The QCD spectrum: mixing, strong decays and the role of sea quarks

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Abstract

The light hadron spectrum as computed in nonperturbative QCD is reviewed and compared to lattice data and experiment. The mixing of mesons, hybrids and glueballs is calculated in the Field Correlator Method. The strong decay mechanisms are found out in the method and compared to the known phenomenological models. The role of sea quarks (unquenched approximation) is studied analytically using radially excited mesons as an example, and compared to experiment.

1 Introduction

In the last decade there has been a substantial progress in understanding the QCD spectrum, both in analytical methods [1], and lattice calculations [2]. On the analytic side the most economic and promising turns out the Field Correlator Method (FCM) [3], which starts from the basic principles and correct description of the QCD vacuum with the help of gauge-invariant field correlators. It was proved [4] that confinement can be described by the lowest, quadratic correlator \( \langle F_{\mu\nu}(x)F_{\lambda\sigma}(y) \rangle \) which contains two scalar formfactors \( D(x - y) \) and \( D_1(x - y) \). Lattice data [5] show that \( D \) and \( D_1 \) describe Q\( \bar{Q} \) static potential with a few percent accuracy, and the string tension \( \sigma \) is obtained directly from \( D(x) \): \( \sigma = \frac{1}{2} \int D(x) d^2 x \).

In addition \( D(x) \equiv D(x/T_g) \) contains another important parameter – the gluon correlation length \( T_g \), which was found on the lattice [6] and analytically [7] to be very small, \( T_g \leq 0.2 \text{ fm} \).
This circumstance allows to develop the local Hamiltonian and Lagrangian methods for the description of the $q\bar{q}$ and 3$q$ bound states, which will be discussed in the next section.

However, this is not the whole story since there are valence gluons which can also form bound states by themselves (glueballs) and with the $q\bar{q}$ (hybrids). To introduce these states one should define the valence gluons in contrast to the nonperturbative background, and one can do it unambigiously in the framework of the background perturbation theory [8]. In this way one gets the local Hamiltonian also for hybrids [9, 10] and for glueballs [11] and calculate the corresponding spectra in good agreement with lattice data. In doing so one realizes that a gluon excitation "costs" around 1 GeV, which allows to disregard these states in the first approximation when computing the lowest meson or baryon states.

However the exact treatment requires the introduction of Fock tower of states, and consideration of mixing between meson and hybrid states, which is done in Section 3. The effects of sea quarks on the spectrum and strong decays is considered in section 4, while Section 5 is devoted to conclusions.

2 Hamiltonian

There are two possible approaches to incorporating nonperturbative field correlators in the quark-antiquark (or 3$q$) dynamics. The first has to deal with the effective nonlocal quark Lagrangian containing field correlators [12]. From this one obtains first-order Dirac-type integro-differential equations for heavy-light mesons [12, 13], light mesons and baryons [14]. These equations contain the effect of chiral symmetry breaking [12] which is directly connected to confinement.

The second approach is based on the effective Hamiltonian for any gauge-invariant quark-gluon system. In the limit $T_g \to 0$ this Hamiltonian is simple and local, and in most cases when spin interaction can be considered as a perturbation one obtains results for the spectra in an analytic form, which is transparent.

For this reason we choose below the second, Hamiltonian approach [15, 16]. We start with the exact Fock-Feynman-Schwinger Representation for the $q\bar{q}$ Green’s function (for a review see [17]), taking for simplicity nonzero
flavor case

\[ G^{(x,y)}_{q\bar{q}} = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz)_{xy} (D\bar{z})_{xy} e^{-K_1 - K_2} \times \]  

\[ \langle tr \Gamma_{in}(m_1 - D_1) W_\sigma(C) \Gamma_{out}(m_2 - D_2) \rangle_A \]  

where \( K_i = \int_0^{s_i} d\tau_i (m_i + \frac{1}{4}(\dot{z}_i)^2) \). \( \Gamma_{in, out} = 1, \gamma_5, ... \) are meson vertices, and \( W_\sigma(C) \) is the Wilson loop with spin insertions, taken along the contour \( C \) formed by paths \( (Dz)_{xy} \) and \( (D\bar{z})_{xy} \),

\[ W_\sigma(C) = P_F P_A \exp(ig \int_C A_\mu dz_\mu) \times \]  

\[ \times \exp(g \int_0^{s_1} \sigma^{(1)}_\mu F_{\mu\nu} d\tau_1 - g \int_0^{s_2} \sigma^{(2)}_\mu F_{\mu\nu} d\tau_2). \]  

(2)

The last factor in (2) defines the spin interaction of quark and antiquark.

