Heuristic Model Of The Composite Quality Index Of Environmental Assessment

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Abstract. The goal of the paper is to present the heuristic model of the composite environmental quality index based on the integrated application of the elements of utility theory, multidimensional scaling, expert evaluation and decision-making. The composite index is synthesized in linear-quadratic form, it provides higher adequacy of the results of the assessment preferences of experts and decision-makers.

1. Introduction

The peculiarity of the assessment the ecological status of the territorial development programs is the need to analyze large amounts of heterogeneous information, conflicting goals of different actors involved in the industrial activities, and the complexity of evaluating alternative projects. The solution of problems like these is impossible without using modern computer-based decision support systems that contain databases, mathematical models, interactive and graphical user interfaces, analytics, data mining tools, etc.

Decision support systems provide the possibility to generate or choose the desired alternative from a variety of options when making decisions, give timely and comprehensive information. Such systems are a class of integrated intelligent systems that combine rigorous mathematical methods and solution search models with heuristic search models, logical-linguistic models and knowledge-based expert techniques.

2. Methods used to assess the quality of ecological status

So far, the quantitative and expert methods can be used to assess the environmental quality. Quantitative methods include differential, integrated and mixed methods. Differential method of quality assessment lies in a separate comparison of individual indices (indicators) of the environmental quality with the reference “benchmark” indicators. For this purpose we determine relative indicators $q$ using the formula:

$$ q = \frac{P_i}{P_{i\delta}}, $$

where $P_i$ – individual indicator of the considered state; $P_{i\delta}$ – individual “benchmark” indicator.
However, the application of the differential method does not reduce the amount of information analyzed by the decision maker (DM) and, therefore, it is ineffective in assessing the ecological status quality.

The integrated method of quality assessment involves the use of composite, generalized indices. While the individual quality indicators of the considered state are expressed in scores and are assigned some weighting coefficients $K$, a composite quality index is determined by the formula:

$$Q = \sum_{i=1}^{n} P_i K_i,$$

where $P_i$ – individual indicator of the considered state; $n$ – number of individual indicators under consideration.

In the integrated method of quality assessment quality is expressed as ratio of calculated composite index to predetermined reference index.

The mixed method of quality assessment is applied in cases when composite quality index isn’t sufficient enough to take into account all the essential characteristics of the ecological status. Then the obtained composite indices and distinguished individual indices are considered using the differential method of quality assessment.

Expert evaluation methods used to assess the quality of ecological status are based on the experts’ knowledge, experience and intuition.

In recent years, in assessing the quality of the ecological status, methods based on the use of mathematical apparatus of the choice and decision-making theory are used. Integrated mathematical means of solving practical problems of environmental quality assessment in interactive mode on the computer using the multi-criteria decision making methods [1, 2] are being developed now.

The proposed heuristic model of the composite quality indices is based on the convolution of the individual indicators after the exclusion of non-competitive solutions, through unconditional preference criterion (the Pareto principle). To represent the system of individual quality indicators in the form of a convolution of a multi-purpose index components, we use empirical reasoning that leads to the functional relationships.

The system of quality indices for the heuristic model is formed according to nomenclature of indices groups, used for the environmental quality assessment, on the basis of study of documents and expert polls. In this case, the heuristic model reflects the system as a research object.

At the first step, the individual quality indicators are normalized. To equalize metric scales of heterogeneous quality indicators, methods of rationing and centering, or the procedure of the conditional utility function construction has been applied [3, 4]. The procedure of balancing individual indicators allows not only to formally come to the dimensionless parameters but also to take into account the decision-makers (DM) preferences. Besides, there can also be determined the values of the utility of non- numeric qualitative indicators; it makes possible to use such indicators in the environmental assessment procedures.

Next, the problem of determining composite indices by groups of indices is being solved. For that purpose we apply the methods of multidimensional scaling [5] and the "ideal point" model (Figure 1).
Figure 1 - Geometrical interpretation of the “ideal point” model

To quantitatively determine the proximity of the objects to the “ideal point” in a coordinate space we use metrics.

The following metrics are used [3].

The Heider metric:

\[ u = 1 - \left( \sum_{i=1}^{n} \lambda_i |x_i^* - x_i|^p \right)^{1/n}, \quad n \geq 1, \]

where \( x_i^* \) – value of the "ideal state"; \( x_i \) – value of indicators of the compared states; \( \lambda_i \) – coefficient of relative importance (weight) of the \( i \)-th indicator; \( n \) – number of the considered individual indicators.

