On the use of evidence theory in belief base revision

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Abstract. This paper deals with belief base revision that is a form of belief change consisting of the incorporation of new facts into an agent’s beliefs represented by a finite set of propositional formulas. In the aim to guarantee more reliability and rationality for real applications while performing revision, we propose the idea of credible belief base revision yielding to define two new formula-based revision operators using the suitable tools offered by evidence theory. These operators, uniformly presented in the same spirit of others in [9], stem from consistent subbases maximal with respect to credibility instead of set inclusion and cardinality. Moreover, in between these two extremes operators, evidence theory let us shed some light on a compromise operator avoiding losing initial beliefs to the maximum extent possible. Its idea captures maximal consistent sets stemming from all possible intersections of maximal consistent subbases. An illustration of all these operators and a comparison with others are investigated by examples.

Keywords: belief base revision · evidence theory · credibility · rationality · knowledge representation and reasoning.

1 Introduction

One of the important research topics in Artificial Intelligence is dynamics (or change) beliefs. In many applications, such as image processing, reliability expert opinions, robotics, radar detection and relational databases, intelligent agents face incomplete, uncertain and inaccurate information, and often need a revision operation so as to manage their beliefs change in presence of a new and reliable information. When this information contradicts the agent’s current beliefs, the revision then deals with remaining consistency in order to integrate the new information while modifying the initial beliefs as little as possible. During the last thirty years, the question of how to perform revision gave rise to numerous works according to the representation of beliefs.
The first works on the revision of beliefs come from subjective probabilities, mainly with the works of Richard Jeffrey [21]. In this context, the beliefs of an agent are represented by a measure of probability, and the beliefs revision is what Jeffrey called *probabilistic kinematics*. Shortly thereafter, many logical approaches to revision have been developed. Among these approaches, there are the so-called *syntactic approach* [17,19,28,34] where a great importance was given to revision of finite belief bases, i.e. finite propositional formulas sets. This approach was formalized in terms of postulates (*AGM postulates*) [1] and several operators have been proposed in the literature. Some of them are based on construction of maximum consistent subbases according to different criteria [310,22]. Recently, Creignou and colleagues in [9] focused particularly on two operators, namely RSRG and RSRW, that are respectively similar to Ginsberg’s one [17] and Widtio [34], using set cardinality instead of set inclusion as maximality criterion.

An important issue is to introduce efficient tools that fulfill the needs of others in their investigations. Indeed, in many real applications, belief bases are quite large and choosing, as a revision result, all maximal consistent subbases like Ginsberg’s approach can be an expensive, exhaustive and even explosive solution. On the one hand, keeping only beliefs that are not questioned (streaming from the intersection of maximal coherent subbases) can be a plausible strategy, but not always reliable and can cause, in most the cases, a lot of loss of information since there is no guarantee to not have the empty set as an intersection between maximal consistent subbases. On the other hand, the selection of some maximal consistent subbases according to determined criteria, can be shown also plausible, and the choice of set cardinality like as in [9] seems rather reasonable, because it respects the minimality change criterion of belief revision.

To deal with this issue and in order to be reliable and close to the non-monotony of human reasoning mainly in the case of large belief bases, the selection of consistent subbases maximal with respect to set cardinality is not always a guarantee to select the most relevant information and can consequently neglect potential formulas from initial agent’s beliefs (as shown in Example 2). This paper goes one step further in this context by investigating a more natural criterion in the selection of consistent subbases respecting the minimality change criterion of belief revision in the sense of *credibility* while performing change by capturing the most valuable or potential information from initial beliefs. This can be accomplished by using different tools offered by evidence theory, commonly also known as Dempster-Shafer or belief functions theory [11,30].

To the best of our knowledge, although belief revision in probability theory is fully studied, belief revision strategies in evidence theory has not been addressed so far, except the paper [26] where authors have taken revision from a different angle representing agent’s beliefs by mass functions in the aim to generalize Jeffreys rule from probability to belief functions. In the present paper and from another point of view to revision, we shed light on the use of evidence theory in the context of formula-based belief revision through two main contributions. Following the same spirit of the work [9], we propose at first two formula-based
operators similar to RSRG and RSRW, namely CSRG and CSRW, based on the selection of the most credible consistent subbases. The computation of credibility is assured using the suitable tools offered by evidence theory. Then, we present a new revision strategy instantiated by a compromise operator CSIR between CSRG and CSRW. This strategy captures the most credible consistent sets stemming from all the possible intersections of maximal consistent subbases. It can supersede other strategies in many real applications for the reason that it avoids losing original beliefs as far as possible. A compact representation of all these operators is given within the unified framework already developed in [9].

This work is organized as follows: after a preliminary section (section 2) which introduces some preliminaries on propositional logic and evidence theory and gives a reminder on formula-based (syntactic) revision operators we are interested in, we formally define in Section 3 our new belief operators stemming from consistent subbases maximal with respect to credibility degree and we show their specificities and advantages comparing with others. In section 4 we provide a new (compromise) revision strategy based likewise on credibility degree capturing beliefs with a mind to be prudent. Both contributions are illustrated with examples.

2 Preliminaries

2.1 Propositional Logic

In this section, we assume familiarity with the basic notions of propositional (classical) logic. So, we (very briefly) present the background and terminology used in this paper. Let \( \mathcal{L} \) be the language of propositional logic built on an infinite countable set of variables (atoms) denoted by \( \mathcal{V} \) and equipped with standard connectives \( \neg, \land, \lor, \rightarrow \), the exclusive or connective \( \oplus \), and constants \( \top, \bot \).

We remind that a literal \( a \) is an atom (positive literal) or the negation of an atom \( \neg a \) (negative literal). A clause is a disjunction of literals. We say that a formula is in CNF if it is a conjunction of clauses. For a set \( A \) of formulas, \( Cn(A) \) denotes the closure of \( A \) under the consequence relation \( \models \). A theory \( A \) is a deductively closed set of formulas if \( A = Cn(A) \). Let \( B \) be a finite set of propositional formulas, \( B = \{ \varphi_1, \varphi_2, \ldots, \varphi_n \} \) is identified to \( \bigwedge B \) the conjunction of its formulas, \( \varphi_1 \land \varphi_2 \land \ldots \land \varphi_n \). Given a family of finite sets of formulas \( \mathcal{W} = \{ B_1, \ldots, B_n \} \), we use \( \bigvee_{i=1}^n \bigwedge B_i \) for \( \bigvee_{i=1}^n \bigwedge_{\varphi \in B_i} \varphi \).

