TeV scale partial mirage unification and neutralino dark matter

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Abstract

We study the TeV scale partial mirage unification scenario, where the gluino and wino masses are degenerate around a TeV scale, but the bino mass is not degenerate. This scenario has phenomenologically interesting aspects. First, because of the degeneracy between the gluino and wino masses, this scenario does not have the little hierarchy problem, that is, the higgsino mass is around 150 GeV. The lightest superparticle is a mixture of the bino and higgsino, and can lead to a right amount of thermal relic density as a dark matter candidate.

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1 Introduction

Supersymmetric extension of the standard model (SM) is one of the most promising candidates for a new physics at the TeV scale. In particular, the minimal supersymmetric standard model (MSSM) is interesting from the viewpoint of its minimality. The MSSM has several attractive aspects. The MSSM realizes the unification of three gauge couplings at the grand unified theory (GUT) scale \( M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV} \). Supersymmetry can stabilize the huge hierarchy between the weak scale and the GUT/Planck scale. Supersymmetric models with R-parity have a good candidate for dark matter, that is, the lightest superparticle (LSP).

However, these attractive aspects are not perfectly satisfying. First of all, there is still a fine-tuning problem as follows. By minimizing the Higgs scalar potential, the \( Z \) boson mass is obtained as

\[
\frac{1}{2} M_Z^2 \sim -\mu^2(M_Z) - m_{H_u}^2(M_Z),
\]

where \( \mu \) is the SUSY mass of up- and down-sector Higgs fields and \( m_{H_u} \) is the soft SUSY breaking mass for the up-sector Higgs field. Thus, natural values of \(|\mu^2|\) and \(|m_{H_u}^2|\) would be of \( O(M_Z^2) \). Otherwise, we need fine-tuning between \(|\mu^2|\) and \(|m_{H_u}^2|\) to cancel them and to lead to \( M_Z \). The soft mass \( m_{H_u} \) receives a large radiative correction between the weak scale and the cut-off scale \( \Lambda \),

\[
\Delta m_{H_u}^2 \sim -\frac{3 y_t^2}{4 \pi^2} m_{\tilde{t}}^2 \ln \frac{\Lambda}{m_{\tilde{t}}},
\]

where \( y_t \) is the top Yukawa coupling and \( m_{\tilde{t}} \) is the stop mass. The cut-off scale may be the GUT scale or Planck scale, and we may have \( \Delta m_{H_u}^2 \sim -2m_{\tilde{t}}^2 \) or \(-3m_{\tilde{t}}^2\). On the other hand, the theoretical upper bound for the lightest CP-even Higgs mass is obtained as \[1\]

\[
m_h^2 \leq M_Z^2 + \frac{3 m_t^4}{4 \pi^2 v^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2} + \cdots.
\]

The experimental bound \( m_h \geq 114.4 \text{ GeV} \) requires \( m_{\tilde{t}} \gtrsim 500 \text{ GeV} \). This value of \( m_{\tilde{t}} \) leads to a quite large correction \( \Delta m_{H_u}^2 \). Hence, in order to realize \( M_Z = 91 \text{ GeV} \), we need a few percent fine-tuning of the SUSY mass \( \mu \) and the soft SUSY breaking mass \( m_{H_u} \) at the GUT scale, although these two masses are, in general, independent parameters. This fine-tuning problem is sometimes called as the little-hierarchy problem between the weak scale and a TeV scale [2].

Several types of models have been proposed to solve the little hierarchy problem. Among them, the TeV scale mirage mediation [3][4][5] is one of most interesting scenarios, because the field content in the visible sector is the same as one of the MSSM. In the mirage mediation, the modulus mediation and anomaly mediation [6] are comparable [7][8], and such situation can be realized in the KKLMT-type of moduli stabilization [9].

