Universality of multiplicity distribution in proton-proton and electron-positron collisions

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Abstract

It is argued that the multiplicity distribution in proton-proton (pp) collisions, which is often parameterized by the negative binomial distribution, results from the multiplicity distribution measured in electron-positron (e⁺e⁻) collisions, once the fluctuating energy carried by two leading protons in pp is taken into account.

1. Introduction

The charged particle multiplicity distribution is one of the most basic observables in high energy collisions. Although there is an abundance of experimental results, see e.g., [1], on the theory side this problem is poorly understood.

The multiplicity distribution measured in proton-proton (pp) collisions is often parameterized by the negative binomial (NB) distribution [1–4], which is characterized by two parameters: the mean number of particles \( \langle n \rangle \), and \( k \), which measures the deviation from the Poisson distribution. NB distribution works well, with certain limitations [5], for a broad range of energies and in total and limited phase-space bin rapidity bins. For completeness we add that \( k \) is a decreasing function of energy.

Interestingly, similar experimental observations were made in electron-positron (e⁺e⁻) collisions, see e.g., [1, 6]. NB works relatively well for total and limited phase-space bins in rapidity and \( k \) decreases with energy.

There are many similarities between pp and e⁺e⁻ but there are also important differences. At the same \( \sqrt{s} \), the mean number of particles and \( k \) are significantly larger in e⁺e⁻ than in pp. That is, the mean number of particles in pp at a given \( \sqrt{s} \) is given by the mean number of particles in e⁺e⁻ at 0.35 \( \sqrt{s} \), plus two leading protons. This formula works surprisingly well from 30 to 1800 GeV (see Fig. 10 in Ref. [1]) and it remains to be verified at the LHC energy. More recently, certain similarities between e⁺e⁻ and ultra-relativistic Au+Au collisions at RHIC were reported [12].

Eq. (1) is suggestive of a universal mechanism of particle production in both systems, controlled mainly by the actual energy deposited into particle creation [7]. In e⁺e⁻ all available energy is consumed by produced particles, whereas in pp the effective en-
energy coming into particle production is given by\(^3\)
\[
E_{\text{eff}}^2 = (p_1 + p_2 - q_1 - q_2)^2 \\
\approx s (1 - x_1) (1 - x_2),
\]
(2)
where \(p_i\) and \(q_i\) are the incoming and the leading proton momenta, respectively. \(x_i\) is a fraction of the longitudinal momentum carried by a leading proton, \(x_i = q_i \cdot p_i\), and \(s = (p_1 + p_2)^2\).

It this Letter we show that Eq. (1) can be extended to the whole multiplicity distribution. We demonstrate that NB with small values of \(k\) in \(pp\) results from \(e^+e^-\) once the effective energy in \(pp\) is taken into account.

2. Leading protons

The problem of multiplicity distribution is naturally more complicated than the mean number of particles. To proceed we need to specify the leading proton \(x\) distribution. We choose the beta distribution
\[
f(x) \propto x^\lambda (1 - x)^\mu.
\]
(3)
It is supported by rather limited experimental results \([13, 14]\), however it seems a natural first choice. Having (3) we obtain
\[
\langle x \rangle = \frac{1 + \lambda}{2 + \lambda + \mu},
\]
(4)
which is the average momentum taken by a leading proton.

Next we would like to clarify how \(f(x)\) is related to Eq. (1). We obtain\(^4\)
\[
N_{pp}(\sqrt{s}) = \int f(x_1) f(x_2) N_{ee}(E_{\text{eff}}) dx_1 dx_2 + 2.
\]
(5)
Taking
\[
N_{ee}(\sqrt{s}) = a + b \cdot s^\alpha,
\]
(6)
where \(\alpha \approx 0.17\) \([1]\) we arrive at
\[
N_{pp}(\sqrt{s}) = N_{ee}(\langle (1 - x)^\alpha \rangle^{1/\alpha} \sqrt{s}) + 2.
\]
(7)
It means that 0.35 from Eq. (1) is not related to \(\langle 1 - x \rangle\), as naively expected, but to \(\langle (1 - x)^\alpha \rangle^{1/\alpha}\). The latter can be calculated analytically leading to the following equation
\[
\langle (1 - x)^\alpha \rangle = \frac{\Gamma(1 + \alpha + \mu) \Gamma(2 + \lambda + \mu)}{\Gamma(1 + \mu) \Gamma(2 + \alpha + \lambda + \mu)} = 0.35^\alpha,
\]
(8)
which constrains the possible parameters of the beta distribution. Assuming \(\langle x \rangle = 0.4\) \([14]\), see Eq. (4), we obtain \(\lambda \approx -0.8\) and \(\mu = -0.7\), which fully determines \(f(x)\).

3. Calculations and results

The multiplicity distribution in \(pp\) collisions, \(P_{pp}(n)\), is related to the multiplicity distribution in \(e^+e^-\) interactions, \(P_{ee}(n)\), as
\[
P_{pp}(n; \sqrt{s}) = \int f(x_1) f(x_2) P_{ee}(n - 2; E_{\text{eff}}) dx_1 dx_2,
\]
(9)
where \(n \geq 2\). This equation is a straightforward generalization of Eq. (5). Instead of directly calculating the integral (5) we performed our calculations as follows.

First we sampled \(x_1\) and \(x_2\) of two leading protons from the beta distribution, \(f(x)\), with \(\langle x \rangle = 0.4\) and \(\lambda = -0.8\). This allows to calculate the effective energy\(^5\), \(E_{\text{eff}}\), available for particle production in \(pp\). Our choice of \(\langle x \rangle\) and \(\lambda\) ensures that Eq. (1) is satisfied with the right coefficient 0.35.

