**CP Violation in Ω⁻ Decays**

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We estimate the CP-violating rate asymmetry for the decay $\Omega^- \rightarrow \Xi \pi$. Within the standard model, we find that it could be as large as $2 \times 10^{-5}$. This is significantly larger than the corresponding rate asymmetries for other non-leptonic hyperon decays, which are typically less than $10^{-6}$.

The origin of CP violation remains a mystery in particle physics. So far, CP-odd signals have been observed only in the kaon systems. To determine the source and nature of CP violation, it is necessary to observe it in several different processes. For a number of years theoretical efforts have been made to study it in the weak decays of hyperons belonging to the baryon octet $[1–3]$. Here, we present the results of a recent study exploring CP violation in the nonleptonic decays of the $\Omega^-$ hyperon $[4]$.

The CP-violating observable that we are considering is the rate-asymmetry for $\Omega^- \rightarrow \Xi \pi$ decays. It is given by

$$\Delta(\Xi^0 \pi^-) = \frac{\Gamma(\Omega^- \rightarrow \Xi^0 \pi^-) - \Gamma(\Omega^- \rightarrow \Xi^0 \pi^+)}{\Gamma(\Omega^- \rightarrow \Xi^0 \pi^-) + \Gamma(\Omega^- \rightarrow \Xi^0 \pi^+)}.$$  \hspace{1cm} (1)

In order to evaluate this quantity in more detail, we parametrize the decay amplitude in the form

$$iM_{\Omega^- \Xi \pi} = G_F m_\pi^2 \frac{\alpha^{(P)}_\Omega}{\sqrt{2} f_\pi} \bar{u}_\Xi k_\mu u^0_\Omega,$$  \hspace{1cm} (2)

where the $u$’s are baryon spinors, $k$ is the outgoing four-momentum of the pion, $f_\pi$ is the pion-decay constant, and only the dominant P-wave piece of the amplitude is included. We will consider only the P-wave because, experimentally, the asymmetry parameter in these decays is small and consistent with zero $[5]$, indicating that they are dominated by a P-wave. This amplitude has both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ components which are, in general, complex. We write the amplitudes for $\Omega^- \rightarrow \Xi^0 \pi^-, \Xi^- \pi^0$ as

$$\alpha^{(P)}_{\Omega^- \Xi^0} = \frac{1}{\sqrt{3}} \left( \sqrt{2} \alpha_1^{(Q)} e^{i\delta_1 + i\phi_1} - \alpha_3^{(Q)} e^{i\delta_3 + i\phi_3} \right), \hspace{1cm} \alpha^{(P)}_{\Omega^- \Xi^-} = \frac{1}{\sqrt{3}} \left( \alpha_1^{(Q)} e^{i\delta_1 + i\phi_1} + \sqrt{2} \alpha_3^{(Q)} e^{i\delta_3 + i\phi_3} \right),$$  \hspace{1cm} (3)

where $\alpha_{1,3}^{(Q)}$ are real quantities, strong-rescattering phases of the $\Xi \pi$ system with $J = 3/2$, P-wave and $I = 1/2, 3/2$ quantum numbers are denoted by $\delta_1, \delta_3$, respectively, and CP-violating weak phases are labeled $\phi_1, \phi_3$. The corresponding expressions for the antiparticle decay $\Omega^- \rightarrow \Xi \pi$ are obtained by changing the sign of the weak phases $\phi_{1,3}$ in $[3]$. It follows that, to first order in the small ratio $\alpha_3^{(Q)}/\alpha_1^{(Q)}$,

$$\Delta(\Xi^0 \pi^-) = \sqrt{2} \frac{\alpha_3^{(Q)}}{\alpha_1^{(Q)}} \sin(\delta_3 - \delta_1) \sin(\phi_3 - \phi_1).$$  \hspace{1cm} (4)

Similarly, $\Delta(\Xi^- \pi^0) = -2\Delta(\Xi^0 \pi^-)$.

Let’s evaluate the three factors in $[3]$ one by one. Using the measured decay rates $[5]$ and ignoring all the phases, we can extract $\alpha_3^{(Q)}/\alpha_1^{(Q)} = -0.07 \pm 0.01$ $[6]$. Final-state interactions enhance this value, but the enhancement is not significant for the values of the scattering phases that we estimate below. This ratio is higher than the corresponding ratios in other hyperon decays $[7]$, which range from 0.03 to 0.06 in magnitude, and provides an enhancement factor for the CP-violating rate asymmetry in this mode.

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We now turn to the $\Xi \pi$-scattering phases, $\delta_{1,3}$. There exists no experimental information on these phases, and so we will estimate them at leading order in heavy-baryon chiral perturbation theory. The lowest-order chiral Lagrangian for the strong interactions of the octet and decuplet baryons with the pseudoscalar octet-mesons \[8\] generates the diagrams shown in Fig. 1 for $\Xi^0 \pi^- \to \Xi^0 \pi^-$ and $\Xi^- \pi^0 \to \Xi^- \pi^0$. From the resulting scattering amplitudes, we can construct amplitudes for the $I = 1/2$ and $I = 3/2$ channels using the relations
\[
M_{I=1/2} = 2M_{\Xi^0 \pi^- \to \Xi^0 \pi^-} - M_{\Xi^- \pi^0 \to \Xi^- \pi^0},
\]
\[
M_{I=3/2} = -M_{\Xi^0 \pi^- \to \Xi^0 \pi^-} + 2M_{\Xi^- \pi^0 \to \Xi^- \pi^0},
\]
and project out the partial waves in the usual way. Calculating the $J = 3/2$ P-wave phases, and evaluating them at a center-of-mass energy equal to the $\Omega^-$ mass, yields \[8\]
\[
\delta_1 = -12.8^\circ, \quad \delta_3 = 1.1^\circ.
\]

