Mathematical simulation of the process of putting a connective insert into the orifice in a three-layered composite structure

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Abstract. This article provides the reader with the mathematical model determining the dependence of design parameters of embedding of a connective insert in a three-layered composite structure on its strained state at the outer diameter. Insert material is assumed to be a plastic body with nonlinear hardening. The model is adapted for implementing in the standard finite-element software ANSYS Mechanical used for numerical simulation of the process of embedding of the connective insert.

1. Introduction

One of the most significant issues in designing and manufacturing of three-layered composite structures is their joining with the other structural elements. It is related to the fact that these three-layered panels cannot withstand high concentrated loads typical for mechanical point joints [1, 2]. Moreover, when the structure of attaching element materials is incorrect (impossibility of technological realization or insufficient taking into account of strength properties), the assembled unit will not be able to withstand design loads or will be heavier than the unit manufactured from the panels of the other type, even though the honeycomb panel itself (without taking into account joining elements and reinforcements) is lighter than the panels of the other type.

Figure 1 shows the structures of standard junctions of three-layered panels. If a honeycomb core structure made of nonmetallic material is correctly realized, it takes up well both uniformly distributed concentrated loads. In the latter case, provision is made for local reinforcements in the structures (Figure 2). The typical structural solution involved in the units made of three-layered structures is a wide application of specific filling substances and foam plastic that influence the bearing capacity of the structure. At the same time, this local reinforcement greatly increases the labor intensiveness of the manufacturing of three-layered panels and their mass.

Under working loads, honeycomb core takes lateral forces and provides combined operation of bearing layers under bending and the action of normal and tangential forces as well. However, the honeycomb core itself does not take bending moments and forces in the middle surface, because it has modest rigidity in comparison with the outer layers. Thus, concentrated loads applied to the single area of honeycomb structures are transferred employing special inserts and bushings [1, 3]. For these classes of embeddings under load transfer, the relation of force point is necessary to transfer concentrated force to the three-layered panels with coverings.
There is some information [3,4] (Figure 3a) on using welding as a technological method of insert embedding. However, this method is labor-intensive, and it is used only for three-layered structures with metal coverings.

One of the most common methods of embedding a reinforcing insert is [1, 2, 3] filling it with glue substance. It provides a necessary bond of a holding element transferring the concentrated load to the three-layered panel directly with the filler and the covering. Besides, a cylindrical insert is the common structure. (Figure 3b). Before embedding the insert, the hole is drilled in the top covering of the panel, and its diameter is some little less of its outer diameter. The blind hole is filled with epoxy adhesive. When the insert is embedded, this epoxy adhesive fills the hole on the outside diameter developing sufficient strength of the joint. At the same time, this method is of low efficiency, and the force point produced by this method has increased mass.

To embed the holding element the telescopic bushings are used (Figure 3c), and the elements are screwed one into another. These bushings have flanges to tighten panels in height to reduce tearing forces that act on the covering when bolts are loaded. Walls of telescopic bushing are twice as much as thick then the walls of ordinary bushings that is why they are heavier.

2. Formulation of the problem.

Bushings with grooving (Figure 4) has been suggested to increase the efficiency of the forming process of ‘insert - three-layered structure’ and decreasing their mass. In course of press-fitting of the insert at its outer contour toroidal surface is formed. It provides the joining of the holding element between upper and lower coverings.

To form a reliable ‘insert – three-layered structure’ joint it is necessary to know exactly the size of the toroidal surface. This is due to the fact, that with the smaller size toroidal surface does not touch upper covering and the insert will not be joined between the coverings. If the toroidal surface is too large, the tearing of the upper covering from the filler can happen. This greatly reduces the bearing capacity of the joint. So when the insert is embedded, it is necessary to know the influence of initial geometry parameters of the insert on the toroidal surface size, work, and upset force.

