Large transverse momentum direct photon production in the coherent diffractive processes at hadron colliders

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Abstract

Direct photon production at large transverse momentum in the coherent diffractive processes at hadron colliders is calculated in the two-gluon exchange model. We find that the amplitude for the production process is related to the differential off-diagonal gluon distribution function in the proton. We estimate the production rate at the Fermilab Tevatron by approximately using the usual gluon distribution function. Because of the clean signature, this process can be used to detailed study the small-x physics, and the coherent diffractive processes at hadron colliders.

PACS number(s): 12.40.Nn, 13.85.Ni, 14.40.Gx
In recent years, there has been a renaissance of interest in diffractive scattering. These
diffractive processes are described by the Regge theory in terms of the Pomeron (\(IP\)) exchange
\[1\]. The Pomeron carries quantum numbers of the vacuum, so it is a colorless entity in QCD
language, which may lead to the "rapidity gap" events in experiments. However, the nature
of the Pomeron and its interaction with hadrons remain a mystery. For a long time it had
been understood that the dynamics of the "soft Pomeron" was deeply tied to confinement.
However, it has been realized now that much can be learned about QCD from the wide
variety of small-\(x\) and hard diffractive processes, which are now under study experimentally.

On the other hand, as we know that there exist nonfactorization effects in the hard
diffractive processes at hadron colliders \[2–5\]. First, there is the so-called spectator effect \[4\],
which can change the probability of the diffractive hadron emerging from collisions intact.
Practically, a suppression factor (or survive factor) \(S_F\) is used to describe this effect.
Obviously, this suppression factor can not be calculated in perturbative QCD, which is now
viewed as a nonperturbative parameter. Typically, the suppression factor \(S_F\) is determined
to be about 0.1 at the energy scale of the Fermilab Tevatron \[5\]. Another nonfactorization
effect discussed in literature is associated with the coherent diffractive processes at hadron
colliders \[3\], in which the whole Pomeron is induced in the hard scattering. It is proved in
\[3\] that the existence of the leading twist coherent diffractive processes are associated with
a breakdown of the QCD factorization theorem.

In this paper, we will calculate the direct photon production at large transverse mo-
mentum in the coherent diffractive processes at hadron colliders under the framework of the
two-gluon exchange parametrization of the Pomeron. As shown in Fig.1, the whole Pomeron
represented by the color-singlet two-gluon system emits from one hadron and interacts with
another hadron to produce photon plus a quark jet. The large transverse momentum is re-
quired to ensure the validity of the application of perturbative QCD. In the leading order of
QCD, there is only the quark initiated partonic process contributing to the coherent diffrac-
tive production of large transverse momentum photon. The partonic process \(qp \rightarrow q\gamma p\) is
shown in Fig.2.
To calculate the cross section for the partonic diffractive process, we use the two-gluon exchange parametrization of the Pomeron model. The two-gluon exchange model has been used to calculate diffractive photoproduction processes including the productions of vector meson \( J/\psi \), open charm \( \pi \), large \( p_T \) di-jet \( \gamma \). This model has gained some success in the description of these processes at ep colliders \( \gamma \). Recently, we have extended this model to calculate the diffractive processes at hadron colliders. We have calculated the diffractive \( J/\psi \) \( \gamma \) production, charm jet \( \gamma \) production, and massive muon pair and W boson productions \( \gamma \) at hadron colliders. An important feature of this two-gluon exchange model is the relation of the production amplitude to the off-diagonal gluon distribution function \( l_T \). The results of \( \gamma \) indicate that we can explore much low \( x \) gluon distribution in the proton by studying the diffractive processes in hadron collisions.

In the two-gluon exchange model, the leading order contribution to the partonic diffractive process \( qp \to \gamma qp \) comes from the four diagrams shown in Fig.2. These four diagrams are the same as those calculated in \( \gamma \) except the difference on the virtuality of the photon. In Ref. \( \gamma \) the produced photon is virtual and time-like, while here in this paper we will calculate the production of real photon in the diffractive process at large transverse momentum. Also, these four diagrams imply that the partonic process \( qp \to \gamma qp \) is related by crossing to diffractive di-quark jets photoproduction process \( \gamma p \to q\bar{q}p \).

