A LMM-based seismic fragility analysis method for bridges

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Abstract. The common seismic fragility analysis methods are mainly inaccurate due to assumptions made in derivation process or time consuming because of large sample analysis. A new approach based on lognormal mixture model (LMM) is presented for seismic fragility analysis of long-span bridges, where LMM is used to calculate the structural demand distributions and the seismic fragility is obtained through cumulative distribution function. To demonstrate the feasibility of the present approach, seismic fragility analysis is carried out on a long-span suspension bridge with a main span of 1200m. The results show that the present approach has higher accuracy than the classical approach.

1. Introduction
Fragility curves describe the probabilities that a bridge is beyond a specific damage state under given levels of earthquake intensity [1]. During the last two decades, many methods have been proposed for generating fragility curves from seismic response data, such as cloud approach [2], incremental dynamic analysis (IDA) [3] and maximum likelihood method [4]. However, these methods assume a lognormal distribution for the demand or a lognormal shape for the curve to develop fragility curves. Recently, the problems caused by these assumptions have been studied in some papers [5]-[7], indicating that the assumptions may introduce a considerable error in the fragility curves.

In order to guarantee the accuracy of the whole fragility curve as well as not lead to large computational cost, lognormal mixture model (LMM) [8] is an appropriate way to estimate the demand distribution under given ground motion intensity instead of direct MCS. On one hand, compared with the lognormal distribution used in classical parametric methods, LMM has significant applicability in fitting demand distributions, which can be lognormal or non-lognormal distributions [8]. On the other hand, LMM could accurately predict the failure probability, whether it is high or low, with few demand samples, and has higher efficiency than direct MCS [9].

The purpose of this study is to present a new approach for seismic fragility analysis by using the lognormal mixture model. The procedure of the present approach is provided for better understanding and applying, which include the basic theory of LMM. And the seismic fragility analysis on a suspension bridge with a main span of 1200 m is conducted to validate the feasibility of the present approach.

2. Seismic fragility analysis method
A new approach to establish fragility curves is proposed, where the lognormal mixture model (LMM) is used to fit demand distribution and the demand distributions under various IM levels are estimated independently. Meanwhile, the synthetic ground motions are used as seismic inputs and the ETDM-
based dimension-reduced iteration algorithm[10] presented is applied to nonlinear time-history analysis (NLTHA) within the present approach.

2.1. Lognormal mixture model

The lognormal mixture model (LMM) uses a set of lognormal distributions to form a complex distribution with linear combination. By a change of variables, the lognormal mixture model (LMM) is in fact equivalent to the Gaussian mixture model in the logarithmic space [11], whose parameters can be estimated using the expectation-maximization algorithm[8], and this model has been proven to model any arbitrary statistical distribution [12].

Assume that the probability density function of a seismic demand \(D\) of a structure at each level of \(IM\) can be approximated by LMM, then it can be written as

\[
f_c(D; \Psi) = \sum_{j=1}^{N} \phi_j f(D; \mu_j, \sigma_j)
\]

\[
f(D; \mu_j, \sigma_j) = \left\{ \begin{array}{ll}
1 & \text{if } D > 0 \\
0 & \text{if } D \leq 0
\end{array} \right.
\]

(2)

in which \(N\) denotes the number of mixture model components; \(f(D; \mu_j, \sigma_j)\) denote the lognormal distributions with parameters \( \mu_j \) and \( \sigma_j \); \( \phi_j \) denotes the corresponding weight factor, and \( \sum_{j=1}^{N} \phi_j = 1 \); and \( \Psi = [ \phi_1, \phi_2, \ldots, \phi_N, \mu_1, \mu_2, \ldots, \mu_N, \sigma_1, \sigma_2, \ldots, \sigma_N ]^T \) denotes the vector grouped by all model parameters to be determined.

With \(N_s\) demand samples, the vector \( \Psi \) can be estimated by using maximum likelihood method as

\[
\Psi^* \equiv \arg\max_{\Psi} \frac{1}{N_s} \sum_{i=1}^{N_s} \ln f_c(D_i; \Psi)
\]

(3)

where \( \Psi^* \) denotes the vector \( \Psi \) that makes \( \sum_{i=1}^{N_s} \ln f_c(D_i; \Psi) / N_s \) maximum, and it can be calculated through EM algorithm.

