A CLOSER LOOK AT THE MOND NO-GO STATEMENT FOR PURELY METRIC FORMULATIONS

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Abstract
We reexamine the assumptions made in arriving at a no-go statement for purely metric formulations of MOND. Removing the requirement of gravitational stability at appropriate scales gives life to the possibility of a purely metric theory of MOND.

1 Introduction
Milgrom’s MOND (Modified Newtonian Dynamics) is imminently falsifiable. It was empirically designed to explain the fact that galactic satellites do not experience a Keplerian fall off of their velocities outside the central galactic bulge – rather they asymptote to a constant value\(^1\). Therefore, it suffices to find one galaxy which does not exhibit MOND behavior to reject it as a candidate explanation.

In physics, theories which can be disproved should be welcomed. To many MOND is a pariah, a quaint but irrelevant approach to a phenomenon which can be described by deferring to the more “natural” idea of dark matter. This vantage point may very well be ultimately justified, however its current acceptance is not. Anyone who seriously considers the problem of rotation curves must allow for different possibilities until experimental evidence forces us to discard any or all of them. At this time, there has been no definitive data which can rule out MOND.

One of MOND’s greatest shortcomings has been its resistance to being made fully relativistic. Indeed, if we are to take it seriously as an alteration of the gravitational force at low accelerations, it must be somehow incorporated into a relativistic modification of Einstein’s equation. The impetus for such a construction is partly esthetics; however, it is phenomenology which serves as the greatest source of motivation. MOND in its original formulation is assumed as an alternative to dark matter. If so, the relativistic version of MOND has an effect on the amount of gravitational lensing observed – it must account for the deficiency in the General Relativity with no dark matter prediction\(^2\).
As of yet, there has been no completely satisfactory relativistic theory divided. Attempts to this end can be roughly divided into two theoretical categories – scalar-tensor and purely metric. Of course, it is often possible to write the latter using the former. However, the scalar-tensor models of Bekenstein, Milgrom, and Sanders are quite different in spirit from the purely metric one we constructed. These particular scalar-tensor models all introduce real degrees of freedom versus the purely gravitational degrees of freedom of the purely metric approach. Further, a distinction is made between the “Einstein” and “physical” metrics. The former is responsible for dynamics of the gravitational field, the latter determines the geodesics followed by test particles. The pure metric theory makes no distinction between the two, the strong principle of equivalence being invoked.

The scalar-tensor models suffered from acausal propagation of gravitational waves, the removal of which unfortunately caused the amount of gravitational lensing predicted to be less than that of General Relativity alone. One can get a phenomenologically viable model at the expense of Lorentz invariance by introducing a nondynamical vector field. The preferred-frame effects are locally suppressed; however, they are at least as large or larger than experimental limits.

The purely metric theory suffers from conformal invariance of the field equations at linearized order in perturbation theory. The result of conformal invariance in a covariant theory is the decoupling of light. Therefore, to linearized order, there is no enhanced lensing, but simply the amount predicted by General Relativity. This is the basis of our no-go statement – and even though our original formulation of this statement was specific to our model, it turns out that this result is true for any purely metric theory. The purpose here is to review the assumptions and arguments which lead to the no-go statement for the purely metric theory and to comment on how one may achieve a phenomenologically viable theory of MOND by removing the assumption of gravitational stability.

2 The no-go statement

Here we review the basic no-go argument previously considered. It is as follows:

1. Any purely metric theory of gravity will have ten field equations in four spacetime dimensions of the form,

   \[ G_{\mu\nu}[g] = 8\pi G T_{\mu\nu}, \]

   where \( G \) (which we will call the Gravity tensor from here on) is a function of the metric which for ordinary General Relativity is simply the Einstein tensor. It is obtained by the variation of the gravitational action with respect to the metric. \( T_{\mu\nu} \) is the usual stress-energy tensor obtained from inserting matter sources into the physical system in question.

2. The Gravity tensor is covariant by construction and thus is covariantly conserved.

3. The Gravity tensor can be expanded in weak-field perturbation theory, \( G_{\mu\nu}[\eta + h] \).

4. The MOND force law scales as the square root of the mass,

   \[ F_{\text{MOND}} = \frac{\sqrt{GMa_0}}{r}, \]

   where \( a_0 \) is a constant determined by fitting to nine well measured rotation curves – the value of which is \((1.20 \pm 0.27) \times 10^{-10} \text{ m s}^{-2}\). This follows from the nonrelativistic MOND force law which gives the required constant asymptotic velocities of satellites in circular (or nearly circular) orbits.

