Quantum entanglement, fair sampling, and reality:

Is the moon there when nobody looks?

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Abstract

In 1981, David Mermin\(^1\) described a cleverly simplified version of Bell’s theorem\(^2\). It pointed out in a straightforward way that interpreting entanglement from a local realist point of view can be problematic. I propose here an extended version of Mermin’s device that can actually be given a simple local realist interpretation through a sample selection bias, and I argue that we still have no scientific reason to believe that the moon could possibly not be there when nobody looks.\(^4\)
Entanglement is arguably the greatest mystery brought forth by Quantum mechanics: it has such drastic consequences on the possible physical models that can explain its features that the philosophical representation of the world itself is at stake.

Describing the mystery of entanglement can be difficult. It is sometimes misrepresented as a miraculously perfect correlation between two distant objects. This picture is however insufficient, as the same perfect correlation can be obtained with a simple local realistic model—which would not violate any Bell inequality. It is only when one deviates from identical measurements that the strangeness of entanglement reveals itself. David Mermin illustrated this in a simple version of Bell’s theorem that stripped the mystery of entanglement to the core.

Let us start first by recalling the way Mermin’s device works. It consists of three unconnected black boxes. The first two boxes, labelled A and B, are detectors. Each detector has a switch with three possible positions (1, 2, or 3), and two lamps (a green one labelled G, and a red one labelled R). Whenever a particle reaches a detector, one of the two lamps fires, and a measurement result is recorded accordingly (G or R).

The third black box, labelled C, is the source: it sends pairs of particles going in opposite directions: the first particle going to detector A, the second going to detector B. The detectors can be arbitrarily far from each other and are not connected in any way, so that the detectors have no influence upon one another.

The switches on detectors A and B are randomly and independently selected after a pair of particles has left the source and before either particle has arrived at its detector. There are thus nine equally probable settings for the pair of detectors: 11, 12, 13, 21, 22, 23, 31, 32, and 33 (the first and second digits represent the position of the switch for detectors A and B respectively). There are similarly four possible pairs of measurement results: RR, RG, GR, and GG. A measurement of a pair of particles is thus a quadruple of two digits.
and two letters (like 21GR for instance).

The source then fires an arbitrary large number of pairs and the behaviour of Mermin’s device can be summarized in the following way:

- **Case a.** In those runs in which the detectors have the same settings (11, 22, or 33), the lights always flash the same colour, RR and GG appearing with an equal frequency.

- **Case b.** In those runs in which the detectors have different settings (12, 13, 21, 23, 31, 32), the lights flash the same colour with a frequency of only 1/4.

The challenge proposed by Mermin is to provide a model that can explain both *Case a* and *Case b*, given that the detectors A and B have no known connection between each other.

The perfect correlation observed in *Case a* is a strong constraint on all possible models. The only simple way to explain this perfect correlation is to assume that each particle carries instructions specifying which colour will flash for each of the three possible settings. Furthermore that particles belonging to the same pair carry identical instructions. Instruction sets must provide an instruction for each of the three possible settings 1, 2, and 3, because the settings are not chosen until after the particles have separated, and each of the pairs of settings 11, 22, and 33 has 1/9 of a chance of being chosen in any given run. There are eight possible instruction sets given to each particle: RRR, RRG, RGR, RGG, GRR, GRG, GGR and GGG. Mermin shows that fulfilling *Case a* with such instruction sets introduces an excess of “same colour” results in *Case b*. In order to see this, let us consider the instruction set GGR. This instruction set means that a particle carrying it will flash G for settings 1 and 2, and R for setting 3. In *Case a* the same measurement results are observed on both sides since the particles from one pair carry the same instructions (see Fig. 2).

Let us now consider *Case b*, that is, when the switches A and B are set on different positions, which is graphically represented as diagonal lines. There are six possible ways to draw such diagonal lines, four short ones (see Fig. 3a), and two long ones (see Fig. 3b). There are two ways to obtain the same measurement results with diagonal lines, that is 12GG and 21GG. The remaining four possible diagonals give opposite results, that is, 13GR, 31RG, 23GR, and 32RG.

Hence the totals are two out of six pairs of switch positions with the same colour results, and four out of six with opposite colour results. Therefore, if the source was sending
FIG. 2: *Case a* (same settings) always result in the same colour result (GG or RR).

FIG. 3: *Case b* (different settings) resulting in (a) same colour results, or (b) opposite colour.

only instruction set GGR, the proportion of same colour in *Case b* would be 1/3, and the proportion of opposite colour results would be 2/3.

