Development of a Mathematical Model for Multi-user Coded-Cooperation Based Cognitive Radio System and Its Outage Probability Analysis

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Abstract
In this paper, mathematical system model for a single and multi-user coded-cooperation-based cognitive radio system is developed. In this model, both source and relay are communicating to a single destination with the help of each other. Further, all possible multi-user scenarios are developed and their end-to-end outage probability ($P_{\text{out}}$) is calculated for the underlay mode of cognitive radio. The performance of the system is analyzed in the form of channel gain and interference temperature constraint for the Rayleigh fading channel. The proposed system concludes that the coded cooperation with cognitive radio outperforms the available techniques in the form of bandwidth, diversity, spectrum utilization efficiency and thus improving the quality of communication. Furthermore, the theoretical analysis of the outage probability for both system models is validated by asymptotic analysis. The proposed system can be set as a standard for all those cognitive radio applications which require better spectrum efficiency even if there is a scarcity of multiple physical antennas.

Keywords Coded-cooperation · Cognitive radio · Single relay · Multi-user · Interference temperature constraint · Outage probability

1 Introduction

Today’s biggest challenge is to fulfil the spectrum requirements of the next generation bandwidth-hungry wireless applications. The only possible approach is to utilize the existing frequency spectrum more smartly. For such type of applications cognitive radio (CR) is the best solution which offers an optimized way of spectrum allocation [1–4]. This is because, it can be configured and programmed to use the best possible vacant frequency spectrum in its vicinity which leads to utilization of the same spectrum in a much efficient
way [5, 6]. Next-generation wireless applications are not limited to point-to-point or point-to-multipoint applications but require wide coverage through wireless communication. Also, these applications require transmission diversity which can be achieved by using more than one antenna [7]. But, the next-generation wireless system can support only one antenna due to their small size and less complex architecture.

In recent times, a new technique i.e. cooperative communication has been proposed in which a single antenna can work as a relay as well as virtual antenna and create a useful multi-antenna environment. This relay will generate multiple replicas of the transmitted signal which ultimately provides transmission diversity. So, cooperative communication can provide high spectral efficiency, wide network coverage even with minimum outage probability. It has many types of cooperative protocols like amplify-and-forward (A&F), decode-and-forward (D&F), and coded cooperation (CC), etc. Among all these protocols, the coded cooperation is the only bandwidth-efficient technique because it sends some additional redundancy bits in its user’s data for partner or relay and avoids repetition of received bits.

In this research paper, we are going to propose a hybrid system where the benefits of coded cooperation have been exploited and merged with cognitive radio to develop a spectrum as well as bandwidth-efficient multi-user system model.

Many researchers are working on cooperative communication as well as on cognitive radio systems. In [8], an A&F relay selection technique was discussed for underlay based cognitive radio network. In [9, 10], the working of A&F based cooperative relaying system in spectrum sharing mode has been evaluated. While in [11], A&F based multi-hop system in spectrum-sharing mode was analyzed. Exact outage probability has been calculated for partial A&F based relay selection scheme by Chen et al. in [12] for cognitive relay networks with imperfect CSI. In [13], the Zhong et al. had analyzed the outage capacity of D&F based dual-hop cognitive systems under Nakagami-m fading. But, the above mentioned cooperative schemes are not bandwidth-efficient and have transmission diversity constraint. In [14], Moualeu et al. have calculated the BER for underlay based cognitive coded cooperation system under interference power constraint using turbo codes. The authors in [15] have developed a cognitive two-user radio network using Amplify and Forward cooperative technique and have calculated the gain in asymptotic agility. In [16] authors have developed a OFDM-based cognitive radio system model to increase spectrum efficiency and the results shows the accuracy in channel allocation has also increased significantly. The above literature has shown that all the researchers have mainly focused on the system where the only secondary user can send its data over the channel through the relay. In [17], we already proposed the cognitive radio-based multi-relay coded cooperation concept in which data of only secondary source is transmitted with the help of relay. The issue of the system was that the relay could not transmit its own data when the source was utilizing it as a relay. So, this work is an extension of our previous work [17] where a multi-user communication system is proposed in which both the source and relay can transmit their data simultaneously to the single destination by acting as a relay to each other without any interference.

