CSMA Local Area Networking
under Dynamic Altruism

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Abstract

In this paper, we consider medium access control of local area networks (LANs) under limited-information conditions as befits a distributed system. Rather than assuming “by rule” conformance to a protocol designed to regulate packet-flow rates (e.g., CSMA windowing), we begin with a non-cooperative game framework and build a dynamic altruism term into the net utility. The effects of altruism are analyzed at Nash equilibrium for both the ALOHA and CSMA frameworks in the quasi-stationary (fictitious play) regime. We consider either power or throughput based costs of networking, and the cases of identical or heterogeneous (independent) users/players. In a numerical study we consider diverse players, and we see that the effects of altruism for similar players can be beneficial in the presence of significant congestion, but excessive altruism may lead to underuse of the channel when demand is low.

I. INTRODUCTION

Flow and congestion control are fundamental networking problems due to the distributed, information-limited nature of the decision making process in many popular access technologies. Various distributed mechanisms have been implemented to cooperatively desynchronize demand, e.g., TCP, ALOHA, CSMA. Typically, when congestion is detected, all end-devices are expected

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to slow down their transmission rates and then increase again slowly hoping to find a fair and efficient equilibrium\(^1\). The fact that this process is not incentive compatible (a user/player could selfishly benefit by not following the prescribed protocol) raises two important issues.

First, the fact that users could have access to alternative implementations of the prescribed (“by rule”) protocols, e.g., ones that slow down less than they should, could lead to an unfair allocation or even congestion collapse (see e.g., [16], [47]). To address such threats, there is a steadily growing literature that analyzes the equilibria of different distributed network resource allocation games [3], [16], [20], [29]–[31], [35], [37], [38], [42], [51], [52]. The theoretical models considered ideally would allow for more informed choices in the implementation of the corresponding flow and congestion control protocols (e.g., by associating a utility function to end-devices, which can then be the basis of actions by rationally selfish players).

A “fair” resource allocation may not be optimal from an economic point of view. So, a second important challenge in the design of distributed flow and congestion control protocols is to enable users to optimize their utility of bandwidth while simultaneously maximizing the total welfare in the system. One simple approach is to set a pricing mechanism on resource consumption (or account for the cost of networking) and thus allow users to express their net utility through their willingness to pay. Otherwise, there may not be a trustworthy way for heterogeneous users that need more to get more, thus increasing the incentives of certain users to bypass the prescribed protocol.

For a random-access LAN, several authors have recently considered the problem of distributed optimization of a global objective (total throughput, social welfare) subject to a fairness constraint. For example, in [20], a utility function design problem is studied considering estimation errors of the network state. In a Markovian setting without fictitious play\(^2\) [38] introduces a cooperation parameter (a probability to stop transmitting), but follows a detection and punishment methodology regarding selfish behavior. In [23], a window-update algorithm that tries to directly minimize the average idle time of the channel is proposed. As our main contribution, we formulate and analyze a novel CSMA medium access control game with conditionally altruistic players. Altruistic tactics in evolutionary/mean-field games have long been considered, see [26].

\(^1\)RED [40] was intended to anticipate congestion and desynchronize TCP backoff actions.

\(^2\)i.e., without steady-state estimates of certain quantities.
as a recent reference. For example, altruism has been modeled as a user’s *statically* personalized weight on the utility of others in games of: network formation [8], packet forwarding in delay tolerant networks [27], routing [10], and medium access control by us in [33]. See also [18], [19], [24] for examples assuming a fixed set of altruism parameters that characterize each user or pair of users.

In the following, we consider a fictitious play model where altruism by one user is based on perceived mean throughput of the other players modulated (i.e., made “dynamic”) by factoring the estimated mean total channel idle time [23]. Large idle time may be a signal that competing devices are also behaving in a socially sensitive manner, expressing the current “social norm,” or it could simply reflect low traffic demand. So, we model the altruistic motivations of a user in a simpler framework of heterogeneous users in which the user will employ less than her “fair” share when she doesn’t really need it, but under the constraint that others do the same. The resulting equilibria that such altruistic devices could reach are studied in terms of stability and under assumptions of asynchronous/multi-rate user/players. Finally, we do not assume that the users share information and act in a coordinated fashion, i.e., so as to form a player coalition.

