Abstract

We revisit the implications of the $R$-invariant New Inflation model to the supersymmetric standard model in light of recent discussion of gravitino production processes by the decay of the inflaton or the supersymmetry breaking field. We show that the models with supergravity mediation do not go well with the $R$-invariant New Inflation model, where the gravitino abundance produced by the decay of the inflaton or the supersymmetry breaking field significantly exceeds the bounds from cosmological observations without fine-tuning. We also show that the models with gauge mediation can go together with $R$-invariant New Inflation model, where the dark matter and the baryon asymmetry are consistently explained without severe fine-tuning.
1 Introduction

The Supersymmetric Standard Model (SSM) is considered as one of the most promising candidates for physics beyond the Standard Model (SM), which will be tested at the coming Large Hadron Collider (LHC) experiments. Once the supersymmetric particles are discovered, the next important task will be to determine how the supersymmetry (SUSY) breaking occurs and how the breaking effects are mediated to the SSM sector. So far, variety of mediation mechanisms have been proposed, and they are roughly classified into three categories. The first class is called models with “gravity mediation (SUGRA)”, where the communications between a SUSY breaking sector and the SSM sector are suppressed by the Planck scale ($M_{PL}$) [1, 2]. The second class is called models with “gauge mediation (GMSB)”, where the breaking effects are mediated at the lower energy scale than $M_{PL}$ via gauge interactions of the SSM [3, 4, 5, 6]. The final class is models with “anomaly mediation (AMSB)” in which the breaking effects mediated to the SSM sector are suppressed by more than $M_{PL}$ [7, 8]. Since the characteristic scale of the SUSY breaking (or the size of the gravitino mass) is different among the above categories, the SUSY breaking scale (or of the gravitino mass) can represent the mediation mechanisms.

Fortunately, there are already some evidences that constrain the size of the gravitino mass from cosmology. For example, the late time decay of the unstable gravitino produced after inflation may spoil the success of the Big Bang Nucleosynthesis (BBN) depending on the reheating temperature of the universe $T_R$ (for recent works, see [9, 10] and reference therein). On the other hand, the abundance of stable gravitino is also constrained not to exceed the observation of the dark matter density [11, 12, 13, 14]. Furthermore, recent works on the gravitino abundance produced by the decay of moduli [15, 16, 17, 18] and inflatons [19, 20, 21, 22, 23] have shown that there are much more sever constraints on the gravitino mass depending on models of inflation.

In this paper, we further pursue the constrains on the mediation mechanisms (i.e. the sizes of the gravitino mass) based on a class of New Inflation model which is dubbed $R$-invariant New Inflation model [24, 25]. The $R$-invariant New Inflation model has many attractive features. First attractive feature is the simpleness of the model. The model consists of only one chiral-superfield, and the inflation dynamics are determined by only three parameters. Another attractive feature is that it predicts the spectral index $n_s$ of the
cosmic microwave background radiation as $n_s \simeq 0.95$ in a large parameter space \cite{26,27}, which is well consistent with the WMAP observation \cite{28}. Finally, the most interesting feature from the viewpoint of the SSM model building is that the gravitino mass is determined by the energy scale of the inflation, i.e. the Hubble parameter during inflation.

In Ref. \cite{27}, we showed that the $R$-invariant New Inflation model is well compatible with the SUGRA models (i.e. $m_{3/2} = O(1)$ TeV), while providing the right amount of the baryon asymmetry of the universe by leptogenesis \cite{29} via the decay of the inflaton into right-handed (s)neutrinos \cite{30,24,31,32}. In Ref. \cite{33}, we also showed that the $R$-invariant New Inflation model eludes the Polonyi-induced gravitino problem in the SUGRA models. As we will show, however, such compatibility with the SUGRA model is tainted by a large amount of gravitino produced by the decay of the inflaton or the SUSY breaking field unless we require, which cannot be avoided without fine-tuning. On the other hand, we also show that the $R$-invariant New Inflation model can go well with GMSB models even if we take into account of the gravitino production by the decay of the inflaton or SUSY breaking fields.

The construction of this paper is as follows. We summarize relevant features of the $R$-invariant New Inflation model in the next section. In section 3, we study the consistency of the inflation model with the SSM based on the SUGRA model, in light of the gravitino production from the decay of the inflaton or the SUSY breaking field. In section 4, we study the gravitino production for the gravitino mass scale characteristic for GMSB.

