Effects of Neutrino Oscillation on the Supernova Relic Neutrino Background

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We investigate to what extent the oscillation or conversion of neutrinos enhances the expected event rate of the supernova relic neutrino background (SRN) at the Super-Kamiokande detector (SK). The SRN $\bar{\nu}_e$'s can be almost completely exchanged with $\nu_\mu$-like neutrinos by the MSW oscillation under the inverse mass hierarchy with $\Delta m^2 \sim 10^{-8} - 10^5$ [eV$^2$], or by the magnetic moment of Majorana neutrinos with $\mu_\nu \gtrsim 10^{-12} \mu_B$ and $\Delta m^2 \sim 10^{-4} - 10^0$ [eV$^2$]. In the standard calculation of the SRN flux, the event rate of the SRN $\bar{\nu}_e$'s at the SK in the observable energy range of 15–40 MeV can be enhanced from 1.2 yr$^{-1}$ to 2.4 yr$^{-1}$ if all $\bar{\nu}_e$'s are exchanged with $\nu_\mu$-like neutrinos. The enhancement is prominent especially in the high energy range ($\gtrsim 25$ MeV). In the astrophysically optimistic calculation, the event rate becomes as high as 9.4 yr$^{-1}$. Because the theoretical upper bound of the SRN events without oscillation is about 5 yr$^{-1}$ taking account of the various astrophysical uncertainties, we might have to resort to the neutrino oscillation if more than 5 events in a year, as well as a significantly harder spectrum, were observed in the SK.

There are two astrophysical neutrino sources which have already been detected until present: the Sun and the Supernova 1987A, and the source which is expected to become the third by the observation of the Super-Kamiokande detector (SK) \cite{1} is the supernova relic neutrino background (SRN). The SRN is the accumulation of the neutrinos emitted from past supernovae which have ever occurred in the universe \cite{2,3}. The detection of the SRN would provide us valuable information on the history of the supernova rate and hence the evolution of galaxies, as well as the properties of the supernova neutrinos. In the earlier paper \cite{10}, we calculated the SRN flux and spectrum by using the standard model of the galaxy evolution and predicted the realistic event rate of the SRN $\bar{\nu}_e$'s at the SK. The expected event rate is unfortunately very low: 1.2 yr$^{-1}$ as the central value and 4.7 yr$^{-1}$ as the optimistic value considering the astrophysical uncertainties; both are the rate in the observable energy range of recoil positrons, 15–40 MeV. The supernova rate is much higher in the early phase of the galaxy evolution than in the present, but neutrinos emitted from such old supernovae are considerably redshifted in energy and fall below the observable energy range.

However, neutrino oscillation or conversion of the supernova neutrinos may enhance the expected event rate, and such effects were not considered in the previous paper \cite{10,11}. We consider here to what extent the SRN events can be enhanced by the neutrino oscillation or conversion. The observable energy window is relatively high compared with the average energy of supernova neutrinos, and the reaction cross section of the detection ($\bar{\nu}_e p \rightarrow e^+ n$) is proportional to $E_\nu^2$, where $E_\nu$ is...
the neutrino energy. Therefore, if $\bar{\nu}_e$’s are exchanged with $\nu_\mu(\bar{\nu}_\mu)$’s or $\nu_\tau(\bar{\nu}_\tau)$’s, the SRN event rate becomes larger because of the higher temperature (or average energy) of $\nu_\mu$-like neutrinos than $\bar{\nu}_e$’s [12]. (In collapse-driven supernovae, $\nu_\mu$’s, $\nu_\tau$’s and their antiparticles can approximately be treated as identical particles.) The first of such possibilities is the vacuum oscillation of $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$, which occurs if the vacuum mixing angle is quite large compared with that of the quark sector. Although such large mixing is suggested from some of the candidates for the solution of the solar neutrino problem, such as the ‘just-so’ solution [13] or the large-angle MSW solution [14], the conversion probability of $\bar{\nu}_e$’s and $\bar{\nu}_\mu$’s would at most be 1/2, with the maximum vacuum mixing, after the phase averaging. On the other hand, more efficient conversion process exists, that is, the resonant oscillation or conversion induced by the effective matter potential in a star (well known as the MSW effect [15]). However, the ordinary MSW oscillation induced by the mixing of the flavor eigenstates and the mass eigenstates is relevant only to neutrinos and not to antineutrinos, under the direct effect [15]). However, the ordinary MSW effect under the inverse mass hierarchy, and another is the resonant spin-flavor conversion of $\bar{\nu}_e \leftrightarrow \nu_\mu$ induced by the flavor changing magnetic moment of Majorana neutrinos.

