Probing AdS$_3$/CFT correspondence via world-sheet methods and 2d gravity like scaling arguments

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Abstract

We show how some features of the AdS/CFT correspondence for AdS$_3$ can easily be understood via standard world-sheet methods and 2d gravity like scaling arguments. To do this, we propose a stringy way for perturbing two-dimensional CFT’s around their critical points. Our strategy is to start from a stringy (world-sheet) representation of 2d CFT in space-time. Next we perturb a world-sheet action by some marginal operators such that the space-time symmetry becomes finite dimensional. As a result, we get a massive FT in space-time with a scale provided by two-dimensional coupling constant. It turns out that there exists a perturbation that leads to string theory on AdS$_3$. In this case the scale is equivalently provided by the radial anti-de-Sitter coordinate.

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1 Introduction and Review

Relations between gauge fields and strings present an old, fascinating and unanswered question (see, e.g. [1]). In the last two years there has been much progress in understanding some sort of holographic correspondence, the so-called AdS/CFT correspondence, between superconformal Yang-Mills theory and supergravity or string theory on anti-de-Sitter spaces AdS$_{d+1}$ (for a review and refs., see, [2]). Equivalence between theories in different dimensions raises questions about how detailed bulk information in one theory can be completely coded in lower dimensional degrees of freedom. Despite the large amount of evidence for the AdS/CFT correspondence, there is not yet any direct translation of the configuration of one theory to the other. At the present time it is not known whether the situation may be taken under control. The purpose of this paper is to provide more evidence in favour of such the correspondence for string propagation on curved space-time manifolds that include AdS$_3$.

There is good motivation for specializing to AdS$_3$ [3]. First, in this case CFT is two-dimensional, so the corresponding conformal symmetry is infinite dimensional. In general, two dimensional CFT’s and perturbations around them are better understood than their higher dimensional analogues and one may hope that this will also be the case here. Second, string theory on AdS$_3$ can be defined without turning

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4. It should be noted that the issue of the string propagation on AdS$_3$ is an old story (for a recent review of this issue see [3] and references therein).
on RR fields and thus should be more amenable to standard world-sheet methods. So we are bound to learn something if we succeed.

The features of the AdS/CFT correspondence that are most relevant to our discussion are the following:

1.1 AdS geometry. We use the Euclidean version of AdS$_3$. In this case the metric is given by

$$ds^2 = \frac{l^2}{r^2}(dr^2 + d\gamma d\bar{\gamma})$$

(1.1)

where $l$ is the radius of AdS$_3$. Another convenient set of coordinates is $(\varphi, \gamma, \bar{\gamma})$ with $r = e^{-\varphi}$. In these coordinates the metric is

$$ds^2 = l^2(d\varphi^2 + e^{2\varphi}d\gamma d\bar{\gamma})$$

(1.2)

The boundary consists of a copy of $\mathbb{R}^2$ at $r \to 0$, which in the $(\varphi, \gamma, \bar{\gamma})$ coordinates corresponds to $\varphi \to +\infty$, together with a single point at $r \to +\infty$, or $\varphi \to -\infty$. Thus, in this representation, the boundary of AdS$_3$ is obtained by adding a point at infinity to $\mathbb{R}^2$, which is nothing but a sphere $S^2$.

1.2 World-sheet description. Let $S^2$ be a world-sheet whose coordinates are $(z, \bar{z})$ and $(\varphi, \gamma, \bar{\gamma})$ be sigma model quantum fields on it\footnote{We use bold letters for the space-time notation here and below.}. After introducing a NS $B_{\mu\nu}$ field that is necessary for conformal invariance the sigma model world-sheet Lagrangian with the metric $G_{\mu\nu}$ defined by (1.2) is given by

$$\mathcal{L} \sim \frac{l^2}{l_s^2}(\partial \varphi \bar{\partial} \varphi + e^{2\varphi} \partial \gamma \bar{\partial} \gamma)(z, \bar{z})$$

(1.3)

Here $l_s$ is the fundamental string length. In fact, the above construction defines the embedding: $S^2 \to$ AdS$_3$. From the sigma model point of view one can consider the zero modes of these fields as the coordinates in space-time, i.e. the zero modes of $(\varphi, \gamma, \bar{\gamma})$ are $(\varphi, \gamma, \bar{\gamma})$. The latter has a further consequence that we will exploit in section 2.

It is convenient to introduce a pair of auxiliary fields $(\beta, \bar{\beta})$, and rewrite the Lagrangian as

$$\mathcal{L} \sim \frac{l^2}{l_s^2}(\partial \varphi \bar{\partial} \varphi + \beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma} - \beta \bar{\beta} e^{-2\varphi})(z, \bar{z})$$

(1.4)

Taking into account a proper measure renormalization as well as rescaling of the fields, at the quantum level one gets the SL(2) WZW action with $k = l^2/l_s^2$\footnote{Strictly speaking, it is the product of the holomorphic and anti-holomorphic screening operators.}.

At this point, it is necessary to make a couple of remarks. First, at the quantum level the last term in (1.4) becomes the screening operator of the SL(2) WZW model\footnote{Strictly speaking, it is the product of the holomorphic and anti-holomorphic screening operators.}. Second, at the classical level this term drops in the limit $\varphi \to +\infty$ that results in the free actions. On the other hand, we have seen that this limit is nothing but the definition of the boundary component $\mathbb{R}^2$ for AdS$_3$. So at the classical level we have for the space-time theory the following picture: string theory in the background whose geometry coincides with the boundary of AdS$_3$ is described in terms of the world-sheet free fields while string theory in the background whose geometry coincides with the bulk of AdS$_3$ can be thought of as a perturbation around this free theory. Is this the case for the first quantized theory? To answer this question, it is desirable to have new ways of understanding the description of boundary in space-time in terms of the world-sheet path integral whose world-sheet is a closed surface. In the present paper, we will suggest a way to do so.

1.3 AdS/CFT correspondence. Maldacena conjectured\footnote{Strictly speaking, it is the product of the holomorphic and anti-holomorphic screening operators.} (see\footnote{Strictly speaking, it is the product of the holomorphic and anti-holomorphic screening operators.} for a review and refs.) that string theory on AdS times a compact space is dual to a CFT at the boundary of AdS. Following this work,
methods for calculating correlation functions of various operators in CFT’s were proposed by Gubser, Klebanov, Polyakov, and Witten [5]. The basic idea is to identify the generating functional of connected Green functions in the CFT with the minimum of the classical string (supergravity) theory action, subject to some boundary conditions. To be more precise, let \( \phi(\vec{\gamma}, r) \) be a scalar field in AdS\(_3\) obeying the Laplace equation \( \hat{\Delta} + m^2 \phi = 0 \). Let \( \phi_0 \) be the restriction of \( \phi \) to the boundary of AdS\(_3\). The AdS/CFT correspondence assumes that the \( \phi_0 \) should be considered to couple to a conformal operator \( O \) via a coupling \( \int \phi_0 O d^2 \gamma \). The ansatz is given by

\[
\langle \exp \int \phi_0 O \rangle_{\text{CFT}} = \exp(-I_s(\phi)).
\]

Here \( I_s \) is the classical supergravity action.