## 2 $R$-invariant New Inflation model

Let us summarize the $R$-invariant New Inflation model considered in Ref. \cite{24,25}. The model is defined by the following superpotential and Kähler potential of an inflaton chiral superfield $\phi$,

$$W_{\text{inf}} = v^2 \phi - \frac{g}{n+1} \phi^{n+1}, \quad (1)$$

and

$$K_{\text{inf}} = |\phi|^2 + \frac{k}{4} |\phi|^4 + \cdots. \quad (2)$$

Here, $v^2$ denotes a dimensionful parameter, $g$ and $k$ dimensionless coupling constants, and $n$ is integer. We can take the parameters $v^2$ and $g$ positive without loss of generality.
Hereafter, we take the unit where the reduced Planck scale, $M_{\text{PL}} \simeq 2.4 \times 10^{18}$ GeV, equals to one unless we specify. The above superpotential is generic under a discrete $Z_{2n}$ $R$-symmetry with $\phi$’s charge $2$.

By taking account of supergravity effects, the effective scalar potential of the inflaton $\varphi = \sqrt{2} \text{Re}[\phi]$ is well approximated by

$$V(\varphi) \simeq v^4 - \frac{k}{2} v^4 \varphi^2 - \frac{g}{2} \frac{v^2}{1} \varphi^n + \frac{g^2}{2} \frac{v^2}{2n} \varphi^{2n}, \quad (3)$$
during inflationary period (i.e. $\varphi \sim 0$). The potential becomes very flat for $n \geq 3$ and $|k| \ll 1$, and it serves as a New Inflation potential with the Hubble parameter $H_{\text{inf}} \simeq v^2/3$ for $k > 0$. From the COBE normalization of the amplitude of the primordial density fluctuation, the Hubble parameter can be expressed as a function of $g$ for $k \lesssim 10^{-2}$,

$$H_{\text{inf}} \simeq 10^{5.4} \text{GeV} \times \frac{1}{g}, \quad (n = 4), \quad (4)$$

$$H_{\text{inf}} \simeq 10^{8.6} \text{GeV} \times \frac{1}{g^{1/2}}, \quad (n = 5), \quad (5)$$

$$H_{\text{inf}} \simeq 10^{9.9} \text{GeV} \times \frac{1}{g^{1/3}}, \quad (n = 6), \quad (6)$$

and $H_{\text{inf}}$ increases for larger $n$. Here, we are also assuming that the e-folding number $N_e$ at the horizon crossing to be 50, although our discussion barely depends on this assumption as long as $N_e = O(10)$. The dependence of $H_{\text{inf}}$ on $k$ is also weak as long as $k \lesssim 10^{-2}$ (see Refs. [27] for details).

The remarkable feature of the present model is that the inflation scale $H_{\text{inf}}$ (or $v$) is directly related to the gravitino mass [24]. As we see from Eq. (1), the superpotential develops a non-vanishing vacuum expectation value (VEV), i.e. $\langle W_{\text{inf}} \rangle \neq 0$, once the inflaton settles to its VEV at $\phi_0 \simeq (v^2/g)^{1/n}$ after inflation. On the other hand, we cannot introduce a large constant term in the superpotential, since a constant term of $O(\langle W_{\text{inf}} \rangle)$ results in a small e-folding number, $N_e \ll O(10)$. Thus, we have no free parameter for the VEV of the total superpotential, and the gravitino mass is given by $\langle W_{\text{inf}} \rangle$,

$$m_{3/2} = \langle W_{\text{inf}} \rangle \simeq \frac{m v^2}{n + 1} \left( \frac{v^2}{g} \right)^{\frac{1}{n}}. \quad (7)$$

Therefore, the gravitino mass has an one-to-one correspondence with the Hubble parameter, $H_{\text{inf}} = v^2/3$ (see Eqs. (4)–(6)).
Figure 1: Left) The $g$ dependence of the gravitino mass for a given value of $n$. The shaded region represents the typical gravitino mass regions for GMSB ($m_{3/2} \lesssim 30$ GeV), SUGRA ($m_{3/2} = O(100)$ GeV – $O(1)$ TeV), and AMSB ($m_{3/2} = O(10) – O(100)$ TeV). Right) The $g$ dependences of the mass and the VEV of the inflaton for $n = 4$.

