Local Search for Policy Iteration in Continuous Control

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Abstract

We present an algorithm for local, regularized, policy improvement in reinforcement learning (RL) that allows us to formulate model-based and model-free variants in a single framework. Our algorithm can be interpreted as a natural extension of work on KL-regularized RL and introduces a form of tree search for continuous action spaces. We demonstrate that additional computation spent on model-based policy improvement during learning can improve data efficiency, and confirm that model-based policy improvement during action selection can also be beneficial. Quantitatively, our algorithm improves data efficiency on several continuous control benchmarks (when a model is learned in parallel), and it provides significant improvements in wall-clock time in high-dimensional domains (when a ground truth model is available). The unified framework also helps us to better understand the space of model-based and model-free algorithms. In particular, we demonstrate that some benefits attributed to model-based RL can be obtained without a model, simply by utilizing more computation.

1. Introduction

Stable policy optimization in high-dimensions, and continuous action spaces, can be a challenge even in simulation. In recent years, a variety of deep RL algorithms have been developed, both for the model-free and model-based setting, that aim to tackle this challenge. In continuous control, recent progress on scalable (distributed) algorithms (Schulman et al., 2017b; Song et al., 2019) now allows us to solve problems with high-dimensional observation and action spaces end-to-end, provided adequate computation for simulation and learning is available. At the other end of the spectrum, there exist off-policy algorithms (Hafner & Riedmiller, 2011; Lillicrap et al., 2015; Heess et al., 2015; Abdolmaleki et al., 2018; Haarnoja et al., 2018) that have achieved remarkable data-efficiency, and raise hopes that applications of RL in robotics are within reach. For problems in continuous control, recent results that additionally employ learned models of the environment have promised further data efficiency gains (Byravan et al., 2019; Hafner et al., 2019a). Analogously, in domains with discrete actions, the combination of model based search, e.g. Monte Carlo Tree Search (MCTS), with RL has recently been shown to be a powerful approach (Silver et al., 2017; Schrittwieser et al., 2019; Anthony et al., 2017).

Compared to a more simplistic policy gradient or actor-critic method all of these model-based techniques have one thing in common: they use additional computation (by performing a search or gradient based optimization) either at the time of action selection or in the policy improvement step. From this observation three questions naturally arise: 1) If data-efficiency can be improved via additional computation how should this computation be spent (i.e. can an off-policy algorithm be as efficient as a model-based algorithm by performing more policy updates)? 2) If a model of the environment is available, can we conclude that additional search during action selection is beneficial? 3) Can model-based search and RL be cast into a consistent framework applicable to domains with continuous actions?

In this paper we make an attempt to understand these questions better. In particular we aim to understand how data efficiency and scalability of algorithms for continuous control can be influenced through the use of additional compute during acting or learning. We build on a class of KL-regularized policy iteration schemes (Rawlik et al., 2012; Abdolmaleki et al., 2018) that separate acting, policy improvement, and learning and thus allow us to flexibly employ parametric policies, value functions, exact or approximate environment models, and search based methods in combination. In this framework we instantiate variants that differ in their reliance on learned approximations (e.g. for Q-function / policy / model) and how they use compute. Among these, our framework allows us to instantiate an off-policy model-free variant, that recovers the MPO algorithm (Abdolmaleki et al., 2018), as well as a new algorithm dubbed TreePI .

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This approach can utilize additional compute by searching with an (potentially approximate) environment model – in a way that is akin to MCTS for decision making in discrete domains. We find that it works robustly with exact and approximate environment models even in high-dimensions.

Using our approach we make a number of observations regarding the questions 1)-3) posed above:

1. The literature on model-free algorithms in continuous control has underestimated their data efficiency. Through additional updates of the policy and value-function (additional compute), we can achieve significant improvements.

2. At the expense of additional compute a learned predictive model (of rewards and values) can be used during learning for model-based policy optimization; providing a stronger policy improvement operator than a model-free algorithm.

3. The approximate model can also be used for local policy improvement at action selection time. This leads to better decision making which consequently improves the data that is collected. Compared to model use during learning, any model bias does not directly affect the parametric policy and the approach provides a further advantage in data efficiency.

Finally, when an accurate environment model is available, the approach provides a further advantage in data efficiency. Model bias does not directly affect the parametric policy and the latter can converge to the optimum of the expected reward objective (e.g. Rawlik et al., 2012; Abdolmaleki et al., 2018) and are also referred to as EM policy search algorithms (Deisenroth et al., 2013).

Here, we consider the alternating optimization of $J$ wrt. $\mu$ and $\pi$. The optimization occurs iteratively, repeating the following two steps (referred to as E-and M-step respectively, in analogy to the EM algorithm for statistical inference):

- **E-step** Optimization of $\mu^{(t)} = \arg \max_{\mu} J(\mu, \pi^{(t)})$ given a fixed $\pi^{(t)}$.
- **M-step** Optimization of $J$ with respect to $\pi$ given $\mu^{(t)}$; which amounts to minimizing the average KL.

3. KL regularized K-step Improvement

We make several observations about $J$ – from which we will then derive a model based policy optimization scheme:

A relation to the RL objective. When $\pi = \mu$ then $J(\pi, \pi) = \mathbb{E}_{T \sim \rho_\pi} \left[ \sum_t \gamma^t r_t \right]$, since the KL term vanishes.

Optimality conditions from existing work. Maximizing $J$ with respect to $\mu$ gives the optimal policy $\mu^*(a|s) \propto \pi(a|s) \exp(\mathbb{Q}^*_T(s,a)/\alpha)$ (see e.g. Rawlik et al., 2012; Fox et al., 2015). We thus have $J(\mu^*(\cdot|s), \pi) \geq J(\mu, \pi)$ $\forall \mu$.