Euclidian metric:

\[ u = 1 - \left( \sum_{i=1}^{n} \lambda_i |x_i^* - x_i|^p \right)^{1/2}, \]

Using these metrics is useful for groups that includes quality indicators of unequal importance, however, considerable difficulty is the assignment of corresponding non-negative weight to each of the indicators; the weight represents a coefficient of relative importance of the indicator from the point of view of the objectives of the evaluation.

Hamming metric:

\[ u = 1 - \sum_{i=1}^{n} \lambda_i |x_i^* - x_i| \]

This metric is used mainly to differentiate objects that are measured in scales of names and order.

Introduction of weighted metrics into the space of individual indicators sets the task of determining the relative importance of the indicators; the task can be solved by one of the methods of expert evaluation.

The comparative analysis of these methods in characteristics of the labor intensity, the required qualification of experts and reliability of the results has shown that the pairwise comparison and pairwise substitution methods are the most preferable for determining the coefficients of relative importance.

We use the method of pairwise comparisons, its essence is as follows.
There are \( P \) indicators in the group. Experts compare all pairs of indicators in the group to each other, for example, in the form of statements "more − less", "better − worse" \[6\]. Using the symbol of preference \( > \), we can write \( (x_i > x_j) \) - \( x_i \) is more preferable than \( x_j \).

The \( \delta \) - function is assigned to each pair \( (x_i, x_j) \):

\[
\delta_{ij} = \begin{cases} 
1, & \text{if } x_i > x_j, i,j = 1,p, i \neq j \\
0, & \text{otherwise} \\
0.5, & \text{if } x_i \sim x_j 
\end{cases}
\] \hspace{2cm} (6)

where \( \sim \) means equivalency.

It is required to determine the ranking of objects by each expert, to determine mathematical expectation \( (l_{ij}) \) of a random variable \( \delta_{ij} \). On the basis of \( L = |l_{ij}| \) mathematical expectations of the pairs of indicators to determine the relative importance coefficients of the indicators and get a normalized ranked list of indicators in the group according to the results of expert poll.

The analysis of existing polling methods showed that to improve the reliability of the results automation of the process is needed, through the use of computing technologies during all stages of the process because the traditional techniques base on the questionnaires followed by the computer processing, and don’t take into account some important psychological characteristics of the experts.

It is known that there are two factors that significantly affect the expert assessments during the pairwise comparison of indicators. The order of presenting alternatives in a pair obviously affects the assessment of their relative importance. Influence like this is called a spatial effect.

It is considered that spatial effects are balanced for the given alternative if in one half of the pairs that include this alternative it goes first, and in the other it goes second. The effects associated with the ordering of pairs of alternatives in the list of ranked pairs are called temporary. It is believed that the temporary effects for the given alternative are balanced if pairs that include this alternative, are arranged in the list evenly.

It should be noted that the widely used form of questionnaires for the survey of experts like the "tournament style" table, maximize the negative impact of the spatial and temporary effects on the reliability of the polling results.

To compensate for the spatial and temporary effects, an algorithm of presenting the pairs of the alternatives to the experts \[7\] was developed and computer-implemented. Its scheme is shown on Figure 2. During the procedure the expert poll is executed using of a multi-user or personal computer system in several rounds according to a technique similar to Delphi method \[8\].

The following procedures were automated: the generation of the expert poll, processing of polling data, management of a multi-round expert survey, preparing and mailing the results of the next round to the experts’ terminals, messaging, access by the experts to additional and reference background information on the subject of the poll.

In order to improve the reliability of the expert poll results the experts’ answers are automatically checked on internal consistency. With this purpose the each set of three alternatives (triplets) \( (x_i, x_j, x_k) \) /irrespective of the order/ is checked for correspondence of the obtained expert estimates to the rules of logical decision:

\[
\begin{align*}
(x_i > x_j) \cap (x_j > x_k) & \Rightarrow x_i > x_k; \\
(x_i > x_j) \cap (x_j \sim x_k) & \Rightarrow x_i > x_k; \\
(x_i \sim x_j) \cap (x_j \sim x_k) & \Rightarrow x_i \sim x_k,
\end{align*}
\] \hspace{2cm} (7)

where \( > \) - greater preference of alternatives; \( \sim \) - equivalence of alternatives.
Preferences for the triplets in which contradictions are revealed, are presented for joint correlation. Using automated interactive polling procedure increases the accuracy and efficiency of determining relative importance coefficients in the groups. The polling results are processed by the computer in the following way.

