New optical soliton solutions for the variable coefficients nonlinear Schrödinger equation

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Abstract
This paper is devoted to seek new optical soliton solutions of nonlinear Schrödinger equation (NLSE) with time-dependent coefficients which describes the dispersion decreasing fiber. To achieve optical soliton solutions of NLSE, the basic idea of homogenous balance approach has been used to propose Bernoulli \((G'/G)\)-expansion method, where \(G = G(\zeta)\) satisfies Bernoulli equation, which is easier to solve than previous studies. By applying some transformations and using this method, some periodic wave, bright and dark soliton solutions are successfully obtained. Moreover, 3D surfaces, standard deviation line plots and contour maps graphs of the obtained results under effect of different values of coefficients are illustrated to have acceptable image of dynamic structures and to find the relation between the parameters and wave behaviors.

Keywords Optical solitons · Bernoulli \((G'/G)\)-expansion method · Nonlinear Schrödinger equation · Variable coefficients · Dynamic structures

1 Introduction

The variable coefficients nonlinear Schrödinger equation (VcNLSE) is one of the main topic in modern physics to pragmatically describe many nonlinear behaviors in nonlinear optical fibers, Bose-Einstein condensation, plasma and water waves (Ponomarenko and Agrawal 2007; He et al. 2009; Yu and Yan 2014; Yan et al. 2015; Yao et al. 2016; Kaur and Wazwaz 2019). Several studies have been done to peruse on solutions of VcNLSE; for example, Pérez-García et al. (2006) used similarity transformations connecting NLSE involved time-varying coefficients with the autonomous cubic NLSE. He et al. (2014) studied rogue wave solutions in NLSE with variable coefficients. Kedziora et al. (2015) by using Lax pair and Darboux transformation formalisms could find the solutions
of VcNLSE. Liu et al. (2019) achieved the distinguished types of nonautonomous complex wave solutions, which include bright and dark soliton solutions. Guo and Liu (2020) by analyzing the dynamical properties of the related Hamiltonian obtained some new dark solitons, bubble solutions and periodic solutions. El-Shiekh (2019) modified direct similarity reduction method to find Jacobi, hyperbolic and periodic wave solutions of VcNLSE.

In the past few years studying on different types of VcNLSE with coefficients which depend on the evolution variable has been of great interest to scientists. For instance, stabilized solitons appear in multidimensional VcNLSE equations when the coefficient is controlled appropriately, that exists in optical applications (Bergé et al. 2000), according to these projects, it was realized that there is a similar phenomenon arisen in the context of mean field models of Bose-Einstein condensation (Abdullaev et al. 2003; Saito and Ueda 2003; Montesinos 2004). The studies of the nonlinear waveforms for the nonlinear Schrödinger equations with external potentials are of great significance the rapid development of the Bose-Einstein condensates (Wazwaz 2009; Bao and Cai 2012; Feng 2016; Feng and Zhang 2018). For the nonlinear Schrödinger equations with external potential, Zhang (2000) studied the existence of the condensations in a critical value. Bao and Cai (2012) accomplished numerical studies on the nonlinear Schrödinger equations with singular potential modelling the dipolar Bose-Einstein condensation for ground states.

To study the nonlinear evolution equation (NLEE), a lots of powerful methods are developed such as Hirota bilinear method (Hossen et al. 2018; Ullah et al. 2021; Özkan et al. 2020; Rizvi et al. 2020), exp-function method (Roshid et al. 2014b; Hosseini et al. 2020), double-subequation method (Khatun et al. 2017), complex method (Gu et al. 2018; Gu and Aminakbari 2020; Gu et al. 2020), sine-Gordon expansion method (Bulut et al. 2018), exp(-φ(ξ))-expansion method (Roshid and Rahman 2014), improved Kudryashov method (Khan and Akbar 2016; Rahman et al. 2021), enhanced modified simple equation method (Roshid et al. 2020), extended modified auxiliary equation mapping method (Seadawy et al. 2019b; Seadawy and Cheemaa 2019c, b, a), generalized direct algebraic method (Seadawy et al. 2019; Younas et al. 2020) and generalized exponential rational function method (Younas et al. 2021) and so on.

