Application of Finite Difference Domain Decomposition Method in Heat Conduction in Three-Layer Insulation Suit

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Abstract. When working in a high temperature environment, people need to wear special clothing to avoid burns. The special clothing is usually composed of three layers of fabric materials, which are referred to as layers I, II and III. Layer I is in contact with the external environment directly. And there is also a gap between layer III and the skin. We considered this gap as the layer IV. Firstly, it is assumed that heat balance exists between the air layer and the I layer, and between the IV layer and the dummy skin respectively. According to the experimental data of the outer skin temperature of the dummy and the principle of heat transfer, the heat exchange coefficient between the IV layer and the outer side of the skin is 0.0047. Using the heat balance relationship, the inner wall temperature of layer I is deduced, and the heat exchange coefficient between the air and the layer I is 0.0049. Secondly, the boundary conditions of the heat conduction model are obtained based on these heat exchange coefficients. And the one-dimensional heat conduction model of this special clothing is established. Since the medium of each layer of the special clothing is different, the diffusion coefficient of the equation is a piece-wise function. Combined the boundary conditions, we uses the implicit algorithm to obtain the discrete difference equations and ensure the stability of the numerical format. Finally, the temperature values of different space-time distributions are obtained.

1. Introduction
As an intermediate between human and environment, clothing is one of the most basic guarantees for human beings to engage in material production activities. With the changes in production activities and the working environment of various industries, the requirements for clothing performance for different environments are also increasing. Research on heat-protecting materials and functions for special clothing for high-temperature operation is an important technical requirement for national security development and improvement of textile industry technology.

Workers in petroleum, chemical, metallurgy, shipbuilding, fire protection, and related places with open flames, sparks, molten metals, and flammable materials are suffering from high-temperature environments such as high-temperature liquids and vapors. Intense heat transfer may burn the skin. At present, the evaluation of the protective performance of thermal protective materials mainly relies on a large number of thermal protection performance tests, and the thermal protection performance value of the materials is used as a criterion for evaluating the performance of the materials. However, a large...
number of experimental tests based on high-temperature working environment cannot be repeated and costly, which will inevitably lead to excessive waste of resources. Therefore, most studies tend to be more theoretical model research.

Since the 1950s, the United States, Europe and other western developed countries have begun research on special clothing for high-temperature operation, and have developed a series of advanced and perfect standards for special clothing for high-temperature operation, and the insulation and stretch performance of special clothing for high-temperature operation. The aspect has achieved certain results. In the late 1980s, the establishment of mathematical models for the study of high-temperature heat source transfer and heat source transfer was gradually carried out, mainly including drying models and coupled models of heat and humidity. On this basis, many scholars studied the transfer of energy and temperature in different media under high temperature conditions. In the actual application, the transfer of energy caused by the temperature difference is the heat transfer. If the temperature at each point in an object in space is different, heat flows from a point where the temperature is higher to a point where the temperature is lower, which is a phenomenon of heat conduction [1]. The heat propagation is carried out according to Fourier's law of experiment: the heat flux density of heat conduction (heat flow per unit area per unit time) is proportional to the temperature gradient there [2]. The research on heat transfer process mainly includes partial differential equation method (heat conduction method), finite difference method, multi-layer flat wall model and cylindrical wall model. It can be seen from the literature that some models only consider thermal conduction heat transfer [3], and less heat transfer processing for thermal protective clothing systems. If this is taken into consideration, either heat conduction or heat transfer are considered to be uncoupled, or heat transfer is handled in a very simple form. Even the most widely used model [4, 5] considers fabrics as homogeneous boards with average thermal properties. In the past, some scholars have done some work to simulate the coupled heat conduction in fabrics [6, 7]. However, Mell and Lawson [6] completely ignore the changes in scattering, self-emission and thermal conductivity caused by thermal decomposition of fabrics. Jiang et al. [7] considered their self-emission, but ignored the thermal properties and thermal conductivity changes caused by thermal degradation of the fabric scattering.

Therefore, there is a need to create a more realistic and accurate numerical model of fabric coupling for heat transfer and heat transfer, especially considering the absorption, emission and scattering of the various layers of the fabric. Unfortunately, there is currently no literature that provides the necessary radiation characteristics (absorption coefficient, scattering coefficient) for fabrics needed to solve the heat transfer equation. In order to more accurately simulate the heat transfer of the heat shield, the main job is to obtain the heat transfer performance of the fabric or garment sample. So this paper made experimental data to study the thermal conductivity of special clothing for high temperature operation.