Note that our system with heterogeneous users will respond similarly to a selfish user with low throughput demand and a more altruistic one with high throughput demand. Moreover, excessive altruism will simply result in an underused channel. To address this, a possible improvement would be to include a measure on the expected congestion levels based on the number of users sharing the same channel and to limit the altruism factor in end-user utilities so that channel underuse does not result. We leave these issues, as well as applications to other MAC scenarios not considered here, to future work.

This paper is outlined as follows. In Section II we give a brief background on altruistic behavior. A fictitious-play model with dynamic altruism for a slotted-ALOHA LAN is given in Section III. In Section IV, variations of the LAN model are considered. Numerical studies are given in Sections V and VI, the latter considers player diversity. A discussion of TCP flow control and congestion avoidance, in the context of player altruism, is given in Section VII. Finally, in Section VIII we conclude with a summary and discussion of future work.
II. BACKGROUND ON ALTRUISTIC BEHAVIOR

Economists are often criticized for the common assumption that humans are “rational” (i.e., purely self-interested), which leads to a pessimistic view of the outcome of various formulated game-theoretic models. In reality, many people act “altruistically”, defined as an “unselfish concern for or devotion to the welfare of others” [5]. In fact, despite this selfishness stereotype for economics, certain branches, such as behavioral economics and experimental economics, do incorporate social, cognitive, and psychological factors in their models of human behavior (see [17] for a historical overview), in a way not typically captured in cooperative game-theoretic frameworks. Especially since the early 1980s, there is a very rich and constantly growing literature on experimental economics, whose ambitious objective is to devise (in particular, rational) economic models that express accurately the expected altruistic behavior of humans in certain settings through the analysis of experimental data, e.g., [34], [36].

Two common scenarios in which altruistic behavior consistently appears is in ultimatum bargaining and public-good contribution games. Ultimatum bargaining reveals the altruistic instincts of humans in a resource-sharing problem in which one player decides how to share a fixed amount of money and the other decides whether to accept or reject sharing: here rejection leaves both with zero profit. Experiments show that people altruistically sacrifice their own profit to punish unfair’ decisions by others. Analysis of more traditional public-good experiments, where players determine their individual contribution toward the construction of a pure public good, similarly challenges the assumption that free riding is always the dominant strategy.

Of course, it is well accepted that many factors can affect human behavior, from the nature of the game itself to small details of the experiment implementation. Nevertheless, there are numerous interesting building blocks of human motivation that have been identified in the process and can help us build more realistic economic models. For example, Andreoni [7] shows that there are two different fundamental ways to capture altruism in a utility model based on whether the source of “irrational” satisfaction is the utility of the other users (satisfaction from the benefit of others), or one’s own contribution (satisfaction from being good).

An important lesson of experimental economics is that altruism does not seem to be a static and hardwired characteristic of humans but depends on many aspects of the environment. In other words, the level of altruism of an individual is dynamic and could change over time depending
on the context and the behavior of the group. Indeed, the cooperation rate in many experiments has been proven to be much higher if subjects know that there is a possibility of meeting the same partners again in future periods [21], when their perception on the overall level of altruism in their group is high [14], or even just by a positive framing of the experiment [6].

From these and many other contextual factors that can affect the cooperation levels in a group, social norms is perhaps the most influential (see [13], [45]) but complex to incorporate in a simple economic model. To this end, Fehr and Schmidt [22] have proposed an utility function explaining the altruistic behavior of people in ultimatum experiments, which incorporates a measure of fairness (or “inequity aversion”) in a static way, i.e., its main parameters are indifferent to the dynamics of the system. As a more realistic but less tractable alternative, H. Margolis argues in favor of a more dynamic and complex model, called “neither selfish nor exploited” [39], which proposes a dual utility model which takes into account the history of one’s actions, the current overall behavior, the effect of altruistic action, and the developed norms in a society.

In our scenario, the high complexity of human nature and the surrounding social environment plays a less important role since the cooperation game that we study is limited in time, the identity of the players are hidden, the stakes are relatively low, and the decisions of users are mediated through a programmed device. So, in our model of the following section we incorporate in a simple utility function the effect of the external manifestation of altruistic behavior, that is a statistical norm as termed in [39], or simply “what others do” [14]. To perceive this, the availability of reliable information about the group’s statistical behavior is critical. Our use of the mean idle time per active player to determine the level of altruism in the system is realistic in terms of information availability since it can be easily measured by the different users, though, again, low demand could be mistakenly taken for altruistic behavior and vice versa, cf. Section VIII.

