FIELD-ANTIFIELD FORMALISM IN A NON-ABELIAN THEORY
WITH ONE AND TWO FORM GAUGE FIELDS
COUPLED IN A TOPOLOGICAL WAY

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We make a systematic development of the non-Abelian formulation of two-form gauge fields with topological coupling with the Yang-Mills one-form connection. An analysis of the gauge structure, reducibility conditions and physical degrees of freedom is presented. We employ the Batalin-Vilkovisky formalism to quantize the resulting theory.
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1. Introduction

There has been an increasing interest for gauge theories with rank higher than one, specially for the case of rank two. This is so, first because these theories have an interesting structure of constraints, related to its reducibility condition, where quantization deserves some additional care comparing with the usual gauge theories of rank one. We mention that antisymmetric tensor fields also appear as one of the massless solutions of string theories, in company with photons, gravitons etc. Another interesting aspect of these theories is to provide a possible mechanism of mass generation for gauge fields when vector and tensor fields are coupled in a topological way.

The general idea of this mechanism resides in the following: tensor gauge fields are antisymmetric quantities and consequently in $D = 4$ they exhibit six degrees of freedom. By virtue of the massless condition, the number of degrees of freedom goes down to four. Since the gauge parameter is a vector quantity, this number would be zero if all of its components were independent. This is nonetheless the
case because the system is reducible (which means that the gauge transformations are not all independent) and we mention that the final number of physical degrees of freedom is just one. It is precisely this degree of freedom that can be absorbed by the vector gauge field in the vector-tensor gauge theory in order to acquire mass. The problem is that the reducibility condition is a property that is naturally present just in the Abelian case. So, a directly non-Abelian extension of this theory would not make sense because it would exhibit no physical degrees of freedom.

Freedmann and Townsend 7 have presented an interesting model for a non-Abelian formulation of two-form gauge fields, where the reducibility condition is kept on-shell in a sense that the curvature based on the non-Abelian vector field is null. This model has been widely analyzed by many authors, mainly due to its peculiar structure of constraints 8. Unfortunately, this particular condition of zero curvature avoids the use of this model in connection with a full Yang-Mills theory and, consequently, the possibility of having alternative mechanisms of mass generation for non-Abelian vector fields.

In recent papers, it has been shown that there is a way of getting a non-Abelian formulation for two-form gauge fields without the condition of zero curvature 9,10,11,12. This is achieved by introducing an auxiliary field, that plays the role of a kind of Stückelberg field, which makes a suitable transition between non-Abelian and Abelian limits to avoid the problem of zero degrees of freedom. The consistency of this formulation can be verified by using it in the mass generation of the Salam-Weinberg theory. The obtained mass for the vector fields are effectively the same as the one given by the spontaneous symmetry breaking 13.

The purpose of the present paper is twofold: First we make a systematic development of the non-Abelian formulation of two-form gauge fields with topological coupling with the Yang-Mills one-form connection. We also show that the model presented in Refs. 9,10,11 have a reducible gauge structure. This is important because it assures that the number of degrees of freedom of the non-Abelian theory is just the dimension of the algebra times the number of degrees of freedom of its Abelian limit. Later on we use the field-antifield formalism due to Batalin and Vilkovisky (BV) 14,15 to quantize the resulting model.

Our paper is organized as follows: In Sec. 2 we make a general and brief review of the one-form gauge field in order to fix the notation and prepare for transition for the two-form case. In Sec. 3, we consider the Abelian two-form case and analyze the difficulties we have in passing to the non-Abelian formulation. In Sec. 4, we present a revised version of the model introduced at Ref. 9,10,11 to obtain the non-Abelian two-form gauge field theory, explore its interesting gauge algebraic structure and develop its BV quantization. We left Sec. 5 for some concluding remarks.
2. Brief review of the the one-form gauge field theory

Let us start by reviewing the Yang-Mills gauge theory, here described in terms of Lie algebra valued $p$-forms, in order to establish conventions and general definitions. They will be useful in the next sections to properly describe two-form non-Abelian gauge field theories. Let

$$A = A^a_{\mu} T^a dx^\mu$$  \hspace{1cm} (2.1)

be the one-form connection, with values in the Lie algebra of $SU(N)$, whose generators are assumed to satisfy

$$[T^a, T^b] = i f^{abc} T^c$$
$$\text{Tr} (T^a T^b) = \delta^{ab}$$  \hspace{1cm} (2.2)

Although (2.1) is defined in relation to some coordinate basis $\{dx^\mu\}$, it can be obviously referred to any other non-holonomic basis.

