Two-dimensional Mathematical Model of Oil-bearing Materials in Extrusion-type Transportation over Rectangular Screw Core

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Abstract. A comparative analysis of alternative models for plastic flow in extrusive transportation of oil-bearing materials was conducted; the research was directed at determining the function describing the screw core throughput capacity of the press (extruder). Transition from a one-dimensional model to a two-dimensional model significantly improves the mathematical model and allows using two-dimensional rheological models determining the throughput of the screw core.

1. Introduction
The most important moment in a description of a real-life process is when a model is selected accounting for physical and chemical processes taking place in the operating zone. Due to that, a comparative analysis of alternative models of plastic flow was performed for extrusive transportation of oil-bearing materials [1-5]. A rheological flow in a spiral screw is determined by a selection of boundary conditions of the rheological flow equation, which determines the volume efficiency of the extruder [6, 7]. Thus, the study was aimed at determination of dependency between the throughput capacity of a screw core of a press [8] and selection of boundary conditions in the two-dimensional Poisson's problem [9].

2. Materials and methods
Let us consider the plane \((x, y)\) of an arbitrary section of a zeroth flight of the screw (without the extracting box), where region \(G\) is defined as being limited with rectangular boundary \(\Gamma\). Let us consider Poisson's equation:

\[
\frac{\partial^2}{\partial x^2} V(x, y) + \frac{\partial^2}{\partial y^2} V(x, y) = P(x, y)_{\text{b} G}, \quad V(x, y)|_{\Gamma} = \mu(x, y)
\]  

(1)

determining a dynamic head of a material in this section. The relaxation function, \(\text{relax} (A, B, C, D, E, P, \mu, r_\gamma)\), solves an equation in partial derivatives (1) using a modified numerical Gauss-Seidel method (\(r_\gamma\) is a spectral radius of Jacobi iterations). This number (\(r_\gamma\)) between 0 and 1 controlling the convergence of the relaxation process is defined by a formula:
\[ r_n = 1 - \frac{2 \cdot \pi}{n} \]  

(2)

where \( n \) is the number of points in each direction of the lattice; \( P \) is a matrix determining the source intensity in every point of the region (it is equals zero when there is not pressing).

3. Results

The solution is obtained by consecutive approximations to the nodes of the lattice, which approximates the derivative of Poisson's equation with difference expressions in the square region:

\[
\frac{\partial^2 V(x, y)}{\partial x^2} \approx \frac{V(x + h_x, y) - 2 \cdot V(x, y) + V(x - h_x, y)}{h_x^2}
\]

\[
\frac{\partial^2 V(x, y)}{\partial y^2} \approx \frac{V(x, y + h_y) - 2 \cdot V(x, y) + V(x, y - h_y)}{h_y^2}
\]

(3)

where \( h_x \) is the incremental step along the \( x \) coordinate; \( h_y \) is the incremental step along the \( y \) coordinate. Laplacian \( \Delta V(x, y) \) of equation (1) in this case is substituted with difference operator (3) on a five-point stencil ("cross"), consisting of points \( a(x+h_x, y), b(x-h_x, y), c(x, y+h_y), d(x, y-h_y), e(x, y) \). Equation (1) in this case is replaced with a difference equation of the following form:

\[
a \cdot v_{i+1,j} + b \cdot v_{i-1,j} + c \cdot v_{i,j+1} + d \cdot v_{i,j-1} + e \cdot v_{i,j} = p_{i,j}
\]

(4)

The left part of equation (1) is the laplacian; thus, the ratio of coefficients in the left part of equation (4) for a rectangular region shall correspond to the following expression:

\[ a + b + c + d + e = 0 \]

(5)

From (5) it follows that for a square region, coefficients \( a, b, c, d \) are equal (-\( e/4 \)). Solving equation \( \Delta v = 0 \) with respect to \( v_{i,j} \) in case of rectangular region (3), the ratios of the coefficients of the equation (4) are determined with the following equation:

\[
v_{i,j} = \frac{h_x^2}{2 \cdot h_x^2 + 2 \cdot h_y^2} \cdot v_{i+1,j} + \frac{h_x^2}{2 \cdot h_x^2 + 2 \cdot h_y^2} \cdot v_{i-1,j} + \frac{h_y^2}{2 \cdot h_x^2 + 2 \cdot h_y^2} \cdot v_{i,j+1} + \frac{h_y^2}{2 \cdot h_x^2 + 2 \cdot h_y^2} \cdot v_{i,j-1}
\]

(6)

Approximation of the coefficients of equation (6) corresponds to equation (5). Let us consider a flight of the press with the following geometric parameters: the depth of the flight is 62.5 mm \( (H_d) \); the diameter of the extracting box is 250 mm \( (D_s) \); the flight pitch is 290 mm \( (S_s) \); the width of the lower foil is 32 mm \( (T_a) \); the width of the upper foil is 22 mm \( (T_u) \). Then, the width of the flight is defined by the formula:

\[ W_g = S_s \cdot \cos \left( \arctan \left( \frac{S_s}{\pi \cdot (D_s - 2 \cdot \delta_g)} \right) \right) \approx 272 \text{mm} \]

