Expanding quasiperiodicity in soft matter: Supramolecular decagonal quasicrystals by binary giant molecule blends

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The quasiperiodic structures in metal alloys have been known to depend on the existence of icosahedral order in the melt. Among different phases observed in intermetallics, decagonal quasicrystal (DQC) structures have been identified in many glass-forming alloys yet remain inaccessible in bulk-state condensed soft matters. Via annealing the mixture of two giant molecules, the binary systems assemble into an axial DQC superlattice, which is identified comprehensively with meso-atomic accuracy. Analysis indicates that the DQC superlattice is composed of mesoatoms with an unusually broad volume distribution. The interplays of submesoatomic (molecular) and mesoatomic (supramolecular) local packings are found to play a crucial role in not only the formation of the metastable DQC superlattice but also its transition to dodecagonal quasicrystal and Frank–Kasper ε superlattices.

As the formation of quasicrystalline ordering in both metal alloys and soft matters reinforces the scale-invariance principle, however, it remains unknown that why metallic quasicrystals prefer decagonal quasicrystalline (DQC, 10-fold) ordering, and the condensed soft matter systems exhibit dodecagonal quasicrystalline (DDQC, 12-fold) ordering. By tuning the self-sorting ability in a pair of giant molecules, the well-mixed binary blends generate a metastable DQC phase and corresponding unprecedented phase sequence (DQC → DDQC → Frank–Kasper ε) during thermal annealing. These peculiar self-assembling behaviors result from interplays of submesoatomic and mesoatomic packings. Reducing the self-sorting strength has induced an extra complexity in the submesoatomic packings of giant molecules, which further influences the clustering on the mesoatomic level.

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discrete sizes. Depending on the radius ratio of the mesoatom pair, these binary mixtures replicate the metallurgy of MgZn and MgCu alloys. However, at the eutectic composition (i.e., the marginal composition between the intermetallic phase and the primary phase), besides the decomposition into two phases, a peculiar formation of DDQC-SLs was identified. Accordingly, the DDQC-SLs are only formed by mesoatoms with much smaller radius ratio (31, 35), which suggests the failure of self-sorting behavior in this special case. Such failure could be due to the partial miscibility among different giant molecules and enable the formation of various hybrid mesoatomic species, adding another degree of complexity for packings on the submesosatomic level. This observation sparked the interesting question whether further reducing the self-sorting strength can indeed enable the formation of various hybrid mesoatomic species, adding another degree of complexity for packings on the submesosatomic level, leading the formation of unexplored complex structures in condensed soft matters.

Results and Analysis
To tackle this challenge, we designed a pair of multibranched giant molecules—OP8 and OP14 (Fig. 1A)—by tethering eight and 14 aliphatic isoctyl-polyhedral oligomeric silsesquioxane (OPOSS) cages, respectively, to the core multifunctionalized POSS. The final step in the synthesis was the coupling the core and the cages via the copper-catalyzed azide–alkyne [3 + 2] cycloaddition. Differing from the previously reported binary systems (34), these two giant molecules have similar chemical structures in both the core and periphery groups, and only the numbers of periphery OPOSS cages vary. Upon annealing, self-aggregation will take place, forming mesoatoms with aliphatic OPOSS cages located at periphery and aromatic linkers at the core. Annealed separately, OP8 and OP14 self-assemble into the Frank–Kasper σ (FK σ, P4/mmm) phase of different sizes. OP8 has a lattice with \( a = 22.4 \text{ nm}, c = 11.8 \text{ nm} \), and by mesoatoms with average radius \( r_1 = 3.70 \text{ nm} \) (Fig. 1B). OP14 possesses a lattice with \( a = 18.9 \text{ nm}, c = 9.93 \text{ nm} \), and by mesoatoms with average radius \( r_2 = 3.03 \text{ nm} \) (Fig. 1C). These detailed mesoatom structures were further analyzed by molecular dynamic simulation (details in SI Appendix). Interestingly, though OP8 and OP14 have similar chemical structures and molecular radii, they aggregate by different packing modes.

