A Note on Supersymmetry in Noncommutative Field Theories

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Abstract

We show that a solution of the type IIB supergravity representing D3-branes in the presence of a 2-form background has 16 supersymmetries by explicitly constructing the transformation parameter of unbroken supersymmetry. This solution is dual to a noncommutative Yang-Mills theory in a certain limit.

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1. Introduction

The local supersymmetry in supergravities is directly connected to other local symmetries, such as the gauge symmetry and the general covariance as well as the local Lorentz symmetry, as can be seen in the commutator algebra of the transformations. On the other hand, noncommutative theories can have supersymmetry while the Lorentz symmetry is broken by the presence of a constant 2-form background $B_{\mu\nu}$. It is conjectured that a noncommutative super Yang-Mills theory with constant $B_{23}$ has 16 supersymmetries \[1, 2\]. Thus, it is interesting to see the relation between the supersymmetry and the local Lorentz symmetry in the supergravity, which is dual to the noncommutative field theory with the Lorentz symmetry intuitively broken.

In this paper we show that a solution of the type IIB supergravity representing D3-branes in the presence of non-vanishing $B_{01}, B_{23}$ backgrounds has 16 supersymmetries, although the Lorentz symmetry is broken on the world volume. This solution was obtained in ref. \[3\] by a Wick rotation from the Euclidean solution representing smeared bound states of D$(-1)$, F1, D1 and D3 branes. It originally appeared in the context of the T-duality map \[4\], and was discussed as D-brane bound states \[5, 6\]. In order to study supersymmetry we use the Lorentzian solution rather than the Euclidean one since it is not clear how to define supersymmetry in the Euclidean space.

We substitute this solution into the supertransformations of the fermionic fields in the type IIB supergravity and obtain the conditions on the transformation parameter of unbroken supersymmetry. The general form of the parameter satisfying these conditions contains an arbitrary constant spinor which has 16 independent components. Therefore, we find that 16 supersymmetries exist in four dimensions. This remains true even when we turn off $B_{01}$ or when we take the decoupling limit \[3\]. We will see that the parameter of unbroken supersymmetries explicitly depends on the scalar field $\tau$, the angles of the D-branes tilted in the 01 and 23 directions and the harmonic functions appearing in the metric.

The number of supersymmetries may be partly explained by the isometry SO(6) = SU(4) of S$^5$ contained in the solution. The solution with vanishing 2-form background has $\mathcal{N} = 4$ Poincaré supersymmetry in four dimensions \[7\], whose four-component spinor transformation parameters are independent of the four-dimensional coordinates $x^\mu$. It also has the conformal supersymmetry with $x^\mu$-dependent trans-
formation parameters. We will see that the solution with non-vanishing 2-form background has the same number of supersymmetries with $x^\mu$-independent parameters, but the spinor transformation parameters satisfy a certain condition and have only two independent components. There is no conformal supersymmetry in this case.

In refs. [8, 9] the D-brane effective action was obtained as a solution to the Hamilton-Jacobi equation for the type IIB supergravity compactified on $S^5$. The ADM formalism for the 5-dimensional space-time was used by treating the radial coordinate $r$ as time. Although only the bosonic part of the effective action has been studied, it brought about remarkable results to find new supergravity solutions. The effective action obtained there includes a broad family of super Yang-Mills theories by only assuming that 2-form fields are constant at the surface of constant radius $r$. The geometries dual to the field theories described by this effective action are not known in general. However, there are some known geometries which are dual to the field theories brought by the Hamilton-Jacobi formalism. The noncommutative Yang-Mills theory and the ordinary super Yang-Mills theory are solutions to the Hamilton-Jacobi equation. The dual geometry of the noncommutative Yang-Mills theory is a near horizon limit of a supergravity solution of $N$ D3-branes in a non-vanishing $B_{23}$ background. The dual geometry of the ordinary super Yang-Mills theory is obtained in the limit of vanishing $B_{23}$ [10].