The proposed system efficiently utilises the channel resources in a practical manner where the coded cooperation is used as cooperative relaying protocol. A single relay-based cognitive radio system is developed which is further extended to an advanced multi-user system. For the rigorous analysis of an advanced multi-user system, we have developed four different scenarios with their mathematical model, explaining the secondary user and relay transmission process. In the proposed model, the code-word of each user is partitioned using coded cooperation and the portions of these code-words are transmitted
independently over the fading channels by utilizing minimum bandwidth. Here, an end-to-end outage probability for all possible scenarios is calculated to conclude about the system model performance which outperforms the existing techniques in the form of channel gain and outage probability. Hence, providing the better communication quality for devices which cannot afford multiple antennas due to small size. Along with this, we have also worked on a high signal-to-noise ratio (SNR) regime to validate the research asymptotically.

Remaining of the article is outlined in a manner that, section II contains the introduction of the system model. In section III, the outage probability of the primary model for cognitive radio based coded cooperation is analytically derived and extended by an enhanced model in section IV with high SNR discussion. Moreover, the case-wise outage probability of multi-user transmission scenario is explained in section V. Numerical results with discussion for the proposed system are provided in section VI followed by the conclusion.

2 System Model

In this research work, a hierarchical mathematical model for coded cooperation based cognitive radio system is proposed where source and relay cooperate by transmitting their bits over the two consecutive bit periods. The bits can be transmitted by a user during the first interval and the partner can detect and retransmit it again during the second interval. The system model consists of three nodes i.e. (1) licensed/primary node ‘p’, (2) unlicensed/secondary source node ‘s’ and (3) secondary destination node ‘d’. Secondary source and secondary destination are communicating with each other directly and also through coded cooperation based relay node symbolize by ‘r’ as shown in Fig. 1. One way of limiting the use of the primary user’s spectrum by the secondary user is to incorporate an interference threshold i.e.
interference temperature constraint, ‘$Q$’ at the primary user. The interference temperature constraint corresponds to the maximum interference power level allowed by the primary node for transmitting the data of secondary node and this method of using the spectrum of primary user is called underlay mode of cognitive radio. The transmitter $i \in \{s, r, 1, 2\}$ and the receiver $j \in \{d, p, r, 1, 2\}$ are using Rayleigh flat fading independent and identically distributed (i.i.d.) channel of gain $g_{ij}$, for their transmission having zero-mean and unit variance. The channel gain is inversely proportional to average channel gain, $|h_{ij}|^2$ and distributed exponentially with parameter $\lambda_{ij}$. The transmitted power, $P_t$, between any two nodes is adjusted $P_t|g_{ij}| \leq Q$ to remain up to the level of interference constraint, $Q$ [17]. Therefore, $Q/g_{ij}$ is the maximum allowable instantaneous power of the secondary source and relay node.

Every secondary source message is encoded into a rate $R_s$ of code-word of length $L$, represented as $y = \{y[0], \ldots, y[L−1]\}^T$. Two phases of cooperative communication are used to transmit this code-word. In Phase I, first $L_1$ symbols of code-word i.e. $y^{(1)} = \{y[0], \ldots, y[L−1]\}^T$ are transmitted and are encoded by the rate $R_1 > R_s$ while the remaining portion of the code-word i.e. $y^{(2)} = \{y[L_1], \ldots, y[L_1+L_2−1]\}^T$ with $L_2 = L - L_1$, symbols will be decoded automatically by using the redundancy bits sent along with the $y^{(1)}$. The integration of $y^{(1)}y^{(2)}$ it as one will give a dominant code-word of rate $R\ln$ the first phase, the source will transmit the whole encoded code-word to the destination but the relay will just listen to the first portion of code-word i.e. $y^{(1)}$ with a code rate of $R_1$ and this signal will be received at relay node given by Eqs. (1) and (2) respectively, where $n = 0, \ldots, L_1 − 1$ and $w_r, w_d$ are Additive White Gaussian Noise (AWGN) at the relay node and secondary destination respectively. At this point, we define $\beta = L_1/L$ such that the Phase I code rate will be $R_1 = R_s/\beta$. The relay node is competent enough to effectively decode the remaining $L_2$ symbols i.e. $y^{(2)}$ of the code-word and transmit that to the secondary destination successfully in Phase II, which is stated as

