Non-random structures in universal compression and the Fermi paradox

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Abstract. We study the hypothesis of information panspermia assigned recently among possible solutions of the Fermi paradox (“where are the aliens?”). It suggests that the expenses of alien signaling can be significantly reduced, if their messages contained compressed information. To this end we consider universal compression and decoding mechanisms (e.g. the Lempel-Ziv-Welch algorithm) that can reveal non-random structures in compressed bit strings. The efficiency of the Kolmogorov stochasticity parameter for detection of non-randomness is illustrated, along with the Zipf’s law. The universality of these methods, i.e. independence from data details, can be principal in searching for intelligent messages.

Introduction

The Fermi paradox [1] outlines the puzzling silence of the universe regarding intelligent life. It is a widely discussed topic that connects to various disciplines, from astrophysics to linguistics. The concept of information panspermia introduced in [2] and listed in [1] as Solution 23 to the Fermi paradox, implies the transmission of terrestrial life via compressed strings. The idea is based on the estimation [2] that Earth organisms—including humans, and the entire terrestrial life up to bacteria—have common parts in their genomes. Hence the entire genome can be efficiently compressed—the theoretical, but uncomputable, lower bound of the compression degree is given by the Kolmogorov complexity [3,4]—and the resulting bit string can be transmitted to over Galactic distances, e.g. by Arecibo type radio telescope. After decoding, signals can represent themselves as traveling life streams. One can imagine that the terrestrial life is a result of such a transmitted package.

The principal difficulty with checking this hypothesis is that the decoding of such signals has to be based on criteria that differ from those employed in intra-terrestrial communication. For instance, the spectral (monochromaticity) and temporal (modulations) features are likely to be not efficient for extra-terrestrial messages [1]. Indeed, a well-compressed signal has to behave as white noise by its statistical properties and hence it cannot be distinguished from natural signals, except in the cases when the precise code is known in advance [5]. When searching for intelligent signals one looks for “artificial” features such as monochromaticity. The latter is however costly, especially at semi-isotropic transmission.

Thus traveling bit strings require different strategies for distinguishing them from known physical mechanisms and eventually for their decoding. Our aim is to outline several strategies based on universal information compression and decoding features.

Conjecture: Once intelligent bit string signals will be compressed, we have to look at and reveal universal features of information compression which are absent in compressed signals originated via natural physical mechanisms.

Below we suggest schemes that possibly enable to reveal non-random structures in bit strings. We focus on universal schemes, i.e. those applying to the large classes of situations. This is consistent with Minsky’s methodology on the priority of universal concepts—in computer science and (more generally) mathematics—when searching for intelligent signals [6].

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1 Kolmogorov’s complexity

We start with the most fundamental notion of compression.

The Kolmogorov’s complexity $K(x)$ of a bit string $x$ with length $n$ is defined to be the length (in bits) of the shortest program that —starting from some fixed initial state— will generate $x$ and halt [3,4]:

$$K(x) = \min l(x).$$  \hfill (1)

$K(x)$ depends on a specific computer that executes candidate programs. This dependence is weak in the following sense: there exists a universal computer $\mathcal{U}$ such that for any other computer $C$ [3,4]

$$K_\mathcal{U}(x) < K_C(x) + C,$$  \hfill (2)

where $C$ depends only on $C$ (it does not depend on $x$), and $C_C/n \to 0$ for $n \to \infty$. The validity of (2) stems from the fact that any computer operates with finite means, hence a program to be executed on $\mathcal{U}$ can be represented as $\pi_C p_C$, where $\pi_C$ is the index (coordinate) of $C$, while $p_C$ is a program of $C$. Since $\pi_C$ is finite, we are back to (2).

Note that the notion of the computation time is not involved in definition (1).

Thus the Kolmogorov’s complexity does provide a universal lower bound for the length of possible compressions of $x$ that can recover $x$ uniquely.