One of interesting aspects in the mirage mediation is that the anomaly mediation effect and renormalization group (RG) effects cancel each other at the so-called mirage scale \( M_{\text{mir}} \). That is, soft SUSY breaking terms at \( M_{\text{mir}} \) appear equivalent to the pure modulus mediation, although there is no physical threshold at \( M_{\text{mir}} \). Therefore, the model with
\( M_{\text{mir}} = O(\text{TeV}) \), i.e., the TeV scale mirage model, is interesting as a solution of the little hierarchy problem. In the TeV scale mirage model, the superparticle spectrum derived from the pure modulus mediation appears at the TeV scale through the cancellation between the anomaly mediation and RG effects. In particular, the modulus mediation leading to

\[
|m_{H_u}| \sim \mu \sim M_Z, \quad m_{\tilde{t}} = O(1) \text{TeV}
\]

is interesting. Indeed, concrete models realizing the above spectrum have been studied in Ref. [3, 5]. In those models, gauge kinetic functions for three MSSM vector multiplets are universal and three gaugino masses are universal at \( M_{\text{mir}} \). In addition, the universal gaugino mass is of \( O(1) \text{ TeV} \).

One of interesting aspect in SUSY models is that they have a good candidate for the dark matter as the LSP. In the above TeV scale mirage scenario, the value of \( \mu \) is of \( O(M_Z) \) to avoid fine-tuning, while the gaugino masses are universal around a TeV scale and it is of \( O(1) \text{ TeV} \). Thus, the LSP is higgsino-like in the TeV scale mirage scenario. In this case, the thermal relic density of the LSP is much lower than cosmological observation [10].

Recently, the bottom-up analysis [11] showed that the degeneracy between the wino and gluino masses is most important to avoid fine-tuning in the Higgs sector, but the bino mass can vary with keeping the same degree of fine-tuning. When the bino mass varies, several phenomenological aspects would change. The LSP is a mixture of the higgsino and bino, and its thermal relic density would be totally different from one of higgsino-like LSP. Hence, in this paper we study the TeV scale partial mirage unification, where the wino and gluino masses are degenerate around \( O(1) \text{ TeV} \), but the bino mass is different. We study phenomenological aspects of this scenario, in particular the thermal relic density and direct detection possibility for the neutralino LSP.

Several authors have investigated phenomenological and cosmological aspects of mirage mediation [14, 15, 16, 17, 18, 10, 19, 20]. In particular, it has been noticed that moduli decay in the early universe can produce so many gravitinos and neutralino LSPs that successful Big Bang nucleosynthesis might be ruined and/or too large dark matter abundance would be obtained [16]. A possible way out of the cosmological moduli problem is to dilute the primordial moduli and the subsequently produced gravitinos and LSPs, through some mechanism such as the thermal inflation [21]. In this work, we assume that such a mechanism is realized and the neutralino dark matter is generated through the conventional thermal production mechanism.

This paper is organized as follows. In section 2, we study a concrete model leading to the TeV scale partial mirage unification. In section 3, we study phenomenological aspects of our scenario, in particular, the thermal relic density of the LSP. Section 4 is devoted to conclusion and discussion.

\footnote{See Ref. [12, 13] for other studies on a connection between naturalness of electroweak symmetry breaking and dark matter phenomenology.}
2 TeV scale partial mirage unification

2.1 Moduli stabilization in the generalized KKLT scenario

Indeed, our model is quite similar to the model for the TeV scale mirage [5], which is a generalization of the KKLT scenario for moduli stabilization [9, 23, 24].

We consider the IIB string model with the dilaton $S$, a single Kähler modulus $T$ and complex structure moduli $Z_{\alpha}$. First, we assume that the dilaton $S$ and complex structure moduli $Z_{\alpha}$ are stabilized by the flux-induced superpotential $W_{\text{flux}}(S, Z_{\alpha})$ [25], that is, they have heavy masses of $O(M_P)$, where $M_P$ is the Planck scale. At this stage, the Kähler modulus $T$ is not stabilized. To stabilize $T$, we introduce a $T$-dependent non-perturbative effect in the superpotential. In the original KKLT model, a simple term is considered as

$$W_{\text{np}} = Ae^{-aT},$$

where $A = O(M_P^3)$ and $a$ is a constant. Here and hereafter we use the unit, where $M_P = 1$. Such non-perturbative effect can be generated by a gaugino condensation of the hidden gauge sector on D7 branes, whose gauge kinetic function is proportional to $T$. In general, the gauge kinetic function is a linear combination of $S$ and $T$ as,

$$f_a = k_a T + \ell_a S,$$

e.g. on magnetized D-branes, where $k_a$ and $\ell_a$ are rational numbers [26]. The gaugino condensation in the hidden gauge sector may generate a non-perturbative term like $W_{\text{np}} \sim e^{-8\pi^2(k_h T + \ell_h S)}$. Thus, we consider the superpotential [23]

$$W = \langle W_{\text{flux}} \rangle - A_h e^{-8\pi^2(k_h T + \ell_h S)},$$

where $A_h = O(1)$ and $8\pi^2k_h = O(10)$. In the second term of the right hand side, the dilaton $S$ is replaced by its vacuum expectation value (VEV) $S_0$, because $S$ is assumed to be stabilized with a mass of $O(M_P)$ by the flux-induced superpotential $W_{\text{flux}}$.

