Design and Simulation of PI Controller with Two Degrees of Freedom

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Abstract. Aiming at the fully coupled problem of steady state and dynamic performance of a PI controller with one degree of freedom, two kinds of two DOF PI controllers were designed based on the setpoint filter structure and the setpoint feed-forward structure, and the design method of compensation link was introduced. On this basis, a variable structure two-degree-of-freedom PI controller was designed with reference to the idea of variable gain PI controller. Finally, the simulation results show that the controller can not only simplify the process of setting control parameters, but also take into account the steady-state performance, dynamic performance and anti-jamming performance of the system, and realize the function of output fast, no overshoot, no error tracking input signal changes.

1. Introduction
The parameter adjustment of traditional one-degree-of-freedom PI controller can affect the dynamic and steady-state performance of the system at the same time. If the parameters are adjusted according to the optimal dynamic performance, the steady-state performance of the system will become worse. If the parameters are tuned according to the optimal steady-state performance, the dynamic performance of the system will deteriorate, so we often use trial-and-error or compromise method to solve [1] [2] when tuning the parameters of conventional PI controllers. Based on this, if we can design another compensation link, and use this link to ensure that the system can obtain better dynamic and dynamic performance at the same time, we can realize the two-degree-of-freedom control of the system [3]. The variable structure two-degree-of-freedom PI controller designed in this paper optimizes many problems existing in traditional PI controller. It is simple, practical and reliable. It is very suitable for high performance speed servo system and provides a practical and effective new method for the field of speed servo control.

2.2-DOF PI Controller
The typical two-degree-of-freedom set-value filter controller structure and set-value feed-forward structure are combined with PI control. Taking the first-order system as an example, the two-degree-of-freedom PI control structure diagrams are obtained as shown in figure 1 and 2 below.

Figure 1. Two-degree-of-freedom set value filter PI controller
As can be seen from Figure 1, the output of the two-degree-of-freedom set-value filter PI control system is as follows:

\[ x(s) = \frac{H(s)C(s)P(s)}{1+C(s)P(s)} v(s) + \frac{P(s)}{1+C(s)P(s)} w(s) - \frac{C(s)P(s)}{1+C(s)P(s)} \delta_p(s) \]

\[ = \left( k_p s + k_i \right) H(s) v(s) + \frac{s}{s^2 + (k_p + a)s + k_i} w(s) - \frac{k_p s + k_i}{s^2 + (k_p + a)s + k_i} \delta_p(s) \]  

(1)

In particular, when \( a = 0 \), the output of the control system can be simplified to:

\[ x(s) = \frac{k_p s + k_i}{s^2 + k_p s + k_i} H(s) v(s) + \frac{s}{s^2 + k_p s + k_i} w(s) - \frac{k_p s + k_i}{s^2 + k_p s + k_i} \delta_p(s) \]  

(2)

For the same reason, Figure 2 shows that the output of the set-point feedforward system is as follows:

\[ x(s) = \frac{H(s)C(s)P(s)}{1+C(s)P(s)} v(s) + \frac{P(s)}{1+C(s)P(s)} w(s) - \frac{C(s)P(s)}{1+C(s)P(s)} \delta_p(s) \]

\[ = \frac{k_p s + k_i + s H(s)}{s^2 + (k_p + a)s + k_i} v(s) + \frac{s}{s^2 + (k_p + a)s + k_i} w(s) - \frac{k_p s + k_i}{s^2 + (k_p + a)s + k_i} \delta_p(s) \]  

(3)

In particular, when \( a = 0 \), the output of the control system can be simplified to:

\[ x(s) = \frac{k_p s + k_i + s H(s)}{s^2 + k_p s + k_i} v(s) + \frac{s}{s^2 + k_p s + k_i} w(s) - \frac{k_p s + k_i}{s^2 + k_p s + k_i} \delta_p(s) \]  

(4)

From the above analysis, it can be seen that the steady-state performance will not be affected when the dynamic performance is adjusted by the compensation link \( H(s) \). Therefore, the two-degree-of-freedom PI controller first uses the requirements of steady-state performance to design parameters, and then uses the requirements of dynamic performance to design function \( H(s) \) [4][5].