Preliminary experimental research has shown that the shape of grooves influences on the value and the form of expansion of the toroidal surface. In this connection research dealing with the influence of the shape of grooves on the value of radial expansion of the toroidal surface has been carried out. During the research, semicircular, trapezoidal and rectangular grooves have been made on the inner surface of the insert (Figure 5). Research has been carried out using finite-element software ANSYS.
Figure 2. Variants of local reinforcement of three-layered structures: a) using overlays; b) using a packed honeycomb filler; c) using a beam; d) using a beam; e) using a filler; f) using foam plastic.

Figure 3. Methods of embedding of inserts in the hole of three-layered structures: a) welding; b) glued joint; c) telescopic joint.

Figure 4. Insert design (a) and the design of ‘insert – three-layered structure’ (b).

Figure 5. Shapes of grooves on the inner surface of the insert.
Finite-element simulation has shown that the largest radial shift at the outer contour of the insert takes place in the insert with the rectangular groove (Figure 6).

3. Statement of a mathematical model.

Preliminary experimental research has shown that kinematics of the embedding process is greatly determined by the design parameters of the embedded element and technological parameters of the axial plastic compression process. Under embedding of the holding element with the groove on the inner surface, a large number of factors influence the form of the toroidal surface, namely:

1. ratio of groove width to its height t/p (Figure 4);
2. ratio of groove width to the thickness of a tube billet t/b;
3. degree of material strain of a tube billet at the area of the groove S/S₀.

The significance of the influence of each factor on the form of the toroidal surface is determined according to the results of experimental research using the methodology of mathematical planning of the experiment [5]. Ur/m (Figure 4) is accepted as an optimization parameter, where Ur - the value of insert radial expansion along the outer diameter, m – half of the toroidal surface. Toroidal surface slope and stiffness of insert attachment between the coverings in the three-layered structure depends on the value of an optimization parameter. Preliminary research has allowed determining the values of technological factors during the experiment (Table 1).

For the approximation of response surface, the methodology of three-factor mathematical planning of the second-order [4] was used; in this case, the regression equation is the following:

\[ y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3. \] (1)

### Table 1. Technological factors of insert embedding.

| Factors               | t/p     | t/b     | S/S₀  |
|-----------------------|---------|---------|-------|
| Code                  | X₁      | X₂      | X₃    |
| Zero level (0)        | 0.7     | 0.7     | 0.5   |
| Variability interval  | 0.2     | 0.2     | 0.3   |
| Upper level (+1)      | 0.9     | 0.9     | 0.8   |
| Lower level (-1)      | 0.5     | 0.5     | 0.2   |

The information matrix determinant was used as an accuracy factor of the evaluation of regression equation parameters

\[ \det A = \det\left(N^{-1}X^TX\right), \] (2)

where X – matrix of independent variables of the size N×m; N – number of all measurements of the experiment; m – number of parameters in the regression equation.
Magnitude detA is inversely proportional to the concentration ellipsoid volume. Concentration ellipsoid is fully determined by the elements of covariance matrix $A^{-1}$ and evaluations of regression coefficients $\beta$. Plans maximizing concentration ellipsoid volume are D-optimal. Consequently, the more detA, the closer the plan to D-optimal.

Evaluation of value dispersion predicted by the regression equation is the following:

$$S^2\{y_{calc}\} = S^2_{repr} N^{-1} f'(x) A^{-1} f(x). \quad (3)$$

Evaluation of reproducibility dispersion depends on measurement accuracy and process stationarity. It does not depend on the point coordinates of the plan. So, as the plan characteristics, we consider the value

$$d(x) = N^{-1} f^*(x) A^{-1} f(x). \quad (4)$$

Maximum, minimum, and average model evaluation dispersion within the range $-1 < x_i < 1$ are the following:

$$d_{max} = \max N^{-1} f^*(x) A^{-1} f(x); \quad -1 \leq x_i \leq 1;$$

$$d_{min} = \min N^{-1} f^*(x) A^{-1} f(x); \quad -1 \leq x \leq 1;$$

$$d_{mid} = \frac{\int_{-1}^{1} \int_{-1}^{1} N^{-1} f^*(x) A^{-1} f(x)}{V_x},$$

where $V_x$ – the experimental area volume.

