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Outage probability analysis of overlay cognitive two-way relaying scheme with opportunistic relay selection

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Abstract
The authors investigate an overlay cognitive two-way relaying system where max-min (MM) relay selection scheme is employed for improving the reliability of the considered system. Furthermore, they design the process of the data forwarding and relay selection in the overlay cognitive two-way relaying system. Based on the MM relay selection criterion, the exact close-form expression of outage probability is derived. From numerical simulations, it is observed that there is an outage probability floor appearing in the high signal-to-noise ratio (SNR) region, and the reason for its occurrence is stated. Besides, the approximate result of outage probability in high SNR is provided, which shows that the ratio of interference channel's transmit power and main channel's transmit power will not affect the outage probability in low SNR region, but it will significantly affect it in the high SNR region. Although each individual channel state information (CSI) will not be able to control the outage performance of this considered system in the high SNR region, the ratio of each channel's CSI will be able to do the same.

1 | INTRODUCTION

With the rapid development of wireless communication technology, the demand for wireless spectrum resources is increasing greatly, and the spectrum resources become scarce day by day. Yet most of time in urban areas, much of the usable radio frequencies are idle, which means that the spectrum shortage is not only caused by the lack of usable frequencies but also results from inappropriate spectrum management [1,2]. In order to make full use of the spectrum holes [3], two approaches of spectrum sharing, namely spectrum underlay and spectrum overlay have been considered in the cognitive radio network (CRN). Specifically, underlay CRN allows secondary users (SUs, also been known as the cognitive users) to access the primary networks only when the licensed spectrum is unoccupied by primary users (PUs). In contrast, overlay CRN allows SUs to access the entire licensed spectrum without affecting PUs' quality of service (QoS). Obviously, cognitive radio technology has the potential to address challenges associated with spectrum access [4]. In [5], the authors studied two cooperative communication schemes for secure communication in CRNs where a set of eavesdroppers are trying to tap information transmitted by primary base station (BS) and SUs, and a more practical case was considered where the channel state informations (CSIs) of eavesdroppers are not available. [6] formulated a green coexistence paradigm for underlay multi-hop energy harvesting (EH) CRNs to promote total throughput, in which battery-free SUs capture both the spectrum and the energy of PUs to enhance spectrum efficiency and green energy utilisation.

Emerging as a promising technology, cooperative communication was proposed for promoting the channel capacity and for achieving the diversity gain [7]. Although cooperative relay brings many benefits, such as expanding the communication range and strengthening the reliability of communication systems [7–9], its disadvantage is the reduction in spectral efficiency since much of the spectrum resources are consumed by relay. To deal with this problem, an opportunistic relaying technique has been investigated where only the 'best' relay is selected for assisting sources forwarding the message to destinations [10–12]. Moreover, the 'best' relay selection scheme is to choose the relay with the best condition, which can be measured by the minimum of the channel gains between the first and the second hops under decode-and-forward (DF) [13] and amplify-and-forward (AF) protocols [14].

For the sake of better spectral efficiency and considerable diversity gain, cooperative relays are always been exploited in CRN. Furthermore, as an efficient cooperation strategy, two-way relaying is widely investigated for promoting the channel
capacity and spectral efficiency [15–18]. In [16], an accurate result of outage probability of a DF-based underlay cognitive two-way relaying system is derived. In addition, the max-min (MM) opportunistic relay selection strategy is used in the article to combat the channel fading. Based on [16], the authors considered the interference from the primary transmitter (PT) to SU s and also analysed outage performance of the DF-based underlay cognitive two-way relaying system in [17]. Besides, the performance of an underlay cognitive network is analysed in [18] by utilising the MM opportunistic selection and two-way full-duplex (FD)-based AF relays. Also, physical-layer security (PLS) is considered as a solution for protecting users against eavesdropping attacks by exploiting the physical characteristics of wireless channels [19–21]. In addition, as an effective strategy to prolong the lifetime of communication systems, simultaneous wireless information and power transfer (SWIPT) based green communication has become an attractive research area in recent years [22–24]. [25] investigates a two-way relaying system, where an EH capable relay is employed for exchanging information between two sources and a friendly jammer is exploited for confusing the eavesdropper. What's more, an artificial noise (AN)-aided opportunistic relay selection scheme, called generalized MM relay selection, is proposed in [26] to enhance PLS for the underlay cognitive two-way relaying system. [27] investigates a FD two-way relaying system with SWIPT capabilities and the authors propose a more complex system where multiple pairs of FD users exchange information through an FD relay with multiple-antenna in [28]. Although there are many researches on cognitive cooperative communication networks, in view of the diversity and complexity of the network, these researches cannot meet the needs of practical application development, and performance analysis is not perfect, so there is a large space for research. The present study investigates an overlay two-way relaying system, and has the following contributions:

- At first, a two-way cognitive relaying system is established, in which the popular relay selection scheme, namely MM relay selection, is employed for improving the reliability.
- Then, the process of the data forwarding and relay selection in two-way cognitive relay system is designed.
- The exact closed-form results of outage probability are derived based on the MM relay selection scheme.
- Furthermore, in order to observe the impact of each parameter on the system performance, the approximate approximation results of the outage probability is derived and the reason why outage probability floor appears in high SNR region is figured out.

