Energy Efficiency of Massive MIMO in Co-located Antenna Systems

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Abstract. In this paper, we discuss the energy efficiency (EE) of co-located massive multiple-input-multiple-output (MIMO) system. After analysis, we obtain the EE expressions of the idealistic and the realistic power consumption models, respectively. Through the numerical results, we detail relationship between EE and the maximum transmit power, and the relationship between EE and the number of the base station (BS).

1. Introduction

Information and wireless communication are acting as important parts in global greenhouse gas emissions because the energy consumption of communication and data transmission is increased with the exponential growth of service requirements [1]. Energy saving is very important in wireless communications because mobile devices suffer from the battery life. Moreover, the battery technology evolves much slower than the advanced smartphones or laptops. So how to improve the energy efficiency (EE) has become an vital standard in the next generation wireless networks. It has been demonstrated that co-located antennas (CA) systems exploiting multiple-input multiple-output (MIMO) communication techniques are much better than the single-input single-output (SISO) systems in multipath fading environments [2]. There are also some alternative architecture about CA system such as those with orthogonally polarized antennas [3] and co-located orthogonally antennas [4].

While most of the works on co-located massive MIMO system has focused on the ergodic capacity under different strategies. For example, in [5], author considered the ergodic sum rate on the condition of base station (BS) always has perfect channel state information (CSI). Ergodic sum rate has also been addressed with consideration of correlated fading and shadowing in [6]. In [7], authors have showed that the global mobile data will increase with beyond 100 times speed than now in the next few years. So improving EE will become more and more important in the wireless communication networks. But there are few papers talk about the EE of co-located massive MIMO system [8],[9].

In this paper, we consider the zero forcing beamforming (ZFBF) precoding scheme in the co-located massive MIMO system. We investigate the relation of EE and the maximum transmit power, and the relation of EE and the number of BS in the idealistic and the realistic power consumption models, respectively.
Notation: In this paper, $(\cdot)^T$, $(\cdot)^\dagger$ represent the transpose and the conjugate transpose. $\circ$ denotes the Hadamard product. $\mathbf{I}_{k \times L}$ denotes a $1 \times k$ matrix with all entries one. $\|f\|$ denotes the Euclidean norm of vector $f$. $X \sim W_k(L, \Sigma)$ denotes a $K \times K$ Wishart matrix $X$ with $L$ degrees of freedom and covariance matrix $\Sigma$. The operator $E(\cdot)$ denotes expectation.

2. System Model

2.1 CA System Model

The co-located massive MIMO systems contains $K$ mobile stations (MSs) and $L$ base stations (BSs), $K, L \geq 1$. Suppose that each of the MS and BS is equipped with a single antenna, all the BSs are sited in the center of cell and all the MSs are uniformly located within this cell. We set the radius of the circular cell is 1. We put attention on the downlink performance of the $k$ th user. The received signal of user $k$ can be given by

$$y_k = h_k x_k + \sum_{i \neq j, k} h_k x_i + n_k$$

where $n_k \sim C N(0, N_0)$ is the additive white Gaussian noise (AWGN) at MS $k$. $x_k$ is $L \times 1$ dimensional and $h_k$ is $1 \times L$ dimensional vector, as the transmitted signal vector and the channel gain vector from the BS to MS $k$, respectively. $h_k$ includes small scale and large scale fading channel, which can be modeled as

$$h_k = \gamma_k \circ g_k$$

where $g_k$ denotes the $1 \times L$ small scale fading vector and modeled as independent and identically distributed complex Gaussian random variables with zero mean and unit variance [10]. $\gamma_k$ is the $1 \times L$ dimensional large scale fading vector [11]. For simplification, the large scale fading coefficient as

$$\gamma_{k,l} = d_{k,l}^{-\alpha}$$

where $\alpha$ is the path loss exponent and the value is in [3,5] [12]. $d_{k,l}$ denotes the distance between MS $k$ and BS $l$. In the CA system, we can know $\gamma_{k,1} = \ldots = \gamma_{k,L}$. So the normalized channel gain vector can be written as

$$h_k = \beta_k \circ g_k$$

where $\beta_k$ is the normalized large scale fading vector, which can be written as

$$\beta_{k,l} = \frac{\gamma_{k,l}}{\|\gamma_k\|}$$
where \( 1 \leq l \leq L \), and we easily know \( \sum_{l=1}^{L} \beta_{k,l}^2 = 1 \) for all the MSs.

We assume that the channel state information is all known at both the transmitter and the receiver sides. With the orthogonal linear precoding scheme, ZFBF [13], the transmitted signal of user \( i \) can be written as

\[
x_i = w_i s_i \quad (6)
\]

For all the MSs, \( s_i \sim CN(0, P_i) \) denotes the information bearing signal. With the ZFBF, the precoding vector \( w_k \) of each user is selected such that \( \bar{h}_k w_k = 0 \) for \( i \neq k, 1 \leq i \leq K \). Let \( H = [h_1^T, h_1^T, \ldots, h_K^T]^T \), and the pseudo-inverse of \( H \) can be represented as \( F = H^\dagger \left( HH^\dagger \right)^{-1} \). The precoding vector \( w_k \) can be modeled as

\[
w_k = \frac{f_k}{\| h_k \|} \quad (7)
\]

where \( 1 \leq k \leq K \), \( f_k \) is the column vector of \( F \). Note that the ZFBF requires the number of transmit antennas is no smaller than the number of receive antennas. In the following subsections, we will focus on the case of \( L > K \). The overall transmit power of the BS is assumed to be fixed at \( P_t \), which is equally allocated to all the MSs. It can be modeled as

\[
\bar{P}_i = \frac{P_t}{K}, 1 \leq i \leq K \quad (8)
\]

