A cycle-jumping method for multicyclic Hubbert modeling of resource production

Bolorchimeg N. Tunnell1 | James A. Conder2 |
Ken B. Anderson2 | Marek Locmelis1

Abstract
The amount of ultimately recoverable resources and/or timing of peak production have been the central purpose of numerous studies. One broadly applied method is Hubbert modeling, subsequently extended as multicyclic Hubbert modeling. This paper explores a modification to conventional multicyclic Hubbert modeling that we term “cycle-jumping” wherein the overlap of multiple curves is limited and explicitly accounted for. The model is designed in a way that each curve is described by the same three parameters as a multicyclic Hubbert model, and every two curves are connected through an explicit transition. The transition width indicates the time of the shift from one curve to the next and is controlled by a weighting parameter for the respective curves. Cycle-jumping provides a superior data fit compared to the conventional cycle-addition model and, more important, reflects historical production data more realistically as socioeconomic and political factors important to resource production vary in time.
Recommendations for Resource Managers

- Conventional multicyclic Hubbert modeling poorly reflects transitions in production trends.
- Cycle-jumping with a finite transition period practically and mathematically provides a superior model for historical resource production by limiting the overlap of multiple curves.
- Cycle-jumping with a finite transition period reflects more realistically the production profile affected by external factors including capturing inherent asymmetry in different cycles.

KEYWORDS
asymmetric curve, cycle-adding, cycle-jumping, Hubbert modeling, multicycle, transition width, transition year

1 INTRODUCTION

Modeling of energy resource production with different types of bell-shaped curves has been the central purpose of many studies that attempted to predict the timing of peak production and/or ultimately recoverable resources (URR; Campbell & Laherrère, 1998; Ebrahimi & Ghasabani, 2015; Maggio & Cacciola, 2009, 2012; Nashawi et al., 2010; Reaver & Khare, 2014; Saraiva et al., 2014; Wang et al., 2018). Hubbert modeling, developed by and named after geophysicist Marion K. Hubbert, is the most widely used method for modeling historical energy resource production and subsequent forecasting by extrapolation (Maggio & Cacciola, 2009; Sorrell & Speirs, 2010). Hubbert (1956, 1959) empirically fitted a bell-shaped curve to the U.S. lower 48 states’ oil production data and predicted that oil production would peak at some point between the late 1960s and early 1970s by extrapolating the curve, which occurred as predicted. Hubbert’s modeling was based on simple assumptions, primarily that the production of a finite resource starts at zero, reaches the peak production level when half of the resource is extracted, and then decreases to zero. The area under the bell-shaped curve represents the URR. Hubbert used the first derivative of a logistic (Hubbert, 1980, 1982), but the precise form of the bell curve is less important than the basic inherent assumptions above that the bell curve contains. A variety of other curves such as Gaussian (Bartlett, 2000; Semenychev et al., 2014) and Gompertz (Höök et al., 2011) curves have also been applied to historical production data of energy resources and have proven to have similar descriptive power (Bartlett, 2000; Patzek, 2008).

As noted by numerous authors, historical production trends of energy resources are generally characterized by more than one production cycle due to external factors such as technological advances or civil unrest (Laherrère, 2000; Reynolds, 2014; Sorrell & Speirs, 2010). To address multiple production cycles in the same time series, Al-Fattah and Startzman (1999, 2000) proposed multicyclic Hubbert modeling in which distinct Hubbert cycles are added together to model variability in production, which we term here as “cycle-adding.” Some variations have been used to better fit the production data, such as incorporating an additional
parameter to widen and/or narrow the peaks within the individual curves characterizing their models (Ebrahimi & Ghasabani, 2015; Maggio & Cacciola, 2009, 2012; Saraiva et al., 2014). While the cycle-adding method offers a quantitative way of addressing temporal factors affecting production curves, it fails to reasonably address the transition of one cycle to another. Simply stacking bell-curve production cycles to get a full production time series results in the infinitely long tails of individual curves having unrealistic influence on earlier and later cycles, which necessarily influences goodness-of-fit to data and future projections.

The undue influence future cycles can have on previous cycles in a cycle-adding formulation is illustrated in Figure 1a, which is fitted to the historical coal production of the Eastern U.S. The curve is characterized by a tricyclic production profile. The production profile results from the technological advancement in which the coal is utilized and in conjunction with socioeconomic and political factors (Milici & Dennen, 2009). The first cycle reflects the coal demands of direct consumption by consumers, for example, for railway locomotives and stationary steam engines. This consumption pattern shifted in the early 20th century due to increasing use of petroleum-derived fuels and socioeconomic factors including the Great Depression in the 1930s (Höök & Aleklett, 2009; Milici & Dennen, 2009). The second cycle, which peaks in the mid-1940s, reflects increased wartime industrial demands for coal providing large amounts of war material and goods to European countries and later the war economy when the United States entered World War II (Milici & Dennen, 2009). The third cycle reflects coal demand for the electrical-power-generating-industry that began in earnest post-war through the early 1960s (Milici & Dennen, 2009; Schweinfurth & Finkelman, 2002). While the cycle-adding method fits the data reasonably well (Figure 1a), there are inherent problematic aspects to describing production cycles in this way. For example, because of its relative size cycle 3 is implied to be contributing to net production as early as the late 19th century. Cycle 3 also implies that more than half of the coal produced during wartime was utilized for electricity generation. Clearly neither is the case.

