Phase lags of quasi-periodic oscillations across source states in the low-mass X-ray binary 4U 1636–53

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Abstract. The majority of attempts to explain the origin and phenomenology of the quasi-periodic oscillations (QPOs) detected in low-mass X-ray binaries invoke dynamical models, and it was just in recent years that renewed attention has been given on how radiative processes occurring in these extreme environments gives rise to the variability features observed in the X-ray light curves of these systems. The study of the dependence of the phase lags upon the energy and frequency of the QPOs is a step towards this end. The methodology we developed here allowed us to study for the first time these dependencies for all QPOs detected in the range of 1 to 1300 Hz in the low-mass X-ray binary 4U 1636–53 as the source changes its state during its cycle in the colour-colour diagram. Our results suggest that within the context of models of up-scattering Comptonization, the phase lags dependencies upon frequency and energy can be used to extract size scales and physical conditions of the medium that produces the lags.

1. Introduction

Low-mass X-ray binaries containing both, neutron stars (NS-) and black holes (BH-), show rapid (seconds to milliseconds) X-ray variability components in the power spectral density (PSD) of their X-ray light curves. In the case of NS-LMXBs besides a power-law component at very low frequencies, we identify quasi-periodic oscillations (QPOs) ranging from 0.01 to 1300 Hz (see [1] for a review). The frequencies of most of the QPOs correlate with each other in a given source and other of their properties like the fractional amplitude (rms) and the quality factor (Q), depend also on the spectral state of the source (see, for example, [2, 3]). In particular frequency-frequency correlations of some timing features in X-ray binaries containing black holes, neutron stars or white dwarfs (e.g. [4, 5, 6, 7]) suggests that the origin of these features does not depend on the nature of the central compact object, but are due to a physical component that is common to all these systems, likely the accretion disc or a corona.

Renewed attention has been given to the energy- and frequency-dependence of the time (or phase) lags of the X-ray signal (e.g. [8, 9, 10, 11, 12]), since it is believed that they possibly encode geometrical properties of the medium that produces the variability and the radiative processes occurring around the system. Time/phase lags are defined as Fourier-frequency-dependent measurements of the time (phase) delays between two concurrent and correlated signals, i.e. two light curves of the same source, in two different energy bands (see [8] for more details on the subject).

Compton up- and down-scattering are the most common mechanism to explain time/phase lags of X-ray photons in LMXBs. In this case, photons produced in the accretion disc of BH-
Figure 1. Left panel: the colour-colour diagram of 4U 1636–53 with the 27 originally defined boxes. Right panel: the boxes that were further subdivided; each one displays groups of observations (highlighted by different colour points) with similar PDSs grouped together creating the subdivisions. Each point corresponds to a single RXTE observation. We only include in this diagram the 511 observations with known kHz QPOs. For each box we fitted the QPOs in the average PSD. The dark line is the spline used to parametrise the position of the source on the CCD.

and NS-LMXBs, or even in the surface of the neutron star and/or its boundary layer in the case of NS-LMXBs, are scattered in a corona of hot electrons (see, for example, [13, 14, 15, 9, 16, 12] for a discussion about the models).

Here we study of the phase lags of all QPOs detected above 1 Hz in a large dataset of RXTE observations of the NS-LMXB 4U 1636–53 as a function of the position of the source on the colour-colour diagram (CCD). In §2 we describe our observations and methodology; in §3 we present the results for the frequency dependence of the phase lags (3.1) and for the energy dependence (3.2). We discuss our results and conclude in §4. For a more detailed discussion of the results presented here we kindly refer the reader to our paper [17].

2. Observations and data analysis
We defined in total seven narrow energy bands and two broad bands. The mean energies of the narrow bands are 4.2 keV, 6.0 keV, 8.0 keV, 10.2 keV, 12.7 keV, 16.3 keV and 18.9 keV and of the broad bands are 7.1 keV and 16.0 keV. These are the same energy bands defined in [10]. Additional details in [17].

To study the phase lags of all QPOs as a function of the position of the source in the CCD\(^1\), we first defined boxes on it following two criteria: i) the boxes should be small to avoid changes in the physical conditions of the source, which would imply changes in the shape of the PSDs but with enough observations within each box in order to build the average spectra and ii) we then compared the shapes of the PSDs of each individual observation within each box both, visually and statistically\(^2\). If we identified, within a box, groups of observations with similar PSDs, then we further subdivided the box by grouping these similar observations. We ended up with 37 boxes as depicted in Figure 1.

