Fragmentation fractions of two-body $b$-baryon decays

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Abstract

We study the fragmentation fractions ($f_{B_b}$) of the $b$-quark to $b$-baryons ($B_b$). By the assumption of $f_{\Lambda_b}/(f_u + f_d) = 0.25 \pm 0.15$ in accordance with the measurements by LEP, CDF and LHCb Collaborations, we estimate that $f_{\Lambda_b} = 0.175 \pm 0.106$ and $f_{\Xi^-_b} = 0.019 \pm 0.013$. From these fragmentation fractions, we derive $B(\Lambda_b \to J/\psi \Lambda) = (3.3 \pm 2.1) \times 10^{-4}$, $B(\Xi^-_b \to J/\psi \Xi^-) = (5.3 \pm 3.9) \times 10^{-4}$ and $B(\Omega^-_b \to J/\psi \Omega^-) > 1.9 \times 10^{-5}$. The predictions of $B(\Lambda_b \to J/\psi \Lambda)$ and $B(\Xi^-_b \to J/\psi \Xi^-)$ clearly enable us to test the theoretical models, such as the QCD factorization approach in the $b$-baryon decays.
I. INTRODUCTION

The LHCb Collaboration has recently published the measurements of the $b$-baryon ($B_b$) decays \[^{[1,3]}\], such as the charmful $\Lambda_b$ decays of $\Lambda_b \rightarrow \Lambda_c^+(K^-, \pi^-)$, $\Lambda_b \rightarrow \Lambda_c^+(D^-, \bar{D}^-)$, $\Lambda_b \rightarrow D^0 p(K^-, \pi^-)$, and $\Lambda_b \rightarrow J/\psi p(K^-, \pi^-)$, which are important and interesting results. For example, while the $p\pi$ mass distribution in $\Lambda_b \rightarrow J/\psi p\pi^-$ \[^{[2]}\] suggests the existence of the higher-wave baryon, such as $N(1520)$ or $N(1535)$, a peaking data point in the $Dp$ mass distribution in $\Lambda_b \rightarrow D^0 p(K^-, \pi^-)$ \[^{[3]}\] hints at the resonant $\Sigma_c(2880)$ state. On the other hand, it is typical to have the partial observations for the decay branching ratios, given by \[^{[4]}\]

$$
\mathcal{B}(\Lambda_b \rightarrow J/\psi \Lambda) f_{\Lambda_b} = (5.8 \pm 0.8) \times 10^{-5},
$$

$$
\mathcal{B}(\Xi_b^- \rightarrow J/\psi \Xi^-) f_{\Xi_b^-} = (1.02^{+0.26}_{-0.21}) \times 10^{-5},
$$

$$
\mathcal{B}(\Omega_b^- \rightarrow J/\psi \Omega^-) f_{\Omega_b^-} = (2.9^{+1.1}_{-1.8}) \times 10^{-6},
$$

(1)

where $f_{B_b}$ are the fragmentation fractions of the $b$ quark to $b$-baryons $B_b = \Lambda_b, \Xi_b^-$ and $\Omega_b^-$. The partial observations in Eq. (1) along with the measurements of the $\Xi_b^0$ decays \[^{[3,4]}\] are due to the fact that $f_{\Lambda_b, \Xi_b^-, \Omega_b^-}$ are not well determined. In the assumption of $f_{\Lambda_b} \simeq f_{\text{baryon}}$ with $f_{\text{baryon}} \equiv \mathcal{B}(b \rightarrow \text{all } b\text{-baryons})$, it is often adopted that $f_{\Lambda_b} = 0.1$ \[^{[6,7]}\]. However, according to the recent observations of the relatively less decays associated with $\Xi_b^{-, 0}$ and $\Omega_b^-$ \[^{[8]}\], $f_{\Lambda_b} \simeq f_{\text{baryon}}$ is no longer true. As a result, it is urgent to improve the value of $f_{\Lambda_b}$ and obtain the less known ones of $f_{\Xi_b^{-, 0}}$.

