Understanding Beth, the Particulate Mass Functional

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ABSTRACT: A geometric relationship between loop quantum gravity and partitioned (triangulated) string theory is discussed. Combinatorial analysis reveals that three spatial and three curvature dimensions, intrinsic to the partitioned string, are necessary to replicate Standard Model particles and interactions. This analysis has established that particulate mass is determined by a functional relationship involving these six extra dimensions. The combinatorial analysis involves non-commutative 3D-matrix algebra which forms the mathematical underpinnings of Dirac notation. The functional relationship (symbolized by Beth) requires exponential, Randall-Sundrum, scaling to compute mass. Through the proper interpretation of complex gravity a cyclic cosmological model is developed. This formulation of cyclic cosmology inherently involves observed dark energy. Thus, a comprehensive theory is constructed from geometric fundamentals which models both massive, oscillating neutrinos and the current epoch of mini-inflation.

KEYWORDS: Models of Quantum Gravity, Black Holes in String Theory, M(atrix) Theories, Cosmology of Theories beyond the SM.
1. Introduction

Physical insight, regardless of the technical area, often takes the form of diagrams upon which appropriate mathematical formulations are applied. The appropriateness of the mathematics is determined by its ability to produce calculable predictions which may be confirmed by experiment. From Newton to Feynmann, many deep understandings of physics have been developed and communicated using diagrams constructed from fundamental principles. A great deal of promising mathematical exploration and tool-building has occurred recently in high energy physics and cosmology. These efforts have yet to reveal the fundamental principles upon which to build a successful, comprehensive theory.

This paper takes the approach that particle theory must be constructed from objects which are fundamental (or minimal) in four-space, much as a construct of three points is minimal in two-space. Further, Standard Model particle properties are expected to be identifiable with geometric properties intrinsic to the theoretical constructs representing quarks and leptons. Gravitation, or particulate mass, must also be intrinsic to the theoretical construct, as well as separable from the other particle properties. This requires that the theory provide a comprehensive explanation of cosmology and astrophysical objects, particularly black holes. The resulting theoretical architecture would thus constitute a 'theory of everything'. The approach is not expected to yield a wholly new extension of mathematics, but rather insight into how (and which) mathematical tools are applicable and how to assemble them with reasonable expectation of determining that they are appropriate.

Extrapolating from the minimal object in two-space, one observes that a tetrahedral construction of four points is minimal in three-space - an object that doesn’t experience time in the usual sense. A triangular construction, spinning about an axis, is minimal in four-space. Such a construction is identified with a gravitational quantum, by noting its geometrical relation to the basis for loop Quantum Gravity (QG). Such a simple construct would yield a theory which is clearly insufficient to explain the large variety of fundamental particles necessary to (at least) replicate the Standard Model. A spinning triangle can be quantized by considering orthogonal spin axes, one along an edge and one perpendicular to it. Thus any spinning triangle could be resolved as a superposition, or mixing, of these two fundamental objects. One could consider a construction of pairs of such objects, although that architecture also is insufficient to (at least) replicate the Standard Model.

The correct combinatorial algebra was determined by associating the two fundamental 4D-geometric objects with Rishons [1]: "T", spinning perpendicular to an edge, and "V", spinning along an edge. This association exploits the Rishon theory’s ability to replicate the Standard Model combinatorially. The geometry of the composite object is named tripartite, meaning ”having three partitions”. The perimeter is considered string-like, however, in order to intrinsically incorporate particulate mass into the theoretical architecture, the construct must have a calculable cross-sectional area. The construct is, more precisely, a 1-brane whose mass may be determined through application of quantum gravity.

Careful selection of a symbol which can be used to annotate the functional relationship
between the intrinsic dimensions of the tripartite string and its computable mass was called for. The Hebrew character "Beth" was chosen because (a) it implies a new, "constructive", line of mathematical reasoning; and (b) its meaning, "bountiful", implies that a large number of fundamental particles originate through it.

The theoretical architecture and its relationship to major areas of mathematical exploration is discussed beginning with the applicable combinatorial algebra. Much of the language describes diagrams which, of necessity, could not be included. Diagrams of these ideas were presented at conferences held by the Division of Particles and Fields of the American Physical Society in August 2000 and May 2002. The associated transparencies were posted on the DPF conference websites.

