Pairing of Cooper pairs in a Josephson junction network containing an impurity

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Abstract – We show how to induce pairing of Cooper pairs (and, thus, 4\textit{e} superconductivity) as a result of local embedding of a quantum impurity in a Josephson network fabricable with conventional junctions. We find that a boundary double sine-Gordon model provides an accurate description of the dc Josephson current patterns, as well as of the stable phases accessible to the network. We point out that tunneling of pairs of Cooper pairs is robust against quantum fluctuations, as a consequence of the time reversal invariance, arising when the central region of the network is pierced by a dimensionless magnetic flux $\varphi = \pi$. We find that, for $\varphi = \pi$, a stable attractive finite coupling fixed point emerges and point out its relevance for engineering a two-level quantum system with enhanced coherence.

There is a large number of physical systems that can be mapped onto quantum impurity models in one dimension [1]. Embedding a quantum impurity in a condensed matter system may alter its responses to external perturbations [2], and/or induce the emergence of non--Fermi-liquid, strongly correlated phases [3]. In quantum devices with tunable parameters impurities may be realized by means of point contacts, of constrictions, or by the crossing of quantum wires or Josephson junction chains [4–7]. For instance, novel quantum behaviors have been recently evidenced in the analysis of $Y$-junctuons of quantum wires [6] and of Josephson junction (JJ) chains [7]. Here, we show how embedding a pertinent impurity in a JJ chain may lead to the emergence of nontrivial symmetry protected quantum phases associated [8–11] with the emergence of 4\textit{e} superconductivity in the network.

While a standard perturbative approach works fine when impurities are weakly coupled to the other modes of the system (the “environment”), there are situations in which the impurities are strongly coupled to the environment, affecting its behavior through a change of boundary conditions: when this happens, it is impossible to disentangle the impurity from the rest of the system, the perturbative approach breaks down, and, consequently, one has to resort to nonperturbative methods, to study the system and the impurity as a whole. Such nonperturbative tools are naturally provided by boundary field theories (BFT) [1]: BFTs allow for deriving exact, nonperturbative informations from simple, prototypical models which, in many instances, provide an accurate description of experiments on realistic low-dimensional systems [12]. In particular, BFTs have been succesfully used to describe the dc Josephson current pattern in Josephson devices, such as chains with a weak link [13,14], and SQUIDs [15,16].

In this letter, we analyze a Josephson junction network (JIN), whose BFT description is given by a boundary double sine-Gordon model (DSGM) [17]. The device is made by two half JJ chains, joined through a weak link to a central Aharonov-Bohm cage $C$ [18], pierced by a (dimensionless) flux $\varphi = \Phi/\Phi_0 (\Phi_0 = hc/(2e))$ (see fig. 1). For simplicity, we connect the outer ends of the chains to two bulk superconductors at fixed phase difference $\alpha$.

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The central region is described by

\[ H_C = \frac{E_c}{2} \sum_{j=0}^{3} |Q_j|^2 - 2J \sum_{j=0}^{3} \cos(\chi_{j,j+1}) + J^2 \sum_{j=0}^{3} Q_j Q_{j+1}, \]

with \( Q_j = [-i \frac{\partial}{\partial x_j} - V_g] \), and \( \chi_{j,j+1} = \chi_j - \chi_{j+1} + \frac{\pi}{2} \).

\( \chi_j \) is the phase of the superconducting order parameter at the island \( j \), and \( V_g \) is a gate voltage applied to each superconducting island, \( J \) is the Josephson energy of each junction, \( E_c \) is the charging energy at each site, and \( J^2 \) accounts for Coulomb repulsion between charges on nearest-neighbor junctions. In the charging regime, that is, \( \frac{E_c}{2J^2} \gg 1 \), and tuning \( V_g \) so that \( V_g = N + \frac{1}{2} \), with integer \( N \), one may write \( H_C \) as a spin-1/2 XXZ-model [14], whose Hamiltonian is given by

\[ H_{\text{JJN}} = H_{\text{LL}}[[\Phi_a]] + H_T[[\Phi_a], \{\Theta_a\}] + H_K[S_1, S_3], \]

