New Insights into White-Light Flare Emission from Radiative-Hydrodynamic Modeling of a Chromospheric Condensation.

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Abstract The heating mechanism at high densities during M dwarf flares is poorly understood. Spectra of M dwarf flares in the optical and near-ultraviolet wavelength regimes have revealed three continuum components during the impulsive phase: 1) an energetically dominant blackbody component with a color temperature of $T \sim 10^4$ K in the blue-optical, 2) a smaller amount of Balmer continuum emission in the near-ultraviolet at $\lambda \leq 3646\AA$ and 3) an apparent pseudo-continuum of blended high-order Balmer lines between $3646\AA$ and $\sim 3900\AA$. These properties are not reproduced by models that employ a typical solar-type flare heating level of $\lesssim 10^{11}$ erg cm$^{-2}$ s$^{-1}$ in nonthermal electrons, and therefore our understanding of these spectra is limited to a phenomenological three-component interpretation. We present a new 1D radiative-hydrodynamic model of an M dwarf flare from precipitating nonthermal electrons with a large energy flux of $10^{13}$ erg cm$^{-2}$ s$^{-1}$. The simulation produces bright near-ultraviolet and optical continuum emission from a dense ($n > 10^{15}$ cm$^{-3}$), hot ($T \sim 12,000–13,500$K) chromospheric condensation. For the first time, the observed color temperature and Balmer jump ratio are produced self-consistently in a radiative-hydrodynamic flare model. We find that a $T \sim 10^4$ K blackbody-like continuum component and a small Balmer jump ratio result from optically thick Balmer ($\infty \rightarrow n = 2$) and Paschen recombination ($\infty \rightarrow n = 3$) radiation, and thus the...
properties of the flux spectrum are caused by blue ($\lambda \sim 4300\text{Å}$) light escaping over a larger physical depth range compared to red ($\lambda \sim 6700\text{Å}$) and near-ultraviolet ($\lambda \sim 3500\text{Å}$) light. To model the near-ultraviolet pseudo-continuum previously attributed to overlapping Balmer lines, we include the extra Balmer continuum opacity from Landau-Zener transitions that result from merged, high order energy levels of hydrogen in a dense, partially ionized atmosphere. This reveals a new diagnostic of ambient charge density in the densest regions of the atmosphere that are heated during dMe and solar flares.

1. Introduction

It is notoriously difficult to explain the origin of heating at high atmospheric densities during solar and stellar flares. The broadband color (white-light) distribution of the ultraviolet and optical emission during large and small flares on chromospherically active M dwarf (dMe) stars exhibits the general shape of a $T=8500−9500\text{ K}$ blackbody (Hawley and Fisher, 1992; Hawley et al., 2003) without an indication of a significant Balmer jump at $\lambda = 3646\text{ Å}$. A recent homogeneous analysis of flare spectra around the Balmer jump has confirmed the presence of a hot blackbody (or blackbody-like) component with a color temperature of $T \gtrsim 10,000\text{ K}$ that contributes most of the radiated near-ultraviolet (NUV) and optical energy in the impulsive phase (Hawley and Pettersen, 1991; Kowalski et al., 2013, hereafter, K13). Intriguingly, the presence of hot blackbody emission with $T \sim 9000\text{ K}$ has recently also been inferred from Sun-as-a-Star observations during large (X-class) and small (C-class) solar flares (Kretzschmar, 2011). The hot (hereafter, “hot” refers to temperatures between 8500 and 15,000 K) blackbody continuum component is often interpreted as evidence of impulsive heating in a region with a high density of $n_e \sim 10^{15}\text{ cm}^{-3}$ or more, which has been inferred from phenomenological modeling of dMe flares (Cram and Woods, 1982; Houdebine, 1992; Christian et al., 2003). A recent observation that directly supports this idea is the Vega-like flare spectrum detected during the January 16th, 2009 Megaflare event on the dM4.5e star YZ CMi (Kowalski et al., 2010, 2013), which indicates that photospheric-level densities can be produced and maintained at sufficiently high temperatures during dMe flares. However, heating high densities requires a large amount of volumetric energy deposition such that the heating per unit mass is non-negligible. Direct heating of the photosphere by nonthermal (NT) deka-keV electrons is problematic because the electrons experience rapid energy loss as soon as they impact the top of the “thick-target” chromosphere.

A possible mechanism for increasing the density in the upper chromosphere was proposed in the early gas dynamic simulations of Livshits et al. (1981). These authors showed that a shock from precipitating nonthermal electrons (with an energy flux of $10^{12}\text{ erg cm}^{-2}\text{ s}^{-1}$) produces a rapid downflow and compression of material to a very high ($>10^{15}\text{ cm}^{-3}$) density. The development of downward moving “chromospheric condensations” (hereafter, CC) is accompanied by an upward moving “evaporation” of chromospheric material as it is heated beyond a million degrees (Fisher, Canfield, and McClymont, 1985). Chromospheric...
evaporation and CCs are thought to be the fundamental processes in solar and stellar flares driving the frequently observed Neupert effect (Dennis and Zarro, 1993; Hawley et al., 1995; Guedel et al., 1996). Several studies have explored the role of explosive CCs as the source of white-light emission in solar and dMe flares (Livshits et al., 1981; Gan et al., 1992; Katsova, Boiko, and Livshits, 1997), but the formation threshold of CCs is strongly dependent on the initial atmospheric structure, the energy distribution (power law index, low-energy cutoff, and flux) of the precipitating NT electrons (Fisher, 1989), and the time-dependent ionization of helium (Abbett and Hawley, 1999; Allred et al., 2005, 2006), thus necessitating an accurate and realistic description of the NT electron distribution, the pre-flare atmosphere structure, and the level populations of important cooling agents.

Recent radiative-hydrodynamic (RHD) simulations of dMe (Allred et al., 2006) flares with the RADYN code (Carlsson and Stein, 1995, 1997) explored the atmospheric response to an actual NT electron spectrum that was obtained from the peak of an X-class solar flare inferred from RHESSI data (Holman et al., 2003). A constant, solar-type flare heating level of the NT electrons was employed with injected energy fluxes of $10^{10}$ erg cm$^{-2}$ s$^{-1}$ (F10) and $10^{11}$ erg cm$^{-2}$ s$^{-1}$ (F11). These simulations produced explosive mass motions indicative of chromospheric evaporation and a CC, but with densities far too low to produce strong, blackbody-like white-light emission. Moreover, the prominent spectral component in these simulations was a hydrogen recombination spectrum with strong Balmer recombination continuum emission and a smaller amount of Paschen recombination continuum emission. In these models, the heating at high density was achieved through Balmer and Paschen continuum backwarming of the photosphere, which increased in temperature by 1200 K in the F11 run and produced additional amounts of optical/red (blackbody-like) emission. For the range of nonthermal electron energy fluxes thus far explored in RHD models (F9 - F11; see also Hawley and Fisher, 1994; Abbett and Hawley, 1999), a flare spectrum with hot blackbody emission has not been produced through the development of a CC, through photospheric heating from Balmer and Paschen continuum backwarming (Allred et al., 2006), or through photospheric heating from X-ray and EUV backwarming (Hawley and Fisher, 1992; Allred et al., 2006; see also the extensive comparison of RHD model predictions and recent observations in Section 9 of Kowalski et al. (2013)).

In addition to hot blackbody emission, the recent spectral observations of dMe flares also show evidence for a secondary continuum component in the NUV at $\lambda \lesssim 3700\AA$. A jump in flux at these wavelengths is not accounted for by an extrapolation of a single hot blackbody component that can be fit to the redder wavelengths at $\lambda > 4000\AA$. The jump in flux relative to the blackbody flux is smaller in the peak phase than in the gradual decay phase, when the color temperature of the hot blackbody component also decreases. A comparison to the F11 RHD model spectra from (Allred et al., 2006) led to the conclusion that Balmer recombination continuum emission was needed to account for the additional flux in the NUV (Kowalski et al., 2010). However, high resolution (R~40,000) echelle flare spectra show that the NUV continuum from $\lambda \lesssim 3700\AA$ can
exhibit the color temperature of a hot blackbody (~11,000 K) while not exhibiting any signature of a Balmer discontinuity or edge feature [Fuhrmeister et al., 2008] that is indicative of Balmer recombination continuum emission. Instead, the highest order Balmer lines are broadened and appear to diminish as they converge into a “pseudo-continuum” at wavelengths between λ = 3646 Å and the bluest identifiable Balmer line, which is typically H15 λ3712 or H16 λ3704 for both large solar and dMe flares [Donati-Falchi, Smaldone, and Falciani, 1984; Hawley and Pettersen, 1991; Kowalski et al., 2010].

The broadening and merging of the high order Balmer lines in flares is thought to be due to the well-known Stark effect from ambient charges [Svestka, 1963; Donati-Falchi, Falciani, and Smaldon, 1985]. The Balmer edge itself is also thought to be broadened, but thermodynamically self-consistent (where the internal partition function converges) modeling of this effect was not available until occupational probability theory of pressure ionization was developed by Hummer and Mihalas (1988). This improved description of pressure ionization from Stark broadening at the Balmer edge is currently employed in white dwarf atmospheric modeling codes (Tremblay and Bergeron, 2009), but has not yet been implemented in models of solar and stellar flares to determine if the theory can explain the NUV/blue continuum, and in particular, the lack of a Balmer edge. The Stark broadening of the higher order Balmer lines is potentially a useful tool for constraining the environment (i.e., charge density) where the (blackbody) continuum emission is produced in stellar flares [Cram and Woods, 1982].

What atmospheric conditions and atomic processes at high density produce a flare radiation field that self-consistently exhibits all the important continuum components and line properties in the blue and NUV? First, we must relate the observables in blue/NUV spectra to physical parameters of the atmosphere. Then, the spectra can be used as powerful tools to constrain the heating mechanisms that produce these atmospheric conditions. In this paper, we explore the possible role of a very dense CC for producing white-light emission during dMe flares, which reveals a meaningful interpretation of the spectra that will be used to constrain targeted modeling of individual solar and stellar flares. These simulations are timely due to new spectral constraints in the blue/NUV and optical wavelength regimes [Kowalski et al., 2013] and modern computational facilities enabling calculation of detailed spectra with sophisticated radiative transfer codes. Section 2 describes our method of solution and the starting atmospheric conditions, Section 3 describes the atmospheric dynamics and a new interpretation of the 10^4 K blackbody color temperature; Section 4 describes the improved modeling of the Balmer edge, Section 5 explores the viability of the model, Section 6 summarizes the study and presents several conclusions, and Section 7 describes future modeling work.
2. Method of Solution & Initial Atmosphere

2.1. NLTE Radiative-Hydrodynamic Modeling with RADYN

We performed 1-D radiative-hydrodynamic modeling using the RADYN code (Carlsson and Stein, 1992, 1994, 1995, 1997, 2002). RADYN solves the equations of mass, momentum, internal energy, non-equilibrium level populations, charge conservation, and radiative transfer (see also Abbett and Hawley, 1999). The electron and ion temperatures are equal and an adaptive grid (Dorfi and Drury, 1987) with 250 grid points resolves shocks in the atmosphere. The equation of radiative transfer is solved in RADYN using the technique of Scharmer (1981) and Scharmer and Carlsson (1985). The NLTE level populations for three atoms are calculated: a hydrogen atom (6 level with continuum), a helium atom (9 level with continuum), and a singly ionized calcium ion (6 level with continuum), giving a total of 22 b-b and 19 b-f transitions calculated in detail (see Allred et al., 2005). Continua from elements other than H, He, and Ca II are treated as background continua in LTE using the Uppsala atmospheres program (Gustafsson 1973). A radiative cooling function from CHIANTI (Dere et al., 1997; Young et al., 2003) at 42 temperature points between $10^4$ K and $10^8$ K is included to account for optically thin cooling. Radiation from several ionization states (e.g., Mg II, Fe II, and Si II) that are likely optically thick at $10^4$ K (V. Hansteen, priv. communication, 2012) may be important but is not yet included in the models (see Section 7).

Each line profile is calculated with up to 100 frequency points across the transition using a Voigt profile and complete redistribution. For Lyman transitions, the effects of partial redistribution are approximated by truncating the line profiles at 10 Doppler widths. A change to the line broadening due to the Stark effect is implemented in RADYN compared to the models of Allred et al. (2006). Here, we use the analytic formulae (Method #1) from Sutton (1978) to model the Stark broadening as a Voigt profile for hydrogen. The Stark broadening of calcium and helium transitions remains the same as in previous versions of the code.

Backwarming from coronal X-ray and extreme-ultraviolet (XEUV) radiation is treated according to the method of Hawley and Fisher (1994), Gan and Fang (1994), and Abbett and Hawley (1999), which approximates the radiation source using a plane-parallel geometry within 1 Mm and as a point-source beyond this distance. The emissivity at 41 temperature points from $10^4 - 10^8$ K in 14 wavelength bins from $\lambda = 1 - 2500$Å, is generated from APEC (Smith et al., 2001). The hydrogen, helium, and metal cross sections are obtained from CHIANTI.

