Production of $\sigma$ in $\psi(2S) \to \pi^+\pi^- J/\psi$

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Using 14M $\psi(2S)$ events accumulated by BESH at the BEPC, a Covariant Helicity Amplitude Analysis is performed for $\psi(2S) \to \pi^+\pi^- J/\psi$, $J/\psi \to \mu^+\mu^-$. The $\pi^+\pi^-$ mass spectrum, distinctly different from phase space, suggests $\sigma$ production in this process. Two different theoretical schemes are used in the global fit to the data. The results are consistent with the existence of the $\sigma$. The $\sigma$ pole position is determined to be $(5440^{+230}_{-210})$ MeV.
1. Introduction

We report here a study of the process $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$, which is the $\psi(2S)$ decay mode with the largest branching fraction [11], using very clean $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ ($J/\psi \rightarrow \mu^+\mu^-$) events. Early investigations of this decay by Mark I [2] found that the $\pi^+\pi^-$ mass distribution is strongly peaked toward higher mass in contrast to what is expected from phase space. Furthermore, angular distributions favored $S$-wave production between the $\pi\pi$ system and $J/\psi$, as well as an $S$-wave decay of the dipion system.

BESI studied this process with much higher statistics (3.8 million $\psi(2S)$ events) and found that an additional small $D$-wave component was required in the decay of the dipion system [3]. Also various heavy quarkonium models were fitted, and the parameters for these models determined [3].

Here, we fit the $\pi^+\pi^-$ system from $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ decays with the $J^{PC} = 0^{++}$ $\sigma$ meson. In this decay, the interaction between the $\pi\pi$ system and $\psi(2S)$ or $J/\psi$ is small since these charmonium states are very narrow, so the dipion system is a quasi-isolated system [4].

The $\sigma$ meson was introduced theoretically in the linear $\sigma$ model [4], and its existence was first suggested in a one-boson-exchange potential model of nuclear forces [9]. The $\sigma$ meson is important due to its relation with dynamical chiral symmetry breaking of QCD [7].

There was evidence for a low mass pole in early DM2 [8] and BESI [9] data on $J/\psi \rightarrow \omega\pi\pi$. A huge event concentration in the $I = 0$ $S$-wave $\pi\pi$ channel was observed in a $pp$ central production experiment in the region from $m_{\pi\pi} = 500$ to 600 MeV/$c^2$ [10]. This peak is too large to be explained as background [11]. Many studies on the possible resonance structure in $\pi\pi$ scattering have appeared in the literature [12]. It was proved that the existence of a light and broad resonance is unavoidable even with nonlinear realization of chiral symmetry [13]. Careful theoretical analyses were made to determine the pole location, which was found to be $M - i\Gamma/2 = (470 \pm 30) - i(295 \pm 20)$ MeV/$c^2$ [14] and $M - i\Gamma/2 = (470 \pm 50) - i(285 \pm 25)$ MeV/$c^2$ [15].

Renewed experimental interest arose from E791 data on $D^+ \rightarrow \pi^+\pi^-\pi^+$ [16], where it was found that $M = 478_{-23}^{+24} \pm 17$ MeV/$c^2$, $\Gamma = 324_{-40}^{+42} \pm 21$ MeV/$c^2$. In the recent partial wave analysis of the decay $J/\psi \rightarrow \omega\pi\pi$ [17] by BESII, the pole position of $\sigma$ was determined to be $(541 \pm 39) - i(252 \pm 42)$ MeV/$c^2$. All these experimental results still have large uncertainties.

Fig. 1 shows the decay mechanism of $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ in the $S$-matrix formalism. There are three main contributions including an $S$-wave resonance $(\sigma)$, a $D$-wave term $(2^+)$, and a contact term which is the destructive background required by chiral symmetry [18]. The total amplitude is the sum of these three components. The decay $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ can also be described with an effective Lagrangian for the vector pseudo-scalar pseudo-scalar (VPP) vertex, along with the $\pi\pi$ final state interaction (FSI) obtained from $\pi\pi$ scattering data in a Chiral Unitary Approach (ChUA) [19]. In such an approach, the $\sigma$ resonance is generated dynamically as a pole of the unitarized $t$-matrix, and the pole position is $469 - i203$ MeV/$c^2$ [20]. A fit to the BESI $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ data shows that the $\pi\pi$ FSI plays an important role in this process [19]. A similar result was obtained in Ref. [21] with a comparison of the cases with and without the $\pi\pi$ FSI. We fit our data with both the $S$-matrix and ChUA schemes.