The average \( \langle W_\sigma \rangle_A \) in (1) can be computed exactly through field correlators \( \langle F^{(1)}...F^{(n)} \rangle_A \), and keeping only the lowest one, \( \langle F^{(1)} F^{(2)} \rangle \), which yields according to lattice calculation [5] accuracy around 1% [4], one obtains

\[ \langle W_\sigma(C) \rangle_A \simeq \exp(-\frac{1}{2} \int_{S_{min}} ds_{\mu\nu}(1) \int_{S_{min}} ds_{\lambda\sigma}(2) + \]  

\[ + \sum_{i,j=1}^2 \int_0^{s_i} \sigma^{(i)}_\mu d\tau_i \int_0^{s_j} \sigma^{(j)}_{\lambda\sigma} d\tau_j \langle F^{(1)}_{\mu\nu}(1) F^{(2)}_{\lambda\sigma}(2) \rangle). \]  

(3)

Here \( ds_{\mu\nu}(i) \) is the surface element, \( i = 1, 2 \), and the double integration in [3] is performed over the minimal area surface \( S_{min} \) inside the contour \( C \).

The Gaussian correlator \( \langle F^{(1)}_{\mu\nu}(1) F^{(2)}_{\lambda\sigma}(2) \rangle \equiv D_{\mu\nu,\lambda\sigma}(1,2) \) can be rewritten identically in terms of two scalar functions \( D(x) \) and \( D_1(x) \) [3], which have been computed on the lattice [6] to have the exponential form \( D(x), D_1(x) \sim \exp(-|x|/T_g) \) with the gluon correlation length \( T_g \approx 0.2 \) fm.

As the next step one introduces the einbein variables \( \mu_i \) and \( \nu_i \); the first one to transform the proper times \( s_i, \tau_i \) into the actual (Euclidean) times \( t_i \equiv z_4^{(i)} \). One has [16]

\[ 2\mu_i(t_i) = \frac{dt_i}{d\tau_i}, \int_0^{s_i} ds_i (D^4 z^{(i)})_{xy} = \text{const} \int D\mu_i(t_i) (D^3 z^{(i)})_{xy}. \]  

(4)

The variable \( \nu \) enters in the Gaussian representation of the Nambu-Goto form for \( S_{min} \) and its stationary value \( \nu_0 \) has the physical meaning of the
energy density along the string. In case of several strings, as in the baryon case or the hybrid case, each piece of string has its own parameter $\nu^{(i)}$.

To get rid of the path integration in (1) one can go over to the effective Hamiltonian using the identity

$$G_{\bar{q}q}(x, y) = \langle x | \exp(-HT) | y \rangle$$

(5)

where $T$ is the evolution parameter corresponding to the hypersurface chosen for the Hamiltonian: it is the hyperplane $z_4 = \text{const}$ in the c.m. case [16].

The final form of the c.m. Hamiltonian (apart from the spin and perturbative terms to be discussed later) for the $q\bar{q}$ case is [16, 18]

$$H_0 = \sum_{i=1}^{2} \left( \frac{m_i^2 + \mathbf{p}_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + \frac{\tilde{L}^2 / r^2}{2\mu_i(1 - \zeta)^2 + \mu_2 \zeta^2 + \int_0^1 d\beta (\beta - \zeta)^2 \nu(\beta)} +$$

$$+ \frac{\sigma^2 r^2}{2} \int_0^1 \frac{d\beta}{\nu(\beta)} + \int_0^1 \frac{\nu(\beta)}{2} d\beta.$$  

(6)

Here $\zeta = (\mu_1 + \int_0^1 \beta \nu d\beta)/(\mu_1 + \mu_2 + \int_0^1 \beta \nu d\beta)$ and $\mu_i$ and $\nu(\beta)$ are to be found from the stationary point of the Hamiltonian

$$\frac{\partial H_0}{\partial \mu_i} \bigg|_{\mu_i = \mu_i^{(0)}} = 0, \quad \frac{\partial H_0}{\partial \nu} \bigg|_{\nu = \nu^{(0)}} = 0.$$  

(7)

Note that $H_0$ contains as input only $m_1, m_2$ and $\sigma$, where $m_i$ are current masses defined at the scale 1 GeV. The further analysis is simplified by the observation that for $L = 0$ one finds $\nu^{(0)} = \sigma r$ from [17] and $\mu_i = \sqrt{m_i^2 + \mathbf{p}_i^2}$, hence $H_0$ becomes the usual Relativistic Quark Model (RQM) Hamiltonian

$$H_0(L = 0) = \sum_{i=1}^{2} \sqrt{m_i^2 + \mathbf{p}_i^2} + \sigma r.$$  

(8)

