Aspects of $\kappa$–symmetry

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ABSTRACT

We review various aspects of a fermionic gauge symmetry, known as the $\kappa$–symmetry, which plays an important role in formulations of superstrings, supermembranes and higher dimensional extended objects. We also review some aspects of the connection between $\kappa$–symmetric theories and their twistor-like formulations.

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1. Introduction

κ–symmetry [1] is a remarkable symmetry which plays an important role in the manifestly spacetime supersymmetric formulation of string theories [2]. It is also crucial for the existence of super p–brane theories [3][4][5]. While for \( p = 1 \) (string) there exists a formulation which does not have manifest spacetime supersymmetry but has a world-sheet local supersymmetry (the NSR formulation), such a formulation is forbiddingly complicated for higher super \( p \)–branes.

κ–symmetry is a fermionic symmetry, and in most models where it occurs, interestingly enough, a gauge field is not necessary for its realization. Furthermore, κ–symmetry is typically an on-shell symmetry and consequently, there is no simple way to construct higher order κ–invariants. Whether κ–symmetry is powerful enough to improve dramatically the finiteness behaviour of higher super p–branes is probably one of the most interesting and important open questions. A formidable obstacle to answering this question is the absence of a well defined covariant quantization scheme for κ–symmetry and this is basically due to the fact that the BRST quantization of the symmetry requires an infinite number of ghosts [6].

The aim of this relatively short review is to outline some of the salient features of κ–symmetric field theories, and some of the outstanding open problems. This article is based on a talk given at the conference in honor of Professor Abdus Salam. It is a great pleasure to make this contribution. Professor Abdus Salam has made so many important contributions in so many areas that I did not have any difficulty in choosing a subject which owes greatly to his work. As we will see below, κ–symmetric field theories are essentially sigma models formulated in a superspace, and the background fields occurring in these models are described by superfields. While these models have a very simple and elegant geometric form in superspace [7][8], it would be a nightmare to describe them in component formalism, in presence of a curved background. The notions of superspace and superfields, which play such a significant role in these theories, were invented by Salam and Strathdee in 1974 [9]. The concluding remark in their paper is rather interesting and it reflects their admirable approach to doing physics and their humility even when they present an important result [9]:

“*The approach discussed in this paper [the superspace approach] may not provide the most serviceable one available but, with the present hazy understanding of this curious and potentially important symmetry [supersymmetry], it seems worthwhile to examine every avenue*”.

2. Massless Superparticle in Curved Superspace

As mentioned above, κ–symmetric actions are basically sigma models in which the target space is a superspace. We take it to be a Salam-Strathdee type superspace which can be
viewed as the supercoset $G/H$ where $G$ is the super Poincaré group and $H$ is the Lorentz group in $d$ dimensions [9]. It would be very interesting to consider more general supermanifolds. Focusing on the usual scenario, we consider a target superspace with coordinates $Z^M = (X^m, \theta^\mu)$. It is useful to define the pull-back supervielbein as

$$E^A_\tau = \partial_\tau Z^M E_M^A,$$  \hspace{1cm} (2.1)$$

where $\partial_\tau$ denotes differentiation with respect to the worldline time variable. The tangent space index splits as $A = (a, \alpha)$, where $a = 0, 1, \ldots, d - 1$ labels a Lorentz vector and $\alpha$ labels the fundamental representation of the Lorentz group $\times$ the automorphism group of the super Poincaré algebra in $d$ dimensions. The simplest action for massless superparticle is given by

$$S = \frac{1}{2} \int d\tau e^{-1} \phi E^a_\tau E^a_\tau,$$  \hspace{1cm} (2.2)$$

where $e$ is the einbein field, $\phi(Z)$ is the dilaton superfield and it is understood that the contraction of the Lorentz indices is with the Lorentz metric $\eta_{ab}$. Note that this action describes the coupling of a massless particle to the target space fields that are contained in the superfields $E_M^A$ and $\phi$. (In flat target space, this action reduces to the Brink-Schwarz superparticle action [10]). The remarkable property of this action is that, with suitable constraints imposed on the target superspace torsion, it is invariant under the so-called $\kappa$–symmetry transformations which take the form [1][8]

$$\delta Z^M = \kappa_\alpha E^a_\tau \Gamma_{a \beta}^\alpha E^M_\beta,$$  \hspace{1cm} (2.3)$$

$$e^{-1} \delta e = \kappa_\alpha S^\alpha,$$  \hspace{1cm} (2.4)$$

where $S^\alpha$ is a function of the superfields to be determined by the $\kappa$–symmetry of the action and $E_M^A$ is the inverse supervielbein satisfying $E_M^A E_B^M = \delta^B_A$. The torsion tensor is defined by $T^A = dE^A + E^B \wedge \omega_B^A = \frac{1}{2} E^B \wedge E^C T_{CB}^A$ where the basis one form is $E^A = dZ^M E_M^A$ and the spin connection one form is $\omega_A^B = dZ^M \omega_{MA}^B$. In standard superspace the tangent space group is Lorentzian so that $\omega^\alpha_a = 0$ and $\omega_a^\alpha = 0$. Keeping this in mind, and making use of the following lemma

$$\delta E^A_\tau = \partial_\tau (\delta Z^M E^A_M) + \delta Z^M E_M^B E^C_\tau (-T_{CB}^A + \omega_C^A) B - (-1)^{BC} \omega_B^A C$$  \hspace{1cm} (2.5)$$

we find that the invariance of the action (2.2) under the $\kappa$–symmetry transformations (2.3) requires the following torsion constraints

$$T_{\alpha \beta}^c = 2(\Gamma^c)_{\alpha \beta}, \quad T_{\alpha (bc)} = u^\beta (b \Gamma^c)_{\beta \alpha} + \eta_{bc} v_{\alpha},$$  \hspace{1cm} (2.6)$$

and fixes the function $S^\alpha$ to be

$$S^\alpha = -4 E^\alpha_\tau + E^\alpha_\tau (2u^\alpha_a + 2\Gamma^\alpha_\beta v_{\beta} + \Gamma^\alpha_\beta \phi^{-1} D_{\beta} \phi),$$  \hspace{1cm} (2.7)$$

where $u^\alpha_a$ denotes the dual of the Lorentz metric $\eta_{ab}$.
where $u^\alpha a$ is an arbitrary $\Gamma$-traceless vector-spinor superfield $^\dagger$ and $v^\alpha$ is an arbitrary spinor superfield.