The left panel of Fig. 1 shows the $g$ dependence of the gravitino mass for a given value of $n$. From the figure, we see that the predicted gravitino mass for $n \geq 5$ is too large for all mediation mechanisms listed above, while the gravitino mass for $n = 4$ is compatible with all the three mediation mechanisms. Notice that it is rather difficult to obtain the spectral index $n_s$ which is consistent with the observed spectral index, $n_s = 0.951^{+0.015}_{-0.019}$ [28] for $n = 3$. Thus, we do not pursue the case with $n = 3$ further. For these reasons, we concentrate on the case of $n = 4$ in the following argument, where the gravitino mass can be well approximated by,

$$m_{3/2} \simeq 300 \text{ GeV} \times \frac{1}{g^{3/2}}.$$  \hspace{1cm} (8)

In the right panel of Fig. 1, we also plot the $g$ dependences of the mass and the VEV of the inflaton field $\phi$ for $n = 4$ which are given by,

$$m_\phi \simeq n g \phi_0^{n-1} \simeq n v^2 \left( \frac{v^2}{g} \right)^{-\frac{n}{2}} \simeq 3 \times 10^9 \text{ GeV} \times \frac{1}{\sqrt{g}},$$  \hspace{1cm} (9)

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \phi_0 \simeq \left( \frac{v^2}{g} \right)^{\frac{1}{n}} \simeq 2 \times 10^{15} \text{ GeV} \times \frac{1}{\sqrt{g}},$$  \hspace{1cm} (10)

respectively. From the figure, we see that $m_\phi \simeq 10^{8-10}$ GeV and $\langle \phi \rangle = 10^{14-16}$ GeV for a wide range of parameter space.

Before closing this section, we comment on the possible range of the parameter $g$. Since the Kähler potential receives radiative corrections, we need to require at least $g < O(10)$
to keep perturbativity of the model. Thus, in the following argument, we simply assume \( g \lesssim 10 \) which corresponds to,

\[
m_{3/2} \gtrsim 10 \text{ GeV.}
\]  

We should, however, keep in mind that we need some degree of fine-tuning between the tree level contribution and the radiative corrections to the quartic coupling in the Kähler potential in Eq. (2) to keep the effective quartic coupling small, i.e. \( |k| \lesssim 10^{-2} \), when the coupling constant \( g \) is \( O(1) \).

## 3 Gravitino production in SUGRA model

The most distinguished property of the SUGRA models is that they require a SUSY breaking field which is neutral under any symmetry to obtain gaugino masses of the SSM comparable to the sfermion masses. One problem caused by such a singlet SUSY breaking field is so-called Polonyi problem [34, 35] and Polony-induced gravitino problem [33]. In the paper [33], we showed that, thanks to its relatively small Hubble parameter, the \( R \)-invariant New Inflation model is free from the Polonyi problem and the Polonyi-induced gravitino problem as long as the mass of the SUSY breaking sector field is heavy enough.

The existence of a singlet SUSY breaking field, however, causes another cosmological problem, that is, the enhancement of the branching ratio of the inflaton into a pair of gravitinos [19]. When the SUSY breaking field is a singlet, the mixing between the SUSY breaking field and the inflaton after inflation can be enhanced via the supergravity effects. In our case, the relevant terms which enhance the decay rate of inflaton into a pair of gravitinos are,

\[
K_{\text{mix}} = (C_1^1 Z + C_1^i Z^i) |\phi|^2 + \cdots,
\]

\[
W_{\text{mix}} = C_2 v^2 \phi Z + C_3 g^5 Z + \cdots,
\]

where \( Z \) is the SUSY breaking field which has a non-vanishing \( F \)-term, \( C_i \) \( (i = 1, 2, 3) \) constant parameters, and the ellipses the higher dimensional terms. Since we have no symmetry to suppress the constants \( C_i \), we naively expect them to be of the order of one. Through the supergravity effects, these terms lead to a considerable mixing between the SUSY breaking field \( Z \) and inflaton field \( \phi \).
The mixing between the inflaton and the SUSY breaking field leads to an effective coupling of the inflaton to gravitinos, $G_{\phi}^{\text{eff}}$, with which the decay rate of the inflaton into a pair of the gravitinos is given by,

$$\Gamma_{3/2} = \frac{|G_{\phi}^{\text{eff}}|^2 m_\phi^5}{288\pi m_{3/2}^2 M_{\text{pl}}^2}. \quad (14)$$

