The Implementation of the Minimal Supersymmetric Standard Model in *FeynArts* and *FormCalc*

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June 1, 2001

**Abstract**

We describe the implementation of the MSSM in the diagram generator *FeynArts* and the calculational tool *FormCalc*. This extension allows to perform loop calculations of MSSM processes almost fully automatically. The actual implementation has two aspects: The MSSM Feynman rules are specified in a new model file for *FeynArts*. The computation of the parameters in the MSSM Lagrangian from the input parameters is realized as a Fortran subroutine in the framework of *FormCalc*. The model file does not depend on the latter, however, and can be used even if one does not want to continue the calculation with *FormCalc*. The Feynman rules have been entered in a very generic way to allow e.g. scenarios with complex parameters, and have been tested extensively by reproducing known results for several non-trivial scattering processes.

PACS numbers: 12.60.Jv, 02.70.–c.

**1 Introduction**

One of the main problems of Feynman-diagrammatic computations is the enormous growth of the number of Feynman diagrams, not only with the loop order, but also with the number of particles in a model. While many precision calculations in the Standard Model (SM) could still be performed by hand exactly, the same is very difficult in models like the Minimal Supersymmetric Standard Model (MSSM). Yet it is highly desirable to perform unabridged calculations in the MSSM, too, since also the MSSM allows to make precise predictions in terms of a set of input parameters.

With the availability of powerful software packages, the basic problem of bookkeeping and calculation of the diagrams has been solved for many common cases. Still, it is not entirely trivial to code a model of the complexity of the MSSM in such a system, since this has to be done in a reasonably general way (i.e. not only for special cases of the parameters) and many checks have to be performed to test all sectors of the model.

The present paper documents the implementation of the MSSM in the *FeynArts* and *FormCalc* packages. Other programs for which an MSSM model file exists are GRACE and CompHEP. Conceptually, the *FeynArts–FormCalc* system works in three stages, as sketched in the following diagram.

| Diagram generation | Algebraic simplification | Numerical evaluation |
|--------------------|--------------------------|----------------------|
| *FeynArts*         | *FormCalc*               | *FormCalc*/LoopTools |
| *Mathematica*      | *Mathematica*/FORM       | Fortran              |
A more detailed discussion of the interplay between *FeynArts* and *FormCalc* can be found in [7].

This paper is organized as follows. Corresponding to the three stages, Diagram generation, Algebraic simplification, and Numerical evaluation, there are three sections which describe the modifications to each stage that are necessary for calculations in the MSSM.

Sect. 2 describes the changes in *FeynArts*, namely the new MSSM model file. The model file declares the properties of the fields, their propagators, and their couplings. It contains the parameters of the MSSM Lagrangian. The Feynman rules have been entered in a very generic way to allow e.g. scenarios with complex parameters.

Sects. 3 and 4 describe the changes in *FormCalc*. Sect. 3 outlines the changes in the algebraic simplification and Sect. 4 explains the calculation of the parameters that are used in the model file. The model-file parameters are derived from a (reasonably small) set of input parameters. This is realized as a model-initializing Fortran subroutine in the framework of *FormCalc*. The model file does not depend on the latter, however, and can be used even if one does not want to continue the calculation with *FormCalc*.

## 2 Diagram generation

### 2.1 Algorithms

Supersymmetric theories contain fermion-number-violating couplings, e.g. quark–squark–gluino, so that the usual method of ordering the Dirac matrices oppositely to their occurrence along the arrows on fermionic lines breaks down since one cannot define a fermion-number flow.

*FeynArts* uses the “flipping-rule” algorithm [8]. This algorithm was invented precisely to solve the problem of fermion-number-violating couplings and works as follows: Instead of traversing the fermion lines along the fermion-number flow imposed from the outside, *FeynArts* chooses a direction for each fermion chain. If it turns out later that, for a Dirac fermion, the chosen direction is opposite to the actual fermion flow, *FeynArts* “flips” the coupling, i.e. it derives the coupling appropriate for the reversed fermion flow from the known coupling. The flipping of a coupling is in fact nothing but a charge (as opposed to hermitian) conjugation.

### 2.2 The MSSM model file

The model file is the source of all physics information in *FeynArts*. It declares the properties of the fields, their propagators, and their couplings. In the model file the parameters of the Lagrangian are used, not a restricted set of input parameters.

There are two versions of the MSSM model file in *FeynArts*, both of which follow the conventions of [9, 10, 11]. The file `MSSMQCD.mod` defines the complete (electroweak and strong) MSSM, whereas `MSSM.mod` contains only the electroweak subset, defined as everything except the gluon, its ghost, and the gluino. The four-sfermion couplings appear in `MSSM.mod` although they have both electroweak and strong parts. Counter-terms are not yet entered in the model files.

Table 1 gives the names for the fields and corresponding masses defined in `MSSM.mod` and `MSSMQCD.mod`. The symbols used for the MSSM parameters are specified in Table 2. The complete list of couplings is too long to be included here, but is contained in the *FeynArts* distribution as a PostScript file.

The MSSM model files further declare a number of restrictions with which certain groups of particles can be excluded in the diagram generation. These restrictions are listed in Table 3.

## 3 Algebraic simplification

### 3.1 Algorithms

The algebraic simplification has to address two problems that arise in one-loop calculations in supersymmetric theories in general and in the MSSM in particular. The first is the conceptual
leptons $f = f^\dagger$

| field | mass | sleptons $f = f^\dagger$ | field | mass |
|------|------|----------------|------|------|
| $\nu_g$ | $F[1, \{g\}]$ | 0 | $\tilde{\nu}_g$ | $S[11, \{g\}]$ | MSf |
| $\ell_g$ | $F[2, \{g\}]$ | MLE | $\tilde{\ell}_g$ | $S[12, \{s, g\}]$ | MSf |

quarks $f = f^\dagger$

| field | mass | squarks |
|------|------|--------|
| $u_g$ | $F[3, \{g, o\}]$ | MQU |
| $d_g$ | $F[4, \{g, o\}]$ | MQD |

gauge bosons $f = f^\dagger$

| field | mass | neutralinos, charginos |
|------|------|-------------------------|
| $\gamma$ | $V[1]$ | 0 | $\chi^0_n$ | yes | $F[11, \{n\}]$ | MNeu |
| $Z$ | $V[2]$ | MZ | $\tilde{\chi}^-_c$ | yes | $F[12, \{c\}]$ | MCha |
| $W^-$ | $V[3]$ | MW |

