Influence of Anomalous Dispersion on Optical Characteristics of Quantum Wells

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Frequency dependencies of optical characteristics (reflection, transmission and absorption of light) of a quantum well are investigated in a vicinity of interband resonant transitions in a case of two closely located excited energy levels. A wide quantum well in a quantizing magnetic field directed normally to the quantum-well plane, and monochromatic stimulating light are considered. Distinctions between refraction coefficients of barriers and quantum well, and a spatial dispersion of the light wave are taken into account. It is shown that at large radiative lifetimes of excited states in comparison with nonradiative lifetimes, the frequency dependence of the light reflection coefficient in the vicinity of resonant interband transitions is defined basically by a curve, similar to the curve of the anomalous dispersion of the refraction coefficient. The contribution of this curve weakens at alignment of radiative and nonradiative times, it is practically imperceptible at opposite ratio of lifetimes. It is shown also that the frequency dependencies similar to the anomalous dispersion do not arise in transmission and absorption coefficients.

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Optical methods are widely used at research of electronic properties of low-dimensional semiconductor systems during last decades. It is connected mainly that after interactions with such system, the light wave contains an information about electronic processes, in particular, about an electronic spectrum, lifetimes of excited states and scattering mechanisms. Interesting results turn out, when energy levels of the electronic system are discrete that takes place in quantum dots and quantum wells. Discreteness of energy levels in a quantum well is provided by excitonic states (if incident light is perpendicular to the quantum-well plane), or by quantizing magnetic field directed perpendicularly to the quantum-well plane. In high-quality quantum wells, a radiative broadening of absorption lines at low temperatures and weak doping can be comparable with the contribution of nonradiative relaxation mechanisms or to exceed them. In this case, it is impossible to be limited by linear approximation on interaction of electrons with electromagnetic field, and it is necessary to consider all the orders of this interaction.

Reflection, absorption and transmission of an electromagnetic wave, which interacts with discrete energy levels of electronic system in a quantum well in the region of frequencies corresponding to interband transitions, were considered in References 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24. In these works, a stimulating light could be monochromatic or pulse irradiation. One, two and large number of excited energy levels were considered. Results of these works are applicable for narrow quantum wells, when the inequality

\[ \kappa d \ll 1 \]  

is carried out, where \( d \) is the quantum well width, \( \kappa \) is the module of the light wave vector. Actually above mentioned works are true in a zero approximation on the parameter \( \kappa d \).

On the other hand, for wide quantum wells the parameter \( \kappa d \) can be \( \approx 1 \). For example, for the GaAs hetero-laser radiation (wave length 0.8\( \mu \)m and a quantum-well width \( d \approx 500 \text{Å} \), the parameter \( \kappa d \approx 1.5 \). In this case, it is necessary to take into account the spatial dispersion of the electromagnetic wave, since its amplitude strongly varies inside of the quantum well. Besides, for wide quantum wells, an inequality \( d \gg \alpha_0 \) (\( \alpha_0 \) is the lattice constant) is very strong, that allows to use Maxwell equations for continuous matters at determination of the electromagnetic field. Such approach allows to take into account distinction in refraction coefficients of barriers and quantum wells.

In References 20, 21, the theory considering spatial dispersion of an electromagnetic wave at its passage through a quantum well is developed. One excited energy level was considered (i.e., one interband transition) and alongside with the spatial dispersion, different refraction coefficients of barriers and of quantum well were introduced for monochromatic and for pulse irradiation. Work 22 is devoted to the account of a spatial dispersion of an electromagnetic wave in the case of two closely located interband resonant transitions. That corresponds to the magnetopolaron state in a quantum well 23.

At calculation of optical characteristics of a quantum well in the resonance, the contribution of resonant transitions is allocated from dielectric permeability. It is considered separately. These resonant transitions lead to occurrence of a high-frequency current, in which alongside with the contribution, frequency dependence of which corresponds to absorption, there is the contribution, frequency dependence of which is similar to a curve of the
anomalous dispersion of the refraction coefficient. Below, the role of these two contributions in formation of frequency dependencies of the light reflection, transmission and absorption by a quantum well is investigated as an example of two closely located resonant transitions. It is generalization of two previous works of authors: Unlike of\textsuperscript{20}, two excited energy levels are taken into account, and results of\textsuperscript{22} are generalized on a case, when refraction coefficients of barriers and of quantum well are different.

\textbf{I. BASIC RELATIONSHIPS}

The system consisting of a semiconductor quantum well, located in an interval $0 \leq z \leq d$, and of two semi-infinite barriers is considered. A constant quantizing magnetic field is directed perpendicularly to the semi-infinite barriers is considered. A constant quantization vector is carried out. Thus, wave functions of the electron-hole pair and energy levels do not depend on indices $I$ and $II$.

