Numerical Simulation for Dispersion Characteristics of Lamb Wave Propagation in Composite Laminates with Anti-Symmetric Surface Excitation

Qi Liu¹,²,*, Mengxue Liu¹,², Jindong Li¹,², Teng Wang¹,² and Wensheng Xiao¹,²

¹College of Mechanical and Electronic Engineering, China University of Petroleum, Qingdao, China
²National Engineering Laboratory for Marine Geophysical and Exploration Equipment, Qingdao, China

*Corresponding author: liuqi@s.upc.edu.cn

Abstract. For the nondestructive testing based on ultrasonic Lamb waves, the center frequency of the excitation signal should be appropriately selected to excite the specific Lamb wave mode in composite laminates according to the dispersion curves. However, it is difficult to achieve the dispersion curves of Lamb waves in composite laminates by using analytical methods. In this paper, the three-dimensional finite element model with the anti-symmetric surface excitation is established to analyze the dispersion characteristics of Lamb wave propagation in a composite laminate. The fundamental anti-symmetric (A0) Lamb wave mode is excited in the composite laminate by applying a concentrated force load and an anti-symmetric concentrated force load. The characteristics of Lamb wave propagation in the composite laminate are studied. Moreover, the group velocities of the A0 Lamb wave mode propagating in the composite laminate are calculated under the excitation signals with twelve different center frequencies. The group velocity dispersion curve of the A0 Lamb wave mode obtained by the established model is consistent with that of the guided wave software GUIGUW, which demonstrates the effectiveness and feasibility of the three-dimensional finite element model on analyzing the dispersion characteristics of Lamb wave propagation in the composite laminate.

Keywords: Lamb wave, composite laminates, finite element method, group velocity, dispersion curves.

1. Introduction

The composite materials have been widely applied to different engineering fields, such as aerospace industries and ship manufactures. However, various barely visible damages, which are caused by impacts and fatigues, can seriously decrease the strength and reliability of composite materials [1-3]. The nondestructive testing (NDT) based on ultrasonic Lamb waves has attracted more and more attention in the detection of these visible damages.

To apply Lamb waves to NDT, it is crucial to select a suitable frequency for the excitation signal to excite the specific Lamb wave mode in composite laminates due to the dispersion characteristics and
multi-mode characteristics of Lamb waves [4-6]. The center frequency of the excitation signal is generally obtained by using the dispersion curves of Lamb waves in the composite laminates, whereas it is difficult to achieve the dispersion curves by analytical methods.

In addition to analytical methods, the standard finite element software such as ANASY and ABAQUS, which provides numerical tools able to study complex structure, has been used to obtain the dispersion curves. Sorohan et al. [7] extracted the dispersion curves of Lamb waves in the composite laminates by using the finite element (FE) method based on two-dimensional solid structural elements. Edara [8] conducted the numerical finite element simulation for the study of the fundamental Lamb wave propagation in composite laminates. The dispersion curves were numerically extracted for the fundamental Lamb wave modes in the composite laminate by establishing finite element models based on shell and brick elements. Many research simplified the problem into two-dimensional finite element models to increase the computing efficiency [9, 10], whereas these simplified models may result in inexistent numerical modes due to the invalid assumptions [7].

The semi-analytical finite element (SAFE) method has been also developed for investigating the characteristics of Lamb wave propagation in composite laminates. Compared with the FE method, the SAFE method is more efficient for solving dispersion curves in composite laminates [11]. The GUIGUW software based on the SAFE method is a powerful tool to analyze the ultrasonic propagation characteristics of the guided wave in the isotropy materials and anisotropic materials [12]. By applying the SAFE method, Hayashi et al. [13] described the leaky Lamb wave and the dispersion curves which are agreed well with the previous theoretical studies. To obtain the exponential expansion along the propagating direction of Lamb waves, Gravenkamp et al. [14] proposed a scaled boundary finite element (SBFE) method in which only the discretization of the boundary was required. Samarutunga et al. [15] provided a wavelet spectral finite element model based on the first order shear deformation theory to analyze the Lamb wave propagation in anisotropic composite laminates.

In this paper, the three-dimensional finite element model with the anti-symmetric surface excitation is established to simulate the propagation of Lamb wave in a composite laminate. To investigate the dispersion characteristics of Lamb wave propagation in the composite laminate, only the fundamental anti-symmetric (A0) Lamb wave mode is excited by applying a concentrated force load and an anti-symmetric concentrated force load. The group velocity dispersion curve of the A0 Lamb wave mode propagating in the composite laminate are calculated and compared with that of the GUIGUW software.