Higgs bosons $f = f^\dagger$

| field | mass | ghosts |
|------|------|--------|
| $h^0$ | $S[1]$ | $u_\gamma$ | $U[1]$ | 0 |
| $H^0$ | $S[2]$ | $u_Z$ | $U[2]$ | MZ |
| $A^0$ | $S[3]$ | $u_+$ | $U[3]$ | MW |
| $G^0$ | $S[4]$ | $u_-$ | $U[4]$ | MW |
| $H^-$ | $S[5]$ | $u_{\tilde{g}}$ | $U[5, \{u\}]$ | 0 |
| $G^-$ | $S[6]$ | MGP |

where the following indices are used:

- $g = \text{Index[Generation]} = 1 \ldots 3,$
- $s = \text{Index[Sfermion]} = 1 \ldots 2,$
- $o = \text{Index[Colour]} = 1 \ldots 3,$
- $n = \text{Index[Neutralino]} = 1 \ldots 4,$
- $u = \text{Index[Gluon]} = 1 \ldots 8,$
- $c = \text{Index[Chargino]} = 1 \ldots 2.$

Table 1: The particle set-up in MSSM.mod and MSSMQCD.mod. The gluon, its ghost, and the gluino, which are defined only in the latter, are written in grey.
| Symbol        | Description                                           |
|--------------|-------------------------------------------------------|
| Mh0, MHH, MA0, MG0 | neutral Higgs masses                                  |
| MHp, MGp     | charged Higgs masses                                  |
| CB, SB, TB   | \(\cos \beta, \sin \beta, \tan \beta\)             |
| CA, SA       | \(\cos \alpha, \sin \alpha\)                       |
| C2A, S2A, C2B, S2B | \(\cos 2\alpha, \sin 2\alpha, \cos 2\beta, \sin 2\beta\) |
| CAB, SAB, CBA, SBA | \(\cos(\alpha + \beta), \sin(\alpha + \beta), \cos(\beta - \alpha), \sin(\beta - \alpha)\) |
| MUE          | Higgs-doublet mixing parameter \(\mu\)               |
| MG1          | gluino mass                                           |
| MNeu[n]      | neutralino masses                                     |
| ZNeu[n, n’]  | neutralino mixing matrix                              |
| MCha[c]      | chargino masses                                       |
| UCha[c, c’], VCha[c, c’] | chargino mixing matrices                          |
| MSf[s, t, g] | sfermion masses                                       |
| USf[t, g][s, s’] | sfermion mixing matrix                               |
| Af[t, g]     | (scalar) soft-breaking \(A\)-parameters               |

The indices enumerate the following properties:

sfermion generation: \(g = 1 \ldots 3\),

sfermion type: \(t = \{\)

1. neutrinos,
2. sleptons,
3. up-type squarks,
4. down-type squarks,

\(s = 1 \ldots 2, \quad s’ = 1 \ldots 2, \quad n = 1 \ldots 4, \quad n’ = 1 \ldots 4, \quad c = 1 \ldots 2, \quad c’ = 1 \ldots 2\).

Table 2: Symbols representing the MSSM parameters introduced by \texttt{MSSM.mod} and \texttt{MSSMQCD.mod}.

| Symbol                        | Description                                      |
|-------------------------------|--------------------------------------------------|
| NoGeneration1                 | exclude generation-1 fermions \((\nu_e, e, u, d)\) |
| NoGeneration2                 | exclude generation-2 fermions \((\nu_\mu, \mu, c, s)\) |
| NoGeneration3                 | exclude generation-3 fermions \((\nu_\tau, \tau, t, b)\) |
| NoElectronHCoupling           | exclude all couplings involving electrons and any Higgs field |
| NoLightFHCoupling             | exclude all couplings between light fermions (all fermions except the top) and any Higgs field |
| NoSUSYParticles               | exclude the particles not present in the SM: sfermions, charginos, neutralinos, and the Higgs fields \(H^0, A^0, H^\pm\) |
| THDMParticles                 | exclude the particles not present in the two-Higgs-doublet model: the sfermions, charginos, and neutralinos |

Table 3: Pre-defined restrictions in \texttt{MSSM.mod} and \texttt{MSSMQCD.mod}.
problem of implementing a supersymmetry-preserving regularization scheme, the second is the technical problem of dealing with more diagrams and larger expressions.

The default regularization scheme employed by FormCalc is dimensional regularization. Unfortunately, this scheme is known to break supersymmetry [12]. FormCalc has therefore been equipped with an alternative scheme suited for calculations in supersymmetric theories. It is the constrained differential renormalization scheme (CDR) [13] which is equivalent to regularization by dimensional reduction at the one-loop level [2]. The schemes are chosen with the Dimension option of the CalcFeynAmp function: Dimension -> D selects dimensional regularization (the default), Dimension -> 4 switches to CDR.

The second problem is a technical one. In the SM, both the number of diagrams and the comparatively simple coupling structure allow to perform calculations with “brute force.” To wit, the diagrams are generated with all indices explicitly written out, and then calculated.

The situation is less favourable in the MSSM, however: not only is the number of diagrams considerably higher in most cases, but also the coupling structure is more involved. This is because particles like sfermions, or gauginos and higgsinos in general mix to form mass eigenstates and hence their couplings contain plenty of mixing matrices and can become rather lengthy. This means that the algebraic simplification has to be performed more carefully in order to maintain a decent performance and keep the size of the results as small as possible.

The way in which FormCalc proceeds is already suggested by the level structure of the FeynArts amplitudes. The lowest (generic) level completely determines the kinematical structure of a diagram, so FormCalc first performs the time-consuming simplifications involving kinematical quantities, like the tensor reduction, on the far fewer generic diagrams only. For each diagram at the next higher (classes) level, it then substitutes the generic coupling constants and masses by their actual values. The diagrams are mostly specified by then, except that index summations, e.g. over fermion generations, are not yet carried out. These index summations are performed only in the Fortran code, which means that FormCalc has to keep track of all indices and write out the corresponding loop instructions in Fortran. For a model like the MSSM, the savings incurred with this method of simplification may easily amount to an order of magnitude in both CPU time and size of the Fortran code.

