Study on the Damping Problem in Circular Plates of Radial Phonon Crystals

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Abstract. In order to perfect the theoretical framework of the radial phonon crystal circular plate, a damping type radial circular plate is constructed. The localization factor of damping is solved by transfer matrix method, and the finite element simulation and experimental verification are carried out. Furthermore, the effect of elastic modulus on the bandgap is investigated from the perspective of its frequency nonlinearity. The results show that the damping effect of viscoelastic materials has a significant effect on the bandgap, which is mainly reflected in the fact that the cut-off frequency of bandgap moves to the high frequency with the increase of material loss factor, thus broadening the bandgap. The damping effect of damping alloy has no obvious influence on the bandgap characteristic, which is related to the inherent impedance characteristic of the alloy.

1. Introduction

Phonon crystal, as a kind of periodic composite structure, plays an irreplaceable role in the field of vibration and noise reduction because of its unique band gap vibration characteristics[1-11]. In recent years, some researches have been made on the influence of the damping effect of phononic crystals on bandgap. In 2017, Chunyang Du et al. [12] designed a class of X-shaped locally resonant phononic crystal beams to solve the problem of over-damping, and pointed out the attenuation inhibition effect of such structures on the low-frequency band. In 2014, Siwen Zhang et al. [13] constructed a class of rod-shaped phonon crystals for viscoelastic damping, pointing out the regulating effect of loss modulus of viscoelastic materials on the energy band structure. Weikai Xu et al. [14] analyzed the bandgap characteristics of one-dimensional phonon crystals with damping. Frazier and Hussein [15] studied the overdamping phenomenon of viscoelastic materials. However, most of these studies focus on rod, beam and other structures, and do not involve the radial phonon crystal plate. In fact, our research group has conducted in-depth theoretical research on radial phononic crystal circular plates, including one-dimensional and two-dimensional structures, and made some research achievements [16-21]. In order to simplify the analysis, the damping problem of materials is generally ignored. In fact, damping is the inherent characteristic of all materials. For general metal materials, the material loss factor is very small, and the damping energy dissipation effect can be neglected in the research. However, for various damping materials widely used for vibration and noise reduction, such as damping alloy (Zn-Al alloy, Mn-Cu alloy) and viscoelastic material (epoxy resin, plastic), their values are large, respectively located at 0.05~0.2 and 0.1~5. As necessary, we must investigate the effect of damping energy dissipation.

On the basis of the above research, this paper further constructs a damped radial phonon crystal circular plate, deduces the corresponding transfer matrix, and analyzes the influence of damping on the
bandgap characteristics of a radial circular plate from the perspective of local factor. The finite element method is used to analyze the frequency nonlinearity of elastic modulus. The results show that the damping energy dissipation characteristics of viscoelastic materials mainly affect the cut-off frequency of the bandgap and thus broaden the bandwidth.

2. Theoretical Analysis of Damped Radial Circular Plate

Figure 1 shows the model of 1D radial phononic crystal circular plate. Material 1 is damping material, while material 2 is the matrix material ignoring damping effect. $E$, $\rho$, $\nu$ respectively represent young's modulus, density and Poisson's ratio of the two materials. As shown in the figure, $a_1$, $a_2$ are the radial width of damping material and matrix material, and $r_0$, $h$ are the inner diameter of the innermost ring plate and the thickness of the thin circular plate. In this paper, the damping effect of material 1 on the bandgap is studied.

![Figure 1. One dimensional radial phononic crystal circular plate.](image)

Damping, usually represented by loss factor $\beta$, is a parameter to measure the energy dissipation in the process of elastomer vibration. Because it has a nonlinear functional relationship with strain, stress, temperature, and other variables, it cannot be described by an accurate mathematical model. Therefore, a variety of damping theoretical models has been proposed. In this paper, the viscous damping theory is adopted, and its core expression is:

$$\sigma = E(\varepsilon + \beta \frac{\partial \varepsilon}{\partial t}).$$  

(1)