According to the analysis given in Ref. [18], the effective coupling resulting from Eqs. (12) and (13) is approximately given by,

$$|G_{\phi}^{\text{eff}}|^2 \simeq 3 \langle \phi \rangle^2 \left[ C_1 + \frac{1}{16} (C_2 + C_3) \right]^2 \left( \frac{m_Z^2}{\text{Max}(m_\phi^2, m_Z^2)} \right)^2. \quad (15)$$

Here, $m_Z$ denotes the mass of the SUSY breaking field, which is expected to range from $m_Z = O(m_{3/2})$ to $m_Z = O(\sqrt{m_{3/2}})$.

Then, assuming that the inflaton decays mainly into the SSM particles with the reheating temperature $T_R$, we obtain the gravitino-entropy ratio (yield) as,

$$Y_{3/2}^{\text{inf}} = 2 \frac{\Gamma_{3/2} 3 T_R}{\Gamma_R 4 m_\phi},$$

$$\simeq 4.5 \times |G_{\phi}^{\text{eff}}|^2 \left( \frac{m_\phi}{10^9 \text{GeV}} \right)^4 \left( \frac{1 \text{TeV}}{m_{3/2}} \right)^2 \left( \frac{10^7 \text{GeV}}{T_R} \right),$$

$$\simeq 2.3 \times 10^{-6} C^2 \left( \frac{\langle \phi \rangle}{10^{15} \text{GeV}} \right)^2 \left( \frac{m_\phi}{10^9 \text{GeV}} \right)^4 \left( \frac{1 \text{TeV}}{m_{3/2}} \right)^2 \left( \frac{10^7 \text{GeV}}{T_R} \right) \times \text{min} \left[ \frac{m_Z^2}{m_\phi^2}, 1 \right], \quad (16)$$

where $C$ is defined by $C = |C_1 + (C_2 + C_3)/16|$. In the second equality, we have used Eq. (14) and the relation between the reheating temperature and the total decay rate of the inflaton $\Gamma_R$,

$$\Gamma_R = \left( \frac{\pi^2 g_*}{10} \right)^{1/2} T_R^2. \quad (17)$$

Here $g_*$ denotes the effective number of massless degrees of freedom during the reheating process, and we use $g_* \simeq 230$ which corresponds to the number of the SSM particles.

As we saw in the previous section, the VEV and the mass of the inflaton can be expressed in terms of the gravitino mass. Thus, the above yield of the gravitino is determined by three parameters, the gravitino mass $m_{3/2}$, the mass of the SUSY breaking field $m_Z$, and the reheating temperature $T_R$. Since the successful BBN requires $T_R \lesssim 10^6 - 7$ GeV for
Figure 2: The yield of gravitinos for $m_{3/2} = 1\, \text{TeV}$ (left), $100\, \text{GeV}$ (right). Solid (red) lines denote the yields of gravitinos produced by the inflaton decay in Eq. (16) and dashed (blue) lines denote the one produced by the decay of SUSY breaking field in Eq. (23). Here, we have taken $C$ and $C_0$ to be 1, and $T_R$ to be $10^7\, \text{GeV}$.

$m_{3/2} = O(1)\, \text{TeV}$ to suppress the unstable gravitino abundance produced by the thermal scattering processes [9], we fix the reheating temperature $T_R = 10^7\, \text{GeV}$ in the following of this section.

In Fig. 2 we show the yield of the gravitino produced by the inflaton decay as a function of the mass of the SUSY breaking field as solid (red) lines. As we see from the figure, the yield of the gravitino is suppressed for $m_Z < m_\phi$, while $m_Z$ dependence disappears for $m_\phi < m_Z$.

