Understanding $B \to J/\psi \phi$ in the Standard Model

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Abstract

The rare decay $\bar{B}_d^0 \to J/\psi \phi$ can proceed via four distinct mechanisms: (i) production of the $\phi$ via tri-gluon fusion, (ii) photoproduction of the $J/\psi$ or $\phi$, (iii) final-state rescattering of $D_1^{(*)}D_2^{(*)}$ produced in the $\bar{B}_d$ decay to $J/\psi \phi$, and (iv) production of the $\phi$ via $\omega - \phi$ mixing. In this work, we examined the contributions of photoproduction and final-state rescattering to $\bar{B}_d^0 \to J/\psi \phi$ and found that the corresponding branching ratios were of the orders $10^{-11}$ and $10^{-9}$, respectively. Hence, this decay is dominated by the $\omega - \phi$ mixing effect.

1. The observation of $B$ decays to charmonium provides important evidence for the Cabbibo-Kabayashi-Maskawa model, as well as an important advance in our understanding of the Standard Model and QCD dynamics. Recently, Belle reported an upper limit $9.4 \times 10^{-7}$ for the branching ratio of $B^0 \to J/\psi \phi$ at the 90% confidence level [1]. This process is expected to be suppressed by the Okubo-Zweig-Iizuka (OZI) rule [2] disfavoring disconnected quark diagrams.

The main processes for $\bar{B}_d^0 \to J/\psi \phi$ can be sorted into four different classes: (i) the neutral vector meson $\phi$ is produced through tri-gluon fusion (Fig. 1), which is formally the reason why this channel is OZI-suppressed, (ii) the $J/\psi$ or $\phi$ arises from a photon emission, followed by fragmentation (Fig. 2), (iii) the decay particles $J/\psi$ and $\phi$ are produced through long-distance final-state interactions (FSI) (see Fig. 3), and (iv) the $\phi$ comes from the decay of $B \to J/\psi \omega$ followed by $\omega - \phi$ mixing; that is, $\phi$ is not a pure $s \bar{s}$ state and contains a tiny $q \bar{q}$ component.

In [3], Gronau and Rosner pointed out that the major contribution to the decay $\bar{B}_d^0 \to J/\psi \phi$ arises from $\omega - \phi$ mixing. Neglecting isospin violation and the admixture with the $\rho^0$ meson, one

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can parameterize $\omega-\phi$ mixing in terms of an angle $\delta$ such that the physical $\omega$ and $\phi$ are related to the ideally mixed states $\omega' \equiv (u\bar{u}+d\bar{d})/\sqrt{2}$ and $\phi' \equiv s\bar{s}$ by

$$
\begin{pmatrix}
\omega \\
\phi
\end{pmatrix} = 
\begin{pmatrix}
\cos \delta & \sin \delta \\
-\sin \delta & \cos \delta
\end{pmatrix}
\begin{pmatrix}
\omega' \\
\phi'
\end{pmatrix},
$$

(1)

and the mixing angle is approximately $\delta = -(3.34 \pm 0.17)^\circ$ \[4\]. Within this mechanism, the authors estimated the rates of this decay mode and other similar processes in $B^0$ and $B_s^0$ decays, and found that the Belle’s upper limit is about a factor of five above their estimation. Also, they argued that the final-state rescattering contributions to this decay mode are very small and can be neglected.

Let us make crude estimates of the various contributions to $B \to J/\psi \phi$ by the aforementioned four mechanisms. Due to the complicated QCD dynamics, it is difficult to calculate the tri-gluon
fusion reliably. Roughly, the tri-gluon fusion contribution gives

\[ \mathcal{B}(\bar{B}^0 \to J/\psi \phi)_{\text{tri-gluon}} = \mathcal{B}(\bar{B}^0 \to J/\psi \omega) \alpha_s^3 \approx 2.7 \times 10^{-5} \times (0.3)^3 \sim 2.4 \times 10^{-8}, \]  

