Nuclear surface dynamics and pairing correlations

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Abstract. We solve the Nambu-Gor'kov equations in $^{120}\text{Sn}$, demonstrating that the coupling of quasiparticles with collective surface vibrations renormalizes the low-lying part of the nuclear spectrum and influences pairing correlations in a substantial way.

1. Introduction

It is well known that single-particle properties in closed shell nuclei are strongly renormalized by the coupling to collective modes [1, 2]. In this paper we study renormalization effects in superfluid nuclei [3, 4, 5]. We shall present results of calculations carried out within the framework of Nambu-Gor’kov equations. The formalism is consistent with the basic rules of Nuclear Field Theory (NFT) which have been extensively applied to the case of normal nuclei (see [6] and refs. therein; see also [7]). It takes into account the essential many-body processes which renormalize the self-energy of the quasiparticles and fragment their strength, increasing the effective mass and the level density close to the Fermi energy, and affecting pairing correlations in a substantial way.

2. Formalism

A convenient formalism for the calculation of the properties of quasiparticles in superfluid nuclei taking into account effects beyond mean field was given by Van der Sluys et al. [8]. The derivation of the formalism was based on the equation of motion method, which has a very close relation to the Green’s functions formalism used to derive the Nambu-Gor’kov equations, commonly used to study superconductivity in condensed matter physics [9]. We start from a Hartree-Fock (HF) +BCS calculation. The HF basis is calculated making use of an effective interaction, while the BCS calculation is performed with a nucleon-nucleon bare pairing force. In this way one obtains unperturbed quasiparticle energies $E_a$ and amplitudes $u_a, v_a$, associated with a given spherical orbital $a \equiv \{nlj\}$. We then add the coupling between quasiparticles and vibrations, by solving the energy-dependent Nambu-Gor'kov equations

\[
\left( \begin{array}{cc}
(\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{11}^{(n)} & \tilde{\Sigma}_{12}^{(n)} \\
\tilde{\Sigma}_{12}^{(n)} & (\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{22}^{(n)}
\end{array} \right) \left( \begin{array}{c}
\tilde{u}_a(n) \\
\tilde{v}_a(n)
\end{array} \right) = \tilde{E}_a(n) \left( \begin{array}{c}
\tilde{u}_a(n) \\
\tilde{v}_a(n)
\end{array} \right), 
\]

where $\epsilon_a$ and $\epsilon_F$ denote the HF single-particle and the Fermi energy. Eq. (1) has several solutions, which are labeled by the index $n$, leading to a fragmentation of the original
quasiparticle strength; the renormalized energies and amplitudes associated with a given fragment are denoted by $\tilde{E}_\alpha(n)$ and by $\tilde{u}_\alpha(n)$ and $\tilde{v}_\alpha(n)$, while $\tilde{\Sigma}_{\alpha(n)}^{11}$ and $\tilde{\Sigma}_{\alpha(n)}^{12}$ denote the energy-dependent normal and anomalous self-energies. In the following we shall focus on the anomalous self-energy. It can be separated into two terms, writing $\tilde{\Sigma}_{\alpha(n)}^{12} = \Delta_\alpha^{BCS} + \tilde{\Sigma}_{\alpha(n)}^{12,pho}$.

The first term, $\Delta_\alpha^{BCS}$, is the pairing gap associated with the bare interaction obtained in the previous HF+BCS calculation, while the second term is associated with the phonon-induced interaction and is given by

$$\tilde{\Sigma}_{\alpha(n)}^{12,pho} = -\sum_{b,m} \frac{(2j_h + 1)}{2} V_{ind}(a(n)b(m)) \tilde{u}_b(m) \tilde{v}_b(m),$$  

(2)

where we have introduced the induced pairing interaction:

$$V_{ind}(a(n)b(m)) = \sum_\lambda,\nu \frac{2\hbar^2(ab\lambda\nu)}{2j_h + 1} \times \left[ \frac{1}{\tilde{E}_\alpha(n) - \tilde{E}_b(m) - \hbar\omega_{\lambda\nu}} - \frac{1}{\tilde{E}_a(n) + \tilde{E}_b(m) + \hbar\omega_{\lambda\nu}} \right].$$  

(3)

