Cold, dense nuclear matter in a SU(2) parity doublet model

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We study dense nuclear matter and the chiral phase transition in a SU(2) parity doublet model at zero temperature. The model is defined by adding the chiral partner of the nucleon, the N’, to the linear sigma model, treating the mass of the N’ as an unknown free parameter. The parity doublet model gives a reasonable description of the properties of cold nuclear matter, and avoids unphysical behaviour present in the standard SU(2) linear sigma model. If the N’ is identified as the N’(1535), the parity doublet model shows a first order phase transition to a chirally restored phase at large densities, \( \rho \approx 10\rho_0 \), defining the transition by the degeneracy of the masses of the nucleon and the N’. If the mass of the N’ is chosen to be 1.2 GeV, then the critical density of the chiral phase transition is lowered to three times normal nuclear matter density, and for physical values of the pion mass, the first order transition turns into a smooth crossover.

I. INTRODUCTION

The description of the properties of nuclear matter in effective models of the nuclear interactions, such as in Quantum HadroDynamics, has been quite successful [1]. Early attempts to incorporate the basic symmetries of the underlying fundamental theory of the strong interactions, QCD, however, have experienced severe difficulties. Lee and Wick studied the standard SU(2) linear sigma model for nuclear matter, and showed that there is a state where chiral symmetry is effectively restored by a nearly vanishing nucleon mass [2]. It was, however, discovered immediately by Kerman and Miller, that the solution is unstable in the standard SU(2) linear sigma model, and that the model cannot describe nuclear matter saturation [3].

Boguta extended the linear sigma model by introducing a dynamically generated vector meson in a chirally invariant way [4]. However, the model was unable to generate a chiral phase transition at finite density or finite temperature [5] as the chirally restored phase was mechanically unstable. With the inclusion of an additional scalar field, the dilaton field, the unphysical bifurcations could be avoided, although the compression modulus turned out to be unphysically high [6]. A chiral phase transition is present for either high temperatures or high densities. In the temperature-density plane, however, the critical line of the chiral phase transition was open, so that there was no chiral phase transition present for intermediate temperatures and densities [7]. In particular, when the vector meson masses were generated only by the coupling to the sigma field, chiral symmetry restoration could not be reached, as the vector meson mass vanishes. Also, first attempts to use a linear sigma model to describe nuclei failed [8, 9].

Different effective interaction potentials were introduced to cure the apparent caveats of the linear sigma model. A logarithmic potential term was examined in [10], and successfully applied to the description of nuclear matter and nuclei. On the other hand, studies using the standard Mexican hat potential were not able to adequately describe nuclei. Admittedly, the logarithmic potential could not be applied for studying the chiral limit, where the pion mass becomes exactly massless in the Nambu-Goldstone phase.

A nonlinear realization of chiral symmetry was used in [11, 12] to successfully describe the properties of nuclear matter and nuclei. The essential ingredient of the successful approach was that the scalar field was explicitly kept as a dynamical degree of freedom to describe nuclear matter. Extensions to the linear and nonlinear realization in chiral SU(3) symmetry were performed in [13, 14] which included hyperon degrees of freedom and the description of hypernuclei. Effects from the strange quark condensate were found to be important for arriving at a reasonable compression modulus of nuclear matter and a reasonable description of nuclei.

In all of these investigations, it appears that the scalar sigma field and its vacuum expectation value can not possibly serve, simultaneously, as the chiral partner of the pion, the generator of the nucleon mass, and the mediator of scalar attraction for nucleons. This problem emerged to be particular evident when extending the model to strange baryons, in trying to describe both the masses and the potentials for hyperons. Inclusion of an additional scalar field, the dilaton field, does not remedy the situation [14].

An alternative way of looking at the role of the sigma field for the generation of the hadron masses is established by two seemingly different chiral approaches: a hidden local symmetry for the vector mesons, and a par-
ity doublet model for nucleons.