The procedure for estimating \( \Psi^* \) is as follows:

1. Initialize the undetermined parameter vector \( \Psi \) of the model, denoted as \( \Psi^{(0)} \). Calculate the initial value \( ce^{(0)} = -\sum_{i=1}^{N_s} \ln f_c(D_i; \Psi^{(0)}) / N_s \), and set the iteration step \( i = 1 \).

2. Update the vector \( \Psi \) through the following equation, denoted as \( \Psi^{(i)} \)

\[
\phi_j^{(i)} = \frac{\sum_{k=1}^{N_s} w_{k,j}}{N_s}, \quad \mu_j^{(i)} = \frac{\sum_{k=1}^{N_s} w_{k,j} \ln D_k}{\sum_{k=1}^{N_s} w_{k,j}}, \quad \sigma_j^{(i)} = \sqrt{\frac{\sum_{k=1}^{N_s} w_{k,j} (\ln D_k - \mu_j^{(i)})^2}{\sum_{k=1}^{N_s} w_{k,j}}}
\]

(4)

where \( w_{k,j} \) denotes the posterior probability when the sample \( D_k \) is taken from the \( j \)th component \( f(D_k; \mu_j, \sigma_j) \), which can be written as

\[
w_{k,j} = \frac{\phi_j^{(i-1)} f(D_k; \mu_{j}^{(i-1)}, \sigma_{j}^{(i-1)})}{\sum_{j=1}^{N} \phi_j^{(i-1)} f(D_k; \mu_{j}^{(i-1)}, \sigma_{j}^{(i-1)})}
\]

(5)

3. Calculate \( ce^{(i)} = -\sum_{k=1}^{N_s} \ln f_c(D_k; \Psi^{(i)}) / N_s \). If \( |ce^{(i)} - ce^{(i-1)}| / ce^{(i)} \leq Tol \), where \( Tol \) denotes the convergent tolerance error, then \( \Psi = \Psi^{(i)} \). Otherwise, repeat steps (2) and (3).

After the vector \( \Psi^* \) is obtained following the above steps, the probability density function \( f_c(D; \Psi^{*}) \) of the seismic demand in Eq.(1) is determined. And then the probability that the seismic demand \( D \) exceeds the structural capacity \( C \) of a particular damage state at \( IM \) can be expressed as
\begin{equation}
    P_i(D \geq C | IM) = \sum_{j=1}^{N_m} \left[ 1 - \phi_j \Phi \left( \frac{\ln C - \mu_j^*}{\sigma_j^*} \right) \right]
\end{equation}

where the asterisk superscript of the parameters $\phi_j$, $\mu_j$, and $\sigma_j$ ($j=1,2,\cdots,N_m$) denotes the estimated result of LMM.

### 2.2. Component and system fragility

For a single component, the failure mode is controlled by a single critical response $r_i$, and the component demand $D_{com}$ under a ground motion is generally defined as the peak absolute value $r_{p}$ of the critical response time-history $r(t)$, which can be expressed as

\begin{equation}
    D_{com} = r_p = \max_{t \in [0,T]} |r(t)|
\end{equation}

where $T$ denotes the time duration of the ground motion. When the damage limit state is given, the component fragility can be obtained through Eq. (6).

The general assessment of seismic fragility for the bridge as a whole must be made by combining the effects of the various bridge components. Assume that a bridge has $N_c$ component failure modes. And then the bridge will involve $N_c$ critical responses $r_i$ ($i=1,2,\cdots,N_c$) to control these modes, with $N_c$ corresponding boundary values $z_i$ at a particular damage limit state. For the sake of simplicity, the bridge is usually assumed to operate like a serial or parallel system. If the bridge is assumed as a serial system, with each component performing an indispensable role independently, then any component damage will result in system damage at the same level. And the system demand $D_{sys}$ under a ground motion can be defined as

\begin{equation}
    D_{sys} = \max_{i=1,2,\cdots,N_c} \left( \frac{r_{p}}{z_i} \right) = \max_{i=1,2,\cdots,N_c} \left( \frac{\max_{t \in [0,T]} r_i(t)}{z_i} \right)
\end{equation}