2. Principle of Lamb waves

2.1. Lamb waves

Lamb waves can travel over a long distance in composite laminates. The ultrasonic guided waves propagate by the reflection of waves and the waveform changing on the surface of composite laminates. Various Lamb wave modes, which are the anti-symmetric modes (A0, A1, A2, ...) and symmetric modes (S0, S1, S2, ...), are formed during the guided wave propagation in composite laminates [16]. The phenomenon that there are at least two Lamb wave modes at the same frequency is called the multi-modal characteristics. Moreover, the phenomenon that the wave velocity varies with the frequency or wavelength is called the dispersion characteristics. The dispersion characteristics of Lamb waves are presented by the wave number, wavelength, phase velocity, and group velocity [17-19]. The Lamb wave propagation in composite laminates is complicated due to the dispersion characteristics and multi-modal characteristics.

According to the harmonic displacement solutions for waves propagating in an in-plane direction and the free boundary conditions, the eigenvalue problem is presented. The dispersion characteristics of Lamb waves in the isotropic plate are derived by Rayleigh Lamb equation, which are formulated as follows [20-22]:
\[
\frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2pq}{(q^2-k^2)^2} \quad \text{(symmetric mode)}
\]  
\[
\frac{\tan(qh)}{\tan(ph)} = -\frac{(q^2-k^2)^2}{4k^2pq} \quad \text{(anti-symmetric mode)}
\]

Where \( p^2 = \frac{\omega^2}{c_p^2} - k^2 \), \( q^2 = \frac{\omega^2}{c_i^2} - k^2 \), \( k = \frac{\omega}{c_p} \), \( \omega = 2\pi f \). Here, \( k \) is the wave number, \( f \) is the frequency of Lamb waves, \( c_p \) is the phase velocity, \( c_i \) is the transverse shear wave velocity, \( c_l \) is the longitudinal wave velocity, \( \omega \) is the circular frequency, and \( h \) is the thickness of the plate.

The transverse shear wave velocity and longitudinal wave velocity depend on the physical parameters of materials. For isotropic materials, the formulae are given as follows:

\[
c_i^2 = \frac{\lambda + 2\mu}{\rho}
\]

\[
c_i^2 = \frac{\mu}{\rho}
\]

From the classical laminate theory:

\[
\mu = \frac{E}{2(1+\nu)}
\]

\[
\lambda = \frac{\nu E}{(1-2\nu)(1+\nu)}
\]

Where \( E \) is the elastic modulus of materials, \( \nu \) is the Poisson's ratio of materials, and \( \lambda \) is the first order Lamet's constant of materials. \( \mu \) is the second order Lamet's constant of materials, which is known as the shear modulus of materials.

The relationship between group velocity \( c_g \) and phase velocity \( c_p \) is given by Eq. (7) [23]:

\[
c_g = c_p^2 \left[ c_p - fd \frac{dc_p}{d(fd)} \right]^{-1}
\]

Where \( fd \) is the frequency-thickness product.

2.2. Dispersion curve modeling in composite laminates

The second order Mindlin theory, in which the influence of the spatial derivative of the transverse shear deformation and plane rotation angle is considered, has been widely applied to the dynamic analysis of composite material structures [24]. The displacements of composite laminates are defined as follows:
\[
\begin{align*}
\mathbf{u}(x, y, z, t) &= u_0(x, y, t) + z\psi_x(x, y, t) + \frac{z^2}{2}\phi_x(x, y, t) \\
\mathbf{v}(x, y, z, t) &= v_0(x, y, t) + z\psi_y(x, y, t) + \frac{z^2}{2}\phi_y(x, y, t) \\
\mathbf{w}(x, y, z, t) &= w_0(x, y, t) + z\psi_z(x, y, t)
\end{align*}
\] (8)

Where \( u_0, v_0, \) and \( w_0 \) are the displacements of the neutral plane in \( x, y, \) and \( z \) directions. \( \psi_x \) and \( \psi_y \) represent the local rotation angles in \( x \) and \( y \) directions of the line that is perpendicular to the neutral plane before the deformation, and \( \psi_z \) is the strain in \( z \) direction. \( \phi_x \) and \( \phi_y \) are the derivatives of the rotation angles in \( x \) and \( y \) directions with respect to \( z \) direction.

