Optimizing Pulsar Timing Array Observational Cadences for Sensitivity to Low-frequency Gravitational-wave Sources

M. T. Lam1,2

1 Department of Physics and Astronomy, West Virginia University, White Hall, Morgantown, WV 26506, USA; michael.lam@mail.wvu.edu
2 Center for Gravitational Waves and Cosmology, West Virginia University, Chestnut Ridge Research Building, Morgantown, WV 26505, USA

Received 2018 August 29; revised 2018 September 26; accepted 2018 September 27; published 2018 November 15

Abstract

Observations of low-frequency gravitational waves (GWs) will require the highest possible timing precision from an array of the most spin-stable pulsars. We can improve the sensitivity of a pulsar timing array (PTA) to different GW sources by observing pulsars with low timing noise over years to decades and distributed across the sky. We discuss observing strategies for a PTA focused on a stochastic GW background such as from unresolved supermassive black hole binaries as well as focused on single continuous-wave sources. First, we describe the method to calculate a PTA’s sensitivity to different GW-source classes. We then apply our method to the 45 pulsars presented in the North American Nanohertz Observatory for the GW 11 year data set. For expected amplitudes of the stochastic background, we find that all pulsars contribute significantly over the timescale of decades; the exception is for very pessimistic values of the stochastic-background amplitude. For individual single sources, we find that a number of pulsars contribute to the sensitivity of a given source, but that which pulsars contribute is different depending on the source, or versus an all-sky metric. Our results seem robust to the presence of red noise in pulsar arrival times. It is critical to obtain more robust pulsar-noise parameters as they heavily affect our results. Our results show that it is also imperative to locate and time as many high-precision pulsars as possible, as quickly as possible, to maximize the sensitivity of next-generation PTA detectors.

Key words: gravitational waves – methods: observational – pulsars: general

1. Introduction

As the detection of low-frequency gravitational waves (GWs) nears (Taylor et al. 2016), pulsar timing array (PTA) collaborations must begin looking toward the future characterization of the GW sky. As with ground-based detectors, we must begin planning for the next-generation of PTA detectors, optimized for these observations. The North American Nanohertz Observatory for Gravitational Waves (NANOGrav; McLaughlin 2013) collaboration, one of several efforts worldwide (e.g., Desvignes et al. 2016; Reardon et al. 2016; Verbiest et al. 2016), is currently observing over 70 high-precision millisecond pulsars (MSPs) in its PTA detector for the purpose of low-frequency GW detection from both a stochastic background and from single sources. Without a detection, we have placed constraints on the environments of supermassive black hole binary (SMBHB) mergers, cosmic strings, and inflationary era GWs (Arzoumanian et al. 2014, 2015, 2018a).

Detector sensitivity depends on the GW signal we wish to observe. A stochastic background requires observations of many MSPs (Siemens et al. 2013; Vigeland & Siemens 2016), whereas the sensitivity to single sources such as from a single binary or merger event requires the highest timing precision from a few of the best-timed MSPs (Ellis et al. 2012). PTA observations require years to decades of a timing baseline to detect nanohertz-regime GWs, and therefore large amounts of telescope time are required. In theory, with enough observing time we could observe enough pulsars with adequate timing precision to cover both science targets (stochastic background versus single source), but practical limitations apply. NANOGrav currently observes its pulsars on a monthly cadence, except for a handful of the highest-precision pulsars that are observed weekly, with the goal of covering both stochastic background and single continuous-wave (CW) source characterization. However, the efficacy of the approaches to maximizing GW sensitivity has been unclear so far.

In Lam et al. (2018b), we examined pulse arrival-time uncertainty for MSPs as a function of the radio frequencies observed by specific telescopes, taking into account a wide variety of effects. The requirements per pulsar vary, but large bandwidths covering much of the radio spectrum typically used for high-precision pulsar timing (~GHz) are needed to obtain the best possible arrival-time estimates. We found that while changing the observing frequencies yielded modest improvements in the timing precision for nearby pulsars, distant pulsars affected more by interstellar propagation effects could be improved more substantially. However, we did not discuss or quantify the timing improvements to GW sensitivity.

