I. INTRODUCTION

In nature every system is immersed in an environment, the problem about the system interacting with the environment is a hot topic in quantum information and quantum optics. To describe system-environment evolution Takahashi-Umezawa introduced thermo field dynamics (TFD) \cite{1,2}, with which one may convert the calculations of ensemble averages at finite temperature to equivalent expectation values with a pure state \(\rho\), where

\[
\langle A \rangle = \frac{\text{tr} (A \rho)}{Z (\beta)} , \quad \rho = e^{-\beta H} ,
\]

is the partition function; \(Z (\beta) = \text{tr} \rho = \text{tr} e^{-\beta H}\) is the partition function; \(H\) is the system's Hamiltonian. Then how to find the explicit expression of \(\langle 0 (\beta) \rangle\)? If one expands \(\langle 0 (\beta) \rangle\) in terms of the energy eigenvector set of \(H\), \(\langle 0 (\beta) \rangle = \sum_n \langle n\rangle \hat{f}_n (\beta)\), and then substituting it into Eq. (2), which results in

\[
\hat{f}_n (\beta) f_m (\beta) = Z^{-1} (\beta) e^{-\beta E_n} \delta_{n,m} \quad \text{(after comparing with Eq. (1))}.
\]

By introducing a fictitious mode, \(\langle \tilde{n} | \tilde{m} \rangle = \delta_{n,m}\), then Takahashi-Umezawa obtained

\[
\langle 0 (\beta) \rangle = Z^{-1/2} (\beta) \sum_n e^{-\beta E_n / 2} |n, \tilde{n}\rangle .
\]

Thus the worthwhile convenience in Eq. (2) is at the expense of introducing a fictitious field (or called a tilde-conjugate field, denoted as operator \(\hat{\tilde{a}}\)) in the extended Hilbert space, i.e., the original optical field state \(|n\rangle\) in the Hilbert space \(\mathcal{H}\) is accompanied by a tilde state \(|\tilde{n}\rangle\) in \(\tilde{\mathcal{H}}\). A similar rule holds for operators: every Bose annihilation operator acting on \(\mathcal{H}\) has an image \(\tilde{\hat{a}}\) acting on \(\tilde{\mathcal{H}}\), \([\hat{a}, \hat{\tilde{a}}] = 1\). These operators in \(\tilde{\mathcal{H}}\) are commutative with those in \(\mathcal{H}\).

For a harmonic oscillator the Hamiltonian is \(\hbar \omega a a^\dagger\), \(|n\rangle = a^n \sqrt{n!} |0\rangle\), Takahashi-Umezawa obtained the explicit expression of \(\langle 0 (\beta) \rangle\) in doubled Fock space,

\[
\langle 0 (\beta) \rangle = \text{sech} \beta \exp \left[ a^\dagger a \tan \beta \right] |00\rangle = S (\beta) |00\rangle ,
\]

which is named thermo vacuum state, and \(S (\beta)\) thermo operator,

\[
S (\beta) = \exp \left[ \theta \left( a^\dagger a - a a^\dagger \right) \right] ,
\]

which is similar in form to the a two-mode squeezing operator except for the tilde mode. \(\theta\) is a parameter related to the temperature by \(\tanh \theta = \exp (-\frac{\hbar \omega}{2kT})\).

An interesting question thus challenges us: For the Hamiltonian being \(H = \omega a a^\dagger + \kappa a^2 + \kappa a^\dagger a\), then what is the corresponding thermo vacuum state? One may wonder if this question is worth of paying attention since this \(H\) can be diagonalized by the Bogoliubov transformation as a new harmonic oscillator, correspondingly, the thermo vacuum state for \(H\) can be obtained by acting the same transformation on \(\langle 0 (\beta) \rangle\) in \(\mathcal{H}\) (see Eq. (A9) in the Appendix). To make this issue worthwhile, we emphasize that we shall adopt a completely new approach to construct thermo vacuum state and our result is simpler in form than that in Eq. (A10). Our work is arranged as follows. In Sec. 2 by re-analyzing Eqs. (1)-(2) we shall introduce a new method (the partial trace method) to find the explicit expression of \(\langle 0 (\beta) \rangle\) in \(\mathcal{H}\). Then using this method, we obtain the expression of \(\langle 0 (\beta) \rangle\) in Eq. (4) in Sec. 3. For the degenerate parametric amplifier, we derive a generalized thermal vacuum state \(|\phi (\beta)\rangle\) in Sec. 4. Section 5 is devoting to presenting some applications of \(\langle \phi (\beta) \rangle\).