2. Beyond Dirac Notation

Following Dirac,[2] it is conventional to express $\alpha$ in terms of generalized Pauli matrices, $\sigma_1, \sigma_2, \sigma_3$, and a set of anti-commuting matrices, $\rho_1, \rho_2, \rho_3$. The time component of Dirac 4-vectors shall be set aside, or separated, in order to establish a non-commutative algebra applicable to Quantum Chromodynamics (QCD). Examination of $SU(3)$ by developing an applicable 3D-matrix algebra representation allows one to define appropriate algebraic restrictions, equivalent to $X SU(2) X U(1)$, which replicate QCD objects and interactions. Let $\sigma_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\rho_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Operating on a generalized 3x3 matrix gives

$$\alpha_1 \begin{pmatrix} abc \\ def \\ ghi \end{pmatrix} = \rho_1 \sigma_1 \begin{pmatrix} abc \\ def \\ ghi \end{pmatrix} = \begin{pmatrix} ghi \\ def \\ abc \end{pmatrix},$$

which has the transpose $\begin{pmatrix} gda \\ heb \\ ifc \end{pmatrix}$. Recognizing this as a 90° rotation of the original matrix leads to the definition $R \equiv \alpha_1^T \equiv (\rho_1 \sigma_1)^T$.

The 3x3x3 matrix algebra notation is oriented on the cubical QCD charge and color symmetry diagram by assigning $R$ to the face opposite the red quark’s vertex and $B, G$ to the faces in a clockwise fashion looking down from the top. Anti-colors are simply labeled $M =$ magenta (anti-green), $C =$ cyan, $Y =$ yellow. The usual rules for employing Dirac notation apply, except that complex notation is not required: antiparticles are modeled by preserving the twist of vertex elements.

An $r\bar{g}b$ proton, $|H_{r\bar{g}b}\rangle$, is then explicitly modeled by $|(G^-CGC^- \cdot C^-YCY^- \cdot Y^-GYG^-)^2\rangle$. A path-integral approach was developed [3] which produces explicit formulae for modeling both strong and weak interactions. The result was an algebra which is less generic than the [4]. The operator $G_{gb} = \langle BR^- | (CB^-C-B)^2 | RB^- \rangle$, ap-

![Figure 1: Charge and QCD symmetries applied to a 3x3x3 matrix.](image)
plied to $|H_{rgb}\rangle$ from the right produces an $rg\bar{b}$ proton. The 3D-matrix notation captures weak interactions just as explicitly as strong interactions, by noting that the 3D-matrix represents only one particle at a time. In this way, the intermediate vector boson may be modeled separately from the particles it interacts with. The general Dirac notation $\langle \alpha | r \rangle \langle r | \beta \rangle$ becomes $\langle CR^{-}MG^{-}R^{2}|(YR^{-}Y^{-}R)^{2}\rangle \langle (RM^{-}R^{-}M)^{2}R^{2}GM^{-}R C^{-}\rangle$ explicitly, one of three similar 3D-matrix operators representing $W^{+}$, $W^{-}$, $Z^{0}$.

The basic elements of 3D-matrix algebra are thus established, with symmetry-breaking restrictions employed to replicate SU(3) X SU(2) X U(1). These restrictions involve operators of the generic form: $(RM^{-}R^{-}M)^{2}$, which would permit a simpler notation to be defined. The notation $D_{3x3x3}$ is defined to mean any Standard Model quantum object or interaction written in the foregoing Dirac-like algebra. Although $D_{3x3x3}$ serves to put QCD and QED on the same mathematical basis, it reveals no ‘new physics’. It simply establishes the noncommutative algebra of quantum particulate states, allowing a ’separable’ commutative algebra to describe mass, time and energy.

3. Extra Dimensions

One can now demonstrate that $D_{3x3x3}$ corresponds one-to-one with the fundamental, tripartite, modes of a string and its interactions. An orthonormal orientation vector is defined such that the $R, G, B$ partitions establish a particle’s chirality. In this regard it is more precisely a 1-brane, closely related to a Typo IIA Orientifold Each string partition intrinsically has both a radial, $(r_{R}, r_{G}, r_{B})$, and a curvature, $(\phi_{R}, \phi_{G}, \phi_{B})$, dimension. The color of a quark is defined in accordance with the orientation of its spin in relation to the colored partitions. Since, by construction, there is two of one type ‘Rishon’ and one of another, it is the odd one which establishes QCD color for the construct. In this way a red up quark could be labeled VTT, by defining the positions in this variant of Rishon notation in RGB order. One can now complete the correspondence to the tripartite string’s intrinsic geometry as discussed in the introduction. The electric charge of quarks and leptons in the construct is easily replicated by identifying the ”$T$” partition as having a 1/3 electric charge. Identifying mass with the conformal area of the 1-brane completes the identification of particle properies as intrinsic properties of the construct.

