The Spin Distribution of Millisecond X-ray Pulsars

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Abstract. The spin frequency distribution of accreting millisecond X-ray pulsars cuts off sharply above 730 Hz, well below the breakup spin rate for most neutron star equations of state. I review several different ideas for explaining this cutoff. There is currently considerable interest in the idea that gravitational radiation from rapidly rotating pulsars might act to limit spin up by accretion, possibly allowing eventual direct detection with gravitational wave interferometers. I describe how long-term X-ray timing of fast accreting millisecond pulsars like the 599 Hz source IGR J00291+5934 can test the gravitational wave model for the spin frequency limit.

Keywords: Neutron stars, Pulsars, Gravitational waves

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INTRODUCTION

This proceedings volume (and the meeting it arises from) celebrates the decadal anniversary of the discovery of the first accretion-powered millisecond pulsar SAX J1808.4−3658 [1, 2], and more generally the discovery of millisecond X-ray variability tracing (or at least coupled to) the spins of accreting neutron stars in low-mass X-ray binaries [3, 4, 5]. These discoveries, all made with the Rossi X-ray Timing Explorer (RXTE; [6]), finally verified the proposal that the old, weak-field neutron stars that comprise the population of millisecond radio pulsars were spun up (“recycled”) by sustained accretion [7, 8]. The detailed history of these ideas is reviewed by Alpar in these proceedings [9].

The accreting millisecond X-ray pulsars (AMXPs) are obviously an ideal laboratory for studying the physics underlying the spin and magnetic evolution of neutron stars at the end of their lives, and particularly for understanding the pulsar recycling process in detail. One approach is to measure the short-term spin evolution of individual AMXPs in order to study the action of magnetic accretion torques. Spin frequency derivatives have been reported during transient outbursts of several accretion-powered AMXPs, with spin-up/spin-down rates of order $\sim 10^{-14}$ Hz s$^{-1}$ [10, 11, 12, 13, 14], consistent with the scale predicted by standard magnetic accretion torque theory for the observed luminosities [15, 16]. However, some pulsars are subject to substantial pulse shape variability over an outburst, which can potentially mimic spin evolution (see [17]). This conference witnessed significant debate about how serious this problem is and how to best mitigate it. In this paper, I discuss a second approach: using the underlying spin distribution of the entire ensemble of known AMXPs to explore what governs their spin evolution.
How fast can recycled pulsars spin? We certainly expect a strict upper limit from centrifugal break-up, with the spin frequency where this occurs depending upon the equation of state for the ultradense matter in the neutron star core. A firm upper limit of $\sim 3$ kHz is set by the requirement that the sound speed in the neutron star interior not exceed the velocity of light, while the most “favored” models for the equation of state have maximum spins in the 1500–2000 Hz range \cite{18, 19, 20}.

**Radio pulsars; accretion-powered AMXPs.** Ideally, one would use the large population ($\sim 200$) of known millisecond radio pulsars to infer the underlying spin distribution of recycled pulsars. However, there are a number of significant selection biases against the detection of the fastest millisecond radio pulsars (see, e.g., \cite{21}), making it difficult to accurately estimate the underlying population from the observed sample. What about the X-ray systems? Among the AMXPs, we must distinguish between the accretion-powered pulsars (in which the persistent accretion flux is modulated due to magnetic channeling) and the nuclear-powered pulsars (in which oscillations are detected during thermonuclear X-ray bursts originating at a discrete point on the stellar surface). Seven of the ten known accretion-powered millisecond X-ray pulsars are low-luminosity X-ray transients with short orbital periods (and thus significant Doppler smearing of the pulsation), raising multiple concerns about selection biases against detection, as well as the possibility that the observed population represents only a narrow (and perhaps unrepresentative) evolutionary subset of the low-mass X-ray binary population. The other three accretion-powered millisecond pulsars show only intermittent pulsations with low duty cycle \cite{22, 23, 24, 25}, again raising questions about a selection bias against detection.

