Measuring the spin polarization of a ferromagnet: An application of time-reversal invariant topological superconductor

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Abstract – The spin polarization (SP) of the ferromagnet (FM) is a quantity of fundamental importance in spintronics. In this work, we propose a quasi–one-dimensional junction structure composed of a FM and a time-reversal invariant topological superconductor (TRITS) of un-spin-polarized pairing type to determine the SP of the FM. We find that due to the topological property of the TRITS, the zero-bias conductance (ZBC) of the FM/TRITS junction which is directly related to the SP is a non-quantized quantity but of topological nature. The ZBC only depends on the parameters of the FM, it is independent of the interface scattering potential and the Fermi surface mismatch between the FM and the super conductor, and is robust against the magnetic proximity effect; therefore, compared to the traditional FM/s-wave superconductor junction, the topological property of the ZBC makes this setup a much more direct and simplified way to determine the SP.

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Introduction. – Because of hosting exotic non-Abelian zero modes [1,2] which have great potential in topological quantum computation (TQC) [3–5], topological superconductors (TSs) in every dimension have raised strong and lasting interest for more than a decade [6–22]. Due to the non-trivial topology of the energy bands, TSs have many properties that are fundamentally different from the normal superconductors (NSs). One of the most remarkable difference is that the zero-bias conductance (ZBC) of a normal metal (NM)/TS junction is a quantized quantity of topological nature [23–25], while for a NM/NS junction, the ZBC is parameter dependent and can be greatly suppressed by the interface scattering potential [26].

As the ferromagnet (FM) plays a crucial role in spintronics, the spin polarization (SP) of the FM is of fundamental importance [27,28]. To determine the SP, a general approach is to detect the tunneling spectroscopy of the FM/s-wave superconductor junction [29–33]. The underlying mechanism is based on the fact that for a ballistic NM/s-wave superconductor junction, an electron with Fermi energy injected from the NM to the superconductor will be completely reflected as a hole with opposite spin, which is known as spin-opposite Andreev reflection [33]; however, when the metal is a FM, due to the mismatch of the Fermi surface between the two spin degrees, some of the majority spin electrons cannot undergo the spin-opposite Andreev reflection [34], instead, they are reflected as themselves, which is known as normal reflection; consequently, compared to the NM/s-wave superconductor junction, the conductance of the FM/s-wave superconductor junction is decreased, and the decrement monotonically increases with the mismatch increasing. As a result, the SP can be quantitatively determined by the tunneling spectroscopy.

Although the idea of the above mechanism is generally applied for every FM/s-wave superconductor junction, the concrete decrement can also be induced by other factors, such as the interface scattering potential and the Fermi surface mismatch between the FM and the superconductor [35]. As a result, for a general FM/s-wave superconductor junction, the tunneling spectroscopy may involve many parameters, and consequently, the SP is very hard to be precisely resolved from the tunneling spectroscopy [36,37]. However, in this work, we find that if the normal s-wave superconductor is substituted by a
time-reversal invariant (TRI) TS of un-spin-polarized pairing type ($S_z = 0$, $\hat{S}$ is the total spin angular momentum of the Cooper pair), then as the ZBC turns out to be a quantity of topological nature in the sense that it is only related to the parameters of the FM, but independent of the interface scattering potential and the parameters of the topological superconductor, the process to determine the SP becomes much more direct and simplified.

**Theoretical model.** — So far, the greatest experimental progress made on the transport study of TS is in one dimension [38–43]. For generality, in this work we consider the FM is a quasi–one-dimensional wire, with length $L$ in the $x$-direction and width $W$ in the $y$-direction, and $L \gg W$. Correspondingly, for the TS, the width is also given by $W$, and the length is assumed to be infinite for simplicity. Then the Hamiltonian describing the junction under the representation $\hat{\Psi}^\dagger (x, y) = (\psi_1^\dagger (x, y), \psi_2^\dagger (x, y), \psi_3^\dagger (x, y), \psi_4^\dagger (x, y))$ is given by

$$\hat{H} = \tau_z \left[ -\frac{\hbar^2}{2m} \vec{\nabla}^2 - \mu(x, y) + V(x, y) \right] + \tau_x \Delta(x, y),$$

