It is shown that parity operator plays an interesting role in Dirac equation in (1+2) dimensions and can be used for defining chiral currents. It is shown that the “anomalous” current induced by an external gauge field can be related to the anomalous divergence of an axial vector current which arises due to quantum radiative corrections provided by triangular loop Feynman diagrams in analogy with the corresponding axial anomaly in (1+3) dimensions. It is shown that the non-conservation of “chiral charge” due to anomaly is related with the topological Chern-Simons charge.

As an application pair creation of massless fermions is discussed in electric field.

The Dirac equation for a massless particle in (1+2) dimensions is [2]
\[ i(\gamma^\mu \partial_\mu)\psi = 0 \]  
(1)
where
\[ \partial_0 = \frac{1}{c} \frac{\partial}{\partial t} \]  
(2)

[we may replace \( c \) by fermi velocity \( v_f \) for a possible application to graphene] and
\[ \hat{\phi} = \gamma^\mu \partial_\mu = \gamma^0 \partial_0 + \gamma^1 \partial_1 + \gamma^2 \partial_2 \]  
(3)

We will use natural units and put \( c = 1 \)

Now it is known [3] that in 3 space-time dimensions there exists two inequivalent representations for \( \gamma \)-matrices (this is true for any odd number of space-time dimensions), which corresponds to the choice of sign in \( \gamma^0 \gamma^1 \gamma^2 = \pm iI_2 \) where \( I_2 \) is the two dimensional identity matrix. An explicit matrix realization of these two representations is given by
\[ \gamma^0 = \sigma^3, \ \gamma^1 = i\sigma^1, \ \gamma^2 = i\sigma^2 \]
\[ \gamma^0 = \sigma^3, \ \gamma^1 = i\sigma^1, \ \gamma^2 = -i\sigma^2 \]  
(4)

Using the first representation Eq. (1) takes the form
\[ i (\sigma^3 \partial_0 + i\sigma^1 \partial_1 + i\sigma^2 \partial_2) \psi_+ = 0 \]  
(5)
One can take the second representation for the Dirac equation at a space-time point which is obtained by the parity operation:
\[ x \rightarrow x^p = (x^0, x^1, -x^2) \]
to obtain
\[ i\hat{\phi}\psi_- = 0 \]  
(6)
where
\[ \bar{\psi} = \sigma^3 \partial_0 + i \sigma^1 \partial_1 - i \sigma^2 \partial_2 \]

If Eq.(5) is to be invariant under parity, we should be able to write it as
\[ i [ \sigma^3 \partial_0 + i \sigma^1 \partial_1 + i \sigma^2 \partial_2 ] \psi^P_+(x^P) = 0 \]

(7)

Now since there is no matrix M that simultaneously anti-commute with \( \sigma^2 \) and commute with \( \sigma^1 \) and \( \sigma^3 \), we can not find a relation of type
\[ \psi^P_+(x^P) = M \psi_+(x) \]

so as to get Eq. (7) back to Eq. (5). One might therefore argue that the theory can not be invariant under P. However comparing Eq.(7) with Eq.(6), namely,
\[ i [ \sigma^3 \partial_0 + i \sigma^1 \partial_1 - i \sigma^2 \partial_2 ] \psi - = 0 \]

(8)

one sees that a very natural assumption would be that
\[ \psi_A - (x) = - \eta_\mu \psi_+ (x) \]

\[ \psi_B - (x) = - \eta_\mu \psi_+(x) \]

(9)

Thus to preserve parity invariance, we have to take two representations together; in that case the parity conserving Lagrangian is
\[ \mathcal{L} = \bar{\psi}_+ (i \partial) \psi_+ + \bar{\psi}_- (i \bar{\partial}) \psi_- \]

(10)

It is convenient to transform to new fields[2]
\[ \psi_A = \psi_+ \]
\[ \psi_B = i \gamma^2 \psi_+ \]

(11)

