Numerical Simulation of Heat Transfer during Microwave Heating of Magnetite

Zhiwei PENG,1) Jiann-Yang HWANG,1) Matthew ANDRIESE,1) Wayne BELL,1) Xiaodi HUANG1) and Xinli WANG2)

1) Department of Materials Science and Engineering, Michigan Technological University, Houghton, MI 49931, USA. E-mail: jhwang@mtu.edu, zpeng@mtu.edu, mdandrie@mtu.edu, wmbell@mtu.edu, xihuang@mtu.edu
2) School of Technology, Michigan Technological University, Houghton, MI 49931, USA. E-mail: xinlwang@mtu.edu

Received on January 4, 2011; accepted on February 28, 2011

Numerical simulation of heat transfer during the microwave heating process of a one-dimensional (1-D) magnetite slab subjected to convective, radiative boundary conditions was performed. The governing equations representing the heating process in the slab were discretized using an explicit finite-difference approach, and a computer code was developed to predict the temperature distributions inside the slab. The heat generation from microwave irradiation dominates the initial temperature rise in the heating and the heat radiation heavily affects the temperature distribution, giving rise to a temperature peak in the predicted temperature profile. As heating continues, the temperature peak migrates inward. The microwave power level is crucial to obtain a high temperature increase rate in the initial heating period (i.e. < 60 s for magnetite). Microwave heating at 915 MHz exhibits better heating homogeneity than 2450 MHz due to larger microwave penetration depth. To minimize/avoid temperature non-uniformity during the microwave heating the optimization of the object dimension should be considered.

KEY WORDS: heat transfer; microwave processing; magnetite; heating homogeneity.

1. Introduction

Microwave heating has gained popularity in various applications involving iron and steel making.1–5) The distinguishing characteristic of this technique is attributed to its special heating behaviors. It delivers heat instantly throughout the materials with volumetric heat generation resulting in a faster heating rate than conventional heating. Energy saving and less processing time are thus easy to achieve. Although microwave has shown its superiority in materials heating, a major drawback known as non-uniform temperature distribution inside materials has also been observed by many researchers.6,7) To address the problem, accurate temperature determination inside the materials under microwave irradiation is quite necessary. However, exact temperature measurement in microwave heating has been identified as a hard work since most common temperature measurement tools like thermocouple and pyrometer may not provide precise measurement data. The interaction between thermocouple and microwave lowers the accuracy of data measured while the complexity of emissivity considerations required to properly apply optical pyrometry heavily limits its extensive application.8

In comparison with direct temperature measurement, temperature prediction by analytical and numerical methods seems to offer a promising solution to this problem. Both analytical and numerical methods are required to solve the heat transfer differential equation coupled with Maxwell’s equations, but the former is found to be much more difficult since the heat generation from microwave heating and complex boundary conditions including convective and radiative heat transfer need to be considered simultaneously to obtain closed-form mathematical solution.9–11) Conversely, numerical modeling has been proved as an efficient and accurate method to predict the temperature of materials during microwave heating in the past 20 years.12–16) Most of those works focused on the utilization of microwave in the field of food processing where only heat diffusion and/or convection were considered. Meanwhile, the variations of dielectric properties of materials during the heating were generally ignored due to relatively low temperature range investigated (generally < 120 °C). It is obvious that the same assumption cannot be applied at high temperature where heat radiation becomes quite strong and the dielectric properties may change dramatically. Therefore, to accurately simulate the heat transfer for high temperature microwave processing of materials, radiation effect and the temperature dependency of dielectric properties of materials have to be considered.

The aim of this study is to monitor the heat transfer process in microwave heating by predicting the temperature distribution inside a 1-D magnetite slab using explicit finite-difference approach with “full” consideration of heat diffusion, convection and radiation effect as well as temperature dependences of thermophysical properties and dielectric properties.

