On groups with unbounded Cayley graphs

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Abstract. We show that every non-trivial compact connected group and every non-trivial general or special linear group over an infinite field admits a generating set such that the associated Cayley graph has infinite diameter.

A group $G$ is said to have the Bergman property if for any symmetric generating subset $S \subseteq G$ containing the identity of $G$ there exists $k \in \mathbb{N}$ such that $S^k = G$, where $S^k := \{s_1 \cdots s_k \mid s_1, \ldots, s_k \in S\}$. The smallest such $k$ is called the width of $S$ or the diameter of the associated Cayley graph $\text{Cay}(G, S)$. If there is no such $k$, $S$ is said to have infinite width.

Examples of such groups include finite groups, where the study of worst case diameters has a long history, but also infinite groups such as $\text{Sym}(\mathbb{N})$ (a result due to Bergman [1]). The first example of an infinite group with uniformly bounded width with respect to any generating set has been constructed by Shelah [8, Theorem 2.1].

In [3] Dowerk shows that unitary groups of infinite-dimensional von Neumann algebras admit strong uncountable cofinality. A group $G$ admits this property if for any exhausting chain $W_0 \subseteq W_1 \subseteq \cdots \subseteq G = \bigcup_{n=0}^{\infty} W_n$ of subsets there exist $n, k \in \mathbb{N}$ such that $W_n^k = G$ (see [7, Definition 1.1]). It is apparent that, setting $W_n := S^n$, where $S$ is any symmetric generating set containing the identity, strong uncountable cofinality implies the Bergman property. Indeed, for uncountable groups it is actually equivalent to admitting both the Bergman property and uncountable cofinality (see [4, Proposition 2.2] or [5]). Regarding his result, Dowerk asked whether unitary groups of finite-dimensional von Neumann algebras have the Bergman property. In this note we answer this question in the negative by showing that any non-trivial compact connected group fails to have this property. We also show this for groups of type $\text{GL}_n(K)$ or $\text{SL}_n(K) \neq \mathbf{1}$ ($n \geq 1$), where $K$ is an infinite field.

Note that all of the above groups also fail to have uncountable cofinality. This is shown for $\text{SL}_n(K)$ over an uncountable field in [9, Proposition] and can easily be extended to all groups admitting a finite-dimensional representation with uncountable image. Such groups encompass compact groups
by the Peter–Weyl theorem. This was pointed out by Cornulier in private communication.

Our main result is the following.

**Theorem 1.** The following are true

(i) Any non-trivial compact connected group does not admit the Bergman property.

(ii) Any group of type \( \text{GL}_n(K) \) \((n \geq 1)\) or \( \text{SL}_n(K) \) \((n \geq 2)\), for an infinite field \( K \), does not admit the Bergman property.

The proofs use a few non-trivial facts from the theory of fields, for which we refer the reader to [6]. For facts like the existence of a transcendence basis of \( \mathbb{R} \) over \( \mathbb{Q} \) we need to assume the Axiom of Choice.

1. **Proof of Theorem 1**

This section is devoted to the proof of Theorem 1.

**Proof of Theorem 1(i):** Let \( |\cdot| : \mathbb{C} \to \mathbb{R}_{\geq 0} \) be a norm such that the following hold

(i) \( |x| = 0 \) if and only if \( x = 0 \) (identity of indiscernibles);

(ii) \( |a + b| \leq |a| + |b| \) (subadditive);

(iii) \( |ab| \leq |a||b| \) (submultiplicative);

(iv) \( |x^2| = |x|^2 \) (compatible with squaring);

(v) there exists \( x \in [0,1] \) such that \( |x| > 1 \) (small elements with large norm).

Examples of such norms are presented in Lemma 1 below.

We now consider the matrix group \( \text{SO}(2, \mathbb{R}) \). Set \( C := 4|1/2| \). Then \( C \geq 2|1| = 2 \) by (ii) (as \( |1| = 1; \) see below). Define \( S \) to be the set of elements of \( \text{SO}(2, \mathbb{R}) \) with coefficients in \( B := \{ x \in \mathbb{R} \mid |x| \leq C \} \). Observe that \( 1 \in S \), as (i) and (iv) imply that \( |1| = 1 \), and \( S = S^{-1} \), as the inverse of \( g \in \text{SO}(2, \mathbb{R}) \) is just \( g^\top \). We claim that \( S \) is a generating set and that \( \text{SO}(2, \mathbb{R}) \) has infinite diameter with respect to \( S \).