When altruism can bring future concrete benefits, one could also see altruistic behavior as a long-term net utility optimization. A characteristic example is the notion of direct and indirect reciprocity and the related work in evolutionary game theory that tries to explain the source of cooperative behavior in nature [9], [44]. We leave to future work the study of such evolutionary extensions of the LAN systems we formulate below, again cf. Section VIII.

Finally, some networking mechanisms presume cooperation “by rule” to affect communal benefit, such as flow and congestion control. Notwithstanding a tendency to altruism in the
typical user, these mechanisms may be easily exploited by unpoliced, greedy individuals, cf. the discussion of Section VII.

III. SLOTTED-ALOHA RANDOM-ACCESS LAN WITH DYNAMIC ALTRUISM

A. Altruistic framework for with power based cost and concave utility of throughput

Consider a slotted ALOHA random-access LAN wherein the $N \geq 2$ participating nodes control their access probability parameter, $q$. A basic assumption is that nodes’ control actions are based on observations in steady-state, i.e., “fictitious play” [15], resulting in a quasi-stationary dynamical system [28], [29], [51] based on the mean throughputs:

$$\gamma_i(q) = q_i \prod_{j \neq i} (1 - q_j).$$

Another basic assumption in the following is that the source of a successful transmission is evident to all other participating nodes. We further assume that the degree of altruism $\alpha_i$ of each node $i$ depends on the activity of the other users as:

$$\alpha_i(q_{-i}) = \prod_{j \neq i} (1 - q_j) = \frac{\gamma_i(q)}{q_i} = \gamma_i(q) + \prod_{j} (1 - q_j),$$

where the second term is just the mean idle time of the channel; thus, every node can easily estimate its (dynamic) altruism. By using its control action (strategy), $q_i$, each $i$ seeks to maximize its net utility:

$$V_i(q) = C_i \log(\gamma_i(q)) + A_i \alpha_i(q_{-i}) \overline{\gamma_{-i}}(q) - M_i q_i \quad (1)$$

where: the dynamic altruism factor $\alpha$ modulates the contribution of the mean service of all other players to the net utility of player $i$,

$$\overline{\gamma_{-i}}(q) = \frac{1}{N - 1} \sum_{j \neq i} \gamma_j(q); \quad (2)$$

the utility derived by one’s own throughput is modulated by a concave function [28]–[30] as modeled here in the form of a logarithm (for tractability); and we have assumed a power based cost $Mq$. Note that because we assume that the source of each successfully transmitted packet is

$^3$Power based costs are borne whether or not the transmission is successful.
evident to all nodes, each node $i$ can easily estimate $\bar{\gamma}_i$. Again, though each player $i$ optimizes $V_i$ in a non-cooperative fashion, the game is called altruistic to reflect the second term in (1).

A single-play slotted ALOHA game between two identical players is similar to the game chicken. If $\xi < 1$ is the cost of transmission and the (normalized) payoff of successful transmission is 1, then the following table gives net payoffs for collective action (transmit (Tx) or not) by the players (P1,P2):

|       | P1      |       |       |
|-------|---------|-------|-------|
|       | Tx      | no Tx |       |
| P2    |         |       |       |
| Tx    | $(-\xi, -\xi)$ | $(0, 1 - \xi)$ |       |
| no Tx | $(1 - \xi, 0)$ | $(0, 0)$ |       |

The single-play game has three Nash equilibria: two “pure” strategies, (Tx,no-Tx) and (no-Tx,Tx), and one mixed strategy: Tx with probability $q^*$ (and don’t Tx with probability $1 - q^*$), where $q^* = 1 - \xi$ jointly minimizes the expected net gains, $(1 - \xi)q_k(1 - q_{3-k}) - \xi q_k q_{3-k}$, of players $k \in \{1, 2\}$.

In the following, we consider an iterated version of this game where players pursue mixed strategies based on observations of throughput $\gamma_i$ observed in steady-state. Note that if we further assume that nodes are aware of the $C, M$ parameters of other nodes, then we can replace $\bar{\gamma}$ with the net utility of the other players as in [33] (particularly for throughput based costs $M\gamma$).

