Gravity in Randall-Sundrum two D-brane model

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We analyse Randall-Sundrum two D-brane model by linear perturbation and then consider the linearised gravity on the D-brane. The qualitative contribution from the Kaluza-Klein modes of gauge fields to the coupling to the gravity on the brane will be addressed. As a consequence, the gauge fields localised on the brane are shown not to contribute to the gravity on the brane at large distances. Although the coupling between gauge fields and gravity appears in the next order, the ordinary coupling cannot be realised.

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I. INTRODUCTION

Recently the model construction of the inflation using D-brane has been initiated in braneworld context. See Refs. 2 3 4 for other related issues. However, the self-gravity of D-brane, which would be essential in considering D-brane cosmology, was not seriously considered there. On the other hand, Randall-Sundrum(RS) model based on D-brane action has been considered in Refs. 5 6 7 8 9. In these papers, the bulk spacetime is described by type IIB supergravity compactified on $S^5$ and the brane action is the Born-Infeld plus Chern-Simons action. Then the long wave approximation was employed to discuss the low energy effective theory on the D-brane. Although the gauge field was assumed to be localised on the brane, the gravity on the D-brane was shown not to couple to it.

In this paper we will reexamine the gravity on the branes by investigating the linear perturbation. So far, in the previous series of paper on RS D-braneworld 5 6 7 8 9, the gradient expansion (long wave approximation) has been employed to derive the effective theory on the brane. Furthermore, the form fields $B_2$ and $C_2$ are assumed to be closed, that is, $d B_2 = d C_2 = 0$, for simplicity. However, such assumption kills the transverse tensor part of the form fields. In this paper, on the other hand, we will not impose such an assumption. Then we derive the linearised gravity on the D-branes. Our linear perturbation analysis follows Refs. 10 11 12 13. As seen below, the contribution from the zero mode of gauge fields does not have the usual form and is negligible at large distances. This is consistent with the result from long wave approximation discussed in the previous paper 10 and the appendix of this paper.

The rest of this paper is organised as follows. In Sec. II, we describe the model which we consider here. In Sec. III, we formulate the ADM formalism for the current model. In Sec. IV, the linear perturbation analysis will be done and then derive the linearised gravitational equation on the brane. The contributions from massive modes of the form fields are also considered. Finally we will give summary and discussion in Sec. V. In the appendix, for a comparison with the result obtained in linear perturbation analysis, we rederive the gravitational equation on the brane using the gradient expansion without assumption of $d B_2 = d C_2 = 0$.

II. MODEL

We consider the Randall-Sundrum model in type IIB supergravity compactified on $S^5$. The brane is described by Born-Infeld and Chern-Simons actions. So we begin with the following action

$$
S = \frac{1}{2 \kappa^2} \int d^5 x \sqrt{-g} \left[ (5) R - 2 \Lambda - \frac{1}{2} |H|^2 \right. \\
- \frac{1}{2} (\nabla \chi)^2 - \frac{1}{2} |F|^2 - \frac{1}{2} |\tilde{G}|^2 \\
\left.+ S_{\text{brane}}^{(+) \,+} + S_{\text{brane}}^{(-)\,-} + S_{\text{brane}}^{(+)\,-} + S_{\text{brane}}^{(-)\,+} \right],
$$

where $H_{M NK} = \frac{1}{2} \partial_{[M} B_{N K]}$, $F_{M NK} = \frac{1}{2} \partial_{[M} C_{N K]}$, $G_{K_1 K_2 K_3 K_4 K_5} = \frac{1}{5!} \partial_{[K_1} D_{K_2 K_3 K_4 K_5]}$, $\tilde{F} = F + \chi H$ and $\tilde{G} = G + C \wedge H$. $M, N, K = 0, 1, 2, 3, 4$. $B_{M N}$ and $C_{M N}$ are 2-form fields, and $D_{K_1 K_2 K_3 K_4}$ is a 4-form field. $\chi$ is a scalar field. $g_{MN}$ is the metric of five dimensional spacetime.

$S_{\text{brane}}^{(\pm)}$ is given by Born-Infeld action

$$
S_{\text{brane}}^{(\pm)} = \gamma_{(\pm)} \int d^4 x \sqrt{-\det(h + F^{(\pm)})},
$$

$$
S_{\text{brane}}^{(-)} = \gamma_{(-)} \int d^4 x \sqrt{-\det(q + F^{(-)})},
$$

where $h_{\mu \nu}$ and $q_{\mu \nu}$ are the induced metric on the $D_{\pm}$-brane and

$$
F^{(\pm)} = B_{\mu \nu}^{(\pm)} + (\gamma_{(\pm)})^{-1/2} F_{\mu \nu}^{(\pm)}.
$$

$F_{\mu \nu}$ is the U(1) gauge field on the brane. Here $\mu, \nu = 0, 1, 2, 3$ and $\gamma_{(\pm)}$ are $D_{\pm}$-brane tension.
$S_{CS}^{(±)}$ is Chern-Simons action

$$S_{CS}^{(+)} = γ(+)
\int d^4x \sqrt{-h} \mu^{μρσα} \left[ \frac{1}{4} \mathcal{F}^{(+)\,μρσα} C^{(+)\,ρσ} + \frac{χ}{8} \mathcal{F}^{(+)\,μρσα} \mathcal{F}^{(+)\,ρσ} \right] + \frac{1}{24} D_{μρσα}^{(+)} \right],$$

(5)

$$S_{CS}^{(-)} = γ(-)
\int d^4x \sqrt{-q} \mu^{μρσα} \left[ \frac{1}{4} \mathcal{F}^{(-)\,μρσα} C^{(-)\,ρσ} + \frac{χ}{8} \mathcal{F}^{(-)\,μρσα} \mathcal{F}^{(-)\,ρσ} \right] + \frac{1}{24} D_{μρσα}^{(-)} \right].$$

(6)

Here the brane charges are set equal to the brane tensions. Therefore, our model contains BPS state of D-branes.

