Enhancing students’ mathematical communication ability through problem-centered learning (PCL) approach with scaffolding strategy

Tedy Machmud
Mathematics Department, Faculty of Mathematics and Science, Gorontalo State University, Indonesia
*tedy_m@ung.ac.id

Abstract. The aim of experimental research is to examine the enhancement of students’ mathematical communication abilities after they worked under Problem-Centered Learning approach with scaffolding strategy (PCLSS). Three schools are selected by stratified sampling technique in the Gorontalo City junior high school student, as a sample representing school level: high, medium and low. In each school two classes selected: one group as an experiment group treated by PCLSS approach, and one group as a control group with conventional treatment. The research instrument was students’ mathematical communication ability (MCA) pretest and posttest. Through parametric and non-parametric statistical analysis, informations obtained were: (1) students’ MCA who received PCLSS obtain higher enhancement than students who received conventional teaching approach, both in terms of the level of schools, mathematical prior ability (MPA) and the overall students. All categories show a significant difference in term of communication skill, (2) there is an interaction between learning approaches and school levels to ward the enhancement of student’s MCA, (3) there is no interaction between MPA and learning approach toward the enhancement of student’s MCA.

1. Preliminary
Mathematical communication is one of the six standard process described by NCTM, 2000 [1], so mathematical communication ability (MCA) is important to developed in schools especially in teaching and learning activities in mathematics classrooms. Lindquist and Elliot [2] explained that mathematical communication played an important role in forming the framework for a mathematically literate workforce, lifelong learning.

In addition, mathematical communication is an essential part of learning process of mathematics and mathematical problem solving. This is quite reasonable because during the learning of mathematics, students will be facing language and mathematics symbols, as well as solving mathematics problems. Students need to relate information ranging from concrete to abstract situation. Through effective communication students can organize, consolidate and clarify their mathematical thinking and explain to other students, teachers and others. In addition, students are able to analyze and evaluate mathematical thinking and strategies presented by other people. One of the efforts made to improve students’ MCA is the implementation of the Problem-Centered Learning approach along with scaffolding strategy (PCLSS).
2. Mathematical Communication Ability and Problem-Centered Learning (PCL) Approach with Scaffolding Strategy

2.1. The Importance of Mathematical Communication Ability

Baroody [2] proposes two more reasons for focusing on mathematical communication. Firstly, mathematics is essentially a language by itself. Mathematics is not only a thinking tool that helps us to discover patterns, solving problems and drawing conclusions, but also a tool for communicating our thought, a variety of ideas clearly, precisely and succinctly. In fact, mathematics is considered “the universal language” with its unique symbols and structures. People all over the world can use it to communicate mathematical information despite differences in their native languages.

Secondly, mathematics teaching and learning are social activities that involve at least two parties, teachers and pupils. In the process of teaching and learning, it is crucial that thought and ideas are communicated to others through language. Communicating with peers is essential for the development of communication skills so as to talk about their ideas is an excellent way for them to discover gaps, inconsistencies, or lack of clarity in their thinking. Suryadi [3] describes the activity of discussion in mathematics learning will allow for the exchange of experience, the emergence of problem-solving strategies, the emergence of other findings which they produce, as well as to listen to what other people found and discussed, students are allowed to increase their own learning strategies.

Communication is important to be developed in learning mathematics because if not then it will become an obstacle for the development of mathematical activities (doing match) and can be a source of students’ failure and unhappiness in learning mathematics. Even Sfard [4] stated that thinking is internalized communication. In other words, it can be said that the person who thinks, he is actually learning or communicating.

The importance of communication skills can also be explained from the perspective of social constructivism of Vygotsky [5] which states that learning can takes place when the learner collaborate with an adult or peer who has competences over his or her zone of proximal development (ZPD). The results obtained from the social context is then internalized into shape as individual knowledge acquisition. This process can occur through discussion activities with adults (teachers) and peers (students), so that students can gain a better understanding of the mathematical concepts that he/she learned from solving problem specifically solving mathematical activities.

2.2. Problem-Centered Learning (PCL) Approach with Scaffolding Strategy

Wheatley [6] divided PCL, into three components, namely tasks, group activities, and sharing. This approach begins with setting up a class with assignments such as problem-solving task for the student. The next activity is groups of students, each working in small group and encouraging them to collaborate. After a discussion in each group, the learning activity is followed by class discussions. In this discussion each group presents the material that has been discussed in the group discussion. Hopefully, through this class discussions student can share their opinions to produce a solution to solve the problem at hand can occur.

