A framework for the optimisation of glazed gridshells in the early design stage under structural and energy-based criteria

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Abstract. While it is of primary importance to deliver high performance architecture within constraints of time and money, the integration between structural and energy-based aspects presents significant challenges for the organisation of the design process, especially in the early stages. In the case of complex envelopes, such as glazed gridshells, these engineering-related aspects should be considered from multiple points of view from the very beginning of the concept development, exploiting the potential conflicts between architects and engineers to let creative solutions emerge, and thus avoiding expensive and time-consuming design modifications downstream. This paper aims to define the framework required to integrate a multi-objective optimisation capable of combining such aspects into an integrated flow of information. Such an integrated analysis presents difficulties because, in the early design phases, conceptual changes happen faster than computational capacity; so, to overcome this obstacle it is necessary to define an iterative flow of information between structural and energy-based procedures, while at the same time taking into account the aesthetic requirements. This particular flow not only guarantees a correct passage of technical data among different software tools, but it also allows for a better communication and comprehension of information between diverse actors, such as architects and engineers. Based on this specific plan, a conceptual framework for optimising gridshells under structural, geometrical and energy-based criteria is developed and presented. The final goal of such a procedure is not, clearly, to replace the designer, but to give guidance to transform potential conflict into creative discussion and improve the efficiency of the later phases of the design process.

1. Introduction

The aim of this paper is to lay the ground for an integrated framework to design complex envelopes, defining actors, inputs and degree of precision needed and providing guidance to the design team during the early project phase. While structural and geometrical aspects occupies a more important role in this article, energy-based aspects will be partially treated with the intention to provide a better overview on the conceptual structure of the framework.

Free forms represents one of the most challenging discussion points in today's architectural community. The strive to create iconic buildings, combined with an intense development of CAD software, defined a new relation between structural engineering and architecture producing a new range of geometrical and constructional implications [1]. The first result of this contamination can be found in the decomposition of such free shapes into load-bearing structures and manufacturable panels without
increasing costs excessively, but such complex interactions need to be treated not only the basis of a pure geometrical discretisation, but also on numerical grounds [2]. The interaction between shape and structural aspects becomes even more challenging when dealing with glazed envelopes. Realising and installing doubly curved glass is possible, as the Innsbruck Hungerburg stations show, but this implies a high fabrication cost [3]. Using free form glass to approximate the architectural intention cannot be pursued for big envelopes in which minimisation of costs plays an important role [4] and energy performance is also one of the design priorities.

There are several solutions to transform a free form shape into a manufacturable one, from the triangulation applied in the Great Court Roof of the British Museum in London to the creation of single curvature stripes like in the Strasbourg train station [5], but all these procedures come from a discretisation of the initial architectural surface.

When dealing with glazed roofs we often talk about gridshells: structures able to reach large spans with a minimal use of material. A mandatory step in the design of such light-weight envelopes is form finding, which is a calculation method do define a shape, based on the absence of bending moment, obtaining a pure tension/compression structure [6]. This procedure generates, from a flat design space, a double curved form which needs to be then adapted with the structure and discretised with panels. The subdivision of a free-formed surface presents challenges in terms of panelling, and normally the most common ways to subdivide a shape use triangles, quadrangles or hexagons. Each solution presents different aspects that can influence design choices, having advantages and disadvantages in both detailing and construction procedures. Curved glasses are expensive and have limits in coat applications, while a triangular subdivision allows to have planar glasses and a good shape approximation. Despite the advantages, this method has a strong construction downside for having six bars entering in one node: this makes the connections very complicated to fabricate and check. Hexagonal subdivisions represent the exact opposite of triangulated meshes, having only three bars in one node. However, when trying to fix all hexagon vertexes on a single plane we increase the level of rigidity, greatly deviating from the original shape. On the other side, quadrangular divisions have many interesting advantages due to a reduction of glass waste and a reduction of the number of bars in a single node, but this procedure has a strong influence on the form finding process because of the Quads planarity [7].

