Analytic theory of discontinuities in current-carrying cosmic strings

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Abstract

We formulate an analytic method to study the discontinuities in superconducting cosmic strings. Equations of discontinuities and conditions of their existence are derived from the intrinsic and extrinsic equations of motion. It is the fundamental for research of particular solutions, associated with kinks, cusps and shocks.

1 Introduction

Cosmic strings are 1+1-dimensional topological defects which are believed to be formed in the era of cosmological phase transitions and which are responsible for the global structure of the Universe [1]. Although their experimental recognition is not evident, the physical presence of cosmic strings is reflected in several astrophysical phenomena associated with gravitational lensing, gravitational waves, particle acceleration, cosmic microwave background and gamma-ray bursts.

When the curvature of a string is small with respect to its characteristic width (core), the surface Lagrangian $\Lambda$ is determined through integration over extrinsic coordinates. The starting point is the Goto-Nambu model $\Lambda = -m^2$, and its extension is required when a cosmic string acquires non-trivial internal structure, associated with actual particles (bosons or fermions) which are trapped in the defect core [2]. The Lagrangian of this string $\Lambda = \Lambda(\chi)$
becomes dependent on the magnitude of the current $\chi = -\partial^a \psi \partial_a \psi$ where the gradient is taken over the worldsheet coordinates and the phase $\psi$ originates from the wave function of the field, condensed in the string core.

The equations of motion of such current-carrying or "superconducting" string include a pair of "intrinsic" equations \[ \eta_\mu^\nu \nabla_\nu (\mu v^\mu) = 0 \] (1)

\[ \eta_\mu^\nu \nabla_\nu (n u^\mu) = 0 \] (2)

and a pair of "extrinsic" equations

\[ \perp^\mu_\rho (u^\nu \nabla_\nu v^\rho - v^\nu \nabla_\nu u^\rho) = 0 \] (3)

\[ \perp^\mu_\rho (U u^\nu \nabla_\nu u^\rho - T v^\nu \nabla_\nu v^\rho) = 0 \] (4)

where

\[ \eta^\nu_\mu = v^\nu v^\mu - u^\nu u^\mu \quad \perp^\nu_\mu = g^\nu_\mu - \eta^\nu_\mu \] (5)

are the parallel and orthogonal projective tensors, composed of mutually orthogonal time-like and space-like unit vectors

\[ u^\mu v_\mu = -v^\mu v_\mu = -1 \quad u^\mu v_\mu = 0 \] (6)

and quantities $U, T, \mu = dU/dn, n = -dT/d\mu$ are dependent on the current $\chi$ and obey relations $\mu^2 = K^2 n^2 = \bar{K}^{-2} n^2$ at space-like currents ("magnetic" regime) and $\mu^2 = \bar{K}^{-2} n^2 = K^2 n^2$ at time-like currents ("electric" regime) where parameter $K = \bar{K}^{-1} = 2d\Lambda/d\chi$ is determined by explicit functional dependence $\Lambda (\chi)$ or $\mu (n)$, called as equation of state (EOS) \[4\].

The "intrinsic" equations (1)-(2) admit infinitesimal perturbations within the worldsheet, which are similar to longitudinal waves, propagating at the speed \[ c^2_L = -\frac{dT}{dU} = \frac{n}{\mu} \frac{d\mu}{dn} \] (7)

The "extrinsic" equations (3)-(4) admit infinitesimal perturbations of the string worldsheet, which are similar to transversal waves, propagating at the speed

\[ c^2_E = \frac{T}{U} \] (8)

Finite-amplitude perturbations of the string worldsheet are known as kinks and cusps \[5\] \[6\] \[7\] \[8\]. A kink is known as abrupt changes of the
The curvature of the string $\kappa$. A cusp is another type of geometric phenomenon when a part of the string is inflected and doubled on itself, moving at the speed of light. Most research of these perturbations is devoted to the Goto-Nambu strings. The current-carrying strings may admit not only kinks and cusps but a principally new class of perturbations called as shocks, predicted earlier [9], investigated numerically [10, 11] and explained analytically [12]. In contrast to the kinks and cusps, the shocks do not deal with the string geometry and occur within the worldsheet. A shock originates from a longitudinal (or ”sound”) wave [7] when its amplitude grows up and the current $\chi$ becomes discontinuous that can be described by explicit formulas [12]. The kink formation in superconducting string is studied numerically [10, 11] but no analytic formula has been proposed for explanation of its physical nature.