**Nuclear-powered AMXPs.** On the other hand, the nuclear-powered millisecond X-ray pulsars (the burst oscillation sources) are an ideal probe of the spin distribution. Their oscillations (and their bursts) are bright and easily detected throughout the Galaxy, the signals are sufficiently short-lived to make orbital Doppler smearing irrelevant, their host systems sample a wide variety of orbital periods and evolutionary histories among the low-mass X-ray binaries, and the RXTE/PCA instrument has no significant selection biases against detecting oscillations as rapid as $\sim 2$ kHz.

**Measuring the spin distribution.** The spin frequencies of the known AMXPs (both accretion-powered and nuclear-powered) are plotted in Figure 1. The spins are consistent with a uniform distribution with the observed frequency range (45–619 Hz). The absence of any pulsars above 619 Hz is extremely statistically significant, given that there is no significant loss of RXTE sensitivity out to at least 2 kHz. Using only the subset of nuclear-powered pulsars in order to minimize selection bias, and assuming a uniform distribution up to some maximum frequency, the observed distribution implies a maximum spin frequency of 730 Hz (95% confidence; \cite{5, 30}). While the exact value for the maximum spin frequency depends upon the particular prior assumption chosen for the underlying distribution, the existence of a cutoff frequency is robust. However, the detailed shape of the high-frequency distribution and the shape of the cutoff (flat out to a sharp cutoff, “pile up” up to a sharp cutoff, tail of a Gaussian distribution, etc.) cannot be constrained from the available sample, although numerical simulations suggest that a mere factor of two increase in sample size would already be revealing in this regard.

Assuming that the known AMXPs (or at least the nuclear-powered ones) are a valid
FIGURE 1. The spin frequency distribution of accreting millisecond X-ray pulsars. There is a sharp
cutoff in the population for spins above 730 Hz. RXTE has no significant selection biases against detecting
oscillations as fast as 2000 Hz, making the absence of fast rotators extremely statistically significant
[5, 30].

proxy for the general population of recycled millisecond pulsars, it appears that recycled
pulsars have a spin frequency limit well below the centrifugal break-up rate for most
equation of state models. Moreover, submillisecond pulsars are evidently very rare, if
they exist[1]. The 716 Hz spin frequency of the fastest known millisecond radio pulsar,
PSR J1748−2446ad in Ter 5 [27], is consistent with these conclusions, as are the results
of recent attempts to infer the underlying spin distribution from radio pulsar observations
[28, 29, 21].

HOW TO EXPLAIN THE SPIN DISTRIBUTION

A very low breakup spin rate for neutron stars? It is unlikely that centrifugal breakup
is the physics behind the observed spin frequency cutoff. Figure 2 shows that a breakup
limit as low as 730 Hz excludes most equation of state models for neutron stars, allowing
only those with very large radii [18, 20]. In order to accomodate the generally accepted

1 There was a recent report of evidence for a 1122 Hz burst oscillation in XTE J1739−285 [26], but
independent analysis by several other groups (including ours) indicates that the statistical significance of
the reported signal is marginal.
FIGURE 2. Neutron star break-up and equation of state. The solid curves are theoretical mass-radius relations for a variety of models for the equation of state for ultradense matter from [18]. The dashed curves show the limits arising from breakup spin rates of 730 Hz and 2000 Hz; the allowed phase space is to the right of the appropriate dashed curve. We see that while a 2000 Hz breakup rate is consistent with most equations of state, a 730 Hz breakup rate is inconsistent with most models and excludes the 8–12 km radius range usually inferred for a 1.4 $M_\odot$ neutron star.

an 8–12 km radius range for a 1.4 $M_\odot$ neutron star a breakup limit above $\sim$1500 Hz is required.

