Mathematical Evaluation on the Control of Mining-Induced Ground Subsidence in Thick Loose Strata

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ABSTRACT: Coal mining under thick loose strata in North China leads to ground subsidence, which is a natural result of hydro-mechanical coupling (fluid flow coupled with solid deformation). Therefore, the land surrounding the mining areas is greatly damaged. In this study, the combined weight (CW) method and the fuzzy matter-element analysis (FMEA) method were used to analyze and evaluate the control effect of subsiding land. Overall, 20 sets of geological samples were collected from this area. The influencing factors and the corresponding weights for the control effect of subsiding land were comprehensively analyzed, and an FMEA model was built to predict and verify the results. The results showed that (1) the two evaluation indicators with the most significant impact on land reclamation were the degree of integration and the economic and social benefits and (2) among the 20 sets of samples selected, the predicted conclusions of 17 sets were consistent with the actual engineering situations, which led to an accuracy of 85%. In other words, the CW-FMEA model showed good reliability for evaluating the control effect of subsiding land, which can provide scientific references for control and quality evaluations of land subsidence due to coal mining.

1. INTRODUCTION

Underground mining activities often lead to strata collapse and water flow, causing surface subsidence.1−4 For coal mining under thick loose strata in the North China Plain, damage to the land is more severe.5 However, the North China Plain is known to produce a high grain yield (Figure 1a). This region is characterized by high underground phreatic water levels, thick loose strata, and large areas of ponding, resulting in extensive damage across wide areas of cultivated land (Figure 1b).6 In recent years, as investment by the Chinese government into measures to protect the ecological environment has gradually increased, various coal mine groups have been in active and close contact with relevant scholars and experts, who have subsequently developed methods aimed at controlling subsiding lands that are appropriate to their own local conditions.7 These control methods include shallow-leveling, deep-digging and shallow-filling, deposition-filling and bulge-cutting, and preset-filling methods. These methods have played an important role in controlling subsiding land processes and have achieved satisfactory results. They are conducive to the rapid restoration of the ecological environment in lands that have undergone subsidence, the establishment of a sound mechanism for controlling subsiding land due to coal mining, and sustainable socioeconomic development. (Figure 1c).8

Guo et al.9 collected values for various influencing factors using ArcGIS software and combined it with a fuzzy analytical hierarchy process (AHP) to comprehensively evaluate the geological environment of the Baoding coal mine. Xiao et al.10 established a comprehensive ecological risk assessment model for coal mines using ecological risk assessment theory and proposed appropriate risk prevention measures with application to the Dongtan coal mine in Shandong province. Bi et al.11 conducted field research in the Daliuta coal mine with different inoculation technologies using artificial ecological engineering theory and determined the optimal plants to use for soil restoration of land that has undergone subsidence. Wang et al.12 observed that drainage is difficult in farmlands where land subsided due to coal mining and also noted the low utilization of solid waste produced by coal mining; they performed permeability and compressive strength experiments using concrete produced from different proportions of waste coal gangue and fly ash, which developed concrete materials that could be applied to farmland drainage ditches. Furthermore, Hu et al.13 used unpolluted Yellow River sediments for the land

Received: September 8, 2021
Accepted: November 10, 2021
Published: December 8, 2021
reclamation of cultivated land in subsiding land and, by studying corn growth in greenhouse experiments, determined that the optimal soil cover thickness was 70 cm. Yu et al.\(^{14}\) collected the necessary monitoring data of the surface subsidence in the Hanjiawan coal mine in the Shendong mining area, which was the preparatory work for the experiment. In addition, the comprehensive benefits of land reclamation were studied in this coal mine using the fuzzy comprehensive evaluation method. After the corresponding control measures were proposed, the environmental and economic benefit indexes of the site were increased significantly.

However, most of the research focuses on land reclamation evaluation and ecological restoration. The existing evaluation models also lack important social factors such as the local urbanization level and government investment, which are indexes that cannot be ignored. When calculating the weight value of indexes, the pure qualitative analysis methods were also interfered by human factors. Therefore, many new weight methods should be introduced to make the results more reasonable.

In addition, many new control measures have appeared in the area of surface subsidence in the thick loose strata of the North China Plain. For example, areas with fewer collapsed pits are restored to cultivated land, areas with ponding areas more than 1.5 m deep are used for fishery breeding, and the remaining areas are transformed into wetland parks or woodland.\(^{15,16}\) Thus, land that has subsided is gradually being developed according to an ecological model that integrates agriculture, livestock breeding, tourism, and sightseeing. By doing this, both economic development and the happiness index of residents can be effectively improved, and the “green lung” function of local areas can also be enhanced. The control practices used vary and have their own characteristics, which are of important practical significance for guiding work to control the land reclamation of subsiding land.\(^{17,18}\)

The evaluation indexes generated during the control of subsidence land have not been used at the same time, and so are the social factors in this case.