The second step was to parametrise the position on the CCD with the quantity \(S_a\) as in

\(^1\) The hard colour (HC) was defined as the 9.7-16.0 keV/6.0-9.7 keV count rate ratio and the soft colour (SC) as the 3.5-6.0 keV/2.0-3.5 keV count rate ratio. See [3].

\(^2\) By statistically we mean that we analysed the average of two probabilities of similarity, KS ([19]) and MTT ([20]), and we say that two PSDs are similar if the average of the probabilities are greater than 65%.
The time lags of \( L_{b2}, L_{h}, L_{LF}, L_{h}, L_{hHz}, L_{hHz-harm}, L_{l}, L_{u} \) as in [3].

For the energy-dependence of the lags, we calculated the phase lags of the photons of each narrow energy band relatively to the 10.2 keV band\(^3\). For the frequency-dependence of the lags, we calculated the phase lags of all photons in the broad band above 12 keV relatively to all photons in the broad band below 12 keV.

Finally, to calculate the lags of the QPOs we average the energy- and frequency-dependent lags over a frequency interval of one FWHM around the centroid frequency of the QPO, except in the cases where two QPOs are broad enough to overlap each other. In these cases we averaged over a fraction of the FWHM ranging from one half to one fifth. All phase lags are given in units of 2\( \pi \) rad.

3. Results

3.1. Frequency dependence of the phase lags

In Figure 2 we plot the phase lags of six out of the eight QPOs detected in the PSD as a function of the frequency of the QPO for photons with mean energy \( \simeq 16.0 \) keV relatively to photons with mean energy \( \simeq 7.1 \) keV. Uncertainties are given at a 68\% confidence level. In Figure 3 we plot the phase lags of the QPO specified in each of the panels as a function of the quantity \( S_a \).

In Table 1 we show constant fits of the phase lags as a function of \( S_a \) of all QPOs in order to compare them with each other. We see from Table 1 that the average phase lags are soft in the cases of \( L_{b2} \) and \( L_{l} \). The phase lags of \( L_{b2} \) are significantly different from the phase lags of \( L_{b}, L_{h} \) and \( L_{u} \), the phase lags of \( L_{b} \) are significantly different from the lags of \( L_{b}, L_{h} \) and \( L_{l} \), the phase lags of \( L_{l} \) are significantly different from the lags of \( L_{l} \), and the phase lags of \( L_{l} \) are significantly different from the lags of \( L_{u} \). The phase lags of \( L_{b}, L_{hHz} \) and \( L_{u} \) are consistent with each other within 3\( \sigma \). Also given in this Table are the average time lags as a function of \( S_a \) for each QPO, which we will use later.

The time lags of \( L_{b2} \) are inconsistent with the time lags of all other QPOs by more than 3\( \sigma \). The time lags of \( L_{h} \) are consistent within 3\( \sigma \) with the time lags of all other QPOs but \( L_{h} \). The time lags of \( L_{h}, L_{hHz} \) and \( L_{u} \) are different from the time lags of \( L_{l} \) and \( L_{u} \). At last, the time lags of \( L_{l} \) are different from the time lags of \( L_{u} \) by more than 3\( \sigma \), a result previously known [10].

3.2. Energy dependence of the phase lags

The results in Section 3.1 show that the phase lags do not depend strongly on the frequency of the QPO or \( S_a \), except maybe for the lower kHz QPO (bottom left panel of Figure 2 and 3, respectively). Based on this, we averaged the phase lags over frequency to calculate the phase lags versus energy for each QPO under consideration (the details can be found in [17]). Notice for \( L_{l} \) this procedure is not completely correct since there is a dependence of the lags upon the frequency. The results of this procedure are shown in Figure 4. As with the frequency

\(^3\) Being relative quantities, we chose the 10.2 keV band as the reference band that the lags of photons with other energies are calculated relatively to, because in this band both the variability and the source intensity are high, which reduces the uncertainties of the of time-lag measurements ([22]).
Figure 2. Phase lags, in units of $2\pi$, of the different QPO components of 4U 1636–53 as a function of the frequency of the corresponding QPO. The lags represent the delay of photons with mean energy $\simeq 16$ keV relative to the photons with mean energy $\simeq 7$ keV. Left top panel: the QPO where the PDS shows a second break. Right top panel: the QPO at the break frequency. Left middle panel: the hump QPO. Right middle panel: the hecto-hertz (hHz) QPO. Left bottom panel: the lower kHz QPO. Right bottom panel: the upper kHz QPO. The blue lines are the best fit constant to the lags. The errors correspond to the 1σ confidence level.

