Information contents and physical structures of “ready” states of apparatus and observers

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August 9, 2009

Abstract

Measurement interactions are initiated from “ready” states of apparatus and observers. Observers must be able to report both that they are in their ready states and certain information contents thereof; hence such states must encode classical information. It is shown that such information is encoded by a component of the observer state; an “amplifying” projection is defined that uniquely identifies this classical information-encoding component. Components thus defined exhibit a symmetry formally analogous to envariance; this symmetry is employed to derive the Born Rule for the reportably observable states of a quantum system. A minimal interpretation of quantum measurement is constructed, under which the joint state of all systems involved in the measurement interaction remains a superposition evolving with unitary dynamics throughout the measurement process. This minimal interpretation provides an alternative to traditional interpretations of measurement that require either wave-function collapse or branching.

1 Introduction

Consider an observer \( O \) who seeks to determine the state of a system of interest \( S \) using an apparatus \( A \) embedded in an environment \( E \). This situation is traditionally represented by a von Neumann chain:

\[
(\sum_i \lambda_i |s_i\rangle |A^{\text{ready}}\rangle |E_{\text{init}}\rangle |O^{\text{ready}}\rangle \rightarrow \sum_i \lambda_i |s_i\rangle |a_i\rangle |e_i\rangle |o_i\rangle)
\]  

(1)

where the \( |s_i\rangle \), \( |a_i\rangle \), \( |e_i\rangle \) and \( |o_i\rangle \) are basis vectors for \( S \), \( A \), \( E \) and \( O \) respectively, \( \lambda_i \) is a complex coefficient, \( |E_{\text{init}}\rangle \) is the initial state of the environment, and \( |A^{\text{ready}}\rangle \) and \( |O^{\text{ready}}\rangle \) are “ready” states of \( A \) and \( O \) respectively (cf. Eqn. 9.1 of [1]). The “measurement problem” is the problem of explaining how one gets from the superposition represented by the right-hand side of Eq. 1, which is required by the unitary dynamics of the coupled \( S - A - E - O \) system, to a
situation in which the states of $S$, $A$, $O$, and by implication $E$ as well, have taken on determinate values, i.e. how one obtains:

$$\sum_i \lambda_i |s_i \rangle > |a_i \rangle > |e_i \rangle > |o_i \rangle \rightarrow |s_n \rangle > |a_n \rangle > |e_n \rangle > |o_n \rangle$$  

(2)

for some fixed $n$, as seems to be required in order to say that a “measurement” of $|S \rangle$ by $O$ yielding classical information about the state of $S$ has occurred [1]. As is well known, answers to this question fall into two broad families distinguished by their commitment to unitary time evolution: “orthodox” answers reject unitary time evolution and postulate a “collapse” of the superposition, while “relative-state” answers maintain unitary time evolution and postulate “branching” of the superposition into irreversibly mutually independent components, one for each possible value of $n$ (e.g. [1, 2]).

This paper reconsiders the question of measurement, starting from the assumptions that observers do conduct reliable measurements, and that they do obtain and report classical information indicating that systems of interest are, or at least briefly were, in determinate states. It first asks what information $|O_{\text{ready}} \rangle$ must encode for a reliable, reportable measurement of the state of a system $S$ to occur, either from the perspective of $O$ or from that of a knowledgeable and competent third party $W$ (“Wigner”), and shows that this a priori information must include criteria sufficient to identify both $|A_{\text{ready}} \rangle$ and the pointer states of $A$. It then asks what physical structure $|O_{\text{ready}} \rangle$ must have in order to encode this information in a way that renders it reportable, as classical information encoded in a language, to $W$. It shows that classical reportability requires the information content of $|O_{\text{ready}} \rangle$ to be encoded not by an individual quantum state $|O \rangle$ of $O$, but by one or more long-lived components that are effectively amplified by the dynamics $|O(t) \rangle$, and constructs a projection that produces such components from $|O(t) \rangle$. It then shows that such long-lived, amplified components exhibit a swap symmetry formally analogous to envariance [3, 4], and uses this symmetry to derive the Born Rule for states of $S$ observable by $O$. Finally, it uses the swap symmetry of amplified components to develop a representation of measurement that clarifies Eq. 1 by replacing $|O_{\text{ready}} \rangle$ and $|A_{\text{ready}} \rangle$ with quantum states that project, respectively, to $|O_{\text{ready}} \rangle$ and $|A_{\text{ready}} \rangle$. This elaborated representation of measurement supports a minimal interpretation that avoids both wave-function collapse and branching.

Two preliminaries must be dealt with before proceeding. First, it is commonplace to represent the states of physical systems with names that explicitly reflect their classical information content, e.g. to say that “$| \uparrow \rangle$” represents a state that can be reported by an observer as having “spin up”. Such a notation can be problematic in the present context, as the informational description of states with a given physical description is what is at issue. The notion of an “interpretation” $I$ that maps states or components described physically to classical information contents specified in some language will be borrowed from formal semantics to fill this notational gap; thus for example $I : | \uparrow \rangle \mapsto \text{‘spin up’}$ indicates that $| \uparrow \rangle$ is interpreted as encoding the classical information ‘spin up’ in a way that is accessible to observation. Classical information contents specified in a language must, however, also be physically encoded; an interpretation $I$ is therefore a mapping between physical states, i.e. $I : \psi \mapsto I(\psi)$, where $I(\psi)$ is physically encoded by the system executing the mapping $I$. In contexts with the potential for ambiguity, an interpretation will be explicitly characterized as implemented by a system $X$, and hence as yielding information contents from the perspective of $X$ as an interpreting agent; in such cases the interpretation mapping will be denoted
The superscripted notation $|X^{I_X}(\psi)\rangle$ will be used to denote the state of $X$ that encodes the classical information $I_X(\psi)$. Agents are assumed to be capable of reporting information to each other using the language that forms the range of $I$. A report by agent $X$ to agent $Y$ is assumed to reliably carry information $I_X(\psi)$ interpreting some state $\psi$ from $X$ to $Y$.

Second, it is necessary to clarify what is meant by an “observer.” Observers are typically discussed informally as human-like conscious agents, but are then treated formally, as in Eqs 1 and 2, simply as quantum systems representable in some basis. Zurek [3] remarks that observers differ from other systems, apparatus in particular, only in their ability to “readily consult the content of their memory” (p. 759), where ‘consult’ evidently means more in this context than simply ‘access’ in the mechanical sense. Autonomy and an ability to report back results are typical characteristics of *gedankenexperiment* observers, such as the quantum-scale observer Zeh [5] assumes to have “elementary awareness” and utility as a reporter as it passes through a double-slit apparatus (p. 232). In order to accommodate these various intuitions without introducing a bias as to scale or any specific theory of “awareness” or “experience”, an “observer” will be taken here to be a physical (and hence, quantum) system that can 1) be provided with information describing what is to be measured and how, 2) on the basis of this given information, obtain and record the results of a measurement, and 3) report the results as described above. Such an observer must, as a physical system, have the memory capacity to physically encode any information in its possession.

2 Measurement as a communication process

An informal grasp of the information encoded by $|O_{\text{ready}}\rangle$ for any competent observer can be obtained by thinking of the questions one would expect any collaborator to be able to answer at the outset of an experiment. Any competent collaborator would be expected, for example, to be able to describe both the apparatus to be used in the experiment and the procedure for reading data from the apparatus. “Observations” made by someone who is unable to produce this information when asked are unlikely to be treated as reliable.

