t_3) = \frac{q_2(t_3)}{C_2(t_3)} = \frac{V_s \left[ C_p + C_0 (1 - \frac{X_s}{x_0}) \right] + \Delta Q}{C_p + C_0 (1 + \frac{X_s}{x_0})} = \frac{V_s \left[ C_p + C_0 (1 - \frac{X_s}{x_0}) \right] + V_s C_0 \left( \frac{X_s + x_3}{x_0} \right)}{C_p + C_0 (1 + \frac{X_s}{x_0})} = V_s.
\]

As the condition \( V_2 = V_s \) is fulfilled, \( D_1 \) also starts to conduct at \( t_3 \). Substituting (4.3) to (2.10), we get

\[
q_1(t_3) = V_s \left[ C_p + C_0 (1 + \frac{x_3}{x_0}) \right].
\]

**Stage IV:**
From \( t_3 \) to \( t_4 \), \( D_1 \) is conducting and \( \Delta Q \) is transfered into \( C_1 \) from \( C_2 \). At \( t_4 \), \( x(t_4) = -X_s = x(t_0) \) and the state of the doubler circuit is the same as when \( t = t_0 \), leading to

\[
q_1(t_4) = q_1(t_0)
\]

where

\[
q_1(t_4) = q_1(t_3) + \Delta Q = V_s \left[ C_p + C_0 (1 + \frac{X_s + 2x_3}{x_0}) \right].
\]

From the equations (2.2), (4.6) and (4.7), the displacement at \( t_3 \) is given by \( x_3 = 0 \). As the consequence

\[
\Delta Q = V_s C_0 \frac{X_s}{x_0}.
\]
Substituting this result back into (2.10) and (2.11), the voltages across \( C_1 \) and \( C_2 \) at \( t_2 \) are obtained

\[
V_1(t_2) = \frac{q_1(t_2)}{C_1(t_2)} = \frac{V_s(C_p + C_0)}{C_p + C_0(1 - \frac{X_v}{x_0})},
\]

(4.9)

\[
V_2(t_2) = \frac{q_2(t_2)}{C_2(t_2)} = \frac{V_s(C_p + C_0)}{C_p + C_0(1 + \frac{X_v}{x_0})}.
\]

(4.10)

At \( t_2 \), \( D_3 \) starts to stop conducting since \( V_1 \) is slightly less than \( V_2 + V_s \). This relation can be approximated as \( V_1 \approx V_2 + V_s \). Similarly as equation (4.1), we get

\[
\frac{1 + \frac{C_0}{C_p}}{1 + \frac{C_0}{C_p}(1 - \frac{X_v}{x_0})} = 1 + \frac{1 + \frac{C_0}{C_p}}{1 + \frac{C_0}{C_p}(1 + \frac{X_v}{x_0})}.
\]

(4.11)

The solution of the maximum displacement at steady state is

\[
X_s = \frac{3(\sqrt{2} - 1)}{2} x_0.
\]

(4.12)

Substituting (4.12) back into (4.1), the proof mass displacement at \( t_1 \) is determined by

\[
x_1 = \frac{3}{2} \left( \sqrt{4 - 2\sqrt{2}} - 1 \right) x_0.
\]

(4.13)

Therefore, the peak values of \( V_1 \) and \( V_2 \) are

\[
V_I = V_1(t_2) = V_s \left( 1 + \frac{1}{\sqrt{2}} \right),
\]

(4.14)

\[
V_{II} = V_2(t_1) = V_s \sqrt{1 - \frac{1}{\sqrt{2}}},
\]

(4.15)

\( V_1 \) and \( V_2 \) are then approximated by

\[
V_1 = \frac{V_I + V_s}{2} + \frac{V_I - V_s}{2} \sin(\omega t) = V_s(1 + \frac{1}{2\sqrt{2}}) + V_s \frac{1}{2\sqrt{2}} \sin(\omega t),
\]

(4.16)

\[
V_2 = \frac{V_{II} + V_s}{2} - \frac{V_s - V_{II}}{2} \sin(\omega t) = \frac{1 + \sqrt{1 - \frac{1}{\sqrt{2}}}}{2} - V_s \frac{1 - \sqrt{1 - \frac{1}{\sqrt{2}}}}{2} \sin(\omega t)
\]

