Abstract

The paper presents the QCD description of the small $x$ behaviour of parton distribution functions in the leading twist of Wilson operator product expansion. The smooth transition between the cases of the soft and hard Pomerons is obtained. The results are in qualitative agreement with deep inelastic experimental data.

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1 Introduction

The recent measurements of the deep-inelastic (DIS) structure function (SF) $F_2$ by the H1 [1] and ZEUS [2] collaborations open a new kinematical range to study proton structure. The new HERA data show the strong increase of $F_2$ with decreasing $x$, that contradicts to experimental data of the NMC [3] and E665 collaboration [4] at lower $Q^2$ ($Q^2 \sim 1 GeV^2$), which are quite flat at $x \sim 10^{-2}$. These lower $Q^2$ data are in the good agreement with the standard Pomeron or with the Donnachie-Landshoff picture [5] where the Pomeron intercept $\alpha_p = 1.08$, is very close to standard one $\alpha_p = 1$. However, the HERA data extracted at the larger $Q^2$ ($Q^2 > 10 GeV^2$) are fitted very well (see [3, 6]) by parametrizations of parton distributions (PD) containing the supercritical (or Lipatov) Pomeron. The interpretation of the fast changing of the intercept in the $Q^2$ region between $Q^2 = 1 GeV^2$ and $Q^2 = 10 GeV^2$ (see [3]) is yet absent. There are arguments (see [3]) in favour of one intercept or a superposition of two different Pomeron trajectories, one having an intercept of 1.08 and the one of 1.5 (see discussions of this problem in [3]).

The aim of this letter is a possible “solution” of this problem in the framework of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [10]. We will seek the “solution” of DGLAP equation imposing the Regge-type behaviour (we use the parton distributions (PD) multiplied by $x$ and neglect the nonsinglet quark distribution at small $x$):

$$f_a(x, Q^2) \sim x^{-\delta} \tilde{f}_a(x, Q^2), \quad (a = q, g) \quad (\alpha_p \equiv 1 + \delta)$$

where $\tilde{f}_a(x, Q^2)$ is nonsingular at $x \to 0$ and $\tilde{f}_a(x, Q^2) \sim (1 - x)^\nu$ at $x \to 1$\footnote{We use the term “solution” because we will work in the leading twist approximation in the range of $Q^2$: $Q^2 > 1 GeV^2$, where the higher twist terms may give the sizeable contribution. Moreover, our “solution” is the Regge asymptotic [10] with unknown parameters rather then the solution of DGLAP equation. The parameters are found from the agreement of the r.h.s. and l.h.s. of the equation.}. Of course, we understand that the Regge behaviour [10] contradicts to the well-known double-logarithmical solution: $\sim \exp \sqrt{\phi(Q^2) \ln(1/x)}$, where $\phi(Q^2)$ is a known $Q^2$-dependent function\footnote{Consideration of the more complicate behaviour in the form $x^{-\delta} (\ln(1/x))^{1/2} \sqrt{\phi \ln(1/x)}$ is given in [11]}. However, we think that it is not possible to glue the both Regge-type behaviour: Donnachie-Landshoff picture at low $Q^2$ and Lipatov Pomeron at large $Q^2$, having the double-logarithmical (non Regge-type) solution at immediate $Q^2$.

In the case of the large $\delta$ values (i.e. $x^{-\delta} \gg 1$) similar investigations were already done (see [12-14]) and the results are well known (see [12] for the first three orders and [14] containing a resummation of all orders, respectively):

$$\frac{f_a(x, t)}{f_a(x, t_0)} = \frac{M_a(1 + \delta, t)}{M_a(1 + \delta, t_0)},$$

where $t = \ln(Q^2/\Lambda^2)$, $t_0 = t(Q^2 = Q_0^2)$ and $M_a(1 + \delta, t)$ is the analytical continuation of the PD moments $M_a(n, t) = \int_0^1 dx x^{n-2} f_a(x, t)$ to the noninteger value “$n = 1 + \delta$”. Note \footnote{More correctly, $\phi$ is $Q^2$-dependent for the solution of DGLAP equations with the boundary condition: $f_a(x, Q_0^2) = Const$ at $x \to 0$. In the case of the boundary condition: $f_a(x, Q_0^2) \sim \exp \sqrt{\ln(1/x)}$, $\phi$ is lost (see [11]) its $Q^2$-dependence.}
that recently the fit of HERA data was done in [8] with the formula for PD $f_q(x, t)$ very close to (2) and a very good agreement (the $\chi^2$ per degree of freedom is 0.85) is found for $\delta = 0.40 \pm 0.03$. There are also fits [14] of the another group using equations which are similar to (2) in the LO approximation.

In this article we expand these results to the range where $\delta \sim 0$ (and $Q^2$ is not large) following to the observed earlier (see [12, 13] and Appendix) method [4] to replace the Mellin convolution by a simple product.

Consider DGLAP equations and apply the method from [13] to the Mellin convolution in its r.h.s. (in contrast with standard case, we use below $\alpha(Q^2) = \alpha_s(Q^2)/(4\pi)$):

$$
\frac{d}{dt}f_a(x, t) = -\frac{1}{2} \sum_{i=a,b} \tilde{\gamma}_{ai}(\alpha, x) \otimes f_i(x, t) \quad (a, b) = (q, g)
$$

$$
= -\frac{1}{2} \sum_{i=a,b} \tilde{\gamma}_{ai}(\alpha, 1 + \delta) f_i(x, t) + O(x^{1-\delta}) \quad \left(\gamma_{ab}(\alpha, n) = \alpha \gamma_{ab}(0)(n) + \alpha^2 \gamma_{ab}(1)(n) + \ldots\right),
$$

where $t = \ln(Q^2/\Lambda^2)$. The $\tilde{\gamma}_{ab}(\alpha, x)$ are the splitting functions corresponding to the anomalous dimensions (AD) $\gamma_{ab}(\alpha, n) = \int_0^1 dx x^{n-2} \tilde{\gamma}_{ab}(\alpha, x)$. Here the functions $\gamma_{ab}(\alpha, 1 + \delta)$ are the AD $\gamma_{ab}(\alpha, n)$ expanded from the integer argument “$n$” to the noninteger one “$1 + \delta$”. The functions $\tilde{\gamma}_{ab}(\alpha, 1 + \delta)$ (marked below as AD, too) can be obtained from the functions $\gamma_{ab}(\alpha, 1 + \delta)$ replacing the term $1/\delta$ by the one $1/\tilde{\delta}$:

$$
\frac{1}{\delta} \rightarrow \frac{1}{\tilde{\delta}} = \frac{1}{\delta} (1 - \varphi(\nu, \delta)x^\delta)
$$

