AN ITERATIVE SCHOOL DECOMPOSITION ALGORITHM FOR SOLVING THE MULTI-SCHOOL BUS ROUTING AND SCHEDULING PROBLEM

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ABSTRACT
Servicing the school transportation demand safely with a minimum number of buses is one of the highest financial goals for school transportation directors. To achieve that objective, a good and efficient way to solve the routing and scheduling problem is required. Due to the growth of the computing power, the spotlight has been shed on solving the combined problem of the school bus routing and scheduling. A recent attempt tried to model the routing problem by maximizing the trip compatibilities with the hope of requiring fewer buses in the scheduling problem. However, an over-counting problem associated with trip compatibility could diminish the performance of this approach. An extended model is proposed in this paper to resolve this issue along with an iterative solution algorithm. This extended model is an integrated model for multi-school bus routing and scheduling problem. The result shows better solutions for 8 test problems can be found with a fewer number of buses (up to 25%) and shorter travel time (up to 7% per trip).

Keywords: routing and scheduling, integrated model, trip compatibility, over-counting, school decomposition algorithm
INTRODUCTION

Transferring students from their homes to schools on a daily basis is an important and expensive task. The major contributor to the transportation costs is the number of buses. In order to calculate the number of buses, two sub-problems, namely trip generation and scheduling, need to be solved. To better explain the problem, we first define a few terminologies:

A Trip: An afternoon trip starts from a school, sequentially goes through a set of school stops associated with this school while satisfying the capacity constraint, and maximum ride time constraints;

A Bus: Starts from the depot, sequentially serves a set of trips and goes back to the depot; the trips that are served by one bus are compatible with each other;

Potential compatible trip pair: (trip 1, trip 2) is a compatible trip pair if a bus has enough time (deadhead) to travel from the last stop in trip 1 to the first stop of trip 2 before trip 2’s start time after serving trip 1;

Realized compatible trip pair: trip 1 is served right after trip 2 by the same bus in the scheduling plan.

Trip generation is the process of constructing trips. Given a set of stops, the distances between them, and the students at each stop, trip generation models construct trips that achieve certain goals. Then, these generated trips become the input for the scheduling problem. The scheduling problem groups potential compatible trips and serves them using minimum number of buses. Those compatible trip pairs that are actually served in the scheduling plan are the realized compatible trip pairs. This routing and scheduling decomposition procedure introduces a problem, that is due to the lack of knowledge about the realized compatible trip pair, we cannot directly minimize the number of buses in the trip generation stage. Hence, the adoption of surrogate objective in the trip generating stage (minimizing the number of trips, and/or minimizing the total travel time), inheritably, makes the decomposed solution worse than the solution from the integrated routing and scheduling model.

A few attempts have been made to solve the integrated school bus routing and scheduling problem. One of them came from (1). Due to an over-counting issue which will be discussed in section 3, although they introduced trip compatibility into the routing problem, it is not an integrated model. In this study, we showed that by adding a few constraints into the SWH model in (1), the routing and scheduling problem can be solved simultaneously, which makes it an integrated model. An iterative school decomposition solution algorithm is proposed to solve this integrated model.

The remaining of paper is structured as follows. We first present a literature review for the school bus routing problem and scheduling problem. Then we present the integrated model which simultaneously does the routing and scheduling. Later, we present an iterative solution algorithm that is used for solving the model, and the results are discussed. Finally, the conclusions are presented, and some future research steps are suggested.

LITERATURE REVIEW

A thorough examination on the classification of the school bus routing problem is presented in (2) based on the problem characteristics and solution methods. Some of the recent work on the school bus routing and scheduling (SBRS) is listed in TABLE 1. As it can be seen, the most widely-used objective of the routing problem is the minimization of the total travel time or travel distance while the most prominent objective of the scheduling problem is the minimization of the number of buses.
TABLE 1 Literature Review of school bus routing problem