with the Luttinger-liquid Hamiltonian

\[ H_{\text{LL}}[[\Phi_a]] = \frac{g}{4\pi} \int_0^L dx \sum_{a=L, R} \left[ \frac{1}{u} \left( \frac{\partial \Phi_a}{\partial t} \right)^2 + u \left( \frac{\partial \Phi_a}{\partial x} \right)^2 \right], \]

and the interaction Hamiltonian

\[ H_T[[\Phi_a], \{\Theta_a\}; S_1, S_3] = -J \sum_{a=L, R, i=1,3} \left[ e^{i\phi_a(0)} e^{i\phi_a} S_i^- + \text{h.c.} \right] + \frac{J_z}{2\pi} \sum_{a=L, R, i=1,3} \frac{\partial \Theta_a(0)}{\partial x_i} S_i^\pm (\epsilon L = 1, \epsilon R = 1), \]

and \( H_K[S_1, S_3] = K[S_1^z S_3^z + S_1^+ S_3^- + K_c S_1^0 S_3^0] \). \( S_1, S_3 \) are defined as in fig. 1. \( \Phi_a \) is the collective plasmon field of the half chains, while \( \Theta_a(x, t) \) is its dual field, that is, \( \frac{\partial \Theta_a(x, t)}{\partial x} = \frac{1}{g} \frac{\partial \Phi_a(x, t)}{\partial t} \), and \( \frac{\partial \Phi_a(x, t)}{\partial t} = \frac{i}{g} \frac{\partial \Theta_a(x, t)}{\partial x} \).

The LL parameters are defined as \( g = \pi/[2(\pi - \arccos(\Delta/2))] \),

\[ u = v_f/2(1 - (\Delta J)^2)/(\arctan(\Delta J)), \]

with \( \Delta = E^2/E_J, \ v_f = 2aE_J \), where \( E_J \) and \( E^2 \) are the Josephson energy and the Coulomb repulsion energy of the half chains, and \( a \) is the lattice step [20]. In deriving eq. (2), it is assumed that \( J/E_J \ll 1 \) and \( J^2/E_J \ll 1 \) (i.e., that \( C \) is weakly coupled to the chains); this allows to use Neumann boundary conditions at \( x = 0 \), that is \( \frac{\partial \Phi_a(0)}{\partial x} = \frac{\partial \Phi_a(0)}{\partial x} = 0 \).

The couplings \( K, K_c \) in \( H_T[[\Phi_L, \Phi_R, \Theta_L, \Theta_R]; S_1, S_3] \) are dynamically generated by the interaction between \( C \) and the half chains, as it may be easily inferred from the renormalization group (RG) equations for the dimensionless couplings \( K = LK, K^z = LK^z, \) and \( J = L^{-1/2} J, J^z = J^z \).
Indeed, employing standard BFT techniques [21,22], one obtains
\[
\frac{d\mathcal{K}}{d \ln(L/\eta)} = \mathcal{K} + [1 + \cos(\varphi)] (\mathcal{J})^2,
\]
\[
\frac{d\mathcal{K}^z}{d \ln(L/\eta)} = \mathcal{K}^z + (\mathcal{J}^z)^2,
\]
and
\[
\frac{d\mathcal{J}}{d \ln(L/\eta)} = \left[1 - \frac{1}{2g}\right] \mathcal{J}, \quad \frac{d\mathcal{J}^z}{d \ln(L/\eta)} \approx 0,
\]
showing that both \(\mathcal{K}\) and \(\mathcal{K}^z\) are dynamically generated whenever \(\varphi \neq \pi\).

Integrating the RG equations one sees that \(\mathcal{J}(L) \sim L^{1 - \frac{1}{2g}}\), while \(\mathcal{K}(L) \sim L^{2 - \frac{1}{2g}}\), that is, for \(\varphi \neq \pi\), \(\mathcal{K}\) is always more relevant than \(\mathcal{J}\). At variance, for \(\varphi = \pi\), no \(\mathcal{K}\)-coupling is generated and \(\mathcal{J}\) is the only relevant coupling strength. For \(g > 1/2\), the BFT description of the JJJN allows to make very general statements regarding the regimes accessible to a network of finite size \(L\). Namely, there will be a perturbative weak coupling regime, accessible for small \(\mathcal{K}\) and \(\mathcal{J}\), and a nonperturbative strong coupling regime, accessible when \(\mathcal{K}\) or \(\mathcal{J}\) becomes \(\sim L\). Most importantly, there will be a renormalization group invariant length scale \(L_* \sim J^{-\frac{1}{1-2g}}\), such that for \(L < L_*\), the JJJN is in the perturbative weak coupling regime, while it is in the nonperturbative strong coupling regime for \(L > L_*\).