In this research, we explain the optical soliton solutions of the following form VcNLSE with time-dependent coefficients (Serkin and Hasegawa 2000; Kruglov et al. 2005; Zeng et al. 2016) by proposing Bernoulli \((G'/G)\)-expansion method:

\[
i\psi_t = \frac{1}{2} \beta(t) \psi_{xx} + i \gamma(t) \psi - g(t)|\psi|^2 \psi,
\]

where \(\psi(x, t)\) is a complex wave function of \(x\) and \(t\), the dispersion parameter \(\beta(t)\), the nonlinearity parameter \(g(t)\) and gain/loss parameter \(\gamma(t)\) \((> 0, < 0)\) are real functions of time. One of physical significance of Eq. (1) is that it is used as a model for the Bose-Einstein condensate dynamics, which is considered by the mean field approximation when the nonlinear coefficient \(g(t)\) is controlled by applying Feshbach resonances. The proposed method of this paper is inspired by the \((G'/G)\)-expansion and the generalized \((G'/G)\)-expansion methods (Wang et al. 2008; Zhang et al. 2008), which is based on the ideas that traveling wave solutions of NLEE can be explained by a polynomial in \((G'/G)\), and \(G = G(\xi)\) satisfies a second order linear ordinary differential equation with constant coefficients, while in Bernoulli \((G'/G)\)-expansion method, we express \(G = G(\xi)\) as a Bernoulli equation with variable coefficients. Although many useful methods are developed to investigate NLEEs, our method enrich the studies of this area and the results show that the proposed method is effectual to search exact solutions of many other nonlinear differential equations with variable coefficients.
This paper is structured as: In Sect. 2, the Bernoulli \((G'/G)\)-expansion method is described. In Sect. 3, we give the application of this method by applying to Eq. (1) in two different conditions. In Sect. 3 dynamic behaviors of the begotten results are illustrated. Some conclusion and explanation are expressed at the end.

2 Description of the Bernoulli \((G'/G)\)-expansion method

A NLEE with dependent and independent variables, respectively \(u\) and \(X = (x, t)\), is given by

\[
F(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0.
\]  

(2)

By using transformation, Eq. (2) can be reduced to an ordinary differential equation (ODE), which the solutions can be expressed by

\[
u = \sum_{i=1}^{n} a_i(X) \left( \frac{G'}{G} \right)^i + a_0(X), \quad a_n(X) \neq 0,
\]  

(3)

where \(a_0(X), a_i(X) (i = 1, 2, \ldots, n)\) should be determined, and \(\xi = \xi(X)\) which in this paper we consider it as \(\xi = b(x - c t)\), where \(b\) and \(c\) are constants. Function \(G = G(\xi)\) satisfies the following Bernoulli equation:

\[
G' + p(\xi)\ G = q(\xi)\ G^2,
\]  

(4)

where \(p(\xi)\) and \(q(\xi)\) should be specified.

By balancing the highest order nonlinear term and the highest order derivative of \(u\) in ODE, integer \(n\) will be achieved. Substituting (3) along with Eq. (4) into ODE and collecting the same order of \(G\) together will get a set of algebra equations of variable coefficients that can be determined by use of Maple, while general solutions of \((G'/G)\) are given as

\[
\left( \frac{G'}{G} \right) = - \frac{\left( \cosh \left( \int p(\xi)\ d\xi \right) - \sinh \left( \int p(\xi)\ d\xi \right) \right) q(\xi)}{\int \left( \cosh \left( \int p(\xi)\ d\xi \right) - \sinh \left( \int p(\xi)\ d\xi \right) \right) q(x)\ d\xi - C_1} - p(\xi),
\]  

(5)

where \(C_1\) is an arbitrary constant. Substituting these results into (3), we have exact solutions of Eq. (1).

3 Applications

The first step to find exact solutions of Eq. (1) is done by applying the following transformation

\[
\psi(x, t) = \phi(\xi) e^{ikx-i\mu t},
\]  

(6)

where \(\phi\) is amplitude function, \(k\) and \(\mu\) are phase constants. By substituting (6) into (1), for real and imaginary parts respectively, we have
\[ \mu \phi - \frac{1}{2} k^2 \beta(t) \phi + \frac{1}{2} \beta(t) \phi_{xx} + g(t) \phi^3 = 0, \quad (7) \]

\[ \phi_t + k \beta(t) \phi_x - \gamma(t) \phi = 0. \quad (8) \]

Balancing between \( \phi_{xx} \) and \( \phi^3 \) in (7), can get \( n = 1 \) for (3), which leads to \( \phi \) as follows:

\[ \phi = a_0(t) + a_1(t) \left( \frac{G'}{G} \right), \quad a_1(t) \neq 0, \quad (9) \]

where \( G = G(\zeta) \) satisfies Eq. (4). In this research, we have studied on two different conditions to derive exact solutions of Eq. (1):