As indicated in Refs. \( \gamma \), in the calculations of the amplitude for the massless particle production in the diffractive processes by using the two-gluon exchange model there is no contribution from the region of \( l_T^2 < k_T^2 \), where \( l_T \) and \( k_T \) are the transverse momenta of the loop momentum and the final state photon momentum as shown in the diagrams of Fig.2. This is contrast to the results of Refs. \( \gamma \), where the dominant (large logarithmic) contribution comes from the small \( l_T^2 \) region. Therefore, the expansion method used in of \( \gamma \) in which \( l_T^2 \) is viewed as a small parameter is not further applicable for the calculation of the real photon production in the diffractive process \( qp \to \gamma qp \). So, in the following we directly calculate the cross section by squaring the amplitude and picking up the dominant terms.
Due to the positive signature of these diagrams (color-singlet exchange), we know that the real part of the amplitude cancels out in the leading logarithmic approximation. To evaluate the imaginary part of the amplitude, we must calculate the discontinuity represented by the crosses in each diagram of Fig. 2.

In our calculations, we express the formula in terms of the Sudakov variables. We select $q$ and $p$ as the light cone momenta, where $q$ and $p$ are the momenta of the incident quark and the diffractive proton respectively. For high energy hadron scattering, we know that the light quark and the proton masses are much smaller than the hard scattering scale. So, we have $q^2 = 0$, $p^2 = 0$, and we set $2pq = s$, where $s$ is the total c.m. energy of the quark-proton system, i.e., the invariant mass of the partonic process $qp \rightarrow \gamma qp$. Thus, every four-momenta $k_i$ can be decomposed as

$$k_i = \alpha_i q + \beta_i p + k_{iT}, \quad (1)$$

Where $\alpha_i$ and $\beta_i$ are the momenta fractions of $q$ and $p$ carried by $k$. $k_{iT}$ is the transverse momentum, and it satisfies

$$k_{iT} \cdot q = 0, \quad k_{iT} \cdot p = 0. \quad (2)$$

All of the Sudakov variables for every momentum are determined by the on-shell conditions of the external lines and the crossed lines in each diagram. For example, the Sudakov variables associated with the momenta $k$ and $u$ are,

$$\alpha_u = 0, \quad \beta_u = x_{IP} = \frac{M^2_x}{s}, \quad u_T^2 = 0 \quad (3)$$

$$\alpha_k(1 + \alpha_k) = -\frac{k_T^2}{M_X^2}, \quad \beta_k = -\alpha_k \beta_u, \quad (4)$$

where $k_T$ is the transverse momentum of the outgoing photon, $M^2_x$ is the invariant mass of the diffractive final state (including the large transverse momentum photon and the quark jet).

For the high energy diffractive process, we know that $M^2_x \ll s$, i.e., we have $\beta_u (x_{IP})$ as a small parameter. In the following calculations of the cross section for the partonic process,
we take the leading order contribution and neglect the high order contributions which are proportional to \( \beta_u = \frac{M^2}{s} \).

By using the above Sudakov variables, we can evaluate the diffractive cross section formula for the partonic process \( qp \rightarrow \gamma qp \) as

\[
\frac{d\hat{\sigma}(qp \rightarrow \gamma qp)}{dt}\bigg|_{t=0} = \frac{dM^2_T d^2k_T d\alpha_k}{16\pi s^2 16\pi^3 M_X^2} \delta(\alpha_k(1 + \alpha_k) + \frac{k_T^2}{M_X^2}) \sum |A|^2,
\]

where \( A \) is the amplitude of the process. We know that the real part of the amplitude does not contribute. And the imaginary part of the amplitude \( A \) has the following form for every diagram of Fig.2,

\[
\text{Im}A = C_F \int \frac{d^2l_T}{(l_T^2)^2} F \times \bar{u}_i(u - k) \Gamma_{\mu} v_i(q),
\]

where \( C_F = \frac{4}{9} \) is the color factor for the four diagrams. \( F \) is defined as

\[
F = \frac{3}{2s} g_s^2 e\bar{e} f(x', x''; l_T^2),
\]

where

\[
f(x', x''; l_T^2) = \frac{\partial G(x', x''; l_T^2)}{\partial \ln l_T^2}.
\]

The function \( G(x', x''; l_T^2) \) is the off-diagonal gluon distribution function in the proton [10], where \( x' \) and \( x'' \) are the momentum fractions of the proton carried by the outgoing and returning gluons of each diagrams of Fig.2.

