We study some mesoscopic properties of electron transport by employing one-dimensional chains and Anderson tight-binding model. Principal attention is paid to the resistance of finite-length chains with disordered white-noise potential. We develop a new version of the transfer matrix approach based on the equivalency of a discrete Schrödinger equation and a two-dimensional Hamiltonian map describing a parametric kicked oscillator. In the two limiting cases of ballistic and localized regime we demonstrate how analytical results for the mean resistance and its second moment can be derived directly from the averaging over classical trajectories of the Hamiltonian map. We also discuss the implication of the single-parameter scaling hypothesis to the resistance.

I. INTRODUCTION.

In recent years, the study of one-dimensional (1D) tight-binding models with diagonal disorder has led to new important results. The growing interest in the 1D disordered models is mainly due to two reasons. First, it was found that specific correlations in (random) potentials give rise to delocalized states. Early demonstrations of this effect are related to the so-called dimer model \[1\] that is specified by short-range correlations. It was shown that for particular discrete values of the electron energy the localization length diverges, in contrast to a common belief that any randomness in the potential leads in 1D geometry to an exponential localization of all eigenstates. Recently, delocalized states have been experimentally observed in random dimer superlattice \[2\]. Similar effects of delocalization due to correlated disorder have been also predicted for some special 1D models, see \[3\] and references in \[4\].

Further study of 1D random models with correlated potentials produced even more exciting results. It was shown \[5,6\] that specific long-range correlations may result in an appearance of the mobility edges. This means that a continuous range of energy arises on one side of the mobility edge, where the eigenstates are extended. In Ref. \[6\] the method for constructing such correlated random potentials has been proposed. Using this method, one can relatively easily construct specific random potentials that result in energy bands of a complete transparency for finite samples. The position and the width of such windows of transparency can be controlled by the form of the binary correlator of a weak random potential.

The role of long-range correlations has been studied in details for the tight-binding Anderson-type model \[6\], and for the Kronig-Penney model with randomly distributed amplitudes \[7\] and positions of delta-peaks \[4\]. These results have been also extended to a single-mode waveguide with random surface profiles \[8\]. The predictions of the theory \[6\] have been verified experimentally \[9\], when studying transport properties of a single-mode electromagnetic waveguide with point-like scatterers. The latter were intentionally inserted into the waveguide in a way to provide a random potential with slowly decaying binary correlator. Very recently \[10\] the existence of mobility edges was predicted for waveguides with a finite number of propagating channels (quasi-1D system) with long-range correlations in a random surface scattering potential.

Another reason for the revival of the interest in 1D disordered models is due to the revision of the famous single parameter scaling hypothesis (SPS) \[11\] for transport characteristics of disordered conductors. As is shown in \[12\], the assumption of a random character of fluctuations of the finite-length Lyapunov exponent, originally used \[13\] in order to justify the SPS, turns out to be incorrect. Specifically, it was found that in the vicinity of the band edges the SPS is violated \[12\]. This result is important both from the theoretical and experimental viewpoint (see discussion and references in \[12\]).

One of the effective tools to analyze 1D tight-binding models is the one based on the Hamiltonian map (HM) approach that has been developed in Refs. \[14–16\]. The key point of this approach is a transformation that reduces a discrete 1D Schrödinger equation to the classical two-dimensional Hamiltonian map. The properties of trajectories of this map are related to transport properties of a quantum model. The geometrical aspects of the HM approach turn out to be helpful in qualitative
The canonical variables \(x\) and \(E\) kick depends on the electron energy at the position and momentum of a linear oscillator subjected to a classical Hamiltonian map for canonical variables \(\psi\) diagonalization of a tridiagonal matrix [17] or to a classical Hamiltonian map for canonical variables \(\psi\) potential. It is known that this equation is equivalent to the original equation (1). The effectiveness of the treatment of the map (2), instead of (1), is clearly manifested if the action-angle variables \((r, \theta)\) are introduced according to the following transformation, \(x = r \cos \theta\) and \(p = r \sin \theta\). Then the map Eq. (2) takes the following form,

\[
\sin \theta_{n+1} = D_n^{-1} \sin (\theta_n - \mu) - A_n \sin \theta_n \sin \mu, \\
\cos \theta_{n+1} = D_n^{-1} \cos (\theta_n - \mu) + A_n \sin \theta_n \cos \mu
\]

where

\[
D_n = \frac{r_{n+1}}{r_n} = \sqrt{1 + A_n \sin (2\theta_n) + A_n^2 \sin^2 \theta_n}.
\]