In our work, the specific objective of this paper is to build a new comprehensive evaluation system, including the main factors related to the economic, environmental, and social conditions. Among them, the urbanization level and government investment, which are helpful to improve the lives of nearby residents, are the first important factors added to the evaluation system. The calculation process of this paper is divided into the following steps: First, from the perspective of both subjective and objective analyses and incorporated knowledge from land reclamation experts, several key factors influencing the land subsidence control around coal mines in the North China Plain were selected. Then, the comprehensive weights of factors that influence land subsidence control were calculated using the combined weight (CW) method. These values were then used to develop an overall planning strategy. Finally, the fuzzy matter-element analysis (FMEA) theory was used to identify correlations among the various influencing factors. The main contribution of this paper is that it can provide some measures or directions for the control of other coal mine subsidence areas.

### 2. METHODOLOGY

#### 2.1. Entropy Weight (EW) Method

The EW method\(^{19}\) is an objective analysis method used to calculate the weights of various evaluation indicators. If the degree of variation is larger and the information entropy is smaller, the weight of the corresponding indicator is larger. The method is based on actual values of the factors influencing the evaluation model, so it is able to produce more reliable results.

The single entropy weight method has little effect on the evaluation of objective things. The application of the improved entropy weight method is increasingly used in research.
Nowadays, the entropy weight TOPSIS method,20 entropy weight and fuzzy C-mean weight method,21 entropy weight and fuzzy matter-element analysis method,22 and AHP-entropy weight method23 are widely used in many aspects of life. These applications can help solve complex problems, such as accident hazards, underground mining risks, and public facility management.

Suppose $m > 1$ evaluation samples and $n$ evaluation indicators are used to establish the evaluation matrix. The matrix is shown as follows

$$
B_{m \times n} = \begin{bmatrix}
    x_{11} & x_{12} & \cdots & x_{1n} \\
    x_{21} & x_{22} & \cdots & x_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{m1} & x_{m2} & \cdots & x_{mn}
\end{bmatrix}
$$

(1)

(1) Standardization of indicators

In a multiple indicator evaluation system, it is difficult to calculate the results due to inconsistencies in the units, dimensions, and other factors of each indicator. Thus, to ensure the reliability of the evaluation results, all evaluation indicator parameters should be standardized.

Equation 2 shows the standardization of a positively correlated (beneficial) indicator for column $j$

$$
x_j = \frac{x_j - \min_{1 \leq i \leq m} x_{ij}}{\max_{1 \leq i \leq m} x_{ij} - \min_{1 \leq i \leq m} x_{ij}} \geq 0
$$

and eq 3 shows the standardization of a negatively correlated (cost) indicator for column $j$

$$
x_j = \frac{\max_{1 \leq i \leq m} x_{ij} - x_{ij}}{\max_{1 \leq i \leq m} x_{ij} - \min_{1 \leq i \leq m} x_{ij}} \geq 0
$$

(2)

(2) Weight assignment of each indicator in the matrix according to the following equation

$$
Y_j = \frac{x_j}{\sum_{i=1}^{m} x_{ij}}
$$

Thus, $Y_j$ represents the weight of the $j$th evaluation indicator of the $i$th evaluation sample in the matrix ($i$ is from 1 to $m$ and $j$ is from 1 to $n$). Obviously, $Y_j$ satisfies $0 \leq Y_j \leq 1$ and $\sum_{i=1}^{m} Y_j = 1$ for any $j$.

(3) Calculation of the entropy value of each indicator using the following equation ($k$ is from 1 to $n$)

$$
Z_k = -\frac{1}{\log m} \sum_{i=1}^{m} Y_{ik} \log Y_{ik}
$$

From a basic convex analysis (Lagrange multiplier method for affine constrained maximization), the range of $Z_k$ is between 0 and 1. Note in the actual calculation, the $Y_{ik}$ might be zero or one, and the corresponding $Y_{ik} \log Y_{ik}$ simply vanishes.

(4) Calculation of the entropy weight of each indicator using the following equation ($k$ is from 1 to $n$)

$$
\xi_k = \frac{1 - Z_k}{n - \sum_{k=1}^{n} Z_k} \geq 0
$$

It is easy to see that $\sum_{k=1}^{n} \xi_k = 1$, and the smaller the entropy value $Z_k$, the greater the entropy weight $\xi_k$, which means the greater the importance of this evaluation indicator. In this work, the entropy weight vector given by eq 6 is denoted as $\eta \in \mathbb{R}^n$.