(2)

where use of \( \mathcal{B}(\bar{B}^0 \to J/\psi \omega) \approx \mathcal{B}(\bar{B}^0 \to J/\psi \rho) = (2.7 \pm 0.4) \times 10^{-5} \) [5] has been made. The contribution of photoproduction is calculable to the leading power of the \( 1/m_b \) expansion and is of order

\[ \mathcal{B}(\bar{B}^0 \to J/\psi \phi)_{\text{photoproduction}} = \mathcal{B}(\bar{B}^0 \to J/\psi \gamma) \alpha_{\text{em}}^2 \sim 10^{-7} \times (1/137)^2 \sim 10^{-11}. \]  

(3)

In the final-state rescattering picture, the \( B \to J/\psi \phi \) decay proceeds via a \( B \) meson decay into \( D_s^{(*)}+D_s^{(*)} \) through \( W \)-exchange followed by a rescattering of \( D_s^{(*)}+D_s^{(*)} \) to \( J/\psi \phi \) through \( D_s^{(*)} \pm \) exchange. It is anticipated that

\[ \mathcal{B}(\bar{B}^0 \to J/\psi \phi)_{\text{FSI}} = \mathcal{B}(\bar{B}^0 \to D_s^{(*)}+D_s^{(*)})(10^{-3} - 10^{-4}) \sim 3 \times 10^{-8} - 3 \times 10^{-9}, \]  

(4)

where the analysis of final-state interactions in \( B \to \phi K^*, \rho K^* \) suggests that the rate of the \( B \)-meson decay into the final state under consideration (for example, \( \bar{B}^0 \to J/\psi \phi \)) is suppressed relative to that of the intermediate state \( (\bar{B}^0 \to D_s^{(*)}+D_s^{(*)}) \) in this example) by three to four orders of magnitude [6]. Finally, the production of \( J/\psi \phi \) through \( \omega - \phi \) mixing is expected to be

\[ \mathcal{B}(\bar{B}^0 \to J/\psi \phi)_{\omega - \phi \text{ mixing}} = \mathcal{B}(\bar{B}^0 \to J/\psi \omega) \sin^2 \delta \approx 2.7 \times 10^{-5} \times (0.08)^2 \sim 1.7 \times 10^{-7}. \]  

(5)

Therefore, the rare decay \( B \to J/\psi \phi \) is indeed dominated by the \( \omega - \phi \) mixing effect.

In this letter, we will study the effects of photoproduction and final-state rescattering in more detail even though they are not the main contributions to \( B \to J/\psi \phi \). We wish to have quantitative results to confirm the above crude estimates.

2. First, let us evaluate the photoproduction, which plays an important role in decay modes such as \( B \to \rho K^*, \rho \phi \) [7]. In this mechanism, \( \bar{B}_d^0 \to J/\psi \phi \) can be regarded as the cascade process \( \bar{B}_d^0 \to J/\psi \gamma \to J/\psi \phi \) or \( \bar{B}_d^0 \to \phi \gamma \to J/\psi \phi \). The radiative decay \( B \to V \gamma \) has been well studied in the frameworks of the QCD factorization approach [8], the perturbative QCD approach (pQCD) [9] and soft-collinear effective theory [10]. Due to the suppression of Wilson coefficients, we will neglect the contribution from \( \bar{B}_d^0 \to \phi \gamma \to J/\psi \phi \).

According to the Feynman diagrams depicted in Fig. 2, the amplitude of \( \bar{B}_d^0 \to J/\psi \phi \) can be written as

\[ M \sim \mathcal{A}^{\mu}(\bar{B}_d^0 \to J/\psi \gamma) - ig_{\mu \nu} q^2 \left( -\frac{1}{3} e \right) \langle 0 | \bar{s} \gamma^\nu s | \phi \rangle \]

\[ \approx \left( \frac{f_\phi \sqrt{4\pi \alpha_{\text{em}}}}{3 m_\phi} \right) \mathcal{A}^{\mu}(\bar{B}_d^0 \to J/\psi \gamma) e^{\ast}_\mu, \]

(6)
where we have used \( \langle 0 | \bar{s} \gamma^\nu s | \phi \rangle = -m_\phi f_\phi e^{\nu s} \) and \( f_\phi \) and \( m_\phi \) are the decay constant and mass of the \( \phi \) meson, respectively. Therefore, we obtain the result

\[
\mathcal{B}(\bar{B}_d^0 \to J/\psi \phi) \simeq R_\phi \mathcal{B}(\bar{B}_d^0 \to J/\psi \gamma),
\]

\[
R_\phi = \left| \frac{f_\phi \sqrt{4\pi \alpha_{em}}}{3m_\phi} \right|^2 \simeq 0.0003,
\]

