varstan: An R package for Bayesian analysis of structured time series models with Stan

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Abstract

varstan is an R package (R Core Team 2017) for Bayesian analysis of time series models using Stan (Stan, Development. Team 2017). The package offers a dynamic way to choose a model, define priors in a wide range of distributions, check model’s fit, and forecast with the m-steps ahead predictive distribution. The users can widely choose between implemented models such as multiplicative seasonal ARIMA, dynamic regression, random walks, GARCH, dynamic harmonic regressions, VARMA, stochastic Volatility Models, and generalized t-student with unknown degree freedom GARCH models. Every model constructor in varstan defines weakly informative priors, but prior specifications can be changed in a dynamic and flexible way, so the prior distributions reflect the parameter’s initial beliefs. For model selection, the package offers the classical information criteria: AIC, AICc, BIC, DIC, Bayes factor. And more recent criteria such as Widely-applicable information criteria (WAIC), and the Bayesian leave one out cross-validation (loo). In addition, a Bayesian version for automatic order selection in seasonal ARIMA and dynamic regression models can be used as an initial step for the time series analysis.

Keywords: Time series, Bayesian analysis, structured models, Stan, R.

1. Introduction:

Structured models such as ARIMA Box and Jenkins (1978), GARCH Engle (1982) and Bollerslev (1986), Random walks and VARMA models are widely used for time series analysis and forecast. Several R packages (R Core Team 2017) such as forecast (Hyndman and Khandakar 2008) and astsa (Stoffer 2019), have been developed for estimating the models with classic inferences methods. Although a Bayesian approach offers several advantages such as incorporating prior knowledge in parameters, estimating the posterior distribution in complex models is a hard task and an analytical solution may not be feasible. A Markov chain Monte Carlo (MCMC) approach using Gibbs sampler limits prior selection to be conjugate to the likelihood or Metropolis-Hasting algorithms struggles with slow convergence in high dimensional models. The No U-Turn Sampler Hoffman and Gelman (2014) algorithm provided by Stan (Stan, Development. Team 2017) offers a fast convergence, prior flexibility and its own programming language for modeling (for a further discussion of samplers and algorithms BÃijrkner (2017)). The package varstan, is an R interface of Stan’s language for time series modeling, offering a wide range of models, priors choice and methods making Bayesian time series analysis feasible.

The aim of this article is to give a general overview of the package functionality. First,
general definitions of structured time series models and priors selection. Then, a discussion of the estimating process is given. Also, package’s functionality is introduced; as well as a presentation of the most important functions. Finally, an analysis of the monthly live births in the U.S.A (1948 -1979) is given as an example of the package modeling process.

2. Structured Time series models

A time series model is just a sample of a stochastic process \( Y = \{Y_t\}_{t=1}^{\infty} \), where every observation \( Y_t \) describes the random variable response in a particular time \( t \). Let’s say the process follows a location-scale model Migon, Gamerman, and Louzada (2014) with normal distribution \( Y_t \sim N(\mu_t, \sigma_t) \), where the mean \( (\mu_t) \) and variance \( (\sigma_t) \) are considered the location and scale parameters with a time dependency. In other words, every observation can be written as follows:

\[
Y_t = \mu_t + \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1) \text{ iid.}
\]

The basic ARIMA model proposed by Box and Jenkins (1978), can be seen as a location-scale model, where the time dependency structure is for the location parameter.

\[
(1 - B)^d Y_t = \mu_0 + \sum_{i=0}^{p} \phi_i (1 - B)^d Y_{t-i} - \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \sigma_0 \epsilon_t \tag{1}
\]

where the previous equation is written as:

\[
(1 - B)^d Y_t = \mu_t + \sigma_t \epsilon_t
\]

notice that:

- \( B \) is the back-shift operator, where \( B^d Y_t = Y_{t-d} \);
- \( d \) is the number of differences needed so the process is stationary;
- \( p \) is the number of considered lags in the auto-regressive component;
- \( q \) is the number of considered lags in the mean average component;
- \( \mu_t = \mu_0 + \sum_{i=0}^{p} \phi_i (1 - B)^d Y_{t-i} - \sum_{i=1}^{q} \theta_i \epsilon_{t-i} \) clearly has a time dependency;
- \( \sigma_t = \sigma_0 \) does not have a time dependency because is constant in time;
- \( \epsilon_t \sim N(0, 1) \) are the independent identically distributed (iid) random errors.