Using action-angle variables (3), (4), the inverse localization length is written in a quite simple form (see details in [16]),

\[
\frac{1}{l_{\infty}^2} \equiv \Lambda = \lim_{L \to \infty} \frac{1}{L} \sum_{n=1}^{L} \ln \frac{\psi_{n+1}}{\psi_{n}} = \lim_{L \to \infty} \frac{1}{L} \sum_{n=1}^{L} \ln \frac{r_{n+1}}{r_n}
= \frac{1}{2} \langle \ln (1 + A_n \sin 2\theta_n + A_n^2 \sin^2 \theta_n) \rangle_n.
\]

Here the brackets \(\langle \ldots \rangle_n\) stand for averaging over \(n\), i.e. along the trajectory of the map (3), (4). It is important to note that the above expression for \(\Lambda\) depends only on the angle \(\theta_n\) and not on the radius \(r_n\). The above relation is correct for the energies \(E\) not very close to the band edges \(E = \pm \mu\) where \(\mu = 0, \pi\). At the band edges, there is additional contribution to the localization length that depends on the ratio \(\sin \theta_{n+1} / \sin \theta_n\) [16]. In this paper we consider the case when the energy \(E\) is inside the allowed band, \(|E| < 2\). Moreover, we also exclude the band center \(E = 0\) where the localization length has a singular behavior that requires specific treatment (see [16] and references therein).

Apart from the abovementioned restrictions, the relation (5) is valid for any potential \(\epsilon_n\). However, analytical treatment of the localization length is possible in the two limit cases of a weak or strong potential. Since in the following we consider the case of weak disorder only, \(|A_n| \ll 1\), let us demonstrate how the localization length can be derived from (5). We specify that the distribution function of site energies \(\epsilon_n\) is flat, \(P(\epsilon_n) = 1/W\), within the region \(|\epsilon_n| \leq W/2\), with the variance \(\langle \epsilon_n^2 \rangle = W^2/12\). Then, for \(W \ll 1\) the logarithm in (5) can be expanded, and by taking into account that in the lowest approximation with disorder the values of \(A_n\) and \(\theta_n\) are statistically independent, one can easily obtain,
For the first time this expression was derived by Thouless [19]. It works quite well over the whole range of energies apart from the band center \( E = 0 \). The analytical treatment of the expression (5) was the main interest for different cases, including correlated disorder. Unlike our previous studies [4,6,7,14,16], in what follows we address a question about the resistance of finite samples of size \( L \). Therefore, we are interested in the properties of the map (2) on the finite time scale, for \( n = 1, \ldots, L \).

Our further consideration is based on the expression for the transmission coefficient \( T_L \) in terms of classical trajectories of the map (2) [15],

\[
T_L = \frac{2}{1 + \frac{1}{2} (r_{1,L}^2 + r_{2,L}^2)}.
\]

Here \( r_{1,L} \) and \( r_{2,L} \) stand for the radii of the two complementary trajectories obtained by iterating the map (3) up to the last site of the sample, \( n = L \). Each of the trajectories is specified by the initial conditions at \( L = 0 \), namely, \( r_{1,0} = r_{2,0} = 1 \), \( \theta_{1,0} = 0 \), and \( \theta_{2,0} = \pi/2 \).

