Online Auction-Based Incentive Mechanism Design for Horizontal Federated Learning with Budget Constraint

Jingwen Zhang, Yuezhou Wu and Rong Pan

Abstract

Federated learning makes it possible for all parties with data isolation to train the model collaboratively and efficiently while satisfying privacy protection. To obtain a high-quality model, an incentive mechanism is necessary to motivate more high-quality workers with data and computing power. The existing incentive mechanisms are applied in offline scenarios, where the task publisher collects all bids and selects workers before the task. However, it is practical that different workers arrive online in different orders before or during the task. Therefore, we propose a reverse auction-based online incentive mechanism for horizontal federated learning with budget constraint. Workers submit bids when they arrive online. The task publisher with a limited budget leverages the information of the arrived workers to decide on whether to select the new worker. Theoretical analysis proves that our mechanism satisfies budget feasibility, computational efficiency, individual rationality, consumer sovereignty, time truthfulness, and cost truthfulness with a sufficient budget. The experimental results show that our online mechanism is efficient and can obtain high-quality models.

1 Introduction

Federated Learning (FL) is a distributed machine learning framework that satisfies privacy protection, data security, and government laws [Yang et al., 2019]. By sharing model parameters instead of data, all parties collaborate to train the model, effectively breaking down data silos [Bonawitz et al., 2019]. Sufficient high-quality workers are the key to the success of FL. Existing researches assume that workers serve for free [Yu et al., 2020]. However, resource consumption and privacy leakage risks make workers reluctant to participate in FL. Restrictions on data exchange leave task publishers with no effective means of selecting and paying high-quality workers [Zhang et al., 2021]. Therefore, an incentive mechanism is necessary for FL.

Existing researches on incentive mechanisms [Le et al., 2021; Zeng et al., 2020] for federated learning are applied to offline scenarios, meaning that workers are selected before the task, and once the task starts, the task publisher no longer accepts bids from newly arrived workers. However, it is practical that workers arrive online at different times [Zhao et al., 2014]. Waiting for all interested workers to arrive before the task can result in task delay and impairment of benefit.

We design an online reverse auction-based incentive mechanism for horizontal federated learning with budget constraint and leverage the reputation proposed by Zhang [2022] to indirectly reflect the quality and reliability of workers. We divide FL task into multiple time steps according to global iterations. Workers arrive at different time steps in different orders before or during tasks. The first time step, the publisher selects arrived workers by combining their bids and reputations through the offline proportional share mechanism [Singer, 2010]. At other time steps, arrived workers are divided into two groups, which are used to mutually estimate payment density thresholds. Workers whose unit reputation bid prices do not exceed the corresponding payment density threshold are selected. Theoretical analysis proves that our mechanism satisfies budget feasibility, computational efficiency, individual rationality, consumer sovereignty, time truthfulness and cost truthfulness with sufficient budget. The experimental results show that it helps to obtain high-quality models.

The rest of the paper is organized as follows. Section 2 introduces related work. Section 3 describes the system model and problem definition. Section 4 designs the mechanism in detail. Section 5 conducts theoretical analysis and Section 6 shows the simulation results.

2 Related Work

Jiao et al. [2020] develops a mechanism based on multidimensional reverse auction, which takes into account the data quantity and distribution of workers. Ying et al. [2020] proposes a framework SHIELD based on reverse auction and differential privacy. The probability of being selected is based on the bid price. Roy et al. [2021] considers the cost, QoE, and reputation to select workers through reverse auction and reputation. Zhang et al. [2021] prioritizes workers with lower unit reputation bid prices. Zhou et al. [2021] selects workers using a greedy algorithm with the goal of minimizing social costs through the reverse auction. Seo et al. [2021] proposes an auction-based method to help task publishers dynamically select resource-efficient workers. To improve model accuracy and communication efficiency, Pandey et al. [2020] models
the interaction between the publisher and workers as a two-stage Stackelberg game. Feng et al. [2019] uses Stackelberg game. Workers decide the unit price of data, while the publisher decide the amount of data. Ding et al. [2020] uses a multi-dimensional contract as an incentive mechanism, taking into account training costs and communication delays. Lim et al. [2021] establishes multiple contract items, and the worker selects a contract item and gets paid. All of the above are applied to offline scenarios rather than online scenarios.

3 System Model and Problem Definition

3.1 System Model

A federated learning system includes a task publisher and many workers. Since his data is not enough, the publisher recruits workers to train a high-quality model through FL. The data of the task publisher can be used as the validation set and test set. Workers may be users of smart devices with data and computing power. Due to daily life, workers have accumulated lots of data. The publisher with a budget of $B$ publishes a FL task with $T$ rounds global iterations and scheduled start time. Each interested worker $i$ that arrives at different times, submits a sealed bid price to the task publisher. Each worker $i$ has a true arrival time $a_i \in \{1, ..., T\}$, the true cost $c_i$ of a global iteration, as well as the quantity and quality of data, but all are private. Only the budget $B$, the number of global iterations $T$, and the reputation $R_{ei}$ are public.

