Interaction of the magnetic quadrupole moment of a non-relativistic particle with an electric field in the background of screw dislocations with a rotating frame

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Abstract
In this study, we considered a moving particle with a magnetic quadrupole moment in an elastic medium in the presence of a screw dislocation. We assumed a radial electric field in a rotating frame that leads a uniform effective magnetic field perpendicular to the plane of motion. We solved the Schrödinger equation to derive wave and energy eigenvalue functions by employing analytical methods for two interaction configurations: in the absence of potential and in the presence of a static scalar potential. Due to the topological defect in the medium, we observed a shift in the angular momentum quantum number which affects the energy eigenvalues and the wave function of the system.

1. Introduction
In recent decades, the studies on the defects in solid continua considering the theory of general relativity have gained great attention. According to these studies, the deformations of the elastic medium due to a topological defect in continuous media can be related to the differential geometry [1]. For example, Katanaev et al gave a theoretical description of a continuous distribution of deformations such as disclinations and dislocations according to the Riemann-Cartan geometry [2]. Puntigam et al explored the Voltera process and presented a geometrical description of the topological defect via a cosmic string [3]. De Assis et al investigated Berry’s phase in a chiral cosmic string background [4]. Furtado et al examined the curvature effects on the dynamics of the particles in a two-dimensional quantum dot [5]. Bakke et al discussed the neutral particle’s Landau quantization in the presence of the topological defects [6].

The elastic deformation in a solid can occur as a linear topological defect caused by mechanical working [7]. The screw dislocation and the spiral dislocation are called as the models of the linear topological defects. These kinds of topological defects have widely been used in the literature with the intention of studying quantum dynamics in a medium with distortion. For instance, Dantas et al examined the quantum dynamics of a rotating electron/hole in an existing screw dislocation within a confined ring potential energy [8]. Furtado et al considered an external magnetic field and explored the dynamics of the electrons in a medium with the presence of screw dislocation [9]. Da Silva et al considered a non-relativistic particle in an elastic medium and discussed the Aharonov–Bohm-type effects in the background of a distortion of a vertical line into a vertical spiral [10].

Some of the other recent researches that have been carried on can be found in [11–21].

Over the past few years, many researchers examined the interaction between curved spacetime and quantum fields. For example, Bezerra explored the wave functions and the energy spectra of scalar and spinor quantum particles in the spacetime background which is assumed by generated by a chiral cosmic string [22]. Marques et al examined the hydrogen atom’s energy spectra by solving the Dirac equation in the background spacetimes which are assumed to be induced from a cosmic string and global monopole [23]. Fonseca et al considered an
atom with a magnetic quadrupole moment and investigated the rotating effect while the atom is interacting with an electric field radially in a two-dimensional quantum ring potential [24]. In one of two very recent papers, Hassanabadi et al solved the Schrödinger equation analytically for a moving particle in a rotating frame that interacts with a radial external electric field by its own magnetic quadrupole moment [25]. In the other one, Vieira et al considered a moving particle with a magnetic quadrupole moment and investigated the bound state solutions for the electromagnetic field interactions within a uniformly rotating frame by using a semiclassical approach [26]. Furthermore, the rotating effects have been surveyed by researching in many subjects of physics, such as the Sagnac effect [27–29], the relativistic quantum mechanics [30, 31], the quantum rings [8, 32, 33], the quantum spintronics [34, 35], and the quantum Hall effect [36]. It should be noted that rotating effects have been studied in the Dirac [37] and scalar field theories [38, 39]. Also rotating effects in the context of the condensed matter physics, in particular in Bose-Einstein condensation in ultracold dilute atomic gases, have been investigated [40]. For further reading, we remark on some of the other studies on the subject in [41–51].

In this study, we examine the interaction of a radial electric field with the magnetic quadrupole moment of a moving particle system by solving the corresponding Schrödinger equation in the presence of a screw dislocation in an elastic medium. We investigate the effect of rotation on this non-relativistic system by considering a rotating frame with a constant angular velocity since rotating frame effects are giving rise to the geometric quantum phases such as the Aharonov-Crami phase [52, 53] and the Mashhoon effect [54].

