Blockwise Phase Rotation-Aided Analog Transmit Beamforming for 5G mmWave Systems

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Abstract—In this letter, we propose a blockwise phase rotation-aided analog transmit beamforming (BPR-ATB) scheme to improve the spectral efficiency and the bit-error-rate (BER) performance in millimeter wave (mmWave) communication systems. Due to the phase angle optimization issues of the conventional analog beamforming, we design the BPR-ATB for reducing the rotated beamspace of the equivalent channel and improving the minimum Euclidean distance. To verify the effectiveness of the proposed BPR-ATB scheme, we employ an Alamouti coding technique at the transmitter and evaluate the bit-error-rate performance for mmWave multiple-input and single-output systems. The simulation results show that the proposed BPR-ATB scheme outperforms the conventional discrete Fourier transform-based ATB scheme.

Index Terms—5G-millimeter-wave systems, blockwise phase rotation, Alamouti coding, spectral, and BER performance.

I. INTRODUCTION

The millimeter-wave (mmWave) technology plays a major role in the fifth-generation (5G) wireless communications owing to the large bandwidth [1] and spectral efficiency [2]. The mmWave technology operates in the 30 to 300 GHz band [1], [2], hence the large spectral resource in contrast with microwave technologies operating in the sub 6 GHz band [4]. Typically, mmWave system requires massive antenna arrays, which are equipped with the base station (BS) for achieving a highly directive beamforming [5]. For deployment of this system, the leading barriers are the hardware limitations, the channel sparsity, the free-space path loss, beamforming construction, and phase angle optimization.

The sparse nature of the channel and the discrete Fourier transform-based analog beamforming (DFT-ATB) schemes have been investigated in [1], [3], [5–9]. The authors designed a joint antenna selection based transmit beamforming in [7]. A phase control DFT based hybrid precoding scheme is presented in [8], [9]. Particularly, the traditional analog beamforming incurs a quantization error in communication systems owing to their low minimum Euclidean distance [4], [8], [10]. In addition, the conventional DFT-ATB scheme shows a ‘beam squint’ challenge with a wideband channel [6], [11]. The ‘beam squint’ leads a higher channel spreading factor due to the structural leakage of the conventional analog beamforming. To get a better minimum Euclidean distance of analog precoding, the authors proposed a Golden-Hadamard (GH) based precoding in [12]. Although the GH scheme achieved a remarkable bit-error-rate (BER) performance in microwave systems, the scheme shows a phase angle optimization problem in highly directive wireless systems due to their wide rotated beamspace. Hence, we design a blockwise phase rotation-aided analog transmit beamforming (BPR-ATB) for mmWave communication systems.

In this article, we propose a BPR-ATB scheme to minimize the rotated beamspace of the equivalent channel and improve the minimum Euclidean distance of traditional analog beamforming such as the DFT-ATB scheme. To this end, we seek to obtain an efficient rotated beamspace of the equivalent channel and improve the minimum Euclidean distance. We first run back [12, eq. (9)] and then design a BPR-ATB scheme to get the effective rotated beamspace and generate a satisfactory spectral efficiency of the mmWave communications. After that, we implement the proposed BPR-ATB scheme with Alamouti code and set a power factor-based parameter $\kappa$ in the BER performance metric. Finally, we show the superiority of the proposed BPR-AB scheme over the DFT-ATB scheme in terms of a downlink mmWave multiple-input and single-output (MISO) systems through computer simulations.

II. CHANNEL AND SIGNAL MODELS

We consider a downlink mmWave MISO system with $N_t$ transmit antennas and a single antenna receiver. Then the received signal vector $y \in \mathbb{C}^{1 \times T}$ can be modeled as

$$y = \sqrt{\frac{PN_t}{L}}h^Hx + z,$$

where $P$ denotes the transmit power, $L$ is the number of paths, $X$ is the $N_t \times T$ space-time codeword matrix, $T$ is the number of time slot, and $z \sim \mathcal{CN}(0, 1)$ is additive white Gaussian noise vector with zero-mean and unit variance. The narrow-band mmWave channel $h \in \mathbb{C}^{N_t \times 1}$ with $L$ propagation paths [1, 5], that is

$$h = \sum_{l=1}^{L} \alpha_l \mathbf{a}(\theta_l),$$

where $\alpha_l$ is the complex gain of the $l$-th path, $\theta_l$ represents the angle of departure of the $l$-th path, $\mathbf{a}(\theta_l)$ denotes the transmit steering vector of the $l$-th path, which is given by

