Testing Bell’s Inequality with Ballistic Electrons in Semiconductors

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We propose an experiment to test Bell’s inequality violation in condensed-matter physics. We show how to generate, manipulate and detect entangled states using ballistic electrons in Coulomb-coupled semiconductor quantum wires. Due to its simplicity (only five gates are required to prepare entangled states and to test Bell’s inequality), the proposed semiconductor-based scheme can be implemented with currently available technology. Moreover, its basic ingredients may play a role towards large-scale quantum-information processing in solid-state devices.

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The introduction of Quantum Information Processing (QIP) has led, on the one hand, to unquestionable intellectual progress in understanding basic concepts of information/computation theory; on the other hand, this has stimulated new thinking about how to realize QIP devices able to exploit the additional power provided by quantum mechanics. Such novel communication/computation capabilities are primarily related to the ability of processing entangled states \(\ket{\psi}\). To this end, one should be able to perform precise quantum-state synthesis, coherent quantum manipulations (gating) and detection (measurement). The unavoidable interaction of any realistic quantum system with its environment tends to destroy coherence between quantum superpositions. Thus, decoherence modifies the above ideal scenario and imposes further constraints on QIP. Indeed, mainly due to the need of low decoherence rates, the only experimental realizations of QIP devices originated in atomic physics \(\ket{\psi}\) and in quantum optics \(\ket{\psi}\). It is however generally believed that any large-scale application of QIP cannot be easily realized with such quantum hardware, which does not allow the scalability of existing microelectronics technology. In contrast, in spite of the relatively strong decoherence, a solid-state implementation of QIP can benefit synergistically from the recent progress in single-electron physics \(\ket{\psi}\) as well as in nanostructure fabrication and characterization \(\ket{\psi}\).

As already mentioned, the key ingredient for computational speed-up in QIP is entanglement. Einstein-Podolsky-Rosen (EPR) pairs \(\ket{\psi}\) and three-particle Greenberger-Horne-Zeilinger (GHZ) states \(\ket{\psi}\) are at the heart of quantum cryptography, teleportation, dense coding, entanglement swapping and of many quantum algorithms. Experimentally, two-particle entangled states have been prepared using photons \(\ket{\psi}\) and trapped ions \(\ket{\psi}\); only recently a photonic three-particle entangled state (GHZ) has been also measured \(\ket{\psi}\). A few proposals for the generation of entangled states in solid-state physics have been recently put forward \(\ket{\psi}\) but up to date there are no experimental implementations.

In this Letter we propose an experiment to test Bell’s inequality violation in condensed-matter physics. More specifically, we shall show how to generate, manipulate and measure entangled states using ballistic electrons in coupled semiconductor quantum waveguides (quantum wires). As we shall see, our scheme allows for a direct test of Bell’s inequality in a solid-state system. To this end, a relatively simple gating sequence (five gates only) is identified.

The proposed experimental setup is based on the semiconductor quantum hardware of the earlier proposal for quantum computation with ballistic electrons by Ionicioiu et al. \(\ket{\psi}\). We summarize in the following the main features of this proposal, which has been recently analyzed and validated through numerical simulations by Bertoni et al. \(\ket{\psi}\).

The main idea is to use ballistic electrons as flying qubits in semiconductor quantum wires (QWRs). In view of the nanometric carrier confinement reached by current fabrication technology \(\ket{\psi}\), state-of-the-art QWRs behave as quasi one-dimensional (1D) electron waveguides. Due to the relatively large intersubband energy splittings as well as to the good quality of semiconductor/semiconductor interfaces, electrons within the lowest QWR subband at low temperature may experience extremely high mobility. In such conditions their coherence length can reach values of a few microns; therefore, on the nanometric scale electrons are in the so-called ballistic regime and the phase coherence of their wave functions is preserved. This coherent-transport regime is fully compatible with existing semiconductor nanotechnology \(\ket{\psi}\) and has been the natural arena for a number of interferometric experiments with ballistic electrons \(\ket{\psi}\). Such fully coherent regime is the basic prerequisite for any QIP.

