We study the semileptonic decays $B \to X_s \ell^+ \ell^-$ in SUSY

$B \to X_s e^+ e^-$, $B \to X_s \mu^+ \mu^-$ in generic supersymmetric extensions of the Standard Model. SUSY effects are parameterized using the mass insertion approximation formalism and differences with MSSM results are pointed out. Constraints on SUSY contributions coming from other processes (e.g. $b \to s\gamma$) are taken into account. Chargino and gluino contributions to photon and $Z$-mediated decays are computed and non-perturbative corrections are considered. We find that the integrated branching ratios and the asymmetries can be strongly modified. Moreover, the behavior of the differential Forward-Backward asymmetry remarkably changes with respect to the Standard Model expectation.

---

Co-authors: Antonio Masiero, Ignazio Scimemi and Luca Silvestrini.
1 Introduction

In this work we want to investigate the relevance of new physics effects in the semileptonic inclusive decay $B \to X_s l^+ l^-$. This decay is quite suppressed in the Standard Model; however, new $B$-factories should reach the precision requested by the SM prediction and an estimate of all possible new contributions to this process is compelling.

Because of the presence of so many unknown parameters (in particular in the scalar mass matrices) it is very useful to adopt the so-called “Mass Insertion Approximation” (MIA). In this framework one chooses a basis for fermion and sfermion states in which all the couplings of these particles to neutral gauginos are flavor diagonal. Flavor changes in the squark sector are provided by the non-diagonality of the sfermion propagators which are expanded around these off-diagonal entries (Mass Insertions). A keen analysis of the different Feynman diagrams involved will allow us to isolate the few insertions really relevant for a given process. In this way we see that only a small number of the new parameters is involved and a general SUSY analysis is made possible.

We consider all possible contributions to charmless semileptonic $B$ decays coming from chargino-quark-squark and gluino-quark-squark interactions and we analyze both $Z$-boson and photon mediated decays. Contributions coming from penguin and box diagrams are taken into account; moreover, corrections to the MIA results due to a light $\tilde{l}_R$ are considered. A direct comparison between the SUSY and the SM contributions to the Wilson coefficients is performed. Once the constraints on mass insertions are established, we find that in generic SUSY models there still enough room in order to see large deviations from the SM expectations for branching ratios and asymmetries. For our final computation of physical observables we consider NLO order QCD evolution of the coefficients and non-perturbative corrections ($O(1/m_b^2), O(m_c^2), \ldots$), each in its proper range of the dilepton invariant mass.

2 General framework

We follow the notation and the conventions of ref. for what regards the effective Hamiltonian and the operator basis. With those definitions the differential branching ratio and the forward-backward asymmetry can be written as

$$R(s) \equiv \frac{1}{\Gamma(B \to X_s l^+ l^-)} \frac{d \Gamma(B \to X_s l^+ l^-)}{ds}$$

$$= \frac{\alpha_s}{4\pi^2} \frac{K_{ts}}{K_{cb}} \left( \frac{1-s}{f(z)k(z)} \right) \left[(1 + 2s)\left(\bar{C}_9^{\text{eff}}(s) + |C_{10}^{\text{eff}}|^2\right) + 4\left(1 + \frac{2}{s}\right)|C_7^{\text{eff}}|^2 + 12 \text{Re} \left[ C_7^{\text{eff}} \bar{C}_9^{\text{eff}}(s) \right] \right]$$

$$A_{FB}(s) \equiv \frac{\int_{-1}^{1} d\cos\theta \frac{d^2\Gamma(B \to X_s l^+ l^-)}{d\cos\theta ds} \text{Sgn}(\cos\theta)}{\int_{-1}^{1} d\cos\theta \frac{d^2\Gamma(B \to X_s l^+ l^-)}{d\cos\theta ds}}$$

$$= -\frac{3 \text{Re} \left[ \bar{C}_{10}^{\text{eff}}(s) \bar{C}_9^{\text{eff}}(s) + 2C_7^{\text{eff}} \right]}{(1 + 2s)\left(\bar{C}_9^{\text{eff}}(s) + |C_{10}^{\text{eff}}|^2\right) + 4\left(1 + \frac{2}{s}\right)|C_7^{\text{eff}}|^2 + 12 \text{Re} \left[ C_7^{\text{eff}} \bar{C}_9^{\text{eff}}(s) \right]}$$

where $s = (p_{l^+} + p_{l^-})^2/m_b^2$, $\theta$ is the angle between the positively charged lepton and the $B$ flight direction in the rest frame of the dilepton system, $f(z)$ and $k(z)$ can be found in refs.

In the literature the energy asymmetry is also considered but it is easy to show that these two kind of asymmetries are completely equivalent; in fact a configuration in the dilepton c.m.s. in which $l^+$ is scattered in the forward direction kinematically implies $E_{l^+} < E_{l^-}$ in the $B$ rest frame (see for instance ref.).

It is worth underlying that integrating the differential asymmetry given in eq. (2) we do not obtain
the global Forward–Backward asymmetry which is by definition:

\[
\frac{N(l^+_\rightarrow) - N(l^+_\leftarrow)}{N(l^+_\rightarrow) + N(l^+_\leftarrow)} = \frac{\int_{-1}^{1} \cos \theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d \cos \theta ds} \text{Sgn}(\cos \theta)}{\int_{-1}^{1} \cos \theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d \cos \theta ds}}
\]

where \(l^+_{\rightarrow}\) and \(l^+_{\leftarrow}\) stand respectively for leptons scattered in the forward and backward direction. To this extent it is useful to introduce the following quantity

\[
\bar{A}_{FB}(s) \equiv \frac{\int_{-1}^{1} \cos \theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d \cos \theta ds} \text{Sgn}(\cos \theta)}{\int_{-1}^{1} \cos \theta \int ds \frac{d^2\Gamma(B \rightarrow X_s l^+ l^-)}{d \cos \theta ds}} = A_{FB}(s) \frac{R(s)}{\int ds R(s)}
\]

whose integrated value is given by eq. (3).