We ran flare simulations with the starting atmospheric parameters and boundary conditions similar to those used in Allred et al. (2006), in order to facilitate comparison. A preflare, plane-parallel 1-D atmosphere with log $g = 4.75$ was relaxed to have velocities $\lesssim 10$ m s$^{-1}$ and converged NLTE populations. The upper temperature boundary was initially fixed at $6 \times 10^6$ K ($n_e \sim 2.6 \times 10^{10}$ cm$^{-3}$), and constant non-radiative heating was applied at column mass greater than 3.16 g cm$^{-2}$ ($\tau_{5000} = 0.06$) to emulate convective energy flux. The atmosphere was relaxed using a plane-parallel approximation of the X-ray backwarming, as
would be appropriate from extended overlying active region loops. The effective
temperature of the relaxed atmosphere is 3630 K, corresponding to that of an
early-mid M dwarf. The height coordinate \( z \) of the atmosphere lies along the
length of a semi-circular magnetic loop of half-length \( L_{1/2} = 10^9 \) cm (10 Mm),
with \( z = 0 \) at the photosphere \( (75000 = 1.0, \log \text{ col mass} = 1.14 \text{ g cm}^{-2}) \).
A modification to the gravitational acceleration is incorporated near the top of
the loop \( \text{[Abbett and Hawley, 1999]} \); otherwise, the geometry is assumed to be a
vertical tube of constant cross-sectional area. The imposed 1D geometry assumes
the plasma motion is confined to the direction of the magnetic field, as would be
appropriate for a low \( \beta \) plasma, but is not necessarily satisfied in the photosphere
of the preflare atmosphere (the equipartition magnetic field strength is 4.2 kG
at the photosphere, consistent with observations \( \text{[Johns-Krull and Valenti, 1996]} \)
or in the chromosphere during the flare. Detailed molecular transitions that are
important for flux redistribution in an M dwarf photosphere have not yet been
included in the opacity.

2.2. Flare Heating Prescription

We model the atmospheric heating from a double power law, nonthermal (here-
after, NT) electron beam injected at the top of the atmosphere. In this study, we
seek to directly connect to the work of \( \text{[Allred et al., 2006]} \), which used the thick-
target parameters inferred from RHESSI \( \text{[Lin et al., 2002]} \) hard X-ray spectra
during the peak of the 2002 July 23 X4.8 GOES class solar flare \( \text{[Holman et al.,}
2003 \text{; see also Ireland et al., 2013]} \). A double power law of electron energies
is parameterized by \( \delta_l = 3.0 \) (the power law index from the cutoff energy to
the break energy), \( \delta_u = 4.0 \) (the power law index at energies greater than the
break energy), \( E_c = 37 \) keV (the “cutoff energy” or lowest energy of the NT
electron spectrum), and \( E_B = 105 \) keV (the break energy between \( \delta_l \) and \( \delta_u \)).
The value of \( E_c \) derived from observations is an upper limit, and \( E_c = 37 \) keV is
moderately large compared to typically assumed or inferred values on the order of
20 keV. Note also that the break in hard X-ray photon spectra (and thus also the
inferred double power law shape of the NT electron spectrum) can be the result of
the non-uniform ionization of the ambient atmosphere \( \text{[Su, Holman, and Dennis,}
2011]} \) or from energy losses to the return current electric field \( \text{[Holman, 2012]} \).
The parameters of the NT electron spectrum are poorly constrained during dMe
flares because the hard X-ray emission is too faint to observe except during the
largest flares \( \text{[Osten et al., 2010]} \). A powerful superflare on the RS CVn HR
1099 is consistent with a hard NT electron spectrum with \( \delta \sim 3 \) \( \text{[Osten et al.,}
2007]} \), similar to the relatively hard double power law spectrum employed in our
simulations.

Free parameters which may not be well-constrained by RHESSI observations
alone are the energy flux \( \text{[Krucker et al., 2011]} \) and the precise time profile
(duration) of the flare heating in any given magnetic loop. We extend the range
of heating fluxes from \( \text{[Allred et al., 2006]} \) to also consider those with \( 10^{12} \) erg
cm\(^{-2}\) s\(^{-1}\) (F12) and \( 10^{13} \) erg cm\(^{-2}\) s\(^{-1}\) (F13), in addition to \( 10^{11} \) erg cm\(^{-2}\)
s\(^{-1}\) (F11). The F13 flux is not much larger than the largest values that have
been derived during solar flares \( \text{[Neidig, Grosser, and Hrovat, 1994]} \), and such...
a level has recently been suggested to be relevant to the brightest solar flare kernels (Krucker et al., 2011; Milligan et al., 2014; Gritsyk and Somov, 2014).

The duration of heating in a single magnetic loop in solar flares is constrained in some cases to \( \leq 25 - 60 \) s (Wang, Fang, and Ming–DeDing, 2007; Schrijver et al., 2006) and in other cases may be several minutes (Qiu et al., 2010; Warren, 2006). A wavelet analysis of high time resolution hard X ray light curves for 647 solar flares observed with the Compton Gamma Ray Observatory indicates a range in the duration of individual bursts from 0.1 s to 50 s (Aschwanden et al., 1998). The shortest burst durations from high time resolution constraints observed in the NUV in dMe flares range from \( \sim 2 \) s (Robinson et al., 1995) to 10 s (Mathioudakis et al., 2006). For our heating profile, we use a constant energy flux for a duration of 2.3 s, after which the NT electron energy deposition decreases to 0, and the cooling phase is modeled up to 5.1 s for the F13 model (and 30 s for the models with lower beam fluxes). Short heating bursts represent the heating of a single loop, which can be superposed in sequence to emulate arcade development as has been done for soft X-ray data of solar flares (Warren, 2006).

The elastic collisions between NT electrons and the ambient charged and neutral components of the atmosphere heat the gas through Coulomb interactions. Following Hawley and Fisher (1994), Abbett and Hawley (1999), and Allred et al. (2005, 2006), the NT electron energy deposition rate is included as an extra term in the conservation of energy equation. The Coulomb heating rate as a function of height is modeled here using the analytic, non-relativistic test-particle approach of Emslie (1978), which was modified to include the height dependent ionization fraction of the atmosphere in Hawley and Fisher (1994). This prescription is based on the collisional thick target model (Brown, 1971), but it also includes an approximation for pitch angle scattering of the NT electrons (Emslie, 1978). The inelastic collisions between NT electrons and the ambient neutral hydrogen of the atmosphere cause direct excitation and ionization (Mott and Massey, 1965, Chapter xvi); we follow Kašparová et al. (2009) and add these NT collisional rates to the thermal rates for hydrogen according to the prescription in Fang, Henoux, and Gari (1993). The results from Dalgarno and Griffin (1958) are included in these rates to account for secondary ionizations of hydrogen by electrons that are ionized by the beam from the ground state of hydrogen.

3. Properties of the NUV and Optical Continuum Predictions

3.1. Comparison to Observables

Continuum predictions from radiative-transfer modeling are constrained with two simple observational parameters: 1) the Balmer jump ratio and 2) the color

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A comprehensive study of NT rates was presented in the calculations of Ricchiard (1985), Ricchiard and Canfield (1985), who found that the \( \text{H} \, n = 1 \rightarrow \infty \) and \( n = 1 \rightarrow n = 2 \) rates are important, but that the \( n = 2 \rightarrow \infty \) rate was negligible compared to the \( n = 2 \rightarrow \infty \) thermal rates for their range of beam fluxes.
temperature of the continuum at wavelengths longer than the Balmer edge. These two parameters were referred to as $\chi_{\text{flare}}$ and $T_{BB}$, respectively, in a recent homogeneous analysis of twenty flares on dMe stars from K13. A blackbody function is fit to the “BW” windows (see K13) of the flux spectrum from $\lambda = 4000 - 4800\text{Å}$ (the blue-optical wavelength regime) to obtain $T_{BB}$, while $\chi_{\text{flare}}$ is the ratio of flux at $\lambda = 3615\text{Å}$ (blueward of the Balmer jump) to the flux at $\lambda = 4170\text{Å}$ (redward of the Balmer jump). The distribution of these parameters was found to fall between $\chi_{\text{flare}} = 1.5 - 4$ and $T_{BB} = 9000 - 14000\ K$ during the peak phases of nearly all of the flares in the sample from K13. The value of $\chi_{\text{flare,peak}}$ varied among flare type, with the most impulsive flares exhibiting a narrow range of $1.5 - 2.2$ and the gradual flares exhibiting larger values. We measure these same parameters from the flux spectra for the peak phases of the model continua for the range of beam fluxes studied here. The model continua are calculated at a fixed number of points across the NUV and optical, and so we interpolate the continua to the wavelengths used in the observations. We also compute the ratio of the line-integrated, continuum-subtracted $H\gamma$ emission line flux to the value of the continuum flux at $\lambda = 4170\text{Å}$ (hereafter, $H\gamma_{\text{EW}4170}$; note, this was referred to as $H\gamma/C_{4170}$ in K13).

The model predictions for $\chi_{\text{flare}}$, $T_{BB}$, $H\gamma_{\text{EW}4170}$ are given for each NT electron flux in Table I. For the F13 simulation, the table gives the coarse time evolution of these parameters for the impulsive and gradual phases. The evolution of the flux spectrum for F13 is shown in Figure II. At early times, the F13 flux spectrum exhibits a large Balmer jump ratio and a cool color temperature in the blue-optical. However, from $t = 1.2\ s$ to $2.2\ s$, the continuum at $\lambda > 3646\text{Å}$ becomes bluer as the flux at $\lambda = 4300\text{Å}$ increases significantly while the flux at $\lambda \sim 6690\text{Å}$ increases only marginally. This results in an increase in the blue-optical color temperature, $T_{BB}$, of $2000\ K$. Also, the Balmer jump ratio decreases from $3.3$ to $2$ as the flux at $\lambda \sim 3550\text{Å}$ decreases while the flux at $\lambda = 4300\text{Å}$ increases; the continuum at wavelengths shorter than the Balmer edge also becomes bluer over this time interval as the flux at $\lambda < 3000\text{Å}$ increases while the flux at $\lambda \sim 3550\text{Å}$ decreases. In Section 3.3, we will explain how these changes in the flux spectrum are related to the increasing hydrogen b-f opacities in the atmosphere.

At $t = 2.2\ s$, the F13 simulation produces a flux spectrum that falls into the regime of the observations for the peak phase of dMe flares with a relatively small Balmer jump ratio, $\chi_{\text{flare}} = 2.0$, and a blue-optical color temperature of $T_{BB} = 9300\ K$. The $H\gamma_{\text{EW}4170}$ also falls within the range of the observed values at flare peak (cf. Figures 11–12 of K13). Blue-optical emission with a color temperature higher than $\sim 6000\ K$ has not previously been self-consistently reproduced in RHD models of dMe flares (K13). The F11 and F12 simulations do not produce properties that are consistent with observations of the impulsive phase: the Balmer jump ratios are too large ($> 7$) and the color temperatures are too low ($< 6000\ K$). From Table I it can be seen that the time-evolution of the characteristics of the F13 simulation is consistent with the general time-evolution observed in dMe flares: the Balmer jump and $H\gamma_{\text{EW}4170}$ decrease in the rise phase.

\footnote{The flux spectra for the F13 simulation are available upon request.}
and increase in the decay phase. The timescale of the burst (5 s), however, is much shorter than the timescales of most dMe flares (minutes to hours); adding up sequentially heated magnetic strands as in the spatial development of a flare arcade (e.g., Warren, 2006; Qiu et al., 2010) is outside the scope of this work, but we intend to pursue this in a future paper.

3.2. Dynamics and Energy Balance in the F13 Atmosphere

The evolution of the F13 flux spectrum in Figure 1 is tied to the dynamical evolution of the atmosphere. We discuss here general properties of the hydrodynamics.

The NT electrons initially deposit their energy over a broad extent, with significant volumetric heating from 275 km (log col mass = \(-2.1\) g cm\(^{-2}\)) to the top of the chromosphere at 450 km (log col mass = \(-4.05\) g cm\(^{-2}\)), and there is non-negligible NT electron energy deposition (300 erg s\(^{-1}\) cm\(^{-3}\)) directly in the corona. The maximum energy deposition from the NT electrons (\(8 \times 10^{15}\) erg s\(^{-1}\) cm\(^{-3}\)) initially occurs at \(z_o = 390\) km (log col mass = \(-3.05\) g cm\(^{-2}\)), which is 99% neutral but with a significant electron density \(n_e = 10^{12}\) cm\(^{-3}\). Most of the NT energy is deposited initially into NT hydrogen ionization at \(z_o\). As the electron density increases at \(z_o\) an increasing fraction of the NT energy goes into raising the temperature since heating of electrons occurs more efficiently than heating of neutrals (note, the thermalization of electrons, protons, and neutrals is assumed to happen instantaneously in these models).

Hydrogen becomes completely ionized by \(t = 0.002\) s and thermal ionization increases over NT ionization as the temperature exceeds 20,000 K. He I then becomes the primary cooling agent as the temperature reaches 40,000 K. At \(t = 0.003\) s, the temperature approaches 65,000 K and most helium at this height is now singly ionized. From \(t = 0.003\) s to \(t = 0.01\) s the He III fraction increases from 2% to 99.6% at \(z_X = 425\) km as the temperature rapidly increases to 140,000 K. At \(t = 0.016\) sec, a localized temperature maximum of 180,000 K forms at \(z_X\), and the thermal pressure generates upward velocities of 4 km s\(^{-1}\) and downward velocities of 2 km s\(^{-1}\). These mass motions are the seeds to the explosive condensation and evaporation which follow from runaway heating as material is evacuated from this region of the atmosphere.