**First condition:** Substituting (10) into (7) and (9), collecting coefficients with the same order of \( G' \), \( j = 0, 1, 2, 3 \) to zero, yields a system of differential equations as follows:

\[
\begin{align*}
3 \ g(t) a_0(t) a_1^2(t) p^3(\zeta) - a_1^3(t) p^3(\zeta) g(t) - 3 \ g(t) a_0^2(t) a_1(t) p(\zeta) + 0.5 \ a_1(t) p(\zeta) \beta(t) k^2 \\
- 0.5 \ \beta(t) a_1(t) p''(\zeta) b^2 + a_0(t) g(t) - 0.5 a_0(t) \beta(t) k^2 - a_1(t) p(\zeta) \mu + a_0(t) \mu = 0,
\end{align*}
\]

\[
\begin{align*}
a_1(t) q(\zeta) \beta(t) p^2(\zeta) b^2 - a_1(t) q(\zeta) \beta(t) k^2 - a_1(t) q(\zeta) \beta(t) p'(\zeta) b^2 - 2 \ a_1(t) \beta(t) p(\zeta) q'(\zeta) b^2 \\
+ 6 a_1^3(t) p^2(\zeta) q(\zeta) g(t) - 12 a_1^2(t) p(\zeta) q(\zeta) a_0(t) g(t) + 6 a_1(t) q(\zeta) a_0^2(t) g(t) \\
+ 2 a_1(t) q(\zeta) \mu + a_1(t) \beta(t) q(\zeta) b^2 = 0,
\end{align*}
\]

\[
\begin{align*}
3 q'(\zeta) a_1(t) q(\zeta) \beta(t) b^2 - 3 p(\zeta) a_1(t) q^2(\zeta) \beta(t) b^2 - 6 p(\zeta) g(t) a_0^3(t) q^2(\zeta) \\
+ 6 a_0(t) g(t) a_1^2(t) q^2(\zeta) = 0,
\end{align*}
\]

\[
\begin{align*}
g(t) a_1^2(t) q^2(\zeta) + \beta(t) a_1(t) q^2(\zeta) b^2 = 0,
\end{align*}
\]

\[
\begin{align*}
a_1(t) \beta(t) k > - \beta(t) a_1(t) p'(\zeta) b k + a_1(t) p(\zeta) \gamma(t) - a_0(t) \gamma(t) + a_0'(t) = 0,
\end{align*}
\]

\[
\begin{align*}
q(\zeta) a_1(t) p(\zeta) b c - q(\zeta) a_1(t) p(\zeta) b k + a_1(t) q'(\zeta) \beta(t) b k - a_1(t) q'(\zeta) b c - \gamma(t) q(\zeta) a_1(t) \\
+ a_1'(t) q(\zeta) = 0,
\end{align*}
\]

\[
\begin{align*}
\beta(t) a_1(t) q^2(\zeta) b k - a_1(t) q^2(\zeta) b c = 0,
\end{align*}
\]

which have solutions

\[
\begin{align*}
a_0(t) &= \pm \frac{1}{2} C_2 \left( \pm 2 C_4 c b + \sqrt{-2 c^2 k^2 + 4 c k \mu} \right) \ e^{\int \gamma(t) \ dt}, \\
a_1(t) &= C_2 \ e^{\int \gamma(t) \ dt} = \frac{c}{k}, \\
g(t) &= - \frac{c b^2}{k C_2^2 \ e^{\int \gamma(t) \ dt}}, \quad p(\zeta) = \pm \frac{C_4 c b + \sqrt{-2 c^2 k^2 + 4 c k \mu}}{c b}, \quad q(\zeta) = C_3 \ e^{C_4 \zeta},
\end{align*}
\]

where \( C_2, C_3 \) and \( C_4 \) are arbitrary constants. \( \gamma(t) \) is an arbitrary function. In this study, it is considered as follows

\[ \gamma(t) = \gamma_0 e^{\rho t}, \]

with \( \gamma_0 \) and \( \rho \) as constants.
Substituting the above results into (10) and by use of (6) and (5), we have:

\[
\psi_1(x, t) = \pm \frac{C_3 \left( \cosh \left( \frac{\sqrt{-2c^2k^2+4ck\mu}}{cb} \right) \zeta \right) \pm \sinh \left( \frac{\sqrt{-2c^2k^2+4ck\mu}}{cb} \right) \zeta \right) \pm \frac{\sqrt{-2c^2k^2+4ck\mu}}{cb} C_1}{C_3 \left( \cosh \left( \frac{\sqrt{-2c^2k^2+4ck\mu}}{cb} \right) \zeta \right) \pm \sinh \left( \frac{\sqrt{-2c^2k^2+4ck\mu}}{cb} \right) \zeta \right) + \frac{\sqrt{-2c^2k^2+4ck\mu}}{cb} C_1}
\]
\[
\times \frac{\sqrt{-2c^2k^2+4ck\mu}}{2cb} C_2 e^{\int_{t_0}^{t} e^{j\kappa x - j\mu t} dt}
\]

where \(\zeta\) is defined by \(b(x - ct)\).

