Abstract. We present a model independent and non-parametric reconstruction with a Machine Learning algorithm of the redshift evolution of the Cosmic Microwave Background (CMB) temperature from a wide redshift range \( z \in [0, 3] \) without assuming any dark energy model, an adiabatic universe or photon number conservation. In particular we use the genetic algorithms which avoid the dependency on an initial prior or a cosmological fiducial model. Through our reconstruction we constrain new physics at late times. We provide novel and updated estimates on the \( \beta \) parameter from the parametrisation \( T(z) = T_0(1+z)^{1-\beta} \), the duality relation \( \eta(z) \) and the cosmic opacity parameter \( \tau(z) \). Furthermore we place constraints on a spatial varying fine structure constant \( \alpha \), which would have signatures in a broad spectrum of physical phenomena such as the CMB anisotropies. Overall we find no evidence of deviations within the 1\( \sigma \) region from the well established \( \Lambda \)CDM model, thus confirming its predictive potential.
1 Introduction

Our current knowledge for the evolution of the Universe as a whole from the first fraction of a second to our present day, about 13.6 billion years later, rests upon the successful hot Big Bang cosmological model [1]. It is built on the robust theoretical framework of General Relativity (GR) and based on well tested observations such as the expansion of the Universe [2], the relative abundance of light elements [3] and the cosmic microwave background (CMB) [4]. The presence of the latter is considered to be the best indication for a primordial expanding state of the Universe originating from an initial high density state to become an almost perfect isotropic blackbody radiation at a temperature of about 3K and whose emission we receive around 380,000 years after the Big Bang.

The hot Big Bang model predicts that the CMB photon energy is redshifted with the cosmic expansion. In other words, the Universe has a hot and dense past and cools as it expands adiabatically according to the linear average temperature-redshift relation (TRR) of the CMB, \( T_{\text{CMB}}(z) = T_0(1 + z) \) where \( T_0 = (2.72548 \pm 0.00057)\text{K} \) is the local measurement of the CMB temperature today i.e. at \( z = 0 \) [5] and \( T_{\text{CMB}}(z) \) represents the temperature measured by an observer at redshift \( z \). This relation is not confined to a specific metric theory, holding in the framework of GR and the electromagnetic theory of Maxwell under the assumption that photons are massless, the CMB is thermal radiation, the first law of thermodynamics is true and that the expansion of space is isotropic [6]. Although this linear temperature relation is well established [7] and departures from it would require important distortions in the Planck spectrum of the CMB [8], it can be modified for example [9] by adding extra components such as a decaying vacuum energy density or some process of quantum gravitational origin that could affect the adiabatic photon production (or destruction), due to late inflationary models induced by a scalar field, in string theory models where axions and photons could be mixed [10] or theories with deviations from homogeneity and isotropy.
To constrain deviations from adiabatic evolution, the following power-law \( T_{\text{CMB}}(z) = T_0 (1 + z)^{1-\beta} \) is commonly used, where \( \beta \) is the parameter of the theory and \( \beta > 0 \) or \( \beta < 0 \) would be compatible with net photon production or destruction respectively [8]. It is also of great interest models where some fundamental constants are not space-time invariant, such as the fine structure constant \( \alpha \). This effect can be found for example in theories with extra compact dimensions (aiming to unify gravity and other fundamental forces), where the cosmic evolution of the scale factor will have a time dependence on the coupling constants [11]. A different possibility is the inclusion of a new scalar field with couplings to the Maxwell scale factor \( F_{ab}F^{ab} \) whose evolution involves a spatial variation of \( \alpha \) [12].