4 Numerical evaluation

In the FormCalc framework, the Mathematica expressions resulting from the algebraic simplification are translated to Fortran code for numerical evaluation. The generated Fortran code of course has to be provided with the proper numerical values for the parameters appearing in the model, i.e. the variables in Table 2. This is solved by a subroutine which is called at the beginning of the calculation to initialize all model parameters.

Technically, the input parameters are specified in common blocks defined in model.h. Some less commonly changed inputs, notably some breaking parameters in the sfermion sector, are realized as preprocessor variables which take default values if not defined by the user. Preprocessor variables are also used for switches, such as whether squark mixing should be turned on or off.

4.1 Parameters of the MSSM

Supersymmetry (SUSY) completely determines the supersymmetric part of the MSSM Lagrangian once the SM parameters are known. The complete MSSM, however, with softly broken SUSY in its general form, necessarily introduces a large number of masses, phases, and mixing angles to parametrize the SUSY breaking. At a final count, 105 of these degrees of freedom cannot be absorbed in some other quantity or rotated away [4]. In order to get a handle on so many parameters, several assumptions have been made in mssm_ini.F to arrive at a moderate number of input parameters while retaining reasonable generality.

The first assumption is that the SUSY-GUT relation holds, which relates the U(1), SU(2), and
SU(3) gaugino mass parameters $M_1$, $M_2$, and $M_3$ according to

$$M_1 = \frac{5}{3} s_W^2 M_2 \quad \text{and} \quad M_3 = \frac{\tan \beta}{\tan \alpha} s_W M_2 \quad \text{with} \quad m_{\tilde{g}} \equiv |M_3|,$$

(1)

where $\sqrt{s}$ is the CMS energy, $c_W = M_W / M_Z$, and $s_W = \sqrt{1 - c_W^2}$. The running couplings $\bar{\alpha}$ and $\bar{\alpha}_s$ are MS values; we use the CERNlib function $\text{ALPHAS2}$ for $\bar{\alpha}_s(s)$ and

$$\frac{1}{\alpha(s)} = \frac{1}{\alpha(M_Z^2)} - \frac{1}{3\pi} \sum_{j \neq t} Q_j^2 N_c \ln \frac{s}{M_Z^2} = \frac{1}{\alpha(M_Z^2)} - \frac{20}{9\pi} \ln \frac{s}{M_Z^2}$$

(2)

with $1/\alpha(M_Z^2) = 127.934$.

Secondly, we assume the SUSY-breaking parameters in the sfermion sector to be flavour-blind and distinguish only two $A$-parameters for different isospin values, hence the sfermion sector is governed by seven scalar parameters:

$$
M_Q^2 = M_Q^2 \mathbb{I}, \quad M_U^2 = M_U^2 \mathbb{I}, \quad A_U = A_u \mathbb{I}, \\
M_D^2 = M_D^2 \mathbb{I}, \quad A_D = A_d \mathbb{I}, \\
M_L^2 = M_L^2 \mathbb{I}, \quad M_E^2 = M_E^2 \mathbb{I}, \quad A_L = A_d \mathbb{I},
$$

(3)

where in addition the various $M_{\tilde{X}}$ get the default value $M_{\text{SUSY}}$, a common SUSY-breaking mass, i.e.

$$M_Q = M_L = M_U = M_D = M_E = M_{\text{SUSY}} \quad \text{(by default)}.$$  

(4)

The remaining MSSM input parameters thus have 10 degrees of freedom:

1. quotient of vev’s: tan $\beta$ (real),
2. Higgs mass: $M_A$ (real),
3. breaking parameters: $A_u, A_d$ (complex), $M_{\text{SUSY}}, M_2$ (real),
4. mixing parameter: $\mu$ (complex).

The inputs to $\text{mssm\_ini.F}$ are summarized in Table I. All other variables in the MSSM Lagrangian (see Table II) are determined once this reduced set of MSSM input parameters and the SM inputs are specified and are calculated by $\text{mssm\_ini.F}$. This is the reason why $\text{mssm\_ini.F}$ is much more involved than its companion file for the SM, $\text{sm\_ini.F}$. The calculation of the parameters can be directed by several switches, which are also listed in Table III.

When the particle masses are calculated from these input parameters, $\text{mssm\_ini.F}$ checks whether they are consistent with the current exclusion limits and automatically omits already-excluded points with a warning. The following bounds are used:

$$
\begin{align*}
m_t & \geq 80 \text{ GeV} \quad [17], \\
m_b & \geq 70 \text{ GeV} \quad [17], \\
m_{\tilde{q}, \tilde{l}} & \geq 150 \text{ GeV} \quad [17], \\
m_h & \geq 91 \text{ GeV for real input parameters} \quad [19], \\
& \geq 85 \text{ GeV for complex input parameters} \quad [20], \\
m_{\tilde{t}} & \geq 70 \text{ GeV} \quad [18], \\
m_{\tilde{\chi}} & \geq 90 \text{ GeV} \quad [21], \\
\Delta \rho_{l,b} & \leq 3 \times 10^{-3} \quad [16], \\
m_{\tilde{\chi}_0} & \geq 30 \text{ GeV} \quad [21], \\
m_{\tilde{\chi}} & \geq 175 \text{ GeV} \quad [22].
\end{align*}
$$

(6)

The detailed formulas built into $\text{mssm\_ini.F}$ for computing the various parameters are presented in the following sections, organized according to the various sectors of the MSSM.

