USING GALAXY TWO-POINT CORRELATION FUNCTIONS TO DETERMINE THE REDSHIFT DISTRIBUTIONS OF GALAXIES Binned BY PHOTOMETRIC REDSHIFT

MICHAEL SCHNEIDER,1 LLOYD KNOX,1 HU ZHAN,1 AND ANDREW CONNOLLY2

Received 2006 June 6; accepted 2006 July 13

ABSTRACT

We investigate how well the redshift distributions of galaxies sorted into photometric redshift bins can be determined from the galaxy angular two-point correlation functions. We find that the uncertainty in the reconstructed redshift distributions depends critically on the number of parameters used in each redshift bin and the range of angular scales used, but not on the number of photometric redshift bins. Using six parameters for each photometric redshift bin, and restricting ourselves to angular scales over which the galaxy number counts are normally distributed, we find that errors in the reconstructed redshift distributions are large; i.e., they would be the dominant source of uncertainty in cosmological parameters estimated from otherwise ideal weak lensing or baryon acoustic oscillation data. However, either by reducing the number of free parameters in each redshift bin or by (unjustifiably) applying our Gaussian analysis into the non-Gaussian regime, we find that the correlation functions can be used to reconstruct the redshift distributions with moderate precision; e.g., with mean redshifts determined to ~0.01. We also find that dividing the galaxies into two spectral types, thereby doubling the number of redshift distribution parameters, can result in a reduction in the errors in the combined redshift distributions.

Subject headings: cosmology: observations — galaxies: distances and redshifts

Online material: color figures

1. INTRODUCTION

There are many different techniques for determining the distance-redshift relation and/or growth-redshift relation motivated by the desire to understand the dark energy. Those that rely on the distances to a relatively small number of objects, such as the Type Ia supernova method (e.g., Riess et al. 1998), can use spectroscopic redshift determinations and thus avoid redshift error as a significant source of uncertainty. However, when the distance (and/or growth) constraints are derived from measurement of very large numbers of objects, spectroscopy can be a practical impossibility. In such cases one must rely on “photometric redshifts”; i.e., redshifts estimated from photometry in multiple broad bands (e.g., Loh & Spiller 1986; Connolly et al. 1995; Sawicki et al. 1997).

The relatively low cost per object of imaging surveys compared to spectroscopic surveys is a great advantage and provides significant motivation for pursuing the technique of estimating photometric redshifts. Imaging surveys can potentially constrain dark energy via a variety of techniques, including cluster counting (e.g., Haiman et al. 2001), cosmic shear (e.g., Hu 2002; Huterer 2002), and baryon acoustic oscillations (BAO; e.g., Seo & Eisenstein 2003; Blake & Glazebrook 2003; Padmanabhan et al. 2006). It may even be possible for imaging surveys to use Type Ia supernovae, without spectroscopic follow-up, to constrain cosmology (Barris & Tonry 2004).

But abandoning spectroscopy has its disadvantages too. In general, there is some tolerance of redshift error, but less tolerance for uncertainty about the probability distribution of those errors. The impact of redshift uncertainties on dark energy constraints has been studied for supernovae (Huterer et al. 2004), cluster number counts (Huterer et al. 2004), weak lensing (Bernstein & Jain 2004; Huterer et al. 2006; Ishak 2005; Ma et al. 2006), and baryon oscillations (Zhan & Knox 2006; Zhan 2006).

All of the studies cited in the above paragraph model the error distribution as Gaussian. However, photometric redshift error distributions, due to spectral-type/redshift degeneracies, often have bimodal distributions, with one smaller peak separated from a larger peak by $\Delta z$ of order unity (e.g., Benítez 2000; Fernández-Soto et al. 2001, 2002). Thus, a fraction of galaxies have photometric redshifts that are “catastrophically” wrong. Here we study how well the coarse properties of the true redshift distribution of galaxies in a given photometric redshift bin can be reconstructed from galaxy two-point correlation functions.

The idea is that catastrophic photometric redshift (“photo-z”) errors introduce additional correlations between galaxies in different redshift bins. In general, such errors will alter both the amplitude and shape of the binned angular correlation functions. Measurements of the correlation functions over a range of angular scales would thus provide valuable information to unravel the effects of large photo-z errors.

We emphasize that we are not attempting a forecast of the photo-z errors achievable given all possible information. In particular, we neglect information from spectroscopic calibration of the photo-z error distribution. Spectroscopy, possibly combined with a “super” photometric (12 or more bands) photo-z training set, will play a critical role. In this sense our forecasts here are highly conservative. Furthermore, the catastrophic errors are likely to be avoided by use of luminosity function and surface brightness priors. For recent results on spectroscopic calibration of photo-z measurements for weak lensing, see Ilbert et al. (2006).

The outline of this paper is as follows. In §2 we describe our model for the photo-z errors, the Fisher matrix we use to constrain the parameters of this model, and our model for the galaxy

1 Department of Physics, University of California, 1 Shields Avenue, Davis, CA 95616; schneider@ucdavis.edu, lknox@ucdavis.edu, zhan@physics.ucdavis.edu
2 Department of Physics and Astronomy, University of Pittsburgh, 3941 O’Hara Street, Pittsburgh, PA 15260; ajc@phyast.pitt.edu

3 The current plan for photo-z calibration is described in http://www.lsst.org/Science/Phot-z-plan.pdf.
angular power spectra. We present our results in § 3, including the details of our fiducial model and its impact on the resulting Fisher matrix constraints. We discuss some implications of our results in § 4 and draw conclusions on the feasibility of constraining photo-z errors with galaxy angular correlation functions.

2. METHOD

In this section we first introduce our model for the catastrophic photo-z errors and then describe the Fisher matrix formalism (Jungman et al. 1996; Tegmark et al. 1997) that we use to forecast how well the parameters of this error model can be constrained from observations of the galaxy angular correlation function, binned in redshift. We restrict ourselves to forecasting here and leave for later work the development of a practical algorithm for constraining the photo-z errors in a galaxy survey.

2.1. Model for Catastrophic Photo-z Errors

To focus on the gross mislabeling of galaxy redshifts introduced by catastrophic photo-z errors, we bin the galaxy distribution in redshift and model the errors as a linear mixing of the values of the galaxy number density in each bin. In terms of this distribution in redshift and model the errors as a linear mixing of the values of the galaxy number density in each bin. In terms of this model, our goal is then to constrain the number of galaxies from each true-z bin that contribute to the observed number in a given photo-z bin.