2.2 Belief base revision

In this paper, we focus on formula-based (syntactic) revision operators already presented within a unified framework in [9]. Each operator, denoted by \( * \), is a function that takes a belief base \( B \) and a formula \( \mu \) representing new information as input and returns a new belief base \( B * \mu \). Many formula-based operators stem from \( \mathcal{W}(B, \mu) \), the set of maximal subbases of \( B \) consistent with \( \mu \). They then
make use of this set to define the revised belief base according to a given strategy. The maximality criterion as well as the strategy can vary. In the literature, maximality was first considered in terms of set inclusion, and thus the following set was considered

$$\mathcal{W}_\subseteq(B,\mu) = \{B'_i \subseteq B \mid \bigwedge B'_i \not\models -\mu \text{ and for all } B'_j, j \neq i \text{ s. t. } B'_i \subset B'_j \subseteq B, \bigwedge B'_j \models -\mu\}.$$  

In [9], authors consider then two different strategies. The first one is “permissive” by considering that all maximal subbases are equally plausible and the second one is more “drastic” and stems from the intersection of consistent maximal subbases, i.e. it only keeps beliefs that are not questioned. Thus, these two strategies provide two well-known operators, namely Ginsberg’s operator, $*_G$, [17] and Widtio operator, $*_\text{wid}$, [34] defined respectively as

$$B *_G \mu = \bigvee_{B' \in \mathcal{W}_\subseteq(B,\mu)} \bigwedge (B' \cup \{\mu\})$$

and

$$B *_{\text{wid}} \mu = \bigwedge \bigcap_{B' \in \mathcal{W}_\subseteq(B,\mu)} (B' \cup \{\mu\}).$$

Authors in [9] focus then on maximality defined in terms of cardinality. This is a quite natural issue since in various applications the cardinality criterion is used because information acquisition is expensive. So, they consider the set of consistent subbases maximal w.r.t. cardinality $\mathcal{W}_{\text{card}}(B,\mu)$ instead of $\mathcal{W}_\subseteq(B,\mu)$.

Formally, we present $\mathcal{W}_{\text{card}}(B,\mu)$ as

$$\mathcal{W}_{\text{card}}(B,\mu) = \{B'_i \subseteq B \mid \bigwedge B'_i \not\models -\mu \text{ and for all } B'_j \subseteq B, j \neq i \text{ s. t. } |B'_i| < |B'_j|, \bigwedge B'_j \models -\mu\}.$$  

Analogously and respectively to Ginsberg’s and Widtio operators, the two strategies presented above provide two operators RSRG [2] and RSRW. Indeed, the notation RSR comes from the expression “Removed Sets Revision” qualifying operators stemming from the removal of the smallest number of formulas from the initial belief base [2]. Formally, we have

$$B *_{\text{RSRG}} \mu = \bigvee_{B' \in \mathcal{W}_{\text{card}}(B,\mu)} \bigwedge (B' \cup \{\mu\})$$

and

$$B *_{\text{RSRW}} \mu = \bigwedge \bigcap_{B' \in \mathcal{W}_{\text{card}}(B,\mu)} B' \cup \{\mu\}.$$  

Moreover, note that these formula-based operators are sensitive to the syntactic form of the knowledge representation. The following example illustrates this idea.
Example 1. Consider $B_1 = \{a, b\}$, $B_2 = \{a, a \rightarrow b\}$ two belief bases and a formula $\mu = \neg b$ representing the new information. The bases $B_1$ and $B_2$ are equivalents. The unique subset of $B_1$ which is consistent with $\mu$ is $\{a\}$, while there are two maximal (in terms of set inclusion) subsets of $B_2$ which are consistent with $\mu$, namely, $\{a\}$ and $\{a \rightarrow b\}$. Consequently, $B_1 \ast_{\mu} \mu = a \land \neg b$ and $B_2 \ast_{\mu} \mu = \neg b$.

Therefore, in order to get rid of syntax dependency, Hansson [19] has shown that it seems natural to revise explicitly defined belief bases and then extend these operations to belief sets, considering the deductive closure of the result of revision. Otherwise, it is possible to define, from a revision operator $\ast$, a new one denoted by $\boxplus$ whose the result is a set of beliefs (or theory) such as

$$B \boxplus \mu = Cn(B \ast \mu).$$

We adopt this point of view in the present paper and we obtain correspondingly the following operators.

$$B \boxplus_G \mu = Cn(\bigvee_{B' \in W_G(B, \mu)} \bigwedge B' \cup \{\mu\})$$

$$B \boxplus_{\mu \text{d}} \mu = Cn(\bigwedge_{B' \in W_{\mu \text{d}}(B, \mu)} \bigcap B' \cup \{\mu\})$$

$$B \boxplus_{\text{RRG}} \mu = Cn(\bigwedge_{B' \in W_{\text{RRG}}(B, \mu)} \bigvee B' \cup \{\mu\})$$

$$B \boxplus_{\text{RSSW}} \mu = Cn(\bigcap_{B' \in W_{\text{RSSW}}(B, \mu)} \{B' \cup \{\mu\}\})$$

2.3 Evidence theory

Evidence theory has been considered as a convenient framework dealing with imperfect information. It was initially introduced by Arthur Dempster in 1967 [11] and then formalized by Glenn Shafer in 1976 [30] as a generalization of subjective probability theory. It has been the starting point of several theoretical developments especially the transferable belief model [33]. In addition, it has been applied in several fields such as artificial intelligence [14], clustering [27, 13], multicriteria decision aid [57, 18], etc.

