CALCULATION OF SUB-BANDS \{1,2,5,6\} FOR 64-POINT COMPLEX FFT AND ITS EXTENSION TO N (= 2^N)-POINT FFT

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ABSTRACT

FFT algorithm is one of the most applied algorithms in digital signal processing. Digital signal processing has gradually become important in biomedical application. Here hardware implementation of FFTs have found useful applications for bio- wearable devices. However, for these devices, low-power and low-area are of utmost importance. In this report, we investigate a sub-structure of decimation-in-frequency (DIF) FFT where a number of sub-bands are of interest to us. Specifically, we divide the range of frequencies into 8 sub-bands (0-7) and calculate 4 of them (1,2,5,6). We show that using concepts like pushing and radix \(2^2\), the number of complex multiplications can be drastically reduced for 16-point, 32-point and 64-point FFTs while computing those specific bands. Later, we also extend it to \(N = 2^n\)-point FFT based on optimized 64-point FFT structure. The number of complex multiplications is further reduced using merge-FFT. Our results show that the number of multiplications (and hence power) can be reduced greatly using our optimized structure compared to an unoptimized structure. This can find application in biomedical signal processing \{1,2,3,4,5,6,7,8\} specifically while computing power spectral density of a physiological time series where reducing computational power is of utmost importance.

1. INTRODUCTION

Fast fourier transform (FFT) \[9\] is a widely used technique in digital signal processing. This algorithm is specifically useful for applications in telecommunications, biomedical signal processing \{10,11,12,13,14,15,16,17,18,19,20\}. One of the main application of FFT based algorithms in biomedical signal processing is their usefulness in computing power spectral density (PSD). A number of methods for the computation of PSD on hardware have been proposed \[21\]. In these algorithms, the main bottleneck is to compute the FFT efficiently.

While computing FFT on hardware, the reduction of power and area is of utmost importance. Power consumption is directly related to number of complex multipliers in the structure. Hence, if we can reduce the number of multipliers in the hardware structure, the power consumption will also be less.

Another important observation from biomedical signal processing is that only a few of the sub-bands from available frequency range are required for computation. These bands contain most information. For example, in case of EEG data of human brain, the range of frequencies can be divided into \(\alpha (8−15 \text{ Hz}), \beta \left(16−31 \text{ Hz}\right), \gamma \left(32−50 \text{ Hz}\right), \delta (< 4 \text{ Hz})\), \(\theta \left(4−7 \text{ Hz}\right), \mu \left(8−12 \text{ Hz}\right)\) band waves. These bands relate to a number of separate physiological functions. For example, \(\alpha\) wave indicates a relaxed, reflexive state of brain and is responsible for inhibition control. On the other hand, \(\beta\) band may refer to a stressed, mildly obsessive behavior. These sub-bands and their power spectral density have been shown to be useful to create biomarkers for a number of pathological conditions including prediction of seizure onset \[22\].

In this report, we explore the ways to derive efficient structures for FFT when we are only concerned about a few sub-bands. Efficient FFT structures are well known in literature. Most of them use real valued signals to reduce the redundancy in architecture. Apart from that, twiddle factor transformation techniques like pushing, modulation and architectures like radix-\(2^3\), \(2^4\) are also used to reduce the power consumption. We explore these transformations and use merge-FFT \[21\] to reduce number of multiplications required for computing specific sub-bands. The whole FFT band is divided into 8 sub-bands (0 - 7). In this report, we are only concerned about computing \{1,2,5,6\} sub-bands. We consider the Decimation in Frequency (DIF) FFT structure for our experiment. But the same can be extended to Decimation in Time (DIT) structures as well.

The paper is structured as follows. In section 2, we revisit some techniques to reduce the number of twiddle factors. Next, in section 3, we discuss how these techniques can be used to reduce number of multiplications in computing sub-bands of 16-point, 32-point and 64-point FFTs. In section 4, we generalize our results for \(N = 2^n\) point FFTs. Finally, in section 5, we further reduce the number of multiplier in FFT-structure using concepts from merge-FFT \[21\].
2. TWIDDLE FACTOR TRANSFORMATION

The N-point discrete Fourier Transform of sequence $x[n]$ is given by

$$X(k) = \sum_{n=0}^{N-1} x[n]W_N^{nk}$$  \hspace{1cm} (1)

Where $W_N = e^{-j\frac{2\pi}{N}}$. For notational convenience we refer $W_N^{nk}$ as $W_k$ or twiddle factor.

