D-centro dominating sets in graphs

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Abstract
In this paper a new concept D-centro dominating set in graphs is introduced and graphs are characterized with some results. A subset $S \subset V(G)$ of a connected graph $G$ is said to be D-centro dominating set of $G$, if for every $v \in V - S$, there exists a vertex $u$ in $S$ such that $D(u,v) = Rad(G)$. The minimum cardinality of the D-centro dominating set is called D-centro domination number, denoted by $DC_r(G)$. The D-centro dominating set with cardinality $DC_r(G)$ is called $DC_r$-set of $G$. Some bounds for the D-centro domination number are determined. An important realization result on D-centro domination number is proved that for any integers $a$ and $b$ with $2 \leq a \leq b$, there exists a connected graph $G$ such that $DC_r(G) = a$ and $DC(G) = b$.

Keywords
Detour distance, detour eccentricity, detour radius, D-centro sets.

AMS Subject Classification
05C12, 05C69.

1. Introduction
By a graph $G = (V,E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of $G$ are denoted by $n$ and $m$ respectively. For basic graph terminology we refer to Harary [7]. For vertices $r$ and $s$ in a connected graph $G$, the detour distance $D(r,s)$ is the length of the farthest $r$-$s$ path in $G$. For any vertex $r$ of $G$, the detour eccentricity of $r$ is $e_D(r) = \max\{D(r,s) : s \in V\}$. A vertex $s$ of $G$ such that $D(r,s) = e_D(r)$ is called a detour eccentric vertex of $r$. The detour radius $R$ and detour diameter $D$ of $G$ are defined by $Rad(G) = \min\{e_D(s) : s \in V\}$ and $Diam(G) = \max\{e_D(s) : s \in V\}$ respectively. An $r$-$s$ path of length $D(r,s)$ is called an $r$-$s$ detour path. These concepts were studied by Chartrand et al [6]. If $e_D(s) = Rad(G)$ then $s$ is called a detour central vertex of $G$ and the subgraph induced by all detour central vertices of $G$ is called detour center of $G$ and is denoted by $CD(G)$. Next we study the following definitions given in [1]. For any vertex $p$ in $G$, a set $S$ of vertices of $V$ is an $p$-D-centro set if $D(p,s) = Rad(G)$ for every $s \in S$, that is, $p$ and $s$ are said to be D-centro to each other. It is denoted by $DC_p(G)$. Let $p$ be a vertex of $G$ and $S$ be the $p$-D-centro set of $G$. Then $p$ is said to be the D-centro vertex of $G$ with respect to $S$ if the cardinality of $S$ is the maximum among all $S$. The maximum $p$-D-centro set is denoted as $S_p$. The set of all D-centro vertices of $G$ is called D-set of $G$ and the cardinality of D-set is said to be D-centro number of $G$ and it is denoted by $Dn(G)$. A set $S$ is said to be D-centro set of $G$ if $D(r,s) = Rad(G)$ for every pair of vertices of $S$. That is, $r$ and $s$ are D-centro to each other in $S$. The maximum cardinality among all D-centro sets is called DC-set. It is denoted by $DC(G)$.

2. D-centro dominating set
Next we define and study the properties of D-centro dominating set.

Definition 2.1. A subset $S \subset V(G)$ of a connected graph $G$ is said to be D-centro dominating set of $G$ if for every $v \in V - S$, there exists a vertex $u$ in $S$ such that $D(u,v) = Rad(G)$. The minimum cardinality of the D-centro dominating set is called D-centro domination number, denoted by $DC_p(G)$. The D-centro dominating set with cardinality $DC_p(G)$ is called $DC_p$-set.
set of $G$.

Sometimes, there exists no $u$-$D$-centro vertex in $G$ for a vertex $u$. Next we study these types of vertices in $G$.

**Definition 2.2.** A vertex $u \in G$ has no $u$-$D$-centro vertex is called null $D$-centro vertex. The collection of null $D$-centro vertices is called as null $D$-centro set of $G$.