In this work, we will consider the time-allocation optimization for various pulsars in the array to maximize overall GW sensitivity. Lee et al. (2012) examined this problem first by developing the methodology to optimize a specific stochastic-background detection statistic, the signal-to-noise ratio (S/N) based on the cross correlation between pulsar residuals. They then developed the algorithm for mathematical optimization of the integration time per pulsar given the constraints of fixed total observing time for one or more telescopes. They applied their methods to a theoretical PTA observed over a 5 year time baseline as per Jenet et al. (2005), along with measured (but simplified) noise parameters from different sets of PTA pulsars. The amplitudes of the background they considered3 have nearly been ruled out by current PTA observations (e.g., Arzoumanian et al. 2018a), and the time to detection is now known to take

---

3 In Figure 4 of Lee et al. (2012), where the authors considered amplitudes that have yet to be reached, the x-axis labels must, at a minimum, be flipped in order to agree with the conclusions made in the paper.
longer than 5 years total even for next-generation telescopes (Rosado et al. 2015; Taylor et al. 2016).

Here, we will expand upon this previous work and develop the methodology for time optimization for the goals of increasing the sensitivity of a PTA to both a stochastic background from a population of SMBHBs and individual CW sources. Since the GW sensitivity metric we use is a function of the sky locations between two pulsars and the CW source (Anholm et al. 2009) versus just the angular separation between the two pulsars, and the signal spectrum differs from a stochastic background, the answer obtained for the time allocation is expected to be different; one must consider these differences depending on the project goals. The cross-correlation statistic used as our GW sensitivity metric for both source classes is described in Section 2. In Section 3, we apply this formalism to the 45 pulsars presented in the NANOGrav 11 year data set (NG11; Arzoumanian et al. 2018b), providing specific prescriptions for allocating the observing time per pulsar depending on the two science goals. We include both individual white- and red-noise parameters for calculating our S/N metric, though note that our results may vary greatly given uncertainties in these noise parameters. We look at the effect on the time allocation as a function of changing the total observational time baseline and the amplitude of the stochastic background. We also consider the effect on the GW S/N caused by removing an individual pulsar from the array and reoptimizing the time allocation. For CW sources, we look at both individual sources and a collection of sources distributed across the sky, and the time allocation depends on many more parameters than for a stochastic background. We describe several future directions for improving the sensitivity of a PTA to low-frequency GWs in Section 4.

2. The Cross-correlation Statistic

For the S/N metric that we wish to maximize, we use the cross-correlation statistic derived in Siemens et al. (2013). We first break it into its component terms for an individual pulsar pair (pulsars labeled with subscripts i, j), given as

$$\rho_{ij} = \left( 2 T_{ij} \chi_{ij}^2 \int_{f_L}^{f_H} df \frac{P_x^2(f)}{P_i(f) P_j(f)} \right)^{1/2},$$

(1)

where $T_{ij}$ is the overlapping time span between the two pulsars; $\chi_{ij}$ is the overlapping reduction function (e.g., the Hellings–Downs correlation function for an isotropic background) that depends on pulsar angular separations (Hellings & Downs 1983; Chamberlain & Siemens 2012); $f$ is the frequency, with $f_L$ and $f_H$ as the low- and high-frequency spectral cutoffs, respectively; $P_x$ is the “signal” GW power spectrum\(^4\) (“Earth term”); and $P_i, P_j$ are the total pulsar-noise spectra that include the effect of GWs at the pulsar (“pulsar term”). Since power is absorbed by parameter fits in the timing models, we assume a low-frequency cutoff $f_L \approx 1/T_{ij}$, due to the spin-period and spin-period-derivative quadratic subtraction from the TOAs to good approximation (Siemens et al. 2013). Once we compute our individual $\rho_{ij}$ values, the average S/N statistic is given simply by

$$\langle \rho \rangle = \left( \sum \rho_{ij}^2 \right)^{1/2}.$$

(2)

We will drop the angular brackets denoting the average going forward for brevity.