We model the interaction between the publisher and workers as an online reverse auction model. At his arrival time $\hat{a}_i$, worker $i$ submits the bid price $b_i$ to the publisher, which represents the claimed cost of a global iteration. Since in reality the worker is strategic and tries to maximize his utility, his arrival time $\hat{a}_i$ and bid price $b_i$ may be different from his true arrival time $a_i$ and cost $c_i$. We assume that workers will not claim an arrival time $\hat{a}_i$ earlier than the true arrival time $a_i$, that is, $\hat{a}_i \geq a_i$. That is because if a worker lied about an earlier arrival time and was selected, the publisher could find that he did not upload a local model. We also assume that the workers are independent and cannot collude with each other. Due to the uneven abilities, the quantity and quality of data of different workers may be different. To obtain a higher-quality model with a limited budget, the publisher will select high-quality workers from the workers who arrive online. Since his data is not enough, the publisher requires each worker to train a local model and upload it.

Figure 1 shows the workflow. First, the publisher publishes a FL task, announcing the budget $B$ and the number of global iterations $T$, the scheduled start time, the model structure, data type, and other requirements. According to the strategy, the interested workers submit their arrival time $\hat{a}_i$ and bid price $b_i$ to the publisher one by one online. At the scheduled start time, the task publisher collects all bids of workers with $\hat{a}_i \geq 1$ and selects them through the offline reverse auction mechanism by combining the bid prices and reputation. If the number of selected workers is less than the minimum number required to start the task, the publisher will delay the start time and re-execute the worker selection phase at the new start time. The winning workers participate in all remaining global iterations. The unselected workers will not leave but wait for the next worker selection phase before the next global iteration. The winning worker starts a round of global iteration. The task publisher notifies the winning workers and distributes the initial global model to them. The winning worker uses his data to train the local model based on the downloaded global model and then uploads the local model. The publisher aggregates all local models to obtain a new global model. At this point, a global iteration is over. Before each remaining global iteration $t > 1$, the publisher collects the bids of newly arrived workers and previously unselected workers, combined with the reputation, and selects new ones through an online reverse auction mechanism with budget constraint. Then the global iteration $t$ starts. After $T$ global iterations, the task publisher updates the reputation of the winning worker $i$ and pays him $p_i$. 

3.2 Problem Definition

To select cost-effective workers online with a limited budget $B$ and obtain a higher-quality model after $T$ global iterations, it is necessary to design an incentive mechanism $M(f, p)$, including an online selection mechanism $f$ and a payment mechanism $p$. Suppose the selected workers form the set $S$. Define the selected time step of the worker $i$ as $t_i = t$, if the worker $i$ is selected at the $t$-th time step. The utility $u_i$ of the worker $i$ is

$$u_i = \begin{cases} 0, & i \notin S, \\ p_i - c_i \cdot (T - t_i + 1), & i \in S. \end{cases}$$

Since the budget of the task publisher is limited, $\sum_{i \in S} p_i \leq B$ needs to be met. The task publisher hopes to select as many high-quality workers as possible with the limited budget. Since reputation indirectly reflects the quality and reliability of workers, the utility $U$ of the publisher is

$$U = \sum_{i \in S} R_{ei}(T - t_i + 1).$$

The online mechanism $M$ is designed to maximize the utility $U$ of the publisher by determining the set $S$ with the budget constraint. At the same time, the mechanism needs to meet the following six critical economic properties.
• **Individual Rationality**: The worker cannot be paid less than his true cost.

• **Budget Feasibility**: The sum of the rewards paid by the task publisher to the workers cannot exceed the budget.

• **Computational Efficiency**: The time complexity of the mechanism is polynomial.

• **Consumer Sovereignty**: The publisher cannot arbitrarily exclude any worker. As long as their bids are low enough, workers have chances to be selected and paid.

• **Cost Truthfulness with Sufficient Budget**: When the remaining budget is sufficient, the worker can maximize his utility by submitting his true cost.

• **Time Truthfulness with Sufficient Budget**: When the remaining budget is sufficient, the worker can maximize his utility by submitting his true arrival time.

### 4 Online Incentive Mechanism Design

Designing an online incentive mechanism for federated learning requires overcoming many challenges. The worker may lie to the publisher about his arrival time and cost for greater benefit. Thus, the mechanism should be designed to motivate workers to report true information, which helps the publisher make decisions. Moreover, the total rewards paid to workers cannot exceed the budget. Furthermore, the mechanism needs to be applied to the scenario where workers arrive online in different orders. In the offline scenario, the publisher has collected bids submitted by all workers, so it is easy to make a selection decision. In the online scenario, the publisher cannot collect bids from all workers in advance but only receives online bids from different workers before or during the task. With incomplete information, it is difficult to make a decision on whether to select the arrived worker or not.