On any Lie algebra valued $p$-form $\omega$,

$$\omega = \omega^a T^a$$  \hspace{1cm} (2.3)

it is possible to define the exterior covariant derivative by

$$D \omega = d \omega - i A \wedge \omega + i (-1)^p \omega \wedge A$$  \hspace{1cm} (2.4)

where $d$ represents the usual exterior derivative.

The curvature two-form

$$F = dA - i A \wedge A$$  \hspace{1cm} (2.5)

is such that the Biachi identities

$$DD \omega = i \omega \wedge F - i F \wedge \omega$$
$$\equiv i [\omega, F]$$  \hspace{1cm} (2.6)
$$DF = 0$$  \hspace{1cm} (2.7)
are satisfied for any gauge connection $A$ and algebra-valued p-form $\omega$. A fundamental consequence of (2.6) is that if we define the gauge variation of the one-form connection to be given by

$$\delta A = D\chi$$  \hfill (2.8)

where $\chi$ is an arbitrary algebra valued parameter, the curvature two-form transforms as

$$\delta F = d\delta A - i\delta A \wedge A - i A \wedge \delta A = D\delta A = DD\chi = i[\chi, F]$$  \hfill (2.9)

This implies that the action

$$S_0 = +\frac{1}{2} \text{Tr} \int F \wedge^* F$$  \hfill (2.10)

is invariant under (2.8), due to the cyclic property of the trace operation. In (2.10), the symbol $*$ represents the Hodge duality operation, so the integrand is proportional to the oriented volume element in $M_4$, the Minkowski space-time. To be more precise, the duality operation maps the p-form coordinate basis $\{1, dx^\mu, dx^\mu \wedge dx^\nu, dx^\mu \wedge dx^\nu \wedge dx^\rho, dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma\}$ into the basis $\{\eta, \eta^\mu, \eta^{\mu\nu}, \eta^{\mu\nu\rho}, \eta^{\mu\nu\rho\sigma}\}$. In these expressions, $\eta$ is the four-form oriented volume element, $\eta^\mu$ is a three-form, $\eta^{\mu\nu}$ is a two-form and so on. They satisfy relations such as $dx^\mu \wedge \eta_\nu = \delta^\mu_\nu \eta$, $dx^\mu \wedge \eta_{\nu\rho} = 2\delta^\mu_{[\nu} \eta_{\rho]}$ and $dx^\mu \wedge \eta_{\nu\rho\sigma} = 3\delta^\mu_{[\nu} \eta_{\rho\sigma]}$. As $F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$, $*F = \frac{1}{2} F_{\mu\nu} \eta^{\mu\nu}$ and consequently $F \wedge^* F = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \eta$.

An arbitrary variation of $F$, due to (2.8), takes the form $\delta F = D\delta A$ (see Eq. 2.9). This implies that $\delta S = -\text{Tr} \int D^* F \wedge \delta A$. So the Hamilton principle applied to (2.10) implies in the equation of motion $D^* F = 0$.