To know more about the general super Yang-Mills theories which is derived as a solution to the Hamilton-Jacobi equation, and to look for the corresponding dual geometries, it is useful to consider the solution with a non-vanishing 2-form extended in the 01 and 23 directions and non-vanishing 0-forms, and to see how supersymmetries survive. It is also important to look at the correspondence between the symmetries of the supergravity and those of the field theory in the light of the AdS/CFT correspondence, as the maximal supersymmetric case shows that the local symmetries in the supergravity become the global symmetries of the field theory on the boundary [11].

We write down the supertransformations of the type IIB supergravity in the string frame in Section 2, and give the explicit form of the solution with non-vanishing 2-form fields in Section 3. In Section 4 we substitute the solution into the supertransformations and find the parameter of unbroken supersymmetry.
2. Type IIB supergravity in ten dimensions

The field content of the type IIB supergravity in ten dimensions \[12, 13\] is a vielbein \( e^A_M \), a complex Rarita-Schwinger field \( \psi_M \), a real fourth-rank antisymmetric tensor field \( D_{MNPQ} \) with a self-dual field strength, two real second-rank antisymmetric tensor fields \( B_{MN} \), \( C_{MN} \), a complex spinor field \( \lambda \) and a complex scalar field \( \tau = \chi + ie^{-\phi} \). We use the string frame rather than the Einstein frame. In the string terminology \( \chi, C_{MN} \) and \( D_{MNPQ} \) are 0, 2, 4-form fields in the R-R sector, while \( B_{MN} \) and \( \phi \) are a 2-form field and a dilaton in the NS-NS sector. We denote ten-dimensional world indices as \( M, N, \ldots = 0, 1, \ldots, 9 \) and local Lorentz indices as \( A, B, \ldots = 0, 1, \ldots, 9 \). The fermionic fields satisfy chirality conditions

\[
\bar{\Gamma}^{10} D \psi_M = -\psi_M, \quad \bar{\Gamma}^{10} D \lambda = \lambda,
\]

where \( \bar{\Gamma}^{10} D = \Gamma^0 \Gamma^1 \cdots \Gamma^9 \) is the ten-dimensional chirality matrix. We choose the ten-dimensional gamma matrices \( \Gamma^A \) to have real components. The gamma matrices with the world indices are denoted as \( \hat{\Gamma}_M = \Gamma^A e^A_M \).

The local supersymmetry transformations of the fermionic fields in the string frame can be obtained from those in the Einstein frame \[12, 13\] and are given by

\[
\kappa \delta \lambda = -e^{\frac{1}{4} \phi} \left( \hat{\Gamma}^M e^* P_M + \frac{1}{24} e^{\frac{1}{2} \phi} \hat{\Gamma}^{MNP} e \mathcal{G}_{MNP} \right),
\]

\[
\kappa \delta \psi_M = \left[ \partial_M + \frac{1}{4} \left( \omega_M^{AB} \Gamma_{AB} + \frac{1}{2} \partial_N \phi \hat{\Gamma}^N_M \right) - \frac{1}{2} i Q_M \right] \epsilon
\]

\[
+ \frac{1}{16 \cdot 5!} ie^{\phi} \hat{F}_{P_1 \ldots P_5} \hat{\Gamma}^{P_1 \ldots P_5} \hat{\Gamma}_M \epsilon
\]

\[
- \frac{1}{96} e^{\phi} \left( \hat{\Gamma}_M^{NPQ} \mathcal{G}_{NPQ} - 9 \hat{\Gamma}^{PQ} \mathcal{G}_{MPQ} \right) e^* \epsilon,
\]

where \( \kappa \) is the gravitational coupling constant and

\[
P_M = \frac{1}{2} ie^{\phi} \frac{1 + i \tau^*}{1 - i \tau} \partial_M \tau,
\]

\[
Q_M = -\frac{1}{4} e^{\phi} \left( \frac{1 - i \tau^*}{1 - i \tau} \partial_M \tau + \frac{1 + i \tau^*}{1 + i \tau} \partial_M \tau^* \right),
\]

\[
\mathcal{G}_{MNP} = i \left( \frac{1 + i \tau^*}{1 - i \tau} \right)^\frac{3}{2} (F_{MNP} + \tau H_{MNP}),
\]

\[
\hat{F}_{MNPQR} = F_{MNPQR} + 10 C_{[MN} H_{PQR]}.
\]