In existing incentive mechanisms for federated learning or others [Deng et al., 2021], a certain indicator of selected workers will not exceed a payment density threshold. For example, in RRAFL [Zhang et al., 2021] the payment density threshold is \( \frac{b_{k+1}}{R_{k+1}} \), while in the proportional share mechanism [Singer, 2010] it is \( \min(\frac{\sum_{i \in U' \cap R_0} b_i}{\sum_{i \in U' \cap R_0} R_i}, \frac{b_{k+1}}{R_{k+1}}) \). Inspired by this, in the online scenario, the publisher can make decisions by learning a payment density threshold.

A two-stage sampling-accepting process is utilized to learn the payment density threshold [Babaioff et al., 2008]. The first stage rejects workers and only collects their bids as samples to learn the threshold. The second stage leverages this threshold for worker selection. However, this mechanism does not satisfy consumer sovereignty since the workers who bid in the first stage cannot win no matter how they bid. Workers are more inclined to delay arrivals, which will lead to task starvation. In addition, since the threshold is only learned using samples from the first stage and not updated in later stages, it may lack accuracy.

Zhao et al. [2014] proposes a multi-stage sampling-accepting process to solve the above problem. The task is divided into multiple stages. At each stage, bids from workers who have left in previous stages are added to the sample set. The payment density threshold is updated by dynamically increasing the sample set size \( U' \) and the sample budget \( B' \). This approach satisfies consumer sovereignty without causing task starvation while making the threshold more accurate by dynamically updating. However, we assume that once the worker submits a bid, whether he is selected or not, he will stay until the end of the task. There is no way to update the threshold by adding departing workers to the sample set as above. Therefore, this approach cannot be applied directly.

In federated learning, the workers selected at different stages participate in different numbers of global iterations. Only a sufficient number of workers are selected, can the FL task start. In the crowdsensing task, workers selected in different stages perform the same independent task. Their workload is the same. Moreover, workers can work immediately when they are selected. The incentive mechanism for crowdsensing cannot be directly applied to federated learning.

We propose an online incentive mechanism \( M(f, p) \) according to the nature of FL and design a method to learn the payment density threshold. To obtain a higher-quality model with a limited budget \( B \), on the one hand, the publisher selects more workers with lower bid prices \( b_i \), and on the other hand, selects higher-quality workers with higher reputation \( R_0 \). Balancing the worker’s bid price \( b_j \) and the reputation \( R_0 \), define the cost density \( \rho_i \) of worker \( i \) as

\[
\rho_i = \frac{b_j}{R_0}.
\]

We naturally use the offline proportional share mechanism \( M(f, p) \) according to the nature of FL and design a method to learn the payment density threshold based on the sample set and sample budget, as shown in Algorithm 1. Algorithm 1 adopts a greedy strategy, which first arranges the workers in the sample set \( U' \) in the order of increasing cost density, that is \( \rho_1 \leq \ldots \leq \rho_k \leq \rho_{k+1} \leq \ldots \leq \rho_{|U'|} \). According to the proportional share allocation rule, we find the last worker \( k \) in the sequence that satisfies \( \rho_k \leq \frac{B'}{\sum_{i \in U'} R_i} \) from front to back. The first \( k \) workers in the sequence form the set \( S' \). The learned payment density threshold is \( \rho^* = \min(\frac{\sum_{k \in S'} R_i}{\sum_{k \in S'} b_{k+1}}) \).

We design the online selection mechanism \( f \) and payment mechanism \( p \) using a modified multi-stage sampling-accepting process, as shown in Algorithm 3. Federated learning consists of multiple rounds (\( T \) rounds) of global iterations, which correspond to multiple stages (multiple time

| Algorithm 1 GetPaymentDensityThreshold |
|-----------------------------------------|
| **Input**: Sample budget \( B' \); Sample worker set \( U' \); |
| **Output**: Density threshold \( \rho^* \); |
| 1: Sort the workers in \( U' \) such that \( \frac{b_1}{R_0} \leq \ldots \leq \frac{b_{|U'|}}{R_0} \); |
| 2: \( S' = \phi; i = 1 \); |
| 3: while \( \frac{b_i}{R_0} \leq \frac{B'}{\sum_{j \in S'} R_j} \) do |
| 4: \( S' = S' \cup \{i\}; i = i + 1 \); |
| 5: end while |
| 6: \( k = i - 1 \); |
| 7: \( \rho^* = \min(\frac{B'}{\sum_{j \in S'} R_j} \cdot \frac{b_{k+1}}{R_{k+1}}) \); |
| 8: Return \( \rho^* \); |
Algorithm 2 SelectWorkersFromGroup
Input: Worker set $U$; Payment density threshold $\rho^*$; Winner Worker set $S$.