Although it is possible to estimate $f_{\Lambda_b}$ by the ratio of $f_{\Lambda_b}/(f_u + f_d)$ with $f_{u,d,s} \equiv \mathcal{B}(b \rightarrow B^-, \bar{B}^0, \bar{B}^0_s)$, different measurements on $f_{\Lambda_b}/(f_u + f_d)$ are not in good agreement, given by

$$
\frac{f_{\Lambda_b}}{f_u + f_d} = 0.281 \pm 0.012(\text{stat})^{+0.058}_{-0.056}(\text{sys})^{+0.128}_{-0.087}(\text{Br}) \quad \text{[9]},
$$

$$
\frac{f_{\Lambda_b}}{f_u + f_d} = 0.125 \pm 0.020 \quad \text{[4]},
$$

(2)

with the uncertainty related to Br due to the uncertainties on the measured branching ratios, where the first relation given by the CDF Collaboration \[^{[9]}\] is obviously two times larger than the world averaged value of the second one \[^{[4]}\], dominated by the LEP measurements on $Z$ decays. Moreover, since the recent measurements by the LHCb Collaboration also indicate this inconsistency \[^{[10,12]}\], it is clear that the values of $f_{\Lambda_b}$ and $f_{\Xi_b^-}$ can not be experimentally

\[^{1}\] $f_{\text{baryon}} \sim 0.1$ was also taken in the previous versions of the PDG.
determined yet. In this paper, we will demonstrate the possible range for \( f_{\Lambda_b}/(f_u + f_d) \) in accordance with the measurements by LEP, CDF and LHCb Collaborations and give the theoretical estimations of \( f_{\Lambda_b} \) and \( f_{\Xi^-} \), which allow us to extract \( B(\Lambda_b \to J/\psi \Lambda) \), \( B(\Xi^-_b \to J/\psi \Xi^-) \), and \( B(\Omega^-_b \to J/\psi \Omega^-) \) from the data in Eq. (1). Consequently, we are able to test the theoretical approach based on the factorization ansatz, which have been used to calculate the two-body \( B_b \) decays [7, 13–19].

II. ESTIMATIONS OF \( f_{\Lambda_b} \) AND \( f_{\Xi^-} \)

Experimentally, in terms of the specific cases of the charmful \( \Lambda_b \to \Lambda^+_c \pi^- \) and \( \bar{B}^0 \to D^+ \pi^- \) decays or the semileptonic \( \Lambda_b \to \Lambda^+_c \mu^- \bar{\nu} X \) and \( \bar{B} \to D \mu^- \bar{\nu} X \) decays detected with the bins of \( p_T \) and \( \eta \), where \( p_T \) is the transverse momentum and \( \eta = -\ln(\tan \theta/2) \) is the pseudorapidity defined by the polar angle \( \theta \) with respect to the beam direction [9–11], the ratio of \( f_{\Lambda_b}/(f_u + f_d) \) can be related to \( p_T \) and \( \eta \). This explains the inconsistency between the results from CDF and LEP with \( p_T = 15 \) and \( 45 \) GeV, respectively. While \( f_s/f_u \) is measured with slight dependences on \( p_T \) and \( \eta \) [20], \( f_{\Lambda_b}/(f_u + f_d) \) is fitted as the linear form in Ref. [11] with \( p_T = 0 - 14 \) GeV and the exponential form in Refs. [11, 12] with \( p_T = 0 - 50 \) GeV, respectively, for the certain range of \( \eta \).

A. The present status of \( f_{\Lambda_b}/(f_u + f_d) \)

With the semileptonic \( \Lambda_b \to \Lambda^+_c \mu^- \bar{\nu} X \) and \( \bar{B} \to D \mu^- \bar{\nu} X \) decays, the LHCb Collaboration has shown the dependence of \( f_{\Lambda_b}/(f_u + f_d) \) on \( p_T \) in the range of \( p_T = 0 - 14 \) GeV to be the linear form, given by [11]

\[
f_{\Lambda_b}/(f_u + f_d) = \left( 0.404 \pm 0.017(\text{stat}) \pm 0.027(\text{syst}) \pm 0.105(\text{Br}) \right) \left( 1 - [0.031 \pm 0.004(\text{stat}) \pm 0.003(\text{syst})]p_T \right),
\]

where \( \text{Br} \) arises from the absolute scale uncertainty due to the poorly known branching ratio of \( B(\Lambda^+_c \to pK^-\pi^+) \). By averaging \( f_{\Lambda_b}/(f_u + f_d) \) with \( p_T = 0 - 14 \) GeV, we obtain

\[
\bar{f}_{\Lambda_b} = (0.316 \pm 0.087)(f_u + f_d),
\]

which agrees with the first relation in Eq. (2) given by the CDF Collaboration with \( p_T \simeq 15 \) GeV. On the other hand, with the charmful \( \Lambda_b \to \Lambda^+_c \pi^- \) and \( \bar{B}^0 \to D^+ \pi^- \) decays, another
analysis by the LHCb Collaboration presents the exponential dependence of \( f_{\Lambda_b}/f_d \) on \( p_T \) [11, 12]:

\[
f_{\Lambda_b}/f_d = (0.151 \pm 0.030) + \exp\{-(0.57 \pm 0.11) - (0.095 \pm 0.016)p_T\},
\]