With this superpotential and the Kähler potential,

$$K_0 = -3\ln(T + T^*),$$

we can write the scalar potential,

$$V_F = e^{K_0} \left[K_0^{TT^*}|D_T W|^2 - 3|W|^2\right],$$

where

$$D_T W \equiv (\partial_T K_0)W + \partial_T W.$$  

The Kähler modulus $T$ is stabilized at the SUSY point $D_T W = 0$, where we can estimate $\langle W \rangle \approx \langle W_{\text{flux}} \rangle$ because $8\pi^2k_h = O(10)$. At this SUSY point, the vacuum energy is negative,

$$V_F = -3m^2_{3/2},$$

We can replace $S$ by its VEV only when the flux-induced superpotential $W_{\text{flux}}$ includes its supersymmetric mass, which is heavier than the mass of $T$ and the gravitino mass. Otherwise, such analysis is not valid [27, 28].
where $m_{3/2}$ denotes the gravitino mass, $m_{3/2} = e^{K_0/2}W$. The modulus $T$ has a mass of $O(8\pi^2 m_{3/2})$, which is much larger than the gravitino mass.

To obtain a de Sitter (Minkowski) vacuum, we add the uplifting potential,

$$V_{\text{lift}} = e^{2K_0/3}P_{\text{lift}}.$$  \hspace{1cm} (9)

Such potential can be generated by putting anti D3-brane at a tip of warp throat \cite{9}, and the warp factor leads to a suppressed value of $P_{\text{lift}}$ \cite{25},

$$P_{\text{lift}} \sim e^{-32\pi^2 K \text{Re}(S_0)/3M},$$  \hspace{1cm} (10)

where $K$ and $M$ are integer-valued NS and R 3-form fluxes. We tune our parameters to realize almost vanishing vacuum energy, i.e.

$$V_F + V_{\text{lift}} \approx 0.$$  \hspace{1cm} (11)

Since $V_F \approx -3e^{K_0}|W_{\text{flux}}|^2$, the above fine-tuning requires

$$|W_{\text{flux}}|^2 \sim P_{\text{lift}} \sim e^{-32\pi^2 K \text{Re}(S_0)/3M}.$$  \hspace{1cm} (12)

Hence, we can parameterize $W_{\text{flux}}$ as

$$W_{\text{flux}} = A_0e^{-8\pi^2 \ell_0 S_0},$$  \hspace{1cm} (13)

where $\ell_0$ is a rational number and $A_0 = O(1)$. We consider the low-energy SUSY breaking, i.e. $m_{3/2} = O(10)$ TeV. That requires

$$8\pi^2 \ell_0 \text{Re}(S_0) \simeq \ln(M_P/m_{3/2}) \sim 4\pi^2.$$  \hspace{1cm} (14)

At the minimum of $V_F + V_{\text{lift}}$, the values of $T$ and the F-term $F^T$ are obtained as

$$k_0 T \simeq (\ell_0 - \ell_h)S_0,$$

$$\frac{F^T}{T + T^*} \simeq \frac{\ell_0}{\ell_0 - \ell_h \ln(M_P/m_{3/2})}.$$  \hspace{1cm} (15)

When $\ell_0/(\ell_0 - \ell_h) = O(1)$, $F^T/(T + T^*)$ is smaller than $m_{3/2}$ by a factor of $O(4\pi^2)$. That implies that the modulus mediation due to $F^T$ and the anomaly mediation are comparable in this scenario \cite{7,8}.

We have put anti D3-brane at the tip of warp throat. Instead of that, a similar uplifting can be realized by adding a spontaneous SUSY breaking sector, i.e. the F-term uplifting \cite{29,30}.

