Gravitational waves from gauge-invariant perturbations of spherically symmetric spacetimes

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Abstract. One difficulty associated with perturbations of spherical collapse models in General Relativity is attributed to the junction conditions required at the interface of the interior matter-filled region and the exterior vacuum region. This implies extracting information about gravitational waves at spacelike infinity is also a difficult task. In this talk, I present a method which eliminates the need for junction conditions in both the background and perturbed spacetimes, thereby allowing relatively simple modelling of gravitational waveforms. This is achieved by using a recently developed method that enables a single line element to be expressed for the entire spherically symmetric background spacetime. Perturbing this spacetime in a gauge-invariant manner implies junction conditions are not required at any stage of the perturbation. Wave equations are derived for the Newman-Penrose Weyl scalars which hold in both the matter filled regions of the spacetime as well as the vacuum exterior regions.

1. Introduction
The direct detection of gravitational waves is one of the most eagerly anticipated events in physics, and will usher in a new era in gravitational wave astrophysics. The detection of such waves relies on an a priori knowledge of the expected signature, which in turn relies on knowledge of the source of the gravitational wave. Whilst in recent years the numerical relativity community has made amazing progress in modeling such sources – which include the coalescence of binary systems of black holes and neutron stars (e.g. [1, 2, 3]) and the gravitational collapse of isolated sources induced by supernovae (e.g. [4, 5, 6]) – the accurate extraction of gravitational waveforms is fraught with difficulties. The primary reason is that weaker gravitational waves have amplitudes of the same order as gauge modes and intrinsic numerical errors [7].

A common solution to this problem is to analyse non-spherical gravitational collapse using linear perturbations of spherically symmetric spacetimes. This was studied in detail by Gundlach and Martín-García [8, 9] who looked at the perturbations of spherically symmetric perfect fluids with an exterior vacuum spacetime. These two papers highlight the difficulties one faces with the perturbations at the interface between the interior, fluid filled region and the exterior vacuum. This point is emphasized with the recent work of Mars, Mena & Vera [10] who presented a critical review of various formulations of the problem. The difficulty is essentially associated with the identification of the various coordinate systems used either side of the boundary in both the background and perturbed spacetimes.
In this work I develop a method that avoids the problem of matching perturbed spacetimes. This is done by establishing the entire background spacetime in a single coordinate system as first achieved by Adler et al. [11] and later developed by Lasky & Lun [12, 13, 14, 15]. Both regions of the background spacetime being in a single coordinate system implies the perturbed manifold is also expressed in a single coordinate system. That is, no matching is required across the interface in either the background or perturbed manifold.

This work is set out as follows; in section 2, I describe the spherically symmetric background spacetime. In the present work this is done for a perfect fluid interior region with vacuum exterior. For the extension of the background formulation to more complicated fluids, including a general decomposition of the energy momentum tensor and a relativistic plasma, see Lasky & Lun [14] and [15] respectively. In section 3 perturbation equations are presented which are derived using the Geroch-Held-Penrose (GHP) formulation [16] of the Newman-Penrose equations [17]. I further use these perturbation equations to derive a single wave equation on the perturbed $\Psi_4$ component of the Weyl tensor.

2. The Background Spacetime

The critical aspect of the method outlined in this article is the background spacetime. Once the background is established in the correct way the perturbations can be handled with relative ease. Therefore, for clarity, I shall only consider the simplified background spacetime of a spherically symmetric perfect fluid [12]. Note that the method for setting up the background is more robust than a simple perfect fluid; for instance it has been successful with both a completely general decomposition of the energy-momentum tensor [14], as well as adapting to the case of Einstein-Maxwell plasmas [15]. In these two cases the physics is more complicated as the exterior spacetimes are not necessarily Schwarzschild due to the presence of outgoing radiation. However, even with more complicated physics, the basis of the method remains the same.

