Disoriented Sleptons

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Abstract

We discuss some fundamental concerns regarding the recent proposal of Dimopoulos and Giudice for dynamically aligning the soft masses of the sfermions in the minimal supersymmetric standard model (MSSM) with the corresponding fermion masses to suppress flavor changing neutral currents. We show that the phenomenologically-favored presence of right-handed neutrinos in the theory, even if only at very high scales, generically disaligns the slepton mass matrices. Further suppression is then needed to meet the current upper bound on the rate for $\mu \rightarrow e\gamma$. Planned improvements in the search for $\mu \rightarrow e\gamma$ should easily detect this rare mode. (With improved sensitivity $\mu \rightarrow 3e$ may also be seen.) By measuring the helicity of the amplitude for $\mu \rightarrow e\gamma$ such experiments could distinguish between unified and non-unified models at very high energies; by inserting the various MSSM parameters as they become available, the mixing in the leptonic Yukawa couplings can be extracted; and by combining the results with those of various neutrino experiments some information about the right-handed neutrino Majorana matrices can also be gained.
The most promising candidates for a fundamental theory underlying the standard model have been supersymmetric (SUSY) models. There are compelling theoretical and phenomenological reasons to believe that nature is supersymmetric on microscopic scales, and that the observed asymmetry at low energies between bosons and fermions is due to spontaneous SUSY breaking. Much attention has been focused on the highly-successful minimal supersymmetric extension of the standard model, the MSSM. In this paper we address some aspects of this model and of its extension to include neutrino masses. In particular, we analyze the implications of neutrino masses to a new mechanism recently introduced by Dimopoulos, Giudice and Tetradis (DGT) \cite{1} to ameliorate the flavor problem of the MSSM. In this analysis we present their mechanism somewhat differently from their original work, and then focus on its implications to rare leptonic processes such as $\mu \to e\gamma$. If the DGT mechanism is operative at a very large momentum scale $\Lambda$, then data about such rare processes can be combined with results from direct SUSY searches and from various neutrino experiments to reveal important information about the leptonic couplings at the scale $\Lambda$.

The long-standing problem which DGT have sought to solve is that of flavor-changing neutral currents (FCNC) in the MSSM. If the soft mass matrices of squarks and sleptons are—as expected—not too far above the electroweak scale, and if they are neither proportional to unit matrices nor aligned with the corresponding fermion mass matrices, then they induce unacceptably large contributions to various FCNC processes, in particular neutral kaon oscillations and $\mu \to e\gamma$. Various mechanisms have been suggested to overcome this difficulty: if the gauginos are somewhat heavier than expected, they would raise the squark masses and make them roughly proportional to the unit matrix (though the difficulties in the leptonic sector would be harder to overcome); if the soft masses start out universal—proportional to the unit matrix—at a very high scale, they typically remain so in their first- and second-generation entries, and so contribute little to the two sensitive FCNC processes mentioned above; and if some flavor symmetries align the squarks with the quarks and the sleptons with the leptons, then once again the FCNC contributions can be suppressed. But the first two solutions have serious shortcomings: gaugino dominance requires unnaturally heavy gauginos and moreover is not very effective for $\mu \to e\gamma$; and universal soft masses seem an unlikely outcome of various theories at the highest scales. The third approach postulates a set of approximate symmetries to explain both the observed fermion mass matrices and their alignment with the squark and slepton masses \cite{2}. It is somewhat similar in spirit to the approach of DGT, in that the same suppression mechanism which works in the quark and lepton sectors is applied to the SUSY-breaking sector, and generically yields similar FCNC suppression. The actual amount of suppression, though, varies considerably depending on the horizontal symmetries used, and can be stronger or weaker than in the DGT scenario. In any case, a thoroughly novel mechanism for suppressing FCNC’s is very welcome. The recent proposal of DGT introduces just such a mechanism: a model, or more correctly a paradigm, in which the squark and slepton mass matrices are dynamically aligned with those of the corresponding fermions.

In this letter we first present the idea and the assumptions of DGT somewhat differently than in the original proposal, focusing on the intrinsic link between any such dynamics and various fundamental concerns about the vacuum energy. We then show that even if such
a mechanism is viable, and can greatly improve the situation in the quark sector, we do not expect it to be nearly sufficient in the lepton sector. In the paper of DTG individual lepton numbers were conserved, and therefore not surprisingly the slepton and lepton mass matrices were exactly aligned and no FCNC processes such as $\mu \rightarrow e\gamma$ could occur. However, realistically we expect the lepton number symmetries to be violated in the neutrino sector for a variety of phenomenological reasons. As we show below, such violations will induce a misalignment between leptons and sleptons. Though we can not at present predict the degree of misalignment precisely, we expect the effect to be phenomenologically important, and to yield valuable information about the leptonic flavor violations at very high energies.

In the standard model, flavor violation comes about exclusively through the Cabibbo-Kobayashi-Maskawa mixing matrix. We can choose a basis for the quark fields such that the gauge interactions are flavor-conserving, as are the Yukawa couplings of the leptons $Y_E = \hat{Y}_E$ (the hat indicates that the matrix is diagonal) and of the down-type quarks $Y_D = \hat{Y}_D$, but then there is no more freedom to diagonalize the Yukawa couplings of the up-type quarks $Y_U$: they are given by $Y_U = K^\dagger \hat{Y}_U$, where $K$ is the CKM matrix. Since the Yukawa couplings of the first two quark generations are small and the mixing with the third generation is small, the standard model exhibits very feeble FCNCs. In the lepton sector, flavor is exactly conserved.

The minimal extension of this standard model introduces eight more potentially flavor-violating matrices: the five scalar mass matrices $\tilde{m}^2_Q, \tilde{m}^2_U, \tilde{m}^2_D, \tilde{m}^2_L,$ and $\tilde{m}^2_E$, and the three trilinear scalar coupling matrices $A_E, A_D,$ and $A_U$. We will choose once again to keep the gauge and gaugino interactions flavor-diagonal, and to do so we always rotate superpartners together. If we then stay with the above choice of basis for quark and lepton fields, we have no more freedom to diagonalize the eight new soft-SUSY-breaking matrices. If their off-diagonal terms are not suppressed relative to the diagonal ones, unacceptably large FCNCs can arise, as discussed above. We will concentrate first on the scalar masses, and then return to a discussion of the $A$ terms.

