Longitudinal and transverse form factors from \( ^{65}\text{Cu} \) and \( ^{71}\text{Ga} \) nuclei

Sarah M. Obaid\(^1\) · Fouad A. Majeed\(^2\)

Received: 24 June 2022 / Revised: 24 October 2022 / Accepted: 21 November 2022 / Published online: 14 December 2022
© The Korean Physical Society 2022

Abstract
In the present work, the inelastic electron scattering for longitudinal and transverse form factors of \( ^{65}\text{Cu} \) and \( ^{71}\text{Ga} \) nuclei lies in the \( fp \)-shell region are studied in the framework of the shell model. The calculation is performed in the \((1f_{5/2}, 2p_{1/2}, 2p_{3/2}, 1g_{9/2})\) model space using \( \text{jun}45 \) effective interaction. The wavefunctions employed to conduct the shell model calculations are extracted from the \( \text{jun}45 \) effective interaction for these nuclei with the \( jj44 \) shell model space and \( (\text{Sk}35−\text{Skzs} ∗) \) residual interaction to evaluate the interactions matrix element between initial and final states. The effective charges used to account for the core-polarization (CP) effect are created using calculations of microscopic perturbations that include intermediate one-particle, one-hole excitation from the core and the model space (MS) orbits into all upper orbits with \( n\hbar\omega \) excitations following the same approach done in [Radhi et al. in Euro. Phys. J. A 50:1–9, 2014]. To account for the (CP) effects contribution, the inelastic form factor is obtained by employing the shape of Tassie and Bohr–Mottelson models with appropriate proton and neutron effective charges. The calculated form factors were compared with available experimental data.

Keywords Inelastic electron scattering · Longitudinal and transverse form factors · Shell model

1 Introduction
Shell model calculations performed within a model space in which nucleons are constrained to occupying a few orbits are capable of reproducing reported transition strengths by taking into account proper effective charges for protons and neutrons. The form factors for the stable \( fp \)-shell nuclei have been explored using \( fp \)-shell wave functions with phenomenological effective charges [1]. The inelastic form factors were fitted 30 years ago by fitting polynomials to describe the form factors and many studies in the last 5 years were able to calculate the form factors for lower and mid of \( fp \)-shell nuclei [2, 3]. Majeed studied inelastic longitudinal form factors for the excitation of \( 2^+ \) in \( ^{58}\text{Ni} \) nucleus through the nuclear shell model by utilizing the configurations \((p_{3/2}, p_{1/2}, \text{and} f_{5/2})\) [4]. The C2 inelastic form factors for \( ^{54}\text{Fe} \) and \( ^{56}\text{Fe} \) in the framework of the shell model were conducted by Salman and Kadhim, with M3Y and MSDI chosen for N–N interaction. Their results including core-polarization till \( 6\hbar\omega \) enhance the calculations of the form factors compared to the observed data [5]. Hamoudi and Abbas investigated the influence of short-range correlations between two bodies on the calculations of the elastic form factors for some selected \( fp \)-shell nuclei. They proved that when the correlation is considered shows remarkable importance especially at the high-momentum transfer region [6]. Salman et al. [7] studied the longitudinal form factors for the \( 2^+ \) state for the isotopes \( ^{64,66,68}\text{Zn} \) in the model space F5P by utilizing the interaction F5PVH and their results for the reduced transition probabilities and the form factors were in reasonable agreement in comparison to the observed data. The core-polarization effect is considered using a microscopic theory that allows the excitation from the core nucleons and projects them to the model space with exciton energy 2 and \( 4\hbar\omega \) were conducted by Majeed and his collaborators, for some \( p, sd, \) and \( fp \) nuclei, show great success in describing the form factors compared to the observed data [8–11]. Abbas et al. conducted a study to investigate the form factors and the related distribution of the charge density for the inelastic Coulomb C2 for \( ^{58,60,62}\text{Ni} \) and \( ^{64,66,68}\text{Zn} \) isotopes by utilizing the method of SHF with the parametrization of type (\( \text{Sk}35−\text{Skzs} ∗ \)). They discussed their results and compared them with the observed data [12].

