Analysis of Turbulent Coherent Structures in a Flow over an Escarpment using Proper Orthogonal Decomposition

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Abstract. An analysis of turbulent coherent structures was conducted on the flow over a 1:25 scale model of the Bolund Hill escarpment by means of Proper Orthogonal Decomposition (POD) using Particle Image Velocimetry (PIV) data. Mapping of flow energy at various modes in the vicinity of the escarpment, provided an insight into the influence of inflow conditions on the underlying behaviour of turbulent coherent structures.

1. Introduction
With wind turbines increasingly being placed amidst complex topography, identified by abrupt changes in elevation, such as hills, mountains, ridges and valleys, a new set of challenges has emerged for the wind farm developer. In these environments, the wind speed and turbulence characteristics are modified by the terrain, and hence, linearized models, commonly used in the wind industry, are often not as reliable for these applications as compared to their use in flat or gently sloping terrain.

In order to improve parameterizations used in wind farm models and thus to produce more accurate predictions of wind characteristics in these regions, a better understanding of the mean and turbulent wind behaviour is vital. One area that is not fully understood is the relationship between inflow conditions and the resulting turbulent flow behaviour over a complex terrain.

The present study examines the flow over a 1:25 scale model of the Bolund Hill escarpment, a well-known test case for validation of numerical models in complex terrain \cite{1}. Wind tunnel flow measurements were taken using Particle Image Velocimetry (PIV) and tests were conducted using two different inflow wind speed profiles. Subsequently, the Proper Orthogonal Decomposition (POD) technique was used to analyze the turbulent coherent structures present in the flow and the relative influence of the inflow conditions.

2. Methodology
The present experiment was conducted at the Wind Engineering, Energy, and Environment (WindEEE) dome at the Western University in London, Ontario, Canada \cite{2}. PIV measurements were conducted along the 270\degree wind direction, in a two-dimensional vertical plane above the Bolund Hill scale model surface, in the vicinity of the escarpment.
2.1. Experimental setup

For this study, the WindEEE test chamber was operated in the wind tunnel mode, whereby only the 60 fans mounted on one wall of the chamber, arranged in four rows of 15 fans each, were used (see Figure 1). A contraction section was placed immediately downstream of the fan wall to increase wind speed and uniformity within the test section. A 1:25 scale model of Bolund Hill was constructed from Expanded Polystyrene (EPS) by means of CNC milling according to topographical data. The model was secured to the floor of the test chamber such that the model was aligned with the 270° wind direction, and the escarpment leading edge was roughly 12.4 m downstream of the fan wall.

To conduct PIV measurements, the flow field was illuminated with a Litron Nano Piv Series dual cavity Nd:YAG laser that has 532 nm wavelength, 425 mJ/pulse energy and 18 Hz frequency. A 50° optical lens was used to convert the beam into a two-dimensional sheet in the plane formed by the longitudinal and vertical directions. Three 12 Mega Pixel cameras (IO Industries Flare 12M125-CL), each with resolution of 4096×3072 pixels, were used to capture image pairs at a rate of 9 Hz. The cameras were positioned in a row parallel to the 270° wind direction, at a distance of roughly 3.55 m from the measurement plane (see Figure 1). Each camera had field of view of roughly 0.78 m wide by 0.58 m high, with at least 10% overlap between camera fields, resulting in an overall field of view of roughly 2.09 m wide and 0.58 m high (see Figure 1). IO Industries Coreview software was used to control image capture. Seeding of the chamber was accomplished by means of a commercial fog system (Ultratec CLF-4460) located in the dome’s upper plenum.

The WindEEE facility allows for customized control of fan speed and roughness element height. As part of the overall experiment, various combinations of wind speeds, upstream roughness, inflow profiles and model geometries were tested. This paper presents a direct comparison of two such cases, whose key parameters are indicated in Table 1. The two cases, denoted “uniform” and “shear”, were tested with the same model geometry and same upstream roughness configuration, at roughly the same Reynolds number. The cases differed only in the shape of the inflow wind profile: for the uniform case, each of the 60 fans was operated at the same speed (50% of their maximum RPM), resulting in a mean streamwise upstream wind speed at hill height, $U_h$, of 14.6 m/s (see Table 1). For the shear case, fans in row 3 were set to 75% of maximum RPM, while fans in the other three rows were held at 50%. For reference, row 1 is closest to the floor and row 4 closest to the ceiling (see Figure 1). The modified shear profile case is illustrative of a non-classical boundary layer which could for example be used to experimentally simulate non-neutral atmospheric conditions, and furthermore can provide insight into the merits of using a customized inflow profile to approximate full-scale wind conditions in a multi-fan wind tunnel setup.