The action is also invariant under the worldline reparametrizations, and provided that $u^\beta a = 0$ and $\phi v_\alpha + \frac{1}{2} D_\alpha \phi = 0$, also under the local $\lambda$-symmetry transformations

$$\delta Z^M = \lambda E^\alpha a E^M_\alpha, \quad \delta e = 0,$$

(2.7)

where $\eta(\tau)$ and $\lambda(\tau)$ are arbitrary parameters. (For flat target space version of (2.7), see ref. [2]). Furthermore, provided that the target superspace admits a superconformal Killing vector $k^M$, the action is also invariant under the following target space rigid superconformal transformations [11]

$$\delta Z^M = k^M, \quad \delta e = 2e(U + \frac{1}{2} k^M \partial_M \phi),$$

(2.8)

where $U$ is a scalar superfield which occurs in the superconformal Killing equation $L_k E^a_M = U E^a_M + E^a_M b L^b a$, where the second term is a Lorentz transformation. Superconformal Killing vectors can be found at least in certain spacetimes of dimension $d \leq 7$. In higher dimensions one can always find the super Killing vectors which obey the super Poincaré algebra. The action will be invariant under such transformations, and hence its manifest spacetime supersymmetry.

The algebra of the symmetry transformations described above closes on-shell, i.e. modulo the equations of motion which take the form

$$E^a_\tau E^a_\tau = 0,$$

$$E^a_\tau \Gamma^a_{\alpha\beta} S^\beta = 0,$$

$$e\phi^{-1} \partial_\tau (e^{-1} \phi E^a_\tau) + E^b_\tau E^c_\tau (-T_{abc} + \omega_{bca}) + E^b_\tau E^a_\tau (T_{\alpha ab} - \omega_{\alpha ab}) = 0.$$

(2.9)

Instead of deriving the algebra of the $\kappa$-transformations directly in the Lagrangian formalism, following [12], we shall consider the algebra of first class constraints which generate the $\kappa$-transformations. To this end let us define the conjugate momenta

$$P_A = E^M_A \frac{\delta S}{\delta \partial_\tau Z^M},$$

$$= e^{-1} \phi E^a_\tau \delta^a_A.$$

(2.10)

The constraints which follow from the form of the Lagrangian are $P_a P^a \approx 0$ and $P_\alpha \approx 0$. However, these do not form a set of first class constraints and therefore do not form a closed algebra. Instead, let us define the combinations [12][13]

$^\dagger$ The freedom for having a vector-spinor in the $\kappa$-symmetry imposed constraint on $T_{\alpha (ab)}$ is usually overlooked, but as one allows field redefinitions afterwards to arrive at a standard set of constraints, this omission becomes inconsequential.
\[ A = \frac{1}{2} P^a P_a + A^\alpha P_\alpha, \quad B^\alpha = A^\alpha\beta P_\beta, \]

where \(A^\alpha\) and \(A^{\alpha\beta}\) are functions of superfields which are to be determined by the closure of the algebra of the above constraints. For definiteness let us consider the ten dimensional spacetime. It turns out that \(A^\alpha = 0\) and \(A^{\alpha\beta} = \Gamma^{\alpha\beta}_a P_a\). Furthermore, in [12] it has been shown that various sets of constraints can be obtained from those required by the closure of the constraint algebra plus the Bianchi identity

\[ \sum_{(ABC)} (D_A T_{BC} - R_{ABC} + T_{ABC} E T_{ECD}) = 0, \]

where \(R_{ABC}\) is the Riemann curvature. In addition, if one allows the local Lorentz transformations \(\delta E^a = E_b^a \Lambda_b\), and transformations of the form \(\delta E^\alpha = E^b \Lambda_b^\alpha + E^\beta \Lambda^\alpha_\beta\), it has been shown that [12] the set of constraints proposed by Nilsson [14]

\[ T_{\alpha\beta} = 2 (\Gamma^a)_{\alpha\beta}, \quad T_{\alpha a}^b = v_\alpha \delta^b_a, \quad T_{ab}^c = 0, \quad T_{\alpha\beta}^\gamma = \left( \Gamma^{abcde} \right)_{\alpha\beta} (\Gamma_{abcde})_{\gamma\delta} v_\delta, \]

or the set proposed by Witten [8]

\[ T_{\alpha\beta}^a = 2 (\Gamma^a)_{\alpha\beta}, \quad T_{\alpha a}^b = 0, \quad T_{a(bc)} = 0, \quad T_{\alpha\beta}^\gamma = 0, \quad T_{\alpha a} = -\frac{1}{24} (\Gamma_a \Gamma^{bcd})_{\alpha} \beta T_{bcd}, \]

can be derived. In obtaining (2.13) scale transformations of the form \(\delta E^\alpha = -\frac{1}{2} u E^\alpha\) and \(\delta E^a = -u E^a\) must also be allowed.

The two sets of constraints (2.12) and (2.13), as well as many other possible choices of constraints are all equivalent via appropriate field redefinitions. It is interesting to see how one such particular set, which differs from (2.12) and (2.13), emerges from the principle of light-like integrability in loop superspace [15]. In fact, one expects a very close connection between light-like integrability and \(\kappa\)–symmetry [8][12][15].

Neither set of the constraints (2.12) and (2.13) are sufficient to imply the \(N = 1, d = 10\) supergravity equations of motion. To describe them, one has to introduce a 3-form superfield in Nilsson’s case, while in Witten’s case, as \(T_{abc}\) happens to be totally antisymmetric, one can construct a super 3-form \(H\) in terms of \(T_{abc}\) and the dilaton superfield \(\phi\) [8][16]. Either set of constraints turn out to arise in superstring theory where the string couples to the two form field \(B\) whose field strength is \(H = dB\).

The massless superparticle can also be coupled to background Maxwell and Yang-Mills fields. For simplicity let us take the gauge group to be \(SO(N)\). Introducing fermionic worldline fields \(\psi^I(\tau), I = 1, \ldots, N\), the action can be written as

\[ S = \int d\tau \left( \frac{1}{2} e^{-\phi} E_\tau^a E_\tau^a + \partial_\tau Z^M B_M + \psi^I D_\tau \psi^I \right), \]
where $D_I \psi^I = \partial_\tau \psi^I + \partial_\tau Z^M A^I_M \psi^J$ and $B_M(Z)$, $A^I_M(Z)$ are the background Maxwell and Yang-Mills superfields. The $\kappa$–symmetry transformations again take the form (2.3) with the additional transformation rule

$$
\delta \psi^I = -\delta Z^M A^I_M \psi^J .
$$

The $\kappa$–symmetry transformations leave the action invariant provided that in addition to the constraints (2.5), the following ones are imposed

$$
H_{\alpha \beta} = 0, \quad H_{\alpha a} = \phi (\Gamma_a)_{\alpha \beta} h^\beta , \quad F^I_{\alpha \beta} = 0, \quad F^I_{\alpha a} = (\Gamma_a)_{\alpha \beta} \chi^\beta_{IJ} ,
$$

and that the function $S^\alpha$ is given by

$$
S^\alpha = -4 E^\alpha_T + E^a_T \left( 2u^a_{\alpha} + 2 \Gamma^\alpha_\beta v_\beta + \Gamma^\alpha_\beta \phi^{-1} D_\beta \phi \right) + \left( h^\alpha + \phi^{-1} \chi^\alpha_{IJ} \psi^I \psi^J \right) ,
$$

where $h^\alpha$, $\chi^\alpha_{IJ}$ are arbitrary superfields and $H = dB$, $F = dA + A \wedge A$. It should be noted that while the $B$–term in (2.14) is not necessary for $\kappa$–symmetry of the action, it will become necessary in the case of a massive superparticle action, as we will see in Sec. 4.

