Bremsstrahlung Energy Losses for Cosmic Ray Electrons and Positrons

A. Widom and J. Swain
Physics Department, Northeastern University, Boston MA USA

Y.N. Srivastava
Physics Department, University of Perugia, Perugia IT

Recently cosmic ray electrons and positrons, i.e. cosmic ray charged leptons, have been observed. To understand the distances from our solar system to the sources of such lepton cosmic rays, it is important to understand energy losses from cosmic electrodynamic fields. Energy losses for ultra-relativistic electrons and/or positrons due to classical electromagnetic bremsstrahlung are computed. The energy losses considered are (i) due to Thompson scattering from fluctuating electromagnetic fields in the background cosmic thermal black body radiation and (ii) due to the synchrotron radiation losses from quasi-static domains of cosmic magnetic fields. For distances to sources of galactic length proportions, the lepton cosmic ray energy must be less than about a TeV.

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I. INTRODUCTION

Measurements of the energy distribution of cosmic rays incident on earth bound detectors has previously been restricted to baryons[1,2]. More recently, experimental observations of electrons and positrons as a charged lepton components to cosmic rays have also been observed[3].

It is thereby of great interest to understand the distances from our solar system to possible sources of these charged leptons. The understanding relies on possible energy losses due to interactions with cosmic electromagnetic fields. In what follows we consider energy losses due to bremsstrahlung radiation from charged particle accelerations[4].

These measurements give rise to issues of how much radiative damping of these lepton energies exist. Such energy losses would be important for estimating how distant may be the sources of lepton cosmic rays. For cosmic ultra-relativistic leptons of energy \( E = mc^2 \gamma \) with \( \gamma \gg 1 \) we estimate in Secs. III and IV that within the distance scales of our galaxy, energy losses from classical bremsstrahlung can be neglected if \( 1 \ll \gamma \ll 10^7 \).

II. CLASSICAL RADIATION DAMPING

When a non-relativistic \(|v| \ll c\) classical particle with charge \( e\), mass \( m\) and acceleration \( \mathbf{a}\) moves through a classical electromagnetic field, it radiates electromagnetic power \( P \) determined by

\[
P = \frac{2e^2|\mathbf{a}|^2}{3c^3} = \frac{2e^4}{3mc^2c^2}\mathbf{E}^2,
\]

wherein the non-relativistic acceleration \( \mathbf{a} = (e\mathbf{E}/m) \) has been employed. The incoming electromagnetic energy flux \( S \) of a plane wave incident on a charged particle per unit time per unit area is given by

\[
S = \frac{e}{4\pi}\mathbf{E}\mathbf{E}^\prime.
\]

Thompson employed Eqs. (1) and (2) to derive the total cross section for the classical charged particle to scatter an electromagnetic wave; i.e. \( \sigma = P/S \) yields

\[
\sigma = \left(\frac{8\pi r_c^2}{3}\right) \quad \text{wherein} \quad r_c = \left(\frac{e^2}{mc^2}\right).
\]

Numerically,

\[
\sigma_{\text{electron}} \approx 6.652446 \times 10^{-25} \text{ cm}^2,
\]

\[
\sigma_{\text{proton}} \approx 1.973104 \times 10^{-31} \text{ cm}^2.
\]

To understand the manner in which radiation gives rise to energy losses of classical relativistic charged particles it is merely necessary to generalize Thompson’s above argument to the fully relativistic form of classical electromagnetic theory.

A. Relativistic Notation

The proper time of a moving classical particle will be written as

\[
-c^2 d\tau^2 = \eta_{\mu\nu}dx^\mu dx^\nu
\]

with the metric signature \((+,+,+,-)\). The four velocity is then

\[
\frac{dx^\mu}{d\tau} = (d\mathbf{r}, cdt), \quad \text{and} \quad \mathbf{v} = \frac{dx}{dt} = (\gamma \mathbf{v}, \gamma c) \quad \text{wherein} \quad \gamma = \frac{1}{\sqrt{1 - |\mathbf{v}/c|^2}}.
\]

so that

\[
\mathbf{v}^\mu v_\mu = -c^2.
\]