2.1. Stochastic GW Background

Here, we will consider the the form of a stochastic GW background of the power-law form (Jenet et al. 2006)

$$P_{SB}(f) = \frac{A_{SB}^2}{12\pi^2} \left( \frac{f}{1\text{ yr}^{-1}} \right)^{2\alpha} f^{-3} \equiv bf^{-\beta},$$

(3)

where $A_{SB}$ is the strain amplitude at a frequency of 1 yr\(^{-1}\) and $\alpha$ is the spectral index of the characteristic strain, when cast in terms of $\beta = 13/3$ for SMBHBs. Other values exist for primordial GWs or cosmic strings (Jenet et al. 2006). Siemens et al. (2013) provide scaling relations for different regimes of the strength of the signal power spectra versus the noise power spectra, but we consider the full form of Equation (1) to be integral. The pulsar-noise spectra are given by three terms,

$$P_i(f) = P_{W,i}(f) + P_{R,i}(f) + P_{SB}(f),$$

(4)

the sum of the uncorrelated-in-time white noise $P_{W,i}(f)$, correlated-in-time red noise $P_{R,i}(f)$, and time-correlated but spatially uncorrelated GW pulsar term with a power spectrum still given by $P_{SB}(f)$. For a single white-noise rms $\sigma_i$, the white-noise term can be written as $P_{W,i}(f) = 2\sigma_i^2 \Delta t_i = 2\sigma_i^2/c_i$, where $\Delta t_i$ is the time between observations and $c_i$ is the equivalent cadence that we use as our per-pulsar model parameter discussed later. Pulsars with red noise were modeled in the form of a power law, $A_{R,i}(f/1\text{ yr}^{-1})^{-\gamma}$; otherwise, we assumed $P_{R,i}(f) = 0$. Practically, the white-noise levels change over time due to changes in observing bandwidths, integration times, etc., in which case more generally we have (dropping the subscript $i$)

$$P_{W,\text{total}}(f) = 2\sigma_n^2/c_n \approx \sum_n N_n T_n.$$
2.2. CW Sources

A common method for CW analyses is to perform searches over a multidimensional likelihood function that includes intrinsic source parameters (e.g., chirp mass, GW frequency, strain/distance) along with parameters describing the orientation of the binary with respect to the Earth (Arzoumanian et al. 2014; Babak et al. 2016). Frequentist approaches maximize either an effective matched-filter-type spatially incoherent ($F_p$) or coherent statistic ($\mathcal{F}_p$; Ellis et al. 2012). In the upcoming NG11 analysis of CW sources (K. Aggarwal et al. 2018, in preparation), we use a Bayesian approach to compute the evidence via a Bayes factor for the parameters while accounting for the proper angular-correlation patterns. We assume here that the correlation must be considered for a detection, as sinusoidal-type waveforms can be present in individual pulsar timing data but due to other systematic effects, such as clock errors or errors in the solar system ephemerides.\(^5\) Therefore, the conservative statistic we maximize over will take a different form than previous calculations (e.g., the $F_p$-statistic).

Using the cross-correlation statistic (Equation (1)), we can again calculate the average S/N, but for CWs. Note that we need to modify the equation by replacing $\chi_q$ with $\bar{\chi}_q$, the sky-location-dependent overlap reduction function (Anholm et al. 2009). The signal spectrum for a source with GW frequency $f_{CW}$ is\(^6\)

$$P_{CW}(f) = \frac{A_{CW}^2}{4\pi^2} \delta_T(f - f_{CW}) f^{-3},$$

where the approximating function

$$\delta_T(f) = \frac{\sin(\pi f T)}{\pi f}$$

(7)

tends to $\delta(f)$ for infinite time and $A_{CW}$ is the amplitude of the GW. Source frequencies are expected to evolve over time such that the pulsars experience lower frequencies than at the Earth (Arzoumanian et al. 2014). Since the signal term (Equation (6)) will cause $\bar{\chi}_q$ to include power mostly from $f_{CW}$, we do not include $P_{PW}(f)$ in the pulsar-noise term, though we do include the $P_{SB}(f)$ term.

3. Application to the NANOGrav 11 Year Data Set

We applied our formalism to the 45 MSPs discussed in NG11. Each pulsar-noise model contained several white-noise components and a red-noise power-law model when significant. For simplicity, we used the noise parameters directly reported in the paper (see Table 2 in NG11). While work has been done to study the many contributions to the TOA uncertainties (Shannon et al. 2014; Lam et al. 2016), there remain components that have not been well quantified, e.g., from radio-frequency interference or polarization miscalibration (Lam et al. 2018b). We took the conservative approach and used the directly measured values from the full timing of each pulsar.