But $H_0$ is not the whole story, one should take into account 3 additional terms $H_{self}, H_{spin}$ and $H_{Coul}$, namely, spin terms in (3) which produce two types of contributions: self-energy correction [19]

$$H_{self} = \sum_{i=1}^{2} \frac{\Delta m_q^2(i)}{2\mu_i}, \quad \Delta m_q^2 = -4\sigma \pi \eta(m_i), \quad \eta(0) \cong 1 \div 0.9,$$  

(9)

where $\eta(m_i)$ is a known function of current mass $m_i$ [19], and spin-dependent interaction between quark and antiquark $H_{spin}$ [11, 20] which is entirely described by the field correlators $D(x), D_1(x)$, including also the one-gluon exchange part present in $D_1(x)$. 

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Finally one should take into account gluon exchange contributions [3], which can be divided into the Coulomb part $H_{\text{Coul}} = -\frac{4}{3} \alpha_s(r)$, and $H_{\text{rad}}$ including space-like gluon exchanges and perturbative self-energy corrections (we shall systematically omit these corrections since they are small for light quarks to be discussed below). In addition there are gluon contributions which are nondiagonal in number of gluons $n_g$ and quarks (till now only the sector $n_g = 0$ was considered) and therefore mixing meson states with hybrids and glueballs [21]; we call these terms $H_{\text{mix}}$ and refer the reader to [21] and the cited there references for more discussion. Assembling all terms together one has the following total Hamiltonian in the limit of large $N_c$ and small $T_g$:

$$H = H_0 + H_{\text{self}} + H_{\text{spin}} + H_{\text{Coul}} + H_{\text{rad}} + H_{\text{mix}}.$$ (10)

We start with $H_0 = H_R + H_{\text{string}}$ where $H_{\text{string}}$ is the term proportional to $\hat{L}^2$ in (6) and $H_R$ - all the rest terms in (6). The eigenvalues $M_0$ of $H_R$ can be given with 1% accuracy by [22]

$$M_0^2 \approx 8\sigma L + 4\pi\sigma(n + \frac{3}{4})$$ (11)

where $n$ is the radial quantum number, $n = 0, 1, 2, \ldots$. Remarkably $M_0 \approx 4\mu_0$, and for $L = n = 0$ one has $\mu_0(0, 0) = 0.35$ GeV for $\sigma = 0.18$ GeV$^2$, and $\mu_0$ is fast increasing with growing $n$ and $L$. This fact partly explains that spin interactions become unimportant beyond $L = 0, 1, 2$ since they are proportional to $d\tau_1 d\tau_2 \sim \frac{1}{4\mu_1 \mu_2} dt_1 dt_2$ (see [3] and [4] [24]). Thus constituent mass (which is actually "constituent energy") $\mu_0$ is "running". The validity of $\mu_0$ as a socially accepted "constituent mass" is confirmed by its numerical value given above, the spin splittings of light and heavy mesons [23] and by baryon magnetic moments expressed directly through $\mu_0$, and being in agreement with experimental values [24].

We now come to the gluon-containing systems, hybrids and glueballs. Referring the reader to the original papers [9]-[11] one can recapitulate the main results for the spectrum. In both cases the total Hamiltonian has the same form as in (10), however the contribution of corrections differs.

For glueballs it was argued in [11] that $H_0$ (6) has the same form, but with $m_i = 0$ and $\sigma \rightarrow \sigma_{adj} = \frac{3}{2}\sigma$ while $H_{\text{self}} = 0$ due to gauge invariance.

We now coming to the next topic of this talk: hybrids and their role in hadron dynamics. We start with the hybrid Hamiltonian and spectrum. This topic in the framework of FCM was considered in [9] [10] The Hamiltonian
$H_0$ for hybrid looks like \[1,9,10\]

$$H_0^{(hyb)} = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1 + \mu_2 + \mu_4}{2} + \frac{\mathbf{p}_\xi^2 + \mathbf{p}_\eta^2}{2\mu} + \sigma \sum_{i=1}^{2} |\mathbf{r}_g - \mathbf{r}_i| + H_{str}. \quad (12)$$

Here $\mathbf{p}_\xi, \mathbf{p}_\eta$ are Jacobi momenta of the 3-body system, $H_{self}$ is the same as for meson, while $H_{spin}$ and $H_{Coul}$ have different structure \[10\]; $H_{str}$ is the string term similar to $H_{string}$ in (10).