| Author            | Year | Objective | Constraints | # of School | Fleet Mix | Data                                                                 |
|-------------------|------|-----------|-------------|-------------|-----------|----------------------------------------------------------------------|
| Fügenschuh        | 2009 | NOB       | SCH         | M           | HO        | Five counties in Germany, up to 102 schools, 490 trips               |
| Díaz-Parra et al. | 2012 | TTD, NOT  | LOG, C, MRT, SBL | S        | HO        | Artificial, 50 problems, each problem has 200 bus stop              |
| Kim et al.        | 2012 | NOB       | LOG, SCH    | M           | HO/HT     | Artificial, up to 100 schools, 562 trips, and 28175 students        |
| Park et al.       | 2012 | NOT       | LOG, C, MRT, TW | M        | HT        | Artificial, up to 100 schools, 2000 stops, 32048 students           |
| Schittekat et al. | 2013 | TTD       | LOG, C, SBL | S           | HO        | Artificial, up to 80 stops, 400 students                            |
| Caceres et al.    | 2014 | TTD, NOT  | LOG, C, MRT, SBL | M        | HO        | Williamsonsville Central School District, 13 schools, up to 177 stops and 1237 student per school |
| Faraj et al.      | 2014 | TTD       | LOG, C, MRT | M           | HT        | 944 students, 23 schools, Brazilian city                           |
| Kinable et al.    | 2014 | TTD       | LOG, C, MRT | M           | HT        | Artificial, up to 40 stops and 800 students                         |
| Bögl et al.       | 2015 | TTD, PLT  | LOG, C, MRT, SBL | M        | HO        | Artificial, up to 8 schools, 500 students                           |
| Chen et al.       | 2015 | TTD, NOB  | LOG, SCH    | S           | HO/HT     | Benchmark problems from Park, Tae and Kim 2012; Kim, Kim and Park, 2012 |
| Kang et al.       | 2015 | NOB, TTD  | LOG, C, MRT | M           | HT        | 26 students, six schools, three buses                              |
| Kumar and Jain    | 2015 | TTD       | LOG, LOG    | S           | HO        | Artificial, up to 40 schools, 235 trips, 11600 students             |
| Mushi et al.      | 2015 | TTD       | LOG, C      | S           | HO        | 58 stops, 456 students, Dar es Salaam, Africa                      |
| Santana et al.    | 2015 | TTD       | LOG, C, TW  | S           | HO        | 600 students, 440 nodes, Bogota, Colombia                           |
| Silva et al.      | 2015 | TTD       | LOG, C, SBL | M           | HT        | 716 students, 23 schools, Brazilian city                           |
| Yan et al.        | 2015 | TTD, PLT  | LOG, LOG    | S           | HO        | Six universities (treated as one school), 400 students, Taiwan      |
| Yao et al.        | 2016 | TTD       | LOG, C, MRT | M           | HO        | Artificial, up to 2 schools, 116 stops, and 1088 students           |
| de Souza Lima et al. | 2017 | TTD, NOT, BAL | LOG, C | M       | HT        | Artificial, up to 20 schools, 150 stops; Benchmark from Park, Tae and Kim 2012 |
| Shafahi et al.    | 2017 | TTD, NOT, TC | LOG, C, SBL, TCC, TW | M       | HO        | Artificial, up to 25 schools, 200 stops, 3656 students              |

Note: NOT: Number of trips; NOB: Number of buses; TTD: Total travel distance (or travel time); MRT: Maximum ride time; PLT: Penalty; BAL: Balance between each trip; TC: Trip compatibility; LOG: Logistic constraints (formulate the trip, same as defined in (3)); C: Capacity Constraint; TW: Time window constraint; MWD: Maximum walk distance; SBL: Sub-tour elimination constraint; TSF: Transfer; SCH: Scheduling; TCC: Trip compatibility constraint; S: Single-school; M: Multi-school; HO: Homogeneous fleet; HT: Heterogeneous fleet

Given the fact that school bell time is usually predetermined by the school authorizations, the most important tasks in the school bus planning problem are stop generation, routing, and scheduling. Some of the more recent works have been focusing on the combination of two or more tasks, which are summarized in FIGURE 1. (3) and (4) tried to solve the routing problem with time window constraints. Although such models do not directly provide the scheduling plan, the scheduling problem becomes trivial given the routing plan with start time of each trip. In practice, to the best of the authors knowledge, not a lot of the research has been implemented to design
school bus transportation systems. Most school districts would make minor changes to their existing routing and scheduling plan due to new conditions. Some others use commercial software that do not reveal their algorithms. A nice formulated mathematical model and a practical algorithm can be helpful.

Note: papers listed in TABLE 1 but not shown in FIGURE 1 solve only routing problem.

FIGURE 1 Problem coverage of recent school bus papers

MODEL DESCRIPTION

(1) provided a new alternative to solve school bus routing problem by considering the trip compatibility. However, only considering the potential trip compatibility in the routing problem instead of the realized ones in the scheduling is not necessarily beneficial. One example is depicted in FIGURE 2. In this example, trip a is compatible with trips b and c, but trip b and c are incompatible with each other. In such a case, one bus can serve either trip b or trip c after trip a, but not both of them. The total potential compatible trip pairs are two but only one is realized in the schedule plan. This over-counting compatibility is the difference between the potential and realized compatible trip pairs. The SWH model developed in (1) counted the total potential compatibility and not the realized ones. The consequence is that there is no clear relationship between the number of trips, the number of realized compatible trip pairs and number of buses, which is essential in the extended model and will be explained in Lemma 1.
FIGURE 2 Illustration of over-counting (compatibility) problem

We approach fixing this shortcoming of the SWH model by adding additional constraints. The new proposed model can resolve the over-counting (compatibility) problem and becomes the integrated model for school bus routing and scheduling problem. The extended model is shown in Equations (1) – (22), with all notations summarized in TABLE 2. Some terminologies are defined as follows:

**Minimum number of trips (for each single school):** the smallest ceiling integer of the ratio of the total number of students divided by the bus capacity. It is the minimum number of trips due to bus capacity that are needed to transport all students;

**Additional allowed trips (A):** the maximum additional number of trips beyond the minimum number of trips (for each single school) that can be used. The combination of the minimum number of trips and the additional allowed trips defines the potential trip set for each school. (e.g. if school k has 180 students and bus capacity is 40, then the minimum number of trips is 5. If the additional allowed trips parameter is set to be two, at most 7 trips can be used for school k, and potential trip set for school k is the integer index from 1 to 7).

