Partial Order Reduction for Security Protocols

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Abstract
Security protocols are concurrent processes that communicate using cryptography with the aim of achieving various security properties. Recent work on their formal verification has brought procedures and tools for deciding trace equivalence properties (e.g. anonymity, unlinkability, vote secrecy) for a bounded number of sessions. However, these procedures are based on a naive symbolic exploration of all traces of the considered processes which, unsurprisingly, greatly limits the scalability and practical impact of the verification tools.

In this paper, we overcome this difficulty by developing partial order reduction techniques for the verification of security protocols. We provide reduced transition systems that optimally eliminate redundant traces, and which are adequate for model-checking trace equivalence properties of protocols by means of symbolic execution. We have implemented our reductions in the tool Apte, and demonstrated that it achieves the expected speedup on various protocols.

1 Introduction
Security protocols are concurrent processes that use various cryptographic primitives in order to achieve security properties such as secrecy, authentication, anonymity, unlinkability, etc. They involve a high level of concurrency and are difficult to analyse by hand. Actually, many protocols have been shown to be flawed several years after their publication (and deployment). This has lead to a flurry of research on formal verification of protocols.

A successful way of representing protocols is to use variants of the π-calculus, whose labelled transition systems naturally express how a protocol may interact with a (potentially malicious) environment whose knowledge increases as more messages are exchanged over the network. Some security properties (e.g., secrecy, authentication) are then described as reachability properties, while others (e.g., unlinkability, anonymity) are expressed as trace equivalence properties. In order to decide such properties, a reasonable assumption is to bound the number of protocol sessions, thereby limiting the length of execution traces. Even under this assumption, infinitely many traces remain, since each input may be fed infinitely many different messages. However, symbolic execution and dedicated constraint solving procedures have been devised to provide decision procedures for reachability [23,14] and, more recently, equivalence properties [31,10]. Unfortunately, the resulting tools, especially those for checking equivalence (e.g., Apte [7], Spec [30]), have a very limited practical impact because they scale very badly. This is not surprising since they treat concurrency in a very naive way, exploring all possible symbolic interleavings of concurrent actions.

Our contributions. We develop partial order reduction (POR) techniques [26,17,20] for trace equivalence checking of security protocols. Our main challenge is to do it in a way that is compatible with symbolic execution: we should provide a reduction that is effective when messages remain unknown, but leverages information about messages when it is inferred by the constraint solver. We achieve this by refining interleaving semantics in two steps, gradually eliminating redundant traces. The first refinement, called compression, uses the notion of polarity [3] to impose a simple strategy on traces. It does not rely on data analysis at all and can easily be used as a replacement for the usual semantics in verification algorithms. The second one, called reduction, takes data into account and achieves optimality...
in eliminating redundant traces. In practice, the reduction step can be implemented in an approximated fashion, through an extension of constraint resolution procedures. We have done so in the tool Apte, showing that our theoretical results do translate to significant practical optimisations.

Outline. We consider in Section 2 a rich process algebra for representing security protocols. It supports arbitrary cryptographic primitives, and even includes a replication operator suitable for modelling unbounded numbers of sessions. Thus, we do not restrict to a particular fragment for which a decision procedure exists, but show the full scope of our theoretical results. We give in Section 3 an annotated semantics that will facilitate the next technical developments. We then define our compressed semantics in Section 4 and the reduced semantics in Section 5. In both sections, we first restrict the transition system, then show that the restriction is adequate for checking trace equivalence under some action-determinism condition. We finally discuss how these results can be lifted to the symbolic setting in Section 6. Specifically, we describe how we have implemented our techniques in Apte, and we present experimental results showing that the optimisations are fully effective in practice. We discuss related work in Section 7, and conclude in Section 8. Complete proofs are given in appendices.

2 Model for security protocols

In this section we introduce our process algebra, which is a variant of the applied π-calculus that has been designed with the aim of modelling cryptographic protocols. Processes can exchange complex messages, represented by terms quotiented by some equational theory.

One of the key difficulties in the applied π-calculus is to model the knowledge of the environment, seen as an attacker who listens to network communication and may also inject messages. One has to make a distinction between the content of a message (sent by the environment) and the way the message has been created (from knowledge available to the environment). While the distinction between messages and recipes came from security applications, it is naturally of much broader interest, as it gives a precise, intentional content to labelled transitions that we exploit to analyse data dependencies.

We study a process algebra that may seem quite restrictive: we forbid internal communication and private channels. This is however reasonable when studying security protocols faced with the usual omnipotent attacker. In such a setting, we end up considering the worst-case scenario where any communication has to be made via the environment.

2.1 Syntax

We assume a number of disjoint and infinite sets: a set $C$ of channels, whose elements are denoted by $a, b, c$; a set $N$ of private names or nonces, denoted by $n$ or $k$; a set $X$ of variables, denoted by $x, y, z$ as usual; and a set $W$ of handles, denoted by $w$ and used for referring to previously output terms. Next, we consider a signature $\Sigma$ consisting of a finite set of function symbols together with their arity. Terms over $S$, written $T(S)$, are inductively generated from $S$ and function symbols from $\Sigma$. When $S \subseteq C$, elements of $T(S)$ are called messages. When $S \subseteq W$, they are called recipes and written $M, N$. Intuitively, recipes express how a message has been derived by the environment from the messages obtained so far. Finally, we consider an equational theory $E$ over terms to assign a meaning to function symbols in $\Sigma$.

Example 1. Let $\Sigma = \{\text{enc}/2, \text{dec}/2, \text{h}/1\}$ and $E$ be the equational theory induced by the equation $\text{dec}(\text{enc}(x, y), y) = x$. Intuitively, the symbols $\text{enc}$ and $\text{dec}$ represent symmetric
We only consider processes that are normal w.r.t. the environment. Given a configuration \( \Pi \) of \( P \), called the \textit{frame}, is a substitution mapping handles to messages that have been made available to the environment. Given a configuration \( A, \Phi(A) \) denotes its second component. Given a frame \( \Phi \), \( \text{dom}(\Phi) \) denotes its domain.

\[
\begin{align*}
\text{IN} & \quad (\{\text{in}(c,x).Q\} \uplus \mathcal{P}; \Phi) \xrightarrow{\text{in}(c,M)} (\{Q\{M\Phi/x\}\} \uplus \mathcal{P}; \Phi) & \quad M \in \mathcal{T}(\text{dom}(\Phi)) \\
\text{OUT} & \quad (\{\text{out}(c,u).Q\} \uplus \mathcal{P}; \Phi) \xrightarrow{\text{out}(c,w)} (\{Q\} \uplus \mathcal{P} \cup \{w \mapsto u\}) & \quad w \in W \text{ fresh} \\
\text{REPL} & \quad (\{\Pi_{i=1}^{\alpha}.P\} \uplus \mathcal{P}; \Phi) \xrightarrow{\text{ass}(\alpha, \overline{\pi})} (\{P_0; \ldots; P_{\alpha}.P\} \uplus \mathcal{P}; \Phi) & \quad (\overline{\pi}, \overline{\pi} \text{ fresh}) \\
\text{PAR} & \quad (\{\Pi_{i=1}^{\alpha}.P_i\} \uplus \mathcal{P}; \Phi) \xrightarrow{\cdot} (\{P_1, \ldots, P_{\alpha}\} \uplus \mathcal{P}; \Phi) \\
\text{ZERO} & \quad (\{0\} \uplus \mathcal{P}; \Phi) \xrightarrow{\cdot} (\mathcal{P}; \Phi)
\end{align*}
\]

 encryption and decryption, whereas \( h \) is used to model a hash function. Now, assume that the environment knows the key \( k \) as well as the ciphertext \( \text{enc}(n, k) \), and that these two messsages are referred to by handles \( w \) and \( w' \). The environment may decrypt the ciphertext with the key \( k \), apply the hash function, and encrypt the result using \( k \) to get the message \( m_0 = \text{enc}(h(n), k) \). This computation is modelled using the recipe \( M_0 = \text{enc}(h(\text{dec}(w'), w)) \).

**Definition 2.** Processes are defined by the following syntax where \( c, a \in C \), \( x \in X \), \( u, v \in \mathcal{T}(\mathcal{N} \cup \mathcal{X}) \), and \( \overline{\pi} \) (resp. \( \overline{\pi'} \)) is a sequence of channels from \( C \) (resp. names from \( \mathcal{N} \)).

\[
P, Q ::= 0 \mid (P \mid Q) \mid \text{in}(c,x).P \mid \text{out}(c,u).P \mid \text{if } u = v \text{ then } P \text{ else } Q \mid !_a^{\pi} P
\]

The last construct combines replication with channel and name restriction: \( !_a^{\pi} P \) may be read as \( \{\nu \mathcal{C}.\text{out}(a, \mathcal{C}).w \overline{\pi}(P)\} \) in standard applied \( \pi \)-calculus. Our goal with this compound construct is to support replication in a way that is not fundamentally incompatible with the action-determinism condition which we eventually impose on our processes. This is achieved here by advertising on the public channel \( a \) any new copy of the replicated process. At the same time, we make public the new channels \( \mathcal{C} \) on which the copy may operate — but not the new names \( \overline{\pi} \). While it may seem restrictive, this style is actually natural for security protocols where the attacker knows exactly to whom he is sending a message and from whom he is receiving, e.g., via IP addresses.

We shall only consider \textit{ground} processes, where each variable is bound by an input. We denote by \( fc(P) \) and \( bc(P) \) the set of free and bound channels of \( P \).

**Example 3.** The process \( P_0 \) models an agent who sends the ciphertext \( \text{enc}(n, k) \), and then waits for an input on \( c \). In case the input has the expected form, the constant \( \text{ok} \) is emitted.

\[
P_0 = \text{out}(c,\text{enc}(n,k)).\text{in}(c,x).\text{if } \text{dec}(x,k) = h(n) \text{ then } \text{out}(c,\text{ok}).0 \text{ else } 0
\]

The processes \( P_0 \) as well as \( !_a^n P_0 \) are ground. We have that \( fc(P_0) = \{c\} \) and \( bc(P_0) = \emptyset \) whereas \( fc(!_a^n P_0) = \{a\} \) and \( bc(!_a^n P_0) = \{c\} \).

**2.2 Semantics**

We only consider processes that are normal w.r.t. internal reduction \( \rightarrow \) defined as follows:

\[
\begin{align*}
\text{if } u = v \text{ then } P \rightarrow Q & \quad \text{when } u \equiv_{E} v & P \mid Q \rightarrow P' \mid Q \quad \text{when } P \rightarrow P' \\
\text{if } u = v \text{ then } P \rightarrow Q & \quad \text{when } u \neq_{E} v & P \mid Q \rightarrow Q \mid P' \quad \text{when } P \rightarrow P' \\
(P_1 \mid P_2) & \rightarrow (P_1 \rightarrow P_1) \mid (P_2 \rightarrow P_2) & P \mid 0 \rightarrow P \mid 0 \quad P \rightarrow P
\end{align*}
\]

Any process in normal form built from parallel composition can be uniquely written as \( P \mid (P_1 \mid (\ldots \mid P_n)) \) with \( n \geq 2 \), which we note \( \Pi_{i=1}^n P_i \), where each process \( P_i \) is neither a parallel composition nor the process 0.

We now define our labelled transition system. It deals with configurations (denoted by \( A, \Phi \)) which are pairs \( (\mathcal{P}; \Phi) \) where \( \mathcal{P} \) is a multiset of ground processes and \( \Phi \), called the \textit{frame}, is a substitution mapping handles to messages that have been made available to the environment. Given a configuration \( A, \Phi(A) \) denotes its second component. Given a frame \( \Phi \), \( \text{dom}(\Phi) \) denotes its domain.
Rule In expresses that an input process may receive any message that the environment can derive from the current frame. In rule Out, the frame is enriched with a new message. The last two rules simply translate the parallel structure of processes into the multiset structure of the configuration. As explained above, rule repl combines the replication of a process together with the creation of new channels and nonces. The channels $\overline{c}$ are implicitly made public, but the newly created names $\overline{r}$ remain private. Remark that channels $\overline{c}$ and names $\overline{r}$ must be fresh, i.e., they do not appear free in the original configuration. As usual, freshness conditions do not block executions: it is always possible to rename bound channels $\overline{c}$ and names $\overline{r}$ of a process $\Pi_{\overline{c},\overline{r}}^{t} P$ before applying repl. We denote by $bc(tr)$ the bound channels of a trace $tr$, i.e., all the channels that occur in second argument of an action $\text{sess}(a, \overline{c})$ in $tr$, and we consider traces where channels are bound at most once.

Example 4. Going back to Example 3 with $\Phi_{0} = \{w_{1} \mapsto k\}$, we have that:

$$\left(\{\{^{1}_{0} P_{0}\}; \Phi_{0}\} \xrightarrow{\text{sess}(a, c)} \text{out}(c, w_{2}) \xrightarrow{\text{enc}(n, k)} \{\{\text{out}(c, 0) ; 1^{n}_{0} P_{0}\}; \Phi}\right)$$

where $\Phi = \{w_{1} \mapsto k, w_{2} \mapsto \text{enc}(n, k)\}$ and $M_{0} = \text{enc}(h(\text{dec}(w_{2}, w_{1})), w_{1})$.

2.3 Equivalences

We are concerned with trace equivalence, which is used to model anonymity, untraceability, strong secrecy, etc. Finer behavioural equivalences, e.g., weak bisimulation, appear to be too strong with respect to what an attacker can really observe. Intuitively, two configurations are trace equivalent if the attacker cannot tell whether he is interacting with one or the other. To make this formal, we introduce a notion of equivalence between frames.

Definition 5. Two frames $\Phi$ and $\Phi'$ are in static equivalence, written $\Phi \sim \Phi'$, when $\text{dom}(\Phi) = \text{dom}(\Phi')$, and: $M\Phi =_{E} N\Phi \Leftrightarrow M\Phi' =_{E} N\Phi'$ for any terms $M, N \in T(\text{dom}(\Phi))$.