DGT have proposed that these scalar mass matrices be promoted to dynamical fields rather than be treated as mere parameters. The advantage is that there may then exist a dynamical relaxation mechanism which would align these matrices with the Yukawa matrices and thereby minimize the flavor-changing interactions. Such a situation may arise in string theory, where the low-energy field theory parameters are often dynamically determined by the vacuum expectation values of certain fields. The fundamental, microscopic theory and the low-energy effective field theory are matched at a scale $\Lambda$ which we take to be of order the string or Planck scales, but could also be some lower scale. The Yukawa couplings are assumed to be fixed by the fundamental theory, perhaps by expectation values of fields with very large masses, so they are simply parameters of the effective theory. As for the scalar mass matrices, it is conceivable that their eigenvalues and orientations are determined by different mechanisms. We will only consider the “disoriented” scenario in which the eigenvalues are first fixed by some dynamics responsible for supersymmetry breaking, and then the orientations are dynamically determined by a set of light moduli fields. (We call them “moduli", with an abuse of language, because they would correspond to flat directions when either the Yukawa couplings vanish or supersymmetry is unbroken.) If we denote the scalar masses by $3 \times 3$
matrices $\hat{m}_I^2$ where $I$ runs over the five fields $Q$ (squark doublets), $U$ (up-type antisquark singlets), $D$ (down-type antisquark singlets), $L$ (slepton doublets) and $E$ (charged slepton singlets), and diagonalize them by means of unitary matrices $U_I$ as in Refs. [1],

$$\hat{m}_I^2 = U_I^\dagger \hat{m}_I^2 U_I, \quad I = Q, U, D, L, E,$$

(1)

where $\hat{m}_I^2$ is a real diagonal matrix, then the disoriented assumption amounts to promoting $U_I$ to dynamical fields, whose expectation value is determined by minimizing an effective potential. One could further expand the set of dynamical fields to include the eigenvalues $\hat{m}_I^2$, resulting in the "plastication" scenario of DGT, but we limit our discussion of this scenario to a brief remark towards the end of this work.

To determine what is the physics which fixes the alignment of the $U_I$, we need to examine the exact effective potential of the theory, which includes the effects of quantum fluctuations from all scales. After allowing for possible new physics beyond the MSSM at some intermediate SUSY-invariant scale $M$ between the cutoff $\Lambda$ and the effective SUSY-breaking scale $\tilde{m} \sim m_Z$ in the observable sector, we may write the effective potential generically as

$$V_{\text{eff}} = c_4 O(\Lambda^4) + c'_4 O(\Lambda^2 M^2) + c''_4 O(M^4) + c_2 O(\Lambda^2 \tilde{m}^2) + c'_2 O(M^2 \tilde{m}^2) + c_0 O(\tilde{m}^4) + \ldots$$

(2)

where we have omitted a constant and any terms smaller than $\sim \tilde{m}^4$. Since the first three terms do not involve any supersymmetry breaking, they must vanish inasmuch as the vacuum energy of a supersymmetric theory vanishes: $c_4 = c'_4 = c''_4 = 0$. Assuming a hierarchy $\Lambda \gg M \gg \tilde{m}$, the dominant term would then be the $O(\Lambda^2 \tilde{m}^2)$ part, which appears as a quadratic divergence in the low-energy effective theory. We will discuss such divergences, and the possibility that they are absent, further below, and argue that not only are they expected, but that even if they vanish the main results of our work will not change. Therefore, we will assume that the $O(\Lambda^2 \tilde{m}^2)$ terms dominate and determine the orientation of the scalar masses. We note in passing that whatever mechanism is ultimately responsible for cancelling the cosmological constant, namely the constant term in $V_{\text{eff}}$, may have implications for the other terms in this potential, but we cannot yet speculate on what those implications may be.

The direct consequence of this assumption, as noted already in Ref. [1], is that the physics which determines the alignment is the physics at the cutoff scale $\Lambda$, namely, just at the scale in which the effective low-energy theory breaks down. All higher-dimension “irrelevant” operators are in principle as relevant as the renormalizable “relevant” ones. (For instance, operators with four derivatives and a non-trivial flavor structure contribute to $c_2$ already at 1-loop order, potentially competing in an important way with the aligning “force” determined by the low energy Yukawa couplings.) Therefore it is really the entire fundamental theory at the scale $\Lambda$, rather than just its low-energy sector (the MSSM plus any additional new physics below $\Lambda$), which sets the dynamics of the scalar mass orientations. Since our experimental knowledge is limited to the low-energy sector while our theoretical understanding of the fundamental theory is not sufficiently advanced to calculated $V_{\text{eff}}$, we cannot proceed further without some strong assumptions. This is a serious and apparently inherent weakness of this approach to solving the flavor problem. However, it can also be viewed favorably as affording us a window into the fundamental theory at the scale $\Lambda$: sensitivity to such scales means that our
predictions are not independent of this unknown realm, and therefore that they may be used to experimentally probe it.

What, then, can we say about the $O(\Lambda^2 \tilde{m}^2)$ terms? First, consider the radiative contribution of the MSSM modes. The MSSM would possess a global $U(3)^5$ flavor symmetry if not only the dynamical fields $\tilde{m}^2$ would transform under this symmetry but also the Yukawa couplings would transform appropriately. Since the Yukawa couplings are in fact fixed parameters, they are the spurions which carry the information about $U(3)^5$ breaking. Hence, to lowest order in these Yukawa couplings, the MSSM modes contribute

$$V_{\text{MSSM}}^{\text{eff}} = \frac{\Lambda^2}{(16\pi^2)^2} \left[ c_Q \text{Tr} \tilde{m}_Q^2 \left( K^\dagger \tilde{Y}_U \tilde{Y}_U^\dagger K + k_Q \tilde{Y}_D \tilde{Y}_D^\dagger \right) + c_U \text{Tr} \tilde{m}_U^2 \tilde{Y}_U \tilde{Y}_U^\dagger + c_D \text{Tr} \tilde{m}_D^2 \tilde{Y}_D \tilde{Y}_D^\dagger + c_L \text{Tr} \tilde{m}_L^2 \tilde{Y}_E \tilde{Y}_E^\dagger + c_E \text{Tr} \tilde{m}_E^2 \tilde{Y}_E \tilde{Y}_E^\dagger \right]$$

(3)

to the full effective potential. The $c_I$ and also $k_Q$ are numerical (scalar) coefficients which can only be calculated once the matching conditions are specified at the cutoff scale. Fortunately, we only need to assume that they do not vanish. We also expect $k_Q$ to be of order one; indeed in the low energy MSSM, $k_Q = 1$ to lowest order in the Yukawa couplings and up to small hypercharge effects. Then these MSSM contributions align $\tilde{m}^2_U$ with the diagonal mass-squared matrix $\sim Y_U^\dagger \tilde{Y}_U$ of the up-type quarks, $\tilde{m}^2_D$ with the diagonal mass-squared matrix $\sim Y_D^\dagger \tilde{Y}_D$ of the down-type quarks, and $\tilde{m}^2_Q$ with the non-diagonal linear combination $K^\dagger \tilde{Y}_U \tilde{Y}_U^\dagger K + k_Q \tilde{Y}_D \tilde{Y}_D^\dagger$. In the leptonic sector, there is only one spurion, the diagonal Yukawa matrix $Y_E$, so both $\tilde{m}^2_L$ and $\tilde{m}^2_E$ align with it and become diagonal: individual lepton numbers are conserved.