Table 1. Weighted average phase lags and time lags as function of $S_a$ (see Figure 3) for the $L_{b2}$, $L_b$, $L_h$, $L_{hHz}$, $L_l$ and $L_u$ QPOs of the NS-LMXB 4U 1636–53 of all photons in the broad band 12-20 keV relatively to all photons in the broad band 4-12 keV.

| QPO    | phase lag [2π rad] | $\chi^2$ | dof | time lag [msec] |
|--------|--------------------|----------|-----|-----------------|
| $L_{b2}$ | $-0.010 \pm 0.002$ | 25.6     | 5   | $-8.7 \pm 2.1$  |
| $L_b$   | $0.005 \pm 0.002$  | 40.7     | 11  | $0.05 \pm 0.09$ |
| $L_h$   | $0.018 \pm 0.002$  | 4.1      | 5   | $0.8 \pm 0.1$   |
| $L_{hHz}$ | $0.004 \pm 0.004$ | 13.5     | 10  | $0.008 \pm 0.035$ |
| $L_l$   | $-0.017 \pm 0.001$ | 13.5     | 8   | $-0.020 \pm 0.002$ |
| $L_u$   | $0.008 \pm 0.002$  | 19.7     | 14  | $0.010 \pm 0.002$ |
Figure 3. Phase lags, in units of $2\pi$, of the QPOs in Figure 2 as a function of the parameter $S_a$ which is representative of the position of the source in the CCD. The blue lines are the best fit constant to the lags.

dependence, the results for the energy dependence of the phase lags of the kHz QPOs are consistent with the ones obtained in [10].

It is interesting to note, based on the shapes of the lag vs E plots, that the trend with energy, being marginal or not, is an increasing trend for $L_b$, $L_h$, $L_{hHz}$, and $L_u$, but is decreasing for $L_l$ and possibly for $L_{b2}$. In the case of $L_{b2}$ the question arises if the phase lags of $L_{b2}$ may be linearly correlated with the phase lags of $L_l$, although the F-test probability of 0.026 indicates the contrary. The same question arises about a possible anti-correlation between the phase lags of $L_h$ and $L_l$ (F-test probability 0.005).

Comparing the slopes of the lag vs E linear fits for each QPO, we found that we can separate the components into two groups having slopes consistent with each other: On the one hand $L_{b2}$ and $L_l$, and on the other hand $L_b$, $L_h$, $L_{hHz}$, and $L_u$. To further investigate the slopes we also calculated the Pearson Product Moment Correlation (PPMC) coefficient for all 15 possible combinations of lag vs lag. This correlation coefficient is a statistic that calculates the actual relationship between two variables.

We have not found any correlation between the lags except in the case of the phase lags of $L_h$ in relation to the phase lags of $L_{hHz}$. Even so, we still can separate the lags in two groups as mentioned above.
Figure 4. Phase lags, in units of $2\pi$, of the different QPO components of 4U 1636–53 as a function of energy. The lags represent the delay of photons in the bands whose average energies are given by the values in the x-axis relative to the photons in the band whose average energy is 10.2 keV. The panels show the same QPOs as in Figure 2.

4. Discussion
4.1. Frequency dependence
We see from Figure 3 that the phase lags are practically independent of $S_a$, except for $L_l$ (for which we see a complex behaviour) and maybe for $L_{hHz}$. A scenario where the accretion flow has two components ([23, 24, 25]) could explain the fluctuations of the phase lags of $L_l$ to softest values and the hard lags of the $L_u$ that would “keep its constancy” through compensations in the properties of the coronal component.

We derived upper limits to the size of the medium in which the time lags are produced by light travel time arguments in the context of models for the lags that involve reflection off the disc or Comptonization. From the values in Table 1 we can roughly estimate the scale size, $a$, of the medium where the lags of each QPO are produced with the expression $a \sim c\Delta t (k_BT_e)^{4\pi} (\ln(E_2/E_1))^{-4}$ (see, e.g. [13]). Here, $\tau \equiv 5$ is the optical depth and $k_BT_e \equiv 5$ keV is the plasma temperature (typical values in modelling the spectra of 4U 1636–53, e.g. [26]), $\Delta t$ are the time lags, $E_2 = 16.0$ keV and $E_1 = 7.1$ keV are the energies of the photons in the broad energy bands we chose. The electron density comes from $n_e = \tau/(a\sigma_T)$. See Table 2.