### 4.2 The Higgs sector

The neutral Higgs sector is fixed by choosing a value for $\tan \beta = v_2 / v_1$ (the ratio of the vacuum expectation values of the two Higgs doublets) and for the mass $M_A$ of the CP-odd neutral Higgs boson $A^0$. For the CP-even Higgs masses, which receive sizable radiative corrections, we use the approximation formula of [23] which agrees with the full two-loop calculation [24] to within less than 2 GeV.
| parameter | Fortran name | type                | default value |
|-----------|--------------|---------------------|---------------|
| tan β     | TB           | double precision    |               |
| M_A       | MA0          | double precision    |               |
| A_u       | Au           | double complex      |               |
| A_d       | Ad           | double complex      |               |
| M_SUSY    | MSusy        | double precision    |               |
| M_2       | M_2          | double precision    |               |
| μ         | MUE          | double complex      |               |
| M_Q       | MSQ          | preprocessor variable | MSusy        |
| M_L       | MSL          | preprocessor variable | MSusy        |
| M_U       | MSU          | preprocessor variable | MSusy        |
| M_D       | MSD          | preprocessor variable | MSusy        |
| M_E       | MSE          | preprocessor variable | MSusy        |

| switch         | action                                                                 |
|----------------|------------------------------------------------------------------------|
| NO_SQUARK_MIXING | Sets $A_u = \mu^* \cot \beta$ and $A_d = \mu^* \tan \beta$, so that the off-diagonal entries of the sfermion mass matrices vanish (cf. Eq. (37)), i.e. makes sfermion mass eigenstates = sfermion gauge eigenstates. |
| COMPLEX_PARAMETERS | Uses a simpler (one-loop) approximation for the Higgs masses which is valid for all parameters. This switch must be set if complex input parameters are used because otherwise the more precise (two-loop) approximation for the Higgs masses is taken, which is valid only for real parameters. |
| SM_ONLY        | Calculates $M_h$ as usual, but then reverse-engineers the mixing in the Higgs sector ($\alpha$ and $\beta$) such that the MSSM Higgs sector looks like a SM Higgs sector (see [11], p. 356), with the light CP-even Higgs boson $h$ figuring as the SM Higgs boson. |
| NO_EXCLUSION_LIMITS | Ignores the experimental bounds, i.e. does not exclude points in parameter space if the bounds in Eq. (37) are violated. |

Table 4: The inputs and switches for `mssm_ini.F`. Note: A preprocessor variable is defined with “#define var value” and a switch is set with “#define switch”. The #define must stand at the beginning of the line.
4.2.1 The neutral CP-even Higgs bosons $h$ and $H$

In this section we discuss the case of real MSSM parameters, for which a more sophisticated computation of the Higgs masses is implemented. The case of complex parameters is treated in Sect. 4.2.2.

The gauge eigenstates $\phi_1$ and $\phi_2$ of the neutral CP-even Higgs bosons mix via the mass matrix

$$M^2_{\text{Higgs}} = \begin{pmatrix} M_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta - \hat{\Sigma}_{\phi_1} & -(M_A^2 + M_Z^2) \sin \beta \cos \beta - \hat{\Sigma}_{\phi_1 \phi_2} \\ -(M_A^2 + M_Z^2) \sin \beta \cos \beta - \hat{\Sigma}_{\phi_1 \phi_2} & M_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta - \hat{\Sigma}_{\phi_2} \end{pmatrix}. \quad (7)$$

Diagonalizing this matrix yields the Higgs masses

$$M^2_{H,h} = \frac{M_A^2 + M_Z^2}{2} - \hat{\Sigma}_{\phi_1} - \hat{\Sigma}_{\phi_2} \pm \left[ \frac{(M_A^2 + M_Z^2)^2 + (\hat{\Sigma}_{\phi_1} - \hat{\Sigma}_{\phi_2})^2}{4} - M_A^2 M_Z^2 \sin^2 2\beta \right]^{1/2} + \frac{1}{2} (M_A^2 - M_Z^2) \cos 2\beta (\hat{\Sigma}_{\phi_1} - \hat{\Sigma}_{\phi_2}) + (M_A^2 + M_Z^2) \sin 2\beta \hat{\Sigma}_{\phi_1 \phi_2} + \hat{\Sigma}_{\phi_1 \phi_2}^2 \right]^{1/2}. \quad (8)$$

The mixing angle $\alpha$ of the CP-even Higgs doublet is hence

$$\alpha = \arctan \left( \frac{-(M_A^2 + M_Z^2) \sin \beta \cos \beta - \hat{\Sigma}_{\phi_1 \phi_2}}{M_Z^2 \cos^2 \beta + M_A^2 \sin^2 \beta - \hat{\Sigma}_{\phi_1} - M_h^2} \right). \quad (9)$$

The presence of renormalized self-energies $\hat{\Sigma}$ indicates that the mass matrix contains significant radiative corrections which must be taken into account to make quantitatively correct predictions. The self-energies of course receive contributions from all sectors of the MSSM, but not all are numerically of equal relevance. We take into account only the following terms:

- $\hat{\Sigma}^{(1,t/\bar{t})}(0)$: the one-loop $t/\bar{t}$-contributions at zero momentum transfer up to $m_t^4$ [
- $\hat{\Sigma}^{(2,t/\bar{t})}(0)$: the dominant two-loop $t/\bar{t}$-contributions of $\mathcal{O}(\alpha \alpha_s)$ at zero momentum transfer and the leading two-loop Yukawa correction of $\mathcal{O}(\alpha^2)$ [
- $\hat{\Sigma}^{(1,\text{rest})}$: the one-loop leading-log contributions from all other sectors [

In the following, we give the explicit expressions for these contributions as implemented in mssm.ini.F.

- The one-loop $t/\bar{t}$-contributions at zero momentum transfer up to $m_t^4$:

  $$\hat{\Sigma}^{(1,t/\bar{t})}_{\phi_1}(0) = \frac{G_F \sqrt{2}}{\pi^2} M_Z^2 A \cos^2 \beta \ln \frac{m_t^2}{M_S^2},$$
  $$\hat{\Sigma}^{(1,t/\bar{t})}_{\phi_1 \phi_2}(0) = -\frac{G_F \sqrt{2}}{\pi^2} M_Z^2 \cot \beta \left( -\frac{3}{8} m_t^2 + M_Z^2 \Lambda \sin^2 \beta \right) \ln \frac{m_t^2}{M_S^2}, \quad (10)$$
  $$\hat{\Sigma}^{(1,t/\bar{t})}_{\phi_2}(0) = \frac{G_F \sqrt{2}}{\pi^2} \frac{m_t^4}{8 \sin^2 \beta} \left[ -2 \frac{M_Z^2}{m_t^2} + \frac{11 M_A^4}{10 m_t^2} \right] \ln \frac{m_t^2}{M_S^2} + \left( -12 + \frac{M_Z^2 \sin^2 \beta + 8 M_A^4 \Lambda \sin^2 \beta}{m_t^2} \right) \ln \frac{m_t^2}{M_S^2} + \left( 12 - \frac{m_t^2}{5 M_S^2} - 2 \frac{m_t^2}{M_S^2} \right) \frac{(M_t^{LR})^2}{M_S^2} + \left( M_t^{LR} \right)^4 \frac{(M_t^{LR})^6}{M_S^2} + \left( \frac{3 m_t^4}{5 M_S^2} - \frac{12 m_t^4}{5 M_S^2} + 2 m_t^2 \right) \frac{(M_t^{LR})^8}{M_S^2} \quad (12)$$

