OBSERVATIONAL CONSTRAINTS ON MODELS OF INFLATION FROM THE DENSITY PERTURBATION AND GRAVITINO PRODUCTION

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Present data require a spectral index \( n \sim > 0.95 \) at something like 1-\( \sigma \) level. If this lower bound survives it will constrain ‘new’ and ‘modular’ inflation models, while raising it to 1.00 would rule out all of these models plus many others. After inflation, gravitinos are created by the oscillating field until the ‘intermediate’ epoch when the Hubble parameter falls below the gravitino mass, or reheating, whichever is earlier. In a wide range of parameter space, these gravitinos are more abundant than those from thermal collisions, leading to stronger cosmological constraints.

1 Introduction

Inflation is supposed to do two quite different jobs. Starting at the Planck scale, inflation should protect our patch of the universe against collapse, and against invasion by the presumably hostile region around it. Then, much later, when the rate of expansion is at least five orders of magnitude below the Planck scale, inflation is supposed to generate the specific initial conditions, that are required for the subsequent Hot Big Bang if it is to produce the observed Universe. The second job is done during the last 70 (or fewer) \( e \)-folds of inflation. Only that era is directly accessible to observation, and it is the focus of this article. The initial conditions include the following

- An extremely homogeneous and isotropic Universe
- A spatially flat Universe
- A clean Universe: no relics which would spoil nucleosynthesis, overclose the Universe, or otherwise contradict observation.
- A primordial curvature perturbation, whose spectrum is rather flat on cosmological scales.
Inflation sets the first and second conditions in a completely straightforward fashion. The same is true of the fourth condition, provided that the inflation is of the slow-roll variety. The third condition, though, might be problematic because the slow-roll inflation which generates the curvature perturbation may also generate light relics with gravitational-strength interactions, such as moduli with spin 0, and the gravitino with spin 3/2. To get rid of these one might need a separate bout of late inflation, lasting only a few e-folds. According to present thinking, the late inflation would not be of the slow-roll variety, but rather what is called thermal inflation.

This article is in two parts. In the first part, I focus on the primordial density perturbation, and in particular on the spectral index which specifies the scale-dependence of its spectrum. The spectral index is a potentially very powerful discriminator between different inflation models. Already, the constraint on the spectral index from a best fit to relevant data is on the verge of ruling out hitherto popular models.

In the second part of the article, I discuss the production of gravitinos after slow-roll inflation. On the basis of the rather complete formalism recently presented by Kallosh et al., it has been shown recently that the conjecture of late-time gravitino creation is likely to be correct: gravitinos are created at the ‘intermediate epoch’ when the Hubble parameter falls below the gravitino mass, or at reheating, whichever is earlier. This bout of late-time creation usually swamps the bout of creation just after inflation, leading to stronger cosmological constraints.
2 Inflation and the spectral index of the primordial curvature perturbation

Let us begin by recalling the history of the Universe, as summarized in Table 1. The curvature perturbation is generated when cosmological scales leave the horizon during inflation. Until these scales re-enter the horizon, long after inflation, it is time-independent (frozen in); this is the object that I am calling the primordial curvature perturbation. The freezing-in of the curvature perturbation on super-horizon scales is a direct consequence of the lack of causal interactions on such scales, under the sole assumption of energy conservation, and independently of whether Einstein gravity is valid. This is extremely fortunate, since we know essentially nothing the Universe while cosmological scales are outside the horizon.

The spatial Fourier components of the primordial curvature perturbation are uncorrelated (Gaussian perturbation), which means that its stochastic properties are completely determined by its spectrum \( P_\mathcal{R}(k) \), defined essentially as the mean-square value of the spatial Fourier component with comoving wavenumber \( k \). The spectral index

\[
n(k) \equiv 1 + \frac{\text{d} \log P_\mathcal{R}}{\text{d} \log k}
\]

defines the shape of the spectrum.

A special case, predicted by most inflation models, is that of a practically scale-invariant \( n \), giving \( P_\mathcal{R} \propto k^{(n-1)} \). The most special case, predicted only by rather special models of inflation, is that of a spectral index practically indistinguishable from 1, giving a practically scale-invariant \( P_\mathcal{R} \).