It should be noted that the asymptotic behaviour of the classical solution is given by

\[
\phi(\vec{\gamma}, r) \to r^{2-\Delta}\left(\phi_0(\vec{\gamma}) + O(r^2)\right) + r^\Delta\left(A(\vec{\gamma}) + O(r^2)\right),
\]

where \( \Delta \) is one of the roots of

\[
\Delta(\Delta - 2) = m^2.
\]

\( \phi_0(\vec{\gamma}) \) is regarded as a “source” function while \( A(\vec{\gamma}) \) describes physical fluctuations.

It is easy to see that there are two roots of (1.7) for \(-1 < m^2 < 0\). Explicitly,

\[
\Delta_{\pm} = 1 \pm \sqrt{1 + m^2}.
\]

It was understood by Breitenlohner and Freedman in the early eighties that for this mass range there exist two possible quantizations for the scalar field while for \( m^2 > 0 \) there exists a unique admissible boundary condition for the scalar field in AdS\(_3\) leading to a unique AdS-invariant quantization [4]. In the framework of the AdS/CFT correspondence this issue is raised and discussed in [7, 8, 9]. Based on their experience with two-dimensional quantum gravity, where the generating functional corresponding to the theory with one branch of gravitational dressing is the Legendre transformation of the generating functional corresponding to the other branch [10], Klebanov and Witten suggested that the two different theories are also related by the Legendre transform that interchanges the roles of \( \phi_0(\vec{\gamma}) \) and \( A(\vec{\gamma}) \) [8]. Thus the two theories are not independent but are, in fact, related to each other by the Legendre transformation. They also proposed to interpret \( A(\vec{\gamma}) \) as the expectation value of the conformal operator \( O(\vec{\gamma}) \) namely,

\[
A(\vec{\gamma}) \sim \langle O(\vec{\gamma}) \rangle.
\]

It is natural to ask whether the above analogy with two-dimensional quantum gravity or, equivalently, two-dimensional string theory is deeper. If so, this could lead us to a better understanding of the AdS/CFT correspondence by our understanding of 2d gravity. In the present paper, we will show that 2d gravity methods are indeed appropriate for some purposes.

The outline of the paper is as follows. We start in section 2 by describing how the world-sheet methods and 2d gravity like scaling arguments can be used to describe the AdS\(_3\)/CFT correspondence in the bosonic case. Our strategy is to start from a world-sheet representation of CFT in space-time and, then, to perturb such a theory via marginal perturbations of a world-sheet action. The role of these perturbations is to break infinite dimensional symmetry in space-time to finite dimensional one. It turns out that one of the perturbations is the screening operator of the SL(2) WZW model; i.e. the world-sheet action has a form [1.4]. The latter allows us to interpret this case as string on AdS\(_3\) and consider some features of the AdS/CFT correspondence in the framework of our formalism. We then go on in section 3 to generalize the results of section 2 for the supersymmetric case. This is done by simply adding free world-sheet fermions. Finally, section 4 will present our conclusions and directions for future work.
2 World-sheet description for AdS$_3$/CFT correspondence: Bosonic string

Let us now show how some basic features of the AdS$_3$/CFT correspondence can be caught via world-sheet methods. To do this, we propose a stringy way for perturbing 2d CFT’s around their critical points. Our strategy is to start with a stringy representation of CFT. The latter means that there are two Virasoro algebras: one acts on the world-sheet and another acts in space-time which is two-dimensional (for example, the AdS$_3$ boundary). Next we perturb a world-sheet action by some marginal perturbations such that the space-time symmetry becomes finite dimensional. As a result, we obtain a massive FT in space-time. Note that a scale for such a theory is introduced by a two-dimensional coupling constant or in the framework of the AdS$_3$/CFT correspondence by the radial anti-de-Sitter coordinate, i.e. by $\varphi$. It turns out that the scaling argument of David, Distler and Kawai [11] applied for 2d gravity is also useful to study a scaling limit of this FT.

2.1 More on the world-sheet description of space-time CFT. Following the picture we sketched in subsection 1.2 it seems natural to start with the same set of free fields that is used to describe the SL(2) WZW model. Since the Hilbert space of any two-dimensional conformal field theory decomposes into holomorphic and anti-holomorphic sectors it makes sense to consider one of them, say holomorphic. Thus what we have is a free boson $\varphi$ coupled to the background charge and a first order bosonic ($\beta, \gamma$) system of weight $(1,0)$. The two-point functions of these fields are normalized as

$$\langle \varphi(z_1)\varphi(z_2) \rangle = -\log z_{12} \quad , \quad \langle \beta(z_1)\gamma(z_2) \rangle = \frac{1}{z_{12}} .$$

The stress tensor of the free fields coincides with the Sugawara stress tensor of the SL(2) WZW model at the level $k$ and is written as

$$T(z) = \beta \partial \gamma - \frac{1}{2} \partial^2 \varphi - \frac{1}{a} \partial \varphi(z) \quad ,$$

where $a = \sqrt{2(k-2)}$. It is well-known that it provides the world-sheet Virasoro algebra with

$$L_n = \oint_{C_0} dz \gamma^{n+1} T(z) \quad , \quad n \in \mathbb{Z} ,$$

and the central charge $c = \frac{3k}{k-2}$. The contour $C_0$ surrounds 0.

On the other hand, having a pair of the complex space-time coordinates ($\gamma, \bar{\gamma}$), it is straightforward to write down the differential operator realization for the space-time Virasoro algebras e.g., for the holomorphic sector $L_n \sim \gamma^{n+1} \frac{\partial}{\partial \gamma}$. At the quantum level the most obvious generalization of that is $L_n = -\oint_{C_0} dz \gamma^{n+1} \beta(z)$. However, the world-sheet reparametrization (conformal) invariance of string theory dictates that the vertices should have dimension 1 in $z$ so that the integral over $dz$ is invariant. In other words, the $L_n$’s have to obey

$$[L_n, L_m] = 0 \quad , \quad n, m \in \mathbb{Z} .$$

In the framework of the SL(2) WZW model Giveon, Kutasov and Seiberg [13] proposed to modify the formula for $L_n$ to

$$L_n = -\oint_{C_0} dz \gamma^{n+1} \beta + \frac{1}{2} a(n+1) \gamma^n \partial \varphi(z) \quad , \quad n \in \mathbb{Z} .$$

We refer to [12] for a review of two-dimensional gravity.
Conformal techniques may be used to check that at least for the free fields such $L_n$’s indeed generate the Virasoro algebra whose central charge is given by $c = 6k$ with $k = \oint_{C_0} dz \partial \gamma^{-1}(z)$.

