Chiral Majorana Interference as a Source of Quantum Entanglement

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Interferometry is a powerful tool for entanglement production and detection in multiterminal mesoscopic systems. Here we propose a setup to produce, manipulate and detect entanglement in the electron-hole degree of freedom by exploiting Andreev reflection on chiral one-dimensional channels via interferometry. We study the best possible case in which two-particle interferometry produces superpositions of maximally entangled states. This is achieved by mixing chiral Dirac channels through chiral Majorana modes. We show that it is possible to extract entanglement witnesses through current cross-correlation measurements.

Introduction – Entanglement is at the core of Quantum Theory and represents a key resource for quantum information and computation. Generation, manipulation, and detection of entangled electrons is at the basis of quantum computing with integrated solid-state devices. Among many possibilities, a great attention has been devoted to entanglement in multiterminal mesoscopic conductors (see Refs. [1, 2] for a review) and the most noticeable schemes rely on Cooper pair emission from superconducting contacts [3, 4], correlated electron-hole entangled states by tunnel barriers [5], and integrated single-particle emitters [6].

Here, we suggest to produce entangled electron-hole (e-h) pairs through Andreev reflection on chiral electronic states [7–10] by promoting the e-h degree of freedom (DoF) to an internal interferometric state, analogous to spin in co-propagating spin-resolved edge states in the IQHE [11–14]. In a chiral channel, electrons and holes co-propagate. A particle state that scatters off a superconductor (SC) results in a coherent e-h superposition that co-propagate on the same channel, allowing for e-h interferometry in a controlled way. Interferometry requires a set of linear elements such as beam-splitters (BSs) and phase shifters as building blocks for single- and two-particle elements, such as a Mach-Zehnder (MZ) and a Hanbury-Brown-Twiss (HBT) interferometer. These elements make possible to generate, manipulate and detect entanglement in the e-h and channel DoF.

Recent experiments with chiral one-dimensional (1D) channels in contact with s-wave superconductors opened the way to exploring Andreev reflection on 1D chiral channels [5, 10]. At the same time, no proposal for controlled BS, MZ or HBT e-h interferometer is currently available with ordinary superconductors. The situation is different for topological superconductors (TSCs) hosting chiral Majorana modes at their boundary [15], where interferometric elements were first introduced in Refs. [16, 17]. Remarkably, in these systems two-particle interferometry produces superpositions of maximally entangled states. In particular, as pointed out in Ref. [18], post-selecting states with one fermion per lead yields maximally entangled pairs in the electron-hole space. This would be a quite fortunate coincidence in an ordinary interferometer characterized by arbitrary transmission and reflection coefficients. The advantage of using chiral Majorana channels is that the production of exactly maximally entangled states is guaranteed.

Detection of entangled fermions in the context of quantum transport was first proposed as a particular consequence of anti-bunching in current cross-correlation measurements for a subclass of states (spin-entangled particles propagating along different channels) by using a BS analyzer [19–21]. This was later generalized to the case of multiple mode and occupancy entanglement [22, 24], whereby current cross-correlations can provide entanglement witnesses [25, 26]. Here, we review these proposals in view of single- and two-particle interferometry via chiral Majorana modes and introduce a fundamental phase gate that allows to implement a phase shift between electron and hole states at zero energy. The combined action of a $\mathbb{Z}_2$ MZ and a phase gate allows for arbitrary rotations in the e-h DoF to perform local operations at will on each propagating channel, boosting the entanglement witness power. Our approach makes it possible to exploit single- and two-particle interferometry in the e-h DoF as a platform for quantum computation in dual-rail architectures [27, 28].

