Matrix Models, Emergent Spacetime and Symmetry Breaking†

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Abstract. We discuss how a matrix model recently shown to describe emergent gravity may contain extra degrees of freedom which reproduce some characteristics of the standard model, in particular the breaking of symmetries and the correct quantum numbers of fermions.

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1. INTRODUCTION

One of the recurrent themes of this conference is the fact that at the Planck scale ordinary geometry may no longer be valid, and that a quantized theory of gravity could be based on this new, more general, geometry. The guiding analogy is with the transition from the classical to the quantum phase space. The former is described by ordinary geometry, that made of points, lines, tangents etc. The latter requires a noncommutative geometry [1, 2, 3] better described by operators on an Hilbert space.

In this case the coordinates and in general the observables of phase space become noncommuting operators on an Hilbert space. The commutator of the coordinates is a constant (Planck’s constant) and therefore this kind of noncommutative geometry is in some sense the simplest. A noncommutative geometry of spacetime (as opposed to phase space) is then the one described by the commutator $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is constant quantity of the order of the square of Planck’s length. This has led to the study of field theories on noncommutative space, where the noncommutativity of the space is implemented by the use of the Grönewold-Moyal $\star$ product, for a review see [4]. While this is not the place to describe successes and problems of this approach, it is clear, at

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least to me, that the implementation of noncommutativity with a constant $\theta$ is at best an approximation of a more general theory.

In these proceedings we will describe how the matrix model introduced in [7, 8], and which was shown in [9] to describes emergent gravity, may lead to an understanding of aspects of the standard model, somehow along the line of the Connes vision of the standard model [1, 5, 6].

2. NONCOMMUTATIVE SPACETIME AND THE MATRIX MODEL

The general programme is to describe noncommutative spacetime as a matrix model with a simple action, inserting fermions, and have gravity, and other physical characteristic, emerge as fluctuations around a semiclassical vacuum. We will be very impressionistic and concise for because of length restrictions, but not only, it should be born in mind that the work present is still in progress. More details can be found in [10].

The rationale behind this approach is quite simple and can be very heuristically (and therefore incorrectly) be stated as follows:

• A noncommutative geometry is described by a noncommutative algebra, deformation of the commutative algebra of function on some space
• Any noncommutative ($C^*$)-algebra is represented as operators on some Hilbert space
• Operators on a Hilbert space are just infinite matrices

Consider a $U(1)$ gauge theory in a space described by the Grönewold-Moyal $\star$ product. The theory is noncommutative (also in the $U(1)$ case), due to the noncommutativity of the product. In this sense we can talk of a noncommutative geometry, because we substitute the commutative algebra of functions on spacetime by the algebra obtained deforming the product as:

\[
(f \star g)(x) = e^{\frac{i}{2} \theta_{\mu\nu} \partial_\mu \partial_\nu f(y)g(z)}\bigg|_{x=y=z}
\]  

(2.1)

so that the noncommutativity of the coordinates is encoded on the commutation relation

\[
x^{\mu} \star x^{\nu} - x^{\nu} \star x^{\mu} = [x^{\mu}, x^{\nu}]_\star = i\theta^{\mu\nu}
\]

(2.2)

and derivation become inner, i.e. they can be expressed by a commutator:

\[
\frac{\partial}{\partial x^{\mu}} f = i(\theta)^{-1}_{\mu\nu}[x^{\nu}, f]
\]

(2.3)

The field strength of the $U(1)$ theory is modified by

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]_\star
\]

(2.4)

Consider as usual the action to be the square of the curvature:

\[
S = -\frac{1}{4} \int dx F_{\mu\nu} \star F^{\mu\nu}
\]

(2.5)
The theory is invariant for the gauge transformation $F \rightarrow U \star F \star U^\dagger$ where $U$ is $\star$-unitary: $U \star U^\dagger = 1$. A nonabelian (and noncommutative) Yang-Mills gauge theory is obtained considering $A_\mu = A_\mu^\alpha \lambda^\alpha$ for $\lambda^\alpha$ generators of $U(n)$.