where $\vec{\tau} = (\tau_x, \tau_y, \tau_z)$ are Pauli matrices in particle-hole space, $V(x, y)$ is the potential induced by disorder, external field, etc.; here we assume it takes the form $-M\tau_z \sigma_y \Theta(\vec{x}) + V\delta(y)$, where the former term denotes the magnetization of the FM, $\sigma_y$ is a Pauli matrix acting on the spin space, $\Theta(x)$ is the Heaviside function, the latter term denotes the scattering potential at the interface. $\mu(x, y)$ is the chemical potential, we set $\mu(x, y) = \mu_F$ (or $E_F$) for the ferromagnetic part ($x < 0$) and $\mu(x, y) = \mu_s$ for the superconductor ($x > 0$). $\Delta(x, y) = -i \Delta(x) \delta(x)$ is the pairing potential, which is assumed to be of $p$-wave type and homogeneous at $x > 0$ and vanishes at $x < 0$ for the sake of theoretical simplicity. The mass $m$ of the particle is assumed to be positive and the same throughout the system.

As the system is strongly confined in the $y$-direction, the system will form a series of subbands with band index $n$ a good quantum number. Then the field operator can be expressed as $\psi_n(x, y) = \sum_{n} \psi_{n\sigma}(x) \chi_n(y)$, where $\chi_n(y) = \sqrt{\frac{2}{W}} \sin(k_n y)$, with $k_n = n \pi / W$. By a Fourier transformation $\hat{\psi}_n(x) = \int \frac{d^2 \vec{k}}{2\pi} e^{i \vec{k} \cdot \vec{x}} \hat{\psi}_{n\sigma}(x)$, the Hamiltonians for the ferromagnetic part and the superconducting part under the representation $\hat{\Psi}^\dagger_{nk} = (\hat{c}^\dagger_{n\uparrow,k}, \hat{c}_{n\downarrow,-k}, \hat{c}^\dagger_{n\downarrow,k}, \hat{c}_{n\uparrow,-k})$ are given as

$$\hat{H}_F(k) = \left[ \frac{\hbar^2 k^2}{2m} + \epsilon_n - \mu_F \right] \tau_z + M \tau_0 \sigma_z,$$

$$\hat{H}_S(k) = \left[ \frac{\hbar^2 k^2}{2m} + \epsilon_n - \mu_s \right] \tau_z + \Delta(k) \tau_x,$$

respectively, where $\epsilon_n = n^2 \hbar^2 \pi^2 / (2m W^2)$, $\Delta(k) = \Delta_0 k$. As $\tau_z \hat{H}_S(k) \tau_z = \hat{H}_S(-k)$ (time-reversal symmetry), $\tau_x \hat{H}_S(k) \tau_x = -\hat{H}_S(-k)$ (particle-hole symmetry), $\tau_y \hat{H}_S(k) \tau_y = -\hat{H}_S(k)$ (chiral symmetry), and $(\tau_x K)^2 = 1$ ($K$ is the complex conjugate operator), $\hat{H}_S(k)$ belongs to the BDI class [44,45]. From eq. (2), it is direct to obtain the excitation energy spectra of the FM, $E_{F,n\uparrow(\downarrow)} = \hbar^2 k^2 / 2m + \epsilon_n - \mu_F + M$, then the particle number partition $(N_F, N_s)$ can be directly obtained as $N_F = (L/\pi) \sum_{n} \sqrt{2m(\mu_F - M - \epsilon_n)}$, $N_s = (L/\pi) \sum_{n} \sqrt{2m(\mu_F - M - \epsilon_n)}$, where the two super-scripts mean that the two summations are limited by two upper limits $n'$ and $n''$, respectively. $n'$ satisfies $(\mu_F - M - \epsilon_{n'}) > 0$ and $(\mu_F - M - \epsilon_{n'+1}) < 0$, similarly $n''$ satisfies $(\mu_F - M - \epsilon_{n''}) > 0$ and $(\mu_F - M - \epsilon_{n''+1}) < 0$. If the particle number partition $(N_F, N_s)$ is known, the SP, which is defined as [46]

$$P = \frac{\sum_n N_{n\uparrow}(E_F) \psi_{n\uparrow}^2 - \sum_n N_{n\downarrow}(E_F) \psi_{n\downarrow}^2}{\sum_n N_{n\uparrow}(E_F) \psi_{n\uparrow}^2 + \sum_n N_{n\downarrow}(E_F) \psi_{n\downarrow}^2} = \frac{N_F - N_s}{N_F + N_s},$$