The four acceleration is defined as

\[
\mathbf{a}^\mu = \frac{dx^\mu}{d\tau} \quad \text{and} \quad \mathbf{a} = \frac{d(\gamma \mathbf{v})}{dt},
\]
\[ w^\mu = (\gamma a, \gamma v \cdot a/c), \]
\[ w^\mu w_\mu = \gamma^2 \left[ \left| a \right|^2 - \frac{|v \cdot a|^2}{c^2} \right], \tag{8} \]
so that
\[ v^\mu w_\mu = 0. \tag{9} \]
The four momentum
\[ p^\mu = mv^\mu \text{ obeys } p^\mu p_\mu = -m^2 c^2 \tag{10} \]
in virtue of Eq. (7).

### B. Classical Electrodynamic Fields

In terms of the vector potential \( A^\mu = (A, \Phi) \), the electrodynamical fields are described by
\[ E = -\frac{1}{c} \frac{\partial A}{\partial t} - \text{grad} \Phi \text{ and } B = \text{curl} A. \tag{11} \]
Equivalently, as a tensor
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{12} \]
The Lorentz force plus the classical radiation reaction force on a charge may be written as
\[ mw^\mu = \frac{e}{c} F^{\mu\nu} v_\nu + F^\mu, \tag{13} \]
wherein \( F^\mu \) is the radiation damping force,
\[ F^\mu = -\left( \frac{2e^4}{3mc^2} \right) \times \left[ F_\beta^{\mu\alpha} \left( \frac{v_\alpha}{c} \right) + F^{\nu\alpha} \left( \frac{v_\alpha}{c} \right) F_\beta^{\nu}\left( \frac{v_\beta}{c} \right) \right]. \tag{14} \]
It is worthy of note that the damping force may be written in terms of the Maxwell electromagnetic energy-pressure tensor,
\[ T_{\mu\nu} = \frac{1}{4\pi} \left[ F_\mu^\lambda F_{\nu\lambda} - \frac{1}{4} \eta_{\mu\nu} (F^{\alpha\beta} F_{\alpha\beta}) \right]. \tag{15} \]
In virtue of Eqs. (14) and (15) we have
\[ r_c = \left( \frac{e^2}{mc^2} \right) \text{ and } \sigma = \left( \frac{8\pi r_c^2}{3} \right), \]
\[ F^\mu = -\sigma \left( T^{\mu\nu} \left( \frac{v_\nu}{c} \right) + F_{\mu\nu} \left( \frac{v_\alpha}{c} F^{\nu\beta} \frac{v_\beta}{c} \right) \right). \tag{16} \]
Eq. (16) is the completely relativistic version of the Thompson cross section \( \sigma \) outlined in the above Sec. II Eqs. (11), (2) and (3). The Thomson cross section \( \sigma \) fully determines the strength of the classical radiation damping force.

#### C. Particle Orbits

In virtue of Eq. (13), the orbits of a charged particle in space is determined by the force on a charge
\[ \frac{dp}{dt} = ma = e \left[ E + \frac{v \times B}{c} \right] + f. \tag{17} \]
wherein \( f \) is the radiation damping retarding force determined by Eqs. (13) and (16). One requires the Maxwell electromagnetic energy-pressure tensor which in general has the form
\[ \{T^{\mu\nu}\} = \left( \frac{P}{S/c} \frac{S^{1/c}}{u} \right). \tag{18} \]
In Eq. (18), we have (i) the Maxwell pressure tensor
\[ P = \frac{1}{8\pi} \left[ (|E|^2 + |B|^2) (1 - 2(EE + BB)) \right], \tag{19} \]
(ii) the energy flux per unit time per unit area
\[ S = \frac{c}{4\pi} [E \times B], \tag{20} \]
and (iii) the energy density
\[ u = \frac{1}{8\pi} \left[ |E|^2 + |B|^2 \right]. \tag{21} \]
The radiation retardation force is thereby
\[ f = \sigma \left( \left( \frac{S - P \cdot v}{c} \right) - \left( \frac{\gamma^2 v}{c} \right) \bar{u} \right), \tag{22} \]
wherein
\[ \bar{u} = u + \left[ \frac{v \cdot P \cdot v}{c^2} \right] - 2 \left[ \frac{v \cdot S}{c^2} \right]. \tag{23} \]
This completes our theoretical derivation of classical radiation damping retardation forces.