The interaction between panelisation and structural aspects during a form finding procedure has been studied and can be visible in projects such as the Hippo house at the Berlin zoo from Schlaich Bergermann & Partners or the Eiffel Tower pavilions from RFR [4, cit]. These examples show that conflicting parameters, such as structural performance and planarity, can still coexist in an optimisation process, but need to be analysed closely to find a correct result.

It is also necessary to consider that glazed gridshells are extremely transparent structures that, by design, let radiation through. For this reason, their design often requires to integrate shading systems to control light. This can be expressed in multiple and disparate ways: the Fondation Pathé by RPBW shows a way of proceeding by creating a multi-layered structure each one covering a function. This last example is obviously not the only way to approach the topic: the Esplanade domes in Singapore show how outside shading can control radiation preserving, at the same time, the gridshell transparency. On the other side, buildings such as the Cervantes Theater in Mexico City use the roof structure to create self-shading without the introduction of additional elements. Different strategies have distinct architectural expression and, for this reason, it cannot be said a priori which one is the best. However, a way to guide and ease the project development would be to develop an integrated decision-making process including different design options. The aim of this article is to establish an integrated design framework for complex envelopes, defining actors, inputs and degree of precision needed providing guidance to the design team during the early project phase.
2. State of the Art

One of the main characteristics of a design process is the growing quantity of available information the more we proceed thorough the project phases. The very beginning of any design, also in the built environment, starts with a large amount of solutions on the table, which is then reduced and synthesised during the different phases. Variations and alternatives are analysed from complementary points of view until one or a small group of options are detailed and further implemented, allowing the final solution to emerge. In the building sector a normal process starts from architectural ideas which are then developed and engineered up to the construction site, but it often happens that the final solution does not represent the optimised one. The reason for this is a strong connection between aesthetics and technicality inside the decision-making process but, as in other fields, this procedure is divided in phases or stages. In this respect, the Royal Institute of British Architects (RIBA) produced in 2013 the Plan of Work 2013 [8]: this is a document describing the main stages of a project. From these it is possible to extract information about the required inputs and outputs. According to the RIBA Plan of Work there are 8 phases going from the production of the requirements brief to the use of the building. The design itself occupies three of the stages, which are: Concept Design, Developed Design, Technical Design. During these phases, the information exchange leads to a conclusion of the project.

2.1. Time-Cost diagram

The main target of the design process is to determine the (potentially) best solution combining all the different actors’ requests: looking from this perspective, the ability to influence the design is mandatory. B. Paulson in 1976 developed a diagram correlating the level of influencing and the cost of the project through the consecutive phases [9]. It is evident from figure 2 that, regardless of the name of the project phase, the first section is the one in which more freedom is possible because document production did not start yet, so any change has a minimal impact on the overall costs. At the beginning of 2000 there was a further step on Paulson’s theory, when the CEO of HOK Patrick MacLeamy reviewed the diagram implementing it with new curves – later renamed MacLeamy’s curves [10]. The new Gaussian shapes aimed to explain the difficulties during a building project and show a potential way of redistributing effort to reduce expense. Such curves show how the traditional work in a project occurs during a phase in which the ability to influence the design is already far too expensive. Based on this consideration, it is clearly most effective to move the efforts to an early stage of the design process.

Even if MacLeamy’s idea is indeed interesting, this particular approach can have an effective application only when actors do not change during the different project phases and, therefore, a continuous and consistent flow of information is ensured. Unfortunately, in the building sector it is frequent to have different actors for distinct phases: engineers who deals with the schematic design are rarely also the ones consulting construction companies for the execution planning.
It has to be also said that when MacLeamy developed his curve in 2001, parametric software was not yet deeply integrated in the design process. This last fact changed dramatically the way of intending concept and design both for the architects and engineers. The intensive use of these software inside projects is evident, but it is still mainly adopted to optimize local processes, speeding up the general work flow no matter if architectural or engineering procedures. As a consequence of this, MacLeamy’s idea of shifting the effort in the early phases can increase the effectiveness of the process. Most of the effort would be made when the ability to impact the design is still high. Nevertheless, the relocation of the productivity requires a different approach to the work and a flexible and iterative workflow. The answer to this adaptability is parametric design, which is able to give a fast response and adapt to several design variations. Such way of intending the process allows designers to investigate more solutions upstream, anticipating issues already at an early stage.

