Modeling of Sedimentation Process in the Irrigation Channel

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Abstract. Irrigation has been a central feature of agriculture for over 5,000 years and is the method in which water is supplied to plants at regular intervals for agriculture. Channel irrigation allows irrigation over large areas, with large volumes of water. The content of the water in channel from the river generally contains a lot of material that can precipitate during the water flood the area of agriculture. This paper is to derive a mathematical model of sedimentation processes in the irrigation channel. The model is analyzed using Finite Element Method with respect to the geometry of the channel in the district Galang, Sumatera Utara Province. From a computational point of view, results have shown the importance streamlines of the mixture velocity and the dispersed phase volume fraction.

1. Introduction
Agriculture is one of the priority economic sectors in North Sumatra after plantation. According to statistical data of the Ministry of Agriculture of the Republic of Indonesia, land area of rice in 2015 reached 781.769Ha (5.54% of the rice land area of the entire Indonesian) with a growth rate last reached 8.98%. The production of rice in 2015 from the province of North Sumatra reached 4.044.829Ton (5.37% of rice production throughout Indonesia) with the last growth reached 11.40% [1]. Even though the land area is still low compared to several provinces in Java, but the rate of growth is still high.

Building settling basin is one of the complementary buildings irrigation works to be poured basic charge (bed load) back to the river. Traditionally, settling basins were built based on retention time. [2]. One way to increase agricultural output multiplier is to implement irrigation construction techniques, which use the river as a natural resource.

Analysis on a physical phenomenon that occur in a real situation can be performed using computer-aided technology through mathematical modeling. The process of sedimentation in a building settling basin can be analyzed to obtain the optimal parameters, so it can produce more efficient performance. Sil, B.S. and Choudhury [3] has produced the application of characteristics of water flow and sediment transport based simulations to predict sediment transport and water in the river system. Some authors have derived the profile shape model of the river, and sedimentation models on a tube and have also conducted experimental studies some basic changes in a canal and formulate conditions channel geometry and characterize the effects of geometry and flow [4, 5, 6].

This presents the dynamic model adopted for describing the flow in channel, sediment transport and morphological evolution. The computation using finite element method (FEM) is performed by using COMSOL scheme, employed for solving the adopted mathematical model, with some
considerations on the treatment of the source terms. The results of the research can be used to describe the sedimentation process of basin settling for agriculture.

2. Phenomenological Theory of Sedimentation

The derivation of the phenomenological theory for settling of sedimentation with compression was conducted by Concha and Bustos [7]. A suspension is a mixture of solid particles and a liquid. In this theory, a suspension was modeled as a mixture of two superimposed continuous media. The dynamics of a suspension can be modelled by a momentum transport equation for the mixture, a continuity equation, and a transport equation for the solid phase volume fraction. The Mixture Model, Laminar Flow interface automatically sets up these equations. It uses the following equation to model the momentum transport:

\[
\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p - \nabla \cdot \left( \rho c_s(1 - c_s) \mathbf{u}_{\text{slip}} \mathbf{u} + \nabla \cdot \left[ \eta \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \rho \mathbf{g} \right)
\]  

(1)

where \( \mathbf{u} \) is the mass averaged mixture velocity (m/s), \( p \) denotes the pressure (Pa), \( \mathbf{g} \) refers to the acceleration of gravity (m/s\(^2\)), \( c_s \) is the dimensionless particle mass fraction, and \( \mathbf{u}_{\text{slip}} \) gives the relative velocity between the solid and the liquid phases (m/s). Further, \( \rho = (1 - \phi_s)\rho_f + \phi_s \rho_s \) is the mixture density, where \( \rho_f \) and \( \rho_s \) are the pure-phase densities (kg/m\(^3\)) of liquid and solids, respectively, and \( \phi_s \) is the solid-phase volume fraction (m\(^3\)/m\(^3\)). Finally, \( \eta \) represents the mixture viscosity (Ns/m\(^2\)) according to the Krieger-type expression

\[
\eta = \eta_f \left( 1 - \frac{\phi_s}{\phi_{\text{max}}} \right)^{-2 \phi_{\text{max}}}
\]

(2)

where \( \eta_f \) is the dynamic viscosity of the pure fluid and \( \phi_{\text{max}} \) is the maximum packing concentration.

