Trap modulation spectroscopy of the Mott-insulator transition in optical lattices

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It is now possible to study strongly interacting condensed matter systems using cold atoms trapped in optical lattices 1, 2. Recent experiments have led to the realization of a Tonks-Girardeau gas in one-dimension (1D) 3, 4 and to the detection of the quantum phase transition from a superfluid (SF) to a Mott-insulator (MI) in bosonic systems 2, 5, 6, to name just two. Equilibrium (ground state) properties of the various condensed phases can be extracted from the interference pattern of the gas after expansion (see for example 5). Dynamical properties, however, cannot be characterized in this way. In order to have access to the excitation spectrum, and therefore to the dynamical properties, additional spectroscopic tools such as Bragg spectroscopy 8 have been designed. Stöferle et al. 6 introduced modulation spectroscopy based on a periodic change of the optical lattice depth and analyzed the energy absorption using time-of-flight techniques.

In this Letter we introduce a new technique which enables a direct probing of the phase diagram and excitation spectrum of an interacting cold atomic gas. Excitations in the system are produced by periodically modulating the strength of an external harmonic potential that confines the atoms. The width of the expanded atomic cloud in time-of-flight experiments is then used as a measure of the increase in energy of the system due to the trap modulation. We use this technique to study the superfluid to Mott insulator quantum phase transition obtained upon increasing the depth of the optical lattice. We observe a noticeably different behavior in the 1D and 3D results. While the 3D results show a sharp transition consistent with mean-field predictions, the 1D results show a broad transition between the superfluid and insulating phases. A comparison with exact time-dependent DMRG results in 1D for a model system allows us to identify the region of formation of a "welding-cake" structure in the atomic density profile in 1D, with coexisting superfluid and insulating regions. The latter has been recently observed experimentally in 3D 4.

The trap modulation spectroscopy presented here can be interpreted as a dynamical compressibility measurement, performed in a regime where the modulation frequency is much larger than the inter-well tunneling frequency. This is in contrast to recent proposals of measuring the compressibility of the system in the sense of a volume reduction following an adiabatic compression 10, as recently performed in a system of interacting fermions 11. We show below that our technique is well suited to characterizing dynamical quantities such as, e.g., the change of the excitation spectrum across the superfluid to Mott insulator quantum phase transition.

FIG. 1: (color online) Trap modulation spectroscopy of a Bose gas in an optical lattice. a) In the superfluid regime repeated compressions of the trap lead to a compression of the Bose gas and hence to excitations, whereas in the Mott insulator regime (b) the Bose gas becomes incompressible and no excitation takes place. In a) and b) the upper diagram represents the initial condition before applying the modulation.

The working principle of our method is illustrated in Fig. 1. A Bose-Einstein gas is held in the combined potential of a harmonic trap of mean frequency ω and an optical lattice of depth V0. The frequency of the harmonic trap is then increased and decreased in a sinusoidal fashion, with the instantaneous mean trap frequency given by

\[
\omega(t) = \omega + \delta \omega \sin^2(\Omega t/2),
\]

(1)
where $\delta \omega$ is the peak-to-peak modulation depth and $\Omega$ the modulation frequency. Figure 2 shows results of exact time-dependent density-matrix-renormalization-group (tDMRG) calculations of this scheme for a model system of $N = 15$ particles in one dimension. The system's dynamics is modeled using the Bose-Hubbard (BH) Hamiltonian [12], with a second order Trotter expansion of the Hamiltonian and time-steps of 0.01 $J$ [13] (where $J$ is the inter-well tunneling energy), in which we take advantage of the conserved total number of particles $N$ by projecting onto the corresponding subspace. The truncated Hilbert space dimension is up to $m = 100$, while the allowed number of particles per site is $D = 8$.

The trap modulation Eq. (1) is accounted for by a term $\frac{1}{2} m \omega(t)^2 d_L^2 \sum_i i^2 \hat{n}_i$ in the BH-Hamiltonian, where $m$ is the atomic mass, $d_L$ the lattice constant and $\hat{n}_i$ the number operator on site $i$.[14] The BH-Hamiltonian describes well the microscopic dynamics in the experiment (see below), and Fig. 2 shows the salient features of the physical picture described above. In the superfluid regime (Fig. 2(a)), after 8 modulation cycles of the trap frequency the distribution of atoms inside the lattice is markedly different compared to the initial situation as the trap modulation has led to a compression of the superfluid towards the central lattice sites. In the Mott insulator regime, on the other hand, the initial and final distributions are indistinguishable from each other, demonstrating that the system has not been excited by the repeated trap compressions.

