Generalized Relativistic Meson Wave Function

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Abstract

We study the most general, relativistic, constituent $q\bar{q}$ meson wave function within a new covariant framework. We find that by including a tensor wave function component, a pure valence quark model is now capable of reproducing not only all static pion data ($f_\pi$, $\langle r^2/\pi \rangle$) but also the distribution amplitude, form factor ($F_\pi(Q^2)$), and structure functions. Further, our generalized spin wave function provides a much better detailed description of meson properties than models using a simple relativistic extension of the $S = L = 0$ nonrelativistic wave function.

12.40.Aa, 14.40.Aq, 13.60.-r
I. INTRODUCTION

Since an exact solution to a bound state problem in QCD is still unavailable many approximate treatments have been developed. Among them, the constituent quark model has perhaps received the most attention and is widely regarded as a very efficient and effective tool in description of hadronic phenomena. The complex low energy structure of QCD currently precludes an unambiguous identification of the complete degrees of freedom and it is therefore important to continue to advance, refine and test the valence quark dominance approximation. Prospects are encouraging that new, precision data provided by future CEBAF experiments will significantly clarify this situation and also detail the role of exotic quark and/or gluon configurations. However, before the relative importance of valence versus exotic configurations can be established, uncertainties in the description of hadronic amplitudes due to valence quark model approximations must be reduced. In a previous study we investigated alternative model approaches by comparing different relativistic formulations for the light pseudoscalar mesons. In particular, we developed and examined a covariant variable front approach which permitted quantitatively assessing the relevance of Lorentz covariance for any constituent formulation. We also established that pion and the kaon static properties can be well described by relativistic models utilizing constituent $q\bar{q}$ meson wave functions represented by the product of a noninteracting spinor component and a momentum space orbital amplitude. It is believed that such models will be able to describe low energy mesonic data to within a 10-20%. Unfortunately, there is still a large energy gap between this region and the energy-momentum scale where asymptotic freedom dominates and a clear need exists for an improved, QCD related quark model to describe this intermediate regime. This is especially true for analyzing electromagnetic form factors with $Q^2 > 2 - 3$ GeV$^2$ for which all quark models described in Ref. fail to describe the data. As mentioned above, however, before making any exotic extensions of the quark model an improved framework for the $q\bar{q}$ system must be developed. The purpose of the present paper is to report one such attempt which utilizes a more general quark meson wave
In this article we extend our covariant variable front quark model by incorporating a more general spinor wave function with tensor components. Although not an observable, the wave function is constrained by results from QCD studies of moments of the distribution amplitudes [6–9]. Additional information comes from meson structure functions measured in Drell-Yan experiments [10–12]. In the next section we briefly review the basic assumptions of the covariant quark model, and detail our extended valence meson wave function. In Sec. III we analyze the quark distribution amplitudes and structure functions for the low lying mesons and present numerical results. Finally, we discuss and summarize our major results in Sec.IV

II. VALENCE QUARK MESON WAVE FUNCTION

Following our previous paper we specify the quantization surface \( \Sigma \) by a timelike four vector \( n^\mu \) with \( n^2 = 1 \), although the analysis is also appropriate for the case when \( n^\mu \) is null-like, \( n^2 = 0 \). The transverse and longitudinal components of an arbitrary four vector \( A^\mu \) are denoted as \( A_T \) and \( A_L \), respectively, \( A^\mu = (A_L, A_T) \) and are defined by

\[
A_L \equiv n \cdot A, \quad A_T^\mu \equiv A^\mu - A_L n^\mu.
\]

Since \( n \cdot A_T = 0 \), \( A_T^\mu \) has only three independent components which we label \( (A_T^\perp, A_T^3) \) and henceforth identify by \( A_T \). We define the wave function \( \Psi^\alpha(k_{T_i}, \lambda_i; \tau_i), i = 1 \ldots N \) as the probability amplitude for finding \( N \) constituents (quarks, antiquarks, gluons) with transverse momenta \( k_{T_i} \), helicities \( \lambda_i \) and flavor-color components \( \tau_i \) in a meson state \( \alpha \), \( (\alpha = 1 \ldots 8 \) for the pseudoscalar octet) with momentum \( P_T = \sum_i k_{Ti}, P_L = \sqrt{M^2 + |P_T^2|} \) by the following matrix element

\[
\langle k_{T_i}; \lambda_i; \tau_i | P_T; \alpha \rangle = (2\pi)^3 \sqrt{2P_L} \delta^3(P_T - \sum_i k_{Ti}) \left[ \prod_i \sqrt{\frac{k_{iL}}{m_i}} \right] \Psi^\alpha(k_{Ti}; \lambda_i; \tau_i).
\]  

