An Efficient Observer Design for Multi Leak Detection and Isolation in Water Supply Networks

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(2nd International Conference on Access to Recent Advances in Engineering and Digitalization (ARACONF)-10–12 March 2021)

\textit{(DOI: 10.31590/ejosat.903122)}

Abstract

This paper introduces an efficient observer design for leak detection and isolation of pipeline networks. The designed observer can be applied to the multistage pipeline networks with distributed leaks. The position and flow rate of the several leaks can be estimated efficiently. The designed observer is based on the discretized model of the system so called "Discretization Based Observer". For computer applications, two leaks with different positions and flow rates have been considered in a 150 meter pipe. After 5 seconds measurements and estimations, accurate leak estimations have been obtained. Comparing to the literature observers, simultaneous state and parameter estimations are obtained with fast convergence by the designed observer.

Keywords: Observer design, Pipeline networks, Leak detection.

Su Şebekelerinde Çoklu Sızıntı Tespiti ve İzolasyonu için Etkin Bir Gözetleyici Tasarımı

Öz

Bu çalışma, boru hattı ağlarının sııntı tespiti ve izolasyonu için verimli bir gözlemevi tasarıımı sunmaktadır. Tasarlanan gözlemevi, dağıtık şekilde bulunan sııntıtlara sahip çok katmanlı boru hattı aşırlarına uygulanabilir. Birbirinden farklı sııntıların konumü ve akış hızı verimli bir şekilde tahmin edilebilir. "Ayrıklaştırma Temelli Gözetleyici" olarak adlandırılan tasarlanan gözlemevi sistemin ayrıklarınımlı modeline dayanmaktadır. Bilgisayar uygulamaları için, 150 metrelık bir boruda farklı komularda ve akış hızlarına sahip iki sııntı dikkate alınmıştır. Ölçüm ve kestirimlerden 5 saniye sonra doğru sııntı tahminleri elde edilmiştir. Literatür gözlemevicileri ile karşılaştırıldığında, tasarlanan gözlemevi tarafından hızlı yakınsama ile eşzamanlı durum ve parametre tahminleri elde edilir.

Anahtar Kelimeler: Gözetleyici tasarımı, Boru hattı aşırları, Sızıntı tespiti.

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1. Introduction

With the development of large pipelines, the leaks that may occur in the networks. The detection and isolation of leaks have become problems that need to be investigated since they can not only lead to losses, but can also lead to serious environmental problems. For this reason, the leak detection and isolation is an important for multistage pipeline networks. In pipelines, leaks can be detected by calculating physical effects from the mathematical model of pipelines and monitoring their measurements directly. There are many studies in the literature that investigate fault detection and analysis or isolation processes using model-based approaches [Isermann, 1984, Gertler, 1988, Chen and Patton, 2012].

During the past 30 years, effective methods have been developed to detect and isolate leaks using model-based methods. A leakage detection mechanism based on a non-linear model with very high sensitivity to the uncertainty of the leak location was proposed in [Billmann and Isermann, 1987]. Nonlinear observers were used to generate residuals in [Shields et al., 2001], although it is disadvantageous from a practical point of view. A model-based approach has been proposed to locate leaks in a pipeline using flow and pressure sensors in [Verde, 2005]. Aamo has designed a leak detection system in which flow dynamics are managed with an adaptive observer [Aamo, 2016]. In [Navarro et al., 2017], real-time leak isolation with fitting loss coefficient calibration in a pipeline was designed.

The detection and isolation problem of leaks may be solved with a nonlinear observer that is robust and sensitive to the leaks. The results obtained in [Ashton et al., 1998] show that the problem of detecting a leak can also be solved by a nonlinear observer. Most of the results obtained through the model-based approach are based on observer design. In many studies, observer design is usually performed either based on the linearized model of the pipeline or based on the nonlinear model of the networks [Verde, 2004]. In [Aamo et al., 2006], an adaptive observer using heuristic methods for updating laws was proposed. This Luenberger-type observer is capable of determining the flow rate and location of a single-point leak. This method was refined further in [Hauge et al., 2009].