Intuitions about the prior knowledge of the measurement context required of observers can be made more precise by modelling measurement as a communication process in which the apparatus $A$ plays the role of Alice and the observer $O$ plays the role of Bob. Alice encodes messages into the environment $E$; Bob passively receives and decodes these messages. Alice follows four rules: 1) all messages consist of finite strings of finite words in a language $\mathcal{L}(A)$ chosen by Alice, 2) messages either specify Alice’s status (in particular, whether she is in $|A^{\text{ready}}\rangle$) or specify the value of a variable $P$ (Alice’s “pointer”), 3) Alice communicates in real time and never falsely, and 4) Alice communicates a value of $P$ if and only if it results from an interaction with $S$. In order to model a realistically open laboratory environment, other agents are assumed to be encoding messages into $E$ simultaneously with Alice; these agents behave unpredictably and may encode the same message contents as Alice does, but do not use the language $\mathcal{L}(A)$ and hence cannot maliciously substitute their own messages for Alice’s. Our questions are 1) what information must Bob possess at the outset of a session of interactions with Alice in order to reliably recognize and decode Alice’s messages? and 2) can this information be obtained from Alice’s messages alone?

First, and most fundamentally, Bob must assume that he himself exists through time, and that he can reliably re-identify at a time $t_2$ his own stored record of a message received at a previous time $t_1$. Without this information, Bob would be unable to distinguish current from previous
messages, and hence would be unable to compare the contents of current and previous messages; indeed without this information, Bob would have no way of distinguishing $t_2$ from $t_1$. Second, Bob must assume that the states of the environment with which he interacts encode messages, i.e. that 1) they originate from agents that themselves exist through time (or from ensembles of agents that can be treated as identical for the purposes of measurement), and 2) they encode information regarding the states of agents or other entities that exist through time. Bob must, in other words, can be treated as identical for the purposes of measurement), and 2) they encode information they originate from agents that themselves exist through time (or from ensembles of agents that must assume that states of the environment with which he interacts encode messages, i.e. that 1) message, $Y$ presence of third-party messages, Bob must have or devise a rule for recognizing $A$ both from \[C\] and from \[I\] $X$ is Bob’s name for the sender of the message, $Y$ is Bob’s representation of the content of the message, and $t$ is the receipt time of the message. This $R$ is the minimal structure necessary to distinguish, for example, \[A,|A\text{ready} >,t_1\] both from \[A,|A\text{ready} >,t_2\] and from \[C,|A\text{ready} >,t_1\] as communicated by some third-party agent $C$. Bob cannot obtain the data structure $R$ or the mapping $I^B$ from observation of messages, since they are required in order to treat environmental states as messages. Third, Bob must have the information that the environmental encoding of messages is at least approximately stable, that distortion of message content during transmission is at least approximately detectable, and that, in particular, message contents are not arbitrarily re-arranged during transmission. Bob can only establish authentication protocols to protect message content on the assumption that some channel for interactions with other agents is reliable; hence reliability of transmission must be assumed from the outset.

Armed with the information that messaging agents exist and that $E$ does not distort message contents, Bob can confront the problem of identifying Alice as an agent that sends messages, i.e. the problem of recognizing $L(A)$. In the limit of no third-party messages, Bob must know in advance that there are no third-party messages in order to treat $L(A)$ as a single language. In the presence of third-party messages, Bob must have or devise a rule for recognizing $L(A)$. A classic theorem of Moore [6], however, shows that Bob cannot devise a rule that reliably distinguishes states of $E$ that encode messages in $L(A)$, even if $E$ is a deterministic finite-state machine and Bob can test $E$ with finite inputs; any such rule that distinguishes a finite number of words of $L(A)$ may still fail to distinguish future words of $L(A)$. Hence Bob must have a criterion for identifying messages in $L(A)$, i.e. identifying messages from Alice, in advance of the interaction.

Given a criterion for identifying messages in $L(A)$, Bob is able to map messages from Alice to records $[A,Y,t]$, but still faces the problem of interpreting the content $Y$ of these messages to determine the state of Alice. This is equivalent to the problem of constructing a functional model of Alice as a message-generating machine. Moore’s theorem shows that no finite sequence of observations is sufficient to construct such a rule, even if Alice is a deterministic finite-state machine and Bob is allowed to interact with Alice by manipulating finite inputs. Hence Bob must possess sufficient information to interpret $L(A)$, and in particular to distinguish messages indicating Alice’s status from messages indicating the value of $P$, from the outset. It is interesting that Moore already noted the applicability of his result to the characterization of measurement in 1956.

Bob’s final challenge is to relate decoded values of $P$ to states of the system of interest $S$. Bob cannot, however, perform experiments to do this without independent access to the states of $S$. Without such access, and without a sufficiently detailed model of both $A$ and the $S-A$ interaction $H_{SA}$, Bob can only refer to states of $S$ as “correlate of $|P^i\rangle$” for some value $|P^i\rangle$ of $P$.

To summarize, Bob, or the observer $O$, must stably encode in $|O\text{ready}\rangle$ at least the following reportable and hence classical information:
1. The existence through time of $O$ himself, the apparatus $A$, the encoding environment $E$, and as a contingent generator of messages from $A$, the system of interest $S$. $|O_{\text{ready}}\rangle$ must, in other words, encode the re-identifiable existence through time of the elements of a von Neumann decomposition $\{S, A, E, O\}$.

2. The assumption that $E$ transmits messages without significant, undetectable distortion.

3. An interpretation $I : |E\rangle \rightarrow R(\mathcal{L}(A))$ that identifies states $|E\rangle$ that encode information in $\mathcal{L}(A)$, and that maps those states to structures $[A, |A^n\rangle, t] \in R(\mathcal{L}(A))$, where $|A^n\rangle$ is $O$’s encoding of a reportable state of $A$. The shorthand notation $\langle O \rangle_{A^n, t} = |O_{A^n, t}\rangle$ will be used to indicate a state of $O$ encoding a reportable data structure instance $[A, |A^n\rangle, t]$. $O$ can report the information $[A, |A^n\rangle, t]$ to $W$ only if $O$ can interpret $|O_{A^n, t}\rangle$; the notation $\langle IO : O_{A^n, t} \rightarrow [A, |A^n\rangle, t]\rangle$ will be used to indicate this fact.

4. The rule that distinct $|P^i\rangle$ indicate distinct $|S^i\rangle$, even in the absence of any independent characterization of $S$.

None of this information can be obtained by non-destructive observations of $A$ during the course of the measurement process.

