Backreaction I: The Torus

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Abstract

We use wrapped D-brane probes to measure position dependent perturbations of compactification moduli. Due to the backreaction of the D-branes on the local geometry, we suspect that measuring the fluctuations of one modulus to high precision will generically affect the others. These considerations lead us to conjecture a novel uncertainty principle on the Calabi-Yau moduli space. We begin our investigation with a gedanken experiment on a torus.

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1 Motivation

Among the most fascinating and longstanding puzzles that physicists face in this new century is: “What is the nature of spacetime geometry?” The classical, (pseudo-)Riemannian geometry that served us during the last century does not adequately describe the universe as seen by strings and D-branes. Classical geometry fails to capture the smooth behavior of conformal field theories at orbifold singularities and flop transitions [1, 2], as well as the smooth physics of conifold singularities that are resolved [3] or subject to discrete torsion [4, 5] in the presence of D-branes. Every shortcoming of classical geometry is a clue to this puzzle, and as these pieces are assembled, a picture of a new stringy geometry [1, 2, 5, 6] is only beginning to emerge.

Consider a spacetime consisting of a noncompact $D$-dimensional manifold, $M$, with a compact Calabi-Yau manifold over each point. Classically, each simply-connected, compact Calabi-Yau manifold is described by some topological characteristics (Hodge numbers, an intersection form, etc.) and also a point in moduli space, which corresponds to a choice of Kähler and complex structure moduli. In stringy geometry there are smooth transitions which connect the moduli spaces of topologically distinct Calabi-Yau, forming an extended moduli space. Thus our spacetime is described by associating a point in the extended moduli space with each point in $M$. Even in the simplest case, where the moduli are chosen identically everywhere, many new features of stringy geometry have been discovered.

However some phenomena only appear when the moduli are allowed to vary. For example, near large numbers of BPS D-branes wrapped around non-trivial cycles in a Calabi-Yau, we know that the moduli experience an attractor flow [7, 8, 9] as one moves toward the branes. To better understand the nature of stringy geometry, we will consider such variations in moduli as seen by D-brane probes. In particular, we will pose the question, “How well can D-branes measure local variations in moduli?”

We will present arguments to motivate the following conjecture.

**Conjecture 1.** A measurement of a variation in a period $p$ which is localized to a region of diameter $d$ in the noncompact directions will generically affect the values of the other periods. Furthermore this effect increases as the precision of the measurement of $p$ increases and as $d$ decreases.

In particular we expect that for $d$ very large, corresponding to measuring a nearly con-

\[^1\]The periods $p_i$ are a set of coordinates on moduli space that roughly correspond to the volume of cycles $C_i$ in the Calabi-Yau.
stant modulus, the conjectured effect disappears. Assuming that this phenomenon is not an artifact of our measurement scheme, we can interpret it as an uncertainty principle.

![Diagram showing brane probes wrapped around cycles C1 and C2 and moduli deformed in between.]

Figure 1: Branes measure period $p_1$ then $p_2$. Do these operations commute?

This conjecture is plausible for the following reason. Imagine that we measure a local variation of period $p_1$ by wrapping branes around the corresponding cycle $C_1$ and using these branes to probe the region with the variation (see Fig. 1). To obtain an accurate measurement of the deformation of $p_1$, we slowly send many branes very close to the region to be measured. However this is likely to cause a backreaction on the geometry, which (using attractor flows as a guide) we suspect will alter the other periods in much the same way as an accurate position measurement alters momentum in the case of the Heisenberg uncertainty principle.

Being mere mortals, in this paper we will only begin an analysis of the validity of this conjecture. We will consider an illustration of our proposed experiment in the case of toroidal compactification. In the case of the two-torus, we find a variant of the space-space uncertainty expected in any theory of quantum gravity, which upon dimensional reduction becomes a Heisenberg uncertainty principle. We conclude with preliminary evidence for our conjecture on the four-torus.