A hidden local symmetry is a type of gauged linear sigma model: the overall mass scale of the vector and axial vector fields is fixed by a new constant, while their splitting is due to a nonvanishing vacuum expectation value for the sigma field. This model automatically exhibits vector meson dominance and gives a good description of the vacuum properties of the vector and axial vector mesons \[ \frac{m}{2} \]. The model was extended by the Minnesota group to describe low-energy pion-nucleon scattering, the properties of nuclei, and nuclear matter at finite temperature and density \[ \frac{m}{2} \]. Note that the chiral symmetry demands only that the spectral functions of the vector and axial vector mesons are degenerate in the chirally restored phase, and not that the vector and axial vector meson masses drop to zero (see e.g. \[ \frac{m}{2} \]).

In a parity doublet model for nucleons, the chiral partner of the nucleon, the N’, is added to the linear sigma model. This possibility was raised by Sakurai, with the first realistic model given by DeTar and Kunihiro \[ \frac{m}{2} \]. In a certain assignment, to be presented below, the sigma field is responsible for causing the mass splitting between the nucleon and the N’, while the overall mass scale of the nucleons is given by a new parameter, which couples in a chirally invariant fashion \[ \frac{m}{2} \]. The N’, which is a negative parity state, is usually taken to have the mass of the N’(1535). The parity doublet model was extended and for the chiral phase transition. We demonstrate that solutions for nuclear and neutron matter, and to study the chiral phase transition in cold, dense systems were studied \[ \frac{m}{2} \]. For this model, nuclear matter and the chiral symmetry is due to a nonvanishing vacuum expectation value, as well as that for strange D-mesons, is avoided when the N’ is introduced as the chiral partner of the nucleon. It turns out that a smaller mass for the N’ reduces the critical density for the chiral phase considerably.

This paper is organized as follows: first we introduce the chiral SU(2) parity model and fix its parameters. For comparison, we study the pressure of cold nuclear matter in the standard linear sigma model, and discuss the apparently unphysical nature of its stable solutions. We then show how a parity doublet model gives a reasonable and stable description for the properties of nuclear matter. We then extend the model to high density, and study the chiral phase transition in cold nuclear matter.

II. THE SU(2) PARITY MODEL

There are two ways of assigning chiral transformations for parity doubled nucleons. In the naive assignment, the two nucleons belong to different multiplets, while in the mirror assignment they belong to the same multiplet, and so are true chiral partners \[ \frac{m}{2} \]. We adopt the latter.

In the mirror model, under \[ SU_L(2) \times SU_R(2) \] transformations \( L \) and \( R \), the two nucleon fields \( \psi_1 \) and \( \psi_2 \) transform as

\[
\begin{align*}
\psi_{1R} & \rightarrow R \psi_{1R} , \\
\psi_{2L} & \rightarrow L \psi_{2L} ,
\end{align*}
\]

This allows for a chirally invariant mass, \( m_0 \):

\[
m_0 ( \bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2 ) = m_0 ( \bar{\psi}_{2L} \gamma_5 \psi_{1R} - \bar{\psi}_{1L} \psi_{2R} + \bar{\psi}_1 \psi_{2L} ) .
\]

The chiral Lagrangian in the mirror model is

\[
\mathcal{L} = \bar{\psi}_1 i \partial \psi_1 + \bar{\psi}_2 i \partial \psi_2 + m_0 ( \bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2 ) + \bar{\psi}_1 ( \sigma + i \gamma_5 F \cdot \gamma_5 \psi_1 + b \bar{\psi}_2 ( \sigma - i \gamma_5 \gamma_5 F \cdot \gamma_5 \psi_2 - g_\omega \bar{\psi}_1 \gamma_5 \omega^\mu \psi_1 - g_\omega \bar{\psi}_2 \gamma_5 \omega^\mu \psi_2 ) + \mathcal{L}_M ,
\]