The extended model solves the multi-school bus routing and scheduling problem simultaneously with predetermined school dismissal time, bus stops location and student demand at each stop under the assumption of homogenous fleet and single-load trips. Single-load trips mean that each trip is dedicated to one school. Picking up students from other schools within a trip is prohibited but buses can accommodate trips from different schools. Each stop can be visited several times until all the student demands are satisfied. The maximum number of trips one bus can accommodate and the maximum travel time for buses are not constrained.
### TABLE 2 Notations

#### Variables for school bus routing

| Variable name | Description |
|---------------|-------------|
| $s_2t_{s,t}$  | Binary decision variable that equals 1 if stop $s$ is assigned to trip $t$ |
| $t_2s_{t,k}$  | Binary decision variable that equals 1 if trip $t$ is assigned to school $k$ |
| $p_{4t_{s,t}}$ | Portion of the capacity of the bus doing trip $t$ that is filled at stop $s$ |
| $x_{s_{1,s_{2},t}}$ | Binary variable that equals 1 if in trip $t$ the bus goes directly from stop $s_{1}$ to stop $s_{2}$ |
| $tt_{t}$       | Travel time (duration) of trip $t$ |
| $start_{t}$    | The start time of trip $t$ |
| $end_{t}$      | The end time of trip $t$ |
| $b_{s_{1},t_{2}}$ | Binary variable that equals 1 if trips $t_{2}$ can be served after trip $t_{1}$ (they are compatible) |
| $dd_{s_{1},t_{2}}$ | The travel deadhead duration from the last stop of trip $t_{1}$ to the first stop of trip $t_{2}$ |
| $l_{s,t}$      | Binary variable that equals 1 if the last stop of trip $t$ is stop $s$ |
| $c_{s_{1},s_{2}}$ | Variable used for sub-tour elimination – The units of “artificial commodity” that is shipped from stop $s_{1}$ to $s_{2}$ using trip $t$ |

#### Variables for bus blocking

| Parameter name | Description |
|----------------|-------------|
| $y_{t_{1},t_{2}}$ | A binary variable that equals 1 if trip $t_{2}$ follows trip $t_{1}$ within the same block (tour). This variable is only created for compatible pair of trips. |
| $a_{t}$ | Binary variable that equals 1 if a trip belongs to a tour of size 1 (no trip goes before or after it) |
| $m_{t}$ | A binary variable that equals 1 if trip $t$ has a trip that is being served before it and also another trip that is being served afterwards |
| $f_{t}$ | A binary variable that is equal to 1 if trip $t$ is served as the first trip in a tour of size greater than 1 |

#### Parameters for School bus routing and bus blocking

| Parameter name | Description |
|----------------|-------------|
| Schools        | Set of schools |
| $Trips_{School_{k}}$ | Set (index) of potential trips dedicated to school $k$ |
| $Trips$        | Set (index) of all potential trips |
| $Stops_{School_{k}}$ | The set of stops in which students for school $k$ should go to / come from |
| $Stops$        | Set of all stops |
| $Bus_{Cap}$    | The capacity of each bus (=48 in our experiment) |
| $Students_{s,k}$ | The number of students at stop $s$ for school $k$ |
| $O_{k}$        | Origin stop for school $k$ |
| $D_{s_{1},s_{2}}$ | The duration to drive from stop $s_{1}$ to $s_{2}$ plus the dwell time required at the stops |
| $SchoolEnd_{k}$ | Time which school $k$ closes |
| $Buffer$       | The allowable buffer for picking up students from the schools = 0 for cases solved due to the trips being PM trips with no flexibilities. |
| $M$            | A large value (big-M) |
| $C_T$          | Coefficient for total travel time = 1 for the cases solved |
| $C_{c}$        | Coefficient of trips = 1000 for the cases solved |
| $C_{p}$        | Coefficient of compatibility = changing in different cases |
| $A$            | The number of allowable trips over the minimum required trips for each school |
The extended model is defined as follows:

$$\min z = C_T \sum_{t \in \text{Trips}} tt_t + C_C \sum_{k \in \text{Schools}} \sum_{t \in \text{Trips}_k} t2s_{t,k} - C_B \sum_{t1 \in \text{Trips}} \sum_{t2 \in \text{Trips}} b_{t1,t2}$$

Subject to:

Constraints (2) to (19) are the same as in SWH model, shown in FIGURE 3.

$$\sum_{j \in \text{Trips} | j \neq t} b_{t,j} \leq 1 \forall t \in \text{Trips}$$

$$\sum_{i \in \text{Trips} | i \neq t} b_{i,t} \leq 1 \forall t \in \text{Trips}$$

$$tt_t \leq MRT \forall t \in \text{Trips}$$

The objective (1) and constraints (2) to (19) are the same as in SWH model. The objective is to minimize the sum of the number of trips and total travel time while maximizing compatible trip pairs. The constraints contain the logistic constraints, capacity constraints, sub-tour elimination constraints, conservation of flow, the minimum and the maximum number of trips for each school, eliminating symmetries and identifying the compatible trips. The precise formulation and discussion of these constraints can be found in (1). From here on, ‘the model’ is referred to the extended model with constraints (20) to (22).

For the newly added the constraints, constraints (20) and (21) make sure that a trip has at most one successive and one preceding trip respectively. These two constraints are the realization of compatible trip pairs in the scheduling problem. Constraints (22) limit the maximum ride-time for each trip. Note that the objective function (1) is an objective in scheduling problem that minimizes both the number of buses and the aggregate travel time due to lemma 1.

