Calculation of active earth pressure against retaining walls under translation mode considering soil arching effect in cohesive soil

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Abstract. In this study, the contour of the soil arch is not assumed as a certain curve, and the soil stress analysis is conducted on a differential element with two dimensions. Herein, analytical expressions of the active earth pressure, active earth force are derived based on the static equilibrium of the differential element. Furthermore, the effects of wall-soil friction angle and soil internal friction angle on the active earth pressure are investigated. Eventually, comparisons among the proposed method of this study and the previous measured data and other analytical approaches yield satisfactory results.

1. Introduction
In engineering practice, the Rankine and Coulomb theories, presuming that earth pressure is distributing linearly, are commonly used. However, many experimental results [1–3] have verified that earth pressure distribution is nonlinear, for which the soil arching effect is responsible [4].

To date, approaches to calculating the active earth pressure considering the soil arching effect have been formulated [4–9]. However, these methods suffer from some shortcomings, among which the most obvious drawback is assuming the profile of the soil arch in spite of a discrepancy with the factual shape of the soil arch. Besides, under translation mode, stress analysis in the soil is conducted in a flat differential element, where there is an assumption that the shear stress between such differential elements is zero although shear stress does exist between such differential elements due to the deflection of the principal stresses. In addition, in most of these methods, the state of stress at any point of the soil in the sliding wedge is deemed to reach a limit-equilibrium state, which is not correct [10].

In this study, to address the aforementioned issues, the stress solution of any point in the sliding soil is obtained by solving equilibrium differential equations of a two-dimensional differential element without presetting the silhouette of the soil arch. Then, the theoretical formulae are derived to determine the active earth pressure, active earth force as well as the angle of the rupture surface. The outcomes of the proposed method are verified against the results of field tests, as well as the theoretical values from previously published methods.

2. Theoretical derivation

2.1. Analytic model
The following derivation process is based on the assumptions that the retaining wall is vertical under translation mode, the back of the wall is rough and the backfill is cohesive and homogeneous.
The details of the analytic model are as follows: the height of the rough retaining wall is $H$; the wall-soil friction angle is $\delta$; the internal friction angle, unit weight and cohesion of backfill are $\varphi$, $\gamma$, and $c$, respectively, as shown in Fig. 1. When the wall shifts away from the backfill until the soil mass behind the wall attains an active limit-equilibrium state, an active rupture surface ($BC$) at an angle of $\beta$ to the horizontal will be triggered. The stresses on a differential element with a thickness of $dz$ and a width of $dx$ at any depth ($z$) and at any distance ($x$) in the backfill are as shown in Fig. 1.

![Fig. 1 A differential element of stress analysis in cohesive backfill behind a wall](image1)

Given a rough wall, the directions of the minor principal stresses on the differential element in the sliding cohesive backfill during an active limit-equilibrium state deflect due to frictional resistance at the wall [5], and accordingly, yields soil arch, which was defined as the trajectory of the minor principal stresses [4]. According to this definition, the soil arching effect can be considered by introducing an angle ($\alpha$) between the direction of minor principal stress and the horizontal is introduced when analyzing the stress of the soil differential element instead of presupposing the profile of the soil arch, as shown in Fig. 2 and Fig. 3.

![Fig. 2 Mohr stress circle at the back of the wall](image2)

![Fig. 3 Mohr stress circle at the sliding surface](image3)
2.2. Equilibrium differential equations

Establishing the rectangular coordinate system, as shown in Fig. 1, according to the differential element, the equilibrium differential equations can be obtained by

\[ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \] (1)

\[ \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} = \gamma \] (2)

where \( \sigma_x \) is the horizontal stress on a differential element; \( \sigma_z \) is the vertical stress on a differential element; \( \tau_{xz} \) and \( \tau_{zx} \) are the shear stress on a differential element; and \( \gamma \) is the unit weight of the backfill.

2.3. Boundary conditions

The relationship between the lateral stress (\( \sigma_x \)) and the principal stress on the differential element can be described by the Mohr stress circle, as illustrated in Fig. 2. According to the trigonometric function relationship and Rankine earth pressure theory, the following equation can be obtained:

\[ (\frac{\sigma_x - \sigma_z}{2})^2 + (\frac{\tau_{xz}}{\tan \phi})^2 = \left( \frac{c}{\tan \phi} \right)^2 \] (3)

\[ (\frac{\sigma_z - \sigma_x}{2})^2 + (\frac{\tau_{zx}}{\tan \phi})^2 = \left( \frac{c}{\tan \phi} \right)^2 \] (4)

where \( c \) is the cohesion of the backfill; \( K_a \) is the coefficient of the active earth pressure from Rankine earth pressure theory; \( \sigma_x \) is the lateral stress at any point in the backfill; \( \alpha \) is the angle between the direction of the minor principal stress and the horizontal, namely, the deflection angle of the minor principal stress \( \sigma_3 \); \( \sigma_1 \) is the major principal stress on a differential element.