1: Sort all workers in $U$ such that $Re_1 \leq \ldots \leq Re_{|U|}$;
2: $i = 1$;
3: while $i \leq |U|$ do
4: $\rho_i = \frac{b_i}{Re_i}$;
5: if $\rho_i \leq \rho^*$ then
6: if $i \notin S$ and $(T - t + 1) \cdot Re_i \cdot \rho^* \leq \frac{B}{T} - \sum_{j \in U} p_j$ then
7: $S = S \cup \{i\}$; $p_i = (T - t + 1) Re_i \cdot \rho^*$; $\rho_i^* = \rho^*$;
8: else if $i \in S$ then
9: $p_i = p_i + (\rho^* - \rho_i^*) \cdot Re_i \cdot (T - t + 1)$;
10: if $p_i > p_i^*$ then
11: $p_i = p_i^*$;
12: $\rho_i = \rho_i^*$;
13: end if
14: end if
15: end if
16: $i = i + 1$;
17: end while

Algorithm 3 Online Selection And Payment Mechanism
Input: Budget $B$; Total rounds of global iterations $T$; First-round budget ratio $ratio$;

1: /*Before the task*/
2: $B_1 = B \cdot ratio$; $t = 1$; $U_1 = \phi$; $U_2 = \phi$;
3: while The scheduled start time has not been reached do
4: Worker $i$ arrives; $p_i = 0$;
5: Add worker $i$ to $U_1$ or $U_2$ according to his $gid_i$;
6: end while
7: /*Select workers at the first time step through an offline reverse auction.*/
8: $U = U_1 \cup U_2$; $S = \phi$; $i = 1$;
9: Sort the workers in $U$ such that $\frac{b_i}{Re_i} \leq \ldots \leq \frac{b_{|U|}}{Re_{|U|}}$;
10: while $\frac{T \cdot b_1}{Re_1} \leq \frac{B_1}{Re_1 + \sum_{j \in U} Re_j}$ do
11: $S = S \cup \{i\}$; $i = i + 1$;
12: end while
13: $k = i - 1$; $\rho^* = \min(\frac{\rho_1}{\sum_{j=2}^{k} Re_j}; \frac{\rho_{k+1}}{\sum_{j=k+2}^{n} Re_j})$;
14: for each worker $i \in S$ do
15: $p_i = T \cdot Re_i \cdot \rho^*$; $\rho_i^* = \rho^*$;
16: end for
17: /*Select workers at other time steps through an online reverse auction.*/
18: $t = 2$;
19: while $t \leq T$ do
20: Add each newly worker $i$ arrived in $(t - 1, t]$ to $U_1$ or $U_2$ according to his $gid_i$, and set $p_i = 0$;
21: $B^t = (B_1 + \frac{(B - B_1)(t - 1)}{T - 1}) / T$;
22: $\rho_U^t = GetPaymentDensityThreshold(\frac{B^t}{T}, U_1)$;
23: $\rho_U^t = GetPaymentDensityThreshold(\frac{B^t}{T}, U_2)$;
24: SelectWorkersFromGroup($U_1, \rho_U^t, S$);
25: SelectWorkersFromGroup($U_2, \rho_U^t, S$);
26: $t = t + 1$;
27: end while

Unselected workers are paid 0 and wait for the next worker selection process at the next time step. The reward for the selected worker $i \in S$ is temporarily $p_i = T \cdot Re_i \cdot \rho^*$. The maximum payment density threshold he encounters after being selected is $\rho_i^* = \rho^*$. At other time steps $t > 1$, all newly arrived workers join the corresponding group, and their rewards are initialized to 0. First, update the sample budget $B^t = (B_1 + \frac{(B - B_1)(t - 1)}{T - 1}) / T$. Then, with $U_1$ and $U_2$ as the sample set respectively, and $\frac{B^t}{T}$ as the sample budget, compute the payment density threshold $\rho_U^t$. Then, $U_2$ through Algorithm 1. $\rho_U^t$ and $\rho_U^t$ are used to make decisions on workers with higher reputation. There are both unselected and selected workers in $U_1$. If the unselected worker $i$ satisfies $p_i \leq \rho_U^t$, and $\rho_i^* = \rho_U^t$. If the selected worker $i$ satisfies $p_i \leq \rho_U^t$ and $\rho_i^* < \rho_U^t$, and $U_1$ has remaining budget, then $p_i = \rho_U^t$.

steps). Since only a sufficient number of workers are selected can the FL task start, we design different worker selection methods for the first time step and other time steps, respectively. The first-round budget ratio is ratio, and the first-round budget is $B_1 = B \cdot ratio$. The task publisher sets up two arrived worker groups $U_1$ and $U_2$, which is used as the sample set for each other to calculate the payment density threshold. A worker’s bid does not affect the payment density threshold used to decide on him, thus ensuring cost truthfulness. The result of id,%2 is as gid i of worker i, which determines which group to join. Workers cannot join other groups by modifying their bids. Because of the randomness of the hash value, the distribution of bids in the two groups is similar, thus ensuring the validity of the payment density threshold.

