New $[SU(3)]^4$ Realization of Lepton/Dark Symmetry

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Abstract

Extending the well-known $SU(3)_C \times SU(3)_L \times SU(3)_R$ model of quarks and leptons to include a fourth $SU(3)_N$ gauge factor, a new realization is obtained, different from leptonic color, which contains a lepton/dark symmetry with the help of an input $Z_4$ symmetry. It is seen to encompass a previous extension of the standard model to $SU(2)_N$ lepton symmetry.
**Introduction**: It is well-known that the standard model (SM) of quarks and leptons may be embedded in $SU(5)$ with the fermions as $\mathbf{5}^*$ and $\mathbf{10}$ representations per family. Adding the right-handed neutrino, they form a single $\mathbf{16}$ representation of $SO(10)$. It is also well-known that this $\mathbf{16}$ may be embedded in the $\mathbf{27}$ of $E_6$. This last set of fermions has an interesting realization, using the maximal $SU(3)_C \times SU(3)_L \times SU(3)_R$ subgroup of $E_6$, i.e.

\begin{align}
q &\sim (3,3^*,1) \sim \begin{pmatrix} d \\ u \\ h \end{pmatrix}, \\
q^c &\sim (3^*,1,3) \sim \begin{pmatrix} d^c \\ d^c \\ d^c \end{pmatrix},
\end{align}

(1)

\begin{align}
f &\sim (1,3,3^*) \sim \begin{pmatrix} N_1^0 & E^c & \nu \\ E & N_2^0 & e \\ \nu^c & e^c & S^0 \end{pmatrix},
\end{align}

(2)

where

\[ Q = I_{3L} - \frac{1}{2} Y_L + I_{3R} - \frac{1}{2} Y_R. \]

(3)

The columns in the $3 \times 3$ fermion matrices denote 3 representations of $SU(3)$ with $(I_3,Y) = (1/2,1/3), (-1/2,1/3), (0,-2/3)$ from top to bottom. The rows denote $3^*$ representations with $(I_3,Y) = (-1/2,-1/3), (1/2,-1/3), (0,2/3)$ from left to right. The scalar

\[ \lambda_0 \sim (1,3,3^*) \sim \begin{pmatrix} \eta_1^0 & \eta_2^+ & \phi_L^0 \\ \eta_1^- & \eta_2^0 & \phi_L^+ \\ \phi_R^0 & \phi_R^+ & \zeta^0 \end{pmatrix}, \]

(4)

transforms as $f$, with allowed Yukawa couplings

\[ Tr[q\lambda_0 q^c], \quad \epsilon_{abc}\epsilon_{\alpha\beta\gamma}f_{ab}f_{c\beta}(\lambda_0)_{\alpha\gamma}. \]

(5)

To generalize the above, $[SU(N)]^k$ in a moose chain, i.e. fermions of the form $(N,N^*,1...1), (1,N,N^*,...1),...$ to $k$ copies, may be considered. In general, supersymmetric $[SU(N)]^k$ has the intriguing property that it is a finite field theory [1] with three families, for any $N$ or $k$. For $N = 3$ and $k = 4$, the fourth $SU(3)$ [2] [3] [4] [5] could be leptonic color [6] [7] or not [8]. For $N = 3$ and $k = 6$, the fermions of $[SU(3)]^3$ are separated by three additional $SU(3)$ factors [9] [10] to allow for chiral color [11] [12].
In this paper, a new choice of the fourth $SU(3)$ is studied for an $[SU(3)]^4$ model. It will be shown that it contains a previously proposed $SU(2)_N$ lepton symmetry. Furthermore, residual conserved global baryon number $B$ and lepton number $L$ may be defined, and dark symmetry is derivable from lepton symmetry, with vector gauge bosons in the dark sector.