The action (2.14) is also invariant under the worldline reparametrizations, and provided that $A^I_M$ and $B_M$ are symmetric tensors with respect to the superconformal Killing vectors, also invariant under the target space rigid superconformal symmetry (2.8). The $\lambda$–symmetry (2.7) and (2.15), however, imposes the constraints $h^\alpha = 0$ and $\chi^\alpha_{IJ} = 0$, in addition to the previous ones: $u^a_{\alpha} = 0$, $\phi^\alpha + \frac{1}{2} D_\alpha \phi = 0$.

Considering again the ten dimensional spacetime, the constraints (2.16) describe super Maxwell and super Yang-Mills equations of motion. The coupled supergravity plus super Maxwell/super Yang-Mills system does not arise from the constraints implied by the $\kappa$–symmetry of the superparticle. Superstring theory however does describe such a system as we shall see later. In passing we note that coupling of Yang-Mills fields to the massless superparticle can be achieved also by using bosonic coordinates [12] instead of the fermionic ones used above.

**Flat Target Superspace**

It is useful to consider the flat target superspace limit of the actions described above. For simplicity let us consider the action (2.2) with the dilaton field set equal to a constant, e.g. $\phi = 1$. In a Salam-Strathdee superspace the spin connection is vanishing but the torsion does not completely vanish. Its only nonvanishing component is $T_{\alpha \beta}^a = 2(\Gamma^a)_{\alpha \beta}$. Thus from the definition of the torsion one finds the components of the supervielbein in flat superspace to be $E^a_m = \delta^a_m$, $E^\alpha_m = 0$, $E^a_\mu = (\Gamma^a \theta)_\mu$, $E^\alpha_\mu = \delta^\alpha_\mu$. Substituting this into (2.2) gives

$$
S = \frac{1}{2} \int d \tau e^{-1} \left( \partial_\tau X^a - \bar{\theta} \Gamma^a \partial_\tau \theta \right) \left( \partial_\tau X^b - \bar{\theta} \Gamma^b \partial_\tau \theta \right) \eta_{ab} .
$$

(2.18)
This is the Brink-Schwarz superparticle action [10]. The \( \kappa \)–transformation rules (2.3) become

\[
\delta X^a = \bar{\theta} \Gamma^a \delta \theta , \quad \delta \theta = \Pi^a \Gamma_a \kappa , \quad e^{-1} \delta e = -4 \bar{\kappa} \partial \tau \theta .
\] (2.19)

where \( \Pi^a = (\partial \tau X^a - \bar{\theta} \Gamma^a \partial \tau \theta) \). Since on-shell \( \Pi^a \Pi_a = 0 \) and that \( Tr \, \Pi^a \Gamma_a = 0 \), the matrix \( \Pi^a \Gamma_a = 0 \) has half as many zero eigenvalues, and therefore using \( \kappa \)–symmetry one can gauge away only half as many degrees of freedom \( \theta^a \).

This phenomenon of halving the fermionic degrees of freedom is rather similar to a phenomenon discovered long ago by Mack and Salam [17] in their study of conformally invariant field theories formulated on a five dimensional cone. From manifestly \( SO(4,2) \) invariant field theories on the cone, by suitably restricting the fields, and in the case of fermions by a gauge symmetry similar to \( \kappa \)–symmetry, they could obtain the 3+1 dimensional description of correct degrees of freedom. For example, let us define the cone by \( y^A y_A = 0, \, A = 1, ..., 6 \), where \( y^A \) are the embedding coordinates of a 4 + 2 dimensional plane. Consider the manifestly \( SO(4,2) \) invariant field equation for a fermionic field \( \psi \) to be of the form:

\[
(\Gamma^{AB} y_A \partial_B - 2) \psi = 0 , \quad y^A \partial_A \psi = -2 \psi ,
\] (2.20)

where the second equation is the restriction imposed on the field, representing its homogeneity degree on the cone. The above field equation is actually invariant under the following transformations

\[
\delta \psi = y^A \Gamma_A \kappa , \quad y^A \partial_A \kappa = -3 \kappa ,
\] (2.21)

where, the second equation expresses the condition which has to be imposed on the symmetry parameter \( \kappa \) in the form of homogeneity degree on the cone [18]. Since \( y^A y_A = 0 \) and \( Tr \, y^A \Gamma_A = 0 \), indeed the field \( \psi \) has as many degrees of freedom in 3+1 dimension. This phenomenon and the form of the transformation rules is similar to the Siegel’s \( \kappa \)–symmetry transformation rules [1]. Given that superparticle actions in \( d = 3, 4, 6 \) dimensions have also superconformal symmetry, one wonders if there is a deeper connection between the two phenomenon and possibly with the remarkable representations of the superconformal groups known as the supersingletons [18].

Quantization

In covariant BRST quantization of the massless superparticle, or indeed any \( \kappa \)–symmetric system, two features arise. Firstly, the \( \kappa \)–transformation rules close on-shell, and second, the \( \kappa \)–symmetry is an infinitely reducible type symmetry, in the terminology of Batalin and Vilkovisky [19]. Both of these features can be handled in the Batalin-Vilkovisky quantization procedure. However, the fact that the system is infinitely reducible means that an infinite tower of ghost fields are needed. This leads to some problems, such as the problem of regularizing infinite sums and the problem of Stueckelberg type residual gauge symmetries of the final BRST invariant action. A period of intense activity in these area occurred in
1989, resulting in a number of papers to which we refer the reader for a detailed discussion of these problems [6][20].

More recently, the idea of trading the $\kappa$–symmetry for world-line local supersymmetry has been put forward [21] in attempt not only to understand the origin of $\kappa$–symmetry as a special supersymmetry transformation, but also to achieve the covariant quantization in a way which may avoid the problems mentioned briefly above, as well as achieving an off-shell formulation which would then enable one to construct higher order invariants. (Some progress has already been made in [22] towards the quantization of these theories). The approach of ref. [21], which is currently gaining popularity, makes use of twistor-like variables, which are essentially commuting fermionic variables arising as superpartners of the fermionic coordinates of the target superspace. We now turn to a brief discussion of this approach.

