Bell’s theorem, the measurement problem, Newton’s self-gravitation and its connections to violations of the discrete symmetries $\mathcal{C}, \mathcal{P}, \mathcal{T}$

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Abstract. About 50 years ago John St. Bell published his famous Bell theorem that initiated a new field in physics. This contribution discusses how discrete symmetries relate to the big open questions of quantum mechanics, in particular:

(i) how correlations stronger than those predicted by theories sharing randomness (Bell’s theorem) relate to the violation of the $\mathcal{CP}$ symmetry and the $\mathcal{P}$ symmetry; and its relation to the security of quantum cryptography,
(ii) how the measurement problem (“why do we observe no tables in superposition?”) can be polled in weakly decaying systems,
(iii) how strongly and weakly interacting quantum systems are affected by Newton’s self-gravitation.

These presented preliminary results show that the meson-antimeson systems and the hyperon-antihyperon systems are a unique laboratory to tackle deep fundamental questions and to contribute to the understand what impact the violation of discrete symmetries has.

1. Introduction
Bell’s famous theorem is a statement about how Nature is not functioning. It shows that correlations stronger than those based on two main assumptions, Bell’s locality assumption and a notion of reality, exist. These correlations are known to provide the security against any eavesdropping when generating keys at two spatially separated locations. Entanglement, the impossibility to factorize a density matrix with respect to a certain algebra, can be proven for pairs of neutral meson-antimeson systems and pairs for hyperon-antihyperon systems. However, due to the decay property of these systems it is not trivial to find experimental setups for which the existence of correlations useful for outperforming classical devices, such as a secure key generation, can be verified. After introducing Bell’s theorem and its connection to quantum cryptography (Section 2) we show that the violation of the discrete symmetry $\mathcal{CP}$ (C...charge conjugation, $\mathcal{P}$...parity) is crucial for the security of a sifted key from a quantum cryptographic protocol (Section 3). Then we discuss weakly decaying systems with half-integer spins, i.e. hyperons. We discuss how entanglement can be tested for these systems violating the discrete symmetry $\mathcal{P}$ and whether Bell’ theorem can be tested. In the Section 4 we show that these systems typically produced at accelerator facilities are a unique laboratory to help to unfuzz the measurement problem and in Section 5 we argue that the assumption a possible self-gravitation in neutral $K$-meson systems leads to conceptual ambiguities. Last but not least we investigate...
how a potential violation of the $CPT$ symmetry ($T$...time reversal) would impact the entangled $K$-meson system.

2. Bell’s theorem and the security of quantum cryptography

John St. Bell was a physicist working at CERN and contributing intensively and sustainably to the development of Particle Physics and Collider Physics. His famous 1964-theorem, known nowadays as Bell’s theorem, shows that predictions of local realistic theories are different to those of quantum theory. As, e.g. one application it has been found that the violation of Bell’s theorem is a necessary and sufficient criterion for generating a secure key for cryptography at two distant locations, which we sketch in the following, after repeating the basic concepts behind Bell’s theorem.

In the famous Einstein-Podolsky-Rosen scenario a source generates two particles, which are separated and independently measured by two experimenters, Alice and Bob. Let us assume that both parties can choose among two different measurement alternatives $i = n, n'$ for Alice and $j = m, m'$ for Bob. These settings yield either the outcomes $k, l = +1$ or $k, l = -1$. Any classical or quantum correlation function can be defined, e.g., by

$$E(i, j) = \sum_{k,l} (k \cdot l) P_{kl}^{ij}(i, j),$$

where $P_{kl}^{ij}(i, j)$ is the joint probability for Alice obtaining the outcome $k$ and Bob obtaining the outcome $l$, when they chose measurements $i$ and $j$, respectively. Taking up the argument of Einstein, Podolsky and Rosen Bell’s locality assumption imposes a factorization of the joint probabilities into individual ones. Bell inequalities are then tests for correlations that can be simulated using only local resources and shared randomness (a modern terminology for local hidden variables) and have, therefore, at hitherto nothing to do with quantum theory.

Applying instead the probabilities derived by the rules of quantum mechanics, however, the inequality is sometimes violated. This is Bell’s great achievement having found a contradiction between predictions of local hidden variable theories and quantum theory!