\( \text{jun}45 \) effective interaction

Biomedical Engineering Department, Al-Mustaqbal University College, Babylon, Iraq

Department of Physics, College of Education for Pure Sciences, University of Babylon, Babylon, Iraq
The present study aimed to investigate the inelastic longitudinal and transverse electron scattering form factors for $^{65}$Cu and $^{76}$Ga nuclei laying in the fp-shell region. Shell model calculations will be performed using the effective \textit{jun4S} [13] interaction for the FP model space. The shell model calculation will be performed without restrictions imposed on the model space using the code NushellX@MSU [14]. The inelastic form factors calculated by employing the Bohr–Mottelson and Tassie models by means of effective proton and neutron charges. The theoretical findings will be compared to the existing experimental data.

\[ E_{\text{kin}} = \sum_{i=1}^{A} \frac{\hbar^2}{2m_i} \int \tau_i d^3r. \]  

(3)

\[ E_{\text{Sky}} \] is the Skyrme force energy functional given by [16]

\[ E_{\text{Sky}} = \int d^3r \left[ \frac{b_0}{2} \rho^2 - \frac{b_0}{2} \sum_{q} \rho_q^2 + \frac{b_1}{3} \rho^{a+2} \right] \]

\[ - \frac{b_0}{2} \sum_{q} \rho_q \Delta \rho - b_4 \rho \nabla J - b_3 \sum_{q} \nabla J_q \]

(4)

\[ \rho_q \] is the local neutron and proton densities which are \( q \)-dependent, \( \rho \) is the total density, \( \tau_q \) is the neutrons and protons densities of the kinetic energy, and \( J_q \) is the current density of the spin–orbit interaction. These relationships are the ones who provide them [16]

\[ \rho_q(r) = \sum_{i \sigma} \left| \phi_i(r, \sigma, q) \right|^2, \]

\[ J_q(r) = -i \sum_{i \sigma} \nabla^* \phi_i \left( \sigma | \tilde{\sigma} \right), \]

(5)

\[ \] The parameters of the energy Skyrme equation are listed below [15, 16]

\[ b_0 = b_0 \left(1 + \frac{1}{2} b_0 \right), \]

\[ b_i = b_i \left(1 + \frac{1}{2} b_i \right) \]

(6)

The nucleon densities \( \rho_n(r) \) and \( \rho_p(r) \) may be employed to calculate the wavefunction of the single-particle for the HF method. The most general product wave functions (\( \psi_{\beta} \)) in (SHF) theory utilize the Skyrme forces, consisting of discrete particles that move individually. In neutron and proton densities in this technique, it is given as [17]

\[ \rho_q(r) = \sum_{i} |\varphi_i(r)|^2 = \sum_{\beta} \omega_\beta \left( 2 j_\beta + 1 \right) \left( \frac{R_\beta}{r} \right)^2, \]

(7)

where the state is \( \beta \), the state probability is \( \omega_\beta \), \( j_\beta \) is the state angular momentum, and \( R_\beta \) is the harmonic-oscillator radial part of the wavefunction.

Here, \( L_n^{(3/2)} \) is the polynomials of the associated Laguerre and \( b \) is the width parameter for the harmonic oscillator.

The Coulomb energy is the third element of the total energy equation. The Coulomb energy is aided by the exchange component, which provides just a small amount of energy. Because the Coulomb contact has an unlimited range, it makes a significant contribution. The energy of Coulomb is determined as [15, 16]
\[ E_{\text{Coul}} = \frac{e^2}{2} \int \frac{\rho_q(\vec{r}) \rho_q(\vec{r}')}{r} d^3r d^3r' + E_{\text{Coul,exch}} \]  
(8)

\[ E_{\text{Coul,exch}} = -\frac{3}{4} e^2 \left( \frac{3}{\pi} \right)^{1/3} \int \rho_q^2(\vec{r}) d\vec{r} \]  
(9)

\[ E_{\text{Coul,exch}} = -\frac{\langle \overrightarrow{p}_m^2 \rangle}{2 Am}, \]  
(10)

where \( P_{cm} = \sum \hat{p}_i \) is the total momentum operator, \((A)\) is the number of nucleons, and \( m \) the nucleon average mass.