Where, $\sigma = \beta \frac{\partial \varepsilon}{\partial t}$ represents the damping force, which indicates that the damping force in the solid structure is proportional to the strain velocity. Thus, in the physical equation of bending vibration of the circular plate, equation (1) is introduced to obtain the corresponding stress component expression as follows:

$$\sigma_r = -\frac{E}{1-\nu^2}z[(\kappa_r + \nu\kappa_\theta) + \beta(\kappa_r \frac{\partial w}{\partial t} + \nu \kappa_\theta \frac{\partial w}{\partial t})].$$  

(2)

$$\sigma_\theta = -\frac{E}{1-\nu^2}z[(\kappa_\theta + \nu\kappa_r) + \beta(\kappa_\theta \frac{\partial w}{\partial t} + \nu \kappa_r \frac{\partial w}{\partial t})].$$  

(3)

$$\tau_{r\theta} = -2z\frac{E}{2(1+\nu)}(\kappa_{r\theta} + \beta \frac{\partial \kappa_{r\theta}}{\partial t}).$$  

(4)
Combined with previous research work [22] and substituting equations (2)–(4), the differential vibration equation of the damped radial circular plate can be derived as follows:

\[ \nabla^2 \nabla^2 w + \frac{\rho h}{D} \frac{\partial^4 w}{\partial t^4} + \frac{\beta}{D} \frac{\partial w}{\partial t} = \frac{q(r, \theta, t)}{D}. \]  

(5)

Here and now, in its shape solution \( k^4 = \omega^2 \frac{\rho h}{D} + \omega \frac{\beta}{D}, \) \( D \) represents bending stiffness. Moreover, it is not difficult to derive the transfer matrix, as shown in formula (6). The specific derivation process is detailed in the literature [22], which will not be repeated here.

\[ T_i = R_i^{-1} L_i R_i^{-1} L_i. \]  

(6)

Where, \( R_i, L_i, R_2, \) and \( L_2 \) are all square matrices of the second order.

The viscous damping model is used to derive the transfer matrix of a damped radial circular plate. For the analysis in Section 4.2 below, Maxwell mechanical model of viscoelasticity is introduced. First, it is necessary to understand the damping complex modulus model, whose functional relationship is described as:

\[ E(\omega) = E_s(\omega) + jE_i(\omega) = E_s(\omega)(1 + j\beta). \]  

(7)

Where \( E_s \) and \( E_i \) are the energy storage modulus and loss modulus respectively, reflecting the actual energy storage and energy consumption characteristics of viscoelastic materials. It is not difficult to see that they are both functions of frequency. The relation between \( E_s, E_i, \beta \) and frequency \( \omega \) satisfies the following function:

\[ E_s = \frac{E_0 \omega^2 \tau^2}{1 + \omega^2 \tau^2}. \]  

(8)

\[ E_i = \frac{E_0 \omega \tau}{1 + \omega^2 \tau^2}. \]  

(9)

\[ \beta = \frac{1}{\omega \tau}. \]  

(10)

Where \( \tau = \frac{\eta}{E_0}, \) represents the relaxation time, and \( \eta \) is a coefficient of viscosity.

3. Numerical Analysis of Damped Radial Circular Plates

According to the theoretical analysis results of damping, viscoelastic materials and damping alloys are introduced to construct two types of damped radial phononic crystal circular plates for solving the numerical solution. Material types and material parameters are shown in the following two tables.

**Table 1.** Material parameters of the first type of damping circular plate.

| Material      | \( \rho \) [kg \cdot m\(^{-3}\)] | \( E \) [Pa]   | \( \nu \) |
|---------------|----------------------------------|----------------|--------|
| Epoxy resin   | 1800                             | 0.4le10        | 0.38   |
| STEEL         | 7850                             | 2le10          | 0.3    |

**Table 2.** Material parameters of the second type of damping circular plate.

| Material   | \( \rho \) [kg \cdot m\(^{-3}\)] | \( E \) [Pa]   | \( \nu \) |
|------------|----------------------------------|----------------|--------|
| Mn-Cu      | 8800                             | 10.8le10       | 0.35   |
| PMMA       | 1062                             | 0.32le10       | 0.3333 |
The period number was selected as $n = 4$ and the frequency range was 0–5000Hz. The localization factor of bending vibration of the radial circular plate with different loss factors $\beta$ was calculated. The relevant geometric parameters are $r_0 = 10\text{mm}$, $a_1 = a_2 = 40\text{mm}$, $h = 4\text{mm}$. The calculation results are shown in figure 2.