Two things are clear from this analysis of the information contents of $|O_{\text{ready}}\rangle$. First, Zurek’s question [7] as to “what the ‘systems’ which play such a crucial role in all the discussions of the emergent classicality are” (p. 1818) can be provisionally answered: they correspond to reportable sets of re-identification criteria encoded by the ready states of competent observers. With the exception of the unknown system $S$, these systems are represented in $|O_{\text{ready}}\rangle$ as effectively classical; in particular, $O$ and $A$ are explicitly taken (by $O$) to have determinate re-identifiable states ($|O_{\text{ready}}\rangle$ and the $|O_{A^n, t}\rangle$ for $O$; $|A_{\text{ready}}\rangle$ and the $|P^i\rangle$ for $A$), and $E$ is taken to decohere and hence dis-entangle all encoded messages. Hence $|O_{\text{ready}}\rangle$ satisfies the basic Copenhagen Interpretation requirement that apparatus, and implicitly the observer and environment as well, be taken to be classical $ab\ iniito$. Second, because $O$ must know the values available to $P$ in advance, $O$ can represent the raw data of any measurement by an annotated one-dimensional histogram, with successive occurrences of each of the $|P^i\rangle$ ticked off with their arrival times.

Hence from $O$’s perspective, $|O_{\text{ready}}\rangle$ acts as a filter that collapses Eq. 1 and Eq. 2, i.e.

$$
(\sum_i \lambda_i |s_i\rangle) |A_{\text{ready}}\rangle |E_{\text{init}}\rangle |O_{\text{ready}}\rangle \rightarrow |S^n\rangle |A^n\rangle |E^n\rangle |O_{A^n, t}\rangle >
$$

where the reportable joint state $|S^n\rangle |A^n\rangle |E^n\rangle |O_{A^n, t}\rangle >$ at a particular time $t$ replaces the entangled but determinate joint state $|s_n\rangle |a_n\rangle |e_n\rangle |o_n\rangle >$ in the right-hand side of Eq. 2. Given the information in $|O_{\text{ready}}\rangle$, $O$ has no choice but to regard $A$ as having “internal” components that interact with $S$, and to apply an analogue of Tegmark’s [8] analysis of decoherence in the human brain to conclude that the states of these internal components that interact with $S$ are decohered by the internal environment of $A$. Hence from $O$’s perspective, there is no wave-function collapse (only the “virtual collapse” of decoherence), no branching of the wave function, and no measurement problem. This perspective is self-consistent and relies only on known physics, i.e. the physics of decoherence acting within $A$ [9].

The question that cannot be answered from $O$’s perspective is the question of how the information $O$ possesses, e.g. the information encoded by $|O_{\text{ready}}\rangle$, relates to $O$’s physical state. $O$ cannot
examine $|O_{\text{ready}}>\text{ at the outset of the measurement}; O$ is *occupying* $|O_{\text{ready}}>$ at the outset of the measurement. Addressing this question requires an external perspective, that of $W$ interacting with $O$ not as a colleague, but as an experimental system of interest. Unlike $O$, $W$ is able to experimentally distinguish physical states of $O$ from the information that $O$ reports such states as encoding. Hence $W$ can, by assuming that $E$’s reaction onto $A$ and $O$’s reaction onto $E$ can be neglected, represent the measurement process perturbatively as:

$$
(\sum_i \lambda_i |s_i> |A_{\text{init}}> |E_{\text{init}}> |O_{\text{init}}> \rightarrow (\sum_i \lambda_i |s_i> |a_i> |e_i>) |O_{\text{init}}> \rightarrow |S_{\text{final}}> |A_{\text{final}}> |E_{\text{final}}> |O_{\text{final}}>
$$

and frame the measurement problem as the question of how $|A_{\text{init}}>\text{ relates to } |A_{\text{ready}}>\text{ as reported by } O$, how $|O_{\text{init}}>\text{ relates to } |O_{\text{ready}}>\text{ as reported by } O$, and how $|O_{\text{final}}>\text{ relates to } |O_{\text{final}}>\text{ other than that they are quantum states of } A \text{ and } O \text{ respectively. Note that to do this, } W \text{ cannot treat } |A_{\text{init}}>\text{ as identical to } |A_{\text{ready}}>\text{ or } |O_{\text{init}}>\text{ as identical to } |O_{\text{ready}}>\text{; the physical structures of } |A_{\text{ready}}>\text{ and } |O_{\text{ready}}>\text{, and how } |A_{\text{ready}}>\text{ and } |O_{\text{ready}}>\text{ can be reported by } O \text{ as effectively classical are what is at issue.}

### 3 Encoding of classical information: The reportable quasi-state space $\mathcal{R}^0(O)$

The effective classicality of $|O_{\text{ready}}>\text{ and } |O^{[A,][A^n>]}\text{ results from the operational requirement that they be reportably re-identifiable by } O \text{ over the course of the measurement process, in particular, over the course of multiple rounds of observation that yield multiple reportable instances of } |O_{\text{ready}}>\text{, } |O^{[A,][A^n>]}\text{, and } |O^{[A,][P^n>]}\text{ for multiple pointer values } |P^n>\text{ at distinct observation times. The full information required to re-identify a quantum state, however, constitutes a full specification of the state in some basis, and hence is sufficient to reproduce the state } de \text{ novo, i.e. in isolation. Hence if } |O_{\text{ready}}>\text{ and } |O^{[A,][A^n>]}\text{ are quantum states, } O \text{ cannot have sufficient information to re-identify them, nor can } W [10]. \text{ If it is assumed that } O \text{ does re-identify them, which the considerations above indicate that } O \text{ must do and must be regarded by } W \text{ as doing in order to carry out reliable measurements, then } |O_{\text{ready}}>\text{ and } |O^{[A,][A^n>]}\text{ cannot be quantum states. They must, therefore, be and be regarded by } W \text{ as } components \text{ of quantum states, in particular, components of the quantum states with which } W \text{ specifies } O\text{'s time evolution during the measurement process. The question of how } |O_{\text{ready}}>\text{ and } |O^{[A,][A^n>]}\text{ are involved in the physical process of measurement therefore becomes the question of how, from } W\text{'s perspective, particular components of quantum states of } O \text{ that are re-identifiable and hence reportable by } O \text{ are involved in the physical process of measurement. } W \text{ can only address this question by examining } O\text{'s dynamics at finite time resolution and finite sensitivity.}

Let $\mathcal{H}^Y$ be the Hilbert space of a system $Y$, spanned by a Schmidt basis $\{|y_i>\}$. Let $Y$ evolve over time with some dynamics $|Y(t)>\text{, under which } Y \text{ typically occupies a given state } |Y>\text{ for a time } t_c$. Consider the evolution $|Y(t)>\text{ for a time period } t_{\text{obs}} \gg t_c \text{ as a sequence of non-overlapping intervals of length } \Delta t \lesssim t_{\text{obs}}, \text{ and consider the following projection } Q:$

\[ Q(\langle Y(t) \rangle)_{\Delta t} = \beta_k |y_k \rangle \text{ if } \beta_k^2 \gtrsim 1, \text{ where } \beta_k^2 = \left( \int_{\Delta t} |\lambda_k(t)|^2 \, dt \right) / \left( \sum_{j \neq k} \int_{\Delta t} |\lambda_j(t)|^2 \, dt \right); \]

\[ Q(\langle Y(t) \rangle)_{\Delta t} = 0 \text{ otherwise.} \]