Operators with higher powers of the Yukawa couplings will not significantly change the minimum configuration of the sfermion masses. This observation, based upon direct inspection of such operators, is independent of any additional suppressions from powers of $1/16\pi^2$. One underlying reason is that the minimum configuration corresponds to points of enhanced flavor symmetry, thus off diagonal entries in the configuration will always be proportional to the appropriate CKM mixings violating those symmetries. Additional suppressions arise because the Yukawa coupling eigenvalues are hierarchical and mostly $\ll 1$. For example, the $i, j$ entry of the matrix to which $\tilde{m}^2_U$ aligns will be of order $(m_u)_i (m_u)_j \theta_{ij}$ (in order to account for the chiral quantum numbers of the $U$ multiplet); this matrix is close to being diagonal because when $i < j$, $(m_u)_i (m_u)_i \ll (m_u)_i (m_u)_j \ll (m_u)_j (m_u)_j$, and also $\theta_{ij} \ll 1$.

Of course, the MSSM modes are but a small part of the theory at the cutoff scale, and we have already seen that it is this full fundamental theory which determined the alignment of the scalar masses. To make progress, therefore, we must make some strong assumption about the remaining physics. If it is completely arbitrary, the partial alignment which would result from the low-energy modes is generically destroyed. On the other hand, if the remaining physics is related to the Yukawa couplings, in the sense that it preserves the same approxi-
mate symmetries, then the alignment can be preserved. This will therefore be our working assumption:

- the explicit (rather than spontaneous) flavor violations in the theory at the cutoff are entirely parametrized by the Yukawa couplings, treated as spurions.

It follows from this minimality assumption that the complete effective action has the same form as Eq. 3 but with different coefficients, $c_i \rightarrow c'_i$ and $k_Q \rightarrow k'_Q$. The effective action will be minimized when the soft masses are as closely aligned as possible with the Yukawa couplings, that is, when the approximate flavor symmetries are maximized. (The ground state of many physical systems is the state of enhanced symmetry, so our results may have quite a wide range of applicability.) The minimality assumption itself is very restrictive, and hence quite predictive. Deviations from our predictions would indicate that there are new flavor violations in the cutoff theory which would normally be almost inaccessible to experimental probes. We will make this assumption throughout this work, and derive some quantitative phenomenological consequences. It should be noted, however, that even if there are other sources of flavor violation which misalign the scalar masses, they are unlikely to cancel the ones we can calculate from our effective potential, and hence our predictions serve very generally as rough lower bounds on the expected experimental signals.

Under the minimality assumption, the mass matrix of the squark doublets aligns with the linear combination $K^\dagger \hat{Y}_U \hat{Y}_U^\dagger K + k'_Q \hat{Y}_D \hat{Y}_D^\dagger$. If we assume $k'_Q \sim \mathcal{O}(1)$ as suggested by $V_{\text{MSSM}}^\text{eff}$, then the second term in the linear combination may be ignored relative to the first, and $m^2_Q$ aligns approximately with the up-type Yukawa couplings, that is, it is misaligned by the CKM matrix $K$ relative to the diagonal down-type quark masses. Thus the strength of FCNCs in the quark-squark sector is suppressed by the small off-diagonal matrix elements of $K$, and this is the extent to which the disorientation mechanism alleviates the flavor problem in this sector. (Recall that, unlike the orientations, the eigenvalues of the squark mass matrices are fixed parameters in our current non-plasticated discussion.) Disorientation sets the generic values of such quantities as $\epsilon_k$, $\epsilon'/\epsilon_K$ and $B\bar{B}$ and $D\bar{D}$ mixings at (or below) their experimental values or bounds. On the other hand for the $K_L - K_S$ mass difference it is less effective\[1\], requiring a further suppression

\[
\frac{\tilde{m}^2_{Q2} - \tilde{m}^2_{Q1}}{\tilde{m}^2_Q} < 0.1 \left(\frac{\tilde{m}_Q}{300 \text{ GeV}}\right).
\]  

(4)

We see that the disorientation mechanism suffices for satisfying most phenomenological bounds, but that some other solution—such as an accidental approximate degeneracy, an approximate universality, or perhaps plastication [1]—is still needed, at least for $\Delta m_K$. (The usual bounds on supersymmetric parameters from $K_L - K_S$ mass difference are not greatly alleviated by disorientation simply because disorientation can only suppress flavor-changing neutral currents by a factor of the appropriate CKM angle, but a much greater suppression is mandated by the experimental value of $\Delta m_K$, and is provided in the Standard Model by the lightness of the charm quark. Other quantities, such as $\epsilon_K$, which are sufficiently suppressed in the
standard model by small CKM mixing angles, are indeed similarly suppressed in a disoriented scenario.)

What of the leptonic sector? It is well-known that for generic soft masses the bounds on FCNCs from the radiative $\mu$ decay process $\mu \to e\gamma$ are considerably stronger than the $K_L - K_S$ bounds. Under the minimality assumption, the alignment is essentially determined by the Yukawa couplings of the low-energy modes, which in the MSSM conserve individual lepton numbers. Therefore all FCNCs in the lepton sector vanish. However, there is considerable evidence that individual lepton numbers are not in fact conserved in nature, and therefore that there is indeed low-energy physics beyond the MSSM. The main indication comes from the solar neutrino flux, whose observed deficit can be explained by resonant (MSW) neutrino oscillations favoring neutrino masses in the $10^{-3}$ eV range. Other, perhaps less compelling, indications come from the atmospheric neutrino problem and from the density fluctuations at large scales measured by COBE. This latter observation can be more easily explained provided a substantial fraction of the dark matter is hot. With the MSSM particle content the only way to obtain that is to have some neutrino mass (presumably the $\nu_\tau$) in the eV range. Once we allow for flavor violation in the leptonic sector, then it is bound to show up in the slepton mass matrices. Almost all models for neutrino masses involve a Majorana mass matrix $M_N$ for the singlet ("right-handed") neutrino states, which also interact with the lepton doublets through a new set of Yukawa couplings $Y_N = K_L^\dagger \hat Y_N$; here we use the basis defined above in which $Y_E = \hat Y_E$ is diagonal, so $K_L$ is the leptonic analog of the CKM matrix $K$. In the quark sector there is a natural choice for which up-type quark to group with a given down-type quark in a single generation: one defines the three generations so as to make the CKM matrix close to the identity. In the leptonic sector we will find it convenient to define $K_L$, in the above basis, as the matrix which brings $Y_N$ into a diagonal form with increasing diagonal entries. As we will see below, the slepton masses will be aligned with $K_L$, so to avoid excessive flavor-changing processes $K_L$ will have to be close to the identity up to a permutation matrix. For simplicity we will assume that this permutation matrix is the identity. The eigenvalues of $M_N$ are much larger than the weak scale, making the observed neutrinos mostly left-handed and very light via the see-saw mechanism. Leptonic flavor violation is parametrized by the misalignment matrix $K_L$ and by $M_N$. We will assume, in the spirit of our previous minimality assumption and in agreement with phenomenological expectations, that $M_N \ll \Lambda$ (where again $\Lambda$ is of order the Planck or string scale). Therefore the effective potential for the slepton masses will be of the form (to lowest order in the Yukawa couplings)

$$V_{\text{leptonic}}^{\text{eff}} = \frac{\Lambda^2}{(16\pi^2)^2} \left[ c_L' \text{Tr} \tilde m_L^2 (K_L^\dagger \hat Y_N \tilde Y_N^\dagger K_L + k_L' \hat Y_E \hat Y_E^\dagger) + c_N' \text{Tr} \tilde m_N^2 \hat Y_N \hat Y_N + c_E' \text{Tr} \tilde m_E^2 \hat Y_E \hat Y_E^\dagger \right].$$