2.2. Analytical Hierarchy Process (AHP) Method. The AHP method is a subjective analysis method, proposed by Saaty,24 to quantify qualitative factors. It is a method that converts nonquantitative factors into numerical values, followed by a series of pairwise comparisons on the importance of each influencing factor, which are used to calculate the weight of each factor.25,26

The basic procedures are as follows:

(1) A hierarchical structure system is established, which can generally be divided into three layers, namely, the target layer, the intermediate classification layer, and the factor layer (Figure 2).

(2) A judgment matrix (suppose the size is $d$ by $d$, $d \geq 2$ could be smaller than $n$ due to the hierarchy) is established through mutual comparisons among all of the factors. The element $(i, j)$ in this matrix represents the level of importance of factor $i$ to factor $j$. The larger the number, the higher the level that factor $i$ is more important than the
factor $j$ (i.e., escalating differences), and the element $(j, i)$ is the reciprocal of the element $(i, j)$. In solving practical problems, the importance analysis table proposed by Saaty had many limitations. The results calculated were often quite different from those calculated by the entropy weight method or gray relational degree analysis, so it cannot be used to solve problems effectively. After several conversations with other experts, the following proposed Table 1 will be more convincing.

### Table 1. Degree of Importance between Two Factors

| comparison of the importance of two factors | corresponding quantized value |
|-------------------------------------------|------------------------------|
| two factors are of equal importance       | 1                            |
| one factor is a little bit more important than the other | 2                            |
| the importance of one factor is much larger compared with that of the other | 3                            |
| the importance of one factor is extremely greater than that of the other | 4                            |
| intermediate values of the above comparisons such as $3/2, 5/2, 7/2, 9/2$ | 5                            |

(3) The weight of each factor is then calculated as the non-negative standardized (by $\pm \| \|_1$) eigenvector corresponding to the largest eigenvalue of the previous judgment matrix.

(4) Consistency test of each (single) judgment matrix. The AHP method requires that the judgment matrix has general consistency, which will ensure a basically reasonable calculated result. The calculation formulae are shown as follows:

\[
R_C = \frac{I_C}{I_R} \quad (7)
\]

\[
I_C = \frac{\lambda_{\text{max}} - d}{d - 1} \quad (8)
\]

where $R_C$ is the consistency ratio, $I_C$ is the consistency index, $I_R$ is the random consistency index whose value is determined according to Table 2 (empirical values provided by Saaty), $\lambda_{\text{max}}$ is the largest eigenvalue, and $d$ is the size of the judgment matrix. The consistency test requires $R_C$ should be smaller than 0.1, otherwise, the judgment matrix should be revised.

(5) Total consistency ratio test. Due to the hierarchical architecture, the total consistency ratio $R_C^{\text{total}}$ is also required to satisfy the certain criterion. The $R_C^{\text{total}}$ is calculated as follows:

\[
R_C^{\text{total}} = \frac{\sum_{i=1}^{P} w_{i} I_{C,i}}{\sum_{i=1}^{N} w_{i} I_{R,i}} \quad (9)
\]

where $p$ is the number of factors in the intermediate classification layer and $w_i$ is the weight of factor $i$ in the intermediate classification layer to the target layer. The total consistency ratio should be smaller than 0.1, otherwise, the judgment matrix should also be revised.

(6) Calculation of the final integrated weights of $n$ factors (evaluation indicators), which are known as $\eta_{n} \in \mathbb{R}^n$.

### 2.3. Combined Weight (CW) Method

The CW method was designed to comprehensively analyze the subjective (AHP) and objective (EW) weight information that influenced the subsiding land control, which enabled a more comprehensive and scientific calculation of the weights of the evaluation indicators.

In the most general case, suppose $N$ weight vectors are obtained using $N$ different methods, which are denoted as $\eta_1, \eta_2, ..., \eta_N$. They form a basis weight vector set, and a combined weight vector $\eta^*$ is simply a linear combination of this set $\eta^* = \sum_{k=1}^{N} \alpha_k \eta_k$. The objective is to minimize the deviation between each of the $N$ weight vectors and $\eta^*$. The previous "game theory" (GT) used in other research papers constructed $N$ minimization objectives, and for each objective $\| \eta^* - \eta_k \|^2$, they only take derivative with respect to the $\alpha_k$ and set it to zero, which is not consistent with the necessary condition of a stationary point of a multivariate function. Furthermore, the variables $\alpha_1$ to $\alpha_N$ are not arbitrary but constrained, thus it is not mathematically correct to do an unconstrained minimization problem (with no specific scalar objective) first and normalize the result later. In this work, a complete convex optimization problem is proposed to resolve the abovementioned issue. The problem is defined as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{N} \beta_k \| \eta^* - \eta_k \|^2 \\
\text{s.t.} & \quad \sum_{k=1}^{N} \alpha_k = 1 \\
& \quad \alpha_1 \geq 0, \alpha_2 \geq 0, ..., \alpha_N \geq 0
\end{align*}
\]

(10)

where $\beta_1$ to $\beta_N$ are the prescribed (they were assigned before solving the optimization problem) scalars for deviations, and users could tune them to get satisfying results. In general, if one method is dominant or is preferred by users, the corresponding $\beta$ should be larger.