2 | SYSTEM MODEL AND RELAY SELECTION SCHEME

2.1 | System model

As shown in Figure 1, we consider an overlay cognitive network consisting of one primary network and one secondary network. In the primary network, there is one PT trying to communicate with the primary receiver (PR) through licensed spectrum, and the secondary network includes two secondary sources SA and SB, which are attempting to exchange their information with each other with the help of N DF-based relays K in an opportunistic way. Specifically, in order to avoid the interfering from PT, each node should first detect the spectrum hole with the aid of spectrum sensing technology and then determine whether to utilise the licensed spectrum to transmit their signals. When the spectrum hole is detected, secondary network starts to work. It is assumed that each node is equipped with single antenna and all links obey independent flat Rayleigh fading. Due to the shadowing, there is no direct link between SA and SB. As discussed earlier, the entire information exchange process with the help of relays can be divided into three phases as shown in Figure 2. In the first phase, the secondary source SA broadcasts its signal xA to all relays with power P_A over the detected spectrum hole. Similarly, in the second phase, after the spectrum hole is detected, SB transmits its signal xB to all relays with power P_B. Upon that, the signals received by relay K in the first and the second phase can be respectively expressed as

\[ y_{AK} = \sqrt{P_A}b_{AK}x_A + \sqrt{\alpha_A}P_AB_{PK}^{(1)}x_P + n_{K1}, \]

(1)

\[ y_{BK} = \sqrt{P_B}b_{BK}x_B + \sqrt{\alpha_B}P_Bb_{PK}^{(2)}x_P + n_{K2}, \]

(2)

where \( b_{AK} \) and \( b_{BK} \) represent channel fading coefficients from SA and SB to relay K with zero mean and variance \( \sigma_{AK}^2 \) and \( \sigma_{BK}^2 \), respectively. \( b_{PK}^{(1)} \) and \( b_{PK}^{(2)} \) represent the channel fading coefficient from PT to relay K with zero mean and variance \( \sigma_{PK}^2 \) during the first phase and the second phase. \( n_{K1} \) and \( n_{K2} \) are the additive white Gaussian noise (AWGN) with zero mean variance N_0 at relay K during the first phase and the second phase. P_P is the transmitting power of PT. Furthermore, \( \alpha_A \) and \( \alpha_B \) are random variables (RV) during the first phase and the second phase, which can be defined as.
where $H_0$ and $H_1$ respectively represent the events, in which the licensed spectrum is unoccupied and occupied by PT that leads to $a = 0$ and $a = 1$. Moreover, $H_i$, $i \in \{A, B, R\}$ denotes the status of licensed spectrum detected during three different phases, e.g., $H_A = H_0$ means $S_A$ detects the licensed spectrum is unoccupied in the first phase, while $H_A = H_1$ indicates the licensed spectrum is considered occupied. In the first and second phases, as discussed before, only when the spectrum hole is detected, that is, $H_A = H_0$ and $H_B = H_0$, cognitive users begin to transmit data to relays. If not, cognitive users stay unchanged and wait for the next round of detection. Similarly, in the third phase, if the signals transmitted by $S_A$ and $S_B$ are decoded successfully, the selected relay re-encodes $x_A$ and $x_B$ by bit-level Exclusive-Or (XOR) operation and then transmits $x_A \oplus x_B$ to $S_A$ and $S_B$ at the same time. If the spectrum hole is not detected, the selected relay keeps the signal and waits for the next spectrum hole to transmit the data. Finally, $S_A$ and $S_B$ can decode each other’s information by one more XOR operation. The signal received by $S_A$ and $S_B$ in the third phase can be written as

$$
\gamma_A = \sqrt{P_A b_{KA} x_{A\theta B} + \sigma_A^2} + n_A, \\
\gamma_B = \sqrt{P_B b_{KB} x_{A\theta B} + \sigma_B^2} + n_B,
$$

where $P_K$ is the transmitting power of the selected relay $K$. $b_{KA}$ and $b_{KB}$ represent channel fading coefficients from relay $K$ to $S_A$ and $S_B$ with zero mean and variance $\sigma_{KA}^2$ and $\sigma_{KB}^2$, $p_A$ and $p_B$ represent channel fading coefficients from PT to $S_A$ and $S_B$ with zero mean and variance $\sigma_{PA}^2$ and $\sigma_{PB}^2$, respectively. According to channel reciprocity, we have $\sigma_{KA}^2 = \sigma_{AK}^2$, $\sigma_{KB}^2 = \sigma_{BK}^2$. $n_A$ and $n_B$ are additive white Gaussian noise (AWGN) with zero mean variance $N_0$ at $S_A$ and $S_B$. To simplify the performance analysis in the proposed method, it is assumed that the fading coefficients of each channel are independent and identically distributed RVs during different phases [26,29].

### 2.2 Relay selection scheme

In the first and the second phase, it is well known that the massage can be considered decoding successfully as long as the channel capacity is higher than a target rate $R$. From (1) and (2), the signal-to-noise ratio (SNR) at relay $K$ can be obtained as:

$$
\gamma_{AK} = \frac{\gamma_{AK}}{a_{TFK} + 1}, \gamma_{BK} = \frac{\gamma_{BK}}{a_{TFK} + 1},
$$

where $\gamma_{AK} = P_A |h_{AK}|^2/N_0, \gamma_{BK} = P_B |h_{BK}|^2/N_0$. $P_A = P_p |b_{PK}^1|^2/N_0, P_B = P_p |b_{PK}^2|^2/N_0$. Let $D_{AB}$ denote the set of relays who can decode $x_A$ and $x_B$ successfully. Specifically, $D_{AB}$ can be described as