where \( a \) and \( b \) are the coupling constants of the mesons fields \( (\sigma, \pi, \omega) \) to the baryonic fields \( \psi_1 \) and \( \psi_2 \). Note that we assume the same vector coupling strength for both parity partners. The mesonic Lagrangian \( \mathcal{L}_M \) contains the kinetic terms of the different
meson species, and potentials for the scalar and vector fields. The potential for the spin zero fields is the same
as in the ordinary SU(2) linear sigma model. Kinetic and potential terms are added for an isoscalar vector meson,
ω, as in the σ-ω model of nuclear matter [29]:
\[
\mathcal{L}_M = \frac{1}{2} \partial_\mu \sigma^0 \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi_\mu - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \\
+ \frac{1}{2} m^2_\sigma \omega^\mu \omega^{\mu} + g_4^2 (\omega_i \omega^i)^2 \\
+ \frac{1}{2} \mu^2 (\sigma^2_\mu + \pi^2_\mu) - \frac{\lambda}{4} (\sigma^2 + \pi^2)^2 \\
+ \epsilon \sigma, \tag{5}
\]
where \( F_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) represents the field strength tensor of the vector field. As usual, the parameters \( \lambda, \mu \) and \( \epsilon \) can be related to the sigma and pion masses, and the pion decay constant, in vacuum:
\[
\lambda = \frac{m^2_\sigma - m^2_\pi}{2 \sigma_0}, \\
\mu^2 = \frac{m^2 - 3m^2_\pi}{2}, \\
\epsilon = m^2 f_\pi, \tag{6}
\]
with \( m_\pi = 138 \) MeV, \( f_\pi = 93 \) MeV and \( \sigma_0 = f_\pi \) the vacuum expectation value of the sigma field. Since the mass of the \( \sigma \) meson in the vacuum cannot be fixed precisely by experiment, we will treat it as a free parameter. The vacuum mass of the \( \omega \) field is \( m_{\omega} = 783 \) MeV while the \( g_4 \) term for the \( \omega \) field also represents a fit parameter, with finite values of this parameter causing a softening of the equation of state.

To investigate the properties of dense nuclear matter and the chiral phase transition at zero temperature, we adopt a mean-field approximation [30]. The fluctuations around constant vacuum expectation values of the mesonic field operators are neglected, while the nucleons are treated as quantum-mechanical one-particle operators. Only the time-like component of the vector meson \( \langle \omega \rangle \equiv \omega_0 \) survives, assuming homogeneous and isotropic infinite nuclear matter. Additionally, parity conservation demands \( \langle \pi \rangle = 0 \).

The mass eigenstates for the parity doubled nucleons, the \( N^+ \) and \( N^- \), are determined by diagonalizing the mass matrix, Eq. (4), for \( \psi_1 \) and \( \psi_2 \):
\[
\begin{pmatrix}
N^+ \\
N^-
\end{pmatrix} = \frac{1}{\sqrt{2 \cosh \delta}} \begin{pmatrix}
\cos \delta/2 & \sin \delta/2 \\
\sin \delta/2 & \cos \delta/2
\end{pmatrix} \begin{pmatrix}
\psi_1 \\
\psi_2
\end{pmatrix} \tag{7}
\]
where \( \sinh \delta = -(a + b)\sigma/2m_0 \). In the basis of Eq. (7), the masses of \( N^+ \) and \( N^- \) are given by
\[
m_i = m_{N_{\pm}} = \frac{1}{2} \left( (a + b)^2 \sigma^2 + 4m_0^2 \mp (a - b)\sigma \right). \tag{8}
\]
If chiral symmetry is completely restored, i.e. \( \sigma = 0 \), the two nucleonic parity states become degenerate in mass with \( m_{N^+} = m_{N^-} = m_0 \). Thus, if the value of \( m_0 \) is large, the nucleon masses are primarily generated by the explicit mass term, while the spontaneous breaking only generates the mass splitting [21]. In contrast, in the naive assignment, as well as in the standard linear sigma model (and also the mirror model model with \( m_0 = 0 \)), spontaneous symmetry breaking generates the nucleonic masses.

The grand canonical partition function is
\[
\frac{\Omega}{V} = \mathcal{V}_M + \sum_i \frac{\gamma_i}{(2\pi)^3} \int_0^{k_{F_i}} d^4k \left( E^*_i(k) - m^*_i \right), \tag{9}
\]
where \( i \in \{N^+, N^-\} \) denotes the nucleon type, \( \gamma_i \) is the fermionic degeneracy, \( E^*_i(k) = \sqrt{k^2 + m^2_i} \) the energy, and \( m^*_i = m_i - g_\omega \omega_0 = \sqrt{k^2 + m^2_\pi} \) the corresponding effective chemical potential. The single particle energy of each parity partner \( i \) is given by \( E_i(k) = E^*_i(k) + g_\omega \omega_0 \).