There are actually some particle-physics models which give the inverse mass hierarchy [16–18], and some possible phenomenological consequences have also been discussed [13–20]. The MSW resonance condition,

$$\frac{\Delta m^2}{2E_\nu} \cos 2\theta = \sqrt{2}G_F Y_e \frac{\rho}{m_p}, \quad (1)$$

can be satisfied with $\Delta m^2 \lesssim 10^5$ [eV^2] above the neutrino sphere in supernovae, where $\Delta m^2 = m_{\bar{\nu}_e}^2 - m_{\nu_\mu}^2$ (defined as positive under the inverse mass hierarchy), $\theta$ is the vacuum mixing angle, $Y_e$ the electron number fraction per nucleon, $m_p$ the proton mass, and $\rho$ the density. We use here $\rho \sim 10^{11}$ [g/cm^3] and $Y_e \sim 0.37$ at the neutrino sphere [21], and also use $E_\nu \sim 20$ MeV and $\cos 2\theta \sim 1$. The another condition necessary for the conversion, i.e., the adiabaticity condition is given by

$$\frac{\Delta m^2 \sin^2 2\theta}{2E_\nu \cos 2\theta} \geq \frac{1}{n_e} \left| \frac{dn_e}{dr} \right| \sim \frac{1}{\rho} \left| \frac{d\rho}{dr} \right|, \quad (2)$$

at the resonance layer, where $r$ is the radius from the center of the star, and $n_e$ the number density of electrons. This condition is satisfied around the neutrino sphere when $\sin^2 2\theta \gtrsim 3 \times 10^{-9}$, where we assume the right-hand side of Eq. (2) is $\sim (30 \text{ km})^{-1}$, i.e., the typical radius of the neutrino sphere. When $\Delta m^2$ is smaller than $\sim 10^5$ [eV^2], the resonance occurs in more outer region in the star, and necessary mixing, $\sin^2 2\theta$ for the adiabatic condition becomes larger. In order to show this, we give here a rough estimate of the adiabaticity. Let us assume the density profile is described by a power-law: $\rho \propto r^{-k}$. If we neglect the change in $Y_e$, the resonance layer, $r_{\text{res}}$, scales as $r_{\text{res}}^{-k} \propto \Delta m^2$, from Eq. (3). On the other hand, the right-hand side of the adiabaticity condition, Eq. (2), scales as $\propto r^{-1}$. Therefore, from the relation of $r_{\text{res}}$ and $\Delta m^2$, the right-hand side of Eq. (2) scales as $\propto (\Delta m^2)^{1/k}$. Then it is easy to see that the necessary mixing, $\sin^2 2\theta$ scales as $\propto (\Delta m^2)^{1/k-1}$, i.e.,
\[ \sin^2 2\theta \gtrsim 3 \times 10^{-9} \left( \frac{\Delta m^2}{10^5 \text{[eV}^2]} \right)^{k-1} . \]  

The numerical calculation \cite{22} shows that the value of \( k \) lies in the range of 2–3, and hence the lower bound for \( \Delta m^2 \) to keep the right-hand side of Eq. (3) below \( \sim 1 \) is \( \Delta m^2 > \sim 2 \times 10^{-8} \text{[eV}^2] \) \((k = 3)\) or \( 10^{-12} \text{[eV}^2] \) \((k = 2)\). Therefore the mass range relevant to the MSW oscillation in supernovae is \( \Delta m^2 \sim 10^{-8} - 10^5 \text{[eV}^2] \), and with these values of the parameters, the conversion of \( \bar{\nu}_e \)’s and \( \bar{\nu}_\mu \)’s in the SRN spectrum is possible.

