Simulation of a gas turbine engine rotor assembly

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Abstract. The quality of aircraft and rocket engines depends primarily on the geometric accuracy of assembly units and parts. Mathematical models implemented in the form of computer models are used to predict quality indicators (in particular, assembly parameters). The work presents a numerical model for predicting assembly parameters of the rotors based on the use of actual surfaces of the parts obtained as a result of measurement on coordinate measuring machines and of mathematical modeling. Aircraft engine turbine rotor parts surfaces are measured on a coordinate measuring machine. Experimental assemblies of three parts were carried out. The measurement error of the parts does not exceed 5 µm, of assembly – 10 µm. Face and radial run-outs were measured for four positions of the disc and retainer in the shaft assembly, with run-outs ranging from 0.08 mm up to 0.1 mm. Calculations of face and radial run-outs of the disc with respect to the base surfaces of the shaft were made using the developed digital model. Relative deviations of the calculated values from the experimental values do not exceed 10%.

1. Introduction
Complex industrial and knowledge-intensive products are characterized by high requirements to geometrical accuracy of the parts and assembly units. These products include modern aircraft engines which are subject to high requirements for reliability, minimum weight, cost effectiveness and durability. In manufacturing aircraft engines, the product assemblies, which are necessary to determine optimal assembly method that provides the specified quality, are tested. Performing test assemblies of the parts involves significant labor costs and time resources at enterprises. Test part assemblies may be excluded using digital prediction of geometric assembly parameters based on the results of contact, laser [1] and optical scanning of the separate parts. Digital prediction requires development of mathematical model [2] of the assembly process. Digital prediction and determination of optimal parameters shall allow the assembly of separate aircraft engine units (compressor and turbine) without using test assemblies.

The assembly model choice depends on the stiffness requirements. Some models are based on the solid state hypothesis, for example, the T-Map model [3]. Other models, such as the Skin form model and the Deviation Area Model (DD), can also simulate a flexible part or assembly [4]. These models can be either point-based or feature-based. Compared to the features that simultaneously characterize position and direction information, the position of a point in space is described by its location rather than orientation, with variations that vary depending on the choice of different points [5].

The authors have previously considered a model based on the use of measured data on surface deviations, made in the CAE-package ANSYS [6], which allows to calculate on displacement of the parts in the assembly. The disadvantage of using such an approach is the high labor intensity and poor
accuracy, which do not allow to obtain data quickly in the production process. The big advantage is the registration of surface deflections in the course of assembling, modeling for tightening the bolted-type connections is being finished [7]. In order to reduce the labor intensity for calculations and to increase the autonomy, an algorithm and model for calculating the assembly positions of the measured parts, not taking the rigidity into account, is developed and implemented [8]. In work [9], application of the model, which does not take the rigidity into account, is considered for assembling of artificially generated cone rings having deviations of the form, i.e. natural experiments were not carried out to check the developed model.

The purpose of this article is to research according to the assessment error in the calculation of the assembly parameters using the model previously developed that does not take the rigidity into account. The requirements to the model authenticity are determined by the requirements to the measurement accuracy of the concerned parameters. In order to achieve this goal, experiments have been carried out to measure and assemble the turbine rotor parts.

2. Experimental part

This turbine rotor assembly has three parts: a disc, a retainer and a shaft. The assembly model is shown in figure 1. Cylindrical surfaces $A$ of the shaft form a reference axis, the parts are interconnected along the face and cylindrical surfaces. In the assembled state between the parts, gaps occur that may be characterized with functions of distances between the points of surfaces $G$ [8]. Correspondingly, let us denote between the face surfaces of the part pairs: shaft-retainer and retainer-disc of the gap function by $G_{1,v-s}$ and $G_{1,s-d}$. Between the cylindrical surfaces of the corresponding pairs of parts, the gap functions are denoted as $G_{2,v-s}$ and $G_{2,s-d}$.

![Figure 1. CAD-model for assembly of parts: disc (1), retainer (2), and shaft (3).](image)

Face $K$ and cylindrical surface $L$ on the disc are control ones in the assembly, face runout of $\delta_{l,r}$ surface $K$ and radial runout of $\delta_{r,r}$ surface $L$ are calculated. The basis of the developed model is information on the actual geometry of the parts. In order to obtain this information, mating and controlled surfaces have been measured. In order to verify the adequacy of the model, it is necessary to carry out a series of experiments [10]; for this reason, runout values have been measured in several assembly options.
2.1. Measuring
Parameters of separate parts, and then of the assembled parts have been measured.