2.1. Pushing

Twiddle factors can be transformed by a method called pushing. In this technique, a twiddle factor at the left of the butterfly diagram can be pushed to the right. This reduces the total number of twiddle factors in the FFT structure if some of the FFT points are not required for computation. We have shown this method in Fig. 1. For further clarification please refer [23].

2.2. Radix-$2^k$

Using FFT structure corresponding to radix-$2^k$ can significantly reduce the number of multiplications in computation. The main idea in radix-$2^k$ is derived from pushing where the twiddle factors are pushed to the right of butterfly structure. Radix-$2^k$ means that the complex multiplications will be there after each $k$-stages in the FFT structure.

3. SUB-BAND CALCULATION (16-POINT, 32-POINT, 64-POINT)

In this section, we apply the previously described techniques for twiddle factor transformation in the cases of 16-point, 32-point and 64-point FFT.

3.1. 16-Point FFT

3.1.1. Single Bands

In case of 16-point decimation in frequency FFT structure, the FFT points corresponding to each of the 4 sub-bands $\{1,2,5,6\}$ are given by $\{X(2), X(3), X(4), X(5), X(10), X(11)\}$. For calculating the number of complex multiplications, we assume that twiddle factors corresponding to $1, j, -1, -j$ do not require any multiplier.

Using only pushing, the number of complex multiplication gets reduced by more than half while calculating either of $\{1,2,5,6\}$. In this case, bands $\{1,5\}$ are similar whereas bands $\{2,6\}$ are similar. We see that Band-1 and Band-5 require the same number of multiplications after optimization. On the other hand, Band-2 and Band-6 requires same number of multiplications after optimization. The signal flow diagrams are shown in Figs. 2 and 3 respectively.

- **Band-1** $X(2)$ and $X(3)$: Without any optimization the number of multiplications required is 10. However using transformation of twiddle factors the complex multiplications is reduced to 4.
- **Band-2** $X(4)$ and $X(5)$: Without any optimization the number of multiplications required is 6. However using transformation of twiddle factors the complex multiplications is reduced to 3.
- **Band-5** $X(10)$ and $X(11)$: Without any optimization the number of multiplications required is 10. However using transformation of twiddle factors the complex multiplications is reduced to 4.
- **Band-6** $X(12)$ and $X(13)$: Without any optimization the number of multiplications required is 6. However using transformation of twiddle factors the complex multiplications is reduced to 3.

3.1.2. Double Bands

When we want to compute two different bands, the minimum overlaps in terms of number of multiplications are between
Fig. 4: Signal Flow diagram for calculating bands-1, 2, 5, 6 using pushing

bands $1 - 2$, $1 - 6$, $2 - 5$ and $5 - 6$. The number of multiplications required for computing any of the above pair of bands is $3 + 4 = 7$. However, computing bands $1 - 5$ requires only 4 multiplications and computing bands $2 - 6$ requires 3 multiplications.

3.1.3. All Sub-Bands
The number of multipliers required is 7 in this case. The optimized signal flow diagram is shown in Fig. 4.

3.1.4. Using $2^2$ Structure
Here the number of multipliers required is one more than in the previous part i.e., 8. The signal flow graph is shown in Fig. 5.

3.2. 32-Point FFT
Based on the optimized 16-point FFT structure, 32-point FFT structure for bands $\{1,2,5,6\}$ can be easily calculated by adding one extra stage in the beginning. In this subsection, we compare the un-optimized, 16-point optimized FFT, optimized 32-point FFT (using radix-$2^2$ and pushing) for computing the sub-bands of 32-point FFT.

Fig. 5: Signal Flow diagram for calculating bands-1, 2, 5, 6 using $2^2$

Fig. 6: Signal Flow diagram for calculating each of bands-1, 2, 5, 6 using $2^2$ and pushing

Fig. 7: Signal Flow diagram for calculating bands-1, 2, 5, 6 using $2^2$ and pushing

3.2.1. Regular FFT Structure
For computing single and two bands, it requires $14 + 16 = 30$ multiplications. However, to compute all of the sub-bands, the number of multipliers needed is 34.