Here we propose an alternative approach for multicyclical Hubbert modeling tentatively termed “cycle-jumping,” wherein overlap of multiple cycles is limited. The cycle-jumping

![Figure 1](image-url)
method illustrated in Figure 1b addresses this problem by limiting the permissible overlap between production cycles. In Figure 1b, three cycles are fit to the data with a best fitting transition time for moving from one cycle to the next. This paper investigates the usefulness of the cycle-jumping method. The goal is to determine whether, and to what degree, this method is equivalent, superior, or inferior to the conventional multicyclic Hubbert model in terms of accurately and practically modeling historical resource production data. We show that the method is indeed superior, particularly when the transition from one curve to the next is allowed to proceed over a finite time-window emulating the transitioning from one set of economic drivers to another.

2 | METHODOLOGY

The mathematical definition of the multicyclic Hubbert model presented by Al-Fattah and Startzman (1999, 2000) is given as Equation (1):

\[ P_t = \sum_{i=1}^{k} (R_i) = \sum_{i=1}^{k} 4(P_{\text{max}}) \left\{ \frac{e^{-a(t-t_{\text{max}})}}{1 + e^{-a(t-t_{\text{max}})}} \right\}, \]

where \( P_t \) is the production rate at time \( t \), \( t_{\text{max}} \) is the time when production peaks in each Hubbert cycle, \( P_{\text{max}} \) is the maximum production rate of each production cycle, \( a \) is the inverse decay period, and \( k \) is the number of production cycles. When \( k \) equals 1, it becomes the unicyclic Hubbert curve, as introduced by Hubbert (1980, 1982). Individual production cycles described by the same three parameters are added to form the multicyclic Hubbert curve defined by 3\( k \) parameters.

Our multicyclic Hubbert modeling with cycle-jumping is configured in a way that each production cycle is determined by the same three parameters: \( P_{\text{max}} \), \( a \), and \( t_{\text{max}} \) and every two abutting cycles are connected through a transition width. The transition width is a time range that indicates the shift from one cycle to the next and is defined as weighted coaddition of neighboring two cycles. A weighting function that smoothly transitions from one cycle to the next can be formulated as follows (Conder, ):

\[ w = \frac{1 - \exp (\gamma x)}{1 - \exp (\gamma)}, \]

where \( w \) is a weight applied to the second cycle within the transition width and \( 1 - w \) is the weight of the first. \( \gamma \) is a parameter describing the form of the weighting, and \( x \) is a normalized transition width (Figure 2). The transition window is described by three parameters: transition year, transition width, and \( \gamma \), that shows the combined effect of lingering of the curve 1 and starting of the next curve. A zero transition width collapses the three parameters into a single parameter: a transition year, which represents a sudden and complete shift from one curve to the next. For the analysis presented here, curve-describing parameters were derived using MATLAB software and the methodology described in Conder (2015) with modifications in the G matrix, where the coaddition of curves is limited to the transition windows. The transition year, transition width, and parameter \( \gamma \) were determined using a grid search.
For both cycle-adding and cycle-jumping methods, the goodness-of-fit is optimized by minimizing the difference between the model data and the historical data and this can be directly evaluated by the root mean square error (RMSE) which is expressed as follows:

\[
\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (P_{\text{act}} - P_{\text{pre}})^2}{N}} = \sqrt{\frac{\text{SSE}}{N}},
\]

where \(P_{\text{act}}\) represents actual historical data and \(P_{\text{pre}}\) represents predicted production derived from curve-fitting. The lower the RMSE value, the closer the predicted data is to the actual historical data. Of course, reducing the RMSE value by adding additional parameters does not guarantee that it is a superior model, as RMSE can be reduced by including even unwarranted parameters. The significance of having additional parameters that represent the transition window in the proposed cycle-jumping method can be evaluated in multiple ways. One objective evaluation is performing a statistical F-test (Stein & Gordon, 1984), which is expressed as follows:

\[
F = \left\{ \frac{X_1 - X_2}{(k_2 - k_1)X_2} \right\}(N - k_2 - 1),
\]

\(X\) represents the sum of squared error (SSE) for each model, \(N\) represents the number of historical data, \(k_1\) represents the number of parameters used in model 1, and \(k_2\) represents the number of parameters used in model 2 where \(k_2 > k_1\). The degrees of freedom for constructing
the f-distribution are \((k_2 - k_1)\) and \((N - k_2 - 1)\). The probability of having additional parameters improve the goodness-of-fit simply by chance is tested by integrating the tail of the f-distribution above \(F\) (Anderson & Conder, 2011; Reaver & Khare, 2014). The tables for 90% and 95% probability thresholds are easily found in standard statistical texts or online.

Another way of comparing goodness-of-fit for various models with different complexities is to apply the corrected Akaike Information Criterion (AICc), which is based on the information theory (Motulsky & Christopoulos, 2004). The equation for which is expressed as follows:

\[
\text{AICc} = N \log\left(\frac{\text{SSE}}{N}\right) + 2K + \frac{2K(K + 1)}{N - K - 1},
\]

where \(K\) equals the number of model parameters plus 1. The first term in the right-hand side of Equation (5) evaluates the model misfit to the actual data set while the rest describes the complexity of the model. The combination of these two expressions is therefore used to identify the simplest model that provides the least misfit to the actual data set (Motulsky & Christopoulos, 2004). The difference between AICc of any two models shows the extent of information loss for given models with different complexities. The one with the lowest AICc has the greatest-likelihood of being closest to the “true” model (Burnham & Anderson, 2004; Conder, 2015).