**Model**: The gauge symmetry is $SU(3)_C \times SU(3)_L \times SU(3)_N \times SU(3)_R$. The fermions transform as

$$ q \sim (3, 3^*, 1, 1) \sim \begin{pmatrix} d & u & h \end{pmatrix}, \quad l \sim (1, 3, 3^*, 1) \sim \begin{pmatrix} N_1 & \nu & E^c_1 \\ E_1 & e & N^c_1 \end{pmatrix}, \quad l^c \sim (1, 1, 3^*) \sim \begin{pmatrix} N^c_2 & E^c_2 & S^c_2 \\ \nu^c & c^c & S^c_1 \\ E_2 & N_2 & E_0 \end{pmatrix}, \quad q^c \sim (3^*, 1, 1, 3) \sim \begin{pmatrix} d^c & d^c & d^c \\ h^c & h^c & h^c \end{pmatrix},$$

with the electric charge given by

$$ Q = I_{3L} - \frac{1}{2} Y_L + I_{3R} - \frac{1}{2} Y_R + Y_N. $$

Again, the columns in the $3 \times 3$ fermion matrices denote $3$ representations of $SU(3)$ with $(I_3, Y) = (1/2, 1/3), (-1/2, 1/3), (0, -2/3)$ from top to bottom. The rows denote $3^*$ representations with $(I_3, Y) = (-1/2, -1/3), (1/2, -1/3), (0, 2/3)$ from left to right. The $SU(2)_N$ of Refs. is clearly embedded in $l, l^c$.

The scalars transform as

$$ \lambda_0 \sim (1, 3, 1, 3^*) \sim \begin{pmatrix} \eta^0_1 & \eta^0_2 & \phi^0_L \\ \eta^-_1 & \eta^-_2 & \phi^-_L \end{pmatrix}, \quad \lambda_L \sim (1, 3, 3^*, 1) \sim \begin{pmatrix} \phi^0_2 & \phi^0_1 & \phi^+_3 \\ \phi^-_2 & \phi^-_1 & \phi^0_3 \\ \phi^-_2 & \phi^-_1 & \phi^+_3 \\ \lambda_2 & \lambda_1 & \zeta^+_L \end{pmatrix}, \quad \lambda_R \sim (1, 1, 3, 3^*) \sim \begin{pmatrix} \phi^0_4 & \phi^+_4 & \chi_4 \\ \phi^-_4 & \phi^-_5 & \chi_5 \\ \phi^-_6 & \phi^-_6 & \zeta^-_R \end{pmatrix}. $$

The allowed Yukawa terms are

$$ Tr[q^c q \lambda_0], \quad Tr[l^c l], \quad \epsilon_{abc} \epsilon_{\alpha\beta\gamma} l_{a\alpha} b_{\beta} (\lambda_L)_{c\gamma}, \quad \epsilon_{abc} \epsilon_{\alpha\beta\gamma} l^c_{a\alpha} b_{\beta} (\lambda_R)_{c\gamma}. $$
Two other scalars are added: $\lambda'_L \sim (1, 3, 3^*, 1)$ and $\lambda'_R \sim (1, 1, 3, 3^*)$, together with a $Z_4$ symmetry. Under $Z_4$,

\[
q, q^c, \lambda_0 \sim 1, \quad \lambda_{L,R} \sim -1, \quad l, \lambda'_L \sim i, \quad l^c, \lambda'_R \sim -i.
\] (12)

Hence Eq. (11) remains valid and $\lambda'_{L,R}$ do not couple to $l, l^c$. Allowed trilinear scalar couplings are $Tr[\lambda_0^\dagger \lambda_L \lambda_R]$ and $Tr[\lambda'_0 \lambda'_L \lambda'_R]$, $\epsilon_{abc} \epsilon_{a\beta\gamma}(\lambda_0)_{aa}(\lambda_0)_{b\beta}(\lambda_0)_{c\gamma}$, $\epsilon_{abc} \epsilon_{a\beta\gamma}(\lambda'_L)_{aa}(\lambda'_L)_{b\beta}(\lambda'_L)_{c\gamma}$, and $\epsilon_{abc} \epsilon_{a\beta\gamma}(\lambda'_R)_{aa}(\lambda'_R)_{b\beta}(\lambda'_R)_{c\gamma}$. Note that cubic terms of the form $(\lambda_L)^3$, $(\lambda_R)^3$, $(\lambda'_L)^3$, or $(\lambda'_R)^3$ are all forbidden by $Z_4$. The absence of these terms will lead to a residual lepton/dark symmetry as discussed in the next section.