2. Modeling

A 1-D object of homogeneous solid having dimension of 2L (Fig. 1) heated with microwaves was considered. Microwave energy was assumed to be of uniform intensity and
parallel polarization, impinging on both faces of the object. It was delivered in a transverse electric and magnetic (TEM) mode at 915/2.450 MHz and the microwave dissipation in the object followed the Lambert’s law (a satisfactory approximate alternative to Maxwell’s equations applied in microwave heating provided no obvious standing wave pattern forms in materials). Since the same energy was delivered into both sides of the object, giving rise to a temperature distribution with mirror symmetry; thus only one-half of the object needed to be considered.

The mathematical analysis pertinent to microwave heating process was based on Fourier’s law of heat conduction. The shrinkage or deformation of the object during the heating was assumed to be negligible and the surrounding air temperature was considered as a constant.

The mathematical heat transfer equation governing the microwave heating process in 1-D (x direction) slab object was given as: \( \frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{P(x)}{\rho c_p} \) ................................ (1)

where \( T, \rho, c_p, k \) are the temperature, density, specific heat capacity, and thermal conductivity, respectively; \( P(x) \) is heat generation term by microwave absorption. According to Lambert’s law, \( P(x) \) can be expressed in terms of microwave power flux \( P_0 \) and penetration depth \( (D_p) \) as follows:

\[
P(x) = \frac{P_0}{D_p} e^{-(x-L)/D_p} \] ................................ (2)

The following initial and boundary conditions were proposed:

\[
t = 0 \quad T = T_0 \quad 0 \leq x \leq L \] ................................ (3)

\[
x = 0 \quad -k \frac{\partial T}{\partial x} = 0 \quad t > 0 \] ................................ (4)

\[
x = L \quad -k \frac{\partial T}{\partial x} = h(T - T_\infty) + \varepsilon \sigma \left( T^4 - T_{\infty}^4 \right) \] \( t > 0 \) ................................ (5)

where \( t, T_0, h, T_\infty, \varepsilon, \) and \( \sigma \) are the time, initial temperature, heat transfer coefficient, environmental temperature, emissivity, and Stefan-Boltzmann constant, respectively.

3. Methodology

The method used in this study was the explicit finite-difference approximation, where the governing equations were transformed into difference equations by dividing the domain of solution to a grid of points in the form of mesh and the derivatives were expressed along each mesh point, referred as a node. The spatial domain \([0, L]\) was divided into \( m \) sections, each of length \( \Delta x = L/m \). Meanwhile, the time domain \([0, t]\) was divided into \( n \) segments, each of duration \( \Delta t = t/n \). The index \( i \) represents the mesh points in the \( x \) direction, starting with \( i = 0 \) being one boundary (slab center) and ending at \( i = m \) (slab surface). Specifically, the following difference equations were used:

\[
\frac{\partial T}{\partial t} = \frac{t_{i+1}^{n+1} - T_i^n}{\Delta t} \] ................................ (6)

\[
\frac{\partial T}{\partial x} = \frac{T_{i+1}^n - T_{i-1}^n}{2\Delta x} \] ................................ (7)

\[
\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1}^n + T_{i-1}^n - 2T_i^n}{(\Delta x)^2} \] ................................ (8)

To evaluate the conductivity spatial derivative in Eq. (1), the following equation was applied:

\[
\frac{\partial k}{\partial x} = \frac{k_{i+1}^n - k_{i-1}^n}{2\Delta x} \] ................................ (9)

By substituting above difference equations into the heat transfer equation and the initial and boundary conditions, the temperature of the sample at a given time could be determined. The solution was found by developing a computer code in a Mathematica 7.0 program.