Under the assumption that the failure surface is planar, it is impossible that the sliding wedge of soil slides along the back of the wall as well as the rupture surface simultaneously. When the retaining wall translates away from the backfill, the soil on the wall reaches a limit-equilibrium state earlier than the soil on the sliding surface [10], which means that the deflection angle of the minor principal stress at the back of the wall is not the same as the deflection angle of the minor principal stress at the sliding surface. Therefore, the lateral stress (\( \sigma_{wx} \)) on the back of the retaining wall and the vertical stress (\( \sigma_{wz} \)) on the back of the retaining wall can be obtained by replacing \( \alpha \) in Eq. (4) with \( \alpha_w \) as follows

\[ \sigma_{wx} = \sigma_x + \frac{c}{\tan \phi} (K_a \cos^2 \alpha + \sin^2 \alpha) - \frac{c}{\tan \phi} \] (5)

\[ \sigma_{wz} = \sigma_x + \frac{c}{\tan \phi} (K_a \sin^2 \alpha + \cos^2 \alpha) - \frac{c}{\tan \phi} \] (6)

Where \( \alpha_w \) is the deflection angle of the minor principal stress at the back of the wall; according to the trigonometric function relationship in the Mohr stress circle in Fig. 2, \( \alpha_w \) can be obtained by

\[ \alpha_w = \frac{1}{2} \arcsin \left( \frac{\sin \delta}{\sin \phi} \right) - \frac{1}{2} \delta \] (7)

From Eq. (5) and Eq. (6), a new ratio (\( K_{sw} \)) which is the sum of the lateral stress on the wall and \( c/\tan \phi \) to the sum of the vertical stress on the wall and \( c/\tan \phi \) is obtained by

\[ K_{sw} = \frac{\sigma_{wx} + \frac{c}{\tan \phi}}{\sigma_{wz} + \frac{c}{\tan \phi}} = \frac{K_a \cos^2 \alpha + \sin^2 \alpha}{K_a \sin^2 \alpha + \cos^2 \alpha} \] (8)

By substituting Eq. (7) into Eq. (8), \( K_{sw} \) can be expressed as
Based on the Mohr stress circle shown in Fig. 2, the shear stress ($\tau_{xz}$) on a differential element can be obtained by

$$
K_{aw} = \frac{1 - 2\sin \varphi \cos^2 \left( \frac{1}{2} \arcsin \frac{1}{\sin \varphi} - \frac{1}{2} \delta \right)}{1 - 2\sin \varphi \sin^2 \left( \frac{1}{2} \arcsin \frac{1}{\sin \varphi} - \frac{1}{2} \delta \right)}
$$

(9)

By substituting Eq. (7) into Eq. (10), the shear stress ($\tau_{w}$) on the wall can be obtained by

$$
\tau_{w} = \frac{1 - K_{aw}}{2} \left( \sigma_{r} + \frac{c}{\tan \varphi} \right) \sin 2\alpha
$$

(10)

Define a new ratio ($k_{1}$) of $\tau_{w}$ to the sum of $\sigma_{wz}$ and $c/\tan \varphi$ as follows

$$
k_{1} = \frac{\tau_{w}}{\sigma_{wz} + \frac{c}{\tan \varphi}} = \frac{\sin \varphi \sin \left( \arcsin \frac{\sin \delta}{\sin \varphi} - \delta \right)}{1 + \sin \varphi \cos \left( \arcsin \frac{\sin \delta}{\sin \varphi} - \delta \right)}
$$

(12)

From Eq. (12), one boundary condition for Eqs. (1) and (2) is given by

$$
\tau_{xz}(x, z) = k_{1} \left( \sigma_{wz} + \frac{c}{\tan \varphi} \right) \quad (x = 0)
$$

(13)

According to the Mohr stress circle in Fig. 3, the deflection angle ($\alpha_{b}$) of the minor principal stress at the sliding surface can be obtained by

$$
\alpha_{b} = \frac{\pi}{4} + \frac{\varphi}{2} - \beta
$$

(14)

Where $\beta$ is the active failure angle.

Substituting Eq. (14) into Eq. (4), the vertical stress ($\sigma_{bz}$) at the sliding surface can be obtained by

$$
\sigma_{bz} = \left( \sigma_{r} + \frac{c}{\tan \varphi} \right) \left( K_{s} \sin^{2} \alpha_{b} + \cos^{2} \alpha_{b} \right) - \frac{c}{\tan \varphi}
$$

(15)

Substituting Eq. (14) into Eq. (10), the shear stress ($\tau_{b}$) on the sliding surface can be obtained by

$$
\tau_{b} = \frac{1 - K_{s}}{2} \left( \sigma_{r} + \frac{c}{\tan \varphi} \right) \sin 2\alpha_{b}
$$

(16)

Define a new ratio ($k_{2}$) of $\tau_{b}$ to the sum of $\sigma_{bz}$ and $c/\tan \varphi$ as follows

$$
k_{2} = \frac{\tau_{b}}{\sigma_{bz} + \frac{c}{\tan \varphi}} = \frac{\sin \varphi \cos (2\beta - \varphi)}{1 + \sin \varphi \sin (2\beta - \varphi)}
$$

