Effect of exchange correlation potential on dispersion properties of lower hybrid wave in degenerate plasma

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Abstract. The dispersion properties of lower hybrid wave are studied in electron-ion degenerate plasma with exchange effect in non-relativistic regime. It is found that the combined effect of Bohm potential and exchange correlation potential significantly modifies the dispersion properties of lower hybrid wave. The graphical results explicitly show the influence of degeneracy pressure, Bohm force and exchange correlation potential on the frequency of the lower hybrid mode. Present work should be of relevance for the dense astrophysical environments like white dwarfs and for laboratory experiments.

1. Introduction

Recently, there has been a great deal of interest in investigating the properties of electrostatic waves [1-6] in plasmas incorporating quantum effects. The study of such plasma waves provides important insights about instabilities in various laboratory fusion and astrophysical plasma environments [7-11]. Several authors have studied the propagation characteristics of electrostatic electron and ion waves in quantum plasmas [4-7]. When the de-Broglie wavelength \( \lambda_B \) of the particles becomes comparable to the average inter-particle distance \( \lambda_B \), these quantum effects cannot be ignored. It is also evident that; when density is high the exchange correlation potential can even dominate the role of particle dispersion effects [11-13]. The general idea that produces this exchange correlation potential is essentially the density fluctuation theory (DFT) [11]. Several authors have studied the influence of exchange potential and Bohm force on the propagation characteristics of different waves [11-13] in plasmas and showed the relevance of exchange correlation potential in context with solid-state plasmas, inertial confinement fusion plasmas and in an astrophysical region such as white dwarf stars. Therefore, it is important to study the combine influence of both the dispersion effects and exchange potential on the dynamics of plasma waves.

The fundamental process responsible for heating and accelerating plasma around white dwarfs, neutron stars and in tokamaks [14] is reconnection. The investigation regarding the electron acceleration and the magnetic reconnection phenomena (via. lower hybrid plasma waves) in laboratory, space, and astrophysical plasmas is widely studied by many researchers [14, 15]. But none of the authors have investigated the instability of the lower hybrid waves in electron-ion quantum plasma taking into account the particle dispersion effects and exchange-correlation potential; as the lower hybrid waves provide the necessary electron acceleration that leads to plasma heating mechanism. These waves are simultaneously in resonance with both the magnetized electrons and un-magnetized ions. The lower-hybrid resonance frequency is given as \( \omega_L = \sqrt{\omega_i \Omega_e} \) (where \( \Omega_i \) and \( \omega_e \) are the ion and electron gyro frequencies, respectively). Lower hybrid waves have also their importance in space [14] and fusion plasmas [14, 16].
So in the present letter we have studied the effect of exchange correlation potential, Fermi velocity and Bohm force on the propagation characteristics and phase velocity of lower hybrid waves. The present investigation is done using quantum hydrodynamic (QHD) model. This paper is organized as follows. In Section 2 we have presented QHD set of equations for dense quantum plasmas. In Section 3 we describe contribution of Bohm force and exchange potential on the dispersion of transverse waves in magnetized plasmas. In Section 4 a brief summary of obtained results is presented.

2. Theoretical model

We consider two-fluid quantum plasma system consisting of electron and ion species. We have assumed that the electrons are degenerate, as the Fermi temperature of electrons is much greater than the plasma electron temperature while the ions are assumed to be non-degenerate and are unaffected by quantum corrections. The dynamics of electrons and ions are governed by following set of QHD equations