4. Results and Discussion

The material considered in the simulation is magnetite derived from magnetite concentrate in Tilden Mine, Michigan. The thermophysical properties of the material and modeling parameters are tabulated in Table 1. (19–22)

4.1. Heating Time

The temperature profiles for different heating time periods ranging from 1 s to 60 s (at 915 MHz) are shown in Fig. 2. The highest temperatures inside the object are around 36°C, 122°C, 294°C, and 767°C for 1 s, 10 s, 30 s, and 60 s, respectively. Temperature in the object increases rapidly with time due to the increase of thermal energy transformed from the microwave irradiation. Continued microwave heating creates non-uniform temperature distribution in the slab. The temperature of slab center (\( L = 0 \) m) stays colder (37°C) after heating for 60 s, giving an indication that the thermal runaway may occur during the microwave heating. Additionally, the surface of the object (\( L = 0.2 \) m, \( L/\Delta x = 400 \)) is found to be the position with the highest temperature in the initial periods (~ 1 s). Longer heating (> 60 s) leads to a temperature peak, which migrates inward with time, as represented in Fig. 3. It is mainly attributed to the effects of microwave heat generation and thermal radiation. In the initial heating, the thermal contribution from microwave generation dominates the temperature rise in the sample and weak thermal radiation effect could be expected due to relatively low temperature of the object. As the heating continues, the temperature of object increases considerably, leading to a high radiation effect. Thus, an obvious temper-
Table 1. Thermophysical properties and modeling parameters used in the simulation.

| Parameter | Value                                      | Units                             |
|-----------|--------------------------------------------|-----------------------------------|
| k         | 3.8558–1.37 × 10⁻³T⁻¹                         | W/K m                            |
| c_p       | 611.84+1.384T**                             | J/kg °C                          |
| p         | 2.800***                                   | Kg/m³                            |
| α         | (0.001377–4.8929 × 10⁻⁷T⁻¹) / (611.84 +1.384 T) | m²/s                             |
| D_p (915 MHz) | 0.0471 – 0.453 × 10⁻⁷T + 4.184 × 10⁻¹⁷T² – 4.845 × 10⁻¹⁷T³ + 1.298 × 10⁻¹⁷T⁴ – 1.366 × 10⁻¹⁷T⁵ + 5.099 × 10⁻¹⁷T⁶   | m                                 |
| D_p (2450 MHz) | 0.0161 – 1.863 × 10⁻⁷T + 2.182 × 10⁻¹⁷T² – 1.603 × 10⁻¹⁷T³ + 0.395 × 10⁻¹⁷T⁴ – 0.405 × 10⁻¹⁷T⁵ + 1.502 × 10⁻¹⁷T⁶   | m                                 |
| h         | 10                                          | W/m² °C                          |
| ε         | 0.96****                                   | None                             |
| T₀         | 25 T                                       | °C                               |
| T∞         | 25 °C                                       | °C                               |

* Value calculated based on the data reported in Ref. 19.
** Value calculated based on the data reported in Ref. 20.
*** Values taken from Ref. 21.
**** Value taken from Ref. 22.

Fig. 2. Temperature distributions in magnetite slab for different microwave heating periods at 915 MHz: a–1 s, b–10 s, c–30 s, and d–60 s. Power: 1 MW/m²; Dimension (L): 0.2 m.

Fig. 3. Temperature distributions in magnetite slab for different microwave heating periods at 915 MHz: a–60 s, b–300 s, c–600 s, and d–1 200 s. Power: 1 MW/m²; Dimension (L): 0.2 m.

Fig. 4. Temperature dependences of magnetite thermal diffusivity (α) and microwave penetration depth (D_p).

Fig. 5. Temperature distributions in magnetite slab under different microwave heating powers at 915 MHz: a–0.5 MW/m², b–1 MW/m², c–2 MW/m², and d–4 MW/m². Heating time: 60 s; Dimension (L): 0.2 m.

A temperature peak is formed inside the object after relatively long heating time. Note that heat diffusion and convection also contribute to the heat transfer in microwave processing of materials. But for the magnetite in this study (actually, for many ceramic materials), their contributions are quite small, especially the heat diffusion. The heat diffusivity (α in Table 1 and Fig. 4) is found to be in the order of 10⁻⁶ m²/s and decreases with increasing temperature.