The building block of our quantum hardware is a pair of adjacent QWR structures. The qubit state is defined according to the quantum-mechanical state of the electron across this two-wire system. More precisely, we shall use the so-called dual-rail representation for the qubit: we define the basis state \(|0\rangle\) by the presence of the electron in one of the wires (called the 0-rail) and the basis state \(|1\rangle\) by the presence of the electron in the other one (the 1-rail). Saying that the electron is in a given wire we mean that: (i) its wave-function is localized on that QWR and (ii) its free motion along the wire is well described in terms of a quasi-monoeenergetic wave-packet within the lowest QWR electron subband (with central kinetic energy \(E\) and central wave-vector \(k = \sqrt{2m^*E}/\hbar\)).

An appealing feature of the proposed scheme is the mobile character of our qubits: using flying qubits we can transfer entanglement from one place to another, without the need to interconvert stationary into mobile qubits. In the case of stationary qubits (e.g. electron spins in quantum dots) this is not
easily done.

Any QIP device can be built using only single- and two-qubit gates \[23\]. We choose the following set of universal quantum gates: \( \{ H, P_\varphi, P^C_\varphi \} \), where \( H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \) is a Hadamard gate, \( P_\varphi = \text{diag}(1, e^{i\varphi}) \) is a single-qubit phase shift, and \( P^C_\varphi = \text{diag}(1, 1, 1, e^{i\varphi}) \). We now briefly describe the physical implementation of the universal quantum gates in terms of the previously introduced dual-rail representation. The Hadamard gate can be implemented using an electronic beam-splitter, also called waveguide coupler \[24\] \[25\] \[26\]. The idea is to design the two-wire system in such a way to spatially control the inter-wire electron tunneling. For a given inter-wire distance, a proper modulation (along the QWR direction) of the inter-wire potential barrier can produce a linear superposition of the basis states \( |0\rangle \) and \( |1\rangle \). More specifically, let us consider a coupling window, i.e., a tunneling-active region, of length \( L_c \) characterized by an inter-wire tunneling rate \( \omega = 2\pi/\tau \). As it propagates, the electron wave-packet oscillates back and forth between the two waveguides with frequency \( \omega \). Let \( v = \hbar k/m^* \) be the group velocity of the electron wave-packet along the wire; then, the state \( |0\rangle \) goes into the superposition \( \cos \alpha |0\rangle + \sin \alpha |1\rangle \) with \( \alpha = \omega t = 2\pi L_c/v \). Let \( L_t \) be the length necessary for the complete transfer of the electron from one wire to the other, \( \alpha = \pi \), \( L_t = \pi \tau/2 \). For a transfer length \( L_c = L_t/2 \) the device is equivalent to a beam-splitter and hence, up to a phase shift, to a Hadamard gate. By a proper modulation of the inter-wire potential barrier we can vary the tunneling rate \( \omega \) and therefore the rotation angle \( \alpha \). As a result, this structure can operate as a NOT gate by adjusting the inter-wire potential barrier such that \( L_c = L_t \) (\( \pi \)-rotation). Similarly, the gate can be turned off by an appropriate potential barrier for which the electron wave-packet undergoes a full oscillation period, returning back to its original state \( (L_c = 2L_t, 2\pi \text{-rotation}) \). Another way of turning the \( H \) gate off is to suppress inter-wire tunneling by applying a strong potential bias to the coupled QWR structure.

The phase shifter \( P_\varphi \) can be implemented using either a potential step (with height smaller than the electron energy \( V < E \)) or a potential well along the wire direction; the well is preferred since the phase-shift induced is more stable under voltage fluctuations. In order to have no reflection from the potential barrier, the width \( L \) of the barrier should be an integer multiple of the half wavelength of the electron in the step/well region, \( L = n\lambda/2 \), \( n \in \mathbb{N} \).

We finally describe the two-qubit gate. In our scheme the controlled phase shifter \( P^C_\varphi \) is implemented using a Coulomb coupler \[27\]. This quantum gate exploits the Coulomb interaction between two single electrons in different QWR pairs (representing the two qubits). The gate is similar in construction to the beam splitter previously introduced. In this case the multi-wire structure (see Fig. \[3\]) needs to be tailored in such a way (i) to obtain a significant Coulomb coupling between the two \( I \)-rails only and (ii) to prevent any single-particle interwire tunneling. Therefore, only if both qubits are in the \( |1\rangle \) state they both experience a phase shift induced by the two-body Coulomb interaction. In contrast, if at least one qubit is in the \( |0\rangle \) state, then nothing happens.