For bipartite entangled particles with two degrees of freedom a tight Bell inequality is the famous Clauser-Horne-Shimony-Holt (CHSH) Bell inequality [9], i.e.

$$S(n, m, n', m') := E(n, m) - E(n, m') + E(n', m) + E(n', m')$$

for all realistic hidden variable theories.

$$\leq 2.$$ (2)

In quantum mechanics the $S(n, m, n', m')$-function is given by deriving the four quantum mechanical expectation values $E_{QM}^{ij}(n, m') = Tr(O_n \otimes O_{m'} \rho)$ (where $O_i$ are appropriate operators and $\rho$ is the density matrix of the bipartite state). It is straightforward to prove that only entangled states can violate CHSH-Bell inequality, but not all entangled states violate the inequality. The maximum violation

$$S_{QM} = 2\sqrt{2}$$

is reached for a maximally entangled state, e.g., the antisymmetric Bell state

$$|\psi^-\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle \right).$$ (4)
A quantum cryptographic protocol: Let us extent the Einstein-Podolsky-Rosen scenario such that Alice and Bob measure randomly and independently one out of three specifically chosen observables each, where two of those observables are equal. Moreover, we assume that the source produces without loss of generality the maximally entangled antisymmetric Bell state (4). Via a fully public open channel Alice and Bob announce their observable choices but not their measurement outcomes. Then we have two cases: Alice and Bob have chosen by chance the same or unequal observables. In case, Alice and Bob have chosen the same observable, since the source produces the antisymmetric Bell state, their measurement outcomes are perfectly anti-correlated and they can use these outcomes to both obtain a fully identical and random string of “0” and “1” (before, they have decided which measurement outcome is labeled “0” and which one is labeled “1”). In the remaining case, they announce their outcomes (even in public) and use this data to compute the four quantum mechanical correlations functions of the CHSH-Bell inequality. The protocol is also summarized in Fig. 1 for the case of neutral $K$-mesons.

If there is no eavesdropping, the CHSH-Bell inequality should be maximally violated. As proven in detail in Ref. [10] any violation of Bell’s theorem guarantees that an attack by an eavesdropper, even including the manipulation of the source, cannot reveal enough bit’s of the string of the sifted key of Alice and Bob. Consequently, Alice and Bob can be sure based on the quantum laws that there generated key is secure! These correlations that violate Bell’s theorem outperform any classical device!

Now we are prepared to connect both results, the existence of correlation stronger than those by classical physics and the violation of the discrete symmetries, i.e. to learn that the security of a sifted key depends on analyzing matter or antimatter.

3. Testing Bell’s theorem with $CP$ symmetry violating systems

Only, since 2012 a promising proposal for conclusively testing Bell’s 1964-theorem for systems usually produced at accelerator facilities, so called neutral $K$-mesons, is on the market [1]. From the theoretical point of view these systems at high energies are of great interest since — as will be presented in this contribution— a puzzling relation between the information theoretic content and the violation of discrete symmetries exists [2, 3, 4]. Discrete symmetries and their violation play an important role in the understanding of the four forces ruling the universe and may have played or possible will play an important role in the development of our universe. As is well known the violation of the $CP$ symmetry ($C$... charge conjugation, $P$... parity) is a key ingredient to understand why matter slipped off the map.

The main problems in testing Bell’s theorem conclusively are limitations that arise from the experimental side. These are in particular that only the antisymmetric Bell state (compare with Eq.(4)),

$$|\psi^-\rangle = \frac{1}{\sqrt{2}} \left\{ |K^0\rangle \otimes |\bar{K}^0\rangle - |\bar{K}^0\rangle \otimes |K^0\rangle \right\}, \quad (5)$$

