The Age of the Universe

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Abstract. In this article I review the main methods for determining the age of the Universe. I describe how to determine the age of the oldest known systems at $z = 0$, the system of galactic globular clusters, using different techniques. I also describe how to date the Universe using the decay of radioactive elements (Cosmochronology). Finally, I focus on how to determine the age of the Universe at different redshifts and specially the age of radio-quiet galaxies at high redshift. I finish by arguing that the most probable age for the Universe is $14 \pm 2$ Gyr.

1. Introduction

One of the most important questions in Cosmology is to know the age of the Universe. Unfortunately, a definitive answer is still to be found.

Once the values of $\Omega_0$ and $H_0$ have been established for our preferred cosmological model, the age of the Universe is unequivocally determined. In the case of an Einstein-de Sitter Universe $t = (2/3)H_0^{-1}$ (see e.g. Padmanabham 1993 for cases where $\Omega \neq 1$ and $\Lambda \neq 0$). On the contrary, if we are able to estimate the age of the Universe by other methods (e.g. dating stellar systems) then we can constrain the cosmological model and determine which kind of Universe we are living in. We can repeat the above argument at different redshifts, in this way we will gain information not only about $H_0$ but also about $\Omega_0$ since, e.g., at $z = 1.5$ the age of the Universe is $t = 1.6h^{-1}$ Gyr if $\Omega_0 = 1$ and $t = 2.6h^{-1}$ Gyr if $\Omega_0 = 0.2$, where $h = H/100\text{km}s^{-1}\text{Mpc}^{-1}$. Therefore, an accurate knowledge of the age of the Universe at different redshifts can tell us what kind of Universe we live in.

Stars are the only clocks in the Universe available to us that are “cosmology” independent. The evolution of stars depends only on the rate at which the different nuclear burnings take place. So a good strategy to measure the age of the Universe is to find the oldest stars at every redshift.

This article is organised as follows: In section 2, I review the stellar evolution of low mass stars. A detailed review on how to estimate globular clusters (GC) ages follows in the next section. Cosmochronology is the subject of the next section. In section 3, I describe how to date high-redshift galaxies. I finish with a summary of the different ways to date the Universe.
2. Stellar evolution of low-mass stars

This section is devoted to reviewing the principles of stellar evolution for stars whose mass is below about 2.0 \(M_\odot\) and above 0.5 \(M_\odot\). These stars are the ones of interest when dating the oldest objects at \(z < 4\) since their lifetimes are between 1 and 20 Gyr. The evolution of a low mass star has the following stages.

- Fragments of the interstellar medium collapse until they reach hydrostatic equilibrium.

- The star now suffers a quasi-static gravitational contraction until the free fall is stopped by hydrogen burning at the center. The so-called main sequence starts and H is transformed into He. This is the longest period in the life of the stars and it lasts until H is exhausted in the core.

- After H has been exhausted at the centre of the star, it continues to be burned in a very thin shell around the ashes of the previous H burning core. The He core contracts and becomes degenerate and the envelope expands. The star goes into the red giant phase.

- As the star moves up in luminosity and the surface cools, mass is lost from the uppermost layers. The degeneracy at the core is removed suddenly when He to C/O burning starts. This is the helium flash and the red giant branch (RGB) evolution is finished at this point.

- After the helium core flash has taken place the star goes into the horizontal branch (HB) where it will burn He into C/O until He is exhausted in the He-core.

- The star now initiates the second ascent to the giant branch, this time with a double shell burning around a degenerate core of C/O. These two shells: H \(\rightarrow\) He, He \(\rightarrow\) C/O, continue to burn until rapid detachment of a considerable fraction of the remaining envelope is produced (this is the Planetary Nebula phase). After this the core contracts until it becomes degenerate yet again and cannot contract anymore (the star becomes a white dwarf with a degenerate C/O core) and the star will follow the white dwarf cooling sequence.

