Hawking Effect of AdS$_2$ Black Holes in the Jackiw-Teitelboim Model

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It might be tempting to consider that the two-dimensional anti-de Sitter black hole in the Jackiw-Teitelboim model is thermally hot by invoking the non-vanishing surface gravity, which raises a natural question of “where is the observer to measure the temperature?”, asymptotically at infinity or in a finite region outside the horizon? In connection with this issue, one might expect that the local temperature would also be blue-shifted near the horizon while it would vanish at infinity because of the Tolman factor in the local temperature. In this paper, the local temperature will be shown to vanish and to respect the equivalence principle everywhere as long as a consistent Stefan-Boltzmann law is required. The essential reason for the vanishing local temperature will be discussed on various grounds.

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I. INTRODUCTION

The thermal property of a Schwarzschild black hole has been characterized by the Hawking temperature at infinity [1,2]. In the Israel-Hartle-Hawking state [3,4], one can also define the local Tolman temperature derived from the Stefan-Boltzmann law to relate the local energy density and the local temperature in local proper frames [5,6].

Let us consider a two-dimensional black hole for a simple argument. Then, the local temperature of the black hole can be obtained from the Stefan-Boltzmann law

$$\rho = \gamma T^2 = T_H/\sqrt{g},$$  (1)

where $\rho$ is the local energy density and $T$ is the Tolman temperature, which consists of the Hawking temperature $T_H$ and the redshift factor. The issues are as follows: Firstly, in thermal equilibrium, the local energy density is generally negative finite near the horizon up to a finite distance [7]; thus, the Tolman temperature is plagued by imaginary values in that region as long as the usual Stefan-Boltzmann law is employed. This negative energy density also appears in the same manner in the four-dimensional Schwarzschild black hole [8]. Secondly, the Tolman temperature is divergent at the horizon despite the finite local energy density, which means that the usual form of the Stefan-Boltzmann law is inconsistent [9]. Thirdly, in the Israel-Hartle-Hawking state of the equilibrium system, there are no effective excitations at the horizon so that the equivalence principle can be naturally restored there [9–11], which implies that the thermal temperature must vanish on the horizon.

To resolve these issues, one should note that the Tolman temperature from the Stefan-Boltzmann law has been derived from the assumption that the energy-momentum tensor is traceless [5,6]. Hence, the usual Stefan-Boltzmann law should be extended to the anomalous case of an energy-momentum tensor with a non-vanishing trace, where the Hawking process of radiation in the black hole systems is actually responsible for the conformal anomaly for matter fields [12]. For an energy-momentum tensor with a non-vanishing trace, one can show that the local temperature is effectively zero at the horizon so that the above issues can be naturally resolved for two and four-dimensional Schwarzschild black holes [9,13] and for the Schwarzschild anti-de Sitter (AdS) black hole [14]. (For a recent review, see Ref. 15.)

However, this still raises a question on the behavior of the local temperature of an AdS black hole with a constant curvature. The essential difference from the above asymptotically flat black holes is that the local temperature for AdS black holes does not reduce to the Hawking temperature at infinity because of the AdS boundary. Moreover, the AdS geometry is locally equivalent and it is indistinguishable. Regarding the calculations of the local temperature for the AdS black hole, one may use the local temperature identified with the Unruh temperature for an accelerating observer in a higher-dimensional Minkowski spacetime [16–18]; however, the local temperature suffers from an imaginary value.

Thus, we would like to consider an amenable two-
dimensional AdS black hole with a constant curvature in order to study whether the local temperature of the AdS black hole can be well-defined without imaginary values and eventually justify whether the black hole is thermally hot or not. In Sec. II, we consider a soluble AdS2 black hole in the Jackiw-Teitelboim (JT) model [19, 20] and find a clue to the behavior of the local temperature from the fact that the general covariance requires a vanishing energy-momentum tensor related to the absence of the Hawking flux. This implies that the local temperature vanishes everywhere. In order to confirm this result, in Sec. III, we derive the local temperature in the presence of an energy-momentum tensor with a non-trivial trace. We will show that the effective local temperature for the AdS2 black hole becomes zero without any imaginary values. In Sec. IV, the claims in the Sec. III will be clarified by pointing out some differences from the result of an earlier work [21]. Finally, a conclusion will be given in Sec. V.