### III. BASIC EQUATIONS

In this section we write down the basic equations and boundary conditions. Let us perform (1+4)-decomposition

$$ds^2 = G_{MN} dx^M dx^N = e^{2φ(y,x)} dy^2 + g_{μν}(y,x) dx^μ dx^ν,$$

where $y$ is the coordinate orthogonal to the brane. $D_+ -$brane and $D_- -$brane are supposed to locate at $y = y^{(+)} = 0$ and $y = y^{(-)} = y_0$.

The spacelike “evolutional” equations to the $y$-direction are

$$e^{-φ} ∂_y K = (- R - K^2) T^{(5)} + \frac{4}{3} T^{(5)} M - K^2$$

$$e^{-φ} D_ν e^φ,$$

(8)

$$e^{-φ} ∂_ν K = (4) R - K^2$$

$$e^{-φ} \left[ D^α D_ν e^φ \right] \text{traceless},$$

(9)

$$∂_y φ + D^α φ K ∂_y φ - \frac{1}{2} H_{γαβ} \tilde{F}^{γαβ} = 0,$$

(10)

$$∂_y X^{γμ} + e^φ K X^{γμ} + D_α φ H^{αγμ} + D_α H^{αγμ} + \frac{1}{2} F^{γαβ} \tilde{G}^{αβ} = 0,$$

(11)

$$∂_y \tilde{F}^{γμ} + e^φ K \tilde{F}^{γμ} + D_α φ \tilde{F}^{αμ} + D_α \tilde{F}^{αμ} - \frac{1}{2} H_{γαβ} \tilde{G}^{γαβ} = 0,$$

(12)

$$∂_y \tilde{G}^{γαβ} = e^φ K \tilde{G}^{γαβ},$$

(13)

where $X^{γμ} := H^{γμ} + χ \tilde{F}^{γμ}$ and the energy-momentum tensor is

$$κ^2 \left( R - \frac{2}{4} K^2 + \tilde{K}_ν^{μ} \tilde{K}^{νμ} \right) = κ^2 (T_{γμ} e^{-2φ},$$

(15)

$$D_ν K^{νμ} = D_μ K = κ^2 (T_{γμ} e^{-2φ},$$

(16)

$$D_α (e^φ X^{γαμ}) + \frac{1}{6} e^φ F_{α1α2α3} \tilde{G}^{α1α2α3} μ = 0,$$

(17)

$$D_α (e^φ \tilde{F}^{γαμ}) - \frac{1}{6} e^φ H_{α1α2α3} \tilde{G}^{α1α2α3} μ = 0,$$

(18)

Under $Z_2 -$symmetry, the junction conditions at the brane located $y = y^{(±)}$ are

$$\left[K_ν^{μ} - g_{μν} K\right]_{y = y^{(±)}} = ± \frac{κ^2}{2} \gamma(±)(g_{μν} - T^{(±)μν}) + O(T^{(±)}^2),$$

$$\left[H_{γμν} (y^{(±)}, x) = ± \frac{κ^2}{2} \gamma(±) e^φ T^{(±)μν},$$

(21)

$$\left[F_{γμν} (y^{(±)}, x) = ± \frac{κ^2}{2} \gamma(±) e^φ T^{(±)μν},$$

(22)

$$\left[G_{μνμν} (y^{(±)}, x) = ± \frac{κ^2}{2} \gamma(±) e^φ T^{(±)μν},$$

(23)

$$\left[∂_y φ (y^{(±)}, x) = ± \frac{κ^2}{4} \gamma(±) e^φ T^{(±)μν},$$

(24)

In the above

$$T^{(±)μν} = F^{(±)μν} F^{(±)μν} - \frac{1}{4} δ^{(±)μν} F^{(±)γμ} F^{(±)γν},$$

(25)

From the junction condition for $χ$, we can omit the contribution of $χ$ to the gravitational equation on the brane in the approximations which we will employ. Moreover, we omit the quadratic term in Eq. (20).
IV. LINEARISED GRAVITY

A. Background

The background bulk spacetime is five dimensional anti-deSitter spacetime and its metric is given by
\[ \gamma^{(g)}_{\mu\nu} = a^2(y) \eta_{\mu\nu} = e^{-2 \frac{a}{l}} \eta_{\mu\nu}, \] (26)
where
\[ \frac{1}{l} = -\frac{1}{6} \kappa^2 \gamma(+) = \frac{1}{6} \kappa^2 \gamma(-) := -\frac{1}{6} \kappa^2 \gamma, \] (27)
and
\[ 2\Lambda + \frac{5\kappa^4}{6} \gamma^2 = 0. \] (28)

\( l \) is the curvature radius of anti-deSitter spacetimes. Eqs. (27) and (28) represent the Randall-Sundrum tuning and then the tension \( \gamma(+) \) and \( \gamma(-) \) have the same magnitude with opposite signature, \( \gamma(+) < 0 \) and \( \gamma(-) > 0 \).