Here the single particle states \( |k_T; \lambda; \tau \rangle \) describe effective, massive constituents quantized on
the spacelike surface $\Sigma$ perpendicular to $n^\mu$ and the longitudinal momenta are constrained by the on shell condition,

$$k_{iL} = \sqrt{m_i^2 + |k_{iT}^2|}. \quad (2.2)$$

The single particle states are normalized according to

$$\langle k'_T, \lambda', \tau' | k_T, \lambda, \tau \rangle = \frac{P_L}{2(2\pi)^3} \delta^3(k'_T - k_T) \delta_{\lambda' \lambda} \delta_{\tau' \tau} \quad (2.3)$$

for fermions and

$$\langle P'_T, \alpha | P_T, \beta \rangle = 2(2\pi)^3 P_L \delta^3(P'_T - P_T) \delta_{\alpha \beta} \quad (2.4)$$

for bosons.

The set of wave functions defined by Eq. (2.1) constitutes a Lorentz group representation basis. In a formalism with a fixed quantization surface, Lorentz transformations depending on interactions do not leave $\Sigma$ invariant interactions do not conserve particle number and therefore as the system evolves different Fock sectors mix [14]. This in general leads to complicated transformation properties for the wave functions under the action of interaction dependent generators of the Lorentz group. A simplification is usually made by using an interaction free transformation rule for the wave function in a given Fock sector [15]. The valence $q\bar{q}$ wave functions describing a meson state with 4-momenta $P$ and $P' = \Lambda P \equiv \mathcal{L}(P \rightarrow \Lambda P)P$ respectively are thus related by

$$\frac{1}{\sqrt{2P_L'}} \left[ \prod_i \sqrt{\frac{k_i^L}{m_i}} \right] \Psi^\alpha(k_{Ti}; \lambda_i; \tau_i) = \prod_i \sum_{\lambda'_i} D_{\lambda_i \lambda'_i}(R_W) \frac{1}{\sqrt{2P_L'}} \left[ \prod_i \sqrt{\frac{k'_i^L}{m_i}} \right] \Psi^\alpha(k'_{Ti}; \lambda'_i; \tau_i) \quad (2.5)$$

with $k'_i = \Lambda k_i$ and $D_{\lambda \lambda'}(R_W)$ denoting the matrix representations of the Wigner rotation $R_W$ in the spin space expressed in terms of $q\bar{q}$ momentum variables,

$$R_W = R_W(k_{iT}) = \mathcal{L}(\Lambda P \rightarrow \hat{P}) \mathcal{L}(P \rightarrow \Lambda P) \mathcal{L}(\hat{P} \rightarrow P),$$

and $\hat{P} = (M, 0)$ being the meson rest frame momentum. Using this approximation the meson state is defined in a standard way and contains only $q\bar{q}$ valence component. Eqs. (2.1), (2.3) and then lead to the following wave function normalization
\[ \sum_{\lambda, \tau} \int [dk_{iT}]^P \Psi^\dagger \alpha(k_{iT}; \lambda; \tau) \Psi^\beta(k_{iT}; \lambda; \tau) = \delta^\alpha \beta, \]  
\[ [dk_{iT}]^P = [dk_{iT}]_P \equiv 2 \prod_i \frac{d^3k_{iT}}{(2\pi)^3} \delta^3(P_T - \sum_i k_{iT}). \]  

(2.6)

In general, approximations and specifically Fock space truncations destroy covariance. Here the truncation generates noncovariance by the emergence of an unphysical dependence of matrix elements upon \( n^\mu \). In \[4\] we describe a method for restoring covariance by allowing the quantization surface \( \Sigma \), or equivalently the quantization vector \( n^\mu \), to transform actively under Lorentz transformations by relating \( n^\mu \) to the meson external momenta.

