Multiphonon nuclear response in $^{16}\text{O}$: A microscopic treatment equivalent to shell model

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Abstract. We have reformulated an equation of motion method for generating iteratively a basis of multiphonon states built of Tamm-Dancoff phonons. Represented in such a basis, the Hamiltonian matrix has a simple structure and can be easily brought to diagonal form. The method is here adopted to compute the energy levels and the electromagnetic responses in $^{16}\text{O}$, chosen as testing ground because of its complex structure.

1. Introduction
The increasing evidence of multiphonon collective modes at low-energy [1], and of multiple giant resonance at high energy [2] requires theoretical studies which go beyond Tamm-Dancoff (TDA) or random-phase approximations (RPA). An extension of these, basically harmonic, approximations, is also needed in order to describe the widths and damping of giant resonances.

The existing extensions, known as second RPA (SRPA), derive equations where the RPA particle-hole ($\text{ph}$) are coupled to the $2\text{ph}$ configurations. We mention here a recent calculation using a Skyrme force [3] and a relativistic RPA (RRPA) plus phonon coupling (PC) approach applied to the giant dipole resonance (GDR) and to the pygmy giant resonance in spherical open shell nuclei [4].

All the above approaches include up to $2\text{ph}$ configurations and, therefore, are more suited to describe the fragmentation of collective modes. Moreover, they rely on the quasiboson approximation, valid in the small amplitude vibrational limit.

Here we report of a reformulation of an equation of motion multiphonon approach (EMPM) [5, 6] which generates iteratively a multiphonon basis built of Tamm-Dancoff phonons and solves the eigenvalue problem in such a basis. The method is basically free of approximations and, therefore, equivalent to shell model.

In order to illustrate its potential we solved the eigenvalue problem for $^{16}\text{O}$ in a space which includes up to three phonons. The isovector $E1$ response will be discussed.

2. The method
We consider a two-body Hamiltonian $H$ in the second quantized form

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \frac{1}{4} \sum_{ijkl} V_{ijkl} a_i^\dagger a_j^\dagger a_k a_l ,$$

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where \( \epsilon_i \) are the single-particle energies, \( V_{ijkl} \) the antisymmetrized matrix elements of the nucleon-nucleon interaction, \( a_i^\dagger \) (\( a_i \)) the creation (annihilation) particle operators with respect to the physical vacuum.

We intend to construct \( n \)-phonon states \(|n; \beta \rangle\) having the following structure

\[
|n; \beta \rangle = \sum_{\gamma} C_{\lambda}^{(\beta)} O_{\lambda}^\dagger |n - 1; \gamma \rangle,
\]

(2)

where \(|n - 1; \gamma \rangle\) is a \((n - 1)\)-phonon state upon which acts the particle-hole \((p - h)\) Tamm-Dancoff phonon operator

\[
O_{\lambda}^\dagger = \sum_{ph} c_{\lambda}(ph) a_p^\dagger a_h^\dagger
\]

(3)

and \( a_p^\dagger \) (\( a_h \)) creates a particle (hole) with respect to the \( ph \) vacuum \(|0\rangle\).

The starting point of the procedure consists in writing the equations of motion

\[
<n, \beta || [H, O_{\lambda}^\dagger] || n - 1, \alpha > = \left( E^{(n)}_{\beta} - E^{(n-1)}_{\alpha} \right) <n, \beta | O_{\lambda}^\dagger | n - 1, \alpha >.
\]

(4)

After expanding the commutator and making several manipulations [7], we get, for the \( n \)-phonon subspace, the eigenvalue equation

\[
ADC = EDC,
\]

(5)

where \( D(\lambda \alpha, \lambda' \alpha') = <n - 1, \alpha' || O_{\lambda'} O_{\lambda}^\dagger || n - 1, \alpha > \) is the metric matrix and

\[
A(\lambda \alpha, \lambda' \alpha') = (E_{\lambda} + E_{\alpha}) \delta_{\lambda' \lambda} \delta_{\alpha' \alpha} + V_{\lambda \alpha, \lambda' \alpha'}.
\]

(6)

Here, \( E_{\lambda} \) and \( E_{\alpha} \) are the energies of the TDA phonon \(|\lambda \rangle\) and the \((n - 1)\)-phonon state \(|(n - 1)\alpha \rangle\). They interact through a potential of the form

\[
V_{\lambda \alpha, \lambda' \alpha'} = \sum_{rs \lambda' q} \rho^{(n-1)}_{\alpha \alpha'}(rs) V_{r \alpha s \lambda' q}(tq),
\]

(7)

where \( \rho^{(n)}_{\alpha \alpha'}(rs) = <n, \alpha' || a_r^\dagger a_s || n, \alpha > \) are either particle \((r = p, s = p')\) or hole \((r = h, s = h')\) one-body densities.

All quantities are given by recursive formulas, which allows to solve the eigenvalue Eq. (5) iteratively once the TDA phonons are generated. The redundant states are eliminated by the procedure outlined in [5, 6], based on the Clolesky decomposition method.

A set of orthonormal multiphonon states \( \{|0\rangle, |n = 1, \lambda \rangle, \ldots |n, \alpha \rangle \ldots \} \) is thus generated. In each \( n \)-phonon block, the Hamiltonian is diagonal. Only subspaces differing by at most two-phonons are coupled. The matrix elements \(<n', \beta || H || n, \alpha >\) \((n' = n + 1, n + 2)\) are given by simple recursive formulas so that it is a simple matter to diagonalize the Hamiltonian in the full space.

3. A numerical testground: \(^{16}\text{O}\)

We have chosen \(^{16}\text{O}\) as testing ground for our method. We used realistic Hamiltonians and adopt either a Nilsson or Hartree-Fock single particle basis. All the shells up to the \( N = 3 \) \((p, f)\) have been include in order to construct the TDA phonons.

A full calculation up to three phonons is too lengthy due to the huge number of the three-phonon states. We have therefore truncated such a subspace by including the three phonon space up to the energy \( \simeq 45 \text{ MeV} \).
Figure 1. Isovector $E1$ strength distribution in $^{16}$O.

The $E1$ strength distribution computed in either Nilsson or HF basis is plotted in Fig. 1. The fragmentation is practically not affected by the number of phonons. It is, instead, strongly sensitive to the basis adopted. In fact, it increases considerably in going from Nilsson to HF.

The position of the peaks depend on both the basis and the number of phonons. In the Nilsson basis, the TDA main peak is at $\sim 22$ MeV, consistently with experiments. In HF, it is about 7 MeV higher.

If we include the states up to two-phonons, the peaks are shifted upward by 5 MeV in the HF basis and by $\sim 18$ MeV in Nilsson. This is simply due to the depression of the ground state induced by the coupling between the $ph$ vacuum and the two-phonon subspace. The ground state is depressed by 20.8 MeV in Nilsson case and 5.5 MeV in HF basis. Indeed, the coupling between states differing by two-phonons is the most effective one. One should, therefore, expect a corresponding shift for the excited states once the three-phonons are included.

Indeed, the coupling between one and three phonons shifts downward the peaks but not not enough. The main peak, in fact, is at $\leq 30$ MeV in both bases, too high with respect to the experimental value $\sim 21$ MeV. It is to be said, however, that only a fraction of the three phonon states have been included. We expect that, if the number of three-phonon states is incremented, the one- to three-phonon coupling would be more effective and, eventually, would bring the peaks close to the experimental ones.

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