**Second condition:** Considering \(b = 1, c = 0\) and \(k = 0\), and substituting (10) into (7) and (9), collecting coefficients with the same order of \(G^j, (j = 0, 1, 2, 3)\) to zero, yields a system of differential equations as follows:

\[
a_0(t) \mu - a_1(t) p(x) \mu - 0.5 \beta(t) a_1(t) p''(x) - a_1^3(t) p^3(x) g(t) + 3 a_1^2(t) p^2(x) a_0(t) g(t)
\]

\[-3 a_1(t) p(x) a_0^2(t) g(t) + a_0^3(t) g(t) = 0,
\]

\[2 a_1(t) q(x) \mu + p^2(x) a_1(t) q(x) \beta(t) - 2 p(x) q'(x) a_1(t) \beta(t) + 6 a_1^3(t) g(t) a_1(t) q(x)
\]

\[+ q''(x) a_1(t) \beta(t) + 6 p^2(x) g(t) a_1^3(t) q(x) - 12 p(x) a_0(t) g(t) a_1^2(t) q(x)
\]

\[+ p'(x) a_1(t) q(x) \beta(t) = 0,
\]

\[3 q'(x) a_1(t) q(x) \beta(t) - 3 p(x) a_1(t) q^2(x) \beta(t) - 6 p(x) g(t) a_1^3(t) q(x)
\]

\[+ 6 a_0(t) g(t) a_1^3(t) q^2(x) = 0,
\]

\[g(t) a_1^3(t) q^3(x) + \beta(t) a_1(t) a_1^2(t) q^3(x) = 0,
\]

\[\gamma(t) a_1(t) p(x) - a_1'(t) p(x) - \gamma(t) a_0(t) + a_0'(t) = 0,
\]

\[a_1'(t) q(x) - \gamma(t) a_1(t) q(x) = 0.
\]

Solving this system of differential equations, gives us two different cases:

**Case 1.**

\[p(x) = p, \quad q(x) = q, \quad a_1(t) = C_2 e^{\int_{t_0}^{t} \gamma(t) dt}, \quad a_0(t) = \frac{1}{2} p a_1(t),
\]

\[\beta(t) = \frac{4 \mu}{p^2}, \quad g(t) = - \frac{4 \mu}{p^2 a_1^2(t)},
\]

where \(p, q, \mu\) and \(C_2\) are arbitrary constants.

Substituting the above results into (10) and by use of (6) and (5), for wave function, we have:

\[
\psi_2(x, t) = -\frac{1}{2} \frac{C_2 p C_1 - (\cosh (p x) - \sinh (p x))q}{p C_1 + (\cosh (p x) - \sinh (p x))q} e^{\int_{t_0}^{t} e^{j\kappa x - j\mu t} dt}.
\]

**Case 2.**
\[ p(x) = \frac{1}{2} \sinh \left( \frac{1}{2} \frac{C_4 + \frac{x}{C_3}}{C_3} \right) + C_5, \quad q(x) = \frac{C_6 e^{C_3} e^{C_2 x}}{\cosh \left( \frac{1}{2} \frac{C_4 + x}{C_3} \right)}, \]
\[ a_1(t) = C_2 e^{\int \gamma(t) \, dt}, \quad a_0(t) = C_5 \, a_1(t), \quad \beta(t) = 4 \mu C_3^2, \quad g(t) = -\frac{4 \mu C_3^2}{a_1(t)}, \]

where \( C_2, C_3, C_4, C_5, C_6 \) and \( \mu \) are arbitrary constants. By setting these results into (10) and by use of (6) and (5), wave function can be obtained as:

\[ \psi_3(x, t) = -\frac{1}{2} \frac{C_2}{C_3} \left( -2 C_3 C_6 e^{C_3} \cosh \left( \frac{1}{2} \frac{C_3+x}{C_3} \right) + C_1 \sinh \left( \frac{1}{2} \frac{C_3+x}{C_3} \right) \right) e^{\int \gamma(t) \, dt} e^{-i\mu t}. \]

(12)

4 Dynamic structures

In this section we will demonstrate the dynamic structures of our results to get better understanding of wave solutions. Figure 1 illustrates the wave propagation of wave function \( \psi_1(x, t) \), by using \( \mu = 1.2, k = 1.1, p = 2, C_1 = -1.1, C_2 = 0.2, C_3 = 1.1, b = 1.5 \)

![Fig. 1](image-url)