The main feature of the present approach based on the Background Perturbation Theory (BPTh), is that valence gluon in the hybrid is situated at some point on the string connecting quark and antiquark, and the gluon creates a kink on the string so that two pieces of the string move independently (however connected at the point of gluon). This differs strongly from the flux-tube model where hybrid is associated with the string excitation as a whole.

Results for light and heavy exotic $1^{-+}$ hybrids also given in \[1\] and are in agreement with lattice calculations. Typically an additional gluon in the exotic ($L = 1$) state “weights” $1.2 \div 1.5$ GeV for light to heavy quarks, while nonexotic gluon ($L = 0$) brings about 1 GeV to the mass of the total $q\bar{q}g$ system.

## 3 Hamiltonian and Fock states

As was mentioned above the QCD Hamiltonian is introduced in correspondence with the chosen hypersurface, which defines internal coordinates $\{\xi_k\}$ lying inside the hypersurface, and the evolution parameter, perpendicular to it. Two extreme choices are frequently used, 1) the c.m. coordinate system with the hypersurface $x_4 = const.$, which implies that all hadron constituents have the same (Euclidean) time coordinates $x_4^{(i)} = const, i = 1,...,n$, 2) the light-cone coordinate system, where the role of $x_4$ and $x_4^{(i)}$ is played by the $x_+, x_+^{(i)}$ components, $x_+ = \frac{x_0 + x_3}{\sqrt{2}}$.

To describe the structure of the Hamiltonian in general terms we first assume that the bound valence states exist for mesons, glueballs and baryons consisting of minimal number of constituents. To form the Fock tower of states starting with the given valence state, one can add gluons and $q\bar{q}$ pairs keeping the $J^{PC}$ assignment intact. At this point we make the basic simplifying approximation assuming that the number of colors $N_c$ is tending to
infinity, so that one can do for any physical quantity an expansion in powers of $1/N_c$. Recent lattice data confirm a good convergence of this expansion for $N_c = 3, 4, 6$ and all quantities considered \cite{25} (glueball mass, critical temperature, topological susceptibility etc.).

Then the construction of the Fock tower is greatly simplified since any additional $q\bar{q}$ pair enters with the coefficient $1/N_c$ and any additional white (e.g. glueball) component brings in the coefficient $1/N_c^2$. In view of this in the leading order of $1/N_c$ the Fock tower is formed by only creating additional gluons in the system, i.e. by the hybrid excitation of the original (valence) system. Thus all Fock tower consists of the valence component and its hybrid equivalents and each line of this tower is characterized by the number $n$ of added gluons. Then, the internal coordinates $\{\xi\}_n$ describe coordinates and polarizations of $n$ gluons in addition to those of valence constituents.

We turn now to the Hamiltonian $H$, assuming it to be either the total QCD Hamiltonian $H_{QCD}$, or the effective Hamiltonian $H^{(\text{eff})}$, obtained from $H_{QCD}$ by integrating out short-range degrees of freedom. We shall denote the diagonal elements of $H$, describing the dynamics of the $n$-th hybrid excitation of $s$-th valence state ($s = m \{ff\}, gg, 3g, b\{f_1f_2f_3\}$) for mesons, 2-gluon and 3-gluon glueballs and baryons respectively with $f_i$ denoting flavour of quarks) as $H_{nn}^{(s)}$. For nondiagonal elements we need only the lowest order operators $H_{n,n+1}^{(s)}$ and $H_{n-1,n}^{(s)}$ describing creation or annihilation of one additional gluon, viz.

\begin{align}
H_{qqg} &= g \int \bar{q}(\mathbf{x}, 0)\hat{a}(\mathbf{x}, 0)q(\mathbf{x}, 0)d^3x \\
H_{g2g} &= \frac{g}{2} \int_{abc} (\partial_\mu a_\nu^a - \partial_\nu a_\mu^a) a_\mu^b a_\nu^c d^3x. \quad (14)
\end{align}

As it is clear from (13), (14), the first operator refers to the gluon creation from the quark line, while the second refers to the creation of 2 gluons from the gluon line. In what follows we shall be mostly interested in the first operator, which yields dominant contribution at large energies, and physically describes addition of one last cross-piece to the ladder of gluon exchanges between quark lines, while (14) corresponds in the same ladder to the $\alpha_s$ renormalization graphs.