The next step in this analysis would be to apply the lessons learned below to the measurement of moduli on $T^4$, $T^6$, $T^2 \times K3$, as well as some simple orbifolds and multi-parameter Calabi-Yau three-folds if possible.

2 The Experiment

We consider Type II string theory at small $g_s$ and in a spacetime of the form $\mathbb{R}^{1,D-1} \times T^2$. A critical string theory may be obtained by adding a superconformal field theory with central
charge $c = 12 - (3D/2)$. We perturb spacetime so that the two radii of the torus vary in the noncompact directions, which yields a torus fibration rather than a direct product. Specifically, the tori have constant radii

$$R_1 = R_2 = R \gg \sqrt{\alpha'}$$

(2.1)
everywhere in $\mathbb{R}^{1,D-1}$ except for a small, spherical region in space of diameter $d$. In the interior of this sphere the radii of the torus are slightly different from $R$, each by a small amount (not necessarily the same amount for both radii) of order $\delta R \ll R$. We describe this variation perturbatively as a wavepacket consisting of a superposition of closed strings.

The moduli describing these variations are massless fields in the low energy effective theory on $\mathbb{R}^{1,D-1}$ so this wavepacket travels at the speed of light and is composed of strings with momenta on the order of $1/d$. As these strings interact with each other, the wavepacket will tend to disperse. This dispersion is expected, as the wavepacket that we are considering is a finite energy perturbation of the interacting theory which does not, a priori, correspond to a stable deformation of the string background.

There are perturbative arguments that lead us to believe that these variations in moduli are sufficiently stable for the experiment to be sensible. To see this, note that the leading contribution to this dispersion comes from the four point function on the sphere, which is of order $g_s^2$ and not from the on-shell three point amplitude, which is kinematically forbidden. One advantage of using D-brane probes is that they interact with the moduli with an amplitude of order $g_s$, and therefore to leading order in $g_s$, we may neglect the dispersion. In addition, for small $\delta R$, the density of closed strings in the packet is small. This suppresses the dispersion further because the dispersion scales as the density squared, while our measurement process scales only as the density.

To measure the variation $\delta R$, let us place an advanced experimental physicist in the path of the wavepacket. He knows both the packet’s trajectory and the order of magnitude of its size, but wants to determine $\delta R$ more precisely. The experimenter wants to understand spacetime geometry as the branes see it, and so he is only interested in the scattering of the wavepacket by branes and not by any other probes at his disposal. Fortunately he has a reservoir of branes of various dimensions and wrappings in his lab which he can arrange in any configuration before the wavepacket arrives. After reading Refs. [11, 12, 13, 14, 15], he

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2However, in Ref. [10] it was shown that smooth variations of the moduli of the torus which admit a timelike killing isometry can be lifted to exact classical string vacua by adding an appropriate compensating variation of the dilatonic and light-cone gauge metric, at least if $D = 2$. 

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figures out how many branes of each type he will need so that on the order of one brane will be scattered by the incoming wavepacket.

![Figure 2: A variation knocks brane probes into a detector.](image)

The branes are placed in a line, along the path of the variation (see Fig. 2). The tree-level scattering calculations [14, 15] are only valid when spacetime is reasonably flat, which is achieved by separating the brane probes sufficiently. Eventually the packet passes through the assortment of branes, knocking some of them into a detector. After counting hits and reconstructing the time of the scattering from the brane velocities, our experimenter extrapolates $\delta R$. However in order to obtain a very accurate measurement, the experimenter would need a large number of branes, and this would deform the modulus variation that he is trying to measure so much that his results would be useless. For example, the first few branes would see a very different wavepacket from the last few.

That is to say, there is an uncertainty bound characteristic of theories of quantum gravity (not the uncertainty we have conjectured) on how accurately he can perform this measurement. For the duration of this paper we will describe just what the branes do to these moduli and what kind of uncertainty relation this implies.