3. Variable Structure Two Degree of Freedom PI Controller

For a two-degree-of-freedom PI controller with set-value filter, when the input is a time-varying given signal, in order to obtain good steady-state performance, the transfer function of the system can be designed according to equation (2).

\[ \Phi_s(s) = \frac{x(s)}{v(s)} = \frac{k_p s + k_i}{s^2 + k_p s + k_i} H_1(s) = 1 \]  

(5)

Thus, the set value filter function \( H_1(s) \) can be obtained as:

\[ H_1(s) = \frac{s^2 + k_p s + k_i}{k_p s + k_i} \]  

(6)

In order to make the input and output of the system without overshoot under step action, the transfer function is designed as a first or second order low pass filter link.

\[ \Phi_s(s) = \frac{x(s)}{v(s)} = \frac{k_p s + k_i}{s^2 + k_p s + k_i} H_2(s) = \frac{mk_p}{s + mk_p} \]  

or

\[ \frac{mk_p s + k_i}{s^2 + k_p s + k_i} \]  

(7)
Thus, the set-value filter function $H_2(s)$ can be obtained as:

$$
H_2(s) = \frac{\left(s^2 + k_p s + k_i\right)m}{s^2 + \left(mk_p + k_i / k_p\right)s + mk_i} \text{ or } \frac{mk_p s + k_i}{k_p s + k_i}
$$

(8)

In the formula, $m$ is a positive constant and can be set artificially.

Similarly, for a set-value feedback controller, when the input of the system is a time-varying given signal, in order to obtain good steady-state performance [6], according to equation (4) the transfer function is designed as:

$$
\Phi_i(s) = \frac{x(s)}{v(s)} = \frac{k_p s + k_i + sH_i(s)}{s^2 + k_p s + k_i} = 1
$$

(9)

Thus, the corresponding set-value filter function $H_1(s)$ can be obtained as follows:

$$
H_1(s) = s
$$

(10)

In order to make the output of the system without overshoot under step action, the transfer function is designed as a first-order or second-order low-pass filter link.

$$
\Phi_i(s) = \frac{x(s)}{v(s)} = \frac{k_p s + k_i + sH_2(s)}{s^2 + k_p s + k_i} = \frac{mk_p s + k_i}{s + mk_p} \text{ or } \frac{mk_p s + k_i}{s^2 + k_p s + k_i}
$$

(11)

Thus, the set-value filter function $H_2(s)$ can be obtained as:

$$
H_2(s) = \frac{(m-1)k_p s - k_i}{s + mk_p} \text{ or } (m-1)k_p
$$

(12)

From the previous calculation, the compensating links $H(s)$ corresponding to the two types of controllers under the action of two input signals can be obtained as shown in Table 1 below [7].

**Table 1** $H(s)$ corresponding to two types of controllers under the action of two input signals

| Given Type | Time-varying Given | Step giving |
|------------|--------------------|-------------|
| Design Transfer Function | $\Phi_i(s) = \frac{x(s)}{v(s)} = 1$ | $\Phi_i(s) = \frac{x(s)}{v(s)} = \frac{mk_p s + k_i}{s + mk_i}$ | $\Phi_i(s) = \frac{x(s)}{v(s)} = \frac{mk_p s + k_i}{s^2 + k_p s + k_i}$ |
| $H(s)$ in Set Value Filter Controller | $s^2 + k_p s + k_i$ | $\frac{mk_p s + k_i}{s^2 + \left(mk_p + k_i / k_p\right)s + mk_i}$ | $\frac{mk_p s + k_i}{k_p s + k_i}$ |
| $H(s)$ in Set Value Feedforward Controller | $s$ | $\frac{(m-1)k_p s - k_i}{s + mk_i}$ | $(m-1)k_p$ |

In order to make the system have good dynamic performance under the action of two different types of input signals, a variable structure two-degree-of-freedom PI controller is designed based on the two-degree-of-freedom PI controller, which can distinguish the input type according to the given change rate, and then according to the results of table 1, the appropriate compensation link $H(s)$ is selected. Its control structure is shown in figure 3 and 4 [8][9].