The effective Hamiltonian in the one-hadron sector can be written as follows

$$
\hat{H} = \hat{H}^{(0)} + \hat{V} \quad (15)
$$
where $H^{(0)}$ is the diagonal matrix of operators,

$$H^{(0)} = \{ H^{(s)}_{00}, H^{(s)}_{11}, H^{(s)}_{22}, ... \}$$  \hspace{1cm} (16)

while $\hat{V}$ is the sum of operators (13) and (14), creating and annihilating one gluon. In (16) $H^{(s)}_{nn}$ is the Hamiltonian operator for what we call the "$n$-hybrid", i.e. a bound state of the system, consisting of $n$ gluons together with the particles of the valence component. In this way the $n$-hybrid for the valence $\rho$-meson is the system consisting of $q\bar{q}$ plus $n$ gluons "sitting" on the string connecting $q$ and $\bar{q}$.

Before applying the stationary perturbation theory in $\hat{V}$ to the Hamiltonian (15), one should have in mind that there are two types of excitations of the ground state valence Fock component: 1) Each of the operators $H^{(s)}_{nn}, n = 0, 1, ...$ has infinite amount of excited states, when radial or orbital motion of any degree of freedom is excited, 2) in addition one can add a gluon, which means exciting the string and this excitations due to the operator $\hat{V}$ transform the $n$−th Fock component $\psi^{(s)}_n$ into $\psi^{(s)}_{n+1}$.

The wave equation for the Fock tower $\Psi_N \{ P, \xi \}$ has the standard form

$$\hat{H} \Psi_N = (\hat{H}^{(0)} + \hat{V}) \Psi_N = E_N \Psi_N,$$  \hspace{1cm} (17)

where $N$ numerates energy eigenvalues, and $\xi$ is a set of internal quantum numbers in the $n$-hybrid, or in the integral form

$$\Psi_N = \Psi_{N}^{(0)} - G^{(0)} \hat{V} \Psi_N$$  \hspace{1cm} (18)

where $G^{(0)}$ is diagonal in Fock components,

$$G^{(0)}(E) = \frac{1}{\hat{H}^{(0)} - E}, \quad G^{(0)}_{nm}(E) = \delta_{nm} \frac{1}{H^{(s)}_{nn} - E},$$  \hspace{1cm} (19)

and $\Psi_{N}^{(0)}$ is the eigenfunction of $\hat{H}^{(0)}$,

$$\hat{H}^{(0)} \Psi_{N}^{(0)} = E_{N}^{(0)} \Psi_{N}^{(0)}$$  \hspace{1cm} (20)

and since $\hat{H}^{(0)}$ is diagonal, $\Psi_{N}^{(0)}$ has only one Fock component, $\Psi_{N}^{(0)} = \psi_n(P, \{ \xi \}_n)$, $n = 0, 1, 2, ...$, and the eigenvalues $E_{N}^{(0)}$ contain all possible excitation energies of the $n$-hybrid, with the number $n$ of gluons in the system fixed,

$$E_{N}^{(0)} = E_{n}^{(0)}(P) = \sqrt{P^2 + M_n^2(k)}.$$  \hspace{1cm} (21)
Here \( \{k\} \) denotes the set of quantum numbers of the excited \( n \)-hybrid.

From (18) one obtains in the standard way corrections to the eigenvalues and eigenfunctions.

As a first step one should specify the unperturbed functions \( \Psi_N^{(0)} \), introducing the set of quantum numbers \( \{k\} \) defining the excited hybrid state for each \( n \)-hybrid Fock component \( \psi_n(P, \{\xi\}_n) \); we shall denote therefore:

\[
\Psi_N^{(0)} = \psi_{n\{k\}}(P, \{\xi\}_n), \quad n = 0, 1, 2, ...
\]

(22)

The set of functions \( \psi_{n\{k\}} \) with all possible \( n \) and \( \{k\} \) is a complete set to be used in the expansion of the exact wave-function (Fock tower) \( \Psi_N \):

\[
\Psi_N = \sum_{m\{k\}} c_N^{m\{k\}} \psi_{m\{k\}}.
\]

(23)

Using the orthonormality condition

\[
\int \psi_{m\{k\}}^+ \psi_{n\{p\}} d\Gamma = \delta_{mn} \delta_{\{k\}\{p\}}
\]

(24)

where \( d\Gamma \) implies integration over all internal coordinates and summing over all indices, one obtains from (17) an equation for \( c_{m\{k\}} \) and \( E_N \),

\[
c_N^{n\{p\}}(E_N - E_N^{(0)}) = \sum_{m\{k\}} c_N^{m\{k\}} V_{n\{p\},m\{k\}}
\]

(25)

where we have defined

\[
V_{n\{p\},m\{k\}} = \int \psi_{n\{p\}}^+ \hat{V} \psi_{m\{k\}} d\Gamma.
\]

(26)

and \( E_N^{(0)} \) is the eigenvalue of the wave function component \( \psi_{n\{p\}} \).