After a straightforward calculation, and neglecting the higher order contributions which are proportional to \( \beta_u = \frac{M^2}{s} \), we get the amplitude squared as

\[
\sum |A|^2 = \frac{128\pi^5 \alpha_s^2 \alpha e \bar{e}}{9} s^2 (1 + \alpha^2) k_T^2 \frac{1}{\alpha_k(M_X^2)^2} \int \frac{d^2l_T}{(l_T^2)^2} \frac{d^2l_T'}{(l_T'^2)^2} f(x', x''; l_T^2) f(y', y''; l_T'^2)
\]

\[
\left(1 + \alpha_k\right) k_T^2 - \left(1 + \alpha_k\right)(k_T, l_T) \left(1 + \alpha_k\right) l_T^2 - \left(1 + \alpha_k\right)(k_T, l_T')
\]

\[
\left(1 + \alpha_k\right) l_T'^2 - \left(1 + \alpha_k\right)(k_T, l_T') \right),
\]

where \( f(x', x''; l_T^2) \) and \( f(y', y''; l_T'^2) \) are the differential off-diagonal gluon distribution functions associated with the loop momenta \( l_T \) and \( l_T' \) respectively.
From the above equations, we can see that the amplitude for the partonic process $qp \to \gamma qp$ is related to the off-diagonal gluon distribution in the proton. By now there is no parametrization of the off-diagonal parton distributions, and it is expected that at small $x$ the off-diagonal gluon distribution is not far away from the usual diagonal gluon distribution $[15]$. So in the following, we approximate the off-diagonal gluon distribution by the usual gluon distribution, i.e.,

$$G(x', x''; Q^2) = G(x; Q^2)$$ and

$$f(x', x''; Q^2) = f_g(x; Q^2),$$

where $x = x_{IP} = \frac{M_X^2}{s}$. $f_g(x; Q^2)$ is the usual differential gluon distribution in the proton.

So, after integrating over the azimuth angle of $\vec{l}_T$, we get

$$\sum |A|^2 = 128 \pi^5 \alpha_s^2 \alpha_e^2 q_9 s_2 (1 + \alpha_k^2) k_T^2 \alpha k (M_X^2)^2 |I|^2,$$

(10)

where the integration $I$ is defined as

$$I = \int \frac{dl_T^2}{(l_T^2)^2} \frac{1}{2} |1 - \frac{k_T^2}{k_T^2} - (1 + \alpha_k)^2 l_T^2 | f_g(x; l_T^2).$$

(11)

From the above results of the integration, we can see that the amplitude $A(qp \to q\gamma p)$ will be zero in the integral region of $l_T^2 < k_T^2/(1 + \alpha_k)^2$. So, the dominant contribution to the integration of the amplitude over $l_T^2$ will come from the region of $l_T^2 \sim k_T^2/(1 + \alpha_k)^2$. This behavior of the amplitude integration over $l_T^2$ is similar to the results of Refs. [8,14]. Therefore, if we neglect the evolution effects of the gluon distribution function in this region, the integration will then be

$$I = \frac{(1 + \alpha_k)^2}{k_T^2} f_g(x; \frac{k_T^2}{1 + \alpha_k^2}).$$

(12)

Under this approximation, the differential cross section for the partonic process $qp \to \gamma qp$ will then be

$$\frac{d\hat{\sigma}}{dt}(qp \to \gamma qp)|_{t=0} = \int_{M_X^2 > 4k_T^2} dM_X^2 dk_T^2 d\alpha_k \frac{\pi^2 \alpha_s^2 e_9^2}{18} \delta[\alpha_k (1 + \alpha_k) + \frac{k_T^2}{M_X^2}]$$

$$\frac{(1 + \alpha_k^2)(1 + \alpha_K)^2}{\alpha_k (M_X^2)^2 k_T^2} (f_g(x; \frac{k_T^2}{1 + \alpha_k^2}))^2.$$
is similar to the results of \cite{8,13}. However, in the processes of Refs. \cite{6,7,11–13}, the cross section is proportional to the integrated gluon distribution function. This is because in the processes of \cite{6,7,11–13}, there exists large logarithmic contribution in the region of $l_T^2 \ll M^2_x$, which will lead to the gluon distribution after integrating over $l_T^2$ in the small $l_T^2$ region. On the other hand, in the processes of \cite{8,14} and the direct photon production process calculated in this paper, in the smaller $l_T^2$ region, $(l_T^2 < \frac{k_T^2}{(1+\alpha^2)})$, the integral is equal to zero, so there is no large logarithmic contribution to the amplitude. The contribution from the integration of large $l_T^2$ region only leads to the differential gluon distribution terms, because the integral decreases rapidly as $l_T^2$ increases.