The Lagrangian (10) can then be written as
\[ \mathcal{L} = \bar{\psi}_A (i \gamma^\mu \partial_\mu) \psi_A + \bar{\psi}_B (i \gamma^\mu \partial_\mu) \psi_B \]

(12)

It is instructive to put mass term in the Lagrangian (12), which one can always put equal to zero:
\[ \mathcal{L} = \bar{\psi}_A (i \gamma^\mu \partial_\mu) \psi_A + \bar{\psi}_B (i \gamma^\mu \partial_\mu) \psi_B - m ( \bar{\psi}_A \psi_A - \bar{\psi}_B \psi_B ) \]

(13)

where under the parity operation
\[ \psi^P_{A,B}(x^P) = \eta_\mu \sigma^2 \psi_{B,A}(x) \]

(14)

The peculiarity of the parity transformations (14) is that it changes A states into B states. In fact this doubling of two components spinors was noted in a pioneering work on gauge theories in (1+2) dimensions (e.g. QED3) [4,5].

The Hamiltonian density is
\[ \mathcal{H} = [ \bar{\psi}_A (- i \gamma^\mu \partial_\mu) \psi_A + \bar{\psi}_B (- i \gamma^\mu \partial_\mu) \psi_B + m ( \bar{\psi}_A \psi_A - \bar{\psi}_B \psi_B ) ] \]

(15)

The Hamiltonian density has the so called conjugate symmetry [6], \( \psi_A \leftrightarrow \sigma^3 \psi_B \), in the sense that \( \mathcal{H} \rightarrow - \mathcal{H} \).

It may be noted that the Lagrangian (12) is invariant, even in the presence of the mass term, under two independent transformations
\[ \psi_A \rightarrow e^{i \alpha_A} \psi_A, \quad \psi_B \rightarrow e^{i \alpha_B} \psi_B \]

(16)

where \( \alpha_A \) and \( \alpha_B \) are real, and has thus \( U_A(1) \otimes U_B(1) \) symmetry. The corresponding conserved currents are
\[ J^\mu_A = \bar{\psi}_A \gamma^\mu \psi_A, \quad J^\mu_B = \bar{\psi}_B \gamma^\mu \psi_B \]

(17)

One can form even (odd) combination corresponding to "vector" ("axial vector") under parity
\[ J^\mu_\pm = 1/2 [ \bar{\psi}_A \gamma^\mu \psi_A \pm \bar{\psi}_B \gamma^\mu \psi_B ] \]

(18)
An external gauge field $A_\mu$ (electromagnetic) can be introduced by replacing the ordinary derivative by the covariant derivative ($-e, e > 0$ is the electronic charge, we have put $c=1$)

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu$$

So the gauge invariant Lagrangian obtained from Eq. (13) is

$$L = \overline{\psi}_A (i\gamma_\mu D_\mu) \psi_A + \overline{\psi}_B (i\gamma_\mu D_\mu) \psi_B - m(\overline{\psi}_A \psi_A - \overline{\psi}_B \psi_B)$$

(19)

This gives the Dirac equation in (1+2) dimensions

$$[i\gamma_\mu D_\mu \mp m] \psi_{A, B} = 0$$

(20)

To get physical insight of Eq. (20) we multiply on left by $(-i\gamma_\nu D_\nu \mp m)$ to put the resulting equation in the Pauli form

$$[D_\mu D_\mu - e^2 \sigma_{\mu\nu} F_{\mu\nu} + m^2] \psi_{A, B} = 0$$

(21)

This equation differs from the Klein Gorden equation in the term $e\sigma_{\mu\nu} F_{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Using

$$\sigma_{\mu\nu} = \epsilon^{\lambda\mu\nu} \gamma_\lambda$$

(22)

one can write

$$-\frac{e}{2} \sigma_{\mu\nu} F_{\mu\nu} = 1/2 f^\lambda \gamma_\lambda$$

(23)

where $f^\lambda$ is current induced by the external gauge field $A_\mu$ [4, 6, 7]

$$f^\lambda = -e\epsilon^{\lambda\mu\nu} F_{\mu\nu}$$

(24)

and has abnormal parity. The corresponding induced charge is

$$Q = \int d^2 x \ f^0(x)$$

(25)

where

$$f^0(x) = -e\epsilon^{ij} F_{ij} = 2e(\vec{\nabla} \times \vec{A})^3 = 2eB$$

(26)

Here $B$ is the magnetic field perpendicular to the $x-y$ plane. Thus $Q = 2e\Phi$, where $\Phi$ is the magnetic flux.