The field strengths are defined as

\[
F_{MNPQR} = 5 \partial_{[M} D_{NPQR]}, \quad F_{MNP} = 3 \partial_{[M} C_{NP]}, \quad H_{MNP} = 3 \partial_{[M} B_{NP]}.
\]
The 5-form field strength $\tilde{F}_{MNPQR}$ must satisfy the self-duality condition $*\tilde{F}_{M_1\cdots M_5} = \tilde{F}_{M_1\cdots M_5}$. The transformation parameter $\epsilon$ is a complex spinor satisfying the chirality condition $\bar{\Gamma}_D\epsilon = -\epsilon$. The unusual term proportional to $\partial_N \phi$ in $\delta \psi_M$ is due to the use of the string frame.

For later convenience, we redefine the transformation parameter as

$$\tilde{\epsilon} = \left(\frac{1 + i\tau^*}{1 - i\tau}\right)^{-\frac{1}{4}} \epsilon, \quad \tilde{\epsilon}^* = \left(\frac{1 + i\tau}{1 - i\tau}\right)^{-\frac{1}{4}} \epsilon^*. \quad (4)$$

It is easy to show that in terms of $\tilde{\epsilon}$ the derivative term in $\delta \psi_M$ looks simpler

$$\left(\partial_M - \frac{1}{2} iQ_M\right) \epsilon = \left(\frac{1 + i\tau^*}{1 - i\tau}\right)^{\frac{1}{2}} \left(\partial_M + \frac{1}{4} ie^\phi \partial_M \chi\right) \tilde{\epsilon}. \quad (5)$$

3. The solution with non-vanishing 2-form fields

The noncommutative super Yang-Mills theory is described by a solution of the type IIB supergravity with non-vanishing $B_{MN}$ field [3] in the Seiberg-Witten limit [1]. Before taking the limit the solution has the metric

$$ds_{\text{string}}^2 = G_{MN} dx^M dx^N = f^{-1/2} h' \left[ - (dx^0)^2 + (dx^1)^2 \right] + f^{-1/2} h \left[ (dx^2)^2 + (dx^3)^2 \right] + f^{1/2} \delta_{ij} dx^i dx^j, \quad (6)$$

where

$$f = 1 + \frac{\alpha' R^4}{r^4}, \quad r^2 = \delta_{ij} x^i x^j,$$

$$h^{-1} = \sin^2 \theta f^{-1} + \cos^2 \theta, \quad h'^{-1} = - \sinh^2 \theta' f^{-1} + \cosh^2 \theta'. \quad (7)$$

The radius $R$ is given by $R^4 = 4\pi gN (\cos \theta \cosh \theta')^{-1}$. We have decomposed the world indices as $M = (\mu, i)$ ($\mu = 0, 1, 2, 3; i = 4, 5, \cdots, 9$). Other non-vanishing fields are

$$e^{2\phi} = g^2 hh', \quad \chi = - g^{-1} \sin \theta \sinh \theta' f^{-1},$$

$$B_{01} = \tanh \theta' f^{-1} h', \quad B_{23} = \tan \theta f^{-1} h,$$

$$C_{01} = g^{-1} \sin \theta \cosh \theta' h f^{-1}, \quad C_{23} = - g^{-1} \cos \theta \sinh \theta' h f^{-1},$$

$$D_{0123} = g^{-1} \cos \theta \cosh \theta' h h' f^{-1}. \quad (8)$$
and $D_{ijkl}$ determined from the above $D_{0123}$ by the self-duality condition of $\tilde{F}_{MNPQR}$.