**Theorem 2.5.** If $DC_r(G) = p - 1$ where $p$ is the order of $G$. Then $G$ has $p - 2$ null $D$-centro vertices.

**Proof.** Let $S$ be a $D$-centro dominating set of $G$ with order $p$. Since $|S| = p - 1$, there is only one vertex $r$ in $V - S$. By the definition of $DC_r(G)$, this vertex $r$ is $D$-centro to any one of the vertices in $S$ say $s$. Suppose that the vertex is $D$-centro to two or more vertices in $G$. Then $D(v_1, r) = D(v_2, s) = Rad(G)$ and $D(v_1, r) \neq Rad(G)$, where $i = 3, \ldots , p - 1$. Since, $v_1, v_2$ are the $D$-centro vertices of $r$, it is enough to take the vertex $r$ instead of $v_1, v_2$ in $S$ and $DC_r(G) \leq p - 2$, which is a contradiction by our hypothesis. Therefore, there are only two vertices are null $D$-centro vertices. Hence, the cardinality of null $D$-centro vertices is $p - 2$.

**Theorem 2.6.** A graph $G$ with no cycles does not contains the null $D$-centro vertices.

**Proof.** Let $G$ be a graph with no cycles. Suppose that $G$ contains a null $D$-centro vertex $w$ and so $DC_w(G) = \phi$. Clearly $D(w, r) < Rad(G)$ for all $r \in G$ is not possible since no pair of vertices have detour distance less than detour radius. Therefore $D(w, r) > Rad(G)$ for all $r \in G$. Now, consider $D(w, r) = Rad(G) + 1$ where $r \in G$. Since no vertex in the path $w-r$ has detour length from $w$ is equal to $R$, $G$ contain a cycle. It is a contradiction and so it completes the proof.

Next we develop a bound for $DC_r(G)$.

**Theorem 2.7.** Let $G$ be a graph with $k$ null $D$-centro vertices. Then $1 \leq DC_r(G) - k \leq \frac{n}{2}$.

**Proof.** Let $K$ be the null $D$-centro set of $G$ with $k$ number of vertices. By theorem 2.3, the null $D$-centro vertices lie in $D$-centro dominating set. Therefore $DC_r(G) \geq k$. But $Rad(G) \geq 1$. Then there exists atleast a path of detour length $2$ such that the set $DC_r(G)$ must contain atleast one non null $D$-centro vertex. Therefore $DC_r(G) > k$ and so $DC_r(G) \geq k + 1$. Obviously, the set $V - K$ contains atmost $\frac{n}{2}$ vertices. Hence $DC_r(G) \leq \frac{n}{2} + k$. Thus, $k + 1 \leq DC_r(G) \leq \frac{n}{2} + k$ and so $1 \leq DC_r(G) - k \leq \frac{n}{2}$.
Theorem 2.8. Let $P$ be a diametral path in a Tree $T$ and $S$ be the D-centro dominating set. Let $a, b \in P$ are the vertices D-centro to each other. Then $DC_T(T) = R$ if and only if for each vertex in the set $S$ satisfies one of the stated conditions holds. It is notice that the cardinality of D-centro dominating set increases when $1 \leq DC_b(G) \leq DC_a(G)$ and $DC_a(G) \cap V(P - \{a_i\}) = \emptyset$ for some $x \in DC_b(G)$. Therefore by the hypothesis, it is obvious that $DC_T(T) = R$. This proof is similar when the diametral path is odd.

Theorem 2.9. For a Tree $T$, $R \leq DC_T(T) \leq D - 1$, where $R$ and $D$ be radius and diameter of $T$.