2. BESII Experiment

The data sample used for this analysis is taken with the BESII detector at the BEPC storage ring operating at the $\psi(2S)$ resonance. The number of $\psi(2S)$ events is $14.0 \pm 0.6$ million [22], determined from the number of inclusive hadrons.

The Beijing Spectrometer (BES) detector is a conventional solenoidal magnet detector that is described in detail in Ref. [23]; BESII is the upgraded version of the BES detector [24]. A 12-layer vertex chamber (VC) surrounding the beam pipe provides track and trigger information. A 40-layer main drift chamber (MDC), located radially outside the VC, provides trajectory and energy loss $(dE/dx)$ information for charged tracks over 85% of the total solid angle. The mo-
momentum resolution is \( \sigma_p/p = 0.017\sqrt{1+p^2} \) (p in GeV/c), and the \( dE/dx \) resolution is \( \sim 8\% \). An array of 48 scintillation counters surrounding the MDC measures the time-of-flight (TOF) of charged tracks with a resolution of \( \sim 200 \) ps for hadrons. Radially outside the TOF system is a 12 radiation length, lead-gas barrel shower counter (BSC). This measures the energies of electrons and photons over \( \sim 80\% \) of the total solid angle with an energy resolution of \( \sigma_E/E = 22\%/\sqrt{E} \) (E in GeV). Outside the solenoidal coil, which provides a 0.4 Tesla magnetic field over the tracking volume, is an iron flux return that is instrumented with three double layers of counters (MUID) that identify muons of momentum greater than 0.5 GeV/c.

A GEANT3 based Monte Carlo (MC) simulation program [25] with detailed consideration of detector performance (such as dead electronic channels) is used to simulate the BESII detector. The consistency between data and MC simulation has been carefully checked in many high purity physics channels, and the agreement is quite reasonable [25].

3. Event Selection

Events with \( \pi^+\pi^-\mu^+\mu^- \) final states and with the invariant mass \( m_{\mu^+\mu^-} \) constrained to the \( J/\psi \) mass are selected for analysis. Each track, reconstructed using hits in the MDC, must have a good helix fit in order to ensure a correct error matrix in the kinematic fit, and the number of tracks are required to be between 4 and 7.

To select a pair of muons, the muon pair candidates tracks are required to have net charge zero: \( p_{\mu^+} > 1.3 \) GeV/c or \( p_{\mu^-} > 1.3 \) GeV/c or \( p_{\mu^+} + p_{\mu^-} > 2.4 \) GeV/c; \( |\cos \theta_\mu| < 0.6 \) to ensure that tracks are in the sensitive region of the MUID; the cosine of the angle between these two tracks in their rest frame \( \cos \theta_{\mu^+\mu^-} < -0.975 \) to guarantee the collinearity of the tracks; the sum of the MUID hits \( N_{MUID}^+ + N_{MUID}^- \geq 3 \) to ensure that the tracks are muons; and the invariant mass of two candidate tracks \( m_{\mu^+\mu^-} \) within 0.35 GeV/c\(^2 \) of the \( J/\psi \) mass.

