Test of the hyperon-nucleon interaction within leading order covariant chiral effective field theory

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Motivated by the recent experimental measurements of differential cross sections of the Σ−p elastic scattering in the momentum range of 470 to 850 MeV/c by the J-PARC E40 experiment, we extend our previous studies of S = −1 hyperon-nucleon interactions to relatively higher energies up to 900 MeV/c for both the coupled-channel Λp → (Λp, Σ+n, Σ0p), Σ−p → (Λn, Σ+n, Σ−p) and single-channel Σ+p → Σ+p reactions. We show that although the leading order covariant chiral effective field theory is only constrained by the low energy data, it can describe the high energy data reasonably well, in particular, the J-PARC E40 differential cross sections. The predicted cusp structure close to the ΣN threshold in the Λp → Λp reaction agrees with the latest ALICE observation as well as with the results of the next-to-leading order heavy baryon chiral effective theory. On the other hand, the comparison with the latest CLAS data on the Λp cross sections between 0.9 and 2.0 GeV/c clearly indicates the need of higher order chiral potentials for such high momenta. This is also the case for the latest J-PARC data on the Σp → Λn differential cross sections. Nevertheless, even for these cases, the predictions are in qualitative agreement with the data, albeit with large uncertainties, implying that the predicted total and differential cross sections are of relevance for ongoing and planned experiments.

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I. INTRODUCTION

Baryon-baryon interactions play an important role in ab initio studies of nuclear structure, hypernuclei, as well as neutron stars [1–3]. In contrast to the nucleon-nucleon (NN) interaction, where the wealth of experimental data has allowed for the construction of high-precision potentials [4–9], hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions still remain poorly constrained because of the lack of high-quality YN and YY scattering data [10–12].

For the YN and YY interactions, the old data were mainly either total cross sections or threshold parameters, which have relatively large uncertainties [13–19]. Only in the past two decades, differential cross sections were measured. With a scintillating fiber block (SCIFI) technique, the KEK-PS E251 experiment [20] measured the Σ+p differential cross sections in the momentum range of 300 ≤ PΣ+ ≤ 600 MeV/c for two angles −0.4 ≤ cosθcm ≤ 0.1 and 0.1 ≤ cosθcm ≤ 0.6. Afterwards, the KEK-PS E289 experiment [21] obtained the differential cross sections of the Σ−p elastic scattering in the momentum range of 400 ≤ PΣ− ≤ 700 MeV/c. This is the first measurement of the Σ−p elastic scattering in the momentum region where the contributions of Π- and higher partial waves are notable. In 2005, the KEK-PS E289 experiment [22] updated their study of the Σ−p scattering in the momentum region of 350 ≤ PΣ− ≤ 750 MeV/c with three times more data than the KEK-PS E251 experiment [20]. Recently, a new measurement on Σ−p scattering with high statistics was performed at the J-PARC Hadron Experimental Facility by the E40 experiment [23]. Differential cross sections of the Σ−p elastic scattering were extracted with a drastically improved accuracy for the Σ− momentum ranging from 470 to 850 MeV/c. They also performed the first precise measurement of the differential cross sections of the Σ−p → Λn reaction in the momentum range of 470-650 MeV/c [24]. The CLAS Collaboration studied the Λp → Λp elastic differential cross sections in the incident Λ momentum range of 0.9–2.0 GeV/c, which are the first data on this reaction since the 1970s [25]. In addition, with high precision correlation techniques, the ALICE Collaboration studied the coupling strength of the ΛΣ ↔ ΛΛ in the pΛ system [26]. The opening of the inelastic ΛΣ channel is clearly visible in the extracted correlation function as a cusp-like structure occurring at a relative momentum of 289 MeV/c. All these new measurements, though of relatively higher energy, could impose strong constraints on theoretical hadron-hadron interactions.

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As discussed in Ref. [45], 12 independent combinations of the symmetry. They are collected in Ref. [48].

where \( a \) short summary and outlook in Sec. IV.