![Figure 2. Localization factor of bending vibration of damped thin plate.](image)

It is not difficult to see from figure 2 (a) that, with the increase of $\beta$, the band gap initial frequency of the first type of damped thin plate is stable at about 1700Hz, while the cut off frequency is obviously shifted to the high frequency with an amplitude of about 400Hz, which widens the band gap. This shows that the damping effect of viscoelastic materials does have a certain impact on the bandgap characteristics of radial phonon crystal circular plates. Obviously, we can effectively use it in the field of vibration and noise reduction and take targeted measures for noise in certain frequency bands to effectively suppress the propagation of these waves. However, figure 2 (b) shows that the band gap characteristic of the radial circular plate composed of damping alloy is very little affected, which is largely related to the inherent damping characteristic of damping alloy itself, and its small loss factor (ranging from 0.05 to 0.2) makes its role in special occasions and applications far less than that of viscoelastic materials.

4. Simulation Verification of Damped Radial Circular Plate

4.1. Finite Element Analysis of Damped Circular Plate

In order to verify the authenticity and accuracy of the numerical results, it is necessary to carry out finite element simulation analysis. This section adopts the multi-physics field software COMSOL5.3 for analysis, which is different from the relevant settings in literature [22]. In order to objectively reflect the damping effect of materials, we load the "viscoelastic" module in the "linear elastic model" of the solid mechanics' module and set its relaxation time to reflect its damping characteristics.

Two types of damped circular plates are also simulated. In view of the damping problem, according to the relation between viscoelastic damping and relaxation parameters in literature [13], the loss factor $\beta$ is converted into the corresponding relaxation time $\tau$. The calculation results are shown in figure 3.
Figure 3. Simulation calculation of damping radial circular plates.

The simulation results in figure 3 have a discrepancy with the numerical calculation results. One of the reasons for this error is that the exact value cannot be obtained when the relaxation time $\tau$ corresponding to the loss factor $\beta$ is calculated. Another reason is that there is a certain gap between the internal calculation model carried by COMSOL and our actual requirements, which leads to some defects in the simulation settings. However, the overall comparison shows that the results of simulation and numerical calculation can confirm each other.

4.2. Frequency Nonlinearity of Elastic Modulus and its Influence

The above starts with the inherent damping characteristics of the material, the viscous damping model is used to investigate the influence of damping on the bandgap characteristics of the radial circular plate, and it is found that the damping effect of viscoelastic material on the band gap is more obvious and more practical than that of the damping alloy. Therefore, in this section, finite element analysis is conducted again from the perspective of frequency nonlinearity of damping and elastic modulus for viscoelastic damping materials (taking epoxy resin as an example).

Taking the relaxation time $\tau=10^{-2}$ s, we defined two analytic functions respectively in COMSOL5.3 to represent the energy storage modulus $E_\varepsilon$ and loss modulus $E_\mu$ through the formula (8)–(9) and obtained the expression of the complex modulus $E^*$. The following results are obtained through calculation:

Figure 4. Influence of complex modulus on bandgap characteristics.
It can be seen from figure 4 that the band gap characteristics obtained when the elastic modulus $E$ is complex and constant are quite different. In particular, when $E$ is taken as complex modulus $E^*$ and is taken as storage modulus $E_s$, the band gap characteristics obtained are very close, which can be clearly seen from figures (a) and (b). When the loss modulus $E_l$ is taken, a series of adjacent narrow band gaps appear in the frequency range 0–6000hz, as shown in (c). This shows that the band gap characteristics of the generalized phononic crystal thin plate are mainly affected by the loss modulus. In order to deeply analyze this influence, the displacement transmissibility when the relaxation time of viscoelastic materials is $-2 \times 10^2$, $-3 \times 10^2$ and $-4 \times 10^2$ is calculated, and the results are shown in figure 5.