Models with more complex structure, such as multiplicative seasonal ARIMA models

\[
SARIMA(p, d, q) \times (P, D, Q)_S
\]

where \( S \) is the periodicity/frequency, \( D \) the seasonal differences, \( P \) auto-regressive and \( Q \) mean average seasonal components Tsay (2010), and dynamic regression Kennedy (1992), are location-scale models with additional changes in the basic ARIMA structure. Let’s say \( Y = \{Y_t \mid Y_t \sim N(\mu_t, \sigma)\} \) follows an ARIMA(p,d,q) model as in (1), then a dynamic regression is just adding independent terms to the location parameter

\[
(1 - B)^d Y_t = \mu_t + (1 - B)^d X_t b
\]
$X_t$ are the additional variables (no time dependence considered), $b$ are the regression parameters, and every variable in $X_t$ has the same differences as in the ARIMA model. A GARCH model proposed by Bollerslev (1986) as a generalization of an ARCH (Engle 1982), the location-scale structure is easier to be noticed, but is fair to recall that the time dependency structure is for the scale parameter.

$$Y_t = \mu_t + \sigma_t \epsilon_t$$

$$\sigma_t^2 = \sigma_0 + \sum_{i=1}^{s} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{k} \beta_i \sigma_{t-i}^2.$$  

In this model, the location parameter is constant in time, and $\sigma_0$ is the arch constant parameter\(^1\). An additional variation of the garch model is the student-t innovations with unknown degrees of freedom. Which implies adding latent parameters to the GARCH structure:

$$Y_t = \mu + \epsilon_t \left(\frac{v-2}{v} \sigma_t^2 \lambda_t\right)^{1/2}$$

where the unknown degrees of freedom have an inverse gamma distribution ($\lambda_t \sim IG(v/2, v/2)$) and the hyper-parameter $v$ is unknown. A further discussion is given by Fonseca, Cerqueira, Migon, and Torres (2019).

2.1. Prior Distribution

By default, varstan declares weakly informative normal priors for every lagged parameter\(^2\). Other distributions can be chosen and declared to every parameter in a dynamic way before the estimation process starts. For simplicity, varstan provides parameters() and distribution() functions, that prints the defined parameters for a specific model, and prints the available prior distributions for a specific parameter respectively.

The priors distributions for the constant mean ($\mu_0$) and regression coefficients, can be chosen between, normal, t student, Cauchy, gamma, uniform, and beta. For the constant scale parameter ($\sigma_0$) a gamma, inverse gamma (IG), half normal, chi square, half t-student, or half Cauchy distributions.

In SARIMA models, non stationary or explosive process Shumway and Stoffer (2010) could cause divergences in Stan’s estimation procedure. To avoid this, the ARMA coefficients are restricted to a $\Phi = [-1, 1]$ domain, and the available prior distributions are uniform, normal, and beta\(^3\).

For VARMA models, the covariance matrix $\Sigma_0$ is factorized in terms of a correlation matrix $\Omega_0$ and a diagonal matrix that has the standard deviations on the non zero values, through:

$$\Sigma_0 = D \Omega_0 D$$

\(^1\) Wuertz, Setz, Chalabi, Boudt, Chausse, and Miklovac (2020) denoted $\sigma_0$ as $\alpha_0$ or $\omega_0$

\(^2\) Lagged parameters are the ones in the ARMA or GARCH components, in (1) the $\phi_i$’s are the lagged parameters of the auto-regressive part

\(^3\) In SARIMA we define in $\Phi$ domain

\(^4\) If $\theta \sim beta(\alpha, \beta)$ in $[0, 1]$ then $\theta_1 = 2(\theta - 1) \sim beta(\alpha, \beta)$ in $[-1, 1]$
where \( D = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_d) \) denotes the diagonal matrix with \( \sigma_i \) elements that accepts priors just like scale constant parameter (\( \sigma_0 \)). Following the Stan recommendations, for \( \Omega_0 \) a LKJ-correlation prior with parameter \( \zeta > 0 \) by Lewandowski, Kurowicka, and Joe (2009) is proposed. For a further discussion of why this option is better than a conjugated inverse Wishart distribution see BÃijrkner (2017) and Natarajan and Kass (2000).