On the other hand, if a bridge is assumed as a parallel system, then it will reach a specific damage state when all components have reached that damage state. And the system demand $D_{sys}$ under a ground motion can be defined as

\begin{equation}
    D_{sys} = \min_{i=1,2,\cdots,N_c} \left( \frac{r_{p}}{z_i} \right) = \min_{i=1,2,\cdots,N_c} \left( \frac{\max_{t \in [0,T]} r_i(t)}{z_i} \right)
\end{equation}

Since the probability density function of the demand is acquired by LMM, the bridge system fragility of a particular damage state at IM can be expressed as

\begin{equation}
    P_i(D_{sys} \geq 1 | IM) = \sum_{j=1}^{N_m} \left[ 1 - \phi_j \Phi \left( \frac{-\mu_j}{\sigma_j} \right) \right]
\end{equation}

### 3. Numerical example

#### 3.1. A suspension bridge model

A 2040m long suspension bridge now being built in South China is used to verify the feasibility of the present approach. The bridge has a main span of 1200m, leading to a rise-span ratio of 1:9.5, as shown in Fig. 1.
Figure 1. Elevation of a suspension bridge

The 3-D finite element model of the bridge is established using the general-purpose finite element software ANSYS, as shown in Fig. 2, which consists of 475 beam elements and 374 truss elements, leading to a total number of 5028 degrees of freedom for the whole structure. To take into account the large-displacement and stress-stiffening effects under the dead load of the bridge, the seismic analysis model is established based on the geometric configuration and the internal force status of the completion state of the bridge, which can be obtained through form-finding static analysis. It has been observed that the newly induced geometric nonlinear effects due to seismic excitations can be neglected as compared with those induced by the dead load of the bridge during erection [13]. Therefore, the above model can be used to conduct the subsequent inelastic seismic response analysis of the suspension bridge after completion.

Figure 2. Finite element model of suspension bridge after completion

Under strong ground motions, the bottom sections of the main towers may first enter into inelastic state, for the sake of simplicity, in this study, only the bottom parts of the main towers are modelled as nonlinear fiber element[14]. Remaining elements of the bridge are considered as linear. The Menegotto-Pinto model [15] and the modified Kent-Park model [16] are used to simulate the stress-strain relations of the steel bars and the concrete materials in the fiber elements, respectively, and the yield strength of steel bars and the compressive strength of concrete are taken as 330MPa and 22.4MPa, respectively.

3.2. Ground motions
A significant step in the fragility analysis is the selection of earthquake motion set that represents the variability of earthquakes. Actual ground motion records could be preferred, while many of the available records are not well identified as the site soil classification and are not free field time-history but rather modified by the structural response. Therefore, the number of available and usable ground motion records is not large enough to obtain sufficiently accurate results [17]. Due to these reasons, artificially generated ground motions compatible with the site characterization are used in this study.

The non-stationary ground acceleration process \(X(t)\) is assumed to be a uniformly modulated random process expressed as

\[
X(t) = g(t)x(t)
\]  

(11)
where \( g(t) \) is the modulation function of ground acceleration, which can be expressed as

\[
g(t) = \begin{cases} 
(t / t_a)^2 & 0 \leq t < t_a \\
1 & t_a \leq t < t_b \\
e^{-\alpha(t-t_b)} & t_b \leq t
\end{cases}
\]  

(12)

in which \( t_a = 3.2s \), \( t_b = 17.0s \) and \( \alpha = 0.15 \); and \( x(t) \) is a stationary random process with zero mean, whose power spectrum can be taken as the Clough-Penzien spectrum [18] as follows

\[
S(\omega) = \frac{\omega_0^4 + 4\zeta_s^2\omega_0^2\omega^2}{\left(\omega_0^2 - \omega^2\right)^2 + 4\zeta_s^2\omega_0^2\omega^2 \left(\omega_0^2 - \omega^2\right) + 4\zeta_i^2\omega_i^2\omega^2}S_0
\]  

(13)

in which \( \omega_0 = 10\text{rad/s} \) and \( \zeta_s = 0.6 \) denotes the predominant circular frequency and damping ratio of the subsoil, respectively; \( \omega_i \) and \( \zeta_i \) can simulate the variation in low-frequency energy of ground motion, \( \zeta_i = \zeta_s \); \( \omega_i = 0.1 - 0.2\omega_0 \); and \( S_0 \) denotes the spectral intensity factor.