Equation (1) easily generalizes to the conditional complexity $K(x|y)$, where during the (conditional) minimization all programs are assumed to start with $y$, i.e. the latter is known. In particular, one can consider $K(x|l(x))$: the conditional complexity given the knowledge of the string length.

It is important that almost all long bit strings $x$ are incompressible or algorithmically random [3,4]:

$$K(x|l(x)) \geq l(x),$$  \hfill (3)

i.e. their (conditional) complexity is not much shorter than their length [3,4]. This follows from the fact that there are $2^n$ bit strings of length $n$, while the amount of shorter strings (i.e. shorter programs) is much less, i.e. there are $2^{n-k} \ll 2^n$ bit strings of length $n-k$. It thus is contradictory to assume that almost all strings of length $n$ can be generated by such programs. An example of an arbitrary long string that can be generated by a short program is the $n$-digit rational approximation to $\pi$ as $\frac{2^n}{n} \equiv 0.146723\ldots$.

On the other hand, we have

$$K(x) < K(x|l(x)) + \log_2 l(x) + \text{const},$$  \hfill (4)

where

$$\log_2 l(x) \equiv \log_2 \log_2 \log_2 \log_2 \log_2 l(x) + \ldots,$$  \hfill (5)

and iterated logarithms continue as far as the logarithm is well-defined. Indeed, knowing $l(x)$ we can just order the computer to print $x$: the knowledge of $l(x)$ is obligatory, since the computer should know where to halt. This knowledge demands $\log_2 l(x)$ bits, while the knowledge of the latter string demands $\log_2 \log_2 l(x)$ bits etc.

However, it is impossible to prove that some single bit string of length $n$ is not compressible, if $n$ is large enough. Thus the corresponding function is not computable [4,3]. To understand this point, order the strings in a lexicographic way (i.e. 0 comes before 1, 00 comes before 01, etc.) and try to look for the first string of length $n$ whose complexity is not smaller than $n$. If such a string can be found, then the very description “first string of length $n$ whose complexity is not smaller than $n$” constitutes a rather short program for that string, hence this string is compressible to a large extent. The resulting contradiction shows that although most of strings are incompressible, one can never show that an individual string is incompressible, because showing that would immediately mean that the string is compressible. This undecidability is the major hindrance for practical applications of the Kolmogorov’s complexity. Another implication of the undecidability is that one can never compute after how many steps $n$ a given (even a short) program is going to halt. For, if we can compute $n$ for any such program, we shall sequentially check all short programs and eventually determine that certain strings are not compressible.

Non-compressible strings hold all feasible (computable) features of probability theory. The precise definition of randomness (which is equivalent to the algorithmic randomness) was given by Martin-Löf [7]. It does improve upon the earlier concept of random “kollektiv” by von Mises, which does not support all features of the probability theory (e.g. the law of the iterated logarithm) [7].

For an ergodic, finite-memory random process $(X_1, \ldots, X_n)$, the Kolmogorov’s complexity of almost any realization $(x_1, \ldots, x_n)$ converges for $n \to \infty$ to the entropy of the random process [4]:

$$|H(X_1, \ldots, X_n) - C(x_1, \ldots, x_n)| = O(\log_2 n),$$  \hfill (6)

and

$$H(X_1, \ldots, X_n) \equiv - \sum_{x_1, \ldots, x_n} P(x_1, \ldots, x_n) \log_2 P(x_1, \ldots, x_n),$$
where \( P \) is the probability. Note that

\[
H(X_1, \ldots, X_n) = \mathcal{O}(n), \quad C(x_1, \ldots, x_n) = \mathcal{O}(n).
\]

Equation (6) is consistent with the (first) Shannon’s theorem that determines \( H(X_1, \ldots, X_n) \) to be the minimal number of bits necessary for describing (with a negligible error) realizations of the random process [4].