At the first time step $t = 1$, once the scheduled start time is reached, the task publisher selects workers through the offline proportional share mechanism. First arranges the workers in the arrived workers set $U$ in the order of increasing cost density, that is

$$\rho_1 \leq \ldots \leq \rho_k \leq \rho_{k+1} \leq \ldots \rho_{|U|}. \quad (4)$$

Then, find the last worker $k$ in the worker sequence that satisfies $T \rho_k \leq \frac{B_1}{\sum_{j=1}^{k} Re_j}$. If the number of workers selected is lower than the minimum number to start the first global iteration, delay the start time and repeat the above process. Otherwise, the first $k$ workers in the sequence form the winning worker set $S$ to participate in all global iterations. According to the worker selection result of the first time step $t = 1$, the payment density threshold $\rho^*$ is

$$\rho^* = \min(\frac{B_1}{T \sum_{j \in S} Re_j}; \frac{b_{k+1}}{Re_{k+1}}). \quad (5)$$
\begin{align*}
\min(p_i + (\rho U_t - \rho^*_t) \cdot R e_i, (T - t + 1) \cdot \frac{B}{T} - \sum_{j \in U_t} p_j + p_i) \quad \text{and} \quad \rho^*_t = \rho U_t. \quad \text{Do the same for } U_2 \text{ as } U_1, \text{ using } \rho U_1 \text{ instead of } \rho U_2. \quad \text{Workers in } S \text{ start the next global iteration.}
\end{align*}

5 Theoretical Analysis

We will prove that our mechanism satisfies individual rationality, budget feasibility, computational efficiency, consumer sovereignty, cost truthfulness and time truthfulness with a sufficient budget.

Theorem 1. The mechanism satisfies individual rationality.

Proof. If worker \( i \) is selected at \( t = 1 \), indicating \( b_i \leq \min(\sum_{j \in S} R e_j \cdot B_{b_i} / R e_{b_i, +1}) \cdot R e_i / T \), his temporary reward is \( p_i = \min(\sum_{j \in S} R e_j \cdot B_{b_i} / R e_{b_i, +1}) \cdot R e_i / T \cdot b_i \). If worker \( i \) is selected at \( t > 1 \), indicating \( b_i \leq \rho^* \), his temporary reward is \( p_i = \rho^* \cdot R e_i / T \cdot (T - t + 1) \geq \sum_{j \in S} R e_j \cdot B_{b_i} / R e_{b_i, +1} \cdot R e_j \). The final reward of the selected worker is not lower than the temporary reward, indicating \( u_i \geq 0 \). The utility of unselected workers is \( u_i = 0 \).

Theorem 2. The mechanism satisfies budget feasibility.

Proof. At \( t = 1 \), the total temporary reward is \( \sum_{i \in S} R e_i \cdot \min(\sum_{j \in S} B_{b_i} / R e_{b_i, +1}) \cdot \frac{B_{b_i}}{R e_{b_i, +1}} \leq \sum_{i \in S} R e_i \cdot \min(\sum_{j \in S} B_{b_i} / R e_{b_i, +1}) = B_1. \) At \( t > 1 \), lines 24 and 25 of Algorithm 3 ensure that the total reward of workers selected from \( U_1 \) and \( U_2 \) does not exceed \( \frac{B}{T} \) respectively, which means the total reward does not exceed \( B \).

Theorem 3. The mechanism satisfies computational efficiency.

Proof. For Algorithm 1, the time complexity of sorting the workers in \( U' \) (line 1) is \( O(|U'| \log_2 |U'|) \) and that of selecting workers from \( U' \) (lines 3-5) is \( O(|U'|) \). Therefore, the time complexity of Algorithm 1 is \( O(|U| \log_2 |U'|). \) For Algorithm 2, the time complexity of sorting the workers (line 1) is \( O(|U| \log_2 |U'|) \) and that of selecting or updating the payment (lines 3-19) is \( O(|U|) \). Therefore the time complexity of Algorithm 2 is \( O(|U| \log_2 |U'|). \) Suppose the number of workers bidding during \((t - 1, t)\) is \( n_t \). At \( t = 1 \), the time complexity of sorting the workers (line 9) is \( O(n_1 \log_2 n_1) \), that of selecting workers (lines 10-12) is \( O(|S_1|) \) and that of calculating the temporary reward (lines 14-16) is \( O(n_1) \). At \( t > 1 \), the time complexity of computing the payment density thresholds for \( U_1 \) and \( U_2 \) (lines 22 and 23) is \( O((\sum_{j = 1}^{t} n_j) \log_2 (\sum_{j = 1}^{t} n_j)) \) and that of making a decision on \( U_1 \) and \( U_2 \) is \( O((\sum_{j = 1}^{t} n_j) \log_2 (\sum_{j = 1}^{t} n_j)). \) The time complexity of Algorithm 3 is \( O((\sum_{i = 1}^{T} ((\sum_{j = 1}^{t} n_j) \log_2 (\sum_{j = 1}^{t} n_j)))) \).