This replacement (4) appears very naturally in the consideration of the Mellin convolution at $x \rightarrow 0$ (see [13] and Appendix) and preserves the smooth and nonsingular transition to the case $\delta = 0$, where

$$
\frac{1}{\delta} = \ln \frac{1}{x} - \varrho(\nu)
$$

The concrete form of the functions $\varphi(\nu, \delta)$ and $\varrho(\nu)$ depends strongly on the type of the behaviour of the PD $f_a(x, Q^2)$ at $x \rightarrow 0$ and in the case of the Regge regime (4) they are (see Appendix):

$$
\varphi(\nu, \delta) = \frac{\Gamma(\nu + 1)\Gamma(1 - \delta)}{\Gamma(\nu + 1 - \delta)} \quad \text{and} \quad \varrho(\nu) = \Psi(\nu + 1) - \Psi(1),
$$

where $\Gamma(\nu + 1)$ and $\Psi(\nu + 1)$ are the Euler $\Gamma$- and $\Psi$-functions, respectively. As it can be seen, there is a correlation with the PD behaviour at large $x$.

If $\delta$ is not small (i.e. $x^{-\delta} >> 1$), we can replace $1/\tilde{\delta}$ by $1/\delta$ in the r.h.s. of Eq.(3) and obtain its solution in the form (2). The case of small $\delta$ values will be considered lower.

The new points in our investigations are as follows. Note that the $Q^2$-evolution of $M_a(1 + \delta, t)$ contains the two: “+” and “−” components, i.e. $M_a(1 + \delta, t) = \sum_{i=\pm} M_i^a(1 + \delta, t)$, and in principle each component evolves separately and may have independent (and not equal) intercept.

5The used formula (Eq.(2) from [3]) coincides with (2) in the leading order (LO) approximation, if we save only $f_q(x, Q^2)$ in the r.h.s. of [3] (or put $\gamma_{qq} = 0$ and $\gamma_{qg} = 0$ formally). Eq.(2) and Eq.(3) from [3] have some differences in the next-to-leading order (NLO), which are not very important because they are corrections to the $\alpha$-correction.

6The method is based on the earlier results [7]
2 Leading order

Consider DGLAP equations for the “+” and “−” parts (hereafter \( s = \ln(lnt/lnt_0) \)):

\[
\frac{d}{ds}f^\pm_a(x,t) = -\frac{1}{2\beta_0}\tilde{\gamma}_\pm(\alpha, 1 + \delta_\pm)f^\pm_a(x,t) + O(x^{1-\delta}),
\]

(7)

where

\[
\gamma_\pm = \frac{1}{2}\left[ (\gamma_{gg} + \gamma_{qq}) \pm \sqrt{(\gamma_{gg} - \gamma_{qq})^2 + 4\gamma_{gq}\gamma_{gq}} \right]
\]

are the AD of the “±” components (see, for example, [18]).

The AD \( \tilde{\gamma}_-(\alpha, 1 + \delta_-) \) does not contain the singular term (see [12, 14] and below) and \( f^-_a(x,t) \) have the solution in the form:

\[
f^-_a(x,t) = e^{-d_- (1+\delta_-) s}, \text{ where } d_- = \frac{\gamma_\pm (1 + \delta_\pm)}{2\beta_0}
\]

(8)

The AD \( \tilde{\gamma}_+(\alpha, 1 + \delta_+) \) contains the singular term and \( f^+_a(x,t) \) have the solution similar (8) only for \( x^{-\delta_+} \gg 1 \):

\[
f^+_a(x,t) = e^{-d_+ (1+\delta_+) s}, \text{ if } x^{-\delta_+} \gg 1
\]

(9)

Both intercepts \( 1 + \delta_+ \) and \( 1 + \delta_- \) are unknown and should be found, in principle, from the analysis of the experimental data. However there is the another way. From the small \( Q^2 \) (and small \( x \)) data of the NMC [3] and E665 collaboration [4] we can conclude that the SF \( F_2 \) and hence the PD \( f_a(x,Q^2) \) have flat asymptotics for \( x \to 0 \) and \( Q^2 \sim (1 \div 2) GeV^2 \). Thus we know that the values of \( \delta_+ \) and \( \delta_- \) is approximately zero at \( Q^2 \sim 1 GeV^2 \).

Consider Eqs.(7) with \( \delta_\pm = 0 \) and with the boundary condition \( f_a(x,Q^2_0) = A_a \) at \( Q^2_0 = 1 GeV^2 \). For the “−” component we already have the solution: Eq.(8) with \( \delta_- = 0 \) and \( d_-(1) = 16f/(27\beta_0) \), where \( f \) is the number of the active quarks and \( \beta_i \) are the coefficients in the \( \alpha \)-expansion of QCD \( \beta \)-function. For its “+” component Eq.(7) can be rewritten in the form (hereafter the index \( 1 + \delta_+ \) will be omitted in the case \( \delta \to 0 \)):

\[
ln\left(\frac{1}{x}\right)\frac{d}{ds}\delta_+(s) + \frac{d}{ds}ln(A^+_a) = -\frac{1}{2\beta_0}\left[ \hat{\gamma}_+(ln\left(\frac{1}{x}\right) - \varrho(\nu)) + \overline{\gamma}_+ \right]
\]

(10)

where \( \hat{\gamma}_+ \) and \( \overline{\gamma}_+ \) are the coefficients of the singular and regular parts at \( \delta \to 0 \) of AD \( \gamma_+(1+\delta) \):

\[
\gamma_+(1+\delta) = \hat{\gamma}_+ + \overline{\gamma}_+, \quad \hat{\gamma}_+ = -24, \quad \overline{\gamma}_+ = 22 + \frac{4f}{27}
\]

The solution of Eq.(10) is

\[
f^+_a(x,t) = A^+_a x^{d_+ s} e^{-\overline{d}_+ s},
\]

(11)

where

\[
\hat{d}_+ \equiv \frac{\hat{\gamma}_+}{2\beta_0} \simeq \frac{4}{3}, \quad \overline{d}_+ = \frac{1}{2\beta_0} \left( \overline{\gamma}_+ - \hat{\gamma}_+ \varrho(\nu) \right) \simeq \frac{4}{3} \varrho(\nu) + \frac{101}{81}
\]
Hereafter the symbol $\simeq$ marks the case $f = 3$.