It turns out that the following is also true: for any $\pi$ and $\mu_1(a|s) \propto \pi(a|s) \exp(\mathbb{Q}^*_T(s,a)/\alpha)$ we have $J(\mu_1, \pi) \geq J(\pi, \pi)$ (see e.g. Abdolmaleki et al., 2018). We can think of $\mu_1$ as picking the action that is soft-optimal under the assumption that at the next time step we resort to acting according to the current policy $\pi$. $\mu_1$ thus amounts to a local improvement over $\pi$, and we refer to it below as one-step improved. This insight is exploited e.g. in the algorithm of Abdolmaleki et al. (2018), which iterates three steps (1) estimating $Q^*_T$, (2) optimizing for $\mu_1$, and (3) moving $\pi$ towards $\mu_1$; corresponding to a policy iteration scheme that repeatedly performs one-step improvements of the policy.

From one-step to K-step policy improvement. Going beyond one-step improvements we can consider a sequence of $K$ local policies $\mu_{1:K} = \{\mu_{K}(\cdot|s_1), \ldots, \mu_{1}(\cdot|s_K)\}$ that are individually optimized based on the result of the following
nested K-step optimization adapted from Equation (1):
$$\mu_{1:K}^*(a|s) = \arg \max_{\mu_{1:K}} J^K(\mu_{1:K}, \pi)$$
$$= \arg \max_{\mu_{1:K}} \mathbb{E}_{\mu_{1:K}} \left[ \sum_{t=1}^{K} \gamma^{t-1} (r_t - \alpha KL t) \right]$$
where \(s_1 = s, a_1 = a\), and using the short-hand notation \(r_t = r(s_t, a_t)\), \(KL_t = KL(\mu(\cdot|s_{t+1}) \| \pi(\cdot|s_t))\). The solution to this optimization corresponds to the policy that acts soft-optimally for the next \(K\) steps, and subsequently resorts to acting according to \(\pi\); and it bears resemblance to the \(K\)-step greedy policy defined in (Efroni et al., 2018) albeit for a regularized objective. Here, too, we find that \(J(\mu_{1:K}^*, \pi) \geq J(\pi, \pi) \forall \pi\), since each local policy in \(\mu_{1:K}^*\) improves on \(\pi\) by at least as much as the one-step improvement.

**Observation.** We can express \(\mu_{1:K}^*\) recursively via a K-step soft-optimal Q-function (see supplementary for details):
$$\mu_K^*(a|s) \propto \pi(a|s) \exp \left( \frac{Q_{\pi}^{K-1}(s, a)}{\alpha} \right)$$
$$Q_{\pi}^{K}(s, a) = \begin{cases} r(s) + \gamma \mathbb{E}_{s'\sim p_{\pi}} \left[ V_{\pi}^{K-1}(s') \right] & \text{if } k > 1 \\ r(s) + \gamma \mathbb{E}_{s'\sim p_{\pi}} \left[ V_{\pi}^{K}(s') \right] & \text{else} \end{cases}$$
$$V_{\pi}^{K}(s) = \alpha \log \sum_{a} \pi(a|s) \exp \left( \frac{Q_{\pi}^{K}(s, a)}{\alpha} \right) da.$$  
(3)

In addition, if all \(\{Q_{\pi}^{k}\}_{k=1}^{K}\) are given then it is also easy to sample approximately from an estimate \(\mu_K \approx \mu_K^*\), e.g. via the following self-normalized importance sampling scheme:
$$\alpha \sim \mu_K(\cdot|s_t) = \text{Categorical}(a^1 : M; w_1^M / \sum_{i=1}^M w_i^M),$$
$$w_i^M = \{\exp(Q_{\pi}^{k}(s, a^j)/\alpha)\}_{j=1}^M,$$
$$a^1 : M = \{a^1, \ldots, a^M\} \sim \pi(\cdot|s).$$
(4)

Thus, to sample from \(\mu_K^*(\cdot|s)\) we simply sample \(M\) actions from \(\pi(\cdot|s)\) and importance weight with the exponentiated soft-Q values \(Q_{\pi}^{k}(s, a)\), which is possible both for discrete and continuous actions without discretization.

A number of different schemes can be used to estimate \(\mu_{1:K}\). If \(\mu\) is parametric we can directly follow the gradient of \(J(K(\mu_{1:K}, \pi))\) with respect to \(\mu_{1:K}\) resulting in a K-step KL regularized policy gradient (related to e.g. Schulman et al., 2017a; Haarnoja et al., 2018; Bhardwaj et al., 2020), as described in more detail in the supplementary. Alternatively we can attempt to directly estimate \(Q_{\pi}^{K}\) using the recursive form of Equation (3), from which \(\mu_K^*\) follows immediately as above. This choice allows us to instantiate a family of policy iteration schemes in which we alternate between estimating \(\mu_{1:K}\) for the current \(\pi\) and updating \(\pi\) towards \(\mu_K^*\):  
**i) perform an E-Step** by locally estimating \(\mu_{1:K}^{(i)} \approx \mu_{1:K}^*\) for reference policy \(\pi = \pi^{(i)}\) according to the recursion in Eq. (3);  
**ii) perform an M-Step** by minimizing \(\min_{\pi} J^K(\mu_{1:K}^{(i)}, \pi)\) which amounts to fitting \(\pi^{(i+1)}\) to samples from \(\mu_K^{(i)}\):
$$\pi^{(i+1)} = \arg \min_{\pi} \mathbb{E}_{s \sim p_{\pi}} [KL(\mu_K^{(i)}(\cdot|s) \| \pi(\cdot|s))].$$
(5)
and we fit a parametric approximation to \(Q_{\pi}^{(i+1)}\) for bootstrapping in the next E-step.

For such a policy iteration scheme to be effective, a lightweight estimate of \(\mu_K^*\) is desirable. In a model free setting this is largely impractical for \(K > 1\) since a Monte-Carlo estimator of Eq. (3) may be very hard to construct (it would at least require privileged access to the environment; i.e. the ability to reset to any given state to perform the required K-step rollouts). For \(K = 1\) we recover the method from Abdolmaleki et al. (2018) which uses a parametric \(Q_{\pi}^*\) to construct a Monte Carlo estimate of \(\mu_{1}\) in the E-step.

In the case where a model of the environment is available, or can be learned, sample based estimation of \(Q_{\pi}^{K}(s, a)\) from Eq. (3) becomes practical – as we will show below. The only caveat in this case is that the naive approach would require the evaluation of a full tree of trajectories (up to depth \(K\)).

