Chapter 11

Super-Eddington accretion; flow regimes and conditions in high-z galaxies

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We review and discuss theoretical studies addressing the possibility of gas accretion onto black holes occurring at rates exceeding the Eddington limit. Our focus is on the applications to the growth of black hole seeds at high redshift. We first present the general notion of Super-Eddington accretion, and then summarize the different models and numerical simulations developed to study such regime. We consider optically thick flows in accretion disks as well as in spherically symmetric envelopes, and devote particular attention to the widely adopted model based on the SLIM disk solution. While attractive for its simplicity, the SLIM disk solution is challenged by the latest generation of three-dimensional radiation (magneto)-hydrodynamical simulations, in which radiative losses can be an order of magnitude higher, and the mechanisms of radiation transport is more complex than straight advection as it takes place in a complex turbulent regime. We then discuss the gas supply rate to the sub-pc scale accretion disk or envelope from larger scales, revisiting gas inflow rates in protogalaxies under various conditions. We conclude that in the dense gaseous nuclei of high-z galaxies the conditions necessary for the onset of Super Eddington accretion regimes, such as a high optical depth and high gas supply rates from large scales, should be naturally met. Feedback from the growing BH seed should not alter significantly such conditions according to the results of radiation magneto-hydrodynamical simulations of super-critical flows in accretion disks. Furthermore, based on the required nuclear gas inflow rates and the tendency of stellar feedback to remove efficiently gas in low mass halos, we argue that super-critical accretion will be more easily achieved in relatively sizable halos, with virial masses $M_{\text{vir}} > 10^{10} M_\odot$, which become more common at $z < 15$. 

Preprint of a review volume chapter to be published in Lattif, M., & Schleicher, D. R. G., “Super-Eddington accretion; flow regimes and conditions in high-z galaxies”, Formation of the First Black Holes, 2018 © Copyright World Scientific Publishing Company, www.worldscientific.com/worldscibooks/10.1142/10652
1. Introduction

In this chapter we will cover and summarize work carried out to model Super-Eddington flows on massive black hole seeds. These are flows in which the accretion rate onto a massive seed black hole occurs at a rate higher than predicted by the Eddington limit. We refer to Chapter 10 for the basic concepts of accretion onto black holes and the general understanding of massive black hole growth on cosmic timescales. Here we recall that for a black hole of mass $M_{BH}$, the Eddington limit on the mass accretion rate $\dot{M}_{Edd}$ is set by imposing perfect balance between the gravitational force pulling matter towards the black hole and radiation pressure of the emerging photons exerting a force in the direction opposite to the accretion flow. It is obtained under the assumption of spherical symmetry and is given by $\dot{M}_{Edd} = 4\pi GM_{BH}m_p/\epsilon c \sigma_T$, where $G$ is the gravitational constant, $m_p$ is the proton mass, $\epsilon$ is the radiative efficiency of the accretion process, and $\sigma_T$ is the Thomson scattering cross section for the emerging photons by which radiation pressure is communicated to matter. We also recall that the characteristic accretion timescale expressed as a function of the Eddington luminosity $L_{Edd} = \epsilon \dot{M}_{Edd} c^2$ reads:

$$t_{acc} = \left(\frac{\epsilon}{1-\epsilon}\right) \left(\frac{L_{Edd}}{L}\right) t_E = (4.3 \times 10^7 \text{yr}) \left(\frac{L_{Edd}}{L}\right),$$  \hspace{1cm} (1)

where $t_E = M_{BH} c^2/L_{Edd}$ is the Eddington timescale, and the last equality assumes $\mu_e = 1.15$ (valid for primordial gas) and a radiative efficiency of $\epsilon = 0.1$.

As mentioned in Chapter 10, unless the formation of massive black hole seeds by e.g. direct collapse is invoked, Super-Eddington accretion phases are required in order to grow light seeds from Pop. III stars, with typical masses $\sim 100 M_\odot$ or lower, to the gargantuan masses of bright QSOs, $10^8 - 10^{10} M_\odot$ [Mortlock et al. 2011; Bañados et al. 2018] by $z = 6$. It is thus of crucial interest to understand if Super-Eddington accretion can occur under realistic conditions.

For astrophysically relevant models of Super-Eddington accretion there are at least two key aspects that warrant consideration. One is, of course, under which conditions an accretion disk or, more generically, a gaseous envelope, can supply gas to a black hole seed at a rate higher than Eddington, and whether this can be achieved in a nearly steady-state regime or only episodically [Pezzulli et al. 2016]. The other aspect concerns understanding the conditions under which the accretion disk/envelope can
be fed at a rate well above Eddington via gas inflows initiated at much larger scales. Tackling both requires studying and modeling the gas flow along with the various physical processes governing its behaviour across a prohibitively wide range of scales, from several kiloparsecs to centi-parsecs. Such complexity is conceptually and computationally challenging, and is absent in the case of stellar mass black holes for which only the scale of the accretion disk is relevant as external gas flows are negligible (unless the black hole is part of a binary system, in which case other considerations may apply). The problem of studying gas inflows at galactic scales is al-
ready by itself a multi-scale process since its natural boundary conditions are determined by gas dynamics in the circumgalactic medium. This is especially true when a galaxy is still in the early assembly phase and thus is hosted in a dark matter halo vigorously fed by cold filaments originating from the cosmic web (the so-called "cold accretion" mode, see e.g. Dekel and Birnboim (2006); Keres et al. (2009). The timescales of gas inflows may vary from scale to scale as different phenomena drive them, e.g. nuclear bars and spiral density waves in galactic nuclei, tidal interactions, mergers and bar-instabilities on galactic scales, cold flows and feedback-driven galactic fountains on larger scales. Ultimately, the duty cycle of gas feeding to the black hole seed will be determined by the concerted action of all these processes, hence a characteristic timescale is hard to define. Because of that the simplest approach will have to assume a boundary value for $\dot{M}$, constant or time-dependent, and explore whether or not Super-Eddington regimes can persist. This is indeed the approach followed by numerical simulations that model directly a accretion in a disk/envelope surrounding the black hole.

While the possibility of super-critical accretion onto black holes has been considered since a long time (Begelman, 1979; Abramowicz et al., 1988), it is only recently that hydrodynamical simulations have become capable of studying its feasibility in realistic astrophysical environments, including in the specific case targeted by this chapter, namely massive black holes and black hole seeds. Pioneering numerical work over a decade ago has been carried out by Ohsuga et al. (2005), who identified very clearly two key factors leading to super-critical accretion: photon trapping and the anisotropy of the radiation field. These two aspects play a different role in the various super-critical simulations of accretion disks or accreting spherical envelopes that were developed later, as we will discuss in this chapter.