As it can be seen from (11) the flat form $\delta_+ = 0$ of the “$+$”-component of PD is very nonstable from the (perturbative) viewpoint, because $d(\delta_+/ds) \neq 0$, and for $Q^2 > Q_0^2$ we have already the nonzero power of $x$ (i.e. Pomeron intercept $\alpha_p > 1$). This is in agreement with the experimental data. Let us note that the power of $x$ is positive for $Q^2 < Q_0^2$ that is in principle also supported by the NMC [3] data, but the use of this analysis to $Q^2 < 1GeV^2$ is open question.

Thus, we have the DGLAP equation solution for the “$+$” component at $Q^2$ is close to $Q_0^2 = 1GeV^2$, where Pomeron starts in its movement to the supercritical (or Lipatov [19, 20]) regime and also for the large $Q^2$, where Pomeron have $Q^2$-independent intercept. In principle, the general solution of (7) should contain the smooth transition between these pictures but this solution is absent [1]. We introduce some “critical” value of $Q^2$, $Q_c^2$, where the solution (9) is replaced by the solution (11). The exact value of $Q_c^2$ may be obtained from a fit of experimental data. Thus, we have in the LO of the perturbation theory:

\[
\begin{align*}
    f_a(x, t) &= f_a^-(x, t) + f_a^+(x, t), \\
    & f_a^-(x, t) = A_a^- \exp(-d_-s), \\
    & f_a^+(x, t) = \begin{cases}
                A_a^+ x^d_+ \exp(-\mu^+ s), & \text{if } Q^2 \leq Q_c^2 \\
                f_a^+(x, t_c) \exp(-d_+(1 + \delta_c)(s - s_c)), & \text{if } Q^2 > Q_c^2
            \end{cases}
\end{align*}
\]

where

\[
\begin{align*}
    t_c &= t(Q_c^2), 
    s_c &= s(Q_c^2), 
    A_a^- = A_a - A_a^+ & \text{and} \\
    A_a^+ = (1 - \rho_0)A_a^+ - \rho_0 A_g, 
    A_g^+ = \rho_0 A_g - \varepsilon A_g
\end{align*}
\]

and the values of the coefficients $\rho_0$, $\alpha$ and $\varepsilon$ may be found, for example, in [18].

Using the concrete AD values at $\delta = 0$ and $f = 3$, we have

\[
A_a^+ \approx \frac{1}{27} \frac{4A_q + 9A_g}{\ln(\frac{1}{x}) - \rho(\nu) - \frac{85}{108}}, 
A_a^+ \approx A_a + \frac{4}{9} A_q - \frac{4}{243} \frac{9A_g - A_q}{\ln(\frac{1}{x}) - \rho(\nu) - \frac{85}{108}}
\]

Thus, the value of the “$+$” component of the quark PD is suppressed logarithmically and this is in qualitative agreement with the HERA parametrizations of SF $F_2$ (see [21, 22]) (in the LO $F_2(x, Q^2) = (2/9) f_a(x, Q^2)$ for $f = 3$), where the magnitude connected with the factor $x^{-\delta}$ is $5 \div 10\%$ from the flat (for $x \to 0$) magnitude.

3 Next-to-leading order

By analogy with the previous section and knowing the NLO $Q^2$-dependence of PD moments, we obtain the following equations for the NLO $Q^2$-evolution of the both: ”$+$” and ”$-$” PD components (hereafter $\tilde{s} = ln(\alpha(Q_0^2)/\alpha(Q^2)), p = \alpha(Q_0^2) - \alpha(Q^2)$):

\[
\begin{align*}
    f_a(x, t) &= f_a^-(x, t) + f_a^+(x, t), \\
    & f_a^-(x, t) = \bar{A}_a^- \exp(-\tilde{d}_-\tilde{s} - \tilde{d}_-p), \\
    & f_a^+(x, t) = \begin{cases}
                \bar{A}_a^+ x^{d_+} \exp(-\tilde{d}_+\tilde{s} - \tilde{d}_+p), & \text{if } Q^2 \leq Q_c^2 \\
                f_a^+(x, t_c) \exp(-d_+(1 + \delta_c)(\tilde{s} - \tilde{s}_c) - d_+(1 + \delta_c)(p - p_c)), & \text{if } Q^2 > Q_c^2
            \end{cases}
\end{align*}
\]

\[
\text{The form } \exp\left(-s\gamma_+(1 + \delta_\nu)/2\beta_0\right) \text{ coincides with both solutions: Eq.}(11) \text{ if } x^d_+ >> 1 \text{ and Eq.}(11) \text{ when } \delta = 0 \text{ but it is not the solution of DGLAP equation.}
\]
where

\[
\bar{s}_c = \bar{s}(Q_c^2), \quad p_c = p(Q_c^2), \quad \alpha_0 = \alpha(Q_0^2), \quad \alpha_c = \alpha(Q_c^2)
\]

\[
\bar{A}_q^+ = (1 - \alpha_0 K^a_\pm) A_q^+ + \alpha_0 K^a_\pm A_q^-
\]

\[
d_{++}^a = \bar{d}_{++}^a \left( \ln \left( \frac{1}{x} \right) - g(\nu) \right) + \bar{d}_{++}^q, \quad d_{++}^q = \frac{\gamma^\pm}{2 \beta_0} - \frac{\gamma^\pm \beta_1}{2 \beta_0^2} - K^q_\pm
\]

and \n
\[
K^q_\pm = \frac{\gamma^\pm}{2 \beta_0 + \gamma^\pm - \gamma^+}, \quad K^q_\pm = K^q_\pm \frac{\gamma^+ - \gamma_{q0}^{(0)}}{\gamma^+ - \gamma_{q0}^{(0)}}
\]

(16)

The NLO AD of the “±” components are connected with the NLO AD \(\gamma_{ab}^{(1)}\). The corresponding formulae can be found in [12].