In addition,
\[ \tilde{G}_{y_{a_1a_2a_3a_4}} = -a^4 \kappa^2 \epsilon_{a_1a_2a_3a_4}, \] (29)
where \( \epsilon_{a_1a_2a_3a_4} \) is the Levi-Civita tensor with respect to the induced metric \( g_{\mu\nu} \) on the brane. Other form fields vanish in this order.

B. Linear perturbation

First we consider the linear perturbation for the evolutionary equation of \( B_{\mu\nu} \) and \( C_{\mu\nu} \). In linear order Eqs. (11) and (12) become
\[ \partial_y H_{\mu\nu} + a^{-2} \partial_\alpha H_{\mu\nu}^\alpha + \frac{3}{l} F_{\alpha\beta\gamma} \epsilon_{\mu\nu}^{\alpha\beta} = 0, \] (30)
and
\[ \partial_y F_{\mu\nu} + a^{-2} \partial_\alpha F_{\mu\nu}^\alpha - \frac{3}{l} H_{\alpha\beta\gamma} \epsilon_{\mu\nu}^{\alpha\beta} = 0, \] (31)
where \( F_{\mu\nu} = \eta_{\sigma\beta} F_{\beta\mu\nu} \).

The constraint equations (17) and (18) are rewritten as
\[ F_{\mu\nu} = \frac{l}{6} \epsilon_{\mu\nu}^{\alpha\beta} \partial^\alpha H_{y\beta}, \] (32)
and
\[ H_{\mu\nu} = -\frac{l}{6} \epsilon_{\mu\nu}^{\alpha\beta} \partial^\alpha F_{y\beta}. \] (33)

Here note that we can impose the following gauge conditions
\[ B_{y\mu} = C_{y\mu} = 0, \] (34)
using the gauge transformations \( B_{MN} \rightarrow B'_{MN} = B_{MN} + \partial_M \int_0^y dy' B_{N}(y', x) - \partial_N \int_0^y dy' B_{M}(y', x) \) and \( C_{MN} \rightarrow C'_{MN} = C_{MN} + \partial_M \int_0^y dy' C_{N}(y', x) - \partial_N \int_0^y dy' C_{M}(y', x) \). Then
\[ H_{y\mu\nu} = \partial_y B_{\mu\nu} \text{ and } F_{y\mu\nu} = \partial_y C_{\mu\nu}. \] (35)

\( B_{\mu\nu} \) and *\( C_{\mu\nu} \) are decomposed to
\[ B_{\mu\nu} = B_{\mu\nu}^{TT} + \partial_\mu B^T_\nu - \partial_\nu B^T_\mu, \] (36)
and
\[ \ast C_{\mu\nu} := \frac{1}{2} \ell^2 \epsilon_{\mu\nu}^{\alpha\beta} C_{\alpha\beta} \]
\[ = \ast C_{\mu\nu}^{TT} + \partial_\mu \ast C^{TT}_\nu - \partial_\nu \ast C^{TT}_\mu, \] (37)
where \( \partial_\mu B_{\mu\nu}^{TT} = \partial_\mu \ast C_{\mu\nu}^{TT} = \partial_\mu B^{TT}_\mu = \partial_\mu C^{TT}_\mu = 0 \).

In momentum space, the field equations are
\[ \partial_y^2 B_{\mu\nu}^{(m)TT} - a^{-2} k^2 B_{\mu\nu}^{(m)TT} + \frac{6}{l} \partial_y \ast B_{\mu\nu}^{(m)TT} = 0, \] (38)
\[ \partial_y^2 C_{\mu\nu}^{(m)TT} + \frac{6}{l} \partial_y \ast C_{\mu\nu}^{(m)TT} = 0, \] (39)
\[ \partial_y^2 B_{\mu\nu}^{(m)T} + \frac{6}{l} \partial_y \ast B_{\mu\nu}^{(m)T} = 0, \] (40)
and
\[ \partial_y^2 C_{\mu}^{(m)T} - a^{-2} k^2 C^{(m)T}_\mu + \frac{6}{l} \partial_y B^{(m)T}_\mu = 0. \] (41)

The constraint equations become
\[ B_{\mu\nu}^{(m)TT} + \frac{\ell}{6} \partial_y \ast C_{\mu\nu}^{(m)TT} = 0, \] (42)
and
\[ \ast C^{(m)T}_\mu + \frac{\ell}{6} \partial_y B^{(m)T}_\mu = 0, \] (43)
which are consistent with Eqs. (39) and (40).