II. THE PARTIAL TRACE METHOD

Following the spirit of TFD, for a density operator \(\rho = e^{-\beta H} / Z (\beta)\) with Hamiltonian \(H\), we can suppose that the ensemble averages of a system operator \(A\) may be calculated as

\[
A = \text{tr} (\rho A) = \langle \psi (\beta) | A | \psi (\beta) \rangle ,
\]

where \(|\psi (\beta)\rangle\) corresponds to the pure state in the extended Hilbert space.

Let \(T\) denote the trace operation over both the system freedom (expressed by \(\text{tr}\)) and the environment freedom
by \( \tilde{\text{tr}} \), i.e., \( \text{Tr} = \text{tr} \tilde{\text{tr}} \), then we have
\[
\langle \psi(\beta) | A | \psi(\beta) \rangle = \text{Tr} [A | \psi(\beta) \rangle \langle \psi(\beta) |] = \text{tr} [A \tilde{\text{tr}} | \psi(\beta) \rangle \langle \psi(\beta) |].
\] (7)

Note that
\[
\tilde{\text{tr}} | \psi(\beta) \rangle \langle \psi(\beta) | \neq | \psi(\beta) \rangle \langle \psi(\beta) |,
\] (8)
since | \psi(\beta) \rangle involves both real mode and fictitious mode.

Comparing Eq. (7) with Eq. (1) we see
\[
\tilde{\text{tr}} | \psi(\beta) \rangle \langle \psi(\beta) | = e^{-\beta H} / Z(\beta).
\] (9)

Eq. (9) indicates that, for a given Hamiltonian \( H \), if we can find a density operator of pure state | \psi(\beta) \rangle in doubled Hilbert space, whose partial trace over the tilde freedom may lead to density operator \( e^{-\beta H} / Z(\beta) \) of the system, then the average value of operator \( A \) can be calculated as an equivalent expectation value with a pure state | \psi(\beta) \rangle, i.e., \( \langle A \rangle = \text{tr} (A e^{-\beta H} / Z(\beta)) = \langle \psi(\beta) | A | \psi(\beta) \rangle. \)

In particular, when \( H = \hbar \omega a \dagger a \), a free Bose system, Eq. (9) becomes
\[
\text{tr} | 0(\beta) \rangle \langle 0(\beta) | = (1 - e^{-\beta \hbar \omega}) e^{-\beta \hbar \omega a \dagger a} \equiv \rho_c, \tag{10}
\]
\( \rho_c \) is the density operator of chaotic field. This equation enlightens us to have a new approach for deriving | 0(\beta) \rangle \langle 0(\beta) | in doubled Hilbert space should be such constructed that its partial trace over the tilde freedom may lead to density operator \( \rho_c \) of the system. In the following we shall employ the technique of integration within an ordered product (IWOP) of operators \( \hat{A}, \hat{A} \dagger \) to realize this goal.

