Superfluid critical temperature in 3D Fermi gas with repulsion

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Abstract

The critical temperature of a superfluid phase transition in a Fermi gas with repulsive interaction is found. The influence of a magnetic field on the transition is analyzed. The estimates for the critical temperature for a trapped gas of \textsuperscript{6}Li atoms and \textsuperscript{3}He–\textsuperscript{4}He mixtures are presented.

Keywords: Superfluidity in neutral Fermi systems; \textsuperscript{3}He–\textsuperscript{4}He mixtures; trapped Fermi gases

One of the most important questions in connection with nonconventional superconductivity is the origin of an attractive interaction. In this paper we show that a nonideal Fermi gas with purely repulsive bare interaction is unstable towards Cooper pairing with orbital momentum \(l = 1\). This instability exists due to Kohn-Luttinger mechanism based on many-body effects [1].

1. Theoretical model

We consider a nonideal Fermi-gas described by the Hamiltonian:

\[
\hat{H} = \sum_{\sigma = \uparrow, \downarrow, \mathbf{p}} \xi_{\mathbf{p}} \hat{a}_{\mathbf{p}\sigma} \hat{a}^\dagger_{\mathbf{p}\sigma} + g \sum_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4} \hat{a}^\dagger_{\mathbf{p}_1 \uparrow} \hat{a}^\dagger_{\mathbf{p}_2 \downarrow} \hat{a}_{\mathbf{p}_3 \downarrow} \hat{a}_{\mathbf{p}_4 \uparrow},
\]

where \(\xi_{\mathbf{p}} = \mathbf{p}^2/2m - \mu\), \(\mu\) is the chemical potential, \(g\) the constant of a bare point-like repulsive interparticle interaction, and different spin components, \(\sigma = \uparrow, \downarrow\), are assumed to have equal masses \(m\) and concentrations \(n\).

As it was shown in [2,3], an effective interparticle interaction originated from both the bare one and many-body effects, is attractive when two particles have a nonzero relative angular momentum \(l\). This attractive interaction is maximal for \(l = 1\) and, therefore, results in a \(p\)-wave triplet Cooper pairing with the critical temperature:

\[
T_{c1} = \tilde{\varepsilon} \exp \left\{ -1/\nu_F |V_{1}^{eff}| \right\} = \tilde{\varepsilon} \exp \left\{ -12.9/\lambda^2 \right\},
\]

where \(\nu_F = m p_F/2\pi^2\) is the density of states at the Fermi energy \((p_F = (6\pi^2n)^{1/3}\) the Fermi momentum \), \(V_{1}^{eff} = (2\ln 2 - 1)g^2 \nu_F/5\) the \(p\)-wave harmonic of the effective interaction in the Cooper channel (see [2] for details), \(a\) the scattering length \((a = mg/4\pi\) in the Born approximation\), \(\lambda = 2a p_F/\pi\) the gas parameter \((\lambda \ll 1\) for the considered case\), and \(\tilde{\varepsilon}\) the cutoff parameter of the order of the Fermi-energy, \(\tilde{\varepsilon} \sim \varepsilon_F = p_F^2/2m\).
To fix the parameter $\tilde{\varepsilon}$ and find the critical temperature one has to keep all contributions up to $\lambda^4$ in the Bethe-Salpeter equation, that defines the critical temperature (see [5] for more details). These contributions originate from the effective interaction, retardation effects (momentum and frequency dependence of the effective interaction) and renormalization of Green functions ($Z$-factor and $m^*$).

The corresponding critical temperature, found numerically, is

$$T_{c1} \approx \frac{2}{\pi} C \varepsilon_F \exp \left\{ -\frac{12.9}{\lambda^2 (1 + 4.3\lambda)} + \frac{13.4}{(1 + 4.3\lambda)^2} \right\},$$

where $C = 0.577$ is the Euler constant, and neglected terms are of order $\lambda$. (Note, that this formula extrapolates the expression for $T_{c1}$ from $\lambda \ll 1$ to $\lambda < 1$.)

Magnetic field dependence of the critical temperature $T_{c1}$ can be analyzed in the same way. We present the results for $T_{c1}$ as a function of polarization $\alpha = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$ on Fig 1. It turns out, that the nonmonotonic dependence of $T_{c1}$ on $\alpha$ is a result of a competition between the increase of the angular dependence of the effective interaction $V^{eff}$ and decrease of its amplitude. (The former increases $|V_1^{eff}|$ and, hence, $T_{c1}$, while the later decreases.)

2. Conclusions.

In conclusion, let us mention two possible experimental applications of the presented theory. The first one is to $^3$He-$^4$He mixtures, providing that the concentration $x$ of $^3$He is more than 3%. (In this case the interaction between $^3$He atoms is repulsive.) The estimate of the critical temperature in zero magnetic field gives the value $T_{c1} \approx 5 \cdot 10^{-6}$K for the maximal concentration $x \approx 9.5\%$ (at pressure 10 bar). By applying the magnetic field this value can be increased by a factor of 6 (at polarization $\alpha \sim 40\%$), that gives a hope to observe the transition experimentally.

The second application is to trapped neutral Fermi gases. For these systems the critical temperature $T_{c1}$ is estimated to be of the order of $10^{-7} \div 10^{-6}$K for densities $n \sim 10^{14}$cm$^{-3}$.

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