Theorem 4. The mechanism satisfies consumer sovereignty.

Proof. At each time step, as long as the worker’s bid is low enough and the remaining budget is sufficient, that worker is selected and paid. The task publisher does not automatically reject either worker. Therefore, the mechanism satisfies consumer sovereignty.

Theorem 5. The mechanism satisfies cost truthfulness with a sufficient budget.

Proof. Consider the scenario where the worker wins with \( c_i \).

Case 1: If worker \( i \) with \( c_i \) is selected at \( t \geq a_i \) and he with \( b_i \) is still selected at \( t \), then \( u(b_i, b_{-i}) = u(c_i, b_{-i}). \)

Case 2: Suppose worker \( i \) with \( c_i \) is selected at \( t > a_i \). If he with \( b_i > c_i \) is selected at \( t' < t \), then he will get extra reward \( R e_i \cdot \rho^* \) at each time step during \([t', t)\) \( (\rho^* \text{ may change at different time steps}). \) Worker \( i \) with \( c_i \) is not selected in \([t', t)\), indicating \( \frac{u_i}{R e_i} > \rho^* \), that is, \( c_i > R e_i \cdot \rho^* \). This means that the utility of worker \( i \) with \( b_i \) is negative during \([t', t)\). Thus \( u(b_i, b_{-i}) < u(c_i, b_{-i}) \).

Case 3: If worker \( i \) with \( b_i \) does not win, then \( u(c_i, b_{-i}) > u(b_i, b_{-i}) \).

Case 4: Suppose worker \( i \) with \( c_i \) is selected at \( t \) and he with \( b_i > c_i \) is selected at \( t' > t \). Worker \( i \) with \( c_i \) will get extra reward \( R e_i \cdot \rho^* \) at each time step during \([t, t')\) \( (\rho^* \text{ may change at different time steps}). \) Worker \( i \) with \( c_i \) is not selected, indicating \( \frac{u_i}{R e_i} > \rho^* \), that is, \( c_i > R e_i \cdot \rho^* \). This means that the utility of worker \( i \) with \( c_i \) is not negative during \([t, t')\). Thus \( u(b_i, b_{-i}) \leq u(c_i, b_{-i}). \)

In summary, the mechanism satisfies cost truthfulness.

Theorem 6. The mechanism satisfies time truthfulness with a sufficient budget.

Proof. If a worker is selected at time step \( t \), he will be paid a maximum reward that can be earned during \([t, T]\). If reporting the true arrival time \( a_i \) makes worker \( i \) to be selected at time step \( t \), and reporting a later arrival time \( \tilde{a}_i \) makes him to be selected at time step \( \tilde{t} \), then \( t \leq \tilde{t} \). Reporting \( a_i \) allows worker \( i \) to receive an extra reward during \([t, t')\), but reporting \( \tilde{a}_i \) does not. As long as \((a_i, c_i)\) are submitted, the utility of worker \( i \) is greater than that of reporting \( \tilde{a}_i \). Therefore, the mechanism satisfies time truthfulness with a sufficient budget.

6 Experiments

We use the MNIST dataset with a two-layer fully connected model which has a hidden layer of 50 cells, and the Fashion MNIST dataset with a LeNet model for experiments. The task publisher has a validation set and a test set of size 5000 each. Each worker has a training set of size 1000. The dataset is iid, but its accuracy may not be the same, which is achieved by modifying the labels to others. Workers train the model for 1 epoch with a learning rate of 0.05 and a batch size of 128 at each global iteration. Each task contains 10 global iterations. We set the minimum number of workers to start a task to be 1, and the first-round budget ratio to be 0.35.
We set up multiple benchmarks. The first, Fixed Threshold, is online using a fixed threshold $\rho^2 = 0.75$. The second is RRAFL, proposed by Zhang et al. [2021]. The third, Vanilla FL, randomly selects workers as long as the budget remains. The fourth, Bid Greedy, prefers workers with low bids. The fifth is Proportional Share, proposed by Singer et al. [2010]. The sixth is Approximate Optimal, where the task publisher has prior information on all workers, such as $c_i$. The above five are offline mechanisms.