### 4. Policy improvement with Tree-PI

In this section we propose an algorithm for approximately sampling from \(\mu_K^*\) that removes the impractical requirement of building a full tree up to depth \(K\).

We assume deterministic system dynamics and the availability of a model \(s_{t+1} = f(s_t, a_t)\). In this case we can obtain a particle approximation to \(\mu_K^*\) using a form of self-normalized importance sampling that bears similarity to MCTS. We first describe the procedure in its exact form and then a practical, approximate algorithm.

**Unbiased solution.** For brevity of notation, and without loss of generality, let \(\alpha = 1\). For deterministic systems we can estimate \(\exp Q_{\pi}^{K}(s, a)\) by building a tree, of depth \(K\) in the following way: starting from \(s\) we recursively sample \(M\) actions per state according to \(\pi\), i.e. \(a_j \sim \pi(\cdot|s)\), up to a depth of \(K\) steps. Arrows are labeled with actions. Leaves are set to \(Q_{\pi}^0(s, a)\). We can then compute an approximation to \(\exp Q_{\pi}^{K}(s, a)\) recursively using the approximate value function \(\hat{V}^{d+1}(s) = \log \frac{1}{M} \sum_{j=1}^M \exp(\epsilon(f(s, a_j)) + \gamma \hat{V}(f(s, a_j))\) starting from the leaves \(\hat{V}^1(s) = \frac{1}{M} \sum_{j=1}^M \exp(Q_{\pi}(s, a_j))\). Then \(\hat{Q}_{\pi}^{K}(s, a) = r(s) + \gamma \hat{V}^{K-1}(f(s, a))\). It is easy to see that \(\mathbb{E}[\exp Q_{\pi}^{K}(s, a)] = \exp Q_{\pi}^{K}(s, a),\) i.e. \(\exp Q_{\pi}^{K}(s, a)\) is an unbiased estimate of \(\exp Q_{\pi}^{K}(s, a)\). We can then sample from \(\hat{\mu}_K\) as described in eq. (4) using \(\hat{Q}^{K}\) instead of \(Q_{\pi}^{K}\).

**Practical algorithm.** Building the full tree of depth \(K\), with branching factor \(M\), would be computationally ex-
5. Learning parametric estimators

In the general RL setting it is not always reasonable to assume knowledge of the environment model \( f \). For the K-step policy improvement operator from Section 4 to be applicable in these settings a model has to be learned jointly to recent work (Schrittwieser et al., 2019); although we here learn Q-predictions in an off-policy manner. We focus on assessing the difference between search-based policy improvement and model-free learning only. We hence do not aim to model future observations to prevent corroboration of our experimental results with issues due to modelling high-dimensional physical systems (e.g. predicting the dynamics of a humanoid walking). On the other hand, it is possible that in some domains learning an observation model could lead to further improvement; both for the model-free and search based instances, as argued in some recent works (Lee et al., 2019; Byravan et al., 2019; Hafner et al., 2019b).

For training, we assume access to a replay buffer containing trajectory snippets, \( \mathcal{B} = \{\tau_1^{1:M}, \ldots, \tau_M^{1:M}\} \), of observations and actions, \( \tau_t = (s_t, a_t, r_t, s_{t+1}) \). We denote by \( s_t f_{\phi_{\text{enc}}}(o_t) \) a parameterized encoding function, mapping from observations to a “latent” state and by \( f_{\phi_{\text{tran}}} = (s_t, a_t) \) a learned transition model. Let \( s_t = f_{\phi_{\text{tran}}}(o_t, a_{1:t-1}) \) be the prediction of the state at time \( t \) after encoding observation \( o_t \) and applying the approximate transition model, conditioned on observed actions, for \( t \) time-steps. We train a predictive model of rewards (loss \( \mathcal{L}_r \)), Q-values (with loss \( \mathcal{L}_Q \)) and actions (with policy loss \( \mathcal{L}_{\pi_\theta} \)). This amounts to finding the minimum of the combined loss:

\[
\mathcal{L} = \mathbb{E}_{\tau \sim \mathcal{B}} \left[ \sum_{t=1}^T \mathcal{L}_r + \mathcal{L}_Q + \mathcal{L}_{\pi_\theta} \mid s_t = f_{\phi}(o_t, a_{1:t-1}) \right],
\]

\[
\mathcal{L}_r = \left( r_t - \phi_r(s_t) \right)^2,
\]

\[
\mathcal{L}_Q = \left( r_t + \gamma \mathbb{E}_{\pi(\cdot)}[\hat{Q}_{\pi_\theta}^\tau(s_t, \cdot; \phi_Q)] - \hat{Q}_{\pi_\theta}^\tau(s_t, a_t; \phi_Q) \right)^2,
\]

\[
\mathcal{L}_{\pi_\theta} = \text{KL}[\hat{\mu}^K(\cdot|s_t) \| \pi_\theta(\cdot|s_t)] + R(\pi_\theta, \pi(\cdot), s_t)
\]

\[
\approx \sum_{j=1}^M - \log \pi_\theta(a_t^j|s_t) w_j^t + R(\pi_\theta, \pi(\cdot), s_t)
\]

where \( a_t^{1:M}, w_t^{1:M} = \text{TreePI}(s_t, \pi(\cdot), \hat{Q}_{\pi(\cdot)}^\tau, M, N, K, \alpha) \),

(6)

with \( \phi_r, \phi_Q, \theta \) in the above denoting the parameters of the learned reward predictor, Q-value function and policy re-
spective. Note that the above slightly abuses notation, using $s_i$ to refer to the latent states, to simplify the presentation; and we assume $s_i = f_{enc}(o_1)$. We use a target network (with parameters $\phi_Q^i$ that are copied every periodically from $\phi_Q$) to stabilize the TD loss $L_Q$. Further, $R$ denotes an additional regularization term that prevents over-fitting of the policy $\pi_\theta$ to the M samples. This can be crucial for avoiding collapse of the policy distribution in the low sample and small temperature regime (i.e. small $M = 20$, small $\alpha$) that we typically find ourselves in. While the form of regularization can be chosen freely, recent work in RL has found KL regularization via a trust-region constraint to be particularly effective (Schulman et al., 2015; Abdolmaleki et al., 2018). We thus let $R(\pi_\theta, \pi_{(i)}(s_i), s_i) = \eta [KL(\pi_{(i)}(\cdot|s_i)\|\pi_\theta(\cdot|s_i)) - \epsilon_{KL}]$, set $\epsilon_{KL} = 0.005$ to a small value and optimize for the Lagrangian multiplier $\eta$ together with other parameters via gradient descent / ascent, solving for: $\arg\max_\eta \min_\pi_\theta L$. Finally, after 500 steps we set $\pi_{(i+1)} = \pi_\theta$ and start the next round of optimization – i.e. we perform partial optimization for computational efficiency. A listing of the procedure is given in Algorithm 1 in the supplementary material.

