We briefly report on our method [1] of simplifying the equations of motion of charged particles in an electromagnetic (EM) field that is the sum of a plane travelling wave and a static part; it is based on changes of the dependent variables and the independent one (light-like coordinate $\xi$ instead of time $t$). We sketch its application to a few cases of extreme laser-induced accelerations, both in vacuum and in plane problems at the vacuum-plasma interface, where we are able to reduce the system of the (Lorentz-Maxwell and continuity) partial differential equations into a family of decoupled systems of Hamilton equations in 1 dimension. Since Fourier analysis plays no role, the method can be applied to all kind of travelling waves, ranging from almost monochromatic to so-called impulses”.

Here we summarize a new approach [1] that is especially fruitful if in the spacetime region $\Omega$ of interest (i.e., where we wish to follow the charged particles’ worldlines) $\mathbf{E}, \mathbf{B}$ can be decomposed into a static part and a plane transverse travelling wave propagating in the $z$ direction:

$$
\mathbf{E}(x) = \frac{\epsilon^+(ct-z)}{\sqrt{m^2c^2+p^2}} + \mathbf{E}_s(x), \quad \text{pump=travelling wave} \quad \text{static}
$$
$$
\mathbf{B}(x) = k \wedge \epsilon^+(ct-z) + \mathbf{B}_s(x), \quad \text{travelling wave} \quad \text{static}
$$

$x=x_1+y_1+z_1k$, $\epsilon^+ \perp \mathbf{k}$. We decompose vectors as $\mathbf{u} = \mathbf{u}^+ + u^2 \mathbf{k}$. We assume only that $\epsilon^+(\xi)$ is piecewise continuous and

1. $\epsilon^+$ has a compact support $[0, l]$, or $\mathbf{a}$: $\epsilon^+ \in L^1(\mathbb{R})$,

or $\mathbf{a'}$: $\epsilon^+ \in L^1(\mathbb{R})$,

$$
\Rightarrow \alpha^+(\xi) \equiv -\int_{-\infty}^{\xi} dy \epsilon^-(y) \rightarrow 0 \quad \text{as} \ \xi \rightarrow -\infty;
$$

2. $\alpha^+$ is the travelling-wave part of the transverse EM potential $\mathbf{A}^+$. $\mathbf{a'} \Rightarrow \alpha^+(\xi) = 0$ if $\xi \leq 0$, $\alpha^+(\xi) = \alpha^-(l)$ if $\xi \geq l$. We can treat on the same footing all such $\epsilon^+$, in particular,

1. A modulated monochromatic wave fulfilling (3):

$$
\epsilon^+(\xi) = \epsilon(\xi) \left( \begin{array}{c} i \alpha_1 \cos(k\xi + \varphi) + i \alpha_2 \sin(k\xi) \\ \text{modul.} \\
\end{array} \right) \quad \epsilon^+_s(\xi)
$$

$$
\Rightarrow -\alpha^+(\xi) = \frac{\epsilon(\xi)}{k^2} \epsilon^+_s(\xi) + O \left( \frac{1}{k} \right) \approx \frac{\epsilon(\xi)}{k^2} \epsilon^+_s(\xi):
$$

3. $\text{An ‘impulse’ (few cycles, or even a fraction of)}$. 