In order not to spoil the success of the BBN, the gravitino abundance produced by the decay of the inflaton must satisfy the constraints in Ref. [9],

$$Y_{3/2} \lesssim Y_{3/2}^{\text{upper}}, \quad (18)$$

$$Y_{3/2}^{\text{upper}} = \begin{cases} 
1 \times 10^{-16} - 5 \times 10^{-14} & \text{for } m_{3/2} \simeq 0.1 - 1\, \text{TeV} \\
2 \times 10^{-14} - 5 \times 10^{-14} & \text{for } m_{3/2} \simeq 1 - 3\, \text{TeV} \quad (B_h \simeq 10^{-3}), \\
3 \times 10^{-14} - 2 \times 10^{-13} & \text{for } m_{3/2} \simeq 3 - 10\, \text{TeV} 
\end{cases} \quad (19)$$

where we have taken the hadronic branching ratio of the gravitino decay to be $B_h \simeq 10^{-3}$ for conservative discussion. The red (solid) lines in Fig. 2 show that we need to require the coefficient $C$ in Eq. (16) to be very small, i.e. $C \ll 1$ to satisfy the above bound, unless the mass of the SUSY breaking field to be much smaller than the mass of inflaton, i.e. $m_Z \ll m_\phi$.

Unfortunately, however, the later option, $m_Z \ll m_\phi$, brings back the other cosmological problem, the Polonyi-induced gravitino problem [33]. Since the SUSY breaking field is neutral under any symmetry, there is no reason to forbid the linear term in the Kähler
potential,
\[ K_{\text{shift}}(Z) = C_0^\dagger Z + C_0 Z^\dagger, \]
with the order one coefficient \( C_0 \). As discussed in Ref. [33], the above linear term leads to a large linear term in the scalar potential of the SUSY breaking field during inflation,
\[ V(Z) \simeq m_Z^2 |Z|^2 + 3H^2_{\text{inf}}(C_0^\dagger Z + C_0 Z^\dagger) + \cdots, \]
where we have assumed that \( H_{\text{inf}} \ll m_Z \). By this linear term, the SUSY breaking field is shifted from its VEV by
\[ \langle Z_{\text{inf}} \rangle = \frac{3H^2_{\text{inf}}C_0}{m_Z^2}, \]
during inflation, and it begins coherent oscillation after inflation with an amplitude of \( O(\langle Z_{\text{inf}} \rangle) \).

Once the SUSY breaking field begins its coherent oscillation, it dominantly decays into gravitinos. By solving the Boltzmann equation, we obtain the yield of the gravitino from the SUSY breaking field as,
\[ Y_{3/2}^{\text{hidden}} \simeq 3 \frac{T_R}{m_Z} \frac{H^2_{\text{inf}}^2}{2m_Z} \langle Z_{\text{inf}} \rangle^2 \]
\[ \simeq 1.4 \times 10^{-11} C_0^2 \left( \frac{m_{3/2}}{1\text{TeV}} \right)^{4/3} \left( \frac{10^{10}\text{GeV}}{m_Z} \right)^3 \left( \frac{T_R}{10^7\text{GeV}} \right), \]
where we have expressed \( H_{\text{inf}} \) in terms of \( m_{3/2} \) by using the result of the previous section. The dashed (blue) lines in Fig. 2 shows the yield of the gravitino produced by the decay of the SUSY breaking field for a given gravitino mass. As we expected, the yield increases when the mass of the SUSY breaking field gets smaller.

As a result, we find that in order not to spoil the success of the BBN the parameters \( C \) and \( C_0 \) must be suppressed severely, i.e. \( C, C_0 \lesssim 10^{-4} \). Thus, the \( R \)-invariant New inflation model with \( n = 4 \) suffers from a fine-tuning problem to evade the gravitino problem, since we have no reason for such parameters to be suppressed. Therefore, we find that the \( R \)-invariant New Inflation model is not successful with the SUGRA models with \( m_{\text{gaugino}} \simeq m_{\text{scalar}} \simeq m_{3/2} \).