As it is well known, variation (2.8) closes in an algebra. Actually, it is easy to verify that

$$[\delta_2, \delta_1] A = \delta_3 A$$  \hfill (2.11)

if one defines
\[
\chi_3 = i [\chi_1, \chi_2]
\] (2.12)

The field-antifield quantization of YM theory can be easily constructed once we consider the classical action (2.10), the gauge variation (2.8) and the algebra structure contained in (2.12). We get the field-antifield functional generator

\[
Z[J] = \int D\Phi^A D\Phi_A^\dagger \delta \left[ \Phi_A^\dagger - \frac{\partial \Psi}{\partial \Phi_A^\dagger} \right] \exp i \left( S + \text{Tr} \int J \wedge A \right)
\] (2.13)

where

\[
S = \text{Tr} \int \left( \frac{1}{2} F \wedge * F - A^\dagger \wedge Dc - i c^\dagger c^2 + \Bar{c}^\dagger \wedge g \right)
\] (2.14)

and \( J \) is an external three-form current. In (2.14), \( c \) is the ghost corresponding to the parameter \( \chi \), \( \Bar{c} \) is its corresponding four-form antifield and \( \Bar{A} \) is the three-form antifield corresponding to \( A \). \( \Phi^A \) and \( \Phi_A^\dagger \) represent the components of all the fields and antifields appearing in \( S \). The form degrees of the antifields introduced in (2.14) are defined in such a way that the integrand is proportional to the oriented volume element. They also have opposite Grassmanian parity when compared to the corresponding fields. The pair \( \Bar{g} \) and \( b \) is necessary for the gauge-fixing procedure. This is also done with the aid of the gauge-fixing fermion \( \Psi \). According to Eq. (2.13), we must restrict the antifields by the condition \( \Phi_A^\dagger = \frac{\partial \Psi}{\partial \Phi^\dagger} \). For non-degenerated choices, the generator functional is independent of \( \Psi \). A convenient gauge-fixing can be given by \( \Psi = \text{Tr} \int \Bar{c} d^r A \). The usual Faddeev-Popov expression for the functional generator is recovered after integrating over the antifields and the auxiliary pairs.

It is convenient to introduce a fundamental structure in the field-antifield procedure which is given by the so-called antibracket. Let \( X \) and \( Y \) be algebra valued forms. The antibracket between \( X \) and \( Y \) is given by

\[
(X, Y) = \frac{\partial_r X}{\partial\Phi^A} \frac{\partial_l Y}{\partial\Phi_A^\dagger} - \frac{\partial_r X}{\partial\Phi_A^\dagger} \frac{\partial_l Y}{\partial\Phi^A}
\] (2.15)

We observe that the Witt notation of sum and integration over internal variables are being understood. The BRST variation of any functional is defined through

\[
s X = (X, S)
\] (2.16)

It is not difficult to verify that \( s \) is nilpotent, as a consequence of the master equation \( s S = 0 \), which is satisfied as the theory is anomaly free. It is a mere exercise to
derive the BRST variations for all fields and antifields and verify that indeed they act as (nilpotent) right differentials.

3. Two-form gauge field theories

Let us start from the Abelian two-form case in order to become clear what are the difficulties we have to pass to its non-Abelian counterparts. To do this, we see that in analogy to the one-form gauge field theory, one can introduce a two-form gauge-field

\[ B = \frac{1}{2} B_{\mu\nu} \, dx^\mu \wedge dx^\nu \quad (3.1) \]

We note that this is not a connection. In spite of this fact we define a geometric quantity \( H \) in a similar way to the Abelian curvature \( F \), that is to say, by using the exterior derivative,

\[ H = dB = \frac{1}{6} H_{\mu\nu\rho} \, dx^\mu \wedge dx^\nu \wedge dx^\rho \quad (3.2) \]

where

\[ H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu} \quad (3.3) \]

Concerning the gauge transformation of \( B \), we assume that it has a similar transformation to the Abelian version of that one of \( A \) (see expression (2.8)):

\[ \delta B = d\xi \quad (3.4) \]

Here, \( \xi \) is a one-form gauge parameter. We directly notice a characteristic of the Abelian two-form formulation. Since \( \xi \) is a one-form parameter, one may rewrite it in terms of a exterior derivative of some zero-form parameter, say

\[ \xi = d\alpha \quad (3.5) \]