We assumed that during the entire time span encompassing NG11 each pulsar was observed monthly per telescope, except for weekly observations of two pulsars at the Green Bank Telescope (GBT) starting in 2013 (PSRs J1713+0747 and J1909–3744 for 30 minutes each) and five pulsars at the Arecibo Observatory (AO) starting in 2015 (PSRs J0030+0451, J1640+2224, J1713+0747, J2043+1711, and J2317+1439 for one hour each); see NG11 for more details. The goal of these observations was to increase the sensitivity of the PTA to CWs (Arzoumanian et al. 2018b). For the high-cadence campaigns, we assumed a time increase of 30 minutes during the non-monthly observation weeks at GBT or 60 minutes weekly at AO; the total time was 864 hr per year. Recall that the reported white-noise rms of these pulsars will not be affected because we separate out the observational cadence in $P_{W,i}(f)$.

For our optimization analysis, we assumed that the pulsar white-noise rms values were fixed over all epochs, as well as the parameters describing the red-noise power law; future mitigation of timing effects, for example, from radio-frequency-dependent pulse-propagation delays, may alter the measured parameters. We note that after the first several years of observing, we switched to wider-bandwidth telescope backends, causing the effective white-noise rms to decrease (see Equation (5)), which we ignore here for simplicity and assume that the wider-bandwidth backends were consistently used over the whole time span. Again, the reduction in white noise versus backends requires measurements of all white-noise components, which will involve a more in-depth analysis in the future (see Section 4.1).

For some pulsars where the time baseline was relatively short, the rms noise was very small, largely because the timing-model fit removes more power at low frequencies/longer timescales, e.g., from red noise, than for longer baselines where low-frequency power is still absorbed but less so (alternatively since the low-frequency cutoff $f_c \approx 1/T$ decreases and the fit removes less of the total spectral power; Blandford et al. 1984; Madison et al. 2013). For example, PSR J2234+0611 was listed with an rms of 30 ns, the lowest of any pulsar, yet it was only observed for two years and neither contains the lowest median template-fitting TOA uncertainties (Arzoumanian et al. 2018b) nor does it have the lowest rms from other white-noise sources (Lam et al. 2018a). Nonetheless, we used all NG11 pulsar-noise parameters as reported with the important caveat that future estimates will likely change.

The global-maximum optimization over pulsar cadences was performed using the Nelder–Mead method (Nelder & Mead 1965) implemented in the Python SCIPY package (Gao & Han 2012) via a basin-hopping algorithm (Wales & Doye 1997). To simplify the parameter search, we started with a several-pulsar array and iteratively added pulsars into the array along with the appropriate number of hours per year per source for each telescope, except for PSRs J1713+0747 and B1937+21 where the appropriate time was added to both. We then found the global maximum for that subarray and used the solution as a starting guess when adding a new pulsar. This algorithm was robust against the order in which pulsars were added. We assumed no new pulsars were to be added to the array and no changes in total observing time from the program at the end of NG11 would be made (both currently

\(^5\) In current NANOGrav work, the ephemerides are accounted for using BAYESEPHEM. The errors are expected to be reduced in the future (Arzoumanian et al. 2018a).

\(^6\) The boxcar function of length $T$ acts as a finite observing span, which is a multiplication in the Fourier domain of a delta function and normalized sinc function; this procedure can be used to calculate other signal spectra as well. Note the factor of 3 in the denominator versus Equation (3) from the lack of sky averaging (Anholm et al. 2009; Thrane & Romano 2013).
untrue). For PSRs J1713+0747 and B1937+21, we assumed for simplicity that they were jitter- and scintillation-noise dominated versus S/N-limited and so the base $\sigma_i$ was the same at both GBT and AO; this is a good approximation for a majority of observations (Lam et al. 2016; Arzoumanian et al. 2018b), but should be revisited in the future.