If while estimating the preference $(x_i, x_j), m_i$ experts expressed in favor of the preference $x_j > x_i, m_j$ experts were against, and $m_k$ experts were for the equivalence of indicators, then the mathematical expectation of a random variable:

$$ l_{ij} = M[\sigma] = \frac{1m_i + 0.5m_k + 0m_j}{m_i + m_k + m_j} \quad (8) $$

The total number of respondents

$$ m = m_i + m_k + m_j \quad (9) $$

From (8) and (9) we obtain

$$ l_{ij} = \frac{m_i + 0.5m_k}{m} \quad (10) $$

It is obvious that $l_{ij} = 1 - l_{ij}$.

The set of quantities $l_{ij}$ forms a matrix $L$, of dimension $p \times p$.

The relative importance coefficient vector of order is determined by the formula:

$$ \lambda^t = \frac{1}{K^t} L K^{t-1}, t = \frac{1}{p} \quad (11) $$

where $L = \|l_{ij}\|, K^0 = (1, 1, \ldots, 1); K^t = (K_1^t, K_2^t, \ldots, K_p^t)$. Then $\lambda^t = \sum_{i=1}^{p} \sum_{j=1}^{p} l_{ij} K_j^t$.

We confine ourselves to the definition of the first-order relative importance coefficient ($t = 1$), which gives a sufficient approximation accuracy of the results of pairwise comparisons and simplifies
all the calculations. These coefficients represent the relative sums of the elements of the matrix $L$ rows.

Indeed, when $t = 1$, we obtain from (11):

$$
\lambda_i^1 = \frac{\sum_{j=1}^p l_{ij}}{\sum_{i=1}^p \sum_{j=1}^p l_{ij}}
$$

(12)

where

$$
\sum_{i=1}^p \sum_{j=1}^p l_{ij} = \frac{P \cdot P}{2}.
$$

Note that using the matrix $X = \|\omega_i\|$ made of the $K$-th expert answers as matrix $L$, we obtain the relative importance coefficients offered for the indicators by the given expert.

The experts' opinion consistency is evaluated with the concordance coefficient $W$; to test the significance of the polling results, the Pearson criterion $\chi^2$ is determined.

The determined after experts' survey relative importance coefficients of the individual environmental quality indicators are used to calculate metrics values by the (3), (4), (5) formulas. Using metrics makes it possible to come from a set of individual quality indicators to composite group indices.

3. Synthesis of heuristic composite environmental quality index in a linear-quadratic form

Unlike other earlier papers where the additive form of objective function was used, we suggest synthesis of heuristic composite environmental quality index in a linear-quadratic form:

$$
K = \sum_{i=1}^m (\alpha_i\gamma_i u_i(x_i) + \frac{1}{2}\alpha_i\alpha_i\gamma_i^2(x_i)) + \sum_{i,j=1}^m \alpha_i\alpha_i u_i(x_i)u_j(x_j),
$$

(13)

where $\gamma_i$ – relative importance coefficients for the groups of indicators; $\alpha_i, \alpha_i, \alpha_j$ – scaling coefficients; $u_i, u_j$ – metric functions of the the groups of indices.

While synthesizing the composite quality index $K$ it is necessary to take into account that the decision about correspondence of ecological status to a certain quality level is usually taken by a group of people (expert group) [9] but the final decision belongs to the decision-maker. With this consideration in mind, the procedure of synthesizing must be based on the decision-maker preferences [10] with due consideration to the opinion of other experts about the importance of indicators of one or another group.

Considering the experts' opinions can be reached by introducing the relative importance coefficients $\gamma$ of the groups of indices in the first-order terms (13). The problem of determining the relative importance coefficients of the groups of indicators is solved by the method of pairwise comparisons.

The pairwise comparison of group indices differs from that of individual indicators because in the latter case the expert has to deal not with tangible, physically perceivable indicators but with generalized composites, like hazardous chemical content in the atmosphere due to storage, transportation and consumption of products; possibility of emission into the atmosphere of pollutants (into water, air, earth, closed or ventilated room); harmful radiation level (high-frequency radiation, light radiation, etc.) while manufacturing, storage, transportation, consumption of products; hazardous noise- and vibration level, etc. In situations like these rating with “better – worse – equal”-scale becomes difficult.