2.1.1. Measuring separate parts. Part surfaces have been measured using a DEA Global Performance Coordinate Measuring Machine (CMM). The details have been installed rigidly on the CMM plate (figure 2) using special devices. Measurements have been carried out under normal conditions of temperature, vibration and relative humidity.

During the measurement, the following number of measured points has been obtained: 200-300 points on mating planes; 100 points on each control surface \( K \) and \( L \). The part panes have been measured by sections. In the case of cylindrical surfaces, the sections are like intersection lines of the surface and planes perpendicular to the rotation axis. For face surfaces, sections are like intersection lines of the surface and cylindrical surfaces, the axis and center of which coincide with the normal plane vector. Therefore, the sections are like circumferences. The coordinates of the measured points have been saved in *.txt files for further analysis in the MATLAB system. Figure 3 shows the measured points of the disc face surface.

![Figure 2. Installing part ‘Disc on a CMM plate’.

Figure 2. Installing part ‘Disc on a CMM plate’.

Figure 3. View of the measured face points in the MATLAB environment.

2.1.2. Assembly measurement. The parts are connected to the assembly in series: the shaft is the base part, the retainer is attached to it, and the disc is attached to the retainer with stud bolts. The disc and retainer rotate in regard to the shaft forming various options for the rotor assembly. Due to the fact that the rotor dimensions in the assembly (shaft length) are larger than the dimensions of the Dea Global CMM working space, it is impossible to measure the reference and control surfaces of the assembly at the same time. For this reason, assembly parameters have been controlled using a specialized device equipped with a dial gauge. The shaft has been mounted on the reference surfaces in a specialized device (on rollers). Four positions of the disc and retainer regarding the shaft have been measured: in position at 0, 90, 180 and 270 degrees.

3. Methods and materials
Creating an assembly model includes two components: actual models of part surfaces and a model for mating part surfaces.
3.1. Creating valid models of part surfaces

Creating valid models begins with processing of the measured data. In order to process the measured data, the coordinates of the measured surface points have been loaded into the MATLAB software package. Processing coordinates of the points includes: smoothing outliers in measurements, arranging a point grid on surfaces with sufficiently high form deviations. After entering of adjustments, mathematical basing is carried out. Let us consider these three subtasks of the analysis in more detail.

The coordinates after measuring may have ‘outliers’ due to availability of surface contaminants, improper operation of the sensor, etc. [11]. On a certified measuring device, such values occur on rare occasions. For example, when measuring 200 points, ten of such values may be accumulated, the value of which may be on average from 5 up to 30 µm and more [12]. Furthermore, there is a random measurement error which is a small value up to 2-3 µm and may be filtered. In order to filter such values on cylindrical and flat surfaces, at first, it is necessary to add substitute elements (respectively, “cylinder” and “plane”) into the array of measured points according to the least squares method (LSM) [13].

In order to analyze the resulting deviations in the form of the holes and shafts at the first stage, the axis of the substitute element of the part pane coordinate is aligned in the direction of axis \( Z \). At the next stage, the coordinates of the points (in the section plane) are transferred into the polar coordinate system. In the case of cylindrical panes, the coordinate system center is transferred into the center of the inscribed circle of the corresponding section. The third stage is filtering the random components of the deviation and “outliers”. The moving average method has been used for filtering. The values after filtration are numerically equal to the arithmetic average for the values of the original function for the specified period which is calculated by the formula:

\[
\text{SMA}_t = \frac{1}{n} \sum_{i=0}^{n-1} p_{t-i},
\]

being \( \text{SMA}_t \) is the value of the moving average at the point \( t \);

\( n \) – the number of values of the original function for calculating the moving average (smoothing interval);

\( p_{t-i} \) – the value of the original function at the point \( t-i \).

In the case of cylindrical panes, the filtering operation is carried out for values of polar radii, and in the case of end panes, the coordinates of the points are filtered along axis \( Z \). In the case of pane \( B \), the points with outliers have been simply deleted from the calculations.