For \( \pi^+\pi^- \) pair selection, the two candidate tracks are also required to have net charge zero. Each track is required to have momentum \( p_e < 0.5 \) GeV/c, polar angle \( |\cos \theta| < 0.75 \), and transverse momentum \( p_{\pi\pi} > 0.1 \) GeV/c to reject tracks that spiral in the MDC. The \( dE/dx \) measurement of each track must be within three standard deviations of the \( dE/dx \) expected for the pion hypothesis, and the cosine of the laboratory angle between the candidate tracks must satisfy \( \cos \theta_{\pi\pi} < 0.9 \) to eliminate \( e^+e^- \) pairs from \( \gamma \) conversions. The mass recoiling against the candidate \( \pi^+\pi^- \) pair, \( m^{\text{recoil}}_{\pi^+\pi^-} \), is shown in Fig. 4. In order to get well reconstructed signal events and to suppress background, \( |m^{\text{recoil}}_{\pi^+\pi^-} - m_{J/\psi}| < 20 \) MeV/c\(^2 \), corresponding to three times the mass resolution, is required.

Figure 1. Decay mechanisms for \( \psi(2S) \rightarrow \pi^+\pi^- J/\psi \) in the S-matrix formalism. The final amplitude is the superposition of \( \psi(2S) \rightarrow \sigma J/\psi, \psi(2S) \rightarrow 2\pi^+ J/\psi, \) and \( \psi(2S) \rightarrow (\pi^+\pi^-)_{\text{cont}} J/\psi \) S-matrix elements.
With the above selection criteria, about 40,000 $\psi(2S) \to \pi^+\pi^- J/\psi \to \pi^+\pi^- \mu^+\mu^-$ candidate events are obtained. Fig. 2, shows the $\pi^+\pi^-$ invariant mass distribution for these events, where the dots with error bars are data, and the histogram is Monte Carlo simulation with the PPG++GEN generator, which is based on chiral symmetry arguments and partially conserved axial vector currents [26]. It describes the low mass $\pi\pi$ spectrum reasonably well but not the high mass region; the inconsistency between data and Monte Carlo will be considered in the systematic errors.

In the second, considering the VPP vertex and the S-wave $\pi\pi$ FSI, while neglecting the D-wave FSI, the amplitude is [19] 

$$A = V_0 + V_{0S} \cdot G \cdot 2i_{t_{\pi\pi}^{\pi\pi}} ,$$

where $G$ is the two-pion loop propagator, $V_{0S}$ is the S-wave part of $V_0$, and $t_{\pi\pi}^{\pi\pi} = 0$ is the full S-wave $I = 0 \, \pi\pi \to \pi\pi$ t-matrix, which is the same as those defined in Refs. [19 20]; $p_1$ and $p_2$ are the four momenta of the two pions, and $p_1^0$ and $p_2^0$ are their energies in the lab frame; $g_1, g_2$, and $g_3$ are free parameters to be determined by data.

The normalized probability density function used to describe the whole decay process is 

$$f(x, \alpha) = \frac{d\sigma/d\Omega}{\sigma} ,$$

where $x$ represents a set of quantities which are measured by experiment, and $\alpha$ represents unknown parameters to be determined. The total cross section, $\sigma$, can be expressed as 

$$\sigma = \int \epsilon(\Omega) \frac{d\sigma}{d\Omega} d\Omega ,$$

where $\epsilon(\Omega)$ is the detection efficiency which is usually a function of detector performance. The total cross section can be determined by MC integration. Re-weighting a total of $N$ generated events based on simulated $\psi(2S) \to \pi^+\pi^- J/\psi$ using a phase space generator, the total cross section is then 

$$\sigma = \frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} \frac{d\sigma}{d\Omega} \bigg|_i .$$

where $N_{mc}(< N)$ is the number of MC simulated events after applying the selection criteria.

The maximum likelihood function [27 28] is given by the joint probability density of the selected $\psi(2S) \to \pi^+\pi^- \mu^+\mu^-$ events, 

$$\mathcal{L} = \prod_{i=1}^{N_{evt}} f(x, \alpha) ,$$

and a set of values, $\alpha$, is obtained by minimizing the function $S$, 

$$S = - \log \mathcal{L} .$$

For the amplitudes in the first model, the amplitudes for the cascade two-body decay process can be expanded with helicity amplitudes as: 

$$A_s = F_m^J \cdot D_{M,\lambda-\nu}^{sJ} BW_X(S_{\pi\pi}, m_X, \Gamma_X) F_{00}^{s} D_{\lambda,0}^{s} ,$$

where $G$ is the two-pion loop propagator, $V_{0S}$ is the S-wave part of $V_0$, and $t_{\pi\pi}^{\pi\pi} = 0$ is the full S-wave $I = 0 \, \pi\pi \to \pi\pi$ t-matrix, which is the same as those defined in Refs. [19 20]; $p_1$ and $p_2$ are the four momenta of the two pions, and $p_1^0$ and $p_2^0$ are their energies in the lab frame; $g_1, g_2$, and $g_3$ are free parameters to be determined by data.