By the time that cosmological scales re-enter the horizon, long after nucleosynthesis, we know the content of the Universe: there are photons, three types of neutrino with (probably) negligible mass, the baryon-photon fluid, the (non-baryonic) dark matter, and the cosmological constant. The primordial curvature perturbation is associated with perturbations in the densities of each of these components, which all vanish on a common spatial slicing (an adiabatic density perturbation). It is also associated with anisotropies in the momentum distributions. Using well-understood coupled equations, encapsulated say in the CMBfast package, the perturbations and anisotropies can be evolved forward to the present time, if we have a well-defined cosmological model. Here we will make the simplest assumption, namely the \( \Lambda \)CDM cosmology; the Universe is spatially flat, and the non-baryonic cold dark matter is cold (CDM). Flatness is the naive prediction of inflation, and there is no definite evidence against CDM.
I would like to report the result of a recent fit of the parameters of the LCDM model. The data set consisted of the following.

- The normalization \((2/5)P_{R}^{1/2} = 1.94 \times 10^{-5}\) from COBE data on the cmb anisotropy.
- Boomerang and Maxima data at the first and second peaks of the cmb anisotropy.
- Hubble parameter \(h = 0.65 \pm 0.075\), total density \(\Omega_0 = 0.35 \pm 0.075\), baryon density \(\Omega_B h^2 = 0.019 \pm 0.002\).
- Slope of galaxy correlation functions \(\tilde{\Gamma} = 0.23 \pm 0.035\)
- RMS matter density contrast \(\tilde{\sigma}_8 = 0.56 \pm 0.059\) in sphere of radius \(8h^{-1}\) Mpc.

The epoch of reionization was calculated, assuming that a fraction \(f \gtrsim 10^{-4}\) has collapsed.

The result (for \(f \simeq 10^{-2}\)) is

\[ n = 0.99 \pm 0.05 \]  

This is higher than that of Kinney et al. \(\text{[11]}\) (\(n = 0.93 \pm 0.05\)) and of Tegmark et al. \(\text{[12]}\) (\(n = 0.92 \pm 0.04\)). Probably, this is because the former do not include \(\tilde{\sigma}_8\) or \(\tilde{\Gamma}\), while the latter do not include \(\tilde{\sigma}_8\), and have also a lower \(\tilde{\Gamma}\). Also, both have reionization redshift \(z_R \simeq 0\). We shall see that the tighter lower bound on \(n\) implied by our analysis is significant, in the context of some models of inflation. (These are the only two analyses so far which include most of the relevant data, including the crucial nucleosynthesis constraint. A recent analysis \(\text{[13]}\) omitting the latter gives \(n = 1.03 \pm 0.08\).)

### 3 Comparison with models of slow-roll inflation

The near scale-independence of the primordial curvature perturbation presumably requires slow-roll inflation, in which the potential \(V\) satisfies flatness conditions \(M_P |V'/V| \ll 1\) and \(M_P^2 |V''/V| \ll 1\). (We do not consider the possibility of a break in the spectrum, associated with temporary failure of slow-roll; see for instance \(\text{[14]}\)) Assuming a single-component inflaton and Einstein gravity, the prediction depends mainly on the inflaton potential \(V(\phi)\), and the number of e-folds \(N_{\text{COBE}}\) of slow-roll inflation after the scales explored by COBE leave the horizon. (It depends also on the inflaton field value \(\phi_{\text{end}}\).)
when slow-roll ends, but in an interesting class of models this dependence is very weak.) The prediction of slow-roll inflation is

$$\frac{4}{25} P_R(k) = \frac{1}{75\pi^2 M_P^2} \frac{V'^3}{V'}$$

$$\frac{n(k) - 1}{2} = M_P^2 \frac{V''}{V} - 3M_P^2 \left(\frac{V'}{V}\right)^2$$