is typically produced with high enough intensity and, secondly, only the strangeness content of neutral $K$-mesons can be measured by an “active” measurement procedure. Active measurements are a crucial requirement for any conclusive test of Bell’s theorem since obviously if Alice and Bob have no control over their measurement settings, it is straightforward to construct a local realistic theory resulting in the observed correlations. In particular, a decay event is a “passive” measurement procedure, i.e. no experimenter has control over into which particles the meson will decay nor at which time this decay will occur. Though—as shown in Refs. [11, 12]—a decaying system can be viewed as an open quantum system, in particular modeled by a Markovian Lindblad master equation, the decay property cannot be ignored. This, in particular, means that one is not allowed to normalize to only surviving pairs. These are all subtle points that need to be taken into account for testing Bell’s theorem conclusively in the domain of high
Figure 1. (Color online) The figure and table sketches the protocol how a key can be shifted where the security is based on physics laws. Alice and Bob choose to insert in the neutral kaon beam a piece of matter block randomly on three different distances (corresponding to a certain time $t_n$), where two distances are chosen to be the same and the other ones such that Bell’s inequality for instable systems [1] is violated. The times of Alice side are denoted by $t_n, t_n', t_m$ and the times of Bob side $t_m, t_m', t_n$. In the due of the protocol Alice and Bob exchange there basis choices (=time choices). With probability $\frac{1}{3}$ both experimenters choose the same direction and since the initial state is assumed to be anti-correlated, they know that their outcomes are anti-correlated. The two individual outcomes, Yes and No, are in this case not revealed. In the other cases also the outcome results are revealed and these data sets are used to test Bell’s inequality. The security is given by the fact that no eavesdropper can know which pairs are used for the key and which for Bell’s test.

energy systems. The requirement of an “active” measurement procedure rules out all other meson system due to short decay constants, except the neutral $\kappa$-meson system. The second requirement that all information available has to be considered makes it hard to find a Bell inequality that is violated for the observed constants in the $\kappa$-meson system. These problems were overcome by the new Bell theorem in Ref. [1].

**Actively measuring the strangeness content of neutral $\kappa$-mesons:** The experimenter places at a certain distance from the source a piece of matter that forces the incoming neutral $\kappa$-meson beam to interact with the material and to reveal the strangeness content, i.e. being at that distance in the state $|\kappa^0\rangle$ or in the state $|\overline{\kappa}^0\rangle$. Since Bell’s theorem tests against all local realistic theories one is not allowed to ignore the fact that the neutral kaon could have decayed before. Therefore the question that one has to raise has to include that information, i.e. one
has to ask: “Are you at a certain distance from the source in the state $|\mathcal{K}^0\rangle$ or not?”, which is obviously different to the question “Are you at a certain distance from the source in the state $|\bar{\mathcal{K}}^0\rangle$ or in the state $|\mathcal{K}^0\rangle$?”. 

To test Bell’s theorem given by the $S$-function, Eq.(2), one has to compute four expectation values for such active measurements of strangeness given for the antisymmetric Bell state for different distances (that one can always convert in proper times since the velocity for a given experimental setup is known). Surprisingly, an optimization over all possible distances (times) does not show any value higher than 2 and $-2$, i.e. no contradiction to local realistic theories. Why is this the case?

The point is that the oscillation in comparison to the two decay constants is too slow or, equivalently, the decay is too fast in comparison to the oscillation. Since we cannot obviously change the natural constants of elementary particles, we have to search for a different Bell inequality. Unfortunately, the CHSH-Bell version is already the most tight one. In Ref. [1] the authors derived a new type of Bell’s inequality for decaying system by including the decay property into the derivations of the bounds from local realistic theories. They assumed that any local realistic theory must also describe the well experimentally tested time evolution of single mesons correctly. This is not a strong requirement since in a typical accelerator experiment $\mathcal{K}$-mesons are only generated in pairs, in big contrast to typical photon experiments. Observing only a single event on one side, one knows with very high probability that the other one existed but due to purely experimental reasons was not detected.

With this new Bell inequality [1] taking the decay property into account without spoiling the conclusiveness, the authors show which time regions have to be investigated experimentally to reveal correlations that are stronger than those allowed by classical physics. Surprisingly, though investigating strangeness oscillation the $CP$ symmetry violation plays a crucial role! Asking the question “Are you at a certain distance from the source in the state $|\mathcal{K}^0\rangle$ or not?” or “Are you at a certain distance from the source in the state $|\bar{\mathcal{K}}^0\rangle$ or not?” makes the difference, i.e. leading in one case to a violation in the other one not!

Consequently, the security of cryptography protocols depends in a given setup on analyzing the particle or the antiparticle content (see Fig. 1)! How odd is Nature!