Proof. Let $G$ be a tree $T$ with radius $R$ and diameter $D$. Suppose $DC_T(T) < R$. Let $P$ be any diametral path of even vertices in $T$. Let $S$ be the $DC_T$-set. Since $P$ is a diametral path, the branches of $T$ does not have length greater than $R$. So it is dominated by any one of the vertex in $P$ with respect to D-centro domination. Therefore it is enough to choose $S$ in $V(P)$. Each vertex of $P$ has only one D-centro vertex in $P$ and the central vertices of odd path contains two end vertices as D-centro vertex and vice versa. Suppose $DC_T(T) \leq R - 1$. Then there are at least three or more vertices in the diametral path as D-centro vertices to any vertex in the diametral path which of them, two vertices forms a cycle with any vertices of $T$, which is a contradiction. Therefore $DC_T(T) \geq R$. Now take the path $(P_N)$ where $n$ is even. Partitioned $V(P)$ into two subsets $V_1 = \{a_1, a_2, \ldots, a_n\}$ and $V_2 = \{b_1, b_2, \ldots, b_n\}$ where $a_n$ and $b_1$ are the central vertices of $P$. Start with the vertex $a_1$ which is D-centro to $b_1$. For each vertex in $S$ has the following condition. (i) If $DC_a(G) > 1$ and $DC_b(G) = 1$ where $1 \leq i \leq n$, then $a_i \in S$. (ii) If $1 \leq DC_b(G) \leq DC_a(G)$ and $DC_a(G) \cap V(P - \{a_i\}) = \emptyset$ for some $x \in DC_b(G)$, then both $a_1$ and $b_1$ belongs to the set $S$. (iii) If $1 \leq DC_b(G) \leq DC_a(G)$ and $DC_a(G) \cap V(P - \{a_i\}) = \emptyset$ for some $x \in DC_b(G)$, then there exists a vertex $b \in DC_a(G) \cap V(P - \{a_i\})$ and $DC_a(G) = \{b\}$. Where $y$ and $b$ are D-centro to each other and $y \in V(P)$, then $a_i \in S$. From the above conditions, the set $S$ contains $n - 1$ vertices. Therefore $DC_T(T) > D - 1$. Then the graph $G$ requires $n$ number of vertices to dominate all other vertices with respect to D-centro domination where $n$ is the total number of vertices in the diametral path $P_N$. Therefore, $DC_T(G) \leq D - 2$. This is a contradiction since $S$ is not minimum. This proof is similar when the diametral path is odd.

Theorem 2.10. Every vertex except end vertices in a diametral path $P$ of a tree $T$ is a support vertex. Then $DC_T(T) = |S(T)|$ where $S$ is the D-centro dominating set.

Theorem 2.11. (i) For a complete graph $G = K_n$, $DC(G) = 1$.

Proof. Let $G = K_n$ and let $V(G) = v_i; 1 \leq i \leq n$. The detour length of any two vertices is $n - 1$. Every singleton set $v_i$ ($1 \leq i \leq n$) forms a $DC_T$-set and so $DC_G(G) = 1$.

(ii) Let $G = K_{m,n}$ and be partitioned into two sets $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_m\}$ such that every edge of $G$ joins a vertex of $V_1$ with a vertex of $V_2$.

Case(i): If $m < n$, then the detour distance between two vertices from $V_1$ is $2m$ and that of two vertices from $V_2$ is $2m - 1$. That is, $e_D(u) = 2muv \in V_1$ and $e_D(v) = 2m - 1 \forall v \in V_2$. Therefore $Rad(G) = 2m - 1$. The D-centro vertices of each element of $V_1$ is $V_2$ and the set $V_2$ is $V_1$. Therefore the D-centro set contains only two elements. That is, an element from $V_1$ and an element from $V_2$. And also by the definition, it is enough to take one element from $V_1$ and one element from $V_2$ to satisfy the minimum D-centro dominating set. Hence $DC(G) = DC_T(G) = 2$.

Case(ii): If $m = n$, the proof is same as case(i).

Theorem 2.12. (i) For a path $P_n$.

$DC_T(P_n) = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n-1}{2}, & n \text{ is odd} \end{cases}$

(ii) For a path $P_n$, $DC(P_n) = 2$.