$$+ \left( 3 \frac{m_t^4}{7 M_S^2} - \frac{12 m_t^4}{7 M_S^2} + 3 \frac{m_t^2}{2 M_S^2} \right) \frac{(M_t^{LR})^8}{M_S^2} \quad (12)$$
with
\begin{equation}
M_t^{LR} = A_u - \mu^* \tan \beta, \tag{13}
\end{equation}
\begin{equation}
M_S = \left[ M_Q^2 M_U^2 + m_H^2 (M_Q^2 + M_U^2) + m_\tilde{t}^4 \right]^{1/4}, \tag{14}
\end{equation}
\begin{equation}
\Lambda = \frac{1}{8} \frac{1}{3} s_W^2 + \frac{4}{9} s_W^4. \tag{15}
\end{equation}

- The dominant two-loop $t/\tilde{t}$-contributions of $O(\alpha \alpha_s)$ at zero momentum transfer and the leading two-loop Yukawa correction of $O(\alpha^2)$:
\begin{equation}
\tilde{\Sigma}^{(2,t/\tilde{t})}_{\phi_1,\phi_2}(0) = 0, \tag{16}
\end{equation}
\begin{equation}
\tilde{\Sigma}^{(2,t/\tilde{t})}_{\phi_2}(0) = \frac{G_F \sqrt{2} \pi a(m_t^2) \pi}{m_t^2} \sin \beta \left[ 3 \ln \frac{m_t^2}{M_S^2} - 6 \ln \frac{m_t^2}{M_S^2} - 6 \frac{M_t^{LR}}{M_S^2} \right. \\
\left. - \frac{3 (M_t^{LR})^2}{M_S^2} \ln \frac{m_t^2}{M_S^2} + \frac{3 (M_t^{LR})^4}{4 M_S^4} \right] \\
- \frac{9 G_F^2 \pi a_0^2}{16 \pi^2 \sin \beta} \left[ \tilde{X}_t \ln \frac{m_{\tilde{t}}^2 m_{\tilde{t}}^2}{m_t^2} + \ln \frac{m_{\tilde{t}}^2 m_{\tilde{t}}^2}{m_t^2} \right], \tag{17}
\end{equation}
where
\begin{equation}
\tilde{X}_t = \left( \frac{m_{\tilde{t}}^2 \sin \beta}{m_t^2} \right) \left( U_{11}^t U_{12}^t \right)^2 \left( 2 - \frac{m_{\tilde{t}}^2 + m_{\tilde{t}}^2}{m_{\tilde{t}}^2 - m_{\tilde{t}}^2} \ln \frac{m_{\tilde{t}}^2}{m_{\tilde{t}}^2} \right) + \frac{m_{\tilde{t}}^2 - m_{\tilde{t}}^2}{m_t^2} \left( U_{11}^t U_{12}^t \right)^2 \ln \frac{m_{\tilde{t}}^2}{m_{\tilde{t}}^2}. \tag{18}
\end{equation}
and $U^t$ is the stop mixing matrix defined in Eq. [37]. We use the $\overline{\text{MS}}$ top mass
\begin{equation}
m_t = m_t(m_t) \approx \frac{m_t}{1 + \frac{1}{\pi^2} a(m_t)}, \tag{19}
\end{equation}
instead of the pole mass to include also the leading $t/\tilde{t}$-contributions beyond $O(\alpha \alpha_s)$.

- The one-loop leading-log contributions from all other sectors:
\begin{equation}
\tilde{\Sigma}^{(1,\text{rest})}_{\phi_1} = - \frac{G_F M_Z^4}{12 \sqrt{2} \pi^2} \left[ \left( \frac{12 N_c m_b^4}{M_Z^4 \cos^4 \beta} - \frac{6 N_c m_b^2}{M_Z^2 \cos^2 \beta} \right) + P_b + P_f + P_g + P_{2H} \right] \ln \frac{M_{\tilde{t}}^2}{M_Z^2} \tag{20}
\end{equation}
\begin{equation}
\left. + \theta(M_A - M_Z)(P_{1H} - P_{2H}) \ln \frac{M_{\tilde{t}}^2}{M_Z^2} \right],
\end{equation}
\begin{equation}
\tilde{\Sigma}^{(1,\text{rest})}_{\phi_1,\phi_2} = - \frac{G_F M_Z^4}{12 \sqrt{2} \pi^2} \left[ \left( \frac{3 N_c m_b^2}{M_Z^2 \cos^2 \beta} - P_b + P_f + P_g + P_{2H} \right) \ln \frac{M_{\tilde{t}}^2}{M_Z^2} \tag{21}
\end{equation}
\begin{equation}
\left. + \theta(M_A - M_Z)(P_{1H} + P_{2H}) \ln \frac{M_{\tilde{t}}^2}{M_Z^2} \right],
\end{equation}
\begin{equation}
\tilde{\Sigma}^{(1,\text{rest})}_{\phi_2} = - \frac{G_F M_Z^4}{12 \sqrt{2} \pi^2} \left[ (P_b + P_f + P_g + P_{2H}) \ln \frac{M_{\tilde{t}}^2}{M_Z^2} \right], \tag{22}
\end{equation}
\[ + \theta (M_A - M_Z)(P_{1H} - P_{2H}) \ln \frac{M_Z^2}{M_A^2} \sin^2 \beta \]
\[ + \frac{G_F N_c m_b^2}{4\sqrt{2}\pi^2 M_{\text{SUSY}}^2} \left[ \frac{m_b^2 \mu^2 (M_b^{L,R})^2}{3M_{\text{SUSY}}^2 \cos^2 \beta} + M_Z^2 \tan \beta \left( M_b^{L,R} + \frac{1}{3} \mu \tan \beta \right) \right] \] (22)