**Proposition 3.1:** If the game is synchronous-play and all users $i$ have the same (normalized) parameters

$$c := C_i/M_i < 1 \quad \text{and} \quad a := A_i/M_i,$$

then there is a symmetric Nash equilibrium $q^* = q^*1$, where $0 < q^* < 1$ is a solution to

$$f(q) := aq^2(1 - q)^2 + q - c = 0. \quad (3)$$

**Proof:** When $q_i = q$ for all $i$, the first-order necessary conditions of a symmetric Nash
equilibrium,

\[ 0 = \frac{\partial V_i}{\partial q_i}(q^1) = -\frac{M}{q} f(q), \]

i.e., equivalent to (3). Note that \( f(0) = -c < 0 \) and \( f(1) = 1 - c > 0 \), the latter by hypothesis. So, by the continuity of \( f \) and the intermediate value theorem, a root of \( f \) exists in \((0, 1)\).

All such solutions \( q^*1 \) correspond to Nash equilibria because \( \partial^2 V_i(q)/\partial q_i^2 = -C_i/q_i^2 < 0 \) for all \( i,q \).

The following corollary is immediate.

**Corollary 3.1:** There is a unique symmetric Nash equilibrium point (NEP) if \( \min_{q \in (0,1)} f'(q) > 0 \) (i.e., \( f \) is strictly increasing), a condition on parameters \( N \) and \( a \).

Note that there may be non-symmetric Nash equilibria in these games, even for the case of homogeneous users, e.g., [32]. Also, it is well known that Nash equilibria of iterative games are not necessarily asymptotically stable, e.g., [2], [49], [53]. In [28], [29] for a slotted-ALOHA game with throughput based costs \( M\gamma \), using a Lyapunov function for arbitrary \( N \geq 2 \) players, a non-cooperative two-player ALOHA was shown to have two different interior\(^4\) Nash equilibria only one of which was locally asymptotically stable (see also [41]).

For stability analysis of our altruistic game, consider the discrete-time \((n)\), synchronous-play gradient-ascent dynamics,

\[ q_i(n) = \arg \max_{q_i} V_i(q_i; q_{-i}(n-1)) \forall i. \quad (4) \]

In a distributed system\(^5\) the corresponding continuous-time Jacobi iteration approximation is:

\[ \dot{q}_i(t) = \frac{\partial V_i}{\partial q_i}(q(t)) \forall i, \quad (5) \]

and is motivated when players take small steps toward their currently optimal response, i.e., better-response dynamics [50]. That is, for positive step-size \( \varepsilon \ll 1 \) (5) approximates the discrete-time

\(^4\) i.e., not including the stable boundary deadlock equilibrium at \( q = 1 \).

\(^5\) cf. Section IV-C for a discussion of asynchronous play.
better-response dynamics,
\[
q_i(n) = q_i(n-1) + \varepsilon \frac{\partial V_i}{\partial q_i}(q(n)) \quad \forall i,
\]  
(6)
which is a kind of distributed gradient ascent. The Jacobi iteration is also motivated by the desire to take small steps to avoid regions of attraction of undesirable boundary NEPs, particularly those corresponding to the capture strategy \(q_i = 1\) for some \(i\). Note that when more than one player selects this strategy, the result is a bad outcome for the game \textit{chicken} or a deadlocked “tragedy of the commons.” Additionally the players avoid the opt-out strategy \(q_i = 0\) for some \(i\). In summary, (6) represents a repeated game in which players adjust their transmission parameters \(q_i\) to (locally) maximize their net utility \(V_i\).

To find conditions on the parameters of net utilities (1) for local stability of the equilibria, we can apply the Hartman-Grobman theorem [46] to (5), i.e., check that the Jacobian is negative definite. The following proposition uses the conditions of [48] for stability (and uniqueness) for a special case.

\textit{Proposition 3.2:} In the case where players have the same parameters \(C\) and \(A\), the symmetric NEP \(q^*\) is locally asymptotically stable under the dynamics in (5) when the normalized parameters satisfy
\[
C > 2(N-1)A.
\]  
(7)
\textit{Proof:} By [48], the result follows if the symmetric \(N \times N\) matrix \(H(q)\) is negative definite, where
\[
H_{ij} = \frac{\partial^2 V_i}{\partial q_i \partial q_j} + \frac{\partial^2 V_j}{\partial q_j \partial q_i}.
\]
First note that, for all \(i\),
\[
H_{ii}(q) = -C \frac{q_i^2}{q_i^2} < -C.
\]
For \( l \neq i \),
\[
\frac{\partial^2 V_i}{\partial q_i \partial q_l} = \frac{\partial}{\partial q_l} \left( \frac{C}{q_i} - A\alpha(q_{-i}) \frac{1}{N-1} \sum_{j \neq i} q_j \prod_{k \neq i,j} (1 - q_k) \right)
= A \prod_{j \neq i,l} (1 - q_j) \frac{1}{N-1} \sum_{j \neq i} q_j \prod_{k \neq i,j} (1 - q_k)
+ A\alpha(q_{-i}) \frac{1}{N-1} \left( \sum_{j \neq i,l} q_j \prod_{k \neq i,j,l} (1 - q_k)
- \prod_{k \neq i,l} (1 - q_k) \right).
\]