### 3 The Calculation

To analyze the preceding gedanken experiment we have calculated the relevant scattering amplitudes using the results of Refs. [14, 15]. Specifically, for the case of flat space and thus
toroidal compactifications, the authors calculated tree-level amplitudes for closed strings scattering off D-branes. We will outline those calculations here with a view towards the analogous calculations in non-trivial Calabi-Yau string compactifications.

### 3.1 The Scattering

We are interested in the interaction of a closed string comprising the modulus and a D-brane completely wrapped around some toroidally compactified dimensions. The relevant tree-level calculation is the absorption and re-emission of a closed string from an open string attached to the brane seen schematically in Fig. 3. Thus we are interested in a disk with two punctures, corresponding to the vertex operators of the incoming and outgoing closed strings. The incoming modulus is a massless NS-NS closed string with polarizations in the compact directions and momentum only in the noncompact direction. The outgoing state, at lowest order in $g_s$, may, a priori, be any massless (in the $D + 2$-dimensional sense) closed string with arbitrary polarization and momenta in either the compact or noncompact directions (although we will see that, as expected, some final states are forbidden) or a pair of open strings attached to the brane.

As an example, consider the case in which both the initial and final states are NS-NS closed strings. Their vertex operators are of the form

$$V = \epsilon_{\mu\nu}(\partial X^\mu + ip \cdot \psi \bar{\psi}^\mu)(\bar{\partial} X^\nu + ip \cdot \bar{\psi} \bar{\psi}^\nu)e^{ip \cdot X}$$  \hspace{1cm} (3.1)
where $p$ is the momentum and $\epsilon$ a polarization tensor.

We always assume that the incoming string has no compact momentum. The compact momentum of the outgoing string is quantized, $p_m = N_m/R$, where the index $m$ runs over the compact coordinates. To specify the vertex operators for the incoming moduli, we only need to identify their polarization tensors. The Kähler modulus is a complex combination of the volume of the torus and the integral of the NS-NS two form over the torus, while the complex structure modulus can be identified by writing the metric on the torus in standard form as

$$ds^2 = |dx_D + \tau dx_{D+1}|^2.$$  \hfill (3.2)

Thus, we can arrive at the appropriate form for the polarization tensors by writing small deformations of these moduli in terms of variations of the metric, dilaton and antisymmetric tensor fields. We then find that polarization tensors corresponding to incoming excitations of Kähler and complex moduli can be written as

$$\epsilon^K = \begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix}, \quad \epsilon^C = \begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix}.$$  \hfill (3.3)

in the compact directions and zero in the noncompact directions.

Since toroidal compactifications are described by free field theories on the world-sheet, the vertex operators were easy to construct. However, in a general Calabi-Yau compactification the vertex operators we are interested in correspond to truly marginal deformations of the SCFT and are more difficult to construct. Fortunately many Calabi-Yau moduli spaces have distinguished points where the nonlinear sigma model is a rational $(2,2)$ superconformal field theory, such as the Gepner point on the quintic hypersurface. Near such points we can construct these deformations from the (chiral, chiral) and (chiral, antichiral) primaries as described in Ref. [6]. Boosting the resulting vertex operator in a noncompact direction yields our wavepacket. One can explicitly check that in the case of toroidal compactification this procedure yields the polarizations given in Eq. (3.3).

Once the appropriate vertex operators have been constructed, the amplitude for the disk with two closed string insertions (and one superconformal ghost, as required on the disk) is calculated in Refs. [14, 15] using the covariant formulation of Ref. [11]. By imposing Neumann boundary conditions in $p + 1$ directions and Dirichlet conditions on the rest, the authors obtain an amplitude for the scattering of an NS-NS string by the p-brane in flat space. We will not reproduce their formulas here, but instead refer the interested reader to Refs. [14, 15] for details. Although the final state may be in the R-R sector (or even a pair
of open string excitations of the brane when \( d < R \), for our present purpose it will suffice to restrict our attention to NS-NS outgoing states.

