Critical Speed and Forced Response Analysis of an Integrated Rotor with Ti 6-2-4-6 Alloy

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Abstract: Titanium 6-2-4-6 alloy is establishing as the favourite material for the turbine rotors of all the new age commercial aero engines because of quieter operation due to high strength to weight ratio. This paper is mainly focused on prediction of the rotor dynamic characteristics of the Steel and Ti 6-2-4-6 alloy in the sense of weight, critical speeds and forced response analysis of both the materials. Standard Nelson rotor supported by two orthotropic bearings is used as the reference rotor to carry out the rotor dynamic analysis on both Steel and Ti 6-2-4-6 alloy. Finite element modelling, modal analysis and harmonic analysis for the Ti 6-2-4-6 and steel are performed using ANSYS Parametric Design Language. Critical speeds are obtained by plotting the Campbell diagram for the modal frequencies obtained modal analysis. Ti 6-2-4-6 alloy offers 40% weight reduction compared to the steel for the same volume of the material. Whereas there is only 15% decrease in the critical speed. Peak amplitude mode obtained from the forced response analysis is 2% away from the critical speed obtained from the Campbell diagram.

Keywords: Ti 6-2-4-6, Campbell diagram, Critical speed, Forced response analysis, modal frequencies, Orthotropic bearings.

I. INTRODUCTION

Rotor dynamics is a branch within Engineering mechanics, in which we study about the dynamic behavior of rotating structures like gas turbine, steam turbines, compressors, pumps etc., Generally these rotating structures are referred as rotor which consists of rotating shafts, associated with discs and supported on bearings. Rotor dynamics includes following types analysis. Critical speeds are the natural frequencies of the rotor system where resonance occurs. As a first step in design of any rotating equipment, an analysis is performed to determine the system modal frequencies, critical speeds, mode shapes and energy distribution among the system. The evaluation of the rotor, bearing strain energy, for example, can give us important insight as to whether we may expect to encounter any stability issues and unbalance response at a particular mode. Usually the critical speeds are must have to be 10% to 20% above or below the operating speed range. However, there are many rotors that operate on top of the second critical speed due to sufficient bearing damping. Often attempts to elevate the second critical speed by increased bearing stiffness leads to serious 1st mode stability problems. When bearing and seal damping is included, we can compute the damped natural frequencies or complex eigen values of the system. This analysis helps to identify the peak amplitude, phase at the particular mode frequency. The force which is applied may be the unbalance caused by any misalignment of the rotor system, offset of the disk from the centre or any kink caused due to self weight or thermal expansion of the system. The unbalance of the mass can be calculated as U = Rotor Mass Unbalance = Wu R.Where U represents a small weight Wu placed at a certain distance R away from the geometric centre C.

In an industrial rotor, the actual unbalance is always unknown. It is usually measured in terms of units such as ounce-inch, gram-inch or gram-mm. It also has an angle relative to the reference mark.

Alloy steels are the most widely used material for the turbine rotor shaft. Rotor shafts used in aero engine turbines need to withstand extremely high levels of temperature, pressure, and rotation. As Thrust capacity range increases, high speeds or larger shafts are required. The high specific strength of titanium alloy composites, over a wide temperature extend, make them extremely suitable for general aero engine applications and not least for rotors but rather additionally for structures. The remarkably great consumption properties likewise make titanium alluring for the compound and nourishment industry.

An important property of titanium-based materials is that reversible transformation of the crystal structure from alpha (\(\alpha\), hexagonal close-packed) structure to beta (\(\beta\), body-cantered cubic) structure when the temperatures reaches certain level. This allotropic behaviour, which depends on the type and amount of alloy contents, allows complex variations in microstructure and more diverse strengthening opportunities than those of other nonferrous alloys such as copper or aluminium. By alloying, the character, in terms of the \(\alpha/\beta\) - ratio, can be adjusted to allow for special properties by the processing. Alloys of interest for aerospace are Ti-5Al-2Sn-2Zr-4Mo-4Cr (commonly called Ti-17) and Ti-6Al-2Sn-4Zr-6Mo for high strength in heavy sections at elevated temperatures and Ti6242 (Ti-6Al-2Sn-4Zr-2Mo) for creep resistance.
II. OBJECTIVES

The objectives of this paper are to construct the Campbell diagram from the modal frequencies to find the critical speed of both steel and Ti 6-2-4-6 material, Unbalance response analysis to validate the critical speed obtain from the Campbell diagram to the frequency at the peak response from the Harmonic response analysis.