### 2.2 Tassie and Bohr–Mottelson

The following equations were used to calculate the effective proton and neutron charges for Bohr–Mottelson [18]:

\[ e_{\text{eff}}(t_x) = e(t_x) + e\delta e(t_x) \]
\[ e\delta e(t_x) = Z/A - 0.32(N-Z)/A - 2t_x \left[ 0.32 - 0.3(N-Z)/A \right], \]  
(11)

where \( t_x(p) = 1/2 \) and \( t_x(n) = -1/2 \).

The combination of the reduced single-particle matrix elements and the one-body density matrix (OBDM) components yields the reduced matrix element of the electron scattering operator for an n-particle model space function of multipolarity \( \lambda \), which is represented as [19]

\[ \langle \ell \parallel \hat{d}^{\lambda} \parallel b \rangle = \sum_{ab} \text{OBDM}(f,i,a,b,\lambda)(a||\hat{d}^{\lambda}||b). \]  
(12)

The core polarization matrix element and the model space are both portions of the many-particle reduced matrix elements of the longitudinal operator [8]

\[ \langle \ell \parallel \hat{d}^{\lambda} \parallel b \rangle = \left( \langle \ell \parallel \hat{d}^{\lambda,\text{ms}} \parallel b \rangle \right) - \left( \langle \ell \parallel \hat{d}^{\lambda,\text{cp}} \parallel b \rangle \right), \]  
(13)

where the matrix elements of the model space (MS) and the core-polarization (CP) in Eq. 13 are given as [19]

\[ \langle \ell \parallel \hat{d}^{\lambda,\text{ms}} \parallel b \rangle = \int_0^\infty drr^2j_x(qr)\rho_{\lambda,a}^{\text{MS}}(r) \]  
(14)

\[ \langle \ell \parallel \hat{d}^{\lambda,\text{cp}} \parallel b \rangle = \int_0^\infty drr^2j_x(qr)\Delta \rho_{\lambda,a}^{\text{CP}}(r). \]  
(15)

In terms of transition charge density, the matrix element of Coulomb interaction can be represented as the sum of the (MS) and (CP) elements [19]

\[ O(C,\lambda, q) = q \int_0^\infty drr^2j_x(qr)\rho_{\lambda,a}^{\text{MS}}(r) + \int_0^\infty drr^2j_x(qr)\Delta \rho_{\lambda,a}^{\text{CP}}(r). \]  
(16)

where the momentum transfer is \( q \) and \( j_x(qr) \) is spherical Bessel function. For the initial \( i \) and final \( f \) nuclear states, the nucleons ‘charge density \( \rho_{\lambda,a}^{\text{MS}}(r) \) of the transition is described using the one-body density matrix [19]

\[ \rho_{\lambda,a}^{\text{MS}}(r) = \sum_{k_a,k_b} F(i,f,k_a,k_b,\lambda,u)\langle j_{a} \parallel Y_{\lambda} \parallel j_{b} \rangle R_{n_{a}l_{a}}(r)R_{n_{b}l_{b}}(r), \]  
(17)

where \( k \) stands for \((nlf)\) the single-particle (s.p.) states and \( u \) is the index which refers to either neutrons or protons and \( F(i,f,k_a,k_b,\lambda,u) \) is the one body matrix element.

The transition density for the CP valence model \( V \) is given by [19]

\[ \Delta \rho_{\lambda,a}^{T}(r) = \delta e_p \rho_{\lambda,a}^{\text{MS}}(r) + \delta e_n \rho_{\lambda,a}^{\text{MS}}(r); \]  
(18)

\[ \delta e_p, \delta e_n \] are the charges associated with the neutron and protons to account for polarization.