Using of Eq. (42), Eq. (38) becomes
\[ \partial_y^2 B_{\mu\nu}^{(m)TT} - a^{-2} k^2 B_{\mu\nu}^{(m)TT} - \frac{36}{\ell^2} B^{(m)TT}_\mu = 0. \] (44)

In the same way, Eq. (39) with Eq. (43) give us
\[ \partial_y^2 C^{(m)T}_\mu - a^{-2} k^2 C^{(m)T}_\mu - \frac{36}{\ell^2} C^{(m)T}_\mu = 0. \] (45)

The junction conditions are
\[ \partial_y B_{\mu\nu}^{(T)}(y(\pm), x) = -\partial_y \ast C^{(TT)}_{\mu\nu}(y(\pm), x) = -\kappa^2 \gamma B^{(TT)}_{\mu\nu}(\pm), \] (46)
and
\[ \partial_y B^{(T)}_{\mu}(y(\pm), x) = -\partial_y \ast C^{(TT)}_{\mu}(y(\pm), x) = -\kappa^2 \gamma A^{(T)}_{\mu}(\pm). \] (47)
where \( F^{(\pm)}(x) \) is decomposed to

\[
F^{(\pm)}(x) = B^{(\pm)TT}_\mu + \partial_\nu B^{(\pm)T}_\nu + B^{(\pm)T}_\mu + F_{\mu\nu} = B^{(\pm)TT}_\mu + \partial_\nu A^{(\pm)}_\nu - \partial_\nu A^{(\pm)}_\mu, \tag{48}
\]

where \( A^{(\pm)}_\mu := A_\mu + B^{(\pm)}_\mu \) and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \).

Next we focus on the perturbed metric

\[
ds^2 = (1 + 2\phi)dy^2 + (\gamma_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = (1 + 2\phi)dy^2 + (a^2\eta_{\mu\nu} + h_{\mu\nu}^{TT} - \gamma_{\mu\nu}\psi)dx^\mu dx^\nu \tag{49}
\]

where \( h_{\mu\nu}^{TT} \) is transverse-traceless part of \( h_{\mu\nu} \). Since the bulk background spacetime is anti-deSitter spacetime, the Green function is exactly the same with that of the Randall-Sundrum model. The difference is just presence of the bulk form fields. Therefore we follow the argument of Ref. [13, 14] and then the perturbation \( h_{\mu\nu} \) in the Gaussian normal coordinate can be computed as

\[
\bar{h}_{\mu\nu} = h_{\mu\nu}^{TT} - \gamma_{\mu\nu}\left(\psi - \frac{2}{\ell}T^5(x)\right), \tag{50}
\]

where \( T^5(x) \) is a brane-bending mode (radion field) and \( \bar{h}_{\mu\nu} \) come from the \( D_+ \) and \( D_- \) brane, respectively. The third one is the contribution from the bulk fields. In the above \( \delta^{(5)}T_{\mu\nu} \) is the projected bulk stress tensor in the linear order. \( G_R \) is the five dimensional retarded Green function

\[
G_R(y, x; y', x') = G_R^{(0)}(y, x; y', x') + G_R^{(KK)}(y, x; y', x'), \tag{53}
\]

where

\[
G_R^{(0)}(y, x; y', x') = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')} \frac{1}{\ell(1-a_0^2)} \frac{a(y)^2a(y')^2}{\kappa^2 - (\omega + i\ell)^2}, \tag{54}
\]

and

\[
G_R^{(KK)}(y, x; y', x') = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')} \int dm \frac{u_m(y)u_m(y')}{m^2 + k^2 - (\omega + i\ell)^2}. \tag{55}
\]

\( G_R^{(0)} \) is the truncated retarded Green function for zero mode. \( u_m(y) \) are the mode functions which are expressed by Bessel functions, \( u_m(y) \propto J_1(m\ell)N_2(mz) - N_1(m\ell)J_2(mz). \)

In the present model, the equation for \( T^5 \) becomes

\[
\partial^2 T^5(x) = \frac{\kappa^2}{6} T^{(+)} = 0, \tag{56}
\]

that is, the radion is a massless scalar field.

The equation for \( \psi \) comes from the Hamiltonian constraint equation:

\[
-\frac{3}{2} \frac{1}{a^2} \partial^2 \psi = \kappa^2 \delta^{(5)} T^y_y, \tag{57}
\]

where

\[
\kappa^2 \delta^{(5)} T^y_y = \frac{1}{2} (H_{ga\alpha} H_{y}{}^{\alpha\beta} + \tilde{F}_{ga\alpha} \tilde{F}_{y}{}^{\alpha\beta})
- \frac{1}{24} (H_{\mu\alpha\beta} H^{\mu\alpha\beta} + \tilde{F}_{\mu\alpha\beta} \tilde{F}^{\mu\alpha\beta}). \tag{58}
\]

The relation between \( \phi \) and \( \psi \) comes from the traceless part of \( (\mu, \nu) \)-component of five dimensional Einstein equation

\[
\psi - \phi \sim \kappa^2 (1/\partial^2)^2 \partial^\mu \partial^\nu [\delta^{(5)} T_{\mu\nu}] \text{traceless}. \tag{59}
\]
C. Zero-mode truncation

In order to see the low energy effective (gravitational) theory on the D-brane, we will truncate zero-mode carefully. Let us focus on the zero mode for $B_{\mu \nu}$ and $C_{\mu \nu}$. Introducing new variables

$$\Psi_{\mu \nu}^{(\pm)} := P_{\mu \nu}^{(0)} \pm C_{\mu \nu}^{(0)},$$

we obtain two linearly independent equations

$$\frac{\partial^2}{\partial y^2} \Psi_{\mu \nu}^{(\pm)} \pm \frac{6}{\ell} \frac{\partial}{\partial y} \Psi_{\mu \nu}^{(\pm)} = 0.$$  \hspace{1cm} (61)

The junction conditions are written by

$$\partial_y \Psi_{\mu \nu}^{(+)}(y^{(\pm)}, x) = 0,$$  \hspace{1cm} (62)

$$\partial_y \Psi_{\mu \nu}^{(-)}(y^{(\pm)}, x) = -2\kappa^2 \gamma F_{\mu \nu}^{(\pm)}.$$  \hspace{1cm} (63)