### III. DERIVATION OF | 0(\beta) \rangle IN EQ.(11) VIA THE NEW APPROACH

Using the normally ordered expansion formula \( \hat{A} = : \exp \{ e^{-\beta \hbar \omega} - 1 \} a \dagger a : \),
\[
e^{-\beta \hbar \omega a \dagger a} = : \exp \{ e^{-\beta \hbar \omega} - 1 \} a \dagger a : , \tag{11}
\]
(where the symbol : : denotes the normal ordering form of operator), and the IWOP technique we have
\[
: \exp \{ e^{-\beta \hbar \omega} - 1 \} a \dagger a : = \int \frac{d^2 z}{\pi} e^{-|z|^2 + z^* \hat{a} e^{-\beta \hbar \omega} + z \hat{a} e^{-\beta \hbar \omega}} a \dagger a. \tag{12}
\]

Remembering the ordering form of vacuum projector operator | 0 \rangle \langle 0 | = e^{-a^\dagger a} : , we can rewrite Eq. (12) as
\[
: \exp \{ e^{-\beta \hbar \omega} - 1 \} a \dagger a : = \int \frac{d^2 z}{\pi} e^{z^* \hat{a} e^{-\beta \hbar \omega} / 2 | 0 \rangle \langle 0 | e^{z \hat{a} e^{-\beta \hbar \omega} / 2} \langle z | \langle 0 | \langle 0 | z \rangle \rangle. \tag{13}
\]

where | \tilde{z} \rangle is the coherent state \( \tilde{a}, \tilde{a} \dagger \) in fictitious mode
\[
| \tilde{z} \rangle = \exp (z \tilde{a}^\dagger - z^* \tilde{a}) | \tilde{0} \rangle, \tilde{a} | \tilde{z} \rangle = z | \tilde{z} \rangle, \langle \tilde{0} | \tilde{z} \rangle = e^{-|z|^2 / 2}. \tag{14}
\]

Further, multiplying the factor \( (1 - e^{-\beta \hbar \omega}) \) to both sides of Eq. (13) and using the completeness of coherent state \( \int d^2 z | \tilde{z} \rangle \langle \tilde{z} | = 1 \), we have
\[
(1 - e^{-\beta \hbar \omega}) \times \text{Eq. (13)} = (1 - e^{-\beta \hbar \omega}) \int \frac{d^2 z}{\pi} | \tilde{z} \rangle e^{z^* \hat{a} e^{-\beta \hbar \omega} / 2 | 0 \rangle \langle 0 | e^{z \hat{a} e^{-\beta \hbar \omega} / 2} \langle z | \approx (1 - e^{-\beta \hbar \omega}) \int \frac{d^2 z}{\pi} | \tilde{z} \rangle e^{z^* \hat{a} e^{-\beta \hbar \omega} / 2 | 0 \rangle \langle 0 | e^{z \hat{a} e^{-\beta \hbar \omega} / 2} \langle z | \tag{15}
\]

where
\[
\langle 0(\beta) | = \sqrt{1 - e^{-\beta \hbar \omega}} \exp \left[ a \dagger a e^{-\beta \hbar \omega} / 2 \right] | 0 \rangle, \tag{16}
\]
which is the same as Eq. (4). Thus, according to Eq. (11), from the chaotic field operator we have derived the thermo vacuum state, this is a new approach, which has been overlooked in the literature before.

### IV. GENERALIZED THERMO VACUUM STATE | \phi(\beta) \rangle

Now, consider a degenerate parametric amplifier whose Hamiltonian is
\[
H = \omega a \dagger a + \kappa \gamma a \dagger a^2 + \kappa a^2, \tag{17}
\]
whose normalized density operator \( \rho \) is
\[
\rho \left( e^{-\beta H} \right) = e^{-\beta H} = e^{-\beta (\omega a \dagger a + \kappa \gamma a \dagger a^2 + \kappa a^2)}. \tag{18}
\]

Recalling that \( \frac{1}{2} \left( a^\dagger a + \frac{\gamma}{\kappa} \right), \frac{1}{2} a^\dagger a \) and \( \frac{1}{2} a^2 \) obey the SU(1,1) Lie algebra, thus we can derive a generalized identity of operator \( \left[ 9, 10 \right] \) as follows:
\[
\exp \left[ a^\dagger a + g a^2 + k a^2 \right] = e^{-f^2 / 2} e^{f g^2 / 2} e^{f k / 2} e^{-f^2 / 2} \tag{19}
\]