with
\[ M_b^{L,R} = A_u - \mu^* \cot \beta, \]
\[ P_b = N_c (1 + 4Q_b s_W^2 + 8Q_b^2 s_W^4), \]
\[ P_f = N_c (N_g - 1) (2 - 4s_W^2 + 8(Q_t^2 + Q_b^2) s_W^4) + N_g (2 - 4s_W^2 + 8s_W^4), \]
\[ P_g = -44 + 106s_W^2 - 62s_W^4, \]
\[ P''_g = 10 + 34s_W^2 - 26s_W^4, \]
\[ P_{1H} = -9 \cos^2 2\beta + (1 - 2s_W^2 + 2s_W^4) \cos^2 2\beta, \]
\[ P_{2H} = -10 + 2s_W^2 - 2s_W^4, \]
\[ P''_{2H} = 8 - 22s_W^2 + 10s_W^4, \]
\[ Q_t = \frac{2}{3}, \quad Q_b = -\frac{1}{3}, \quad N_c = 3, \quad N_g = 3. \]

4.2.2 The case of complex parameters

The sophisticated calculation of the Higgs masses as in the preceding section is valid only for real parameters. When complex parameters are used, the preprocessor variable COMPLEX_PARAMETERS must be defined (see Table 4), which makes mssm.in.F use a simple (one-loop) approximation for the Higgs masses that is valid also for complex parameters.

For the Higgs masses the dominant one-loop contributions are the \( m_t^4 \) terms [25]. Neglecting the splitting of the \( t \) masses, the \( \phi_2 \) self-energy of Eq. (3) becomes
\[ \Sigma_{\phi_2(t, \bar{t})}(0) = \frac{3G_F}{2\sqrt{2}\pi^2} \frac{m_t^4}{\sin^2 \beta} \ln \left( 1 + \frac{M_Q^2 M_U^2}{m_t^2} + \frac{M_Q^2 + M_U^2}{m_t^2} \right) \] (24)
with the running top mass \( \bar{m}_t \) as defined in Eq. (19). All other radiative corrections in the Higgs mass matrix are set to zero.

4.2.3 The charged Higgs bosons

There are only small radiative corrections for the charged Higgs masses. We use the following relation, valid for \( M_A \sim O(M_Z) \) [23], up to \( M_A \)-values of 250 GeV:
\[ M_{H^\pm}^2 = M_A^2 + M_W^2 + \frac{\alpha M_W^2}{12\pi s_W} \left[ N_c (N_g - 1) + N_g - 9 + 15s_W^2 \right] \ln \frac{M_{\text{SUSY}}^2}{M_W^2} \]
\[ + \frac{N_c \alpha}{8\pi s_W M_W^2} \left[ \frac{2\sqrt{2}m_b^2}{\sin^2 \beta \cos^2 \beta} - \frac{M_W^2}{\sin^2 \beta} \left( \frac{m_b^2}{\sin^2 \beta} + \frac{m_t^2}{\cos^2 \beta} \right) + \frac{2}{3} M_W^4 \right] \ln \frac{M_{\text{SUSY}}^2}{m_t^2}, \] (25)

with \( N_c = N_g = 3 \) and the running top mass \( \bar{m}_t \) as defined in Eq. (19). Above 250 GeV, this formula reduces to \( M_{H^\pm}^2 = M_A^2 + M_W^2 \).

4.3 The chargino sector

The charged gauginos and higgsinos mix via the mass matrix [13, 14]
\[ X = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}. \] (26)
X is diagonalized with two unitary matrices $U$ and $V$ according to

$$U^* X V^\dagger = \text{diag}(m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}),$$

which yields the chargino masses

$$m_{\tilde{\chi}_{1,2}}^2 = \frac{M_\tilde{f}^2 + |\mu|^2 + 2M_W^2}{2} \mp \sqrt{\left(\frac{M_\tilde{f}^2 + |\mu|^2 + 2M_W^2}{4}\right)^2 - |M^0_{\tilde{f}}|^2 \sin 2\beta - |\mu|^2}.$$  \hfill (28)

For the numerical diagonalization, the LAPACK subroutine ZGESVD \cite{28} is used.

### 4.4 The neutralino sector

The neutral gauginos and higgsinos mix via the mass matrix \cite{9, 10}

$$Y = \begin{pmatrix}
M_1 & 0 & -M_Z s_W \cos \beta & M_Z s_W \sin \beta \\
0 & M_2 & M_Z c_W \cos \beta & -M_Z c_W \sin \beta \\
-M_Z s_W \cos \beta & M_Z c_W \cos \beta & 0 & -\mu \\
M_Z s_W \sin \beta & -M_Z c_W \sin \beta & -\mu & 0
\end{pmatrix}.$$  \hfill (29)

$Y$ is diagonalized with a unitary matrix $N$ according to

$$N^* Y N^\dagger = \text{diag}(m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}, m_{\tilde{\chi}_3}, m_{\tilde{\chi}_4}),$$

which yields the four neutralino mass eigenstates. For the numerical diagonalization, the LAPACK subroutine ZGESVD \cite{28} is used. Note that we work with a general complex matrix $N$ and also check that the computed mass values are nonnegative.