As one can see, all statistical properties of the transmission are entirely determined by the evolution of the two complementary trajectories of the classical map. Some analytical and numerical analysis of the expression (7) have been recently performed in Ref. [20]. The question under study was the statistical distribution of the transmission coefficient \( T_L \). In particular, it was shown that the correlations between the two classical trajectories are different for the ballistic and localized regimes, giving rise to the different distribution functions of \( T_L \). In next sections we address the question of global properties of the resistance \( R_L = T_L^{-1} \), by paying main attention to the mean values \( \langle R_L \rangle \) and \( \langle R_L^2 \rangle \).

III. RESISTANCE.

A. First moment.

According to Eq. (7) the resistance \( R_L = T_L^{-1} \) of a sample of length \( L \) is given by the following formula,

\[
R_L = \frac{1}{2} + \frac{1}{4} r_{1,L}^2 + \frac{1}{4} r_{2,L}^2.
\]

Here the final radii \( r_{i,L}^2 (i = 1, 2) \) are expressed through the kicks amplitudes \( A_n \) and the phases \( \theta_n^{(i)} = \theta_{i,n} \) of the two trajectories. Using Eq. (4) we get,

\[
\langle r_{i,L}^2 \rangle = \prod_{n=0}^{L-1} \left( 1 + A_n^2 \sin^2 \theta_n^{(i)} + A_n \sin 2\theta_n^{(i)} \right).
\]

It is convenient to represent the mean value of \( r_{i,L}^2 \) in the equivalent form

\[
\langle r_{i,L}^2 \rangle = \left\langle \exp \left\{ \sum_{n=0}^{L-1} \ln \left( 1 + A_n^2 \sin^2 \theta_n^{(i)} + A_n \sin 2\theta_n^{(i)} \right) \right\} \right\rangle.
\]

Then, in the limit of weak disorder, \( |A_n| \ll 1 \), one can expand the logarithm in Eq. (10). Separating quadratic and linear terms, we represent \( \langle r_{i,L}^2 \rangle \) in a form of the product of two factors,

\[
\langle r_{i,L}^2 \rangle = \left\langle \exp \left\{ \sum_{n=0}^{L-1} A_n^2 \sin^2 \theta_n^{(i)} - \frac{1}{2} \sum_{n=0}^{L-1} A_n^2 \sin^2 2\theta_n^{(i)} \right\} \right\rangle \times \left\langle \exp \left\{ \sum_{n=0}^{L-1} A_n \sin 2\theta_n^{(i)} \right\} \right\rangle.
\]

The first factor contains the sums \( \sum_{n=0}^{L-1} A_n^2 \sin^2 \phi_n^{(i)} \) where \( \phi_n^{(i)} \) stands for either \( \theta_n^{(i)} \) or \( 2\theta_n^{(i)} \). Since \( L \gg 1 \) these sums are self-averaged quantities, with small variance. Thus, they can be substituted by their mean values,

\[
\lim_{L \to \infty} \sum_{n=0}^{L-1} A_n^2 \sin^2 \phi_n^{(i)} = L \left\langle A_n^2 \right\rangle \left\langle \sin^2 \phi_n^{(i)} \right\rangle = 4\lambda,
\]

with

\[
\lambda = \frac{L \left\langle A_n^2 \right\rangle}{8} = \frac{L}{l_\infty}.
\]

Therefore, the first factor of Eq.(11) takes the form,

\[
\left\langle \exp \left\{ \sum_{n=0}^{L-1} A_n^2 \sin^2 \theta_n^{(i)} - \frac{1}{2} \sum_{n=0}^{L-1} A_n^2 \sin^2 2\theta_n^{(i)} \right\} \right\rangle \approx \exp (2\lambda).
\]

The second factor in Eq.(11) exhibits strong fluctuations. We can calculate its mean value using the distribution function \( P(S_A) \),

\[
\langle f \rangle = \int_{-\infty}^{\infty} f(S_A) P(S_A) dS_A,
\]

where

\[
f(S_A) = \exp(S_A)\text{ with } S_A = \sum_{n=0}^{L-1} A_n \sin \theta_n^{(i)}.
\]

