Andreev tunneling into a one-dimensional Josephson junction array

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Abstract

In this letter we consider Andreev tunneling between a normal metal and a one dimensional Josephson junction array with finite-range Coulomb energy. The $I-V$ characteristics strongly deviate from the classical linear Andreev current. We show that the non linear conductance possesses interesting scaling behavior when the chain undergoes a $T=0$ superconductor-insulator transition of Kosterlitz-Thouless-Berezinskii type. When the chain has quasi-long range order, the low lying excitation are gapless and the $I-V$ curves are power-law (the linear relation is recovered when charging energy can be disregarded). When the chain is in the insulating phase the Andreev current is blocked at a threshold which is proportional to the inverse correlation length in the chain (much lower than the Coulomb gap) and which vanishes at the transition point.

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In the past few years there has been a renewed interest in the properties of metasuperconductor (NS) interfaces \[1\]. At low temperature and voltage the transport through a NS interface is due to Andreev tunneling \[2\]: a quasi-electron in the normal metal is reflected as a quasi-hole and a Cooper pair is added to the condensate on the superconducting side. Interference effects have been predicted for the subgap conductivity in NS tunnel junctions \[3\].

Kim and Wen studied the effect of collective modes below the superconducting gap on the NS tunneling \[4\]; due to the excitation of the low-lying modes in the tunneling process, the \(I-V\) characteristics become non-linear. If the electrostatic energy is large enough, charging effects suppress transport at low temperature and voltage: this is the so called Coulomb blockade (see Ref.\[5\] for reviews). Tunneling can take place only if electromagnetic modes with energy of the order of charging energy can be excited whose effect on Andreev tunneling has been studied by Hekking et al. \[6\].

In this letter we will study Andreev tunneling from a normal metal electrode to a one dimensional Josephson Junction Array (JJA). The system is depicted in Fig. 1. The transport in the JJA is due to Josephson tunneling. The Josephson energy \((\propto E_J)\) is minimized for a phase coherent superconducting state. If charging effects are relevant, the physics at low temperatures is dominated by its competition with the electrostatic energy \((E_0 = e^2/2C_0\) where \(C_0\) is the capacitance of the islands to the ground). The latter favours quantum fluctuations of the phase \(\varphi\) of the superconducting order parameter which can destroy superconductivity \[7\]. In one dimensional JJA (which the case we will consider) this results in a \(T = 0\) phase transition \[8\] of the Kosterlitz-Thouless-Berezinskii type at \(\sqrt{E_J/8E_0} = 2/\pi\).

We will show that Andreev tunneling into a JJA is a powerful tool to perform spectroscopy of low lying modes of JJA. Moreover, it offers another way to study the critical properties of strongly interacting superconducting systems. Infact, quantum fluctuations of the phases in the JJA produce a nonlinear Andreev current which is very sensitive to the critical properties of the JJA. Here we point out some features of the phase diagram of the device which are reflected in the two-particle tunneling.

Very recently it has been proposed that two electron tunneling can probe the spectrum of an \(S-S\) junction \[10\]. The physics of this system is contained also in our setup in the limit of very small \(E_J\). In this limit, the correlation length \(\xi\) of the JJA will not exceed few lattice constants and the transport properties are mainly determined by the junctions closest to the N-S boundary.

We start from the Hamiltonian:

\[
H = \sum_\sigma \int_\mathbf{r} d^3r \psi_\sigma^\dagger(\mathbf{r})\epsilon(-i\nabla)\psi_\sigma(\mathbf{r}) + \frac{1}{2} \sum_{ij} Q_i C_{ij}^{-1} Q_j - E_J \sum_i \cos(\varphi_i - \varphi_{i+1}) + t \left[ e^{i\varphi_0} \psi_\uparrow(0) \psi_\downarrow(0) + h.c. \right].
\]

It includes the Hamiltonians of the metal particle, that of the one dimensional JJA and the tunneling Hamiltonian which describes Andreev processes between the
normal metal and the first superconducting island respectively. In Eq. (1) \( \psi^\dagger_\sigma \) are the creation operators for the electrons of the normal metal particle and \( \epsilon(-i\nabla) = -\frac{\nabla^2}{2m} - \mu \) is the relative kinetic energy, measured from the Fermi energy. The integral is extended to the normal metal and \( \sigma \) labels the spin index. \( Q_i \) and \( \varphi_i \) denote the charge and the phase of the i-th island of the JJA and are canonically conjugated, \([Q_i, \varphi_j] = 2e\delta_{ij}i\). We will consider a model for the capacitance matrix \( C_{ij} \) where only the ground capacitance \( C_0 \) and the nearest neighbor capacitance \( C \) are present.