2 Universal coding on the example of Lempel-Ziv-Welch algorithm

Two related aspects of the Kolmogorov’s complexity is that it does not involve the computation time — hence the search for a shorter description can take indefinitely long — and that it is not computable [3,4]. It is useful for establishing bounds [2], but it cannot be employed for quantifying the degree of compression/complexity in practice.

At this point it is useful to employ the notion of a universal, asymptotically optimal data-compression code that is to large extent inspired by the notion of the Kolmogorov’s complexity [4]. Here asymptotically optimal means that when applied to random, ergodic processes, the average code-length of the code approaches for long sequences the entropy of the process, i.e. the analogue of (6) holds. And universal means that for achieving this optimal performance, the code need not be given the frequencies of the symbols to be coded [4]. Normally, the instantaneously decodable (i.e. prefix-free) feature is also assumed within the definition of universality. Recall that prefix-free codes can be decoded on-line, i.e. without knowing the whole (long) message [4]. (Otherwise, the knowledge of the space-symbol is required for the decoder [4].)

Recall that several standard examples of optimal data-compression codes — e.g. the Huffman’s code or the arithmetic code— do demand a priori knowledge of symbol frequencies, since they operate by coding more frequent symbols via shorter code-words (the same idea is employed already in the Morse code) [4].

The most known (but not the only) example of a universal, asymptotically optimal data-compression code is the Lempel-Ziv-Welch algorithm [8–10]; see [4] for a review. The algorithm and its modifications have been widely used in practice: although newer schemes improve upon them, they provide a simple approach to understanding universal data compression algorithms. The rough idea of this method is as follows [8–10,4]: the string to be coded is parsed (e.g. by commas) into substrings that are selected according to the following criteria. After each comma one selects the shortest substring that has not occurred before. Hence the prefix of this substring did occur before, and the parsed substring can be coded via the coordinate of this occurrence and the last symbol (bit) of the substring.

The Lempel-Ziv-Welch algorithm presents an example of simple and robust scheme that is relatively easy to uncover. For a given string the code-length of its Lempel-Ziv-Welch compression provides an estimate for the algorithmic complexity. This Lempel-Ziv complexity [11] is also widely employed in practice, in particular in biomedical research [12].

For our purposes it is important to stress that the Lempel-Ziv complexity can be employed for detecting non-random structure inherent in noisy data, e.g. it can detect random permutations of words in a text [13].

3 The Kolmogorov’s stochasticity parameter

Now we will recall that non-random structures in strings can be also efficiently detected by means of the Kolmogorov’s stochasticity parameter. This parameter is defined for \( n \) independent, ordered, real-valued random variables \( X_1 \leq X_2 \leq \ldots \leq X_n \). Assuming the cumulative distribution function of \( X \),

\[
F(x) = P\{X \leq x\},
\]

one defines an empirical distribution function \( F_n(x) \)

\[
F_n(x) = \begin{cases} 
0, & x < X_1; \\
k/n, & X_k \leq x < X_{k+1}, \quad k = 1, 2, \ldots, n - 1; \\
1, & X_n \leq x. 
\end{cases}
\]

The Kolmogorov’s stochasticity parameter \( \lambda_n \) is defined as [14–17]

\[
\lambda_n = \sqrt{n} \sup_x |F_n(x) - F(x)|.
\]

Kolmogorov proved [14] that for any continuous cumulative distribution function \( F \):

\[
\lim_{n \to \infty} P\{\lambda_n \leq \lambda\} = \Phi(\lambda),
\]
where the limit converges uniformly, and where \( \Phi(\lambda) \) is the fourth elliptic function

\[
\Phi(\lambda > 0) = \psi_4 \left( 0, e^{-\lambda^2} \right) = \sum_{k=-\infty}^{+\infty} (-1)^k e^{-2k^2\lambda^2},
\]

(11)

where \( \Phi(0) = 0 \). Now \( \Phi(\lambda) \) is invariant with respect to \( F \), hence it is universal. As a function of \( \lambda \), \( \Phi(\lambda) \) monotonously grows from 0 to 1. It is close to zero for \( \lambda < 0.4 \), where \( \Phi(0.4) = 0.0028 \), and close to 1 for \( \lambda > 1.8 \), where \( \Phi(1.8) = 0.9969 \).

