A deterministic reformulation of quantum mechanics is thought to be able to bypass the usual philosophical interpretations of probability and stochasticity of the standard quantum mechanical scenarios. Recently 't Hooft proposed a different perspective based on the ontological formulation of quantum mechanics, obtained by writing the Hamiltonian of a quantum system in a way to render it mathematically equivalent to a deterministic system. The ontological deterministic models consist of elementary cells, also called cellular automata, inside which the quantities describing the dynamics oscillate in periodic orbits, extending and replacing the quantum mechanical classical language based on harmonic oscillators. We show that the structure of the cellular automaton sets finds a clear physical interpretation with the Majorana infinite-component equation: the cellular automata are elementary building blocks generated by the Poincaré group of spacetime transformations with positive-definite energy down to the Planck scales, with a close relation to the Riemann Hypothesis.

Keywords: Majorana tower, quantum mechanics, ontological quantum mechanics

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1. Introduction

The never-ending debate whether quantum mechanics (QM) is stochastic or has deep roots that are fully deterministic in their intrinsic nature takes its birth from the historical Bohr–Einstein debate in 1935 [1]; certain interpretations of QM may also involve a more subtle level of the discrete–deterministic type, giving stochastic behavior as output. In 1964, Bell’s theorem [2]–[4] put an end to many scenarios where hidden variables were the only engine for randomness, ideally favoring models where nonlocality takes place [5], [6].

The ontological quantum mechanics (OQM) considered here, in the new formulation proposed by ’t Hooft [7], [8], is a reformulation of QM slightly different from the classical probabilistic interpretations that can be found in the literature [9]–[11]. In OQM, the Hamiltonian of a quantum system is rendered mathematically equivalent to that of a deterministic system characterized by a novel mathematical language.
that describes physical structures evolving deterministically. This language can be used to describe the evolution of both a quantum and a classical system. This is a step forward in the debate started with the discussions between Bohr and Einstein, where Einstein did never accept the intrinsic stochasticity present in the language of quanta, whilst Bohr did. Differently from the well-known hidden variable theories falsified by both the theory and the experiments on Bell’s inequalities [12], this new approach is robust with respect to the Einstein–Podolsky–Rosen paradox and the problem of hidden variables [13], [14].

The QM ontological states are represented by sets of orthonormal unit vectors in the Hilbert space of support that can be either finite or infinite dimensional. By definition, a system is ontological if it evolves in time into other ontological states, with no difference between the usual quantum and deterministic states. Locally, ontological and deterministic systems can be constructed in such a way that can feature QM properties including quantum entanglement and the violation of Bell’s inequalities.

A classical dynamical system varies on time scales much shorter than the time scale related to the energy exchanged in any interaction considered there, \( \Delta t \ll 1/\Delta E_{\text{int}} \). The system is deterministic if it evolves from ontological states into other ontological states and any state, either classical or quantum, is identified by a ket vector \(|n \rangle\). Of course, a generic system can be described by a continuous or a cyclic dynamics. As usual, if the evolutionary time step is discrete, then the Hamiltonian is periodic in its eigenvalues, and hence introducing the concept of a “beable,” a vector state, as proposed by Bell, so defined to replace the traditional term “observable” that can be usually found in QM and that might imply the interaction with an observing device or imply a measurement process. In addition, one defines the terms “changeable” and “superimposable” and nonlocal phenomena associated with cellular automata (CA) in Hilbert spaces [15].

### 2. Deterministic systems

Here, we analyze two main classes of deterministic systems that lead to the formulation of an ontological deterministic representation of QM. The continuous systems have a set of equations describing a continuous dynamics whose QM-type indeterminism is due to a discretization in time or, equivalently, to a tessellation of the phase space, that can go down the Planck scales, as occurs in the search of the distribution of prime numbers with the Hilbert–Pólya approach, where Hermitian Hamiltonians are sought with eigenvalues that describe the distribution of the zeros of the Riemann zeta function [16].

The other class is instead built with periodic models with an \( SU(2) \) structure that can be described and characterized by using the infinite-component Majorana equation [17].

#### 2.1. Continuous deterministic systems

The deterministic nature of any given physical system is formally revealed through the analysis of the eigenvalue spectrum of its Hamiltonian, which can be written as

\[
H = T(p) + V(x) + A(x) \cdot p,
\]

where \( x \) and \( p \) are the usual coordinates and momenta, for which the usual relation between coordinate and momentum from Heisenberg’s indetermination principle \([x_i, p_j] = i\delta_{ij}\) holds. Different relations should be applied, like in the relativistic formulation of the Heisenberg principle by Landau and Peierls [18] and, as an example, when structured electromagnetic fields are present [19].