6. Experiments

To understand when and how TreePI can improve performance we consider several experimental scenarios. We start with experiments on the DeepMind control suite (Tassa et al., 2018) which explore the utility of our algorithm in combination with a learned model as described in Section 5. We focus on data efficiency and compare to several strong off-policy baselines for learning from scratch.

In a second set of experiments, we attempt to disentangle the effects of search from the problem of learning predictive models. We assume access to the true model $f$ of the environment; inserting true states in the calculation of Eq. 6 and replacing the learned reward $r_{\phi_r}$ with the true reward function $r$. The focus here is on the best use of a fixed compute budget to achieve fast learning.

6.1. Control Suite with a Learned Predictive Model

Experimental Setup We experiment with four domains from the DeepMind control suite: i) the ‘Cartpole (swingup)’ task, where the control is one-dimensional and the goal is to swing-up and balance a pole attached to a cart, ii) the ‘Finger (hard)’ task where the goal is to spin a rotating object to a target orientation with a two-joint robot, iii) the ‘Humanoid (stand)’ task where a humanoid with 21 degrees of freedom should stand up from a randomly initialized position, iv) The ‘Walker-2d (run)’ task where a simpler, two-dimensional, body should run as fast as possible.

Model setup and Baselines We use feed-forward neural networks to represent the different components $f_{\phi_{loc}}, f_{\theta_{run}}, r_{\phi_r}, Q_{\phi_Q}, \pi_\theta$ in TreePI. Full details on the network setup and hyperparameters are given in the supplementary material. The reward, Q and policy networks $r_{\phi_r}, Q_{\phi_Q}, \pi_\theta$ operate on the shared representation obtained from $f_{\phi_{loc}}$ and $f_{\theta_{run}}$; and we use a network predicting mean and variance of a Gaussian distribution as the policy $\pi_\theta$. The hyperparameters for all methods were tuned based on initial experiments on the Walker-2D domain and then fixed for the remaining experiments. If not otherwise noted we set $M = 20$, $K = 10$ and $N = 200$ in the experiments for TreePI. We use two versions of TreePI : (a) we execute the current policy $\pi$ at action selection time (and hence only perform search when updating the policy) denoted by TreePI/$\pi$; and (b) at action selection time, we perform an additional search to draw a sample from $\mu_{K}$ for the current state (cf. Eq (4)), denoted by TreePI/search.

We consider two sets of off-policy RL baselines to disentangle improvements due to the network (and additional predictive losses) from improvements due to the policy optimization procedure itself. First, we compare to two state-of-the-art RL algorithms: MPO (Abdolmaleki et al., 2018) and SAC (Haarnoja et al., 2018). We align their network architecture with the TreePI setup to the extent possible (using separate policy and Q-function networks; see the

![Figure 1. Comparison of TreePI (with depth K=10 and executing the search policy during at decision time) against state-of-the-art algorithms on control suite domains. Search improves slightly over MPO with 10 updates per step and a predictive model. MPO without predictive model loss – as well as SAC – is less data efficient. We plot the median performance (over 5 runs). All algorithms reach similar performance at convergence (overlapping stars denote median after 20k episodes).](image-url)
Results The main results are depicted in Figure 1. First, we observe that simply adding a predictive model of rewards and values to a standard off-policy learner (MPO+Predictive) results in improved data-efficiency in the Cartpole and Finger domain when compared to well tuned versions of state-of-the-art agents (SAC, MPO). Second, TreePI with $K = 10$ (TreePI(10)/Search) results in improved data-efficiency in three of the four domains; remarkably achieving near optimal performance in all domains in less than 2000 episodes.

To analyze this improvement further, we test how much TreePi gains from the model-based search and how much from the fact that it uses additional compute (via model rollouts). To isolate these effects, we varied the number of Policy and Q-function updates performed per environment step for different methods. Results are presented in Figure 2, where we plot the performance against updates per environment step; displayed at a specific time during training (150 episodes). In addition we also plot the performance for TreePi(10)/π – i.e. using TreePi but executing π, thus no search during action selection. All methods gain in data-efficiency as the number of updates – and thus the use of compute – grows up to 10 (see e.g. Popov et al., 2017 for related results). Additional updates result in premature convergence of the policy (overfitting), reducing performance – and also resulting in sub-optimal behavior at convergence; not shown in the plot. Interestingly, MPO+Predictive gains in data-efficiency at a larger rate with more updates, almost catching up to TreePi(10)/π at 10 updates per step. To better appreciate this result we also plot MPO+Predictive in Figure 1 which obtains performance much closer to TreePi.

In addition, running TreePi with a depth of $K = 1$ and executing π (TreePi(1)/π Figure 1) recovers MPO+Predictive as expected (except for differences in setting $\alpha$, see supplementary). Thus, even though some of the improvement can be attributed to TreePi being a better policy improvement operator – it is consistently better than MPO at lower numbers of updates – spending more compute on policy updates can partially alleviate this difference. Some of the data-efficiency gains attributed to model-based methods in the literature may thus be obtained in a model-free setting: a result similar to recent observations for RL in discrete domains (van Hasselt et al., 2019).