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Figure 2. Distributions of candidate $\psi (2S) \rightarrow \pi^+ \pi^- J/\psi (J/\psi \rightarrow \mu^+ \mu^-)$ events. (a) is the $\pi^+ \pi^-$ recoil mass spectrum, fitted with a double Gaussian function, and (b) is the $\pi^+ \pi^-$ invariant mass spectrum. The histogram in (a) is data, the curve is the fit, the events between arrows are selected; in (b) dots with error bars are data, and the histogram is MC simulation.

where $F_{\chi^0}^J$ is the helicity amplitude, which can be found in Ref. \cite{28}, $D^{J,M,\lambda-\nu}_M(\phi, \theta, 0)$ is the D-function, and $BW_X(S_{\pi\pi}, m_X, \Gamma_X)$ is the Breit-Wigner propagator of $X$, defined as:

$$ BW_X(s_{\pi\pi}, m_X, \Gamma_X) = \frac{1}{s_{\pi\pi} - m_X^2 + i m_X \Gamma_X(s_{\pi\pi})}. \quad (10) $$

The $\sigma$ particle, a broad structure in the low $\pi^+ \pi^-$ mass region, is not a typical Breit-Wigner resonance. In the first model, four types of Breit-Wigner parameterizations are used to describe it:

1) Constant width

$$ \Gamma_X(s) = \Gamma. \quad (11) $$

2) Width containing a kinematic factor, which was used by the E791 Collaboration \cite{10}

$$ \Gamma_X(s) = \rho \frac{s}{m_X^2} \Gamma = \sqrt{1 - \frac{4 m_X^2}{s}} s \Gamma. \quad (12) $$

3) P.K.U. ansatz \cite{29}, which removes the spurious singularity hidden in Eq. (12)

$$ \Gamma_X(s) = \rho \frac{s}{m_X^2} \Gamma = \sqrt{1 - \frac{4 m_X^2}{s}} s \Gamma. \quad (13) $$

4) Zou and Bugg’s approach \cite{30}, where the form includes explicitly into $\Gamma_X(s)$ the Adler zero at $s = m_X^2/2$.

$$ \Gamma_X(s) = g_1 \frac{\rho_{\pi\pi}(s)}{\rho_{\pi\pi}(m_X^2)} + g_2 \frac{\rho_{4\pi}(s)}{\rho_{4\pi}(m_X^2)} \quad (14) $$

$$ g_1 = f(s) \frac{s - m_X^2/2}{m_X^2 - m_{\pi\pi}^2/2} \frac{e^{-m_X^2/a}}{a}, $$

where the definitions of $\rho_{\pi\pi}$, $\rho_{4\pi}$, and $f(s)$ are the same as Ref. \cite{17}.

5. Partial wave analysis

The minimization used in the partial wave analysis and to obtain the pole parameters of the $\sigma$ is based on MINUIT \cite{31}. For the first model, the components considered include amplitudes of $\sigma(0^+)$, a $D$-wave term, and a contact term. The
tail of the $f_0(980)$ has been tried in the fit. However, it has similar behavior to the contact term in this mass region, and therefore it is ascribed to the contact term. All four $\sigma$ Breit-Wigner parameterizations fit the data well, but have strong destructive interference with the contact term, especially in the low $\pi^+\pi^-$ invariant mass region. The $D$-wave contribution is only 0.3 to 1%, in agreement with the BESI result [3] based on a different analysis method. Fig. 3 shows the projections of the fit results compared with data for the Breit-Wigner parameterization for the P.K.U. ansatz; other parameterizations give similar results.