(7)

with \( f_\phi = 0.237 \text{GeV} \). In the literature, it has been estimated that \( \mathcal{B}(\bar{B}_d^0 \to J/\psi \gamma) = 7.7 \times 10^{-9} \) \[11\] in QCD factorization and \( \mathcal{B}(\bar{B}_d^0 \to J/\psi \gamma) = 4.5 \times 10^{-7} \) \[12\] in perturbative QCD. Therefore, the predictions of QCDF and pQCD differ by one to two orders of magnitude. The possible reason for this huge discrepancy was explained in Ref. \[12\]. Roughly speaking, this is due mainly to the use of different \( J/\psi \) wave functions in Ref. \[11\] and Ref. \[12\]. If the charm quark is heavy, the wave function of \( J/\psi \) will be symmetric under \( x \leftrightarrow 1 - x \) and sharply peaked around \( x = 0.5 \). However, the cross section of \( e^+e^- \to \eta_c + J/\psi \) calculated within the NRQCD approach is much smaller than the experimental data. Bondar and Chernyak \[13\] have pointed out that the origin of the discrepancy is due to the fact that the charm quark is not heavy enough and, as a result, the charmonium wave functions are not sufficiently narrow for a reasonable application of NRQCD to the description of charmonium production. Using more realistic models, these authors have proposed a new wave function for \( J/\psi \), which can be used to explain the data well. This new wave function is employed in Ref. \[12\], while the delta function is used in Ref. \[11\].

Even taking the pQCD result for \( \bar{B}_d^0 \to J/\psi \gamma \), the photoproduction mechanism leads to a very small branching ratio for \( \bar{B}_d^0 \to J/\psi \phi \) of order \( 10^{-11} \), which is not accessible even at the future Super-B factories. Since the \( \phi \) is produced from a virtual photon which is transversely polarized mostly, the longitudinal polarization of the decay \( B \to J/\psi \phi \) via photoproduction will be very small.

3. As mentioned above, \( \bar{B}_d^0 \to J/\psi \phi \) receives long-distance contributions from a \( B \) meson decay into \( D_s^{(*)} + D_s^{(*)} \) followed by a rescattering of \( D_s^{(*)} + D_s^{(*)} \) to \( J/\psi \phi \). The \( D_s^{(*)}D_s^{(*)} \) states from \( \bar{B}_d^0 \) decays can rescatter to \( J/\psi \phi \) through the \( t \)-channel \( D_s^{(*)} \) exchange in the triangle diagrams depicted in Fig. 3. Before proceeding, we would like to remark briefly on the motivation for considering the rescattering mechanism with \( D_s^{(*)} \) exchange. At the hadron level, final-state interactions manifest as the rescattering processes with \( s \)-channel resonances and one particle exchange in the \( t \)-channel. Due to the lack of the existence of resonances at energies close to the \( B \) meson mass, we will therefore model FSIs as rescattering processes of some intermediate two-body state with one particle exchange in the \( t \)-channel. We will compute the absorptive part via the optical theorem \[6\]. We consider charm intermediate states based on the idea that if the intermediate states are CKM more favored than the final state, then the absorptive part of the final-state rescattering amplitude can easily give rise to large strong phases and make significant contributions to the rates. It has been shown in Ref. \[6\] that the direct \( CP \)-violating partial rate asymmetries in charmless \( B \)
decays to $\pi \pi / \pi K$ and $\rho \pi$ are significantly affected by final-state rescattering and their signs are generally different from those predicted by the short-distance approach. Especially, the calculated $CP$ asymmetry $A_{CP}(K^+ \pi^-) = -0.14^{+0.01}_{-0.03}$ for $B^0 \to K^+ \pi^-$ via rescattering agrees with experiments in both magnitude and sign, whereas the QCD factorization prediction $A_{CP}(K^+ \pi^-) \approx 0.045$ is wrong in sign. This example illustrates that the rescattering approach gives a reasonable description of FSIs.