(4.17)

yielding

\[
V_1^2 - V_2^2 = V_s^2 (\alpha + \gamma) \left[ (\alpha - \gamma) + (\beta + \lambda) \sin(\omega t) \right] \left( 1 + \frac{\beta - \lambda}{\alpha + \gamma} \sin(\omega t) \right)
\]

(4.18)

where

\[
\alpha = 1 + \frac{1}{2\sqrt{2}}, \quad \beta = \frac{1}{2\sqrt{2}}, \quad \gamma = \frac{1 + \sqrt{1 - \frac{1}{\sqrt{2}}}}{2}, \quad \lambda = \frac{1 - \sqrt{1 - \frac{1}{\sqrt{2}}}}{2}.
\]

Since \( \frac{\beta - \lambda}{\alpha + \gamma} \approx 0.058 \ll 1 \) is negligible and \( \alpha - \gamma = \beta + \lambda \), the electrostatic force can be given by

\[
F_e = \frac{1}{2} \frac{C_0}{x_0} V_s^2 (\alpha^2 - \gamma)^2 \left( 1 + \sin(\omega t) \right) = \frac{1}{2} \frac{C_0}{x_0} V_s^2 \frac{5(1 + \sqrt{2}) - 2\sqrt{4 - 2\sqrt{2}}}{8} \left( 1 + \sin(\omega t) \right)
\]

(4.20)

which is represented as

\[
F_e = F_0 \left( 1 + \sin(\omega t) \right)
\]

(4.21)

where \( F_0 = \frac{5(1 + \sqrt{2}) - 2\sqrt{4 - 2\sqrt{2}}}{16} \frac{C_0}{x_0} V_s^2 \).
Figure 9. Q-V diagram of Bennet’s doubler at steady-state for both variable capacitors.

(4.22) \[ V_s = \sqrt{\frac{16}{5(1 + \sqrt{2}) - 2\sqrt{4 - 2\sqrt{2}}} \frac{mA - \frac{3}{2}(\sqrt{2} - 1)x_0b\omega_0}{x_0} \approx \sqrt{1.61 \frac{mA - \frac{3}{2}(\sqrt{2} - 1)x_0b\omega_0}{x_0}}. \]

Based on those analysis above, the completed Q-V diagram combined by both transducers is summarized in Figure 9.

Table 2. Diodes parameters: reverse saturation current \( I_s \), zero-bias junction capacitance \( C_j \) and built-in junction voltage \( V_j \)

| Diode   | \( I_s \) [nA] | \( C_j \) [pF] | \( V_j \) [V] |
|---------|---------------|---------------|--------------|
| 1N6263  | 3.87          | 1.77          | 0.39         |
| BAS716  | 3.52e-6       | 1.82          | 0.65         |
| BAT41   | 10.00         | 5.76          | 0.37         |

4.2. Numerical validations. Figure 10a shows the saturation voltages for different acceleration amplitudes, where the simulation results with use of the mathematically idealized diode and the
analytical solution expressed by formula (3.8) are compared. The figure also exhibits that the low-losses diode BAS716 performs very close to that of mathematically idealized diode. In the same manner, Figure 10b presents the comparison of the analytical solution obtained from (4.22) against the numerical simulations using different Schottky diodes. Despite of disparities in reverse current, junction capacitance and built-in junction voltage, both diodes 1N6263 and BAT41 give almost the same saturation voltages. The agreement between theoretical and numerical results in both cases verifies the predictions of our analytical approach and solutions. Diode parameters used on the simulations are listed in Table 2.

4.3. Effect of diode operation on mechanical dynamics. The Q-V cycle is a useful geometrical tool that enables us to realize the operation of voltage doubler circuit at steady state. However, the harvesting system performance in reality is more sophisticated, especially in transient time.

Based on dynamic simulations, we observe that the phases of the external force $F(t)$, the proof mass displacement $x(t)$ and the electrostatic force $F_e(t)$ are initially different. However, those differences gradually decrease due to effect of the diode states (i.e. blocked and conducting). This variation process leads to the negligible phase shift at steady state. Such a clarification supports the assumption that we made in theoretical analysis sections. In other words, the dynamic motion of the proof mass also strongly depends on both of the transducing force and the diode operation mechanism. This statement is valid when different diode models such as the mathematically idealized diode and the Schottky diodes are utilized.