(5)

with the wider range of \( p_T = 0 - 50 \) GeV. By averaging the value in Eq. (5) with \( p_T = 0 - 50 \) GeV, we find

\[
\bar{f}_{\Lambda_b} = (0.269 \pm 0.040)f_d = (0.135 \pm 0.020)(f_u + f_d),
\]

(6)

with \( f_u = f_d \) due to the isospin symmetry, where the error has combined the uncertainties in Eq. (5). It is interesting to note that, as the relation in Eq. (5) with \( p_T = 0 - 50 \) GeV overlaps \( p_T \approx 45 \) GeV for the second relation from LEP in Eq. (2), its value of \( \bar{f}_{\Lambda_b} = (0.135\pm0.020)(f_u+f_d) \) is close to the LEP result of \( f_{\Lambda_b} = (0.125\pm0.020)(f_u+f_d) \). Apart from the values in Eqs. (4) and (6), the reanalyzed results by CDF and LHCb Collaborations give \( f_{\Lambda_b}/(f_u + f_d) \) to be 0.212 \( \pm \) 0.058 and 0.223 \( \pm \) 0.022 with the averaged \( p_T \approx 13 \) and 7 GeV, respectively [12]. We hence make the assumption of

\[
R_{\Lambda_b} \equiv f_{\Lambda_b}/(f_u + f_d) = 0.25 \pm 0.15,
\]

(7)

to cover the possible range in accordance with the measurements from the three Collaborations of LEP, CDF and LHCb, which will be used to estimate the values of \( f_{\Lambda_b} \) and \( f_{\Xi_{b}^{-}} \) in the following.

**B. Theoretical determination of \( f_{\Xi_{b}^{-}}/f_{\Lambda_b} \)**

In principle, when the ratios of \( f_{\Lambda_b}/(f_u + f_d) \) and \( f_{\Xi_{b}^{-}}/f_{\Lambda_b} \) are both known, by adding the relations of [4, 20]

\[
f_u + f_d + f_s + f_{\text{baryon}} = 1,
\]

\[
f_{\text{baryon}} \simeq f_{\Lambda_b} + f_{\Xi_{b}^{-}} + f_{\Xi_{0}^{b}},
\]

\[
f_s = (0.256 \pm 0.020)f_d,
\]

(8)

and \( f_u = f_d \) as well as \( f_{\Xi_{b}^{-}} = f_{\Xi_{0}^{b}} \) due to the isospin symmetry, we can derive the values of \( f_u, f_d, f_s, f_{\Lambda_b}, f_{\Xi_{b}^{-}}, \) and \( f_{\Xi_{0}^{b}} \). For \( f_{\Xi_{b}^{-}}/f_{\Lambda_b} \), it was once given that

\[
f_{\Xi_{b}^{-}}/f_{\Lambda_b} \simeq f_s/f_u \quad [8, 21],
\]

\[
f_{\Xi_{0}^{b}}/f_{\Lambda_b} \simeq 0.2 \quad [22],
\]

(9)
FIG. 1. The $B_b \to B_n J/\psi$ decays via the internal $W$-boson emission diagram.

where the first relation from Refs. [8, 21] requires the assumption of $R_1 \equiv \mathcal{B}(\Xi_b^- \to J/\psi \Xi^-)/\mathcal{B}(\Lambda_b \to J/\psi \Lambda) \simeq 1$ [11], while the second one from Ref. [22] uses $R_2 \equiv \mathcal{B}(\Xi_b^0 \to \Xi_c^+ \pi^-)/\mathcal{B}(\Lambda_b \to \Lambda_c^+ \pi^-) \simeq 1$ along with $R_3 \equiv \mathcal{B}(\Xi_c^+ \to pK^-\pi^+)/\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+) \simeq 0.1$ from the naive Cabibbo factors. However, we note that the theoretical calculations provide us with more understanding of $b$-baryon decays, such as the difference between the $\Lambda_b \to \Lambda$ and $\Xi_b^- \to \Xi^-$ transitions, based on the $SU(3)$ flavor and $SU(2)$ spin symmetries. As a result, the assumption of $R_1 = R_2 \simeq 1$ might be too naive. Since the theoretical approach with the factorization ansatz well explains $\mathcal{B}(\Lambda_b \to p\pi^-)$ and $\mathcal{B}(\Lambda_b \to pK^-)$, and particularly the ratio of $\mathcal{B}(\Lambda_b \to p\pi^-)/\mathcal{B}(\Lambda_b \to pK^-) \sim 0.84$ [23], it can be reliable to determine $f_{\Xi_b^-}/f_{\Lambda_b}$.