The leading-order (LO) Lagrangian for the contact terms is

\[
\mathcal{L}_{\text{CT}} = \sum_{i=1}^{5} \left[ \frac{c_1}{2} \text{tr} \left( \bar{B}_1 B_2 (\Gamma_i B_1) \right) + \frac{c_2}{2} \text{tr} \left( \bar{B}_1 B_2 (\Gamma_i B_1) \right) B_2 (\Gamma_i B_1) \right] + \frac{c_3}{2} \text{tr} \left( \bar{B}_1 B_2 (\Gamma_i B_1) \right) B_2 (\Gamma_i B_1) \right],
\]

where \( c_i \) are constants that need to be determined by fitting to either experimental or lattice QCD data and \( \Gamma_i \) are the elements of the Pauli algebra,

\[
\Gamma_1 = 1, \quad \Gamma_2 = \gamma^\mu, \quad \Gamma_3 = \sigma^{\mu\nu}, \quad \Gamma_4 = \gamma^\mu \gamma_5, \quad \Gamma_5 = \gamma_5.
\]

As discussed in Ref. [45], 12 independent combinations of the 15 LECs survive in the \( S = -1 \) system, assuming strict SU(3) symmetry. They are collected in Ref. [48].

To construct the OME potentials, we need the following LO meson-baryon Lagrangian:

\[
\mathcal{L}^{(1)}_{MB} = \text{tr} \left( \bar{B} (i \gamma_\mu D^\mu - M_B) B - \frac{D}{2} \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] - \frac{F}{2} \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \right),
\]
where \( D^\mu B = \partial_\mu B + [\Gamma_\mu, B] \) is the derivative with \( \Gamma_\mu \) and \( u_\mu \) defined as
\[
\Gamma_\mu = \frac{1}{2} \left( u^1 \partial_\mu u + u \partial_\mu u^1 \right), \quad u_\mu = i(u^1 \partial_\mu u - u \partial_\mu u^1)
\]
with \( u^2 = U = \exp \left( i \frac{\pi \Lambda}{f_0} \right) \). The values of the coupling constants are \( D + F = 1.277, F/(F + D) = 0.4 \), and the meson decay constant is \( f_0 = 92.2 \text{ MeV}/c^2 \).

From these Lagrangians, one can straightforwardly obtain the contact and OME potentials. The scattering amplitudes can then be obtained by solving the coupled-channel Kadyshevsky equation [54].

\[
T^{\prime\prime\prime\prime \prime}(p', p; \sqrt{s}) = V^{\prime\prime\prime\prime \prime}(p', p) + \sum_{\rho', \nu', \nu} \int_0^\infty \frac{dp'\rho'\nu'\nu'\nu'}{(2\pi)^3} \frac{M_{B_\rho\nu}M_{E_{\nu'\nu'}\nu}}{E_{\nu'\nu'} E_{\nu'\nu'} \left( \sqrt{s} - E_{\nu'\nu'} - E_{\nu'\nu'} + i\epsilon \right)} \cdot \sum_{\rho'\nu', \nu, \nu} \rho'\nu'\nu'\nu',
\]

where \( \sqrt{s} \) is the total energy of the two-baryon system in the center-of-mass frame and \( E_{\nu'\nu'} = \sqrt{p'^2 + M_{B_n^{-n}}^2} \), \( n = 1, 2 \). The labels \( \nu, \nu', \nu'' \) denote the particle channels, and \( \rho, p', \rho'' \) denote the partial waves. In practice, the potentials in the scattering equation are regularized with an exponential form factor of the following form,
\[
f_{\Lambda_F}(p, p') = \exp \left[ -\left( \frac{p}{\Lambda_F} \right)^4 - \left( \frac{p'}{\Lambda_F} \right)^4 \right].
\]

