Quantum fluctuations in the shape of exotic nuclei

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(Dated: September 9, 2009)

Quantum fluctuations concerning the shape of nuclei are treated within the framework of covariant density functional theory. Long range correlations beyond mean field are taken into account by configuration mixing of wave functions with triaxial shapes and the restoration of spontaneously broken rotational symmetries through three-dimensional angular momentum projection. The controversial nucleus $^{16}$C is treated as an example and it is found that its ground state has a triaxial shape but with large shape fluctuations. They are of crucial importance for a proper description of the spectroscopic properties of such nuclei.

PACS numbers: 21.10.Dr, 21.10.Re, 21.30.Fe, 21.60.Jz, 27.20.+n

Atomic nuclei are highly correlated and strongly interacting many-body systems of mesoscopic character. On the one side they are so small that quantum effects are dominant, on the other the number of their constituents is finite and large enough so that classical concepts such as their shape and orientation play an important role for an understanding of their structure. Only in heavy nuclei are the deformation parameters well defined quantities. In transitional nuclei, and in light systems, quantum fluctuations have to be taken into account.

The rapid development in radioactive nuclear beam facilities and gamma ray detectors have in recent years allowed one to study exotic nuclei far from stability and many new phenomena have been observed and predicted in this context. In order to investigate the shape degrees of freedom it is not sufficient to only study the ground state properties of these nuclei, one also needs information about the spectroscopy of the low-lying excited states. Therefore, presently much interest is focused on the measurement of the energies of the first $2^+$, or $4^+$ states and of the reduced transition probabilities ($B(E2)$-values) from the first $2^+ (2_1^+)$ to the ground state ($0_1^+$)\textsuperscript{21}. These are fundamental quantities which reveal rich information about nuclear shapes and shell structure.

Recently, the structure of the nucleus $^{16}$C having two neutron particles and two proton holes above the doubly magic nucleus $^{16}$O has become a very interesting and challenging topic. In lifetime measurements using the recoil shadow method (RSM)\textsuperscript{22} an anomalously hindered $B(E2 : 2_1^+ \to 0_1^+)$ value of 2.6(9) e$^2$fm$^4$ was found, contradicting the low excitation energy of E($2_1^+$) = 1.766 MeV (see Ref.\textsuperscript{22} for an earlier measurement). Furthermore, inelastic proton scattering\textsuperscript{4,12} indicated a large quadrupole deformation $\beta_{20} \sim$ 0.47(5). The quenched $B(E2)$ value, combined with the large nuclear deformation, led to the suggestion that the neutron motion plays a predominant role in the $2_1^+$ state of $^{16}$C\textsuperscript{4,12}.

Although the structure of $^{16}$C has not yet been fully understood. The more recent lifetime measurement of the $2_1^+$ state using the recoil distance method (RDM) after a fusion-evaporation reaction gave a similar, but slightly larger, $B(E2)$ value of 4.15(73) e$^2$fm$^4$\textsuperscript{23}.

On the theoretical side, very different models have been used to describe the quenched $B(E2)$ value in $^{16}$C, including antisymmetrized molecular dynamics (AMD)\textsuperscript{8}, three-body models\textsuperscript{9,10,11} and the shell model\textsuperscript{12}. In AMD, the unusually small $B(E2)$ value derived from the lifetime measurement with RSM was interpreted as the coexistence of an oblate proton and a prolate neutron shape. In the three-body models or in the shell model, a careful adjustment of an effective charge or modifications of the Hamiltonian were required to reproduce the data.

The self-consistent mean field approach derived from a global density functional theory (DFT) provides a vivid way to study macroscopically defined quantities, e.g. ground state energy, nuclear radius and deformation. Exotic shapes of $^{16}$C have been investigated using non-relativistic and relativistic DFT calculations, constrained by deformation parameters\textsuperscript{13,14}. However, a very flat energy surface has been found both on the prolate and oblate side indicating that this is a transitional nucleus. Therefore, substantial effects from configuration mixing connected with shape fluctuations are expected which play an important role for quantities of a quantum nature, e.g., discrete energy spectra and transitions probabilities\textsuperscript{15}. In order to understand such matrix elements, e.g. $B(E2)$ values, a microscopic approach going beyond the mean-field level is required which is able to treat shape quantum fluctuations properly.

