Optimized Secure Position Sharing
with Non-trusted Servers

Pavel Skvortsov, Björn Schembera, Frank Dürr, Kurt Rothermel

Institute of Parallel & Distributed Systems (IPVS), Universität Stuttgart
Universitätstrasse 38, 70569 Stuttgart, Germany
Tel.: +4971168565802; Fax: +4971168565832

Abstract

Today, location-based applications and services such as friend finders and geo-social networks are very popular. However, storing private position information on third-party location servers leads to privacy problems. In our previous work, we proposed a position sharing approach for secure management of positions on non-trusted servers [1, 2], which distributes position shares of limited precision among servers of several providers. In this paper, we propose two novel contributions to improve the original approach. First, we optimize the placement of shares among servers by taking their trustworthiness into account. Second, we optimize the location update protocols to minimize the number of messages between mobile device and location servers.

Keywords: Location-based service, location privacy, obfuscation, position sharing, map-awareness, location update, placement

1. Introduction

Driven by the availability of positioning systems such as GPS, powerful smartphones such as the iPhone or Google Android phones, and cheap flat rates for mobile devices, location-based services enjoy growing popularity. Advanced location-based applications (LBAs) such as friend finders or geo-social networks are typically based on location server (LS) infrastructures that store mobile user positions to ensure scalability by enabling the sharing of user positions between multiple applications. This principle relieves the mobile device from sending position information to each LBA individually. Instead, the mobile device updates the position at the LS, and LBAs query the LS for the positions of mobile objects.

Although certainly useful from a technical point of view, storing data on LSs raises privacy concerns if the LSs are non-trusted. Multiple incidents in the past have shown that even if the provider of the LS is not misusing the data, private information can be...
revealed due to attacks, leaking, or loss of data \[^3, 4, 5\]. Therefore, the assumption of a trusted LS is questionable, and technical concepts are needed to ensure location privacy without requiring a trusted third party (TTP).

Various location privacy approaches have been proposed in the literature. Many approaches such as \(k\)-anonymity (e.g., \[^6\]), rely on a TTP and are, therefore, not applicable to non-trusted environments. Approaches without the need for a TTP mainly rely on the concept of spatial obfuscation, i.e., they reduce the precision of position information stored on the LS. However, this severely impacts the quality of service of LBAs since they can only be provided with coarse-grained positions, depending on the degree of obfuscation.

To solve this conflict between privacy and quality of service, we have proposed the concept of position sharing in our previous work \[^1, 2\]. The basic idea of this concept is to split up the precise user position into position shares of limited precision. These shares are distributed among multiple LSs such that each LS has a limited, coarse-grained view onto the user position. LBAs are provided with access rights to a certain number of shares on different LSs. By using share fusion algorithms, a position of higher precision can be calculated. Therefore, LBAs can be provided with different, individual precision levels (privacy levels) although each LS only manage less precise information. This approach does not require a TTP. Furthermore, it provides graceful degradation of privacy, where no LS is a single point of failure with regard to privacy. Instead, the precision of positions revealed to an attacker increases with the number of compromised LSs.

The concept of position sharing can be applied to various settings and use cases. For instance, it can be used to implement a secure personal data vault for location information of individual users over a non-trusted server infrastructure. The concept of a personal data vault has been first proposed in \[^7\]. A data vault is a data repository controlled by the user for storing personal data and controlling the access to data from different services. With the advent of cloud computing, it seems to be attractive to implement data vaults atop cloud computing infrastructures, relieving the user from operating her own dedicated servers. However, the providers of these cloud infrastructures might be non-trusted. Position sharing offers the possibility to avoid storing all location data of the data vault at a single provider. Instead, the data vault can be distributed among servers of different cloud providers, where each provider only has a well-defined limited view onto the personal location information.

As another use case for position sharing, consider a startup company that wants to provide location-based services to its customers. However, as typical for many startups, the startup company does not own a dedicated server infrastructure but instead utilizes an infrastructure as a service (IaaS) of a third-party IaaS provider. Assume that the customer trusts the startup company to handle his private location information securely. However, although the customer trusts the startup company, this does not imply that the startup company trusts the IaaS provider operating the physical servers. Position sharing enables the startup company to distribute its valuable private customer data to several third-party IaaS providers to avoid a single point of failure and provide a trustworthy virtual service to its customers over non-trusted IaaS infrastructures.

In this work, we present two novel contributions to extend the original position sharing approach: (1) an algorithm for optimizing the placement of position shares on
location servers of different trustworthiness; (2) optimized share update protocols that significantly reduce the communication overhead of the original approach.

In our previous approach, we made the simplified assumption that every LS provider is equally trustworthy, i.e., each LS has the same risk of being compromised. However, a user might trust certain providers such as big companies with good reputation more than others. Therefore, it seems reasonable that LSs of providers of higher trust levels should store more precise information, i.e., either position shares of higher precision or more shares. For instance, in the examples above IaaS providers operated by companies like Google, Microsoft or Amazon might be considered more trustworthy than servers of a cheaper but not so well-known IaaS providers. Still it might be reasonable to utilize such cheaper providers for monetary reasons. With our extended position sharing approach, we can balance the risk of revealing data by considering the individual trustworthiness of providers.

Therefore, the first contribution of this paper is an improved position sharing approach that takes individual trust levels of LS providers into account. To this end, we optimize the placement of shares on LSs to increase the protection of privacy. We propose a suitable privacy metric and scalable share placement algorithms that (a) flexibly select \( n \) of LSs, and (b) balance the risk among providers such that the risk of disclosing private information stays below a user-defined threshold. Moreover, we aim to meet the user-defined privacy requirements with a minimum number of LSs to minimize the overhead of updating and querying several LSs.

Moreover, in our previous work we did not consider how multiple position updates affect the communication overhead. This factor can negatively affect the scalability of our approach, since each update includes \( n \) messages (where \( n \) is the number of LSs). Therefore, as the second contribution of this paper, we propose a position update algorithm which improves scalability by minimizing the number of transmitted messages and does not change the user’s desired location privacy levels. This improvement is achieved by omitting updates of shares that can remain unchanged after the given position change. Our evaluations show that the proposed method can save up to 60% of messages compared to the previous version of this approach.

The rest of this paper is structured as follows: In Section 2, we give an overview of the related work. In Section 3, we describe our system model and privacy metric. In Section 4, we present the basic position sharing approach including the share fusion and share generation algorithms. In Section 5, we propose a share placement algorithm which distributes position shares among LSs depending on their individual trustworthiness. Then in Section 6, we present position update algorithm which optimizes the communication overhead caused by multiple position updates. Finally, in Section 7, we conclude this paper with a summary.

2. Related Work

In this section, we will discuss existing approaches for location privacy. For a more in-depth analysis and classification of location privacy techniques, we refer to our survey paper [8].

A classic solution employed to preserve location privacy is cryptography. However, if user positions stored on servers are encrypted, server-side query processing of
advanced queries such as range queries over the encrypted data becomes impossible, or it is possible only at higher cost [9].

Another example of a cryptography-based approach for location privacy was proposed by Mascetti et al. [10] to implement proximity services for geo-social networks. The authors assume that service providers are non-trusted and consider the scenario where mobile users want to notify their friends called buddies in their proximity. The main idea is that the secret keys are shared with the selected buddies in a distributed fashion and remain unknown to the service providers. The authors use a precision metric which is defined through the union of multiple discrete space cells called granules. A drawback of this approach is that it requires a complex implementation of the encryption functionalities. Similarly to the work of Zhong et al. [11], this approach only considers specific friend-finder and proximity calculation scenarios.

Another method to preserve location privacy is to send dummy positions to LBAs together with the actual user position [12]. The problem with this approach is, however, that dummy positions can be easily distinguished if the attacker has some background information such as database of real user movements [13].

The idea of mix zones [14] is to select privacy-sensitive areas called mix zones in which users do not send position updates while they are visible outside the mix zones. Some extensions try to avoid the threat of analyzing possible user trajectories based on the known entry and exit points on the borders of a mix zone [15]. The mix zones approach lacks flexibility since it needs a pre-defined division of space into zones, and does not allow for various privacy levels in different zones.