The mean meson fields \( \bar{\sigma} \) and \( \bar{\omega} \) are determined by extremizing the grand canonical potential \( \Omega/V \):
\[
\frac{\partial (\Omega/V)}{\partial \bar{\sigma}} \bigg|_{\bar{\sigma}, \bar{\omega}} = -\mu^2 \bar{\sigma} + \lambda \bar{\sigma}^3 - \epsilon + \sum_i \rho^*_i (\bar{\sigma}, \bar{\omega}) \frac{\partial m_i}{\partial \bar{\sigma}} \bigg|_{\bar{\sigma}} = 0, \\
\frac{\partial (\Omega/V)}{\partial \bar{\omega}} \bigg|_{\bar{\sigma}, \bar{\omega}} = -m_\omega \bar{\omega} - 4g_\omega \omega_0 + \sum_i \rho_i (\bar{\sigma}, \bar{\omega}) = 0. \tag{10}
\]
The scalar density \( \rho^*_i \) and the baryon density \( \rho_i \) for each chiral partner are given by the usual expressions
\[
\rho^*_i = \gamma_i \int_0^{k_{F_i}} \frac{d^3k}{(2\pi)^3} \frac{m_i}{E^*_i} \\
= \frac{\gamma_i m_i}{4\pi^2} \left[ k_F E^*_i - m_i^3 \ln \left( \frac{k_F + E^*_i}{m_i} \right) \right], \\
\rho_i = \gamma_i \int_0^{k_{F_i}} \frac{d^3k}{(2\pi)^3} \frac{\gamma_i k_F^3}{6\pi^2}. \tag{11}
\]
The basic nuclear matter saturation properties we impose can be formulated in the following way. The stable minimum of the grand canonical potential for \( \mu_B = 923 \) MeV has to meet two conditions:
\[
E/A(\mu_B = 923\text{MeV}) - m_N = -16\text{MeV} \\
\rho_0(\mu_B = 923\text{MeV}) = 0.16 \text{fm}^{-3}. \tag{12}
\]
Altogether we have four parameters to be related to nuclear matter properties: \( m_0, g_\omega, m_\pi \) and \( g_4 \). The parameters \( a, b \) are related to the vacuum masses of the parity partners. The mass of the positive parity state will always be the nucleon mass \( m_{N^+} = 939 \) MeV. The negative parity state will be a free parameter. Here we will consider two cases: \( m_{N^-} = 1.5 \) GeV, which corresponds to the conventional choice of \( N'(1535) \) as the parity partner and, as an alternative, \( m_{N^-} = 1.2 \) GeV. We will investigate if a pair \( (g_\omega, m_\pi) \) exists which fulfills Eq. (12) for a given choice of \( m_0, g_4 \) and \( m_{N^-} \). Then, by varying \( m_0 \) and \( g_4 \), we check how the fit parameters or observables like the incompressibility
\[
K = 9\rho_0^2 \frac{\partial^2 (E/A)}{\partial \rho^2} \bigg|_{\rho = \rho_0} = 9 \frac{P}{\rho} \bigg|_{\rho = \rho_0} = 9 \frac{\mu_B}{\rho} \bigg|_{\rho = \rho_0}. \tag{13}
\]
III. NUCLEAR MATTER IN LINEAR SIGMA MODELS

A. Nuclear matter in the standard linear sigma model

As was already shown by Kerman and Miller, it is not possible to reproduce stable nuclear matter properties in the standard linear sigma model by 'just adding' vector mesons. The same holds for the parity model with $m_0 = 0$. As already mentioned above, for a successful description of nuclear matter, the minimum requirements are: A binding energy of 16 MeV and a nuclear matter density of approximately 0.16 fm$^{-3}$ at $\mu_B = 923$ MeV. Furthermore, one would expect this state to be different from the vacuum or the chirally restored phase. But in the standard linear $\sigma - \omega$ model, as well as in the parity model with $m_0 = 0$, for $\mu_B = 923$ MeV only two possible phases exist: the vacuum state, characterized by $\tilde{\sigma} = f_\pi$ and $\rho_B = 0$, and the chirally restored phase with $\tilde{\sigma} \approx 0$. Which of these phases is stable, depends on the choice of parameters. Neither of these phases can represent saturated nuclear matter.