Following the rationale of the book we will focus on host galaxies at very high redshift, between $z = 20$ and $z = 6 - 7$, namely up to as late as the epoch of appearance of the first QSOs (as early as $z = 7.5$ according to Bañados et al. (2018). We recall that the first motivation for studying Super Eddington accretion at such early epochs is to find out if light BH seeds formed by the collapse of primordial metal-free Pop. III stars could grow rapidly and become as massive as required to explain the population of bright high-$z$ QSOs at $z > 6$ discovered in the last decade (see Chapter 12). It is indeed the only plausible alternative route to postulating a direct collapse scenario, a topic covered in other chapters of this book. As we
will see, though, simulations are revealing that galaxy assembly is highly dynamical and complex at such high redshifts, and has wide range of flow conditions. Therefore, the nature and strength of gas inflow rates at scales above the accretion disk/envelope depend on galaxy mass as well as on the nature of gasdynamics in the host galaxy. In other words, galaxies at such high redshifts are by no means simple. Observational evidence on the nature of gas flows and stellar components of galaxies is still lacking, so this is still mostly theorists' playground. Yet the advent of ALMA, JWST, WFIRST, EUCLID and large ground based telescopes such as E-ELT will change the landscape soon, not only by providing enough sensitivity to detect emission from the early stages of massive BH seed growth, but also allowing to distinguish, in principle, between different scenarios for their initial birth and subsequent growth, including eventually the regime of super-critical growth (see chapter 14).

The chapter is organized as follows. In section 2 we begin by discussing theoretical scenarios for the realization of Super-Eddington accretion, starting from relatively idealized models such as the popular SLIM disk model, and we will then move on to discuss the results of complex three-dimensional simulations that include radiation, the effect of the magnetic and , in some cases, also solve the set of equations in the relativistic regime. A major distinction, in this context should be done between accretion occurring in a rotationally supported configuration, namely an accretion disk, and accretion occurring in a spherically symmetric envelope. For the latter reason two separate subsections are devoted to the two different flow regimes. Section 3 concerns the conditions of the large scale gas inflows in protogalaxies and galaxies at high-z, namely focusing on how we can inform the boundary condition for the accretion flow onto a black hole seed in a galactic/proto-galactic nucleus. Finally, section 4 describes the first attempts made to combine models of super-critical accretion such as those discussed in section 2 with the galactic astrophysics framework covered in section 3. hydrodynamical simulations of galactic/proto-galactic nuclei. We conclude with section 5, which summarizes the whole chapter.

2. Super-Eddington flows: direct models of the accretion process

This section is devoted to discuss theoretical models and simulations carried out at scales smaller than the Bondi radius of the black hole to study directly the accretion process, eventually down to the radius of the last
stable orbit. The works summarized here represent the backbone of our current understanding of Super-Eddington accretion. There is, however, no direct information on nuclear and galactic-scale gas dynamics as these models, due to their computational complexity, cannot cover a wide range of scales. We have divided the section into two main parts, one focusing on flows in accretion disks and one on flows in extended envelopes that are not rotationally supported.

2.1. Super-Eddington flows in accretion disks: from the SLIM disk models to 3D MHD simulations

For accretion flows occurring in rotating disk-like configurations the SLIM disk model (Begelman, 1979; Abramowicz et al., 1988) has is a well-established framework yielding well understood super-critical flow solutions. Analogously to Shakura-Sunyaev "alpha" disks, in SLIM disks dissipation is described by a viscous stress tensor that is proportional to pressure. Slim disk solutions are obtained by solving vertically-averaged radial momentum equations. Later Sadowski (2011) improved the model introducing a vertical structure by solving the the fully relativistic, axisymmetric hydro-

Fig. 2. In the left panel we show the history of the black hole accretion rate at 10 Schwarzschild radii for four representative simulations of accreting SMBHs in Jiang et al. (2017), in units of the accretion rate corresponding to the Eddington limit. In the left panel the radiative and kinetic luminosity in units of the Eddington luminosity, time-averaged and plotted as a function of distance, are shown for two of such runs. Time is shown in a dimensional units, the same in each run.
dynamical equations in a Kerr metric augmented with hydrostatic equilibrium, a vertical energy profile and energy transport differential equations. In this steady-state model accretion can occur in a Super-Eddington regime if the medium is optically thick, so that radiation is advected with the flow rather than escaping. In order for the latter condition to be satisfied the photon diffusion timescale has to be much longer than the radial advection timescale, which is governed by viscous transport. By construction this requires a very high optical depth, and indeed steady-state models such as those in Sadowski [2011] have an optical depth $\tau > 10^3$ beyond ISCO (the innermost stable circular orbit around a Kerr black hole). Note that, because of the high optical depth, the vertical energy transport equations can be solved using the diffusion approximation (as in Sadowski, 2011). The timescale constraints in the SLIM disk model imply that no outflows are possible because, for that to happen, radiation pressure has to overcome bulk thermal and turbulent pressure. This result is at variance with what will happen in a standard radiative accretion disk in which, as the inflow rate rises above the Eddington limit, a radiation pressure-driven outflow would occur. As we will see later, realistic 3D MHD simulations suggest that neither picture is correct, namely while advection is important and partial photon trapping can occur in a dense, heavily mass loaded accretion disk, outflows can also occur. The extent by which outflows are important, and how luminous above Eddington can the disk be, is dependent on the nature of radiative transport. The jury is still out on which numerical approach to radiative transport is capturing the energy flow via radiation most realistically. Therefore, this is an area of research in which new insight is strongly tied to advances in simulation methods.

A straightforward way to characterize the SLIM disk model is to state that accretion is highly radiatively inefficient compared to the standard efficiency in Shakura-Sunyaev accretion disks around a Kerr black hole. This is because the photon diffusion timescale becomes much longer than the other characteristic timescales in the disk, namely the sound crossing timescale and the viscous timescales associated with the mechanism of mass transport. Indeed SLIM disk solutions imply a radiative efficiencies low as low as $< 0.1\%$ [Sadowski et al. 2013; Sadowski and Narayan 2016] as opposed to $\sim 40\%$ in a standard viscous disk around a spinning BH. As a result, $\dot{M}$ can be 1-2 orders of magnitude above $M_{\text{EDD}}$ while the emitted luminosity remains close to the Eddington limit. This has important consequences on the energetic feedback onto the surrounding interstellar medium, as discussed below (see section 4). A low radiative efficiency also
characterizes the advection dominated flow (ADAF) model, except that in
that the medium is optically and geometrically thin, which can only occur
with very low accretion rates (Sub-Eddington). The latter is the oppo-
site situation of what is needed in the SLIM disk model, which requires a
medium that is optically thick, which yields long diffusion timescales, and
thus relatively hot. An attractive feature of the SLIM disk model is that
a simple fitting formula for the radiative efficiency, weakly dependent on
the spin of the Kerr black hole, can be found (see Sadowski et al. 2009; Madau
et al. 2014). The latter was readily employed in large-scale MBH accretion
models in place of the standard viscous disk radiative efficiency, as we will
discuss in section 4.