What we actually need in practice to do physics in space-time is not only space-time symmetry generators defined in terms of world-sheet fields but a world-sheet path integral representation for space-time correlators. For example, a possible way to do this is

$$\langle \ldots \rangle_{\text{CFT}} = \left| \int [d\beta d\gamma]_p e^{-S_0[\beta,\gamma]} \right|^2 \int [d\varphi] e^{-S_0[\varphi]} \ldots ,$$

where $S_0[\beta, \gamma]$ and $S_0[\varphi]$ are the standard free actions that provide the two-point functions (1.1). $[d\beta d\gamma]_p$ means that the $(\beta, \gamma)$ system has the Bose-sea level $-p$. Note that up to a sign $k$ coincides with the Bose-sea charge $Q_B$ of the $(\beta, \gamma)$ system, so $p$ is an eigenvalue of $k$ (14).

At this point, it is necessary to make a remark. The ansatz (2.6) means that the balance of charges for the free fields is simply

$$\#\beta = \#\gamma , \quad \#\bar{\beta} = \#\bar{\gamma} , \quad \sum \alpha_i = 0 .$$

Here $\alpha_i$ is given by $e^{\alpha_i \varphi}$. This is nothing but the so-called Feigin-Fuchs representation used to compute correlators of the $SL(2)$ WZW model. The background charge of the field $\varphi$ is compensated by inserting the identity operator (its conjugate representation $I = \exp(-\frac{2}{\pi} \varphi)$) at infinity. It is natural to use such an ansatz because we do not know any conjugate representation of the Virasoro generators (2.5). As a simple check, one can compute correlators of the $L_n$’s (14).

Thus, as promised in the introduction, the space-time CFT is described in terms of the free world-sheet fields.

2.2 Perturbations. So far we have been studying the space-time CFT by the world-sheet path integral. It is natural to ask whether the discussion can be extended to its perturbations. Since we are interested in the AdS$_3$/CFT correspondence it is reasonable to consider this space-time CFT as the conformal theory which resides on the boundary of AdS$_3$ and try to relate a scale of the perturbed theory with the radial anti-de-Sitter coordinate. In doing so, what we need is to first realize which “space-time boundary conditions” within the path integral. Indeed, one can start from the world-sheet action (14) and try to directly define the boundary CFT. We have already discussed this issue in (14) where it was proposed to require that string vertex operators for the boundary theory be independent of $\varphi$. It is amusing that one cannot define the Virasoro and $N = 2$ generators in a $\varphi$-independent way but one can do so for the $N = 4$ algebra that is in harmony with the Maldacena conjecture. However, such a construction of the bulk theory remains inconsistent because, while it says how to describe the vertex operators of physical states, it says nothing about the symmetry breaking. Indeed, one of the reasons why AdS$_3$ is special is that in this case the boundary CFT is two-dimensional, so the corresponding conformal symmetry is infinite dimensional. On the other hand, string theory on AdS$_3$ has $SL(2) \times SL(2)$ symmetry which corresponds to the global part of the two-dimensional conformal symmetry. After this is noted, it immediately comes to mind to realize this symmetry breaking by the perturbations. It is natural to ask whether all such perturbations results in AdS$_3$. It turns out that this is not the case, so we have to be careful at this point.

Following the ideas we sketched, modify our ansatz (2.6) to

$$\langle \ldots \rangle = \langle e^{S_{\text{int}}} \ldots \rangle_{\text{CFT}} = \left| \int [d\beta d\gamma]_p e^{-S_0[\beta,\gamma]} \right|^2 \int [d\varphi] e^{-S_0[\varphi]+S_{\text{int}}} \ldots .$$

Actually, it was assumed in (14) that the free actions are perturbed by $S_{\text{int}}$. However, because of the space-time conformal invariance, $S_{\text{int}}$ has to commute with the $L_n$’s otherwise the Ward identities are broken. So we need to be more careful with perturbations. We will return to this point in the next subsection.
So what we have done was to perturb the free actions by $S_{\text{int}}[\varphi; \beta, \gamma; \bar{\gamma}, \bar{\beta}]$. By analogy with the SL(2) WZW model, we require

$$[S_{\text{int}}, L_n] = [S_{\text{int}}, \bar{L}_n] = 0 \quad , \quad n \in \mathbb{Z} \quad .$$

In other words, $S_{\text{int}}$ is marginal for the world-sheet theory. This is, however, not the whole story. A novelty is to require

$$[S_{\text{int}}, L_n] = [S_{\text{int}}, \bar{L}_n] = 0 \quad , \quad n \in \mathbb{Z} \quad .$$

only for $n = 0, \pm 1$. What this means in practice is as follows. We have no longer the conformal theory in space-time. To be more precise, what we got is the so-called quasi-conformal theory \[17\]. Even though we have broken the infinite dimensional symmetry to its global part $\text{SL}(2) \times \text{SL}(2)$, it is instructive to ask whether we got string theory on $\text{AdS}_3$. However, before answering this question, let us give a few examples of $S_{\text{int}}$ obeying (2.9)-(2.10).

(1) Following the ideas sketched in subsection 1.2, it is natural to try the screening operator of the $\text{SL}(2)$ WZW model. For simplicity, let us restrict ourselves to its holomorphic part, namely, $S_+ = \oint \! dz \, \beta \exp(-\frac{a}{2} \varphi)$. Then, a simple algebra shows that it indeed obeys (2.9)-(2.10). The latter means that the operator

$$O_0 = \int d^2 z \, \beta \bar{\beta} \, e^{-\frac{a}{2} \varphi}(z, \bar{z})$$

(2.11)

can be used as $S_{\text{int}}$.

(2) We now want to reconsider the above derivation. The idea is to modify $S_+$ to $\oint \! dz \, \beta \exp(y \varphi)$. The analysis proceeds as above and leads to a new solution $S_- = \oint \! dz \, \beta^{-2} \exp(-a \varphi)$. Thus we can use

$$O_1 = \int d^2 z \, (\beta \bar{\beta})^{-2} e^{-a \varphi}(z, \bar{z})$$

(2.12)

as the perturbation of the free theory. It is interesting to note that $S_-$ is nothing but the second screening operator of the $\text{SL}(2)$ WZW model found by Dotsenko \[18\].

(3) Alternatively, it is possible to find a solution of the constraints (2.9)-(2.10) in a pure algebraic way. In doing so, one has to keep in mind that the $L_n$’s and $\bar{L}_m$’s commute. It automatically follows, then, that any polynomial in the $L_n$’s is a solution of (2.9). Thus the problem is reduced to finding polynomials that obey (2.10). It is straightforward to write down a solution of the problem. It is simply given by the quadratic Casimir operator of $\text{sl}(2)$ namely,

$$C_2 = -L_0^2 + \frac{1}{2}(L_1 L_{-1} + L_{-1} L_1) \quad .$$

(2.13)

This means that a formal expression $O_2 = C_2 C_2$ may be used as the perturbation $S_{\text{int}}$. However, it is not clear whether such an expression may be written in a local form like $\int \! d^2 z \, V(z, \bar{z})$.