Because of the connection between commutator with the coordinates and derivatives (2.3), one is led [11] to the definition of covariant coordinates

$$X^\mu = x^\mu + \theta^{\mu
u} A_\nu$$

and consequently

$$D_\mu f = i\theta^{-1}_\mu [X^\mu, f]_\star = \partial_\mu f - i[f, A_\mu]_\star$$

and therefore we have

$$F^{\mu
u} = [D^\mu, D^\nu]_\star = [X^\mu, X^\nu]_\star + \theta^{\mu
u}$$

The constant $\theta$ can be reabsorbed by a field redefinition and the action is the square of this quantity, integrated over spacetime.

The objects we have defined are elements of a noncommutative algebra and we can always represent them as operators on a Hilbert space, in this case the integral becomes a trace and this suggests the use of the matrix following action

$$S = -\frac{1}{4g} \Tr [X^\mu, X^\nu] [X^\mu', X^\nu'] g_{\mu\mu'} g_{\nu\nu'}$$

where the $X$’s are operators (matrices) and the metric $g_{\mu\mu'}$ is the flat Minkowski (or Euclidean) metric. The important characteristic of this action is that gravity emerges naturally from it [9].

The equations of motion corresponding to the action (2.9) are

$$[X^\mu, [X^\nu, X^\nu']] g_{\mu\mu'} = 0$$

A possible vacuum (which we may call the $U(1)$ Grönewold-Moyal vacuum) is given by a set of matrices $X_0$ such that

$$[X_0^\mu, X_0^\nu] = i\theta^{\mu\nu}$$

with $\theta$ constant. This corresponds to the Grönewold-Moyal case and in this case the vacuum is just the deformation of spacetime described earlier. Now let fluctuate these “coordinates” and consider $X^\mu = X_0^\mu + A^\mu$ so that $[X^\mu, X^\nu] = i\theta(X)$ and we are considering a nonconstant noncommutativity.

Gravity emerges as nontrivial curvature considering the coupling with a scalar field $\Sigma$. The (free) action is then

$$\Tr [X^\mu, \Sigma] [X^\nu, \Sigma] g_{\mu\nu} \sim \int \diff x (D_\mu \Sigma)(D_\nu \Sigma) \theta^{\mu\mu'} \theta^{\nu\nu'} g_{\mu\nu} = \int \diff x (D_\mu \Sigma)(D_\nu \Sigma) G^{\mu\nu}$$

which describes the coupling of the scalar field to a curved background defined by the new (non flat) metric $G^{\mu\nu}(x) = \theta^{\mu\mu'} \theta^{\nu\nu'} g_{\mu\nu'}$. A curved background emerges in an effective way and the gravitational action is recovered as an effective action at one loop, for details see [9].
3. ALTERNATIVE VACUA AND NONABELIAN SYMMETRY

An alternative vacuum, still solution of the equations of motion, is

$$\bar{X}_0^\mu = X_0^\mu \otimes 1_n$$

(3.1)

In this case the Moyal-Weyl limit is given by matrix valued functions on $\mathbb{R}^{D\theta}$ and the gauge symmetry is given by unitary elements of the algebra of $n \times n$ matrices of functions of the $\bar{X}_0$. This theory therefore has a noncommutative $U(n)$ gauge symmetry because in the semiclassical limit it corresponds to a nonabelian gauge theory. However the $U(1)$ degree of freedom is the one which couples gravitationally for the emergent gravity, hence the gauge theory in this case is a nonabelian $SU(n)$ theory. In fact consider the fluctuations of $\bar{x}_0$ to be

$$\bar{X} = \bar{x}_0 + A_0 + A_\alpha \lambda_\alpha$$

(3.2)

Where in the fluctuations we have separated the traceless generators of $SU(n)$ from the trace part ($A_0$). The $U(1)$ trace part of the fluctuation gives rise to the gravitational coupling, while the remaining $A_\alpha$ describe an $SU(n)$ gauge theory.