This paper is organized as follows. In section 2, we consider a non-relativistic moving particle with a magnetic quadrupole moment in an elastic medium in the presence of screw dislocation and derive the Schrödinger equation to examine quantum dynamics while the moving particle is in a rotating frame and interacts with the external electric field. In section 3, we solve the Schrödinger equation and obtain the energy eigenvalue and wave functions for two configurations: in the absence of potential energy, and in the presence of a static scalar potential energy by using the Nikiforov-Uvarov (NU) method. Finally, in section 4 we conclude the manuscript.

2. Magnetic quadrupole moment of a moving particle in the presence of a screw dislocation

Before describing the dynamic of a non-relativistic scalar particle with a magnetic quadrupole moment that is located in a region that possesses a uniform effective magnetic field and an external electric field, we would like to touch on a screw dislocation in an elastic medium. It is worth noting that dislocations as the topological defects in a lattice can be classified as spiral dislocations and screw dislocations. When a considerable number of bonds are broken between atoms along the line of a crystalline material, so part of that lattice is offset by a bit relative to the other part of the lattice. In fact, it causes distortion in a crystalline, so-called screw dislocation. We employ natural units, $\hbar = c = 1$, and express the line element which is associated with a screw dislocation oriented along the $z$-axis in the spacetime with the kind of topological defects. Indeed, this topological defect that carries torsion (but no curvature) can be described by the following line element [8, 9, 13, 55]:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\rho^2 + (\mathrm{dz} + \beta \mathrm{d}\phi)^2 + \rho^2 \mathrm{d}\phi^2,$$

where $\phi$ and $\rho$ are the azimuthal and radial coordinates, respectively. The ranges of the line coordinate elements are $0 < t < \infty$, $0 < \rho = \sqrt{x^2 + y^2}$, $0 < \phi < 2\pi$, and $-\infty < z < \infty$. The covariant metric tensor $g_{ij}$ associated with the spatial part of the line element equation (1) is given by

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \rho^2 + \beta^2 & \beta \\ 0 & \beta & 1 \end{pmatrix}, \quad i, j = \rho, \phi, z.$$

Here, $\beta$ is a parameter associated with the Burgers vector $b_z$ by $\beta = \frac{b_z}{2\pi}$. It is worth noting that the Burgers vector is an important concept in linear defects in order to describe the character of a dislocation in a crystalline material. Such as, the Burgers vector related to a screw dislocation is contained in the defect region while the Burgers vector related to a spiral dislocation is found to be perpendicular to the defect region [3].

Next, let us present the magnetic multipole expansion since our focus lies on the magnetic quadrupole polarizability tensor. Under an external magnetic field, $\vec{B}$ (whose components are determined by $B_i$), the magnetic potential energy of a localized distribution of moving charged particles or currents in a medium in the rest frame of the particle can be written as