OP8 with fewer branches adopts a double-shell packing (Fig. 1B), inserting several OPOSS cages into the core of the mesoatoms. In contrast, OP14 with more branches can only aggregate into the smaller single-shell mesoatoms (Fig. 1C). The completely different packing mode of OP8 and OP14 drives us to explore the self-assembly behavior in their binary blends and particularly the volume distribution of the resultant mesoatoms.

We prepared freeze-dried OP8/OP14 binary mixtures of different compositions, annealed them, and probed these mixtures in situ by synchrotron small-angle X-ray scattering (SAXS, detailed sample preparations and characterizations, see Materials and Methods). During heating, a special phase transition behavior was captured for the blend OP8/OP14 at mass ratio = 1:2. This specimen exhibits an unknown SAXS pattern at 120 °C (red solid curve in Fig. 2E) with two previously identified DDQC-SL and FK σ structure at higher temperatures following a phase sequence of the unknown phase → DDQC → FK σ (Fig. 2E). With different compositions, a structure diagram (Fig. 1D) has been obtained by probing the assembling structures in situ during heating, and the unknown phase is shown in red area. In the following analysis and further discussions, we focus on the blend of OP8/OP14 at mass ratio = 1:2 to study this unknown structure and refer to this blend as “the sample” if not otherwise specified.

Transmission electron microscopy (TEM) was conducted to resolve this unknown structure (see Materials and Methods and SI Appendix). Fig. 3A shows a unique pattern that demonstrates an overall 10-fold rotational symmetry as indicated by fast Fourier transform (FFT) results and Fourier filtered patterns (Fig. 3 B and C). These results suggest the quasiperiodicity on the TEM imaging plane (we here defined as the x-y plane). Detailed tiling features were further analyzed with the raw TEM image (Fig. 3D). First, the mesoatoms’ positions were assigned to the centers of each dark spots (red dots in Fig. 3D). Then, the Delaunay triangulation was performed based on mesoatoms’ position, presenting the connecting vectors of each pair of neighboring mesoatoms (partially presented in Fig. 3D and SI Appendix, Fig. S9). Finally, these vectors were further summarized as a density heat map present in Fig. 3 E and F. Three sets of characteristic vectors (i.e., \( a, b, \) and \( c \) as indicated in Fig. 3E) distribute evenly with \( 2\pi/10 \) angular intervals, validating the overall 10-fold symmetry. If only delineating the major characteristic vector \( a, \) the quasiperiodic patterns are

![Fig. 1. Self-assembling of giant molecule OP8, OP14, and their binary blends OP8/OP14. (A) Chemical structures and molecular components of OP8 and OP14. (B) Molecular model of OP8 and its hierarchical self-assembling procedure to FK σ superlattice. (C) Molecular model of OP14 and its hierarchical self-assembling procedure to FK σ superlattice. (D) Composition temperature structure diagram of binary blends OP8/OP14 probed by in situ SAXS during the first heating (note: the phase diagram for the final structures after ex situ long-time annealing is shown in SI Appendix, Fig. S7). (E) DQC → DDQC → FK σ structure evolution of the binary blends OP8/OP14 of mass ratio = 1:2 at 120 °C.](https://doi.org/10.1073/pnas.2115304119)
reckoned as a tessellation of fat rhombus and flattened hexagons (Fig. 3A). While the major feature length \( a \approx 8.7 \text{ nm} \) characterize the size length of the tiles, the other two lengths \( b = 10.3 \text{ nm} \) and \( c = 13.5 \text{ nm} \) (\( a^2 + b^2 \)) represent the other two characteristic lengths in these two tiles (Fig. 3 A, Inset). Interestingly, such tessellations highly resemble the axial DQC observed in Mg-Zn-Y alloys (36–38).

To probe its axial periodicity (along the \( z \)-direction), grazing incident small-angle X-ray scattering (GISAXS) was applied on the spin-coated thin film samples. The sample annealed at \( 120^\circ \text{C} \) affords a GISAXS pattern (Fig. 2A, Upper) of three major features: 1) three sets of diffractions peaks are separated by an identical vertical interval \( \Delta q_z = 0.056 \text{ Å}^{-1} \); 2) the axial diffraction \( q_z \sim \Delta q_z \) coincides with the front peak in SAXS of the unknown structure, and it also matches the \( \{00002\} \) peak of DQC and \( \{0002\} \) peak of FK \( \sigma \) along the phase transition; and 3) the diffraction peak of \( \{00001\} \) is extinct (corresponding to the peak at \( q_y = 0, q_x = \Delta q_z \)).

