We study the physical reach of a Neutrino Factory in the 2+2 and 3+1 Four Neutrino mixing scenarios, with similar results for the sensitivity to the mixing angles. Huge CP-violating effects can be observed in both schemes with a near, $O(10)$ Km, detector of $O(10)$ Kton size in the $\nu_\mu \rightarrow \nu_\tau$ channel. A smaller detector of 1 Kton size can still observe very large effects in this channel.

1 Introduction

Indications in favour of neutrino oscillations have been obtained both in solar neutrino\[1\] and atmospheric neutrino\[2\] experiments with $\Delta m_{\text{sol}}^2 \leq 10^{-4}$ eV$^2$ and $\Delta m_{\text{atm}}^2 \sim 10^{-3}$ eV$^2$. The LSND data\[3\] would indicate a $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation with a third neutrino mass difference: $\Delta m_{\text{LSND}}^2 \sim 0.3 \rightarrow 6$ eV$^2$. If MiniBooNE\[4\] confirms the LSND results we would therefore face three independent evidence for neutrino oscillations characterized by squared mass differences quite well separated. To explain the whole ensemble of data at least four different light neutrino species are needed.

There are two classes of four neutrino spectra: three almost degenerate neutrinos and an isolated fourth one (the 3+1 scheme), or two pairs of almost degenerate neutrinos divided by the large LSND mass gap (the 2+2 scheme). Although the latter is still favoured\[5\], the new analysis of the experimental data\[3\] results in a shift of the allowed region towards smaller values of the mixing angle, $\sin^2(2\theta)_{\text{LSND}}$, reconciling the 3+1 scheme with exclusion bounds\[6\],\[7\].

Four neutrino oscillations imply a Maki-Nakagawa-Sakata (MNS) $4 \times 4$ mixing matrix, with six rotation angles $\theta_{ij}$ and three phases $\delta_i$ (for Dirac-type neutrinos). This large parameter space is actually reduced to a smaller subspace whenever some of the mass differences become negligible. Consider the measured hierarchy in the mass differences, $\Delta m_{\text{sol}}^2 \ll \Delta m_{\text{atm}}^2 \ll \Delta m_{\text{LSND}}^2$ and

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\[1\]Indications in favour of neutrino oscillations have been obtained both in solar neutrino experiments and atmospheric neutrino experiments.

\[2\]The LSND experiment is an example of an atmospheric neutrino experiment.

\[3\]MiniBooNE is another experiment that has confirmed the LSND results.

\[4\]The Maki-Nakagawa-Sakata (MNS) mixing matrix is a key element in the theory of neutrino oscillations.

\[5\]The 3+1 scheme is preferred when the LSND mass gap is large.

\[6\]Exclusion bounds refer to limits on the mixing angles that prevent certain neutrino oscillation scenarios.

\[7\]The measured hierarchy in the mass differences is a crucial aspect of understanding neutrino oscillations.
define $\Delta_{ij} = \Delta m^2_{ij} L/(4E_\nu)$. At short distance, $L = O(1)$ Km, for neutrinos up to $O(10)$ GeV, $(\Delta_{\text{sol}}; \Delta_{\text{atm}}) \ll 1$ and $\Delta_{LSND} = O(1)$ and we can safely neglect the solar and atmospheric mass differences. In the 2+2 scheme the rotation angles in the $(1-2)$ and $(3-4)$ planes become irrelevant in oscillation experiments, together with two CP-violating phases, and the parameter space reduces to 4 rotation angles and 1 phase only. In the 3+1 scheme the rotations in the whole three-dimensional subspace $(1-2-3)$ are irrelevant for oscillation experiments, and the physical parameter space contains three rotation angles and no phases. When considering CP-violating phenomena at least two mass differences should be taken into account. In this approximation, regardless of the scheme, the parameter space contains 5 angles and 2 phases.

In the 2+2 scheme, the following parametrization was adopted [12] :

$$U_{MNS} = U_{14}(\theta_{14}) U_{13}(\theta_{13}) U_{24}(\theta_{24}) U_{23}(\theta_{23}, \delta_3) \times U_{34}(\theta_{34}, \delta_2) U_{12}(\theta_{12}, \delta_1)$$ \hspace{1cm} (1)

in the 3+1 scheme, the following parametrization [10] shares the same virtues of eq. (1):

$$U_{MNS} = U_{14}(\theta_{14}) U_{24}(\theta_{24}) U_{34}(\theta_{34}) \times U_{23}(\theta_{23}, \delta_3) U_{13}(\theta_{13}, \delta_2) U_{12}(\theta_{12}, \delta_1).$$ \hspace{1cm} (2)

In the one-mass dominance approximation, the unphysical angles and phases automatically decouple in both cases.