The same reasoning for *Case b* applies to any of the instruction sets RRG, RGR, RGG, GRR, and GRG, since they all share with GGR that one colour appears in the instruction set once and the other colour twice. Hence, if the source is sending any of these 6 possible states, the *Case b* proportion of same colour results should be 1/3, and the proportion of opposite colour results should be 2/3 (independently of the specific probability distribution
of the states).

The two remaining instruction sets, RRR and GGG, always give the same colour results, independently of the switch settings. If we thus want to minimize the amount of same results in Case b (while keeping the Case a feature), the source C should never send any of the homogeneous states RRR and GGG. Even so, in the long run the observed frequency of same results in Case b should be close to 1/3, which is significantly higher than the 1/4 specified by Mermin’s device. This leaves us with a conundrum, as the only reasonable way to account for Case a is incompatible with an account for Case b.

As stressed by Mermin, experiments done since Bell’s paper are in agreement with quantum-theoretic predictions, so that Mermin’s device could in principle be constructed. However, this device would not behave entirely according to Mermin’s expectation. Indeed, all experiments performed so far share a common feature: most particles sent to the detectors in order to exhibit the conundrum remain undetected[4,5,6].

The fact that most particles remain undetected implies that no direct and complete information on the population of all emitted particles is possible. The issue that it raises is not specific to experiments on quantum entanglement. It is quite general in statistical analysis as soon as obtaining data from the entire population under study proves impossible. The statistics are then restricted to a sample of that population. In order to make any inference on the target population, one has to make sure that no selection bias enters in the sampling procedure: the sample must fairly represent the population.

To start with, the selection method itself may introduce a bias. Take for example a survey research that for practicality is only selecting households with landline telephones. It will automatically exclude individuals having no telephone or only mobile phones, which represents a non negligible and distinctive part of the population.

A selection bias may also arise due to the type of question asked in the survey, inducing some self selection of the individuals in the sample (non-response error). Exemplifying this would be a survey on the negative or positive perception of surveys in general. It would be likely to be biased favorably towards surveys, simply because individuals with a negative opinion would be more likely to refuse to participate in the survey.

In the context of EPR-Bell experiments, the possibility that a sample selection bias could account for the observed correlations was raised first in 1970 by Pearle[7], and has been discussed many times since[8,9,10,11,12,13,14,15,16]. It is often referred to as the “detection
efficiency loophole”, but it is however a rather misleading terminology, as it already contains an implicit interpretational choice: the blame is implicitly put on the detectors imperfections. As we will see, this interpretation leaves the conundrum intact. Another interpretation of these non-detections in terms of a selection bias is however possible, and we will see that it can resolve the conundrum.

We thus consider an extended version of Mermin’s device, in which whenever a particle enters a detector, it can either induce a green flash (G), a red flash (R), or no flash at all (N). In addition to the four possible flash results (RR, RG, GR and GG), we then have four possible results inducing only one flash (RN, NR, GN, and NG), and one with no flash at all (NN). Whenever no flash is recorded (N) the whole pair is simply discarded from the statistics since no flash correlation can be established. This might seem a rather questionable way to deal with no-flash events, but this is precisely the way non-detected pairs are treated in real experiments, as the statistics are computed exclusively on the sub-ensemble of pairs actually detected.

There are basically two simple ways to account for the occurrence of no-flash events in the framework of Mermin’s device: they can be either carried by the detectors (independently of the incoming instruction set), or by the particles themselves.

Let us first consider the case where the absence of flash is due to the detectors. It means that for some reason the detectors are not very efficient and that they fail to fire now and then when they should have, independently of the incoming instruction set. Whatever causes this unreliability of the detectors, it can be symbolize by adding an additional possible switch position, that we conveniently label 0, and by stating that the position of the switch is now randomly chosen between those four positions, instead of the three positions in Mermin’s original device (see Fig. 4). As a matter of detail, let p be the probability that the switch is on position 0 then the probability associated to any of the remaining three switches is equal to (1-p)/3. Whenever the switch turns out to be set on this additional position (0), none of the lamps can flash, whatever the incoming particle.