**Pulsars not at magnetic spin equilibrium?** One possible alternative is that most of the systems in the sample are not at their magnetic spin equilibrium. While spin equilibrium is often assumed for these systems since their accretion spin-up time scale is much shorter than their X-ray active evolutionary lifetime, some authors argue that many systems may not have reached equilibrium, or may depart significantly from it on short time scales [31]. This is primarily an evolutionary question that should properly be examined on a case by case basis, involving the long-term accretion and evolutionary history of the binary as well as issues of persistent versus transient activity. One could attempt to evaluate these issues using short-term accretion torque measurements as well as binary evolution modeling of individual systems. However, even allowing for departures from spin equilibrium on time scales much shorter than the spin-up time, the basic time scale argument given above suggests that, for a sufficiently large sample, a statistical assumption of spin equilibrium should be valid. Since many of the nuclear-powered AMXPs are persistent X-ray sources, it is likely that at least significant number of these systems is at or near spin equilibrium.
Pulsars are at magnetic spin equilibrium? Another alternative is that the observed spin distribution simply reflects the distribution of magnetic field strengths and mass accretion rates for the observed systems, as related through the spin equilibrium equation. This is certainly the most conservative assumption (in that it invokes no new physics beyond what is already used to explain X-ray pulsations), but it requires that the AMXPs all have magnetic field strengths of order $\sim 10^8$ G. However, fields this strong should be dynamically important and lead to magnetically channeled pulsations from the persistent emission from all the systems. Nonetheless, most of the burst oscillation sources do not show detectable pulsations in their non-burst emission. While this might be due to the same sort of intermittency effects observed in Aql X-1 [25], possibly arising from fluctuations in the accretion flow geometry [32], this may also reflect a wider range of effective magnetic field strengths, with many systems not having dynamically important fields [33, 5, 30]. The most direct way to address this question would be to develop some means for accurately estimating magnetic field strengths in the burst oscillation sources, possibly using burst oscillation drift time scales [5]. However, a more detailed measurement of the spin distribution (i.e. with a larger sample) may also help, since the shape expected for spin equilibria spanning a narrow range of magnetic field strengths should look different than one affected by some competing torque mechanism besides accretion torques.

Gravitational wave emission? An intriguing last alternative is that accretion spin-up torques in the AMXPs may compete with angular momentum losses due to gravitational radiation [34, 35]. A variety of mechanisms for the emission of gravitational waves from rapidly rotating accreting neutron stars have been discussed in the literature, including $r$-modes [34, 36], accretion-induced crustal quadrupoles [35, 37], toroidal magnetic fields [38], and magnetically confined “mountains” [39]. Irrespective of the specific emission mechanism, an attractive aspect of the gravitational wave model is that the resulting spin-down torque scales very steeply with spin frequency, $\propto \Omega^5$. Thus, this model provides a natural way to produce a sharp cutoff in population at high spin frequency, while not affecting the usual magnetic spin equilibrium picture at lower frequencies.

Testing gravitational radiation torques

Another attractive aspect of the gravitational radiation model is the possibility of testing this hypothesis through the direct detection of gravitational waves. Although the intrinsic gravitational wave strain amplitude expected from any these pulsars is small ($h < 10^{-26}; [35]$), these sources would be continuous sources of gravitational wave emission in a frequency band accessible by current and planned ground-based gravitational wave interferometer experiments like LIGO and VIRGO, with the expected frequency being an order unity multiple of the spin frequency (with the exact factor depending upon the specific mechanism). Thus, one could, in principle, integrate the gravitational wave signal to obtain a detection, in stark contrast to the short-lived transient “chirp” signals expected from binary merger events.

However, the known X-ray timing properties of the AMXPs will limit the ability to integrate a gravitational wave signal coherently. For example, let us consider the effect
of an accretion torque near magnetic spin equilibrium,
\[ \dot{\nu} = 4 \times 10^{-14} \left( \frac{\dot{M}}{0.01M_{\text{Edd}}} \right) \left( \frac{\nu}{600 \text{Hz}} \right)^{-1/3} \text{ Hz s}^{-1}, \]
where \( \dot{M}_{\text{Edd}} \) is the Eddington mass accretion rate, and \( \nu \) and \( \dot{\nu} \) are the spin frequency and its derivative. Assuming steady accretion, this corresponds to a decoherence time scale of
\[ \tau_{\text{dec}} = \sqrt{\frac{1}{\dot{\nu}}} \approx 60 \left( \frac{\dot{M}}{M_{\text{Edd}}} \right)^{-1/2} \left( \frac{\nu}{600 \text{Hz}} \right)^{-1/6} \text{ d}. \]