$$
\Rightarrow \text{modul.} \quad \text{carrier wave} \epsilon^+(\xi)
$$

$$
\Rightarrow \text{modul.} \quad \text{carrier wave} \epsilon^+(\xi)
$$

$$
\Rightarrow \text{modul.} \quad \text{carrier wave} \epsilon^+(\xi)
$$

$$
\Rightarrow \text{modul.} \quad \text{carrier wave} \epsilon^+(\xi)
$$
the unknown $\beta$ is the EM potential 1-form, $E \cdot x$ is the particle proper time) by
\[ \hat{\lambda} \equiv \frac{\partial H}{\partial \dot{\gamma}} \] while the energy gain (normalized to $mc^2$) is
\[ E \equiv \frac{\hat{H}(\xi)}{mc^2} = \int_{\xi_0}^{\xi_1} dq^+ \frac{\dot{\gamma}}{s}. \tag{13} \]
in the interval $[\xi_0, \xi_1]$. Once solved for $E_s, B_s = \text{const}$ then eq. (12) are solved by
\[ \hat{u}^+ = \frac{q}{mc^2} [K - \alpha(\xi) + \xi \beta E_s^+ + \beta B_s], \]
\[ \hat{s} = \frac{-q}{mc^2} [K^+ + \xi E_s^+ - \hat{\gamma} \hat{E}_s^+ + (\hat{\gamma} \hat{B}_s^+)^2] \tag{14} \]
$(K^\pm)$ are integration constants) whereby (9) become three 1st order ordinary differential equations (ODE) rational in the unknown $\hat{x}(\xi)$. Contrary to (9)[12], (11) is a transcendental system, and the unknown $z(t)$ appears in the argument of the rapidly varying functions $e^{+}, \alpha^{+}$ in (11), which now reads:
\[ \frac{1}{q} p(t) = E_s + \beta B_s + \epsilon [-ct + z(t)] (\beta k + 1 - \beta z). \]
Also determining $E(t)$ is more complicated.

A. Dynamics under $A^\mu = A^\mu(t,z)$

This applies in particular to [2] if $E_s = E_s^+ z|k, B_s = B_s^+ (z)$, choosing e.g. $A^0 = -\int dz E_s^+(\xi), A^+ = \alpha^+$.
\( k \wedge \int d\zeta B^\perp(\zeta), \ A^2 = 0. \) As \( \partial \hat{H}/\partial \bar{x}^\perp = 0, \) we find \( \hat{H}^\perp = qK^\perp = \text{const, i.e. the known result} \) \( \frac{mc^2}{q}\bar{u}^\perp = K^\perp - \hat{A}(\xi, \bar{z}). \) Setting \( \nu := \bar{u}^\perp \) and replacing in \( [6], [12] \) we obtain

\[
\bar{s}' = \frac{1 + \dot{\bar{v}}}{2s^2} - \frac{1}{2} \quad \bar{s}' = -\frac{q}{mc^2} E_s^\perp(\bar{z}) - \frac{1}{2s^2} \frac{\theta}{s^2} = 1. \tag{15}
\]

Once solved the system \( [15] \) in the unknowns \( \bar{z}(\xi), \bar{s}(\xi), \) the other unknowns are obtained from

\[
\bar{x}(\xi) = x_0 + \hat{\bar{y}}(\xi), \quad \hat{\bar{y}}(\xi) = \int_0^\xi \frac{dy}{\bar{s}(y)}. \tag{16}
\]

If in addition \( B_s = 0, \) then \( A_s = 0 \) (in the Couboum gauge), \( \bar{u}^\perp(\xi) = \frac{q}{mc^2} [K^\perp - \alpha^\perp(\xi)] \) and \( \bar{v} = \bar{u}^\perp \) are already known. The system \( [15] \) to be solved simplifies to

\[
\bar{s}' = \frac{1 + \dot{\bar{v}}}{2s^2} - \frac{1}{2} \quad \bar{s}' = -\frac{q}{mc^2} E_s^\perp(\bar{z}). \tag{17}
\]

Some remarkable properties of the solutions are \[11]:

1. Where \( \epsilon^\perp(\xi) = 0 \) then \( \dot{\bar{v}}(\xi) = v_c = \text{const, } \hat{\bar{H}} \) is conserved, \( \bar{v} \) is solved by quadrature.

2. The final transverse momentum is \( mcu^\perp(\xi_f) \). If \( \epsilon \) of \( [9] \) varies slowly and \( \bar{u}(0) = 0 \), then \( \bar{u}^\perp(\xi_f) \approx 0 \).

3. \( \bar{s}(\xi) \) is insensitive to fast oscillations of \( \epsilon^\perp \), contrary to \( u, \gamma, \beta \), which can be reobtained via \[8\].

