Tooth contact analysis of spur and helical gears with end relief profiles

S Cazan and S Crețu

Mechanical Engineering Department, “Gheorghe Asachi” Technical University of Iasi, Iasi, Romania

E-mail: stelian.cazan09@gmail.com

Abstract. The meshing process of two gears is altered by elastic deformations of the machine parts involved in the power transmission chain. The edge effect appears and some additional manufacturing procedures are necessary to be done. The paper presents a computerized program used to establish the suitable value for the profile correction according to the particular running parameters as torques and misalignments. Semi-analytical methods were used to obtain the elastic state, developed during the meshing process, around the concentrated contact of pinion and gear. For spur gears the separation is possible to be obtained analytically, while for helical gears a numerical algorithm has been developed to obtain the matrix of separations. The two approaches were validated and some numerical results regarding the 2D and 3D pressures distributions as well as the contact areas, are presented. For operating conditions without misalignment, the lead modification by end relief could lead to a very favourable, quasi-Hertzian, pressures distribution. The presence even of a quite small value of misalignment changes the quasi-Hertzian distribution to a skewed one with detrimental effect on gear’s life.

1. Introduction

The determination of contact pressures on the gear flank surfaces can be accomplished by making a Finite Element Analysis (FEM) or using semi-analytical methods (SAM) In some particular conditions, the determination of stress tensor can be accomplished by using Hertz analytical formulas. To do this, all Hertz assumptions must be fulfilled. But his assumptions are ideal ones. Unlike Hertz method, when FEM or SAM are used, the increasing of the contact pressures at the edge zones are identified. This paper is oriented on solving the issue that appears because of the edge effect. So, an additional manufacturing stage is necessary in order to eliminate the unwilling effect. To obtain a suitable lead’s profile modification the values for parameters which characterized modified profile have to be adopted. Some general value are mentioned in like ISO 6336. To obtain the best results these values have to be established considering the particular operating parameter of gears, especially the torque and misalignment. The practical experience provides some indications. A simulation program to provide the optimum values for profile modification parameters is developed for both spur and helical gears.

2. Semi-analytical method

2.1 Short presentation of SAM algorithm used to solve non-Hertzian contacts

The following equations define the elastic model of surface deformation:

a) geometric equation of elastic contact:
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\[ g(x, y) = h(x, y) + w(x, y) - \delta_0 \]  \hspace{1cm} (1)

b) integral equation of the normal displacement of elastic half-space frontier:

\[ w(x, y) = \frac{1}{\pi} \left( \frac{1 - u_I^2}{\pi E_I} + \frac{1 - u_{II}^2}{\pi E_{II}} \right) \int \int \frac{p_I(\xi, \eta)}{\sqrt{(x - \xi)^2 + (y - \eta)^2}} \, d\xi \, d\eta \]  \hspace{1cm} (2)

c) static equation:

\[ \int \int p(x, y) \, dx \, dy = F \]  \hspace{1cm} (3)

Non-adhesion and non-penetration conditions:

\[ g(x, y) = 0, \quad p(x, y) > 0, \quad (x, y) \in A_r \]  \hspace{1cm} (4)

\[ g(x, y) > 0, \quad p(x, y) = 0, \quad (x, y) \notin A_r \]  \hspace{1cm} (5)

Except the cases when Hertz assumptions are satisfied, the above equations have no analytical solutions and consequently in such a case it is necessary to use a discrete approach to obtain a workable solution. So, the above equations become:

\[ g_{ij} = h_{ij} + w_{ij} - \delta_0 \]  \hspace{1cm} (6)

\[ w_{ij} = \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} K_{i-k,j-l} p_{kl} \]  \hspace{1cm} (7)

\[ \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} p_{ij} = F \]  \hspace{1cm} (8)

\[ g_{ij} = 0, p_{ij} > 0, (i, j) \in A_r \]  \hspace{1cm} (9)

\[ g_{ij} > 0, p_{ij} = 0, (i, j) \notin A_r \]  \hspace{1cm} (10)

In (1), \( h(x, y) \) is the matrix of separations between two bodies in contact, \( w(x, y) \) is the elastic displacement measured along the normal displacement and \( \delta_0 \) is the rigid displacement. A computer program has been done. This project consists of the following subroutines:

- a subroutine with initial data definition;
- a subroutine with matrix of separations calculation;
- a subroutine with pressures matrix calculation, according to the upper equations;
- a subroutine of von Misses stresses calculation (if needed);
- a subroutine with pressures distribution in case of lubrication and roughness (if needed);
- a subroutine with profile modification geometry definition (like this paper which proposes an end relief profile modification).

2.2 Matrix of separations. Flanks geometry

Matrix of separations represents the matrix consisting of the distances between each homologous points between two flanks. These distances are calculated before the normal force is applied, so before the deformation of the teeth. The equations of the pinion flank surface, as presented by Litvin and Fuentes [1] are:
\[
\begin{align*}
    x_1 &= r_b \cos(\theta_1 + \mu_1) + u_1 \cos \lambda_b \sin(\theta_1 + \mu_1), \\
    y_1 &= r_b \sin(\theta_1 + \mu_1) - u_1 \cos \lambda_b \cos(\theta_1 + \mu_1), \\
    z_1 &= -u_1 \sin \lambda_b + p_1 \theta_1
\end{align*}
\]  
(11)

and the equations of the gear surface are:

\[
\begin{align*}
    x_2 &= \frac{r_b}{\cos \alpha_0} [m_{12} \cos \phi_2 - \sin \alpha_0 \sin(\phi_2 + \phi_0)] + u_1 \cos \lambda_b \sin(\phi_2 - \phi_0) \\
    y_2 &= -\frac{r_b}{\cos \alpha_0} [m_{12} \sin \phi_2 + \sin \alpha_0 \cos(\phi_2 + \phi_0)] + u_1 \cos \lambda_b \cos(\phi_2 - \phi_0) \\
    z_2 &= -u_1 \sin \lambda_b + p_1 (m_{12} \phi_2 + \phi_0 - \phi_1)
\end{align*}
\]  
(12)

The parameters meaning exposed in (11) and (12) are explained in [1].

