Strong anisotropy of cuprate pseudogap correlations: implications for Fermi arcs and Fermi pockets

Mike Guidry, Yang Sun and Cheng-Li Wu

1 Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996-1200, USA
2 Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
3 Physics Department, Chung Yuan Christian University, Chung-Li, Taiwan 320, Republic of China

E-mail: sunyang@sjtu.edu.cn

New Journal of Physics 11 (2009) 123023 (12pp)
Received 9 August 2009
Published 21 December 2009
Online at http://www.njp.org/
doi:10.1088/1367-2630/11/12/123023

Abstract. A full Fermi surface exists in underdoped high-temperature superconductors if the temperature \( T \) lies above the pseudogap (PG) temperature \( T^* \). Below \( T^* \) the Fermi surface is anomalous. Some experiments indicate that only arcs of Fermi surface survive. Others indicate that the Fermi surface is closed but consists of pockets with a small surface area. We show that generalizing the Bardeen–Cooper–Schrieffer theory of normal superconductivity to include \( d \)-wave pairs and antiferromagnetism on a two-dimensional lattice leads to a \( T^* \) having a pronounced \( k \) dependence because of strong anisotropy in the PG correlations. We discuss implications for gapping of the Fermi surface within the Fermi arc and Fermi pockets picture of underdoped cuprates, and propose that related phenomena arising from anisotropic PG correlations may also occur in underdoped iron arsenide superconductors.

4 Author to whom any correspondence should be addressed.
1. Introduction

High-temperature superconductors were discovered more than 20 years ago [1], but there is no uniformly accepted explanation for their properties ([2] and references therein). They exhibit pseudogaps (PGs) at lower hole doping: partial energy gaps occurring below a temperature $T^*$ but above the superconducting critical temperature $T_c$ [3], with $T^*$ and $T_c$ exhibiting strong and different doping dependence for low hole-doping. It is widely believed that understanding the PG states and the $T^*$ scale is central to understanding the high-$T_c$ mechanism [2, 4].

For normal superconductors—described well by the Bardeen–Cooper–Schrieffer (BCS) theory [5]—the Fermi surface (the highest occupied fermion level) is the key to the understanding of superconductivity. In optimally and overdoped high-$T_c$ compounds, a ‘normal’ Fermi surface exists, and the BCS theory utilizing $d$-wave singlet hole pairs seems applicable [2]. But in underdoped cuprates PG states (lying between $T^*$ and $T_c$) have anomalous Fermi surfaces, with indications that the large Fermi surface found in the overdoped region has been reduced to arc-like vestiges, or to small pockets in $k$-space.

2. Fermi arcs

Angle-resolved photoemission spectroscopy (ARPES) [6, 7] probes electronic properties [6, 8] of states in high-temperature superconductors. ARPES data suggest a decreased state density near the Fermi energy for temperature $T < T^*$ that is anisotropic in momentum, with strong $T$-dependence. A full Fermi surface is observed for $T > T^*$; as the temperature decreases below $T^*$, only arcs centered on the $d$-wave nodal lines (diagonals in momentum space) survive [9]–[11], scaling as $T / T^*$ to zero length for $T \rightarrow 0$ [11]. Thus gapping is anisotropic in
momentum space, with an ungapped Fermi surface surviving only in the form of temperature-dependent Fermi arcs. The issue received renewed focus because of results by Kanigel et al [11] that place the strongest constraints yet on the nature of Fermi arcs.

Earlier theoretical work on Fermi arcs has employed a variety of approaches, including precursor pairs [12, 13], the perturbative renormalization group [14], high-$T$ expansions in the $t-J$ model [15], resonating valence band (RVB) models with strong gauge coupling [16], and time-reversal violating phases [17], but there is no agreed-upon explanation of Fermi arcs. Competing ideas about the explanation of Fermi arcs have been reviewed recently by Norman et al [18].