We chose three values for the strain amplitude $A_{SB}$ of the SMBHB background: $1 \times 10^{-15}$ (optimistic), $6 \times 10^{-16}$ (moderate), and $2 \times 10^{-16}$ (pessimistic). The most pessimistic lower limit for the background is $\approx 1 \times 10^{-16}$ (Dvorkin & Barausse 2017; Bonetti et al. 2018), though most estimates suggest that $A_{SB}$ will be at least several times larger (Ravi et al. 2015). We also looked at two different values of the total time span observed after the end of NG11, $T_1$ = 10 or 20 years, for calculating $\rho$. In Table 1, we show our results in terms of the amount of time per pulsar per year allocated given different values of $A_{SB}$ and $T_1$.

### 3.1. Maintaining the PTA: The Time Allocation per Pulsar

Table 1 shows that our best pulsars do not overwhelmingly dominate the stochastic-background time allocation. This makes sense when we consider a hypothetical PTA where we ignore all $T_i$ and $\chi_{ij}$ and only have pulsars with white noise. Following Siemens et al. (2013) but including the differences in white noise and cadences, we have $\rho_{ij} \propto \{c_i \sigma_i \} / \{\sigma_j^2 \sigma_{ij}^2\}^{1/2}$. Then, one can easily show that the total $\rho$ maximizes when the “quality” of each pulsar, the white-noise spectrum $2\sigma_j^2/c_i$, is similar, and thus one should spend more time on pulsars with higher white-noise rms. Lee et al. (2012) reached a similar result. Since the optimization performed is over complex nonlinear equations (Equations (1), (2)), the results we obtain for an individual pulsar can differ compared with that of the scaling relations. For real PTAs, one should consider red noise, varying time baselines, changes in the relative contributions of white-noise parameters over time, and sky positions, which will modify the results accordingly, though the basic principle still applies. We see that the time allocation is of similar magnitude for all pulsars, i.e., no pulsar overwhelmingly dominates the time allocation, though several pulsars do not contribute significantly for certain assumed parameters and thus are not listed (when $c_i < 1$ hr/year).

Comparing against the value of $\rho$ determined from a continuation of the NG11 observing strategy versus the optimizations found here, we see a $\sim 5\%$ increase in $\rho$ for $A_{SB} = 1 \times 10^{-15}$, but a nearly $\sim 8\%$--$10\%$ increase for $A_{SB} = 3 \times 10^{-16}$. NANOGrav’s general strategy of observing many high-precision pulsars with roughly equal time (along with high-cadence programs) therefore currently provides close to optimal stochastic-background sensitivity.

### 3.2.Trimming the PTA: Removing Pulsars to Improve Sensitivity

Figure 1 shows the effect of removing a pulsar from the array and whether reallocating its observing time will increase or decrease sensitivity to the stochastic background. We find that all 45 pulsars contribute significantly for decades-long experiments and that the relative importance of continuing to time these pulsars also tends to improve. However, note in Table 1 where certain pulsars become significant for $T = 20$ years, whereas they are unlisted for earlier times. Formally, we find values for these pulsars, though they are too small to report given the practical necessity of requiring enough observations to generate a timing solution. We find that while it may be beneficial to remove pulsars in the short term, e.g., PSR J1453+1902, all pulsars begin to contribute significantly for longer experiments; the amount of time baseline required does depend on the amplitude of the background. In addition, the GW significance seems robust to red noise, as for PSR J0030+0451 the red-noise index $\gamma = 4.0$, which is steeper than the background $\beta = 13/3$.

### 3.3. Directional PTA Tuning

For CW sources, we looked at specific potential galaxies that may host SMBHBs. Mingarelli et al. (2017) examined the detectability of individual local SMBHBs by simulating binaries within nearby host galaxies; they estimated that a single source will likely be detectable within $\sim 10$ years from now. Assuming different noise statistics of pulsars observed by the International Pulsar Timing Array (IPTA; Verbiest et al. 2016), different potential sources may be detectable. Assuming only white noise in the IPTA pulsars, they found that M104 (NGC 4594) may likely host a detectable CW source. In a more updated analysis, they find that M104 is the most likely detectable SMBHB source regardless of the assumed black hole/galaxy host scaling relation, followed consistently by M84 (NGC 4374; C. Mingarelli 2018, private communication). NGCs 3115 and 1316 also feature prominently as possible SMBHB-host candidates regardless of pulsar red-noise properties. We looked at these four specific sources independently, shown for reference in Figure 2, and assumed $T = 20$ years, a source strain of $A_{CW} = 10^{-15}$ and frequency $f_{CW} = 0.5$ yr$^{-1}$, and a stochastic-background pulsar term of $A_{SB} = 10^{-15}$. For comparison, we also considered sources uniformly distributed across the sky with the same assumed parameters and maximized the $S/N$ over all sky positions $\theta_n$ as