While 2D radiation-hydro simulations of accretion disks or spherically
symmetric envelopes have been available since the early 2000s (see Ohsuga
et al. 2005 and section 1), it is only recently that simulations that include
the third dimension as well as MHD to study the Super-Eddington regime
have appeared in the literature (see example in Figure 1, from Jiang et al.
2017). Currently there a just a handful of 3D super-critical accretion disk
simulations. They are carried out by different research groups employing
different numerical techniques and approaches, each with its own advan-
tages and disadvantages. Overall they can achieve \( \dot{M} \) in the range several
tens to a few hundred times \( \dot{M}_{\text{Edd}} \) (Figure 2 and 3). A large, super-critical
\( \dot{M} \) is imposed as a boundary inflow condition at the beginning of the sim-
ulation (Sadowski and Narayan 2016; Jiang et al. 2017). Modern MHD
simulations are carried out at high resolution with low-diffusivity numeri-
cal schemes that can capture accurately the magneto-rotational instability
(MRI). The latter is found to be the main angular momentum and mass
transport mechanism in the accretion disk, in agreement with the findings
for the more conventional regime of accretion in presence of low rates of
mass loading. MRI is thus the main source of viscous transport, although
evidence is mounting that in dense optically thick disks other transport
mechanisms are also at play.

One group of simulations, primarily by Sadowski and collaborators,
comprises fully relativistic 3D MHD calculations with a finite difference
code combined with a relatively simple treatment of the radiation part, ei-
ther using flux-limited diffusion (FLD, e.g. Sadowski et al. 2013), or, more
recently, the M1 approximation (Sadowski and Narayan 2016). These sim-
ulations so far have focused on accretion onto stellar mass black hole rather
SMBHs, although there is one single attempt to study the accretion around
a 1000 \( M_\odot \) MBH in Sadowski and Narayan (2016) which yielded qualita-
tively similar results.

A second type of simulations (Jiang et al. 2014, Jiang et al. 2017) comprises non-relativistic 3D MHD calculations with finite volume hydro methods (ATHENA) using a highly sophisticated radiative transport schemes such as the variable Eddington tensor (VET) scheme coupled with solving the steady-state radiative transfer equation at each timestep. With the latter approach the very inner region of the disk, including the generation of a relativistic jet, cannot be properly modeled, but radiation transport is more accurate and realistic as it takes into account the actual flow configurations. This can have significant effects on the nature of energy and momentum transport through the disk as well as on the net energy loss. Very recently the latter simulation method has been applied to studying accretion onto a SMBH of mass $10^8 M_\odot$ (Jiang et al. 2017).

Perhaps not surprisingly given the differences in the numerical methods, the results of simulations carried out by different groups are quite different. Both the radiative efficiency and the relative proportion between the radiative and mechanical energy output generated by accretion differ significantly. In particular, the simulations with the VET method produce higher radiative efficiencies, at least one order of magnitude above those of Sadowski and Narayan (2016). The latter result is still mostly based on studying accretion onto stellar mass black holes. Jiang et al. (2017) have suggested that the cause of the difference might be the more pronounced anisotropy of the radiative flux captured with the VET method as opposed to the M1 method. The M1 approximation is indeed still a moment-based approach to the radiative transfer equation, which means it still integrates out the information on directionality of photon propagation to some extent. A higher anisotropy in the propagation pattern allows radiation to percolate more efficiently through a clumpy, turbulent medium with large inhomogeneities. MRI, as expected, gives rise to a turbulent flow regime, which naturally creates a highly inhomogeneous flow (Figure 1).

When compared to the standard SLIM disk model, the simulations of Jiang et al. (2017), which are the only ones to focus exclusively on accretion onto supermassive black holes, are those that show the most striking differences. Photon transport occurs via advection, not diffusion as in the SLIM disk model, and disks evolve in a regime in which the advection timescale is shorter than the sound crossing timescale. Advection is primarily generated by MRI turbulence. Advection of photons along with the gas flow is also the main mechanism of radiative emission in the simulations of Sadowski and collaborators. Despite higher luminosities, because accretion and emitted
radiation are both highly anisotropic, high mass accretion rates exceeding 100 times the Eddington rate do occur (Figure 2).

In addition, Jiang et al. (2017) show that radiation can leak out also via other forms of turbulent transport, in which, while mass is not advected, radiation can be advected due to effects such as magnetic buoyancy. In this case radiation transport is not through diffusion nor through advection, rather it occurs in a third way which is of course not captured by simple models. On the contrary, Sadowski and Narayan (2016) found turbulent transport of radiation to be negligible compared to advection in their simulations. The reason of the latter discrepant conclusion is still unclear. While Sadowski and collaborators find a higher kinetic luminosity, especially in the outer parts of the disks (Figure 4), the total luminosity in their most recent three-dimensional simulations (Sadowski and Narayan, 2016), hence the sum of kinetic and radiative luminosity, is found to be only a factor of 2 lower than that expected in the standard viscous thin disk accretion theory for both zero and high spin BHs, being, respectively, 3% and 9%\(M c^2\). In Jiang et al. (2017) the kinetic energy luminosity is always 15% – 30% of the radiative luminosity, and increases with higher accretion rates.

Finally, while there is discrepancy in the predicted radiative luminosity, both groups find radiative efficiencies higher than predicted by the SLIM disk model in the regime of very high accretion rates, corresponding to at least 10 times the accretion rate at the Eddington limit. For lower accretion rates the radiative efficiencies of Sadowski and Narayan (2016) agree with the predictions of the SLIM disk model, while those of Jiang et al. (2017) are in the range 5 – 7%, hence significantly higher. In Jiang et al. (2017), the radiative efficiency first increases with increasing accretion rate, then, as the accretion rate becomes higher than 1000\(M/\text{yr}\), it starts to decrease to \(\sim\) 1%, a finding that is further supported by additional numerical simulations being analyzed as we write (Y. Jiang, private communication). As a reference, Sadowski and Narayan (2016) find typically a radiative efficiency of 0.006 and 0.01 – 0.02 for, respectively, zero spin and near maximally rotating black holes. This is about an order of magnitude lower than the standard radiative efficiency in a thin viscous disk.

2.2. Super-Eddington flows in spherically symmetric envelopes

In this section we will discuss spherically symmetric accretion flows, without dynamically important angular momentum. This is the configuration
in which the conventional Eddington limit can be applied more naturally, as this assumes no directionality in the gas flow or in the radiation field. Nevertheless, the Super-Eddington accretion regime is possible under certain conditions. Radiation hydrodynamics simulations have been used since more than a decade to show that, if the BH is embedded in an optically thick medium, efficient photon trapping can reduce the radiative force enough to allow super-critical accretion from an envelope (Ohsuga et al. 2005). As in the case of accretion disks, though, the problem has always been how to maintain a sustained super-critical regime. The problem often documented in the literature is that, as soon as radiative feedback from the accreting hole is accounted for, the emerging hot ionizing flux stifles gas accretion from larger scales effectively (Milosavljević et al. 2009; Alvarez et al. 2009; Park and Ricotti 2011; Park and Ricotti 2012; Park and Ricotti 2013). Therefore, the time-dependent infall rate at the edge of the envelope, a boundary condition that in principle could be set based on gas inflow rates found at larger scales (e.g. in protogalaxy simulations), and the anisotropy of the radiation field, are both important factors. Inayoshi and Haiman (2016) have developed a self-consistent model in which radiative heating and ionization is accounted for in 1D radiation-hydro simulations. They found a steady-state solution corresponding to Super-Eddington accretion at the remarkably high rate of more than 5000$L_{\text{edd}}/c^2$, which they dub as Hyper-Eddington accretion (Figure 5). The configuration is that of a radiation-dominated core where photon trapping occurs surrounded by an accreting optically-thin envelope. The core thus shields the envelope from the effect of radiation pressure. The BH needs thus to be embedded in a dense gas cloud satisfying the following condition:

$$M_{\text{BH}} \geq 10^4 M_\odot \left( \frac{n_\infty}{10^5 \text{cm}^{-3}} \right)^{-1} \left( \frac{T_\infty}{10^4 \text{K}} \right)^{3/2},$$

where $n_\infty$ and $T_\infty$ are density and temperature of the ambient gas, respectively. The condition above corresponds to the Bondi radius being larger than the size of the ionizing region (the Stromgren sphere). If this is satisfied accretion occurs essentially at the Bondi rate. We recall that the Bondi accretion rate, which is derived under the assumption of spherical symmetry and an infinitely large radius at which gas begins to acquire a net inward radial motion, is expressed as:

$$\dot{M}_B \equiv \pi \rho_\infty \frac{G^2 M_{\text{BH}}^2}{c^3_\infty},$$

where $c_\infty$ is the sound speed at infinity and $M_{\text{BH}}$ is the black hole mass.
where an adiabatic index $\gamma = 1$ was assumed. The Bondi rate can be much larger than the Eddington rate given sufficiently high density and temperature, but also significantly smaller, as in the case of mini-halos discussed in Chapter 10. As the temperature in galactic nuclei should not be much different from the mean temperature of the galactic ISM, namely in the range $10^3 - 10^4$ K (molecular gas would be colder but it should be dissociated efficiently by the warm ionizing flux resulting from accretion onto the Bh seed), the ambient gas density $n_\infty$ becomes the most important variable. For example, for light BH seeds formed by Pop. III remnants, with masses around $100 M_\odot$, the corresponding density must be $> 10^7$ atoms/cm$^2$ for the above condition to be satisfied. Qualitatively, the condition of a dense, highly optically thick flow yielding very long photon diffusion timescales reminds of the conditions in the SLIM disk model, except that here there is no role of advection. Furthermore, the seemingly high gas densities required to initiate sustained Super-Eddington accretion are commonly found in the center of protogalaxies or circumnuclear regions of high-$z$ gas rich galaxies in hydrodynamical simulations (see e.g., next two sections and Lupi et al. 2016).

Takeo et al. (2018) have improved further such model by studying the effect of the anisotropy of the radiation field emitted around the black hole, to mimic the effect of an unresolved accretion disk, for which one expects radiation to be primarily emitted around the rotation axis of the disk (Ohsuga et al. 2005, Jiang et al. 2014, Sadowski and Narayan, 2016). The starting point is still a spherically symmetric gas cloud, and a large-scale gas infall rate is given as a boundary condition. They used a 1D multi-frequency radiative transfer code coupled with a 2D hydrodynamical simulation and a non-equilibrium primordial chemistry network. Self-gravity of the gas cloud, assumed to be initially of uniform density, was neglected, and a finite difference approach to solve the hydro equations was employed. The cloud is initially static. They consider a range if BH seed masses ($10^3 - 5 \times 10^5 M_\odot$) along with different geometries for the radiation field. The radiation field geometry was chosen as an ansatz in the initial conditions by projecting the radiative flux vector over a fraction of the solid angle. The underlying assumption is that the accretion flow will be organized in a disk or torus, or, more generically, would have an anisotropic pattern, thus leaving funnels of low density gas along which radiative losses would preferentially occur. Furthermore, only the radial component of the radiation force was considered. Because of anisotropic emission, matter was found to accrete preferentially in the plane perpendicular to the emission direction. The
main difference with the calculations for isotropic radiation fields is that, for the same envelope density, lower mass BH seeds, down to $10^3 M_\odot$, can enter a sustained super-critical accretion (see left panel of Figure 5). This is interesting since this is in the mass range of Pop. III remnants. Another novel result found by such calculations is that, as the mass of the BH seed increases, a transition is observed in the qualitative evolution of the gaseous envelope. Indeed above a BH mass of $5 \times 10^5 M_\odot$ the ionized gas around the hole collapse due to super-sonic non-radial motions of neutral gas and radiative recombination. The neutral gas thus absorbs the momentum carried by photons and is accelerated outward in funnels along the poles triggering warm outflows with $T \sim 8000$ K. The latter finding could interesting observational implications that may tell the model apart from other that have been proposed for Super-Eddington accretion.

3. Large scale flow conditions in primordial galaxies: Super-Eddington gas supply rates?

In order to sustain Super-Eddington accretion an necessary condition is that the accretion disk/envelope itself is fed at its edge from large scales with gas impinging at very high rates, as assumed indeed in the Jiang et al. (2017) and Sadowski and Narayan (2016) simulations. We can ask what is the current evidence of high inflow rates at scales just above the accretion disk. We can also ask if such flow has a high angular momentum or not, and thus whether or not accretion will occur indeed through a disk-like interface.

First of all let us set the spatial scale of interest. While that specifically will depend on the mass of the MBH under consideration, we can proceed conservatively and decide that any result concerning the flow properties below pc scales, possibly below 0.1 pc, would be relevant as it would provide the natural boundary condition for smaller scale models/simulations starting at the edge of the accretion disk. Indeed the accretion disk around an MBH of a few thousand solar mass or larger, hence ranging from the mass of a Pop. III remnant to the mass of a full-fledged MBH, will have a physical radius in the range $10^{-3}$ and $10^{-2}$ pc roughly. The most interesting case is that of an MBH weighing the few hundred to a few thousand solar mass, which would be the expected mass for the remnant of a Pop. III star. The latter scenario has been considered in recent works modeling accretion from a spherically symmetric envelope as in Inayoshi et al. (2017).