$$
D_{AB} = \{K | \gamma_{AK}^a > T, \gamma_{BK}^a > T, K \in \{1, 2, \cdots, N\}\},
$$

where $T = 2^{3R} - 1$, the existence of factor 3 is owing to the fact that the process of transmission is divided into three phases. In order to guarantee the reliability of this system, during the third phase, the combined signal is forwarded by the optimal relay $K^*$, which is selected by the following criterion

$$
K^* = \arg \max_{K \in D_{AB}} \min(\gamma_{KA}, \gamma_{KB}).
$$

Thus, the optimal relay can be selected by distributed relays’ timer competition. Here, we propose the progress of data transmission and relay selection for the two-way cognitive relaying system in Figure 3. Specifically, after receiving the data from $S_A$, relays can obtain their own related CSI through pilot-based or blind channel estimation (which beyond the scope of this article and we omit the illustration of the progress for brevity), and then decode the received signal. Later, those relays that successfully decode the signal begin to send a positive acknowledgment (ACK) to $S_B$, and $S_B$ will send the data to relays as soon as it receives the ACK, thereby the relays can also acquire the relevant CSI and decode the signal from $S_B$. Next, the relays that successfully decode the signals transmitted by $S_A$ and $S_B$ set the initial value $T_K = \lambda/\min\{\gamma_{KA}, \gamma_{KB}\}$ ($\lambda$ is a constant) of the timer by using the CSIs and start their own timers immediately. While the timers are approaching zero, all relays are in listening mode. If the relay does not receive any signal when its timer has reduced to zero, this relay will immediately use XOR coding scheme to re-encode the receiving signals of $S_A$ and $S_B$, and then transmits them. Once other relays detect data packet transmitted by a relay, they back off. Here, we must set RIFS (Relay Interframe Space) $\leq$ SIFS (Short Interframe Space), so that the relay transmissions will always begin before the regular data transmissions. In fact, the collision probability in the progress of relays competition can also be very small by choosing the correct $\lambda$, so the collision among different relays can be ignored in the considered system. In addition, it is worth noting that the data frame is much longer than the control frame in the actual transmission. So that the transmission time of ACK, SIFS, and RIFS can be ignored in the performance analysis.
Hence, in the third phase, from (4) and (5), we can obtain the SNR at $S_A$ and $S_B$ as

$$
\gamma_{KA}^{\text{out}} = \frac{\gamma_{KA}}{a_{RPA}^{-1}} + 1, \gamma_{KB}^{\text{out}} = \frac{\gamma_{KB}}{a_{RPB}^{-1}} + 1.
$$

where $\gamma_{KA} = P_K |K_{KA}|^2 / N_0, \gamma_{KB} = P_K |K_{KB}|^2 / N_0, \gamma_{PA} = P_p |P_{PA}|^2 / N_0, \gamma_{PB} = P_p |P_{PB}|^2 / N_0$.

### 3 PERFORMANCE ANALYSIS

#### 3.1 Exact outage performance for all SNR

This section derives the exact outage probability of the MM scheme under the threshold rate $R$. Notably, the outage event occurs when channel capacity is lower than $R$. In this system, the outage event can be divided into two cases: one is $|D_{AB}| = 0$, where $|D_{AB}|$ represents the cardinality of the set $D_{AB}$, the other is $|D_{AB}| \neq 0$. It is worth noting that the above discussion is based on the assumption that the status of licensed spectrum detected by each specific node is available in each phase. In addition, we must emphasize that in the actual transmission process, the transmission time of data frames is much longer than that of RIFS, so the time of the relay selection can be ignored in the process of performance analysis.

For the first case, $|D_{AB}| = 0$ implies no relay decodes $X_A$ and $X_B$ successfully at the same time. In this way, no relay forwards $X_A$ and $X_B$, then the outage happens. So the outage probability can be described as

$$
P_{\text{out}} = \Pr \left( |D_{AB}| = 0 | \hat{H}_A = H_0, \hat{H}_B = H_0 \right).
$$

Moreover, utilising Bayes’ theory and the conclusion in [30], the term $\Pr \left( H_i = H_0 | \hat{H}_i = H_0 \right)$ and $\Pr \left( H_i = H_1 | \hat{H}_i = H_0 \right)$ can be obtained as

$$
\Pr \left( H_i = H_0 | \hat{H}_i = H_0 \right) = \frac{\Pr \left( \hat{H}_i = H_0 | H_i = H_0 \right) \Pr \left( H_i = H_0 \right)}{\sum_{i \in \{0, 1\}} \Pr \left( H_i = H_0 | H_i = H_i \right) \Pr \left( H_i = H_i \right)}
= \frac{P_0 (1 - P_f)}{P_0 (1 - P_f) + (1 - P_0) (1 - P_f)} = \theta_0.
$$
where $P_0 = \Pr\{H_1 = H_0\}$ is the probability of that licensed spectrum is unoccupied by PT, $P_d = \Pr\{\hat{H}_1 = H_1 | H_1 = H_1\}$ is the successful detection probability (SDP), in contrast, $P_f = \Pr\{\hat{H}_1 = H_1 | H_1 = H_0\}$ is the false-alarm probability (FAP). Owing to the background noise and the characteristics of channel fading, it is impossible for us to perfectly detect the spectrum hole without false alarm. Furthermore, the missed detection of the occupied spectral band will result in interference between PU and SUs. To prevent interference between primary network and secondary network, it is important to achieve reliable spectrum sensing. Therefore, $P_d$ and $P_f$ should be limited in a meaningful range for the sake of primary users' QoS. For instance, the IEEE 802.22 standard requires $P_d > 0.9$ and $P_f < 0.1$ [31].