Once we are convinced that we can reproduce a $SU(n)$ gauge theory the next objective is to reproduce the standard model, including the symmetry breaking, or at least some gauge theory which closely resembles resembles it. Therefore the game becomes to find a noncommutative geometry, described by a matrix model, with a bosonic action like (even if possibly not identical to) (2.9), with the insertion of fermions transforming properly under the gauge group and a symmetry breaking mechanism. We should not be shy of making as many assumptions as are needed. The game is not to find the standard model, but rather to find a noncommutative geometry which “fits” it. The programme in this sense is similar to the one started by Alain Connes and collaborators [1, 5, 6]. The main difference is the fact that in Connes’ work spacetime is still commutative, while in the present case spacetime emerges in the limit as a Groënewold-Moyal space. Other differences is the fact that we are naturally Minkowskian and that the action is not the same. The model I will present is incomplete and can be considered as a first approximation. Remarkably however some key characteristics of the standard model emerge naturally, which makes us confident that a fully viable (and predictive) model is within reach.

Consider a another extra dimension which we call $\mathcal{X}^\Phi$ of the form

$$\mathcal{X}^\Phi = \begin{pmatrix} \alpha_1 I_2 \\ \alpha_2 I_2 \\ \alpha_3 I_3 \end{pmatrix}$$

(3.3)

Where the $\alpha$’s are constants all different among themselves. This new coordinate is still solutions of the equations of the motion because $[X^\mu, \mathcal{X}^\Phi] = 0$, this signifies that $\theta^\mu\Phi = 0$, and in turn that $G^{\Phi\Phi} = G^{\mu\Phi} = 0$, whatever the value of the metric $g_{\mu\Phi}$ and $g_{\Phi\Phi}$. Therefore the extra coordinate is not geometric and does not correspond to propagating degrees of freedom from the four dimensional point of view. The new coordinate is not invariant for the transformation

$$\mathcal{X}^\Phi \to U \mathcal{X}^\Phi U^\dagger \neq \mathcal{X}^\Phi$$

(3.4)
for a generic \( U \in SU(7) \). It is invariant only for a subgroup of it. The traceless part of it is \( SU(2) \times SU(2) \times SU(3) \times U(1) \times U(1) \).

If we consider the gauge bosonic action we have that the spacetime \((\mu \nu)\) part of action remains unchanged, while for the \(\mu \phi\) components we have that Moyal-Weyl limit is

\[
\begin{align*}
[\bar{X}^\mu + A^\mu, \chi^\phi] &= \theta^{\mu \nu} D_\nu \chi^\phi = \theta^{\mu \nu} (\partial_\nu + iA_\nu) \chi^\phi,
-(2\pi)^2 \text{Tr} [\bar{X}^\mu, \chi^\phi][\bar{X}^\nu, \chi^\phi] g_{\mu \nu} = \int d^4 \chi G^{\mu \nu} (\partial_\mu \chi^\phi \partial_\nu \chi^\phi + [A_\mu, \chi^\phi][A_\nu, \chi^\phi])
\end{align*}
\]

because the mixed terms, assuming the Lorentz gauge \( \partial_\mu A_\mu = 0 \), vanish:

\[
\int \partial_\mu \chi^\phi [A_\mu, \chi^\phi] = -\frac{1}{2} \int \chi^\phi [\partial_\mu A_\mu, \chi^\phi] = 0
\]