In the remainder we will summarize the gas inflow rates in the range
kiloparsecs to 0.1 pc found in simulations of protogalaxies and high-z galaxies. First we cover the results concerning metal-free protogalaxies, namely galactic-scale objects forming in mini-halos with a virial mass $< 10^{10} M_\odot$, most typically in the range $10^8 - 10^9 M_\odot$, at $z \sim 15 - 20$. These are the typical target of the simulations designed to study the possibility of massive BH seed formation by direct gas collapse in metal-free halos, which were thoroughly described in Chapter 5 of this book. In this case inflow rates are in the range $0.1 - 1 M_\odot/yr$ (Latif et al. 2016) at scales of 0.1 pc. If we assume that a light BH seed would be present in such a protogalaxy, such inflow would be well above the Eddington limit for BHs of a hundred to several thousand solar masses. Indeed the accretion rate at the Eddington limit, $\dot{M}_{\text{Edd}}$ can be conveniently expressed as:

$$\dot{M}_{\text{Edd}} = \frac{(4\pi G/\epsilon \kappa c)}{M_{\text{BH}}} = 2.2 \times 10^{-8} \left(\frac{M_{\text{BH}}}{M_\odot}\right) M_\odot \text{yr}^{-1}$$

from which it is clear that the Eddington limit for a BH with mass of $1000 M_\odot$ corresponds to $2.2 \times 10^{-5} M_\odot/yr$. In the equation above we have assumed an opacity $\kappa = 0.4 \text{ cm}^2/\text{g}$ and $\epsilon = 0.1$, as in standard Eddington-limited accretion flows. However, Super-Eddington accretion models discussed in the last section predict rates that are in the range a few tens to several thousand $\dot{M}_{\text{Edd}}$ for both super-critical flows in accretion disk and in extended envelopes (see section 2). For a BH of $1000 M_\odot$ these numbers would imply an inflow rate of $10^{-3} - 0.1 M_\odot \text{yr}^{-1}$ to feed the nuclear region at the corresponding rate, which are still well within the range of the pc-scale inflow rates in the simulations of metal-free protogalaxies. The mass inflow rate of gas with negligible pressure support pulled in by the gravitational action of the halo, and neglecting the effect of angular momentum, is $\dot{M} \sim \alpha V_{inf}^3/G$, where $V_{inf}$ is the infall velocity and $\alpha$ is the coefficient of viscous drag relenting the inflow in an eventual disk-like envelope (Mayer and Bonoli, 2018). To first order inflow rates should thus scale as $\alpha V_{vir}$, where we have taken the characteristic infall velocity to be of order the virial velocity scale of the halo. Note $\alpha \sim 0.01 - 0.1$ if self-gravitating instabilities are the main transport mechanism and they are treated as a local effective viscosity acting on the flow (Lodato and Rice 2004). By assuming $\alpha = 0.01 - 0.1$ one recovers inflow rates in the range $0.1 - 1 M_\odot/yr$, comparable to those in the protogalaxy simulations (Latif et al. 2013, 2015), which have halos masses $\sim 10^9 M_\odot$. Based on the scaling $\dot{M} \propto M_{vir} \sim V_{vir}^{-3}$ one expects much higher rates in more massive halos, 100-1000 times larger indeed in $10^{10} - 10^{11} M_\odot$ halos, hence in the range.
100-1000 $M_\odot$/yr. Equation (4) indicates that an inflow rate of 1000 $M_\odot$/yr would be super-critical even for a $10^5 M_\odot$ BH, suggesting that in these larger galaxies super-critical accretion is possible throughout all stages of black hole growth, across many decades in mass. Halos with virial masses in the range $10^{11} - 10^{12} M_\odot$ appear below $z = 12$, hence in an epoch in which gas in such biased halos will be already metal-enriched, rendering radiative cooling is more efficient, which also can boost gas inflow further (Mayer et al., 2015). Both observations, which are just beginning to probe this phase (Grazian et al., 2015; Vito et al., 2018) and simulations (Feng et al., 2014, 2016), agree on the point of fast metal enrichment in such biased high density peaks at high-z.

The MassiveBlack (Di Matteo et al., 2012), MassiveBlackII (Khandai et al., 2015), and BlueTides (Feng et al., 2016) simulations have studied gas infall in these massive high-z galaxies with the goal of assessing the growth history of putative BH seeds at their centers. All these simulations modeled large cosmological volumes in order to be able to identify highly biased density fluctuations that should correspond to the hosts of high-z QSOs (Li et al., 2006). Given the large volumes adopted they limited their analysis to scales of order a kiloparsec, corresponding to their resolution limit. Due to the same resolution limit they assumed initial BH seeds with already large masses, $> 10^5 M_\odot$. Therefore, they mainly assessed the rates of gas infall from the cosmic web onto the nascent galaxies in very massive halos. Di Matteo et al. (2012) were the first to show that the feeding occurs mainly in the form of cold gas flows ($T \sim 10^4 K$) accreting directly from the cosmic web, although a circumnuclear disk, perhaps unstable and clumpy, could be present at smaller unresolved scales (Gabor and Bournaud, 2013). Infall rates of order 100$M_\odot$/yr sustained on Gyr timescales were observed in these simulations, even in the presence of AGN feedback (starting from an initial BH seed with mass $\sim 10^5 M_\odot$ implanted in each halo). While in the simulations the sub-resolution accretion onto BH seeds was capped at the Eddington limit (except in the BlueTides suite, in which it was allowed to reach 3 times the Eddington value, see Feng et al., 2016) these prominent inflows would be highly Super-Eddington if they could continue unimpeded all the way to edge of the accretion disk/ envelope. Whether or not they can really continue unimpeded to smaller scales will depend on the gas dynamics inside the galaxy, hence on the nature of the galactic potential and of the density distribution.

Only very recently it has become possible to tackle directly this question using the zoom-in technique by increasing many-fold the mass and spatial
resolution in a selected object within a large cosmological volume. The MassiveBlackHR simulation (Capelo, Mayer, Di Matteo & Feng, in prep.) achieves this in Halo 3 of the precursor MassiveBlack cosmological volume. The selected halo has clustering properties consistent with those of bright $z \sim 6$ QSOs (Di Matteo et al., 2012). The simulation, which employed the unprecedented resolution of 500 million fluid elements (SPH particles in this scale) reaches a spatial resolution of a few tens of parsecs in the gas phase. Preliminary analysis shows that a prominent gas disk a few kpc in size forms at the center of the main dark matter halo at $z \sim 8$, following a merger between two massive proto-galaxies. The disk, with regular grand-design spiral structure and a central bar-like distortion, is reminiscent of low-z disks in galaxies, has a mass comparable to the disk of our own Milky way but it is a factor of 10 more compact size, has a much higher gas content relative to the stellar content (50%) and thus a much higher density. This simulation shows that not only high gas inflow rates ($> 100 M_\odot/yr$) persist down to sub-kpc scales, but they occasionally increase further, peaking at $1300 M_\odot/yr$, mostly in coincidence with mergers and flybies. Vigorous bars and spiral density waves do aid angular momentum transport via non-axisymmetric torques at sub-kpc scales, confirming the findings for much smaller simulated protogalaxies at $z > 10$ (Chapter 10). If such inflows continue to pc scales at similar rates they would, of course, be highly Super-Eddington accretion.