Theoretically, we use the factorization approach to calculate the two-body $b$-baryon decay, such that the amplitude corresponds to the decaying process of the $B_b \to B_n$ transition with the recoiled meson. Explicitly, as shown in Fig. 1, where the $W$-boson emission is internal, the amplitude via the quark-level $b \to c \bar{s} s$ transition can be factorized as

\[
A(B_b \to B_n J/\psi) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 \langle J/\psi | \bar{c} \gamma^\mu (1 - \gamma_5) c | 0 \rangle \langle B_n | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_b \rangle ,
\]

for $\Lambda_b \to \Lambda J/\psi$ or $\Xi_b^- \to \Xi^- J/\psi$, where the parameter $a_2$ is given by [24, 25]

\[
a_2 = c_{2,s}^{\text{eff}} + \frac{c_{1,s}^{\text{eff}}}{N_c} ,
\]

with the effective Wilson coefficients $(c_{1,s}^{\text{eff}}, c_{2,s}^{\text{eff}}) = (1.168, -0.365)$. Note that the color number $N_c$ originally being equal to 3 in the naive factorization, which gives $a_2 = 0.024$ in Eq. (11), should be taken as a floating number from $2 \to \infty$ to account for the non-factorizable effects in the generalized factorization. The matrix element for the $J/\psi$ production is given by $\langle J/\psi | \bar{c} \gamma_\mu c | 0 \rangle = m_{J/\psi} f_{J/\psi} \varepsilon_\mu^*$ with $m_{J/\psi}$, $f_{J/\psi}$, and $\varepsilon_\mu^*$ as the mass, decay
constant, and polarization vector, respectively. The matrix elements of the $B_b \to B_n$ baryon transition in Eq. (10) have the general forms:

\[
\langle B_n | q \gamma_\mu | B_b \rangle = \bar{u}_{B_n} [f_1 \gamma_\mu + \frac{f_2}{m_{B_b}} i \sigma_{\mu \nu} q^\nu + \frac{f_3}{m_{B_b}} q_\mu] u_{B_b}, \\
\langle B_n | q \gamma_\mu \gamma_5 | B_b \rangle = \bar{u}_{B_n} [g_1 \gamma_\mu + \frac{g_2}{m_{B_b}} i \sigma_{\mu \nu} q^\nu + \frac{g_3}{m_{B_b}} q_\mu] \gamma_5 u_{B_b},
\]

(12)

where $f_j (g_j)$ ($j = 1, 2, 3$) are the form factors, with $f_{2,3} = 0$ and $g_{2,3} = 0$ due to the helicity conservation, as derived in Refs. [7, 14, 26]. It is interesting to note that, as the helicity-flip terms, the theoretical calculations from the loop contributions to $f_{2,3} (g_{2,3})$ indeed result in the values to be one order of magnitude smaller than that to $f_1 (g_1)$, which can be safely neglected. In the double-pole momentum dependences, one has that

\[
F(q^2) = \frac{F(0)}{(1 - q^2/m_{B_b}^2)^2}, \quad (F = f_1, \ g_1).
\]

(13)

We are able to relate different $B_b \to B_n$ transition form factors based on $SU(3)$ flavor and $SU(2)$ spin symmetries, which have been used to connect the space-like $B_n \to B'_n$ transition form factors in the neutron decays [27], and the time-like $0 \to B_b \to B'_n$ baryonic as well as $B \to B_n \to B'_n$ transition form factors in the baryonic $B$ decays [28–32]. As a result, we obtain $(f_1(0), g_1(0)) = (C, C), (-2\sqrt{2}/3C, -2\sqrt{2}/3C)$, and $(0, 0)$ with $C$ a constant for $\langle p | u \gamma_\mu (\gamma_5) b | \Lambda_b \rangle$, $\langle \Lambda | s \gamma_\mu (\gamma_5) b | \Lambda_b \rangle$, and $\langle \Sigma^0 | s \gamma_\mu (\gamma_5) b | \Lambda_b \rangle$, which are the same as those in Ref. [26] based on the heavy-quark and large-energy symmetries for the $\Lambda_b \to (p, \Lambda, \Sigma^0)$ transitions, respectively. In addition, we have $f_1(0) = g_1(0) = C$ for $\langle \Xi^- | s \gamma_\mu (\gamma_5) b | \Xi^- \rangle$. To obtain the branching ratio for the two-body decays, the equation is given by