The normalization Eq. (2.6) is identical to the corresponding nonrelativistic expression in the meson rest frame, since the relativistic wave function is constructed to reduce to the nonrelativistic one in this frame. For the ground state pseudoscalar octet the form of the relativistic wave function is then derived from Eq. (2.5) to be

\[ \Psi^\alpha(k_{iT}; \lambda_i; \tau_i) = \chi^\alpha_{\tau_1 \tau_2} \xi(k_{iT}; \lambda_i) \Phi(\mathcal{M}^2), \]
\[ \chi^\alpha_{\tau_1 \tau_2} \equiv i \left[ \frac{\lambda^\alpha}{\sqrt{2}} \otimes \frac{I}{\sqrt{3}} \right], \]
\[ \xi(k_{iT}; \lambda_i) \equiv \sqrt{2} \frac{\sqrt{m_1 m_2}}{\sqrt{\mathcal{M}^2 - (m_1 - m_2)^2}} \pi(k_{iT}; \lambda_1) \gamma_5 v(k_{2T}; \lambda_2) \]  

(2.7)

where \( \lambda^\alpha \) are the Gell-Mann \( SU(3) \) flavor matrices, \( I \) is the identity matrix in the color space, \( m_1, m_2 \) are the quark and antiquark constituent masses respectively, \( \mathcal{M} \) is the \( q \bar{q} \) invariant mass,

\[ \mathcal{M}(k_i) \equiv (k_1 + k_2)^2 = (k_{1L}(k_{1T}) + k_{2L}(k_{2T}))^2 + (k_{1T} + k_{2T})^2, \]  

(2.8)

with \( k_{iL}(k_{iT}) \) given be Eq. (2.2) and \( \Phi(\mathcal{M}) \) being the spin independent orbital wave function usually assumed gaussian,

\[ \Phi(\mathcal{M}) = N \exp \left[ -\frac{\mathcal{M}^2}{8\beta^2} \right]. \]  

(2.9)

The overall normalization constant \( N \) is determined from Eq. (2.6).

As explained above the explicit form of the transverse variables will depend on the choice for \( n^\mu \) which in turn will be specified after selecting which matrix element is to be calculated.
with the wave function of Eq. (2.7). The wave function specified by Eq. (2.7) has also been extensively studied in [4] for fixed quantization schemes. Here we study the extension of Eq. (2.7) to the most general Dirac structure for the $q\bar{q}$ system. We write the spinor component, $\xi(k_{iT}; \lambda_i)$ of the wave function in the general form,

$$\xi(k_{iT}; \lambda) = \sum_p \Gamma_p u(k_{T1}, \lambda_1) v(k_{T2}, \lambda_2).$$

(2.10)

with the Lorentz $4 \times 4$ matrices $\Gamma_p$ represented by combinations of Dirac matrices and the constituent momenta, $k_i$. Using the Dirac equations for the free $u$ and $v$ spinors it is easily shown that sum in Eq. (2.10) reduces to two terms involving either $\gamma_5$ or $[k_1, k_2] \gamma_5$. The most general wave function $\Psi^\alpha$ for a pseudoscalar meson can thus be written in the form

$$\Psi^\alpha(k_{iT}; \lambda, \tau; \tau) = \chi^\alpha \gamma_5 \Phi_P(\mathcal{M}) + \sqrt{2 \frac{m_1 m_2}{M^2 - (m_1 - m_2)^2}} \left[ [k_1, k_2] \gamma_5 \Phi_T(\mathcal{M}) \right] \sqrt{2 \frac{m_1 m_2}{M^2 - (m_1 - m_2)^2}} \Phi(\mathcal{M}).$$

(2.11)

We shall assume that the two spin independent wave functions appearing in Eq. (2.11) can be parameterized with a gaussian shape of Eq. (2.9) having the same momentum size parameter $\beta$

$$\Phi_P(\mathcal{M}) = \Phi(\mathcal{M}), \Phi_T(\mathcal{M}) = r_{PT} \frac{1}{M \beta} \Phi(\mathcal{M}).$$

(2.12)

Since the tensor term in Eq. (2.11) explicitly involves higher powers of the constituent longitudinal and transverse variables whose ranges are related to $\mathcal{M}$ and $\beta$, respectively, we have chosen the $\mathcal{M} \beta$ factor as the relative normalization between the $\Phi_P$ and $\Phi_T$ terms in Eq. (2.12). The dimensionless, numerical coefficient $r_{PT}$ will then be determined by fitting various properties of the pion. In the $r_{PT} = 0$ limit the wave function of Eq. (2.11) reduces to the one of Eq. (2.7) and corresponds to the $S = L = 0$ state in the meson rest frame. The tensor term introduces an $S = L = 1$ orbital component which mixes the $S = L = 1$ lower with $S = L = 0$ upper components of the Dirac spinors to give an overall $J^P = 0^-$ state.