(5)

The SU(2)-singlet charged slepton masses align to lowest order with the mass-squared matrix of the charged leptons, and hence are diagonal in the basis we have chosen:

$$\tilde m_E^2 \simeq \tilde m_E^2.$$  

(6)
But the presence of the right-handed neutrinos at the cutoff scale, and therefore of the Yukawa couplings \( Y_N \) as spurions violating leptonic flavor symmetries, is enough to misalign the SU(2)-doublet slepton mass matrices. The misalignment is frozen in at the scale \( \Lambda \), and so it remains even after the right-handed modes are integrated out of the low-energy theory. The sensitivity to the theory at the cutoff scale implies an insensitivity to the details of the decoupling of the right-handed neutrinos and to the flavor violation in \( M_N \). Moreover, as in the quark sector, we will assume that \( k_L' \sim O(1) \) and that the Yukawa couplings of the neutrinos are larger than those of the charged leptons: then \( \tilde{m}_L^2 \) aligns to a good precision with the Yukawa couplings of the right-handed neutrinos and therefore is misaligned by the leptonic CKM matrix \( K_L \) relative to the charged-lepton masses,
\[
\tilde{m}_L^2 \simeq K_L^\dagger \tilde{m}_L^2 K_L ,
\]
which is much as \( \tilde{m}_Q^2 \) was misaligned by the CKM matrix \( K \) relative to the down-type quark masses. Consequently, the strength of leptonic FCNCs is sensitive only to the matrix \( K_L \) and not to the overall unknown size of the neutrino Yukawa couplings. By measuring processes such as \( \mu \rightarrow e \gamma \) (and \( \mu \rightarrow 3e \)) and \( \tau \rightarrow \mu \gamma \), we can directly probe these leptonic mixing angles. We stress that the above result does not necessarily require Yukawa couplings \( \sim 1 \) in the neutrino sector—having \( Y_N > \sim Y_E \) suffices.

Another potential source for flavor violations is the set of \( A \)-term matrices. As discussed in Ref. [1], various assumptions could be made about the \( A \) terms: they are at once similar to the Yukawa couplings, since they couple “left”- and “right”-handed modes, and to the scalar masses, since they break supersymmetry softly. If the orientation as well as the eigenvalues of the \( A \) terms were fixed at the cut-off scale (like the Yukawa couplings), then to satisfy phenomenological bounds they would need to either be negligibly small or closely aligned with the Yukawa couplings; this would also follow from our minimality hypothesis (which would require \( A_i \propto Y_i \) or \( A_i = 0 \)), and is also one component of the commonly-made universality assumption. (Notice that fixed \( A \)-term orientations allow the \( U_I \) soft-masses orientations to be regarded as pseudo-Goldstone bosons of the flavor symmetries.) Alternatively, the \( A \) terms may have fixed eigenvalues but dynamically-determined orientations (like the soft masses):
\[
A_i = V_i^\dagger \hat{A}_i W_i , \quad i = U, D, N, E.
\]
Without loss of generality, we may arrange the (diagonal) entries of \( \hat{A}_i \) in ascending order. We will assume in particular that the \( V_i \) and \( W_i \) fields are independent of the \( U_I \) fields. (If they were all regarded strictly as pseudo-Goldstone bosons of flavor symmetries then there would be fewer independent fields—and consequently insufficient freedom to align both the sfermion masses and the \( A \) terms.) What of the fixed eigenvalues \( \hat{A}_i \)? If they are too large, undesirable minima develop [13] in the full MSSM scalar potential which spontaneously break the electromagnetic gauge symmetry. The limiting values for \( \hat{A}_i \) (unless the sfermions are extremely heavy) are roughly the corresponding fermion masses:
\[
\hat{A}_i \lesssim m_i , \quad i = u, d, n, e.
\]
Thus we must suppose—as is always done—that the $A_i$ terms have sufficiently small eigenvalues, at most comparable to the hierarchical masses of the charged leptons. If in fact the eigenvalues are comparable to the hierarchical charged lepton masses, then they must also be at least roughly aligned, rather than antialigned, with the corresponding leptons in order to satisfy Eq. (3). We now turn to the issue of alignment, namely the expectation values of $V_i$ and $W_i$.

The alignment of the $A_i$ is determined under the minimality assumption by a spurionic analysis similar to that for the sfermion masses. To proceed, we further postulate that the only parameters breaking the $U(1)_R$ symmetry of the MSSM are the soft ones, namely $A_i$, the bilinear Higgs coupling $B$, and the gaugino mass parametrized by $M_{1/2}$, all of which transform in the same way under $R$. This symmetry will allow only terms involving $AA^\dagger$ and $A^\dagger M_{1/2}$ but not any $AA$ terms. Then the operators determining the alignment of $A_E$ are given to lowest order by

$$V_{\text{eff}}^A \sim c_{EN}^A M_{1/2}^\dagger \text{Tr } A_N Y_N^\dagger + c_{AE}^A M_{1/2}^\dagger \text{Tr } A_E Y_E^\dagger + c_{EN}^A \text{Tr } A_E Y_E^\dagger Y_N A_N^\dagger + c_{EE}^A \text{Tr } A_E A_E^\dagger Y_N Y_N^\dagger + h.c.$$  

(10)

What alignment do these terms induce? The first operator, $\text{Tr } A_N Y_N^\dagger$, will likely dominate the alignment of $A_N$. It can be positive or negative, and has its largest magnitude when $W_N \simeq 1$ (the unit matrix) and $V_N \simeq K_L$; hence for $c_{EN}^A$ of any sign, the first term in the potential is always minimized when $A_N \simeq K_L^\dagger \tilde{A}_N$ (up to an overall sign). Similarly, the second operator is minimized when $V_E \simeq W_E \simeq 1$ so it favors $A_E \simeq \tilde{A}_E$. Next, to analyze the third term, we recall that $V_N \simeq K_L$ and $W_N \simeq 1$, that $\tilde{A}_N$ and $\tilde{Y}_N$ are arranged in increasing order, and that $\tilde{A}_E$ and $\tilde{Y}_E$ are strongly hierarchical (unless the former is negligibly small); then, up to terms of order $K_L m_i/m_j$ for $i < j$, the third term is minimized when $A_E \simeq K_L^\dagger A_E$. Finally, the last operator in this potential is positive semidefinite, and attains its largest magnitude when $V_E \simeq K_L$. So if the coefficient $c_{EE}^A$ is nonnegligible, it must be negative in order to lead to approximate alignment (rather than antialignment) of the charged sleptons with the charged leptons.