The right hand side is to be evaluated at the epoch of horizon exit $k = aH$, given by

$$N(k) \equiv \ln(k_{\text{end}}/k) = M_P^2 \int_{\phi_{\text{end}}}^{\phi} V(r) \, dr$$

$$N(\text{COBE}) = 60 - \ln \frac{10^{10} \text{ GeV}}{V^{1/4}} - \frac{1}{3} \ln \frac{V^{1/4}}{T_R} - N_0$$

The number $N_0$ (non-negative in any reasonable cosmology) parameterizes our ignorance about the history of the Universe between the end of slow-roll inflation and nucleosynthesis. It is zero in standard cosmology, but one bout of thermal inflation could generate $N_0 \sim 10$, and two or more bouts are quite feasible.

### 3.1 Models of slow-roll inflation

The easiest way of satisfying the flatness conditions is to have field values $\phi \gg M_P$; then the flatness conditions are satisfied by $V = V_0 f(\phi/M_P)$, where $f$ is any function whose value and derivatives are of order 1. In a non-hybrid model, $f$ should becomes steep so that inflation ends, but that still leaves a lot of freedom. The simplest choice is a monomial $V \propto \phi^2$ or $\phi^4$ (usually called chaotic inflation), but in the large-field regime monomials have no special significance. The reason is that all of the coefficients in a power series for $V$ are expected to have coefficients of order 1 in Planck units.

In principle, string theory presumably determines these coefficients. This idea does yield some proposals for the potential of special fields such as moduli or fields corresponding to the distance between D-branes. In the former case one might have inflation with the potential in the last row of Table 2. With this possible exception, it seems that if Nature has chosen to inflate at large field values, there is at present no theoretical guidance about the form of the potential.

For this reason, models of inflation based on current theoretical ideas should invoke $\phi \lesssim M_P$ and preferably $\phi \ll M_P$. Then it is justified to
focus on the renormalizable terms of the potential (quartic and lower). (One or two non-renormalizable terms might be invoked for special purposes.) By inflating along a flat direction of global supersymmetry, the flatness conditions are marginally satisfied. To have them well-satisfied, without fine-tuning, one can invoke an approximate global symmetry $\phi \rightarrow \phi + \text{const}$ (shift symmetry). With any such symmetry, the potential is completely flat in the limit of exact symmetry, and (provided that the potential does not vanish in this limit) the approximate flatness required for inflation can be ascribed to the approximate symmetry.

The difference between the large- and small field cases is neatly illustrated by a proposal reported at this meeting. It seeks to justify the potential $V \propto \phi^2$ at $\phi \gg M_P$ by invoking a shift symmetry, but to achieve this a particular symmetry-breaking term is invoked. If instead all symmetry-breaking terms were allowed, with coefficients of order 1 in Planck units, the potential would be given by the generic power-series expansion mentioned earlier. While the shift symmetry justifies the flatness of the potential, it does not suggest any particular form for it because we are in the large-field regime.

Returning to the small-field regime, field theory with non-renormalizable terms essentially ignored allows only a few different types of term for the variation of $V$. With the reasonable assumption that one such term dominates over the relevant range of $\phi$, and with the restriction $n < 1$, we arrive at essentially the models displayed in Table 2. Details of these models, with extensive references and possible complications, are given in [1]. One of these complications is the possibility, considered by several authors, that two terms need to be kept over the relevant range of $\phi$. While this can happen, it is clear that the dominance of one term is the generic situation in the sense that it will hold over most of the potential’s parameter space.

When the COBE normalization is imposed on the prediction, the small-field requirement can generally be satisfied with physically reasonable values of the parameters. The only significant exceptions are the logarithmic potential with $c \sim 1$ (as in D-term inflation), and the quadratic potential in the last line (more below on the latter), which both require $\phi \sim M_P$.

