An axion-like scalar field environment effect on binary black hole merger

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Abstract The environment, such as an accretion disk, could modify the signal of the gravitational wave from astrophysical black hole binaries. In this article, we model the matter field around intermediate-mass binary black holes by means of an axion-like scalar field and investigate their joint evolution. In detail, we consider equal mass binary black holes surrounded by a shell of axion-like scalar field both in spherically symmetric and non-spherically symmetric cases, and with different strengths of the scalar field. Our result shows that the environmental scalar field could essentially modify the dynamics. Firstly, in the spherically symmetric case, with increase of the scalar field strength, the number of circular orbits for the binary black hole is reduced. This means that the scalar field could significantly accelerate the merger process. Secondly, once the scalar field strength exceeds a certain critical value, the scalar field could collapse into a third black hole with its mass being larger than that of the binary. Consequently, the new black hole that collapses from the environmental scalar field could accrete the binary promptly and the binary collides head-on with each other. In this process, there is almost no quadrupole signal produced, and, consequently, the gravitational wave is greatly suppressed. Thirdly, when the scalar field strength is relatively smaller than the critical value, the black hole orbit could develop eccentricity through accretion of the scalar field. Fourthly, during the initial stage of the inspiral, the gravitational attractive force from the axion-like scalar field could induce a sudden turn in the binary orbits, hence resulting in a transient wiggle in the gravitational waveform. Finally, in the non-spherical case, the scalar field could gravitationally attract the binary moving toward the center of mass for the scalar field and slow down the merger process.

Key words: stars: black holes — gravitational waves — methods: numerical

1 INTRODUCTION

Gravitational waves (GWs) are an important prediction of Einstein’s general relativity (GW). Their direct detection, first made by the LIGO-VIRGO collaboration (Abbott et al. 2016a,b,c,d), not only provides a unique method to test Einstein’s theory of GR in the strong field domain, but will also open a new era in astronomy (for reviews see Sathyaprakash & Schutz 2009; Cai et al. 2017). Signals from the reported events GW150914 and GW151226 have been shown to match the waveforms predicted by GR for merging stellar-mass black holes (BHs) (Abbott et al. 2016d). The corresponding initial masses of the merging BHs are $36_{-4}^{+5} M_\odot$ and $29_{-4}^{+4} M_\odot$ for GW150914 (Abbott et al. 2016a,c) and $14_{-4}^{+8} M_\odot$ and $8_{-2}^{+2} M_\odot$ for GW151226 (Abbott et al. 2016a,b). These events directly demonstrate the existence of astrophysical BHs, the formation of binary BHs (BBHs) in nature and their consistent behavior as GR predicts. Besides, the source parameters estimated from comparison of numerical simulations with the GW signal and observations can give new insight into their astrophysical formation (Abbott et al. 2016a,e). Many more stellar-mass BHs are expected to be detected by LIGO in the next few years.

A typical stellar-mass BH, with mass ranging from about 3 to several tens of solar masses, is assumed to be formed by the gravitational collapse of a single massive star. This process is inevitable at the end
of the life of such a star, when all stellar energy sources are exhausted. If the collapsing mass is above the Tolman–Oppenheimer–Volkoff limit, roughly 3 solar masses (Bombaci 1996), a stellar-mass BH will form. In addition, there is observational evidence for two other types of BHs, which are much more massive than the stellar-mass case. They are intermediate-mass BHs and supermassive BHs. For the former, its typical mass ranges between $10^2$ and $10^5$ solar masses. One of the possible candidates is believed to be located in the center of globular clusters (Gebhardt et al. 2005). For the latter, it is the largest type of BH, on the order of hundreds of thousands to billions of solar masses, and is found in the centers of almost all currently known massive galaxies (Antonucci 1993; Urry & Padovani 1995).

As mentioned above, a stellar-mass BH is formed via the collapse of a single massive star. After its formation, most of the material composing the massive star has already been accreted into the BH. Outside of the BH, there is almost nothing. Hence, in the numerical relativity computation of the binary stellar-mass BH merger, we can use the vacuum solution outside their event horizon. On the other hand, intermediate-mass BHs are too massive to be formed by the collapse of a single star. There are three possible formation scenarios. The first is the merging of stellar mass BHs or other compact objects by means of accretion. The second is the runaway collision of massive stars in dense stellar clusters. The third is the primordial BHs formed deep in the radiation dominated era. In the first two channels, a BH is likely to be surrounded by matter. As for a supermassive BH, its origin remains an open question. Astrophysical observations suggest that they are located in the centers of galaxies and can grow by accretion of matter and by merging with other BHs. All these suggest that, when simulating the merger of intermediate-mass or supermassive BH binaries, we need to consider the effect of environment.