If this is done, no gauge transformation is obtained for \( B \). This means that the components of the gauge parameter \( \xi \) are not all independent (that is why the theory is said to be reducible). This is a welcome result because if this was not so,
the theory written in terms of $B$ would have zero degrees of freedom. Actually, the action

$$S_0 = -\frac{1}{2} \int H \wedge^* H$$

describes a reducible theory with only one degree of freedom, being equivalent to a massless scalar field. Actually, the numer of degrees of freedom is $n (= 6$: number of components of $B$) $- n_1 (= 2$: due to the massless condition of the field $B$) $- m_0 (= 4$: number of gauge parameters) $+ m_1 (= 1 = number of reducibility conditions$), which gives $n = 1$. The study of scalar particles by means of this involved theory is natural in the context of some string and supergravity formulations. An interesting feature of such a theory is that it can effectively generate mass for one-form gauge fields, without obstructing their gauge invariance. In the Abelian case, this goal can be done if one adopts the action

$$S_0 = \int \left( \frac{1}{2} F \wedge^* F - \frac{1}{2} H \wedge^* H + m F \wedge B \right)$$

which presents a general topological coupling between the one-form and the two-form gauge fields. Here, the number of degrees of freedom is 3, one for $B$ and two for $A$. It is easy to verify that when one eliminates $B$ from the two coupled equations of motion derived from (3.7), one verifies that $F$ satisfies a massive wave equation. The quantization of such a theory will not be discussed here, but it is well known that the field $A$ presents a massive pole in its propagator. For details, see Ref. [6].

We now pass to consider the non-Abelian case. When compared to the one-form gauge field theory, the non-Abelian extension of the two-form case is more subtle. The first problem arises in the gauge transformation. Since $B$ is not a connection and we do not have a precise definition of its geometrical nature, it appears to be reasonable to infer that its gauge transformation is the non-Abelian generalization of (3.4), i.e.

$$\delta B = D\xi$$

(3.8)

The point we would like to emphasize here is that the transformation (3.8) is not reducible. If one rewrites the one-form parameter $\xi$ in terms of a zero-form, say $\alpha$, as $\xi = D\alpha$, one obtains

$$\delta B = DD\alpha = i [\alpha, F]$$

(3.9)
where in the last step we have used the first Bianchi identity (2.6). We then notice that the theory can be considered reducible just on-shell if the curvature $F$ vanishes identically.

A second problem concerns the definition of $H$. In the Abelian case, the Maxwell curvature two-form is obtained by just taking the exterior derivative of the connection. So, in that case, it was very natural the obtainment of the Abelian $H$ by means of the exterior derivative of $B$. However, the non-Abelian Yang-Mills curvature is not the covariant derivative of the connection. Thus, we have to be carefully in defining what is the non-Abelian $H$.

It is usually considered the definition

$$H = DB = dB - i[A,B]$$

which has the correct Abelian limit and also takes values in the SU(N) algebra if $B$ does. Now, under (3.8), for arbitrary $A$ variations, we have

$$\delta H = D D\xi - i[\delta A, B] = i[\xi, F] - i[\delta A, B]$$

So the non-Abelian generalization of (3.6) will only be gauge invariant if we impose that not only $F$ must vanish, but also that $A$ must not transform. The action

$$S_0 = \frac{1}{2} \text{Tr} \int (F \wedge B^* + \frac{1}{2} A \wedge^* A)$$

presents these features. It is invariant under $\delta B = D\xi$ when one restricts $\delta A$ to vanish. As $F$ also vanishes as a consequence of the equation of motion of $B$, it describes a pure gauge given by $A = ig^{-1} d g$, where $g$ represents a $SU(N)$ group element. It is easy to show that it presents $1 \times (N^2 - 1)$ degrees of freedom, where $N^2 - 1$ gives the dimension of the algebra. Actually (3.10) is completely equivalent to a SU(N) $\sigma$-model. Also, after eliminating $A$, it has (3.6) as the Abelian limit. The point here is that in a theory like that, $B$ is coupled to a one-form gauge field which is a pure gauge. This fact forbids a mechanism of mass generation for non-vanishing curvature gauge one-form fields, that can be constructed, for instance, from (3.7). This is so because there the curvature two-form has a dynamical role and can not be set to zero.
4. Coupling two-forms with non pure-gauge one-forms

The solution of the problems quoted in the end of the last section can be achieved by redefining the quantity $B$ by

$$\tilde{B} = B + D\Omega$$

(4.1)

where the one-form $\Omega$ plays the role of a St"uckelberg field. In Ref. the same procedure is applied in the context of the BF Yang-Mills theory.