(7)

where \( \delta_g = 1.5 \text{ mm} \) – a gap between the flight and the extracting box. In this case the sectional geometry may be represented as a trapezoid with area \( S_T = 0.016978 \text{ m}^2 \); and perimeter \( P_T = 0.668698 \text{ m} \). Taking into account the geometry of the region where solution (1) exists, let us determine its dimensions \( (X, Y) \) from the following expressions:
\[
\begin{align*}
X \cdot Y &= S_r \\
2 \cdot X + 2 \cdot Y &= P_r
\end{align*}
\tag{8}
\]

Considering (7) and (8), the dimensions of the rectangular region are: \(X = 272\) mm; \(Y = 62\) mm. The geometry of this region and location of nodes for 9 points along each of the coordinate axes are shown in Figure 1.

\textbf{Figure 1.} Nodes of the rectangular lattice

Taking into account the location of the points in the solution region (Fig. 1), the authors determined matrices for the relax\( (A, B, C, D, E, P, \mu, r_0) \) function, which for the example in question look like this:

\[
A = \{a_{i,j}\} = B = \{b_{i,j}\} = 0,475
\]

\[
C = \{c_{i,j}\} = D = \{d_{i,j}\} = 0,025
\]

\[
E = \{e_{i,j}\} = (-1)
\tag{9}
\]

\(P\) is a square matrix containing the values of the right part of equation (1) in each point through the solution region (Figure 1). There is no pressure at the entrance into the zeroth flight, thus, one may assume that the elements of matrix \(P\) are identically zero, \(\mu\) - a square matrix of boundary conditions \(V(x, y) = \mu(x, y)\) and initial approximation \(V(x, y)\) to the solution. Let us consider two cases of boundary conditions. The first one is when the cover of the channel moves, while its walls are stationary. Then, boundary condition and initial approximation matrix \(\mu_1\) looks like this:
Using matrix (10) while taking (9) as the arguments of the \( \text{relax}(A, B, C, D, E, P, \mu_1, r_\beta) \) function, let us obtain a solution for the Poisson's equation (1) for the moving cover case (Table 1).

\[
\mu_1 = \begin{bmatrix}
0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
0.5 & 0 & \cdots & 0 & \cdots & 0 & 0.5 \\
1 & 1 & \cdots & 1 & \cdots & 1 & 1
\end{bmatrix}
\]

Using matrix (10) while taking (9) as the arguments of the \( \text{relax}(A, B, C, D, E, P, \mu_1, r_\beta) \) function, let us obtain a solution for the Poisson's equation (1) for the moving cover case (Table 1).

### Table 1. Moving cover, stationary walls

| V(x, y) | x, mm | 0 | 34 | 68 | 102 | 136 | 170 | 204 | 238 | 272 |
|---------|-------|---|----|----|-----|-----|-----|-----|-----|-----|
| 0.0000  | 0     | 0 | 0  | 0  | 0   | 0   | 0   | 0   | 0   | 0   |
| 7.75    | 0.08703 | 0.11577 | 0.12285 | 0.12411 | 0.12285 | 0.11577 | 0.08703 | 0   | 0   | 0   |
| 15.5    | 0.1771 | 0.23266 | 0.246 | 0.24835 | 0.246 | 0.23266 | 0.1771 | 0   | 0   | 0   |
| 23.25   | 0.27347 | 0.35175 | 0.36973 | 0.37284 | 0.36973 | 0.35175 | 0.27347 | 0   | 0   | 0   |
| 31      | 0.37999 | 0.47398 | 0.49422 | 0.49764 | 0.49422 | 0.47398 | 0.37999 | 0   | 0   | 0   |
| 38.75   | 0.50138 | 0.60003 | 0.61959 | 0.62282 | 0.61959 | 0.60003 | 0.50138 | 0   | 0   | 0   |
| 46.5    | 0.64368 | 0.7302 | 0.74582 | 0.74832 | 0.74582 | 0.7302 | 0.64368 | 0   | 0   | 0   |
| 54.25   | 0.81493 | 0.86405 | 0.87272 | 0.87409 | 0.87272 | 0.86405 | 0.81493 | 0.5 | 0   | 0   |
| 62      | 1     | 1 | 1  | 1  | 1   | 1   | 1   | 1   | 1   | 1   |

In the second case, boundary condition and initial approximation matrix \( \mu_2 \) looks like this:

\[
\mu_2 = \begin{bmatrix}
1 & 1 & \cdots & 1 & \cdots & 1 & 1 \\
1 & 0 & \cdots & 0 & \cdots & 0 & 1 \\
1 & 0 & \cdots & 0 & \cdots & 0 & 1 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
1 & 0 & \cdots & 0 & \cdots & 0 & 1 \\
0.5 & 0 & \cdots & 0 & \cdots & 0 & 0.5 \\
0 & 0 & \cdots & 0 & \cdots & 0 & 0
\end{bmatrix}
\]

Using matrix (11) while taking (9) as the arguments of \( \text{relax}(A, B, C, D, E, P, \mu_2, r_\beta) \), let us obtain a solution for the Poisson's equation (1) for the case with a moving screw core and a stationary cover (Table 1).
Comparing numerical solutions (Table 1) and (Table 2) for different boundary conditions $\mu_1, \mu_2$, one gets different characteristics of speed even for the simplest case of Newtonian liquid flow in the screw core, which are different by more than 30% (Figure 2).

