Transient tunneling current of single electron transistors

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The transient tunneling current of single electron transistors (SETs) is theoretically investigated. The time-dependent current formula given by Jauho, Wingreen and Meir [Phys. Rev. B 50, 5528 (1994)] is applied to study the temperature effect on the transient current through a single quantum dot embedded in asymmetry barrier. It is found that the tunneling rate ratio significantly influences the feature of transient current. Finally, the oscillation structures on the exponential growth transient current of single hole transistors composed of germanium quantum dots is analyzed.

I. INTRODUCTION

The transport properties of single electron transistors (SETs) have been extensively studied theoretically and experimentally.\(^1\)\(^-\)\(^6\) The main structure of SETs consists of a single quantum dot (QD) and three electrodes (source, drain and gate). The manipulation of SETs is based on Coulomb blockade effect arising from the particle interactions of QD. From the practical point of view, it is important for SETs to operate at room temperature. Therefore, the size of QD is required to be less than 10 nm for silicon (Si) or germanium (Ge) semiconductor QDs. In such size range of QD, the quantum confinement effects and charging energies of QDs are larger than the thermal energy of room temperature. Recently, the Coulomb oscillation and staircase features of tunneling current of room temperature SETs have been reported by several groups, where Si or Ge QDs are embedded into SiO\(_2\) matrix.\(^4\)\(^-\)\(^6\)

Even though it is difficult to align a single QD of nanometer with electrodes in the fabrication of individual SETs, several methods were used to solve this difficulty. Nevertheless, those methods still can not precisely control the barrier thickness, which significantly influences the tunneling time for electrons to access QDs. Further understanding for the location of QDs, the implementation technology of SETs using thermal oxidation method\(^6\) can be improved. The measurement of transient current can provide above information. In this study we apply the current formula of Ref. [2] to examine the transient current of the SET with asymmetrical tunneling rates at room temperature. It is found that the transient current exhibits exponential decay when electrons are injected into the QDs from the left electrode and \(\Gamma_L/\Gamma_R < 1\), where \(\Gamma_L/\Gamma_R\) represents the tunneling rate for electrons from the left (right) electrode to the QD, on the other hand the transient current exhibits the exponential growth as \(\Gamma_L/\Gamma_R > 1\). In addition to theoretical studies of transient current, we report the experimental measurement of transient current of single hole transistor (SHT).

II. FORMALISM

The transient current of a SET consisted of a single QD and three electrodes was theoretically derived by Jauho, Wingreen and Meir.\(^2\) When applied voltage is insufficient to overcome the charging energies arising from electron-electron repulsion interactions in the QD, the time-dependent tunneling current through the ground state of QD can be expressed as (ref.[2])

\[
J(t) = \frac{1}{2} [J_L^{in}(t) + J_R^{out}(t) - J_L^{out}(t) - J_R^{in}(t)],
\]

where

\[
J_{L/R}^{in}(t) = -\frac{e}{h} \Gamma_{L/R} N(t)
\]

and

\[
J_{L/R}^{out}(t) = -\frac{e}{h} \Gamma_{L/R} \int \frac{d\epsilon}{\pi} \Gamma_{L/R}(\epsilon) ImA_C(\epsilon, t).
\]

According to Eq. (1), there are four components for the net current from the left electrode to the right electrode. The current of Eq. (2), \(J_{L/R}^{out}(t)\), denotes the current flowing out from the QD to the left (right) electrode. This current results from the time-dependent electron occupation number of QD, which is given by

\[
N(t) = \int \frac{d\epsilon}{2\pi} (\Gamma_L f_L(\epsilon) + \Gamma_R f_R(\epsilon)|A_C(\epsilon, t)|^2.
\]

\(J_{L/R}^{in}(t)\) denotes the current flowing into the QD from the left (right) electrode. Obviously, \(J_{L/R}^{in}(t)\) is determined by the time-dependent spectrum function \(ImA_C(\epsilon, t)\) where \(Im\) means taking the imaginary part of \(A_C(\epsilon, t)\). Notations \(e\) and \(h\) denote, respectively, the electron charge and Plank’s constant. For the simplicity, we assume that the tunneling rates \(\Gamma_L/\Gamma_R\) are bias and energy independent. \(\Gamma_L\) and \(\Gamma_R\) denote, respectively, the tunneling rates from the left and right electrodes to the QD.
where \( E_1 \) is the ground state energy level of QD, \( \Gamma = \Gamma_L + \Gamma_R \). In the absence of gate voltage \( V_g \) the \( A_C(\epsilon) \) becomes time-independent retarded Green’s function \( A_C(\epsilon) = 1/\epsilon - E_1 + i\Gamma/2 \). The resonant energy level \( E_1 \) is shifted to \( E_1 - eV_g \), that is \( A_C(\epsilon) = A_C(\epsilon) = 1/\epsilon - (E_1 - eV_g) + i\Gamma/2 \), when the system goes into steady state. In the transient process the density of states of QD depends on time. Consequently, \( J(t) \) displays time-dependent behavior in the transient process.

