Macro-coherent two photon and radiative neutrino pair emission

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ABSTRACT

We discuss a possibility of detecting a coherent photon pair emission and related radiative neutrino pair emission from excited atoms. It is shown that atoms of lambda- and ladder-type three level system placed in a pencil-like cylinder give a back to back emission of two photons of equal energy $\Delta/2$, sharply peaked with a width $\propto 1/(\text{target size})$ and well collimated along the cylinder axis. This process has a measurable rate $\propto (\text{target number density})^2 \times \text{target volume}$, while a broader spectral feature of one-photon distribution
separated by \((\text{mass sum of a neutrino pair})^2/(2\Delta)\) from the two photon peak may arise from radiative neutrino pair emission, with a much smaller rate.

**Introduction**

Superradiance proposed by Dicke [1],[2] is a remarkable effect, giving a large rate, at its maximum \(\propto N^2\) with \(N\) the number of target atoms, much larger than the spontaneous emission rate \(\propto N\). It may give rise to an effective enhancement of weaker rates of forbidden transitions such as M1, E2 and even weak interaction process of neutrino pair emission, thus giving a possibility of measuring these small rates.

In the present work we consider two photon emission \(|1\rangle \rightarrow |3\rangle + \gamma + \gamma\) and radiative neutrino pair emission \(|1\rangle \rightarrow |3\rangle + \gamma + \nu_i\nu_j\) from coherently excited targets of three level atomic system; \(|1\rangle, |3\rangle\) and some intermediate state \(|2\rangle\). (We denote three neutrino mass eigenstates by \(\nu_i, i = 1, 2, 3\).) The latter neutrino process is related to laser irradiated neutrino pair emission discussed in [3] aiming at the neutrino mass spectroscopy. It is shown below that these processes have rates \(\propto n^2V\) \((n\) the target number density and \(V\) the target volume) and striking kinematical features of angular correlation and energy spectrum, hence may be detectable, if the rate is large enough.

A very sharp single photon peak of two photon pair emission is located at the half of the energy difference \(\Delta/2\), \(\Delta = E_1 - E_3\), and its angular distribution is well collimated, for a large aspect ratio, to the cylinder axis. It can thus be used for a precise determination of \(\Delta/2\). This means that we do not need an independent experiment for measurement of the value \(\Delta\) from atoms in a complex environment. Moreover, the photon pair is highly correlated (back to back, and so on), and has a spin correlation with atomic angular momentum involved. Thus, these pairs are ideal entangled states.
On the other hand, the photon energy distribution arising from radiative neutrino pair emission has a threshold in the vicinity of the two photon peak, ranging in the continuous energy region \( h\omega \leq \Delta/2 - (m_i + m_j)^2/(2\Delta) \), with \( m_i \) three neutrino mass values. Numerically, \( (2m_3)^2/2\Delta = 5\text{meV} (m_3/50\text{meV})^2(0.1\text{eV}/\Delta) \) for the heaviest neutrino \( \nu_3 \). The threshold rise of the rate thus provides a critical information of neutrino masses, giving their sum \( m_i + m_j \), if the process has a measurable rate. It might even be possible in a distant future to detect relic neutrino of 1.9 K \[4\], if the neutrino mass spectroscopy works ideally.

As usual, we use the natural unit such that \( \hbar = 1 \) and \( c = 1 \) throughout the present paper. We abbreviate the new phenomenon of coherent two photon pair emission as macro-coherent two photon emission (MCTPE).

### Macro-coherent two photon emission

We consider a coherent collection of excited atoms having three level structure of lambda(Λ)- and ladder-type. An example of Ba atom levels is depicted in Figure 1. In this example both of the two types coexist; Λ-type, \( ^1D_2(|1\rangle) \rightarrow P_1(|2\rangle) \rightarrow S_0(|3\rangle) \), and the ladder-type, \( ^1D_2(|1\rangle) \rightarrow ^3D_{2,1,0}(|2\rangle) \rightarrow S_0(|3\rangle) \), if one prepares \( ^1D_2 \) as the initial state. Other candidates are metastable states of noble gas atoms.

