Neutrino Mixing and Leptonic CP Violation
from $S_4$ and Generalised CP Symmetries

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We consider a class of models of neutrino mixing with $S_4$ flavour symmetry and generalised CP symmetry, broken to $Z_2$ and $Z_2 \times CP$ residual symmetries in the charged lepton and neutrino sectors, respectively. In this scheme, and up to discrete ambiguities, the neutrino mixing matrix is determined by two angles and one phase. We classify the phenomenologically viable mixing patterns, deriving predictions for the Dirac and Majorana CPV phases and for the effective Majorana mass in neutrino-less double beta decay.

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1 Introduction

The patterns that have emerged from neutrino oscillation data in recent years (see e.g. [1]) offer a potential window into the origins of flavour. Extensions of the Standard Model with non-Abelian discrete flavour symmetries (see e.g. [2, 3]) have been considered extensively in attempts to understand the flavour problem. While the flavour symmetry may determine the neutrino mixing angles and/or the Dirac phase, a generalised CP (gCP) symmetry [4], implemented in a consistent way [5, 6], allows also to constrain also Majorana CP violating (CPV) phases.

In the flavour+gCP approach, a fundamental symmetry described by a group $G_{\text{CP}} = G_f \rtimes H_{\text{CP}}$ is assumed to be realised at some high-energy scale and to be broken at lower energies to residual symmetries $G_e$ and $G_\nu$, in the charged-lepton and neutrino sectors, respectively. Here, $G_f$ is a flavour symmetry group admitting a 3D irreducible representation $\rho$, while $H_{\text{CP}}$ denotes a group of gCP transformations.

The present contribution is based on the work of Ref. [7], in which we take $G_f = S_4$, $G_e = Z_2$ and $G_\nu = Z_2 \times \text{CP}$. After briefly reviewing our approach, we summarise the phenomenological consequences of this simple breaking pattern.

2 Framework

The residual flavour symmetries are associated to the group elements $g_e$ and $g_\nu$. The residual gCP transformation in the neutrino sector is instead described by a matrix $X_\nu$ in flavour space. These residual symmetries constrain the charged-lepton and neutrino mass matrices, $M_e$ and $M_\nu$, which satisfy

$$
\rho(g_e)^\dagger M_e M_e^\dagger \rho(g_e) = M_e M_e^\dagger, \quad \rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu, \quad X_\nu^T M_\nu X_\nu = M_\nu^*.
$$

Additionally, the consistent combination of flavour and gCP symmetries mandates that the matrix $X_\nu$ must satisfy the condition:

$$
X_\nu \rho^* (g_\nu) X_\nu^{-1} = \rho(g_\nu).
$$

Eqs. (1) constrain the form of the unitary rotations diagonalizing the neutrino and charged-lepton mass terms, and therefore shape the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [8] which reads [7]:

$$
U_{\text{PMNS}} = P_e U_{23}(\theta^e, \delta^e) \Omega_{e}^\dagger \Omega_{\nu} R_{23}(\theta^\nu) P_\nu Q_\nu.
$$

Here, $P_{e,\nu}$ are permutation matrices, $Q_\nu = \text{diag}(1, i^{k_1}, i^{k_2})$ with $k_{1,2} = 0, 1$, and

$$
U_{23}(\theta^e, \delta^e) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta^e & \sin \theta^e e^{-i\delta^e} \\
0 & -\sin \theta^e e^{i\delta^e} & \cos \theta^e
\end{pmatrix},
R_{23}(\theta^\nu) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta^\nu & \sin \theta^\nu \\
0 & -\sin \theta^\nu & \cos \theta^\nu
\end{pmatrix}.
$$

*We consider the standard parametrisation of the PMNS, see e.g. [9].
The matrices $\Omega_{e,\nu}$ are fixed by the choice of $G_f$ and of the specific residual symmetries. Thus, apart from discrete ambiguities, the PMNS matrix is determined by three real parameters: 2 angles, $\theta^e, \theta^\nu \in [0, \pi)$, and 1 phase, $\delta^e \in [0, 2\pi)$.

3 Application to $G_f = S_4$

$S_4$ is the non-Abelian symmetric group of permutations of four objects (e.g. the vertices of a tetrahedron). It has 24 elements, admits 5 irreducible representations $^1 1, 1', 2, 3, 3'$, and is conveniently described by the generators $S, T, U$, satisfying $S^2 = T^3 = (ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$.