In control applications, besides state and parameter estimation, many issues such as fault detection, fault isolation, disturbance and unknown input estimation are performed on the basis of observers. The requirement of an observer design is often needed in the lack of sensors or physical constraints. In the literature, there are many linear/nonlinear observers developed for such problems and based on the mathematical model of the system. First, observers used for state estimation in linear systems have been introduced in [Luenberger, 1966]. Then, these observers were detailed and extended for nonlinear systems [Thau, 1973]. Research on the state estimation problems in nonlinear systems continued with extended Luenberger observer [Birk and Zeitz, 1988], Kalman-type filters [Cox, 1964, Simon, 2006], sliding-mode observers [Drakunov, 1983, Slotine et al., 1987], high-gain observer [Gauthier et al., 1992], Takagi-Sugeno fuzzy observers [Tanaka and Wang, 1997], discretization-based gradient observer [Iplikci, 2013, Beyhan, 2013] and etc.

In this paper, an efficient nonlinear observer namely "discretization based nonlinear gradient observer" (DBGO) [Beyhan, 2013, Cetin, 2017] is designed for the estimation of leak position and flow-rate. Designed with the discretized nonlinear model of the system, DBGO provides fast convergence of error dynamics. Therefore, DBGO achieves better state and parameter estimation results [Beyhan, 2013, Cetin et al., 2017]. In numerical applications, two cascaded leaks are investigated where the positions and flow-rates of cascaded leaks are estimated very efficiently. In nonlinear dynamics of the pipeline, the leak position is a parameter and the flow-rate is a state. Therefore, DBGO provides state/parameter estimation very efficiently. The rest of the paper is organized as follows. The mathematical modeling of the pipeline networks are given in Section 2. Section 3 presents the proposed observer dynamics. The application results of the observer is given in Section 4. Section 5 concludes the paper.

2. Distributed and Lumped Parameter Models of Pipeline

Considering that for a pipeline of length L, convective changes in velocity, compressibility, variations in cross-sectional area or fluid density are neglected, the dynamics of the fluid in this pipeline can be defined as [Chaudhry, 1979]:

$$\frac{\partial H}{\partial t} = -\frac{c^2}{gA} \frac{\partial Q}{\partial z}$$

$$\frac{\partial Q}{\partial t} = -gA \frac{\partial H}{\partial z} - fQ|Q|$$

where $H$ is the piezometric pressure head, $Q$ is the flow rate, $t$ is the time coordinate, $z$ is the length coordinate, $A$ is the sectional flow area, $D$ is the diameter of pipe, $c$ is the pressure wave velocity, $g$ is the gravity acceleration, $f$ is the Darcy-Weisbach friction factor, respectively. A leakage flow ($Q_{zi}$) that may occur at any coordinate of the pipeline ($z_i$) can be associated with the square root of the piezometric pressure head at the related coordinate as,

$$Q_{zi}(t) = \lambda \sqrt{H(t)}$$

where $\lambda$ is the leak coefficient and the model of the leakage flow at point $i$ in the pipeline can be defined as

$$Q_{zi} = Q_{zi}^2 - Q_{zi}^2$$

where $Q_{zi}$ is the leakage flow at point $i$, $Q_{zi}^2$ and $Q_{zi}^2$ are the flow before and after the point $i$ respectively. To figure out the location of the leakage, the pipeline can be divided into sub partitions in space. The discretized form of the pipeline is seen in Figure 1. In this form, time is kept as a continuous variable and there are 7 variables, $H_1, ..., H_4, Q_1, ..., Q_3$ have to taken into account. Here, one can be consider that $Q_1, Q_3$ are measured directly and $H_1, H_4$ are the control variables.