The values in Table 2 suggest that at least two components, one for $L_b$, $L_{hHz}$, $L_l$ and $L_u$ with density $\sim 10^{19}$–$20$ cm$^{-3}$ and another for $L_{b2}$ and $L_b$ with density $\sim 10^{17}$–$18$ cm$^{-3}$, are needed
The trends with energy show resemblance if we group together $L_{b2}$, $L_b$, $L_h$, $L_{hHz}$, $L_l$ and $L_u$ QPOs of the NS-LMXB 4U 1636–53 are produced. Here, $c\Delta t$ gives the light travel time corresponding to the measured time lags (see Table 1), $a$ is the size scale and $n_e$ is the electronic density of the medium.

| QPO  | $c\Delta t$ [km] | $a$ [km]   | $n_e$ [$10^{20}$ cm$^{-3}$] |
|------|------------------|------------|----------------------------|
| $L_{b2}$ | 2610 ± 630       | 628.6 ± 151.7 | 0.0012 ± 0.0003 |
| $L_b$   | 15 ± 27          | 3.6 ± 6.5   | 0.21 ± 0.37            |
| $L_h$   | 240 ± 30         | 57.8 ± 7.2  | 0.013 ± 0.002          |
| $L_{hHz}$ | 2.4 ± 10.5      | 0.58 ± 2.53 | 1.3 ± 5.7              |
| $L_l$   | 6.3 ± 0.6        | 1.52 ± 0.14 | 0.50 ± 0.05            |
| $L_u$   | 3.0 ± 0.6        | 0.72 ± 0.14 | 1.04 ± 0.21            |

for producing the measured phase lags. Notice the scale sizes for the first set are quite small (0.5 to 3.5 km), while for the latter are much bigger, 60 km and 630 km.

On one hand, the phase lags of $L_{b2}$ and $L_{hHz}$ could have the same mechanism of the lags seen in AGNs, given the similarity of their lag spectrum (see [27]). On the other hand, it is tempting to compare our results on $L_l$ and $L_u$ (bottom panels of Figure 2) with those phase lags measured by [28]. It is possible that the same mechanism that produces the soft lags of the QPO at 35 Hz and the hard lags of the QPO at 67 Hz also applies to the lower and upper kHz QPOs of 4U 1636–36, since they also have opposites signs and are inconsistent with each other (see [28] for additional discussion about this).

Based on our results and on the findings of [29], for example, we suggest that the time lags of the upper kHz QPO encode the properties of the medium at the magnetospheric radius (where the upper kHz QPO would be produced, at 16 to 21 km in our estimations4, and where the scale size for the lags is $\sim 0.72$ km), while the time lags of the lower kHz QPOs encode the properties of the medium at the boundary layer and nearby (where the lower kHz QPO would be produced, near the surface of the neutron star, where the scale size for the lags is $\sim 1.52$ km). This idea would explain why the time lags of these two QPOs are inconsistent with each other and of opposite sign.

4.2. Energy dependence

The trends with energy show resemblance if we group together $L_b$, $L_h$, $L_{hHz}$ and $L_u$ on one hand and $L_{b2}$ and $L_l$ on the other. The lags of the first group become harder and those of the second group become softer as the energy increases. These trends may be a hint of a general mechanism behind the production of the lags of the QPOs of each group.

Recently, [12] developed a new model for the energy dependence of the time lags based on a thermal Comptonizing plasma. They were able to obtain soft lags (of the same magnitude as the ones reported here for 4U 1636–53 and the ones reported by [11] for 4U 1608–52) only when there is a variation in the heating rate of the corona and a significant fraction, $\eta = 0.4$, of the photons impinging back into the photon source. They constrained the size of the Comptonizing region to $L = 1.0$ km. The same mechanism could apply to $L_{b2}$ since it also shows soft lags. However, the hard lags found in the other QPOs are beyond this model. This model also explains satisfactorily the increase of the fractional rms with energy, although cannot explain the flattening at energies above 10 keV seen in the data.

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4 We assumed the neutron star mass in the range $M \sim 1.6 - 1.9 M_\odot$, fixed the radius of the neutron star in 10 km, fixed the magnetic field at $B \sim 10^8$ G and assumed that the source accretes at a rate $0.03 - 0.06 L_{edd}$. 
More recently [30] performed an extensive study of the spectral-timing properties of the lower and upper kHz QPOs in the source 4U 1728–34 and they found that the QPO spectrum is compatible with the one of a Comptonized black body with temperature higher than the temperature of the continuum, which implies that a more compact inner region of the boundary layer is producing the QPO. They also found that the lag-energy spectrum of the lower kHz and the upper kHz QPO are systematically different in the same way we have found in [10] and that they also depend weakly on frequency. In a scenario where the lags of the upper kHz QPO are dominated by reverberation, we see the upper kHz QPO signal as a response of the boundary layer to the variations in the accretion rate. Evidence that broader and lower frequency noise components are also generated by the same kind of mechanism ([31, 32]) would then be corroborated by our results based on the general shapes of the lag-energy spectrum, except for \( L_2 \) which is similar to the lower kHz QPO \( L_1 \). Then, the more coherent signal of the lower kHz QPO may have its origin in a more compact region of the boundary layer itself due to internal oscillations in the heating rate of the boundary layer (as in [9, 12], for example). Our results for the lag energy spectrum of the QPOs seem to point also in that direction.