3. Magnetic quantum oscillations

The arc-wise disappearance of the Fermi surface implied by ARPES measurements has been called into question by recent measurements of Shubnikov–de Haas (SdH) and de Haas–van Alphen (dHvA) effects that probe the cuprate Fermi surface directly [19]–[23]. These experiments indicate that rather than disconnected arcs the Fermi surface consists of small closed regions (‘pockets’). Although these measurements cannot determine the location of the Fermi surface in momentum space, they do determine its area precisely and indicate that in the underdoped cuprates the Fermi surface is small. This differs substantially from the large Fermi surface found for ordinary superconductors and for overdoped cuprate samples. The exact state that the quantum oscillation experiments are seeing under high magnetic field conditions is uncertain, but it is generally expected that it corresponds to the normal state that is a precursor to the superconductivity.

These findings raise the issue of whether the ARPES interpretation can be reconciled with the quantum oscillation interpretation, or whether the two classes of measurements are revealing information about different things. ARPES is sensitive to surfaces, typically under conditions of relatively high temperature and a low magnetic field. Quantum oscillation measurements are done at relatively low temperature and a high magnetic field, and are sensitive to the bulk of the crystal. Thus, it is possible that ARPES and quantum oscillations are probing different things. Another possibility is that ARPES experiments are resolving only one side of closed Fermi pockets suggested by quantum oscillation experiments, thus giving the illusion that the Fermi surface has become arc-like. (It is generally argued that the quantum oscillation experiments are inconsistent with the open Fermi surfaces implied by Fermi arcs, because the origin of the observed oscillations is expected to be cyclotron resonances of electrons in closed orbits in the magnetic field.)

In this paper, we address these issues in terms of a solution to a minimal model of spin, charge, $d$-wave superconductivity (SC) and antiferromagnetism (AF) on a two-dimensional (2D) lattice with no double occupancy. We shall demonstrate that this solution implies that the correlation energy associated with the opening of the PG necessarily has a strong angular dependence in the $k$-space, and that this angular dependence leads to strong temperature-dependent and doping-dependent restrictions on regions of the Brillouin zone, where ungapped Fermi surfaces can exist. We shall demonstrate explicitly that this prescription produces a description of the length of Fermi arcs as a function of temperature that is in quantitative agreement with the ARPES results of Kanigel et al [11], if one assumes a full hole Fermi surface as a starting point. Conversely, if one assumes the Fermi surface to correspond to the closed pockets suggested by quantum oscillation experiments, we shall argue that our results
place strong restrictions on possible location and size of those pockets. Finally, we shall suggest that ARPES and quantum oscillation results both probe the strong $k$-space anisotropy of the PG correlations, perhaps with a different emphasis.

4. The SU(4) model

In earlier work we proposed an SU(4) model for the ground state properties of cuprate systems. Generalized SU(4) coherent states are the simplest Hartree–Fock–Bogoliubov variational solutions that incorporate AF competing with $d$-wave superconductivity on a lattice with no double occupancy [25]–[28]. They generalize BCS to incorporate AF self-consistently. If AF correlations vanish, the SU(4) gap equations reduce to the BCS gap equations; if instead singlet pairing vanishes, the SU(4) gap equations describe an AF spin system with the Néel order; for finite AF and SC correlations, the SU(4) gap equations describe the evolution with doping ($P$) and temperature ($T$) of $d$-wave Cooper pairs interacting with strong AF correlations.

SU(4) states forbid double occupancy by symmetry, not by projection [26]. The exact $T = 0$ ground state at half filling is an AF Mott insulator that evolves rapidly with hole doping into a superconductor with strong AF correlations in the underdoped region, and finally into a singlet, $d$-wave superconductor beyond a critical hole doping $P \approx 0.16–0.19$ [27]. At finite $T$ for $P > P_q$, pairing is reduced by thermal fluctuations and the pairing gap vanishes at $T_c$. In the underdoped region ($P < P_q$), there are quantum fluctuations associated with AF correlations in addition to thermal fluctuations. The interplay between AF, pairing and thermal fluctuations produces PG states above $T_c$ in which the fermionic pairs interact through AF correlations. Thus, the energy gap $\Delta_1$ opened by the AF correlations in the paired basis is the SU(4) PG, while the (singlet and triplet fermion) pairs in these states may be viewed as $d$-wave preformed pairs.

The SU(4) model describes both the PG and pairing gap with doping dependence quantitatively in agreement with data (see [28]). The SC transition temperature $T_c$ and PG transition temperature $T^*$ are also well reproduced, as illustrated in figure 1. We conclude that SU(4) coherent states give descriptions of pairing gaps and PGs that are in quantitative agreement with cuprate data.