II. ADs2 BLACK HOLE IN THE JT MODEL

Let us start with the JT model described by the action [19, 20]

\[ S_{JT} = \frac{1}{2\pi} \int d^2x \sqrt{-g} \Phi \left[ R + \frac{2}{\ell^2} \right], \]  

(2)

where \( \Phi \) is an auxiliary field. This model has been extensively studied from various points of view [22–26]. The actions for classical and quantum matter can be written in the forms of

\[ S_{CL} = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ \frac{1}{2} (\nabla f)^2 \right], \]  

(3)

\[ S_{QT} = -\frac{1}{96\pi} \int d^2x \sqrt{-g} \left[ R \frac{1}{\sqrt{R}} \right], \]  

(4)

where \( f \) is a classical scalar field and Eq. (4) is the Polyakov action [27]. In the conformal gauge

\[ ds^2 = -e^{2\rho(\sigma^+,\sigma^-)} d\sigma^+ d\sigma^-, \]  

(5)

the equations of motion for \( \rho \), \( \Phi \), and \( f \), and the constraint equations are given as

\[ \partial_+ \partial_- \rho + \frac{1}{4\ell^2} e^{2\rho} = 0, \]  

(6)

\[ \partial_+ \partial_- \Phi - \frac{1}{4\ell^2} e^{2\rho} \left( \frac{1}{24} - \Phi \right) = 0, \]  

(7)

\[ \partial_+ \partial_- f = 0, \]  

(8)

\[ \partial_+^2 \Phi - 2\partial_+ \rho \partial_- \Phi = T^M_{++}. \]  

(9)

The energy-momentum tensor for matter fields consists of two parts:

\[ T^M_{\pm \pm} = T^\text{CL}_{\pm \pm} + T^\text{QT}_{\pm \pm}, \]  

(10)

where

\[ T^\text{CL}_{\pm \pm} = \frac{1}{2} (\partial_\pm f)^2, \]  

(11)

\[ T^\text{QT}_{\pm \pm} = T_{\pm\pm}^\text{bulk} + T_{\pm\pm}^\text{boundary}, \]  

(12)

with \( \kappa = 1/(12\pi) \). For later convenience, the energy-momentum tensor \( T^\text{QT}_{\pm \pm} \) can be divided into two parts as

\[ T_{\pm\pm}^\text{bulk} = -\kappa \left[ (\partial_\pm \rho)^2 - \partial_\pm^2 \rho \right], \]  

(13)

and the boundary term becomes

\[ T_{\pm\pm}^\text{boundary} = -\kappa t_{\pm}, \]  

(14)

where \( t_{\pm} \) reflects the nonlocality of the Polyakov action in Eq. (4). Solving the equations of motion in the conformal gauge, we obtain

\[ e^{2\rho} = \frac{M}{\sinh^2 \left[ \frac{\sqrt{M}(\sigma^+ - \sigma^-)}{2\ell} \right]}, \]  

(15)

\[ \Phi^{-1} = -\frac{1}{M} \tanh \left[ \frac{\sqrt{M}(\sigma^+ - \sigma^-)}{2\ell} \right], \]  

(16)

\[ f_i = f_i^{(+)}(\sigma^+) + f_i^{(-)}(\sigma^-), \]  

(17)

where \( M \) is an integration constant that plays the role of the mass of the black hole. The curvature scalar related to the trace of the energy-momentum tensor \( T^\text{QT}_{\mu\nu} = R/(24\pi) \) is given as

\[ R = -\frac{2}{\ell^2}, \]  

(18)

which is independent of the mass parameter. As \( M \to 0 \), the solution Eq. (15) exactly comes down to the AdS2 vacuum

\[ e^{2\rho} = \frac{4\ell^2}{(x^+ - x^-)^2}. \]  