If extended to include all the other QPOs, these models provide an opportunity to study the dynamic and physical conditions of the Comptonising corona in neutron-star low-mass X-ray binaries.

References

[1] van der Klis M 2006 Compact stellar X-ray sources (Cambridge Astrophysics Ser. vol 39) ed Lewin W and van der Klis M (Cambridge: Cambridge University Press) p 39
[2] van Straaten S, van der Klis M, di Salvo T and Belloni T 2002 Astrophys. J. 568 912
[3] Altamirano D, van der Klis M, Méndez M, Jonker P, Klein-Wolt M and Lewin W H G 2008 Astrophys. J. 685 436
[4] Wijnands R A D and van der Klis M 1999 Astrophys. J. 514 939
[5] Psaltis D, Belloni T and van der Klis M 1999 Astrophys. J. 520 262
[6] Mauche C W 2002 Astrophys. J. 580 423
[7] Warner B and Woudt P A 2002 Mon. Not. R. Astron. Soc. 335 84
[8] Nowak M A, Vaughan B A, Wilms J, Dove J B and Begelman M C 1999 Astrophys. J. 510 874
[9] Lee H C, Misra R and Taam R E 2001 Astrophys. J. 549 L229
[10] de Avellar M G B, Méndez M, Belloni T and Sanna A 2013 Mon. Not. R. Astron. Soc. 433 3453
[11] Barret D 2013 Astrophys. J. 770 9
[12] Kumar N and Misra R 2014 Mon. Not. R. Astron. Soc. 445 2818
[13] Sunyaev R A and Titarchuk L G 1980 Astron. & Astrophys. 168 121
[14] Payne D G 1980 Astrophys. J. 237 951
[15] Lee H C and Miller G S 1998 Mon. Not. R. Astron. Soc. 299 479
[16] Falanga M and Titarchuk L 2007 Astrophys. J. 661 1084
[17] de Avellar B, Méndez M, Altamirano D, Sanna A and Zhang G 2016 Mon. Not. R. Astron. Soc. 461 79
[18] Zhang G, Méndez M, Belloni T M and Homan J 2013 Mon. Not. R. Astron. Soc. 436 2276
[19] Kolmogorov A 1933 Giornale dell’ Instituto Italiano degli Attuari 4 83
[20] Melo I, Tomásik B, Torrieri G, Vogel S, Bleicher M, Koróny S and Gintner M 2009 Phys. Rev. C 80 024904
[21] Méndez M, van der Klis M, Ford E C, Wijnands R and van Paradijs J 1999 Astrophys. J. 511 L49
[22] Vaughan B A and Nowak M A 1997 Astrophys. J. 474 L43
[23] Esin A A, McClintock J E and Narayan R 1997 Astrophys. J. 489 865
[24] Meyer-Hofmeister E, Liu B F and Meyer F 2005 Astron. & Astrophys. 432 181
[25] Done C, Gierliński M and Kubota A 2007 Astron. & Astrophys. Rv15 1
[26] Luu M, Méndez M, Zhang G and Keek L 2015 Mon. Not. R. Astron. Soc. 454 541
[27] Zoghbi A, Fabian A C, Uttley P, Miniutti G, Gallo L C, Reynolds C S, Miller J M and Ponti G 2010 Mon. Not. R. Astron. Soc. 401 2149
[28] Méndez M, Altamirano D, Belloni T and Sanna A 2013 Mon. Not. R. Astron. Soc. 435 2132
[29] Bulik P and van der Klis M 2015 Astrophys. J. Lett. 798 L29
[30] Peille P, Barret D and Uttley P 2015 Astrophys. J. 811 109
[31] Uttley P 2004 Mon. Not. R. Astron. Soc. 347 L61
[32] Uttley P, Wilkinson T, Cassatella P, Wilms J, Pottschmidt K, Hanke M and Böck M 2011 Mon. Not. R. Astron. Soc. 414 L60