The variable \( S_A \) is a sum of random independent numbers, therefore, according to the central limit theorem \( P(S_A) \) is a Gaussian function with variance \( 4\lambda \). The distribution of \( S_A \) is shown in Fig. 1 and one can see quite good fitting of the numerical data by a Gaussian function. As a result, the second factor of Eq.(11) can be calculated explicitly,

\[
\langle \exp(S_A) \rangle = \frac{1}{\sqrt{8\pi \lambda}} \int_{-\infty}^{\infty} \exp(S_A - 8\lambda^{-1}S_A^2) dS_A = \exp(2\lambda).
\]
that lead to distinct values of conductance and resistance are not self-averaged functions as 1. The only condition that we have used here is the condition of weak disorder.

In the metallic regime, the fluctuations of the transmission coefficient are weak and it is a self-averaged function. Therefore, the mean value of conductance \( \langle G \rangle \) is the same as \( 1/\langle R \rangle \). However, in the localized regime, both the conductance and resistance are not self-averaged functions that lead to distinct values of \( \langle G \rangle \) and \( 1/\langle R \rangle \). Indeed, it is well-known that the mean conductance has a pre-exponential factor \( \sim (L/l_s)^{3/2} \). Derivation of this factor requires some special efforts [18]. This can be understood from Eq. (7) which contains the random radii in the denominator. Unlike this, in the case of resistance (8) these random variables reside in the nominator and strongly simplify the calculations.

**B. Second moment.**

From Eq. (8) the second moment of the resistance can be expressed as follows,

\[
R_L^2 = \frac{1}{4} + \frac{1}{4} R_{1,L}^2 + \frac{1}{4} R_{2,L}^2 + \frac{1}{16} R_{1,L}^4 + \frac{1}{16} R_{2,L}^4 + \frac{1}{8} R_{1,L}^2 R_{2,L}^2.
\]

One can see that apart from the second and fourth moments of \( r_{i,L} \), this expression contains a product \( r_{1,L}^2 r_{2,L}^2 \). The mean value of this term depends on the correlations between the two complementary trajectories of the classical map (3). This fact strongly complicates analytical treatment. In what follows, we consider the two limiting cases of the ballistic and localized regimes separately.

First, let us start with the evaluation of the mean value of the fourth moments \( r_{i,L}^4 \). This can be done by using Eq.(9) in the same way as described above, directly and without restrictions,

\[
\langle r_{i,L}^4 \rangle = \left\langle \exp \left( 2 \sum_{n=0}^{L-1} A_n^2 \sin^2 \theta_n^{(i)} - \sum_{n=0}^{L-1} A_n^2 \sin^2 2\theta_n^{(i)} \right) \right\rangle \\
\times \left\langle \exp \left( 2 \sum_{n=0}^{L-1} A_n \sin 2\theta_n^{(i)} \right) \right\rangle.
\]

Here we have used the weak disorder condition \( A_n^2 \ll 1 \) and have kept only terms up to \( O(A_n^2) \). We have also presented the mean value in the form of the product of two factors, one with a small variance (first factor) and another that reveals strong fluctuations (second factor). Inside the first factor, the sum \( \sum_{n=0}^{L-1} A_n^2 \sin^2 \phi_n^{(i)} \) with \( \phi_n^{(i)} \) as \( \theta_n^{(i)} \) or \( 2\theta_n^{(i)} \), is a self-averaged quantity. Therefore, after substitution by its mean value we get,

\[
\left\langle \exp \left( 2 \sum_{n=0}^{L-1} A_n^2 \sin^2 \theta_n^{(i)} - \sum_{n=0}^{L-1} A_n^2 \sin^2 2\theta_n^{(i)} \right) \right\rangle \approx \exp (4\lambda) .
\]