4.2. Heating Power

The temperature profiles for different microwave powers (P₀) in the range of 0.5 MW/m² to 4 MW/m² are given in Fig. 5. The temperature of the object increases with increasing microwave power. The highest temperatures after microwave heating for 60 s are 278°C, 767°C, 1 047°C, and 1 143°C for 0.5 MW/m², 1 MW/m², 2 MW/m², and 4 MW/m², respectively. It demonstrates that a suitable power applied in microwave heating is crucial to obtain high heating rate in short time. Moreover, it is interesting to note that these highest temperatures locate at different mesh positions (L/Δx): 396, 396, 390, and 385, respectively. It shows the temperature peak shifts to the center of the object with increasing...
power as the contribution of microwave heat generation from higher power to temperature increase becomes even considerable with comparison to heat convection and diffusion. Owing to slower heat diffusion and strong heat radiation to environment at high temperature the peak migrates inward to keep heat balance between the object and surrounding.

4.3. Microwave Frequency

In microwave processing of materials, the dissipation of microwave power in materials highly relies on the microwave frequency. It is known that two frequencies, 915 MHz and 2450 MHz, are commonly assigned for industrial and domestic applications. To evaluate the effect of microwave frequency on temperature distribution in magnetite, the temperature profiles in the object for different microwave heating periods at frequency of 2450 MHz are shown in Fig. 6 to compare with 915 MHz in Fig. 2. The comparison indicates there is negligible temperature difference between 915 MHz and 2450 MHz in the initial heating periods (1 s). As heating time extends to 60 s, the maximum temperature of the object at 2450 MHz is found to be much higher than that at 915 MHz (996°C and 767°C, respectively) and the heating rate is consistent with the experimental data reported in literature.\(^\text{4,5,23}\) The heating rate difference between two frequencies is attributed to the different microwave wave-lengths and microwave absorption properties (permittivity and permeability) of the material at 915 MHz and 2450 MHz. Their effects on the heating can be indicated by the change of microwave penetration depth \(D_p\) in materials:\(^\text{21}\)

\[
D_p = \frac{\lambda_0}{2\sqrt{2\pi}} \left[ \varepsilon_r'\mu_r' - \varepsilon_r\mu_r + \left( \varepsilon_r\mu_r' - \varepsilon_r'\mu_r \right)^2 + \left( \varepsilon_r'\mu'_r + \varepsilon_r''\mu_r' \right)^2 + (\mu_r'\varepsilon_r' + \mu_r''\varepsilon_r'')^2 \right]^{1/2} \quad \cdots (10)
\]

where \(\lambda_0\) is the microwave wavelength in free space; \(\varepsilon_r'\) and \(\varepsilon_r''\) are the real and imaginary parts of complex relative permittivity, respectively; \(\mu_r'\) and \(\mu_r''\) are the real and imaginary parts of complex relative permeability, respectively.

In this simulation, the temperature dependences of microwave penetration depths at 915 MHz and 2450 MHz were determined via cavity perturbation technique, as shown in Table 1 and Fig. 4. The microwave penetration depth at 2450 MHz is found to be much smaller than that at 915 MHz below 500°C, mainly due to their different microwave wavelengths. At higher temperature, the microwave penetration depth is also greatly affected by the permittivity and permeability. The magnetite permittivity increases with temperature, while the permeability decreases apparently around Curie point. Note that the magnitude of permittivity is much larger than that of permeability. Thus, the change of permittivity dominates the variation of microwave penetration depth in magnetite at high temperatures.