More details about the covariant ChEFT can be found in Refs. [43–48, 55, 56]. It has been customary to use the variation of the cutoff as an error estimator (see, e.g., Ref. [40]). In recent years, it was proposed that one can treat the difference between the optimal results obtained at different orders as the estimate of truncation uncertainties [52]. This approach can be briefly described as follows. The expansion parameters for ChEFT read
\[
Q = \Max \left\{ \frac{p}{\Lambda_b}, \frac{m}{\Lambda_b}, \frac{m}{\Lambda_b} \right\},
\]
where \( p \) is the baryon momentum in the c.m. frame and \( \Lambda_b \) is the cutoff or the chiral symmetry breaking scale. In our numerical study, \( \Lambda_b \) is fixed at the optimal cutoff of 600 MeV/c. One can estimate the next to leading order (NLO) truncation uncertainties as
\[
\Delta^{\text{NLO}} = \Max \left\{ Q^2 \left| O^{\text{LO}} \right|, Q \left| O^{\text{LO}} - O^{\text{NLO}} \right| \right\},
\]
where \( O \) represent either total cross sections or differential cross sections in our present case. As we did not have the NLO results, we could use either the experimental data as the NLO results (assuming that at NLO, we can fully reproduce the data), or simply use the first term, i.e., \( Q^2 \left| O^{\text{LO}} \right| \). It should be noted that in our previous works, we have used the cutoff variation to estimate theoretical uncertainties, while in the present work, we use Eq. (5) to estimate truncation uncertainties. One should keep in mind that because we are missing the NLO results, our estimate of theoretical uncertainties can only be trusted for the observables where our LO results can describe reasonably well the data.

### III. RESULTS AND DISCUSSION

In this work we study in detail the following \( YN \) reactions for which experimental data exist: 1) the coupled-channel reaction \( \Lambda p \rightarrow \Lambda p, \Sigma^+ n, \Sigma^0 p \); 2) the coupled-channel reaction \( \Sigma^+ p \rightarrow \Lambda n, \Sigma^0 n, \Sigma^- p \); and 3) the single-channel reaction \( \Sigma^+ p \rightarrow \Sigma^+ p \).

**Table I:** Experimental \( YN \) total cross sections used in the fitting procedure. Momenta are in units of MeV/c and cross sections in mb.

| \( \Lambda p \rightarrow \Lambda p \) [15] | \( \Lambda p \rightarrow \Lambda p \) [57] | \( \Sigma^- p \rightarrow \Lambda n \) [13] | \( \Sigma^- p \rightarrow \Sigma^0 n \) [13] | \( \Sigma^- p \rightarrow \Sigma^- p \) [18] | \( \Sigma^+ p \rightarrow \Sigma^+ p \) [18] |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( P_{\text{lab}}^{\Lambda \Lambda} \) | \( \sigma_{\text{exp}} \) | \( P_{\text{lab}}^{\Lambda \Lambda} \) | \( \sigma_{\text{exp}} \) | \( P_{\text{lab}}^{\Lambda \Sigma} \) | \( \sigma_{\text{exp}} \) | \( P_{\text{lab}}^{\Lambda \Sigma} \) | \( \sigma_{\text{exp}} \) |
| 135 ± 15        | 209 ± 58        | 145 ± 25        | 180 ± 22        | 110 ± 5         | 174 ± 47        | 110 ± 5         | 396 ± 91        | 135 ± 2.5       | 184 ± 52        | 145 ± 5         | 123 ± 62        |
| 165 ± 15        | 177 ± 38        | 185 ± 15        | 130 ± 17        | 120 ± 5         | 178 ± 39        | 120 ± 5         | 159 ± 43        | 142.5 ± 2.5     | 152 ± 38        | 155 ± 5         | 104 ± 30        |
| 195 ± 15        | 153 ± 27        | 210 ± 10        | 118 ± 16        | 130 ± 5         | 140 ± 28        | 130 ± 5         | 157 ± 34        | 147.5 ± 2.5     | 146 ± 30        | 165 ± 5         | 92 ± 18         |
| 225 ± 15        | 111 ± 18        | 230 ± 10        | 101 ± 12        | 140 ± 5         | 164 ± 25        | 140 ± 5         | 125 ± 25        | 152.5 ± 2.5     | 142 ± 25        | 175 ± 5         | 81 ± 12         |
| 255 ± 15        | 87 ± 13         | 250 ± 10        | 83 ± 13         | 150 ± 5         | 147 ± 19        | 150 ± 5         | 111 ± 19        | 157.5 ± 2.5     | 164 ± 32        |                     |                 |
| 300 ± 30        | 46 ± 11         | 290 ± 30        | 57 ± 9          | 160 ± 5         | 124 ± 14        | 160 ± 5         | 115 ± 16        | 162.5 ± 2.5     | 138 ± 19        |                     |                 |