### 2.2 Soft SUSY breaking terms in the visible sector

Now, we consider soft SUSY breaking terms in the visible sector. We assume that the compactification scale is close to the GUT scale. Thus, the following initial values are obtained at the GUT scale.

First, we study the gaugino masses of the visible gauge sector, i.e. $SU(3) \times SU(2) \times U(1)_Y$. Here, these gauge groups are denoted by $G_a$, ($a = 1,2,3$), i.e. $G_1 = U(1)_Y$, $G_2 = SU(2)$ and $G_3 = SU(3)$. Suppose that the SU(3) and SU(2) gauge kinetic functions are given as

$$f_{\nu} = T + \ell S,$$  \hspace{1cm} (16)

where $f_{\nu}$ is the gaugino mass.
where $\ell$ is a rational number. The gauge coupling unification in the MSSM, $g_{\text{GUT}}^2 \simeq 2$, requires $\text{Re}(T) + \ell \text{Re}(S_0) \simeq 2$. The modulus-mediated contributes to the gluino and wino masses are obtained as

$$M_0 = F^T \partial_T \ln(\text{Re}(f_v)) = \frac{F^T}{T + T^*} \left( \frac{\ell_0 - \ell_h}{\ell_0 - \ell_h + k_h \ell} \right).$$

(16)

Since $F^T/(T + T^*) = O(m_{3/2}/(4\pi^2))$, the contributions due to the anomaly mediation are comparable. Thus, just below $M_{\text{GUT}}$ the gluino mass $M_3$ and wino mass $M_2$ are obtained as

$$M_a = M_0 + \frac{b_a}{16\pi^2} g_{\text{GUT}}^2 m_{3/2},$$

(17)

with $b_a = 1, -3$ for $a = 2, 3$. Then, at the energy scale $Q$, these gaugino masses are given as

$$M_a(Q) = M_0 \left[ 1 - \frac{1}{8\pi^2} b_a g_a^2(Q) \ln \left( \frac{M_{\text{mir}}}{Q} \right) \right],$$

(18)

where the so-called mirage scale $M_{\text{mir}}$ is defined as

$$M_{\text{mir}} = \frac{M_{\text{GUT}}}{(M_P/m_{3/2})^{\alpha/2}},$$

(19)

with

$$\alpha = \frac{m_{3/2}}{M_0 \ln(M_P/m_{3/2})} = \frac{\ell_0 - \ell_h + k_h \ell}{\ell_0}. \quad (20)$$

When $\alpha = 2$, we have $M_{\text{mir}} \sim 1 \text{ TeV}$, that is, the gluino and wino masses are unified around 1 TeV. Note that there is no physical threshold around $M_{\text{mir}}$. Here we consider the model with $\alpha = 2$.

If we consider the same gauge kinetic function for the $U(1)_Y$ group as Eq. (15), the bino mass is also unified at $M_{\text{mir}}$. However, the degeneracy between the bino and gluino masses is less important to reduce the degree of fine-tuning in the Higgs sector, although the degeneracy between the wino and gluino masses are important [11]. Hence, we consider a generic case for the gauge kinetic function of the $U(1)_Y$ group as

$$f_Y = k_Y T + \ell_Y S,$$

(21)

in the $U(1)_Y$ charge normalization, which can be embedded into the $SU(5)$ GUT. We assume that $k_Y \text{Re}(T) + \ell_Y \text{Re}(S) \simeq 2$, because of the gauge coupling unification, $g_{\text{GUT}}^2 \simeq 2$. Then, the modulus-mediated contribution to the bino mass is obtained as $k_Y M_0$. The bino mass also has a contribution due to the anomaly mediation, and at $M_{\text{GUT}}$ the bino mass is obtained as

$$M_1 = k_Y M_0 + \frac{b_1}{16\pi^2} g_{\text{GUT}}^2 m_{3/2},$$

(22)

where $b_1 = 33/5$. Obviously the bino mass $M_1$ is not degenerate at $M_{\text{mir}}$ unless $k_Y = 1$.

