Kwong, Kwok-Kun; Lee, Hojoo
Higher order Wirtinger-type inequalities and sharp bounds for the isoperimetric deficit. (English) Zbl 1478.28002
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Summary: Using Fourier analysis, we derive Wirtinger-type inequalities of arbitrary high order. As applications, we prove various sharp geometric inequalities for closed curves on the Euclidean plane. In particular, we obtain both sharp lower and upper bounds for the isoperimetric deficit.

MSC:
28A75 Length, area, volume, other geometric measure theory
26D10 Inequalities involving derivatives and differential and integral operators
42A38 Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type
53A04 Curves in Euclidean and related spaces
53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)

Full Text: DOI arXiv

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