Now let us turn to a geometrical interpretation of the above results. Heuristically, the idea is to integrate away the auxiliary $\beta, \bar{\beta}$ fields in the world-sheet path integral in order to get a term in the sigma model action that corresponds to $G_{\mu \nu} \partial X^\mu \partial X^\nu$. The last would allow us to reconstruct the space-time geometry. It is well known that it works fine for $O_0$ where one easily finds the $\text{AdS}_3$ metric. But it fails for $O_1$ and $O_2$. The problem is that we have no longer a linear dependence on $\beta \bar{\beta}$ in $S_{\text{int}}$. So it is desirable to have new ways of understanding string theory on curved spaces to find the geometrical interpretation for these perturbations.

2.3 More on string theory on $\text{AdS}_3$. As we have seen above, string theory on $\text{AdS}_3$ can be described by perturbing the free field actions within the world-sheet path integral. The result of such a perturbation is
breaking the infinite dimensional conformal symmetry in space-time. From a local observer point of view
who lives on the \((\gamma, \bar{\gamma})\) plane, what we got is a two-dimensional Field Theory in a vicinity of its critical
point. A simple idea behind some of the recent advances in studying the AdS/CFT correspondence
is that the anti-de-Sitter coordinate \(\varphi\) is responsible for a scale in this Field Theory \([2]\). We will now
attempt to make more precise this statement in the case of AdS\(_3\). Moreover, we will show that it is in
harmony with what we proposed in the previous subsection.

Our ansatz for the Field Theory partition function is that

\[
Z[r] = \langle \delta \left( \int d^2 z \beta \bar{\beta} e^{-\frac{2}{a} \varphi(z, \bar{z})} - r^{-1} \right) \rangle_{\text{CFT}} ,
\]

where \(0 \leq r \leq \infty\).

As a preliminary check, note that the Laplace transformed partition function

\[
Z[t_0] = LZ[r] = \int_0^\infty dr e^{-t_0/r} Z[r]
\]

has the expected form \([2.8]\) with the perturbation \(S_{\text{int}} = t_0 \int d^2 z \beta \bar{\beta} e^{-\frac{2}{a} \varphi}\).

Now we will recall how physical states (primary conformal operators) appear in the space-time CFT.
Several different proposals are known to define them. One is based on the idea that the \(L_n\)'s defined in
\([2.5]\) are in fact the coefficients of the Laurent expansion

\[
L_n = \oint_{C_0} d\gamma \gamma^{n+1} T(\gamma) ,
\]

where \(T(\gamma)\) is the stress tensor of the boundary theory. The realization of the primary fields (their
holomorphic parts) via this definition is then \([1]\)

\[
[L_n, \Phi^\Delta_m] = (n(\Delta - 1) - m) \Phi^\Delta_{n+m} . \tag{2.17}
\]

Here the \(\Phi^\Delta_m\)'s are the Laurent modes of the primary field whose conformal weight (dimension) is \(\Delta\) i.e.,
\(\Phi^\Delta(\gamma) = \sum_n \gamma^{-n-\Delta} \Phi^\Delta_m\). An example of such a realization within string theory on AdS\(_3\) times some
compact manifold \(X\) was proposed in \([13]\). Giveon and co-workers simply form a vertex operator for
\(\Phi^\Delta\) by dressing a spinless operator \(V(z, \bar{z})\) of CFT on \(X\) by the SL(2) WZW primary field. Explicitly,

\[
\Phi^\Delta_{m\bar{m}} = \int d^2 z \gamma^{j+m} \bar{\gamma}^{j+m} e^{\frac{2}{a} \varphi} V(z, \bar{z}) . \tag{2.18}
\]

Then it follows from Eq. \([2.17]\) that for the primary field \(\Phi^\Delta\) the conformal weight is given by \(\Delta = j+1\).

Now we are ready to compare our ansatz \([2.14]\) to "experiment". First, let us to define the scaling
dimension \(\Delta(\Phi)\) of the physical operator as it is done within 2d gravity \([1]\) namely,

\[
\frac{1}{Z[r]} \langle \delta \left( \int d^2 z \beta \bar{\beta} e^{-\frac{2}{a} \varphi(z, \bar{z})} - r^{-1} \right) \rangle_{\text{CFT}} \sim r^{-1+\Delta(\Phi)} , \quad r \to 0 . \tag{2.19}
\]

We can use a simple scaling argument to evaluate the one-point functions of the operators \([2.18]\). Indeed,
shifting the zero mode of the field \(\varphi\)

\[
\varphi \to \varphi + \frac{a}{2} \log r \tag{2.20}
\]

\(\text{Alternatively, one can think about the } L_n\text{'s as } L_n \Phi^\Delta(\gamma) = \oint_{C_\gamma} d\gamma' (\gamma' - \gamma)^{n+1} T(\gamma') \Phi^\Delta(\gamma). \text{ See } [13] \text{ for a discussion of this approach within string theory on AdS}_3.
and assuming that the path integral measure $[d\varphi]$ is invariant under such a shift we find that the scaling dimension is given by $\Delta(\Phi) = j + 1$.

Moreover, taking into account the contribution from the $\delta$-function we get for the non-normalizable one-point functions

$$\langle \delta(\int d^2z \beta \bar{\beta} e^{-\frac{2}{\alpha}s}(z, \bar{z}) - r^{-1})\Phi_{m\bar{m}}^{\Delta} \rangle_{\text{CFT}} \sim r^{\Delta(\Phi)} .$$

(2.21)

Now we can easily read off some interesting conclusions. One of the first observations is that the conformal weights (dimensions with respect to $L_0$) of the operators (2.18) coincide with their scaling dimensions namely, $\Delta = \Delta(\Phi)$. This gives us a hint that one should try to catch the dynamics of such operators via the effective action for scalar fields in the AdS$_3$ background. Further evidence in favor of this suggestion is provided by comparing Eq. (1.6) to Eq. (2.21). Indeed, we can interpret the last operators via the effective action for scalar fields in the AdS$_3$ background. So we would like to relate the asymptotic behaviour (1.6) that is responsible for a “source”. A question remains, however, as to what is a good world-sheet approximation for it. A possible answer to this question immediately comes to mind just by looking at the ansatz (1.5) for the AdS/CFT correspondence. Thus we have the explicit example of the translation of physical states of one theory to another, i.e. the example of the ’t Hooft Holographic principle [13, 14].

2.4 Towards RG analysis. Up to now our discussion has not been sensitive to the second term in the asymptotic behaviour (1.6) that is responsible for a “source”. A question remains, however, as to what is a good world-sheet approximation for it. A possible answer to this question immediately comes to mind just by looking at the ansatz (1.5) for the AdS/CFT correspondence. Thus we have the explicit example of the translation of physical states of one theory to another, i.e. the example of the ’t Hooft Holographic principle [13, 14].