\[
\chi^2 = 4.2, \quad \Delta = 2.6, \quad \Delta = 6.4, \quad \Delta = 2.3, \quad \Delta = 0.3
\]

\( \Sigma^{-} p \) inelastic capture ratio at rest [58], \( r_R = 0.468 \pm 0.010 \). (\( \chi^2 = 0.032 \))
In Table II we collect the low-energy data with $P_{\text{lab}} \leq 350$ MeV/c, which consist of total cross sections for the following reactions: $\Lambda p \to \Lambda p$ from Ref. [15] (six data points) and Ref. [57] (six data points), $\Sigma^- p \to \Lambda n$ [13] (six data points), $\Sigma^- p \to \Sigma^- p$ [18] (seven data points), $\Sigma^+ p \to \Sigma^+ p$ [18] (four data points). In addition to the low energy $YN$ scattering data, the $\Sigma N$ inelastic capture ratio at rest, $r_R$ [58], is also considered. For reference, we also present the corresponding $\chi^2$ obtained with $\Lambda p = 600$ MeV/c. Note that the $\Sigma^+ p \to \Sigma^+ p$ and $\Sigma^- p \to \Sigma^- p$ cross sections were obtained by incomplete angular coverage $\cos \theta \in [-0.5, 0.5]$ experimentally [18].

| $\Lambda p \to \Lambda p$ [19] | $\Lambda p \to \Lambda p$ [17] | $\Sigma^- p \to \Lambda n$ [59] | $\Sigma^- p \to \Sigma^0 n$ [59] | $\Sigma^- p \to \Sigma^- p$ [21] | $\Sigma^+ p \to \Sigma^+ p$ [22] |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $P^0_{\text{lab}} \sigma_{\text{exp}}$ | $P^0_{\text{lab}} \sigma_{\text{exp}}$ | $P^0_{\text{lab}} \sigma_{\text{exp}}$ | $P^0_{\text{lab}} \sigma_{\text{exp}}$ | $P^0_{\text{lab}} \sigma_{\text{exp}}$ | $P^0_{\text{lab}} \sigma_{\text{exp}}$ |
| 350 ± 50 17.5 ± 9 | 500 ± 100 9 ± 2 | 175 ± 25 59 ± 13.5 | 175 ± 25 101 ± 18 | 450 ± 50 19.10 ± 5 | 450 ± 50 19.10 ± 5 |
| 450 ± 50 27 ± 8 | 650 ± 50 16.5 ± 3.5 | 225 ± 25 60 ± 12 | 225 ± 25 75 ± 12 | 550 ± 50 29.14 ± 10 | 550 ± 50 29.14 ± 10 |
| 550 ± 50 7 ± 4 | 750 ± 50 10.5 ± 2.5 | 275 ± 25 42 ± 8 | 275 ± 25 44 ± 8 | 650 ± 50 15.23 ± 8 | 650 ± 50 15.23 ± 8 |
| 650 ± 50 9 ± 4 | 850 ± 50 10 ± 2.5 | 325 ± 25 24.5 ± 6 | 325 ± 25 42 ± 8 | 650 ± 50 15.42 ± 6 | 650 ± 50 15.42 ± 6 |

In Table III we collect the high energy $\Lambda p \to Ap, \Sigma^\pm p \to \Sigma^\pm p$ cross sections for $P_{\text{lab}} \geq 350$ MeV/c and $\Sigma^- p \to \Lambda n$ and $\Sigma^- p \to \Sigma^0 n$ for $P_{\text{lab}} \geq 175$ MeV/c. We stress that these data are not fitted, and therefore they serve as nontrivial tests on the $YN$ interaction by ChEFT.