III. NUMERICAL RESULTS
A. \( \pi \) electromagnetic form factor

Before detailing the quark distributions given by the generalized wave functions including the additional tensor component \( \Phi_T \) we shall present new results for the pion electromagnetic form factor. The form factor calculation for both noncovariant and covariant variable front models has been previously summarized \([4]\). For simplicity we maintain the same quark masses and oscillator size parameter \( \beta \) used in our previous calculations with \( m_q = \beta = 250 \text{ MeV} \) and vary \( r_{PT} \) to optimize the form factor description. It is significant to note that the tensor term drastically modifies the form factor behaviour especially in the high momentum transfer region \( Q^2 > 2 \text{ GeV}^2 \). Further as shown in Fig. 1 the generalized model provides an agreement with the data in this region without altering the correct low \( Q^2 \) behaviour. As shown below the importance of the tensor term is further demonstrated in the analysis of the distribution amplitude and the structure functions.

B. Distribution amplitude

The quark distribution amplitude for a meson \( \alpha \), \( \phi^\alpha(\xi) \), can in principle be obtained from the moments \( \langle \xi^n \rangle \), \([3]\)

\[
\langle \xi^n \rangle = \int d\xi \xi^n \phi^\alpha(\xi) \quad (3.1)
\]

which are formally defined by the matrix elements

\[
\langle 0 | \psi(0) \gamma^\mu \gamma_5 \frac{\lambda^\beta}{2} \partial_{\mu_1} \cdots \partial_{\mu_n} \psi(0) | P; \alpha \rangle = if_\alpha \langle \xi^n \rangle \delta_{\alpha\beta} P^\mu P_{\mu_1} \cdots P_{\mu_n} \quad (3.2)
\]

where the trace terms corresponding to higher twist operators have been omitted. Here \( f_\alpha \) is the meson decay constant and the normalization is such that \( \langle \xi^0 \rangle = 1 \). The expansion of the quark field operators, \( \psi, \overline{\psi} \) in terms of constituent \( q\overline{q} \) creation and annihilation operators Eqs. (2.11), (2.12) leads to

\[
\langle \xi^n \rangle = \frac{\sqrt{6}}{f_\alpha} \int \frac{d^3 q_T}{(2\pi)^3} \frac{1}{\sqrt{M_\alpha}} \frac{m_1 m_2}{\sqrt{k_{1L} k_{2L} M^2 - (m_1 - m_2)^2}} \left( \frac{2q_T^2}{M_\alpha} + \frac{k_{1L} - k_{2L}}{M_\alpha} \right)^n \Phi(M) \quad (3.3)
\]
\[
\left[ \frac{k_1 m_2 + k_2 L m_1}{m_1 m_2} - r_{PT} \frac{2((k_1 L + k_2 L)^2 - (m_1 + m_2)^2)}{\beta(m_1 + m_2)} \left(1 - \frac{(m_1 - m_2)(k_1 L m_2 - k_2 L m_1)}{(k_1 L + k_2 L)m_1 m_2}\right) \right] \\
\] 

(3.3)

with \( q_T = (q_T^3, q_T^\perp) \),

\[
k_i L = \sqrt{m_i^2 + |q_T^2|},
\]

\[
\mathcal{M} = k_1 L + k_2 L,
\]

(3.4)

and \( M_\alpha \) being the meson mass. The decay constant \( f_\alpha \) is calculated from Eq. (3.3) using \( \langle \xi^0 \rangle = 1 \) (see also Sec.V in Ref. [4]). In QCD, it can be shown that the physical part of the distribution amplitude \( \phi^\alpha(\xi) \) is restricted for \(-1 < \xi < 1 \) and for large \( n \) the moments in Eq. (3.2) behave as \( \langle \xi^n \rangle \to 1/n^2 \) implying that \( \phi^\alpha(\xi \to \pm 1) \to 0 \). In order to reproduce this feature in our model calculation we make the replacement

\[
M_\alpha \to \mathcal{M} = k_1 L + k_2 L = \sqrt{m_1^2 + |q_T^2|} + \sqrt{m_2^2 + |q_T^2|}.
\]

(3.5)

