Joint Learning of Probabilistic and Geometric Shaping for Coded Modulation Systems

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Abstract—We introduce a trainable coded modulation scheme that enables joint optimization of the bit-wise mutual information (BMI) through probabilistic shaping, geometric shaping, bit labeling, and demapping for a specific channel model and for a wide range of signal-to-noise ratios (SNRs). Compared to probabilistic amplitude shaping (PAS), the proposed approach is not restricted to symmetric probability distributions, can be optimized for any channel model, and works with any code rate \( k/m \), \( m \) being the number of bits per channel use and \( k \) an integer within the range from 1 to \( m-1 \). The proposed scheme enables learning of a continuum of constellation geometries and probability distributions determined by the SNR. Additionally, the PAS architecture with Maxwell-Boltzmann (MB) as shaping distribution was extended with a neural network (NN) that controls the MB shaping of a quadrature amplitude modulation (QAM) constellation according to the SNR, enabling learning of a continuum of MB distributions for QAM. Simulations were performed to benchmark the performance of the proposed joint probabilistic and geometric shaping scheme on additive white Gaussian noise (AWGN) and mismatched Rayleigh block fading (RBF) channels.

Index Terms—coded modulation, probabilistic shaping, geometric shaping, deep learning, autoencoder

I. INTRODUCTION

Constellation shaping refers to the optimization of the transmitted signal distribution to maximize the information rate. This directly follows from the definition of the channel capacity as \( C := \max_{p(x)} I(X;Y) \), where \( I(X;Y) \) is the mutual information of the channel input \( X \) and output \( Y \), and \( p(x) \) is the distribution over the channel input. Typical communication systems involve well-known constellation geometries such as quadrature amplitude modulation (QAM), amplitude-shift keying (ASK), and phase-shift keying (PSK), with uniform probabilities of occurrence of the individual constellation points. Shaping of constellations can be achieved by improving either the locations of the points or their probabilities of occurrence, referred to as geometric and probabilistic shaping, respectively.

Probabilistic shaping was shown to enable higher communication rates as well as smooth adaptation of the spectral efficiency (SE) [1] compared to the coarse granularity imposed by traditional schemes. These latter schemes can only operate at a fixed number of SEs determined by the available code rates and modulation orders. However, probabilistic shaping requires distribution matching (DM) to reversibly map an incoming stream of independent and uniformly distributed bits to a stream of matched bits, such that, once modulated to channel symbols, the probabilities of occurrence of these symbols match a target distribution. The integration of probabilistic shaping into practical bit-interleaved coded modulation (BICM) systems [2] is not straightforward. On the one hand, performing DM before channel coding leads to suboptimal rates, as coding alters the bit distribution. On the other hand, performing channel coding and then DM results in high error rates as matching breaks the code structure. Probabilistic amplitude shaping (PAS) [1] was introduced to integrate probabilistic shaping with BICM systems at reasonable complexity. However, PAS assumes that the target distribution is symmetric around the origin. While this holds for the additive white Gaussian noise (AWGN) channel, it might not be necessarily the case for other channel models. Moreover, PAS is only compatible with code rates of \( \frac{m-2}{m} \) (in 2D) or higher, \( m \) being the number of bits per channel use. Some approaches that partly alleviate this constraint were recently proposed [3]. However, they come at the cost of higher implementation complexity.

In this work, a new architecture for BICM systems is proposed which leverages the recent idea of trainable communication systems [4]. The proposed approach makes no assumption on the target distribution and is compatible with any code rate \( \frac{k}{m} \), \( k \) being an integer within the range from 1 to \( m-1 \). Furthermore, by optimizing the proposed end-to-end system made of the modulator, channel, and demapper on the bit-wise mutual information (BMI) [5], which is an achievable rate for BICM systems [6], joint optimization of geometric shaping, probabilistic shaping, bit labeling, and demapping is performed for a given channel model, code rate, and for a wide range of signal-to-noise ratios (SNRs). Moreover, the conventional PAS architecture with QAM and Maxwell-Boltzmann (MB) as shaping distribution was extended with a neural network (NN) that controls the MB shaping according to the SNR. This enables joint optimization of the MB distribution and demapper for any channel model and for a wide range of SNRs. Being able to optimize the shaping distribution \( p(x) \) for any channel model is an important benefit as finding the optimal shaping distribution using conventional approaches is typically a difficult problem even when assuming knowledge of the channel distribution \( p(y|x) \).