As is known, the electron-hole pair in a magnetic field is free, if two conditions are satisfied. First, the Lorentz’s force must be large in comparison with Coulomb and exchange forces of electron - hole interactions in pairs. Then, the wave function of an electron-hole pair can be represented in the form of a product of two functions: $\Phi(z)$ and the function depending on $r$, in the $xy$ plane of the quantum well. Estimations show\textsuperscript{27,28} that in GaAs for a magnetic field corresponding to formation of magnetopolarons, this condition is carried out. The second condition is some excess of a size-quantization energy over the Coulomb and exchange energy of interaction in the electron-hole pair. Then, it is possible to consider the electron-hole pair as free and to use the approximation of infinitely high barriers, which is used below. The wave function describing dependence on $z$ is as follows:

$$\Phi(z) = \frac{2}{d} \sin \left(\frac{\pi m_{e} z}{d}\right) \sin \left(\frac{\pi m_{v} z}{d}\right); \quad 0 \leq z \leq d \tag{4}$$

and $\Phi(z) = 0$ in barriers. The index $\lambda = (m_{e}, m_{v})$ depends on quantum numbers of size-quantization of electrons $m_{e}$ and holes $m_{v}$.

Let us note that in GaAs at $\kappa d \geq 1$, the second condition of existence of free electron-hole pairs is not carried out and the function $\Phi(z)$ will be another there. However, the approximation (4), essentially simplifying calculations, does not affect qualitatively (as it is shown in Section 4) on frequency dependence of optical characteristics of a quantum well.

In the present work, monochromatic light of frequency $\omega_I$ and normal incidence on a quantum-well plane is considered. According to these assumptions, an electric field of stimulating light wave looks like:

$$E_0 = e_I E_0 e^{-i(\omega_I t - \kappa_1 z)} + c.c., \quad \kappa_1 = \nu_I \omega_I/c, \tag{5}$$

where $E_0$ is the complex amplitude, $\nu_I$ is the real material refraction coefficient in barriers. It is supposed also that there are two closely located excited energy levels in a quantum well, and the others excited energy levels in the quantum well are located far enough. Such situation is possible, for example, for a magnetopolaron state\textsuperscript{19,23}.

In case of a direct interband transition, the average induced charge density in a quantum well $\rho(z,t) = 0$, that allows to introduce the gauge $\varphi(z,t) = 0$, where $\varphi(z,t)$ is the scalar potential. Then, the amplitude of electric field in barriers is determined by the equation:

$$\frac{d^2 E}{dz^2} + \kappa_1^2 E = 0, \quad \kappa_1 = \nu_I \omega_I/c; \quad z \leq 0, z \geq d, \tag{6}$$
and in a quantum well (0 ≤ z ≤ d), it is determined by the equation

\[
\frac{d^2 E}{dz^2} + \kappa^2 E = \frac{4\pi i\omega_i}{c^2} J(z); \quad \kappa = \frac{\nu_i}{c}.
\]  

(7)

e is the light velocity in vacuum; \(\nu_i, \nu\) are the refraction coefficients in barriers and in the quantum well, respectively. \(J(z)\) is the Fourier-component of the current density (averaged on the system ground state), which is induced in the quantum well by a monochromatic plane wave. In case of two excited energy levels, \(J(z)\) for a quantum well with infinitely high barriers looks like (The general formula is represented, for example, in20,21):

\[
\tilde{J}(z) = \frac{ivc}{4\pi} \sum_{j=1}^{2} \frac{\gamma_{rj}}{\omega_j} \int_0^d d' \Phi_j(z') E(z'),
\]

(8)

and \(\tilde{J}(z) = 0\) in barriers. Here, \(\gamma_{rj}\) is the radiative broadening of the excited states of an energy doublet in the case of narrow quantum wells.

If the doublet is formed by a magnetopolaron \(A\) to which the Landau quantum number of a hole \(n = 1\) (according to the magnetopolaron classification22), then,

\[
\gamma_{rj} = \gamma_r Q_{0j},
\]

(9)

where

\[
\gamma_r = \frac{2e^2 \lambda}{\hbar c \nu_0 \omega_g m_0 c}
\]

(10)

\((\hbar \omega_g\) is the energy gap, \(m_0\) is the free electron mass, \(H\) is the magnetic field, \(e\) is the electron charge). The factor

\[
Q_{0j} = \frac{1}{2} \left(1 + \frac{\lambda}{\sqrt{\lambda^2 + (\Delta F_{pol})^2}}\right),
\]

(11)

determines a change of radiative lifetime at deviation of the magnetic fields from the resonant value defined by equality \(\Omega_c = \omega_{LO}\). \(\Delta F_{pol}\) is the polaron splitting22, \(\Omega_c, \omega_{LO}\) are the cyclotron frequency and longitudinal optical phonon frequency, respectively. In the resonance, \(\lambda = 0\), \(Q_{0j} = 1/2\) and \(\gamma_{r1} = \gamma_{r2}\).

Resonant denominators in (8) are:

\[
\omega_j = \omega_1 - \omega_j + i\gamma_j/2,
\]

(12)

where \(\omega_j\) are the frequencies of resonant transitions of the doublet energy levels, \(\gamma_j\) are the radiative broadenings of these levels. In (8), resonant denominators are taken into account only. Subscripts \(j = 1\) and \(j = 2\) of the function \(\Phi_j(z)\) correspond to pairs of quantum numbers of the size-quantization, which constitute the expression \(n\), and \(m_c, m_v\) correspond to subscript \(j = 1\); \(m_c, m_v\) correspond to subscript \(j = 2\). The Landau quantum number \(n\) is kept at the direct transition. The total field \(E\) enters on the right-hand side of the equation (8), that is connected with refusals from the perturbation theory on the coupling constant \(e^2/\hbar c\).