The constants $\theta$ and $\theta'$ parametrize the asymptotic values of $B_{23}$ and $B_{01}$ for $r \to \infty$. The existence of non-vanishing $\chi$, $B_{01}$ and $C_{01}$ means that $D(-1)$, F1 and D1 branes enter into the bound state.

We also need the spin connection $\omega_{AB}$ for the metric (6). From the torsionless condition 
\[\partial [\eta_{A} e_{B}] + \omega_{[A} e_{B]} = 0\]
the non-vanishing components are obtained as
\[\omega^{aI}_{\mu} = -\frac{1}{4} \left( 1 + 2 \sinh^2 \theta' f^{-1} h' \right) h'^{\frac{1}{2}} f^{-\frac{1}{2}} \partial_{i} f \delta^{iI} \delta_{a}^{\mu} \quad \text{(for } \mu = 0, 1),\]
\[\omega^{aI}_{\mu} = -\frac{1}{4} \left( 1 - 2 \sin^2 \theta f^{-1} h \right) h^{\frac{1}{2}} f^{-\frac{1}{2}} \partial_{i} f \delta^{iI} \delta_{a}^{\mu} \quad \text{(for } \mu = 2, 3),\]
\[\omega_{IJ}^{i} = -\frac{1}{2} f^{-1} \partial_{j} f \delta^{[I} \delta_{j}^{J]}, \quad (9)\]
where we have decomposed the local Lorentz indices as $A = (a, I)$ ($a = 0, 1, 2, 3$; $I = 4, 5, \cdots, 9$). For later convenience, we define $\hat{\Gamma}^{r} = \frac{x^{i}}{r} \hat{\Gamma}^{i}$ as the gamma matrix for the radial ($r$) direction.

**4. Supersymmetry with non-vanishing 2-form fields**

The final task is to substitute the solution (6), (8) into the supersymmetry transformations (11) and obtain the conditions on the transformation parameter $\epsilon$ for unbroken supersymmetry. Since the fields in eqs. (6), (8) depend only on the radial coordinate $r$, some expressions in the supertransformations become simpler. For example, we obtain
\[G_{\mu\nu r} = i \left( \frac{1 + i \tau^{\epsilon}}{1 - i \tau} \right)^{\frac{1}{2}} (\partial_{r} C_{\mu\nu} + \tau \partial_{r} B_{\mu\nu}) \quad (10)\]
and other components of $G_{MNP}$ vanish. Only the $M = r$ components of $P_{M}$, $Q_{M}$ are non-vanishing. The vanishing of the supertransformation of $\lambda$ in eq. (11) then requires
\[\epsilon^{*} = -\frac{1}{8} e^{\phi} G_{\mu\nu r} \hat{\Gamma}^{\mu\nu} (P_{r})^{-1}. \quad (11)\]
Substituting (12) we obtain more explicit condition
\[\epsilon^{*} = -f^{\frac{1}{2}} \left( \sin \theta h^{\frac{1}{2}} + i \sinh \theta' h'^{\frac{1}{2}} \right)^{-1} \left( h^{\frac{1}{2}} \cos \theta \Gamma^{0123} - i h'^{\frac{1}{2}} \cosh \theta' \right) \Gamma^{01} \bar{\epsilon}. \quad (12)\]
We then consider the condition from \( \delta \psi_M = 0 \). By using the self-duality of \( \tilde{F}_{M_1 \cdots M_5} \) and the chirality condition of \( \epsilon \) we find that the 5-form field strength term in \( \delta \psi_M \) becomes

\[
\frac{1}{5!} \tilde{F}_{NPQRS} \tilde{\Gamma}^{NPQRS} \tilde{F}^{0123} \tilde{\Gamma}^r \tilde{\Gamma}^r M \epsilon, \tag{13}
\]

where \( \tilde{F}_{0123r} = \partial_r D_{0123} + C_{01} \partial_r B_{23} + C_{23} \partial_r B_{01} \). The \( \mathcal{G} \) terms are simplified as