Simulations were performed on the AWGN and mismatched Rayleigh block fading (RBF) channels. On the AWGN channel and when considering a code rate compatible with PAS, results show that the proposed scheme achieves performance...
competitive with QAM shaped with the MB distribution using the extended PAS architecture. For a lower code rate that PAS does not support, the proposed scheme achieves higher rates than geometric shaping and unshaped QAM. On the RBF channel, joint probabilistic and geometric shaping leads to higher rates than QAM shaped with MB as well as geometric shaping.

The rest of this paper is organized as follows. Section II details the architecture of the proposed system as well as the joint probabilistic and geometric shaping optimization algorithm. The numerical results are presented in Section III. Section IV concludes this paper.

II. JOINT GEOMETRIC AND PROBABILISTIC SHAPING

Let us denote by $n$ the code length and by $r$ the code rate. It is assumed that $r = \frac{k}{m}$, where $2^m$ is the modulation order, and $k \in \{1, \ldots, m - 1\}$. We denote by $q = \frac{n}{m}$ the number of channel symbols per codeword, with $n$ assumed to be divisible by $m$. The overall architecture of the proposed system is shown in Fig. 1. A source generates independent and uniformly distributed bits $\tilde{c}_I$, which are fed to a matcher that implements a DM algorithm [7]. Given a target shaping distribution $p_\theta$ of dimension $2^k$ provided by an NN with trainable parameters denoted by $\theta$, the matcher maps $\tilde{c}_I$ to a bit vector $c_I = [b_1^{(i)} \ldots b_k^{(i)}]$ of length $rn$, where $b_k^{(i)}, i \in \{1, \ldots, q\}$, are bit vectors of dimension $k$ distributed according to $p_\theta$ (note that the distribution $p_\theta$ is over the bit vectors themselves and not over the individual bits). Because of the shaping redundancy introduced by the matcher, $\tilde{c}_I$ has a smaller length than $rn$. The matched information bits $c_I$ are fed to a channel encoder that generates a vector of parity bits $c_P = [c_p^{(1)} \ldots c_p^{(q)}]$ of length $(1 - r)n$, where $c_p^{(i)}, i \in \{1, \ldots, q\}$, are bit vectors of dimension $m - k$. Assuming a systematic code, the codeword $c = [c_I c_P]$ is mapped to a vector of channel symbols $x \in \mathbb{C}^q$ by a mapper and according to a constellation $C_\theta$ also provided by the NN with trainable parameters $\theta$. The constellation $C_\theta$ consists of a set of $2^m$ points in the complex plane, numbered from 0 to $2^m - 1$. The mapping operation will be explained in detail in Section II-A.

On the receiver side, a differentiable demapper computes log-likelihood ratios (LLRs) $l \in \mathbb{R}^n$ from the received symbols $y \in \mathbb{C}^q$. The LLRs are fed to a decoding algorithm that reconstructs the matched information bits. Because DM is reversible, the unmatched information bits can be retrieved.