![Figure 1: Discretized model of the pipeline.](image)

e-ISSN: 2148-2683
The state space system can be defined as \( x = [Q_1, H_2, Q_2, H_3, Q_3] \), \( u = [H_1, H_4] \), \( y = [Q_1, Q_3] \) and the model [Besançon, 2007]:

\[
\begin{align*}
\dot{Q}_1 &= -\mu Q_1^2 + \frac{a_1}{z_1} (u_1 - H_2) \\
\dot{H}_2 &= \frac{a_2}{z_2} (Q_1 - Q_2) \\
\dot{Q}_2 &= -\mu Q_2^3 + \frac{a_1}{z_2} (H_2 - H_3) \\
\dot{H}_3 &= \frac{a_2}{z_2} (Q_2 - Q_3) \\
\dot{Q}_3 &= -\mu Q_3^2 + \frac{a_1}{z_3} (H_3 - u_2) \\
y &= \begin{bmatrix} Q_1 \\ Q_3 \end{bmatrix}
\end{align*}
\]

where \( z_1, z_2, z_3 \) are lengths of each sections in Figure 1, \( a_1 = gA \), \( a_2 = b^2/gA \) and \( \mu = f/(2DA) \). In this paper, a pipeline network was considered with the following physical parameters: length: \( L = 100m \), diameter: \( D = 0.1m \), friction factor: \( f = 0.005 \), speed of sound: \( b = 1250m/s \), sectional area: \( A = 0.08659m^2 \), gravity: \( g = 9.81m/s^2 \), respectively. Assume that a pipeline having two leaks with unknown location and magnitude. These leaks divide the pipeline into three subsections as same as the pipeline in the Figure 1. So it can be considered that the first leak occurred at a distance \( Z_1 \) and the second \( Z_1 + Z_2 \). While the total length of the pipeline is \( L = Z_1 + Z_2 + Z_3 \). Thus (4) becomes:

\[
\begin{align*}
\dot{Q}_1 &= -\mu Q_1^2 + \frac{a_1}{z_1} (u_1 - H_2) \\
\dot{H}_2 &= \frac{a_2}{z_2} (Q_1 - Q_2 - Q_{21}\sqrt{H_2}) \\
\dot{Q}_2 &= -\mu Q_2^3 + \frac{a_1}{z_2} (H_2 - H_3) \\
\dot{H}_3 &= \frac{a_2}{z_2} (Q_2 - Q_3 - Q_{22}\sqrt{H_3}) \\
\dot{Q}_3 &= -\mu Q_3^2 + \frac{a_1}{L-Z_1-Z_2} (H_3 - u_2) \\
y &= \begin{bmatrix} Q_1 \\ Q_3 \end{bmatrix}
\end{align*}
\]

where \( Z_1, Z_2 \) correspond to unknown locations and \( Q_{21}, Q_{22} \) unknown flow-rates. Now these four parameters can be estimated by using an observer to detect the leaks and then isolate them.

### 3. Discretization Based Gradient Observer

In this paper, it is considered to design an observer for the pipeline network whose mathematical model is known. Generally, a multi-input multi-output (MIMO) nonlinear system can be expressed in state space representation as:

\[
\begin{align*}
x &= f(x, u, \theta), \\
y &= g(x, u, \theta) \\
u \in U, \ x \in X, \ \forall t \geq 0
\end{align*}
\]

where \( x \in \mathbb{R}^n, u \in \mathbb{R}^r, \theta \in \mathbb{R}^q \) and \( y \in \mathbb{R}^q \) denote the state vector, input vector, parameters and output measurement vector of the MIMO system, respectively. In this study, the nonlinear system is controllable and it is assumed that \( f(\cdot) \) and \( g(\cdot) \) functions are known. In addition, the dynamics of the system can be derived according to states, control signals and parameters.

In the process of designing the nonlinear observer, the Runge-Kutta integration method was used to obtain the discrete values of the system states and parameters. If the nonlinear system in (6) is sampled in \( T_s \) sampling intervals, the discretized model at the \((n+1)\)th discrete-time index can be rewritten as follows

\[
\begin{align*}
x[n+1] &= f(x[n], u[n], \theta) \\
&= x[n] + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4), \\
y[n+1] &= g(x[n], u[n], \theta)
\end{align*}
\]

where

\[
\begin{align*}
k_1 &= T_s f(x[n], u[n], \theta) \\
k_2 &= T_s f(x[n] + 0.5k_2, u[n], \theta) \\
k_3 &= T_s f(x[n] + 0.5k_3, u[n], \theta) \\
k_4 &= T_s f(x[n] + k_4, u[n], \theta).
\end{align*}
\]