In the case of atoms coherently excited by a pulsed laser the single photon superradiance follows the stochastic spontaneous decay \[1\], \[2\]. Hence its time profile of evolution is somewhat complicated.

Instead, we imagine in the present work that the initial excited state is prepared by two laser irradiation, forming the dark state \[5\], a pure mixture of two quantum states \(|1\rangle \) and \(|3\rangle \). In the example of Ba the mixture of \(^1D_2 \) and \(^1S_0 \) is formed by two lasers of wavelengths, 554 nm and 1500 nm for Ba, corresponding to transitions, \(|1\rangle \rightarrow |2\rangle \) and \(|3\rangle \rightarrow |2\rangle \). The time of the dark
Figure 1: Ba energy level. The transition time $69 \text{ s}$ of $^3D_2 \rightarrow ^1S_0$ (E2) is taken from a theoretical calculation of [10].

state formation is of order, the larger of these E1 decay times, a few $\times 4\mu$ sec, which is shorter than two photon superradiance time we consider below. After the dark state formation we switch off laser irradiation and measure two photon emission until the collisional relaxation time ($\sim 0.1 \text{ sec}$ for gas targets). This cycle is repeated as many times as possible. We may thus expect for computations below that the dark state is present as the initial state of our time development, and may assume that a fraction of the initial state is in the metastable excited state $|1\rangle$ with a probability $\sim (\text{ratio of Rabi frequencies})^2$. This means that for rate computation at its maximum we may ignore a complicated time profile, and follow the S-matrix approach based on states on the mass-shell [6].

In more complicated situations in which two time scales of dark state formation and superradiance are comparable, one needs time integration of the optical Bloch equation [5].

The emission rate, summed over target atom positions $\vec{r}$ of two photon emission at $\vec{r}$ and detected at $\vec{r}_0$, is given by (atoms distributed uniformly by
a constant number density $n$)

$$\Gamma = \int \left( \Pi_i \frac{d^3 k_i}{(2\pi)^3} \right) 2\pi \delta(\Delta - \sum \omega_i) |n \int_V d^3 e^i \sum_i \langle \vec{r} - \vec{r}_0 | \mathcal{M}(\vec{k}_i) |^2, \tag{1}$$

where $\mathcal{M}(\vec{k}_i)$ is the probability amplitude of emitting two photons of momenta $\vec{k}_i$ ($\omega_i = |\vec{k}_i|$) from a single atom. Dependence on the detection point $\vec{r}_0$ disappears in the rate, and one may use a shape factor defined by $F(\vec{K}) = \int_V d^3 e^i e^{i \vec{K} \cdot \vec{r}}$ (in general $\propto$ Fourier transform of the number density).

A cylindrical target is the standard example, since preparation of excited atoms via laser irradiation often gives a coherent region of this type. The shape factor for a cylinder of area $\pi d^2$ and length $l$ \cite{7} is given by

$$F(\vec{K}) = \frac{(4\pi \sin(K z/l/2)/K z)}{(e^{i K \rho d}/(i K \rho) - (1 - e^{i K \rho d})/K^2 \rho)}.$$  

This function approaches the volume of the target, $F(\vec{K}) \to \pi d^2 l$ in the long wavelength, $|\vec{K}|^{-1} \ll d, l$. For subsequent discussion it is convenient to factor out volume related quantity and introduce a dimensionless function $\mathcal{H}$ defined by $\mathcal{H}(d K_\rho, l K_z) = |F(\vec{K})|^2/\pi d^2 l)^2$, which turns out

$$\mathcal{H}(x, y) = \left( \frac{4 \sin(y/2)}{x^2 y} \right)^2 (2 - 2 x \sin x - 2 \cos x + x^2). \tag{2}$$

For small arguments of $x, y$, $\mathcal{H}(x, y) \sim 1 - x^2/18 - y^2/12$ to the second order.