Meanwhile at \(t = 0.016\) s, between \(z = 450\) and \(z = 460\) km (at the preflare transition region heights), there is a strong upward flow of 30 km s\(^{-1}\) resulting from the increase in temperature to 90,000 K of lower density material (previously the upper chromospheric material) between the site of maximum beam heating (425 km) and the transition region (450 km). At \(t = 0.2\) s, the evaporation consists of two fronts: a high density chromospheric front with maximum velocity of 125 km s\(^{-1}\) and a low density transition region front with maximum velocity of 250 km s\(^{-1}\). The downward front is a high density “chromospheric condensation” (CC), with a maximum velocity of 95 km s\(^{-1}\). The maximum densities of the chromospheric fronts have increased from \(n_H \sim 2 \times 10^{14}\) cm\(^{-3}\) (\(t = 0.2\) s) to \(n_H \sim 5 \times 10^{14}\) cm\(^{-3}\) (\(t = 0.4\) s) and the temperatures have decreased from 50,000 K to 37,000 K. At the location of \(z_X\), the temperature has reached nearly 10 MK by \(t = 0.4\) s from the runaway NT electron heating.
as the density has decreased (material is evacuated) from $n_H = 5 \times 10^{13}$ cm$^{-3}$ ($t = 0$ s) to $n_H = 6 \times 10^{12}$ cm$^{-3}$ ($t = 0.4$ s). At the upper and lower ends of the transition from very hot ($T \sim 400,000$ K – 10 MK) plasma to $T \sim 35,000$ K plasma, upward and downward propagating shock fronts are located at the very steep (nearly discontinuous) jumps in gas pressure, where the He III fraction changes from 100% to $< 30\%$.

At $t = 0.33$ s, the timesteps of the simulation become unmanageably small ($\Delta t \sim 10 – 100$ ns) due to the precision requirements of RADYN in calculating the population levels of He I and He II in regions of strong temperature gradient, especially at the upper evaporation fronts. At $t = 0.478$ s, we decrease the accuracy of the minor level populations, with accurate solutions provided only for level population densities that are $> 10^{-9}$ of the total elemental population density (initially, the parameter is $10^{-12}$), while the 2nd derivative of the grid weights is increased to weight more strongly toward the higher density (CC) front. The result is a dramatic increase in the time step size until $t = 5.1$ s as the low-density (upper) chromospheric evaporation front is smoothed over (the number of grid points here drastically decreases and the temperature is no longer maintained at 35,000 - 50,000 K). The results for the simulation at $z > 450$ km are thus less accurate, but this approximation is required in order to compute 1-D simulations with very steep and dynamic temperature and density gradients. This problem may ultimately find better resolution in 3-D modeling, since confinement of material to the vertical dimension is not imposed in the upper evaporation fronts. However, we focus only on the radiating material in the CC and below, since this higher density material produces most of the optical emission; the model above 450 km should not affect the analysis at the lower elevations.

Figure 2 summarizes the dynamical evolution of the lower atmosphere at $t = 2.2$ s, when maximum white-light emission is present. The top panel shows the velocity of the downward moving CC. At this time in the CC, the maximum hydrogen density is $7.6 \times 10^{15}$ cm$^{-3}$, or $17.4 \times 10^{-9}$ g cm$^{-3}$, which is over a factor of 12 greater than the hydrogen density at this height ($z = 255.4$ km) before the flare. This mass density is comparable to the preflare density at $z \sim 145$ km, which lies between the temperature minimum and the lower chromosphere. The evolution of the speed and temperature of the maximum density zone in the CC is shown in the bottom panels. By $t = 2.2$ s, the speed of the downflowing material has decreased to $\sim 50$ km s$^{-1}$ as it encounters denser material and higher gas pressure lower in the atmosphere, the equation of motion for which was derived analytically by [Fisher 1989]. Over the simulation, the temperature of the region of maximum density in the CC decreases steadily; this is a result of larger hydrogen emissivity for lower temperatures while increasing the density in the CC gives a larger volumetric cooling rate. Eventually the temperature nearly settles between 12,500 and 13,000 K at 2.2 s as continuous injection of NT electrons deposit energy in the CC. After the NT electron energy deposition decreases to 0 at $t = 2.3$ s, the CC continues to propagate downward, increasing

\[^3\]The speed of sound in the CC at $t = 0.4$ s is 30 km s$^{-1}$ whereas material is traveling at speeds nearly 100 km s$^{-1}$.\]
its density to $1.3 \times 10^{16} \text{ cm}^{-3}$ at $t = 4.0 \text{ s}$. The temperature drops to 7000 K at this time (when the NT energy input is cut off), which is much larger than the preflare temperature of $\sim 5000 \text{ K}$ at this height (196.5 km).

At $t = 2.2 \text{ s}$, the $E \sim 65 - 200 \text{ keV}$ NT electrons encounter a thick-target “wall” at the CC and produce a maximum energy deposition rate of $5 \times 10^{7} \text{ erg cm}^{-3} \text{ s}^{-1}$ (compared to a maximum of $< 10^{6} \text{ erg cm}^{-3} \text{ s}^{-1}$ initially) at $z = 256.9 \text{ km}$; the full width at half maximum (FWHM) of the energy deposition rate is only 0.46 km (compared to 75 km initially). The simulation becomes computationally problematic shortly after this time and therefore the NT heating rate is cut off at 2.3 s. The energy budget around the CC is shown in Figure 3, which gives the individual contributions to the change in total internal energy per unit mass. Positive quantities contribute toward heating, excitation, and ionization; negative quantities contribute toward cooling, recombination, and de-excitation. The lower end of the CC from Figure 2 at 239.4 km is indicated by a vertical dotted line in Figure 3. In the CC, the NT electron energy deposition is balanced by the thin and thick (detailed) radiative losses, resulting in a net decrease in the internal energy (recombination and cooling).

At deeper layers than the CC, $E \gtrsim 200 \text{ keV}$ NT electrons can still penetrate and heat the atmosphere. At $z \sim 200 \text{ km}$, there is net ionization and heating from both Balmer continuum backwarming (red diamonds) and NT electron energy deposition. At these depths, radiation from the optically thin loss function and from the detailed transitions does not counterbalance these sources of energy. There are also some contributions from compressional and viscous work where the CC impacts and “accretes” the stationary material below it.

### 3.3. Properties of Contribution Function for Continuum Emission in the Flare Atmosphere

The CC decreases in temperature and increases in density over time (Figure 2). By $t = 2.2 \text{ s}$ the CC has attained temperatures $>12,000 \text{ K}$ and increased in density over 12 times the preflare density, thus producing a large amount of radiative cooling from hydrogen and other elements (Figure 3). In this section, we explore the height dependent properties of the emergent intensity from the CC at three continuum wavelengths calculated in detail: $\lambda = 3550 \text{ Å}$ in the NUV, $\lambda = 4300 \text{ Å}$ in the blue-optical, and $\lambda = 6690 \text{ Å}$ in the red-optical, which are indicated on Figure 1. We use the contribution function to the emergent intensity $\frac{dI_\lambda}{dz}$ (Leenaarts et al., 2013) for a plane-parallel atmosphere:

$$ C_I = \frac{dI_\lambda}{dz} = \frac{\chi_\nu}{\mu} S_\nu e^{-\tau_\nu/\mu} \frac{e}{\lambda^2} $$

4The differential $dI_\lambda/dz$ in the definition of the contribution function is the same as the differential describing the variation of the specific intensity with height in the radiative transfer equation; however, these differentials are not equivalent.
where $\chi_\nu$ is the height-dependent extinction coefficient (cm$^{-1}$; not to be confused with $\chi_{\text{flare}}$, the Balmer jump ratio), $\tau_\nu$ is the optical depth ($\int_0^{10^9 \text{cm}} \chi_\nu dz$), and $S_\nu$ is the height-dependent source function. More simply, this equation is $j_\nu = \frac{\mu}{\mu_c}e^{-\tau_\nu}/\lambda^2$, or the plasma emissivity ($j_\nu$) at a given height and frequency multiplied by the attenuation of the radiation (determined by $e^{-\tau}$) as it propagates outward in the direction of $\mu$ from that height.

We discuss the atomic processes that contribute to the continuum emissivity in Section 3.4.

In Figure 4, we show the contribution function to the emergent vertical intensity ($\mu \sim 1$) for $\lambda = 3550\text{Å}$, $4300\text{Å}$, and $6690\text{Å}$ at $t = 2.2$ s. The temperature profile (dashed line) of the flare atmosphere is qualitatively similar to that of the quiescent atmosphere, except the flare chromosphere is hotter and has moved to higher column mass. Radiation escapes from two principal regions of the flare atmosphere: a stationary component (see Figure 2) produces a wide maximum in $C_I$ at $z \sim 200$ km in the mid/upper flare chromosphere below the CC, while the CC produces a very thin component peaking at $z \sim 256.3$ km with maxima in $C_I$ of 950, 140, and 120 erg cm$^{-2}$ s$^{-1}$ Å$^{-1}$ sr$^{-1}$ cm$^{-1}$ (extending above the top of the figure) for $\lambda = 3550$, 4300, and 6690 Å, respectively. Smaller values of $z$ (defined in Section 2) refer to larger physical depths, whereas larger values of $\Delta z$ refer to larger physical depth ranges in the atmosphere. We measure the FWHM of the narrow component of $C_I$ to compare the physical depth ranges ($\Delta z_{\text{FWHM}}$) over which radiation escapes from the CC. The $\Delta z_{\text{FWHM}}$ is the largest for $\lambda = 4300\text{Å}$: the $\Delta z_{\text{FWHM}}$ values are 1 km, 1.75 km, and 1.25 km for $\lambda = 3550$, 4300, and 6690 Å, respectively. The contribution function in the stationary component of the flare atmosphere is also markedly different among these three wavelengths. Whereas $\lambda = 4300\text{Å}$ has a local maximum at 200 km, and 20 – 25% of the emergent intensity originates from layers below the CC at $z < 230$ km, the other two wavelengths have very little emergent intensity originating from these depths, with $\lambda = 3550\text{Å}$ showing the least. The arrows on the top of the figure indicate the layers at which $\tau_\lambda = 1$ for each wavelength. The $\tau_\lambda = 1$ layer occurs in the CC for $\lambda = 3550$ and 6690 Å, and in the stationary flare layers for $\lambda = 4300$ Å. Therefore, the intensity at $\lambda = 4300$ Å originates over the largest physical depth range (largest $\Delta z$) and from the lowest atmospheric heights due to the smallest optical depth in the CC.

At times earlier than 2.2 s, a larger fraction of the emergent intensity at $\lambda = 3550$ Å and 6690 Å originates from the stationary component of the flare atmosphere because the CC has a lower density and optical depth. The evolution of the optical depth for $\lambda = 4300$ Å and 3550 Å and the evolution of the column densities for hydrogen in $n = 2$ and $n = 3$ in the CC (where $v < -5$ km s$^{-1}$) are shown in Figure 5. As the CC becomes denser and attains temperatures that excite hydrogen to $n = 2$ and $n = 3$, the probability of photoionization by light with these wavelengths increases. The values of the column densities at $t = 2.2$ s of $> 3 \times 10^{17}$ cm$^{-2}$ ($n = 2$) and $> 1 \times 10^{17}$ cm$^{-2}$ ($n = 3$) define useful

\footnote{$j$ is often used to describe the emissivity in units of photons whereas $\eta$ is used for the emissivity in units of ergs; here, we use $j$ to describe the emissivity in units of ergs.}
requirements for large optical depths ($\tau_{4300} = 0.8, \tau_{3550} = 5$) in any future flare model of CCs with material near temperatures of 12,000 K.

The evolution of the contribution function results from increasing, wavelength-dependent optical depths at optical/NUV wavelengths and can explain the evolution of the flux spectrum in Figure 1 (Section 3.1). The very large and continuously increasing optical depth at $\lambda = 3550$ Å causes a decrease in the flux at this wavelength between $t = 1.2$ s and $t = 2.2$ s because the radiation can only escape from a very thin layer. At $t = 2.2$ s, the radiation at $\lambda = 4300$ Å is still relatively optically thin in the CC ($\tau < 0.8$), and therefore has a higher probability of escape, giving rise to a larger FWHM of the contribution function in the CC ($\Delta z_{\text{FWHM}}$) and also a larger fraction that can escape from the stationary component of the flare atmosphere. The flux at 4300 Å increases from $t = 1.2$ s to 2.2 s in Figure 1 and results from the increasing density but a lower optical depth due to the lower photoionization cross section and smaller population density for $n = 3$ compared to $n = 2$. The larger optical depth at $\lambda = 6690$ Å relative to $\lambda = 4300$ Å is caused by a larger photoionization cross section (but same population density), which leads to a much smaller increase in flux at this wavelength in Figure 1 from $t = 1.2 - 2.2$ s. The NUV flux at $\lambda < 3000$ Å significantly increases over this time interval due to similar reasoning for the increase at $\lambda = 4300$ Å.

