On the heterogeneous rock mass stress condition at the unsteady temperature condition

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Abstract. The unsteady thermoelastic problem of the rock mass stress state in the vicinity of a spherical cavity in the temperature field presence is considered. The temperature field around the cavity, partially filled with coolant, is calculated by the explicit difference scheme. Such tasks arise during the creation and operation of the underground cavities for the various products’ long-term storage. Moreover, the tendency to self-heating of the stored substances leads to the appearance of high-temperature fields, which in turn can cause the significant temperature stresses’ occurrence. A feature of the work is to take into account the two-dimensional heterogeneity of the massif material, which is due to both the fracture of the rock and the dependence of the deformation characteristics on temperature. In the article, the problem of thermo-elasticity is solved as quasi-stationary. Obtaining analytical solutions in the problems of thermo-elasticity of inhomogeneous bodies is associated with great difficulties. Such solutions are obtained, as a rule, for the case of one-dimensional heterogeneity of an object’s material. The obtained temperature field was used to solve the problem of determining the stress state of the array, and the finite element method was used. The results obtained indicate the need for calculations and design of underground cavities to take into account the real properties of the rock mass materials, as well as their change under the influence of various physical processes.

Introduction
Calculation of stresses in a rock mass containing a spherical cavity with a heated fluid is of great practical importance.
The existing methods for creating underground cavities lead to technological heterogeneity of the rock mass. It is believed that heterogeneity in the rock mass is continuous. The presence of a heated liquid in a spherical cavity partially filling it, as well as taking into account the soil pressure asymmetry to the array surrounding the cavity (Figure 1), lead to a two-dimensional quasi-stationary unconnected problem of thermo-elasticity.

**Materials and methods**

The elastic modulus of the rock mass, presented in the form [1,2]:

\[
E(r) = E_0 e^{-\delta T(r,\theta)} \left[ 1 + (k_1 - 1) \left( \frac{a}{r} \right)^m \right]
\]  

(1)

takes into account how the temperature field and the rock fracturing on the physical and mechanical characteristics of the massif influence \(T(r,\theta,t)\). Here \(\delta, k_1, m\) – are the empirical constants, \(a\) – is the spherical cavity radius, \(E_0\) – is the modulus of elasticity of the rock mass at infinity, \(t\) – denotes time.

The temperature field calculation for any time \(t\) is necessary to calculate the mechanical characteristics of the rock mass, in particular, the elastic modulus according to the formula (1).

We introduce the fill factor of the spherical cavity with a heated liquid \(\eta = l/2a\) (Fig.1).

The temperature field around the cavity at any given time \(t\) determined by the formula from [3,4] is:

\[
T(r,t) = \frac{Q_0}{8\pi \sqrt{\pi} \gamma c \alpha r \sqrt{a t}} e^{-\frac{(r-a)^2}{4a t}}
\]  

(2)

When the spherical cavity is partially filled with hot liquid with an initial temperature \(T_0\), unsteady heat flow to the rock mass leads to the different non-uniform temperature fields for different moments of heating time, which are described by the unsteady heat equation [5]:

\[
\frac{\partial T(r,\theta,t)}{\partial t} = a_t \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2 \tan \theta} \frac{\partial T}{\partial \theta} \right)
\]  

(3)

here \(a_t\) – defines the thermal diffusivity.

The boundary condition on the wetted surface of the spherical cavity follows from the condition of the fluid flowing into the rock mass and changes in the amount of heat in the fluid. It is believed that there is an ideal heat insulator above the surface of the liquid medium in the cavity. This assumption is justified by the fact that, as a rule, the coefficient of the gaseous medium thermal conductivity above
the liquid medium in the cavity is an order of magnitude lower than the thermal conductivity of the rock mass. The temperature of the cavity surface wetted part and the liquid medium $T(t)$ for any moment of time are the same, and the temperature on the wetted part of the surface will be determined by the heat flow in the meridional direction and is calculated as follows.

The boundary condition on the wetted surface of the cavity has the form:

$$\frac{\partial T(r, \theta, t)}{\partial t} = \alpha \frac{\partial T(r, \theta, t)}{\partial r} \bigg|_{r=a}$$

(4)

where $\alpha$ – is the constant determined by the physical characteristics of the liquid medium and rock mass.

On the non-wetted surface, we have the condition for the heat absence:

$$\frac{\partial T(r, \theta, t)}{\partial r} \bigg|_{r=a} = 0$$

(5)

The second boundary condition is formulated on a sufficiently remote external boundary, which is a spherical cavity. As a rule, the radius of this spherical surface is selected as $b \geq 10a$.

On this border:

$$T(b, t) = T_\infty$$

(6)

where $T_\infty$ – is the rock mass temperature at infinity.