The kinetic term \( T(p) \sim p^2/2 \) and the classical potential \( V(x) \) responsible for the change of the geometry of the trajectories (and of spacetime, see [20]) represent the usual constituents of a standard Hamiltonian that can be found in both continuous and quantum systems where interference patterns are present.

A route to chaos and randomness from a continuous deterministic system is clearly provided if one considers, for example, only the so-called magnetic term of the Hamiltonian, \( A(x) \cdot p \); this term alone can describe a route to chaos when a Heisenberg-like texture is introduced in the phase space of a system.
described by the Hamiltonian $H = A(x) \cdot p$. In the simplest case, one can set $A(x) = x$, assuming a lattice geometry for the time coordinate, and the Hamiltonian eigenvalues can also become periodic. These properties of the Hamiltonian and lattice structure present similarities with the semiclassical dynamics based on the class of $H = xp$ Hamiltonians that have already been used in the attempts to solve the Riemann hypothesis from the Hilbert–Pólya approach initiated in [21]–[23]: the Riemann Hypothesis is true if there exists a Hermitian or unitary operator whose eigenvalues are distributed like the zeros of Riemann’s $\zeta(z)$. Along this line, space, time, and also, often, momentum, can be considered discrete quantities. As described in the literature on the prime number distribution, the magnetic-term dominated Hamiltonians cannot always be Hermitian, they mainly represent the properties of PT-symmetric quantum systems [24], [25] unless, after some modifications, ad-hoc assumptions are made for the phase space, which becomes rigged [26]. Following the idea by Hilbert and Pólya, Hermitian Hamiltonians describing this type of dynamical systems can describe the distribution of the zeros of Riemann’s zeta function and thus of primes [20], connecting two apparently distant worlds: the fabric of spacetime and the fabric of the atoms of mathematics, the prime numbers.

The limit to the lattice size, for both continuous and periodic dynamical systems, finds its roots down to the Planck scale, where the problems of an undefined time coordinate below the Planck time $\tau_p$ or their equivalence with both spatial and temporal coordinates are instead described by an indetermination relation directly derived from Einstein’s equations. This relation is derived from the relativistic scalar proper energy $E$, averaged over a proper volume $L^3$, and the corresponding interval of time $\tau$ [27]. In this case, the lattice structure is directly provided by the fluctuations in the fabric of spacetime. Importantly, Einstein’s equations and deterministic continuous dynamical systems can hold their validity down to the Planck scale, with a dynamics recalling that in Minkowski spacetime with a lattice structure.

The lattice-like structure is given by the indetermination relation between the proper energy $E$ averaged over a given proper volume $L^3$ in General Relativity (GR),

$$\langle E \rangle = E \sim \frac{g^2}{L} R_{(4)} = L \left( \Delta \left( \frac{\Delta g}{g} \right) + \left( \frac{\Delta g}{g} \right)^2 \right),$$

where $g$ is the metric tensor, $\Delta g$ is the corresponding fluctuation, and $R_{(4)}$ is the rank-four Riemann curvature tensor $R_{sikl} \in \otimes^4 \hat{T}$, i.e., a rank-four tensor defined in the cotangent bundle $\hat{T}$ of a given manifold $(M, g)$.

If we rescale this relation down to the Planck scale, imposing a characteristic size, the Planck length $L_p$, after having defined the light crossing time as $\tau = L$ and the Planck time $\tau_p$, Einstein’s equations for the gravitational field are shown to retain their formal validity down to the Planck scale, even if metric fluctuations over a scale larger than $L_p$ are expected to occur, extending the approach used in Minkowski spacetime to a more general scenario. In this way, it is possible to generalize the structure of the CA and base their characteristics on the geometric properties of spacetime.

We find that these fluctuations can give rise to a relation between the curvature tensor and spacetime fluctuations that holds down to the Planck scales. Once a characteristic spatial or temporal scale, $L$ or $\tau$, is fixed, as in the building of a lattice structure or a spacetime foam, this corresponds to the introduction of fluctuations of the proper energy $\bar{E}$ averaged over $L^3$. If we set $\bar{E} = \Delta E$ and $\tau = \Delta t$, considering fluctuations as large as the energy and time values considered, we can write an indetermination relation that involves the Riemann tensor and the proper time:

$$\Delta E \cdot \Delta t = h \left( \frac{\tau}{\tau_p} \right)^2 g^2 L^3 R_{(4)}.$$
Equivalently, we can write \( \tau / \tau_P = L / L_P \). At Planck scales, Eq. (3) is written as

\[
\Delta E \cdot \Delta t = \hbar L^2 g^2 \mathcal{R}(4) = \hbar \left( \Delta \left( \frac{\Delta g}{g} \right) + \left( \frac{\Delta g}{g} \right)^2 \right),
\]

(4)

where \( \Delta E \) is averaged over the volume \( L^3 \) of a 3D space-like hypersurface \( \sigma \), preserving the continuity of Einstein’s equations down to the Planck scale, including the equivalence between Einstein–Rosen bridges and Einstein–Podolsky–Rosen states (ER = EPR) and graviton exchanges, as described in [27].