Many existing location privacy approaches are based on $k$-anonymity, e.g., [16]. The main idea is to send a set of $k$ different positions of real mobile users ($k$-set) to the LBA, such that the actual user position is indistinguishable from $k - 1$ other positions. Many extensions of $k$-anonymity aim to make the $k$-set more robust to various attacks—mostly against attacks based on the analysis of user attributes, e.g., [17, 18, 19, 20, 21]. However, in order to select a $k$-set, a trusted anonymizer with global view is needed, which requires trust to a TTP, introducing a single point of failure.

Obfuscation approaches such as [22] deliberately decrease the precision of user positions stored on servers. A TTP is not required in this case since the cloaked region can be generated by the user independently, but the queries over obfuscated locations can result in imprecise or probabilistic answers. Our approach is also based on spatial obfuscation, but it gracefully degrades position precision depending on the number of missing position shares, and it supports multiple obfuscation levels (i.e., privacy levels).

Marias et al. [23] propose to apply the concept of secret sharing [24] to position information to distribute the information about single positions among several servers. Their secret sharing approach relies on cryptographic techniques, which means that all shares are required in order to retrieve a position. In order to overcome this drawback, we proposed the position sharing approach based on spatial obfuscation [1], map-aware spatial obfuscation [2], and cryptographic multi-secret sharing techniques (PShare) [25]. However, in all these works, LSs are assumed to be equally trustworthy, and, therefore, each LS stores one position share of equal precision. In this paper, we will show that optimized share placement based on the individual trust levels of
providers significantly increases privacy if providers are of different trustworthiness and opens up the possibility to minimize the number of required LSs. Moreover, we will extend our previous work by introducing an optimized location update algorithm, which reduces the number of transmitted messages to the LSs.

3. System Model and Privacy Metric

This section introduces our conceptual system model, operational system model and privacy metrics of the position sharing approach.

3.1. System Model

Our system model is shown in Figure 1. It consists of four components: mobile objects, location servers, location-based applications, and a trust database.

The user is represented by the mobile object (MO), which knows its precise position $\pi$, for instance, determined through GPS. A position of certain precision is defined by a circular area which we call obfuscation area, where radius $r$ of this circular area defines the precision $\text{prec}(\pi) = \phi = r$ of position $\pi$. A smaller radius corresponds to a higher precision: if $r_1 = \text{prec}(\pi_1)$, $r_2 = \text{prec}(\pi_2)$ and $r_1 < r_2$, then the precision of $\pi_1$ is higher than the precision of $\pi_2$.

The MO executes a local component to perform the generation of position shares on the mobile device. We assume that this component can be implemented in a trustworthy way, e.g., by using a Trusted Computing Platform [26]. The MO generates one master share $s_0$ with the minimal acceptable precision $\phi_{\text{min}}$, chosen such that there are no problems with regard to privacy, and a set of $n$ refinement shares $S = \{s_1, \ldots, s_n\}$:

$$\text{generate}(\pi, n, \phi_{\text{min}}) = \{s_0, S\}$$

(1)

Given a subset $S' \subseteq S$ of refinement shares, its fusion with $s_0$ (which is known to everyone) results in a position $\pi'$ of a certain well-defined precision:

$$\text{fuse}(s_0, S') = \pi'$$

(2)
Figure 2: (a) Distributed Location Data Vault scenario; (b) Virtual location-based service provider scenario

with $\text{prec}(\pi) < \text{prec}(\pi')$, i.e., $\phi \leq \phi'$.

The fusion of $s_0$ with the set $S$ of all refinement shares obtained from the LSs provides the exact position $\pi$ of precision $\phi_{\text{max}}$.

We say that shares are **heterogeneous** if each share $s_i$ increases the position precision by an individual amount $\Delta \phi_i$. Typically, for heterogeneous shares, share fusion has to be performed in a certain fixed order, in contrast to **homogeneous** shares that can be fused in any order. If a share generation algorithm produces homogeneous shares, only the number of different shares defines the resulting precision level $\phi_k$. In this case, the precision increase is equal for each share: $\Delta \phi_1 = \Delta \phi_2 = \ldots = \Delta \phi_n = \phi_{\text{max}} / n$.

A **location server (LS)** stores and delivers location data of users to LBAs. Each LS has a standard authentication mechanism, which allows to specify access rights for the LBAs (given by a user) to access shares stored by this LS. The maximal allowed precision of a user position is defined individually depending on the concrete LBA by specifying a certain set of LSs accessible for each LBA.

We assume that each provider operates one LS. Internally, this LS can be implemented by a number of physical servers, e.g., running in a data center. Also, an LS can be implemented based on the “virtual provider” model on top of an Infrastructure-as-a-Service (IaaS). For instance, to implement a cloud-based personal Data Vault storing and filtering locations of individual users as already motivated in Section 1, each LS could be implemented atop a virtual machine operated by an individual IaaS provider as depicted in Figure 2. The set of LS then implements the distributed personal Data Vault offering a well-defined interface to LBAs. The LBAs need to be aware of the interface of the distributed Data Vault in order to query data from the Data Vault, which is a typical assumption of the Data Vault concept.

Another operational model is the implementation of a virtual LBA offered by a startup company (LBA provider) not owning a dedicated server infrastructure as motivated in Section 1. Here, we assume that the customer actually trusts the LBA provider, however, the LBA provider does not trust the IaaS providers operating the virtual machines running the LS as depicted in Figure 2. Although in this setting there is only a single LBA which is trustworthy from the point of view of the mobile object (customer) and thus can be provided with position information of maximum precision, the position sharing concept is still useful since it allows the trusted LBA provider to im-
plement its service atop a non-trusted IaaS infrastructure to manage positions of the whole population of all customers. The LBA logic is implemented by a trusted app running on the mobile device, so the non-trusted IaaS providers cannot influence how data is distributed among LS.

Since the Data Vault scenario is the more general case with several non-trusted LBAs, which should be provided with location information of different precision, we will further on focus on this scenario only.

Each LS is non-trusted and can be compromised with probability $p_i$. Risk value $p_i$ represents the probability of LS$_i$ to behave maliciously, i.e., to misuse the user’s private position information, or to be compromised by an external attacker. The concrete concepts for calculating $p_i$ are beyond the scope of this paper. For instance, we can use the generic probabilistic trust model described in [27]. This model is generic in the sense that it allows mapping of various representations of trust values to the probabilistic interval $[0; 1]$. Different LSs might have different risks depending, for instance, on the reputation of their provider. Moreover, different users might have individual trust in the same LS (and/or its provider).

The trust database manages the trust in different LSs by providing the probabilities $p_i$ that LS$_i$ can be compromised. Based on the obtained risk values, the user can determine the number and set of LSs needed to satisfy his security requirements, as we will show in the following sections. We assume that the trust database is given, and it is filled with data, for example, by analyzing the feedback of other users through a reputation system [28, 29].

Location-based applications (LBAs) query or track MO’s position and obtain multiple shares from different LSs depending on the access rights given by user. Then, the LBA fuses the obtained shares by using function $\text{fuse}(\ldots)$ (Equation 2) in order to get the user position with a certain level of precision.

### 3.2. Privacy Metric

The user’s privacy levels are primarily defined through precision levels $\phi_k$, which are pre-defined by the user for each $0 \leq k \leq n$ as radii $r_k$ of a circular obfuscation areas. Additionally, we use a probabilistic privacy metric since the precision of a position obtained by an attacker depends on the probabilities of compromising LSs as well as on the ability of an attacker to derive higher position precisions by analyzing the obtained shares (as shown in [12]). The following distribution $P_{k,\text{attack}}$ defines the probability of an attacker obtaining a position $\pi_{k,\text{attack}}$ of a certain precision $\phi_{k,\text{attack}} = \text{prec}(\pi_{k,\text{attack}})$ depending on the number $k$ of compromised LSs:

$$P_{k,\text{attack}}(\phi_k) = \Pr[\phi_{k,\text{attack}} \leq \phi_k]$$  \hspace{1cm} (3)

This metric can be used by the MO to define the acceptable probabilistic guarantees represented as a set of probability thresholds $P_k(\phi_k)$ corresponding to various precision levels $\phi_k$. For example, an MO can specify that an attacker must not be able to obtain a position of precision $\phi_1 \leq 1$ km with probability $P_{1,\text{attack}} > 0.2$, and $\phi_2 \leq 2$ km with $P_{2,\text{attack}} > 0.1$, etc.
4. Basic Position Sharing Approach

In this section, we will present the basic principle and two basic versions of the position sharing approach: (a) for open space models (with no map knowledge) and (b) for spatial constrains (taking into account map knowledge as explained later). Each of them includes an algorithm for LBAs to fuse position shares, and an algorithm for MOs to generate the shares.