The metric representing a spherically symmetric spacetime can be expressed in $3+1$ form as

$$ds = -\alpha^2 dt^2 + \frac{(\beta dt + dr)^2}{1 + E} + r^2 d\Omega^2. \quad (1)$$

Here, $\alpha(t, r) > 0$ is the lapse function, $\beta(t, r)$ is the radial component of the shift vector, $E(t, r) > -1$ and $d\Omega^2 := d\theta^2 + \sin^2\theta d\phi^2$. The function $E$ shall be referred to as the energy function as in the dust case it reduces to the energy function of the Lemaître-Tolman-Bondi spacetime [18, 19, 20], which is a measure of the energy of a shell at a radius $r$ [21].

The energy-momentum tensor for a perfect fluid is given by

$$T^{\mu\nu} = (\rho + P) n^\mu n^\nu + Pg^{\mu\nu}, \quad (2)$$

where $\rho(t, r)$ and $P(t, r)$ are respectively the energy-density and isotropic pressure as measured by an observer $n^\mu$. It is worth stressing that $n^\mu$ is not a comoving observer, rather it is the observer whose vector field is everywhere orthogonal to the spacelike hypersurfaces given by the line element (1). The vector field is required to be timelike and normalized, $n^\alpha n_\alpha = -1$, and hypersurface forming, $n_\mu \nabla_\nu n^\nu = 0$ (where square brackets denote antisymmetrization). Using these conditions, one can express the normal vector in component form

$$n_\mu = [-\alpha, 0, 0, 0] \quad \text{and} \quad n^\mu = \alpha^{-1} [1, -\beta, 0, 0]. \quad (3)$$

Conservation of energy-momentum is given by $\nabla_\alpha T^{\alpha\mu} = 0$. In spherical symmetry this provides two independent equations, however one of these equations is trivially satisfied by the Einstein equations (for details see [12]). The remaining conservation equation is Euler’s equation which relates the pressure gradient to the lapse function

$$-\frac{1}{\rho + P} \frac{\partial P}{\partial r} = \frac{1}{\alpha} \frac{\partial \alpha}{\partial r}. \quad (4)$$
where $\rho \neq -P$. This equation, coupled to a suitable equation of state, implies that the pressure gradient determines the lapse function uniquely (note that integration of equation (4) introduces an arbitrary function of the temporal coordinate, however this can always be scaled away by a suitable coordinate transformation $t \rightarrow t'$). The standard method in numerical relativity is to choose a gauge (i.e. choose $\alpha$ and $\beta$) and solve for the remaining metric coefficients. However, the method utilized here is to let the physics dictate the choice of gauge. That is, rather than making an arbitrary gauge choice, equation (4) is solved dependent on the equation of state, thus prescribing the lapse function.

The remaining equations determining the background spacetime are found by reducing the Einstein Field Equations, $G_{\mu\nu} = 8\pi T_{\mu\nu}$. I provide here only a brief summary of the results; for a full derivation the interested reader is referred to Lasky & Lun [12]. Begin by defining a function which is referred to as the “mass",

$$M(t, r) := 4\pi \int_0^r \rho r' dr'.$$  \hspace{1cm} (5)

It is noted that this is not necessarily a physical mass as it does not properly take into account the volume element associated with the metric. However, in the derivation this quantity arises in a very natural way [12]. Moreover, in the dust limit this function becomes the familiar mass function of the Lemaitre-Tolman-Bondi dust solutions, and in the vacuum limit it also becomes the Schwarzschild mass, suggesting that this quantity is of considerable importance.

After much work, an algebraic relation can be found between the mass and energy functions and the lapse and shift functions

$$\frac{\beta}{\alpha} = \left(\frac{2M}{r} + E\right)^{1/2},$$  \hspace{1cm} (6)

where a positive square root has been taken. Taking the negative root gives an expanding model rather than a contracting model which is evidenced by the fact that changing the sign of $\beta$ in the metric is equivalent to mapping $t \rightarrow -t$. This equation implies explicit dependance on the shift function in the metric is no longer required, and everywhere the shift function is substituted using the above relation.