### A. Total cross sections

We show the predicted total cross sections for all the $S = -1$ channels below the laboratory momentum about 900 MeV/c in Figs. 2–4, in comparison with the available experimental data. It should be noted that the covariant ChEFT results were obtained with the optimal cutoff of 600 MeV/c, while the uncertainties are obtained using Eq. (5). In these figures, the cross section data [13–15, 18, 57] included in the fitting procedure are denoted by filled symbols, while for the high energy data [17–21, 22, 59, 60] open symbols are used. For the sake of comparison, we also show the results from the J"ulich’04 model (orange solid lines) [23] and the Nijmegen NSC97f potential (red dashed lines) [27], if available. It is clear that the ChEFT results agree with the experimental data reasonably well not only at low energies but also at high energies. Note that the high energy results have to be considered as genuine predictions because none of them were included in the fitting procedure and, therefore, the agreement with data reasonably well not only at low energies but also at high energies. Note that the high energy results have to be considered as genuine predictions because none of them were included in the fitting procedure and, therefore, the agreement with data reasonably well not only at low energies but also at high energies. Note that the high energy results have to be considered as genuine predictions because none of them were included in the fitting procedure and, therefore, the agreement with data reasonably well not only at low energies but also at high energies. Note that the high energy results have to be considered as genuine predictions because none of them were included in the fitting procedure and, therefore, the agreement with data reasonably well not only at low energies but also at high energies. Note that the high energy results have to be considered as genuine predictions because none of them were included in the fitting procedure and, therefore, the agreement with data reasonably well not only at low energies but also at high energies. Note that the high energy results have to be considered as genuine predictions because none of them were included in the fitting procedure and, therefore, the agreement with data reasonably well not only at low energies but also at high energies.

From Figs. 2 and 3 we observe that the $\Lambda p \to \Lambda p$ and $\Lambda n \to \Lambda n$ total cross sections show a cusp structure when the $\Sigma^+ n$ and $\Sigma^0 n$ thresholds open. The former shows a pronounced cusp of almost 50 mb at the $\Sigma^+ n$ threshold, while the magnitude of the latter is relatively smaller. Because the cusps occur over a very narrow momentum range, it is hard to observe them experimentally. Nonetheless, there is experimental evidence for an enhancement in the $\Lambda p \to \Lambda p$ cross section near the $\Sigma N$ threshold [14, 57, 61, 66], as shown in Fig. 2.

The cusp structure at the $\Sigma N$ threshold in the total $\Lambda N \to \Lambda N$ cross sections can be traced back to the strong $\Lambda N - \Sigma N (I = 1/2) \ 3S_1 - 3D_1$ coupling induced by the tensor force. This can bechecked by examining Fig. 7 of Ref. [45], where the $\Sigma N - \Sigma N 3S_1$ phase shifts show a prominent cusp structure at the $\Sigma N$ threshold, while the cusp in the $3S_0$ channel is much smaller. In fact, a cusp-like structure at the $\Sigma N$ threshold has been observed in the $\Delta p$ correlation function, which represents the first direct experimental observation of the $\Delta N - \Sigma N$ coupled-channel effect in the $\Lambda p$ system [26]. The study based on the nonrelativistic ChEFT shows that the LO potential predicts a smaller $\Sigma N$ cusp with respect to the NLO potential, while the latter one is more consistent with the experimental data [26]. In other words, the measured correlation function confirms the strength of the coupled-channel $\Delta N - \Sigma N$ interaction in the nonrelativistic NLO potential. By comparing the $\Delta N$ cross sections at the $\Sigma N$ threshold predicted by the relativistic and nonrelativistic ChEFT [41, 67], it is interesting to note that the relativistic LO result is comparable with the nonrelativistic NLO result, which indicates the reliability of the leading order relativistic $\Delta N - \Sigma N$ interaction.