4. Testing Bell’s theorem with parity violating systems

In the last section we discussed neutral mesons which are spinless particles. Hyperons are half-integer spin particles that are baryons containing in addition to up or down quarks also one or more strange quarks. They decay via the weak interaction violating the $P$ symmetry. The Standard Model of elementary particles predicts also tiny contribution of $CP$ violating processes, however, no violation of the $CP$ symmetry has been up to now experimentally found. In this section we discuss the quantum information theoretic content of weakly decaying hyperons and discuss whether Bell’s theorem can be tested for these weakly decaying systems.

Any closed quantum system’s dynamic is given by the Schrödinger equation, i.e. by a unitary evolution. Some times one is only interested in a part of the closed quantum system or has only access to a part of the system, for instance a spin in a heat bath. The dynamics of the system of interest, the open quantum system, can be derived by the unitary evolution of the total system, system of interest plus environment, and then taking the partial trace of the environmental degrees of freedom (for an introduction to open quantum systems consult e.g. Ref. [13]). On the other hand, if the total Hamiltonian is not known, one can study the dynamics of open quantum systems by a proper parametrization of the dynamical map. Any time evolution of a quantum state $\rho$ can always be written in the form [15]

$$\rho(t) = \sum_i \mathcal{L}_i(t, t_0, \rho(t_0)) \rho(t_0) \mathcal{L}_i^d(t, t_0, \rho(t_0))$$  

(6)
where the operators \( \mathcal{L}_i \), called Kraus operators, are in general depending on the initial time \( t_0 \) and state \( \rho(t_0) \). In particular, the dynamical map defines a universal dynamical map if it is independent of the state it acts upon. This is only the case if and only if the map is induced from an extended system with the initial condition \( \sigma_{\text{total}}(t_0) = \rho(t_0) \otimes \rho_{\text{environment}}(t_0) \) where \( \rho_{\text{environment}}(t_0) \) is fixed for any \( \rho(t_0) \).

In Ref. [16] it has been shown that any hyperon decay process can be modeled efficiently by an open quantum formalism, i.e. via these Kraus operators. Typically, the directions of the momentum of the daughter particles of a hyperon are measured and depend on the initial spin state of the hyperon [14]. In the weakly decay process there are two interfering amplitudes, one conserves and one violates the parity \( P \) symmetry.

A typical momentum distribution of the daughter particle of a decaying hyperon computes to \( \langle \theta, \phi \rangle \) are the angular coordinates of the momentum direction of one daughter particle and \( \rho_{\text{spin}} \) the density operator corresponding to the spin degrees of freedom of the decaying hyperon) [16]

\[
I(\theta, \phi) = \text{Tr}_{\text{spin}}(\mathcal{L}_+ \rho_{\text{spin}} \mathcal{L}_+ + \mathcal{L}_- \rho_{\text{spin}} \mathcal{L}_-)
\]

where the Kraus operators \( \mathcal{L} \) have the conceptually simple form \( (\omega_+ > 0) \)

\[
\mathcal{L}_\pm = \sqrt{\omega_\pm} (\mathcal{J}_1 \pm \mathcal{J}_2)
\]

with \( \omega_+ + \omega_- = 1 \). The two Blochvectors \( \mathcal{J}_{1,2} \) have to be orthogonal, \( \mathcal{J}_1 \cdot \mathcal{J}_2 = 0 \), since the transition is completely positive and are chosen such that they have maximal length \( |\mathcal{J}_{1,2}|^2 = s(2s + 1) \) \( (s \ldots \text{spin number}) \).

A Blochvector expansion of a density matrix is generally given by \( \rho = \frac{1}{d} \{ I_d + \vec{b} \cdot \vec{\Gamma} \} \) where \( d \) is the dimension of the system [17]. Since we are dealing with spin-degrees of freedom we have \( d = 2s + 1 \) and we can choose as a set of orthonormal basis the generalized Hermitian and traceless Gell-Mann matrices \( \vec{\Gamma} \) \( (s = \frac{1}{2} \text{ they correspond to the Pauli matrices}) \). Given this structure we can reinterpret the weak decay process as an incomplete spin measurement of the decaying particle

\[
I(\theta, \phi) = \omega_+ \text{Tr}(\Pi_{\mathcal{J}_1 \pm \mathcal{J}_2} \rho_{\text{spin}}) + \omega_- \text{Tr}(\Pi_{\mathcal{J}_1 \mp \mathcal{J}_2} \rho_{\text{spin}})
\]

\[
= \frac{1}{(2s + 1)} \{ 1 + (\omega_+ + \omega_-) \mathcal{J}_2 \cdot \vec{s} \} \]

