Numerical Simulation of Particle Motion in a Curved Channel

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Abstract. In this work the lattice Boltzmann method (LBM) is used to numerically study the motion of a circular particle in a curved channel at intermediate Reynolds numbers (Re). The effects of the Reynolds number and the initial particle position are taken into account. Numerical results include the streamlines, particle trajectories and final equilibrium positions. It has been found that the particle is likely to migrate to a similar equilibrium position irrespective of its initial position when Re is large.

1. Introduction
In microfluidic devices, manipulation and separation of particles are usually necessary in the processes of enzymatic analysis, DNA analysis and sample separation. However, it is necessary to focus the samples in a tight stream before separation, sorting or analysis in order to ensure these samples passing through the micro-channels quickly. Therefore, understanding the behavior and characteristics of particle suspensions in microfluidics is helpful to provide insight into the design of microfluidic channels [1].

Inertial focusing is usually adopted to align the particles along a tight stream in micro-channels, which has the advantage over other methods because it does not require external forces or multiple streams to focus particles. The most famous phenomena of inertial focusing may be the Segré–Silberberg effect [2], which is a fluid dynamic separation effect where a dilute suspension of neutrally buoyant particles flowing in a tube equilibrates at a distance ~0.6R (tube radius) from the tube center. Later a full analytical solution of the forces that dominate particles in Poiseuille flow was provided by Ho and Leal [3]. They [3] showed that particles migrate from the center of a channel towards the wall due to shear-induced lift forces, and are rejected from the channel perimeter by wall-induced lift forces creating a stable equilibrium at a distance of 0.6R from the center of the channel.

So far this phenomena has numerous applications in micro-particle manipulation ranging from microfluidic cell sorting to particle separation and ordering [4–7]. However, attempts to study the flow characteristics as well as the motion of particles in a curved channel are rarely reported in the past due to the complex nature of flow, most of which are involving experimental work. Little effort has been paid to the study of the migration of particles in a curved channel from a numerical aspect. Therefore, a more complete understanding of the migration of particles in these channels is needed to provide help with the design of microfluidic channels and to further enhance the focusing of particles. This motivates the present work.

2. Numerical Model
In this work, lattice Bhatnagar-Gross-Krook Boltzmann method proposed is used to solve the fluid flow [8],

\[ f_i(x + e_i \Delta t, t + \Delta t) - f_i(x, t) = \frac{1}{\tau} [ f_i(x, t) - f_i^{(eq)}(x, t) ] \]  

(eq)1

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where \( f_i(x, t) \) and \( f_i^{(eq)}(x, t) \) are the distribution function and corresponding equilibrium distribution function associated with the \( i \)-th discrete velocity direction \( c_i \). \( \Delta t \) is the time step and \( \tau \) is the relaxation time, respectively. In the two-dimensional nine-velocity lattice (D2Q9) model proposed by Qian et al [8] is adopted. The lattice speed is \( c = \Delta x / \Delta t \), where \( \Delta x \) is the lattice spacing. \( f_i^{(eq)}(x, t) \) for this lattice is,

\[
f_i^{(eq)} = w_i \rho_f \left[ 1 + \frac{c_i \cdot u}{c_i^2} + \frac{(c_i \cdot u)^2}{2c_i^4} - \frac{u^2}{2c_i^4} \right],
\]

where \( \rho_f \) is the fluid density, \( c_i \) is the sound speed, and \( w_i \) is the weight coefficient given by,

\[
w_i = \begin{cases} 4/9, & c_i^2 = 0 \\ 1/9, & c_i^2 = c_s^2 \\ 1/36, & c_i^2 = 2c_s^2 \end{cases}, \quad c_s = \sqrt{3} c.
\]

The fluid density \( \rho_f \) and velocity \( u \) can be calculated by the following formula,

\[
\rho_f = \sum_i f_i \cdot \rho_f u = \sum_i c_i f_i.
\]

The Navier-Stokes equations can be obtained from the lattice Boltzmann equation (LBE) through a Chapman-Enskog expansion proposed by He and Luo [9].

In this work we aim to numerically study the migration of a circular particle in a curved channel. The physical model is shown in Fig.1. The width of the channel is denoted as \( h \). Other parameters such as \( L_1, L_2, L_3 \) and \( H \) are shown in Fig. 1, which are set to be \( L_1 = L_2 = L_3 = 5h \) and \( H = 3h \). The diameter and density of the particle are expressed by \( d \) and \( \rho_p \), respectively. At the inlet, a parabolic flow with the maximum velocity of \( U_0 \) is applied, while the fully-developed condition is applied at the outlet. No slip boundary is used on all the channel walls. In the simulations, the parameters are chosen as follows: \( \rho_f = 1, d = 10, h = 8d \) and \( U_0 = 0.05 \) (in lattice unit). The Reynolds number is defined through \( Re = U_0 d / h \).