\( Q \) is clearly non-unitary. It projects each \( \Delta t \)-long interval in the evolution \( \langle Y(t) \rangle \) to its maximum-power component in the basis \( \{ | y_i \rangle \} \), re-normalized by its partial power integral, if a single component exists with significantly greater power than any competitor. If \( |\psi_k \rangle = \beta_k |y_k \rangle \), \( Q(\langle Y(t) \rangle)_{\Delta t} \) during some chosen interval \( \Delta t \), \( |\psi_k \rangle \) will appear to be stable over that interval if observed with a time resolution on the order of \( \Delta t \). Observed with a time resolution on the order of \( \Delta t \) using a procedure sensitive only to a component with maximal integrated power, even though \( \lambda_k(t) \) may equal zero sparsely during the interval, and the full state \( \langle Y(t) \rangle \) is completely unknown at every time \( t \) during the evolution. Considered as a function of time, \( Q(\langle Y(t) \rangle) \) yields a sequence of states that are pure in the basis \( \{ | y_i \rangle \} \), each of which persists for a time interval on the order of \( n \Delta t \) for integer \( n \), possibly but not necessarily separated by intervals during which \( Q(\langle Y(t) \rangle) = 0 \). \( Q \) can be though of as a macroscopic analogue of a Copenhagen observable for the basis \( \{ | y_i \rangle \} \), and approaches such an observable as \( \Delta t \to \Delta t \) in the special case of states \( | Y \rangle \) that are approximately pure in \( \{ | y_i \rangle \} \). \( Q \) is not, however, a macroscopic limit of a Copenhagen observable. The behaviour of \( Q \) does not approach the Born Rule as \( \Delta t \to \Delta t \); \( Q \) projects significantly mixed states to zero. The behaviour of \( Q \) thus corresponds not to observation of the state \( | Y(t) \rangle \), but to observation of a component of \( | Y(t) \rangle \) that has been amplified by \( Y \)'s internal dynamics.

The single-component amplifying projection \( Q \) can be generalized to capture significant local maxima in the power spectrum of \( | Y(t) \rangle \). With \( Y \), the basis \( \{ | y_i \rangle \} \) and the evolution \( | Y(t) \rangle \) as above, consider a case in which, during an interval \( \Delta t \), there is a collection of \( N \) non-overlapping subspaces \( \{ Y \}_{\Delta t} \), where each \( Y \) is spanned by a subset \( K_i \) of basis vectors, such that \( Y = \bigoplus K_i \), and such that for every \( K_i \), there is a component \( \lambda_k | y_k \rangle \), where \( | y_k \rangle \in K_i \), such that:

\[ \int_{\Delta t} |\lambda_k(t)|^2 \, dt \geq \alpha_i \sum_{j \neq k} \left( \int_{\Delta t} |\lambda_j(t)|^2 \, dt \right) \text{ where } | y_j \rangle \in K_i \text{ and } \alpha_i > 1. \]  

In such a case, call \( \phi_i | \Delta t = \beta_i | y_k \rangle \) the \( i \)th "quasi-component" of \( | Y(t) \rangle \) for the interval \( \Delta t \) and call \( \phi | \Delta t = \frac{1}{\sqrt{\sum_i \phi_i | \Delta t}} \) a "quasi-state" of \( Y \) for the interval \( \Delta t \), where the normalization is given by:

\[ \beta_i^2 = \int_{\Delta t} |\lambda_k(t)|^2 \, dt \text{ and } \beta^2 = \sum_i \beta_i^2. \]  

(7)

Quasi-states as so defined clearly correspond to sets of \( N \) local maxima, in the representation defined by the basis \( \{ | y_i \rangle \} \), of the power spectrum of \( | Y(t) \rangle \) during the interval \( \Delta t \). For a given dynamics \( | Y(t) \rangle \) and interval \( \Delta t \) different choices of the \( Y \) yield different sets of quasi-states; however:
Lemma 1: For each interval $\Delta t$ there is a unique “maximal” quasi-state with maximum dimension $N$ and minimal dispersion among the values of the $\alpha_i$.

Proof: Let $\phi|_{\Delta t}$ be maximal. Then within $\Delta t$, $\mathfrak{Y}$ includes exactly $N$ subspaces $\mathfrak{y}$, each spanned by a subset $K_i$ of basis vectors, for which $\alpha_i > 1$, namely $K_1...K_N$ where the values of $\alpha_i$ for these $K_i$ have minimal dispersion. Now suppose some $\phi'|_{\Delta t} \neq \phi|_{\Delta t}$ is also maximal. $\phi'|_{\Delta t}$ cannot require a basis subset different from the $K_1...K_N$ with $\alpha_i > 1$, because no subsets other than $K_1...K_N$ have sufficiently large values of $\alpha$. But $\phi'|_{\Delta t}$ cannot assign different values of $\alpha_i$ to the $K_1...K_N$ and achieve a lower dispersion, since the $\alpha_i$ already have minimal dispersion. So $\phi'|_{\Delta t}$ is based on the same subsets $K_1...K_N$ and assigns the same values to the $\alpha_i$, hence $\phi'|_{\Delta t} = \phi|_{\Delta t}$. □

The generalized amplifying projection $Q^y : \mathfrak{Y} \rightarrow \mathfrak{Y}$ for the basis $\{|y_i>\}$ can now be defined:

$$Q^y(|Y(t)>|_{\Delta t} = \text{the maximal } \phi|_{\Delta t} \text{ if any } \phi|_{\Delta t} \text{ are defined for } \{|y_i>\} \text{ and } \Delta t;$$

$$Q^y(|Y(t)>|_{\Delta t} = 0 \text{ otherwise.}$$

As in the single-component case, $Q^y(|Y(t)>)$ acting over time will yield a sequence of quasi-states, each of which persists for a time interval on the order of $\Delta t$ for integer $n$, possibly but not necessarily separated by intervals during which $Q(|Y(t)> = 0$. These quasi-states will appear stable if observed with time resolution on the order of $\Delta t$ using a procedure sensitive only to components with relatively high integrated power, even though the underlying state $|Y(t)>$ remains completely unknown. Call the image of $Q^y(|Y(t)>$ in $\mathfrak{Y}$, i.e. the set of maximal quasi-states for a process $|Y(t)>$ plus the vector $|0>$, the “quasi-space” $\mathfrak{Y}(Y)$ of $|Y(t)>$.

As an example of the action of $Q^y(|Y(t)>$, consider an array of $M$ identical spin-$\frac{1}{2}$ systems evolving over time, described in the $z$-component basis $\{|\uparrow>_i, |\downarrow>_i\}$. If the state of each spin evolves independently and randomly with a characteristic time $t_c \ll \Delta t$, then $Q^y(|Y(t)>)$ acting over intervals of length $\Delta t$ projects every spin to zero. Similarly, if every spin remains in a state $(\frac{1}{\sqrt{2}})(|\uparrow> \pm |\downarrow>)$ over the interval $\Delta t$, $Q^y(|Y(t)>)$ acting over $\Delta t$ projects every spin to zero. If every spin is in either pure state $|\uparrow>$ or $|\downarrow>$ throughout $\Delta t$, $Q^y(|Y(t)>)$ acting over $\Delta t$ projects $|Y>$ to itself. If $N$ spins are in pure states and the remaining spins are sufficiently near $(\frac{1}{\sqrt{2}})(|\uparrow> \pm |\downarrow>)$ to have a total integrated power less than $N/M$, $Q^y(|Y(t)>$ will pick out the $N$ pure states as quasi-components of an $N$-dimensional quasi-state. The dimension $N$ of the quasi-state projected by $Q^y(|Y(t)>$ thus measures, in this binary system, the number of bits of classical information that can be obtained from $|Y(t)>$ by interacting with $|Y(t)>$ with a time resolution on the order of $\Delta t$ and a procedure sensitive only to components with relatively high integrated power. The projected quasi-state for an interval $\Delta t$ can be known, as a result of such an interaction, without knowing the state $|Y(t)>|_t$ at any particular time $t$ within $\Delta t$.