Wheatley [7] highlights some of the activities that characterize specific PCL, among other things as follows:

a. PCL focuses on the process of inquiry and reasoning in problem solving rather than on getting the correct result of an experiment or the right answer to a problem question;

b. PCL focuses on the importance of communication in learning because all activities are carried out by students working in cooperative and collaborative groups;

In the context of PCL students are exposed to the real problems that they may initially find it difficult to understand the concepts involved in solving problems/issues, such as to solve the problem/issue which required high the creativity of the students. In these circumstances the presence of scaffolding is necessary, for opening the understanding and solving problems/issues. Teachers’ scaffolding can be implemented in each session of PCL approach component activity.
In a learning context there are some form of scaffolding that can be done through: modeling, bridging, and schema building [8]. The definition of modeling is scaffolding in the form of examples or solving mathematical models. From a given sample or model, students can then compare, analyze, interpret and evaluate the context of mathematical problems that it faces. Bridging is a form of scaffolding undertaken by reviving students' knowledge and understanding of the concept of something that already exists. Schema building is the scaffolding in the form of schematic, diagram illustration of the problem situation, it can also be a network of concepts related to the problem situation.

Other forms of scaffolding is also expressed by Roehlar and Cantlon [9] in their study, which include: Inviting student participation, offering explanation, and verifying and clarifying students' understanding. Inviting student participation is a form of scaffolding that is given with the intention to invite students to actively participate in learning through the efforts of teachers to student participation. Offering explanation is a form of scaffolding that refers to a statement corresponding to the understanding of the emerging concept of what the student has learned, why and when the concept was used and how the concept is used. Verifying and classifying students' understandings is a form of scaffolding related to the verification of the emerging understanding of the students. If the emerging understanding makes sense, the teacher verifies that response, but if the understanding seems unreasonable, teacher does a clarification.

Associated with PCL approach, mathematical communication is an important focus in the PCL approach. This is because the characteristics of the PCL approach is the negotiation of meaning, discussion, group work in a cooperative and collaborative setting. Negotiating the meaning would encourage students to try to understand and interpret mathematical situations and tasks at hand and communicate the meaning of the situation and the task to other friends and teachers.

The problems to be revealed in this paper is whether the students' MCA receiving PCLSS learning approach is better than the students who received learning with the conventional learning (CL) approach, in terms of schools, Mathematical Prior Ability (MPA) and the students as a whole? Whether there is interaction between factors of learning with the school factor for student MCA, and whether there is interaction between factors of learning with students' MPA factors MCA students?

3. Method
This research is a quasi experimental design research. In this design, the experimental group was given PCLSS approach, and the control group were given a Conventional Learning (CL). Each class was given a pretest and posttest (O), and no special treatment was given to the control class. To see the impact of learning approaches more specifically to MCA then in this study two control variables namely School Level (top, middle, bottom) and students' MPA (high, medium, low) are observed.

The subject population in this study were all eighth grade junior high school students Gorontalo city. By using stratified sampling techniques, a top, a middle, and a low school level were determined. Each school is designated as a subject class VIII student sample, and the random technique group (cluster random sampling) selected an experimental class and a control class. All students who were involved in this research as a whole is 151 students.

The independent variable in this study is PCLSS and CL approaches, while the dependent variable is mathematical communication skills on Pythagorean Theorem and Geometry topics for eight grade class, measured through indicators of student abilities including, expressing, illustrating and explaining ideas, situations, relationships and mathematical representations in writing, or vice versa. The indicators of mathematical communication activity can be traced, among others through the description evoking responses of students when trying to understand and explain an idea, situation, relationships and create mathematical representations in writing, or otherwise, in the form of a communication rubric for mathematics subjects. A rubric for mathematics subject that has been developed is Math Communication Rubric [10]. It uses a holistic rubric scale (score 0-4) with emphasis on levels of effectiveness, accuracy and completeness in the use of mathematical language (terms, symbols, signs and or representation) to illustrate the ideas, situations, relationships, concepts and processes of the problem/ question in written.
Data obtained from the pretest and posttest results were analyzed to determine the magnitude of the enhancement of experiment and control class students’ mathematical communication ability. The enhancement is calculated by the normalized gain formula, namely:

\[
g = \frac{\text{posttest score} - \text{pretest score}}{\text{maximum possible score} - \text{pretest score}}
\]