\[
\mathcal{B}(B_b \to J/\psi B_n) = \frac{\Gamma(B_b \to J/\psi B_n) \tau_{B_b}}{6.582 \times 10^{-25}},
\]

(14)

with $\tau_{B_b}$ the life time, where

\[
\Gamma(B_b \to J/\psi B_n) = \frac{|\bar{P}_{J/\psi}|^2}{8\pi m_{B_b}^2} |A(B_b \to J/\psi B_n)|^2,
\]

(15)

with $|\bar{P}_{J/\psi}| = |\bar{P}_{B_n}| = \{[m_{B_b}^2 - (m_{J/\psi} + m_{B_n})^2][m_{B_b}^2 - (m_{J/\psi} - m_{B_n})^2]\}^{1/2}/(2m_{B_b})$. As a result, we obtain

\[
\frac{\mathcal{B}(\Xi^-_b \to J/\psi \Xi^-)}{\mathcal{B}(\Lambda_b \to J/\psi \Lambda)} = \frac{\tau_{\Xi^-_b}}{\tau_{\Lambda_b}} \frac{C^2}{(-\sqrt{2}/3C)^2} = 1.63 \pm 0.04,
\]

(16)
with $\tau_{\Xi^-}/\tau_{\Lambda_b} = 1.089 \pm 0.026 \pm 0.011$ \cite{33}. We note that, theoretically, $R_1 = 1.63$ apparently deviates by 63% from $R_1 = 1$ in the simple assumption. To determine $f_{\Xi^-}/f_{\Lambda_b}$, we relate Eq. (16) to (1) to give

$$f_{\Xi^-} = (0.108 \pm 0.034)f_{\Lambda_b},$$

which is different from the numbers in Eq. (9).

C. Determinations of $f_{\Xi^-}$ and $f_{\Lambda_b}$

According to Eqs. (4), (7), (8) and (17), we derive the values of $f_u$, $f_d$, $f_s$, $f_{\Lambda_b}$, $f_{\Xi^-}$ and $f_{\Xi^0}$ in Table I which agree with the data in the PDG \cite{4}. Note that $f_{\Omega^-} < 0.108$ is from the error in $f_{baryon}$. In addition, $f_{baryon} = 0.213 \pm 0.108$, which overlaps $0.089 \pm 0.015$ from Z-decays \cite{4} and $0.237 \pm 0.067$ from Tevatron \cite{4}, is due to the assumption of $R_{\Lambda_b} = 0.25 \pm 0.15$ in Eq. (7) to cover the possible range from the data. Similarly, $f_{\Lambda_b} = 0.175 \pm 0.106$ overlaps $f_{\Lambda_b} = 0.07$ from the LEP measurements \cite{34}, while $f_{\Xi^-} = f_{\Xi^0} = 0.019 \pm 0.013$ is consistent with $f_{\Xi^-} = 0.011 \pm 0.005$ from the measurement \cite{35}. We hence convert the data in Eq. (1) to be

$$B(\Lambda_b \to J/\psi \Lambda) = (3.3 \pm 2.1) \times 10^{-4},$$
$$B(\Xi^- \to J/\psi \Xi^-) = (5.3 \pm 3.9) \times 10^{-4},$$
$$B(\Omega^- \to J/\psi \Omega^-) > 1.9 \times 10^{-5},$$

\hspace{1em} (18)

TABLE I. Results of $f_i \ (i = u, d, s, \text{baryon, } \Lambda_b, \Xi^-^{0,}, \text{and } \Omega^-_{\text{b}})$, compared with those from Z-decays and Tevatron in PDG \cite{4}.