The parity conserving Lagrangian which gives Eq. (21) is

$$L = \overline{\psi}_A (D_\mu D_\mu + m^2) \psi_A - \overline{\psi}_B (D_\mu D_\mu + m^2) \psi_B + 1/2[\overline{\psi}_A \gamma_\mu \psi_A - \overline{\psi}_B \gamma_\mu \psi_B] f_\mu$$

(27)

so that $f_\mu$ is coupled with the current $J_\mu$ given in Eq. (18) [6]. The Lagrangian is invariant under $U_A(1) \otimes U_B(1)$.

We now discuss the dynamics responsible for the generation of the mass introduced above. We consider a model [8] similar to Nambu-Jona-Lasino, which has the Lagrangian density invariant under the chiral transformations (16) and parity:

$$L = \overline{\psi}_A (i\gamma_\mu \partial_\mu) \psi_A + \overline{\psi}_B (i\gamma_\mu \partial_\mu) \psi_B + \frac{G}{2}[\overline{\psi}_A \psi_A - \overline{\psi}_B \psi_B]^2 + (\overline{\psi}_A \psi_B + \overline{\psi}_B \psi_A)^2 + (i(\overline{\psi}_A \psi_B - \overline{\psi}_B \psi_A))^2$$

(28)

Note that the 1st term in square bracket is invariant by itself but the other two terms must come together to preserve the invariance under the transformations (16) [we now treat $\alpha_A$ and $\alpha_B$ as infinitesimal]. We now break the chiral symmetry by giving non-zero expectation values to operators $\overline{\psi}_A \psi_A - \overline{\psi}_B \psi_B$ and $(i(\overline{\psi}_A \psi_B - \overline{\psi}_B \psi_A))$ [the parity conservation forbids the vacuum expectation value of $[(\overline{\psi}_A \psi_B + \overline{\psi}_B \psi_A)]$, thereby generating the mass term

$$-m(\overline{\psi}_A \psi_A - \overline{\psi}_B \psi_B)$$

(29)

and the transition mass

$$-m_T i(\overline{\psi}_A \psi_B - \overline{\psi}_B \psi_A)$$

(30)
Here

\[
m = -G(\bar{\psi}_A \psi_A - \bar{\psi}_B \psi_B)
\]
\[
m_T = -G(\bar{i} \psi_A \psi_B - \bar{\psi}_B \psi_A)
\]  

(31)

It is the term (30) which breaks the chiral symmetry spontaneously. Then \(J_-^\mu\) is no longer conserved and

\[
\partial_\mu J_-^\mu = -m_T(\bar{\psi}_A \psi_B + \bar{\psi}_B \psi_A)
\]  

(32)

Next we discuss whether \(f_\mu\) can be related to "anomalous" divergence of some axial current, which arises due to quantum corrections. Here analogy with the axial vector "anomalous" divergence in \((1+3)\) dimensions, namely [9]

\[
\partial_\mu J_5^\mu = \frac{e^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}
\]
\[
= \frac{e^2}{4\pi^2} \vec{E}.\vec{B}
\]  

(33)

is useful. As is well known this divergence arises from quantum corrections provided by triangle graph which has two vector vertices and one axial vector vertex or that provided by its divergence

\[
\partial_\mu J_5^\mu = m(\bar{\tau}_5 u - \bar{d} t\gamma_5 d)
\]

(34)

where \(u\) and \(d\) denote up and down quarks (\(m\) is the quark mass), which provide the internal legs of the triangle. Note that although quark mass \(m\) appears above, but Eq. (33) is independent of quark mass. Indeed in \((1+2)\) dimensions if we compare Eq. (32) with Eq. (34) we have corresponding \(\psi_A\) and \(\psi_B\) fields, which appear in the Lagrangian (12) or Hamiltonian (15).