Using the concrete values of the LO and NLO AD at \(\delta = 0\) and \(f = 3\), we obtain the following values for the NLO components from (15),(16) (note that we keep only the terms \(\sim O(1)\) in the NLO terms)

\[
d_{-}^q \simeq \frac{16}{81} \left[ 2 \zeta(3) + 9 \zeta(2) - \frac{779}{108} \right] \approx 1.97, \quad d_{-}^q \simeq \frac{28}{81} \simeq 2.32
\]

\[
d_{+}^q \simeq \frac{2800}{81}, \quad \bar{d}_{++}^q \simeq 32 \left[ \zeta(3) + \frac{263}{216} \zeta(2) - \frac{372607}{69984} \right] \approx -67.82
\]

\[
d_{+}^q \simeq \frac{1180}{81}, \quad \bar{d}_{++}^q \simeq \frac{953}{27} - 12 \zeta(2) \approx -52.26
\]

(17)

and

\[
\bar{A}_q^+ \simeq \frac{20}{3} \alpha_0 \left[ A_q + \frac{4}{9} A_q \right] + \frac{4}{27} A_q \left( \frac{9 A_q - A_q}{27 \ln(\frac{1}{x}) - g(\nu) - \frac{85}{108}} \right) (1 - \frac{692}{81} \alpha_0)
\]

(18)

It is useful to change in Eqs. (15)–(18) from the quark PD to the SF \(F_2(x, Q^2)\), which is connected in NLO approximation with the PD in the following way (see [18]):

\[
F_2(x, Q^2) = \left( 1 + \alpha(Q^2)B_q(1 + \delta) \right) \delta^2 f_q(x, Q^2) + \alpha(Q^2)B_q(1 + \delta) \delta^2 f_q(x, Q^2),
\]

(19)

where \(\delta^2 = \sum_{i=1}^f / f \equiv< e_i^2 >\) is the average charge square of the active quarks: \(\delta^2 = (2/9\) and \(5/18\) for \(f = (3\) and \(4\), respectively. The NLO corrections lead to the appearence in the r.h.s. of Eqs. (15) of the additional terms \(\left( 1 + \alpha_B \pm \right)/\left( 1 + \alpha_0 B \pm \right)\) and the necessity to transform \(\bar{A}_q^+\) to \(C^\pm \equiv F_2^\pm(x, Q^2)\) into the input parts. The final results for \(F_2(x, Q^2)\) are in the form:

\[
F_2(x, t) = F_2^-(x, t) + F_2^+(x, t)
\]

\[
F_2^-(x, t) = C^- \exp \left( -d^-_\bar{s} - d^-_{-}p \right)(1 + \alpha B^-)/(1 + \alpha_0 B^-)
\]

\[
F_2^+(x, t) = C^+ \exp \left( -d^+_{+} p - d^+_\bar{s} - d^+_q \right)(1 + \alpha B^+)/(1 + \alpha_0 B^+), \quad \text{if} \ Q^2 \leq Q^2_c
\]

\[
F_2^+(x, t_c) \exp \left( -d^+_{+} (1 + \delta_c)(\bar{s} - \bar{s}_c) - d^+_q (p - p_c) \right) \left( 1 + \alpha B^+ (1 + \delta_c) \right)/(1 + \alpha_0 B^+ (1 + \delta_c)), \quad \text{if} \ Q^2 > Q^2_c
\]

(20)
where

\[ B^\pm = B_q + \frac{\gamma_\pm}{\gamma_{0q}} B_g, \quad C^\pm = \tilde{A}_q^\pm (1 + \alpha_0 B^\pm) \]

with the substitution of \( A_q \) by \( C \equiv F_2(x, Q_0^2) \) into Eq.(18) \( \tilde{A}_q^\pm \) according

\[ C = (1 + \alpha_0 B_g) \delta_s^+ A_q + \alpha_0 B_q \delta_s^2 A_g, \quad (21) \]

For the gluon PD the situation is more simple: in Eq.(18) it is necessary to replace \( A_q \) by \( C \) according to Eq.(21).

For the concrete values of the LO and NLO AD at \( \delta = 0 \) and \( f = 3 \), we have for \( Q^2 \)-evolution of \( F_2(x, Q^2) \) and the gluon PD:

\[
F_2(x, t) = F_2^-(x, t) + F_2^+(x, t), \quad f_g(x, t) = f_g^-(x, t) + f_g^+(x, t)
\]

\[
F_2^-(x, t) = C^- \exp \left(-\frac{32}{81} \bar{s} - 1.97 p \right)(1 - \frac{8}{9} \alpha)/(1 - \frac{8}{9} \alpha_0)\left [ C^+ x \left(\frac{-\frac{4}{3} + \frac{2800}{81} \rho}{\rho} \right) \exp \left(-\frac{4}{3} (\rho(\nu) + \frac{101}{108} \bar{s}) + \frac{2800}{81} \rho(\nu) + 67.82 \right)p \right] \]

\[
F_2^+(x, t) = \left \{ \begin{array}{ll}
(1 + 6[ln(\frac{1}{x}) - \rho(\nu) - \frac{101}{108} \alpha]/(1 + 6[ln(\frac{1}{x}) - \rho(\nu) - \frac{101}{108} \alpha_0]), & \text{if } Q^2 \leq Q_c^2
\end{array} \right.
\]

\[
f_g^+(x, t) = \left \{ \begin{array}{ll}
(1 + \alpha B^+(1 + \delta_c))/\left(1 + \alpha_c B^+(1 + \delta_c)\right), & \text{if } Q^2 > Q_c^2
\end{array} \right.
\]

where

\[ C^- = C - C^+, \quad \tilde{A}_g^- = A_g - \tilde{A}_g^+ \quad \text{and} \]

\[ C^+ \simeq \frac{2}{27} \left( 26 \alpha_0 [A_g + 2C] + \frac{A_g(1 - 10.5\alpha_0) + 2C(1 - 8.55\alpha_0)}{ln(\frac{1}{x}) - \rho(\nu) - \frac{85}{108}} \right)
\]

\[ \tilde{A}_g^+ \simeq A_g \left( 1 - \frac{88}{9} \alpha_0 \right) + 2C \left( 1 - \frac{692}{81} \alpha_0 \right) - \frac{2}{27} \frac{2A_g(1 - \frac{674}{81} \alpha_0) - C(1 - \frac{692}{81} \alpha_0)}{ln(\frac{1}{x}) - \rho(\nu) - \frac{85}{108}} \quad (24) \]

Let us give some conclusions following from Eqs.(22)-(24). It is clearly seen that the NLO corrections reduce the LO contributions. Indeed, the value of the supercritical Pomeron intercept, which increases as \( ln(\alpha_0/\alpha) \) in the LO, obtains the additional term \( \sim (\alpha_0 - \alpha) \) with the large (and opposite in sign to the LO term) numerical coefficient. Note that this coefficient is different for the quark and gluon PD and this is in agreement with the recent MRS(G) fit in [4] and the data analysis by ZEUS group (see [22]). The intercept of the gluon PD is larger than the quark PD one (see also [3, 22]). However, the effective reduction of the quark PD is smaller (which is in agreement with W.-K. Tung analysis in [23]), because the quark PD part increasing at small \( x \) obtains the additional \( \sim \alpha_0 \) but not \( \sim 1/lnx \) term, which is important at very small \( x \).