To evaluate Fig. 3 we note that the effective Lagrangian for $\phi D_s^{(*)} D_s^{(*)}$ vertices can be found in [6], and the effective Lagrangian for $J/\psi D_s^{(*)} D_s^{(*)}$ vertices is given by

$$\mathcal{L}_{\psi D_s D_s} = ig_{\psi D_s D_s} \psi_{\mu} \left( \partial_{\mu} D_s D_s^{\dagger} - D_s \partial_{\mu} D_s^{\dagger} \right),$$

$$\mathcal{L}_{\psi D_s^s D_s} = -2g_{\psi D_s D_s} \epsilon_{\mu \nu \alpha \beta} \partial_{\mu} \psi_{\nu} \left( \partial_{\alpha} D_s^{\dagger} D_s^{\dagger} + D_s \partial_{\alpha} D_s^{\dagger} \right),$$

$$\mathcal{L}_{\psi D_s^s D_s^s} = -ig_{\psi D_s D_s} \left\{ \psi^\dagger \left( \partial_{\mu} D_s^{\dagger} D_s^{\dagger} - D_s^{\dagger} \partial_{\mu} D_s^{\dagger} \right) + \psi^{\dagger} D_s^{\dagger} \partial_{\mu} D_s^{\dagger} - \psi_{\nu} \partial_{\mu} D_s^{\dagger} D_s^{\dagger} \right\}.$$  

The coupling constants for the $\phi D_s^{(*)} D_s^{(*)}$ vertices can be related to the parameters $g_{\psi}, \beta$ and $\lambda$ appearing in the effective chiral Lagrangian describing the interactions of heavy mesons with low momentum vector mesons [20] in the following manner

$$g_{\phi D_s D_s} = \frac{\beta g_{\psi}}{\sqrt{2}} = 3.75, \quad f_{\phi D_s D_s} = \frac{\lambda g_{\psi}}{\sqrt{2}} = 2.30 \text{ GeV}^{-1},$$

$$f_{\phi D_s D_s} = \frac{\lambda g_{\psi}}{\sqrt{2} m_{D_s}} = 4.85, \quad g_{\psi D_s D_s} = 4 g_{\psi D_s D_s} = 4 f_{\psi D_s D_s} / m_{D_s} = 10,$$  

where we have assumed $\beta = 0.9$ and $\lambda = 0.56 \text{ GeV}^{-1}$ [21] and the relation $g_{\psi} = m_{\rho} / f_{\pi} [20]$. The couplings for $J/\psi D_s^{(*)} D_s^{(*)}$ are taken from Ref. [22] based on an effective field theory of quarks and mesons. Note that the same $\phi D_s^{(*)} D_s^{(*)}$ vertex also appears in the rescattering contribution to $B \to \phi K^*$. A study in [6] shows that the rescattering mechanism via $D_s^{(*)}$ exchange can enhance the rate and yield a large transverse polarization in $B \to \phi K^*$.

In total, there are eight different FSI diagrams in Fig. 3. The $B^0 \to D_s^{(*)} - D_s^{(*)}$ amplitudes via $D_s^{(*)}$ exchange are similar to the $B \to D_s^{(*)} D_s^{(*)}$ amplitudes via $D_s^{(*)}$ exchange that have been studied in Ref. [6]. Therefore, the amplitudes of the former can be obtained from the latter through the replacements $\tilde{K}^+ \to J/\psi$ and $D^{(*)} \to D_s^{(*)}$. For example, the absorptive part contributions of $B^0 \to D_s^{(*)} D_s^{(*)} \to J/\psi \phi$ amplitudes via $D_s$ exchange is given by

$$\text{Abs} \left( D_s^{(*)} D_s^{(*)}; D_s \right) = \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4 (p_B - p_1 - p_2) A_{B^0 \to D_s^{(*)} D_s^{(*)}}$$

$$\times (2i) g_{D_s D_s} \phi F(p_1, k) F(p_2, k) \frac{m_{D_s}^2}{t - m_{D_s}^2} (\epsilon_5 \cdot p_1) (\epsilon_4 \cdot p_2),$$

where $k = p_1 - p_3 = p_4 - p_2$ is the momentum of the exchanged particle. Since the particle exchanged in the $t$ channel is off shell and since final state particles are hard, form factors or cutoffs
must be introduced to the strong vertices to render the calculation meaningful in perturbation theory. The form factor $F(p,k)$ for the off-shell effect of the exchanged particle can be parametrized as

$$F(p,k) = F(t,m_{\text{exc}}) = \left( \frac{\Lambda^2 - m_{\text{exc}}^2}{\Lambda^2 - t} \right)^n,$$