(19)

In fact, the black hole and the vacuum geometry described by Eqs. (15) and (19), respectively, are related through the coordinate transformation

\[ x^\pm = \frac{2\ell}{\sqrt{M}} \tanh \left[ \frac{\sqrt{M}(\sigma^\pm)}{2\ell} \right]. \]  

(20)

Furthermore, the dynamical equation of motion Eq. (6) is decoupled from the matter field and the constant curvature is independent of the energy density of infalling matter, so we set \( f = 0 \) from now on.

Let us now consider the general coordinate transformation for the energy-momentum tensor in order to calculate the Hawking flux for the AdS2 black hole in the JT model. The Hawking radiation in two dimensions can be given by the anomalous transformation of the boundary term in the energy-momentum tensor. We assume that the constraint Eq. (9) should be Virasoro anomaly
free in such a way that $T_{\pm \pm}^{QT}$ can be transformed as the primary operator in conformal field theory as [28–31]

$$T_{\pm \pm}^{QT}(\sigma^\pm) = \left( \frac{\partial_x x^\pm}{\partial \sigma^\pm} \right)^2 T_{\pm \pm}^{QT}(x^\pm),$$

(21)

then, the boundary term in Eq. (12) is determined by

$$-t_\pm(\sigma^\pm) = -\frac{1}{2} \{x^\pm, \sigma^\pm\}$$

$$= \frac{M}{4\ell^2},$$

(22)

where $\{x^-, \sigma^+\}$ is a Schwartzian derivative. At first sight, one might conclude that the Hawking temperature for the CGHS black hole is given by [32]. Explicitly, the Hawking radiation for the CGHS black hole that is asymptotically flat is in contrast with the asymptotically flat case, for in this case because the bulk and the boundary terms contribute to the Hawking radiation simultaneously unlike in the asymptotically flat case,

$$T_{\text{bulk}}^{\pm} = -\frac{\kappa M}{4\ell^2}, \quad T_{\text{boundary}}^{\pm} = \frac{\kappa M}{4\ell^2};$$

(24)

thus, the radiation $h$ vanishes as

$$h(\sigma^+, \sigma^-) = T_{\text{bulk}}^{\pm}(\sigma^+, \sigma^-) = T_{\text{bulk}}^{\pm} + T_{\text{boundary}}^{\pm} = 0.$$  

(25)

The negative contribution from the bulk part of Eq. (24) is in contrast with the asymptotically flat case, for instance, a CGHS black hole that is asymptotically flat [32]. Explicitly, the Hawking radiation for the CGHS black hole is given by

$$h(\sigma^-) = T_{\text{bulk}}^{\pm}(\sigma^+, \sigma^-)|_{\sigma^+ \to \infty} + T_{\text{boundary}}^{\pm}(\sigma^-)|_{\sigma^+ \to \infty}$$

$$= -\kappa t_\pm(\sigma^-).$$

(26)

In the limit of asymptotic null infinity ($\sigma^+ \to +\infty$), the Hawking radiation originates only from the boundary term $t_\pm$ because the bulk contribution vanishes at null infinity and the Stefan-Boltzmann law can be well-defined as $-t_\pm = \pi^2 T_H^2$ at infinity. In this respect, we can also expect the local temperature in the proper frame to be zero for the AdS$_2$ black hole, which will be clarified by calculating it explicitly in the next section.

III. EFFECTIVE LOCAL TEMPERATURE

The argument in the previous section will be elaborated by calculating the local temperature directly. For this purpose, we recapitulate the derivation of the Stefan-Boltzmann law in order to figure out the local temperature for the AdS black hole where there is no asymptotic flat region. As a matter of fact, the conventional local temperature relying on the Tolman’s form is based on the traceless energy-momentum tensor. However, the energy-momentum tensor of matter fields is anomalous because of the trace anomaly [33], so the usual Tolman temperature should be generalized to the case of a non-vanishing trace because the trace of the energy-momentum tensor is, indeed, non-vanishing for an AdS$_2$ black hole [12].