Equations (8)–(11) are the base for the standard Kolmogorov-Smirnov test. A simpler implementation of this test was proposed by Arnold [16]. Let \( \lambda^* \) be the empirically observed value of \( \lambda_n \). It was shown that even for a single bit string, \( \Phi(\lambda^*) \approx 0 \), e.g. \( \lambda^* < 0.4 \), is a good indicator of randomness [16,17].

To illustrate the power of this method in revealing the degree of randomness of sequences, we will consider the following example. Let us consider sequences as superposition of random and regular sequences. Namely, we will consider one-parameter sequences defined as

\[
z_n = \alpha x_n + (1 - \alpha)y_n,
\]

(12)

where \( x_n \) are random sequences and

\[
y_n = \frac{an \mod b}{b},
\]

(13)

are regular sequences defined within the interval \((0,1)\), and \( a \) and \( b \) are mutually fixed prime numbers, parameter \( \alpha \) while varying within 0 and 1 is indicating the variation in the fraction of random and regular sequences.

Thus we have \( z_n \) with a cumulative distribution function

\[
F(X) = \begin{cases} 
0, & X < 0 \\
\frac{X^2}{2\alpha(1-\alpha)}, & 0 < X \leq \alpha \\
\frac{2\alpha X - X^2}{2\alpha(1-\alpha)}, & \alpha < X \leq 1 - \alpha \\
1 - \frac{(1 - X)^2}{2\alpha(1-\alpha)}, & 1 - \alpha < X \leq 1 \\
1, & X > 1.
\end{cases}
\]

(14)

We will inquire into the stochastic properties of \( z_n \) vs. the parameter \( \alpha \) varying within the interval \([0,1]\) for fixed \( a \) and \( b \), in order to follow the transition of the sequences from regular to random ones.

For each value of \( \alpha \) for fixed \( a \) and \( b \), namely, \( a = 611, b = 2157 \), 100 sequences \( z_n \) were generated, of 10000 elements each sequence. The latter then were divided into 50 subsequences and for each subsequence parameter \( \Phi(\lambda_n) \) was obtained and hence the empirical distribution function \( G(\Phi) \) of them was determined \((m \text{ varied within } 1,\ldots,50)\). In accord to Kolmogorov’s theorem, for example, for purely random sequences, the empirical distribution should be uniform. So, we estimated \( \chi^2 \) of the functions \( G(\Phi) \) and \( G_0(\Phi) = \Phi \) as an indicator for randomness. Grouping 100 \( \chi^2 \) values per one value of \( \alpha \), we constructed mean and error values for \( \chi^2 \). The dependence of \( \chi^2 \) vs. \( \alpha \) for given values of \( a \) and \( b \) is shown in fig. 1, revealing the informativity of Kolmogorov’s parameter approach in quantifying the degree of non-randomness in the sequences.

The Kolmogorov’s parameter has been applied, for example, in revealing of non-random structures in genomic sequences [18]. Here is a sample piece of the latter:

\[
ACAGAGCTGAGTCACGTGGTGAAT \ldots,
\]

(15)

where \( A, G, C \) and \( T \) stand for adenine, guanine, cytosine, and thymine, respectively. Such non-random structures were employed for detecting mutations in human genome sequences, because mutations can be related to their regions with locally higher randomness [18]. In particular, it was shown that only 3-base nucleotide (codon) coding enables to distinguish somatic mutations.