II. ELECTRIC FIELD OF AN ELECTROMAGNETIC WAVE

The further calculation is spent in the assumption that

\[
m_c(1) = m_c(2) = m_c, \quad m_v(1) = m_v(2) = m_v,
\]

(13)

that, in particular, corresponds to the magnetopolaron \(A\). In this case, the equality

\[
\Phi_1(z) = \Phi_2(z) = \Phi_{m_c, m_v}(z) \equiv \Phi(z)
\]

(14)

takes place and the formula (8) becomes

\[
\tilde{J}(z) = \frac{ivc}{4\pi} \left(\frac{\gamma_{r1}}{\omega_1} + \frac{\gamma_{r2}}{\omega_2}\right) \Phi(z) \int_0^d d' \Phi(z') E(z').
\]

(15)

The solution of the equation (6), determining the amplitude of the field \(E(z)\) in barriers, is

\[
E_{r}(z) = E_0 e^{i\kappa z} + C_R e^{-i\kappa z}; \quad z \leq 0,
\]

(16)

\[
E_{r}(z) = C_T e^{i\kappa z}; \quad z \geq d,
\]

(17)

\(C_R\) determines the amplitude of the reflected wave, \(C_T\) determines the amplitude of the wave which passed through the quantum well. It is convenient to represent the integro-differential equation (7) for the field amplitude in a quantum well in the form of the Fredholm integral equation of the second type20

\[
E(z) = C_1 e^{i\kappa z} + C_2 e^{-i\kappa z} - \frac{i}{2} \left(\frac{\gamma_{r1}}{\omega_1} + \frac{\gamma_{r2}}{\omega_2}\right) F(z) \int_0^d d' \Phi(z') E(z'),
\]

(18)

where

\[
F(z) = F_{m_c, m_v} = e^{i\kappa z} \int_0^z d' e^{-i\kappa z'} \Phi(z')
\]

\[+ e^{-i\kappa z} \int_z^d d' e^{i\kappa z'} \Phi(z').
\]

(19)

For arbitrary \(m_c\) and \(m_v\), \(F(z)\) is equal

\[
F(z) = iB \left\{ d\Phi(z) - e^{i\kappa z} - (-1)^{m_c + m_v} e^{i\kappa(z - d)} \right\}
\]

\[+ \frac{d}{2} \left[ \frac{m_c^2 + m_v^2}{m_c m_v} - \frac{(\kappa d)^2}{\pi^2 m_c m_v}\right] \Phi(z) \right\}.
\]

(20)

\(\Phi(z)\) is defined in (4),

\[
\Phi(z) = \frac{2}{d} \cos \left(\frac{\pi m_c z}{d}\right) \cos \left(\frac{\pi m_v z}{d}\right),
\]

\[
B = 4\pi^2 m_c m_v \kappa d
\]

\[\times \left[ \pi^2 (m_c + m_v)^2 - (\kappa d)^2 \right]^{-1}
\]

\[\times \left[ (\kappa d)^2 - \pi^2 (m_c - m_v)^2 \right]^{-1}.
\]

(21)
It follows from (19) and (20) that
\[
F(0) = iB[1 - (-1)^{m_e + m_v} e^{i\kappa d}],
\]
\[
F(d) = (-1)^{m_e + m_v} F(0). \tag{22}
\]

Multiplying the equation (18) on \(\Phi(z)\) and integrating on \(z\) from 0 up to \(d\), we obtain
\[
\int_0^d dz \Phi(z) E(z) = \frac{h\tilde{\omega}_1\tilde{\omega}_2}{\tilde{\omega}_1\tilde{\omega}_2 + (i\varepsilon/2)\gamma_{1r}\tilde{\omega}_1 + \gamma_{1r}\tilde{\omega}_2}, \tag{23}
\]
where designations are introduced:
\[
\varepsilon = \varepsilon' + i\varepsilon'' = \int_0^d dz \Phi(z) F(z), \tag{24}
\]
\[
h = \int_0^d dz \Phi(z) \left(C_1 e^{i\kappa z} + C_2 e^{-i\kappa z}\right)
= F(0)\left[C_1 + (-1)^{m_e + m_v} e^{-i\kappa d} C_2\right]. \tag{25}
\]
As a result, complex amplitude of the electric field in a quantum well becomes
\[
E(z) = C_1 e^{i\kappa z} + C_2 e^{-i\kappa z}
- \frac{(i/2)h F(z)(\gamma_{1r}\tilde{\omega}_2 + \gamma_{1r}\tilde{\omega}_1)}{\tilde{\omega}_1\tilde{\omega}_2 + i(\varepsilon/2)(\gamma_{1r}\tilde{\omega}_2 + \gamma_{1r}\tilde{\omega}_1)}. \tag{26}
\]

Parameter \(\varepsilon\), as well as in the case of one excited energy level, determines renormalization of the radiative broadening \(\varepsilon'\) and shift \(\varepsilon''\) for each of two excited levels. It follows from (24) and (20) that
\[
Re \varepsilon' = 2B[1 - (-1)^{m_e + m_v} \cos \kappa d],
\]
\[
Im \varepsilon' = \varepsilon''.
\]

(At \(\kappa d \to 0\) \(\varepsilon' \to 1, \varepsilon'' \to 0\) \((m_e = m_v)\) and \(\varepsilon' \to 0, \varepsilon'' \to 0\) \((m_e \neq m_v)\)). Thus, the real radiative broadening of energy levels of the doublet is determined by values
\[
\varepsilon'_{\gamma_{1r}} = \tilde{\gamma}_{1r}, \quad i = 1, 2. \tag{28}
\]

That fact that \(\varepsilon'\) and \(\varepsilon''\) are identical for both energy levels of the doublet is connected with the assumption of equality of the size-quantization quantum numbers \(m^{(1)}_e = m^{(2)}_e\).