After identifying redundancies – i.e. some residual symmetries lead to the same PMNS – and excluding phenomenological outliers (degenerate neutrino masses, texture zeros in the PMNS), we find 4 possible forms of the PMNS up to permutations of its rows and columns $^7$:

$$U^A_{\text{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}} e^{-i\pi/6} & \frac{1}{2} e^{i\pi/3} & \frac{1}{2} e^{i\pi/3} \\ * & * & * \\ * & * & * \end{pmatrix}, \quad U^B_{\text{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}} e^{i\pi/3} & * e^{i\pi/3} & * e^{i\pi/3} \\ * & * & * \\ * & * & * \end{pmatrix},$$

$$U^C_{\text{PMNS}} = \begin{pmatrix} \frac{1}{2} e^{i\pi/3} & * e^{-i\pi/6} & * e^{-i\pi/6} \\ * e^{-i\pi/6} & * & * \\ * & * & * \end{pmatrix}, \quad U^D_{\text{PMNS}} = \begin{pmatrix} \frac{1}{2} e^{i5\pi/6} & * & * \\ * & * & * \\ * & * & * \end{pmatrix},$$

where each star represents a different (and often lengthy $^7$) function of $\theta^e, \theta^\nu$, and $\delta^e$. The predicted absolute values and relative phases are instead written explicitly.

Not all permutations are viable if one takes into account the $3\sigma$ ranges of neutrino oscillation parameters obtained in the global analysis of Ref. $^8$. There are in fact 15 surviving viable cases, which we denote A1, A2, B1-B4, C1-C5, D2-D5 $^7$. Four of these (A1, A2, D4 and D5) are however strongly disfavoured by data ($\chi^2_{\text{min}} \gtrsim 15$).

4 Results: Correlations and $0\nu\beta\beta$-decay

From the constraints on the form of the PMNS, correlations between angles and phases follow. For instance, in case C5, the following sum rule for $\cos \delta$ is satisfied:

$$\cos \delta = \frac{1 - 4 \cos^2 \theta_{12} \sin^2 \theta_{23} - 4 \sin^2 \theta_{12} \cos^2 \theta_{23} \sin^2 \theta_{13}}{2 \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13}}.$$  

This case, for which $\chi^2_{\text{min}} = 0.5$, is obtained by permuting the rows and columns of $U^C_{\text{PMNS}}$ so that the fixed-magnitude element is brought to the 3-2 position. Apart from cases B3, B4, and C1, all other cases satisfy a bona fide sum rule for $\cos \delta$.  

$^1$Our conclusions are independent of the choice of 3D representation.  

$^2$ i.e., with $\cos \delta$ given as a function of the $\theta_{ij}$, with no explicit dependence on $\theta^e, \theta^\nu, \delta^e$.  

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For purposes of illustration, the constraints on the plane $(\sin^2 \theta_{23}, \sin^2 \theta_{12})$ for each of the viable cases with $\chi^2_{\text{min}} < 15$ are collected in Figure 1.

Additionally, CPV phases are constrained to lie in specific intervals, and their values are strongly correlated. It is then possible to derive predictions for the neutrinoless double beta ($0\nu\beta\beta$)-decay effective Majorana mass observable $|\langle m \rangle|$. Case C1 in particular provides rather sharp predictions, since not only are mixing angles constrained but also CPV phases are fixed by symmetry to be $\alpha_{21} = k_1 \pi$ and $\alpha_{31} - 2\delta = k_2 \pi$. Predictions for $|\langle m \rangle|$ in this case are shown in Figure 2.

5 Summary and Outlook

We have studied a class of models based on the breaking of $S_4 \rtimes H_{\text{CP}}$ flavour and generalised CP symmetry. We have found strong correlations between CPV phases and correlations between mixing angles and phases, including sum rules for $\cos \delta$ in some of the viable cases. We have additionally derived predictions for the $0\nu\beta\beta$-decay effective Majorana mass. Future data (Daya Bay, JUNO, T2K, T2HK, DUNE) will allow to test and discriminate between different symmetry predictions.
Figure 2: Predictions for the effective Majorana mass in case C1 (see text) as a function of the smallest neutrino mass, $m_{\text{min}}$. Blue and red bands correspond to $k_1 = 0$, while black and purple bands refer to $k_1 = 1$. The KATRIN bound $m_{\text{min}} < 0.2$ eV is a prospective one.

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