The CP for Tassie model transition density is provided by [19]

\[ d \rho_{0,p}^{\text{core+MS}}(r) = \sum_{k_a,k_b} F(i,f,k_a,k_b,0,p)\langle j_{a} \parallel Y_{0} \parallel j_{b} \rangle R_{n_{a}l_{a}}(r)R_{n_{b}l_{b}}(r). \]  
(19)

At the photon point, the proportionality constant \( N \) is given by the matrix elements of gamma transitions \( M(E\lambda) \), where \( E_{\gamma} \) is the energy due to excitation [19]

\[ M(E\lambda) = \left\{ e \int_0^\infty drr^2r^l\rho_{\lambda,a}^{\text{MS}}(r) + \int_0^\infty drr^2r^l\frac{d\rho_{0,p}^{\text{core+MS}}(r)}{dr} \right\}. \]  
(20)

The matrix of gamma transition elements can be described as MS matrix elements with effective charges

\[ M(E\lambda) = e_{\text{eff}}(E) \int_0^\infty drr^2r^l\rho_{\lambda,a}^{\text{MS}}(r) + e_{\text{eff}}(E) \int_0^\infty drr^2r^l\frac{d\rho_{0,p}^{\text{core+MS}}(r)}{dr}. \]  
(21)

Equating Eq. (21) with Eq. (22) yields the constant of proportionality \( N \) by means of the effective charges. A detailed discussion of these above-mentioned models for effective neutrons and protons can be found in Ref. [20].
The electron scattering form factor between the initial (i) and final (f) nuclear shell model states, involving angular momentum and momentum transfer \( q \), is given by [21]

\[
|F_{nf}(q)|^2 = \frac{4\pi}{Z^2} \frac{1}{2j_i + 1} |\langle C\lambda, q \rangle|^2 |F_{cm}(q)F_{j_f}(q)|^2, \tag{23}
\]

with \( q \) taken as longitudinal Coulomb (C) and transverse electric (E) and magnetic (M) form factors, where \( F_{cm}(q) = e^{q^2/4A} \) is the center of mass correction due to the lack of transitional invariance in the shell model and \( F_{j_f}(q) = \left[ 1 + (q/4.33)^2 \right]^{-2} \).

3 Results and discussion

This section will be devoted to the calculations and discussion of the predicted results and the comparison with the measured data. Since there are various theoretical attempts to explain the characteristics of \( fp\)-shell nuclei and the utilized model space and the involved parameters, besides the approach on how the two-body matrix elements were fitted to the observed data from the work of Ref [17] to describe the charge density and form factors. According to the many-particle shell model, \( fp\)-shell nuclei are considered a core of \( ^{56}\text{Ni} \) nucleons distributed over the \( f_{5/2}, p_{3/2}, p_{1/2}, \) and \( g_{9/2} \) shell (this shell forms the model space). The wavefunctions used in the shell model calculations are extracted from the \( j\text{un}45 \) effective interaction for this nucleus with the jj44 shell model space and (Sk35–Skzs*) residual interaction to evaluate the interactions matrix element between initial and final states. Since we have proved previously in our published papers (see Refs. [8–11]) that the core-polarization must be considered by either introducing effective charges for proton and neutron or by include it by means of microscopic theory; therefore, in this study for Tassie model, the proton and neutron effective charges are created using calculations of microscopic perturbations that include intermediate one-particle, one-hole excitation from the core and the MS orbits into all upper orbits with \( n\text{hoo} \) excitations as stated in Ref. [20].

It is important here to mention that for both of the studied nuclei, the full model space was used and we did not impose any restriction on the configuration mixing and all orbits and coupling of angular momentum were considered.

In all of the following figures, the solid red lines give the results obtained using Tassie. Solid blue lines show the result, including Bohr–Mottelson. The Sk35–Skzs* parameters used to perform the present study are listed in Table 1.