This sequence is well represented when the evolution of a star is plotted in the plane \(T_{\text{eff}}\) vs Luminosity \((L)\). In Fig. 1 this evolution is shown for a set of stellar masses in the range 1.0 to 0.55 \(M_\odot\). The most remarkable fact is that the location of the main sequence turn off (MSTO) changes with mass (i.e. with age). Therefore it should be possible in principle to measure the age of a coeval stellar population if its MSTO is well defined. In the figures we show the evolution from the zero age main sequence (ZAMS) up to the red giant branch tip (RGT). All tracks have been computed using the latest version of JMSTAR (Jimenez & MacDonald 1997). A very comprehensive and detailed review of stellar evolution is given by, e.g. Kippenhahn & Weigert (1990) and Hansen & Kawaler (1994).
Figure 1. The effect of different choices of the total mass on the evolutionary tracks is shown for metallicity $Z = 0.0002$, helium content $Y = 0.24$ and mixing length $\alpha = 1.5$. The turnoff is markedly affected and its position depends on the mass of the star, and therefore on its age.

The important point to notice here is that stars are a natural clock in the Universe and that they experience noticeable changes during their life-time and therefore provide an independent method to compute the age of the Universe.

3. Globular Cluster Ages

We have to focus our attempts to compute stellar ages in suitable stellar populations (i.e. suitable clocks). By suitable we mean the following:

- All the stars must be born at the same time.
- The population must be chemically homogeneous.
- There must have been no further episodes of star formation that gave birth to new stars which could cover up the oldest population.

In the Milky Way there is only one set of such systems: the Globular Clusters (GC).

GCs are the first places where stars are born in a galaxy (also the bulk of the halo stars). There are several mechanisms proposed to explain the origin of GCs (Fall & Rees 1985, Harris & Pudritz 1994, Padoan, Jimenez & Jones 1997) but all agree that their origin was primordial. Whatever their origin, the important feature of GCs is that they fulfill the above requirements about a suitable clock.

There is strong observational evidence that the oldest GCs are chemically homogeneous. One of the strongest arguments for this is the thin red giant
Evolutionary tracks are shown for three values of the metallicity ($Z = 0.001, 0.0005$ and $0.0002$) for the same value of the mass ($0.8 \ M_\odot$) and helium content ($Y = 0.24$). The effect of the metallicity is to shift the horizontal position of the track. The higher the metallicity, the higher the opacity and therefore the cooler the atmosphere of the star.

Support for GCs being old comes from two facts: their metallicity is as low as $1/100$ of solar and the characteristics of their CMD are those corresponding to ages larger than $10$ Gyr.

3.1. The Isochrone fitting method

The first (and more obvious method) to compute the age of a GC is to exploit the fact that the locus of the MSTO in the plane $T_{\text{eff}}$ vs. $L$ changes with age (mass). In this way one computes different isochrones, i.e. tracks in the plane $T_{\text{eff}}$ vs. $L$ at the same time for all masses, with the chemical composition of the GC and finds the better fit to the MSTO region. To do this a very important step is needed: the distance to the GC is needed in order to transform the theoretical luminosity into observed magnitudes in different bands, and thus is where trouble starts. If the distance to the GC is unknown, there is a degeneracy between age and distance. In this way, we can simulate a different age by simply getting the GC closer or more distant to us.
Problem [1] Use any set of published isochrones (e.g. the Yale isochrones) to demonstrate the previous statement. Also give the error that propagates in the age determination if the distance modulus is not known better than 0.1 mag.