### 3.2 Results

We first consider the case \( d > R \), so that the moduli in the wavepacket have insufficient energy to excite any internal modes of the brane or Kaluza-Klein modes on the torus. In this case, we found that 0 and 2 branes interact with Kähler moduli, while 1 branes interact with complex structure moduli. This is expected since the Kähler modulus on a torus is described by the period of a closed (1,1) form while the complex structure modulus can be described by the periods of closed (0,1) and (1,0) forms. Generally, the branes scatter these moduli either to R-R states or to gravitons and dilatons with both polarizations along the noncompact directions, but never mix the Kähler and complex structure moduli. When the final polarizations of the NS-NS states are both along the noncompact directions, one of the outgoing polarizations must (using the standard \( \epsilon \cdot p = p \cdot \epsilon = 0 \) gauge) lie along the plane of the scattering and be orthogonal to the outgoing momentum. We will use this fact in Sec. 5 when we consider measurements on \( T^4 \). This and other possible scattering processes are summarized in the table below.

| Brane Dim. | \( d \) vs. \( R \) | Incoming Modulus | Possible Outgoing (NS-NS) States |
|------------|-----------------------|------------------|---------------------------------|
| 0 or 2     | \( d > R \)           | Kähler           | Dilaton, Graviton, Kähler Modulus |
| 0 or 2     | \( d > R \)           | Complex Str.     | No Interaction                  |
| 1          | \( d > R \)           | Kähler           | No Interaction                  |
| 1          | \( d > R \)           | Complex Str.     | Dilaton, Graviton, Complex Str. Modulus |
| 0,1 or 2   | \( d < R \)           | Kähler           | Any                             |
| 0,1 or 2   | \( d < R \)           | Complex Str.     | Any                             |

**Table 1: Possible Scattering Processes**

When the size of the wavepacket, \( d \), is less than the largest radius of the torus we can excite Kaluza-Klein modes. In this case we find that the scattering proceeds as one would expect in the flat space case, with the branes interacting generically with all massless closed string states. Outgoing closed strings with non-vanishing compact momenta can have any transverse polarization, and at the same order in \( g_s \) the final state can consist of two open strings bound to the brane probe with opposite compact momenta.
4 Space-Space Uncertainty Principle

In the previous section we saw that trying to measure $\delta R$ generically transformed the polarizations of the closed strings in our wavepacket, thus changing the value of the modulus. Below we argue that this indicates the existence of a version of the space-space uncertainty relation present in theories of quantum gravity. We will derive our version of this relation in three different ways. For simplicity we will restrict our attention to dilations of the area of the torus, the quanta of which we will refer to as dilatons.

Our experimenter has chosen the number of branes so that of order one brane will scatter. This means that, because the wavepacket is assumed to be dilute, of order one dilaton will scatter. Even for the case when the Kaluza-Klein modes are suppressed ($d > R$), the outgoing string will generically have polarizations along the noncompact directions. Thus $\delta R$ is changed by the loss of one incoming dilaton. As a result the smallest $\delta R$ that our experimentalist can measure at fixed $d$, corresponds to a wavepacket containing of order one dilaton. To learn what this $\delta R$ is, we will count the number of dilatons in a packet.

We know that the dilatons have energies of order $1/d$, thus it will suffice to calculate the energy of a given packet. The energy of a packet $E_{\text{packet}}$ should depend on its size $d$ and its amplitude $\delta R$ and so we can solve for the smallest measurable $\delta R$ as follows:

$$1 \sim \text{# of dilatons} = \frac{E_{\text{packet}}}{\text{energy per dilaton}} \sim d \times E_{\text{packet}}.$$  \hspace{1cm} (4.1)

We will now calculate the total energy as a function of $d$ and $\delta R$ in three ways.

4.1 Counting Dilatons

We will calculate the energy of the variation using the effective field theory, Einstein’s equation, as well as a model of space as a stretchable sheet with constant tension equal to the inverse Newton’s constant, $T = 1/G_N$.