Example 6. Continuing Example 4, consider $\Phi' = \{w_{1} \mapsto k', w_{2} \mapsto \text{enc}(n, k)\}$. The test $\text{enc}(\text{dec}(w_{2}, w_{1}), w_{1}) = w_{2}$ is true in $\Phi$ but not in $\Phi'$, thus $\Phi \not\sim \Phi'$.

We then define $\text{obs}(tr)$ to be the subsequence of $tr$ obtained by erasing $\tau$ actions.

Definition 7. Let $A$ and $B$ be two configurations. We say that $A \sqsubseteq B$ when, for any $A \xrightarrow{\alpha} A'$ such that $bc(tr) \cap fc(B) = \emptyset$, there exists $B \xrightarrow{\alpha} B'$ such that $\text{obs}(tr) = \text{obs}(tr')$ and $\Phi(A') \sim \Phi(B')$. They are trace equivalent, written $A \approx B$, when $A \sqsubseteq B$ and $B \sqsubseteq A$.

In order to lift our optimised semantics to trace equivalence, we will require configurations to be action-deterministic. This common assumption in POR techniques is also reasonable in the context of security protocols, where the attacker knows with whom he is communicating.

Definition 8. A configuration $A$ is action-deterministic if whenever $A \xrightarrow{\alpha} (P; \Phi)$, and $P, Q$ are two elements of $\mathcal{P}$, we have that $P$ and $Q$ cannot perform an observable action of the same nature (in, out, or sess) on the same channel (i.e., if both actions are of same nature, their first argument has to differ).

3 Annotated semantics

We shall now define an intermediate semantics whose transitions are equipped with more informative actions. The annotated actions will notably feature labels $\ell \in \mathbb{N}^{*}$ indicating from which concurrent processes they originate. A labelled action will be written $[\alpha]^{\ell}$ where $\alpha$ is an action and $\ell$ is a label. Similarly, a labelled process will be written $[P]^{\ell}$. When reasoning
We will see that, when checking well labelled configurations in the rest of the paper. Under this assumption, we obtain the 3.0.0.1 Symmetries of trace equivalence.

In other words, labels and detailed non-observable actions require that well labelling is preserved by the following fundamental lemma.

Definition 9. Two labels are dependent if one is a prefix of the other. We say that the labelled actions \( \alpha \) and \( \beta \) are sequentially dependent when their labels are dependent, and recipe dependent when \( \{\alpha, \beta\} = \{[\text{in}_c, M]; [\text{out}(c', w)]\} \) with \( w \) occurring in \( M \). They are dependent when they are either sequentially or recipe dependent. Otherwise, they are independent.

Definition 10. A configuration \((P; \Phi)\) is well labelled if \( P \) is a multiset of labelled processes such that two elements of \( P \) have independent labels.

Obviously, any unlabelled configuration may be well labelled. Further, it is easy to see that well labelling is preserved by \( \rightarrow_a \). Thus, we shall implicitly assume to be working with well labelled configurations in the rest of the paper. Under this assumption, we obtain the following fundamental lemma.

Lemma 11. Let \( A \) be a (well labelled) configuration, \( \alpha \) and \( \beta \) be two independent labelled actions. We have \( A \xrightarrow{\alpha \beta} A' \) if, and only if, \( A \xrightarrow{\alpha \beta} A' \).

3.0.0.1 Symmetries of trace equivalence.

We will see that, when checking \( A \approx B \) for action-deterministic configurations, it is sound to require that \( B \) can perform all traces of \( A \) in the annotated semantics (and the converse). In other words, labels and detailed non-observable actions zero and \( \text{par}(\sigma_1, \ldots, \sigma_n) \) are actually relevant for trace equivalence. Obviously, this can only hold if \( A \) and \( B \) are labelled...

Figure 1 Annotated semantics

about trace equivalence between two configurations, it will be crucial to maintain a consistent labelling between configurations along the execution. In order to do so, we define skeletons of observable actions, which are of the form \( \text{in}_c, \text{out}_c, \text{in}^0 \) or \( !a \) where \( a, c \in C \), and we assume a total ordering over those skeletons, denoted \( < \), with \( \leq \) its reflexive closure. Any process that is neither 0 nor a parallel composition induces a skeleton corresponding to its top level connective, and we denote it by \( \text{sk}(P) \).

We define in Figure 1 the annotated semantics \( \rightarrow_a \) over configurations whose processes are labelled. In \( \text{PAR} \), note that \( \text{sk}(P_i) \) is well defined as \( P_i \) cannot be a zero or a parallel composition. Also note that the label of an action is always that of the active process in that transition. More importantly, the annotated transition system does not restrict the executions of a process but simply annotates them with labels, and replaces \( \tau \) actions by more descriptive actions.

We now define how to extract sequential dependencies from labels, which will allow us to analyse concurrency in a trace without referring to configurations.

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Assuming we perform actions that decompose negative non-replicated processes. This is done using the right label. Such mismatches can actually be systematically used to show that \( \text{skl}(A) = \text{skl}(B) \), i.e., the configurations have the same set of labelled skeletons. This technical condition is obviously not restrictive in practice.

**Example 12.** Let \( A = ((\text{in}(a,x),(\text{out}(b,m),P_1) \mid P_2])^0; \Phi) \) with \( P_1 = \text{in}(c,y).0 \) and \( P_2 = \text{in}(d,z).0 \), and \( B \) the configuration obtained from \( A \) by swapping \( P_1 \) and \( P_2 \). We have \( \text{skl}(A) = \text{skl}(B) = ([\text{in}_a]^0) \). Consider the following trace:

\[
\text{tr} = [\text{in}(a,ok)]^0.\text{par}([\text{out}_b; \text{in}_d])^0.\text{out}(b,w)]^0.1.\text{in}(c,w)]^0.1.\text{in}(d,w)]^0.2
\]

Assuming \( \text{out}_b < \text{in}_d \) and \( \text{ok} \in \Sigma \), we have \( A \not\approx \text{skl}(A) \). However, there is no \( B' \) such that \( B \not\approx B' \), for two reasons. First, \( B \) cannot perform the second action since skeletons of sub-processes of its parallel composition are \( \text{out}_b; \text{in}_d \). Second, even if we ignored that mismatch on a non-observable action, \( B \) would not be able to perform the action \( \text{in}(c,w) \) with the right label. Such mismatches can actually be systematically used to show \( A \not\approx B \), as shown next.

**Lemma 13.** Let \( A \) and \( B \) be two action-deterministic configurations such that \( A \approx B \) and \( \text{skl}(A) = \text{skl}(B) \). For any execution

\[
A \xrightarrow{[\alpha_1]^1} A_1 \xrightarrow{[\alpha_2]^2} \ldots \xrightarrow{[\alpha_n]^n} A_n
\]

with \( \text{bc}(\alpha_1, \ldots, \alpha_n) \cap \text{fc}(B) = \emptyset \), there exists an execution

\[
B \xrightarrow{[\alpha_1]^1} B_1 \xrightarrow{[\alpha_2]^2} \ldots \xrightarrow{[\alpha_n]^n} B_n
\]

such that \( \Phi(A_i) \sim \Phi(B_i) \) and \( \text{skl}(A_i) = \text{skl}(B_i) \) for any \( 1 \leq i \leq n \).

## 4 Compression

Our first refinement of the semantics, which we call compression, is closely related to focusing from proof theory \cite{Baader2013}. We will assign a polarity to processes and constrain the shape of executed traces based on those polarities. This will provide a first significant reduction of the number of traces to consider when checking reachability-based properties such as secrecy, and more importantly, equivalence-based properties in the action-deterministic case.

**Definition 14.** A process \( P \) is positive if it is of the form \( \text{in}(c,x).Q \), and it is negative otherwise. A multiset of processes \( P \) is initial if it contains only positive or replicated processes, i.e., of the form \( \text{in}(c,x).Q \).

The compressed semantics (see Figure 2) is built upon the annotated semantics. It constrains the traces to follow a particular strategy, alternating between negative and positive phases. It uses enriched configurations of the form \( (P;F;\Phi) \) where \( (P;\Phi) \) is a labelled configuration and \( F \) is either a process (signalling which process is under focus in the positive phase) or \( \emptyset \) (in the negative phase). The negative phase lasts until the configuration is initial (i.e., unfocused with an initial underlying multiset of processes) and in that phase we perform actions that decompose negative non-replicated processes. This is done using the Neg rule, in a completely deterministic way. When the configuration becomes initial, a positive phase starts: we choose one process and start executing the actions of that process (only inputs, possibly preceded by a new session) without the ability to switch to another process of the multiset, until a negative subprocess is released and we go back to the negative phase. The active process in the positive phase is said to be under focus. Between any...
4.1 Reachability

We now formalise the relationship between traces of the compressed and annotated semantics. In order to do so, we translate between configuration and enriched configuration as follows:

$$\langle [P; \Phi] \rangle = \langle P; \emptyset; \Phi \rangle, \quad \langle (P; \emptyset; \Phi) \rangle = \langle P; \Phi \rangle \quad \text{and} \quad \langle (P; P; \Phi) \rangle = \langle P \cup \{P\}; \Phi \rangle.$$

Similarly, we map compressed traces to annotated ones:

$$[\epsilon] = \epsilon, \quad [\text{foc}(\alpha).\text{tr}] = \alpha.[\text{tr}], \quad [\text{rel}.\text{tr}] = [\text{tr}] \quad \text{and} \quad [\alpha.\text{tr}] = \alpha.[\text{tr}] \quad \text{otherwise.}$$

We observe that we can map any execution in the compressed semantics to an execution in the annotated semantics. Indeed, a compressed execution is simply an annotated execution with some annotations (i.e., foc and rel) indicating the start of a positive/negative phase.

\begin{lemma}
For any configurations \(A, A'\) and \(\text{tr}\), \(A \xrightarrow{\text{tr}} A'\) implies \([A] \xrightarrow{[\text{tr}]} [A']\).
\end{lemma}

Going in the opposite direction is more involved. In general, mapping annotated executions to compressed ones requires to reorder actions. Compressed executions also force negative actions to be performed unconditionally, which we compensate by considering complete executions of a configuration, i.e., executions after which no more action can be performed except possibly the ones that consist in unfolding a replication (i.e., rule Repl). Inspired by the positive trunk argument of [24], we show the following lemma.

\begin{lemma}
Let \(A, A'\) be two configurations and \(\text{tr}\) be such that \(A \xrightarrow{\text{tr}} A'\) is complete. There exists a trace \(\text{tr}_c\), such that \([\text{tr}_c]\) can be obtained from \(\text{tr}\) by swapping independent labelled actions, and \([A] \xrightarrow{[\text{tr}_c]} [A']\).
\end{lemma}
Proof sketch. We proceed by induction on the length of a complete execution starting from $A$. If $A$ is not initial, then we need to execute some negative action using $\text{NEG}$: this action must be present somewhere in the complete execution, and we can permute it with preceding actions using Lemma 11. If $A$ is initial, we analyse the prefix of input and session actions and we extract a subsequence of that prefix that corresponds to a full positive phase.

4.2 Equivalence

We now define compressed trace equivalence ($\approx_c$) and prove that it coincides with $\approx$.

Definition 17. Let $A$ and $B$ be two configurations. We say that $A \subseteq_c B$ when, for any $A \triangleright_{\alpha} A'$ such that $bc(tr) \cap fc(B) = \emptyset$, there exists $B \triangleright_{\alpha} B'$ such that $\Phi(A') \sim \Phi(B')$. They are compressed trace equivalent, denoted $A \approx_c B$, if $A \subseteq_c B$ and $B \subseteq_c A$.

Compressed trace equivalence can be more efficiently checked than regular trace equivalence. Obviously, it explores less interleavings by relying on $\rightarrow_c$, rather than $\rightarrow$. It also requires that traces of one process can be played exactly by the other, including details such as non-observable actions, labels, and focusing annotations. The subtleties shown in Example 12 are crucial for the completeness of compressed equivalence w.r.t. regular equivalence. Since the compressed semantics forces to perform available outputs before e.g. input actions, some non-equivalences are only detected thanks to the labels and detailed non-observable actions of our annotated semantics.

Theorem 18. Let $A$ and $B$ be two action-deterministic configurations with $\text{skl}(A) = \text{skl}(B)$. We have $A \approx B$ if, and only if, $[A] \approx_c [B]$.

Proof sketch. $(\Rightarrow)$ Consider an execution $[A] \triangleright_{\alpha} A'$. Using Lemma 15 we get $A \triangleright_{\alpha} [A']$. Then, Lemma 13 yields $B \triangleright_{\alpha} B'$ for some $B'$ such that $\Phi([A']) \sim \Phi(B')$ and labelled skeletons are equal all along the executions. Relying on those skeletons, we show that positive/negative phases are synchronised, and thus $[B] \triangleright_{\alpha} B''$ for some $B''$ with $[B''] = B'$.

$(\Leftarrow)$ Consider an execution $A \triangleright_{\alpha} A'$. We first observe that it suffices to consider only complete executions there. This allows us to get a compressed execution $[A] \triangleright_{\alpha} [A']$ by Lemma 16.

Since $[A] \approx_c [B']$, there exists $B'$ such that $[B] \triangleright_{\alpha} B'$ with $\Phi([A']) \sim \Phi(B')$. Thus we have $B \triangleright_{\alpha} [B']$ but also $B \triangleright_{\alpha} [B']$ thanks to Lemma 11.

Improper blocks. Note that blocks of the form $\text{foc}(\alpha)$.tr$^+$.rel.zero do not bring any new information to the attacker. While it would be incorrect to fully ignore such improper blocks, we can show that it is sufficient to consider them at the end of traces. We can thus consider a further optimised compressed trace equivalence that only checks for proper traces, i.e., ones that have at most one improper block and only at the end of trace. We have also shown that this optimised compressed trace equivalence actually coincides with $\approx_c$.

5 Reduction

Our compressed semantics cuts down interleavings by using a simple focused strategy. However, this semantics does not analyse data dependency that happen when an input depends on an output, and is thus unable to exploit the independency of blocks to reduce interleavings. We tackle this problem now.