For the second factor of Eq. (20) we proceed in the same way as we did in the derivation of Eq. (16). We use the fact that the quantity \( S_A = \sum_{n=0}^{L-1} A_n \sin 2\theta_n^{(i)} \) is distributed according to the Gaussian (see demonstration in Fig.1) with the variance \( 4\lambda \). Therefore, the mean value of the second factor in Eq. (20) is given by,

\[
\langle \exp (2S_A) \rangle = \frac{1}{\sqrt{8\pi\lambda}} \int_{-\infty}^{\infty} \exp (2S_A - 8\lambda^{-1} S_A^2) \, dS_A = \exp (8\lambda) .
\]

Substituting Eqs. (21) and (22) into Eq. (20) we obtain the following result for the mean value of \( r_{i,L}^4 \),

\[
\langle r_{i,L}^4 \rangle = \exp (12\lambda) .
\]
\[
\langle r_{1,L}^2 r_{2,L}^2 \rangle = \sum_{n=0}^{L-1} \ln \left( 1 + A_n \sin^2 \theta_n^{(1)} + A_n \sin 2\theta_n^{(1)} \right) \times \sum_{n=0}^{L-1} \ln \left( 1 + A_n \sin^2 \theta_n^{(2)} + A_n \sin 2\theta_n^{(2)} \right) .
\]  

(24)

Using the weak disorder condition we can expand the logarithms and present the correlator \( \langle r_{1,L}^2 r_{2,L}^2 \rangle \) as follows,

\[
\langle r_{1,L}^2 r_{2,L}^2 \rangle = \sum_{n=0}^{L-1} A_n \left( \sin 2\theta_n^{(1)} + \sin 2\theta_n^{(2)} \right) \times \langle A_n^2 \rangle .
\]

(25)

\[
\approx \exp \left\{ L \langle A_n^2 \rangle \right\} \times \langle \exp (S_P) \rangle .
\]

(26)

where

\[ Z_n = \left( \sin^2 \theta_n^{(1)} + \sin^2 \theta_n^{(2)} \right) - \frac{1}{2} \left( \sin^2 2\theta_n^{(1)} + \sin^2 2\theta_n^{(2)} \right) .
\]

(27)

Here we introduced a random variable \( S_P \),

\[ S_P = \sum_{n=0}^{L-1} A_n \left( \sin 2\theta_n^{(1)} + \sin 2\theta_n^{(2)} \right) .
\]

(28)

and splited the mean value \( \langle r_{1,L}^2 r_{2,L}^2 \rangle \) into two factors, one with small variance and another with strong fluctuations respectively, see Eq.(25). Now, we have to take into account the correlations between the phases \( \theta_n^{(1)} \) and \( \theta_n^{(2)} \). In the first factor we have the following self-averaged quantities: \( \sum_{n=0}^{L-1} A_n^2, \sum_{n=0}^{L-1} \left( \sin^2 \theta_n^{(1)} + \sin^2 \theta_n^{(2)} \right), \) and \( \sum_{n=0}^{L-1} \left( \sin^2 2\theta_n^{(1)} + \sin^2 2\theta_n^{(2)} \right). \) The mean value of \( \langle A_n^2 \rangle \) is \( 8/L_\infty \), while the mean values \( \langle \sin^2 \theta_n^{(1)} + \sin^2 \theta_n^{(2)} \rangle \) and \( \langle \sin^2 2\theta_n^{(1)} + \sin^2 2\theta_n^{(2)} \rangle \) are very close to 1. Substituting these mean values in the first factor of Eq.(26) we get,

\[ \langle r_{1,L}^2 r_{2,L}^2 \rangle = \exp (4\lambda) \langle \exp (S_P) \rangle .
\]

(29)

Now, we have to take into account the correlations between the phases \( \theta_n^{(1)} \) and \( \theta_n^{(2)} \). These correlations is the main problem for further analytical treatment. It turns out that the correlations between phases \( \theta_n^{(1)} \) and \( \theta_n^{(2)} \) are very different in the ballistic (\( \lambda < 1 \)) and localized (\( \lambda > 1 \)) regimes.