First it is easy to see that the solutions to the equations for gauge fields are

$$\Psi_{\mu \nu}^{(+)}(y, x) = \alpha_{\mu \nu}(x),$$  \hspace{1cm} (64)

and

$$\Psi_{\mu \nu}^{(-)}(y, x) = \frac{2}{\ell} y^{-6}(y) F_{\mu \nu}^{(+)}(x) + \beta_{\mu \nu}(x),$$  \hspace{1cm} (65)

using the junction condition at $y = 0$. $\alpha_{\mu \nu}(x)$ and $\beta_{\mu \nu}(x)$ are constant of integrations which is not arisen in $H_{y \mu \nu}$ and $F_{y \mu \nu}$.

$$H_{y \mu \nu}(y, x) = \partial_y P_{\mu \nu}^{(0)}(y, x) = -\kappa^2 \gamma a^{-6}(y) F_{\mu \nu}^{(+)},$$  \hspace{1cm} (66)

$$F_{y \mu \nu}(y, x) = \partial_y C_{\mu \nu}^{(0)}(y, x) = -\kappa^2 \gamma a^{-6}(y) \epsilon_{\mu \alpha \nu \beta} F_{\alpha \beta}^{(+) \parallel}.$$  \hspace{1cm} (67)

The remaining junction condition then implies the relation between gauge fields on the two branes

$$a_0^6 \tau_{\mu \nu}^{(-)} = F_{\mu \nu}^{(+)},$$  \hspace{1cm} (68)

and

$$T_{0 \mu \nu}^{(-)} = a_0^{-14} T_{0 \mu \nu}^{(+)}.$$  \hspace{1cm} (69)

We also can compute the bulk stress tensor as

$$\kappa^2 \delta^{(5)} T_{\mu \nu} = \frac{1}{2} a^{-2} \left( H_{\mu \rho \alpha} H_{\nu \alpha \rho} - \frac{1}{4} \eta_{\mu \nu} H_{\alpha \beta \rho} H_{\alpha \beta \rho} \right) + \frac{1}{2} a^{-2} \left( \tilde{F}_{\mu \rho \alpha} \tilde{F}_{\nu \alpha \rho} - \frac{1}{4} \eta_{\mu \nu} \tilde{F}_{\alpha \beta \rho} \tilde{F}_{\alpha \beta \rho} \right)$$

$$+ \frac{1}{4} a^{-4} \left( H_{\mu \rho \alpha \beta \rho} - \frac{1}{6} \eta_{\mu \nu} H_{\alpha \beta \rho} H_{\alpha \beta \rho} \right) + \frac{1}{4} a^{-4} \left( \tilde{F}_{\mu \rho \alpha \beta \rho} - \frac{1}{6} \eta_{\mu \nu} \tilde{F}_{\alpha \beta \rho} \tilde{F}_{\alpha \beta \rho} \right)$$

$$= \left( \frac{6}{\ell} \right)^2 a^{-14} T_{\mu \nu}^{(+)} + \frac{1}{4} a^{-4} \left( H_{\mu \rho \alpha \beta \rho} - \frac{1}{6} \eta_{\mu \nu} H_{\alpha \beta \rho} H_{\alpha \beta \rho} \right) + \frac{1}{4} a^{-4} \left( \tilde{F}_{\mu \rho \alpha \beta \rho} - \frac{1}{6} \eta_{\mu \nu} \tilde{F}_{\alpha \beta \rho} \tilde{F}_{\alpha \beta \rho} \right)$$  \hspace{1cm} (70)

where $H_{\alpha \beta \rho} = \eta_{\mu \nu} \eta_{\rho \sigma} H_{\mu \nu \sigma}$. From Eq. (60) with Eqs. (56), (69) and (70) we finally obtain the following linearised equation on branes

$$\partial^2 \tilde{h}_{\mu \nu}(y, x) = \partial^2 \tilde{h}_{\mu \nu}^{TT} - \gamma_{\mu \nu} \partial^2 \psi$$

$$= -2\kappa^2 \frac{a^2(y)}{\ell(1 - a_0^2)} \gamma |T_{\mu \nu}^{(+)}| + 2\kappa^2 \frac{a^2(y) a_0^2}{\ell(1 - a_0^2)} \gamma |T_{\mu \nu}^{(-)}| - \frac{6}{\ell^2} \frac{a^2(y) (a_0^2 - 1)}{1 - a_0^2} T_{\mu \nu}^{(+)}$$

$$= -2\kappa^2 \frac{a_0^2}{\ell(1 - a_0^2)} \int_0^{y_0} \frac{dy}{4} a^{-2} \left[ H_{\mu \rho \alpha \beta \rho} \gamma + \tilde{F}_{\mu \rho \alpha \beta \rho} \gamma \right] + \frac{2}{3} \kappa^2 \gamma \eta_{\mu \nu} \delta^{(5)} T_{\gamma} \right]^{(0)}$$

$$= -2\kappa^2 \frac{a_0^2}{\ell(1 - a_0^2)} \int_0^{y_0} \frac{dy}{4} a^{-2} \left[ H_{\mu \rho \alpha \beta \rho} \gamma + \tilde{F}_{\mu \rho \alpha \beta \rho} \gamma \right] \text{traceless} + \frac{2}{3} \kappa^2 \gamma \eta_{\mu \nu} \delta^{(5)} T_{\gamma} \right]^{(0)}$$