\[ \frac{\partial}{\partial t} n_e + n_e \nabla \cdot \vec{v}_e = 0 \]  
\[ m_e \frac{\partial}{\partial t} \vec{v}_e = q_e n_e \left( \vec{E} + \frac{1}{c} \left[ \vec{v}_e, \vec{B} \right] \right) - \frac{\hbar^2}{4m_e} \nabla \left( \nabla^2 n_e \right) - V_{ex} \nabla n_e \]  
\[ V_{ex} = -0.985 \frac{n_e}{\epsilon} \left[ 1 + \frac{0.034}{\alpha_{Bohr} n_e^{1/3}} \left( 1 + 18.37 \alpha_{Bohr} n_e^{1/3} \right)^3 \right] \]  
\[ \frac{\partial}{\partial t} n_i + n_i \nabla \cdot \vec{v}_i = 0 \]  
\[ m_i \frac{\partial}{\partial t} \vec{v}_i = q_i n_i \left( \vec{E} + \frac{1}{c} \left[ \vec{v}_i, \vec{B} \right] \right) - \nabla P_i \]  

where \( n_e, n_i, v_e, \) and \( q_e, q_i \) represents the number density, fluid velocity, mass and charge of species (electrons and ions) respectively. Equation (1) and (2) are the continuity and momentum equations for electrons. Equation (3) shows the expression for exchange correlation potential, where \( \epsilon \) is the dielectric constant of material and \( \alpha_{Bohr} = e \hbar^2 / m_e \) is the effective Bohr atomic radius of the species. Since the electrons are fermions they follow Fermi-Dirac statistics. Therefore, the temperature of electrons considered here is Fermi temperature \([\text{Sharma and Chhajalani 10}]\) which is given as \( T_{Fe} = \frac{\hbar^2}{3 \pi^2 n_e^3} \left( \frac{3}{2} m_e \right)^{3} \). Fermi velocity of fermions is \( v_{Fe} = \frac{\hbar}{m_e} \left( \frac{3}{2} m_e \right)^{1/2} \), and the corresponding Fermi pressure of degenerate electrons is given as \( P_e = \frac{\hbar^2}{3 \pi^2 n_e^3} \left( \frac{3}{2} m_e \right)^{3} \left( \frac{5}{2} m_e \right) \). Further the dynamics of non-degenerate ions are governed by following set of continuity and momentum equations (4) and (5) respectively. The ion thermal pressure is given via. equation of state i.e., \( P_i = k_i T_i n_i \) in which \( T_i \) is temperature of ions and \( k_i \) is Boltzmann constant.

3. Dispersion relation and graphical discussion

Let the solution of the system of equation be of the form \( \exp \left[ ik \cdot \hat{x} - i \omega t \right] \), where \( \omega \) is the frequency of perturbation and \( k \) is the x-component of perturbed wave vector. The perturbation in physical quantities can be ruled according to [10],

\[ n = n_0 + n_1, \hat{v} = \hat{v}_0 + \hat{v}_1, \hat{B} = \hat{B}_0 + \hat{B}_1, \hat{E} = \hat{E}_0 + \hat{E}_1 \]  

where the subscript “1” denotes the perturbed part and subscript “0” denotes an unperturbed part. At equilibrium, \( \hat{v}_0, \hat{E}_0 = 0 \). We restrict ourselves to \( \hat{E} = E \hat{e}_x \) and \( \hat{B}_0 = B_0 \hat{z} \). After linearizing the equations and applying quasi-neutrality condition the dispersion relation for lower hybrid wave in dense astrophysical plasmas can be obtained as
\[ \omega^2 = \omega_c \Omega_e + k^2 \left[ 1 + \frac{m_e}{m_i} \right]^{-1} \left[ v_{fe}^2 - \frac{\hbar^2 k^2}{4m_i^2} - \frac{V_{exc}}{m_e} \right] + \left[ 1 + \frac{m_e}{m_i} \right]^{-1} k^2 v_{ti}^2 \]  

(7)

where \( \omega_c = eB_0 / m_e c \) and \( \Omega_e = eB_0 / mc \) are electron and ion cyclotron frequencies respectively. Equation (7) represents the dispersion relation for modified lower hybrid wave. The modification is due to exchange correlation potential, degeneracy pressure and Bohm force of degenerate electrons and thermal pressure of non-degenerate ions.