Twistor-like Formulation

In order to introduce the twistor-like formulations, it is convenient to pass to the first order form of the action, which for the massless superparticle in flat target superspace is given by

$$S = \int d\tau \left[ P_a (\partial_\tau X^a - \bar{\theta} \Gamma^a \partial_\tau \theta) - \frac{1}{2} e P_a P^a \right].$$

This action is invariant under the $\kappa$–symmetry transformations

$$\delta X^a = \bar{\theta} \Gamma^a \delta \theta , \quad \delta \theta = \Gamma^a P_a \kappa , \quad \delta e = -4 \bar{\kappa} \partial_\tau \theta , \quad \delta P_a = 0 .$$

The constraints which follow from the action (2.22) are the reparametrization constraint $T := P_a P^a \approx 0$, and the fermionic constraint $d_\alpha := P_\alpha - P_a \Gamma^a_{\alpha \beta} \theta^\beta \approx 0$, where $P_\alpha$ is the conjugate momentum associated with $\theta_\alpha$ (see eq. (2.10)). The fermionic constraint is a combination of first class constraints generating the $\kappa$–symmetry transformations and second class constraints which, of course, have no such an interpretation.

The main idea of the twistor-like formulation is to replace the momentum variable $P_m$ with a suitable combination of commuting fermions, which are the twistor-like variables, to reformulate the action (2.3) in a world-line locally supersymmetric fashion [21]. The word *twistor-like* is used to avoid confusion with the supertwistor which consists of a multiplet of fields forming a multiplet of superconformal groups which are known to exists in dimensions $d \leq 6$. In fact such variables have been used previously in a twistor formulation of superparticles and superstrings in $d = 3, 4, 6$ [23]. A similar, but not quite the same, multiplet of variables were introduced in [24] to give a twistor-like formulation of these models in $d = 10$ as well. However, the twistor-like formulation we shall briefly review below, which is due to [21], differs from this formulation, in that it involves a different set of variables yet. The advantage of this formulation is that it allows a natural generalization to curved superspace, as well as higher super p-branes [25][26]. It should be noted that in all these three different
formulations, there is a common variable, namely the commuting spinor mentioned earlier which is used to replace the momentum variable $P_a$.

For superparticles and superstrings the twistorlike formulation works in $d = 3, 4, 6, 10$. To simplify matters, let us focus our attention to the $d = 3$ case. The twistor-like formulation of the massless superparticle in $d = 3$ is given by [21]

$$S = \int d\tau P_a (\partial_\tau X^a - \bar{\theta} \Gamma^a \partial_\tau \theta + \bar{\lambda} \Gamma^a \lambda) , \quad (2.24)$$

where $\lambda$ is the twistor-like variable, a two component commuting Majorana spinor in $d = 3$. Its equation of motion is $P_a \Gamma^a \lambda = 0$, whose solution can be shown to be

$$P_a = e^{-1} \bar{\lambda} \Gamma_a \lambda , \quad (2.25)$$

thanks to the identity

$$\Gamma^a (\alpha \beta \Gamma^a \gamma)^\delta = 0 . \quad (2.26)$$

Note that $P_a P^a = 0$ as well, due to this identity. In this action $\kappa$-symmetry has been replaced by $n = 1$ local world-line supersymmetry generated by $Q := \lambda^a \delta_a$ [21], were we recall that $d_\alpha = P_a - (P_a \Gamma^a \theta)_\alpha$.

To close the $n = 1$ supersymmetry off-shell, it is convenient to pass to a superfield formalism which will naturally introduce the necessary auxiliary fields. To this end, let us consider the superline with coordinates $(\tau, \eta)$ and the following superfields

$$P_a(\tau, \eta) = P_a + \eta \rho_a , \quad X^a(\tau, \eta) = X^a + \eta \lambda^a , \quad \theta(\tau, \eta) = \theta + \eta \lambda , \quad (2.27)$$

where $\eta$ is the single anticommuting coordinate, and the auxiliary fields $\rho$ and $\chi$ have been introduced. Note that the twistor variable $\lambda$ has been paired with the target space fermionic variable $\theta$. In terms of these superfields the off-shell supersymmetric action is [21]

$$S = \int d\tau d\eta P_a (D X^a + \bar{\theta} \Gamma^a D \theta) , \quad (2.28)$$

where $D = \frac{\partial}{\partial \eta} + \eta \frac{\partial}{\partial \tau}$. Note that this action has the form of a Wess-Zumino term.

To formulate the massless superparticle action in $d > 3$, and to put the above formulation in a somewhat more geometrical form that will enable us to make the curved superspace and higher p-brane generalizations, it is useful to introduce the following notation. For definiteness let us focus on $d = 10$, in which case the $\kappa$ symmetry can be replaced by $n = 8$ world-line supersymmetry. The coordinates of $n = 8$ worldline superspace will be denoted by $Z^M = (\tau, \mu)$ and the coordinates of the target superspace by $Z^\alpha_M = (m, \mu)$, where $\mu = 1, \ldots, 8$, $m = 0, 1, \ldots, 9$, $\alpha = 1, \ldots, 16$. We take the $d = 10$ spinors to be Majorana-Weyl. The world-line supervielbein will be denoted by $E_M^A$ and the target space supervielbein
by $E_M^A$, where the tangent space indices split as $A = (0, r)$ and $A = (a, \alpha)$, respectively, with $r = 1, \ldots, 8$, $a = 0, 1, \ldots, 9$, $\alpha = 1, \ldots, 16$. Note that the un-underlined indices always refer to the world-line superspace quantities, while their underlined versions always refer to the corresponding target space quantities.

The twistor-like formulation of massless superparticle and superstrings in $d > 3$ with $\kappa$ symmetry completely replaced by an $n$–extended worldline supersymmetry requires the introduction of $n$ twistor-like variables $\lambda_r^a$ satisfying the constraint [27]

$$\bar{\lambda}_r \Gamma^a \lambda_s = \frac{1}{8} \delta_{rs} \left( \bar{\lambda}_q \Gamma^a \lambda_q \right). \quad (2.29)$$

To express this constraint in a geometrical form, it is convenient to introduce the notation

$$E_A^A = E_A^M \left( \partial_M Z^M \right) E_M^A. \quad (2.30)$$

Let us furthermore make the identifications

$$E_r^a |_{\theta=0} = \lambda_r^a, \quad E_0^a |_{\theta=0} = \mathcal{E}_0^a, \quad (2.31)$$

where $\mathcal{E}_0^a = \partial_\tau X^a - \bar{\theta} \Gamma^a \partial_\tau \theta$, in flat target superspace. Thus, the fermionic coordinates $\theta^a$ have been elevated to a superfield whose expansion is of the form $\theta^a(\tau, \theta) = \theta^a(\tau) + \lambda_r^a(\tau) \theta^r + \cdots$. Furthermore, for definiteness and to simplify matters we shall characterize, both, the worldline and target superspace geometries. Namely, we shall take the worldline superspace to be characterized by the torsion constraints

$$T_{rs}^0 = 2 \delta_{rs}, \quad T_{0r}^0 = 0, \quad T_{s0}^r = 0, \quad T_{rs}^q = 0, \quad (2.32)$$

and we shall take the target superspace geometry to be characterized by the torsion constraints

$$T_{\alpha \beta}^c = 2 \gamma^c_{\alpha \beta}, \quad T_{ba}^a = 0, \quad T_{\alpha \beta}^\alpha = 0. \quad (2.33)$$

Of course, all consequences of these constraints which follow from the Bianchi identities are understood to hold.