Next, we consider soft SUSY breaking scalar masses as well as A-terms. Such SUSY breaking terms are determined by the kinetic term of chiral superfield $\Phi^i$,

$$\int d^4 \theta C C^* e^{-K_0/3} Z_i \Phi^i \Phi^i,$$

(23)
where $Z_i$ is the Kähler metric of the matter field $\Phi^i$. Here, $C$ denotes the chiral compensator superfield, i.e. $C = C_0 + F^C \theta^2$, and its F-component is obtained as $F^C/C_0 = m^2_{4/2}$ in our model. Then, the modulus-mediated contributions to A-terms and soft scalar masses are obtained as

\begin{align*}
\tilde{A}_{ijk} &= a_{ijk} M_0 = F^T \partial_T \ln(e^{K_0} Z_i Z_j Z_k), \\
\tilde{m}^2_i &= c_i M_0^2 = -|F^T|^2 \partial_T \partial_T \ln(e^{-K_0/3} Z_i),
\end{align*}

where we have assumed that holomorphic Yukawa couplings are independent of the modulus $T$. Here we take the following form,

\begin{equation}
e^{-K_0/3} Z_i = (T + T^*)^{n_i}, \tag{26}\end{equation}

where $n_i$ is a rational number. Then, $a_{ijk}$ and $c_i$ are obtained as

\begin{align*}
a_{ijk} &= (n_i + n_j + n_k) \left( \frac{\ell_0 - \ell_h + k_h \ell_h}{\ell_0 - \ell_h} \right), \\
c_i &= n_i \left( \frac{\ell_0 - \ell_h + k_h \ell_h}{\ell_0 - \ell_h} \right)^2. \tag{28}\end{align*}

A-terms and soft scalar masses have contributions due to the anomaly mediation. Thus, these values at $M_{GUT}$ are given as

\begin{align*}
A_{ijk} &= \tilde{A}_{ijk} - \frac{1}{16 \pi^2} (\gamma_i + \gamma_j + \gamma_k) m_{3/2}, \\
m^2_i &= \tilde{m}^2_i - \frac{1}{32 \pi^2} \frac{d \gamma_i}{d \ln Q} m_{3/2}^2 + \left[ \sum_{jk} \frac{1}{4} |y_{ijjk}|^2 \tilde{A}_{ijk} - \sum_a g^2_a C^a_2(\Phi^i) r_a M_0 \right] m_{3/2}, \tag{30}\end{align*}

with $r_{2,3} = 1$ and $r_1 = k_Y$, where $\gamma_i$ denotes the anomalous dimension of $\Phi^i$ and $y_{ijjk}$ is Yukawa couplings. In addition, $C^a_2(\Phi^i)$ denotes quadratic Casimir of the field $\Phi^i$ under the gauge group $G_a$. We include RG effects to obtain A-terms and soft scalar masses at the energy scale $Q$. If $k_Y = 1$ and the following relations

\begin{equation}a_{ijk} = c_i + c_j + c_k = 1, \tag{31}\end{equation}

are satisfied for large Yukawa couplings $y_{ijjk}$, RG effects and the anomaly mediation contributions cancel each other at $M_{\text{mir}}$. Then, we have $A_{ijk}(M_{\text{mir}}) = \tilde{A}_{ijk}$ and $m^2_i(M_{\text{mir}}) = \tilde{m}^2_i$. Even for $k_Y \neq 1$, this spectrum is realized approximately because RG effects due to $U(1)_Y$ are not important except for right-handed slepton masses unless $|k_Y| \geq O(1)$. We take

\begin{equation}c_{H_u} = 0, \quad c_{t_L} = c_{t_R} = \frac{1}{2}. \tag{32}\end{equation}

Then we can realize the little hierarchy between $m_{H_u} = O(M_0^2/(4 \pi^2))$ and $m^2_t = M_0^2/2$. We consider $M_0 = O(1)$ TeV and a moderate value of $\tan \beta$, e.g. $\tan \beta = 10$. Then, we neglect all of Yukawa couplings except the top Yukawa coupling. For the down-sector Higgs field, we take $c_{H_d} = 1/2$. 

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With the above assignment of $a_{ijk}$ and $c_i$, we have a smaller higgsino mass $\mu = 100-200$ GeV. Thus, a small value of $|k_Y|$, $|k_Y| < 1$, is interesting because in such a case the LSP would be a mixture between the higgsino and bino. If $k_Y$ and the bino mass are quite small, right-handed slepton masses for $c_{\ell} = 1/2$ may become tachyonic at the weak scale. Thus, we take $c_{\ell} = 1$ for both left-handed and right-handed slepton masses. At any rate, slepton masses are irrelevant to the fine-tuning problem of the Higgs sector.