(17)

From Eq. (17), another boundary condition for Eqs. (1) and (2) is derived as

$$
\tau_{xz}(x, z) = k_{2} \left( \sigma_{bz} + \frac{c}{\tan \varphi} \right) \quad (x = \frac{H - z}{\tan \beta})
$$

(18)

2.4. Active earth pressure

The soil arching effect is verified to be solely on the direction of the principal stress but not on its value [5,11]. Therefore, it can be assumed that the vertical principal stresses are constant at the same depth, and from Eq. (2) the following equation can be obtained as

$$
\frac{\partial^{2} \tau_{xz}}{\partial x^{2}} = 0
$$

(19)
According to the boundary conditions by Eq. (13) and Eq. (18), the solution to Eq. (19) can be expressed as

\[
\tau_{xz}(x,z) = \left( \sigma_z(x) + \frac{c}{\tan \phi} \right) \left( (k_2 - k_1) \tan \beta \frac{x}{H - z} + k_1 \right)
\]  

(20)

Substituting Eq. (20) into Eq. (2), and under the assumption that the vertical principal stresses are constant at the same depth, the following equation can be obtained as

\[
\frac{d\sigma_z}{dz} + \frac{(k_2 - k_1) \tan \beta}{H - z} \sigma_z = \gamma - \frac{(k_2 - k_1)c \tan \beta}{(H - z) \tan \phi}
\]  

(21)

The solution to Eq. (21) is

\[
\sigma_z(z) = \left( \frac{\gamma H}{1 - m} + \frac{c}{\tan \phi} + q \right) \left( 1 - \frac{z}{H} \right)^m - \frac{\gamma(H - z)}{1 - m} - \frac{c}{\tan \phi}
\]  

(22)

where \( m = (k_2 - k_1) \tan \beta \); \( q \) is a uniformly distributed load acting on the surface of the backfill.

By substituting Eq. (22) into Eq. (20), the shear stress \( \tau_{xz} \) can be obtained by

\[
\tau_{xz}(x,z) = \left( \frac{\gamma H}{1 - m} + \frac{c}{\tan \phi} + q \right) \left( 1 - \frac{z}{H} \right)^m - \frac{\gamma(H - z)}{1 - m} - \frac{c}{\tan \phi}
\]  

(23)

By substituting Eq. (23) into Eq. (1), and integrating the resulting equation with respect to \( x \), the lateral active earth pressure \( \sigma_{wa} \) at any level on the wall back can be derived as

\[
\sigma_{wa} = K_m \left( \frac{\gamma H}{1 - m} + \frac{c}{\tan \phi} + q \right) \left( 1 - \frac{z}{H} \right)^m - \frac{\gamma(H - z)}{1 - m} - \frac{c}{\tan \phi}
\]  

(24)

3. Parametric analysis

3.1. Lateral active earth pressure

As shown in Fig. 4 and Fig. 5, the peak point of the active earth pressure distribution curve moves down with the increase in the backfill internal friction angle and the wall-soil friction angle while the earth pressure converges to zero at the bottom of retaining wall, which leads to that the curvature of the active earth pressure distribution curve increases with the increase in the backfill internal friction angle and wall-soil friction angle. At the same depth of the retaining wall (except the bottom), the larger backfill internal friction angle and wall-soil friction angle correspond to the larger earth pressure while the sensitivity of earth pressure toward the backfill internal friction and the wall-soil friction angle is different. Fig.4 shows the sensitivity of earth pressure increases towards the increase of wall-soil friction angles and Fig.5 shows it remains unchanged under different backfill internal friction angles.
3.2. Comparison with existing methods and measured data

To verify the accuracy and applicability of the proposed formula, the results from the proposed equation are compared with the model experimental results [3] and other analytical results calculated by the previously issued methods [4,5,11], as shown in Fig. 6.

In terms of the active earth pressure above the wall bottom, the values from Paik et al. [5] and Goel et al. [11] are much larger than the measured ones, and the values from Handy [4] are slightly larger than the measured ones, whereas values from the proposed formula are in good agreement with the measured values. For the active earth pressure at the bottom of the wall, the values predicted by Paik et al. [5], Goel et al. [11] and Handy [4] all gradually approach zero, which is the same case as the values derived from the proposed equation. Therefore, it is found that the proposed formulae in this study are in good agreement with the measured values.

4. Conclusion

This paper presents an analytical approach for calculating the active earth pressure against a rough retaining wall subjected to the translation mode in cohesive backfill. The effect of soil arching is
considered by introducing the deflection angle of the minor principal stress into the stress analysis of the soil differential element with two dimensions. According to the static equilibrium of the differential element, a formula for the lateral active earth pressure is obtained. Finally, the effects of major parameters in the formula on the lateral active earth pressure are investigated in detail. Comparisons between the proposed method with experimental data and existing methods indicate that the equations proposed by this study can yield satisfactory results.

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