It is necessary to stress that one main purpose of the present work is to predict cross sections for many coupled channels that have not been measured yet such that they could be checked by future experiments at J-PARC [68, 69], BEPC [70], LHC [71], or HIAF [72].
Lately, the CLAS Collaboration reported the first measurement of the $\Lambda p \rightarrow \Lambda p$ elastic scattering cross section in the incident $\Lambda$ momentum range of 0.9–2.0 GeV/c, which are the first data on this reaction since the 1970s [25]. Although the momentum range is much larger than what one expects a leading order ChEFT study can cover, it is interesting and instructive to check how they compare with the data. In Fig. 5, we compare the covariant ChEFT results with the CLAS data. For the sake of comparison, we also show the results of the Jülich model [23], the Nijmegen model [27], and those of the LO heavy baryon ChEFT. Clearly, none of them can reproduce the data for such high energies, which are far away from the region where all these models and EFTs were calibrated. Nonetheless, the covariant ChEFT results are not particularly worse either. It is interesting to note that somehow the covariant ChEFT results reach the maximum around the same momentum as the data do. Of course, for such higher energies, one should not trust too much the LO results, and even the NLO results. In addition, as the expansion parameter $Q$ is close to unity when the laboratory momentum reaches 1500 MeV/c, the chiral expansion breaks down for this and higher energies. On the other hand, it is gratifying to see that the predictions at least provide a reasonable estimate of order of magnitude of the data. We leave a more careful and systematic study to a future work.

B. $\Sigma^+ p$ and $\Sigma^- p$ differential cross sections

In Fig. 6, we show the predicted differential cross sections below $P_{lab} \leq 900$ MeV/c in comparison with the available data [13, 18, 20–23], which were not included in the fitting procedure. Here, the experimental measurements are: (a) $\Sigma^- p \rightarrow \Lambda n$ differential cross sections at $P_{\Sigma^-} = 135$ MeV/c and $P_{\Sigma^-} = 160$ MeV/c [13], (b) $\Sigma^+ p$ elastic scattering at $P_{\Sigma^+} = 170$ MeV/c [18] and $P_{\Sigma^+} = 450$ MeV/c [20, 22], (c) $\Sigma^- p$ elastic scattering at $P_{\Sigma^-} = 160$ MeV/c [13], $P_{\Sigma^-} = 400$-700 MeV/c [21], and $\Sigma^- p$ elastic scattering at $P_{\Sigma^-} = 470$-850 MeV/c [23]. For comparison, we also plot the $\Sigma^- p$ elastic scattering differential cross sections from the one boson exchange model (Jülich 04) [28], the quark-cluster model of the Kyoto–Niigata group (fss2) [73], and the meson exchange model from the Nijmegen group (ESC08c) [49].

As can be seen from Fig. 6, the flat theoretical differential cross sections for $\Sigma^- p \rightarrow \Lambda n$ at $P_{lab} = 135$ MeV/c and $P_{lab} = 160$ MeV/c are in reasonable agreement with the data [13] within uncertainties. One should note that the experimental data shown in Fig. 6(a–e) are averages over different momentum intervals. Specifically, for $\Sigma^- p \rightarrow \Lambda n$ the data are averages over the intervals $100 \leq P_{\Sigma^-} \leq 170$ MeV/c and $150 \leq P_{\Sigma^-} \leq 170$ MeV/c [13], respectively. In view of the large experimental uncertainties we refrain from averaging our theoretical results and, following common practice, present our predictions at the central value of the momenta. The same is also true for the data of Ref. [18] which represent averages over $150 \leq P_{\Sigma^-} \leq 170$ MeV/c for $\Sigma^- p \rightarrow \Sigma^- p$ and $160 \leq P_{\Sigma^+} \leq 180$ MeV/c for $\Sigma^+ p \rightarrow \Sigma^+ p$, respectively.