The mixture model uses the following form of the continuity equation

\[
\left( \rho_f - \rho_s \right) \left[ \nabla \cdot \left( \phi_s(1 - c_s) \mathbf{u}_{\text{slip}} \right) \right] + \rho_s \left( \nabla \cdot \mathbf{u} \right) = 0
\]

(3)

The transport equation for the solid-phase volume fraction is

\[
\frac{\partial \phi_s}{\partial t} + \nabla \cdot \left( \phi_s \mathbf{u} \right) = 0
\]

(4)

The solid-phase velocity, \( \mathbf{u}_s \), is given by \( \mathbf{u}_s = \mathbf{u} + (1 - c_s) \mathbf{u}_{\text{slip}} \). Consequently, Equation 4 is equivalent to

\[
\frac{\partial \phi_s}{\partial t} + \nabla \cdot \left( \phi_s \mathbf{u} + \phi_s(1 - c_s) \mathbf{u}_{\text{slip}} \right) = 0
\]

(5)

Rao and others formulate the continuity equation and the particle transport in a slightly different way. Instead of the slip velocity, \( \mathbf{u}_{\text{slip}} \), they define a particle flux, \( \mathbf{J}_s \) (kg/(m\(^2\)·s)), and write the continuity equation as
\[ \nabla \cdot \mathbf{u} = \frac{\rho_s - \rho_f}{\rho_s \rho_f} (\nabla \cdot \mathbf{J}_s) \]  \hspace{1cm} (6)

and the solid phase transport according to

\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u}) = -\nabla \cdot \mathbf{J}_s \frac{1}{\rho_s} \]  \hspace{1cm} (7)

By comparing Equation 6 and Equation 7 with Equation 3 and Equation 5, it is clear that they are equivalent if

\[ u_{\text{dip}} = \frac{J_s}{\phi_s \rho_s (1 - c_s)} \]

Following Rao et al. [7] and others, the particle flux is

\[ \frac{J_s}{\rho_s} = - \left[ \phi D_\phi \nabla (\gamma \phi) + \phi^2 \gamma D_\mu \nabla (\ln \mu) \right] + f_n u_{\text{ref}} \phi \]  \hspace{1cm} (8)

Here, \( u_{\text{ref}} \) is the settling velocity (m/s) of a single particle surrounded by fluid and \( D_\phi \) and \( D_\mu \) are empirically fitted parameters (m²) given by

\[ D_\phi = 0.41a^2 \]  \hspace{1cm} (9)

\[ D_\mu = 0.62a^2 \]  \hspace{1cm} (10)

where \( a \) is the particle radius (m).

The shear rate tensor, \( \dot{\gamma} \) (1/s), is given by

\[ \dot{\gamma} = \nabla \mathbf{u} + (\nabla \mathbf{u})^T \]  \hspace{1cm} (11)

and its magnitude by

\[ \dot{\gamma} = \sqrt{\frac{1}{2} \left( \dot{\gamma} : \dot{\gamma} \right)} \]  \hspace{1cm} (12)

which for a 2-dimensional problem is

\[ \dot{\gamma} = \sqrt{\frac{1}{2} \left( 4u_x^2 + 2(u_y + v_x)^2 + 4v_y^2 \right)} \]  \hspace{1cm} (13)
The settling velocity, \( u_{st} \), for a single spherical particle surrounded by pure fluid is given by

\[
 u_{st} = \frac{2 a^2 (\rho_s - \rho_f)}{9 \eta_0} g 
\]  \hspace{1cm} (13)

For several particles in a fluid, the settling velocity is lower. To account for the surrounding particles, the settling velocity for a single particle is multiplied by the hindering function, \( f_h \), defined as

\[
 f_h = \frac{\eta_f (1 - \phi_{av})}{\eta} 
\]  \hspace{1cm} (13)

where \( \phi_{av} \) is the average solid phase volume fraction in the suspension, \( \eta_f \) is the dynamic viscosity of the pure fluid (Ns/m\(^2\)), and \( \eta \) is the mixture viscosity (Equation 2).

