Dynamics of superconducting strings with chiral currents

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We rederive, using an elementary formalism, the general solution to the equations of motion for a superconducting string with a chiral (null) neutral current, earlier obtained by Carter and Peter. We apply this solution to show that the motion of such string loops is strictly periodic and analyze cusp-like behavior and vorton solutions of arbitrary shape. We argue that this solution can be used to approximately describe the dynamics of superconducting strings with small non-chiral currents. We use this description to estimate the electromagnetic radiation power from such strings.

I. INTRODUCTION

Cosmic strings are linear topological defects that may have been created in the early Universe [1]. They have been extensively studied in connection with several problems in cosmology (for reviews see [2,3]). Assuming that the radius of curvature of a string is always much larger than the string core thickness, the string dynamics can be described by the Nambu-Goto action [4,5]. In this approximation the equations of motion are simple, and can be easily solved.

In 1985, Witten [6] showed that strings could behave as superconducting wires in certain particle physics models. This new internal degree of freedom opened up a variety of interesting effects. In particular, superconducting strings may have stable configurations, vortons [7] and springs [8], which could contribute to the dark matter in the universe, or put constraints on the particle physics models that give rise to those strings [10,11]. Superconducting cosmic strings have also been considered as sources for structure formation [12], gamma ray bursts [13–15], and ultra-high energy cosmic rays [15–17].

The general problem of a superconducting string coupled to the electromagnetic field cannot be solved analytically. However, if the charge carriers are not coupled to any long-range field (so-called neutral superconducting strings) [18] the situation is significantly simpler. Such strings are of interest in their own right, and may also be considered as approximations to the full theory including electromagnetic coupling.

If in addition, we consider the case that the charge and current are equal in magnitude, then it was shown by Carter and Peter [19] that the equations of motion can be solved exactly. In this case, the charge-current 4-vector is lightlike, and the current is said to be chiral or null. It consists of charge carriers which are all moving in the same direction. Such currents arise automatically in certain supersymmetric theories in which a zero mode can travel only in one direction along the string [20,1]. They could also result from evolution of a loop with an arbitrary distribution of charge and current [21,22]. Left- and right-moving charge carriers can scatter off the string, and if the numbers of left- and right-movers are not equal, the string can be driven towards the chiral limit. This will happen particularly near a cusp, where the string is contracted by a large factor, resulting in a much higher density of charge carriers than elsewhere along the string and in a great enhancement of the scattering rate. One can expect therefore that the current will be very nearly chiral in the vicinity of cusps.

In the case of electrically charged currents, the electromagnetic radiation from oscillating loops is dominated by powerful bursts emitted from near-cusp regions [23,21]. One can therefore use the solution for strings with chiral currents to estimate the radiation power from such loops.

To avoid confusion, we note that cusps were originally defined [21] as points of infinite contraction, where the string momentarily reaches the speed of light. Strictly speaking, such cusps can be formed only on idealized Nambu-Goto strings. For realistic strings, the cusp development is truncated either by the annihilation of overlapping string segments at the tip of the cusp [23,24] or for superconducting strings, by the back reaction of charge carriers or of the electromagnetic radiation. However, unless the string current is very large, so that the energy of the charge carriers is comparable to that of the string itself, the truncation occurs at a very large Lorentz factor and the string exhibits cusp-like behavior. Below we shall use the word “cusps” to refer to such ultra-relativistic string segments.

Carter and Peter derived their solution [19] using Carter’s formalism [25,26] which is not familiar to most cosmologists. In view of the importance of this result, in the present paper we shall give an alternative, elementary derivation. We shall also give an explicit solution for the initial value problem and discuss several physical implications of the result.

We begin in Section II by reviewing the well-known solution of string equations of motion in the case of non-
superconducting Nambu-Goto strings. Our derivation of the solution for chiral superconducting strings is presented in Section III. In Section IV, we show how the motion of such strings can be found from given initial conditions. Some physical implications of the solution are discussed in Section V, where we show that the motion of string loops with chiral currents is strictly periodic and calculate the maximum Lorentz factor reached at the cusp. In Section VI we argue that the chiral string solution can be used as an approximate description of strings with small non-chiral currents. In Section VII we use this approximate description to estimate the electromagnetic radiation from oscillating loops with charged currents.

After this paper was submitted we learned about independent work by Davis et al. [30] which gives a derivation of the chiral string solution similar to ours.