The initial condition for $t=0$ has the form:

$$T(r, \theta, t) = T_0$$

at $r=a$ and $\theta_0 \leq \theta \leq \pi$

and $T(r, \theta, t) = T_\infty$ in the rest of the area

(7)

Here $\theta_0$ – defines the angle corresponding to the boundary between the wetted and non-wetted parts of the cavity surface.

Temperature field calculated for time $t=0.5$ years, is used in solving the boundary value problem of thermo-elasticity for a closed axisymmetric area with a variable elastic modulus [6,7].

The axisymmetric region is bounded by two spherical surfaces with radii $r=a$ and $r=b$, which replaces the elastic half-space weakened by a spherical cavity. Distributed load on the external spherical cavity [8] can be calculated:

$$\rho_h(\theta) = -\gamma(H-b\cos \theta) \left( \frac{v}{1-v} \sin^2 \theta + \cos^2 \theta \right)$$

$$q_h(\theta) = \frac{\gamma}{2} (H-b\cos \theta) \frac{1-2v}{1-v} \sin 2\theta$$

(8)

and corresponds to the medium repulse reaction. Here $\gamma$ – is the specific weight of the rock mass, $H$ – is the cavity depth, $v$ – is the Poisson’s ratio, $\rho_h(\theta)$ – is the normal pressure and $q_h(\theta)$ – defines the tangential force along the contour of the outer spherical surface. There are no loads on the inner spherical surface $r=a$.

For an approximate solution of equation (3) under boundary conditions (4), (5), (6) and the initial condition (7), we use the finite difference method in the form of an explicit difference scheme [9].

As a result of this solution, a temperature field was obtained in the vicinity of the spherical cavity after half a year of the liquid medium cooling with the cavity filling factor $\eta=0.5$ (Fig.2).

The geometric dimensions of the cavity and the physical characteristics of the soil were as follows:

$a=25$ m, $b=250$ m, $H=1200$ m – define the cavity depth, $c=0.96$ kJ / kN, $a_1=1.67$ $10^4$ m$^2$/c, $\gamma=250$ kN / m$^3$ – define the soil density.

It is seen that when the liquid cools in the cavity, a part of the rock mass located in the depth is heated.

The temperature field shown in Figure 2 was used to study the soil heterogeneity effect on its stress-strain state.
The stresses in the soil mass around the cavity were calculated by the finite element method [10, 11]. The self-weight of the soil and the reaction of repulse of the medium at the outer boundary were $r=b$ (Fig. 1).

When modeling a closed annular spherical region by finite elements, its axisymmetry is taken into account. The annular finite elements in cross section form an arbitrary quadrangle. The finite elements are isoperimetric [12].

**Discussion and Results**

The results of the stresses calculation at $\eta=0.5$ after half a year, the liquid cooling is shown in Fig. 3. The stresses around the spherical cavity for a uniform ($\delta=0; k_1=1$) and heterogeneous ($\delta=0.01; k_1=0.5$) arrays are shown.

**Figure 3.** Maximum compressive stresses in $t=0.5$ years of array heating at $\eta=0.5$.

1 – homogeneous array; 2 – heterogeneous array

Figure 4 shows stress plots $\sigma$ at three angles $\theta$. 
The calculations were carried out with the following initial data: $E_0 = 2 \cdot 10^4$ MPa, $\nu = 0.23$, $a = 25$ m, $b = 250$ m, $H = 1200$ m, $\gamma = 250$ kN/m$^3$, $\eta = 0.5$, $t = 0.5$ of the year, $k_1 = 0.5$, $\delta = 0.01$.

The solid curves show the results for the inhomogeneous material, and the dotted line shows the results for the homogeneous material. It can be seen that the heterogeneity effect is very significant. If in a homogeneous array the maximum stresses in the warmer zone take place on the cavity contour, then, taking into account the inhomogeneity, the maximum shifts deeper into the array (Figure 4.2 and 4.3), while the circuit stress decreases significantly. Stress reduction is also observed in a less heated zone (at $\theta = 0^\circ$, Figure 4.1).

**Figure 4.** The maximum compressive stresses dependence on the radius at: $t = 0.5$ year and $\eta = 0.5$. 1– homogeneous array; 2 – heterogeneous array.

The various types influence heterogeneity analysis due to fracturing of the massif and the dependence of the elastic modulus on temperature shows that, when these types of heterogeneity are taken into account, the maximum compressive stresses decrease [13,14,15]. When both types of heterogeneity are taken into account, the maximum compressive stresses decrease by more than three times in comparison with a homogeneous rock mass.

**Summary**

The results of the study indicate the need to take into account when designing the underground cavities’ real properties of the rock mass material, as well as their changes under the influence of various physical processes.
Heterogeneity factors are favorable in the sense of the stress state around the spherical cavity. This allows to increase the cavity depth and temperature fields while maintaining the strength and stability of the cavity.

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