Definition 19. Two blocks $b_1$ and $b_2$ are independent, written $b_1 \parallel b_2$, when all labelled actions $\alpha_1 \in b_1$ and $\alpha_2 \in b_2$ are independent. Otherwise they are dependent, written $b_1 \parallel b_2$. 
An easy induction on the compressed trace \( \text{tr} \) allows us to map an execution w.r.t. the reduced semantics to an execution w.r.t. the compressed semantics.

\[ A \xrightarrow{\downarrow_{\text{ec}}} A' \text{ implies } A \xrightarrow{\downarrow_{\text{c}}} A'. \]
Next, we show that our reduced semantics only explores specific representatives. Given a frame \( \Phi \), a plausible trace \( tr \) is \( \Phi \)-minimal if it is minimal\(^1\) in its equivalence class modulo \( \equiv_{\Phi} \).

**Lemma 24.** Let \( A \) be an initial configuration and \( A' = (P; \emptyset; \Phi) \) be a configuration such that \( A \trianglelefteq_{c} A' \). We have that \( tr \) is \( \Phi \)-minimal if, and only if, \( A \trianglelefteq_{r} A' \).

**Proof sketch.** In order to relate minimality and executability in the reduced semantics, let us say that a trace is **bad** if it is of the form \( tr.b_0 \ldots b_n.b'.tr' \) where \( n \geq 0 \), there exists a block \( b'' \) such that \( (b'' \equiv_{E} b')\Phi \), we have \( b_i \parallel b'' \) for all \( i \), and \( b_i < b'' < b_0 \) for all \( i > 0 \). This pattern is directly inspired by the characterisation of lexicographic normal forms by Anisimov and Knuth in trace monoids \([4]\). We note that a trace that can be executed in the compressed semantics can also be executed in the reduced semantics if, and only if, it is not bad. Since the badness of a trace allows to swap \( b' \) before \( b_0 \), and thus obtain a smaller trace in the class \( \equiv_{\Phi} \), we show that a bad trace cannot be \( \Phi \)-minimal (and conversely). □

### 5.2 Equivalence

The reduced semantics induces an equivalence \( \approx_{r} \) that we define similarly to the compressed one, and we then establish its soundness and completeness w.r.t. \( \approx_{c} \).

**Definition 25.** Let \( A \) and \( B \) be two configurations. We say that \( A \sqsubseteq_{r} B \) when, for every \( A \trianglelefteq_{r} A' \) such that \( bc(tr) \cap fc(B) = \emptyset \), there exists \( B \trianglelefteq_{r} B' \) such that \( \Phi(A') \sim \Phi(B') \). They are reduced trace equivalent, denoted \( A \approx_{r} B \), if \( A \sqsubseteq_{r} B \) and \( B \sqsubseteq_{r} A \).

**Theorem 26.** Let \( A \) and \( B \) be two initial, action-deterministic configurations.

\[
A \approx_{r} B \text{ if, and only if, } A \approx_{c} B
\]

**Proof sketch.** We first prove that \( tr \equiv_{\Phi} tr' \) iff \( tr \equiv_{\Psi} tr' \) when \( \Phi \sim \Psi \). (⇒) This implication is then an easy consequence of Lemma 24 (⇐) We start by showing that it suffices to consider a complete execution \( A \trianglelefteq_{c} A' \). Since \( A' \) is initial, by taking \( tr_{m} \) to be a \( \Phi(A') \)-minimal trace associated to \( tr \), we obtain a reduced execution of \( A \) leading to \( A' \). Using our hypothesis \( A \approx_{r} B \), we obtain that \( B \trianglelefteq_{r} B' \) with corresponding relations over frames. We finally conclude that \( B \trianglelefteq_{r} B' \) using Lemma 24, and the result stated above. □

**Improper blocks.** Similarly as we did for the compressed semantics in Section 4, we can further restrict \( \approx_{r} \) to only check proper traces (see Appendix B).

## 6 Application

We have developed two successive refinements of the concrete semantics of our process algebra, eventually obtaining a reduced semantics that achieves an optimal elimination of redundant interleavings. However, the practical usability of these semantics in algorithms for checking the equivalence of replication-free processes is far from immediate: indeed, all of our semantics are still infinitely branching, because each input may be fed with arbitrary messages. We now discuss how existing decision procedures based on symbolic execution \([23, 14, 31, 10]\) can be modified to decide our optimised equivalences rather than the regular one, before presenting our implementation and experimental results.

---

\(^1\) Note that minimal traces are not unique, since only labelled skeletons are taken into account when comparing actions. However, the redundancy induced by the choice of recipes is not a concern in practice as it does not arise with the current constraint-based techniques for deciding trace equivalence.
6.1 Symbolic execution

Our compressed semantics can easily be used as a replacement of the regular one, in any tool whose algorithm is based on a forward exploration of the set of possible traces. This modification is very lightweight, and already brings a significant optimisation. In order to make use of our final, reduced semantics, we need to enter into the details of constraint solving. In addition to imposing the compressed strategy, symbolic execution should be modified to generate dependency constraints in order to reflect the data dependencies imposed by our predicate \( \mathsf{tr} \triangleright b \). Actually, the generation of dependency constraints can be done in a similar way as shown in Fig. \( \mathbf{3} \) (even if the class of processes considered in Fig. \( \mathbf{3} \) is more restrictive). We simply illustrate the effect of these dependency constraints on a simple example.

**Example 27.** We consider roles \( R_i := \mathsf{in}(c_i, x). \mathbf{if} \ x = \mathsf{ok} \ \mathbf{then} \ \mathsf{out}(c_i, \mathsf{ok}) \) where \( \mathsf{ok} \) is a public constant, and then consider a parallel composition of \( n \) such processes: \( P_n := \Pi_{i=1}^n R_i \).

Thanks to compression, we will only consider traces made of blocks, and obtained a first exponential reduction of the state space. Now, assume that our order prioritizes blocks on \( c_i \) w.r.t. those on \( c_j \) when \( i < j \), and consider a trace starting with \( \mathsf{in}(c_j, \cdot). \mathsf{out}(c_j, w_j) \). Trying to continue the exploration with an input action on \( c_i \) with \( i < j \), the dependency constraint added will impose that the recipe \( R_i \) used to feed the input on \( c_i \) makes use of the previous output to derive \( \mathsf{ok} \). Actually, this dependency constraint will impose more than that. Indeed, imposing that \( R_i \) has to use \( w_j \) is not very restrictive since in this example there are many ways to rely on \( w_j \) to derive \( \mathsf{ok} \), e.g. \( R_i = \mathsf{dec}(\mathsf{enc}(\mathsf{ok}, w_j), w_j) \) or even \( R_i = w_j \).

Actually, according to our reduced semantics, this step will be possible only if the message stored in \( w_j \) is mandatory to derive the message expected in input on channel \( c_i \). In this example, the conditional imposes that the recipe \( R_i \) leads to the message \( \mathsf{ok} \) (at least to pursue in the then branch), and even if there are many ways to derive \( \mathsf{ok} \), in any case the content of \( w_j \) is not mandatory for that. Thus, on this simple example, all the traces where the action \( \mathsf{in}(c_j, \cdot) \) is performed after the block on \( c_j \) with \( i < j \) will not be explored thanks to our reduction technique.

The constraint solver is then modified in a non-invasive way: dependency constraints are used to dismiss configurations when it becomes obvious that they cannot be satisfied. The modified verification algorithm may explore symbolic traces that do not correspond to \( \Phi \)-minimal representatives (when dependency constraints cannot be shown to be infeasible) but we will see that this approach allows us to obtain a very effective optimisation. Note that, because we may over-approximate dependency constraints, we must ensure that constraint resolution prunes executions in a symmetrical fashion for both processes being checked for equivalence.

**Remark.** A subtle point about compression is that it actually enhances reduction in a symbolic setting. Consider the process \( P = \mathsf{in}(c, x). \mathsf{out}(c, n_1). \mathsf{out}(c, n_2) \) in parallel with \( Q = \mathsf{in}(c', x) \). If \( \prec \) gives priority to \( Q \), then \( Q \) can only be scheduled after \( P \) if its input message requires the knowledge of one of the nonces \( n_1 \) and \( n_2 \) revealed by \( P \). Thus we have two symbolic interleavings, one of which is subject to a dependency constraint. Now, we could have applied the ideas of reduction directly on actions rather than blocks but we would have obtained three symbolic interleavings, reflecting the fact that if the input on \( c' \) depends on only the first output nonce, it should be scheduled before the second output.
6.2 Experimental results

The optimisations developed in the present paper have been implemented, following the above approach, in the official version of the state of the art tool Apte [11]. We now report on experimental results; sources and instructions for reproduction are available [21]. We only show examples in which equivalence holds, because the time spent on inequivalent processes is too sensitive to the order in which the (depth-first) exploration is performed.

*Toy example.* We consider again our simple example described in Section 6.1. We ran Apte on \( P_n \approx P_n \) for \( n = 1 \to 22 \), on a single 2.67GHz Xeon core (memory is not relevant). We performed our tests on the reference version and the versions optimised with the compressed and reduced semantics respectively. The results are shown on the left graph of Figure 3 in logarithmic scale: it confirms that each optimisation brings an exponential speedup, as predicted by our theoretical analysis.

![Figure 3](image-url) Impact of optimisations on toy example (left) and Denning-Sacco (right).

*Denning-Sacco protocol.* We ran a similar benchmark, checking that Denning-Sacco ensures strong secrecy in various scenarios. The protocol has three roles and we added processes playing those roles in turn, starting with three processes in parallel. The results are plotted on Figure 3. The fact that we add one role out of three at each step explains the irregular growth in verification time. We still observe an exponential speedup for each optimisation.

*Practical impact.* Finally, we illustrate how our optimisations make Apte much more useful in practice for investigating interesting scenarios. Verifying a single session of a protocol brings little assurance into its security. In order to detect replay attacks and to allow the attacker to compare messages that are exchanged, at least two sessions should be considered. This means having at least four parallel processes for two-party protocols, and six when a trusted third party is involved. This is actually beyond what the unoptimised Apte can handle in a reasonable amount of time. We show below how many parallel processes could be handled in 20 hours by the different versions of Apte on various use cases of protocols.

| Protocol                  | ref | comp | red |
|---------------------------|-----|------|-----|
| Needham Schroeder (3-party) | 4   | 6    | 7   |
| Private Authent. (2-party) | 4   | 7    | 7   |
| Yahalom (3-party)         | 4   | 5    | 5   |

| Protocol                  | ref | comp | red |
|---------------------------|-----|------|-----|
| Denning-Sacco (3-party)   | 5   | 9    | 10  |
| WMF (3-party)             | 6   | 12   | 13  |
| E-Passport PA (2-party)   | 4   | 7    | 9   |

7 Related Work

The techniques we have presented borrow from standard ideas from concurrency theory, trace theory and, perhaps more surprisingly, proof theory. Blending all these ingredients,
and adapting them to the demanding framework of security protocols, we have come up with partial order reduction techniques that can effectively be used in symbolic verification algorithms for equivalence properties of security protocols. We now discuss related work, and there is a lot of it given the huge success of POR techniques in various application areas. We shall focus on the novel aspects of our approach, and explain why such techniques have not been needed outside of security protocol analysis. These observations are not new: as pointed out by Baier and Katoen [7], “[POR] is mainly appropriate to control-intensive applications and less suited for data-intensive applications”; Clarke et al. [12] also remark that “In the domain of model checking of reactive systems, there are numerous techniques for reducing the state space of the system. One such technique is partial-order reduction. This technique does not directly apply to [security protocol analysis] because we explicitly keep track of knowledge of various agents, and our logic can refer to this knowledge in a meaningful way.”

We first compare our work with classical POR techniques. Then, we discuss more specifically previous works that use POR in a symbolic execution setting. Finally we comment on previous work in the domain of security protocol analysis.

7.1 Classical POR

Partial order reduction techniques have proved very useful in the domain of model checking concurrent programs. Given a Labelled Transition System (LTS) and some property to check (e.g., a Linear Temporal Logic formula), the basic idea of POR [26, 20, 7] is to only consider a reduced version of the given LTS whose enable transitions of some states might be not exhaustive but are such that this transformation does not affect the property. POR techniques can be categorized in two groups [20]. First, the persistent set techniques (e.g., stubborn sets, ample sets) where only a sufficiently representative subset of available transitions is explored. Second, sleep set techniques memoize past exploration and use this information along with available transitions to disable some provably redundant transitions. Note that these two kinds of techniques are compatible, and are indeed often combined to obtain better reductions. Theoretical POR techniques apply to transition systems which may not be explicitly available in practice, or whose explicit computation may be too costly. In such cases, POR is often applied to an approximation of the LTS that is obtained through static analysis. Another, more recent approach is to use dynamic POR [17, 20, 2] where the POR arguments are applied based on information that is obtained during the execution of the system.

Clearly, classical POR techniques would apply to our concrete LTS, but that would not be practically useful since this LTS is wildly infinite, taking into account all recipes that the attacker could build. Applying most classical POR techniques to the LTS from which data would have been abstracted away would be ineffective: any input would be dependent with any output (since the attacker’s knowledge, increased by the output, may enable new input messages). Our compression technique lies between these two extremes. It exploits a semi-commutation property: outputs can be permuted before inputs, but not the converse in general. Further, it exploits the fact that inputs do not increase the attacker’s knowledge, and can thus be executed in a chained fashion, under focus. The semi-commutation is reminiscent of the asymmetrical dependency analysis enabled by the conditional stubborn set technique [20], and the execution of inputs under focus may be explained by means of sleep sets. While it may be possible to formally derive our compressed semantics by instantiating...
abstract POR techniques to our setting, we have not explored this possibility in detail\(^2\). As mentioned earlier, the compressed semantics is inspired from another technique, namely focusing \(^3\) from proof theory. Concerning our reduced semantics, it may be seen as an application of the sleep set technique (or even as a reformulation of Anisimov’s and Knuth’s characterization of lexicographic normal forms) but the real contribution with this technique is to have formulated it in such a way that it can be implemented without requiring an *a priori* knowledge of data dependencies: it allows us to eliminate redundant traces on-the-fly as data (in)dependency is discovered by the constraint resolution procedure (more on this in the next sections) — in this sense, it may be viewed as a case of dynamic POR.