### 2.4. Fuzzy Matter-Element Analysis (FMEA) Method

The theoretical bases of this method are matter-element theory and extension set theory. Using the qualitative analysis of matter-elements and combining with the quantitative calculation of the correlation functions, each indicator that influences an evaluation object is converted into a compatible problem so that a comprehensive evaluation model, which is more in line with objective reality is obtained. The calculation process of the FMEA is described as follows:

(1) Determination of the classical domain. The FMEA model is generally composed of three parts, which are the elements of matter, character, and quantity. If there are $L$ final classes (evaluation levels) and $n$ evaluation indicators, the following $n$-dimensional matter-element of the $k$th class will be defined, as shown in the following equation:

### Table 2. Random Consistency Index Value, $I_R$

| $d$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 |
|-----|----|----|----|----|----|----|----|----|----|----|----|
| $I_R$ | 0  | 0  | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 |


3. MODEL PREPARATION

3.1. Investigation Process of the Evaluation Model. A CW method was used to optimize the evaluation indicator weight vectors obtained by the EW method and the AHP method to provide a more scientific and rational evaluation of the control effect of subsiding land around coal mines. These weight vectors were then introduced into the FMEA model for the comprehensive evaluation of the control effect of each subsiding land.

3.2. Determination of the Evaluation Indicators. By studying the scientific subsiding control methods around the world and the experiences obtained during the subsiding land control at some coal mines, it was concluded that the control processes of subsiding land are influenced by a variety of factors. These factors can be broadly divided into geographical factors (yi) and social factors (yj).

3.3. Determination of the Evaluation Classes. To date, there have been few studies on land subsidence control in coal mines. Existing studies have mainly focused on the control of soil on cultivated land or water quality in areas of ponding where subsidence has occurred. A comprehensive evaluation model has been developed to analyze the overall situation of land control that has subsided due to coal mining. To rectify this situation, the results of relevant studies and opinions from land reclamation experts were analyzed. This enabled us to partition efforts to restore subsiding land due to coal mining into five classes, as follows: class I: restoration of subsiding land at a coal mine has not yet started or has only just started. Class II: restoration of subsiding land at a coal mine is limited, and most evaluation indicators are poor (i.e., only land reclamation or simple aquaculture has been carried out). Class III: restoration of subsiding land at a coal mine is poor, and most evaluation indicators are average (i.e., land subsidence has been controlled, but reclaimed land, aquaculture production, forested land, and wetlands have not yet been restored). Class IV: restoration of subsiding land at a coal mine is good, and most evaluation indicators are good (i.e., land subsidence has been controlled, but reclaimed land, aquaculture production, forested land, and wetlands have been restored). Class V: restoration of subsiding land at a coal mine is excellent, and most evaluation indicators are excellent (i.e., land subsidence has been controlled, and all aspects of land, aquaculture production, forested land, and wetlands have been restored).

Table 3. Evaluation of the Preresearch Degree of Subsiding Land

| preresearch level | poor | less poor | ordinary | good | excellent |
|-------------------|------|----------|---------|------|-----------|
| assignment        | 0–20 | 20–40    | 40–60   | 60–80| 80–100    |

Table 4. Evaluation of the Degree of Integration of Subsiding Land

| degree of integration effect level | poor | less poor | ordinary | good | excellent |
|-----------------------------------|------|----------|---------|------|-----------|
| assignment                        | 0–10 | 10–20    | 20–30   | 30–40| 40–50     |

Table 5. Evaluation of Economic and Social Benefits of Subsiding Land

| economic and social benefit level | poor | less poor | ordinary | good | excellent |
|----------------------------------|------|----------|---------|------|-----------|
| assignment                       | 0–10 | 10–20    | 20–30   | 30–40| 40–50     |

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wetlands are independent of one another). Class IV: restoration of subsiding land at a coal mine is relatively good, and most evaluation indicators are good (i.e., land subsidence has been comprehensively controlled; there are complementary effects between reclaimed land, aquaculture, forested land, and wetlands, and a sustainable pattern of development of the social ecosystem has been initiated). Class V: restoration of subsiding land at a coal mine is very good; there is dense vegetation and a variety of bird species in the area, and the economic and social benefits show strong sustainability.41

To better analyze the factors influencing the effects of land subsidence control, parameter values were hierarchically structured to establish these evaluation indicator classification intervals for the control effect of subsiding land around coal mines (Table 6). These effects were classified by quantitatively analyzing the data to provide scientific guidance and suggestions for coal mines where there is poor land subsidence control and ensuring sustainable land control.