The Standard Cosmological Model, which contains a cosmological constant \( \Lambda \), Cold Dark Matter (CDM) and is built on the hot Big Bang Theory and the Inflationary Paradigm, has become the best phenomenological description for the current accelerating expansion of the Universe. Yet there exists also plenty of other models that could account for the accelerated expanding Universe without the need of a cosmological constant \( \Lambda \), whose nature still remains unknown, for example through the inclusion of scalar fields in Dark Energy (DE) models or through Modified Gravity (MG) theories. Even though DE and MG theories are driven by different physical backgrounds, it has been shown that both kind of models can be studied on the same ground in an effective fluid approach [13, 14]. One of the first convincing evidences for a cosmic expansion came in 1998 as a result of an unanticipated dimming through the observed light of type Ia supernovae (SNIa) [15]. Although the cosmic acceleration has been asserted through other independent probes like the CMB [16], baryon acoustic oscillations (BAO) [17] or the Hubble parameter [18], the presence of some cosmic opacity that can contribute to astronomical photometric measurements of distant SNe Ia is still an open possibility [19]. As mentioned in Ref. [20], opacity sources could come from the non-conservation of the photon number density, which in turns changes the temperature-redshift and the distance duality relation, or from MG theories with non-minimal couplings between the electromagnetic Lagrangian and a new scalar field [21]. If there is some extra dimming contribution, this would have an imprint in the cosmological parameters and the expansion rate inferred from SNe Ia measurements. Hence, testing the cosmic opacity parameter denoted as \( \tau(z) \) and in turn the duality relation \( \eta(z) \) where both are related through the parametrization \( e^{\tau(z)/2} = \eta(z) \) [20] is of great interest.

Machine Learning algorithms are successful at processing and extracting crucial information from large amounts of data and can remove the problem of model bias [22]. They are also very useful to test the consistency of the dataset in a model independent approach and also to search for tensions or systematics. In this paper we will use a particular Machine Learning (ML) algorithm, the genetic algorithms (GA). The robustness of the GA resides in the fact that is a non-parametric method and does not require an initial prior or a cosmological fiducial model [23, 24]. Even though the temperature-redshift relation (TRR) appears to be well established, measurements of the connection between the redshift and the CMB temperature serves as an important cosmological probe. Among others, it can help to confirm the assumption of photon number conservation, the CMB is thermal radiation, entropy conservation and that the expansion of space is isotropic. It provides also a way to discriminate alternative cosmologies [25, 26]. Measurements of the TRR can be also used to measure the local expansion rate \( H_0 \) through the time evolution of the background \( T_{\text{CMB}}(z) \) [6].
In this paper we implement the GA which is a model independent and non-parametric algorithm to reconstruct the evolution of the CMB temperature from a wide redshift range $z \in [0, 3]$ without assuming any dark energy model, an adiabatic universe or photon number conservation. We then provide novel and updated estimates on the $\beta$ parameter from the parametrisation $T_{\text{CMB}}(z) = T_0(1 + z)^{1-\beta}$, the duality relation $\eta(z)$ and the cosmic opacity $\tau(z)$. Furthermore we place constraints on a spatial varying fine structure constant $\alpha$, which could affect among others the CMB anisotropies [27].

This paper is organized as follows. In Section 2 we present the notation and methodology of our analysis with the minimal assumptions made for the reconstruction of the data. In Section 3 we present our constraints and results and in Section 4 we present our Conclusions. Finally, in Appendix A we compare our error analysis of the GA with the Fisher matrix approach and in Appendix B we describe the data used and our implementation for the error analysis.

2 Analysis

Genetic algorithms (GA), which are a particular ML method, implement adaptive heuristic search approaches based on grammatical evolutionary algorithms and inspired by biological evolution operations of mutation and crossover [28]. It has been proven to be remarkably useful and robust in a wide range of areas such as particle physics [29–31], cosmology [23, 32], astronomy and astrophysics [33, 34] and other fields like computational science, economics, medicine and engineering [35, 36]. For further details on the GA and more applications to cosmology see [23, 24].

Mock datasets have already tested the reconstruction method of the GA approach [24, 37] and recently they have been used to reconstruct the deceleration parameter $q(z)$, which quantifies the acceleration of the Universe, making a $\sim 4.5\sigma$ model independent confirmation of the accelerated expansion [38]. Moreover, through a unified GA analysis using data from the Hubble expansion $H(z)$, Baryon Acoustic Oscillations (BAO), the growth-rate and $E_g$ data [39] there appears to be hints for the existence of an important non-adiabatic contribution to the dark energy (DE) sound speed or the presence of DE anisotropic stress, which if not considered can bias the cosmological parameters deduced from the data [40], thus pointing out to possible deviations from the ΛCDM model.

On the other hand, Gaussian Processes (GP), which likewise have been applied to cosmology [41–44], are also non-parametric methods for data reconstruction and assumes that the stochastic data is characterized by a Gaussian process that can be mapped to a cosmological function of concern. Although GP requires the choice of a kernel function and a fiducial model, normally ΛCDM, it has been asserted that this doesn’t have an effect in the reconstruction [41].