Narrowing the discussion a bit more, we now focus on the fact that our techniques are designed for the verification of equivalence properties. This requirement turns several seemingly trivial observations into subtle technical problems. For instance, ideas akin to compression are often applied without justification (e.g., in \([27, 29, 25]\)) because they are obvious when one does reachability rather than equivalence checking. To understand this, it is important to distinguish between two very different ways of applying POR to equivalence checking (independently of the precise equivalence under consideration). The first approach is to reduce a system such that the reduced system and the original systems are equivalent. In the second approach, one only requires that two reduced systems are equivalent iff the original systems are equivalent. The first approach seems to be more common in the POR literature (where one finds, e.g., reductions that preserve LTL-satisfiability \([7]\) or bisimilarity \([22]\) though there are instances of the second approach (e.g., for Petri nets \([19]\) ). In the present work, we follow the second approach: neither of our two reduction techniques preserves trace equivalence. This allows stronger reductions but requires extra care: one has to ensure that the independencies used in the reduction of one process are also meaningful for the other processes; in other words, reduction has to be symmetrical. This is the purpose of our annotated semantics and its “strong symmetry lemma” (Lemma \([13]\) ) but also, for the reduced semantics, of Lemma \([24]\) and Proposition \([41]\) . We come back to these two different approaches later, when discussing specific POR techniques for security.

### 7.2 Infinite data and symbolic execution

Symbolic execution is often used to verify systems dealing with infinite data, e.g., recipes and messages in security; integers, list, etc. in program verification. In many works combining POR and symbolic executions (e.g., \([27, 28, 29]\) ) the detection of redundant explorations does not rely on the data, and can thus be done trivially at the level of symbolic executions. In such cases, POR and symbolic execution are orthogonal. For instance, in \([27]\), two actions are *data-dependent* if one actions is a *send* action of some message \(m\) to some process \(p\) and the other is a *receive* action of process \(p\). This is done independently of \(m\), which is meaningful when one considers internal reduction (as is the case that work) but would be too coarse when one considers labelled transitions representing interactions with an environment that may construct arbitrary recipes from previous outputs (as is the case in our work). Due to this omnipotent attacker, POR techniques cannot be effective in our setting unless they really take data into account. Further, due to the infinite nature of data, and the dynamic

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\(^2\) Although this would be an interesting question, we do not expect that any improvement of compression would come out of it. Indeed, compression can be argued to be maximal in terms of eliminating redundant traces without analyzing data: for any compressed trace there is a way to choose messages and modify tests to obtain a concrete execution which does not belong to the equivalence class of any other compressed trace.

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Leibniz International Proceedings in Informatics (LIPIcs)

Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany
extension of the attacker’s knowledge, POR and symbolic execution must be integrated rather than orthogonal. Our notion of independence and trace monoid are tailored to take all this into account, including at the level of trace equivalence (cf. again Lemma 24). And, although reduction is presented (Section 5) in the concrete semantics, the crucial point is that the *authorised* predicate (Definition 22) is implemented as a special *dependency* constraint (Section 6) so that our POR algorithm fundamentally relies on symbolic execution. We have not seen such uses of POR outside of the security applications mentioned next [25, 6].

7.3 Security applications

The idea of applying POR to the verification of security protocols dates back, at least, to the work of Clarke *et al.* [12, 13]. In this work, the authors remark that traditional POR techniques cannot be directly applied to security mainly because “[they] must keep track of knowledge of various agents” and “[their] logic can refer to this knowledge in a meaningful way”. This led them to define a notion of semi-invisible actions (output actions, that cannot be swapped after inputs but only before them) and design a reduction that prioritizes outputs and performs them in a fixed order. Compared to our work, this reduction is much weaker (even weaker than compression only), only handles a finite set of messages, and only focuses on reachability properties checking.

In [18], Fokkink *et al.* model security protocols as labeled transition systems whose states contain the control points of different agents as well as previously outputted messages. They devise some POR technique for these transition systems, where output actions are prioritized and performed in a fixed order. In their work, the original and reduced systems are trace equivalent modulo outputs (the same traces can be found after removing output actions). The justification for their reduction would fail in our setting, where we consider standard trace equivalence with observable outputs. More importantly, their requirement that a reduced system should be equivalent to the original one makes it impossible to swap input actions, and thus reductions such as the execution under focus of our compressed semantics cannot be used. The authors leave as future work the problem of combining their algorithm with symbolic executions, in order to be able to lift the restriction to a finite number of messages.

Cremers and Mauw proposed a reduction technique based on the *sleep set* idea. Basically, when their exploration algorithm chooses to explore a specific action (an output or an input with its corresponding message), it will also add all the other available actions that have priority over the chosen one to the current *sleep set*. An action in this sleep set will never been explored. In this work, only reachability property are considered, and the reduction cannot be directly applied to trace equivalence checking. More importantly, the technique can only handle a finite set of messages. The authors identify as important future work the need to lift their method to the symbolic setting.

Earlier work by Mödersheim *et al.* has shown how to combine POR technique with symbolic semantics [25] in the context of reachability properties for security protocols, which has led to high efficiency gains in the OFMC tool of the AVISPA platform [5]. While their reduction is very limited, it brings some key insight on how POR may be combined with symbolic execution. In a model where actions are sequences of outputs followed by inputs, their reduction imposes a *differentiation* constraint on the interleavings of $\text{in}(c,x).\text{out}(c,m) | \text{in}(d,y).\text{out}(d,m')$. This constraint enforces that the symbolic interleaving $\text{in}(d,M').\text{out}(d,w').\text{in}(c,M).\text{out}(c,w)$ should only be explored for instances of $M$ that depend on $w'$. Our reduced semantics constrains patterns of arbitrary size (instead of just size 2 diamond patterns as above) by means of the *authorised* predicate (Definition 22). Moreover, our POR technique has been designed to be sound and complete for trace equivalence checking.
as well.

In a previous work [6], we settled the general ideas for the POR techniques presented in the present paper, but results were much weaker. That earlier work only dealt with the restrictive class of simple processes, which does not feature nested parallel composition or replication, and made heavy use of specific properties of processes of that class to define reductions and prove them correct. In the present work, we show that our two reduction techniques apply to a very large class of processes for reachability checking. For equivalence checking, we only require the semantic condition of action-determinism. Note that the results of the present paper are conservative over those of [6]: the reductions of [6] are obtained as a particular case of the results presented here in the case of simple processes. Finally, the present work brings a solid implementation in the state of the art tool Apte [9], whereas [6] did not present experimental results — it mentioned a preliminary implementation, extending SPEC, which we abandoned since it was difficult to justify and handled a more restricted class than APTE.

8 Conclusion

We have developed two POR techniques that are adequate for verifying reachability and trace equivalence properties of action-deterministic security protocols. We have effectively implemented them in Apte, and shown that they yield the expected, significant benefit.

We are considering several directions for future work. Regarding the theoretical results presented here, the main question is whether we can get rid of the action-determinism condition without degrading our reductions too much. Regarding the practical application of our results, we can certainly go further. We first note that our compression technique should be applicable and useful in other verification tools, not necessarily based on symbolic execution. Next, we could investigate the role of the particular choice of the order $\prec$, to determine heuristics for maximising the practical impact of reduction. Finally, we plan to adapt our treatment of replication to bounded replication to obtain a first symmetry elimination scheme, which should provide a significant optimisation when studying security protocols with several sessions.

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\section*{A Annotated semantics}

\subsection*{A.1 Reachability}

\begin{itemize}
  \item \textbf{Proposition 28.} Well labelling is preserved by \(\rightarrow_a\).
  \begin{proof}
    For all transitions except PAR and REPL, the multiset of labels of the resulting configuration is a subset of labels of the original configuration. Thus, well labelling is obviously preserved in those cases.

    Consider now a PAR transition, represented below without the permutation, which does not play a role here:
    \[
    ([\{P_1^{n_1}\}] \cup \mathcal{P}; \Phi) \xrightarrow{\text{par}(\tau_1, \ldots, \tau_n)} ([\{P_1^{\ell_1}, \ldots, [P_n^{\ell_n}]\}] \cup \mathcal{P}; \Phi)
    \]
    We check that labels are pairwise independent. This is obviously the case for the new labels \(\ell \cdot i\). Let us now consider a label \(\ell'\) from \(\mathcal{P}\) and show that it is independent from any \(\ell \cdot i\). It cannot be equal to one of them (otherwise it would be a suffix of \(\ell\), which contradicts the well labelling of the initial configuration) and it cannot be a strict prefix either (otherwise it would be a prefix of \(\ell\) too). Finally, \(\ell \cdot i\) cannot be a prefix of \(\ell'\) because \(\ell\) is not a prefix of \(\ell'\).

    As far as labels are concerned, REPL transitions are a particular case of PAR where there are only two sub-processes. Thus REPL preserves well labelling.
  \end{proof}
  \end{itemize}

\begin{itemize}
  \item \textbf{Lemma 11.} Let \(A\) be a (well labelled) configuration, \(\alpha\) and \(\beta\) be two independent labelled actions. We have \(A \xrightarrow{\alpha} A'\) if, and only if, \(A \xrightarrow{\beta} A'\).
  \begin{proof}
    By symmetry it is sufficient to show one implication. Assuming \(\ell_1\) and \(\ell_2\) to be independent, we consider a transition labelled \(\alpha = [\alpha']^{\ell_1}\) followed by one labelled \(\beta = [\beta']^{\ell_2}\).

    We first observe that a transition labelled \(\ell_1\) can only generate new labels that are dependent with \(\ell_1\). Thus, \(\ell_2\) must be present in the original configuration and our execution is of the following form, where we write \(P_\alpha\) (resp. \(P_\beta\)) instead of \([\alpha']^{\ell_1}\) (resp. \([\beta']^{\ell_2}\)):
    \[
    A = (\mathcal{P} \uplus \{P_\alpha, P_\beta\}; \Phi) \xrightarrow{\text{lo}(\ell_1)} (\mathcal{P} \uplus \mathcal{P}_\alpha \uplus \{P_\beta\}; \Phi_\alpha) \xrightarrow{\text{lo}(\ell_2)} (\mathcal{P} \uplus \mathcal{P}_\alpha \uplus \mathcal{P}_\beta; \Phi_\beta)
    \]
    It remains to check that \(\beta\) can be performed by \(P_\beta\) in the original configuration, and that doing so would not prevent the \(\alpha\) transition to happen next. The only thing that could prevent \(\beta\) from being performed is that the frames \(\Phi\) and \(\Phi_\alpha\) may be different, in the case where \(\alpha\) is an input. In that case, the recipe independence hypothesis guarantees that \(\beta\) does not rely on the new handle introduced by \(\alpha\) and can thus be played with only \(\Phi\). Finally, performing \(\alpha\) after \(\beta\) is easy. We only detail the case where \(\beta = \text{out}(c, w)\) and \(\alpha\) is an input of recipe \(M\). In that case we have \(\Phi_\alpha = \Phi, \Phi_\beta = \Phi_\alpha \uplus \{w \mapsto m\}\), and \(M \in \mathcal{T}(\text{dom}(\Phi))\). We observe that \(M \in \mathcal{T}(\text{dom}(\Phi_\beta))\) and we construct the execution:
    \[
    A = (\mathcal{P} \uplus \{P_\alpha^{f_1}, [P_\beta]^{f_2}\}; \Phi) \xrightarrow{\text{lo}(\ell_1)} (\mathcal{P} \uplus \mathcal{P}_\beta \uplus \{P_\beta\}; \Phi_\beta) \xrightarrow{\text{lo}(\ell_2)} (\mathcal{P} \uplus \mathcal{P}_\alpha \uplus \mathcal{P}_\beta; \Phi_\beta)
    \]
  \end{proof}
  \end{itemize}

\subsection*{A.2 Equivalence}

\begin{itemize}
  \item \textbf{Definition 29.} Given a process \(P\), we define the set of its \textit{enabled skeletons} as
    \[
    \text{enable}(P) = \begin{cases} \{\text{sk}(P)\} & \text{if } P \text{ starts with an observable action} \\ \bigcup \{\text{sk}(P_i)\} & \text{if } P = \Pi_i P_i \\ \emptyset & \text{if } P = 0 \end{cases}
    \]
\end{itemize}
We may consider skeletons, labelled skeletons and enabled skeletons of a configuration by taking the set of the corresponding objects of all its processes.

- **Property 30.** For any configurations $A$, $A'$ and non-observable action $\alpha$, if $A \xrightarrow{\alpha} A'$ or $A \xrightarrow{\alpha} A'$ then $\text{enable}(A) = \text{enable}(A')$.

- **Property 31.** Let $A$ be an action-deterministic configuration and $P$, $Q$ two of its processes. We have that $\text{enable}(P) \cap \text{enable}(Q) = \emptyset$.

- **Lemma 32.** Let $A$ be an action-deterministic configuration. If $A \xrightarrow{tr_1} A_1$ and $A \xrightarrow{tr_2} A_2$ for some traces $tr_1, tr_2$ such that $\text{obs}(tr_1) = \text{obs}(tr_2)$ then $\text{enable}(A_1) = \text{enable}(A_2)$ and $\Phi(A_1) = \Phi(A_2)$.

**Proof.** We first prove a stronger result when the configurations $A_1$ and $A_2$ are canonical, i.e. only contain processes that are neither 0 nor a parallel composition. Actually, in such a case, we prove that $A_1 = A_2$.