Therefore the leptonic $A$ term relevant at low energies has the form

$$A_E \simeq \tilde{V}_E^\dagger \tilde{A}_E$$

(11)

where the fixed matrix $\tilde{V}_E$ is, up to phases, $\simeq 1$ if the second operator in $V_{\text{eff}}^A$ dominates over the second and third, and otherwise has entries comparable to $K_L$. We comment below on the possible CP-violating effects of these phases.

We should add that the above form for $A_E$ is valid at the cut-off scale $\Lambda$. RG evolution to low energies will add two types of terms to the cut-off expressions: one from the gauge sector and the other from the Yukawa sector. The gauge contribution to $A_E$ is a diagonal matrix proportional to $M_{1/2}^2 Y_E$ (we will use an approximate proportionality constant of $-0.3$). The gauge contribution to the soft masses adds universal terms $\propto M_{1/2}^2 1$, but these will not change the form of the mass matrices. Note that, in accordance with low-energy measurements of the gauge couplings and with the MSSM RG equations, we have assumed that the three gauge couplings approximately unify near the cut-off scale and that the gauginos have a common
mass $M_{1/2}$ at that scale. The Yukawa sector contributions were first partially analyzed in Ref. \cite{4} and were recently analyzed in detail in Ref. \cite{9}. We will comment on these contributions briefly below.

Before discussing the phenomenology let us recollect our assumptions. First, we assumed that the slepton mass matrix orientations were dynamical degrees of freedom fixed by a cut-off scale effective potential in which the flavor violations are entirely due to the Yukawa couplings (the minimality assumption), and second, when right handed neutrinos are added to the MSSM to account for neutrino masses, we assume that $Y_N$ are at least as large (in a matrix sense) as $Y_E$. The result is Eqs. \eqref{1} and \eqref{4} for slepton masses: namely, the “left-handed” [that is, SU(2)-doublet] soft slepton masses $\tilde{m}_L^2$ are misaligned relative to the charged lepton masses by the leptonic CKM matrix $K_L$, while the “right-handed” soft slepton masses $\tilde{m}_E^2$ are closely aligned with the charged leptons. If the $A$ terms are not negligible, then we further assumed that their orientations are dynamical according to Eq. \eqref{8}, that their eigenvalues satisfy Eq. \eqref{9}, that alignment rather than antialignment results from the effective potential, and that $K_L$ is close to the identity. The last three requirements simply allow the stability of the electroweak vacuum. The resulting $A$ terms are misaligned on their left-hand side by at most $\sim K_L$ and are aligned with the charged leptons on their right-hand side.

With the low-energy soft SUSY-breaking parameters at hand, we may study the expected phenomenology and compare the results to current experimental bounds and to future experimental potential. We will concentrate on the radiative flavor-changing muon decay $\mu \to e\gamma$ since it furnishes perhaps the most sensitive probe of these parameters, and since we anticipate its sensitivity to be greatly improved in the near future. With the above form for the soft terms, the radiative decay is completely dominated by one helicity amplitude $A_{1\to L}e\gamma$, while the other helicity amplitude vanishes to lowest order in $m_e/m_\mu$. We have independently and fully computed the dominant helicity amplitude in the MSSM, neglecting terms of order $m_e/m_\mu$, and compared our results with previous calculations\cite{3} We assume for brevity that mixing between the first two generations dominates; the generalization to full three-generation mixing is straightforward. Our result for the branching ratio is $\text{BR}(\mu \to e\gamma) = \frac{\gamma_\mu m_\mu^3}{16\pi^2} |A_{R\to L}|^2$, where $A_{R\to L} = \frac{e}{16\pi^2} m_\mu K_{L}^{1\mu} K_{L}^{e} [A_A + A_B + A_C + A_D + A_E]$, and

\begin{align}
A_A &= \frac{1}{2} \sum_{i=1}^{4} \left( g^2 |U_{i1}|^2 + g_2^2 |U_{i2}|^2 + 2g_2 g_2 \Re U_{i1} U_{i2}^* \right) \left[ \frac{f(M_{0e}^2/\tilde{m}_{L1}^2)}{\tilde{m}_{L1}^2} - (L1 \leftrightarrow L2) \right] \tag{12}
A_B &= -g_2^2 \left( c_\tau^2 \left[ \frac{g(M_{+2}^2/\tilde{m}_{L1}^2)}{\tilde{m}_{L1}^2} - (L1 \leftrightarrow L2) \right] + s_\tau^2 \left[ \frac{g(M_{-2}^2/\tilde{m}_{L1}^2)}{\tilde{m}_{L1}^2} - (L1 \leftrightarrow L2) \right] \right) \tag{13}
A_C &= \frac{g_2^2}{v_D} \left( C_1 \left[ \frac{j(M_{-1}^2/\tilde{m}_{L1}^2)}{\tilde{m}_{L1}^2} - (L1 \leftrightarrow L2) \right] + C_2 \left[ \frac{j(M_{+2}^2/\tilde{m}_{L1}^2)}{\tilde{m}_{L1}^2} - (L1 \leftrightarrow L2) \right] \right) \tag{14}
\end{align}

\textsuperscript{3} Much of the existing literature omits parts of the amplitude, and in particular the important contribution $A_C$ arising from chargino propagation with a mass insertion on the internal higgsino-wino line. In comparing our results with two of the complete calculations, we found one minor sign and normalization discrepancy with Ref. \cite{10} and agreement with Ref. \cite{9}.
\[ \mathcal{A}_D = -\frac{g_2}{\sqrt{2} v_D} \sum_{i=1}^{4} U^*_{L_i} U_{L_2} M_{0i} \left[ h(M^2_{0i}/m^2_{L_1}) - (L1 \leftrightarrow L2) \right] \] (15)

\[ \mathcal{A}_E = -\sum_{i=1}^{4} \left( U^*_{L_i} g^2 + U_{L_2} g' g_2 \right) M_{0i} \times \left\{ \begin{array}{c}
K_- \left[ \frac{1}{m^2_{L_1} - m^2_{E_2}} \left( h(M^2_{0i}/m^2_{L_1}) - h(M^2_{0i}/m^2_{E_2}) \right) \right] - (L1 \leftrightarrow L2) \\
K_+ \left[ \frac{1}{m^2_{L_1} - m^2_{E_2}} \left( h(M^2_{0i}/m^2_{L_1}) - h(M^2_{0i}/m^2_{E_2}) \right) \right] + (L1 \leftrightarrow L2) \end{array} \right\} \] (16)