2. Methodology
In this paper, we use the experimental data of Ahmed Ghazy et al [8]. The dummy whose body temperature is controlled at 37°C is placed in the high temperature environment of the temperature chamber (75 °C) to ensure the constant temperature of the environment. The temperature change outside the skin of the dummy was measured under the condition of a working time of 90 minutes. Fit the experimental data as shown in Fig. 1.
It can be seen from Fig. 1 that the temperature outside the skin of the dummy rises sharply with the increase of time, but the temperature tends to be stable after 1000s. Temperature change is a continuous heat transfer process that can be simulated using the "High Temperature Environment - Apparel - Skin" system. The heat of the external environment is sequentially transmitted to the outer skin of the dummy through the layers I, II, III, and IV, as shown in Fig. 2. In this paper, the distribution of temperature at different time and space is obtained under the condition of known external ambient temperature $T_1$ and artificial outer skin temperature $T_2$. Since the thermal conductivity of each layer of material is different and the temperature variation inside each layer is unknown, so the nonlinear situation is discussed.

For the nonlinear method, this paper derives the one-dimensional heat conduction equation and performs heat propagation according to Fourier's law. The heat $Q_1$ from time $t_1$ to $t_2$ is equal to the heat $Q_2$ from temperature $u_{1_1}$ to $u_{2_1}$. Combining the Gaussian formula for mathematical reasoning, a partial differential equation can be constructed for solving. Considering the problem that the explicit equation results do not converge, this paper converts the explicit solution into an implicit solution to ensure the convergence of the results.

Combined with the principle of heat transfer, the process of heat transfer gradually transitions from non-steady state to steady state. In this paper, for the steady-state and non-steady-state conditions, the heat transfer process is analyzed and solved, and the temperature distribution is calculated.
3. Establishment of heat transfer model

3.1. Establishment of temperature fitting model for I and IV layers

Since there is heat conduction between different media, the mutual heat conduction relationship between the air and the special clothing for high temperature operation is considered here. It is not difficult to find that the curve presented in Fig. 1 is similar to the sigmoid function image. So this article uses a function like \( y = c_1(1+c_2e^{-c_3x})^{-1} \) to fit the curve. The coefficient \( c_1, c_2, c_3 \) is obtained, thereby obtaining a temperature fitting model and a heat exchange coefficient \( k_4 = c_3 \).

In the same way, the high temperature external environment and the I layer are analyzed. We must know the temperature distribution of the I layer in the steady state. Therefore, based on the experimental data, the temperature distribution of the I layer is deduced. The derivation process is as follows:

The relationship between the temperature of the layer IV and the skin temperature of the dummy is obtained by Fourier's law:

\[
q = \lambda_4 (u_4 - u_3) \cdot \delta_4^{-1}
\]  

(1)

Then, the thermal resistance of each layer can be expressed as:

\[
\begin{align*}
  u_4 &= \delta_4 \cdot \lambda_4^{-1} \cdot q + u_3 \\
  u_3 &= \delta_2 \cdot \lambda_2^{-1} \cdot q + u_5 \\
  u_1 &= \delta_1 \cdot \lambda_1^{-1} \cdot q + u_2
\end{align*}
\]

(2)

By using equation (2) and the temperature distribution of \( u_4 \), the temperature distribution of \( u_1 \) can be obtained, and then the heat exchange coefficient \( k_1 \) can be fitted.

3.2. Establishment of one-dimensional heat conduction model.

This model investigates the heat transfer problem of a three-layer fabric material for a garment for high-temperature work and an air layer with a gap between the layer III and the skin. Firstly, function \( u(x, y, z, t) \) is used to represent the temperature of the special clothing for high-temperature operation at position \((x, y, z)\) and time \(t\). The propagation of heat follows the Fourier's law of experiment: the heat \( dQ \) of an infinitely small area \( dS \) flowing in an infinite hour section \( dT \) is proportional to the direction derivative \( \frac{\partial u}{\partial n} \) of the high-temperature work-specific clothing temperature along the normal direction of the curved surface \( dS \), namely:

\[
dQ = -k(x, y, z) \frac{\partial u}{\partial n} dS dt
\]