The system – The main ingredients are chiral Dirac modes ($\chi$DMs) and chiral Majorana modes ($\chi$MMs) in quantum anomalous Hall insulator/SC structures, as those proposed in Refs. [16, 17]. The recent experimental detection of $\chi$MMs in these systems [29] makes the present proposal feasible and particularly appealing. The system consists in the two-dimensional (2D) surface of a topological insulator (TI) on top of a substrate divided in ferromagnetic (FM) and SC regions. The TI surface hosts a single 2D fermionic Dirac cone described by $H_0 = -iv(\nabla \times \mathbf{s})_z$, with $\mathbf{s}$ a vector of spin Pauli matrices. A FM domain wall acts as $H_{\text{sw}} = M(r)s_z$ and gaps the system everywhere apart from a line where the domain wall changes sign. Along this line a 1D $\chi$DM forms, analogous to the edge states of the IQHE, and can be used as an electronic wave guide. Similarly, SC proximity induces singlet pairing described by $H_{\text{sc}} = \Delta \sum_{\bm{k}} c_{\bm{k}, \uparrow} c_{\bm{-k}, \downarrow}^\dagger + \text{H.c}$.
that opens a topological gap, thus realizing a 2D TSC. Gapless $\chi$MMs form along the border between the SC and the FM regions. By properly arranging these regions it is possible to realize an interferometric setup composed by several linear elements. We now characterize the transport in terms of scattering matrices in the Landauer-Blüttiker formalism adapted to describe $\chi$MMs [30].

The interferometric setup – The setup is illustrated in Fig. 1. We follow the notation of Ref. [18] and denote $\chi$DMs with double arrow lines and $\chi$MMs with single arrow lines. Given that $\chi$DMs have $(s_y) = -1$, we can regard the fundamental excitations as spinless Dirac fermions described by fermionic operators $a(\epsilon)$ at energy $\epsilon$. For energies $0 < \epsilon \ll \Delta$ we define electron- and hole-like states in channel $i$ at energy $\epsilon$ as $a_{i, \uparrow}(\epsilon) = a_{i}(\epsilon)$ and $a_{i, \downarrow}(\epsilon) = a_{i}^\dagger(-\epsilon)$, and introduce a e-h DoF $\gamma = \pm = e, h$. Analogously, at the boundary of the TSC a single $\chi$MM flows, either clockwise $\gamma_1(\epsilon)$ at the boundary between the TSC and the $M_T$ magnetic domain, or anti-clockwise $\gamma_2(\epsilon)$ at the boundary between the TSC and the $M_M$ magnetic domain.

Current is injected in the system upon biasing contacts 1 and 2 and it is collected in contacts 3 and 4. The resulting current and noise in contacts 1 and 2 and it is collected in contacts 3 and 4, that is, the magnetic domain wall and a TSC forces the incoming particle to split into two $\chi$MMs, $\gamma_1$ and $\gamma_2$ [16, 17]. For energies much smaller than the SC gap, we can assume the BS scattering matrix to be energy independent, reading [16, 17]

$$
\begin{pmatrix}
\gamma_1(\epsilon) \\
\gamma_2(\epsilon)
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \begin{pmatrix} a_{i, \uparrow}(\epsilon) \\
a_{i, \downarrow}(\epsilon) \end{pmatrix}.
$$

(2)

Beam splitter – In this context, we denote a BS as an element that takes an incoming $\chi$DM and produces two outgoing $\chi$MMs. The tri-junction between a magnetic domain wall and a TSC forces the incoming particle $a_{+}$ and hole $a_{-}$ states to split into two $\chi$MMs, $\gamma_1$ and $\gamma_2$ [16, 17]. For energies much smaller than the SC gap, we can assume the BS scattering matrix to be energy independent, reading [16, 17]

$$
\begin{pmatrix}
a_{+} \\
a_{-}
\end{pmatrix} = S_{bs} \begin{pmatrix} e^{i\pi n_o + ik L_1} & 0 \\ 0 & e^{ik L_2} \end{pmatrix} S_{bs}^\dagger \begin{pmatrix} a_{+} \\
a_{-} \end{pmatrix},
$$

(3)

where $k(L_1 - L_2) = \epsilon \delta L/v_M$ is the phase difference gathered at energy $\epsilon$ and $n_o$ is the number of vortices in the SC, with $v_M$ the velocity of the $\chi$MMs. The scattering matrix thus mixes incoming chiral electron and hole states in the left ($L$) lead to outgoing chiral electron and hole states in the right ($R$) lead. At finite energy, by varying the path length difference $\delta L$ any arbitrary scattering matrices between incoming and outgoing states in a given channel can be generated. At zero energy this element is expressed as a $\tau_x$ scattering matrix in the e-h space, that represents a $\mathbb{Z}_2$ MZ interferometer being able only to change a particle into a hole, and viceversa.