### 4.5 The sfermion sector

The sfermion gauge eigenstates are connected via the mass matrix \cite{9, 10}

$$Z = \begin{pmatrix}
M_f^{LL} + m_f^2 & m_f (M_f^{LR})^* \\
 m_f M_f^{LR} & M_f^{RR} + m_f^2
\end{pmatrix}$$

where

$$M_f^{LL} = M_Z^2 (I_3^f - Q_f s_W^2) \cos 2\beta + \begin{cases} M_Q^2 & \text{for left-handed squarks}, \\
M_L^2 & \text{for left-handed sleptons}, \end{cases}$$  \hfill (32)

$$M_f^{LR} = A_f - \mu^* \begin{cases} \cot \beta & \text{for } u\text{-type sfermions} (I_3^f = +1/2), \\
\tan \beta & \text{for } d\text{-type sfermions} (I_3^f = -1/2), \end{cases}$$  \hfill (33)

$$M_f^{RR} = M_Z^2 Q_f s_W^2 \cos 2\beta + \begin{cases} M_U^2 & \text{for right-handed, } u\text{-type squarks}, \\
M_D^2 & \text{for right-handed, } d\text{-type squarks}, \\
M_E^2 & \text{for right-handed sleptons}. \end{cases}$$  \hfill (34)

The mass eigenstates are obtained by diagonalizing $Z$ with a unitary matrix $U_f$, viz.

$$U_f^* Z U_f^\dagger = \text{diag}(m_{\tilde{f}_{1,2}}^2),$$

which results in the sfermion masses

$$m_{\tilde{f}_{1,2}}^2 = m_f^2 + \frac{1}{2} \left( M_f^{LL} + M_f^{RR} \mp \sqrt{\left( M_f^{LL} - M_f^{RR}\right)^2 + 4m_f^2 |M_f^{LR}|^2} \right).$$  \hfill (36)

For the numerical diagonalization the LAPACK subroutine ZHEEV \cite{28} is used.

Sfermion mixing can be switched off by defining the preprocessor variable NO_SQUARK_MIXING (see Table 3). This causes the breaking parameters $A_u$ and $A_d$ to be set to the values

$$A_u = \mu^* \cot \beta, \quad A_d = \mu^* \tan \beta$$

so that the off-diagonal terms in the mass matrix $Z$ vanish.
4.5.1 Supersymmetric contributions to $\Delta \rho$

In addition to the experimental bounds on the squark masses, mssm_ini.F also checks whether the MSSM contributions to the $\rho$-parameter are consistent with the current exclusion limits (see Eq. (3)). This bound becomes relevant mainly when parameters are chosen to achieve a large mass splitting between $\tilde{t}_1$ and $\tilde{t}_2$ or $\tilde{b}_1$ and $\tilde{b}_2$.

We have implemented the calculation of the MSSM contributions to $\Delta \rho$ according to [29]. The sfermion contributions are significant only when the masses of the isospin partners are very different, and since this is governed by the quark masses (see Eq. (36)), only corrections from $t/b$-loops have been included. Contributions from gluino exchange, which are very lengthy and vanish for large $m_{\tilde{g}}$, have similarly been neglected.

These contributions to $\Delta \rho$ are given by

$$
\Delta \rho_{t,b} = U_{11}^t U_{12}^b U_{12}^{t*} U_{21}^t F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + U_{11}^b U_{12}^t U_{12}^{b*} U_{21}^b F(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2)
$$

$$
+ |U_{11}^t|^2 |U_{11}^b|^2 F(m_{\tilde{t}_1}^2, m_{\tilde{b}_1}^2) + |U_{11}^b|^2 |U_{21}^b|^2 F(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2)
$$

$$
+ |U_{21}^t|^2 |U_{21}^b|^2 F(m_{\tilde{t}_2}^2, m_{\tilde{b}_1}^2) + |U_{21}^t|^2 |U_{21}^b|^2 F(m_{\tilde{t}_2}^2, m_{\tilde{b}_2}^2),
$$

where the $U^f$ are the sfermion mixing matrices defined in Eq. (33) and the function

$$
F(x, y) = \frac{3 G_F}{8 \sqrt{2} \pi^2} F_1(x, y) + \frac{G_F}{4 \sqrt{2} \pi^2} \frac{\pi_i(m_t^2)}{\pi} F_2(x, y)
$$

includes the one- and two-loop contributions

$$
F_1(x, y) = x + y - \frac{2xy}{x-y} \ln \frac{x}{y},
$$

$$
F_2(x, y) = x + y - \frac{2xy}{x-y} \left( 2 + \frac{x}{y} \ln \frac{x}{y} \right) \ln \frac{x}{y} + \frac{x^2(x+y)}{(x-y)^2} \ln^2 \frac{x}{y} - 2(x-y) \text{Li}_2 \left( 1 - \frac{x}{y} \right).
$$

The two-loop contribution is of the order of 10 to 15% of the one-loop result.

5 Tests

The model files MSSM.mod and MSSMQCD.mod have been checked against results known from the literature for a variety of scattering processes. In cases where the Fortran programs of the original authors were available, we found perfect agreement for all differential cross-sections. In the other cases we could reproduce qualitatively the figures. The list of processes we checked together with the agreement we achieved is given in Table 3.

6 Availability, Requirements

The FeynArts package including the MSSM.mod and MSSMQCD.mod model files can be downloaded from http://www.feynarts.de. The program itself requires Mathematica 3 or above. For the topology editor, which is not necessarily invoked, a Java VM and the J/Link package are needed, both of which can be obtained free of charge (see the instructions on the web site).

The FormCalc package is available from http://www.feynarts.de/formcalc and includes the initialization file mssm_ini.F. It runs on Unix-like platforms and requires Mathematica 3, FORM 3, and the GNU C compiler (gcc). To link the Fortran code generated by FormCalc, one needs in addition the LoopTools library http://www.feynarts.de/looptools and the CERNlib http://wwwinfo.cern.ch/asd/index.html.

FeynArts and FormCalc each include a comprehensive manual which explains installation and usage. Both are open-source programs and stand under the GNU library general public license.
| process          | agreement | reference | scope              |
|------------------|-----------|-----------|--------------------|
| $qar{q} \rightarrow t\bar{t}$ | 11 digits | 30        | SUSY-QCD only      |
| $qar{q} \rightarrow t\bar{t}$ | 10 digits | 31        | full MSSM          |
| $gg \rightarrow t\bar{t}$       | qualitative | 32 | SUSY-QCD only      |
| $gg \rightarrow t\bar{t}$       | 10 digits | 31        | full MSSM          |
| $gg \rightarrow H^+H^-$         | 10 digits | 33        | full MSSM          |
| $gg \rightarrow W^-H^+$         | qualitative | 34 | full MSSM          |
| $e^+e^- \rightarrow t\bar{t}$   | 11 digits | 35        | full MSSM          |
| $e^+e^- \rightarrow W^+W^-$     | qualitative | 36 | sfermion contributions |
| $e^+e^- \rightarrow H^+H^-$     | 7 digits  | 37        | full MSSM          |
| dipole moments       | 11 digits | 38        | full MSSM          |

Table 5: Checks performed with *FeynArts* and *FormCalc* to test the implementation of the MSSM.