Arbitrary constants \(C_1\) and \(C_2\) enter, according to (25), in the function \(h\). Constants \(C_1, C_2, C_R\) and \(C_T\) were determined from continuity conditions of the magnetic field and the tangential projection of the electric field on borders \(z = 0\) and \(z = d\). Normal projections of an electric field are equal to zero. Arbitrary constants are equal:
\[
C_1 = \frac{2E_0}{\Delta} e^{-i\kappa d}[1 + \zeta + (1 - \zeta)N],
\]
\[
C_2 = -\frac{2E_0}{\Delta} (1 - \zeta)[e^{i\kappa d} + (1)^{m_e + m_v} N], \tag{29}
\]
\[
C_R = \frac{E_0\rho}{\Delta}, \quad C_T = \frac{4E_0}{\Delta}
\times \zeta e^{-i\kappa d\left[1 + (-1)^{m_e + m_v} e^{-i\kappa d} N\right]}, \tag{30}
\]
\[
\Delta = (\zeta + 1)^2 e^{-i\kappa d} - (\zeta - 1)^2 e^{i\kappa d} - 2(\zeta - 1)
\times [((\zeta + 1)e^{-i\kappa d} + (1)^{m_e + m_v}(\zeta - 1)]N. \tag{31}
\]

\[
\rho = 2i(\zeta^2 - 1) \sin \kappa d
+ 2[(\zeta^2 + 1)e^{-i\kappa d} + (1)^{m_e + m_v}(\zeta^2 - 1)]N. \tag{32}
\]

In (29) - (32), the designation
\[
\zeta = \kappa/\kappa_1 = \nu/\nu_1 \tag{33}
\]
is introduced and the function \(N\) looks like:
\[
N = -i(-1)^{m_e + m_v} e^{i\kappa d}
\times (\tilde{\gamma}_{1r}\tilde{\omega}_2 + \tilde{\gamma}_{1r}\tilde{\omega}_1)/2.[\tilde{\gamma}_{1r}\tilde{\omega}_2 + i(\varepsilon/2)(\gamma_{1r}\tilde{\omega}_2 + \gamma_{1r}\tilde{\omega}_1)]. \tag{34}
\]
At obtaining of this formula, the equality
\[
F^2(0) = (-1)^{m_e + m_v} e^{i\kappa d} \varepsilon'. \tag{35}
\]
is used.

Uncertain coefficients (29) - (30) on their form coincide with obtained in (20) for one excited energy level. Distinction consists in the function \(N\), which in case \(\gamma_{1r}\) or \(\gamma_{1r}\) \(\to 0\) passes in \(N\) (the formula (41) in (20)). The curve \(N(\omega_{\pm})\) is a function with two extrema, etch of which corresponds to the resonant transition. It may be represented more evidently, namely,
\[
N = -i(-1)^{m_e + m_v} e^{i\kappa d}
\times \left[\frac{\tilde{\gamma}_{1r}/2}{\Omega_1 + i\Gamma_1/2} + \frac{\tilde{\gamma}_{1r}/2}{\Omega_2 + i\Gamma_2/2}\right]. \tag{36}
\]
Here, the renormalized resonant frequencies \(\Omega_1\) and \(\Omega_2\) and total broadening of each energy level \(\Gamma_1\) and \(\Gamma_2\) are equal:
\[
\Omega_j = \omega_l - \omega_j - \varepsilon''_{\gamma_{1r}}/2 - \beta'_j,
\]
\[
\Gamma_j = \gamma_j + \beta_{j'} + 2\beta'_j. \tag{37}
\]

Values \(\beta'_{1(2)}\) and \(\beta''_{1(2)}\) make complex parameters
\[
\beta_1 = \beta_1' + i\beta_1'' = \frac{\varepsilon \gamma_{1r}}{2} \tilde{\omega}_1,
\]
\[
\beta_2 = \beta_2' + i\beta_2'' = \frac{\varepsilon \gamma_{1r}}{2} \tilde{\omega}_2. \tag{38}
\]
They determine mutual influence of energy levels. In the function (36), two peaks are presented in the explicit form, but they are not of the Lorentz-type, since parameters $β_1$ and $β_2$ ( bringing the contribution in broadening, and in shift of peaks) depend on frequency. It is naturally that representation of the function $N$ in the form of the sum Lorentzian’s (as it is made in 19, 22) and non-Lorentzian’s curves leads to identical results. Let us represent also an obvious form of parameters $β_j’$ and $β_j^\prime$:

$$β_1’ = \frac{(γ_r2/2)σ_1 - e^γ(γ_r2/2)σ_2}{(ω_1 - ω_2)^2 + γ_2^2/4},$$

$$β_2’ = \frac{(γ_r1/2)σ_1 + e^γ(γ_r1/2)σ_2}{(ω_1 - ω_1)^2 + γ_2^2/4},$$

$$β_1^\prime = \frac{(γ_r2/2)σ_2 + e^γ(γ_r2/2)σ_1}{(ω_1 - ω_2)^2 + γ_2^2/4},$$

$$β_2^\prime = -\frac{(γ_r1/2)σ_2 + e^γ(γ_r1/2)σ_1}{(ω_1 - ω_1)^2 + γ_2^2/4},$$

$$σ_1 = (ω_1 - ω_1)(ω_1 - ω_2) + γ_1γ_2/4,$$

$$σ_2 = (γ_1/2)(ω_1 - ω_2) - (γ_2/2)(ω_1 - ω_1).$$

In a quantum well in case of plane monochromatic exciting wave in time representation, the vector of the electric field looks like

$$E(z,t) = e_t e^{-iωt} E(z) + c.c.,$$

where

$$E(z) = [e^{iκz} + (F(z)/F(0))N]C_1 + [e^{-iκz} + (-1)^m e^{-iκz} (F(z)/F(0))N]C_2 \quad (40)$$

is the sum of plane waves $\exp(±iκz)$, which are connected with reflection from quantum-well borders. Besides, (as it follows from a kind of $F(z)$) the field in a quantum well contains oscillating terms, describing coordinate dependence of wave functions of electrons and holes. In time representation, field vectors to the left ($E^l(z,t)$) and the right ($E^r(z,t)$) of the quantum well are:

$$E^l(z,t) = e_t e^{-iωt} \left[ E_0 e^{iκz} + C_R e^{-iκz} \right] + c.c.,$$

$$E^r(z,t) = e_t C_R e^{-i(ωt - κz)} + c.c. \quad (41)$$

In a limiting case of a homogeneous environment ($ν = ν_1, ζ = 1$), the electric field (after substitution (29) and (30)) becomes ($κd = 0$):

$$E^l(z,t) = e_t E_0 e^{-iωt} \left[ e^{iκz} - i(-1)^m e^{iκz} \right. \left. \times \left( \frac{γ_r1/2}{Ω_1 + iΓ_1/2} + \frac{γ_r2/2}{Ω_2 + iΓ_2/2} \right) e^{iκ(d+z)} \right] + c.c., \quad (42)$$

$$E(z,t) = e_t E_0 e^{-iωt} \left[ e^{iκz} - i(-1)^m e^{-iκz} \right. \left. \times \left( \frac{γ_r1/2}{Ω_1 + iΓ_1/2} + \frac{γ_r2/2}{Ω_2 + iΓ_2/2} \right) F(z) \right] + c.c.. \quad (43)$$

If, on the contrary, to neglect by the spatial dispersion of the light wave ($κd = 0$), but to consider environment as not uniform ($ζ \neq 1$), for allowed transitions $m_e = m_v$,

$$E^l(z,t) = e_t E_0 e^{-iωt} \left[ e^{iκz} + \frac{ζN}{1(ζ - 1)N} \right] + c.c.,$$

$$E^r(z,t) = e_t E_0 e^{-iωt} \left[ \frac{1 + N}{1(ζ - 1)N} \right] e^{iκz} + c.c.,$$

$$E(t) = e_t E_0 e^{-iωt} \left[ \frac{1 + N}{1(ζ - 1)N} \right] + c.c. \quad (45)$$

Functions $ζN/(1-(1-ζ)N)$ and $(1+N)/(1-(1-ζ)N)$ in a considered limiting case look like:

$$\frac{ζN}{1-(1-ζ)N} = -\frac{iζ(γ_r1Ω_2 + γ_r2Ω_1)/2}{ω_1Ω_2 + iζ(γ_r1Ω_2 + γ_r2Ω_1)/2}$$

and

$$\frac{1+N}{1-(1-ζ)N} = \frac{ω_1Ω_2 + iζ(γ_r1Ω_2 + γ_r2Ω_1)}{2}. \quad (46)$$

It is visible that $γ_r1$ and $ζ$ enter only in the form of product $γ_r1ζ$. It means that at $ν \neq ν_1$ in case of narrow quantum wells, the refraction coefficient of barriers $ν_1$ enters in radiative damping, and a refraction coefficient of quantum well $ν$ does not appear anywhere. Physical sense of this result is clear: At $κd \ll 1$, it is possible to pass to the limit $d \to 0$, when the substance of a quantum well is absent, but the induced current, corresponding to transitions with an exciton creation, is kept. Thus, it is proved that obtained earlier results for narrow quantum wells are true and at $ν \neq ν_1$, since formulas include only refraction coefficients of barriers.

In this limiting case, the field in a quantum well does not depend on coordinate, since in dipole approximation, the phase of a light wave does not vary in a quantum well.

### III. OPTICAL CHARACTERISTICS OF A QUANTUM WELL

In this Section, formulas for the reflection, transmission and absorption of a plane monochromatic electromagnetic wave by a quantum well are represented in the
general case, when \( \nu \neq \nu_1 \) and \( \kappa d \neq 0 \), and in limiting cases of a homogeneous environment and \( \kappa d = 0 \).