On the other hand, a scalar field has already been introduced to study scalar radiation from BH binaries in scalar-tensor theory, triggered either by non-trivial boundary conditions (Berti et al. 2013) or by non-trivial potential (Healy et al. 2012). In Cao et al. (2013), GW radiation in $f(R)$ theory was studied by solving the equivalent GR equations coupled to a real scalar field. In this article, we study the BBH environment by modeling the matter via an axion-like scalar field.

The rest of the paper is organized as follows. In Section 2, we will briefly introduce the axion-like scalar field. In Sections 3 and 4, the evolution of the system with an initially spherically distributed scalar field is considered. The former is the case with large scalar field strength and the latter corresponds to cases with medium and small field strength. The non-spherically symmetric case is presented in Section 5. Our final conclusion is drawn in Section 6. The associated mathematical equations and numerical method are presented in Appendix A.

In most of the paper we will stay in geometric units\(^1\), in which the speed of light $c$ and the gravitational constant $G$ are normalized to 1. So only one dimension is left, which we choose to be length. We also define a new unit $M$, which is related to a meter via $1M = 10^{11}$ m. We employ the following notation: Greek indices ($\mu, \nu, ...$) refer to four-dimensional spacetime indices and take values from 0 to 3. Latin indices ($i, j, k, l, ...$) refer to three-dimensional space indices and take values from 1 to 3.

## 2 AXION-LIKE SCALAR FIELD

In this section, we model the matter field which surrounds a BBH by means of a spherical shell composed by an axion-like scalar field. Several possible candidates exist for the extraordinarily light bosons in fundamental physics, such as the axion field for fuzzy dark matter (Hu et al. 2000). In this work, we consider the Hilbert–Einstein action with the axion-like scalar field written as

$$S = \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right],$$

where $V(\phi)$ is the potential of the axion-like field. In natural units it can be written as

$$V(\phi) = m^2 f^2 \left( 1 - \cos \left( \frac{\phi}{f} \right) \right).$$

For our purpose of considering the interaction of an axion-like scalar field and BBHs, we will fix the mass and decay constant of the particle at $m = 10^{-21}$ eV and $mf = 0.5$ GeV respectively in the calculation, as suggested in Hui et al. (2017).

We will consider the evolution of an equal-mass BBH system with an axion shell around it. Initially, each of the BHs has a mass parameter of $0.5 M^2$, corresponding to an Arnowitt-Deser-Misner (ADM) mass of 0.990473 $M$, and the separation between the two BHs is $11 M$ along the $y$-axis. For the axion shell, we will always set it to be spherical with a radius of $120 M$. The initial condition for this axion shell along the radial direction is

$$\phi(r) = \phi_0 e^{- (r-r_0)^2/2 \sigma^2}, \quad \phi(r)/dt = 0,$$

\(^1\) The transformation from natural to geometric units is presented in Appendix B.

\(^2\) Here we consider an intermediate-mass BBH and hence choose the mass of the BH to be $0.5 M \simeq 100 M_{\odot}$. 
where $r_0$ is the center of the initial Gaussian profile and $\sigma$ is the width parameter; we have chosen that $r_0 = 120\, M$ and $\sigma = 2\, M$.

For the simplest case, in which the whole system is spherically symmetric, we place the center of the axion shell at the center of mass for the BBH system, which is just the origin of numerical domain $(x, y, z) = (0, 0, 0)$. We will consider evolution of the system with three different $\phi_0$s, i.e. small, medium and large with $\phi_0 = 10^{-2}, 10^{-3}, 10^{-4}$ respectively.

To get a clear physical picture and make an estimation of how much impact the axion field will have on the dynamics of the BBH system, we can calculate the total energy of the axion field and compare it with the total mass of the BHs. The energy density of the axion field is given by

$$E := n_\mu n_\nu T^{\mu\nu} = \frac{1}{2} D_i \phi D^i \phi + \frac{1}{2} \phi^2 + \frac{c}{\hbar} m^2 f^2 \left[ 1 - \cos \left( \phi / \left( \sqrt{\frac{\hbar}{c}} f \right) \right) \right].$$

(4)

We can integrate the energy density of the initial profile over the whole space and make an estimation of the total energy of the axion shell. We performed this simple integration and found the total energy is 4.108, 0.04108 and 0.0004108 for large, medium and small $\phi_0$ respectively, which is about 4, 0.04 and 0.0004 times the total mass of the BBHs.

For $\phi_0 = 10^{-3}$ we will also consider a non-spherically symmetric case, in which we place the center of the shell at $(x, y, z) = (0, 0, 50)$, i.e., the shell is translated $50\, M$ along the positive direction of the $x$-axis, while the center of mass for the two BHs is still located at the coordinate origin, and the profile of the spherical axion shell along the radial direction is not changed. So, the binary is located inside the axion shell $50\, M$ away from its center.