For Sco X-1 (the brightest persistent low-mass X-ray binary and the most promising gravitational wave target), the mean accretion rate is \( \sim 0.5\dot{M}_{\text{Edd}} \), corresponding to a decoherence time of only \( \tau_{\text{dec}} \approx 10 \text{ d} \), complicating long integrations. Moreover, long-term monitoring of SAX J1808.4–3658 has shown rapid orbital evolution \([17, 40]\); if this is typical of AMXPs, then it will lead to significant uncertainties in the Doppler corrections necessary for coherent integration of a gravitational wave signal, unless there is contemporaneous X-ray timing of the target. A recent detailed assessment by Watts et al. \([41]\) concludes that direct LIGO detection of the predicted gravitational wave signals from AMXPs will be prohibitively difficult given the current state of X-ray timing models for the source population and the current sensitivity of the LIGO detectors.

However, indirect detection of gravitational wave emission may be possible through X-ray timing! Accretion-powered AMXPs with pulsations detected at multiple transient outburst epochs may be used to study the long-term spin and orbital evolution of the pulsar. Since one is measuring the mean spin frequency at widely separated epochs, this analysis is relatively unaffected by the pulse shape variability that can interfere with short-term timing studies. The best case for this sort of study is SAX J1808.4–3658, which has been observed in outburst by RXTE in 1998, 2000, 2002, and 2005. Precise X-ray timing of these outbursts has revealed that the pulsar is steadily spinning down, and that most of the torque is applied during X-ray quiescence when accretion is shut off \([17]\). Accretion torques are clearly not responsible. In this particular case, the observed spin down is consistent with magnetic dipole torques due simply to the magnetized neutron star’s rotation, given the known constraints on the magnetic field strength. These robust constraints come from applying magnetic accretion torque theory to the observed range of X-ray luminosities over which accretion-powered pulsations are detected \([16, 17]\). Thus, for this 401 Hz pulsar, gravitational wave torques are not required to explain the observed long-term spin down.

However, if gravitational wave torques are responsible for limiting accretion spin-up to \( \sim 730 \text{ Hz} \), then we would not expect any significant gravitational wave torques at 401 Hz, given their very steep \( \sim \Omega^5 \) spin frequency dependence; we note that \( (401 \text{ Hz}/730 \text{ Hz})^5 \approx 0.05 \). On the other hand, one would expect gravitational wave emission to become more important for significantly faster rotators, in which case magnetic dipole torques alone would presumably be inadequate to explain any observed long-term spin down.

The ideal target is thus the 599 Hz pulsar IGR J00291+5934, the fastest known accretion-powered millisecond pulsar \([42]\); we note that \( (599 \text{ Hz}/730 \text{ Hz})^5 \approx 0.37 \), a
factor of 7 improvement over SAX J1808.4−3658. This target has only been timed with RXTE during a single outburst in 2004. However, an analysis of archival RXTE/ASM data suggests that the source has a recurrence time of $\sim 3.2 \text{ yr}$, so that the source should go into outburst in the very near future. If this outburst can be observed with RXTE, then precise comparison of the spin frequency between 2004 and the new outburst will yield a crude estimate of (or limit on) the long-term spin-down rate. The magnetic field strength of the pulsar can be tightly constrained by examining the dynamic range of luminosity over which X-ray pulsations are detected, and this can be used to compute the expected contribution of magnetic dipole torques to the spindown rate just as in SAX J1808.4−3658. If the observed spindown rate significantly exceeds the expected magnetic dipole spindown value, then that would be strong indirect evidence that gravitational wave emission is affecting the pulsar’s spin evolution. This is probably the most promising avenue for testing gravitational wave spin down in AMXPs for the foreseeable future.

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