Also for the magnetic moment of neutrinos, there are some practical particle-physics models \cite{23,24} which give the magnetic moment of about \( \mu_{\bar{\nu}} \sim 10^{-12} \mu_B \), and the phenomenological consequences of the resonant conversion in the Sun or supernovae have been discussed \cite{22,31}. The conversion of \( \bar{\nu}_e \)’s and \( \nu_\mu \)’s (or \( \nu_\tau \)’s) is especially effective in the isotopically neutral region of massive stars, i.e., above the iron core and below the hydrogen envelope \cite{31}. For supernovae with the solar metallicity, the relevant mass range \cite{31} is \( \Delta m^2 \sim 10^{-4} - 10^9 \text{[eV}^2] \), and necessary magnetic interaction is \( \mu_{\bar{\nu}_e,\nu_\mu} \gtrsim 10^{-12} (10^9 \text{G}/B_0) \mu_B \), where \( B_0 \) is the magnetic field at the surface of the iron core and \( 10^9 \text{[Gauss]} \) is an reasonable value inferred from the observed magnetic fields of white dwarfs \cite{22}. (Conversion may occur also with \( \Delta m^2 \) larger than \( 10^9 \text{[eV}^2] \), but this corresponds to the resonance in the collapsed iron core, and the detailed calculation is quite difficult.) The metallicity of the progenitor affects the conversion probability \cite{31}, and the conversion can occur also with smaller \( \Delta m^2 \) than the above range in lower metallicity stars. Such effects may be important in supernovae in the early phase of galaxy evolution. However, it should be noted that the magnetic moment of neutrinos interacts only with the transverse magnetic fields. Considering the global structure of magnetic fields in the collapsing star, the conversion probability changes with the direction of neutrinos. The conversion probability after averaging over direction, however, can be close to the unity, with some appropriate parameters. In Fig. 1 the examples of the evolution of the conversion probability due to this mechanism are shown, by using the precollapse model of Nomoto and Hashimoto (1988) \cite{33}.

Considering these possibilities, we calculate the SRN flux and event rate at the SK under the condition of the complete conversion, i.e., \( P = 1 \), where \( P \) is the conversion probability of \( \bar{\nu}_e \)’s and \( \nu_\mu \)-like neutrinos. Because the SRN is the linear superposition of the (redshifted) spectrum of supernova neutrinos, it is easy to get the SRN spectrum with any values of \( P \):

\[ F(E_\nu) = (1 - P)F_{\bar{\nu}_e}(E_\nu) + PF_{\nu_\mu}(E_\nu) , \]  

where \( F \) is the differential flux of the SRN and \( E_\nu \) is the neutrino energy. We assume that \( P \) is independent of \( E_\nu \) throughout this paper. In fact, in the both mechanisms described above, the conversion probability generally depends on \( E_\nu \). Because only the neutrinos with high energy \( (\gtrsim 15 \text{ MeV}) \) can be observed by the SK, the conversion in the whole energy range is not necessary to enhance the event rate of the SK, but the conversion above 15 MeV is sufficient.

It should also be noted that strong conversion of \( \bar{\nu}_e \)’s and \( \nu_\mu \)-like neutrinos is disfavored by the data of neutrinos from SN1987A, observed in Kamiokande II \cite{34} and IMB \cite{35}. Smirnov, Spergel, and Bahcall \cite{36} concluded that \( P > 0.35 \) is excluded by the 99 % confidence level. Although Kernan and Krauss \cite{37} arrived at the opposite conclusion that a significant oscillation is favored by the data, the obtained parameters of supernova neutrino spectrum are quite different from the theoretically plausible parameters. Jegerlehner, Neubig, and Raffelt \cite{38} also concluded by
using the maximum likelihood method that the fitted temperature of SN1987A data should be
less than 4.7 MeV with the confidence level of 95.4 %, which is considerably small compared with
the theoretically predicted $\nu_\mu$ temperatures. However, because of the statistical uncertainties,
one cannot completely exclude the possibility of the full conversion. Actually, the above values
of the confidence level are determined by the Kolmogolov-Smirnov test in Ref. [34] and by the
difference of the value of the likelihood function in Ref. [38], both of the two methods suffer
considerable uncertainty when the sample size is small. Therefore, we consider the case of the
complete conversion of $\bar{\nu}_e$’s and $\nu_\mu$-like neutrinos, as the simplest model.