Using Eq. (13), we can estimate the production rate of large $p_T$ direct photon in the diffractive processes at the Fermilab Tevatron. We adopt the value of $S_F = 0.1$ to describe the spectator effect at this machine. The numerical results are displayed in Fig.3 and Fig.4. In the calculations, we set the scale of the running coupling constant identical to the scale of the gluon distribution function, i.e, $Q^2 = \frac{k_T^2}{(1+\alpha^2)}$. For the parton distribution functions, we select the GRV NLO set \cite{16}.

In Fig.3, we plot the differential cross section as a function of the lower cut of the transverse momentum of the produced photon, $k_{T\text{min}}$. Approximately, we can derive the dependence on $k_{T\text{min}}$ from Eq. (13), in which the integration over $M^2_x$ mainly comes from the region of $M^2_x \sim 4k_T^2$. So the differential cross section behaves as,

$$\frac{d^2\hat{\sigma}}{dk_T^2 dt}\bigg|_{t=0} \sim \frac{1}{(k_T^2)^2(f_g(x; 4k_T^2))^2}$$

Where $x = 4k_T^2/M^2_x$. Similarly, integration over $k_T^2$ dominantly comes from the region of $k_T^2 \sim k_{T\text{min}}^2$. So we get the approximate dependence of the cross section on $k_{T\text{min}}$ as

$$\frac{d\hat{\sigma}}{dt}\bigg|_{t=0} \sim \frac{1}{(k_{T\text{min}}^2)^2(f_g(x; 4k_{T\text{min}}^2))^2}$$

This dependence can be seen from Fig.3. This is a distinctive feature of the calculation of our model for this process.

In Fig.4, we plot the dependence of the differential cross section on $x_{1\text{min}}$, $x_1$ is the longitudinal momentum fraction of the proton carried by the incident quark. $x_{1\text{min}}$ is the
lower bounder of $x_1$ in the integration of the cross section. From this figure we can see that the dominant contribution to the cross section comes from the region of $x_1 \sim 10^{-1}$, which is similar to the case of the diffractive massive muon pair and $W$ boson productions [13]. However, if we compare this result to that of the diffractive $J/\psi$ and charm jet productions [11,12], we will find that the dominant contribution region of $x_1$ to the cross section of direct photon production here is some orders of magnitude larger than that of $J/\psi$ and charm jet productions. This is because the direct photon production is the quark initiated process which is sensitive to large-$x$ quark distribution in the proton. However, the productions of $J/\psi$ and charm jet are the gluon initiated processes, so they are sensitive to the small-$x$ gluon distribution in the proton. So, the $x_1$ dependence of these two types of processes are distinctive different.

In conclusion, we have shown that the large transverse momentum direct photon diffractive production can provide a test for the two-gluon exchange model of the Pomeron, and can also be viewed as a compensate to the $J/\psi$ and charm jet productions for the study of the coherent diffractive processes at hadron colliders. And furthermore, because of the clean signature, this process can be used to detailed study the small-$x$ physics, and the coherent diffractive processes at hadron colliders. The photon production process is a quark induced process, and is sensitive to the large-$x$ quark distribution function in the proton. However, all of the coherent diffractive processes calculated in the two-gluon exchange model are related to the off-diagonal gluon distribution function in the proton. So, the measurement of these processes at hadron colliders may provide a lot of information about the off-diagonal gluon distribution function.

This work was supported in part by the National Natural Science Foundation of China, the State Education Commission of China, and the State Commission of Science and Technology of China.
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Figure Captions

Fig.1. Sketch diagram for the diffractive direct photon production at hadron colliders in perturbative QCD.

Fig.2. The lowest order perturbative QCD diagrams for partonic process $qp \rightarrow \gamma qp$.

Fig.3. The differential cross section $d\sigma/dt|_{t=0}$ for the large transverse momentum direct photon production at the Fermilab Tevatron as a function of $k_{T\text{min}}$, where $k_{T\text{min}}$ is the lower bound of the transverse momentum of the produced photon.

Fig.4. The differential cross section $d\sigma/dt|_{t=0}$ as a function of $x_{1\text{min}}$, where $x_{1\text{min}}$ is the lower bounder of $x_1$ in the integration of the cross section.
Fig. 1
Fig. 2
Fig. 3

The figure shows the graph of $d|\sigma|/dt_{\text{had}}$ (in $10^6 \text{GeV}^4$) against $k_{\text{min}}$ (GeV). The graph depicts a decreasing trend as $k_{\text{min}}$ increases.
Fig. 4

\[ \frac{d\sigma}{dt}|_{t=0} (10^8 \text{GeV}^4) \]