The second subtask comes out from the need for a spline description of the surfaces. The curves and surfaces of the complex form in CAD-systems and metrological support of measuring equipment are described by spline equations and presented in a portioned form, i.e. like a patchwork quilt. In order to describe the surfaces of the part with geometric form deviations, the surfaces formed from Coons bicubic portions have been used. The surface to be described is like a segment corresponding to the parameter values \( 0 \leq u \leq 1, \quad 0 \leq v \leq 1 \). The Coons portion is formed due to mating of boundary spline curves and expressed as:

\[
P(u, v) = \sum_{i=0}^{1} \sum_{j=0}^{1} a_{ij} u^i v^j,
\]

being \( P(u, v) \) is the bicubic surface point;

\( a_{ij} \) are algebraic vector coefficients with components \( x, y, z \).

When scanning the points in sections, their number in sections is not the same. Furthermore, areas where measurements could not have been carried out (fastening spots or spots where the measuring anvil could not have been brought to) might be available. For mating end and cylindrical panes, it is required to transmit information about deviations at each point, for formation descriptive surfaces and further calculating for the assembly state. Therefore, sections have been made with the same number of points for each pane using available measured data pre-filtered. In order to solve the second subtask of the analysis, piecewise-polynomial interpolation (and, if necessary, extrapolation) with cubic
splines of the coordinate values along axis \( Z \) of the cross sections for end panes and the radius vector values for cylindrical panes have been used. The pane points filtered and ordered are transferred back from the polar to the Cartesian coordinate system, and the center is returned to its original position.

After filtering, the coordinate system is required to be corrected because the outliers of the points on the reference surfaces of the parts have distorted the calculated parameters of the substitute elements and introduced errors into the calculated center of the coordinate system and the direction of the reference axes. For this reason, substitute elements have been inserted into the points of the reference elements according to the least squares method and all the measured points of the parts have been aligned. Mathematically defined (1) surfaces are like actual models of the measured surfaces of the parts and are used in further calculation of searching for the mating state.

3.2. Mating model of part surfaces

The developed mating model of curves or surfaces has been based on the best alignment procedure with the availability of a set of constraints on the surface intersection. Mating of two surfaces having form deviations may be characterized as a size of the gap between the surfaces. In case of surface intersection, a negative gap is formed – an interference fit. The spatial gap function characterizes achievement of mating state of the part surfaces and depends on the vector of mutual arrangement of surfaces \( \vec{V} \). In the general, the vector of mutual arrangement contains three linear and three angular parameters for the corresponding coordinate axes. Let us denote the gap function as \( G(\vec{V}) \). Contact parameters for connections have been specified: plane to plane (gap \( G_1 \)) and cylinder to cylinder (gap \( G_2 \)).

In order to solve the contact task using models of the measured surfaces, an iterative algorithm has been developed that allows to calculate parts mating without taking into account deformation of the parts during the assembly process. Algorithm for finding the mating state assumes iterative movement of one mating surface in regard to another one with the stress application vector of the surface assembly \( \vec{D} \). In order to calculate the gap function \( G(\vec{V}) \), the best alignment of mating surfaces is performed at each stage. Iteration algorithm (ICP) of the nearest points is used to perform the best alignment procedure [14,15]. According to this algorithm, at each iteration, the rotation and movement angles along the coordinate axes of the corresponding points on the surfaces to reduce the distance between the points are calculated with the non-linear optimization search methods. The number of mating points was 120 for end faces and 40 for cylindrical surfaces. In order to exclude intersections of two surfaces, the system of inequalities presented in work [16] is used, which imposes limitations on the gap function \( G(\vec{V}) \). In consequence of the algorithm, the rotation matrix and movement vector of the moving part are calculated determining the transformation of its initial coordinate system into the coordinate system in assembled state:

\[
\vec{p}_{as} = \vec{p}_d \cdot R_{as} + \vec{t}_{as},
\]

being \( \vec{p}_{as}, \vec{p}_d \) – vector of coordinates of points \( (x, y, z) \) of the part respectively after the assembly and in the initial state;

\( R_{as}, \vec{t}_{as} \) – the rotation matrix and the point coordinate movement vector that \( \vec{p}_d \) specify the movement of the part during the assembly.