A look at Fig. 5 representing this new situation shows that Mermin’s line of thought, as used above, is still valid. The no-flash events N always occur on the additional column corresponding to the failure of the detector, but the statistics on the pairs producing a flash concerns only the first three columns, and thus remain the same as obtained previously. Hence, with this additional switch position, the conundrum remains intact.
FIG. 4: No-flash events carried by a fourth switch (labelled 0) in the detectors.

|       | Detector A | Detector B |
|-------|------------|------------|
| **Instruction** | G G R | G G R |
| **Observed Result** | G G R N | G G R N |

FIG. 5: Non-detection with carried by the detectors. Any result with a no-flash event (N) is discarded. The statistics of the observed results remain the same, as they concern only the first three columns. The possible results of *Case b* are displayed here, and remain the same as in Fig. 3. More runs are necessary to achieve the same statistical relevance, but the conundrum is intact.

If however the no-flash events are carried by the particles themselves, the conundrum can be resolved. Consider for instance the same GGR pair we considered above, but this time with the freedom to put an instruction of the type “do not flash”, that we label N, at a specific place in the instruction sets. We know that for this particular state we have an excess of the diagonal GG correlations (same colour results, in *Case b*). Let us then put a N in place of one of the G instruction for the first particles, say the G corresponding the switch 2, so that the particle going to A has the instruction GNR. The second particle, the one going to detector B, carries the initial GGR instruction set. Since the particles from one pair no longer carry the same instruction, we now use an explicit notation for each particle, and we denote for instance this particular pair state by GNR-GGR, meaning that the particle going to detector A has instruction set GNR, while the second has instruction set GGR.
FIG. 6: Non-detection carried by the instruction sets in Case a. Any result with a no-flash event (N) is discarded.

Let us first consider Case a. As can be seen from Fig. 6, it still gives a perfect correlation for the detected pairs, since the results are the same colour in two cases: (11GG, 33RR), while the third is discarded because detector A does not flash (22NG).

If we now consider Case b, we see (Fig. 7 a) that we retain only one possibility to obtain the same colour result (12GG), while the other one is discarded because the particle entering in detector A produces no flash (21NG). Similarly, there are three possibilities (Fig. 7 b) to obtain opposite colour results (13GR, 31RG, 32RG), while the remaining one is discarded because the particle entering in detector A produces no flash (23NR).

Thus, out of the 4 possible ways to register a Case b with the GNR-GGR state, we then have one possibility with same colour, and three possibilities with opposite colours. In other words, if the source was sending only this state, the lights would flash the same colour with a frequency of 1/4 in Case b, which is exactly the specified frequency of Mermin’s device.

We can use the same logic for any of the remaining instruction sets RRG, RGR, RGG, GRR, and GRG. All we need to do is put a no-flash instruction N in place of one of the colour that appear twice in the instruction set, either for the particle going to detector A or for the one going to detector B. The list of instruction sets displayed on Table I was built following this idea (the last six instruction sets are the same as the first six instruction sets after particle exchange):
FIG. 7: Non-detection carried by the instruction sets in Case b. Any result with a no-flash event (N) is discarded.

| Instruction Set | 0 | R |
|-----------------|---|---|
| Detector A      |   |   |
| Observed Result | G | N | R |
| Detector B      |   |   |
| Observed Result | G | G | R |
| Instruction     | G | G | R |

| Instruction Set | 0 | R |
|-----------------|---|---|
| Detector A      |   |   |
| Observed Result | G | N | R |
| Detector B      |   |   |
| Observed Result | G | G | R |
| Instruction     | G | G | R |

TABLE I: A list of instruction sets satisfying both Case a and Case b specifications.

| Alice | Bob |
|-------|-----|
| NRG   | GRG |
| NGR   | RGR |
| RNG   | RRG |
| GNR   | GGR |
| RGN   | RGG |
| GRN   | GRR |
| GRG   | NRG |
| RGR   | NGR |
| RRG   | RNG |
| GGR   | GNR |
| RGG   | RGN |
| GRR   | GRN |

Hence, if the source sends a uniform distribution of these twelve states, the proportion of same colour results in Case b will therefore be close to 1/4 in the long run, and the proportion of opposite colour results close to 3/4, while keeping at the same time the perfect correlation of Case a. The conundrum is resolved.
Note that each switch position (each column) will in the long run receive the same number of instructions N, G and R, so that the detectors are equivalent and balanced. The three switches of a detector and the twelve possible states being random and independent, there are thirty-six possible combinations of both, among which only six no-flash instructions N per detector, so a detector will flash 5/6 of the time (circa 83%). Interesting enough, this efficiency is very similar to the known bound on detectors efficiency required to validate a genuine violation of Bell inequalities\textsuperscript{10,13}.