Since we have that \( \bar{X}^\mu \) and \( \chi^\phi \) commute, this means \( \chi^\phi = \text{const} \) and the first term in the integral above vanish. We can therefore separate the fluctuations of this extra dimension which are a field, the (high energy) Higgs field. In the action the first term is nothing but the covariant derivative of it. The second term instead is

\[
[A_\mu, \chi^\phi] = \begin{pmatrix}
0 & (\alpha_2 - \alpha_1) A^\mu_{12} & (\alpha_3 - \alpha_1) A^\mu_{13} \\
(\alpha_1 - \alpha_2) A^\mu_{21} & 0 & (\alpha_3 - \alpha_2) A^\mu_{23} \\
(\alpha_1 - \alpha_3) A^\mu_{31} & (\alpha_2 - \alpha_3) A^\mu_{32} & 0
\end{pmatrix}
\]

(3.7)

where we consider the block form of \( A^\mu \)

\[
A^\mu = \begin{pmatrix}
A^\mu_{11} & A^\mu_{12} & A^\mu_{13} \\
A^\mu_{21} & A^\mu_{22} & A^\mu_{23} \\
A^\mu_{31} & A^\mu_{32} & A^\mu_{33}
\end{pmatrix}
\]

(3.8)

If we now assume that the differences \( \alpha_i - \alpha_j \) are large, say of the grand unification scale, it is easy to see that that all non diagonal blocks of \( A^\mu \) acquire large masses, thus effectively decoupling. In this way we have reduced the symmetry to a smaller group, resembling more the standard model.

We now assume that a this first stage of breaking has reduced the symmetry and indicate the spacetime coordinates \( X \) as

\[
X^\mu = \begin{pmatrix}
X^\mu_0 \otimes I_2 \\
X^\mu_0 \\
X^\mu_0 \otimes I_3
\end{pmatrix}
\]

(3.9)

The smaller group contains the electroweak \( SU(2) \) group and the colour \( SU(3) \) groups defined by:

\[
W^\mu = \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]

(3.10)
and

\[ G^\mu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g^\mu \end{pmatrix} \] (3.11)

Where \( w \) and \( g \) are in the adjoint representations of the respective groups. There are also some \( U(1) \) symmetries, apart from the one generated by the trace which as we said couples to the gravitational degrees of freedom. In order to recognize the physically relevant factors let us now introduce the fermions.

### 4. FERMIONS AND THE GAUGE CHARGE PROBLEM

The action for fermions in matrix models has been introduced in [8] and discussed in this context in [12] and is

\[ S_F = \text{Tr} \bar{\Psi} \gamma^a [X^a, \Psi] \sim \int d^4x \bar{\Psi} \gamma^a D^a \Psi \] (4.1)

Here \( \Psi \) is a \( 7 \times 7 \) whose entries are spinors, i.e. they carry an extra index on which the \( \gamma \) matrices act, these are represented diagonally on the matrix space. In this note we will choose \( \Psi \) to be upper triangular and its components to be Dirac spinors. This choice is not unique (and in the long run it may turn out not to be the best one), for more details see [10]. We accommodate all known fermions (except right handed neutrinos) as:

\[ \Psi = \begin{pmatrix} 0 & 0 & 0 & \nu_L & u_{L1} & u_{L2} & u_{L2} \\ 0 & 0 & 0 & 0 & d_{L1} & d_{L2} & d_{L3} \\ 0 & 0 & e_R & d_{R1} & d_{R2} & d_{R1} \\ 0 & 0 & 0 & 0 & u_{R1} & u_{R2} & u_{R3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \] (4.2)

Where the numerical labels (1,2,3) for quarks indicate colour. Right handed neutrino can be inserted as diagonal elements in the \((4,4)\) position in this matrix. We do not insert them for the present discussion because it commonly believed that their origin lies beyond the standard model, a belief confirmed in the present model by the fact that they will not fit in the triangular scheme. The empty spaces may be filled by extra particles, for example there is room for another weak doublet with the quantum numbers of the Higgsino.