III. LITERATURE SURVEY

H.D. Nelson, J.M McVaugh [1] discussed about the finite element modeling procedure for the shaft elements, rigid disk and different bearings such as iso-tropic bearing, Ortho-tropic bearings for modal analysis and response analysis. Inertia effects due to the rotation of shaft and gyroscopic effects also included in their work. A comparison also made between the critical frequencies and frequency obtained from the unbalance response analysis.

Ritesh Fegade., Vimal Patel [2] studied on Unbalance response analysis using ANSYS APDL with various bearings located at two different locations. Harmonic analysis option is used to perform the unbalance response analysis in the work. The FE modeling approach shows the good accuracy with analytical calculations.

Rosyid et al [3] utilized Sub structuring technique for the investigation of rotor bearing framework utilizing model decrease procedure. The adequacy of diminished model was assessed by looking at basic velocities, damping proportion, first regular frequencies and the reaction of both the full framework and diminished framework. The productivity of the diminished model relies on the assurance of the number and area of ace hub. In any case, the drawback was that the decrease was less exact in insecure or almost precarious framework.

Khulief and Mohiuddin [4] exhibited a limited component elastodynamic model of rotor bearing framework which represented gyroscopic minute and anisotropic orientation. Two modular truncation plans were presented for planar (undamped) mode and complex (damped) mode. For both modular decrease plans modular attributes and dynamic reactions of two rotor framework were assessed. Creators inferred that both had a similar degree of precision. Be that as it may, it was extremely advantageous to utilize modal transformation for gyroscopic matrices.

Nicoara Dumitru, Eugenia Secară [5] discussed about the computational technique for the rotating structures with bearings. They developed mass matrix, gyroscopic matrix, damping matrix and stiffness matrix for rotor system with bearings in order to solve the equations of motion to have the fundamental frequencies.

Ulrich Werner [6] presented the advanced mathematical model to analyze the system level threshold stability. System level comprises of the load path from the rotors, bearings and stator parts. Causation of instability from internal rotor damping considered as the source.

Yogesh Verma [7] discussed about various model reduction methods to reduce size of the full model to the optimum level without effecting the dynamic characteristics of the system. He obtained Campbell diagram and unbalance response and compared against the regular methods where he got good accuracy between these approaches.

Erik Swanson, Chris D. Powell, Sorin Weissman [8] presented an excellent approach towards the practical understanding of the rotor dynamics terminology and behavior of the rotor when it vibrates. In their work they explained different rotor modes which will generate when any rotating structure is in operation. And also with the help of Campbell diagram and speed vs response chart the shown the both maximum amplitude mode and critical speed are identical to each other.

Mohammad T. Ahmadian, Omid Ghasemalizadeh, Mohammad Bonakdar [9] discussed about the calculation of natural frequencies with the help of transfer matrix method (TMM) of rotor bearing system. For all the components such as Shaft elements, disk and bearings they derived the elemental matrices which can be incorporated in to the equation of motion to compute the natural frequencies.

Basim A. Khidhir, Ksm Sahari [10] discussed about the titanium and its alloy in the sense of the crystal structure of the material, effect of the alloying materials and its classification. The effect of the heat treatment on the titanium also presented along with the specific properties. Titanium alloy composition with respect grades and its mechanical properties are also presented along with applications.

Sani A. Salihua, Y.I. Suleimanb, A. I. Eyninavia, Abdullahi Usmana [11] presented the review on the classification of the Titanium alloys, its composition, and mechanical properties. Discussed about applications of the titanium alloys in the various industries such as aerospace, automotive, medical industries etc.,
IV. METHODOLOGY

Nelson rotor which is a 355 mm long overhanging steel shaft of 14 various cross sections. The rotor carries a disc of mass 1.401 kg and eccentricity 1 mm at 88.9 mm from left end and is supported by firstly two bearings at a distance of 165.1 mm and 287 mm from the left end respectively. The geometry details of the nelson rotor as follows

| Cross section No | Inner Radius (mm) | Outer Radius (mm) |
|------------------|-------------------|-------------------|
| 1                | 0                 | 2.55              |
| 2                | 0                 | 5.1               |
| 3                | 0                 | 3.8               |
| 4                | 0                 | 10.15             |
| 5                | 0                 | 16.5              |
| 6                | 7.6               | 16.5              |
| 7                | 8.9               | 12.7              |
| 8                | 0                 | 12.7              |
| 9                | 0                 | 6.35              |
| 10               | 0                 | 7.6               |
| 11               | 0                 | 6.35              |
| 12               | 0                 | 19.05             |
| 13               | 0                 | 10.15             |
| 14               | 7.6               | 10.15             |

Rotor is built and analyses are performed using the ANSYS APDL. BEAM188, MASS21 and COMBI214 elements are adopted.