![Figure 1: Schematic of experimental setup (not to scale). Contraction not shown. Mast positions (M0, M7, M6, M3) from the full-scale measurements [1] shown for reference.](image-url)
Inflow parameters were measured using an array of eight Cobra Probes at a reference position located 2.25 m upstream of the escarpment leading edge. Streamwise wind speed $U_h$, Reynolds number $Re$, streamwise turbulence intensity at hill height $I_{u'h}$, and friction velocity $u_*$ were similar between the two cases (see Table 1). Friction velocity was calculated according to $u_* = (-\bar{\overline{u'w'}})^{1/2}$, where the reference value indicated in Table 1, $u_{*0.5}$, was taken as the $u_*$ value at the full-scale height of 5 m. This is analogous to the method used in the analysis of the full-scale measurements [1], and is used here for consistency and comparison purposes. Upstream roughness $z_0$ was roughly ten times higher for the uniform case compared to the shear case, however both cases are within the fully aerodynamically rough regime, where $u_*z_0v^{-1} \gg 1$ [3]. $z_0$ was computed using the well-known logarithmic law, using the subset of data points within the overlap layer. Values of $z_0$ are shown in Table 1 in full-scale units.

| Table 1. Inflow parameters |
|-----------------------------|
|                            |
| **Uniform**                 |
| $U_h$ (m/s)                 | 14.6 |
| Re                          | $4.57 \times 10^5$ |
| $u_{*0.5}$ (m/s)            | 0.856 |
| $z_0$ (m)                   | $2.72 \times 10^{-3}$ |
| $I_{u'h}$                   | 0.120 |
| Fan configuration           | All fans 50% |
|                            |
| **Shear**                   |
| $U_h$ (m/s)                 | 15.6 |
| Re                          | $5.21 \times 10^5$ |
| $u_{*0.5}$ (m/s)            | 0.992 |
| $z_0$ (m)                   | $2.87 \times 10^{-4}$ |
| $I_{u'h}$                   | 0.137 |
| Fan configuration           | Fan rows 1,2,4: 50% |
|                            | Fan row 3: 75% |

Inflow profiles for the uniform and shear cases at the upstream reference position are shown in Figure 2. Figure 2a shows the high degree of similarity in the streamwise wind speed profiles. Normalized TKE $\overline{k}$ is slightly higher for the uniform case (Figure 2b), while Reynolds shear stress profiles (Figure 2c) are generally similar, apart from the divergence seen just below $y/h = 1$. This higher shear stress for the shear profile case near hill height represents an increase in momentum transport, and appears to be initiated at the interface between fan rows 2 and 3 (see Figure 1) and mixed into the flow. A region of roughly constant shear stress can be observed from $0.2 < y/h < 0.5$, encompassing the full-scale height of 5 m, and thus providing validity to the method of estimating friction velocity described above [4]. Integral length scales in the longitudinal direction $L_u^x$ (Figure 2d) indicate that the average eddy size for the shear case is smaller than that of the uniform case.

2.2. Data post-processing

Instantaneous velocity fields were computed from the PIV image pairs using an in-house algorithm running through image processing software Heurisko® (AEON Verlag & Studio GmbH & Co. KG). Mean velocity fields were obtained by time-averaging instantaneous velocities at each grid point. Turbulent velocity was obtained by subtracting the mean velocity from the instantaneous velocity at each grid point in each velocity field.

The underlying features of the turbulent flow field were analyzed using the Proper Orthogonal Decomposition (POD) technique, which provides the distribution of flow energy content at different orthogonal modes, enabling detection and characterization of underlying energetic flow patterns. POD was undertaken specifically for this dataset in order to better understand the turbulent structures and energy distributions in escarpment flows.

In a separate paper recently submitted for publication [5], the mean and turbulent flow behaviour over the escarpment was characterized using the PIV data, and the influence of different inflow conditions, including the uniform and shear profiles, was examined. In this study, an increase in TKE...
of over 200% was observed for the shear case, compared to the uniform profile case, in the region close to the hill surface, just upstream of mast position M6 (see Figure 1).

This large increase occurred despite what can arguably be described as only minor differences in the inflow profiles for the two cases (see Figure 2). Moreover, the TKE measurements for the shear case were significantly closer to the full-scale TKE measurements in the same region than the uniform case [5], thus providing further incentive to investigate what inflow conditions may yield better representations of actual wind flow behaviour, and how the turbulent mechanisms and structures in the resulting flow field are affected.

Through the POD technique, the turbulent velocity field is broken down (i.e. decomposed) into orthogonal basis functions in time and space [6]. The method is optimized such that the energy of the signal is represented using the fewest number of modes [7]. Lower modes correspond to larger, more energetic structures, while higher modes are associated with smaller structures with lower energy. Previous efforts have documented relationships between the POD modes and the underlying physical turbulent structures present in the flow (Régert et al [8]; Basley et al. [9] and Podvin et al. [10]).