$$U_{\vec{B}} = -\sum_i B_i(0) \mu_i - \frac{1}{6} \sum_{ij} \frac{\partial^2 B_i(0)}{\partial x_j} \mathbf{M}_{ij} - \frac{1}{24} \sum_{ijk} \frac{\partial^2 B_i(0)}{\partial x_j \partial x_k} R_{ijk} + \ldots,$$
where \( \hat{\mu} \) is the total magnetic dipole moment defined by
\[
\hat{\mu} = \frac{1}{2} \sum_{j,k} \epsilon_{jk} \int x_j x_k \, d^3x.
\]
Here, \( \epsilon_{jk} \) is the Levi-Civita tensor, \( x_j \) and \( x_k \) are components of the position, \( \vec{r} \), and current-density distribution vectors, \( \vec{j} \), respectively. In the second term of the right-hand side of equation (3), \( M_{ij} \) is the magnetic quadrupole moment tensor, and it is defined in the form of
\[
M_{ij} = \sum_{k,l} \epsilon_{ikl} \int (3x_l x_k - r^2 \delta_{kl}) x_j \, d^3x,
\]
in which, \( x_j \) is the \( j \)th component of the position vector \( \vec{r} \), such that the magnitude of the \( \vec{r} \) is denoted by \( r \), the \( j \)th component is denoted as the components of the \( \vec{J} \), and the Kronecker delta function is indicated by \( \delta_{kl} \). Meanwhile, the tensor \( M_{ij} \) is traceless, that is, \( \sum_i M_{ij} = 0 \), it is a symmetric tensor as well. As regards the third term of the right-hand side of equation (3), the magnetic octupole moment tensor \( R_{ijk} \) is defined as
\[
R_{ijk} = 6 \sum_{\ell,m} \epsilon_{ikl} \epsilon_{\elljm} \int x_l x_j x_k x_m \, d^3x.
\]
Hereafter, we use the natural units, \( q = c = h = 1 \). We follow the references [49, 50, 56–58], and consider a non-relativistic moving particle that possesses only a magnetic quadrupole moment. Note that, in the frame of the moving particle, the particle experiences a different magnetic field, let us call it by \( \vec{B} \), which can be obtained via the Lorentz transformation as \( \vec{B} = \vec{B} - \vec{v} \times \vec{E} \) (up to \( O(v^2) \)) meanwhile, the velocity vector of a charged particle is given by \( \vec{v} \) terms. Here, \( \vec{E} \) and \( \vec{B} \) are the electric and magnetic fields in the laboratory frame, respectively. Thus, the lagrangian of this system can be written as
\[
L = \frac{1}{2}mv^2 + \sum_{ij} M_{ij} \frac{\partial \vec{B}_j}{\partial x_i},
\]
in which \( m \) is the mass of a neutral particle. Note that the second term of the right-hand side of equation (7) has been supposed as the potential energy of a neutral particle with a magnetic quadrupole moment (as defined in [49, 50, 56–58]). Furthermore, \( M_{ij} \) is a symmetric and traceless tensor which presents the magnetic quadrupole moment tensor \( M_{ij} \) and \( \vec{B} \) is the magnetic field whose components are determined by \( \vec{B}_j \). Now, let us rewrite the lagrangian equation (7), according to \( \vec{B} \), as
\[
L = \frac{1}{2}mv^2 + \vec{M} \cdot \vec{B} + \vec{\nu} \cdot (\vec{M} \times \vec{E}),
\]
in which \( \vec{M} \), according to equations (7) and (8), is a vector whose components are defined by
\[
M_i = \sum_{j} M_{ij} \partial_\nu,
\]
now, let us present the non-null components of the magnetic quadrupole moment tensor \( M_{ij} \) as
\[
M_{xz} = M_{yz} = M,
\]
where \( M \) is a positive constant. Then, we obtain the canonical momentum [59]
\[
\vec{p} = mv^2 + \vec{M} \times \vec{E},
\]
to introduce the Hamiltonian of this system with
\[
H = \frac{1}{2m} |\vec{p} - \vec{M} \times \vec{E}|^2 - \vec{M} \cdot \vec{B},
\]
where \( \vec{p} = -i\hbar \vec{\nabla} \) is the momentum operator. If we compare the above momentum with \( \vec{p} = q\vec{A}^eff \), we conclude that \( \vec{A}^eff = \vec{M} \times \vec{E} \) and \( V^eff = -\vec{M} \cdot \vec{B} \), as in [58, 60]. Note that we do not have any external magnetic field in this configuration; therefore, we get \( V^eff = 0 \).

Thus, we express the Schrödinger equation for a quantum particle with a magnetic quadrupole moment in a rotating frame in the presence of a static scalar potential energy \( V(\rho) \) [8, 25, 26].
\[
\left( \frac{\hbar^2}{2m} - \vec{\Omega} \cdot \vec{L} + V(\rho) \right) \Psi(t, \vec{r}) = i\partial_t \Psi(t, \vec{r}).
\]
Here, \( \vec{L} \) is the angular momentum operator and \( \vec{\Omega} \) is the constant angular velocity which is taken as \( \vec{\Omega} = \omega \hat{z} \), since we assume the rotating frame along the \( z \)-axis. In references [8–10, 22], it is shown that the presence of the torsion influenced the angular momentum. Particularly, it is noted that the \( z \)-component of the angular momentum, \( L_z = -i\partial_\nu \), is being modified by the torsion effect with an additional term.
Before we derive the term associated with the rotating frame, we recall the definition of the angular momentum, \( \vec{L} = r \times \vec{P} \), where \( r = \rho \hat{\rho} \) and \( \rho \) is the radial coordinate of the cylindrical coordinate system. Note that \( \hat{\rho} \) is the unit vector that points to the radial direction. We use the definition of the generalized momentum operator

\[
\hat{\Pi} = \hat{p} - \hat{M} \times \hat{E}.
\]  