Based on the Mg-Zn-Y DQC model, an ABAC-type mesoatomic decoration was assigned to the rhombic and flat hexagon QC structural unit as presented in Fig. 2C. A super unicell containing 5,000+ mesoatoms was reconstructed following the tilting rules (partially shown in Fig. 2D and SI Appendix, Fig. S10). Direct 3D Fourier transformations were applied to simulate the structure’s diffractions in 3D reciprocal space. Different integration processes were then adopted to afford the simulated GISAXS and TEM FFT results (Fig. 2A and B, and for details, see SI Appendix). The SAXS result was otherwise simulated by specialized powder diffraction simulation programs (Fig. 2E, for details, see SI Appendix). The simulated and experimental diffractions can be properly assigned according to axial DQC model and these simulated results agree well with experimental data (Fig. 2 A, B, and E). These results prove the unknown structure adopts an Mg-Zn-Y-type four-layer DQC-SL.

The phase sequence was further investigated to reveal the thermodynamic property of the associated phases. At \( 120^\circ \text{C} \), the SAXS pattern of DQC has been well developed (Fig. 2E). Raising the temperature to \( 135^\circ \text{C} \), a SAXS pattern of DDQC was recorded. Carefully increasing the temperature to \( 140^\circ \text{C} \) results in a distinctive scattering pattern of an FK \( \sigma \) phase following a sequence of DQC \( \rightarrow \) DDQC \( \rightarrow \) FK \( \sigma \). The specimen reached a disordered state above \( 145^\circ \text{C} \). The sample retained disorder cooling from \( 160^\circ \text{C} \) to room temperature at the rate of \( -30^\circ \text{C}/\text{min} \), and the same phase sequence recurs upon reheating. In addition, when annealed for 1 wk at \( 120^\circ \text{C} \), the DQC gradually evolves into the FK \( \sigma \) phase (Fig. 1E and SI Appendix, Fig. S8). The phase sequence of DQC \( \rightarrow \) DDQC \( \rightarrow \) FK \( \sigma \) during the heating or long-time annealing demonstrates a unique transition of two metastable QC states, which is rarely seen.

**Discussion**

To further investigate the packing details, we reconstruct the Wigner–Seitz (WS) cells for each assembled structure (FK \( \sigma \) by unblended OP8 and unblended OP14; DQC, DDQC, and FK \( \sigma \) by OP8/OP14 at mass ratio = 1:2.). The WS cells can well approximate the sizes and shapes of the mesoatoms in their associated SL (Fig. 4 B–E). Further aided by molecular dynamic (MD) simulations (for details, see SI Appendix), these cells provide valuable insights for the mesoatom formation in various supramolecular structures.

By this method, we first study the mesoatoms in two unblended assemblies (FK \( \sigma \) by OP8 and OP14, individually), and two packing modes were identified. The FK \( \sigma \) structure by OP14 contains WS cells with volumes in between mesoatoms [3OP14] and [4OP14] (Figs. 1C and 4 B, Left), both adopting a single-shell packing mode (i.e., aromatic linkers aggregate inside forming the core, while all aliphatic OPOSS are purged outside forming the shell). The FK \( \sigma \) superlattice of OP8 contains WS cells having similar volumes with mesoatoms [8OP8].
to [10OP8] (Figs. 1B and 4B, Right). Differed from the single-shell OP14 mesoatoms, the less branched OP8 giant molecules can aggregate via a double-shell mode since the single-shell mode for more crowded OP14 suffers great entropic penalty, leaving a few aliphatic OPOSS in the center of the aromatic core as indicated by MD simulation (SI Appendix, Fig. S13). This double-shell packing increases the radius of the mesoatoms and lowers the overall surface volume ratio (for detailed calculations, see SI Appendix) to reduce its overall free energy.