5. Momentum dependence

The 15 generators $\{ G_i \}$ of SU(4) are $M = \frac{1}{2}(n - \Omega)$ and

\[
D^\dagger = \sum_k s_{gk} c_{k+}^\dagger c_{-k \downarrow}, \quad \pi_{ij}^\dagger = \sum_k s_{gk} c_{k+q,i}^\dagger c_{-k,j}^\dagger,
\]

\[
Q_{ij} = \sum_k c_{k+q,i}^\dagger c_{k,j}, \quad S_{ij} = \sum_k c_{k,i}^\dagger c_{k,j} - \frac{1}{2} \Omega \delta_{ij}, \tag{1}
\]

and Hermitian conjugates, where $\uparrow, \downarrow, i$ and $j$ are spin indices, $q \equiv (\pi, \pi, \pi)$, the maximum number of doped holes or particles that can form coherent pairs is $\Omega$ (assuming the half-filled normal state as vacuum), $s_{gk}$ is the sign of the $d$-wave pair form factor $g(k) = g(k_x, k_y) = \cos k_x - \cos k_y$, the charge is $M$ with $n = \sum_k n_k$ the electron number operator, and $\pi_{ij}^\dagger, Q_{ij}$ and $S_{ij}$ are tensor components of triplet pair, staggered magnetization and spin operators $\vec{\pi}, \vec{Q}$ and $\vec{S}$, respectively.

New Journal of Physics 11 (2009) 123023 (http://www.njp.org/)
Figure 1. SU(4) cuprate phase diagram compared with data. Strengths of the AF and singlet pairing correlations were determined by global fits to cuprate data [27, 28]. The PG temperature is $T^*$ and the SC transition temperature is $T_c$. The AF correlations vanish, leaving a pure singlet $d$-wave condensate, above the critical doping $P_q$. Dominant correlations in each region are indicted by italic labels. Data from [30] (arrows indicate lower limits).

These generators are summed over $k$. They are appropriate for data that are not momentum selected (for example, figure 1). However, ARPES Fermi-arc data exhibit explicit dependence on $k$. The SU(4) formalism may be extended to deal with this case by viewing the individual $k$ components of the operators defined in equation (1) as the symmetry generators $G(k): \{G(k)\} \equiv \{D(k), D^\dagger(k), \vec{p}(k), \vec{Q}(k), \vec{S}(k), n_k\}$, where, for example, the singlet pair generator $D^\dagger$ in equation (1) is related to the $k$-dependent generators $D^\dagger(k)$ by $D^\dagger = \sum_{k>0} D^\dagger(k)$, and $k > 0$ means either $k_x > 0$ or $k_y > 0$.

Instead of the global SU(4) symmetry generated by equation (1), the symmetry now is a direct product of $k$-dependent SU(4) groups, $\prod_{k>0} \otimes SU(4)_k$. The corresponding Hamiltonian is

$$H = \sum_{k>0} \epsilon_k n_k - \sum_{k,k'>0} \left\{ \chi_{kk'} \vec{Q}(k) \cdot \vec{Q}(k') + G_{kk'}^{(0)} D^\dagger(k) D(k') + G_{kk'}^{(1)} \vec{Q}^\dagger(k) \cdot \vec{Q}(k') \right\},$$

which $\epsilon_k$ and $n_k$ are single-particle energies and occupation numbers, respectively, and the interaction strengths are $\chi_{kk'} = \chi^0 |g(k)| g(k')$ and $G_{kk'}^{(i)} = G^{(i)} |g(k)| g(k')$ $\quad (i = 0, 1)$, with $|g(k)| = |\cos k_x - \cos k_y|$. Note that while the definition of $\chi_{kk'}$ is the same as that in [29], the definition of $G_{kk'}^{(i)}$ is now different. In [29], $G_{kk'}^{(i)}$ was assumed according to rather limited experimental information at the time, which led to unphysical predictions in figure 7 of [29]. The above definition of $G_{kk'}^{(i)}$ eliminates the unnecessary assumption in equation (10) of [29] for the pair form factor and corrects the mistake introduced in figure 7 of that paper.
Figure 2. The \( k \)-anisotropic factor, PG correlation energy and graphical Fermi-arc solution, assuming a hole-like Fermi surface centered on \((k_x, k_y) = (\pi, \pi)\).