The term $\hbar(ab\lambda\nu)$ denotes the basic particle-vibration coupling vertex, which will be calculated following the collective model by Bohr and Mottelson (cf. [10], Eq. (6.209)):

$$\hbar(ab\lambda\nu) = (-1)^{j_a - j_b} \chi_\lambda <a| \frac{\partial U}{\partial r_i}|b> <j_b||Y_\lambda||j_a> \left[ \frac{1}{(2j_a + 1)(2j + 1)} \right]^{1/2}. $$  

(4)

The coefficient $\chi_\lambda$ multiplies the self-consistent value of the coupling constant introduced in [10], in order to reproduce the experimental deformation parameters $\beta_\lambda$ and the energies $\hbar\omega_{\lambda\nu}$ of the lowest $2^+, 3^-, 4^+$ and $5^-$ modes in our QRPA calculation with a separable force; typically $\chi_\lambda \approx 0.9$.

Furthermore, we can symmetrize the matrix (1) in order to get a $2 \times 2$ eigenvalue equation which is formally identical to the BCS eigenvalue equation. This can be achieved multiplying Eq. (1) by the $Z_{\alpha(n)}$ energy-dependent function, $[Z_{\alpha(n)}]^{-1} = 1 - \tilde{\Sigma}_{\alpha(n)}^{odd}/\tilde{E}_{\alpha(n)}$, where $\tilde{\Sigma}_{\alpha(n)}^{odd}$ is the odd part of $\tilde{\Sigma}_{\alpha(n)}^{11}$. It is then possible to rewrite Eq. (1) as

$$\begin{pmatrix} (\tilde{\epsilon}_{\alpha(n)} - e_F) - \tilde{\Delta}_{\alpha(n)} & \tilde{\Sigma}_{\alpha(n)}^{11} \\ -(\tilde{\epsilon}_{\alpha(n)} - e_F) & \tilde{\epsilon}_{\alpha(n)} - e_F \end{pmatrix} \begin{pmatrix} \tilde{u}_{\alpha(n)} \\ \tilde{v}_{\alpha(n)} \end{pmatrix} = \begin{pmatrix} \tilde{E}_{\alpha(n)} \\ \tilde{E}_{\alpha(n)} \end{pmatrix} \begin{pmatrix} \tilde{u}_{\alpha(n)} \\ \tilde{v}_{\alpha(n)} \end{pmatrix},$$  

(5)

where $\tilde{\epsilon}_{\alpha(n)} - e_F = Z_{\alpha(n)} [ (\epsilon_a - e_F) + \tilde{\Sigma}_{\alpha(n)}^{even} ]$, and where $\tilde{\Sigma}_{\alpha(n)}^{even}$ is the even part of $\tilde{\Sigma}_{\alpha(n)}^{11}$. The term $\tilde{\epsilon}_{\alpha(n)}$ in Eq. (5) represents the renormalized single-particle energy, and one can now identify the pairing gap with the term $\tilde{\Delta}_{\alpha(n)}$ [9]:

$$\tilde{\Delta}_{\alpha(n)} = Z_{\alpha(n)} \tilde{\Sigma}_{\alpha(n)}^{11} = Z_{\alpha(n)} (\Delta_\alpha^{BCS} + \tilde{\Sigma}_{\alpha(n)}^{12,pho}) \equiv \tilde{\Delta}_{\alpha(n)}^{bare} + \tilde{\Delta}_{\alpha(n)}^{pho}.$$  

(6)

3. Results

Summarizing, our calculations are carried out in the following steps:

(a) We perform a HF calculation to obtain the single-particle levels which are going to be renormalized. We make use of the effective interaction SLy4, whose effective mass $m_e \approx 0.7m$ leads to a single-particle density in reasonable agreement with experiment, after renormalization effects are taken into account.