A. Mapper architecture

The mapper maps each codeword $c = [c_I c_P]$ to a vector of channel symbols $x \in \mathbb{C}^q$ by mapping each bit vector $[b_1^{(i)} b_2^{(i)}], i \in \{1, \ldots, q\}$, of length $m$ to a channel symbol $x \in \mathbb{C}^q$ according to the constellation $C_\theta$. The key idea behind the mapper architecture is to partition the constellation $C_\theta$ into $2^m - k$ sub-constellations each of size $2^k$, i.e., $C_\theta = [C_\theta^{(1)} \ldots C_\theta^{(2^m-k)}]$. All sub-constellations are normalized to form the normalized constellation $C_\theta = [C_\theta^{(1)} \ldots C_\theta^{(2^m-k)}]$. In order to map a vector of bits $[b_1^{(i)} b_2^{(i)}], i \in \{1, \ldots, q\}$ to a channel symbol, the sub-constellation $C_\theta^{(j)}$ such that $j$ has $b_2^{(i)}$ as binary representation is chosen. Next, the channel symbol $x_k \in C_\theta^{(j)}$ such that $k$ has $b_1^{(i)}$ as binary representation is selected to modulate the bit vector $[b_1^{(i)} b_2^{(i)}]$, as illustrated in Fig. 1. Note that the bit vectors $b_1^{(i)}$ are shaped according to $p_\theta$, whereas the parity bits $b_2^{(i)}$ are not as channel encoding does not preserve shaping. The parity bits are assumed to be uniform and independent and identically distributed (i.i.d.) [1]. Therefore, using the proposed mapper, the constellation consists of a set of $2^m - k$ sub-constellations, all probabilistically shaped according to $p_\theta$, but with possibly different geometries. The normalization (1) ensures the power constraint $\mathbb{E} \left[ |x|^2 \right] = 1$.

This approach is similar to PAS [1], in which $k = m - 2$ and the signs of the constellation point components are selected according to the two parity bits, leading to symmetric constellations. Our approach can be seen as a generalization of PAS, where $k$ can take any value from the range from 1 to $m - 1$, and the sub-constellations can be freely optimized, subject only to a power constraint.

B. System training

At training, only the mapper, channel, and demapper are considered, as channel encoding and decoding are not required to perform probabilistic and geometric shaping on the BMI. The system considered for training is shown in Fig. 2. Joint
probabilistic and geometric shaping consists of jointly optimizing $p_\theta$ and $C_\theta$ to maximize the BMI, i.e., to find the vector of parameters that solves

$$\arg\max_\theta R(\theta) \quad \text{subject to} \quad \sum_{x \in C(\theta)} p_\theta(x)|x|^2 = 1 \quad \text{(3)}$$

where (3) is an average power constraint, ensured by the normalization step performed by the mapper. Optimizing on the BMI is relevant as it is an achievable rate for BICM systems [6]. The BMI is defined as

$$R(\theta) := \left[H_\theta(X) - \sum_{i=1}^m H_\theta(B_i|Y)\right]^+ \quad \text{(4)}$$

where $[.]^+ := \max (., 0)$, $X$ is the random variable corresponding to the channel input, $Y$ the random variable corresponding to the channel output, and $B_1, \ldots, B_m$ the random variables corresponding to the bits transmitted in a single channel use (which consist of information and parity bits). The first term in (4) is the entropy of the channel input $X$,

$$H_\theta(X) = -\sum_{x \in C_\theta} p_\theta(x) \log(p_\theta(x)) + (m-k) \quad \text{(5)}$$

and depends only on the shaping distribution $p_\theta$. In (5), any sub-constellation $C_\theta^{(j)}$, $i \in \{1, \ldots, 2^{m-k}\}$, could be used in the sum as they all share the same probabilistic shaping $p_\theta$. The second term in (4) is the sum of the transmitted bit entropies conditioned on the channel output $Y$, i.e.,

$$H_\theta(B_i|Y) = -E_{b_i} \log(p(b_i|y)) \quad \text{(6)}$$

where $p(b_i|y)$ is the posterior probability of $b_i$ given $y$. Note that this conditional entropy depends on $\theta$ through the distribution of $Y$ and $B_i$.

