State determination for composite systems of two spatial qubits

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Abstract. In a recent letter [Phys. Rev. Lett. 94, 100501 (2005)], we presented a scheme for generating pure entangled states of spatial qudits using transverse correlations of parametric down-converted photons. Here we show how the modification of this scheme can be used to generate mixed states and we investigate the state determination for composite systems of two spatial qubits, motivated by the fact that quantum information protocols may be easier to be implemented for this case. By means of local operations on the twin photons we were able to perform the quantum tomography process to reconstruct the density matrix of a mixed state of two spatial qubits.

1. Introduction
The quantum state is considered to be the most complete description available for an individual physical system. The statistical distributions for the observables of a given system are completely characterized by its state. Therefore, the development of techniques to perform the state determination is of the utmost importance because they allow us to predict the results that are more likely to happen for any further possible measurement. Beside this fact, the explorations of new technological fields is bringing more motivations for the study of these techniques. In Quantum Information, for example, protocols like teleportation [1] and superdense coding [2] require an initial know quantum state to be implemented. Another special case is the use of reconstructed density matrices to calculate quantities such as the concurrence [3] of a composite system.

Several techniques have been used for the state estimation of different physical systems. In the field of atomic physics, quantum endoscopy was used to determine the state of ions and atoms [4, 5]. In quantum optics, the Wigner function of multi mode fields could be measured using the homodyne detection [6–8] and the technique of quantum tomographic reconstruction (QTR) was used for measuring the polarization state of the down-converted photons [9].

In general, these methods are based on a linear inversion of the measured data. In the case of QTR, the data is acquired with a series of measurements performed on a large number of identically prepared copies of a quantum system. The fact that this transformation being linear, make it strongly dependent of any experimental errors that may occur while recording the data. They can appear as a consequence of the experimental noise or misalignment and, therefore, the reconstructed state is only a reasonable approximation of the real quantum state. The density matrices obtained may have properties that are not fully compatible with a quantum state. Another alternative that has been considered for the state
determination is the numerical technique called maximum likelihood estimation [10,11]. It is based on a relation between the observed data and the quantum state that could have generated them. Even though it generates only the possible density matrices, it has the drawback of enhancing the uncertainty on the state estimation.

We have recently demonstrated that it is possible to use the transverse correlations of the twin photons, produced during the process of spontaneous parametric down conversion (SPDC), to generate pure entangled states of higher dimensional quantum systems, that is known as qudits [12, 13]. Because the spaces of these photons are defined by the number of different available ways for their transmission through apertures where they are sent to, spatial qudits. In the present work, we first discuss, how a modification of the setup considered to create pure entangled states can be used to generate mixed states. As the study of mixed states is an important field of research because it allows one to consider more realistic experimental situations where a pure state due to interactions with its environment becomes a mixture of quantum systems. Following this investigate the state determination for a system compose of two spatial qubits. We emphasis on this type of system because we plan to use them first for quantum communication. Therefore, a procedure for their characterization is of upmost importance.

We show that it is possible to perform the QTR for a mixed state of two spatial qubits, that is, we have experimentally reconstructed the density matrix of a system composed of two spatial qudits. The quality of the reconstruction is also discussed. Even though we had considered just this special case it is straightforward to show that the technique used can be generalized to a system composed of two spatial qudits.

2. Controlled generation of mixed states
The studies of references [12, 13] show that the state of down-converted photons when transmitted through identical multi slits, with \(d\) being the distance between two consecutive slits and \(a\) as the half width of the slits, can be written as

\[
|\Psi\rangle \propto \sum_{l=-l_D}^{l_D} \sum_{m=-l_D}^{l_D} W_{lm} e^{i \frac{k d^2}{8 z_A} (m-l)^2} |l\rangle_s \otimes |m\rangle_i,
\]

where \(l_D = (D - 1)/2\) and \(D\) is the number of slits in these apertures. The function \(W_{lm}\) is related with the spatial distribution of the pump beam at the plane of the multi slits \((z = z_A)\) and is given by

\[
W_{lm} = W \left[ \frac{(l + m)d}{2} ; z_A \right].
\]