\[ \text{The overall factor } K_L^{1\mu \nu} K_L'^{1\epsilon} \text{ is the off-diagonal mixing in the leptonic CKM matrix; } U \text{ is the matrix which diagonalizes the neutralino mass matrix } M_0 \text{ via} \]

\[ U M_0 U^\dagger = U \begin{pmatrix}
M_1 & 0 & -g_1 v_D / \sqrt{2} & g_1 v_U / \sqrt{2} \\
0 & M_2 & g_2 v_D / \sqrt{2} & -g_2 v_U / \sqrt{2} \\
-g_1 v_D / \sqrt{2} & g_2 v_D / \sqrt{2} & 0 & \mu \\
g_1 v_U / \sqrt{2} & -g_2 v_U / \sqrt{2} & \mu & 0
\end{pmatrix} U^\dagger \] (17)

\[ = \begin{pmatrix}
M_{01} & 0 & 0 & 0 \\
0 & M_{02} & 0 & 0 \\
0 & 0 & M_{03} & 0 \\
0 & 0 & 0 & M_{04}
\end{pmatrix} \] (18)

\[ \tilde{m}_{L_i} \text{ and } \tilde{m}_{E_i} \text{ are the mass eigenvalues of the left-handed sleptons; } v_U \text{ and } v_D \text{ are the up- and down-type Higgs boson mass parameters, satisfying } v_U^2 + v_D^2 = v^2 = (174 \text{ GeV})^2 \text{ and } v_U / v_D = \tan \beta \text{ (so the standard model fermions have Dirac masses } m_i = Y_i v_{U,D}) \text{; the chargino mass matrix is diagonalized via} \]

\[ \begin{pmatrix}
M_2 & g_2 v_D \\
g_2 v_U & -\mu
\end{pmatrix} = \begin{pmatrix}
c_+ & -s_+ \\
+s_+ & c_+
\end{pmatrix} \begin{pmatrix}
M_{01} & 0 \\
0 & M_{02}
\end{pmatrix} \begin{pmatrix}
c_- & s_- \\
-s_- & c_-
\end{pmatrix} \] (19)

from which we obtain the useful parameter combinations \[ C_1 = M_{1+} s_- c_- / g_2 \text{ and } C_2 = -M_{1+} s_- c_- / g_2 \] we also use \( K_- = \mu \tan \beta - 0.3 M_{1/2} + (\tilde{A}_{E_{2,1}} + \tilde{A}_{E_{2,2}}) / 2 \) and \( K_+ = (\tilde{A}_{E_{2,1}} - \tilde{A}_{E_{2,2}}) / 2 \equiv \tilde{A}_{12} \) in which we expect \( \tilde{A}_{E_{2,j}} \equiv \tilde{A}_{E_{2}/Y_{\mu}}(K_L V_L^\dagger)^{j\mu \nu}/K_L^\mu \) to be between zero and the SUSY-breaking scale, as discussed above; and the four loop functions are defined via \( f(x) = (2 x^3 + 3 x^2 - 6 x + 1 - 6 x \ln x) / [12(1 - x)^4] \), \( g(x) = (x^3 - 6 x^2 + 3 x + 6 x \ln x) / [12(1 - x)^4] \), \( h(x) = (-x^2 + 2 x \ln x) / [2(1 - x)^3] \), and \( j(x) = (x^2 - 4 x + 3 + 2 \ln x) / [2(1 - x)^3] \).

Various contour plots of the calculated branching ratio for \( \mu \to e \gamma \) are shown in Fig. 1. In all the plots we have used a slepton degeneracy \( \Delta \tilde{m}_L^2 = \tilde{m}_{L_2}^2 - \tilde{m}_{L_1}^2 = 0.1 \tilde{m}_L^2 \) and a leptonic mixing \( |K_L^{1\mu \nu} K_L'^{1\epsilon}| = 0.04 \). The horizontal axis spans values of the \( \mu \) parameter between \(-500 \text{ GeV} \) and \( 500 \text{ GeV} \), while the vertical axis spans the same range of the approximately-unified gaugino mass \( M_{1/2} \) at the cut-off scale. For various values of \( \tan \beta \) and \( \tilde{m}_L \), the figures show contours of constant branching ratio normalized to the current experimental
upper bound of $\text{BR}_{\exp} = 4.9 \times 10^{-11}$: the black, dark gray, light gray, and white regions indicate $\text{BR}/\text{BR}_{\exp} < 0.1$, $0.1 < \text{BR}/\text{BR}_{\exp} < 1$, $1 < \text{BR}/\text{BR}_{\exp} < 10$, and $10 < \text{BR}/\text{BR}_{\exp}$, respectively. Also shown, as cross-hatched regions, are those parameter ranges excluded by LEP bounds on the lightest chargino and neutralino masses. In the top row of plots, in which $\tan \beta = 2$ while $\tilde{m}_L$ varies between 100 GeV and 500 GeV, we have used $\tilde{A}_{E_{2,1}} = 0$ (which would result from $\tilde{V}_L = K_L$) and $\tilde{A}_{E_{2,2}} = 100$ GeV. Making $\tilde{A}_{E_{2,1}} \sim \tilde{A}_{E_{2,2}}$ would not change the results significantly. For the remaining two rows of the figure, in which $\tan \beta = 2$ or $\tan \beta = 5$, we have set $\tilde{A}_{E_{2,2}} = 0$. Making $\tilde{A}_{E_{2,2}} = 100$ GeV does not make much difference when $\tan \beta = 5$ so we omit the corresponding figure. To properly interpret these contours for any mixing, their scaling behavior is needed:

$$\text{Br}(\mu \rightarrow e\gamma) = 4.9 \times 10^{-11} \left[ \frac{|\Delta \tilde{m}_L^2 + f M_0 \tilde{A}_{12}|/\tilde{m}_L^2}{0.1} \right]^2 \left[ \frac{K_L^{1\mu \nu} K_L^{2\mu \nu}}{0.04} \right]^2 \left[ \frac{300 \text{ GeV}}{\tilde{m}_L} \right]^4 F \quad (21)$$

where $F$ and $f$ arise from loop functions. Under different assumptions about the mixing angles and degree of degeneracy, the allowed regions will correspond to different contours in our plots. The value of $F$ is $\sim 1$ when $\mu \sim M_{1/2} \sim \tilde{m}_L$, but can be one or two orders of magnitude larger when $\mu$ or $M_{1/2}$ are hierarchically lower than the slepton mass. Thus, while light sleptons result as expected in very large branching ratios (unless $\mu$ is accurately tuned to produce a cancellation), simply raising the slepton masses without raising $\mu$ and $M_{1/2}$ does not immediately lower the branching ratio: to quickly suppress the branching ratio, the entire SUSY-breaking scale must be raised, which necessitates fine-tuning the electroweak scale. The amplitude responsible for this behavior is the oft-neglected $A_C$, which is never negligible and which depends logarithmically on this hierarchy, dominating the amplitude by a factor of $\sim 10$ when the sfermions are $\sim 3$ times as heavy as the charginos. There is also a significant enhancement in the branching ratio when $\tan \beta$ is large, as in many other processes which then require suppression to agree with experiment [11]. Finally, the $A$ term contribution are often significant, especially when the “left-handed” soft-breaking masses are very nearly degenerate ($\Delta \tilde{m}_L^2 \ll \tilde{m}_L$).