In (12), $\Pr\{|D_{AB}| = 0 \{H_B = H_0, H_A = H_0, \hat{H}_B = H_0, \hat{H}_A = H_0\}$ indicates the probability of no relay decodes successfully in the case that the status of licensed spectrum detected in the first and the second phase is both idle and the detection is correct. To simplify the subsequent derivation, we assume that all relays are close to each other and forming a cluster, then the transmitting power of all relays is the same. Therefore, the probability can be derived as

$$
\Pr\{|D_{AB}| = 0 \{H_B = H_0, H_A = H_0, \hat{H}_B = H_0, \hat{H}_A = H_0\} = 
\prod_{k=1}^{N} (1 - \Pr(\gamma_{AK} > T, \gamma_{BK} > T)) = 
\left(1 - \exp\left(-\frac{T}{\bar{\gamma}_{AK}}\right)\exp\left(-\frac{T}{\bar{\gamma}_{BK}}\right)\right)^N.
$$

Similarly,

$$
\Pr\{|D_{AB}| = 0 \{H_B = H_1, H_A = H_1, \hat{H}_B = H_0, \hat{H}_A = H_0\} = 
\prod_{k=1}^{N} \left(1 - \Pr(\gamma_{AK} > T, \gamma_{BK} > T)\right) = 
\left[1 - \frac{\bar{\gamma}_{AK}}{\bar{\gamma}_{AK} + \bar{\gamma}_{BK}} \exp\left(-\frac{T}{\bar{\gamma}_{AK}}\right) \frac{\bar{\gamma}_{BK}}{\bar{\gamma}_{BK} + \bar{\gamma}_{PK}} \exp\left(-\frac{T}{\bar{\gamma}_{BK}}\right)\right]^N,
$$

For the second case, $|D_{AB}| \neq 0$ indicates one or more relays decode the signal from $S_A$ and $S_B$ successfully. Therefore, the outage happens in the third phase when $S_A$ or $S_B$ fails to decode the signal from the selected relay $K^*$. Without loss of generality, here let $|D_{AB}| = k$, and there are $\binom{N}{k}$ situations satisfying $|D_{AB}| = k$. Thus, in this case, the outage probability can be defined as (20).

$$
P_{out}^k = \sum_{k=1}^{N} \binom{N}{k} \left[ \Pr\{\gamma_{K^*A}^B < T, |D_{AB}| = k \} \Pr\{H_A = H_0, \hat{H}_B = H_0, \hat{H}_R = H_0\} + \Pr\{\gamma_{K^*B}^B < T, |D_{AB}| = k \} \Pr\{H_A = H_0, \hat{H}_B = H_0, \hat{H}_R = H_0\} \right]
= \sum_{k=1}^{N} \binom{N}{k} \left[ \Pr\{|D_{AB}| = k \} \Pr\{H_A = H_0, \hat{H}_B = H_0\} \times \Pr\{\gamma_{K^*A}^B < T | \hat{H}_R = H_0\} + \Pr\{|D_{AB}| = k \} \Pr\{H_A = H_0, \hat{H}_B = H_0\} \times \Pr\{\gamma_{K^*B}^B < T | \hat{H}_R = H_0\} \right].
$$
Similar to the derivation in case 1, the term $\Pr\{ |D_{AB}| = k | \hat{H}_A = H_0, \hat{H}_B = H_0 \}$ can be obtained as

$$
\Pr\{ |D_{AB}| = k | \hat{H}_A = H_0, \hat{H}_B = H_0 \} = \pi_1^2 \exp \left( \frac{kT}{\gamma_{AK}} \right) \exp \left( \frac{kT}{\gamma_{BK}} \right) \exp \left( -1 \exp \left( -\frac{T}{\gamma_{AK}} \right) \exp \left( -\frac{T}{\gamma_{BK}} \right) \right) N^{-k} + \pi_1 \exp \left( \frac{kT}{\gamma_{AK}} \right) \exp \left( \frac{kT}{\gamma_{BK}} \right) \exp \left( -1 \exp \left( -\frac{T}{\gamma_{AK}} \right) \exp \left( -\frac{T}{\gamma_{BK}} \right) \right) N^{-k} 
$$

(21)

Then by considering the second term in (20), we have

$$
\Pr\{ \gamma_{K_A}^{\text{d}} < T, \hat{H}_R = H_0 \} = \Pr\{ \gamma_{K_A}^{\text{d}} > T, \gamma_{K_B}^{\text{d}} < T, \hat{H}_R = H_0 \} 
$$

As mentioned in [13], it is worth to note that although $\gamma_{K_A}^{\text{d}}$ and $\gamma_{K_B}^{\text{d}}$ are independent for all relays, $\gamma_{K_A}^{\text{d}}$ and $\gamma_{K_B}^{\text{d}}$ cannot be treated as independent. However, exploiting the corollary 1 in [32], we can rewrite $\Pr\{ \gamma_{K_A}^{\text{d}} > T, \gamma_{K_B}^{\text{d}} < T \}$ as

$$
\Pr\{ \gamma_{K_A}^{\text{d}} > T, \gamma_{K_B}^{\text{d}} < T \} \approx \left( 1 - F_{\gamma_{K_A}^{\text{d}}} (T) \right) F_{\gamma_{K_B}^{\text{d}}} (T),
$$