(a) The \( k \)-anisotropic factor \(|g(k)|\) (blue lines) and contours of equal hole doping \(P\) (dashed gray lines), in the first Brillouin zone. (b) Correlation energy \(\alpha(\theta)\) for unit \(\alpha = kT^*\) evaluated along the Fermi surface (curves of constant \(P\) in part (a)) as a function of the angle \(\theta\) (defined in inset). Curves corresponding to doping \(P = 0\)–0.3 lie almost on top of each other, indicating that \(\gamma(\theta)\) is largely insensitive to doping. (c) Graphical solution for Fermi arcs. The curve defines the PG correlation energy and horizontal lines correspond to constant thermal energy scales \(k_B T\) associated with a given temperature \(T\). Their intersections (black dots) represent points in the momentum space where the PG is just closed by thermal fluctuations; these bracket arcs \(\Delta\theta\) of the surviving Fermi surface.

6. Anisotropy of the PG

We shall term this \( k \)-dependent formalism the SU(4)\(_k\) model \cite{29}. All results of the original \( k \)-independent SU(4) model \cite{25}–\cite{28} (including figure 1) are recovered as a special case of the SU(4)\(_k\) model for observables that are dominated by contributions from near the Fermi surface \((\tilde{k} = k_f)\) and averaged over \(k\) directions. However, general solutions of the SU(4)\(_k\) model give new \( k \)-anisotropic properties. Of direct relevance to the present discussion is that the temperature for the PG closure \(T^*(k)\) becomes anisotropic in \(k\); specifically, we derive in the SU(4)\(_k\) model

\[
T^*(k) \equiv T^*(k_f, \theta) = T^* |g(k_f, \theta)/g_0(k_f)|,
\]

where \(T^*\) is the gap closure temperature measured by ARPES along the antinodal \((\theta = 0, \pi/2)\) direction, which is the maximum possible value of \(T^*\) and should be somewhat larger than \(T^*\) inferred from experiments that average over \(k\) (see \cite{29} for details). Equation (3) defines the full temperature and doping dependence of the PG closure temperature. The ultimate physical reason for this result is that the singlet and triplet pairs interacting by AF interactions in the SU(4) PG state each carry a \(g(k)\) form factor, which introduces a \(k\) dependence in the effective AF coupling and thus in \(T^*\). The behavior of \(g(k)\) as a function of \((k_x, k_y)\) is illustrated in figure 2(a).

The strong \(k\) dependence of the PG temperature \(T^*\) implied by equation (3) and figure 2(a) is our most important result. This represents a theoretical prediction with a number of potential observational implications that is independent of the experimental controversy over whether
the Fermi surface of underdoped cuprates consists of small close pockets or disconnected arcs. Nevertheless, we now examine the possible implications of this finding for these competing pictures of the Fermi surface.

7. Implications for Fermi arcs

In \( g(k) \), the components \((k_x, k_y)\) or \((k_f, \theta)\) are constrained by the Fermi surface \((\tilde{k}^2 = k_f^2)\). Assuming an isotropic hole surface centered around \((\pi, \pi)\) (dashed gray lines in figure 2(a) and inset to figure 2(b)), we have \((\pi - k_x)^2 + (\pi - k_y)^2 = k_f^2 = 2\pi(1 + P)\). For a given doping \(P\) (thus \(k_i\)) and temperature \(T\), with \(T^*(k) = T\) by virtue of equation (3) under the above constraint, we obtain the angles \(\theta_1\) and \(\theta_2\) at which the PG closes (the angle \(\theta\) is defined in the inset to figure 2(b)), and the length of the surviving Fermi arc is \(k_0|\theta_2 - \theta_1| = k_1\Delta\theta\).