### III. RESULTS AND DISCUSSION

Due to the complicate spectrum function, the transient current lacks analytic form. To numerically calculate \( J(t) \), we set the Fermi energy level of electrodes and the ground state energy level \( E_F = 50 \, meV \) and \( E_1 = 90 \, meV \), respectively. Therefore, the energy levels of QD are empty at zero temperature under zero bias. First of all, we plot the time-dependent tunneling current through a symmetric double-barrier tunneling structure with \( \Gamma_L = 0.5 \, meV \) and \( \Gamma_R = 0.5 \, meV \) in response to a step-like modulation for different applied voltages at zero temperature in Fig. 1: solid line \((V_a = 30 \, mV)\), dashed line \((V_a = 25 \, mV)\) and dotted line \((V_a = 20 \, mV)\). The tunneling current is zero at \( t = 0 \) since the Fermi level of electrodes is below the resonant state \( E_1 \). When the gate voltage is added into the system, the left electrode is injected into the new resonant energy level \( E_1 - eV_g \). Consequently, the tunneling current jumps instantly and finally reaches the steady state. Current also displays an interesting ringing behavior, which was pointed out in Ref.[2]. The oscillations of the curve do not maintain constant frequency in the transient process. In addition, the frequency of oscillations is increased, when the separation between the Fermi energy of left electrode and the resonant level \( E_1 \) is decreased. Due to symmetry tunneling rates, the behavior of \( J(t) \) can be understood by the analysis of \( J_L(t) = J_{L,1}(t) + J_{L,2}(t) - J^R(t) \), where

\[
J_{L,1}(t) = \frac{-e}{\hbar} \Gamma_L \int_{\epsilon V_a}^{\epsilon V_a + \epsilon F} \frac{d\epsilon}{\pi} \{ \text{Im} \left[ \frac{1}{\epsilon - (E_1 - eV_g) + i\Gamma/2} \right] + \text{Re} \left[ \frac{1}{\epsilon - (E_1 - eV_g) + i\Gamma/2} \right] \} \cdot \sin((\epsilon - E_1 + eV_g) t) \exp \left( -\frac{L}{\hbar} \right).
\]
of temperature is more complicated due to the enhancement of the tunneling rate ratio $\Gamma_L/\Gamma_R$ in each other.

In addition, the magnitude of $J(t)$ is suppressed with increasing temperature since the right electrode provides current $J_R^{out}(t)$ into the QD, which is opposite to $J_L^{in}(t)$. Consequently, the net current becomes small with increasing temperature. We also show the charge density $N(t)$ in Fig. 7: solid line ($k_BT = 25$ meV) and dashed line ($k_BT = 10$ meV). We see that $N(t) = N_L(t) + N_R(t)$ approaches the same value for $k_BT = 10$ meV and $k_BT = 25$ meV in the steady state. To understand this feature, we plot $N_L$ and $N_R$, which are defined as the charge density provided from the left and right electrodes, respectively. In the steady state (or a large time), $N_L$ declines with increasing temperature, on the other hand $N_R$ increases with increasing temperature. We see that they compensate in each other.

