Static characteristics of a simple rotor-bearing system: modelling and experiment

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Abstract. The dynamics of rotating machinery have been extensively investigated during the past century. To date, longer rotating shafts, higher rotating velocity, higher fluid pressure, and generally high performances are required. For this purpose, a deep understanding and a more accurate modelling of the rotating machinery dynamical behaviour is needed, especially in terms of the resultant vibrations and stability issues. In this paper, a mathematical model was developed for a simple rigid rotor-bearing system using finite element method. Several natural frequencies and corresponding mode shapes of the systems were presented. In order to validate the model, some experimental tests were performed using a small test rig. The response of the system in the frequency domain and the direct orbit of the shaft were plotted as well.

1. Introduction

The dynamics of rotating machinery have been extensively investigated during the past century. To date, longer rotating shafts, higher rotating velocity, higher fluid pressure, and generally high performances are required. For this purpose, a deep understanding and a more accurate modelling of the rotating machinery dynamical behaviour is needed, especially in terms of the resultant vibrations and stability issues.

During the operation of a rotational mechanical system supported on hydrodynamic bearings, hydrodynamic pressure distribution is generated within the bearing oil film, which bears both weight and unbalanced forces of the rotor and has a direct influence on the dynamic behaviour of the rotor i.e., system stability, bending critical speed, vibration modes.

Many literatures can be found about identification methods of rotor-bearing dynamic parameters [1]-[4]. R. Tiwari [5] reported the development of algorithms for the simultaneous identification of both the unbalance and the bearing dynamic parameters; the proposed algorithms were compared against experimental data. Ref. [6] proposed a technique known as algebraic identification, which allows the development of identifiers aimed to determination on-line of unknown parameters, based on both the differential algebra and the operational calculus from the system mathematical model. Refs [7] proposed identifiers using algebraic identification to estimate the rotor dynamic system parameters; both numerical and experimental results showed that the identification was obtained in a short time and exhibits high robustness before parametric uncertainty. One of the main advantages of the algebraic identification is to provide identification relationships completely independent from the
initial conditions of the system since only requires the system response signal. The estimation of the parameters is carried out online in either continuous or discrete time. In this paper, a mathematical model was developed for a simple rigid rotor-bearing system using finite element method. Several natural frequencies and corresponding mode shapes of the systems were presented. In order to validate the model, some experimental tests were performed using a small test rig. The response of the system in the frequency domain and the direct orbit of the shaft were plotted as well.

2. The mathematical model

It is well-known that the dynamic behaviour of rotating machines is strongly influenced by the bearings properties since the stiffness of the rotor-bearing system is the result of the bearing stiffness and the shaft stiffness addition. Likewise, the damping of the system is mainly the result of the bearing damping properties. A simple rigid rotor-bearing system is shown in Figure 1. The rotor is supported on two different flexible bearings.

![Rotor-bearing system](image)

Figure 1. Rotor-bearing system.

The finite element method was used for the numerical simulation with 4 degree of freedoms (DOFs) for one node i.e., radial displacement \((u, w)\) and angular displacement \((\theta, \psi)\) in \(y\) and \(z\) direction (see Figure 2). Considering \(j\)-th element, the displacement respond vector as follow:

\[
x_j = \begin{bmatrix} u_j \\ \theta_{sj} \\ \psi_{zj} \\ u_{j+1} \\ \theta_{sj+1} \\ \psi_{zj+1} \end{bmatrix}
\]

(1)

![Numerical model for rotor-bearing system](image)

Figure 2. Numerical model for rotor-bearing system.

The mathematical model corresponding to the two DOFs rotor-bearing system showed in, considering constant frequency is given by:

\[
[M]\ddot{x} + ([C] + \Omega[G])\dot{x} + [K]x = W + F(t)
\]