**Residual B and L Symmetries**: Under $SU(3)_L \times SU(3)_N \times SU(3)_R$, the following neutral scalars have vacuum expectation values (VEVs), as shown in Table 1.

| Scalar  | $I_{3L}$ | $Y_L$  | $I_{3N}$ | $Y_N$  | $I_{3R}$ | $Y_R$  | VEV |
|---------|----------|--------|----------|--------|----------|--------|-----|
| $\zeta^0$ | 0        | $-2/3$ | 0        | 0      | 0        | $2/3$  | $u_0$          |
| $\eta_0^0$ | 1/2      | 1/3    | 0        | 0      | $-1/2$  | $-1/3$ | $v_1$          |
| $\eta_2$ | $-1/2$  | 1/3    | 0        | 0      | 1/2      | $-1/3$ | $v_2$          |
| $\phi^0_L$ | 1/2      | 1/3    | 0        | 0      | 0        | 2/3    | $v_L$          |
| $\phi^0_R$ | 0        | $-2/3$ | 0        | 0      | $-1/2$  | $-1/3$ | $v_R$          |
| $\chi_1$ | 0        | $-2/3$ | $1/2$    | $-1/3$ | 0        | 0      | $u_L$          |
| $\chi_5$ | 0        | 0      | $-1/2$  | $1/3$  | 0        | 2/3    | $u_R$          |
| $\phi^0_{3'}$ | $-1/2$  | 1/3    | 0        | 2/3    | 0        | 0      | $v_3$          |
| $\phi^0_{6'}$ | 0        | 0      | $-2/3$ | $1/2$  | $-1/3$  | 0      | $v_6$          |

Table 1: Scalars with vacuum expectation values.

From the allowed Yukawa couplings

\[
d^d d h^0, \quad u^c u h^0, \quad h^c h c^0, \quad d^e h \phi^0_L, \quad h^c d \phi^0_R,
\] (13)

it is seen that the $u$ quarks get masses from $v_2$, and the $d, h$ quarks get diagonal masses from $v_1$ and $u_0$, with mixing terms from $v_{L,R}$. The $l$ and $l^c$ fermions have the allowed Yukawa
couplings

\begin{align*}
(\nu \nu^c + N_1 N_2^c + E_1^c E_2) \eta^0_1, & \quad (e e^c + N_1^c N_2 + E_1 E_2^c) \eta^0_2, & \quad (S_1 S_1^c + S_2 S_2^c + E_0^c E_0) \zeta^0, \\
(\nu S_1^c + N_1 S_2^c + E_1^c E_0^c) \phi^0_L, & \quad (S_1 \nu^c + S_2 N_2^c + E_0^c E_2) \phi^0_R, \quad (E_1 E_1^c - N_1 N_1^c) \chi_1, & \quad (E_2 E_2^c - N_2 N_2^c) \chi_5.
\end{align*}

It is seen that the charged leptons get masses from \( v_2 \), whereas \( S_{1,2} \) and \( E_0 \) get masses from \( u_0, (N_1, E_1) \) from \( u_L \), \( (N_2, E_2) \) from \( u_R \), with mixing among them (except \( S_1 \)) from \( v_{L,R} \) as well as \( v_{1,2} \). The neutrinos have diagonal masses from \( v_1 \), but they also mix with \( S_1 \) from \( v_{L,R} \). Note that \( v_{3,6} \) are not involved in fermion masses because \( \lambda_{L,R}^c \) do not couple to \( l, l^c \).

From the above, two global residual symmetries may be defined. (1) Baryon number \( B = 1/3 \) for \( q \) and \( B = -1/3 \) for \( q^c \). (2) Lepton number \( L = 1 \) for \( \nu, e, S_1 \), and \( L = -1 \) for \( \nu^c, e^c, S_1^c \). Under \( L \),

\begin{align*}
l & \sim \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}, & \quad l^c & \sim \begin{pmatrix} 0 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_L & \sim \begin{pmatrix} -1 & 0 & -1 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{pmatrix}, & \quad \lambda_R & \sim \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{pmatrix}.
\end{align*}

The \( \lambda_0 \) scalars all have \( L = 0 \).