Given the restriction on $\phi$, the flatness conditions require that $V_0$ dominates the potential in all of the models, leading to simple expressions for $\epsilon$ and $\eta$. The contribution of gravitational waves is negligibly small in all of them, and the formula for $n$ is well approximated by

$$n - 1 = 2\eta. \tag{3}$$

There are models giving $n - 1$ both positive and negative, but in the former case an observational value for $n$ does not tell us much. For $n < 1$, in contrast,
Table 2. Some field-theory models of inflation predicting a spectral index \( n < 1 \).

| model      | potential                        | spectral index n value of N |
|------------|----------------------------------|----------------------------|
|            |                                   |                            |
| 'mutated'  | \( V_0(1 + c \ln \phi) \)       | 1 - \frac{1}{3} N \   | 0.98 0.95 |
| 'new'      | \( V_0(1 - \phi^{-2}) \)       | 1 - \frac{2}{3N} \     | 0.97 0.93 |
| 'new'      | \( V_0(1 - \phi^3) \)          | 1 - \frac{3}{N} \     | 0.94 0.85 |
| 'modular...' | \( V_0 - \frac{1}{2}m^2\phi^2 \) | 1 - \frac{m^2 M^2}{V_0} | 0.92 0.80 |

\( n \) is a good discriminator between models. Some predictions are listed in Table 2. Except in the last row, the prediction depends on \( N \) and is therefore scale-dependent. However, since \( n \) is constrained to be close to 1, the scale-dependence is negligible over the cosmological range \( \Delta N \sim 4 \), and accordingly one may set \( N = \tilde{N}_{\text{COBE}} \). In the ‘new’ inflation models, Eq. (2) gives a non-trivial lower bound on \( N \), which would almost exclude the \( p = 3 \) model if this 1-\( \sigma \) bound were taken seriously.

Another case of interest is the potential \( V = V_0 - \frac{1}{2}m^2\phi^2 + \cdots \). More or less independently of the additional terms which stabilize the potential, the vev of \( \phi \) is \( \langle \phi \rangle \sim \sqrt{2V_0/m^2} = [2/(1 - n)]^{1/2} M_P \). Depending on the nature of \( \phi \), this kind of inflation has been termed ‘natural’, ‘topological’ and ‘modular’ (see for instance [15] for a recent espousal of modular inflation). In all cases the model is regarded as implausible if \( \langle \phi \rangle \) is much bigger than \( M_P \), which means that it is viable only if \( n \) is not too close to 1. Our 2-\( \sigma \) bound \( n > 0.9 \) implies \( \langle \phi \rangle > 4.5 M_P \), which may perhaps be regarded as already disfavoring these models.

4 Running-mass models of inflation

So far we focussed on models giving a practically scale-independent spectral index. This seems to be a generic prediction of inflation models based on spontaneously broken (global) supersymmetry. As Stewart pointed out some years ago, the opposite is the case for models based on softly broken supersymmetry [19]. In such models, the inflaton mass runs with scale, and in the linear log approximation the potential is

\[
V = V_0 - \frac{1}{2} M_P^2 c \left( \ln \frac{\phi}{\phi_*} - \frac{1}{2} \right) \phi^2
\]
This leads to

\[ \frac{n(k) - 1}{2} = se^{\Delta N(k) - c} \]

\[ \Delta N \equiv \ln(k/k_{\text{COBE}}) \] (5)

If \( c \) is a gauge coupling, its expected magnitude is

\[ |c| \sim 10^{-2} \text{ to } 10^{-1} \] (6)

With \( c \) in the upper part of this range, the spectral index can change very significantly over the range \( \Delta N \sim 4 \) or so which corresponds to cosmological scales. (The other parameter \( s \) controls end of inflation, and to avoid severe fine-tuning it should satisfy \( \phi_{\text{end}} \simeq \phi_\star \).)

The fit mentioned earlier determines the region of \( c \) and \( s \) allowed by observation. A gauge coupling \( c \sim 0.1 \) for the inflaton is allowed, giving potentially observable scale-dependence of the spectral index.

5 Gravitino creation from the vacuum

There are strong cosmological constraints on the abundance of the gravitino, over most of the expected mass range. The light, practically stable gravitino typically predicted by gauge-mediated models of supersymmetry breaking must not overclose the Universe, while the heavier gravitino of gravity-mediated models must not interfere with nucleosynthesis. Only the very heavy gravitino predicted by anomaly-mediated supersymmetry breaking seems to be free from cosmological constraints.