Let $\Theta = \{S_1, \ldots, S_n\}$ be a finite set of mutually exclusive and exhaustive statements called frame of discernment and $2^\Theta$ be the power set of $\Theta$. A Basic Belief Assignment (BBA) [30] is the basic function used in evidence theory for modeling imperfect data. It is a mapping $m$ defined from $2^\Theta$ to $[0, 1]$ such as $m(\emptyset) = 0$ and $\sum_{A \subseteq \Theta} m(A) = 1$. The quantity $m(A)$ represents the belief mass of subset $A$, i.e., the belief committed exactly to $A$. When $m(A) \neq 0$, $A$ is called a focal element or a focal set.

The function $m$ constitutes a flexible tool in evidence theory that models every state of belief. A BBA is said to be Bayesian if all its focal elements are
singleton and consonant if all these elements are nested. It is called vacuous if the total belief is assigned only to Θ (total ignorance case) and simple if it has two focal elements and Θ is one of these focal sets. In the latter case, m(Θ) reflects an ignorance level since it is the belief mass which is not assigned to any subset A ≠ Θ and transferred to Θ.

A BBA can be also represented by two functions called credibility (or belief) and plausibility, denoted in the literature respectively by Bel and Pl [30]. Formally, these two functions are defined from \( 2^\Theta \) to \([0, 1]\) as follows:

\[
Bel(A) = \sum_{X \subseteq A, X \neq \emptyset} m(X)
\]

\[
Pl(A) = \sum_{A \cap X \neq \emptyset} m(X)
\]

Bel(A) is the total belief of subsets X which are included in A whereas Pl(A) is the total belief of subsets X having a non-empty intersection with A, i.e., the subsets that are included in A and those having a partial intersection with A. Bel(A) and Pl(A) are therefore the minimal and maximal total beliefs committed to A. They are also connected by the relation \( Pl(A) = 1 - Bel(\overline{A}) \) where \( \overline{A} \) is the complement of A in \( \Theta \).

The combination is a fundamental notion in evidence theory allowing the aggregation of imperfect information given by several sources and modeled by BBAs. Several combination rules have been developed in this context [32,12,25]. Among them, Dempster’s rule [30] remains the most commonly-used operator in the combination of independent BBAs. It is given by

\[
m(A) = (1 - k)^{-1} \sum_{X \cap Y = A} m_1(X) \cdot m_2(Y),
\]

where \( m = m_1 \oplus m_2 \) is the BBA deduced from the combination of \( m_1 \) and \( m_2 \) (called orthogonal sum) and \( k = \sum_{X \cap Y = \emptyset} m_1(X) \cdot m_2(Y) \) is the belief mass that the combination assigns to the empty set. The ratio \( (1 - k)^{-1} \) is a normalization factor guaranteeing that no belief mass is given to the empty set and that the total belief is equal to one.

Dempster’s rule is a conjunctive operator, i.e., the resulting focal elements are intersections of those related to \( m_1 \) and \( m_2 \). It can be proved to be both commutative and associative. Thus, the combination result of several BBAs is independent of the order in which they are considered.

The decision-making is also an important notion of evidence theory that aims to choose the ”best” statement of \( \Theta \). Among other rules, one can cite the maximum of credibility rule that selects the most credible \( S_i \) [14], the maximum of plausibility rule that chooses the most plausible \( S_i \) [14], and the maximum of pignistic probability [31]. The latter operator is based on the idea of transforming a BBA into a function having similar properties of a probability distribution called pignistic probability function BetP. The decision is therefore to choose the statement having the maximum of pignistic probability.

\[6\]
3 Credible belief base revision

In this section, we investigate the idea of belief base revision considering maximality in terms of set credibility (instead of set inclusion and set cardinality) denoted throughout this paper by CSR (Credible Sets Revision). Recall that our goal is to define belief base revision operators requiring rationality when revising so as to avoid losing valuable beliefs.

3.1 Credible belief operators

As described hereafter, the credible belief base revision leads to define two new formula-based revision operators using the suitable tools offered by evidence theory. To ensure uniformity with the RSRG and RSRW operators, we denote these operators by CSRG (referring to the permissive strategy) and CSRW (referring to the drastic strategy) that stem from \( W_{Bel}(B,\mu) \), the set of consistent subbases maximal w.r.t. credibility. Formally, we have

\[
W_{Bel}(B,\mu) = \{ B'_i \subseteq B \mid \bigwedge B'_i \not\models \neg \mu \text{ and for all } B'_j \subseteq B, j \neq i \text{ s.t. } Bel(B'_i) < Bel(B'_j), \bigwedge B'_j \models \neg \mu \}.
\]

Let us turn to explain the computation of the credibility degree of each maximal consistent subbase \( Bel(B'_i) \). As presented below, the CSRG and CSRW operators work in three major steps: the definition of BBAs, the combination and the decision-making.

Indeed, starting from \( W(B,\mu) \) (i.e., the set of maximal consistent subbases \( B'_i \) with \( 1 \leq i \leq n \)), the first step consists in representing each \( B'_i \) by a simple BBA denoted \( m_i \). This function takes into account the cardinality of \( B'_i \) in order to reflect its importance with regard to the other subbases and compared to the initial agent’s belief base \( B \). Formally, this BBA is given for each \( B'_i \) as follows:

\[
\begin{align*}
    m_i(B'_i) &= \frac{|B'_i|}{|B|} \\
    m_i(B) &= 1 - \frac{|B'_i|}{|B|}
\end{align*}
\]

where \( |B'_i| \) is the cardinality of \( B'_i \) and \( |B| \) is the cardinality of \( B \). As one can remark, \( m_i(B'_i) \) represents the proportion of formulas belonging to \( B'_i \) with regard to \( B \). Note also that \( m_i(B) \) is interpreted as an ignorance level that reflects the belief mass which is not assigned to \( B'_i \) and therefore transferred to \( B \).