Based on the chosen plan $B_3$ and performed calculations, the experiment matrix plan is transferred to the natural values of factors at the coding stage.

The experiment took place according to the measurement procedure after their randomization and using table of random numbers [5].

When checking measurement accuracy equality, analysis of experimental results to find out gross errors in measurements was carried out. Analysis of reproducibility dispersion evaluation of experimental data was also done.

Calculations for the embedding process have allowed finding out the value of coefficients of regression equation and determining the regression equation (mathematical model)

$$y = 0.2549 - 0.0247x_1 - 0.0189x_2 + 0.0283x_3 + 0.0341x^2 -$$

$$-0.0159x^2 - 0.0159x^2 - 0.0194x_1x_2 + 0.0189x_1x_3 + 0.0466x_2x_3. \quad (6)$$

Verification of this regression equation was done using Fisher variance relation:

$$F_{exp} = \frac{S^2_{ad}}{S^2_{repr}}. \quad (7)$$

Dispersion evaluation determining inadequacy of experimental results with degrees of freedom $f_1 = N-m-f_{repr}$ was calculated using formula:

$$S^2_{ad} = \frac{1}{f_1} \sum_{1 \leq u \leq N} \left(\frac{y_u}{\bar{y}_u} - C_u\right)^2,$$

where $N$ – total number of measurements in the plan, $m$ – a number of coefficients in the regression equation, $\bar{y}_u$ - arithmetical mean value of response in $u$ experiment, $C_u$ - value of response in $u$-experiment calculated using regression equation. Consequently:

$$S^2_{ad} = \frac{3*0.00205599}{42-10-28} = 0.001542$$

with degree of freedom $f_1 = 4$. Experimental value of Fisher variance relation is:

$$F_{exp} = \frac{0.001542}{0.001825} = 0.845$$
And degree of freedom \( f_1 = 4 \). Theoretical value \( F^T \) of statistics at significance level 0.05 and degrees of freedom \( f_1 = f^\text{aov} = 4 \) and \( f_2 = f^\text{rep} = 28 \) equals \( F^T_{0.05;4,28} = 2.7 \).

Thus, \( F^\text{exp} < F^T_{0.05;4,28} \).

Consequently, with probability \( P = 1 - \alpha = 1 - 0.05 = 0.95 \) hypothesis of adequacy of obtained regression equation is not rejected.

Embedding force for the insert is found from the equation (2.2)

\[
P = \frac{A_D}{U}.
\]

Strain forces work \( A_D \) was determined \([6, 7, 8, 9, 10]\) taking into account boundary conditions (Figure 5) at the insert outer contour. At the upper insert end, there is a B region that is not subjected to plastic strain. The height of this region equals the thickness of the covering of the component. Plastic region A is located lower; the formation of the toroidal surface takes place there. In the lower insert part, there is region C that remains unstrained. Insert material is assumed to be a plastic body with nonlinear hardening.

Constant work of strain is a combination of (Figure 7) work of internal forces \( A\text{int} \) to deform the insert form, work of friction forces on the inner surface \( A\text{fr.int.} \), work of shear forces \( A\text{sht} \) between plastic and hard regions.

\[
A_D = A\text{int} + A\text{fr.int.} + A\text{sht}.
\]

![Figure 7. Scheme of insert strain.](image)

Taking into account components of equation (10), final equation for strain force work was determined:

\[
A_D = \frac{\pi \sigma_{\text{int}} U^{1+n}}{(1-\varepsilon_z)\varepsilon_z^n} \left[ \frac{(R-r)^2}{(1+n)h_X^2} \right]^{\frac{\gamma r_0 h_X^{1-n}}{\sqrt[3]{3h_X^{1+n}}} + \frac{2}{3} \frac{(R-r)^3}{r_0^2 (R-r_0)}}
\]

where \( \sigma_{\text{int}} \) – ultimate strength of the insert material; \( n = \varepsilon_z / (1-\varepsilon_z) \); \( \varepsilon_z = (h_0 - h) / h_0 \); \( h_0 \) - initial insert height with segments; \( h \) – bushing height after strain; \( \Psi \) - empirical coefficient taking into account state of frictional surfaces and the shape of strain region; \( R \) and \( r_0 \) - inner and outer radii of the insert respectively; \( u \)-value of axial shift, \( \text{mm.} \)