(normalized to unity at $t = m_{\text{exc}}^2$, where $m_{\text{exc}}$ is the mass of the exchanged particle. The cutoff $\Lambda$ in the form factor $F(t)$ should be not far from the physical mass of the exchanged particle. To be specific, we write

$$\Lambda = m_{\text{exc}} + \eta \Lambda_{\text{QCD}},$$

where the parameter $\eta$ is expected to be of order unity and it depends not only on the exchanged particle but also on the external particles involved in the strong-interaction vertex. As we do not have first-principles calculations for form factors, we shall use the measured decay rates to fix the unknown cutoff parameters. Although the strong couplings are large in magnitude, the rescattering amplitude is suppressed by a factor of $F^2(t) \sim (m^2 \Lambda_{\text{QCD}}^2/t^2)^n$. Consequently, the off-shell effect will render the perturbative calculation meaningful. It is also evident from Eq. (12) that the final-state rescattering contributions vanish in the heavy quark limit, as it should be.

As discussed in Ref.[6], the FSI contribution from the $\bar{B} \to D_s^- D_s^+$ decay will affect both $A_L$ and $A_\parallel$ amplitudes of the $\bar{B} \to J/\psi \phi$ decay, whereas both $\bar{B} \to D_s^0 D_s$ and $\bar{B} \to D_s D_s^*$ will affect only the $A_\perp$ term of the $\bar{B} \to J/\psi \phi$ decay amplitude. Finally, the FSI effect from the decay $\bar{B} \to D_s^0 D_s^*$ contributes to all three polarization components $A_{L,\parallel,\perp}$.

In order to perform a numerical study of the long-distance contributions, we need to specify the short-distance $A(\bar{B}^0 \to D_s^{(*)} D_s^{(*)})$ amplitudes. This decay proceeds only through $W$-exchange, and it can be calculated in pQCD effectively without introducing any new parameters [15] [16]. Numerically, we have (in units of $V_{cb} V_{cd}^* \text{ GeV}$)

$$\mathcal{A}(\bar{B}^0 \to D_s^+ D_s^-) = 7.93 \times 10^{-6} + i 0.94 \times 10^{-6},$$

$$\mathcal{A}(\bar{B}^0 \to D_s^{(*)} D_s^{(*)}) = 0.98 \times 10^{-6} + i 1.12 \times 10^{-7},$$

Figure 3: Long-distance contribution to $\bar{B}^0 \to J/\psi \phi$
and

\[
\begin{align*}
   a &= 1.9 \times 10^{-6} - i1.4 \times 10^{-7}, \\
   b &= -6.5 \times 10^{-9} + i4.7 \times 10^{-8}, \\
   c &= 6.7 \times 10^{-9} - i4.9 \times 10^{-8}, 
\end{align*}
\]

(16)

for the \( B^0 \to D^{s+}_s D^{s-}_s \) amplitude given by

\[
\mathcal{A}(B \to D^{s+}_s(p_1, \xi_1)D^{s-}_s(p_2, \xi_2)) = a(\xi_1^+ \cdot \xi_2^+) + b(\xi_1^+ \cdot p_2)(\xi_2^+ \cdot p_1) + ic\epsilon_{\alpha\beta\mu\nu}\epsilon_1^\alpha\epsilon_2^\beta p_1^\mu p_2^\nu. \quad (17)
\]

It follows that the branching ratios of \( B \to D^{(s+)}_s D^{(s-)}_s \) read

\[
\begin{align*}
   \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) &= (3.3 \pm 1.1) \times 10^{-5}, \\
   \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) &= (2.6 \pm 1.0) \times 10^{-5}, \\
   \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) &= (1.2 \pm 0.4) \times 10^{-5}. 
\end{align*}
\]