We use the momentum vectors $\vec{k}_1, \vec{k}_2$ of 2 photons which define a plane, not necessarily containing the cylinder axis. The total momentum component parallel to the cylinder axis is denoted by $K_z$ and its orthogonal by $K_\rho$:

$$K_z = (\vec{k}_1 + \vec{k}_2) \cdot \vec{e}_z = \omega \cos \theta + (\Delta - \omega) \cos \theta_2, \tag{3}$$

$$K_\rho = K \sin \theta_{12} = \left( [(\vec{k}_1 + \vec{k}_2) \cdot \vec{e}_x]^2 + [(\vec{k}_1 + \vec{k}_2) \cdot \vec{e}_y]^2 \right)^{1/2}$$

$$= (\omega^2 \sin^2 \theta + (\Delta - \omega)^2 \sin^2 \theta_2 + 2\omega(\Delta - \omega) \sin \theta_2 \cos \theta_2 \cos \phi_{12})^{1/2}, \tag{4}$$

where the energy conservation $\Delta = \omega_1 + \omega_2$ is used. The angles, $\theta, \theta_2, \phi_{12}$, are defined relative to the cylinder axis.
Two photon emission for the transition $^{1}D_{2} \rightarrow ^{1}S_{0}$, may proceed via intermediate state $|2\rangle = ^{1}P_{1}$ ($\Lambda$-type, hence $E_{2} > E_{1} > E_{3}$), and the double differential spectrum of a single photon is given by

$$\frac{d^{2}\Gamma_{2\gamma}}{d\omega d\cos\theta} = \int_{0}^{2\pi} d\varphi_{12} \int_{-1}^{1} d\cos\theta_{2} \frac{d^{4}\Gamma_{2\gamma}}{d\omega d\cos\theta_{d} d\cos\theta_{2} d\varphi_{12}},$$

(5)

$$\frac{d^{4}\Gamma_{2\gamma}}{d\omega d\cos\theta_{d} d\cos\theta_{2} d\varphi_{12}} = \frac{(np^{2}l)^{2}|d_{12}|^{2}|d_{23}|^{2}\omega^{3}(\Delta - \omega)^{3}}{4(2\pi)^{2}} \mathcal{H}(dK_{\rho}, lK_{z}) \times \left( \frac{1}{(\omega + \Delta_{21})^{2} + \gamma^{2}/4} + \frac{1}{(\omega - \Delta_{23})^{2} + \gamma^{2}/4} \right),$$

(6)

with $\Delta_{ij} = E_{i} - E_{j}$. The dipole matrix element squared $|d_{ij}|^{2}$ for the transition $|i\rangle \leftrightarrow |j\rangle$ may be replaced by the measurable E1 natural width $\gamma_{ij}$, using $\gamma_{ij} = |d_{ij}|^{2}\Delta_{ij}^{3}/(3\pi)$. The width factor in the formula (6) $\gamma \approx \gamma_{12} + \gamma_{23}$.

The rate becomes large when the arguments $x, y$ of $\mathcal{H}(x, y)$ are of order unity or less. This occurs only when two energy factors are close; $\omega \approx \Delta - \omega$. Hence a sharp peak appears at the half of the available atomic energy; $\omega = \Delta/2$.

It is trivial to extend our result to the ladder-type of atomic system such as Ba levels of $^{1}D_{2} \rightarrow ^{3}D_{2,1,0} \rightarrow ^{1}S_{0}$. What is needed is to replace a positive $\Delta_{21}$ by a negative value; $\Delta_{21} = -\Delta_{12} < 0$.

**Event rate and angular distribution**

The integrated rate over the photon energy including the peak value is of order, the central peak rate times the width factor around it, which gives $O[\Delta\omega/(\pi dl\Delta^{2})]$, $1/(dl\Delta^{2})$ arising from the Jacobian of variable change $(\cos\theta_{2}, \cos\varphi_{12}) \rightarrow (x, y)$. With the expected $\Delta\omega = O[1/d]$, this factor becomes $O[1/(V\Delta^{2})]$, with $V$ the volume of target region. Hence the integrated rate may be written as $\Gamma_{2\gamma} = O[An^{2}V/\Delta^{2}]$. More precisely, a slight departure from the linearity $\propto V$ is observed and is of logarithmic type; $V(1 + O[\ln d])$.