### III. SOME EXACT SOLUTIONS FOR \( B_s, E_s = \text{CONST} \)

Let \( b^\perp + bk \equiv qB_s/mc^2, \ c^\perp \equiv qE_s/mc^2 \) (constants), \( w(\xi) \equiv q[K - \alpha^\perp(\xi) + \xi E_s]/mc^2 \) (all dimensionless); \( [14] \) take the more explicit form

\[
\begin{align*}
\dot{\bar{u}}^x &= (e^x - b^y)\bar{z} + b^y\bar{w}^x(\xi), \\
\dot{\bar{u}}^y &= (e^y - b^x)\bar{z} - b^x\bar{w}^y(\xi), \\
\bar{s} &= (e^x - b^y)\bar{x} + (e^y - b^x)\bar{y} - w^z(\xi),
\end{align*}
\]

For any \( E_s^\perp, B_s^\perp, c_s^\perp, \) if \( B_s^\perp = k \wedge E_s^\perp \), setting \( \kappa = \frac{gE_s^\perp}{mc^2} \), we find the following exact solutions (parts of them are new):

\[
\begin{align*}
(\dot{x} + i\bar{y})(\xi) &= (1 - \kappa \xi) e^{ib(\xi)}(1 - \kappa \xi)^{1/2} i \int_0^\xi \frac{d\zeta}{(1 - \kappa \zeta)^{1/2}} (w^x + i\bar{w}^y)(\zeta), \\
\dot{z}(\xi) &= \frac{1}{2} \left( 1 - \kappa \xi \right)^{-2} + i\bar{x}^2(\xi) - 1, \quad \dot{s}(\xi) = 1 - \kappa \xi, \tag{19}
\end{align*}
\]

\[
\begin{align*}
\bar{u}^x(\xi) &= (1 - \kappa \xi) \bar{x}^x(\xi), \quad \bar{u}^y(\xi) = (1 - \kappa \xi) \bar{x}^y(\xi), \\
\bar{z}^x(\xi) &= \frac{1}{2(1 - \kappa \xi)} + (1 - \kappa \xi) \left( \bar{x}^x(\xi) - 1 \right)/2;
\end{align*}
\]

here we have adopted the initial conditions \( x(0) = 0 = u(0) \). We next analyze a few special cases.

#### A. Case \( E_s = 0, \ B_s = 0 \) (zero static fields)

Then \( [19] \) becomes \( [7], [8] \):

\[
\begin{align*}
\dot{s} &= 1, \quad \bar{u}^x = -\frac{q}{mc^2}, \quad \bar{u}^y = \frac{\bar{u}^z}{2}, \quad \bar{z} = 1 + \bar{u}^z \tag{20}
\end{align*}
\]

The solutions \( [20] \) induced by two \( x \)-polarized pulses and the corresponding \( \epsilon^\perp \) trajectories in the \( zx \) plane are shown in fig. 2. Note that:

- The maxima of \( \gamma, \alpha^\perp \) coincide and (approximately also of \( \epsilon(\xi) \), if \( \epsilon(\xi) \) is slowly varying).

- Since \( u^\perp \geq 0 \), the \( z \)-drift is positive-definite. Rescaling \( \epsilon^\perp \rightarrow \alpha^\perp, \bar{z}, \bar{u}^\perp \) scale like \( a \), whereas \( \bar{z}, \bar{u}^\perp \) scale like \( a^2 \) (hence the trajectory goes to a straight line in the limit \( a \rightarrow \infty \)). This is due to magnetic force \( qB \wedge k \).

#### Corollary

The final \( v_\perp \) and energy gain read

\[
\begin{align*}
\bar{u}^x = \bar{u}^\perp \rightarrow \infty, \quad u_f^\perp = E_f = \frac{1}{2} u_f^2 = \gamma_f - 1; \tag{21}
\end{align*}
\]

Both are very small if the pulse modulation \( \epsilon \) is slow [extremely small if \( \epsilon \in S(\mathbb{R}) \) or \( \epsilon \in C^\infty(\mathbb{R}) \)].