In the second step, the BBAs describing the maximal coherent subbases are combined using Dempster’s rule. The combined BBA is defined as the orthogonal sum of these BBAs. It is given formally by :

\[
m = m_1 \oplus ... \oplus m_n
\]

Since Dempster’s rule is a conjunctive operator and the focal elements \( m_i \) of each BBA are \( B'_i \) and \( B \), the focal sets \( F(B,\mu) \) of \( m \) (the combined BBA) are
therefore all the subbases $B'_i$, all the sets derived from their possible combinations (non-empty intersections) denoted by $F \cap (B, \mu)$ and the initial base $B$. This is due to the fact that $B$ is a common focal element defined in each $m_i$. Thus, it is clear that $B$ plays a central role in the combination since it allows appearing all the $B'_i$ and their potential intersections. The sets $F(B, \mu)$ and $F \cap (B, \mu)$ are formally defined respectively as follows:

\[
F(B, \mu) = W(B, \mu) \cup F \cap (B, \mu) \cup \{B\}
\]

\[
F \cap (B, \mu) = \{ \bigcap_{i,j \in \{1...n\}} (B'_i, B'_j) \cup \bigcap_{i,j,k \in \{1...n\}} (B'_i, B'_j, B'_k) \cup ... \cup (B'_1, ..., B'_n) \} \setminus \{\emptyset\}
\]

At this point, let us note that the combination allows deducing intersections of subbases with belief masses. If an intersection (or several) supports completely a subbase $B'_i$, it is therefore a focal set affirming $B'_i$. Thus, its belief mass can be added to $m(B'_i)$ which allows obtaining an overall degree characterizing $B'_i$. This measure is nothing else than its credibility degree $Bel(B'_i)$. Formally, we have

\[
Bel(B'_i) = m(B'_i) + \sum_{X \subset B'_i, X \in F \cap (B, \mu)} m(X).
\]

In the last step, the credibility degrees of all the $B'_i$ are exploited for the decision. Remind that $W_{Bel}(B, \mu)$ will contain the most credible consistent subbases among all the $B'_i$ of $W(B, \mu)$. Hence, the CSRG operator $^\ast_{CSRG}$ takes into account all the subbases in $W_{Bel}(B, \mu)$ considering them equally fair and favorable. This operator can be captured as follows:

\[
B ^\ast_{CSRG} \mu = \bigvee_{B'_i \in W_{Bel}(B, \mu)} \bigwedge (B'_i \cup \{\mu\})
\]

As for the CSRW operator $^\ast_{CSRW}$, it stems from the intersection of the most credible consistent subbases. The CSRW operator $^\ast_{CSRW}$ can be defined as

\[
B ^\ast_{CSRW} \mu = \bigwedge_{B'_i \in W_{Bel}(B, \mu)} \bigcap B'_i \cup \{\mu\}.
\]

Consequently, the associated operators $^\oslash_{CSRG} ^\oslash_{CSRW}$ (returning a theory) respectively to $^\ast_{CSRG}$ and $^\ast_{CSRW}$ are the following:

\[
B ^\oslash_{CSRG} \mu = Cn(\bigwedge_{B'_i \in W_{Bel}(B, \mu)} \bigvee B'_i \cup \{\mu\})
\]
\[ B \oplus_{\text{CSR}} \mu = \text{cn}(\bigcap_{B' \in \text{Bel}(B, \mu)} \{B'^* \cup \{\mu\}\}) \]

We can now define the notion of logical consequence in each of these formalisms. A formula \( \psi \) is a logical consequence of the revision result \( B * \mu \) if:

- In the case of CSRG operator, it is a logical consequence of each subbase \( B'_i \) in \( \text{WBel}(B, \mu) \) augmented with the new information \( \mu \). Formally, we have
  \[ B *_{\text{CSRG}} \mu \models \psi \text{ if and only if for each } B'_i \in \text{WBel}(B, \mu), B'_i \cup \{\mu\} \models \psi. \]

- In the case of CSRW operator, it is a logical consequence of the intersection of all subbases in \( \text{WBel}(B, \mu) \) augmented with the new information \( \mu \). Formally, we have
  \[ B *_{\text{CSRW}} \mu \models \psi \text{ if and only if } (\bigcap_{B'_i \in \text{WBel}(B, \mu)} B'_i) \cup \{\mu\} \models \psi. \]

As far, we can conclude that an interpretation \( I \) is a model of the revised belief base \( (I \models B * \mu) \) if and only if \( I \) satisfies \( \mu \) and satisfies at least one set of \( \text{WBel}(B, \mu) \) in the case of CSRG operator, and every formula occurring in all maximal consistent subbases (i.e., in all sets of \( \text{WBel}(B, \mu) \)) in the case of CSRW operator.

Finally, it is interesting to note that the maximum of credibility is generally used within evidence theory to select the most credible element of the frame of discernment. In this work, we have adapted the use of this rule according to the studied context since the objective is not to select one formula from the initial set \( B \). The decision should rather be taken on the different subbases (which are subsets of formulas) not on the formulas composing \( B \). In addition, it is important to notice that we have not hope to investigate the maximum of plausibility in the context of CSRG and CSRW operators since the plausibility considers even the focal elements having a partial intersection with \( B'_i \). These focal elements are not the results of combining \( B'_i \) with other subbases. They are rather induced by the combination of other sets. Similarly, the maximum of pignistic probability (basically used to select the most likelihood element of the frame of discernment) is inappropriate in our context. Indeed, the pignistic transformation cannot be exploited correctly in this case since the objective is to select a maximal consistent subbase not a unique formula of \( B \).

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4. \( B \) constitutes the frame of discernment and the formulas are the statements.
3.2 Illustration of CSRG and CSRW operators

In what follows, we provide a simple example illustrating the CSRG and the CSRW operators.