Distances to GCs are very poorly known since it is impossible to get the parallax of individual stars and therefore ages of GCs are not accurately known using the isochrone fitting method. Usually, there are different methods to compute distances to GCs: the RR-Lyrae method; the subdwarf fitting method; the tip of the red giant branch; and the luminosity function. The RR-Lyrae method consists in using the known Period-Luminosity relation for the RR-Lyrae pulsators in the HB, it gives an uncertainty of 0.25 mag in the distance modulus determination, which translates to 3 Gyr error in the age determination. The subdwarf method uses the nearby low metal subdwarfs to calibrate the distance of GCs; again its uncertainty is about 0.2 mag. The tip of the RGB method uses the fact that stars at the tip of the RGB flash at a well defined luminosity (Jimenez et al. 1996); therefore the tip of the RGB is well defined and can be used as a distance indicator. The luminosity function method is explained later on. It gives more precise distance determination, and the error in the distance is only 0.05 mag.

In order to circumvent the need for the distance determinations in computing the age, Iben & Renzini (1984) proposed an alternative method for deriving ages using the MSTO, the so-called $\Delta V$ method. The method exploits the fact that the luminosity of the MSTO changes with mass (age) and not only its $T_{\text{eff}}$ (see Fig. 1), and also that the luminosity of the (HB) does not change since the core mass of the He nucleus is the same independently of the total mass of the stars (provided we are in the low mass range, see section 1), and the luminosity in the HB is provided by the He core burning. Since the method is based on a relative measure (the distance between the HB and the MSTO), it is distance independent. Of course, the method needs the knowledge of at least one GC distance in order to be zero calibrated. Unfortunately, the method has a serious disadvantage: the need to know accurately the location of the MSTO point. This turns out to be fatal for the method since it has associated an error of 3 Gyr in the age determination.

Furthermore, all the above methods are affected by three main diseases: the calibration colour-$T_{\text{eff}}$; the calibration of the mixing-length parameter; and the need to fit morphological features in the CMD (i.e. the MSTO). See Table 1 for a detailed review of all errors involved in the different methods.

The most common ages obtained for the oldest GCs using the MSTO method are in the range 14-16 Gyr. Nevertheless, an error bar of 3.5 Gyr is associated with all age determinations using the MSTO methods described above.

3.2. The Horizontal Branch Morphology Method

A method that is independent of the distance modulus can be developed using the fact that the spread of stars along the HB is mainly due to previous mass loss which varies stochastically from one star to another. The range of colours in which zero age horizontal branch stars (ZAHB) are found is a function of metallicity (the ‘first parameter’) and of the range of ZAHB masses. More precisely, the ZAHB colour at a given metallicity depends on both the total
mass of the star and the ratio of core mass to total mass, but the core mass is essentially fixed by the physics of the He flash and is quite insensitive to the mass and metallicity. For a given average mass loss, the average final mass is thus a decreasing function of age, which is therefore a popular candidate for the ‘second parameter’ (Searle & Zinn 1978), although other candidates such as CNO abundance have also been suggested. A strong case for age as the chief (though perhaps not necessarily the only) second parameter has been made by Lee, Demarque & Zinn (1994), who find a tendency for the clusters to be younger in the outer Galactic halo. Jimenez et al. (1996) used analytical fits to a variety of RGB models and followed evolution along the RGB with mass loss treated by the Reimers (1975) formula. They showed that, for clusters with narrow RGBs (the majority), star-to-star variations in initial mass, metallicity, mixing-length parameter or delayed He core flash at the RGT can be ruled out as a source of the spread along the HB. This leaves variations in the Reimers efficiency parameter $\eta$ as the only likely alternative. Jimenez et al. (1996) demonstrated that, in fact, the origin of the spread of the HB is star-to-star variations in $\eta$ (or some equivalent parameter).

It is therefore meaningful to proceed to an analysis of both the RGT and the HB and to link them together to deduce general properties from morphological arguments.

The procedure that we use to analyse the morphology of the RGB and the HB together and constrain the mass of the stars at the RGB is as follows.

- The mass on the upper part of the RGB is determined from the average mass of the HB, the average mass-loss efficiency and its dispersion. Cal-
calculating the average mass and then the 2σ value of the distribution will give the range of masses along the HB.