In the higher energy region, the experimental differential cross sections for $\Sigma^- p \rightarrow \Sigma^- p$ are averages over $400 \leq P_{\Sigma^-} \leq 700$ MeV/c [21], while those for $\Sigma^+ p \rightarrow \Sigma^+ p$ are averages over $300 \leq P_{\Sigma^+} \leq 600$ MeV/c [20] and $350 \leq P_{\Sigma^+} \leq 750$ MeV/c [22]. The predictions at the central momenta are depicted in Fig. 6(f–i). The J-PARC E40 experiment measured the differential cross sections of $\Sigma^- p \rightarrow \Sigma^- p$ at four-momentum intervals, i.e., 470–550 MeV/c, 550–650 MeV/c, 650–750 MeV/c, and 750–850 MeV/c in [23]. Clearly, the covariant ChEFT results agree with the E40 results reasonably well. The quark-cluster model can also describe the experimental data quite well but not those of ESC08c. As a result, one can conclude that the latest $\Sigma^- p \rightarrow \Sigma^- p$ differential cross section data indeed impose strong constraints on the theoretical $BB$ interactions. \(^1\)

In Ref. [24], the J-PARC E40 Collaboration reported the inelastic differential cross sections of $\Sigma^- p \rightarrow \Lambda n$ for two momentum intervals, i.e., 470–550 MeV/c and 550–650 MeV/c. They are compared with predictions of the LO covariant ChEFT, those of the LO [39] and NLO [41, 51] HB ChEFT, as well as those of the quark cluster model [49, 73] in Fig. 7. It is clear that at LO neither the covariant ChEFT nor the HB ChEFT can reproduce the data at a quantitative level, but the fss2 and NLO HB ChEFT results are in much better agreement with the data. Considering that the LO covariant ChEFT can describe the elastic channel reasonably well (see Fig. 6), the poor performance for the inelastic channel is a bit unexpected. Nonetheless, the good performance of the NLO HB ChEFT clearly suggests that one need to perform higher order studies for this inelastic channel.

IV. CONCLUSION AND OUTLOOK

We predicted the total $S = -1$ N cross sections in a wide energy range up to $P_{lab} = 900$ MeV/c and for all the allowed channels based on the covariant chiral effective field theory. In particular, we showed that the predicted $\Sigma^+ p$ and $\Sigma^- p$ differential cross sections are in reasonable agreement with the latest J-PARC E40 data. The comparison with other models showed that the differential cross sections can help better constrain theoretical models. It should be noted that although the qualitative agreement with data supports the purpose of the present work, i.e., predicting predictions that can be used as (rough) guidance to plan future experiments, the comparison with either the J-PARC $\Sigma^- p \rightarrow \Lambda n$ differential cross sections or the CLAS $\Lambda p$ cross section data

\(^1\) It should be mentioned that in Ref. [46], it was shown that the NSC97f $\Sigma N$ phase shifts for $^3S_1$ with $I = 3/2$ are not consistent with the lattice QCD data.
show that higher order ChEFT studies are needed. On the other hand, the present study showed clearly how the measurement of different cross sections can help better constrain theoretical descriptions of baryon-baryon interactions, which otherwise cannot be distinguished between each other using only the total cross section data.

The hyperon-nucleon potentials are receiving much attention in recent years because they play an important role in our understanding of hypernuclear physics and dense neutron stars. In addition, there are ongoing and planned experimental efforts to measure them at facilities such as J-PARC, BEPC, LHC, JLab, and HIAF. In recent years, lattice QCD simulations have also greatly advanced our understanding of the hyperon-nucleon interactions and will achieve more in the near future. We hope that the results presented in this work will stimulate more future experimental, theoretical, and lattice QCD studies.