In the framework of DFT, shape fluctuations and angular momentum projection have been treated using non-relativistic density functionals of Skyrme\textsuperscript{16,17} and Gogny\textsuperscript{18}, as well as covariant functionals\textsuperscript{19}. In all of these cases, however, intrinsic triaxiality was neglected.
which will be found to be crucial for a reproduction of the data. Only very recently, a mixing of all five quadrupole degrees of freedom has been attempted within the context of triaxial Skyrme calculations.

The present investigation is based on a similar idea using covariant density functional theory (CDFT). Based on Lorentz invariance, this method connects the spin and spatial degrees of freedom of the nucleus in a consistent way. Numerous investigations have shown that the experimental data for the ground and excited states can be nicely interpreted within a relativistic framework. Recently, we have developed three-dimensional angular momentum projection (3DAMP) for relativistic point coupling models to incorporate correlations related to the restoration of broken rotational symmetries. This concept has now been further extended to include fluctuations for triaxial deformations within the framework of the Generator Coordinate Method (GCM).

In this letter, we use this method to study the effects of quantum fluctuation for triaxial shapes in the controversial nucleus $^{16}$C. It turns out that the ground state of $^{16}$C corresponds to a triaxial shape having large shape fluctuations along the $\gamma$ degree of freedom, which describes the triaxiality. Such a new picture for the shape of $^{16}$C is essential to reproduce quantitatively the experimental $B(E2 : 2^+_1 \rightarrow 0^+_3)$ value of Ref. [7] that is consistent with the results from inelastic $(p,p')$ scattering [5].

The set of deformed intrinsic wave functions $|q\rangle$ with the quadrupole deformations in the Hill-Wheeler coordinates $q = (\beta, \gamma)$ [15] is generated by constrained RMF+BCS calculations using the parameter set PC-F1 of Ref. [22]. Further details are given in Ref. [21]. In particular, the strength parameters of the zero-range pairing force are $V_n = 308$ and $V_p = 321$ MeV-fm$^3$.

In the left panel of Fig. 1 we plot the potential energy surface (PES) in $\beta$-$\gamma$ plane obtained with these mean field calculations. This PES is rather soft, which is consistent with the results found in Ref. [14]. Of course, for all the non-spherical points with $\beta \neq 0$ the intrinsic wave functions $|\beta \neq 0, \gamma\rangle$ have a certain orientation and rotational symmetry is spontaneously broken. To restore this symmetry, 3DAMP [21] is carried out and the right panel of Fig. 1 shows the projected PES for $J^\pi = 0^+$. Of particular interest is the observation of an obvious triaxial minimum with $\beta = 0.6, \gamma \sim 20^\circ$. To demonstrate the shapes at the various points in the ($\beta$-$\gamma$)-plane, in Fig. 2 we plot the corresponding intrinsic density distributions for neutrons and protons in the $y$-$z$ plane integrated over the $x$-coordinate. Fig. 2a refers to the axially symmetric minimum in the unprojected PES of Fig. 1a and Fig. 2b corresponds to the minimum in the projected PES of Fig. 1b. Here we observe rather different deformation parameters for neutrons ($\beta_n = 0.69, \gamma_n = 15.48^\circ$) and protons ($\beta_p = 0.46, \gamma_p = 31.47^\circ$), which shows us another kind of decoupled structure for the density distribution of neutron and proton, different to that indicated in Ref. [8]. A careful analysis shows that the difference between neutron and proton deformation is due to the special $2p-2h$ configurations $\nu(1d_{5/2})^2 \otimes \pi(1p_{3/2})^{-2}$. For large prolate deformations ($\beta > 1.2$), the deformation driving orbit $1d_{5/2}$ dives into the Fermi sea of protons as well. In this case, neutrons and protons have almost the same deformation and the decoupled structure ceases.