In general, a quasi-state $\phi|_{\Delta t}$ is a projection of the state vector $|Y(t)>$ during any given period $\Delta t$, and hence encodes less state information than is encoded in $|Y(t)>$. However, because the components of $\phi|_{\Delta t}$ are time integrals, $\phi|_{\Delta t}$ also encodes information about the dynamics of $Y$.

In particular,

Theorem 1: A quasi-state $\phi|_{\Delta t}$ encodes classical information during a period of observation $t_{obs} \gtrsim \Delta t$ that the physical state $|Y(t)>|_t$ does not encode at any time $t$ contained within the interval $t_{obs}$. 

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Proof: To determine the value of $\phi|_{\Delta t}$ at a given time $t$ within the interval $\Delta t$, a state $|Y(t)\rangle > |t\rangle$ would have to encode the values of the partial power integrals given in Eq. 6, and hence would have to encode the values of the its components for the entire interval $\Delta t$. This can be true only in the limit as the characteristic time of the dynamics $t_c \to \Delta t$. But $t_c \ll \Delta t$ by assumption. □

Theorem 1 shows that while a quasi-state $\phi|_{\Delta t}$, and by extension the quasi-space $\Phi^q(Y)$, encodes less information than the evolving state vector $|Y(t)\rangle >$, it also encodes information that $|Y(t)\rangle >$ does not encode at any given time, and indeed does not encode in any sparse sample of times. Reportable measurement, however, is a sparse sampling process; at most one value is reportable for each consecutive, non-overlapping interval $\Delta t$. Hence reportable measurements of quasi-states yield classical information, in particular integrated power-spectrum information, that could not be obtaining by sampling $|Y(t)\rangle >$ directly at the same sparsity.

It is useful at this point to consider a familiar non-abstract example of such a quasi-space and the classical information that can be reported from it: the binary-state representation of a conventional, classical digital computer, and the binary state-vector of such a computer as it evolves through time to define an execution trace. This example is most illustrative if one considers a computer from the pre-integrated circuit era, i.e. one based on relatively noisy, slow individual transistors, for which $\Delta t$ was on the order of ms. If observed with ms time resolution, such a computer appears to occupy well-defined binary states, and to undergo discontinuous state transitions between such states. The actual dynamics, however, are continuous; the binary representation is possible because quasi-stable components $|1\rangle >$ and $|0\rangle >$ dominate the power spectrum at the ms time scale. These components are quasi-states in the basis $\{|V_{i,j}\rangle >\}$, where $V_{n,m}$ is the voltage on the wire connecting gate $n$ to gate $m$. The amplification that maintains the stability of these quasi-states is achieved by externally biasing the transistors.

If one observes only the quasi-states of a conventional, classical digital computer, one can formulate an hypothesis as to the algorithm it is executing, and use the hypothesized algorithm as a predictive theory of device behaviour. However, Theorem 2 of Moore [6] shows that no finite sequence of observations of the quasi-states can produce a unique correct algorithmic description; hence theories of device behaviour based on specific algorithms always require a priori assumptions regarding proper functioning and rational design. Moreover, such algorithmic descriptions are not dynamical theories, even in the limit of $\Delta t$ time resolution; all information regarding the underlying physics of the device is completely lost in the discrete quasi-state transition representation. This loss of information about the underlying physics effectively defines the level of description at which one can characterise distinct physical implementations of a single virtual machine as computationally equivalent.

With this example as background, the concept of a quasi-space of quasi-states can now be applied to the observer $O$ and the reportable information-encoding states $|O^{ready}\rangle >$ and $|O^{[A,|A^nA^\dagger]\rangle} >$. For human observers, the time required to reportably re-identify an object embedded in a background, as deduced from experiments measuring the time required to reportably notice changes in the visual properties or positions of objects in commonplace scenes (e.g. [11, 12]), is on the order of 100s of ms; re-identification times extend to 10s of s for objects that are not foci of attention. Human re-identification times exceed by at least an order of magnitude the time required for a visual stimulus to elicit a behavioural response (10s of ms, e.g. [13, 14]), exceed by at least two orders of magnitude the typical firing times of neurons, and exceed by over 15 orders of magnitude the decoherence times of the molecular environments of neurons estimated by Tegmark [8], which can be taken to provide an estimate of the characteristic time $t_c$ of the physical dynamics.
underlying observation. Hence the time scale of reportable information-bearing states of human observers is far longer than any relevant dynamic time scales, suggesting that these reportable “states” are in fact quasi-states. This observation depends only on ratios of characteristic time scales, not on any specific features of humans as observers; hence it can be generalized:

**Lemma 2:** For any observer \( O \), the components of \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \), for any system \( A \) and encoded state \( |A^n\rangle \), form at most a partial basis of \( |O(t)\rangle \).

**Proof:** The time intervals \( \Delta t \) over which \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) are re-identifiable by \( O \) and reportable to a third party \( W \) are at least long enough for \( O \) to conduct multiple observations of \( A \); hence \( \Delta t \gg t_c \), where \( t_c \) is the time scale at which entanglements \( |A > |E > |O > \) corresponding to individual observations of \( A \) by \( O \) are established and evolve. Hence if \( |O_{\text{ready}}\rangle \) and \( |O[A,|A^n>,t]\rangle \) were states of \( O \) in any complete basis, re-identification and reporting would require that they be cloned by \( O \). This is forbidden [10]; hence \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) cannot be full quantum states, i.e. their components cannot span \( |O(t)\rangle \). □

Lemma 2 formalizes the observation made at the beginning of this section, and shows that the \( |O_{\text{init}}\rangle \) required by Eq. 4 must have components that are absent from \( |O_{\text{ready}}\rangle \). Hence \( |O_{\text{init}}\rangle \) must replace \( |O_{\text{ready}}\rangle \) in Eq. 1 if Eq. 1 is to describe a physical interaction. Given that \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) are not full quantum states of \( O \), however, it is clear that they must be quasi-states:

**Theorem 2:** For any observer \( O \), a basis \( \{|o_i\rangle\} \) exists such that \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \), for any system \( A \) and encoded state \( |A^n\rangle \), are quasi-states of \( O \).

**Proof:** A report by \( O \) to \( W \) can be represented by an effective interaction \( H_{OW} \). From \( W \)’s perspective, \( H_{OW} \) is a non-destructive measurement of \( |O > \) that, in particular, does not alter \( |O_{\text{ready}}\rangle \) or the \( |O[A,|A^n>,t]\rangle \). Let \( \{|o_i\rangle\} \) be the basis for \( O \) in which \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) are eigencomponents of \( H_{OW} \). \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) must be quasi-states of \( O \) in the basis \( \{|o_i\rangle\} \), i.e. comprise components that are local maxima in the power spectrum, since otherwise the components that were local maxima would swamp \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \), and \( W \)’s observation of \( O \) would yield the quasi-states formed by these other components, not \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) as results. □

**Corollary 2.1:** Reportable records of \( O \) are quasi-states of \( O \).

**Proof:** If a record \( R \) is reportable, it must be re-identifiable by \( O \); therefore it is a quasi-state by the reasoning above. □

**Corollary 2.2:** The information encoded in \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) could not be obtained by direct observation of a sparse set of individual states \( |O_i\rangle \) during the time required to report \( |O_{\text{ready}}\rangle \) and \( |O[A,|A^n>,t]\rangle \).