Recall the Lawson-Woodward Theorem \[10, 13\] (an outgrowth of the original Woodward-Lawson Theorem \[14, 15\]): in spite of large energy variations during the interaction, the final energy gain \( E_f \) of a charged particle \( P \) interacting with an EM field is zero if:

- i) the interaction occurs in R^3 vacuum (no boundaries);
- ii) \( E_s = B_s = 0 \) and \( \epsilon^\perp \) is slowly modulated;
- iii) \( v^\perp \approx c \) along the whole acceleration path;
- iv) nonlinear (in \( \epsilon^\perp \)) effects \( qB \wedge k \) are negligible;
- v) the power radiated by \( P \) is negligible.

Our Corollary, as Ref. \[9\], states the same result if we relax iii), iv), but the EM field is a plane travelling wave.

To obtain a non-zero \( E_f \) one has to violate some other conditions of the theorem, as e.g. we see in next cases.

#### B. Case \( E_s = 0, \ B_s = B_s^\perp k \)

Then \( [19] \) becomes \( \dot{s} = 1 \) and

\[
\begin{align*}
(\dot{x} + i\bar{y})(\xi) &= \int_0^\xi d\zeta e^{ib(\zeta)}(w^x + i\bar{w}^y)(\zeta), \quad \bar{u}^x = \bar{x}^x, \tag{22}
\end{align*}
\]

\[
\begin{align*}
\bar{u}^x &= \frac{\bar{u}^z}{2}, \quad = E = \bar{e} = 1, \quad \bar{z}(\xi) = \int_0^\xi d\zeta \bar{u}^z(\zeta). \tag{22}
\end{align*}
\]

\( [22] \) reduces to the solution of \( [16], [17] \) for monochromatic \( \epsilon^\perp \). This leads to cyclotron autodopamine if \( -b = k = \frac{2\pi}{\lambda} \gg 1 \); for circular polarization \( w^x(\xi) + i\bar{w}^y(\xi) \approx e^{i\kappa \xi} w(\xi) \)

\[
\begin{align*}
(\dot{x} + i\bar{y})(\xi) &\approx iW(\xi)e^{i\kappa \xi}, \quad W(\xi) = \int_0^\xi d\zeta w(\zeta) > 0
\end{align*}
\]
FIG. 2. Solutions (20) and $e^-$ trajectories in the zx plane induced by two x-polarized pulses with carrier wavelength $\lambda = 0.8 \mu m$, gaussian modulation $\epsilon(\xi) = a \exp[-\xi^2/2\sigma^2]$, $\sigma = 20 \mu m^2$, $ea\lambda/mc^2 = 4.15$ (left, right).

where $w(\xi) \equiv q\epsilon(\xi)/kmc^2$; clearly $W(\xi)$ grows with $\xi$. In particular if $\epsilon^+(\xi) = 0$ for $\xi \geq l$, then for such $\xi$

$$W'(\xi) \approx \frac{k^2}{2} W^2(l) \approx 2E_f, \quad \frac{|\dot{x}'(\xi)|}{\dot{x}'(\xi)} \approx \frac{2}{kW(l)} \ll 1;$$

C. Case $E_s = E_z^+ k, B_s = 0$

Then the solution (19) reduces to $\dot{s}(\xi) = 1 - \kappa \xi,$

$$(\dot{x} + i\dot{y})(\xi) = \int_0^{\xi} dy \frac{w^+ (w^+ w^+)(y)}{1 - \kappa y}, \quad \dot{z}(\xi) = \int_0^{\xi} dy \left\{ \frac{1 + \dot{z}(y)}{1 - \kappa y^2} - 1 \right\};$$