We will describe the evolution of these systems in the following sections.

### 3 EVOLUTION OF THE SYSTEM IN THE SPHERICALLY SYMMETRIC LARGE $\phi_0$ CASE

In this section, we will present the numerical results of the evolution of the system in the spherically symmetric large $\phi_0$ case, i.e. $\phi_0$ equals $10^{-2}$. The dynamics of the axion field and the BBH will be discussed. In addition, we will also examine the GWs emitted.

We will firstly look at the evolution of the axion shell in this case. Since the system is spherically symmetric, it is sufficient to look at the axion field in the $x - y$ plane, so all the diagrams in this section and subsequent sections will be presented in the $x - y$ plane.

In Figure 1 we show the profile of the axion field at $t = 30.3\, M$. We can see that very soon after the system starts to evolve, the Gaussian shaped axion shell splits into two parts and evolves inward and outward separately. This is expectable, since the solution of the scalar field wave equation derived from Equation (1) for our initial condition, given by Equation (3), is just a combination of an ingoing and outgoing Gaussian shaped wave. The two parts of the axion shell have the same mass, which is consistent with the fact that the initial condition we have chosen is symmetric with respect to the center of the initial Gaussian profile. The outgoing part will go out of the numerical boundary in some time and will not interact with the BBHs.

In Figure 2 we show the evolution of the ingoing shell together with the BH orbits. It can be seen from the plots that as the ingoing shell falls into the center, it grows denser, and when it finally impacts the BHs, the center value has reached about 20 times the initial value. It is clear that the axion field is attracted by the BBH, and changes from spherical to an almond shape and rotates with the BBH. Since the mass of the axion field is much larger than the two BHs in this case, a large amount of the BH’s angular momentum has turned into angular momentum of the axion field, and the total angular momentum is conserved. More careful examination of the axion field shows that, after a sign change at the center, most of the scalar field is absorbed. There are no outgoing waves afterwards.
The joint evolution of the scalar field and the BBH system in the large $\phi_0$ case is shown. The orbits of the two BHs are given in white and green lines. Different subplots represent different timesteps, which are chosen to be $t = 116.15, 126.25, 131.30, 136.35, 146.45, 151.50, 156.55, 181.10, 207.05$ M respectively. The orbits of the BHs quickly bend to the center as soon as the axion shell passes, and change from a semi-circle to a head-on collision.

$\alpha - 1$ in the large $\phi_0$ case is shown, where $\alpha$ is the lapse function. Left panel: a detailed sketch of $\alpha - 1$ at $t = 171.70$ M, in which a third BH can be roughly seen in the center (marked by the white circle in the figure). Right panel: sketch of $\alpha - 1$ in different timesteps, which are $t = 75.75, 101.00, 136.35, 141.40, 146.45, 151.50, 161.60, 176.75, 202.00$ M respectively. The final BH is comparable to the two initial BHs.
The axion field does not implode through the center. This motivated us to check if there is a third BH that formed in the center by the axion field before the two BHs collide with each other. We can check this by looking at the lapse function \( \alpha \), which is related to the \( tt \) component of the spacetime metric under 3+1 decomposition

\[
g_{\mu \nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt) \cdot (dx^j + \beta^j dt).
\] (5)

We can roughly say a third BH has formed when the lapse function \( \alpha \) reaches zero at some point. The data on \( \alpha - 1 \) are shown in Figure 3, so we are actually looking for the \(-1\) point in these figures. In the righthand sub-panel of Figure 3, we show how the lapse function evolves through different timesteps, and the lefthand side is a detailed sketch of the data at \( t = 171.7 \, M \). A third BH is roughly seen in the center, the mass of the which is about \( 2 \, M \), equal to the mass of the infalling axion shell. After the collision happens, a BH that is much heavier than the original binary is formed.

We have estimated the total energy of the axion shell at the beginning of this section, and found that for the large \( \phi_0 \) case, the energy of the axion field is four times the total mass of the BBH, so we expect the ingoing shell will have an obvious impact on the evolution of the BHs.

In Figure 4, we depict the orbit of the BBHs in the left panel and their radius as a function of time in the right panel. Considering also the joint evolution of the BBH and the axion field shown in Figure 2, we can see the orbit has a sharp turn when the axion field passes through, and changed from a semi-circle inspiralling into a head-on collision. The radial position plot in Figure 4 also shows that the BHs are slightly attracted by the axion shell as it approaches the region around \( t = 80 \, M \) to \( t = 130 \, M \), and the turning point is located around \( t = 135 \, M \) when the ingoing shell starts to collide with the BHs, as depicted in the third subplot in Figure 2. The waveforms emitted in this process will be discussed in the next section.

