What is the optimum stellar rotation rate for a collapsar?

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Summary. — We consider low angular momentum, neutrino cooled accretion flows onto newborn black holes in the context of the collapsar model for long Gamma Ray Bursts, and find a considerable energy release for rotation rates lower than those usually considered. The efficiency for the transformation of gravitational binding energy into radiation is maximized when the equatorial angular momentum $l_0 \simeq 2 r_g c$.

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1. – Introduction

The collapsar model for GRBs [1, 2] has proved quite successful for interpreting the association of long GRBs with star forming regions and with observed SNe at low redshift. It is not entirely clear, however, that stellar cores (or which fraction of them) will actually lead to the production of a successful GRB at the end of their evolution, since known mechanisms (mass loss, magnetic fields) act to spin down the inner regions of the star in various ways [3]. We consider here the outcome of accretion onto a newborn black hole following core collapse in the case where the stellar rotation rate is low by the standards usually considered in collapsar calculations.

2. – Ballistics

It is an obvious, but sometimes overlooked fact, that in Newtonian theory, a test particle in orbit about a massive point body can be placed in a stable circular orbit for any given value of its orbital angular momentum. The condition gives the circularization radius as

$$r_c = \frac{\ell^2}{GM_*},$$

where $\ell$ is the specific angular momentum and $M_*$ is the mass of the central body. The orbit is stable precisely because $\ell(r)$ is a monotonically increasing function of radius. A
For a cloud rotating as a rigid body, where the specific angular momentum varies with the polar angle as $\ell = l_0 \sin^2 \theta$, three possible types of trajectories are shown: (a) direct accretion onto the central black hole; (b) impact with a symmetrical flowline from the opposite hemisphere (without which direct accretion onto the black hole would also occur); (c) encounter with the centrifugal barrier and settling at the circularization radius in the presence of dissipation.

Perturbation conserving angular momentum will merely induce small, epicyclic oscillations about the equilibrium radius $r_c$.

In general relativity (GR) this no longer holds, and a qualitatively different effect appears, allowing for the presence of capture orbits even at a finite value of $\ell$. This is the unique “pit-in-the-potential” feature of GR, dictating that at low enough values of $\ell(r)$, one need not lose angular momentum in order to fall onto the central object. In terms of stability, the equilibrium angular momentum, $J_{eq}(r)$, for test particle motion exhibits a minimum at a particular radius, $r_{ms}$, termed the marginally stable orbit. For $r < r_{ms}$, stable circular orbits are no longer possible since $dJ_{eq}(r)/dr < 0$.

The location of the marginally stable orbit is a function of the spin of the central object (a black hole if we are to consider a relativistic scenario following the collapse of an iron core inside a massive star), but is in all cases a modest number of gravitational radii, $r_g = 2GM/c^2$. In particular, $r_{ms} = 3r_g; 3/2r_g$ for a non–spinning (Schwarzschild) and maximally spinning (Kerr) hole, respectively. Thus for this effect to have any impact upon the evolution of infalling test masses, we must consider a situation in which the circularization radius and that of the marginally stable orbit are comparable, $r_c \approx r_{ms}$. This immediately translates into the condition

$$\ell = f \times (r_g c),$$

where $f$ is a factor of order unity.
A meridional projection of flowlines for infalling particles in rigid (slow) rotation around a black hole is shown in Figure 1. Given the rotation law, polar inflow is entirely radial, while along the equator the centrifugal barrier is felt most importantly. It is evident as well that if we consider the behavior of an accreting gas cloud, purely ballistic considerations will not tell us the entire story, since the flow lines at small radii and in the plane of the equator clearly cross. A shock may thus form, and a hydrodynamical study is necessary if one wishes to proceed any further.

The general context here can be thought of as GRBs, but it has various astrophysical applications, namely (and originally to our knowledge) to the case of wind–fed neutron stars in High–Mass X-Ray Binaries [4].

3. – Hydrodynamics

We have thus carried out a series of two–dimensional simulations of low angular momentum accretion onto black holes, with some simplifications and assumptions keeping collapsars in mind.

First, we do not perform fully GR calculations but use instead the potential proposed by Paczyński & Wiita [5], \( \Phi_{PW} = -GM_\star/(r - r_g) \). This reproduces the position of the marginally stable orbit correctly for a Schwarzschild hole, albeit at a slightly different value of the specific orbital angular momentum. Second, we assume azimuthal symmetry in order to compute a decent number of dynamical times in the inner regions of the flow (several hundred). Third, we assume an initially free-falling flow in rigid body rotation, parametrized by the equatorial angular momentum, \( l_0 \), and the accretion rate, \( \dot{M} \). Finally, we make use of fairly detailed thermodynamics, incorporating an ideal gas of free nucleons and α particles, \( e^\pm \) pairs and photons and neutrino cooling (due to pair captures by free nucleons and \( e^\pm \) annihilation), while using as well the viscosity prescription of Shakura & Sunyaev [6] to allow for angular momentum transport. We refer the reader to [7] for further details of the implementation.

For low angular momentum, \( l_0 < 2r_g c \), the resulting configuration is completely independent of the strength of the viscosity, and we see the formation of a dwarf disk in the equatorial plane. This material accretes simply because it does not have enough angular momentum to remain in orbit around the black hole (it does not have a circularization radius). For high angular momentum \( l_0 > 2r_g c \), the outcome depends somewhat on the actual value of \( \alpha \), but is generically different in the sense that the dwarf disk is replaced by a hot, shock–heated toroidal bubble that surrounds the black hole. The qualitative nature of this transition is independent of the assumed mass accretion rate (we explored values between \( 10^{-3} \) and \( 0.5M_\odot s^{-1} \)), and we show in Figure 2 representative snapshots for the two classes of solutions.

4. – Maximizing the power output

A common concern when addressing the issue of progenitors for long GRBs has always been their rotation rate [3,9,8]. If the stellar core is not endowed with sufficient angular momentum, mostly radial inflow will ensue, and this is not a favorable situation in terms of the potential energy release. On the other hand, a problem that is not so frequently addressed is that if too much angular momentum is present, the circularization radius of the gas may be too large to produce an efficient neutrino cooled accretion flow [10]. A large fraction of the energy may then simply produce outflows not directly emanating from the central region and thus not capable of producing the required power output.
Fig. 2. – Accretion flow morphologies for low (left) and high (right) specific angular momentum calculations (in units of $r_g c$) without the effects of viscosity ($\alpha = 0$). The dwarf disk in near free fall and the large toroidal bubble are clearly seen in either case. The axes are labeled in units of $r_g$, with density contours and the velocity field shown in each case.

The maximum power is actually obtained by first lowering the gas as much as possible in the gravitational potential well and simultaneously shocking it to dissipate as much of its energy as possible, which can then be extracted, possibly to produce a GRB. We argue here that this may occur precisely at the transition when the flow morphology changes from that of a hot torus to a thin, efficiently cooled dwarf disk, when $l_0 \approx 2 r_g c$, or $5 \times 10^{16} \text{cm}^2 \text{s}^{-1}$ for a hole of $3M_\odot$. The efficiency is maximized in two different ways: on one hand, the raw neutrino luminosity, $L_\nu$, is highest close to this threshold, and on the other, the most energetic neutrinos are simultaneously produced, thus giving a relatively high efficiency for annihilation.

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