Assume that \( g = (\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}) \in \text{SO}(2, \mathbb{R}) \) corresponds to an element \( z \in \text{U}(1) \subseteq \mathbb{C} \) via the isomorphism \( (\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}) \mapsto a + bi \). Take \( k \in \mathbb{N} \) large enough such that

\[
|z^{1/2^k}|, |z^{-1/2^k}| \leq 2.
\]

This is possible by (iv). Set \( a' := \text{Re}(z^{1/2^k}) \) and \( b' := \text{Im}(z^{1/2^k}) \). By property (iv) we have \( |i| = 1 \) as \( |i^4| = |1| = 1 \). From (ii), (iii) we derive that

\[
|a'| = \left| \frac{1}{2}(z^{1/2^k} + z^{-1/2^k}) \right| \leq \frac{1}{2} \left( |z^{1/2^k}| + |z^{-1/2^k}| \right) \leq C.
\]

Similarly, using that \( |i| = 1 \), we obtain

\[
|b'| = \left| \frac{1}{2^k}(z^{1/2^k} - z^{-1/2^k}) \right| \leq \frac{1}{2^k} \left( |z^{1/2^k}| + |z^{-1/2^k}| \right) \leq C.
\]
The following norms set witnessing that $H$ to the group $K := \pi$ for some $g \in \mathbb{R}$. Since $g$ was arbitrary, $S$ follows to be a generating set of $SO(2, \mathbb{R})$.

It remains to show that $SO(2, \mathbb{R})$ does not have finite diameter with respect to $S$. Indeed, for any $R \in \mathbb{R}_{\geq 0}$ on large enough such that, if $x$ is an element as in (v), $|x^{2k}| = |x|^{2^k} \geq R$. Now putting $a := x^{2^n}$ and $b := \sqrt{1-a^2}$, we obtain an element $g = a + bi \in U(1) \cong SO(2, \mathbb{R})$ which needs arbitrarily many factors in $S$ to be represented (taking $R$ large enough). Indeed, by induction, all coordinate entries of an element in $S^{2k}$ have norm at most $2^{k-1}C^k$ ($k \in \mathbb{Z}_+$).

Now let $H$ be a non-trivial compact connected Lie group represented as a matrix subgroup of $O(n)$ for some $n \in \mathbb{N}$. We observe that finitely many copies $H_i (i = 1, \ldots, m)$ of $SO(2, \mathbb{R})$ in $H$ generate $H$ as a group, e.g., see [2] Theorem 2. This allows us to extend our argument for $SO(2, \mathbb{R})$ above to the group $H$ as follows. Assume that $H$ is generated by

$$H_i = \left\{ g_i^{-1} \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \oplus \text{id}_{n-2} \right\} g_i \mid a, b \in \mathbb{R}, \ a^2 + b^2 = 1$$

for some $g_i \in O(n)$ ($i = 1, \ldots, m$). After conjugating by $g_i^{-1}$ we may assume that $g_1 = 1$. As a generating set $S$ for $H$ it now suffices to take the set of elements in $H_i (i = 1, \ldots, m)$ with coefficients $a, b \in B$, where $B$ is defined as above. Apparently, $S$ generates $H_i (i = 1, \ldots, m)$ and hence $H$. Let $D$ be the maximum of the norms of the matrix entries of the $g_i (i = 1, \ldots, m)$; and so it is the maximum of the norms of the entries of the $g_i^{-1} = g_i^\top$. Then any coordinate entry of an element in $S \cap H_i$ has norm bounded by $c := n^2D^2C$. Thus, by induction, for $g \in S^{2k}$, all coordinate entries of $g$ have norm at most $n^{k-1}c^k$ ($k \in \mathbb{Z}_+$). Therefore, $S$ cannot generate $H$ in finitely many steps, since, as above, the coordinate entries of $H_i \cong SO(2, \mathbb{R})$ are unbounded with respect to the norm.

If $H$ is now any non-trivial compact connected group, then by the Peter–Weyl theorem $H$ has a non-trivial finite-dimensional unitary representation, say $\pi: H \to U(n) \subseteq O(2n)$. Let $S$ be a generating set in $O(2n)$ witnessing that $\pi(H)$ does not have the Bergman property. Then $\pi^{-1}(S)$ is a generating set witnessing that $H$ does not have the Bergman property.

It remains to construct norms satisfying (i)–(v). This is done in the subsequent lemma.

**Lemma 1.** Let $T$ be a transcendence basis of $\mathbb{R}$ over $\mathbb{Q}$. Consider $K := \mathbb{Q}(T)$ and let $G$ denote the Galois group of the field extension $\mathbb{C}/K$. The following norms $|\cdot|: \mathbb{C} \to \mathbb{R}_{\geq 0}$ satisfy properties (i)–(v) from the proof Theorem [2] (ii).