Alternatively, in order to increase slepton masses we could consider the scenario with an extra (anomalous) $U(1)$ gauge group. We assume that such $U(1)$ sector is separated away from the SUSY breaking anti D3 brane, and $U(1)$ is broken at a certain scale, e.g. $M_{GUT}$. Such breaking induces another source of soft scalar masses, which are proportional to $U(1)$ charge $q_i$ of the fields $\Phi^i$,

$$m_i^{2(D)} = q_i D.$$  \hspace{1cm} (33)

The size of $D$ is model-dependent.\(^3\) This type of contribution could also increase slepton masses.

The size of slepton masses are important for analysis on the thermal relic density of the LSP as shown in the next section. Hence, in the following section we consider two cases for slepton masses, 1) the case that slepton masses are determined from $c_{\ell} = 1$ and 2) the case that slepton masses vary. The latter case can be realized by the D-term contributions.

### 3 Neutralino Dark Matter

In this section, we consider neutralino dark matter phenomenology. Recent WMAP and other observations imply that the cold dark matter abundance is \(^{34}\)

$$0.085 < \Omega_{DM} h^2 < 0.119 \text{ (95\% CL)},$$ \hspace{1cm} (34)

where $h \simeq 0.7$ is the scaled Hubble constant. We assume that the neutralino LSPs were in thermal equilibrium when the temperature of the Universe is larger than the LSP mass $m_\chi$. As the temperature drops below $m_\chi$, the number density of the LSP is exponentially suppressed. At some point neutralino LSP annihilation rate becomes smaller than the Hubble expansion rate. Then the neutralino LSPs fall out of equilibrium and the LSP number density in a comoving volume remains constant \(^{35}\). We also assume that the neutralino LSP constitutes all the cold dark matter in the Universe at the current epoch.

In the TeV scale mirage mediation, in which all three gaugino masses are unified at TeV scale, it turns out that the neutralino LSP is higgsino-like \(^{10}\). This is because the gluino mass $M_3$ is smaller than bino mass $M_1$ (and wino mass $M_2$) at higher energy scales. (Notice that $M_3 : M_2 : M_1 \simeq (1 - 0.3\alpha)g_3^2 : (1 + 0.1\alpha)g_2^2 : (1 + 0.66\alpha)g_1^2$, and $\alpha \sim 2$ for the TeV scale mirage mediation.) Such a small $M_3$ gives a small stop mass squared and in turn leads to a small $|m_{H_u}^2|$ and thus $|\mu|$ at the weak scale, compared to bino mass $M_1$ and wino mass $M_2$.

For higgsino-like LSP, the lighter chargino $\chi^+_1$ and the two light neutralinos $\chi^0_1$, $\chi^0_2$ are nearly degenerate. In this case dominant annihilation processes for the neutralinos and

\(^3\)See Ref. \(^{24, 31}\) for the D-term contributions in the KKLT scenario and Ref. \(^{32, 33}\) for the D-term in dilaton-moduli mediation of heterotic string and type I string theory.
Figure 1: (a) sparticle masses at the weak scale and (b) thermal relic density $\Omega_{\chi} h^2$ of neutralino LSP, as a function of $M_1(M_{\text{GUT}})$.

Chargino are neutralino pair annihilation into gauge bosons, and the neutralino-neutralino and neutralino-chargino coannihilations into fermion pair \(^{[36]}\). These annihilation processes are so effective that the thermal relic density of the neutralino LSP is too small unless neutralino LSP is rather heavy ($m_{\chi_1^0} \sim |\mu| \sim 1$ TeV). Therefore, the higgsino LSP with $|\mu| \sim O(M_Z)$, which avoids fine-tuning in the Higgs sector, cannot provide the correct amount of thermal relic density in Eq. (34).