$$\mathbf{a}(\theta_l) = [1, e^{j\frac{2\pi}{\lambda} \sin \theta_l}, ..., e^{j\frac{2\pi}{\lambda} \frac{d}{\sin \theta_l}}]^T,$$

the wavelength, $\lambda = c/f_c$, $c$ is the speed of light, $f_c$ is the carrier frequency, and $d = \lambda/2$ is the antenna spacing.
III. BLOCKWISE PHASE ROTATION-AIDED ANALOG TRANSMIT BEAMFORMING (BPR-ATB) SCHEME

A. BPR-ATB matrix design

Due to the phase angle optimization and beam squint issues in the high dimensional ATB scheme, we first run back [12, eq. (9)] and then we design the Golden Hadamard based BPR-ATB scheme in this section. Let the number of total transmit antennas \( N_t = 2^q \) and \( \xi = n \left\{ (1 + n)^q - (1 - n)^q \right\} / 2^q \) where \( q = \log_2 N_t \) and \( n \) denotes the root of the geometric number. Consider the space-time codeword matrix \( X \) as

\[
X = F_{2^q} S,
\]

where \( S \) is a \( 2^{q-1} \times T \) orthogonal space time block code matrix, \( F_{2^q} \) be a propose \( 2^q \times 2^{q-1} \) BPR-ATB matrix constructed by \( 2^{q-1} \) columns of a \( 2^q \times 2^q \) recursive Golden-Hadamard matrices as follows

\[
F_{2^q} = \frac{g}{\sqrt{\xi}} \begin{bmatrix}
W_{2^q-1} A_{2^q-1} & W_{2^q-1} B_{2^q-1} \\
W_{2^q-2} A_{2^q-1} & -W_{2^q-2} A_{2^q-1}
\end{bmatrix},
\]

where \( g \) denotes the golden number [12, 13]. \( W_{2^q-1} \) is a \( 2^{q-1} \times 2^{q-1} \) block Hadamard matrix, \( A_{2^q-1} = \text{diag}(e^{j \theta_{1}}) \) and \( B_{2^q-1} = \text{diag}(e^{j \theta_{2}}) \) are the \( 2^{q-1} \times 2^{q-1} \) block-diagonal phase rotation matrix, \( \nu_1 \) and \( \nu_2 \) are the block-order of \( A_{2^q-1} \) and \( B_{2^q-1} \). By substituting (3) in (11), the system can achieve a spectral efficiency for MISO system given as

\[
R = \log_2 \left\{ 1 + \frac{P}{\sigma^2} H^H F_{2^q} F_{2^q}^H h \right\}.
\]

B. Problem formulation

Particularly, the phase rotation on the transmitted signals is effectively equivalent to rotating the phases of the corresponding channel coefficients. It should be noted that, the conventional DFT-ATB scheme generates a satisfactory array gain with equivalent channel [7, 8], but this scheme suffers a phase angle optimization problem, which leads to a wide beamspace of the equivalent channel as shown in Fig. 1. Using by (6) and (7), the optimization problem can be formulated

\[
f_{2^q}^{opt} = \arg \max_{f_{2^q}} R_{f_{2^q}},
\]

s.t. \( f_{2^q} \in \{ F, 0 \}^{2^q} \),

\[
(f_{2^q})_n = 0, \quad n = 0, 1, \ldots, 2^q - 1,
\]

where \( f_{2^q} \) denotes the vector of \( F_{2^q} \). We observe that the (7) maximizes the spectral efficiency but it has a non-convex objective function, which conducts a phase angle optimization problem. In addition, the spectral efficiency (6) is a correctly monotone enhancing function of beamforming gain \( |h F_{2^q}| \). To simplify this problem, we formulate the phase angle optimization as below:

Let \( h_{\nu} \) be the \( \nu \)-th element of \( h \) and \( f_{2^q, \nu} \in F \) be the effective analog beamforming vector. Thus, the phase angle optimization problem is given by

\[
\phi_{\nu}^{opt} = \arg \max_{\phi_{\nu}} \left| \sum_{\nu=1}^{2^q} h_{\nu}^{*} f_{2^q, \nu}(\phi_{\nu}) \right|
\]

s.t. \( \phi_{\nu} \in \left\{ \frac{2 \pi b}{2^q} \left| b = 0, 1, \ldots, 2^q - 1 \right\} \right. \),

\[
\nu = 1, \ldots, 2^q,
\]

where \( f_{\nu}(\phi_{\nu}) = e^{j \phi_{\nu}} / \sqrt{2^q} \). We observe the (8) is still leading an optimization problem due to the global phase angle, which generates an extensive beamspace with a mmWave channel. As a result, the user suffers from a high computational burden to optimize the global phase angle. To overcome the phase angle optimization problem, we reformulate (8) and propose the BPR-ATB based algorithm in the Subsection C.