For the binary assemblies, since the chemical structures of OP8 and OP14 are highly alike, OP8 and OP14 tend to mix to a certain degree during their aggregation into mesoatoms. Then the hybrid mesoatoms [xOP8/yOP14] of various sizes will be formed via either single-shell or double-shell packing modes, depending on the detailed hybrid composition. By simulating the stable packing models of mesoatoms [xOP8] and [yOP14] at different compositions (Fig. 4A and SI Appendix, Fig. S13), we identified three mesoatomic volume zones, favoring the single-shell, transitional, and double-shell packing methods (as colored by blue, white, and red zones in Fig. 4B–E, respectively). Within this frame, the mesoatomic compositions in the blending assemblies are further studied: the DQC-SLs demonstrate an unusually wide volume distribution (Fig. 4C) containing both single-shell and double-shell mesoatoms. Moreover, this distribution only overlaps slightly with the unblended OP8 and OP14 FK σ assemblies (Fig. 4B), suggesting the minority of discrete single component [xOP8] or [yOP14] mesoatoms and the majority of hybrid mesoatom [xOP8/yOP14] compositions. Meanwhile, these hybrid [xOP8/yOP14] mesoatoms also possess a volume polydispersity of which is bounded by the volumes between aforementioned single-shell and double-shell mesoatoms. This set of mesoatoms finally constructs the DQC-SL; as it later transforms into the DDQC-SL, the majority of double-shell mesoatoms diminishes, affording the DDQC structure with a narrower mesoatomic volume distribution (Fig. 4D). As the structure evolves into the final stable FK σ phase, the distribution further unifies (Fig. 4E).

As the emergence of QC ordering suggest the existence of nonequilibrium but kinetically facilitated particles clustering in both condensed soft matters (31) and metallic alloys (29, 39), additional kinetically favored packing from the submesoatomic level has been spotted in our case—namely, those later-diminished larger double-shell mesoatoms. Furthermore, according to the valuable insights from other soft matter systems, the interplays of local packings on both submesoatomic and mesoatomic level are critical for these intriguing assembling behaviors in our case (Fig. 4F): 1) Starting from the amorphous binary blends, the local configurations of the mixed giant molecules will transit to either single-shell or double-shell mesoatoms upon annealing. 2) Though the double-shell mesoatoms are in a nonequilibrium state, for certain local configurations, they are kinetically more favored within the reach of minimum rearrangement (40). Thus, these double-shell mesoatoms form once annealed at low temperature and persist because of the limited diffusion of the giant molecules (41, 42). 3) With the emergence of mesoatoms of such broad volume distribution, the DQC lattice with 10-fold symmetry becomes the optimized structure to include all the mesoatoms via tetragonally close packing with minimum rearrangement. 4) When annealed for a longer time or at higher temperature, giant molecules within double-shell mesoatoms gradually redistribute, forming smaller...
but more stable single-shell species (Fig. 4F, Middle). This redistribution on a submesoatomic level further change the packings on the mesoatomic level, affording the characteristic “triangle square” tiling elements for DDQC and FK σ structures (23). Inherited from the random tiling in DQC-SLs, the random distribution of these “triangle square” tiling elements leads to a metastable DDQC-SL (20, 43). 5) When annealed for an even longer time, the DDQC structure gradually reforms into its crystalline approximant—the FK σ structures—potentially reaching its global minimum. The details of the redistribution dynamic between single-shell and double-shell mesoatoms remain to be explored.

As for the question why DQC is seldom observed in conformed soft matters and DDQC is rare for metal alloys, we would share some inspiring thoughts based on the deep analysis of the aforementioned DQC system in binary giant molecule blends. The formation of DQC-SLs usually requires a wider volume distribution of the constructing mesoatoms, which is hard to achieve for a “single-component” supramolecular assembly. However, at the atomic scale, similar criterion can be easily fulfilled by multielement metallic alloys (12). Therefore, DQC structures are seldom seen in soft matters but widely exist in alloys. On the contrary, 12-fold DDQC structures possess a relatively narrower volume distribution (nearly identical with FK σ structures). For the metallic atoms, they can easily nucleate into the stable FK σ (or other crystalline approximants) crystals instead of being trapped in metastable DDQC state (16). For supramolecular assemblies in soft matter, since rearranging the mesoatoms needs either higher kinetic energy or longer time, less stable local configurations can preserve at more general conditions (44). Thus, DDQC structures are more frequently observed in condensed soft matters and less in alloys.