- Since the vertical position of the RGB depends only on metallicity and α, once the metallicity is known α is the only free parameter. Therefore, we can find a fit for the best value of α, using the vertical position of the RGB.

- Now we have all the necessary parameters to model the RGB and the HB. With these data we can calculate a track and give the age at the RGT, and therefore the age of the GC itself.

In Jimenez et al. (1996) we analysed eight GCs using the above method and found that the oldest GCs were not older than 14 Gyr.

### 3.3. The Luminosity Function Method

When the MSTO method is used (both isochrone fitting and ∆V), a different distance modulus can be mimicked with a different mass for the MSTO, and therefore with a different age. This age-distance degeneracy in general leads to an uncertainty in the age of 3 Gyr. On top of this, other uncertainties in the stellar physics lead to an additional uncertainty in the age of about 2 Gyr.

In order to tackle this problem, an alternative method that is independent of the distance modulus has been proposed by Jimenez et al. (1996). Here I review the use of the stellar luminosity function (LF) of GCs, in order to break the age-distance degeneracy.

The LF seems to be the most natural observable to try to constrain both age (Paczynski 1984, Ratcliff 1987) and distance modulus at the same time. The LF is a natural clock because the number of stars in a given luminosity bin decreases with time, since more massive stars evolve more rapidly than less massive ones. The fact that small differences in stellar masses corresponds to large differences in evolutionary time explains the power of the LF clock, rather than being a source of uncertainty in getting GC ages (as it is in the MSTO method).

The LF is also a natural distance indicator, because the number of stars in a given luminosity bin depends on the position of the bin.

Stars of different mass evolve along the main sequence at different speed: the more massive the faster. This means that the number of stars inside a fixed luminosity bin decreases with time. This effect is particularly strong around the subgiant region, so that the whole shape of the LF is changing with time, and not only its normalization. In other words the ratio between the number of stars in two different bins in the LF can be used as a clock for GC ages (Jimenez & Padoan 1996).

For a determination of both distance and age, one needs to get from the LF at least two independent constraints, which means three bins in the LF, since one is required for the normalization. A forth bin is also very useful in order to check for the completeness of the stellar counts (see Fig. 4).

Therefore the LF method for determining age and distance of GCs consists in the production of 4-bin theoretical LFs for GCs, to be compared with their
Figure 4. The figure shows the bins used in the LF method (see text) to determine age and distance of a GC observational counterparts. This comparison is made for a given chemical composition, that is assumed to be known from other methods, like spectroscopy.

The number of bins should not be larger than necessary (four), since each bin should be as wide as possible, in order to reduce the statistical errors in the stellar counts, due to uncertainty in the photometry and to the stochastic nature of the stellar mass function. The bins we use are all 1 mag wide, apart from the first one, at the RGB, that is used for the normalization, and may be extended as luminous as possible along the RGB.

The second bin, that is the main constraint for the distance modulus, is positioned between the RGB and the SGB (sub-giant branch), in order to partially contain the steepest section of the LF (this gives the sensitivity to a translation in magnitude.). The third bin, that is the main constraint for the age, contains the SGB, because this is the part of the LF that is most sensitive to age. The fourth bin is just next to the third one, and will typically include the upper part of the main sequence.

The procedure to obtain the LF from evolutionary stellar tracks is illustrated in Jimenez and Padoan (1996). A power law stellar mass function is assumed here, as in that work.

We have shown that a careful binning of the stellar LF allows a very precise determination of age and distance of GCs, at the same time.

If stellar counts with 5% 1σ uncertainties in 1 mag wide bins are available, the age can be determined with an uncertainty of 0.5 Gyr, and the distance modulus with an uncertainty of 0.06 mag.