4 EVOLUTIONS OF THE SYSTEM IN SPHERICALLY SYMMETRIC MEDIUM AND SMALL \( \phi_0 \) CASES

In this section, we will present the numerical results of the evolution of the system in the spherically symmetric medium and small \( \phi_0 \) cases, i.e. \( \phi_0 \) equals \( 10^{-3} \) and \( 10^{-4} \) respectively.

For the medium \( \phi_0 \) case, the qualitative behavior of the axion field in the early time is the same as the large \( \phi_0 \) case in that the shell splits into two parts and evolves inward and outward separately. The evolution of the ingoing shell together with the BH orbits is shown in Figure 5. We can see that after the ingoing shell falls to the center it escapes, propagates outward and then disappears outside the numerical boundary after some time. This behavior is different from the large \( \phi_0 \) case.

Since the axion shell escapes from the center, we can guess that there will not be a third BH formed in this process. This can again be confirmed by looking at the lapse function, which is shown in Figure 6. As before, we show the evolution of \( \alpha - 1 \) through different timesteps in the righthand side of Figure 6, and in the lefthand side a detailed sketch is shown of \( \alpha - 1 \) at \( t = 1232.20 \, M \). This time, the data show no sign of a third BH in the center, and the axion shell could escape from the center to infinity due to energy conservation, as the ingoing axion shell has a very large initial velocity, which dominates over the negative gravitational potential. As the axion shell has escaped, the final BH is not as large as the first case, but is comparable to the initial BHs.

The orbits of the BHs and the radial position as a function of \( t \) are shown in Figure 7. We can see, in this case, the orbits of the BHs are not so strongly affected by the axion shell as in the large \( \phi_0 \) case. But it is clear that the binary orbits developed a large amount of eccentricity (the wiggles in the right panel of Fig. 7) through the accretion of the axion shell, and turned again into round orbits at the late stage of the evolution. A turning point can been seen in the right panel of Figure 7 around \( t = 126 \, M \). At this time, the ingoing axion shell has all gathered inside the orbits of the two BHs, as is depicted in the fourth subplot in Figure 5. The two BHs are attracted by the axion field to fall faster to the center so that the turning point is formed in the panel, which marks the change from semi-circular orbits to elliptic orbits. This is different from the large \( \phi_0 \) case in that the orbits turn sharply as soon as the axion starts to collide with the BBH. So, we can see that when the axion field is smaller, it interacts with the BBH mainly through gravitational attraction, not through collision.

We see that in both large and medium \( \phi_0 \) cases, the axion shell has an obvious influence on the motion of the BHs. It needs to be pointed out that there is no inconsistency between this and Birkhoff’s theorem, which states that any spherically symmetric solution of the vacuum field equations must be static and asymptotically flat, i.e. the exterior solution must be given by the Schwarzschild metric. In fact, Birkhoff’s theorem is no longer valid in the process described here, as the influence of the BHs on the axion shell and spacetime must also be taken into account. Firstly, the axion shell is attracted by the BHs...
Fig. 4 Left panel: the orbits of the two BHs in the large $\phi_0$ case. Two different colors represent two BHs. Right panel: the radial position of one of the BHs as a function of time in the large $\phi_0$ case.

Fig. 5 The joint evolution of the scalar field and the BBH system in the medium $\phi_0$ case is shown. The orbits of the two BHs are given in white and green lines. Different subplots represent different timesteps, which are chosen to be $t = 111.10, 116.15, 121.20, 126.25, 131.30, 136.35, 141.40, 146.45, 166.65$ $M$ respectively. The ingoing axion shell escapes from the center.
Fig. 6 \( \alpha - 1 \) in the medium \( \phi_0 \) case is shown, where \( \alpha \) is the lapse function. *Left panel:* a detailed sketch of \( \alpha - 1 \) at \( t = 1232.20 \ M \); there is no sign of a third BH in the center. *Right panel:* sketch of \( \alpha - 1 \) in different timesteps, which are \( t = 1105.95, 1146.35, 1191.80, 1217.05, 1222.10, 1227.15, 1232.20, 1242.30, 1262.50 \ M \) respectively. The final BH is much larger than the two initial BHs.

Fig. 7 *Left panel:* the orbits of the two BHs in the medium \( \phi_0 \) case. Two different colors represent two BHs. *Right panel:* the radial position of one of the BHs as a function of time in the medium \( \phi_0 \) case. The orbits gain eccentricity through the passage of the axion shell, and turn into round orbits again at the late stage of the evolution.

and cannot always stay spherically symmetric. Secondly, as the BHs are inspiralling, the spacetime metric cannot stay static and spherically symmetric. So in our cases, Birkhoff’s theorem breaks down, and both the BHs and the axion shell can influence each other.