Today’s design process is an iterative flow in which all actors work in their domain and it is very rare that boundaries, specified in contracts, are exceeded. If this approach might work for straightforward projects, it is extremely counterproductive when talking about complex glazed envelopes, where the integration between different domains becomes fundamental. Designing gridshells is not a linear process due to the strict integration of many different aspects. If glass planarity and load-bearing elements can be treated by the structural engineer alone, the integration of energy performance requires at least two different actors besides the architect and the client. When dealing with such complex interactions, the traditional logic-based engineering can be too weak and, therefore, a more explorative approach is required which is able to produce and evaluate multiple solution of the same problem with a limited computational effort [11].

3. Integrated Design Methodology
Taken individually, structure and energy are the two pillars of every project, with each one presenting many specific challenges. Combining these two aspects with complex envelopes, such as gridshells, requires a great amount of knowledge in both fields to reach acceptable solutions, each presenting a considerable amount of issues during the design. So far, such complex envelopes have been treated focusing on single aspects such as structural slenderness or direct radiation, but rarely combining them in the initial design phase. Even if this division is a possible approach to the topic, the building sector is moving more and more toward an integration between actors and fields, trying to merge them inside a single flow of information. The urge for zero-emission buildings in today’s architecture requires to have a pre-evaluation of the building performances during an early project phase in order to control and adjust the design [12]. On the other hand, the inability of calculation software to keep up with architectural modifications cause a strong delay in the generation of a computational flow, which can deeply affect the project and even cause partial redesign.
3.1. Design tools
The development of a process with a quick response to the designer inputs is clearly crucial to have a pre-evaluation of their effects on the project. This is made possible by working in a parametric environment such as Grasshopper for Rhinoceros [13]. Being able to generate a result quickly brings out the other strength of parametric tools, which is creating multiple design variations both in the architectural and engineering domains. Inside the Grasshopper environment, it is possible to find tools for structural calculation, energy simulation, and geometry adjustment such as Karamba, DIVA, and Kangaroo [14] [15] [16]. Each of these tools works inside the common space of Grasshopper and can be interconnected to the others with algorithms which are able to explore the problem in a fast and effective way, generating many solutions from the given problem.

The production of several results is called Design Optioneering [17] and is a useful approach thanks to its ability of stimulating design and guiding it toward acceptable options. Architects can explore different shapes, while the structural engineer can follow them defining specific static systems. Nevertheless, this approach is mostly used to optimise sectoral problems such as structural calculations or energy performance [18] and is rarely adopted to combine conflicting design aspects. The reason for such lack of integration has its roots in a strong task division between actors, but can also be traced back to a partial absence of information, e.g. structural engineers are generally not prepared to discuss energy-based topics and vice versa. To have a combination of several topics, it is necessary to adopt a multidisciplinary method able to include very different aspects and to associate them into an integrated flow: unfortunately, this is no ordinary task and requires not only knowledge of the topic, but also some optimisation expertise.

3.2. Multidisciplinary design
Multi-objective optimisation is a very wide topic and approaching it blindly can be, to some extent, dangerous given the many parameters of different sectors which need to be understood to create a fruitful interaction between them. The mathematical definition of optimisation is to determine the maximum or the minimum of a function. This can be quite evident when looking at a general curve function (single objective optimisation), but things can be different when looking at multi-objective optimisation. As said, the fact of maximising or minimising when more parameters are involved is not straightforward, because the chosen attributes of the function need to be interconnected. A good example is the relation between the offer of some products and their quality: the more a certain device is available on the market, the less it is performing. In this particular case, there is not a single minimum of our function, but two, meaning that there is a gradient of possible solutions in between these peaks. A single objective optimisation in the structural or energy domain is something that today is done on a regular basis, enabled by parametric tools such as Karamba or DIVA which are embedded in Grasshopper, a plug-in for the modelling software Rhinoceros. If on one side such routines are efficiently treated, the combination of them is not at all integrated. The reason for this lack of combination resides in the fact that while in other sectors, such as the automotive industry, processes are well structured in terms of information and organisation, the building sector has still a strong "human variable" in its workflow. The amount of information during a building project is large and for this reason tasks are split between actors who provide their input based on their knowledge.