Note that there is the fourth quark threshold at \( Q_{th}^2 \sim 10 \text{GeV}^2 \) and the \( Q_{th}^2 \) value may be larger or smaller to \( Q_c^2 \) one. Then, either the solution in the r.h.s. of Eqs. [21, 22, 23]...
before the critical point $Q_c^2$ and the one for $Q^2 > Q_c^2$ contain the threshold transition, where the values of all variables are changed from ones at $f = 3$ to ones at $f = 4$. The $\alpha(Q^2)$ is smooth because $\Lambda_{\overline{MS}}^{F = 3} \rightarrow \Lambda_{\overline{MS}}^{F = 4}$ (see also the recent experimental test of the flavour independence of strong interactions into [24]).

For simplicity here we suppose that $Q_{th}^2 = Q_c^2$ and all changes initiated by threshold are done automatically: the first (at $Q^2 \leq Q_c^2$) solutions contain $f = 3$ and second (at $Q^2 > Q_c^2$) ones have $f = 4$, respectively. For the “−” component we should use $Q_{th}^2 = Q_c^2$, too.

Note only that the Pomeron intercept $\alpha_p = 1 - (d_+ s + d_- + p)$ increases at $Q^2 = Q_{th}^2$, because that agrees

$$\alpha_p - 1 = \begin{cases} \frac{4}{3} s(Q_{th}^2, Q_0^2) - \frac{2800}{81} p(Q_{th}^2, Q_0^2), & \text{if } Q^2 \leq Q_c^2 \\ 1.444s(Q_{th}^2, Q_0^2) - 38.11p(Q_{th}^2, Q_0^2), & \text{if } Q^2 > Q_c^2 \end{cases}$$

with results [25] obtained in the framework of dual parton model. The difference

$$\Delta \alpha_p = 0.11s(Q_{th}^2, Q_0^2) - 3.55p(Q_{th}^2, Q_0^2)$$

depends on the values of $Q_{th}^2$ and $Q_0^2$. For $Q_{th}^2 = 10 GeV^2$ and $Q_0^2 = 1 GeV^2$ it is very small:

$$\Delta \alpha_p = 0.012$$

4 Discussion

Let us summarize the obtained results. We have got the DGLAP equation “solution” having the Regge form (1) for the two cases: at small $Q^2$ ($Q^2 \sim 1 GeV^2$), where SF and PD have the flat behaviour at small $x$, and at large $Q^2$, where SF $F_2(x, Q^2)$ fastly increases when $x \rightarrow 0$. The behaviour in the flat case is nonstable with the perturbative viewpoint because it leads to the production of the supercritical value of Pomeron intercept at larger $Q^2$ and the its increase (like $4/3 \ln(\alpha(Q_0^2)/\alpha(Q^2))$ in LO) when the $Q^2$ value increases. The solution in the Lipatov Pomeron case corresponds to the well-known results (see [24, 25, 26]) with $Q^2$-independent Pomeron intercept. The general “solution” should contain the smooth transition between these pictures. Unfortunately, it is impossible to obtain it in the case of the simple approximation (1), because the r.h.s. of DGLAP equation (7) contains both: $\sim x^{-\delta}$ and $\sim const$ terms. As a result, we used the two above “solutions” gluing them at some point $Q_c^2$.

Note that our “solution” is some generation (or an application) of the solution of the DGLAP equations in the momentum space. The last one has two: ”+” and “−” components. Our conclusions are related to the “+” component, which exhibits the basic Regge asymptotic behaviour. The Pomeron intercept corresponding to “−” component, is $Q^2$-independent and this component is the subasymptotical one at large $Q^2$. However, the magnitude of the “+” is suppressed like $1/\ln(1/x)$ and $\alpha(Q_0^2)$, and the subasymptotical “−” component may be important. Indeed, it is observed experimentally (see [21, 22]). Note, however, that the suppression $\sim \alpha(Q_0^2)$ is really very slight if we choose a small value of $Q_0^2$.

\footnote{The Pomeron intercept value increasing with $Q^2$ was obtained also in [20, 27].}
Our “solution” in the form of Eqs. (22)-(24) is in very good agreement with the recent MRS\( (G)\) fit [4] and with the results of [7] at \( Q^2 = 15 GeV^2 \). As it can be seen from Eqs. (22), (23), in our formulae there is the dependence on the PD behaviour at large \( x \). Following [28] we choose \( \nu = 5 \) that agrees in the gluon case with the quark counting rule [29]. This \( \nu \) value is also close to the values obtained by CCFR group [30] (\( \nu = 4 \)) and in the last MRS\( (G)\) analysis [6] (\( \nu = 6 \)). Note that this dependence is strongly reduced for the gluon PD in the form

\[
f_g(x, Q_0^2) = A_g(\nu)(1 - x)^\nu,
\]

if we suppose that the proton’s momentum is carried by the gluon, is \( \nu \)-independent. We used \( A_g(5) = 2.1 \) and \( F_2(x, Q_0^2) = 0.3 \) when \( x \to 0 \).

For the quark PD the choice \( \nu = 3 \) is more preferable, however the use of two different \( \nu \) values complicates the analysis. Because the quark contribution to the “+” component is not large, we put \( \nu = 5 \) to both: quark and gluon cases. Note also that the variable \( \nu(Q^2) \) has (see [31]) the \( Q^2 \)-dependence determined by the LO AD \( \gamma_{NS}^{(0)} \). However this \( Q^2 \)-dependence is proportional to \( s \) and it is not important in our analysis.