C. Proposed BPR-ATB based algorithm

Let \( \phi_{\nu} \in \{ \phi_{\nu_1}, \phi_{\nu_2} \} \), where \( \phi_{\nu_1} \) and \( \phi_{\nu_2} \) are the block phase angle of \( A_{2^q-1} \) and \( B_{2^q-1} \). We set \( \phi_{\nu_1} \) and \( \phi_{\nu_2} \) in the designed transmit beamformer of \( F_{2^q} \). Then the optimal block phase angle is given by

\[
\phi_{\nu}(\phi_{\nu_1}^{opt}, \phi_{\nu_2}^{opt}) = \arg \max_{\phi_{\nu_1}, \phi_{\nu_2}} \left| \sum_{\nu=1}^{2^q} h_{\nu}^{*} e^{j \phi_{\nu_1}} e^{j \phi_{\nu_2}} \right|
\]

s.t. \( \phi_{\nu_1} \in \left\{ \frac{2 \pi b_1}{2^q-1} \left| b_1 = 0, 1, \ldots, 2^q - 1 \right\} \right. \),

\[
\phi_{\nu_2} \in \left\{ \frac{2 \pi b_2}{2^q-1} \left| b_2 = 2^q - 1, \ldots, 2^q - 1 \right\} \right. \),

\[
\nu_1 = \nu_2 \quad \text{and} \quad b_1 \neq b_2.
\]

Consider \( C \) as the set of indexes of useful antennas and \( \nu \in \{ S_{\nu_1}, S_{\nu_2} \} \), where \( S_{\nu_1} \) and \( S_{\nu_2} \) are the subset of beam indices. Based on (9), we demonstrate the proposed BPR-ATB scheme in Algorithm 1.

![Fig. 1. The rose diagram of the rotated beamspace for the effective equivalent mmWave channel (a) the conventional DFT-ATB scheme [7, 8] in solid lines, (b) and (c) the effective equivalent mmWave channel with the proposed BPR-ATB scheme in dotted lines.](image-url)
Algorithm 1 Proposed BPR-ATB based Algorithm

1: **Input parameters:** $h, b_1, b_2, q$.
2: **Output:** $f_{opt}^v, v$.
3: Obtain $\varphi_\nu(\phi_{v1}^opt, \phi_{v2}^opt)$.
4: Begin $C := \{\nu\}, \nu \in \{S_{v1}, S_{v2}\}$, and $\nu_{v1} = \nu_{v2} = \emptyset$.
5: for $\nu_{v1} = 1 : 2^{q-1}$ do
6: Find $\nu_{opt1} = \arg\max_{\nu \in C} |\sum_{\nu_1 \in S_{v1}} (h_{\nu_1}^* + h_{\nu_1}^* e^{j\phi_{v2}}) e^{j\phi_{v1}}|$.
7: $\nu_{v1} := \nu_{v1} \cup \{\nu_{opt1}\}$.
8: $C := C \mod \{\nu_{opt1}\}$.
9: end for
10: for $\nu_{v2} = 1 : 2^{q-1}$ do
11: Find $\nu_{opt2} = \arg\max_{\nu \in C} |\sum_{\nu_2 \in S_{v2}} (h_{\nu_1}^* + h_{\nu_1}^* e^{j\phi_{v2}}) e^{j\phi_{v2}}|$.
12: $\nu_{v2} := \nu_{v2} \cup \{\nu_{opt2}\}$.
13: $C := C \mod \{\nu_{opt2}\}$.
14: end for
15: Obtain $f_{opt}^v = (\Im e^{j\phi_{v1}} e^{j\phi_{v2}}) / \sqrt{\zeta}$ according to (9).