The developed surface models and the iterative algorithm are like an assembly model of parts.

As in the case of a real assembly, for the concerned parts, mating of the surfaces for ‘shaft and retainer’ is simulated at first. The ‘shaft’ part is limited in all six degrees of freedom. After calculating the position of the ‘retainer’ part, mating of ‘retainer’ and ‘disc’ is calculated. At the same time, details “shaft” and “retainer” are already limited according to all six degrees of freedom.

4. Results and discussion

The developed assembly model of the concerned assembly unit has been used for calculating parameters of the face runout \( \delta_{r,r} \) and radial runout of disc \( \delta_{r,r} \) (surfaces \( K \) and \( L \) in figure 1) in
regard to the base surfaces of the shaft. As referred in Section 2, the retainer and disc parts have rotated in regard to the shaft part in increments of 90 degrees.

Let us proceed to comparing the assembly results during the experiment and simulation (in a hard position). Repeatability the simulation results with the actual parameters obtained during the assembly have been evaluated by calculating absolute deviations:

\[ \Delta_a = P_{\text{meas}} - P_m, \]  

and relative deviations:

\[ \Delta_{\text{rel}} = \frac{\Delta_a}{P_{\text{meas}}} \cdot 100\%, \]  

being \( P_m \) is the parameter calculated as a simulation result; \( P_{\text{meas}} \) is a parameter measured during assembly.

Table 1 shows the results of measurement and simulation of the turbine rotor assembly parameters. The measurement results have been obtained with an accuracy of 0.01 mm; that is why the simulation data in the table are rounded to two decimal places. When calculating the \( \Delta_{\text{rel}} \) parameter, values \( P_m \) have not been rounded off.

| Angle | 0   | 90  | 180 | 270 |
|-------|-----|-----|-----|-----|
| Parameter | \( \delta_{\text{i,r}} \) | \( \delta_{\text{s,r}} \) | \( \delta_{\text{i,r}} \) | \( \delta_{\text{s,r}} \) |
| \( P_{\text{meas}} \), mm | 0.10 | 0.08 | 0.09 | 0.10 |
| \( P_m \), mm | 0.09 | 0.08 | 0.08 | 0.08 |
| \( \Delta_a \), mm | 0.01 | 0.00 | 0.01 | -0.01 |
| \( \Delta_{\text{rel}} \), % | 6.79 | 5.58 | 6.73 | -7.30 |

Analyzing the results in Table 1, we can conclude that the simulation error does not exceed the measurement error of the assembly parameters. The maximum relative error value was 9%. Figures 4 and 5 show diagrams of parameters \( \delta_{\text{i,r}} \) and \( \delta_{\text{s,r}} \) obtained during measurement and simulation.

**Figure 4.** Values \( \delta_{\text{i,r}} \) while measuring (1) and simulating (2).

**Figure 5.** Values \( \delta_{\text{s,r}} \) while measuring (1) and simulating (2).
Based on the diagrams in figures 2 and 3, we can conclude that the nature of changes in the values of run-outs is the same for experimental and simulated data. The value of the angle at which the minimum exists for the parameter $\delta_{t,r}$ does not match the angle for the parameter $\delta_{r,r}$. As noted above, the relative deviations of the parameter values do not exceed 10% (Table 1). It should be noted that this tolerance includes both simulation errors and the errors of measuring instruments for the actual parameter (the dial gauge has a division value of 0.01 mm). The results show that the parameter $\delta_{t,r}$ is minimum at a position of 90 degrees, and $\delta_{r,r}$ minimum at 0 degrees.

5. Conclusion
A model based on the use of the best alignment algorithm that allows predicting the assembly parameters of turbine rotor parts when mating them on flat and cylindrical panes. The model uses part surfaces split into finite elements. An assumption has been made that rigid bodies are assembled. Due to this fact, the model allows to calculate assembly parameters with less labor intensity than in available CAE-packages. The approach under consideration can be used to assign tolerances to parts [17] at the design stage. In the future, the developed model may be extended to deformable surfaces and used to optimize geometric deviations (runouts) and vibrational characteristics (imbalances) of the assembly.

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