Although there is an obvious minimum in the projected PES shown in Fig. 1 this minimum is still very soft along the $\gamma$ direction. One has, therefore, to allow for superpositions of wave functions with different deformations and to perform a GCM calculation in the full $\beta$-$\gamma$ plane (for details see Ref. [21]).

$$|\Psi_M^J\rangle = \sum_K d^2 q f_{K}(q) P_{MK}^{JM}|q\rangle$$

including correlations due to restoration of broken symmetries and fluctuations of the deformation coordinates.

To illustrate the importance of shape fluctuations in the $\gamma$ degree of freedom, in a first step, we restrict ourselves to axially symmetric shapes and choose the generator coordinates as $\beta = 0, 0.1, \cdots, 1.5$ for $\gamma = 0^\circ$, and $\gamma = 180^\circ$. As the result, the full 3DAMP+GCM calculation is simplified into a 1DAMP+GCM calculation and all components with $K \neq 0$ vanish. In Fig. 3a, we plot energies and average quadrupole moments of the lowest GCM states for each angular momentum $0^+, 2^+, 4^+$ together with the mean-field PES and the projected PES as a function of $\beta$. The corresponding probability distributions $|g_\beta^J(\beta)|^2$ are shown in Fig. 3b, where $g_\beta^J(\beta)$ is the solution of the Hill-Wheeler-Griffin equation in the "natural basis" (for details see Ref. [13]).

Fig. 3 shows two minima on the projected PES with $J^\pi = 0^+$, one on the prolate and one on the oblate side having similar probabilities. Therefore, 1DAMP+GCM calculations lead to a nearly vanishing quadrupole moment for the ground state with a rather small $B(E2 : 2^+_1 \rightarrow 0^+_3) = 0.11e^2$fm$^4$. According to Fig. 1 it is es-

![FIG. 1: (Color online) Potential energy surfaces in $\beta$-$\gamma$ plane for the nucleus $^{16}$C obtained by triaxial relativistic mean-field calculations (left panel) and with projection onto angular momentum $J = 0$ after the variation (right panel). The energy gap between two neighbor contour lines is 0.5 MeV.](image-url)
Intrinsic density distributions of protons and neutrons in the y-z plane (the x-axis has been integrated over) (a) for the minimum of the mean-field potential energy surface (PES) of Fig. 1a at $\beta = 0.4$, $\gamma = 60^\circ$; (b) for the minimum of the projected PES $(0^+_1)$ in Fig. 1b with $\beta = 0.6$, $\gamma = 20^\circ$; (c) for the deformation of the projected GCM state $0^+_1$ with $\langle \beta \rangle = 0.44$, $\langle \gamma \rangle = 24.27^\circ$.

The deformations $\langle \beta \rangle$, $\langle \gamma \rangle$ for the ground state $(0^+_1)$ obtained from 3DAMP+GCM calculations. The quantities $\beta_{min}$, $\gamma_{min}$ and $\beta_{MF}$, $\gamma_{MF}$ are the deformations at the minima of the projected $(J = 0)$ and mean-field potential energy surfaces.

|                | $(\beta)$ | $(\gamma)$ | $\beta_{J=0}$ | $\gamma_{J=0}$ | $\beta_{MF}$ | $\gamma_{MF}$ |
|----------------|-----------|-------------|---------------|---------------|--------------|--------------|
| neut.          | 0.50      | 21.41$^\circ$ | 0.69          | 15.48$^\circ$ | 0.48         | 0$^\circ$    |
| prot.          | 0.34      | 31.30$^\circ$ | 0.46          | 31.47$^\circ$ | 0.27         | 0$^\circ$    |
| total          | 0.44      | 24.27$^\circ$ | 0.60          | 20.00$^\circ$ | 0.40         | 0$^\circ$    |

For comparison with the mean-field results, we define the quadrupole moments for the ground state

$$
\langle Q_{2\mu} \rangle \equiv \int d^2q |g_{\alpha=0}^{J}(q)|^2 q_{2\mu}(q), \quad \mu = 0, 2.
$$