$$x_d^{(2)}[n] = \begin{cases} h_{r,d} \sqrt{P_t}y[L_1 + n] + w_d^{(2)}[n], & \text{if } \delta \geq 2R_s/\beta - 1 \\ h_{s,d} \sqrt{P_t}y[L_1 + n] + w_d^{(2)}[n], & \text{otherwise} \end{cases}$$

(3)

where, $n = 0, \ldots, L_2 − 1$ the message at the secondary destination will be decoded based on the integration of symbols received in phase I and II, i.e.$x_d = \left[\left(\begin{array}{c} x_d^{(1)} \\ x_d^{(2)} \end{array}\right) \right]$, where

$$x_d^{(1)} = \left[x_d^{(1)}(0) \ldots x_d^{(1)}(L_1 - 1)\right]^T,$$

(4)

$$x_d^{(2)} = \left[x_d^{(2)}(0) \ldots x_d^{(2)}(L_2 - 1)\right]^T.$$

(5)

If the coded cooperation based relay is not able to decode the message send by secondary source effectively, then it is supposed to be in an outage, corresponding to

$$C_{ij} = \log_2 \left(1 + \delta_{ij}\right) < R_s/\beta,$$

(6)

where $\delta_{ij}$ is the SNR received at node $\{i,j\}$ given by [18]
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\[ \delta_{ij} = \frac{Qg_{ij}}{\sigma^2 g_{ip}} , \]  

(7)

where \( \sigma^2 \) represent the variance of white Gaussian noise at the secondary receiver.

The SNR received at relay/secondary destination has probability density function (PDF) is given by [18]

\[ f_{r,y}(x) = \int_{0}^{\infty} yf_{Z}(yx)f_{y}(y)dy = \int_{0}^{\infty} y\frac{\lambda_{ij}}{Q/\sigma^2} \exp \left(-\frac{\lambda_{ij}y}{Q/\sigma^2}\right) \lambda_{ip} \exp \left(-\lambda_{ip}y\right)dy \]  

(8)

\[ f_{r,i}(x) = \frac{\lambda_{ij}\lambda_{ip}Q/\sigma^2}{(\lambda_{ij}x + \lambda_{ip}Q/\sigma^2)^2} . \]  

(9)

And the CDF i.e. cumulative distribution function as

\[ F_{r,y}(x) = \int_{0}^{x} f_{r,y}(z)dz = \int_{0}^{x} \frac{\lambda_{i,k}\lambda_{ij}Q/\sigma^2}{(\lambda_{ij}x + \lambda_{ip}Q/\sigma^2)^2}dz = \frac{\lambda_{ij}x}{\lambda_{ij}x + \lambda_{ip}Q/\sigma^2} \]  

(10)

The proposed system works in two models i.e. primary and enhanced respectively. In the primary model, the only source can transmit its data to the destination through the relay and in the enhanced model relay can also transmit its data with the secondary source to the destination. Both models are explained in the next sections.

3 Primary Model

This section explains the basic transmission scheme where the source will transmit its data to the destination using coded cooperation based relay node. Frames transmitted in both Phase I and II will be received by the destination, for a given secondary source as shown in Fig. 1. The first frame possesses \( \beta \) fraction of the total allocated \( N \) bits, while the second frame utilizes the rest of \( 1 - \beta \) fraction of the total bits. In the proposed system, the hybridized time-sharing method is used between two independent channels. In this way, we can define an outage event as [7]

\[ C_{s,d}(\delta_{s,d}) = \beta \log_2 \left(1 + \delta_{s,d}\right) + (1 - \beta) \log_2 \left(1 + \delta_{s,d}\right) < R_r \]  

(11)

The average outage probability at the secondary destination is explained by two cases i.e.

