Impact of a first-order phase transition on neutron star merger simulations

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Abstract. In 2017 gravitational waves from a neutron star merger were detected for the first time. The analysis of these gravitational waves provides insights in the still not fully known neutron star equation of state enhancing our knowledge on the behavior of matter at extreme densities beyond nuclear saturation. Here, I discuss results of neutron star merger simulations with equations of state including a first order phase transition to quark matter. Such a transition is expected to occur at some density, however the exact value of this density remains unknown. I will point out how future gravitational-wave detections of neutron star merger may help to decide whether such a phase transition occurs at densities reached in neutron star mergers.

1. Introduction
Neutron stars offer the possibility to study the properties of extremely dense, cold matter (neutron star matter). Currently, the composition and exact behavior of matter under these extreme conditions is not fully understood.

The fundamental theory of strong interactions, quantum chromodynamics (QCD), predicts a phase transition from nuclear matter (protons and neutrons) to deconfined quark matter (see e.g. [1] for the QCD phase diagram). However, it is not possible to employ perturbative QCD methods for neutron star matter. Therefore, the nature (first-order phase transition or smooth crossover) as well as the onset density of the phase transition in the low temperature regime at finite baryon chemical potential remain unknown. The possibility that this phase transitions occurs at densities reached in the core of heavy neutron stars has been considered for a while (see e.g. [2] or [3]). The advent of gravitational-wave astronomy helps increasing our knowledge on neutron star matter properties. It also offers the prospect to gain some insights in the presence and nature of a phase transition in the remnants of neutron star mergers.

2. Neutron stars
Neutron stars are possible remnants of core-collapse supernovae of stars with at least $8 \, M_\odot$. Typically, they have radii of about 10-15 km and their masses range from roughly $1 \, M_\odot$ to $2 \, M_\odot$. They offer the possibility to study properties of extremely dense, cold matter.

In order to model a neutron star in hydrostatic equilibrium one has to balance the internal pressure of matter with the gravitational attraction. Assuming a static, spherically symmetric star the general relativistic equations of stellar structure are the so-called Tolman-Oppenheimer-Volkoff-equations (TOV-equations) (see [4] and [5]).

In order to solve these equations and to obtain the mass and radius of a star the microphysical
equation of state (EoS), i.e. the pressure of neutron star matter as a function of its total energy density, is needed. Generally, the pressure is also a function of the matter composition and the temperature, however for neutron stars zero temperature and beta-equilibrium composition are good approximations. With a given EoS model the structure of a neutron star can be modeled by choosing a central density and numerically integrating the TOV-equations until the pressure drops to zero. This marks the surface of the star and the total mass contained within this yields the stellar mass. Repeating this procedure with several different central densities yields the mass-radius relation of a specific EoSs. An example of such a mass-radius relation and the corresponding EoS are shown in Fig. 1 and Fig. 2. Note that Fig. 1 shows the pressure as a function of the rest-mass density and not the total energy density. Further analysis (see e.g. chapter 3.18 of [3]) shows that a star in the region of the mass-radius curve with \( \frac{dM}{dR} > 0 \) is unstable and hence collapses to a black hole implying that a maximum neutron star mass \( M_{\text{max}} \) exists. The unstable neutron star branch is represented by the dashed line in Fig. 2 while the solid line depicts the stable neutron star branch. Because the true EoS of matter in the high density region around and above nuclear saturation density is still not completely understood many different EoS models exist. Each of these models leads to a different mass-radius relation and hence to a different value of \( M_{\text{max}} \). It must be stressed here, that there is only one true neutron star EoS. Determining simultaneously the masses and the radii of several neutron stars with different masses could therefore reveal this true EoS. However, although some masses can be measured with high precision by timing radio signals of pulsars in binary systems (see e.g. [7]) radius measurements are still not precise enough to unambiguously pin down the true EoS although there are ongoing efforts trying to measure radii such as the NICER mission [8]. The determination of a neutron star mass alone can only yield a lower limit on \( M_{\text{max}} \) immediately ruling out all EoS that predict lower values of \( M_{\text{max}} \). One way to gain further constraints on the true neutron star EoS is by studying neutron star mergers.

