The PBD Model Based Simulation for Soft Tissue Deformation in Virtual Surgery

Yanan Liu, Chengbin Guan*, Jianhui Li, Xueli Yu and Shuang Zhang

School of Information, Management and Engineering, Neusoft Institute Guangdong, Foshan, Guangdong, China
Email: liuyanan@nuit.edu.cn; guanchengbin@nuit.edu.cn

Abstract. With the development of computer technology, surgical simulators based on virtual reality have been widely studied, and the key technology of virtual surgery simulation is deformation. The simulation of soft tissue deformation mainly includes two aspects: (1) Using the appropriate physical model for deformation calculation; (2) The virtual surgery simulation has collision detection, force feedback and so on. In the design of deformation algorithm, the above two problems should be considered at the same time to ensure that the system has high real-time and authenticity. Aiming at the above problems, this paper used the method of position based dynamics (PBD), which can effectively simulate the effect of soft tissue deformation.

Keywords. Soft tissue; deformation; PBD; simulation.

1. Introduction
The object of virtual surgery simulation is usually soft tissue of 3D. Researchers usually use finite element method (FEM) or mass spring method (MSM) as computational methods to simulate deformation. The FEM is difficult to satisfy the real-time requirement because of its large computation, and the MSM is easy to produce shaking, which reduces the stability and authenticity of the system [1].

PBD model is a new method for simulating deformable objects. This method was originally used to simulate 2D objects such as cloth and so on. It has been packaged by NVIDIA in the physics engine PhysX [2] which used to simulate 3D objects. The computational complexity of the method is much smaller than that of the FEM, and the authenticity and maneuver ability of the method are higher than that of the MSM. However, the proposed method does not support topology changed at present, so no researchers have used this method in virtual surgery systems which support cutting operations.

In this paper, PBD model is used to simulate deformation of the soft tissue of 3D. On the basis of traditional PBD model, it used tetrahedral model, which is prepared for the simulation of the cutting operation in the future. The system realizes the soft tissue deformation simulation and improves the authenticity of the simulation.

2. Related Work and Methods
Deformation simulation is critical in virtual surgery. Although many people have done in-depth research, deformation simulation is still a challenging problem so far. In the research of deformable object simulation, force-based dynamics and position-based dynamics are the two most widely used models [3]. The dynamic model based on force mainly includes the MSM and the FEM, which would cause instability in the case of explicit integration. However, the PBD model is becoming more and
more popular because of its robustness and high efficiency. It is characterized by directly control position of vertices, eliminating the instability of dynamic model based on force, and simplifying the calculation process.

2.1. Mass-spring Model
MSM is the simplest and fastest physical deformation method [4]. The model consists of many particles, each of which has its mass, position and velocity. In the spatial position, adjacent particles are connected by a spring that combines all the particles in the model into a network, as shown in figure 1 [5].

Each spring is considered to be weightless and follows Hooke’s law. In the simulation, the external force and the elastic tensile force generated by the spring are calculated for each particle, and the acceleration of the particle is calculated by Newton’s second law of motion. Finally, the number of particles and the new position of the particle are obtained.

According to different methods of solving position and velocity, the particle spring model is divided into explicit Euler method, Runge-Kutta methods, implicit Euler method. In short, the mass spring model has a simple structure and high computational efficiency.

![Figure 1. A network of particle spring model.](image)

2.2. Finite Element Model
The FEM is a kind of continuous parameterized model, so it has high precision, the adaptability of the finite element algorithm is strong, and it can be applied to the simulation of various geometric structures and material characteristics. And it can accurately describe the deformation of the tissue in the case of large deformation [6]. The FEM discretized the soft tissue into small cells and calculated the stress and strain of each cell. Stress measures the force of the cell in each direction, and the strain describes the change of the vertex position in the cell. The base of the change of the position is the position of the static state. The FEM has high precision, but the large amount of calculation becomes the biggest obstacle to its application in real-time simulation [7].

2.3. Meshless Model
The method is suitable for large deformation and it is an extension of the FEM. Steinmann et al. [8] proposed a cutting algorithm based on meshless discretization of deformation field, which used visibility graph to update shape. The deformation method to preserve physical properties is an important research content of virtual surgery.

In 2005, Muller et al. [9] used a particle system based on meshless and it is without any pre-treatment or auxiliary data structure, with highly efficient calculation and unconditional stability. Zou et al. [10] used meshless matching algorithm to calculate the position of each particle at each time step, which can realize real-time simulation of linear, quadratic and plastic deformation of soft tissue.

2.4. PBD Model
This model was first proposed by Muller et al. in 2007 [3]. This method ignored the velocity in the deformation based force method and directly acted on the position of particles in the deformed body, thus result of soft tissue deformation simulation achieving relatively stable.
In a word, the PBD model compared with particle spring model improves the system instability and particle caused by superelastic. Compared with other models, the advantages of PBD model is small calculation and real deformation simulation and is widely adopted by the industry.