The differential equation of Lamb wave propagation in composite laminates is solved by the principle of virtual work:

\[
0 = \int_{t_1}^{t_2} (\delta U + \delta V - \delta K) dt
\] (9)

Where \( \delta U \) is the energy of the virtual strain, \( \delta V \) is the virtual work generated by the external forces, and \( \delta K \) is the energy of the virtual work.

Suppose the formulae of the displacement solution are:

\[
\begin{align*}
\mathbf{u}_0 &= U_0 e^{i(k_1 x + k_2 y) - \omega t} \\
\mathbf{v}_0 &= V_0 e^{i(k_1 x + k_2 y) - \omega t} \\
\mathbf{w}_0 &= W_0 e^{i(k_1 x + k_2 y) - \omega t}
\end{align*}
\] (10)

\[
\begin{align*}
\psi_x &= \Psi_x e^{i(k_1 x + k_2 y) - \omega t} \\
\psi_y &= \Psi_y e^{i(k_1 x + k_2 y) - \omega t} \\
\phi_x &= \Phi_x e^{i(k_1 x + k_2 y) - \omega t} \\
\phi_y &= \Phi_y e^{i(k_1 x + k_2 y) - \omega t}
\end{align*}
\] (11)

Where \( k_1 \) and \( k_2 \) are the shear revision coefficients of composite laminates. Here, \( k_1^2 = k_2^2 = \pi^2/12 \).

The dispersion equations can be solved by the second order Mindlin theory and the principle of virtual work. However, it is difficult to express the formulae of the dispersion curves because the solutions are a set of transcendental equations [25].

3. Finite element simulation setup

3.1. Finite element model
The three-dimensional finite element model of a composite laminate is established in the ABAQUS software and given in Figure 1, which is composed of four unidirectional laminates with a symmetric stacking sequence \([0/90]_s\). The size of this finite element model is 200 mm × 200 mm × 2 mm. The material is T700/3234 carbon fiber epoxy, and its mechanical properties are given in Table 1.
Figure 1. Three-dimensional finite element model of the composite laminate with anti-symmetric surface excitation.
Table 1. Mechanics properties of the three-dimensional finite element model.

| Mechanics properties       | Values   |
|----------------------------|----------|
| Elastic modulus $E_{11}$ (Gpa) | 128      |
| Elastic modulus $E_{22}$ (Gpa) | 8.4      |
| Elastic modulus $E_{33}$ (Gpa) | 8.4      |
| Poisson's ratio $V_{12}$    | 0.32     |
| Poisson's ratio $V_{13}$    | 0.32     |
| Poisson's ratio $V_{23}$    | 0.45     |
| Shear modulus $G_{12}$ (Gpa) | 4        |
| Shear modulus $G_{13}$ (Gpa) | 4        |
| Shear modulus $G_{23}$ (Gpa) | 3.6      |
| Density $\rho$ (kg/cm$^3$)  | 1650     |

Additionally, six piezoelectric (PZT) wafers are bonded on the composite laminate. Two PZT wafers ($A$ and $B$), which are used as the actuators to provide an excitation signal, are placed on the centers of double surfaces of this composite laminate. When the same input burst signals are excited in the same direction by the two PZT wafers (which is the anti-symmetric surface excitation), the symmetric Lamb wave mode is enhanced, whereas the anti-symmetric Lamb wave mode is weakened. In this work, to only generate the fundamental anti-symmetric (A0) Lamb wave mode, a concentrated force load is applied to the upper surface of the composite laminate, and an anti-symmetric concentrated force load is applied to the lower surface of the composite laminate.

The velocities of Lamb waves propagating in the composite laminate are different in all directions due to its anisotropic characteristics. Thus, three PZT wafers ($C$, $D$, and $E$) are used as receivers to receive the displacement signals in the thickness direction of the composite laminate. A reference coordinate system is established based on the excitation point as the origin. The positions of the three PZT wafers are $C$ (0 mm, 10 mm), $D$ (5$\sqrt{2}$ mm, 5$\sqrt{2}$ mm), and $E$ (10 mm, 0 mm).