The size of leptonic mixings we have inserted is consistent with that suggested by the MSW solution of the solar neutrino problem. Actually, the $K_L$ mixings and the neutrino mixings observed at low energies (via neutrino oscillations of various sorts) are in general only indirectly related, via the Majorana mass matrix $M_N$. However, they are essentially equal when the eigenvalues of $M_N$ are all of the same order while those of $Y_N$ are strongly hierarchical. With such mixings, Fig. 1 and Eq. (21) indicate that we need significant degeneracy in the slepton masses and a somewhat high SUSY-breaking scale to suppress $\mu \rightarrow e\gamma$ below its experimental bound—in fact, roughly the same degeneracy and SUSY-breaking scale as were needed to satisfy the neutral kaon mixing constraints. Thus the lepton sector fares no better (and no worse) than the quark sector in a disoriented scenario when neutrinos have sizeable Yukawa couplings at the cut-off scale. Admittedly, we have the freedom in the leptonic sector to assume that for some unknown reason the charged and neutral leptons are very closely aligned at the cut-off scale, in which case the mixing needed for the MSW scenario must be provided by the Majorana mass matrix of the right-handed neutrinos. Or we could assume that the Yukawa
couplings of the neutrinos are much smaller than those of the charged leptons, though this seems unlikely. Otherwise, the SUSY-breaking scale must be at least several hundred GeV, implying the usual fine-tuning problems for the Z boson mass, or the left-handed slepton mass eigenvalues must be made highly degenerate. If the leptonic mixings $|K_L^{1e}K_L^{1e}|$ are larger, say $\sim \sqrt{m_e/m_\mu} \approx 0.07$ or even $\sim \theta_e \approx 0.2$, then the SUSY-breaking scale must be raised further or the sleptons be made more degenerate to accommodate the experimental bounds.

These arguments will be greatly strengthened by the planned improvement in the experimental searches for $\mu \rightarrow e\gamma$. Current proposals call for sensitivity to branching ratios as low as $10^{-14}$ \cite{12}. If the disoriented scenario is correct, we certainly expect that $\mu \rightarrow e\gamma$ will be observed in this next generation of experiments. By measuring that rate we would gain some direct information about the leptonic CKM matrix. Of course, the other parameters affecting the branching ratio must be measured as well, but they will probably be determined within the coming decade.

The above predictions of the disoriented scenario should be contrasted with the effects of RG evolution in the universal scenario \cite{3, 4}. In the latter flavor violating slepton masses vanish by fiat at the cut-off scale, but are induced at lower scales by the neutrino Yukawa couplings via RG evolution: the diagonal slepton masses $\tilde{m}_\nu^2$ are augmented by $\delta\tilde{m}_\nu^2 = (3/8\pi^2)\tilde{m}_\nu^2 Y_N Y_N^\dagger \ln(\Lambda/M_N)$, leading to a branching ratio given by Eq. (21) but with a mass splitting $\Delta\tilde{m}_\nu^2/\tilde{m}_\nu^2 = (3/8\pi^2)Y_{\nu\mu}^2 \ln(\Lambda/M_N)$. Thus, unless the neutrino Yukawa coupling is of order one, the slepton masses are highly degenerate and hence this rare $\mu$ decay is greatly suppressed. The disoriented scenario in effect allows $\Delta\tilde{m}_L^2$ to be a free observable parameter while keeping the degree of misalignment between leptons and sleptons small, namely $\approx K_L$.

What does the disoriented scenario predict when $\tilde{m}_{L1}^2 = \tilde{m}_{L2}^2$ for some reason, such as plastication \cite{1}? The only contributions to $\mu \rightarrow e\gamma$ are those proportional to off-diagonal $A$ terms [namely the $K_+ = (\tilde{A}_{E2,1} - \tilde{A}_{E2,2})/2$ term in $A_E$] and those involving the third family left-handed sleptons. When $\tilde{A}_{E2,1} - \tilde{A}_{E2,2} \approx 100$ GeV and the slepton mass is $\approx 250$ GeV the resulting rate is just below the present bound; for fixed gaugino mass, the rate decreases with the eighth power of the slepton mass, so heavy sleptons would only allow detection at the next generation of experiments. To account for mixing with the third family, the relevant mixing angle $K_L^{3e}K_L^{3e}$ should be substituted for $K_L^{1e}K_L^{1e}$ in the above calculation. Assuming these have the same size as their quark sector counterpart, this contribution alone yields a branching ratio roughly an order of magnitude below the current bound even if the slepton masses are not degenerate and are $\sim 100$ GeV.

Our discussion so far has assumed that the orientations of squarks and sleptons are independent dynamical degrees of freedom. In the context of a grand-unified theory, the larger symmetry would typically reduce the number of independent orientations. As discussed in Ref. \cite{11}, the result in a unified disoriented model is a slepton mixing angle of order the Cabibbo angle $\theta_e \approx K^{1e}K^{1d}$, or perhaps of order $\sqrt{m_e/m_\mu}$ in a more detailed and realistic model. Since the mixing angle is larger than the 0.04 we used above, the branching ratio of $\mu \rightarrow e\gamma$ is also larger. As a consequence we expect that in a disoriented GUT scenario the superpartners are quite heavy or the charged sleptons of the first two generations are highly degenerate. Third-generation and $A$ term effects are then important, and may dominate if
\( \tilde{m}_{L_1}^2 - \tilde{m}_{L_2}^2 \) is sufficiently small. In very simple disoriented GUT models the misalignment may be only in the “right-handed” sector, but generically it is present in both sectors. In fact, a disoriented GUT scenario and a conventional GUT (see Ref. [14] for a detailed analysis) have similar predictions. They both differ qualitatively, however, from the disoriented non-unified scenario in an important way: the helicity of the amplitudes. As we have shown, in a non-unified disoriented scenario with only the MSSM fields plus right-handed neutrinos at high scales, the process is completely dominated by the single helicity amplitude \( \mu_R \to e_L \gamma \). On the other hand, in a realistic unified theory of flavor we expect flavor violations of comparable order in both the left- and right-handed sectors, while the minimal (and unrealistic) SU(5) model produces only right-handed mixing. Therefore in any unified theory we expect an amplitude for \( \mu_L \to e_R \gamma \) at least as large as \( \mu_R \to e_L \gamma \). Fortunately, in the planned experiments the decaying muon is polarized, so if sufficiently many \( \mu \to e \gamma \) are observed, the angular distribution of the emitted electrons would reveal the helicity of the amplitude. A pure \( \mu_R \to e_L \gamma \) result would be difficult to understand in a generic unified theory (disoriented or otherwise), but would be expected in a disoriented scenario if \( SU(3) \times SU(2) \times U(1) \) is the gauge group up to the cut-off scale.