Example 2. Consider the belief base $B = \{a \rightarrow \neg b, \neg b \rightarrow c, a \rightarrow d, a \rightarrow \neg c, a \rightarrow \neg d, b, a \rightarrow b, a \rightarrow e, \neg b \rightarrow e\}$, and a new information $\mu = a \land (b \leftarrow e)$. So, we have

$$\mathcal{W}(B, \mu) = \{B'_1 = \{a \rightarrow \neg b, \neg b \rightarrow c, a \rightarrow d\}; B'_2 = \{a \rightarrow \neg b, a \rightarrow \neg c, a \rightarrow d\}; B'_3 = \{a \rightarrow \neg b, a \rightarrow \neg d\}; B'_4 = \{b, a \rightarrow b, a \rightarrow e, \neg b \rightarrow e\}\}.$$  

It is clear that $\mathcal{W}_{\text{card}}(B, \mu) = \{B'_4 = \{b, a \rightarrow b, a \rightarrow e, \neg b \rightarrow e\}\}$. Therefore,

$$B \star_{\text{RSRG}} \mu = b \land (a \rightarrow b) \land (a \rightarrow e) \land (\neg b \rightarrow e) \land (a \land (b \leftarrow e)) = \bot.$$ 

As $\mathcal{W}_{\text{card}}(B, \mu)$ contains a unique set (i.e. $B'_4$), we have

$$\bigcap_{B' \in \mathcal{W}_{\text{card}}(B, \mu)} B' = \{\{b, a \rightarrow b, a \rightarrow e, \neg b \rightarrow e\}\}.$$ 

Therefore, $B \star_{\text{RSRW}} \mu = B \star_{\text{RSRG}} \mu = \bot$. Consequently, we obtain

$$B \bowtie_{\text{RSRG}} \mu = B \bowtie_{\text{RSRW}} \mu = \text{Cn}(\bot).$$  

Let us now consider the CSRG and CSRW operators. Thus, we determine at first the BBA related to each subbase $B'_i$ ($1 \leq i \leq 4$) as follows:

$$\begin{align*}
\{ & m_1(B'_1) = 3/9 \\
& m_1(B) = 6/9 \}
\end{align*}$$  

$$\begin{align*}
\{ & m_2(B'_2) = 3/9 \\
& m_2(B) = 6/9 \}
\end{align*}$$  

$$\begin{align*}
\{ & m_3(B'_3) = 2/9 \\
& m_3(B) = 7/9 \}
\end{align*}$$  

$$\begin{align*}
\{ & m_4(B'_4) = 4/9 \\
& m_4(B) = 5/9 \}
\end{align*}$$
Hence, we obtain the outcome using the CSRG operator although it has the maximal cardinality with each subbase. Dempster’s rule is then applied to combine \( m_1, m_2, m_3 \) and \( m_4 \). This leads to the following BBA:

\[
\begin{align*}
  m(B'_1) &= 0.1354 \\
  m(B'_2) &= 0.1354 \\
  m(B'_3) &= 0.0774 \\
  m(B'_4) &= 0.2166 \\
  m\{a \rightarrow \neg b, a \rightarrow d\} &= m(B'_1 \cap B'_2) = 0.0677 \\
  m\{a \rightarrow b\} &= m(B'_4) + m(B'_3) = 0.0967 \\
  m(B) &= 0.2708 
\end{align*}
\]

Based on the combined BBA, we compute the credibility degree \( Bel \) for each subbase \( B'_i \). Since Dempster’s rule allows appearing the focal element \( \{a \rightarrow \neg b, a \rightarrow d\} \) which is common between \( B'_1 \) and \( B'_2 \), \( m\{a \rightarrow \neg b, a \rightarrow d\} \) should be considered in the computation of the credibility degrees \( Bel(B'_1) \) and \( Bel(B'_2) \). Similarly, since the focal element \( \{a \rightarrow b\} \) is common between \( B'_1, B'_2 \) and \( B'_3, B'_4 \), \( m\{a \rightarrow b\} \) should be also added to the credibility degrees \( Bel(B'_1), Bel(B'_2) \) and \( Bel(B'_3) \). As a result, we obtain the following values:

\[
\begin{align*}
  Bel(B'_1) &= m(B'_1) + m\{a \rightarrow \neg b, a \rightarrow d\} + m\{a \rightarrow b\} \\
  &= 0.1354 + 0.0677 + 0.0967 = 0.2998 \\
  Bel(B'_2) &= m(B'_2) + m\{a \rightarrow \neg b, a \rightarrow d\}m\{a \rightarrow b\} \\
  &= 0.1354 + 0.0677 + 0.0967 = 0.2998 \\
  Bel(B'_3) &= m(B'_3) + m\{a \rightarrow b\} = 0.0774 + 0.0967 = 0.1741 \\
  Bel(B'_4) &= m(B'_4) = 0.2166 
\end{align*}
\]

This yields to have

\[
W_{Bel}(B, \mu) = \{B'_1 = \{a \rightarrow \neg b, a \rightarrow d\}; B'_2 = \{a \rightarrow \neg b, a \rightarrow \neg c, a \rightarrow d\}\}.
\]

Observe that \( W_{Bel}(B, \mu) \neq W_{\text{card}}(B, \mu) \) and a fortiori neither \( B *_{\text{CSRG}} \mu \neq B *_{\text{RSRG}} \mu \), nor \( B *_{\text{CSRW}} \mu \neq B *_{\text{RSRW}} \mu \). As a consequence,

\[
B *_{\text{CSRG}} \mu = ((a \rightarrow b) \land \neg b \rightarrow c \land (a \rightarrow d)) \lor ((a \rightarrow \neg b) \land (a \rightarrow \neg c) \\
\land (a \rightarrow d)) \lor (a \land (b \leftarrow \neg c)) \\
= a \land \neg b \land d \land e.
\]

Hence, \( B *_{\text{CSRG}} \mu = Cn(a \land \neg b \land d \land e) \). Furthermore, we have

\[
\bigcap_{B^* \in W_{\text{inc}}(B, \mu)} B^* = \{a \rightarrow \neg b, a \rightarrow d\}.
\]

Therefore, \( B *_{\text{CSRW}} \mu = (a \rightarrow \neg b) \land (a \rightarrow d) \land (a \land (b \leftarrow \neg c)) = a \land d \land e \). Consequently, we obtain \( B *_{\text{CSRW}} \mu = Cn(a \land d \land e) \).

As one can remark, the subbase \( B'_4 \) was not considered in the belief revision outcome using the CSRG operator although it has the maximal cardinality with
regard to $B'_1$ and $B'_2$. This is due to the fact that it is less credible than the other subbases. This result can be explained by the lack of $(a \rightarrow \neg b)$ and $(a \rightarrow d)$ in $B'_4$ which are pertinent formulas in $B$ giving a considerable advantage to $B'_1$ and $B'_2$ and involving a loss of credibility of $B'_4$ in favor of these subsets.