Starting from \( Q_0^2 = 1 GeV^2 \) (by analogy with [26]) and from \( Q_0^2 = 2 GeV^2 \), and using two values of the QCD parameter \( \Lambda \): the more standard one (\( \Lambda_{f=4}^{MS} = 200 \ MeV \) and \( \Lambda_{f=4}^{MS} = 255 \ MeV \) obtained in [3]) [4], we have the following values of the quark and gluon PD “intercepts” \( \delta_a = - (\hat{d}_s + \tilde{s} + \hat{d}_a) \) (here \( \Lambda_{f=4}^{MS} \) is marked as \( \Lambda \)):

| \( Q^2 \) | \( \delta_q(Q^2) \) | \( \delta_g(Q^2) \) | \( \delta_q(Q^2) \) | \( \delta_g(Q^2) \) |
|---------|---------|---------|---------|---------|
| \( \Lambda = 200 \ MeV \) | \( \Lambda = 255 \ MeV \) | \( \Lambda = 200 \ MeV \) | \( \Lambda = 255 \ MeV \) | \( \Lambda = 255 \ MeV \) |
| 4 | 0.191 | 0.389 | 0.165 | 0.447 |
| 10 | 0.318 | 0.583 | 0.295 | 0.659 |
| 15 | 0.367 | 0.652 | 0.345 | 0.734 |

if \( Q_0^2 = 1 GeV^2 \)

| \( Q^2 \) | \( \delta_q(Q^2) \) | \( \delta_g(Q^2) \) | \( \delta_q(Q^2) \) | \( \delta_g(Q^2) \) |
|---------|---------|---------|---------|---------|
| \( \Lambda = 200 \ MeV \) | \( \Lambda = 255 \ MeV \) | \( \Lambda = 255 \ MeV \) | \( \Lambda = 255 \ MeV \) | \( \Lambda = 255 \ MeV \) |
| 4 | 0.099 | 0.175 | 0.097 | 0.198 |
| 10 | 0.226 | 0.368 | 0.227 | 0.410 |
| 15 | 0.275 | 0.438 | 0.278 | 0.486 |

if \( Q_0^2 = 2 GeV^2 \)

These values of \( \delta_a \) are above at \( 4 GeV^2 \) those from [4]. Because we have the second (subasymptotical) part, the our effective “intercepts” have smaller values.

Note that as input we can use the Eq. (1) with \( \delta \equiv \varepsilon = 0.08 \), which corresponds to Donnachie-Landshoff value [33] of the Pomeron intercept. For this purpose we should represent the value \( 1/\varepsilon \) (see Eq. (4)) as the series (here \( \Psi(\nu) \equiv \frac{d}{d\nu} \Psi(\nu) \)):

\[
\frac{1}{\varepsilon} = \frac{1}{\varepsilon} \left[ 1 - \frac{\Gamma(\nu + 1)\Gamma(1 - \varepsilon)}{\Gamma(\nu + 1 - \varepsilon)} x^\varepsilon \right] = \ln \frac{1}{x} - \left[ \Psi(\nu + 1) - \Psi(1) \right] + \\
\frac{\varepsilon}{2} \left[ \left( \ln \frac{1}{x} - \left[ \Psi(\nu + 1) - \Psi(1) \right] \right)^2 - \left[ \Psi'(\nu + 1) - \Psi'(1) \right] \right] + O(\varepsilon^2)
\]
and save only the first term in the r.h.s. This is possible if the second term in the r.h.s. of Eq.(25) is negligibly small, i.e.

\[ \frac{\varepsilon}{2} \left[ \left( \ln \frac{1}{x} - \left[ \Psi(\nu + 1) - \Psi(1) \right] \right)^2 - \left[ \Psi(\nu + 1) - \Psi'(1) \right] \right] \ll \ln \frac{1}{x} - \left[ \Psi(\nu + 1) - \Psi(1) \right], \]

or for \( \nu = 5 \)

\[ 1.5 \cdot 10^{-12} \ll x \ll 9.6 \cdot 10^{-2} \]

This \( x \) range corresponds to the range considered here. The right boundary slowly depends on the \( \delta \) value: it is determined by the large \( x \) SF behaviour, i.e. by the \( \nu \) value. The left boundary strongly increases with increasing of the \( \delta \) value, that leads to the impossibility to apply the expansion (25) for the large \( \delta \) values.

In the case \( \delta = \varepsilon \) all our conclusions are not changed except the relation \( \alpha_p = 1 + \delta(Q^2) \), which transforms to the form \( \alpha_p = 1 + \varepsilon + \delta(Q^2) \), i.e. the value of the Pomeron intercept slightly increases.

In support of our analysis we represent the Fig.3 of paper [8] and add it by our results for \( \delta_q(Q^2) \) of above table at \( Q^2_0 = 1 \, GeV^2 \). The dashed-dotted curve represents the intercept of the Pomeron trajectory \( \alpha_p(Q^2) \) which was obtained in [26] as the result of a fit of experimental data. This analysis was done in the framework of Regge-type behaviour of DIS SF, that corresponds to the start point of our consideration, too. The authors of [26] obtained very strong \( Q^2 \)-dependence of the Pomeron intercept in the region of \( 1 \, GeV^2 < Q^2 < 10 \, GeV^2 \) and approximate \( Q^2 \)-independent its values \( \alpha_p \approx 1.05 \) and \( \alpha_p \approx 1.4 \) at \( Q^2 < 1 \, GeV^2 \) and \( Q^2 > 10 \, GeV^2 \), respectively. The solid curves represent our values of \( Q^2 \)-dependence of Pomeron intercept

\[
\alpha_p(Q^2) = \left\{ \begin{array}{ll}
1.05 + \delta_q(Q^2), & \text{if } Q^2 \leq Q^2_c \\
1.05 + \delta_q(Q^2_c), & \text{if } Q^2 > Q^2_c,
\end{array} \right.
\]

where we choose \( \alpha_p(Q^2_0) = 1.05 \) and \( Q^2_c = 15 \, GeV^2 \). As can be seen in the Figure both the results are in very good agreement.

As a conclusion, we note that BFKL equation (and thus the value of Lipatov Pomeron intercept) was obtained in [19] in the framework of perturbative QCD. The large-\( Q^2 \) HERA experimental data are in the good agreement with Lipatov’s trajectory and thus with perturbative QCD. The small \( Q^2 \) data agrees with the standard Pomeron intercept \( \alpha_p = 1 \) or with Donnachie-Landshoff picture: \( \alpha_p = 1.08 \). Perhaps, this range requires already the knowledge of nonperturbative QCD dynamics and perturbative solutions (including BFKL one) should be not applied here directly and should be corrected by some nonperturbative contributions (see [34]).