The \(|l\rangle\) (or \(|m\rangle\)) state, is a single-photon state defined, up to a global phase factor, by

\[
|l\rangle_j \equiv \sqrt{\frac{\alpha}{\pi}} \int dq_j e^{-i q_j ld} \text{sinc} (q_j a) |1q_j\rangle,
\]

and represents the photon in mode \(j\) transmitted by the slit \(l\). \(q_j\) is the transverse component of wave vector of the down-converted photons. The base \(\{|l\rangle_j\}\) satisfies \(\langle l | l' \rangle_j = \delta_{ll'}\). We use these states to define the logical states of the qudits and thus, it is clear that Eq. (1) represents a composite system of two qudits. The spaces of these photons are defined by \(D\) because the degree of freedom of each photon are the paths available for their transmission through the multi slits.

It can be seen from Eq. (1) and Eq. (2), that it is possible to create different pure states of spatial qudits if one knows how to manipulate the pump beam in order to generate distinct transverse profiles at the plane of the multi slits \((W(\xi ; z_A))\). In Ref. [13], we showed experimentally that a maximally entangled state of spatial ququarts (\(D = 4\))

\[
|\Psi\rangle = \frac{1}{2} \sum_{l=-\frac{D}{2}}^{\frac{D}{2}} e^{i k \frac{d^2}{4 z_A}} |l\rangle_i \otimes |-l\rangle_2.
\]
Figure 1. Schematic diagram of the experimental setup used for generating and for characterizing mixed states of spatial qubits. $A_s$ and $A_i$ are the double-slits at signal and idler propagation paths, respectively. $D_s$ and $D_i$ are detectors and C is a photon coincidence counter. The configuration used to determine the diagonal elements is represent in (b). (c) and (d) were used at the second type of measurement and (e) at the third type.

can be generated when the pump beam is focused at the plane of two identical four-slits in such a way that it is non vanishing except at region smaller than the dark part of these apertures. The state in Eq. (4) describes the correlation between the photons such that, when the photon in mode $s$ is transmitted by the slit $l$ the photon in mode $i$ will pass through the symmetrically opposite slit $-l$. In this experiment, a non-linear crystal was directly pumped by a Krypton laser. A lens having with a small focal length was used before this crystal to focus the pump beam.

Let us assume that, before reaching the crystal, the pump beam pass through an unbalanced Mach-Zehnder interferometer where the transverse profile of the laser beam is modified variedly in each arm. If the difference between the lengths of these arms is set larger than the laser coherence length, we will obtain an incoherent superposition of the states generate by each arm. It is interesting to note that this generation of mixed states of spatial qudits can be completely controlled. Beside the fact that we can control the probabilities for generating them by placing attenuators at the interferometer arms.

In the following section, we will show how is possible to use the QTR technique to determine the density matrices of these composite systems. The state whose density matrix was reconstructed is a mixed state of spatial qubits. This state was generated with the experimental setup represented in Fig.1(a). A 5 mm $\beta$-barium borate crystal was pumped by a 500 mW Krypton laser emitting at $\lambda = 413$ nm for generating SPDC. Before being incident at the crystal, the pump beam crossed an unbalanced Mach-Zehnder interferometer. The difference between the lengths of each arm (200 mm) was set larger than the laser coherence length (80 mm). Two identical double slits $A_s$ and $A_i$ were aligned in the direction of the signal and idler beams, respectively, at a distance of 200 mm from the crystal ($z_A$). The slit’s width was $2a = 0.09$ mm and their separation $2d = 0.18$ mm. At the arm 1 of the interferometer, we placed a lens that focused the laser beam at the plane of these double slits, into a region smaller than $2d$. In arm 2, we used a set of lenses that increased the transverse width of the laser beam at $z_A$. The transverse profiles that were generated are illustrated in Fig. 1(a). The photons transmitted through the double-slits were detected in coincidence between detectors $D_s$ and $D_i$. Two identical single slits of dimension 5.0 x 0.1 mm and two interference filters of 8 nm full width at half maximum (FWHM) bandwidth were placed in front of the detectors.

