Sum-Rate Analysis of Intelligent Reflecting Surface Aided Multi-User Millimeter Wave Communications System

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Abstract. Intelligent reflecting surface (IRS) is emerging as one of the potential solutions for the future generation communication system. By exploiting a controllable large number of passive elements, IRS is envisioned to provide a high spectral and energy-efficient network. For successful implementation of IRS, it is required to have optimization at a different level and here, this paper analyzed the sum-rate performance of a IRS assisted multi-user (MU) millimeter wave (mmWave) system by exploiting the joint optimization of the active and passive beamforming. Here in this paper, we have considered zero-forcing (ZF) and minimum mean square error (MMSE) based linear precoder. Through the simulation, we have demonstrated the achievable gain due to optimization, and also a relative performance comparison between the precoders is also presented.

Keywords: IRS, MIMO, MMSE, ZF, Sum-Rate, Optimization.

1. INTRODUCTION

From the perceptive of the future generation communication system, MIMO and mmWave technologies play an important role. And MIMO technology has improved the spectral efficiency of the wireless communication system significantly. But the complexity of the system is still a bit concerned and regarding this IRS is emerging as one of the promising solutions to fulfill the demand of high data rate communication but with low complexity. And also it is considered to be a prime contender for the future generation communication system[1, 2, 3]. Therefore design and analysis of a IRS assisted mmWave MIMO system is highly demanding research topic. IRS is basically a meta-surface consist of a large number of passive reflecting elements and all the elements can be controlled by using a smart controller. Therefore, one can configure the reflection coefficient corresponding to each element in such a way that it can produce a significant boost in signal strength in the desired direction or suppressed the power from the undesired direction[4]. As it consist of passive materials the power consumption is also significantly low.

Lots of works have been carried out and still going on for the maximization of channel capacity for IRS assisted networks[5, 6, 7, 12]. Various approaches have been taken to enhance the IRS assisted system performance. Researchers are extensively engaged to provide an
optimized solution for beamforming, precoding, channel estimation, resource management, and deployment, etc. Here in this paper, the authors have studied the performance of the IRS assisted mmWave MU system by exploiting the joint optimization of active beamforming (ABF) and passive beamforming (PBF). In [8], authors have proposed a joint optimization technique to reduce the transmit power without sacrificing the signal to interference plus noise ratio (SINR) requirement. Authors in [9] have worked on joint phase optimization to enhance the outage probability and ergodic rate for IRS-MIMO system with multiple IRSs. Extending further in [10], authors have exploited the joint active and passive beamforming optimization algorithm for the realization of Simultaneous wireless information and power transfer (SWIT). As in [11], authors proposed low complex ZF based beam formation to improve the sum rate in comparison to MRC based beam formation. Lots of work is going on to address the issue of beamforming optimization and related applications.

In this paper, the authors have addressed the joint optimization of PBF and ABF. And authors have considered the IRS-assisted network as an application in mmWave. Now under low-rank mmWave channel conditions and also in absence of a direct path between the base station (BS) and MU, the implementation of IRS is becoming a game changer. This motivates the authors to address the issue of enhancing the sum-rate of the network under such volatile channel conditions. Here in this paper authors have compared the performance of a IRS assisted MU network considering optimized and non-optimized active/passive beamforming. And also authors have considered ZF and MMSE based precoders and the corresponding optimization to demonstrate the relative performance comparison.

The rest of the paper is organized as follows: Section 2 describes the IRS-assisted mmWave system model. And also this section represents the theoretical framework regarding the optimization of precoder and phase. Next, Section 3, demonstrated the numerical results and corresponding analysis. Finally, the overall conclusion is reported in Section 4.

2. SYSTEM MODEL

![Figure 1. Representation of IRS assisted Network.](image)

In this paper we have considered a IRS assisted downlink mmWave MISO system as shown in figure 1. Here the BS is consists of $N_b$ number of antenna elements. And also we have considered that there is a single IRS having $M$ number of elements. The total system is aiming to provide service to $K$ MUs having single-antenna. As presented in figure 1, the communication link BS
and MUs can be made possible via the reflected path from the IRS as the direct path is blocked because of the presence of the obstacles.

The main role of the IRS is to manipulate the wireless channel to have favorable condition. Each element in IRS is responsible for reflecting the incident signal towards the desired direction with the help of individual reflection coefficients ($\alpha_m$). Here it is assumed that, $|\alpha_m| = 1, \forall m = 1, ..., M$ and $\angle \alpha_m$ can be adjusted in $[0,2\pi]$. The main idea is to improve the system capacity by optimizing the phases by exploiting the channel state information. The optimized ABF and PBF not only improve the channel capacity but also reduce the transmit power requirements, thereby making the network more energy efficient.