1. Ballistic regime.

In the ballistic regime the exponential \( \exp S_P \) is close to one, and we expand it up to quadratic terms,

\[ \exp (S_P) \approx 1 + S_2 
+ \frac{1}{2} \left[ \left( \sum_{n=0}^{L-1} A_n^2 \sin^2 2\theta_n^{(1)} \right) + \left( \sum_{n=0}^{L-1} A_n^2 \sin^2 2\theta_n^{(2)} \right) \right] .
\]

(30)

Here the term \( S_2 \) has the following form,

\[ S_2 = \sum_{n=0}^{L-1} A_n^2 \sin \left( 2\theta_n^{(1)} \right) \sin \left( 2\theta_n^{(2)} \right) .
\]

(31)

This term describes the correlations between the phases \( \theta_n^{(1)} \) and \( \theta_n^{(2)} \) of the two classical trajectories that start from complementary initial conditions. It should be stressed that the correlation term keeps track of the change in the correlations along a finite sample, when running from \( n = 0 \) to \( n = L - 1 \). Taking into account that for the white-noise sequence \( \epsilon_n \) the fluctuations of \( A_n \) and \( \theta_n \) are statistically independent [6], we can express \( S_2 \) as follows

\[ S_2 = 8L_\infty \sum_{n=0}^{L-1} R_2(\lambda_n) .
\]

(32)

Here we introduced a binary correlator \( R_2 \),

\[ R_2 = \left\langle \sin \left( 2\theta_n^{(1)} \right) \sin \left( 2\theta_n^{(2)} \right) \right\rangle
\]

(33)

that is expected to depend on the scaling parameter \( \lambda_n = n/L_\infty \) only. Note that the average in Eq.(32) is taken over different realizations of the disordered potential for a fixed value of \( n = 1, ..., L - 1 \). Due to the ergodicity, this average is performed over different realizations of \( \epsilon_n \) after running along the trajectory up to a fixed \( n \).

![FIG. 2. Numerical data for the correlator \( R_2 \) versus scaling parameter \( \lambda_n = n/L_\infty \) shown for the transition from the ballistic to localized regime for \( E = 1.5 \) and \( W = 0.1 \). The average was done over \( 10^4 \) realizations of the disorder with an additional “window moving” average, in order to reduce large fluctuations.](image-url)
A specific character of the correlator $R_2$ is clearly seen from the data reported in Fig. 2. As one can see, the correlator $R_2$ changes from $-1/2$ in the ballistic regime, $\lambda_n \ll 1$, to $1/2$ in the localized regime, $\lambda_n \gg 1$. Taking into account that $\langle \sin^2 \theta_n \rangle = 1/2$, the latter means synchronization of the phases in the localized regime, $\theta_n^{(1)} \approx \theta_n^{(2)}$, see details in [20]. In the ballistic regime the phases are anti-synchronized, $\theta_n^{(1)} \approx -\theta_n^{(2)}$, however, this holds only for very small values of $\lambda$ as one can see from the graph in Fig. 2. This behavior is a manifestation of peculiar statistical correlations between the phases $\theta_n^{(1)}$ and $\theta_n^{(2)}$.

In order to evaluate analytically the correlator $R_2$ in the ballistic regime, we use the approximate map for the phase $\theta_n$ that can be directly obtained from Eq. (3) in the limit of weak disorder,

$$\theta_{n+1} = \theta_n - \mu + \epsilon_n \frac{\sin^2 \theta_n}{\sin \mu}. \quad (34)$$