Consider now the Fock tower built on the valence component \( \psi_{\nu\{\kappa\}} \), where \( \nu \) can be any integer. For \( \nu\{\kappa\} = 0\{0\} \) this valence component corresponds to the unperturbed hadron with minimal number of valence particles. For higher values of \( \nu\{\kappa\} \) the Fock component \( \psi_{\nu\{\kappa\}} \) corresponds to the hybrid with \( \nu \) gluons which after taking into account the interaction is "dressed up" and acquires all other Fock components, so that the number \( N \) in (23) contains the "bare number" \( \nu\{\kappa\} \) as its part \( N = \nu\{\kappa\}, ... \) (at least for small perturbation \( \hat{V} \)).

One can impose on \( \Psi_N \) the orthonormality condition

\[
\int \Psi_N^+ \Psi_M d\Gamma = \sum_{m\{k\}} c_N^{m\{k\}} c_M^{m\{k\}} = \delta_{NM}.
\]

(27)
Expanding in powers of $\hat{V}$, one has

$$c_{m\{k\}}^N(\nu\{\kappa\}) = \delta_{mv}\delta_{\{k\}\{\kappa\}} + c_{m\{k\}}^N(1) + c_{m\{k\}}^N(2) + ...$$  \hspace{1cm} (28)

$$E_N(\nu\{\kappa\}) = E_{\nu\{\kappa\}}^{(0)} + E_N^{(1)} + E_N^{(2)} + ...$$  \hspace{1cm} (29)

It is easy to see that $E_N^{(1)} \equiv 0$, while for $c^{(1)}$ one obtains from (25) the standard expression

$$c_{n\{p\}}^N(\nu\{\kappa\}) = \frac{V_{\nu\{\kappa\}}}{E_{\nu\{\kappa\}} - E_{n\{p\}}}.$$  \hspace{1cm} (30)

In what follows we shall be interested in the high Fock components, $\nu + l, \{k\}$, obtained by adding $l$ gluons to the valence component $\nu\{\kappa\}$. Using (25) and (28) one obtains

$$C_{\nu + l,\{k\}}^N(\nu\{\kappa\}) = \sum_{\{k_1\}...\{k_l\}} \frac{V_{\nu + l,\{k_1\},\nu + l - 1,\{k_2\}}}{E_{\nu\{\kappa\}}^{(0)} - E_{\nu\{\kappa\}}^{(0)}} \frac{V_{\nu + l - 1,\{k_1\},\nu + l - 2,\{k_2\}}}{E_{\nu\{\kappa\}}^{(0)} - E_{\nu\{\kappa\}}^{(0)}} ...$$

$$\frac{V_{\nu + 1,\{k_1\},\nu\{\kappa\}}}{E_{\nu\{\kappa\}}^{(0)} - E_{\nu\{\kappa\}}^{(0)}} + O(V^{l + 2}).$$  \hspace{1cm} (31)

Since $\hat{V}$ is proportional to $g$, one obtains in (28) the perturbation series in powers of $\alpha_s$ for $c^N$ and hence for $\Psi_N$ (23). One should note that $\alpha_s(Q^2)$ is the background coupling constant, having the property of saturation for positive $Q^2$ [3, 26] and the background perturbation series has no Landau ghost pole and is defined in all Euclidean region of $Q^2$.

The estimate of the mixing between meson and hybrid was done earlier in the framework of the potential model for the meson in (27). In (21) the mixing between hybrid, meson and glueball states was calculated in the framework of the present formalism and we shortly summarize the results. One must estimate the matrix element (26) between meson and hybrid wave functions taking the operator $\hat{V}$ in the form of (13), where the operator of gluon emission at the point $(x,0)$ can be approximated as

$$a_\mu(x, t) = \sum_{k, \lambda} \frac{1}{\sqrt{2}\mu(k)V} \times$$

$$\times \left[ \exp(i k \cdot x - i \mu t) c_{\mu}^{(\lambda)}(k) + e^{(\lambda)} c_\lambda^{+}(k) \exp(-i k \cdot x + i \mu t) \right]$$  \hspace{1cm} (32)
Omitting for simplicity all polarization vectors and spin-coupling coefficients which are of the order of unity, one has the matrix element

\[ V_{Mh} = \frac{g}{\sqrt{2} \mu_g} \int \varphi_M(r)^\mu \psi_h^+(r) \, d^3r \]  

(33)

where \( \varphi_M(r) \), \( \mu \psi_h(r_1, r_2) \) are meson and hybrid wave functions respectively, and in (33) it is taken into account that the gluon is emitted (absorbed) from the quark position.