Next we sampled the number of particles from the multiplicity distribution measured in \(e^+e^-\) collisions at \(\sqrt{s} = E_{\text{eff}}\). We take NB with the mean given by Eq. (6), where \(a = -2.65\), \(b = 5.01\), \(\alpha = 0.17\) and \(k^{-1} = c + d \ln(\sqrt{s})\), where \(c = -0.066\) and \(d = 0.024\) \([1]\).\(^6\)

On top of that we add two particles corresponding to the leading protons.

We performed our calculations at \(\sqrt{s} = 30, 200, 900, 1800, 7000\) and \(14000\) GeV. The results are

\(^3\)In our calculations we use the exact formula for \(E_{\text{eff}}^2\) given by \(\sqrt{s} - \sqrt{m^2 + (x_1 p_1)^2} - \sqrt{m^2 + (x_2 p_2)^2} = p_1^2 (x_1 - x_2)^2\), where \(p_2^2 = s/4 - m^2\) and \(m\) is a proton mass.

\(^4\)In Ref. \([9]\) it was found that \(x\)'s of two leading protons are uncorrelated.

\(^5\)We accept only those events where \(E_{\text{eff}} > 0.3\) GeV so that at least two pions can be produced.

\(^6\)Negative \(k\) is rounded to the nearest integer value. NB with a negative integer \(k\) becomes binomial distribution with the number of trials \(-k\) and the Bernoulli success probability \(1 - N_{ee}/k\). We also checked that the Poisson distribution for \(k < 0\) leads to practically the same results.
shown in Fig. 1, where the calculated multiplicity distributions in \( pp \) collisions (open symbols) are compared with NB fits. It is worth mentioning that Eq. (1) is satisfied by construction so our multiplicity distributions have the correct mean values, see Tab. 1. The crucial test of our approach is the value of \( k \), which we calculate as

\[
k = \frac{\langle N \rangle^2}{\langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle},
\]

where \( \langle N \rangle = N_{pp} \). In Tab. 2 we list the obtained values of \( k \) and compare them with the data. Taking into account the simplicity of our approach, the agreement is satisfactory.

4. Comments

Several comments are in order.

(i) We do not offer any explanation of multiplicity distributions in \( e^+e^- \) collisions. Our goal was to demonstrate that the problem of multiplicity distributions in \( pp \) could be reduced to \( e^+e^- \) once the fluctuating energy carried away by two leading protons is taken into account.

(ii) In this paper we focused on the total phase-space multiplicity distributions. It is plausible that, as assumed in this Letter, the total number of particles is determined (mostly) by the amount of available energy. This is not obvious for limited phase-space bins since the distribution of particles in transverse momentum or rapidity may be modified by some nontrivial dynamics. This problem is under investigation.

(iii) The main uncertainty of our approach is the leading proton \( x \) distribution given in Eq. (3). This form is partly supported by existing data, however

| \( \sqrt{s} \) [GeV] | \( N_{pp} \) (model) | \( N_{pp} \) (data) |
|-------------------|-------------------|-------------------|
| 30.4              | 11.4              | 10.54 ± 0.14      |
| 200               | 21.1              | 21.4 ± 0.6        |
| 900               | 34.6              | 35.6 ± 1.1        |
| 1800              | 43.7              | 45 ± 1.5          |
| 7000              | 69.1              | –                 |
| 14000             | 87.5              | –                 |

Table 1: Calculated mean number of charged particles in \( pp \) collisions compared with the experimental data.

| \( \sqrt{s} \) [GeV] | \( k \) (model) | \( k \) (data) |
|-------------------|----------------|----------------|
| 30.4              | 12.7           | 9.2 ± 0.9      |
| 200               | 5.8            | 4.8 ± 0.4      |
| 900               | 4.2            | 3.7 ± 0.3      |
| 1800              | 3.8            | 3.1 ± 0.1      |
| 7000              | 3.3            | –              |
| 14000             | 3.0            | –              |

Table 2: \( k \) parameters of calculated multiplicity distributions in \( pp \) collisions from Fig. 1 compared with the experimental data.
at rather limited energies and ranges of $x$. Thus it should be considered more as an educated guess. In addition, we assumed that $f(x)$ is energy independent, which is not proven experimentally. A mild energy dependence is indicated by theoretical studies of [10].

(iv) As seen in Fig. 1 the NB fits overestimate calculated multiplicity distributions for higher values of $N$. Interestingly, similar trend is seen in experimental data, see, e.g., Fig. 6 in Ref. [1] and Figs. 3−5 in Ref. [15].

(v) In this paper we extrapolate $N_{el}(\sqrt{s})$ into higher energies using 3NLO QCD result [1, 16], which is almost identical to NLO QCD fit and Eq. (6). For $k^{-1}$ we assume it is a linear function of $\ln(\sqrt{s})$ up to the LHC energies.

(vi) It would be interesting to extend our study on different colliding systems including proton-nucleus and nucleus-nucleus collisions [17].

5. Conclusions

In conclusion, we argued that the multiplicity distribution in $pp$ collisions is directly related to the multiplicity distribution in $e^+e^-$ interactions, once the leading proton effect in $pp$ is properly accounted. In $pp$ a large fraction of initial energy, roughly 0.5 on average, is carried away by two leading protons and is not available for particle production. This component fluctuates from event to event, which results in a broader multiplicity distribution in $pp$ than in $e^+e^-$. We provide a new argument in favor of a common mechanism of particle production in both systems, which is mainly controlled by the amount of energy available for particle production.

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