We may remark here that usually an anomaly arises when the quantum calculation breaks a classical symmetry. In perturbative calculations this occurs when the regulator breaks the symmetry in some way. So, for instance, for the triangle anomaly in \((1+3)\) dimensions, the Pauli-Villars regularization breaks chiral invariance due to mass term for regulator field [reflected also in Eq. (34)]. We may also mention here that if the Lagrangian (19) is replaced by

\[
\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu)\psi - m\bar{\psi}\psi
\]

(35)

then it has been shown [10] that the mass term and Chern-Simons term induce each other : \(<m\bar{\psi}\psi>=<J_\mu>_m A_\mu\) with \(<J_\mu>_m=\frac{m e^2}{4\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\nu\rho}\), giving rise to parity violation in \((1+2)\) dimensions. However, if one uses the Lagrangian (19), there is no Chern Simons term since the sign of fermion mass terms cancel between them any contribution in this term [11].

Coming back to our case if \(m_T\) were absent there would be no anomaly. It is the presence of \(m_T\) which gives rise to anomaly as we now show. Noting from Eq. (32) that \(\partial_\mu J_5^\mu\) involves \(m_T\bar{\psi}_A \psi_B\) and \(m_T\bar{\psi}_B \psi_A\), the relevant Feynman graphs are shown in Fig 1.

![Triangle diagrams](image)

FIG. 1: Triangle diagrams for "anomalous" current divergence. The \(\times\) denote the vacuum expectation value \(<i(\bar{\psi}_A \psi_B - \bar{\psi}_B \psi_A)>\).

Note that it is essential to put mass transitions shown (see analogy with Majorana neutrinos), such mass transitions are provided by Eq.(30). Noting that A and B propagators involve opposite masses [we take \(m_T = m\)], the matrix elements are given by

\[
T_{\mu\nu} = m e^2 \int \frac{d^Dl}{(2\pi)^D} \left[ Tr \left\{ \frac{1}{l+k_1-m} \gamma_\mu \frac{1}{l-m} \gamma_\nu (\frac{1}{l-k_2-m} \frac{1}{l-k_2+m}) + \frac{1}{l+k_1+m} \gamma_\mu \frac{1}{l+m} \gamma_\nu (\frac{1}{l-k_2+m} \frac{1}{l-k_2-m}) \right\} \right]
\]
\[
= m^2 e^2 \int \frac{d^Dl}{(2\pi)^D} \left[ Tr \left\{ \frac{1}{(l+k_1+m)\gamma_\mu (l+m)\gamma_\nu -(l+k_1-m)\gamma_\mu (l-m)\gamma_\nu}{(l+k_1)^2-m^2}(l^2-m^2)(l-k_2)^2-m^2} \right\} \right] + \frac{k_1 \leftrightarrow k_2}{\mu \leftrightarrow \nu}
\]  

(36)
The numerator in the integral which contributes is,
\[
N^{\mu\nu} = 2m \, Tr [\gamma^\rho \gamma^\mu \gamma^\nu (l + k_1)_\rho + \gamma^\rho \gamma^\nu (l + k_1)_\rho]
\]
\[
= -4mi [\epsilon^{\mu\nu} (l + k_1)_\rho + \epsilon^{\mu\nu} l_\rho]
\]
\[
= -4mi \epsilon^{\mu\nu} k_1 \rho
\]  
(37)