**Conclusion**

The discovery of DQC structures and its intriguing transition (i.e., DQC → DDQC → FK σ) in our binary giant molecule blends is unprecedented. Complex interplays of packings on two hierarchical levels—the submesoatomic level and mesoatomic level—are found to play the central role in the peculiar assembling behaviors. In our case, reducing the self-sorting strength in the binary giant molecule blends has introduced an extra degree of complexity on submesoatomic packings. The resultant mesoatoms with a broad volume distribution induce unusual local packings on mesoatomic level, affording the DQC-SL. Moreover, with the redistributions of giant molecules, packings on both submesoatomic and mesoatomic level change simultaneously, leading the transitions toward the stable FK σ structure via an intermediate DDQC state. While such interplays might widely exist in various hierarchical assemblies of soft matters, these specifically designed giant molecules have offered us valuable insight to the hidden complexity behind different hierarchical levels.

**Materials and Methods**

**General Methods and Equipment.** 1H and 13C experiments were measured on a Varian Mercury 500 M NMR or 300 M NMR spectrometer. Gel permeation chromatography was measured in tetrahydrofuran at 35 °C on Tosoh EcoSEC instrument with three columns (TSKgel SuperH3000 × 2, TSKgel SuperH5000), a Guard Column (TSKgel SuperH1000-4000), and an ultraviolet detector. A JEM-1230 TEM with an accelerating voltage of 120 kV was utilized to record the bright field images of thin sections of different samples. Fast Fourier transformation results and FFT-filtered images were performed on ImageJ software. The density of the samples was measured by the floatation method. The experimental measured density of these phases formed by the giant shape amphiphiles is \( \rho = 1.07 \) (g/cm³). SAXS and GISAXS were collected at beamline BL16B1 of the Shanghai Synchrotron Radiation Facility (SSRF). The one-dimensional circular integrations of 2D SAXS patterns were performed by the Igor Pro-6.37 (with Nika 2D SAS macros) software package. The GISAXS results were analyzed by GIXSGUI (45). As the identical methods and settings were also used in several of our previous reports, more detailed descriptions are documented in ref. 34.

**Giant Molecular Monomer Sample Preparation.** The detailed synthetic procedures are recorded in detail in SI Appendix. All pure giant molecule product was transferred in to separate vials and dried under vacuum before further operation.
Giant Molecular Blends Preparation. Both the pure OP8 and OP14 were transferred into separate vials and dried under vacuum before further operation. Equal mass (20 mg) of each giant molecule was separately dissolved in 7.5 mL methylene chloride, and then, the calculated volume of stock solution (for example 100, 200...900 μL) was taken and mixed to afford target compositions. The mixtures were dried under vacuum, redissolved in benzene (0.5 mL), and further freeze dried to afford sticky white solids.

SAXS sample. A small piece of blended sample was placed into an aluminum sample holder and sealed by the Kapton tape. For ex situ SAXS, a Mettler Toledo FPHT hot stage was used for sample annealing. All thermal annealing process were performed under the nitrogen atmosphere. The samples of different blending mass ratios were annealed at target temperatures for 12 h, and at least three pieces of samples were annealed at the same time to confirm the reproducibility. For in situ SAXS results were obtained by annealing the specimen in line at different temperatures. At each targeted temperature, the specimen was measured at least three times with 5-min intervals until no change was detected in diffraction patterns.

TEM Sample Preparation. The samples for TEM experiments were prepared by dropping 5 μL sample solution (1 mg/mL in dichloromethane) onto carbon-coated copper grids. After the solvent evaporation, the copper grids were annealed at 120 °C for 3 h, then cooled down to room temperature. The annealed samples were further stained by RuO4.

GISAXS Sample Preparation and Characterization. The samples for GISAXS experiments were prepared by spin coating the sample solution (10 mg/mL in dichloromethane) onto a clean silicon wafer. After the solvent evaporation, the specimen was then annealed at 120 °C for 3 h, then cooled down to room temperature. The annealed samples were directly characterized by GISAXS.

Data Availability. SAXS, GISAXS, TEM and diffraction simulation data are publicly available at http://dx.doi.org/10.17605/OSF.IO/ES43X. All other study data are included in the article and/or SI Appendix.

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