where $N_{n\sigma}(E_F)$ is the density of states at the Fermi energy and $\psi_{n\sigma}(E_F)$ is the Fermi energy, can be directly obtained. Note that in the quasi–one-dimensional case, as $N_{n\uparrow}(E_F) \psi_{n\uparrow} = N_{n\downarrow}(E_F) \psi_{n\downarrow} = 0$, the ballistic definition $P = (N_F(E_F) \psi_{F,n\uparrow} - N_s(E_F) \psi_{F,n\downarrow})/(N_F(E_F) \psi_{F,n\uparrow} + N_s(E_F) \psi_{F,n\downarrow})$ does not apply [29]. For the superconducting part, the quasi-particle energy spectra is given as $E_{\nu n} = \sqrt{(\hbar^2 k^2 / 2m + \epsilon_n - \mu_s)^2 + \Delta^2(k^2)}$. When $\mu_s - \epsilon_n > 0$, the bands with index smaller than $n + 1$ are all of non-trivial topology [47]. In this work, we first consider $\epsilon_1 < \mu_s < \epsilon_2$, in other words, only bands with index $n = 1$ are of non-trivial topology.

**Relation between ZBC and SP.** To simplify the calculation of the ZBC, in the following, we set $L \rightarrow \infty$. Due to the orthogonality of $\{\chi_n(y)\}$, if an electron with spin-up, excitation energy $E$ and band index $n$ is injected from the FM, the wave function in the FM is given as $\psi_{F,n}(x, y) = c_{1} e^{i \vec{k}_1 \cdot \vec{r}} \hat{c}_{n\uparrow}\chi_n(y) + c_{n+1} e^{i \vec{k}_{n+1} \cdot \vec{r}} \hat{c}_{n\downarrow}\chi_n(y)$, where $c_1 = (1, 0, 0, 0)^T$, $c_2 = (0, 1, 0, 0)^T$, $c_3 = (0, 0, 1, 0)^T$, and $c_4 = (0, 0, 0, 1)^T$. $\psi_{n\uparrow}(E) = \sqrt{2m(\mu_F - M - E - \epsilon_n)}$, $\psi_{n\downarrow}(E) = \sqrt{2m(\mu_F - M - E + \epsilon_n)}$, and $\psi_{n\uparrow}(E) = \sqrt{2m(\mu_F + M - E - \epsilon_n)}$, $\psi_{n\downarrow}(E) = \sqrt{2m(\mu_F + M - E + \epsilon_n)}$. $b_{n\uparrow}$ and $b_{n\downarrow}$ denote the amplitudes corresponding to spin-equal and spin-opposite normal reflection, respectively. $a_{n\uparrow}$ and $a_{n\downarrow}$ denote the amplitudes corresponding to spin-equal and spin-opposite Andreev reflection, respectively. In this work, we only are interested in the special case with $E = 0$. When $E = 0$, the wave function in the superconductor is very simple. If $n = 1$, corresponding to the band of non-trivial topology, $\psi_{n\uparrow}(x > 0) = c_{n\uparrow} e^{i \vec{k}_n \cdot \vec{r}} - e^{-\hbar k_n x} + c_{n+1} e^{i \vec{k}_{n+1} \cdot \vec{r}} - e^{-\hbar k_{n+1} x} + c_{n+2} e^{i \vec{k}_n \cdot \vec{r}} + e^{-\hbar k_n x} - d_{n\uparrow} e^{i \vec{k}_n \cdot \vec{r}} + e^{-\hbar k_n x}$, where $\vec{c}_7 = (i, 1, 0, 0)^T$ and $\vec{c}_8 = (0, 0, 1, 0)^T$. While for $n \geq 2$, corresponding to the bands of trivial topology, $\psi_{n\uparrow}(x > 0) = c_{n\uparrow} e^{i \vec{k}_n \cdot \vec{r}} - e^{-\hbar k_n x} + c_{n+1} e^{i \vec{k}_{n+1} \cdot \vec{r}} - e^{-\hbar k_{n+1} x} + c_{n+2} e^{i \vec{k}_n \cdot \vec{r}} + e^{-\hbar k_n x} - d_{n\uparrow} e^{i \vec{k}_n \cdot \vec{r}} + e^{-\hbar k_n x}$, where $\vec{c}_7 = (-i, 1, 0, 0)^T$ and $\vec{c}_8 = (0, 0, 1, 0)^T$.
\( \bar{c}_k = (0, 0, -i, 1)^T \). As \( k_{n^+} \) and \( k_{n^-} \) will not show up in the results, we do not write down their expressions explicitly here.