3. Neutron star mergers
A neutron star binary system continuously loses energy and angular momentum through the radiation of gravitational waves (GWs) (more information on GWs can be found in any standard textbook on General Relativity e.g. [9]). Therefore, the two neutron stars inspiral towards each
Figure 3. Semi-logarithmic plot of a GW spectrum of the cross polarization at a distance of 20 Mpc along the polar axis calculated from the simulated merger of two $1.35M_\odot$ neutron stars using the DD2F EoS (see [6]). On can clearly see the dominant peak $f_{\text{peak}}$ at about 3 kHz caused by the fundamental quadrupole oscillation mode of the remnant.

Neutron star mergers are thought to be possible progenitors of short gamma-ray bursts (see [10]). They are also expected to be an important site of the rapid neutron capture process (r-process) to explain the origin of many heavy elements beyond iron in the universe [11]. These views were strengthened by the recent first unambiguous neutron star merger observation GW170817 [12]. During the late inspiral phase of a neutron star binary tidal effects begin to show up as the neutron stars are deformed by the gravity of their companions. These tidal effects manifest themselves in the GW signal of a neutron star merger and contain information on the underlying EoS since the EoS determines how easily a neutron star can be deformed. This deformation is described by the so-called tidal deformability $\Lambda = k_2[(2R\cdot c^2)/(3M\cdot G)]^5$. Here $R$ and $M$ are the radius and the mass of a neutron star respectively, $c$ is the speed of light, $G$ the gravitational constant and $k_2$ is the so-called tidal Love number [13]. $\Lambda$ as well as the binary mass can be inferred from the inspiral GW signal of a neutron star merger with some precision to give constraints on the EoS.

Once the stars have merged the total mass of the systems determines the fate of the formed remnant. Although the mass of the remnant will most likely exceed $M_{\text{max}}$, the additional thermal pressure and the fast differential rotation can temporarily stabilize the remnant against the gravitational collapse. This stabilization is possible up to a threshold mass to prompt black hole collapse $M_{\text{thres}}$. If the mass of the remnant is below $M_{\text{thres}}$, it does not suffer from an immediate collapse to a black hole, although it is still expected to eventually collapse. The strong oscillations of the remnant produce a postmerger GW signal (see e.g. [14]). An example of a postmerger GW spectrum is shown in Fig 3. It contains a very prominent peak at a few kHz. This peak is caused by the fundamental quadrupole fluid mode (see e.g. [14] for an analysis of neutron star merger spectra) of the remnant and is a robust feature occurring in all simulations. The frequency of this peak will be named $f_{\text{peak}}$ for the rest of this work. $f_{\text{peak}}$ is connected to the mean density of the remnant which is ultimately determined by the stellar structure and thus the high density part of the microphysical EoS. Therefore, a measurement of $f_{\text{peak}}$ gives a constraint on the true neutron star EoS [15].
Figure 4. Dominant postmerger GW frequency $f_{\text{peak}}$ as a function of the tidal deformability $\Lambda$ for $1.35-1.35 M_\odot$ mergers. Black symbols display results from hadronic models. The solid black line shows a least-square fit to the black symbols and the grey shaded area marks the largest observed deviation from the fit. The green symbols represent results from simulations with the hybrid DD2F-SF models. They appear as clear outliers. Arrows mark the DD2F-SF models 3, 6 and 7 which have roughly the same onset density and stiffness of the quark phase but differ in the density jump at the phase transition $\Delta n$ (in fm$^{-3}$). Figure taken from Ref. [17].