| $f_i$ ($i = u, d, s, \text{baryon, } \Lambda_b, \Xi^-^{0,}, \text{and } \Omega^-_{\text{b}}$) | our result | Z-decays \cite{4} | Tevatron \cite{4} |
|---|---|---|---|
| $f_u = f_d$ | $0.349 \pm 0.037$ | 0.404 ± 0.009 | 0.330 ± 0.030 |
| $f_s$ | $0.089 \pm 0.018$ | 0.103 ± 0.009 | 0.103 ± 0.012 |
| $f_{\text{baryon}}$ | $0.213 \pm 0.108$ | 0.089 ± 0.015 | 0.237 ± 0.067 |
| $f_{\Lambda_b}$ | $0.175 \pm 0.106$ | — | — |
| $f_{\Xi^-} = f_{\Xi^0}$ | $0.019 \pm 0.013$ | — | — |
| $f_{\Omega^-}$ | < 0.108 | — | — |
with $\mathcal{B}(\Xi_b^- \to J/\psi \Xi^-) \simeq 1.6\mathcal{B}(\Lambda_b \to J/\psi \Lambda)$ to be in accordance with Eq. (16). With the use of $f_{\Xi_b^-}$, we can also estimate the $\Xi_b^0$ decays [4, 5], given by

$$\mathcal{B}(\Xi_b^- \to \Xi^- \ell^- \bar{\nu}_\ell X) = (2.1 \pm 1.5) \times 10^{-2},$$
$$\mathcal{B}(\Xi_b^0 \to \bar{K}^0 p \pi^-) = (1.1 \pm 1.5) \times 10^{-5},$$
$$\mathcal{B}(\Xi_b^0 \to \bar{K}^0 p K^-) = (1.1 \pm 1.1) \times 10^{-5},$$
$$\mathcal{B}(\Xi_b^0 \to D^0 p K^-) = (9.5 \pm 9.4) \times 10^{-5},$$
$$\mathcal{B}(\Xi_b^0 \to \Lambda_c^+ K^-) = (4.2 \pm 4.7) \times 10^{-5}. \quad (19)$$

### D. Test of the non-factorizable effects

To numerically test the non-factorizable effects, the CKM matrix elements in the Wolfenstein parameterization are taken as $(V_{cb}, V_{cs}) = (A\lambda^2, 1 - \lambda^2/2)$ with $(\lambda, A) = (0.225, 0.814)$ [4], while $f_{J/\psi} = 418 \pm 9$ MeV [35]. The constant value of $C$ in Ref. [23] is fitted to be $C = 0.136 \pm 0.009$ to explain the branching ratios and predict the CP violating asymmetries of $\Lambda_b \to p(K^-, \pi^-)$, which is also consistent with the value of $0.14 \pm 0.03$ in the light-cone sum rules [26] and those in Refs. [7, 14].

To explain the branching ratios of $\Lambda_b \to J/\psi \Lambda$ and $\Xi_b^- \to J/\psi \Xi^-$ in Eq. (18), the floating color number $N_c$ is evaluated to be

$$N_c = 2.15 \pm 0.17, \quad (20)$$

which corresponds to $a_2 = 0.18 \pm 0.04$, in comparison with $a_2 = 0.024$ for $N_c = 3$. Note that since $N_c = 2.15$ in Eq. (20) is not far from 3, we conclude that the non-factorizable effects are controllable. As a result, the theoretical approach based on the factorization ansatz is demonstrated to be reliable to explain the two-body $B_b$ decays.

### III. CONCLUSIONS

In sum, we made the assumption of $R_{\Lambda_b} = f_{\Lambda_b}/(f_u + f_d) = 0.25 \pm 0.15$, which is in accordance with the the measurements by LEP, CDF and LHCb Collaborations. We have estimated that $f_{\Lambda_b} = 0.175 \pm 0.106$ and $f_{\Xi_b^-} = 0.019 \pm 0.013$, which can be used to extract the branching ratios of the anti-triplet $b$-baryon decays. Explicitly, we have found $\mathcal{B}(\Lambda_b \to J/\psi \Lambda) = (3.3 \pm 2.1) \times 10^{-4}$, $\mathcal{B}(\Xi_b^- \to J/\psi \Xi^-) = (5.3 \pm 3.9) \times 10^{-4}$ and $\mathcal{B}(\Omega_b^- \to J/\psi \Omega^-)$...
$1.9 \times 10^{-5}$. We have also demonstrated that the predictions of $\mathcal{B}(\Lambda_b \to J/\psi \Lambda)$ and $\mathcal{B}(\Xi^-_b \to J/\psi \Xi^-)$ would help us to test the theoretical models, such as the factorization approach.

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