To account for the last term of eq. (2), one may resort to a Schrieffer-Wolff transformation [23], which amounts to projecting over the ground state of \(\mathbf{C}\), by summing over its high energy states. At \(\varphi = \pi\), the ground state of \(\mathbf{C}\) is twofold degenerate, while such a degeneracy disappears at \(\varphi \neq \pi\). As a result, for \(\varphi \neq \pi\), the central region \(\mathbf{C}\) is effectively described by the boundary Hamiltonian \(\mathbf{H}_B\), given by

\[
\mathbf{H}_B[\varphi] = -E_W^{(2)}(\varphi) \cos[\Phi_-^a(0)] - E_W^{(4)}(\varphi) \cos[2\Phi_-^a(0)],
\]
with
\[
E_W^{(2)}(\varphi) = \cos\left(\frac{\varphi}{2}\right) \frac{2\mathcal{J}^2}{K + K^z} + \sin^2\left(\frac{\varphi}{2}\right) \cos\left(\frac{\varphi}{2}\right) \frac{2\mathcal{J}^4}{K(K + K^z)^2},
\]
\[
E_W^{(4)}(\varphi) = \sin^2\left(\frac{\varphi}{2}\right) \frac{2\mathcal{J}^4}{K(K + K^z)^2},
\]
where \(\Phi_-^a = [\Phi_L^a - \Phi_R^a]/\sqrt{2}\). At variance, for \(\varphi = \pi\), \(\mathbf{H}_B[\pi] \sim -E_W^{(2)}(\varphi) \cos[2\Phi_-^a(0)]\).

\(I_J(\alpha)\) may be perturbatively computed as \(I_J(\alpha) = \lim_{\varphi \to \infty} -\frac{1}{2} d\ln \mathcal{Z}\), where \(\mathcal{Z}\) is the partition function of the JJJN, given by \(\mathcal{Z} = \mathcal{Z}_0 - \int_0^\beta d\tau \mathcal{H}_B(\tau)\), \(\mathcal{H}_B(\tau)\) being the boundary interaction Hamiltonian in the (imaginary time) interaction representation, while \(\mathcal{Z}_0\) is the partition function at \(\mathcal{H}_B = 0\). From eq. (3), one obtains
\[
I_J(\alpha) = E_W^{(2)}(\varphi) \sin(\alpha) + 2E_W^{(4)}(\varphi) \sin(2\alpha),
\]
with
\[
E_W^{(2)}(\varphi) = \left(\frac{a}{L}\right)^\frac{1}{3} E_W^{(2)}(\varphi), \quad E_W^{(4)}(\varphi) = \left(\frac{a}{L}\right)^\frac{1}{3} E_W^{(4)}(\varphi).
\]
The ratio \(E_W^{(2)}(\varphi)/E_W^{(4)}(\varphi)\), measures the relative weight of SCP vs. PCP tunneling rate. One notices that, for \(\varphi = \pi\), \(E_W^{(2)}(\pi) = 0\), while \(E_W^{(4)}(\pi) \neq 0\). Thus, at \(\varphi = \pi\) PCP tunneling is the only allowed mechanism for charge transfer across \(\mathbf{C}\). In fig. 3, we plot \(I_J(\alpha)\) vs. \(\alpha\) for different values of \(\varphi\). For \(\varphi \neq \pi\), \(I_J(\alpha)\) is periodic, with period \(\Delta \alpha = 2\pi\), while, for \(\varphi \sim \pi\), the period shrinks to \(\Delta \alpha = \pi\).