In this section we apply the GA to reconstruct the background temperature of the CMB $T_{\text{CMB}}(z)$ given in Table 1. Hereafter we will express $T(z) \equiv T_{\text{CMB}}(z)$. In our analysis we use 37 points of the compilation from Table 1 which spans over a wide redshift range of $z \in [0, 3]$. For the reconstruction of the data we use the following parametrization of the CMB temperature:

$$T_{\text{CMB}}(z) = T_0(1 + z)^{1-\beta},$$

where $T_0$ is the today's CMB temperature, $\beta$ is the parameter we aim to estimate, and $z$ is the redshift. The duality relation $\eta(z)$ and the cosmic opacity $\tau(z)$ also play a crucial role in our analysis. Furthermore, we place constraints on the spatially varying fine structure constant $\alpha$, which could affect among others the CMB anisotropies [27].

This paper is organized as follows. In Section 2 we present the notation and methodology of our analysis with the minimal assumptions made for the reconstruction of the data. In Section 3 we present our constraints and results and in Section 4 we present our Conclusions. Finally, in Appendix A we compare our error analysis of the GA with the Fisher matrix approach and in Appendix B we describe the data used and our implementation for the error analysis.
0 ≤ z ≤ 3.025. The data is in the form \((z_i, T_i, \sigma_{T_i})\). Since our \(\chi^2\) has a quadratic form

\[
\chi^2_T = \sum_{i}^{N_T} \left( \frac{T_i - T^{th}(z_i)}{\sigma_{T_i}} \right)^2,
\]

where \(T^{th}(z) = T_0 \tilde{T}(z)\) and \(\tilde{T}(z)\) is the dimensionless temperature, we can minimize the \(\chi^2\) analytically over \(T_0\) finding

\[
\chi^2_T = A - \frac{B^2}{\Gamma},
\]

\[
T_0 = \frac{B}{\Gamma},
\]

where the parameters \(A\), \(B\) and \(\Gamma\) are defined as

\[
A = \sum_{i}^{N_T} \left( \frac{T_i}{\sigma_{T_i}} \right)^2,
\]

\[
B = \sum_{i}^{N_T} \frac{T_i \tilde{T}(z_i)}{\sigma_{T_i}^2},
\]

\[
\Gamma = \sum_{i}^{N_T} \left( \frac{\tilde{T}(z_i)}{\sigma_{T_i}} \right)^2,
\]

and we denote the theoretical value \(T^{th}(z)\) of the background temperature of the CMB obtained from the GA as \(T^{th}(z) = T^{GA}(z)\). Then \(\tilde{T}(z) = T^{GA}(z)/T_0\) and we set \(N_T = 37\). Our best-fit function found is

\[
T^{GA}(z) = T_0 \left( 1 + z \left( e^{0.00123z^2} - 0.03581z + 0.00678z^2 \right) \right),
\]

and our own assumption is that the value of the background temperature of the CMB \(T(z)\) today is given by \(T(z = 0) = T_0\) where \(T_0\) is obtained directly from Eq. (2.3). The best-fit \(\chi^2\) for the GA is \(\chi^2 = 28.816\), which is smaller than that of the \(\Lambda\)CDM model with a \(\chi^2\) of \(\chi^2 = 29.176\) or that with the Fisher matrix approach with \(\chi^2 = 28.892\), see Appendix A. In the left panel of Fig. 1 we present the \(T(z)\) data compilation shown as grey points along with the \(\Lambda\)CDM best-fit (dashed line) and the GA best-fit (solid black line). The shaded gray regions corresponds to the 1σ errors of the GA. For the evaluation of the GA errors on the reconstructed quantities we make use of the path integral approach, first derived in [24, 45], where one calculates analytically a path integral over the functional space that can be scanned by the GA. We have also tested our GA approach leaving free the function \(f_{GA}(z)\) as \(T^{GA}(z) = T_0(1 + z)^{1-f_{GA}(z)}\) finding a similar \(\chi^2\) with Eq. (2.7).