### 3.1 The \( ^{65}\text{Cu} \) nucleus

The nucleus \( ^{65}\text{Cu} \) consists of 29 protons and 36 neutrons, and in the conventional shell model, the core is taken at \( ^{56}\text{Ni} \) with 1 proton and 8 neutrons outside the core. The effective charges are calculated at the photon point using the microscopic theory as done in Ref. [20], and we get the values \( e_p^{\text{eff}} = 1.28 \) for protons and \( e_n^{\text{eff}} = 0.32 \) for neutrons, which is used with Tassie model calculations. The effective charges calculated from the formula derived from Bohr–Mottelson gives \( e_p^{\text{eff}} = 1.17 \) for protons and \( e_n^{\text{eff}} = 0.41 \) for neutrons.

#### 3.1.1 Longitudinal C2 form factors

Figure 1 panels a, b, c, d, e, f, g, and h depicts the comparison of the observed and predicted C2 form factors for the states 1/2+, 1/2−, 3/2+, 3/2−, 5/2+, 5/2−, 7/2+, and 7/2− and available data are taken from [22] which span the range of momentum transfer \( q = (0–3) \text{fm}^{-1} \). The neutron and proton charges used to perform the calculations are 0.5 and 1.5, respectively. The calculated C2 form factor for the state 1/2+ is shown in panels (a) and (b); from this figure panel (a), we can see our theoretical calculations for the longitudinal form factors of Tassie Bohr–Mottelson models give good agreement with the observed data. In contrast, calculations underestimate the measured data in panel (b). The form factors for the 1.725 MeV (3/2−→7/2) and 2.327 MeV for (3/2−→7/2) states which have experimentally spin and parity assignments are shown in Fig. 1 panels (c) and (d), respectively. Theoretical prediction reproduces the observed data in all the regions of the momentum transfer. Comparing the longitudinal form factors for the states (5/2−→7/2) and (5/2−→7/2), the results are shown in Fig. 1 panels (e) and (f). Tassie and Bohr–Mottelson models’ calculations for the form factors of C2 show enhancement at the first maxima and second maxima, which agrees with the observed data in panel (e). In contrast, in panel (f), Tassie and Bohr–Mottelson’s predictions underestimate the observed data at the location of maxima in the first and second loops. The C2 form factors for the states 1.482 MeV (7/2−→7/2) and 2.094 MeV (7/2−→7/2) are displayed in the panels (g) and (h) Fig. 1. In panel

| Table 1 Parameters for Sk35–Skzs* employed to perform the calculations [17] |
|-------------------------------------------------|
| **Force** | **Sk35–Skzs** |
| \( t_s \) (MeV.fm\(^3\)) | −1446.759 |
| \( t_t \) (MeV.fm\(^3\)) | 250.852 |
| \( t_r \) (MeV.fm\(^3\)) | −132.993 |
| \( t_j \) (MeV.fm\(^3\)) | 12.127.649 |
| \( W_f \) | 153.054 |
| \( x_0 \) | 0.329 |
| \( x_1 \) | 0.518 |
| \( x_2 \) | 0.139 |
| \( x_3 \) | 0.018 |
| \( a \) | ½ |
Fig. 1 Calculated longitudinal C2 form factor of $^{65}\text{Cu}$
(g) of Fig. 1, the C2 form factor is very well described and spans all q values.

3.1.2 Longitudinal C4 form factors

Figure 2 panels a, b, c, and d depicts the calculation of the C4 form factors for the states 7/2, 7/2, 5/2, 7/2 in comparison to the experimental data taken from Ref. [22]. The Tassie and Bohr–Mottelson predicted the C4 form factors which are in agreement with the observed data except for the state (7/2, 7/2) where the experimental data shows two locations of diffraction maxima and one diffraction minima and theoretical calculations of Tassie and Bohr–Mottelson predicted three diffraction maxima and two diffraction minima which is not in agreement with the observed data. Tassie and BM calculations in panel (b) of Fig. 2 overestimated the first diffraction for the state (7/2, 7/2) at 2.278 MeV and agreed in the second maxima. The C4 for the states (5/2, 7/2) and (7/2, 7/2) at 2.593 MeV and 2.645 MeV is well estimated by both Tassie and BM calculations.