To explore the applicability of multicyclic Hubbert modeling with cycle-jumping we apply the method to coal production of (a) Canada, of which production could reasonably be represented by two cycles; and (b) the United States, combining the Eastern and Western U.S. coal production which also could reasonably show a tricyclic pattern. Since this study focuses on a transitioning from one cycle to the next, therefore determining the optimal number of cycles for modeling Canadian and U.S. coal production data will not be discussed here. Instead, we refer to Anderson and Conder (2011), who present a detailed discussion on the statistically justifiable ideal number of cycles applied to past production and its correlation with the objective data that reflect true historical causes and circumstances to avoid overfitting due to increased number of parameters that describe the model. It is noted that the Pennsylvania anthracite production is excluded from the U.S. coal production data, as the historical production of Pennsylvania anthracite is adequately modeled on its own with a unicyclic Hubbert curve and the cycle-jumping method proposed here is to solve the inherent problems in multicyclic cycle-adding method.

Here, we also emphasize that we do not test the forecasting ability of the newly proposed model, but simply examine how accurately and realistically the new method can model the past production compared to the cycle-adding method. The historical coal production data of these regions until 2009 are taken from Rutledge (2011). The United States coal production data from 2010 to 2017 are complemented from the Energy Information Administration (EIA), whereas Canadian production data from 2010 to 2018 are retrieved from British Petroleum (BP). For both regions, historical coal production was modeled by cycle-adding, cycle-jumping with zero transition width, and cycle-jumping with a finite transition width.

3 | RESULTS

3.1 | Canada

Canadian coal production can reasonably be modeled with two production cycles. Optimized multicyclic modeling with the best goodness-of-fit, using the cycle-adding method of this
system, is described by six parameters that define the two curves that best represent the historical production data (Figure 3).

A cycle-jumping model with zero transition width for this case is described by seven parameters, including a transition year (Table 1 and Figure 4a). The transition year is identified as 1974 when the coal production shifts from curve 1 to curve 2 and the predicted coal production rate shows an abrupt increase from 9.6 to 32.2 Mt. Using the cycle-jumping method with a finite transition width results in a model with nine parameters including transition year, transition width, and $\gamma$ parameter (Figure 4b). In this case, after optimization, the transition year is identified as 1966, which corresponds to the initiation of the second production cycle (i.e., the start of curve 2 and the phase out of curve 1). The transition width is calculated as 22 years bracketing the zero transition width year of 1974. Mathematically, the relative importance of the two associated cycles are defined by adding a decaying curve 1 to an increasing in importance curve 2 using the weighting method in Equation (2). The $\gamma$ parameter for the weighting is 0.7. In this model, there is no effect due to the second production cycle until the transition year, but there is a decaying effect of the curve 1 across the transition window, which likely reflects time necessary for decommissioning of the existing infrastructure associated with utilization described by the first production cycle. Without influence of the later curve on the behavior of the first, the peak year of curve 1 is deferred by 10 years when compared to the cycle-addition model in which the overlap of the curves shifts the peak of curve 1 to earlier. Curve-describing parameters and RMSEs for all three cases are given in Tables 1 and 2.

**FIGURE 3** Historical coal production of Canada modeled with optimized multicyclic Hubbert cycle-adding method. Curve-describing parameters and misfits are given in Tables 1 and 2. Source: Rutledge (2011) and BP
The cycle-jumping method reduces misfit by 25% for zero transition width and 75% for finite transitions when compared to cycle-adding (Table 1). Of course, the number of parameters that describes the model also increases. We test whether the better fit provided by the additional parameters truly reflects a superior model using both a statistical $F$-test and the corrected AIC.

| Parameters          | Optimized multicyclic Hubbert curve with cycle-adding method | Optimized multicyclic Hubbert curve with cycle-jumping method (zero transition width) | Optimized multicyclic Hubbert curve with cycle-jumping method (finite transition width) |
|---------------------|---------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| $N$                 | 6                                                             | 7                                                                                   | 9                                                                                   |
| $P_{\text{max}}$    | 15.0                                                          | 15.7                                                                                | 15.9                                                                                |
| $a$                 | 0.08                                                          | 0.05                                                                                | 0.06                                                                                |
| $t_{\text{max}}$    | 1928                                                          | 1943                                                                                | 1938                                                                                |
| Transition year     | NA                                                            | 1974                                                                                | 1966                                                                                |
| Transition width    | NA                                                            | NA                                                                                  | 22                                                                                  |
| $\gamma$ parameter  | NA                                                            | NA                                                                                  | 0.7                                                                                 |
| SSE                 | 2,828                                                         | 2,097                                                                               | 717                                                                                 |
| RMSE                | 4.2                                                           | 3.6                                                                                  | 2.1                                                                                 |

Abbreviations: RMSE, root mean square error; SSE, sum of squared error.