Following, we present some theoretical context for the remaining derived quantities we will reconstruct and in Fig. 2 we present a flowchart of our fitting process for illustration purposes. We stress that the robustness of the GA approach resides in the fact that is a non-parametric method and does not require an initial prior or a cosmological fiducial model, obtaining constraints in a model independent approach.
2.1 $\beta$ parameter

If we assume that the expansion of the Universe is adiabatic, then the hot Big Bang model predicts that the CMB temperature evolves proportional to $(1 + z)$. One can parameterize possible deviations to this linear law as

$$T(z) = T_0 (1 + z)^{1 - \beta},$$

(2.8)

where $\beta$ is a parameter that would account for adiabatic photon production $\beta > 0$ or destruction $\beta < 0$. This phenomena can occur for example in decaying dark energy models [46] where DE interacts with matter by the creation of photons, affecting in turn the CMB spectrum [47]. From Eq. (2.9) we find that we can write $\beta$ as the logarithmic derivative of $T(z)$

$$\beta(z) = 1 - (1 + z) \frac{d \ln (T(z)/T_0)}{dz},$$

(2.9)

where $T(z)$ represents our best-fit reconstructed function, Eq. (2.7), and $T_0$ our derived parameter, Eq. (2.3). In the right panel of Fig. 1 we present our reconstruction of the $\beta$ parameter where the dashed line is the prediction from $\Lambda$CDM and the GA best-fit is the solid black line. The shaded gray regions corresponds to the 1$\sigma$ errors of the GA. We find that our model independent approach is consistent with an adiabatic universe and the conservation of photon number.

2.2 Duality relation and the cosmic opacity

The distance duality relation (DDR) defines a connection between the luminosity distance $d_L$ and the angular diameter distance $d_A$ in the following way

$$\eta(z) = \frac{d_L}{(1 + z)^2 d_A} = 1,$$

(2.10)

where any deviations from $\eta(z) \neq 1$ would be a hint for new physics, e.g. that the Universe is opaque. This relation is valid under the condition of the conservation of photon number in cosmic evolution, gravity must be described by a metric theory and the travel of photons along null geodesics [48] holding true for all curved space-times. The DDR has been tested.
from different datasets ranging from radio galaxies and ultra compact radio sources [49], The CMB [50–52], Baryon Acoustic Oscillations (BAO) [53, 54], H 21cm signal from disk galaxies [55], Gamma Ray Bursts [56] and high redshift quasars [57] among others. If we assume the temperature-redshift relation of the CMB from the ΛCDM model, i.e. \( T(z) = T_0 (1 + z) \), then the factor \( 1 + z \) can be written as \( 1 + z = T(z)/T_0 \). Inserting this relation into the rhs of Eq. (2.10) we find that

\[
\frac{dL}{dA} = (1 + z)^2 = \left( \frac{T(z)}{T_0} \right)^2 .
\]  

(2.11)

Substituting the ratio \( \frac{dL}{dA} = \left( \frac{T(z)}{T_0} \right)^2 \) from Eq. (2.11) in Eq. (2.10) we see that the DDR can also be written in terms of the redshift temperature relation of the CMB as

\[
\left( \frac{T(z)}{T_0 (1 + z)} \right)^2 \equiv \eta(z),
\]  

(2.12)

as it is also shown in Ref. [58] and which should be equal to unity in the ΛCDM model. The above relation is directly connected to the cosmic opacity \( \tau(z) \) as we will show below and therefore we can use our GA reconstruction on \( \eta(z) \) and \( \tau(z) \) to constrain the transparency of the universe.

If we have an opaque universe, the photon flux collected by the observers is lowered by a factor \( e^{-\tau(z)} \), and the observed luminosity distance \( d_{L,\text{obs}} \) can be expressed as [20]

\[
d_{L,\text{obs}}(z) = d_{L,\text{true}}(z) e^{\tau(z)/2},
\]  

(2.13)

where \( \tau(z) \) denotes the opacity parameter between an observer at \( z = 0 \) and a source at \( z \), and physically it gives us information about how transparent is the universe or in other words it connotes the optical depth associated to the cosmic absorption. This parameter can mimic a dark energy behaviour [56] and reconstructions for the parameter \( \tau(z) \) have been done in the past [59–61] and recently it has been tested from Gravitational Waves mock data from the third generation of the Einstein Telescope and using Gaussian Processes [20]. From Eq. (2.10) and Eq. (2.13) we can see that

\[
e^{\tau(z)/2} = \eta(z),
\]  

(2.14)

then using Eq. (2.12) we see that \( \tau(z) \) and our reconstruction for \( T(z) \) are connected in the following way

\[
\tau(z) = 4 \ln \left( \frac{T(z)/T_0}{(1 + z)} \right).
\]  