3.1.3 Transverse E2 form factors

The form factor for the E2 transition state (1/2−, 7/2) at $E_x = 0.771$ MeV is displayed in Fig. 2 panel (a); the theoretical calculations of the Tassie model coincide with the Bohr–Mottelson model. The observed data are scattered with a high error bar in the location of first maxima. Theoretical results within the error bar in the first maxima while reproducing the observed data in the second maxima up to $q = 1.5$ fm$^{-1}$. The second panel (b) presents the calculations of the (1/2−, 7/2) state, where the observed data are scattered with a high error bar and do not shown clearly the diffraction maxima and minima, while theoretical calculation predicts three maxima and two minima. The transverse form factors for the 1.725 MeV (3/2−, 7/2) and 2.329 MeV for (3/2−, 7/2) states are displayed in panels (c) and (d). The Tassie model coincides with the Bohr–Mottelson model. The observed data in panel (c) are described by the Tassie and Bohr–Mottelson models in the first and third maxima. The transverse magnetic form factor of $^{65}$Cu 2.329 MeV for the (3/2−, 7/2) state is calculated and presented in Fig. 2d; Tassie and Bohr–Mottelson models could not describe the observed data in the first and second maxima except for the third maxima, theory, and...
Fig. 3 Calculated transverse E2 form factor of $^{65}$Cu
observed data agree. The E2 transverse form factor for the states (1.116 MeV, 1.624 MeV) $\frac{5}{2}^-_{1}$ and $\frac{5}{2}^-_{2}$ calculations are manifested in (e) and (f), respectively. The results of Tassie and the Bohr–Mottelson models are not able to reproduce the observed data for the transverse form factor for 1.482 MeV ($\frac{7}{2}^-_{1}$, $\frac{7}{2}^-_{2}$) and 2.094 MeV ($\frac{7}{2}^-_{2}$, $\frac{7}{2}^-_{2}$) are depicted in Fig. 2 panels (g) and (h).

3.1.4 Transverse E4 form factors

The transverse E4 form factor for the states $\frac{7}{2}^-, \frac{5}{2}^-, \frac{5}{2}^+$ is depicted in Fig. 4 panels (a), (b), (c) and (d), respectively. The predicted E4 form factor for the state ($\frac{7}{2}^-, \frac{7}{2}$) at 2.099 MeV underestimated the experimental data in all the momentum transfer regions, as shown in panel (a). The experimental data for the state ($\frac{7}{2}^-, \frac{7}{2}$) at 2.278 MeV have high error bar and the predicted E4 form factors by Tassie and BM are identical and describe the observed data within the error bars. Bohr–Mottelson and Tassie calculations show three diffraction maxima with lowest peak in the middle diffraction minima for the state ($\frac{5}{2}^-, \frac{7}{2}$) at 2.593 MeV in panel (c) of the figure, where the experimental data are described well in all the regions of the momentum transfer. Tassie and BM are almost the same for the calculation of the transverse E4 form factor for the state with two diffraction maxima underestimated the measured data and falls within the high error bars.

3.2 $^{71}$Ga nucleus

The nucleus $^{71}$Ga consists of 31 protons and 40 neutrons, and in the conventual shell model, the core is taken at $^{56}$Ni with 3 proton and 8 neutrons outside the core. The effective charges calculated at the photon point using the microscopic theory as done in Ref. [17], and we get the values $e_{p}^{\text{eff}} = 1.29$ for protons and $e_{n}^{\text{eff}} = 0.45$ for neutrons, which is used with Tassie model calculations. The effective charges calculated from the formula derived from Bohr–Mottelson give $e_{p}^{\text{eff}} = 1.19$ for protons and $e_{n}^{\text{eff}} = 0.51$ for neutrons.