As a preliminary check, let us consider the perturbation $t_0 \int d^2z \beta \bar{\beta} e^{-\frac{2}{\alpha}s}$. As before, we can shift $\varphi$ to determine the scaling dimension of the operator $\int d^2z \beta \bar{\beta} e^{-\frac{2}{\alpha}s}$. Thus $\Delta_0 = 0$. On the other hand, a scaling argument shows that $Z[r] \sim r$ as $r \to 0$. Using the Laplace transform (2.15), we find that the partition function $Z[t_0]$ scales as $t_0^2$. So for $\langle t_0 \rangle$, we have $t_0^2$. In terms of $r$ it corresponds to $r^2$. This is the expected form of the asymptotic behavior for zero conformal dimension.

We want now to repeat this analysis in a general case. As before, the first step is to find the scaling law for $\langle t_i \rangle$ in terms of $t_0$. In fact, the ansatz (1.5) assumes that all $\langle t_i \rangle$ should scale in the same way, i.e. like $t_0^2$. The latter means that $t_i$ behaves as $t_0^{1-\Delta_i}$. So what we find for $\langle t_i \rangle$ is then $t_0^{\Delta_i-\Delta_i}$. In terms of $r$ it is replaced by $r^{2-\Delta_i}$. This is exactly what we need to push our interpretation of the world-sheet coupling constants as the space-time “sources”, namely, in the scaling limit

$$\phi_{0i}(\gamma) \sim r^{\Delta_i-2}L^{-1}\langle t_i \rangle , \quad r \to 0 .$$

(2.23)

We would like to make a few comments:

1. Although the representations for $A$ and $\phi_0$ given by (2.22)-(2.23) look in many ways attractive, we have to stress their speculative character. They rest on the scaling argument only, and so further

(1) corresponds to the partition function $Z$ of the perturbed theory. So $\langle t_0 \rangle$ is simply $t_0Z$. 

8
work is needed to prove them strictly. Moreover, we do not know any similar representation for the product \( \phi_0 \mathcal{O}(\vec{\gamma}) \) in Eq. (1.3). What we can only refer to the examples of subsection 2.2.

2. As we saw in the previous subsection, the scale of the perturbed theory is provided by the anti-de-Sitter coordinate \( \varphi \). This means that the coupling constant (source) in Eq. (1.5) becomes running, i.e. \( \phi_0(\vec{\gamma}, r) \). The latter allows to define the corresponding \( \beta \)-function as

\[
\beta_i = r \partial_r \phi_0(\vec{\gamma}, r) .
\]  

(2.24)

It is natural to suggest that the scaling limit of the running coupling is described via \( L^{-1} \langle t_i \rangle \). Thus the linearized \( \beta \)-function is simply

\[
\beta_i = (2 - \Delta_i) \phi_0(\vec{\gamma}, r) .
\]  

(2.25)

A simple observation is, then, that \( \beta_i \) vanishes for \( \Delta_i = 2 \), i.e. for the operator whose dimension equals 2. This gives us a hint that one should catch dynamics of such the operator via the effective action for the massless scalar field. This is in harmony with the formula (1.7).

3. Note that the situation that we are considering here is more subtle than the one in two-dimensional gravity in which the Legendre transformation of the generating functional corresponds to the other branch of gravitational dressing. The crucial point is that in the problem at hand a transformation has to know about the unitary bound. In other words, it should be defined only for the range \( 0 \leq \Delta \leq 2 \). So a naive attempt to adopt the 2d gravity analysis [10] fails.

3 Inclusion of Supersymmetry

According to the Maldacena conjecture, in the situation that we are considering the theory on the boundary of AdS\(_3\) has to possess the \( N = 4 \) superconformal algebra as the symmetry algebra. Thus, in this section we will generalize and apply the previous results to the supersymmetric case.

3.1 World-sheet description of space-time SCFT. Following the original analysis of [13], the world-sheet description of \( N = 4 \) SCFT on the boundary of AdS\(_3\) is as follows.

The AdS\(_3\)-part (its holomorphic sector) is described by the same set of the free fields we used in subsection 2.1 namely, \( (\varphi, \beta, \gamma) \). The only difference is that the level \( k \) is shifted as \( k \rightarrow k + 2 \). The latter is due to free fermions with the two-point functions

\[
\langle \psi^i(z_1) \psi^j(z_2) \rangle = \frac{\eta^{ij}}{z_{12}} ,
\]  

(3.1)

where \( i, j = 0, \pm; \eta^{00} = -1, \eta^{+-} = \eta^{-+} = 2 \). It is well known that such fermions are needed to define the world-sheet \( N = 1 \) superconformal algebra that is standard within the Neveu-Schwarz-Ramond (NSR) formulation of superstring theories.

The \( S^3 \)-part (its holomorphic sector) can be described in a similar way. So we introduce a free boson \( \varphi_c \) coupled to a background charge, the first order bosonic \( (\beta_c, \gamma_c) \) system of weight \((1,0)\) and three fermionic fields \( \psi^i_c \). The two-point functions of these fields are normalized as

\[
\langle \varphi_c(z_1) \varphi_c(z_2) \rangle = -\log z_{12} , \quad \langle \beta_c(z_1) \gamma_c(z_2) \rangle = \frac{1}{z_{12}} , \quad \langle \psi^i_c(z_1) \psi^j_c(z_2) \rangle = \frac{\eta^{ij}}{z_{12}} ,
\]  

(3.2)

where \( i, j = 0, \pm; \eta^{00}_c = 1, \eta^{+-}_c = \eta^{-+}_c = 2 \).

The stress tensor of the bosonic fields is given by

\[
T_c(z) = \beta_c \partial \gamma_c - \frac{1}{2} \partial \varphi_c \partial \varphi_c + \frac{i}{a} \partial^2 \varphi_c(z) ,
\]  

(3.3)
where $a = \sqrt{2k}$. It coincides with the Sugawara stress tensor of the SU(2) WZW model at the level $k - 2$

$$ T_c(z) = \frac{1}{k} \eta_{cij} J_c^i J_c^j(z) , $$

(3.4)

such that the currents are

$$ J_c^-(z) = -i\beta_c(z) , \quad J_c^0(z) = \beta_c \gamma_c + \frac{i}{2} a \partial \varphi_c(z) , \quad J_c^+(z) = -i\beta_c \gamma_c^2 + a \gamma_c \partial \varphi_c + i(k - 2) \partial \gamma_c(z) . $$

(3.5)

In a similar way, the stress tensor of the free fermions coincides with the Sugawara stress tensor of the SU(2) WZW model at the level 2. The corresponding currents are given by

$$ j_c^-(z) = i\psi^-_c \psi^0_c(z) , \quad j_c^0(z) = \frac{1}{2} \psi^+_c \psi^-_c(z) , \quad j_c^+(z) = i\psi^+_c \psi^0_c(z) . $$

(3.6)

As to a manifold $X$, it is usually associated with $T^4$ or $K3$. The explicit choice is not crucial for what follows; however, to be more precise let us take the four torus as $X$. Then, the $T^4$-part (its holomorphic sector) can be described by four scalar fields $X^\mu$ without background charges together with their fermionic partners $\chi^\mu$, $\mu = 1, \ldots, 4$. Their two-point functions can be normalized as it was done in (3.2). Finally, one should keep in mind the corresponding superconformal ghosts that are needed to cancel the conformal anomaly \[13, 20\].