(2)
where $\Omega$ is the shaft rotational speed; $W$ is rotor weight and $F$ is external force; $[M]$ and $[M_s]$ are mass and secondary effect of rotatory inertia matrix; $[C]$ is damping matrix; $[G]$ is gyroscopic matrix and $[K]$ is stiffness matrix, with:

$$[M]_{j\alpha\beta} = \frac{\rho S_m L}{420}
\begin{bmatrix}
156 & 0 & 0 & -22L & 54 & 0 & 0 & 13L \\
0 & 156 & 22L & 0 & 0 & 54 & -13L & 0 \\
0 & 22L & 4L^2 & 0 & 0 & 13L & -3L^2 & 0 \\
-22L & 0 & 0 & 4L^2 & -13L & 0 & 0 & -3L^2 \\
54 & 0 & 0 & -13L & 156 & 0 & 0 & 22L \\
0 & 54 & 13L & 0 & 0 & 156 & -22L & 0 \\
0 & -13L & -3L^2 & 0 & 0 & -22L & 4L^2 & 0 \\
13L & 0 & 0 & -3L^2 & 22L & 0 & 0 & 4L^2
\end{bmatrix}$$

(3)

$$[G]_{j\alpha\beta} = \frac{\rho I}{15L}
\begin{bmatrix}
0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\
36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\
3L & 0 & 0 & -4L^2 & -3L & 0 & 0 & L^2 \\
0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\
0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\
-36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\
3L & 0 & 0 & L^2 & -3L & 0 & 0 & -4L^2 \\
0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0
\end{bmatrix}$$

(4)

$$[K]_{j\alpha\beta} = \frac{EI_x}{(1+\alpha)L^3}
\begin{bmatrix}
12 & 0 & 0 & -6L & -12 & 0 & 0 & -6L \\
0 & 12 & 6L & 0 & 0 & -12 & 6L & 0 \\
0 & 6L & (4+\alpha)L^2 & 0 & 0 & -6L & (2-\alpha)L^2 & 0 \\
-6L & 0 & 0 & (4+\alpha)L^2 & 6L & 0 & 0 & (2-\alpha)L^2 \\
-12 & 0 & 0 & 6L & 12 & 0 & 0 & 6L \\
0 & -12 & -6L & 0 & 0 & 12 & -6L & 0 \\
0 & 6L & (2-\alpha)L^2 & 0 & 0 & -6L & (4+\alpha)L^2 & 0 \\
-6L & 0 & 0 & (2-\alpha)L^2 & 6L & 0 & 0 & (4+\alpha)L^2
\end{bmatrix}$$

(5)

where $\rho$, $S_m$, $L$, $E$, $I_x$ and $\alpha$ are the density, cross-section, length, Young’s modulus, moment of inertia of the cross section and shear coefficient of the $j$-th element.

3. Experiment setup

Figure 3 presents an overview of the experimental test-rig using for the experimental tests. Basically, it is encompassed by a flexible rotor-bearing system. In this test rig, a rigid shaft of 10 mm diameter which is supported by two rolling element bearings at the ends of shaft is driven by an electric motor through a flexible coupling. The coupling is modified from a normal flexible cross coupling in order to minimize the coupling constraints on the rotor lateral movement. The DC motor can run at a maximum speed of 5000 rpm. One or two disks with a diameter and thickness of 75 mm and 25 mm respectively can be installed on the shaft. Two proximity probes of 90° phase angle are installed at each bearing position in the horizontal, named as P1X and P2X, and vertical direction, named as P1Y and P2Y, to measure the vibration of the shaft (see Figure 4). One additional proximity probe is used as a key phasor for tacho signal and synchronous averaging technique [9]-[10].
In this study, the shaft was run-up gradually from 250 rpm up to 4000 rpm first. Next, it was remained at the 4000 rpm about 20 second after run-down slowly to 250 rpm. During the test, signal from 4 proximity probes were acquired with the sampling frequency of 10 kHz.

![Figure 3. Photo of the test rig using for the experiment.](image)

![Figure 4. Installed proximity sensors for vibration measurement [8].](image)

4. Results

In the present section, the natural frequencies and mode shapes of the system obtained from the numerical simulation have been presented. In order to validate the model, some experimental tests were performed using a small test rig which were described in the previous section. With no running and neglecting the weight of rotor, Eq. (2) can be expressed as:

\[
[M]x + [K]x = 0
\]

By re-written Eq. (6), we have:

\[
-[M]^{-1}[K]x = \omega^2 x
\]

Then, the natural frequencies can be obtained by Eq. (7)

\[
\omega = \sqrt{[M]^{-1}[K]}
\]

First-six natural frequencies of the system are listed in Table 1.

| Mode | #1     | #2     | #3     | #4     | #5     | #6     |
|------|--------|--------|--------|--------|--------|--------|
| Natural frequency (Hz) | 25,8078 | 25,8458 | 110,4473 | 110,7843 | 347,1279 | 357,4829 |
Figure 5 shows the 3D mode shape of the rotor-bearing system. Because of the sake of simplicity, only mode shapes of the system at the first and third natural frequency were plotted. It can be seen that the maximum displacement of the shaft at mode #1 and mode #3 is approximately 200 μm and 100 μm, respectively.