A. Shaft of the Rotor
Geometry of the 14 cross-sectional variations of shaft has been built in ANSYS APDL using BEAM188 which is a linear/quadratic two node beam element in 3-D with six DOF at each node, which includes three translational and three rotational movements. This element facilitates of cross sectional properties definition of the shaft.

B. Rotor Disk
DISK is modeled using MASS21 is a single point element. The degree of freedom of the MASS21 element can be up to 6 directions, which are three translational and three rotational movements. With the KEYOPT (3) option, the rotary inertia effects of the disk can be included or excluded and also the element can be reduced to a 2D capability. If the element has only one mass input, it is assumed that mass acts in all coordinate directions. Properties of the disk which is modeled as MASS21 elements are like mass 1.401e-3 Kg, Polar moment of inertia is 2 Kg-m^2 and Diametrical moment of inertia is 1.36 Kg-m^2.

C. Orthotropic Bearings
Symmetric orthotropic bearings are modeled with COMBI214 is a 2D spring damper bearing element with longitudinal tension and compression capability. It is defined by two nodes and has two degrees of freedom at each node: translations in any two nodal directions (x, y, or z). It does not represent any bending or torsional behavior. The element has stiffness (K) and damping (C) characteristics that can be defined in straight terms (K11, K22, C11, C22) as well as in cross-coupled terms (K12, K21, C12, C21). The Properties of the Orthotropic bearings are as Kxx=Kyy=4.378E4 N/mm nad Kxy=kyx= -8.756E3 N/mm.

D. Material Properties
To perform the modal analysis and unbalance response analysis for the rotor, the following material properties are used
Table 4.2: MATERIAL PROPERTIES

| Property                  | Ti-6-2-4-6 | Steel   |
|---------------------------|------------|---------|
| Young’s Modulus (Mpa)     | 114000     | 208000  |
| Poison Ratio              | 0.33       | 0.30    |
| Density (tonne/mm³)       | 4.65E-09   | 7.806E-09 |

The below geometry shows the finite element geometry of the rotor bearing system

![Finite Element Model](image)

Fig1. Finite Element Model

E. Modal Analysis
Modal analysis of the rotor is performed for both Ti-6-2-4-6 and steel to find out Modal frequencies at angular velocities of 100 rad/sec, 1000 rad/sec, 2000 rad/sec and 3141 rad/sec. Gyroscopic effects are included in this analysis by switching on the coriolis components about the stationary reference frame. The following APDL commands are used to perform the modal analysis:

ANTYPE,2 **MODAL
MODOPT,QRDAMP,20 *** QR DAMPED, No of modes = 20
MXPAND,20, , ,0
LUMP,1
PSTRES,0
MODOPT,QRDAMP,20,5,0,1,OFF
/STATUS,SOLU
SOLVE

F. Critical Speed Analysis
Critical speeds are obtained from constructing the Campbell diagram from the results or modal frequencies obtained from the Modal analysis performed above. Campbell diagram consists of rotor spin speed on X-axis and modal frequencies on the Y-axis. The intersection point of the Engine order line and the modal frequencies obtained in the modal analysis is called as Critical Speed.

G. Unbalance Forced Response Analysis
Forced response analysis is performed with unit static unbalance located at the 1 mm offset from the center of the shaft at 88.9 mm of length from the left end. This analysis performed with ANSYS APDL by selecting Harmonic analysis option. The excitation frequency range used for this analysis is ranging from 0 to 480 Hz. Time history plot consisting of the frequency (Ω) in Hz on the X-axis and Amplitude on the Y-axis. The following APDL code is used to perform the Forced response analysis.
V. RESULTS

A. Modal Frequencies

Modal frequencies are extracted from the modal analysis. These frequencies indicate or help in understanding the dynamic characteristics of the rotor bearing system. The mode shape at the particular modal frequency indicates the energy distribution among the components in the rotor system. Below Table 36 shows the modal frequency of the steel.