The $n = 8$ locally supersymmetric action can now be written as [28]

$$S = \int d\tau d^8 \theta P_\alpha^r E_r^\alpha, \quad (2.34)$$

where $P_\alpha^r$ is a Lagrange multiplier superfield. The constraints (2.32) leave enough room for worldline diffeomorphisms and local $n = 8$ supersymmetry. (See ref. [28] for a detailed description of these transformations).

The field for the Lagrange multiplier is $E_r^\alpha = 0$. Taking the curl of this equation, and recalling (2.31), yields an integrability condition whose $\theta = 0$ component is the twistor
constraint (2.29). The classical equivalence of this theory to the usual \( \kappa \)-symmetric one has been shown in [28].

Analogous twistor-like formulations have also been proposed for massive superparticle [29][26], superstrings [28][30], supermembranes [25] and all higher super p-branes [26]. The questions of covariant quantization, and higher order invariants have been so far addressed only in limited cases. It has been shown in [22] that the quantization of the superstring in which two of the eight \( \kappa \)-symmetries are replaced by \( n = 2 \) world-sheet local supersymmetry, a semi-light cone gauge, while not covariant, avoids a problem encountered in the usual light-cone gauge formulation due to the global issues that arise in choosing such a gauge. As for the construction of higher order \( \kappa \)-invariants, results have been obtained for the case of massive superparticle in \( d = 2, 3 \) [29] by using the twistor-like formulation. (See the references in [29] for some other approaches to \( \kappa \)-symmetry calculus). We next turn to the description of the \( \kappa \)-symmetric superstring theory in curved superspace.

3. Superstring in Curved Superspace

For definiteness, let us focus our attention on the heterotic string propagating in \( d = 10 \) supergravity plus \( SO(32) \) Yang-Mills background. Let the coordinates of the world-sheet be \( \sigma^i, i = 0, 1 \), and the pulled-back supervielbein

\[
E^A_i = \partial_i Z^M E^A_M ,
\]

where the tangent space index splits as \( A = (a, \alpha) \) with \( a = 0, 1, ..., 9 \) and \( \alpha = 1, ..., 16 \). We shall be dealing with sixteen component Majorana-Weyl spinors in \( d = 10 \). The ingredients for the action are the Kalp-Ramond super 2-form \( B = dZ^M \wedge dZ^N B_{NM} \), the Yang-Mills superfield \( A_M \), the dilaton superfield \( \phi \) and the heterotic fermions \( \psi^I, I = 1, ..., 32 \) which are Majorana-Weyl spinors on the world-sheet. In terms of these building blocks, the heterotic string action can be written as

\[
S = \int d^2 \sigma \left[ -\frac{1}{2} \phi \sqrt{-g} g^{ij} E_i^a E_j^a + \frac{1}{2} \epsilon^{ij} \partial_i Z^M \partial_j Z^N B_{NM} + \frac{1}{2} \sqrt{-g} \psi^I \gamma^i D_i \psi^I \right] ,
\]

where \( g_{ij} \) is the world-sheet metric of signature \((-1, +1)\), \( g = \det g_{ij} \) and \( D_i \psi^I = (\partial_i \delta^{IJ} + \partial_i Z^M A^{IJ}_M) \psi^J \). We shall require that the action be invariant under the \( \kappa \)-symmetry transformations of the form

\[
\delta Z^M = \kappa_{i\alpha} P^i_+ E^a_j \Gamma^{a\alpha\beta} E^M_{\beta} , \quad \delta \psi^I = -\delta Z^M A^{IJ}_M \psi^J , \quad \epsilon_{ir} \delta e^{jr} = P_{+im} P^j_+ S^{ma} \kappa_{na} ,
\]

where \( \kappa_{i\alpha} \) is the transformation parameter. Note that unlike in the particle case, this parameter is a world-sheet vector in addition to being a target space (Majorana-Weyl) spinor. Note also that only its self-dual projection occurs. The quantity \( S^{ma} \) is to be determined by
the $\kappa$–symmetry of the action. The duality projectors are defined as

$$P_{\pm}^{ij} = \frac{1}{2} (g_{ij} \pm \sqrt{-g} \epsilon_{ij}),$$

satisfying the relation $P^+_i P^+_j = 0$. Note that the Levi-Civita symbols $\epsilon_{ij}$ and $\epsilon^{ij}$ are both constants, satisfying $\epsilon^{ij} \epsilon_{jk} = \delta^i_k$.

The $\kappa$–symmetry of the action imposes the constraints

$$T_{\alpha\beta}^c = 2(\Gamma^c)_{\alpha\beta}, \quad T_{\alpha(bc)} = u^\beta (\Gamma^c)_{\beta\alpha} + \eta_{bc} v_\alpha,$$

$$H_{\alpha\beta\gamma} = 0, \quad H_{a\alpha\beta} = -2\phi (\Gamma_{a\beta})_{\alpha\beta} h_\beta + 2\phi u^\beta_{[a} \Gamma_{b]\beta\alpha},$$

$$F_{\alpha\beta}^I = 0, \quad F_{\alpha\alpha}^I = (\Gamma_a)_{\alpha\beta} \chi^I_{\alpha\beta},$$

$$h_\alpha = v_\alpha + \frac{1}{2}\phi^{-1} D_\alpha \phi.$$

Furthermore, the quantity $S^\alpha_i$ is determined to be

$$S^\alpha_i = -4E^\alpha_i + 2E_{i\alpha} \left(-u^\alpha_a + (\Gamma^\alpha_a h_\beta) - \frac{1}{2}\phi^{-1} \psi^I \gamma_i \psi^J \chi^\alpha_{IJ} \right).$$

Concerning the occurrence of $u^\alpha_a$ in these formulae, see the footnote below eq. (2.6). The action (3.2) is also invariant under the world-sheet reparametrization and Weyl scalings. While the rigid spacetime superconformal symmetries are no longer possible [11], the action is of course invariant under the rigid spacetime Poincaré supersymmetry.

Assuming the $\kappa$–symmetry constraints (3.5), the action (3.2) is also invariant under the following local bosonic $\lambda$–symmetry transformations

$$\delta Z^M = P^{ij}_+ E^\alpha_i \epsilon^j M, \quad \delta e_{ir} = 0,$$

provided that $u^\alpha_a = 0$, $h_\alpha = 0$ and $\chi^\alpha_{IJ} = 0$. (For the variation of $\psi^I$ we adopt the same rule as in (2.15)). The flat target space version of this symmetry has been given in ref. [2].