Proof. Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$. In a path $P_n$, $\text{Rad}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor$ and $\text{Diam}(P_n) = n - 1$. Let $S$ be the D-centro dom set. Case(i): Suppose that $n$ is odd. We take $n = 2k + 1$, where $k$ is a positive integer. Now take the vertex $v_{k+1}$. Then $v_{k+1}$ has the minimum eccentricity $R$, where $R$ is the eccentricity of radius of $P_n$. Since $P_n$ is a path, the two end vertices $v_1$, $v_n$ are the D-centro vertices of $v_{k+1}$ and the detour distance
of these two vertices \(v_1, v_n\) from the vertex \(v_k+1\) is equal to detour radius and so \(v_k+1 \in S\). In the remaining vertices 
\[v_2, \ldots, v_k, v_{k+1}, \ldots, v_{n-1}\] for \(i\) from 2 to \(k\), \(v_i\) and \(v_{k+1}\) are D-centro to each other. In the set there are \(\frac{n-3}{2}\) vertices, which are also in D-centro dominating set. Hence, 
\[DC_G(G) = 1 + \left(\frac{n-3}{2}\right) = \left(\frac{n-1}{2}\right)\].

Case (ii): Suppose that \(n\) is even and so \(n = 2k\) for every positive integer \(k\). Each vertex has only one vertex as D-centro vertex. Therefore \(DC_G(G) = \left(\frac{2}{n}\right)\).

Theorem 2.13. For a cycle \(G = C_n\) where \(n \geq 3\), 
\[DC_G(C_n) = \left\lceil \frac{n}{3} \right\rceil\]

Proof. Consider this cycle, \(G = C_n\). By Theorem 2.5 in [1], 
\[N(x) = DC_G(G)\] for all \(x \in C_n\). That is, neighborhood vertices of every vertex of \(G\) are D-centro vertices. Therefore, by the definition of D-centro dominating set, 
\[DC_G(G) = \left\lceil \frac{n}{3} \right\rceil\] \(\square\)

Theorem 2.14. For any wheel graph \(W_n\), 
\[DC_G(W_n) = 1\]

Proof. Let \(V(W_n) = \{u, v_1, v_2, \ldots, v_{n-1}\}\) with \(u\) as its central vertex. Since \(u\) is adjacent to all other vertices 
\[v_1, v_2, \ldots, v_{n-1}\], the detour distance between any pair of vertices of \(V(W_n)\) is \(n - 1\). Therefore any one vertex of \(V(W_n)\) is a D-centro dom set. Since it is minimum, 
\[DC_G(W_n) = 1\] \(\square\)

Theorem 2.15. For a double star \(G = S_{m,n}\), 
\[DC_G(G) = 2\]

Proof. Consider the graph \(G = S_{m,n}\) whose vertex set is 
\[\{r, s, u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}\]. Now the eccentricity, 
\[e_D(x) = 2\] if \(x \in r, s\) and 
\[e_D(x) = 3\] if \(x \in V(S_{m,n} - r, s)\) and 
\[Rad(G) = 2\]. Therefore r-D-centro set of \(G\) is \(\{v_1, v_2, \ldots, v_n\}\) and s-D-centro set of \(G\) is \(\{u_1, u_2, \ldots, u_m\}\). The \(u_1\)-D-centro set, 
\[DC_{u_1}(G) = \{s, u_1, u_2, \ldots, u_{m-1}, u_{m+1}, \ldots, u_m\}\] and the \(v_1\)-D-centro set, 
\[DC_{v_1}(G) = \{r, v_1, v_2, \ldots, v_{n-1}, v_{n+1}, \ldots, v_n\}\]. Now \(S = r, s\). Then it is enough to take \(S\) as D-centro dominating set. Hence 
\[DC_G(G) = 2\]. Now we see that every pair of vertices between the sets \(\{r, v_1, v_2, \ldots, v_n\}\) and \(\{s, u_1, u_2, \ldots, u_m\}\) are D-centro to each other. Therefore by the definition, 
\[DC_G(G) = 1 + m\] where \(m \geq n\) \(\square\)

3. Realization Results

Next we develop three realization results on \(DC(G)\) and 
\[DC_G(G)\]

Theorem 3.1. For every consecutive pair \(k, n\) of integers with 
\(3 \leq k < n\), there exists a connected graph \(G\) of order \(n\) such that 
\[DC(G) = k\].

