How many different parties can join into one stable government?

Dietrich Stauffer
Institute for Theoretical Physics, Cologne University,
D-50923 Köln, Euroland

Abstract: Monte Carlo simulations of the Sznajd model with bounded confidence for varying dimensions show that the probability to reach a consensus in $d$-dimensional lattices depends only weakly on $d$ but strongly on the number $Q$ of possible opinions: $Q = 3$ usually leads to consensus, $Q = 4$ does not.

In democracies where not just two parties dominate in elections, the government often is formed by a coalition of several parties, since no single party won more than 50 percent of the seats in parliament. According to (West) German experience of the last half-century, coalitions with up to three parties appear often, those with four and more happen only rarely. Obviously, the more parties a government has, the less stable it is in general. However, if parties need 5 percent of the vote to get into parliament (as is usually the case in Germany), the total number of parties is limited anyhow, and the above rule of up to three parties per coalition could simply come from the fact that typically only five are represented. Thus we want to find a simple computer model to check if a consensus is indeed difficult to reach if more than three parties try to form a government.

The Sznajd model [1, 2] (see [3] for a recent review) has been shown to agree well with election statistics in Brazil and India [4, 5] and thus is a natural choice for the present question. We assume that there are $Q$ different opinions $q = 1, 2, \ldots Q$ (= parties) possible for each of $L^d$ sites (= politicians) on a hypercubic lattice in $d$ dimensions; if and only if two neighbouring sites have the same opinion $q$, they convince their $4d-2$ neighbours to join party $q$. However, politicians are assumed to switch only to politically neighbouring parties, i.e. from opinion $q \pm 1$ to opinion $q$: bounded confidence [6, 7]. The initial opinions are randomly distributed. Random sequential updating and free boundary conditions are used throughout; neighbours are always nearest neighbours. Opinions 1 and $Q$ are not taken as neighbours. The program sznajd31.f is available from stauffer@thp.uni-koeln.de.

The number of political leaders is much more limited than the number of voters, and thus we work with rather small lattices, like $L = 19, 7, 5$
Figure 1: Variation with dimensionality $d$ of the fraction of cases where no consensus is reached. The upper data correspond to $Q = 4$ parties (which usually fail), the lower to $Q = 3$ parties which usually keep together. For larger lattices at $Q = 3$ the failure probability goes to zero.

and 5 in two to five dimensions. Simulations are stopped if after 10,000 sweeps through the lattice no consensus is reached, if a fixed point without consensus is reached, or if a consensus is reached with all $L^d$ sites having the same opinion $q$. Fig.1 shows that consensus is the rule for $Q = 3$ but rare for $Q = 4$, from 1000 separate simulations for each point. ($d = 2.5$ corresponds to the two-dimensional triangular lattice with six neighbours for each site. With $Q = 2$, always a consensus was found, which does not agree with the break-up of the West Germany federal government in 1982; India is a counterexample where many more parties are mostly kept together since years in one government.) This transition from consensus ($Q = 3$) to no consensus ($Q = 4$) is similar to that in [7, 8].

[Ref. [2] claimed that for the triangular lattice the border between consensus and no consensus is shifted to $Q = 5$; however, there a consensus was also counted if the opinions separated into a fixed point with well separated opinions, like only $q = 1$ and 3 for $Q = 3$: “agree to disagree”. Our figure now shows that indeed a true consensus for $Q = 4$ is easier for the triangular
than for the square or simple cubic lattices, but still failures occur much more often than consensus even on the triangular lattice.]

Thus not just the limited total number of parties in parliament is responsible for keeping limited the number of parties in a government; it is also very difficult to reach a consensus among four parties.

This work started with a question of J. Liu from Harvard at [3] and profitted from comments by J. Kertész, D. Chowdhury and G. Weisbuch.

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