To avoid mirror fermions we choose

\[ \gamma^5 \Psi = \begin{pmatrix} 0 & 0 & 0 & \nu_L & u_{L1} & u_{L2} & u_{L2} \\ 0 & 0 & 0 & 0 & d_{L1} & d_{L2} & d_{L3} \\ 0 & 0 & -e_R & -d_{R1} & -d_{R2} & -d_{R1} \\ 0 & 0 & 0 & 0 & -u_{R1} & -u_{R2} & -u_{R3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \] (4.3)
The fermions which appear in the matrix $\Psi$ in the present case are chiral Dirac fermions, in the sense that they are in the $(1/2, 0) \oplus (0, 1/2)$ representation of $SL(2, \mathbb{C})$, but only the left or right component is different from zero. That is, we have manually set to zero the “mirror sector” which appears naturally. This is not new in noncommutative geometry, a similar phenomenon of fermion doubling appears in Connes’ approach \cite{13}.

The correct hypercharge, electric charge and baryon number are then reproduced by the following generators (the constant in front of the unit matrix is chosen in order to make them traceless)

\[
Y = \begin{pmatrix} 0_{2 \times 2} & -\sigma_3 \\ -\frac{1}{3} I_{3 \times 3} & 0 \\ \end{pmatrix} - \frac{1}{7} \mathbf{1} \quad (4.4)
\]

\[
Q = T_3 + \frac{Y}{2} = \frac{1}{2} \begin{pmatrix} \sigma_3 & -\sigma_3 \\ -\sigma_3 & \sigma_3 \\ \end{pmatrix} - \frac{2}{7} \mathbf{1} \quad (4.5)
\]

\[
B = \begin{pmatrix} 0_{2 \times 2} & 0 \\ 0 & -\frac{1}{3} I_{3 \times 3} \\ \end{pmatrix} - \frac{1}{7} \mathbf{1} \quad (4.6)
\]

which act in the adjoint. Now we can write

\[
\left[ Q - \frac{1}{2} B \right] = \frac{1}{2} \begin{pmatrix} \sigma_3 & 0 & 0 \\ 0 & -\sigma_3 & 0 \\ 0 & 0 & 0 \\ \end{pmatrix} = \frac{1}{2} \sigma_3 \otimes \sigma_3 \quad (4.7)
\]

where by $[\cdot]$ we indicate the $4 \times 4$ matrix obtained dropping the colour part, which plays no role in the following. It is always understood that the remaining blocks of the $7 \times 7$ matrix are zero.

The present formulation solves an outstanding problem in noncommutative gauge theories which we will call the charge quantization problem \cite{16,17}. In the usual formulation of gauge theory (on commutative spaces) it is of course possible to have different fermions transforming under different representations of a particular gauge group. For example there are particles with electric charge $0, \pm 1, \pm 2/3, \pm 1/3$. This is possible because the gauge group is the tensor product of a finite dimensional group (or algebra) times the functions on the spacetime manifold. Different particles will belong in turn to the tensor product of a vector belonging to the appropriate module (vector space on which the corresponding representation of the group act) times again the space of functions on the spacetime manifold.

In conventional noncommutative gauge theory with a $\star$ product, the space time and the internal symmetry are intimately intertwined, in fact the theory is noncommutative also in the case of $U(1)$ gauge group, the noncommutativity being given by the product. The representations of this large gauge group are much less than the ones of the tensor product (commutative space) case. In the latter case it is possible to have functions transforming in their internal components, according to any representation of the gauge group. In the the former case only fundamental, adjoint and singlet cases are possible. In
particular it is impossible to obtain charges which are different from 0, ±1. The problem can be solved at the price of adding one extra $U(1)$ for each different charge, and these extra gauge degrees of freedom have to be later spontaneously broken.