The prior distribution for \( \alpha \) and \( \beta \) parameters in a GARCH model can be chosen between \textit{normal, uniform or beta distribution}, this is due to their similarity to an auto-regressive coefficient constrained in \([0,1]\). For the MGARCH parameters a \textit{normal, t-student, Cauchy, gamma, uniform or beta} distributions can be chosen. Finally, for the \( \nu \) hyper-parameter in a GARCH model with unknown degree freedom innovations, a \textit{normal, inverse gamma, exponential, gamma, and a non-informative Jeffrey’s prior} are available Fonseca, Ferreira, and Migon (2008).

3. Estimation process

Just like \texttt{brms} (BÃijrkner 2017) or \texttt{rstanarm} (Goodrich, Gabry, Ali, and Brilleman 2020) packages, \texttt{varstan} does not fit the model parameters itself. It only provides a \texttt{Stan} (Stan, Development. Team 2017) interface in \texttt{R} (R Core Team 2017) and works exclusively with the extended Hamiltonian Monte Carlo Duane (1987), No U-Turn Sampler algorithm of Hoffman and Gelman (2014). The main reasons for using this sampler are its fast convergences and its less correlated samples. BÃijrkner (2017) compares between Hamiltonian Monte Carlo and other MCMC methods, and Betancourt (2017) provides a conceptual introduction of the Hamiltonian Monte Carlo.

After fitting the model, \texttt{varstan} provides functions to extract the posterior residuals and fitted values, such as the predictive m-steps ahead and predictive errors distribution. For model selection criteria, \texttt{varstan} provides posterior sample draws for the pointwise log-likelihood, Akaike Information Criteria (AIC), corrected AIC (AICc) and Bayesian Information Criteria (BIC) Bierens (2006). For a better performance in the model selection, an adaptation of the \texttt{bayes_factor} of the \texttt{bridgesampling} (Gronau, Singmann, and Wagenmakers 2020) package, the Bayesian leave one out (loo), and the Watanabe Akaike information criteria (waic) from the \texttt{loo} (Vehtari and Gabry 2017) package are provided Vehtari, Gelman, and Gabry (2016) and Kass and Raftery (1995).

The \texttt{bayes_factor()} approximates the model’s marginal likelihood using the \texttt{bridgesampling} algorithm, see Gronau, Sarafooglou, Matzke, Ly, Boehm, Marsman, Leslie, Forster, Wagenmakers, and Steingroever (2017) for further detail. The \texttt{waic} proposed by Watanabe (2010) is an improvement of the Deviance information criteria (DIC) proposed by Spiegelhalter, Best, Carlin, and Van Der Linde (2002). The \texttt{loo} is asymptotically equivalent to the \texttt{waic} Watanabe (2010) and is usually preferred over the second one Vehtari \textit{et al}. (2016).

3.1. Automatic order selection in arima models

Selecting an adequate order in a seasonal ARIMA model might be considered a difficult task.
In Stan, an incorrect order selection might be considered an ill model, producing multiple divergent transitions. Several procedures for automatic order selection have been proposed Tsay (2010), Hannan and Rissanen (1982) and Gomez (1998). A Bayesian version of Hyndman, Athanasopoulos, Bergmeir, Caceres, Chhay, O’Hara-Wild, Petropoulos, Razbash, Wang, and Yasmeen (2020) algorithm implemented in their forecast (Hyndman and Khandakar 2008) package is proposed. This adaptation consists in proposing several models and select the 'best' one using a simple criteria such as AIC, BIC or loglik. Finally, fit the selected model.

In the proposed function, the BIC is used as selection criteria for several reasons: it can be fast computed, and it is asymptotically equivalent to the bayes_factor. After a model is selected, the function fits the model with default weak informative priors. Even so, a BIC is a poor criteria for model selection. This methodology usually selects a good initial model with a small amount of divergences (usually solved with more iterations) delivering acceptable results. For further reading and discussion see Hyndman and Khandakar (2008).

4. Package structure and modeling procedure

Similar to brms (BÄijkner 2017) and rstanarm (Goodrich et al. 2020), varstan is an R interface for Stan, therefore, the rstan (Stan Development Team 2020) package and a C++ compiler is required, the https://github.com/stan-dev/rstan/wiki/RStan-Getting-Started vignette has a detailed explanation of how to install all the prerequisites in every operative system (Windows, Mac and Linux). We recommend to install R-4.0.0.0 version or higher for avoiding compatibility problems. The current development version can be installed from GitHub using the next code:

```r
R> if (!requireNamespace("remotes")) install.packages("remotes")
R> remotes::install_github("asael697/varstan",dependencies = TRUE)
```

The varstan dynamic is different from other packages. First, the parameters are not fitted after a model is declared, this was considerate so the user could select the parameter priors in a dynamic way and call the sampler with a satisfactory defined model. Second, all fitted model became a varstan S3 class, the reason of this is to have available summary, plot, diagnostic and predict methods for every model regardless of its complexity. The procedure for a time series analysis with varstan is explained in the next steps:

1. Prepare the data: varstan package supports numeric, matrix and time series classes (ts).

2. Select the model: the version() function provides a list of the current models. Their interface is similar to forecast and R’s stats packages. These functions return a list with the necessary data to fit the model in Stan.