### Acknowledgements

We thank A. Kraft for his Fortran code which provided the basis for MSSM.mod, S. Berge, O. Brein, and D. Wackerroth for helping us cross-check our programs, W. Hollik and S. Heinemeyer for proofreading the manuscript and valuable suggestions on the implementation of the Higgs masses, and G. Jahn for some checks of the four-sfermion couplings.

T.H. has been supported by the Deutsche Forschungsgemeinschaft (Forschergruppe “Quantenfeldtheorie, Computeralgebra und Monte-Carlo Simulation”) under contract number Ku 502/8–1.

### References

[1] J. Küblbeck, M. Böhm, and A. Denner, *Comp. Phys. Commun.* **60** (1990) 165; T. Hahn, hep-ph/0012260.

[2] T. Hahn and M. Pérez-Victoria, *Comp. Phys. Commun.* **118** (1999) 153.

[3] F. Yuasa et al., *Prog. Theor. Phys. Suppl.* **138** (2000) 18; T. Kaneko, *Comp. Phys. Commun.* **92** (1995) 127.

[4] M. Kuroda, hep-ph/9902340.

[5] E. Boos, M. Dubinin, V. Ilyin, A. Pukhov, V. Savrin, hep-ph/9503280; A. Pukhlov et al., hep-ph/9908288.

[6] A. Belyaev, A. Gladyshev, and A. Semenov, hep-ph/9712303.

[7] T. Hahn, *Nucl. Phys. Proc. Suppl.* **89** (2000) 231.

[8] A. Denner, H. Eck, O. Hahn, and J. Küblbeck, *Nucl. Phys.* **B387** (1992) 467; A. Denner, H. Eck, O. Hahn, and J. Küblbeck, *Phys. Lett.* **B291** (1992) 278.

[9] H. Haber, G. Kane, *Phys. Rep.* **117** (1985) 75.

[10] J. Gunion, H. Haber, *Nucl. Phys.* **B272** (1986) 1.

[11] J. Gunion, H. Haber, G. Kane, and S. Dawson, *The Higgs Hunter’s Guide*, Frontiers in Physics Vol. 80, Addison-Wesley, 1990.
[12] D.M. Capper, D.R.T. Jones, and P. van Nieuwenhuizen, Nucl. Phys. B167 (1980) 479.

[13] F. del Aguila, A. Culatti, R. Muñoz Tapia, and M. Pérez-Victoria, Nucl. Phys. B537 (1999) 561.

[14] S. Dimopoulos and D. Sutter, Nucl. Phys. B452 (1995) 496.

[15] H. Plothow-Besch, PDFLIB: User's Manual, CERN-ETT-TT 2000.04.17
http://wwwinfo.cern.ch/asdoc/pdflib.ps.gz.

[16] D. Groom et al., Eur. Phys. J. C15 (2000) 1.

[17] L3 Collaboration, Phys. Lett. B471 (1999) 308.

[18] L3 Collaboration, Phys. Lett. B471 (1999) 280.

[19] S. Andringa et al., ALEPH 2000-074 CONF 2000-051, DELPHI 2000-148 CONF 447, L3 Note 2600, OPAL Technical Note TN661
http://lephiggs.web.cern.ch/LEPHIGGS/papers/osaka_note.ps).

[20] G. Kane, L. Wang Phys. Lett. B488 (2000) 383.

[21] L3 Collaboration, Phys. Lett. B472 (2000) 420;
OPAL Collaboration, hep-ex/9909051.

[22] L3 Collaboration, Phys. Lett. B471 (1999) 308.

[23] S. Heinemeyer, W. Hollik, and G. Weiglein, Phys. Lett. B455 (1999) 179 and hep-ph/0002213.

[24] S. Heinemeyer, W. Hollik, and G. Weiglein, Eur. Phys. J. C9 (1999) 343.

[25] A. Dabelstein, Ph.D. thesis, Munich, 1993, MPI-Ph/93-64;
A. Dabelstein, Z. Phys. C67 (1995) 495.

[26] M. Carena, M. Quirós, and C. Wagner, Nucl. Phys. B461 (1996) 407.

[27] H. Haber, R. Hempfling, and A. Hoang, Z. Phys. C75 (1997) 539.

[28] E. Anderson et al., LAPACK user's guide, SIAM Press, Philadelphia (1999).

[29] A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, and G. Weiglein, Phys. Rev. Lett. 78 (1997) 3626 and Phys. Rev. D57 (1998) 4179.

[30] S. Berge, Diploma thesis, Karlsruhe, 1999.

[31] W. Hollik, W.M. Mösle, and D. Wackeroth, Nucl. Phys. B516 (1998) 29;
D. Wackeroth, hep-ph/9807558
D. Wackeroth et al., in preparation.

[32] Y. Zeng-Hui, H. Pietschmann, M. Wen-Gan, H. Liang, and J. Yi, Eur. Phys. J. C9 (1999) 463.

[33] O. Brein and W. Hollik, Eur. Phys. J. C13 (2000) 175.

[34] O. Brein, W. Hollik, and S. Kanemura, Phys. Rev. D63 (2001) 095001.

[35] W. Hollik and C. Schappacher, Nucl. Phys. B545 (1999) 98.

[36] S. Alam, K. Hagiwara, S. Kanemura, R. Szalapski, and Y. Umeda, Phys. Rev. D62 (2000) 095011.

[37] A. Kraft, Ph.D. thesis, Karlsruhe, 1999;
J. Guasch, W. Hollik, A. Kraft, Nucl. Phys. B596 (2001) 66.

[38] W. Hollik, J. Illana, S. Rigolin, C. Schappacher, and D. Stöckinger, Nucl. Phys. B551 (1999) 3,
E: Nucl. Phys. B557 (1999) 407.

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