Next, we study the transient current for the case of different tunneling rate ratio $\Gamma_L/\Gamma_R$ at room temperature $k_BT = 25$ meV. Comparing with the results at zero temperature, the oscillation structure of $J(t)$ at room temperature is more complicated due to the enhancement of $J_R^{in}(t)$. For $\Gamma_L = 0.7$ meV and $\Gamma_L = 0.3$ meV, the former exhibits the exponential decay, the latter exhibits the exponential growth. This indicates that the location of QD exists a considerable effect on the transient transport properties of SETs. Even though the above theoretical analysis is for SETs, we attempt to report the experimental measurement of the transient current of Ge SHT. For multivalley conduction band of germanium semiconductors, the intervalley interaction effect cannot be ignored. However, such effect is not included in Eq. (1). The more realistic system described by Eq. (1) is a Ge SHT, where valence band is a single valley. In Figure 9 we show the transient current of Ge SHT for two different applied voltages $V_a = 20$ mV and $V_a = 30$ mV at room temperature and $V_g = -3$ V. The detailed fabrication process of Ge SHTs and measurement technique of transient current will be discussed in elsewhere. The solid lines are the experimental curves. The dashed lines are the fitting curves. According to fitting curves, the transient currents display exponential growth feature. Based on previous theoretical analysis, the Ge SHT has the characteristic of $\Gamma_L/\Gamma_R < 1$. Two fitting curves also provide the bias-dependent tunneling times 70 sec and 60 sec for the applied bias $V_a = 20$ mV and $V_a = 30$ mV. This very long tunneling time is arising from a fact of very high barrier of $SiO_2$ which the Ge QD is embedded into. Besides, we observe the small oscillation structures on the exponential growth curves. The oscillation structures vanish for sufficient long time. This indicates that this oscillation structures are not the measurement error. Owing to the lack of information about the detailed size and shape of QD, there are some difficulties to do the detailed comparison between the theoretical calculation and experimental measurement.

IV. SUMMARY

In this study the transient current of a SET is investigated by using the formula derived by Jauho, Wingreen and Meir. The tunneling rate ratio exists a considerable influence on the transient current of SETs. The tunneling current of a single Ge SHT at room temperature displays the oscillation structures in the transient process. This feature is attributed to time-dependent density of states of QD in a switching on the gate voltage.

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Figure Captions

Fig. 1. Transient current through the symmetry barrier for different applied voltages at zero temperature and $V_g = 25$ mV: solid line ($V_a = 30$ mV), dashed line ($V_a = 25$ mV) and dotted line ($V_a = 20$ mV), where $J_0 = e \text{meV}/h$ and $t_0 = h/(\text{meV})$.

Fig. 2. Transient current through the symmetry barrier for $V_g = 25$ mV, $V_a = 30$ mV and zero temperature. Solid line is duplicated from that of Fig. 1. Dashed line and dotted line denote $J_L^{in}(t)$ and $J_L^{out}(t)$. Dash-dotted line represents $J_R^{in}(t)$.
Fig. 3. Transient current through the symmetry barrier for different applied gate voltages at zero temperature and $V_a = 30 \text{ mV}$: solid line ($V_g = 25 \text{ mV}$), dashed line ($V_g = 20 \text{ mV}$), dotted line ($V_g = 15 \text{ mV}$) and dash-dotted line ($V_g = 10 \text{ mV}$), where $J_0/e = \text{meV}/h$ and $t_0 = h/(\text{meV})$.

Fig. 4. Transient electron occupation number for different applied gate voltages at zero temperature and $V_a = 30 \text{ mV}$. The curves are one to one corresponding to those curves of Fig. 2.

Fig. 5. Transient current for different tunneling rate ratio at zero temperature, $V_a = 30 \text{ mV}$ and $V_g = 25 \text{ mV}$: the curves from the bottom to the top (for $t/t_0 < 1$) correspond, respectively, $\Gamma_L = 0.3, 0.4, 0.5, 0.6$ and $0.7 \text{ meV}$. Meanwhile, $\Gamma = \Gamma_L + \Gamma_R = 1 \text{ meV}$. Here $J_0/e = \text{meV}/h$ and $t_0 = h/(\text{meV})$.

Fig. 6. Transient current through the symmetry barrier for different temperatures at $V_a = 30 \text{ mV}$ and $V_g = 25 \text{ mV}$: solid line ($k_B T = 0 \text{ meV}$), dashed line ($k_B T = 15 \text{ meV}$), dotted line ($k_B T = 20 \text{ meV}$) and dash-dotted line ($k_B T = 25 \text{ meV}$), where $J_0/e = \text{meV}/h$ and $t_0 = h/(\text{meV})$.

Fig. 7. Transient electron occupation number for two different temperatures: solid line ($k_B T = 25 \text{ meV}$) and dashed line ($k_B T = 10 \text{ meV}$).

Fig. 8. Transient current for different tunneling rate ratio at temperature $k_B T = 25 \text{ meV}$, $V_a = 30 \text{ mV}$ and $V_g = 25 \text{ mV}$. $\Gamma = \Gamma_L + \Gamma_R = 1 \text{ meV}$. Here $J_0/e = \text{meV}/h$ and $t_0 = h/(\text{meV})$.

Fig. 9. Transient current of Ge single hole transistor for different applied voltages at room temperature and $V_g = 3 \text{ V}$. The curves from the bottom to the top correspond to $V_a = 20 \text{ mV}$ and $30 \text{ mV}$, respectively.