Phase shifter – A fundamental ingredient appearing in the setup of Fig. 1 is the phase shifter between electrons and holes in a given Dirac channel. This can be easily accomplished by a top gate that locally shifts the chemical potential. For a gate voltage $V_g$ such that $|V_g| \ll M$, where $M$ is the magnitude of the Zeeman splitting of the magnetic domains, electrons and holes will in general acquire an opposite phase,

$$
a_{+}(\epsilon) \to e^{i\varphi_g} a_{+}(\epsilon), \quad a_{-}(\epsilon) \to e^{-i\varphi_g} a_{-}(\epsilon),
$$

(4)

with $\varphi_g = (e/v) \int_0^L dx V_g(x)$. Importantly, this phase is energy-independent so that also carriers at $\epsilon = 0$ acquire it. The scattering matrix associated to the phase shift is $P = \exp(i\varphi_g \tau_z)$. This element is of paramount importance in that, combined with the $\mathbb{Z}_2$ MZ, it provides a
way to rotate the states in the e-h space and generate any superposition state. Moreover, a channel-dependent shift can be obtained by modifying the path length of a given channel. This can be achieved by moving the domain wall through a magnetic field.

Four-terminal element – Finally, the core of the setup in Fig. 3 is a four-terminal device that mixes two incoming \( \chi \) DMs into two outgoing \( \chi \) DMs. In terms of electron and hole channels, the element mixes four incoming states into four outgoing states. This four-terminal element was introduced in Refs. [16–18] and it is described by the scattering matrix

\[
\begin{pmatrix}
    b_{1+} \\
    b_{1-} \\
    b_{2+} \\
    b_{2-}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
    1 & 1 & 1 & 1 \\
    1 & -1 & -1 & -1 \\
    1 & 1 & -\eta & -\eta \\
    1 & 1 & \eta & \eta
\end{pmatrix} \begin{pmatrix}
    a_{1+} \\
    a_{1-} \\
    a_{2+} \\
    a_{2-}
\end{pmatrix},
\]

with \( \eta = (-1)^{n_e e^{i\delta L}/h c s} \) a phase due to the propagation of the \( \chi \)MMs in the interferometer (here \( \delta L \) is the path length difference between the two-particle trajectories [16–18]). At zero energy the phase can be only \( \eta = \pm 1 \), depending on the number of vortices \( n_e \) in the system. A more generic four-terminal element can be obtained by allowing for a cross talk between the \( \chi \)MMs in the SC region.

Cross-correlations as Entanglement witness – The general idea developed in Refs. [19–23] is that, given an unknown initial state that is possibly entangled in the e-h and channel DoF, it is possible to establish the presence of entanglement via measuring current cross-correlations after mixing the channels through a QPC and relate the cross-correlator to an entanglement witness. We now show that witnessing entanglement in a e-h system is not only possible, but also much more effective, thanks to the possibility to insert MZs before and after the QPC.

The most general unknown two-particle state at the input of the QPC can be cast in the generic form

\[
|\Psi\rangle = \sin \theta |\phi(\Phi_{11})\rangle + \sin \phi |\phi(\Phi_{22})\rangle + \cos \theta |\phi(\Phi_{12})\rangle,
\]

with \( \theta, \phi \in [0, \pi/2] \) and \(|\Phi_{ij}\rangle\) two particle states at energy \( \epsilon \) in lead \( i \) and \( j \), \(|\Phi_{11}\rangle = \sum_{\alpha,\beta} \Phi_{\alpha,\beta} a_{\alpha,\beta}^\dagger(E) a_{\alpha,\beta}(E)|0\rangle\), where \(|0\rangle\) is the grounded Fermi sea. The states satisfy \( \Phi_{\alpha,\beta}^\dagger = -\Phi_{\beta,\alpha} \) and the normalization conditions \( \sum_{\alpha,\beta} |\phi(\Phi_{11})_{\alpha,\beta}|^2 = 1 \) and \( \sum_{\alpha,\beta} |\phi(\Phi_{12})_{\alpha,\beta}|^2 = 1/2 \). The state described by Eq. (6) displays entanglement in the occupation number and channel DoF [23–24].