$$= O(H_{\mu \nu}^{(5)}).$$  \hspace{1cm} (71)

that is, $\partial^2 \tilde{h}_{\mu \nu}(x) = O(h_{\mu \nu}^{(5)}) = O((\partial_{\mu \nu} F_{\rho \sigma})^2)$. The gauge fields localised on the branes does not appear as usual. The appropriate contribution from the boundary stress tensor is exactly cancelled out by that from the bulk stress tensor. In the above $(\cdots)^{(0)}$ represents the zero
mode part. For example, \((\delta^{(5)}T_{yy})^{(0)}\) is
\[ (\delta^{(5)}T_{yy})^{(0)} = -\frac{1}{24} (H_{\mu\alpha\beta}H^{\mu\alpha\beta} + \tilde{F}_{\mu\alpha\beta}\tilde{F}^{\mu\alpha\beta}). \] (72)

\section*{D. Massive modes of form fields}

The mode functions for \(B_{\mu\nu}^{(m)TT}\) etc. satisfying the junction condition at \(y = y^+(\epsilon) = 0\) is
\[ \psi_m = \sqrt{m\epsilon y/\ell} \frac{\alpha_m J_6(m\epsilon y/\ell) - \beta_m N_6(m\epsilon y/\ell)}{\sqrt{\alpha_m + \beta_m}}, \tag{73} \]
where
\[ \alpha_m = m\epsilon N_5(m\epsilon) - 6N_6(m\epsilon), \tag{74} \]
and
\[ \beta_m = m\epsilon J_5(m\epsilon) - 6J_6(m\epsilon). \tag{75} \]
\(m\) should be quantised by the junction condition at \(y = y^{(-)}(\epsilon) = y_0\). For \(m\ell \gg 1\) and \(m\epsilon y^{(-)}/\ell \gg 1\), we obtain the mass spectrum of
\[ m_n^{(\pm)} = \frac{n\pi}{\ell (1 - e^{-y_0/\ell})}. \tag{76} \]
After determination of correct normalisation, for \(m\ell \ll 1\), we can evaluate the contribution from massive modes to the right-hand side of Eq. (61) as
\[ |\kappa^2 \delta^{(5)}T_{\mu\nu}| \lesssim \frac{1}{\ell^2} \left( \frac{r_0}{r} \right)^3 \left( \frac{\ell}{r} \right)^{12} |T_{\mu\nu}^{(+)}| \lesssim \frac{1}{\ell^2} |T_{\mu\nu}^{(+)}|, \tag{77} \]
where \(r_0\) is spatial scale of support of form field. Thus even if we consider the contribution from the massive modes, they will be negligible at low energy scale.

\section{V. SUMMARY AND DISCUSSION}

In this paper we derived the linearised gravitational equation on the D-brane and then it turned out that the gauge fields do not couple to the gravity on the brane at zero modes in a usual way. Instead, an unusual couplings appear and it is negligible at large distances. We also discussed the contribution from the Kaluza-Klein modes which was shown to be also negligible at large distances.

The model which we considered is minimum extension of Randall-Sundrum type model to the supergravity-like one. Therein \(Z_2\) symmetry and RS tuning are assumed. In this model, RS tuning corresponds to the condition of equality of brane tension and charge. It is likely that D-brane in BPS state does not provide us the realistic model for the braneworld. As analysed in Ref. [3], on the other hand, it was shown that the coupling of the gravity to the gauge fields will appears and the coupling constant is proportional to the cosmological constant. Therefore non-BPS state will be important for braneworld cosmology.

We can also consider the cases without \(Z_2\)-symmetry. For example, a model considered in Ref. [1] does not have \(Z_2\)-symmetry. So the careful study of the self-gravitational effect for such a mode will be important.

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\section*{APPENDIX A: LONG-WAVE APPROXIMATION}

In this appendix we approximately solve the bulk field equations by long wave approximation (gradient expansion) and derive the effective theory on the brane. The equation obtained here will include the non-linear effect. Thus we can obtain the same result with one obtained in Sec. IV if we linearise the equation. This appendix can be regarded as the extension of previous work [10] into the general cases where we do not impose \(dC_2 = dB_2 = 0\).

In the case with bulk fields we must carefully use the geometrical projection method [17] because the projected Weyl tensor \(E_{\mu\nu}\) contains the leading effect from the bulk fields.

The bulk metric is written again as,
\[ ds^2 = e^{2\phi(x)} dy^2 + g_{\mu\nu}(y, x) dx^\mu dx^\nu. \tag{A1} \]
The induced metric on the brane will be denoted by \(h_{\mu\nu} := g_{\mu\nu}(0, x)\) and then
\[ g_{\mu\nu}(y, x) = a^2(y, x) \left[ h_{\mu\nu}(x) + g^{(1)}_{\mu\nu}(y, x) + \cdots \right]. \tag{A2} \]
In the above \(g^{(1)}_{\mu\nu}(0, x) = 0\) and \(a(0, x) = 1\). In a similar way, the extrinsic curvature is expanded as
\[ K^{(0)}_{\mu\nu} = K^{(1)}_{\mu\nu} + K^{(2)}_{\mu\nu} + \cdots. \tag{A3} \]
The small parameter is \(\epsilon = (\ell/L)^2 \ll 1\), where \(L\) and \(\ell\) are the curvature scale on the brane and the bulk anti-deSitter curvature scale, respectively.