This result may be interpreted graphically. In figure 2(b), we show \(\gamma(\theta) \equiv |g(k_f, \theta)/g_0(k_i)|\) versus angle \(\theta\) along different Fermi surfaces (dashed gray lines in figure 2(a)). We see that \(\gamma(\theta)\) is almost independent of doping \(P\). Thus, solving equation (3) with the Fermi surface constraint is equivalent to solving

\[ k_B T^*(k) = \alpha \gamma(\theta), \quad \alpha = k_B T^*, \]

with \(k_B\) the Boltzmann constant. The PG correlation energy \(\alpha \gamma(\theta)\) depends on the \(k\)-direction \(\theta\) (figure 2(c)). The PG closes when the thermal excitation energy \(k_B T\) is comparable to the PG correlation energy. The intersections of horizontal lines of fixed \(k_B T\) with the correlation energy curve (black dots in figure 2(c)) define Fermi-arc solutions \(\theta_1\) and \(\theta_2\). Outside those points (solid blue curve), the correlation energy is larger than the energy of thermal fluctuations (shaded region), the PG opens, and the Fermi surface is gapped. Inside these points (dotted red curve), the thermal energy exceeds the correlation energy, the PG closes, and the Fermi surface exists in an arc \(\Delta\theta\) between the dots. When \(T > T^*\), the PG is closed in all directions and there is a full Fermi surface since \(k_B T > \alpha\), and \(\alpha\) is the maximum PG correlation energy.

For a given temperature \(T\), the arc solution exemplified graphically in figures 2(b) and (c), or algebraically in equation (3), implies an anisotropy of \(\gamma(\theta)\) that partitions the \(k\)-space uniquely into regions that can have a Fermi surface \((T > T^*(k_1, \theta))\) and those that cannot \((T < T^*(k_1, \theta))\). As figure 2(c) suggests, arc lengths decrease with \(T\) at fixed doping, with a full Fermi surface at \(T = T^*\), but only the nodal points at \(T = 0\). Doping dependence enters primarily through the maximum PG temperature \(T^*\) and the Fermi momentum \(k_1\); the temperature dependence enters through \(T/T^*\). For fixed \(T\), arcs shrink toward the nodal points with decreased doping because \(T^*\) in \(T/T^*\) increases at smaller doping (figure 1). However, if Fermi-arc length is measured by the fraction \(\Delta\theta/(\pi/2)\) and temperature is scaled by \(T^*\), the weak doping dependence of \(\gamma(\theta)\) ensures that \(\Delta\theta/(\pi/2)\) versus \(T/T^*\) is almost independent of doping and compound.

8. Temperature dependence of Fermi arcs

Kanigel et al [11] report fractional arc lengths versus \(T/T^*\) for Bi2212 that we plot in figure 3. Our theoretical solution for the fractional arc lengths (obtained from figure 2(c) or equation (3)) is the solid line in figure 3. Agreement between data and theory is remarkable, given that the theoretical curve has no adjustable parameters (it is determined completely by the parameters fixed previously in figure 1) and that we predict the scale \(T^*\) implicit in the data with the same
Figure 3. Fermi arc length versus temperature. Experimental arc length is displayed as a percentage of full Fermi surface length versus $T/T^*$ for underdoped Bi2212 [11]; the solid curve is our prediction for an isotropic Fermi surface (inset upper left); the dashed curve assumes a Fermi surface with flatter antinodal segments (inset lower right). No parameters (beyond those already fit to gaps in figure 1) were adjusted in either calculation, and the curves are almost independent of doping.

The dashed curve in figure 3 repeats the analysis using the flatter Fermi surface shown (inset lower right). The similarity of dashed and solid curves indicates that arc solutions depend only weakly on differing curvature in nodal and antinodal regions. Absolute arc lengths depend on doping and temperature. However, the scaled arc lengths of figure 3 are near-universal functions only of the ratio $T/T^*$, largely independent of compound, doping and Fermi surface details, as the data suggest.

The quantitative agreement between theoretical prediction and data without parameter adjustment in figure 3 is unlikely to be an accident. This suggests that ARPES experiments are seeing a real temperature-dependent and a $k$-dependent effect associated with the anisotropy of the PG correlation energy, irrespective of the experimental controversy over whether this effect should be interpreted as evidence for arc-like Fermi surfaces.