Figure 1 shows the pressure as a function of the condensate in the parity model, at $\mu_B = 923$ MeV and $m_0 = 0$, for different values of the vector coupling. The remaining parameters are chosen as $m_\sigma = 1$ GeV, $g_4 = 0$ and $m_{N'} = 1.5$ GeV. For small values of $g_\sigma$, the pressure is maximal at $\sigma \approx 0$, i.e. the chirally restored phase is stable. For larger $g_\sigma$ the pressure of this phase is reduced and finally falls below zero. Then the vacuum has the greatest pressure, and thus represents the stable phase. For any $g_\sigma$, a stable, intermediate phase, which could represent ordinary nuclear matter, does not exist. This situation does not change if the value of the sigma mass is varied. Figure 2 shows the mean sigma value corresponding to the stable state (maximum pressure) of the system as a function of $g_\sigma$ and $m_\sigma$. All other parameters remain the same as before. Still the stable state is either the vacuum or the phase with $\tilde{\sigma} \approx 0$. Note that changing the values of $m_{N'}$ or $g_4$ does not alter these findings.

B. Nuclear matter with the parity doublet model

The situation changes if we allow for finite values of the mass term $m_0$. As shown in Figure 3 when $m_0 = 800$ MeV, it is possible to find values of $g_\sigma$ and $m_\sigma$ such that there is an intermediate phase which might represent nuclear matter. In this intermediate phase, $\tilde{\sigma} \sim 30$ MeV.

Figure 4 shows the pressure as a function of the $\sigma$-field for $m_0 = 800$ MeV, $m_\sigma = 400$ MeV, $g_4 = 0$ and $m_{N'} = 1.5$ GeV. For small values of $g_\sigma$ and $m_\sigma$, the stable phase has $\tilde{\sigma} \approx 30$ MeV; for large values of these parameters, again the vacuum is the stable state.

It turns out that, for these intermediate phases, which exist for a wide range of finite $m_0$ values, a reproduction of nuclear matter properties is possible. Figure 5 shows the $m_\sigma$ values as resulting from such fits as a function of $m_0$ for $m_{N'} = 1200, 1500$ MeV and $g_4 = 0, 3.8$. Each choice of $m_{N'}$ and $g_4$ corresponds a minimum value of $m_0$, for which a nuclear matter fit exists. These minimum $m_0$ values lie between 300 and 500 MeV, with a higher $N'$ mass allowing for smaller values. The maximum possible values for $m_0$ are in the range of 800 MeV for all cases considered. The corresponding sigma vacuum masses are in the range of 300 to 550 MeV. The larger the sigma vacuum mass is, the larger the $N'$ mass is chosen and it decreases with increasing $g_4$ coupling.
FIG. 3: Expectation value $\bar{\sigma}$ of the chiral condensate in the stable phase as a function of $m_\sigma$ and $g_\omega$ for $m_0 = 800$. Note the appearance of an intermediate phase, with $\bar{\sigma} \sim 30$ MeV.

For high $m_0$ values the sigma mass turns out to be rather low. The value of the corresponding nuclear incompressibility is depicted in Fig. 6 as a function of $m_0$. Only for large values of $m_0$ it lies in a reasonable range. The same holds for finite $g_4$ values, although finite values lead to lower incompressibility. Thus, reasonable values for the incompressibility are obtained for high values of the mass parameter $m_0$ and that leads to a small vacuum sigma mass. The situation could change if the model is extended to SU(3), as shown in [13, 14].