5 Conclusions

Thus, in our analysis Eq.(1) can be considered as the nonperturbative (Regge-type) input at \( Q^2_0 \sim 1 \, GeV^2 \). Above \( Q^2_0 \) the PD behaviour obeys DGLAP equations. Under the action of perturbative QCD the Pomeron splits into two components. The intercept of the “−” component is \( Q^2 \) independent and this component is the subasymptotical one at larger \( Q^2 \) values. The Pomeron corresponding to the “+” component moves to
the supercritical regime and tends to its perturbative value. Above some $Q_c^2$, where its perturbative value is already attained, the Pomeron intercept keeps a constant value. Our analysis supports the idea [26, 27] about the one effective Pomeron having a $Q^2$-dependent intercept, however the character of the $Q^2$-dependence is different.

The application of this approach to analyse small $x$ data and taking into account a resummation of all $\alpha_s$-orders of perturbative QCD (by analogy with [15]) invite further investigation and will be considered in future.

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6 Appendix

Here we present the illustration of the method to replace the convolution by simple product at small $x$. More detailed analysis can be found in [13].

1. Consider the basic integral

$$J_\delta(a, x) = x^a \ast \varphi(x) \equiv \int_x^1 \frac{dy}{y} y^a \varphi\left(\frac{x}{y}\right),$$

where $\varphi(x) = A x^{-\delta} (1 - x)^\nu \equiv x^{-\delta} \tilde{\varphi}(x)$. Expanding $\tilde{\varphi}(x)$ near $\tilde{\varphi}(0)$, we have

$$J_\delta(a, x) = x^{-\delta} \int_x^1 \frac{dy}{y} y^{a+\delta-1} \left[ \tilde{\varphi}(0) + \frac{x}{y} \tilde{\varphi}^{(1)}(0) + \ldots + \frac{1}{k!} \left(\frac{x}{y}\right)^k \tilde{\varphi}^{(k)}(0) + \ldots \right]$$

(A1)

The second term on the r.h.s. of eq.(A1) can be summed, and $J_\delta(a, x)$ has the following form

$$J_\delta(a, x) = x^{-\delta} \left[ \frac{1}{a+\delta} \tilde{\varphi}(0) + O(x) \right] + x^a \frac{\Gamma\left(-(-a+\delta)\right)\Gamma(1+\nu)}{\Gamma(1+\nu-a-\delta)} \tilde{\varphi}(0)$$

Consider two important cases:

a) $a \geq 1$

$$J_\delta(a, x) = x^{-\delta} \frac{1}{a+\delta} \tilde{\varphi}(x) + O(x^{2-\delta})$$

b) $a = 0$

$$J_\delta(0, x) = x^{-\delta} \left[ \frac{1}{\delta} \tilde{\varphi}(0) + O(x) \right] + \frac{\Gamma(-\delta)\Gamma(1+\nu)}{\Gamma(1+\nu-a-\delta)} \tilde{\varphi}(0) = x^{-\delta} \frac{1}{\delta} \tilde{\varphi}(x) + O(x^{1-\delta})$$
where
\[
\frac{1}{\delta} = \frac{1}{\delta} \left( 1 - \frac{\Gamma(\nu + 1)\Gamma(1 - \delta)}{\Gamma(\nu + 1 - \delta)} x^\delta \right)
\]

2. Consider the integral
\[
I_\delta(x) = \hat{K}(x) * \varphi(x) \equiv \int_x^1 \frac{dy}{y} \hat{K}(y) \varphi(x/y)
\]
and define the moments of the kernel \(\hat{K}(y)\) in the following form
\[
K_n = \int_0^1 dy \; y^{n-2} \hat{K}(y)
\]

In analogy with subsection 1 we have
\[
I_\delta(x) = x^{-\delta} \int_x^1 dy \; y^{\delta-1} \hat{K}(y) \left[ \tilde{\varphi}(0) + \frac{x}{y} \tilde{\varphi}^{(1)}(0) + \ldots + \frac{1}{k!} \left( \frac{x}{y} \right)^k \tilde{\varphi}^{(k)}(0) + \ldots \right]
= x^{-\delta} \left[ K_{1+\delta} \tilde{\varphi}(0) + O(x) \right]
- x^a \left[ N_{1+\delta}(x) \tilde{\varphi}(0) + N_\delta(x) \tilde{\varphi}^{(1)}(0) + \ldots + \frac{1}{k!} N_{1+\delta-k}(x) \tilde{\varphi}^{(k)}(0) + \ldots \right],
\]
where
\[
N_\eta(x) = \int_0^1 dy \; y^{\eta-2} \hat{K}(xy)
\]

The case \(K_{1+\delta} = 1/(a + \delta)\) corresponds to \(\hat{K}(y) = y^a\) and has been already considered in subsection 1. In the more general cases (for example, \(K_{1+\delta} = \Psi(\delta) + \gamma\)) we can represent the "moment" \(K_{1+\delta}\) as the combination of singular and regular (for \(\delta \to 0\)) parts, i.e. \(K_{1+\delta} = -1/\delta + \Psi(1+\delta) + \gamma\). For the singular term the analysis from subsection 1 may be repeated. As the regular part can be represented by series of the sort \(\sum_{m=1}^\infty 1/(a+\delta+m)\), then any additional contributions from term \(N_{1+\delta}(x)\varphi(0)\) to any term of the series, are not necessary.

So, for the initial integral at small \(x\) we get the simple equation:
\[
I_\delta(x) = x^{-\delta} \tilde{K}_{1+\delta} \tilde{\varphi}(x) + O(x^{1-\delta})
\]
where \(\tilde{K}_{1+\delta}\) coincides with \(K_{1+\delta}\) after the replacement \(1/\delta \to 1/\tilde{\delta}\).

References

[1] H1 Collab.: T.Ahmed, S.Aid, A.Akhundov et al., Nucl.Phys. B439, 471 (1995); B407, 515 (1993).

[2] ZEUS Collab.: M.Derrick, D.Krakauer, S.Magill et al., DESY preprint 94-143 (1994), submitted in Zait.Phys.; Phys.Lett. B316, 412 (1993).