(2.15)

### 2.3 Fine structure constant

Fundamental constants, which we assume to be constant over space-time, are described operationally, meaning that nature does not force it to be constant. They are not given by the theory and must be obtained experimentally. For a review on the variation of fundamental constants see [62]. Here we will probe the interesting case where the fine structure constant \( \alpha = \frac{e^2}{\hbar c} \) is not space invariant and we will express the relative spatial variation as \( \Delta \alpha / \alpha \). If there are eventually signatures of a variation it would have imprints in different physical mechanisms such as the CMB anisotropies [27]. Constraints on this variation have been performed in the past [12, 63, 64], but to the best of our knowledge, this is the first time
that spatial variations on the fine-structure constant $\Delta \alpha / \alpha$ are constrained in a model independent and non parametric approach. This variation can be produced for example through an evolving scalar field which is coupled to the electromagnetic Lagrangian [12] producing violations in the photon number conservation. For this type of models, assuming adiabaticity, the relation between the evolution of the CMB temperature $T(z)$ and the spatial variation of the fine structure constant is expressed as [63]

$$T(z)/T_0 \sim (1 + z) \left(1 + \varepsilon \frac{\Delta \alpha}{\alpha}\right), \quad (2.16)$$

where the coefficient $\varepsilon$ depends on the specific model under consideration, but it is commonly assumed to be of order unity [64], hence we will consider as a test case $\varepsilon = 1$. Eq. (2.16) can be seen as a general phenomenological relation which can be tested through observations. Writing $\Delta \alpha / \alpha$ as a function of $T(z)$ we find

$$\frac{\Delta \alpha}{\alpha} \sim -\frac{(1 + z) - T(z)/T_0}{\varepsilon (1 + z)}. \quad (2.17)$$

### 3 Results

In this Section we present our best-fit reconstructions for the parameter $\beta$, the duality relation $\eta(z)$, the opacity parameter $\tau(z)$ and spatial variations on the fine structure constant $\Delta \alpha / \alpha$. Inserting our reconstructed $T(z)$ function Eq. (2.7) in Eqs. (2.9),(2.12),(2.15) and (2.17) we derived the following constraints at redshift $z = 0$

$$\beta(z = 0) = 0.0000 \pm 0.0224, \quad (3.1)$$
$$\eta(z = 0) = 1.0000 \pm 0.0002, \quad (3.2)$$
$$\tau(z = 0) = 0.0000 \pm 0.0004, \quad (3.3)$$
$$\frac{\Delta \alpha}{\alpha}(z = 0, \varepsilon = 1) = 0.0000 \pm 0.0001, \quad (3.4)$$
and at redshift \( z = 3.025 \) where we have our last data point

\[
\begin{align*}
\beta(z = 3.025) &= -0.0309 \pm 0.1475, \\
\eta(z = 3.025) &= 0.9483 \pm 0.1986, \\
\tau(z = 3.025) &= -0.1062 \pm 0.4188, \\
\Delta \frac{\alpha}{\alpha}(z = 3.025, \varepsilon = 1) &= -0.0262 \pm 0.1020.
\end{align*}
\] (3.5) (3.6) (3.7) (3.8)

From the numbers given above, we can see that both at low and high redshifts our constraints are consistent with the ΛCDM model.

In the left and right panel of Fig. 3 we show the reconstruction of the duality relation \( \eta(z) \) and the cosmic opacity parameter \( \tau(z) \) respectively. In both cases the expected value from ΛCDM corresponds to the dashed line and the GA best-fit to the solid black line along with the 1σ errors (gray regions). Both cases are consistent with photon number conservation and a transparent universe and hence with the ΛCDM model. Finally, in Fig. 4 we show our reconstruction of the variation of the fine structure constant for \( \varepsilon = 1 \) which is consistent with a non varying constant within the 1σ region. Overall, for all our reconstructions we find no evidence of deviations within the 1σ region from the well established ΛCDM model.