It is easy to obtain the zeroth order solutions. Without derivation they are given by the Randall-Sundrum set up; Eqs. (27) and (28). Then
\[ K^{(0)}_{\mu\nu} = -\frac{1}{\ell^2} \delta_{\mu\nu}, \tag{A4} \]
and
\[ g_{\mu\nu}^{(0)} = a^2(y, x) h_{\mu\nu}(x) = e^{-\frac{2\phi(y, x)}{3}} h_{\mu\nu}(x). \tag{A5} \]
where
\[ d(y, x) = \int_0^y dy e^{\phi(x)}. \] (A6)

\( \tilde{G}_{\alpha\gamma,\alpha_2\gamma_2\alpha} \) is also given by Eq. (21).

Next we consider the first order equations. The first order equations for \( \tilde{F}_{y\mu\nu} \) and \( H_{y\mu\nu} \) are
\[ \partial_y \tilde{F}_{y\mu\nu} - \frac{1}{2a^4} \tilde{G}_{y\mu\beta} \tilde{G}_{\nu\sigma\rho\alpha} h^{\alpha\rho} h^{\beta\sigma} = 0, \] (A7)
and
\[ \partial_y H_{y\mu\nu} + \frac{1}{2a^4} \tilde{G}_{y\mu\beta} \tilde{G}_{\nu\sigma\rho\alpha} h^{\alpha\rho} h^{\beta\sigma} = 0. \] (A8)

Together with the junction conditions on \( D_+ \)-brane the solutions are given by
\[ H_{y\mu\nu} (y, x) = -\kappa^2 \gamma a^{-6} e^{\phi} \mathcal{F}_{\mu\nu}^{(+)} , \] (A9)
and
\[ \tilde{F}_{y\mu\nu} (y, x) = -\frac{\kappa^2}{2} \gamma a^{-6} e^{\phi} \epsilon_{\mu\nu\rho\sigma} \mathcal{F}_{\rho\sigma}^{(+)} h^\alpha h^\beta. \] (A10)

The remaining junction conditions on \( D_- \)-brane imply the relation between \( \mathcal{F}_{\mu\nu}^{(+)} \) and \( \mathcal{F}_{\mu\nu}^{(-)} \) as
\[ \mathcal{F}_{\mu\nu}^{(-)} = a_0^{-6} \mathcal{F}_{\mu\nu}^{(+)} , \] (A11)
and then
\[ T_{\mu\nu}^{(-)} = a_0^{-14} T_{\mu\nu}^{(+)} . \] (A12)

where \( a_0 = a(y_0, x) = e^{-d_0(x)/\ell} \) and \( d_0(x) := d(y_0, x) \).

Let us first substitute the junction conditions for \( H_{y\mu\nu} \) and \( \tilde{F}_{y\mu\nu} \) on the \( D_+ \) brane into the constraint equations of Eqs. (17) and (18). Then we see
\[ D^\mu \left( \mathcal{F}_{\mu\nu}^{(+)} - \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} C_{\rho\sigma}^{\alpha\beta} \right) = 0, \] (A13)
\[ \epsilon_{\mu\nu\rho\sigma} D_\nu (\mathcal{F}_{\rho\sigma}^{(+)} - B_{\alpha\beta}) = 0, \] (A14)
where \( D_\mu \) is the covariant derivative with respect to \( h_{\mu\nu} \).

Using these results the evolutional equation for the traceless part of the extrinsic curvature is
\[ e^{-\phi} \partial_y \tilde{K}_\nu^{(+)} = -\tilde{K} \tilde{K}_\nu^{(+)} + \tilde{R}_\nu^{(+)} - \kappa^2 \gamma a^{-16} T_{\nu}^{(+)} - e^{-\phi} [D^\mu D_\nu e^{\phi}]_{\text{traceless}}, \] (A15)
where
\[ \tilde{R}_\nu^{(+)} = \frac{1}{a_0^2} \left[ R_\nu^{(+)}(h) + \frac{2}{\ell} D^\mu D_\nu d + \frac{2}{\ell^2} D^\mu D_\nu d^2 \right]_{\text{traceless}} \] (A16)
and
\[ D^\mu D_\nu e^{\phi} = \frac{1}{a^2} D^\mu D_\nu e^{\phi} + \frac{1}{a^2 \ell} \left( D^\mu e^{\phi} D_\nu d + D^\mu d D_\nu e^{\phi} - \delta^\mu_\nu D^\rho D_\rho d e^{\phi} \right). \] (A17)

\( R_\nu^{(+)}(h) = h^{\mu\nu} R_{\mu\nu}(h) \) is the Ricci tensor with respect to \( h_{\mu\nu} \) and \( T_\nu^{(+)} = h^{\mu\nu} T_{\mu\nu} \).

The solution is summarised as
\[ \tilde{K}_\nu^{(+)} (y, x) = -\frac{\ell}{2a^2} \tilde{R}_\nu^{(+)}(h) + \frac{1}{2} \kappa^2 \gamma a^{-16} T^{(+)}_\nu - e^{-\phi} [D^\mu D_\nu d + \frac{1}{\ell} D^\mu d D_\nu d]_{\text{traceless}} + \frac{\chi_\nu^\mu(x)}{a_0^4} , \] (A18)

where \( \chi_\nu^\mu \) is the “integration of constant”.