The global fits determine the best estimation of the Breit-Wigner parameters for each parameterization. The pole position in the complex energy plane is related to the mass and width of the resonance by

$$\sqrt{s_{\text{pole}}} = m_\sigma - \frac{i\Gamma_\sigma}{2}. \quad (15)$$

The best fit results and the corresponding pole positions for all the parameterizations are listed in Table 1. The statistical error of the resonance mass (width) is determined by a decrease of $1/2$ in the log-likelihood from its maximum value with all other parameters fixed to their best solutions.

For the second model, where the VPP vertex is represented by an effective Lagrangian and the $\pi\pi$ S-wave FSI is included, the $\pi\pi$ mass spectrum can also be reproduced well. Here the $\sigma$ requires a much smaller interference between the S-wave FSI and the contact term. In this case, the fit is worse than the fits of the first model; this may due to the fixed pole position of the $\sigma$ and the neglected $D$-wave contribution. For the second model, the pole is not measured in the fit, but taken from Ref. [20], which was determined from $\pi\pi$ scattering data.

To check the goodness of fit in our analysis, we construct a variable

$$\chi^2_{\text{obs}} = \sum_{i=1}^{N} \left( \frac{N_i^{DT} - N_i^{MC}}{\sqrt{N_i^{DT}}} \right)^2, \quad (16)$$

where $N$ is the number of cells, $N_i^{DT}$ and $N_i^{MC}$ are the numbers of events in the $i$'th cell of the Dalitz plot with axes $m_{\pi^+\pi^-}^2$ and $m_{J/\psi\pi^+}^2$ for data and MC simulation, respectively. Such a variable should be distributed according to the $\chi^2$ distribution with $n = N - K$ degrees of freedom, where $K = 12$ is the number of parameters to be determined in our Maximum Likelihood fit. In our case, 15 bins in both $m_{\pi^+\pi^-}^2$ and $m_{J/\psi\pi^+}^2$ give 225 cells. To ensure proper $\chi^2(n)$ behavior, cells with less than five events have been merged into adjacent ones. The number of cells becomes $N = 208$, and the number of degrees of freedom $n = 196$. From the observed $\chi^2$ value determined using Eq. (16) for each parameterization, the confidence levels (C.L.) are calculated and listed in Table 1.

### 6. Systematic Errors

For the first model, the systematic error of the $\sigma$ pole position arises from the uncertainties of the strength of the $2^+$ component, the form of the contact term, and the inconsistency between data and of MC simulation. For the $2^+$ component, we conservatively remove it from the fit, and the difference of the fitted values from the nominal values are taken as systematic errors. Two contact terms, namely, constant amplitude and $\alpha_1 + i\alpha_2\rho$, where $\alpha_1$ and $\alpha_2$ are two parameters to be fitted, are adopted in the fit; the difference is considered as the systematic error. The MC simulation and data have different mass resolutions in the high mass region of the $\pi^+\pi^-$ system. A modification of $\pi^+\pi^-$ mass resolution is made to improve the fit, and the difference of the fitted pole positions with and without this modification is taken as the systematic error. The systematic error from non-signal backgrounds is neglected.

In order to obtain the $m_\sigma$ and $\Gamma_\sigma$ errors in Eq. (16), we set the denominator in Eq. (16) equal to zero and obtain the pole position and corresponding errors by taking into account the mass and width errors of the Breit-Wigner parameterizations. This is done using a MC sampling method, where the correlation between the mass and width is ignored. Table 2 summarizes the systematic errors from all sources, and Table 1 lists the parameters of pole position and their total errors.
Figure 3. Fit results of $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ (P.K.U. ansatz). Dots with error bars are data and the histograms are the fit results. (a) and (b) are the $\pi^+\pi^-$ invariant mass, (c) and (d) the cosine of the $\sigma$ polar angle in the lab frame, and (e) and (f) the cosine of the $\pi^+$ polar angle in the $\sigma$ rest frame. The upper plots are the detected distributions, while the bottom ones are the distributions after efficiency correction.