The procedure to calculate the separations matrix is represented graphically in the following. There are chosen two values for \( \theta \) on the plane \( Z = 0 \). Then, using the previous equations, two points are determined. Starting with these points, the parameter \( u \) will be varied, so that a number of points will be taken into account for meshing for each straight line that form the flank. Note that each flank is a ruled surface. Between these values for \( \theta \), there will be established another points, so that the number of columns of the separation matrix needed to correspond with the number of \( \theta \) parameters. Each straight line is going to be a bit longer because, after the deformation of the flanks, these lines that form the curved surface of the teeth becomes a flat surface, comprised into a rectangle. The rectangle formed after deformation is chosen large enough to comprise the whole flank, no matter the position on the tooth.

![Figure 1. Schematic representation of meshing flanks and the separations between the pinion and gear contacting surfaces.](image-url)
The rectangle is taken with a length about 1.5 times longer that the pinion/gear width. It is necessary to bring in discussion this rectangle because the real contact area after the deformation is unknown, so this area must be approximated with a rectangle large enough.

3. Validation and results

3.1 Validation
The subroutine for contact pressures determination reviled a very correlation with FEM results. The validation is presented in [2] for the contact performed by a cylindrical roller with its inner raceway. The pressures distribution subroutine works in the same way in all cases of concentrated contact. What differs from a case to another is the subroutine of separation matrix. In this context a subroutine was developed to obtain the separation for the general case of helical gears case. First, the results will be validated with the results presented in [3] for a spur gear case. It can be accomplished this goal, as long as the helix angle can be equalized to 0 in the initial data subroutine.

Table 1. The initial data for the spur gear set

|                  | Pinion | Gear  |
|------------------|--------|-------|
| No. teeth        | 23     | 51    |
| Module           | 4 mm   | 4 mm  |
| Correction Coefficients | \( x_1 = 0 \) | \( x_2 = 0 \) |
| Power            | 15 kW  |       |
| Angular velocity | \( \omega_2 = 53.77 \text{ rad/s} \) |       |
| Gear width       | \( B = 32 \text{ mm} \) | \( B = 32 \text{ mm} \) |
| Helix angle      | \( \beta = 0^0 \) | \( \beta = 0^0 \) |
The second case results will be for a helical gear, with a helix angle greater than 0. Note that the end relief radius had the value $0.1 \cdot B$ and the drop had the value $c = 5 \mu m$. The modified profile was accomplished only for the pinion profile.

### 3.2 Results

The set of results are the 3D and 2D distributions of the contact pressures, figure 3 and figure 4 respectively, and the contact areas during flanks’ elastic deformations, figure 5.

![3D Pxy pressures distribution](image1)

Figure 3. 3-D pressures distribution (spur gears).

![2D Pxx pressures distribution](image2)

Figure 4. 2-D distribution of maximum pressures along flanks’ width

![Contact area](image3)

Figure 5. The contact area during elastic deformation
Comparisons between the current results and the results from [3] were done. So, the maximum pressure value in [3] is 665.76 MPa, while the current method indicates a maximum pressure with a value of 662 MPa. The maximum pressures values were selected at the edge zones. The middle pressure value in [3] is 590 MPa, while the current method indicates a middle pressure of 594 MPa. As long as the results almost coincide, the method has received a first validation. The specific end relief values were adopted taking into account the recommendations from ISO 6336 [4].

The following results are obtained for the initial data presented in table 2. The initial data correspond with the initial data from [5].

**Table 2.** The initial data for the helical gear set

|                     | Pinion | Gear |
|---------------------|--------|------|
| No. teeth           | 21     | 49   |
| Module              | 4 mm   | 4 mm |
| Correction Coefficients | $x_1 = 0$ | $x_2 = 0$ |
| Load                | 1000 N | 1000 N |
| Gear width          | B = 32 mm | B = 32 mm |
| Helix angle         | $\beta = 15^0$ | $\beta = 15^0$ |

The afferent values for end relief correction are the same as for the first analysed case. Similar to the above procedure, the second result set consists of 3D and 2D distribution of the contact pressures and the elastic contact area, developed during gears meshing.

![3D Pressure Distribution](image_url)

**Figure 6.** 3D pressures distribution developed during meshing in helical gears.
4. Conclusions

Based on a SAM approach of the concentrated contact problem a computerized project has been developed in order to calculate the contact pressures developed during meshing of two helical gear flanks. The validity of the program was checked by comparing the results obtained by the authors’ project and open literature results for a spur gear case. To obtain the particular case of spur gear, the null value has been considered for the helix angle in the initial data subroutine resulting a very accurate correlation between the results. The second group of results were provided by the computerized program for a helical gear. Unlike spur gear cases, for a pair of helical gears, the spatial pressures distribution is asymmetrical. In the left side, the value of the maximum pressure is greater than the value of the right maximum pressure. This happens because of the helix angle. For a pair of helix gear with the same number of teeth for both pinion and gear, the shape of the spatial distribution of the contact pressures is similar to that obtained for a pair of spur gear.

5. References

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