**Proof:** The Corollary follows from Theorem 1 and the identification of \( |O_{\text{ready}}\rangle \) and the \( |O[A,|A^n>,t]\rangle \) as quasi-states of \( O \). Theorem 2 of Moore [6] shows that the conclusion follows for any finite set of observations, even if these are interspersed with tests using finite classical inputs. □

Theorem 2 entails a significant empirical prediction, namely that the “states” of human observers that encode classical information that can be reported to third parties are quasi-states, i.e. vectors of wave-function components with relatively high integrated power over the time intervals required for reporting. This prediction is consistent with the re-identification time data considered above,
with the fact that human observers require continuous energy input to make and report observations, with phenomenological models of attention, short-term memory, and reportable awareness based on competition between disparate patterns of neural activity (e.g. [15, 16]), and with the utility and near-universal application of algorithmic models of cognitive processes (e.g. [17, 18]).

Given Theorem 2, the mapping of states of $O$ to the classical information that $O$ interprets and can report them as encoding can be represented as:

$$I^O(Q^o(H_O)) \rightarrow I^O(Q^o(|O(t) > |\Delta t_i))) \rightarrow I^O(Q^o(|O(t) > |\Delta t_2)))$$

$$I^O(Q^o(|O(t) > |\Delta t_i))) \rightarrow Q^o(|O(t) > |\Delta t_i)) \rightarrow Q^o(|O(t) > |\Delta t_2))$$

$$|O(t) > |\Delta t_i) \rightarrow H_O \rightarrow |O(t) > |\Delta t_2)$$

**Diagram 1**

where $|O(t) > |\Delta t_i$ represents an evolution $|O(t) >$ during the interval $\Delta t_i$ satisfying Eqs 6 - 8, $I^O$ represents the semantic interpretation function mapping quasi-states of $O$ to the information they encode from $O$’s perspective and $R^o(O)$ is the quasi-space of “reportable” quasi-states of $O$. Given this notation, it is possible to define a notion of consistency for observers:

**Definition 1**: $O$’s interpretation map $I^O$ is consistent if and only if Diagram 1 commutes for all intervals $\Delta t$ of the dynamics $|O(t) >$, i.e. if and only if $I^O(Q^o(H_O)) \circ I^O \circ Q^o = I^O \circ Q^o \circ H_O$ where the composition notation “$f \circ g$” represents function $g$ acting, followed by function $f$. If $I^O$ is consistent in this sense, $I^O(Q^o(H_O))$ is a reliable inferential operator acting on information contents, i.e. the information $O$ infers from $I^O(Q^o(|O(t) > |\Delta t_i)))$ using his interpretation of the physical dynamics $H_O$ matches his interpretation $I^O(Q^o(|O(t) > |\Delta t_2)))$ of the physical result $|O(t) > |\Delta t_2$ of those dynamics. Consistency in this sense is clearly required for reliable measurements.

Given this notion of consistency, the induced mapping $Q^o(H_O)$ can be interpreted, from $W$’s perspective, as an apparent dynamics or “quasi-process” acting on quasi-states of $O$. Quasi-states of $O$ are vectors of components of $|O >$ in the basis $\{|o_i >\}$ that are stable at the macroscopic time scale $\Delta t$; the induced quasi-process $Q^o(H_O)$ therefore appears classical, and can be considered as analogous to a sequence of state transitions in a conventional, classical digital computer. $W$ can investigate the quasi-process of $O$ directly by using instruments with time resolution on the order of or greater than $\Delta t$; in the case of human observers, functional magnetic resonance imaging (fMRI) systems probe from this scale up in time [12], while transcranial magnetic stimulation (TMS, e.g. [19]) and event-related potential (ERP, e.g. [20]) methods probe from an order of
magnitude below to an order of magnitude above this scale. All such instruments are, of course, sensitive only to relatively high-power components of $|O(t)>$.

4 O-induced quasi-states and the Born Rule

Suppose that $A$, $E$ and $O$ are all quantum systems evolving on a characteristic time scale $t_e \ll \Delta t$, that these systems interact to form one or more einselected [3,7] entangled states $|A> |E> |O> = \sum_i \lambda_i |a_i> |e_i> |o_i>$ during a period $t_{obs} \gtrsim \Delta t$, and that $O$ is in the quasi-state $|O[A,|A^n>,\Delta t]>$, i.e. the quasi-state encoding the classical information that $A$ is in $|A^n>$ during the interval $\Delta t$ of $t_{obs}$. This quasi-state can be written as $|O[A,|A^n>,\Delta t]> = (\frac{1}{\sqrt{\beta}}) \sum_i \phi_i |\Delta t>$ where each $\phi_i |\Delta t>$ is a particular basis vector $|a_k>$ with an amplitude $|\lambda_k|^2$ that satisfies Eq. 6 for the interval $\Delta t$, i.e. that is a local maximum in the power spectrum of $O$ over $\Delta t$. Einselection associates each such $|a_k>$ with a particular $|a_k>$ in the einselected basis $\{|a_i>\}$. As Eq. 6 only stipulates requirements on the magnitudes of the $\lambda_i$, it is clear that Eq. 6 is true of each of these basis vectors $|a_k>$. Hence the vector $\varphi |\Delta t> = (\frac{1}{\sqrt{\beta}}) \sum_k \beta_k |a_k>$ with the coefficients $\beta_k$ as defined by Eq. 7 can be considered an “O-induced” quasi-state of $A$ during the interval $\Delta t$.

**Theorem 3:** The ready state $|A^{\text{ready}}>\text{ and the encoded pointer states }\{|P^i>\}\text{ of an apparatus }A\text{ under observation by an observer }O\text{ are }O\text{-induced quasi-states during the intervals of their observation by }O.

**Proof:** $|A^{\text{ready}}>$ and the $\{|P^i>\}$ are non-destructively observable by $O$, so they must correspond to sets of einselected states of $A$. By Theorem 2, the information that $A$ is occupying such states is re-identifiably encoded by $O$ during the observation process by a quasi-state $|O[A,|A^n>,\Delta t]>$. That $|A^{\text{ready}}>$ and the $\{|P^i>\}$ are O-induced quasi-states during the intervals of their observation then follows from Eq. 6 by the reasoning above. $\square$

The $|P^i>$ will to be called “pointer states” of $A$ below, but with the understanding that they are $O$-induced quasi-states of $A$ during the intervals of their observation.