3. Finite element analysis for Sedimentation

Building settling basin contained in weir irrigation area of the Snake River at Village Serbajadi, Deli Serdang regency is very important in the analysis again because of extensive services that are large enough about 18,500 hectares, far less than the irrigated areas more only a maximum of 1000 hectares. On the other side, the Snake River is very productive for the movement of sediment (sediment transport). Sediment deposition problems on the channel need to be handled appropriately. The deposition of sediments in these channels will cause problems in the development of agricultural production and the cultivation of paddy fields. Based on the above ideas, it will be evaluated the building settling basin. The real channel of the building settling basin that considered in this paper is depicted in Figure 1. The analysis in this paper considers the 2D as the vertical slice of the channel.

![Figure 1. Building settling basin at Galang](image)

To analyze the sedimentation process, it is considered the equation of momentum transport from the equations (1), (3), (4) and (5) to give the dynamic of the problem. So, the following governing equations are considered to the problem of mixture model in turbulent flow.

\[
 \rho \frac{\partial u}{\partial t} + \rho (u \cdot \nabla)u = \nabla \cdot [\sigma (\mu + \mu_T) (\nabla u + \nabla u^T)] - \nabla \cdot \left\{ \eta c_d (1 - c_d) u_{slip} \cdot u_{slip} \right\} + \rho g + F \]  \hspace{1cm} (14)
\[(\rho - \rho_d) \left\{ \nabla \cdot \left[ \phi_d \left( 1 - c_d \right) u_{\text{slip}} - D_{md} \nabla \phi_d \right] + \frac{m_k}{\rho} \right\} + \rho \left( \nabla \cdot u \right) = 0 \] (15)

\[
\frac{\partial \phi_d}{\partial t} + \nabla \cdot N_{\phi_d} = - \frac{m_k}{\rho}, \quad \phi_d = \phi_{id}
\] (16)

\[N_{\phi_d} = \phi_d u_d, \quad u_d = u + \phi_d \left( 1 - c_d \right) u_{\text{slip}} - \frac{D_{md}}{\phi_d} \nabla \phi_d
\] (17)

\[\rho \frac{\partial \epsilon}{\partial t} + \rho (u \cdot \nabla) \epsilon = \nabla \cdot \left[ \left( \mu + \frac{\mu_f}{\sigma} \right) \nabla \epsilon \right] + C_{e1} \frac{\epsilon}{k} P_{\epsilon} - C_{e2} \rho \frac{\epsilon^2}{k}
\] (18)

\[\mu_f = \rho C_{\mu} \frac{k^2}{\epsilon}, \quad P_{\epsilon} = u_r \left[ \nabla u \cdot \left( \nabla u + (\nabla u)^T \right) \right]
\] (19)

\[\frac{\partial \rho}{\partial t} + \rho \left( u \cdot \nabla \right) \phi_f = \frac{\phi_f \rho_d}{\rho}, \quad c_d = \frac{\phi_f \rho_d}{\rho}, \quad D_{md} = \frac{\mu_f}{\rho \sigma_f}
\] (20)

The Equation (18)-(20) describe the types of slip model and turbulence model, that are Hadamard-Rybczynski model and RANS k-ε model respectively.

The boundary conditions are considered with respect to the real condition of the model, that are wall, slip wall, inlet, and outlet, as follows.

**Boundary condition 1 Wall function:**

\[u \cdot n = 0, \quad \left[ \left( \mu + \mu_f \right) \left( \nabla u + \nabla u^T \right) - \frac{2}{3} \rho k \right] n = -\rho \frac{u_r}{\delta_u} u_{\text{tang}}
\] (22)

\[u_{\text{tang}} = u - (u \cdot n)n \] (23)

\[\nabla k \cdot n = 0, \quad \epsilon = -\rho \frac{C_{\mu} k^2}{\kappa_2^2} \mu
\] (24)

\[-n \cdot N_{\phi_d} = 0, \quad \nabla \cdot n = 0
\] (25)

**Boundary condition 2 Wall slip:**

\[u \cdot n = 0, \quad K - (K \cdot n)n = 0
\] (26)

\[K = \left[ \left( \mu + \mu_f \right) \left( \nabla u + \nabla u^T \right) - \frac{2}{3} \rho k \right] n \] (27)

\[\nabla k \cdot n = 0, \quad \nabla \epsilon \cdot n = 0
\] (28)

\[-n \cdot N_{\phi_d} = 0 \] (29)