Having the above set of the free fields, we now wish to realize the superconformal algebra of the boundary SCFT. This can be done as in \[13\]. Let us restrict ourselves to the holomorphic sector. The Virasoro generators (2.5) are modified to

$$ L_n = -\frac{1}{2} \oint_{C_0} dz \gamma^n \left( 2\gamma \beta + a(n+1) \partial \varphi - (n^2 - 1) \psi^+ \psi^- + i(n^2 + n) \gamma^{-1} \psi^0 + i(n^2 - n) \gamma \psi^- \psi^0 \right)(z) $$

(3.7)

where $a$ is now $\sqrt{2k}$.

The SU(2) generators are provided by the $S^3$-part or, equivalently, by the supersymmetric SU(2) WZW model. Explicitly,

$$ T_n^i = \oint_{C_0} dz \gamma^n \left( J_c^i + j_c^i \right)(z) , \quad n \in \mathbb{Z} . $$

(3.8)

A simple algebra shows that such defined $L_n$’s and $T_n^0$’s obey the commutation relations of the $N = 4$ superconformal algebra whose central charge is $c = 6kk$.

It is known that one of the drawbacks of the NSR formalism is that it is not manifestly supersymmetric. In the problem at hand it is rather difficult to explicitly write down all fermionic generators. For simplicity, we will restrict ourselves to the global generators. In the NSR sector they are simply \[13\]

$$ Q_{\pm}^{\alpha} = \oint_{C_0} dz e^{-\frac{1}{2} \phi} S^\alpha(z) , \quad \alpha = \pm, \pm . $$

(3.9)

Here $\phi$ is the scalar field which appeared by the bosonization of the superconformal ghosts. $S^\alpha$ are the so-called spin fields of the $N = 1$ world-sheet superconformal algebra. Note that the spin fields are built only via the fermions $\psi, \psi_c, \chi$. 


Finally, let us generalize our ansatz (2.3). For example, this can be done as
\[
\langle \ldots \rangle_{\text{SCFT}} = \left| \int [d\beta d\gamma] e^{-S_0[\beta, \gamma]} \right|^2 \left| \int [d\varphi] e^{-S_0[\varphi]} \right|^2 \left| \int [d\beta_c d\gamma_c] e^{-S_0[\beta_c, \gamma_c]} \right|^2 \left| \int [d\varphi_c] e^{-S_0[\varphi_c]} \right| \ldots ,
\] (3.10)

Above, we have omitted the additional scalars fields as well as the fermions and the superconformal ghosts. The path integral measures for these fields are standard (see e.g., [20]). We use the Feigin-Fuchs representation again, so we require the following balance of charges
\[
\#\beta = \#\gamma , \quad \#\bar{\beta} = \#\bar{\gamma} , \quad \sum \alpha_i = 0 , \quad \#\beta_c = \#\gamma_c , \quad \#\bar{\beta}_c = \#\bar{\gamma}_c , \quad \sum \alpha_{ci} = 0 ,
\] (3.11)

where \(\alpha_i\) and \(\alpha_{ci}\) are given by \(e^{\alpha_i \varphi}\) and \(e^{\alpha_{ci} \varphi c}\), respectively. It should be noted that unlike the \((\beta, \gamma)\) system of the \(\text{AdS}_3\)-part, the \((\beta_c, \gamma_c)\) system of the \(S^3\)-part has a zero Bose sea level.

3.2 The world-sheet description of superstring theory on \(\text{AdS}_3 \times S^3 \times X\). To get superstring theory from SCFT, we follow the same strategy as we proposed in section 2. So we perturb the free world-sheet actions by marginal perturbations such that the space-time symmetry becomes finite dimensional while the \(G_{\mu\nu} \partial X^\mu \partial X^\nu\) term with the \(\text{AdS}_3\) metric appears in the world-sheet action. As a result, we get a supersymmetric FT in space-time. Again, a scale for this theory is provided by a world-sheet coupling constant or, equivalently, by the radial anti-de-Sitter coordinate \(\varphi\).

Let us first modify the ansatz (3.1) to
\[
\langle \ldots \rangle = \langle e^{\hat{S}_{\text{int}}} \ldots \rangle_{\text{SCFT}} .
\] (3.12)

We now require that \(\hat{S}_{\text{int}}\) obeys
\[
[\hat{S}_{\text{int}}, \hat{L}_n] = [\hat{S}_{\text{int}}, \hat{\bar{L}}_n] = 0 , \quad n \in \mathbb{Z} ,
\] (3.13)

where \(\hat{L}_n\) means a total world-sheet Virasoro generator. So \(\hat{S}_{\text{int}}\) is marginal within the world-sheet theory.

To extend our bosonic analysis to the supersymmetric case, we need, in addition to
\[
[\hat{S}_{\text{int}}, \hat{L}_n] = [\hat{S}_{\text{int}}, \hat{\bar{L}}_n] = 0 , \quad \text{only for} \quad n = 0, \pm 1 ,
\] (3.14)

that \(\hat{S}_{\text{int}}\) obeys
\[
[\hat{S}_{\text{int}}, \hat{T}^n_r] = [\hat{S}_{\text{int}}, \hat{\bar{T}}^n_r] = 0 , \quad \text{for} \quad n = 0 \quad \text{and} \quad [\hat{S}_{\text{int}}, \hat{Q}^0_r] = [\hat{S}_{\text{int}}, \hat{\bar{Q}}^0_r] = 0 , \quad \text{for} \quad r = \pm \frac{1}{2} .
\] (3.15)

Thus we have no longer a superconformal theory in space-time. To be more precise, what we got is the global \(N = 4 \times N = 4\) algebra. In the above, we restrict ourselves to the Neveu-Schwarz sector.

However, the generalization to others is straightforward.

Let us now give a few examples of \(\hat{S}_{\text{int}}\) obeying (3.13)-(3.15).

(1) As in section 2, let us try the screening operator of the \(\text{SL}(2)\) WZW model. A precisely analogous computation shows that it indeed obeys (3.13)-(3.15). Thus the operator
\[
\hat{\mathcal{O}}_0 = \int d^2 z \beta \bar{\beta} e^{-\frac{2}{\alpha} \varphi(z, \bar{z})}
\] (3.16)

\(^*\)Strictly speaking, we also need to require that \([\hat{S}_{\text{int}}, \hat{G}_r] = [\hat{S}_{\text{int}}, \hat{\bar{G}}_r] = 0\), where the \(\hat{G}_r, (\hat{\bar{G}}_r)\)’s are the \(N = 1\) fermionic generators. However, it proves irrelevant for what follows.
can be used as $\hat{S}_{\text{int}}$.