First, we set up 100 workers, whose accumulated reputation $Re_i$, bid price $b_i$ and internal reputation $re_i$ are randomly generated from $[0, 1], \frac{1}{3} Re_i + \frac{1}{15}, \frac{1}{3} Re_i + \frac{1}{15}, \max(0, Re_i - 0.1), \min(1, Re_i + 0.1)$, respectively, and the probability of arriving at each time step $t$ is $\frac{1/t}{\sum_{i=1}^{T} 1/t}$. Figure 2 shows the effect of budget on the unit payment utility of the task publisher. Figure 3 shows the impact of the number of workers when the budget is 125. The offline Approx. Optimal mechanism outperforms ours because of holding all prior information. Our mechanism achieves cost and time truthfulness by sacrificing utility, but its result is still close to the offline RRAFL and Proportional Share mechanisms, and significantly outperforms other benchmarks. We can infer that our mechanism can improve the unit payment utility of the task publisher as much as possible in the online scenario.

Then we up set 30 workers. 15, 5, 5, and 5 workers are with data accuracy $dacc$ of 1.0, 0.7, 0.4, and 0.1, respectively. Their bids are generated from $[\frac{1}{3} dacc + \frac{1}{15}, \frac{1}{3} dacc + \frac{1}{15}]$. Run 70 tasks for the MNIST dataset and Fashion MNIST dataset respectively, and use the last 65 tasks to evaluate the mechanism. The budget in each task is 80. From Table 1, we can observe that the proportion of workers with a data accuracy of 1.0 selected by our mechanism remains high whether it is the MNIST task or the Fashion MNIST task. We can infer that our mechanism can help select high-quality workers in online scenarios.

### Table 1: Proportion of workers with a data accuracy of 1.0 among the selected workers

| Mechanism     | MNIST | Fashion MNIST |
|---------------|-------|---------------|
| Our Mechanism | 1.0000| 0.9411        |
| Fixed Threshold | 1.0000| 0.9175        |
| RRAFL         | 1.0000| 0.8956        |
| Vanilla FL    | 0.4770| 0.4770        |
| Bid Greedy    | 0.3296| 0.3296        |

Table 2 illustrates that the average loss of the global model in our mechanism is close to the offline mechanism with the best results, and substantially lower than that in the offline Vanilla FL and Bid Greedy mechanisms. This suggests that our mechanism can help to improve the model quality in the online scenarios as in the offline scenario.

### Table 2: Average loss of global models with different mechanisms

| Mechanism     | MNIST | Fashion MNIST |
|---------------|-------|---------------|
| Our Mechanism | 0.4393| 1.1162        |
| Fixed Threshold | 0.4993| 1.4971        |
| RRAFL         | 0.4389| 1.1376        |
| Vanilla FL    | 0.5022| 1.4566        |
| Bid Greedy    | 0.5452| 1.6925        |

7 Conclusion

We designed an online auction-based incentive mechanism for horizontal federated learning to help the task publisher select and pay workers who arrive one by one online. The task is divided into multiple time steps. At the first time step, workers are selected through the proportional share mechanism. At other time steps, two worker sets are mutually used as sample sets to calculate the payment density thresholds, which are used to make decisions on the two worker sets respectively. Finally, theoretical analysis proves that our online mechanism satisfies six economic properties. The experimental results show its effectiveness.
References

[Babaioff et al., 2008] Moshe Babaioff, Nicole Immorlica, David Kempe, and Robert Kleinberg. Online auctions and generalized secretary problems. *ACM SIGecom Exchanges*, 7(2):1–11, 2008.

[Bonawitz et al., 2019] Keith Bonawitz, Hubert Eichner, Wolfgang Grieskamp, Dzmitry Huba, Alex Ingerman, Vladimir Ivanov, Chloe Kiddon, Jakub Konečný, Stefano Mazzocchi, H Brendan McMahan, et al. Towards federated learning at scale: System design. *arXiv preprint arXiv:1902.01046*, 2019.

[Deng et al., 2021] Yongheng Deng, Feng Lyu, Ju Ren, Yichao Chen, Peng Yang, Yuezhi Zhou, and Yaoxue Zhang. Fair: Quality-aware federated learning with precise user incentive and model aggregation. In *IEEE INFOCOM 2021-IEEE Conference on Computer Communications*, pages 1–10. IEEE, 2021.

[Ding et al., 2020] Ningning Ding, Zhixuan Fang, and Jianwei Huang. Incentive mechanism design for federated learning with multi-dimensional private information. In *2020 18th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOPT)*, pages 1–8. IEEE, 2020.

[Feng et al., 2019] Shaohan Feng, Dusit Niyato, Ping Wang, Dong In Kim, and Ying-Chang Liang. Joint service pricing and cooperative relay communication for federated learning. In *2019 International Conference on Internet of Things (iThings) and IEEE Green Computing and Communications (GreenCom) and IEEE Cyber, Physical and Social Computing (CPSCom) and IEEE Smart Data (SmartData)*, pages 815–820. IEEE, 2019.