4.1. Position Sharing: Basic Principle

In general, our position sharing approach is based on geometric transformations, where imprecise geometric positions are defined by circular areas $c_i$ with radii $r_i$. Each share is a vector shifting the center of the current obfuscated position represented as a circle, whose radius is decreased after every shift. An example of precision increase through such share fusion is shown in Figure 3. The share defines the precision increase $\Delta_i$ after the corresponding $i$th shift of the center of the circle and the radius decrease.

After generating the shares, the MO distributes them among multiple LSs and updates them continuously. The master share is publicly available to anyone. In order to control the access to refinement shares, the MO defines access rights to a certain subset of shares for each individual LBA.

The concrete share generation and share fusion algorithms depend on whether we consider the availability of map knowledge or not. Next, we first assume that no map knowledge is available before we present an extended approach taking map knowledge into account.

4.2. Open Space Position Sharing (OSPS)

In [1], we presented the position sharing approach for open space, referred to as OSPS (Open Space Position Sharing). The open space model weakens the attacker model by assuming that no map is known to an adversary. In other words, we assume a uniform probability distribution of MO positions. The share fusion algorithm is executed on the LBA, which knows the number of LS providers resp. the total number of refinement shares $n$, the obtained refinement shares

Figure 3: Basic position sharing approach: after getting each new share (i.e., a shift vector plus a radius decrease), the precision is increased until we get the exact target position $\pi$. 

\[ \text{Figure 3: Basic position sharing approach: after getting each new share (i.e., a shift vector plus a radius decrease), the precision is increased until we get the exact target position } \pi. \]
Algorithm 1 OSPS: fusion of shares

1: function fuse_k_shares(OSPS(n, s_0, s_1, ..., s_k))
2:   Δr ← r_0/n; \( \vec{p} \) ← \( \vec{p}_0 \); r ← r_0
3: for i = 1 to k do
4:   \( \vec{p} \) ← \( \vec{p} \) + \( \vec{s}_i \);
5:   r ← r − Δr
6: return \( c_k = \{\vec{p}, r\} \)

Figure 4: OSPS: same set of shares fused in an arbitrary order (\( n = 4 \), \( k = 3 \), \( r_0 = 20 \) km, \( ∆r = 5 \) km)

The share fusion algorithm of OSPS is shown in Algorithm 1 (cf. Figure 4). Starting from the initial obfuscation circle \( c_0 \) (lines 2, 5), step-by-step for \( k \) shares (line 3) each vector \( \vec{s}_i \) shifts the center \( p_i \) of the current obfuscation circle \( c_i \) (line 4) while reducing the radius \( r_i \) (line 5) of the current obfuscation circle by a pre-defined value \( ∆r = r_0/n = ∆\phi \) (line 2). Note that in OSPS, \( ∆\phi \) has the same value \( ∆\phi \) for each \( i \). The resulting obfuscation circle is \( c_k \) (line 6).

In Figure 4 we show an example of share fusion for \( n = 4 \), \( k = 3 \), \( r_0 = 20 \) km, \( ∆r = 5 \) km. As shown, the order of fusing the refinement shares can be arbitrary, while the precision (radius) of every obfuscation circle \( c_k \) is well defined. Note that our algorithm is not dependent on the absolute values of \( r_0 \) and \( ∆r \); i.e., it works for any selected size of \( r_0 \).

The generation of shares in OSPS is presented in Algorithm 2. The input parameters are the user-defined radius \( r_0 \) of the initial obfuscation circle \( c_0 \), the total number of shares \( n \), and the precise user position \( \pi = p_n \). First, we determine the maximum shift length \( ∆r = Δ_\phi = r_0/n \) (line 2). In the second step, position \( p_0 \) of the initial circle \( c_0 \) is selected randomly according to a uniform distribution such that \( \pi = p_n \) is inside \( c_0 \) (line 3). The set of the refinement shift vectors \( S = \{s_1, ..., s_{n-1}\} \) is generated with random direction and length (lines 4-6), starting from the center of \( c_0 \). All shift vectors of \( S \) are concatenated such that the resulting point \( \pi = p_n \) (with \( r_n = 0 \)
Algorithm 2 OSPS: generation of shares

1: function generate_n_shares_OSPS(s₀, n, π)
2: \( \Delta r \leftarrow r_0 / n \)
3: select randomly \( p_0 \) such that \( \text{distance}(\vec{p}_0, \pi) \leq r_0 \)
4: do
5: for \( i = 1 \) to \( n - 1 \) do
6: select rand. \( \vec{s}_i \) with \( |\vec{s}_i| \leq \Delta r \) such that \( \pi \in c_i \)
7: while \( \text{distance}(\vec{p}_0 + \sum_{i=1}^{n-1} \vec{s}_i, \pi) > \Delta r \)
8: \( \vec{s}_n \leftarrow \pi - (\vec{p}_0 + \sum_{i=1}^{n-1} \vec{s}_i) \)
9: return \( \vec{s}_0 \ldots \vec{s}_n \)

Figure 5: CSPS: a) intersection of 3 circles \( c_0, c_1, c_2 \) and the map representation \( M_u \) (yellow); b) adjustment of intersection area through radius increase for \( c_1 \): \( A_1 = \text{area}(M_u \cap c_0 \cap c_1) = \text{area}(c'_1) \)

correspondingly) coincides with the user’s position \( \pi \) within \( c_0 \) (line 8). The vector lengths are selected from the interval \([0; \Delta r]\) uniformly at random. Finally, MO sends the position information to \( n \) LSs, including the master share \( s_0 \), the radius decrease after every shift \( \Delta r \) (in OSPS, \( \Delta r \) is constant for all shifts), and one share \( \vec{s}_i \) for each LS. The master share is stored on every LS, while each LS stores only one refinement share.

4.3. Constrained Space Position Sharing (CSPS)

The obvious limitation of OSPS is that it is not applicable for constrained space models. If an attacker has map knowledge, the attacker is able to reduce the size of the generated obfuscation areas. By excluding areas where the user cannot be located such as water surfaces, ravines, and agriculture fields, it is possible for the attacker to intersect an obfuscation circle \( c_k \) with areas such as roads, squares, buildings, etc. Since the total area of possible user locations in \( c_k \) is smaller than the total area of \( c_k \), the target privacy guarantees—based on the size of obfuscation circles, i.e., precision—are not preserved. To resolve these problems, we present an improved version of the position sharing approach next, which was first introduced in [2].

In the map-aware position sharing approach for constrained space (CSPS), we introduce the binary map representation \( M_u \) (cf. Figure 5): \( M_u \) shown in yellow. Each location is marked by “true” if the MO can be possibly located there, or “false” if it
is impossible that MO is located in this area. We define the obfuscation area $A_k$ for $k$ shares of precision $\phi_k$ through the intersection of $k$ circles $c_1 \ldots c_k$, and $M_u$:

$$A_k = \text{area}(M_u \cap c_0 \cap c_1 \cap \ldots \cap c_k) = \pi \cdot r_k^2 \quad (4)$$

Before share generation, the user has to select a map representation $M_u$, which defines the map regions where the user can possibly be located. $M_u$ is individual for each user, since different users can be possibly located in different map regions.

The share fusion algorithm of CSPS is shown in Algorithm 3 (cf. Figure 5a). It is similar to the OSPS share fusion (Algorithm 1). The main difference is that $c_0$ as well as each $c_i$ are intersected with the map representation $M_u$ (lines 2, 7). Note that the radii are not pre-defined as in OSPS, but each obfuscation has its individual radius defined by the map-aware share generation algorithm.