(18)

In the above calculation, we have included the errors coming from the the hadronic wave functions that are dominated by the \( D^{(s)}_s \) meson distribution amplitude rather than the \( B \) meson, as the latter is more or less fixed by the well measured channels such as \( B \to K\pi, \pi\pi \). Since we employ the updated \( D^{(s)}_s \) distribution amplitude [17]

\[
\phi_D(x, b) = \frac{3}{\sqrt{6}} f_{D(s)} x(1-x) \left[ 1 + a_{D(s)}(1-2x) \right] \exp \left( \frac{-\omega^2 b^2}{2} \right), \quad (19)
\]

with \( a_{D(s)} = 0.5 \text{ GeV} \) and \( \omega = (0.6 - 0.8) \text{ GeV} \), our predictions are slightly smaller than the ones in [15] but consistent with the current experimental limits [18,19]

\[
\begin{align*}
   \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) &< 1.0 \times 10^{-4} \text{ (BaBar)}, \quad < 3.6 \times 10^{-5} \text{ (Belle)}, \\
   \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) &< 1.3 \times 10^{-4} \text{ (BaBar)}, \\
   \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) &< 2.4 \times 10^{-4} \text{ (BaBar)}. 
\end{align*}
\]

(20)

For the parameter \( \eta \) in Eq. (14), we shall use the one \( \eta = 0.80 \) extracted from \( B \to \phi K^+ \) [6]. With the \( B \to D^{(s+)}_s D^{(s-)}_s \) amplitudes given before and the parameters (11), the decay rate and the longitudinal polarization fraction \( f_L \) of \( B \to J/\psi \phi \) due to final-state rescattering turn out to be

\[
\mathcal{B}(B^0 \to J/\psi \phi)_{\text{FSI}} = (3.7^{+5.8}_{-2.5}) \times 10^{-9}, \quad f_L = 0.41 \pm 0.02. \quad (21)
\]

\[1\] Our estimate of \( \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) \) is smaller by more than a factor of two than a value of \((7.8^{+2.0}_{-1.6}) \times 10^{-5}\) obtained in [15] using the same PQCD approach. This is mainly due to the additional exponential term \( \exp(-\omega^2 b^2/2) \) in the revised \( D^{(s)}_s \) distribution amplitude, Eq. (19). Based on the diagrammatic approach, an estimate of \( \mathcal{B}(B^0 \to D^{+}_s D^{-}_s) = (4.0^{+1.3}_{-1.1}) \times 10^{-6} \) was obtained in Ref. [23], which is smaller than the PQCD result by one order of magnitude. This should be checked by experiment.
Here we only show the major errors stemming from the uncertainties in the parameter $\eta$ and the cutoff scale $\Lambda$ (see Eq. (14)) where we have assigned a 15% error to $\Lambda_{\text{QCD}}$ and an error of 0.01 to $\eta$. As in Ref.[6], we have assumed monopole behavior [$n = 1$ in Eq. (13)] for the form factor $F(t, m_{D_s})$ and a dipole form ($n = 2$) for $F(t, m_{D_s}^*)$. It should be stressed that the estimate of the FSI contributions is model-dependent as it depends on how we model the final-state rescattering. In view of this point and the theoretical discrepancy between PQCD and the topological diagram approach for the rate of $B \to D_s^{(*)} + D_s^{(*)}$, it is conceivable that the actual theoretical uncertainties are considerably larger than those given in Eq. (21). At any rate, it is evident that the final-state rescattering contribution to $B_d^0 \to J/\psi \phi$ is smaller than the effects of $\omega - \phi$ mixing by two orders of magnitude. We thus confirm the argument by Gronau and Rosner [3] that a significant enhancement of this mode by rescattering is unlikely.

4. In this work we have examined the contributions from photoproduction and final-state rescattering to $B_d^0 \to J/\psi \phi$ and found that the corresponding branching ratios are of order $10^{-11}$ and $10^{-9}$, respectively. Hence, this decay is dominated by the $\omega - \phi$ mixing effect as advocated by Gronau and Rosner.

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