3. Change the priors: varstan package defines by default, weak informative priors. Functions like set_prior(), get_prior and print() aloud to change and check the models priors.
Other useful functions are `parameters()` that prints the parameter's names of a specified model, and `distribution()` prints the available prior distributions of a specified parameter.

4. **Fit the model:** the `varstan()` function call Stan, and fit the defined model. Parameters like number of iterations and chains, warm-up, and other Stan's control options are available. The `varstan()` class contains a `stanfit` object returned from the `rstan` package, that can be used for more complex Bayesian analysis.

5. **Check the model:** `summary()`, `plot()`/`autoplot()` and `extract_stan()` methods are available for model diagnostic and extract the parameter's posterior chains.

   *The plot and summary methods will only provide general diagnostics and visualizations, for further analysis use the bayesplot (Gabry and Mahr 2019) package.*

6. **Select the model:** For multiple models, `varstan` provides `loglik()`, `posterior_residuals()`, `posterior_fit()`, `AIC()`, `BIC()`, `WAIC()`, `loo()` and `bayes_factor` functions for model selection criteria.

7. **Forecast:** the `posterior_predict()` function samples from the model's n-steps ahead predictive distribution.

5. **Case study: Analyzing the monthly live birth in U.S. an example**

   As an example, a time series modeling for the monthly live births in the United States 1948-1979, published in astsa (Stoffer 2019) package is provided. In Figure 1, the data has a seasonal behavior that repeats annually. The series waves in the whole 40 years period (superior part). In addition, the partial (pacf) and auto-correlation (acf) functions are far from zero (middle part), and have the same wave pattern as birth series. Indicating non stationary and a strong periodic behavior. After applying a difference to the data, the acf and pacf plots still have some non-zero values every 12 lags (inferior part).
For start, a seasonal ARIMA model could give a good fit to the data. Following Tsay (2010) recommendations for order selection using the auto-correlation functions, a $p = 1$, $d = 1$, $q = 1$ and for the seasonal part $P = 1$, $D = 1$, $Q = 1$. The model is defined in varstan as follows

```r
R> model1 = Sarima(birth,order = c(1,1,1),seasonal = c(1,1,1))
R> model1
```

```
y ~ Sarima(1,1,1)(1,1,1)[12]  
373 observations and 1 dimension  
Differences: 1 seasonal Differences: 1  
Current observations: 360
```

Priors:
Intercept:
$\mu_0 \sim t$ (loc = 0, scl = 2.5, df = 6)

Scale Parameter:
$\sigma_0 \sim \text{half}_t$ (loc = 0, scl = 1, df = 7)

$\text{ar}[1] \sim \text{normal}$ (mu = 0, sd = 0.5)
$\text{ma}[1] \sim \text{normal}$ (mu = 0, sd = 0.5)
Seasonal Parameters:

\[
\text{sar}\[ 1 \] \sim \text{normal} (\mu = 0, \sigma = 0.5) \\
\text{sma}\[ 1 \] \sim \text{normal} (\mu = 0, \sigma = 0.5) \\
\text{NULL}
\]

The function \text{Sarima} generates a Seasonal ARIMA model ready to be fitted in \text{Stan} (Stan, Development. Team 2017). As the model is printed, all the important information is shown: the model to be fit, the total observations of the data, the seasonal period, the current observations that can be used after differences, and a list of priors for all the model’s parameters. Using the information provided by the \text{acf-plot} in Figure 1 (middle right), the partial autocorrelations are not that strong, and a normal distribution for the auto-regressive coefficient (\text{ar}[1]) could explore values close to 1 or -1, causing the prior to be too informative. Instead beta distribution in \([-1, 1]\)^5 centered at zero, could be a more proper prior. With the functions \text{set_prior()} and \text{get_prior()} any change is automatically updated and checked.