Using Eq. (1) and Eq. (2), we can show that the two-photon state, after the double slits, when only
arm 1 is open is given by

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|+\rangle_i|+\rangle_s - |+\rangle_i|-\rangle_s + |−\rangle_i|−\rangle_s).$$  \hspace{1cm} (5)$$

To simplify, we used the state $|+\rangle_j$ and $|−\rangle_j$ to represent the $j$ photon being transmitted by the upper (lower) slit of the respective double slit. The state shown in Eq. (5) is a maximally entangled state of two spatial qubits. However, if the laser beam cross only arm 2, the state of the twin photons transmitted by the apertures will be a state of the type given by

$$|\Psi_2\rangle = \frac{1}{2}e^{i\theta}(|−\rangle_s |+\rangle_i + |+\rangle_s |−\rangle_i) + \frac{1}{2}(|−\rangle_s |−\rangle_i + |+\rangle_s |+\rangle_i),$$  \hspace{1cm} (6)$$

which has a smaller degree of entanglement. Therefore, the two-photon state generated in our experiment, when the two arms are liberated, is a mixed state of the spatial maximally entangled state of Eq. (5) and the state of Eq. (6). It is described by the density operator

$$\rho_{the} = A|\Psi_1\rangle_1\langle\Psi_1| + B|\Psi_2\rangle_2\langle\Psi_2|,$$  \hspace{1cm} (7)$$

where $A$ and $B$ are the probabilities for generating the states of arm 1 and arm 2, respectively.

3. Reconstruction

Now, we will show how QTR can be experimentally implemented to reconstruct the density matrix of the state generated in the first part of our experiment, given by Eq. (7), without the use of any information about this generation.

The process of quantum tomography, which is described in Refs. [14, 15], is easy to be understood. The first thing to remark is the fact that it is always possible to measure the diagonal elements of any density operator, directly. Therefore, the quantum tomographic reconstruction is just a protocol to determine the non diagonal ones. It consists in the use of local operations at the subsystems to allow their detection in different basis. Since the form of the global density operator depends on which basis of the subsystem detected at, the effect of these local operations is the generation of new density operators. Because we know which were the change of basis performed by the local operations, we can relate the diagonal elements (that are measurable) of a new density matrix, say $\tilde{\rho}$, with the non diagonal ones (that are not measurable) of the original density operator, $\rho$. If we repeat this procedure to obtain more density operators and measure their diagonal, we will create a set of independent equations which allows the determination of the non diagonal elements of $\rho$.

To characterize the state generated in our experiment, we first adopt a general form for its density matrix

$$\rho = \begin{bmatrix}
\rho_{++++} & \rho_{+++-} & \rho_{+-+-} & \rho_{++-+} & \rho_{+-++} & \rho_{++-} & \rho_{+-+} & \rho_{++-}
\rho_{++-} & \rho_{+-+-} & \rho_{++-} & \rho_{+-+} & \rho_{++-} & \rho_{+-+} & \rho_{++-} & \rho_{++-}
\rho_{+-+-} & \rho_{++-} & \rho_{+-+} & \rho_{++-} & \rho_{+-+} & \rho_{++-} & \rho_{+-+} & \rho_{++-}
\rho_{++-} & \rho_{+-+} & \rho_{++-} & \rho_{+-+} & \rho_{++-} & \rho_{+-+} & \rho_{++-} & \rho_{++-}
\end{bmatrix},$$  \hspace{1cm} (8)$$

where $\rho_{j_{s,j_{i,k_{s,k_{i}}}k_{i}}} = \langle j_{s}j_{i} |\rho| k_{s}k_{i}\rangle$ and $j_{s},k_{i} = \pm$.