For the small \( \phi_0 \) case, the qualitative behavior of the axion field is the same as in the medium \( \phi \) case: the shell splits into two parts, evolving inwardly and outwardly separately. The ingoing shell implodes through the center and then escapes to infinity. The axion field is so small that it has little impact on the BBH. The orbits of the BHs are shown in Figure 8, and they are nearly the same as in the vacuum GR case.

The waveform of all three spherically symmetric cases is plotted in Figure 9. For the large \( \phi_0 \) case, we can see the radiated wave is greatly reduced since head-on collision suppresses the quadrupole and produces less gravitational radiation than the spherical orbits. For the small \( \phi_0 \) case, the GW emitted is nearly the same as the vacuum GR case. For the medium \( \phi_0 \) case, the peak value
**Fig. 8** Left panel: the orbits of the two BHs in the small $\phi_0$ case. Two different colors represent two BHs. Right panel: the radial position of one of the BHs as a function of time in the small $\phi_0$ case. The orbits of the BHs in the small $\phi_0$ case are nearly the same as the vacuum GR case.

**Fig. 9** The waveform of all the three cases with spherical symmetry detected at $r = 50M$. For the medium $\phi_0$ case (blue curve), the initial burst produced around $t = 126M$ is detected around $t = 180M$ at $r = 50M$. For the large $\phi_0$ case (red curve), the initial burst produced around $t = 135M$ is detected around $t = 220M$ at $r = 50M$. The wiggle located at $t = 50M$ is from artificial numerics.

is a little higher than the small case since an elliptic orbit produces more radiation, and as a result, we can see the BHs merge faster than in the small case.

In Figure 9 we also plot the beginning part of the waveforms. A small burst can be seen in the medium $\phi_0$ case. From the detection time, we find the small burst has resulted from the sudden change in the BH orbits induced by the attraction of the axion field around $t = 126M$, which has been described before in the radius-time plot of Figure 7, but did not result from the collision of the
axion field and the BBH. This peak is not seen in the small $\phi_0$ case, since the axion field is so small and impact of the orbits of the BHs is too tiny to produce this kind of small burst. For the large $\phi_0$ case, the shell reaches the BH a little bit slower than the medium case, and is followed very quickly by the collision of the two BHs, so only a burst of GW radiation caused by the collision can be seen in the waveform.

5 EVOLUTIONS OF THE SYSTEM IN NON-SPHERICALLY SYMMETRIC MEDIUM $\phi_0$ CASE

In this section we will look at the numerical results in the non-spherically symmetric case. We will present the evolution of the axion field, the orbits of the BHs and the corresponding gravitational radiation emitted.

Initially, the qualitative behavior of the axion field is the same as the spherically symmetric case: the shell splits into two parts and they propagate inwardly and outwardly separately. But when the ingoing shell impacts the binary it will be pulled back by the BHs and a sharp point will form in the axion shell. We plot the evolution of the ingoing shell together with the orbits of the BHs in Figure 10. It is clear that the axion field is dragged back by the BH every time it passes them. The shell implodes through center and escapes to infinity, just the same as the medium spherically symmetric case, so there is no new BH formed.

In Figure 11 we plot the orbits of the two BHs. We can see that, under the influence of the axion shell, the two BHs keep moving to the positive direction of the $x$-axis, even after the collision. So although the axion shell passes the BBH twice in total, the net impact of the axion field on the BBH is to the positive direction of the $x$-axis. This again confirms that at this value of the axion field, the BHs interact with the axion field mainly through gravitational attraction, but not through collisions. So when the axion shell first passes the BBH, the BBH is outside the axion shell, and the shell attracts the BHs to the positive direction of the $x$-axis. After the axion shell implodes through the center and passes the BBH for the second time, the BBH is again inside the shell, so there will not be much attractive force applied by the axion shell. This is consistent with Birkhoff’s theorem.

In Figure 12, we plot the waveform produced in this process compared to the small $\phi_0$ and medium $\phi_0$ spherically symmetric cases. The waveform is very similar to the small $\phi_0$ case with spherical symmetry, only that the two BHs merge a little earlier. From the subplot showing the initial part of the waveforms, we can see the small burst is not present in this non-spherically symmetric case, since in this case, the net effect of the axion field on the BBH is a translation to the collapsing direction of the axion shell, and there is no sudden fall in the initial part of the orbits as in the spherically symmetric Medium $\phi_0$ case, which can be seen in the right panel of Figure 11. This again confirms that the small burst comes from the sudden change of orbits but not from the collisions between the axion field and the BHs.

6 CONCLUSIONS

In this paper, we studied how environment could modify the motion and GW signals of astrophysical BH binaries. We considered the joint evolution of an intermediate-mass BBH system and a shell with axion-like scalar field around it. Both cases with and without spherical symmetry were studied. We performed full numerical simulations and showed the numerical results concerning the evolution of the axion field, the orbits of the BHs and the GW emitted in different cases.