| Mode No | Angular Velocity (rad/sec) |
|---------|-----------------------------|
|         | 0   | 1000 | 2000 | 3015 |
| 1       | 327.6 | 275.1 | 227.6 | 185.8 |
| 2       | 340.5 | 403.4 | 480.1 | 569.7 |
| 3       | 962.3 | 959.5 | 956.8 | 954.0 |
| 4       | 962.9 | 966.2 | 970.3 | 975.9 |
| 5       | 1188.2 | 1162.9 | 1138.8 | 1115.3 |
| 6       | 1194.4 | 1225.5 | 1267.6 | 1297.9 |
| 7       | 2784.1 | 2767.1 | 2749.2 | 2729.7 |
| 8       | 2788.0 | 2806.1 | 2827.6 | 2854.1 |

| Mode No | Angular Velocity (rad/sec) |
|---------|-----------------------------|
|         | 100 | 1000 | 2000 | 3141 |
| 1       | 269.0 | 213.5 | 167.3 | 130.4 |
| 2       | 283.3 | 355.4 | 446.7 | 555.2 |
| 3       | 913.8 | 911.4 | 909.1 | 906.7 |
| 4       | 914.4 | 917.3 | 921.1 | 926.7 |
| 5       | 1087.2 | 1067.8 | 1049.2 | 1030.9 |
| 6       | 1092.0 | 1116.3 | 1150.0 | 1201.4 |
| 7       | 2620.1 | 2605.4 | 2589.8 | 2572.7 |
| 8       | 2623.5 | 2639.0 | 2657.2 | 2679.4 |
B. Campbell Diagram

A Campbell diagram is the most traditional way of graphical representing the interference margin between the natural frequency and the external excitation frequency. It is usually constructed between the frequency vs the rotational speed of the shaft. The mode shapes and excitation lines are plotted on the Campbell for the complete speed range. Campbell diagram is one of the most important tool for understanding the dynamic behavior of the rotating machines, Campbell diagram can be drawn to find the wheel section points of the whirl speeds with one revolution excitations. This diagram provides the projection of compressor disk wheel from axial vibration. The critical speed of the rotor after the instabilities which cause resonance conditions due to critical speed of the rotor system can be expressed in analytical form. Evaluation of critical speed of the rotor system is great important for controlling the resonance by providing necessary damping. The below two figudges indicates the Campbell diagram for the steel and Ti6-2-4-6 alloy respectively. The intersection point of the Engine order line (Black color line) with I st BW whirl frequency line indicates the critical speed of the rotor bearing system where resonance occurs. 250 Hz and 210 Hz are the critical speeds obtained for the steel and Ti6-2-4-6 alloy. From the above results we can conclude that there is only 15% reduction in the critical speed for the Ti 6-2-4-6 alloy with 40% reduction in the weight.

C. Frequency at Peak Response

Frequency response plots are obtained from the static unbalance forced response analysis. Down table shows the comparison of the Critical speeds from the Campbell diagram vs frequency at the peak response obtained from the forced response analysis as well as comparison of the critical speeds and frequency from forced response analysis between the steel and Ti 6-2-4-6 alloy materials.

| Material         | Critical Speed | Frequency @Peak Response | % of Difference (Critical speed vs frequency at peak) |
|------------------|----------------|--------------------------|-----------------------------------------------------|
| Steel            | 248            | 252                      | -1.61%                                              |
| Ti 6-2-4-6       | 210            | 214                      | -1.90%                                              |
| % of difference  | 15%            | 15%                      |                                                     |

VI. CONCLUSION AND FUTURE SCOPE

A. Conclusions

Critical speed and Unbalance forced response analysis of the both steel and Ti 6-4-2-6 leads to two important conclusions. Ti 6-4-2-6 alloy is 40% less weight compared to steel with only 15% reduction in the critical speeds for the same volume of the materials. So it can be best suitable material for the rotors of the aero engine turbines which demands less weight and better rotor dynamics capabilities. Another conclusion that can be drawn is less than 2% deviation in the frequencies obtained from the Campbell diagram and Unbalance forced response analysis for both steel and Ti 6-4-2-6 alloy within the given operating speed range. So we can directly consider the resonant frequency or critical speed from the Unbalance forced response analysis which gives many other insights like the deflections or the sensitivity of the key locations due to the applied unbalances without constructing the Campbell diagram.
B. Future Scope

In future this work can be extended to assess the stability condition of the Ti 6-4-2-6 alloy in comparison with the steel. Can also perform studies on whirl orbit plots, sensitivity of bearing locations, thinner sections, mid section of the rotor subjected to various loading conditions such as dynamic unbalance, thermal loads etc...Further the above work can be studied using the speed dependent bearings, different types of dampers to suite all the industrial needs.

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