**Boundary condition 3 Inlet:**

\[u = u_0 \] (30)

\[
\frac{(\rho - \rho_d)}{\rho_c} \left\{ \nabla \cdot \left[ \phi_d \left( 1 - c_d \right) u_{\text{slip}} - D_{md} \nabla \phi_d \right] + \frac{m_k}{\rho} \right\} + \rho \left( \nabla \cdot u \right) = 0
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\[\nabla k \cdot n = 0, \quad \nabla \epsilon \cdot n = 0
\] (28)

\[-n \cdot N_{\phi_d} = 0 \] (29)

**Boundary condition 3 Inlet:**

\[u = u_0 \] (30)
\[ k = \frac{3}{2} \left( \| u_0 \| I_T \right)^2, \quad \varepsilon = C_{\mu}^{3/4} \frac{k^{3/2}}{L_T} \]  

(32)

\[ \phi_u = \phi_{u0} \]

(33)

**Boundary condition 4 Outlet using velocity:**

\[ u = u_0 \]

\[ \nabla k \cdot n = 0, \quad \nabla \varepsilon \cdot n = 0 \]

(34)  

(35)

**Boundary condition 4 Outlet using pressure, no viscous stress:**

\[ p = p_0, \quad \left(\mu + \mu_r \right) \left( \nabla u + \nabla u^T \right) - \frac{2}{3} \rho k l \right) n = 0 \]

\[ \nabla k \cdot n = 0, \quad \nabla \varepsilon \cdot n = 0 \]

(36)  

(37)

The 2D geometric model and the boundary conditions are designed as depicted in Figure 2. The water inlet is applied at the left side, and the water outlet is applied at the right side and the right bottom side. Wall slip is applied at top side, and the rest are set as the wall.

![Figure 2. Geometry and boundary conditions](image)

By using Finite Element Method, the mesh is generated using triangle elements as depicted in Figure 3. A careful selection of the mesh type and number is considered before the simulation.

An initial-boundary value problem was derived by introducing constitutive assumption, simplifying the balance, and restricting to one space dimension. The problem was of the form of a second order partial differential equation for volumetric solid concentration. The governing equation of the phenomenological theory of sedimentation is its mixed hyperbolic-parabolic nature. The mixed type nature corresponds to the interface between the compression zone, where the solid effective stress varies, and the hindered settling zone.
The parameters used in the computation are considered as a description for the material properties of water and also the averaged diameter of particle dispersed in water as described in Table 1. On the other hand, some values are given in initial values of the computation with deal to the condition in the channel as listed in the Table 2.

Table 1. Specification of the material properties

| No | Parameter                                | Symbol | Value | Unit  |
|----|------------------------------------------|--------|-------|-------|
| 1  | Continuous phase density                 | $\rho_c$| 1000  | kg/m³ |
| 2  | Continuous phase viscosity               | $\mu_c$| 0.001 | Pa·s  |
| 3  | Dispersed phase density                  | $\rho_d$| 1100  | kg/m³ |
| 4  | Dispersed phase particle diameter        | $D_d$  | 2E-4  | m     |

Table 2. Some variables that used in the computation

| No | Variables                        | Symbol | Value                  | Unit                   |
|----|----------------------------------|--------|------------------------|------------------------|
| 1  | Inlet velocity                   | $v_{in}$| 1.25*step function    | m/s                    |
| 2  | Outlet velocity                  | $v_{out}$| 1.25*step function    | m/s                    |
| 3  | Inlet dispersed phase volume fraction | $\phi_{din}$| 0.003          | -                      |
| 4  | Dispersed phase mass-out flux    | $q_{d,out}$| $2\pi r_{d,in}$       | kg/(m·s)               |

4. Results and Discussions
This section will show the distribution of volume fraction of the dispersed phase in the channel in some units of time. Figure 4 shows streamlines of the mixture velocity and the dispersed phase volume fraction at several times of computation. From the figures it can be seen that the distribution of volume fraction is suitable with the velocity field. Opposing effects of gravity settling and turbulence-induced particle dispersion produce volume-fraction gradients in the interior. The magnitude of the mixture strain rate (and hence the turbulence production) decreases with the distance from the inlet side.
The streamlines in Figure 4 show the differences of mixture fraction in some areas. At time 0.0, the form of the streamline is relatively uniform. The streamlines change from time to time. In times of 0.1 s to 0.4 s, the solid fall to channel bed according to the form of the streamlines.