Firstly, when the shell of the scalar field is spherically symmetric with respect to the BBH center of mass, we studied three cases with different $\phi_0$s, i.e. large, medium and small with $\phi_0 = 10^{-2}, 10^{-3}, 10^{-4}$ correspondingly. Our results show that with increasing scalar field strength, the number of circular orbits of the BBH is reduced. It means that the scalar field could significantly accelerate the merger process. Once the scalar field strength exceeds some certain critical value, the scalar field could collapse into a third BH with its mass being larger than the binary. In this case the orbits of the two BHs are greatly affected by the scalar field such that they are turned from quasi-circular to a head-on collision right after the pass of the scalar shell. As a result, there is almost no quadrupolar signal produced, and, consequently, the GW is greatly suppressed. When the field strength of the scalar field is smaller than the critical value as in the medium $\phi_0$ case, the scalar field is not strong enough to form a third BH, so the axion shell implodes through the center and then escapes to infinity. The two BH orbits develop eccentricity through the accretion of the axion field, which could accelerate the merger process compared to the semi-circular orbit. The eccentricity disappears again at the late stage of the evolution.

In the case when the axion field is non-spherically symmetric with respect to the BBH system, we found that the orbits of the two BHs are constantly attracted to the center of mass for the scalar field, and the existence of the scalar field slows down the merger process compared to the spherically symmetric case.
The joint evolution of the scalar field and the BBH system in the medium $\phi_0$ case is shown. The orbits of the two BHs are marked by white and green lines. Different subplots represent different timesteps, which are chosen to be $t = 55.55, 75.75, 90.90, 101.00, 116.15, 161.60, 176.75, 191.90, 202.00 \, M$ respectively. It is clear that the axion shell is dragged by the BHs every time it passes.

The orbits of the two BHs in the medium $\phi_0$ non-spherically symmetric case. Two different colors represent two different BHs. The radial position of the BH as a function of time in the medium $\phi_0$ non-spherically symmetric case. The BHs are attracted to the positive directions of the $x$-axis under the influence of the axion shell, even after the collision happens. The final velocity is $0.0014c$. 
We also found that the behaviors in the initial part of the waveform can be affected by the strength of the scalar field and the degree of symmetry in the system. In the spherically symmetric case, when the scalar field is very small, the interaction between the scalar field and the BHs is too weak to produce any initial burst. As the strength of the scalar field gets stronger, a small burst can be found in the initial part of the waveform induced by the sudden alteration of BH orbits, which is caused by the sudden amplified attraction force to the center when the scalar field passes the two BHs and gathers inside the BH orbits. When the strength of the scalar field gets even stronger, the collision of the BHs starts much earlier, so the initial burst is buried in the burst of radiation produced by the collision of the two BHs, and cannot be seen in the waveform. In the non-spherically symmetric case, although the strength of the scalar field is the same as the medium $\phi_0$ spherically symmetric case, the initial burst does not exist in the waveform. This is because the net effect of the scalar field on the BBH is a translation along the direction of collapse of the axion, and there is no sudden turn to the center in the orbits as in the spherically symmetric case. This confirms that the small initial burst comes from the sudden change of orbits but not from the collisions between the scalar field and the BHs at this strength of the scalar field.

In summary, our result showed that the axion-like scalar field in the environment could essentially modify the dynamics, and that it is possible to investigate the environment around BBH systems through the GWs they emit. The abundance and distribution of an axion-like scalar field around a BBH system can be constrained through studying the overall behavior and also the initial part of the waveform.

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Appendix A: MATHEMATICAL BACKGROUND
AND NUMERICAL METHOD

In this appendix we will describe the mathematical background and numerical method used in the numerical calculations. We apply the extended AMSS-NCKU code used by Zhoujian Cao et al. in their previous works Cao et al. (2008) and Cao et al. (2013). The code is based on the BSSN formalism, which is a conformal-traceless “3+1” formulation of the Einstein equations. In this formalism, the spacetime is decomposed into a set of three-dimensional spacelike slices \( \Sigma_i \), and can be described by a three-metric \( \gamma_{ij} \). The extrinsic curvature \( \psi \) and the conformal factor \( \omega \) are defined as two independent variables, and \( \gamma_{ij} \) is subjected to the constraint that it is unimodular. The same can be done to the extrinsic curvature, that is, in place of \( K_{ij} \) we evolve

\[
K = \gamma^{ij} K_{ij}, \quad A_{ij} \equiv \gamma_0^{-1} K_{ij} - \frac{1}{3} \gamma_{ij} K. \tag{A.2}
\]

Similar to the conformal metric, \( A_{ij} \) is subjected to a constraint that \( A_{ij} \) is traceless. We further introduce another evolution variable, the conformal connections

\[
\Gamma^i \equiv -\hat{\Gamma}^{ij}_j, \tag{A.3}
\]

which are defined in terms of the contraction of the spatial derivative of the inverse conformal three-metric \( \gamma_{ij} \).