We assign the same numerical values for redshift intervals to photo-z bins and true-z bins. The parameters of our error model are then defined as:

\[ \bar{N}_{\alpha} \equiv \text{mean number of galaxies per steradian of spectral type } \alpha \text{ in photo-z bin } \iota \text{ that come from true-z bin } \alpha. \]

By considering only the mean number of galaxies mixing between photo-z bins, we are ignoring possible angular fluctuations in the mixing. We expect this to be a good approximation on scales large enough for the fractions of different galaxy types to be uniform and in the limit of homogeneous noise. The separate index for galaxy subpopulations is to allow for the possibility of different photo-z errors for different galaxy spectral types. However, for most of our results we ignore any information about galaxy types and consider just the parameters \( \bar{N}_{\alpha} \equiv \sum_{\alpha} \bar{N}_{\alpha} \) of the entire sample of galaxies.

Using these parameters, we construct the redshift distribution of galaxies in photo-z bin \( \iota \):

\[ \frac{dN_a}{dzd\Omega}(z, \theta) = \sum_{\alpha} \bar{N}_{\alpha} \left[ \frac{1}{\bar{N}_{\alpha}} dN_a/dz/d\Omega(z, \theta) \psi_0(z) \right], \]

where \( dN_a/dz/d\Omega \) is the number of galaxies of spectral type \( a \) in redshift interval \( dz \) and angular interval \( d\Omega \), \( \psi_0(z) \) is a top-hat window function defining the true-z bin \( \alpha \), and

\[ \bar{N}_{\alpha} = \frac{1}{\Omega} \int d\Omega \int_{0}^{\infty} dz \frac{dN_a}{dzd\Omega}(z, \theta) \psi_0(z) \]

is the mean number of galaxies (per steradian) in true-z bin \( \alpha \).

Because we are binning in redshift, our model cannot tell us anything about the shape of the redshift distribution within each bin (given by the term in parentheses in eq. [1]). We therefore assume that this is known. However, the normalization of the redshift distribution in each bin is determined by the parameters \( N_{\alpha} \).

If we integrate equation (1) over redshift, we get an expression for the total number of galaxies (per steradian) in each photo-z bin,

\[ N_{\iota}(\theta) = \sum_{\alpha} \bar{N}_{\alpha} N_{\alpha}(\theta). \]

We take the set of \( N_{\iota}(\theta) \) as our data set that we use to constrain the mean parameters \( \bar{N}_{\alpha} \).

2.2. Fisher Matrix

Rephrased in more abstract terms, our problem is to figure out how well a set of parameters \( \{a_p\} \) can be constrained from a data set \( \{N_{\iota}\} \), through the influence of \( a_p \) on the statistical properties of the data.

On large scales, the values of \( N_{\iota} \) are Gaussian distributed, and their statistical properties are completely described by the mean, \( \bar{N}_{\iota} \), and covariance, \( \Sigma_{\iota} \). However, on small scales, where nonlinear clustering becomes important, the galaxy number density becomes significantly non-Gaussian and higher order correlations are required to completely describe the statistics of the density field. To avoid the complexities of non-Gaussianity, we limit ourselves to a range in \( \theta \) in which the data are Gaussian to a good approximation (see § 3.1 for details). Note, however, that there is more information in the data in smaller scales than we are considering here, which could improve the parameter constraints beyond those shown below.

For Gaussian data, the Fisher matrix is given by (Tegmark et al. 1997),

\[ F_{\pi \pi'} = \frac{\partial \bar{N}_{\iota}}{\partial a_p} w^{-1} \frac{\partial \bar{N}_{\iota}}{\partial a_{p'}} + \frac{1}{2} \text{tr} \left( \frac{\partial w}{\partial a_p} w^{-1} \frac{\partial w}{\partial a_{p'}} w^{-1} \right) \]

where \( \bar{N} \) is the mean of the data and \( w \) is the covariance matrix of the data, defined by

\[ N = \bar{N} + \delta N, \]

with \( N \equiv \{N_{\iota}\} \), and

\[ w = \langle \delta N \delta N^T \rangle \]

so that \( w = \{w_{ij}(\theta, a_p)\} \). In the approximation that the likelihood is only quadratic in the parameters, the inverse Fisher matrix is equal to the covariance of the parameters \( a_p \).

Our parameter set for the Fisher matrix is the set of \( \{N_{\iota}\} \) plus the linear galaxy bias with respect to the dark matter in each redshift bin (\( \{b_{\iota}\} \)) and the amplitude of the power spectrum at \( z = 0 \) (which is degenerate with the galaxy bias at \( z = 0 \)).

The expression for the Fisher matrix in equation (4) simplifies considerably when we Fourier-transform over the variable \( \theta \). In this case the covariance matrix becomes block diagonal (with one block for each value of the conjugate variable \( \ell \) due to isotropy. The second term in equation (4) then becomes,

\[ F_{\pi \pi'}^{(2)} = \frac{1}{2} f_{\text{sky}} \sum_{\ell, (ai), (aj), (bj)} (2\ell + 1) \frac{\partial C_{(ai)(aj)}^{-1}}{\partial a_p} \frac{\partial C_{(bj)(bj)}^{-1}}{\partial a_p} \frac{\partial C_{(aj)(bj)}^{-1}}{\partial a_p}, \]

as our data set that we use to constrain the mean parameters \( \bar{N}_{\alpha} \).
where $f_{\text{sky}}$ is the fractional angular area of the sky covered by the survey, $C_{(ai)(bj)}(\ell)$ is the Fourier transform of $w_{ij}^{\text{photo-z}}(\theta)$, and we have written out the matrix multiplications explicitly.\(^5\)

When we Fourier transform the first term in equation (4), only the monopole ($\ell = 0$) contributes to the mean, so that

$$F_{pp'}^{(1)} = \sum_{(ai)(bj)} \frac{\partial N^a_i}{\partial p} \left[ C^{-1}(\ell = 0) \right] \frac{\partial N^b_i}{\partial p'}.$$ \hspace{1cm} (8)

This term tells us what information the mean of the data contributes to the parameter constraints. Because $N$ does not depend on the galaxy bias, equation (8) is nonzero only for the parameters $\{N^a_i\}$.

While formally the cosmological monopole of the power spectrum cannot be determined (because of unknown contributions from superhorizon-sized modes), for a survey that covers only a fraction of the sky, this term will be aliased and will contain contributions from the power spectrum at small (but nonzero) $\ell$. We therefore define an "effective" monopole variance:

$$C_{(ai)(bj)}(\ell = 0) = \frac{\delta^K_{(ai)(bj)}}{\sigma^2_{(ai)}}.$$ \hspace{1cm} (9)

From the predicted amplitude of the angular power spectrum at small $\ell$ we set $\sigma^2_{(ai)} = 10^{-3} \times N^a_i$. We consider this a conservative upper bound. To avoid confusion in the strict interpretation of this term, we point out that it is identical in form to adding extra Fisher information from an independent measurement of $N$.