3.1. Diagonal Elements

The diagonal elements can be determined by coincidence measurements with the detectors just behind the double slits [13] or at the plane of image formation of these apertures when two lenses are placed in the signal and idler paths, as showed by Fig. 1(b) [16]. In these measurements, one detector is kept fixed behind one slit while the other detector scans the $x$ direction over the entire region of the double slit. Two measurements of this kind, outlined in Fig. 2, with detector $D_{i}$ going from slit “+” to the slit “−” were performed. After normalization, we obtain the diagonal of Eq. (8): $\rho_{++++} = 0.042$, $\rho_{+-+-} = 0.468$, $\rho_{+---} = 0.462$ and $\rho_{++--} = 0.028$. 
3.2. Change of Basis

As mentioned above, for the reconstruction process it is necessary that local operations are applied in the subsystems to change the base used for writing the global density operator. What is interesting in the use of the spatial qubits is the fact that this local operations occur naturally while they freely evolve in the space after the double slits. Just by varying the detector transverse position one can detect these spatial qubits in different basis. To understand this, lets first consider the expression for the two-photon state in a transverse plane at a distance $z - z_A$ on the double-slit’s plane

$$|\Psi\rangle_z \propto W(d; z_A)|g_+\rangle_s|g_+\rangle_i + W(-d; z_A)|g_-\rangle_s|g_-\rangle_i + W(0; z_A) (|g_+\rangle_s|g_-\rangle_i + |g_-\rangle_s|g_+\rangle_i),$$

(9)

where the normalized state $|g_\pm\rangle_j$ (or $|g_\mp\rangle_j$) is given by

$$|g_\pm\rangle_j \equiv \sqrt{\frac{a}{\pi}} \int dq_j e^{[-q_j^2(z-z_A)^2]/2}\{e^{+i q_j d}\text{sinc}(q_j a)|1q_j\rangle},$$

(10)

with $j = s, i$ and represents the photon in mode $j$, at the plane-$z$, when it was transmitted by the slit “+” (or “-”) of the double slit. Since the photon at this plane was certainly transmitted by one of these two slits and since the states $|g_\pm\rangle$ form a orthonormal base, we can use them to define the Hilbert space of this photon. For example, if we focus the laser beam at $z_A$, such that $W(-d; z_A) = W(d; z_A) = 0$, the state of Eq. (9) will be

$$|\Psi\rangle_z = \frac{1}{\sqrt{2}}(|g_+\rangle_s|g_-\rangle_i + |g_-\rangle_s|g_+\rangle_i),$$

(11)

which is a maximally entangled state. In this case, each photon of the pair will be described as a mixture of the states $|g_\pm\rangle$ at the plane-$z$.

However, suppose if the detectors with small transverse apertures are placed at this plane. These apertures will then select a photon in a certain state to be detected. This happens in the same way as the
polarizers do for the entangled states of polarized photons to test Bell inequalities [17, 18]. The state of the selected photon, which will be detected later, can be described as

\[ |h_j⟩ = f_+ (x, z) |g_+⟩_j + f_- (x, z) |g_-⟩_j, \]

where

\[ |j⟩ \langle g_± | h_j⟩|^2 = |f_± (x, z)|^2, \]

are the probabilities that the selected photon in the point \((x, z)\), have crossed the apertures “+” or “-”, respectively.

The amplitude probability \(f_+ (x, z)\) is calculated by using the electric field operator, in the paraxial approximation [19]

\[ \mathbb{E} = \int dq \hat{a} (q) e^{i (q x - q^2 (z - z_A)^2 / 2k)} , \]

and by calculating \( \langle vac | \mathbb{E}(x, z) | ± \rangle \). After normalizing the state of Eq. (12), we have

\[ f_± (x, z) = \frac{\exp \left( \frac{k (x ± d)^2}{2 (z - z_A)} \right) \text{sinc} \left( \frac{k (x ± d)}{z - z_A} \right)}{\sqrt{\text{sinc}^2 \left( \frac{k (x ± d)}{z - z_A} \right) + \text{sinc}^2 \left( \frac{k (x ± d)}{z - z_A} \right)}}. \]