In the earlier calculation which did not take account of the possibility of neutrino oscillation
[11], the spectrum of the neutrino emission from a supernova is assumed to obey the Fermi-Dirac
distribution with zero chemical potential [12]. All stars in the mass range of 8–60 $M_\odot$ are assumed
to end their life with a collapse-driven supernova, and the region is divided into the three sub-
regions: 8–12.5, 12.5–20, and 20–60 $M_\odot$. The total energy ($E_{\bar{\nu}_e}$) and temperature ($T_{\bar{\nu}_e}$) of $\bar{\nu}_e$’s
emitted from a supernova in these three mass regions are represented by those of the supernovae
with the mass of 10, 15, and 25 $M_\odot$, respectively, by using the results of Woosley, Wilson, and
Mayle [7]. Their calculation gives ($E_{\bar{\nu}_e}$, $T_{\bar{\nu}_e}$) = (4.8, 4.0), (6.0, 5.0), and (11, 5.3) in units of
$(10^{52}$ ergs, MeV), for 10, 15, and 25 $M_\odot$ stars, respectively. In addition to this modeling, here
we assume that the total energy of neutrinos is the same for $\bar{\nu}_e$’s and $\nu_\mu$-like neutrinos while the
temperature of $\nu_\mu$-like neutrinos is higher than $T_{\bar{\nu}_e}$ by a factor of 1.4 for all supernovae, in order
to include the effect of neutrino oscillation. This simple model is inferred from the simulation of
neutrino emission from a supernova of about 20 $M_\odot$ up to 18 [sec] after the collapse by Wilson
and Mayle [39] (see also Ref. [40]). Following their calculation, the total energy and the average
energy of each neutrino are (4.6, 15.3) for $\bar{\nu}_e$’s and (4.9, 21.7) for $\nu_\mu$-like neutrinos, again in units
of $(10^{52}$ ergs, MeV). With these assumptions, we can calculate the SRN flux and spectrum in the
case that all or a part of $\bar{\nu}_e$’s are exchanged with $\nu_\mu$-like neutrinos. Fig. 2 shows the calculated
SRN differential number flux. The thick-solid line is the standard SRN spectrum obtained in the
previous work [1] without any neutrino oscillation or conversion. The thick-dashed line is also
the standard SRN spectrum, but when all $\bar{\nu}_e$’s are completely exchanged with $\nu_\mu$-like neutrinos
($P = 1$). The thick-dotted lines are the same with the thick-dashed line, but with different values
of $T_{\nu_\mu}/T_{\bar{\nu}_e}$ ratio: 1.2 (lower line) and 1.6 (upper line). By these lines, one can estimate how the
uncertainty of $T_{\nu_\mu}/T_{\bar{\nu}_e}$ ratio affects the SRN spectrum. The thin-solid line is the ‘optimistic’ SRN
spectrum (no conversion), with a larger value of the Hubble constant and luminosity density of
galaxies, and also using a different model of galaxy evolution which enhances the SRN flux in high
energy region (see Ref. [11]). The thin-dashed line is the same with the thin-solid line, but with
the condition of $P = 1$. As expected, the neutrino oscillation increases the neutrino energy and
the spectrum becomes harder. This hardening is expected to enhance the event rate at the SK
detector.

Fig. 3 shows the expected event rate at the SK as a function of kinetic energy of recoil positrons,
which are produced by the reaction $\bar{\nu}_e p \rightarrow n e^+$. All the lines correspond to those of the SRN flux
shown in Fig. 2. We use $9.72 \times 10^{-44} E_{e} p_e$ cm$^2$ as the cross section of the $\bar{\nu}_e$ absorption reaction,
where $E_e$ and $p_e$ are the energy and momentum of recoil positrons, measured in MeV. The fiducial
volume of the SK for the SRN observation is 22,000 tons (same with the solar neutrino observation),
and the detection efficiency is also taken into account, which is provided by Totsuka [1]. It is
apparent from this figure that the event rate is enhanced by the neutrino oscillation, especially in the high energy region ($\gtrsim 25$ MeV). In Table I, the energy-integrated event rate in the observable energy range of 15–40 MeV is shown for the models calculated in this paper. With $T_{\nu_\mu}/T_{\bar{\nu}_e} = 1.4$, the event rate of the standard calculation is enhanced up to $2.4 \, \text{yr}^{-1}$ from 1.2, and the optimistic calculation up to $9.4 \, \text{yr}^{-1}$ from 4.7, if the conversion is complete. The important property of the effect of oscillation is that the enhancement of the event rate is prominent in the higher energy range. Therefore, the event rate in the range of 25–40 MeV is also shown in the table. About four events in a year are possible in this high energy range if we use the optimistic model.