As the peak ground acceleration (PGA) is the most widely used seismic intensity measure (IM) [19], PGA is selected as IM in this study. The range of mean PGA is given as 0.1g~1.0g, where \( g \) is the gravity acceleration, and the corresponding range of the spectral intensity factor \( S_0 \) is 0.0016~0.1642m²/s³. 10 values of \( S_0 \) are selected uniformly within the range of \( S_0 \). For each value of \( S_0 \), a sufficient number of samples of \( x(t) \) can be generated using the spectral representation method with power spectrum \( S(\omega) \). Substitution of the samples of \( x(t) \) and the modulation function \( g(t) \) into Eq.(11) yields the samples of non-stationary ground acceleration process \( X(t) \).

3.3. Damage limit states
In fragility analysis, the damage of structural component or system under earthquakes is described in terms of damage index, which are often measured by means of curvature ductility, displacement ductility, residual displacement, etc. Since the bottom regions of the towers are the most vulnerable part of the suspension bridge under severe ground motions, the curvature ductility ratios of the bottom sections are selected as the damage indexes in this study, which are the ratio of the curvature at a section to the yield curvature at the same section. The suspension bridge has two main towers, with two legs of each tower, so 4 critical sections are involved to control the damage state of the bridge. The capacity of the bridge components is defined in terms of limit states. 4 damage limit states defined by HAZUS-MHMR3 [21] are commonly adopted, namely slight, moderate, extensive and complete damages, and the corresponding values of the curvature ductility ratio \( \lambda \) are 1.0, 1.58, 3.22 and 6.84, respectively [22].

3.4. Seismic fragility analysis
Seismic fragility analysis of the suspension bridge is now conducted using the new approach presented in Section 2. To demonstrate the accuracy and efficiency of the present approach, seismic fragility analysis is also carried out using the classical cloud approach and direct Monte-Carlo simulation (MCS). At each PGA point, 100 samples of ground acceleration process are generated as seismic input for the three approaches, and the corresponding demands under these ground acceleration samples are calculated using the ETDM-based dimension-reduced iteration algorithm.
3.4.1 Component fragility curves.
The fragility curves of the west and east towers at 4 damage states are shown in Figs. 3 and 4, respectively. It can be seen from these figures that, for slight and moderate damages, the fragility curves of the cloud approach are close to those of the direct MCS, however, for the extensive and complete damages, the fragility curves of the cloud approach are far from those of the direct MCS. Good agreements can be observed in Figs. 3 and 4 between the results of the present approach and direct MCS, indicating that the calculation accuracy of the present LMM-based approach is close to that of the direct MCS.

![Fragility curves for the west tower by the cloud approach, LMM and MCS](image-url)
3.4.2. System fragility curves

The structure is assumed as a series system, and the system fragility curves at 4 damage states are calculated by the present approach and direct MCS, as shown in Fig. 5. As the PSDM is built for single structural response, the classical cloud approach could not directly calculate system fragilities, however the upper and lower bounds of system fragilities could be obtained by the approach based on the series system, which are also illustrated in the figure. As can be seen from the figure, the results of the present approach agree well with those of the direct MCS, which are close to the lower bounds of the slight and moderate fragility curves but beyond the upper bounds of the extensive and complete fragility curves of the cloud approach. It is indicated that the present approach has higher accuracy than the classical cloud approach in calculating system fragility curves.
4. Conclusions

A new approach based on LMM is presented for seismic fragility analysis of long-span bridges. The seismic fragility analysis of a long-span suspension bridge with a main span of 1200m is carried out using the cloud approach, the direct MCS and the present approach, respectively. The results show that, the present approach has higher calculation accuracy than the cloud approach.

