Optimal polarized observables for model-independent new physics searches at the linear collider

N. Paver

Dipartimento di Fisica Teorica, Università di Trieste and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Trieste, Italy

Abstract
We discuss the sensitivity to four-fermion contact interactions of the process of fermion-pair production at the $e^+ - e^-$ Linear Collider (LC), with $E_{CM} = 0.5$ TeV and longitudinally polarized electron beam. The analysis is based on polarized integrated cross sections with optimal cuts. Beam polarization and optimization are shown to have a crucial role in the derivation of model-independent constraints on the new interactions from the data.

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1 Introduction and polarized cross sections

The concept of effective contact interaction represents a convenient framework to parameterize physical effects of some new dynamics active at a very high scale $\Lambda$, in reactions among the 'light', Standard Model, degrees of freedom such as quarks and leptons, $W$, $Z$, etc., at 'low energy' $E \ll \Lambda$. These effects are suppressed by an inverse power of the large scale $\Lambda$, and should manifest by deviations of experimentally measured observables from the SM predictions. We consider here the process, at LC energy:

$$e^+ + e^- \rightarrow \bar{f} + f,$$

and the relevant $SU(3) \times SU(2) \times U(1)$ symmetric, lowest-dimensional $eff$ contact-interaction Lagrangian with helicity conserving, flavor-diagonal fermion currents $[1]$:

$$\mathcal{L} = \sum_{\alpha, \beta} \frac{g_{\alpha \beta}^2}{\Lambda_{\alpha \beta}^2} \eta_{\alpha \beta} (\bar{e}_\alpha \gamma^\mu e_\alpha) (\bar{f}_\beta \gamma^\mu f_\beta).$$

In Eq. (2), generation and color indices have been suppressed, $\alpha, \beta = L, R$ indicate left- or right-handed helicities, and the parameters $\eta_{\alpha \beta} = \pm 1, 0$ specify the independent, individual, interaction models. Conventionally, $g_{\alpha \beta}^2 = 4\pi$ as a reminder that the new interaction, originally proposed for compositeness, would become strong at $E \sim \Lambda$. Thus, in practice, the scales $\Lambda_{\alpha \beta}$ define a standard to compare the reach of different new-physics searches. For example, a bound on $\Lambda$ in the case of a very heavy $Z'$ exchange with couplings of the order of the electron charge would translate into a constraint on the mass $M_{Z'} \sim \sqrt{\alpha} \Lambda$, and the same is true for leptoquarks or for any other heavy object exchanged in process $[1, 2]$.

Constraints on $\mathcal{L}$ can be obtained by looking at deviations of observables from the SM predictions in the experimental data. In general, such sought-for deviations can simultaneously depend on all four-fermion effective coupling constants, that cannot be easily disentangled. A commonly adopted possibility is to assume non-zero value for only one parameter at a time, with the remaining ones set equal to zero. In this way, one would test the individual models mentioned above, and current bounds from a global analysis of the relevant data are of the order of $\Lambda_{\alpha \beta} \sim \mathcal{O}(10)$ TeV $[3]$. However, for the derivation of model-independent constraints, a procedure that allows to simultaneously take into account the terms of different chiralities, and at the same time to disentangle the contributions of the different individual couplings to avoid potential cancellations that can weaken the bounds, is highly desirable. To this purpose, initial beam longitudinal polarization would offer the possibility of defining polarized cross sections, that allow to reconstruct from the data the helicity cross sections depending on the individual $eeff$ chiral couplings of Eq. (2), and consequently to make an analysis in terms of a minimal set of free independent parameters $[4, 5]$. Also, integrated cross sections would be of advantage in the case of limited statistics, and some optimal choice of the kinematical region may further improve the sensitivity to the new interaction.
For $f \neq e, t$ and $m_f \ll \sqrt{s} \equiv E_{CM}$, the differential cross section for process (1) is determined in Born approximation by the s-channel $\gamma, Z$ exchanges plus $L$ of Eq. (2). With $P_e, P_{\bar{e}}$ the initial beams longitudinal polarization [6]:

$$\frac{d\sigma}{d\cos \theta} = \frac{3}{8} [ (1 + \cos \theta)^2 \tilde{\sigma}_+ + (1 - \cos \theta)^2 \tilde{\sigma}_- ] ,$$

where, in terms of helicity cross sections

$$\tilde{\sigma}_+ = \frac{1}{4} [(1 - P_e)(1 + P_{\bar{e}}) \sigma_{LL} + (1 + P_e)(1 - P_{\bar{e}}) \sigma_{RR}] ,$$

$$\tilde{\sigma}_- = \frac{1}{4} [(1 - P_e)(1 + P_{\bar{e}}) \sigma_{LR} + (1 + P_e)(1 - P_{\bar{e}}) \sigma_{RL}] ,$$

and ($\alpha, \beta = L, R; N_C \approx 3(1 + \alpha_s/\pi)$ for quarks and $N_C = 1$ for leptons):

$$\sigma_{\alpha\beta} = N_C \frac{4\pi \alpha^2_{em}}{3s} |A_{\alpha\beta}|^2 .$$