Algorithm 3 CSPS: fusion of shares

1: function $\text{fuse}_k$\_shares\_CSPS($M_u, n, c_0, s_1, \ldots, s_k, r_1, \ldots, r_k$)  
2: $A_k \leftarrow M_u \cap c_0$  
3: $p \leftarrow p_0$  
4: for $i = 1$ to $k$ do  
5: $p \leftarrow p + s_i$  
6: $c_i \leftarrow \{p, r_i\}$  
7: $A_k \leftarrow A_k \cap c_i$  
8: end for  
9: return $A_k$

Our goal is to preserve the required privacy level by providing an obfuscation area of a given size. To this end, we adapt the radius such that the intersection area of $M_u$ and obfuscation circles $c_0 \ldots c_k$ is not smaller than this size.

Algorithm 4 shows the CSPS share generation algorithm with map knowledge (cf. Figure 5b). In Algorithm 4, the radius $r_k$ is increased until the area of $M_u \cap (c_0 \cap c_1 \cap \ldots \cap c_k)$ is equal or greater than the value of the non-intersected area of $c_k$. By applying this condition, we ensure that the precision difference between areas $A_k$ and $A_{k+1}$ corresponds to the required $\Delta \phi$.

In lines 4-6, the radius $r_0$ of the initial circle $c_0$ is increased considering the map representation $M_u$, in order to adjust the size of $A_0 = M_u \cap c_0$. Also, to adjust the radii of shares $s_1 \ldots s_{n-1}$, $M_u$ is included in the condition, which defines the target area size (lines 11-13). The remaining steps of the share generation algorithm for CSPS are similar to Algorithm 2.

If we used a deterministic algorithm for the area adjustment (lines 5, 12), an attacker could calculate the inverse function to decrease the size of the obfuscation area. Therefore, the resulting center of circle $c_i$ must be randomly shifted together with the radius increase, so that its original center $c_i'$ cannot be determined (cf. Figure 6b).

Algorithm 5 shows the $\text{increase}(\ldots)$ function for increasing $r_i$, together with shifting the center $p_i$ (cf. Figure 6b). First, for the given $p_i$ we increase the radius $r_i$ until its value provides the required size of the obfuscation area (lines 4-6). Then $p_i$ is randomly shifted, such that the shift is not greater than $r_i - r_i'$ (lines 7-9). Next, we check again whether the current radius $r_i$ still provides the required size of intersection area.
Algorithm 4 CSPS: generation of shares

1: function gen_n_shares_CSPS($n, M_u, r_0, \pi$)
2: select randomly $p_0$ with $\text{distance}(p_0, \pi) \leq r_0$
3: $A_0 \leftarrow \text{area}(c_0)$
4: while $\text{area}(M_u \cap c_0) < A_0$ do
5: $r_0 \leftarrow \text{increase}(r_0, p_0, \Delta r)$
6: end while
7: for $i = 1$ to $n - 1$ do
8: $r_i \leftarrow r_0 \ast \frac{(n - i)}{n}$
9: select rnd. $\vec{s}_i$ with $|\vec{s}_i| \leq 2 \ast r_{i-1}$ and $\pi \in c_i$
10: $A_i \leftarrow \text{area}(c_i)$
11: while $\text{area}(M_u \cap \bigcap_{j=1}^{i}(c_j)) < A_i$ do
12: $r_i \leftarrow \text{increase}(r_i, p_i, \Delta r)$
13: end while
14: end for
15: $\vec{s}_n \leftarrow \pi - (\vec{p}_0 + \sum_{i=1}^{n-1} \vec{s}_i)$
16: return $\vec{s}_0 \ldots \vec{s}_n, r_0 \ldots r_n$

Figure 6: Adjustment of $p_i$ during radius increase: (a) no adjustment of $p_i$; (b) randomized adjustment of $p_i$

(line 10). If this area is not large enough, we call the function $\text{increase}(r_i, \ldots)$ recursively (line 11). If the intersection area now exceeds the target value $A_i$, we decrease the current radius $r_i$ until the intersection area hits its limit (lines 12-16).

In Figure 6, we show that after the adjustment of $p_i$, the target position $\pi$ can be located anywhere within $c_i$; it is not restricted to the obfuscation area which corresponds to the smaller radius $r'_i$. Thus, it is not possible for an attacker to reduce the obfuscation area $A_i$ knowing the share generation algorithm and $r'_i$.

5. Optimization of Share Placement

So far, we assumed that all servers are equally trustworthy. Now, we consider the case when servers have various trust levels. Since each LS has an individual trust value, the user’s position privacy highly depends on the number of selected LSs and
the placement of shares on different LSs. In our previous work \cite{1,2}, we used an equal share placement strategy, i.e., each LS stored shares of the same precision increase $\Delta \phi = \Delta \phi_i$. Now, we want to make sure that an LS with a higher trust level can store more precise position information than an LS that has a higher risk of being compromised.

### 5.1. Problem Statement: Share Placement under Individual Trust Levels

The problem of share placement on LSs with individual trust levels can be defined as a constrained optimization problem. The constraint is that an attacker cannot derive a position $\pi_k\text{attack} = \text{fuse}(s_0, S_k)$ of precision $\text{prec}(\pi_k\text{attack}) > \phi_k$ with a probability higher than $P_k(\phi_k)$, where $S_k$ denotes the set of compromised refinement shares. That is, the user defines probabilistic guarantees $P_k$ for different precision levels $\phi_k$.

The optimization goal is to provide the specified privacy levels and their probabilistic guarantees by minimizing the number of LSs. By minimizing the number of required LSs, we limit the induced overhead of updating (communicating) and storing shares at multiple servers.

We assume the following values to be given:

- master share $s_0$,
- set $S$ of $n$ refinement shares $\{s_1, \ldots, s_n\}$ to provide the precision (privacy) levels $\phi_k$ for the LBAs,
- candidate set $L$ of $m_0$ available LSs, which can store shares $L = \{LS_1, \ldots, LS_{m_0}\}$,
- set of risk values $\{p_1, \ldots, p_{m_0}\}$ providing the probabilities for each LS $i$ of $L$ to be compromised ($p_i \in [0; 1]$),
probability distribution \( P_k(\phi_k) \) specifying the required probabilistic guarantees \( P_k \) for each precision (privacy) level \( \phi_k \).

Problem: Find a share placement \( \text{place}(\ldots) \) of \( n \) shares to a set \( L' \leq L \) of LSs:

\[
\text{place}(\{s_1, \ldots, s_n\}, L) : S \rightarrow L' \subseteq L,
\]

such that \( |L'| \) is minimal, and the user’s security requirements are fulfilled:

\[
\forall \phi_{k,\text{attack}} : P_k(\phi_k) > \Pr[\phi_{k,\text{attack}} \leq \phi_k]
\]

5.2. General Selection & Placement Algorithm

In our work, we made two assumptions. The first assumption is that increasing the number \( m \) of LSs leads to higher security with regard to probabilistic guarantees of privacy levels. At the same time, a large \( m \) is not desired, since it would increase storage and communication overhead. Therefore, it is beneficial to incrementally increase \( m \) only until the security requirements are fulfilled. The second assumption is that we can increase security by optimizing the distribution of shares for a given \( m \). For the validation of both assumptions, we refer to our work \[30\].

The algorithm for optimal share placement (Algorithm \[6\]) consists of two major steps: (a) selection of a minimum number \( m = |L'| \) of location servers \( L' = \text{LS}_1, \text{LS}_2, \ldots, \text{LS}_m \) required to fulfill the given privacy constraint; (b) optimization of the placement \( n \) shares among these \( m \) LSs.

The basic idea is to start with the smallest set of LSs and incrementally increase \( m \) (lines 2 and 7) until the security constraints (Equation \[6\]) are fulfilled. In each step, we greedily add the next most trusted LS to set \( L' \), since the subset of the most trusted LSs provides the highest security. Therefore, the available LSs must be initially sorted by ascending risks \( p_i \) (line 3).

For each number of LSs, we first check whether a uniform placement (line 8) where each LS stores an equal number of shares, independent of its individual risk value, fulfills the user-defined probabilistic guarantees of privacy levels (lines 9-11). If the non-optimized solution (i.e., uniform placement) already represents a solution with regard to the constraint, we skip the optimization algorithm to save energy resources of the mobile device, which executes this algorithm.