(14)

Solving equation (13) begins by finding \( \hat{\Pi} \Psi(t, \vec{r}) \), therefore, it can be written as follows

\[
\hat{\Pi} \Psi(t, \vec{r}) = (-\nabla^2 + 2i(\hat{M} \times \hat{E}) \nabla + (\hat{M} \times \hat{E})^2) \Psi(t, \vec{r}).
\]  

(15)

We proceed further with the introduction of the radial electric field vector \( \hat{E} \):

\[
\hat{E} = \frac{\lambda}{2} \rho^2 \hat{\rho},
\]  

(16)

where \( \lambda \) is a constant that denotes a non-uniform distribution of electric charges inside the non-conducting cylinder.

The selection of the electric field for the laboratory frame as given in equation (16), let us define a uniform effective magnetic field out of the interaction between the radial electric field and magnetic quadrupole moment in the form of

\[
\hat{B}^{\text{eff}} = \nabla \times (\hat{M} \times \hat{E}) = 2M\lambda \hat{z}.
\]  

(17)

According to equations (16) and (17), let us now rewrite equation (15) as follows

\[
\hat{\Pi} \Psi(t, \vec{r}) = (-\nabla^2 + 2iM\lambda \rho \nabla \rho + M^2 \lambda^2 \rho^2) \Psi(t, \vec{r}).
\]  

(18)

Then, we use the line element that is given in equation (1), to derive the Laplacian operator [61]. We find

\[
\nabla^2 = \frac{1}{\rho} \partial_\rho (\rho \partial_\rho) + \frac{1}{\rho^2} (\partial_\rho - \beta \partial_\varphi)^2 + \partial^2_\varphi.
\]  

(19)

Therefore we conclude that the angular momentum operator is being modified via \( \partial_\rho \to \partial_\rho - \beta \partial_\varphi \) in the elastic medium with the screw dislocation.

\[
\hat{L}^{\text{eff}}_z = -i(\partial_\rho - \beta \partial_\varphi).
\]  

(20)

Note that this operator shows the effect of the torsion on the \( z \)-component of the angular momentum. In [62] it is shown that the topology of a screw dislocation that corresponds to the distortion of a circular curve into a vertical spiral makes such transformation and also showed that the effective operator of the \( z \)-component of the angular momentum does not commute with the Hamiltonian. Next, the effects of rotation on this nonrelativistic system are investigated by \( \hat{\Omega} \hat{L} \Psi(t, \vec{r}) \) (the second term of the left-hand side of equation (13)); thereby, it can be found as

\[
\hat{\Omega} \hat{L} \Psi(t, \vec{r}) \equiv \omega \hat{L}^{\text{eff}}_z \Psi(t, \vec{r}) = [-i\omega (\partial_\rho - \beta \partial_\varphi) - \omega M \lambda \rho^2] \Psi(t, \vec{r}).
\]  

(21)

To solve equation (13), we consider a particular solution. Since the Hamiltonian commutes with the operators \( \hat{p}_z \) and \( \hat{L}^{\text{eff}}_z \), we make an ansatz and assume the wave function in the following form.

\[
\Psi(t, \vec{r}) = e^{-i\ell z + i\ell z + i\ell \hat{z} \psi(\rho)}.
\]  

(22)

Here, \( \mathcal{E} \) is the energy of the system, \( k \) denotes a constant that is so-called the wave number along the \( z \)-axis, and \( \ell \) is a quantum number where \( \ell = 0, \pm 1, \pm 2, ... \). By employing the wave-function solution in equation (13), we find the second-order time-independent radial Schrödinger equation as

\[
\frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} - \frac{1}{\rho^2} (\ell - \beta k)^2 - k^2 + 2(M\lambda + m\omega)(\ell - \beta k) + 2m\mathcal{E} = 2mV(\rho)
\]  

\[
- (M^2 \lambda^2 + 2mM\lambda \omega) \rho^2 \psi(\rho) = 0.
\]  

(23)