Now because \( 0 < q_i < 1 \) for all \( i \) and the triangle inequality,
\[
|H_{ij}(q)| \leq 2A \quad \forall j \neq i.
\]

So, by the Gershgorin circle (disc) theorem (see p. 344 of [25]), all of \( H(q) \)'s eigenvalues are less than \(-C + (N - 1)2A\). So, if (7) holds, then all the eigenvalues of \( H(q) \) are negative, and so \( H(q) \) is negative definite.

\[\blacksquare\]

**B. The marginal effect of altruism**

In this section, we will write \( q^* \) (of the symmetric NEP \( q^* \) in symmetric users case) as a function of the normalized altruism parameter \( a := A/M, q^*(a) \). Note that \( q^*(0) = c := C/M \).

Recall that the total throughput for slotted ALOHA, \( Nc(1-c)^{N-1} \), is maximal when \( c = 1/N \).

The maximum total throughput is \( (1-1/N)^{N-1} \approx e^{-1} \) for large \( N \), i.e., the maximum throughput per player is \( 1/(Ne) \) in this cooperative setting without networking costs.

So, if \( c > 1/N \), i.e., total throughput is less than \( e^{-1} \) because of excessive demand (overloaded system), then a marginal increase in altruism from zero \( (0 < a \ll 1) \) will cause a marginal decrease in \( q^* \downarrow 1/N \), resulting in an increase in throughput per user \( \gamma \uparrow 1/(Ne) \). Also, if \( c < 1/N \), i.e., total throughput is less than \( e^{-1} \) because of a lack of demand (an underloaded system), then a marginal increase in altruism from zero will again cause a marginal decrease in \( q^* \), but here resulting in a decrease in throughput \( \gamma \) (further away from the optimum \( e^{-1} \)). See Section V-D below.
IV. MODEL VARIATIONS

In this section, we discuss model variations, which we subsequently analyze. An analytically straightforward variation is to simply use \((N-1)\overline{\gamma}\) (i.e., just mean total channel idle-time), instead of \(\gamma\) given by (2), thus not requiring an estimate of the the number of active users, \(N\). However, the mean idle-time per active user \(\overline{\gamma}\) better captures the current levels of altruistic behavior in the system and is more consistent with ideas of inequity aversion [22]. So we will not explore this simple variation further here as, again, we are herein neither interested in maximizing social welfare nor related “efficiency” issues captured by a global criterion (accommodating users with very different magnitudes of demand in a typically additive way); rather, we are interested in the effect of altruism on distributed, non-cooperative network-access games. More ambitious model variations than those discussed in this section are mentioned in the concluding Section VIII.

Note how our model of altruism leads to neither selflessness nor full cooperation, but is closer to a (rationally) selfish model, i.e., again as H. Margolis characterized it, neither selfish nor exploited [39]. Also in our model, altruism needs to accommodate the limited and potentially unreliable information of a distributed network.

A. Throughput based costs

In [33] we considered throughput based costs with a static altruism parameter and with utility proportional to throughput. Instead of (1), for throughput based costs with dynamic altruism and utility being a concave (log) function of throughput, we can model the net utility as:

\[
\tilde{V}_i(q) = C_i \log(\gamma_i(q)) + A_i \alpha_i(q_{-i}) \overline{\gamma}_{-i}(q) - M_i \gamma_i(q) - M_i \gamma_i(q).
\] (8)

Proposition 3.1 can easily be adapted for power based costs. Instead of (3), the first-order necessary condition for a symmetric Nash equilibrium \(q_1\) under throughput based cost is

\[
\tilde{f}(q) := a q_2 (1-q)^{2N-3} + q (1-q)^{N-1} - c = 0.
\] (9)

All solutions \(q\) for (9) correspond to NEPs \(q_1\) because \(\partial^2 \tilde{V}_i(q)/\partial q_i^2 = -C_i/q_i^2 < 0\) for all \(i, q\) (as for power based cost). Note that \(\tilde{f}(0) = \tilde{f}(1) = -c < 0\), so we cannot simply use the intermediate value theorem as we did for Proposition 3.1 to establish existence of a symmetric Nash equilibrium when \(c < 1\). Here, existence requires

\[
\max_{0 < q < 1} \tilde{f}(q) \geq 0,
\] (10)
a condition on $N, c, a$. Note that if the inequality in (10) strictly holds then there will be an even number of symmetric NEPs, again by the intermediate value theorem. If the maximum equals zero then there may be a unique symmetric NEP.