(23)

where $F_{\gamma_{K_A}^{\text{d}}} (\cdot)$ and $F_{\gamma_{K_B}^{\text{d}}} (\cdot)$ denote the cumulative density functions (CDF) of $\gamma_{K_A}^{\text{d}}$ and $\gamma_{K_B}^{\text{d}}$, respectively. Thus, we can further rewrite (22) by

$$
\Pr\{ \gamma_{K_A}^{\text{d}} < T, \hat{H}_R = H_0 \} = \Pr\{ \gamma_{K_A}^{\text{d}} > T, \gamma_{K_B}^{\text{d}} < T, \hat{H}_R = H_0 \} 
$$

(24)

where the CDF of $\gamma_{K_A}^{\text{d}}, \gamma_{K_B}^{\text{d}}, \frac{\gamma_{K_A}^{\text{d}}}{\gamma_{K_B}^{\text{d}} + 1}$ and $\frac{\gamma_{K_A}^{\text{d}}}{\gamma_{K_B}^{\text{d}} + 1}$ has been derived in Appendix. Therefore, by substituting (33)–(36) into (24), we get the closed-form result as (25).
we define an interference to signal radio (ISR) floor occurs in high SNR regions. In order to better explain it, From Figure 4, one can observe that an outage probability as follows:

\[ P_{\text{out}} = \frac{1}{\pi} \sum_{i=1}^{k} \left( -k \right)^{(i-1)} \frac{\tilde{y}}{\bar{y}_{KB}(\bar{y}_{KB} - \tilde{y})} \left( \bar{y}_{KB} \left( 1 - \exp \left( -\frac{T}{\bar{y}_{KB}} \right) \right) - \frac{\tilde{y}}{T} \left( 1 - \exp \left( -\frac{iT}{\bar{y}_{KB}} \right) \right) \right) \times \left( \bar{y}_{KB} \left( 1 - \frac{T}{\bar{y}_{KB}} \right) - \frac{\tilde{y}}{T} \left( 1 - \exp \left( -\frac{iT}{\bar{y}_{KB}} \right) \right) \right) \]

(25)

Substituting (21) and (25) into (20), the closed-form result of \( P_{\text{out}}^k \) can be derived. Finally, we get the total outage probability as follows:

\[ P_{\text{out}} = P_{\text{out}}^0 + P_{\text{out}}^k. \]

(26)

### Approximate outage performance in high SNR region

From Figure 4, one can observe that an outage probability floor occurs in high SNR regions. In order to better explain it, we define an interference to signal radio (ISR) \( \alpha = P_B/P_A \). Also, we assume \( P_A = P_B = P_K \).

Notably, when \( x \to 0 \), we have \( \exp(x) \approx 1 \) by ignoring the higher order terms. Therefore, under High SNR, defining \( \lambda_1 = \sigma_{KB}/\sigma_{KA}^2, \lambda_2 = \sigma_{PA}/\sigma_{KA}^2, \lambda_3 = \sigma_{PB}/\sigma_{KA}^2, \) and \( \lambda_4 = \sigma_{PK}/\sigma_{KA}^2 \), (19) can be rewritten as

\[ P_{\text{out}}^0 = \pi_1^2 \left( -1 \right)^N \left( \frac{\lambda_1}{\lambda_1 + \lambda_4 T} \right)^k \]

and

\[ P_{\text{out}}^k = \pi_1^2 \left( -1 \right)^N \left( \frac{\lambda_1}{\lambda_1 + \lambda_4 T} \right)^k \]

(27)

Similarly, (21) and (25) can be rewritten as (28) and (29).

\[ \Pr \left\{ |D_{AB}| = k | \hat{H}_A = H_0, \hat{H}_B = H_0 \right\} \]

\[ = \pi_0^2 \cdot 0^{N-k} + \pi_1^2 \left( \frac{1}{1 + A \lambda_4 T} \right)^k \left( \frac{\lambda_1}{\lambda_1 + \lambda_4 T} \right)^k \]

\[ \times \left[ 1 - \left( \frac{1}{1 + A \lambda_4 T} \left( \frac{\lambda_1}{\lambda_1 + A \lambda_4 T} \right) \right) \right]^{N-k} \]

\[ + \pi_0 \pi_1 \left( \left( \frac{\lambda_1}{\lambda_1 + A \lambda_4 T} \right)^k \left( \frac{\lambda_1}{\lambda_1 + \lambda_4 T} \right)^N \right) \]

(28)

Consequently, combine (26)–(29), the total outage probability in high SNR can be obtained as (30).
The numerical simulations are provided to validate the derived expressions and to obtain insights into performance, and the main notations and symbols are listed in Table 1.

From (30), it can be observed that the outage probability is a constant determined by \( \alpha, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) and \( T_i \), which means in high SNR region, it is not the value of each node’s transmit power and each channel's CSI affects the result of outage probability, but their ratios.

4 | Numerical simulation results

Here, the numerical simulations are provided to validate the derived expressions and to obtain insights into performance, and the main notations and symbols are listed in Table 1.

Figure 4 depicts the outage probability \( P_{out} \) versus the SNR for different \( N \). One can observe that with a certain target transmission rate \( R = 1 \text{bps/Hz} \), which is a common assumption in TWR system [13,14], the outage performance of this proposed system improves as the number of relays increases. Moreover, it can be seen from Figure 4 that an outage probability floor occurs in each curve under high SNR regions.
which is caused by the fact that when SNR increases, interference from PT becomes the dominant factor in the outage event. Therefore, the CSI and the transmit power ratio of main channel and interference channel (i.e. $\alpha$ and $\lambda_2$, $\lambda_3$, $\lambda_4$) will significantly impact the outage performance to a certain extent, which is shown in Figures 5–7. Further, the Mont-Carlo simulation results match the analytical results well.