3.1 Case (a)

If the code-word sent by the secondary source is successfully decoded by relay then conditional outage probability can be written as

\[ P_{out}^a = Pr(\beta \log_2(1 + \delta_{s,d}) + (1 - \beta) \log_2(1 + \delta_{s,d}) < R_r) = Pr((1 + \delta_{s,d})^\beta(1 + \delta_{s,d})^{1-\beta} < 2^R_r) \]  

(12)
3.2 Case (b)

If the relay fails to decode the secondary source’s code-word successfully then conditional outage probability is stated as

\[ P_{out}^{(b)} = \Pr \left( \log_2 \left( 1 + \delta_{s,d} \right) < R_r \right) = \Pr \left( \delta_{s,d} < 2^{R_r} - 1 \right) \]  

For the above two cases, the average outage probability is

\[ P_{out} = P_{out}^{(a)} + P_{out}^{(b)} \]

\[ P_{out} = \left( 1 - \Pr(\delta_{s,r} \leq 2^{R_r/\beta} - 1) \right) H + \Pr \left( \delta_{s,r} < 2^{R_r/\beta} - 1 \right) \Pr \left( \delta_{s,d} < 2^{R_r} - 1 \right) \]

\[ = \left\{ \begin{array}{ll}
\frac{2^{R_r/\beta} - 1}{2^{R_r/\beta} - 1} &
\int_0^{2^{R_r/\beta} - 1} f_{r,s}(x) dx \\
\int_0^{2^{R_r/\beta} - 1} f_{r,s}(x) dx &
\int_0^{2^{R_r/\beta} - 1} f_{r,s}(x) dx
\end{array} \right\} \]

\[ P_{out} = \left[ \left( 1 - \frac{\lambda_{s,d}(2^{R_r/\beta} - 1)}{\lambda_{s,d}(2^{R_r/\beta} - 1) + \lambda_{s,p} Q/\sigma^2} \right) H \right] \]

where \( H \) is given as

\[ H = \Pr(1 + \delta_{s,d})^\beta(1 + \delta_{r,d})^{(1-\beta)} < 2^{R_r} \]

\[ H = \int_0^{2^{R_r/\beta} - 1} f_{r,s}(x) \int_0^{a(x)} f_{r,s}(y) dy dx \]

where

\[ a(x) = \frac{2^{R_r/(1-\beta)}}{(1+x)^{\beta/(1-\beta)}} - 1 = \frac{2^{R_r}}{(1+x)^{\beta} - 1} \]
\[ \rho = \frac{1}{(1 - \beta)} \]

\[ \theta = \frac{\beta}{(1 - \beta)} \]

\[ \varphi = \frac{A_{r,d}a(x)}{(a(x) \frac{\lambda_{r,d}}{Q/\sigma^2} + 1)} \] (22)

where

\[ A_{r,d} = \frac{\lambda_{r,d}}{\lambda_{r,p}} \]

Substitute the value of \( \varphi \) together with the result of Eq. (9) in Eq. (20) and then expand it through binomial expansion

\[ (t - 1)^q = \sum_{r=0}^{q} \binom{q}{r} (-1)^{q-r} (t)^r \]

We can rewrite the Eq. (20) as

\[ H = \left[ \frac{\lambda_{r,d}}{Q/\sigma^2} \sum_{r=0}^{q} \binom{q}{r} 2^{r} \phi (-1)^{-r} (2^{r} \sqrt[\beta]{t} - 1) \right] \]

\[ F_1 \left( \begin{array}{c} 1 : 0, 2; 1 : \left( 2^{r} \sqrt[\beta]{t} - 1 \right), \frac{-\lambda_{r,d}}{Q/\sigma^2} \end{array} \right) \] (23)

where \( F_1(.) \) is the Appell hypergeometric function [19]. By using the above-obtained value of \( H \) in Eq. (18) the final value of outage probability can be calculated.

### 3.3 III(a). Asymptotic Analysis for Primary Model

The asymptotic behaviour of the outage probability obtained in the above section is studied by considering high mean SNR value i.e. \( \overline{\delta} \rightarrow \infty \).