On the other hand, the TeV scale partial mirage unification scenario can provide not only a solution of the little hierarchy problem but also a right amount of thermal relic density. As a specific numerical example, we choose a parameter set:

$$\alpha = 2, \ M_0 = 800 \text{ GeV}, \ c_{H_u} = 0, \ c_{H_d} = 1/2, \ c_q = 1/2, \ c_l = 1, \ \tan\beta = 10, \quad (35)$$

while varying the bino mass at the GUT scale within some range, $100 \text{ GeV} < M_1(M_{\text{GUT}}) < 2$ TeV. Figures 1 show (a) sparticle masses at the weak scale and (b) thermal relic density $\Omega_{\chi} h^2$ of the neutralino LSP, as a function of $M_1(M_{\text{GUT}})$. One can notice that $\mu$ values at weak scale remain small i.e., $130 \lesssim \mu \lesssim 160$ GeV so that there is no little hierarchy problem in this case. Our model leads to the CP even Higgs mass, $m_h \sim 116$ GeV.

For a large $M_1(M_{\text{GUT}})$ value, $\mu$ is much smaller than $M_1$ at the weak scale implying higgsino-like LSP. It leads to a very small relic density $\Omega_{\chi} h^2 \sim O(10^{-3})$. The bino mass at the weak scale decreases as $M_1(M_{\text{GUT}})$ decreases, and becomes similar to $\mu$ value at the weak scale when $M_1(M_{\text{GUT}}) \sim 350$ GeV. In the bino-higgsino mixed region of LSP, the relic density $\Omega_{\chi} h^2$ increases rapidly as $M_1(M_{\text{GUT}})$ decreases, due to the enhanced bino-component of neutralino LSP. When $M_1(M_{\text{GUT}}) \sim 300$ GeV, $\Omega_{\chi} h^2 \sim 0.1$, thus providing a right amount of relic density which is consistent with the WMAP bound on the dark matter density.

As $M_1(M_{\text{GUT}})$ further decreases, the neutralino LSP becomes bino-like and the relic density $\Omega_{\chi} h^2$ gets too large and increases until $M_1(M_{\text{GUT}}) \sim 200$ GeV. Below this point,
an interesting annihilation channel for neutralino LSP is open. For the region around 
$M_1(M_{\text{GUT}}) \simeq 160$ GeV, the mass of the neutralino LSP is equal to the half of the light 
CP even higgs mass, i.e., $m_{\chi^0} \sim m_h/2$. In this case the neutralino pair annihilation through 
s-channel higgs exchange becomes very efficient so that the relic density $\Omega_b^h$ is reduced 
to a very small values $O(10^{-3})$, passing acceptable ones $O(10^{-1})$. When $m_{\chi^0} \sim m_Z/2$, Z 
resonance effect is dominant for reducing the relic density.

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Figure 2: (a) slepton masses at the weak scale and (b) thermal relic density $\Omega_b^h$ of 
neutralino LSP, as a function of $m_{\text{slepton}}(\text{GUT})$.

As we discussed in the last section, for the scenario with an extra (anomalous) $U(1)$ 
gauge group, additional $D$-term contributions to soft terms would make slepton mass a 
free parameter in practice. In order to see possible effects of the $D$-term contributions, 
we fix $M_1(M_{\text{GUT}}) = 200$ GeV with the parameter set (35) while varying $m_{\text{slepton}}(M_{\text{GUT}})$, the 
slepton mass at the GUT scale. Notice that in this case, the neutralino LSP is bino-
like and $m_{\chi^0} \simeq 75$ GeV. Figures 2 show (a) slepton masses at weak scale and (b) the 
thermal relic density $\Omega_b^h$, as a function of $m_{\text{slepton}}(M_{\text{GUT}})$. When the slepton mass is large 
($m_{\text{eR}} \simeq 500$ GeV), the relic density is quite large ($\Omega_b^h \simeq 0.6$), as expected for the bino-
like LSP with rather heavy sparticle mass spectrum. As $m_{\text{slepton}}(M_{\text{GUT}})$ decreases, however, 
slepton masses at weak scale decrease. Accordingly, the relic density $\Omega_b^h$ decreases and 
gets close to the WMAP bound (34) when $m_{\text{eR}} \sim 150$ GeV. It is known that in this case, 
the LSP relic density is mainly determined from neutralino pair annihilation into lepton 
pair through $t$-channel exchange of SU(2) singlet sleptons [37].