Finding a local solution to (2)-(3) is done by performing stochastic gradient descent (SGD) on a loss function that serves as a proxy for $R$. As noticed in [5], the BMI is closely related to the total binary cross-entropy (BCE) defined by

$$\hat{\mathcal{L}}(\theta) := -\sum_{j=1}^m E_{y,b_j} \log(p(b_j|y)) \quad \text{(7)}$$

where $\tilde{p}(b_j|y)$ is the posterior probability of $b_j$ given $y$ approximated by the demapper. One can rewrite $\mathcal{L}$ as

$$\hat{\mathcal{L}}(\theta) = \sum_{i=1}^m H_\theta(b_i|y) + \sum_{i=1}^m E_{y} [D_{KL}(p(b_i|y)||\tilde{p}(b_i|y))] \quad \text{(8)}$$

$$= H_\theta(X) - R(\theta) + \sum_{i=1}^m E_{y} [D_{KL}(p(b_i|y)||\tilde{p}(b_i|y))] \quad \text{(9)}$$

where $D_{KL}$ is the Kullback–Leibler (KL) divergence. Because we are performing probabilistic shaping, training on $\hat{\mathcal{L}}$, as done in, e.g., [5], would lead to the minimization of $H_\theta(X)$, which is not desired. To avoid this issue, we train on the loss

$$\mathcal{L}(\theta) := \hat{\mathcal{L}}(\theta) - H_\theta(X) \quad \text{(10)}$$

$$= -R(\theta) + \sum_{i=1}^m E_{y} [D_{KL}(p_i(b_i|y)||\tilde{p}(b_i|y))]. \quad \text{(11)}$$

Therefore, by minimizing $\mathcal{L}$, one maximizes the BMI. Moreover, assuming the demapper is trainable, it would be jointly optimized with the transmitter to minimize its KL divergence to the true posterior distribution of $B_i$ given $Y$.

A challenge of optimizing the shaping distribution with SGD is to compute the gradient of $\mathcal{L}$ with respect to (w.r.t.) to $p_\theta$. This difficulty was addressed in [8] by leveraging the Gumbel-Softmax trick [9] to implement a trainable sampling mechanism as the source of information bits. However, the Gumbel-Softmax trick requires extra hyper-parameters to be set, and can lead to numerical instabilities at training. In this work, we avoid the need of implementing a trainable sampling mechanism by implementing the loss in a different manner at training. More precisely, let us denote by $h$ the channel vector state, which captures all the random elements of the channel, e.g., the noise realization of an AWGN channel or the fading coefficient and noise realization of a fading channel. Then, without loss of generality, (6) can be rewritten as

$$H(B_i|Y) = -E_{b_i,x,h} \log(p(b_i|y(x,h)))$$

$$= -E_h \left[ \sum_{j=1}^{2^{m-k}} \sum_{x \in C_\theta^{(j)}} \sum_{b_i \in \{0,1\}} p(b_i|x)p_\theta(x) \cdot \log(p(b_i|y(x,h))) \right] \quad \text{(12)}$$

where the last equality comes from the fact that $h$ is independent of $x$ and $b_i$. In (12), $p(b_i|x)$ is set by the labeling of the constellation point, and equals to either 0 or 1. More precisely, each point $x$ of the constellation $C_\theta$ is associated to a bit vector of size $m$. In the rest of this work, the labeling of the constellation is set to natural labeling, i.e., the first element of $C_\theta$ is associated to 0, the second one to 1, etc. In other words, the bit vector associated to each constellation point is fixed, however the points can freely move within the complex plane during the optimization process, being only subject to the power constraint (3). When optimizing the constellation geometry $C_\theta$, i.e., the point positions, on the BMI, their
corresponding labeling is considered as it impacts the BMI value. As a result, joint optimization of the constellation geometry and labeling is achieved.

The key idea to enable optimization of the shaping distribution is to sample \( \hat{h} \) to estimate the outer expectation in (12), but to explicitly implement the inner expectation over \( x \) and \( b_i \). This is not as in, e.g., [5], where both the channel and the source are sampled. This trick avoids the need for a trainable sampling mechanism as the gradient of \( \mathcal{L} \) is correctly computed with respect to \( p_B \). In practice, at training, each batch example consists of sampling a channel realization \( h \), and transmitting all the points forming the constellation \( C_\theta \), as shown in Fig. 2. The loss \( \mathcal{L} \) is then estimated by

\[
\mathcal{L}(\theta) \approx - \left( H_\theta(X) + \sum_{i=1}^{m} \frac{1}{B} \sum_{l=1}^{B} \sum_{j=1}^{2^{m-k}} \sum_{x \in \mathcal{X}_j} p_B(x) \cdot \log \left( \tilde{p} \left( b_i | y \left( x, \hat{h}^{(l)} \right) \right) \right) \right)
\]

where \( B \) is the batch-size, and \( \hat{h}^{(l)} \) the \( l \)th sample of channel realization.