(7)

where \( \vec{s} \) is the Bloch vector representation of \( \rho_{\text{spin}} \), i.e. \( \vec{s} = \text{Tr}(\vec{\Gamma} \rho_{\text{spin}}) \). With probability \( \omega_+ \) the spin state of the hyperon is projected onto direction \( \mathcal{J}_1 + \mathcal{J}_2 \) or with the remaining probability \( \omega_- \) the initial spin state is measured along the direction \( \mathcal{J}_1 - \mathcal{J}_2 \). Thus the weak process can be associated to a spin measurement with an imperfect Stern-Gerlach apparatus (switching with probability \( \omega_\pm \) the magnetic field). The imperfection has two causes: Firstly, the difference \( (\omega_+ - \omega_-) \) equals an asymmetry (denoted in the following by \( \alpha \)) and is a typical measurable constant for each hyperon. The asymmetry corresponds to interference contrast (visibility) times the cosine of the phase shift of the two interfering amplitudes, one is parity \( P \) conserving and one violates the symmetry \( P \). Secondly, the two directions \( \mathcal{J}_1 \pm \mathcal{J}_2 \) are typical for the spin number \( s \). Indeed, for \( s = \frac{1}{2} \) the Blochvector \( \mathcal{J}_1 \) is zero, thus only two directions, \( \pm \mathcal{J}_2 \), are chosen by Nature.

Entangled hyperons can be produced, e.g. by proton-antiproton annihilations. The introduced open quantum formalism allows for a straightforward extension by the tensor product of the Kraus operators [16]. Let us assume that (i) there is no initial correlation between the momentum degrees of freedom and the spin degrees of freedom and (ii) there is no entanglement between the momentum degrees of freedom. Experiments [18, 19, 20], e.g. for the spin-\( \frac{1}{2} \) \( \Lambda \) hyperon and \( \Lambda \) anti-hyperon, suggest that the initial spin state is a maximally entangled Bell
state (except for backward scattering angles). Therefore without loss of generality we can assume that (iii) the spin degrees of freedom of the particle and antiparticle are produced in the antisymmetric Bell state (compare with Eq.(4))
\[ |\psi^-\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow \Lambda \rangle \otimes |\downarrow \bar{\Lambda} \rangle - |\downarrow \Lambda \rangle \otimes |\uparrow \bar{\Lambda} \rangle \right). \]

Then the computation of the angular distribution of the momenta of the two daughter particles of the \( \Lambda \) and \( \bar{\Lambda} \) results in
\[ I(\theta_\Lambda, \phi_\Lambda; \theta_{\bar{\Lambda}}, \phi_{\bar{\Lambda}}) = \frac{1}{4} \left\{ 1 - \alpha_\Lambda \alpha_{\bar{\Lambda}} \vec{n}_\Lambda \cdot \vec{n}_{\bar{\Lambda}} \right\}. \]

Since the Bloch vectors
\[ n_{\Lambda/\bar{\Lambda}} = \begin{pmatrix} \sin \theta_{\Lambda/\bar{\Lambda}} \cos \phi_{\Lambda/\bar{\Lambda}} \\ \sin \theta_{\Lambda/\bar{\Lambda}} \sin \phi_{\Lambda/\bar{\Lambda}} \\ \cos \theta_{\Lambda/\bar{\Lambda}} \end{pmatrix} \]
are multiplied, \( \vec{n}_\Lambda \cdot \vec{n}_{\bar{\Lambda}} \), by the constants \( \alpha_\Lambda \cdot \alpha_{\bar{\Lambda}} \), Törnqvist [21] concluded that the hyperon \( \Lambda \) decays “as if it had a polarization \( \alpha_\Lambda \) tagged in the direction of the \( \pi^+ \) (coming from the \( \bar{\Lambda} \)) and vice versa”. The knowledge of how one of the \( \Lambda \)'s decayed – or shall decay (since time ordering is not relevant) – reveals the polarization of the second \( \Lambda \). He concludes that this is the well-known Einstein-Podolsky-Rosen scenario.