The cycle-jumping method reduces misfit by 25% for zero transition width and 75% for finite transitions when compared to cycle-adding (Table 1). Of course, the number of parameters that describes the model also increases. We test whether the better fit provided by the additional parameters truly reflects a superior model using both a statistical $F$-test and the corrected AIC.

**FIGURE 4** (a) Canadian coal production modeled by optimized multicyclic Hubbert curve with zero transition width cycle-jumping method. The transition year is found at 1974. (b) Canadian coal production modeled by optimized multicyclic Hubbert curve with a finite transition width cycle-jumping method. Transition year is 1966 and transition width is 22. $\gamma$ parameter for the weighting is 0.7. Curve parameters are given in Table 1. **Source:** Rutledge (2011) and BP
Table 2 shows the results of the F-test and the corrected AIC. When the models with six and seven parameters are compared, the F value is substantially higher than the F critical value and therefore, the seven-parameter model is better than the six-parameter model. The improvement of the model by adding an extra parameter is guaranteed at a >99.9% confidence level that the improvement is not due to chance. A similar statement can be made for the corrected AIC test in which the AICc value by the seven-parameter model is lower than that of the six-parameter model and gives a greater than 99.9% probability that it is closer to the “true” model. When the models described by seven and nine parameters (cycle-jumping without and with an included transition period) are compared, the improvement by adding two extra parameters is further warranted at a >99.9% confidence level. This is also supported by the corrected AIC test in which the AICc value for the nine-parameter model is lower than that of the seven-parameter model at a greater than 99.9% likelihood that it is closer to the true model than either of the other two models. This demonstrates that the cycle-jumping method with finite transition width provides a mathematically superior model to the cycle-adding method and cycle-jumping method with zero transition width as an outcome of implicitly using more realistic assumptions.

3.2 The United States

The U.S. coal production, excluding Pennsylvania anthracite, can be reasonably modeled with three cycles. An optimized multicyclic Hubbert model with cycle-addition described by nine parameters is illustrated in Figure 5. Again, as seen in Figure 5, cycle-addition suggests that curve 3 affected curves 1 and 2 well before the economic engines driving curve 3 likely began.

A multicyclic Hubbert model with zero transition width cycle-jumping method provides a better fit compared to that of the cycle-adding method (Table 4 and Figure 6a). The transition years that give the least RMSE are found at 1932 and 1960 when the coal production shifts from one cycle to the next, exiting the previous cycle completely.

A multicyclic Hubbert model with a finite transition width cycle-jumping method described by 15 parameters (three cycles with two transitions) is illustrated in Figure 6b. In this model, the first transition period starts at 1930 with a duration of 12 years that is estimated as a weighted sum of the first and the second curves with a γ parameter of 0.5. The second transition width starts at 1945 and continues for 20 years, which is calculated with a γ parameter of 0.1. Model parameters and RMSEs for all three cases are presented in Tables 3 and 4.

As before, as the number of parameters that describes the multicyclic Hubbert model increases, the corresponding goodness-of-fit increases. The multicyclic Hubbert model with finite transition width yields the best fit, whereas the cycle-addition model provides the highest RMSE and the cycle-jumping model with zero transition width gives an RMSE in between.

| Number of parameters (N) | SSE  | AICc | Fvalue/Fcritical | Confidential level (%) |
|--------------------------|------|------|------------------|------------------------|
| 6                        | 2,828| 476  | 53.3/3.9         | >99.9                  |
| 7                        | 2,097| 430  | 145.3/3.1        | >99.9                  |
| 9                        | 717  | 262  |                  |                        |

Abbreviations: AICc, Akaike information criterion; SSE, sum of squared error.
Whether the improvement in the goodness-of-fit provided by additional parameters yields a superior mathematical model was again tested with $F$-test and AICc methods (Table 4). When the models with 9 and 11 parameters are compared, the model with 11 parameters (instantaneous transitions) yields a higher AICc value than that of the nine-parameter model, suggesting a roughly one in four chance that the additional parameters were warranted. This is also shown by the $F$-test in which the $F$ value gives only a 52% confidence level that the improvement is not due to chance, suggesting that including the additional parameters cannot be done with high confidence. However, substantial improvement is found using finite transition widths. When the models with 11 and 15 parameters are compared, the addition of four parameters decreases the RMSE and the AIC value. The $F$ value implies that the improvement to the misfit is achieved with a 98.9% confidence level. When the models with 9 and 15 parameters are compared, we see a similar result. A direct comparison shows that the improvement over the cycle-adding model by the additional six parameters describing the two transitions is warranted from the $F$-test with a 97.6% confidence level. The AICc value gives the finite transition width model a >83% likelihood