\[
- \frac{e^\phi}{96} \left( 3 \tilde{\Gamma}_{\mu} \nu_{\rho i} \mathcal{G}_{\nu_{\rho i}} - 18 \tilde{\Gamma}^i \mathcal{G}_{i \mu \nu} \right) = - \frac{e^\phi}{16} f^i f^j \left( h^{-1} \mathcal{G}_{0123} + 3 h^{-1} \mathcal{G}_{01r} \right) \Gamma^01 \tilde{\Gamma}^r \mu \tag{14}
\]

for \( \mu = 0, 1 \),

\[
- \frac{e^\phi}{96} \left( 3 \tilde{\Gamma}_{\mu} \nu_{\rho i} \mathcal{G}_{\nu_{\rho i}} - 18 \tilde{\Gamma}^i \mathcal{G}_{i \mu \nu} \right) = - \frac{e^\phi}{16} f^i f^j \left( h^{-1} \mathcal{G}_{01r} + 3 h^{-1} \mathcal{G}_{23r} \Gamma^0123 \right) \Gamma^01 \tilde{\Gamma}^r \mu \tag{15}
\]

for \( \mu = 2, 3 \), and

\[
- \frac{e^\phi}{96} \left( 3 \tilde{\Gamma}_i \nu_{\rho j} \mathcal{G}_{\nu_{\rho j}} - 9 \tilde{\Gamma}^i \mathcal{G}_{i \mu \nu} \right) = - \frac{e^\phi}{16} f^i f^j \left( h^{-1} \mathcal{G}_{01r} + h^{-1} \mathcal{G}_{23r} \Gamma^0123 \right) \Gamma^01 \left( \tilde{\Gamma}_i j - 3 \delta_i j \right) \tag{16}
\]

for \( i = 4, \cdots, 9 \). Note that the summations of \( \nu, \rho \) on the left-hand sides run 0 to 3.

We use these formulae and substitute eq. (11) into \( \epsilon^* \) in \( \delta \psi_M \). For \( \mu = 0, \cdots, 3 \) the spin connection term and the \( \partial_j \phi \) term cancel with the real part of the 3-form terms in eqs. (14), (15), whereas the 5-form term cancels with the imaginary part of the 3-form terms. Thus, we find a simple result

\[
\kappa \delta \psi_\mu = \partial_\mu \epsilon. \tag{17}
\]

Therefore, \( \delta \psi_\mu = 0 \) requires that the transformation parameter \( \epsilon \) is independent of \( x^\mu \).

Substituting eqs. (11), (13), (16) into eq. (11) the supertransformation of the Rarita-Schwinger field for \( i = 4, \cdots, 9 \) becomes

\[
\left( \frac{1 + i \tau^+}{1 - i \tau^-} \right)^{-\frac{1}{4}} \kappa \delta \psi_i = \left( \partial_i + \frac{1}{4} i e^\phi \partial_i \chi \right) \epsilon + \left( \frac{1}{4} \omega_{ij} \Gamma^r \Gamma_{ij} + \frac{1}{8} \partial_j \phi \tilde{\Gamma}^r \right) \epsilon
\]

\[
- \frac{i}{8} x^j f^i f^j \tilde{F}_{0123r} \left( hh' \right)^{-1} \Gamma^0123 \tilde{\Gamma}^r \tilde{\Gamma}^r \epsilon
\]

\[
- \frac{1}{64} e^{2\phi} f \left( P_r \right)^{-1} \frac{x^j}{r} \left( \tilde{\Gamma}_i j - 3 \delta_i j \right)
\]

\[
\times \left[ 2 \left( hh' \right)^{-1} \Gamma^0123 \mathcal{G}_{01r} \mathcal{G}_{23r} + h'^{-2} \mathcal{G}_{01r}^2 - h^{-2} \mathcal{G}_{23r}^2 \right] \epsilon. \tag{18}
\]
There are four kinds of gamma matrix structures on the right-hand side: $\Gamma_{ij}^{0123}$, $\delta_j^i$, and $\delta_j^i \Gamma_{0123}$. The terms with the first two structures are shown to vanish. The remaining terms are