$$\rho_{sky} = \left( \sum_n \rho(\theta_n) \right)^{1/2}. \quad (8)$$

The results of our CW analysis are given in Table 2. We find that many pulsars contribute to the sensitivity toward different sources though typically different pulsars are preferred; this differs from an analysis using the $F_p$-statistic without sky correlations included in which only one or two pulsars dominate the $S/N$ (Arzoumanian et al. 2014; Babak et al. 2016). Tests using lower $f_{CW}$ show a preference for a greater number of pulsars with similar $c_i$, i.e., these sources suggest an observing program more similar to the stochastic background because the low-frequency waves mimic components of red noise akin to a stochastic background. Recall that the $\rho$ statistic is conservative since it includes correlations but suboptimal since it does not include signal-waveform matched-filtering. Again, note the caveats above on the uncertainty in the pulsar-noise properties, which is why PSRs J1911+1347 and J2234+0611 with low amounts of white noise contribute significantly in the all-sky analysis, which is what one would expect from an $F_p$-statistic analysis as well. Since M104 and M84 are $\sim 25^\circ$ separated on the sky, and both are $\sim 40^\circ$ from NGC 3115, we see significant overlap for the pulsars that contribute to $\rho$. For these sources, PSRs J1744--1134 and J1918--0642 take up significant portions of observing time in part due to the long time spans and strong correlations with PSR J1713+0747.
It is possible to “tune” PTA observations to efficiently improve the S/N toward multiple targets. In general, one can construct a weighted metric that maximizes the sensitivity to a number of possible sources across the sky, potentially in a form similar to Equation (8), and likely should given that any one individual galaxy host is a detectable CW source is not guaranteed. Current and future PTA experiments may wish to increase their sensitivity to specific sky locations in order to hasten a detection and also improve later characterization, for example, in studying the galaxy members of the nearby Virgo cluster in addition to the sources analyzed here. However, for an individual source in a particular sky direction, one should consider some prior distribution of GW amplitudes \( A_{\text{GW}} \), frequencies \( f_{\text{GW}} \), and even the stochastic-background amplitude \( A_{\text{SB}} \). Given these factors in addition to the particular sky distribution of CW sources and weighting used (certain galaxies are more likely to host detectable SMBHBs than others), optimizing an overall metric for CWs is a complex procedure that should be looked at more closely in future work.

4. Future Applications

We briefly discuss several next steps for improving GW sensitivity via this framework. One obvious next step is to determine methods of combining the metric provided here that
Figure 1. Metric $(\rho_{\text{total}} - \rho_{\text{subset}})/\rho_{\text{total}}$ shows the fractional difference between the S/N of the whole PTA $\rho_{\text{total}}$ and the S/N when the pulsar listed is removed and the time is reallocated and reoptimized $\rho_{\text{subset}}$. The larger the metric, the more important the individual pulsar is in contributing to the PTA’s GW sensitivity. Here, we assume $A_{\text{B}} = 1 \times 10^{-15}$. The left panel shows the metric per pulsar when observing for an additional $T_s = 10$ years, the right panel for an additional $T_s = 20$ years. The total time observed for each pulsar in NG11 is shown in parentheses (in years). The colors of the pulsar names are the same as of the point in the left panel, which is correlated with the metric.