The solution to the trace part of the extrinsic curvature is
\[ \tilde{K} (y, x) = -\frac{\ell}{6a^2} \tilde{R}(h) - \frac{1}{a^2} D^2 d + \frac{1}{a^2 \ell} (Dd)^2 . \] (A19)

On the \( D_+ \)-brane Eqs. (A18) and (A19) becomes
\[ \tilde{R}_\nu^{(+)}(h) = \frac{2}{\ell} \chi_\nu^\mu(x) \] (A20)
and
\[ 0 = \tilde{K} (0, x) = -\frac{\ell}{6} \tilde{R}(h) \] (A21)

They correspond to the Einstein equation on the brane obtained in Ref. [17] and \( \chi_\nu^\mu \) is projected Weyl tensor \( E_{\mu\nu} \). For the moment, \( \chi_\nu^\mu(x) \) is unknown term.

On \( D_- \)-brane, Eq. (A14) becomes
\[ \frac{\kappa^2}{2} \gamma T^{(-)}_\nu = -\frac{\ell}{2a_0^2} \tilde{R}_\nu^{(-)}(h) + \frac{\kappa^2}{2} \gamma a^{-16} T^{(+)}_\nu - e^{-\phi} [D^\mu D_\nu d_0 - \frac{1}{\ell} D^\mu d_0 D_\nu d_0]_{\text{traceless}} + \frac{\chi_\nu^\mu(x)}{a_0^4} . \] (A22)

and
\[ 0 = \tilde{K} (y_0, x) = -\frac{1}{a_0^2} D^2 d_0 + \frac{1}{a_0^2 \ell} (Dd_0)^2 . \] (A23)

All together we obtain the Einstein equation on \( D_+ \) brane
\[ (a_0^2 - 1) G_{\mu\nu}(h) = \frac{2}{\ell} \left[ D_\mu D_\nu d_0 - \frac{1}{\ell} D_\mu d_0 D_\nu d_0 \right]_{\text{traceless}} \] (A24)
The equation for radion becomes

\[ D^2 d_0 - \frac{1}{\ell} (Dd_0)^2 = 0. \]  
\( (A25) \)

Defining

\[ \Psi = 1 - e^{-2d_0/\ell} \quad \text{and} \quad \omega(\Psi) = \frac{3}{2} \frac{\Psi}{\Psi - 1}, \]  
\( (A26) \)

we rewrite down the Einstein equation as

\[ G_{\mu\nu}(h) = \left[ \frac{1}{\Psi} D_\mu D_\nu \Psi + \frac{\omega}{\Psi^2} D_\mu \Psi D_\nu \Psi \right] \text{traceless}. \]  
\( (A27) \)

Thus the contribution from the gauge fields to gravity on the brane does not exist at low energy scale.

[1] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, Liam McAllister and S. P. Trivedi, JCAP 0310, 013(2003).
[2] C. P. Burgess, P. Martineau, F. Quevedo and R. Rabadan, JHEP 06, 037(2003); C. P. Burgess, N. E. Grandi, F. Quevedo and R. Rabadan, JHEP 0401, 067(2004); K. Takahashi and K. Ichikawa, Phys. Rev. D69, 103506(2004).
[3] T. Shiromizu, T. Torii and T. Uesugi, Phys. Rev. D67, 123517(2003); M. Sami, N. Dadhich and T. Shiromizu, Phys. Lett. B568, 118(2003); E. Elizalde, J. E. Lidsey, S. Nojiri and S. D. Odintsov, hep-th/0307177; T. Uesugi, T. Shiromizu, T. Torii and K. Takahashi, Phys. Rev. D69, 043511(2004).
[4] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D66, 106006(2002); O. DeWolfe and S. B. Giddings, Phys. Rev. D67, 066008(2002).
[5] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999).
[6] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999).
[7] T. Shiromizu, K. Koyama, S. Onda and T. Torii, Phys. Rev. D68, 063506(2003).
[8] S. Onda, T. Shiromizu, K. Koyama and S. Hayakawa, Phys. Rev. D69, 123503(2004).
[9] T. Shiromizu, K. Koyama and T. Torii, Phys. Rev. D68, 103513(2003).
[10] T. Shiromizu, Y. Himemoto and K. Takahashi, hep-th/0405071.
[11] T. Wiseman, Class. Quant. Grav. 19, 3083(2002); S. Kanno and J. Soda, Phys. Rev. D66, 043526(2002); ibid, 083506,(2002); T. Shiromizu and K. Koyama, Phys. Rev. D67, 084022(2003); S. Kanno and J. Soda, Gen. Rel. Grav. 36, 689(2004).
[12] M. Sato and A. Tsuchiya, Prog. Theor. Phys. 109, 687(2003).
[13] J. Garriga and T. Tanaka, Phys. Rev. Lett. 84, 2778(2000).
[14] T. Tanaka and X. Montes, Nucl. Phys. B582, 259(2000).
[15] H. J. Kim, L. J. Romans and P. van Nieuwenhuizen, Phys. Rev. D32, 389(1985).
[16] H. Hata and Y. Tanii, Phys. Lett. B624, 283(2002).
[17] T. Shiromizu, K. Maeda and M. Sasaki, Phys. Rev. D62, 024012(2000).