3.3. Responsibility division
Liability is the key aspect of the building industry: no one wants to take responsibility for a duty which is not paid or for which there is no insurance. This simple fact generates tension due to a dismissal of responsibilities and this deeply impacts the design quality with a lack of integration between actors. For simple geometries, this absence does not produce particular conflict, but it generates several issues when different fields overlap and need to be treated with an integrated approach. During the different project phases, specific actors work with disparate intensities: the architectural work is more acute when the design starts, while the engineering comes later, when the architectural decisions are almost consolidated. The reason for this partition resides in the amount of time needed to engineer an architectural solution compared to a change in the design. While this does might still be acceptable for
straightforward buildings, it is a matter of discussion when dealing with glazed roofs, in which structure and energy are strictly connected. Gridshells, in particular, require particular care in their structural design due to their slenderness, and difficulties increase if light and energy aspects need to be included as well.

During the different design phases, the project evolves and, often, architectural choices clash with engineering constraints, provoking late modifications to the initial concept. In order to reduce such phenomenon, an early analysis of the different aspects should take place before document production starts and with this intent, predesign and schematic design are the most suitable phases to do so.

The previously described McLeamy curves show that the initial phases of a project are the ones in which design changes have minimal impacts on the overall costs, therefore it is important to act at the beginning instead of procrastinating decision further on. Concentrating the effort in the early phases, it is possible to start operating before the end of pre-design, establishing a workflow which covers the entire schematic design. The passage between these two phases is the most suitable time to launch a discussion, because the different actors involved bring their proposals to the table and start the interaction between aesthetics and technical solutions. When dealing with complex envelopes such as glazed roofs, it is even more important to anticipate design inaccuracies because of the high number of factors, spanning from structural to energy design. In this phase, geometry is still subject to important variations and input parameters have high fluctuations that can still change during the optimisation. Integrated design is necessary for complex geometries and, in order to have a fluid process, it is important that the different actors and tools have an affinity between each other. Parametric design goes in this direction aiming to associate different topics inside a single workspace and implementing the software communication with the use of algorithms able to repeat and reiterate procedures.

3.4. Algorithm optimisation

Setting up an integrated optimisation, such as the one between structure and energy, requires a combination of many inputs from different fields, presenting a consistent challenge in terms of design [19]. In addition to this, when different domains are treated by separate actors there is a strong chance of creating an unproductive conflict between the parts which can reduce the design quality drastically. Going against this direction, many architectural and engineering offices tried to integrate different disciplines, starting to deal with contrasting topics already in the competition or early design stage [20]. Combining many different inputs coming from separate domains requires a tool capable of repeating commands in a precise sequence and evaluating each solution comparing it to the others. To do so a computer is necessary and a specific algorithm, able to deal with many variables, needs to be set up. Genetic Algorithms (GA) are widely used in multi-objective optimisations because of their capability of mutating and randomly explore the mathematical function in search of its limit values. GA are powerful tools especially when dealing with different domains that intersect each other, making the design space unclear to visualize. Due to their ability of searching complex space functions, they provide a range of optimal solutions known as the Pareto Front [21].

The very fact that multidisciplinary design does not provide a unique solution is the base of the Design Support System presented further in this article: the use of algorithms to combine separate themes in the building. Multidisciplinary design does not give a single answer to the problem but it provides a spectrum of alternatives, which need to be then evaluated by the design team. In this sense GA are capable of providing options to stimulate the designers and help them into the design phases, but do not have the purpose of replacing the human factor.