If the uniform (non-optimized) share placement on LSs does not satisfy the user’s privacy requirements, we optimize the placement by relocating shares from less trusted to more trusted LSs (line 12). The share placement algorithm invoked in line 12 in the major contribution of this section, and will be presented in detail in Section 5.3.

If \( m \) reaches the total number of available LSs \( m_0 \), but no solution has been found, the given security requirements are too hard. Therefore, the user should relax the constraints (probabilistic guarantees of privacy levels) given in Equation \[6\] step by step, and execute the algorithm again.

In lines 9 and 13 of Algorithm \[6\] we determine the probabilistic guarantees of privacy levels of a placement. We calculate \( P_{k,\text{attack}} \) for different numbers \( k_{\text{attack}} \) of compromised LSs for a given placement as follows:

\[
P_{k,\text{attack}} = \sum_{k = k_{\text{attack}}}^{m} \sum_{i=0}^{\binom{m}{k}} p_{i}\text{incl} \cdot p_{i}\text{excl},
\]
Algorithm 6: General Selection & Placement Algorithm

1: function place($P_k(\phi_k), S, L, m_0, m_{\text{min}}, n$)  
2: $m \leftarrow m_{\text{min}} - 1$  
3: sort by ascending $p_i(L)$  
4: $L' \leftarrow$ get selected set $(L, m$)  
5: solution found $\leftarrow$ false  
6: repeat  
7: $m \leftarrow m + 1$  
8: distribute equal($P_k(\phi_k), S, L', m$)  
9: if $\forall \phi_k : P_k < P_k,\text{attack}(\phi_k)$ then  
10: solution found $\leftarrow$ true  
11: else  
12: place optimized($S, L', m$)  
13: if $\forall \phi_k : P_k < P_k,\text{attack}(\phi_k)$ then  
14: solution found $\leftarrow$ true  
15: end if  
16: end if  
17: until $(m = m_0) || (\text{solution found})$  
18: return $S \rightarrow L'$

\[
p_{i,\text{incl}} = \prod_{j=0}^{m} p_j, \forall p_j \in P_{k,i} \quad (8)
\]

\[
p_{i,\text{excl}} = \prod_{j=0}^{m} (1 - p_j), \forall p_j \notin P_{k,i} \quad (9)
\]

where $P_{k,i}$ is the set of risks of the $i$th $k$-combination out of $m$ LS risks.

There are $\binom{m}{k}$ combinations of LS to compromise exactly $k$ LSs. Each combination has the probability defined by multiplying the risks $p_j$ of (included) $k$ LSs and the inverse risks $1 - p_j$ of the rest (excluded) $m - k$ LSs. To get a probability of exactly $k$ compromised LSs, we have to sum up the probability of each $k$-combination. Finally, to get a probability of at least $k$ compromised LSs, we sum up the probabilities corresponding to $\{k, k+1, ..., m\}$ compromised LSs.

5.3. Optimized Share Placement Algorithm

In this section, we present an algorithm to solve the optimized share placement problem, which is called from Algorithm 6 through function place optimized(...). Here, we consider the situation when a set $L'$ of $m = |L'|$ LSs with lowest risks has been selected and is fixed. Thus, an optimized placement of $n$ shares to these LSs in $L'$ must be found, after the uniform placement strategy did not satisfy the user’s security constraints. First, we show that this problem is NP-hard. Then, we propose a heuristic solution based on a genetic algorithm.

We selected a strategy that minimizes the risk of revealing the positions of high precision by applying the principle of risk balancing. This principle is well-known from risk-based capital allocation and is commonly applied if the risk values are available, and there are no correlations between them [31][32], as is the case in our system.
model, where we do not know any relations between different LS providers. We call this placement problem the Balanced Risk Placement Problem (BRPP).

Formally, a share placement \( S \rightarrow L' \subseteq L \) has balanced risk if the proportion of position precisions \( \phi_{i1,j} \) and \( \phi_{i2,j} \) stored by LS\(_{i1}\) and LS\(_{i2}\) respectively \((j = 1 \ldots n)\) is inversely proportional to the corresponding risks \( p_{i1} \) and \( p_{i2} \) of LS\(_{i1}\) and LS\(_{i2}\):

\[
S \rightarrow L' \subseteq L : \forall i1, i2 \in m, j \in n : \frac{\sum_{j=1}^{n} \Delta \phi_{i2,j}}{\sum_{j=1}^{n} \Delta \phi_{i1,j}} = \frac{p_{i1}}{p_{i2}} \tag{10}
\]

If the exact equality of proportions is not feasible due to the given risk values and other parameters, the goal is to find a share placement solution which is close to the best possible solution:

\[
\minimize \max_{i=1}^{m} \sum_{j=1}^{n} p_{i} \Delta \phi_{i,j} - \min_{i=1}^{m} \sum_{j=1}^{n} p_{i} \Delta \phi_{i,j}, \tag{11}
\]

under the restrictions of probabilistic guarantees of precision levels given in Equation 6. This problem defines how the function place\(_{\text{optimized}}(\ldots)\) of Algorithm 6 must be implemented.

BRPP is NP-hard, which can be shown by reducing the Agent Bottleneck General Assignment Problem (ABGAP), which is known to be NP-hard \[33, 34\], to BRPP. ABGAP is defined as:

\[
\minimize \max_{i=1}^{m} \sum_{j=1}^{n} p_{i} \Delta \phi_{i,j} \tag{12}
\]

ABGAP is equivalent to our placement problem, since one can be polynomially transformed into another: If we simplify our problem by adding an LS with 0 risk, we can exclude the second term from Equation 11. This means that in order to solve our problem, we must also solve ABGAP. Thus, our problem is at least NP-Hard.

The total number of possible placement combinations for distributing \( n \) shares among \( m \) LSs is \( O(m^n) \). Since this number grows exponentially with the number of shares, an exhaustive search is very costly for larger \( m \) and \( n \). Even relatively small numbers such as \( m = 5 \) and \( n = 15 \) require the analysis of more than \( 3 \times 10^{10} \) combinations. To reduce the computational complexity, we implement a linear-time heuristic. Our goal is not to find the best placement among all possible combinations, but to find a placement which satisfies the required probabilistic guarantees. To this end, we need a strategy that guides our search for a secure placement in a reasonable (linear) time.

In general, problem-specific heuristics or meta-heuristics can be used to find heuristic solution. We applied the meta-heuristic of genetic algorithms \[35\]. Genetic algorithms reproduce the process of biological evolution. They work on multiple solution candidates by combining and mutating them into new possible solutions. Each new solution (in our case, a share placement) is rated according to a fitness (objective) function defined by Equation 11. Then, the best placements in terms of the objective functions are selected, and the cycle can repeat until the goal is reached or the limit of cycles is achieved.

We implemented a genetic algorithm for share placement as shown in Algorithm 7. The input parameters are the probabilities \( P_k(\phi_k) \), the set \( L \) of size \( m \), and the fixed set
Algorithm 7 Genetic Algorithm for Share Placement

```
1: function place_optimized(S, L', m)
2:     t ← 0
3:     Popul[1...10] ← RandomPlacement(S, L', m)
4:     while t < 200 and ∀ Pk < Pk_attack do
5:         for p = 1 to 40 do
6:             i1 ← RandomInteger(m); i2 ← RandomInteger(m)
7:             u ← RandomBoolean()
8:             if u then
9:                 PopulTemp[p] ← Cross(PopulTemp, i1, i2)
10:             end if
11:         end for
12:         Evaluate(PopulTemp)
13:     Popul ← Select10Best(PopulTemp)
14:     P_attack(φ) ← BestLevels(Popul)
15:     t ← t + 1
16: end while
```

of shares $S$ of size $n$. First, we define the initial population as 10 random placements (line 3). Then, we build a population of 40 new placements by recombining two placements with a uniform crossover (with a probability of 50%) (lines 5-10). Afterwards, the placement is mutated by changing one assignment randomly (line 11), ensuring that theoretically all possible placements could be created.

Next, the 40 created placements are rated according to the objective function, and 10 best placements are selected (lines 13-14). This cycle is iterated 200 times or stopped if the conditions of Equation 11 are satisfied (lines 4-17). The value of 200 iterations is selected such that it ensures convergence. Our experiments have shown that we already achieve a near-optimal placement solution after about 20 iterations. If after all cycles the probabilistic guarantees are still not satisfied (line 4), we say that the solution cannot be found for the given input parameters.