For all cases studied, Fig. 7 shows that if a fit to nuclear matter is possible, at the saturation point of nuclear matter, the effective nucleon mass is almost identical to the value of $m_0$. Thus, high values of $m_0$ correspond to small incompressibilities, and large effective nucleon mass. This could be a problem for the spin-orbit splitting, as shown by Furnstahl et al [31], though, such a problem can be solved by adjusting the corresponding tensor coupling.

The results for the fits which give low incompressibilities and which are used in the next section to investigate dense matter are shown in Table 1.
FIG. 7: The effective nucleon mass at saturation, $m_N^*(\rho_0)$, versus $m_0$.

| $m_N^*$ [MeV] | P1 | P2 | P3 | P4 |
|---------------|----|----|----|----|
| $g_4$         | 0  | 3.8| 0  | 3.8|
| $m_0$ [MeV]  | 790| 790| 790| 790|
| $m_\sigma$ [MeV] | 318.56| 302.01| 370.63| 346.59|
| $g_{N\omega}$ | 6.08| 6.77| 6.79| 7.75|
| $g_B$         | 9.16| 9.16| 13.00| 13.00|
| $b$           | 6.35| 6.35| 6.97| 6.97|
| $\bar{\mu}$ [MeV] | 147.50| 128.93| 199.26| 176.29|
| $\lambda$     | 4.75| 4.16| 6.82| 5.82|
| $m_{N^+}(\rho_0)/m_{N^+}$ | 0.86| 0.86| 0.84| 0.83|
| $m_{N^-}(\rho_0)/m_{N^-}$ | 0.79| 0.78| 0.73| 0.72|
| $K$ [MeV]     | 436.41| 374.75| 510.57| 440.51|

TABLE I: Fit parameter and nuclear matter properties for the four fits mainly used. For all parameter sets: $E/A(\rho_0) - m_N = -16$ MeV, $\rho_0 = 0.16$ fm$^{-3}$ and the vacuum nucleon mass $m_N = 939$ MeV.

IV. THE CHIRAL PHASE TRANSITION

We now investigate dense hadronic matter within the parity doublet model. We want to use the same $m_0$ value for all cases, with the best possible values for the incompressibility. Thus we choose $m_0 = 790$ MeV, which is the maximum value allowing for a fit to nuclear matter in all of the cases considered: $g_4 = 0$ and $g_4 = 3.8$, and $m_{N^-} = 1.2$ and $m_{N^-} = 1.5$ GeV. In Figure 8 we show the expectation value of the chiral condensate as a function of the chemical potential. At $\mu_q = \mu_B/3 \approx 308$ MeV the condensate jumps from its vacuum value down to values of around 40 MeV, depending on the parameter set. This is the liquid gas phase transition, which is present in all cases. As the chemical potential is increased, the scalar condensate decreases, until a chiral phase transition occurs.

In the chiral limit of zero pion mass, drawn as thin lines in Figure 8, the chiral transition is always of first order, as the condensate jumps from $\approx 30$ MeV down to zero.

The case of a physical pion mass is drawn as thick lines in Figure 8. The order of the transition, and the value of the critical chemical potential, depend on the values of the $m_N^-$ and $g_4$, although it is always in the range 330 MeV $\leq \mu_q^c < 600$ MeV.

When $m_{N^-} = 1500$ MeV, a first order chiral transition occurs, with the critical chemical potential $\mu_q^c \approx 1300$ MeV for $g_4 = 0$ and $\mu_q^c \approx 410$ MeV for $g_4 = 3.8$. For $m_{N^-} = 1200$ MeV, the chiral “transition” becomes a smooth crossover. In this case we define the “critical” chemical potential as the value $\mu_q^c$ at which the change in the sigma field with chemical potential is the largest. The resulting values for the choice $m_{N^-} = 1200$ MeV are: $\mu_q^c \approx 300$ MeV for $g_4 = 0$ and $\mu_q^c \approx 340$ MeV for $g_4 = 3.8$, i.e. considerably smaller than in the case with $m_{N^-} = 1500$ MeV. The critical chemical potentials do not change considerably comparing physical and vanishing pion mass for given $m_{N^-}$ and $g_4$.

