Centre-of-Mass Energy of Colliding Particles in the Background of Charged Black Strings

IBRAR HUSSAIN

School of Electrical Engineering and Computer Science
National University of Sciences and Technology
H-12, Islamabad, Pakistan

E-mail: ibrar.hussain@seecs.nust.edu.pk

Abstract. The centre-of-mass (CM) energy of collision for two particles falling freely from rest at infinity is investigated in the background of a charged black string. It is found that like black holes, the CM energy of collision is arbitrarily high in the vicinity of the horizon of the black string if one of the colliding particles has critical charge and the other particle takes any different value of the charge.

Key words: Charged black strings; Particles collision; Centre-of-mass energy

1. Introduction

Bañados, Silk and West (BSW) found that infinite centre-of-mass (CM) energies can be produced during the particles collision in the vicinity of the event horizon of the extremal Kerr black hole [1]. A kinematic explanation of acceleration of particles by black holes can be found in [2]. A general explanation for the effect of unbound acceleration of particles by black holes is available in [3]. After the seminal work of BSW, their mechanism was then applied to a series of black hole spacetimes [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. Berti et al. and Jacobson and Sotiriou obtained some practical limitations on the achievable CM energy from the astrophysical limitations [24, 25].

Following the same sprite, in the next section we study the CM energy of collision for two charged particles falling freely from rest at infinity in the background of a charged black string. We show that like black holes, the charged black string can also accelerate the colliding particles to an unlimited CM energy if one of the colliding particles has critical charge and the other particle takes any different value of the charge.

We conclude our discussion in the last section. Here we use the system of units $G = c = 1$. 

Published under licence by IOP Publishing Ltd
2. Charged Black Strings and Collision Energy in the CM Frame

The Hoop conjecture, proposed by Kip Thorne in 1972 [26], states that horizons form when and only when mass $M$ of an object gets compressed into a region whose circumference in every direction is less than its Schwarzschild circumference, $4\pi M$. This excluded the possibility of the formation of cylindrical black holes. However, the Hoop conjecture holds only if the cosmological constant vanishes. Thus in the presence of a negative cosmological constant cylindrical black holes (also called black strings) [27], have been investigated with great interest. They represent asymptotically anti-de Sitter (AdS) spaces in the transverse and axial directions. Here we discuss the collision process for the charged black string. The line element of the charged black string is given by [28]

$$ds^2 = -f dt^2 + \frac{1}{f} dr^2 + r^2 d\phi^2 + \alpha^2 r^2 dz^2,$$  

where

$$f = \alpha^2 r^2 - \frac{4M}{\alpha r} + \frac{4Q^2}{\alpha^2 r^2},$$  

and $\alpha^2 = -\Lambda/3 > 0$, denotes the negative cosmological constant. In (2) $M$ and $Q$ are the Arnowitt-Deser-Misner (ADM) mass and charge per unit length in the $z$ direction. The event horizon can be obtained by putting $f = 0$, as [29]

$$r_{\pm} = \frac{(4M)^{\frac{1}{2}}}{2\alpha} \left[\sqrt{s} \mp \sqrt{2\sqrt{s^2 - Q^2(\frac{2}{M})^{\frac{3}{2}}} - s}\right].$$  

Here $r_{\pm}$ represents the outer and inner horizons and the quantity $s$ in (3) is defined by

$$s = \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{64Q^6}{27M^4}}\right)^{\frac{1}{2}} + \left(\frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{64Q^6}{27M^4}}\right)^{\frac{1}{2}}.$$  