\( \dot{f} \) is the discretized model of the continuous-time system in (6). Several studies using the same discretization-based model in real-time applications as observer or controller have been proposed in the literature [Beyhan, 2013, Cetin et al., 2017, Cetin and Ipliki, 2015, Ipliki and Ballityar, 2016]. In this paper, discretized model of the continuous-time system in (7) is used to derive estimations of the unmeasurable pipeline network states and unknown system parameters in the NMPC framework. The equation (7) is available for observer (simultaneously state/parameter estimation) in discretization based model of a dynamic process. The states of the nonlinear system are estimated as follows on the estimation errors

\[
e = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \vdots \\ \hat{e}_N \end{bmatrix} = \begin{bmatrix} x_1[n+1] - \hat{x}_1[n+1] \\ x_2[n+1] - \hat{x}_2[n+1] \\ \vdots \\ x_N[n+1] - \hat{x}_N[n+1] \end{bmatrix}_{N \times 1}.
\]

When the performance index \( F \) to be minimized is defined as

\[
\begin{align*}
F(x[n], u[n]) &= e^T [n+1] e[n+1]. \\
&= e^T_1 [n+1] + \cdots + e^T_N [n+1].
\end{align*}
\]

Levenberg-Marquardt rule is used to minimize the performance index \( F \) in Eq. (9). Then, the update rule for the state estimation can be written as

\[
x[n+1] \Leftarrow x[n] - (J_x^T x + \mu_x I)^{-1} J_x^T e
\]

where \( \mu_x > 0 \) is a constant switching parameter, \( I \) is an identity matrix and \( J_x \) is the Jacobian matrix for the state estimation given by

\[
J_x = \frac{\partial e_j}{\partial x_i} = -\frac{\partial \hat{y}_i[n+1]}{\partial \hat{x}_i[n]}, \quad (i, j = 1, ..., N)
\]
In Eq. (11), $J_k$ is consisted of the partial derivatives related to the unmeasurable dynamics of the nonlinear system. \[
\frac{\partial y_{[n+1]}}{\partial \theta [n]} \]
term can be obtained by Eq. (7) with vector form as
\[
\frac{\partial y[n+1]}{\partial x[n]} = \left[ \sum_{i=1}^{N} \frac{\partial^T y[n+1]}{\partial x_i[n+1]} \frac{\partial x_i[n+1]}{\partial x_j[n]} \right]|_{x=x[n]}.
\] (12)

where
\[
\frac{\partial x_i[n+1]}{\partial x_j[n]} = \frac{\partial x_i[n]}{\partial x_j[n]} + \frac{1}{\tilde{K}_i} \frac{\partial K_1}{\partial x_j[n]} + \frac{2}{\tilde{K}_2} \frac{\partial K_2}{\partial x_j[n]} + \frac{2}{\tilde{K}_3} \frac{\partial K_3}{\partial x_j[n]}.
\] (13)

For the discretization-based model of the system, the terms in (13) are determined through the chain rule:
\[
\frac{\partial K_1, [n]}{\partial x_i[n]} = T_s \sum_{i=1}^{N} \frac{\partial f_i}{\partial x_i[n]} |_{u=u[n]}
\]
\[
\frac{\partial K_2, [n]}{\partial x_i[n]} = T_s \sum_{i=1}^{N} \frac{\partial f_i}{\partial x_i[n]} |_{u=u[n]} + \frac{1}{\tilde{K}_i} \frac{\partial K_1}{\partial x_i[n]} |_{u=u[n]}
\]
\[
\frac{\partial K_3, [n]}{\partial x_i[n]} = T_s \sum_{i=1}^{N} \frac{\partial f_i}{\partial x_i[n]} |_{u=u[n]} + \frac{2}{\tilde{K}_2} \frac{\partial K_2}{\partial x_i[n]} |_{u=u[n]}
\]
(14)