Under the transformations

$$\delta B = i[\chi, B] + D\xi + i[\eta, F]$$
$$\delta \Omega = i[\chi, \Omega] - \xi - D\eta$$

(4.2)

we immediately verify that

$$\delta \tilde{B} = i[\chi, \tilde{B}]$$
$$\delta \tilde{H} = i[\chi, \tilde{H}]$$

(4.3)

where $\tilde{H} = D\tilde{B}$. Transformations (4.2) generalize (3.8) and is reducible, as we are going to see soon. The sector associated with the 0-form parameter $\eta$ is an additional symmetry related to the reducibility of the theory. Of course, we are keeping the transformations (2.8)-(2.9) for $A$ and $F$ respectively.

Due to the cyclic property of the trace operation, the non-Abelian generalization of action (3.6) now becomes easy. It is enough to use $\tilde{H}$ instead of $H$, and there is no necessity of imposing pure gauge for the Yang-Mills sector. Furthermore, in the Abelian limit, $\tilde{H}$ becomes identical to (3.10), since $d^2$ vanishes identically.

The inclusion of a topological term does not break the gauge invariance. One can verify that the action

$$S_0 = \text{Tr} \left( \frac{1}{2} F \wedge^* F - \frac{1}{2} \tilde{H} \wedge^* \tilde{H} + m F \wedge \tilde{B} \right)$$

(4.4)

is indeed invariant under (2.8) and (4.2). From (4.4) one can extract the equations of motion, associated respectively with the variations of $B$, $\Omega$ and $A$, as
\[ D^* \tilde{H} + m F = 0 \]
\[ \left[ F^* \tilde{H} \right] = 0 \]
\[ D(\ast F - i[\Omega, \ast \tilde{H}] - m \tilde{B}) + i[B, \ast \tilde{H}] = 0 \]  \hspace{1cm} (4.5)

We observe that the second of the equations above gives the integrability condition for the first of them. As \( \Omega \) does not appear dynamically in the equations of motion, it can be seen simply as an auxiliary field, with no dynamics. This means that quantically there will be present no modes associated to them. It is trivial to verify that equations (4.5) imply, in the Abelian limit, that
\[ d^* d^* F + m^2 F = 0 \]  \hspace{1cm} (4.6)

which means that the free theory is massive and that does not contain the presence of the St"uckelberg field \( \Omega \).

Let us now consider the quantization of the theory described by the action (4.4). First we need to derive the algebraic structure of its gauge transformations. We can verify that the usual Yang-Mills structure given by (2.12) is kept, but it is also necessary to consider that there is a mixing in the composition rules for the other parameters. We get
\[ \xi_3 = i [\chi_1, \xi_2] - i [\chi_2, \xi_1] \]
\[ \eta_3 = i [\chi_1, \eta_2] - i [\chi_2, \eta_1] \]  \hspace{1cm} (4.7)

and it is a mere exercise to show that the algebra closes on all the fields \( \phi^i \) belonging to the theory: \([\delta_2, \delta_1] \delta \phi^i = \delta_3 \phi^i \).

Now we observe that the gauge transformations (2.8) and (4.2) are reducible. To show this, let us use a compact notation where the fields \( \phi^i \), given here by \( A, B \) and \( \Omega \) are represented in a column matrix
\[ (\phi) = \begin{pmatrix} A \\ B \\ \Omega \end{pmatrix} \]  \hspace{1cm} (4.8)

Consistently, the gauge parameters can also be written in the same way
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\[ (\epsilon) = \begin{pmatrix} \chi \\ \xi \\ \eta \end{pmatrix}. \]  

(4.9)

In matrix form, the gauge transformations (2.8) and (4.2) are then just given by

\[ \delta (\phi) = (R)(\epsilon) \]  