**Fig. 1** |\( \psi_1 \)| is solution of Eq. (1) for \( c = -2.5 \) when (a1-a3) \( \gamma_0 = -0.2, \rho = 0.11 \), (b1-b3) \( \gamma_0 = 0, \rho = 0 \), (c1-c3) \( \gamma_0 = 0.2, \rho = 0.11 \)
and $c = -2.5$ with three different values of $\gamma_0$ and $\rho$ as follows: Fig. 1 (a1- a3) describe 3D, line and contour maps images for $\gamma_0 = -0.2$, $\rho = 0.11$. Fig. 1 (b1- b3) describe 3D, line and contour maps images for $\gamma_0 = 0$, $\rho = 0$, which show us the results when $\gamma(t) = 0$. We also represent in Fig. 1 (c1- c3), 3D, line and contour maps images when $\gamma_0 = 0.2$, $\rho = 0.11$. In these figures, we can see periodic behaviors of the wave that is by considering negative or positive values of $\gamma_0$ respectively, the height of wave decreases or increases over time.

Figures 2 and 3 are displayed to show the important role of constant $c$ in dynamic behaviors of wave function $\psi_1(x, t)$, respectively by setting $c = 0.1$ and $c = 2.5$ while other parameters are same as Fig. 1. In Fig. 2 for $c = 0.1$ we have solitary waves that include several peaks, which for different values of $\gamma_0$, they show decreasing or increasing behaviors across the time. In Fig. 3 by setting $c = 2.5$, compared to Fig. 1, we have a different kind of periodic wave plots with milder waves and these waves have diagonal motion across the $x$ and $t$ coordinates.

Figure 4 exposes the solution of $\psi_2(x, t)$ in Eq. (1) by using $\mu = 1.2$, $p = 2$, $q = 1.1$, $C_1 = 0.01$ and $C_2 = 1.41$ with three different values of $\gamma_0$ and $\rho$ as follows: Fig. 4 (a1- a3) describe 3D, line and contour maps graphs for $\gamma_0 = -0.2$, $\rho = 0.11$. Fig. 4 (b1- b3) describe 3D, line and contour maps graphs for $\gamma_0 = 0$, $\rho = 0$, which show us the results when $\gamma(t) = 0$. We also show in Fig. 4 (c1- c3), 3D, line and contour maps graphs by setting $\gamma_0 = 0.2$, $\rho = 0.11$. As we can see, these figures demonstrate dark soliton solutions with different heights in different $\gamma_0$ values.

In Fig. 5, wave function $\psi_3(x, t)$ is displayed, by setting $\mu = 1.2$, $C_1 = 0$, $C_2 = 1.1$, $C_3 = 1.1$, $C_4 = 0.01$, $C_5 = 0.01$ and $C_6 = 1.3$ with three different values of $\gamma_0$ and $\rho$ as follows: Fig. 5 (a1-a3) describe 3D, line and contour maps

\begin{figure}
\centering
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{a1}
\caption{(a1)}
\end{subfigure}
\hfill
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{a2}
\caption{(a2)}
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\hfill
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{a3}
\caption{(a3)}
\end{subfigure}
\hfill
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{b1}
\caption{(b1)}
\end{subfigure}
\hfill
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{b2}
\caption{(b2)}
\end{subfigure}
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\begin{subfigure}{0.3\textwidth}
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\includegraphics[width=\textwidth]{b3}
\caption{(b3)}
\end{subfigure}
\hfill
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{c1}
\caption{(c1)}
\end{subfigure}
\hfill
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{c2}
\caption{(c2)}
\end{subfigure}
\hfill
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{c3}
\caption{(c3)}
\end{subfigure}
\caption{$|\psi_1|$ is solution of Eq. (1) for $c = 0.1$ when (a1-a3) $\gamma_0 = -0.2$, $\rho = 0.11$, (b1-b3) $\gamma_0 = 0$, $\rho = 0$, (c1-c3) $\gamma_0 = 0.2$, $\rho = 0.11$}
\end{figure}
graphs for $\gamma_0 = -0.2$, $\rho = 0.11$. Fig. 5 (b1-b3) show 3D, line and contour maps graphs for $\gamma_0 = 0$, $\rho = 0$, which give us the results when $\gamma(t) = 0$. Also, Fig. 5 (c1-c3), illustrete by considering $\gamma_0 = 0.2$, $\rho = 0.11$. Bright soliton solutions are well represented in these Figures with decreasing or increasing slope in negative or positive values of $\gamma_0$.

For these figures to have good point of view, $+/\bar{}$ standard deviation are represented in line plots.

5 Conclusion

In this article, we have presented Bernoulli ($G'/G$)-expansion method to find optical soliton solutions of VcNLSE, based on expressing exact solutions of NLEE by a polynomial, which is defined as a fractional function of the Bernoulli equation with $p$ and $q$ as variable coefficients. Due to the complexity of the computation, by use of Maple we could get the explicit output of solutions. To our knowledge, the begotten results have not been
reported in another literature. This method could easily explain exact solutions of VcN-LSE in two conditions, and also it can be applied to the other NLEEs with space-dependent or time- and space-dependent coefficients.