**Figure 2.** Two-dimensional diagrams of the dynamic head for moving cover $V_k$ and a moving screw core $V_c$.  

**Table 2.** Stationary cover, moving walls

| x, mm | 0  | 34 | 68 | 102 | 136 | 170 | 204 | 238 | 272 |
|-------|----|----|----|-----|-----|-----|-----|-----|-----|
| 0     | 1  | 1  | 1  | 1   | 1   | 1   | 1   | 1   | 1   |
| 7.75  | 1  | 0.91296 | 0.88423 | 0.87715 | 0.87588 | 0.87715 | 0.88423 | 0.91296 | 1 |
| 15.5  | 1  | 0.8229 | 0.76733 | 0.75399 | 0.75164 | 0.75399 | 0.76733 | 0.8229 | 1 |
| 23.25 | 1  | 0.72652 | 0.64824 | 0.63026 | 0.62715 | 0.63026 | 0.64824 | 0.72652 | 1 |
| 31    | 1  | 0.62  | 0.52601 | 0.50577 | 0.50234 | 0.50577 | 0.52601 | 0.62  | 1 |
| 38.75 | 1  | 0.49862 | 0.39996 | 0.38039 | 0.37717 | 0.38039 | 0.39996 | 0.49862 | 1 |
| 46.5  | 1  | 0.35632 | 0.26979 | 0.25418 | 0.25167 | 0.25418 | 0.26979 | 0.35632 | 1 |
| 54.25 | 0.5 | 0.18506 | 0.13595 | 0.12728 | 0.1259 | 0.12728 | 0.13595 | 0.18506 | 0.5 |
| y, mm | 0  | 0  | 0  | 0   | 0   | 0   | 0   | 0   | 0   |
| 62    | 0  | 0  | 0  | 0   | 0   | 0   | 0   | 0   | 0   |

Comparing numerical solutions (Table 1) and (Table 2) for different boundary conditions $\mu_1, \mu_2$, one gets different characteristics of speed even for the simplest case of Newtonian liquid flow in the screw core, which are different by more than 30% (Figure 2).
As a result, the simplifications assumed in a number of works taken from a one-dimensional model of the dynamic head give only a crude estimate for a real-life screw core [10, 11]. For a quantitative assessment of this deviation, the authors used a simplex approximation of numerical solutions (Table 1) and (Table 2) accounting for the geometry (Fig. 2) of the given region $G$, a surface patch with the rectangular boundary $\Gamma$ solution of Poisson's equation (1). In the first case, the throughput was calculated with the equation:

$$q_k = \int_0^{62\, \text{mm}} \int_0^{272\, \text{mm}} V_{S_k}(x, y) \, dx \, dy$$

(12)

where $V_{S_k}(x, y)$ is a spline approximation of the numerical solution for the moving cover. In the second case, the throughput was assessed following a similar formula:

$$q_c = \int_0^{62\, \text{mm}} \int_0^{272\, \text{mm}} V_{S_c}(x, y) \, dx \, dy$$

(13)

where $V_{S_c}(x, y)$ is a spline approximation of the numerical solution for the moving screw core.

The calculations have shown that the ratio of the throughput values calculated with equations (12) and (13) is:

$$1 - \frac{\int_0^{62\, \text{mm}} \int_0^{272\, \text{mm}} V_{S_k}(x, y) \, dx \, dy}{\int_0^{62\, \text{mm}} \int_0^{272\, \text{mm}} V_{S_c}(x, y) \, dx \, dy} = 22\%$$

(14)

4. Conclusion
Consequently, transition from a one-dimensional model [12-17] to a two-dimensional model is not only improving significantly the mathematical model, but it is also linked largely to selection of a variant of such transition.

As a result of numerical modeling, a necessity for application of two-dimensional rheological screw core flow models was determined. At that, different qualitative variants of such transition shall be considered. Speed characteristics of the flows in the considered variants differ by 30%, while the throughput values of the screw core vary by 20%, depending on a model selected, even for the unchanged geometry of the core and Newtonian flow rheology. As a result of comparison analysis of two-dimensional models boundary conditions, significant differences were discovered in the dynamic head of these flows, suggesting taking into consideration influence on the material flow from the side walls of the core, as it is done in both Couette's problem and Poisson's problem. Thus, a transition from a one-dimensional model to a two-dimensional model significantly improves the mathematical model and allows using two-dimensional rheological models determining the screw core throughput.

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