Before closing this section, we comment on some possible cures of this problem. It is logically possible to assume that only the gaugino mass terms break a symmetry under which the SUSY breaking field is charged, while the terms in Eqs. (12), (13) and (20)
are suppressed at the tree level by the symmetry (see Ref. [37] for related discussion). In that case, however, the constants $C_0$ and $C_1$ are generated via at least one- and two-loop diagrams respectively, in which SSM particles circulate. Thus, the SUGRA models with $m_{\text{gaugino}} \simeq m_{\text{scalar}} \simeq m_{3/2}$ get $C_0 = O(0.1)$ and $C_1 = O(0.01)$ even if they are suppressed at the tree level. Therefore, if we try to solve the fine-tuning problem by this assumption, we further need to assume that the gaugino mass is suppressed compared with the scalar masses, although such hierarchy requires a fine-tuning for the correct electroweak symmetry breaking.\footnote{The above assumption of “soft” symmetry breaking by only the gaugino mass terms might work in the large cutoff supergravity proposed in Ref. [38]. In the large cutoff supergravity, the gaugino masses are suppressed compared with the scalar masses while the fine-tuning in the electroweak symmetry breaking is not required, thanks to the focus point mechanism [39, 40, 41].}

4 Gravitino production in Gauge Mediation

In this section, we consider the GMSB models, where the gravitino is the LSP and stable. As we discussed in the previous section, the gravitino cannot be much lighter than $O(10)$ GeV due to the perturbativity of the inflation model. Hence, in the following, we concentrate on the case of $m_{3/2} = 10$ GeV as an example. Besides, we also assume that the SUSY breaking field is charged under some symmetries, since there is no need to assume it to be neutral in the GMSB models. (For a neutral SUSY breaking field, we have checked that the gravitino abundance produced by the inflaton decay significantly exceeds the observed dark matter abundance for $m_{3/2} = 10$ GeV.)

Before going to discuss the gravitino abundance, let us make an assumption about the reheating process of the new inflation model. Although there are many possibilities for the reheating mechanism, the $R$-invariant New Inflation model has an attractive reheating scenario which leads to non-thermal leptogenesis [27]. By introducing the interaction between the inflaton and the right-handed neutrino $N$'s,

$$W_{\text{neutrino}} = \frac{h}{6}\phi^3 N^2,$$

we can make the inflaton mainly decay into the right-handed neutrino with the reheating temperature,

$$T_R \simeq \left(\frac{10}{g_*\pi^2}\Gamma_R\right)^{1/4} \simeq 1.5 \times 10^6 h^{-5/4} \, \text{GeV}. \quad (25)$$
The attractive feature of this reheating process is that the produced right-handed neutrinos immediately decay into the SSM particles and results in leptogenesis. In Ref. [33, 47], we showed that this specific reheating mechanism reproduces the observed baryon asymmetry of the universe only for $T_R = 10^6-7$ GeV for a wide range of the parameter $g$. Thus, for the purpose of finding a cosmologically consistent scenario, we assume this reheating mechanism with the reheating temperature $T_R = 10^6-7$ GeV. We should also mention that this mechanism provides Majorana masses of the right-handed neutrinos which is required by the see-saw mechanism [42],

$$m_N = \frac{h}{3} \langle \phi \rangle^3 \approx \frac{h}{12g} m_\phi.$$  

(26)

For $m_{3/2} = 10$ GeV and $T_R = 10^6-7$ GeV, the thermally produced gravitino abundance is not enough to explain the observed dark matter density as long as $m_{\text{gaugino}} \leq O(1)$ TeV [11, 12, 13, 14]. Therefore, to explain the observed dark matter density by gravitino, we need to have other sources of gravitino such as inflaton or the SUSY breaking field as we discussed in the previous section.

First, let us consider the gravitino production from the decay of the SUSY breaking field $Z$. Notice that there is no linear term in the Kähler potential during inflation as in Eq. (20), since we are assuming that the SUSY breaking field is charged under some symmetries. The dynamics of inflation, however, still shifts the field value of the SUSY breaking field during inflation via gravitational effect, when the SUSY breaking field has a non-vanishing VEV. That is, during inflation, the SUSY breaking field obtains a so-called Hubble mass term around its origin,

$$V(Z) \simeq m_Z^2 |Z - \langle Z \rangle|^2 + H_{\text{inf}}^2 |Z|^2 + \cdots,$$

(27)

while it also has a mass term around the VEV $\langle Z \rangle$. Hence, the field value of the SUSY breaking field is shifted from $\langle Z \rangle$ by,

$$\Delta Z = \frac{3H_{\text{inf}}^2}{3H_{\text{inf}}^2 + m_Z^2} \langle Z \rangle,$$

(28)
during inflation. Then, as we discussed in the case of SUGRA models, the SUSY breaking field starts to oscillate around $\langle Z \rangle$ after inflation and produces gravitino when it decays.