A Neutrino Factory [10] is perfectly suited to explore this large parameter space, hopefully including the discovery of leptonic CP violation [12]. A comparison of the physical reach of a Neutrino Factory in the 2+2 and 3+1 schemes has been extensively presented elsewhere [10] and will be summarized here. We shall consider in what follows as a “reference set-up” a neutrino beam resulting from the decay of $n_\mu = 2 \times 10^{20}$ unpolarized positive and/or negative muons per year. The collected muons have energy $E_\mu$ in the range 10 – 50 GeV.

As the dominant signals are expected to peak at $L/E_\nu \sim 1/\Delta m^2_{LSND}$, most of the CP-conserving parameter space can be explored in short baseline experiments ($L \sim 1$ Km) with a small size detector with $\tau$ tracking and ($\mu, \tau$) charge identification capability. We consider an hypothetical 1 ton detector with constant background $B$ at the level of $10^{-5}$ of the expected number of charged current events and a constant reconstruction efficiency $\epsilon_\tau = 0.5$ for $\mu^\pm$ and $\epsilon_\tau = 0.35$ for $\tau^\pm$ (neutrinos with $E_\nu \leq 5$ GeV have not been included). To extend our analysis to the CP-violating parameter space we consider an hypothetical 10 Kton detector, located a bit farther from the neutrino source, at $L = O(10 - 100)$ Km.

2 Sensitivity reach of the Neutrino Factory

We follow a conservative (or even “pessimistic”) hypothesis [10] and consider the four gap-crossing angles in the 2+2 scheme, $\theta_{13}, \theta_{14}, \theta_{23}$ and $\theta_{24}$, to be equally small (i.e. less than $10^{-6}$), with the possible exception of one angle free to vary in some interval. The remaining angles $\theta_{12}$ and $\theta_{34}$ are the solar and atmospheric mixing angles in the two-family parametrization, respectively. In the 3+1 scheme we restrict to one of the allowed regions [10], $\Delta m^2_{34} = 0.9 \text{ eV}^2$, $\sin^2(2\theta)_{LSND} \simeq 2 \times 10^{-3}$, for simplicity, and we take equally small gap-crossing angles $\theta_{14}$. The remaining angles, $\theta_{12}, \theta_{23}$ and $\theta_{13}$ can be obtained by the combined analysis of solar and atmospheric data in the three-family parametrization [13].

Our results [10] show that the considered set-up can severely constrain the whole four-family model CP-conserving parameter space, both in the 2+2 scheme and 3+1 scheme. In the former, the sensitivity reach to all gap-crossing angles in the LSND-allowed region is at the level of $\sin^2\theta \geq 10^{-6} - 10^{-4}$, depending on the specific angle considered. In the latter the sensitivity reach is at the level of $\sin^2\theta \geq 10^{-5} - 10^{-3}$, slightly less than in the 2+2 case.

This results can be easily understood in terms of a simple power counting argument [10]. In the third column of Tab. [10] we report the leading order in $\epsilon$ for the CP-conserving oscillation
Table 1: Small angles suppression in the CP-conserving and CP-violating oscillation probabilities, and in the signal-to-noise ratio of the CP asymmetries, in the three-family model and in both four-family model mass schemes.

| Scheme       | Transition       | \(P_{CP}\) | \(P_{\overline{CP}}\) | \(A/\Delta A\) |
|--------------|------------------|-------------|------------------------|---------------|
| Three-family | \(\nu_e \rightarrow \nu_\mu\) | \(\epsilon^2\) | \(\epsilon\) | \(O(1)\) |
|              | \(\nu_e \rightarrow \nu_\tau\) | \(\epsilon^2\) | \(\epsilon\) | \(O(1)\) |
|              | \(\nu_\mu \rightarrow \nu_\tau\) | 1           | \(\epsilon\) | \(O(epsilon)\) |
| 2+2         | \(\nu_e \rightarrow \nu_\mu\) | \(\epsilon^2\) | \(\epsilon^2\) | \(O(epsilon)\) |
|              | \(\nu_e \rightarrow \nu_\tau\) | \(\epsilon^2\) | \(\epsilon^2\) | \(O(epsilon)\) |
|              | \(\nu_\mu \rightarrow \nu_\tau\) | \(\epsilon^2\) | \(\epsilon^2\) | \(O(1)\) |
| 3+1         | \(\nu_e \rightarrow \nu_\mu\) | \(\epsilon^4\) | \(\epsilon^3\) | \(O(epsilon)\) |
|              | \(\nu_e \rightarrow \nu_\tau\) | \(\epsilon^4\) | \(\epsilon^3\) | \(O(epsilon)\) |
|              | \(\nu_\mu \rightarrow \nu_\tau\) | \(\epsilon^4\) | \(\epsilon^2\) | \(O(1)\) |