Note that, in our notation, Ma et al. (2006) and Zhan (2006) have omitted the term in equation (8) and instead fixed $\bar{N}_i = \bar{N}_i$. By letting $\bar{N}_i$ vary freely, we are assuming no prior knowledge of the true redshift distribution. And by including the term in equation (8), we are taking into account the fact that the mean number of galaxies in each photo-z bin can be determined from the data. Huterer et al. (2006) and Zhan & Knox (2006) use a method more similar to ours in this respect; i.e., they fix the total number of galaxies in each photometric redshift bin and allow $\bar{N}_i$ to vary. Huterer et al. (2006) find that there is very little difference between the two approaches for the case of weak lensing. This may not be the case, however, for galaxy power spectra.

2.3. Galaxy Angular Power Spectrum with Photo-z Errors

Using equation (3), the observed angular power spectrum is related to the "true" angular power spectrum (i.e., without photo-z errors) by

$$C_{(ai)(bj)}(\ell) = \sum_{\alpha \alpha'} \bar{N}^a_{\alpha i} \bar{N}^b_{\alpha' j} P_{\alpha \alpha'}(\ell),$$ \hspace{1cm} (10)

where bars denote quantities averaged over a redshift bin and $P_{\alpha \alpha'}(\ell)$ is the power spectrum of the normalized density fluctuations $\delta N^a_{\alpha i}/\bar{N}^a_i$ [while $C_{(ai)(bj)}(\ell)$ is the power spectrum of $\delta N^{ab}_{\alpha \alpha'}$]. We also add shot noise to the model for the observed power spectrum,

$$C^\text{noise}_{(ai)(bj)} = \frac{\delta^K}{A_{\text{survey}}},$$

where $A_{\text{survey}}$ is the angular area of the survey in square arcminutes and $\delta^K$ is the Kronecker delta function. We plot fiducial power spectra given by equation (10) in Figure 1, along with the variation of the power spectra when one of the parameters $N^{ai}_{\alpha i}$ is changed by 1 $\sigma$.

We use Figure 1 to justify our neglect of other cosmological parameters that can affect the galaxy power spectrum. We show there the effect on the galaxy power spectra of a 1 $\sigma$ variation of one of the $N^{ai}_{\alpha i}$ photo-z error parameters (where the 1 $\sigma$ variation is computed with the Fisher matrix from § 3). We find that the cross-power spectra are very sensitive to changes in the photo-z error parameters, but the auto-power spectrum remains nearly unchanged under the parameter variation. From this we conclude that the constraints we obtain on the photo-z error parameters must come largely from the cross-correlations between the photo-z bins. Because it is unlikely that the cross-correlations induced by photo-z errors are degenerate with variations in the power spectrum due to changing cosmological parameters, we consider our parameter set to be complete for the conclusions we wish to draw in this paper.

We use the halo model to obtain analytic expressions for the linear galaxy bias and nonlinear three-dimensional galaxy power
specification as described in the Appendix. We then use the Limber approximation (Limber 1953) to project this into the binned angular galaxy power spectrum,

\[
P^{ab}_{\alpha}(\ell) = \frac{\delta^k}{\ell^2} \int_0^\infty dz \frac{d\chi}{dz} (\Delta^2)_{\alpha \beta} \left[ \frac{\ell}{D(\chi)} \right] D(\chi) W^{a}_{\alpha}(\chi) W^{b}_{\beta}(\chi),
\]

(11)

where \(\chi(z)\) is the comoving distance as a function of redshift, \(D(\chi)\) is the comoving angular diameter distance (equal to \(\chi\) for zero spatial curvature), \(\Delta^2(k)\) is the three-dimensional variance of galaxy number density fluctuations per logarithmic interval in \(k\), and

\[
W^{a}_{\alpha}(\chi) \propto \frac{d\bar{N}^a}{d\chi} \psi_\alpha(\chi)
\]

is the probability distribution for finding a galaxy of type \(a\) in \(z\)-bin \(\alpha\) at a comoving distance \(\chi\) in the survey, in the absence of photo-z errors, with \(\psi_\alpha(\chi)\) a top-hat window function for the \(z\)-bin \(\alpha\), as defined in equation (2), and normalization \(\int W^{a}_{\alpha}(\chi) d\chi = 1\). To simplify the computation, we ignore the redshift evolution of \(\Delta^2(k, z)\) in equation (11); evaluating it instead at the mean redshift of the bin.

Note the delta function in equation (11), so that our model neglects any intrinsic cross-correlation between redshift bins. For \(\ell = 50\) (near the peak of the angular power spectrum) and redshift bins with width of 0.5, we have checked that the cross-correlations between bins are less than 1% of the autocorrelations. However, with our fiducial model, the catastrophic photo-z errors induce correlations at the level of \(\sim 5\%\) of the autocorrelation (see Fig. 1, left). We therefore expect that neglecting intrinsic cross-correlations should not alter our Fisher matrix errors by more than \(\sim 1\%\), which is completely unimportant for the conclusions we draw here.

An important effect that we have neglected here is magnification of galaxies by weak lensing from intervening large-scale structure (Turner 1980; Turner et al. 1984). Lensing can induce correlations between different photo-z bins beyond those that would be found in an unlensed survey. The amplitude of the weak-lensing induced correlations depends on the luminosity function of the galaxies in the survey but could be of similar magnitude to the photo-z error–induced correlations (Villumsen 1995). We expect that this effect can be calculated with sufficient accuracy that residual uncertainties will not lead to significant confusion with photo-z–error induced correlations.

2.4. Galaxy Bias Parameterization

If we restrict ourselves to linear theory, then the galaxy power spectrum factors into a product of the dark matter power spectrum times a constant (scale independent but redshift dependent) bias:

\[
P^{\text{gal, lin}}_{(\alpha \beta)(\alpha \beta)}(\ell) = b^{a}_{\alpha} b^{b}_{\beta} P_{\text{DM, lin}}(\ell).
\]

(12)

At small scales, the bias \(b^a_{\alpha}\) becomes scale dependent, and this factorization no longer holds. Therefore, when we include the parameters \(\{b^a_{\alpha}\}\) in our Fisher matrix analysis, we are approximating the galaxy power spectrum with the linear power spectrum. Below we evaluate the effects of this assumption.