3.2.1 Longitudinal C2 form factors

Figure 3 panels a, b, c, d, and e displays the comparison of the observed and predicted C2 form factors for the states 1/2, 3/2, and 5/2 and available data are taken from [22]. The calculated C2 form factor for the state 1/2 is shown in Fig. 3; from this figure panel (a), the theoretical results do not reproduce the experimental data in the second maxima. The form
Longitudinal and transverse form factors from $^{65}$Cu and $^{71}$Ga nuclei

3.2.2 Transverse E2 form factors

Figure 4 panels (a, b, c, d, and e) shows the transverse E2 form factors for the states 1/2, 3/2, 3/2, 5/2, and 5/2, predicted theoretically and plotted with momentum transfer $q$ without comparison, because there are no available measured data for the transverse form factors.

The shell model makes the assumption that effective charge values are constant, which could not be appropriate in the case of particularly neutron-rich nuclei. Therefore, the global dependence on $(N-Z)/A$ which is introduced by Bohr–Mottelson (BM) particle–vibration coupling model for effective charges and the quenching effects for weakly bound particles were adopted by Ogawa et al. [23] which showed that the experimental $Q$ moments were well reproduced.

Although the shell model calculations with $jj44$ model space using the effective charges for proton and neutron are successful in reproducing the longitudinal and transverse form factors for $^{65}$Cu, it is still not able to reproduce the experimental data for the longitudinal form factors in reasonable way for $^{71}$Ga. The use of effective charges seems not enough for the calculations of the form factors of $^{71}$Ga and a microscopic theory should be used to perform the calculations, which might improve the calculations (Figs. 5 and 6).

4 Conclusion

In this study, the model space $f_{5/2}g_{9/2}$ labeled as $jj44$ in NushellX@MSU and the effective interaction $jun45$ were utilized with the shell model to investigate the inelastic longitudinal and transverse electron scattering form factors for $^{65}$Cu and $^{71}$Ga nuclei. The Skyrme Hartree–Fock method was employed with $(Sk35–Skzs^d)$ for calculating of the form factors. The effect of core polarization is found to be very needful for the longitudinal C2 and C4 form factors using
effective charge that is considered for both Tassie and the Bohr–Mottelson models that enhance the calculations of the form factors and to match the observed data. Since the effective charge plays no role in the calculations of the transverse $E_2$ and $E_4$ form factors and the calculations were performed solely on the model space wave function. General theoretical predictions of the form factors agree with the corresponding measured data. For $^{71}$Ga nucleus, the extracted data for low $q$ values and more data are required to determine the shapes of the transverse and longitudinal form factors. Once these data are available and compared with the present calculations, a conclusion can make clearer about the ability of the present calculations to reproduce the measured form factors.

Acknowledgements The first author S. M. Obaid acknowledges the financial support from Al-Mustaqbal University College. This work was supported by Al-Mustaqbal University College (Grant No. MUC-G-0322).