In Table 1 we list values for both \( \pi \) and \( K \) mesons decay constants calculated from Eq. (3.3) using spin averaged meson masses \( M_\pi = 610 \text{ MeV} \) and \( M_K = 790 \text{ MeV} \) and compare with results using dynamical masses for \( M_\alpha \) determined by Eq. (3.5). Note that the experimental values lie almost midway between the two methods. The sensitivity to the mass prescription for normalized quantities like moments of the distribution amplitude \( \langle \xi^n \rangle \) or structure functions, which we analyze in the following subsection, is even smaller. Using Eq. (3.5) in Eq. (3.3) we make the following change of variables

\[
q_T^3 \to \xi(q_T) \equiv \frac{2q_T^3}{\mathcal{M}} + \frac{k_1 L - k_2 L}{\mathcal{M}}.
\]

(3.6)

For fixed \( q_T^\perp \) the variable \( \xi(q_T) \) maps the domain of the variable \( q_T^3 \), \(-\infty < q_T^3 < \infty \) into the finite interval \(-1 < \xi(q_T) < 1 \). The expression for the distribution amplitude can now be obtained from Eqs. (3.1), (3.3), (3.5) and (3.6) and is given by

\[
\phi^\alpha(\xi) = \frac{\sqrt{6}}{f_\alpha} \int \frac{d^2q_T^\perp}{(2\pi)^3} \frac{1}{\sqrt{\mathcal{M}}} J(q_T^\perp, \xi) \frac{m_1 m_2}{\sqrt{k_1 L k_2 L \mathcal{M}^2 - (m_1 + m_2)^2}} \Phi(\mathcal{M}) \left[ \frac{k_1 L m_2 + k_2 L m_1}{m_1 m_2} - r_{PT} \frac{2((k_1 L + k_2 L)^2 - (m_1 + m_2)^2)}{\beta(m_1 + m_2)} \left(1 - \frac{(m_1 - m_2)(k_1 L m_2 - k_2 L m_1)}{(k_1 L + k_2 L)m_1 m_2}\right) \right]
\]

(3.7)
with $k_{iL}$ and $\mathcal{M}$, now functions of $q_T^\perp, \xi$, obtained from Eq. (3.4) using Eq. (3.6) to express $q_T^3 = q_T^3(\xi)$. $J(q_T^\perp, \xi)$ is the Jacobian of the transformation from $(q_T^3, q_T^\perp)$ to $(\xi, q_T^\perp)$. The explicit forms of the functions, $k_{iL}(q_T^\perp, \xi)$, $\mathcal{M}(q_T^\perp, \xi)$ and $J(q_T^\perp, \xi)$ are given in the Appendix.

In Figs. 2 and 3 we plot the function $\phi(\xi)$ for $\pi$ and $K$ mesons respectively. In Fig. 2 the solid line gives our prediction for the set of parameters $m_q = \beta = 250$ MeV, $r_{PT} = 1.0$ which best describe the form factor shown in Fig. 1. The dashed line is the prediction for the pion distribution amplitude without the tensor term in the wave function of Eq. (2.11), i.e. with $r_{PT} = 0$. The wide, camel like shape of the distribution amplitude obtained for $r_{PT} = 1.0$, a value that optimizes the form factor description, is similar to that provided by the QCD sum rule approach for the matrix element of Eq. (3.2), however, the predictions for the lowest moments $\langle \xi^2 \rangle = 0.27, \langle \xi^4 \rangle = 0.13$, are smaller then the ones of Chernyak and Zhitnitsky ($\langle \xi^2 \rangle_{CZ} = 0.40, \langle \xi^4 \rangle_{CZ} = 0.24$) yet closer to those obtained in Ref. and lattice calculations. The asymmetry in the $K$ distribution amplitude in Fig. 3 is due to the large $SU(3)$ breaking due strange quark mass ($m_s = 480$ MeV). Notice the sensitivity of the pion distribution amplitude to $r_{PT}$ which is shown in Fig. 4. The largest sensitivity is observed for $r_{PT} \sim 0.5$ where the interference between the pseudoscalar and tensor terms dominate.

C. Structure functions

Structure functions contain important hadronic information and are obtained from inelastic processes. Accordingly we wish to further test our approach by computing the meson structure functions and comparing with available data usually extracted from Drell-Yan lepton [10,11] or charged hadron production [12] on nuclear targets using mesonic beams. The extraction from hadron production experiments is typically more complicated and somewhat model dependent due to uncertainties in the hadronization mechanism. In this paper the experimental pion structure function was extracted from the Drell-Yan muons produced by 252 GeV pions on tungsten [11]. The theoretical cross section for muon production as a
function of longitudinal momentum fraction $\xi_F$ of the muon pair is given by