The dimensionless quantity $A$ is intrinsic to the target atomic system, and
Figure 2: Energy spectrum of MCTPE and radiative neutrino pair emission. The rate of radiative neutrino pair emission is rescaled up with a factor $1.0 \times 10^{41}$. The size factors assumed are $d = 100\text{eV}^{-1}$ and $l/d = 100$. 
of order $1 \times 10^{-15}$ for the $\Lambda$--type Ba transition, $^1D_2 \rightarrow ^1P_1 \rightarrow ^1S_0$. Numerically, the rate at $n = 10^{10} \text{cm}^{-3}$ and $V = 1 \text{cm}^3$ is $\sim 1.7 \times 10^5 \text{sec}^{-1}$. We show in Figure 2 the calculated energy spectrum along with one-photon spectrum arising from radiative neutrino pair emission (see below). The two photon process appears to fall into the range of measurable region.

Dependence of the rate on target factors $\propto n^2 V$ differs from that for the single photon superradiance $\propto n^2 d^2 \lambda$ \[8\], since the wavelength $\lambda$ limits the coherent region in this case. In the case of MCTPE there is no such limitation of wavelength, because the photon pair wave function $e^{i(\vec{k}_1 + \vec{k}_2) \cdot \vec{r}}$ can become of order unity for the collinear momentum configuration $\vec{k}_1 + \vec{k}_2 = 0$. The ratio of the single to the two photon superradiance is $O[0.02](\text{cm}/l)$ for $^1D_2$ of Ba atom \[9\]. Thus, one may be able to simultaneously observe both of the single superradiance and MCTPE in a single experiment \[10\].

It is interesting that for the ladder-type of system there is a further enhancement if the resonance condition, $\Delta/2 = \Delta_{23}$, namely $\Delta_{12} = \Delta_{23}$, is met. This might open a new possibility of detecting weak transition rates between magnetic levels equally spaced energetically. Ideal targets for the neutrino mass spectroscopy might be atoms having the level structure of ladder-type of $\Delta_{12} = \Delta_{23}$ such as vibrational levels of molecules.

The angular distribution of one photon is symmetric under $\theta \leftrightarrow \pi - \theta$. There are two angular components; one isotropic and the other axial. For the pencil-like cylinder $d \ll l$ the axial component is dominant and we focus on this component hereafter. The axial component is limited to a narrow angular region of $\theta \leq O[2d/l]$ in the forward and the backward direction, as shown in Figure 3. Both of them show the back to back correlation of two photons.

Radiative neutrino pair emission
Figure 3: Angular distribution of a photon from MCTPE at $\omega = \Delta/2$ and from radiative neutrino pair emission at $\omega = (0.3, 0.5, 0.7)\Delta$, both measured from the cylinder axis $\theta = 0$ and taking arbitrary rate units. The size factors assumed are $d = 100\text{eV}^{-1}$ and $l/d = 100$. 
We now consider radiative neutrino pair emission; $|1\rangle \rightarrow |3\rangle + \gamma + \nu_i \nu_j$. The neutrino pair emission arises from the vertex of four Fermi interaction of the kind, $\nu_i^\dagger \nu_j^\dagger e^\dagger e$ and $\nu_i^\dagger \bar{\sigma} \nu_j^\dagger \cdot e^\dagger \bar{\sigma} e$, using two component spinor fields of $\nu_i$ and $e$. Its precise form is determined by the standard electroweak theory, and written out in [3]. The neutrino pair current involves both scalar $\nu_i^\dagger \nu_j^\dagger$ and spin vector $\nu_i^\dagger \bar{\sigma} \nu_j^\dagger$ parts. In the electron side the rate is largest for a spin-flip magnetic type transition of $e^\dagger \bar{\sigma} e$, when the orbital wave function overlap is of order unity.

Thus, it is best in the Ba case to take the spin triplet-singlet transition. A convenient three level is $6s5d^1D_2 - 6s5d^3D_3 - 6s^2^1D_2$ (ladder-type). Other example includes fine structure (FS) split levels of Yb; $6s6p^3P_2 - 3P_1 - 6s^2^1S_0$ and other alkali-earth atoms. Since the possible background MCTPE contains a M1 transition, the background rate is much smaller than two E1 case in the preceding section. In the following computations we ignore the wave function overlap factor, which is of order unity in magnetic type transitions.