The indetermination relation here discussed thus becomes an equivalence in a Minkowski-like manifold and at the same time defines a lattice structure as required for the OQM formulation. Quantum indetermination can arise from the lattice-like effects of spacetime fluctuations applied to deterministic continuous systems down to the Planck scales, as occurs to Einstein’s equations, for which continuity holds. This is of course compatible with the holographic principle where any cell occupies a volume \( L \cdot L_P^2 \) and any spatial region with magnitude \( L \) cannot contain more than \( L^3 / (L_L^2) = L^2 / L_P^2 \) cells and the maximum of bit numbers stored in a region with a characteristic length \( L \) is \( L^2 / L_P^2 = \tau / \tau_P \). One then finds agreement with indetermination relation (3), which can be rewritten as

\[
\Delta E \cdot \Delta t = \hbar L^2 g^2 \mathcal{R}(4) N_{bM}
\]

(5)

and \( N_{bM} = L^2 / L_P^2 \) is the maximum number of bits stored there.

In this way, with the Holographic Principle, QM appears as emerging from a lattice structure (see, e.g., [28]). The term \( N_{bM} \) can be described by a general function that generates, according to a more generic Hamiltonian, all the particles described by a given model like the Standard Model.

This information can be the core of an interpretation of the physics of CA in the periodic deterministic systems we discuss below or it can be stored as a particle, according to the Hamiltonian of the system considered, such as the Standard Model in a lattice system [29]–[31], which can provide the characteristic levels of the energy exchanges, interactions, and time intervals of its quanta.

2.2. Periodic deterministic systems. When one considers a periodic model, it has an \( SU(2) \) symmetry related to the rotation group [7], which is a subgroup of the Poincaré group. The elementary building blocks here considered to consist of a CA system that updates itself at every time step, of duration \( dt \), and then, after a period \( T = N dt \), returns to its initial position. The CA essentially behave like gears that, cyclically rotating, concur to generate the perceived randomness of QM when the dynamics has support in a lattice, and the time coordinate of the manifold is divided into discrete intervals \( \tau \). In this way, one can extend this construction to the hypothesis of a countably infinite lattice where the Hamiltonian eigenvalues are in any case periodic [7]. Each single element of this construction characterized by a finite number of states can thus be assumed to be periodic in time and obey the \( SU(2) \) symmetry in a discrete-time quantized manifold. When one extends this procedure to the continuum, the Hamiltonian has to be linearly dependent on \( p \), the linear momentum, as occurs in the \( H = xp \) class of dynamical systems already discussed.

Deterministic models can be seen as consisting of elementary cells inside which the data just oscillate along periodic orbits with their \( SU(2) \) symmetry. Rotation implies angular momentum, because the main invariant in the Poincaré group corresponding to rotation is the angular momentum. The energy eigenstates can be interpreted as the eigenstates \( |m \rangle \) of \( L_3 \), the so-called \( z \)-component in a three-dimensional rotor. The distribution of the eigenstates of these cells overlaps with those produced by the infinite-component Majorana equation, also known as “Majorana Tower” [17], [32], [33], generated by the group of Lorentz boosts belonging to the Poincaré group of spacetime transformations. Of course, finite groups of rotors
correspond to finite subgroups of the Majorana Tower, where the matrix elements \( \langle r | s \rangle^p \) can be deduced from recursion relations, in this simple example with \( H = \omega n \) and \( x = s/\sqrt{\ell} \), \( p = s/\sqrt{\ell} \), given by

\[
2r^x \langle r | s \rangle^p = (\ell, s | a_x - ia_y | \ell, s + 1)^x \langle r | s - 1 \rangle^p - 2(\ell, s | b_x + ib_y | \ell - 1, s - 1)^x \langle r | s + 1 \rangle^p.
\] (6)