Nonetheless, additional search at action selection time

6.2. Humanoid Domains with a Known Model

For the experiments with a known environment model and reward we consider more challenging domains and attempt to answer the question: can TreePI help us to allocate computational budget more efficiently to ‘solve’ them faster?
Experimental setup We consider three difficult, high-dimensional domains, using humanoid bodies. First, we perform experiments with the ‘Humanoid (run)’ task from the Control suite (see above). Second, we consider the problem of reproducing highly agile motion-capture data with the the complex humanoid body of Merel et al. (2019). Deep RL approaches are currently popular for producing controllers that track motion-capture data, but the training process can be quite slow (e.g. Peng et al., 2018). Earlier research has used model-based search to control humanoids (Tassa et al., 2012; Hämäläinen et al., 2014) and match reference movements (Liu et al., 2010). Our approach is able to interpolate between model-free deep RL and sample-based planning. Concretely, we evaluate a setup similar to Peng et al. (2018); Merel et al. (2019) and train a policy to reproduce a cartwheel and backflip motion from the CMU Motion Capture Database1 (see the supplement for a full description of the task). We note that this task is non-trivial since the simulation is physically accurate and the humanoid’s actuation, weight and size are different from the people that executed the recorded motion (in fact, exact replication of the reference motion may not be possible).

We use a fast, distributed, implementation of TreePI (with the search written in C++) and compare to high-performance distributed model-free RL algorithms. We use a distributed actor-learner setup analogous to Espeholt et al. (2018). To keep the comparison as fair as possible, we restrict TreePI to 16 asynchronous actors using 32 threads for expanding the tree. We use up to 6000 actors for the RL algorithms, at which point they send 100x more data back to the learner and perform 10-50x more total environment interactions than TreePI (counting all transitions within the search) at 10x the compute cost. We set the branching factor to $M = 50$, and the depth to $K = 10$ and experiment with varying $N$ for TreePI, $\alpha = 0.1$ was used throughout. We use both an off-policy algorithm MPO as well as an on-policy algorithm PPO (Schulman et al., 2017b) as baselines. We keep network architectures as similar as possible. Full details of the experimental setup are given in the supplementary material. For TreePI search on the learner could become a computational bottleneck. Fortunately, with a larger number of actors that all perform search to choose high-quality actions, this data is of sufficient quality so that we can simply maintain a small replay buffer (containing 100k time-steps) and train the policy to match actors action choices directly. We thus adjust policy learning to simply maximize the likelihood of actions sampled from this buffer. That is, we change the policy loss in Equation (6) to $L_{\pi^0} = \log \pi(a_t | s_t) + R(\pi^0, \pi^{(i)}, s_t)$ for $a_t, s_t \in \tau$. The rest of the losses are kept unchanged.

Results The performance in terms of wall-clock time after 3h/6h and 48h is presented in Figure 4. We can observe that TreePI in combination with the true environment model results in significantly faster convergence than any of the model-free algorithms. In addition, as we vary $N$ (the number of rollouts) we observe that for few rollouts TreePI behaves similar to a standard RL algorithm. As $N$ increases the tree search starts to find significantly improved policies, resulting in faster convergence. Interestingly, the RL methods (which do not make use of an environment model) fail to reach a similar speed-up even when thousands of actors are used. We speculate that for RL algorithms with an increasing number of actors the policy improvement step (which happens only on the learner) becomes a bottleneck. In contrast, TreePI off-loads this step to the actors. With knowledge of the true model actors can locally improve the policy and produce high-quality actions to be consumed by the learner. Note that at the end of learning the high-performing policy will have been distilled into a network and can be executed without reliance on the model. The training speed-up can thus be interesting in situations where a simulation of the environment is available at training time.

7. Discussion and Related work

Policy optimization schemes based on the KL regularized objective have a long history in the RL and optimal control literature – see e.g. Kappen (2005); Toussaint & Storkey (2006); Todorov (2007); Rawlik et al. (2012) for different perspectives on this objective. A number of different approaches have been considered for its optimization. These include both policy iteration schemes similar to the ones considered in this paper (e.g. Toussaint & Storkey, 2006; Rawlik et al., 2012; Montgomery & Levine, 2016; Levine & Koltun, 2013; Theodorou et al., 2010; Abdolmaleki et al., 2018) as well as algorithms that optimize the regularized objective that we consider in the E-step (e.g. Ziebart, 2010; Haarnoja et al., 2017; Schulman et al., 2017a; Hausman et al., 2018; Fox et al., 2015; Haarnoja et al., 2018), often via some form of regularized policy gradient. Recently, algorithms in this class have regained considerable attention, including both model-free (e.g. Schulman et al., 2017a; Hausman et al., 2018; Fox et al., 2016; Haarnoja et al., 2018; Maddison et al., 2017; Abdolmaleki et al., 2018) and model-
| Algorithm / Task       | Humanoid (run) | Cartwheel | Backflip |
|-----------------------|----------------|-----------|----------|
| TreePI (2k roll.)     | 786.5/840.7    | 48.7 / 61.2 / 61.2 | 40.2 / 54.6 / 58.9 |
| TreePI (500 roll.)    | 756.8/839.7    | 32.9/49.2/60.5   | 29.7/42.8/58.2   |
| TreePI (10 roll.)     | 374.7/843.5    | 13.8/30.3/58.7   | 11.2/29.6/56.4   |
| MPO (64 actors)       | 374.6/837.3    | 7.8/15.3/54.3    | 9.1/18.7/49.7    |
| MPO (6k actors)       | 324.9/842.8    | 9.4/23.6/60.3    | 12.6/27.4/57.7   |
| PPO (6k actors)       | 526.3/839.2    | 6.3/28.4/59.9    | 14.7/29.1/58.4   |

Figure 4. Left: Performance wrt. wall-clock time for solving the three high-dimensional humanoid control tasks (average over 20 test episodes). TreePI results in significantly faster convergence. Statistically equivalent results marked in bold. Right: Comparison of TreePI and MPO for the Cartwheel task both in terms of wall-clock time. A clear advantage for the search based method can be observed.

Local Search for Policy Iteration in Continuous Control

Among model-based control for high-dimensional continuous action spaces work by Levine & Koltun (2013; Montgomery & Levine, 2016; Piché et al., 2019) approaches. In contrast to our work, however, most model-based approaches make use of local dynamics models combined with known reward functions.