4 Conclusions

We have presented a model independent and non-parametric reconstruction of data coming from the redshift evolution of the CMB temperature which spans over a redshift range of \( 0 \leq z \leq 3.025 \) with a Machine Learning algorithm without assuming any dark energy model, an adiabatic universe or photon number conservation. In particular we used the genetic algorithms which avoids the dependency on an initial prior or a cosmological fiducial model. From our reconstruction we have provided constraints and updated estimates in a novel approach on the β parameter from the parametrisation \( T(z) = T_0(1 + z)^{1-\beta} \), the duality relation \( \eta(z) \) and the cosmic opacity parameter \( \tau(z) \). Furthermore we place constraints on a spatial varying fine structure constant \( \alpha \), which would have signatures in a broad spectrum
of physical phenomena such as the CMB anisotropies. Within uncertainties, our model independent approach is consistent with the standard view of $T \propto (1 + z)$ having found no strong discrepancies within the 1\(\sigma\) region with the \(\Lambda\)CDM model. Finally, our results evidence that a transparent universe is preferred at 1\(\sigma\).

**Numerical Analysis File**: The Genetic Algorithm code used by the author in the analysis of the paper will be released upon publication of the paper.

**Acknowledgments**

It is a pleasure to thank S. Nesseris for useful discussions. We also acknowledge support from the Research Projects FPA2015-68048-03-3P [MINECO-FEDER], PGC2018-094773-B-C32 and the Centro de Excelencia Severo Ochoa Program SEV-2016-0597.

**A Fisher matrix approach**

To evaluate the rigor of the path integral approach for the error analysis of the GA we compare it numerically with the Fisher matrix approach. We chose the following function which could be used to test deviations from the \(\Lambda\)CDM model

$$f(z; a, b) = T_0 (1 + z)^{1 + ax + bx^2}, \quad (A.1)$$

where \(z\) is the redshift and \(a\) and \(b\) are constant numbers. Then we fitted the model \(f(z; a, b)\) of Eq (A.1) by minimizing the $\chi^2$

$$\chi^2(a, b) = \sum_i \left( \frac{y_i - f(z_i; a, b)}{\sigma_{y_i}} \right)^2. \quad (A.2)$$
The best-fit value is given by \((a, b)_{\text{min}} = (a = -0.0264 \pm 0.0502, b = 0.0106 \pm 0.0231)\) with a 
\(\chi^2_{\text{min}} = 28.892\). The shaded gray region from Fig. 5 is the 1\(\sigma\) error following a Fisher Matrix 
approach \[24\]. The error of our best-fitted function \(f(z; a, b)\) is obtained from 
\[
\sigma_f(z)^2 = \sum_{i,j} C_{ij} \partial_i f(z; a, b) \partial_j f(z; a, b) \big|_{\text{min}}, \quad (A.3)
\]
which is evaluated at the best fit \[65\] and the dummy variables \((i, j)\) correspond to our 
parameters \((a, b)\). The covariance matrix \(C_{ij}\) is obtained from the inverse of the Fisher 
matrix \(C_{ij} = F_{ij}^{-1}\) where 
\[
F_{ij} = \frac{1}{2} \partial_i \partial_j \chi^2(a, b) \big|_{\text{min}}, \quad (A.4)
\]
evaluated at the best-fit. Comparing the shaded gray regions from the Fisher matrix method 
see Fig. 5 and the GA approach, see Fig. 1 we see that the path integral approach \[24, 45\] is 
robust.

**B Data compilation and error analysis**

In our analysis we use 37 points and the compilation can be found in Table 1. The main 
advantage of our compilation is that it spans over a wide redshift range of \(0 \leq z \leq 3.025\), 
thus testing the \(\beta\) parameter, the duality relation \(\eta(z)\), the cosmic opacity parameter \(\tau(z)\) 
and variations on the fine structure constant \(\frac{\Delta \alpha}{\alpha}\) up to high redshifts.

The background temperature of the CMB can be measured at both high and low red-
shifts. For the former, it can be recovered through fine-structure transitions of atomic or 
molecular species in cool absorption-line systems along the line of sight to high redshift 
quasars \[66\]. For low redshifts it can be obtained from Sunyaev-Zel’dovich (SZ) effect in
Table 1. Compilation of the CMB temperature-redshift relation $T(z)$ measurements used in this analysis and related references. We used 37 data points.