It has long been known that gravitinos are efficiently produced by thermal collisions after reheating, and that to make these thermal gravitinos cosmologically safe usually requires a low reheat temperature and/or sufficient late-time entropy production. More recently, it has been noticed that gravitinos may be produced even more efficiently between the end of inflation and reheating, through the oscillation of the field that was responsible for the inflationary energy density. These gravitinos are created from the vacuum, through the amplification of the vacuum fluctuation.

The abundance of gravitinos created from the vacuum is determined by the evolution equation of the relevant mode function (the function multiplying the creation operator). There are separate mode functions for helicity 1/2 and 3/2, as seen by a comoving observer in the expanding Universe. The evolution of the helicity 3/2 mode function is essentially the same as for a spin 1/2 particle, whose effective mass is the field-dependent gravitino mass \( m_{3/2}(\phi) \)
appearing in the Lagrangian \( \mathcal{L} \). (We shall denote its vacuum value by simply \( m_{3/2} \).) This means that the creation of helicity 3/2 gravitinos from the vacuum takes place just after inflation, with number density \( n \sim 10^{-2}m_{3/2}^2(\phi) \) just after creation. Barring an unforeseen cancellation, the supergravity expression for the potential requires that in the early Universe \( |m_{3/2}(\phi)| \lesssim H \). As a result creation of helicity 3/2 gravitinos from the vacuum is insignificant compared with gravitino production from thermal collisions.

The evolution of the helicity 1/2 mode function is far more complicated. It was given first \(^7\), \(^8\) in the case that only a single chiral superfield is relevant. This one-field case was at first investigated \(^7\), \(^8\) in the approximation of unbroken supersymmetry in the vacuum. Under the reasonable approximation that the oscillating field has a quadratic potential derived from global supersymmetry, it was found that the creation of the helicity 1/2 gravitino is the same as the creation of the inflatino. This may be traced to the fact that in this approximation there is gravitino-goldstino equivalence, the inflatino being the goldstino of spontaneously broken global supersymmetry. The number density is now \( n \sim 10^{-2}M^3 \) where \( M \) is the mass of the oscillating field. This is bigger than the abundance of helicity 3/2 gravitinos, since \( M > H \) is required for the field to oscillate.

The problem with the approximation of unbroken supersymmetry in the vacuum is that it makes the gravitino massless in the vacuum, leaving the inflatino as a physical particle. For this reason, the one-field model was next studied \(^6\) under the assumption that the oscillating field also breaks supersymmetry in the vacuum, making it presumably a modulus of string theory. The result is now very different; unless reheating intervenes, gravitino creation continues until the ‘intermediate’ epoch, defined as the epoch at which the energy density is of order

\[
M_S^4 \equiv 3M_P^2m_{3/2}^2
\]  

(7)

(corresponding to Hubble parameter \( H = m_{3/2} \). (It was assumed that inflation ends before the intermediate epoch, as is the case in a wide class of inflation models.)

The number density of gravitinos, just after creation ends, is again \( n \sim 10^{-2}M^3 \) (we discount for the moment the case that the energy density of created gravitinos becomes significant). If \( M \) is bigger than \( m_{3/2} \), this late-time gravitino creation is more efficient than creation just after the end of inflation. However, because the oscillating field is now required to break supersymmetry in the vacuum, \( M \) cannot be many orders of magnitude bigger than \( m_{3/2} \). As a result, it turns out that even late-time gravitino creation cannot be as efficient as thermal gravitino production. This one-field case is
therefore of only academic interest.

Recently, a formalism has been given which describes the helicity 1/2 gravitino in the presence of any number of chiral and gauge supermultiplets. Using this formalism, it has recently been confirmed that late-time gravitino creation occurs in the generic case, leading to a gravitino abundance at least as big as the one found in the one-field model. I briefly explain how this comes about.