3.4. Opacity Components in the Optical Depth

3.4.1. The Optically Thick Hydrogen Recombination Flare Spectrum for the F13

In this section, we quantitatively demonstrate how the properties of the flux spectrum at $t = 2.2$ s in Figure 1 primarily arise from spontaneous hydrogen recombination continuum emission by reconstructing the contribution function to the emergent intensity from atomic emissivity processes. The b-f opacity ($\chi_{\nu}$) corrected for stimulated emission is given by Mihalas (1978) Equation 7-1:

$$\chi_{\nu} = \Sigma_i \left\{ [n_i - n_i^* e^{-h\nu/kT}] \sigma_i(\nu) \right\}$$

(2)

where $\sigma_i$ is the photoionization cross-section of an atom with an electron in level $i$ for light of frequency $\nu$; $n_i^* = n_e \times (n_i/n_e)^* \sqrt{1 - \frac{T}{T_{\text{LTE}}}}$ where $n_e$ is the number density of protons and $(n_i/n_e)^*$ is the density ratio in LTE which can be calculated using the Saha equation. We calculate the hydrogen b-f and hydrogen f-f (bremsstrahlung) opacity at $\lambda = 3550$ Å, 4300 Å, and 6690 Å for the flare atmosphere at $t = 2.2$ s. The total hydrogen continuum (b-f+f-f) opacity values for $\lambda = 4300$ and $3550$ Å are shown in the top left of Figure 6. The hydrogen b-f opacity dominates for both wavelengths, but the hydrogen f-f is non-negligible in the CC at optical wavelengths. At $\lambda = 4300$ Å, the Paschen b-f ($n = 3 \rightarrow \infty$) continuum opacity is significantly smaller than the Paschen b-f ($n = 2 \rightarrow \infty$) continuum opacity.

See Section 4.2 there are additional opacities from photoionization of the higher levels hydrogen, hydrogen free-free absorption, and opacity from the H$^{-}$ ion that can also contribute towards the optical depth at these wavelengths.
largest, whereas at $\lambda = 3550\text{Å}$ the Balmer b-f ($n = 2 \rightarrow \infty$) continuum opacity dominates. The b-f and f-f opacities from the H$^-$ ion are not shown explicitly in Figure [2] but are included in RADYN and in the contribution function (Figure [4]). The H$^-$ opacity is small ($\sim 3.5\%$) compared to the hydrogen b-f and f-f opacities in the CC but dominates at the lower temperatures ($T < 9000$ K) in the flare atmosphere at $z \lesssim 130$ km. Although the electron density is high in the CC, Thompson scattering is relatively very small.

Most of the CC is so dense that the LTE condition $S = B$ applies and the source function is only dependent on local quantities. The hydrogen b-f and f-f continuum emissivity, $j_\lambda$, is shown in the top right panel of Figure [6]. The hydrogen emissivity can be approximated as 

\[ \{\chi_{b-f} + \chi_{f-f}\} B_{\nu} \lambda^2 \]

through most of the CC, but to account for departures from LTE at the surface of the CC, we use the NLTE hydrogen b-f emissivity from Equation 7-2 of Mihalas (1978).

The ratio of emissivities ($j_\lambda = 3550 / j_\lambda = 4300$) in the CC is about 9.5, as we expect for hydrogen recombination (Balmer and Paschen) continuum emission at these temperatures. When $\tau$ is non-negligible, the wavelength dependent attenuation becomes important in the contribution function (Equation 1). The attenuation factor, $e^{-\tau_\lambda}$, is shown in the bottom left panel, and illustrates how the $\lambda = 3550\text{Å}$ emissivity is preferentially attenuated at $z = 180 - 200$ km, where $\tau_{3550} = 20$. Here, $\tau_{4300} \sim 1$ which is large enough to attenuate some of the radiation from these heights (compare the shift in peaks at this height indicated by vertical dashed lines in the top right and bottom right panels). The $e^{-\tau_\lambda}$ factor gives rise to the wavelength-dependent changes in the FWHM of $C_I$ ($\Delta z_{\text{FWHM}}$) and in the fraction of the emergent intensity that originates from the stationary component (Section 3.3 and Figure 4).

The contribution function, $C_I$, is shown again in the bottom right panel of Figure [6] (note logarithmic y-scale as compared to Figure 4 which is on a linear scale). Integrating $C_I$ over height gives the emergent intensity at $\mu = 0.95$. The integration over height reveals that the Balmer jump intensity ratio ($I_{\lambda=3550} / I_{\lambda=4300}$, similar to $\chi_{\text{flare}}$) is $\sim 2.2$, a value indicative of the small Balmer jump in emission in the flux spectrum in Figure 1. The emergent optical intensity ratio $I_{\lambda=4300} / I_{\lambda=6690}$ of 2.3 is a value consistent with the color temperature of a hot ($\sim 9000$ K) blackbody. We have thus shown that hydrogen recombination (and bremsstrahlung) continuum emissivity between $11,000 - 14,000$ K integrated over heights where optical depths become large ($\tau_{4300} \gtrsim 0.5$) and are wavelength dependent (from the hydrogen cross-section dependence and thermal population density variation) can self-consistently generate the color temperature ($T_{BB}$) of the blue-optical emission and the Balmer jump ratio ($\chi_{\text{flare}}$) in agreement with observations of the peak phases of dMe flares.

\[ \text{SOLA: kowalski_ms.tex; 25 March 2015; 2:21; p. 14} \]
The contribution functions for the F11 model at $t = 2.2$ s are shown in Figure 7(a,b). At this time, the atmosphere has not yet experienced explosive evaporation, which is consistent with the F11 simulation from Allred et al. (2006). In contrast to the F13 simulation, the $C_I$ profiles for $\lambda = 3550$Å and $4300$Å during the F11 simulation extend over a large portion of the flare chromosphere. The $C_I$ profiles also have second peaks near the preflare photosphere, which is not shown here because the important molecular species and opacities in M dwarf photospheres have not yet been included in RADYN (Section 2). The optical depths for $4300$Å and $3550$Å do not reach values of 1 until heights near the preflare photosphere; the values of $\tau_{3550}$ in the flare chromosphere range between 0.001 and 0.1, with $\tau_{4300}$ about an order of magnitude less. Therefore, the NUV and optical radiation from the flare chromosphere is dominated by optically thin hydrogen Balmer and Paschen continua emission from recombination due to the increased electron density (and temperature). The emergent F11 NUV and optical spectrum is a combination of mostly unattenuated hydrogen recombination emission from $z = 250 - 400$ km, combined with any increased photospheric radiation from backwarming (Allred et al., 2006). The value of $\chi_{\text{flare}}$ is large ($\sim 9$; Table 1) and $T_{\text{BB}}$ is low ($\sim 5300$ K; Table 1), as would be expected for hydrogen recombination at $T \sim 10,000$ K with negligible wavelength dependent attenuation. If we use the formula for spontaneous thermal hydrogen recombination emissivity (Equation 7-2 of Mihalas, 1978, see also Wiese, Kelleher, and Paquette (1972) and Aller (1963)) in Equation 1, we obtain an approximate formula for the ratio of the contribution function at two wavelengths, $\lambda_1$ and $\lambda_2$:

$$
\frac{C_I(\lambda_1)}{C_I(\lambda_2)} \approx \frac{\lambda_2^3 g_{bf,1} n_1}{\lambda_1^3 g_{bf,2} n_2} e^{\frac{\lambda_1}{T_{\text{e}}}(\frac{1}{n_1^2} - \frac{1}{n_2^2})} e^{1.578 \times 10^5 / T_{\text{e}}(\frac{1}{n_1^2} - \frac{1}{n_2^2})} e^{(\tau_2 - \tau_1) / \mu}
$$

(3)

at height $z$ with $(T_e, n_e)$, where $g_{bf}$ are the Gaunt factors and $n_1$ and $n_2$ are the principal quantum numbers that dominate the opacity in Equation 2 for $\lambda_1$ and $\lambda_2$, respectively. When $\tau_{1,2}$ are negligible (as for the F11 in the flare chromosphere), $C_I(\lambda_1 = 4300)/C_I(\lambda_2 = 6690)$ for $\mu = 1$ is $\sim 0.8$ at $T = 10000$ K and $e^{\tau_2 - \tau_1}$ is not much larger than 1, which causes an optically thin Paschen recombination continuum to exhibit a color temperature of $\sim 5000$ K (e.g., Section 9 of K13, Kunkel, 1970, Kerr and Fletcher, 2014). When $\tau_{1,2}$ are non-negligible (as for the F13 at $t = 2.2$ s; Section 3.4.1), the ratio of the attenuation factors is $e^{\tau_2 - \tau_1} > 1$ which causes the optical continuum to appear hotter. Therefore, increasing the optical depth makes the Paschen continuum bluer with a higher value of $T_{\text{BB}}$ (apparently hotter), as can be seen by approximating the atmosphere as a slab with constant $(T_e, n_e)$ and integrating over height. Note, the correction of the b-f opacity for stimulated emission (second term in Equation 2) also affects the color temperature because it decreases the opacity at $\lambda = 4300$Å by a smaller factor than at $\lambda = 6690$Å. However, the actual emergent intensity

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9The value $1.578 \times 10^5$ in the second exponential is $13.598 \text{ eV} / k_B$. 

SOLA: kowalski_ms.tex; 25 March 2015; 2:21; p. 15
is an integration over height with varying \((T_e, n_e)\), which is why we model the atmosphere gradients, which are apparent in Figure 2. The emergent radiative flux is a Gaussian integration of intensity over five \(\mu\) values: the intensity at a smaller value of \(\mu\) gives an even bluer optical continuum and smaller Balmer jump ratio due to the \(e^{(\tau_2-\tau_1)/\mu}\) factor.

4. Stark Broadening at the Balmer Edge

Stark broadening is thought to be an important symmetric broadening mechanism of the hydrogen Balmer lines during solar and stellar flares (Svestka, 1963; Worden et al., 1984; Hawley and Pettersen, 1991; Johns-Krull et al., 1997; Paulson et al., 2004; Allred et al., 2006), in addition to possible turbulent (Eason et al., 1992; Doyle et al., 1988) and thermal contributions. The linear (first order) Stark effect is the splitting of the degenerate orbital angular momentum \((l)\) states of each principal quantum state \((n)\) of hydrogen, due to a net electric microfield from the surrounding distribution of charges. To date, the most complete theories of the linear Stark effect are those of Vidal, Cooper, and Smith (1970, 1971, 1973; Seaton, 1990; Stohle and Jacquemod, 1993), and (Kepple and Griem, 1968), which take into account the perturbations from the slowly moving (quasi-static) protons and from the rapidly moving (dynamic) electrons. The Vidal et al. treatment is known as the unified theory because it provides an accurate treatment of the electron perturbations from the line core through the far wings. Theories of Stark broadening typically only present results for the lowest order Balmer transitions, but Lemke (1997) extended the results from the Vidal et al. unified theory to \(n = 22\). Therefore, the Vidal et al. unified theory (which is considered the best treatment of the non-overlapping far line wings and of the higher order Balmer lines; see discussion in Johns-Krull et al., 1997, and in Vidal, Cooper, and Smith (1970)), is most useful to the interpretation of flare spectra in the Balmer jump region.\(^{19}\)

In future work (Section 7), we will implement the Lemke (1997) extension of the Vidal et al. results (see Section 4.1). In RADYN, the analytic approximations of Sutton (1978) are used to add a Stark damping parameter in quadrature with the other damping parameters in the Voigt profile (Section 2). Solar modeling of infrared hydrogen lines has shown that this is adequate (Carlsson and Stein, 1992).

4.1. Occupational Probability of Level \(n\)

Here, we address the atomic physics necessary to interpret the merging of the rapidly converging higher order Balmer lines and the apparent lack of a Balmer discontinuity (or edge) at \(\lambda = 3646\) Å in flare spectra.

\(^{19}\) Bengston, Tannich, and Kenpöl (1970) provided results up to H12 using the theory of Kepple and Griem (1968) for one temperature and electron density, and Bengston et al. 1996 provided higher order line profiles for the study of Johns-Krull et al. (1997) via priv. communication, but these results have not been made publicly available to our knowledge.
To model the Stark broadening of the Balmer edge, we apply the occupational probability theory developed in Hummer and Mihalas (1988) and Dappen, Anderson, and Mihalas (1987), which is summarized thoroughly in the recent modeling study of cool DA white dwarf atmospheres of Tremblay and Bergeron (2009). This theory provides a thermodynamically self-consistent approach to pressure ionization, which allows the internal partition function to converge. Unlike procedures that truncate the internal partition function, occupational probability theory includes a continuous transition from bound to free states as upper levels of hydrogen converge and Stark energy shifts cause them to overlap. Electric field ionization experiments with sodium have shown that enhanced ionization rates of electrons in level \( n \) occur when there is a critical electric microfield strength causing the highest Stark state of \( n \) to cross with the lowest Stark state of \( n + 1 \) (Pillet et al., 1984). At a level crossing, electrons can undergo a Landau-Zener transition from \( n \) to \( n + 1 \) as the electric field fluctuates with time above and then below the critical value (Zener, 1932). For electric field perturbations that cause level crossing of \( n \) and \( n + 1 \), all states \( \geq n \) become dissolved or destroyed, and an electron excited to \( n \) can undergo a reverse-cascade to the continuum through a series of Landau-Zener transitions.