### III. CLASSICAL BREMSSTRAHLUNG

Let us now consider explicit cases of electrons moving through random electromagnetic fields. The induced electron acceleration gives rise to classical electrodynamical radiation damping forces via bremsstrahlung. Due to the very small Thompson cross section in Eq. (13), the strength of the classical radiation damping retardation forces are also very small for known cosmic electromagnetic fields. We seek to calculate the energy loss when the charged particle moves a distance \( d\ell \), i.e.
\[ -\frac{d\mathcal{E}}{dt} = -mc^2 \left( \frac{d\gamma}{dt} \right) = |\mathbf{F} \cdot \mathbf{n}|, \tag{24} \]
wherein \( \mathbf{n} = \mathbf{v} / |\mathbf{v}| \) is a unit vector in the direction of the particle velocity. Eq. (24) may computed by employing Eq. (22).
A. Background Thermal Radiation

For the cosmic background radiation, the Maxwell energy-pressure tensor has the form

\[
\{T^{\mu\nu}\} = \begin{pmatrix} P_T & 0 & 0 & 0 \\ 0 & P_T & 0 & 0 \\ 0 & 0 & P_T & 0 \\ 0 & 0 & 0 & u_T \end{pmatrix},
\] (25)

wherein the trace condition for the electromagnetic tensor \( T_{\mu}^{\mu} = 0 \) holds true. One finds the relations

\[
3P_T = u_T, \\
\lambda_T = \left[ \frac{\hbar c}{k_B T} \right], \\
u_T = \frac{\pi^2}{15} \left[ \frac{k_B T}{\lambda_T^3} \right].
\] (26)

The classical electrodynamic retardation force for a charged particle moving through black body radiation is thereby

\[
f = -4\sigma P_T \gamma^2 \left[ \frac{v}{c} \right],
\] (27)
or equivalently

\[
-\frac{d\mathcal{E}}{d\ell} = -mc^2 \gamma \frac{d\gamma}{d\ell} = 4\sigma P_T \gamma^2 \sqrt{1 - \frac{1}{\gamma^2}}.
\] (28)

In the high energy limit \( \gamma \gg 1 \),

\[
-\frac{d\mathcal{E}}{d\ell} \approx -mc^2 \gamma \frac{d\gamma}{d\ell} \approx \left( \frac{4}{3} \right) \gamma^2 \sigma u_T.
\] (29)

The numerical values of the cosmic background radiation parameters are as follows:

\[ T \approx 2.725 \text{ }^\circ\text{K}, \]

\[ k_B T \approx 3.762 \times 10^{-16} \text{ erg} \approx 2.348 \times 10^{-4} \text{ eV}, \]

\[ \lambda_T \approx 8.403 \times 10^{-2} \text{ cm}, \]

\[ u_T \approx 0.2604 \left[ \frac{\text{eV}}{\text{cm}^3} \right], \]

\[ \sigma_{\text{electron}} u_T \approx 1.732 \times 10^{-25} \left[ \frac{\text{eV}}{\text{cm}^3} \right], \]

\[ -\left[ \frac{d\mathcal{E}}{d\ell} \right]_{\text{electron}} \approx 2.309 \times 10^{-26} \left[ \frac{\text{eV}}{\text{cm}^3} \gamma^2, \right], \]

\[ \frac{d}{d\ell} \left( \frac{1}{\gamma} \right) \approx 4.520 \times 10^{-32} \left[ \frac{1}{\text{cm}} \right], \]

\[ \frac{d}{d\ell} \left( \frac{1}{\gamma} \right) = \frac{1}{L}. \] (30)

Thus,

\[
\gamma(\ell) \approx \frac{\gamma(0)}{1 + [\gamma(0)\ell/L]}.
\] (32)

The classical bremsstrahlung energy losses due to the thermal cosmic background black body radiation can be neglected if the electron path length \( \ell \) obeys

\[
\ell \ll (L/\gamma).
\] (33)

The length scale associated with the size of our milky way galaxy is

\[
\ell_G \sim 10^{23} \text{ cm} \sim 30 \text{ kpc}.
\] (34)

For cosmic thermal radiation damping to be totally neglected on electron path lengths of the size of our galaxy it is sufficient that \( 1 \ll \gamma \ll 10^7 \).