Using realistic Gaussian approximation for the wave functions in (33) one obtains the estimate [21]

\[ V_{Mh} \approx g \cdot 0.08 \text{ GeV}. \]  

(34)

A similar estimate is obtained in [21] for the hybrid-glueball mixing matrix element, while the meson-glueball mixing is second-order in (34).

Hence the first-order hybrid admixture coefficient (30) for the meson is

\[ C_{Mh} = \frac{V_{Mh}}{E_M^{(0)} - E_h^{(0)}} = \frac{V_{Mh}}{\Delta M_{Mh}} \]  

(35)

and for the ground state low-lying mesons when \( \Delta M_{Mh} \sim 1 \) GeV it is small, \( C_{Mh} \sim 0.1 - 0.15 \), yielding a 1-2\% probability. However for higher states in the region \( M_M \gtrsim 1.5 \) GeV, the mass difference \( \Delta M_{Mh} \) of mesons and hybrids with the same quantum numbers can be around 200 MeV, and the mixing becomes extremely important, also for meson-glueball mixing, which can be written as

\[ C_{MG} = \sum_h \frac{V_{Mh}V_{hG}}{\Delta M_{Mh}\Delta M_{hG}} \]  

(36)

and \( V_{Mh} \sim V_{hG} \). A good example is given by the three \( f_0 \) mesons: \( f_0(1390) \), \( f_0(1500) \) and \( f_0(1710) \) studied on the lattice in [28].

The authors [28] arrive at the following result of careful lattice studies:

\[ |f_0(1710) > = 0.859|g > + 0.302|s\bar{s} > + 0.413|n\bar{n} > , \]

\[ |f_0(1500) > = -0.128|g > + 0.908|s\bar{s} > - 0.399|n\bar{n} > , \]

\[ |f_0(1390) > = -0.495|g > + 0.290|s\bar{s} > + 0.819|n\bar{n} > . \]  

(37)

From (37) it is clear that a strong mixing occurs between states in the region 1.4–1.7 GeV, however the dominant valence component in all three cases is clearly visible: it is \( (n\bar{n}) \) for \( f_0(1390) \), \( (s\bar{s}) \) for \( f_0(1500) \) and the two-gluon glueball \( (g) \) for \( f_0(1710) \).
4 Sea quarks in the spectrum and strong decays

In all discussion above the dynamics in the $q\bar{q}$ system was described by the Wilson loop $W_\sigma(C)$ in (1). However the formalism in (1)-(3) is correct only in the large $N_c$ limit, which holds for the lower part of the spectrum with accuracy of the order of 10%. For higher excited states with the radius exceeding 1.3-1.4 fm one should take into account the admixture of quark pairs. This can be accomplished formally replacing $W_\sigma(C)$ in (1) by the product $W_\sigma(C)\prod_f\det(m_f + \hat{D})$. Using the heat kernel (Fock-Feynman-Schwinger) representation for the determinant one has

$$\text{Re } \ln \det(m + \hat{D}) = \frac{1}{2} \ln \det(m^2 + \hat{D}^2) =$$

$$= \int d^4x \int_0^\infty \frac{ds}{s} \exp(-m^2s)(Dz)_{xx} \exp(-K)W(C_x).$$

(38)

where $W(C)$ is the Wilson loop without spin factors, present in (2), and $C_x$ is the closed contour beginning and ending at the point $x$. In this way the determinant can be expanded as a series in powers of number of sea-quark loops and the averaging yields

$$\langle W_\sigma(C)\det(m + \hat{D})\rangle = \langle W_\sigma(C)\rangle\langle \det(m + \hat{D})\rangle + \frac{a_1}{N_c}W_1(C,C_x) + ...$$

(39)

where

$$W_1(C,C_x) \equiv \langle W_\sigma(C)W(C_x)\rangle - \langle W_\sigma(C)\rangle\langle W(C_x)\rangle.$$ (40)

It was shown in [29] that the interaction of Wilson loops given by (40) effectively produces holes in the original world sheet of the string in $W_\sigma(C)$. It was argued in [30], that for unstable states with the life-time $T \sim 1$ fm and average radius larger than 1.4 fm the holes due to the $q\bar{q}$ pairs can be in metastable equilibrium which can be called ”the predecay state”. In this state the linear potential between quarks starts to saturate, and the resulting meson masses calculated in [30] are strongly decreased by 200-500 MeV, which is in good agreement with experimental data. The resulting radial Regge trajectories are of surprisingly good linear form, and are given in [31].