As a part of the system modeling, we have considered, $H_{bi}$, represents the mmWave channel between BS and IRS unit, having dimension $(M \times N_{bs})$. And $H_{iu} \in \mathbb{C}^{M \times 1}$ is the channel between the IRS unit and the $k^{th}$ MU. Here we have considered that there is no direct link between the IRS and MU. And also for simplicity, in our analysis we have excluded all of those signal which are reflected by the IRS multiple times. This valid as in mmWave the path loss is significantly high and thus can be ignored. Therefore, considering all the aspect the overall channel matrix between the BS and MU can be expressed as $\mathbf{H} = H_{iu}\phi H_{bi}$. The received signal vector $\mathbf{y}$ is given by

$$\mathbf{y} = (H_{iu}\phi H_{bi})\mathbf{s} + \mathbf{n}. \quad (1)$$

Where, $\mathbf{s}$ denote the transmitted signal vector and the corresponding covariance matrix can be defined as $Q = \mathbb{E}[\mathbf{s}\mathbf{s}^H]$. The received signal at the $k^{th}$ MU can be expressed as

$$\mathbf{y}_k = H_{iu}^H \phi^H H_{bi} \mathbf{p}_k \mathbf{s}_k + H_{iu}^H \phi^H H_{bi} \sum_{j=1,j\neq k}^{K} \mathbf{p}_j \mathbf{s}_j + \mathbf{n}_k. \quad (2)$$

Where, reflection matrix $\phi = \text{diag} \{\phi_1, ..., \phi_M\}$ and $s_k$ is the transmitted signal from the BS to $k^{th}$ users with normalized power. Here, $n_k$ is the noise vector that follows complex Gaussian distribution. Here $P = [p_1, ..., p_k]$ represents the precoding matrix and $p_k$ is the precoding vector corresponding to the signal from BS to $k^{th}$ user. The constraint related to the transmit power at the BS can be considered as $tr(PP^H) \leq P_{\text{max}}$. Under the above setup, the channel capacity can be expressed as

$$C = \log_2 \det(\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{H}\mathbf{Q}\mathbf{H}^H) \quad (3)$$

As in (3), for the IRS assisted communication system, the capacity is depends on $\phi$. This is because of the fact that the overall channel matrix $\mathbf{H}$ is greatly influenced by the phase distribution. At the same time, the capacity is also a function of the transmit covariance matrix $\mathbf{Q}$.

Here, in this paper, the maximization of the downlink sum-rate corresponding to the network as in figure 1, is the prime focus. And for that authors have looked for the joint optimization of ABF and PBF.

Here, authors have considered ZF and MMSE based precoder to understand the relative performance gain. For the network as presented in figure 1, the signal to interference noise ratio for the $k^{th}$ users can be given as

$$\text{SINR}_k = \frac{|H_{ik}^H \phi^H H_{bi} \mathbf{p}_k|^2}{\sum_{j=1,j\neq k}^{K} |H_{ik}^H \phi^H H_{bi} \mathbf{p}_j|^2 + \sigma_n^2} \quad (4)$$

As in [12], the weighted sum rate problem can be defined as

$$\max_{\mathbf{P}, \phi^f(\mathbf{P}, \phi)} = \sum_{k=1}^{K} \omega_k \log_2 (1 + \text{SINR}_k) \quad (5)$$
The continuous phase set

\[ \theta_m = \exp(j\phi_m) \quad \phi_m \in [0, 2\pi) \] (7)

Here \( \omega_k \) is the weight applied to the \( k^{th} \) users. Now to enhance the performance of the proposed system optimization of the \( P \) and \( \phi \) is very much required. Here in this paper authors have taken ZF and MMSE based precoder. And also demonstrated the performance gain that can be achieved by optimizing the precoder.

The ABF optimization problem can be addressed as in [12] for a given sets of \( \phi \). The precoding matrix optimization problem under the power constraint can be expressed as

\[
\max_P f(P) = \sum_{k=1}^{K} \frac{\overline{\alpha}_k |\overline{H}_k^H P_k|^2}{\sum_{j=1}^{K} |\overline{H}_k^H P_j|^2 + \sigma_n^2} \] (8)

Where, \( \overline{H}_k^H = H_{tu}^H \phi^H H_{bi} \) And the optimized precoding matrix corresponding to each user can be obtained[12] as

\[
P_{k_{opt}} = \sqrt{\alpha_k} \beta_k \left( \mu I_N + \sum_{i=1}^{K} |\beta_i|^2 \overline{H}_i \overline{H}_i^H \right)^{-1} \overline{H}_k \] (9)

Where, \( \alpha = [\alpha_1, \ldots, \alpha_K]^T \) is the auxiliary vector resulted from the Lagrangian dual transformation [12, 13] and \( \overline{\alpha}_k = \omega_k (1 + \alpha_k) \). And \( \beta = [\beta_1, \ldots, \beta_K]^T \) is the auxiliary vector introduced by the use of quadratic transform (QT) technique [13]. And \( \mu \) is the Lagrange multiplier for the power constraint as mentioned in (6).