Consider the low energy effective theory for the NS-NS sector of Type II string theory compactified on a two-torus. We assume that the non-compact space is flat $\mathbb{R}^{1,D-1}$ (with $D = 8$ in the critical case). In this theory, the moduli are realized as combinations of the various scalar fields arising from the components of the metric and NS-NS antisymmetric tensor field along the torus. The Kähler modulus is a complex scalar field which is a combination of the determinant of the metric on the torus $\sqrt{\det(g_{mn})}$ and the antisymmetric tensor field component $B_{mn} = -B_{nm}$ along the torus, while the complex structure modulus is a combination of the metric components $g_{mn}$. Explicitly, the Kähler and complex structure
moduli can be identified as the complex scalars \( \rho = \rho_1 + i \rho_2 \) and \( \tau = \tau_1 + i \tau_2 \) which are defined by (take \( D=8 \) for ease of notation):

\[
\rho = \frac{R^2}{\alpha'} \left( B_{89} + i \sqrt{\det(g_{mn})} \right), \quad \tau = \frac{1}{g_{88}} \left( g_{89} + i \sqrt{\det(g_{mn})} \right).
\]  

(4.2)

Note that the imaginary part of \( \rho \) is related to area of the torus by

\[
\rho_2 = \frac{A}{4\pi^2 \alpha'}.
\]  

(4.3)

Thus, to consider the energy associated to a localized perturbation of the area of the torus moving along the noncompact \( x_1 \) direction, with some magnitude associated with a radius perturbation \( \delta R \), we can take

\[
\rho_2 = \frac{A_0}{4\pi^2 \alpha'} \left( 1 + \frac{\delta R}{\sqrt{A_0}} \exp \left[ -\frac{(x_1 - t)^2 + |x|^2}{d^2} \right] \right)
\]  

(4.4)

(where \( A_0 \) is the area of the unperturbed torus) and calculate the energy of the configuration in the effective theory.

To do this, we first note that the effective action for the massless fields in the NS-NS sector in a toroidally compactified type II string theory, expressed in Einstein frame, takes the form

\[
S = \frac{1}{16\pi G_N^D} \int d^Dx (-G_D)^{1/2} \left( R_D - \frac{1}{2} \partial_\mu \tau \partial^\mu \tau - \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + \cdots \right).
\]  

(4.5)

Here \( G_N^D \) is the \( D \)-dimensional Newton’s constant, while \( G_D \) and \( R_D \) are the effective \( D \)-dimensional spacetime metric and Ricci scalar respectively. To compute the energy of the wavepacket, we make the field redefinition \( \rho_2 = e^{-K} \). The action for \( K \) has the standard free field form. As we are in flat \( D \) dimensional space with no other background field fluctuations, the Hamiltonian is the standard one and the energy of the field configuration (4.4) is

\[
E = \frac{1}{32\pi G_N^D} \int d^{D-1}x \left( (\partial_t K)^2 + (\partial_x K)^2 \right) \propto \frac{1}{G_N^D A_0} (\delta R)^2 d^{D-3}.
\]  

(4.6)

The Newton constant in \( D + 2 \) dimensions is \( G_N = G_N^D A_0 \), and thus we have

\[
E_{\text{packet}} \sim \frac{(\delta R)^2 d^{D-3}}{G_N}.
\]  

(4.7)

Taking the energy of each dilaton to be on the order of \( 1/d \), the number of dilatons comprising the wavepacket is

\[
\text{# dilatons} \sim d \times E_{\text{packet}} \sim \frac{(\delta R)^2 d^{D-2}}{G_N}.
\]  

(4.8)
When $D = 2$ we note that the number of dilatons in the packet is independent of the size of the variation $d$. This follows from the fact that for a one dimensional packet we can always boost the size of the variation $d$ to any arbitrary value by, for example, giving the brane probes an initial momentum.