Another phenomenon, where sea quarks play the crucial role is the strong decay process. As it was shown in [32] one can distinguish three possible mechanisms of strong decays, where nonperturbative QCD is involved: 1)
chiral mechanism, which appears after bosonization \cite{33}, \cite{34} 2) string breaking mechanism and 3) pair creation via the intermediate hybrid or glueball formation.

Taking into account valence (perturbative) gluon field $a_\mu(x)$, one has after averaging over background gluon field and bosonization the following effective quark-meson Lagrangian \cite{32}-\cite{34}

$$L_{QML} = \int d^4x \int d^4y \left\{ \frac{1}{2} \psi^+_{a\alpha}(x) \left[ \left[ i(\hat{\partial} - ig\hat{a}) + im_f \right] _{\alpha\beta} \delta(x-y) \hat{\delta}_{fg} + iM_S \hat{U}_{\alpha\beta}^{(fg)}(x,y) \right] \psi_{a\beta}(y) - 2n_f [J(x,y)]^{-1} M_{S\alpha}^2(x,y) \right\}$$ (41)

where $\hat{U} = \exp(i\hat{\phi}(x,y)\gamma_5)$, and $\hat{\phi} = t^a \phi_a/f_\pi$ is the pionic field, $n_f$ is number of flavours, and $J(x,y)$ is the kernel proportional to the integral of field correlator, $f, g$ – flavour indices, while $M_S(x,y)$ is the effective quark mass operator containing the string connecting quark to the closest antiquark position, for details and notations see \cite{32}-\cite{34}.

All decay mechanisms listed above are contained in the Lagrangian (41), e.g. the chiral mechanism obtains by expanding $\hat{U}(\phi)$ in powers of $\phi_a$, as for the details and comparison with experiment, see \cite{33} and the literature cited in there.

The nonperturbative string breaking mechanism is described in (41) by the term $\psi^+ M_S \psi$, where it is essential that the mass operator $M_S(x,y) = M_{S\alpha}^{(br)}$ enters at the vertex of the quark-antiquark formation (rather than the string mass operator in the quark propagation). The explicit computation of $M_{S\alpha}^{(br)}$ in the string breaking term was done in \cite{32}, and yields

$$\Delta L^{(br)} = \frac{2T_g \sigma}{\sqrt{\pi}} \int d^4x \bar{\psi}(x)\psi(x).$$ (42)

As it is clear from (42) one obtains the $^3P_0$ mechanism with the predicted coefficient, which agrees with the generic phenomenologically fitted value.

Finally, the mechanism 3) is given by the combination of the $q\bar{q}$ generation by the perturbative gluon, due to the term $\psi^+ \hat{a}\psi$, with the subsequent string world sheet enveloping the gluon and quark trajectories. In this way one obtains the Hybrid- Mediated Decay (HMD) and – for the OZI violating processes – the Glueball-Mediated Decay (GMD), suggested in \cite{32}. The relative role and theoretical estimates of all three mechanisms was not elaborated in \cite{32}.
5 Conclusions

It was shown above that the Vacuum Correlator Method (VCM) is a powerful tool for the investigation of all effects in the QCD spectrum. In particular the low-lying part of the spectrum, where the string breaking is not essential, is well described by the Hamiltonian \((\text{10})\) containing the minimal number of input parameters: current quark masses, string tension and \(\alpha_s\). Here effects of mixing and decay are inessential (less than 10\%) and the leading large \(N_c\) approximation is valid. However for higher masses (and for high-energy processes) one should take into account the Fock-tower structure of hadrons, where in particular hybrids contribute significantly. Superficially these Fock towers are similar to the light-cone Fock states considered in \([35]\), however the (essential) difference is that in our case all Fock components are bound states – mesons or hybrids in the leading large \(N_c\) limit, while in \([35]\) only perturbative dynamics is present.

It was demonstrated above that also the \(1/N_c\) effects, namely glueball admixture in mesons and decay amplitudes, are easily computed in the framework of the VCM, and moreover no new parameters are introduced. However only first step is done in \([32]\) for the construction of decay and production amplitudes.

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