5. Circuit topologies to improve the saturation voltage

5.1. A new voltage doubler with single switch. Although the diode $D_2$ plays an vital role for initially charging $C_1$, in principle, it could be removed after a few transient vibration cycles. This also enlarges the charging current through the storage capacitor due to the relation $I_{Cs} = I_{D3} - I_{D2}$. Therefore, it is worthwhile to investigate performance of the harvester when $D_2$ is disconnected. An electronic switch $SW$ in series with $D_2$ can be used for this function, as shown in Figure 11.

In the simulation, $SW$ is only ON in the first several vibration cycles, then turned OFF to eliminate effect of $D_2$ on $I_{Cs}$. Figure 12a shows evolution of the output voltage in two cases without and with presence of $SW$. Saturation voltage in the latter case is about $\sim 15.60 \, V$. This is a significant improvement over the 13.84 $V$ achieved for the circuit topology in Figure 2. Similar results are obtained with different acceleration amplitudes in Figure 12b.

5.2. Cockcroft-Walton generator applied to MEMS device. A common topology of voltage doubler further developed from the Greinacher circuit [28] is depicted in Figure 13, in which the feedback diode $D_2$ is added to connect the storage capacitor and the two transducers. Both theoretical operation analysis and simulation results show that performances of the Bennet’s doubler and the
Figure 12. (a) The time evolution of output voltage at $A = 2 \text{ g}$ and (b) The saturation voltage versus acceleration amplitudes, comparison of two cases: with and without the switch.

Figure 13. An adapted configuration of the Greinacher’s doubler.

Figure 14. The two-stage Cockcroft-Walton multiplier.

Greinacher configuration are completely identical. The roles of three diode $D_1$, $D_2$ and $D_3$ are the same as they do in Figure 2.

Based on the Greinacher doubler circuit, a well-known voltage cascade was early proposed by the British and Irish physicists John D. Cockcroft and Ernest T. S. Walton in 1932 [29, 30]. The Cockcroft-Walton generator (i.e., named after the two authors) was proved to be able to generate a high DC voltage from a low-voltage AC, which therefore is interesting to be utilized for the micro-scale harvesters. Figure 14 shows the circuit diagram of the two-stage Cockcroft-Walton, in which the voltage across two capacitor $C_{s1}$ and $C_{s3}$ is the output voltage, called $V_{out}$. The simplified operation of such a multi-stage voltage doubler is depicted in Figure 15. Similar to the Bennet’s
configuration, operation of the Cockcroft-Walton multiplier can also be divided into a sequence of four stages. At first, all diodes are blocked. In the second stage, $D_1$ and $D_3$ are simultaneously conducting and charges are transferred to $C_2$ and $C_{s2}$. All diodes are reverse-biased in the third stage. In the final stage, $D_2$ and $D_4$ are conducting, transferring the scavenged energy to $C_{s1}$ and $C_{s3}$. $D_5$ is mainly used for pre-charging $C_1$ and its conduction during operation is insignificant and negligible.

Figure 16 shows a remarkable increase of the saturation voltage when the Cockcroft-Walton multiplier and the Bennet’s doubler are compared. Since the topology discussed in Section 5.1 requires a control unit for controlling the switch, the Cockcroft-Walton multiplier is much more convenient to keep the simplicity in practical implementation. Furthermore, our simulations reveal that this circuit topology is capable of operating with very low ratio of capacitance variation $\eta < 2$. In particular, its minimum value is found $\eta_{\text{min}} = 1.52$, making such a circuit attractive for further investigation in future work.

6. Conclusion

This study presented a theoretical analysis of MEMS electrostatic energy harvesters configured as Bennet’s doubler at saturation regime, based on combination of Q-V diagram and dynamic simulations. The steady state operation of voltage doubler was approximately determined as a
right-angled trapezoidal conversion cycle. Mathematically idealized and non-ideal diode models were investigated, resulting in different analytical solutions of the saturation voltages. The theoretical approach was verified by circuit simulation results obtained from a complete model of the harvesting system. An essential effect of the diode operation mechanism to the in-phase behavior of the input mechanical vibration and the electrostatic force was discussed. A similarity of Bennet’s doubler and resistive fly-back charge-pump circuit is realized by comparing their Q-V diagram. An alternative circuit using a single switch was introduced, where the saturation voltage was significantly improved in comparison with the conventional topologies. The Cockcroft-Walton multiplier is another promising solution since it shows a potential to work with MEMS harvesters that have small varying capacitance ratio.

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