In addition, to complete our research, we have demonstrated the dynamic structures of the obtained wave functions by providing an arbitrary function with different values of the free parameters. We could also show some periodic behavior of waves, bright and dark soliton solutions and utilize MATLAB to help us to clearly depict the curvature of the waves. These illustrations showed us the relation between parameters and waves behaviors, in which, considering the negative values of \( \gamma_0 \), the wave functions decrease over time and for positive values of \( \gamma_0 \), the wave functions increase with time.
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References

Abdullaev, F.K., Caputo, J.G., Kraenkel, R.A., Malomed, B.A.: Controlling collapse in Bose-Einstein condensates by temporal modulation of the scattering length. Phys. Rev. A 67(1), 013605, 1–10 (2003). doi.org/10.1103/physreva.67.013605

Bao, W., Cai, Y.: Mathematical theory and numerical methods for Bose-Einstein condensation. Kinetic Related Models 6, 1–135 (2012). doi.org/10.3934/krm.2013.6.1

Bergé, L., Mezentsev, V.K., Rasmussen, J.J., Christiansen, P.L., Gaididei, Y.B.: Self-guiding light in layered nonlinear media. Opt. Lett. 25(14), 1037, 1–3 (2000). doi.org/10.1364/ol.25.001037

Bulut, H., Sulaiman, T.A., Baskonus, H.M., Aktürk, T.: On the bright and singular optical solitons to the (2+1)-dimensional NLS and the Hirota equations. Opt. Quant. Electron. 50(3), 134, 1–12 (2018). doi.org/10.1007/s11082-018-1411-6

El-Shiekh, R.M.: Classes of new exact solutions for nonlinear schrödinger equations with variable coefficients arising in optical fiber. Results Phys. 13, 102214, 1–5 (2019). doi.org/10.1016/j.rinp.2019.102214

Feng, B.: Sharp threshold of global existence and instability of standing wave for the Schrödinger-Hartree equation with a harmonic potential. Nonlinear Anal. Real World Appl. 31, 132–145 (2016). doi.org/10.1016/j.nonrwa.2016.01.012
New optical soliton solutions for the variable coefficients…

Feng, B., Zhang, H.: Stability of standing waves for the fractional Schrödinger-Choquard equation. Comput. Math. Appl. 75(7), 2499–2507 (2018). https://doi.org/10.1016/j.camwa.2017.12.025

Gu, Y., Aminakbari, N.: Two different systematic methods for constructing meromorphic exact solutions to the KdV-Sawada-Kotera equation. AIMS Math. 5(4), 3990–4010 (2020). https://doi.org/10.3934/math.2020257

Gu, Y., Yuan, W., Aminakbari, N., Lin, J.: Meromorphic solutions of some algebraic differential equations related Painlevé equation IV and its applications. Math. Methods Appl. Sci. 41(10), 3832–3840 (2018). https://doi.org/10.1002/mma.4869

Gu, Y., Wu, C., Yao, X., Yuan, W.: Characterizations of all real solutions for the KdV equation and WR. Appl. Math. Lett. 107, 106446, 1–8 (2020). https://doi.org/10.1016/j.aml.2020.106446

Guo, Q., Liu, J.: New exact solutions to the nonlinear Schrödinger equation with variable coefficients. Results Phys. 16, 102857, 1–5 (2020). https://doi.org/10.1016/j.rinp.2019.102857

He, X.-G., Zhao, D., Li, L., Luo, H.-G.: Engineering integrable nonautonomous nonlinear Schrödinger equations. Phys. Rev. E 79(5), 056610, 1–9 (2009). https://doi.org/10.1103/physreve.79.056610

He, J., Charalambidis, E., Kevrekidis, P., Frantzeskakis, D.: Rogue waves in nonlinear Schrödinger models with variable coefficients: Application to bose-einstein condensates. Phys. Lett. A 378(5–6), 577–583 (2014). https://doi.org/10.1016/j.physleta.2013.12.002

Hosseini, K., Mirzazadeh, M., Rabiei, F., Baskonus, H.M., Yel, G.: Dark optical solitons to the Biswas-Arshad equation with high order dispersions and absence of the self-phase modulation. Optik 209, 164576, 1–6 (2020). https://doi.org/10.1016/j.ijleo.2020.164576