(3)

Where \( k(x, y, z) \) is the heat conduction coefficient of the special clothing for high-temperature operation at point \((x, y, z)\), and it takes a positive value? When one of the special clothing for high temperature operation is uniform and isotropic, \( k \) is a constant, and \( n \) is the normal of the curved surface \( dS \) along the direction of the heat flow.
In order to derive the equation satisfied by the temperature $u$. We take a curved surface $\Gamma$ consisting of a closed curve in a special clothing for high-temperature work, and the area it contains is denoted as $\Omega$. And the surface $\Gamma$ flows from the time $t_1$ to the time $t_2$. The heat is:

$$Q_1 = \int_{t_1}^{t_2} \left( \int_{\Gamma} k \frac{\partial u}{\partial n} \, dS \right) \, dt \quad (4)$$

The inflow of heat changes the internal temperature of the zone $\Omega$. The heat required to change the temperature of the special clothing from $u(x, y, z, t_1)$ to $u(x, y, z, t_2)$ in the time interval $(t_1, t_2)$ is:

$$Q_2 = \iiint_{\Omega} c(x, y, z) \rho(x, y, z) \left[u(x, y, z, t_2) - u(x, y, z, t_1)\right] \, dv \quad (5)$$

Where $c$ is the specific heat of a layer of clothing for high-temperature work, and $\rho$ is the density of a layer of clothing for high-temperature work.

Assume that there is no heat source in the special clothing for high-temperature operation under investigation. According to the heat conservation formula, we obtain $Q_2 = Q_1$, namely

$$\iiint_{\Omega} c(x, y, z) \rho(x, y, z) \left[u(x, y, z, t_2) - u(x, y, z, t_1)\right] \, dv = \int_{t_1}^{t_2} \left( \int_{\Gamma} k \frac{\partial u}{\partial n} \, dS \right) \, dt \quad (6)$$

Let function $u$ have a second-order continuous partial derivative for variables $x, y, z$, and have a first-order continuous partial derivative for $t$, namely:

$$\frac{\partial u}{\partial n} = \frac{\partial u}{\partial x} \cos \alpha + \frac{\partial u}{\partial y} \cos \beta + \frac{\partial u}{\partial z} \cos \gamma \quad (7)$$

According to Gauss formula, Eq (4) can be transformed as:

$$Q_1 = \int_{t_1}^{t_2} \left\{ \iiint_{\Omega} \left[ \frac{\partial}{\partial x} \left( k \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial u}{\partial z} \right) \right] \, dv \right\} \, dt \quad (8)$$

According to Gauss formula, Eq (5) can be also transformed as

$$Q_2 = \iiint_{\Omega} c(x, y, z) \rho(x, y, z) \left( \int_{t_1}^{t_2} \frac{\partial u}{\partial t} \, dt \right) \, dv = \int_{t_1}^{t_2} \left( \iiint_{\Omega} c \rho \frac{\partial u}{\partial t} \, dv \right) \, dt \quad (9)$$

Substituting Eq (8) and Eq (9) into Eq (6), we have:

$$\int_{t_1}^{t_2} \left\{ \iiint_{\Omega} c \rho \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left( k \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left( k \frac{\partial u}{\partial y} \right) - \frac{\partial}{\partial z} \left( k \frac{\partial u}{\partial z} \right) \right\} \, dt = 0 \quad (10)$$
Since the $t_1$, $t_2$ and $\Omega$ regions are arbitrary and the integrand is continuous, we get:

$$\rho c \frac{\partial u}{\partial t} = \rho \frac{\partial}{\partial x} \left( k \frac{\partial u}{\partial x} \right) + \rho \frac{\partial}{\partial y} \left( k \frac{\partial u}{\partial y} \right) + \rho \frac{\partial}{\partial z} \left( k \frac{\partial u}{\partial z} \right)$$

(11)

The formula (11) is a heat conduction equation of a non-uniform isotropic body. Since each layer of the special clothing for high-temperature operation is composed of a uniform substance, at this time, $k$, $c$ and $\rho$ are constant, definition $k / \rho c = a^2$. The heat conduction equation of the special clothing for high-temperature operation is:

$$\frac{\partial u}{\partial t} = a^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

(12)