The proposed quantum hardware has some advantages. Firstly, the QIP device needs not to be “programmed” at the hardware level (by burning off the gates), as it may appear. Programming is done by switching on/off the gates and this way any quantum algorithm can be implemented \[23\]. Secondly, we use cold programming, i.e., we set all the gates before “launching” the electrons, so we do not need ultrafast (i.e., subdecoherent) electronics for gate operations. This property is essential and is a distinct advantage of the proposed quantum architecture over other solid-state proposals \[29\]. Therefore, the gating sequence needed for the proposed experiment can be pre-programmed using static electric fields only.

One important requirement of our quantum hardware is that electrons within different wires need to be synchronized at all times in order to properly perform two-qubit coupling (the two electron wave-packets should reach simultaneously the Coulomb-coupling window). It is thus essential to have highly monoenergetic electrons launched simultaneously. This can be accomplished by properly tailored energy filters and synchronized single-electron injectors at the preparation stage.

We now turn to the proposed experimental setup for testing Bell’s inequality. Two-particle entangled states (Bell states) can be generated using three Hadamard gates and a controlled-sign shift (see dashed box in Fig. \[1\] the controlled sign shift plus the lower two Hadamards form a CNOT gate). Consider the correlation function for two (pseudo)spins \( P(a, b) = \langle \sigma^{(1)}_a \sigma^{(2)}_b \rangle \) (here, \( \sigma = \sigma_x, \sigma_y, \sigma_z \) is the pseudo-spin projection along the unit vector \( a \) \[10\]. Any local, realistic hidden-variable theory obeys the Bell-CHSH \[31\] \[52\] inequality:

\[
|P(a, b) + P(a', b) + P(a', b') - P(a, b')| \leq 2 \tag{1}
\]

This inequality is violated in quantum mechanics. For the singlet \( |\Psi^-\rangle \), a standard calculation gives the result

\[
P(a, b) \equiv \langle \Psi^- | \sigma^{(1)}_a \sigma^{(2)}_b | \Psi^- \rangle = -a \cdot b \tag{2}
\]

Choosing \( a = b \cdot a', b' = -b', a = \sqrt{2}/2 \), we obtain

\[
2\sqrt{2} \leq 2, \text{ violating thus Bell inequality} \[4\].
\]

Let us now focus on the correlation function \( P(a, b) \). In the EPR-Bohm gedankenexperiment we need to measure the spin component of one particle along a direction \( n \). However, in our setup this is not possible directly, since we can measure only \( \sigma_z \), i.e., whether the electron is in the \( 0 \)- or in the \( 1 \)-rail. The solution is to do a unitary transformation \( |\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle \), such that the operator \( \sigma_n \) is diagonalized to \( \sigma_z \), \( \langle \psi' | \sigma | \psi' \rangle = \langle \sigma_z | \psi' \rangle | \psi' \rangle \). We are looking for a unitary transformation \( U \) which satisfies \( U^+ \sigma_z U = \sigma_n \), with \( n = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \) a unit vector. In terms of our elementary gates we obtain \( U(\theta, \varphi) = H P_\varphi H P_\varphi H P_\varphi H P_\varphi H P_\varphi H P_\varphi H P_\varphi \).

Thus, measuring the spin (in the EPR-Bohm setup) along a direction \( n \) is equivalent to performing the unitary transformation \( U(\theta, \varphi) \) followed by a measurement of \( \sigma_z \). Going back to our entangled pair, we now apply on each qubit a local transformation \( U(\theta_1, \varphi_1) \) and \( U(\theta_2, \varphi_2) \), respectively. Here, \( a = (\theta_1, \varphi_1) \) and \( b = (\theta_2, \varphi_2) \) are the two directions dis-
Suppose that instead of having an ideal singlet state \(|\Psi^{-}\rangle\), we realize a perfect singlet \(|\Psi_{\alpha}\rangle\) by measuring the dependence of the correlation function \(S(\alpha, \theta)\) for \(\alpha = \pi\) and \(\theta = \pm \theta_{2}\). For what values of \(\alpha\) we obtain

\[
S(\alpha, \theta) \equiv \langle \Psi_{\alpha} | \sigma_{a}^{(1)} | \sigma_{b}^{(2)} | \Psi_{\alpha} \rangle = -\sin \frac{\alpha}{2} \sin \left( \theta + \frac{\alpha}{2} \right)
\]

which is a superposition of the singlet and of a separable state.