The reflection coefficient \( R \) is defined by standard way as the ratio of the module of the reflected energy flux to the module of the incident energy flux:

\[
R = \frac{|C_R|^2}{|E_0|^2}.
\]

Similarly, the transmission coefficient \( T \) is defined as

\[
T = \frac{|C_T|^2}{|E_0|^2}.
\]

The dimensionless absorption coefficient \( A \) (a share of energy absorbed by a quantum well), according to (46) and (47), looks like

\[
A = 1 - R - T.
\]

Using (31), (32), and (30), it is possible to represent the reflection \( R \) in the form

\[
R = \frac{v_1 + (L^2 + G^2)X_1 - YL - Z_1G}{|\Delta|^2}.
\]

The denominator \( \Delta \), defined by (21), is been transformed to

\[
|\Delta|^2 = v + (L^2 + G^2)X - YL + ZG.
\]

Functions \( L \) and \( G \) determine the frequency dependence of reflection in the region of interband transitions into two excited energy levels:

\[
L = \sum_{j=1}^{2} \frac{\tilde{\gamma}_r j \Omega_j}{\Omega_j^2 + \Gamma_j^2/4}, \quad G = \sum_{j=1}^{2} \frac{\tilde{\gamma}_r j \Gamma_j/4}{\Omega_j^2 + \Gamma_j^2/4},
\]

\[
L^2 + G^2 = \sum_{j=1}^{2} \frac{(\tilde{\gamma}_r j)^2}{\Omega_j^2 + \Gamma_j^2/4} + \sum_{j=1}^{2} \frac{(\tilde{\gamma}_r j/2)^2}{\Omega_j^2 + \Gamma_j^2/4} + \frac{2(\tilde{\gamma}_r j/2)(\tilde{\gamma}_r j/2)(\Omega_j \Omega_j + \Gamma_j \Gamma_j)}{(\Omega_j^2 + \Gamma_j^2/4)(\Omega_j^2 + \Gamma_j^2/4)}. \tag{52}
\]

Coefficients \( v, v_1, X, X_1, Y, Z \) and \( Z_1 \) depend on parameters \( \zeta = \nu/\nu_1 \) and \( \kappa d \):

\[
v = 4|4\zeta^2 \cos^2 \kappa d + (\zeta^2 + 1) \sin^2 \kappa d|,
\]

\[
v_1 = 4(\zeta^2 - 1)^2 \sin^2 \kappa d,
\]

\[
X = 8(\zeta - 1)^2[\zeta^2 + 1 + (-1)^{m_e + m_r}(\zeta^2 - 1) \cos \kappa d],
\]

\[
X_1 = 8[\zeta^4 + 1 + (-1)^{m_e + m_r}(\zeta^4 - 1) \cos \kappa d],
\]

\[
Y = 8(\zeta^2 - 1)(-1)^{m_e + m_r}(\zeta^2 + 1)
+ (\zeta^2 - 1) \cos \kappa d \sin \kappa d,
\]

\[
Z = 8(\zeta - 1)[(-1)^{m_e + m_r}(\zeta^2 + 1) + (\zeta^2 - 1) \cos \kappa d]
\times [1 - (-1)^{m_e + m_r}(\zeta - 1) + (\zeta + 1) \cos \kappa d]
Z_1 = 8(\zeta^2 - 1) \sin^2 \kappa d.
\]

If, in a quantum well, there is one excited energy level, it is necessary to pass in the formula (49) to the limit \( \gamma_{r2} \to 0, \gamma_{r1} = \gamma_r, \gamma_1 \to \gamma, \omega_1 = \omega_0 \). In this case, the reflection coefficient looks like:

\[
R = \frac{1}{|\Delta|^2} \left[ v_1 + \frac{(\tilde{\gamma}_r/2)^2X_1 - (\tilde{\gamma}_r/2)(\Omega \Omega + Z \Gamma_1/2)}{\Omega^2 + \Gamma^2/4} \right],
\]

\[
|\Delta|^2 = v + \frac{(\tilde{\gamma}_r/2)^2X - (\tilde{\gamma}_r/2)(\Omega \Omega - ZT/2)}{\Omega^2 + \Gamma^2/4},
\]

\[
\Omega = \omega_L - \omega_0 - \varepsilon \gamma_r/2,
\]

\[
\Gamma = \gamma + \tilde{\gamma}_r, \tilde{\gamma}_r = \varepsilon \gamma_r.
\]

Since refraction coefficients of barriers and quantum well, as a rule, do not differ enough (i.e., \( \zeta \approx 1 \)), in the denominator \( |\Delta|^2 \), \( v \approx 4 \) plays a dominating role. Other multipliers contain factors \( \zeta^2 - 1 \) or \( (\zeta - 1)^2 \) and are small in comparison with \( v \) even in the resonance, when \( L \approx 1 \) and \( G \approx 1 \). In numerator \( R \), the value \( v_1 \ll 1 \) and the contribution of other terms is significant. In particular, a sign-variable term \( \sim L \), whose frequency dependence is similar to a curve of an anomalous dispersion, plays the essential role. Such dependence takes place in the refraction coefficient in the region of frequencies corresponding to absorption peaks.