In the above heat conduction equation, the independent variables $x$, $y$ and $Z$ of the space coordinates are depicted, so it is a three-dimensional heat conduction equation. However, we abstract each layer of high-temperature operation special clothing as a uniform thin rod, and the side adiabatic. The temperature distribution on the same section is the same. So it is easy to get the one-dimensional heat conduction equation of each layer of high-temperature operation special clothing:

$$\frac{\partial u}{\partial t} = a^2 (x) \frac{\partial^2 u}{\partial x^2}$$

$$k_1 \frac{\partial u}{\partial n} \bigg|_{x=0} = h(u_\infty - u) \bigg|_{x=0}$$

$$k_4 \frac{\partial u}{\partial n} \bigg|_{x=L} = h(u - u_0) \bigg|_{x=L}$$

$$u_\infty = 75^\circ C; u_0 = 37^\circ C$$

(13)

4. Model solving

4.1. Solution of temperature fitting model for me and IV layers.

Import the experimental data into MATLAB 2016 for programming and get the parameter: $c_1 = 48.0800$, $c_2 = 0.3412$, $c_3 = 0.0047$.

The IV layer temperature fitting function can be obtained as:

$$y' = \frac{48.08}{1 + 0.3412e^{-0.0047x}}$$

(14)

The heat exchange coefficient between the layer IV and the dummy skin is $k_4 = 0.0047$.

From the temperature distribution of the first layer, the parameters can be obtained by using MATLAB2016 programming: $c_1 = 0.53913$, $c_2 = 0.52217$, $c_3 = 0.0049438$

The I layer temperature fitting function can be obtained as:
The heat exchange coefficient between the inner wall of the first layer and the external environment is as follows: \( k_i = 0.0049438 \).

4.2. Solution of one-dimensional heat conduction model.

The finite difference region decomposition algorithm (FDM) is used to solve the one-dimensional heat conduction equation of each layer of the special clothing for high-temperature operation. The basic idea is to replace the continuous solution region with a grid of finite discrete points, and approximate the continuous function on the region with discrete variables to obtain the difference quotient derivative. Then the original differential equation is transformed into an algebraic equation system to obtain a discrete solution. Finally, the interpolation method can be used to obtain the approximate solution on the solution area of the original problem from the discrete solution. The specific finite difference domain decomposition algorithm steps are shown in Fig. 3.

![Fig. 3 FDM algorithm steps](image)

The main process of FDM algorithm is represented as follows. Firstly, select \( u(x,t) \) for the forward difference quotient of \( t \) and the second-order center difference quotient for \( x \):

\[
\begin{align*}
\frac{u^{n+1}_j - u^n_j}{\Delta t} & \approx \frac{\partial u}{\partial t} \\
\frac{u^{n+1}_{j+1} - 2u^{n+1}_j + u^{n+1}_{j-1}}{\Delta x^2} & \approx \frac{\partial^2 u}{\partial x^2}
\end{align*}
\]

(16)

The one-dimensional heat conduction equation of each layer of the high-temperature work clothing is discretized, and the discrete equation is:

\[
\frac{u^{n+1}_j - u^n_j}{\Delta t} = a^2 \frac{u^{n+1}_{j+1} - 2u^{n+1}_j + u^{n+1}_{j-1}}{\Delta x^2}
\]

(17)

Finishing is available:

\[
-\frac{a^2 \Delta t}{\Delta x^2} u^{n+1}_{j-1} + \frac{(1 + 2a^2 \Delta t)}{\Delta x^2} u^{n+1}_j - \frac{a^2 \Delta t}{\Delta x^2} u^{n+1}_{j+1} = u^n_j
\]

(18)

Its matrix form is:
5. Results and discussion

The unsteady condition, the temperature-thickness variation trend in steps of 5 mm is shown in Fig. 4(Non-steady state conditions); under steady state conditions, the change trend of temperature and thickness in steps of 200 mm is shown in Fig. 4(Steady-state conditions).