Does the imperfection of the spin measurement allow for detection of entanglement?

In general entanglement is detected by a certain observable that can witness the entanglement content, i.e. a Hermitian operator \( W \) for which holds \( \text{Tr}(W \rho) < 0 \) for at least one state \( \rho \) and \( \text{Tr}(W \rho_{\text{sep}}) \geq 0 \) for all separable states \( \rho_{\text{sep}} \). For the antisymmetric Bell state such an optical entanglement witness is given by \( W = \frac{1}{2} \left( I \otimes I + \sum_i \sigma_i \otimes \sigma_i \right) \) (any other witness can be obtained by local unitary transformations). Since the weak interaction only allows for an imperfect spin measurement we have to multiply the spin part by \( \alpha_\Lambda \alpha_{\bar{\Lambda}} \). Thus the entanglement witness for the \( \Lambda \bar{\Lambda} \) system results in
\[ \frac{1}{3} - \alpha_\Lambda \alpha_{\bar{\Lambda}} \geq 0 \text{ for } \rho_{\text{sep}}, \]
which is clearly violated since \( \alpha_\Lambda \alpha_{\bar{\Lambda}} = 0.46 \pm 0.06 \) [22]. Therefore, the measurement of the correlation functions \( \langle \sigma_i \otimes \sigma_i \rangle \) in \( x, x \) and \( y, y \) and \( z, z \) directions of the \( \Lambda \) and \( \bar{\Lambda} \) reveals entanglement. Let us here emphasize that a re-normalization (dividing by \( \alpha_\Lambda \alpha_{\bar{\Lambda}} \)) is not proper since also an mixed separable state may give the value up to \( \frac{1}{3} \). Generally, one can say that the asymmetries lead to imperfect spin measurements which shrink the observable space. Equivalently, we can say that the given interferometric device leads to a shrinking of the Hilbert space of the accessible spin states.

However, does the imperfection of the spin measurement allow for detection of correlations stronger than those of classical physics?

For that we have to investigate Bell’s inequalities and in principle all its variants. The CHSH-Bell type one, Eq. (2), leads to [16]
\[ \alpha_\Lambda \alpha_{\bar{\Lambda}} \leq \frac{1}{\sqrt{2}}. \]

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This is clearly not violated since \( \alpha_4 \approx (0.46 \pm 0.06)! \) However, here we anyway missed a requirement for any conclusive test of Bell’s theorem: active measurements! The weak interaction chooses the quantization axes \( \pm \vec{\alpha}_2 \) spontaneously, we just know the probabilistically which one (with the probabilities \( \omega_{\pm} \)). Thus Bell’s theorem cannot be tested in this way!

5. Testing a solution of the measurement problem with strange particles

Depending on the preferred interpretation of the quantum theory there exists the so called “measurement problem” or an explanation to the questions “why do we see no macroscopic objects -like the proceedings you are holding in your hands- in superposition?” or “is there a transition from a quantum world to a classical world?”.

Ghirardi, Rimini and Weber [23] constructed the first mathematical concise modification of the Schrödinger equation that breaks for macroscopic systems the linearity, i.e. the superposition principle. Such models modeling physically the collapse are nowadays called collapse models. According to collapse models a particle undergoes spontaneously a localisation with a certain mean frequency. The effect adds up for quantum objects consisting of several constituents such that the effect gets stronger and, subsequently, no superpositions survive for observable time periods for systems that are typically called macroscopic systems (such as a chair). Thus collapse models provide a theoretical framework for understanding how a classical world emerges from quantum mechanics. Since these models propose a deviation of the dynamics of physical systems from the one of standard quantum mechanics these models are experimentally testable!

The most popular model is the CSL model (Continuous Spontaneous Localisation). It introduces two new terms to the Schrödinger equation breaking the linearity and over a stochastic average super-luminal signaling is avoided. Two new phenomenological constants are introduced, a coherence length \( r_c = 10^{-5} \text{cm} \) and collapse strength \( \gamma_{\text{GRW}} = 10^{-30} \text{cm}^3/\text{s} \) [23] or \( \gamma_{\text{Adler}} = 10^{-22} \text{cm}^3/\text{s} \) [24], and would play the role of natural constants if collapse models turn out to be a good description of Nature. These values are in agreement with all known experimental data. Much larger values are ruled out because the collapse would become so strong to be detectable also for isolated microscopic systems, contrary to experimental evidence. Much smaller values are also ruled out, because in such cases the collapse would become so weak that the localization of the wave function of macroscopic objects would not be guaranteed anymore. Without this, collapse models would lose their immediate interest.