Iterating this map, one can express an angle $\theta_n$ in terms of the amplitudes $\epsilon_0, \epsilon_1, \ldots, \epsilon_{n-1}$ of all previous kicks for a fixed value of $\mu$ (energy $E$). Taking into account the initial conditions $\theta_0^{(1)} = 0$ and $\theta_0^{(2)} = \pi/2$, the following explicit formulas are derived for the phases,

$$\theta_n^{(1)} \approx -n\mu + \sin^{-1} \mu \sum_{i=0}^{n-1} \epsilon_i \sin^2(i\mu) - \sin^{-2} \mu \sum_{i=0}^{n-1} \epsilon_i \sin^3(i\mu) \cos(i\mu)$$
$$- \sin^{-2} \mu \sum_{i=0}^{n-1} \epsilon_i \epsilon_k \sin^2(i\mu) \sin(2k\mu) + O(\epsilon^3), \quad (35)$$

and

$$\theta_n^{(2)} \approx \frac{\pi}{2} - n\mu + \sin^{-1} \mu \sum_{i=0}^{n-1} \epsilon_i \cos^2(i\mu)$$
$$- \sin^{-2} \mu \sum_{i=0}^{n-1} \epsilon_i \cos^3(i\mu) \sin(i\mu)$$
$$- \sin^{-2} \mu \sum_{i=0}^{n-1} \epsilon_i \epsilon_k \cos^2(i\mu) \sin(2k\mu) + O(\epsilon^3) \quad (36)$$

We substitute the above expansions into Eq. (33) for the correlator $R_2$. Neglecting the contributions of the fast oscillating terms $\sin m (kn\mu)$ with positive integers $k$ and $m$, after some algebra we get,

$$R_2(\lambda_n) = -\frac{1}{2} + 4\lambda_n - 16\lambda^2 + O(\lambda^3). \quad (37)$$

Numerical data in Fig. (2) confirm the validity of this estimate in the ballistic regime, $\lambda_n \ll 1$ (see also [20]). Then the two-point correlator $S_2$ takes the form,

$$S_2 = -4\lambda + 16\lambda^2 + O(\lambda^3). \quad (38)$$

Now we come back to Eqs. (29), (30), and (31). The mean values of the two sums in Eq. (30) can be easily evaluated and for the correlator $\langle r_{1,2,\infty}^2 \rangle$ we obtain,

$$\langle r_{1,2,\infty}^2 \rangle = \left[ 1 + 16\lambda^2 + O(\lambda^3) \right] \exp(4\lambda). \quad (39)$$

Now the second moment of the resistance in the ballistic regime can be expressed as,

$$\langle R^2_2 \rangle = \frac{1}{8} \left[ 2 + (5 + 16\lambda^2) \exp(4\lambda) + \exp(12\lambda) \right]. \quad (40)$$

Since this expression is valid in the quadratic approximation with $\lambda^2$, we expand the exponential terms and obtain the final formula for the second moment of resistance in the ballistic regime,

$$\langle R^2_2 \rangle = 1 + 4\lambda + 16\lambda^2 + O(\lambda^3). \quad (41)$$

There is a good agreement between this analytical result and numerical curve shown in Fig. 3 up to $\lambda \approx 0.3$.

![Fig. 3. Plot of Eq. (41) (solid line) against numerical data (open circles) within the ballistic regime. Numerical data are obtained for $E = 1.5, W = 0.1, L = 4200$ and different lengths $L$, with the average over 50,000 realizations of disorder.](image)

2. Strongly localized regime.

Let us now consider strongly localized regime, $\lambda \gg 1$. As was mentioned, in this regime the phases fluctuate coherently, i.e. $\theta_n^{(1)} \approx \theta_n^{(2)}$. This property helps to evaluate the mean value $\langle S_P \rangle$, see Eq. (28). Using the same procedure that was used for the ballistic regime, we find that in the localized regime the distribution function of random variable $S_P$ is the Gaussian with zero mean value and variance

$$\langle S_P^2 \rangle - \langle S_P \rangle^2 = 16\lambda. \quad (42)$$

Numerical data in Fig. 4 confirm this analytical result.
FIG. 4. Numerical data (solid line) for the distribution of $S_P$ plotted against the Gaussian (dotted line) with the zero mean and variance $\sigma^2 = 16$. The histogram for the distribution of $S_P$ fits well by a Gaussian curve. Numerical data are shown for $E = 1.5, W = 0.1, L = 42000$ and $L/l_\infty = 10$, with additional average over $10^4$ realizations of the random potential.