where we have set \( D^2 = f^2 - 4kg \). Thus Comparing Eq. (18) with Eq. (19) we can recast Eq. (18) into the following form
\[
(\text{tr} e^{-\beta H}) \rho = \sqrt{\lambda} e^{\beta \omega} \exp \left[ E^* a^2 \right] \exp \left[ a^\dagger a \ln \lambda \right] \exp \left[ E a^2 \right], \tag{20}
\]
where we have set
\[
D^2 = \omega^2 - 4 | \kappa |^2, \quad \lambda = \frac{D}{\omega \sinh \beta D + D \cosh \beta D}, \tag{21}
\]
\[
E = -\frac{\lambda}{D} \kappa \sinh \beta D.
\]
Further, using the formula in Eqs. (11) and (13), we have
\[
\langle \text{tr} e^{-\beta H} \rangle = \int \frac{d^2 z}{\pi} e^{\beta \omega} e^{E_e a^\dagger a} |0\rangle \langle 0 | e^{E_a^2} e^{\sqrt{\lambda}S} |\tilde{z}\rangle \langle \tilde{z}| \langle \tilde{z}| \langle \tilde{z}|\rangle.
\]

Using the formula in Eqs. (11) and (13), we have
\[
\langle \text{tr} e^{-\beta H} \rangle = \int \frac{d^2 z}{\pi} e^{\beta \omega} e^{E_e a^\dagger a} |0\rangle \langle 0 | e^{E_a^2} e^{\sqrt{\lambda}S} |\tilde{z}\rangle \langle \tilde{z}| \langle \tilde{z}|\rangle.
\]

Thus the normalized state for Eq. (18) in doubled Fock space is given by
\[
|\phi (\beta)\rangle = \frac{\sqrt{\lambda e^{\beta \omega} e^{E_e a^\dagger a} |0\rangle \langle 0 | e^{E_a^2} e^{\sqrt{\lambda}S} |\tilde{z}\rangle \langle \tilde{z}| \langle \tilde{z}|\rangle}}{Z (\beta)}.
\]

V. APPLICATIONS OF GENERALIZED THERMO VACUUM STATE

A. Internal energy distribution of the system

As an application of Eq. (27), we can evaluate the each term’s contribution to energy in Hamiltonian. Based on the idea from Eq. (11) to (2), the system operator A can be calculated as \(\langle A \rangle = \langle \phi (\beta) | A | \phi (\beta) \rangle\). Thus using the completeness of coherent state and the integral formula Eq. (25) as well as noticing \(\langle \phi (\beta) | A | \phi (\beta) \rangle = 1, (1 - \lambda)^2 - (2 \langle E \rangle)^2 = 4\lambda \sinh^2 (\beta D/2),\) then we have
\[
\langle \omega a^\dagger a \rangle = \omega \langle \phi (\beta) | (a a^\dagger - 1) | \phi (\beta) \rangle
\]
\[
= 2\omega \lambda^{1/2} \sinh (\beta D/2) \frac{\partial}{\partial \lambda} \frac{1}{\sqrt{(1 - \lambda)^2 - 4EE^*}}
\]
\[
= \frac{\omega}{2} \frac{\langle \omega a^\dagger a \rangle}{D \coth \beta D/2},
\]

as well as
\[
\langle \kappa a^\dagger a \rangle = -\frac{\kappa^2}{\coth \beta D/2},
\]

From Eqs. (31) and (32) we see that the two items \(\langle \kappa a^\dagger a \rangle\) have the same energy contributions to the system, as expected. Combing Eqs. (30)-(32) we can also check Eq. (28).