It can be seen from Fig. 4 that in the unsteady state, as the thickness deepens, the temperature change shows a downward trend and gradually reaches equilibrium, and the falling curve shows a linear relationship piece by piece. Compared with the non-steady state, the "temperature-thickness" curve in the one-dimensional heat conduction model approaches gentleness, which is similar to the cases where the inverse proportional function takes the first quadrant, and the fitting result is more ideal. As the thickness increases, the temperature gradually decreases, and the degree of change becomes slower and gentler until the temperature reaches equilibrium. Keep the thickness constant, and as the heat transfer time increases, the temperature gradually rises until it reaches equilibrium. The longer the heat transfer time, the faster the temperature reaches the equilibrium point, and the temperature remains constant after reaching equilibrium.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & L & 0 \\
-\frac{a^2\Delta t}{\Delta h^2} & \left(1 + \frac{2a^2\Delta t}{\Delta h^2}\right) & -\frac{a^2\Delta t}{\Delta h^2} & 0 & L & 0 \\
0 & -\frac{a^2\Delta t}{\Delta h^2} & \left(1 + \frac{2a^2\Delta t}{\Delta h^2}\right) & -\frac{a^2\Delta t}{\Delta h^2} & L & 0 \\
M & M & M & M & M & M \\
0 & 0 & L & -\frac{a^2\Delta t}{\Delta h^2} & \left(1 + \frac{2a^2\Delta t}{\Delta h^2}\right) & -\frac{a^2\Delta t}{\Delta h^2} \\
0 & 0 & 0 & 0 & L & 1
\end{bmatrix}
\begin{bmatrix}
u_{1}^{n+1} \\
u_{2}^{n+1} \\
u_{3}^{n+1} \\
u_{4}^{n+1} \\
u_{n-1}^{n+1} \\
u_{n}^{n+1}
\end{bmatrix}
= \begin{bmatrix}
u_{1}^{n} \\
u_{2}^{n} \\
u_{3}^{n} \\
u_{4}^{n} \\
u_{n-1}^{n} \\
u_{n}^{n}
\end{bmatrix}
\]

Fig. 5 shows the three-dimensional "temperature-time-thickness" trend from the 465s to 1084s and the trend of the whole process. As can be seen from Fig. 5, as time and thickness increase, the temperature gradually decreases and approaches a steady state. When the time is determined, the temperature gradually decreases with the increase of the thickness, the slope gradually decreases, and the temperature surface tends to be gentle. When the thickness is determined, as time increases, the temperature gradually rises until it is stable.
6. Conclusion
In real life, high-temperature environmental work clothing is widely used. The heat conduction process of each layer needs further research. In the process of solving the heat conduction model in this paper, because of the instability and non-convergence of the results of the partial differential equation. The result of the program running and the experimental data are greatly different. Therefore, the explicit solution of partial differential equation is transformed into implicit solution to ensure the convergence of the operation. The approach is innovative. In addition, it can better reflect the changing rule of temperature in the special clothes for high-temperature operation, so as to save the production cost.

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References
[1] K.W. Graves, Fire fighter's exposure study, Technical Report, AGFSRS 71-2, Wright-Patterson Air Force Base, Ohio, December, 1970.
[2] Sun Jinhai. Equations of mathematical physics and special functions/Department of Mathematics, Huazhong University of Science and Technology [M]. Beijing: Higher Education Press, 2001.
[3] G.N. Mercer, HS. Sidhu. Mathematical modeling of the effect of fire exposure on a new type of protective clothing, ANZIAM, Vol. 49, pp. C289-C305, 2007.
[4] D.A. Torvi, J.D. Dale, Heat transfer in thin fibrous materials under high heat flux, Fire Technol.35 (3)(1999) 210-231.
[5] Y. Su, J. He, J. Li, Modeling the transmitted and stored energy in multilayer protective clothing under low-level radiant exposure, Appl. Therm. Eng. 93(2016)1295-1303.
[6] W.E. Mell, J.R. Lawson. A heat transfer model for fire fighter’s protective clothing. NIST Report, NISTIR 6299, 1999.
[7] Y.Y. Jiang, E. Yanai, K. Nishimura, H. Zhanga, N. Abe, M. Shinoharab, et al., An integrated numerical simulator for thermal performance assessments of firefighters’ protective clothing, Fire Saf. J. 45 (5) (2010) 314–326.
[8] Ahmed Ghazy, Donald J. Bergstrom. Numerical Simulation of Heat Transfer in Firefighters' Protective Clothing with Multiple Air Gaps during Flash Fire Exposure [J]. Numerical Heat Transfer, Part A: Applications,2012,61(8).