![Figure 5. Influence of loss modulus on bandgap characteristics.](image)

It can be clearly seen from figure 5 that the influence law of loss modulus $E_l$ on bandgap under different relaxation time. It is found that the band gap order in the frequency from 0 Hz to 6000Hz decreases with the relaxation time $\tau$ decreasing from $-2 \times 10^2$ to $-4 \times 10^2$, that is, the band gap of each order not only has an increasing frequency, but also has an increased bandwidth. Because of the dissipation factor $\beta = 1 / \omega \tau$, the smaller the relaxation time, the larger the dissipation factor and the stronger the damping effect. According to figure 5, with the decrease of relaxation time, the loss factor of viscoelastic materials becomes larger and larger, which affects the band gap to move to the high frequency and increases the bandwidth, which is completely consistent with the numerical results and further verifies the accuracy of the theory and numerical simulation.

5. Experimental Verification
The influence of natural damping effect of viscoelastic materials on the bandgap of curved wave in a circular plate of generalized phonon crystal is analyzed by numerical simulation and finite element simulation. In this section, a reasonable and feasible experimental scheme will be designed to fabricate typical damped generalized phonon crystal samples for transverse vibration transfer rate experimental verification.

First of all, it is difficult to directly verify the influence of different damping effects of viscoelastic materials on elastic bandgap characteristics through experiments. It is well known that epoxy resin is a typical viscoelastic material, and it is also a material specially used as the damping layer, while plexiglass is a material whose damping effect can be almost ignored. Meanwhile, it can be seen from table 1 and table 2 that the medium parameters of the two materials are extremely close. Based on the detailed analysis of the influence of various parameters in the reference [22] on the bandgap characteristics, in order to verify the rationality of the experiment, the finite element simulation transmissivity curves in two cases are given first, as shown in figure 6. It can be seen that when the scatterer is epoxy resin with large damping effect, the cut-off frequency of the band gap obviously
increases, while the initial position fluctuates a little bit but not much, and the maximum attenuation within the band gap is at the same horizontal line, which is completely consistent with the influence law of damping on the characteristics of the band gap mentioned above.

Therefore, it is reasonable to use these two materials as scatterers and steel as the matrix to construct and process radial phonon crystal samples for experimental analysis.

Figure 6. Comparison of the transmissibility curves of the two materials.

Figure 7 shows the layout of the experiment table. The DH5930 dynamic signal test and analysis system designed by Donghua company was used as the signal collection and analysis instrument in the experiment, and the data obtained in the test were saved for subsequent analysis. In the experiment, the damping type circular plate sample is first suspended on the support, and then the micro acceleration sensor is placed on the test point, and the input and output signals are collected by striking the force hammer from the center of the circular plate.

Figure 7. Experimental sample and experimental table layout.

Acceleration vibration data were extracted and the Fourier transform was performed to transform time-domain data into frequency-domain data, and transmissibility curve was obtained after data processing:
In figure 8, (a) represents the transmissibility curve measured with epoxy resin, and (b) represents the transmissibility curve measured with plexiglass. By contrast, it is obvious that the starting frequency of the bandgap is basically the same, while the cut-off frequency has a significant change, and the maximum attenuation within the bandgap is the same. This shows that the change of damping effect on the characteristics of the band gap mainly focuses on the cut-off frequency, and the increase of damping has the effect of broadening the bandwidth, and makes the bandgap move to high frequency, which verifies the correctness of the theoretical analysis and simulation results.

6. Conclusion
Through theoretical analysis, numerical analysis and simulation verification of a one-dimensional damped radial phonon crystal circular plate, the following conclusions are drawn:

1. The damping effect of viscoelastic materials has a great influence on the characteristics of the band gap. With the increase of damping intensity (i.e., the increase of loss factor $\beta$), the cut-off frequency of the band gap keeps moving towards the high frequency, while the initial frequency remains basically unchanged, thus broadening the width of the band gap. Obviously, this characteristic of viscoelastic materials can be effectively used in the field of vibration and noise reduction, and targeted measures can be taken for noise in certain frequency bands to effectively inhibit the propagation of these waves.

2. The damping effect of damping alloy has no significant influence on the bandgap, which is related to the inherent damping characteristic of damping alloy itself. Its small loss factor (ranging from 0.05 to 0.2) makes its role in special occasions and applications far less than that of viscoelastic materials.

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