This LF method is therefore an excellent clock for relative ages of GCs, and also a very good distance indicator. In other words, its application will provide very strong constraints for the theory of the formation of the Galaxy.
In Fig. 5 we show the result of applying the LF method to the metal poor galactic globular cluster M55. The plot shows contour plots for the error in the determination of the distance modulus and age of M55 simultaneously. The contour plots correspond to different values for the uncertainty in the number of stars in the luminosity function. As stated above, if stellar counts are within an uncertainty of 5% then the age is determined with an uncertainty of 0.5 Gyr, and the distance modulus with an uncertainty of 0.06 mag.

The age obtained for M55 (12 Gyr) confirms the conclusion of the HB morphology method that GCs are not older than 14 Gyr.

Table 1. The table shows the errors associated with the different methods described in the text to compute the age of the oldest globular clusters. The first column lists the main uncertainties when computing GCs ages.
4. Cosmochronology

Cosmochronology uses the radioactive decay of nuclear species to date the age of the elements and therefore the life of stellar populations. The key point is the use of nuclear species that are only produced through the r-process, and therefore produced only in massive stars that live for around $10^8$ Gyr. The idea is to look at very metal-poor stars and find in their spectrum elements that are only produced in the r-process. Then measuring the radioactive decay of these elements, it is possible to find the age of these very metal-poor stars on the assumption that the radioactive elements were produced by massive (short-lived) stars in the very early evolution of the Galaxy. In this way the r-process is entirely responsible for the synthesis of $^{187}$Re, $^{232}$Th, $^{235}$U, $^{238}$U and $^{244}$Pu. Knowing the short-lived progenitors in alpha decay chains of the above isotopes, one can compute the production rates of the pairs $^{232}$Th/$^{238}$U and $^{235}$U/$^{238}$U. Using these chronometers it is possible to get the age of the Galaxy in the following way:

- Compute production ratios for the entirely r-process long lived isotopes.
- Get meteoritic abundances ratios of the same isotopes to obtain the duration of nucleosynthesis since the formation of the Galaxy until the formation of the solar system.
- Use a model for the chemical evolution of the Galaxy.

Then a variety of predictions are drawn due to the different models used in the previous three points. We will simply note here that the latest investigations in this subject (Truran, private communication) give values between 11 and 14 Gyr for the age of the oldest stars in the galaxy. Nevertheless, it should be noted that Cosmochronology has an entirely different type of model dependence as compared with the uncertainties in GC ages; the results thus are an independent check on the ages of the GCs. This therefore adds to the evidence that the most likely age of the Universe is around $14 \pm 2$ Gyr.

5. Ages of High Redshift Galaxies

In the previous sections we have described two independent methods to compute the age of the universe at $z = 0$. Usually the literature refers to the age of the Universe as the age at $z = 0$, nevertheless it is possible to find at higher redshifts (due to the new 10m class telescopes and the Hubble Space Telescope) suitable objects where it is possible to measure the age of their stars in a reliable and accurate way. As we have pointed out previously, the age of the Universe at different redshifts give us an independent measure of $\Omega$. The first point to answer is: are GCs good “clocks” at high redshift?

Unfortunately not. The reason is obvious; we cannot resolve individual stars at high redshift. Therefore the strategy is to find analogues to GCs at high redshift bearing in mind that we will have only the integrated light of the high redshift population.
The disks of spirals galaxies are very bad suitable clocks since we know that stars are born continuously (it would make no sense trying to ask what is the age of a city by measuring the age of its citizens). Much better candidates are elliptical galaxies. Their stars are believe to form in a first initial burst, after which there is no further significant amount gas left to be processed into new born stars. Once the stars in a elliptical are born, they evolve passively, aging and becoming redder. Therefore, ellipticals that have no significant activity from active galactic nuclei are the best suitable clocks for measuring the age of the Universe at high redshifts.

In this section I describe a method to find ellipticals with no significant AGN contribution at high redshift and how to date its population from their integrated spectrum.

Finding distant galaxies and analyzing their starlight remains one of the only direct methods of studying the formation and evolution of galaxies. Recent studies with the Keck and HST have provided strong evidence that the majority of star formation took place at relatively recent redshifts \((z \sim 1)\) (Lilly et al. 1995). However this does not mean that all galaxies formed at that redshift, and in fact it seems clear that many galaxies were formed much earlier at a higher redshift (Driver et al. 1995).