II. NAMBU-GOTO STRINGS

We first review the equations of motion and their solution in the case of non-superconducting strings. We will use similar techniques in the next section to solve the equations of motion for superconducting strings, in the case that the current is chiral.

For an infinitely thin non-superconducting relativistic string, the flat spacetime equations of motion are

$$\partial_a \left[ \sqrt{-\gamma} \gamma^{ab} x^b_x \right] = 0$$

(1)

where $a$ and $b$ take the values 0 and 1 (denoting the worldsheet coordinates $\tau$ and $\sigma$ respectively), $x^b(\sigma, \tau)$ is the position of the string, and $\gamma_{ab}$ is the induced metric on the worldsheet,

$$\gamma_{ab} = x^d_a x^{\mu b}$$

(2)

and $\gamma = \det(\gamma_{ab})$. We are free to choose a particular parameterization of the worldsheet, i.e. gauge condition. In this case, it is convenient to use the conformal gauge, namely

$$\gamma_{ab} = \Omega(\sigma, \tau) \eta_{ab}$$

(3)

where $\eta_{ab}$ is the two-dimensional Minkowski metric. Then $\sqrt{-\gamma} = \Omega$ and thus $\sqrt{-\gamma} \gamma^{ab} = \eta^{ab}$. In this gauge the equation of motion for the string is just the two-dimensional wave equation,

$$x^{\mu \prime \prime} - x^\sigma' = 0,$$

(4)

where $\sigma'$ denotes $\partial x / \partial \sigma$ and $x'$ denotes $\partial x / \partial \tau$.

It can be seen that the constraints written above in Eq. (3), do not fix completely our gauge, and we can choose the condition

$$x^0 = \tau$$

(5)

so the equations of motion become

$$x^{\prime \prime} - \ddot{x} = 0.$$

(6)

The general solution has the form

$$x = \frac{1}{2} [a(\sigma - \tau) + b(\sigma + \tau)].$$

(7)

In order for the metric to have the form of Eq. (3), we must impose the conditions

$$|a|^2 = |b|^2 = 1.$$  

(8)

III. CHIRAL SUPERCONDUCTING STRINGS

We consider a superconducting string with a neutral current (i.e., one not coupled to the electromagnetic field). We can describe the current via an auxiliary scalar field $\phi$, in terms of which the conserved worldsheet current is

$$J^a = \frac{1}{\sqrt{-\gamma}} \epsilon^{ab} \Phi_b.$$  

(9)

The current is chiral if $\Phi_a$ is a null worldsheet vector, i.e.,

$$\gamma^{ab} \Phi_b \Phi_a = 0,$$

(10)

in which case $J_a J^a = 0$. In this case, the equations of motion can be written

$$\partial_a \left( \epsilon^{ab} x^b \right) = 0$$

(11a)

$$\partial_a \left( \sqrt{-\gamma} \gamma^{ab} \Phi_b \right) = 0$$

(11b)

where

$$\mathcal{T}^{ab} = \sqrt{-\gamma} \left( \mu \gamma^{ab} + \theta^{ab} \right),$$

(12)

$\mu$ is the energy per unit length of the string, and $\theta^{ab}$ is the worldsheet energy-momentum tensor of the charge carriers,

$$\theta^{ab} = \gamma^{ac} \gamma^{bd} \phi_c \phi_d$$

(13)

in the chiral case.

As above, we would like to have a gauge in which the $\mathcal{T}^{ab}$ has the form

$$\mathcal{T}^{ab} = \mu \eta^{ab}.$$  

(14)

If we can accomplish this, the equation of motion will be the wave equation, Eq. (4), we can choose $x^0 = \tau$, and the general solution will be given by Eq. (7), as before. However, we note that since $\mathcal{T}$ is a $2 \times 2$ matrix,

$$\det \mathcal{T} = (-\gamma) \det \left( \mu \gamma^{ab} + \theta^{ab} \right)$$

$$= - \det \left[ \gamma_{ab} \left( \mu \gamma^{bc} + \theta^{bc} \right) \right] = - \det \left( \mu \delta^c_a + \theta^c_a \right)$$

(15)
which is gauge-invariant. Since the matrices are $2 \times 2$, the determinant is easily expanded,
\[
\det T = -\mu^2 - \mu \text{Tr} \theta_a - \det \theta_a. \tag{16}
\]

Since $\theta$ is traceless, $\det T$ can only be $-\mu^2$ as required by Eq. (14), if $\det \theta_a = 0$. But for (and only for) a chiral current, from Eq. (13),
\[
\theta_a = \phi^c \phi_a, \tag{17}
\]
which is the outer product of two vectors, and thus has vanishing determinant.