### III. Numerical Results

To assess the performance of the proposed scheme, referred to as PS-GS as it performs joint probabilistic and geometric shaping, we compared it to the conventional unshaped QAM, to an optimized geometric shaping (GS) baseline with no probabilistic shaping, and to QAM shaped according to a MB distribution by PAS, referred to as MB-QAM. For fairness, the PAS architecture [1] was extended with an NN that computes the MB shaping from the SNR, in order to find an optimized MB distribution for each SNR value (see Section III-A). The GS baseline consists of an NN-based mapper with uniform probability of occurrence of the points, as in, e.g., [5].

To evaluate the BMI achieved by the different schemes, no DM algorithm was used, and the information bit vectors were sampled according to the shaping distributions (or to the uniform distribution in the cases of uniform GS and uniform QAM). This is equivalent to assuming the use of a perfect DM algorithm. AWGN and mismatched RBF channels were considered (see Section III-B and III-C, respectively). The number of bits per channel use was set to \( m = 6 \) and the SNR was defined as \( 1/N_0 \), where \( N_0 \) is the noise spectral density. The SE was defined as the number of information bits transmitted per channel use, formally \( \text{SE} := H(X) - m(1 - r) \). Regarding the proposed PS-GS approach, two code rates were considered, 1/2 and 2/3. The code rate constrains the partitioning of the constellation \( C_\theta \) in the mapper as well as the size of the shaping distribution \( p_B \) (see Section II). To see how gains in BMI translate into gains in bit error rate (BER), a standard IEEE 802.11n code was used, with a length of \( n = 1944 \) bits and a code rate of 2/3. This code rate was chosen as it is compatible with PAS for \( m = 6 \). However, PAS cannot operate with a code rate of 1/2. A conventional sum-product belief-propagation decoder with 100 iterations was leveraged for channel decoding.

Regarding the proposed scheme, the NN that generates the shaping distribution \( p_B \) and the constellation \( C_\theta \) was made of two dense layers with 64 units each and hyperbolic tangent activation function, followed by two parallel dense layers with linear activation, one with \( 2^k \) units to generate the shaping distribution, and the other with \( 2^{m+1} \) units to generate the real and imaginary parts of the constellation points. This architecture is shown in Fig. 3a. When considering the AWGN channel, the true posterior distribution on bits was implemented by the demapper. For the mismatched RBF channel, as no efficient implementation of the optimal demapper is known to the best of our knowledge, the demapper was implemented by an NN whose architecture is shown in Fig. 3b. The NN-based demapper was jointly optimized with the transmitter. In Fig. 3b, \( \hat{y} \) is the equalized received symbol and \( \hat{h} \) the linear minimum mean square error (LMMSE) channel estimate. For fairness, an NN-based demapper was leveraged for all considered schemes when considering the mismatched RBF channel, including the QAM baseline for which the transmitter includes no trainable parameters.

Training was done with batches of size 1000, using the Adam optimizer [10] with the learning rate set to \( 10^{-3} \), and by uniformly sampling the SNR from the range from 0 to 20 dB for the AWGN channel, and from the range from 5 to 25 dB for the mismatched RBF channel. For all the schemes
that required training, the best of five seeds is shown.

