Nambu mass hierarchies in low energy string models

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Abstract

This paper explores a recent idea of Nambu to generate hierarchies among Yukawa couplings in the context of effective supergravity and superstrings models. The Yukawa couplings are homogeneous functions of the moduli and a geometrical constraint between them with a crucial role in the Nambu mechanism is found in a class of models of no-scale type. The Yukawas are dynamical variables at low energy to be determined by a minimization process.

1. Nambu mass hierarchies

A mystery of the Standard Model is the difference between the mass of the top quark and the mass of the other fermions. The top quark mass is roughly of the order the electroweak scale $v \approx 250\text{GeV}$, whereas in a first approximation all the other fermions are massless. No definite solution of this puzzle was found and so new ideas are necessary.

An interesting idea was recently proposed by Nambu [1]. The Yukawa couplings are regarded as dynamical variables to be determined by minimizing the vacuum energy density. All the other parameters are held fixed, including the vev’s of the scalar fields. If $\Lambda$ is the cut-off and $\mu$ is a typical mass of the theory, the vacuum energy density can be written as

$$<V>=V^{(0)}\Lambda^4+V^{(2)}\Lambda^2+V^{(4)}\ell\frac{\Lambda}{\mu}.$$ (1)

Nambu imposes the vanishing of quartic $V^{(0)}$ and quadratic $V^{(2)}$ divergences. The first condition is automatic in supersymmetric theories and the second gives a constraint between the Yukawas known as the Veltman condition.

In the example chosen by Nambu, for two couplings $\lambda_1$ and $\lambda_2$, the vacuum energy reads

$$<V>=-A(\lambda_1^4+\lambda_2^4)+B(\lambda_1^2\ell n\lambda_1^2+\lambda_2^2\ell n\lambda_2^2)$$ (2)

and the Veltman condition is

$$\lambda_1^2+\lambda_2^2=a^2.$$ (3)

A picture in the case $B=0$ is shown in Fig.1. The shaded regions are excluded by the Veltman condition, eq.(3). For $B \neq 0$ new minima for $<V>$ appear close to the boundary. They correspond to the configurations $(\lambda_1, \lambda_2) = (a, a e^{-\frac{Aa^2}{B}})$ or $(a e^{-\frac{Aa^2}{B}}, a)$. The hierarchy is obtained if $\frac{Aa^2}{B} \gg 1$. The mechanism is easily generalized to more couplings, in which case one coupling $\lambda_{i_0} \simeq a$ and all the others are exponentially suppressed $\frac{\lambda_i}{\lambda_{i_0}} < 1$, $i \neq i_0$. The applicability of this idea to the Standard Model is under study [2].

An analog of the Veltman condition can be obtained in superstring models, imposing the vanishing of the quadratic divergences coming from supergravity [3].

2. Yukawas as dynamical variables in low-energy supergravity and superstrings.

The question of the dynamical nature of the Yukawa couplings find a natural answer in superstrings. In
these models, the Yukawa couplings have a non-trivial dependence on the moduli fields which characterize the complex structure of the compact manifold. In the effective supergravity theory, the existence of moduli manifests usually in flat directions in the scalar potential. They appear due to some approximate, non-compact duality symmetries acting on the moduli. As a consequence, the vev's of the moduli are not determined at the tree level of supergravity and the Yukawas can be considered as dynamical variables. The duality symmetries form an $SL(2, \mathbb{Z})$ group and are described by

$$T_\alpha \rightarrow \frac{a_\alpha T_\alpha - i b_\alpha}{i c_\alpha T_\alpha + a_\alpha}, \quad a_\alpha d_\alpha - b_\alpha c_\alpha = 1, \quad (4)$$

where $a_\alpha \cdots d_\alpha$ are integer numbers and $T_\alpha$ are the moduli. There are two distinct possibilities:

i) The number $n$ of undetermined $T_\alpha >$ The number $M$ of Yukawas +1. In this case we can freely perform the minimization with respect to all the Yukawas.

ii) The number $n$ of undetermined $T_\alpha <$ The number $M$ of Yukawas +1. In this case, we will have generically geometrical constraints.

The most simple and interesting case is to have only one constraint, corresponding to the same number of Yukawas and moduli. This is the case which will be investigated in the next section.

Consider a model containing the dilaton-type field $S$ and the moduli $T_\alpha$, specific to the superstring effective supergravities. The Kähler potential and the superpotential read

$$K = \frac{3}{n} \sum_{\alpha=1}^{n} \ln(T_\alpha + T_\alpha^+) - \ln(S + S^+) + \frac{1}{2} K^i_{ij} \hat{\phi}^i \hat{\phi}^j + \cdots$$

$$W = \frac{1}{3} \lambda_{ijk} \hat{\phi}^i \hat{\phi}^j \hat{\phi}^k, \quad (5)$$

where the dots stand for the higher order terms in the fields $\phi^i$. The Kähler metric depends on the moduli $T_\alpha$, and eventually on $S$. The low-energy spontaneously broken theory contains the normalized fields $\hat{\phi}^i$ defined by $\phi^i = (K^{-1/2})^I_{ij} \hat{\phi}^j$ and the Yukawas $\hat{\lambda}_{ijk}$. In order to obtain the relation between $\lambda_{ijk}$ and $\hat{\lambda}_{ijk}$, consider the scalar potential $\hat{V}$, which contains the piece

$$V \supset e^K (K^{-1})^i_{ij} D_i W \bar{D}^j \bar{W} = \bar{W}_i \bar{W}^i, \quad (6)$$

where $D_i W = \partial W / \partial \phi^i + K_i W$ and $\bar{W}_i = \frac{1}{3} \lambda_{ijk} \hat{\phi}^i \hat{\phi}^j \hat{\phi}^k$. From eq.(6) we get the required relation

$$\hat{\lambda}_{ijk} = e^K (K^{-1/2})^i_{ij} (K^{-1/2})^j_{jk} (K^{-1/2})^k_{kl} \lambda_{ijk}, \quad (7)$$

which mathematically express the dependence of $\hat{\lambda}_{ijk}$ on the moduli through the Kähler potential $K$.