The Einstein equation and equations of motion for the scalar field can be derived from the Lagrangian in Equation (1), which are

\[
G_{\mu \nu} = 8\pi T_{\mu \nu}, \quad \Box \phi = \frac{dV}{d\phi}, \tag{A.4}
\]

where \( G_{\mu \nu} \) is the Einstein tensor, \( V \) is the potential of the axion field given in Equation (2) and \( T_{\mu \nu} \) is the energy-momentum tensor of the scalar field. For the scalar field equation (A.5), we can decompose it using the 3+1 formalism described above. Defining an auxiliary variable \( \phi = \mathcal{L}_n \phi \), where \( \mathcal{L}_n \) denotes the Lie derivative along the normal to the hypersurface \( \Sigma_i \), the evolution equation for the scalar field \( \phi \) and the Lie derivative \( \varphi \) in terms of the lapse function \( \alpha \) and the shift vector \( \beta^j \) can be expressed as

\[
\partial_t \phi = \alpha \varphi + \beta^j \partial_j \phi, \quad \tag{A.6}
\]

\[
\partial_t \varphi = \alpha \psi_0^{-1} \left( \xi^{ij} \partial_j \partial_i \phi - (\Gamma^i - 2 \xi^{ij} \psi_0 \partial_j \psi_0) \partial_i \phi \right) + \psi_0^{-4} \xi^{ij} \partial_i \alpha \partial_j \phi + \alpha \varphi, \tag{A.7}
\]

where the BSSN metric conformal transformation (A.1) and the following relationships are used

\[
K = -\frac{\gamma^{ij}}{2\alpha} \frac{\partial \gamma_{ij}}{\partial t}, \quad \Gamma^i = -\frac{1}{\sqrt{\gamma}} \partial_i \left( \sqrt{\gamma} \bar{\gamma}^{ij} \right). \tag{A.8}
\]

With the 3+1 formalism, the different components of the scalar field energy-momentum tensor are given by

\[
E := n_t n_b T^{ab} = \frac{1}{2} D_i \phi D^i \phi + \frac{1}{2} \varphi^2 + V, \tag{A.10}
\]

\[
s_k := -\gamma_{l} n_t n_b T^{ab} = -\varphi D_l \phi, \tag{A.11}
\]

\[
s_{ij} := \gamma_{l} n_b T^{ab} = D_i \phi D_j \phi - \psi_0^{-4} \xi^{ij} \left( \frac{1}{2} D_k \phi D^k \phi \right) - \frac{1}{2} \varphi^2 + V, \tag{A.12}
\]

So, the Einstein equation in (A.4) can be decomposed into the five evolution equations for the dynamical variables defined above, together with the gauge conditions, and the evolution equations for the system read:

\[
\partial_t \psi_0 = \beta^i \psi_0 - \frac{1}{6} \alpha K + \frac{1}{6} \beta^j \beta^i, \tag{A.13}
\]

\[
\partial_t \bar{\gamma}_{ij} = \beta^k \bar{\gamma}_{ij,k} - 2 \alpha A_{ij} + 2 \bar{\gamma}_{k(ij} \beta^k_{j)}, \tag{A.14}
\]

\[
\partial_t K = \beta^i K_{i} - D^2 \alpha + \alpha \left[ A_{ij} A^{ij} + \frac{1}{3} K^2 + 4\pi (\rho + s) \right], \tag{A.15}
\]

\[
\partial_t A_{ij} = \beta^k A_{ij,k} + \psi_0^{-4} \left( \alpha (R_{ij} - 8\pi s_{ij}) - D_i D_j \alpha \right) \tag{A.16}
\]

\[
\partial_t \Gamma^i = \beta^j \Gamma^i_{j} - 2 A^{ij} \alpha_{j} + 2 \alpha (\bar{\Gamma}^i_{j} A^{kj} - \frac{2}{3} \bar{\gamma}^{ij} K_{j}) - 8\pi \bar{\gamma}^{ij} s_{j} + 6 A^{ij} \phi_{j} - \bar{\Gamma}^i \beta^j + \frac{2}{3} \bar{\gamma}^{ij} \beta^j \tag{A.17}
\]
\[ \partial_t \phi = -2 \alpha K, \quad \text{(A.18)} \]
\[ \partial_t B^i = \frac{3}{4} B^i, \quad \text{(A.19)} \]
\[ \partial_t \Gamma^i - 2 B^i, \quad \text{(A.20)} \]
where “TF” means the trace free part.