If the superconductor shows only weak pairing which means that only when the band minimum is lower than \( \mu_s \), the band is metallic and has states to pair to be superconducting [47], we only need to consider the \( n = 1 \) bands. However, for generality, here we consider that all bands are paired to be superconducting.

Again due to the orthogonality of \( \{ \chi_n(y) \} \), the boundary conditions of the wave functions at the interface are given as [48]

\[
\begin{align*}
\psi_{f,n}(0) &= \psi_{s,n}(0), \\
v_s \psi_{s,n}(0^+) - v_f \psi_{f,n}(0^-) &= -iZ \tau_z \sigma_0 \psi_{s,n}(0),
\end{align*}
\]

where \( v_s = \partial_\delta H_S/h \), \( v_f = \partial_\delta H_F/h \), \( Z = 2V/h \). Based on eq. (4), all coefficients can be directly obtained, and then according to the Blonder-Tinkham-Klapwijk formula [26], the ZBC is given as

\[
G(0) = \frac{e^2}{h} \sum_{n_{\pm}} (1 + A_{n_{\pm}+} + A_{n_{\pm}^-} - B_{n_{\pm}+} - B_{n_{\pm}^-}),
\]

where \( + (-) \) indicates that the injected electron is spin-up (spin-down). The summation on the majority spin band number \( n_+ \) (minority spin band number \( n_- \) goes from 1 to \( n' \) (\( n'' \)).

\[
A_{n_{\pm}+} = q_{n_{\pm}+} h(0)|a_{n_{\pm}+}^\dagger|q_{n_{\pm}+} c(0)
\]

\[
B_{n_{\pm}+} = |b_{n_{\pm}+}|^2,
\]

\[
A_{n_{\pm}^-} = q_{n_{\pm}^-} h(0)|a_{n_{\pm}^-}^\dagger|q_{n_{\pm}^-} c(0)
\]

\[
B_{n_{\pm}^-} = |b_{n_{\pm}^-}|^2.
\]

Due to the current conservation, these quantities satisfy the constraint \( A_{n_{\pm}+} + A_{n_{\pm}^-} + B_{n_{\pm}+} + B_{n_{\pm}^-} = 1 \). This constraint can simplify the conductance formula as

\[
G(0) = \frac{e^2}{h} \sum_{n_{\pm}} (A_{n_{\pm}+} + A_{n_{\pm}^-}).
\]

Based on eq. (4), a direct calculation shows that \( A_{n_{+}+} \) and \( A_{n_{-}+} \) always vanish and

\[
A_{n_{+}+} = A_{n_{-}+} = \frac{4 q_{n_{+}} q_{n_{-}}}{(q_{n_{+}} + q_{n_{-}})^2}, \quad n = 1,
\]

\[
0, \quad n \geq 2,
\]

where \( q_{n_{+}+} = q_{n_{-}+} = q_{n_{+}} h(0) = q_{n_{-}} h(0) \). \( A_{n_{+}+} \) and \( A_{n_{-}+} \), both denoting the spin-parallel Andreev reflection, taking value zero is a natural result since the superconductor is of un-spin-polarized pairing type. The non-vanishing quantities only depend on the parameters of the FM, they are independent of the scattering potential and the parameters of the superconductor, which suggests that they are of topological nature. Substituting eq. (7) into eq. (6), it is direct to obtain

\[
\bar{G}(0) \equiv \frac{hG(0)}{e^2} = \frac{16 q_{1+} q_{1-}}{(q_{1+} + q_{1-})^2}.
\]

The zero-bias conductance is only related to the lowest spin-up and spin-down subband of the FM. As a result, it is found that only in the strict one-dimensional limit, \( G(0) \) has enough information to directly determine the polarization of the FM. In the strict one-dimensional limit, i.e., \( \epsilon_1 < \mu_f \) and \( \mu_f - \epsilon_1 \ll \epsilon_2 \), the particle number for each spin is given as \( N_{\uparrow} = Lq_{1\uparrow}/\pi, \quad N_{\downarrow} = Lq_{1\downarrow}/\pi \). As a result, \( \bar{G}(0) = 16 N_{\uparrow} N_{\downarrow}/(N_{\uparrow} + N_{\downarrow})^2 \). Combining this result with eq. (3), it is direct to obtain

\[
P = \sqrt{1 - \frac{\bar{G}(0)}{4}}.
\]