Let us start with the first law of thermodynamics, which is assumed to be $dU = TdS - pdV$, where $U$, $S$, and $V$ are the thermodynamic internal energy, temperature, and volume in the proper frame, respectively, and $U = \int p dV$. Using the Maxwell relation $\delta S/\delta V|_T = \partial p/\partial T|_V$, we can get the first-order differential equation for the energy density as

$$2\rho = T \frac{\partial \rho}{\partial T}|_V - T_{\mu}^\mu,$$

(27)

where we have used the fact that the conformal anomaly is independent of the temperature [34]. Solving Eq. (27), we obtain the proper energy density as [9]

$$\rho = \gamma T^2 - \frac{1}{2} T_{\mu}^\mu,$$

(28)

which reduces to the conventional Stefan-Boltzmann form for $T_{\mu}^\mu = 0$. In this case, the two-dimensional Stefan-Boltzmann constant is identified with $\gamma = \pi/6$ for a massless scalar field [12].

Let us perform the coordinate transformation from tortoise coordinates to Schwarzschild coordinates by using the relation $r^* = -\ell(\sqrt{M} \tanh^{-1} r/\ell \sqrt{M})$ in order to specify the static local observer easily. Then, the two-dimensional line element Eq. (5) can be rewritten as

$$ds^2 = -g(r)dt^2 + \frac{1}{g(r)} dr^2,$$

(29)

where $g(r) = -M + (r/\ell)^2$. The horizon of the black hole is $r_H = \sqrt{M} \ell$, and the Hawking temperature Eq. (23) is given by the surface gravity. The one-loop quantum-mechanical energy-momentum tensors Eq. (4) can be rewritten in the static coordinates as

$$T_{\pm \pm}^{QT} = \frac{1}{96\pi} \left[ g_{\mu \nu} \frac{1}{2} (g')^2 - 8t_\pm \right] = -\frac{\kappa M}{4\ell^2} - \kappa t_\pm,$$

(30)

$$T_{\text{boundary}}^{\pm} = -\frac{\kappa}{4} g_{\mu \nu} = \frac{\kappa}{4\ell^2}.$$

(31)

In the Israel-Hartle-Hawking state [3,4], there are no inward and outward fluxes at the past and the future event horizon, so the boundary condition should be determined by

$$-t_\pm = \frac{M}{4\ell^2}.$$

(32)
Note that the Boulware state is the state appropriate to the vacuum around a static gravitational systems and contains no radiation at spatial infinity [35]; however, the state is indistinguishable from either Israel-Hartle-Hawking state or the Unruh state [36] because the energy-momentum tensor Eq. (30) has no spatial dependence in the static Schwarzschild-like coordinates so that three vacua share the same boundary condition Eq. (32). This feature is in contrast to the asymptotically flat black holes in that all the vacuum states are degenerate for the AdS case. The absence of the Unruh vacuum has something to do with the stability of the AdS black hole without evaporation of the black hole [37]. In fact, if one wanted to consider an evaporating black hole, the boundary would be required to become absorptive by allowing energy to be transferred between bulk fields and external fields [38].

For the proper frame dropped from rest, the velocity from the geodesic equation of motion can be written as

$$u^\mu = \frac{dx^\mu}{d\tau} = \left(\frac{1}{\sqrt{g(r)}}, 0\right),$$

and the local energy density is given as

$$\rho = T^\mu_\mu u^\mu u^\nu T_{\nu\nu} = \frac{1}{g} \left[ T^{QT}_{++} + T^{QT}_{--} + 2T^{QT}_{+-} \right].$$