Using Einstein’s equation, we can model the wavepacket (see Fig. 2) as a gravitational wave corresponding to a Gaussian profile in the spacetime metric. For example when $D = 2$ with noncompact coordinates $x$ and $t$, the metric along the compact directions (the metric is $\eta_{\mu\nu}$ along all $D$ noncompact directions) is

$$g_{mn} = \begin{bmatrix} A + he^{-\frac{(x-t)^2}{d^2}} & 0 \\ 0 & A + he^{-\frac{(x-t)^2}{d^2}} \end{bmatrix}.$$ \hspace{1cm} (4.9)

The background metric on the torus is $A\delta_{m,n}$ and the amplitude of the wavepacket variation is parametrized by $h$. Treating, as usual, the Einstein tensor of this gravitational wave as an effective stress-energy tensor and then integrating over all space we learn that

$$E_{\text{packet}} = \frac{\sqrt{\pi}h^2}{2^{5/2}G_NAd}$$ \hspace{1cm} (4.10)

when $D=2$. For general $D$, we find that the energy of the wavepacket is

$$E_{\text{packet}} \sim \frac{h^2}{G_NA}d^{D-3} \sim \frac{(\delta R)^2}{G_N}d^{D-3},$$ \hspace{1cm} (4.11)

which agrees with Eq. (4.7).

A simpler and more intuitive derivation of the results in Eqs. (4.7) and (4.8) can be obtained by considering spacetime to be a $D$-dimensional sheet with constant tension $1/G_N$ embedded in $\mathbb{R}^{1,D}$. When spacetime is Minkowskian, the flat sheet describes the unperturbed vacuum. We can deform this sheet by fixing a $D-2$ sphere in space of diameter $d$ and pulling the center of this sphere away a distance $\delta R$ in the transverse direction. This locally stretches the sheet into a cone over $S^{D-2}$ (see Fig. 4).

In the case $D = 2$ this sheet is just a string and the energy of this deformation is just $1/G_N$ times how far we have stretched the string. When $\delta R << d$ this is

$$E_{\text{packet}} = \frac{1}{G_N}(\sqrt{(d/2)^2 - (\delta R)^2} - d/2) \sim \frac{(\delta R)^2}{G_N d}$$ \hspace{1cm} (4.12)

in agreement with Eq. (4.7). A similar calculation can be carried out for arbitrary dimension $D$ and clearly each extra noncompact dimension contributes one more power of $d$ because we integrate tension over a cone of one higher dimension.
4.2 Uncertainty Principle

Having discussed the wavepacket we are now ready to return to the scattering process. Recall that we have placed just enough branes in the path of this wavepacket so that on the order of one dilaton is scattered. Thus, setting the number of dilatons in the packet to be of order one in Eq. (4.8) gives a lower bound on how small of a $\delta R$ we can measure, or equivalently how small $\delta R$ can be for a packet of width $d$:

\[(\delta R)^2 \, d^{D-2} \geq G_N \, . \quad (4.13)\]

We interpret this result as a space-space uncertainty. For example, for a single dilaton, $d$ is just the uncertainty in its position and so Eq. (4.13) assumes a familiar form. In the dimen-

Figure 4: Space as a stretched string or sheet when D=2 or 3.
tionally reduced theory, the variation in moduli appears to be a packet of energy moving at the speed of light in the non-compact directions with some momentum \( p \sim \delta R/d \) and so Eq. (4.13) resembles the Heisenberg uncertainty relation. As such space-space uncertainty principles are expected in all theories of quantum gravity, we will need to learn to distinguish this effect from our conjecture in the case of more general Calabi-Yau compactifications.

Notice that this uncertainty principle tells us that any local measurement of a period in some region \( U \subset \mathbb{R}^{1,D-1} \) will affect its value in that region. However, it does not imply that this measurement affects the values of other periods in \( U \) and thus cannot imply our conjecture.