In particular, it is the reddest galaxies at high redshifts which provide the best constraints on the earliest epochs of galaxy formation and evolution, since their colour is most likely due to an evolved stellar population. In a recent publication (Dunlop et al. 1996) we have reported the discovery of the oldest known galaxy at \(z = 1.55\) (53W091).

Once the spectrum of the galaxy has been obtained it is necessary to build synthetic stellar populations models in order to compute the age of the stellar system.

The necessary steps to compute a synthetic integrated spectrum are the following:

- The first step is to specify the initial mass function and the star formation rate of the population that we want to synthesize. Usually a Miller-Scalo initial mass function (IMF) (Miller & Scalo 1979) is used (even though the use of a physical IMF is more suitable (see Padoan et al. 1997)). The star formation rate is different depending on the population. For ellipticals, (and GCs) since there is no further episodes of star formation apart from the initial burst, an instantaneous burst is a quite good approximation.

- Select a library of stellar interiors that cover a large range in masses (from \(0.1 \, M_\odot\) to \(120 \, M_\odot\)).

- Create an isochrone using the previous IMF and star formation rate (SFR).

- Using a library of theoretical stellar atmospheres (or observed stellar spectra) associate them which every point in the isochrone in order to add them all and produce a synthetic spectrum of the population.

A more better and accurate way is to join the last two steps and use self consistent stellar evolution models. These models compute the interior and the photosphere in one step and therefore for every point in the track the proper
atmospheric model is known. Unfortunately, they are not so easy to compute (see Jimenez et al. 1997 for a detailed description of these models).

In Fig. 6 we have plotted the spectrum of 53W091 and three different synthetic stellar population models. The effect with age on the spectral features is quite well known and the best fit is obtained for an age of 3.5 Gyr.

How robust is this age estimation? We were very careful to choose a “suitable” clock at that redshift in order to make a meaningful age determination. 53W091 is a “clean” elliptical (i.e. no contamination from any AGN contribution (Dunlop & Peacock 1993) and no new star formation episodes), so the only evolution is due to aging stars. Therefore, it is only a question of having accurate stellar models (interior and atmospheric) in order to make accurate predictions for the integrated spectrum. Unfortunately, the evolution of post-main-sequence stages is not so well understood, and many of the differences between different synthetic population models come from their different treatment of post-main-sequence stages.

Nevertheless, there are some other features in the spectrum that can help us to circumvent these problems and make a robust prediction for the age of 53W091.

**Problem [2]** Use the breaks around 2600 Å and 2900 Å in the spectrum of 53W091 and compute the most probable age of this galaxy. To do so build a set of synthetic stellar spectra as described in the text. The main advantage of the breaks is that they are reddening independent.

What are the implications for cosmology of a galaxy aged 3.5 Gyr at $z = 1.55$? We need to know the mass of 53W091 in order to make our cosmological implications much stronger.

In order to compute the mass of an object knowing its apparent magnitude we just have to integrate the IMF and scale it until we match the observed apparent magnitude. For the case of 53W091 the mass is about $10^{12} \, M_\odot$ (see Kashlinsky & Jimenez 1997).

We assume in what follows that the data on 53W091 imply that the galaxy at $z=1.55$ has mass in excess of $10^{12} M_\odot$ and its stellar population has an age of $\approx 3.5$ Gyr. What then are the cosmological implications of at least one object in the Universe having collapsed (formed galaxy) on mass scale of $> 10^{12} M_\odot$ at least 3.5 Gyr before the redshift of 1.55?