We have seen that massive halos with $M > 10^{10} M_\odot$ are in principle capable of sustaining super-critical accretion over a range of masses, from light Pop. III seeds to black holes with masses $\sim 10^8 M_\odot$. There is, though, another important consideration that goes in the direction of favouring halos with masses above $10^{10} M_\odot$ as natural environments for super-critical accretion; this is the effect of stellar and SN feedback as a function of halo mass. At moderate redshifts, $z < 5$, it is now fairly established that dwarf galaxies with a virial mass $M_{\text{vir}} < 10^{10} M_\odot$ cannot preserve large amounts of dense gas in their nuclei because of the strong effect of winds/outflows driven by SNe (Governato et al., 2010; Shen et al., 2014; Tollet et al., 2016; Habouzit et al., 2017), see also chapter 10. While no specific studies have been carried out to explore the detailed conditions of nuclear gas flows in realistic high-z dwarf galaxy hosts, one expects a qualitatively similar behaviour. Moreover, feedback-driven outflows should be widespread in low mass galaxies and become even stronger at high redshift reflecting the notion that specific star formation rates increase at a fixed galaxy stellar (as indeed verified in the simulations of Fiacconi et al. (2017b)). As for mini-
halos at even higher-z, episodic accretion of the central BH would contribute further to heating and expelling the gas (see Chapter 10). Therefore the consistently large gas inflows necessary for sustained Super-Eddington accretion should be more prevalent in such galaxies hosted in massive halos since these are more resilient to the effect of feedback-driven outflows. We also note that, typically, in simulations of high-z metal-free protogalaxies (see Chapter 5) normal star formation beyond the stage of Pop. III stars, and thus its resulting feedback is not taken into account, hence the the effect of feedback is underestimated. It follows that halos with $M_{\text{vir}} > 10^{10} M_\odot$ are the natural candidates to support sustained super-critical accretion onto their central BHs.

Episodic Super-Eddington accretion might occur for a wider range of halo masses, especially in the presence of strong tidal forces promoting temporarily gas inflows during mergers (major and minor), or in strong tidal interaction events, all particularly frequent at high-z. Episodic intense accretion might still be capable of growing some BHs to large sizes even starting from lights seeds, especially in highly biased regions such as those that will later host bright QSOs since these are by construction growing their mass faster [Pezzulli et al. 2016, 2017]. Pezzulli et al. (2017) found only a 50% decrease in the overall population of MBHs that grow efficiently via super-critical accretion at $z > 7−8$ in their semi-analytical model when the effect of stellar and SNe feedback are taken into account. However it is not known how much bigger such effects would be in halos with $M_{\text{vir}} < 10^{10} M_\odot$ since the latter work focuses on halos that reach a mass as large as $10^{13} M_\odot$ at $z = \sim$. In any case, in their semi-analytical model mergers play a major role in revitalizing inflows even in low mass halos, an aspect that should be tested with detailed high-resolution cosmological hydrodynamical simulations.

4. Models of Super-Eddington growth of massive BH seeds in high redshift galaxies

In principle the growth of BH seeds in the early Universe should be studied using multi-scale simulations that capture the accretion disk flow as well as the larger scale feeding of gas from the surrounding interstellar medium to the disk. However this is not computationally feasible at the moment. as the range of densities and timescales is prohibitive. The difficulty is exacerbated further by the need of treating radiative processes at all scales, and eventually MHD and relativistic effects at the scale of the accretion disk.
This is a daunting task with current hydro codes even on large parallel supercomputers as the multiple timescales introduce load balancing bottlenecks that prevent efficient scaling on large node counts. Already at the scale of the accretion disk itself the inclusion of advanced radiative transfer schemes in a global 3D simulations, such those of Jiang et al. (2017) described in section 2, results in a computational challenge. Therefore connecting such simulations to the protogalaxy simulations starting from cosmological initial conditions, the topic of the previous section, requires some modeling interface to simplify the computation. This interface can be realized in the form of a sub-grid model of disk accretion applied to the BH seeds that evolve in simulations of the protogalactic environment. Similar sub-grid approaches are routinely adopted to study the growth of MBHs at lower redshift in cosmological simulations, which describe mass growth as well as radiative and momentum feedback from the accreting MBH onto the surrounding ISM (the so-called AGN feedback, see Chapter 10).

The main difference is that in conventional cosmological/galaxy-scale simulations accretion rates are capped at the Eddington limit (Di Matteo et al., 2005; Hopkins and Quataert, 2011), whereas in this Chapter we want to discuss how the gas flow behaves when one drops such constraint. The SLIM disk model lends itself to be cast in the form of a sub-grid model given its relative simplicity, in that its main features can be encapsulated in the resulting radiative efficiency (see section 2). This indeed has been done in Lupi et al. (2016), who have studied the growth of light BH seeds in the nuclear regions of protogalactic disks using hydro simulations by allowing accretion to be come super-critical, as well as in Pezzulli et al. (2016, 2017), who have modeled super-critical accretion in a semi-analytical model of galaxy and black hole formation. In a preceding work by Madau et al. (2014) the same SLIM disk model has been used in a semi-analytical code to show that even very light seed BHs originating from Pop. III stars can grow to billion solar masses by $z \sim 6 – 7$. In such work the seed BH was considered to be isolated and with an arbitrary large gas reservoir to feed onto. This was shown to be the case for both steady and intermittent super-critical accretion. The accretion rate was only a few times above Eddington (Figure 6), which is a very conservative assumption in light of the results of the small-scale simulations described in section 2. Madau et al (2014) proposed the following spin-dependent fitting equation to the numerical results of Sadowski (2009):
Super-Eddington accretion

\[ \frac{L}{L_E} = A(a) \left[ \frac{0.985}{\dot{m}_E/\dot{m}} + \frac{0.015}{\dot{m}_E/\dot{m} + C(a)} \right], \quad (5) \]

where the functions \( A, B, \) and \( C \) scale with the spin of the black hole as

\[ A(a) = (0.9663 - 0.9292a)^{-0.5639}, \quad (6) \]
\[ B(a) = (4.627 - 4.445a)^{-0.5524}, \quad (7) \]
\[ C(a) = (827.3 - 718.1a)^{-0.7060}. \quad (8) \]

Because photon-trapping occurs in Super-Eddington accretion the emitted luminosity is not linearly proportional to the accretion rate anymore, \[ \text{Madau et al.} (2014) \] also propose a convenient way to recast the accretion timescale equation (see above equation (1)):

\[ t_{\text{acc}} = \frac{t_E}{16(1 - \epsilon)} \left( \frac{\dot{m}_E}{\dot{m}} \right) \left( 8.4 \times 10^6 \text{yr} \right) \left( \frac{3\dot{m}_E}{\dot{m}} \right), \quad (9) \]

where the last inequality holds for moderate Super-Eddington rates independent of the value of the black hole spin \[ \text{Madau et al.}(2014) \]. While low radiative efficiency of the SLIM disk model is accretion-rate dependent, the equation shown above shows that it is very mildly dependent on the spin, at stark variance with what occurs for the standard viscous disk model for which spin dependence is important. The next key question is whether or not the conditions for Super-Eddington accretion, namely sustained high gas accretion rates, are physically possible in protogalaxies. \[ \text{Lupi et al.} (2016) \] indeed addressed this aspect using idealized simulations that only model the circumnuclear region of dense gaseous protogalaxies (the circumnuclear disk, hereafter CND). This approach allows to reach very high resolution and thus study the nature of the gas flow at scales even as small as 0.02 pc. Note, though, that this is still an order of magnitude larger than the sphere of influence of a stellar black hole coming from the collapse of even the largest Pop. III stars (of order a few thousand solar masses). Controlled experiments with isolated systems allow to survey the parameter space extensively at high resolution, which would require prohibitive computational costs with cosmological simulations. \[ \text{Lupi et al.} (2016) \] performed their simulations using two different numerical hydrodynamics techniques, the Lagrangian finite mass meshless method (MFM) in the GIZMO code \[ \text{Hopkins}(2013) \] and the adaptive mesh refinement code RAMSES \[ \text{Teyssier}(2002) \]. Star formation and feedback from Supernovae
were included, adopting the so-called blast wave model by Stinson et al. (2006) in both codes, in which cooling of the gas heated by SN ejecta is temporarily suspended to mimic unresolved turbulence and momentum deposition. Black holes were treated as massive collisionless particles that accrete at a rate determined by their radiative efficiency. Initially black holes with a range of masses were distributed at random locations inside the CND.