The occupational probability, \( w_{n} \), is the probability of an atom being perturbed by a net electric microfield that is below the critical field strength that would dissolve/destroy level \( n \):

\[
w_{n}(\text{charged}) = \int_{0}^{\beta_{\text{crit}}} P(\beta) \, d\beta
\]

where \( P(\beta) \) is the probability distribution of net electric microfield strength \( (\beta) \) and \( \beta_{\text{crit}} \) is the critical net electric microfield that dissolves/destroys level \( n \) (Seaton, 1990). Both the high frequency electron collisions and quasi-static proton collisions contribute to level destruction, but \( w_{n} \) is primarily determined from the ambient protons (Hummer and Mihalas, 1988; Tremblay and Bergeron, 2009). The ambient electrons contribute to the cascade+ionization process through their high-frequency collisions with the electrons that are excited to the dissolved states and through the mixing of Stark states allowing the reverse-cascade of Landau-Zener transitions to proceed (see Tremblay and Bergeron, 2009). Following Tremblay and Bergeron (2009), we adopt the Q-fit electric microfield model (Nayfonov et al., 1994) of the Hooper distribution to represent the probability distribution \( P(\beta) \). In our calculations, we use the total occupational probability obtained by multiplying Equation 4 by the probability of level destruction from neutral collisions, which are modeled using the same method described in Tremblay and Bergeron (2009) and Hummer and Mihalas (1988). For the conditions in flare atmospheres, the total occupational probability is approximately equal to \( w_{n}(\text{charged}) \), given by Equation 4.

4.2. Continuous Opacities at the b-f Edge

Dappen, Anderson, and Mihalas (1987) developed the method for incorporating the occupational probability theory into b-f and b-b opacities for hydrogen to
account for the effects of Landau-Zener transitions and the subsequent cascade to the continuum.

The Landau-Zener $b-b$ opacity for a Balmer line, $H_n$, is the nominal $b-b$ opacity multiplied by the occupational probability $w_n/w_{n=2}$, where $w_2 = 1$ for the conditions in flare atmospheres. The Landau-Zener $b-f$ opacity is the nominal $b-f$ opacity (Equation 2) extrapolated to wavelengths longer than the standard $b-f$ edge ($\lambda_o$), multiplied by the probability that an atom experiences a critical microfield strength or greater, causing the upper level of the transition to be dissolved. For any wavelength $\lambda > \lambda_o$, this probability is obtained by using a “pseudo” quantum number $n^*$:

$$n^* = \left(\frac{1}{n_i^2} - \frac{hc}{\lambda Z^2 E}\right)^{-1/2}$$

where $E = 13.598 \text{ eV} = 2.1787102 \times 10^{-11} \text{ erg}$ and $Z$ is the atomic number ($Z = 1$ for hydrogen). The dissolved probability (or dissolved fraction) is

$$D(\lambda) = 1 - \frac{w_{n^*}}{w_i}$$

where $w_{n^*}$ is the occupational probability for level $n^*$ and $i = 2$ for the Balmer lines. As discussed by Dappen, Anderson, and Mihalas (1987) the pseudo levels ($n^*$) are simple extrapolation tools between the limits of $w_n \sim 1$ and $w_n << 1$ that are useful for estimating the critical microfield strength (where the highest pseudo-stark state of $n^*$ and lowest of $n^* + 1$ overlap) that can cause level dissolution through a cascade of Landau-Zener transitions. Also, note, there are several different ways to estimate the critical field strength, and we use the popular method from Seaton (1990).

Now accounting for Landau-Zener transitions, wavelengths longer than the Balmer edge ($\lambda_o = 3646 \text{ Å}$) allow there to be a fraction of atoms in $n = 2$ that can be photoionized by $\lambda > 3646 \text{ Å}$, whereas without Landau-Zener transitions, photons with $\lambda > 3646 \text{ Å}$ could photoionize atoms only with electrons in $n > 2$. The total $b-f$ opacity, with and without Landau-Zener transitions, in a representative layer of the CC in the F13 model is shown in Figure 8. Due to the high ambient proton density ($5 \times 10^{15} \text{ cm}^{-3}$), the dissolved fraction is $>99\%$ (the occupational probability, $w_n$, is $<1\%$) for the upper levels that contribute opacity at $\lambda \lesssim 3730 \text{ Å}$. The nearly complete level dissolution results in a direct extrapolation of the Balmer $b-f$ opacity to these wavelengths. As $\lambda$ increases, the fraction of $n = 2$ hydrogen atoms that can be photoionized decreases, due to the larger critical microfield required to cause destruction of the upper level of an atomic transition. This leads to a continuous transition from the Balmer continuum opacity into the Paschen continuum opacity, which dominates at $\lambda > 4200 \text{ Å}$. The hydrogen $f-f$ opacity (without Landau-Zener transitions) is also shown in this figure, illustrating that it is small but non-negligible ($\chi_{f-f}/\chi_{\text{tot}} \sim 0.2$) at redder wavelengths.

Hereafter, we refer to these new opacities as Landau-Zener opacities, and the pseudo-continuum resulting from this opacity as the Landau-Zener continuum. Note, Dappen, Anderson, and Mihalas (1987) referred to this as the “dissolved
level pseudo-continuum." In Figure 9, we show how the Landau-Zener b-f opacity from \( \lambda = 3646 \, \text{Å} \) to \( \lambda = 4200 \, \text{Å} \) changes as a function of \( n_p (\approx n_e) \). Self-consistent atmospheric parameters were obtained from representative heights in the F11 and F13 models, at various times, and covering a range of proton densities, \( n_p = 10^{13} - 5 \times 10^{15} \, \text{cm}^{-3} \). As the ambient charge density increases, the Landau-Zener b-f opacity maximum shifts from \( \lambda = 3660 \, \text{Å} \) to \( \lambda = 3730 \, \text{Å} \). Also shown here are the occupational probabilities, \( w_n/w_2 \) (open circles, right axis, \( w_2 = 1 \)), that are multiplied by the nominal b-b opacity for each of the Balmer b-b transitions. Thus the opacity from the lines is transferred to the Landau-Zener continuum.

4.3. Landau-Zener Opacity Modeling Using the RH Code

We use the NLTE RH code \cite{Uitenbroek2001} to model the Landau-Zener b-f and b-b opacities and emissivities between \( \lambda = 3646 \, \text{Å} \) and \( \lambda = 5000 \, \text{Å} \) for the F13 model at \( t = 2.2 \, \text{s} \). The RH code is well-suited for modeling the Balmer jump region due to the ability to calculate overlapping transitions \cite{RybickiHummer1992} with more wavelength points than in the original dynamic (RADYN) simulation. By assuming statistical equilibrium and hydrogen number conservation, we can use an arbitrarily large hydrogen atom to model the merging of higher order lines near the Balmer b-f edge. Statistical equilibrium is a poor assumption during the early phases of the flare simulation, but at \( t = 2.2 \, \text{s} \), the resulting error is small\(^{11}\). As in RADYN, the linear Stark broadening in RH is treated with a Voigt profile and the approximate formulae from \cite{Sutton1978} are used; a prescription for quadratic Stark broadening is also included.

We obtain the atmospheric parameters (column mass, temperature, electron density, and velocity) at \( t = 2.2 \, \text{s} \) from the RADYN simulation with a 6-level hydrogen atom, and we calculate the spectrum with the RH code using a 20-level hydrogen atom. In the RH calculation, Ca II and He are treated as background (LTE) elements. At each depth in the atmosphere, the effect of Landau-Zener transitions is modeled by multiplying the Balmer b-f cross section by the dissolved fraction (Equation 6) and the Balmer b-b line profile by the occupational probability (\( w_n/w_2 \)), as in \cite{TremblayBergeron2009}. The NLTE opacity and emissivity is then calculated in RH with these modifications. We show the result of the RH flux calculation in Figure 11, compared to several observed flare spectra during the mid rise (S#27), late rise (S#28), and peak phases (S#31) of a large flare on the dM4.5e star YZ CMi from K13. The observed spectra have been normalized to the flux at \( \lambda = 4700 \, \text{Å} \) to show the range of properties at \( \lambda < 3900 \, \text{Å} \) and the lack of color temperature evolution at \( \lambda > 4000 \, \text{Å} \) (see K13 for the evolution of the flare flux without scaling). Because the flare area is unresolved, the continuum brightness is not constrained by observations. Thus, the RH model surface flux spectrum has been scaled by the factor \( R_{\text{flare}}^2/d^2 \) to match the \( \lambda = 4700 \, \text{Å} \) flux of each observed flare spectrum. An instrumental convolution with a Gaussian of FWHM = 13.5 Å has also been applied to the

\(^{11}\)We compared the RH and RADYN predictions at \( t = 2.2 \, \text{s} \) for the Balmer lines and continua that overlap, and we found satisfactory agreement.
model. We are not attempting to model this flare here, but we use these spectra to illustrate that the model spectrum characteristics are similar to the data at least at some phases of flare evolution.

First, the blue-optical color temperature between the Balmer lines at $\lambda > 4000\,\text{Å}$ is well-fit to a blackbody with a temperature of $T_{\text{BB}} = 10,400\,\text{K}$, which is shown in the figure as the dashed line. Note that this color temperature is larger by 1000 K compared to that in Table 1 due to the extension of the Landau-Zener continuum and Balmer wings between $\lambda = 4000\,\text{Å}$ and $\lambda = 4060\,\text{Å}$. Although the color temperature is represented by a blackbody with $T = 10,400\,\text{K}$, the F13 model radiative flux is larger by a factor of 1.7 compared to the surface flux of a blackbody with this temperature. The radiation temperature ($T_{\text{rad,}\mu=1,\lambda=4300} \sim 11,600\,\text{K}$) is also significantly larger than the color temperature ($T_{\text{BB}} = 9300 - 10,400\,\text{K}$). Hereafter, we refer to the hot blackbody component as the hot “blackbody-like” continuum component, since we have identified the emission as predominantly due to optically thick hydrogen (Paschen) recombination emission in Section 3.4.1. Second, a jump in flux occurs above an extrapolation of the blackbody to the bluest wavelengths, and this jump in flux is the same in both the F13 model and the observations of the late rise phase spectrum. Third, there is no Balmer discontinuity feature present in the model spectrum at $\lambda = 3646\,\text{Å}$: the model calculation produces a featureless Landau-Zener continuum connecting the Balmer continuum emission at $\lambda < 3646\,\text{Å}$ to the bluest significant Balmer line in emission, $\text{H}10\lambda3798$. This effect originates from the direct extrapolation of Balmer continuum opacity in Figure 8 to these wavelengths where the opacity is due to the upper levels of hydrogen that have nearly 100% probability of being dissolved.

The RH flux spectrum with the opacity effects from Landau-Zener transitions (and without instrumental convolution) is shown in Figure 11 around the Balmer edge region; no Balmer lines are at all apparent between $\lambda = 3646 - 3740\,\text{Å}$, and $\text{H}11$ is barely identifiable. The RH prediction using a 20-level hydrogen atom without the effects from Landau-Zener transitions (gray line, “no L-Z”) is shown and definitively illustrates that the overlap/merging of the Balmer wings indeed form a pseudo-continuum between the Balmer lines at wavelengths shorter than 4100Å, but less than the calculation with the opacity effects from Landau-Zener transitions (black line, “L-Z”). The two calculations disagree most concerning the appearance and nature of the pseudo-continuum between 3646Å and 3720Å. In particular, the “no L-Z” calculation exhibits a “blue continuum bump” at $\lambda \sim 3720\,\text{Å}$ and a large positive slope at wavelengths between 3646Å and 3720Å, whereas the “L-Z” calculation predicts a continuum with the same slope as the observed Balmer continuum at all $\lambda < 3720\,\text{Å}$. The “no L-Z” calculation also shows that the H10-H19 lines do not merge enough to conceal the Balmer discontinuity, which is just as apparent if we apply a large instrumental broadening. Using a hydrogen atom with more levels (we also tried a 35 level hydrogen atom) does not improve the modeling result: the opacity effects from Landau-Zener transitions are necessary to completely conceal the Balmer edge at these wavelengths.

12We use the same continuum “BW” windows and weighting as in [Kowalski et al. (2013)].
densities. As discussed by Dappen, Anderson, and Mihalas (1987), the slope of the pseudo-continuum in the “no L-Z” calculation between 3646 Å and the blue continuum bump is the result of the rapidly decreasing oscillator strength density of the higher-order transitions. In the “L-Z” calculation, Stark broadening decreases the oscillator strength density of the b-b transitions while replacing oscillator strength density with the Landau-Zener continuum.

In Figure 12 we show how the contribution function at \( \lambda = 3870 \) Å (between H8 \( \lambda 3889 \) and H9\( \lambda 3835 \)) and \( \lambda = 4300 \) Å compare with and without opacity effects from Landau-Zener transitions. As expected, the contribution at \( \lambda = 4300 \) Å does not change. For \( \lambda = 3870 \) Å without Landau-Zener transitions, the opacity is dominated by Paschen b-f opacity. This wavelength is more optically thin than \( \lambda = 4300 \) Å. Hence there is more contribution from the stationary component of the flare atmosphere at \( z = 150 - 230 \) km. In the case with Landau-Zener transitions, the Landau-Zener opacity increases the optical depth beyond that of the optical depth at 4300 Å (the contribution at \( z = 150 - 230 \) km is less than in the calculation without Landau-Zener transitions) and more \( \lambda = 3870 \) Å photons escape from higher in the atmosphere in the CC (at \( z = 251 \) km) due to recombination of electrons from the \( n^* = 8.3 \) pseudo-level (Equation 5) to the \( n = 2 \) level of hydrogen.