A big difference from MCTPE is in the way how momenta are balanced; the emitted photon is anti-parallel to the neutrino pair. Since neutrinos are unobserved, all neutrino momenta are integrated. This gives a different size dependence, and one expects from dimensional grounds a total maximal rate of order, $G_F^2 \Delta^3 N^2 / [(2\pi)^4 d l] \sim 4 \pi^{-1} (n/10^{20} cm^{-3})^2 d^3 l / cm^4 (\Delta / 10 eV)^3$, where we took as an example $\Delta \geq O[10] eV$ relevant to noble gas atoms. This estimate of rate is valid for the ladder-type of levels for which the resonance enhancement is present, while for the $\Lambda$–type there is a suppression due to the off-resonance effect.

A straightforward computation similar to MCTPE leads to a single photon differential rate of the form,

$$\frac{d^2 \Gamma_{\gamma\nu\nu}}{d\omega d \cos \theta} = \frac{4 (n \pi d l)^2 G_F^2 |d_{23}|^2 \omega^3}{(2\pi)^7 ((\omega - \Delta_{23})^2 + \gamma^2 / 4) \int_0 \sqrt{(\Delta - \omega)^2 - (m_i + m_j)^2} \ dK \times}$$
\[ \int_0^{2\pi} d\varphi_{\mathbf{K}} \int_{-1}^1 d\cos\theta_{\mathbf{K}} \mathcal{H}(x, y) \frac{K^2}{(\Delta - \omega)^2 - K^2} \times \]

\[ \sqrt{((\Delta - \omega)^2 - K^2 - (m_i + m_j)^2)((\Delta - \omega)^2 - K^2 - (m_i - m_j)^2)} \times \]

\[ \left( k_{ij}^{(0)} f^{(0)}(\omega, K) + m_i^2 k_{ii}^{(M)} \delta_{ij} \right), \]

\( x = d \left( \omega^2 \sin^2 \theta + K^2 \sin^2 \theta \right) + 2K \omega \sin \theta \sin \theta \cos \varphi_{\mathbf{K}} \right)^{1/2}, \]

\( y = l \omega \cos \theta + 1K \cos \theta \), \]

\[ f^{(0)}(\omega, K) = \left( \frac{(\Delta - \omega)^2}{4} - \frac{K^2}{12} - \frac{K^2}{6(\Delta - \omega)^2 - K^2} (m_i^2 + m_j^2) \right) \]

\[ \frac{1}{12} \frac{3(\Delta - \omega)^2 + K^2}{(\Delta - \omega)^2 - K^2} (m_i^2 - m_j^2)^2 \right), \]

\[ k_{ij}^{(0)} = |c_{ij}^{(0)}|^2 + 3|c_{ij}^{(s)}|^2, \quad k_{ij}^{(M)} = |c_{ij}^{(0)}|^2 - 3|c_{ij}^{(s)}|^2, \]

where \( k_{ij}^{(0)} = |c_{ij}^{(0)}|^2 + 3|c_{ij}^{(s)}|^2, \quad k_{ij}^{(M)} = |c_{ij}^{(0)}|^2 - 3|c_{ij}^{(s)}|^2 \) and \( c_{ij}^{(0)} = U_{ei}^* U_{ej}, \quad c_{ij}^{(s)} = U_{ei}^* U_{ej} - \frac{1}{2} \delta_{ij}, \) and \( U_{ei} \) is the neutrino mass mixing matrix element. The second term in \( f^{(0)}(\omega, K) \) represents the interference term proper to the identical Majorana particle.