For \(g > 1\), \(\mathbf{H}_B\) is a relevant operator. Thus, when \(L/L_* > 1\), the boundary Hamiltonian in eq. (3) is the dominating potential term, and the field \(\phi_\tau(x, \tau)\) takes values corresponding to minima of \(\mathbf{H}_B\); it obeys Dirichlet boundary conditions at both boundaries, yielding the mode expansion \(\phi_\tau(x, \tau) = \alpha + \left(\frac{\mathcal{J}^2}{L^2}\right) \pi P + \sqrt{\frac{2}{g}} \sum_{n \neq 0} \sin\left(\frac{2\pi n}{P}\right) \frac{a_n}{n} e^{-2\pi n^2/a}\) [14]. At the strongly interacting fixed point, instanton trajectories are the leading quantum fluctuations; they are described by imaginary time trajectories \(P(\tau)\), with \(P(\tau \to -\infty)\) and \(P(\tau \to \infty)\) corresponding to nearest-neighbor minima of \(\mathbf{H}_B\).

The instanton profile is derived from \(\frac{dS_{\text{Eff}}}{dP(\tau)} = 0\), where \(S_{\text{Eff}}[P] = \int \frac{P(\tau) D\alpha n}{e^{-S_E[\phi_\tau]}}\) is obtained from the Euclidean action \(S_E[\phi_\tau]\) for a spinless LL with parameters \(g\) and \(u\), with a boundary interaction Hamiltonian given by \(\mathbf{H}_B\), after integrating over the oscillator modes of \(\phi_\tau(x, \tau)\). As a result, one gets
\[
S_{\text{Eff}}[P] = \int_0^\beta d\tau \left\{ \frac{M}{2} \left(\frac{\dot{P}}{\sqrt{\mathcal{P}}}\right)^2 + \frac{g\pi}{4L} P^2 + V[\pi P]\right\},
\]
where the “instanton mass” \(M \sim \ln(uT/a)\), \(T\) being the “instanton size”, while \(V(x) = -E_W^{(2)}(\varphi) \cos(x) - E_W^{(4)}(\varphi) \cos(2x)\). From the effective action in eq. (5), one

Fig. 3: Josephson current vs. \(\alpha\) for different values of \(\varphi\) and for \(J^*/K^2 \approx 0.3\). Top left panel: \(\varphi = \pi\); bottom left panel: \(\varphi = 1.01\pi\); top right panel: \(\varphi = 1.1\pi\); top right panel: \(\varphi = 2\pi\).
gets the equation of motion $M\ddot{\phi} - \frac{e^2}{4\pi^2} P + \pi E_W^{(2)}(\phi) \sin[\pi P] + 2\pi E_W^{(4)}(\phi) \sin[2\pi P] = 0$ which, neglecting the “inductive term” $\propto \frac{1}{\tau^2}$, describes soliton solutions of the double-sine Gordon model \cite{17}. These are given by $P(\tau) = \sum_{n=\pm 1}^{\infty} \tan^{-1}[\exp(\frac{L}{V} \omega (\tau + R(\phi)))], \quad \text{with} \quad R(\phi) \text{ defined by} \quad \frac{1}{4} \sinh^2([|E_W^{(2)}(\phi)| + \frac{E_W^{(4)}(\phi)}{2})R(\phi)] = 2E_W^{(2)}(\phi)/[E_W^{(2)}(\phi)].$

In fig. 4, we plot $H_B[\Phi]$ vs. $\Phi$ for various values of $\phi \in [\pi, 2\pi]$ and for $-2\pi \leq \Phi(0) \leq 2\pi$. We see that, for $\phi \neq \pi$, the minima are separated by a distance $2\pi$. The corresponding instanton trajectories correspond to a “long jump” from $P(-\infty) = -1$ to $P(\infty) = 1$ (represented by a solid arrow in fig. 4). This may be regarded as a sequence of two “short instantons” (dashed arrows), separated by a distance $2R(\phi)$ (see fig. 5). As $E_W^{(2)}(\phi)$ becomes smaller (that is, as $\phi$ gets closer to $\pi$), $R(\phi)$ increases, and eventually diverges, as $\phi \to \pi^\pm$.