We can now associate the model flux spectrum in Figure 10 to several atomic emission processes. At \( \lambda < 3646 \) Å the flux is almost entirely Balmer continuum emission from free electrons recombining to \( n = 2 \). This part of the spectrum also exhibits a color temperature \( T_{BB} > 10,000 \) K, with a peak in the NUV between \( \lambda = 2500 - 2600 \) Å. The Landau-Zener continuum flux between \( \lambda = 3646 - 3760 \) Å is bona-fide Balmer continuum recombination radiation resulting from free electrons recombining from the \( n^* \) pseudo (dissolved) upper levels of hydrogen to \( n = 2 \) (Dappen, Anderson, and Mihalas, 1987); i.e., undergoing reverse Landau-Zener transitions from the continuum to \( n = n^* \), at which point they transition to \( n = 2 \). The flux between the rapidly converging Balmer lines from H10 to H\( \epsilon \) is mostly Landau-Zener continuum emission from electrons recombining from the dissolved upper levels to \( n = 2 \), but at these wavelengths there is a smaller fraction of atoms in the atmosphere that experience a critical field strength, and thus the Landau-Zener continuum emission is less than at \( \lambda = 3646 - 3760 \) Å. There is also some contribution from the far wings of the Balmer b-b transitions. The far wings of H\( \delta \) and H\( \epsilon \) and the redmost extent of the Landau-Zener continuum emission produce additional emission above the nominal Paschen recombination spectrum as red as \( \lambda = 4200 \) Å (compare solid and dotted red lines). At \( \lambda > 4200 \) Å, Paschen continuum emission from recombination of free electrons to \( n = 3 \) dominates, with a smaller (~20%) contribution from thermal bremsstrahlung (free-free) emission. Paschen recombination radiation also contributes toward the radiation at \( \lambda < 4200 \) Å but to a much smaller degree than the calculation without including the Landau-Zener continuum, as can be seen by the attenuation of radiation at \( z = 160 - 230 \) km in Figure 12. An interesting observational effect of including the Landau-Zener continuum emission is that the measured color temperature of the flux spectrum increases from 9300 K (6 level, without Landau-Zener transitions, dotted red line, Table 1) to 10 400 K (20 level, with Landau-Zener transitions, thick red line), improving further the
applicability of the F13 model to the measured $T_{BB}$ distribution at peak for a variety of flares (K13).

We note that the lower order Balmer lines (Hα-Hδ) can be used to constrain the models. The CC in the F13 model generates broad profiles, large surface fluxes, and a reverse decrement (more Hγ flux compared to Hα flux) in the lower order Balmer lines. The large widths of the lower order Balmer lines in Figure 10 result from Stark broadening. However, improvements to the implementation of the Stark effect (see Section 7) are required for a detailed comparison. Although the F13 model produces $\sim 18x$ the surface flux in Hγ compared to the F11 model (and $\sim 6x$ that in the F12 model), the ratio of Balmer line to continuum values is notably lower than in the smaller NT electron flux simulations (Table 1). At $t = 2.2$ s, the relative amount of Balmer line to continuum emission produced by the CC is consistent with typical observed values in the impulsive phase: $H_{\lambda EW 4170} \sim 20$ in the F13 model compared to $H_{\lambda EW 4170} \sim 15 - 30$ in the late rise and peak phase spectra in Figure 10. Additional flaring areas that are either impulsively heated with smaller NT electron flux or gradually decaying from a large electron flux may be required to more accurately reproduce the time evolution of the emission lines in these individual spectra.

4.4. A New Diagnostic of Ambient Charge Density in Chromospheric Flares

The opacity effects from Landau-Zener transitions result in Balmer recombination radiation longward of 3646 Å and explain the NUV/blue “pseudo-continuum” previously attributed to the merging of Stark-broadened Balmer lines. The occupational probability theory as employed in white dwarf atmospheric models provides a self-consistent quantum mechanical method of modeling the opacity effects from cascading Landau-Zener transitions. This new modeling explains the lack of an observed Balmer discontinuity (or edge) in flare spectra, while still allowing a Balmer jump in flux above an extrapolation of a blackbody that is fit to the blue-optical wavelengths in the Paschen continuum ($\lambda > 4000$ Å). Accounting for Landau-Zener transitions gives an emergent flux spectrum from the F13 model at $t = 2.2$ s with continuum emission bridging Balmer continuum emission at $\lambda \leq 3646$ Å to the highest-order Balmer line that is detectable, which is H10 or H11 in the case of the F13 model. The microscopic picture that is reinforced here is that slowly moving thermal electrons are able to recombine to $n = 2$ and produce a photon with wavelength >3646 Å (a Balmer continuum photon) due to the nearly complete dissolution of the upper levels: $1 - w_n / w_2 \geq 0.85$ for levels $n \geq 11$, in a dense plasma with $n_p = 5 \times 10^{15}$ cm$^{-3}$. The success of modeling the “pseudo-continuum” emission from $\lambda = 3646 - 3760$ Å in Figure 10 as a red wavelength extension of Balmer continuum emission is consistent with our conclusion that the white-light emission, in this RHD model, is dominated by a combination of hydrogen Balmer b-f, hydrogen Paschen b-f, and hydrogen f-f emission.

In solar flares, a “pseudo-continuum” component between the Balmer lines from $\sim 3700 - 3870$ Å was first noted by Zirin (1980) and Zirin and Neidig (1981). The current, best treatment of the modeling of the Balmer jump region and pseudo-continuum was applied to several solar flares by Donati-Falchi, Falciani, and Smaldone.
who used static, isothermal models with $n_e = 10^{13.2} - 10^{13.6} \text{ cm}^{-3}$ and $T_e = 7000 - 10,000$ K. These models produced a “blue continuum bump” at $\lambda = 3675\AA$, which shifts to redder wavelengths and increases in brightness for higher electron density, and has been discussed in light of a new solar flare observation with broad wavelength coverage spectra in the NUV (Kowalski, Cauzzi, and Fletcher 2014). The pseudo-continuum bump at $\lambda \sim 3720\AA$ in Figure 11 produced in our calculation without Landau-Zener opacity effects (Section 4.3) is very likely analogous to the NUV/blue bump feature at $\lambda \sim 3675\AA$ produced in the Donati-Falchi, Falciani, and Smaldone (1985) lower density slab models. As noted by these authors, this feature is due to the merging of Stark broadened high order Balmer lines. Another feature in the Donati et al. models is a continuum “dip” at 3646Å due to the smearing of the Balmer edge. Donati-Falchi, Falciani, and Smaldone (1985) truncated the model hydrogen atom at $n = 84$ and broadened the Balmer edge using the Stark width of the highest Balmer line. For $n = 84$, the Stark damping parameter is 30Å (Svestka 1963; Johns-Krull et al. 1997) which results in a flattening and smearing of the edge in their models. Since the Landau-Zener continuum modeling from occupational probability theory does not predict a continuum bump at $\lambda \sim 3720\AA$ or a dip near the Balmer edge (for very high proton densities), this modeling may also provide a more self-consistent explanation for the range in Balmer jump characteristics observed in solar flare spectra (as in the compilation by Neidig 1983), while also improving the diagnostic capabilities of these spectra.

Intuitively, occupational probability theory provides insight beyond a model that considers pseudo-continuum from merged lines as just a superposition of b-b opacities. In the classical (Donati et al.) framework, the opacity of a b-b transition is spread over a larger spectral range by the Stark broadening, and is then added to the opacity of other Stark-broadened b-b opacities at a given wavelength. In the occupational probability framework, the Stark-broadened b-b opacity is replaced by a larger b-f opacity at a given wavelength when hydrogen atoms experience a critical microfield perturbation. The larger b-f opacity results from the fact that free states are not degenerate (in orbital angular momentum), and therefore the oscillator strength density of the continuum is not re-distributed over a superposition of Stark states as for b-b transitions. This physical difference explains the large discrepancy between the two model spectra in Figure 11 from $\lambda = 3646 - 3730\AA$ (see also the discussion in Dappen, Anderson, and Mihalas 1987). In summary, the opacity sources from Donati-Falchi, Falciani, and Smaldone (1985) are continuum opacity from a Stark-broadened b-f edge and a superposition of Stark-broadened b-b opacities whereas in occupational probability theory, the opacity sources are continuum opacity from Stark-dissolved levels and a superposition of the remaining undissolved Stark-broadened b-b opacities.

Modeling the opacity effects from Landau-Zener transitions provides a new physical interpretation of several dMe flare spectra in the literature. We’ve already shown how this modeling with a very large density ($\sim 5 \times 10^{15} \text{ cm}^{-3}$) can account for the continuum and line properties in a flare with H10 or H11 as the bluest identifiable Balmer line (Figure 10). In the continuum peak (UT 1:15) spectrum from the dMe flare studied in García-Alvarez et al. (2002), H9 or
H10 also appear as the bluest Balmer line; however, their gasdynamic modeling using the Jevremovic et al. (1998) fit to the Balmer decrement indicates densities nearly two orders of magnitude lower than the occupational probability modeling indicates based on the bluest identifiable Balmer line. The bluest identifiable Balmer lines in most dMe flares with moderate spectral resolution are typically H15 and H16 both in the impulsive phase (Hawley and Pettersen, 1991) and in the gradual decay phase (Kowalski et al., 2010, 2013). Even \( R = 40,000 \) echelle spectra of dMe flares sometimes do not show emission in Balmer lines bluer than about H15 and instead exhibit a nearly featureless continuum with a color temperature of \( 9,000 \) – \( 11,000 \) K at \( \lambda \lesssim 3700\AA \) (Figures 2 and 5 of Fuhrmeister et al., 2008). It is also notable that one peak flare spectrum of a dMe flare showed Balmer lines as blue as H19 (Fuhrmeister et al., 2011), which the authors concluded was a result of only a small enhancement of the chromospheric electron density in the flare. Since the Landau-Zener continuum resulting from occupational probability theory can fully account for similar characteristics of this spectral region in the flare spectra in Figure 10, we suggest the range of characteristics observed around the Balmer jump in stellar flares may be explained with the range of Landau-Zener opacities in Figure 9.

Future modeling of particular flares using occupational probability theory can be used to infer the magnitude and gradient of the ambient charge (proton) density in the densest regions of the flare atmosphere that are significantly heated and produce white-light emission. In Section 4.3 and Figure 6 we showed how the Landau-Zener continuum opacity maximum changes and occupational probabilities dissolve the hydrogen Balmer transitions over 2.5 orders of magnitude in charge density. We present a simple method using the Balmer decrement to estimate the charge density for an optically thick flare atmosphere in LTE (as for the F13 atmosphere studied here). We introduce the Landau-Zener Balmer decrement, which is the flux in a Balmer line transition (e.g., H10 or H11 which are relatively unaffected by strong metallic or helium lines) that experiences a large change in the dissolved fraction of the upper level of this transition, divided by the flux of a lower order line that is not affected by level dissolution (e.g., H\(_\gamma\)). In Table 2 we show how the Landau-Zener Balmer decrement H11/H\(_\gamma\) changes for different temperatures and over the density range of \( n_p = 10^{13} \) to \( n_p = 5 \times 10^{15} \) cm\(^{-3}\). This Balmer decrement is obtained by multiplying \( w_{n=11}/w_{n=5} \) (where \( w_5 \approx 1 \)) by the Balmer decrement formula for the optically thick, isothermal, LTE limit in Appendix B of Drake and Ulrich (1980). The decrement sharply drops for \( n_p > 10^{15} \) cm\(^{-3}\) as \( n = 11 \) becomes dissolved (see also Figure 9). The observed decrements for H11/H\(_\gamma\) in the two peak spectra from Hawley and Pettersen (1991) are 0.26 and 0.22, which suggests a charge density of not much larger than \( 10^{15} \) cm\(^{-3}\) but also not smaller than \( 10^{14} \) cm\(^{-3}\). This method could be used as an alternative to the Inglis-Teller relation, which

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\(^{13}\)Aside from many faint metallic and helium emission lines.

\(^{14}\)Assuming that the Balmer line emission does not originate at a different spatial location, for example from a larger area with a lower density, than the continuum emission.

\(^{15}\)See the discussion in Hummer and Mihalas (1983) about how the theory of occupational probability differs from the Inglis-Teller relationship: the Landau-Zener transitions require fluctuations of the microfield around the critical field value.
requires data with high spectral resolution. However, since lines and continua are formed at different physical depths and depth ranges (as for the F13 atmosphere studied here), we plan to improve the accuracy of this spectral diagnostic using a grid of NLTE flare atmospheres in future work.

5. The Viability of the F13 Model

M dwarf flares are more energetic than solar flares (in total). Typical white-light energies of dMe flares are $10^{30} - 10^{32}$ erg [Hawley et al. 2014], with occasional events exceeding $10^{34}$ erg in the $U$-band alone [Kowalski et al. 2010]. The areal extent of flare footpoints cannot be measured directly [16] and hard X-ray emission is far too faint except during the largest dMe flares [Osten et al. 2010]. Although we cannot determine if NT electron fluxes on M dwarfs are indeed as high as $10^{13}$ erg cm$^{-2}$ s$^{-1}$, typical values of $\sim 10^{11}$ erg cm$^{-2}$ s$^{-1}$ in strong solar flares cannot explain the properties of white-light emission during dMe flares, and thus we infer that the energy flux is higher. The atmospheric response to an F13 NT electron burst provides insight into the radiative transfer processes that could explain white-light emission, but is this very high flux a realistic model for dMe flares? In this section, we evaluate its viability and conclude that some issues still remain.