If the superconductor is a normal \( s \)-wave superconductor, the ZBC in the strict one-dimensional limit is given as

\[
\bar{G}(0) = \frac{e^2}{h} \frac{16 q_{1+} q_{1-}}{(\kappa^2 + q_{1+} q_{1-} + Z^2)^2 + Z^2(q_{1+} - q_{1-})^2},
\]

where \( \kappa \approx \sqrt{2m\mu_s} \), and \( Z = mZ/h \). As \( \bar{G}(0) \) involves parameters of all three parts, i.e., the FM, the superconductor, and the interface, the resolution of SP from \( \bar{G}(0) \), if not possible, is very complicated [36,37]. It is found that only in the clean limit and without mismatch of the Fermi surface between the FM and the superconductor, SP can be directly determined by \( \bar{G}(0) \) through the formula

\[
P = \left( 1 - \frac{\bar{G}(0)}{4} \right)^{\frac{1}{2}},
\]

where \( \bar{G}(0) \equiv h\bar{G}(0)/e^2 \). Any of the two ideal conditions in real junctions is in fact hardly to satisfy. All of these suggest that compared to the FM/s-wave superconductor junction, the topological property of \( \bar{G}(0) \) makes the resolution of SP from the FM/TRITS junction much more direct and simplified, simultaneously with an improvement of the precision.

When the number of subbands for spin-up and that for spin-down are both larger than one, the simple formula (9) is obviously no longer valid. However, \( \bar{G}(0) \) can still provide important information about the SP. By defining a quantity as \( \eta \equiv M/(\mu_f - \epsilon_2) \), which is the relative strength of the magnetization, it is direct to find

\[
\eta = \frac{4 \sqrt{4 - \bar{G}(0)}}{4 + (4 - \bar{G}(0))}.
\]

If we further define a quantity as \( \lambda = \epsilon_1/(\mu_f - \epsilon_1) \), then the particle number for each spin can be expressed as \( N_{\uparrow} = (Lk_f/\pi) \sum_{n} \sqrt{1 + \eta - (n^2 - 1)\lambda}, \quad N_{\downarrow} = (Lk_f/\pi) \sum_{n} \sqrt{1 - \eta - (n^2 - 1)\lambda} \), with \( k_f = \sqrt{2m(\mu_f - \epsilon_1)} \), therefore, if the value of \( \lambda \) is known, the
upper limit (shown in fig. 1(b), what can be precisely determined is the approach to determine $\lambda$ the ZBC will be given as two voltage probes on the FM and

$$\text{SP can also be deduced from } \tilde{G}(0).$$

As $\eta$ can easily be determined by measuring the width of the ferromagnetic metal (if $m$ is known), the only residual challenge is to determine $\mu_f$. However, if there are at least two subbands occupied by the minority spin electrons, we can easily determine $\lambda$ in the same setup by just tuning $\mu_s$.

As $\lambda$ enters into $N_1$ and $N_2$ through terms with $n \geq 2$, we can tune $\mu_s$ from the region $(\epsilon_1, \epsilon_2)$ to $(\epsilon_2, \epsilon_3)$. When $\mu_s$ goes across $\epsilon_2$, there is a jump in ZBC, with $\delta \tilde{G}(0) = 16q_21q_21/(q_21 + q_22)^2$, then a direct calculation gives

$$\lambda = \frac{1}{3} \left[ 1 - \sqrt{\frac{\delta \tilde{G}(0)}{8 - \delta \tilde{G}(0)}} + \eta^2 \right].$$

In fig. 1(a), it is shown that when $\lambda \ll 1$ (many bands occupied), the formula (13) is almost valid in the whole region of $\eta$, and $P$ is well approximated by $\eta$, i.e., $P \approx \eta$, this is a very useful result for experiments.

