Anisotropic plasticity-damage material model for sheet metal —
Regularised single surface formulation

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Sheet metal forming as well as mechanical joining demand increasingly accurate and efficient material modelling to capture large deformations, the inherent sheet orthotropy and even process-induced damage, which is expected to be influential. To account for large strains the additive logarithmic strain space is utilised that enables a straightforward incorporation of plastic anisotropy, herein modelled by a Hill48 yield function. A gradient-enhancement is used to equip the ductile damage model with an internal length scale curing the damage-induced localisation. An affine combination of the local and non-local softening variable is derived enabling a more efficient single surface formulation for the regularised plasticity-damage material model.

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1 Introduction

The dual surface geometrically linear plasticity–gradient-damage material model from [1] has already been extended to finite strains in [2] based on a multiplicative decomposition of the deformation gradient $F$ and in [3] based on the additive logarithmic strain space from [4]. The latter additive decomposition enables a straightforward extension of the given model to capture anisotropic plasticity as outlined below, as well as further enhancements.

The properties of the matrix logarithm and the favourable characteristics of the additive logarithmic strain space enable us to use the logarithmic Hencky strain $H = \frac{1}{2} \ln (F^\top \cdot F)$ together with the simple additive elastic-plastic split to obtain the work-conjugate logarithmic stress $T(H)$ [4]. Hence, only a geometric pre-processing (computation of $H$) and post-processing are needed to obtain, for instance the second Piola-Kirchhoff stress and the consistent tangent modulus in the “real/finite world”.

Secondly, we propose a single surface formulation for the plasticity-damage model. Similar to [5], a naive regularisation in which only a partial differential equation (PDE) is added to govern a global damage variable proved to be not sufficient. However, herein we derive the necessary modification of the material model directly from a dual surface model and obtain a convenient expression that achieves an affine combination of the local and corresponding non-local softening variable.

2 Single-surface anisotropic finite plasticity–gradient-damage

The anisotropic plasticity is modelled by HILL’s 1948 yield function as $\Phi^p = \sqrt{T^\top : H : T - \sqrt{2/3} [\sigma_y - R(\alpha)]} \leq 0$ with the initial yield stress $\sigma_y$, the hardening stress $R(\alpha)$, and the fourth order Hill tensor $H$ [4]. Damage is induced by the local damage variable $d$ that affects the material via the degradation functions $f_1(d) = \exp(-\eta_d d)$ with the damage parameters $\eta_d$.

The effect of damage on the material is modelled by the concept of effective stress $\tilde{T} = f_d^{-1} T$ on plasticity and by degrading the elastic response, for instance the bulk modulus $\kappa$ and shear modulus $\mu$. The evolution equations for the internal variables can be derived following standard thermodynamic considerations with the plastic Lagrange multiplier $\lambda^p$ and the evolution direction $n$, for the plastic strain tensor $H^p = 1/f_4(d) \lambda^p n$ and the internal plastic hardening variable $\dot{\alpha} = \sqrt{2/3} \dot{\lambda}^p$. The local damage variable $d$ on the other hand, can be directly computed from the underlying damage model exemplary stated in Eq. (2). Applying a backward Euler time integration leads to the time-discrete expression for the “damaged” stress in Eq. (1).

$$T_{n+1} = \left[ 4 + 2f_2/f_4 \mu \Delta \lambda^p / \sqrt{2/3} [\sigma_y - R] \right] H^p : \left[ f_1(d_{n+1}) \kappa \text{tr} (H_{n+1}) I + 2f_2(d_{n+1}) \mu \left[ H^\text{dev}_{n+1} - H^p \right] \right]$$

3 Damage regularisation through gradient-enhancement

Ductile damage is introduced in the framework of continuum damage mechanics and fully coupled to elasto-plasticity. This captures the softening effect of ductile damage, but leads to mesh-dependent localisation of local damage models requiring regularisation strategies [1]. We see the interplay of the local $d$ and non-local damage $\varphi$ as crucial for proper regularisation. The local variable can control its non-local counterpart via an additional partial differential equation stated below. However, the non-local variable also needs a handle on the local variable to limit localisation and spread the damage into areas without local damage contribution. The latter relationship is herein accomplished by a more efficient single surface approach.