B. Wigner function and quantum tomogram

The Wigner function plays an important role in studying quantum optics and quantum statistics. It gives the most analogous description of quantum mechanics in the phase space to classical statistical mechanics of Hamilton systems and is also a useful measure for studying the nonclassical features of quantum states. In addition, the Wigner function can be reconstructed by measuring several quadratures \(P (x, y) = \hat{x} \cos \theta + \hat{y} \sin \theta\) with a homodyne detection and then applying an inverse Radon transform—quantum homodyne tomography. Using Eq. (26) one can calculate conveniently the Wigner function and quantum tomogram. Recalling that the single-mode Wigner operator \(\Delta (z)\) in coherent
state representation is given by \[16, 17\]
\[
\Delta (\alpha) = e^{2|\alpha|^2} \int \frac{d^2 z}{\pi^2} |z\rangle \langle -z| e^{-2(z\alpha^* - z^* \alpha)},
\]
thus the Wigner function is
\[
W (\alpha) = \langle \phi (\beta) | \Delta (\alpha) | \phi (\beta) \rangle = e^{2|\alpha|^2} \int \frac{d^2 z}{\pi^2} \langle \phi (\beta) | z \rangle \langle -z| \phi (\beta) \rangle e^{-2(z\alpha^* - z^* \alpha)}
\]
\[
= \frac{\tanh (\beta D/2) e^{-\frac{1}{2}(\alpha^2 + |\alpha|^2)}}{\pi} \tanh (\beta D/2)
\]
where we have noticed \((1 + \lambda)^2 - 4|E|^2 = \frac{4D\cosh^2 \beta D/2}{4 \sinh \beta D/2 \cosh \beta D/2}\) and used the integral formula \[25\]. In particular, when \(\kappa = 0\), Eq. (31) reduces to
\[
W (\alpha) = \frac{\tanh (\beta D/2)}{\pi} \exp \left\{ -2 |\alpha|^2 \tanh (\beta D/2) \right\},
\]
which is just the Wigner function of thermo vacuum state \(|0(\beta)\rangle\).

On the other hand, we can derive the tomography (Radon transform of Wigner function) of the system by using Eq. (26). Recalling that, for single-mode case, the Radon transform of the Wigner operator is just a pure state density operator \[18\].

\[
\int \delta (q - f q' - g p') \Delta (\alpha) dq' dp' = |q\rangle f, g \langle q|,
\]
where \(\alpha = (q + ip)/\sqrt{2}\), and \((f, g)\) are real,
\[
|q\rangle f, g = C \exp \left[ \frac{\sqrt{2}}{A} q a^\dagger - \frac{e^{2z}}{2} a^2 \right] |0\rangle,
\]
and \(C = \left[ \pi \left( f^2 + g^2 \right) \right]^{-1/4} \exp \left( -q^2 / 2 (f^2 + g^2) \right)\). Eq. (37) is named as the intermediate coordinate-momentum representation \[18\]. From Eq. (36) and Eq. (26) it then follows that the tomogram can be calculated as
\[
\mathcal{R} (q) f, g = \langle \phi (\beta) | q \rangle f, g \langle q| \phi (\beta) \rangle = \int \frac{d^2 z}{\pi} |f, g \rangle \langle q| \hat{z} | \phi (\beta) \rangle | q\rangle^2.
\]
Then submitting Eqs. (37) and (26) into Eq. (38), we obtain
\[
\mathcal{R} (q) f, g = \frac{2 \sinh (\beta D/2)}{C} \sqrt{\lambda + |G|^2 / \lambda}
\]
\[
\times \exp \left\{ 2q^2 \left[ \frac{1 - \lambda \text{Re}G^{-1}}{|A|^2 \left( \lambda + |G|^2 / \lambda \right)} + \text{Re} \frac{2E}{A^2 G} \right] \right\}
\]
where we have used Eq. (29) and set \(G = 1 + 2 e^{2z} E\). Eq. (30) is the positive-definite tomogram, as expected. As far as we are concerned, this result has not been reported in the literature before.

In sum, by virtue of the technique of integration within an ordered product (IWOP) of operators we have presented a new approach for deriving generalized thermo vacuum state which is simpler in form that the result by using the Umezawa-Takahashi approach, in this way the thermo field dynamics can be developed.