Figure 2. Sky positions of our pulsars (circles) in relation to the four CW sources (stars). Circle sizes are proportional to the time span observed $T_s$; the colors are proportional to the white-noise rms $\sigma_i$. The curve denotes the Galactic plane.
### Table 2
Optimal Pulsar Cadences for Continuous-wave Sources

| Pulsar          | Telescope | Time Span (year) | $\sigma_i$ (μs) | $A_R$ (μs) | $\gamma$ | Cadence (hours/year) | M84 | M104 | NGC 3115 | NGC 1316 | All Sky |
|-----------------|-----------|------------------|-----------------|------------|----------|----------------------|-----|------|-----------|-----------|---------|
| J0035+0037      | AO/GBT   | 10.3             | 0.103           | 0.021      | 1.6      | 69.1/248.2           | 60.0/60.0 | 35.9/35.9 | 19.4/19.4 | 35.0/35.0 | 19.4/19.4 |
| J1341+0230      | AO       | 4.4              | 0.308           | –          | –        | –                   | 26.6/26.6 | 29.5/29.5 | 29.5/29.5 | 26.6/26.6 | 29.5/29.5 |
| J1306−1325      | GBT      | 4.5              | 0.180           | –          | –        | –                   | 21.0/21.0 | 21.0/21.0 | 21.0/21.0 | 21.0/21.0 | 21.0/21.0 |

| Note. | We assumed $A_{CW} = 1 \times 10^{-15}, f_{CW} = 0.5 \text{ yr}^{-1}, A_{RH} = 1 \times 10^{-15}$, and $T_a = 20$ years. Dashed entries formally have values, but are too small to be reported.

is some weighted combination of different $\rho$ statistics for a stochastic background and specific CW sources, though various project science goals may differ, we will not expand further on this point.

#### 4.1. Recalculation of the Per-pulsar Time Allocation

The methods discussed here can be used generically for any PTA project. Specifically for the NANOGrav collaboration, the upcoming 12.5 year data set includes 48 pulsars (see, e.g., Lam et al. 2018a; a paper describing the data set fully is in preparation), though over 70 are now being observed; the end of 2018 marks the final observations for the 14 year data set, with a significant fraction of the pulsars being analyzed. In conjunction with the increased number of pulsars observed, NANOGrav uses significantly more telescope time per year now than the 864 hr used in this paper, along with observations taken with an additional telescope: the Very Large Array. Given that the state of the NANOGrav PTA is already different than what has been presented here, we recommend recalculating the various metrics for a more up-to-date version of the data set.

In addition to accounting for changes in the observing program, obtaining robust noise parameters using these future data sets will be critical. As mentioned previously, our results for specific pulsars are skewed because some pulsars with short
while baselines have very small rms values. Since the red-noise rms grows with time, it may take many years before a full noise profile is obtained for a given pulsar. Therefore, one can either make a predictive guess of how well a pulsar will perform using various models for red noise (e.g., Lam et al. 2017; note that there is large scatter in scalings for intrinsic-spin or interstellar-medium-related noise) or a postdictive recalculation of the different metrics presented here in an attempt to optimize future GW sensitivity.

4.2. Implementation of Wideband Receivers

While NANOGrav currently observes one pulsar per hour per telescope per epoch, it does so by splitting each observation into two half-hour segments for each frequency receiver. Therefore, 60 minutes of observing time (including overhead) equates to \( \lesssim 30 \) minutes of effective integration time. Wideband receivers will allow for a doubling of the effective integration time for the same on-sky time; in practice, a wideband receiver will have a somewhat higher system temperature and an optimization for the best frequency tuning should be performed for maximizing TOA precision (Lam et al. 2018b). While our framework applies generally, with different observational parameters we expect the results of our analyses to differ and one should account for as many inputs as possible in developing observing strategies.

4.3. International Pulsar Timing Array

Different PTA collaborations pool their resources into the IPTA collaboration. Each member currently has separate observing strategies. With the resources of many radio telescopes, the best possible GW sensitivity will be obtained by performing the optimization we have laid out across the combined network. Given the differing sensitivities, radio-frequency coverages, and cadences of the telescopes, along with practical constraints, it may be preferential for individual telescopes to observe different sources rather than multiple telescopes spending their time observing the same sources. Future analyses must be performed to investigate how best to optimize all of the IPTA telescopes, especially as new potential member collaborations seek to join.

We gratefully thank Xavier Siemens, Maura McLaughlin, Stephen Taylor, and Chiara Mingarelli for discussions enabling this work. M.T.L. and the NANOGrav Project receive support from NSF Physics Frontiers Center award number 1430284. The Arecibo Observatory is operated by SRI International under a cooperative agreement with the NSF (AST-1100968), and in alliance with Ana G. Méndez-Universidad Metropolitana, and the Universities Space Research Association. The National Radio Astronomy Observatory and the Green Bank Observatory are facilities of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.