Plugging the energy density Eq. (35) into Eq. (28), we finally obtain

$$\gamma T^2 = \frac{1}{g} \left[ T^{QT}_{++} + T^{QT}_{--} \right] = 0. (36)$$

Interestingly the effective local temperature Eq. (36) should vanish generically because the inward and the outward fluxes crossing the horizon do not exist at the horizon. In fact, we treated the appropriate limiting procedure because the denominator in Eq. (36) also vanishes on the horizon. As a result, for the AdS case, from Eqs. (30) and (32), the local temperature Eq. (36) naturally vanishes everywhere, and the equivalence principle turns out to be perfectly valid as compared to the asymptotically flat black hole, which satisfies the equivalence principle just on the horizon [10]. If one were to insist on using the conventional Tolman temperature of

$$\rho = \gamma T^2 = T_H / \sqrt{g}$$

rather than Eq. (28), then the fluxes should be defined as

$$T^{QT}_{\pm \pm} = M/\sqrt{4}$$

for $t_{\pm} = M/\sqrt{2}$. Unfortunately, the Tolman temperature for such a case is divergent in spite of the finite local energy density at the horizon, which is inconsistent.

**IV. PARTICLE EXCITATIONS AND TEMPERATURE**

One might wonder the reason for the claims in the previous section looking different from those in an earlier work. Now, we are going to explain some differences between our claims and the results based on previous claims by using the following alternative method: at first sight, the Hawking radiation in the AdS black hole is analogous to that in the Rindler case, because the black-hole geometry Eq. (15) can be obtained through a coordinate transformation from the AdS vacuum Eq. (19) where the energy-momentum tensor is always zero. However, if we employ suitable coordinates Eq. (20), we will observe particle excitations Eq. (23), and we would conclude that the state is thermal [21]. However, this is not the case for the two-dimensional AdS black hole.

Let us first consider the Unruh effect [39], where the particles are excited in the Rindler spacetime. The normal ordered energy-momentum tensor in the Minkowski spacetime is defined by

$$T_{\mu\nu} := T_{\mu\nu} - \frac{\kappa}{\nu} T_{\mu0} T^0_0, \quad (37)$$

where the vacuum expectation value of the energy-momentum tensor gives rise to the quadratic divergence in two dimensions. In general, the normal ordering depending on the coordinates breaks the tensorial property of the energy-momentum tensor in such a way that the normal ordered one does not transform as a tensor under coordinate transformations:

$$T_{\pm \pm}(y^\pm) := \left( \frac{\partial x^\pm}{\partial y^\pm} \right)^2 T_{\pm \pm}(x^\pm) : - \frac{\kappa}{2} (x^\pm, y^\pm). \quad (38)$$

The anomalous term comes from the coordinate transformation of the vacuum expectation value of the energy-momentum tensor in Eq. (37). Note that the Schwartzian derivative is innocent because the coordinate transformation is global. For a linear Lorentz transformation, the energy-momentum tensor is a true tensor because the Schwartzian derivative vanishes. In particular, for the Rindler coordinate transformation of $x^\pm = \pm (1/a) e^{\mp y^0}$, where $x^\pm$ and $y^\pm$ are the Minkowski and the Rindler coordinates, respectively, the vacuum expectation value of the energy-momentum tensor in Rindler coordinates is

$$M(0) : T_{\pm \pm}(y^\pm) : |0\rangle_M = \frac{\kappa}{4} a^2, \quad (39)$$

where $\{x^\pm, y^\pm\} = -a^2/2$, with $a$ being a linear acceleration, and $|0\rangle_M$ is the ordinary Minkowski vacuum. Using $\rho = \gamma T^2$ with $\gamma = \pi/6$ for a single scalar field, where $\rho = T^{00} = (T_{++} + T_{--}) + 2(T_{+-})$ with $T_{++} = 0$ in the Rindler space, the Unruh temperature can be obtained as

$$T = \frac{a}{2\pi}. \quad (40)$$

From this non-tensorial quantity, we obtained the Unruh temperature of the accelerated detector, but it is still physical because the coordinate transformation is global, so general covariance is not involved.
On the other hand, the above calculations can be extended to curved spacetime by adding the bulk contribution to the normal ordered energy-momentum tensor [40]:

\[ T_{\mu\nu} = T_{\mu\nu}^{\text{bulk}} + T_{\mu\nu}^{\text{boundary}}, \]  