4 Zipf’s law

It is natural to look for general features of communication systems, those which will presumably be present in meaning-conveying (non-random) messages, even if it is not (yet) known how decipher this meaning [19]. One general feature
Fig. 1. The $\chi^2$ for the sequence $z_n$ vs. the parameter $\alpha$ reflecting the variation of the ratio of random and regular fluctuations.

of man-created texts —that remains stable across different languages, and to some extent can be traced out as well in certain animal communication systems [19]— is the law discovered by Estoup [20] and independently (and much later) by Zipf [21]; see [22–29] for an incomplete list of references and [30] for a review. This Zipf’s law states that ordered frequencies

$$f_1 \geq f_2 \geq \ldots \geq f_m,$$

(16)
of words display a universal dependency,

$$f_r \propto r^{-1},$$

(17)
of the frequency on the rank $r$, $1 \leq r \leq m$. The law expressed by (17) gets modified for very rare words, those with frequency $O\left(\frac{1}{m}\right)$. This set of rare words is known as hapax legomena. However, the Zipf’s law efficiently generalizes to this situation, so that a single expression describes well both moderate and small frequencies [31].

The main point of looking at such rank-frequency relation is that once they hold the Zipf’s law, this fact will likely indicate on a non-random structure implicit in a message, and allow to identify this message as a text [19,32].

The precise origin of the Zipf’s law is not yet completely clear. Zipf himself believed that this origin is to be sought in specific co-adaptation of the speaker (coder) and hearer (decoder) [21]: the former finds it economical to employ a possibly smaller set of words for denoting possibly larger set of meanings. Hence the speaker tends to minimize the redundancy. In contrast the hearer (decoder) would like to introduce sufficient redundancy, so as to minimize communication errors. The qualitative idea by Zipf was formalized in several derivations (of certain aspects) of the Zipf’s law [22,24,26]. However, these derivations do not explain all the relevant aspects of the law, e.g. its unique extension to the hapax legomena domain. Such aspects are explained by the derivation of the law proposed in [31] that deduces the law from rather general properties of the mental lexicon, the cognitive (mental) system responsible for the organization of speech and thought [31].

The Zipf’s law gets modified in a definite way if one goes out of the realm of alphabetic languages. E.g. in a sufficiently long Chinese text the frequencies of characters (that are more morphemes than words) follow a modified Zipf’s law [33]. However, these modifications concern only relatively low-frequency characters [33]. Moreover, for sufficiently short Chinese texts the Zipf’s law holds fully and is not distinguishable from its standard form of English texts [33,31].

5 Conclusions

We expanded over the information panspermia hypothesis proposed in [2] and reviewed in [1]. The basic implication of this hypothesis is that alien messages may not be rare, but they are compressed. Recognizing such messages may be challenging, since they may look like a noise. In fact, an ideally compressed message should be operationally (algorithmically) random, according to the Kolmogorov’s complexity theory; see sect. 1. However, this ideal compression
is not achievable in principle, because it relates to the undecidability. Thus practically compressed messages can still show certain ordered patterns. In this contribution we considered several methods that can detect those patterns.

Though the ideal compression is not achievable in principle, there are several optimal and universal compression algorithms. Here optimal means that for a long random data the algorithm compresses according to the first Shannon’s theorem. Following, the methodology proposed by Minsky [6], one can conjecture that some of those compression schemes is known to aliens. Here we analyzed one of the most widespread schemes of this kind, the Lempel-Ziv-Welch compression algorithm.

Next, we considered the Kolmogorov’s stochasticity parameter, an approach closely related to the Kolmogorov-Smirnov test [15–17]. This parameter also has a substantial degree of universality and was recently shown to be very effective in detecting genetic mutations [18]. Searching for mutations can be regarded as looking for locally random domains in DNA [18].

Finally, we turned to the Zipf’s law: (again) a universal law that folds for ordered word frequencies of (almost all) human texts. The law does indicate on a non-random structure —though the underlying cause of this structure is not yet clear—and it can survive after compression.

The common thread of these methods is that they are universal, i.e. independent of details of data. This is a principal condition in searching for extra-terrestrial messages.

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