The finite-amplitude perturbations may play important role in the evolution of closed string configurations (vortons) and string networks, they can be responsible for various observable effects [13, 14, 15, 16]. The structures, triggered by finite-amplitude variations of the current, may also appear during reconnection or self-intersection of the string loops, when the current increases beyond the range of the vorton stability, that in all cases will result in visible radiation events. Although a string with finite current ($\chi \neq 0$) may reveal qualitatively new properties, which are absent in the chiral case ($\chi = 0$), a relationship between perturbations of the current and the geometry can be scarcely known without numerical computation. It is still unknown whether the kink velocity depends on the jump of the current $\Delta \chi$, how it is dependent on the jump of the curvature $\Delta \kappa$, can a shock produce changes in the curvature, which perturbations of the curvature are possible besides the kinks, and which perturbations of the current are possible besides the shocks?

Indeed, a universal method at the analytic level is highly desirable but it seems to be extremely complicated, almost like the numerical solution. Although the kinks, cusps and shocks are no more than discontinuous solutions of the equations of motion (1)-(4), and the relevant mathematical theory is developed [17] but it is not easy to adjust it to the string equations of motion. It is the main difficulty and it is the purpose of the present paper. We derive the basic equations of discontinuities in the current-carrying cosmic strings and outline several ideas for their further analysis. Particular solutions and discussions will be given in separate study.
2 Discontinuities in differential equations

The general theory of discontinuous solution of ordinary differential equations and its applications are well known [17]. In application to our problem, the front of a discontinuity is a hypersurface in 4-dimensional space whose equation is given by a scalar function \( \zeta(x) = 0 \), which acquires distinct values \( \zeta_+ \neq \zeta_- \) for the states before and behind the front, labeled by ”−” and ”+”.

As a result, an arbitrary tensor distribution \( \Omega = \Omega_{\alpha... \beta...} \) may reveal a jump \( \Omega^+ \equiv \Omega(x, \zeta^+) \neq \Omega^- \equiv \Omega(x, \zeta^-) \), which depends on the direction of the unit space-like characteristic vector

\[
\lambda_{\mu} = \frac{d\zeta}{dx^\mu} \quad \lambda^\mu \lambda_\mu = 1
\]  

A numerical analysis of the equations of motion can be applied to the process of birth, growth (or decay) and stable phase of discontinuities. The analytic analysis, developed in the present paper, is applied to the stable discontinuities, characterized by a stable front with constant characteristic vector

\[
\lambda_{\mu} = \lambda_{+\mu} = \lambda_{-\mu}
\]  

Appearance of a discontinuity implies that the covariant derivative of an arbitrary tensor \( \Omega(x, \zeta) \) is presented in the extended form [17]

\[
\nabla_\nu \Omega^+ = \nabla_\nu \Omega^- + \lambda_\nu [D_{\lambda} \Omega]
\]  

where differentiation along the characteristic direction is included in square brackets:

\[
[D_{\lambda} \Omega] \equiv \lambda^\mu [\nabla_\mu \Omega] = \lambda^\mu \frac{\partial \Omega}{\partial \zeta} \frac{d\zeta}{dx^\mu} = \frac{d\Omega}{d\zeta}
\]

We expect no gravitational wave and consider the same background metric \( g_{\mu\nu} = g_{+\mu\nu} = g_{-\mu\nu} \) at both sides of the discontinuity. Coming to finite increment \( d\zeta \rightarrow \zeta_+ - \zeta_- \), in the frames of linear approximation, we write

\[
\lambda_\nu [D_{\lambda} \Omega] = \lambda_\nu \frac{\Omega^+ - \Omega^-}{\zeta_+ - \zeta_-}
\]