3. Exact solution of the Schrödinger equation

In this section, we solve the derived time-independent Schrödinger equation for two different choices of the scalar potential energy; with the absence of the static potential energy and with the pseudo-harmonic type potential energy. In both solutions, we employ the Nikiforov-Uvarov (NU) method to obtain the energy eigenvalue and the wave functions. We avoid writing the NU method here to prevent unnecessary repetition. For details, one can see e.g. Appendix in [51].
3.1. The first case
In this case, we solve equation (23) in the absence of static potential energy, that is, \( V(\rho) = 0 \). We find this choice interesting since the magnetic potential energy is also absent. Thereby, this case corresponds to the moving particle’s quantum dynamics in a rotating frame by considering the interaction of its magnetic quadrupole moment with the external radial electric field. We start the solution by dropping the static potential energy terms out of the equation (23). We find

\[
\frac{d^2 \psi_{ne}(\rho)}{d\rho^2} + \frac{1}{\rho} \frac{d \psi_{ne}(\rho)}{d\rho} + \frac{1}{\rho^2} \left[ -(M^2\lambda^2 + 2m\Lambda \omega)\rho^4 + (2M\lambda + m\omega)(\ell - \beta k) - k^2 + 2m\mathcal{E}\right] \rho^2 \\
- (\ell - \beta k)^2 \psi_{ne}(\rho) = 0.
\]

(24)

We proceed by introducing a new changing variable, \( s = \rho^2 \). Thus, we rewrite equation (24) as

\[
\frac{d^2 \psi_{ne}(s)}{ds^2} + \frac{1}{s} \frac{d \psi_{ne}(s)}{ds} + \frac{1}{4s^2} \left[ -(M^2\lambda^2 + 2m\Lambda \omega)s^2 + (2M\lambda + m\omega)(\ell - \beta k) - k^2 + 2m\mathcal{E}\right] s \\
- (\ell - \beta k)^2 \psi_{ne}(s) = 0.
\]

(25)

Then, we compare equation (25) with the NU equation and obtain the wave function with regard to the generalized Laguerre polynomials \( L_n^\alpha(x) \):

\[
\psi_{ne}(\rho) = N_{nl} \rho^{2\alpha + 1} e^{\alpha \rho^2} L_n^{\alpha-\lambda}(\alpha_1, \rho^2),
\]

(26)

where \( N_{nl} \) is the normalization constant. Moreover, we find the coefficients \( \alpha_{10}, \alpha_{11}, \alpha_{12} \) and \( \alpha_{13} \) as follows:

\[
\alpha_{10} = 1 + |\ell - \beta k|, \quad \alpha_{11} = \sqrt{M^2\lambda^2 + 2m\Lambda \omega}, \\
\alpha_{12} = \frac{|\ell - \beta k|}{2}, \quad \alpha_{13} = -\frac{1}{2} \sqrt{M^2\lambda^2 + 2m\Lambda \omega}.
\]

(27)

Then, we follow the NU method and derive the energy eigenvalue function as

\[
\mathcal{E}_{ne} = \frac{k^2}{2m} - \left( \frac{M\Lambda}{m} + \omega \right)(\ell - \beta k) + \frac{1}{m}(1 + 2n + |\ell - \beta k|) \sqrt{M^2\lambda^2 + 2m\Lambda \omega}.
\]

(28)

Henceforth, we discuss the behavior of energy eigenvalues \( \mathcal{E}_{ne} \) in terms of the constant angular velocity \( \omega \). At a particular value of the angular velocity, the energy eigenvalues become zero. This special value of the \( \omega \) is called a critical angular velocity and we denote it with \( \omega_c \). We obtain the critical angular velocity as

\[
\omega_c = \frac{1}{8(\ell - \beta k)} \left\{ \frac{1}{m}(4k^2(\ell - \beta k) + 8M\Lambda(1 + 2n)^2 + 16M\lambda(1 + 2n)|\ell - \beta k|) \\
\pm \left[ \frac{1}{m^2}(4k^2(\ell - \beta k) + 8M\Lambda(1 + 2n)^2 + 16M\lambda(1 + 2n)|\ell - \beta k|) + \right. \\
+ \sqrt{16m^2(\ell - \beta k)^2(4M^2\lambda^2(1 + 2n)^2 - (k^2 - 2\ell\lambda\mu)^2 - 4kM\lambda\beta(k^2 - 2\ell\lambda\mu + k\mu\lambda) \\
+ 8M^2\lambda^2(1 + 2n)|\ell - \beta k| + 4M^2\lambda^2(\ell - \beta k)^2} \right]^{1/2} \left. \right\}.
\]