Thus for a chiral current there is the possibility that Eq. (14) can be satisfied. In that case, from Eqs. (14) and (12) we see that
\[
\sqrt{-\gamma} \gamma^{ab} \phi_{,b} = \frac{1}{\mu} \left[ T^{ab} - \sqrt{-\gamma} \theta^{ab} \right] \phi_{,b} = \eta^{ab} \phi_{,b} \tag{18}
\]
and so Eq. (11) becomes the wave equation,
\[
\ddot{\phi} - \phi'' = 0. \tag{19}
\]

We now contract Eq. (14) with $\phi_{,a} \phi_{,b}$. Since the current is chiral, $\gamma^{ab} \phi_{,a} \phi_{,b} = 0$, and using Eq. (13), $\theta^{ab} \phi_{,a} \phi_{,b} = 0$. Thus to satisfy Eq. (14), we must have $\eta^{ab} \phi_{,a} \phi_{,b} = 0$, or $\phi^2 = \phi^2$. Without loss of generality we take the solution of the form
\[
\phi(\sigma, \tau) = F(\sigma + \tau), \tag{20}
\]
which also satisfies Eq. (13).

Using Eq. (20), the condition for chirality, Eq. (10), becomes
\[
\gamma^{00} + \gamma^{11} + 2 \gamma^{01} = 0. \tag{21}
\]

Since $\gamma^{ab}$ is the inverse of the $2 \times 2$ matrix $\gamma_{ab}$ we have
\[
\gamma^{ab} = \frac{1}{\gamma} \begin{pmatrix} \gamma_{11} & -\gamma_{01} \\ -\gamma_{01} & \gamma_{00} \end{pmatrix}, \tag{22}
\]
so Eq. (21) implies
\[
\gamma_{00} + \gamma_{11} - 2 \gamma_{01} = 0. \tag{23}
\]

Using Eq. (3), this becomes
\[
0 = \dot{x}^{\mu} \dot{x}_{\mu} + x^{\mu} x'_{\mu} - 2 \dot{x}^{\mu} x'_{\mu} = (\dot{x}^{\mu} - x^{\mu'}) (\dot{x}_{\mu} - x'_{\mu}) \tag{24}
\]
which means that $\dot{x}^{\mu} - x^{\mu'} = (1, -a')$ is a null 4-vector, or that
\[
|a'| = 1. \tag{25}
\]

To solve the rest of the problem, we define a matrix
\[
S_{ab} = \frac{1}{\sqrt{-\gamma}} (\mu \gamma_{ab} - \theta_{ab}). \tag{26}
\]

From Eqs. (10) and (13), $\theta_{ab} \theta^{bc} = 0$, and since $\gamma^{ab}$ and $\theta^{ab}$ are symmetrical,
\[
S_{ab} T^{bc} = \mu^2 \delta_a. \tag{27}
\]

and consequently if $T$ has the form of Eq. (14) we have that
\[
S_{ab} = \mu m_{ab}. \tag{28}
\]

Now, comparing the 01 component of Eqs. (27) and (28) gives
\[
\mu \gamma_{01} = \theta_{01} = F'^2. \tag{29}
\]

The metric component is
\[
\gamma_{01} = \dot{x}^{\mu} x'_{\mu} = \frac{1}{4} \left( |a'|^2 - |b'|^2 \right). \tag{30}
\]

From Eqs. (31) and (27), we find
\[
1 - |b'|^2 = \frac{4 F'^2}{\mu} \tag{31}
\]

Since the determinant of $T$ and thus of $S$ is fixed, it remains only to show that one more component of Eq. (23) is satisfied. For example, a sufficient condition is that
\[
\mu \gamma_{00} - \theta_{00} = \mu \sqrt{-\gamma}. \tag{32}
\]

Using Eq. (23) it is easy to show that
\[
\sqrt{-\gamma} = \frac{1}{2} (\gamma_{00} - \gamma_{11}), \tag{33}
\]
and Eq. (32) becomes
\[
\mu \gamma_{00} - F'^2 = \frac{\mu}{2} (\gamma_{00} - \gamma_{11}) \tag{34}
\]
so
\[
\frac{\mu}{2} (\gamma_{00} + \gamma_{11}) = F'^2 \tag{35}
\]
which is satisfied using Eq. (23) and Eq. (29).