6. Optimization of Position Update Algorithm

Our basic approach [1, 2] described in Section 5 allows for sharing user’s position among multiple non-trusted LSs, but for the price of increased communication overhead. A complete set of shares has to be re-generated and sent to the corresponding set of LSs every time a position update event is triggered. More precisely, an MO must send $n$ messages with new position shares to $n$ different LSs, while an LBA must receive $k$ messages from $k$ LSs in order to obtain the position of the $k$th precision level. This principle can produce a high communication overhead, e.g., if the update rate is high and the number of LSs is large. However, in many cases such as sporadic movements of the MO, the re-generation and update of the whole share set causes redundancy. Hence, our goal is to send a smaller number of messages than $n$ after each position change.
In this section, we begin by defining the problem of message reduction. Then, we describe an optimized position update algorithm for position sharing approach.

6.1. Problem Statement: Minimization of Position Update Messages

We formulate the reduction of position updates as a constrained optimization problem. The optimization goal is to reduce the total number of position update messages being sent from MOs to LSs (denoted as the number of messages $N_{MO-LS}$). The constraints are that there should be no change of position precision $\phi_k$ as a result, as well as no reduction of the user’s probabilistic guarantees $P_{k,attack}(\phi_k)$ of precision (privacy) levels.

We assume the following values to be given:
- $n$ location servers,
- the MO’s previous consecutive precise position $\pi_i$, i.e., the position before $\pi_{i+1}$ (the algorithm is run on the MO side, which means that the MO’s own precise positions are available),
- the MO’s next consecutive precise position $\pi_{i+1}$, i.e., the position after $\pi_i$,
- master share $s_0$ generated for $\pi_i$,
- set $S_i$ of $n$ refinement shares $s_1 \ldots s_n$ generated for $\pi_i$,
- probability distribution $P_k(\phi_k)$, which specifies the required probabilistic guarantees for each precision level $\phi_k$.

Problem: Find the set of shares $S_{i+1}^{opt}$ such that $S_{i+1}^{opt}$ requires the minimal number of update messages from MOs to LSs. In other words, $S_{i+1}^{opt}$ ($S_{i+1}^{opt} = S_{i+1}^{opt}$) and $S_i$ should differ in as few shares as possible, i.e., in $S_{i+1}^{opt}$ as many shares as possible should be reused from $S_i$.

The concatenation of all shift vectors of $S_{i+1}^{opt}$ must point to $\pi_{i+1}$:

$$S_{i+1}^{opt} = \{s_0^{i+1} \ldots s_n^{i+1}\} : \sum_{k=0}^{n} s_k^{i+1} = \pi_{i+1}, \quad (13)$$

The precision $\phi_k$ of each imprecise position $p_{i+1}^{k}$ derived by share fusion after obtaining the minimized set $S_{i+1}^{opt}$ has to be the same as the precision of the corresponding imprecise position $p_k^{i}$ derived from the original set of shares $S_i$:

$$\forall S_i^{opt}, \quad S_i \in S^i, \quad S_{i+1}^{opt} = S_{i+1}^{opt} : \phi_k(p_{i+1}^{k}(S_{i+1}^{opt})) = \phi_k(p_k^{i}(S_i)) \quad (14)$$

Finally, the set of shares $S_{i+1}^{opt}$ must also satisfy the current user’s privacy requirements, i.e., each further $k$th share must provide the pre-defined probability $P_k(\phi_k)$:

$$\forall \phi_{k,attack} : P_k(\phi_k) > Pr[\phi_{k,attack} \leq \phi_k]; \quad (15)$$

Note that we do not assume that an MO’s complete trajectory is available. We consider only the neighboring consecutive position updates. Hence, we cannot apply statistical analysis of the past positions and the respective parameters such as speed, and therefore we do not consider approaches for preserving privacy of a complete trajectory.
6.2. Optimized Position Update Algorithm

Depending on the movement scenario, different position update approaches can be beneficial.

The key factor which separates sporadic and continuous update scenarios is the relation of the distance traveled between two consecutive updates and the radii of obfuscation circles.

If MO moves fast or the update rate is very low, the new MO’s master share can be located completely outside the previous master share, as depicted in Figure 7. The condition of having no intersection between two consecutive master shares is:

\[
\text{distance}(p_{i+1}^0, p_i^0) > 2 \times r_0
\]  

In the following, we will only consider sporadic position updates with long time intervals between updates rather than the continuous tracking of users. Thus, we assume that the time between updates is long enough such that there is no relation between consecutive updates, which an attacker could exploit. An optimized position sharing approach focusing on continuous updates using, for instance, dead-reckoning techniques, has been described by us in [36].

The main idea of our optimized position update algorithm is that under the condition of Equation 16 we can recalculate and update only the master share while keeping the refinement shares unchanged.

Now we can estimate the communication costs for the optimized position update approach. Since only one share has to be updated, the number of sent messages between MO and LS is \( N_{\text{MO-LS}}^{\text{opt}} = 1 \). In contrast, the basic position update approach requires all shares to be re-generated and sent, i.e., \( N_{\text{basic}} = n \). The resulting reduction rate of the communication cost is:

\[
R_{\text{opt-LS}} = \frac{n - 1}{n}
\]  

Note that since \( N_{\text{opt-LS}}^{\text{MO}} = 1 \), our optimized position updated algorithm is always optimal w.r.t. the cost of communication between MO and LSs.

The pseudocode for the optimized position update approach run by MO is presented in Algorithm 8. Before sending a location update, the MO determines whether the optimized approach is applicable by checking the condition of Equation 16(line 2). If the condition of Equation 16 is satisfied, only a new master share has to be generated.
and sent (lines 3-4) to the corresponding LS. The refinement shares $s_1 \ldots s_n$ will remain the same without causing any inconsistency during their fusion. This is enabled by the fact that shares are relative shift vectors, while the absolute coordinates are only contained in the master share $s_0$. If the condition of Equation 16 is not satisfied, the basic position update protocol is applied (lines 5-7).

```
Algorithm 8 Optimized Position Update Algorithm
1: function update_shares($\pi_i, \pi_{i+1}, n, s_0 \ldots s_n$)
2:     if distance($p_{i0}^0, p_{i+1}^0$) > $2 \ast r_0$ then
3:         update_shares_opt($\pi_i, \pi_{i+1}, n, s_0$)
4:     send($s_0$)
5:     else
6:         $s_0 \ldots s_n \leftarrow$ regenerate_all_shares($p_{i+1}, n, \phi_{min}, \Delta\phi$)
7:     send($s_0 \ldots s_n$)
8: end if
```

6.3. Security of Position Updates

The first privacy requirement corresponding to our problem statement (Equation 14) is that the position update optimization must not reduce the obfuscation area, i.e., it must not cause an undesired increase in position precision $\phi_k$. Regarding this condition, we can state that the proposed optimized location update algorithm does not reduce the number of shares, i.e., it does not change the precision level available to authorized LBAs. In other words, the smaller number of shares sent from MO to LS does not affect the number of shares provided to LBAs. Therefore, no change of precision occurs.

The second privacy requirement corresponding to our problem statement (Equation 15) is the probabilistic metric $P_k(\phi_k)$. According to the optimized location update algorithm, separate location updates only update master shares $s_{i0}^0$. The remaining refinement shares $s_{i1} \ldots s_{in}$ and the corresponding refined obfuscation circles remain unchanged, i.e., the attacker does not get new knowledge. Thus, the randomness of share generation is preserved by the unchanged share generation algorithms (Algorithm 2, Algorithm 4), and the probabilities $P_k(\phi_k)$ are the same as in the basic position sharing approaches.

7. Evaluation

Next, we present the evaluation of our share placement algorithm and the optimized position update algorithm. We start with an evaluation of the share placement runtime performance, before we compare the probabilistic guarantees of precision levels of our basic approach with the probabilistic guarantees resulting from optimized share placement. Then, we evaluate the communication cost of the optimized position update algorithm.