(i) Since, $K \subseteq \mathbb{C}$ is algebraic, $G$ is a profinite group and each element in $\mathbb{C}$ has a finite orbit under the action of $G$. For $x \in \mathbb{C}$ we define the Galois radius of $x$ to be $\rho(x) := \max_{\sigma \in G} |x^\sigma|$. Then $|\cdot| := \rho$ defines a norm with the desired properties.
(ii) Choose \( t \in T \). Set \( L := \mathbb{Q}(T \setminus \{ t \}) \). Let \( \nu: K \to \mathbb{Z} \) be the degree valuation corresponding to \( t \) on \( K \). Extend \( \nu \) to a valuation \( \omega: \mathbb{C} \to \mathbb{Q} \) (by using Zorn’s lemma). Setting \( |x| := \exp(\omega(x)) \) gives a norm on \( \mathbb{C} \) satisfying the above properties.

**Proof of Lemma 1** In both cases, we verify properties (i)–(v).

(i): Properties (i)–(iv) are immediate from the definition. We now show that also property (v) is fulfilled. Choose arbitrary real numbers \( a \in (0,1) \) and \( b > 1 \). Set \( p(X) := (X - a)(X - b) \in \mathbb{R}[X] \). We claim that by density of \( K \supseteq \mathbb{Q} \) in \( \mathbb{R} \) we can find an irreducible polynomial over \( K \) with coefficients arbitrarily close to those of \( p(X) \). Indeed, by Gauss’ lemma, an irreducible polynomial over \( \mathbb{Q} \) remains irreducible over \( K \) and hence, it suffices to approximate by irreducible rational polynomials. Using Eisenstein’s criterion, we can find an irreducible monic rational polynomial \( q(X) \) which has coefficients arbitrarily close to the coefficients of \( p(X) \). Indeed, Eisenstein’s criterion implies that for \( \alpha, \beta, \gamma \in \mathbb{Z} \), \( \gamma > 0 \), the polynomial \( q(X) = X^2 + (\alpha/\gamma)X + (\beta/\gamma) \) is irreducible if \( p^2 \) does not divide \( \beta \), \( p \) divides \( \alpha \) and \( \beta \) and does not divide \( \gamma \). Choosing \( \gamma \) large enough and coprime to \( p \), we can easily find \( \alpha \) and \( \beta \) with the desired properties such that \( \alpha/\gamma \) is close to \( -(a + b) \) and \( \beta/\gamma \) is close to \( ab \). By the implicit function theorem, the zeroes of \( q \), say \( x \) and \( y \), are arbitrarily close to \( a \) and \( b \), respectively. Hence \( \rho(x) \) is close to \( b > 1 \), as desired.

(ii): Properties (i)–(iv) follow from the definition of a valuation. For property (v) observe that \( |t^{-1}| = \exp(\nu(t^{-1})) = e > 1 \). Also, for \( y \in \mathbb{Q} \) we have \( |y| = \exp(\nu(y)) = 1 \). Taking \( y \) so that \( x := t^{-1}y \in [0,1] \), we obtain that for \( |x| = |y| |t^{-1}| = |t^{-1}| = e > 1 \), as wished.

**Remark 1.** In the proof of Theorem 1(i) and Lemma 1 the field \( \mathbb{R} \) can be replaced by a Euclidean field \( R \) and \( \mathbb{C} \) by \( R[i] \). Then, if \( T = \emptyset \) in Lemma 1(ii) we need to take a \( p \)-adic valuation, instead of the degree valuation, on \( K = \mathbb{Q} \) and extend it. Then, for \( y \), in the above argument, we have to take a suitable element \( r/s \in \mathbb{Q} \) such that \( p \nmid r, s \).

**Proof of Theorem 1(ii).** At first we consider the case \( G = \text{SL}_n(K) \). We distinguish two cases.

Case 1: Assume first that a transcendence basis \( T \) of \( K \) over its prime field \( k \) is not empty. Define for \( \lambda \in K^\times \), \( \mu \in K \) the matrices

\[
D(\lambda) := \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & \lambda \end{pmatrix} \quad \text{and} \quad E_{12}(\mu) := \begin{pmatrix} 1 & \mu \\ 0 & 1 \end{pmatrix} .
\]

Note that

\[
E_{12}(\mu)^{D(\lambda_1)} \cdots D(\lambda_n) = E_{12}(\mu)^{D(\lambda_1 \cdots \lambda_n)} = E_{12}((\lambda_1 \cdots \lambda_n)^2 \mu)
\]