It is also clear that the complex vector gauge bosons in \( SU(3)_N \) which take the \( (1/2, -1/3) \) state under \( (I_{3N}, Y_N) \) to the \( (-1/2, -1/3) \) and \( (0, 2/3) \) states must have \( L = -1 \). For the \( \lambda_{L,R}^c \) scalars, \( \phi_3^0 \) and \( \phi_6^0 \) are required to have \( L = 0 \) for them to acquire VEVs. Together with the nonzero \( L \) assignments in the gauge sector, \( \lambda_{L,R}^c \sim l, l^c \) under \( L \) is then fixed. The input \( Z_4 \) symmetry is thus responsible for \( L \) conservation in this model including \( \lambda_{L,R}^c \).

If \( \lambda_{L,R}^c \) are absent, then the extra \( Z_4 \) symmetry is not necessary. Fermion masses are unaffected because they depend only on \( u_{0,L,R} \) and \( v_{1,2,L,R} \), but without \( v_{3,6} \), the symmetry
breaking of \( SU(3)_L \times SU(3)_N \times SU(3)_R \) would not be realistic. Specifically, \( v_3 \) breaks \( SU(3)_N \) and \( SU(2)_L \), and \( v_6 \) breaks \( SU(3)_N \) and \( SU(2)_R \).

**Gauge Sector**: The \( SU(3)_C \) gauge factor contains the gluon octet. Each of the other \( SU(3) \) factors contains eight vector gauge bosons, i.e.

\[
\frac{g}{2} \begin{pmatrix}
W_3 + W_8/\sqrt{3} & W_1 - iW_2 & W_4 - iW_5 \\
W - 1 + iW_2 & -W_3 + W_8/\sqrt{3} & W_6 - iW_7 \\
W_4 + iW_5 & W_6 + iW_7 & -2W_8/\sqrt{3}
\end{pmatrix}.
\]

Of the nine VEVs, four \( (v_{1,2,3,L}) \) contribute to the mass of \( W_{3L} \). They must be small compared to the other five VEVs, from which four of the five vector fields \( (W_{3N}, W_{3R}, W_{8L}, W_{8N}, W_{8R}) \) obtain mass. If \( v_6 \) is missing, only three would do so. As it is, one linear combination is the analog of the \( U(1)_Y \) gauge boson of the SM and would get a mass from \( v_{1,2,3,L} \). It mixes with \( W_{3L} \) to form the photon and the SM \( Z \) boson in the usual way. In the limit \( g_L = g_R = g_N \), this state is given by \( (\sqrt{3}W_{3R} + 2W_{8N} - W_{8L} - W_{8R})/3 \).

Under \( SU(3)_N \), the gauge boson \( (W_{1N} - iW_{2N})/\sqrt{2} \) has \( L = 1 \) and \( Q = 0 \), \( (W_{4N} - iW_{5N})/\sqrt{2} \) has \( L = 0 \) and \( Q = 1 \), and \( (W_{6N} - iW_{7N})/\sqrt{2} \) has \( L = -1 \) and \( Q = 1 \). Their respective masses are \( g_N \sqrt{(u_L^2 + u_R^2)/2} \), \( g_N \sqrt{(v_3^2 + v_6^2)/2} \), and \( g_N \sqrt{(u_L^2 + u_R^2 + v_3^2 + v_6^2)/2} \). To summarize,

\[
L(W_N) \sim \begin{pmatrix}
0 & 1 & 0 \\
-1 & 0 & -1 \\
0 & 1 & 0
\end{pmatrix}, \quad Q(W_N) \sim \begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 1 \\
-1 & -1 & 0
\end{pmatrix}, \quad (20)
\]

whereas

\[
L(W_{L,R}) \sim 0, \quad Q(W_{L,R}) \sim \begin{pmatrix}
0 & 1 & 0 \\
-1 & 0 & -1 \\
0 & 1 & 0
\end{pmatrix}. \quad (21)
\]

