We investigate flow separation in the air flow over transverse sand dunes. CFD simulations of the air flow over differently shaped dunes are performed. The length of the recirculation region after the brink of the dune is found to depend strongly on the shape of the dune. We find that the nondimensionalised separation length depends linearly on the slope of the dune at the brink within a large interval. A phenomenological expression for the separation length is given.

1 INTRODUCTION

Dunes are naturally occurring, beautifully shaped sand deposits. Since the middle of the previous century, they have attracted the attention of scientists who have been seeking to model them and understand the processes leading to their formation. From the point of view of the physicist, sand dunes constitute a variable boundary problem: The air flow is determined by the shape of the dune and in turn influences the dune shape by transporting sand grains. Therefore the air flow over dunes is of great importance for understanding dune formation and evolution.

Since the start of scientific interest in dunes, there has been some work on this topic, both theoretical (Nelson and Smith 1989; Parsons, Walker, and Wiggs 2004) and experimental (Engel 1981; Sweet and Kocurek 1990). However, due to the difficult nature of the problem, these works have only tackled part of the problem. In several publications, transverse dunes have been modeled as triangular structures (Engel 1981; Parsons, Walker, and Wiggs 2004; Parsons, Wiggs, Walker, Ferguson, and Farvey 2004). Field measurements of air flow over dunes tend to lack measurements of the dune profile (Sweet and Kocurek 1990; Frank and Kocurek 1996).

A recent field measurement (Parteli, Schwämmle, Herrmann, Monteiro, and Maia 2004) suggests that the shape of transverse dunes has significant influence on the length of the recirculation region. Since the sand transport in the recirculation region in the lee of a dune is negligible, the foot of the following dune shape is located at or downwind of the flow reattachment point. Therefore the distance of closely spaced dunes is a measure of the length of the recirculation region.

In (Parteli, Schwämmle, Herrmann, Monteiro, and Maia 2004) the separation length after different dunes was determined in this way.

In this work we will present results for widely spaced or isolated transverse dunes. This is to some extent an idealisation. However, we think this is valid and useful: We want to concentrate on the effect of the dune shape, other things being equal, and the presence and shape of neighbouring dunes would constitute additional parameters.

2 METHOD

Our simulations were performed with the computational fluid dynamics software FLUENT. The wind flow over dunes is fully turbulent. We simulated the Reynolds-averaged Navier-Stokes equations using the k-ε model for closing the equations.

The simulations were two-dimensional, implying a wind direction perpendicular to the dunes. The cross sections of the dune shapes were constructed from two circle segments, a concave one modeling the foot of the dune and a convex one for the crest. We verified that this is realistic enough to describe the transverse dune shapes measured in (Parteli, Schwämmle, Herrmann, Monteiro, and Maia 2004) well. To obtain different shapes, the position of the slip face was varied from the start to the end of the convex part, see Figure [1]. Note that this has the consequence that not all the dunes have the same
Figure 1: The seven different dune shapes investigated. The scale displays the brink position. The crest height of the dunes with negative brink position equals the brink height, the height of those with positive brink position is 3 metres.

Brink angle $\alpha$

Brink height $\Delta$

Reattachment point

Separation length $l$

Figure 2: The geometric variables characterising the dune shapes. The brink angle is positive for dunes with a sharp brink and negative for round dunes as the one shown in the figure.

The velocity profile at the influx boundary of the simulation region was set to the logarithmic profile which forms in flow over a plane in neutral atmospheric conditions. The shear velocity was chosen to be 0.4 m/s, the roughness length $25\mu m$. The size of the roughness elements on the ground, the grain size, was set to $250\mu m$. The roughness length is generally considered to be of the order of the grain size or up to a factor of 30 smaller (Wright, Schaffner, and Maa 1997).

The region around the dune in which the flow was simulated was chosen large enough so that the boundaries did not influence the results. This was verified by choosing larger simulations areas for some dune shapes and comparing the results. From the brink position 0, the simulation region extends 45 m to the left and 70 m to the right (see Figure 3). The height of the simulated region was chosen to be 30 m for all dunes except the one with brink position -15 m, where 20 m was found to be sufficient.

Figure 3: The simulated region around the dune.

The length of flow separation, our quantity of interest, was found to depend slightly on the spacing of the simulation grid. Therefore we performed the simulation of the flow over each dune with three different grids and interpolated the separation lengths to the continuum. The average grid spacings were 10, 7 and 5 cm, respectively.

3 RESULTS

Table 1 shows the results for all dune shapes. The length of flow separation, our quantity of interest, was measured from the slip face brink, where the flow separates, to the flow reattachment point (see Figure 2). The errors were estimated to be one grid spacing for the determination of the flow reattachment point. The separation lengths determined for the different grids and their errors were interpolated to the continuum with the standard linear regression formulas. To the statistical error we added quadratically a systematic error of 0.5 metres. The systematic error accounts for biases which may be inherent in the turbulence model and parameter settings used. We expect it to be strongly correlated.

To nondimensionalise the separation length $l$, it was divided by the height of the slip face. This dimensionless quantity is universal, that is it does not depend on the absolute height of the dune. That is because the flow is already fully turbulent for small dunes. We find that $l/\Delta$ is larger for dunes with a sharp brink than for rounded dunes. It depends linearly on the brink position $d$ respectively the angle of the dune shape at the brink, $\alpha$. As can be seen in Figure 4, the linear relation extends up to an absolute angle of $7.5^\circ$. This amounts to a variation of the dune length between 6.7 and 13.3 times the height. Fitting the relation

$$l(\alpha)/\Delta(\alpha) = A \cdot \alpha + B,$$

we obtain $A = 0.208/\alpha$ and $B = 5.85$. The points with brink position $\pm 15$ were ignored for this fit since they deviate from the linear law.