3.4. Selection of the Sample Data. Coal mining activities have caused extensive damage to the ground surface; the area of cultivated land has been continuously reduced, soil fertility has decreased, and ponding is a serious problem in areas where subsiding land has occurred. The damage caused by the settlement of land that has undergone subsidence at different coal mines varies according to the coal mining method used, different geological conditions, and inconsistent phreatic water levels. In our work, 20 coal mines from Shandong Province, Anhui Province, and Liaoning Province were selected, as shown in Table 7 and Figure 3. Based on these data, the FMEA model is constructed.

### Table 6. Evaluation Indicator Classification Intervals for the Control Effects of Subsiding Lands

| evaluation indicator | indicator type       | I (poor) | II (less poor) | III (ordinary) | IV (good) | V (excellent) | extended range of I–V |
|----------------------|----------------------|----------|----------------|----------------|-----------|--------------|-----------------------|
| x₁                   | positively correlated| 2–3      | 3–4            | 4–5            | 5–6       | 6–7          | 0–7                   |
| x₂                   | positively correlated| 0–10     | 10–20          | 20–30          | 30–40     | 40–80        | 0–80                  |
| x₃                   | positively correlated| 0–6      | 6–12           | 12–18          | 18–24     | 24–30        | 0–30                  |
| x₄                   | positively correlated| 0–8      | 8–16           | 16–24          | 24–32     | 32–40        | 0–40                  |
| x₅                   | positively correlated| 0–8      | 8–16           | 16–24          | 24–32     | 32–40        | 0–40                  |
| x₆                   | positively correlated| 0–20     | 20–40          | 40–60          | 60–80     | 80–100       | 0–100                 |
| x₇                   | positively correlated| 0–10     | 10–20          | 20–30          | 30–40     | 40–50        | 0–50                  |
| x₈                   | positively correlated| 0–10     | 10–20          | 20–30          | 30–40     | 40–50        | 0–50                  |
| x₉                   | positively correlated| 0–10     | 10–20          | 20–30          | 30–40     | 40–50        | 0–50                  |

### Table 7. Sample Data

| shafts               | maximum ponding depth (m) | degree of cultivation | degree of forestation | degree of aquaculture | degree of wetlands | preresearch degree | degree of urban planning | degree of integration | economic and social benefits | evaluation level of control effect |
|----------------------|---------------------------|-----------------------|-----------------------|-----------------------|-------------------|-------------------|-------------------------|-------------------------|-----------------------------|----------------------------------|
| Henghe coal mine     | 5.8                       | 9                     | 8                     | 4                     | 11                | 15                | 5                       | 5                       | 5                           | I                                |
| Xinglonghuang coal mine (early stage) | 5.7                       | 8                     | 12                    | 5                     | 10                | 20                | 5                       | 5                       | 5                           | I                                |
| Yangzhuang coal mine (early stage) | 4.1                       | 10                    | 7                     | 3                     | 10                | 20                | 5                       | 5                       | 5                           | I                                |
| Wugou coal mine      | 4                         | 5                     | 10                    | 2                     | 13                | 20                | 0                       | 5                       | 5                           | I                                |
| Longgu coal mine     | 6                         | 25                    | 9                     | 10                    | 12                | 10                | 10                      | 8                       | 10                          | II                               |
| Beixulou coal mine   | 5.3                       | 7                     | 8                     | 12                    | 21                | 25                | 15                      | 15                      | 15                          | II                               |
| Baodian coal mine    | 5.9                       | 17                    | 5                     | 10                    | 14                | 30                | 5                       | 10                      | 20                          | II                               |
| Xinglonghuang coal mine | 6                         | 15                    | 16                    | 13                    | 15                | 35                | 15                      | 18                      | 15                          | II                               |
| Guotun coal mine     | 6                         | 15                    | 10                    | 26                    | 10                | 35                | 10                      | 15                      | 15                          | II                               |
| Daxing coal mine of Tiemei group | 2.3                       | 75                    | 10                    | 10                    | 5                 | 30                | 5                       | 8                       | 15                          | II                               |
| Chali coal mine      | 5.3                       | 19                    | 6                     | 17                    | 12                | 30                | 5                       | 18                      | 20                          | III                              |
| Xuchang coal mine    | 5.5                       | 35                    | 20                    | 18                    | 20                | 45                | 25                      | 18                      | 20                          | III                              |
| Nantun coal mine     | 6                         | 25                    | 11                    | 30                    | 14                | 60                | 15                      | 30                      | 25                          | III                              |
| Jiangzhuang coal mine | 6.2                       | 27                    | 8                     | 35                    | 10                | 50                | 20                      | 30                      | 25                          | III                              |
| Beixulou coal mine   | 5.5                       | 30                    | 20                    | 29                    | 16                | 50                | 15                      | 30                      | 30                          | III                              |
| Binhu coal mine      | 4.1                       | 16                    | 20                    | 31                    | 28                | 50                | 15                      | 35                      | 25                          | III                              |
| Yangzhuang coal mine | 4.3                       | 24                    | 19                    | 27                    | 30                | 60                | 40                      | 40                      | 35                          | IV                               |
| Henghe coal mine     | 6                         | 20                    | 20                    | 25                    | 35                | 70                | 45                      | 35                      | 40                          | IV                               |
| Jinqiao coal mine    | 3.6                       | 26                    | 15                    | 35                    | 24                | 65                | 40                      | 35                      | 35                          | IV                               |
| Baodian coal mine    | 6.1                       | 25                    | 21                    | 20                    | 33                | 80                | 45                      | 45                      | 40                          | V                                |