Hake [11]

4. Results and Discussions

4.1. Students’ MCA Analysis According to The School Level, The MPA, The Whole Students and The Learning Approaches

Descriptive statistical analysis of data that includes students’ MCA minimum score, maximum score, the mean of pretest, posttest, n-gain, and the standard deviation in terms of school level, students’ mathematical prior ability, and learning approach based on overall students is presented in Table 1.

| Data Group | Learning N | Data | Score Min. | Mean | Standard Deviation |
|------------|------------|------|------------|------|--------------------|
| Top PCLSS 22 | Pre 5.00 20.00 11.932 4.875 | | | |
| | Post 25.00 35.00 28.523 3.421 | | | |
| | n-Gain 0.455 0.778 0.597 0.086 | | | |
| | CL 23 Pre 5.00 17.50 10.870 3.422 | | | |
| | Post 20.00 35.00 24.348 3.632 | | | |
| | n-Gain 0.333 0.800 0.467 0.096 | | | |
| Middle School Level | PCLSS 29 Pre 5.00 15.00 8.917 3.127 | | | |
| | Post 20.00 35.00 25.948 4.402 | | | |
| | n-Gain 0.333 0.800 0.549 0.107 | | | |
| | CL 30 Pre 5.00 15.00 8.917 3.127 | | | |
| | Post 10.00 30.00 17.333 6.789 | | | |
| | n-Gain 0.077 0.636 0.283 0.170 | | | |
| Bottom PCLSS 23 | Pre 5.00 12.50 8.804 3.003 | | | |
| | Post 15.00 27.50 22.609 3.411 | | | |
| | n-Gain 0.286 0.545 0.446 0.066 | | | |
| | CL 24 Pre 5.00 15.00 8.958 3.451 | | | |
| | Post 7.50 27.50 15.938 5.307 | | | |
| | n-Gain 0.000 0.500 0.229 0.125 | | | |
| High PCLSS 17 | Pre 12.5 20 15.441 2.686 | | | |
| | Post 25 35 30.294 3.293 | | | |
| | n-Gain 0.455 0.800 0.611 0.108 | | | |
| | CL 16 Pre 12.5 17.5 14.063 1.548 | | | |
| | Post 20 35 26.406 4.180 | | | |
| | n-Gain 0.200 0.800 0.475 0.162 | | | |
| MPA | PCLSS 38 Pre 5 17.5 9.342 2.768 | | | |
| | Post 20 35 25.132 3.533 | | | |
| | n-Gain 0.333 0.778 0.518 0.097 | | | |
| | CL 46 Pre 5 15 8.967 2.504 | | | |
| | Post 7.5 27.5 17.826 5.313 | | | |
| | n-Gain 0.000 0.500 0.290 0.143 | | | |
| Medium Low PCLSS 19 | Pre 5 12.5 6.316 2.103 | | | |
| | Post 15 27.5 22.632 3.771 | | | |
Referring to Table 1, it can be described that: firstly, by the quality of students 'MCA based on the school level (top, middle and bottom), MPA (high, medium and low) and the students' overall gain PCLSS learning approach, as well as students were obtained with a CL study in early learning is relatively the same treatment, and secondly, in each category, students’ MCA quality who received PCLSS better than who received CL. The results of comparison test to see equality MCA students in each category are described as follows.

The Summary of comparison test to see MCA’s difference between students who received PCLSS learning approach and students who received CL, in terms of school level, is presented in Table 2.

Table 2. Summary of Differences Test Data Output n-Gain MCA Judging from Level Schools and Learning Approach

| School Level | Learning Approach | n-Gain Mean | Statistical Test | Sig. (2-tailed) | Decision |
|--------------|-------------------|-------------|------------------|-----------------|----------|
| Top          | PCLSS             | 0.597       | t-Test           | 4.777           | 0.000    | Rejected |
|              | CL                | 0.467       | Mann-Whitney-Test|                 |          |          |
| Middle       | PCLSS             | 0.549       |                 | 102.000         | 0.000    | Rejected |
|              | CL                | 0.283       |                 |                 |          |          |
| Bottom       | PCLSS             | 0.447       |                 | 30.500          | 0.000    | Rejected |
|              | CL                | 0.229       |                 |                 |          |          |

**H0:** There is no difference MCA’s n-gain mean score between the students who received PCLSS approach and the students who received CL approach on each school level

The Summary of comparison test to see MCA’s difference between students who received PCLSS learning approach and students who received CL, in terms of students’ MPA, is presented in Table 3.