In the following analysis for strange particles, we consider the strongest value of the collapse strength \( \gamma_{\text{Adler}} \). The collapse rate is then for this kind of models typically defined by

\[
\Lambda = \frac{\gamma_{\text{Adler}}}{8\pi^2 r_c^3} \simeq 10^{-9} \text{Hz}.
\] (13)

A very interesting underground experiment in Gran Sasso by the IGEX collaboration [25, 26] searches for X-rays as a signature of the mechanism inducing the spontaneous collapse of the wave function (see also contribution in this Proceedings [27]). Collapse models predict for charged particles an emission of electromagnetic radiation which is not the case for standard quantum mechanics. By measuring the radiation the experiment constrains the mean frequency of a potential spontaneous collapse. Typically collapse tests are put to reality by bringing more and more massive system into interference and searching for a breakdown of the superposition that cannot be explained by decoherence. However, one can also follow a different road by studying systems that naturally oscillate: neutrinos, neutral mesons, and chiral molecules. These systems represent a natural case-study for testing quantum linearity [28, 29]. We focus here on neutral \( K \)-mesons and discrete symmetries in the following.

The first problem that arises when deriving the modified dynamics of the time evolution of neutral \( K \)-mesons comes from the fact that the collapse takes place in position space whereas the strangeness oscillation takes place in the flavor space. The second immediate question is
whether the mass eigenstates couple separately to the noise field or the strangeness eigenstates. In Ref. [29] a certain reasonable ansatz was chosen and after a cumbersome and lengthly computation the following modification due to spontaneous collapse assumed by the CSL model was obtained ($\mathcal{CP}$ violating effects were neglected):

$$P(K^0, t; |K^0|) = \frac{1}{4} \left\{ e^{-\Gamma_s t} + e^{-\Gamma_L t} - \cos(\Delta m t) \cdot e^{-\Gamma_L t} \cdot \frac{e^{\frac{-(\Delta m)^2}{m_0^2} t}}{\text{CSL effect}} \right\}. \quad (14)$$

The effect of a spontaneous collapse would reveal itself by a damping of the strangeness oscillation. Interestingly, the effect is proportional to the mass difference of the weak interaction squared ($\Delta m^2$) giving rise to the strangeness oscillation ($m_0$ is a reference mass, typically 1 amu).

Obviously, certain decoherence models would lead to a similar damping, however, proceeding to entangled $K$-mesons the time dependent would allow in principle a possibility to distinguish the prediction of the CSL model and certain decoherence models. Decoherence models for entangled $K$-mesons have been proposed [30, 31] and have been compared with experimental data for various meson systems [32, 33, 34, 35, 36, 37]. However, since the time resolution is not good enough yet, a direct comparison of environmental decoherence is not possible.

The above result was obtained by neglecting the $\mathcal{CP}$ violating effects. Though this effect is small ($O(10^{-3})$), however, it has a huge effect on the dynamics and as such cannot be safely neglected. Investigations are ongoing.

6. Testing gravity with strange particles

One of the biggest challenges in physics is the search for a theory that consistently combines quantum theory and gravitation. Most physicists believe that—whatever the correct quantum theory of gravity is—in the low-energy limit gravity can be described by a perturbative quantum field theory, in full analogy to the low-energy limit of Quantum Electrodynamics. However, there is no experimental evidence, to date, that rules out a theory in which gravity remains unquantized, even at the fundamental level. Even following the idea that gravity has to be quantized, it was put forward that the naive perturbative quantization of the gravitation field does not need to work out.