where $M$ denotes the constellation size, the operator $Q(\cdot)$ represents the Q-function [16], $\Xi_{k,l} = \|h_{eq}^f e_{k,l}\|_F$ where the operator $\| \|_F$ denotes a Frobenius norm, $e_{k,l} = S_k - S_l$ represents an error matrix between the codewords $S_k$ and $S_l$, $e(S_k, S_l)$ is the Hamming distance between the bit mappings corresponding to the vectors $S_k$ and $S_l$. Invoking the Chernoff upper bound $Q(x) \leq e^{-x^2/2}$ in (11), the pairwise error probability can be upper bounded with an equivalent channel $h_{eq}$ as

$$\Pr(S_k \rightarrow S_l|h_{eq}) \leq e^{-\gamma_0 \Xi_{k,l}^2 / 4},$$

where $h_{eq} = F^H h$, and $\Xi_{k,l}^2$ is the minimum Euclidean distance as

$$\Xi_{k,l}^2 = \arg\min_{S_k \neq S_l} \|h^H F_{2v} e_{k,l}\|_F,$$

where the property of the error matrix $e_{k,l}^H = aI$ for the orthogonal space-time block coding, $a$ is a constant depending on the constellation [see in APPENDIX A].

Throughout the simulations, we assumed the parameters of Table II with different $\kappa$ factor of Table I. In the case of mmWave system, we use a sparse geometric mmWave channel model [8] with $L = 3$ paths and angles of departure uniformly distributed over $[-\pi/2, \pi/2]$. Since the exhaustive search method (8) obviously measures a high complexity, as a result, we designed a sub-optimal BPR-ATB algorithm in Algorithm 1. The total time complexity of the Algorithm 1 is $O(2^{q}\zeta)$, which is adoptable even if $q$ is large and $2^q < \zeta$. We further consider the $2 \times 2$ block-diagonal rotation matrices with

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we compare the proposed BPR-ATB scheme against the conventional DFT-ATB scheme via computer simulations. To show the superiority of the proposed scheme, we employ a complex Alamouti coding technique at the transmitter. For example, if we use the $k$-th entry of a complex alamouti code with time slot $T = 2$ in (4) where we consider $T$ is equal to the number of radio frequency chain, then the codeword matrix $X$ is given by

$$X = \sqrt{\gamma_0 \kappa} F S_k = \sqrt{\gamma_0 \kappa} F \begin{bmatrix} s_{11} & -s_{21}^* \\ s_{21} & s_{11}^* \end{bmatrix},$$

where $S_k$ is the $k$-th entry of $2 \times 2$ complex Alamouti code [14], symbols $s_{11}$ and $s_{21}$ belongs to a quadrature amplitude modulation (QAM) constellation, $\kappa = |F_{i,j}|^2$ is a power factor of the $(i,j)$ element of analog beamforming $F$, and $\gamma_0$ is the received signal-to-noise power (SNR). The $\kappa$ value depends on the structure of $F$ matrix (see in Table I).

Consider $S_k$ and $S_l$ as the transmitted and decoded space-time codewords, respectively, where $k \neq l$. The union bound on the bit-error-rate (BER) is formulated as [16]

$$BER \leq \sum_{k \neq l} \frac{e(S_k, S_l)}{\log_2(M)} Q\left(\Xi_{k,l} \sqrt{\frac{\gamma_0 \kappa}{2}}\right),$$

where $M$ denotes the constellation size, the operator $Q(\cdot)$ represents the Q-function [16], $\Xi_{k,l} = \|h_{eq}^f e_{k,l}\|_F$ where the operator $\| \|_F$ denotes a Frobenius norm, $e_{k,l} = S_k - S_l$ represents an error matrix between the codewords $S_k$ and $S_l$, $e(S_k, S_l)$ is the Hamming distance between the bit mappings corresponding to the vectors $S_k$ and $S_l$. Invoking the Chernoff upper bound $Q(x) \leq e^{-x^2/2}$ in (11), the pairwise error probability can be upper bounded with an equivalent channel $h_{eq}$ as

$$\Pr(S_k \rightarrow S_l|h_{eq}) \leq e^{-\gamma_0 \Xi_{k,l}^2 / 4},$$

where $h_{eq} = F^H h$, and $\Xi_{k,l}^2$ is the minimum Euclidean distance as

$$\Xi_{k,l}^2 = \arg\min_{S_k \neq S_l} \|h^H F_{2v} e_{k,l}\|_F,$$

where the property of the error matrix $e_{k,l}^H = aI$ for the orthogonal space-time block coding, $a$ is a constant depending on the constellation [see in APPENDIX A].