Before the QPC, we induce a phase difference between the electron and hole in each channel by local gate voltages and domain wall displacement. For the moment we do not consider the MZs that are present in the setup of Fig. 1. The QPC mixes the channel of the incoming particles without changing the electron/hole character of the particle injected. The outgoing states after the combined system phase shifter plus QPC is given by

\[
\begin{pmatrix}
    b_{j+} \tau \\
    b_{j-} \tau \\
    b_{j+} \tau' \\
    b_{j-} \tau'
\end{pmatrix} = \sum_{j' = 1,2} S_{j'j,\tau} a_{j',\tau},
\]

where \( b_{j,\tau} \) are the outgoing states after the QPC in lead \( j \) with electron/hole character \( \tau \), where the scattering matrix is given by

\[
S_{\tau} = \begin{pmatrix}
    r e^{i\varphi_\tau} & t e^{-i\varphi_\tau} \\
    r e^{-i\varphi_\tau} & t e^{i\varphi_\tau}
\end{pmatrix}.
\]

The phase \( \varphi_\tau \) accumulated before the QPC has two contributions, the gate contribution \( \varphi_g \), that is opposite for electrons and holes, and a dynamical phase difference due to the different path length between the four-terminal scattering region and the QPC along the two possible paths. This contribution is the same for particles and holes, so that we can write \( \varphi_\tau = \varphi_L + \tau \varphi_g \), with \( \varphi_L = \epsilon \delta L/v \) and \( v \) the velocity of Dirac modes. We now consider the dimensionless current cross-correlator between the output channels 3 and 4,

\[
C_{34} \equiv \frac{h^2 \nu^2}{2 e^2} \lim_{T \to \infty} \int_0^T dt_1 dt_2 \langle I_3(t_1) I_4(t_2) \rangle,
\]

where \( T \) is the measurement time. The current operator of chiral fermions in lead \( i \) is written in terms of particle and hole contributions as

\[
I_i(t) = \frac{e}{\nu} \sum_{\epsilon,\omega,\tau} e^{-i\omega t} b_{i,\tau}^\dagger(\epsilon) b_{i,\tau}(\epsilon + i\omega),
\]

and the average \( \langle \ldots \rangle \) is taken over the incoming state by assuming a discrete spectrum characterised by a density of states \( \nu \) in each lead. Importantly, electrons and holes contribute with different sign to the current, that is accounted for by the \( \tau \) in Eq. (6). In each lead there are incoming and outgoing states. However, due to the chirality of states localized at the domain wall boundary, there is no back scattering and the current can be described only in terms of outgoing channels.

The quantity \( C_{34} \) has the advantage of being a linear function of the input state [23]. Assuming a QPC characterized by \( \tau' = r = \sqrt{1-T} \) and \( \tau = t = i \sqrt{T} \), with \( T \) the transmission probability of the QPC we find

\[
C_{34}(\Psi) = T(1-T) \left[ -\sin^2 \theta + v \sin^2 \theta \sin(2\phi) + w \cos^2 \theta \right] + \frac{1}{2} \cos^2 \theta \sum_{\tau,\sigma} \tau \sigma |\phi(\Phi_{12})_{\tau,\sigma}|^2,
\]

where \( v \) and \( w \) are real quantities satisfying \(|v|, |w| \leq 1 \) that can be expressed in terms of the phases \( \varphi_\tau \) as

\[
v = 2 \Re \sum_{\tau,\sigma} (\Phi_{\tau,\sigma})^* \Phi_{\sigma,\tau} e^{i\varphi_\tau - i\varphi_\sigma},
\]

\[
w = \sum_{\tau,\sigma} \tau \sigma (\Phi_{12})^* \Phi_{12} e^{i(\varphi_\tau - \varphi_\sigma)}.
\]

The correlator \( C_{34} \) is very similar to that of Ref. [23]. Antisymmetry of the \(|\Phi_{ij}\rangle\) states implies that \( \varphi_g \) drops from \( v \). Analogously, the phase \( \varphi_L \) drops from \( w \). By
further redefining $\varphi_g \to \pi/2 + \varphi_g$ we have that $w = \sum_{\tau,\sigma}(\Phi_{12}^{(a)})^*\Phi_{12}^{(b)} e^{i\varphi_{\tau,\sigma}} (\tau-\sigma)$. 