Our modulation scheme allows us to detect signatures of the superfluid to Mott insulator transition. Figure 3 (a) shows the results of the 1D numerical simulation for the relative energy growth rate during the trap modulation (obtained from a linear fit to the energy increase for small $t$ and rescaled by the energy at $t = 0$). The relative growth rate data are plotted as a function of the Hubbard parameter $U/(2dJ)$ for different occupation numbers $n$ at the central lattice site (typically $n = 2$ in our experiments). A weighted average over different occupation numbers was also performed in order to take into account the contributions of different 1D tubes in the deep 2D lattice. The figure shows that the relative growth rate for $N = 15$ and $N = 10$ (triangles and open squares, respectively) changes rapidly in the parameter region $2 \leq U/(2dJ) \leq 8$. We find that this corresponds to the formation of a "wedding-cake" structure in the density profile upon increasing $U/(2dJ)$, as in Fig. 2(a) and (b). This restructuring of the density profile is accompanied by sharp features in the relative growth rate for $N = 15$ and $N = 10$, which are largely washed out by the numerical averaging over the tubes (black dots).

In order to test these predictions experimentally, we created Bose-Einstein condensates of roughly $6 \times 10^4$ rubidium-87 atoms inside a crossed dipole trap [14] of mean frequency $\omega \approx 2 \pi \times 80$ Hz. A three-dimensional optical lattice with lattice constant $d_L = 421$ nm was then adiabatically (within 100 ms) superimposed on the condensate. The final depth $V_0$ of the lattice (in the range between $6 E_{\text{rec}}$ and $22 E_{\text{rec}}$), where the recoil energy $E_{\text{rec}} = \hbar^2 \pi^2 \left( \frac{\mu}{2 m d_L^2} \right)$ determined the Hubbard parameter $U/(2dJ)$, where $U$ is the on-site interaction energy [16]. We realized the superfluid to Mott insulator transition both in 1D ($d = 1$) and 3D ($d = 3$). For the 1D case we ramped two of lattice beams to a maximum depth of $26 E_{\text{rec}}$ (which resulted in a 2D array of one-dimensional tubes [6, 17]) and varied the depth $V_0$ of the third lattice, whereas for the 3D case all three lattices had the same depth. In both cases we performed various tests (visibility of the interference pattern [18], excitation spectrum [6], adiabaticity of the lattice ramps [19]) in order to identify the critical lattice depth for entering the Mott insulating regime and to ensure that the various regimes were reached adiabatically, i.e., without exciting the Bose gas.

Once the final lattice depth was reached, the harmonic trap frequency was modulated by sinusoidally modulating the power of the crossed dipole trap between the extreme values $P_1$ and $P_0(1 + \alpha)$ at frequency $\Omega$ [20]. The resulting variation of the trap frequency $\omega(t) \propto \sqrt{P(t)}$ was reasonably sinusoidal for $\alpha \lesssim 1$, with a modulation parameter $\delta \omega/\omega = \sqrt{1 + \alpha} - 1$. After a few milliseconds the modulation was switched off and the optical lattices were ramped down in 15 ms to $V_0 = 4 E_{\text{rec}}$. The atoms were allowed to thermalize for a further 10 ms, then both the lattice and the dipole trap were suddenly switched off, and the atoms were imaged after a time-of-flight of 23.3 ms. For a fixed lattice depth in the superfluid regime we observed a roughly linear increase with modulation time in the width of the expanded Bose gas for up to 10 ms of trap modulation, and this was independent of the modulation frequency $\Omega$ over a wide range (between

![FIG. 2: Numerical simulation of the effects of trap modulation in a one-dimensional lattice with total atom number $N = 15$ (corresponding to $n = 2$) for (a) $U/(2dJ) = 2$ and (b) $U/(2dJ) = 9$. The filled circles represent the atomic distribution at $t = 0$, while the open circles show the distribution after 8 compression cycles with $\delta \omega/\omega = 1$.](image-url)
$\approx 0.5$ kHz and $\approx 4$ kHz). These findings are consistent with our numerical simulations.