### Table 3

Multicyclic Hubbert curves parameters described by cycle-adding and cycle-jumping methods fitted to the U.S. coal production

| Parameters | Optimized multicyclic Hubbert curve with cycle-adding method | Optimized multicyclic Hubbert curve with cycle-jumping method (zero transition width) | Optimized multicyclic Hubbert curve with cycle-jumping method (finite transition width) |
|------------|-------------------------------------------------------------|----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| $N$        | 9                                                          | 11                                                                               | 15                                                                                |
| $P_{\text{max}}$ | 426.08, 280.85, 993.48                                      | 476.45, 506.43, 989.63                                                          | 482.92, 1,091.5, 995.75                                                          |
| $a$        | 0.103, 0.261, 0.057                                         | 0.091, 0.107, 0.055                                                             | 0.084, 0.037, 0.057                                                              |
| $t_{\text{max}}$ | 1919, 1946, 2001                                           | 1922, 1947, 2001                                                                | 1924, 1992, 2001                                                                 |
| Transition year | NA, NA, 1932                                               | 1960                                                                             | 1930, 1945                                                                        |
| Transition width | NA, NA, NA                                                 | NA                                                                               | 12, 20                                                                            |
| $\gamma$ parameter | NA, NA, NA                                                  | NA                                                                               | 0.5, 0.1                                                                          |
| SSE        | 248,533                                                     | 246,612                                                                          | 227,670                                                                          |
| RMSE       | 35.34                                                       | 35.20                                                                            | 33.82                                                                            |

**Abbreviations:** RMSE, root mean square error; SSE, sum of squared error.

### Table 4

Comparison of multicyclic Hubbert models described by 9, 11, and 15 parameters for the U.S. coal production

| Number of parameters ($N$) | SSE       | AICc    | $F$ value/$F$ critical | Confidence level (%) |
|---------------------------|-----------|---------|------------------------|----------------------|
| 9                         | 248,537   | 1,440   | 0.7/3.0                | 51.7                 |
| 11                        | 246,612   | 1,443   | 3.4/2.4                | 98.9                 |
| 15                        | 229,766   | 1,436   |                        |                      |

**Abbreviations:** AICc, Akaike information criterion; SSE, sum of squared error.
FIGURE 5  Historical U.S. coal production modeled with optimized multicyclic Hubbert curve with cycle-adding method. Curve-describing parameters and misfits are given in Tables 3 and 4. *Source*: Rutledge (2011) and EIA.

FIGURE 6  The U.S. coal production modeled by cycle-jumping. (a) Multicyclic Hubbert model with zero transition width cycle-jumping that gives the least misfit. The transition years are found at 1932 and 1960. (b) Multicyclic Hubbert model with a finite transition width cycle-jumping that provides the lowest misfit. Transition years are 1930 and 1945 and transition widths are 12 and 20. γ parameters for the weighting are 0.5 and 0.1. Curve-describing parameters are given in Table 3. *Source*: Rutledge (2011) and EIA.
of being closer to the “true” model than the other two models combined. In all cases explored, the finite transition width cycle-jumping method provides mathematically superior models compared to both cycle-addition and instantaneous transitions.

4 DISCUSSION

Historical resource production modeling with the multicyclic Hubbert method is essentially a curve-fitting practice with two main purposes: (a) modeling of past production data and (b) subsequent forecasting of the URR by extrapolating. Modeling of historical production trends should not only be mathematically optimal but also practically realistic to produce meaningful results (Wang & Feng, 2016). The prediction of future production trend and the URR estimation using curve-fitting are controversial topics amongst researchers, as the multicyclic Hubbert modeling cannot predict individual future cycles (Anderson & Conder, 2011) and future production trend is largely dependent on an assumed URR (Brandt, 2010). Additionally, the concept of the URR is dynamic due to economic and physical factors (Sorrell et al., 2010) and an excellent goodness-of-fit does not necessarily imply a good forecasting property (Wang & Feng, 2016). Regardless of the methodology used, the accuracy of forecasting can only be justified once the future is unfolded (Höök et al., 2011). Therefore, our discussion will focus less on forecasting than on the modeling ability of the proposed method and its superiority to understanding past cycles and their transitional relationships. While this methodology does not directly lead to improved long-term forecasting, the better accounting of the historical data will inevitably better short-term forecasting and any gains that leads to in the more distant future.

The results above show that cycle-jumping with a finite transition width provides mathematically better modeling in both the Canadian and U.S. cases. Our results show that cycle-jumping with zero transition width provides a mathematically better model compared to the cycle-adding method in the case of Canadian coal production. While it provides a moderately mathematically inferior model in the case of U.S. coal production, the lack of significant improvement can be understood by recognizing the narrow width of the middle curve that naturally limits the influence of its tails on neighboring cycles, which is where RMSE gains between models tend to be greatest. Further, the complete stop of curve 1 at transition year is unrealistic. Moreover, the decline in the first cycle is not due to resource depletion but is rather due to external factors limiting coal production. As the second cycle ramps up, reactivation of previously operated mines implicitly shows the lingering effect of the first cycle. Therefore, the sudden and complete shift from the first cycle to the next is an oversimplification with some lingering effect of the first curve being expected.