The helicity amplitudes are

$$A_{\alpha\beta} = Q_e Q_f + g^e_\alpha g^f_\beta \chi_Z + \frac{s\eta_{\alpha\beta}}{\alpha \Lambda^2_{\alpha\beta}},$$

where $Q$’s and $g$’s are are the fermion electric charges and SM chiral couplings, respectively, and $\chi_Z = s/(s - M_Z^2 + is\Gamma_Z/M_Z)$.

The above relations clearly show that the helicity cross sections, that directly relate to the individual four-fermion contact interaction couplings and therefore allow a model-independent analysis, can be disentangled by the measurement of $\tilde{\sigma}_+$ and $\tilde{\sigma}_-$ with different choices of the initial beam polarizations, and making linear combinations. In particular, one can easily see by integration of Eq. (3) in $\cos \theta$ that the ‘conventional’ observables, the total cross section $\sigma \equiv \sigma_F + \sigma_B = \tilde{\sigma}_+ + \tilde{\sigma}_-$ and the forward-backward difference $\sigma_{FB} \equiv \sigma_F - \sigma_B = \frac{3}{4} (\tilde{\sigma}_+ - \tilde{\sigma}_-)$, depend on all helicity cross sections and therefore do not allow the separation by themselves, unless their measurements at different initial polarizations are combined (a minimum of four measurements is needed).

For the discussion of the expected uncertainties and the corresponding sensitivities to the parameters of $L$, as well as for improving the significance of the resulting bounds on $\Lambda_{\alpha\beta}$, one can more generally define polarized cross sections integrated over the a priori arbitrary kinematical ranges ($-1, z^*$) and ($z^*, 1$):

$$\sigma_1(z^*) \equiv \int_{z^*}^1 \frac{d\sigma}{d\cos \theta} d\cos \theta = \frac{1}{8} \left\{ [8 - (1 + z^*)^3] \tilde{\sigma}_+ + (1 - z^*)^3 \tilde{\sigma}_- \right\} ,$$

$$\sigma_2(z^*) \equiv \int_{-1}^{z^*} \frac{d\sigma}{d\cos \theta} d\cos \theta = \frac{1}{8} \left\{ (1 + z^*)^3 \tilde{\sigma}_+ + [8 - (1 - z^*)^3] \tilde{\sigma}_- \right\} ,$$

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and try to disentangle the helicity cross sections from the general relations, at different values of the polarizations \( P_e \) and \( \bar{P}_e \):

\[
\tilde{\sigma}_+ = \frac{1}{6(1-z^*2)} \left[ (8-(1-z^*)^3)\sigma_1(z^*)-(1-z^*)^3\sigma_2(z^*) \right],
\]

\[
\tilde{\sigma}_- = \frac{1}{6(1-z^*2)} \left[ -(1+z^*)^3\sigma_1(z^*)+(8-(1+z^*)^3)\sigma_2(z^*) \right].
\]

(10)

(11)

In practice, we adopt \( P_e = \pm P \) with \( P < 1 \) and \( \bar{P}_e = 0 \). Then, the basic set of integrated observables are \( \sigma_{1,2}(z^*, P_e) \) and, as a second step, we construct the cross sections \( \tilde{\sigma}_\pm(P_e) \) which finally yield the helicity cross sections \( \sigma_{\alpha\beta} \) by solving the linear system of equations corresponding to the two signs of \( P_e \).

One can easily see that the specific choice \( z^* = 0 \) in Eqs. (10) and (11) leads back to the forward and backward cross sections \( \sigma_F \) and \( \sigma_B \). Instead, the values \( z^* = z^*_\pm = \mp(2^{2/3} - 1) = \mp 0.587 (\theta^*_+ = 126^\circ \text{ and } \theta^*_- = 54^\circ) \) to a very good approximation allow to directly ‘project’ out \( \tilde{\sigma}_\pm \) [3, 7]:

\[
\tilde{\sigma}_+ = \gamma (\sigma_1(z^*_+)-\sigma_2(z^*_+)), \quad \tilde{\sigma}_- = \gamma (\sigma_2(z^*_+)-\sigma_1(z^*_+)),
\]

(12)

where \( \gamma = [3 \left(2^{2/3} - 2^{1/3}\right)]^{-1} = 1.018 \). Finally, \( z^* \) could be taken as an input parameter related to given experimental conditions, that can be tuned to get maximal sensitivity of \( \sigma_{\alpha\beta} \) on \( \Lambda \)'s [8].