The transition coefficient \( T \) is equal

\[
T = \frac{16 \zeta^2 [L^2 + (1 - G)^2]}{|\Delta|^2}
\]

and does not contain sign-variable terms in the numerator. The same takes place and for absorption

\[
A = 16 \zeta [\zeta^2 + 1 + (-1)^{m_e + m_r}(\zeta^2 - 1) \cos \kappa d]
\times \frac{G - (L^2 + G^2)}{|\Delta|^2}. \tag{59}
\]

For one excited energy level, we obtain

\[
T = \frac{16 \zeta^2 \Omega^2 + \gamma_r^2/4}{|\Delta|^2 \Omega^2 + \Gamma^2/4},
\]

\[
A = \frac{16 \zeta [\zeta^2 + 1 + (\zeta^2 - 1) \cos \kappa d]}{|\Delta|^2} \gamma_r^2/4 \Omega^2 + \Gamma^2/4. \tag{60}
\]

In approximation of a homogeneous medium \( (\zeta = 1) \),

\[
R = L^2 + G^2,
\]

\[
T = L^2 + (1 - G)^2,
\]

\[
A = 2(G - (L^2 + G^2)). \tag{61}
\]

There is no the term \( \sim L \) in the reflection, i.e., this approximation does not take into account the contribution into the reflection, originating from sign-variable terms in the current (8).

In a limiting case \( \kappa d = 0 \),

\[
|\Delta|^2 \to |\Delta_0|^2 = 16 \zeta^2 + X_0(L_0^2 + G_0^2) + Z_0G_0, \tag{62}
\]
\[ R = \frac{X_{10}(L_0^2 + G_0^2)}{\left| \Delta_0^* \right|^2}, \]
\[ T = \frac{16\varepsilon^2[L_0^2 + (1 - G_0)^2]}{\left| \Delta_0^* \right|^2}, \]
\[ A = \frac{16\varepsilon^2\zeta^2 + 1 + (-1)^{m_c + m_v}(\zeta^2 - 1)}{\left| \Delta_0^* \right|^2}. \]  

Coefficients \( X_0 \), \( Z_0 \) and \( X_{10} \) originate from (54) and (56) if \( \kappa d = 0 \):
\[
X_0 = 8(\zeta - 1)^2[\zeta^2 + 1 + (-1)^{m_c + m_v}(\zeta^2 - 1)],
\]
\[
X_{01} = 8[\zeta^4 + 1 + (-1)^{m_c + m_v}(\zeta^4 - 1)],
\]
\[
Z_0 = 8(\zeta - 1)[\zeta^2 + 1 + (-1)^{m_c + m_v}(\zeta^2 - 1)].
\]

Functions \( G_0 \) and \( L_0 \) differ from \( G \) and \( L \) by replacement of \( \gamma_j \) by \( \gamma_j \), since at \( \kappa d \to 0 \), \( \varepsilon' \to 1 \). Besides, \( \varepsilon'' = 0 \) in \( \Omega_j \) (37), and \( \varepsilon = 1 \) in \( \beta_j \) (34).

IV. LIGHT REFLECTION, ABSORPTION AND TRANSMISSION FOR ARBITRARY \( \Phi(z) \)

In Sections 2 and 3, coefficients \( R \), \( T \) and \( A \) were obtained for \( \Phi(z) \) in the form of (4), that is true for free electron-hole pairs. In this Section, \( \Phi(z) \) is considered as arbitrary real function, in particular, it may be an excitonic function. Calculation, similar to Section 2, leads to following expressions for constants \( C_R \) and \( C_T \):
\[
C_R = \frac{E_0}{\Delta_1} \left\{ 2i(\zeta^2 - 1) \sin \kappa d + 2[e^{-i\kappa d}w_R + (-1)^{m_c + m_v}(\zeta^2 - 1)]N \right\},
\]
\[
C_T = \frac{4E_0}{\Delta_1} e^{-i\kappa d} \zeta[1 + (-1)^{m_c + m_v}e^{-i\kappa d}N],
\]
where
\[
\Delta_1 = (\zeta + 1)^2 e^{-i\kappa d} - (\zeta - 1)^2 e^{i\kappa d} - 2(\zeta - 1) \times \left[ (\zeta + 1)e^{-i\kappa d}w_R + (-1)^{m_c + m_v}(\zeta - 1)N \right].
\]

In (64) - (66), function \( N \) is defined by the formula (34) (or (36)), and constants \( w \) (real) and \( w_R \) (complex) are:
\[
w = \frac{(-1)^{m_c + m_v}}{2|F(0)|^2} \left[ F^2(0)e^{-i\kappa d} + [F^*(0)]^2 e^{i\kappa d} \right],
\]
\[
w_R = \frac{(-1)^{m_c + m_v}}{2|F(0)|^2 d} \times \left[ (\zeta + 1)^2 F^2(0)e^{-i\kappa d} + (\zeta - 1)^2 [F^*(0)]^2 e^{i\kappa d} \right].
\]

At obtaining of these formulas, relationships \( F(d) = e^{i\kappa d} F^*(0), Re \varepsilon = \varepsilon = |F(0)|^2 \), which follow from (19), were used (\( \Phi(z) \) is considered real since represents one-dimensional movement). If (4) is used for \( \Phi(z) \), then \( w \to 1, w_R \to \zeta^2 + 1 \) and formulas (64) - (66) coincide with (30) and (31).