To prove this intermediate result, we proceed by induction on $\text{obs}(tr_1)$. The base case is trivial. Let us show the inductive case. We assume that $tr_1 = tr_1^\alpha.\alpha.tr_1^\alpha$ with $\alpha$ an observable action and $tr_1^\alpha$ containing only non-observable actions. Since $\text{obs}(tr_1) = \text{obs}(tr_2)$, we have that $tr_2 = tr_2^\alpha.\alpha.tr_2^\alpha$ with $tr_2^\alpha$ containing only non-observable actions and $\text{obs}(tr_2^\alpha) = \text{obs}(tr_1^\alpha)$. Our given executions are thus of the form:

\[
A \xrightarrow{tr_1} A_1^0 \xrightarrow{\alpha} A_1 \quad \text{and} \quad A \xrightarrow{tr_2} A_2^0 \xrightarrow{\alpha} A_2
\]

It may be the case that $A_1^0$ or $A_2^0$ are not canonical. The idea is to reorder some non-observable actions. More precisely, we perform all available non-observable actions of $A_1^0$ and $A_2^0$ before performing $\alpha$. By doing this, we do not change the observable actions of the different sub-traces and obtain

\[
A \xrightarrow{tr_1} A_1^0 \xrightarrow{\alpha} A_1 \quad \text{and} \quad A \xrightarrow{tr_2} A_2^0 \xrightarrow{\alpha} A_2
\]

with $A_1^0$ and $A_2^0$ canonical. By inductive hypothesis, we have that $A_1^0 = A_2^0$. We now must show $A_1 = A_2$. By action-determinism of $A$, there is only one process $P$ that can perform $\alpha$ in $A_1^0 (= A_2^0)$. The resulting process $P'$ after performing $\alpha$ is thus the same in the two executions. Since $A_1$ and $A_2$ are canonical and $tr_1^\alpha$ and $tr_2^\alpha$ contain only non-observable actions, $A_1 = A_2$.

In order to be able to apply our previous result, we complete the executions with all available non-observable actions:

\[
A \xrightarrow{tr_1} A_1 \xrightarrow{\alpha} A_1' \quad \text{and} \quad A \xrightarrow{tr_2} A_2 \xrightarrow{\alpha} A_2'
\]

such that $A_1$ and $A_2$ are canonical and $tr_1^\alpha$ and $tr_2^\alpha$ contain only non-observable actions. We also have that:

- $\Phi(A_1) = \Phi(A_1')$ and $\text{enable}(A_1) = \text{enable}(A_1')$; and
- $\Phi(A_2) = \Phi(A_2')$ and $\text{enable}(A_2) = \text{enable}(A_2')$.

We now conclude thanks to our previous result, and obtain $A_1' = A_2'$ implying the desired equalities.$\blacksquare$

- **Proposition 33.** Let $A$ and $B$ be two action-deterministic configurations such that $A \approx B$. If $A \xrightarrow{tr_A} A'$ and $B \xrightarrow{tr_B} B'$ with $\text{obs}(tr_A) = \text{obs}(tr_B)$ then $\Phi(A') \sim \Phi(B')$ and $\text{enable}(A') = \text{enable}(B')$. 

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Proof. By hypothesis, we know that $A \approx B$, and also that $A \xrightarrow{w} A'$. Moreover, the freshness conditions on channels (i.e., $bc(tr_A) \cap fc(B) = \emptyset$) holds as $B$ is able to perform $tr_B$, and $tr_A$ and $tr_B$ share the same bound channels. Hence, we know that there exist $tr'_B$ and $B''$ such that

$$ B \xrightarrow{w} B'', \quad obs(tr_A) = obs(tr'_B), \quad \text{and} \quad \Phi(A') \sim \Phi(B''). $$

Now, since $B$ is an action-deterministic configuration, applying Lemma 32 on $tr_B$ and $tr'_B$, we obtain that $\text{enable}(B') = \text{enable}(B'')$ and $\Phi(B') = \Phi(B'')$. This allows us to conclude that $\Phi(A') \sim \Phi(B')$.

It remains to show that $\text{enable}(A') = \text{enable}(B')$. By symmetry, we only show one inclusion. Let $\alpha_s \in \text{enable}(A')$, we shall show that $\alpha_s \in \text{enable}(B')$. We deduce from the latter that there is a trace $tr'$ that is either $\alpha$ or $\tau.\alpha$ (where $\alpha$ is an observable action whose the skeleton is $\alpha_s$) such that

$$ A \xrightarrow{\alpha_s} A' \xrightarrow{\alpha} A_0 $$

for some $A_0$. Since $A \approx B$, we know that there exist $tr'_0$, $tr'$, $B'_0$, and $B'$ such that

$$ B \xrightarrow{\alpha_s} B'_0 \xrightarrow{\alpha} B_0 $$

with $\Phi(A_0) \sim \Phi(B_0)$ and $\text{obs}(tr_A) = \text{obs}(tr'_0)$ and $\text{obs}(tr') = \alpha$. we have that $\alpha \in \text{enable}(B'_0)$. In particular, using Property 30, we have that $\alpha_s \in \text{enable}(B'_0)$.

Now, since $B$ is an action-deterministic configuration, applying Lemma 32 on $tr_B$ and $tr'_B$, we obtain $\text{enable}(B') = \text{enable}(B'_0)$, and thus $\alpha_s \in \text{enable}(B')$. △

**Proposition 34.** Let $A$ be an action-deterministic configuration and $P, Q$ two of its processes. If $\text{enable}(P) = \text{enable}(Q)$ then $\text{sk}(P) = \text{sk}(Q)$.

Proof. Let us show that $\text{sk}(P) = \text{sk}(Q)$. If $\text{enable}(P)$ is an empty set then $P = 0$ and thus from $\text{enable}(Q) = \emptyset$ we deduce that $Q = 0$ as well implying the required equality on skeletons. If $\text{enable}(P)$ is a singleton then it must be $\{\text{sk}(P)\}$ — we cannot be in the case where $P$ is a parallel composition, for in that case there would be at least two skeletons in $\text{enable}(P)$ by action-determinism of $A$. The same goes with $Q$ thus we have $\{\text{sk}(P)\} = \{\text{sk}(Q)\}$. Finally, if $\text{enable}(P)$ contains at least two skeletons then it must be the case that $P$ is a parallel composition of the form $P \parallel Q$ and $\text{enable}(P) = \text{un}_1\{\text{sk}(P_1)\}$. Similarly, $Q$ must be of the form $Q \parallel Q_1$ and $\text{enable}(Q) = \text{un}_1\{\text{sk}(Q_1)\}$. Here, we make use of action-determinism to obtain that the number of subprocesses in parallel is the same as the cardinality of the sets of skeletons, and thus the same for $P$ and $Q$: indeed, no two parallel subprocesses can have the same skeleton. We conclude that $\text{sk}(P) = \text{par}(S)$ where $S$ is the ordered sequence of skeletons from $\text{un}_1\{\text{sk}(P_1)\}$, and $\text{sk}(Q) = \text{par}(S)$ where $S$ is the ordered sequence of skeletons from $\text{un}_1\{\text{sk}(Q_1)\}$.

△

**Lemma 13.** Let $A$ and $B$ be two action-deterministic configurations such that $A \approx B$ and $\text{skl}(A) = \text{skl}(B)$. For any execution

$$ A \xrightarrow{[\alpha_1]^i A_1 \ldots \rightarrow_a A_n} $$

with $bc(\alpha_1, \ldots, \alpha_n) \cap fc(B) = \emptyset$, there exists an execution

$$ B \xrightarrow{[\alpha_1]^i B_1 \ldots \rightarrow_a B_n} $$

such that $\Phi(A_i) \sim \Phi(B_i)$ and $\text{skl}(A_i) = \text{skl}(B_i)$ for any $1 \leq i \leq n$. △
Proof. We show this result by induction on the length of the derivation \( A \xrightarrow{\ell} A_n \). The case where \( \ell \) is empty (i.e., no action even a non-observable one) is obvious. Assume that we have proved such a result for all the executions of length \( n \), and we want to establish the result for an execution of length \( n + 1 \).

Consider an execution of \([\alpha_1]^{f_1} \cdots [\alpha_n]^{f_n}\) from \( A \) to \( A_n \), followed by \([\alpha_{n+1}]^{f_{n+1}}\) towards \( A_{n+1} \). By induction hypothesis, we know that there exists an execution

\[
B \xrightarrow{\ell_1} B_1 \xrightarrow{\ell_2} \cdots \xrightarrow{\ell_n} B_n
\]

such that \( \Phi(A_n) \sim \Phi(B_n) \) and \( \skl(A_i) = \skl(B_i) \) for any \( 1 \leq i \leq n \). It remains to establish that there exists \( B_{n+1} \) such that \( B_n \) can perform \([\alpha_{n+1}]^{f_{n+1}}\) towards \( B_{n+1} \), \( \Phi(A_{n+1}) \sim \Phi(B_{n+1}) \) and \( \skl(A_{n+1}) = \skl(B_{n+1}) \). We distinguish several cases depending on the action \( \alpha_{n+1} \).

Case \( \alpha_{n+1} = \text{zero} \). We have that \([\text{zero}]^{f_{n+1}} \in \skl(A_n)\), and thus, since \( \skl(A_n) = \skl(B_n) \), we have also that \([\text{zero}]^{f_{n+1}} \in \skl(B_n) \). We deduce that \( A_n = ([0]^{f_{n+1}} \uplus P_0; \Phi_0) \), and \( B_n = ([0]^{f_{n+1}} \uplus Q_0; \Psi_0) \) for some \( P_0 \), \( Q_0 \), \( \Phi_0 \), and \( \Psi_0 \). Moreover, since \( \skl(A_n) = \skl(B_n) \), we deduce that \( \skl(P_0) = \skl(Q_0) \). Let \( B_{n+1} = (Q_0; \Psi_0) \). We have that:

\[
\begin{align*}
B_n &= ([0]^{f_{n+1}} \uplus Q_0; \Psi_0) \xrightarrow{[\text{zero}]^{{f_{n+1}}}} (Q_0; \Psi_0) = B_{n+1}, \\
\Phi(A_{n+1}) &= \Phi(A_n) \sim \Phi(B_{n+1}) = \Phi(B_{n+1}), \text{ and} \\
\skl(A_{n+1}) &= \skl(P_0) = \skl(Q_0) = \skl(B_{n+1}).
\end{align*}
\]

Case \( \alpha_{n+1} = \text{par}(S) \) for some sequence \( S = (\beta_1, \ldots, \beta_k) \). Note that this sequence is ordered according to our order \( < \) over skeletons (i.e., \( \beta_1 < \ldots < \beta_k \)) and \( \beta_i \)'s are pairwise distinct by action-determinism of \( A \). We have \([\text{par}(S)]^{f_{n+1}} \in \skl(A_n)\), and thus, since \( \skl(A_n) = \skl(B_n) \), we have also that \([\text{par}(S)]^{f_{n+1}} \in \skl(B_n) \). \( A_n = ([\Pi_{i=1}^k P_i]^{f_{n+1}} \uplus P_0; \Phi_0) \), \( \uplus_{i=1}^k \skl(P_i) = \{\beta_1, \ldots, \beta_k\} \) and \( B_n = ([\Pi_{i=1}^k Q_i]^{f_{n+1}} \uplus Q_0; \Psi_0) \) for some \( P_i, Q_i, P_0, Q_0, \Phi_0, \) and \( \Psi_0 \). Further, we have

\[
A_{n+1} = ([\uplus_{i=1}^k ([P_{\pi(i)}]^{f_{n+1}}) \uplus P_0; \Phi_0)
\]

for some permutation \( \pi \) over \([1; k]\) such that \( \skl(P_{\pi(i)}) = \beta_i \) for all \( i \). Moreover, since \( \skl(A_n) = \skl(B_n) \), we deduce that \( \skl(P_0) = \skl(Q_0) \), and

\[
\{\skl(P_i) \mid 1 \leq i \leq k\} = \{\skl(Q_i) \mid 1 \leq i \leq k\} = \{\beta_1, \ldots, \beta_k\}
\]

Remark that, since \( P_i \) (resp. \( Q_i \)) cannot be a zero or a parallel we have that \( \text{enable}(P_i) = \{\skl(P_i)\} \) (resp. \( \text{enable}(Q_i) = \{\skl(Q_i)\} \)) and those sets are singletons. Moreover, by action-determinism of \( A \) and \( B \) we know that all those singletons are pairwise disjoint. From this, we conclude that there exists a permutation \( \pi' \) over \([1; k]\) such that

\[
\forall i, \ \skl(Q_{\pi'(i)}) = \beta_i = \skl(P_{\pi(i)})
\]

and thus

\[
\forall i, \ \skl([Q_{\pi'(i)}]^{f_{n+1}}) = \skl([P_{\pi(i)}]^{f_{n+1}})
\]

We can finally let \( B_{n+1} \) be \( ([\uplus_{i=1}^k ([Q_{\pi'(i)}]^{f_{n+1}}) \uplus Q_0; \Psi_0) \) and we have:

\[
\begin{align*}
B_n &= ([\Pi_{i=1}^k Q_i]^{f_{n+1}} \uplus Q_0; \Psi_0) \xrightarrow{[\text{par}(S)]^{f_{n+1}}} ([\uplus_{i=1}^k ([Q_{\pi'(i)}]^{f_{n+1}}) \uplus Q_0; \Psi_0) = B_{n+1}, \\
\Phi(A_{n+1}) &= \Phi(A_n) \sim \Phi(B_{n+1}) = \Phi(B_{n+1}), \text{ and} \\
\skl(A_{n+1}) &= \skl(P_0) \uplus \uplus_{i=1}^k \skl([P_{\pi(i)}]^{f_{n+1}}) \\
&= \skl(Q_0) \uplus \uplus_{i=1}^k \skl([Q_{\pi'(i)}]^{f_{n+1}}) = \skl(B_{n+1}).
\end{align*}
\]
Case $\alpha_{n+1} = \text{in}(c, M)$ for some $c$, and $M \in \mathcal{T}(\text{dom}(\Phi(A_n)))$. We have that $[\text{in}]^{\ell_{n+1}} \in \text{skl}(A_n)$, and thus, since $\text{skl}(A_n) = \text{skl}(B_n)$, we have $[\text{in}]^{\ell_{n+1}} \in \text{skl}(B_n)$. We deduce that $A_n = \{[\text{in}(c, x_A), P]^{\ell_{n+1}}\} \cup \mathcal{P}_0; \Phi_0$, and $B_n = \{[\text{in}(c, x_B), Q]^{\ell_{n+1}}\} \cup \mathcal{Q}_0; \Psi_0$ for some $x_A, x_B, P, Q, \mathcal{P}_0, \mathcal{Q}_0, \Phi_0,$ and $\Psi_0$. Since $A \approx B$ and thus $\text{dom}(\Phi(A)) = \text{dom}(\Phi(B))$, we have that $\text{dom}(\Phi_0) = \text{dom}(\Psi_0)$. Moreover, since $\text{skl}(A_n) = \text{skl}(B_n)$, we deduce that $\text{skl}(\mathcal{P}_0) = \text{skl}(\mathcal{Q}_0)$. Let $B_{n+1} = \{[Q[u_B/x_B]]^{\ell_{n+1}}\} \cup \mathcal{Q}_0; \Psi_0$ where $u_B = M\Psi_0$. We have that:

- $B_n = [\text{in}(c, M)]^{\ell_{n+1}} \cup \mathcal{P}_0; \Phi_0$, and $\Phi(A_n) \sim \Phi(B_n) \sim \Phi(B_{n+1})$.