![Figure 3](image)

**FIG. 3:** (a) Numerical simulation of trap modulation spectroscopy in 1D with $\delta \omega / \omega = 0.22$ and $\Omega / 2\pi = 1$ kHz. Shown here are the results for boson numbers $N = 15$ (open triangles) and $N = 10$ (open squares), corresponding to occupation numbers $n = 2$ and $n = 1.5$ of the central lattice site, respectively, as well as a weighted average (filled circles) over total atom numbers $N = 5, N = 10$ and $N = 15$. The energy growth rates are extracted by taking the average of two extreme linear fits to the initial growth curve, leading to a relative error of around 15 percent. (b) Experimental results in 1D (open squares) for $\delta \omega / \omega = 0$ relative error of around 15 percent. (b) Experimental results are consistent with the (averaged) 1D numerical simulations shown in Fig. 3 (a) if an overall scaling factor is applied to the theoretical results. Generally, we see a decrease in the excitability of the Bose gas reflected by $\sigma / \sigma_0$ approaching unity. While in 1D this decrease is gradual and even for large values of $U/(2dJ)$ there is a residual excitation of the Bose gas, in 3D we observe a sharp drop in $\sigma / \sigma_0$ around $U/(2dJ) = 4$. For $U/(2dJ) \geq 5$, $\sigma / \sigma_0$ remains constant around 1, indicating that the system is incompressible. This agrees well with the physical picture of a transition from a compressible (and hence excitable) superfluid to an incompressible Mott insulator for $U/(2dJ) \approx 5.8$ (mean-field prediction) or $U/(2dJ) \approx 4.9$ (Quantum Monte-Carlo simulation) [2].

Performing the above experiment in 3D for different values of the modulation depth $\delta \omega / \omega$, we found that the sharp feature around $U/(2dJ) = 4$ was clearly visible for $\delta \omega / \omega \lesssim 0.5$ but became increasingly washed out for larger values. Indeed, for an intermediate value $\delta \omega / \omega = 0.7$ (see Fig. 4 (a)) we saw a structure with two apparent “steps” at $U/(2dJ) \approx 4$ and $U/(2dJ) \approx 9$, and $\sigma / \sigma_0$ approached unity for $U/(2dJ) > 10$. The proximity of the step positions to the theoretical values of $U/(2dJ)$ for the formation of Mott insulator shells with $n = 1$ and $n = 2$ atoms per lattice site, respectively. [3] [22] suggests the following intuitive explanation. For small modulation depths, the formation of the $n = 1$-shell greatly reduces the overall excitability of the system, while for larger values of $\delta \omega / \omega$ the $n = 1$ shell, being located away from the trap center and hence exposed to larger potential gradients when the trap is compressed, remains partly compressible and only when the $n = 2$ shell is formed does the system become insensitive to the trap modulation.

We tested this hypothesis by measuring the Bose gas response as a function of $\delta \omega / \omega$ in the superfluid and in the Mott insulator regime. Figure 4 (b) shows that while in the superfluid regime $\sigma / \sigma_0$ increased roughly linearly with the modulation depth, in the Mott insulator regime we observed that for $\delta \omega / \omega \lesssim 0.6$ there was no dependence on the modulation depth. For larger values of $\delta \omega / \omega$, the Bose gas could again be excited, suggesting that in this regime the trap compression was sufficient to overcome the potential barrier $U$ for moving atoms between adjacent lattice sites. Alternatively, a strong trap modulation might lead to a heating of the thermal fraction of the sample, thus “melting” the Mott insulator [23] and increasing the compressibility of the Bose gas. While we were unable to rule out either of these two scenarios, this remains an interesting question for future experiments and theoretical analysis.

Finally, we investigated the response of our system to trap modulation for adiabatic and non-adiabatic loading of the lattice. In all of the experiments described above, we made sure that the timescales and shapes of the intensity ramps for the lattice beams were such that we reached the final lattice depth adiabatically and hence always prepared the system in its ground state. One can, however, prepare the same final lattice depth non-adiabatically, in which case one expects to find the system in an excited state. If the lattice depth is above the critical value for the Mott insulator transition, this means that for non-adiabatic loading the system will not be in a pure Mott insulator state and should, therefore, retain its compressibility. We tested this assumption by using a two-part ramp for the optical lattice with a 100 ms exponential ramp up to $5 E_{\text{rec}}$ and a linear ramp in 0.1 ms to the final value $V_0$. The second part of the ramp was highly non-adiabatic, which we verified by lowering the lattice after a holding time of 5 ms and checking that the Bose gas was excited, as expected. We then performed
we obtained good agreement with exact numerical calculations. For the three-dimensional case a full theoretical interpretation will require further calculations. As a perspective for future studies we believe that it is of great importance to explore in detail the differences and similarities of our approach with other methods based on a quasi-static compression of the trap [11].

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In summary, we used trap modulation spectroscopy in order to study the superfluid to Mott-insulator phase transition. The method presented here seems to be a promising tool for the characterization of strongly interacting cold atomic systems. For the one-dimensional case...
dipole trap beam that was approximately collinear with the 1D lattice beam and checking the response of the Bose gas as a function of $V_0$.

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