### 2 Numerical analysis, optimization and bounds on \( \Lambda \)

We adopt a \( \chi^2 \) procedure, defined as follows:

\[
\chi^2 = \left( \frac{\Delta \sigma_{\alpha\beta}}{\delta \sigma_{\alpha\beta}} \right)^2,
\]

(13)

where \( \Delta \sigma_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma^\text{SM}_{\alpha\beta} \) are the deviations of helicity cross sections due to the contact four-fermion interaction [2], and \( \delta \sigma_{\alpha\beta} \) are the corresponding expected experimental uncertainties on \( \sigma_{\alpha\beta} \), combining both statistical and systematic uncertainties. Assuming that no deviation from the SM is observed within the experimental accuracy, constraints on the allowed values of \( \Lambda \)'s are obtained by imposing \( \chi^2 < \chi^2_{\text{crit}} \), where the actual value of \( \chi^2_{\text{crit}} \) specifies the desired ‘confidence level’ (\( \chi^2_{\text{crit}} = 3.84 \) as typical for 95% C.L. with a one-parameter fit). For the expected uncertainties on \( \sigma_{1,2} \), we assume the following identification efficiencies (\( \epsilon \)) and systematic uncertainties (\( \delta_{\text{sys}} \)) for the different final states [9]: \( \epsilon = 100\% \) and \( \delta_{\text{sys}} = 0.5\% \) for leptons; \( \epsilon = 60\% \) and \( \delta_{\text{sys}} = 1\% \) for \( b \) quarks; \( \epsilon = 35\% \) and \( \delta_{\text{sys}} = 1.5\% \) for \( c \) quarks. To have an indication on the role of statistics, for the LC with \( \sqrt{s} = 0.5 \text{ TeV} \) we consider time-integrated total luminosities \( L_{\text{int}} = 50 \) and \( 500 \text{ fb}^{-1} \), and assume \( 1/2L_{\text{int}} \) for each values \( P_e = \pm P \). We take the values \( P = 1, 0.8, 0.5 \) as a reasonable variation around \( P = 0.8 \), expected at the LC [10], in order to study the dependence of the results on the initial beam longitudinal polarization. The numerical analysis uses
the program ZFITTER \[11\] along with ZEFIT, with input values \( m_{\text{top}} = 175 \text{ GeV} \) and \( m_H = 100 \text{ GeV} \). It takes into account one-loop SM electroweak corrections in the form of improved Born amplitudes \[12\], as well as initial- and final-state radiation with a cut on the photon energy emitted in the initial state \( \Delta = E_\gamma/E_{\text{beam}} = 0.9 \) to avoid radiative return to the \( Z \) peak.

The results for the reachable mass scales \( \Lambda \) are shown in Tables 1 and 2 \[8\]. The left entries in each box represent the values obtained by the polarized integrated cross sections defined by \( z^*_\pm \), see Eq. (12). As one can see, the best sensitivity occurs for \( b\bar{b} \) production

| process | \( P \) | \( \Lambda_{LL} \) | \( \Lambda_{RR} \) | \( \Lambda_{LR} \) | \( \Lambda_{RL} \) |
|---------|--------|----------------|----------------|----------------|----------------|
| \( \mu^+\mu^- \) | 1.0 | 40 → 41 | 39 → 40 | 26 → 40 | 28 → 41 |
|          | 0.8 | 37 → 38 | 37 → 38 | 25 → 37 | 26 → 37 |
|          | 0.5 | 32 → 32 | 31 → 32 | 21 → 30 | 21 → 30 |
| \( \tau b \) | 1.0 | 41 → 42 | 45 → 47 | 17 → 31 | 34 → 42 |
|          | 0.8 | 40 → 41 | 38 → 39 | 17 → 29 | 29 → 38 |
|          | 0.5 | 36 → 37 | 29 → 29 | 13 → 25 | 22 → 31 |
| \( \tau c \) | 1.0 | 32 → 33 | 36 → 37 | 21 → 32 | 20 → 30 |
|          | 0.8 | 31 → 32 | 32 → 33 | 20 → 31 | 18 → 27 |
|          | 0.5 | 27 → 28 | 26 → 27 | 18 → 27 | 15 → 22 |