4.1 Canadian coal production

In the case of Canadian coal production, the economic engines for the second cycle and the resulting increase in production rate were driven by the signing of long-term contracts with Japan to export metallurgical coal established in the late 1960s, as well as the increasing domestic coal demand for coal-fired power plants (Muise & McIntosh, 1996; Page & Shapiro, 2006). These contracts and domestic demand promoted reactivation of old mines and the development of new coal mines, particularly in Alberta and British Columbia due to proximity to the west coast, and subsequent constructions of ports and railroads
(Muise & McIntosh, 1996; Page & Shapiro, 2006). Moreover, unpredicted increases in the crude oil price in the 1970s led to continued growth in coal extraction by using large open-pit mining techniques until 1990 (Muise & McIntosh, 1996; Page & Shapiro, 2006). After the 1990s, coal extraction remained fairly stable. The decline in coal production in the first cycle was due to a cheaper alternative fossil fuel supply in Canada since 1947 which continued until the late 1960s (Page & Shapiro, 2006). In the cycle-adding case, the second cycle suggests that sizable coal production relevant to this cycle has been ongoing since the 1920s, which does not correspond to the true economic drivers that triggered this cycle (Figure 3). The first curve and its decline represent the direct utilization of coal, primarily for heating and powering locomotives and ships (Muise & McIntosh, 1996). Coal produced from existing populations of coal mines in Canada at that time and decreasing demand of coal (for the reason given above) should apply to all existing mines at that time. The curve describing this production cycle contains no information about the possible future coal mine populations and their production behavior (Patzek, 2008; Patzek & Croft, 2010). The second cycle, having peaked at a different time from the first cycle, corresponds to the development of new subpopulations with coal exporting and domestic thermal energy purposes under different economic condition (Patzek, 2008).

The transition width, 22 years in the case of Canadian coal production, is defined by the lingering of curve 1 and the ramping up of curve 2 (Figure 4b). In this case, rapid increases in demand are driven by the development of new export markets and expansion of the domestic coal demand for coal-fired power plants following the invention of extra-high voltage power lines (Muise & McIntosh, 1996). New and reactivated coal mines would be exploited with large open-pit mines at a faster rate driven by demand created by the new market opportunities and advanced technology (Muise & McIntosh, 1996). The drivers of curve 1 were largely subsumed in curve 2, such as electrical heating in homes displacing coal stove heating.

An interesting consequence of coaddition of the two cycles incorporating a transition period, is an apparent positively skewed asymmetry of the second curve, despite the production behavior of the first cycle being reasonably modeled by symmetric Hubbert curve. Asymmetric growth and decline is commonly observed in real resource production systems (Bardi, 2005; Brandt, 2007). In fact, some authors have argued that symmetric curves are impractical or unrealistic and tend to underestimate URR (Bardi, 2005; Brandt, 2007; Sorrell & Speirs, 2010; Wang & Feng, 2016). While cycle-adding can implicitly account for some asymmetry by summing different portions of the curves together, it is inefficient for more than subtle asymmetry. The cycle-jumping method with sudden jumps to the next cycle explicitly imposes symmetric curves and therefore, cannot account for this inherent asymmetry beyond simple truncation (Figure 4a), although skewness could conceivably be included for an additional parameter per curve. In contrast, finite width transitions can capture fairly substantial asymmetry through the changing weights of the two curves through the transition window reflecting real economic differences in the ramp up and down sides of the individual cycles.

Individual production cycles modeled by multicyclic Hubbert methods should be attributed to some definable objective basis simulating the production path caused by external factors (Patzek, 2008; Patzek & Croft, 2010) and are independent of the others (Reaver & Khare, 2014). Importantly, the model for Canadian coal production using cycle-jumping with a finite transition period described above is not only superior from a data fit perspective, but is better correlated with historical data regarding factors driving changes in coal production than models based on cycle addition or jumping with immediate transitioning from one curve to another.
4.2 The U.S. coal production

The three cycles in the historical U.S. coal production reflect external factors such as economic depression, wartime demand and changes in coal utilization (Milici & Dennen, 2009). Due to the Great Depression, which began in 1929, coal production dropped sharply forming the first cycle and resulted in greater decreasing rate of coal production relative to the growth part of the curve. The multicyclic Hubbert model with cycle-adding fails to illustrate this negative asymmetry in curve 1 largely due to the addition of curve 3’s initial tail (Figure 5). As mentioned in the introduction, the straightforward addition of curve 3 implicitly implies that some of the coal produced during the early 1900s was utilized for the economic drivers of curve 3, such as fueling coal-fired power plants. The inclusion of the tail not only deviates from the true economic conditions and coal utilization but also pushes the form of the first curve toward positive asymmetry (Figure 5). In contrast, multicyclic Hubbert curves with transition period cycle-jumping better capture the negatively skewed asymmetry of the cycle 1, better reflecting the true economic restrictions (Figure 6).

The second cycle corresponds to the demand leading up to and during World War II, which caused large annual variations in coal production rates for approximately 15 years. As outlined above, with cycle-addition the economic engines that drive curves 1 and 3 appear to be contributing to the coal production rate during the World War II, which again fails to reflect economic reality. The multicyclic Hubbert model with zero transition width cycle-jumps models curve 2 with a symmetric pattern (Figure 6a). This also deviates from the real trend and does not account for the inherent asymmetry. The Hubbert curve with finite transition width models only the 4 years, that is 1942–1945, of curve 2 as specific to only that curve (Figure 6b). The 12 years beforehand reflect the combination of rapid drop due to the Great Depression and rapid ramping up for war. The 20 years after 1945 reflect the decrease of wartime demand while transitioning to a new economy with both the Great Depression and WWII behind.