Much remains to be done: we have only sampled a small number of design choices within the presented framework, and we expect that the benefits we have observed might transfer to related algorithms. We hope that the perspective of this work will inspire others to investigate other algorithms that flexibly blend the use of model-based and model-free approaches.
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We provide a derivation of the K-step optimal policy from the right hand side wrt. with which we finally obtain

\[
\mu^*_K(a|s) = \pi(a|s) \exp \left( \frac{Q^*_\pi(s, a)}{\alpha} \right) / Z
\]
with \( Z = \int \pi(a|s) \exp\left( Q^\pi_\pi(s, a)/\alpha \right) \) which gives the desired result for \( K = 1 \).

Further, expanding the definition of the value function and inserting \( \mu^*_1(s) \) we obtain

\[
V^\pi_1(s_t) = \mathbb{E}_{\mu^*_1} \left[ r_t - \alpha KL_t + \gamma \mathbb{E}_{s' \sim p(s', s, a)}[V^\pi_1(s')] \right]
\]

\[
= \mathbb{E}_{\alpha \sim \mu^*_1} \left[ r_t - \alpha \log \frac{\mu^*_1(a|s)}{\pi(a|s)} + \gamma \mathbb{E}_{s' \sim p(s', s, a)}[V^\pi_1(s')] \right]
\]

\[
= \mathbb{E}_{\alpha \sim \mu^*_1} \left[ Q^\pi_1(s, a) - \alpha \log \frac{\pi(a|s) \exp(Q^\pi_1(s, a)/\alpha)}{\pi(a|s)} / Z \right]
\]

\[
= \mathbb{E}_{\alpha \sim \mu^*_1} \left[ \alpha \log Z \right]
\]

\[
= \alpha \log \int \pi(a|s) \exp\left( Q^\pi_1(s, a)/\alpha \right) da,
\]

where we have used \( \mathbb{E}_{\mu^*_1}[c] = c \) in the last step. This proves the expected result.

**Induction from \( K = 1 \) to \( K \).** For any \( K > 1 \) assuming that \( \mu^*_K \) it is easy to see that we can find \( \mu^*_K \) via the maximization

\[
\mu^*_K = \arg \max_{\mu^*_K} J^K(\mu^*_K, \pi)
\]

\[
= \arg \max_{\mu^*_K} \mathbb{E}_{\mu^*_K} \left[ r_t + \gamma \mathbb{E}_{s' \sim p(s', s, a)}[V^\pi_{K-1}(s')] - \alpha KL_t \right],
\]

as per Observation (3). We obtain \( \mu^*_K(a|s) = \pi(a|s) \exp(Q^\pi_{K-1}(s, a)/\alpha)/Z \) via steps analogous to Equations (8)-(11). From this the definition of \( \hat{V}^\pi_\pi(s) \) then immediately follows analogous to Equation (13).

**A.2. Derivation of tree estimator**

We give a more detailed explanation of the TreePI estimator (without approximations) from Section 4 in the main paper.

Recall that we are interested in a procedure for approximately sampling from \( \mu^*_K \). We assume deterministic system dynamics and the availability of a model \( s_{t+1} = f(s_t, a_t) \).

To sample from \( \mu^*_K(a|s) \propto \pi(a|s) \exp(Q^\pi_{K-1}(s, a)/\alpha) \) we need an estimate of \( \exp(Q^\pi_{K-1}(s, a)/\alpha) \). We here show how to obtain such an estimate by building a fully populated tree, of depth \( K \).

**Definition.** Let \( \hat{Q}^K(s, a) \) be the tree estimate build as follows – assuming \( \alpha = 1 \) for brevity of notation. Starting from \( s \) we recursively sample \( M \) actions per state according to \( \pi, \) i.e. \( a_j \sim \pi(\cdot|s) \), up to a depth of \( K \) steps. Arrows are labeled with actions. Leaves are set to \( Q^\pi_0(s, a) \). We can then compute an approximation to \( \exp(Q^\pi_{K-1}(s, a)) \) recursively using the approximate value function \( \hat{V}^{d+1}(s) = \log \frac{1}{M} \sum_{j=1}^M \exp(r(f(s, a_j)) + \gamma \hat{V}(f(s, a_j)) \) starting from the leaves \( \hat{V}^1(s) = \frac{1}{M} \sum_{j=1}^M \exp(Q^\pi_0(s, a_j)) \).

To prove that this results in the required estimate let us first simplify the derivation: we consider the case of trajectories with finite horizon \( |\tau| = T \) and set \( \gamma = 1 \). We then seek to prove the following proposition

**Observation.** For \( \gamma = 1 \) and \( \alpha = 1 \) and deterministic system dynamics the M-sample estimate \( \hat{Q}^K(s, a) \) is unbiased. That is \( \exp(r(s) + \mathbb{E}_{\pi} V^\pi_{K-1}(s_{t+1})) = \mathbb{E}_{\pi}[\exp(\hat{Q}^K(s, a))] = \exp(Q^\pi_{K-1}(s, a)) \).

We start by expanding the definition of \( \exp(Q^\pi_{K-1}(s, a)) \), making use of the fact that we consider purely deterministic dynamics of the form \( s_{t+1} = f(s_t, a_t) \) and assuming a trajectory of length \( T \):

\[
\exp(Q^\pi_{K-1}(s_t, a_t) = \exp \left( r(s_t) + \int \pi(a|s_{t+1}) \exp(Q^\pi_{K-1}(s_{t+1}, a_{t+1})) da_{t+1} \right)
\]

\[
= \mathbb{E}_{\pi(a|s_{t+1})} \left[ \exp(r(s_t)) \exp(Q^\pi_{K-1}(s_{t+1}, a_{t+1})) \right]
\]

\[
= \mathbb{E}_{\pi(a|s_{t+1})} \left[ \exp(r(s_t)) \exp \left( \frac{1}{M} \sum_{j=1}^M \exp(r(f(s_{t+1}, a_{t+1})) \right) \right]
\]

\[
= \mathbb{E}_{\pi(a|s_{t+1})} \left[ \exp(r(s_t)) \cdot \exp(\hat{V}^1(s_{t+1})) \right]
\]