The question arises as to what supergravity theory, if any, do the above constraints describe. In the absence of the Yang-Mills sector, and ignoring the question of $\kappa$–symmetry anomalies, it was shown by Witten that the constraints of pure $d = 10, N = 1$ supergravity are consistent with $\kappa$–symmetry. Whether they are necessary for $\kappa$ symmetry is more difficult to establish. To this end, as in the particle case, one may investigate the closure of first class constraints which may be constructed out of the constraints which follow from the Lagrangian. Then allowing field redefinitions (including the scaling of the supervielbein) and taking into account the Bianchi identities, in [12] it has been shown for the case of pure supergravity that Witten or Nilsson constraints, supplemented by additional constraints needed for the description of $N = 1, d = 10$ supergravity, can be derived.

However, pure $N = 1, d = 10$ supergravity has gravitational anomalies. One has to include an $SO(32)$ or $E_8 \times E_8$ Yang-Mills sector, and utilize the Green-Schwarz mechanism to cancel the gravitational, gauge and mixed anomalies. These anomalies reflect themselves

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\[1\] The study of $\kappa$–symmetric string theory in curved background was pioneered by Witten [8]. Soon afterwards, various extensions of his results were obtained in refs. [31][32].

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in the form of \( \kappa \)-symmetry anomalies, from the world-sheet point of view, which can also be cancelled by a similar mechanism. In trying to understand the consequences of the \( \kappa \)-symmetry constraints (3.4), this anomaly cancellation mechanism must be taken into account. The \( \kappa \)-symmetry anomalies have been discussed in detail in \cite{33}, and previously in ref. \cite{34}. To cancel them one adds a counterterm \( \Gamma \), whose \( \kappa \)-symmetry variation is \[ \delta \Gamma = \int d^2 \sigma \epsilon^{ij} E^A_i E^B_j \kappa^\alpha \kappa_{\alpha BA} , \] (3.6)

where \( \kappa_{\alpha BA} \) are particular components of a super 3-form \( \kappa \) which takes the form

\[ \kappa = \omega_{3Y} - \omega_{3L} + \cdots , \] (3.7)

and \( \omega_{3Y} \) and \( \omega_{3L} \) are the Yang-Mills and Lorentz Chern-Simons forms. Then the \( \kappa \)-variation of the classical action plus the anomalous variation \( \delta \Gamma \) will yield the same result as in the variation of the classical action alone, except that the curvature \( H = dB \) will now be replaced by

\[ \mathcal{H} = dB + \kappa . \] (3.8)

If we wish to impose the so-called standard set of constraints, i.e. those in (3.4) with \( u^\alpha_a = 0 \) and \( v^\alpha = 0 \) set equal to zero (one could consider any equivalent set), then the above replacement is consistent provided that \[ (dK)_{\alpha\beta\gamma\delta} = 0 , \quad (d\kappa)_{a\alpha\beta\gamma} = 0 . \] (3.9)

Such a \( \kappa \) can be found, thanks to the existence of the following relation \[ \text{Tr}(RR) = dX + K , \] (3.10)

for some 3-form \( X \) and a 4-form \( K \) which has the vanishing projections \( K_{\alpha\beta\gamma\delta} = 0 \) and \( K_{a\alpha\beta\gamma} = 0 \). Since \( \text{Tr}(FF) \) has the same vanishing projections as well, it follows that a suitable choice for \( \kappa \) is \[ \kappa = \omega_{3Y} - \omega_{3L} + X . \] (3.11)

With this choice, \( d\kappa = \text{Tr}(FF) - K \) indeed has the vanishing projections in the \((\alpha\beta\gamma\delta)\) and \((a\alpha\beta\gamma)\) directions as required.

In summary, the constraints needed to be imposed are the torsion constraints (3.4) with \( u^\alpha_a = 0 \), \( v^\alpha = 0 \) and with the replacement \( H \rightarrow \mathcal{H} \) to be made. When the consequences of the Bianchi identities are taken into account as well, all the superfields get determined in terms of the totally antisymmetric \( T_{abc}, \chi_\alpha \) and \( \phi \). In the absence of the Lorentz Chern-Simons term, for example, the remaining projection of \( \mathcal{H} \) is found to be: \[ \mathcal{H}_{abc} = -\frac{3}{2} \phi T_{abc} + \frac{1}{4} (\Gamma_{abc})_{\alpha\beta} \text{Tr}(\chi^\alpha \chi^\beta) \] [16]. In this case the resulting supergravity equations are those of the usual \( N = 1, D = 10 \) supergravity plus Yang-Mills system. When the effect of the Lorentz
Chern-Simons term is taken into account as outlined above, the resulting equations describe an anomaly free model which can accommodate stringy corrections, in particular an $R^4$-type correction. For more details, we refer the reader to ref. [35].

So far we have considered the heterotic string in $d = 10$. For a discussion of actions for heterotic strings in $d = 3, 4, 6$, see ref. [32], where the occurrence of various supergravity/matter, as well as off-shell supergravity backgrounds is also discussed. A Green-Schwarz type action for the Type IIA superstring has been discussed in ref. [36], and for the Type IIB string in ref. [31]. In the latter two cases, what has been shown is that the superspace constraints of on-shell Type IIA and Type IIB supergravities are sufficient for the $\kappa$-symmetry of the string actions.

4. Massive Superparticle, Supermembranes and Higher Super $p$–Branes

Just as the action for the massless superparticle is similar to the heterotic string action, the massive superparticle action is similar to the action for supermembranes and higher super $p$–branes. We shall describe the action for them in a unified manner.

A super $p$–brane moving in a superspace of $d$ bosonic dimensions will sweep a worldvolume of $p+1$ dimensions. Denoting the coordinates of the world-volume by $\sigma^i$ and the coordinates of the target superspace by $Z^M$, we again adopt the definition (3.1), with $i = 1, \ldots, p+1$. The tangent space index again splits as $A = (a, \alpha)$ with $a = 0, 1, \ldots, d-1$ and for definiteness, we take $\alpha$ we label the minimum dimensional spinors possible in $d$–dimensions. Thus, it can be said that we are considering Type 1 super $p$–branes, in the sense that we are considering the minimum possible target space supersymmetry. In fact, while the existence of Type 2 super $p$–branes (i.e. target spaces with higher than minimum supersymmetry) has been proposed, at present no action is known for them, and this is one of the most interesting open problems in the theory of super $p$–branes.