2.5. Galaxy Subpopulations

We consider two ways of utilizing the multiband photometric data in a galaxy survey. On the one hand, we imagine assigning photo-z values to each galaxy and binning them according to these values, while ignoring any other features of the galaxies. We can then use the number counts in each bin along with our model for the power spectrum to constrain the true redshift distribution of all the galaxies in each bin. On the other hand, we can sort the galaxies according to their spectral (or morphological) type and repeat the analysis (jointly) for each subpopulation. From this larger data set, we can then infer constraints on the parameters \(N_{\alpha a} \equiv \sum_{z} \bar{N}_{\alpha a}(z)\) for the full galaxy sample by adding the variances of the \(\bar{N}_{\alpha a}(z)\) (including cross-terms).\footnote{This is equivalent to making a formal change of variables in the inverse Fisher matrix from \(\bar{N}_{\alpha a}\) to \(\bar{N}_{\alpha a}(z)\).} We expect the \(\bar{N}_{\alpha a}(z)\) to be different for each subpopulation not only because of different redshift distributions for each galaxy type, but also because of different photo-z error distributions.

While the shot noise increases by dividing the galaxy sample into subpopulations, it may be possible to improve the constraints on the parameters of the total galaxy sample for several reasons. First, we gain knowledge of the mean number density of each subgroup, which contributes to the term in equation (8). Second, we gain extra information from the cross-correlation between the subgroups, which is even more helpful if the redshift distribution of one of the types can be well constrained on its own. In fact, the exposure times and bands for the survey could even be optimized for the best constrained galaxy subpopulation. In the limit of exact redshifts of a subsample of galaxies, J. A. Newman (2006, in preparation) has shown that cross-correlating with the photo-z sample can place significant constraints on the photo-z errors. Third, a given galaxy subgroup may be more biased than the total galaxy sample and could therefore have equivalent or greater signal-to-noise ratio (S/N) than the total sample, even though the noise increases for the subsamples. However, this potential gain in S/N would be limited to those photo-z bins that sit at the peak(s) of the photo-z error distribution for the strongly biased subsample. The gain in parameter constraints from dividing the galaxy sample (if there is any gain at all) could, therefore, be limited to finite patches in photo-z.

2.6. Mean Redshift in Each Photo-z Bin

As discussed in the introduction, we would like to know whether the constraints on the photo-z error parameters shown in Figure 4 will be sufficient to place interesting constraints on dark energy parameters from weak lensing and baryon acoustic oscillation surveys. To properly address this question, we should forecast joint constraints on a suite of cosmological and photo-z parameters with both galaxy and shear data. However, this analysis has already been done for the case of Gaussian photo-z errors (Zhan 2006), and we choose not to repeat that effort here. Instead, we make contact with previous work by reducing the constraints on the \(\bar{N}_{\alpha a}\) to constraints on the mean redshift in each photo-z bin, defined as:

\[
\bar{z}_{\alpha i}^a \equiv \frac{\int dz d\bar{N}_{\alpha a}(z)/dz}{\int dz d\bar{N}_{\alpha a}(z)/dz}.
\]

(13)

The errors on \(\bar{z}_{\alpha i}^a\) are extracted from the inverse Fisher matrix:

\[
(F_{\bar{z}_{\alpha i}^a})_{(\alpha \beta)(\alpha \beta)} = \sum_{(k \beta)(\alpha \beta)} \frac{\partial \bar{z}_{\alpha i}^a}{\partial \bar{N}_{\alpha a}(k \beta)} \frac{\partial \bar{z}_{\alpha i}^b}{\partial \bar{N}_{\alpha a}(k \beta)} (F_{\bar{N}_{\alpha a}}^{-1})_{(k \beta)(k \beta)} (F_{\bar{N}_{\alpha a}}^{-1})_{(k \beta)(k \beta)}^{-1},
\]

(14)

where \(F_{\bar{N}_{\alpha a}}\) is the Fisher matrix in equation (4).
It has been shown in Huterer et al. (2006), Ma et al. (2006), Zhan & Knox (2006), and Zhan (2006) that useful dark energy constraints require the mean redshift in each photo-z bin to be constrained to $\sim 0.003$ near $z \approx 1$ with the required constraint weakening rapidly toward higher redshift. We therefore use 0.003 as a benchmark for assessing our results.

3. RESULTS

In this section we first describe a simple fiducial model for the galaxy survey and photo-z errors and then show the forecasted constraints from the Fisher matrix in equation (7). We then study how our results depend on the fiducial model in order to draw conclusions independent of the many assumptions in our model.

3.1. Fiducial Model

We choose our fiducial model to mimic the Large Synoptic Survey Telescope (LSST). Throughout, we consider a survey covering 20,000 deg$^2$ and bin the galaxies in photo-z over the range 0–3. Our fiducial cosmological parameters are $\Omega_m = 0.24$, $\Omega_b h^2 = 0.022$, $\Omega_{\Lambda} = 0.76$, $h = 0.72$, and $\sigma_8 = 0.74$.

To ensure that our assumption of Gaussianity in equation (4) is reasonable, we limit the $\ell$ range in equation (10) by adding large noise to each element of $P_{\ell} (\theta)$ with $\ell > \ell_{\text{max}} (z)$, where $\ell_{\text{max}} (z) \equiv \chi (z) k_{\text{max}} (z)$ and $\Delta_{\text{DM}} \ell_{\text{max}, \text{z}} = 0.4$. We also set a lower bound on $\ell$ to justify our use of the Limber approximation for the angular power spectrum in equation (11). The Limber approximation relies on the observation that, when projecting the three-dimensional power spectrum into two dimensions, the dominant contribution is from those Fourier modes that do not oscillate significantly along the line of sight. We can quantify this statement by considering only modes with line-of-sight component of the wavevector $k_3 < 2\pi/\Delta \chi (z)$, where $\Delta \chi (z)$ is the comoving width of the $z$-bin under consideration. The Limber approximation also requires $\ell \gg k_3 \chi (z)$. Putting these together, we set $\ell_{\text{min}} (z) \equiv 4 \pi \chi (z)/\Delta \chi$ (with an arbitrary factor of 2 inserted just to be conservative).

In Table 1 we show the values of $\ell_{\text{max}}$ as a function of redshift in our fiducial model for six $z$-bins over the range 0 $\leq z$ $\leq 3$. We evaluate both $\ell_{\text{min}}$ and $\ell_{\text{max}}$ at the centers of the photo-z bins.