Finally, the remaining Einstein equations reduce to just two equations which contain the time evolution of the energy function

$$\frac{\mathcal{L}_n E}{1 + E} = 2 \left(\frac{2M}{r} + E\right)^{1/2} \frac{1}{\rho + P} \frac{\partial P}{\partial r},$$  \hspace{1cm} (7)

and the mass function

$$\mathcal{L}_n M = 4\pi Pr^2 \left(\frac{2M}{r} + E\right)^{1/2}.$$  \hspace{1cm} (8)

Here, $\mathcal{L}_n$ is the Lie derivative operator which, when acting on a scalar, takes the form

$$\mathcal{L}_n = \frac{1}{\alpha} \frac{\partial}{\partial t} - \frac{\beta}{\alpha} \frac{\partial}{\partial r}.$$  \hspace{1cm} (9)

Equations (7) and (8), combined with equations (4), (5) and an equation of state provide all the information required to solve for the metric coefficients, and therefore uniquely determine the evolution of the spacetime. Moreover, it can be shown that the only initial conditions required are the initial energy-density and its time rate of change. With these specified, the equation of state gives the initial lapse function, equation (5) implies the initial mass function is known and equation (8) can be rearranged to find the initial energy function.
So far, the equations have described an interior, fluid filled region of the spacetime. The exterior, vacuum region can now be found by setting $P = \rho = 0$ for all $r \geq r_s(t)$, where $r_s(t)$ is the surface of the collapsing star. Moreover, this is only required to be stipulated on the initial spacelike hypersurface, whereby the subsequent evolution is given by the solution of equations (7) and (8). Setting $P = \rho = 0$ implies $M = M_s$ for $r \geq r_s$, where $M_s$ is the Schwarzschild mass. Moreover, equation (4) implies the lapse function is a function of only the temporal coordinate, which can then be scaled to unity by a suitable coordinate transformation. Equation (8) is now trivially satisfied and equation (7) becomes $\mathcal{L}_n E = 0$. Finally, the metric can be expressed as

$$ds^2 = -dt^2 + \frac{\left(\sqrt{2M_s/r + E} dt^2 + dr^2\right)^2}{1 + E} + r^2d\Omega^2.$$  

It can be shown that equation (10), together with the condition $\mathcal{L}_n E = 0$, is exactly the Schwarzschild spacetime in coordinates that generalize the Painleve-Gullstrand coordinates (which are recovered with the trivial solution $E = 0$).

The perfect fluid system of equations reducing to the Schwarzschild spacetime in this way implies that all regions of the spacetime are expressed in a single coordinate system. Setting up a physically relevant problem involves specifying an initial energy-density distribution on the initial spacelike hypersurface, as well as an equation of state relating the pressure and the energy-density. At some finite radius on the initial hypersurface, $r_s(t = 0)$, the density and pressure tend to zero. Subsequent evolution of the spacetime is achieved by solving equations (4), (7) and (8). Solving these equations will provide a solution for $r_s(t)$ throughout the spacetime, which is the evolution of the boundary of the interior fluid filled region and the exterior vacuum region. The entire spacetime is therefore described by a single line element (1), where the metric coefficients are the solutions of the equations just described.

3. Perturbations
As noted, a single solution of the Einstein Field Equations is now provided for a collapsing spacetime which includes both the interior collapsing region and the exterior vacuum region. Utilizing a single solution, as opposed to matching two distinct solutions together at some interface, implies that junction conditions are not required in the background spacetime. Whilst the benefits of this methods for the background spacetime have been explicitly discussed in Lasky & Lun [12, 13, 14, 15], the discussion of perturbations of such spacetimes has not been discussed prior to this article.

With the background spacetime specified, one is free to choose their favorite perturbation scheme. For the purposes of this discussion, I choose to work with the GHP formalism [16] due to the relative simplicity in the expressions\(^1\). This formalism is an extension of the Newman-Penrose [17] equations where the following differential operators are defined;

$$\bar{\mathbf{b}} := \ell^\alpha \nabla_\alpha - p\epsilon - q\bar{\epsilon}, \quad \bar{\mathbf{b}}' := k^\alpha \nabla_\alpha - p\gamma - q\bar{\gamma},$$

$$\bar{\mathbf{\delta}} := m^\alpha \nabla_\alpha - p\beta - q\bar{\alpha}, \quad \bar{\mathbf{\delta}}' := \bar{m}^\alpha \nabla_\alpha - p\alpha - q\bar{\beta}.$$  

(11)