ORCID iDs

M. T. Lam https://orcid.org/0000-0003-0721-651X

References

Anholm, M., Ballmer, S., Creighton, J. D. E., Price, L. R., & Siemens, X. 2009, PhRvD, 79, 084030
Arzoumanian, Z., Baker, P. T., Brazier, A., et al. 2018a, ApJ, 859, 47
Arzoumanian, Z., Brazier, A., Burke-Spolaor, S., et al. 2014, ApJ, 794, 141
Arzoumanian, Z., Brazier, A., Burke-Spolaor, S., et al. 2015, ApJ, 810, 130
Arzoumanian, Z., Brazier, A., Burke-Spolaor, S., et al. 2018b, ApJS, 235, 37
Babak, S., Petitjean, A., Sesana, A., et al. 2016, MNRAS, 455, 1665
Blundford, R., Narayan, R., & Romani, R. W. 1984, A&A, 5, 369
Bonetti, M., Sesana, A., Barausse, E., & Haardt, F. 2018, MNRAS, 477, 2599
Chamberlain, S. J., & Siemens, X. 2012, PhRvD, 85, 082001
Desvignes, G., Caballero, R. N., Lentati, L., et al. 2016, MNRAS, 458, 3341
Dvorkin, I., & Barausse, E. 2017, MNRAS, 470, 4547
Ellis, J. A., Siemens, X., & Creighton, J. D. E. 2012, ApJ, 756, 175
Gao, F., & Han, L. 2012, Comp. Optimization Appl., 51, 259
Hellings, R. W., & Downs, G. S. 1983, ApJL, 265, L39
Jenet, F. A., Hobbs, G. B., Lee, K. J., & Manchester, R. N. 2005, ApJL, 625, L123
Jenet, F. A., Hobbs, G. B., van Straten, W., et al. 2006, ApJ, 653, 1571
Lam, M. T., Cordes, J. M., Chatterjee, S., et al. 2016, ApJ, 819, 155
Lam, M. T., Cordes, J. M., Chatterjee, S., et al. 2017, ApJ, 834, 35
Lam, M. T., McLaughlin, M. A., Arzoumanian, Z., et al. 2018a, ApJ, submitted (arXiv:1809.03058)
Lam, M. T., McLaughlin, M. A., Cordes, J. M., Chatterjee, S., & Lazio, T. J. W. 2018b, ApJ, 861, 12
Lee, K. J., Bassa, C. G., Jansen, G. H., et al. 2012, MNRAS, 423, 2642
Madison, D. R., Chatterjee, S., & Cordes, J. M. 2013, ApJ, 777, 104
McLaughlin, M. A. 2013, CQGra, 30, 224008
Mingarelli, C. M. F., Lazio, T. J. W., Sesana, A., et al. 2017, NatAs, 1, 886
Nelder, J. A., & Mead, R. 1965, CompJ, 7, 308
Ravi, V., Wyithe, J. S. B., Shannon, R. M., & Hobbs, G. 2015, MNRAS, 447, 2772
Reardon, D. J., Hobbs, G., Coles, W., et al. 2016, MNRAS, 455, 1751
Rosado, P. A., Sesana, A., & Gair, J. 2015, MNRAS, 451, 2417
Shannon, R. M., Osłowski, S., Dai, S., et al. 2014, MNRAS, 443, 1463
Siemens, X., Ellis, J., Jenet, F., & Romano, J. D. 2013, CQGra, 30, 224015
Taylor, S. R., Vallisneri, M., Ellis, J. A., et al. 2016, ApJL, 819, L6
Thrane, E., & Romano, J. D. 2013, PhRvD, 88, 124032
Verbiest, J. P. W., Lentati, L., Hobbs, G., et al. 2016, MNRAS, 458, 1267
Vigeland, S. J., & Siemens, X. 2016, PhRvD, 94, 123003
Vitale, S., Cerdonio, M., Coccia, E., & Ortolani, A. 1997, PhRvD, 55, 1741
Wales, D. J., & Doye, J. P. K. 1997, JPCA, 101, 5111