Another related rare \( \mu \) decay is \( \mu \to 3e \). It was recently observed [9] that, in contrast to previous statements in the literature, the amplitude which dominates this branching ratio is not the box diagram but rather the (photon) penguin diagram. Indeed, while the box contribution would be several orders of magnitude below the experimental bound, the penguin diagram yields a branching ratio

\[
\frac{BR(\mu \to 3e)}{BR(\mu \to e\gamma)} \approx \frac{\alpha}{8\pi} \left( \frac{16}{3} \ln \frac{m_\mu}{2m_e} - \frac{14}{9} \right) \approx 0.36 \frac{BR_{\text{exp. bound}}(\mu \to 3e)}{BR_{\text{exp. bound}}(\mu \to e\gamma)}
\]

which is comparable to the experimental bound when \( \mu \to e\gamma \) is close to its experimental bound. At present, \( \mu \to 3e \) yields slightly weaker constraints than \( \mu \to e\gamma \); only if the precision of \( \mu \to 3e \) experiments keeps pace with the planned improvements in \( \mu \to e\gamma \) searches will the former process remain competitive. As pointed out in Ref. [9], the penguin diagram is enhanced by \( \ln(m_\mu/2m_e) \), which results from phase space integration as an electron and positron become collinear. (The coefficient of the log is just determined by the QED \( \beta \) function by requiring the cancellation of the infrared divergences in the inclusive rate to order \( \alpha \).) We should remark, however, that the experimental resolution may not allow highly collinear \( e^+e^- \) pairs to be distinguished from other processes (including \( \mu \to e\gamma \)), so the denominator in the log should be replaced by the appropriate minimum resolvable energy.

We have assumed throughout our discussion that the effective potential term \( c_2 \tilde{m}_e^2 \Lambda^2 \), which appears as a quadratic divergence in the low-energy theory, dominates and fixes the dynamics which aligns the soft masses. Such quadratic divergences are ubiquitous even in supersymmetric theories, when the supersymmetry is softly broken: while scalar masses are protected from quadratic divergences, the vacuum energy is not. Could \( c_2 \) vanish in a particular theory? Without a symmetry argument, assuming \( c_2 = 0 \) is akin to assuming the Higgs is light in a non-supersymmetric theory. Nevertheless, there have been studies where the vanishing of \( c_2 \) was invoked in order to proceed to a dynamical determination of the effective
low energy parameters \[15, 16\], and in particular of the gravitino mass itself. (Notice that not only \(c_2\) but also any terms \(c_2'\) arising from intermediate scales must vanish, presumably by the same mechanism.) While the implications of this idea are interesting, it is not clear yet how to implement it in an explicit field-theoretic model. As a matter of fact, to date the only available examples \[17\] satisfy \(c_2 = 0\) only at 1-loop order (see \[18\] for an explicit example of its violation at 2-loop order). Moreover, if such an implementation were found, making \(V_{\text{eff}} = \mathcal{O}(\tilde{m}^4)\), there would still be two contributions: one from the MSSM modes, and one which remains as a boundary term from matching the low- and high-energy theories. While the first contribution is determined to lowest order by the 1-loop RG evolution of the Higgs mass and of the cosmological constant, and is \(\sim \tilde{m}^4 \log(\Lambda/\tilde{m})/(4\pi)^2\), the second contribution \(\sim \xi_0 \tilde{m}^4\) is in principle unknown. The first contribution yields a phenomenology similar to the one we have studied throughout most of this paper. The second can only be controlled by making our minimality assumption (or some equivalently strong assumption)—and then, again, similar predictions would be made. We do not know the relative size of the two contributions. If we were to treat \(\xi_0\) as a usual threshold correction arising from 1-loop field-theoretic diagrams, we would expect it to be \(\xi_0 \sim 1/16\pi^2\) and hence subdominant in the limit \(\log(\Lambda/\tilde{m}) \gg 1\), thus weakening the dependence on unknown cut-off physics. On the other hand, the MSSM contributions are suppressed by small Yukawa couplings. In the end, we must plead at least as much ignorance about the \(c_2 = 0\) case as about the \(c_2 \neq 0\) case, and so the minimality assumption is unavoidable.

Finally, we comment briefly on CP violation. In Ref. \[19\] the issue of dynamical CP phases was discussed in the context of the MSSM with universal soft terms. It was shown that, when the phases of \(A, M_{1/2}, \mu\) and \(B_\mu\) are promoted to dynamical variables, the only CP-violating effects have a CKM origin, \(i.e.\) arise from the Jarlskog invariant \(J\), and thus are suppressed. This was under the strong assumption that no new sources of explicit CP violation are present. The same conclusion can be reached in the non-universal case discussed here, under the parallel assumption that the coefficients in the effective potential are real. To understand this observation, consider the limit in which the quark and lepton Jarlskog invariants \(J_{q,\ell}\) vanish, and choose a flavor basis in which the Yukawa matrices are real: in this basis also the soft terms relax to real matrices. In Ref. \[19\], the invariant \(J\) enters the effective potential at higher-loop order, so its effect on the CP-violating phases is further suppressed. In contrast, in a non-universal scenario, CP violation is already present in Eqs. (7) and (11), and has no further loop suppressions. But as long as \(J_\ell\) is not much larger than its quark counterpart \(J_q\), no additional loop suppressions are needed to ensure that CP-violating quantities such as electric dipole moments are sufficiently small.

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Figure Captions

Fig. 1: Contours of constant branching ratio for $\mu \rightarrow e\gamma$ as functions of the $\mu$ parameter and the unified gaugino mass $M_{1/2}$ (approximately the wino mass) for various values of the slepton mass $\tilde{m}_L$, $\tan \beta$ and the $A$ parameter (as defined in the text). The black, dark gray, light gray, and white regions indicate $\text{BR}/\text{BR}_{\text{exp}} < 0.1$, $0.1 < \text{BR}/\text{BR}_{\text{exp}} < 1$, $1 < \text{BR}/\text{BR}_{\text{exp}} < 10$, and $10 < \text{BR}/\text{BR}_{\text{exp}}$, respectively. The hatched regions are those excluded by LEP I lower bounds on the mass of the lightest chargino and neutralino.

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$m_{\tilde{L}} = 100 \text{ GeV}$
$M_{1/2} = 500 \text{ GeV}$

$\tan \beta = 5, A_{E2,2} = 0$
$\tan \beta = 2, A_{E2,2} = 0$
$\tan \beta = 2, A_{E2,2} = 100 \text{ GeV}$