Data availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

References

1. R. Sahu, K.H. Bhat, D.P. Ahalparas, Inelastic electron scattering from $fp$-shell nuclei. J. Phys. G: Nucl. Part. 16, 733 (1990)
2. R.B.M. Mooij. Structure studies on $fp$-shell nuclei. inis.iaea.org (1984)
3. S.J. Ahmad, K.S. Jassim, F.A. Majeed, The effect of core polarization by means of Tassie and Bohr-mottelson models for some $FP$-shell nuclei. J. Adv. Res. Dyn. Control Syst. 12(5), 200–205 (2020)
4. F.Z. Majeed, F.M. Hmood, Inelastic longitudinal electron scattering C2 form factors in $^{58}$Ni. Iraqi J. Phys. 14(29), 15–26 (2016)
5. A.D. Salman, D.R. Kadhim, Longitudinal electron scattering form factors for $^{54,56}$Fe. Int. J. Mod. Phys. E. 23(10), 145054 (2014)
6. A.K. Hamoudi, M.A. Abbas, Electron scattering from some even $fp$- and N50-shell nuclei with implanting the influence of short-range correlation functions. Chin. J. Phys. 60, 623–631 (2019)
7. A.D. Salman, S.A. Al-Ramahi, M.H. Oleiwi, Inelastic electron-nucleus scattering form factors for $^{54,66,68}$Zn isotopes. AIP Conf. Proc. 2144(1), 030029 (2019)
8. F.A. Majeed, The effect of core polarization on longitudinal form factors in $^{10}$B. Phys. Scr. 85(6), 065201 (2012)
9. F.A. Majeed, The role of the core polarization on C2 and C4 form factors of $fp$-shell nuclei. Phys. Scr. 59(1–2), 95–105 (2014)
10. F.A. Majeed, L.A. Najim, Contribution of high energy configurations to longitudinal and transverse form factors in p-and sd-shell nuclei. Indian J. Phys. 89(6), 611–618 (2015)
Longitudinal and transverse form factors from $^{65}$Cu and $^{71}$Ga nuclei

11. F.A. Majeed, S.M. Obaid, Nuclear structure study of $^{22,24}$Ne and $^{24}$Mg nuclei. Indian J. Phys. 65(2), 159–167 (2019)
12. S.A. Abbas, S.H. Salman, S.A. Ebrahiem, H.M. Tawfeek. Investigation of the Nuclear Structure of Some Ni and Zn Isotopes with Skyrme-Hartree-Fock Interaction. Baghdad Sci. J. 0914–0914 (2022).
13. M. Honma, T. Otsuka, T. Mizusaki, M. Hjorth-Jensen, New effective interaction for—shell nuclei. Phys. Rev. C 80(6), 064323 (2009)
14. B.A. Brown, W.D.M. Rae, The shell-model code NuShellX@MSU Nucl. Data Sheets 120, 115118 (2014)
15. D.E.J. Skyrme, The effective nuclear potential. Nucl. Phys. A 9, 615–634 (1958)
16. M. Ghafouri, H. Sadeghi, M. Torkiha, Self-consistent description of the SHFB equations for $^{112}$Sn. Res. Phys. 8, 734–743 (2018)
17. J. Friedrich, P.-G. Reinhard, Skyrme-force parametrization: Least-squares fit to nuclear ground-state properties. Phys. Rev. C 33, 335 (1986)
18. A. Bohr, B.R. Mottelson, Nuclear Structure, vol. 2 (Benjamin, New York, 1975)
19. B.A. Brown, R. Radhi, B.H. Wildenthal, Electric quadrupole and hexadecupole nuclear excitations from the perspectives of electron scattering and modern shell-model theory. Phys. Rep 101(5), 313–358 (1983)
20. R.A. Radhi, Z.A. Dakhil, N.S. Manie, Microscopic calculations of quadrupole moments in Li and B isotope. Euro. Phys. J. A 50(7), 1–9 (2014)
21. B.A. Brown, B.H. Wildenthal, C.F. Williamson, F.N. Rad, S. Kowalski, H. Crannell, J.T. O’Brien, Phys. Rev. C 32, 1127 (1985)
22. T.H.R. Riedeman, K. Allaart, H.P. Blok, M.N. Harakeh, K. Heyde, C.W. de Jager, H. de Vries, Electron scattering off $^{65}$Cu and $^{71}$Ga. Nucl. Phys. A 573, 173–215 (1994)
23. H. Ogawa, K. Asahi, K. Sakai, T. Suzuki, H. Izumi, H. Miyoshi, M. Nagakura, K. Yogo, A. Goto, T. Suga, T. Honda, H. Ueno, Y.X. Watanabe, K. Yoneda, A. Yoshimi, N. Fukuda, Y. Kobayashi, A. Yoshiida, T. Kubo, M. Ishihara, N. Imai, N. Aoi, W.-D. Schmidt-Ott, G. Neyens, S. Teughels, Phys. Rev. C 67, 064308 (2003)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.