(41)

where \( T_{\mu\nu}^{\text{bulk}} = -\kappa \left[ \left( \partial \rho \right)^2 - \left( \partial^2 \rho \right) \right] \) and \( T_{\mu\nu}^{\text{bulk}} = -\kappa \partial_\mu \partial_\nu \rho \) in the conformal gauge Eq. (5). The resulting energy-momentum tensor consists of the vacuum polarization of the scalar field on the curved spacetime and the normal-ordering effect associated with the coordinate system. The conformal coordinate transformation of the bulk part is calculated as

\[ T_{\pm \pm}^{\text{bulk}} (y^\pm) = \left( \frac{\partial x^\pm}{\partial y^\pm} \right)^2 T_{\pm \pm}^{\text{bulk}} (x^\pm) + \kappa \frac{1}{2} \left( x^\pm, y^\pm \right) \]  

(42)

while the coordinate transformation of the normal ordered energy-momentum tensor is implemented by using Eq. (38). Because the Schwartzian derivatives in Eqs. (42) and (38) exactly cancel, the energy-momentum tensor Eq. (41) behaves as a true tensor under the coordinate transformation [29]:

\[ T_{\pm \pm} (y^\pm) = \left( \frac{\partial x^\pm}{\partial y^\pm} \right)^2 T_{\pm \pm} (x^\pm); \]  

(43)

in other words, it is generally covariant. Then, the boundary term Eq. (14) can be identified with the normal ordered part in Eq. (41), so \( T_{\pm \pm}^{\text{boundary}} = : T_{\pm \pm} : = -\kappa t_{\pm \pm} \) [40].

Unlike the Rindler case, in order for general covariance, the contribution of the bulk energy-momentum tensor should no longer be ignored in curved spacetime. Therefore, the physical temperature should be derived in the regime of the generally covariant framework when gravity couples to matter, which is a big difference from the derivation of the Unruh temperature. This is also the reason why the bulk contribution could not be ignored in Sec. III. Of course, a generally covariant quantity such as the energy-momentum tensor cannot be translated directly to the physical temperature because it is still coordinate dependent, except at asymptotic infinity. For an asymptotically flat black hole such as a CGHS black hole, the bulk contribution can be safely ignored, as seen from Eq. (26), so the procedure to get the temperature looks similar to that for the Unruh case. However, our concern is for the AdS\(_2\) black hole without any other flat regions. In that sense, considering a local quantity defined in a local inertial frame, such as \( \rho = T_{\mu\nu}^{\text{QT}} u^\mu u^\nu \) already defined in Eq. (34), is natural.

V. CONCLUSION

The local temperature for the AdS\(_2\) black hole in the JT model was shown to vanish everywhere, which respects the equivalence principle everywhere. If the local temperature of the black hole is identified with the usual Tolman temperature, then the usual Tolman temperature is incompatible with the Stefan-Boltzmann law defined in the presence of the conformal anomaly. Furthermore, the three vacua of the Boulware, Israel-Hartle-Hawking, and Unruh states were found to be degenerate in the JT model so that the black hole is stable without any evaporation.

Let us comment on the behaviors of the local temperatures of asymptotically flat and non-flat black holes. For the Schwarzschild black hole, the equivalence principle could be recovered just at the horizon because the temperature measured by a fixed observer in the gravitational background is generically higher than the Unruh temperature of an accelerating observer; however, they are the same at the event horizon of the black hole so that the equivalence principle in the quantized theory is restored at the horizon [10]. In other words, the local temperature should vanish at the horizon and excited quanta cannot be found on the horizon, which is compatible with the calculations of the local temperature [9,13]. On the other hand, in the AdS\(_2\) black hole, the influx and the outward fluxes are zeros from the boundary conditions so that their coordinate transformation into the local inertial frame is also trivial and the corresponding local temperature is in essence zero.

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