The equation for the evolution of the helicity 1/2 mode function $\theta$ is

$$0 = \left( \partial_0 \partial_0 + k^2 + B^\dagger B + 2B_1 \partial_0 + \dot{B}' - ik\gamma_3\gamma_0 A' \right) \theta + (2B_1 - am_{3/2}(\phi)) \left( \partial_0 + \dot{B} - ik\gamma_3\gamma_0 A \right) \theta - \frac{4ak^2}{\alpha} \Xi$$

(8)

where a prime and $\partial_0$ both denote differentiation with respect to conformal time, $d/d\tau \equiv d/dt$. In this equation, $\dot{A} = A_1 + \gamma_0 A_2$ and $\dot{B} = B_1 + \gamma_0 B_2$, where

$$A_1 \equiv \frac{p - 3M_p^2m_{3/2}^2(\phi)}{\rho + 3M_p^2m_{3/2}^2(\phi)}$$

(9)

$$A_2 \equiv \frac{2M_p^2m_{3/2}(\phi)}{\rho + 3M_p^2m_{3/2}^2(\phi)}$$

(10)

$$B_1 \equiv \frac{3a}{2} \left( -HA_1 + m_{3/2}(\phi)A_2 \right)$$

(11)

$$B_2 \equiv -\frac{a}{2} \left[ 3HA_2 + (1 + 3A_1) m_{3/2}(\phi) \right]$$

(12)

An over-dot denotes $d/dt$, $\rho$ is the energy density, and $p$ is the pressure. The energy density and pressure appear because they determine the Einstein tensor of the Universe, which appears because we are evolving the gravitino mode function in curved spacetime. At least in the cases studied so far, the oscillation of $p$ caused by the oscillating field is the dominant cause of gravitino creation.

The last term involves

$$\alpha = \rho + 3M_p^2m_{3/2}^2(\phi)$$

(13)

and $\Xi$ which is a linear combination of fermion fields, orthogonal to the combination that is eaten by the gravitino to acquire mass. The coefficient of each fermion field is a function of the scalar fields to which it couples.
The equation holds for each momentum $k/a$, where $a$ is the scale factor of the Universe. Flat spacetime field theory holds during an ‘initial’ era and a ‘final’ era. Up to slowly-varying pre-factors, the initial condition is $\theta = \exp(-ikx_0)$, corresponding to the vacuum, and the final occupation number $|\beta|^2$ is read off from (while the gravitino remains relativistic) from $\theta = \alpha \exp(ikx_0) + \beta \exp(-ikx_0)$. There is negligible creation in some adiabatic regime $k \gtrsim k_{\text{max}}$, in which $\theta = \exp(-ikx_0)$ holds at all times.

If the only relevant fields form a chiral supermultiplet, $\Xi$ vanishes. This is the one-field case mentioned earlier, and it gives $k_{\text{max}} \sim a_{\text{int}} M$, where $M$ is the mass of the oscillating field and the subscript denotes the intermediate epoch. If the relevant fields form two chiral supermultiplets, $\Xi$ can be expressed in terms of $\theta$; this case was considered in detail in [5], and shown to lead to the same estimate for $k_{\text{max}}$. In the general case, $\Xi$ is an independent quantity, and the evolution of $\theta$ and the spin 1/2 fields is given by a system of coupled equations. Without solving the system, the fact that $\Xi$ is an independent quantity means that, barring accidental cancellations, a lower bound on the gravitino abundance will be obtained by setting $\Xi = 0$. This leads to the estimate

$$k_{\text{max}} \sim a_{\text{crea}} M$$

where the subscript denotes the epoch when gravitino creation ends; it is the intermediate epoch or the epoch of reheating, whichever is earlier.