For $m_{\text{slepton}}(M_{\text{GUT}}) \lesssim 115$ GeV, the neutralino LSP and the lighter stau are almost degenerate. Then LSP-stau coannihilation [35] becomes very effective to reduce thermal relic density of the neutralino LSP. From the Fig. 2(b), one can notice that the thermal relic density $\Omega_b^h$ reaches the WMAP range and then drops quickly below 0.01 in the small 
$m_{\text{slepton}}(M_{\text{GUT}})$ region.

Our model has also an interesting aspect for the direct detection search of neutralino 
dark matter. For the spin-independent cross section of neutralino-proton scattering, the
Figure 3: (a) spin-independent cross section of neutralino and proton and (b) $b \to s\gamma$ branching ratio, as a function of $M_1(M_{\text{GUT}})$.

Contributions from $t$-channel CP even Higgs exchanges are usually dominant \cite{35}. The cross section $\sigma_{SI}$ would enhance if neutralino LSP is a mixed state of gaugino and higgsino, due to the nature of neutralino-neutralino-Higgs couplings. Figure 3(a) shows the spin-independent scattering cross section as a function of $M_1(M_{\text{GUT}})$ with the parameters \cite{35}. The $\sigma_{SI}$ is quite small ($\sim 5 \times 10^{-9}$ pb) for large $M_1(M_{\text{GUT}})$ region i.e., higgsino-like LSP case. There is, however, about one order of magnitude increase of $\sigma_{SI}$ in the bino-higgsino mixed region. For $M_1(M_{\text{GUT}}) \simeq 300$ GeV, which provides a right amount of thermal relic density $\Omega_\chi^2 \simeq 0.1$, the spin-independent scattering cross section is $\sigma_{SI} \simeq 4 \times 10^{-8}$ pb with $m_{\tilde{\chi}_1^0} \simeq 105$ GeV. This cross section value is quite close to the current limit from XENON experiment \cite{39}, i.e., $8.8 \times 10^{-8}$ pb for a WIMP mass of 100 GeV. Therefore, our model would be explored in the near future experiments on the direct searches.

Before closing this section, we comment on experimental constraints. Our benchmark point \cite{35} satisfy the experimental bounds on particle masses such as $m_{\tilde{\chi}^+} > 104$ GeV for chargino and $m_h > 114$ GeV for light Higgs boson. Figure 3(b) shows $b \to s\gamma$ branching ratio $BR(b \to s\gamma)$ for the parameter choice \cite{35}, as a function of $M_1(M_{\text{GUT}})$. The NLO calculation for $BR(b \to s\gamma)$ gives about $3 \times 10^{-4}$ for our parameter choice, insensitive to $M_1(M_{\text{GUT}})$. These predictions are rather smaller than the current world average of experimental values \cite{40}, $BR(b \to s\gamma)^{\text{exp}} = (3.55 \pm 0.26) \times 10^{-4}$, due to large contribution from chargino-stop loop which adds destructively to Standard Model contribution for our choice on the sign of $\mu$ ($> 0$). Considering theoretical and experimental uncertainties, it turns out that the calculated branching ratio is consistent with the measured one within $2\sigma$ range.
4 Conclusions

We have studied the TeV scale partial mirage unification scenario, where the gluino and wino masses are degenerate, but the bino mass is not degenerate. We have shown an example leading to such a spectrum. This spectrum has phenomenologically interesting aspects. First, there is no fine tuning problem because of the degeneracy of the gluino and wino masses, that is, our model leads to $130 \lesssim \mu \lesssim 160$ GeV. The LSP is the mixture of the bino and higgsino. In the TeV scale partial mirage unification, a right amount of thermal relic density of neutralino LSP can be obtained through various channels for neutralino annihilations. A mixed bino-higgsino LSP, which is available through adjusting the bino mass at the GUT scale, may lead to an appropriate neutralino annihilation rate into gauge bosons and so the right amount of the relic density. The neutralino pair annihilation via s-channel higgs exchange play an important role for obtaining the suitable relic density, when $m_\chi \sim m_h/2$ in bino-like LSP region. Furthermore, if the slepton mass can vary independently, LSP annihilations through $t$-channel SU(2) singlet slepton exchange or LSP-stau coannihilation can make the thermal relic density satisfy the WMAP bound on dark matter density. The TeV scale partial mirage unification scenario also provides a sizable spin-independent scattering cross section between neutralino dark matter and nucleon, which can be explored in near future experiments, when the neutralino dark matter is a mixture of bino and higgsino.

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