9. Gapping of the Fermi surface

Figure 3 has been interpreted [11] as indicating a rapid drop from a full Fermi surface at $T = T^*$, destroying the antinodal Fermi surface at essentially constant temperature, followed by a linear decrease of Fermi-arc length with decreasing $T/T^*$ on the near-nodal region of the
Fermi surface, extrapolating to zero arc length at the nodal points for $T/T^* \to 0$. This suggests that nodal and antinodal Fermi surfaces may be removed differently as $T$ is lowered (see [31]). For example, it has been argued [11] that abrupt removal of the antinodal surface at $T \approx T^*$ may be associated with a lattice vector connecting antinodal surfaces (perhaps associated with charge ordering [31]), while the linearly varying removal of the nodal regions extrapolating to a nodal-point surface at $T = 0$ may support a nodal liquid picture [32]. The present results invite simpler hypotheses. Our unified analysis suggests that figure 3 is consistent with loss of both nodal and antinodal surfaces by the same mechanism. Nor do the data of [11] imply unique support for nodal liquids. We have demonstrated specifically that the SU(4) model accounts quantitatively for Fermi arcs and $T^*$, without invoking nodal liquids.

10. Discriminating among theoretical predictions

We have shown that a theory of Fermi arcs must make two correct predictions: (i) the scale $T^*$ and its doping dependence and (ii) that $T^*(k) \propto \gamma(\theta)$. Any theory having a $T^*(k)$ consistent with equation (4) can describe the scaled data of figure 3, if $T^*$ is taken from data. Scaled ARPES data (figure 3) test whether a PG has a nodal structure similar to that of the superconducting state. But discriminating among different theories meeting this condition requires more: a quantitative, self-consistent description of Fermi-surface loss and of the PG temperature scale $T^*$ and its doping dependence (figure 1). We conclude that only a highly restricted set of models can be consistent with the aggregate properties of Fermi arcs.

11. Implications for Fermi surface pockets

Finally, let us address implications of the present results if the Fermi surface for underdoped cuprates is interpreted in terms of small pockets instead of Fermi arcs, as favored by magnetic quantum oscillation experiments. This discussion must necessarily be more qualitative than the preceding one, since quantum oscillation experiments determine the area of the Fermi surfaces and indicate that they are likely to correspond to closed orbits, but cannot locate the resulting small pockets definitively in $k$-space.

We hypothesize that cuprate normal states deviate from Fermi liquid behavior primarily because of correlations producing the PG, which in the present model represents fluctuating AF correlations in a singlet and triplet pair basis. Thus suppression of AF correlation in cuprates should lead to a normal Fermi surface and BCS-like SC. There are no AF correlations beyond the critical doping $P_c \simeq 0.18$, consistent with experiments suggesting that SC is ($d$-wave) BCS-like with a Fermi liquid normal state. Below the critical doping, the AF correlations compete very strongly with pairing in general, but the present analysis suggests the additional feature that this competition is strongly localized in angle in the $k$-space.

The actual Fermi surface of the physical system depends on how interactions rearrange the occupied and unoccupied orbitals of the non-interacting system. However, the preceding arguments indicate that, independent of details, at low doping the momentum space available to form a normal Fermi surface becomes small and restricted to limited regions of $k$-space by the angular form factors displayed graphically in figure 2.

Thus, we expect large conventional Fermi surfaces near critical doping and beyond, but at low doping the strong anisotropy of the PG correlations severely restricts the volume of $k$-space available to form Fermi pockets, or Fermi surfaces of any form. This argument also suggests
that any ungapped Fermi surface is likely to lie near the nodal region in either the ARPES or quantum oscillation interpretations. This is consistent with the conjecture of [24] for location of the Fermi pockets. Hence, the most important implication of both Fermi arc and quantum oscillation data may be that they provide comprehensive evidence for localized, anisotropic correlations responsible for producing the PG, independent of detailed interpretation.

12. Implications for iron arsenide superconductors

Recently, a new class of high-temperature superconductors based on iron compounds has been discovered, with critical temperatures in the vicinity of 55 K that are surpassed only by the cuprate superconductors [33]–[43]. Although these iron arsenide (FeAs) superconductors differ at the microscopic level from the cuprate superconductors, they exhibit many properties in their collective antiferromagnetic and superconducting structure that are similar to those of the cuprates. We have proposed [44] that the cuprate and FeAs superconductors are both described by an SU(4) emergent symmetry, and that the FeAs superconductors represent the second example (after cuprates) of the new class of non-Abelian superconductors that we proposed in earlier work [26].