The small microwave penetration depth at 2450 MHz results in a quick temperature increase in short time (e.g. < 60 s). This indicates, under the same conditions (power, heating time, object dimension, etc.), most of microwave energy at 2450 MHz would dissipate in the area closer to surface than that at 915 MHz. As heating continues, the temperature of object increases and the radiation effect at the surface of object becomes quite strong. The difference of the highest temperatures between 915 MHz and 2450 MHz decreases, as shown in Fig. 3 and Fig. 7. The highest temperatures at 915 MHz after microwave heating for 60 s, 300 s, 600 s, and 1200 s are 767°C, 1104°C, 1160°C and 1218°C, respectively. At 2450 MHz, the counterparts are 996°C, 1154°C, 1215°C, and 1285°C, respectively. Furthermore, owing to more energy is located in the section close to surface, the temperature inside the object at 2450 MHz is much lower than that at 915 MHz. In other words, in the heating time range studied, temperature distribution at 915 MHz is more uniform than 2450 MHz. Hence, 915 MHz is more suitable for large scale microwave heating of magnetite where maximum temperature uniformity is demanded.

4.4. Object Dimension

Volumetric heating is known as a main advantage of microwave processing of materials due to the propagation behaviors of microwave.\(^\text{24}\) However, this superiority also depends on the object dimension, as demonstrated in Fig. 8. It shows the temperature distributions for slab with different dimensions \(L = 0.2 \text{ m}, 0.15 \text{ m}, 0.1 \text{ m},\) and 0.05 m, respectively) after microwave heating for 60 s at 2450 MHz. As the dimension decreases, the temperature homogeneity in the object is improved. The temperature peak magnitude remains almost constant while its position moves close to the center of the object. The object with dimension \(L\) of 0.05 m under microwave irradiation exhibits better temper-
The temperature distribution inside the object is non-uniform.

(2) In the initial periods, the thermal contribution from microwave generation dominates the temperature rise. As the heating continues, the temperature of object increases considerably, resulting in an apparent radiation effect.

(3) Microwave heat generation and heat radiation from sample surface to environment lead to a temperature peak in the temperature profile, which migrates inward with time.

(4) An optimal microwave power is required to obtain a high temperature increase rate for magnetite in short time.

(5) Microwave heating at 915 MHz exhibits better heat generation and heat radiation from sample surface to environment than the others. This could be clearly demonstrated by the temperature increase at the slab center with decreasing dimension. The temperature at the slab center increases from 25°C to 153°C as the dimension decreases from 0.2 m to 0.05 m. This indicates an optimal dimension of the material is required to obtain the minimum temperature non-uniformity and high heating performance. Also, it should be noted that further reduction of dimension size (e.g. $L = 0.02$ m) would result in apparent standing wave pattern, which may dramatically worsen the heating uniformity.\(^{(1,25)}\)

5. Conclusions

From the results obtained from numerical simulation of the heat transfer of one-dimensional magnetite slab under microwave irradiation with consideration of conduction, convection, and radiation effect, the following conclusions can be drawn:

(1) The temperature distribution inside the object is non-uniform.

(2) An optimal microwave power is required to obtain a high temperature increase rate for magnetite in short time.

(3) Microwave heating and heat radiation from sample surface to environment lead to a temperature peak in the temperature profile, which migrates inward with time.

(4) A reasonable dimension of material is important for minimizing temperature non-uniformity during the microwave heating.

Acknowledgments

The authors wish to express their gratitude to the Michigan Public Service Commission, U.P. Steel, and the United States Department of Energy (DOE) for financial support.