Here, $\ell^\mu$, $k^\mu$ and $m^\mu$ define the null tetrad, $\alpha$, $\beta$, $\epsilon$ and $\gamma$ are spin coefficients and $p$ and $q$ are scalars associated to the spin and boost weights, $s$ and $w$ respectively, according to

$$s = (p - q)/2, \quad \text{and} \quad w = (p + q)/2.$$  

(12)

\(^1\) Another valid perturbation method, given that the background spacetime has been established using a $3 + 1$ formalism, is to use a $2 + 1 + 1$ perturbation scheme [22, 23, 24, 25].
For a consistent description, one must express the background spacetime using this formalism. It is trivial to show that the vanishing spin coefficients, Weyl and energy momentum components on the background are

\[ \kappa = \sigma = \tau = \nu = \lambda = \pi = 0, \]  
\[ \Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = \Phi_{01} = \Phi_{02} = \Phi_{12} = 0. \]

Moreover, at the interface between the two regions of the background spacetime, the remaining energy-momentum terms (\( \Phi_{00}, \Phi_{11}, \Phi_{22} \) and \( \Lambda \)) also tend to zero such that they are equivalently zero for all \( r \geq r_*(t) \).

A linear perturbation can now be performed on the background system of equations such that for any scalar quantity, \( \varphi \), say,

\[ \tilde{\varphi} = \varphi + \varphi_*, \]

where the subscript '⋆' denotes a perturbed quantity. After much work, one can show that the perturbed system of equations is given by

**Perturbed Ricci Identities:**

\[ b' \rho_* + b_* \rho - \bar{\sigma}' \kappa_* = 2 \rho \rho_* + \Phi_{00} \]
\[ b' \mu_* + b_* \mu - \bar{\sigma}' \lambda_* = -2 \mu \lambda_* - \Phi_{22} \]
\[ \bar{b} \kappa_* - \bar{\sigma} \lambda_* = 2 \rho \lambda_* + \Psi_0 \]
\[ \bar{b} \lambda_* - \bar{\sigma} \nu_* = -2 \mu \lambda_* - \Psi_4 \]
\[ \bar{b} \tau_* - \bar{\sigma}' \kappa_* = (\tau_* + \bar{\pi}_*) \rho + \Psi_1 + \Phi_{01} \]
\[ \bar{b} \tau_* - \bar{\sigma}' \lambda_* = - (\pi_* + \bar{\tau}_*) \mu - \Psi_3 - \Phi_{21} \]