The range of the electrostatic interaction between Cooper pairs is \( \lambda = \sqrt{C/C_0} \). In Eq. (1) tunneling occurs with an amplitude \( t \) at a single point of the junction [11]. Finally we disregard quasiparticle tunneling in the JJA since we are dealing with energies far below the superconducting gap.

The Andreev current across the NS boundary when a voltage \( V \) is applied is calculated from the time variation of the electron charge at the metal island

\[
I_{NS} = -4e |t|^2 \Im D_R(-2eV) = 4e |t|^2 \int_{-\infty}^{\infty} dt e^{-2i\omega Vt} i\Theta(t) \langle [A^\dagger(t), A(0)] \rangle
\]  

where \( A(t) = e^{i\varphi_0(t)} \psi_\uparrow(0,t)\psi_\downarrow(0,t) \). The retarded correlator in Eq. (2) can be obtained from the analytic continuation of the corresponding time ordered Green function in imaginary frequencies. The latter is the Fourier transform of \( D_T(\tau) = \langle T_\tau A^\dagger(\tau)A(0) \rangle \). All the key features are contained in the phase–phase correlation function of the first superconducting island, \( g(\tau) = \langle T_\tau e^{i\varphi_0(\tau)}e^{-i\varphi_0(0)} \rangle \). The critical properties of the JJA are reflected in the form of \( g(\tau) \) and can be studied by mapping the “quantum” one dimensional JJA at \( T = 0 \) into a classical 1 + 1 dimensional (space and time) XY-model [8] where \( E_0 \) plays the role of the “temperature”. Vortices in the classical XY model correspond to \( T = 0 \) phase slips in the JJA which occur at a certain junction of the chain and are due to quantum fluctuations modulated by \( E_0 \).

\( \overline{E_0 \gg E_1} \) In this limit phase slips destroy the phase order and the Josephson chain is insulating. In the disordered phase the correlator \( g(\tau) \) decays exponentially in imaginary time. A careful evaluation of \( g(\tau) \) close to the transition point has been done in Ref. [12]. At large times, taking into account the mapping we get

\[
g(\tau) = e^{-\langle\omega_p|\tau|\rangle/\xi} / (\langle\omega_p|\tau|\rangle)^{1/2} \quad \tau \to \infty
\]  

where \( \omega_p = \sqrt{8E_1E_0} \). In the limit \( C \ll C_0 \) the correlation length is given by \( \xi = \exp[b/\sqrt{2/\pi - (E_1/8E_0)^{1/2}}] \), where \( b \sim 1 \). It diverges at the superconductor-insulator transition. Inserting the analytic continuation of Eq. (3) into Eq. (2) we get the tunneling current at small voltages of the array in its insulating phase

\[
I_{NS} = \frac{4G_A\omega_p}{3e\sqrt{\pi}} \text{sign}(V) \Theta \left(2e | V | / \omega_p - \xi^{-1} \right) \left[2e | V | / \omega_p - \xi^{-1} \right]^{3/2}.
\]
Here $G_A = 8\pi e^2 |t|^2 N^2(0)$ is the Andreev conductance for a bulk superconductor ($N(0)$ is the density of states of the metal particle). The typical $I-V$ characteristics is shown in Fig. (2), curve (a). The exponential decay of correlation in the JJA corresponds to the Coulomb blockade of the Andreev tunneling. Interestingly enough, the threshold is not the bare charging energy but it is proportional to the inverse correlation length. Hence it vanishes at the transition with a behavior related to the critical state of the chain,

$$V_{tr} = \omega_p/2e\xi.$$

As long as the correlation length $\xi$ is larger than $\lambda$, the $I-V$ characteristics are mainly determined by the self-capacitance and eq.(4) is valid also for finite $C$. At lower values of the ratio $E_J/E_0$ further away from the transition, the correlation length $\xi$ decreases and eq.(4) does not hold any longer because $C$ cannot be disregarded.

At the critical value $\sqrt{E_J/8E_0} = 2/\pi$ the JJA undergoes a transition to the superconducting phase where the phase-slips of opposite sign are tightly bound in pairs and the voltage drop across the array is zero.