(b) We work out a BCS calculation with the bare nucleon-nucleon interaction Argonne $v_{14}$ in the HF basis. This calculation takes into account the coupling with orbitals lying at very high
Figure 1. Results obtained solving the Nambu-Gor’kov equations in $^{120}$Sn. (a) State-dependent neutron paring gaps (see the text). The values of the Fermi energy $\epsilon_F$ and of the gap obtained from the experimental odd-even mass difference $\Delta_{exp}$ are also indicated. (b) $Z$– factor and quasiparticle strength $N$ associated with the lowest quasiparticle peaks. The values of the experimental quasiparticle strength [13] are indicated by stars, except for the $d_{5/2}$ orbital which shows a pronounced fragmentation. (c) The theoretical quasiparticle spectra obtained at the various steps of the calculation are compared to the experimental data (see the text). (d) Theoretical and experimental strength functions associated with the $d_{5/2}$ orbital.

energy (about 1 GeV) due to the effect of the hard core, but we will be interested in the values of the quasiparticle energies and amplitudes of the few valence orbitals close to the Fermi energy. We remark that the absolute value of the resulting pairing gap is sensitive (within 20-30%) to the momentum dependence of the effective mass at large momenta, which for the time being represents an open theoretical problem [11, 12].

(c) We do a QRPA calculation obtaining a phonon spectrum which will be coupled to the quasiparticles obtained in point (b).

(d) We solve by iteration the Nambu-Gor’kov equations (1), obtaining a renormalized low-lying spectrum, and a redistribution of the quasiparticle strength. At the same time, we obtain the renormalized value of the pairing gap $\tilde{\Delta}$.

The main results are shown in Fig. 1. In Fig. 1(a) we show the pairing gap $\Delta^{BCS}$ obtained at the BCS level (filled dots), which displays a very limited state-dependence and is close to 1.1 MeV. The gap is shown as a function of the unperturbed single-particle energy in the HF field for the five orbitals $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, $d_{3/2}$ and $h_{11/2}$. The value of the BCS gap is
diminished by the spectroscopic $Z$–factor shown in Fig. 1(b) ($Z \approx 0.8$), leading to a value
$\Delta^{\text{bare}} = Z \Delta^{\text{BCS}} \approx 0.8$ MeV (diamonds). We also show in Fig. 1(b) the quasiparticle strength
$N$ associated with the lowest fragments for each orbital, compared to the experimental findings.
The total state-dependent gap (triangles) is obtained by summing $\Delta^{\text{bare}}$ with the phonon-induced
part $\Delta^{\text{pho}} \approx 0.8$ MeV (squares). We then obtain a total gap $\Delta \approx 1.6$ MeV, that can be compared
with the typical estimate of the gap derived from the odd-even mass difference, which is in the
range 1.35–1.45 MeV, depending on the particular formula used. Also displayed in Fig. 1(a)
(dashed line) is the gap obtained after the first iteration of the Nambu-Gor’kov equations.

The low-lying quasiparticle spectrum obtained at the various stages of the calculation is displayed
in Fig. 1(c), where the positions of the lowest fragment associated with each of the five valence
orbitals are indicated. In the first column (labeled HF) we show the quantities $|\epsilon_a - \epsilon_F|$ resulting
from the HF calculation, referred to the value for the level $d_{3/2}$, lying closest to the Fermi
energy. In the second column ($v_{14}$) we report the quasiparticle energies obtained from the BCS
calculation. In the third and fourth column we give the quasiparticle energies obtained taking
into account the renormalization processes by solving the Nambu-Gor’kov equation, after the
first iteration (NFT0) and at convergence (NFT). The theoretical results are compared to the
experimental spectra in the last two columns. It is seen that the particle-vibration coupling
increases the level density, leading to a much better agreement with experiment. While the
quasiparticle strength of the three orbitals lying close to the Fermi energy is well described by
the BCS quasiparticle picture, the $d_{5/2}$ orbital becomes strongly fragmented (cf. Fig. 1(d)).

4. Conclusions and outlook
One can conclude that it is possible, at least in the case of $^{120}$Sn, to draw a picture that is
consistent with the available experimental data concerning the quasiparticle properties close to
Fermi energy. However, several elements should be further investigated, before one can reach
firm quantitative theoretical results, in particular concerning the absolute value of the total
pairing gap. On the one hand, the mean field should be either derived microscopically, or at
least refitted comparing experiment with theoretical results taking into account renormalization
effects; on the other hand, contributions which are expected to provide a repulsive contribution
to the pairing interaction, associated with three-body effects and with the influence of spin
modes, should also be considered in detail. As suggested by P. Schuck at this Conference, it
would be interesting to investigate renormalization effects in finite nuclei in the case of $T = 0$
pairing gap, which has been found to take very large values in nuclear matter [14, 15].

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