It remains to show that $\text{skl}(A_{n+1}) = \text{skl}(B_{n+1})$. Since we have that $\text{skl}(\mathcal{P}_0) = \text{skl}(\mathcal{Q}_0)$, and the label of the new subprocess is the same (namely, $\ell_{n+1}$) on both sides, we only need to show that:

$$\text{skl}(P[M\Phi_0/x_A]) = \text{skl}(Q[M\Psi_0/x_B])$$

In order to improve the readability, we will note $P' = P[M\Phi_0/x_A]$ and $Q' = Q[M\Psi_0/x_B]$. We have that $A$ and $B$ are two action-deterministic configurations such that $A \approx B$. Moreover, they perform the same trace, respectively towards $A_{n+1}$ and $B_{n+1}$. Thus, thanks to Proposition 3[3], we deduce that $\text{enable}(A_{n+1}) = \text{enable}(B_{n+1})$. Moreover, our hypothesis $\text{skl}(\mathcal{P}_0) = \text{skl}(\mathcal{Q}_0)$ implies that $\text{enable}(\mathcal{P}_0) = \text{enable}(\mathcal{Q}_0)$, and thus we deduce that $\text{enable}(P') = \text{enable}(Q')$ (recall that by action-determinism, unions of the form $\text{enable}(A_{n+1}) = \text{enable}(\mathcal{P}_0) \cup \text{enable}(P')$ are actually disjoint unions). We conclude using Proposition 3[3].

Case $\alpha_{n+1} = \text{out}(c, w)$ for some $c$ and some $w$ with $w \notin \text{dom}(\Phi(A_n))$. This case is similar to the previous one. However, during such a step, the frame of each configuration is enriched, and thus the fact that $\Phi(A_{n+1}) \sim \Phi(B_{n+1})$ is now a consequence of Proposition 3[3].

Case $\alpha_{n+1} = \text{sess}(a, \bar{c})$ for some $a$, and some $\bar{c}$. Firstly, we show for later that $\bar{c}$ are fresh in $B_n$. Indeed, we deduce from $bc(\alpha_1, \ldots, \alpha_{n+1}) \cap fc(B) = \emptyset$ that $\bar{c}$ are fresh in $B$ and we know that free channels of $B_n$ are included in $fc(B) \cup bc(\alpha_1, \ldots, \alpha_n)$. Thereby, if there was a $c_i \in \bar{c} \cap fc(B_n)$ it would be in $bc(\alpha_1, \ldots, \alpha_n)$ but this is forbidden because of the freshness condition (in the current trace) over channels, i.e., new channels cannot be introduced twice (once in $\alpha_1, \ldots, \alpha_n$ and once in $\alpha_{n+1}$).

As before, we obtain

$$A_n = \{[[\alpha_1, \ldots, \alpha_n P]^{\ell_{n+1}}]\} \cup \mathcal{P}_0; \Phi_0 \text{ and } B_n = \{[[\alpha_1, \ldots, \alpha_n Q]^{\ell_{n+1}}]\} \cup \mathcal{Q}_0; \Psi_0$$

for some $P, Q, \mathcal{P}_0, \mathcal{Q}_0, \Phi_0,$ and $\Psi_0$. Moreover, since $\text{skl}(A_n) = \text{skl}(B_n)$, we deduce that $\text{skl}(\mathcal{P}_0) = \text{skl}(\mathcal{Q}_0)$.

We have

$$A_{n+1} = \{[[P]^{\ell_{n+1}}, [[\alpha_1, \ldots, \alpha_n Q]^{\ell_{n+1}}], P]^{\ell_{n+1}}\} \cup \mathcal{P}_0; \Phi_0$$

Accordingly, let us pose

$$B_{n+1} = \{[[Q]^{\ell_{n+1}}, [[\alpha_1, \ldots, \alpha_n Q]^{\ell_{n+1}}], Q]^{\ell_{n+1}}\} \cup \mathcal{Q}_0; \Psi_0$$

We have that:

- $B_n \upharpoonright \text{sess}(a, \bar{c})^{\ell_{n+1}} \cup \mathcal{P}_0; \Phi_0$.
- $\Phi(A_n) \sim \Phi(B_n) \sim \Phi(B_{n+1})$.

It remains to show that $\text{skl}(A_{n+1}) = \text{skl}(B_{n+1})$. Since we have that $\text{skl}(\mathcal{P}_0) = \text{skl}(\mathcal{Q}_0)$, and since the labels of corresponding subprocesses are the same on both sides, we only need to show that:

....α
Case 2. Otherwise, when there is at least one process in Case 1.

There exists a trace which allows us to conclude using Proposition 34.

Proof. Let \( A \) and \( B \) be two action-deterministic configurations. We have that \( A \subseteq_a B \) if, for every \( A' \) such that \( A \not\subseteq_\alpha A' \) with \( \text{bc}(tr) \cap \text{fc}(B) = \emptyset \), then there exists \( B' \) such that \( B \not\subseteq_\alpha B' \), and \( \Phi \sim \Psi \). They are in annotated trace equivalence, denoted \( A \approx_a B \), if \( A \subseteq_a B \) and \( B \subseteq_a A \).

Lemma 36. Let \( A \) and \( B \) be two action-deterministic configurations such that \( \text{sk}(A) = \text{sk}(B) \).

\[
A \approx B \text{ if, and only if, } A \approx_a B
\]

Proof. Firstly, \( A \approx_a B \) trivially implies \( A \approx B \). For the other direction, we use Lemma 13 to conclude.

B Compression

B.1 Reachability

Lemma 16. Let \( A \), \( A' \) be two configurations and \( tr \) be such that \( A \not\subseteq_\alpha A' \) is complete. There exists a trace \( tr_c \), such that \( [tr_c] \) can be obtained from \( tr \) by swapping independent labelled actions, and \( [A] \not\subseteq_\alpha [A'] \).

Proof. Let \( A = (P; \Phi) \) be a configuration and \( (P; \Phi) \not\subseteq_\alpha A' \) a complete execution. We proceed by induction on the length of \( tr \), distinguishing two cases.

Case 1. We first consider the case where there is at least one process in \( P \) that is negative and non-replicated. Since we are considering a complete execution, at least one negative action \( \alpha \) is performed on this process in \( tr \). This action may be an output, the decomposition of a parallel composition, or the removal of a zero. If there are more than one such action, we choose the one that can be performed using \( \text{NEG} \), i.e., the one that is minimal according to our arbitrary order on labelled skeletons. Since our action can be performed initially by our process, and by well-labelling, the label of the action is independent with all labels of previously executed actions in \( tr \). Moreover, there cannot be any second-order dependency between \( \alpha \) and one of those actions. Indeed, if \( \alpha \) is an output, no input performed before \( \alpha \) is able to use the handle of \( \alpha \). It can thus be swapped before all the others by using Lemma 11 obtaining an execution of trace \( \alpha.tr' \) ending in the same configuration \( A' \). The rule \( \text{NEG} \) can be performed in the compressed semantics to trigger the action \( \alpha \), and by induction hypothesis on \( tr' \) we can complete our compressed execution towards \( A' \).

Case 2. Otherwise, when \( P \) contains only positive or replicated processes, we must choose one process to focus on, start a positive phase and execute all its actions until we can finally release the focus. As long as all processes are positive or replicated, only input or session actions can be performed. In either case, the action yields a new process (the continuation

\[
\text{sk}(P) = \text{sk}(Q) \quad \text{and} \quad \text{sk}([P; \Phi]) = \text{sk}([Q; \Phi])
\]

As in the previous case, thanks to Proposition 33, we know that \( \text{enable}(A_{n+1}) = \text{enable}(B_{n+1}) \), and we deduce that \( \text{enable}(P) = \text{enable}(Q) \) and \( \text{skl}([P; \Phi]) = \text{skl}([Q; \Phi]) \), which allows us to conclude using Proposition 34.

Finally, we can define the annotated trace equivalence and show that, for action-deterministic configurations, it coincides with trace equivalence.

Definition 35. Let \( A \) and \( B \) be two configurations. We have that \( A \subseteq_a B \) if, for every \( A' \) such that \( A \not\subseteq_\alpha A' \) with \( \text{bc}(tr) \cap \text{fc}(B) = \emptyset \), then there exists \( B' \) such that \( B \not\subseteq_\alpha B' \), and \( \Phi \sim \Psi \). They are in annotated trace equivalence, denoted \( A \approx_a B \), if \( A \subseteq_a B \) and \( B \subseteq_a A \).

Lemma 36. Let \( A \) and \( B \) be two action-deterministic configurations such that \( \text{sk}(A) = \text{sk}(B) \).

\[
A \approx B \text{ if, and only if, } A \approx_a B
\]

Proof. Firstly, \( A \approx_a B \) trivially implies \( A \approx B \). For the other direction, we use Lemma 13 to conclude.
of the input, or the new session) which may be negative or positive. We define the positive prefix of our execution as the prefix of actions for which all but the last transition yield a positive process. It is guaranteed to exist because $A'$ contains only negative processes.

The positive prefix is composed only of input and replication actions. Because session actions are performed by negative processes, and no new negative process is created in the positive prefix, session actions can be permuted at the beginning of the prefix thanks to Lemma 11. Thus, we assume without loss of generality that the prefix is composed of session actions, followed by input actions: we write $\text{tr} = \text{tr}_0,\text{tr}_1,\text{tr}_0$. In the portion of the execution where inputs of $\text{tr}_m$ are performed, there is an obvious bijective mapping between the processes of any configuration and its successor, allowing us to follow execution threads, and to freely permute inputs pertaining to different threads. Such permutations are made possible by Lemma 11. Indeed, they concern actions that are (i) sequentially independent (i.e., labels are independent) since two different threads involve actions performed by processes in parallel and (ii) recipe independent since there is no output action in $\text{tr}_m$.

The last action of the positive prefix releases a negative process $P^-$. Let $P$ be its antecedent (through its corresponding thread) in the configuration obtained after the execution of $\text{tr}$. We have that:

$$\text{tr} = (P; \Phi) \xrightarrow{\text{tr}_0} (P_1; \Phi) \xrightarrow{\text{tr}_1} (P_2; \Phi) \xrightarrow{\text{tr}_2} (P; \Phi) \xrightarrow{\text{tr}_0} A'$$

Now we can write $\text{tr} = \text{tr}_0 \cup P$ where $P$ is either $P$ or a replicated process that gives rise to $P$ in one transition. By permuting actions pertaining to $P$ before all others thanks to Lemma 11 and previous remarks, we obtain an execution of the form

$$\text{tr} = (P_0; \Phi) \xrightarrow{\text{tr}_1} (P'_0 \cup P^-; \Phi) \xrightarrow{\text{tr}_2} (P; \Phi) \xrightarrow{\text{tr}_3} A'$$

where $\text{tr}_0,\text{tr}_2,\text{tr}_3$ is a permutation of $\text{tr}$ of independent labelled actions, $\text{tr}_1 = [\alpha]^\ell \text{tr}'_1$, and $P'_0 = P$ when $P_1 = P$, and $P'_0 = P_0 \cup P_1$ otherwise.

In the compressed semantics, if we initiate a focus on $P_1$ we can execute the actions of $\text{tr}_1$ and release the focus when reaching $P^-$, i.e., we have that (where $\ell'$ is the label of $P^-$):

$$(P; \emptyset; \Phi) \xrightarrow{\text{rel}(\alpha)^{\ell'}_c} (P'_0; \Phi) \xrightarrow{\text{rel}(\alpha)^{\ell'}_c} (P'_0 \cup P^-; \emptyset; \Phi)$$

We can conclude by induction hypothesis on $\text{tr}_2,\text{tr}_3$.

---

### B.2 Equivalence

We prove below the two implications of Theorem 18 dealing first with soundness and then with the more involved completeness result.

**Lemma 37 (Soundness).** Let $A$ and $B$ be action-deterministic configurations such that $\text{skl}(A) = \text{skl}(B)$. We have that $A \approx B$ implies $[A] \approx_e [B]$.

**Proof.** By symmetry it suffices to show $[A] \subseteq_e [B]$. Consider an execution $[A] \xrightarrow{\text{tr}} A'$ such that $\text{bc}([\text{tr}]) \cap \text{fc}(B) = \emptyset$. Thanks to Lemma 15, we know that $A \xrightarrow{\text{tr}} [A']$. Let $[\text{tr}] = [\alpha_1]^\ell_1 \ldots [\alpha_n]^\ell_n$, and we denote $A_1, \ldots, A_n$ the intermediate configurations that are reached during this execution. We have that:

$$A_0 = A \xrightarrow{[\alpha_1]^\ell_1} A_1 \xrightarrow{[\alpha_2]^\ell_2} \ldots \xrightarrow{[\alpha_n]^\ell_n} A_n = (P; \Phi_A).$$

Applying Lemma 13, we deduce that $B$ can perform a very similar execution (same labels, same actions), i.e.,

$$B_0 = B \xrightarrow{[\alpha_1]^\ell_1} B_1 \xrightarrow{[\alpha_2]^\ell_2} \ldots \xrightarrow{[\alpha_n]^\ell_n} B_n = (Q; \Phi_B).$$
with $\Phi(A_i) \sim \Phi(B_i)$ and $\text{skl}(A_i) = \text{skl}(B_i)$ for $0 \leq i \leq n$.