B. Proportional throughput utility

Suppose that utility is simply proportional to throughput and cost is power based, i.e., the net utility is

$$
\hat{V}_i(q) = C_i \gamma_i(q) + A_i \alpha_i(q_{-i}) \gamma_{-i}(q) - M_i q_i. \tag{11}
$$

Note that the net utility $\hat{V}_i$ is linear in $q_i$ (this would also be the case if throughput based costs were involved). This normally leads to candidate “bang-bang” Nash equilibrium play-actions, $q_i \in \{0, 1\}$ for all players $i$; i.e., the players are either out of the game ($q_i = 0$ if $\partial \hat{V}_i / \partial q_i < 0$) or are all in ($q_i = 1$ if $\partial \hat{V}_i / \partial q_i > 0$). Note that the latter play action, potentially leading to the deadlock of “tragedy of the commons”, is not an equilibrium here because if $q_j = 1$ then $\partial \hat{V}_i / \partial q_i = -M < 0$ for all $i \neq j$.

It turns out that for this case, there is a symmetric interior equilibrium $q^1$ for the identical players case with $0 < q < 1$, i.e., where

$$
\hat{f}(q) := \frac{\partial \hat{V}_i}{\partial q_i}(q^1) = c(1-q)^{N-1} - aq(1-q)^{2N-3} - 1 = 0. \tag{12}
$$

If $c > 1$, $\hat{f}(0) = c - 1 > 0$ and $\hat{f}(1) = -1 < 0$ and so there is a solution to $\hat{f}(q) = 0$ for $0 < q < 1$ by the intermediate value theorem. It should be noted, however, that such an interior Nash equilibrium $q^1$ is not stable, i.e., it’s a saddle point in the domain $[0, 1]^N$.

C. Asynchronous/Multirate Players

Asynchronous players were considered previously in [30] using the ideas from [11], [12]. A very similar approach can be used to extend the results herein to account for the effects of asynchronous play. Numerical results for this case are given in Section VI-B below.
V. NUMERICAL STUDIES FOR IDENTICAL PLAYERS AT NASH EQUILIBRIUM

A. Power based costs

For normalized utility parameter $c = 0.5$ and normalized altruism parameter $a = 1$, Figure 1(a) is a plot of $f$ in (3); i.e., for power based costs, for $N = 2, 3, 5, 10$ players. The root at $q = 0.4$ corresponds to $N = 2$ (i.e., corresponding to NEP $q_1$) and, as the first term of $f$ becomes negligible, the root at $\approx 0.5$ corresponds to the $N > 2$ cases. For $c = 0.5$ and $N = 5$, Figure 1(b) is a plot of $f$ for $a = 0.1, 1, 10, 100$. Note that $a = 100$ corresponds to the larger curve which has a the smaller root $q$, i.e., under “excessive” altruism the NEP $q \to 0$.

Fig. 1. Power based costs
B. Throughput based costs

Using the same parameter cases as those for power based costs, Figure 2(a) is a plot of $\tilde{f}$ in (9) for $c = 0.5$, $a = 1$ and $N = 2, 3, 5, 10$. Figure 2(b) is a plot of $\tilde{f}$ in (9) again for $c = 0.5$, $N = 5$ and $a = 0.1, 1, 10, 100$. The larger curve, corresponding to $a = 100$, has two zero-crossings $q$ at approximately 0.1 and 0.4, i.e., has two different symmetric NEPs $q_1$. The other parameter sets do not possess an interior NEP, a situation which will be remedied if we reduce the utility $c$ from 0.5 to zero; that is, increasing $\tilde{f}$.

Fig. 2. Throughput based costs

C. Throughput-proportional utilities and costs

Figure 3(a) is a plot of $\hat{f}$ in (12) for $c = 2$, $a = 1$ and $N = 2, 3, 5, 10$. Figure 3(b) is a plot of $\hat{f}$ for $c = 2$, $N = 5$ and $a = 0.1, 1, 10, 100$. Following intuition, the lower curves (and lower
roots, NEPs) correspond to larger $N$ (larger congestion leading to lower throughput) or larger $a$ (greater altruism again leading to lower throughput), in a monotonic fashion when all other parameters fixed.

