Approximate Quantum Cloning with Nuclear Magnetic Resonance

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Here we describe a Nuclear Magnetic Resonance (NMR) experiment that uses a three qubit NMR device to implement the one to two approximate quantum cloning network of Bužek et al.

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Quantum information processing [1] has been the subject of much recent interest, not only because it offers new modes of computation and communication, but also because quantum information differs from classical information in several fundamental ways. One important example is the fact that it is impossible to accurately clone (copy) an unknown quantum state [2], and so quantum bits (qubits) cannot be duplicated. It is, however, possible to prepare an approximate copy [3], and several schemes for optimal approximate cloning have been developed. Nuclear Magnetic Resonance (NMR) [4, 5, 6] has already been used to demonstrate simple quantum information processing methods [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24], and here we describe an NMR experiment that uses a three qubit NMR device to implement the one to two approximate quantum cloning network [25] of Bužek et al.

We started by slightly modifying the approximate cloning network [25] of Bužek et al. to take advantage of our specific hardware; our version of the network is shown in Fig. 1. This takes in one qubit in an arbitrary state (normally considered to be a pure state, |ψ⟩), and two ancilla qubits in state |0⟩; the two ancilla qubits are prepared into an appropriate initial state, and then the input qubit is copied. At the end of the cloning sequence the three qubits are all entangled with one another, and so the reduced density operator descriptions of each qubit (obtained by tracing out the other two qubits) correspond to mixed states. The two ancilla qubits are both in the state $\frac{1}{\sqrt{6}}|1⟩|ψ⟩ + \frac{1}{\sqrt{6}}|ψ⟩|ψ⟩$ (that is, approximate clones of |ψ⟩ with fidelity 5/6), while the input qubit is now the approximate transpose of |ψ⟩⟨ψ|. Alternatively, using the identity $|ψ⟩⟨ψ| + |ψ⟩⟨ψ| = 1$, the states of the ancilla qubits can be written as $\frac{2}{3}|ψ⟩⟨ψ| + \frac{1}{3}|1⟩$. Cloning is not, of course, confined to pure states, and any mixed state ρ can be cloned to give $\frac{2}{3}ρ + \frac{1}{3}|1⟩$. Such mixed state cloning is particularly easy to study using NMR techniques, as the ensemble averaging inherent in NMR experiments is a help rather than a hindrance.

We implemented this network using a 3 qubit NMR quantum computer based on the single $^{31}$P nucleus (P) and the two $^1$H nuclei (A and B) in E-(2-chloroethyl)phosphonic acid (Fig. 2) dissolved in D$_2$O. Ethynylphosphonic acid diethyl ester was prepared from trimethylsilylacetylene by successive treatment with EtMgBr and diethyl chlorophosphate, followed by cold aqueous

![FIG. 1: Our slightly modified version of the approximate quantum cloning network developed by Bužek et al. The initial “preparation” stage has been replaced by an alternative network, which is simpler to implement with NMR techniques; the second “copy” stage is unchanged. Filled circles connected by control lines indicate controlled π phase shift gates [24], empty circles indicate single qubit Hadamard gates, while grey circles indicate other single qubit rotations. The two rotation angles in the preparation stage are $\theta_1 = \arcsin (1/\sqrt{3}) \approx 35^\circ$ and $\theta_2 = \pi/12 = 15^\circ$.](image1)

![FIG. 2: The three qubit system provided by E-(2-chloroethenyl)phosphonic acid dissolved in D$_2$O and its $^1$H NMR spectrum. The broad peak near ~50 Hz is a folded signal arising from residual HOD.](image2)
et al. the preparation of the initial state in product operator notation \cite{29}. The experiment begins with a nominal temperature of 20°C, and all NMR experiments were run at a 0.08M solution, and all NMR experiments were run at a 0.08M, 20°C. Relaxation times were measured by inversion-recovery methods, while T1 relaxation times were measured using a single spin-echo; all relaxation times are averaged over the four components of the relevant multiplet, as there was little sign of correlated relaxation.

| ν/Hz | T1/s | T2/s | JF/Hz | JA/Hz | JB/Hz |
|------|------|------|-------|-------|-------|
| P    | 104.0| 17.6 | 1.82  | 9.1   | 14.3  |
| A    | −104.0| 16.9 | 1.82  | 11.3  | 14.3  |

Na₂CO₃ to remove the silyl protecting group \cite{22}. HCl was added cis across the triple bond using LiCl in acetic acid \cite{27}, and the Z–E isomerisation was accomplished by heating with PhSH in the presence of AIBN \cite{28}. Acidic hydrolysis of the ester groups gave E-(2-chloroethenyl)-phosphonic acid (overall yield 5%, purity > 95%), from which contaminating metal ions were removed using chelex resin. The NMR sample was prepared by dissolving 7mg of compound into 600µL of D₂O to give a 0.08M solution, and all NMR experiments were run at a nominal temperature of 20°C using a homebuilt 600 MHz (¹H frequency) NMR spectrometer at the OCMS with a homebuilt double resonance (¹H inner, ³¹P outer) probe. The measured NMR parameters are listed in Table I.