**Lemma 1**: The last two components in the objective function (1) with equal coefficients ($C_B = C_C$) yield the minimum number of buses for scheduling problem.

Proof: The total number of buses needed is equal to the number of trips if no trips are tiered. In this case, the number of trips ($\sum_{k \in \text{Schools}} \sum_{t \in \text{Trips}_k} t2s_{t,k}$) equals the number of buses. However, when any two trips are blocked together, a single bus can operate both of them together and therefore each realized compatible trip pair ($b_{i,j} = 1$) would yield in a saving. The total realized compatible trip pairs ($\sum_{t1 \in \text{Trips}} \sum_{t2 \in \text{Trips}} b_{t1,t2}$) is the number of buses that can be saved by blocking. Hence, the last two terms in objective function (1) equals the number of buses at the best scheduling condition.

In SWH model, $b_{t1,t2}$ represents the potential compatible trip pairs while in our work, after adding constraints (21) - (22), $b_{t1,t2}$ is the realized compatible trip pairs in the scheduling plan. In other word, $b_{t1,t2}$ is the solution to the scheduling problem which indicates the assignment of the trips to buses and the service sequence of trips for each bus.
\begin{align*}
    s2t_{s,t} & \leq t2t_{s,t} \quad \forall k \in Schools, s \in \text{Stops}_k, t \in \text{Trips}_k \\
    \sum_{s \in \text{School}_k} p4t_{s,t} & \leq 1 \quad \forall k \in Schools, \forall t \in \text{Trips}_k \\
    \sum_{t \in \text{Trips}_k} p4t_{s,t} = \frac{\text{Students}_{s,t}}{\text{Bus Cap}} \quad \forall k \in Schools, s \in \text{School}_k \\
    p4t_{s,t} & \leq s2t_{s,t} \quad \forall k \in Schools, t \in \text{Trips}_k, s \in \text{Stops}_k \\
    \sum_{s \in \text{Stops}_k} x_{s,t} = s2t_{s,t} \quad \forall k \in Schools, \forall t \in \text{Trips}_k, \forall s \in \text{Stops}_k \\
    \sum_{s \in \text{Stops}_k} x_{s,t} = t2t_{s,t} \quad \forall k \in Schools, t \in \text{Trips}_k \\
    \sum_{j \in \text{Stops}_k} x_{j,s} = \sum_{i \in \text{Stops}_k} x_{i,s} \quad \forall k \in Schools, s \in \text{Stops}_k, t \in \text{Trips}_k \\
    tt_t = \sum_{s1 \in \text{Stops}_k} \sum_{s2 \in \text{Stops}_k} x_{s1,s2} \times D_{s1,s2} \quad \forall k \in Schools, t \in \text{Trips}_k \\
    end_t = start_t + tt_t \quad \forall t \in \text{Trips} \\
    SchoolEnd_k \leq start_t \leq SchoolEnd_k + \text{Buffer} \quad \forall k \in Schools, t \in \text{Trips}_k \\
    end_{t1} + dd_{t1,t2} - M \times (1 - b_{t1,t2}) \leq start_{t2} \quad \forall t1, t2 \in \text{Trips} \\
    dd_{t1,t2} \geq \frac{M}{2} \times (1 - t2s_{t1,k1}) + \frac{M}{2} \times (1 - t2s_{t2,k2}) + \sum_{s1 \in \text{Stops}_k} D_{s1,s2} \times l_{s1,t1} \quad \forall k1, k2 \in Schools, t1 \in \text{Trips}_k1, t2 \in \text{Trips}_k2 \\
    x_{s,t} = l_{s,t} \quad \forall k \in Schools, t \in \text{Trips}_k, s \in \text{Stops}_k \\
    t2s_{t1,k} \geq t2s_{t2,k} \quad \forall k \in Schools, t1, t2 \in \text{Trips}_k \quad |t2 \leq t1| \\
    \sum_{i \in \text{Stops}_k} c_{i,s} = s2t_{s,t} - \sum_{i \in \text{Stops}_k} c_{i,s} \quad \forall k \in Schools, t \in \text{Trips}_k, s \in \text{Stops}_k \\
    c_{s1,s2} \leq M \times x_{s1,s2} \quad \forall k \in Schools, t \in \text{Trips}_k, s1, s2 \in \text{Stops}_k \\
    \left(\sum_{k \in \text{School}_k} \frac{\text{Students}_{k,s,t}}{\text{Bus Cap}}\right) \leq \sum_{k \in \text{School}_k} \sum_{t \in \text{Trips}_k} t2s_{t,k} \quad \forall k \in Schools \\
    \sum_{k \in \text{School}_k} \sum_{t \in \text{Trips}_k} t2s_{t,k} \leq A + \left(\sum_{k \in \text{School}_k} \frac{\text{Students}_{k,s,t}}{\text{Bus Cap}}\right) \quad \forall k \in Schools
\end{align*}

FIGURE 3 Formulations of constraints for school bus routing problem in SWH
ITERATIVE SCHOOL DECOMPOSITION ALGORITHM

Model Relaxation
Adding constraints (20) – (22) make the model an integrated model and even harder to solve using an exact optimal algorithm and commercial solver. Therefore, an Iterative School Decomposition Algorithm (ISDA) is developed that makes the problem solvable by reducing the problem size and focusing on one school at each time. Two model relaxation techniques along with some terminologies are defined here:

- **Trip compatibility relaxation**: Relaxing constraints (20) - (21).
- **Trip compatibility conversion**: trip-to-trip compatibility in the compatibility-check constraints (constraints (12)-(13) in SWH) is replaced by trip-to-school compatibility, which equals to one if a bus can arrive at another school before its dismissal time after serving a trip.
- **Objective Modification**: the objective for each single-school (k) trip generation problem is modified as follows:

\[
\begin{align*}
\min z_k &= C_T \sum_{t \in \text{Trips,School}_k} t t_t + C_c \sum_{t \in \text{Trips,School}_k} t 2 s_{t,k} - C_B \sum_{t \in \text{Trips,School}_k} \sum_{s \in \text{Schools}} \frac{M}{\text{SchoolEnd}_{s} - \text{SchoolEnd}_{r}} b_{t,s} \forall k \in \text{Schools} \\
\min z' &= \sum_{k \in \text{Schools}} \tilde{a}_k
\end{align*}
\]

The difference between the modified objective and original objective is that the modified one uses smaller weights on trip compatibility \((C_c > C_B)\) and that the trip compatibility has a higher weight for schools which dismissal times are closer to the current school. The other modification is due to the problem-size reduction by using trip-to-school compatibility as opposed to trip-to-trip compatibility.

- **School trip capacity update**: We declare a notion of school trip capacity which is equal to the number of the maximum unpaired trips for each school. We iteratively update the maximum number of trips for each school that has not been assigned with compatible trips. This value is the total number of potential trips used for one school minus the realized number of compatible trip pairs that have already been assigned. The initial school trip capacity equals to the minimum number of trips plus the additional allowed trips. As a compatible trip of a preceding school is assigned to a succeeding school, we decrement the school trip capacity of the following school.

- **School decomposition**: divide the original multi-school problem into several single-school sub-problems and sequentially solve them. Note that the single school problems have different stop information, but all share the common information about the other schools’ dismissal times.

To distinguish between the different relaxed models after the application of several relaxation techniques, we would refer to the model after trip compatibility relaxation as the **first-degree-relaxed model**. The model after trip compatibility relaxation and trip compatibility conversion is referred to as the **second-degree-relaxed model**; and the model after trip compatibility relaxation, trip compatibility conversion, and objective modification is referred to as the **third-degree-relaxed model**.

**Lemma 2**: The second-degree-relaxed model is a relaxation of the original model, which provides an integer upper bound.

Proof: trip compatibility relaxation takes off two sets of constraints regarding the trip compatibility. It enlarges the feasible region, becoming a relaxation of the original model. Any feasible range relaxation would yield an upper bound to the original problem. Notice that the integrality constraints remain such that the solution is still an integer solution. Considering that the...
initial locations for afternoon trips are schools, the trip-to-school compatibility is equivalent to the trip-to-trip compatibility. For fixed school dismissal time, trip compatibility conversion makes no changes to the model.

**Lemma 3:** Applying school decomposition will not alter the optimality of the second-degree-relaxed model.

Proof: The objective function and the constraints for the second-degree-relaxed problem are all decomposed by the school. It means that the summation of the optimal solution of subproblems equals to the optimal solution for the non-decomposed problem. The school decomposition does not change the optimality of the solution.

**The Algorithm Procedure**

The procedure for Iterative School Decomposition Algorithm (ISDA) is described as below, and the flowchart is shown in FIGURE 4.

Step 0: Relax the original model by trip compatibility relaxation, trip compatibility conversion, and objective modification and divide it into several single school sub-routing-problems.

Step 1: Randomly generate a permutation of the schools.

Step 2: Calculate the initial school bus trip capacity, which equals to the minimum number of trips (based on the capacity) plus the additional allowed trips.

Step 3: Solve the single-school sub-routing-problem using commercial solvers and assign compatible trips to schools for which dismissal times are after the current school. Solve the school bus scheduling problem using commercial solvers, which assigns trips to compatible schools and then update the school trip capacity.

Step 4: Repeat Step 3 for each school in the list until the last one.

Step 5: Solve the scheduling problem using commercial solvers given the current routing solution to the school-decomposed second-degree-relaxed problem and calculate the number of buses used.

Step 6: Repeat from Step 1 to Step 5 (using a different random permutation) for a finite number of iteration and find the best solution with respect to the minimum number of buses used.

Note that in Step 3 and 5, the school bus scheduling problem is solved using the same scheduling method proposed in (1). The difference is that at step 3, only the trip plans for the solved schools are known and the trip plan for the unsolved schools is unknown. Solving the scheduling problem in step 3 does not give the accurate number of buses. Instead, it aims to assign compatible pairs to schools for updating the left trip capacity. The scheduling problem in step 5, given all the routing plan information, is the one that provides the exact number of buses needed for this multi-school system.
Discussion of the Algorithm
A few things need to be noted in the school decomposition algorithm. First, after trip compatibility relaxation, the trip compatibility over-counting problem occurs again. If we use the same weight
for the potential compatible trip pairs as for the realized ones, we would over-estimate the saving due to realized compatible trip pairs in Lemma 1. Thus the objective cannot calculate the exact number of buses. Objective modification, which adjusts the weight for the potential trip compatibility, aims to solve this issue. Notice that the number of trips and total travel time in the objective remains the same with or without relaxation and that the decomposition does not change the optimality of the second-degree-relaxed problem. The trip compatibility relaxation will only increase the second component of compatible trips in the objective function. By decreasing the weight for the potential trip compatibility (third-degree-relaxed problem), the over-counting problem will be compensated. Sensitivity analysis estimates the appropriate weight for compatibility.

