Measuring the cosmological parameters from cosmic microwave background and gravitational wave observations

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We constrain the cosmological parameters from cosmic microwave background (CMB) and gravitational wave observations. We not only combine LIGO observations with CMB to measure cosmological parameters, but also forecast the potential abilities of the LISA detector and PTA projects. In the $\Lambda$CDM+$r$ model, the constraint from BK15 and FAST projects have a significant impact on tensor-to-scalar ratio.

Since the discovery of temperature anisotropies in the cosmic microwave background, it has become significant to study cosmology and physics of early Universe. The CMB power spectrum indicates cosmological evolution information which can measure the cosmological parameters well. The polarization of CMB from tensor perturbation encodes the information of primordial gravitational waves.

Gravitational waves provide another way to probe cosmological evolution. After LIGO Science Collaboration discovered the first direct detection of gravitational wave from the coalescence of binary black holes [1], many experiments are prepared to measure gravitational wave in a wide range of frequencies. To achieve a better constraint on cosmological parameters, observations should be combined at different frequency bands.

In this paper, we consider the constraints from cosmic microwave background and gravitational wave observations including Planck satellite [2], BICEP2 and Keck array through 2015 reason (BK15) [3], Baryon Acoustic Oscillation (BAO) [4–6], LIGO detector [7, 8], LISA detector and Pulsar timing array (PTA) projects which is necessary in theory and in experiments.

The detection ability for gravitational wave detectors are characterized with signal-to-noise ratio (SNR). For advanced LIGO detectors, SNR is given by [9]

$$\rho = \sqrt{T} \left[ \int df \left( \frac{\Omega_{gw}}{\Omega_n} \right)^2 \right]^{1/2},$$

where $T$ is the observation time, $P_{nI}$ and $P_{nJ}$ are the auto power spectral densities for noise in detectors $I$ and $J$, $S_n(f)$ is the strain power spectral density of a stochastic gravitational wave background. For an autocorrelation measurement in the LISA detector, SNR can be calculated by [9, 10]

$$\rho = \sqrt{T} \left[ \int df \left( \frac{\Omega_{gw}}{\Omega_n} \right)^2 \right]^{1/2},$$

where $\Omega_n$ is related to the strain noise power spectral density $S_n$. For a PTA measurement, SNR can be obtained by

$$\rho = \sqrt{\frac{2}{T}} \left[ \sum_{I,J} \chi_{IJ}^2 \right]^{1/2} \left( \int df \left( \frac{\Omega_{GW}(f)}{\Omega_n(f) + \Omega_{GW}(f)} \right)^2 \right)^{1/2},$$

where $\chi_{IJ}$ is the Hellings and Downs coefficient for pulsars $I$ and $J$ [11]. We consider two PTA projects, namely IPTA [12] and FAST [13], and make the same assumptions for these PTAs as presented in a previous study [14].

In order to characterize the spectral properties of stochastic gravitational wave background, the energy distribution in frequency is defined as follows

$$\Omega_{gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{df} = \frac{2\pi^2}{3H_0^2} f^3 S_h(f).$$

Gravitational wave fractional energy density per logarithmic wavenumber interval today is given by [15]

$$\Omega_{gw} \approx \frac{15 \Omega_m^2 A_s r}{16 H_0^2 \eta_0 k_{eq}^3} \left( \frac{k}{k_s} \right)^{n_t},$$

where $\Omega_m$ is the mass density, $H_0$ is the Hubble constant, $\eta_0 = 1.41 \times 10^9$ Mpc denotes the conformal time today and $k_{eq} = 0.0730 \Omega_m h^2$ Mpc$^{-1}$ denotes the wavenumber when matter-radiation equality, $k_s = 0.05$ Mpc$^{-1}$ denotes the pivot scale, $n_t$ is the tensor spectral index. The parameter $r$, called tensor-to-scalar ratio, quantify the tensor amplitude $A_t$ compared to the scalar amplitude $A_s$ at the pivot scale

$$r \equiv \frac{A_t}{A_s}.$$
about measuring the tilt of primordial gravitational-wave power spectrum from observations [16]. Some relevant works have also been reported in [17–20].

We use the publicly available codes CosmicMC [21] to constrain the cosmological parameters. In the standard ΛCDM model, the six parameters are the baryon density parameter $\Omega_b h^2$, the cold dark matter density $\Omega_c h^2$, the angular size of the horizon at the last scattering surface $\theta_{\text{MC}}$, the optical depth $\tau$, the scalar amplitude $A_s$, and the scalar spectral index $n_s$. We extend this model by adding the parameter $r$ and consider these seven parameters as fully free parameters. The consistency relation is ignored for more inflation models. Our results are given in Table I and Fig. 1.

In the ΛCDM+$r$ model, the Planck18+BAO datasets constrain the standard six parameters accurately, BK15 and gravitational wave observations effect them slightly. However, the constraints on tensor-to-scalar ratio are improved obviously. The constraint on parameter $r$ from Planck18+BAO datasets is given by

$$r < 0.212 \quad (95\% \text{ C.L.}) \quad (7)$$

Combining Planck18+BAO with BK15, the constraint on parameter $r$ becomes

$$r < 0.075 \quad (95\% \text{ C.L.}) \quad (8)$$

The constraint on tensor-to-scalar ratio is significantly improved by considering the BK15 data.

The abilities of gravitational wave observations measuring the cosmological parameters are also tested in this paper. First, we consider LIGO data and combine Planck18+BAO+BK15 with LIGO to measure these cosmological parameters. The constraint on parameter $r$ at 95% C.L. is

$$r < 0.076 \quad (95\% \text{ C.L.}) \quad (9)$$

Then, we assume that stochastic gravitational wave background cannot be detected by future gravitational wave observations, such as LISA, IPTA or FAST, and observe how these data will improve the constraint on cosmological parameters. The constraint on parameter $r$ becomes

$$r < 0.075 \quad (95\% \text{ C.L.}) \quad (10)$$

from Planck18+BAO+BK15+LISA datasets;

$$r < 0.075 \quad (95\% \text{ C.L.}) \quad (11)$$

from Planck18+BAO+BK15+IPTA datasets;

$$r < 0.049 \quad (95\% \text{ C.L.}) \quad (12)$$

from Planck18+BAO+BK15+FAST datasets; FAST project presents better constraint on the tensor-to-scalar ratio which is obvious in Table I and Fig. 1.

In summary, we constrain the cosmological parameters from CMB experiments and gravitational wave observations. We not only combine LIGO observations with CMB to measure cosmological parameters, but also forecast the potential abilities of the LISA detector and PTA projects. We find that BK15 can significantly improve the constraint on tensor-to-scalar ratio. Combining Planck18+BAO+BK15 with FAST project, the constraint on tensor-to-scalar ratio becomes better.

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FIG. 1: The contour plots and the likelihood distributions for the cosmological parameters in the $\Lambda$CDM+$r$ model at the 68% and 95% CL from the combinations of Planck18+BAO, Planck18+BAO+BK15, Planck18+BAO+BK15+LIGO, Planck18+BAO+BK15+LISA, Planck18+BAO+BK15+IPTA and Planck18+BAO+BK15+FAST, respectively.

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