![Streamlines at different times](image)

**Figure 4.** Volume fraction distribution of the dispersed phase

The minimum and maximum of mixture fraction change with respect to time computation. Table 2 show the minimum and maximum of mixture fraction from time 0.0 s to 4.0 s. From the Figure 5, it can be shown that the minimum mixture fraction tends down, from $2.9799 \times 10^{-3}$ to $0.7802 \times 10^{-3}$. On the other hand, the maximum mixture fraction tends up, from $3.0181 \times 10^{-3}$ to $6.5914 \times 10^{-3}$. It means that the mixture fraction in the channel firstly almost uniform, and the range of mixture fraction tends to big. Physically, it means that in some areas of the channel, the solid in mixture fall to ground of the channel.
5. **Conclusions**
A 2D numerical model based on momentum transport equation for the mixture is presented to search the relationship between the sediment movement, and the pattern of volume fraction of the dispersed phase in the channel. From a computational point of view, results have shown the importance streamlines of the mixture velocity and the dispersed phase volume fraction. From a physical point of view, results highlighted the contour of the volume fraction according to the water flow with deal to the inlet and outlet position.

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**Table 2. The minimum and maximum volume fraction**

| Time (sec) | Minimum (x 10^{-3}) | Maximum (x 10^{-3}) | Range (Max-Min (x 10^{-3}) |
|-----------|---------------------|---------------------|----------------------------|
| 0.00      | 2.9799              | 3.0181              | 0.0382                     |
| 0.05      | 2.5906              | 3.4026              | 0.8120                     |
| 0.10      | 2.2579              | 3.7847              | 1.5268                     |
| 0.15      | 1.9667              | 4.1788              | 2.2121                     |
| 0.20      | 1.6639              | 4.6553              | 2.9914                     |
| 0.25      | 1.4013              | 5.1447              | 3.7434                     |
| 0.30      | 1.1651              | 5.6308              | 4.4657                     |
| 0.35      | 0.9594              | 6.1130              | 5.1536                     |
| 0.40      | 0.7802              | 6.5914              | 5.8112                     |

**Figure 5. Minimum and maximum volume fraction versus time**
References

[1] Minister of Agriculture Republic of Indonesia. 2016. Sub Sector Food Crops. [http://www.pertanian.go.id/ap_pages/mod/datatp]. Downloaded 22 Juni 2016.

[2] Gary Fornshell. 2001. Settling Basin Design. WRAC Publication No.106. 06-2001. 1-6. [http://aqua.ucdavis.edu/DatabaseRoot/pdf/WRAC-106.PDF] . Downloaded 22 July 2016.

[3] Sil, B.S. & Choudhury, P. 2016. Muskingum equation based downstream sediment flow simulation models for a river system. International Journal of Sediment Research 31(2016),139–148

[4] Tulus. 1997. The profile of a river in equilibrium state. Research Report HEDS. Jakarta.

[5] Tulus. 2013. Mathematical Modeling of Sedimentation on Thickening Processes. Proceedings of The 2nd International Seminar on Operational Research (InteriOR) 2013, 241-244.

[6] Nazari-Giglou, A., Jabbari-Sahebari, A., Shakibaenia, A. & Borghei, S.M. 2016. An experimental study of sediment transport in channel confluences. International Journal of Sediment Research 31(2016), 87–96.

[7] Rao, R., L. Mondy, A. Sun, and S. Altobelli. 2012. A Numerical and Experimental Study of Batch Sedimentation and Viscous Resuspension. Int. J. Num. Methods in Fluids, vol. 39, 465–483.

[8] F. Concha, M.C. Bustos. 1985. Theory of sedimentation of flocculated fine particles. In B.M. Moudgil, P. Somasundaran (Eds.). Proceedings of the EF Conference on Flocculation, Sedimentation and Consolidation, Sea Island, GA, AIChE, New York, p. 275-284.