Another thing is that although it was mentioned above that trip-to-trip compatibility is equivalent to the trip-to-school compatibility; it only holds when the bus service start (departure) time is equal to the school dismissal time. Any trips with a buffer after the school dismissal time should be prohibited. A simple approach to address this problem is to modify the school dismissal time to include buffer when solving for school k. The actual bus service start time can be updated after solving the problem.

**COMPUTATIONAL RESULT**

**Experiment Setup**

In this set of experiments, the same eight set of mid-size problems generated in (1) is used to test the performance of the Iterative School Decomposition Algorithm (ISDA) using eight combinations of parameters setting.

1) A0TL15: zero additional allowed trip and 15 seconds running time limit;
2) A0TL30: zero additional allowed trip and 30 seconds running time limit;
3) A1TL15: one additional allowed trip and 15 seconds running time limit;
4) A1TL30: one additional allowed trip and 30 seconds running time limit;
5) A1TL120: one additional allowed trip and 120 seconds running time limit;
6) A1TL30MRT: one additional allowed trip, 30 seconds running time limit with maximum ride time constraints;
7) A2TL30MRT: two additional allowed trips, 30 seconds running time limit with maximum ride time constraints;
8) A3TL30MRT: three additional allowed trips, 30 seconds running time limit with maximum ride time constraints;

In the scenario names, A is the additional allowed trips, TL is the maximum running time limit (second) for each single-school sub-routing-problem, and MRT is the maximum ride time parameter (for all trips) set to 40 minutes (Constraints 23). The sensitivity analysis from (1) showed that the best ratio of trip compatibility over the number of trips is 0.12-0.14. In this experiment, the coefficient for the number of trips is set to be 1000, the coefficient for the aggregated travel time (minutes) is 1, and the coefficient for the modified trip compatibility is 125.364. Every approach was tested for 100 iterations and the solution with the fewest number of buses were reported. All the case studies were run a computer with i7 CPU 870 @ 2.93 GHz and 8GB RAM. The commercial solver used to solve single school routing and scheduling problem is FICO Xpress. The configurations and running time for these test problems are shown in TABLE 3. The resulting number of buses, the mean travel time per trip and maximum travel time per trip are listed in TABLE 4.
Solution Quality
As pointed above, the most important criterion for the school bus routing and scheduling (SBRS) is to use the minimum number of buses to provide the student transport service satisfying all the constraints. Apart from that, travel time is another essential Level-of-Service criterion, especially for pupils. Two travel time criteria adopted are the mean travel time per trip (average travel time over all trips) and the maximum travel time per trip (maximum travel time among all the trips). TABLE 4 showed that ISDA always found better or at least equal solutions to the SWH with respect to the number of buses for all test scenarios. This demonstrates the importance of doing trip assignments for dealing with the issue of over-counting compatibilities. The mean travel time for ISDA is usually pretty small, less than 20 minutes (TABLE 4). The underlying reason is that shorter trips are easier to be compatible with other trips and that minimizing total travel time is also in the objective. Therefore, ISDA prefers forming short trips, which is a good practice for school bus problem.

In scenario 1, the mean travel time is increased by 3.5 minutes per trip, which seems undesirable. However, the detailed result in TABLE 4 reveals that such travel time increase is an inevitable side-effect of the bus saving. A1TL30 found solutions with 23 buses compared to the 31 buses from SWH at the expense of the 3.5 minutes and 20 minutes increase of the mean and maximum travel time respectively. When limiting the maximum ride time, the number of buses (from A1TL30MRT) increases to 28 but the average travel time is 0.7 minutes shorter than that from SWH and the maximum travel time is the same with SWH. Considering the high cost of a school bus and a driver and relatively low cost for a small increase in travel time, from a financial point of view, the savings gained by using fewer buses could easily justify the additional travel times. There are still many scenarios that ISDA find solutions that use fewer buses and also have a smaller mean and maximum ride time. It shows the ISDA can find better results than SWH with respect to all criteria (fewer number of buses and shorter mean and maximum ride time). These savings are resulted from the fact that this paper solves the routing and scheduling problem while the SWH model only solves the routing problem. This makes the ISDA heuristic relatively slower as shown in TABLE 3. The average running time for ISDA is 14 times the running time for SWH. However, the run-time is still acceptable (less than 100 minutes for one iteration) given the planning nature of the problem.