To accurately solve the velocities of Lamb waves propagating in the composite laminate, an additional PZT wafer $F$ besides the PZT wafer $E$ is set to receive the displacement signals in the thickness direction of the composite laminate. Since the distance between two PZT wafers ($E$ and $F$) should be as small as possible, the position of the PZT wafer $F$ in the model is $F$ (11 mm, 0 mm).

The point sources are employed to model the six PZT wafers as shown in Figure 1. This type of the model of point sources is described in detail in [26].

3.2. Excitation signal

Generally, there are two types of excitation signals to generate Lamb waves: the narrowband signal and broadband signal. Since the Lamb wave generated by the broadband signal has abundant frequency components, it is difficult to distinguish different modes of signals. Moreover, the long propagation distance of Lamb waves in composite laminates can result in the energy leakage and wave distortion of signals due to the dispersion characteristics [27]. Thus, the narrowband signal is usually used to suppress the dispersion phenomenon. In this work, the hamming windowed tone burst, which is consisted of $N$ cycles at the specific center frequency $f_0$, is selected as the excitation signal, and its formula is given as follows:

$$x(t) = \frac{1}{2} \times (1 - \cos \frac{2\pi f_0 t}{N}) \times \sin 2\pi f_0 t$$  \hspace{1cm} (13)

The time domain waveform of the excitation signal with the tone burst cycles of five at the center frequency of 150 kHz and its frequency spectrum obtained by the fast Fourier transform (FFT) method are shown in Figure 2.
3.3. Element size and time step

To solve the established finite element model, the explicit method is used in this work. A denser mesh and smaller time step can provide more accurate simulation results, whereas they require a longer calculation time and more computer resources. To guarantee the high efficiency along with the accuracy, the element type of SCR8 is selected, and the maximum element size $\Delta x$ and time step $\Delta t$ can be employed by Eq. (14) and Eq. (15) [28]:

$$\Delta x \leq \frac{\lambda_{min}}{20}$$  \hspace{1cm} (14)

$$\Delta t \leq \frac{1}{20f_{max}}$$  \hspace{1cm} (15)

Where $\lambda_{max} = \varepsilon_{p} / f_{max}$. Here, $f_{max}$ is the highest frequency of the excitation signal.

In this paper, the center frequencies of the excitation signal are set to 50kHz, 100kHz, 150kHz, 200kHz, 250kHz, 300kHz, 350KHz, 400kHz, 450kHz, 500kHz, 550kHz, and 600kHz. Thus, the parameter settings of the three-dimensional finite element model are listed in Table 2.

**Table 2. Parameter settings of the three-dimensional finite element model**

| $f_0$ (kHz) | $f_{max}$ (kHz) | $\Delta x$ (mm) | $\Delta t$ (s) |
|-------------|-----------------|-----------------|----------------|
| 50          | 100             | 4               | 5×10^{-7}      |
| 100         | 150             | 3               | 3×10^{-7}      |
| 150         | 200             | 2               | 2.5×10^{-7}    |
| 200         | 250             | 1.6             | 2×10^{-7}      |
| 250         | 300             | 1.4             | 1.5×10^{-7}    |
| 300         | 350             | 1.2             | 1.4×10^{-7}    |
| 350         | 400             | 1.0             | 1.2×10^{-7}    |
| 400         | 450             | 0.9             | 1.1×10^{-7}    |
| 450         | 500             | 0.8             | 1×10^{-7}      |
| 500         | 550             | 0.7             | 9×10^{-8}      |
| 550         | 600             | 0.7             | 8×10^{-8}      |
| 600         | 650             | 0.6             | 7×10^{-8}      |
4. Numerical results and discussions

4.1. Lamb wave propagation in composite laminates

The displacement fields of A0 Lamb wave modes propagating in the composite laminate under the excitation signals with twelve different center frequencies are shown in Figure 3. As can be seen from Figure 3, Lamb waves propagate to all around in an approximate diamond symmetry based on the excitation point as the center. This is because the composite laminate with a symmetric stacking sequence $[0/90]_2s$, has the orthotropic characteristics under the reference coordinate system.

Moreover, the velocity of the A0 Lamb wave mode propagating in the composite laminate increases as the center frequency of the excitation signal increases, and the reflected wave caused by the boundary of the composite laminate begins to appear when the center frequency of the excitation signal is greater than 300 kHz.