For physical pion mass and $g_4 = 3.8$, the critical $N^-$ mass, i.e. the mass where the first order transition turns into a smooth transition, turns out to be $\approx 1490$ MeV. This explains why the first order phase transition for $m_{N^-} = 1500$ MeV is rather weak. In contrast, for $g_4 = 0$, the critical $N^-$ mass is found to be approximately 1370 MeV, i.e. considerably away from the value of 1.5 GeV. Thus, as shown in Fig. 8, the discontinuity in the $\bar{\sigma}$ and thus the transition is considerably stronger.

As can be seen in Figure 8, although the critical chemical potentials vary considerably with the change of the...
quartic coupling, the corresponding “critical density” does not. This is because there is no strong coupling between scalar and vector field in our model. In contrast, the critical density for the chiral transition very strongly depends on the vacuum mass of the parity partner. It is reduced by a factor of three when changing the $N^-$-mass from 1.5 to 1.2 GeV.

The change in the nucleon masses is shown in Figure 11. The effective mass of the nucleon drops at the liquid gas phase transition, and does not change strongly as a function of the chemical potential until the chiral phase transition. The mass of the parity partner also drops at the liquid gas phase transition, and then decreases strongly with increasing chemical potential. In the chiral limit, both nucleon masses jump discontinuously at the chiral transition, to $m_0$. If there is a smooth crossover, the mass of the nucleon and the one of its parity partners smoothly approach $m_0$ asymptotically, the nucleon from below, and the parity partner, from above.

In Figure 11 we show the relative densities of the nucleon and its parity partner. From the figure, one can see that the chiral phase transition occurs once there is any significant population of the $N^-$ states. At asymptotically high densities or chemical potentials, the chiral condensate vanishes, the nucleons are equal in mass, and so each chiral partner contributes half of the total nucleon density.

Figure 12 shows the resulting binding energy per particle for the parameter sets considered before. These are compared to the Walecka model fits NL3 and TM1. At high densities all equations of state (EoS) in the parity doublet model are much softer than the Walecka models, but they show a larger curvature at small densities, which causes the still relatively high incompressibilities. As could be expected, for finite values of $g_4$, the EoS is considerably softened at high densities. A smaller mass of the negative parity state yields a reduced energy per particle at high densities. Finally, we consider the behavior of the effective sigma mass $m_{\sigma}^*$ with density. It is obtained by first determining the sigma-omega mass matrix through the corresponding second derivatives of the thermodynamic potential (or pressure) with respect to the fields at fixed chemical potential and then diagonalizing this matrix. In Figure 13 we show the resulting effective $\sigma$-mass, $m_{\sigma}^*$, as a function density. The different transitions in dense matter cause very significant structures.

First, at the liquid gas phase transition, the sigma mass jumps from its vacuum value down to a value of around 200-300 MeV and then increases again. Right before the chiral transition, it decreases strongly. This takes place when the $N^-$ states start to get populated. This
FIG. 12: Binding energy per particle of nuclear matter for $m_{N^-} = 1200, 1500$ MeV and $g_4 = 0, 3.8$ in comparison with relativistic mean-field calculations TM1 [32] and NL3 [33].

FIG. 13: Effective sigma mass $m_\sigma^*$ vs density. After a jump down at the liquid gas phase transition the mass increases again as a function of density until the $N^-$ states get populated. This causes a significant decrease of $m_\sigma^*$. If the chiral phase transition actually takes place, the mass increases again.

decrease continues until the chiral transition. If the transition is of first order ($m_{N^-} = 1.5$ GeV), the sigma mass jumps as well as the density and then starts to increase again. If the transition is a crossover ($m_{N^-} = 1.2$ GeV), this increase happens continuously.

V. SUMMARY

We have shown that it is possible to obtain successful fits of saturated nuclear matter in a SU(2) parity doublet model with $\sigma$ and $\omega$ mesons. Agreement with current estimates of the nuclear incompressibility favors large values of the explicit mass term parameter $m_0$. Furthermore, we found that nuclear matter fits are possible for different values of the vacuum mass of the $N^-$. At higher densities, chiral restoration takes place, where the order of the transition and the critical density depend on $N^-$’s mass.