Conformal area of the partitioned or triangulated string is naturally a functional relation of the extra spatial and curvature dimensions. Exponential scaling of mass has been examined by Randall-Sundrum theory and is incorporated in applicable QG models. Thus it is logical to consider the functional relation $\lambda \equiv \langle Beth(r_{R}, r_{G}, r_{B}, \phi_{R}, \phi_{G}, \phi_{B}) \rangle$ when calculating particulate mass. Further, the explicit formulation of $D_{3x3x3}$ requires that standard calculations include a sum over all chromodymanic states of hadrons as well as standard tree-level diagrams to account for interactions. This then leads to the relation:

$$m = \sum_{trees} \sum_{states} G_{\mu \nu} e^{-\lambda} | D_{3x3x3} \rangle,$$  \hspace{1cm} (3.1)
which is valid for particles and condensed matter where $\sum |D_{3x3x3}| \rightarrow 1$.

4. Quantum Gravity

The method of calculating particulate mass using quantum gravity triangulation of a 2D surface is naturally applicable [8] to the three partitions of the string when it is regarded as the edge of a zero-thickness membrane surface. In this construct, energy is represented by string tension, which is altered when a photon is added. Thus a photon must be an open (massless) string whose energy causes a change in string tension when it interacts with any particle. The photon can now be considered to mediate gravity as follows: a change in string tension calls for equivalent, quantized change to partition radii ($r_R, r_G, r_B$) and intrinsic curvature angles ($\phi_R, \phi_G, \phi_B$). The mass or energy of the interacting particle may be affected depending on whether the altered quanta combine to affect the conformal area of the closed 1-brane.

A given particle will have a unique 'shape', ($r_R, r_G, r_B, \phi_R, \phi_G, \phi_B$), or low-energy string mode, as determined by its spin and substructure. The intrinsic shape is analogous to determining the electron orbital shapes from QED and spherical harmonics. Analysis to resolve this nonperturbative string shape exactly must be accomplished in light of the fundamental geometric construction as outlined above. The case of weak interactions in which the identity, and mass, of the interacting particles is changed, requires that the mass functional Beth be commutative. Thus Beth is also dependant on the non-commutative algebra required to model QC/ED interactions. An intermediating particle also acquires mass from its intrinsic geometry, although the geometry is quite different in the case of a gluon.

Electro-weak symmetry requires that massive bosons interact much like the photon in that they cause a quantum change in each interacting strings intrinsic curvature. They also must change the sign of the intrinsic curvature, $\phi$, in relation to spin axis, since a quantum change is required to change a particle's identity. Dimensional analysis reveals that weak interactions are mediated by bosons comprised of two tripartite string quarks in an intrinsically bound state. Such a tightly bound state doesn't require another intermediating particle, so the compositeness is hidden.

Strong interactions of the tripartite string would require that each quark change its spin axis in order to change color-identity. The partitioned string model calls for such interacting quarks to stretch into a relatively long and thin 'dog-bone' configuration, since the dimensions of the quarks are much less than their interaction range. Considering that the area of such a configuration is dominated by the thin ribbon-like area attributed to the gluon supports the observations that the proton's mass is mostly attributable to gluons. The architecture of this theory is directly comparable to the modified Lund theory, which uses an open string with thickness to model mass.