5 Possible Evidence on the Four-Torus

The allowed scattering channels described in Sec. \( \S \) provide preliminary evidence for our conjecture on the four-torus. Consider now a ten dimensional Minkowskian spacetime with global coordinates \( x_\mu \), compactified on \( T^4 \), which extends along dimensions numbered 6 through 9. Among the moduli of the four-torus are the complex structure moduli of the two subtori that span the 6-7 and 8-9 directions. Imagine that a wavepacket consisting of a variation of the 8-9 torus complex structure (that is, gravitons polarized along the 8 and 9 directions) is propagating through space, and consider measurements of the two complex structures using 1-branes wrapped around a cycle of the subtorus to be measured. Notice that when a 1-brane wraps a cycle of one of the tori, the other torus sees this 1-brane as a 0-brane and thus its complex structure is not effected by this measurement.

More precisely, choose coordinates such that the graviton scattered by the first measurement has momentum

\[
\hat{p}_2 = \hat{x}_1.
\] (5.1)

and so the incoming wavepacket has initial momentum and polarization

\[
\hat{p}_1 = \cos(\theta)\hat{x}_1 + \sin(\theta)\hat{x}_2, \quad \epsilon_1 = \begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix}
\] (5.2)

where the polarization lies along the 8 and 9 directions. To exhibit our conjecture we will measure the complex structure variation of the subtori in both orders and compare the results. These measurements will be performed in a finite region \( U \) and so the space-space uncertainty principle (4.13) will apply in both cases.
Let us first measure the complex structure modulus on the 6-7 torus using 1-brane probes wrapped around the 6 direction. Using our results from the two-torus, we know that a complex structure variation with these momenta can only scatter to its original polarization, or to a dilaton or graviton with at least one polarization along the $\hat{x}_2$ direction. These amplitudes are invariant under time reversal, and so only a 6-7 graviton or a dilaton or graviton with a polarization component along the $\hat{x}_2$ direction can scatter with our brane to produce 6-7 gravitons, thereby creating a variation in the complex structure of this torus. However, no such strings are present and so this measurement does not affect the complex structure at string tree level (although the space-space uncertainty limits the accuracy to which this measurement is possible).

Alternately, let us measure the complex structure of the 8-9 torus first (see Fig. 5), using 1-brane probes wrapped around the 9 direction. While most of our 8-9 gravitons will pass by the brane without interacting, those that scatter may change polarization to become, for example, 2-3 gravitons. Later, when we use 1-brane probes wrapped around the 6 direction to measure the complex structure of the other torus, this brane will scatter some of the 2-3 gravitons created in the first measurement into 6-7 gravitons. Thus, as the complex structure of the 6-7 torus is measured, it will change. This implies that the result of this measurement and all subsequent measurements of the 6-7 complex structure will be altered if we first measure the 8-9 complex structure, which is exactly our conjecture. Like the space-space uncertainty principle, this effect requires the region $U$ to be finite. However, unlike the space-space uncertainty, this effect grows with the number of 8-9 gravitons.

This measurement process can be reinterpreted as follows. Two necessarily non-parallel branes exchange closed strings, altering the moduli of the branes. The necessity of making them non-parallel, or in the general Calabi-Yau case of wrapping the branes around different cycles, is likely to destroy the supersymmetry. Thus one may expect both nontrivial interactions between the branes and that the moduli flow as the constraints of supersymmetry are removed.

Several issues still need to be addressed before this evidence should be taken seriously. For example, the branes are necessarily close to each other, and so one must check that the perturbative treatment is valid. Also, there may be a better way to perform this experiment, perhaps a way to filter out the 2-3 gravitons between measurements. If all goes well, these and other issues will be resolved in a sequel.

\footnote{In fact, if we alternate wrappings on the brane probes we believe that the complex structure variations of the two subtori will approach each other. In the case of the six-torus, this may be seen, perhaps coincidentally, as a flow towards the attractor point identified in Ref. \cite{9}.}
Figure 5: An interaction between two complex structure moduli in $T^4$.

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