Here, the average SNR can be stated as

\[ \overline{\delta}_{ij} = \frac{Q \lambda_{i,u}}{\sigma^2 \lambda_{i,p}}. \] (24)

The PDF and CDF expression can be stated as

\[ f_{\overline{\delta}_{ij}}(x) = \frac{\lambda_{i,j}}{\lambda_{i,p}Q/\sigma^2} \] (25)

\[ F_{\overline{\delta}_{ij}}(x) = \frac{\lambda_{i,j}x}{\lambda_{i,p}Q/\sigma^2} \] (26)

The final equation for the asymptotic outage probability \( (P_{\text{asy}}) \) can be illustrated as
where $\lambda_{r,p}$: Channel gain between relay and primary user, $\lambda_{s,p}$: Channel gain between source and primary user, $\lambda_{s,d}$: Channel gain between source and destination, $\lambda_{r,d}$: Channel gain between relay and destination, $Q$: is the interference temperature constraint, $R_r$: is the rate at which each code-word of length ‘$L$’ is encoded and $\beta$: $L1/L$, $L1$ length of first code word; $\sigma^2$: represent the variance of white Gaussian noise at the secondary receiver.

The above Eq. (27) shows that, the asymptotic outage probability depends mainly on channel gain parameter of all the nodes, interference temperature constraint and the code-word rate. A comparative study of both numerical and asymptotic outage probability is described in the result section.

4 Enhanced Models

The cooperative schemes present in literature describes the scenarios where either source or relay can transmit its information to the destination at any single instant of time as explained in the above section. However, in our enhanced model a pair-wise coded cooperation system is developed where user and relay both can act as source simultaneously and transmit their data to the single destination. To make cooperative channel interference-free, the data of user and relay is multiplexed by hybridizing FDMA/TDMA. In our proposed system both users i.e. user 1 and 2 can act as a source as well as coded cooperation based relay to each other for transmitting their data. The transmission will take place in two phases. In phase I, the user will transmit their data through orthogonal channels to other user and the destination simultaneously. If a user is capable enough to decode the other user’s message successfully then the second portion of code-word will be transmitted to the destination. If not, then the second portion of the code-word will be transmitted by its own in phase II.

This cooperative protocol is much easier to apply realistically because this requires no feedback from the relay or knowledge of the $s-r$ channel. Both can transmit their data simultaneously over the same channel in an optimal way which enhances the efficiency of the system and also improves the spectrum utilization. To discuss the proposed system rigorously, all the possible scenarios have been framed for multi-user scheme and their outage probability is calculated in the next section.

5 Case Wise Outage Probability

In this section, we have developed all the possible transmission scenarios with different sources and their corresponding relays. For the mathematical purpose, we define,

$$\Delta_{1,2} \cong \left\{ \log_2 \left( 1 + \delta_{1,2} \right) < R_r/\beta \right\} = \left\{ \delta_{1,2} < 2^{R_r/\beta} - 1 \right\}$$ (28)
as the event which represents the occurrence of outage from user 1 to user 2 over the transmission channel, where

\[ \delta_{1,2} = \frac{Qg_{1,2}}{\sigma_{g_{1,p}}^2} = \frac{Q|h_{1,2}|^2}{\sigma_{g_{1,p}}^2|h_{1,p}|^2} \]  

(29)

and define

\[ \Delta_{2,1} \equiv \{ \log_2 (1 + \delta_{2,1}) < R_r / \beta \} = \{ \delta_{2,1} < 2^{R_r / \beta} - 1 \} \]  

(30)

as the event which represents the occurrence of outage from user 2 to user 1 over the transmission channel, where

\[ \delta_{2,1} = \frac{Qg_{2,1}}{\sigma_{g_{2,p}}^2} = \frac{Q|h_{2,1}|^2}{\sigma_{g_{2,p}}^2|h_{2,p}|^2} \]  

(31)

and

\[ P_b() = P_b(\delta_{1,2} \geq 2^{2R_r} - 1) = 1 - F_{1,2}(2^{2R_r} - 1) \]  

(32)

In this paper, we have developed four scenarios for coded cooperation based multi-user cognitive radio network where both the users can act as a source and a relay to each other virtually and can transmit their data simultaneously. The mathematical models of all these cases are developed and their outage probability is also calculated.