Using the Feynman parametrization, the denominator takes the the form,
\[
[(l + k_1 x - k_2 y)^2 - \Delta]^3
\]
where
\[
\Delta = m^2 - x k_1^2 - y k_2^2 + (x k_1 - y k_2)^2
\]
Making the shift \( l \rightarrow l - (k_1 x - k_2 y) \), the denominator becomes \( (l^2 - \Delta)^3 \) and the dimensional regularization gives \( D=3 \)
\[
T_{\mu\nu} = -4m^3 e^2 i e^{\mu\nu\rho} k_1 \rho \int_0^1 dx \int_0^{1-x} dy \frac{(-1)^3 i}{(4\pi)^{D/2}} \frac{\Gamma(3 - D/2)}{\Gamma(3)} \frac{1}{\Delta}^{3-D/2}
\]
\[
+ \frac{k_1 \leftrightarrow k_2}{\mu \leftrightarrow \nu}
\]  
(38)

For photons on the mass shell
\[
k_1^2 = 0, \quad k_2^2 = 0
\]
and putting \(2k_1 k_2 = q^2, \Delta = m^2 - q^2 xy\). Thus we obtain, neglecting terms of order \(q^2\),
\[
T_{\mu\nu} = \frac{e^2}{16\pi} \frac{m^3}{(m^6)^{1/2}} e^{\mu\nu\rho} k_1 \rho + \frac{k_1 \leftrightarrow k_2}{\mu \leftrightarrow \nu}
\]
\[
= \frac{e^2}{16\pi} (\text{sign } m) e^{\mu\nu\rho} (k_1 - k_2)_\rho
\]  
(39)

Thus finally
\[
\partial_\mu J^\mu = \frac{e^2}{16\pi} (\text{sign } m) \epsilon_{\mu}(k_1) \epsilon_\nu(k_2) e^{\mu\nu\rho} (k_1 - k_2)_\rho
\]  
(40)

It may be noted that in (1+3) dimensions \(\partial_\mu J^\mu\) is independent of \(m\); in (1+2) dimensions, the corresponding quantity \(\partial_\mu J^\mu\) is also independent of the magnitude of \(m\) but does depend on its sign (which is typical for odd space-time dimensions). In both cases mass was used as a regulator. In configuration space Eq. (40) takes the form
\[
\partial_\mu J^\mu = \frac{e^2}{16\pi} (\text{sign } m) A_\lambda f^\lambda = \frac{e^2}{16\pi} (\text{sign } m) e^{\mu\nu\lambda} F_{\mu\nu} A_\lambda
\]  
(41)

Note that \(e^{\mu\nu\lambda} F_{\mu\nu} A_\lambda\) is the Chern-Simons term in (1+2) dimensions. Then for the chiral charge
\[
N = N_A - N_B = \int d^2 x J_0^\mu
\]  
(42)

we have
\[
\Delta N = N(\infty) - N(-\infty) = \int_{-\infty}^{\infty} dt \partial_0 \int d^2 x J_0^\mu(t, \vec{x})
\]
\[
= \int d^3 x \partial_\mu J_\mu
\]  
(43)

where we have used the Gauss’s theorem and set the surface integral equal to zero. Thus
\[
\Delta N \neq 0 = \nu,
\]  
(44)
where
\[ \nu = \frac{e^2}{16\pi} (\text{sign } m) \int d^3x \epsilon^{\mu\nu\lambda} F_{\mu\nu} A_\lambda \] (45)

Note that the integral in Eq. (45) is the Chern-Simons term. Thus the non-conservation of chiral charge is related with the topological Chern-Simons charge. In general the topological connection with Chern-Simons term is discussed in [4,5].