Though the degeneracy between the minima is broken by the $1/L$-term, a pertinent tuning of the phase difference $\alpha$ allows to restore it. Indeed, for $\phi > \pi$, the degeneracy of the minima at $\Phi_{\pm}(0) = 2\pi n \mp \pi$ is restored by choosing $\alpha = 2\pi n, n \in \mathbb{Z}$, while, for $\phi < \pi$, one has to choose $\alpha = \pi(1 + 2n)$ and, at $\phi = \pi, \alpha = \pi(\frac{1}{2} + 2n)$.

In a BFT approach, long instantons are represented by the boundary vertex operators $e^{\pm i \theta (0, \sigma)}$, with scaling dimension $h_1 = g$, while short instantons by the operators $e^{\pm i \theta (0, \tau)}$, with scaling dimension $h_1 = \frac{1}{2}$. For $1 < g < 4$ and $\phi = \pi$, short instantons are relevant perturbations of the strong coupling (Dirichlet) fixed point. As it happens for $Y$-shaped JJNs, also here a new, time reversal invariant, attractive finite coupling fixed point emerges in the boundary phase diagram \cite{7}, as a result of the twofold degeneracy of the ground state of $C$. At variance, for $1 < g$ and $\phi \neq \pi$, no finite coupling fixed point emerges, since, now, a possible departure from the Dirichlet fixed point should be due to long instantons, which are an irrelevant perturbation. For $\phi = \pi$, the twofold degeneracy of the ground state of $C$ is due to a $Z_2$ symmetry of $V[\Phi_-]$ manifesting its invariance under time reversal $\phi \to 2\pi - \phi$, while for $\phi \neq \pi$, time reversal is not anymore a symmetry of $V[\Phi_-]$, as evidenced in figs. 4, 5. Indeed, for $\phi = \pi + \delta$, two nearest-neighboring minima at the top left panel of fig. 4 are splitted by an amount $\alpha \sin(\frac{\delta}{2})$. As a result, for $1 < g < 4$ and for $\phi = \pi$, the $Z_2$ symmetry protects the robustness of PCP tunneling across $C$.

We used quantum impurities as a resource for inducing local 4e superconductivity in a Josephson network, fabricable with conventional Josephson junctions. The proposed tunneling mechanism, realized for $1 < g < 4$ and for $\phi = \pi$, allows for the emergence of 4e superconductivity, as a result of embedding an impurity in a Josephson network. Our analysis evidences that the emergence of 4e superconductivity is the signature of the presence in the de Josephson current across the device of two distinct —and competing— periodicities, whose relative weight is tuned by the magnetic flux $\phi$ piercing the central region. Since, for a generic value of $\phi$, the double sine-Gordon boundary potential has been normalized so that the Cooper pair charge $2e$ is associated to the higher periodicity, one gets that, when $\phi = \pi$: —the period is halved, the charge carriers should have charge $4e$. Thus, the phenomenon analyzed in this paper is basically different from the mere phase difference renormalization taking place, for instance, in a series array of $N$ equal Josephson junctions where, although the Josephson current is $\propto \sin(\alpha/N)$, only a single harmonics is present.

Associated with 4e superconductivity there is, for $1 < g < 4$, a new finite coupling attractive fixed point, which allows for the possibility of using the JJN as a two-level quantum system with enhanced coherence \cite{7, 24}. Furthermore, PCP tunneling is robust, as a consequence of

Fig. 4: Minima of the boundary potential $H_B$ for different values of $\phi$ and for $\Lambda^{(2)}(\pi)/\Lambda^{(4)}(0) \approx 0.3$ (in arbitrary units). The long (short) instanton trajectories are represented as solid (dashed) arrows. Top left panel: $\phi = \pi$; bottom left panel: $\phi = 1.01\pi$; bottom right panel: $\phi = 1.1\pi$; top right panel: $\phi = 2\pi$.

Fig. 5: Profile of the instanton excitations $P(\tau)$ for different values of $\phi$ and for $J^2/K^2 \approx 0.3, M = 1$. Top left panel: $\phi = \pi$; bottom left panel: $\phi = 1.01\pi$; bottom right panel: $\phi = 1.1\pi$; top right panel: $\phi = 2\pi$. 

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the time reversal invariance, realized only for $\varphi = \pi$. The proposed mechanism exhibits intriguing similarities with the electron bunching phenomenon [25], observed in shot noise measurements [26] on quantum dots in the Kondo regime.

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