Therefore, by changing the transverse detector position \(x\), we can do measurements in the subsystems in different orthonormal bases, measure experimentally the diagonal of the different \(\rho'\) operators and then reconstruct the density operator in the detection \(z\)-plane. by reconstructing \(\rho'\), we are also reconstructing \(\rho\), the density operator at the double-slits plane \(z_A\), because the matrix elements of \(\rho'\) and \(\rho\) are exactly the same. We show it below. Suppose a local unitary operator \(U_j\) represents the evolution operator from state \(|l⟩_j⟩\) to \(|g_l⟩_j∧\) for photon \(j\), with \(j = i, s, \)

\[ U_j = e^{-i k (z - z_A)} \int dq e^{i \left( q (x - q^2 (z - z_A)^2 / 2k) \right)} |1_q⟩_{jj} ⟨1_q|. \]

such that

\[ |g_l⟩_j = U_j |L⟩_j, \]

with \(l = +, −.\) If

\[ ρ = \sum_{l, m, l', m'} \rho_{l,m,l',m'} |l, m⟩ ⟨l', m'| \]

and

\[ \rho' = U_s ⊗ U_i ρ U_i^† ⊗ U_s^† = \sum_{l, m, l', m'} \rho_{l,m,l',m'} U_s ⊗ U_i |l, m⟩ ⟨l', m'| U_i^† ⊗ U_s^† = \sum_{l, m, l', m'} \rho_{l,m,l',m'} |g_l,g_m⟩ ⟨g_{l'}, g_{m'}| \]

by doing the reconstruction of \(\rho'\) we are also reconstructing \(\rho\).
3.3. Non diagonal Elements

For simplifying the notation, we rewrite the $|h\rangle$ state, neglecting a global phase, as

$$|h\rangle = \cos \theta |g_+\rangle + e^{i\eta} \sin \theta |g_-\rangle,$$

where

$$\eta = \frac{2kd}{z-z_A}.$$  \hspace{1cm}  \text{(21)}

$\cos \theta \equiv |f_+(x,z)|$ and $\sin \theta \equiv |f_-(x,z)|$. Note that, when we select a given value for $\eta$ angle at the detection plane located at a $z$ distance, the value of $\theta$ angle is completely defined. By considering the value of the experimental parameters: $z-z_A$, $k$, $d$, and $a$, it can be shown that the state $|h'\rangle = \cos \theta' |g_+\rangle + e^{i(\eta+\pi)} \sin \theta' |g_-\rangle$ is orthogonal to the state $|h\rangle$ with a high accuracy, $|\langle h' | h \rangle| \leq 10^{-3}$, when $-\pi \leq \eta \leq \pi$. For practical purposes the base $\{|h\rangle, |h'\rangle\}$ are orthonormal because this value is less than the experimental error for determining the probabilities at the different basis considered.

The second type of coincidence measurements were done by positioning the idler detector behind aperture “+” (“−”) of its double-slit and displacing transversely, the signal detector in the $z$-plane (See Fig. 1(c)) such that the signal photons detected were selected at the $x$-position in which $x/2 = -0.688$ mm) and $x_\pi/2 = 0.688$ mm). The fourth order interference pattern (20, 21), when the idler detector is fixed at “+” aperture, is shown in Fig. 3. We also selected signal photons at $x$-position such that $\eta = 0$ ($|h_2\rangle, x_0 = 0$ mm) and $\eta + \pi = \pi$ ($|h'_2\rangle, x_\pi = 1.376$ mm).

With these measurements, we determined the diagonal elements of the density operators written in basis $\{|+,h_j\rangle, |+,h'_j\rangle, |-,h_j\rangle, |-,h'_j\rangle\}$, with $j = 1$ and 2, which denotes the reconstructing bases with $\eta = \frac{2kd}{2}$ and $\eta = 0$, respectively. By repeating this detection procedure and reversing the roles of the signal and idler detectors (See Fig. 1(d)), we found the diagonal elements of another global density operators written in the basis $\{|h_j, +\rangle, |h_j, -\rangle, |h'_j, +\rangle, |h'_j, -\rangle\}$.