3. Simulation

3.1. The Principle of PBD

The core pseudo-code of PBD is shown in table 1.

Table 1. The core pseudo-code of PBD.

```
1: for all vertices \textit{i} do
2: \texttt{initialize} \texttt{x}_i = x^0, v_i = v^0, \texttt{w}_i = 1/m_i
3: end for
4: loop
5: for all vertices \textit{i} do \texttt{v}_i \leftarrow \texttt{v}_i + \Delta t \texttt{w}_i f_{ext}(x_i)
6: for all vertices \textit{i} do \texttt{p}_i \leftarrow \texttt{+} \Delta t \texttt{v}_i
7: for all vertices \textit{i} do genCollConstraints (x_i \rightarrow p_i)
8: loop solver Iteration times
9: project Constraints (C_1 \cdots C_{M+M_{coll}} \texttt{p}_1 \cdots \texttt{p}_N)
10: end loop
11: for all vertices \textit{i} do
12: \texttt{v}_i \leftarrow (\texttt{p}_i - \texttt{x}_i)/\Delta t
13: \texttt{x}_i \leftarrow \texttt{p}_i
14: end for
15: velocity Update (v_1 \cdots v_N)
16: end loop
```

The initial value of the particle’s position \texttt{x} and velocity \texttt{v} needs to be specified in (1) - (3) before the simulation cycle begins. In line (5) - (6), a simple Explicit Euler Method integration is performed for the velocity and position. The value of \texttt{p}_i is not directly specified, but is used to predict the position of the particles in the next time step. Line (7) sets the external constraint functions such as collision detection. Where \texttt{x}_i and \texttt{p}_i are used to perform continuous collision detection. After the line (8) - (10) through the loop correction \texttt{p}_i position, made it meet external constrains \texttt{M_{coll}} and internal constrains \texttt{M}, it is not hard to see from the above steps, compared with the method based on the mechanics of deformation, PBD is unconditionally stable [11], by modifying the position and velocity of each particle, the particles’ position had been changed.

Adding appropriate damping force in the process of circulation can improve the stability of the system. On the one hand, damping force plays an active role in the system, which increases the stability of the system by reducing the turbulence amplitude of particle position. On the other hand, damping changes the dynamic motion of the simulated object, and the resulting effect is ideal, such as the reduction of the vibration amplitude of the deformable object, or the change of the linear or angular momentum of the deformable object. To maintain the linear and angular moment of a deformable object, a symmetric damping force, commonly called a spring damping force can be used. Ruediger et al. [12] used an iterative spring damping method to further improve the stability of the system.

In the process of solving, the constraint function can be written as follows:

$$C(x + \Delta x) = C(x) + \nabla C(x) \cdot \Delta x + O(|\Delta x|^2) = 0$$  \hspace{1cm} (1)$$

As the introduction of a global variable \( \lambda \) to fix \( \nabla C(x) \) in the same direction
\[ \Delta x = \lambda \nabla x C(x) \] (2)

The \( \lambda \) can be solved

\[ \lambda = -\frac{C(x)}{|\nabla x C(x)|^2} \] (3)

Finally, the expression of \( \Delta x \) is

\[ \Delta x = -\frac{C(x)}{|\nabla x C(x)|^2} \nabla x C(x) \] (4)

For each particle \( i \), its displacement change is

\[ \Delta x_i = -\frac{w_i C(x)}{\sum w_j |\nabla x_j C(x)|^2} \nabla x_i C(x) \] (5)

3.2. Stretch Constraint

The deformation of soft tissue is mainly affected by the stretch constraint, user may make position of particles changed compared with original position, by interacting with virtual soft tissue, in order to restore the deflected particle to its original position, the stretch constraint is shown in figure 2.

\[ C_{\text{stretch}}(x_1, x_2) = |x_1 - x_2| - d \] (6)

where \( d \) is the distance between \( x_1 \) and \( x_2 \) when \( x_1 \) and \( x_2 \) are statically, the constraint function \( C_{\text{stretch}} \) takes the gradient of particle \( x_1 \) and \( x_2 \) is:

\[ \nabla x_1 C(x_1, x_2) = \frac{x_1 - x_2}{|x_1 - x_2|} \] (7)

\[ \nabla x_2 C(x_1, x_2) = \frac{x_2 - x_1}{|x_1 - x_2|} \] (8)

3.3. Volume Constraint

Soft tissue model adopts tetrahedron mesh, for a tetrahedron, it has four vertices, while users pulling and dragging, four vertices’ position has been changed, in order to ensure that deformation effect is more real, kept volume constant is very important, only doing this, avoiding occurring superelastic deformation. As shown in figure 3.