The vertical displacement signals received by three PZT wafers $(C, D,$ and $E)$ under the excitation signal with the center frequency of 150 kHz are shown in Figure 4. The displacement signal obtained by the PZT wafer $C$ is approximately coincided with that of the PZT wafer $E$. The amplitude of the displacement signal and the wave velocity obtained by the PZT wafer $D$ are higher than those of other two PZT wafers. In brief, the reliability of the established finite element model is validated.
4.2. Group velocity dispersion curve of Lamb wave

To further validate the effectiveness and feasibility of the established finite element model on analyzing the dispersion characteristics of Lamb wave propagation in the composite laminate, the group velocity dispersion curve of A0 Lamb wave mode is extracted by the established model. The phase velocity is the propagation velocity of the wave at a certain phase point. Different harmonics propagate at different phase velocities during the Lamb wave propagation in the composite laminate, whereas Lamb wave propagates at the group velocity after the superposition of different wave groups.

The group velocity of A0 Lamb wave mode propagating in the composite laminate is achieved by:

\[ c_g = \frac{s_{EF}}{t_{DOA}} \]  

(16)

Where \( s_{EF} \) is the distance between two PZT wafers E and F, and \( t_{DOA} \) is the time difference between the arrive times of A0 Lamb wave modes received by two PZT wafers E and F.

Figure 5. Displacement signals received by two PZT wafers (E and F).

Figure 5 describes the vertical displacement signals obtained by two PZT wafers E and F under the excitation signal with the center frequency of 150 kHz. As shown in Figure 5, the amplitude of the displacement signal attenuates as the distance of Lamb wave propagation in the composite laminate increases.

| Table 3. Group velocities of A0 Lamb wave modes obtained by ABAQUS and GUIGUW. |
|-------------------------------|-----------------|-----------------|--------|
| Frequency (kHz)    | ABAQUS (m/s) | GUIGUW (m/s) | Relative Error |
| 50                | 826.4         | 877.5          | 5.8%   |
| 100               | 1082.3        | 1073           | 0.9%   |
| 150               | 1142.2        | 1170           | 2.4%   |
| 200               | 1165.5        | 1222           | 4.6%   |
| 250               | 1213.9        | 1259           | 3.6%   |
| 300               | 1238.3        | 1287           | 3.8%   |
| 350               | 1276.2        | 1308           | 2.4%   |
| 400               | 1297.7        | 1325           | 2.1%   |
| 450               | 1308.2        | 1339           | 2.3%   |
| 500               | 1327.6        | 1351           | 1.7%   |
| 550               | 1345.5        | 1361           | 1.1%   |
| 600               | 1352.3        | 1369           | 1.2%   |
According to Eq. (16), the group velocities of A0 Lamb wave modes propagating in the composite laminate are calculated under the excitation signals with twelve different center frequencies. Moreover, the group velocities of A0 Lamb wave modes obtained by the established finite element model in the ABAQUS software are compared with those of the GUIGUW software. The comparative results are recorded in Table 3. Figure 6 depicts the group velocity curves of A0 Lamb wave mode for the composite laminate obtained by two methods.

As shown in Table 3 and Figure 6, the maximum relative error is 5.8% and the minimum relative error is 0.9, which demonstrates that the established finite element model can be used to analyze the dispersion characteristics of Lamb wave propagation in the composite laminate.

5. Conclusions
Aiming at the problem that it is difficult to achieve the dispersion curves of Lamb waves in composite laminates by using analytical methods, the three-dimensional finite element model with the anti-symmetric surface excitation is adopted to analyze the dispersion characteristics of Lamb wave propagation in a composite laminate in this paper. The three-dimensional finite element model is established in the ABAQUS software, which is made of four unidirectional laminates with a symmetric stacking sequence [0/90], and six point sources modeling the PZT wafers. The A0 Lamb wave mode is excited by applying a concentrated force load and an anti-symmetric concentrated force load to the upper and lower surfaces of the composite laminate respectively.

The excitation signals with twelve different center frequencies are employed. The characteristics of Lamb wave propagation in the composite laminate are analyzed, which validates the reliability of the established finite element model. Additionally, the group velocities of A0 Lamb wave modes propagating in the composite laminate are calculated by the established finite element model and compared with those of the GUIGUW software. The results indicate that the group velocity dispersion curve of the A0 Lamb wave mode extracted by the established model is consistent with that of the GUIGUW software. Thus, the three-dimensional finite element model with the anti-symmetric surface excitation is effective and feasible on the analysis of the dispersion characteristics of Lamb waves for composite laminates.

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