In all real EPR-Bell experiments meant to challenge local realism, the detectors had a much lower rate of detection. This can be accounted for in this context by considering mixed process in which not only the detectors have a fourth switch (0) yielding no-flash, but the instruction sets carry no-flash instructions (N) as well (see Fig. 8). This allows in principle to retain the resolution of Mermin’s conundrum even in the case of detectors A and B having distinct efficiencies $\eta^A$ and $\eta^B$. One needs only to distinguish the fair sampling part $\eta_f$ in the probability of a flash from the unfair sampling part $\eta_u$. The former corresponds to the possibility that the detector itself may fail to fire when it should have, independently of the state of the incoming particle, whereas the later corresponds to the possibility that the incoming particle may have a no-flash instruction for the selected measurement setting. The probability $\eta$ that a detector fires is thus the product of these two independent probabilities, that is $\eta = \eta_u \eta_f$. In order to keep the resolution of Mermin’s conundrum, each particle needs to carry only one single instruction set from the list given in Table I, and the unfair sampling efficiency $\eta_u$ should thus remain equal to 5/6. In order to modelize two detectors with different efficiencies, one needs only to consider detectors with two different fair sampling efficiencies $\eta^A_f$ and $\eta^B_f$, so that the corresponding efficiencies of these detectors are $\eta^A = \eta_u \eta^A_f$ and $\eta^B = \eta_u \eta^B_f$. Having detectors with different $\eta_f$ does not change the correlation since the sampling process associated to it is fair: it just reduces the number of pairs detected.

Our extended version of Mermin’s device shows that the proof of the conundrum rely on the validity of an extra assumption. The alternative can be summarized as follows: either the instruction sets carry no-flash instructions and the conundrum is resolved, or they do not, and the conundrum holds. The assumption that the second alternative holds is known in EPR-Bell experiments as the fair sampling assumption. It is tempting to validate this assumption on a theoretical basis, arguing that Quantum mechanics does not predict that the sampling would be unfair (a feature that can however be accounted for in the framework
FIG. 8: Non-detection carried by both the detectors and the instruction sets. The proportion of rejected particles can be arbitrarily large (at least 1/6) and thus reproduce the statistics of a real implementation of Mermin’s device.

of contextual probability theory\cite{17}, but since it is precisely the completeness of Quantum Mechanics that is at stake in EPR-Bell experiments, this would render the argument circular, and thus invalid. Consequently, the fair sampling assumption must be justified empirically. There is however little experimental evidence to support the validity of this assumption. One experimental feature that can be brought forward in support of fair sampling is that the observed size of the detected sample pairs is independent of the measurement settings. This feature is however shared by non-symmetrical models like Larsson’s\cite{13}, and our extended version of Mermin’s device shares this feature as well. As shown elsewhere\cite{18}, the validity of the fair sampling can actually be questioned on the basis of experimental data.

A detailed discussion about photon detection issues and about the possible constraints that a sample selection bias would impose on real detectors is beyond the scope of this article. A valid concern is that the existence of such a selection bias should have been spotted in optical experiments outside the EPR-Bell domain. It should however be noted that two-photon sources used in EPR-Bell experiments are special in other ways than the alleged quantum nonlocality. They can for instance be seen as a conditional source of single photons\cite{19}, so that it is conceivable that such selection bias would be elusive as it would be a characteristic of single photon sources exclusively. It is clear that this should be experimentally testable, and other measurement systems that could detect such a selection bias have already been proposed\cite{20}. Naturally, this would not be necessary if high efficient
EPR-Bell experiments were within reach, but as long as this is not the case, and given that such experiments might be impossible in actuality\textsuperscript{21,22}, the possibility of a selection bias should be investigated through both theoretical and experimental means.

In the meantime, analysis shows that it indeed is still possible to ascribe properties to objects independently of observation, and contrary to David Mermin’s statement\textsuperscript{2}, I would thus argue that Einstein’s attacks against the metaphysical underpinning of quantum theory are still valid today, and do not contradict nature itself.

\footnotesize
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After this paper was submitted, a referee pointed out that the idea of a sample selection bias in the context of Mermin’s device had been made by Mermin himself (on page 168 of his collection of articles\textsuperscript{23}), as well as in the Letters section of Physics Today\textsuperscript{24} (commenting on another version\textsuperscript{25} of Mermin’s demonstration\textsuperscript{1,2}) both in two letters — one by Marshall and Santos, the other by Jordan — and in a commentary on those letters, in which Mermin points out that any correlations whatever can be produced by having two such “don’t fire” instruction in each instruction set.