The singularity at $r = 0$ is surrounded by the horizons $r_{\pm}$ if the inequality $Q^2 \leq \frac{3}{4} M^{\frac{3}{2}}$, holds. The black string will be extremal if

$$Q^2 = \frac{3}{4} M^{\frac{3}{2}}.$$  

The electric potential is defined by

$$A = \frac{2Q}{\alpha r^2}.$$  

Consider two charged particles having the same rest mass $m$ are at rest at infinity and then they start to move toward the charged black sting defined above. Let these particles
collided at some distance $r$. Let $q_1$ and $q_2$ be the charge of the two incoming particles respectively. The collision energy in the CM frame is defined by

$$E_{cm} = m \sqrt{2} \sqrt{1 - g_{\mu\nu}u_1^\mu u_2^\nu},$$

(7)

where

$$u^\mu = \frac{dx^\sigma}{d\tau} \left( \frac{\partial}{\partial x^\sigma} \right)^\mu$$

(8)

is the four-velocity of each of the colliding particle at the point of collision [1]. The motion of particles can be determined by the following Lagrangian [30]

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + \frac{qA^\mu}{m} \dot{x}^\mu,$$

(9)

where $A^\mu$ is the electromagnetic 4-potential and $\dot{x}^\mu = dx^\mu/d\tau$.

For the charged black string (1) the Lagrangian (9) becomes

$$L = \frac{1}{2} \left[ -f \dot{t}^2 + \frac{1}{f} r^2 + r^2 \dot{\phi}^2 + \alpha^2 r^2 \dot{z}^2 \right] - \frac{2Qq}{m\alpha r^2} \dot{t}.$$  

(10)

This Lagrangian does not depends explicitly on $t$, therefore the following conserved quantity exists

$$\frac{\partial L}{\partial \dot{t}} = -f \dot{t} - \frac{2Qq}{m\alpha r^2} = -\frac{E}{m},$$

(11)

where $E$ is the energy of the particle as measured by an observer at rest at infinity. We take for simplicity $\dot{\phi} = 0 = \dot{z}$.

From (11) we write

$$\dot{t} = \frac{1}{f} \left( \frac{E}{m} - \frac{2Qq}{m\alpha r^2} \right).$$

(12)

The normalization condition

$$g_{\mu\nu}u^\mu u^\nu = -1,$$

(13)

yields

$$\dot{r} = -\sqrt{\left( E - \frac{2Qq}{m\alpha r^2} \right)^2 \frac{1}{m^2} - f}.$$  

(14)

The substitution of (12) and (14) in (7) gives

$$E_{cm} = \sqrt{2m} \left[ 1 + \frac{1}{f} \left( \frac{E_1}{m} - \frac{2Qq_1}{m\alpha r^2} \right) \left( \frac{E_2}{m} - \frac{2Qq_2}{m\alpha r^2} \right) \right] - \frac{1}{f} \sqrt{\left( \frac{E_1}{m} - \frac{2Qq_1}{m\alpha r^2} \right)^2 - f^2} \frac{1}{2}.$$  

(15)
This CM energy expressed by (15) blows up at the horizon i.e. at $f = 0$. At $f = 0$ the numerator in (15) also vanishes. Therefore, we calculate the limiting value of the $E_{cm}$ at the event horizon by expanding (15) around $f = 0$ i.e. around $r_+$, as follows

$$E_{cm} = \sqrt{2m} \left[ 1 + \frac{1}{2} \left( \frac{q_2}{q_1} - \frac{E_2 r_+^2 \alpha}{2Q} \right) + \frac{q_1}{q_2} - \frac{E_1 r_+^2 \alpha}{2Q} \right].$$

(16)

The expression (16) clearly show that the CM energy which depends on the charge of the black string will be unlimited in the vicinity of the horizon if one of the colliding particles has the critical charge

$$q_c = \frac{E \alpha r_+^2}{2Q},$$

(17)

and the other colliding particle takes any other value of the charge. This result is similar to the results obtained for the Reissner-Nordstrom and Reissner-Nordstrom de-Sitter black holes [4, 16]. For the colliding particles to reach the horizon of the black string the quantity inside the square root in (14) must be positive.