**Corollary 3.1:** Einselection alone is insufficient to determine $|A^{\text{ready}}>$ or the $\{|P^i>\}$ for $O$.

**Proof:** Einselection establishes that the entangled state $|A> |E> |O> = \sum_i \lambda_i |a_i> |e_i> |o_i>$ can be written in the indicated basis. It does not establish that the $\lambda_i$ meet the requirements of Eq. 6, as these requirements pertain to the internal dynamics of $O$, and by extension to the internal dynamics of any system $A$ that $O$ can employ as an apparatus. $\square$

Corollary 3.1 confirms the result obtained in Sect. 2, namely that $O$ must know how to identify $|A^{\text{ready}}>$ and the pointer states $\{|P^i>\}$ at the outset of an observation. Einselection determines which states of $A$’s pointer are non-destructively observable [3,7], but it does not determine what part of $A$ is to be regarded by $O$ as the pointer [9], or the time required by $O$ to reportably record an observation of a pointer state $|P^n>$. Let $N$ be the number of pointer states $|P^i>$ of $A$ that are reportable by $O$, i.e. to which quasi-states $|O[A,P^n,\Delta t]>$ correspond by einselection for any interval $\Delta t$ with length on the order of $O$’s re-identification time. Each of the $|P^i>$ is the image, under the amplifying projection $Q^O(|A(t)>)$, of the dynamics $|A(t)> |\Delta t$ during some interval $\Delta t$, where the basis $\{p_i>\} \subset \{|a_i>\}$ spans the subspace $\mathcal{H}^P \subset \mathcal{H}^A$ that defines $A$’s pointer. Given $|P^i>$, nothing is required of its inverse image $|A(t)> |\Delta t$ other than compliance with Eq. 6; i.e. any state $|A>$ whatsoever can be traversed
during this interval provided that the other states traversed adequately compensate for $|A>$’s contribution to the integrated power spectrum. The induction of quasi-states by observation thus involves an irreversible loss of reportable information formally analogous to decoherence: if $O$ is in the quasi-state $|O^{[A],|P^n⟩,Δt}>$, $O$’s reportable information concerning $|A(t)>$ is limited to the statement that $A$ is occupying the quasi-state $|P^n>$. In particular:

**Lemma 2:** Any two physical states $|A_j>$ and $|A_k>$ satisfying Eq. 6 can be swapped without altering the resulting quasi-state $|P^n>$. 

**Proof:** Corresponding states $|O_j>$ and $|O_k>$ of $O$ that can be counter-swapped to preserve the quasi-state $|O^{[A],|P^n⟩,Δt}>$ are guaranteed to exist by the einselection of a joint basis for the entangled state $|A> |E> |O>$, and hence the compliance of $|O_j>$ and $|O_k>$ with Eq. 6. □

As emphasized by Zurek [3, 4] in the context of envariance, $O$’s ignorance of $|A(t)>$ is an objective fact about the effective $A−O$ interaction: it results from the dynamic timescale $t_c$ of the $A−O$ interaction being much smaller than $O$’s re-identification time $Δt$. The swap symmetry exhibited by quasi-states is formally analogous to the swap symmetry underlying envariance; it effectively extends the concept of envariance from the decoherence time scale to the re-identification time scale $Δt$.

Consider now the result of $O$ employing $A$ to make measurements of the state of an unknown quantum system $S$ that has been prepared in a state $|S> = ∑_k ξ_k|s_k>$ with unknown coefficients $ξ_k$ and Schmidt basis $|s_k>$. Over an extended period of time, $O$ observes with certainty a sequence of pointer states $|P^n>$, but is objectively ignorant of the physical states $|A(t)>$ or $|S(t)>. A$ pointer state $|P^n> will be observed in an interval $Δt$ in this sequence, i.e. $O$ will report the entangled quasi-state $|O^{[A], P^n⟩,Δt}>$, if and only if entangled states $|S> |E> |O> = ∑_k ξ_k|s_k> |a_k> |e_k> |o_k>$ with components $|o_k>$ satisfying Eq. 6 exist. The requirements of Eq. 6 are independent of the characteristic time scale of the $S−A$ interaction, so it can be assumed without loss of generality that the $S−A$ interaction is instantaneous and occurs exactly once during each interval $Δt$. Provided that $A$ has a pointer state $|P^0>$ that is registered as the pointer value whenever interaction with $S$ is undetectable, the pointer-state probabilities $Prob(|P^i>)$ will sum to unity, i.e. every component of $|S(t)>$ will contribute power to some $|P^i>$. Zurek’s [4] proof of the Born Rule from envariance can then be extended to the case in which the reported pointer states are $O$-induced quasi-states.

**Theorem 4** (Born Rule): A quasi-state $|P^i> of A$ is reportable in the interval $Δt$ with probability $∑_j (∫_Δ |ξ_j|^2 dt)$, where $j$ ranges over those components of $|S> |A> |E> |O> = ∑_k ξ_k|s_k> |a_k> |e_k> |o_k>$ contributing power to $|P^i>$ and hence to $|O^{[A], P^i⟩,Δt}>$ in the interval $Δt$.

**Proof:** The tactic of Zurek’s proof ([4], Eqs. 8a - 10) is to divorce probabilities from individual states $ψ = ∑_i ξ_i|s_i>$. In step 1, an ancillary “counter” state $|C_i>$ comprising $M_i$ orthonormal components $|c_{ij}>$, where $M_i = |ξ_i|^2$, is associated with each basis vector $|s_i>$. The sum $(1/ ∑_i M_i) ∑_{ij} |c_{ij}>^2 = 1$, corresponding to certainty, hence each of the $|c_{ij}>$ “counts” a unit of probability $u = 1/(∑_i M_i)$. In step 2, envariance is used to demonstrate that the partial sum $(∑_j |c_{ij}>|s_i>)$ for each $i$ cannot depend on the identity of $|s_i>$, since any envarient state can be swapped for $|s_i>$ without altering $ψ$. The partial sum $∑_j |c_{ij}> = M_i = |ξ_i|^2$ is then the probability associated with $|s_i>$, as required.

This tactic can be applied directly in the current setting. Let $ψ_i|Δt_i$ be the inverse image of $Q^o(|S(t)> |A(t)> |E(t)> |O(t)>)$ for the interval $Δt_i$. The $N$ pointer states $|P^i>$ are
orthonormal and complete, so \( \sum_i Prob(|P_i>) = \sum_i Prob(\xi_i|\psi_i|\Delta t_i) = 1 \), where the combined amplitude \( \xi_i = \sum_k (\int |\Delta t_i| |\xi_{ik}|^2 dt) \), where \( k \) ranges over those components of \( |S> |A> |E> |O> \) contributing power to \( |P_i> \) and hence to \( |O^{[A,|P_i>|\Delta t_i]}> \) in the interval \( \Delta t_i \). Now associate a \( |C_i> \) as above with each \( \psi_i|\Delta t_i> \), where there are \( M_i = |\xi_i|^2 \) orthonormal components \( |c_{ij}> \) of each \( |C_i> \), each counting a unit of probability \( u = 1/(\sum M_i) \). The swap symmetry of Lemma 2 then shows that the partial sum \( \sum_j |c_{ij}> \psi_i|\Delta t_i> \) for each \( i \) cannot depend on the identity of \( \psi_i|\Delta t_i> \). The partial sum \( \sum_j |c_{ij}> = M_i = |\xi_i|^2 \) is then the probability associated with \( \psi_i|\Delta t_i> \), as required. \( \square \)

Again as emphasized by Zurek [3, 4], the Born Rule in this case states an objective fact consequent to \( O \)'s objective ignorance of both \( |S(t)> \) and \( |A(t)> \).