| $z$  | $T(K)$          | Ref. | $z$  | $T(K)$          | Ref. |
|------|----------------|------|------|----------------|------|
| 0.000| 2.72548 ± 0.00057 | [5]  | 0.072| 2.931 ± 0.017  | [71] |
| 0.023| 2.72 ± 0.10     | [67] | 0.125| 3.059 ± 0.032  | [71] |
| 0.152| 2.90 ± 0.17     | [67] | 0.171| 3.197 ± 0.030  | [71] |
| 0.183| 2.95 ± 0.27     | [67] | 0.220| 3.288 ± 0.032  | [71] |
| 0.200| 2.74 ± 0.28     | [67] | 0.273| 3.416 ± 0.038  | [71] |
| 0.202| 3.36 ± 0.20     | [67] | 0.322| 3.562 ± 0.050  | [71] |
| 0.216| 3.85 ± 0.64     | [67] | 0.377| 3.717 ± 0.063  | [71] |
| 0.232| 3.51 ± 0.25     | [67] | 0.428| 3.971 ± 0.071  | [71] |
| 0.252| 3.39 ± 0.26     | [67] | 0.471| 3.943 ± 0.112  | [71] |
| 0.282| 3.22 ± 0.26     | [67] | 0.525| 4.380 ± 0.119  | [71] |
| 0.291| 4.05 ± 0.66     | [67] | 0.565| 4.075 ± 0.156  | [71] |
| 0.451| 3.97 ± 0.19     | [67] | 0.618| 4.404 ± 0.194  | [71] |
| 0.546| 3.69 ± 0.37     | [67] | 0.676| 4.779 ± 0.278  | [71] |
| 0.550| 4.59 ± 0.36     | [67] | 0.718| 4.933 ± 0.371  | [71] |
| 2.418| 9.15 ± 0.72     | [68] | 0.777| 4.515 ± 0.621  | [71] |
| 1.777| 7.20 ± 0.80     | [69] | 0.870| 5.356 ± 0.617  | [71] |
| 1.973| 7.9 ± 1        | [70] | 0.972| 5.813 ± 1.025  | [71] |
| 2.338| 10 ± 4         | [7]  | 3.025| 12.1$^{+1.7}_{-3.3}$ | [72] |
| 0.037| 2.888 ± 0.039   | [71] |      |                |      |

Clusters of galaxies. The existing measurements at high redshifts still have large error bars and the majority of the points can be only treated as upper limits. However, in the near future, with high resolution spectroscopy with larger telescopes, the precision of these measurements can be competitive with local interstellar data. For the last data point of Table 1 we compute the error as

$$
\sigma_T(z = 3.025) = \sqrt{\frac{\sigma_u^2 + \sigma_d^2}{2}} = 2.562,
$$

where $\sigma_i$ is the error on the temperature estimates and $N$ is the number of the observational data used.

The 1σ errors of $\beta$, $\eta(z)$, $\tau(z)$ and $\Delta z$ were computed following the traditional error propagation, since it has been shown [39] that in agreement with the errors obtained using the definition of the standard deviation $\delta g^2 = \langle g^2 \rangle - \langle g \rangle^2$, where $g$ is a quantity formed by a function $f$. For the $\beta$ parameter, since it is defined as

$$
\beta(z) = 1 - (1 + z)\frac{d\ln(T(z)/T_0)}{dz},
$$

following the aforementioned approach we find that the error of $\beta$, e.g. $\delta \beta$ is

$$
\delta \beta(z) = -(1 + z)\frac{d(\delta T(z)/T(z))}{dz},
$$

(B.1)
where $T(z)$ is our best-fit function given by the GA and its $1\sigma$ error obtained through the path integral approach is $\delta T(z)$. Similarly we can derive the rest of the errors. For the duality relation $\eta(z)$ we have that

$$\eta(z) = \left(\frac{T(z)/T_0}{1+z}\right)^2,$$  

(B.4)

then

$$\delta\eta(z) = \frac{2T(z)\delta T(z)}{T_0^2(1+z)^2}.$$  

(B.5)

For the cosmic opacity parameter defined as

$$\tau(z) = 4\ln\left(\frac{T(z)/T_0}{1+z}\right),$$  

(B.6)

we have

$$\delta\tau(z) = \frac{4}{T(z)/T_0} \ln\left(\frac{\delta T(z)}{T_0}\right),$$  

(B.7)

and finally for the spatial variation of the fine structure constant

$$\frac{\Delta\alpha}{\alpha} = -\frac{(1+z) - T(z)/T_0}{\varepsilon (1+z)},$$  

(B.8)

we found the following

$$\delta\left(\frac{\Delta\alpha}{\alpha}\right) = \frac{1}{(1+z)\varepsilon} \frac{\delta T(z)}{T_0}.$$  

(B.9)

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