This system may be conveniently described using product operator notation \cite{22}. The experiment begins with the preparation of the initial state \( P_z A_0 B_0 \) using a modification of the “cat state” method of Knill et al. \cite{22}. Qubit P can then be set to any desired point on the Bloch sphere using a single radiofrequency (RF) pulse. After the cloning sequence, the reduced density matrices of qubits A and B will correspond to Bloch vectors lying parallel to the original Bloch vector of P, but with lengths only 2/3 that of the original vector; note that the effect of the maximally mixed component is simply to reduce the length of the Bloch vector without affecting its orientation. Detailed pulse sequences are shown in Fig. 3 and described below.

NMR pulse sequences were developed by replacing the abstract gates in Fig. 1 with an idealized sequence of NMR pulses (including z-rotations) and delays; the resulting sequences were then simplified by adsorbing z-rotations into abstract reference frames \cite{22,24} and combining RF pulses when convenient \cite{24}. The detailed implementation of the sequence was chosen so as to avoid ¹H selective pulses as far as possible; those selective pulses in the “echo” sequence that could not be avoided were implemented using a variant on the “jump and return” sequence \cite{15,20}. This sequence incorporates a delay which should naively take the value \( \epsilon_{90} = 1/(4\delta\nu) \), where the two ¹H signals have frequencies ±\( \delta\nu \), but in practice it is better to slightly reduce its length in order to allow for non-idealities arising from the fact that \( \delta\nu \) is not much greater than the J couplings; in our experiments \( \epsilon_{90} \) was reduced to 90% of its nominal value.

The initial “purification” sequence was developed separately, but uses many of the same ideas. Our implementation is based on the “cat state” methods \cite{22} of Knill et al. The core of the sequence comprises a Hadamard gate and a pair of controlled-NOT gates that convert \( P_0 \) to a density matrix containing all terms along the anti-diagonal, a gradient filter that selects the desired triple quantum term, and a second pair of controlled-NOT gates and a Hadamard which act to produce \( P_0 A_0 B_0 \). The novel gradient filter sequence used here uses two gradients with strengths in the ratio 1 : 0.6633 to select the desired three quantum terms while crushing other anti-diagonal terms; note that phase cycling is not required.

Unlike conventional gradient sequences used to select coherence transfer pathways \cite{3,22}, this scheme is largely unaffected by diffusion. It is, however, vulnerable to imperfections in the anticat sequence: these are removed by the final z-filter, which incorporates a variable delay to suppress zero quantum terms \cite{13}. Note that this variable delay can be combined with the standard CYCLOPS phase cycle \cite{1}, and so no additional phase cycling is necessary \cite{13}. In principle this sequence will also suppress terms arising from initial \( A_z \) and \( B_z \) magnetization, as...
An important feature of the approximate cloning network is that all input states are cloned equally well, and so it is necessary to study the behaviour of the pulse sequence when applied to a wide range of points on the Bloch sphere. We have studied a total of 312 input states, arranged on a 13 by 24 rectangular grid of $\theta$ and $\phi$ values, with a spacing of $15^\circ$: this choice does not cover the Bloch sphere uniformly (for example the north and south poles are both sampled 24 times), but is experimentally convenient. For each input state we measured the total NMR signals observed from spins $A$ and $B$, and their real and imaginary components are plotted in Fig. 4. The experimental results clearly show the expected cosine and sine modulations, indicating that the cloning network is effective for all these input states.

Finally, we recall that approximate quantum cloning is unitarily equivalent to another interesting operation, the optimal universal-NOT gate [21], and so our experiment could also be considered as an implementation of universal-NOT; this point is not, however, specifically demonstrated here.

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FIG. 5: Summarised experimental results from cloning the general initial state $P_x \sin \theta \cos \phi + P_y \sin \theta \sin \phi + P_z \cos \theta$; the integrals of the real (a, b, c) and imaginary (d, e, f) parts of the NMR signals from spin $A$ (a, d) and spin $B$ (c, f) are shown as mesh and contour plots as a function of $\theta$ and $\phi$; the theoretical results are shown in (b, e).

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