We use the redshift distribution from Song & Knox (2004) (which is based on Subaru observations with limiting magnitude in $R$ of 26),

$$\frac{d\bar{N}}{dz \, d\Omega} (z) = \bar{N}_{\text{tot}} \exp \left[ - \left( \frac{z}{1.2} \right)^{1.2} \right] \times \begin{cases} z^{1.3}, & z < 1 \\ z^{1.1}, & z > 1 \end{cases}$$

with the normalization, $\bar{N}_{\text{tot}}$ set by $\int dz \, d\bar{N}/dz \, d\Omega = 65$ arcmin$^{-2}$.

When considering galaxy subpopulations, we consider two spectral types, which we label "red" and "blue," roughly depending on the absence or presence of active star formation.

In the absence of a well-motivated model, we generated several redshift distributions for the red and blue subpopulations in an ad hoc fashion and compared the results among them. We require only that the redshift distributions sum to give equation (15) and that the blue distribution dominate at large redshifts ($z \gtrsim 1$).

For the red and blue galaxy subpopulations, we set

$$\frac{d\bar{N}_{\text{red}}}{dz \, d\Omega} = R \bar{N}_{\text{tot}} z^{1.4} \exp (-r_1 z / r_2) \quad (16)$$

and

$$\frac{d\bar{N}_{\text{blue}}}{dz \, d\Omega} = \frac{d\bar{N}}{dz \, d\Omega} - \frac{d\bar{N}_{\text{red}}}{dz \, d\Omega},$$

with $R = 0.8$, $r_1 = 1.3$, $r_2 = 1.4$. These fiducial redshift distributions are shown in Figure 2.

Our fiducial model for $N_{\alpha}^{\text{red}}$ is based on the estimated photo-z values for a simulated random sample of 100,000 galaxies over redshifts from 0 to 4, with colors assigned by filtering spectra from a sample of 10 redshift-evolved spectral energy distributions (SEDs). The simulation assumed photometric data was available in six filters ($ugrizy$), modeled after the LSST, with the data limited in the i-band at $i < 25$ and a S/N of 10–15 at the depth of the survey. The depth of the simulation is what one would achieve after about 400 visits per filter. This is just a fiducial approach, as the errors can be optimized by weighting the exposure times in each band separately. The photo-z of each galaxy was estimated by matching the galaxy colors with a SED template library. No priors on the luminosity function or surface brightness were used, which can significantly improve the photo-z estimates in some cases. In this regard, our fiducial model is therefore a worst-case scenario.

To model the errors for the galaxy subpopulations, we divided the templates for the simulated galaxy SEDs into two groups based on the presence or absence of strong emission lines. The fiducial parameters, $N_{\alpha}^{\text{red}}$, are constructed by first creating the matrix, $E_{\alpha} \equiv N_{\alpha}^{\text{red}} / N_{\alpha}^{\text{blue}}$, by binning the photo-z versus z plane, and normalizing so that $\sum \alpha E_{\alpha} = 1$ (for each $\alpha$ and $\alpha$). This normalization conserves the total number of galaxies in the survey.

### Table 1

| z Range | $\ell_{\text{min}} (z)$ | $\ell_{\text{max}} (z)$ |
|---------|--------------------------|--------------------------|
| 0.0–0.5 | 7                        | 114                      |
| 0.5–1.0 | 23                       | 458                      |
| 1.0–1.5 | 45                       | 1018                     |
| 1.5–2.0 | 71                       | 1875                     |
| 2.0–2.5 | 103                      | 3195                     |
| 2.5–3.0 | 140                      | 5186                     |

*At http://www.lsst.org.*

*“DM” denotes the dark matter power spectrum.*
We then use the fiducial redshift distribution in equation (15) to create $N_{i,a}$ according to equation (2) and multiply with $E_{i,a}$ to get the parameters $N_{i,a}$. An example of our fiducial model for the $N_{i,a}$ is plotted in Figure 3.

### 3.2. Parameter Constraints

Our main results are given in Figure 4, which shows the Fisher constraints on the parameters $N_{i,a}$, assuming a 10% prior on the galaxy bias and 100% prior on the $N_{i,a}$. The model for the open squares includes the red and blue galaxy subpopulations, while the model for the filled squares ignores this information.

In column (2) of Table 2, we show the constraints on the mean redshift in each photo-z bin (eq. [14]) implied by the constraints on the $N_{i,a}$ without galaxy subpopulations (Fig. 4, filled squares). These are 2 orders of magnitude larger than the benchmark value (described in § 2.6) needed for constraining dark energy parameters. In the following subsections, we discuss three ways that the constraints on $z_i$ could possibly be improved.

#### 3.2.1. Adding Galaxy Subpopulations

For a wide range of fiducial models, we find that dividing our galaxy sample into red and blue subpopulations improves the forecasted constraints on the redshift distribution in each photo-z bin, as shown by the open squares in Figure 4 and column (3) of Table 2. We also show forecasted constraints on the redshift distributions of the subpopulations in Figure 5. Comparing the errors on the mean redshift in each photo-z bin in Table 2, we see that there is a significant improvement over the constraints obtained without using information about galaxy subpopulations, but the errors are still much larger than the benchmark for dark energy surveys given in § 2.6.

Because our fiducial models for the biases and redshift distributions of the subpopulations (in the absence of photo-z errors) are rather ad hoc, we have varied the parameters in equations (16) and (A2) over a range of physically reasonable values, and we find no change to the qualitative nature of our results. This is discussed more in § 3.3.2.

#### 3.2.2. Sensitivity to Parameterization

We show in Figure 6 (left) and column (3) of Table 2 the forecasted parameter constraints with a fiducial photo-z error model that only allows mixing between adjacent photo-z bins and with the number of photo-z error parameters reduced to only those that can take nonzero values in the fiducial model. The fiducial errors

![Figure 3](image3.png)

**Fig. 3.** Fiducial redshift distribution with catastrophic photo-z errors: $N_{i,a}$ (for the LSST model; see text). The two plots show the same parameters in different perspectives. On the left is the number density, $N_{i,a}$, as a function of redshift (indexed by $i =$ photometric $z$ and $a =$ spectroscopic $z$). On the right each window shows a different photo-z bin, $i$, while each bar in a given window shows a different spectroscopic index, $a$, for the given $i$. For example, the bar between $z = 0.5$ and $z = 1$ in the window for photo-z bin 1 is the number density of galaxies with spectroscopic redshifts in the range $0.5 < z < 1$ that have been given photometric redshifts in the range $0 < z < 0.5$.