As a particular case we consider incoming states with singly occupied channels 1 and 2 by choosing $\theta = 0$ in the generic input state $|\psi\rangle$. The current correlator is found to be

$$C_{34}(\Phi_{12}) = \frac{1}{2} \sum_{\tau,\sigma} \tau\sigma |\Phi_{12}^{(a)}|^2 + T(1-T)w,$$

(13)

The quantity $w$ captures all the relevant information on the input state. One can show that $w$ is non-negative for separable input states $|\psi\rangle$. The first term in Eq. (13) is nothing but the current correlator before the QPC, $C_{12}(\Phi_{12}) = \frac{1}{2} \sum_{\tau,\sigma} \tau\sigma |\Phi_{12}^{(a)}|^2$. Experimentally one can act on the QPC and switch the tunneling between counter propagating states on and off (by setting $T = 0$ or $T = 1$) and measure separately $C_{34}(\Phi_{12})$ and $C_{12}(\Phi_{12})$. It then follows that the case $C_{34}(\Phi_{12}) - C_{12}(\Phi_{12}) < 0$ witnesses the presence of entanglement in the state $|\psi\rangle$.

For $\theta \neq 0$, i.e., incoming channels 1 and 2 with fluctuating local occupancy in (6), $C_{34}(\Psi)$ can be related to the entanglement of formation $E_f(\Psi)$ [35]: general-ized Werner states [36, 37] are defined by introducing a joint orthonormal basis for ports 1 and 2 formed from the states $|\chi_k\rangle \otimes |\chi_{k'}\rangle$ and $|\Psi_{kk'}^{(\pm)}\rangle = (|\chi_k\rangle_1 \otimes |\chi_{k'}\rangle_2 \pm |\chi_{k'}\rangle_1 \otimes |\chi_k\rangle_2)/\sqrt{2}$ with $k < k'$, where $k$ enumerates all configurations with two or fewer particles per port. It then follows that the entanglement of formation of a state $\rho$ can be lower bounded by the quantity $W(\rho) = \sum_{kk'} \langle \Psi_{kk'}^{(\pm)} | \rho | \Psi_{kk'}^{(\pm)} \rangle /2$. Analogously, we can relate the net correlator $\delta C_{34}(\Psi) = C_{34}(T = 1/2) - C_{34}(T = 0) = C_{34}(\Psi) - C_{12}(\Psi)$, which depends only on the quantities $v$, $w$, and the angle $\theta$, to a lower bound to the entanglement of formation through

$$W(\Psi) = -2\delta C_{34}(\Psi) + \cos^2(\theta)/2.$$

(14)

By noticing that $W(\Psi) > -2\delta C_{34}(\Psi)$ we find that $W(\Psi) > 1/2$ and, consequently, $E_f(\Psi) > 0$ whenever $\delta C_{34}(\Psi) < -1/4$ (see Refs. [22, 23]). Thus, the sign of $-2\delta C_{34}(\Psi) - 1/2$ is sufficient to witness the presence of entanglement in the initial state $\Psi$. We can further post process the data and obtain full information about the state. First of all, we define $v(\varphi_L + \pi/2) = -v(\varphi_L)$. This allows to define $\delta C_{34}^{(\pm)} = (\delta C_{34}(\varphi_L + \pi/2) \pm \delta C_{34}(\varphi_L))/2$ such that

$$\delta C_{34}^{(\pm)}(\varphi_g) = (w(\varphi_g) \cos^2 \theta - \sin^2 \theta)/4,$$

$$\delta C_{34}^{(\pm)}(\varphi_L) = v(\varphi_L) \sin^2 \theta \sin(2\varphi)/4.$$