5.1 Case (a)

Here, in phase, I both of the users can effectively decode the message sent by its partnership with the outage probability.

\[ \ell_a = P_b(\Delta_1^c \cap \Delta_2^c) = P_b(\delta_{1,2} \geq 2^{R_r / \beta} - 1).P_b(\delta_{2,1} \geq 2^{R_r / \beta} - 1) \]  

(33)

For this situation, each user in phase II is cooperating by sending the remaining portion of the code-word along with its partner’s message as shown in Fig. 2. Thus, the user 1 and user 2 have conditional outage probabilities which can be represented as:

\[ p_{\text{out},1}^{(\alpha)} = P_b(\beta \log_2 (1 + \delta_{2,1}) + (1 - \beta) \log_2 (1 + \delta_{2,2}) < R_r) = P_b((1 + \delta_{2,1})^\beta (1 + \delta_{2,2})^{(1-\beta)} < 2^{R_r}) \]  

(34)

and

\[ p_{\text{out},2}^{(\alpha)} = P_b(\beta \log_2 (1 + \delta_{2,2}) + (1 - \beta) \log_2 (1 + \delta_{2,1}) < R_r) = P_b((1 + \delta_{2,2})^\beta (1 + \delta_{2,1})^{(1-\beta)} < 2^{R_r}) \]  

(35)

5.2 Case (b)

In phase I of this case, none of the two users can decode the message of their partner, so the conditional \( P_{\text{out}} \) for this scenario can be represented as
Figure 3 explains that no user can effectively decode the messages sent by their partners. So, both users will send the next segment of their own code words by itself in phase II. For user 1 and user 2 the conditional outage probabilities are given as

\[ p_{\text{out},1}^{(b)} = P_b(\beta \log_2 (1 + \delta_{1,d}) < R_r) \]  \hspace{1cm} (37)

and

\[ p_{\text{out},2}^{(b)} = P_b(\beta \log_2 (1 + \delta_{2,d}) < R_r) \]  \hspace{1cm} (38)

respectively.

5.3 Case (c)

In phase I, user 2 can effectively decode the message sent by user 1, but user 1 itself is unable to successfully decode the message sent by user 2 as given in Fig. 4. The probability of occurrence of this event is given as

\[ \ell_c = P_b(\Delta_{1,2}' \cap \Delta_{2,1}) = [P_b(\delta_{1,2} \geq 2^{R_r/\beta} - 1).P_b(\delta_{2,1} < 2^{R_r/\beta} - 1)] \]  \hspace{1cm} (39)
Then, in phase II, the second part of user 1’s code-word is sent by both users simultaneously on their respective channels. Decoding of user 2’s message at the destination is done only by the message received in phase I because none of the users can transmit the remaining portion of code-word of user 2 in phase II. Thus for this case, the outage probabilities of both the users can be defined as
$$p_{\text{out},1}^{(c)} = P_{b}\left(\beta \log_2 \left(1 + \delta_{1,d}\right) + (1 - \beta) \log_2 \left(1 + \delta_{1,d} + \delta_{2,d}\right) < R_r\right)$$

$$= P_{b}\left((1 + \delta_{1,d})^\beta (1 + \delta_{1,d} + \delta_{2,d})^{(1-\beta)} < 2^{R_r}\right)$$

$$p_{\text{out},2}^{(c)} = P_{b}\left(\beta \log_2 \left(1 + \delta_{2,d}\right) < R_r\right)$$

5.4 Case (d)

Case (d) and Case (c) both are precisely same here. However, the role of user 1 is switched with user 2 and vice-versa as shown in Fig. 5. The probability of occurrence of this case is given by

$$\ell_d = P_{b}\left(\Delta_{1,2} \cap \Delta_{2,1}^c\right) = \left[P_{b}\left(\delta_{1,2} < 2^{R_r/\beta} - 1\right) P_{b}\left(\delta_{2,1} \geq 2^{R_r/\beta} - 1\right)\right]$$ (42)