[Jiao et al., 2020] Yutao Jiao, Ping Wang, Dusit Niyato, Bin Lin, and Dong In Kim. Toward an automated auction framework for wireless federated learning services market. *IEEE Transactions on Mobile Computing*, 2020.

[Le et al., 2021] Tra Huong Thi Le, Nguyen H Tran, Yan Kyaw Tun, Minh NH Nguyen, Shashi Raj Pandey, Zhu Han, and Choong Seon Hong. An incentive mechanism for federated learning in wireless cellular network: An auction approach. *IEEE Transactions on Wireless Communications*, 2021.

[Lim et al., 2021] Wei Yang Bryan Lim, Jianqiang Huang, Zehui Xiong, Jiawen Kang, Dusit Niyato, Xian-Sheng Hua, Cyril Leung, and Chunyan Miao. Towards federated learning in uav-enabled internet of vehicles: A multi-dimensional contract-matching approach. *IEEE Transactions on Intelligent Transportation Systems*, 2021.

[Pandey et al., 2020] Shashi Raj Pandey, Nguyen H Tran, Mehdi Bennis, Yan Kyaw Tun, Aunas Manzoor, and Choong Seon Hong. A crowdsourcing framework for on-device federated learning. *IEEE Transactions on Wireless Communications*, 19(5):3241–3256, 2020.

[Roy et al., 2021] Palash Roy, Sujan Sarker, Md Abdur Razzaque, Md Mamun-or Rashid, Mohmmad Mehedi Hassan, and Giancarlo Fortino. Distributed task allocation in mobile device cloud exploiting federated learning and subjective logic. *Journal of Systems Architecture*, 113:101972, 2021.

[Seo et al., 2021] Eunil Seo, Dusit Niyato, and Erik Elmroth. Auction-based federated learning using software-defined networking for resource efficiency. In 2021 *17th International Conference on Network and Service Management (CNSM)*, pages 42–48. IEEE, 2021.

[Singer, 2010] Yaron Singer. Budget feasible mechanisms. In *2010 IEEE 51st Annual Symposium on foundations of computer science*, pages 765–774. IEEE, 2010.

[Yang et al., 2019] Qiang Yang, Yang Liu, Tianjian Chen, and Yongxin Tong. Federated machine learning: Concept and applications. *ACM Transactions on Intelligent Systems and Technology (TIST)*, 10(2):1–19, 2019.

[Ying et al., 2020] Chenhao Ying, Haiming Jin, Xudong Wang, and Yuan Luo. Double insurance: Incentivized federated learning with differential privacy in mobile crowdsensing. In *2020 International Symposium on Reliable Distributed Systems (SRDS)*, pages 81–90. IEEE, 2020.

[Yu et al., 2020] Han Yu, Zelei Liu, Yang Liu, Tianjian Chen, Mingshu Cong, Xi Weng, Dusit Niyato, and Qiang Yang. A fairness-aware incentive scheme for federated learning. In *Proceedings of the AAAI/ACM Conference on AI, Ethics, and Society*, pages 393–399, 2020.

[Zeng et al., 2020] Rongfei Zeng, Shixun Zhang, Jiaqi Wang, and Xiaowen Chu. Fmore: An incentive scheme of multi-dimensional auction for federated learning in mec. In *2020 IEEE 40th International Conference on Distributed Computing Systems (ICDCS)*, pages 278–288. IEEE, 2020.

[Zhang et al., 2021] Jingwen Zhang, Yuezhou Wu, and Rong Pan. Incentive mechanism for horizontal federated learning based on reputation and reverse auction. In *Proceedings of the Web Conference 2021*, pages 947–956, 2021.

[Zhang et al., 2022] Jingwen Zhang, Yuezhou Wu, and Rong Pan. Auction-based ex-post-payment incentive mechanism design for horizontal federated learning with reputation and contribution measurement. *arXiv preprint arXiv:2201.02410*, 2022.

[Zhao et al., 2014] Dong Zhao, Xiang-Yang Li, and Huadong Ma. How to crowdsource tasks truthfully without sacrificing utility: Online incentive mechanisms with budget constraint. In *IEEE INFOCOM 2014-IEEE Conference on Computer Communications*, pages 1213–1221. IEEE, 2014.

[Zhou et al., 2021] Ruiting Zhou, Jinlong Pang, Zhibo Wang, John CS Lui, and Zongpeng Li. A truthful procurement auction for incentivizing heterogeneous clients in federated learning. In *2021 IEEE 41st International Conference on Distributed Computing Systems (ICDCS)*, pages 183–193. IEEE, 2021.