The action for super $p$–branes requires a Kalp-Ramond type super $(p+1)$–form whose pull-back is

$$B_{i_1 \ldots i_{p+1}} = \partial_{i_1} Z^M_{i_1} \ldots \partial_{i_{p+1}} Z^{M_{p+1}} B_{M_{p+1} \ldots M_1}.$$  \hspace{1cm} (4.1)

An action for super threebrane in $d = 6$ flat target spacetime was first given in ref. [3]. Generalization to all super $p$–branes in curved target superspace was found in ref. [4]. The action takes the form

$$S = \int d^{p+1}\sigma \left[ -\frac{1}{2} \sqrt{-G} g^{ij} \phi E_i^a E_j^a + \epsilon^{i_1 \ldots i_{p+1}} B_{i_1 \ldots i_{p+1}} + \frac{1}{2} (p-1) \sqrt{-g} \right].$$  \hspace{1cm} (4.2)

Note the presence of the worldvolume cosmological term for $p \neq 1$. It is essential for the equation of motion for the metric $g_{ij}$ to yield the nondegenerate form

$$g_{ij} = \phi E_i^a E_j^a.$$  \hspace{1cm} (4.3)
Since this is an algebraic equation it can be used in establishing the \( \kappa \)-symmetry of the action. In this “1.5” formalism we need not vary the metric \( g_{ij} \) and the \( \kappa \)-transformation of the only independent variable \( Z^M \) is given by

\[
\delta Z^M = \kappa^\alpha (1 + \Gamma)_{\alpha}^\beta E^M_{\beta},
\]

(4.4)

where the matrix \( \Gamma \) is given by

\[
\Gamma = \frac{\epsilon_p}{(p+1)! \sqrt{-g}} \varepsilon_{i_1...i_{p+1}} \Gamma_{a_1}^{a_1} ... \Gamma_{a_{p+1}}^{a_{p+1}} E_{i_1}^{a_1} ... E_{i_{p+1}}^{a_{p+1}},
\]

(4.5)

which satisfies \( \Gamma^2 = 1 \) provided that we use (4.3) and choose \( \epsilon_p = (-1)^{(p+2)(p+3)/4} \). Thus, the matrix \( (1 + \Gamma) \) is a projection operator on-shell, and the phenomenon of halving the fermionic degrees of freedom occurs for all super \( p \)-branes.

The action (4.2) is invariant under the \( \kappa \)-symmetry transformation provided that in addition to the torsion constraints (3.4a), the following following constraints on the super \( (p+2) \)-form \( H = dB \) are satisfied [4]

\[
H_{\alpha\beta\gamma A_1...A_{p-1}} = 0,
\]

(4.6a)

\[
H_{\alpha\beta p...a_1} = 2\epsilon_p \phi (\Gamma_{a_1...a_p})_{\alpha\beta},
\]

(4.6b)

\[
H_{\alpha a_{p+1}...a_1} = \epsilon_p \phi (\Gamma_{a_1...a_{p+1}})_{\alpha}^\beta h_\beta + \epsilon_p \phi u_\beta [a_{p+1} \Gamma_{a_1...a_p}] y_{\alpha} \beta,
\]

(4.6c)

\[
h_\alpha = v_\alpha + \frac{1}{2} \phi^{-1} D_\alpha \phi,
\]

(4.6d)

where \( v_\alpha \) and \( h_{\alpha a} \) are arbitrary superfields. (Concerning the occurrence of the vector-spinor \( h_{\alpha a} \) in the constraints, see the footnote below eq. (2.6)). The Bianchi identity \( dH = 0 \) is satisfied provided that the following \( \Gamma \)-matrix identity is satisfied:

\[
\Gamma_{(\alpha\beta\gamma}\Gamma^{ac_1...c_p} = 0.
\]

(4.7)

This identity which is crucial for the \( \kappa \)-symmetry of the action imposes restrictions on possible dimensions \( d \) of spacetime and on possible values of \( p \) for a \( p \)-dimensional extended object. Assuming Lorentzian signature, these restrictions are [5]: \( (p = 1; d = 3, 4, 6, 10), \) \( (p = 2; d = 4, 5, 7, 11), \) \( (p = 3; d = 6, 8), \) \( (p = 4; d = 9), \) \( (p = 5; d = 10). \) It is a very interesting fact that the existence of the \( \kappa \)-symmetry imposes so severe restrictions on both the dimension of spacetime and the extension of the fundamental object. We recall that the restriction \( d \leq 11 \) arises for the existence of supergravities with a single gravitational field, and the restriction \( d \leq 6 \) arises for the existence of scalar supermultiplets. Since in a gauge in which \( \kappa \)-symmetry is fixed, it is expected that a globally supersymmetric field theory in a \( (p+1) \)-dimensional worldvolume emerges, the occurrence of the restrictions \( d \leq 11 \) and \( p \leq 5 \) can be viewed as a consequence of an interesting fusion of worldvolume and targetspace supersymmetries via the \( \kappa \)-symmetry.
Turning to the constraints (3.4a) and (4.6), only in the case of eleven dimensional it is known rigorously that these constraints (with the dilaton superfield redefined away) do imply the unique $N = 1, d = 11$ supergravity equations of motion [4][37]. For the case of super fivebrane in ten dimensions, one expects that the corresponding equations of motion are those of the dual formulation of the $N = 1, d = 10$ pure supergravity, in which instead of a 2-form field, a 6-form field occurs. In the remaining cases one expects that a suitable version of supergravity theories possibly coupled to matter/Yang-Mills supermultiplets will be described by the constraints required for the existence of the super $p$–brane actions. It would be of considerable interest to work out explicitly the consequences of these constraints and to determine exactly which supergravity theories are described by them. (See ref. [38] for a description of supergravity theories in diverse dimensions).

**Heterotic and Type II Super $p$–Branes**

One of the most interesting open problems is to find certain generalizations of the super $p$–brane actions. The possibility of super $p$–branes described by non–Poincaré symmetries of non-Lorentzian target spaces has been discussed in ref. [39]. In particular, the case of a 2+2 worldvolume embedded in 10+2 dimensional target space is rather interesting, but no action has been found for it. There are two other kinds of generalizations whose existence have been established by indirect means and for which no actions have been written down so far. These are the analogs of the heterotic string, which are called the **heterotic $p$–branes** (in the sense that they describe the coupling Yang-Mills to the super $p$–brane) [40][41], and the analogs of the Type IIA and Type IIB superstrings, which are called the **Type II super $p$–branes** (in the sense that the target space supersymmetry is not the minimal $N = 1$ supersymmetry, but instead an $N = 2$ supersymmetry which could be of (2, 0) or (1, 1) type) [42][43]. The existence of these types of super $p$–branes has been deduced from the existence of certain kinds of supersymmetric extended soliton solutions to the anomaly free $N = 1$ supergravity plus Yang-Mills, Type IIA or Type IIB supergravity theories in $d = 10$. These are solutions with $(p + 1)$–dimensional Poincaré symmetry and they are asymptotically flat in the internal directions. For example, the super fivebrane solution of ref. [41] is obtained by solving the equations of motion which follow from the following low energy action for the bosonic degrees of freedom of the heterotic string in $d = 10$:

$$S = \frac{1}{\alpha'^4} \int d^{10}x \sqrt{-g} e^{-2\phi} \left( R + 4(\nabla \phi)^2 - \frac{1}{3} H^2 - \frac{1}{12} \alpha' \text{Tr} F^2 \right),$$  \hspace{1cm} (4.8)