The Josephson Hamiltonian is well approximated by its phase-wave Gaussian limit in which quantum phase slips are neglected. The low energy excitations are modes with dispersion

$$\omega_q^2 = \frac{2E_J(1-\cos q)}{(8E_0)^{-1} + 2(8E_C)^{-1}(1-\cos q)}.$$  \hspace{1cm} (5)

The resulting phase correlator in imaginary time is

$$g(\tau) = \exp \left\{ - (\eta - 1) \int_0^\infty dx \, \frac{1-\cos(x\tilde{\tau})}{x \sqrt{1+x^2}} \right\}$$ \hspace{1cm} (6)

where $\tilde{\tau} = \kappa \omega_p \tau$, $\kappa = (1/4 + C/C_0)^{-1/2}$ and $\eta = 1 + (2\pi)^{-1} \sqrt{8E_0/E_J}$. The system possesses quasi-long range order and the correlations decay with a power-law for $\tau \to \infty$, $g(\tau) \sim |\omega_p \tau|^{\eta-1}$. As a result the $I-V$ characteristics at low voltage have a power-law behavior

$$I_A = G_A \frac{e^{-\gamma(\eta-1)}}{\Gamma(1+\eta)} \frac{\omega_p}{2e} \left( \frac{2e|V|}{\omega_p} \right)^\eta V \ll \omega_p/e$$ \hspace{1cm} (7)

where $\gamma$ is the Euler number. In the classical limit, $\eta \to 1$, the ground state of the chain has all the phases aligned and no collective modes are excited. Then the chain behaves as a bulk superconductor and the Andreev $I-V$ characteristic is linear. When $E_0$ increases phase-wave collective modes become active. In particular they are excited by the Andreev tunneling events themselves and their back-action determines the anomalous nonlinear characteristics of Eq. (7).
The phase-wave Gaussian limit is the fixed point Hamiltonian also when quantum phase slips are included but the parameters are renormalized. In Eq. (7) $\eta$ is replaced by the renormalized value which deviates from the phase-wave value close to the S-I transition (see Ref. [13] for more details).

Comparing eq.(4) with eq.(7) one sees that close to the transition point the $I-V$ curves start as a power law, but with different exponents when approaching the transition from the two sides (see the curves (b) and (c) in Fig. (2)). Approaching the transition from above $\eta^+_{cr} = 5/4$. The jump is

$$\eta^-_{cr} - \eta^+_{cr} = 1/4$$

The jump the $I-V$ curves is related to the jump in the superfluid density at the transition point. A similar behavior, though with a different value, has been predicted and observed long time ago for the conductance in the classical 2D superconducting films [14].

Now we analyze in more detail the Andreev $I-V$ characteristics in the ordered phase, $\sqrt{E_J/8E_0} > 2/\pi$. Eq. (9) is valid at voltages much smaller than $\omega_p/e$. For voltages much larger than $\omega_p/\kappa$ the short time limit of Eq.(6) is needed, which is

$$g(\tau) = \exp \left\{ -\eta - 1 \over 2 \left( \pi \tau - \tau^2 + \pi \tau^3 \right) \right\} \quad \tau \to 0$$

Analytic continuation leads to

$$I_{NS} = G_A \left( V - \pi \eta \left( \eta - 1 \right) \frac{\kappa \omega_p}{e} \right) \quad V \gg \frac{\kappa \omega_p}{e}$$

with corrections which vanish exponentially with the applied voltage. At very large voltages, where eq.(9) applies, we recover the linear behavior of the $I-V$ characteristics, apart from an offset which is related to the average energy cost for exciting the collective phase-wave modes of the chain. The $I-V$ curves are shown in the inset of Fig. (2). We calculated the behavior at intermediate voltages directly in real times from Eq.(2). The excitations of the chain act as bosonic modes of an electromagnetic environment [13] coupled to tunneling pair and their energy spectrum is given by Eq (5). Integrating the Bose operators of the environment we get the real time correlator

$$P(t) = \langle e^{i\varphi_0(t)} e^{-i\varphi_0(0)} \rangle = \exp \left\{ - \int_{-\pi}^{\pi} \frac{dq}{4\pi} \zeta(q) \left[ 1 - \cos(\omega_q t) + i \sin(\omega_q t) \right] \right\}$$

where $\zeta(q) = \sqrt{e^{2C(q)/E_J} \sin^2(q/2)}$. The current takes the form

$$I_{NS} = -4e \mid t \mid^2 N^2(0) \int_0^{2eV} {d\omega \over 2\pi} (2eV - \omega) P(\omega).$$