(29)

We choose the positive sign in the brackets and take \( \ell = \lambda = m = M = 1, k = 0.5 \) to perform numerical calculations. For \( \beta = 0.5 \) from equation (29) we calculate the first four, \( n = 0, 1, 2, 3 \), critical angular velocities as \( 9.71, 48.83, 116.39, 212.39 \), respectively. Then, we extend our analysis by considering two different values of the dislocation parameter, \( \beta = 0 \) and \( \beta = 0.8 \). We use equation (28) and demonstrate the behavior of the energy eigenvalue functions versus the angular velocity in figure 1. It is worth noting that for \( \omega > \omega_c \) values we find the negative energy eigenvalues, while \( \omega < \omega_c \) values we obtain positive energy eigenvalues. We observe that in the absence of dislocation, for a lower value of the angular velocity, more precisely for \( \omega_c = 6.73 \), the particle becomes confined. When the dislocation parameter values increase, higher angular velocity is required for the confinement.

Next, we investigate the variation of the energy eigenvalue versus the dislocation parameter at an arbitrary angular velocity value. We choose \( \omega = 20 \) and for \( n = 0, 1, 2 \) we present the plot of the energy eigenvalue function versus \( \beta \) in figure 2. We observe that in the ground state, with the assigned angular velocity the particle stays in confinement up to 1.43 disorder value. Regarding the first excited states, the confinement is valid up to 0.30 disorder value, when we consider the second excited states, we do not observe confinement. Besides, we observe a critical value of the Burger parameter, \( \beta_c = k/l \). After this critical value, the change in the energy eigenvalue has a different rate.

Before we examine a non-zero scalar potential energy, we would like to discuss the role of the parameters on the degeneracy of the eigenvalue function. If we consider the system does not have a dislocation and rotation, the energy eigenvalue function reduces to
We find that for the $\ell = 0$ case the energy eigenvalue becomes independent of $\ell$ parameter, 

$$E_{nf} = \frac{k^2}{2m} + \frac{M\lambda}{m}(1 + 2n + |\ell| - \ell).$$  

(30)

We observe a degeneracy for any $n + \ell = n' + \ell'$ values.

If we consider the system with a dislocation, $\beta \neq 0$, but without a rotation, $\omega = 0$, then the energy eigenvalue function turns to the following form:

$$E_{nf} = \frac{k^2}{2m} + \frac{M\lambda}{m}(1 + 2n + |\ell - \beta k| - (\ell - \beta k)).$$  

(31)

In this case, for the $\ell - \beta k \geq 0$ values, we obtain

$$E_{nf} = \frac{k^2}{2m} + \frac{M\lambda}{m}(1 + 2n)$$  

(32)

which shows that the $\ell$ parameter dependence is lost. In the other case where $\ell - \beta k < 0$, we arrive at

$$E_{nf} = \frac{k^2}{2m} + \frac{M\lambda}{m}(1 + 2n + 2|\ell - \beta k|).$$  

(33)

We observe a degeneracy in the eigenvalue function for the $n + \ell = n' + \ell'$ quantum numbers. As a result of this analysis, we conclude that the angular velocity and the quantum number $\ell$ removes the degeneracy of the system.

It is worth mentioning that, for $k = 0$, equations (30) and (31) are the same as the equation (12) of [49] if and only if $\Omega = 0$ is considered there. Furthermore, they are very similar to the equation (27) of [50].