probabilities \(P_{CP}\) in the three-family model and in both schemes of the four-family model. In three families, the small parameter is \(s_{13} \sim \epsilon\). In four families we consider equally small LSND gap-crossing angles: \(s_{13} = s_{14} = s_{23} = s_{24} \sim \epsilon\) for the 2+2 scheme; \(s_{14} = s_{24} = s_{34} = \epsilon\) for the 3+1 scheme. In this last case we take \(s_{13} \sim \epsilon\), also. Notice that the appearance transition probabilities in the 2+2 scheme are generically of \(O(\epsilon^2)\), with the only exception of \(\nu_\mu \rightarrow \nu_\tau\). In the 3+1 scheme, on the contrary, they are all \(O(\epsilon^4)\). This explains the (slight) decrease in the sensitivity in the 3+1 scheme with respect to the 2+2 scheme.

3 CP-violating Observables

In the four-family model we can consider CP-violating observables whose overall size does not depend on \(\Delta_{sol}\) (as in three families) but on \(\Delta_{atm}\). Large CP-violating effects are therefore possible in this case. Finally, the CP-violating observables are maximized for \(L/E_\nu \sim 1/\Delta m^2_{SND} = O(10)\) Km (for neutrinos of \(E_\nu = O(10)\) GeV) and matter effects are therefore completely negligible.

We consider the neutrino-energy integrated quantity \[1\]:

\[
\bar{A}^{CP}_{\alpha\beta}(\delta) = \frac{N[l_\tau^-/N_\nu[l_\mu^-]]_+ - (N[l_\tau^+/N_\nu[l_\mu^+]])_-}{(N[l_\tau^+/N_\nu[l_\mu^+]])_+ + (N[l_\tau^-/N_\nu[l_\mu^-]])_-},
\]

where \(N[l_\pm]\) is the number of taus due to oscillated neutrinos and \(N_\nu[l_\pm]\) is the expected number of muons in the absence of oscillations. In order to quantify the significance of the signal, we compare the value of the integrated asymmetry with its error, \(\Delta \bar{A}^{CP}_{\alpha\beta}\), in which we include the statistical error and a conservative background estimate at the level of \(10^{-5}\), and subtract the matter induced asymmetry \(\bar{A}^{CP}_{\alpha\beta}(0^\circ)\).

In Fig. 3 we show the signal-to-noise ratio of the subtracted integrated CP asymmetry in the \(\nu_\mu \rightarrow \nu_\tau\) channel for the 2+2 (left) and the 3+1 (right) schemes, respectively. In both cases, for \(E_\nu = 50\) GeV, \(\sim 100\) standard deviations are attainable at \(L \simeq 30 - 40\) Km. For a detector of size \(M\), a reduction factor \(\propto 1/\sqrt{M}\) should be applied. Therefore, for an OPERA-like \(O(1)\) Kton detector we still expect large CP-violating effects in the \(\nu_\mu \rightarrow \nu_\tau\) channel. The other two channels, \(\nu_e \rightarrow \nu_\mu, \nu_\tau\) give a much smaller significance in both schemes.

The real gain with respect to the three-family model\[14\] is that the small solar mass difference, that modules the overall size of the CP-violating asymmetry, is traded with the much larger atmospheric mass difference. The optimal channel to observe CP violation is the \(\nu_\mu \rightarrow \nu_\tau\) channel. This result can be easily understood looking at Tab. 3. In the fourth and fifth columns we report the leading order in \(\epsilon\) for the different CP-violating oscillation probabilities \(P_{CP}\) and
for the related signal-to-noise ratio of the CP asymmetries (remind that \( A/\Delta A \) is proportional to \( P_{CP}/\sqrt{P_{CP}} \)). Notice that in the three-family model the \( \nu_e \rightarrow \nu_\mu, \nu_\tau \) channels have a signal-to-noise ratio of the corresponding CP asymmetry of \( O(1) \) in the small angles. On the contrary, in both the 2+2 and 3+1 four-family model, it is the \( \nu_\mu \rightarrow \nu_\tau \) channel to be of \( O(1) \) in the small angles, thus justifying a posteriori our results.

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