Thus, Eq. (7) with the constraints given by Eqs. (24) and (31) are a general solution to the equations of motion for a superconducting string with a chiral current. This solution is the same as that found by Carter and Peter in [13], except they did not explicitly specify the relation between $b'$ and $F'$, but gave instead the inequality $|b'|^2 < 1$.

Note that in this gauge $\sigma$ parameterizes the total energy on the string. The energy in a region is
\[
E = \int d^3 x \, T^0_0, \tag{36}
\]
where $T^0_0$ is given by
\[
T^0_0(x) = \int d\tau \, T^{ab} x'^a_{,\nu} x_{,\nu} \delta^4 [x - x(\sigma, \tau)]. \tag{37}
\]

With $x^0 = \tau$, and using Eq. (14), the energy is
\[
E = \mu \int d\sigma = \mu \int dt |\dot{\sigma}| \tag{38}
\]
which means that with this parameterization the energy on the string is $\mu \Delta \sigma$. 

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IV. FINDING THE SOLUTION FROM INITIAL CONDITIONS.

We would now like to find the evolution of a string with a chiral current from given initial conditions. We suppose that we are given the position of the string at some time \( t_0 \), as a function \( \mathbf{x}(l) \) parameterized by arc length in the laboratory frame, with \( l \) increasing in the opposite direction to the current flow. We also need the initial charge and current as the values of the auxiliary scalar field \( \phi(l) \), and the perpendicular component of the string motion, \( \dot{\mathbf{x}}_\perp(l) \). Motion parallel to the string direction is dependent on the choice of parameter and has no physical meaning. From these conditions, we want to find the functions \( a \) and \( b \).

The first step is to reparameterize everything in terms of \( \sigma \). For a stationary string, the linear energy density of the string itself is just \( \mu \), and the energy due to the current is \((d\phi/dl)^2\). Boosting the string in a transverse direction just gives the Lorentz factor \( \Gamma = 1/\sqrt{1-|\mathbf{x}_\perp|^2} \), so
\[
\frac{dE}{dl} = \Gamma \left[ \mu + \left( \frac{d\phi}{dl} \right)^2 \right] \tag{39}
\]
and thus
\[
\frac{d\sigma}{dl} = \Gamma \left[ 1 + \frac{1}{\mu} \left( \frac{d\phi}{dl} \right)^2 \right]. \tag{40}
\]
Using Eq. (40) we can change parameters from \( l \) to \( \sigma \).

Now we need to determine the full form of \( \dot{\mathbf{x}} \). We observe that \( \dot{\mathbf{x}} \cdot \dot{\mathbf{x}} = (|\mathbf{b}'|^2 - |a'|^2)/4 = -\phi'^2/\mu \), so we can write
\[
\dot{\mathbf{x}} = \dot{\mathbf{x}}_\perp - \frac{\phi'^2}{\mu} \frac{\dot{\mathbf{x}}'}{|\dot{\mathbf{x}}'|^2}. \tag{41}
\]
Then
\[
a' = \dot{\mathbf{x}}' - \dot{\mathbf{x}} \tag{42a}
\]
\[
b' = \dot{\mathbf{x}}' + \dot{\mathbf{x}} \tag{42b}
\]
and Eq. (40) gives a complete solution for the future evolution of the string.

V. CHIRAL STRING DYNAMICS

We have obtained the analytic general solution for superconducting strings with chiral currents. For a string with a current determined from,
\[
\phi(\sigma, \tau) = F(\sigma + \tau), \tag{43}
\]
the general solution for the string is given by
\[
x_0 = \tau \tag{44a}
\]
\[
x = \frac{1}{2} [a(\sigma - \tau) + b(\sigma + \tau)], \tag{44b}
\]
with the following constraints for the otherwise arbitrary functions \( a' \) and \( b' \),
\[
|a'|^2 = 1 \tag{45a}
\]
\[
|b'|^2 = 1 - \frac{4F'^2}{\mu}. \tag{45b}
\]

Note that \( F' = d\phi/d\sigma \) is in general not the same as the physical current, which goes as \( d\phi/dl \). Using Eq. (40) we can write
\[
\frac{d\phi}{dl} = \Gamma \left[ 1 + \frac{1}{\mu} \left( \frac{d\phi}{dl} \right)^2 \right] F' \tag{46}
\]
so
\[
F' = \frac{\mu(d\phi/dl)}{|\mu + (d\phi/dl)^2|\Gamma}. \tag{47}
\]

Thus \( F' \) increases with \( d\phi/dl \) for small currents, but it reaches a maximum, \( F' = \sqrt{\mu}/(2\Gamma) \) when \( d\phi/dl = \sqrt{\mu} \). For larger values of \( d\phi/dl \), \( F' \) decreases again. This happens because \( F' \) measures the winding number per unit energy, and the current contribution to the energy density goes as \((d\phi/dl)^2\).