https://doi.org/10.1021/acsomega.1c04970
4. RESULTS AND DISCUSSION

4.1. EW Method Calculation Results. The EW method was used to calculate the weights of the main indicators that influence the control effect of subsiding land. Evaluation indicator data during the process of land subsidence control were collected for typical coal mines. In this study, all evaluation indicators were positively correlated with restoration progress. The calculation results are given in Table 8.

| evaluation indicator                  | entropy value | entropy weight  | importance ranking |
|--------------------------------------|---------------|----------------|-------------------|
| maximum ponding depth (m)            | 0.9688        | 0.0371         | 9                 |
| degree of cultivation                | 0.8906        | 0.1302         | 4                 |
| degree of forestation                | 0.9114        | 0.1055         | 6                 |
| degree of aquaculture                | 0.9111        | 0.1059         | 5                 |
| degree of wetlands                   | 0.9206        | 0.0945         | 7                 |
| preresearch degree                   | 0.9224        | 0.0924         | 8                 |
| degree of urban planning             | 0.8898        | 0.1312         | 3                 |
| degree of integration                | 0.8594        | 0.1674         | 1                 |
| economic and social benefits         | 0.8860        | 0.1358         | 2                 |

As shown in Table 8, the degree of integration has the largest weight, followed by economic and social benefits, degree of urban planning, degree of cultivation, degree of aquaculture, degree of forestation, and degree of wetlands, with the degree of preresearch and the maximum ponding depth. The total weight of the first seven largest evaluation indicators was 0.8705, which plays a decisive role in evaluating the effect of land subsidence control. The weight of the degree of integration is the largest, reflecting that sustainable ecological construction integrating aquaculture and tourism and other industries has positive effects on land subsidence control. The economic and social benefits of land subsidence control are important indicators for us to intuitively evaluate land subsidence control and also a fundamental guarantee for the sustainable development of land subsidence control.

4.2. AHP Method Calculation Results. The AHP method was used to conduct, from a subjective perspective, a qualitative analysis of each evaluation indicator that influenced the land subsidence control. In addition, the degrees of importance of various evaluation indicators were calculated, ranked, and combined with the scores of factors influencing the land subsidence control provided by multiple experts and scholars in the field of land reclamation. The judgment matrices and their corresponding eigenvectors as well as the integrated weight...
vector are summarized in Tables 9–12. Consistency requirements were also met.

| y_i | y_j | y_k | eigenvector |
|-----|-----|-----|-------------|
| 1   | 1   | 0.5 |             |
| 1   | 1   | 0.5 |             |

Table 10. Establishment of the Grade II Judgment Matrix 1 ($R_C = 0.0022$)

| y_j   | x_1  | x_2  | x_3  | x_4  | x_5  | eigenvector |
|-------|------|------|------|------|------|-------------|
| x_1   | 1    | 0.4  | 1/2  | 2/5  | 1/2  | 0.0996      |
| x_2   | 2.5  | 1    | 3/2  | 6/5  | 3/2  | 0.2778      |
| x_3   | 2    | 2/3  | 1    | 4/5  | 1    | 0.1918      |
| x_4   | 5/2  | 5/6  | 5/4  | 1    | 1    | 0.2298      |
| x_5   | 2    | 2/3  | 1    | 1    | 1    | 0.2001      |

Table 11. Establishment of the Grade II Judgment Matrix 2 ($R_C = 0.0023$)

| y_j   | x_6  | x_7  | x_8  | x_9  | eigenvector |
|-------|------|------|------|------|-------------|
| x_6   | 1    | 2/3  | 1/2  | 0.4  | 0.1443      |
| x_7   | 3/2  | 1    | 3/4  | 3/4  | 0.2285      |
| x_8   | 2    | 4/3  | 1    | 1    | 0.3046      |
| x_9   | 2.5  | 4/3  | 1    | 1    | 0.3226      |