(4.10)

where the gauge generator is

\[ (R) = \begin{pmatrix} D & 0 & 0 \\ i [\ , B] & D & i [\ , F] \\ i [\ , \Omega] & -1 & -D \end{pmatrix} \]  

(4.11)

In \((R)\) the lines correspond respectively to \(A, B\) and \(\Omega\) and the columns to \(\chi, \xi\) and \(\eta\). Now we note that \(\det(R) = -D(D^2 - i [\ , F])\), which vanishes identically due to Bianchi identity \((2.6)\). This indicates that it has at least one null vector \((Z)\), that gives the reducibility conditions associated with the gauge symmetries. It is easy to verify that the null vector is given by

\[ (Z) = \begin{pmatrix} 0 \\ D \\ -1 \end{pmatrix} \]  

(4.12)

Incidentally, we mention that in higher dimensions, other linearly independent null vectors can be found.

Following the usual rules for the field-antifield quantization \(\text{[16, 15]}\), we get from \((2.8), (2.11), (4.2), (4.7)\) and \((4.21)\), the functional generator (see Eq. \((2.13)\))

\[ Z[J, Q] = \int D\Phi^A D\Phi^\dagger A \delta \left[ \Phi^\dagger A - \frac{\partial \Phi}{\partial \Phi^A} \right] \exp i \left\{ S + \text{Tr} \int (J \wedge A + Q \wedge B) \right\} \]  

(4.13)

where

\[ S = S_0 + \text{Tr} \left( A^\dagger \wedge Dc + B^\dagger \wedge (ic, B) + Db + i[a, F]) + \Omega^\dagger \wedge (i[c, \Omega] - b - D a) \\
- ic^2 + b^\dagger \wedge (-i\{b, c\} + D\theta) + a^\dagger \wedge (-i\{a, c\} - \theta) \\
+ \bar{a}^\dagger \wedge c + \bar{b}^\dagger \wedge f + \bar{e}^\dagger \wedge g + \bar{d}^\dagger \wedge h + \bar{\Xi}^\dagger \wedge \pi + \ldots \right) \]  

(4.14)
Note that it was not included an external source for \( \Omega \), since it is not dynamical, but we have introduced the two-form external source \( Q \) which couples to \( B \). The action \( S_0 \) is given by \( \{ 4.4 \} \). \( c \) is the usual ghost corresponding to the gauge parameter \( \chi \), \( a \) is the zero-form ghost corresponding to \( \eta \) and \( b \) is the one-form ghost corresponding to the one-form gauge parameter \( \xi \). Since the theory has one reducibility relation, we correspondingly have introduced the ghost-for-ghost \( \theta \). The antifields corresponding to \( B \) and \( \Omega \) are respectively two and three forms. Those corresponding to the \( c \) and \( a \) are four-forms, and that one corresponding to \( b \) is a three-form. Also trivial pairs were introduced in order to fix the degrees of freedom related to the forms \( B \) and \( \Omega \) and their reducibilities. As usual, \( \{ b, c \} \) means that \( b \) and \( c \) are symmetrized.

All the original fields \( \phi^i \) have ghost number zero, the ghosts have ghost number one and the ghosts-for-ghosts have ghost number two. The ghost number of an antifield is minus the ghost number of the corresponding field minus one. Of course, \( S \) has total ghost number zero. Now, the dots in \( \{ 4.14 \} \) represent terms depending on possible higher order reducibility conditions or higher rank gauge structure functions. Instead of calculating them directly from the algebraic gauge structure of the theory, they can be determined from the classical master equation, or equivalently, from the nilpotency of the BRST transformations over all the fields and antifields.