We then notice that $\cos^2 \theta = 2C_{12}(\Psi)/(2\bar{w} - 1)$, where $\bar{w} = \int \frac{d\varphi_g}{2\pi} w(\varphi_g)$. Upon introducing $\delta \bar{C}_{34}^{(\pm)} = \int \frac{d\varphi_g}{2\pi} \delta C_{34}^{(\pm)}(\varphi_g) = ((1 + \bar{w}) \cos^2 \theta - 1)/4$, we can express

$$\cos^2 \theta = \frac{2}{3} \left( 4\delta \bar{C}_{34}^{(\pm)} - C_{12} + 1 \right),$$

(17)

allowing to establish the occupation and channel admixture as a function of measurable quantities. Finally, we notice that the states $|\Phi_{ij}\rangle$ can only be e-h singlets, so that $\varphi$ depends only on the relative phase difference between $\Phi_{11}$ and $\Phi_{22}$. By varying $\varphi_L$, one can then access $\sin(2\varphi) = 2(\max_{\varphi,\varphi_L} \delta C_{34} - \min_{\varphi,\varphi_L} \delta C_{34})/(1 - \cos^2 \theta)$. The analysis allows to fully access the occupation-number (e-h) and channel DoF entanglement by further exploiting the phase before the QPC [23]. This result can be generalized to generic mixed input states $\rho$, as the combination of the maps $C_{12}$ and $C_{34}$ preserves the linearity of $\delta C_{34}$.

**HBT state** – Having established the general entanglement witnessing protocol via current cross-correlation measurements, we now apply it to the output state of the two-particle interferometer in the setup of Fig. 1. Upon biasing only the contacts 1 and 2 the incoming state at energy $\epsilon$ reads $|\Psi_{in}\rangle = a_{i1}^\dagger a_{i2}^\dagger |\epsilon\rangle [0]$ and the outgoing state reads $|\Psi_{out}\rangle = S_{j,\mu,1,2} S_{k,\nu,2,1} b_{j,\mu}^\dagger b_{k,\nu}^\dagger |\epsilon\rangle [0]$ (summed over repeated indexes), with $S$ the scattering matrix in Eq. (5). In Ref. [13] the system was studied as a Hanbury-Brown-Twiss (HBT) two-particle interferometer and it was recognized that post-selecting states with a single fermion per lead yields maximally entangled states. In this case the current cross correlations allow to access $v = -Re[\eta^2 \bar{v}^L]$ and $w = \{1 + \cos(2\varphi_g)\} + Re[\eta[1 - \cos(2\varphi_g)]]/2$, and the associated witness for $T = 1/2$ and $\eta$ real is

$$W(\Psi_{\text{HBT}}) = \frac{1}{8} [3 - \eta + 2\eta \cos(2\varphi_L) - (1 - \eta) \cos(2\varphi_g)].$$

(18)

In particular, for $\eta = -1$, $\varphi_L = \pi/2$ and $\varphi_g = \pi/2$ one can reach $W = 1$, that corresponds to $E_f = 1$. This means that the phases are means to rotate the initial state to have maximum overlap with the generalized Werner states, confirming that the state coming out from the HBT interferometer is a superposition of maximally entangled states.

**Local operations** – The quantity $C_{34}$ differs from that of Ref. [23] in the measured observable: particle current in the original case in contrast to charge current in the present case. This grants a much more powerful characterization of the incoming states. By inserting MZs before the QPC we can rotate the e-h state on each channel and measure any linear combination of the three Pauli matrices $\tau_i$, with $i = 1, 2, 3$. This operation only affects the state $|\Phi_{12}\rangle$ [28] and $C_{12}$ measurements can assess every local single-particle observable. The insertion of MZs and phase shifters together with the possibility of switching the QPC on and off give us the opportunity to cross-correlate the local operation and to perform a full tomography of the input state.

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particle rotation can only affect the global phase of the
state $\Phi_{ij}$. 