[3] NM Collab.: P.Amaudruz, M.Arneodo, A.Arvidson et al., Phys.Lett. B295, 159 (1992); B309, 222 (1993).
[4] E665 Collab.: in the B.Badelek’s report “Low $Q^2$, low $x$ in electroproduction. An overview”. In Proceeding de Moriond on QCD and high energy hadron interactions (1995), Les Arcs.

[5] A.Donnachie and P.V.Landshoff, Nucl.Phys. B244, 669 (1984); B267, 690 (1986).

[6] A.D.Martin, W.S.Stirling and R.G.Roberts, Preprint RAL-95-021, DTP/95/14 (1995).

[7] G.M.Frichter, D.W.McKay and J.P.Ralston, Phys.Rev.Lett. 74, 1508 (1995).

[8] A.Levy, DESY preprint 95-003 (1995).

[9] J.D.Bjorken, In Proceeding of the International Workshop on DIS, Eilat, Izrael, Feb.1994; P.V.Landshoff, Lecture delivered at the PSI school at Zuoz, Aug.1994; A.Donnachie and P.V.Landshoff, Zait.Phys. C61, 139 (1994).

[10] V.N.Gribov and L.N.Lipatov, Sov.J.Nucl.Phys. 18, 438 (1972); L.N.Lipatov, Yad.Fiz. 20, 181 (1974); G.Altarelli and G.Parisi, Nucl.Phys. B126, 298 (1977); Yu.L.Dokshitzer, ZHETF 46, 641 (1977).

[11] V.I.Vovk, A.V.Kotikov and S.J.Maximov, Teor.Mat.Fiz. 84, 101 (1990); A.V.Kotikov, S.I.Maximov and I.S. Parobij, Preprint ITP-93-21E (1993) Kiev, Teor.Mat.Fiz. (1995) in press.

[12] A.V.Kotikov, Yad.Fiz. 56, N9, 217 (1993).

[13] A.V.Kotikov, Yad.Fiz. 57, 142 (1994); Phys.Rev. D49, 5746 (1994).

[14] R.K.Ellis, E.Levin and Z.Kunszt, Nucl.Phys. 420B, 514 (1994).

[15] R.K.Ellis, F.Hautmann and B.R.Webber, Phys.Lett. B348, 582 (1995).

[16] M.Bertini, P.Desgrolard, M.Giffon, L.Jenkovszky and F.Paccanoni, Preprint LYCEN/9366 (1993).

[17] C.Lopez and F.J.Yndurain, Nucl.Phys. 171B (1980) 231; 183B, 157 (1981); A.M.Cooper-Sarkar, G.Ingelman, K.R.Long, R.G.Roberts and D.H.Saxon, Z.Phys. C39, 281 (1988); A.V.Kotikov, JINR preprints P2-88-139, E2-88-422 (1988) Dubna (unpublished).

[18] A.J.Buras, Rev.Mod.Phys. 52, 149 (1980).

[19] E.A.Kuraev, L.N.Lipatov and V.S.Fadin, ZHETF 53, 2018 (1976); 54, 128 (1977); Ya.Ya.Balitzki and L.N.Lipatov, Yad.Fiz. 28, 822 (1978); L.N.Lipatov, ZHETF 63, 904 (1986).

[20] M.Giafaloni, Nucl.Phys. B296, 249 (1987); S.Catani, F.Fiorani and G.Marchesini, Phys.Lett. B234, 389 (1990); Nucl.Phys. B336, 18 (1990); S.Catani, F.Fiorani, G.Marchesini and G.Oriani, Nucl.Phys. B361, 645 (1991).

[21] G.Wolf, DESY preprint 94-022 (1994).
[22] ZEUS Collab.: M.Derrick, D.Krakauer, S.Magill et al., *Phys.Lett.* **B345**, 576 (1995).

[23] W.K.Tung, *Nucl.Phys.* **B315**, 378 (1989).

[24] SLD Collab.: K.Abe et al., preprint *SLAC-PUB-6687* (1995), submitted to *Phys.Rev.Lett.*.

[25] A.Capella, U.Sukhatme, C.-I.Tan and J.Tran Thanh Van, *Phys.Rep.* **236**, 225 (1993); *Phys.Rev.* **D36**, 109 (1987).

[26] H.Abramowitz, E.M.Levin, A.Levy, and U.Maor, *Phys.Lett.* **B269**, 465 (1991).

[27] A.Capella, A.Kaidalov, C.Merino, and J.Tran Thanh Van, *Phys.Lett.* **B337**, 358 (1994); M.Bertini, M.Giffon and E.Predazzi, *Phys.Lett.* **B349**, 561 (1995); M.A.Braun, *Phys.Lett.* **B345**, 155 (1995).

[28] A.Donnachie and P.V.Landshoff, *Phys.Lett.* **B191**, 309 (1987); *Nucl.Phys.* **B303**, 634 (1988).

[29] S.Brodsky and G.Farrar, *Phys.Rev.Lett.* **31**, 1153 (1973); V.Matveev, R.Muradyan and A.Tavkhelidze, *Lett. Nouvo Cim.* **7**, 719 (1973).

[30] CCFR Collab.: R.Z.Quintas et al., *Phys.Rev.Lett.* **71**, 1307 (1993).

[31] D.J.Gross, *Phys.Rev.Lett.* **32**, 1071 (1974).

[32] R.D.Ball and S.Forte, *Phys.Lett.* **B336**, 77 (1994); preprints CERN-TH-7422-94 (1994), CERN-TH-95-1(1995).

[33] A.Donnachie and P.V.Landshoff, *Nucl.Phys.* **B244**, 669 (1984); **B267**, 690 (1986).

[34] P.V.Landshoff and O.Naichmann, *Zait.Phys.* **C35**, 405 (1987); D.A.Ross, *J.Phys.* **G15**, 1175 (1989); N.N.Nikolaev and B.G.Zakharov, *Zait.Phys.* **C53**, 331 (1992); E.Gotsman, E.M. Levin and U.Maor, *Zait.Phys.* **C57**, 677 (1993); J.R.Gudell and B.U.Nguyen, *Nucl.Phys.* **B420**, 669 (1994).
Figure 1. The intercept of the Pomeron trajectory $\alpha_p(Q^2)$ (dashed-dotted line) as obtained from the ALLM parametrization (see \textsuperscript{8, 20}). The dotted lines show the uncertainty of the fit. The solid curves represent the values of $\alpha_p(Q^2)$ from Eq.\textsuperscript{26}.