There is a point we should mention. Integrating the auxiliary fields $\beta, \bar{\beta}$ in the world-sheet path integral away, we get a non-linear term in the sigma model action which corresponds to $G_{\mu\nu} \partial X^\mu \partial X^\nu$ with the metric of AdS$_3$ while the fermionic terms remains quadratic. Here an analogy with the Green-Schwarz (GS) formulation appears because such behavior reminds us of results of gauge fixing the GS action where the fermionic term becomes quadratic (see, e.g., \cite{21, 22}).

(2) It is also not difficult to find one more operator obeying the above constraints. A simple algebra shows that it is given by

$$\hat{O}_1 = \int d^2z \left( \beta \bar{\beta} \right)^k e^{-a\phi(z, \bar{z})}.$$  \hspace{1cm} (3.17)

(3) Obviously, the quadratic Casimir operator of the global $N = 4 \times N = 4$ algebra,

$$\hat{C}_2 = -L_0^2 + \frac{1}{2} (L_1 L_{-1} + L_{-1} L_1) + \eta_{cij} T_i^c T_j^c + 2\epsilon_{\alpha\beta} Q^\alpha_0 Q^\beta_{-\frac{1}{2}}$$  \hspace{1cm} (3.18)

obeys the constraints. However, it is not clear how to rewrite $\hat{C}_2$ in a local form.

It is interesting to note that the screening operators of the SU(2) WZW model $\int d^2z \beta_\alpha \bar{\beta}_\beta e^{-a\phi}$ and $\int d^2z \left( \beta_\alpha \beta_\beta \right)^{-k} e^{-ia\phi}$ are marginal for both space-time and world-sheet theories.

We conclude this subsection with a brief discussion of $\hat{O}_1$. On the one hand, it is the second screening operator of the SL(2) WZW model. Moreover, it is known that it becomes the second screening operator for the Liouville theory under the Drinfeld-Sokolov reduction. The latter is similar to a light-cone-like gauge in the context of the AdS/CFT correspondence \cite{2}. On the other hand, the second screening operator is crucial for a strong coupling regime of 2d gravity \cite{9}. So putting the two facts together, we suggest that the correct perturbation of the free actions is given by

$$\hat{S}_{\text{int}} = \hat{O}_0 + \hat{O}_1.$$  \hspace{1cm} (3.19)

Let us give one more piece of evidence in favor of this suggestion. Obviously, these operators coincide for $k = 1$. In this sense, something should happen as one approaches this value. This problem was first discussed in \cite{24} and later in \cite{25}. However, our description is different because in fact it assumes an analogy with the so-called $c = 1$ barrier in two-dimensional quantum gravity \cite{12}.

### 3.3 More comments on superstring theory

The extension of our analysis of subsections 2.3 and 2.4 to the supersymmetric case is straightforward. Therefore, we only summarize the most relevant formulae.

A new ansatz for the partition function is given by

$$\hat{Z}[^r] = \langle \delta \left( \int d^2z \beta \bar{\beta} e^{-\frac{2}{a}\phi(z, \bar{z})} - r^{-1} \right) \rangle_{\text{SCFT}}.$$  \hspace{1cm} (3.20)

Of course, one can easily get the expected form (3.12) with $\hat{S}_{\text{int}} = -t_0 \int d^2z \beta \bar{\beta} e^{-\frac{2}{a}\phi}$ by the Laplace transformation.

It seems natural to define the scaling dimension $\Delta(\hat{\Phi})$ of the physical operator as

$$\frac{1}{\hat{Z}[^r]} \langle \delta \left( \int d^2z \beta \bar{\beta} e^{-\frac{2}{a}\phi(z, \bar{z})} - r^{-1} \right) \hat{\Phi} \rangle_{\text{SCFT}} \sim r^{-1+\Delta(\hat{\Phi})}, \hspace{0.5cm} r \to 0,$$  \hspace{1cm} (3.21)

and, then, to probe the vertex operators proposed by Giveon and co-workers \cite{13}. In the supersymmetric case these operators look like hatted versions of \cite{2.18}. On the one hand, the commutation relations

\footnote{The interested reader is referred to lectures of Gervais \cite{23}.}
with the Virasoro generators show that their conformal dimension is given by $\Delta = j + 1$. On the other hand, the scaling argument gives the scaling dimension as $\Delta(\hat{\Phi}) = j + 1$. Therefore, as in subsection 2.3, we propose the following representation of $A$

$$A(\vec{\gamma}) \sim r^{-\Delta} \langle \delta(\int d^2 z \beta \beta e^{-\frac{2}{r^2}(z, \bar{z})} - r^{-1})\hat{\Phi}^\Delta(\vec{\gamma}) \rangle_{\text{SCFT}} \quad , \quad r \to 0 \quad ,$$

(3.22)

with $\hat{\Phi}^\Delta(\vec{\gamma}) = \sum_{m, \bar{m}} \gamma^m \bar{\gamma}^{\bar{m}} - \Delta \gamma^m \bar{\gamma}^{\bar{m}} \hat{\Phi}^\Delta_{m, \bar{m}}$. This is the world-sheet representation of the Klebanov-Witten proposal [8].

As to the space-time “sources” $\phi_{0i}$, we again propose that their scaling limit is described via $L^{-1}\langle t_i \rangle$. This allows us to find the linearized $\beta$-function. Explicitly, $\beta_i = (2 - \Delta_i)\phi_{0i}$.

We here conclude this section with some speculations about Eq.(1.7) which gives the relation between the mass $m$ of the effective scalar field in AdS$_3$ and the conformal dimension $\Delta$ of the corresponding boundary operator.

(1) In general, it is not clear how to define the S-matrix within Field Theory in AdS spaces. As a result, we cannot consistently define masses as poles in the scattering amplitudes. So the best that we can do with the problem at hand is to take the flat limit $l \to \infty$. The latter is equivalent to the limit $k \to \infty$. In free fields terms, it means the following rescaling for the scalar field $\varphi \to \sqrt{\frac{2}{k}}\varphi$. Thus the exponent in Eq.(2.18) becomes $e^{j\varphi}$. As to the fields $\gamma$ and $\bar{\gamma}$, it is useful to bosonize them in the standard way, for instance, $\gamma = e^{i\sigma_1 - \sigma_2}$. Now we can define the mass just as the sum of exponents. So we find $m^2 = j^2$. Note that the bosonization makes clear that $\gamma$ and $\bar{\gamma}$ do not contribute. However, there is a contribution of other fields. Since we are only interested in a dependence of $j$ we simply modify the formula to $m^2 = j^2 + \text{const}$. There are two facts that we should recall. The first is that the conformal dimension of the operators (2.18) is given by $\Delta = j + 1$. The second is that the $\beta$-function vanishes for $\Delta = 2$ which signals about massless modes. So putting these facts together, we recover the relation (1.7).