Proof. Suppose that \(3 \leq k < n\). Construct a complete graph \(K_k\) of vertices \(\{u_1, u_2, \ldots, u_k\}\) of order \(k\). By previous results, 
\[DC(K_k) = k\] and 
\[Rad(G) = k - 1\]. Now add a new vertex \(x\) to any one of \(\{u_1, u_2, \ldots, u_k\}\).

Now we join \(x\) to \(u_i \forall (1 \leq i \leq n)\) for some \(i\). It forms a new graph \(G\) of order \(n\) where \(n = k + 1\). Since \(x\) is an end vertex adjacent to \(u_i\), it does not affect the radius. Hence the detour eccentricity of \(u_i\) is \(k - 1\) and \(e_D(v) = \{k/v \neq u_i \forall v \in G\}\). Further since each vertex except \(x\) are adjacent to all other vertices, 
\[D(u,v) = k - 1\], for any pair of vertices \(u, v\). Hence there exists a graph of order \(n\) such that 
\[DC(G) = k\] and 
\[3 \leq k \leq n\]. \(\square\)

Theorem 3.2. For every pair \(r, s\) of positive integers with 
\(2 \leq r \leq s\), there exists a connected graph \(G\) of order \(s\) such that 
\[DC(G) = r\].

Proof. Let \(r\) and \(s\) be positive integers such that \(2 \leq r \leq s\). Case (i): If \(2 = r = s\). Then there exists a path of length 2 such that 
\[DC(G) = 2\].

Case (ii): Let \(s = 3\).

Subcase (i): If \(s = 3\) and \(2 = r < s\). Then there exists a path of length 3 such that 
\[DC(G) = 2\].

Subcase (ii): If \(s = 3\) and \(2 < r < s\), that is \(3 = r = s\). Then there exists a complete graph \(K_3\) such that 
\[DC(G) = 3\].

Case (iii): Let \(s = 4\).

Subcase (i): If \(s = 4\) and \(2 = r < s\), then there exists a path of length 4 such that 
\[DC(G) = 2\].

Subcase (ii): If \(s = 4\) and \(2 < r < s\), that is \(4 = r = s\), then there exists a complete graph \(K_4\) such that 
\[DC(G) = 4\].

Subcase (iii): If \(s = 4\) and \(2 < r < s\), that is \(r = 3\), then there exists a graph \(G = K_3 \cup K_1\) such that 
\[DC(G) = 3\] by previous theorem.

Case (iv): Take \(2 \leq r \leq s\) where \(s \geq 5\). The graph \(G\) has desired properties if \(2 \leq r \leq s\) by the above cases. Now we have to prove \(2 < r < s\) where \(s \geq 5\).

Figure 2 A graph \(K_k\)

Figure 3 A graph \(G\) for case (iv)
Construct a complete graph $G = K_r$ where $V(K_r) = u_1, u_2, \ldots, u_r$. Clearly $r \geq 4$, since $s > 4$. Now add new vertices $u_{r+1}, u_{r+2}, \ldots, u_{s-r}$ to $u_1$. It forms a new graph $G$ of order $s$. Since the vertices $u_{r+1}, u_{r+2}, \ldots, u_{s-r}$ are end vertices adjacent to $u_1$, it does not alter the radius. That is, the eccentricity of $u_1$ is $r - 1$ and $e_D(x) = \{r + 1 \in G \}$. Further, each vertex of $\{u_1, u_2, \ldots, u_l\}$ is adjacent to all other vertices in $G - \{u_{r+1}, u_{r+2}, \ldots, u_{s-r}\}$ and $D(x, y) = r - 1 \forall x, y \in \{u_1, u_2, \ldots, u_l\}$. Hence there exists a graph $G$ of order $s$ such that $DC(G) = r$.

**Theorem 3.3.** For any integers $a$ and $b$ with $2 \leq a \leq b$, there exists a connected graph $G$ of order $n > 2$ such that $DC_G(u) = a$ and $DC_G(b) = b$.