We now show several interesting consequences that we can extract from the result. First of all, we see that there are arbitrarily shaped static solutions for the case in which the current satisfies \( 4F'^2/\mu = 1 \). In this case \( |b'| = 0 \), so the position of the string, up to a constant vector, is given by
\[
x = \frac{1}{2}[a(\sigma - \tau)] \tag{48}
\]
so the set of points traced by the string does not depend on time. These are vortons of any shape, which could have important cosmological consequences.

Another somewhat unexpected consequence of the exact solution is that the motion of a loop with a chiral current is strictly periodic in its rest frame. The period is \( T = E/(2\mu) \), where \( E \) is the total energy of the loop in that frame.

We also see that chiral strings do not have true cusps. In fact, we can easily calculate the Lorentz factor that these strings can reach,
\[
\Gamma = \frac{1}{\sqrt{1-|\mathbf{x}_\perp|^2}} = \sqrt{1 + \left( \frac{|b'| \sin \theta}{1 + |b'| \cos \theta} \right)^2}, \tag{49}
\]
where \( \theta \) is the angle between \( a' \) and \( b' \). This expression has its maximum at \( \cos \theta = -|b'| \), and the maximum Lorentz factor is
\[
\Gamma_{\text{max}} = \frac{\sqrt{\mu}}{2|F'|}. \tag{50}
\]

This result can be seen directly from Eq. (47). At this point, the energy density in the string itself and that in the charge carriers (see Eq. (39)), are equal.
However, this is not the maximum concentration of energy per unit length that can be achieved. Using Eq. (68) we see that the maximum energy density corresponds to the minimum of $|x'|$, i.e. $\theta = \pi$. There,

$$|x'|_{\text{min}} = \frac{1}{2} \left(1 - |b'|\right)$$

and so

$$\left.\frac{dE}{dt}\right|_{\text{max}} = \frac{2\mu}{1 - |b'|} = \frac{2\mu}{1 - \sqrt{1 - 4\Gamma^2/\mu}}.$$ 

(52)

At this point $a'$ and $b'$ are antiparallel, and thus so are $\mathbf{x}$ and $\mathbf{x}'$. Thus $\mathbf{x}_\perp = 0$ and the string is not moving, so this is also the point of maximum energy density in the local rest frame.

VI. STRINGS WITH A SMALL NON-CHIRAL CURRENT

The Carter-Peter solution for chiral strings, Eqs. (43)–(47) can also be used as an approximate description of strings having both right- and left-moving charge carriers in the case when the current is sufficiently small,

$$\phi^2, \phi'^2 \ll \mu.$$ 

(53)

The contribution of charge carriers to the worldsheet energy momentum tensor is then suppressed compared to that of the string itself by a factor $\sim \Gamma^2 \phi'^2$, where $\Gamma$ is the Lorentz factor of the string. (We assume for simplicity that $\phi^2 \sim \phi'^2$.) The effect of charge carriers on the string dynamics will therefore be negligible, except in the vicinity of cusps where $\Gamma$ can be very large.

The string current is also greatly enhanced in near-cusp regions. For strings with bosonic superconductivity, large currents can be unstable with respect to quenching and to quantum tunneling. A large charge density can also destabilize the condensate, resulting in ejection of charge carriers. These effects disappear for a chiral current and tend to drive the string towards the chiral limit. For strings with fermionic superconductivity, the growth of the current leads to enhanced scattering of left- and right-moving charge carriers off the string. The left- and right-moving currents are typically not equal, so the scatterings suppress the minority charge carriers, leaving the string with nearly chiral current in the near-cusp region.

We thus have a situation where away from the cusps the current is non-chiral but small and its effect on the string dynamics is negligible, while near the cusps the current is large and nearly chiral. Hence, both near cusps and away from cusps the string dynamics can be approximately described by the chiral string solution, Eqs. (43)–(47).