A. Extending PAS with deep learning

For fairness, the PAS architecture with MB as shaping distribution and QAM geometry was extended with an NN that controls the MB shaping according to the SNR. More precisely, if we denote by $C_{\text{QAM}}$ the QAM constellation vector, the probability of occurrence of a point $x \in C_{\text{QAM}}$ using MB shaping is

$$p_{\text{MB}}(x) = \frac{\exp(-\mu|x|^2)}{\sum_{x' \in C_{\text{QAM}}} \exp(-\mu|x'|^2)}$$

(14)

where $\mu \geq 0$ controls the constellation shaping. The PAS architecture was extended with an NN that computes $\mu$ from the SNR, as shown in Fig. 3c. This architecture is similar to the one used for joint probabilistic and geometric shaping (see Fig. 1), except that the mapper is implemented following the PAS scheme. Extending PAS as proposed in this work enables the learning of a continuum of MB distributions determined by the SNR for any channel model, constellation geometry, and differentiable demapper. Training is done using the approach described in Section II-B, i.e., by transmitting for each channel realization the entire QAM constellation, and by performing SGD on the loss (13).

B. AWGN channel

Considering the AWGN channel, the true posterior distribution was implemented in the demapper. Therefore, the KL divergence in (11) is null, and the loss function $\mathcal{L}$ equals the BMI $R$ up to the sign. The BMIs achieved by the compared schemes were estimated by Monte Carlo sampling of (11) and are shown in Fig. 4a. The code rate is indicated in parenthesis in the legend. One can see that PS-GS with a code rate of $2/3$ and MB-QAM achieve essentially the same BMI, followed by PS-GS with a code rate of $1/2$. The lower performance of PS-GS with a code rate of $1/2$ can be explained by the higher number of sub-constellations into which $C_0$ must be partitioned by the mapper due to the lower code rate. Indeed, the higher the number of sub-constellations into which the constellation $C_0$ must be partitioned, the stronger the constraint as all sub-constellations must share the same probabilities of occurrence of points. Note that conventional PAS does not operate at a code rate of $1/2$ with $m$ set to 6. However, PS-GS with a code rate of $1/2$ still achieves significantly higher rates than GS alone, which itself outperforms uniform QAM.

Fig. 4b shows the corresponding SE. When considering uniform shaping, i.e., unshaped QAM and GS alone, the SE is constantly equal to $4$ as the SE only depends on the code rate and modulation order. On the other hand, probabilistic shaping enables smooth adaptation of the SE, as opposed to conventional systems for which only coarse adaption is possible as the SE is determined by the available code rates and modulation orders. From this figure, one can see that PS-GS with a code rate of $2/3$ and MB-QAM achieve similar SEs. PS-GS with a code rate of $1/2$ obviously achieves lower SE due to the lower code rate.

![Fig. 4: Results for the AWGN channel](image-url)
Because of space restriction, only the BMI and SE are shown for the mismatched RBF channel. Fig. 5a shows the lower bounds on the BMI achieved by the compared schemes and obtained by Monte Carlo sampling of (11). As one can see, PS-GS with a code rate of 2/3 achieves the highest values, followed by PS-GS with a code rate of 1/2 and GS. As opposed to the AWGN channel, these schemes outperform MB-QAM. Looking at Fig. 5b, one can see that PS-GS achieves a higher SE than MB-QAM for a code rate of 2/3. The gains observed using the proposed scheme in this paper demonstrate the benefits of being able to learn the shaping for any channel model.

IV. Conclusion

We have introduced a novel NN architecture that enables joint probabilistic and geometric shaping for BICM systems in an end-to-end manner. The proposed architecture is compatible with any code rate \( k/m \), where \( m \) is the number of bits per channel use and \( k \) is an integer within the range from 1 to \( m - 1 \). It further enables joint optimization of the geometric shaping, probabilistic shaping, bit labeling, and demapping on the BMI for any channel model and for a wide range of SNRs. Numerical results show that the proposed approach achieves a BMI competitive with QAM shaped according to a MB distribution by PAS at a compatible code rate. At a code rate at which PAS does not operate, it outperforms geometric shaping alone. On the mismatched RBF channel, the proposed scheme achieves higher rates than shaped QAM for all considered rates, showing the benefits of being able to learn the shaping for an arbitrary channel model.

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