If tensor \( \Omega \) obeys the equation of motion

\[
\nabla_\nu \Omega = 0
\]
formula (11) implies $\lambda\nu [D_\lambda \Omega] = 0$. Operating with arbitrary $\Delta \zeta = \zeta_+ - \zeta_-$ in (13), we derive the equation of discontinuity

$$\lambda\nu (\Omega_+ - \Omega_-) = 0$$

(15)

It states that variables $\Omega_+$ and $\Omega_-$ may differ but their projection unto the characteristic direction $\lambda\nu$ must coincide at both sides of the discontinuity. The acoustic limit $\Omega_+ \to \Omega_-$ corresponds to identical states before and behind the front when the discontinuity decays.

Tensor $\Omega$ may obey a more sophisticated equation of motion

$$\Xi \nabla_\nu \Omega = 0$$

(16)

with dual tensor $\Xi = \Xi_\rho^{\nu \cdots \nu}$. Substituting formula (11) in equation (16), we have

$$0 = \Xi_+ \nabla_\nu \Omega_+ = \Xi_+ \nabla_\nu \Omega_+ + \lambda\nu \Xi_+ [D_\lambda \Omega]$$

(17)

$$0 = \Xi_- \nabla_\nu \Omega_- = \Xi_- \nabla_\nu \Omega_- - \lambda\nu \Xi_- [D_\lambda \Omega]$$

(18)

The terms in square brackets in the right side of (17)-(18) yield the equation of discontinuity

$$\lambda\nu \Xi_+ (\Omega_+ - \Omega_-) = \lambda\nu \Xi_- (\Omega_+ - \Omega_-) = 0$$

(19)

under additional condition

$$\Xi_+ \nabla_\nu \Omega_- = \Xi_- \nabla_\nu \Omega_+ = 0$$

(20)

If $\Xi$ is a vector or a scalar, constraint (20) provides the ultimate possibility $\Xi_+ = C\Xi_-$ with scalar constant $C \neq 0$. In general, we must test each solution of equation (19) whether it satisfies requirement (20).

We may also deal with a multi-component system, which includes a set of tensors $\Omega_i$ (and $\Xi_i$), and the equation of motion in the form

$$\Xi_i \nabla_\nu \Omega_i = 0$$

(21)

where summation over index $i$ is performed. Equations (19)-(20), then, are written so

$$\lambda\nu \Xi_{+i} (\Omega_{+i} - \Omega_{-i}) = 0 \quad \lambda\nu \Xi_{-i} (\Omega_{+i} - \Omega_{-i}) = 0$$

(22)

and

$$\Xi_{+i} \nabla_\nu \Omega_{-i} = \Xi_{-i} \nabla_\nu \Omega_{+i} = 0$$

(23)
After all, we should bear in mind an important principle of resolvability [18]. When we are looking for solution of differential equations in the form of discontinuity, any initial perturbation is defined by some number of free parameters. Its evolution is governed by a set of boundary conditions, corresponding to the equations of motions. A perturbation can exist in a stable form if the number of these equations coincides with the number of free parameters in them. If there are too many parameters, the solution is undefined; if there are only few parameters, the system is unresolved. There may also exist a degenerate solution which automatically satisfies all the equations without regard of the free parameters.

3 Discontinuities in cosmic strings

Let us apply the common ideology of discontinuities (16)-(23) to the intrinsic and extrinsic equations of motion of a cosmic string. Applying formula (17) to equation (1), we have

$$\eta^\nu_{\mu+} \nabla_\nu (n^\mu_+ - u^\mu_+) = \eta^\nu_{\mu-} \nabla_\nu (n^\mu_- + u^\mu_-) + \lambda_\nu \eta^\nu_{\mu+} [D_\lambda (nu^\mu)]$$

(24)

so that the equation of discontinuity (19) is written in the form

$$\lambda_\nu \eta^\nu_{\mu+} (n^\mu_+ - n^\mu_-) = \lambda_\nu \eta^\nu_{\mu-} (n^\mu_- + n^\mu_+) = 0$$

(25)

and it is valid under condition

$$\eta^\nu_{\mu+} \nabla_\nu (n^\mu_-) = \eta^\nu_{\mu-} \nabla_\nu (n^\mu_+) = 0$$