Figure 5 shows the curve of outage probability as SNR increases for different $\alpha$ in the case where $R = 1\text{bps/Hz}$, $N = 3$, and $\sigma_{Ka}^2 = \sigma_{KB}^2 = 1$, $\sigma_{PK}^2 = \sigma_{PA}^2 = \sigma_{PB}^2 = 0.1$. From Figure 5, one can observe that the curves for different values of $\alpha$ almost coincides when SNR is below about 20 dB where $\alpha$ is the radio of main channel's and interference channels' transmit power. Also, we can find that the smaller values of $\alpha$, the lower of the outage probability floor, the better performance of this system. As discussed previously, when SNR increases, interference from PT becomes the dominant factor in the outage event. A small $\alpha$ indicates the less interference under high SNR region, which leads to better performance and lower outage probability floor. However, within low SNR region, the transmission failure due to the characteristics of Rayleigh channel becomes the domain factor in the outage event.

Figure 6 shows the variation of outage probability with $\alpha$ for different SNR, which further elaborates how $\alpha$ affects the outage performance in different SNR regions. Specifically, when SNR is low, the nature of Rayleigh fading channel determines the outage performance in this considered system; thereby the total outage probability remains almost unchanged as $\alpha$ increases. Moreover, when SNR is high, the interference from PT takes over the outage performance, then the outage probability increases with $\alpha$. Thus, we draw the conclusion that $\alpha$ can hardly affect the outage probability performance in low SNR region but significantly impact the outage probability floor in high SNR region when the target transmission rate, amount of relays, and each channels' CSIs are determined.

Figure 7 shows the outage probability versus SNR under different $\lambda_4$, $\lambda_3$, $\lambda_2$ in the case that $R = 1\text{bps/Hz}$, $N = 3$, $\lambda_1 = 1$ and $\alpha = 1$. Obviously, the change in the value of $\lambda_4$, $\lambda_3$, $\lambda_2$ impacts the outage probability floor in high SNR region but hardly affects the probability in low SNR region. Specifically, this system performs better when $\lambda_4$ decreases, where $\lambda_4 = \sigma_{PK}^2/\sigma_{Ka}^2$. As discussed before, under high SNR, the decrease of $\lambda_4$ indicates less interference from PT in the first and second phases, upon that, more relays decode the signal.
from $S_4$ and $S_B$ successfully at the same time, which reduces the possibility of outage event. Contrarily, in low SNR regions, the characteristics of Rayleigh channel are the domain factor of outage event, and so the interference can hardly affect the result. Similarly, the decrease of $\lambda_1$ and $\lambda_2$ increases the outage performance of this system, where $\lambda_2 = \sigma^2_D/\sigma^2_{KA}$; $\lambda_3 = \sigma^2_P/\sigma^2_{KA}$. Specifically, the decrease of $\lambda_3$ and $\lambda_2$ indicates less interference from PT during the third phase, which improves the probability of successful decoding.

Figure 8 depicts the variation of outage probability with $d$ for different SNR and also explains the influence of $\lambda_1$ on outage performance from another angle, where $\lambda_1 = \sigma^2_{KB}/\sigma^2_{KA}$. To describe the relative position between $S_A$, $S_B$ and relay $K$, we normalise the distance between $a$ and $b$ to 1, and the parameter $\sigma^2_{KA}$ and $\sigma^2_{KB}$ are modeled as $\sigma^2_{KA} = d^{-3}$, $\sigma^2_{KB} = (1-d)^{-3}$, $d \in (0,1)$, where the constant ‘3’ is the path-loss exponent, which is widely assumed in small-scale wireless communication [12–14]. From Figure 8, one can observe that it performs best when $d = 0.5$, that is, relay $K$ is at the midpoint of $S_A$ and $S_B$. This is because under the MM selection scheme, the channel capacity is determined by the channel with worse condition, and hence, the channel capacity achieves the best when the relay is at the midpoint of $S_A$ and $S_B$ and thereby minimises the outage probability.

Figure 9 shows the outage probability versus SNR under different $P_0$, $P_d$, $P_f$ in the case where $R = 1$ bps/Hz, $\alpha = 1$, $N = 3$ and $\sigma^2_{KA} = \sigma^2_{KB} = 1$, $\sigma^2_{PK} = \sigma^2_{PA} = \sigma^2_{PB} = 0.1$. Obviously, one can observe that outage probability reduces with the increase of $P_0$ under high SNR region, that is, the reliability performance is improved with the spectrum sensing reliability. However, when SNR is under about 20 dB, $P_0$ can hardly impact the outage probability. In contrast, $P_d$ and $P_f$ influence the outage probability in both high and low SNR region, and the decrease of $P_f$ (increase of $P_d$) will lead to an obvious increase in outage probability, suggesting the SDP and FAP can greatly impact the reliability performance of this system. This is because the increase of FAP leads to the increase of interference from PT, and thus it affects the decoding process in the relay, which reduces the total outage probability. Also, the increase of FAP means cognitive users will produce interference and affect the transmission between primary users. Therefore, the FAP must be limited in a proper range.