$$\frac{d^2\sigma}{dQ^2d\xi_F} = K \frac{4\pi\alpha^2}{9Q^4} \left[ f_\nu^\pi(x_1)G^N_\nu(x_2) + f_s^\pi(x_1)G^N_s(x_2) \right] \frac{\xi_F^2 + 4Q^2/s}{(\xi_F^2 + 4Q^2/s)^{1/2}},$$

$$x_{1,2} = [\pm \xi_F + (\xi_F^2 + 4Q^2/s)^{1/2}]/2,$$  \hspace{1cm} (3.8)

where $Q^2$ and $s$ are the mass of the muon pair and the square of the c.m energy respectively, and $\alpha$ is the electromagnetic fine structure constant. $f_\nu^\pi(x_1)$ is the pion valence (sea) quark structure function and $G^N_\nu(x_2)$ parameterizes the nuclear contribution. Assuming isospin symmetry for $\pi^-$ we have

$$f_\nu^\pi(x) = x\overline{\nu}(x) = xd(x),$$

$$f_s^\pi(x) = xu_s(x) = x\overline{u_s}(x) = \ldots = x\overline{s}(x).$$  \hspace{1cm} (3.9)

The nuclear contributions can be similarly parameterized in terms of valence and sea quark distributions of individual nucleons. The normalization ($K$ factor) is measured to be $K \sim 1.75 \pm 0.13$ while a perturbative analysis to first order in $\alpha_s$ gives $K \sim 1.4$. The structure functions are in principle functions of of $x_i$ and $Q^2$, however, since we are describing the average data for $36.0 < Q^2 < 72.3$ GeV$^2$ we have suppressed the explicit $Q^2$ dependence. The meson structure functions can equivalently be defined in terms of a diagonal matrix element involving the commutator of two vector currents \[16\]

$$W_\mu^\nu(x, Q^2) = \frac{1}{4\pi} \int dz e^{iqz} \langle p| [\bar{\psi}_i \gamma^\mu \psi_i(z), \bar{\psi}_i \gamma^\nu \psi_i(0)]|p \rangle$$  \hspace{1cm} (3.10)

with $x = p \cdot q/M^2$, $Q^2 = -q^2 > 0$ and $i$ referring to a particular flavor. Since it is known that the total longitudinal momentum of the hadron is only partially distributed among quarks, the constituent quarks which are assumed to carry the entire momentum of the hadron cannot be identified with the partons contributing to deep inelastic structure functions. In the scaling limit, $Q^2 \to \infty$, Eq. (3.10) can be calculated using a short distance expansion of the bilocal operator. In perturbative QCD scaling violations can be described in terms of a convolution of the partonic distributions defined at a scale of reference $Q_0^2 < \infty$ and the Altarelli-Parisi splitting functions which characterize the single parton response amplitudes
for the change of scale due to radiation of gluons \[17\]. Here we also use the convolution approach \[18\] to relate our constituent quark model to the parton model. In a convolution model the quark distribution function for a hadron \( \alpha \), \( q^\alpha \), is represented by a product of the distribution function of a constituent quark, \( Q^\alpha \), in a hadron and the probability for a constituent quark to fragment into a QCD parton \( i \), \( q/i/v \),

\[
q^\alpha_i(x, Q^2) = \int_x^1 \frac{dy}{y} Q^\alpha_v(x, Q^2_0) q/i/v(y, Q^2/Q^2_0).
\]

The constituent quark distributions are defined through Eq. (3.10) with the QCD fields replaced by an effective constituent quark/gluon basis at \( Q^2_0 \sim 1 \text{ GeV}^2 \). The \( Q^2 \) evolution of \( q/i/v(x/y, Q^2/Q^2_0) \) is governed by perturbative QCD. However for any value of \( Q^2 \) phenomenological input is still required. For the average \( Q^2 \sim 50 \text{ GeV}^2 \) of the Drell-Yan data the valence and sea quark distributions are

\[
q^\pi_v(x) = \int_x^1 \frac{dy}{y} Q^\pi_v(x) q/v_v(y),
q^\pi_s(x) = \int_0^1 \frac{dy}{y} Q^\pi_v(x) q/s_v(y).
\]