There are clearly many avenues for further investigation: including strange quarks, other hadronic resonances, and the like. Probably the outstanding question is to look at decay widths, given that the natural experimental candidate for the nucleon’s parity partner, the N’(1535), likes to decay to $\eta\pi$ so much. We simply found the present exercise most encouraging, in that although the nucleon parity partner is relatively heavy, the properties of nuclear matter change significantly, and in a direction which bring them closer to known experimental values.

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[1] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6, 515 (1997).
[2] T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974).
[3] A. K. Kerman and L. D. Miller, in Proceedings of the Relativistic Heavy Ion Summer Study (Berkeley, 1974), pp. 73–107, preprint LBL-3675.
[4] J. Boguta, Phys. Lett. 120B, 34 (1983).
[5] N. K. Glendenning, Ann. Phys. (N.Y.) 168, 246 (1986).
[6] I. Mishustin, J. Bondorf, and M. Rho, Nucl. Phys. A555, 215 (1993).
[7] P. Papazoglou, J. Schaffner, S. Schramm, D. Zschiesche, H. Stöcker, and W. Greiner, Phys. Rev. C 55, 1499 (1997), nucl-th/9609035.
[8] R. J. Furnstahl and B. D. Serot, Phys. Rev. C 47, 2338 (1993).
[9] R. J. Furnstahl and B. D. Serot, Phys. Lett. B316, 12 (1993).
[10] E. K. Heide, S. Rudaz, and P. J. Ellis, Nucl. Phys. A571,
713 (1994), nucl-th/9308002.
[11] R. J. Furnstahl, B. D. Serot, and H.-B. Tang, Nucl. Phys. A598, 539 (1996), nucl-th/9511028.
[12] R. J. Furnstahl, B. D. Serot, and H.-B. Tang, Nucl. Phys. A615, 441 (1997), nucl-th/9608035.
[13] P. Papazoglou, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C 57, 2576 (1998), nucl-th/9706024.
[14] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C 59, 411 (1999), nucl-th/9806087.
[15] P. Ko and S. Rudaz, Phys. Rev. D 50, 6877 (1994).
[16] G. W. Carter, P. J. Ellis, and S. Rudaz, Nucl. Phys. A603, 367 (1996), nucl-th/9512033.
[17] G. W. Carter, P. J. Ellis, and S. Rudaz, Nucl. Phys. A618, 317 (1997), nucl-th/9612043.
[18] G. W. Carter and P. J. Ellis, Nucl. Phys. A628, 325 (1998), nucl-th/9707051.
[19] R. D. Pisarski, Phys. Rev. D 52, R3773 (1995).
[20] C. DeTar and T. Kunihiro, Phys. Rev. D 39, 2805 (1989).
[21] D. Jido, Y. Nemoto, M. Oka, and A. Hosaka, Nucl. Phys. A671, 471 (2000), hep-ph/9805306.
[22] D. Jido, T. Hatsuda, and T. Kunihiro, Phys. Rev. Lett. 84, 3252 (2000), hep-ph/9910375.
[23] Y. Nemoto, D. Jido, M. Oka, and A. Hosaka, Phys. Rev. D 57, 4124 (1998), hep-ph/9710445.
[24] H.-c. Kim, D. Jido, and M. Oka, Nucl. Phys. A640, 77 (1998), hep-ph/9806275.
[25] D. Jido, H. Nagahiro, and S. Hirenzaki, Phys. Rev. C 66, 045202 (2002), nucl-th/0206043.
[26] H. Nagahiro, D. Jido, and S. Hirenzaki, Phys. Rev. C 68, 035205 (2003), nucl-th/0304068.
[27] T. Hatsuda and M. Prakash, Phys. Lett. B 224, 11 (1989).
[28] S. Eidelman et al., Phys. Lett. B 592, 1+ (2004), URL http://pdg.lbl.gov.
[29] J. D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974).
[30] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).
[31] R. J. Furnstahl, J. J. Rusnak, and B. D. Serot, Nucl. Phys. A632, 607 (1998), nucl-th/9709064.
[32] Y. Sugahara, and H. Toki, Nucl. Phys. A579, 557 (1994).
[33] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).