The unified model of cuprate and FeAs superconductivity foreshadowed in [26] and proposed explicitly in [44] requires only that the observed superconducting and antiferromagnetic collective properties be compatible with SU(4) symmetry. This implies that various underlying microscopic structures are, in principle, capable of generating the observed non-Abelian superconductivity of the cuprate and FeAs high-temperature superconductors. However, the microscopic fermion structure of the SU(4) generators and the nonlinear conditions imposed on them by the non-Abelian algebra imply strong constraints on the permissible microscopic structure underlying the emergent superconductivity and AF. In the FeAs case, we find that requiring compatibility between superconducting properties and AF properties deduced from neutron scattering severely restricts the microscopic form of the pairing form factor, and leads to a unique form factor prediction when coupled with ARPES data. Interestingly, we find that a unified model of cuprate and FeAs superconductors is possible at the emergent degrees of freedom level only if the microscopic pairing form factor for FeAs compounds is different from that for cuprates.

The relevance of these results to the present discussion derives from our contention that the essential physics of Fermi arcs and quantum oscillation results for underdoped cuprates lies not in the underlying Fermi surface itself, but rather in how that Fermi surface is modified by the anisotropic PG correlations. Since we have argued in [44] that FeAs compounds are also described by an SU(4) symmetry, it is natural to assume that PGs occur in those compounds and are anisotropic, just as for the cuprates. Therefore, it is expected that Fermi arc and PG pocket phenomena can occur in the iron arsenide superconductors. Equation (3) should apply to FeAs systems because it follows from the SU(4) PG structure, but since the shape of the Fermi surface and the form factor $g(k)$ (and thus the factor $\gamma(\theta)$) may generally be different in FeAs compounds relative to cuprates, the observed patterns may be different as well. We shall deal with quantitative implications of PG anisotropy for the FeAs superconductors in the future work.

These ideas, and the unification hypothesis of [44], can be tested simultaneously by systematic comparison of cuprate and FeAs Fermi surface data in the underdoped region. If our ideas are correct, both of these systems are described by SU(4) symmetry and thus will
exhibit measurable effects that can be associated with anisotropic PG correlations, even though the underlying Fermi surfaces are likely rather different in the two cases.

13. Conclusions

We have demonstrated that an SU(4) solution for cuprate superconductivity implies a strong $k$-space anisotropy and corresponding localization for correlations associated with the PG, leading to a strongly $k$-dependent PG temperature $T^*$. Because this result represents the most general solution for a minimal model of charge, spin, $d$-wave singlet superconductivity and AF on a 2D lattice with no double occupancy, we believe it to reflect general properties expected for any realistic model of the cuprates. This anisotropy is expected to have significant consequences for gapping of the cuprate Fermi surface, particularly in the underdoped region. Because we have argued that the new FeAs superconductors also correspond to an SU(4) symmetry, we expect related phenomena for them as well. However, the detailed implications of anisotropy in the PG correlations for FeAs compounds may differ from those for the cuprates because the orbital form factors and Fermi surfaces likely differ in the two cases.

We have shown that if the Fermi surface in the underdoped region is interpreted in terms of arcs surviving on a cylindrical hole Fermi surface, as suggested by ARPES experiments, the theory reproduces quantitatively with no parameter adjustment the observed variation of the length of the arc segments with temperature. This suggests that what is being measured in the ARPES experiments is related directly to the anisotropic structure of the PG correlations, irrespective of any additional interpretation. We can make less definitive statements about the Fermi surface pockets favored by quantum oscillation experiments because present data do not allow the proposed small pockets to be localized in $k$-space, or even to be enumerated. However, our findings suggest that an ungapped Fermi surface of any form is unlikely to survive the PG correlations as the temperature is lowered below $T^*$, except in increasingly localized nodal regions of $k$-space.

Thus, our analysis provides theoretical support for the proposal that ARPES and quantum oscillation experiments are seeing (perhaps different aspects of) physics associated with strong $k$-space anisotropy of the PG correlations in underdoped cuprates, and suggests that related phenomena should be observable in the FeAs superconductors. It remains to be seen exactly how these experiments are related to each other, but we may expect that further elucidation of this relationship will provide valuable insight into the nature of the PG state.

Acknowledgments

We thank Elbio Dagotto, Pengcheng Dai and Takeshi Egami for discussion, and Elbio Dagotto for noting an error in our original formulation of these ideas.