For non-relativistic quantum mechanics a nonlinear equation was put forward by Diosi [38] and Penrose [39] that is dubbed Schrödinger-Newton equation

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = \left( -\frac{\hbar^2}{2m} \nabla^2 - Gm^2 \int d^3 \vec{r}' \frac{\rho(\vec{r}', t)}{|\vec{r} - \vec{r}'|} \right) \psi(\vec{r}, t) \quad (15)$$

that takes the self-gravitation of the quantum system under investigation into account. This equation can—as the collapse models (see previous Section)—also be considered as a possible solution of the quantum measurement problem. It has been shown that this equation can be considered as a non-relativistic limit of classical gravity [40, 41, 42, 43].

Thus the question is how does gravity source the quantum system. When considering neutral mesons, in particular neutral $K$-mesons, one immediately runs into conceptual problems:

- Do the two masses of the weak interaction Hamiltonian couple independently to the gravitational field?
- Or is only the rest mass of the neutral $K$-meson (strong interaction processes) the one relevant for gravitational effects?
In Ref. [44] the effect for these two different scenarios were considered and the change in the dynamics of the strangeness oscillation, namely a shift in energy which is half the one for first scenario.

Our current understanding of the non-relativistic limit of particle physics, as well as the foundations of the Schrödinger-Newton equation (provided it is correct), is not sufficient to derive the Schrödinger-Newton effects for flavor oscillating systems unambiguously. In particular, there is no definite unambiguous answer, which are the relevant masses and how to treat the spatial wave-function (as for collapse models). Again, \( CP \) violating effects have not yet been taken into account and are expected to lead to more ambiguities.

7. Testing the \( CPT \) symmetry with strange particles

The \( CPT \) invariance is intrinsic to all known local effective relativistic field theories without gravity, such as those upon which current particle-physics phenomenology is based. This symmetry, for which the most sensitive tests are given by the \( K \)-meson system, has not been found to be violated experimentally. Assuming a violation of this symmetry sounds at first sight dangerous since it is then not clear how particles and antiparticles relate. In the last years, however, several ideas have been developed: \( CPT \) may be violated, e.g., as the result of a breakdown of Lorentz symmetry, as proposed in the Standard Model Extension models [45] or in models with quantum gravity backgrounds [46, 47, 48]. For example, in Ref. [49] \( CPT \) symmetry violations during the lepton genesis in the early universe are considered that may help to understand the problem of the missing antimatter in our present-day universe.

For entangled \( K \)-mesons it was claimed that, irrespective of \( CP \) and possible \( CPT \) symmetry violations, the bipartite state obtained from a \( \phi \)-meson decay is an antisymmetric spin singlet state being a direct consequence of the bose statistics and the assumption of conservation of angular momentum. This, in turn, implies that the neutral meson-antimeson state must be symmetric under the \( C \)\( P \) symmetry. There is an implicit assumption here, any violation of \( CPT \) must occur within quantum mechanics, e.g. due to spontaneous Lorentz symmetry violation for which the operator \( CP \) would then have to be still well defined. If, however, the \( CPT \) is intrinsically violated, e.g. via decoherence scenarios of the space-time foam, the concept of an “antiparticle” can be modified perturbatively. This means that the Hilbert space of the antiparticle will have components that are independent of the particle’s Hilbert space. Such a scenario leads to a different initial state of a decaying \( \phi \)-meson [50, 51], i.e. an additional symmetrical Bell state contribution

\[
|\psi^-\rangle = \frac{1}{\sqrt{2}} \left\{ |K^0\rangle \otimes |K^0\rangle - |\bar{K}^0\rangle \otimes |\bar{K}^0\rangle \right\} + \frac{\omega}{\sqrt{2}} \left\{ |K^0\rangle \otimes |\bar{K}^0\rangle + |\bar{K}^0\rangle \otimes |K^0\rangle \right\},
\]

where \( \omega \) is a complex number quantifying the \( CPT \) symmetry violating effects associated to the different Nature of particles and antiparticles. The derivation of such a scenario predicts that the order of the effect is at most

\[
|\omega| \approx \sqrt{\frac{m_K^2}{m_{\text{Plank}} (\Gamma_s - \Gamma_L)}} \approx 10^{-3}.
\]

The KLOE collaboration has measured the values of \( \omega \) and is starting to reach the interesting region of the Plank scale [52].

8. Outlook

Bell’s theorem has started to conquer the realm of high energy physics giving the hope to solve very long-standing fundamental problems via a new road.
Acknowledgements: The author gratefully acknowledges the Austrian Science Fund projects FWF-P26783.

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