![Figure 4](image4.png)

**Fig. 4.** Forecasted constraints for the parameters $N_{i,a}$ divided by the fiducial $N_i$, in each photo-z bin, i.e., the error on the fraction of outliers within each photo-z bin. The filled squares are the fractional constraints when no galaxy subpopulations are considered, while the open squares are the constraints when the galaxy sample is divided into red and blue spectral types. For the open squares, the parameters for the red and blue subpopulations ($N_{i,a}$) were constrained first, and then these constraints were combined to produce the constraints on $N_{i,a}$ shown here (by summing over the index $a$ in the inverse Fisher matrix components). A 10% prior on the galaxy bias and a 100% prior on the $N_{i,a}$ were imposed. The fiducial model assumed LSST photo-z errors (see text) and sky coverage of 20,000 $\text{deg}^2$.

| $z$ Range   | $\sigma_{\text{LSST}}$ | $\sigma_{\text{sub}}$ | $\sigma_{\text{Gauss}}$ | $\sigma_{\text{inv}}$ |
|------------|-------------------------|------------------------|--------------------------|-----------------------|
| 0.0–0.5     | 0.29                    | 0.059                  | 0.0050                   | 0.0030                |
| 0.5–1.0     | 0.068                   | 0.041                  | 0.010                    | 0.0050                |
| 1.0–1.5     | 0.12                    | 0.036                  | 0.0087                   | 0.0096                |
| 1.5–2.0     | 0.16                    | 0.049                  | 0.0091                   | 0.016                 |
| 2.0–2.5     | 0.23                    | 0.085                  | 0.0079                   | 0.022                 |
| 2.5–3.0     | 0.28                    | 0.61                   | 0.0047                   | 0.025                 |
assume a 5% contribution from each of the adjacent bins to a given photo-z bin. This crudely mimics a Gaussian model for the photo-z errors. We have also tightened the prior on the galaxy bias from 10% to 1%. While the constraints on $\bar{N}_a / C_{11}$ in the left panel of Figure 6 are moderately improved from the default model in Figure 4, the constraints on $\bar{z}_i$ in Table 2 improve by nearly 2 orders of magnitude. This shows that reducing the number of free parameters in each photo-z bin indeed has a large impact on the ability to constrain the redshift distribution in each bin.

### 3.2.3. Sensitivity to Range of Angular Scales

The forecasted constraints are very sensitive to the maximum $\ell$ used in the galaxy power spectrum (but are rather insensitive to the minimum $\ell$ cutoff). In particular, for the lowest photo-z bin, the maximum cutoff at $\ell = 114$ (from Table 1) removes some of the baryon features in the power spectrum that can help in diagnosing photo-z errors (Zhan 2006).

To demonstrate this sensitivity, in the right panel of Figure 6 and column (5) of Table 2 we show the forecasted constraints when $\ell_{\text{max}}(z)$ = 4000 for all $z$. The constraints on the mean redshift in each photo-z bin are 2 orders of magnitude smaller than those with $\ell_{\text{max}}$ from Table 1 (labeled “$\sigma_{\text{r}}$,LSST” in Table 2).

Recall that the maximum $\ell$ cutoff is imposed to validate our assumption of Gaussian data (in, e.g., eq. [4]). Therefore, the constraints presented in this section should be interpreted only up to non-Gaussian corrections, which could be quite large. Our results are an indication that there is much to be gained by developing the appropriate tools for analyzing the non-Gaussian case.

### 3.3. Fiducial Model Dependence

To test the robustness of our conclusions, we recomputed the Fisher matrix in equation (7) while varying the number of redshift bins, the galaxy bias and halo occupation distribution in the non-linear power spectrum, and the fiducial model for the photo-z errors.

#### 3.3.1. Redshift Distribution

For comparison, we use a second fiducial model for $E_a$ with 10% of the galaxies in each photo-z bin uniformly distributed over the remaining photo-z bins. This model, although not...
physically motivated, gives us a reference for determining the sensitivity of the Fisher constraints to the fiducial error model.

We show ratios of the Fisher matrix errors obtained with these two fiducial photo-z error models in Figure 7. For most of the parameters, the different fiducial models lead to differences in the forecasted errors of a factor of 5. The LSST fiducial model for the photo-z errors is actually well motivated by our photo-z estimation simulation, so any uncertainties in the fiducial model will be much smaller than the changes introduced by this artificial “uniform” error model. We therefore conclude that uncertainties in the fiducial photo-z errors will affect our forecasted constraints by factors of less than or about a few.

3.3.2. Galaxy Bias and Nonlinear Power Spectrum

The results we have shown so far model the galaxy power spectrum using only the linear theory prediction. To make sure that this approximation will not affect the qualitative nature of our results, we compare in Figure 8 the forecasted errors for the photo-z error distribution obtained using the linear theory power spectrum to those obtained with the nonlinear model (see the Appendix). We see that our use of the linear power spectrum is supported in part by National Science Foundation grant 0307961.

![Figure 8: Ratios of forecasted constraints for the parameters $\tilde{\eta}_i$ using the nonlinear power spectrum divided by the constraints using the linear theory power spectrum. The fiducial redshift distribution and photo-z errors are the same in each case. Each line shows a different photo-z bin (corresponding to the index $i$ in the parameters). See the electronic edition of the Journal for a color version of this figure.]

We have shown that the ability to constrain general (i.e., non-Gaussian) photo-z error distributions with galaxy two-point correlation functions depends on the parameterization of the photo-z errors, the range of angular scales probed by the correlation function, and prior knowledge of the galaxy bias. Binning the galaxy sample in photo-z, we have presented constraints on the binned redshift distribution and mean redshift in each photo-z bin. Parameterizing the redshift distribution by binned values is insensitive to small scatter from photo-z errors but otherwise assumes no a priori knowledge of the photo-z error distribution. We find that reducing the number of parameters in each photo-z bin can be very helpful, and this could be achieved with improved knowledge of the photo-z errors from, e.g., spectroscopically calibrated samples or luminosity function priors.

We have limited our use of the galaxy correlation function to angular scales at which the galaxy number density is Gaussian distributed. At low redshifts this severely limits the amount of data available to constrain the photo-z error parameters. We hypothesize that including information from correlations on non-Gaussian scales could significantly improve the constraints and demonstrate that the constraints on the mean redshift in each photo-z bin do improve by 2 orders of magnitude with a naive extrapolation of our Gaussian calculation to non-Gaussian scales.