![Graph](image)

(a) Ranging $N$

![Graph](image)

(b) Ranging $a$

**Fig. 3.** Throughput proportional utility and cost

**D. An example comparing altruism and non-cooperation**

In this section, we compare the Nash equilibria under altruistic player action with equilibria in purely non-cooperative scenarios. For all scenarios, we considered the case of power based costs, log-utility of throughput, normalized utility parameter $c = 0.5$, and identical users. For the purely non-cooperative scenario, i.e., $a = 0$, the symmetric Nash equilibrium $q = c = 0.5$ is simply obtained by solving (3). For the scenarios with altruism, the normalized altruism parameter was taken to be $a = 20$. Recall that for static altruism, $\alpha \equiv 1$. At Nash equilibrium $q^* = q^* \frac{1}{2}$, the
throughput \((\gamma^* = q^*(1 - q^*)^{N-1})\) and utility \((1)\) performance per user is given in the following table, in decreasing order of throughput.

| Scenario            | \(N\) | \(q^*\) | \(\gamma^*\) | \(V^*/M\) |
|---------------------|-------|---------|---------------|-----------|
| Dynamic Altruism    | 4     | .22     | .1044         | -0.36     |
| Static Altruism     | 4     | .16     | .0935         | 0.53      |
| Non-cooperative     | 4     | .50     | .0625         | -1.89     |
| Static Altruism     | 8     | .28     | .0277         | -1.52     |
| Dynamic Altruism    | 8     | .50     | .0039         | -3.27     |
| Non-cooperative     | 8     | .50     | .0039         | -3.27     |

Note that for \(N = 8\), the altruistic component of utility at Nash equilibrium is negligible, when comparing dynamic altruism versus non-cooperation, owing to high contention under the assumed parameters when there are eight users. We see that the non-cooperative scenario has poorest throughput performance in the above examples. However, if the level of altruism is too high, under either the static or dynamics mechanisms, the channel may be underused; in this case, the altruism parameters could be adjusted via an “evolutionary” process to avoid channel underuse.

VI. NUMERICAL STUDIES WITH PLAYER DIVERSITY

A. Players with different altruism parameters

Consider the game with power based costs. In this section, we consider players with different normalized altruism parameters \(a\) for \(N = 3\) otherwise identical players with normalized parameter \(c = 0.5\) associated with power-based cost. Specifically, the first player has \(a_1\) ranging from 30 to 70, while the other two players both have \(a = 50\). Note that changing \(a\) in this manner will result in changes in the NEP \(q^*\) and corresponding throughputs \(\gamma^*\) (and utilities \(V^*\) per user, as shown in the following table):

| \(a_1\) | \(q_1^*, q_2^* = q_3^*\) | \(\gamma_1^*, \gamma_2^* = \gamma_3^*\) | \(V_1^*, V_2^* = V_3^*\) |
|---------|--------------------------|--------------------------|--------------------------|
| 30      | 0.15, 0.10               | 0.13, 0.074              | 0.754, 2.37              |
| 40      | 0.12, 0.10               | 0.10, 0.080              | 1.40, 2.24               |
| 50      | 0.10, 0.10               | 0.083, 0.083             | 2.10, 2.10               |
| 60      | 0.091, 0.11              | 0.073, 0.087             | 2.79, 1.83               |
| 70      | 0.079, 0.11              | 0.063, 0.090             | 3.56, 1.82               |
Clearly, increased altruism, \(a_1 > 50\), by player 1 resulted in lower throughput for him and higher throughput for the other two players. Similarly, decreased altruism by player 1, \(a_1 < 50\), resulted in higher throughput for him and lower throughput for the other players.