$$
\kappa \delta \psi_i = \partial_i \tilde{\epsilon} + \frac{i}{4} \sin \theta \sinh (h h')^{1/2} f^{-2} \partial_i f \tilde{\epsilon} + \frac{3}{16} \left( h \cos^2 \theta + h' \cosh^2 \theta' \right) f^{-1} \partial_i f \tilde{\epsilon} + \frac{i}{4} \cos \theta \cosh \theta' (h h')^{1/2} f^{-1} \partial_i \Gamma_{0123} \tilde{\epsilon}.
$$

(19)

The functions on the right-hand side can be written as total derivatives

$$
\frac{i}{4} \sin \theta \sinh \theta' (h h')^{1/2} f^{-2} \partial_i f = -\frac{1}{4} \partial_i \log \left( \frac{h^{1/2} \sinh \theta' - ih^{1/2} \sin \theta}{h^{1/2} \sinh \theta' + ih^{1/2} \sin \theta} \right),
$$

$$
\frac{3}{16} \left( h \cos^2 \theta + h' \cosh^2 \theta' \right) f^{-1} \partial_i f = -\frac{3}{16} \partial_i \log (h h' f^{-2}),
$$

$$
\frac{i}{4} \cos \theta \cosh \theta' (h h')^{1/2} f^{-1} \partial_i f = \frac{i}{4} \partial_i \log \left( \frac{h^{1/2} \cosh \theta' + h^{1/2} \cos \theta}{h^{1/2} \cosh \theta' - h^{1/2} \cos \theta} \right).
$$

(20)

It is easy to see that the condition $\delta \psi_i = 0$ determines the transformation parameter as

$$
\epsilon = (h h' f^{-2})^{1/16} \left( \frac{1 + i \tau^*}{1 - i \tau} \right)^{1/4} \times \left[ \frac{h^{1/2} \sinh \theta' - ih^{1/2} \sin \theta}{h^{1/2} \sinh \theta' + ih^{1/2} \sin \theta} \right]^{1/4} \left[ \frac{h^{1/2} \cosh \theta' - ih^{1/2} \cos \theta \Gamma_{0123}}{h^{1/2} \cosh \theta' + ih^{1/2} \cos \theta \Gamma_{0123}} \right]^{1/4} \tilde{\epsilon}_0,
$$

(21)

where $\tilde{\epsilon}_0$ is an arbitrary constant spinor. Substituting eq. (21) into the condition (12) we find

$$
\tilde{\epsilon}_0^* = \Gamma^{01} \tilde{\epsilon}_0.
$$

(22)

Eq. (21) with a constant $\tilde{\epsilon}_0$ satisfying the condition (22) is our final result on the parameter of unbroken supersymmetry. From eq. (22) half of the 32 supersymmetries are preserved. Thus, we have 16 supersymmetries in four dimensions for the solution with two non-vanishing components of the 2-form fields. Taking $\theta' = 0$ it goes back to the solution for one non-vanishing component, and further taking the decoupling limit $r \to 0$ with some parameters kept constant, it becomes AdS$_5 \times$ S$^5$ in the near horizon limit [3]. These particular cases also have 16 supersymmetries in four dimensions.
This result should be compared with the one in the case of vanishing $B_{MN}$ ($\theta = \theta' = 0$). In this case the $\delta \lambda = 0$ condition is automatically satisfied and one does not obtain eq. (11), which relates $\epsilon^*$ to $\epsilon$. The unbroken supersymmetries are 16 $x^\mu$-independent $\epsilon$ satisfying $i\Gamma^{0123}\epsilon = \epsilon$ and 16 $x^\mu$-dependent $\epsilon$ satisfying $i\Gamma^{0123}\epsilon = -\epsilon$. The former 16 correspond to Poincaré supersymmetry in the four-dimensional viewpoint, while the latter 16 correspond to conformal supersymmetry.

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