In the matrix model formalism, although spacetime remains noncommutative, and in the limit reproduces the one with the Gronewöld-Moyal $\star$ product, the problem does not appear. Fermions are in the adjoint representation, but the various $U(1)$ are generated by a single diagonal traceless matrix. Since the fermions are off diagonal, the fermion matrix retains its form when commuted with the generator of a $U(1)$, such as $Y$ and $Q$ in equations (4.4) and (4.5). For example for the charge $Q$:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & \frac{2}{3}u_{L1} & \frac{2}{3}u_{L2} & \frac{2}{3}u_{L3} \\
0 & 0 & -e_L & -\frac{1}{3}d_{L1} & -\frac{1}{3}d_{L2} & -\frac{1}{3}d_{L3} \\
0 & 0 & -e_R & -\frac{1}{3}d_{R1} & -\frac{1}{3}d_{R2} & -\frac{1}{3}d_{R3} \\
0 & 0 & 0 & \frac{2}{3}u_{R1} & \frac{2}{3}u_{R2} & \frac{2}{3}u_{R3} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\] (4.8)

Which is the correct charge assignment.

Consider now the extra coordinate:

\[
\begin{pmatrix}
0_{2\times2} & \varphi & 0_{2\times1} & 0_{2\times1} & 0_{2\times1} \\
\varphi^\dagger & 0 & 0 & 0 & 0 \\
0_{1\times2} & 0 & 0 & 0 & 0 \\
0_{1\times2} & 0 & 0 & 0 & 0 \\
0_{1\times2} & 0 & 0 & 0 & 0 \\
0_{1\times2} & 0 & 0 & 0 & 0
\end{pmatrix}
\] (4.9)

Where $\varphi$ is the usual 2-component Higgs with vacuum expectation value

\[
\langle \varphi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}
\] (4.10)

Since $[X^\mu, X^\varphi] = 0$ the extra coordinate is still a solution of the equation of motion, with $\theta^\mu \varphi = 0$ which implies that in the semiclassical limit $G^\mu \varphi = 0$. In other words this extra dimension is not dynamical.

The extra coordinate $X^\varphi$ does not commute with the generators of the weak $SU(2)$ and with the hypercharge $Y$, but it does commute with the electric charge (4.5), thus breaking the $SU(2) \times U(1)$ symmetry to $U(1)_Q$.

The computation of the Yukawa couplings is now straightforward, it it the part of the fermionic action involving the extra coordinate $X^\varphi$:

\[
S_Y = \text{Tr} \bar{\Psi}^\dagger \gamma_0 [X^\varphi, \Psi]
\] (4.11)

We will also consider $\Psi$ to be eigenvalue of $\gamma^\varphi$ with eigenvalue 1.

\[
S_Y = v(d^\dagger_R d_L + u^\dagger_R u_L + e^\dagger_R e_L + d^\dagger_L d_R + u^\dagger_L u_R + e^\dagger_L e_R)
\] (4.12)

Those are the correct mass terms, in the sense that left handed fermions couple with the right handed ones, and the extra coordinate has succeeded in breaking the symmetry.
5. CONCLUSIONS

The model presented here is just a “feasibility study” for the construction of more realistic models. At the present we can manage to reproduce some key features of it in a rather simple (and naive) model with a pair of extra coordinates. The list of drawbacks and physically unrealistic features of this simple model is probably too long to be mentioned fully. It comprises the fact that at the bare level the Yukawa couplings are the same for leptons and quarks (but this may change under renormalization), that the unbroken gauge group contains some extra $U(1)$ factors (probably anomalous), and the fact that there are three generations does not appear. Some of the problems are solved considering the internal space not being described by two single coordinates ($X^\Phi$ and $X^\phi$) but having an internal noncommutative structure, typically that of fuzzy spheres [10] along the lines of [14]. As the extra dimension get more structure the model becomes more realistic, and hopefully in a near future it will be possible to make definite predictions. The gravitational aspect is already being investigated from a phenomenological point of view and in fact some cosmological predictions have already appeared [15]. We hope that we have been able to convince that matrix models can have a semiclassical limit describing the standard model and gravity as an emergent phenomenon.

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