After the ABF, the authors have taken the problem of optimization of PBF and aiming for finding out the optimized \( \theta \). Similar to the optimization of \( P \), the optimization problem for \( \theta \) can be defined as follows

\[
\max_{\theta} f(\theta) = \sum_{k=1}^{K} \frac{\overline{\alpha}_k |\theta^H v_{k,k}|^2}{\sum_{j=1}^{K} |\theta^H v_{k,j}|^2 + \sigma_n^2} \] (10)

Now to have the optimal solution, QT is required and it introduced the auxiliary vector \( \rho = [\rho_1, \ldots, \rho_K]^T \). And by utilizing the Lagrange multiplier method, the optimal solution for \( \rho_k \) can be derived[12] as mentioned below

\[
\rho_{k_{opt}} = \frac{\sqrt{\alpha_k} \theta^H v_{k,k}}{\sum_{j=1}^{K} |\theta^H v_{k,j}|^2 + \sigma_n^2} \] (11)

Following the same approach as in[12], for a single IRS assisted network, the optimized \( \theta \) for a given \( \rho \) can be obtained as follows

\[
\theta_{opt} = \left( \sum_{k=1}^{K} |\rho_k|^2 \sum_{j=1}^{K} v_{k,j}^H v_{k,j}^H + \sum_{k=1}^{M} \gamma_k \theta_k \theta_k^H \right)^{-1} \left( \sum_{k=1}^{K} \sqrt{\alpha_k} \rho_k^* v_{k,k} \right) \] (12)

Where \( \theta_k \in \mathbb{R}^{M \times 1} \) is an elementary vector consist of one in the \( k^{th} \) position and zeros elsewhere and \( \gamma_k \) is associated with the constraint \( |\theta_k|^2 \leq 1 \).

Here in this paper authors have used joint optimization techniques and compare the system performance. The system performance has been compared by considering ZF and MMSE based precoder along with and without joint optimization.
3. NUMERICAL RESULTS

In this section, the performance of the IRS-MISO system is evaluated through the numerical simulation in the MATLAB platform. For the simulation, purpose authors have considered the BS with N=32/64 numbers of antennas. The BS is located at the origin with the users are uniformly distributed within a circle of radius 50m. Here we have considered the maximum total transmitted power=30dBm and the noise power as -85dBm. And also to create the MU scenario we have taken K=2 to study the sum-rate performance. And also As mentioned earlier, there is no direct path between the BS and the users. The communication link has been established between the BS and users via a single IRS unit.

The figure 2 shows the sum rate variation with the variation in the number of elements in IRS and also with the variation in the number of antennas in the BS. As presented here, it is clear that with the increase in M, the system sum-rate performance increases. And also the number of BS antennas has signification influence on the system performance. And MMSE precoder based outperforms the ZF in terms of achievable sum rate. Here, particularly for this study, authors have considered unoptimized linear precoders along with random phase distribution for the PBF.

The figure 3 shows the performance gain that can be achieved through the optimization of the precoder and beamforming. As presented here, the joint optimization produces a significant gain over the conventional non-optimized system. As in the case, with M=30 the achievable sum-rates for MMSE precoders are 3.871 b/sec/Hz and 8.088 b/sec/Hz for non-optimized and optimized solutions respectively. It is clear from the above figure that the jointly optimized MMSE based precoder based MISO system outperforms its counterparts. One point that can be observed from the figure 3 is that with the change in M from 30 to 60 the optimized solution provides sum-rate improvement of 1.38 b/sec/Hz whereas the unoptimized solution provides sum-rate improvement of 0.683 b/sec/Hz. Therefore, we can conclude that the variation of M has a larger impact when we have the optimized solution.

A clear performance improvement is visible as in figure 3 and figure 4. As in figure 4, considering the CDF=0.8, the channel capacities are 4.24 b/sec/Hz, 4.45 b/sec/Hz,
Figure 3. The sum-rate variation with IRS Elements (With and Without Optimization).

Figure 4. CDF of Sum-Rate.

8.6 b/sec/Hz and 9.1 b/sec/Hz for IRS-MISO system with ZF precoder (unoptimized), MMSE precoder (unoptimized), ZF precoder (optimized) and MMSE precoder (optimized) respectively. It demonstrates a significant amount of performance gain. Therefore, from the simulation results, it is evident that in the case of a low-ranked mmWave channel jointly optimized IRS assisted network is a promising solution.

4. CONCLUSION

IRS is emerging as a potential solution for the next generation communication system. Here in this paper, the authors have analyzed the performance of a IRS assisted mmWave MU
communication. In this paper, the authors have demonstrated the significance of having joint optimization for ABF and PBF. The simulation results show that a jointly optimized IRS assisted system improves the system performance significantly. As a part of the extension of this work, the authors in like to address the discrete phase optimization problem along with IRS assisted MIMO system.

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