3. Conclusion

Here we have investigated the CM energy of collision for two particles falling freely from rest at infinity in the background of a charged black string. We have found that the CM energy of collision of two charged particles is arbitrarily high at the event horizon of the black string. This arbitrarily high value of the CM energy of collision can be obtained at the horizon if one of the colliding particles has the critical value of charge given by (17) and the other particle takes any different value of the charge. Here we see that, in order to accelerate particles to an arbitrarily high CM energy of collision the black string need not be an extremal one.

Acknowledgments

This work has been supported by Higher Education Commission of Pakistan (HEC) and French Embassy joint split Ph.D. and Post-Doctoral fellowship programme. I would like to thank Laboratoire Jacques-Louis Lions of the Université Pierre et Marie Curie (Paris 6), Paris, where the writing-up of this paper was completed. I would also like to thank the unknown referee for giving useful comments on this work.
References

[1] M. Banados, J. Silk, S.M West, Phys. Rev. Lett. 103, 111102 (2009).
[2] O. B. Zaslavskii, Phys. Rev. D 84, 024007 (2011).
[3] O. B. Zaslavskii, Class. Quantum. Grav. 28, 105010 (2011).
[4] O. B. Zaslavskii, JETP Lett. 92, 571 (2010).
[5] A. A. Grib and Y. V. Pavlov, JETP Lett. 92, 125 (2010).
[6] S. W. Wei, Y. X. Liu, H. Guo and C. E Fu, Phys. Rev. D 82, 103005 (2010).
[7] K. Lake, Phys. Rev. Lett. 104 211102 (2010).
[8] O. B. Zaslavskii, Phys. Rev. D 81, 044020 (2010).
[9] S. W. Wei, Y.X. Liu, H. Guo, Chun-E Fu, Phys. Rev. D 82, 103005 (2010).
[10] S.W. Wei, Y.X. Liu, H. Guo, Chun-E Fu, JHEP 1012 066 (2010).
[11] Y. Zhu, S. Wu, Y. Jiang, G. Yang, arXiv:1108.1843.
[12] A. Williams, Phys. Rev. D 83, 123004 (2011).
[13] M. Patil, A. S. Joshi, K. Nakao, M. Kimura, arXiv: 1108.0288.
[14] S. Gao and C. Zhong, Phys. Rev. D 84, 044006 (2011).
[15] P. J. Mao, R. Li, L. Y. Jia and J. R. Ren, arXiv:1008.2660v3.
[16] C. Zhong and S. Gao, JETP Lett. 94, 631 (2011).
[17] Y. Li, J. Yang, Y. Li, S. Wei, Y. Liu, arXiv:1012.0748v2.
[18] T. Harada and M. Kimura, Phys.Rev. D 83, 024002 (2011).
[19] T. Harada and M. Kimura, Phys.Rev. D 83, 084041 (2011).
[20] Y. Zhu, S. Wu, Y. Liu and Y. Jiang, Phys. Rev. D 84, 043006 (2011).
[21] M. Patil and P. S. Joshi, Phys. Rev. D 84, 104001 (2011).
[22] M. Patil and P. S. Joshi, Class. Quantum Grav. 28, 235012 (2011).
[23] I. Hussain, Mod. Phys. Lett. A (to appear).
[24] E.Berti, V.Cardoso, L.Gualtieri, F.Pretorius, and U.Sperhake, Phys. Rev. Lett. 103, 239001 (2009).
[25] T. Jacobson, T. P. Sotiriou, Phys. Rev. Lett. 104, 021101 (2010).

[26] K. Thorne, Nonspherical Gravitational Collapse: a Short Review in Magic without Magic, ed. J. Klauder (San Francisco: Freeman) (1972).

[27] J. P. S. Lemos, Phys. Rev. D 54, 3840 (1996).

[28] R. G. Cai and Y. Z. Zhang, Phys. Rev. D 54, 4891 (1996).

[29] A. Fatima and K. Saifullah, arXiv:1108.1622v1.

[30] D. Pugliese, H. Quevedo, R. Ruffini, Phys. Rev. D 83, 104052 (2011).