**Proof.** Let $a$ and $b$ be any integers with $2 \leq a \leq b$. Then we can have the following cases.

**Case(i):** Assume that $2 = a = b$. Then there exists a complete bipartite graph $G = K_{m,n}$ for any integer $m, n$ such that $DC_G(u) = DC_G(b) = 2$.

**Case(ii):** Suppose that $2 = a < b$. Take $G$ a double star $S_{m,n}$. Then $G$ satisfies the desired properties.

**Case(iii):** Suppose that $2 < a < b$. Construct a complete graph $G = K_b$ of vertices with $b > 2$. Add a path $P_{a-2}$: $v_1, v_2, \ldots, v_{a-2}$ to $u_1$ for any $i$, between $i$ and $b$ and $a - 2 < b$. Further add a new pendant vertex $x$ to any of the vertices $v_1, v_2, \ldots, v_{a-2}$. It forms a new graph $G$ of order $n = (a + b) - 1$. The subgraph induced by the set of vertices $\{u_1, u_2, \ldots, u_b\}$ is complete and the path $v_1, v_2, \ldots, v_{a-2}$ joined to $u_1$ and join $x$ to $v_2$ as shown in the Figure 4. Hence the eccentricity of $u_i$ does not exceed $b - 1$. That is, $e_D(u_i) = b - 1$. Therefore the new graph $G$ does not alter its radius. Furthermore $e_D(u) = (a + b) - \frac{3}{a} u \in \{u_1, \ldots, u_{a-1}, u_{a+2}, \ldots, u_b\}$. The vertices from $G - \{v_1, v_2, \ldots, v_{a-1}\}$ are $D$-centro to each other. Therefore $DC_G(b) = b$. Further, since $e_D(v) > b - 1$ for any $v$ from the set $\{v_1, v_2, \ldots, v_{a-1}\}$ and $DC_G(u) = 1 + (a - 1)$ and so, $DC_G(u) = a$. Thus there exists a graph $G$ such that $DC_G(u) = a$ and $DC_G(b) = b$.

**Theorem 3.4.** For positive integers $R$, $D$ with $R < D \leq 2R$, there exists a connected graph $G$ with $Rad(G) = R$, $Diam(G) = D$ and $DC_G = R + 1$ and $DC_G = R$.