(26)

Applying formula (17) to intrinsic equation of motion (2), we have

$$\eta^\nu_{\mu+} \nabla_\nu (\mu^\nu_+ v^\mu_+) = \eta^\nu_{\mu-} \nabla_\nu (\mu^\nu_- v^\mu_-) + \lambda_\nu \eta^\nu_{\mu+} [D_\lambda (\mu v^\mu)]$$

(27)

that results in the equation of discontinuity

$$\lambda_\nu \eta^\nu_{\mu+} (\mu^\nu_+ - \mu^\nu_-) = \lambda_\nu \eta^\nu_{\mu-} (\mu^\nu_- - \mu^\nu_+) = 0$$

(28)

which is valid under condition

$$\eta^\nu_{\mu+} \nabla_\nu (\mu^\nu_-) = \eta^\nu_{\mu-} \nabla_\nu (\mu^\nu_+) = 0$$

(29)
Discontinuities in extrinsic equations are determined by the same method. Defining expressions

\[ \Xi_1 = \perp_{\rho} v^\nu \quad \Omega_1 = u^\rho \quad \Xi_2 = - \perp_{\rho} u^\nu \quad \Omega_2 = v^\rho \]  

we put them in equation (3) and, applying formula (22), we obtain equations

\[ \lambda_\nu \perp_{\rho} \left( u^\nu u^\rho - u^\nu v^\rho \right) = 0 \]  
\[ \lambda_\nu \perp_{-\rho} \left( u^\nu u^\rho - u^\nu v^\rho \right) = 0 \]

which are valid under condition (20) that is

\[ \perp_{\rho} \left( u^\nu \nabla_\nu u^\rho - u^\nu \nabla_\nu v^\rho \right) = 0 \quad \perp_{-\rho} \left( v^\nu \nabla_\nu u^\rho - u^\nu \nabla_\nu v^\rho \right) = 0 \]  

Defining expressions

\[ \tilde{\Xi}_1 = U \perp_{\rho} u^\nu \quad \tilde{\Omega}_1 = u^\rho \quad \tilde{\Xi}_2 = -T \perp_{\rho} v^\nu \quad \tilde{\Omega}_2 = v^\rho \]

we put then in equation (4) and, applying formula (22), we obtain equations

\[ \lambda_\nu \perp_{+\rho} \left( u^\nu u^\rho - c^2 E u^\rho v^\rho \right) = 0 \]  
\[ \lambda_\nu \perp_{-\rho} \left( u^\nu u^\rho - c^2 E v^\rho v^\rho \right) = 0 \]

which are valid under condition (23) that is

\[ \perp_{+\rho} \left( u^\nu \nabla_\nu u^\rho - c^2 E u^\rho \nabla_\nu v^\rho \right) = 0 \quad \perp_{-\rho} \left( v^\nu \nabla_\nu u^\rho - c^2 E v^\rho \nabla_\nu v^\rho \right) = 0 \]

\[ \tilde{\Xi}_1 = U \perp_{\rho} u^\nu \quad \tilde{\Omega}_1 = u^\rho \quad \tilde{\Xi}_2 = -T \perp_{\rho} v^\nu \quad \tilde{\Omega}_2 = v^\rho \]

we put then in equation (4) and, applying formula (22), we obtain equations

\[ \lambda_\nu \perp_{+\rho} \left( u^\nu u^\rho - c^2 E u^\rho v^\rho \right) = 0 \]  
\[ \lambda_\nu \perp_{-\rho} \left( u^\nu u^\rho - c^2 E v^\rho v^\rho \right) = 0 \]

which are valid under condition (23) that is

\[ \perp_{+\rho} \left( u^\nu \nabla_\nu u^\rho - c^2 E u^\rho \nabla_\nu v^\rho \right) = 0 \quad \perp_{-\rho} \left( v^\nu \nabla_\nu u^\rho - c^2 E v^\rho \nabla_\nu v^\rho \right) = 0 \]

\[ \lambda_\nu \perp_{+\rho} \left( u^\nu u^\rho - c^2 E u^\rho v^\rho \right) = 0 \]  
\[ \lambda_\nu \perp_{-\rho} \left( u^\nu u^\rho - c^2 E v^\rho v^\rho \right) = 0 \]

\[ \perp_{+\rho} \left( u^\nu \nabla_\nu u^\rho - c^2 E u^\rho \nabla_\nu v^\rho \right) = 0 \quad \perp_{-\rho} \left( v^\nu \nabla_\nu u^\rho - c^2 E v^\rho \nabla_\nu v^\rho \right) = 0 \]