**Appendix:**

As a comparison of our new approach with the usual way of deriving thermo vacuum state in TFD theory, in this appendix, we shall derive the explicit expression of \(|\phi (\beta)\rangle\) by diagonalizing Hamiltonian \[17\]. For this purpose, we introduce two unitary operators: one is a single mode squeezing operator,
\[
S = \exp \left( \frac{\nu a^\dagger a}{\mu} \right) \exp \left( \frac{\nu a^\dagger + 1}{\mu} \ln \frac{1}{\mu} \right) \exp \left( -\frac{\nu a^2}{2} \right),
\]
where \(\mu\) and \(\nu\) are squeezing parameters satisfying the unitary-modulate condition \(\mu^2 - \nu^2 = 1\); and the other is a rotational operator, \(R = \exp \left( \frac{\nu a}{\mu} a^\dagger \right)\), which lead to the following transformations,
\[
Sa^\dagger = \nu a^\dagger - \nu a, \quad S^\dagger a = \nu a + \nu a^\dagger,
\]
and
\[
Ra^\dagger = ae^{-\frac{\nu}{\mu}} \quad \text{and} \quad Ra^\dagger R^\dagger = a^\dagger e^{\frac{\nu}{\mu}}.
\]
Thus, under the unitary transform \(SR\), we have (setting \(\kappa = |\kappa| e^{i\phi}\))
\[
H' = SHR^\dagger S^\dagger = \omega Sa^\dagger S^\dagger + |\kappa| Sa^\dagger S^\dagger + |\kappa| S^\dagger S^\dagger
\]
\[
= (\omega^2 - \nu^2 - 4 |\kappa|^2 \mu) a^\dagger a + (\omega^2 - 2 |\kappa|^2 \mu) \nu
\]
\[
+ (|\kappa|^2 + |\kappa|^2 \nu^2 - \omega^2) (a^2 + a^\dagger a^\dagger).
\]
In order to diagonalize Eq. (A5), noticing \(\mu^2 - \nu^2 = 1\) and making \(|\kappa|^2 (\mu^2 + \nu^2) - \omega \nu \mu = 0\), whose solution is given by
\[
\mu^2 = \frac{\omega^2}{2 \omega^2} + \frac{1}{2}, \quad \nu^2 = \frac{\omega^2 - 1}{2}, \quad \omega' = \sqrt{\omega^2 - 4 |\kappa|^2}.
\]
then Eq. (A5) becomes
\[
H' = \omega' \left( a^\dagger a + \frac{1}{2} \right) - \frac{1}{2} \omega'.
\]
i.e., the diagonalization of Hamiltonian is completed.

According to Eq. (10), the thermal vacuum state corresponding to density operator \(\rho' = e^{-\beta H'} / \text{tr} (e^{-\beta H'}) = e^{-\beta \omega a^\dagger a} / \text{tr} (e^{-\beta \omega a^\dagger a})\) is given by
\[
|0(\beta)\rangle = \sqrt{1 - e^{-\beta \omega'}} \exp \left( a^\dagger a^\dagger e^{-\beta \omega'/2} \right) |0\rangle.
\]
Thus the generalized thermal vacuum state is
\[
|\phi'(\beta)\rangle = R^\dagger S^\dagger |0(\beta)\rangle
\]
\[
= \sqrt{1 - e^{-\beta \omega'}} R^\dagger S^\dagger \exp \left( a^\dagger a^\dagger e^{-\beta \omega'/2} \right) |0\rangle.
\]
Using the transformation in (A3), (A4) and noticing Eq.(A1) as well as $\nu = \sqrt{\omega - \omega'}$, we can finally put Eq.(A9) into the following form

$$|\phi'(\beta)\rangle = \sqrt{1 - e^{-\beta\omega'}}/\mu \exp \left[ \frac{1}{\mu} e^{-\left(\beta\omega' + i\phi\right)/2} a^\dagger a^\dagger \right]^{\nu e^{-i\phi}} \left[ \frac{2}{\nu \mu} e^{\beta\omega'}/2 \right] a_{12}^{\dagger \dagger} |00\rangle. \quad (A10)$$

Comparing Eq.(A10) with Eq.(27), we see that Eq.(27) is simpler in form than that in Eq.(A10).

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