The left box in Fig. 7 shows the redshift $z_{gal}$ at which the galaxy 53W091 must have formed its first stars for $\Omega+\Lambda=1$ Universe. Solid lines correspond to $t_{age} = 3$Gyr, dotted to 3.5 Gyr and dashed to 4 Gyr. Three types of each line correspond to $H_0=60$, 80 and 100 km s$^{-1}$ Mpc$^{-1}$. One can see that the value of $z_{gal}$ decreases as both $\Omega_0$ and $h_0$ decrease. On the other hand, in the low-$\Omega_0$ CDM cosmogonies the small-scale power is also reduced as the product $\Omega h$ decreases; this would at the same time delay collapse of first galaxies until progressively smaller $z$.

To quantify this we proceed in the manner outlined in Kashlinsky (1993). This involves the following steps (see Kashlinsky & Jimenez 1997 for details):

1) Specify the primordial power spectrum, $P(k)$, of the density field at some initial redshift $z_i \gg 1$ when the density field is linear on all scales. The power spectrum depends on the initial power spectrum, assumed to be Harrison-Zeldovich, and the transfer function which accounts for the evo-
Figure 6. The spectrum of the high redshift galaxy 53W091 is plotted with several synthetic stellar population models for different ages. 53W091 is the oldest known galaxy at a redshift of $z = 1.55$. In order to determine its age we have built several synthetic stellar population models at different ages. The best fit is found for an age of 3.5 Gyr, the highest model corresponds to an age of 2 Gyr and it is too blue to fit the observed spectrum of 53W091. The lowest model corresponds to an age of 5 Gyr and the stellar population is too red (old stars) to fit the observed spectrum.
Figure 7. In the left panel the redshift, $z_{gal}$, when star formation in 53W091 was completed is plotted vs $\Omega_0$ for $\Omega_0 + \Lambda = 1$ Universe. Solid lines correspond to $t_{age}=3$, dotted to 3.5 and dashes to 4Gyr. Three lines of each type correspond to $h = 0.6, 0.8, 1$ from bottom up. The middle panel shows $\zeta$, the number of standard deviations of the primordial density field 53W091 should be in the flat $\Lambda$-dominated CDM models is plotted vs $\Omega_0$. $\zeta$ scales $\propto b^{-2/3}$ and is plotted for $b$ normalized to the second year COBE/DMR maps. Same notation as in previous panel. Finally, in the right panel the predicted number density, $n(> M)$, of galaxies like 53W091 with the redshift of formation plotted in the left panel is plotted vs their total mass in units of $10^{10} M_\odot$. Solid lines correspond to $\Omega_0 = 0.1$: they are for $h_0 = 1, 0.8, 0.6$ from top to bottom respectively. Dotted lines correspond to $\Omega_0 = 0.2$ and $h = 0.8, 0.6$ from bottom to top; the line for $h_0 = 1$ lies below the box. Dashes correspond to $\Omega_0 = 0.3$ and $h_0 = 0.6$. 
ution of the shape of the power spectrum in the linear regime. The latter
depends in such models only on the product $\Omega h$ and was adopted from
Bardeen et al. (1986).

2) Compute the amplitude, $\Delta_8$, of that field at $z_i$ on the scale of $8h^{-1}\text{Mpc}$
that produces the observed unity rms fluctuation in galaxy counts today,
or a $1/b$ amplitude in mass fluctuation ($b$ is the bias factor) at $z=0$.

3) Compute the fractional density contrast, $\delta_{col}(z)$, the fluctuation had to
have at $z_i$ in order to collapse at $z$.

4) A convenient quantity to describe 2) and 3) is $Q(z) \equiv \delta_{col}(z)/\Delta_8$. For
$\Lambda$-dominated flat Universe and in the limit of $1 + z_{\text{gal}} > \Omega^{-1/3}$ it can be
approximated as $Q(z) \simeq 3\Omega_0^{0.225}b^{2/3}(1 + z)$.

5) $b$ is determined by normalizing the density distribution given by $P(k)$ to
the COBE-DMR maps (Bennett et al. 1994; Stompor et al. 1995).