**Perturbed Bianchi Identities:**

\[ (b - 4 \rho) \Psi_{1*} = \bar{\sigma}' \Psi_0 - 3 \kappa_* \Psi_2 - (\bar{\sigma}_* + \bar{\pi}_*) \Phi_{00} - \bar{\phi} \Phi_{00} + (b - 2 \rho) \Phi_{01} + 2 \kappa_1 \Phi_{11}, \]
\[ (b' + 4 \mu) \Psi_{3*} = \bar{\sigma}' \Psi_4 + 3 \mu \Psi_2 - (\bar{\sigma}'_* + \bar{\pi}'_*) \Phi_{22} - \bar{\phi}' \Phi_{22} + (b' + 2 \mu) \Phi_{21} - 2 \nu \Phi_{11} \]
\[ \bar{\sigma}' \Psi_{1*} = (b - 3 \rho) \Psi_2 + (b - 3 \rho) \Psi_2 + (b' + \mu) \Phi_{00} - \bar{\phi}' \Phi_{00} + \]
\[ + (b' + \bar{\mu}_*) \Phi_{00} + 2 \rho \Phi_{11} - 2 \rho \Phi_{11} \]
\[ \bar{\sigma}' \Psi_{3*} = (b' + 3 \mu) \Phi_{22} + (b' + 3 \mu) \Psi_2 + (b - \rho) \Phi_{22} - \bar{\phi}' \Phi_{22} + \]
\[ + (b_1 - \bar{\rho}_*) \Phi_{22} + 2 \rho \Phi_{11} + 2 \rho \Phi_{11} \]
\[ \bar{b} - 2 \rho \Phi_{3*} = \bar{\sigma}' \Phi_{00} - (\bar{\sigma}'_* + 2 \pi_* \Phi_2 + (b - 2 \rho) \Phi_{22} - \Phi_{00} + 2 \rho \Phi_{10} + 2 \rho \Phi_{10} + \]
\[ + 2 \pi_1 \Phi_{11} - \bar{\pi}_* \Phi_{11} + 2 \sigma_1 \Phi_{11} \]
\[ \bar{b}' + 2 \mu \Phi_{1*} = \bar{\sigma}' \Phi_2 + (\bar{\sigma}'_* + 3 \pi_* \Phi_2 + (b' + 2 \mu) \Phi_{00} - \Phi_{00} - 2 \rho \Phi_{12} + \]
\[ + 2 \pi_1 \Phi_{11} - \pi \Phi_{00} + 2 \rho \Phi_{11} + 2 \rho \Phi_{11} \]
\[ \bar{b} - \rho \Phi_{4*} = \bar{\sigma}' \Phi_{3*} - 3 \lambda_* \Phi_{2} - (b' + \mu) \Phi_{20} + \bar{\sigma}' \Phi_{21} + \sigma_2 \Phi_{22} - 2 \lambda \Phi_{11}, \]
\[ \bar{b}' + \mu \Phi_{0*} = \bar{\sigma}' \Phi_1 + 3 \sigma_1 \Phi_{2} - (b - \rho) \Phi_{00} + \bar{\sigma}' \Phi_{01} - \lambda \Phi_{00} + 2 \pi_1 \Phi_{11} \]
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Perturbed Contracted Bianchi Identities:
\[
\begin{align*}
\mathbf{p} \Phi_{11*} + \mathbf{p}_* \Phi_{11} + \mathbf{b}' \Phi_{00*} + \mathbf{b}' \Phi_{00} - \partial \Phi_{10*} - \partial \Phi_{01*} + 3 \mathbf{p} \Lambda_* + 3 \mathbf{p}_* \Lambda \\
= 4 \mathbf{p} \Phi_{11*} + 2 (\rho_* + \bar{\rho}_*) \Phi_{11} - 2 \mu \Phi_{00*} - 3 \mathbf{p}_* \Phi_{00}, \\
\end{align*}
\]
(36)

\[
\begin{align*}
\mathbf{b}' \Phi_{11*} + \mathbf{b}' \Phi_{11} + \mathbf{b}' \Phi_{22*} + \mathbf{b}_* \Phi_{22} - \partial' \Phi_{12*} - \partial' \Phi_{21*} + 3 \mathbf{b}' \Lambda_* + 3 \mathbf{b}_* \Lambda \\
= -4 \mathbf{p} \Phi_{11*} - 2 (\mu_* + \bar{\mu}_*) \Phi_{11} + 2 \rho \Phi_{22*} + (\rho_* + \bar{\rho}_*) \Phi_{22}, \\
\end{align*}
\]
(37)

\[
\begin{align*}
\mathbf{p} \Phi_{12*} + \mathbf{b}' \Phi_{01*} - \partial \Phi_{11*} - \partial \Phi_{11} - \partial' \Phi_{02*} + 3 \partial \Lambda_* + 3 \partial_* \Lambda \\
= 3 \rho \Phi_{12*} - 3 \mu \Phi_{01*} - 2 (\tau_* - \bar{\tau}_*) \Phi_{11} - \mu \Phi_{00} - \kappa_* \Phi_{22}, \\
\end{align*}
\]
(38)

\[
\begin{align*}
\mathbf{b}' \Phi_{10*} + \mathbf{b} \Phi_{21*} - \partial' \Phi_{11*} - \partial' \Phi_{11} - \partial \Phi_{20*} + 3 \partial' \Lambda_* + 3 \partial_* \Lambda \\
= -3 \mu \Phi_{10*} + 3 \rho \Phi_{21*} + 2 (\tau_* - \bar{\tau}_*) \Phi_{11} + \kappa_* \Phi_{22} + \nu_* \Phi_{00}. \\
\end{align*}
\]
(39)