If $\epsilon^+$ is slowly modulated the energy gain (13) $E_f$ is negative if $\kappa > 0$, positive if $\kappa \leq 0$ and has a unique maximum point $\kappa_M < 0$ if $\epsilon(\xi)$ has a finite support with a unique maximum. Here is an acceleration device based on this solution: at $t = 0$ the particle initially lies at rest with $z_0 \lesssim 0$, just at the left of a metallic grating $G$ contained in the $z = 0$ plane and set at zero electric potential; another metallic plate $P$ contained in a plane $z = z_p > 0$ is set at electric potential $V = V_p$. A short laser pulse $\epsilon^-$ hitting the particle boosts it into the latter region through the ponderomotive force; choosing $qV_p > 0$ implies $\kappa = -qV_p/z_p mc^2 < 0$, and a backward longitudinal electric force $qE_z^+$. If $qV_p$ is large enough, then $z(t)$ will reach a maximum smaller than $z_p$, then is accelerated backwards and exits the grating with energy $E_f$ and negligible transverse momentum. A large $E_f$ requires extremely large $|V_p|$, far beyond the material breakdown threshold, what prevents its realization as a static field (namely, sparks between $G, P$ would arise and rapidly reduce $|V_p|$). A way out is to make the pulse itself generate such large $|E_z^+|$ within a plasma at the right time so as to induce the it slingshot effect, as sketchily explained at the end of next section.

IV. PLANE PLASMA PROBLEMS

Assume that the plasma is initially in hydrodynamic conditions with all initial data (velocities, densities $n_h$, EM fields of the form (2) not depending on $x^\perp$. Then
also the solutions for $\mathbf{B, E, u_h, n_h, \Delta x_h} \equiv x_h(t, \mathbf{X}) - \mathbf{X}$ (displacements) do not depend on $\mathbf{x^+}$. Here $x_h(t, \mathbf{X})$ is the position at $t$ of the $h$-th fluid material element with initial position $\mathbf{X} \equiv (X, Y, Z)$; $\mathbf{X}_h(t, \mathbf{x})$ is the inverse (at fixed $t$). More specifically, we consider the impact of an EM plane wave with a pump of the type (3a) on a cold plasma at equilibrium (figure below); the initial conditions are:

$$u_h(0, \mathbf{x}) = 0, \quad n_h(0, \mathbf{x}) = 0 \quad \text{if} \quad z \leq 0,$$

$$j^0(0, \mathbf{x}) = \sum_h q_h n_h(0, \mathbf{x}) = 0,$$ 

$$\mathbf{E}(0, \mathbf{x}) = \mathbf{\epsilon}(z), \quad \mathbf{B}(0, \mathbf{x}) = \mathbf{k} \wedge \mathbf{\epsilon}(-z) + \mathbf{B}_0.$$ 

Then Maxwell eqs $\nabla \cdot \mathbf{E} = 4\pi j^0, \partial_t \mathbf{E}/c + 4\pi j^2 = \nabla \times \mathbf{B} = 0$ (the current density is $j = \sum_h q_h n_h \mathbf{\beta}_h = \sum_h q_h n_h u_h/m_c$) imply

$$\mathbf{E}(t, z) = 4\pi \sum_h q_h N_h[Z_h(t, z)],$$

where $N_h(Z) = \int_0^Z d\zeta n_h(0, \zeta)$: we thus reduce by one the number of unknowns, expressing $E^\perp$ in terms of the (still unknown) longitudinal motion. $\mathbf{A}^+\perp$ is coupled to the currents through $\nabla \times \mathbf{A}^+ = 4\pi j^+\perp$ (in the Landau gauges). Including (24) this amounts to the integral equation

$$\mathbf{A}^+ - \mathbf{\alpha}^+ \frac{\mathbf{B}_0}{2 \times c} \wedge \mathbf{x} = 2\pi \int dk d\zeta \theta(\mathbf{ct} - s - |z - \zeta|) \theta(s) j^\perp(s/c, \zeta).$$