The 12-year transition ending curve 1 reflects the period of the Great Depression. The resumption of increasing coal production was forced due to the wartime demand after the rapid decline and continuous economic depression, which cannot be fully illustrated by either of the other two methods. The 20-year-long second transition period starting at 1945 models the coal production trend during the post-war economic adjustment. During this period, petroleum gradually replaced coal as the chief source of energy such as in diesel engines (Schweinfurth & Finkelman, 2002), resulting in the slower decreasing rate of coal production than the increasing rate. The combined transition windows with 4 years of coal demand better illustrates coal production behavior with negative skewness during the WWII and overcomes the symmetric modeling tendency, the central limitation of conventional Hubbert modeling (Wang et al., 2018).

After the economic adjustment, the mid-1960s marks the beginning of curve 3, which also coincides with the commencement of coal-fired power plants acting as the main driving engines for coal demand (Figure 6b). The increasing rate of coal production in curve 3 resulted from and further supported by a growing national demand for electricity. Demand continued for low-cost, low-sulfur steam-coal following the Federal Clean Air Act of 1970 (Milici & Dennen, 2009; Schweinfurth & Finkelman, 2002). Additional driving factors were technological advances in engineering and mining practice, large-scale developments of mining thick, mineable coal resources in the Powder River Basin, and rail infrastructure by the Burlington Northern Railroad (BNSF) and the Union Pacific Railroad (Höök & Aleklett, 2009). Cycle-jumping with finite transition widths more clearly and reasonably reflects the socioeconomic factors
replicated in each of the three U.S. cycles and the transition periods between them compared to the models without finite transition widths.

5 | CONCLUSION

The proposed multicyclic Hubbert model with cycle-jumping is a novel curve-fitting exercise to historical time series data in which overlaps of individual curves are limited. Overlaps of individual curves in the cycle-adding method have inherent problematic aspects which do not truly reflect underlying factors that cause fluctuations in production history leading to multicyclic production behavior. In the newly proposed model here, each curve is described by the same three parameters as in the conventional Hubbert model, but every two curves are connected by an overlapping transition width. The transition width indicates the shift from one curve to the next and is defined by three parameters: transition year, transition width, and a weighting parameter where predicted production in the transition window is a weighted sum of the neighboring two curves. Historical coal production with the newly proposed method is modeled more reasonably and practically by eliminating the inherent problems due to cycle-addition as well as reflecting inherent asymmetry with finite width transitions being able to account for a substantial degree of asymmetry by weighting the two curves, potentially even pointing to explanatory aspects of the asymmetry in the respective weight of the abutting curves. Socioeconomic factors that cause fluctuation in historical coal production rates form the production trends and the shift to the next cycle is clearly reflected by the transition year and the width. Multicyclic Hubbert modeling with the cycle-jumping is mathematically superior when compared to the conventional multicyclic Hubbert model of cycle-addition as demonstrated through statistical tests. This improvement gives not only a better mathematical and historical accounting of the individual curves, but also illuminates how each curve transitioned from one to the next as economic factors change over time.

ACKNOWLEDGMENTS

This paper constitutes part of an MS thesis by B.N. B.N.’s MS degree study at Southern Illinois University was fully funded by the Fulbright scholarship program of the U.S. Department of State. We thank three anonymous reviewers for insightful comments that improved the quality of the manuscript. We thank Dr. Henson for the editorial handling. This is contribution #8 of the Missouri S&T MCTF research group.

AUTHOR CONTRIBUTIONS

B.N.’s MS degree study was supervised by K. B. A and J. A. C. B. N. drafted the manuscript. J. A. C., K. B. A., and M. L. all contributed to interpretation of the data and writing and editing the manuscript.

ORCID

Bolorchimeg N. Tunnell  http://orcid.org/0000-0003-4041-071X
Marek Locmelis  https://orcid.org/0000-0002-9328-0552

REFERENCES

Al-Fattah, S. M., & Startzman, R. A. (1999). Analysis of world natural gas production. In: SPE Eastern Regional Meeting. Society of Petroleum Engineers.
Al-Fattah, S. M., & Startzman, R. A. (2000). Forecasting world natural gas supply. *Journal of Petroleum Technology*, 52, 62–72.

Anderson, K. B., & Conder, J. A. (2011). Discussion of multicyclic Hubbert modeling as a method for forecasting future petroleum production. *Energy & Fuels*, 25, 1578–1584.

Bardi, U. (2005). The mineral economy: A model for the shape of oil production curves. *Energy Policy*, 33, 53–61.

Bartlett, A. A. (2000). An analysis of U.S. and world oil production patterns using Hubbert-style curves. *Mathematical Geology*, 32, 1–17.

BP. Statistical review of world energy-All data 1965–2019. Available at https://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html

Brandt, A. R. (2007). Testing Hubbert. *Energy Policy*, 35, 3074–3088.

Brandt, A. R. (2010). Review of mathematical models of future oil supply: Historical overview and synthesizing critique. *Energy*, 35, 3958–3974.

Burnham, K. P., & Anderson, D. R. (2004). Multimodel inference: Understanding AIC and BIC in model selection. *Sociological Methods & Research*, 33, 261–304.

Campbell, C. J., & Laherrère, J. H. (1998). The end of cheap oil. *Scientific American*, 278, 78–83.

Conder, J. A. (2001). Tectonics and plate boundary processes along the Southeast Indian Ridge and the East Pacific Rise (PhD), Brown University.

Conder, J. A. (2015). Fitting multiple bell curves stably and accurately to a time series as applied to Hubbert cycles or other phenomena. *Mathematical Geosciences*, 47, 663–678.