According to \( \{ 2.16 \} \), the BRST transformations of the fields appearing in action \( \{ 4.14 \} \) are given by

\[
\begin{align*}
s A &= Dc \\
s B &= i [c, B] + D b + i [a, F] \\
s \Omega &= -i [c, \Omega] - b - D a \\
s c &= -ic^2 \\
s b &= -i \{ b, c \} + D\theta \\
s a &= -i \{ a, c \} - \theta
\end{align*}
\]

\[ \text{(4.15)} \]

besides the BRST transformations that come from trivial pairs. It is a kind of directly calculation to verify the nilpotency of the transformations \( \{ 4.14 \} \). We have just to be a little careful with the condition that \( s \) acts as right derivative, which is a consequence of the definition given by expression \( \{ 2.16 \} \). For instance,

\[
\begin{align*}
s^2 A &= s Dc \\
&= s (dc - i [A, c])
\end{align*}
\]
\[ ds \cdot c + i \{ s \cdot A, c \} - i[A, s \cdot c] \tag{4.16} \]

which vanishes identically when one uses again (4.15). The nilpotency condition for the other fields can be verified in the same way. It is not difficult to show that

\[ s^2 b = D (s \theta - i[c, \theta]) \]
\[ s^2 a = -(s \theta - i[c, \theta]) \tag{4.17} \]

which vanishes identically if

\[ s \theta = i[c, \theta] \tag{4.18} \]

and the BRST transformations actually act as a differential over all the fields of the theory. The expressions above show us that we have to add to the integrand of \( S \) given in expression (4.14) the term

\[ i\bar{\theta}[c, \theta] \tag{4.19} \]

which completes the form of the BV action. At the same time, this gives us the expression for the remaining higher order gauge structure functions. It can be verified that this theory is anomaly free and as a consequence the quantum action is just given by \( S \), the quantum master equation being in this way identified with the classical one. This verification can follow the lines presented in Ref.\textsuperscript{17} and the detailed analysis will be presented elsewhere.

Once the BV action (4.14) has been given, we can calculate the number of degrees of freedom. Forgetting the dimension of the algebra, we see that \( n_0 = 4 + 6 \) and \( n_1 = 1 + 2 \), due to the number of components of \( A \) and \( B \) and their massless conditions. \( m_0 = 1 + 4 + 1 \) which is the number of gauge parameters and \( m_1 = 1 \), since we have one reducibility condition. This gives the number of degrees of freedom \( n_0 - n_1 - m_0 + m_1 = 2 \). This seems to be not the same as the one found for the Abelian case. The point here is that the equation of motion coming from the variation of \( \Omega \) is actually a constraint. Analysis of the constraint structure of the theory shows that the number of degrees of freedom actually is 3 as it should be.\textsuperscript{9} To fix the gauge we also need a fermion \( \Psi \). We can choose

\[ \Psi = \text{Tr} \int (\bar{\epsilon} d^* A + \bar{\eta} \wedge d^* B + \bar{\alpha} d^* \Omega + \bar{\theta} \; d^* b - d^* \bar{b} \wedge \Xi) \tag{4.20} \]
With this fermionic gauge-fixing functional, the antifields can be directly calculated. Replacing these values into the expression of $S$ we get

$$S_{\text{eff}} = S_0 + \int \left\{ d^* A g + (d^* B - d^* d^2) \wedge f + d^* \Omega \wedge e \\ - d^* \overline{c} \wedge Dc - d^* \overline{b} \wedge \left( i[c, B] + Db + i[a, F] \right) - d^* a \wedge (i[a, c] + D\theta) \\ + d^* \theta \left( \{b, c\} - D\theta \right) - d^* \overline{b} \wedge \pi + d^* b \wedge h \right\} \right.$$  \hspace{2cm} (4.21)

Integrating over the fields and ghosts belonging to the $B$ sector of the theory, one obtains in the non-interacting limit that the free vector field $A$ acquires a mass $m$ that appears as a pole of the corresponding propagator \cite{6}. It is important to mention that in the particular case of Abelian groups, the field $\Omega$ decouples from the $A, B$ sector, which keeps the canonical form found in \cite{5}.

5. Conclusion

In this work we have studied the BV quantization of a non-Abelian version of a two-form gauge field theory, where there is a topological coupling with a nonpure gauge Yang-Mills connection. The gauge algebraic structure of the theory was derived, pointing also its reducibility character. All of these aspects have been properly considered at the field-antifield functional quantization.

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