**Proof.** We prove this theorem by considering two cases relating this values of $R$ and $D$.

**Case (i):** Assume that $R < D = 2R$. We construct a graph as shown in the Figure 6:

**Case (iv):** Let $2 < a = b$. Construct a complete graph $G = K_b$ of vertices $\{u_1, u_2, \ldots, u_b\}$ with $b > 2$. Add a path $P_{a-1}$: $v_1, v_2, \ldots, v_{a-1}$ to $u_1$ for any $i$, between $i$ and $b$ and $a - 1 < b$. It forms a new graph $G$ of order $n = (a + b) - 1$. The subgraph induced by the set of vertices $\{u_1, u_2, \ldots, u_b\}$ is complete, the path $v_1, v_2, \ldots, v_{a-1}$ join to $u_1$ as shown in the Figure 5. Hence the eccentricity of $u_i$ does not exceed $b - 1$. That is, $e_D(u_i) = b - 1$ and so the new graph $G$ does not alter its radius. Furthermore $e_D(u) = (a + b) - 2$ for every $u$ from the set $\{u_1, \ldots, u_{a-1}, u_{a+1}, \ldots, u_b\}$. The vertices from $G - \{v_1, v_2, \ldots, v_{a-1}\}$ are $D$-centro to each other. Therefore $DC_G(u) = b$. Further, since $e_D(v) > b - 1$ for any $v$ from the set $\{v_1, v_2, \ldots, v_{a-1}\}$ and $DC_G(u) = 1 + (a - 1)$ and so, $DC_G(u) = a$. Thus there exists a graph $G$ such that $DC_G(u) = a$ and $DC_G(b) = b$.
Consider two positive integers $R$ and $D$ such that $R < D < 2R$. Consider a complete graph $K_{R+1}$. Let $K_{(D-R)+1}$ be another complete graph of order $(D-R)+1$ with $R+1 > (D-R)+1$. Let the vertices of $K_{(D-R)+1}$ be $u_1, v_1, \ldots, v_{(D-R)}$. Let $H$ be a graph obtained from $K_{R+1}$ and $K_{(D-R)+1}$ by identifying $u_i$ as the common vertex in $K_{R+1}$ and $K_{(D-R)+1}$. Now add the set $S$ of new pendant vertices $\{x_1, x_2, \ldots, x_{2R-D-1}\}$ to $H$ and join each vertex $x_i(1 \leq i \leq 2R-D-1)$ to the vertex $u_i$ to obtain a new graph $G$ as shown in the Figure 7. The detour eccentricity of $u_i$ is $R$ and that of other vertices $u_1, u_2, \ldots, u_{(R-1)}, u_{(R+1)}, \ldots, u_{(R+1)}$ are equal to $2R$. The detour eccentricity of $v_i(1 \leq i \leq (D-R)+1)$ is $D$ and the detour eccentricity of $x_i(1 \leq i \leq 2R-D-1)$ is $R+1$. Further, $K_{R+1}$ and $K_{(D-R)+1}$ are complete and the detour length of any vertex from $K_{R+1}$ to a vertex $u_i$ is $R$. Hence, by the definition $DC_1(G) = R$. Now, since $K_{(D-R)+1}$ is complete and $S$ contains all pendant vertices, the remaining vertices from $G - K_{(D-R)+1} - \{u_i\} \cup S$ are the null $D$-centro vertices. Therefore, by the definition of $D$-centro dom set, $DC_1(G) = 1 + D - R + 2R - D - 1 = R$. Hence $DC_1(G) = R$. \hfill $\square$

**4. Conclusion**

In this paper, the $D$-centro dominating sets in graphs has been studied. It is simply a dominating set of $G$ with a detour distance $R(G)$. Also a special type of vertex, null $D$-centro vertex has been defined and the bounds for $D$-centro domination number internals of the number of null $D$-centro vertices have been found. The $D$-centro domination number for some special graphs like complete graph, cycle, wheel and star have been determined. Algorithms can be developed for finding the parameter, $D$-centro domination number for arbitrary graphs. This theory can be developed for finding $k$-center with respectomination based detour distance.

**References**

[1] A. Anto Kinsley and P. Siva Ananthi, D-centro sets in graphs, IJSRD - International Journal for Scientific Research and Development— Vol. 4, Issue 01, 2016 — ISSN (online): 2321-0613.

[2] F. Buckley and F. Harary, Distance in graphs, Addison-Wesley, Longman, 1990.

[3] G. Chartrand, T.W. Haynes, M.A. Henning, and Ping Zhang, Detour domination in graphs, Ars Combin. 71 (2004) 149 – 160.

[4] G. Chartrand, David Erwin, G. L. Johns and P. Zhang, On boundary vertices in graphs, J.Combin. Math. Combin.Comput. 48, (2004), 39-53.

[5] G. Chartrand, David Erwin, G. L. Johns and P. Zhang, Boundary vertices in graphs, Disc.Math. 263 (2003), 25-34.

[6] G. Chartrand and P. Zang, Introduction to Graph Theory, Tata McGraw-Hill, (2006).

[7] F. Harary, Graph theory, Addison- Wesley, 1969.

[8] T.N. Janakiraman, M. Bhanumathi and S. Muthammal, Eccentric domination in graphs, International Journal of Engineering Science, advanced Computing and Biotechnology, Volume 1, No.2, pp 1-16, 2010.

[9] KM. Kathiresan, G. Marimuthu and M. Sivanandha Saraswathy, Boundary domination in graphs, Kragujevac J. Math. 33, (2010), 63-70.

[10] A. P. Santhakumaran, P. Titus, The vertex detour number of a graph, AKCE International J. Graphs. Combin., 4(2007), 99-112.