This example illustrates properly the fact that the selection of consistent subbases maximal with respect to set cardinality is not always a guarantee to select the most relevant information since it can neglect potential formulas from the initial agent’s beliefs and thus it can induce a loss of rationality when revising. Additionally, the use of credibility in belief base revision seems reasonable complying with the minimality (not in terms of quantity) criterion of belief revision through capturing relevant information playing a central role in the initial agent’s beliefs. Hence, this credibility criterion can be interpreted as a way to guard as possible as far the general sense of the agent beliefs. So, it would be interesting to study to which extend the credible revision takes the form of syntactic revision, implicitly with some semantic features.

The example presented above can be a proper illustration of the gap between cardinality and credibility as maximality criteria in the selection of consistent subbases. But this cannot deny the fact that the CSRG and RSRG operators can lead to the same revision outcome in some cases.

**Proposition 1.** The CSRG operator leads to the same revision result as the RSRG operator if and only if the combined BBA verifies the following two conditions:

- $\forall B'_i, B'_j \in \mathcal{W}_{Bel}(B, \mu)$:
  
  \[
  \begin{cases} 
  m(B'_i) = m(B'_j) \\
  \sum_{X \subseteq B'_i} m(X) = \sum_{X \subseteq B'_j} m(X)
  \end{cases}
  \]

- $\forall B'_i \in \mathcal{W}_{Bel}(B, \mu)$ and $\forall B'_j \notin \mathcal{W}_{Bel}(B, \mu)$:

  \[
  m(B'_i) - m(B'_j) > \max \left( 0, \sum_{X \subseteq B'_i} m(X) - \sum_{X \subseteq B'_j} m(X) \right)
  \]

**Proof.** As stressed above, the subbases $B'_i$ appear in the combined BBA thanks to the successive intersections with $B$. Moreover, since $m_i(B'_i)$ is defined with respect to its cardinality $|B'_i|$ (i.e. $m_i(B'_i) = \frac{|B'_i|}{|B'_i|}$), the successive combinations of $B'_i$ with $B$ using Dempster’s rule lead to obtain an order between the combined belief masses $m(B'_i)$ that respects the order between the cardinalities $|B'_i|$. As a result:

\[
\begin{cases} 
|B'_i| = |B'_j| \Rightarrow m(B'_i) = m(B'_j) \\
|B'_i| > |B'_j| \Rightarrow m(B'_i) > m(B'_j)
\end{cases}
\]

If the CSRG and RSRG operators reach the same revision result, this means that $\mathcal{W}_{Bel}(B, \mu) = \mathcal{W}_{card}(B, \mu)$. Therefore:
- ∀\( B_i', B_j' \in W_{Bel}(B, \mu) \):

\[
\begin{align*}
\begin{cases}
|B_i'| = |B_j'| \\
Bel(B_i') = Bel(B_j')
\end{cases}
\iff
\begin{cases}
m(B_i') = m(B_j') \\
m(B_i') + \sum_{X \subseteq B_i'} m(X) = m(B_j') + \sum_{X \in F_r(B, \mu)} X \subseteq B_j' m(X)
\end{cases}
\iff
\begin{cases}
m(B_i') = m(B_j') \\
\sum_{X \subseteq B_i'} m(X) = \sum_{X \in F_r(B, \mu)} X \subseteq B_j' m(X)
\end{cases}
\iff
m(B_i') = m(B_j')
\end{align*}
\]

- ∀\( B_i' \in W_{Bel}(B, \mu) \) and ∀\( B_j' \notin W_{Bel}(B, \mu) \):

\[
\begin{align*}
\begin{cases}
|B_i'| > |B_j'| \\
Bel(B_i') > Bel(B_j')
\end{cases}
\iff
\begin{cases}
m(B_i') > m(B_j') \\
m(B_i') + \sum_{X \subseteq B_i'} m(X) > m(B_j') + \sum_{X \in F_r(B, \mu)} X \subseteq B_j' m(X)
\end{cases}
\iff
\begin{cases}
m(B_i') - m(B_j') > 0 \\
m(B_i') - m(B_j') > \sum_{X \subseteq B_i'} m(X) - \sum_{X \in F_r(B, \mu)} X \subseteq B_j' m(X)
\end{cases}
\iff
m(B_i') - m(B_j') > \max \left(0, \sum_{X \subseteq B_i'} m(X) - \sum_{X \in F_r(B, \mu)} X \subseteq B_j' m(X)\right)
\end{align*}
\]

Remark 1. Let us emphasize that if the intersection of each pair of subbases is the empty set, the CSRG and RSRG operators lead to the same revision result. Indeed, if it is the case, the intersection of any other group of subbases is also the empty set. As a result, the focal elements set of the combined BBA consists only of the subbases (without intersections). This implies that there is no intersection supporting totally \( B_i' \); i.e.:

\[
\sum_{X \subseteq B_i'} m(X) = 0
\]

Therefore, \( Bel(B_i') = m(B_i') \) and as a consequence, the revision result of the CSRG operator can be defined as follows:

- ∀\( B_i', B_j' \in W_{Bel}(B, \mu) \):

\[
Bel(B_i') = Bel(B_j') \iff m(B_i') = m(B_j')
\]

- ∀\( B_i' \in W_{Bel}(B, \mu) \) and ∀\( B_j' \notin W_{Bel}(B, \mu) \):

\[
Bel(B_i') > Bel(B_j') \iff m(B_i') > m(B_j') \iff m(B_i') - m(B_j') > 0
\]

These two conditions are nothing else than the conditions exposed in Proposition 1. Hence, the CSRG and RSRG operators lead to the same revision result.
4 Compromise revision strategy

In addition to the CSRG and CSRW operators, we present hereunder another contribution that explores the idea of using evidence theory in belief base revision. More specifically, we propose a new revision strategy based on this theory that captures beliefs with a mind to be prudent.