Figure 10 compares outage probability of the following three algorithms: Algorithm 1– MM selection scheme is
Table 1: Main notations and symbols used

| Notation | Description |
|----------|-------------|
| \(k_{ij}^{(k)}\) | Channel fading coefficient from node \(i\) to node \(j\) in phase \(k\) |
| \(P_{tr}\) | Transmit power of node \(i\) |
| \(D_{AB}\) | Set of relays decode \(x_A\) and \(x_B\) successfully |
| \(H_i\) | Status of licensed spectrum |
| \(\bar{H}_i\) | Status of licensed spectrum detected by node \(i\) |
| \(\gamma_i\) | Mean value of variable \(\gamma_i\) |
| \(A\) | Ratio of \(P_{tr}\) and \(P_X\) |
| \(\tilde{\gamma}_{KA}^{s_{KB}}\) | SNR at \(S_A\) and \(S_B\) |
| \(N\) | Number of relays |
| \(T\) | Threshold of SNR |
| \(P_{tr}^{PA}\) | Probability of that licensed spectrum is unoccupied by PT |
| \(P_{tr}^{PA}\) | Successful detection probability (SDP), false alarm probability (FAP) |
| \(\lambda_1, \lambda_2, \lambda_3, \lambda_4\) | Ratio of \(\sigma_{KB}^{2}, \sigma_{PA}^{2}, \sigma_{PB}^{2}\) and \(\sigma_{KA}^{2}\) |

considered under the three-phase system; Algorithm 2 – MM selection scheme is considered under four-phase system, where the selected relay transmits the superposition signal to \(S_A\) and \(S_B\) in two phases respectively; and Algorithm 3 – random relay selection is considered under the three-phase system. From this figure, we observe that the outage probability of the MM selection scheme with three-phase algorithm outperforms the outage probability with Algorithms 2 and 3. By comparing Algorithms 1 and 2, it can be seen that the three-phase transmission is more efficient than four-phase transmission. It is because in the third phase, the selected relay in Algorithm 1 transmits the superposition signal simultaneously, which improves the spectrum efficiency. Also, we can observe the superiority of MM selection scheme by comparing Algorithms 1 and 3.

5 | CONCLUSION

In this article, MM relay selection scheme is employed for improving the reliability of the cognitive two-way relaying network. On the back of the relay selection criterion, the exact close-form expression of outage probability is derived. Moreover, by analysing the outage probability under high SNR region and the result of MATLAB simulation, one can observe that there is an outage probability floor appearing in high SNR region, which is due to the fact that when SNR increases, the interference from PT becomes the dominant factor in the outage event. In addition, we draw the conclusion that it is not the value of each node's transmit power and each channel's CSI not only affects the result of outage probability, but even their ratios. Finally, it is found that that the transmit power ratio \(\alpha\), CSIs' ratio \(\lambda_1, \lambda_2, \lambda_3, \lambda_4\), successful detection probability \(P_{tr}\), false alarm probability \(P_f\), and the number of relays \(N\) can impact the reliability performance to some extent, which may provide a reference for further study.

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APPENDIX

The PDF of $y_{K^A}$ can be written as follows:

$$f_{y_{K^A}}(x) = \int_0^\infty f_{\min(y_{KA},y_{KB})}(x,z)f_{\min(y_{KA},y_{KB})}(z)dz \quad (31)$$

where the CDF and PDF of $\min(y_{KA},y_{KB})$ can be expressed as:

$$F_{\min(y_{KA},y_{KB})}(z) = \Pr\{\min(y_{KA},y_{KB}) < z\}$$

$$= 1 - \Pr(y_{KA} > z)\Pr(y_{KB} > z)$$

$$= 1 - \exp\left(-\frac{x}{\bar{y}}\right)$$

$$f_{\min(y_{KA},y_{KB})}(z) = \left(\frac{1}{\bar{y}}\right)\exp\left(-\frac{x}{\bar{y}}\right) \quad (33)$$

where $1/\bar{y} = 1/y_{KA} + 1/y_{KB}$.

According to the relay selection scheme in (8), we can get the CDF and PDF of $\min(y_{K^A},y_{KB})$ as:

$$F_{\min(y_{K^A},y_{KB})}(z) = \Pr\{\min(y_{K^A},y_{KB}) < z\}$$

$$= \Pr\{\max\{y_{KA},y_{KB}\} < z\}$$

$$= (1 - \Pr(y_{KA} > z)\Pr(y_{KB} > z))^k$$

$$= \left[1 - \exp\left(-\frac{z}{\bar{y}}\right)\right]^k \quad (34)$$

Refer to the derivation in [10], the joint CDF $F_{\min(y_{KA},y_{KB})}(x,z)$ can be expressed as (36). By observing (32), we note that $F_{\min(y_{KA},y_{KB})}(x,z)$ is not continuous along the x direction. Therefore, it can be obtained that

$$f_{\min(y_{KA},y_{KB})}(z) = \frac{k}{\bar{y}} \exp\left(-\frac{z}{\bar{y}}\right) \left[1 - \exp\left(-\frac{z}{\bar{y}}\right)\right]^{k-1}$$

$$= \sum_{i=1}^k \frac{k}{i} (-1)^{i-1} \frac{z^i}{i!} \exp\left(-\frac{z}{\bar{y}}\right) \quad (35)$$
\[ f_{\gamma_{K',A}}(x) = f_{\gamma_{K'A}}(x)f_{\gamma_{Kb}}(z) \int_{\gamma_{K'a}} f_{\gamma_{K'A}}(z)dz + f_{\gamma_{K'a}}(x)(1 - F_{\gamma_{Kb}}(x)) \int_{\gamma_{Kb}} f_{\gamma_{K'A}}(x)dz \]