20
7.1. Runtime Performance of Placement Optimization

According to the principle of position sharing, share placement has to be calculated on the mobile device of the user, since it is the only trusted entity in our system model. Since mobile devices are typically restricted in terms of processing power and energy, the runtime of our share placement algorithm is crucial. Therefore, we measured the runtime of calculating the placement of a set of shares on a state of the art mobile device (HTC Desire with Android OS, CPU: 1 GHz Qualcomm QSD8250 Snapdragon, memory: 576 MB RAM). We tested the full number of cycles of the genetic algorithm, without terminating the algorithm under the “solution found” condition (i.e., we have tested the worst case scenario, when the solution is not feasible for the given parameters). The number of LSs was given as $m = 5; 10; 20$, and the number of shares $n$ is in the interval $[m; 50]$.

Figure 8a shows the average runtime for placing $n$ shares on $m$ LSs. As our evaluation shows, the proposed Algorithm has linear complexity and is executed in less than one second even for larger input parameters ($m = 20$, $n = 50$). Therefore, we conclude that the algorithm is suited also for resource-poor mobile devices.

7.2. Probabilistic Guarantees of Precision Levels after Placement Optimization

Next, we compare the resulting probabilistic guarantees of precision levels of optimized share placement compared to a basic (non-optimized) placement algorithm. We placed $n = 15$ shares on $m = 5$ LSs with heterogeneous risks. The values of risk were chosen uniform at random from the interval $[0; 0.5]$: $p_1 = 0.4932; p_2 = 0.3292; p_3 = 0.2344; p_4 = 0.1788; p_5 = 0.0925$. The basic algorithm distributes an equal number of shares (3) to each LS, while the optimized placement placed 1, 2, 2, 3, and 7 shares onto the given LSs.

Figure 8b depicts the probabilities $P_{k,\text{attack}}$ for the different precision levels $\phi$. Note that the precision levels $\phi_{k,\text{attack}}$ which correspond to the probability levels $P_{k,\text{attack}}$ are calculated as the weighted average of position precisions of each possible $k$-combination. In Figure 8b, we can see that for the same precision $\phi$, the probabilities $P_{k,\text{attack}}$ are significantly lower for the most $\phi$ values. In other words, the optimized
placement of shares has shifted the peaks of disclosure probability to the left. For example, for \( \phi = 25 \text{ km} \), \( P_{k,\text{attack}} = 10.7\% \) for the optimized share placement and \( P_{k,\text{attack}} = 40.1\% \) for the uniform share placement.

7.3. Communication Cost after Position Update Optimization

Next, we evaluate the communication overhead of the basic and optimized location update protocol. As performance metric we use the number of messages sent to LSs by the different update protocols.

For this evaluation, we used the GeoLife data set [37], including real trajectories of periodic trips to work, or hiking and biking trips. From this data set, we have selected two trips with different update intervals and distances between updates. The first trip has shorter update intervals (up to 15 s) and distances between updates (tens of meters). The second trip has larger update intervals (several minutes to more than one hour) and distances between updates (up to kilometers).

Figure 9 shows the number of update messages sent for each location update (i.e., per position change of the trip) by the basic and optimized update protocol, respectively, for short update intervals. The number of shares is \( n = 5 \). The radius of the master share is \( r_0 = 5 \text{ m} \) (Figure 9), \( r_0 = 50 \text{ m} \) and \( r_0 = 100 \text{ m} \) (Figures 10a, 10b).

The optimized update approach is often selected in Figure 9 due to too small radius of the master share (generating 6 messages per update), while for a larger radius and shorter update intervals the basic approach is applied more often (generating 20 messages per
update). For the given parameters, the basic approach generates 7280 messages in total for the given 364 position updates. The optimized approach generates 4158 messages in total (sending 6 messages instead of 20 in 223 out of 364 position updates), which corresponds to a reduction rate of 42.8%.

Figure 10 shows the number of update messages per location update for the trip with long update intervals. Again the number of shares is \( n = 5 \). In Figure 10a and Figure 10b, we set \( r_0 = 50 \) m and \( r_0 = 100 \) m, respectively. The basic approach generates 420 messages in total, with 20 messages per each update. The optimized position update algorithm with 6 messages per update can be selected often since the distance between the updates is large. The non-optimized updating with 20 messages per update is selected more often with \( r_0 = 100 \) due to a larger number of master share intersections. As a result, the optimized position update algorithm generates 154 for \( r_0 = 50 \) and 224 messages in total for \( r_0 = 100 \) (reduction rate of 63.3% and 46.7% respectively).

In Figure 11, we show the relative communication overhead of the optimized update protocol in relation to the basic update protocol for different radii \( r_0 \). Here, we used the trip with long update intervals. We can see that a larger \( r_0 \) usually causes fewer updates for larger \( r_0 \). At the same time, for smaller \( r_0 \) values (for the given mixed data set: between 0 and 300 m), a smaller \( r_0 \) leads to fewer intersections and, therefore, to a smaller number of update messages.

8. Discussion

Finally, we discuss limitations and possible extensions to our approach presented in this paper.

In this paper, we have only considered sporadic updates of positions rather than continuous updates. This is a reasonable assumption for many systems based on user check-ins to locations. Our basic assumption for sporadic updates is that there is no relation between two consecutive updates. In order for this assumption to hold, there must be sufficient time between consecutive updates depending on the distance between updates since otherwise an attacker can exclude some areas from the obfuscated area to increase the precision of obfuscated positions. In particular, we need to consider
the so-called Maximum Velocity Attack [38]. This attack limits the obfuscation area of subsequent updates by assuming a maximum MO speed and distance travelled since the last update. In order not to be prone to such attacks with our approach, the time interval between consecutive updates must be long enough such that the user could travel between both positions at least two times. Otherwise, we need to suppress the update. This is the well-known counter-measure against such an attack, described in the literature [38, 39].

It is also possible to consider a probability distribution (pdf) based on the movement correlation instead of one based on intersection of the binary movement boundary. However, such an approach requires analysis of the trajectory correlation pattern and is beyond the scope of this paper. We refer to our work [36], which extends the position sharing approach by considering trajectory data, and which estimates the resulting probabilistic privacy guarantees.

9. Conclusion

In this paper, we described our position sharing approach, which distributes position information of a mobile user among multiple location servers of non-trusted providers in the form of separate position shares. This approach has several interesting properties like no single point of failure with respect to privacy and graceful degradation of privacy with the number of compromised location servers.

In this work, we have further improved our basic position sharing approach. The first extension increases the user’s location privacy if the available location servers have different trust levels. Based on a probabilistic security metric, we proposed an approach which improves the user security by selecting the minimal required number of location servers and by optimizing the distribution of position shares among these servers. Our solution has linear complexity and in practice can be executed on resource-poor mobile devices. Our evaluations show that that this optimized share placement provides a 30%–40% lower probability of a user being discovered by an attacker at the same precision (privacy) levels.

The second extension of our basic approach is the optimization of the communication cost caused by position updates. Our calculation of communication cost takes into account the messages sent from mobile users to location servers as well as the messages sent from location servers to location-based applications. We have shown that our position update approach significantly reduces the communication cost achieving a traffic reduction of up to 60% of the non-optimized protocol version.

Acknowledgements

We gratefully acknowledge the German Research Foundation (Deutsche Forschungsgemeinschaft, DFG) for financial support of this research (PriLoc project).
References

[1] F. Dür, P. Skvortsov, and K. Rothermel, “Position sharing for location privacy in non-trusted systems,” in Proceedings of the 9th IEEE International Conference on Pervasive Computing and Communications (PerCom 2011). Seattle, USA: IEEE, March 2011, pp. 189–196.

[2] P. Skvortsov, F. Dür, and K. Rothermel, “Map-aware position sharing for location privacy in non-trusted systems,” in Proceedings of the 10th International Conference on Pervasive Computing (Pervasive 2012), Newcastle, UK, June 2012, pp. 388–405.

[3] “Privacy rights clearinghouse,” http://www.privacyrights.org/data-breach, 2015.

[4] M. F. Mokbel, “Privacy in location-based services: State-of-the-art and research directions,” in MDM, 2007, p. 228.

[5] D. Pedreschi, F. Bonchi, F. Turini, V. S. Verykios, M. Atzori, B. Malin, B. Moelans, and Y. Saygin, “Privacy protection: Regulations and technologies, opportunities and threats,” Mobility, Data Mining and Privacy, pp. 101–119, 2008.