Hossen, M.B., Roshid, H.-O., Ali, M.Z.: Characteristics of the solitary waves and rogue waves with interaction phenomena in a (2+1)-dimensional Breaking Soliton equation. Phys. Lett. A 382(19), 1268–1274 (2018). https://doi.org/10.1016/j.physleta.2018.03.016

Kaur, L., Wazwaz, A.-M.: Bright – dark optical solitons for Schrödinger-Hirota equation with variable coefficients. Optik 179, 479–484 (2019). https://doi.org/10.1016/j.ijleo.2018.09.035

Kedziora, D.J., Ankiewicz, A., Chowdury, A., Akhmediev, N.: Integrable equations of the infinite non-linear Schrödinger equation hierarchy with time variable coefficients. Chaos: Interdiscip. J. Nonlinear Sci. (2015). https://doi.org/10.1063/1.4931710

Khan, K., Akbar, M.A.: Solving unsteady Korteweg-de Vries equation and its two alternatives. Math. Methods Appl. Sci. 39(10), 2752–2760 (2016). https://doi.org/10.1002/mma.3727

Khatun, M.S., Hoque, M.F., Rahman, M.A.: Multisoliton solutions, completely elastic collisions and non-elastic fusion phenomena of two PDEs. Pramana (2017). https://doi.org/10.1007/s12043-017-1390-3

Kruglov, V.I., Peacock, A.C., Harvey, J.D.: Exact solutions of the generalized nonlinear Schrödinger equation with distributed coefficients. Phys. Rev. E 71(5), 056619, 1–11 (2005). https://doi.org/10.1103/physreve.71.056619

Liu, J.-G., Osman, M., Wazwaz, A.-M.: A variety of nonautonomous complex wave solutions for the (2+1)-dimensional nonlinear Schrödinger equation with variable coefficients in nonlinear optical fibers. Optik 180, 917–923 (2019). https://doi.org/10.1016/j.ijleo.2018.12.002

Montesinos, G.: Stabilization of solitons of the multidimensional nonlinear Schrödinger equation: matter-wave breathers. Physica D 191(3–4), 193–210 (2004). https://doi.org/10.1016/j.physd.2003.12.001

Özkan, Y.S., Yaşar, E., Seadawy, A.R.: On the multi-waves, interaction and Peregrine-like rational solutions of perturbed Radhakrishnan-Kundu-Lakshmanan equation. Phys. Scr. 95(8), 085205, 1–13 (2020). https://doi.org/10.1088/1402-4896/ab9a4f

Pérez-García, V.M., Torres, P.J., Konotop, V.V.: Similarity transformations for nonlinear Schrödinger equations with time-dependent coefficients. Physica D 221(1), 31–36 (2006). https://doi.org/10.1016/j.physd.2006.07.002

Ponomarenko, S.A., Agrawal, G.P.: Optical similaritons in nonlinear waveguides. Opt. Lett. 32(12), 1659, 1–3 (2007). https://doi.org/10.1364/ol.32.001659

Rahman, Z., Ali, M.Z., Roshid, H.-O.: Closed form soliton solutions of three nonlinear fractional models through proposed improved Kudryashov method. Chin. Phys. B 30(5), 050202, 1–14 (2021). https://doi.org/10.1088/1674-1056/abd165

Rizvi, S., Seadawy, A.R., Ashraf, F., Younis, M., Iqbal, H., Baaleanu, D.: Lump and Interaction solutions of a geophysical Korteweg-de Vries equation. Results Phys. 19, 103661, 1–8 (2020). https://doi.org/10.1016/j.rinp.2020.103661

Roshid, H., Rahman, M.A.: The exp(–Ψ(ξ))-expansion method with application in the (1+1)-dimensional classical Bousinesq equations. Results. Phys. 4, 150–155 (2014). https://doi.org/10.1016/j.rinp.2014.07.006
Roshid, H.-O., Kabir, M.R., Bhowmik, R.C., Datta, B.K.: Investigation of Solitary wave solutions for Vakhnenko-Parkes equation via exp-function and Exp(-φ(ξ))-expansion method. SpringerPlus. 3(1), 692, 1–10 (2014). https://doi.org/10.1186/2193-1801-3-692

Roshid, M.M., Roshid, H.-O., Ali, M.Z., Rezazadeh, H.: Kinky periodic pulse and interaction of bell wave with kink pulse wave propagation in nerve fibers and wall motion in liquid crystals. Partial Differ. Eq. Appl. Math. 2, 100012, 1–10 (2020). https://doi.org/10.1016/j.padiff.2020.100012

Saito, H., Ueda, M.: Dynamically stabilized bright solitons in a two-dimensional Bose-Einstein condensate. Phys. Rev. Lett. 90(4), 040403, 1–4 (2003). https://doi.org/10.1103/physrevlett.90.040403