Perturbed Commutators:
\[
\begin{align*}
\left[ \mathbf{b}, \mathbf{b}' \right] + \left[ \mathbf{b}_*, \mathbf{b}' \right] = \left( \tau_* + \pi_* \right) \partial + \left( \tau_* + \bar{\pi}_* \right) \partial' - \left( p + q \right) \left( \Psi_{2*} + \Phi_{11*} - \Lambda_* \right), \\
\end{align*}
\]
(40)

\[
\begin{align*}
\left[ \mathbf{b}, \partial_* \right] + \left[ \mathbf{b}_*, \partial \right] = \rho \partial_* + \bar{\rho} \partial + \left( \mu \kappa_* - \Psi_{1*} \right) - q \left( \rho \pi_* - \Phi_{01*} \right), \\
\end{align*}
\]
(41)

\[
\begin{align*}
\left[ \partial, \partial' \right] + \left[ \partial_* , \partial' \right] = \left( \mu_* - \bar{\mu}_* \right) \mathbf{b} + \left( \rho_* - \bar{\rho}_* \right) \mathbf{b}' - \left( \mu_* \rho_* + \mu* \bar{\rho}_* - \Psi_{2*} + \Phi_{11*} + \Lambda_* \right) \\
+ q \left( \rho \bar{\mu}_* + \bar{\rho} \mu - \Psi_{2*} + \Phi_{11*} + \Lambda_* \right). \\
\end{align*}
\]
(42)

Whilst the above system of equations is obviously extremely complicated, it does have the nice feature that the same set of equations holds over the entire spacetime manifold. The standard approach to this subject is to have a background spacetime formulated using two matched solutions of the Einstein field equations. Perturbations are then achieved by perturbing both solutions of the field equations, and then matching the perturbed manifolds together. The boundary between the two regions is then found pointwise in the perturbed regions (for example see \cite{8, 9, 10}). The above equations are a single set of perturbation equations which hold over the entire manifold, implying no matching at the interface is required.

Wave Equation

The Bardeen-Press wave equation \cite{26} can be found by operating on equation (29) with \partial' and equation (34) with \mathbf{b}' + 5 \mu. Adding the resulting equations implies, after much algebra
\[
\Box \Psi_{4*} = S. \\
\]
(43)

Here, \Box is a wave operator defined as
\[
\Box := (\mathbf{b}' + 5 \mu) (\mathbf{b} - \rho) - \partial \partial - 3 \Psi_2 + 2 \Phi_{11}, \\
\]
(44)

and the source for the wave equation is given by
\[
S := 12 \left( \lambda_* \mathbf{b}' - \nu_* \partial' \right) \Lambda - (\mathbf{b}' + 5 \mu) (\mathbf{b}' + \mu) \Phi_{20*} - \partial \partial' \Phi_{22*} - \partial \partial' \Phi_{22} + 3 \mathbf{b}' \Phi_{22*} + 5 \lambda_* \mathbf{b} \Phi_{22} + (\mathbf{b}' + 4 \mu) \partial' \Phi_{21*} + 2 \Phi_{22} (\mathbf{b}' + 3 \partial_* \mu - 3 \lambda_* \rho) \\
+ \Phi_{22} \Phi_{20*} + 4 \Phi_{11} \partial' \Lambda*. \\
\]
(45)

This equation is still highly coupled to both the background and perturbed spin coefficients, and is therefore extremely difficult to solve. Solving it requires stipulating initial conditions for the perturbations. For example, one may wish to initially perturb the interior, fluid filled region of the spacetime. The perturbation will then propagate outwards, through the interface of the
interior and exterior regions, and out to radial infinity. This will be done as a single solution of
the above equation with no extra matching required at the interface. In principal, the interface
between the two regions is found in the same way as is done for the background spacetime; by
solving the full set of equations and finding the point at which all matter terms vanish on each
spacelike slice.

4. Conclusion
This article has introduced a general technique for handling perturbations of spherically
symmetric spacetimes without the need for matching various regions in either the background or
the perturbed manifolds. This was done by utilising the recently developed formalism of Adler
et al. [11] and Lasky & Lun [12, 13, 14, 15] which expresses a spherically symmetric background
spacetime covering both a vacuum exterior region and a fluid filled interior region as a single line
element. This has lead to a single set of perturbation equations covering the entire spacetime.

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