3 Results

The genuine SUSY contributions to the coefficients in the most convenient case \((M_{sq} = M_{gl} = 250 \text{ GeV}, M_2 = 50 \text{ GeV}, \mu = -160 \text{ GeV}, \tan \beta = 2, M_\nu = 50 \text{ GeV}, M_{\tilde{t}_R} \simeq 70 \text{ GeV})\) are

\[
\begin{align*}
C^{MI}_{9}(M_B) &= -1.2(\delta_{23}^u)_{LL} + 0.69(\delta_{23}^u)_{LR} - 0.51(\delta_{23}^d)_{LL} \\
C^{MI}_{10}(M_B) &= 1.75(\delta_{23}^u)_{LL} - 8.25(\delta_{23}^d)_{LR}
\end{align*}
\]

where the \(\delta_s\) are the Mass Insertions normalized with \(M_{sq}^2\) (\(M_{sq}\) is the average squark mass).

The gluino contributions to \(C_7\) are large enough to completely fill the experimental constraints (coming from the measurement of \(BR(b \rightarrow s \gamma)\)):

\[
0.250 < |C^{eff}_7| < 0.445.
\]

Taking into account the experimental and theoretical limits on the \(\delta_s\) we compute the best enhancement and the best depression of the SM predictions for the BR and the asymmetries. In addition we compute the best enhancement compatible with the condition of keeping the SM sign of \(C^{eff}_7\).

The results we obtain are summarized in fig. [4] and in tab. [1]. The experimental best limit for the BR is\(BR_{exp} < 5.8 \times 10^{-5}\).

1. See, for instance, The BaBar Physics Book, SLAC-R-504 and references therein.
2. G. Buchalla and A. J. Buras, Nucl. Phys. B400 (1993), 225;
M. Misiak, Nucl. Phys. B383 (1993) 23, erratum ibid. B439 (1995) 461;
A.J. Buras and M. Münz, Phys. Rev. D52 (1995) 186.
3. G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125.
4. A. Ali, G. Hiller, L. T. Handoko and T. Morozumi, Phys. Rev. D55 (1997) 4105;
   A. Ali and G. Hiller, Phys. Rev D58 (1998) 071501 and Phys. Rev D58 (1998) 074001.
5. P. Cho, M. Misiak and D. Wyler, Phys. Rev. D54 (1996) 3329.
6. L. J. Hall, V. A. Kostolecki and S. Raby, Nucl. Phys. B267 (1986) 415.
7. F. Gabbiani and A. Masiero, Nucl. Phys. B322 (1989) 235.
8. N. Cabibbo and L. Maiani, Phys. Lett. B79 (1978) 109;
   C.S. Kim and A.D. Martin, Phys. Lett. B225 (1989) 186.
9. S. Glenn et al. (CLEO Collaboration), Phys. Rev. Lett. 80 (1998) 2289.
Table 1: Integrated BR, $A_{FB}$ and $\overline{A}_{FB}$ in the SM and in a general SUSY extension of the SM for the decays $B \to X_s e^+ e^-$ and $B \to X_s \mu^+ \mu^-$. The second and third columns are the extremal values we obtain with a positive $C_{7}^{\text{eff}}$ while the fourth one is the $C_{7}^{\text{eff}} < 0$ case. The actual numerical inputs for the various coefficients can be found in the text.

| Observable | SM maximal fraction | SM minimal fraction | SUSY maximal fraction | SUSY minimal fraction | SM (C7 < 0) | SUSY (C7 < 0) | SM fraction |
|------------|---------------------|---------------------|-----------------------|-----------------------|-------------|-------------|-------------|
| $BR(e)$    | $8.6 \times 10^{-6}$| $4.3 \times 10^{-5}$| $5.0$                 | $3.2 \times 10^{-6}$ | $0.37$      | $3.4 \times 10^{-5}$ | $3.9$       |
| $A_{FB}(e)$| $0.23$              | $0.34$              | $1.5$                 | $-0.22$               | $-0.95$     | $0.26$      | $1.1$       |
| $\overline{A}_{FB}(e)$ | $0.077$              | $0.24$              | $3.2$                 | $-0.12$               | $-1.6$      | $0.11$      | $1.5$       |
| $BR(\mu)$ | $5.8 \times 10^{-6}$| $3.8 \times 10^{-5}$| $6.5$                 | $1.4 \times 10^{-6}$ | $0.24$      | $2.8 \times 10^{-5}$ | $4.8$       |
| $A_{FB}(\mu)$ | $0.23$              | $0.34$              | $1.5$                 | $-0.22$               | $-0.95$     | $0.26$      | $1.1$       |
| $\overline{A}_{FB}(\mu)$ | $0.11$              | $0.27$              | $2.4$                 | $-0.17$               | $-1.5$      | $0.16$      | $1.4$       |

Figure 1: $R(s)$ [up-left], $A_{FB}(s)$ [up-right], $\overline{A}_{FB}$ for muons [down-left] and $\overline{A}_{FB}(s)$ for electrons [down-right]. The solid line corresponds to the SM expectation; the dashed and dotted–dashed lines correspond respectively to the SUSY best enhancement and depression; the dotted line is the maximum enhancement obtained without changing the sign of $C_{7}$. 