Since the brink angle $\alpha$ and the brink position $d$ are related by the geometry of the dune, the fit (1) can be reformulated to give the separation length in terms of the brink position:

$$l(\alpha(d)) = (A \cdot \alpha(d) + B) \cdot \Delta(\alpha(d)) = \left(-A \arcsin \frac{d}{R} + B\right) \cdot \left(H_{\text{max}} - d \tan \left(\frac{1}{2} \arcsin \frac{d}{R}\right)\right).$$

This equation contains the height of the round dunes, $H_{\text{max}} = 3$ m, and the radius of the circle segments used to model the shape, $R = 75.75$ m. Remarkably, this
Table 1: Geometry of the simulated dunes and results for length of flow separation. See Figure 2 for a definition of the geometric variables. The brink angle is defined to be positive if the upwind slope is positive at the brink. The first error in the separation length is the statistical error in the determination of the length from the simulation data, the second error is the systematic error of the simulation (see text).

| Brink pos. $d$ [m] | Height $H$ [m] | Brink height $\Delta$ [m] | Angle at brink $\alpha$ [$^\circ$] | Separation length $l$ [m] | Error estimate [m] | $l/\Delta$ |
|---------------------|----------------|---------------------------|-------------------------------|--------------------------|-------------------|-----------|
| -15                 | 1.5            | 1.5                       | 11.4                          | 11.74                    | $\pm 0.026 \pm 0.5$ | 7.83      |
| -10                 | 2.337          | 2.337                     | 7.6                           | 17.33                    | $\pm 0.048 \pm 0.5$ | 7.41      |
| -5                  | 2.835          | 2.835                     | 3.78                          | 18.88                    | $\pm 0.042 \pm 0.5$ | 6.66      |
| 0                   | 3              | 3                         | 0                             | 17.30                    | $\pm 0.065 \pm 0.5$ | 5.77      |
| 5                   | 3              | 2.835                     | -3.78                         | 14.44                    | $\pm 0.050 \pm 0.5$ | 5.09      |
| 10                  | 3              | 2.337                     | -7.6                          | 9.97                     | $\pm 0.041 \pm 0.5$ | 4.27      |
| 15                  | 3              | 1.5                       | -11.4                         | 4.96                     | $\pm 0.021 \pm 0.5$ | 3.31      |

4 DISCUSSION

Comparison of our results to other work is hampered by the fact that the dependence of flow separation on the dune shape has not been investigated before. Therefore, the following comparisons are to be understood as consistency checks.

A recent review of air flow over transverse dunes (Walker and Nickling 2002) cites values of 4–10 for $l/\Delta$. Our results also lie within that range (see Figure 4). Engel (Engel 1981) finds values for the nondimensionalised separation length between 4 and a little over 6, depending on the roughness and the aspect ratio of triangular dunes. In (Parsons, Walker, and Wiggs 2004) a wide range of

Equation 2 can be approximated by using $\arcsin x \approx x$ and $\tan x \approx x$. In this way one obtains a polynomial expression for the dependence of the separation length on the brink position:

$$ l(d) = \left( -A \frac{d}{R} + B \right) \left( H_{\text{max}} - \frac{d^2}{2R} \right) $$

$$ = \frac{A}{2R^2} d^3 - \frac{B}{2R} d^2 - \frac{A H_{\text{max}}}{R} d + B H_{\text{max}} $$

$$ = C d^3 + D d^2 + E d + F $$

(3)

The coefficients are: $C = 0.00103 \text{ m}^{-2}$, $D = 0.0386 \text{ m}^{-1}$, $E = 0.472$, and $F = 17.55 \text{ m}$. The values of this expression are indistinguishable from (2) in the range of $d$ which we investigated. Therefore we do not plot it separately in Figure 5.
between 3 and 15 is given for the same quantity. The values for an aspect ratio of 0.1, which applies to our dune with $\alpha = 0$, are 5.67 and 8.13, depending on the height. This compares well with our value of 5.76. The discrepancy can be explained by the different shape, in particular the fact that our dune shape for $\alpha = 0$ has a horizontal tangent at the brink, whereas the dunes in (Parsons, Walker, and Wiggs 2004) are triangular.

Unlike our simulations, which treated an isolated dune shape, the field measurements (Parteli, Schwämmle, Herrmann, Monteiro, and Maia 2004) were performed in a closely spaced dune field. The authors find that the distance between the brink of each dune and the foot of the following one is typically four times the height or below. Under the assumption that the dune field is stationary, this distance is a measure of the separation length. This is at the lower end of the separation lengths we obtain, for very rounded dunes. The dunes with the smallest separation length in (Parteli, Schwämmle, Herrmann, Monteiro, and Maia 2004) are indeed very round. The small values for other dunes can be put down to the different situation of closely spaced transverse dunes.

5 CONCLUSIONS

We have determined the length of flow separation in the lee of isolated transverse dunes of different shapes. The separation length nondimensionalised by division by the slip face height was found to be larger for dunes with a sharper brink. For a wide range of dune shapes, this growth is well described by the linear relationship (1).

The dependence of the absolute separation length on the position of the slip face is described by the expression (2) more accurately than by a parabola. The maximal separation length does not occur for dune shapes with a horizontal tangent at the brink, but for shapes with a somewhat sharper brink.

These results were obtained for isolated dunes up to 3 metres in height. It would be interesting to see how the flow separation length is influenced by the presence of other dunes nearby, as in a closely spaced dune field. This topic will be treated in a future publication.

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