For the valence quark contributions in \( \pi^- \), \( q_v = q_d = q^\pi_v \), while for the sea \( q_s = q_d = q^\pi_s = \cdots q^\pi \). The number and momentum sum rules are respectively

\[
\int_0^1 dx q^\pi_v(x) = 1,
2 \int_0^1 dx x q^\pi_v(x) + 6 \int_0^1 dx x q^\pi_s(x) = 1 - g_\pi,
\]

where \( g_\pi \) represents the fraction of the gluon momentum in the pion currently measured as \( g_\pi \sim 0.47 \) \[11\]. The parton distributions in a constituent quark, \( q/i/v \), are usually normalized to describe the relevant features of the low-\( x \) hadron scattering phenomenology. The Regge behaviour at small \( x \) motivates the following parameterization \[18\]

\[
q_v/(x) = \frac{\Gamma \left( A + \frac{1}{2} \right)}{\Gamma \left( \frac{1}{2} \right) \Gamma(A)} \frac{1}{\sqrt{x}} (1 - x)^{A-1},
q_s/(x) = \frac{C}{x} (1 - x)^{D-1}
\]

(3.14)
with the parameters $A, D$ and $C$ constrained by the momentum sum rule of Eq. (3.13),

$$\frac{1}{2A+1} + \frac{C}{D} = 1 - g_\pi$$

Each can be determined by comparing the calculated quark distributions $q_\pi^v$ and $q_\pi^s$ to the data. The matrix element which determines the constituent quark distribution functions obtained from the light cone expansion of the current product in Eq. (3.10) has a very similar structure to Eq. (3.2) that defines the quark distribution amplitude. Using the techniques developed in the previous section the following expression for $Q^\alpha_v(x)$ is easily derived,

$$Q^\alpha_v(x) = \int \frac{d^2 q_T^\perp}{(2\pi)^3} J(q_T^\perp, \xi) |\Phi(M)|^2 \left[ 1 + r_{PT} \frac{M^2 - (m_1 + m_2)^2}{\beta M} \right]^2. \quad (3.15)$$

For $\alpha = \pi^-$, $m_1 = m_2$ the integrand is a symmetric function of $\xi = 2x - 1$ yielding $Q^\pi_\pi^\perp(x) = Q^\pi_d^\perp(x)$. For the kaon, $m_2 = m_s > m_1 = m_u = m_d$ and the nonstrange and strange quark distributions correspond to $\xi = 2x - 1$ and $\xi = -2x + 1$ respectively. Again $M = M(q_T^\perp, \xi)$ and $J(q_T^\perp, \xi)$ are specified in the Appendix.

In Fig. 5 we plot valence quark structure functions for the pion calculated in our model with $r_{PT} = 1.0$ (solid line) obtained from the form factor fit and the result without the tensor component having $r_{PT} = 0$ (dashed line) and compare with data. The comparison suggests the importance of the tensor term at large $x$, $0.6 < x < 0.9$ where the sensitivity to the constituent quark distributions is the highest. In Fig. 6 we also compare our results for the sea quark distributions (solid line) with the curve used for experimental fitting (dashed line). The value $C = 0.086$ for the parameter in Eq. (3.14) is obtained by requiring the theoretical and experimental curves to agree at $x = 0$. Fitting $A$ by the valence quark distribution yields $A = 0.75$ and the sum rule of Eq. (3.13) then gives $D = 4.0$. These numbers are in a good agreement with the ones obtained from the unpolarized nucleon structure function fits confirming hadron independence of the splitting functions $q_{i/v}(x)$ [18,19]. In Fig. 7 we also show our predictions for the strange (solid line) and light quark structure functions (dashed line) in kaon.
IV. SUMMARY AND CONCLUSIONS

Within the framework of our covariant variable front quark model, we have generalized the constituent, $q\bar{q}$ pion wave function and have studied the distribution amplitudes and structure functions. The extended model provides excellent agreement with the experimental data for the structure functions and the electromagnetic form factor as well as a good description of the decay constant. The improved description is due to a tensor component in the wave function which is a relativistic correction in the rest frame. We have computed the structure functions and the distribution amplitudes with the wave function of Eq. (2.11) in a noncovariant light cone quantization scheme and find results are similar to the variable front model. This confirms front independence of our results and is consistent with a previous assertion [4] that different front formulations do not lead to significant differences in the predictions for various mesonic properties. The magnitude of the tensor term which optimizes both the form factor and structure function now indicate a camel shape for the distribution amplitude with the dip at $\xi = 0$ although not as profound as the one suggested by the old QCD sum rules, but quite similar to recent nonlocal QCD sum rule calculations and lattice results. Since the shape and the moments of the quark distribution amplitude are very sensitive to the interference between the pseudoscalar and tensor components of the wave function the detailed knowledge of former will provide important information on the structure of the meson wave function. As mentioned in the introduction, the distribution amplitude is not directly measurable, however, information on the magnitude of the lowest moments can be extracted from the meson form factor describing the hadronic part of the $e$ meson $\rightarrow e\gamma$ transitions [6,20]. Finally, distribution amplitude studies provide an effective forum for theoretical comparisons of alternative model formulations which can provide significant insight into QCD dynamics.
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V. APPENDIX