### 3.2. The second case

In this case, we consider a static scalar potential in the form of pseudo-harmonic type potential energy [8].

$$V(\rho) = C_1\rho^2 + \frac{C_2}{\rho^2} + C_3,$$  

(34)
It is worth mentioning that many applications have been reported by this kind of static scalar potential, for example, a quantum ring of a nanosphere [63, 64]. It is a basic example of central potential energies that is constituted with the radial harmonic oscillator, inverse square and constant terms. Its coefficients $C_1$, $C_2$, and $C_3$ are real-valued constants. We start the examination by substituting the considered potential energy into equation (23). We apply a change of the variable, $\bar{\rho} = \rho^2$. We get:

$$\frac{\text{d}^2 \psi_{nr}(\bar{\rho})}{\text{d} \bar{\rho}^2} + \frac{1}{\bar{\rho}} \frac{\text{d} \psi_{nr}(\bar{\rho})}{\text{d} \bar{\rho}} - \frac{1}{4\bar{\rho}^2} \left[ (M^2 \lambda^2 + 2mM \omega + 2mC_1) \bar{\rho}^2 - (2(M\lambda + m\omega)(\epsilon' - \beta k) - k^2 + 2m\epsilon - 2mC_3) \bar{\rho} + (\epsilon' - \beta_k)k^2 + 2mC_1 \right] \psi_{nr}(\bar{\rho}) = 0. \quad (35)$$

Alike the first case, we solve equation (35) with the NU method. We obtain the wave functions as:

$$\psi_{nr}(\rho) = \tilde{N}_{nr} \rho^{2\gamma_{1s}} e^{\gamma_{1s}^2 \rho^2 / 2} \left( \gamma_{1s}^2 \rho^2 \right), \quad (36)$$

where $\tilde{N}_{nr}$ is the normalization constant. From the coefficients

$$\gamma_{10} = 1 + \sqrt{2mC_2} + (\epsilon' - \beta k)^2, \quad \gamma_{11} = \sqrt{2mC_1 + M^2 \lambda^2 + 2mM \omega}, \quad \gamma_{12} = \frac{1}{2} \sqrt{2mC_2} + (\epsilon' - \beta k)^2, \quad \gamma_{13} = -\frac{1}{2} \sqrt{2mC_1 + M^2 \lambda^2 + 2mM \omega}, \quad (37)$$

we derive the energy eigenvalue function as:

$$\mathcal{E}_{nr} = C_3 + \frac{k^2}{2m} - \left( \frac{M \lambda}{m} + \omega \right)(\epsilon' - \beta k) + \frac{1}{m} (1 + 2n + \sqrt{2mC_2} + (\epsilon' - \beta k)^2) \sqrt{2mC_1 + M^2 \lambda^2 + 2mM \omega}. \quad (38)$$

Note that for $C_1 = C_2 = C_3 = 0$, equation (38) reduces to equation (28). As we have done in the first case, we derive the critical angular velocity function, $\tilde{\omega}_c$, out of energy eigenvalue function which is given in equation (38). We find:

$$\tilde{\omega}_c = \frac{1}{8(\epsilon' - \beta k)^2} \left\{ \frac{1}{m} (4(k^2 + 2mC_3)(\epsilon' - \beta k) + 8M \lambda (2mC_2 + (1 + 2n)^2) \right.$$

$$+ 16M \lambda (1 + 2n) \sqrt{2mC_2} + (\epsilon' - \beta k)^2) + \frac{1}{m^2} (4(k^2 + 2mC_3)(\epsilon' - \beta k)$$

$$+ 8M \lambda (2mC_2 + (1 + 2n)^2) + 16M \lambda (1 + 2n)(2mC_2 + (\epsilon' - \beta k)^{1/2})^2$$

$$+ \frac{16}{m^2} (\epsilon' - \beta k)^2 (8mC_2((\epsilon' - \beta k)^2 + 2mC_2 + (1 + 2n)^2) - (k^2 + 2mC_3)^2$$

$$+ 4M \lambda (k^2 + 2mC_3)(\epsilon' - \beta k) + 4M \lambda^2 (2mC_2 + (1 + 2n)^2)^{1/2})$$

$$+ 8(2mC_1 + M^2 \lambda^2)(1 + 2n)(2mC_2 + (\epsilon' - \beta k)^{1/2})^{1/2} \left\} \right\}^{1/2}. \quad (39)$$

Alike the first case, we take the positive sign in the above equation. Then, we assume the following parameters for the numerical calculations: $\epsilon' = \lambda = m = M = C_1 = C_2 = C_3 = 1$, and $k = 0.5$. From equation (39), we calculate the critical angular velocity values for the ground and the first excited states as 26.41, 77.71, 157.39 and 265.50. Respectively. If we consider solely the inverse square energy as the scalar potential energy, we obtain the same results as the first case. If we consider the scalar potential energy as consisted only of the radial harmonic term, that is $C_1 = C_2 = C_3 = 0$, we find the following critical angular velocity values for the first four states: 10.69, 49.82, 117.39 and 213.39. Instead, if we consider solely the inverse square energy as the scalar potential energy, then we find 22.88, 74.09, 153.75, and 261.85. It is worth noting that, when $C_1 = C_2 = C_3$ are taken as zero, we obtain the same results as the first case.