If it is possible to separate the galaxies by spectral type, the constraints on the photo-z errors could perhaps be improved further by including information from the cross-correlation of the galaxy subsamples. We have demonstrated this in Figures 4 and 5 by separating our fiducial galaxy sample into “red” and “blue” spectral types. We expect this procedure to be particularly helpful if there exists a well-populated spectral class of galaxies whose photo-z values can be estimated unusually well.

Our forecasts are limited to parameters of the photo-z error distribution and linear galaxy bias, so we cannot draw any rigorous conclusions about what kind of dark energy constraints can be achieved in weak lensing and baryon acoustic oscillation surveys with the level of photo-z errors forecasted here. However, we make qualitative comparisons with dark energy forecasts in the literature (Huterer et al. 2006; Ma et al. 2006; Zhan & Knox 2006; Zhan 2006) using our constraints on the mean redshift in each photo-z bin given in Table 2. In the Gaussian regime the constraints we forecast of $\sim0.01$ are factors of a few larger than those desired for upcoming dark energy surveys. However, adding non-Gaussian scales in the correlation function may provide the required constraints. The galaxy correlation properties are quite likely to provide at least a powerful consistency test for the redshift distributions as determined via spectroscopic and/or “super” (12 or more band) calibration subsamples.

![Figure 8: Ratios of forecasted constraints for the parameters $\tilde{\eta}_i$ using the nonlinear power spectrum divided by the constraints using the linear theory power spectrum. The fiducial redshift distribution and photo-z errors are the same in each case. Each line shows a different photo-z bin (corresponding to the index $i$ in the parameters). See the electronic edition of the Journal for a color version of this figure.]

3.3.3. Number of Photo-z Bins

We have compared the fractional constraints on the photo-z error parameters when the number of photo-z bins is varied from 2 to 10 and do not find a significant variation. As the number of photo-z bins is increased, there is more information about the photo-z errors from the additional cross-correlations between photo-z bins, but the number of parameters to constrain also increases. Thus, the fractional constraints we show here for six bins should be representative of the constraints that would be obtained for any moderate number of bins. Note, however, that having the same fractional constraints for a larger number of parameters means we have more information about the photo-z error distribution with photo-z bins. Of course, for a sufficiently large number of photo-z bins, the shot noise will begin to dominate.

4. DISCUSSION AND CONCLUSIONS

We have shown that the ability to constrain general (i.e., non-Gaussian) photo-z error distributions with galaxy two-point correlation functions depends on the parameterization of the photo-z errors, the range of angular scales probed by the correlation function, and prior knowledge of the galaxy bias. Binning the galaxy sample in photo-z, we have presented constraints on the binned redshift distribution and mean redshift in each photo-z bin. Parameterizing the redshift distribution by binned values is insensitive to small scatter from photo-z errors but otherwise assumes no a priori knowledge of the photo-z error distribution. We find that reducing the number of parameters in each photo-z bin can be very helpful, and this could be achieved with improved knowledge of the photo-z errors from, e.g., spectroscopically calibrated samples or luminosity function priors.

We have used our limited use of the galaxy correlation function to angular scales at which the galaxy number density is Gaussian distributed. At low redshifts this severely limits the amount of data available to constrain the photo-z error parameters. We hypothesize that including information from correlations on non-Gaussian scales could significantly improve the constraints and demonstrate that the constraints on the mean redshift in each photo-z bin do improve by 2 orders of magnitude with a naive extrapolation of our Gaussian calculation to non-Gaussian scales.

If it is possible to separate the galaxies by spectral type, the constraints on the photo-z errors could perhaps be improved further by including information from the cross-correlation of the galaxy subsamples. We have demonstrated this in Figures 4 and 5 by separating our fiducial galaxy sample into “red” and “blue” spectral types. We expect this procedure to be particularly helpful if there exists a well-populated spectral class of galaxies whose photo-z values can be estimated unusually well.

Our forecasts are limited to parameters of the photo-z error distribution and linear galaxy bias, so we cannot draw any rigorous conclusions about what kind of dark energy constraints can be achieved in weak lensing and baryon acoustic oscillation surveys with the level of photo-z errors forecasted here. However, we make qualitative comparisons with dark energy forecasts in the literature (Huterer et al. 2006; Ma et al. 2006; Zhan & Knox 2006; Zhan 2006) using our constraints on the mean redshift in each photo-z bin given in Table 2. In the Gaussian regime the constraints we forecast of $\sim0.01$ are factors of a few larger than those desired for upcoming dark energy surveys. However, adding non-Gaussian scales in the correlation function may provide the required constraints. The galaxy correlation properties are quite likely to provide at least a powerful consistency test for the redshift distributions as determined via spectroscopic and/or “super” (12 or more band) calibration subsamples.

We thank M. Auger, G. Bernstein, D. Huterer, D. Koo, J. Newman, and J. A. Tyson for useful conversations. This work was supported in part by National Science Foundation grant 0307961.
APPENDIX A

HALO MODEL

The halo model (for a review see Cooray & Sheth 2002) provides an analytic approximation to the nonlinear three-dimensional galaxy power spectrum using the assumptions that all the dark matter is contained in gravitationally bound “halos” of varying mass and that the number of galaxies populating a given dark matter halo is determined solely by the halo mass and redshift. The model for the number of galaxies in a dark matter halo is often referred to as the halo occupation distribution (HOD).

A1. Fiducial HOD Models

Following Hu & Jain (2004), we divide the mean number of galaxies in a dark matter halo of mass $m$ into contributions from a galaxy at the halo’s center ($\bar{N}_c$) and satellite galaxies ($\bar{N}_s$). The mean number of central galaxies is essentially a unit step function parameterized by a minimum threshold mass, $m_{th}(z)$, for a halo to host a galaxy. To allow for scatter in the relation between galaxy luminosity and halo mass, the simple step function is modified to

$$\bar{N}_c(m, z) = \frac{1}{2} f^a(m, z) \text{erfc}\left(\frac{\log(m_{th}(z)/m)}{\sqrt{2}\sigma}\right),$$  \hspace{1cm} (A1)

where $f^a(m, z)$ is the fraction of central galaxies of spectral type $a$. We use equation (7) in Cooray (2006) as our fiducial model for $f^a(m, z)$ and set $\sigma = 0.1$.

The mean number of satellite galaxies is modeled as a power law:

$$\bar{N}_s(m, z) = \left[ \frac{m}{A m_{th}(z)} \right]^b,$$  \hspace{1cm} (A2)

with the two free parameters $A \sim 30$ and $b \sim 1$ (Hu & Jain 2004).