while the worst one is for \( c\bar{c} \), and the decrease of electron polarization \( P \) from 1 to 0.5 worsens the sensitivity by 20-40\%, depending on the final state. As regards the role of the luminosity, the bounds on \( \Lambda \) would scale like \((L_{\text{int}})^{1/4}\) if no systematic uncertainty were assumed\[8\], giving a factor 1.8 of improvement from 50 to 500 \text{ fb}^{-1}\. This is the case of \( \Lambda_{RL} \) and \( \Lambda_{LR} \), where the dominant uncertainty is the statistical one, whereas the bounds for \( \Lambda_{LL} \) and \( \Lambda_{RR} \) depend much more sensitively on \( \delta_{\text{sys}} \). Moreover, it should be noticed that the sensitivity of \( \sigma_{RL} \) and \( \sigma_{LR} \) is considerably smaller than that of \( \sigma_{LL} \) and \( \sigma_{RR} \). Thus, it is important to construct optimal observables to get the maximum sensitivity. Referring to Eq. (13), the uncertainties \( \delta\sigma_{\alpha\beta} \) depend on \( z^* \) through Eqs. (8)-(11), while \( \Delta\sigma_{\alpha\beta} \) are \( z^* \)-independent. This \( z^* \) dependence determines the sensitivity of each helicity amplitude to the corresponding \( \Lambda \). It can be explicitly evaluated, for the statistical uncertainty, from the known SM cross sections and \( L_{\text{int}} \). Then, optimization can be achieved by choosing \( z^* = z^*_{\text{opt}} \) at which \( \delta\sigma_{\alpha\beta} \) becomes minimum, so that the corresponding sensitivity has a maximum. The numerical results, reported in Tables 1 and 2, show that such optimization can allow a substantial increase of the lower bounds on \( \Lambda_{RL} \) and \( \Lambda_{LR} \), and a modest improvement for \( \Lambda_{LL} \) and \( \Lambda_{RR} \).

In conclusion, the measurement of helicity amplitudes of process \( \text{(1)} \) at the LC by means of suitable polarized integrated observables and optimal kinematical cuts to increase the
Table 2: Same as Table 1, but at luminosity 500 fb$^{-1}$.

| process | $P$ | $\Lambda_{LL}$ | $\Lambda_{RR}$ | $\Lambda_{LR}$ | $\Lambda_{RL}$ |
|---------|-----|----------------|----------------|----------------|----------------|
| $\mu^+\mu^-$ | 1.0 | 54 $\rightarrow$ 55 | 55 $\rightarrow$ 56 | 37 $\rightarrow$ 57 | 40 $\rightarrow$ 59 |
|         | 0.8 | 51 $\rightarrow$ 52 | 51 $\rightarrow$ 53 | 35 $\rightarrow$ 54 | 38 $\rightarrow$ 55 |
|         | 0.5 | 44 $\rightarrow$ 45 | 43 $\rightarrow$ 44 | 31 $\rightarrow$ 46 | 31 $\rightarrow$ 47 |
| $\bar{b}b$ | 1.0 | 48 $\rightarrow$ 49 | 64 $\rightarrow$ 67 | 26 $\rightarrow$ 50 | 50 $\rightarrow$ 66 |
|         | 0.8 | 47 $\rightarrow$ 48 | 51 $\rightarrow$ 53 | 25 $\rightarrow$ 48 | 41 $\rightarrow$ 61 |
|         | 0.5 | 43 $\rightarrow$ 44 | 36 $\rightarrow$ 37 | 23 $\rightarrow$ 43 | 30 $\rightarrow$ 47 |
| $\bar{c}c$ | 1.0 | 35 $\rightarrow$ 37 | 42 $\rightarrow$ 43 | 24 $\rightarrow$ 44 | 24 $\rightarrow$ 47 |
|         | 0.8 | 34 $\rightarrow$ 35 | 38 $\rightarrow$ 39 | 23 $\rightarrow$ 43 | 22 $\rightarrow$ 41 |
|         | 0.5 | 30 $\rightarrow$ 32 | 29 $\rightarrow$ 30 | 21 $\rightarrow$ 39 | 18 $\rightarrow$ 32 |

sensitivity, would allow model-independent tests of four-fermion contact interactions, in particular as regards their chiral structure, up to mass scales $\Lambda_{\alpha\beta}$ of the order of 40-100 times the C.M. energy, depending on the final fermion flavor and the degree of initial polarization. Work is in progress to assess the further increase in sensitivity to the new interactions which can be obtained if also a significant positron-beam polarization were available.

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