It is worth noting, however, that absent independent access to \( S \), e.g. by independent observations made with a distinct apparatus \( A' \), \( O \) is equally justified in regarding a sequence of observed quasi-states of \( A \) as resulting from an unknown classical algorithmic process instantiated by \( A \). The underlying dynamics of the \( S-A \) interaction are reduced by \( O \)'s macroscopic re-identification time \( \Delta t \) to classical quasi-state transitions that can be attributed to a classical Turing machine, hence the behaviour of \( S \) as measured by \( O \) can be emulated by such a Turing machine [21].

5 Measurement as quasi-entanglement

Given the foregoing, a measurement interaction with negligible environmental distortion can be represented as in Diagram 2, where the \( |A> |E> |O> \) coupling is represented by the effective interaction \( H_{AO} \):
Here $Q^{ind}_n$ is the induced mapping from states to quasi-states of $A$ and the $n^{th}$ pointer quasi-state $|P^n\rangle$ is represented as being recorded by $O$ as the result of an instantaneous $S-A$ interaction at $t_{int}$. By Theorem 4, the probability with which $|P^n\rangle$ is recorded by $O$, and hence the probability $O$ assigns to $|S^n\rangle$, is determined by the dynamics of the $S-A$ interaction. A measurement is consistent if Diagram 2 commutes, i.e. if $O$ interprets the physical interaction $|S\rangle > |A\rangle > |O\rangle$ as causing, via the map $I^O(Q^c(H_O))$, the transition from $[A, |A^{\text{ready}}\rangle, \Delta t_1]$ to $[A, |P^n\rangle, \Delta t_2]$ for any $|P^n\rangle$.

A measurement as described by Diagram 2 is a physical interaction in which long-lived quasi-states in one of the parties to the interaction, the “observer” $O$, induce long-lived quasi-states in another party to the interaction, the “apparatus” $A$, that appear to a third party $W$ to be entangled by the induced map $Q^{ind}_AO$. Note that this induced mapping is not represented by $O$, who occupies $|O^{\text{ready}}\rangle$ followed by $|O[A, |P^n\rangle, \Delta t_2]\rangle$ and reports that $A$ is ready, and then that $A$ has pointer value $P^n$, but does not observe either of these stages of the action of the interpretation $I^O$. From $O$’s perspective, $A$ is in $|A^{\text{ready}}\rangle$ with certainty and is then in $|P^n\rangle$ with certainty, from which $S$ can be inferred to be in $|S^n\rangle$ with probability given by the Born Rule. From $W$’s perspective, $S, A, O$, and by implication $E$ occupy a mutually-entangled quantum state evolving with characteristic time $t_c$, high-power components of which define stable quasi-states for the duration of the measurement process. These quasi-states appear to be entangled by the induced quasi-process $Q^{ind}_AO$: this quasi-entanglement appears to generate $|P^n\rangle$ from $|A^{\text{ready}}\rangle$. Given sufficient knowledge of the initial states of $S, A, O$ and the relevant Hamiltonians, $W$ could in principle predict these quasi-states and determine the outcome that would be reported by $O$. Such predictions are routinely made by designers of experimental apparatus.

### 6 Discussion: Measurement without collapse or branching

The interpretation of measurement as quasi-entanglement illustrated in Diagram 2 is minimal in the sense of requiring no fundamental physical assumptions beyond minimal no-collapse quantum mechanics, and only two empirical assumptions: that observers reliably conduct measurements and that they report determinate values. These empirical assumptions introduce into the quantum framework the irreducibly classical notion of system re-identifiability. The assumption of re-identifiability allows use of the classical notion of information, and hence use of the classical notion that a quasi-stable physical configuration can encode reportable information. Re-identifiability enables reportable measurement. Without re-identifiability, there is simply physical interaction.

All of the systems involved in a measurement process are physically entangled throughout the measurement, i.e. Eq. 1 describes the measurement process provided that unreportable physical states $|A_{init}\rangle$ and $|O_{init}\rangle$ replace the reportable quasi-states $|A^{\text{ready}}\rangle$ and $|O^{\text{ready}}\rangle$. On the other hand, Eq. 2, the traditional motivation of the measurement problem, reflects an implicit assumption that a single quantum state $|s_n\rangle > |a_n\rangle > |e_n\rangle$ can encode the classical information reported in a measurement; Eq. 2 violates Theorem 1 and must be abandoned. Neither a physical collapse mechanism nor branching of the wave function into irreversibly mutually independent components is required to understand the measurement process as described by Eq. 1 and elaborated in Diagram 2; both of these interpretations are motivated by Eq. 2, and their motivation disappears when Eq. 2 is abandoned. The “virtual collapse” of decoherence is moreover explained: the (classical) information that is “lost” to decoherence is (classical) information that would have
been encoded by components with relatively low integrated power over the measurement interval, had these components had higher integrated power. The observer has no observational access to these components, as they do not contribute to quasi-states that are sufficiently long-lived to encode classical information. The quantum information encoded by these low-power components is never lost; it contributes to the dynamics just as it would have had no measurement been made. The observer’s physical state has access to this quantum information via physical entanglement, but the observer’s records, as re-identified by the observer, do not. Hence the observer’s physical trajectory incorporates information encoded by all components of observed systems, while his reports of his observations do not.

As Schlosshauer [1] emphasizes, all interpretations of measurement come at a philosophical cost, and the philosophical cost of viewing measurement as quasi-entanglement is significant. This cost can be previewed by considering the notational modification that is required of Eq. 1: $|A^{\text{ready}}\rangle$ and $|O^{\text{ready}}\rangle$ are not physical states over which dynamics is defined, so they must be replaced in Eq. 1, on pain of notational incoherence, by the much blander notions of initial apparatus and observer states $|A_{\text{init}}\rangle$ and $|O_{\text{init}}\rangle$ employed in Eq. 4. The cost becomes clearer when one recognizes that $|A_{\text{init}}\rangle$ and $|O_{\text{init}}\rangle$ can only be defined as states that project to $|A^{\text{ready}}\rangle$ and $|O^{\text{ready}}\rangle$; aside from this requirement, these states are completely unknown. Generalization then reveals the full cost: all physical states are completely unknown. Measurements only reveal quasi-states, and hence only reveal the images of amplifying projections. Observers are, at all times, provably ignorant of the states over which dynamics can be consistently defined, not just up to envariance or projection into Copenhagen observables, but up to projection into quasi-states. Because human quasi-states are long-lived with respect even to many biological processes, ignorance of dynamics up to projection into quasi-states is severe. Practical consequences of such ignorance can be avoided by examining systems of interest with apparatus having time resolutions much smaller than the observer’s re-identifiability time $\Delta t$, but only at the price of assuming classicality for the apparatus and hence abandoning the framework of Eq. 1 as a description of the measurement process in favour of the classical observer’s viewpoint, i.e. that of Eq. 3. Hence the central point of the Copenhagen Interpretation: the apparatus must be taken by the observer to be classical.

The approach to measurement advocated here provides an alternative way of formulating and understanding a point consistently raised by Zeh [22], namely that unitary dynamics requires a field-theoretic interpretation, and that discrete entities and events must be regarded as illusory. Here the vague notion of illusion is replaced by the more precise notion of a quasi-state: discrete entities and events must be regarded as quasi-states, as long-lived apparent regularities of a provably unobservable underlying dynamics.

7 References

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