\[
= \mathbb{E}_{\pi(a|s_{t+1})} \left[ \exp(r(s_t)) \cdot \exp(\hat{V}^1(s_{t+1})) \right]
\]

\[
\cdots
\]

\[
\mathbb{E}_{\pi(a|s_{t+1})} \left[ \exp(r(s_t)) \cdot \exp(\hat{V}^1(s_{t+1})) \right]
\]

\[
\mathbb{E}_{\pi(a|s_{t+1})} \left[ \exp(r(s_t)) \cdot \exp(\hat{V}^1(s_{t+1})) \right]
\]

where we recursively expanded the expectation, moving terms outside where possible – which is possible as the exponential cancels with the logarithm due to the determinism of the environment. Any Monte Carlo approximation to the last line here is easily verified to be an unbiased estimator. This includes our tree based estimate \( \exp(Q^\pi_{K-1}(s, a)) \) which recursively samples the inner integrals for each state. We note that a corresponding estimate of \( V^\pi_{K-1} \) would be biased due to the log in front of the expectation.

It should be noted that this analysis hinges on the fact that \( f \) is deterministic and that \( \gamma = 1 \) (as is the case
Algorithm 2 K-Step Regularized Policy Iteration

Input: number of updates per step $updates\_per\_step$, updates per iteration $target\_rate$

Initialize: $\pi_0, Q^0_{\pi_0}, (r_{\phi_1}, f \approx f_\phi$ if needed), $updates = 0, i = 0$

repeat
  reset $s_t$ to start episode
  while not episode ended do
    // sample action with search or $\pi$
    $a \sim \pi^{(i)}(\cdot|s_t)$ or $a \sim $ TreePI (Algorithm 1 in paper)
    
    // collect next transition
    $s_{t+1} = f(s, a)$, add $(s, a, s_{t+1})$ to buffer $B$
    for $updates\_per\_step$ do
      $s_t = s_{t+1}$
      sample $\tau \in B$
      update $\tilde{Q}^\pi_{\rho_\theta}, r_{\phi_1}, f_\phi, \pi_\theta$
      if $updates \mod target\_rate$ equal 0 then
        $\pi^{(i+1)} \leftarrow \pi_0, \tilde{Q}^{\pi^{(i+1)}}_{\pi^{(i+1)}} \leftarrow \tilde{Q}^\pi_{\rho_\theta}$
        $i = i + 1$
      end if
    end for
    $updates += num\_update\_steps$
  end while

until convergence

for e.g. the humanoid motion capture tracking environments we consider for infinite horizons and thus $\gamma < 1$ we would not obtain an unbiased estimate. In that case we would instead obtain a pessimistic estimate due to the relation $\exp(\gamma \log E_{a \sim \pi}[\exp(f(a))]) \geq E_{a \sim \pi}[(\exp(f(a))]^\gamma$ for $0 < \gamma \leq 1$.

A.3. Policy iteration procedure

A full listing of the policy iteration procedure is given in Algorithm 2.

B. Additional details on the model setup

B.1. Hyperparameters for TreePI

We use fully connected neural networks with the following structure: If the model is learned then the state-encoder $f_{\phi_{\text{enc}}}$ consist of two fully-connected layers with 256 units each with exponentiated linear (elu) activation functions (Clevert et al., 2016), layer normalization is used in the first layer of the network (which we find useful for dealing with different observation scales across domains). The transition model similarly contains two layers of 256 units and predicts a delta that is added to the previous state (a standard choice that simplifies the learning problem). The latent state size is assumed to be 256. To perform forward prediction we feed the transition model with both the encoder state (or previous predicted state) as well as the action – which simply concatenate to the latent features.

The reward, Q and policy networks operate either on the ground truth states (when a model of the system is available, i.e. in the humanoid experiments from Section 6.2) or on the shared representation obtained from $f_{\phi_{\text{enc}}}$ and $f_{\phi_{\text{trans}}}$. The networks each contain three layers of 256 units each (with layer normalization after the first layer); followed by a linear layer outputting scalar values for $r_{\phi_1}, \tilde{Q}_{\phi_Q}$ and two layers for prediction policy mean $\mu_\theta(x)$ and log variance $\varsigma_\theta(x)$ of the policy for $\pi_\theta(a|s) = \mathcal{N}(\mu_\theta(x), \text{softplus}(\varsigma_\theta(x)))$ respectively. For the Q-function the action is concatenated to the other inputs.

We tuned other hyperparameters such as $M$ and $\alpha$ on the walker domain and then fixed them for the other experiments. All parameters are listed in Table 1. The algorithm was not overly sensitive to the exact settings of the parameters as long as they were in the right order of magnitude.

B.2. Hyperparameters for the baselines

For the baseline experiments we constructed policy and Q-function networks that are aligned with the above described network architecture. For MPO+Predictive we use the exact same network architecture as outlined in Table 1. For the other baselines (MPO, MPO, PPO) we construct a network without the predictive parts – that is otherwise similar to the TreePI setup. Concretely, we apply an encoder $f_{\phi_{\text{enc}}}$ to the provided observations, followed by networks predicting policy parameters and Q-function (or Value function in the case of PPO). The hyperparameters of individual algorithms are listed in Table 2.
In the distributed setting multiple actors were used to generate experience for the learner. Each actor has generates experience by performing an episode in the environment, data is sent to the learner after each executed transition (either filling a replay buffer or a queue (for PPO)). Parameters are then fetched from the learner every after 100 interactions with the environment.