References

[1] Bednorz J G and Müller K A 1986 Z. Phys. B 64 189
[2] Bonn D A 2006 Nat. Phys. 2 159
[3] Timusk T and Statt B 1999 Rep. Prog. Phys. 62 61
[4] Norman M, Pines D and Kollin C 2005 Adv. Phys. 54 715
[5] Bardeen J, Cooper L N and Schrieffer J R 1957 Phys. Rev. 108 1175

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[6] Norman M R and Pépin C 2003 Rep. Prog. Phys. 66 1547
[7] Damascelli A, Hussain A and Shen Z-X 2003 Rev. Mod. Phys. 75 473
[8] Dagotto E 1994 Rev. Mod. Phys. 66 763
[9] Norman M R et al 1998 Nature 392 157
[10] Zhou X-J et al 2004 Phys. Rev. Lett. 92 187001
[11] Kivel A et al 2006 Nat. Phys. 2 447
[12] Engelbrecht J R, Nazarenko A, Randeria M and Dagotto E 1998 Phys. Rev. B 57 13406
[13] Preosti G, Vilk Y M and Norman M R 1999 Phys. Rev. B 59 1474
[14] Furukawa N, Rice T M and Salmhofer M 1998 Phys. Rev. Lett. 81 3195
[15] Putikka W O, Luchini M U and Singh R R P 1998 Phys. Rev. Lett. 81 2966
[16] Wen X G and Lee P A 1998 Phys. Rev. B 57 13406
[17] Putikka W O, Luchini M U and Singh R R P 1998 Phys. Rev. B 59 1474
[18] Preosti G, Vilk Y M and Norman M R 1999 Phys. Rev. B 57 13406
[19] LeBoeuf D et al 2007 Nature 447 537
[20] Yamaguchi N et al 2007 Phys. Rev. Lett. 98 177004
[21] Takeuchi T et al 2007 Phys. Rev. Lett. 99 177001
[22] Balents L, Fisher M P A and Nayak C 1998 Int. J. Mod. Phys. B 12 1033
[23] Kamihara Y et al 2006 J. Am. Chem. Soc. 128 10012
[24] Watanabe T et al 2007 Inorg. Chem. 46 7719
[25] Kamihara Y et al 2008 J. Am. Chem. Soc. 130 3296
[26] Chen G F et al 2008 Phys. Rev. Lett. 101 057007
[27] Chen G F et al 2008 Phys. Rev. Lett. 101 057007
[28] Chen G F et al 2008 Chin. Phys. Lett. 25 2235
[29] Sun Y, Guidry M and Wu C-L 2008 Chin. Sci. Bull. 53 1617
[30] Guidry M W, Sun Y and Wu C-L 2008 Phys. Rev. B 78 174524
[31] Dai P et al 1999 Science 284 1344
[32] Oda M et al 1998 Physica C 281 135
[33] Watanabe T et al 2000 Phys. Rev. Lett. 84 5848
[34] Campuzano J C et al 1999 Phys. Rev. Lett. 83 3709
[35] Wu A, Moshchalkov V V and Bruynseraede Y 1996 Phys. Rev. B 53 9418
[36] Takigawa M et al 1991 Phys. Rev. B 43 247
[37] Ito T, Takenaka K and Uchida S 1993 Phys. Rev. Lett. 70 3995
[38] McElroy K 2006 Nat. Phys. 2 441
[39] Balents L, Fisher M P A and Nayak C 1998 Int. J. Mod. Phys. B 12 1033
[40] Kamihara Y et al 2006 J. Am. Chem. Soc. 128 10012
[41] Watanabe T et al 2007 Inorg. Chem. 46 7719
[42] Kamihara Y et al 2008 J. Am. Chem. Soc. 130 3296
[43] Chen G F et al 2008 Phys. Rev. Lett. 101 057007
[44] Chen G F et al 2008 Chin. Phys. Lett. 25 2235
[45] Sun Y, Guidry M and Wu C-L 2008 Chin. Sci. Bull. 53 1617
[46] Guidry M W, Sun Y and Wu C-L 2009 Front. Phys. China 4 433

New Journal of Physics 11 (2009) 123023 (http://www.njp.org/)