4 Characteristic vector

We are still unable to solve the equations until we include an explicit expression of the space-like characteristic vector (9) and a link between the physical parameters before and behind the front. Let us present the characteristic vector as a sum

\[ \lambda_\mu^\pm = \alpha_\mu^\pm + \sigma_\mu^\pm \]

of longitudinal component

\[ \alpha_\mu^\pm = \eta_\pm^\mu \lambda_\mu^\nu = \eta_\pm^\mu \alpha_\mu^\nu \]
and transversal component
\[
\sigma_\pm^\mu = \perp_{\pm \nu}^\mu \frac{\alpha_\pm^\nu}{1 - w_\pm^2}
\]
where
\[
\alpha_\pm^2 \equiv \alpha_\pm^\mu \alpha_\pm^\mu = 1 - \sigma_\pm^\mu \sigma_\pm^\mu \equiv 1 - \sigma_\pm^2 \leq 1
\]
(41)
The components (39) and (40) are mutual orthogonal since
\[
\alpha_\pm^\mu + \sigma_\pm^\mu = \alpha_\pm^\mu - \sigma_\pm^\mu = 0
\]
and
\[
\perp_{\pm \nu}^\mu \alpha_\pm^\nu = \eta_{\pm \mu}^\nu \sigma_\pm^\nu = 0 \quad \eta_{\pm \mu}^\nu \perp_{\pm \mu}^\nu = 0
\]
(42)
according to the properties of projective tensor (5). It is also clear that the transversal component (40) is orthogonal to each worldsheet vector \( \sigma_\pm^\mu u_\pm^\mu = \sigma_\pm^\mu v_\pm^\mu = 0 \).

The space-like longitudinal component (39) can be presented as a linear combination of the worldsheet vectors
\[
\alpha_\pm^\mu = \alpha_\pm^\mu \frac{w_\pm^\mu u_\pm^\mu + v_\pm^\mu}{\sqrt{1 - w_\pm^2}}
\]
(43)
so that formula (39) implies
\[
\lambda_{\pm \nu} u_\pm^\nu = -\frac{\alpha_\pm^\mu w_\pm^\mu}{\sqrt{1 - w_\pm^2}} \quad \lambda_{\pm \nu} v_\pm^\nu = \frac{\alpha_\pm^\mu}{\sqrt{1 - w_\pm^2}}
\]
(44)
where \( w_- \) and \( w_+ \) are the velocities before and behind the front, in the preferred reference frame, co-moving the discontinuity. When we operate in the laboratory reference frame, the front of the discontinuity propagates at velocity \( W \) with respect to the string and there is no motion before the discontinuity \( W_- = 0 \) but we do not exclude a possibility of motion behind the front \( W_+ \neq 0 \) (as a result of the discontinuity). When we switch to the co-moving reference frame, the front is at rest \( (W - W \equiv 0) \), and we operate with finite flow before the front \( w_- = -W \) and finite flow behind the front \( w_+ = (W_+ - W) / (1 - W w_+) \). Particularly, \( W_+ = W_- = 0 \) corresponds to \( w_+ = w_- \).