We shall now derive a quantitative criterion for this approximate description to be accurate. Near a cusp, the string gets contracted by a large factor, its rest energy being turned into kinetic energy. The density of charge carriers and the current are enhanced by the same factor. The contraction factor increases as we approach the tip of the cusp. The invariant length of string which attains Lorentz factor at least $\Gamma$ is $\sim L/\Gamma$. Since this Lorentz factor is obtained by compressing the string, the physical length of this region of string is

$$\Delta L_{\Gamma} \sim L/\Gamma^2,$$ 

(54)

and since the physical string motion is perpendicular to the string, this is also the length of string in its rest frame.

$$J_{\Gamma} \sim \Gamma J_0.$$ 

(55)

These values are sustained for a time interval (again in the rest frame)

$$\Delta t_{\Gamma} \sim \Delta L_{\Gamma} \sim L/\Gamma^2.$$ 

(56)

We now have to check whether or not this time interval is sufficient to suppress the minority charge carriers. The answer to this depends on the scattering rate of left- and right-movers and is therefore model-dependent. As an illustration, let us suppose that we have fermionic charge carriers with large masses off the string but which can scatter elastically by exchange of a GUT-scale gauge boson into light particles not bound to the string. Models of this sort have been studied in detail by Barr and Matheson. Their analysis indicates that the time it takes for the current to become nearly chiral is (in the rest frame of the string)

$$\tau \sim \frac{M_X}{J_0^2},$$ 

(57)

where, $M_X$ is the GUT scale. The current will be nearly reduced to one chiral component during a single cusp occurrence, provided that this time is shorter than $\Delta t_{\Gamma}$. Using Eqs. (55), (56) and (57) we find that this condition is satisfied when the Lorentz factor exceeds a certain value $\Gamma^*$,

$$\Gamma \gg \Gamma^* \sim \left( \frac{M_X^4}{J_0^5} \right)^{1/3}.$$ 

(58)

On the other hand, charge carriers have a substantial effect on the string dynamics when the Lorentz factor becomes comparable to $\Gamma_{\text{max}}$ from Eq. (44). Thus, we expect the approximate chiral string description to be accurate when $\Gamma^* \ll \Gamma_{\text{max}}$, or

$$\frac{J_0}{\sqrt{\mu}} \gg \left( \frac{M_X^2}{\mu} \right)^{5/4} (M_X L)^{-1/2}.$$ 

(59)
VII. ELECTROMAGNETIC RADIATION POWER FROM OSCILLATING LOOPS

If the string current is coupled to electromagnetism, then oscillating current-carrying loops emit electromagnetic radiation. Calculations disregarding the effect of charge carriers and of the electromagnetic back-reaction on the motion of the loop give an infinite radiation power. The divergence can be attributed to the infinite Lorentz factor reached at the cusp of a Nambu-Goto string. If the cusp is truncated at a maximum Lorentz factor \( \Gamma^* \), the power can be estimated as \[ P \sim 30 j^2 \Gamma^* \max. \quad (60) \]

Here, \( j \sim q J_0 \) is the electric current away from the cusp, \( q \sim 0.1 \) is the effective charge of the charge-carrying field, and the coefficient comes from numerical calculations.

Later in this Section, we shall argue that the electromagnetic back-reaction in the near-cusp region is sub-dominant compared to the effect of the charge carriers and can therefore be neglected.

As discussed in the preceding Section, for sufficiently small currents the charge-carrier back-reaction is negligible away from the cusps, while near the cusps the current tends to be chiral and the solution Eqs. (43)–(45) can be used. Using this solution we can determine \( \Gamma^* \max \) and therefore we can give an estimate for the electromagnetic power. Substituting Eq. (50) in Eq. (61) we obtain

\[ P \sim qj \sqrt{\mu}. \quad (61) \]

This result is confirmed by more accurate calculations in (27).

Our neglect of the electromagnetic back-reaction can be justified as follows. The energy emitted in a single cusp event is \( \Delta E_{\text{em}} \sim \mu L \), where \( L \) is the length of the loop. The total energy of the string segment in which the maximum Lorentz factor is reached (and which is responsible for most of the radiation) is \( \Delta E_\nu \sim \mu L/\Gamma^* \max \), and the energy of the charge carriers in that region is \( \Delta E_j \sim \Delta E_\nu \). Using Eqs. (46)–(51), we find \( \Delta E_{\text{em}}/\Delta E_j \sim q^2 \ll 1 \). This suggests that the effect of electromagnetic back-reaction on the motion of the string is much smaller than that of the charge carriers.

VIII. ACKNOWLEDGMENTS

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