### 2.2 Total Power Consumption Model

As the related works of [14]-[16], the realistic power consumption in the massive MIMO system consists of three parts that is written as

\[
P_{\text{real}} = \frac{P_t}{\tau} + LP_{\text{dy}} + P_{\text{st}} \quad (9)
\]

where \( \tau \) is the radio frequency power amplifier efficiency, \( P_{\text{dy}} \) and \( P_{\text{st}} \) are the dynamic and the static power consumption, respectively. In the ideal condition, the total power consumption can be expressed as

\[
P_{\text{ideal}} = \frac{P_t}{\tau} \quad (10)
\]

### 2.3 EE Model

As in [17],[18], the EE model of co-located massive MIMO system can be written as
\[ \eta_{EE}(P_l) = \frac{R}{P_{\text{Total}}} \]  

(11)

where \( R \) is the total capacity of the system. \( P_{\text{Total}} \) is equal to (9) when discussing the realistic power consumption model, otherwise \( P_{\text{Total}} \) is equal to (10).

3. EE of CA Massive MIMO System

3.1 EE of CA Massive MIMO System

In this model, we normalize the total system bandwidth into unit. The approximate capacity of MS \( k \) under downlink CA massive MIMO system is simplified as

\[ R_k = \mathbb{E}_{g_k} \left[ \log_2 \left( 1 + \frac{\nu_k}{\|f_k\|^2} \right) \right] \]  

(12)

Set all the BSs coordinates as \( \eta^B = (0,0) \), \( 1 \leq l \leq L \), and the coordinate of MS \( k \) as \( \eta^M_k = (\rho_k, \theta_k) \). According to (3), the large scale fading vector can be written as \( \gamma_k = \rho_k^{-\alpha} 1_{\text{wL}} \), so the average received signal to noise ratio (SNR) \( \nu_k \) is expressed as

\[ \nu_k = \frac{P_l}{KN_0} \rho_k^{-\alpha} \]  

(13)

We can get \( HH^\dagger = \frac{1}{L} GG^\dagger \sim W_k(L, \frac{1}{L} I_K) \), where \( G = [g_1^T, g_2^T, \ldots, g_K^T]^T \). According to [19], the effective channel gain \( 1/\|f_k\|^2 : W_k(L-K+1, 1/L) \). We can get the approximate capacity of MS \( k \) as follows [20]

\[ R_k = P_{\text{Total}}^{\frac{1}{k}} \log_2 \left( \frac{P_l}{N_0} \left( \frac{L}{K} \right) - 1 \right)^{\rho_k^{-\alpha}} \]  

(14)

From [20], the average MS rate of CA model can be written as

\[ R_C = \mathbb{E}[R_k] = \int_0^1 \log_2 \left( \frac{P_l}{N_0} \left( \frac{L}{K} - 1 \right) x^{-\alpha} \right) 2x dx = \log_2 \left( \frac{P_l}{N_0} \left( \frac{L}{K} - 1 \right) \right) \frac{\alpha}{2} \log_2 e \]  

(15)

So according to (11), the EE of the system is expressed as

\[ \eta_{C_{-M\text{-MIMO}_{-EE}}} = \frac{KR_C}{P_{\text{Total}}} \]  

(16)

According to (16), we can obtain the relationship between EE and maximum transmit power in two power consumption models of co-located massive MIMO system, respectively.
4. Simulation Results

In this part, we present the simulation results to evaluate the SE and EE of the downlink co-located massive MIMO system in two power consumption models. We set the dynamic and static power consumption is 30 dBm and 40 dBm, the maximum transmit power is 45 dBm, the BS and MS number are 100 and 50, the path loss exponent is 4 [21] and drain efficiency is 38% [22].

![Fig. 1: Capacity versus the maximum transmit power.](image1)

![Fig. 2: Energy efficiency versus number of BS(L).](image2)

Fig. 1 compares the analytical results with the closed form of capacity in the CA massive MIMO system. It shows that the theory capacity is almost the same as the analytical result. From Fig. 3 (a), it shows that the trend of EE decreases with the growth of the maximum transmit power in the idealistic power consumption model. But from Fig. 3 (b), in the realistic power consumption model, the trend of EE first increases and then decreases with the growth of the maximum power consumption. We can find the maximum EE is at the 33 dBm of maximum transmit power.

![Fig. 3: EE versus the maximum transmit power with two power consumption models.](image3)

(a). Idealistic Power Consumption Model.  
(b). Realistic consumption Model.
Fig. 2 shows the change of EE with the number of the BS in the CA system in the two models with $K = 50$. It shows that EE increases with the growth of the number of BS in the idealistic power consumption model, but decreases with the growth of BS's numbers in the realistic power consumption model.

5. Conclusion

In this paper, we discussed the total capacity and EE of the co-located massive MIMO system. An approximate EE expression was obtained in two power consumption models. Numerical results also have explained the relationship of the system EE and the maximum transmit power, the relationship of EE and the number of the BS. For future research, the condition of multiple cells needs to be considered in the co-located massive MIMO system to conform the real situation.

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