To discuss a possible application of “anomaly” we write it in terms of electric and magnetic fields:
\[ \epsilon^{\mu\nu\lambda} F_{\mu\nu} A_\lambda = -2A^0 B^3 - 2(\vec{A} \times \vec{E})^3 = -2BA_0 - B\vec{E} \cdot \vec{r} \] (46)

where \( B \) is constant and along z-axis and we have used \( \vec{A} = \frac{1}{2} \vec{B} \times \vec{r} \) so that \( \vec{B} = \vec{\nabla} \times \vec{A} \). One can select a gauge in which \( A_0 = 0 \) then Eq. (41) becomes
\[ \partial_\mu J_\mu = -\frac{e^2}{16\pi} (\text{sign } m) B(\vec{E}, \vec{r}) \] (47)

(compare it with second line \( (\vec{E}, \vec{B}) \) of Eq. (33) for axial anomaly in (1+3)). We now discuss the pair creation of massless fermions in an axially symmetric electric flux tube \( \vec{E} \) under a homogeneous magnetic field \( B \) perpendicular to the x-y plane in which the tube lies [12]. Assuming spatial homogeneity of \( \vec{J} \) so that \( \vec{\nabla} \cdot \vec{J} = 0 \), the anomaly Eq. (46) gives
\[ \partial_t (n_B - n_A) = \frac{e^2}{16\pi} (\text{sign } m) \vec{E}(t) \cdot \vec{r} B \] (48)

where \( n_B \) and \( n_A \) denote number density of B type and A type fermions: \( \psi_B^\dagger \psi_B \) and \( \psi_A^\dagger \psi_A \). Further from \( \partial_\mu J_\mu = 0 \) \( \partial_t (n_B + n_A) = 0 \), so that [sign \( m \) is understood]
\[ 2\partial_t n = \frac{e^2}{16\pi} \vec{E} \cdot \vec{r} B \] (49)

where we have used \( \partial_t n_B = -\partial_t n_A = \partial_t n \). Now the Fermi momentum \( p_F \) is given by
\[ p_F = \int \frac{dp_F}{dt'} dt' = -e \int_0^t E(t') dt' \] (50)

The energy density of particles is then
\[ \epsilon(t) = n(t)p_F(t) \] (51)

Using the axial symmetry for the cylindrical coordinates \( (r, \phi, z) \), with \( \vec{B} = B\hat{z}, E = E_r, E_\phi = 0 \), the Maxwell’s equations give
\[ \frac{\partial B}{\partial t} = 0, \quad \frac{\partial E}{\partial t} = -J \] (52)

To proceed further, we notice that in relativistic electrodynamics in (1+2) dimensions, for the action
\[ S = \int d^2x dt L_{\text{em}} \] (53)

to be dimensionless, \( L_{\text{em}} \) has to be [13]
\[ L_{\text{em}} = -\frac{1}{2} (B^2 - \vec{E}^2)^\frac{3}{4} \] (54)

the dimensions (in natural units) of \( A^\mu, \vec{B} \) and \( \vec{E} \) being respectively \( L^{-1}, L^{-2}, L^{-2} \). The electromagnetic energy density is then given by
\[ H_{\text{em}} = \frac{1}{2} (B^2 + \frac{1}{2} E^2)(B^2 - E^2)^{-\frac{3}{4}} \] (55)
which has the right dimensions. The energy conservation then gives [using Eqs. (52)]

\[
\frac{\partial}{\partial t} \int d^2 x \left[ \frac{1}{2}(B^2 + \frac{1}{2}E^2)(B^2 - E^2)^{-\frac{3}{4}} + \epsilon \right] + \frac{\partial E}{\partial t} = 0
\]

We will now assume that \(B >> E\), then

\[
\int d^2 x \left[ -\frac{3}{4}JEB^{-\frac{3}{4}} + \frac{\partial E}{\partial t} \right] = 0
\] (56)

But from Eqs. (49), (50) and (51)

\[
\frac{\partial E}{\partial t} = \partial \kappa n(t)p_F(t) + n(t)\partial p_F(t) = \frac{e^2}{32\pi} rBp_F - n(t)eE(t)
\] (57)

so that from Eq. (56)

\[
\int d^2 x \left[ -\frac{3}{4}JEB^{-\frac{3}{4}} + \frac{\partial E}{\partial t} \right] = 0
\]

implying

\[
J(r, t) = \frac{4}{3}B^{\frac{3}{2}} [-en(r, t) + \frac{e^2}{32\pi} rBp_F]
\] (58)