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TABLE II
SIMULATION PARAMETERS

| Total number of transmit antennas | $N_t = 4$ |
| Total number of receive antenna | $N_r = 1$ |
| Number of time slot | $T = 2$ |
| Channel path | $L = 3$ |
| SNR | $\gamma_0 = 20$ dB |
| Carrier frequency | $f_c = 60$ GHz |
| Wavelength | $\lambda = 5$ mm |
| Antenna spacing distance | $d = \lambda/2$ |
| Modulation scheme | 64 QAM |

TABLE I
DIFFERENT POWER FACTOR OF RF PRECODING

| Power factor of RF precoding, $\kappa = |F_{i,j}|^2$ | DFT [4] | HA [15] | BPR |
|---|---|---|---|
| $1$ | $1$ | $\frac{(1 + \sqrt{5})^2}{4\xi}$ | $\frac{(j + \sqrt{3})^2}{4\xi}$ |

![Fig. 2. Spectral Efficiency versus SNR ($N_t = 4, N_{RF} = T = 2$, and $N_r = 1$ with different $\kappa$ values of the analog transmit beamforming.](image-url)
The proposed BPR-ATB scheme also achieved good spectral efficiency and BER performance in [4], [7], [8] compared with the traditional MISO random channel environment as shown in Fig. 2.

Fig. 3 shows the BER performance of the 4 × 2 BPR-ATB scheme using 2 × 2 Alamouti coding and 64QAM constellations. We see that the DFT-ATB scheme achieves a worse spectral efficiency and BER performance in [4], [7], [8] for both mmWave and the familiar microwave MISO systems owing to their rotated wide beamspace (see in Fig.1) and a teeny κ factor (see in Table I). The proposed BPR-ATB scheme provides at least 2 dB more BER performance with the DFT-ATB scheme. Furthermore, both spectral efficiency and BER performance of a complex case of the proposed BPR-ATB scheme is slightly worsened than that of a real case of the proposed BPR-ATB scheme because of the reduction of the value of the Golden ratio.

V. CONCLUSIONS

We proposed a BPR-ATB scheme to minimize the rotated beamspace of the equivalent mmWave channel and improve the error performance of the systems. We verify the effectiveness of the proposed BPR-ATB scheme by computer simulation and compare the performance with the conventional DFT-ATB scheme. The traditional DFT-ATB scheme exhibited a worse spectral efficiency about 1.8 bits/s/Hz and BER performance difference of 2 dB when compared with the proposed BPR-ATB scheme. Hence, the proposed BPR-ATB scheme can be extended further to the next generation multiple-input and multiple-output non-orthogonal multiple-access (MIMO-NOMA) systems, which will be explored in future studies.

APPENDIX A

From (11), we can measure as a general single-input and single-output case of BER as follows:

$$P_{e,b}(a, \gamma, M) = \frac{(M-1)}{M} \sqrt{\frac{\mu}{2}} \tan^{-1} \left( \sqrt{\mu \cot \frac{\pi}{M}} \right),$$

(18)

where $\mu = (\gamma \sin^2 \frac{\pi}{M})/(1 + \gamma \sin^2 \frac{\pi}{M})$. Similarly, using (14) and (16), we can measure the BER probability for M-ary QAM modulation as

$$P_{e,b}(a, \gamma, M) = 4\zeta^2 \int_0^{\frac{\pi}{2}} \left( \frac{3\gamma}{2(M-1)\sin^2 \theta} \right)^{-1} d\theta,$$

(19)

where $a^2 = 3/(M-1)$ and $\zeta = 1 - (1/\sqrt{M})$. 

It is noted that in (14), $M_X(-s) = \int_0^\infty e^{-sx}p_X(x)dx$ is a moment generating function (MGF) of random variable $X$ where $M_X(s) = \int_0^\infty e^{sx}p_X(x)dx$ is the Laplace transform of $p_X(x)$ with the exponent reversed sign. By using the Rayleigh channel, we can consider $p_X(x) = \exp(-x/\gamma)/\gamma$, where $\gamma \geq 0$ and $\gamma$ denotes the average SNR per bit. Hence, the Laplace transform is given by

$$M_X(-s) = \frac{1}{1+s\gamma}, s > 0.$$
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