Figure 4. The $\pi^+\pi^-$ invariant mass distribution including the components. The left is the P.K.U. ansatz, which has the contributions from $\sigma$, $D$-wave term, contact term, and their sum (the $D$-wave is enlarged by a factor of 20 in the figure). The right is the $\pi^+\pi^-$ invariant mass fitted by the formula from Ref. [19][20], with no explicit $D$-wave. Dots with error bars are data, and the histograms are the fit results.
Table 1
Fit results of the two models and all the Breit-Wigner parameterizations

| Model          | Constant (MeV/c²) | F with ρ (MeV/c²) | P.K.U. ansatz (MeV/c²) | Bugg & Zou’s approach (MeV/c²) | Ref. |
|---------------|-------------------|-------------------|------------------------|-------------------------------|------|
| Pole (MeV/c²) | (553 ± 15 ± 47)   | (559 ± 6 ± 26)    | (554 ± 13 ± 65)        | (541 ± 9 ± 95)                | [19,20] |
| Nσ †          | −i(254 ± 23 ± 54) | −i(179 ± 7 ± 18)  | −i(240 ± 4 ± 19)       | −i(253 ± 8 ± 33)              |      |
| N_contact †   | 1.17e5 ± 6        | 6.33 ± 1.31       | 11.23 ± 0.11           | 15.77 ± 1.01                  |      |
| χ²/νdof, C.L. | 217.83/196        | 227.54/196        | 224.07/196             | 217.88/196                    |      |
|               | 0.136 ± 0.060     | 0.0825            | 0.1357                 | 3 ± 10×10⁻³                   |      |

†: Nσ and N_contact are the numbers of events in the fit.

Table 2
Systematic errors in the pole position (MeV/c²)

|                        | Constant (MeV/c²) | F with ρ (MeV/c²) | P.K.U. ansatz (MeV/c²) | Bugg & Zou’s approach (MeV/c²) |
|------------------------|-------------------|-------------------|------------------------|-------------------------------|
| 2⁺ uncertainty         | σ_m σ_u/2        | σ_m σ_u/2        | σ_m σ_u/2              | σ_m σ_u/2                    |
| form of contact term   | 22 35             | 4 14              | 10 2                   | 3 3                          |
| M.C. imperfection      | 40 38             | 25 10             | 56 17                  | 37 24                        |
| Total Error            | 47 54             | 26 18             | 65 19                  | 95 33                        |

7. Results and discussion

The process $\psi(2S) \rightarrow \pi^+\pi^-J/ψ$, $J/ψ \rightarrow μ^+μ^-$ is studied based on $14 \times 10^6 \psi(2S)$ events collected with the BESII detector. The $\pi^+\pi^-$ invariant mass spectrum of $\psi(2S) \rightarrow \pi^+\pi^-J/ψ$ has a severe suppression near the $\pi^+\pi^-$ threshold, which is distinctly different from phase space and suggests σ production in the process. We fit the data with two different models. For the first model, using four different Breit-Wigner parameterizations, the data can be well fitted, although a strong cancellation between the σ and the contact term is required. In fact, such a large cancellation is dictated by chiral symmetry [26,18]. The pole positions of σ are determined for different Breit-Wigner parameterizations, which are $(553 ± 15 ± 47) - i(254 ± 23 ± 54) \text{ MeV/c}^2$(constant width), $(559 ± 6 ± 26) - i(179 ± 7 ± 18) \text{ MeV/c}^2$(width containing kinematic factor), $(554 ± 13 ± 65) - i(240 ± 4 ± 19) \text{ MeV/c}^2 $(P.K.U. ansatz), and $(541 ± 9 ± 95) - i(240 ± 8 ± 33) \text{ MeV/c}^2 $(Bugg & Zou’s approach). The first Breit-Wigner parameterization may be problematic because the imaginary part does not vanish at threshold. The second parameterization gives a small σ width, and creates a virtual state in the real energy axis below the $\pi\pi$ threshold [29]. The final best esti-
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