To make the polling procedure more precise, the continuous preference scale is used, on which unit 1 corresponds to unconditionally greater preference of the first group, $\frac{1}{2}$ corresponds to equivalency,
and 0 is unconditional preference of the second group. Using the continuous scale allows to more precisely describe expert preferences.

Synthesis of heuristic composite environmental quality index in a linear-quadratic form makes possible to take into account the fact that with the improvement of the individual quality indicators, quality cost increases to a much greater extent than at a lower index value. Taking into consideration this circumstance provides higher adequacy of the results of the experts and DM preference evaluation.

To determine coefficients \( a_1, a_2, a_{ij} \) we use technique based on obtaining additional information from the decision-maker; this leads to the following system of equations and inequations:

\[
\sum_{i=1}^{m} (\alpha_i + \frac{1}{2} \alpha_j) + \sum_{i=j+1}^{m} \alpha_{ij} = 1, \\
\Delta K_S = 0, \\
m_S < \Delta K_S < M_S, \\
\alpha_i + (\delta^k, \alpha^l) > 0,
\]

where \( \Delta K_S \) – the environmental quality assessment; \( m_S, M_S \) – the result of more accurate estimates after obtaining additional information from the decision-maker; \( \delta \) – \( m \)-dimensional vectors with components equal to zero and one.

To solve the system (14), the \( \sum_{s=p+1}^{p} \Delta K_S \) is maximized, it leads to maximum “resolving power” of \( K \)-function.

This statement leads to a linear programming problem:

\[
\sum_{s=p+1}^{p} \Delta K_S (a_1, \ldots, a_m, a_{i1}, \ldots, a_{mn}) \rightarrow \max
\]

To solve the problem of linear programming the simplex method is used. Lack of solution to the linear programming problem means inconsistency of the DM’s preferences and then it will require the DM to be resurveyed.

Then, in order to verify the decisions stability we organize the iteration procedure of correction the function found, based on the analysis by the DM of the quantitative quality scores, as well as the difference between their scores.

As zero iteration of the definition \( K_0 \) the linear programming problem (14), (15) is considered. The result is \( K_0(u) \) function with coefficients \( a_1^0, \ldots, a_m^0, a_{i1}^0, \ldots, a_{mn}^0 \).

On the basis of the analysis of the values \( \Delta K_s = K_0^s(u_t) - K_0^s(u_s) \), \( s = p+1, p, \) the decision-maker must report sets of \( S_S < \) and \( S_S > \) (\( S_S = S^\cup S > \epsilon p+1, p \)) ranking numbers \( S_S \) of those pairs of systems, for which he/she isn’t satisfied with the difference of estimates, or these pairs have insufficiently expressed, from his point of view, distinctions in preferences of alternatives \( u_t, u_s \) (set of \( S_S < \)), or these differences are too large (set of \( S_S > \)).

For each \( s \in S_S < \cup S > \) the procedure of correcting the estimate (15) is applied, where in the capacity of \( m_0^0, M_0^0 \) one should choose \( \Delta K_0^s + \varepsilon, M_0^s \) for \( s \in S_S < \) and \( m_0^s, \Delta K_0^s - \varepsilon \) for \( s \in S_S > \), respectively.

After that we solve the linear programming problem (14) once again with constraints (15), wherein the last group of constraints is replaced with inequalities:

\[
\Delta K_0^s - \varepsilon \leq \Delta K_s \leq \Delta K_0^s + \varepsilon, s \in S < \cup S >; \\
m_s \leq \Delta K_s \leq M_s, s \in S < \cup S >.
\]
The described iterative procedure continues until the decision-maker is satisfied with the obtained differences between the $K$ values for the alternatives $\overline{u}_s$ and $u_s (s = p + 1, p)$.

To improve the quality of preference description with a function the decision-maker is required to indicate as much as possible pairs of alternatives. While correcting the estimates of the alternatives the decision-maker can report new pairs $\overline{u}_s, u_s (s = p + 1, p')$, which at this stage may be compared by preference. Then the next iteration is carried out with additional inequalities with $s = p + 1, p'$ in constraints (14).

4. Conclusion

Synthesized in such a way objective function appears as the composite environmental quality index; it describes in a qualitative manner the preferences of the experts and decision-makers and establishes ranking of consequences which are described by the initial set of individual indicators.

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