[6] P. Kalnis, G. Ghinita, K. Mouratidis, and D. Papadias, “Preventing location-based identity inference in anonymous spatial queries,” IEEE Transactions on Knowledge and Data Engineering, vol. 19, no. 12, pp. 1719–1733, Dec. 2007.

[7] M. Y. Mun, D. H. Kim, K. Shilton, D. Estrin, M. H. Hansen, and R. Govindan, “Pëvloc: A personal data vault for controlled location data sharing,” TOSN, vol. 10, no. 4, pp. 58:1–58:29, 2014.

[8] M. Wernke, P. Skvortsov, F. Dür, and K. Rothermel, “A classification of location privacy attacks and approaches,” Personal and Ubiquitous Computing (Special Issue on Security and Trust in Context-Aware Systems), vol. 18, no. 1, pp. 163–175, 2014.

[9] D. Riboni, L. Pareschi, and C. Bettini, “Privacy in georeferenced context-aware services: A survey,” in Proceedings of the 1st International Workshop on Privacy in Location-Based Applications, October 2008, pp. 151–172.

[10] S. Mascetti, D. Freni, C. Bettini, X. S. Wang, and S. Jajodia, “Privacy in geo-social networks: proximity notification with untrusted service providers and curious buddies,” The VLDB Journal, vol. 20, no. 4, pp. 541–566, Aug. 2011.

[11] G. Zhong, I. Goldberg, and U. Hengartner, “Louis, Lester and Pierre: Three Protocols for Location Privacy,” Privacy Enhancing Technologies, pp. 62–76, 2007.

[12] H. Kido, Y. Yanagisawa, and T. Satoh, “An anonymous communication technique using dummies for location-based services,” in Proceedings of the International Conference on Pervasive Services (ICPS ’05), Jul. 11–14, 2005, pp. 88–97.

[13] P. Shankar, V. Ganapathy, and L. Ifode, “Privately querying location-based services with sybiquery,” in UbiComp, 2009, pp. 31–40.

[14] A. R. Beresford and F. Stajano, “Mix zones: User privacy in location-aware services,” in PerCom Workshops, 2004, pp. 127–131.

[15] B. Palanisamy and L. Liu, “Mobimix: Protecting location privacy with mix-zones over road networks,” in Proc. of the 2011 IEEE 27th Intl. Conf. on Data Engineering, ser. ICDE ’11. Washington, DC, USA: IEEE Computer Society, 2011, pp. 494–505.

[16] M. F. Mokbel, C.-Y. Chow, and W. G. Aref, “The new casper: query processing for location services without compromising privacy,” in Proceedings of the 32nd Intl. Conf. on Very large data bases (VLDB ’06). VLDB Endowment, 2006, pp. 765–774.

[17] A. Machanavajjhala, D. Kifer, J. Gehrke, and M. Venkitasubramaniam, “L-diversity: Privacy beyond k-anonymity,” ACM Transactions on Knowledge Discovery from Data, vol. 1, no. 1, p. 3, 2007.

[18] B. Bamba, L. Liu, P. Pesti, and T. Wang, “Supporting anonymous location queries in mobile environments with privacygrid,” in Proceeding of the 17th Intl. Conf. on World Wide Web (WWW ’08). New York, NY, USA: ACM, 2008, pp. 237–246.

[19] A. Solanas, F. Sebè, and J. Domingo-Ferrer, “Micro-aggregation-based heuristics for p-sensitive k-anonymity: one step beyond,” in Proc. of the 2008 international workshop on Privacy and anonymity in information society (PAIS ’08). New York, NY, USA: ACM, 2008, pp. 61–69.
[20] N. Li, T. Li, and S. Venkatasubramanian, “t-closeness: Privacy beyond k-anonymity and l-diversity,” in *Proc. of IEEE 23rd International Conference on Data Engineering (ICDE 2007)*, Apr. 15–20, 2007, pp. 106–115.

[21] R. C.-W. Wong, J. Li, A. W.-C. Fu, and K. Wang, “(alpha, k)-anonymity: an enhanced k-anonymity model for privacy preserving data publishing,” in *KDD*, 2006, pp. 754–759.

[22] C. Ardagna, M. Cremonini, E. Damiani, S. De Capitani di Vimercati, and P. Samarati, “Location privacy protection through obfuscation-based techniques,” in *Proc. of the 21st IFIP WG 11.3 Working Conf. on Data and Applications Security*, vol. 4602, 2007, pp. 47–60.

[23] G. F. Marias, C. Delakouridis, L. Kazatzopoulos, and P. Samarati, “Location privacy through secret sharing techniques,” in *WOWMOM ’05: Proceedings of the First International IEEE WoWMoM Workshop on Trust, Security and Privacy for Ubiquitous Computing*. Washington, DC, USA: IEEE Computer Society, June 2005, pp. 614–620.

[24] A. Shamir, “How to share a secret,” *Communications of the ACM*, vol. 22, no. 22, pp. 612–613, 1979.

[25] M. Wernke, F. Dur, and K. Rothermel. “PShare: position sharing for location privacy based on Multi-Secret sharing,” in *Proceedings of the 10th IEEE International Conference on Pervasive Computing and Communications (PerCom 2012)*. Lugano, Switzerland: IEEE, March 2012, pp. 153–161.

[26] I. Djordjevic, S. K. Nair, and T. Dimitrakos, “Virtualised trusted computing platform for adaptive security enforcement of web services interactions.” in *ICWS*. IEEE Computer Society, 2007, pp. 615–622.

[27] M. Kinateder, E. Baschny, and K. Rothermel, “Towards a generic trust model - comparison of various trust update algorithms,” in *iTrust*, 2005, pp. 177–192.

[28] A. Gutscher, J. Heesen, and O. Siemoneit, “Possibilities and limitations of modeling trust and reputation.” in *WSPI*, ser. CEUR Workshop Proceedings, vol. 332. CEUR-WS.org, 2008, pp. 1–12.

[29] A. Gutscher, “Reasoning with uncertain and conflicting opinions in open reputation systems,” *Electronic Notes in Theoretical Computer Science*, vol. 244, pp. 67–79, Aug. 2009.

[30] P. Skvortsov, “Position sharing for location privacy in non-trusted systems,” Ph.D. dissertation, Universität Stuttgart, Dissertation, July 2015.

[31] D. Pavlovic, “Dynamics, robustness and fragility of trust,” *CoRR*, vol. abs/0808.0732, 2008.

[32] P. Albrecht, “Risk based capital allocation,” *Encyclopedia of Actuarial Science*, Wiley, New York, USA, vol. 3, 2004.

[33] J. B. Mazzola and A. W. Neebe, “Bottleneck generalized assignment problems,” *Engineering Costs and Production Economics*, vol. 14, no. 1, pp. 61–65, 1988.

[34] S. Arora and M. C. Puri, “A variant of time minimizing assignment problem,” *European Journal of Operational Research*, vol. 110, no. 2, pp. 314–325, 1998.

[35] K. Weicker, *Evolutionare Algorithmen*. Stuttgart: Teubner, 2002.

[36] Z. Riaz, F. Dürr, and K. Rothermel, “Optimized Location Update Protocols for Secure and Efficient Position Sharing,” in *Proceedings of the 2nd International Conference on Networked Systems: NetSys 2015; Cottbus, Germany, March 9-13, 2015*. IEEE Computer Society, March 2015, pp. 1–8.

[37] Y. Zheng, X. Xie, and W.-Y. Ma, “Geolife: A collaborative social networking service among user, location and trajectory,” *IEEE Data Eng. Bull.*, vol. 33, no. 2, pp. 32–39, 2010.

[38] G. Ghinita, M. L. Damiiani, C. Silvestri, and E. Bertino, “Preventing velocity-based linkage attacks in location-aware applications,” in *GIS ’09: Proceedings of the 17th ACM SIGSPATIAL Intl. Conf. on Advances in Geographic Information Systems*. New York, NY, USA: ACM, 2009, pp. 246–255.

[39] M. Wernke, F. Dürr, and K. Rothermel, “PShare: Ensuring location privacy in non-trusted systems through multi-secret sharing.” *Pervasive and Mobile Computing*, vol. 9, no. 3, pp. 339–352, 2013.