The mean value of the normally distributed random variable $S_P$ can be easily evaluated,

$$\langle \exp \left( S_P \right) \rangle = \exp \left( 8\lambda \right).$$  \hspace{1cm} (43)

Then, substituting Eq. (43) into Eq. (29), we get the analytical expression for the mean value of the correlator $r_{1L}^2 r_{2L}^2$,

$$\langle r_{1L}^2 r_{2L}^2 \rangle = \exp \left( 12\lambda \right).$$ \hspace{1cm} (44)

In the localized regime this correlator is exponentially large. This means that the trajectory of the map Eq. (2) extends far away from the origin [14]. Now it is easy to show that the variance $R_L^2$ also grows exponentially in the localized regime,

$$\langle R_L^2 \rangle = \frac{1}{4} \exp \left( 12\lambda \right) + \frac{1}{2} \exp \left( 4\lambda \right) + \frac{1}{4}.$$ \hspace{1cm} (45)

Direct numerical computation of the mean value $\langle R_L^2 \rangle$ gives rise to serious problems, since in a strongly localized regime the variance as well as all the moments of $R_L^2$ fluctuate enormously with realization of the disordered potential. This is a reflection of the well known fact that $R_L$ (as well as the conductance $T_L$) is not a self-averaged quantity. Instead, it is more physical to make an average of the logarithm of the moments.

IV. DISCUSSION AND CONCLUDING REMARKS

To the best of our knowledge, there is no systematic study of the statistical properties of the resistance $R_L$ for the tight-binding models. Only brief discussion of the moments of $R_L$ can be found in Ref. [21], where the analysis was done with the use of the "two-scale approach".

In the spirit of the discussion of the single parameter scaling [12] of the conductance, it is also interesting to consider the first and second moments of $\ln(R_L)$. Due to the simple relation $R_L = T_L^{-1}$ between the resistance and conductance, it is clear that in the strongly localized regime, where the SPS is assumed to work well, the results [20] obtained for $T_L$ are also valid for $R_L$. Indeed, since $\ln(R_L) = -\ln(T_L)$, the following relations hold

$$\langle \ln R_L \rangle = 2\lambda; \quad \langle \ln^2 R_L \rangle = 4\lambda + 4\lambda^2,$$ \hspace{1cm} (46)

$$\text{Var} \ (\ln R_L) = \langle \ln^2 R_L \rangle - \langle \ln R_L \rangle^2 = 2 \langle \ln R_L \rangle.$$ \hspace{1cm} (47)

The last formula corresponds to the single parameter scaling for resistance. We remind that the SPS is applied for conductance $T_L$ [12] and results in the relation,

$$\text{Var} \ (\ln T_L) = -2 \langle \ln T_L \rangle.$$ \hspace{1cm} (48)

In conclusion, in this paper we have analytically studied the main properties of the resistance $R_L$ of finite 1D samples with random potentials. Our analysis is based on the Hamiltonian version of the transfer matrix method. The main equation of this method is the Hamiltonian map that replaces the discrete Schrodinger equation. In this way all statistical properties of resistance can be expressed through two complimentary trajectories of this map. Statistical correlations between these trajectories play an essential role and an analytical evaluation of the moments of the resistance is possible in the situations when the binary correlator of phases is known. At the same time, the mean value of $R_L$ is independent of this correlator. Therefore, the expression for $\langle R_L \rangle$ can be found exactly, in the whole range of the scaling parameter, $0 < \lambda = L/l_\infty < \infty$. In contrast, the mean value of the second moment, $\langle R_L^2 \rangle$, depends on the binary correlator $R_2$. Analytical evaluation of this term was found to be possible in the limiting two cases of ballistic and localized regimes. The derived analytical expressions for the resistance and its second moment correspond quite well to the numerical data obtained in a wide range of the model parameters.

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