4.1 Description of compromise strategy

As explained previously, we have already provoked two extreme approaches for revising belief bases. The former is permissive and allows choosing all the maximal consistent subbases whereas the latter is drastic leading to keep only the beliefs that are not questioned. Between these two extremes, evidence theory let us shed some light on the idea of an intermediary or a compromise strategy. The underlying idea of this approach is to capture the maximal consistent sets stemming from all the possible intersections of maximal consistent subbases. This constitutes obviously an advantage with regard to the drastic strategy which has been intensively criticized in the literature since it is so prudent and can lead in many cases, and particularly with large belief bases, to lose all the initial belief’s agents. In addition, it presents a benefit for the compromise strategy compared to the permissive approach which can lead to an exhaustive revision result especially in the domain of databases repair.

Considering the credibility as the most rational and reliable maximality criterion, we focus on defining a compromise operator between the CSRG and CSRW ones. The proposed operator, called CSIR (Credible Sets Intersections Revision), is also based on the idea of using the information given by the combined BBA since Dempster’s rule allows deducing all the possible intersections between the subbases with their related belief masses. More precisely, the CSIR operator stems from $V_{Bel}(B, \mu)$, the set of the most credible focal sets derived from all the possible intersections of maximal consistent subbases $B_i'$. In other words, $V_{Bel}(B, \mu)$ selects from $F \cap (B, \mu)$ the sets having the highest credibility degree. At this point, let us note that the maximum of credibility implies implicitly the satisfaction of maximal set inclusion criterion in $V_{Bel}(B, \mu)$. Formally, we obtain

$$V_{Bel}(B, \mu) = \{ X_i' \subseteq F \cap (B, \mu) | \bigwedge X_i' \not\models \neg \mu \text{ and for all } X_j' \subseteq F \cap (B, \mu), j \neq i \}
\text{ s. t. } Bel(X_i') < Bel(X_j'), \bigwedge X_j' \models \neg \mu \}.$$  

The operator $*_\text{CSIR}$ is therefore defined as follows:

$$B *_{\text{CSIR}} \mu = \bigvee_{X' \in V_{Bel}(B, \mu)} (X' \cup \{ \mu \})$$

and the associated operator $\oplus_{\text{CSRG}}$ returning a theory is captured by

$$B \oplus_{\text{CSIR}} \mu = Cn(\bigwedge_{X' \in V_{Bel}(B, \mu)} \bigvee X' \cup \{ \mu \}).$$
A formula $\psi$ is a logical consequence of the revision result $B *_{\text{CSIR}} \mu$ if it is a logical consequence of each set $X'$ in $V_{\text{Bel}}(B, \mu)$ augmented with the new information $\mu$. Formally, we have

$$B *_{\text{CSIR}} \mu \models \psi$$

if and only if for each $X' \in V_{\text{Bel}}(B, \mu)$, $X' \cup \{\mu\} \models \psi$.

As far, we can conclude also that in the case of the CSIR operator, an interpretation $I$ is a model of the revised belief base (i.e. $I \models B * \mu$) if and only if $I$ satisfies $\mu$ and satisfies every formula occurring in each maximal (w.r.t. credibility) consistent set derived from all subbases intersections (i.e. in each set of $V_{\text{Bel}}(B, \mu)$).

Finally, it is worth mentioning that if $B_i' \cap B_j' = \emptyset$ for all $B_i', B_j' \in W(B, \mu)$, thus $F_\cap(B, \mu) = \emptyset$ and therefore $B *_{\text{CSIR}} \mu = \mu$. This obvious result can be explained due to the conflicting character of each pair of subbases. Contrary to the drastic approach (Widtio, RSRW and CSRW operators), this case constitutes the unique situation where the compromise strategy (and particularly the CSIR operator) loses all the original beliefs. That is why, it would be also interesting to study other compromise operators complying with cardinality and set inclusion criteria.

### 4.2 Illustration of CSIR operator

We show, in the following example, how compromise strategy and particularly the CSIR operator can be attractive compared to the drastic strategy.

**Example 3.** Let $B$ a belief base such that $B = \{a \rightarrow \neg b, c \rightarrow \neg a, \neg d \rightarrow \neg a, b \rightarrow c, b, a \rightarrow \neg d, d \rightarrow e, c \rightarrow e, b \rightarrow d, d \rightarrow c\}$ and a new information $\mu = a \land \neg e$.

We have

$$W(B, \mu) = \{B_1' = \{a \rightarrow \neg b, c \rightarrow \neg a, \neg d \rightarrow \neg a, b \rightarrow c\};$$
$$B_2' = \{b, a \rightarrow \neg d, d \rightarrow e, c \rightarrow e\};$$
$$B_3' = \{a \rightarrow \neg b, b \rightarrow d, d \rightarrow c\};$$
$$B_4' = \{b, b \rightarrow d, d \rightarrow c\}\}$$

and

$$W_{\text{card}}(B, \mu) = \{B_1' = \{a \rightarrow \neg b, c \rightarrow \neg a, \neg d \rightarrow \neg a, b \rightarrow c\};$$
$$B_2' = \{b, a \rightarrow \neg d, d \rightarrow e, c \rightarrow e\}\}.$$

Let us compute $W_{\text{Bel}}(B, \mu)$ and $V_{\text{Bel}}(B, \mu)$. It is obvious that

$$F_\cap(B, \mu) = \{X_1' = \{a \rightarrow \neg b\}, X_2' = \{b\}, X_3' = \{b \rightarrow d, d \rightarrow c\}\}.$$

The BBAs related to all the subbases are at first determined as follows:

\[
\begin{cases}
  m_1(B_1') = 4/10 \\
  m_1(B) = 6/10
\end{cases}
\]
Formally, we obtain

\[
\begin{aligned}
\{ m_2(B'_2) = 4/10 \\
m_2(B) = 6/10 \\
m_3(B'_3) = 3/10 \\
m_3(B) = 7/10 \\
m_4(B'_4) = 3/10 \\
m_4(B) = 7/10.
\end{aligned}
\]