**B. Sizes of regions of attractions under different play-rates**

In this section, we study how the volume of the regions of attractions of different equilibria are sensitive to players adopting different play-rates, while retaining our assumption of fictitious/quasistationary play. Consider the case of \(N = 3\) players two of whom have the same play rate while the other adopts a play rate that is a multiple, \(r\), of the other two. We consider the case of throughput based costs as in Figure 2(b). That is,

\[
\dot{q}_i(t) = r_i \frac{\partial \tilde{V}_i}{\partial q_i}(q(t)) \quad \forall i,
\]

where \(r_i = r\) for player \(i = 1\), otherwise \(r_i = 1\) and \(\tilde{V}\) is given in Section IV-A. Numerically simulating (13) from different initial points chosen from a grid in the hypercube \([0, 1]^3\), we counted the number of initial points converging to a given NEP so as to estimate the volume of its region of attraction. Note that the introduction of such play-rate parameters \(r_i\) does not change the position of the NEPs. Using normalized parameters \(a = 50\) and \(c = 0.5\), the function \(\tilde{f}\) whose roots are the NEPs is depicted in Figure 4. As can be seen from the following table, the region of attraction is very sensitive to \(r\) in the range 0.1 to 10.

| Volume | \(\text{NEP} = (0.1)_1\) | \(\text{NEP} = (0.75)_1\) |
|--------|-----------------|-----------------|
| \(r = 0.1\) | 0.502           | 0.498           |
| \(r = 0.25\) | 0.507           | 0.493           |
| \(r = 1\)    | 0.556           | 0.444           |
| \(r = 4\)    | 0.839           | 0.161           |
| \(r = 10\)   | 0.841           | 0.159           |

The results are intuitive: a lower \(r\) effectively corresponds to a reluctance to be altruistic and thereby results in a smaller domain of attraction for the more altruistic Pareto equilibrium \((0.1)_1\) (corresponding to throughputs of \(\gamma = 0.081\) and utilities \(V = 1.94\), respectively compared to \(\gamma = 0.047\) and \(V = -1.43\) corresponding to \((0.75)_1\)).
VII. DISCUSSION: END-TO-END TCP FLOW AND CONGESTION CONTROL

Interesting game theoretic models for end-to-end window flow control (as in TCP), based on internal bandwidth bottlenecks have been extensively studied, see, e.g., [1], [43], including modifying backoff parameters. Player diversity depends in part on differences in round-trip times (RTTs) which correspond to responsiveness – players with smallest RTT to a congested bottleneck who follow traditional distributed TCP congestion control will be the most altruistic in that they will back-off first. The equilibrium is a “water-filling” where the player with largest RTT and demand $D_1$ receives $d_1 = \min\{D_1, C\}$, where $C$ is the bottleneck capacity. The player with second largest RTT and demand $D_2$ receives $d_2 = \min\{D_2, \max\{C - D_1, 0\}\}$, and similarly $d_3 = \min\{D_3, \max\{C - D_1 - D_2, 0\}\}$, etc.

Players can reduce the level of their altruism by artificially delaying their response to congestion (i.e., beyond the minimal response time governed by their RTT), in the limit being completely nonresponsive to congestion. Though such manipulation may be more straightforward for end-users or individual applications compared to modifying congestion backoff in layer-2 MAC protocols, a difficulty here is that there may be little knowledge about the behavior of other players (in this highly distributed setting) to form the basis of altruistic action; indeed, it is likely that many players/sessions are only minimally responsive to congestion, e.g., streaming media over UDP running RTCP and p2p file-sharing using utorrent clients.

$^6$Typically with associated memory to buffer packets in times of congestion.
VIII. SUMMARY AND FUTURE WORK

In this paper, we extended a non-cooperative game framework for information-limited MAC of a LAN by adding an altruism term that depended on both the mean throughput of the other players and the mean channel idle time. The cases of heterogeneous or homogeneous users, and of power or throughput based costs, were considered for a quasi-stationary model of the game. A numerical study compares the per-user throughput under dynamic and static altruism with that of purely non-cooperative dynamics, and demonstrates the advantage of altruism under moderate levels of congestion (number of players) in the homogeneous-player setting, and for a heterogeneous user scenario.

In the future, we plan to depart from ideal quasi-stationary dynamics and consider the effects of measurement error (as in \cite{44}, \cite{41}), leading to a more “stochastic” version of the games considered here. We also plan to extend our study of dynamic altruism to the simple power-control based medium access considered in \cite{33} for static altruism.

Finally, we will consider an evolutionary “wrapper” about the dynamics considered here or other factors affecting the parameters of the net utilities \cite{1}, e.g., the desire to avoid under-utilization of the channel or to account for multiple-priority transmission. Indeed, motivated by the prospects of significant reward, defections by a few individual players may cause an evolutionary/slow-cycle of transitions between full cooperation (limited by information availability and associated costs), to complete non-cooperation, via “intermediate” altruistic behavior \cite{44}. So, the altruism considered herein may, in principle, be rationally motivated based on long-term reward considerations.

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