\footnote{A quartic term, $|Z|^2 |\phi|^2$ in the Kähler potential changes the coefficient of the Hubble mass term in Eq. (27), although it does not change our discussion for a wide parameter space.}
By assuming that the SUSY breaking field dominantly decays into gravitinos\(^3\) we obtain the yield of gravitinos,

\[ Y_{3/2}^{\text{hidden}} \simeq \frac{3}{2} \frac{T_R}{m_Z} \frac{m_Z^2 \Delta Z^2}{3 H_{\text{inf}}}. \]  

(29)

Here, we are assuming that the oscillation of the SUSY breaking field does not dominate the energy density of the universe, which is the case for not so large value of \( \langle Z \rangle \) (see also Ref. [43] for the case where the coherent oscillation dominates the energy density of the universe).

The above yield of the gravitino is again determined by the gravitino mass, the mass of the SUSY breaking field, the reheating temperature and the size of the VEV \( \langle Z \rangle \), since the Hubble parameter during inflation can be determined for a given gravitino mass. As discussed above, we take the reheating temperature \( T_R = 10^{6-7} \) GeV which is suitable for non-thermal leptogenesis in the \( R \)-invariant New Inflation model. As for the VEV of the SUSY breaking field, it is non-trivial to obtain a large VEV while keeping the mass of the SUSY breaking field much larger than that of the gravitino. In our discussion, we take \( \langle Z \rangle = (m_{3/2}^3 m_Z^{-3})^{1/5} \) as an example by thinking of the dynamical SUSY breaking sector discussed in Refs. [44, 45, 46] where the SUSY breaking field may have a VEV of the order of the dynamical scale, \( (m_{3/2}^3 m_Z^{-3})^{1/5} \), while the mass of the SUSY breaking field can be high up to \( O(\sqrt{m_{3/2}}) \).

Altogether, we obtain the mass density parameter of the gravitino as

\[ \Omega_{3/2}^{\text{hidden}} h^2 = 0.1 \times \left( \frac{m_{3/2}}{10 \text{ GeV}} \right)^{7/3} \left( \frac{10^5 \text{ GeV}}{m_Z} \right)^3 \left( \frac{T_R}{10^7 \text{ GeV}} \right) \left( \frac{\langle Z \rangle}{10^{12.5} \text{ GeV}} \right)^2, \]  

(30)

for \( m_Z \gg H_{\text{inf}} \). In the right panel of Fig. 3 we plot the mass density parameter of the gravitino for \( \langle Z \rangle = (m_{3/2}^3 m_Z^{-1})^{1/5} \). We also take \( T_R = 10^{6-7} \) GeV for the sake of the baryon asymmetry of the universe. From the figure, we find that the gravitino produced by the SUSY breaking field can explain the observed dark matter density \( \Omega h^2 = 0.1050^{+0.0041}_{-0.0040} \) \((1\sigma)\) [28]. Therefore, the \( R \)-invariant New inflation is not only well consistent with the GMSB models \( (m_{3/2} = 10 \) GeV\), but also naturally provides the dark matter abundance and the baryon asymmetry at the same time for a certain parameter range.

\(^3\)In a class of the GMSB models, the SUSY breaking field can dominantly decay into the SSM particles via the interaction for the gauge mediation [43].
Figure 3: Left) The size of the VEV of the SUSY breaking field, $\langle Z \rangle = (m_{3/2}^4 m_Z^3)^{1/5}$, which we take as an example. Right) The mass density parameter of the gravitino dark matter, $\Omega_{3/2} h^2$, produced by the decay of SUSY breaking field. The shaded region corresponds to the density parameter for $T_R = 10^{6-7}$ GeV. The solid (red) horizontal line shows the observed dark matter density $\Omega h^2 = 0.1050^{+0.0041}_{-0.0040} (1\sigma)$ [28]. In both panels, we have taken $m_{3/2} = 10$ GeV.