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References

[1] Stefanidou, S.P., Kappos, A.J. (2017) Methodology for the development of bridge-specific fragility curves. Earthquake Engineering & Structural Dynamics, 46: 73-93.
[2] Cornell, C.A., Jalayer, F., Hamburger, R.O., Foutch, D.A. (2002) Probabilistic basis for 2000 SAC federal emergency management agency steel moment frame guidelines. Journal of Structural Engineering, 128(4): 526-533.
[3] Vamvatsikos, D., Cornell, C.A. (2002) Incremental dynamic analysis. Earthquake Engineering & Structural Dynamics, 31: 491-514.
[4] Shinozuka, M., Feng, M.Q., Lee, J. (2000) Naganuma T. Statistical analysis of fragility curves. Journal of Engineering Mechanics, 126: 1224-1231.
[5] Karamhou, A., Bochini, P. (2015) Computation of bridge seismic fragility by large-scale simulation for probabilistic resilience analysis. Earthquake Engineering & Structural Dynamics, 44(12): 1959-1978.
[6] Noh, H.Y., Lallemant, D., Kiremidjian, A.S. (2015) Development of empirical and analytical fragility functions using kernel smoothing methods. Earthquake Engineering & Structural Dynamics, 44: 1163-1180.
[7] Mai, C., Konakli, K., Sudret, B. (2017) Seismic fragility curves for structures using non-parametric representations. Frontiers of Structural and Civil Engineering, 11:169–86.
[8] McLachlan, G., Peel, D. (2000) Finite mixture models. New York: John Wiley and Sons.
[9] Xian, J.H., Su, C. (2018) Seismic reliability analysis of energy-dissipation structures with viscous dampers based on dimension-reduced explicit iteration method. The 11th National Conference on Theory and Application of Random Vibration, The 7th National Conference on Stochastic Dynamics of China, Yichang (in Chinese).
[10] Su, C., Liu, X.L., Li, B.M., Huang, Z.J. (2018) Inelastic response analysis of bridges subjected to non-stationary seismic excitations by efficient MCS based on explicit time-domain method. Nonlinear Dynamics, 94(3): 2097-2114.

[11] Cui, Y., Yang, J., Yamaguchi, Y. (2011) CFAR ship detection in SAR images based on lognormal mixture models. In: Proceedings of the 3rd IEEE international Asia-Pacific conference on synthetic aperture radar, Seoul, South Korea.

[12] Buyukcorak, S., Vural, M., Kurt, G.K. (2015) Lognormal mixture shadowing. IEEE Transactions on Vehicular Technology, 64(10):4386-4398.

[13] Dai, X.H., Li, B.M., Su, C. (2017) Study of geometric nonlinearity influences of seismic responses of long span suspension bridge. Bridge. Construction, 47(3), 36–40. (in Chinese)

[14] Taucer, F., Spacone, E., Filippou, F. (1991) A fiber beam-column element for seismic response analysis of reinforced concrete structures. Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, California.

[15] Menegotto, M., Pinto, P. (1973) Method of analysis for cyclically loaded RC frames including changes in geometry and non-elastic behavior of elements under combined normal force and bending. In: IABSE Congress Reports of the Working Commission, vol. 13.

[16] Scott, B., Park, R., Priestley, M. (1982) Stress-strain behavior of concrete confined by overlapping hoops at low and high strain rates. Aci Journal, 79(1): 13-27.

[17] Guneyisi, E.M., Altay, G. (2008) Seismic fragility assessment of effectiveness of viscous dampers in R/C buildings under scenario earthquakes. Structural Safety, 30: 461-480.

[18] Clough, R.W., Penzien, J. (1993) Dynamics of Structures. McGraw-Hill, New York

[19] Simon, J., Vigh, L.G. (2016) Seismic fragility assessment of integral precast multi-span bridges in areas of moderate seismicity. Bulletin of Earthquake Engineering, 14: 3125-3150.

[20] Karmakar, D., Chaudhuri, S.R., Shinozuka, M. (2015) Finite element model development, validation and probabilistic seismic performance evaluation of Vincent Thomas suspension bridge. Structure and Infrastructure Engineering, 11(2): 223-237.

[21] Federal Emergency Management Agency. (1999) Multi-hazard loss estimation methodology. Earthquake model. HAZUS99 User’s Manual. Washington (DC).

[22] Nielson B. (2005) Analytical fragility curves for highway bridges in moderate seismic zones. Atlanta: Georgia Institute of Technology.