Table 3. Summary of Differences Test Data Output n-Gain MCA Judging from students’ MPA and Learning Approach

| MPA Level | Learning Approach | n-Gain Mean | Statistical Test | Sig. (2-tailed) | Decision |
|-----------|-------------------|-------------|------------------|-----------------|----------|
| High      | PCLSS             | 0.611       | 2.837            | 0.008           | Rejected |
|           | CL                | 0.475       |                  |                 |          |          |
| Medium    | PCLSS             | 0.518       | 149.500          | 0.000           | Rejected |
|           | CL                | 0.290       |                  |                 |          |          |
| Low       | PCLSS             | 0.487       | 35.500           | 0.000           | Rejected |
|           | CL                | 0.253       |                  |                 |          |          |

**H0:** There is no difference MCA’s n-gain mean score between the students who received PCLSS approach and the students who received CL approach on each students MPA level

Maximum Score = 40

| CL 15 | n-Gain Pre | 5 | 12.5 | 6.333 | 2.085 |
|-------|------------|---|------|-------|-------|
| Post  | 7.5 | 27.5 | 14.667 | 5.891 |
| PCLSS 74 | n-Gain Pre | 15.00 | 35.00 | 25.676 | 4.443 |
| Post  | 7.5 | 27.5 | 14.667 | 5.891 |
| CL 77 | n-Gain Pre | 5.00 | 17.50 | 9.513 | 3.394 |
| Post  | 7.5 | 27.5 | 14.667 | 5.891 |
The Summary of comparison test to see MCA’s difference between students who received PCLSS learning approach and students who received CL, in terms of the whole students, is presented in Table 4.

| Learning Approach | n-Gain Mean | Mann-Whitney Test | Sig. (2-tailed) | Decision |
|-------------------|-------------|-------------------|-----------------|----------|
| PCLSS             | 0.531       | 844.000           | 0.000           | Rejected |
| CL                | 0.321       |                   |                 |          |

\( H_0: \) There is no difference between n-gain mean score of Students’ MCA between the students who received PCLSS and the students who received CL on the whole students

4.2. Interaction between Learning Approach and School Level on the Enhancement of Students’ MCA

Summary of the analysis is presented in Table 5.

| Data Source       | Sum of Square | Sd | Mean of Sum of Square | F     | Sig.     |
|-------------------|---------------|----|-----------------------|-------|----------|
| Intercept         | 27.338        | 1  | 27.338                | 2005.980 | 0.000   |
| Learning Approach | 1.555         | 1  | 1.555                 | 114.129 | 0.000    |
| School Level      | 0.873         | 2  | 0.437                 | 32.036 | 0.000    |
| Approach* School Level | 0.119 | 2 | 0.060 | 4.376 | 0.014 |
| Error             | 1.976         | 145 | 0.014               |       |          |
| Total             | 31.820        | 151 |                      |       |          |

\( H_0: \) There is no Interaction between Learning Approach and School Level on the Enhancement of Students’ MCA

Based on Table 5 it can be concluded that the learning approach contributes significantly to the differences of students’ MCA. Similarly school level contributes significantly to the differences of students’ MCA. According to the probability value (Sig.) to the interaction of learning approach and school level that less than 0.05, \( H_0 \) is rejected. It means that there is an interaction between learning approach and school level for increasing students’ MCA. Thus the learning approach and school level, together interact significantly to the differences of students’ MCA.

Graphically, the interaction between learning approach and school level on enhancement students’ MCA can be seen in Figure 1.
Figure 1. Interaction between learning approach and School level on the Enhancement of Mathematical Communication Ability

4.3. Interaction between Learning Approach and Students’ MPA on the Enhancement of Students’ MCA

Summary of the analysis is presented in Table 6.