\[ = \frac{1}{\gamma_{K'A}} \frac{1}{\gamma_{Kb}} \exp\left( -\frac{x}{\gamma_{K'A}} - \frac{x}{\gamma_{Kb}} \right) \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{i}{\gamma_{K'A}} \exp\left( -\frac{ix}{\gamma_{K'a}} \right)dz \]

\[ + \frac{1}{\gamma_{K'a}} \exp\left( -\frac{x}{\gamma_{K'A}} \right) \exp\left( -\frac{x}{\gamma_{Kb}} \right) \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{i}{\gamma_{K'A}} \exp\left( -\frac{ix}{\gamma_{K'a}} \right) \]

\[ = \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{i}{\gamma_{PA}} \frac{1}{\gamma_{Kb}(\gamma_{K'A} - \gamma_{Kb})} \exp\left( -\frac{x}{\gamma_{PA}} + \frac{x}{\gamma_{K'A}} \right) - \frac{1}{\gamma_{PA}} \left( \gamma_{PA} - \gamma_{PA} \exp\left( -\frac{ix}{\gamma_{K'A}} \right) \right) \]

\[ \times \left( \gamma_{K'A} \exp\left( -\frac{x}{\gamma_{PA}} + \frac{x}{\gamma_{K'A}} \right) - \frac{1}{\gamma_{PA}} \left( \gamma_{PA} - \gamma_{PA} \exp\left( -\frac{ix}{\gamma_{K'A}} \right) \right) \right) \]

\[ + \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{1}{\gamma_{PA}} \gamma_{K'A} \exp\left( -\frac{ix}{\gamma_{K'a}} \right) \left( \gamma_{PA} - \gamma_{PA} \exp\left( -\frac{ix}{\gamma_{K'A}} \right) \right) \]

\[ \Pr \left\{ \gamma_{K'A} \gamma_{PB}^{-1} \right\} = \Pr \left\{ \gamma_{K'A} \gamma_{PB} < x(\gamma_{PA} + 1) \right\} = \int_{0}^{\gamma_{PA}} \frac{1}{\gamma_{PA}} \exp\left( -\frac{x}{\gamma_{PA}} \right) \Pr \left\{ \gamma_{K'A} < x(t + 1) \right\} dt \]

\[ = \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{i}{\gamma_{PA}} \frac{1}{\gamma_{Kb}(\gamma_{K'A} - \gamma_{Kb})} \exp\left( -\frac{x}{\gamma_{PA}} + \frac{x}{\gamma_{K'A}} \right) - \frac{1}{\gamma_{PA}} \left( \gamma_{PA} - \gamma_{PA} \exp\left( -\frac{ix}{\gamma_{K'A}} \right) \right) \]

\[ \times \left( \gamma_{K'A} \exp\left( -\frac{x}{\gamma_{PA}} + \frac{x}{\gamma_{K'A}} \right) - \frac{1}{\gamma_{PA}} \left( \gamma_{PA} - \gamma_{PA} \exp\left( -\frac{ix}{\gamma_{K'A}} \right) \right) \right) \]

\[ + \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{1}{\gamma_{PA}} \gamma_{K'A} \exp\left( -\frac{ix}{\gamma_{K'a}} \right) \left( \gamma_{PA} - \gamma_{PA} \exp\left( -\frac{ix}{\gamma_{K'A}} \right) \right) \]
where $\delta(\cdot)$ is the impulse function. By substituting (33), (35), (37) into (31), the PDF of $\gamma_{K A}$ can be derived as (38), and the CDF can be written as

$$F_{\gamma_{K A}}(x) = \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{i \bar{\gamma}}{\bar{\gamma}_{KB}(\bar{\gamma}_{KA} - \bar{\gamma})} \times \left( \bar{\gamma}_{KA} \left( 1 - \exp\left(-\frac{x}{\bar{\gamma}_{KA}}\right) \right) - \frac{\bar{\gamma}}{i} \left( 1 - \exp\left(-\frac{ix}{\bar{\gamma}}\right) \right) \right) + \sum_{i=1}^{k} (ki)(-1)^{i-1} \frac{\bar{\gamma}}{\bar{\gamma}_{KA}} \times \left( 1 - \exp\left(-\frac{ix}{\bar{\gamma}}\right) \right).$$

(41)

Then utilising (41), $F_{\gamma_{KB}}(x)$ can be derived as (39).

Similar to the derivation of $F_{\gamma_{K A}}$ and $F_{\gamma_{KB}}(x)$, $F_{\gamma_{KB}}(x)$ and $F_{\gamma_{KB}}$ can be derived as (40) and (42).

$$F_{\gamma_{KB}}(x) = \sum_{i=1}^{k} \binom{k}{i} (-1)^{i-1} \frac{i \bar{\gamma}}{\bar{\gamma}_{KA}(\bar{\gamma}_{KB} - \bar{\gamma})} \times \left( \bar{\gamma}_{KB} \left( 1 - \exp\left(-\frac{x}{\bar{\gamma}_{KB}}\right) \right) - \frac{\bar{\gamma}}{i} \left( 1 - \exp\left(-\frac{ix}{\bar{\gamma}}\right) \right) \right) + \sum_{i=1}^{k} (ki)(-1)^{i-1} \frac{\bar{\gamma}}{\bar{\gamma}_{KB}} \times \left( 1 - \exp\left(-\frac{ix}{\bar{\gamma}}\right) \right).$$

(42)