Generally speaking there are two ways by which a low radiative efficiency can affect BH growth. First of all, for a given accretion rate BHs grow faster with lower radiative efficiency because less mass is converted into radiant energy (by a factor proportional to $1 - \epsilon$). The second way is that BH feedback is reduced with a lower $\epsilon$, once again because the radiant energy emerging from the accretion process is lowered, which implies a larger accretion rate can be sustained. The mechanical component of feedback manifests itself as momentum-driven winds and/or outflows. Its strength does not have a natural scaling behaviour with radiative efficiency, but still has to decrease because of the first effect. Feedback onto the surrounding ISM by the accreting black hole was included in Lupi et al. (2016) by adopting the thermal coupling model in Booth and Schaye (2009), hence neglecting momentum feedback. Lupi et al. (2016) then compared the results obtained with standard viscous disk radiative efficiency ($\epsilon = 0.1$) against those with a reduced efficiency based on the SLIM disk model, and found striking differences concerning the growth rate of the BH seeds. Furthermore, black holes were also allowed to merge, which introduced another channel for growth. The resulting scenario is thus more complex compared to the toy model presented in Madau et al. (2014), in which only growth by gas accretion of a single, isolated black hole seed was considered. Furthermore, as the CNDs were dense and massive self-gravity played an important role, causing fragmentation into clumps which changed the nature of the gas flow (see Figure 8). These very dense clouds were weakly affected by feedback owing to their high density (much higher than present-day Giant Molecular Clouds (GMCs)), and even more so in the case of the SLIM disk accretion regime. This clumpy flow favoured a sustained accretion rate, quite irrespective of hydro method and resolution.

Despite the intrinsic complexity, Lupi et al. (2016) found a recurrent outcome in their simulations, namely that one black hole, which had experienced faster growth since the beginning, was overtaking all the others by capturing competitively most of the available gas in the surroundings as well as merging with the other black holes. This recalls the behaviour
Super-Eddington accretion

in competitive accretion in star formation \citep{Bonellietal2001}. Growth was typically terminated when all the gas was consumed, predominantly into star formation rather than accretion (the mean star formation rate in a sphere of 1 pc was $0.1 M_\odot/yr$ as opposed to $10^{-3} M_\odot/yr$ for the black hole accretion rate). We remark in the simulations there was no gas infall onto the CND from exterior regions that could prolong sustained growth of the black hole seeds. By the end of the simulations, which extended for 3 Myr, when the gas in the CND was depleted, the mass of the major seed BH reached values $10^4 - 10^5 M_\odot$ in the SLIM disk case, while it barely reached $10^3 M_\odot$ when using standard disk accretion efficiency (see e.g. Figure 7). Remarkably, the accretion rates in the SLIM disk case were in the range 100-200 times the Eddington mass accretion rate, which is consistent with the results of the accretion-scale simulations described in section 2. The most massive BH will inevitably migrate to the center via dynamical friction on a few orbital times, explaining why MBHs are found at the center of galaxies.

Finally, it is important to remark that these CND simulations, having a resolution of 0.1 pc, do not resolve the scale of the accretion disk, hence it is not yet possible to connect the gas flow properties at the numerical accretion radius to the gas flow in the MHD disk simulations of Super-Eddington accretion discussed in section 2. Nevertheless, one can measure the angular momentum of the gas that accretes on the BH sink particles. It was found that this is very low, as most of the gas accretes in the dense clumpy phase, namely in a regime in which the gas flow in the CND by nature highly asymmetric and chaotic. As a result in the regime of these simulations accretion, while not spherical and isotropic, ends up having a rate comparable to the Bondi rate for the temperature and density of the local medium around the BHs. Whether this is realistic or not has to be ascertained with cosmological simulations, which provide realistic boundary conditions for the flow in the CND, as well as with improved radiation physics, such as implementing the self-shielding of gas and the local photoionization, which could both suppress cooling, and thus fragmentation.

\citep{Fiacconi2017a} made a first step in addressing the problem of the realism of the initial conditions by studying the nature and geometry of the gas flow in the nuclear region of a $z \sim 5 - 6$ massive galaxy in the state-of-the-art cosmological hydrodynamical simulation dubbed ”PONOS”. They found that the mass of the region within a few 100 pc from the center was varying in the range $10^7 - 10^8 M_\odot$, relatively comparable with the mass assumed in the CND models of \cite{Lupietal2016}. However, gas inflow
rates were observed to be highly variable and anisotropic down to the sub-kpc scale of the nucleus (Figure 9). This is a generic expectation given the highly nonlinear and chaotic gas accretion pattern from the cosmic web to the disks, and is confirmed also in the simulations of more massive z \sim 7-10 protogalaxies in the MassiveBlackHR run (Capelo et al, in prep.). This, perhaps combined with the resolution limit set by the gravitational softening of 40 pc, did not produce a long-lived CND-like structure as that assumed in Lupi et al. (2016). Another potential difference was that the effect of stellar and SN feedback was found to be much stronger than in the CND-scale simulations of Lupi et al. (2016), suppressing fragmentation and leading to a turbulent flow from tens of pc to kpc scales. The highly anisotropic and turbulent nature of the flow down to the nuclear region is qualitatively consistent with the fact that a coherent structure such as CND is not assembled (Figure 9). While the long term evolution of the angular momentum in the turbulent nuclear region was not studied, dissipation through shocks between colliding flows of turbulent eddies is expected to be capable of extracting angular momentum and sustain accretion to smaller (unresolved) scales. The mean inflow rates on scales of a hundred pc are indeed still high enough to be super-critical for a BH seed sitting in the center. The proposed chaotic accretion scenario (King and Pringle, 2006) might apply in this case.

5. Summary

We have reviewed the status of knowledge relevant to Super-Eddington accretion of BH seeds at high redshift. The latest developments of small scale simulations with detailed physics modeling accretion disks and envelopes in super-critical regimes were presented and discussed in the context of massive BH seeds. We the covered gas inflows rates in protogalaxies at z > 15 as well as in more massive galaxies in high sigma peaks at z ~ 10 and below. This is relevant to determine how efficiently the accretion disk or envelope is fed with fresh fuel from the surrounding ISM, and in particular if the inflow rate is compatible with the accretion rates found in small scale simulations of super-critical phases. We then concluded by describing results of simulations that attempt to bridge the scales by treating super-critical accretion with a sub-grid model. Currently only the SLIM disk solution has been considered in such sub-grid models. The main findings that we discussed are:
Super-Eddington accretion in disks and envelopes is feasible and, if it happens, it reaches high amplitudes, from ten times to a few thousand times above the Eddington limit.