Because they are fermions, the abundance of created gravitinos can be estimated in terms of $k_{\text{max}}$, and the occupation number $|\beta|^2$ for that wavenumber,

$$n \sim \frac{1}{2\pi^2} \int_0^{k_{\text{max}}} a^{-3} |\beta_k|^2 dk \sim 10^{-2} |\beta|^2 (k_{\text{max}}/a)^3$$

The corresponding energy density is $\rho \sim (k_{\text{max}}/a)n$, and it cannot be bigger than the total. This leads to the estimate

$$n \sim \min\{10^{-2} (k_{\text{max}}/a_{\text{crea}})^3, \rho_{\text{crea}} (a_{\text{crea}}/k_{\text{max}})^3\} (a_{\text{crea}}/a)^3.$$  

In the case of gravity-mediated supersymmetry breaking, nucleosynthesis requires

$$n/s \lesssim 10^{-13}$$

where $s$ is the entropy density. Thermally produced gravitinos are subject to this constraint, and keeping only them it requires

$$\gamma T_R \lesssim 10^9 \text{GeV}$$
Here, $T_R$ is the reheat temperature, defined as the temperature just after all or most of the energy in the oscillation is converted into radiation, and $\gamma^{-1} \geq 1$ is the entropy increase, if any, after reheating.

Now consider instead the gravitinos created from the vacuum. We can work out $n/s$ at nucleosynthesis, remembering that $n \propto a^{-3}$ is proportional to $\rho$ until reheating, and to $s\gamma$ thereafter. Considering only the non-relativistic regime, we find that gravitinos created from the vacuum lead to the following nucleosynthesis constraint if $T_R \lesssim M_S$

$$\gamma T_R \lesssim \max \{10^{-11} M_S^4/M^3, 10^{-13} M\}$$

If instead $M_S \lesssim T_R \lesssim M$, we find again the second of the previous constraints,

$$\gamma T_R \lesssim 10^{-13} M$$

Finally, if $T_R \gtrsim M_S$ and $T_R \gtrsim M$ we find

$$T_R \gtrsim 10^4 \gamma^{1/3} M$$

The requirements that gravitinos do not spoil nucleosynthesis are Eq. (18), plus the appropriate one of Eqs. (19), (20) and (21). Before considering these constraints, we have to consider the possible entropy increase $\gamma^{-1}$. The most efficient mechanism of entropy release is thermal inflation (or some other type of inflation occurring after the intermediate epoch). One bout of thermal inflation gives huge entropy release, roughly $\gamma^{-1} \sim 10^{15}$, and there could be more than one. If there is no thermal inflation, significant entropy release can come only from an era of matter domination by an unstable particle. (For simplicity we exclude the case of two or more such eras.) Then

$$\gamma^{-1} \sim T_{eq}/T_{\text{decay}}$$

where $T_{\text{decay}}$ is the temperature just after the particle decay (i.e., the final reheat temperature) and $T_{eq}$ is the temperature just before the era of matter domination. In this case, $\gamma T_R \sim T_{\text{decay}}(T_R/T_{eq})$. The era of matter domination must end before nucleosynthesis, corresponding to $T_{\text{decay}} \gtrsim 10$ MeV, which implies

$$\gamma T_R \gtrsim 10 \text{ MeV}.$$
If $M \lesssim 10^7$ GeV, the forbidden region is the same as for thermally produced gravitinos. Otherwise, gravitinos created from the vacuum rule rule out a significant portion of the parameter space, beyond what is ruled out by thermally produced gravitinos.

6  Where are we going with inflation models?

By building and testing models of the early Universe, we obtain a unique window on the nature of the fundamental interactions. This is especially true of inflation model-building, because the crucially important curvature perturbation, once generated, is frozen in until well after nucleosynthesis.

In a few years we shall know $n(k)$ with accuracy $\pm 0.01$. This is the only observable function relating to physics far beyond the Standard Model! The measurement of $n(k)$, plus other constraints like gravitino abundance, will rule out most of the presently existing inflation models. Depending on whether or not $n$ is significantly different from 1, and on how much our understanding of string-derived field theory progresses, one model may have been selected as the best candidate.

The next frontier will be to discover how the inflaton sector talks to the Standard Model sector. There must indeed be communication because the inflaton field must decay into SM radiation (‘reheating’). As a result, given continued progress (which might be the rub), top-down inflation model-builders will eventually meet up with bottom-up extenders of the Standard Model!

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