**Dark Sector**: With the conservation of lepton number \( L \) as defined in the previous sections, a dark parity may be derived \([15]\), i.e. \( \pi_D = (-1)^{L+2j} \). This means that \( \nu, \epsilon, S_1 \) are even, but \( (N, E)_{1,2}, S_2, E_0 \) are odd. The scalars in \( \lambda_0 \) are even, together with \( \Phi_1, \chi_1, \Phi_5, \chi_5 \), in \( \lambda_{L,R} \), whereas \( \Phi_2, \chi_2, \Phi_3, \Phi_4, \chi_4, \Phi_6, \zeta^\pm \) are odd. The scalars in \( \lambda'_{L,R} \) have the opposite \( \pi_D \) as
the ones in $\lambda_{L,R}$. The vector gauge bosons are all even except for $(W_{1N} \pm iW_{2N})/\sqrt{2}$ and $(W_{6N} \pm iW_{7N})/\sqrt{2}$ which are odd. The neutral $(W_{1N} \pm iW_{2N})/\sqrt{2}$ may be dark matter and is analogous to the $X$ boson in Ref. [14]. Similarly, a linear combination of $\chi_{1,5}$ is analogous to the $H$ scalar boson discussed there. The $(N_1, E_1)$ and $(N_2, E_2)$ fermions are analogous to $(N, E)$ and $(N', E')$ respectively. The dark-matter phenomenology is thus the same.

**Phenomenology of $SU(3)_N$:** The breaking of $SU(3)_{L,R}$ to $SU(2)_{L,R}$ is assumed to be at a high scale. The subsequent breaking of $SU(2)_R$ is also assumed to be high. These may be accomplished by $u_0$ and $v_R$ as shown in Table 1. The $SU(3)_N$ breaking to $SU(2)_N$ is through $v_6$ which also breaks $SU(2)_R$. Finally, $SU(2)_N$ breaking is through $u_{L,R}$. This chain allows the neutral vector gauge boson $X = (W_{1N} - iW_{2N})/\sqrt{2}$ to be dark matter as in Ref. [14].

The charged leptons have interactions with the $W_{L,R,N}$ gauge bosons as well as the new fermions and scalars contained in $l, l^c$ and $\lambda_{0,L,R}$. This means that there are many possible one-loop contributions to the muon anomalous magnetic moment, for example. For a study restricted to the simplified $SU(2)_N$ sector, see Ref. [14].

Since the $SU(3)_N$ gauge bosons couple only to $l, l^c$ fermions and $\lambda_{L,R}$, $\lambda'_{L,R}$ scalars, they are not easily produced. The highest energy of the $e^+e^-$ LEP II collider was 209 GeV. Hence the $W_{3N}$ and $W_{8N}$ bosons should be heavier than this value.

There are three $SU(2)_L$ scalar doublets $\Phi_{1,2,3}$ in $\lambda_L$ and three $SU(2)_R$ scalar doublets $\Phi_{4,5,6}$ in $\lambda_R$. They have different $L$ values as shown in Eq. (17), and are connected by $(W_{1N} \pm iW_{2N})/\sqrt{2}$, $(W_{4N} \pm iW_{5N})/\sqrt{2}$, and $(W_{6N} \pm iW_{7N})/\sqrt{2}$. A similar pattern exists also for $\lambda'_{L,R}$. Hence this model predicts many more scalars beyond the lone Higgs boson of the SM.

**Concluding Remarks:** A new realization of $[SU(3)]^4$ gauge symmetry is proposed, embedding the SM quarks and leptons as shown in Eqs. (6) and (7). The new $SU(3)_N$ symmetry has a neutral $SU(2)_N$ subgroup which identifies with the non-Abelian lepton symmetry pro-
posed before [13, 14]. It is shown how all fermions in $q, q^c, l, l^c$ may acquire mass with the breaking of $[SU(3)]^4$ to the SM gauge symmetry, then to $SU(3)_C \times U(1)_Q$. With the help of a $Z_4$ symmetry which applies to $q, q^c, l, l^c$ fermions and the $\lambda_{0,L,R}, \lambda'_{L,R}$ scalars, it is shown that two conserved residual symmetries remain. One is the usual baryon number $B$; the other is generalized lepton number $L$, as shown in Eqs. (16), (17), (19), and (20). Hence two complex vector gauge bosons (one neutral and one charged) have $L \neq 0$. The former may be dark matter, as discussed in Ref. [14], with dark parity $\pi_D = (-1)^{L+2j}$. As most presumed candidates of dark matter are either scalar or fermion, this possibility should not be overlooked.

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