First, is an F13 beam even possible during an M dwarf flare? The energy flux available to accelerate particles at the reconnection site is limited by the free magnetic energy in the corona, which can be estimated by $B_{\text{cor}}^2 v_{\text{inflow}} = 2 \times 10^{13}$ erg cm$^{-2}$ s$^{-1}$, where $v_{\text{inflow}}$ is the rate at which magnetic field flows into the reconnection region and may be between 0.01 - 0.1 times the Alfvén velocity. The energy flux requirement of $2 \times 10^{13}$ erg cm$^{-2}$ s$^{-1}$ into each magnetic footpoint assumes equipartition of energy among protons and electrons and that all free magnetic energy goes into accelerating particles. Using the larger estimate for inflow velocity, $0.1 \times v_{\text{Alfven}}$ gives $B_{\text{cor}} \sim 1.5$ kG as the coronal magnetic field (assuming $v_{\text{inflow}} = 0.01 v_{\text{Alfven}}$ gives 3.3 kG as the coronal magnetic field) necessary to supply this energy flux. Direct (Zeeman broadening) measurements of small-scale magnetic fields of M dwarfs reveal average photospheric magnetic flux, $B_{\text{phot}} f$, of $3 - 4.5$ kG [Johns-Krull and Valenti, 1996; Phan-Bao et al. 2009], but the coronal magnetic field above these starspots has not been directly measured. The most well-studied active corona is that of the dM3.5e star EV Lac [Osten et al. 2006], which exhibits a very hot (10 MK), confined ($L \sim 10^8$ cm), and dense ($n_e \sim 10^{13}$ cm$^{-3}$) component in quiescence. For these structures, the minimum magnetic field required for confinement is estimated to be $\sim 1$ kG. Other values from microwave data of flares and quiescence have suggested coronal magnetic fields on M dwarfs exceeding 1 kG [Osten et al. 2006; Smith, Güdel, and Audard, 2005], which have also been inferred from magnetic confinement estimates using the Haisch Simplified Analysis of EUVE

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[16]Emission model fits, e.g., with a blackbody function or RHD model spectra, give an indirect estimate of the flare area and energy flux requirements.
data of many M dwarf flares (Mullan et al., 2006). Thus, the energy requirement for accelerating an F13 NT electron flux requires relatively high (kilo-Gauss) coronal magnetic fields but not outside the realm of current measurements.

Although the energy flux requirements may be plausible (given large M dwarf coronal magnetic fields), the electron number flux for the F13 model may be problematic due to return current effects. The electron number flux corresponding to our double power law F13 spectrum is $10^{20}$ electrons cm$^{-2}$ s$^{-1}$. A return current electric field is established from the charge displacement of electrons and protons, given that both particles are energized equally in the process of magnetic reconnection and retraction. The initial energy lost per NT electron from the return current electric field is 13 keV Mm$^{-1}$ at the apex of our model loop (Equation 29 of Holman, 2012). The NT electrons with the cutoff energy (37 keV) lose all of their energy and are thermalized after penetrating only 3 Mm into the atmosphere, after which the flux of ongoing NT electrons decreases with distance as more NT electrons are thermalized. Holman (2012) derives the evolution of the NT electron flux distribution accounting for losses by the return current electric field. We use these results and find that only NT electrons with $E_{\text{initial}} > 85$ keV penetrate to the top of the chromosphere without thermalizing. Furthermore, the F13 NT electron density is comparable to the ambient density ($n_e = 2.6 \times 10^{10}$ cm$^{-3}$) at the apex of our model coronal loop; this condition would cause the ambient electrons to be accelerated and would result in instabilities and other effects that are not modeled here.

A more basic issue is that the number flux of electrons available in the corona flowing into the reconnection site is only $\sim 5 \times 10^{18}$ electrons cm$^{-2}$ s$^{-1}$ (using $n_e \times 0.1v_{\text{Alfven}}$, where $B = 1.5$ kG is obtained from the energy flux), whereas the number flux in the F13 beam is $10^{20}$ electrons cm$^{-2}$ s$^{-1}$. Even if electrons in the coronal loop were globally accelerated, there are only an additional $4 \times 10^{19}$ electrons cm$^{-2}$ s$^{-1}$ available. However, this simple assessment does not account for processes that may increase electron density during reconnection and retraction (e.g., Longcope and Guidoni, 2011). Furthermore, high densities between $10^{12} - 10^{13}$ cm$^{-3}$ have been inferred for some (non-flaring) coronal structures on the Sun (Feldman et al., 1994) and on the flare star EV Lac (Osten et al., 2006). In summary, the energy and number flux requirements for an F13 beam could be met under certain conditions (a dense corona with high magnetic field), but the return current losses would be significant.

A final issue with the F13 model is the short presence of the hot, blackbody-like emission, which appears only at the end of the impulsive phase between $t \sim 1.8$ s and $t = 2.3$ s. We expect a coadded burst spectrum to be more directly comparable to spectral observations, which have integration times of 10 s or more. The coadded spectrum over the 5.1 s duration of the impulsive heating and the decay phases (dot-dashed light blue line in Figure 1) exhibits a lower color temperature (7500 K) and larger Balmer jump ratio (2.7) than the instantaneous model spectrum at $t = 2.2$ s (Table 1). The coadded F13 spectrum is not as representative of the peak phases of the M dwarf flares as the instantaneous $t = 2.2$ s spectrum, which exhibits prominent $T_{BB} \sim 10^4$ K blackbody-like emission as observed. A longer duration of NT electron energy deposition for 5 – 10 s may
produce similar coadded and instantaneous spectra, but the simulation becomes very computationally intensive past 2.3 s with sustained heating (Section 3.2).

The coadded 5 s F13 spectrum is more representative of the atmospheric conditions in the gradual decay phase and the very early rise phase. The gradual decay phase of flares exhibits a Balmer jump ratio of $2.6 - 3.3$ and a blue-optical color temperature of about 8000 K (K13), which are similar to the observables obtained from the coadded F13 spectrum (Table 1). The coadded F13 spectrum is also consistent with a spectral observation from the very early rise phase of a flare from K13, which we show in Figure 13. K13 characterized two additional properties of flare spectra that can constrain models of the time-evolution: the slope in the NUV at $\lambda < 3646$ Å and the amount of deviation at red wavelengths (at $\lambda > 5000$ Å) from a blackbody fit to the blue-optical at $\lambda = 4000 - 4800$ Å. Although observations of both the early rise phase and gradual decay phase exhibit a Balmer jump ratio that are larger than during the peak phase, only the early rise exhibits a $T \sim 10,000$ K color temperature at $\lambda < 3646$ Å and a small deviation from the blackbody at redder wavelengths. These two common properties of the average burst model and the early rise phase observation can be seen in Figure 13. However, we expect that flare emission at any particular time (even in the very early rise phase) may consist of a superposition of many individual bursts. In order to conclude that any individual model can explain the time-evolution at any particular time, future modeling work should incorporate a treatment of the spatial evolution of flares, such as the development of two ribbon arcades and sequentially heated white-light kernels.

6. Summary and Conclusions

We have simulated the dynamic and radiative response of an M dwarf atmosphere subject to a short burst of a constant, high flux $(10^{13}$ erg cm$^{-2}$ s$^{-1})$ of NT electrons with a moderately high low-energy cutoff (37 keV). This simulation produces a heated ($T = 12,000 - 13,500$ K), high density ($n_H > 10^{15}$ cm$^{-3}$) “chromospheric condensation” with col $n_{H,n=2} > 3 \times 10^{17}$ cm$^{-2}$, col $n_{H,n=3} > 10^{17}$ cm$^{-2}$, and a total physical depth range of $\sim 20$ km, resulting in large hydrogen b-f and f-f opacities within a short amount of time (2.2 s) after the start of nonthermal electron heating. At this time, the “flare photosphere” ($\tau_{5000} = 1$) is located in the CC at a height that is nearly 250 km higher than the pre-flare photosphere, and the density of the CC is over an order of magnitude larger than the material at this height in the pre-flare. Significant heating to $T > 10,000$ K at high densities (comparable to the density in the very low chromosphere of the pre-flare) is achieved and maintained in the CC and also in the stationary, slightly less dense regions of the atmosphere $\sim 50$ km below the CC (Figures 2, 4).

The atmospheric response in the F13 model provides a new, accurate description of the radiation field in a flare atmosphere with a dense CC, and moreover, shows that such an atmosphere can produce bright white-light emission with properties that are in good agreement with the observations. The flux spectrum from the F13 at $t = 2.2$ s exhibits a blue-optical color temperature of
$T_{BB} \sim 10^4$ K between $\lambda = 4000 - 4800$ Å and a relatively small Balmer jump ratio ($\chi_{\text{flare}} \sim 2.0$; Table 1), both of which are large improvements compared to the range of these parameters in lower beam flux models (F10, F11, and F12 keeping all else the same with the NT electron spectrum). These properties describe typical peak spectra of dMe flares, and we showed the striking match of the instantaneous F13 model at $t = 2.2$ s to the observations of one large dMe flare with time-resolved spectra in the impulsive phase (Figure 10). The F13 model is the first RHD model to reproduce three generally observed characteristics ($T_{BB}$, $\chi_{\text{flare}}$, and $H_{\gamma}$ EW4170; Table 1) self-consistently.

The pioneering gasdynamic simulations of Livshits et al. (1981) and Katsova, Boiko, and Livshits (1997) produced a CC with similar characteristics to those in the F13 CC, and also $\Delta \tau \sim 1$ at $\lambda = 4500$ Å and $T \sim 9000$ K, albeit with $10 - 30\times$ lower NT electron energy fluxes. It is interesting that at least an order of magnitude difference in heating rates gives similar dynamic responses in gasdynamic modeling and radiative-hydrodynamic modeling. Our new radiative-hydrodynamic models use a more realistic starting dMe atmosphere with a NLTE treatment of the entire atmosphere (photosphere, chromosphere, transition region, and corona), a detailed treatment of the helium level populations (which are important for regulating the onset of explosive evaporation; see Section 3.2), and a modern parameterization of the NT electron spectrum from RHESSI data. However, a higher cutoff of the NT electron spectrum (as employed in our model) is expected to require a larger flux in order to produce similar dynamics (Fisher, 1989).

Finally, the RADYN and RH codes provide a detailed spectrum, which we have shown is important for critically testing the models against the observations, for revealing new atomic physics in flares around the Balmer jump, for constraining the charge density in the densest atmospheric regions that are heated during flares (Sections 4.3, 4.4), and for understanding how hydrogen recombination appears as a blackbody-like spectrum with $T \sim 10^4$ K (Section 3.4).

By analyzing the contribution function for the continuum intensity, we conclude that the values of the slope ($T_{BB}$) and Balmer jump ratio ($\chi_{\text{flare}}$) of the emergent optical and NUV continuum intensity in the F13 flare atmosphere are due primarily to hydrogen recombination radiation subject to wavelength dependent attenuation from Balmer and Paschen b-f opacities. Laboratory spectra of hydrogen at high density ($n \sim 10^{16}$ cm$^{-3}$) and temperatures of $T \sim 10^4$ K (Wiese, Kelleher, and Paquette, 1972) do not exhibit the blue-optical color temperature of a hot blackbody with $T \sim 10^4$ K. The necessary ingredient to build up optical depth at this density and temperature, and hence produce a $T \sim 10^4$ K blackbody-like spectrum with a small Balmer jump, is a large physical depth range ($\Delta z$) of emitting material along the line of sight. Thus, the color temperature in flare spectra does not directly relate to the atmospheric temperature so much as it is an indication of how the amount of emitting material along the line of sight varies as a function of wavelength (indeed, material must be heated to $T \sim 10^4$ K for hydrogen to be significantly populated out of the ground state into $n = 2$ and $n = 3$ and generate optical depth). In the CC of the F13 atmosphere, the physical depth range over which continuum emission escapes is $\Delta z_{\text{FWHM}} \sim 1 - 2$ km, which is not present in laboratory experiments. At the blue continuum wavelengths with the lowest opacity (e.g., at $\lambda = 4300$ Å),
the physical depth range over which continuum emission escapes is relatively large in the CC and extends to lower heights at $z \sim 200$ km (50 km below the CC). The amount of emitting material along a line of sight can also change due to variations in the density at the heights where continuum emission escapes; in the F13 flare atmosphere, the density is smaller at $z \sim 200$ km compared to the maximum density in the CC (Figure 2). Thus, the integration of the height-dependent properties (as for the contribution function in Figure 4) is important for determining the emergent intensity and flux spectra.

Another important result is that by applying the occupational probability formulae (Hummer and Mihalas, 1983; Dappen, Anderson, and Mihalas, 1987; Tremblay and Bergeron, 2009) to NLTE b-f and b-b Balmer opacities, we can explain the pseudo-continuum properties in the spectral region redward of the Balmer edge ($\lambda > 3646$ Å) and between the Balmer lines from $\lambda = 3750 - 3900$ Å. These formulae approximate how wavelengths redward of the Balmer jump experience Balmer b-f opacity in addition to Paschen b-f opacity (and f-f opacity); the atomic processes that allow for this additional Balmer continuum opacity are dissolved upper levels of hydrogen from Stark perturbations by ambient protons and Landau-Zener transitions of electrons between dissolved levels of hydrogen.