The $I = 1/2$ P-wave phase is larger than other baryon-pion scattering phases. For instance, the P-wave $\Lambda \pi$-scattering phase has been estimated to be $\delta_\pi \approx -1.7^\circ$ \[9\]. In Fig. 2 we plot the $\Xi \pi$-scattering phases as a function of the pion momentum.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.pdf}
\caption{Diagrams for (a) $\Xi^0 \pi^- \to \Xi^0 \pi^-$ and (b) $\Xi^- \pi^0 \to \Xi^- \pi^0$. A dashed line denotes a pion field, and a single (double) solid-line denotes a $\Xi$ ($\Xi^*$) field.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.pdf}
\caption{$\Xi \pi$-scattering phases as a function of the center-of-mass momentum of the pion. The solid and dashed curves denote $\delta_1$ and $\delta_3$, respectively. The vertical dotted-line marks the momentum in the $\Omega^- \to \Xi \pi$ decay.}
\end{figure}
As for the weak phases $\phi_{1,3}$, within the standard model they arise from the $CP$-violating phase in the CKM matrix. To calculate them, we write the amplitude as

$$\text{Re} \mathcal{M}_{\Omega^{-} \to \Xi^{-}\pi} + i \text{Im} \mathcal{M}_{\Omega^{-} \to \Xi^{-}\pi} = -\langle \Xi^{-}\pi | \mathcal{H}_{\text{eff}} | \Omega^{-} \rangle,$$

where $\mathcal{H}_{\text{eff}}$ the short-distance effective Hamiltonian that contains a set of four-quark operators and describes the $|\Delta S| = 1$ weak interactions in the standard model. The phases are then given by

$$\phi_{1,3} \approx \frac{\text{Im} \mathcal{M}_{\Omega^{-} \to \Xi^{-}\pi}}{\text{Re} \mathcal{M}_{\Omega^{-} \to \Xi^{-}\pi}},$$

where the subscripts refer to the $|\Delta I| = 1/2, 3/2$ components of the amplitudes. Unfortunately, we cannot compute the matrix elements in (7) in a reliable way. In order to estimate them, we employ the vacuum-saturation method used in Ref. [2]. Our calculation yields

$$\alpha_{3}^{(\Omega)} e^{i\delta_{3}} = -0.11 + 2.8 \times 10^{-6} \, i,$$

$$\alpha_{1}^{(\Omega)} e^{i\delta_{1}} = 0.23 + 2.3 \times 10^{-4} \, i.$$

The $|\Delta I| = 3/2$ amplitude predicted in vacuum saturation is comparable to the one we extract from the data, $\alpha_{3}^{(\Omega)} = -0.07 \pm 0.01$. To estimate the weak phase, we can obtain the real part of the amplitude from experiment and the imaginary part of the amplitude from the vacuum-saturation estimate to get $\phi_{3} \approx -4 \times 10^{-5}$. Unlike its $|\Delta I| = 3/2$ counterpart, the $|\Delta I| = 1/2$ amplitude is predicted to be about a factor of four below the fit. Taking the same approach as that in estimating $\phi_{3}$ results in $\phi_{1} \approx 3 \times 10^{-4}$. We can also take the phase directly from the vacuum-saturation estimate (assuming that both the real and imaginary parts of the amplitude are enhanced in the same way by the physics that is missing from this estimate) to find $\phi_{1} = 0.001$.

To summarize, we have

$$\frac{\alpha_{3}^{(\Omega)}}{\alpha_{1}^{(\Omega)}} \approx -0.07,$$

$$|\sin(\delta_{3} - \delta_{1})| \approx 0.24,$$

$$|\sin(\phi_{3} - \phi_{1})| \approx 3 \times 10^{-4} \text{ or } 0.001,$$

where the first number for the weak phases corresponds to the conservative approach of taking only the imaginary part of the amplitudes from the vacuum-saturation estimate and the second number is the phase predicted by the model. The resulting rate asymmetry is

$$|\Delta (\Xi^{0}\pi^{-})| = 7 \times 10^{-6} \text{ or } 2 \times 10^{-5},$$

where the difference between these two numbers can be taken as a crude measure of the uncertainty in the evaluation of the weak phases. For comparison, estimates of rate asymmetries in the octet-hyperon decays [4] result in values of less than $10^{-6}$.

Using the results of a model-independent study of $CP$ violation beyond the standard model in octet-hyperon decays in Ref. [3], we can expect that the $CP$-violating rate asymmetry in $\Omega^{-} \to \Xi^{0}\pi^{-}$ could be ten times larger than our estimate above if new physics is responsible for $CP$ violation. The upper bound in this case arises from the constraint imposed on new physics by the value of $\epsilon$ because the P-waves involved are parity conserving.

In conclusion, we have estimated the $CP$-violating rate asymmetry in $\Omega^{-} \to \Xi^{0}\pi^{-}$. Within the standard model, it is about $2 \times 10^{-5}$, and it could be up to ten times larger if $CP$ violation arises from new physics. Although there are significant uncertainties in our estimates, it is probably safe to say that the rate asymmetry in $\Omega^{-} \to \Xi^{-}\pi$ decays is much larger than the corresponding asymmetries in octet-hyperon decays.
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