Due to this strong symmetry, we are sure that $|B|$ will be able to do this execution in the compressed semantics. In particular, the fact that a given configuration $B_i$ can start a positive phase or has to release the focus is determined by the set $\text{skl}(B_i) = \text{skl}(A_i)$ and the fact that it can keep the focus on a specific process while performing positive actions can be deduce from labels of $\text{tr}$. Finally, we have shown that if $A_i$ can execute an action $\alpha$ using NEG rule then $B_i$ can as well. The only missing part is about the fact that NEG has been made non-branching using an arbitrary order on labelled skeletons. Let say we can use NEG only for actions whose skeleton is minimal among others skeletons of available, negative actions. Using $\text{skl}(A_i) = \text{skl}(B_i)$, we easily show that this is symmetric for $A_i$ and $B_i$. This way, we obtain an execution $[B] \vdash_{\text{skl}} B'$ with $|B'| = B_n$. Finally, we have $\Phi(B') = \Phi_B \sim \Phi_A = \Phi(A')$.

Lemma 38. Let $A$ and $B$ be two action-deterministic configurations such that $A \equiv_c B$. If $A \vdash_{\text{skl}} A'$ and $B \vdash_{\text{skl}} B'$ for a labelled trace $\text{tr}$ then $A' \equiv_c B'$.

Proof. We assume $A \equiv_c B$, $A \vdash_{\text{skl}} A'$ and $B \vdash_{\text{skl}} B'$ for a labelled trace $\text{tr} = \alpha_1 \ldots \alpha_n$. We shall prove $A' \equiv_c B'$. By symmetry, we show one inclusion. Consider an execution $A' \vdash_{\text{skl}} A_2$ such that $bc(\text{tr}_2) \cap \text{fc}(B') = \emptyset$. Let us construct an execution $B' \vdash_{\text{skl}} B_2$ such that $\Phi(A_2) \sim \Phi(B_2)$. Firstly, remark that since $B \vdash_{\text{skl}} B'$, we have that $bc(\text{tr}) \cap \text{fc}(B) = \emptyset$. In order to exploit our hypothesis $A \equiv_c B$, we shall prove that $bc(\text{tr}_2) \cap \text{fc}(B) = \emptyset$. All that remains to show is $bc(\text{tr}_2) \cap (\text{fc}(B) \setminus \text{fc}(B')) = \emptyset$. This is implied by the fact that channels in $\text{fc}(B) \setminus \text{fc}(B')$ must occur in $\text{tr}$ as bound channels and, because of the execution $A \vdash_{\text{skl}} A_2$, those channels cannot appear in the set $bc(\text{tr}_2)$.

We have that $A \vdash_{\text{skl}} A_2$ and thus by hypothesis, $B \vdash_{\text{skl}} B_2$ with $\Phi(B_2) \sim \Phi(A_2)$. Since there can be at most one process in $B$ that has a label that matches the label of $\alpha_1$, there is at most one configuration $B_1$ that satisfies $B \vdash_{\text{skl}} B_1$. By iteration, we obtain the unicity of the configuration $B'$ satisfying $B \vdash_{\text{skl}} B'$. We thus have obtained the required execution $B' \vdash_{\text{skl}} B_2$.

Lemma 39. Let $A$ and $B$ be two action-deterministic configurations. If for any complete execution $A \vdash_{\text{skl}} A'$ with $bc(\text{tr}) \cap \text{fc}(B) = \emptyset$, there exists a trace $\text{tr}'$ and an execution $B \vdash_{\text{skl}} B'$ such that $\Phi(A') \sim \Phi(B')$, then $A \equiv_c B$.

Proof. Let $A \vdash_{\text{skl}} A_0$ be an execution of $A$ with $bc(\text{tr}_0) \cap \text{fc}(B) = \emptyset$. Firstly, we complete the latter execution in an arbitrary way $A \vdash_{\text{skl}} A'$ such that any process of $A'$ is replicated and $bc(\text{tr}_0, \text{tr}_1) \cap \text{fc}(B) = \emptyset$ (it suffices to choose fresh channels for $B$ as well). By hypothesis, there exists an execution $B \vdash_{\text{skl}} B'$ such that $\Phi(A') \sim \Phi(B')$. The latter execution of $B$ is thus of the form $B \vdash_{\text{skl}} B_0 \vdash_{\text{skl}} B'$. It remains to show that $\Phi(A_0) \sim \Phi(B_0)$. For the sake of contradiction, we assume that $\Phi(B_0) \sim \Phi(A_0)$ does not hold. In other words, there is a test of equality over $\text{dom}(\Phi(B_0))$ that holds for $\Phi(A_0)$ and does not for $\Phi(B_0)$ (or the converse). Since $\text{dom}(\Phi(B_0)) \subseteq \text{dom}(\Phi(B')) = \text{dom}(\Phi(A'))$, this same test can be used to conclude that $\Phi(A') \sim \Phi(B')$ does not hold as well leading to an absurd conclusion.

Lemma 40 (Completeness). Let $A$ and $B$ be two action-deterministic configurations satisfying $\text{skl}(A) = \text{skl}(B)$. Then $[A] \equiv_c [B]$ implies $A \equiv B$.

Proof. Assume $[A] \equiv_c [B]$, thanks to Lemma 36 it suffices to show $A \equiv_c B$. Let us show the following intermediate result: for any complete execution $A \vdash_{\text{skl}} A'$ such that $bc(\text{tr}) \cap \text{fc}(B) = \emptyset$, there is an execution $B \vdash_{\text{skl}} B'$ such that $\Phi(A') \sim \Phi(B')$. Thanks to
Lemma 39 and by symmetry of $\approx_n$, this intermediate result implies the required conclusion $A \approx_n B$.

Let $A \triangleleft_{\pi} A'$ be a complete execution with $bc(tr) \cap fc(B) = \emptyset$. We thus have that $A'$ is initial. Applying the Lemma 10 we obtain a trace $tr$ such that $[A] \triangleleft_{\pi} [A']$ and $[tr]$ can be obtained from $tr$ by swapping independent actions. Since we have $[A] \approx_n [B]$, we deduce that $[B] \triangleleft_{\pi} [B']$ with $\Phi(A') \sim \Phi(B')$. Lemma 15 gives us $B \triangleleft_{\pi\alpha\beta} B'$. We can now apply Lemma 11 to obtain $B \approx \approx_n B'$ and conclude.

C Reduction

C.1 Reachability

Lemma 21. Let $A$ and $A'$ be two initial configurations such that $A \triangleleft_{\pi} A'$. We have that $A \not\equiv_{\pi\alpha\beta} A'$ for any $tr' \equiv_{\Phi(A')}$ tr.

Proof. Thanks to Lemma 13 we have that $[A] \not\Downarrow_{\pi\alpha} (P; \Phi)$. We first prove that $tr'$ can be performed using $\rightarrow_n$. For this, it suffices to establish the implication for each of the two generators of $\equiv_{\pi\alpha\beta}$. The first case is given by Lemma 11. The second one is a common property of (derivatives of) the applied $\pi$-calculus that follows from a simple observation of the transition rules. Finally, we must prove that $tr'$ can be played using $\rightarrow_c$. Thanks to initiality of $A$ and $(P; \emptyset; \Phi)$ we know that each block of $tr$ starts when the configuration is initial and after performing it we get another initial configuration. This is still true in $[A] \not\Downarrow_{\pi\alpha} (P; \Phi)$. Finally, labels of blocks of $tr'$ ensures that a single process is used in a positive part of any block. Having proven those two facts, we can easily show that each block of $tr'$ can be performed using $\rightarrow_c$.

For the sake of readability of the following proofs, we now introduce some notations (where $b_1, b_2$ are two blocks and $\Phi$ is a frame):

- we note $b_1 \parallel' b_2$ when $b_1$ and $b_2$ are sequentially independent (i.e., for any $a_1 \in b_1$ and $a_2 \in b_2$, $a_1$ and $a_2$ are sequentially independent);
- we note $b_1 \parallel' b_2$ when $b_1$ and $b_2$ are recipe independent (i.e., for any $a_1 \in b_1$ and $a_2 \in b_2$, $a_1$ and $a_2$ are recipe independent);
- for two traces $tr = b_1 \ldots b_n$ and $tr' = b'_1 \ldots b'_m$, we note $(tr =_{\pi} tr')\Phi$ when $n = m$ and for all $i \in [1; n]$, $(b_i =_{\pi\alpha} b'_i)\Phi$.

Remark. Let $A \triangleleft_{\pi\alpha} A'$ be any compressed execution. If $b_1$ and $b_2$ are two blocks of $tr$ and $a_1$ (resp. $a_2$) is the first labelled action of $b_1$ (resp. $b_2$) we have the following:

$$b_1 \parallel' b_2 \iff a_1 \text{ is sequentially independent with } a_2.$$ 

This is implied by the fact that for any other action $a'_1$ of $b_1$, its label has the label of $a_1$ as a prefix.

Lemma 24. Let $A$ be an initial configuration and $A' = (P; \emptyset; \Phi)$ be a configuration such that $A \triangleleft_{\pi\alpha} A'$. We have that $tr$ is $\Phi$-minimal if, and only if, $A \not\approx_{\pi\alpha\beta} A'$.

Proof. Let $A$ and $(P; \emptyset; \Phi)$ be two configurations such that $A \triangleleft_{\pi\alpha} (P; \emptyset; \Phi)$.

($\Rightarrow$) We first show that if $tr$ is $\Phi$-minimal, then $A \not\equiv_{\pi\alpha\beta} (P; \emptyset; \Phi)$ by induction on the trace $tr$. The base case, i.e., $tr = \epsilon$ is straightforward. Now, assume that $tr = tr_0 \cdot b$ for some block $b$ and $A \not\equiv_{\pi\alpha\beta} (P_0; \emptyset; \Phi_0)$ $\not\equiv_{\pi\alpha\beta} A'$. Since $tr$ is $\Phi$-minimal, we also have that $tr_0$ is $\Phi_0$-minimal and thus we obtain by induction hypothesis that $A \not\equiv_{\pi\alpha\beta} (P_0; \emptyset; \Phi_0)$.

($\Leftarrow$) To
conclude, it remains to show that \( tr_0 \triangleright b' \) for any \( b' \) such that \( (b' =_E b) \Phi_0 \). Assume that it is not the case, this means that for some \( b' \) such that \( (b =_E b') \Phi_0 \), the trace \( tr_0 \) can be written \( tr_0', b_0, \ldots, b_n \) with

\[ b_i \parallel b' \text{ and } b_i \prec b \text{ for any } i > 0, \text{ as well as } b_0 \parallel b' \text{ and } b' \prec b_0. \]

Let \( tr' = tr_0', b' b_0 \ldots b_n \). We have \( tr' \prec_{\text{lex}} tr \) and \( tr' \equiv_{\Phi} tr_0', b' \), which contradicts the \( \Phi \)-minimality of \( tr \).

\((\Leftarrow)\) Now, we assume that \( tr \) is not \( \Phi \)-minimal, and we want to establish that \( tr \) cannot be executed in the reduced semantics. Let \( tr_m \) be the \( \Phi \)-minimal trace of the equivalence class of \( tr \). We have in particular \( tr_m \equiv_{\Phi} tr \) and \( tr_m \prec_{\text{lex}} tr \). Now, we let \( tr^*_m \) (resp. \( tr^r \)) be the “trace of labelled skeletons” associated to \( tr_m \) (resp. \( tr \)). Let \( tr^*_0 \) be the longest common prefix of \( tr^*_m \) and \( tr^r \) and \( tr^*_0 \) be the corresponding prefix of \( tr \) (resp. \( tr_m \)). We have a decomposition of the form \( tr = tr_0 . b . tr_1 \) and \( tr_m = tr'_0 . b_m . tr'_1 \) with \( (tr_0 =_E tr'_0) \Phi \) and \( b_m \prec b \).

Again, since when dropping recipes, the relation \( \equiv_{\Phi} \) only swaps sequentially independent labelled skeletons, block \( b_m \) must have a counterpart in \( tr \) and, more precisely, in \( tr_1 \). We thus have a more precise decomposition of \( tr \): \( tr = tr_0 . b . tr_1 . b'_m . tr_{12} \) such that \( (b'_m =_E b_m) \Phi \).

We now show that \( b'_m \) cannot be executed after \( tr_0 . b . tr_{11} \) in the reduced semantics (assuming that the trace has been executed so far in the reduced semantics). In other words, we show that \( tr_0 . b . tr_{11} \triangleright b'_m \) does not hold. We have seen that \( (b'_m =_E b_m) \Phi \) and \( b_m \prec b \) so it suffices to show:

\[ b_m \parallel b_i \text{ for any } b_i \in b . tr_{11} \]

First, we prove \( b_i \parallel^d b_m \) (i.e., they are recipe independent) for any \( b_i \in b . tr_{11} \). This comes from the fact that \( tr'_0 . b_m . tr'_1 = tr_m \) is plausible, and thus the inputs of \( b_m \) only use handles introduced in \( tr'_0 \) which are the same as those introduced in \( tr_0 \). In particular, the inputs of \( b_m \) do not rely on the handles introduced in \( b . tr_{11} \). Similarly, using the fact that \( tr_0 . b . tr_{11} . b'_m . tr_{12} = tr \) is plausible and \( b'_m = b_m \), we deduce that handles of outputs of \( b_m \) are not used in \( b . tr_{11} \).