In order to examine the highest probability of finding a particle, we plot the probability density functions, $|\psi_{nr}(\rho)|^2$, versus $\rho$ in figure 3. Therefore, at first, we find three different normalized wave-functions from equation (36) for $\beta = 0.5$, where we have calculated the normalization coefficients of the first three wave functions as 2.10, 1.62, and 1.35. From figure 3, we observe that closest peaks to the origin have the largest amplitudes.

To illustrate the effect of the constant angular velocity of the rotating frame on the energy eigenvalues, we plot figure 4. Here, we consider the first three excited states, thus, by using equation (38) we plot $\tilde{\mathcal{E}}_{1s}$, $\tilde{\mathcal{E}}_{2s}$, and $\tilde{\mathcal{E}}_{3s}$ versus the angular velocity for $\beta = 0.5$. In figure 4, initially, we observe a rise in the energy eigenvalues for the increasing values of $\omega_c$; after a certain value of $\omega_c$ we see a decrease in the energy eigenvalues. We calculate the critical angular velocities as 77.71, 157.39, and 265.50, respectively. We observe that these results are in the line with the plots. We see that confinement, the relatively higher excited states need higher critical angular velocity values. This conclusion is also valid for the solely radial harmonic and inverse square cases, where the first three excited states’ critical angular velocities are 49.82, 117.39, 213.39 and 74.09, 153.75, 261.85, respectively.
Next, we investigate the effect of the dislocation of the lattice on the critical angular velocity and energy eigenvalues. We consider the first excited state of the pseudo-harmonic, harmonic, and inverse square cases, and depict their energy eigenvalue functions versus the angular velocity with three Burger parameters in figures 5, 6 and 7, respectively. In figure 5, in other words on the pseudo-harmonic scalar potential energy case, the increase of the dislocation parameter modifies the critical angular velocity values. In the absence of deformation, we calculate the angular velocity as 46.48. This value arises to 117.52 when the dislocation parameter is equal to 0.8. After this angular velocity, the particle becomes confined.

We observe the same characteristic behavior for the other potential energy forms For example, in the harmonic energy and inverse square cases, we obtain the following critical angular velocity values 31.74, 49.82, 71.91; and 43.53, 74.09, 113.24 for the Burger parameters of 0, 0.5 and 0.8, respectively.
4. Conclusion

In this study, we examine the interaction of a magnetic quadrupole moment of a moving particle in an elastic medium with a rotating frame in the presence of a screw dislocation. This medium possesses a radial electric field, therefore non-relativistic particle is influenced by a uniform effective magnetic field. We focus on the magnetic quadrupole polarizability tensor and solve the Schrödinger equation exactly by using the NU method for two particular cases. In the first case, we assume that the interaction in the absence of the static scalar potential energy. Then, in the second case, we consider a pseudo-harmonic oscillator like static potential energy. We acquire the wave-function and energy eigenvalues for both cases. In both cases, we find that the dislocation parameter modifies the energy eigenvalue functions. We show that the particle confinement is based on the critical angular velocity value which is not a constant and depends on the dislocation parameter. We present and discuss the behavior of energy eigenvalues versus the critical angular velocity with different Burger parameters in other forms of the scalar potential energies. Moreover, in the first case, we examine the role of the dislocation and the angular velocity parameter on the degeneracy of the energy eigenvalues. We find that angular velocity removes the degeneracy of the system, while the quantum number $\ell$ effects the degeneracy. In the second case, we demonstrate and discuss the probability density and energy eigenvalues functions that correspond to the wave-function versus angular velocity parameter.

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Figure 6. The energy eigenvalue function for the first excited state of the harmonic scalar potential energy case via three different Burger parameters.

Figure 7. The energy eigenvalue function for the first excited state of the inverse square scalar potential energy case via three different Burger parameters.
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