Magnitude of propagation of plastic strain region \( h_X \) was determined using the principle of minimum of total strain energy for rigid-plastic medium. Differentiating equation (11), we get

\[
\frac{D(1-h)\Psi \cdot r_0}{\sqrt[3]{3h_X^n}} - \frac{D \cdot h \cdot (R-r)^2}{(1+n)h_X^{n+1}} - \frac{2D(1+n)}{\sqrt[3]{3h_X^{n+2}}} \left[ \frac{(R-r_0)^3}{3} - r_0^2 (R-r_0) \right] = 0
\]

where \( D = \frac{\sigma_z \cdot \Psi \cdot U^{1+n}}{(1-\varepsilon_z) \cdot \varepsilon_z^n} \).

Solution of equation (12) is led to the solution of the equation
\[
\frac{D(1-h)\psi \cdot r_0}{\sqrt{3}} h_x^2 - \frac{D \cdot h \cdot (R - r)^2}{(1 + n)} h_x - \frac{2D(1 + n) R - r_0}{\sqrt{3}} \left[ \frac{(R - r_0)^3}{3} - r_0^2 (R - r_0) \right] = 0 .
\] (11)

To find analytical relations defining insert material strain, the principle of minimum of total strain energy and Ritz method were applied. On the assumption of the results of preliminary experimental research, radial shift at the outer insert diameter is the following:

\[
U_r = \frac{1}{2} a R \left( 1 - \frac{y^2}{h^2} \right),
\] (12)

where \( a \) – varying parameter.

Variational equation was composed to find the parameter. Taking into account boundary conditions (Figure 7), it is the following:

\[
\delta A_D = \delta A_{\text{int}} + \delta A_{\text{sh}} + \delta A_{\text{frint}} ,
\] (13)

where, \( \delta A_{\text{cp}} \) and \( \delta A_{\text{pg}} \) - work variations, internal forces, shear forces, shear forces between regions A, B, C, and friction forces along the internal surface respectively.

Applying (14) and the equation for the total system energy based on (15) we get the equation for determining of the parameter \( a \)

\[
a = \left[ \frac{3 \psi \cdot h_x^2 \cdot r_0 \cdot \varepsilon_x^{1+n}}{3 \sqrt{3} \left( 1 - \frac{y^2}{h^2} \right)^{n+1} \cdot (R - r_0)^2 h_x - (R - r_0)^3 (n + 1)} \right]^{\frac{1}{n}}.
\] (14)

Knowing parameter \( a \), and from the equation (14) it is possible to express magnitude \( U_r \). From equation (16) taking into account experimental data, we get the magnitude of straining shift \( U \) of the insert using the magnitude of axial strain \( \varepsilon_x \).

Obtained results of theoretical research have been compared with experimental data. Figure 8 shows experimental research results determining the strained state of the insert and their comparison with the design values. Analysis of obtained results shows that the maximum discrepancy is 3-5%.

![Figure 8](image)

**Figure 8.** Comparison of design and experimental shift magnitudes \( U_r \) from top to bottom: 1 – design magnitude; 2 – experimental magnitude.

4. Conclusion

The mathematical model has been developed permitting to determine work and embedding force of the insert by design parameters of embedded elements. This model also allows determining the strained state of embedded element material at its outer contour. This model is adapted to be implemented in standard finite-element software ANSYS Mechanical. This greatly reduces the amount of numerical simulation; it also greatly decreases the complexity of result analysis when developing the manufacturing process of forming the joint ‘insert-three-layered structure.’

5. References

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