3.5. Vertical Tool

The aim of this paper is to approach an integrated design which can include these topics inside a consistent flow of information. A Vertical Design Tool is presented and outlined in its phases with the goal to give a precise guide of the different inputs and outputs needed to complete the analysis and understand the range of solutions obtained. Based on professional experience and investigations, Pre-design and Schematic design are the suitable project phases for such a tool. During these phases all the different actors develop solutions to match the initial brief intentions and, in this particular section of
the project, a multidisciplinary approach can be successful to improve design and reduce expenses. The proposed tool is defined as "vertical" because it guides the design at the beginning of the project enlarging the pool of potential solutions.

![Diagram](image.png)

**Figure 4.** Vertical Tool in relation to MacLeamy curves.

When dealing with complex geometrical envelopes, structural solutions need to be compatible with energy targets and not entwining the two aspects since the beginning of the project can generate ruptures in the initial design and unwanted frictions between the actors. The following diagram (figure 5) provides a design sequence able to generate optimised solutions dealing with structural, geometrical and energy-based aspects in an early development phase. The proposed design support system would provide orders of magnitude useful to quickly evaluate solutions and guide the process. The aim is not to replace design teams with computers, but instead to have a sequence of rules that can be used to manage complex projects and give a guidance to obtain effective results.

4. **Design Support System**

As said previously, optimisation corresponds to a maximisation or a minimisation of a certain value and, while this is simple when looking at one parameter, it is more and more complicated when adding degrees of constraints. There are mainly three phases during an optimisation process which can be described as follows:

- **Input definition:** choice of entry data for which the system will be optimised.
- **Engine:** numerical procedure able to optimise the given inputs.
- **Output evaluation:** obtained solution which need to be judged.
The above described DSS defines the three macro categories for which a glazed gridshell should be optimised and aim to find the different inputs for each category: geometry, structure and energy. These classes are only representing the general subject, and under each one there are several parameters that can guide the optimisation. Every input is managed by a different actor, e.g. it is the structural engineer who deals with geometry and structure, but it is in the architect’s interest to control the overall gridshell shape. All of these inputs have a different weight inside the optimisation process and, clearly, it is not possible to choose all the parameters at the same time. Therefore a decision need to be made a priori assigning to each input a specific weight based on the importance [22]. Weights are decided by single actors based on their experience and knowledge allowing the design team members to bring into discussion their know-how in a shared workflow.

Once the initial assumptions are made, all the different inputs coming from separate domains are collected by the numerical engine and, with the use of the proper optimisation method, are related one each other obtaining a range of optimal solutions. As the aim of the Design Support System is not to replace the designer but to help him/her, such results are represented by certain indicators describing the
effectiveness of the outcome, e.g. the incident light on the ground level is minimal, but the weight of the structure is too high. These indicators help to judge the solution and allow the designer to adjust the solution by choosing different inputs or by changing the original weights of the optimisation. In this way, not only each actor can evaluate the solution on its own, but the system can be extended to a more articulated discussion including multiple actors.

4.1. Shape and planarity
Dynamic relaxation is one of the potential procedures to perform form-finding. It consists in applying unitised vertical loads on each node of the mesh, deforming it into a catenary shape, with pure tension forces; the obtained form is then flipped obtaining a reverse configuration working only in compression [23]. Clearly this process is based on the fact that gravity is assumed perpendicular to the ground level, allowing to control only the gridshell height and not the overall geometry which, with the goal of a multidisciplinary optimisation, could be interesting to consider. Exiting the strict catenary shapes and increasing the pool to random NURBS could represent a chance to explore different forms which might be more effective from an energy point of view.

Regardless the way a shape is obtained, how it is discretised impacts deeply on construction and design considerations, bringing into discussion different ways of approaching the gridshells. In the case of Quads, planarity and torsion are the main topics and the adjustment of the mesh will follow the principal curvature lines showing already where the load bearing structure will pass and how dense it must be [2, cit.]. These very last parameters can have a deep impact on the initial shape generation and can guide the algorithm toward a better refinement.

4.2. Energy
If on one side geometrical and structural problems proceed on similar tracks, looking at an energy-based topic such as light can generate additional and / or different requirements. Traditional form-found gridshells present many weaknesses in controlling sunlight and they need in many cases louvres or other shading systems to avoid excessive heat accumulation. Technicality apart, it has to be said that covering an entire glazed roof with sun shading might not be the best solution to present to a client, who might wonder why building something in glass if you then need to make it opaque or translucent.