The variable change of Eq. (3.6) gives,

\[
q^3_T(q^1_T, \xi) = \frac{\mu^2_1(q^1_T)(1 - \xi)^2 - \mu^2_2(q^1_T)(1 + \xi)^2}{\sqrt{8(1 - \xi^2)[\mu^2_1(q^1_T)(1 - \xi) + \mu^2_2(q^1_T)(1 + \xi)]}}
\]

where

\[
\mu^2_i(q^1_T) \equiv m_i^2 + (q_T^1)^2,
\]

\[
k_{iL}(q^1_T, \xi) = \sqrt{\mu^2_i(q^1_T) + (q^3_T(q^1_T, \xi))^2},
\]

\[
\mathcal{M}(q^1_T, \xi) = k_{1L}(q^1_T, \xi) + k_{2L}(q^1_T, \xi)
\]

and the Jacobian \( J(q^1_T, \xi) \) is given by

\[
J(q^1_T, \xi) = 2\frac{\xi}{1 - \xi^2}[-\bar{\pi}^2 + (\Delta \mu)^2 - 2\xi \bar{\pi} \Delta \mu] - \frac{2}{1 - \xi^2} \Delta \mu \bar{\pi} - 2\xi \frac{\bar{\pi}^2(\Delta \mu)^2}{\bar{\pi}^2 + (\Delta \mu)^2 - 2\xi \bar{\pi} \Delta \mu} + 2(1 - \xi^2) \frac{\bar{\pi}^3(\Delta \mu)^2}{\bar{\pi}^2 + (\Delta \mu)^2 - 2\xi \bar{\pi} \Delta \mu},
\]

with

\[
\bar{\pi} = \bar{\pi}(q^1_T) \equiv \frac{1}{2}(\mu_1(q^1_T) + \mu_2(q^1_T)),
\]

\[
\Delta \mu = \Delta \mu(q^1_T) \equiv \frac{1}{2}(\mu_1(q^1_T) - \mu_2(q^1_T)).
\]
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FIGURES

FIG. 1a,b. Pion electromagnetic form factor. Solid line shows the result obtained with the $q\bar{q}$ wave function of Eq. (2.12) for $r_{PT} = 1.0$. Dashed line is the result of the covariant model of Ref. [4], dash-doted and doted lines are the results of relativistic models from Refs. [2] and [3] respectively. Data is taken from Ref. [21].

FIG. 2. Pion distribution amplitude with ($r_{PT} = 1$, solid curve) and without ($r_{PT} = 0$, dashed curve) the contribution from the tensor term in the $q\bar{q}$ wave function.

FIG. 3. Same as Fig.2 for the $K$ distribution amplitude.

FIG. 4. $r_{PT}$ dependence of the pion distribution amplitude.

FIG. 5. Valence quark pion structure function $f^\pi_v(x) = xq_v(x)$ (curve convention same as in FIG. 2.). Data is taken from Ref. [11].

FIG. 6. Sea quark pion structure function. Solid line gives our model prediction with $r_{PT} = 1.$, dashed line shows experimental parameterization, $f^\pi_s(x) = 0.173(1 - x)^{8.4}$ of Ref. [11].

FIG. 7. Valence light quark (dashed line) and strange quark (solid line) kaon structure functions.
TABLE I. $\pi$ and $K$ mesons decay constant calculated with the formula of Eq. (3.3). The two set of results correspond to the use of spin averaged, constituent meson masses $M$ and dynamical masses $\mathcal{M}$ respectively, ($m_u = m_d = 250\text{MeV}$, $m_s = 480\text{MeV}$, $\beta = 250\text{MeV}$, $r_{PT} = 1.0$)

| $M$  | $f_\pi\,[\text{MeV}]$ | $f_K\,[\text{MeV}]$ |
|------|-----------------------|----------------------|
|      | 101.                  | 137.                 |
| $\mathcal{M}$ | 75.                  | 104.                 |
| exp. | 93.2                  | 113.                 |
$Q^2 F_\pi(Q^2)[GeV^2]$
\[ \phi_\pi(\xi) \]