The designed observer used for unmeasurable state estimations may also be preferred for estimation of unknown system parameters. Therefore, the equations to be used in parameter estimation process are rewritten for minimize the performance index. To predict the unknown parameters of the $\theta$ in (7), it is assumed that the input and output signals are known. After the system dynamics ($x[n+1]$) are estimated with the discretized model, the estimation error vector of the parameters is updated as in (8). The estimation error vector is used this time in $J_0$ vector for the parameter estimation. $J_0$ is consisted of the partial derivatives with respect to the unknown parameters of the nonlinear system given by
\[
[J_0] = \frac{\partial e_i}{\partial \theta [n]} = \frac{\partial \tilde{y}_i[n+1]}{\partial \theta [n]}, \quad (i = 1, ..., N)
\] (15)

Then, the update rule to be used in parameter estimation can be written below
\[
\theta[n+1] \leftarrow \theta[n] - (J_0^T J_0 + \mu_0 I)^{-1} J_0^T e.
\] (16)

where $I$ is an identity matrix and $\mu_0 > 0$ is a constant. \[
\frac{\partial y_{[n+1]}}{\partial \theta [n]} \]
term can be obtained by Eq. (7) with vector form as
\[
\frac{\partial y[n+1]}{\partial \theta [n]} = \left[ \sum_{i=1}^{N} \frac{\partial^T y[n+1]}{\partial x_i[n+1]} \frac{\partial x_i[n+1]}{\partial \theta [n]} \right]|_{x=x[n]}.
\] (17)

In order to calculate partial derivatives (18) in the definition of $J_0$, the chain rule is used as in Eq. (19).

The designed observer is capable of detecting and isolating unknown leaks by directly estimating the size and location of possible leaks.

4. Numerical Applications

In this section, the estimation results of multistage pipeline network with distributed leaks have been presented. In order to obtain acceptable estimation values of unknown parameters in simulation, all states of the dynamic system are assumed to be measurable and all unknown parameters to be estimated are set to zero initially in model. In case of a leak in the pipeline, the results obtained from the observer will be different from the measurement values. The unknown part of the system dynamics are modelled with the discretization-based observer to adapt the changes in the pipeline networks.

Performance results of leakages that may occur in a 150 meter long pipeline network while the sampling time is 1ms is given in Figure 2. The accurate values of the location estimation of leaks and their magnitudes are obtained in 5 seconds at position 50m for the first leak and 80m for the second leak, respectively. All the states are initialized at zero however they reach true values in a short-time period. The designed observer is applied to a simple pipeline with two leaks. The position and flow rate of the leaks are randomly determined before the algorithm runs. The leaks has positions which are the estimated parameters, and also the unknown leaks by directly estimating the size and location of leaks can be seen in the Figgures.

According to the simulation results, it is presented that the true estimations are simultaneously obtained between 5 – 8 seconds which are better than results in [Guillen et al., 2014]. The estimation errors related to the pipeline state estimations and unknown parameter estimations are given in Figure 3(a) and Figure 3(b), respectively. The fast convergence of the errors can be seen in the Figures.
Figure 2: Parameter estimation results.

a) $Z_1$ leak position estimate.

b) $Z_2$ leak position estimate.

c) $Q_{z1}$ flow rate estimate.

d) $Q_{z2}$ flow rate estimate.

Figure 3: Estimation errors.

a) Parameter estimation errors.

b) State estimation errors.
5. Conclusions and Recommendations

Fault detection and isolation algorithms based on flow dynamics theory can be used to detect the leaks of pipeline networks to prevent safety and environmental problems. Here, discretization based gradient observer is proposed and applied for the simultaneous estimation of states and parameters in pipeline networks. The application results present the efficiency and applicability of the observer. With the designed DBGO observer, it is seen that the observer can predict where the leakage is and whatever the leakage flow-rate is. It is known that the state and parameter observers can be applied in a software such that the algorithms can be run on cheap microprocessors. Provides that there is no need to buy expensive leak detectors. In a future work, the designed observer is planned to be applied in water supply networks of a city.

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