TABLE 3 Configurations and running time comparison between SWH and ISDA

| Scenario | Number of schools | Number of stops | ① | ② | ③ | Running time (min) |
|----------|------------------|----------------|----|----|----|------------------|
|          |                  |                | ① | ② | ③ | SWH              |
| 1        | 20               | 100            | 91.4 | 13 | 0-30 | 0.28 | 4.12 | 14.71 |
| 2        | 20               | 200            | 89.6 | 16 | 0-30 | 3.46 | 99.31 | 28.70 |
| 3        | 20               | 100            | 120.7 | 13 | 0-30 | 5.52 | 23.85 | 4.32 |
| 4        | 20               | 100            | 182.8 | 13 | 0-30 | 25 | 72.90 | 2.92 |
| 5        | 25               | 125            | 90.4 | 13 | 0-30 | 3.5 | 11.93 | 3.41 |
| 6        | 20               | 100            | 91.6 | 13 | 0-90 | 0.29 | 9.52 | 32.81 |
| 7        | 20               | 200            | 89.5 | 16 | 0-90 | 17.4 | 65.55 | 3.77 |
| 8        | 20               | 200            | 91.1 | 14 | 0-16 | 4.18 | 96.10 | 22.99 |

Note: ① Average number of student per school (students per school); ② Maximum number of stops to each school (stops per school); ③ School dismissal time range (min); ④ ISDA is the average running time for ISDA (A0TL15) per iteration; ⑤ Ratio = running time (ISDA) / running time (SWH)
### TABLE 4 Computational Result for School Decomposition Algorithm

| Scenario | NOB  | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
|----------|------|-----|-----|-----|-----|-----|-----|-----|
| MinN     |      |     |     |     |     |     |     |     |
|          | 38   | 38  | 45  | 60  | 41  | 21  | 26  | 45* |
| AvgTT    | 30.42| 24.28|22.53|21.08|37.88|30.42|24.28|136.23|
| MaxTT    | 79.73| 61.89|58.39|69.34|100.58|79.73|61.89|243.69|
| MinTT    |      |     |     |     |     |     |     |     |
|          | 32   | 32  | 38  | 52  | 35  | 17  | 24  | 45* |
| AvgTT    | 14.11| 7.08 |11.72|9.10 |18.57|14.13|11.17|35.15|
| MaxTT    | 35.33| 15.95|33.07|29.56|39.62|35.33|17.26|64.65|
| SWH      |      |     |     |     |     |     |     |     |
|          | 31   | 28  | 39  | 46  | 33  | 17* | 18  | 45* |
| AvgTT    | 14.48| 11.38|11.85|9.58 |19.49|14.61|11.27|35.43|
| MaxTT    | 33.07| 19.99|33.07|29.56|60.51|33.07|19.08|68.42|
| A0TL15   |      |     |     |     |     |     |     |     |
|          | 23*  | 26* | 36  | 45  | 33  | 17* | 16* | 45* |
| AvgTT    | 18.84| 11.30|11.38|9.20 |19.12|14.27|11.07|35.45|
| MaxTT    | 53.11| 24.56|33.07|29.56|60.51|33.07|21.53|68.42|
| A0TL30   |      |     |     |     |     |     |     |     |
|          | 23*  | 26* | 36  | 44  | 33  | 17* | 16* | 45* |
| AvgTT    | 18.84| 11.49|11.41|9.23 |19.12|14.27|11.10|35.45|
| MaxTT    | 53.11| 25.50|33.07|29.56|60.51|33.07|22.77|68.42|
| A1TL15   |      |     |     |     |     |     |     |     |
|          | 23*  | 26* | 34  | 43  | 31* | 17* | 17  | 45* |
| AvgTT    | 19.16| 10.74|10.73|9.38 |18.58|14.27|10.94|35.48|
| MaxTT    | 53.11| 24.72|33.07|29.56|60.51|33.07|24.27|68.42|
| A1TL30   |      |     |     |     |     |     |     |     |
|          | 23*  | 26* | 35  | 42* | 31* | 17* | 18  | 45* |
| AvgTT    | 18.01| 10.60|11.01|8.96 |17.35|14.33|10.45|35.44|
| MaxTT    | 53.11| 23.71|33.07|29.56|39.62|33.07|17.75|68.42|
| A1TL20   |      |     |     |     |     |     |     |     |
|          | 23*  | 27  | 35  | 43  | 31* | 17* | 17  | 45* |
| AvgTT    | 18.18| 10.64|11.14|9.27 |18.17|14.27|10.84|35.43|
| MaxTT    | 53.11| 20.10|33.07|29.56|63.95|33.07|20.14|68.42|
| A1TL30 MRT |    |     |     |     |     |     |     |     |
|          | 28   | 26* | 35  | 42* | 31* | 17* | 17  | 50  |
| AvgTT    | 13.41| 10.45|11.30|8.74 |17.76|14.34|10.92|31.40|
| MaxTT    | 33.07| 22.67|33.07|29.56|39.62|33.07|19.07|39.97|
| A2TL30 MRT |    |     |     |     |     |     |     |     |
|          | 28   | 26* | 33* | 43  | 31* | 17* | 18  | 50  |
| AvgTT    | 13.43| 10.66|10.98|8.65 |17.76|14.35|10.88|31.40|
| MaxTT    | 33.07| 24.75|33.07|29.56|39.62|33.07|19.21|39.97|
| A3TL30 MRT |    |     |     |     |     |     |     |     |
|          | 27   | 26* | 33* | 43  | 31* | 17* | 17  | 50  |
| AvgTT    | 12.60| 10.64|10.67|9.03 |17.76|14.41|11.01|31.40|
| MaxTT    | 33.07| 23.42|33.07|29.92|39.62|33.07|20.85|39.97|