Figure 6 shows the experimental signals of two proximity sensors in the frequency domain when the shaft was run-up from 250 rpm to 4000 rpm. This plot is obtained by performing the Fast Fourier Transform (FFT) technique to change to the signal from the time domain to the frequency domain. It is possible to see from the Figure 6 that the first and second nature frequency of the system are approximately 23 Hz and 108 Hz. These results show a good agreement with the simulation values. This identification is very important to avoid the resonance of the system. In order to obtain exactly the natural frequency of the system, the impact test is usually performed.

Figure 7 shows the orbit the shaft plotted from proximity probes in the non-driven end (NDE) bearing and driven end (DE) bearing at the constant speed of 4000 rpm. It is clear that the orbit at the DE bearing has irregular shape. This fact is probably due to the flexibility of the system. At the NDE position, the orbit has a quite smooth circle shape.

Figure 8 shows the experimental 1X orbit of the shaft at two bearing positions with an elliptical form. These orbits correspond to the vibration of shaft at a frequency of 66.7 Hz (about 4000 rpm). It should be noted that the vibration amplitude at the DE position is nearly two times compared to the NDE position.
5. Conclusions
This paper developed a mathematical model for a simple rigid rotor-bearing system using finite element method. Several natural frequencies and corresponding mode shapes of the systems were presented. In order to validate the model, some experimental tests were performed using a small suitable test rig. The response of the system in the frequency domain and the direct orbit and 1X orbit of the shaft were plotted as well.

6. References
[1] Chatterton S, Pennacchi P, Dang P.V and Vania A 2014. A test rig for evaluating tilting-pad journal bearing characteristics. *Mechanisms and Machine Science* 21, 921-930.
[2] Dang P.V, Chatterton S, Pennacchi P and Vania A 2016. Effect of the load direction on non-nominal five-pad tilting-pad journal bearings. *Tribology International*. 98, 197-211.
[3] Dang P.V, Chatterton S, Pennacchi P and Vania A 2018. Numerical investigation of the effect of manufacturing errors in pads on the behaviour of tilting-pad journal bearings. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*. 232, 480-500
[4] Chatterton S, Pennacchi P, Vania A, Luca A.D and Dang P.V 2019. Tribo-design of lubricants for power loss reduction in the oil-film bearings of a process industry machine: Modelling and experimental tests. *Tribology International* 130, 133-145.
[5] Tiwari R, Chakravarthy V 2009. Simultaneous Estimation of the Residual Unbalance and Bearing Dynamic Parameters from the Experimental Data in a Rotor-Bearing System. *Mechanism and Machine Theory* **44**, 792-812.

[6] Fliess M, Hebertt R 2003. An Algebraic Framework for Linear Identification. *ESAIM: Control, Optimisation and Calculus of Variations* **9**, 151-168.

[7] Manuel A, Carvajal B, Navarro S 2014. On Line Algebraic Identification of Eccentricity Parameters in Active Rotor-Bearing Systems. *International Journal of Mechanical Sciences* **85**, 152-159.

[8] Pham N.Q.H, Dang P.V, Ngo T.N, Tran P.T 2019. Design of a small-scale test rig for rotating machinery characterization. *Lecture Notes in Networks and Systems* **63**, 229-235.

[9] Chatterton S, Dang P.V, Pennacchi P, Luca A.D and Flumian F 2017. Experimental evidence of a two-axial groove hydrodynamic journal bearing under severe operation conditions. *Tribology International* **109**, 416-427.

[10] Dang P.V, Chatterton S, Pennacchi P 2019. The Effect of the Pivot Stiffness on the Performances of Five-Pad Tilting Pad Bearings. *Lubricants* **7**, 61.

**Acknowledgement**

This research is funded by Funds for Science and Technology Development of the University of Danang under project number B2019-DN02-67.