3. Constraints between low energy Yukawas.

The condition to have constraints between $\hat{\lambda}_{ijk}$ (combinations which do not depend on the moduli) is

$$\operatorname{rang} \left( \frac{\partial \lambda_I}{\partial t_\alpha} \right) < \min(M, n), \quad (8)$$

where $I = 1 \cdots M$ replaces the indices $i, j, k$ and $t_\alpha = T_\alpha + T_\alpha^+$. The condition to have just one constraint is (in the case $M = n$)

$$\det \left( \frac{\partial \lambda_I}{\partial t_\alpha} \right) = 0. \quad (9)$$

A natural solution for eq.(9) is $\sum_{\alpha} t_\alpha \frac{\partial \hat{\lambda}_{ij}}{\partial t_\alpha} = 0$, so the Yukawas $\hat{\lambda}_{ijk}$ to be homogeneous functions of the moduli. This homogeneity property translates into a scaling property for the Kähler metric

$$t_\alpha \frac{\partial}{\partial t_\alpha} K^j_{ij} = - K^j_{ij}. \quad (10)$$

This suggest us to consider the no-scale models $\hat{\lambda}$, which were introduced in order to get flat directions on the scalar potential, in connexion with the positivity of the energy in supergravity.

The class of the possible constraints is reduced if the Kähler space spanned by the scalar fields is a
The one-loop effective potential has two pieces (RG) equations and the effective potential approach.

\[ U_{\text{model}} = \left( 1 + \frac{3}{4} \sum_{a=1}^{n} \ell n|ic_aT_a + d_a|^2 \right). \] (11)

Defining \( F_a = \frac{3}{4} \ell n(ic_aT_a + d_a) \) and using eq.(11), we obtain the transformation law of \( \hat{\lambda}_{ijk} \) under (4)

\[ \hat{\lambda}_{ijk} \rightarrow \left( \prod_{a=1}^{n} e^{\frac{F_a}{2}} \right) e^{-\frac{3}{4}(F_i + F_j + F_k + h.c.)} \hat{\lambda}_{ijk}. \] (12)

As a consequence, in this case the only possible constraints are multiplicative, in contrast with the Veltman-type constraint, eq.(3) which is additive.

A very simple example is provided by a model containing two moduli \( T_1, T_2 \), the dilaton \( S \) and two observable fields \( \phi \). The model is defined by

\[ K = -\frac{3}{2} \ell n(t_1 - |\phi_1|^2) - \frac{3}{2} \ell n(t_2 - |\phi_2|^2) - \ell n(S + S^+), \]

\[ W = \frac{8}{3} \lambda_1 \phi_1^3 + \frac{8}{3} \lambda_2 \phi_2^3 + W(S), \] (13)

where \( W(S) \) is a non-perturbative contribution to the superpotential which will fix the value of \( S \) and simultaneously break supersymmetry, as in the usual gaugino condensation scenario. The Kähler potential parametrizes a \([SU(1,2)/U(1) \times SU(2)]^2 \times [SU(1,1)/U(1)] \) manifold and the symmetry of the model is \( U(1)^2 \times \text{diagonal dilatation} \). The low-energy Yukawas \( \hat{\lambda}_i \) as functions of the high-energy \( \lambda_i \) read from eq.(7)

\[ \hat{\lambda}_1^2 = \frac{8}{27} \left[ 1/(s+s^+) \right](t_1/t_2)^2 \lambda_1^2, \]

\[ \hat{\lambda}_2^2 = \frac{8}{27} \left[ 1/(s+s^+) \right](t_2/t_1)^2 \lambda_2^2 \] (14)

and the resulting constraint is obvious from eq.(14)

\[ \hat{\lambda}_1 \hat{\lambda}_2 = \frac{8}{27} \left[ 1/(s+s^+) \right] \lambda_1 \lambda_2 = \text{fixed}. \] (15)

The model is easily generalized to \( n \) couplings and \( n \) moduli. The constraint (15) is valid at the Planck scale and must be run to low-energy in order to be used in the dynamical determination of the couplings.

To compute the vacuum energy at a low-energy scale \( \mu_0 \sim M_{\text{susy}} \) we proceed in the usual way. Using boundary values for the independent model parameters at the Planck scale \( M_P \) (identified here with the unification scale), we evolve the running parameters down to the scale \( \mu_0 \) using the renormalization group (RG) equations and the effective potential approach. The one-loop effective potential has two pieces

\[ V_1(\mu_0) = V_0(\mu_0) + \Delta V_1(\mu_0), \] (16)

where \( V_0(\mu_0) \) is the RG improved tree level potential and \( \Delta V_1(\mu_0) \) summarizes the quantum corrections given by the formula

\[ \Delta V_1(\mu_0) = \frac{1}{64\pi^2} StrM^4 \left( \ell n\frac{M^2}{\mu_0^2} - \frac{3}{2} \right) \] (17)

In (17) \( M \) is the field-dependent mass matrix containing the Yukawa coupling dependence and all parameters are computed at the scale \( \mu_0 \). The vacuum state is determined by the equation \( \partial V_1/\partial \phi_i = 0 \). The vacuum energy is simply the value of the effective potential at the minimum.

A dynamical determination of the couplings and gravitino mass \( m_3/2 \) was also undertaken in [8]. The main difference with respect to our analysis is that in the approach proposed in that paper, the minimization is performed freely, with no constraint for the couplings.

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