Relating the diagonal of the new operators with the non diagonal elements of $\rho$, we determined: $\rho_{++--}(\rho_{--++}), \rho_{+-+−}(\rho_{−−−+}), \rho_{++−−}(\rho_{+++−})$ and $\rho_{−−++}(\rho_{−−−+})$. We show below the explicit expressions that determine $\rho_{++−−}$

$$\Re(\rho_{++−−}) = \frac{(\tilde{\rho}_{\theta_1} + \rho_{++−−} \cos^2 \theta_1 - \rho_{−−++} \sin^2 \theta_1) \cos \eta}{\sin 2\theta_1} - \frac{(\tilde{\rho}_{\theta_2} + \rho_{++−−} \cos^2 \theta_2 - \rho_{−−++} \sin^2 \theta_2) \sin \eta}{\sin 2\theta_2},$$

\hspace{1cm}  \text{(22)}
Figure 4. Fourth order interference pattern as a function of $D_s$ position. It was recorded when the detector idler was fixed at the transverse position which corresponds to $\eta + \pi = \frac{\pi}{2}$ ($x = 0.688$ mm). The solid curve was obtained theoretically.

\[ \Im \left( \rho_{++-} \right) = \frac{\left( \rho_{\theta_1+\theta_1+} - \rho_{+++-} \cos^2 \theta_1 - \rho_{---+} \sin^2 \theta_1 \right) \sin \eta}{\sin 2\theta_1} - \frac{\left( \rho_{\theta_2+\theta_2+} - \rho_{+++-} \cos^2 \theta_2 - \rho_{---+} \sin^2 \theta_2 \right) \cos \eta}{\sin 2\theta_2}, \] (23)

where $\theta_j$ refers to bases with $\eta = \frac{\pi}{2}$ and $\eta = 0$ for $j = 1$ and 2, respectively.

In the third measurement type shown in Fig. 1(f), signal and idler detectors are positioned in the $z$-plane and the fourth-order interference patterns are measured. One of the detectors is kept fixed while the other is scanned transversely for detecting the photon pairs in coincidence (see Fig. 4). Five interference patterns were measured with one of the detectors fixed at the transverse positions related to $\eta = -\pi, -\pi/2, 0, \pi/2,$ and $\pi$. This allows (by means of similar expressions as the described above) to completely determine the density operator $\rho$. These measurements correspond to local operations at both down-converted photons.

3.4. The Reconstructed Density Operator

By performing the quantum tomographic reconstruction, as described above, we found the following form for the density operator of our experiment

\[ \rho = \begin{bmatrix} 0.042 & 0.083 + 0.004i & 0.081 + 0.005i & -0.129 + 0.062i \\ 0.083 - 0.004i & 0.468 & 0.444 - 0.058i & 0.097 - 0.008i \\ 0.081 - 0.005i & 0.444 + 0.058i & 0.462 & 0.096 - 0.006i \\ -0.129 - 0.062i & 0.097 + 0.008i & 0.096 + 0.006i & 0.028 \end{bmatrix}. \] (24)

The elements of a density operator must satisfy the Schwarz inequality, i.e., $|\rho_{jk}| \leq \sqrt{\rho_{jj}\rho_{kk}}$, where $j, k = ++, +-,-+,--$, if it really represents a quantum state. This is not our case for the matrix element $\rho_{++--}$, since it can be seen that $|\rho_{++--}| > \sqrt{\rho_{++}\rho_{--}}$. The reason for that are the fluctuations present in the coincidence measurements which can affect the final result as we discussed in the Introduction. This discrepancy can be reduced by increasing the detection time. Even though our reconstructed density matrix presents properties which are not fully compatible with the quantum state description, it is possible to show that it is coherent with the theory developed in Sec. 2. This is done in the next section, where we also give experimental evidences of the good quality of our reconstruction.
4. Discussion and Conclusion