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FIG. 2: $\Lambda N$ and $\Sigma N$ ($I_3 = 1/2$) cross sections as functions of the laboratory momentum of the initial hyperon for each reaction as specified at the top of each sub figure. The results are obtained with $\Lambda_F = 600 \text{ MeV}/c$, and the bands are theoretical uncertainties estimated using Eq. (5). The experimental data are taken from Sechi-Zorn et al. (filled circles) [15], Alexander et al. (filled squares) [57], Hauptman et al. (open squares) [19], and Kadyk et al. (open circles) [17]. The additional curves are the theoretical predictions of the meson exchange models, NSC97a (red dashed lines) [27] and Jülich’04 (orange solid lines) [28].
FIG. 3: $\Lambda N$ and $\Sigma N$ ($I_3 = -1/2$) cross sections as functions of the laboratory momentum of the initial hyperon for each reaction as specified at the top of each subfigure. The results are obtained with $\Lambda_F = 600$ MeV/c, and the bands are theoretical uncertainties estimated using Eq. (5). The experimental data are taken from Engelmann et al. (filled circles) [13], Petschauer:2016tee (open circles) [59], Eisele et al. (filled squares) [18], Kondo et al. (open squares) [21] and Miwa et al. (open red circles) [24]. The additional curves are the theoretical predictions of the meson exchange models, NSC97a (red dashed lines) [27] and Jülich'04 (orange solid lines) [24].
FIG. 4: \( \Sigma^+ p \) and \( \Sigma^- n \) cross sections as functions of the laboratory momentum of the initial hyperon for each reaction as specified at the top of each sub figure. The results are obtained with \( \Lambda_F = 600 \text{ MeV}/c \), and the bands are theoretical uncertainties estimated using Eq. (5). The experimental data are taken from Eisele et al. (filled squares) [18] and Ahn et al. (open squares) [22]. The additional curves are the theoretical predictions of the meson exchange models, NSC97a (red dashed lines) [27] and Jülich’04 [28] (orange solid lines).

FIG. 5: \( \Lambda p \rightarrow \Lambda p \) cross sections as functions of the laboratory momentum. The results are obtained with \( \Lambda_F = 600 \text{ MeV}/c \), and the bands are theoretical uncertainties estimated using Eq. (5). For comparison, we also present the results of the leading order HB ChEFT (gray shadow band) [39]. The experimental data are taken from Rowley et al. (open red circles) [25]. The additional curves are the theoretical predictions of the meson exchange models, NSC97a (orange dashed line) [27] and Jülich’04 (blue dashed dotted line) [28].

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FIG. 6: $\Lambda N$ and $\Sigma N$ differential cross section $d\sigma/d\cos\theta$ as a function of $\cos\theta$, where $\theta$ is the c.m. scattering angle, for various $P_{lab}$. The results are obtained with $\Lambda F = 600$ MeV/c, and the bands are theoretical uncertainties estimated using Eq. (5). In (a), (b), (c), (d), and (e), the experimental data are taken from Engelmann et al. (filled circles) [13], Eisele et al. (open circles) [18], and Ahn et al. (filled diamonds and open diamonds) [20, 22]. In (f), (g), (h), and (i), the experimental data (filled squares) are taken from the J-PARC E40 experiment [23] for the $\Sigma^- p$ elastic channel of the laboratory momentum from 470 to 850 MeV/c. The open squares are the experimental data obtained in the KEK-PS E289 experiment [21]. The additional curves are the theoretical predictions of the meson exchange models, NSC97a (red dashed lines) [27] and J"ulich’04 (orange solid lines) [28], and ESC08 (magenta dash-dot-dotted lines) [49], and the quark cluster model fss2 (navy dash-dotted lines) [73].
FIG. 7: $\Sigma^- p \rightarrow \Lambda n$ differential cross section $d\sigma/d\cos \theta$ as a function of $\cos \theta$, where $\theta$ is the c.m. scattering angle, for various $P_{\text{lab}}$. The results are obtained with $\Lambda_F = 600$ MeV/$c$, and the bands are theoretical uncertainties estimated using Eq. (5). For comparison, we also present the results of the leading order HB ChEFT (gray shadow bands). The experimental data (open red circles) are taken from the J-PARC E40 experiment [24] for the $\Sigma^- p \rightarrow \Lambda n$ reaction of the laboratory momentum from 470 to 650 MeV/$c$. The additional curves are the theoretical predictions of the meson exchange models, ESC08 (magenta dash-dot-dotted lines) [49], and the quark cluster model fss2 (navy dash-dotted lines) [73]. The solid orange and black dashed lines represent the results of the HB ChEFT model, NLO13 [41] and NLO19 [51], respectively.

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