Table 6. The Summary of Two-way ANOVA Test Results on the Interaction between Learning Approach and Students’ MPA on the Enhancement of Students’ MCA

| Data Source    | Sum of Square | Sd  | Mean of Sum of Square | F       | Sig. |
|----------------|---------------|-----|-----------------------|---------|------|
| Learning Approach | 24.028        | 1   | 24.028                | 1494.132| 0.000|
| MPA            | 1.238         | 1   | 1.238                 | 77.000  | 0.000|
| Approach* MPA  | 0.597         | 2   | 0.298                 | 18.561  | 0.000|
| Error          | 0.058         | 2   | 0.029                 | 1.811   | 0.167|
| Total          | 2.332         | 145 | 0.016                 |         |      |

H0: There is no Interaction between Learning Approach and MPA on the Enhancement of Students’ MCA.

Based on Table 6 it can be concluded that the learning approach contributes significantly to the differences of students’ MCA. Similarly students’ MPA contributes significantly to the differences of students’ MCA. According to the probability value (Sig.) to the interaction of learning approach and students’ MPA that higher than 0.05, H0 is accepted. It means that there is no interaction between learning approach and students’ MPA for increasing students’ MCA. Thus the learning approach and students’ MPA, together don’t contribute significantly to the differences of students’ MCA.

Graphically, the interaction between learning approach and students’ MPA on enhancement students’ MCA can be seen in Figure 2.
5. Discussion
PCLSS learning approach has facilitated the enhancement of students’ mathematical communication ability. PCLSS syntax starts with task step (giving tasks) with schema building scaffolding strategies facilitating students to develop their ideas and to try to understand a given task. For example, when students are asked to define the relationship between the central angle, arc length and wide pie, raised schema building strategies, which students are asked to draw their own circle of a certain size using a compass, plumb, measure and compare. From the fact that they are doing and see themselves, students are guided to come to a conclusion about the relationship between a central angle, arc length and wide pie.

In the second and third phases of the PCLSS syntax namely the group phase and sharing phase, students are facilitated to communicate their ideas by working in groups, followed by class discussion. In the group work through the tasks presented in each study guide, students are led to sharpen their thinking, explore it from the perspective of each group to look at a variety of mathematical material linkages. Through class discussion, groups created atmosphere of negotiations between the ideas developed in each group as they discuss the assignment, students were led to further organize and consolidate their thinking so that they will obtain the same understanding of the tasks discussed in the class discussion.

Figure 2. Interaction between learning approach and students’ MPA on the Enhancement of Mathematical Communication Ability

Figure 3. Groups Discussion
The impact of learning interventions with PCLSS as described above has contributed to the increasing of mathematical communication ability, although in general the range of magnitude of the increase is in the medium category. This conclusion derived from the facts found in the research as described below.

The results of the data analysis (Table 1) when viewed as a whole, shows that students who acquire PCLSS approach apparently getting higher mathematical communication ability enhancement than students who received learning with conventional approach. When viewed from the school level, students who received PCLSS learning obtain higher mathematical communication ability than students who received conventional learning (Table 2). Student for high level school obtained higher mathematical communication ability enhancement. However, in terms of the students’ MPA, apparently high level group (Table 3), obtained higher mathematical communication ability enhancement than the high school level.

If it is traced from the interaction, it turns out learning approach and school-level factors simultaneously interact significantly to the improvement of mathematical communication ability, as well as if it is viewed as a partial learning approach or school-level factors contribute to the differences in mathematical communication ability. However, if it is viewed from the interaction of learning approach and students’ MPA, apparently there is no interaction between learning approach and students’ MPA to improving mathematical communication ability. If it is only partially observed, learning approach and students’ MPA level remains contribute to differences in mathematical communication ability enhancement. From these tests, it is believed that learning PCLSS approach contributed to the increase of mathematical communication ability, while school level factors and the relative level of students’ MPA contribute to the increase in the degree of difference.

The above analysis is in line with the analysis of Qohar [12] who stated that the results of mathematical communication reciprocal teaching students with learning is significantly better than the results of students’ mathematical communication with conventional learning. Similarly, he stated that there is a significant interaction between learning and school levels for students’ mathematical communication ability, but there was no significant interaction between learning and human to score students’ mathematical communication. Note that reciprocal teaching and learning PCLSS developed based on constructivist paradigm that requires students to construct their own understanding and facilitated by the teacher.

Class discussion activity through PCLSS approach has facilitated students to develop the skills of presenting, convincing arguments in logical construction of the situation and the results of mathematical tasks that are being discussed to other students, and at the same time trying to develop the activities of the students, listened to the construction and interpretation of another student of the situation and mathematical tasks.