Seadawy, A.R., Lu, D., Iqbal, M.: Application of mathematical methods on the system of dynamical equations for the ion sound and Langmuir waves. Pramana (2019b). https://doi.org/10.1007/s12043-019-1771-x

Seadawy, A.R., Cheemaa, N.: Propagation of nonlinear complex waves for the coupled nonlinear Schrödinger Equations in two core optical fibers. Physica A 529, 121330, 1–16 (2019). https://doi.org/10.1016/j.physa.2019.121330

Seadawy, A.R., Cheemaa, N.: Applications of extended modified auxiliary equation mapping method for high-order dispersive extended nonlinear Schrödinger equation in nonlinear optics. Mod. Phys. Lett. B 33(18), 1950203, 1–10 (2019). https://doi.org/10.1142/s0217984919502038

Seadawy, A.R., Cheemaa, N.: Some new families of spiky solitary waves of one-dimensional higher-order K-dv equation with power law nonlinearity in plasma physics. Indian J. Phys. 94(1), 117–126 (2019). https://doi.org/10.1007/s12648-019-01442-6

Seadawy, A.R., Ali, A., Albarakati, W.A.: Analytical wave solutions of the (2+1)-dimensional first integro-differential Kadomtsev-Petviashvili hierarchy equation by using modified mathematical methods. Results Phys. 15, 102775, 1–7 (2019). https://doi.org/10.1016/j.rinp.2019.102775

Serkin, V.N., Hasegawa, A.: Novel soliton solutions of the nonlinear Schrödinger equation model. Phys. Rev. Lett. 85(21), 4502–4505 (2000). https://doi.org/10.1103/physrevlett.85.4502

Ullah, M.S., Ali, M.Z., Roshid, H.-O., Seadawy, A.R., Baleanu, D.: Collision phenomena among lump, periodic and soliton solutions to a (2+1)-dimensional Bogoyavlenski’s breaking soliton model. Phys. Lett. A 397, 127263, 1–6 (2021). https://doi.org/10.1016/j.physleta.2021.127263

Wang, M., Li, X., Zhang, J.: The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A 372(4), 417–423 (2008). https://doi.org/10.1016/j.physleta.2007.07.051

Wazwaz, A.-M.: Partial Differential Equations and Solitary Waves Theory. Springer, Berlin Heidelberg (2009)

Yan, Z., Wen, Z., Konotop, V.V.: Solitons in a nonlinear Schrödinger equation with PT-symmetric potentials and inhomogeneous nonlinearity: Stability and excitation of nonlinear modes. Phys. Rev. A 92(2), 023821, 1–9 (2015). https://doi.org/10.1103/physreva.92.023821

Yao, Y.-Q., Li, J., Han, W., Wang, D.-S., Liu, W.-M.: Localized spatially nonlinear matter waves in atomic-molecular Bose-Einstein condensates with space-modulated nonlinearity. Sci. Rep. 6(1), 29566, 1–12 (2016). https://doi.org/10.1038/srep29566

Younas, U., Seadawy, A.R., Younis, M., Rizvi, S.: Dispersive of propagation wave structures to the dullin-Gottwald-Holm dynamical equation in a shallow water waves. Chin. J. Phys. 68, 348–364 (2020). https://doi.org/10.1016/j.cjiph.2020.09.021

Younas, U., Younis, M., Seadawy, A.R., Rizvi, S., Althobaiti, S., Sayed, S.: Diverse exact solutions for modified nonlinear Schrödinger equation with conformable fractional derivative. Results Phys. 20, 103776, 1–10 (2021). https://doi.org/10.1016/j.rinp.2020.103766

Yu, F., Yan, Z.: New rogue waves and dark-bright soliton solutions for a coupled nonlinear Schrödinger equation with variable coefficients. Appl. Math. Comput. 233, 351–358 (2014). https://doi.org/10.1016/j.amc.2014.02.023

Zeng, Z.-F., Liu, J.-G., Jiang, Y., Nie, B.: Transformations and soliton solutions for a variable-coefficient nonlinear Schrödinger equation in the dispersion decreasing fiber with symbolic computation. Fundamenta Informaticae, 145(2), 207–219 (2016) ISSN 0169-2968. https://doi.org/10.3233/FI-2016-1355

Zhang, J.: Stability of attractive Bose-Einstein condensates. J. Stat. Phys. 101(3/4), 731–746 (2000). https://doi.org/10.1023/a:1026437923987

Zhang, S., Tong, J.-L., Wang, W.: A generalized (G'/G)-expansion method for the mKdV equation with variable coefficients. Phys. Lett. A 372(13), 2254–2257 (2008). https://doi.org/10.1016/j.physleta.2007.11.026

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