**Figure 3.** Set-value filtered variable structure two-degree-of-freedom PI controller
Figure 4. Set-point feedforward variable structure two-degree-of-freedom PI controller

After the first control cycle, the given change rate of time-varying is not constant at 0. At this time, $H_1(s)$ is chosen as the compensation link, which can realize the error-free tracking of the system for the given continuous change. The change rate given by the step is 0 after the first control cycle. When the compensation link is $H_2(s)$, the preset trajectory of the step response of the system can be achieved [10] [11].

The structure block diagram of the variable structure two-degree-of-freedom PI controller applied to the speed loop of permanent magnet synchronous motor is shown in figure 5 below.

4. Simulation analysis

Because the step response compensation of the set-value feedforward controller is simpler than that of the filter one, the following simulation analysis is carried out with the feed-forward controller as an example. Next, in order to verify the effectiveness of the proposed controller, the response characteristics and stability of the controller under step signal are simulated and analyzed, and the step response transfer function is designed as a second-order low-pass filter.

The sampling period and calculation step of speed loop and current loop are both 0.1 ms. Both current loops are controlled by PI with a bandwidth of 2000 rad/s and a current limit of 9 A. The PI parameters in the controller are designed in the same way as those in the previous paper, taking $k_p = 2\zeta \omega_n$ and $k_i = \omega_n^2$.

The values of $m = 0$ and $\omega_n$ are 60, 80 and 100 respectively, and the step input is 80 r/min start-up without load. The response waveform is shown in Figure 6 (a).

$\omega_n$ takes 60, $m$ takes 1, 2 and 3, respectively, and the step input is still 80 r/min. The response waveform is shown in Figure 6 (b) below.
From the simulation results of Figure 6, we can see that the dynamic performance of the system is related to the values of $\omega_n$ and $m$.

**Figure 6.** Stepper response waveform of 80r/min no-load starting

In order to verify the anti-jamming performance of the system, under the action of step signal, the motor is first allowed to run without load at 80r/min speed, and then the load is loaded and unloaded as shown in the Figure 7 below. The graph shows that the loading time of the load is longer than the unloading time, and the unloading process is closer to the step change, so we only choose the unloading process to simulate.

When $m$ is 0 and $a$ is 1, 2 and 3, respectively, the response curves are shown in Figure 8 (a).

When $\omega_n$ is 60 and $m$ is 0, 0.5 and 1, respectively, the response waveforms are shown in Figure 8 (b) below.

From the simulation results of Figure 8, it can be seen that the anti-jamming performance of the system is only related to $\omega_n$, but not to $m$.

**Figure 7.** Experimental load curve

**Figure 8.** Response waveforms when load changes

It can be concluded that the dynamic performance of the set-value feedforward two-degree-of-freedom PI controller is related to the values of $\omega_n$ and $m$, and the anti-jamming performance is only related to $\omega_n$, but not to $m$. Similarly, another kind of filter controller has the same properties, which proves that the variable structure two-degree-of-freedom PI controller can make the system have excellent dynamic and anti-disturbance performance under different types of input. The two kinds of
performance can be taken into account, and the tuning process of control parameters becomes simple, and the engineering practicability is strong [12].

5. Concluding Remarks
The variable structure two-degree-of-freedom PI controller designed in this paper can not only simplify the tuning process of control parameters, but also take into account the tracking and anti-disturbance performance. It can also achieve fast step response without overshoot, and can track continuously changing input without error.

The variable structure two-degree-of-freedom PI controller designed in this paper is simple, practical and reliable. It is very suitable for high performance speed servo system, and provides a practical and effective new method for speed servo control field.

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