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Local → non-local — non-local strain energy contribution and additional partial differential equation. A gradient-enhancement is herein introduced on the internal damage variable \( d \) via the Helmholtz-free energy \([1]\), by a non-local energy contribution \( \Psi_{\text{dloc}}(d, \varphi, \nabla_X \varphi) \). The global variable \( \varphi \) is introduced as an additional finite element field and governed by its own PDE as \( c_d \Delta_X \varphi - \beta_d [\varphi - d] = 0 \). The regularisation parameter \( c_d \) contains a length scale, whereas \( \beta_d \) should be regarded as a purely numerical penalty parameter. Both the damage PDE and the balance of linear momentum are strongly coupled and thus solved monolithically in an in-house code based on the FE library deal.II \([7]\).

Non-local → material model giving the non-local variable a handle on its local counterpart. At first, we introduce a second threshold function in addition to the plastic yield function \( \Phi^p \) to govern the "truly" local damage \( d^\text{loc} \) directly via the plastic hardening variable \( \alpha \) as \( \Phi^d(d^\text{loc}, \alpha) = F(\alpha) - \alpha \text{crit} \left[ 1 + \eta_3 d^\text{loc} \right] \). An exemplary generalised function \( F \) is shown in Eq. (2) with the damage initiation threshold \( \alpha \text{crit} \), material parameter \( \eta_3 \) and exponent \( n \). The regularisation effect is incorporated on the material point level as \( \Phi^d = \Phi^d(\alpha, d) + C [\varphi - d] \leq 0 \) by adding the source term scaled by a yet unknown factor \( C \). Because \( \Phi^d \) is still linear in the local damage \( d \), it can be rearranged to give an explicit expression for the damage in Eq. (2) reducing the formulation to a single yield function, where only \( \Phi^d \leq 0 \) needs to be fulfilled.

In contrast, the so-called over-nonlocal approach \([6]\) uses a somewhat opposite combination of a local \( \kappa^{\text{loc}} \) and non-local \( \kappa \) softening variable in the plastic yield function as \( \Phi^p(m \kappa + [1 - m] \kappa^{\text{loc}}) \). However, for successful regularisation this requires an overcompensating (over-nonlocal) formulation \( m > 1 \), and further needs an ad-hoc replacement of the occurrences of the local by its non-local counterpart in the material model. Our affine combination is rigorously derived and does not conflict with a thermodynamically consistent derivation, in contrast to the more pragmatic and ad-hoc over-nonlocal approach.

\[
\Phi^d = 0 \leftrightarrow d = \frac{F(\alpha) - \alpha \text{crit} + C \beta_d \varphi}{C/\alpha \text{crit} \eta_3 + 1} \geq d^\text{loc} \quad \text{with} \quad d^\text{loc} = \left\{ \begin{array}{ll}
\frac{F(\alpha) - \alpha \text{crit} + C \beta_d \varphi}{C/\alpha \text{crit} \eta_3}, \text{e.g.} \quad \left[ \frac{\alpha - \alpha \text{crit}}{\alpha \text{crit} \eta_3} \right]^n 
\end{array} \right.
\]

4 Results and Conclusion

The first numerical example models one-eight of a 3D sheet metal strip under tension differing from \([3]\) only in the hardening law. Fig. 1 shows the force-elongation response for different damage models and parameters each solved on three different progressively locally refined meshes. For each set, the results converge to a mesh-independent and physically sensible response also obtaining a mesh-independent damage distribution in Fig. 2. Q2-Q1 elements are used to especially avoid volumetric locking. The effect of the plastic anisotropy on the damage evolution is shown in the second example of a plane strain plate with a hole under tension in Fig. 3, which is solved as a full model due to the anisotropic evolution. Because of the dependent damage evolution in Eq. (2), we observe how damage follows the anisotropic plasticity, which is especially pronounced for a sheet orientation of \( \theta = 45^\circ \) to the rolling direction. Volumetric locking is prevented here by using Q1\(F^\text{c}\)-Q1 elements.

Fig. 1: Sheet metal strip under tension (full model). Different damage models for three meshes (dashed-dotted-solid) each.

Fig. 2: 3D damage distribution.

Fig. 3: Plane strain plate with a hole under tension. Anisotropic plasticity-damage.

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