Similarly, group work activities through PCLSS, has encouraged students to actively engage, shared and roles in developing ideas, finding and presenting alternative solutions to other mathematical problems.

6. Conclusions and Recommendations

6.1. Conclusions
a. Students who acquire PCLSS approach have mathematical communication ability higher than students who received CL. The differences in students' mathematical communication ability can be observed both in the school level, as well as the overall level of students’ MPA. All categories that compared showed a significant difference.
b. Learning approach and school level, interact significantly toward the differences of students' mathematical communication ability. Similarly, if it is the partial views of the level of school and learning approaches have a significant influence on the differences of mathematical communication ability.
c. Learning approaches and human factors, did not interact significantly to the differences of students' mathematical communication ability. But when viewed as a partial aspect of the learning approach or students’ MPA aspects, there is have a significant differences of mathematical communication ability.

6.2. Recommendations

a. Teachers need to implement this PCLSS learning approach to teach junior high math in an effort to improve students' mathematical communication ability.

b. Teachers need to scrutinize (noticing) the students’ characteristics deeper so that students will be able to provide scaffolding on time and on target.

c. In order to maximize PCLSS implementation in the classroom and maximize the scaffolding and on target, it has to consider the number of students in the class.

d. There are opportunities for other researchers to conduct advanced research in the following matters: How the implementation of this approach in other mathematical topics or at other school levels? How is the impact of the implementation of this approach on improving another mathematical skills?

References

[1] NCTM 2000 Principles and Standards for School Mathematics. [Online] www.nctm.org/standards/overview.htm.

[2] Lim C S & Chew C M 2007 Mathematical Communication in Malaysian Bilingual Classrooms. Paper to be Presented at the 3rd APEC-Tsukuba International Conference: Innovation of Classroom Teaching and Learning through Lesson Study- Focusing on Mathematical Communication, December 9-14, 2007 at Tokyo and Kanazawa, Japan.

[3] Suryadi D 2003 Pengembangan Kemampuan Berpikir Matematika Tingkat Tinggi. SPs UPI Bandung.

[4] Felton M D & Nathan M 2009 Exploring Sfard’s Commognitive Framework: A Review of Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing. Journal for Research in Mathematics Education (JRME), 40 (5): 571-576.

[5] Brenner M E 1998 Development of Mathematical Communication in Problem Solving Groups by Language Minority Students. Bilingual Research Journal, 22:2, 3, & 4 Spring, Summer, & Fall.

[6] Wheatley G H 1993 The Role of Negotiation in Mathematics Learning. Dalam K. Tobin (Ed). The Practice of Constructivism in Science Education 121-134. New Jersey: Lawrence Erlbaum Associates, Inc.

[7] Jakubowski E 1993 Constructing Potential Learning Opportunities in Middle Grades Mathematics. Dalam K. Tobin (Ed). The Practice of Constructivism in Science Education 135-144 New Jersey: Lawrence Erlbaum Associates, Inc.

[8] Walqui A 2006 Scaffolding Instructional for English Language Learners: A Conceptual Framework. The International Journal of Bilingual Education and Bilingualism Vol. 9 No.2.

[9] Roehler L R & Cantlon D J 1997 Scaffolding: A Powerful Tool in Social Constructivist Classroom’s. In K. Hogan & M. Pressley. 1997. (Eds). Scaffolding Student Learning: Instructional Approaches and Issues. Cambridge: Brookline Books, Inc.

[10] Maryland State Department of Education 1991 Sample Activities, Student Responses and Maryland Teachers' Comments on a Sample Task: Mathematics Grade 8, February 1991 Chicago Public Schools Bureau of Student Assessment. [online] http://web.njit.edu/~ronkowitz/teaching/rubrics/samples/math_probsolv_chicago.pdf.

[11] Meltzer D E 2002 The Relationship between Mathematics Preparation and Conceptual Learning Gains in Physics: a Possible “Hidden Variable” in Diagnostic Pretest Scores. Ames, Iowa: Department of Physics and Astronomy. [Online] http://www.physics.iastate.edu/ per/ docs/Addendum_on_normalized_gain.pdf.
[12] Qohar A 2010 *Mengembangkan Kemampuan Pemahaman, Koneksi Dan Komunikasi Matematis Serta Kemandirian Belajar Matematika Siswa SMP Melalui Reciprocal Teaching* Disertation PPS UPI Bandung