If the total mass of a neutron binary system is above $M_{\text{thres}}$ the remnant will promptly collapse to a black hole. In this case the GW signal is expected to ring down very quickly. This means that no peak at frequencies $f_{\text{peak}}$ is present in the GW spectra since the ringing remnant black hole oscillates at higher frequencies. Once postmerger GW signals can be detected it will be possible to clearly distinguish mergers with a prompt collapse of the remnant from mergers with a temporarily stable remnant. The value of $M_{\text{thres}}$ is also determined by the neutron star EoS [16]. Once GW detectors are sensitive enough to measure postmerger GW signals and a large number of merger observations have been done it will be possible to obtain the value of $M_{\text{thres}}$ with reasonable precision. This will help further constraining the neutron star EoS.

4. Impact of a first-order phase transition on neutron star mergers

As stated in the introduction, QCD predicts the occurrence of a phase transition from nuclear to deconfined quark matter at some density in cold matter. Since the high density EoS of matter is uncertain, it cannot be ruled out that this transition takes place at densities reached in neutron star mergers or in the cores of isolated, stable neutron stars. It is also possible that this transition takes place at much higher densities. In order to shed some light on the question of an occurring phase transition in neutron star mergers one can simulate mergers with a EoSs that show such a phase transition and try to identify unambiguous signals of this phase transition. This was done in Ref. [17]. Here the authors performed several $1.35-1.35 M_\odot$ binary merger simulations with many different hadronic EoSs. Additionally, they also simulated merger with seven different variants of the hybrid DD2F-SF model from [18] including a first-order phase transition to deconfined quark matter. They found that the dominant postmerger frequency $f_{\text{peak}}$ as a function of the tidal deformability $\Lambda_{1.35}$ of a single $1.35 M_\odot$ star can be approximated by a second order polynomial for all hadronic EoSs. This is shown in Fig. 4. Black marks show results using EoSs with no first-order phase transition. The solid black cure represents a least square fit to the data points using a second order polynomial while the grey shaded area
depicts the maximum found deviation of a hadronic model from the fit. The results from the hybrid DD2F-SF models clearly deviate from the $f_{\text{peak}}(\Lambda_{1.35})$ relation. They are shown by green crosses in Fig. 4. These models lead to higher densities in the merger remnant due to the phase transition occurring in the early stage of the merger and hence to larger $f_{\text{peak}}$ values. The black arrows in Fig. 4 mark specific values of the density jump at the phase transition for some of the used models. One can see that the size of the deviation from the relation increases with the density jump at the phase transition.

The values of $\Lambda_{1.35}$ however are unaffected by the phase transition because the densities in the 1.35 $M_\odot$ stars prior to the merger are below the transition density of the used hybrid models. The authors conclude that an observed deviation from the empirical $f_{\text{peak}}(\Lambda_{1.35})$-relation (both quantities can be deduced from an observed GW signal) would be a clear, unambiguous indication of a strong first-order phase transition occurring in a neutron star merger. The currently used GW detectors are constantly being upgraded and new detectors with enhanced sensitivity such as the Einstein telescope [19] are planned. Therefore, it is expected that the postmerger GW signals of a neutron star merger will be detectable with high enough precision to compare results to the found $f_{\text{peak}}(\Lambda_{1.35})$ relation at some point in the future.

5. Summary and outlook

The analysis of GW signals from neutron star mergers can give constraints on the still not fully known neutron star equation of state via tidal effects during the late inspiral phase, the collapse behavior at the merger and the postmerger oscillations. In case no prompt collapse occurs a strong order phase transition leads to higher densities in the remnant and hence to larger $f_{\text{peak}}$ values. When comparing $\Lambda_{1.35}$ with $f_{\text{peak}}$, results from simulations with purely hadronic EoSs follow a simple quadratic relation while results from the hybrid DD2F-SF EoSs appear as clear outliers. This means that once GW detectors are sensitive enough, it will be possible to decide, whether a strong first-order phase transition has taken place in the remnant of an observed neutron star merger.

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