3.2.2. Extending 16-Point FFT to 32-Point FFT
To compute all of the sub-bands, number of multiplications required is $14 + 2 \times 7 = 28$

3.2.3. Optimizing 32-Point FFT using $2^2$ and pushing
For computing single and two bands, it requires only 12 multiplications as shown in Fig. 6. However, after optimizing, the structure itself requires 24 multiplications to calculate all of these sub-bands. This is shown in Fig. 7. In this case, both pushing and $2^2$ result in same number of multiplications.
3.3. 64-Point FFT

In order to derive efficient structures for computing 64-point FFT sub-bands (all of \{1,2,5,6\}), we only use efficient structures from 16-point and 32-point. Note that we need to have 2 32-point FFT and one butterfly stage at the beginning.

3.3.1. Regular FFT Structure
The number of multipliers needed is 98

3.3.2. Extending 16-Point FFT to 64-Point FFT
Number of multiplications required is 30+ 2x28 = 86

3.3.3. Extending Optimized 32-Point FFT to 64-Point FFT
Number of multiplications required is 30+ 2x24 = 78

4. EXTENSION TO $2^N$

We know that fast FFT structures require $N^2 \log_2 N$ number of multiplications. As shown in the previous section, by using the optimized 64-point FFT structure, we only require $N^2 \log_2 (N)-6]+78$ number of multiplications to calculate all of the \{1,2,5,6\} sub-bands. However, it can be noted that this is the actual upper bound of number of multipliers required. The actual number of multipliers is even less for real inputs.

We show the number of multipliers in Table 1 and Fig. 8.

| N-Point FFT | #Ms Unoptimized | #Ms Optimized |
|-------------|-----------------|---------------|
| 16          | 32              | 7             |
| 32          | 80              | 24            |
| 64          | 192             | 78            |
| 128         | 448             | 142           |
| 256         | 1024            | 334           |
| 512         | 2304            | 846           |
| 1024        | 5120            | 2126          |

Table 1: Comparison of results

5. MERGE-FFT

The number of multiplications can be further reduced for computing a large $N(= 2^n)$-Point FFT by using merge-FFT [21]. Suppose that we have sequences for two $N/2$ point FFTs. We can merge these two FFT sequences to create a sequence corresponding to $N$-Point as follows. We assume two $N/2$ point sequences are given by $X_1(k)$ and $X_2(k)$. Also, the $N$ point sequence is given by $X(k)$. Then following [21],

$$X(2u) = X_1(u) + X_2(u) \quad (2)$$

$$X(2u + 1) = X_1(u + \frac{1}{2}) - X_2(u + \frac{1}{2}) \quad (3)$$

Implementing equation (2) does not require any multipliers. For implementing equation (3), we require a fractional delay filter of delay $D = \frac{1}{2}$. The ideal response of this filter is given by

$$h_{id}[n] = \frac{\sin(\pi(n-D))}{\pi(n-D)} \quad (4)$$

Equation (5) can be implemented using an $L^{th}$ order least-square FIR filter given by

$$h_{FIR}[n] = sinc(n - D + \frac{L}{2} + 1) \quad 0 \leq n \leq L \quad (5)$$

In our implementation, we used $L = 6$. The filter is then given by $h_{FIR} = \{0.1273, -0.2122, 0.6366, 0.6366, -0.2122, 0.1273\}$. This is a symmetric filter and can be implemented efficiently with 3 real constant multipliers. The implementation is shown in Fig. 9.
5.1. Comparison of FFT for 1,2,5,6 Bands using merge-FFT

We use one EEG channel data from kaggle seizure prediction competition to compare the magnitude of FFT computed for bands 1,2,5,6 using merge-FFT and optimized-FFT from previous section. The result is shown in Fig. 10.

5.2. Comparison between Merge-FFT and Optimized-FFT

As discussed before, using merge-FFT can replace the number of complex multipliers in the first stage using 3-constant real multipliers. Hence we only require $\sum \log_2(N) + 78$ complex multipliers in the FFT structure to compute bands \{1,2,5,6\}. The comparison between number optimized FFT multiplication and number of multiplications for merge-FFT is shown in Fig. 11.

5.3. Conclusion and Future Work

In this report we performed a theoretical analysis of twiddle factor (complex multiplications) reduction while computing 4 sub-bands in case of $N$-point ($N = 2^n$) FFTs. We derived the theoretical upper bound for the number of multipliers needed for computing the sub-bands \{1,2,5,6\}. In future, we plan to implement this structure in VLSI-hardware to estimate the power consumption for large $N$-point FFTs. This implementation will complement our theoretical analysis described in this report.

6. REFERENCES

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