The threshold mass, $m_{th}$, is determined by requiring the HOD model to reproduce the fiducial redshift distribution as follows:

$$\frac{d\bar{N}}{dz d\Omega}(z) = \chi^2(z) \frac{d\chi}{dz} \int dm n(m, z) \left[ \bar{N}_c(m, z) + \bar{N}_s(m, z) \right]$$ \hspace{1cm} (A3)

$$\equiv \chi^2(z) \frac{d\chi}{dz} \bar{n}_g(z),$$ \hspace{1cm} (A4)

where $\chi(z)$ is the comoving distance as a function of redshift and $n(m, z)$ is the halo mass function [we use the Sheth-Tormen model for $n(m, z)$; Sheth & Tormen 1999].

A2. Galaxy Power Spectrum

The power spectrum of galaxies in the halo model is the sum of two terms, as follows:

$$P_g(k, z_1, z_2) = P_{1h}(k, z_1, z_2) + P_{2h}(k, z_1, z_2),$$

where

$$P_{1h}(k, z_1, z_2) \equiv \frac{\delta_{z_1, z_2}}{\bar{n}_g(z_1)} \int dm n(m, z) \left[ \bar{N}_c^2(m, z_1) u_g^2(k|z_1, m) + 2 \bar{N}_c(m, z_1) \bar{N}_s(m, z_1) u_g(k|z_1, m) \right]$$ \hspace{1cm} (A5)

is the contribution to the power from a single halo,\(^9\)

and

$$P_{2h}(k, z_1, z_2) \equiv P_{lin}(k, z_1, z_2) I_2(k, z_1) I_2(k, z_2),$$ \hspace{1cm} (A6)

with

$$I_2(k, z) \equiv \frac{1}{\bar{n}_g(z)} \int dm n(m, z) b_g(m, z) \left[ \bar{N}_c(m, z) + \bar{N}_s(m, z) u_g(k|z, m) \right],$$ \hspace{1cm} (A7)

is the contribution from two different halos. Here $u_g(k|z, m)$ is the Fourier transform of the galaxy number density profile (assumed to follow the Navarro-Frenk-White profile; Navarro et al. 1996), $b_g(m, z)$ is the halo bias (specified in the Sheth-Tormen model along with the mass function), and $\bar{n}_g(z)$ is the mean comoving number density of galaxies defined in equation (A3).

\(^9\) Here $\delta_{z_1, z_2}$ is the Kronecker delta function.
A3. Linear Power Spectrum

The linear power spectrum in equation (A6) is the variance per logarithmic interval in $k$ in linear perturbation theory:

$$P_{\text{lin}}(k, z_1, z_2) = \delta^2_H(0) g(z_1) g(z_2) \left( \frac{ck}{H_0} \right)^{n+3} T^2(k),$$

where $\delta_H(0)$ is the amplitude at $z = 0$, $g(z)$ is the linear growth function, $T(k)$ is the transfer function, and $H_0$ is the Hubble constant. We take a fiducial value of $n = 1$. We calculated the transfer function, $T(k)$, using the publicly available Boltzmann code, cmbfast$^{10}$ (Seljak & Zaldarriaga 1996).

A4. Linear Galaxy Bias

The galaxy bias in the halo model is given by

$$b^a(z_\alpha) = \int dm n(m, z_\alpha) b_h(m, z_\alpha) \left( \frac{N_\alpha}{m_\alpha} \right),$$  \hspace{1cm} (A8)

where the superscript $(a)$ labeling different galaxy subpopulations denotes different values of the parameters $A$ and $b$ in equation (A2) and $m_\alpha$.

10 At http://www.cmbfast.org (vers. 4.5.1).

REFERENCES

Abazajian, K., et al. 2005, ApJ, 625, 613
Barris, B. J., & Tonry, J. L. 2004, ApJ, 613, L21
Benitez, N. 2000, ApJ, 536, 571
Bernstein, G., & Jain, B. 2004, ApJ, 600, 17
Blake, C., & Glazebrook, K. 2003, ApJ, 594, 665
Connolly, A. J., Csabai, I., Szalay, A. S., Koo, D. C., Kron, R. G., & Munn, J. A. 1995, AJ, 110, 2655
Cooray, A. 2006, MNRAS, 365, 842
Cooray, A., & Sheth, R. 2002, Phys. Rep., 372, 1
Fernández-Soto, A., Lanzetta, K. M., Chen, H.-W., Levine, B., & Yahata, N. 2002, MNRAS, 330, 889
Fernández-Soto, A., Lanzetta, K. M., Chen, H.-W., Pascale, S. M., & Yahata, N. 2001, ApJS, 135, 41
Haiman, Z., Mohr, J. J., & Holder, G. P. 2001, ApJ, 553, 545
Hu, W. 2002, Phys. Rev. D, 66, 83515
Hu, W., & Jain, B. 2004, Phys. Rev. D, 70, 043009
Huterer, D. 2002, Phys. Rev. D, 65, 63001
Huterer, D., Kim, A., Krauss, L. M., & Broderick, T. 2004, ApJ, 615, 595
Huterer, D., Takada, M., Bernstein, G., & Jain, B. 2006, MNRAS, 366, 101
Ilbert, O., et al. 2006, A&A, submitted (astro-ph/0603217)

Ishak, M. 2005, MNRAS, 363, 469
Jungman, G., Kamionkowski, M., Kosowsky, A., & Spergel, D. N. 1996, Phys. Rev. D, 54, 1332
Limber, D. N. 1953, ApJ, 117, 134
Loh, E., & Spillar, E. 1986, ApJ, 303, 154
Ma, Z., Hu, W., & Huterer, D. 2006, ApJ, 636, 21
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, ApJ, 462, 563
Padmanabhan, N., et al. 2006, MNRAS, submitted (astro-ph/0605302)
Riess, A. G., et al. 1998, AJ, 116, 1009
Sawicki, M. J., Lin, H., & Yee, H. K. C. 1997, AJ, 113, 1
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Seo, H., & Eisenstein, D. J. 2003, ApJ, 598, 720
Sheth, R. K., & Tormen, G. 1999, MNRAS, 308, 119
Song, Y., & Knox, L. 2004, Phys. Rev. D, 70, 063510
Tegmark, M., Taylor, A. N., & Heavens, A. F. 1997, ApJ, 480, 22
Turner, E. L. 1980, ApJ, 242, L135
Turner, E. L., Ostriker, J. P., & Gott, J. R., III. 1984, ApJ, 284, 1
Villumsen, J. V. 1995, preprint (astro-ph/9512001)
Zhan, H. 2006, JCAP, 08, 008
Zhan, H., & Knox, L. 2006, ApJ, 644, 663