The deformations $\langle \beta \rangle$ and $\langle \gamma \rangle$ are obtained from 3DAMP+GCM calculations. We observe a decoupled structure, where the average neutron deformation $\langle \beta \rangle_n = 0.50$ is obviously larger than that of protons $\langle \beta \rangle_p = 0.34$. This can also be seen in Fig. 2c where

$$
\rho_{r}^{GCM}(r) \equiv \int d^2q |g_{\alpha=0}^{J}(q)|^2 \rho_{r}(r; q)
$$

are the density distributions of neutrons and protons for excited state $(2^+_1)$. The ground state has a triaxial structure with almost uniform probability along the $\gamma$-direction which indicates an obvious quantum shape fluctuation in $\gamma$ direction. For $J = 2$ we have $K$-mixing and the two distributions for $K = 0$ and $K = 2$ separately. Both are concentrated along the axially symmetric configurations with $K = 0$ on the prolate and $K = 2$ on the oblate side. However, the $K = 0$ part exhausts $93.1\%$ of the norm so that the nucleus has a strong prolate deformation in the $(2^+_1)$ state with $\beta \simeq 0.6$.
the calculated spectrum is systematically stretched. It would be far more compressed by the inclusion of a cranking term in the mean-field calculation \[23\].

In summary, starting from covariant density functional theory and including additional correlations by restoring symmetries and by configuration mixing of triaxial shapes we have studied the influence of quantum shape fluctuations on spectroscopic properties in the exotic nucleus $^{16}$C. In contrary to earlier investigations using other methods we find that the ground state has a triaxial shape with large shape fluctuations in $\gamma$. Such a novel picture for the shape of $^{16}$C is essential to reproduce quantitatively $B(E2)$ values derived from a recent lifetime measurement based on the RDM method.

This work has been supported by the Major State Basic Research Developing Program 2007 CB815000, the Asia-Europe Link Project [CN/ASIA-LINK/008 (094-791)] of the EU, the NSFC under Grant Nos. 10775004, 10705004, the DFG Cluster of Excellence “Origin and Structure of the Universe”, and by MEXT KAKENHI 19740115. We thank R. R. Hilton and D. Vretenar for helpful discussions.

\[\begin{align*}
\text{FIG. 5: (Color online) The lowest energy levels of angular momentum} \, J = 0, 2, 4 \, \text{in} \, ^{16}\text{C. The} \, B(E2) \, \text{values are in units of} \, e^2\text{fm}^4. \, \text{The experimental data are taken from Refs.} \, [2, 7].
\end{align*}\]

The predicted GCM state $\left(0_1^+\right)$.

To demonstrate the importance of shape fluctuations in $\gamma$ for spectroscopic properties, we plot in Fig. 5 the lowest energy levels having angular momentum $J = 0, 2, 4$ in $^{16}$C obtained from the 1D and 3D AMP+GCM calculations and compare them with experiment.\[7\]. The inclusion of triaxial states does not change the energy spectrum too much, but it improves the electric quadrupole transitions significantly. Only with a proper treatment of the shape fluctuations in $\gamma$, are the calculated $B(E2 : 2_1^+ \rightarrow 0_1^+)$ $\approx$ 6.50 $e^2\text{fm}^4$ in agreement with recent data.

The predicted ratio $R_{4/2} \equiv E_{4}/E_{2} = 2.63$ is a little larger than the experimental value $R_{4/2} = 2.35$. Both are close to the $R_{4/2} = 2.50$, a typical value for $\gamma$-soft nuclei. The predicted deviations of $R_{4/2}$ from experiment go in the direction of a rotor ($R_{4/2} = 3$). This has its origin in the increasing prolate deformation as angular momentum increases to $J = 2$ and $J = 4$. As observed in many calculations using projection after variation \[10, 11, 12\] the calculated spectrum is systematically stretched. It would be far more compressed by the inclusion of a cranking term in the mean-field calculation \[23\].