A glazed roof has a very poor capacity to control direct radiation, so the treatment of glass to make it less transparent, or the addition of a shading system, are generally required. Nevertheless, by exiting the domain of catenary shapes it can be possible to adjust the gridshell form to minimise the incident light on its surface. This can generate interesting results when considering the overall transparency and can minimise the impact of shading systems on the roof.

5. Outlook of future work
Form-finding procedures are mainly based on the equilibrium of forces; they define the shape based on structural constraints. The main target of such procedures is to exploit geometrical properties of shells in order to cover maximum spans. It is evident that, from the stiffness matrix method to the dynamic equilibrium one, the main focus is the structural aspect, leaving aside the energy part.

This way of approaching gridshells has its roots in the 1960s, when lighting and energy-based aspects were ignored or only mildly considered. On the other hand, progressively stricter regulations on building performance are forcing to consider energy-related aspects already at the early design stage. In this respect, the introduction of solar radiation criteria inside a geometrical and structural optimisation procedure could expand the traditional optimisation boundaries and enlarge the pool of possible shapes.
5.1. Energy
The introduction of energy-based aspects, such as light, inside a form-finding procedure requires a step back from the traditional designing methods. Gridshells are completely independent from the environment because their definition comes from purely structural calculations, but if we want to integrate energy inside the evaluation it is necessary to have a different approach. Clearly this imposes a different way of indenting form-finding and, while this could stimulate the design, it requires a redefinition of input and output information. Structure, geometry and energy become the targets of the design and, being so, they have the ability to change it.

As explained above, equi-tensional approach in traditional form-finding leads inevitably to catenary shapes [24] on which energy has a very limited design impact. A way to include energy-based aspects in the shape definition would be removing the catenary shape restraint to allow a “self-shading” of the gridshell. Figure 6 shows a possible way of reducing the global solar radiation on the shell only by changing its shape.

Future work in this area will include the identification of suitable control parameters, their weighting as part of the multi-criteria optimisation process, and the correlation of numerical analyses with the technical solutions available for solar control through glass properties and shading devices.

5.2. Non equi-tensional shapes
Leaving the domain of uniformly tensioned structures opens the doors to a multitude of potential configurations, which are independent from traditional form-finding methods. Shape definition will not be limited only to catenary forms, but will include the entire set of free-formed shapes. One of the best way to describe such shapes is to use non-uniform rational basis splines surfaces (NURBS), which offer a considerable flexibility in describing complex geometrical definitions [25]. Changing the positions and weights of a NURBS’s control points it is possible to change the surface shape and curvature, describing an infinite range of possible geometries. The very fact that NURBSs are defined through
geometrical inputs is interesting because it could unbind the form-finding process from purely structural-based criteria.

In light of these considerations, the direction of future work will be to define a new shape definition method using the NURBS control points. These geometry variables will be bound to a numerical engine capable of exploring an infinite range of solutions based on the structural and energy related targets. Doing so, the shape definition will be based on a stochastic procedure, which is independent from a structural analysis, allowing to implement different targets for the form-finding. Structural, geometrical and energy-based aspects will be treated uniquely as objectives and, based on their importance in the process, will have an effect on the shape definition.

5.3. Conclusions

The purpose of this paper is to outline an integrated framework to guide multidisciplinary teams through the early design stage of complex envelopes. Structural, geometrical and energy-based aspects are considered individually and then organised in specific input parameters. Such criteria are the base for multidisciplinary discussion and they frame the process not only from the technical, but also from the organisational points of view.

The inputs variety has a strong impact on the shape definition methods: as explained above, this leads to a reconsideration of the form-finding procedure, moving beyond purely structural aspects. The aim is to propose an organic workflow where all the input parameters deemed significant can participate to the optimisation process, while at the same time leaving the final choice to the designers. The future outlook of this research is to identify a numerical form-finding procedure, based on a stochastic approach, where the shape is defined with a geometrical method, rather than with a force-based one.

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