![Figure 2: Inflow conditions measured using Cobra Probes at the upstream reference location.](image)

A POD algorithm developed by Doddipatla [11] following the snapshot method developed by Sirovich [12] was used to decompose the turbulent velocity data into its orthogonal modes or basis functions \( \phi(x) \), where the expansion of the velocity \( \bar{u}(x,t) \) into spatial and temporal coefficients is represented by,

\[
\bar{u}(x,t) = \sum_{n=1}^{N} a^n(t) \phi^n(x)
\]  

(1)

Where \( a^n(t) \) is the temporal coefficient and \( \phi(x) \) is the spatial component or basis function [6]. In the present analysis, the POD data is based on 1200 snapshots.
3. Results
Figure 3 presents the distribution of flow energies at different modes (left axis) as well as the cumulative energy (right axis). It can be seen from the figure that the flow energy is mainly contained in lower modes. The first ten modes cumulatively contained about 42% of the total energy, which was relatively similar for both uniform and shear inflow cases. However, the distribution within these modes differed slightly between the cases. Modes 1-2 contained 19% of the POD energy for the uniform case compared to about 21% for the shear case, whereas modes 4-6 contained 10.5% of the energy for the uniform case and 8.5% for the shear case. On a cumulative basis, the shear case had relatively higher flow energy in 1-6 modes compared to that for the uniform case. Note that the energy distribution presented in Figure 3 encompasses the flow energy over the entire measurement domain. To get a better insight into the local distribution of flow energy, particularly in the vicinity of the escarpment, the flow energy patterns at different modes are presented in Figures 4 and 5 for the uniform and shear cases, respectively.

The investigation area shown in the figures comprises the combined fields of view from the three cameras described earlier. Axes are normalized by hill height \( h \) such that \((x/h = 0, y/h = 1)\) corresponds to the edge of the escarpment, and the POD energy at each location is normalized by the total energy in that mode. Some modes were combined using linear summation, an approach commonly used in the POD analysis. As shown in the figures, for both test cases, the largest structures, and thus the highest POD energy, are observed at the lower modes, whereas at the higher modes, the POD energy patterns become smaller, relatively weaker and more or less indistinguishable from one another (i.e. non-coherent), corresponding to the expected homogenous, isotropic properties associated with the smaller scales of turbulence. The turbulent separation bubble formed at the escarpment is observed to be the most energetic structure and contains roughly one-quarter of the total turbulent energy in that region.

Several notable differences are observed between the two test cases, primarily in the lower POD modes, particularly modes 1 and 2. Mode 1 essentially shows one large feature, where the energy pattern indicates the bulk flow behaviour at the escarpment. While the POD energy for the shear profile case is concentrated near the escarpment leading edge, it is distributed across the measurement window for the uniform case, which may be indicative of bulk motion on a scale larger than that of the measurement window. In the mode range 4-11, the results show that the energy patterns are relatively stronger for the
shear flow and have higher spatial extent. The intermediate modes (i.e. modes 9-11, 14-16 etc.) clearly depict alternating patterns of positive and negative energy, indicating the formation of coherent structures. The contours at higher modes show a series of smaller structures, whose patterns indicate the presence of small scale vortices formed above the surface under the influence of the separation bubble. For most modes, the magnitude of the POD energy is seen to be highest near the escarpment edge, and near the hill surface, and dissipates moving downstream.

**Figure 4**: Streamwise POD energy contours in the vicinity of the escarpment, uniform fan speed case.
Previous studies have shown that the POD energy patterns provide a good representation of the turbulent coherent structures or vortices present in the flow. Hence, the results presented in Figures 4 and 5 can be used to obtain deeper insight into the behavior of turbulent structures in the flow over an escarpment. The results indicate that the larger, energy-containing turbulent vortices grow in size as they move downstream. When a wind turbine is installed in the downstream region, these vortices are expected to dynamically interact with the turbine (blades and tower) and may induce transient loading on the turbine, which may affect the structural integrity of the turbine as well as induce low-frequency noise that may cause discomfort for residents living nearby.
4. Conclusions
The Proper Orthogonal Decomposition (POD) technique was applied to experimental measurements of wind flow over an escarpment to investigate various scales of the turbulent velocity field. Two different inflow profiles were tested: one with uniform fan speed, and one shear profile, representative of a non-classical boundary layer.

The results revealed different scales of turbulent coherent structures present in the flow depending on the inflow wind profile, thus providing a deeper insight into the influence of the inflow conditions on turbulent structures in the vicinity of the escarpment. The most significant differences between the two profiles were observed in the lowest POD modes, corresponding to the largest and most energetic structures. The results also showed that the strongest structures in the measurement region were those associated with the separated flow region at the escarpment leading edge.

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