The conditional $P_{\text{out}}$ for user 1 and user 2 are defined as:

$$p_{\text{out},1}^{(d)} = P_{b}\left(\beta \log_2 \left(1 + \delta_{1,d}\right) < R_r\right)$$

$$p_{\text{out},2}^{(d)} = P_{b}\left(\beta \log_2 \left(1 + \delta_{1,d}\right) + (1 - \beta) \log_2 \left(1 + \delta_{1,d} + \delta_{2,d}\right) < R_r\right)$$

$$= P_{b}\left((1 + \delta_{2,d})^\beta (1 + \delta_{1,d} + \delta_{2,d})^{(1-\beta)} < 2^{R_r}\right)$$ (44)

Fig. 5 Only user 1 is cooperating
Using all the above scenarios the final average $P_{\text{out}}$ of user $i$, for $i\in\{1,2\}$, can be calculated as

$$
P_{\text{out}} = \ell_{a} \cdot P_{\text{out},a}^{(a)} + \ell_{b} \cdot P_{\text{out},b}^{(b)} + \ell_{c} \cdot P_{\text{out},c}^{(c)} + \ell_{d} \cdot P_{\text{out},d}^{(d)}$$

$$= \left[ P_{b}(\delta_{1,2} \geq 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} \geq 2^{R_{c}/\beta} - 1) \right]$$

$$* \left[ P_{b}\left(1 + \delta_{1,a},\delta_{2,a}\right)^{(1-\beta)} < 2^{R_{c}} \right] + P_{b}\left(1 + \delta_{1,a},\delta_{2,a}\right)^{(1-\beta)}$$

$$* \left[ P_{b}(\delta_{1,2} < 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} < 2^{R_{c}/\beta} - 1) \right]$$

$$* \left[ P_{b}(\delta_{1,2} < 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} < 2^{R_{c}/\beta} - 1) \right]$$

$$* \left[ P_{b}(\delta_{1,2} < 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} < 2^{R_{c}/\beta} - 1) \right]$$

$$+ \left[ P_{b}(\delta_{1,2} \geq 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} < 2^{R_{c}/\beta} - 1) \right]$$

$$+ \left[ P_{b}(\delta_{1,2} \geq 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} \geq 2^{R_{c}/\beta} - 1) \right]$$

$$+ \left[ P_{b}(\delta_{1,2} < 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} < 2^{R_{c}/\beta} - 1) \right]$$

$$+ \left[ P_{b}(\delta_{1,2} < 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} \geq 2^{R_{c}/\beta} - 1) \right]$$

$$* \left[ P_{b}(\delta_{1,2} < 2^{R_{c}/\beta} - 1).P_{b}(\delta_{2,1} < 2^{R_{c}/\beta} - 1) \right]$$

(45)

The final expression for $P_{\text{out}}$ will be obtained by substituting the values of Eq. (29) and (31) in Eq. (45) which will finally represent the outage probability of the multi-user system.

5.5 V(a). Asymptotic Analysis for Enhanced Model

To validate the results we have used the asymptotic analysis. Here, we assume that SNR is very high in value i.e.

$$\gamma \rightarrow \infty$$

Then the value of the average SNR is:

$$\bar{\delta}_{u,v} = \frac{Q\lambda_{u,v}}{\sigma^2\lambda_{u,p}}$$

(46)

The corresponding expressions for asymptotic PDF and CDF are derived as

$$f_{\bar{\delta}_{u,v}} = \frac{\lambda_{u,v}}{\lambda_{u,p}Q/\sigma^2},$$

$$F_{\bar{\delta}_{u,v}}(x) = \frac{\lambda_{u,v}x}{\lambda_{u,p}Q/\sigma^2}.$$

(47)

The final equation for the $P_{\text{asy}}$ is obtained by using the above-mentioned PDF and CDF Eq. (47) in the outage probability Eq. (45). To improve the validation of the proposed system, asymptotic analysis has been done and the collective results are compared in the next section.