In combination with \( \langle r | s \rangle^p = p^x \langle s | r \rangle^x \) and with the cyclic relations from [7] and [17], involving the infinitesimal Lorentz transformations in the variables \((c t, x, y, z)\), we have

\[
a_x = i \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0
\end{pmatrix}, \quad a_y = i \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}, \quad a_z = i \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \tag{7}
\]

and

\[
b_x = -i \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad b_y = -i \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad b_z = -i \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix} \tag{8}
\]

implying the Majorana equation that relates the coefficients of the CA to the infinitesimal Lorentz transformations in Eqs. (7) and (8), where the energy \( E = E_0/(j + 1/2) \) depends on the angular momentum that characterizes the CA and is positive-definite,

\[
\left[ W + (\alpha, p) - \frac{E_0}{\ell + 1/2} \right] \Psi = 0, \tag{9}
\]

where \( W \) is the general energy from the Hamiltonian, \( \alpha \) is the set of Dirac matrices, \( p \) is the momentum, \( \ell \) is the angular momentum eigenvalue, \( E \) is the energy considered to build the lattice structure, and \( E_0 \) is the energy of the lowest state. In fact, this family of particles represent an infinite spectrum of excited particle states characterized by the angular momentum parameter \( j \). Thus, any CA can be seen as an excited Majorana state of the fundamental state \( \ell = 0 \).

In the limit \( \delta t \to 0 \), we have the state characterized by the infinite period \( T \) that corresponds to \( \ell = 0 \). This system turns into a point moving continuously along a circle, which behaves just like the standard harmonic oscillator. It is easy to show that down to the Planck scales, Eq. (9) implies the rules dictated by the holographic principle for the energy \( E \) and the information stored there. The larger \( \ell \), the smaller is the energy and information density contained in a 3D hypervolume. To preserve the total information content integrated in the hypervolume, it has to grow linearly with \( \ell \), with their corresponding vacuum and antivacuum states that grow together with their entropy.

When the CA are described by the mathematical structure of the Majorana Tower, this means that any CA depends on or derives from Poincaré transformations of spacetime. The CA behave as quanta of spacetime transformations ruled by the Majorana Tower Hamiltonian. One can obtain an infinite denumerable set of CA that describes the spacetime ruled by GR. By introducing modifications in the Hamiltonian, one can modify the main structure of the original tower of quanta (namely, CA) so as to obtain the particles of the Standard Model, as an example. To conclude, one writes the Majorana Tower in terms of CA (and vice versa), deriving the indetermination relation in energy from GR, and thus from the energy exchanged in any physical process. In this way, one obtains spacetime and quanta from the lattice structure in an ER = EPR interpretation, also following the first ideas given by Landau and Peierls, where fluctuations of energy in time can create particle–antiparticle pairs when enough energy is available.
3. Conclusions

We formulate a Majorana representation of the CA for the ontological formulation of QM and give the interpretation of the CA in terms of symmetries of spacetime. From the Poincaré group of spacetime transformations and the subgroups of the Lorentz transformations and spatial rotations, we obtain a correspondence between the eigenvalues of the CA and those given by subsets of the infinite-component Majorana equation. In this way, we obtain the eigenvalues for the coefficients of the vector states that describe the basic dynamics in a lattice-like structure and that can take its origins in the properties of spacetime at Planck’s scales.

This represents a deep link between the basic fabric constituents of spacetime represented by the transformation groups and corresponding invariants and the structure of CA that represent the fundamental building blocks of OQM. The indetermination relation obtained from Einstein’s equations shows that the scale at which determinism can become or remain manifest is the Planck scale, where OQM interpretation can be obtained form a QM system equivalent to a deterministic dynamical system, also supporting a new interpretation of nonlocality in the ER = EPR scenario, making a parallelism between a deterministic Einstein–Rosen bridge and entangled EPR states [27], where one joins elementary cells into a construction where they interact, again allowing only deterministic interaction laws mathematically closely related to the search for prime numbers through the Pólya-Hilbert approach to the Riemann Hypothesis involving the Majorana infinite-component equation and the properties of Majorana quanta [20].

In other words, in these hypothesis, what is normally thought of as being classical stochastic quantum mechanics can be attributed to the effect of fast, almost hidden, variables that in any case support Bell’s inequalities down to the Planck scale, directly from the texture of energy and spacetime fluctuations. The concept of ontological QM is related to dynamical systems and variables that are rapidly oscillating at the Planck scale, where we are forced to revise the ordinary continuous concepts of space and time, resulting in something that is similar to a set of hidden variables and generates particles. The ontological stance is therefore intended as a global reflection of the languages of physics, classical and quantum, to set the conceptual conditions for their unification.

In this view, the Planck scale becomes a necessary scale where a lattice-like structure naturally arise and where one can find a whole topography of the “nonlocal,” both below and above the Planck scale. Therefore, in a future and more complete formulation of these phenomena, QM, quantum field theory, and GR will have to converge to a common language and set of concepts, including for a better understanding of the black hole information paradox [34]–[36].

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