**Figure 1.** Physical model of the present work

### 3. Results

In the simulations the particle is released after the flow field is fully-developed. To better illustrate the flow field in the curved channel, we present the steady streamlines at different Reynolds numbers (\( Re = 20, 70 \) and 120) in Fig. 2. Only the local enlargement of steady flow field is shown because of large computational domain.

![Figure 2](image-url)
It is hardly to observe the recirculation zones when the Reynolds number is small, such as \( Re = 20 \), as shown in Fig. 2(a). As a result, the distortion of the streamlines is not significant. However, the recirculation zones are becoming more notable for larger \( Re \), as one can see in Fig. 2(b) and (c). Totally speaking, there are two types of corners in the present channel: corners with inwards right angle and those with outwards right angle, which lead to two types of recirculation zone. It is observed that there always exists a recirculation zone for each corner when \( Re \) is large. Furthermore, the rotation of recirculation zones on the upper wall is always counter-clockwise while the opposite is true for those on the bottom wall. Due to the recirculation zones the distortion of the streamlines becomes significant when the Reynolds number is large, such as \( Re = 120 \), which is expected to considerately influence the migration of particle in the channel.

A parameter \( q \) is introduced to describe the initial lateral position of particle in the channel which is defined by the distance of particle to the upper channel wall normalized by the channel width \( h \). As a result, the value of \( q = 0.5 \) indicates that the particle is initially placed on the channel centerline. In what follows, we focus on the particle trajectory as well as the final equilibrium position of particle under different channel Reynolds numbers.

**Figure 2.** Streamlines of steady flow field at: (a) \( Re = 20 \), (b) \( Re = 70 \) and (c) \( Re = 120 \)

**Figure 3.** Particle trajectories at \( Re = 20 \), 70 and 120 for different \( q \): (a) \( q = 0.1 \), (b) \( q = 0.3 \) and (c) \( q = 0.5 \)

Fig. 3 shows the particle trajectories at \( Re = 20 \), 70 and 120 for different initial particle positions. As one can see, the effect of \( Re \) on the particle migration in the channel is significant. To some extent, the particle trajectories are similar to the streamlines shown in Fig. 2. In comparison with the result of \( Re = 20 \), the particle is driven farther away from the channel wall after passing through each bend when increasing \( Re \), which is more significant for small values of \( q \), as shown in Fig. 3. This is due to the fact that the larger the Reynolds number, the larger centrifugal force experienced by the particle.
To further study the migration behavior of particle in the curved channel, we also pay close attention to the evolution of particle orientation $\theta$ which has an initial value of $\pi/2$, as shown in Fig. 4 and Fig. 5, which present the corresponding results of $q = 0.1$-$0.3$ and $q = 0.5$ at different Reynolds numbers, respectively. As we can observe in Fig. 4, the particle always rotates counter-clockwise when traveling in the channel if $q < 0.5$ irrespective of $Re$. In addition, the smaller the value of $q$ is, the faster the particle rotates. This is because the particle experience larger gradient of fluid velocity if it is closer to the channel wall.

![Figure 4](image_url)

**Figure 4.** The orientation of particle for $q = 0.1$, 0.2 and 0.3 at different Reynolds numbers: (a) $Re = 20$, (b) $Re = 70$ and (c) $Re = 120$

Fig. 5 shows a different pattern of particle motion in the serpentine channel when $q = 0.5$. Instead of rotating counter-clockwise, the particle will oscillate if it is initially placed on the channel centerline for the range of $Re$ in this work. Especially, it is observed that the particle is oscillating around $\theta = \pi/2$ when traveling in the channel for large Reynolds numbers, such as $Re = 70$ and 120, as shown in Fig. 5.

Fig. 6 presents the particle velocities when traveling in the channel for different values of $q$. As one can see that the velocities fluctuate when the particle passes through the corners of the channel. In addition, for all cases shown in Fig. 6, the particle velocities are larger for larger Reynolds number, suggesting that the particle is traveling faster when $Re$ is larger.
Figure 5. The orientation of particle for $q = 0.5$ at different Reynolds numbers

Figure 6. The particle velocity for different values of $q$: (a) $q=0.1$, (b) $q=0.2$, (c) $q=0.4$ and (d) $q=0.5$
4. Conclusion
In this work the lattice Boltzmann method based on the momentum exchange scheme has been adopted to numerically study the migration of a particle in a serpentine channel. We focus on the effects of the Reynolds number \((Re)\) as well as the initial position of particle \((q)\) on the migration behavior of particle in the channel. The Reynolds number ranges from 20 to 120. The effect of \(Re\) on the final equilibrium position of particle is significant, which is found to be more sensitive to the initial position of particle when \(Re\) is small. Interestingly, results show that the particle almost stays on the channel centerline for the range of \(Re\) studied when \(q = 0.5\).

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6. References
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