6) Given $P(k)$ we compute the rms fluctuation, $\Delta(M)$, over a region containing
mass $M$.

7) The quantity $\zeta \equiv Q(z_{\text{gal}})\Delta_8/\Delta(M)$ then describes the number of standard
deviations an object of mass $M$ had to be in order to collapse at $z_{\text{gal}}$
in the cosmological model specified by $P(k)$.

The values of $\zeta$ for $M=10^{12}M_\odot$ are plotted versus $\Omega_0$ in the middle box of
Fig. 7 for $\Omega_0+\Lambda=1$ for various values of $t_{\text{age}}$ and $h$. As in the left box solid lines
correspond to $t_{\text{age}}=3$ Gyr, dotted to 3.5 Gyr and dashes to 4 Gyr. Three types
of each line correspond to $h=0.6, 0.8$ and 1 going from bottom to top at large
values of $\Omega$ at the right end of the graph. The line for $t_{\text{age}}=4$ Gyr and $h=1$ lies
above the box. As the figure shows, this galaxy must represent an extremely
rare fluctuation in the density field specified by the low-$\Omega$ flat CDM models.
Note that the total mass of 53W091 must be at least a factor of 10 larger in
which case our conclusion will be much stronger.

The right box shows the expected co-moving number density of such galax-
ies, $n(> M)$, in units of $(h^{-1}\text{Gpc})^{-3}$ vs the total (dark+luminous) mass for
$t_{\text{age}}=3.5$ Gyr. It was computed using the Press-Schechter prescription. The
numbers for the co-moving number density $n(> M)$ were computed for $\Omega_0=0.1, 0.2, 0.3$ and $h=1, 0.8, 0.6$. Solid lines correspond to $\Omega_0=0.1$ and to $h=1, 0.8$
from top to bottom. Dotted lines correspond to $\Omega_0 = 0.2$ and $h=1, 0.8, 0.6$
from top to bottom respectively. The dashed line corresponds to $\Omega_0 = 0.3$ and
$h_0=0.6$. The models not shown lie below the box. One can conclude from Fig. 7
that within the framework of the $\Lambda$ cold dark matter models this object must
be extremely rare in the Universe. There exists a narrow range of parameters
(total mass of $10^{12}M_\odot$, age less than 3 Gyr, $\Omega = 0.1$ and $h \geq 0.8$) for which
one expects to find a few such objects with each horizon, but for most cases the
number density of such objects is less than one per horizon volume.
6. Summary

In this article we have focused our attention on how to calculate the age of the Universe at different redshifts and its cosmological implications. The main conclusions are the following:

- There are well defined stellar clocks in the Universe at different redshifts that allow us to obtain independent measures of the age of the Universe.

- Galactic Globular Clusters are the best cosmological clocks at \( z = 0 \). Several methods have been used to compute their ages. The Luminosity Function is the most accurate method to compute the age and distance of Galactic Globular Clusters. The *age of the oldest globular clusters is \( 14 \pm 2 \) Gyr.*

- Cosmochronology gives a most probable age for the oldest stars of the Galaxy between 11 and 14 Gyr.

- At high redshift quiet radio-galaxies are the best cosmological clocks to probe the early evolution of the Universe. I have described the case of 53W091 and give detailed description on how to compute its age. This galaxy (the oldest at \( z = 1.55 \)) was found to be 3.5 Gyr old. It represents a very rare event in modified cold dark matter models.

Based on the previous arguments, we can say that the Age of the Universe is between 11 and 16 Gyr, with a most probable age of 14 Gyr. This constrains the allowed values for \( \Omega \). If \( H_0 = 65 \) km s\(^{-1}\) Mpc\(^{-1}\) (see this volume) then \( \Omega \lesssim 0.2 \), otherwise \( \Lambda \neq 0 \). In the case \( \Omega = 1 \) then \( H_0 \approx 0.5 \) km s\(^{-1}\) Mpc\(^{-1}\).

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