Quantum wire fracture and discrete-scale invariance

Maiko Kikuchi* and Masanori Yamanaka
Department of Physics, College of Science and Technology, Nihon University,
Kanda-Surugadai 1-8-14, Chiyoda-ku, Tokyo 101-8308, Japan
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I. INTRODUCTION

Discrete-scale invariance is ubiquitously found in catastrophic phenomena. [1] Typical examples are diffusion-limited-aggregation clusters [2], ruptures in heterogeneous systems [3], earthquakes [4], and financial crashes [5]. The invariance is obtained by placing a restriction on the scale invariance, which requires complex critical exponents and log-periodic corrections to scaling. The corrections lead to oscillation in the observables and the periodicity of the oscillation becomes shorter as it approaches the critical point. Phenomenologically, this is regarded as the typical and universal property of discrete-scale invariance.

The strain force during the fracture process of a quantum wire, in the first experiment of this type, have measured experimentally. [6] The magnitude of the force oscillates as a function of time and can be phenomenologically regarded as a sign of discrete-scale invariance. In the theory of discrete-scale invariance, termination of the wire is regarded as a phase transition. We estimate the critical point and exponents.

II. SINGULARITIES AND LOG-PERIODIC CORRECTIONS

In critical phenomena, the observables obey the Power Law near the critical point. This is expressed as

\[ f(x) \propto (x_c - x)^m, \]

where \( f(x) \) is the observable, \( x \) is a parameter, such as temperature, pressure, and so on, \( x_c \) is the critical point, and \( m \) is the critical exponent. The Power Law reflects the scale invariance or self-similarity of the underlying physics. The exponent reflects the dimensionality and symmetry of the system and is used to distinguish the universality class. Discrete scale invariance theory states that critical phenomena can have more general properties than the simple Power Law. There is a complex critical exponent

\[ m = m' + m''i \]

where \( m' \) and \( m'' \) are real numbers, and \( i \) is the imaginary number unit. Putting (2) to (1), we have

\[
\begin{align*}
f(x) & \propto \text{Re}[(x_c - x)^{m'+m''i}] \\
& = \text{Re}[(x_c - x)^{m'} e^{im'' \log(x_c - x)}] \\
& = (x_c - x)^{m'} \cos(m'' \log(x_c - x)) \\
& = (x_c - x)^{m'} [a_0 + \sum_{n>0} \cos(nd \log(x_c - x) + e)]
\end{align*}
\]

where \( \text{Re}[\ ] \) denotes the real part and \( d \) is a constant which is related to the preferred scaling ratio [1]. By neglecting the higher-order terms in the Fourier series, we obtain

\[ f(x) = a + b(x_c - x)^{m'} [1 + c \cos(d \log(x_c - x) + e)] \]

where \( a, b \) and \( c \) are constants. This expresses the log-periodic oscillation superposed on the Power Law.

III. APPLICATION TO THE EXPERIMENTAL DATA

A Power Law distribution of alternation detected in the strain force in a nanowire appears to be a signature of scale invariance, leading to the idea that a rupture in the nanowire can be regarded as a kind of "critical point." In an analogy of the critical point, the rupture of a nanowire can be viewed as a cooperative phenomenon corresponding to the progressive buildup of stress and damage correlations. The rupture interaction increases

*Present address: Department of Materials Science and Engineering, Tokyo Institute of Technology 4259 Nagatsuda, Midori-ku, Yokohama 226-8502, Japan
appears to be no explicit inconsistency between the experimental data and our fitting function at this phenomenological stage. Here we discuss the validity of the assumption.

As shown in Fig. 1, the experimentally-observed rupture time is 2.60 seconds. In contrast, the estimated time by fitting is 3.46 seconds. Some possible reasons for this discrepancy are: (a) In the experiment, the nanowire ruptured before it reached the true critical point. (b) The experimental data does not have sufficient resolution.

For (b), any external noise, such as a small shock to the sample or thermal fluctuation, may force an earlier termination of the wire. If the experiment were performed under ideal conditions, i.e., under adiabatic stretching of the nanowire, and if the atoms were infinitely small, the rupture would be expected to occur at the true critical point. If we find any discrepancy, even in the adiabatic process, it is due to the finite volume effect of the atoms, since atoms cannot be subdivided on this energy scale. In this case, the experimentally-measured strength of the force is identical to that between single-atomic contact. For (b), the eq. (4) has small and rapid oscillation in the limit $x \to x_c$. The amplitude becomes smaller and the period becomes shorter as we approach the critical point. If the amplitude of the oscillation is smaller than that of the resolution in the experiment, we cannot estimate true the critical point by fitting.

In this study, we assumed that discrete-scale invariance is applicable to microscopic systems. In this microscopic system, there are no explicit heterogeneous structures from the viewpoint of classical mechanics. However, from a quantum mechanical point of view, the bonding networks of the wave function of electronic state of atoms may have a heterogeneous structure and would be expected to be reorganized as rupture approached while self-optimizing the total energy of the system.

Repeated experiments are desirable to confirm the validity of discrete-scale invariance and to distinguish quantum fracture from classical fracture. If the exponent takes a universal value, it would further support the assumption of invariance.

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