Next, let us check that the gravitino abundance produced by the inflaton decay does not exceed the observed dark matter density. Since the SUSY breaking field is not neutral, the leading interaction between the SUSY breaking field and the inflaton in the Kähler potential is given by,

$$K_{\text{int}} = b|\phi|^2 |Z|^2.$$ (31)

In this case, the mixing between SUSY breaking field and the inflaton is much suppressed compared with the SUGRA models [17]. As a result, the effective coupling of the inflaton to gravitinos is also suppressed. For $\langle Z \rangle = 0$, it is given by,

$$|G_{\phi}^{\text{eff}}|^2 \simeq 9(1 - b)^2 \langle \phi \rangle^2 m_{3/2}^2 / m_{\phi}^2 \times \left( \frac{m_Z^2}{\text{Max}[m_Z^2, m_{\phi}^2]} \right)^2.$$ (32)

Through this effective coupling, the gravitino is produced at the reheating process of the inflaton. The resulting mass density parameter of the gravitino is given by,

$$\Omega_{3/2} h^2 = 2 \times 10^{-7} (1 - b)^2 \left( \frac{\langle \phi \rangle}{10^{15} \text{GeV}} \right)^2 \left( \frac{m_{3/2}}{10 \text{GeV}} \right)^2 \left( \frac{m_{\phi}}{10^9 \text{GeV}} \right)^2 \left( \frac{10^6 \text{GeV}}{T_R} \right) \times \text{min} \left( \frac{m_Z^2}{m_{\phi}^2}, 1 \right)^2,$$ (33)

where we have used the yield in the first equality in Eq. (16).

In Fig. 4 we plot the gravitino mass density parameter from the inflaton decay for $m_{3/2} = 10$ GeV, $T_R = 10^{6-7}$ GeV, and $b = 0$. From the figure, we see that the gravitino
Figure 4: The mass density parameter of gravitino produced by the decay of the inflaton for $m_{3/2} = 10\text{GeV}$, $T_R = 10^{6-7}\text{GeV}$, and $b = 0$. For simplicity we have assumed $\langle Z \rangle = 0$. The gravitino produced by the decay of the inflaton is much smaller than the observed dark matter density. Thus, the gravitino produced by the inflaton decay with the above specific reheating process is subdominant compared with the gravitino produced by the decay of the SUSY breaking field.

For $\langle Z \rangle \neq 0$, the above expression of the effective coupling in Eq. (32) is changed and becomes complicated. We have checked, however, that the gravitino dark matter density cannot be supplied by the decay of the inflaton as long as the VEV of the SUSY breaking field is within the order of $(m_{3/2}^{-1}m_Z)^{1/5}$.

As a result, we find that the $R$-invariant New Inflation model can be consistent with the GMSB models with $m_{3/2} \simeq 10\text{GeV}$. Furthermore, the observed dark matter density can be explained by the gravitino abundance produced by the decay of the SUSY breaking field while the baryon asymmetry is provided by the non-thermal leptogenesis which is naturally embedded into the $R$-invariant New Inflation model.

5 Summary

In this paper, we revisited the $R$-invariant New Inflation model in light of the recent argument about the gravitino production from the inflaton and the SUSY breaking field.

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4 It may be possible to consider a dynamical SUSY breaking model with the VEV much larger than we considered here. In such cases, the gravitino abundance produced by the inflaton decay may explain the observed dark matter density.

5 The gravitino abundance produced by the decay of the next to LSP can also contributes the dark matter density depending on the details of SSM spectrum [48, 50, 51, 52, 53], which can be a solution to the small scale structure problem of cold dark matter cosmology [54, 55, 56, 57, 58].
As a result, we found that SUGRA models with the $R$-invariant new inflation model suffer from a severe fine-tuning problem where we should have two small parameters of $O(10^{-4})$ which are expected to be $O(1)$ without fine-tuning. On the other hand, we found that the gravitino production from the SUSY breaking field is useful in the GMSB models within the reheating temperature which is consistent with non-thermal leptogenesis. As we have also shown in Ref. [17], the gravitino production in the inflaton decay naturally explains the wino dark matter density in the AMSB models, while non-thermal leptogenesis works properly. Therefore, we conclude that the success of the $R$-invariant New Inflation model to predict the spectral index [26, 27] strongly suggests that the SSM is not realized by the SUGRA where we need a singlet SUSY breaking field, but by models with mediation mechanisms where we do not require a singlet SUSY breaking filed, such as gauge mediation with $m_{3/2} = O(10)$ GeV or anomaly mediation with $m_{3/2} = O(10 - 100)$ TeV.

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