- The radiative efficiency characterizing optically-thick, super-critical flows is low, but whether it is 10 or 1000 times lower than the standard radiative efficiency in the conventional thin "α" disk model is still unclear, and seems to depend on the method adopted by simulations to solve for radiation transport.

- The ratio between radiative and kinetic luminosity emitted in a Super-Eddington accretion phase is also uncertain. In some cases it makes up for most of the emitted energy, which has important implications on how to model feedback from the BH seed or MBH onto the surrounding interstellar medium.

- Pc-scale gas inflow rates in protogalaxies at $z \sim 15 - 20$ or the first massive galaxies at $z < 8 - 10$ are always above the Eddington limit for light BH seeds ($M_{BH} \sim 100 - 1000 M_\odot$), but can be such for a wide range of black hole masses, up to $10^8 M_\odot$, only if we consider galaxies in high sigma peaks with halo masses $10^{11} - 10^{12} M_\odot$. Only in this latter case prolonged Super-Eddington accretion appears feasible. Furthermore, galaxies in low mass halos might not have a long-lasting nuclear gas reservoir due to the strong effect of stellar and SN feedback, which might stifle BH growth via Super-Eddington accretion.

- The results of the SLIM disk accretion model can be easily parametrized as a (very low) modified radiative efficiency, with a weak dependence on the black hole spin. With the latter recipe both semi-analytical models and local, non-cosmological simulations of circumnuclear disks (CNDs) have obtained encouraging results, showing that super-critical accretion can occur steadily as well as episodically depending on the environmental conditions but always leads to rapid growth of light BH seeds, matching naturally the constraints imposed by the rapid emergence of high-z QSOs. However it is still unclear if the dense CND assumed in such simulations occurs in a realistic cosmological setting at high $z$.

- Once the conditions for a super-critical accretion flow are in place the actual growth rate will depend on the gas supply rate to the nuclear envelope/accretion disk as well as on the nature and effectiveness of radiative and kinetic feedback from the growing BH. Other effects such as merging with other black holes in a vigorously
star forming CNDs or inside dense nuclear star clusters, might be important in high-redshift environments.

Future developments will rely heavily on the ability to carry out multi-scale simulations bridging the accretion disk/envelope domain with the galactic domain at nuclear scales to the very minimum. In parallel, small scale accretion simulations have to investigate a much wider range of initial conditions, in terms of black hole masses as well as boundary inflow rates and patterns. They also need to quantify more explicitly the dependence of results, in particular radiative efficiency and mean accretion rate, on the adopted radiation transport model. Finally, combining relativistic simulations with the most sophisticated radiation transport methods, such as VET, is among the technical challenges ahead.
Fig. 3. Accretion rate histories in units of the Eddington accretion rate for four of the 3D relativistic runs of Sadowski and Narayan (2016) with stellar mass black holes (see text). Figure adopted from Sadowski and Narayan (2016), reproduced by permission of Oxford University Press / on behalf of the RAS.
Fig. 4. On the left a slice through the midplane of the accretion disk at the end of one of the simulations of Sadowski and Narayan (2016) shows the radiative flux (orange-scale map) and the density map (grey-scale map). The powerful winds leaking out radiation via advection are evident, as is the photon trapping through inward advection in the high density mid-plane. On the right we show an example of the radiative flux (solid lines) and kinetic flux (dashed lines) in units of the Eddington flux from one of the runs, using blue for quantities measured near the edge of the disks and red for quantities measured near the inner annulus of the computational domain. From the plot it is evident that the kinetic luminosity can supersede the radiative luminosity, in contrast with Jiang et al. (2017) (see text).
Fig. 5. Examples of Hyper-Eddington accreting light (left) and massive BH seeds (right) from Takeo et al. (2018). The mass of the BH is $10^3 M_\odot$ on the left, and the different models differ for how the radiation field geometry is imposed (isotropic for Model A and increasingly anisotropic for Model B and C). Super-critical accretion sets in for anisotropic radiation propagation. For seeds with masses $> 10^5 M_\odot$, shown on the right, the ionization region can recombine rapidly and collapse, leading to the highest accretion rates (after the time marked with the green circle). Figure adopted from Takeo et al. (2018), reproduced by permission of Oxford University Press / on behalf of the RAS.
Fig. 6. The left panel shows the different growth rate of an individual light BH seed, with mass of $100 M_\odot$, for models adopting, respectively, SLIM disk accretion (orange, for an accretion rate constantly at 3 times the Eddington rate) and a standard viscous accretion at the Eddington limit (blue). Solid and dashed lines are for, respectively, a zero spin and a highly rotating black hole (see labels in panel). The right panel shows only super-critical growth curves for the same BH mass but for intermittent accretion with a duty cycle of 0.5 (solid line, for 3 times the Eddington rate) and 0.2 (dashed line, for 4 times the Eddington rate). Redrawn with permission from Madau et al. (2014).
Fig. 7. Mass growth of light black hole seeds in a simulation using the SLIM disk model for accretion (left panel) and in one using the standard 0.1 radiative efficiency for a viscous disk (right panel), from Lupi et al. (2016). The red lines correspond to the most massive BHs at the end of the runs, the black lines correspond to all other BHs in the CND, while the blue dashed lines depict reference accretion histories at fixed Eddington ratios of 500, 400, 300, 200, and 100, respectively. The marked difference in growth rate and absolute growth between standard and SLIM disk-like radiative efficiency is evident. Figure adopted from Lupi et al. (2016), reproduced by permission of Oxford University Press / on behalf of the RAS.
Fig. 8. Density maps of the circumnuclear disk (CND) at large (left) and small scales (right) in simulations carried out with a SLIM disk sub-grid model (top) and with a standard 0.1 radiative efficiency as expected in conventional viscous disk accretion (bottom), from Lupi et al. (2016). These are the same runs analyzed in Figure 7. The maps highlight the different effect of BH (radiative) feedback in the two cases. Indeed in the SLIM disk case, due to much lower radiative efficiency, the bubble created by feedback around the BH is much smaller and weaker (top-right panel), promoting faster sustained growth relative to the standard accretion case (bottom-right panel). Figure adopted from Lupi et al. (2016), reproduced by permission of Oxford University Press / on behalf of the RAS.
Fig. 9. Mollweide projections of gas density through the center showing the inflow and outflow to/from the nucleus at different scales (left to right) and different redshift (from bottom to top redshift decreases) for the "PONOS" cosmological galaxy formation simulation of Fiacconi et al. (2017a). The gas inflow pattern is anisotropic at all scales, down to the nuclear region. The hydrodynamical resolution of the simulation reached a few pc but the gravitational softening was set to 40 pc. Figure adopted from Fiacconi et al. (2017a), reproduced by permission of Oxford University Press / on behalf of the RAS.
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