The QG-partitioned string has a fundamental combinatorial state which serves to model the neutrino (VVV). One should note that any closed loop has a torsion-free, preferred, coiled state which can be called a trecoil. A trecoil is similar to a trefoil knot, except that
a trefoil knot requires that the string be broken and re-attached. In this construction, a
trefoil knot represents a 'sterile' neutrino. Considering the entrained area of the trefoil string
state to model its mass explains why the neutrino’s mass is so small. Further, it is easy
to construct simple variations in which each third-coil is unfolded to create a more massive,
higher-generation, state of the neutrino. In this way, massive, oscillating neutrinos are a
natural part of the model. [9]

The construction of generalized string interaction vertices has been shown to imply that
strings have three partitions [10]. Construction of a four-string interaction involves a tetrahe-
dron at the vertex - which may be interpreted as a Higg’s Boson, given its role in mediating
certain interactions. Causality is preserved in this theory [11], since both mass and time are
derived from a commutative operation.

The membrane surface is also a mathematical discontinuity in space-time: when a particle
passes across it, it actually experiences $R \leftrightarrow \alpha'/R$. The theory thereby includes a natural
model of antimatter, which is created during interactions when a partition of an interacting
matter string is ‘pushed’ across this 4-space discontinuity. The existence and function of this
discontinuity has important implications for both astrophysics and cosmology.

5. Astrophysics

Standard astrophysical theory includes a mathematical singularity at the center of all black
holes. However, M/string theory clearly excludes such singularities from any unified model.
Two distinct lines of reasoning say that a 'physical singularity', constructed as a Planck-
scale tetrahedron, resolves the dilemma. (1) particles impinging on a black hole experience
a 'slowing' of time, due to the extreme curvature of space-time. A central singularity or
discontinuity exists at which time 'stops' altogether. A tetrahedral construction is deduced
such that the singularity within a black hole is a minimal object in three-space. Note that a
tripartite string is not assumed, but necessary as a facet of the central tetrahedral discontinu-
ity. (2) Any standard model of a black hole (with a mathematical singularity) is bi-laterally
symmetric, whereas one with a tetrahedral kernel is not. Matter which is relatively stopped
in time has an observable motion relative to the black hole and thus our space-time. The
Galactic Annihilation Fountain [12] and other artifacts are clearly bi-laterally asymmetric,
and so qualitatively support the construct of a tetrahedral kernel over theories including a
mathematical singularity. Maldacena’s conjecture should be developed, beyond proving con-
sistency with R-S scaling [13], to understand the mathematics of AdS/CFT at a tetrahedral
’interface’.

6. Cosmology

An exponential (R-S) mass scale yields a cyclic cosmological model through consider-
ation of complex gravity [14], which is inherent to string theory. String theory tells us
that the usual gravitational tensor has an anti-symmetric component which is important
over cosmological time-scales. Including the anti-symmetric tensor in equation (3.1) yields: 
\[ \sum_U (G_{\mu\nu} + iB_{\mu\nu})e^{-\lambda}. \]

We now recognize that applying Euler’s relation to this formula yields an equation having the form \( \Omega_m + \Omega_\Lambda = 1; \) where \( \Omega_m \) is again area-like and both terms are scaled exponentially as in R-S theory. This approach yields a cyclic cosmology that incorporates the string theoretic result \( R \leftrightarrow \alpha'/R \) [15]. This important relation helps explain how the initial state of the universe came about. In an ‘inverted’ space-time, the vast scale of the cosmos equates a sub-Planck scale state ‘preceding’ a ‘spherically-inverted’ big bang. This theory also inherently includes the observed [16] cosmological ‘constant’ as an extremely slowly-varying parameter [17]. The theoretical requirement for an imaginary time-like metric, \( e^{i\theta_m,t - \lambda} \), permits cosmological boundary conditions to be established which select the observed set of physical laws. This formulation also circumvents the coincidence problem through a simple mechanism relating \( \Omega_m \) to \( \Omega_\Lambda \) at all times [18].

7. Observation and Experiment

This work qualitatively explains many recent observations [9, 16, 12] in a comprehensive fashion that is consistent with developing modern theories. Although the standard model may be able to be modified or interpreted to explain these observations separately, the unified approach given here has the added benefit of being based on geometric fundamentals. Experimentation with high-energy electron-positron collisions should shed some light on the theorized shape of an electron. Ongoing research and observation programs in the areas of oscillating neutrinos, galactic annihilation radiation and cosmology are likely to support this comprehensive theoretical approach.

8. Conclusion

The formula \( e^{i\theta_m,t - \lambda} \mid D_{3x3x3} \) provides the basis for a comprehensive theory of particle physics and cosmology. Further work to refine the theory and calculate its effects should be within the realm of modern mathematics.
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