The power law behavior of Eq. (7) for small voltages turns very rapidly in the asymptotic form, Eq. (9), at $V \approx \kappa \omega_p/e$. $P(\omega)$ can be interpreted as the probability that
a tunneling event excites a mode of the JJA. This results resembles that obtained in connection with the study of the effect of the electromagnetic environment on single electron tunneling [13]. In this approximation the one-dimensional JJA acts like a line of linear elements with the equivalent impedance

$$Z(\omega) = \frac{h/2e^2}{\sqrt{1 - (\omega/\kappa \omega_p)^2}} \Theta(\kappa \omega_p - \omega). \quad (12)$$

In the phase-wave approximation the function $P(\omega)$ presents an elastic peak, $P(\omega) \sim \omega^{\eta-2}$ for $\omega \to 0$, and a divergence at $\omega = \kappa \omega_p$, $P(\omega) \sim |\kappa \omega_p - \omega|^{\eta-3/2}$. This latter feature is associated with excitations which are almost localized in some junction of the JJA and whose density of states diverges for the infinite chain. It produces some structure near $V = \kappa \omega_p/e$ both in $\partial^2 I / \partial V^2 \sim P(eV)$ and in the differential conductance $\partial I / \partial V$ which is shown in the inset of Fig. (3). Notice that the low voltage behaviour is dominated by the capacitive to the ground $C_0$ since $C$ enters simply in the scaling factor $\kappa$ (see Eqs. (11,12)).

The modifications we expect in going beyond the phase-wave approximations in the ordered state can be qualitatively discussed in terms of the spectrum of the excitations we neglected. Creation and breathing of bound pairs of phase slips produce low energy Gaussian fluctuations which, as already discussed, only renormalize $\eta$ and modify the exponent in Eq. (7) accordingly. They should not affect the structure near $V = \kappa \omega_p/e$ which is due to localized excitations. The latter, however, can be modified due to the interaction of these modes with the phase slips or between themselves. Moreover going to higher voltages phase slips may be induced. Their typical energy scale, away from the transition, is larger than $\kappa \omega_p$ so in this regime source induced phase slips are believed not to be important. They should start to affect the behavior at $V \sim \kappa \omega_p/e$ only close to the transition, so we expect a detectable region in the phase diagram where the behavior of Eq. (11) can be observed. At even higher voltage all the above excitations can be active and this should increase the offset in the $I - V$ curve, Eq. (11).

In conclusion we have shown that Andreev tunneling may be a powerful method to investigate the phase diagram and the excitation spectrum (see also [14]) of JJA. The results derived in this letter are summarized in Figs. (2, 3). When the JJA is in the disordered phase the Andreev tunneling is blocked up to a threshold voltage proportional to the correlation length $\xi$ (Fig. (2)). At the superconductor-insulator transition the $I - V$ curves are power law like with a jump in the exponent at the transition point (Fig. (2)). When the JJA is in the superconducting phase, the characteristics start as a power law and approach rapidly the shifted ohmic behaviour. A richer structure generated by localized excitations of the JJA is seen in the derivatives of the $I - V$ curves (see Fig. (3)), near $V = \kappa \omega_p/e$. We have studied explicitely a one dimensional JJA but other types of arrays can be treated along the same lines. In two dimensions frustration effects due to an applied magnetic field play an important role. The interference patterns manifest in the differential
conductance in a rather peculiar way. Work in this direction is in progress.

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FIGURE CAPTIONS

Figure 1: The system considered in the present work: a normal metal island connected by a tunnel junction to a very long chain of Josephson junctions.

Figure 2: (a) The $I - V$ curve in the limit $E_0 \gg E_J$: $I = 0$ for $V < V_{th}$. At the transition point the current starts as a power law, however there is a jump in the exponent of the characteristics (curves (b) and (c)). For $E_J \to \infty$ the classical linear behavior is recovered (d). In the inset some curves in the $E_J \gg E_0$ regime.

Figure 3: The function $P(\omega)$, which also gives the second derivative of the $I - V$ curves, in the limit $E_J \gg E_0$: $E_0/E_J = 0.05, 0.02, 0.005, 0.0005$, from top to bottom. In the inset the differential conductance, in the same limit, is suppressed for increasing $E_0/E_J$. 