Nomenclature

- $c_p$: specific heat capacity ($\text{J/kg °C}$)
- $D_p$: penetration depth (m)
- $h$: heat transfer coefficient ($\text{W/m}^2\text{°C}$)
- $i$: index of mesh point along x direction (dimensionless)
- $k$: thermal conductivity ($\text{W/K m}$)
- $L$: half slab width (m)
- $m$: number of mesh grid (dimensionless)
- $n$: number of time grid (dimensionless)
- $P_0$: microwave power flux at the surface ($\text{MW/m}^2$)
- $P(x)$: heat generation ($\text{MW/m}^3$)
- $t$: time (s)
- $\Delta t$: time step (s)
- $T$: temperature (°C)
- $T_0$: initial temperature (°C)
- $T_e$: environmental temperature (°C)
- $x$: position (m)
- $\Delta x$: space step (m)
- $\alpha$: thermal diffusivity ($\text{m}^2/\text{s}$)
- $\rho$: density ($\text{kg/m}^3$)
- $\varepsilon$: emissivity (dimensionless)
- $\sigma$: Stefan-Boltzmann constant ($5.6704 \times 10^{-8} \text{ kg s}^{-3} \text{ K}^{-4}$)
- $\varepsilon_r'$: real part of complex relative permittivity (dimensionless)
- $\varepsilon_r''$: imaginary part of complex relative permittivity (dimensionless)
- $\mu_r'$: real part of complex relative permeability (dimensionless)
- $\mu_r''$: imaginary part of complex relative permeability (dimensionless)
- $\lambda$: microwave wavelength in free space (m)

REFERENCES

1) N. Yoshikawa, E. Ishizuka, K. M. ashiko, Y. Chen and S. Taniguchi: ISIJ Int., 47 (2007), 533.
2) D. Malmberg, P. Hahlin and E. Nilsson: ISIJ Int., 47 (2007), 539.
3) Y. Makino: ISIJ Int., 47 (2007), 539.
4) Y. Wang: PhD Dissertation, Michigan Technological University, Michigan, (2005), 110.
5) J. Y. Hwang, X. Huang, S. Yu, Y. Wang, S. Shi and G. Caneda: EPD Cong. Proc., TMS Publications, Warrendale, PA, (2006), 219.
6) Y. C. Ho and K. L. Yen: J. Food Process Pres., 16 (1992), 337.
7) R. Vadivambal and D. S. Jayas: Food Bioprocess Technol., 3 (2010), 161.
8) E. D. Pert, Y. Carmel, A. Birnboim, T. Olorunyolemi, D. Gershon, J. Calame, I. Lloyd and O. Wilson: J. Am. Ceram. Soc., 84 (2001), 1981.
9) J. Dolande and A. Datta: J. Microwave Power E. E., 28 (1993), 58.
10) G. Fleischman: J. Food Eng., 27 (1996), 337.
11) G. Fleischman: J. Food Eng., 40 (1999), 91.
12) L. A. Campanone and N. E. Zaritzky: J. Food Eng., 69 (2005), 359.
13) D. Acierino, A. Barba and M. D’Amore: Heat Mass Transfer, 40 (2004), 413.
14) Y. E. Lin, R. C. Ananthaswaran and V. M. Puri: J. Food Eng., 25 (1995), 85.
15) L. Zhou, V. M. Puri and R. C. Ananthaswaran: J. Food Eng., 25 (1995), 509.
16) K. G. Ayappa, H. T. Davis, E. A. Davis and J. Gordon: AIChe J., 37 (1991), 313.
17) S. Chatterjee, T. Basak and S. Das: J. Food Eng., 79 (2007), 1269.
18) K. G. Ayappa and H. T. Davis: Chem. Eng. Sci., 46 (1991), 1005.
19) J. Molgaard and W. S. Smeltzer: J. Appl. Phys., 42 (1971), 3646.
20) E. Westrum and P. Hahlin: J. Chem. Thermodyn., 1 (1969), 543.
21) Z. Peng, J. Y. Huang, J. Mouria, R. Hutcheou and X. Huang: ISIJ Int., 50 (2010), 1599.
22) A. L. Sprague, T. L. Rouss, R. T. Downs and K. Righter: Icarus, 143 (2000), 409.
23) S. L. McGill, J. W. Walkiewicz and G. A. Smyres: Mat. Res. Symp. Proc., Materials Reserch Society, Warrendale, PA, (1988), 247.
24) Y. Jiang, Y. Zhu and G. Cheng: Cryst. Growth Des., 6 (2006), 2174.
25) H. W. Yang and S. Gunasekaran: J. Food Eng., 64 (2004), 445.

© 2011 ISIJ

888