5. In the deep MOND regime (that is, for accelerations on the order of \( a_0 \)), at least one component of \( h_{\mu\nu} \) must scale as \( \sqrt{GM} \) if rotation curves are to be reproduced.

6. The right hand side of \( F_{\text{MOND}} \) scales as \( GM \).

7. From 6. it must be that in the deep MOND regime at least one of the ten equations is non-zero at order \( h^2 \).
8. A second rank symmetric tensor in four dimensions has two distinguished components – its covariant derivative and its trace. The first is zero by 2. Therefore, the component which begins at quadratic order in the weak-fields in the deep MOND regime is the trace component.

9. The linearized MOND weak-fields are thus traceless.

10. Tracelessness directly implies conformal invariance and henceforth the decoupling from light.

This last statement essentially kills any hope of formulating a phenomenologically viable theory, for the linearized MOND equations give no added lensing to the result of General Relativity given the assumptions of the next section.

3 The Assumptions

The assumptions made in the previous section were:

1. The gravitational force is carried by the metric with its source being the usual stress-energy tensor.
2. Gravity is described by a covariant theory.
3. The MOND force law can be realized in weak-field perturbation theory.
4. The theory of gravity is absolutely stable
5. Electromagnetism couples conformally to gravity.

The third and fifth assumptions are the most rigid. The third, if not true, would bar us from working with any relativistic version of MOND – if there is no region for which the MOND force is weak (or at least as weak as the Newtonian gravitational force), then there is no hope in passing standard phenomenological requirements.

The first, second, and fourth assumptions, however, are capable of undergoing more scrutiny. The first, for example, may be violated if one makes a distinction between a “physical” and “gravitational” metric. In such a case particles would follow geodesics of the former while gravity would behave according to dynamics of the latter.

The second assumption is easily foregone if one specifies a preferred-frame. This can be accomplished by inserting nondynamical vector fields à la Sanders. As mentioned earlier, the effects coming from preferred-frame physics would need to be suppressed at least enough to pass local gravitational tests.

These already mentioned assumption violations, however, do not bear very heavily on a purely metric formulation. The first applies if one wishes rather to consider scalar-tensor theories. The second is contrary to the philosophy of a pure metric approach – that is, we do not want to introduce any new degrees of freedom, regardless of whether they are non-dynamical. The fourth, however, is the most significant to examine if we are interested in remaining close to the original idea of pure metric theories.

Given the choice between a stable and unstable theory, the physicist will always choose the former. However, when doing phenomenology, the latter may be the choice of greater utility. When the instability manifests itself at scales outside or nearly outside the physical scale, the phenomenologist may cautiously accept (or at least consider accepting) the unstable solution as viable.

It is possible to imagine that all of the linearized MOND weak-fields vanish in the equations of motion, in which case there would no longer be a linearized theory, and sufficient bending of light could be realized. If the MOND weak-fields begin at quadratic order in the field equations, they must be cubic in the action and thus possess an inherent instability. This is not necessarily a fatal property. There are two weak-field regimes – the deep MOND (or ultra-weak-field) and the weak-field (or Newtonian). In regions such as the solar system it would be the Newtonian regime which dominates and thus we would experience no deviation from well established physics. At larger scales (galactic and/or cosmological) we would expect the deep MOND regime to enter the fold. The unstable solution would decay then into large wavelength particles (galactic scale at least) diffusing as the universe...
expands. This process could directly result in the return to the Newtonian regime as decay products would build a sufficiently large gravitational potential.

4 Conclusion

To date there has not been a completely satisfactory theory of MOND which is both fully relativistic and phenomenologically viable. The pure metric approach will fail to pass the crucial test of gravitational lensing due to the conformal invariance of the linearized weak-field equations in the deep MOND regime. At the cost of stability, one restores hope that the theory predicts the correct amount of observed lensing. This instability is phenomenologically acceptable if its behavior is detectable at least at galactic scales. We may conclude that although a purely metric approach has some serious issues with which to contend, it is not to be considered dead and gone. If one divises a theory in which all ten of the linearized field equations vanish then there is hope for sufficient lensing. Further, a no-go statement as the one we have made should be taken as a challenge. It only takes one counter example to disprove the statement, just as it takes only one rotation curve which contradicts MOND to reject it as a candidate explanation to one of the most interesting problems currently facing the astrophysics community.

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