The measured density operator of Eq. (24) can be approximately written as

$$\rho = 0.87|\Phi\rangle_1 1|\Phi\rangle + 0.13|\Phi\rangle_2|\Phi\rangle,$$

where the states $|\Phi\rangle$ are given by

$$|\Phi\rangle_1 = 0.077e^{i\phi_1}|++\rangle + 0.704e^{i\phi_2}|+-\rangle + 0.699e^{i\phi_2}|--\rangle + 0.099e^{i\phi_3}|--\rangle$$

and

$$|\Phi\rangle_2 = 0.514|++\rangle + 0.502e^{i\theta}|+-\rangle + 0.501e^{i\theta}|--\rangle + 0.483|--\rangle,$$

with $\phi_1 \simeq \phi_2 \simeq \phi_3 \approx 4.2$ and $\theta = 0.07$.

However, the fact that it is possible to decompose the density operator, $\rho$, in terms of the projectors of a state, $|\Phi\rangle_1$, which has a very high degree of entanglement and a state, $|\Phi\rangle_2$, that is of the form predicted by Eq. (6), is not enough to associate them with the states generated by each arm of the interferometer in our experiment. We still have to give an experimental evidence which corroborates with the expression of Eq. (25) as a reasonable approximation for the quantum state of the twin photons, i.e., we need to show that the values of $A = 0.87$ and $B = 0.13$, obtained mathematically, are reasonable for the probabilities of generating the states of these arms.

We measured the values of $A$ and $B$ by blocking one of the arms of the interferometer and detecting the transmitted coincident photons through the signal and idler double-slits. $A (B)$ is the ratio between the coincidence number when arm 2 (arm 1) is blocked and the total coincidence number when both arms are unblocked. From this measurement we obtained $A = 0.87 \pm 0.03$ and $B = 0.15 \pm 0.03$, indicating a good agreement with the expression of Eq. (25) for the reconstructed density operator.

Another experimental evidence for this high value of $A$ can be found in the fourth order interference patterns recorded in the third steps of the measurements performed (See Fig. 1(e)). Since the state $|\Phi\rangle_1$ is almost a maximally entangled state we would expect to observe conditional interference patterns, which would not be the case for high values of $B$. The conditionality observed in these patterns is showed in Fig. 5. The explanation for having the probability of generating the state in arm 1 superior to the probability of arm 2 is quite simple. The laser beam that cross arm 1 of the interferometer is focused at the slit’s plane, the spatial correlation of the generated photon pairs is larger than when the photon pairs are generated by the pump beam crossing arm 2. These values can be properly manipulated by inserting attenuators at the interferometer.

These experimental observations confirms the good quality of the QTR performed for the two photon state and allow us to consider the states $|\Phi\rangle_1$ and $|\Phi\rangle_2$ as good approximations for the states generated by arm 1 and arm 2 of the interferometer used. Fig. 6 shows a histogram of the real part of the matrix elements for (a) the measured density operator (Eq. (24)), (b) the density operator given by Eq. (25) and (c) the predicted density operator of Sec. 2. The agreement between the predicted and the measured density operator is good within the experimental errors. The largest error for the diagonal elements is only 3.5%. But, for the non-diagonal elements the propagated errors reaches 30% for their real parts and up to 65% for the imaginary parts.

In conclusion, we have demonstrated that it is possible to generate a broad family of mixed states of spatial qudits by exploring the transverse correlation of the down-converted photons. A statistical mixture of spatial qubits were used to show the quantum tomographic reconstruction performed to characterize this type of composite systems. The process was discussed in details and experimental evidences for the
good quality of the reconstruction performed were showed. Even though we had considered the state
determination only for the case of qubits, we believe that it can be generalized and performed in a similar
way for more dimensions. The importance of this work comes form the fact that the use of these systems
in the field of Quantum Communications requires the ability to characterize them.

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Figure 6. Histogram of the real part of the matrix elements for (a) the measured density operator, (b) the density operator given by Eq. (25) and (c) the predicted density operator of Sec. 2.

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