Ebrahimi, M., & Ghasabani, C. N. (2015). Forecasting OPEC crude oil production using a variant Multicyclic Hubbert Model. *Journal of Petroleum Science and Engineering*, 133, 818–823.

EIA. U.S. annual coal report by coal rank and mining method (1949–2019). Available at https://www.eia.gov/coal/data.php#production

Hubbert, M. K. (1956). Nuclear energy and the fossil fuel, *Drilling and Production Practice*. American Petroleum Institute.

Hubbert, M. K. (1959). Techniques of prediction with application to the petroleum industry. In: 44th Annual Meeting of the American Association of Petroleum Geologists. Shell Development Company Dallas, Texas.

Hubbert, M. K. (1980). Oil and gas supply modeling. *NBS Special Publication*, 631, 90.

Hubbert, M. K. (1982). Techniques of prediction as applied to the production of oil and gas. *NBS Special Publication 631*, Washington, DC: U.S. Department of Commerce

Höök, M., & Aleklett, K. (2009). Historical trends in American coal production and a possible future outlook. *International Journal of Coal Geology*, 78, 201–216.

Höök, M., Li, J., Oba, N., & Snowden, S. (2011). Descriptive and predictive growth curves in energy system analysis. *Natural Resources Research*, 20, 103–116.

Laherrère, J. H. (2000). The Hubbert curve: Its strengths and weaknesses. *Oil & Gas Journal*, 98, 63–76.

Maggio, G., & Cacciola, G. (2009). A variant of the Hubbert curve for world oil production forecasts. *Energy Policy*, 37, 4761–4770.

Maggio, G., & Cacciola, G. (2012). When will oil, natural gas, and coal peak? *Fuel*, 98, 111–123.

Milici, R. C., & Dennen, K. O. (2009). Production and depletion of Appalachian and Illinois basin coal resources. In B. S. Pierce & K. O. Dennen (Eds.), *The national coal resource assessment overview: US Geological Survey Professional Paper 1625-F* (p. 18).

Motulsky, H., & Christopoulos, A. (2004). *Fitting models to biological data using linear and nonlinear regression: A practical guide to curve fitting*. Oxford University Press.

Muir, D. A., & McIntosh, R. G. (1996). *Coal mining in Canada: A historical and comparative overview*. Ottawa: National Museum of Science and Technology.

Nashawi, I. S., Malallah, A., & Al-Bisharah, M. (2010). Forecasting world crude oil production using multicyclic Hubbert model. *Energy & Fuels*, 24, 1788–1800.

Page, G. T., & Shapiro, L. (2006). Coal in Canada. Retrieved from https://www.thecanadianencyclopedia.ca/en/article/coal

Patzek, T. W. (2008). Exponential growth, energetic Hubbert cycles, and the advancement of technology. *Archives of Mining Sciences*, 53, 131–159.

Patzek, T. W., & Croft, G. D. (2010). A global coal production forecast with multi-Hubbert cycle analysis. *Energy*, 35, 3109–3122.
Reaver, N. G., & Khare, S. V. (2014). Imminence of peak in US coal production and overestimation of reserves. *International Journal of Coal Geology, 131*, 90–105.

Reynolds, D. B. (2014). World oil production trend: Comparing Hubbert multi-cycle curves. *Ecological Economics, 98*, 62–71.

Rutledge, D. (2011). Estimating long-term world coal production with logit and probit transforms. *International Journal of Coal Geology, 85*, 23–33.

Saraiva, T. A., Szklo, A., Lucena, A. F. P., & Chavez-Rodriguez, M. F. (2014). Forecasting Brazil’s crude oil production using a multi-Hubbert model variant. *Fuel, 115*, 24–31.

Schweinfurth, S. P., & Finkelman, R. B. (2002). Coal—A complex natural resource: an overview of factors affecting coal quality and use in the United States. Report No. 1143.

Semenychev, V. K., Kurkin, E. I., & Semenychev, E. V. (2014). Modelling and forecasting the trends of life cycle curves in the production of non-renewable resources. *Energy, 75*, 244–251.

Sorrell, S., Miller, R., Bentley, R., & Speirs, J. (2010). Oil futures: A comparison of global supply forecasts. *Energy Policy, 38*, 4990–5003.

Sorrell, S., & Speirs, J. (2010). Hubbert’s legacy: A review of curve-fitting methods to estimate ultimately recoverable resources. *Natural Resources Research, 19*, 209–230.

Stein, S., & Gordon, R. G. (1984). Statistical tests of additional plate boundaries from plate motion inversions. *Earth and Planetary Science Letters, 69*, 401–412.

Wang, J., Bentley, Y., & Bentley, R. (2018). Modeling India’s coal production with a negatively skewed curve-fitting model. *Natural Resources Research, 27*, 365–378.

Wang, J., & Feng, L. (2016). Curve-fitting models for fossil fuel production forecasting: Key influence factors. *Journal of Natural Gas Science and Engineering, 32*, 138–149.

---

**How to cite this article:** Tunnell BN, Conder JA, Anderson KB, Locmelis M. A cycle-jumping method for multicyclic Hubbert modeling of resource production. *Natural Resource Modeling*. 2021;34:e12296. https://doi.org/10.1111/nrm.12296