Let us now consider the experimental setup discussed above, with \(a = (0, \sin \theta_{1}, \cos \theta_{1})\) and \(b = (0, \sin \theta_{2}, \cos \theta_{2})\), both in the \(Oyz\) plane. For the correlation function of the ideal singlet

\[
|\Psi_{\alpha}\rangle = |\Psi^{-}\rangle + e^{i\alpha/2} \cos \frac{\alpha}{2} \frac{|1\rangle (|0\rangle - |1\rangle)}{\sqrt{2}}
\]

with \(\theta = \theta_{2} - \theta_{1}\). For \(\alpha = \pi\) we recover the correlation function of the singlet, \(S(\alpha, \theta) = S(\pi, \theta) = -\cos \theta\). The two functions are plotted in Fig. 3. \(S(\theta)\) can be identified by noting that there is no \(\alpha\) dependence.

Experimentally, since the one-qubit gate \(P_{0}\) is easier to control, we can measure the coupling \(\alpha\) of the Coulomb coupler \(P_{C}^{\alpha}\) by measuring the dependence of the correlation function \(S(\alpha, \theta)\) on the phase shift \(\theta\) (which can be accurately determined). This procedure can be used to determine the purity of the singlet, and hence to test/calibrate the Coulomb coupler.

We are now interested to see how small the coupling \(\alpha\) can be in order to still have a violation of Bell’s inequality. The question we ask is: For what values of \(\alpha\) the correlation function \(S(\alpha, \theta)\) in eq. (4) violates Bell’s inequality in Fig. 4? To this end, we have found a numerical solution: the inequality (4) is violated for \(\alpha \in (\pi/2, 3\pi/2)\).

A schematic representation of the proposed experimental setup for measuring Bell’s inequality violation is presented in Fig. 6. It is possible to reduce the number of gates on the 1-rail by using a phase shifter on the 0-rail \(P_{0}^{\alpha} \equiv \text{diag}(e^{-i\theta}, 1)\) instead of the 1-rail one used so far \(P_{1}^{\alpha} \equiv \text{diag}(1, e^{i\theta})\), since the two are equivalent (up to an overall phase) \(P_{1}^{\alpha} = e^{i\theta} P_{0}^{\alpha}\).

In our setup there are two different ways of producing a phase shift \(P_{0}\): (i) electrically, with a potential applied on top of the 0-rail (the quantum well described above); (ii) magnetically, via the Aharonov-Bohm effect, by applying locally a magnetic field on the area between the lower two beam-splitters (this can be done since the \(P_{0}\) and \(P_{C}^{\alpha}\) gates commute). The second method has the advantage of avoiding the no-reflection condition for the potential well (the length of the gate should be a half integer multiple of the electron wavelength). Either way can be used experimentally.

We stress that Aharonov-Bohm rings and quantum interference experiments with ballistic electrons are standard tools in mesoscopic physics. A two-slit experiment with an
Aharonov-Bohm ring having a quantum dot embedded in one arm has been reported in [21],[22]. This experiment is similar to the layout of the lower qubit in Fig. It but the authors do not use beam splitters and Coulomb couplers. In the experimental setup presented here the more difficult part will be to implement the Coulomb coupler (CC) and to perform the experiment at the single electron level. In our case preparation and measurement of the states are done using single electron pumps (SEPs) and single electron transistors (SETs), respectively.

In conclusion, we have proposed the first measurement of Bell’s inequality violation in coupled semiconductor nanostructures using ballistic electrons. Due to the relative simplicity of the proposed experimental setup (only five gates are needed to produce entanglement and to test Bell’s inequality) this measurement scheme is potentially feasible in terms of current semiconductor nanotechnology.

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