Table 12. Integrated AHP Weight Calculation for Each Evaluation Indicator

| evaluation indicator | y_1 (0.5) | y_2 (0.5) | integrated weight $z_f$ |
|----------------------|-----------|-----------|-------------------------|
| x_1                  | 0.0996    | 0         | 0.0498                  |
| x_2                  | 0.2778    | 0         | 0.1389                  |
| x_3                  | 0.1918    | 0         | 0.0959                  |
| x_4                  | 0.2298    | 0         | 0.1149                  |
| x_5                  | 0.2010    | 0         | 0.1005                  |
| x_6                  | 0         | 0.1443    | 0.0721                  |
| x_7                  | 0         | 0.2285    | 0.1142                  |
| x_8                  | 0         | 0.3046    | 0.1523                  |
| x_9                  | 0         | 0.3226    | 0.1613                  |

4.3. CW Vector. In this paper, $\beta_1 = 1$ and $\beta_2 = 1.5$. The subsequently solved optimal values are $\alpha_1 = 0.4$ and $\alpha_2 = 0.6$. The final combined weight vector $\eta^n$ is given in the following equation

$$\eta^n = \alpha_1 R_1 + \alpha_2 R_2 = 0.4 \begin{pmatrix} 0.0371 \\ 0.1302 \\ 0.1055 \\ 0.1059 \\ 0.0945 \\ 0.0924 \\ 0.1312 \\ 0.1674 \\ 0.1358 \end{pmatrix} + 0.6 \begin{pmatrix} 0.0498 \\ 0.1389 \\ 0.0959 \\ 0.1149 \\ 0.1005 \\ 0.0721 \\ 0.1142 \\ 0.1523 \\ 0.1613 \end{pmatrix} = \begin{pmatrix} 0.0447 \\ 0.1354 \\ 0.0997 \\ 0.1113 \\ 0.0981 \\ 0.0802 \\ 0.1210 \\ 0.1584 \\ 0.1511 \end{pmatrix}$$

From the value of $\eta^n$, the weight of the evaluation indicator $x_9$ exceeds the weight of the other evaluation indicators, which is the most important factor. The weight of evaluation indicator $x_7$ is the smallest in the entire model, which can be ignored in future evaluation models. In addition, the total weight of the first seven largest evaluation indicators was 0.8750, which can be used effectively to analyze and evaluate the control effect of subsiding land due to coal mining. Following the effective combination of the EW method and the AHP method, the actual situation in the control process of subsiding land can be accurately interpreted, providing a reliable reference for land subsidence control in other coal mines.

4.4. Discussion. Finally, the comprehensive degrees of correlation were calculated, and the class of each type of subsiding land after land reclamation was determined, as shown in Table 13. The model accuracy was 85%, which indicates the rationality of our CW-FMEA model used in this study. The predicted category also gives insights into the restoration and reclamation measures or directions. For example, it reveals that the ecological restoration $(x_9, x_8, x_7)$ is also very useful in addition to the agricultural reclamation $(x_2)$, by comparing the Daxing coal mine and the Beixulou coal mine. In practice, ecological restoration includes various measures such as planting trees, constructing wetland parks, and carrying out aquaculture. In addition, it is necessary to develop wetland tourism industries such as ecological tourism, leisure vacation, fishing folk culture, wetland science education, and urban cultural theme parks in the coal mine subsidence area. These measures are conducive to the development of local cities and produce more obvious economic, social, and environmental benefits.

However, some coal mines subsiding land of class I and class II have not been restored significantly, with control work carried out only in a few regions. The area of restored cultivated land and aquaculture was small, and the layout was poor. Wetland parks and forestry construction were not included in the plan. The annual income level of residents has not increased a lot and the local living environment has not been improved. All of these factors will inhibit the social urbanization and the development of local villages and towns. These coal mines should, based on their own local damage conditions caused by subsiding land, refer to more highly appraised control methods of land subsidence control.

For coal mines given the wrong predicted category, the comprehensive correlation degrees $K_{ii}$ and $K_{ij}$ are pretty close to each other for the Beixulou coal mine and the Binhu coal mine. This suggests that the true ecological restoration significance should lie between class III and class IV, and there might be other influencing factors that dominate the decision in the “actual category” of Table 13.