In the radiative neutrino pair emission given by \( f^{(0)}(\omega, K) \) the dominant momentum region that gives small \( x, y \) of \( \mathcal{H}(x, y) \) is where the neutrino pair momentum is matched to that of the photon, \( \mathbf{K} \sim -\mathbf{k}_\gamma \). This condition is readily obeyed, if the pair momentum magnitude satisfies an inequality \( K \sim \omega \leq \sqrt{(\Delta - \omega)^2 - (m_i + m_j)^2} \), the upper integration range of \( K \). This gives the threshold photon energy, \( \omega_{ij} = \Delta/2 - (m_i + m_j)^2/(2\Delta) \). There are altogether six neutrino mass thresholds \( \omega_{ij}, i, j = 1, 2, 3 \). A practical method of locating the neutrino mass threshold is to observe the Jacobian peak of the threshold at \( \omega_{ij} \) as in the usual particle physics technique. In Figure 2 the energy spectrum is shown for values of \( d \sim 20 \mu m \) and \( l \sim 2 mm \). For this computation we assumed a single neutrino of mass 50 meV. A successful discovery of the radiative neutrino pair emission would immediately mean a highly sensitive neutrino mass bound approaching \( m_3 \sim 50 \text{meV} \).

The total rate of radiative neutrino pair emission also shows a weak loga-
rithmic dependence on the cylinder size, $\propto V(1+O[\ln(d)])$. Angular distribution of radiative neutrino pair emission is markedly different from MCTPE; it has a broad structure, almost isotropic for $\omega \leq \Delta/2$, around the orthogonal direction to the cylinder axis, as shown in Figure 3. This may be used to distinguish the radiative neutrino pair emission from MCTPE, along with the broader energy spectrum.

**Prospect for neutrino experiments**

Prospect for detecting radiative neutrino pair emission is not bright, as it stands, for gaseous targets such as those in a cell or atomic beam that can be excited by laser irradiation. To have a larger rate for measurement of neutrino pair emission, it seems necessary either to have a further enhancement mechanism or to use denser targets such as atoms implanted within a solid matrix or something similar. The energy shift within solid does not cause a problem, but it is necessary to have small width broadening for successful neutrino mass spectroscopy.

For detection it is mandatory to repeat preparatory laser irradiation in order to avoid the coherence loss caused by collisional relaxation which destroys metastable excited atoms and finally ends at the ground state. Repeated cycles pumping from the ground state into the excited state, of repetition time, typically $\Delta T = O[100 \text{ ms}]$ (relaxation time), are needed. A good energy resolution is also crucial for performing the neutrino mass spectroscopy to precisely locate the thresholds.

Further study on these issues is obviously required prior to actual experiments.
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[5] For example, C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions*, Wiley-VCH(2004).

[6] Alternatively, one may compute the minus of the survival rate of a macroscopic N atomic state $|I\rangle$, $-\frac{d}{dt}\langle I|\rho|I\rangle$ against two photon emission, using the reduced density matrix $\rho$ integrated over environment variables of radiation and neutrino fields out of the entire dynamical variables of atoms + fields. This computation, to the leading order of perturbation theory, yields the real part of time integration of the product, two photon propagators times transition matrix element squared, $\int_{-\infty}^{t}dt' D_1(\vec{r}_a - \vec{r}_b, t - t')D_2(\vec{r}_a - \vec{r}_b, t - t')e^{-i\Delta(t-t')}|\mathcal{M}(\vec{k}_i)|^2$ where $D_i(\vec{r}_a - \vec{r}_b, t - t')$ is the photon propagator of momentum $\vec{k}_i$, namely $ie^{i\vec{k}_i\cdot(\vec{r}_a - \vec{r}_b)-i\omega_i(t-t')}\theta(t-t')/(2\omega_i)$, the photon being emitted at atomic location $\vec{r}_b$ and absorbed at $\vec{r}_a$. The energy conservation factor in eq.(1) $2\pi\delta(\Delta - \sum_i \omega_i)$ and the shape factor in the text are thus derived. This derivation is explained in standard reviews such as [2].

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[8] The total rate, calculated using our $\mathcal{H}(x, y)$, for the superradiant single photon emission is given by $(54d^2n^2\gamma) \ln(2d^2\Delta^2/9)/(\pi\Delta^4)$ with $\gamma$ the E1 or M1 rate $\propto \Delta^3$ from a single atom.

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