The use of Eqs. (49) and (50) allows us to write

\[
\frac{e^2}{32\pi} rBp_F = \frac{e^2}{32\pi} \int_0^t dt' (-e)E(r, t')rB
\]

\[
= -e \int_0^t dt' \partial_t n(r, t')
\]

\[
= -en(r, t)
\] (59)

with \(n(r, 0) = 0\). Thus finally Eq. (58) gives

\[
J(r, t) = -\frac{8}{3}eB^{\frac{3}{2}}n(r, t),
\] (60)

so that

\[
\frac{\partial E}{\partial t} = \frac{8}{3}eB^{\frac{3}{2}}n(r, t)
\] (61)

We have to solve the Eqs. (49) and (61), which give

\[
\frac{\partial^2 E}{\partial t^2} = \frac{e\alpha}{3}B^{\frac{3}{2}}Er
\]

where \(\alpha = \frac{e^2}{4\pi}\), the fine structure constant. This has the solution

\[
E(r, t) = E_0(r) \cos \left( \sqrt{\frac{e\alpha B^{\frac{3}{2}}r}{3}} t \right)
\] (62)

and then from Eq. (61),

\[
en(r, t) = \frac{3}{8} E_0(r) \left( \frac{1}{B^{\frac{3}{2}}} \left( \frac{e\alpha B^{\frac{3}{2}}r}{3} \right) \right)^{\frac{1}{2}} \sin \left( \sqrt{\frac{e\alpha B^{\frac{3}{2}}r}{3}} t \right)
\] (63)
FIG. 2: (a) Electric field $E(r)/E_0$ at $t/t_c=0.2$, 0.6 and 1. (b) $e_n(r)/E_0$ at $t/t_c=0.2$, 0.6 and 1.

We note that even if we start with spatially independent electric field $E_0$ at $t=0$, it develops $r$-dependence subsequently due to the anomaly equation (49). This is the main difference from (1+3) dimensions [12]. The natural length in the system is the magnetic length $l_B = (\frac{1}{eB})^{\frac{1}{2}}$, which also appears in the Landau levels. Then [we take $E_0$ independent of $r$]

$$\frac{E(r,t)}{E_0} = \cos\left(\sqrt{\frac{\alpha}{3e^2 l_B t_B}} \frac{r}{t}\right)$$

and

$$\frac{e_n(r,t)}{E_0} = \frac{3}{8} \left(\frac{\alpha}{3e^2 l_B t_B}\right)^{\frac{1}{2}} \left| \sin\left(\sqrt{\frac{\alpha}{3e^2 l_B t_B}} \frac{r}{t}\right) \right|$$

where $t_B = (\frac{1}{eB})^{\frac{1}{2}} = l_B$ in our units.

We can see from Eqs. (64) and (65) that $E(r,t)$ and $n(r,t)$ oscillate with time for given $r$. If the particles are confined within the radius $R(R >> l_B)$ such that $E(R,t_c) = 0$, where $t_c$ may be roughly defined for the life time of the electric field [12], so that $t_c = \frac{\pi}{2} l_B \left(\frac{\alpha R}{3e^2 l_B}\right)$. Then we can write Eqs. (64)and (65) as follows

$$\frac{E(r,t)}{E_0} = \cos\left(\frac{\pi}{2} \left(\frac{r}{R}\right)^{\frac{1}{2}} \frac{t}{t_c}\right)$$

and

$$\frac{e_n(r,t)}{E_0} = \frac{3}{8} \left(\frac{\pi}{2} \left(\frac{r}{R}\right)^{\frac{1}{2}} \frac{t}{t_c}\right)$$

These are plotted in Figs. 2a and 2b for $\frac{1}{t_c} = 0.2, 0.6$ and 1. The behavior shown in figs (2a) and (2b) may be testable and might find applications in quark gluon plasma when E and B are regarded color electric and magnetic fields in going from QED to QCD [12]. The above considerations might have applications in graphene [13] [where c is to be replaced by $v_f$].

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