\[
R> \text{model1} = \text{set_prior(model = model1,dist = beta(2,2),par = "ar")} \\
R> \text{get_prior(model = model1,par = "ar")}
\]

\[
\text{ar}[ 1 ] \sim \text{beta (form1 = 2, form2 = 2)}
\]

Now that the model and priors are defined, what follows is to fit the model using the \text{varstan()} function. One chain of 2,000 iterations and a warm-up of the first 1,000 chain’s values is simulated.

\[
R> \text{sfit1} = \text{varstan(model = model1,chains = 1,iter = 2000,warmup = 1000)} \\
R> \text{sfit1}
\]

\[
\text{y} \sim \text{Sarima}(1,1,1)(1,1,1)[12] \\
373 \text{ observations and 1 dimension} \\
\text{Differences: 1 seasonal Differences: 1} \\
\text{Current observations: 360}
\]

\[
\begin{array}{cccccc}
\text{mean} & \text{se} & 2.5\% & 97.5\% & \text{ess} & \text{Rhat} \\
\mu0 & 0.0061 & 0.0020 & 0.0020 & 0.0101 & 3941.739 & 1.0028 \\
\sigma0 & 7.3612 & 0.0043 & 7.3528 & 7.3697 & 4001.665 & 1.0000 \\
\phi & -0.2336 & 0.0014 & -0.2362 & -0.2309 & 3594.099 & 1.0007 \\
\theta & 0.0692 & 0.0017 & 0.0658 & 0.0726 & 3808.853 & 1.0019 \\
\phi0 & -0.0351 & 0.0015 & -0.0381 & -0.0321 & 3376.811 & 1.0033 \\
\theta0 & 0.6188 & 0.0017 & 0.6153 & 0.6222 & 3427.074 & 1.0048 \\
\loglik & -1232.2519 & 0.0325 & -1232.3157 & -1232.1882 & 3198.671 & 1.0033 \\
\end{array}
\]

Samples were drawn using \text{sampling(NUTS)}. For each parameter, ess is the effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1).

\[^5\] If \( \theta \sim \text{beta}(\alpha, \beta) \) in \([0, 1]\) then \( \theta_1 = 2(\theta - 1) \sim \text{beta}(\alpha, \beta) \) in \([-1, 1]\)
All fitted models are `varstan` objects, these are S3 classes with the `stanfit` results provided by the `rstan` (Stan Development Team 2020) package, and other useful elements that make the modeling process easier. After fitting the proposed model, a visual diagnostic of parameters, check residuals and fitted values using the plot methods. On figure 2 trace and posterior density plots are illustrated for all the model parameters.

R> plot(sfit1,type = "parameter")

Figure 2: Trace and density plots for all the fitted parameters

In figure 2, all the chains appeared to be stationary, and the posteriors seems to have no multi-modal distributions. Indicating that all chains have mixed and converged. One useful way to assess models’ fit, is by the residuals ($e_i = Y_i - \hat{Y}_i$). The package provides the posterior sample of every residual, but checking all of them is an exhausting task. An alternative, is checking the process generated by the residuals posterior estimate. A white noise behavior indicates a good model fit. The model’s residuals in figure 3, seems to follow a random noise, the auto-correlation in `acf` plots quickly falls to zero, indicating an acceptable model fit.

R> p1 = plot(sfit1,type = "residuals")
R> p2 = plot(sfit1)
R> grid.arrange(p2,p1,ncol = 1)
Because of the sinusoidal pattern that birth series (figure 1) presents, a dynamic Harmonic regression (A fourier transform with arima structure for errors) could also assess a good fit Kennedy (1992). To declare this model, varstan offers a similar declaration structure of forecast (Hyndman and Khandakar 2008) package. A harmonic regression with 4 fourier terms and ARIMA(1,1,1) residuals is declared and fitted to the birth data.

```r
R> model2 = Sarima(birth,order = c(1,1,1),xreg = fourier(birth,K = 2))
R> sfit2 = varstan(model = model2,chains = 1,iter = 2000,warmup = 1000)
R> sfit2
```

y ~ Sarima(1,1,1).reg[4]
373 observations and 1 dimension
Differences: 1, seasonal Differences: 0
Current observations: 372

|       | mean  | se    | 2.5%   | 97.5%  | ess   | Rhat |
|-------|-------|-------|--------|--------|-------|------|
| mu0   | -0.0712| 0.0068| -0.0846| -0.0578| 939.6338| 1.0032|
| sigma0| 10.8085| 0.0124| 10.7841| 10.8328| 994.9722| 1.0003|
| phi   | -0.2705| 0.0019| -0.2742| -0.2668| 1004.4136| 1.0006|
| theta | 0.6242| 0.0015| 0.6212| 0.6272| 921.6062| 0.9992|
| breg.1| -21.6318| 0.0407| -21.7116| -21.5520| 965.9837| 1.0006|
| breg.2| 0.6619| 0.0305| 0.6021| 0.7217| 976.7075| 1.0002|
| breg.3| 4.7937| 0.0207| 4.7531| 4.8344| 1079.1161| 1.0003|