Dempster’s rule is then used to combine these BBAs. The resulting BBA is the following:

\[
\begin{aligned}
m(B'_1) &= 0.169 \\
m(B'_2) &= 0.169 \\
m(B'_3) &= 0.1086 \\
m(B'_4) &= 0.1086 \\
m(X'_1) &= 0.0724 \\
m(X'_2) &= 0.0724 \\
m(X'_3) &= 0.0466 \\
m(B) &= 0.2534.
\end{aligned}
\]

Finally, the credibility degrees of the maximal credible consistent subbases and the credibility degrees of their possible intersections are computed based on the combined BBA as below.

\[
\begin{aligned}
\text{Bel}(B'_1) &= m(B'_1) + m(X'_1) = 0.2414 \\
\text{Bel}(B'_2) &= m(B'_2) + m(X'_2) = 0.2414 \\
\text{Bel}(B'_3) &= m(B'_3) + m(X'_3) + m(X'_4) = 0.2276 \\
\text{Bel}(B'_4) &= m(B'_4) + m(X'_2) + m(X'_3) = 0.2276 \\
\text{Bel}(X'_1) &= m(X'_1) = 0.0724 \\
\text{Bel}(X'_2) &= m(X'_2) = 0.0724 \\
\text{Bel}(X'_3) &= m(X'_3) = 0.0466.
\end{aligned}
\]

This let us to have

\[
\begin{aligned}
W_{\text{Bel}}(B, \mu) &= \{ B'_1 = \{ a \rightarrow \neg b, c \rightarrow \neg a, \neg d \rightarrow \neg a, b \rightarrow c \}; \\
& \quad B'_2 = \{ b, a \rightarrow \neg d, d \rightarrow e, c \rightarrow e \} \}.
\end{aligned}
\]

Once again, remark that although the set \( X'_3 \) has the highest cardinality in comparison with \( X'_1 \) and \( X'_2 \), the above computation yields to have

\[
\begin{aligned}
Y_{\text{Bel}}(B, \mu) &= \{ X'_1 = \{ a \rightarrow \neg b \}, X'_2 = \{ b \} \}.
\end{aligned}
\]

Turn us now to compute revision result with different operators. Observe that

\[
\begin{aligned}
\bigcap_{B'_i \in W(B, \mu)} B'_i &= \bigcap_{B'_i \in W_{\text{Bel}}(B, \mu)} B'_i = \bigcap_{B'_i \in W_{\text{Bel}}(B, \mu)} B'_i = \emptyset
\end{aligned}
\]

and so that we lose all initial beliefs taking into account the drastic strategy. Formally, we obtain \( B \ast_{\text{wid}} \mu = B \ast_{\text{RSR}} \mu = B \ast_{\text{CSRW}} \mu = \mu \) and trivially
This yields automatically $B \mu = Cn((a \rightarrow \neg b) \lor b) \land \mu$. It can be clearly seen that CSIR operator behaves in a careful way as expected.

5 Conclusion

This paper contributes to the current line of research in belief change that has received considerable attention from the AI, database and philosophy communities. In these contexts, revision is considered as the well-known belief change operation remaining consistency in order to integrate the new information while modifying the initial beliefs as little as possible. As far as we know, revision strategies in evidence theory have seldom been addressed. As shown previously, our work deals with the use of evidence theory in belief base revision that was investigated through two extreme revision operators (CSRG and CSRW respectively similar to RSRG and RSRW) and a new compromise revision strategy explicitly instantiated by CSIR operator. In both contributions, we highlighted the potential benefit offered by the different tools of evidence theory in belief base revision. Indeed, the notion of BBA was used to model the information related to each subbase. Dempster’s rule was also applied to combine all these BBAs in order to yield a global BBA synthetizing the information given by the subbases set. In addition, the credibility maximality was used as a selection criterion to choose the most credible subbases intersections (CSIR) instead of cardinality criterion. The presented examples illustrated to which extend the credibility criterion is interesting to avoid losing valuable and relevant beliefs. Let us mention that we can define systematically two other compromise operators in the same spirit of CSIR operator: the former RSIR (Removed sets intersections revision) is an intermediary between RSRG and RSRW operators respecting the cardinality as maximality criterion and the latter SIR is an intermediary between Ginsberg and Widtio operators respecting the set inclusion as maximality criterion.

Belief revision and belief contraction are two sides of a same coin. In fact, unlike belief revision, which allows to incorporate new information into a set of beliefs, belief contraction is the process of rationally removing a given belief from a belief set. It ensures removing from the set what is necessary to no longer imply this information while respecting the principle of minimality of change. Belief contraction is related to belief revision in the sense that belief revision can be defined in term of contraction, that is to say, revising a belief set by new information amounts to first remove from the belief set any belief contradicting the new information, and then to add the new information. Formally, from the works of Levi [23,24] and Harper [20], the correspondence between belief revision and belief contraction has been established in [1], thus providing a useful equivalence for belief change studies. Since then, different concepts and constructions
have undergone significant elaboration and development (15,16,29). Different formula-based contraction operators have been studied in the literature that are classified into two families: the first one includes contraction operators that have been defined in terms of remainder sets, i.e., maximal subsets of formulas that fail to imply a given formula. From a dual point of view, the second family represents the Kernel contraction [18] which is based on the minimal sub-theories implying the formula by which one contracts. Therefore, natural extension of this work is to study the equivalence between formula-based revision operators and their corresponding in the context of belief contraction (such as transitively relational partial meet contraction, full meet contraction, maxichoice contraction [1] and infra contraction [5]).

Besides, the work of Creignou et al. [9] defines PRSRG and PRSRW operators as the extension respectively of RSRG and RSRW operators to stratified belief bases. We remind that a stratified belief base \( B = (S_1, ..., S_n) \) is provided by a partition of the belief base in strata \( S_i \) \((1 \leq i \leq n)\) representing priorities between formulas. It seems so natural to think in the future about the extension of our operators (CRSG, CSRW and CSIR) to stratified belief bases. Finally, future work can include also a thorough investigation of the complexity of these new revision operators in the general case of propositional logic and in some fragments (particularly Horn and Krom fragments).

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