The right-hand side (rhs) is zero for $t \leq 0$, when $t = 0$ is the beginning of the laser-plasma interaction. Within short time intervals $[0, t']$ (to be determined $a\ posteriori$) we can approximate $\mathbf{A}^+(t, z) \simeq \alpha^+(ct - z) + \frac{\mathbf{B}_0}{2 \times c} \wedge \mathbf{x}$; we also neglect the motion of ions with respect to that of electrons. Then the Hamilton equations for the electron fluid with ‘time’ $\xi$ and the initial conditions amount to

$$m_c^2 \ddot{\mathbf{x}}_e(\xi, Z) = 4\pi e^2 \left[ \overline{N}(\mathbf{z}_e) - \overline{N}(Z) \right] + e(\dot{\mathbf{x}}^\perp_e + \mathbf{\beta}_0)^{\perp},$$

$$m_c^2 \ddot{\mathbf{u}}_e(\xi, Z) = e\alpha^+ - e(\dot{\mathbf{x}}^\perp_e + \mathbf{\beta}_0)^{\perp},$$

$$\dot{\varphi}_e(0, \mathbf{X}) = \mathbf{X}, \quad \dot{u}_e(0, \mathbf{X}) = 0 \Rightarrow \dot{s}_e(0, \mathbf{X}) = 1.$$ 

this is a family parametrized by $Z$ of decoupled ODEs which can be solved numerically. The approximation on $\mathbf{A}^+(t, z)$ is acceptable as long as the so determined motion makes $|\text{rhs}(26)| \ll |\alpha^+ + B_0/2 \times c|$: otherwise rhs(26) determines the first correction to $\mathbf{A}^+$ and so on.

If $\mathbf{B}_0 = 0$, again (27) is solved by $\mathbf{u}_e(\xi) = e\epsilon^+(\xi)/m_c^2$, while, setting $v = u^\perp\mathbf{e} - \mathbf{\beta}_0$, (27) take (17)

$$\Delta \dot{\mathbf{s}}_e = \frac{1 + v}{2s^2} - \frac{1}{2}, \quad \dot{s}_e = \frac{4\pi e^2}{m_c^2} \left\{ \overline{N}(\mathbf{z}_e) - \overline{N}(Z) \right\}.$$ 

If $n_e(0, \mathbf{X}) = n_0 = \text{const}$ for $Z \geq 0$, then as long as $\dot{s}_e(\xi, Z) > 0$ (29), (28) reduce to the same Cauchy problem for all $Z$:

$$\Delta' = \frac{1 + v}{2s^2} - \frac{1}{2}, \quad s' = M\Delta,$$

$$\Delta(0) = 0, \quad s(0) = 1$$

with $M = \frac{4\pi e^2 n_0}{m_c^2}$. In fig. 4, we show the solution if $\epsilon^+$ is as in fig. 3 and $n_0 = 2 \times 10^{19} \text{cm}^{-3}$; $s(\xi)$ is indeed insensitive to the fast oscillations of $\epsilon^+$ (see section II A). After the pulse is passed it becomes periodic: a plasma travelling-wave of spacial period $\xi_H \simeq 49\mu m$ follows the pulse. The other unknowns are obtained through (10). Replacing in rhs(26) we find that $\mathbf{A}^+ \simeq \alpha^+$ is verified at least for $t \leq 5\xi_H/c$.

The above results are based on a laser spot size $R = \infty$ (plane wave). When including corrections due to the finite $R$ (based on causality and heuristic estimates), they imply: the impact of a very short and intense laser pulse on the surface of a cold low-density
plasma (or gas, ionized into a plasma by the pulse itself) may induce (for carefully tuned $R$), beside a wakefield propagating behind the pulse \cite{23,24}, also a backward acceleration and expulsion of surface electrons \cite{20,21} (slinghot effect), as schematically depicted in fig. \ref{fig:4}.

For reviews see also \cite{22}.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Solution of \eqref{eq:29} corresponding to the pulse of fig. \ref{fig:3}, initial density $\tilde{n}_0(Z) = n_0\theta(Z)$, $n_0 = 2 \times 10^{18}\text{cm}^{-3}$.}
\end{figure}

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FIG. 5. Schematic stages of the slingshot effect.

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