In figure 2, (a) is the tetrahedral volume in the initial state, (b) is the tetrahedral volume after press point \( x_1 \), \( x_1, x_2, x_3, x_4 \) have represent four vertices of tetrahedron. We assuming that \( m_i \) is mass of the particle, then the function of volume constraint’s expression is that:

\[ C_V(x_1, x_2, x_3, x_4) = \frac{1}{6} \left[ (x_1 - x_2) \times (x_1 - x_3) \right] \cdot (x_1 - x_4) - V_0 \] (9)
The volume of each of the four vertices is that:

\[ \nabla x_2 C = \frac{1}{6} (x_1 - x_3) \times (x_1 - x_4) \] (10)
\[ \nabla x_3 C = \frac{1}{6} (x_1 - x_2) \times (x_1 - x_4) \] (11)
\[ \nabla x_4 C = \frac{1}{6} (x_1 - x_2) \times (x_1 - x_3) \] (12)

Assuming that \( \sum_i \nabla x_i C = 0 \), we can get:

\[ \nabla x_1 C = -\nabla x_2 C - \nabla x_3 C - \nabla x_4 C \] (13)

According to the above that:

\[ \Delta x_i = -\frac{w_i C_2(x_1, x_2, x_3, x_4)}{\sum_{j=1}^4 w_j |\nabla x_j C|} \nabla x_i C \] (14)

\( \Delta x_i \) has been derived based on volume constraint in PBD, this method can improve deformation effect in virtual surgery.

3.4. Bend Constraint

During the process of the soft tissue deformed, not only the stretch constraint of soft tissue, but also the bend constraint, therefore neighbouring triangles must be added bend constraint, such as neighbouring triangles \((x_1, x_2, x_3)\) and \((x_1, x_2, x_4)\), as shown in the figure 4.

As shown in figure 5, \( n_1 \) and \( n_2 \) are two normal of triangle \((x_1, x_2, x_3)\) and \((x_1, x_2, x_4)\), and the initial angle is \( \phi_0 \) between \( n_1 \) and \( n_2 \), then the bend constraint can be defined as follow:

\[ C_{\text{bend}}(x_1, x_2, x_3, x_4) = \arccos(n_1 \cdot n_2) - \phi_0 \] (15)
\[ = \arccos\left(\frac{(x_2-x_1) \times (x_3-x_1), (x_2-x_1) \times (x_4-x_1)}{|x_2-x_1||x_3-x_1||x_4-x_1|}\right) - \phi_0 \] (16)
Figure 5. Diagram of the angle between two triangle normal vectors.

Assuming that $d = \mathbf{n}_1 \cdot \mathbf{n}_2$ then

$$p_3 = \frac{x_2 \times n_2 + (n_1 \times x_2)d}{|x_2 \times x_3|}$$  \hspace{1cm} (17)$$

$$p_4 = \frac{x_2 \times n_1 + (n_2 \times x_2)d}{|x_2 \times x_4|}$$  \hspace{1cm} (18)$$

$$p_2 = -\frac{x_2 \times n_2 + (n_1 \times x_2)d}{|x_2 \times x_3|} - \frac{x_4 \times n_1 + (n_2 \times x_4)d}{|x_2 \times x_4|}$$ \hspace{1cm} (19)$$

$$p_1 = -p_2 - p_3 - p_4$$  \hspace{1cm} (20)$$

According to the above, the displacement of a particle in a neighbouring triangle based on bend constraint is:

$$\Delta x_i = -4w_i \frac{\sqrt{1-d^2} \arccos(d) - \phi_0}{\sum |p_i|^2} p_i$$ \hspace{1cm} (21)$$

4. Results and Analysis

This article is with help of the C++, OpenGL, the experiment platform is NVIDIA GeForce GTX 1070 and Intel(R) Core (TM) i7-6700K CPU @ 4.00 GHz and 8.0 G RAM.

In total, we took three experiments. Respectively are Stanford rabbit and horse. We show two models’ deformation based on PBD method, as figure 6 shown:

Figure 6. The model of horse and bunny deformed after pulling.

According to these pictures we can conclude that the PBD method has the high real time and the authenticity in the virtual surgery.

5. Conclusion

In this paper, a real-time soft tissue deformation algorithm based on PBD was tested and verified, and the tetrahedral mesh model was used to prepare for future cutting. The following conclusions were obtained:
(1) The traditional PBD method can be used to quickly simulate the physical deformation of soft tissue, so as to ensure the real-time operation process simulation.

(2) By comparing with the two groups of experiments, the PBD method can more accurately describe the deformation of soft tissue and ensure the authenticity and stability of the simulator.

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