5 Equations of extrinsic and intrinsic discontinuities

Consider the equations of extrinsic discontinuities (31), (32), (35) and (36). By means of (5) and (17) we get expressions
\[
\perp_{\pm \rho}^\mu v_-^\rho = v_-^\mu - X v_+^\mu + G u_+^\mu \quad \perp_{\pm \rho}^\mu u_-^\rho = u_-^\mu - H v_+^\mu + Y u_+^\mu
\]
(45)
\( \mu \rho \nu \rho \mu v = v^\rho_\nu - X v^\rho_\mu + Hu^\rho_\mu \quad \mu \rho \mu u^\rho = u^\rho_\nu - G v^\rho_\mu + Y u^\rho_\mu \) (46)

where

\[
G = v^\mu_\nu u_\mu \\
H = u^\mu_\nu v_\mu \\
X = v^\mu_\nu v_\mu \\
Y = u^\mu_\nu u_\mu
\] (47)

Substituting formulas (44) and (45)-(46) in equations (31)-(32) and (35)-(36), we have

\[
\alpha + \{ u^\mu_\nu - H v^\mu_\nu + Y u^\mu_\nu + w_+ (v^\mu_\nu - X v^\mu_\nu + G u^\mu_\nu) \} = 0 \quad (48)
\]

\[
\alpha - \{ u^\mu_\nu - G v^\mu_\nu + Y u^\mu_\nu + w_- (v^\mu_\nu - X v^\mu_\nu + H u^\mu_\nu) \} = 0 \quad (49)
\]

\[
\alpha + \{ -w_+ (u^\mu_\nu - H v^\mu_\nu + Y u^\mu_\nu) - c^2 E_+ (v^\mu_\nu - X v^\mu_\nu + G u^\mu_\nu) \} = 0 \quad (50)
\]

\[
\alpha - \{ -w_- (u^\mu_\nu - G v^\mu_\nu + Y u^\mu_\nu) - c^2 E_- (v^\mu_\nu - X v^\mu_\nu + H u^\mu_\nu) \} = 0 \quad (51)
\]

Equations of intrinsic discontinuities are obtained by substituting formula (39) in equations (25)-(28), namely

\[
\alpha_+ u^\mu_\nu = n_+ u^\mu_\nu - n_- u^\mu_\nu = 0 \quad \alpha_- u^\mu_\nu = n_+ u^\mu_\nu - n_- u^\mu_\nu = 0 \quad (52)
\]

\[
\alpha_+ v^\mu_\nu = \mu_+ v^\mu_\nu - \mu_- v^\mu_\nu = 0 \quad \alpha_- v^\mu_\nu = \mu_+ v^\mu_\nu - \mu_- v^\mu_\nu \quad (53)
\]

Taking into account definition (43) and coefficients (47), we immediately obtain

\[
-n_+ \alpha_+ w_+ = n_- \alpha_+ (H + Y w_+) \quad n_+ \alpha_- (G + Y w_-) = -n_- \alpha_- w_- \quad (54)
\]

\[
\mu_+ \alpha_+ = \mu_- \alpha_+ (X + G w_+) \quad \mu_+ \alpha_- (X + H w_-) = \mu_- \alpha_- \quad (55)
\]

### 6 Classes of discontinuities

As we have mentioned above, existence of a stable discontinuous solution is possible when the number of equations of motion coincide with the number of free parameters. A cosmic string is described by four equations of motion. However, the total system of four equations is split into the intrinsic pair (1)-(2) and extrinsic pair (3)-(4), where intrinsic and extrinsic motions are considered separately \[3, 10, 11\]. Therefore, we may expect a 4-parametric
perturbation, which is decomposed in an sum of intrinsic and extrinsic discontinuities. An extrinsic discontinuity is described by 2 extrinsic equations with 2 parameters. An intrinsic discontinuity is described by 2 intrinsic equations with 2 parameters.

The first free parameter is the velocity of the discontinuity $W$ which is determined by the physical state of the string. The second parameter is the increment of the current $\Delta \chi$ or the increment of the curvature $\Delta \kappa$ (but simultaneous change of the current and the curvature is not admitted in a 2-parametric discontinuity). The previous analytical analysis of infinitesimal perturbations have revealed existence of intrinsic perturbations of the current and extrinsic perturbations of the curvature. Intrinsic perturbations of the curvature and extrinsic perturbations of the current do not exist. We cannot expect their appearance at finite amplitude of the perturbation because this threshold amplitude becomes the 3rd parameter which is forbidden. [10, 11]

If we suppose that intrinsic and extrinsic perturbations can form a composite discontinuity it will be a 3-parametric discontinuity in 4-folded system of equation that is also forbidden. Therefore, intrinsic and extrinsic discontinuities cannot co-exist in the same solution.