Note: **MinN**: Solution of bus scheduling problem given the routing trips, which is obtained by minimizing number of trips; **MinTT**: Solution of bus scheduling problem given the routing trips, which is obtained by minimizing aggregated travel time; **SWH**: Number of buses from approach MaxCom+TT(AS) in (1); **AvgTT**: average travel time per trip (minutes); **MaxTT**: maximum travel time per trip (minutes); *: Minimum number of buses among all approaches for each scenario.
A cross-examination of the result from SWH and that from A0TL15 with respect to the bus saving, mean and maximum travel time (per trip) increase percentage are shown in FIGURE 5. Solutions from ISDA have shorter mean travel time in scenarios 2 to 7 than those from SWH. But on the contrary, at scenarios (like 1 and 7) where ISDA can save significant number of buses from SWH, the maximum travel time increases. This shows the tradeoff between the bus saving and travel time increase.

\[ \text{Bus saving} = \text{number of buses (SWH)} - \text{number of buses (A0TL15)}; \]
\[ \text{travel time increase percentage (mean and maximum travel time)} = \frac{\text{travel time (A0TL15)} - \text{travel time (SWH)}}{\text{travel time (SWH)}} \]

**FIGURE 5 Comparison between solution from SWH and ISDA (A0TL15)**

**Maximum Ride Time Constraint**

It can be seen that thanks to minimizing the travel time in the objective, many of the maximum travel time per trip is still under 40 minutes even without the maximum ride time (MRT) constraint (TABLE 4). However, there are certain situations in which merely minimizing total travel time in the objective is not enough. Thus, we need to incorporate MRT constraint. For example, in scenario 8, without MRT, 45 buses can accommodate the school transportation demand with the highest 68 minutes per trip. By adding the MRT constraint, the maximum travel time per trip is significantly reduced (under 40 minutes), but the number of buses increases to 50.

**Running Time Limit**

It is intuitive that the longer running time limit, the higher chance a better solution can be found. This is reflected in TABLE 4. However, there exist many counter examples that increasing time limit does not give a better solution. This might come from the randomness of the school sequence,
the single school routing solution using commercial solver. The good part of this is that once the running time limit exceeds a threshold (15 seconds in most of the test scenarios), the marginal benefits of lengthening the running time limit gets smaller and smaller. A benefit of this phenomenon is that in time-demanding applications, a reasonable running time limit can be set to improve the efficiency without significantly weakening the solution quality.

**Upper Bound and Pseudo-Reduction**

In SBRS, one simple upper bound is the number of trips. The rationale is that if we assign each trip with one bus as if no trips are tiered, the number of trips equals to the number of buses. Clearly, it is a feasible solution, and thus is an upper bound. Based on this idea, a loose upper bound (LUB) is the number of the total potential trips. It can be strengthened (SUB) by finding the actual number of trips which is the output of the routing problem.

Notice that upper bound also gives some insight into the complexity (or the size) of the problem, the larger the upper bound, the more trips are used, and the harder the problem is. Also, we define the pseudo-reduction (PR) in Equation (25) and PRs for different approaches at eight scenarios are shown in TABLE 5.

\[
PR = \left(1 - \frac{\text{number of buses}}{\text{strengthened upper bound}}\right) \times 100\% \tag{25}
\]

**TABLE 5 Pseudo-reduction of different approaches (%)**

| Approach | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 | Scenario 6 | Scenario 7 | Scenario 8 |
|----------|------------|------------|------------|------------|------------|------------|------------|------------|
| MinN     | 7.32       | 19.15      | 16.67      | 20.00      | 10.87      | 48.78      | 44.68      | 10.00      |
| MinTT    | 21.95      | 31.91      | 29.63      | 30.67      | 23.91      | 58.54      | 48.94      | 10.00      |
| MinB     | 26.83      | 36.17      | 33.33      | 40.00      | 21.74      | 60.98      | 48.94      | 12.00      |
| MinB+TT  | 24.39      | 31.91      | 37.04      | 40.00      | 23.91      | 60.98      | 48.94      | 10.00      |
| SWH      | 24.39      | 40.43      | 27.78      | 38.67      | 28.26      | 58.54      | 61.70      | 10.00      |
| ISDA     | 43.90      | 44.68      | 37.04      | 44.00      | 32.61      | 58.54      | 65.96      | 10.00      |

The interpretation of pseudo-reduction is the percentage of trips that are served by multi-trip-serving buses. In other words, it is the percentage of buses that can be saved in comparison to the scenario that none of the trips are compatible. This also gives some insight into the quality of the solution, the higher pseudo-reduction, the better the solution could be. The maximum pseudo reduction is 65.96%, which implies that more than half of the buses can be saved than the worst scenario that each bus can only accommodate one trip. That is an indication of good solution.

**CONCLUSION**

In this paper, an over-counting issue of SWH model is identified and resolved. An Iterative School Decomposition Algorithm is proposed to solve the extended model with two model relaxation techniques. The efficiency of the model and ISDA algorithm is tested on the same eight set of scenarios used by SWH. The result shows that the ISDA can find better results than SWH with a fewer number of buses (up to 25%), and shorter mean and maximum travel time per trip (up to 7%). A few directions for future work can be identified. One of them is a more efficient algorithm to solve each single school routing problem such that it can handle more complicated problems with more stops to each single school. Another one is that a more flexible way to handle bus service
start time can be devised, especially for morning trips. An appropriate time window might be more financially beneficial than a fixed service start time.

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