In the degenerate astrophysical environment like white dwarf stars, neutron stars, pulsar magnetospheres where magnetic field is of the order of \( B \approx 10^5 - 10^{10} \ T \), the number density \( n_o \approx 10^9 - 10^{10} \ m^{-3} \) and temperatures ranges \( T \approx 10^5 \ - 10^8 \ K \) [4, 10-12] the degeneracy effects cannot be ignored. Now in order to analyze the effect of exchange potential and Bohm force in affecting the considered plasma system we write the dispersion relation (7) in normalized form as

\[ \omega^\ast \Omega_k - \omega_e \ast \Omega_e - M \ast \left[ k^2 \left( v_{fe}^\ast - V_{exc}^\ast \right) - H^\ast k^4 \right] - k^2 = 0 \]  

(8)

where we have used \( \left[ 1 + \left( m_i / m_e \right) \right] \approx \left( m_e / m_i \right) \) and \( \left[ 1 + \left( m_i / m_e \right) \right] \approx 1 \). In deriving equation (8) we have used the following dimensionless parameters

\[ \omega_k = \omega / \Omega_e, \ k = k v_e / \Omega_e, \ H = h^2 \Omega_e^2 / 4m_e v_{ti}^2, \ v_{exc}^\ast = V_{exc} / m_e v_e, \ v_{fe}^\ast = v_{fe} / v_e, \ M = m_i / m_e \]  

(9)

The normalized parameters can be chosen in a range as \( H = 0.0 - 0.4, \ V_{exc}^\ast = 0.0 - 0.6, \ v_{fe}^\ast = 0.0 - 0.6 \). Now to analyze the dispersion relation numerically Figure 1 and 2 are plotted to show the influence of exchange correlation potential and Fermi velocity of electrons in affecting the dynamics of lower hybrid waves.

**Figure 1.** Growth rate of instability \( \omega^\ast \) is shown as a function of wave number \( k^\ast \) for arbitrary values of exchange potential parameter \( (V_{exc}^\ast = 0.0, 0.3, 0.5) \), keeping the other parameter to be fixed.

**Figure 2.** Growth rate of instability \( \omega^\ast \) is shown as a function of wave number \( k^\ast \) for arbitrary values of Fermi velocity \( (v_{fe}^\ast = 0.0, 0.3, 0.6) \), keeping the other parameter to be fixed.

Figure 1 shows the influence of exchange correlation potential of electrons on the dispersion characteristics of lower hybrid waves. In figure 1 the growth rate of lower hybrid wave \( \omega^\ast \) is plotted
against the normalized wave number $k'$ in the presence and absence of exchange effects (using equation 8). From figure it is clear that as the value of exchange parameter increases, it decreases the phase velocity of lower hybrid wave. Further, it is quite interesting to study the role of Fermi velocity of electrons in affecting the characteristics of lower hybrid wave. For this purpose we have plotted figure 2 that represents a comparison for higher and lower Fermi velocity of electrons. From figure 2 it is found that the phase velocity of lower-hybrid wave enhances abruptly as Fermi velocity increases. The Fermi velocity of electrons is significantly responsible for the frequent propagation of lower-hybrid waves in degenerate plasmas.

4. Conclusions
In the present work we have studied the role of exchange correlation potential, degeneracy pressure and quantum Bohm force of electrons on the propagation of lower hybrid wave in two-species quantum plasma system. In deriving the dispersion relation we have considered the special case, where the electron-ion plasma is quasi-neutral. The derived dispersion relation is found to be modified due to quantum effect of electrons and thermal effect of ions. It is concluded that the Fermi velocity of electrons and thermal velocity of ions increases the growth rate of lower hybrid waves. The numerical discussion of the results shows that the presence of exchange potential and Bohm force decreases the phase speed of lower hybrid waves while the Fermi velocity of electrons increases it. The present results are applicable to study the propagation characteristics of lower hybrid waves in astrophysical as well as laboratory fusion plasmas.

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