It is visible from comparison of expressions (64) - (66) with coefficients \( C_R, C_T \) and \( \Delta \), obtained in Section 2, that the structure of the function \( N \), which basically determines the frequency dependence, does not depend on type of \( \Phi(z) \). Only values \( Re \varepsilon = \varepsilon' \) and \( Im \varepsilon = \varepsilon'' \) depend on type of \( \Phi(z) \), i.e., the factor at \( \gamma \) and the shift of resonant frequencies. Factors \( w \) and \( w_R \), entering in \( \Delta_1 \) and \( C_R \), can change only a ratio between contributions in optical characteristics of values \( L \) and \( G \). Therefore, changes of optical characteristics will be hardly radical due to changes of \( \Phi(z) \).

V. DISCUSSION OF RESULTS

Frequency dependencies of the light reflection \( R \) and transmission \( T \) are calculated for a case \( m_c = m_v = 1 \) and \( \gamma_1 = \gamma_2 = \gamma ] = \gamma \), that corresponds to the system of the polaron \( A \) and a hole with the Landau quantum number \( n = 1 \) in conditions of the magnetophonon resonance. Calculations were spent with the help of formulas (46) and (47). Functions \( C_R \) and \( C_T \) from (30) and the denominator \( A \) from (31) are used, as well as function \( N \) (34).

In Fig.1 and Fig.2, the reflection \( R \) is shown for small values of the ratio \( \gamma_{r} / \gamma \). It is visible that curves 3 and 4 for the case \( \zeta = \nu \mu_1 \neq 1 \) and \( \kappa d \neq 0 \) differ considerably from the case of \( \zeta = 1 \) (a curve 2) at the same value \( \kappa d \). As it was mentioned above, the formula for the current (8) contains sign-variable terms \( \sim (\omega_k - \omega_1) \) and terms \( \sim \gamma_{ij} [(\omega_k - \omega_1)^2] \), corresponding to absorption curve. Curves 3 and 4 concern to a case, when sign-variable terms dominate. Therefore, \( R(\omega_k) \) is similar to the curve of anomalous dispersion in this case. Alongside with changes of the form of the reflection curve, the increase of \( R \) (in 15 times approximately in Fig.1 and twice in Fig.2) takes place in comparison with cases \( \kappa d \neq 0 \), \( \zeta = 1 \) and \( \kappa d = 0 \), \( \zeta \neq 1 \).

At \( \gamma_{r} / \gamma = 1 \), influence of the absorption dominates, as it is visible in Fig.3. However, influence of sign-variable terms is still noticeable: Curves 3 and 4 (\( \zeta = 1.1 \) and \( \zeta = 0.9 \), respectively) differ from curve 2 (\( \zeta = 1 \)) at the further increase of \( \gamma_{r} / \gamma \). The influence of the anomalous dispersion becomes practically imperceptible and functions \( R, T \) and \( A \) coincide with obtained in. Curves of the transmission \( T \) are shown in Figs.4-6 for the same magnitudes of parameters \( \kappa d \), \( \zeta \) and \( \gamma_{r} / \gamma \), as curves in Figs.1-3. It is visible that transmission is poorly sensitive to changes of \( \zeta \). It happens due to absence in \( T(\omega) \) terms proportional to \( \omega_k - \omega_1 \) and \( \omega_k - \omega_2 \) (see (58)). A small distinction is determined by the denominator \( |\Delta|^2 \) which contains terms linear on \( \omega_k - \omega_j \). The absorption
A, in general, is poorly sensitive to changes of parameter $\zeta$ in all area of changes of $\gamma_r/\gamma$.

$R$ and $T$ for a case $\zeta = 1, kd = 1.5$ (curve 2) are shown also in figures. Some asymmetry of the curve 2 in comparison with the case $kd = 0, \zeta \neq 1$ (curves 1, 5 and 6) attracts our attention. This asymmetry is a consequence of the account in the theory of supreme orders on interaction of an electromagnetic wave with the electronic system that leads to occurrence of shifts of resonant frequencies. These shifts combine with resonant frequencies $\omega_1$ and $\omega_2$. At transition to the limit $kd \to 0$, shifts disappear and symmetry of curves is restored.

Finally, curves 5 and 6 in figures correspond to the case $\kappa d = 0, \zeta \neq 1$. Curves are symmetric here, and influence of parameter $\zeta$ is shown in the form of a parallel shift of curves (if $\gamma_r = \gamma$), or changes in the vicinity of extrema ($\gamma_r \leq \gamma$). The main conclusion of our calculations consists that at greater radiative lifetimes of excited states in comparison with nonradiative lifetimes (which are determined, in particular, by electron and hole scattering on impurities and phonons) frequency dependence of reflection is determined basically by sign-variable terms in the expression for the current density. In this case, it is pos-
sible to neglect by $\gamma_r$ in resonant denominators, i.e., to solve the problem in linear approximation on interaction of an electromagnetic wave with the electronic system.

FIG. 5: Same as Fig.3 for $\gamma_r/\gamma = 0.1$.

FIG. 6: Same as Fig.3 for $\gamma_r = \gamma$.

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