We still need the constraint equations to solve for the initial data. Under a 3+1 decomposition, the momentum constraint and the Hamiltonian constraint equations are

\[ D_j K^i_j - D_i K = 8 \pi p_i, \quad \text{(A.21)} \]
\[ R + K^2 - K_{ij} K^{ij} = 16 \pi E, \quad \text{(A.22)} \]

where \( D_j \) is the covariant derivative with the 3-metric \( \gamma_{ij} \), \( E \) and \( p_i \) are the energy and momentum densities given in (A.10)–(A.12). The constraint equations can be solved with the puncture method. Following the conformal transverse-traceless decomposition approach described above, choose a conformally flat back

\[ \text{The action of the axion field in natural units is given in Equation (1). As mentioned above, we have chosen that } m = 10^{-21} \text{ eV and } m f = 0.5 \text{ GeV. The units of the two parameters and the scalar field in the three different unit systems are summarized in Table B.1.} \]

| Table B.1 Different Dimensions of the Axion Field in Geometric Units |
|---|
| f | s^{-1} | eV | m^{-1} |
| m | kg | eV | m |
| \( \phi \) | \( \sqrt{\text{eV/m}} \) | eV | 1 |

In order to have an expression in geometric units, we will first convert the Lagrangian of the axion field to the SI system. We know that a Lagrangian has a dimension of energy in the SI system, i.e., \( \text{dim} L = ML^2/T^2 \). In order to make each term in the Lagrangian have dimension of energy in the SI system, we should insert back proper combinations of \( h \) and \( c \). It turns out that a factor of \( c/h \) should be inserted in front of the potential term of the scalar field. On the other hand, from the expression of the potential, we know that the argument inside the cosine function should be dimensionless, so a factor of \( \sqrt{c/h} \) should also be inserted into the denominator of this argument. Now we have the Lagrangian in the SI system, which is

\[ L = \int d^3 x \sqrt{g} \left[ \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{c}{h} m^2 f^2 \left[ 1 - \cos \left( \frac{\phi}{\sqrt{c/h} f} \right) \right] \right], \quad \text{(B.1)} \]

where we have chosen

\[ \left( \frac{c}{h} f \right)_{\text{Nat}} = 0.508 M_{\text{pl}}, \]
\[ \left( \frac{c}{h} m^2 f^2 \right)_{\text{Nat}} = 1.486 \times 10^{12} \text{ eV}^4, \quad \text{(B.2)} \]
the subscript “Nat” means natural units and \( M_{pl} = 1/\sqrt{8\pi G_N} = 2.4 \times 10^{18} \text{ GeV} \) is the reduced Planck mass. We can see from the Lagrangian that it is the combination of \( \frac{c}{\hbar} m^2 f^2 \) and \( \sqrt{\frac{\hbar}{c}} f \) that will enter the equations of motion. Dimensions of these two combinations are summarized in Table B.2.

| Table B.2 Different dimensions of \( \frac{c}{\hbar} m^2 f^2 \) and \( \sqrt{\frac{\hbar}{c}} f \), which are the two combinations appearing in the Lagrangian, in three different unit systems. |
|-----------------|-----------------|-----------------|
| \( \frac{c}{\hbar} m^2 f^2 \) | International (SI) | Natural | Geometric |
| \( \sqrt{\frac{\hbar}{c}} f \) | kg/(m \cdot s^2) | (eV)^4 | m^{-2} |

Now we need to get the values of the two combinations in geometric units. Firstly their values in the SI system can be found by inserting back \( \hbar \) and \( c \)

\[
\left( \sqrt{\frac{\hbar}{c}} f \right)_{SI} = \left( \sqrt{\frac{\hbar}{c}} f \right)_{Nat} \times \sqrt{\frac{1}{\hbar c}}
\]

\[
= 2.745 \times 10^{30} \sqrt{\frac{\text{eV}}{\text{m}}},
\]

\[
\left( \frac{c}{\hbar} m^2 f^2 \right)_{SI} = \left( \frac{c}{\hbar} m^2 f^2 \right)_{Nat} \left/ \left( \hbar c^3 \right) \right.
\]

\[
= 1.934 \times 10^{32} \text{eV m}^3,
\]

where the subscript “SI” means SI system. Now we can change to geometric units by dividing proper combinations of \( G \) and \( c \)

\[
\left( \sqrt{\frac{\hbar}{c}} f \right)_{Geo} = \left( \sqrt{\frac{\hbar}{c}} f \right)_{SI} \times \sqrt{\frac{G}{c^4}}
\]

\[
= 0.100,
\]

\[
\left( \frac{c}{\hbar} m^2 f^2 \right)_{Geo} = \left( \frac{c}{\hbar} m^2 f^2 \right)_{SI} \times \frac{G}{c^4}
\]

\[
= 2.560 \times 10^{-20} \text{M}^{-2},
\]

where the subscript “Geo” means geometric units. We will use these values in geometric units in the numerical calculations.

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