### C. Additional details on the Policy Gradient Variant

As described in the main text, we performed an additional ablation where we replaced the search at action selection time with a policy gradient that temporarily changes the network parameters. We now derive the policy gradient used for this experiment. We start by recalling the K-step time with a policy gradient that temporarily changes the parameters of $\pi_\theta^{(i)}$. Defining the KL regularized K-step return as

$$R = \gamma^K V_\pi^\mu(s_{K+1}) + \sum_{t'=t}^K \gamma^{t'-1}(r_{t'} - \alpha KL_{t'})$$

and the corresponding return from time $t$ onwards as

$$R_{t:K} = \gamma^K V_\pi^\mu(s_{K+1}) + \sum_{t'=t}^K \gamma^{t'-1}(r_{t'} - \alpha KL_{t'})$$

we can then write the required derivative as

\[
\nabla_{\theta_\mu} J(\mu_{\theta_\mu}, \pi_\theta^{(i)}) = \nabla_{\theta_\mu} \mathbb{E}_{\pi_\theta^{(i)}} \left[ \gamma^K V_\pi^\mu(s_{K+1}) + \sum_{t=1}^K \gamma^{t-1}(r_t - \alpha KL_t) \right]
\]

\[
= \mathbb{E}_{\mu_{\theta_\mu}} \left[ \sum_{t=1}^K R \nabla_{\theta_\mu} \log \mu_{\theta_\mu}(a_t|s_t) \right]
\]

\[
= \mathbb{E}_{\mu_{\theta_\mu}} \left[ \sum_{t=1}^K R_{t:K} \nabla_{\theta_\mu} \log \mu_{\theta_\mu}(a_t|s_t) \right],
\]

where from the second to third line we have made use of the standard likelihood ratio policy gradient (Williams, 1992) and from the third to fourth line dropped terms not influenced by the decision at decision point $s_t$ (i.e. the action choice at $t$ does not influence the past rewards).

Equation 17 can be estimated by performing rollouts (against the learned model $f$). To change the policy parameters at state $s_t$ during policy execution we then perform 10 gradient steps (using Adam as the optimizer with learning rate 0.0005) estimating the gradient in each step based on 100 rollouts (of depth 10). After these 10 updates we sample an action from $\mu_{\theta_\mu}$ and reset its parameters to $\theta$.

This procedure effectively replaces the action sampling step in Algorithm 1.

### D. Additional details on relations to existing algorithms

As mentioned in the main paper, in the context of EM policy iteration schemes Abdolmaleki et al. (2018) implement the same estimate for $\mu$ as our algorithm if we set $K = 1$. In this setting the main difference is how $\alpha$ is set. While we treat $\alpha$ as a hyper-parameter, MPO starts from a hard-constraint (instead of the KL regularization we employ), performs Lagrangian relaxation to obtain a dual-function and then optimizes this dual for a target maximum KL ($\epsilon_{\text{E-step}} = 0.1$).

Additionally, if we keep $K > 1$ but drop the nested optimization wrt. $\mu_1, \mu_2, \ldots, \mu_{K-1}$ we obtain a different ap-

| Hyperparameters | All methods |
|-----------------|-------------|
| Encoder $f_{enc}$ | 256-LN-256 |
| Policy net | 256-LN-256-256 |
| Activation function | elu |
| Discount factor ($\gamma$) | 0.99 |
| **MPO** | |
| Q function network | 256-LN-256-256 |
| $\epsilon_{\text{E-step}}$ | 0.1 |
| $\epsilon_{\text{mean}}$ | 0.005 |
| $\epsilon_{\text{cov}}$ | 0.0001 |
| Adam learning rate | 0.0003 |
| Replay buffer size | 2000000 |
| Target network update period | 200 |
| Batch size | 256 |
| **SAC** | |
| Q function network | 256-LN-256-256 |
| Target entropy (per action dimension) | -0.5 |
| Adam learning rate | 0.0003 |
| Replay buffer size | 2000000 |
| Target network update period | 200 |
| Batch size | 512 |
| **PPO** | |
| Value function network | 256-LN-256-256 |
| Clipping $\epsilon$ | 0.2 |
| Adam learning rate | 0.0002 |
| Batch size | 512 |

Table 2. Hyperparameters for TreePI. LN corresponds to layer-normalization.
proximate solution to the optimal policy \( \mu^* \) given as
\[
\mu^*_K(a|s_t) \propto \pi(a|s_t) \exp \left( \gamma^K V_\mu^*(s_{t+K}) + \sum_{t'=1}^T \gamma^{t'-1} r_{t'} / \alpha \right),
\]
which corresponds to estimating the K-step optimal path at state \( s_t \) (as estimated via a K-step rollout following \( \pi \)) assuming that \( \pi \) is followed thereafter. This solution can be realized via a simple Monte Carlo estimator (to estimate the path costs in the exponential, for different actions at \( s_t \)) and can be related to the path-integral formulation for policy improvement (Kappen, 2005; Theodorou et al., 2010). Concretely, considering \( \mu^*_K \) corresponds to a path integral with \( \pi \) acting as the prior and (approximate) boot-strapping according to the value function \( V_\mu^* \). As mentioned, we can realize this improved policy by sampling paths simulated by following \( \pi \) and then re-weighting with the exponentiated return. An interesting variant would be to target the same density \( \mu^*_K \) with a more advanced sampler such as a Sequential Monte Carlo (SMC) method – which starts simulating \( M \) paths from \( s_t \) but re-samples after each step according to the exponentiated accumulated reward. Following such a procedure recovers the sampling employed by Piché et al. (2019).

Overall, Our policy \( \pi_{1:K}(a|s) \) from Eq. (3) can thus be seen as an extension of path-integral control where we consider tree-like branching in each state – up to depth \( K \) – rather than linear paths to estimate an improved policy.

E. Additional details on the humanoid Motion Capture domain

We give additional details on the experiments for reproducing motion capture data. In these experiments the observations consist of the humanoid’s proprioceptive state (given by joint positions and velocities) as well as the head position in Euclidean space and time (encoded via a phase variable that starts at 0 at the beginning of an episode and is 1 at the end of the episode).

The setup is thus similar to (Merel et al., 2019) and we use the same complex humanoid body. We note that, Deep RL approaches are currently popular for producing controllers that track motion-capture data but the training process can be quite slow (e.g. Peng et al., 2018). Earlier works have used model-based search to control humanoids (Hämäläinen et al., 2014) and match reference movements (Liu et al., 2010).

We use a time-varying reward that measures the discrepancy between the body-pose of the humanoid and the a reference pose from a motion capture clip. We and train a policy to reproduce a cartwheel and backflip motion extracted from the CMU Motion Capture Database\(^2\) (see the supplementary

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\(^2\)http://mocap.cs.cmu.edu/