In conclusion, the optimistic event rate of $4.7 \, \text{yr}^{-1}$ can be raised up to $9.4 \, \text{yr}^{-1}$ by the complete neutrino oscillation or conversion of $\bar{\nu}_e$’s and $\nu_\mu$-like neutrinos. Because the rate $4.7 \, \text{yr}^{-1}$ in 15–40 MeV is considered to be a theoretical upper bound taking account of various astrophysical uncertainties, we might have to resort to the neutrino oscillation or conversion if more than 5 events were observed in the SK detector in a year. In this case, the events in the high energy range ($\gtrsim 25$ MeV) will be especially enhanced compared with the event rate spectrum without any oscillation or conversion. The theoretical upper bound on the SNR events in the SK becomes $9.4 \, \text{yr}^{-1}$ when the neutrino oscillation is taken into account, but since the complete conversion seems rather unlikely considering the SN1987A data, this upper bound should be considered to be quite conservative one.

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| Astrophysical Model | $T_{\nu_\mu}/T_{\bar{\nu}_e}$ | Event Rate [yr$^{-1}$] |
|---------------------|-------------------------------|------------------------|
|                     |                               | 15–40 MeV              | 25–40 MeV              |
| Standard            | 1                             | 1.2                    | 0.35                   |
|                     | 1.2                           | 1.8                    | 0.66                   |
|                     | 1.4                           | 2.4                    | 1.0                    |
|                     | 1.6                           | 2.9                    | 1.35                   |
| Optimistic          | 1                             | 4.7                    | 1.42                   |
|                     | 1.4                           | 9.4                    | 4.0                    |

Note.— The conversion probability, $P$, is assumed to be the unity, i.e., the complete conversion of $\bar{\nu}_e$’s and $\nu_\mu$-like neutrinos. The energy ranges are those of the kinetic energy of recoil positrons.
FIG. 1. The evolution of the conversion probability of $\bar{\nu}_e \leftrightarrow \nu_\mu$ in the isotopically neutral region of a massive star, due to the magnetic moment of Majorana neutrinos. The 4 $M_\odot$ helium core model of Nomoto and Hashimoto is used, and $\mu_\nu = 10^{-12} \mu_B$ is assumed. The strength of magnetic fields is assumed to be $B_0 = 5 \times 10^9$ [Gauss], where $B_0$ is $B$ at the surface of the iron core, and a global magnetic dipole is also assumed. The values of $\Delta m^2/E_\nu$ are $3 \times 10^{-4}$ (solid line), $1 \times 10^{-4}$ (dashed line), $5 \times 10^{-5}$ (dot-short-dashed line), and $1 \times 10^{-5}$ (dot-long-dashed line), in units of [eV$^2$/MeV].
FIG. 2. The differential number flux of the SRN as a function of neutrino energy. The thick-solid line shows the standard spectrum without any oscillation or conversion of neutrinos. The thick-dashed line is the SRN spectrum when the original $\bar{\nu}_e$’s are completely exchanged with $\nu_\mu$-like neutrinos. The ratio of temperature, $T_{\nu_\mu}/T_{\bar{\nu}_e}$ is assumed to be 1.4, and the thick-dotted lines are the spectra with $T_{\nu_\mu}/T_{\bar{\nu}_e} = 1.6$ (upper) and 1.2 (lower). The thin-solid line is the optimistic SRN flux considering the astrophysical uncertainties (no conversion of neutrinos). The thin-dashed line is the SRN flux using the optimistic model, and with the complete conversion of $\bar{\nu}_e$’s and $\nu_\mu$-like neutrinos, again assuming $T_{\nu_\mu}/T_{\bar{\nu}_e} = 1.4$. 
FIG. 3. The expected event rate of the SRN $\nu_e$'s at the Super-Kamiokande detector, as a function of the kinetic energy of recoil positrons. All the lines correspond to those in Fig. 2 in which the SRN flux is shown. Note that the observable energy range is about 15–40 MeV because of the other background events.