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Fig. 6 Outage probability w.r.t interference temperature constraint, ‘$Q$’

Fig. 7 Outage probability w.r.t channel gain parameter $\lambda_{r,p}$
6 Numerical Results

This portion of the article explains the numerical results of both the primary and enhanced coded cooperation based CR system model. The deviation in $P_{out}$ in the form of channel gain and interference temperature constraint, $Q$ is illustrated in this section. Asymptotic behaviour of both the models is also plotted to validate the numerical results. For the ease of illustration, we assume all the channels to be identically distributed with $\lambda_{s,t}=\lambda_{c,d}=\lambda_{s,d}=1$ and $R=1$ bits/s, $\sigma^2=0$ dB and $K=\beta=1/2$.

The variation of outage probability with interference temperature constraint, $Q$ is shown in Fig. 6. The $P_{out}$ graph is drawn for a wide range of $Q$ i.e. 5 dB to 20 dB for different values of the channel gain parameter. The primary system model having a single relay shows a noticeable improvement in outage probability from $10^{0.8}$ at $Q=5$ dB to $10^{-3.5}$ at $Q=20$ dB. More interestingly a remarked improvement is noted in outage probability of system even for single relay coded cooperation at higher value of interference temperature constraint, $Q$ because secondary user will be allowed to transmit high power for higher value of $Q$. To validate the analysis asymptotic results are also plotted on the same graph which is coinciding at higher values of interference temperature constraint, $Q$ and this proves that the analysis is quite accurate.

The Fig. 7 represents the variation of $P_{out}$ versus relay and primary user channel gain parameter, $\lambda_{r,p}$ for interference temperature, i.e. $Q=20$ dB. The graph shows that for the low value of $\lambda_{r,p}$, the $P_{out}$ is not notably changed, but at higher values of $\lambda_{r,p}$ there is a considerable enhancement and outage probability reduces to $10^{-3.5}$. To validate the analysis asymptotic results are also plotted on the same graph which is coinciding at higher values of channel gain parameter, $\lambda_{r,p}$ that further proves the correctness of the analysis.

The results in Fig. 8 shows the effect of relay and primary user channel gain parameter, $\lambda_{r,p}$ on outage probability for the enhanced model for a wide range of interference

![Fig. 8] Outage probability w.r.t. channel gain parameter $\lambda_{r,p}$ at different interference temperature constraint, $Q$
temperature, i.e. $Q = 20$ dB and 25 dB. Results in Fig. 8 shows that the outage probability improves from $10^{-1}$ to $10^{-2}$ with the increase of $\lambda_{r,p}$ and are constant for the higher value of $\lambda_{r,p}$. So, the result proves that the designed system can enhance the diversity and communication quality even for Rayleigh channel. The plot illustrates that at higher values of $Q$ dB, the numerical, as well as asymptotic results of outage probability, coincide.

In Fig. 9, the results for outage probability are taken for an enhanced model with a wide range of interference temperature, $Q$ dB values i.e. 5 dB to 30 dB for both users. Results demonstrate that the system performance improves with an increase of the interference temperature constraint because higher values of $Q$ dB makes the secondary user transmit more power. As a result, the outage probability improves from $10^0$ at $Q = 5$ dB to $10^{-5}$ at $Q = 30$ dB. This is a marked improvement in bandwidth efficiency and spectrum utilization. Moreover, both the users show the same outage behavior and result is also validated with asymptotic analysis.

7 Conclusion

In this research, we have illustrated the benefits of using coded cooperation as a cooperative protocol with underlay based cognitive radio in a multi-user environment. The result proves coded cooperation technique as an effective protocol for achieving transmission diversity for the single-antenna system. Finally, a hierarchical mathematical model for the purposed system is developed and performance is evaluated in terms of outage probability. The functioning of the system is also analyzed for interference temperature constraint and channel gain and the results are validated through asymptotic analysis. The proposed system model provides diversity along with spectrum and bandwidth gains. The designed system can be referred as benchmark technology for all those cognitive radio applications.
which require better spectrum efficiency even if having the constraint of size and multiple physical antennas.

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