Figure 3: Posterior median residual plot
Samples were drawn using sampling (NUTS). For each parameter, ess is the effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1).

In this scenario both models suggest to be a good choice for birth series analysis. Even so, the harmonic regression fits more parameters. It is an obvious choice for birth’s sinusoidal behavior. As an example of model selection criteria, the bayes_factor in logarithmic scale, that compares the models’ marginal likelihoods is computed. Values above 6 (in logarithmic scale) provides good evidence for selecting the first model. And for birth data, the seasonal arima model (model1) is a better choice.

R> bayes_factor(x1 = sf1,x2 = sfit2,log = TRUE)

Estimated log Bayes factor in favor of model1 over model2: 199.13745

Now, a comparison of the selected model (model1 ~ Sarima(1,1,1)(1,1,1)[12]) and the one given by the auto.sarima() function. For this purpose, a leave of one out cross validation loo() is used; and both looic are compared with the loo_compare() function provided by the loo (Vehtari and Gabry 2017) package.

R> sfit3 = auto.sarima(birth,chains = 1,iter = 4000)
R> sfit3

y ~ Sarima(0,1,2)(1,1,1)[12]
373 observations and 1 dimension
Differences: 1 seasonal Differences: 1
Current observations: 360

| mean     | se   |  2.5%  | 97.5%  | ess   | Rhat   |
|----------|------|--------|--------|-------|--------|
| mu0      | 0.0080| 0.0018 | 0.0045 | 0.0116| 2050.372| 0.9997 |
| sigma0   | 7.3517| 0.0060 | 7.3399 | 7.3634| 1991.938| 1.0008 |
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\begin{verbatim}
theta.1  0.3642  0.0013  0.3616  0.3668 1978.174 1.0006
theta.2  0.1358  0.0011  0.1336  0.1379 2023.769 1.0004
sphi  -0.2465  0.0016  -0.2496  -0.2433 2084.503 1.0005
stheta  0.3040  0.0017  0.3006  0.3073 2167.639 0.9995
loglik  -1231.7452  0.0395  -1231.8225  -1231.6679 1789.987 1.0009
\end{verbatim}

Samples were drawn using sampling (NUTS). For each parameter, ess is the effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat = 1).

Different from model1, the selected one does not contemplate an auto-regressive component, and use 2 mean average components instead. Now what proceeds is to estimate the loo for both models:

\begin{verbatim}
R> loo1 = loo(sfit1)
R> loo3 = loo(sfit3)
R> lc = loo::loo_compare(loo1,loo3)
R> print(lc,simplify = FALSE)
\end{verbatim}

\begin{verbatim}
elpd_diff se_diff elpd_loo se_elpd_loo p_loo se_p_loo looic se_looic
model2  0.0    0.0  -1235.4     15.4  7.1   0.8   2470   30.8
model1 -0.8   5.8  -1236.2     15.6  7.8   0.9   2472   31.2
\end{verbatim}

\texttt{loo_compare()} prints first the best model. In this example is the one provided by the \texttt{auto.sarima()} function, where its \texttt{looic} is 2 units below model1. This \texttt{auto.sarima()} function is useful as starting point. But the reader is encouraged to test more complex models and priors that adjust to the initial beliefs.

\section*{Conclusions}

The paper gives a general overview of \texttt{varstan} package as a starting point of Bayesian time series analysis with structured models, and it offers a simple dynamic interface inspired in the classic functions provided by \texttt{forecast}, \texttt{astsa}, and \texttt{var} packages. The interface functions and prior flexibility that \texttt{varstan} offers, makes Bayesian analysis flexible as classic methods for structured linear time series models. The package’s goal is to provide a wide range of models with a prior selection flexibility. In a posterior version, non-linear models such as wackier GARCH variants, stochastic volatility, hidden Markov, state-space, and uni-variate Dynamic linear models will be included. Along with several improvements in the package’s functionality.

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