The amount of Balmer continuum emission redward of the Balmer jump and the amount of high order b-b Balmer emission vary inversely, and therefore the relative amounts of dissolved and undissolved components in spectra provide a new diagnostic of ambient charge density in the white-light emitting, partially ionized regions in the lower flare atmosphere (Section 4.4, Table 2).

The heating mechanism at high density in dMe flares is still unresolved since the CC required to produce the observed properties of the white-light emission results from a NT electron beam that would experience a considerable loss of energy from the return current electric field. The main results of this work are therefore the new interpretation of the blue-optical color temperature and Balmer jump ratio (as the variations in the physical depth range and optical depth as a function of wavelength) and of the apparent pseudo-continuum between the higher order Balmer lines (as Balmer continuum emission due to Landau-Zener transitions of electrons between dissolved levels). Combined with future modeling efforts using a more plausible heating mechanism (Section 7), these insights will allow the spectral observables from individual flares to directly constrain the atmospheric properties in the dense, lower atmosphere.

7. Future Modeling Improvements

We plan to develop a grid of short flare bursts using a larger low-energy cutoff of the NT electron spectrum, thereby reducing the number flux of NT electrons (and thus return current effects) while allowing a larger fraction of energy to penetrate deeper in the atmosphere. Including additional energy deposition mechanisms, such as NT proton energy deposition (e.g., a neutral beam simulation such as in Karlický et al. (2000) or a combination of energy deposition by other means (e.g., Fletcher and Hudson, 2008) may also be important for producing a similar atmospheric response with a smaller flux of NT electrons.
In addition, we plan to incorporate the spatial development of a flare arcade of sequentially heated footpoints (e.g., as in the observations of the Bastille Day solar flare from Qiu et al. (2010) or the cartoon two-ribbon model of the Megaflare event in Kowalski et al. (2012)) in order to reproduce the observed timescales of dMe flares, which are longer than the 5 s burst modeled here.

In future work with the RH code, we intend to improve the implementation of occupational probability theory and Stark broadening. Hubeny, Hummer, and Lanz (1994) fully extended the occupational probability formalism to NLTE, and we suggest that flare modeling codes developed in the future should similarly include occupational probability formalism in the non-equilibrium rate equation (following Equation 2.23 in Hubeny, Hummer, and Lanz, 1994) in addition to the NLTE b-b and b-f opacity and emissivity. Here, our goal was to demonstrate that large improvements in the NUV model spectrum result from implementing the simple modifications to the opacity and emissivity given in Dappen, Anderson, and Mihalas (1987) and Tremblay and Bergeron (2009). We will also follow the work of Tremblay and Bergeron (2009) and Seaton (1990) and renormalize the Stark profiles of Vidal, Cooper, and Smith (1973) using $\beta_{\text{crit}}$ as an integration limit, which leads to significantly narrower higher order Balmer lines (cf. Figure 5 of Tremblay and Bergeron, 2009). The theory of Vidal, Cooper, and Smith (1973) is only valid to first order when the line wings do not overlap, since the Landau-Zener continuum accounts for larger perturbations. Therefore, too much emission is probably produced between the Balmer lines between $\lambda = 3800 - 4000\AA$ in the model spectrum in Figure 11. Between $\lambda = 3646 - 3800\AA$, the low values of $w_n$ (that multiply into the b-b opacity) lead to only a small amount of blended Balmer line wings compared to the Landau-Zener continuum, so the error is not important. Finally, the linear Stark broadening theory of the Balmer lines using the analytic approximations of Sutton (1978) as Voigt damping parameters should be replaced by the full Stark profiles from Vidal, Cooper, and Smith (1973) and Lemke (1997) for the conditions present in flare atmospheres, in order to give accurate diagnostics on the electron density for unblended Balmer lines.

In future work with the RADYN code (Allred et al. 2015, in preparation), we intend to include the following improvements to the modeled physics: the cooling from Mg II and Fe II ions, an improved optically thin radiative loss function, nonthermal ionization of He I and He II, return current heating, magnetic mirroring of the NT electrons, and the fully relativistic Fokker-Planck solution to energy deposition and pitch angle scattering of NT electrons.
Table 1. Observables calculated from the model flux spectra. These values have been obtained without subtracting the preflare spectrum.

| Model (time step: phase) | $\chi_{\text{flare}}$ | $T_{\text{BB}}$ (K) | $H_{\gamma\text{EW}4170}$ |
|--------------------------|------------------------|----------------------|-----------------------------|
| F11 ($t = 2.2s$; impulsive peak) | 9.2 | 5300 | 150 |
| F12 ($t = 2.2s$; impulsive peak) | 7.4 | 5700 | 85 |
| F13 ($t = 0.2s$; impulsive early, rise 1) | 6.0 | 6300 | 70 |
| F13 ($t = 1.2s$; impulsive late, rise 2) | 3.3 | 7400 | 40 |
| F13 ($t = 2.2s$; impulsive peak) | 2.0 | 9000 | 20 |
| F13 ($t = 4s$; gradual decay) | 3.3 | 5400 | 60 |
| F13 (average burst) | 2.7 | 7500 | 30 |

Table 2. The Landau-Zener Balmer decrement ($H_{11}/H_{\gamma}$) in the optically thick, isothermal, LTE limit. Here, we give only the charged component of $w_n$ (Equation 4, averaged for the range of temperatures), ignoring the minor dependence on the destruction of levels from collisions with other neutrals.

| $n_p$ (cm$^{-3}$) | $w_{n=11}$ | $T = 10,000$ K | $T = 12,500$ K | $T = 15,000$ K | $T = 20,000$ K |
|------------------|------------|----------------|----------------|----------------|----------------|
| $10^{13}$        | 1.00       | 0.30           | 0.33           | 0.35           | 0.37           |
| $10^{14}$        | 0.98       | 0.30           | 0.32           | 0.34           | 0.37           |
| $5 \times 10^{14}$ | 0.85      | 0.26           | 0.28           | 0.30           | 0.32           |
| $10^{15}$        | 0.69       | 0.21           | 0.23           | 0.24           | 0.25           |
| $2.5 \times 10^{15}$ | 0.35      | 0.11           | 0.12           | 0.12           | 0.12           |
| $5 \times 10^{15}$ | 0.15       | 0.05           | 0.05           | 0.05           | 0.05           |
Figure 1. The evolution of the NUV and optical flux spectra during the F13 simulation. The observable parameters extracted from these spectra are given in Table 1. The coadded (average) burst spectrum is discussed in Section 5.
Figure 2. Top: The velocity field in the lower atmosphere at $t = 2.2$ s shows a chromospheric condensation (CC) with a large hydrogen and electron density exceeding $5 \times 10^{15}$ cm$^{-3}$. Bottom left: The time-dependence of the velocity in the highest density zone in the CC. Bottom right: The time-dependence of the temperature in the highest density zone in the CC. The velocity of the CC changes smoothly through the burst while the temperature reflects the cutoff of the NT electron energy deposition at $t = 2.3$ s.
Figure 3. Contributions to the energy balance for $t = 2.2\ s$ in the F13 simulation. These curves show the net amount for each component, and for the detailed radiation, the heating and cooling are integrated over the wavelengths of the transitions. The Balmer continuum (red diamonds) heating/cooling is shown as a separate curve from the total detailed radiation curve (which includes the Balmer continuum). The CC, with velocities less than 5 km s$^{-1}$, is material between $z = 239.4\ km$ (thick vertical dotted black line) and 257.4 km (thick vertical dash-dotted line).
Figure 4. The contribution to the emergent intensity for three continuum wavelengths in the NUV, blue, and red at $t = 2.2$ s compared to the temperature structure (for the same height range as in Figure 2). The column mass scale for $t = 2.2$ s is shown at the top axis. The arrows indicate where $\tau = 1$ occurs for each wavelength. The contribution functions in the CC ($z \sim 239 - 257$ km) are extremely thin and extend off the top of the plot (see text).
Figure 5. The evolution of column density for $n = 2$ and $n = 3$ of hydrogen (left axis, dashed lines) and optical depth (right axis, solid lines) for atmospheric heights in the CC with velocities $<-5 \text{ km} \text{ s}^{-1}$. At $t = 2.2 \text{ s}$, the CC has an extent of 18 km from 239 km to 257 km and has temperatures $>12,000 \text{ K}$. Approximately $3.5 \times 10^{12} \text{ erg cm}^{-2} \text{ s}^{-1}$ of NT electron energy flux is being deposited within the CC at this time.
Figure 6. The contribution function (bottom right) for 4300 Å and 3550 Å is broken up into three parts: opacity (top left), emissivity (top right), and the attenuation (bottom left). The ratios of hydrogen b-f opacity to total hydrogen (b-f and f-f) opacity are shown in the top left panel (right axis). In the stationary flare layers ($z \sim 190$ km), there is a large emissivity at $\lambda = 3550$ Å but also a very large attenuation due to photoionizations of hydrogen from $n = 2$ in the higher layers of the CC. The peaks of the emissivity and contribution function in the stationary layers for $\lambda = 4300$ Å are indicated by vertical dashed lines (see text).
Figure 7. The contribution function for the F11 model at $\mu = 0.95$ and $t = 2.2$ s compared to the (a) electron density and temperature and (b) optical depth. Only heights corresponding to the flare chromosphere, transition region, and lower corona are shown.
Figure 8. The hydrogen b-f continuum opacity accounting for Landau-Zener (L-Z) transitions, at $t = 2.2$ s in the F13 simulation, at a height of 255.4 km (log$_{10}$ col mass = $-2.57$ g cm$^{-2}$) which is the maximum of $C_2(\lambda = 3550) \Delta z \times C_f(\lambda = 4300) \Delta z$ where $\Delta z$ is the vertical extent of a grid cell. The values of $n_p$ and $T_e$ are respectively, $5.6 \times 10^{15}$ cm$^{-3}$ and 12,770 K. At this height, hydrogen is nearly 75% ionized. Also shown are the hydrogen f-f opacity and the hydrogen b-f opacity without accounting for opacity effects from Landau-Zener transitions at this height. At this layer, $\tau_{3550} = 1.1$ and $\tau_{4300} = 0.2$. 
Figure 9. The hydrogen b-f opacity accounting for Landau-Zener transitions at representative times and heights in the F11 and F13 simulations. Each curve corresponds to approximately the ambient charge density at each of the tick marks in the colorbar. The opacity curves have been scaled to a common value at $\lambda = 3600 \text{Å}$ to illustrate the wavelength shift of the opacity maximum compared to the Balmer b-f edge (vertical dashed line) without accounting for Landau-Zener transitions. The occupational probabilities (right axis) of the upper levels of the Balmer transitions are shown as circles (for reference H10 is at $\lambda \sim 3800 \text{Å}$).
Figure 10. RH calculation with a 20-level hydrogen atom and the b-f and b-b opacities including Landau-Zener transitions, during a snapshot of the F13 simulation at $t = 2.2\text{ s}$ (solid red). A $T = 10,400\text{ K}$ blackbody fit to continuum windows at $\lambda = 4000 - 4800\text{ Å}$ is also shown (dashed light blue), along with the RADYN prediction with a 6-level hydrogen atom (red dotted line). Instrumental broadening has been applied to the red spectrum. Several observations during the rise and peak phases of the IF3 flare of K13 are shown: peak spectrum is S#31 from K13 (dark gray), mid rise phase is S#27 from K13 (light gray), and late rise phase is S#28 from K13 (black). All spectra are scaled to the flux at $\lambda = 4690 - 4710\text{ Å}$. 
Figure 11. RH spectra of the Balmer jump region without instrumental convolution applied and with a magnified flux scale. Black line: Electric field effects on b-b opacities and Landau-Zener transition effects on b-b and b-f opacities from critical electric fields. H11 is the bluest identifiable Balmer line. Gray line: Electric field effects on b-b opacities and Landau-Zener transitions not included. H13 is the bluest identifiable Balmer line. Including Landau-Zener transitions in the b-b and b-f opacities predicts more continuum emission between 3646 Å and 3720 Å, whereas not accounting for Landau-Zener transitions produces a “blue continuum bump” at λ ∼ 3720 Å from the blending of Stark-broadened Balmer lines. Instrumental broadening is not shown; it broadens the Balmer lines but does not have a strong effect on the amount of continuum between the lines.
Figure 12. The contribution function at $\lambda = 3870\,\text{Å}$ and $4300\,\text{Å}$ accounting for opacity effects from Landau-Zener transitions (solid) compared to the original calculation (dashed). With L-Z transitions, the additional Balmer opacity at $\lambda = 3870\,\text{Å}$ causes more emission to escape from the CC and less from the stationary flare layers than without the L-Z transitions. The contribution function at $\lambda = 4300\,\text{Å}$ is not affected.
Figure 13. The coadded (average) model F13 spectra (from RADYN; instrumental broadening is not applied) over 5 s compared to the early rise spectrum (S#26) during the IF3 flare from K13. The coadded model spectrum has a larger Balmer jump and lower optical color temperature than the instantaneous model spectrum at $t = 2.2$ s in Figure 10. The flux at $\lambda < 3646\text{Å}$ increases towards bluer wavelengths in the model and observation. Note that the opacity effects from Landau-Zener transitions are not included here because statistical equilibrium is a poor approximation during the early time steps in the model (Section 4.3).

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