As for the degenerate discontinuity, the evident trivial solution

$$\alpha_+ = \alpha_- = 0$$

satisfies automatically both the extrinsic equations (48)-(51) and the intrinsic equations (54)-(55). No change of the curvature and no change of the current is incorporated in this discontinuity because all these parameters remain undetermined since the equations are satisfied automatically. A degenerate discontinuity must propagate at constant velocity (independent on the physical state of the string and its curvature) that can correspond to zero or to the speed of light. Note that equality (56) implies that the characteristic vector (40) is orthogonal to the string worldsheet. We shall see in the next study that these degenerate discontinuities are no more than the cusps.

7 Conclusion

The general form of equation of motion (16) can admit discontinuous solution (19) under condition (20). If equation of motion is given in a multi-
component form \((16)\), the equations of discontinuities \((22)\) are solved under condition \((23)\).

Superconducting cosmic strings can admit two classes of discontinuities. The intrinsic discontinuities of the current are determined by equations \((25)\), \((28)\) under constraints \((26)\), \((29)\). The extrinsic discontinuities of the curvature are determined by equations \((31)\), \((32)\), \((35)\), \((36)\) under constraints \((33)\), \((37)\). Having introduced the characteristic vector \((38)\), \((43)\), we establish equations of extrinsic discontinuities \((48)-(51)\) and equations of intrinsic discontinuities \((54)-(55)\). It is the main result of the present paper.

There is also a degenerate solution \((56)\) which satisfies all the equations of discontinuities and does not concern any change of the current or the curvature.

Looking for explicit solution of equations \((48)-(51)\) and \((54)-(55)\), we should utilize the important condition of stability \((10)\) of the characteristic vector \((38)\), \((43)\). Explicit results will be obtained in the next papers.

References

[1] M.B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. 58, 477 (1995). arXiv:hep-ph/9411342

[2] E. Witten, Nucl. Phys. B 249, 557 (1985).

[3] B. Carter, Phys. Lett. B 228, 466 (1989).

[4] B. Carter and P. Peter, Phys. Rev. D 52, 1744 (1995). arXiv:hep-ph/9411425

[5] A. Vilenkin and E. P. S. Shellard, Cosmic strings and other topological defects, (Cambridge University Press, 2000) p. 159.

[6] J. J. Blanco-Pillado and K. D. Olum, Phys. Rev. D 59, 063508 (1999). arXiv:gr-qc/9810005

[7] E. J. Copeland and T. W. B. Kibble, Phys. Rev. D 80, 123523 (2009). arXiv:0909.1960

[8] C. J Copi and T. Vachaspati, Phys. Rev. D 83, 023529 (2011). arXiv:1010.4030
[9] G. Vlasov, arXiv:hep-th/9905040

[10] X. Martin and P. Peter, Phys. Rev. D 61, 043510 (2000). arXiv:hep-ph/9808222

[11] A. Cordero-Cid, X. Martin, and P. Peter, Phys. Rev. D 65, 083522 (2002). arXiv:hep-ph/0201097

[12] E. Trojan and G.V. Vlasov, Phys. Rev. D 85, 107303 (2012). arXiv:1102.5659

[13] A. Babul, B. Paczynski, D. Spergel, Astrophys. J. 316, L49 (1987).

[14] D. Garfinkle and T. Vachaspati, Phys. Rev. D 36, 2229 (1987).

[15] S. Öhmez, V. Mandic, and X. Siemens, Phys. Rev. D 81, 104028 (2010). arXiv:1004.0890

[16] E. O’Callaghan and R. Gregory, JCAP 1103, 004 (2011). arXiv:1010.3942

[17] A. M. Anile, Relativistic fluids and magneto-fluids, (Cambridge University Press, 1989), p. 215.

[18] L.D. Landau and E.M. Lifshitz, Fluid mechanics, 2nd ed. (Pergamon, Oxford, 1987), p. 331.