If the number of occupied subbands is large, but the magnetization is so strong that it gives $1 > \eta > 1 - 3\lambda$, then as $n'' = 1$, $\delta \tilde{G}(0)$ is always equal to zero, the above approach to determine $\lambda$ breaks down. For this case, as shown in fig. 1(b), what can be precisely determined is the upper limit ($P_u$) and the lower limit ($P_l$) of the polarization. To characterize the uncertainty of the polarization, we define a quantity as $\Delta P = 2(P_u - P_l)/(P_u + P_l)$. When $n'' = 1$, $n' = 2$, in the weak magnetization region, it is found that $\Delta P$ can go beyond 100%. However, with increasing $n'$, $\Delta P$ decreases very fast. When $n' \geq 5$, we can take the boundary value $P_u$ or $P_l$, or $\eta$, as the precise value of the SP. To determine the number of the subbands for the majority spin electrons, we only need to detect the ZBC of the FM (note: only measure the FM part, not the whole FM-TS junction). If the FM is sufficiently clean to guarantee $W \ll \tilde{L} < l$, where $\tilde{L}$ is the distance between two voltage probes on the FM and $l$ is the mean free path, the ZBC will be given as $G(0) = (n'' + n')e^2/h$.

The three cases analyzed above exhaust all possibilities. For most cases, due to the topological nature of $G(0)$ and $\delta \tilde{G}(0)$, the polarization can be easily and precisely determined by this. Only in the parameter region, $n'' = 1$, $2 \leq n' \leq 4$, the precision is not very good. If with the help of other measurements, both $\epsilon_1$ and $\mu_f$ are precisely determined, then $P$ can be directly and precisely determined by the simple quantity $G(0)$ for all cases.

**Magnetic proximity effect.** – When a FM is in proximity of a superconductor, the magnetization of the FM is equivalent to a magnetic field, and it will penetrate into the superconductor and break Cooper pairs within the magnetic penetration depth. However, this pair-breaking effect should not affect the validity of the three formulae (9), (12), (13), because the penetration is a local behavior, it should not affect the topological property of the superconductor. In fact, in real experiments, this pair-breaking effect can be avoided or greatly reduced by adding a finite-thickness insulator between the FM and the superconductor. It is found that no matter how thick the insulator is, when $\epsilon_1 < \mu_s < \epsilon_2$, $G(0)$ is always given by the formula (8). Although $\tilde{G}(0)$ does not depend on the thickness of the insulator, the width of $G(eV)$ will exponentially decrease with the thickness. Therefore, for the purpose of observing the peak and detecting its value, a proper choice of the thickness is needed.

**Experimental realization.** – Compared to the TRI p-wave superconductor of un-spin-polarized pairing type that belongs to the DIII class, a TRI d-wave TS is in fact more experimentally realizable [50,51]. Similarly to the semiconductor-based proposal of TS [16,17], a TRI d-wave TS can also be realized by making a semiconductor wire with intrinsic spin-orbit coupling in proximity of a d-wave superconductor [49]. Both materials are common in reality. As the TRI d-wave TS belongs to the DIII class, it can only host at most one subband (without considering degeneracy) of non-trivial topology. To determine both $\tilde{G}(0)$ and $\lambda$, we can first tune $\mu_s$ to make only the lowest subband be topological and obtain $G(0)$, and then tune $\mu_s$ to make only the second-lowest subband be topological and obtain $G'(0) = \delta \tilde{G}(0)$.

**Discussions and conclusions.** – In the main text, the magnetization direction is chosen along the spin quantization axis of the topological superconductor, this is no doubt a special case. If the magnetization direction does not coincide with the spin quantization axis, but has an angle difference $\theta$, then because the TS we consider does not have spin-rotation symmetry, it is found that eq. (8) becomes

$$\bar{G}(0, \theta) = \frac{16q_1q_1}{(q_1 + q_1')^2 \cos^2 \theta + 4q_1q_1' \sin^2 \theta}.$$  

$G(0, \theta = 0)$ is just the quantity $\tilde{G}(0)$ we obtained previously and it is the quantity that can most directly determine the spin polarization. In reality, there is no doubt that we cannot guarantee that $\theta$ is just equal to zero, but fortunately, the $\theta$-dependence is very simple, and just like that two non-overlapping points determine a line, if we rotate the magnetization direction (by using an external
field or directly rotating the body of FM) and obtain two unequal and non-quantized ($\theta \neq \pi/2$) zero-bias conductance value, we can use the above formula to fit the two data and then we can easily obtain $G(0)$ (the procedure to obtain $\delta G(0)$ is similar). Of course, if the topological superconductor considered has spin-rotation symmetry, the $\theta$-dependence will be absent.

In a word, the independence of $Z$ and the robustness against the magnetic proximity effect make the ZBC an observable that can easily and directly determine the SP of a FM. This points out another potential application of TSs besides their well-known potential application in TQC.

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