Second, we show that \( b_i \parallel^s b_m \) (i.e., they are sequentially independent) for any \( b_i \in b . tr_{11} \). For this, we remark that for any traces \( tr_1 . b . tr_2 \equiv_{\Phi} tr'_1 . b' . tr'_2 \) such that \( (b =_E b') \Phi \), we have that \( b_i \parallel^s b_m \) for all \( b_i \in \text{skl}(tr'_1) \backslash \text{skl}(tr'_1) \) where \( \text{skl}(tr) \) is the multiset of labelled skeletons of blocks of \( tr \) and \( \backslash \) should be read as multiset subtraction. This can be easily shown by induction on the relation \( \equiv_{\Phi} \). By applying this helping remark to \( tr_0 . b_m . tr'_2 \equiv_{\Phi} tr_0 . b . tr_{11} . b'_m . tr_{12} \), we obtain the required conclusion: \( b'_m \parallel^s b . tr_{11} \) and thus \( b_m \parallel^s b . tr_{11} \).

\section{C.2 Equivalence}

\textbf{Proposition 41.} For any static equivalent frames \( \Phi \sim \Psi \) and compressed traces \( tr \) and \( tr' \), we have that \( tr \equiv_{\Phi} tr' \) if, and only if, \( tr \equiv_{\Psi} tr' \).

\textbf{Proof.} The two implications are symmetric, we thus only show one implication. Considering the two generators of \( \equiv_{\Phi} \), the only non-trivial step is to show that \( tr . b_i . tr' \equiv_{\Phi} tr . b_i . tr' \) when \( (b_i =_E b_2) \Phi \). But the latter condition, together with \( \Phi \sim \Psi \), yields \( (b_1 =_E b_2) \Psi \) which allows us to conclude. \hfill \( \blacksquare \)

\textbf{Lemma 42.} Let \( A \) and \( B \) be two action-deterministic configurations. If for any complete execution of the form \( A \overset{bc}{\sqsubseteq_{c}} (P; \emptyset; \Phi) \) with \( bc(tr) \cap \text{fc}(B) = \emptyset \), there exists an execution \( B \overset{bc}{\sqsubseteq_{c}} (Q; \emptyset; \Psi) \) such that \( \Phi \sim \Psi \), then \( A \sqsubseteq B \).

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Leibniz International Proceedings in Informatics

Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany
Proof. Let \( A \) and \( B \) be two action-deterministic configurations, and assume that for any complete execution \( A \xrightarrow{\text{tr}} (P; \emptyset; \Phi) \) with \( \text{bc}(\text{tr}) \cap \text{fc}(B) = \emptyset \), there exists an execution \( B \xrightarrow{\text{tr}} (Q; \emptyset; \Psi) \) such that \( \Phi \sim \Psi \). Now, we have to establish that \( A \sqsubseteq_c B \).

Let \((P'; \emptyset; \Phi')\) be a configuration such that \( A \xrightarrow{\text{tr}} (P'; \emptyset; \Phi') \). First, we can complete this execution to reach a process \((P; \emptyset; \Phi)\) such that each process \( P \in \Pi \) is replicated, i.e.,

\[
A \xrightarrow{\text{tr}} (P'; \emptyset; \Phi') \xrightarrow{\text{tr}} (P; \emptyset; \Phi)\]

Without loss of generality, we can choose \( \text{tr} \) so that it satisfies \( \text{bc}(\text{tr}) \cap \text{fc}(B) = \emptyset \). By hypothesis, we know that there exists an execution \( B \xrightarrow{\text{tr}} (Q; \emptyset; \Psi) \) such that \( \Phi \sim \Psi \). Let \( B' \) be the configuration reached along this execution after the execution of \( \text{tr}' \) and \( \Psi' \) its frame. Similarly to the proof of Lemma [39] we prove that \( \Phi \sim \Psi \) implies \( \Phi' \sim \Psi' \).

\[\boxed{\text{Theorem (26). Let } A \text{ and } B \text{ be two initial, action-deterministic configurations.} \]

\[\text{implies } A \sqsubseteq_c B \text{ if, and only if, } A \sqsubseteq_c B\]

Proof. We prove the two directions separately.

\((\Rightarrow) \) \( A \sqsubseteq_c B \) implies \( A \sqsubseteq_r B \). Consider an execution of the form \( A \xrightarrow{\text{tr}} (P; \emptyset; \Phi) \) with \( \text{bc}(\text{tr}) \cap \text{fc}(B) = \emptyset \). Using Lemma [23] we know that \( A \xrightarrow{\text{tr}} (P; \emptyset; \Phi) \), and Lemma [24] tells us that \( \text{tr} \) is \( \Phi \)-minimal. Since \( A \sqsubseteq_c B \), we deduce that there exists \((Q; \emptyset; \Psi)\) such that:

\[B \xrightarrow{\text{tr}} (Q; \emptyset; \Psi) \text{ and } \Phi \sim \Psi\]

Now, by Proposition [41] we obtain that \( \text{tr} \) is also \( \Psi \)-minimal, and so Lemma [24] tells us that the execution of \( \text{tr} \) by \( B \) can also be performed in the reduced semantics.

\((\Leftarrow) \) \( A \sqsubseteq_r B \) implies \( A \sqsubseteq_c B \). Relying on Lemma [12] it is actually sufficient to show that for any complete execution \( A \xrightarrow{\text{tr}} (P; \emptyset; \Phi) = A' \) with \( \text{bc}(\text{tr}) \cap \text{fc}(B) = \emptyset \), there exists an execution of the form \( B \xrightarrow{\text{tr}} (Q; \emptyset; \Psi) \) such that \( \Phi \sim \Psi \). Note that since the given execution is complete, we have that \( A' \) is initial. Let \( \text{tr}' \) be a \( \Phi \)-minimal trace in the equivalence class of \( \text{tr} \). We have that \( A \) executes \( \text{tr}' \) in the reduced semantics, and so for some \( B' \) we have

\[B \xrightarrow{\text{tr}} B' \text{ and } \Phi(B') \sim \Phi\]

Using Lemma [23] we obtain the same execution in the compressed semantics. Finally, by Proposition [41] we obtain \( \text{tr}' \equiv_{\Phi(B')} \text{tr} \), and by Lemma [21] we obtain:

\[B \xrightarrow{\text{tr}} B' \text{ and } \Phi(B') \sim \Phi\]

\[\boxed{\text{D. Optimization for improper blocks}}\]

Between any two initial configurations, the compressed as well as the reduced semantics execute a sequence of actions of the form of the form \( \text{fc}(\alpha).\text{tr}^+.\text{rel}.\text{tr}^- \) where \( \text{tr}^+ \) is a (possibly empty) sequence of input actions, whereas \( \text{tr}^- \) is a (non-empty) sequence of \( \text{out}, \text{par} \), and \( \text{zero} \) actions. Such sequences are called \emph{blocks}. Now, we also make a distinction between blocks having a null negative phase (i.e., \( \text{tr}^- = \text{zero} \)), and the others. The former are called \emph{improper blocks} whereas the latter are called \emph{proper blocks}. Finally, we say that a trace is \emph{proper} if it contains at most one improper block and only at the end of the trace.

The idea is that, when checking trace equivalence, we do not have to consider all possible traces but we can actually restrict to proper ones. We present below an improved version of the notions of compressed and reduced equivalence.
D.1 Compression

Definition 43. Let A and B be two configurations. We have that \( A \subseteq_{c+i} B \) if, for every \( A' \) such that \( A \rightharpoonup_{c} A' \) and \( \text{bc}(tr) \cap \text{fc}(B) = \emptyset \) for some proper \( tr \), there exists \( B' \) such that \( B \rightharpoonup_{c} B' \) with \( \Phi(A') \sim \Phi(B') \). We write \( A \approx_{c+i} B \), if \( A \subseteq_{c+i} B \) and \( B \subseteq_{c+i} A \).

Operationally, we can obtain \( \approx_{c+i} \) by adding a case to the \textit{RELEASE} rule (and constraining the former rule to not apply in that case):

\[
\text{RELEASE}_{i} \quad (P; [0]^{c}; \Phi) \rightarrow_{(\text{rel})^{c}} (\emptyset; \emptyset; \Phi)
\]

This rule discards exactly the traces containing an improper block that is not at the end: note that having 0 under focus implies that the negative part of the block will be restricted to zero. This is because no negative actions was available before performing this block and consequently there can only be a zero in the negative part of the block.

Proposition 44. Let A and B be two initial action-deterministic configurations.

\[ A \approx_{c} B \text{ if, and only if, } A \approx_{c+i} B \]

Proof. The \((\Rightarrow)\) direction is trivial. We focus on the other one. Assume that \( A \subseteq_{c+i} B \). Let \( A' \) be such that \( A \rightharpoonup_{c} A' \) for some \( tr \) such that \( \text{bc}(tr) \cap \text{fc}(B) = \emptyset \). Let \( b_1, \ldots, b_k \) be the improper blocks that occur in \( tr \). We have that \( b_1, \ldots, b_k \) are pairwise independent. Moreover, we have that there exists \( tr' \) made of proper blocks such that \( tr', b_1, \ldots, b_k \) is obtained from \( tr \) by swapping independent blocks, and thus we have that \( A \rightharpoonup_{c} A' \). There exist initial configurations \( A_0, A_1, \ldots, A_k \) such that \( A \rightharpoonup_{c} A_0 \), and

\[ A_0 \rightharpoonup_{c} A_1, \quad A_0 \rightharpoonup_{c} A_2, \ldots, A_0 \rightharpoonup_{c} A_k. \]

Thanks to our hypothesis, we deduce that there exist configurations \( B_0, B_1, \ldots, B_k \) such that \( B \rightharpoonup_{c} B_0 \) with \( \Phi(A_0) \sim \Phi(B_0) \), and

\[ B_0 \rightharpoonup_{c} B_1, \quad B_0 \rightharpoonup_{c} B_2, \ldots, B_0 \rightharpoonup_{c} B_k. \]

We have also that \( \Phi(A_0) \sim \Phi(B_0) \). Since blocks \( b_1, \ldots, b_k \) are pairwise independent, we deduce that there exist \( B' \) such that \( B \rightharpoonup_{c} B_0 \rightharpoonup_{c} B' \) with \( \Phi(B') \sim \Phi(B_0) \). Then, permutations of blocks can be undone to retrieve \( tr \) (since swapping have been done between independent blocks).

D.2 Reduction

Definition 45. Let A and B be two configurations. We have that \( A \subseteq_{r+i} B \) if, for every \( A' \) such that \( A \rightharpoonup_{r} A' \) and \( \text{bc}(tr) \cap \text{fc}(B) = \emptyset \) for some proper \( tr \), there exists \( B' \) such that \( B \rightharpoonup_{r} B' \) with \( \Phi(A') \sim \Phi(B') \). We write \( A \approx_{r+i} B \), if \( A \subseteq_{r+i} B \) and \( B \subseteq_{r+i} A \).

Proposition 46. Let A and B be two initial action-deterministic configurations.

\[ A \approx_{r} B \text{ if, and only if, } A \approx_{r+i} B \]

Proof. The \((\Rightarrow)\) direction is trivial. We focus on the other one. Assume that \( A \subseteq_{r+i} B \). Let \( A' \) be such that \( A \rightharpoonup_{r} A' \) for some \( tr \) such that \( \text{bc}(tr) \cap \text{fc}(B) = \emptyset \). Lemma 23 tells us that \( A \rightharpoonup_{c} A' \), and thanks to Lemma 24, we have that \( tr \) is \( \Phi(A') \)-minimal.
Let \( b_1, \ldots, b_k \) be the improper blocks that occur in \( \text{tr} \). We have that there exist \( \text{tr}_0, \text{tr}_1, \ldots, \text{tr}_k \) made of proper blocks such that \( \text{tr} = \text{tr}_0, b_1, \text{tr}_1, b_2, \ldots, \text{tr}_{k-1}, b_k, \text{tr}_k \). We have that \( b_1, \ldots, b_k \) are pairwise independent, and also:

\[
\text{tr} \equiv_{\Phi(\text{A}')} \text{tr}_0, \text{tr}_1 \ldots \text{tr}_k, b_1 \ldots b_k
\]

Because there are no dependencies between \( b_i \) and \( \text{tr}_j \) for \( i < j \), and because the \( b_i \) do not have any output, we have that \( A \xrightarrow{\text{tr}_0, \text{tr}_1 \ldots \text{tr}_{k-1}, b_k} A_0 \), and also that:

\[
A \xrightarrow{\text{tr}_0, b_1} A_1, A \xrightarrow{\text{tr}_0, \text{tr}_1, b_2} A_2, \ldots, A \xrightarrow{\text{tr}_0, \text{tr}_1 \ldots \text{tr}_{k-1}, b_k} A_k.
\]

Thanks to our hypothesis, we deduce that there exist \( B_0, B_1, \ldots, B_k \) such that \( B \xrightarrow{\text{tr}_0, \text{tr}_1 \ldots \text{tr}_{k-1}, b_k} B_0 \) with \( \Phi(A_0) \sim \Phi(B_0) \), and also that:

\[
B \xrightarrow{\text{tr}_0, b_1} B_1, B \xrightarrow{\text{tr}_0, \text{tr}_1, b_2} B_2, \ldots, B \xrightarrow{\text{tr}_0, \text{tr}_1 \ldots \text{tr}_{k-1}, b_k} B_k.
\]

We deduce that there exists \( B' \) such that \( B \xrightarrow{\text{tr}_0, \text{tr}_1 \ldots \text{tr}_{k-1}, b_k} B' \). Next, we observe that \( \Phi(A') = \Phi(A_0) \sim \Phi(B_0) = \Phi(B') \). From this we conclude \( \text{tr} \equiv_{\Phi(\text{B}')} \text{tr}_0, \text{tr}_1 \ldots \text{tr}_k, b_1 \ldots b_k \), hence \( B \xrightarrow{\text{tr}_0} B' \). Since \( \text{tr} \) is \( \Phi(A') \)-minimal it is also \( \Phi(B') \)-minimal, and thus \( B \xrightarrow{\text{tr}_0} B' \) by Lemma 24. \( \blacktriangleleft \)