where the notation is self-explanatory. The solution has (1, 0) type supersymmetry on the worldvolume, and it takes the form [41]

$$g_{ab} = \eta_{ab}, \quad g_{\mu \nu} = e^{2\phi} \delta_{\mu \nu}, \quad e^{2\phi} = e^{2\phi_0} + 8\alpha' \left( \frac{x^2 + 2\rho^2}{(x^2 + \rho^2)^2} \right),$$  \hspace{1cm} (4.9)

$$H_{\mu \nu \lambda} = -\epsilon_{\mu \nu \lambda} \nabla_{\rho} \phi, \quad A_\mu = -\frac{2\mu_{\mu \nu} x^\nu}{(x^2 + \rho^2)},$$
where we have used the notation of ref. [41], according to which, $a = 0, 1, ..., 5$ labels the coordinates of the fivebrane worldvolume, $\mu = 5, ..., 9$ labels the four Euclidean internal coordinates $x^\mu$, while $\phi_0$ is the value of the dilaton at spatial infinity, $x^2 = x^\mu x^\nu \delta_{\mu\nu}$, and $\rho$ is the size of the instanton, and $\Sigma_{\mu\nu}$ are the antisymmetric, self-dual 't Hooft matrices which arise in the description of the Yang-Mills instantons in four Euclidean dimensions. The ansatz for $A_\mu$ lies in $SU(2)$ subgroup of $E_8 \times E_8$ or $SO(32)$.

This solution breaks the spacetime symmetries, and in particular the $d = 10$ translation group is broken to that of the worldvolume, and half of the spacetime symmetries are maintained. (There are other interesting solutions which break more than half of the spacetime supersymmetries [44]. Presumably the usual $\kappa$–symmetric formulations can not exist for the super $p$–branes implied by these solutions, but it is tempting to consider that the twistor-like formulations may accommodate them). The unbroken symmetries are linearly realized, while the broken symmetries are nonlinearly realized on the worldvolume. The massless Goldstone bosons and fermions arising in this symmetry breaking turn out to be the position coordinates and the heterotic coordinates of the fivebrane. These fields correspond to the zero modes of the wave operator for the fluctuations around the soliton, but they are easier to determine (at least their number) by using index theorems. In ref. [41], it is argued that there are 240 fermionic zero modes (corresponding to 120 degrees of freedom) and 120 bosonic zero modes. These include the 4 translational and 8 supertranslational zero modes.

The effective action describing the dynamics of the massless fields is presumably a $(1, 0)$ supersymmetric hyperkahler sigma model in $5 + 1$ dimensions. However, its exact form has never been spelled out as yet. Such an action would be the gauge fixed version of an action in which the worldvolume symmetries would be realized covariantly, and in particular the worldvolume supersymmetry would manifest itself in the form of a $\kappa$–symmetry. This covariant form of the action is even more mysterious, and it is one of the major open problems in this subject.

As for the $p$–brane solitons of the Type IIA and Type IIB supergravities, an extensive review about them can be found in ref. [45]. An interesting new feature emerging is that the zero modes implied by these solutions correspond to vector or antisymmetric tensor supermultiplets of $(1, 1)$ or $(2, 0)$ supersymmetry on the worldvolume, unlike in the case of Type I super $p$–branes where the zero modes form only scalar supermultiplets [42]. The values of $p$, the worldvolume supersymmetry $N$ and the nature of the worldvolume supermultiplet which arises, corresponding to the solutions of Type IIA supergravity in $d = 10$ are [42][45]

\[
\{p = 4, N = 2; A_\mu, 4\lambda, 5\phi\}, \quad \{p = 5, N = (2, 0); B^-_{\mu\nu}, 4\lambda, 5\phi\}, \quad \{p = 6, N = 1; A_\mu, \lambda, 3\phi\},
\]

while those corresponding to the solutions of Type IIB supergravity in $d = 10$ [42][45] are

\[
\{p = 3, N = 2; A_\mu, 4\lambda, 5\phi\}, \quad \{p = 5, N = 2; A_\mu, 2\lambda, 4\phi\},
\]

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where $\lambda$ is the fermionic partner of the worldvolume Maxwell field $A_\mu$, and $B_{\mu\nu}^-$ is a worldvolume antisymmetric tensor field with a self-dual field strength.

There is a sense in which some of the super $p$–brane theories may be dual to each other. A super $p$–brane in $d$ dimensions may be considered to be dual to a super $(d-p-3)$–brane in $d$ dimensions, when each one exists as a soliton of the other. In a stronger sense, dual theories describe the strong and weak coupling regimes of the same physics, but this is rather difficult to prove rigorously. Among the duality relations probably the most interesting one is the heterotic string-heterotic fivebrane duality [40][41][46]. The two major open questions are how to construct the heterotic fivebrane action and how to covariantly quantize a $\kappa$–symmetric theory (or its twistor-like version).

As for the actions for super $p$–branes in a physical gauge, in principle the theory of nonlinear realizations applicable to spacetime supersymmetries can be used for their construction. This procedure has been illustrated for some special cases involving string solitons arising in certain globally supersymmetric field theories [47][3]. However, application of this procedure to higher super $p$–branes becomes quickly rather cumbersome [48]. Nonetheless, an interesting proposal has emerged in this area, namely, that the effective action for some of the super $p$–brane actions, in a certain limit, must reduce to certain supersingleton actions which live at the boundary of an appropriate anti de Sitter (AdS) space [49]. (A similar proposal had been made before according to which the action for fluctuations of a super $p$–brane theory compactified on a $AdS^{p+2} \times S^{d-p-2}$ were to be described by supersingleton theories formulated at the $S^p \times S^1$ boundary of the AdS space [50]).

An alternative approach, which may have the additional advantage of possibly yielding a covariant action, is based on the twistor–like formulation of the super $p$–branes [25][26]. In this approach, the theory possesses a rigid target space supersymmetry and a local worldvolume supersymmetry. The $\kappa$–symmetry emerges as a special worldvolume supersymmetry transformation. The classical equivalence of the twistor–like and the usual $\kappa$–symmetric formulation of the heterotic string has been shown to hold [28]. A similar conclusion has been reached in ref. [25] for the case of $p = 2$. However in [26], while we agree with the form of the action of ref. [25] and we generalize it to all super $p$–branes, it is by no means clear to us how this theory could be equivalent to the usual $\kappa$–symmetric one, even for the case of $p = 2$. When the dust settles on this issue, we will probably learn some intriguing and highly nontrivial aspects of these theories.

There exists the possibility that the formulation of ref. [26], after all, is inequivalent to the usual super $p$–brane action, and that it must be taken in its own right as a candidate for a novel super $p$–brane theory. In any event, the fact that we are now dealing with worldvolume supersymmetric field theories may give us a handle on the problem of how to construct the so far elusive heterotic and Type II super $p$–branes actions, as well as the problem of how to covariantly quantize the super $p$–brane theories.
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