(2) There is an equation in 2d quantum gravity that relates the gravitational scaling dimensions with the bare scaling dimensions [13]. It is the so-called Knizhnik-Polyakov-Zamolodchikov (KPZ) equation. Explicitly,

$$\frac{\Delta_{\text{KPZ}}(\Delta_{\text{KPZ}} - 1)}{\gamma_s - 1} - \Delta_{\text{KPZ}} = -\Delta^0 \quad .$$

(3.23)

Here $\gamma_s$ means the string exponent (string susceptibility). In the Liouville theory it is defined via the scaling of the partition function, i.e., $Z[A] \sim A^{\gamma_s - 3}$, where $A$ is the invariant area.

After this is noted, it immediately comes to mind to interpret the relation (1.7) as an analogy of the KPZ equation. First, let us define the string exponent as $\tilde{Z}[r] \sim r^{3 - \gamma_s}$. A motivation for this definition is a simple analogy between $r^{-1}$ and $A$ as they appear in the partition functions. It also assumes that $\int d^2 z \beta \beta e^{-\frac{2}{r^2}}$ can be interpreted as the cosmological operator in the problem of interest. Accepting the above definition, a result we can draw is $\gamma_s = 2$. By substituting this value into Eq.(3.23), we easily find the left hand side of the relation (1.7). However, the missing point of the derivation sketched above is a possible interpretation of the bare dimensions as the masses of the effective scalars in AdS$_3$.

Of course, these conclusions are heuristic and further work is needed to make them more rigorous.

4 Conclusions and Remarks

First let us say a few words about the results.
In this work we have reproduced the basic features of the AdS/CFT correspondence for AdS$_3$ via the world-sheet methods and scaling arguments like in 2d gravity. In doing so, we proposed a stringy way for getting to a vicinity of critical points. In the case of interest we started from the world-sheet description of the CFT in space-time that is the boundary of AdS$_3$. Next we perturbed the world-sheet action by the marginal operator that gave us finite dimensional symmetry in space-time and provided the nonlinear term in the world-sheet action which correspond to $G_{\mu\nu}\partial X^\mu \partial X^\nu$ with the metric of AdS$_3$. We interpreted the result as string theory on AdS$_3$. The scale was introduced via the two-dimensional coupling constant or, equivalently, the radial anti-de-Sitter coordinate. Then we studied the scaling limit via the 2d gravity scaling argument. We found the basic features of the AdS/CFT correspondence within our formalism. Although our procedure looks many ways attractive, we have to stress some its speculative character. It is clear that further work is needed to make this more rigorous.

Let us conclude by mentioning a few problems that are seemed the most important to us.

(i) In fact, all what we found in sections 2 and 3 corresponds to the effective action which is quadratic in the scalar field. It is known that such an action allows one to recover the conformal dimensions of the corresponding operators via their two-point functions. So the open problem is to understand how to compute higher order terms in the effective action within our construction.

(ii) Relations between integrable Field Theories and Conformal Field Theories have been much studied of late. A possible approach to this problem by Zamolodchikov is to perturb CFT by some operators that lead to an integrable FT [26]. The construction we proposed allows us to get to a vicinity of critical points in a stringy way. So it is rather natural to ask whether it may be used to study integrable Field Theories.

(iii) Finally, the problem that obviously deserves more attention is supersymmetry breaking. For instance, $N = 4 \rightarrow N = 2$ or $N = 4 \rightarrow N = 0$. In the framework of our construction it means that we need to find such $\hat{S}_{\text{int}}$ that commutes only with the generators of the rest symmetry.

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References

[1] A. M. Polyakov, Gauge Fields and Strings, Hardwood Academic Publishers, 1987.
[2] O. Aharony, S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, “Large N Field Theories, String Theory and Gravity”, Report No. CERN-TH-99-122, hep-th/9905111.
[3] P. M. Petropoulos, “String Theory on AdS$_3$: Some Open Questions”, Report No. CPTH-PC-732.0899, hep-th/9908189.
[4] J. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231.
[5] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys.Lett. B428 (1998) 105.
   E. Witten, Adv.Theor.Math.Phys. 2 (1998) 253.
[6] P. Breitenlohner, and D. Z. Freedmann, Ann. Phys. 144 (1982) 249.
[7] V. Balasubramanian, P. Kraus, and A. Lawrence, Phys.Rev. D59 (1999) 046003.
[8] I. R. Klebanov and E. Witten, “AdS/CFT Correspondence and Symmetry Breaking,” Report No. PUP-1863, hep-th/9905104.
[9] I. R. Klebanov, “Absorption by Threebranes and the AdS/CFT Correspondence,” presented at Strings ’99, hep-th/9908165.

[10] I. R. Klebanov, Phys.Rev. D51 (1995) 1836; J. L. Barbon, K. Demeterfi, I. R. Klebanov, and C. Schmidhuber, Nucl.Phys. B440 (1995) 189.

[11] F. David, Mod.Phys.Lett.A3 (1988) 819; J. Distler, and H. Kawai, Nucl.Phys. B321 (1989) 509.

[12] A. M. Polyakov, in Les Houches, 1988; I. R. Klebanov, Lectures at the Trieste Spring School, 1991; D. Kutasov, Lectures at the Trieste Spring School, 1991; F. David, in Les Houches, 1992.

[13] A. Giveon, D. Kutasov, and N. Seiberg, Adv.Theor.Math.Phys. 2 (1998) 733.

[14] O. Andreev, Nucl.Phys. B552 (1999) 169.

[15] G. ’t Hooft, “Dimensional Reduction In Quantum Gravity”, in Salamfest 1993, p.284.

[16] L. Susskind, J.Math.Phys. 36 (1995) 6377.

[17] A. B. Belavin, A. M. Polyakov, and A. B. Zamolodchikov, Nucl.Phys. B241 (1984) 333.

[18] Vl. S. Dotsenko, Nucl.Phys. B338 (1990) 747; B 358 (1991) 541.

[19] O. Andreev, “Unitary Representations of Some Infinite Dimensional Lie Algebras Motivated by String Theory on AdS3”, Report No. LANDAU-99/HEP-A2, hep-th/9905002, Nucl. Phys. B. to appear.

[20] D. Friedan, E. Martinec, and S. Shenker, Nucl.Phys. B271 (1986) 93.

[21] R. Kallosh, and A. Tseytlin, JHEP 9810 (1998) 016.

[22] I. Pesando, “On the Quantization of the GS Type IIB Superstring Action on AdS3 × S3 with NSNS Flux”, Report No. DFTT-15-99, hep-th/9903083.

[23] J.-L. Gervais, in Les Houches, 1995.

[24] E. Diaconescu, and N. Seiberg, JHEP 9707 (1997) 001.

[25] N. Seiberg, and E. Witten, JHEP 9904 (1999) 017.

[26] A. B. Zamolodchikov, Adv. Stud. in Pure Math. 19 (1989) 641.