5. CONCLUSIONS

In this study, the CW and FMEA methods were used to comprehensively analyze the main factors influencing the control effect of subsiding land in coal mines, and a model to evaluate the class of the control effect of subsiding land was established. In the CW method, a new minimization objective was developed that fixes the inconsistency in the previous game theory used in other mining engineering projects. The CW method was used to reasonably allocate the weights of the evaluation indicators obtained using the EW and AHP methods, which not only avoided the objective impact of a single factor but also reduced the impact of subjective factors. Through the CW method, the order of importance of influencing factors was determined and the most important factor was the degree of integration. The FMEA model established in this study was characterized by good reliability and can provide scientific references for evaluating the control effect of subsiding land around coal mines. Furthermore, all of the data and calculation processes are shown clearly in tables and mathematical formulae,
Table 13. List of Evaluation Results of the Control Effect of Each Coal Mine Subsiding Land\(^a,b\)

| Coal mine (stage)          | \(K_i\)  | \(K_{II}\) | \(K_{III}\) | \(K_{IV}\) | \(K_v\) | combined weight | actual category |
|---------------------------|---------|-----------|-----------|-----------|--------|----------------|---------------|
| Henghe coal mine (early)  | 0.2322  | -0.2613   | -0.6128   | -0.7061   | -0.7815| I               | I             |
| Xinglonghuang coal mine   | 0.2043  | -0.2849   | -0.5732   | -0.6767   | -0.7648| I               | I             |
| Yangzhuan coal mine       | 0.2159  | -0.2451   | -0.5950   | -0.7287   | -0.7951| I               | I             |
| Wugou coal mine           | 0.1833  | -0.3569   | -0.6272   | -0.7477   | -0.8091| I               | I             |
| Longgu coal mine (early)  | -0.0805 | 0.0026    | -0.3371   | -0.5508   | -0.6519| II              | II            |
| Beixulou coal mine (early)| -0.1937 | 0.2437    | -0.2567   | -0.4769   | -0.6210| II              | II            |
| Baodian coal mine (early) | -0.1132 | 0.0276    | -0.3426   | -0.5289   | -0.6426| II              | II            |
| Xinglonghuang coal mine   | -0.3119 | 0.2575    | -0.2143   | -0.4008   | -0.3544| II              | II            |
| Guotun coal mine          | -0.2763 | 0.2241    | -0.2714   | -0.4103   | -0.5370| II              | II            |
| Daxing coal mine of Tiemei| -0.0882 | -0.0868   | -0.5103   | -0.6622   | -0.6053| II              | II            |
| Chali coal mine           | -0.1909 | 0.0483    | -0.2006   | -0.4389   | -0.5911| II**            | III           |
| Xuchang coal mine         | -0.3922 | -0.1313   | 0.0969    | -0.0748   | -0.3852| III             | III           |
| Nantun coal mine          | -0.4277 | -0.1204   | 0.0283    | -0.1601   | -0.3904| III             | III           |
| Jiangzhuan coal mine      | -0.4246 | -0.1824   | -0.0171   | -0.2558   | -0.3273| III             | III           |
| Beixulou coal mine        | -0.4611 | -0.2168   | -0.0527   | -0.0092   | -0.3553| IV**            | III           |
| Binhu coal mine           | -0.4647 | -0.1675   | -0.0666   | 0.0025    | -0.3651| IV**            | III           |
| Yangzhuan coal mine       | -0.5961 | -0.4594   | -0.1758   | 0.1221    | -0.2144| IV              | IV            |
| Henghe coal mine          | -0.6471 | -0.5144   | -0.3394   | 0.0240    | -0.0940| IV              | IV            |
| Jinxiao coal mine         | -0.5680 | -0.4138   | -0.1251   | 0.0853    | -0.1957| IV              | IV            |
| Baodian coal mine         | -0.6841 | -0.5788   | -0.3066   | -0.1477   | 0.0390 | V               | V             |

\(^a\)Note 1: Numbers in the first five columns of this table (known as the comprehensive degree of correlation) are for the combined weight vector.

\(^b\)Note 2: \(***\) indicates the wrong case of discrimination.

which can be regarded as a benchmark example for other engineering projects that adopt weight modeling and FMEA.

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**Notes**

The authors declare no competing financial interest.

All data, models, or code generated or used during the study are available from the corresponding author by request.

**ACKNOWLEDGMENTS**

This work received support from the National Natural Science Foundation of China (51774199). The authors also thank all of the coal mining enterprises for providing geological subsiding land data for this study.

**LIST OF SYMBOLS**

\(B_{j,k}\) the evaluation matrix
\(Y_{jk}\) weight of the \(j\)th evaluation indicator of the \(i\)th evaluation sample in the matrix
\(Z_i\) the entropy value of each indicator
\(s_i\) the entropy weight of each indicator
\(\eta_i\) the entropy weight vector
\(R_C\) consistency ratio
\(I_C\) the consistency index
\(I_R\) the random consistency index
\(\lambda_{\text{max}}\) the largest eigenvalue
\(d\) the size of the judgment matrix
\(R_{\text{cons}}\) the total consistency ratio
\(p\) number of factors in the intermediate classification layer
\(\eta_s\) the weight vector of AHP method
\(k\) the \(k\)th class of evaluation level
\(d_j\) the \(j\)th evaluation indicator
\(R_{\text{class}}\) the classical domain
\(R_{\text{junct}}\) the joint domain
\(\Gamma(x_{ji})\) the correlation function

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