B. Pomeranchuk Length

In the ultra-relativistic limit \( \gamma \gg 1 \), the energy loss

\[
-\frac{d\mathcal{E}}{d\ell} = -mc^2 \gamma \frac{d\gamma}{d\ell}
\] (35)

can be described by a general Pomeranchuk length scale \( L \) defined by

\[
\frac{d}{d\ell} \left( \frac{1}{\gamma} \right) = \frac{1}{L}
\] (36)

wherein

\[
\frac{1}{L} = \left[ \frac{\sigma}{4\pi mc^2} \right] |n \times E|^2 + |n \times B|^2 - 2n \cdot (E \times B)
\] (37)

and wherein \( n \) is the unit vector in the direction of the velocity. The condition

\[
\ell \ll (L/\gamma)
\] (38)

is sufficient for neglecting classical bremsstrahlung radiation energy losses.

C. Magnetic Bremsstrahlung

Another possible source of radiative damping is a domain of uniform magnetic field \( B \). When electrons move in such fields they exhibit synchrotron radiation due to the rotational angular frequency

\[
\omega_c = -\left( \frac{eB}{mc} \right) \frac{1}{\gamma} = -\left( \frac{eB}{\mathcal{E}} \right),
\]

\[
\frac{e}{mc} \approx 17.6086 \left[ \frac{\text{Hz}}{10^{-6} \text{ Gauss}} \right].
\] (39)
The Pomeranchuk length $L$ in a uniform magnetic field is given by

$$\frac{1}{L} = \left[ \frac{\sigma}{4\pi mc^2} \right] |n \times B|^2 = \left[ \frac{2\sigma u}{mc^2} \right] \sin^2 \vartheta$$  \hspace{0.5cm} (40)

wherein

$$u = \frac{|B|^2}{8\pi}. \hspace{0.5cm} (41)$$

is the energy density of the magnetic field and $\vartheta$ is the angle between the velocity $v$ and the magnetic field $B$. Within the milky way galaxy, the magnetic field energy density $u \sim [eV/cm^3]$ that again yields an enormous Pomeranchuk length

$$L \sim 4 \times 10^{20} \left[ \frac{cm}{\sin^2 \vartheta} \right] \sim 10^7 \left[ \frac{kpc}{\sin^2 \vartheta} \right]. \hspace{0.5cm} (42)$$

However, the helical paths traveled by electrons or positrons have a path length $\ell$ that is more than the distance $s$ traveled along the magnetic field direction, i.e.

$$s = \ell \cos \vartheta. \hspace{0.5cm} (43)$$

The condition that synchrotron radiation losses are negligible then amounts to

$$\gamma \ll \left\langle \left[ \frac{mc^2}{2\sigma u} \right] \frac{\cos \vartheta}{\sin^2 \vartheta} \right\rangle$$  \hspace{0.5cm} (44)

where the average is over the size and angular orientation of the magnetic fields. From this point of view, electron and positrons that must travel distances of more than galactic proportions will have only negligible energy losses for $\gamma \leq 10^6$.

IV. CONCLUSION

Observed cosmic ray charged leptons arriving from distances of more than galactic proportions will exhibit bremsstrahlung energy losses and these in principle will have some effect on energy distributions observed employing detectors in our solar system. The energies $E = mc^2\gamma$ for which the energy distributions will reflect those of the charged leptons emitted from the source are in the range $1 \ll \gamma \ll 10^7$. For distances to sources of galactic length proportions, the lepton cosmic ray energy must be less than about a TeV.

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