for \( \lambda_1, \ldots, \lambda_n \in K^\times \), \( \mu \in K \). Choose \( t \in T \). For \( L := k(T \setminus \{ t \}) \) we have that \( K \) is algebraic over \( L(t) \). Take the normed degree valuation on \( \nu: L(t) \to \mathbb{Z} \) and extend it to a valuation \( \omega: K \to \mathbb{Q} \). Define \( B := \{ b \in K \mid |\omega(b)| \leq 1 \} \).
Set
\[ S := \{ g \in \text{SL}_n(K) \mid \text{all entries of } g \text{ are in } B \}. \]
Then \( S^{*k} \neq \text{SL}_n(K) \), since one sees easily by induction that for \( g \in S^{*k} \), \( |\omega(g_{ij})| \leq k \) (from the strong triangle inequality). We show that \( S \) generates all elementary matrices, and hence \( \text{SL}_n(K) \). Indeed, we may assume that \( n = 2 \), via the embeddings \( \text{SL}_2(K) \hookrightarrow \text{SL}_n(K) \). We show that \( S \) generates any matrix \( M := E_{12}(\alpha) \) for \( \alpha \in K \). Indeed, if \( \alpha = 0 \), then \( M \in S \). In the opposite case \( v := \omega(\alpha) \in \mathbb{Q} \) and we find an integer \( n \in \mathbb{Z} \) such that \( |v-2n| \leq 1 \). Then choose \( \lambda \in K \) such that \( \omega(\lambda) = 1 \) (which exists even in \( L(t) \subseteq K \), since \( \nu \) was normed) and set \( \mu := \alpha \lambda^{-2n} \). By construction, we have \( |\omega(\mu)| \leq 1 \), so that \( D(\lambda), E_{12}(\mu) \in S \) and \( M = E_{12}(\alpha) = E_{12}(\mu)^{D(\lambda)2n} \in \langle S \rangle \) by Equation (1). This completes the proof of Case 1.

**Case 2:** In this case \( K \) is an algebraic extension of its prime field \( k = \mathbb{Q} \) or \( k = \mathbb{F}_p \). In the first case, \( K \) embeds into \( \mathbb{C} \) and we can define
\[ S := \{ g \in \text{SL}_n(K) \mid \text{all entries of } g \text{ are in } B \}, \]
where \( B \) is the unit ball of \( \mathbb{C} \) intersected with \( K = \mathbb{C} \).

Hence we assume that \( k = \mathbb{F}_p \) and \( K \) is an infinite algebraic extension of \( k \). In this case, we construct a set \( B \subseteq K \) with the following properties
(i) \( \langle B \rangle = K \), i.e., \( B \) generates \( K \) as an abelian group;
(ii) \( P(B) := \{ p(b_1, \ldots, b_m) \mid b_1, \ldots, b_m \in B, p \in P \} \neq K \) for each finite set \( P \subseteq \mathbb{Z}[X_1, \ldots, X_m] \) of polynomials over \( \mathbb{Z} \) and \( m \in \mathbb{N} \).

Indeed, if we have such a set \( B \), we can use the same definition for \( S \) as above. This is due to the fact that \( E_{12}(\mu)E_{12}(\lambda) = E_{12}(\mu + \lambda) \) and the elementary matrices generate \( \text{SL}_n(K) \). On the other hand \( S^{*k} \neq \text{SL}_n(K) \) by condition (ii), as the matrix entries are bounded degree polynomials over \( \mathbb{Z} \) in the entries of the matrices.

The set \( B \) is now inductively constructed in Lemma 2 and Corollary 1 below.

In the case \( G = \text{GL}_n(K) \) we add matrices \( \text{diag}(\lambda, 1, \ldots, 1) \) to the generating set \( S \) with \( |\omega(\lambda)| \leq 1 \).

**LEMMA 2.** Fix a set \( P \subseteq \mathbb{F}_p[X_1, \ldots, X_m] \) of non-constant polynomials of total degree at most \( n \), i.e., especially \( P \) is finite. Consider the inclusion of finite fields \( \mathbb{F}_{p^e} \subseteq \mathbb{F}_{p^f} \) for \( e, f \in \mathbb{Z}_+ \) and let \( E \subseteq \mathbb{F}_{p^e} \) be a subset of cardinality \( e \). Define \( P^e \subseteq \mathbb{F}_{p^e}[X_1, \ldots, X_m] \) as the set of non-constant polynomials which arise from the polynomials from \( P \) by substituting a subset of the variables \( X_1, \ldots, X_m \) by elements from \( E \). Then for \( e \) sufficiently large there exists a set \( F \subseteq \mathbb{F}_{p^f} \) such that \( \langle F \rangle_{\mathbb{F}_p} \) is a complement to \( \mathbb{F}_{p^e} \) in \( \mathbb{F}_{p^f} \) as \( \mathbb{F}_p \)-vector spaces and \( r(f_1, \ldots, f_m) \notin \mathbb{F}_{p^e} \) for all \( f_1, \ldots, f_m \in F \) and \( r \in P^e \).

**PROOF.** At first note that the set \( C \) of \( e(f-1) \)-tuples with entries in \( \mathbb{F}_{p^f} \) which span an \( \mathbb{F}_p \)-complement of \( \mathbb{F}_{p^e} \) in \( \mathbb{F}_{p^f} \) is of cardinality \( (p^f - p^e) \cdots (p^{ef} - p^{ef-1}) \), so its portion in the set \( T \) of all \( e(f-1) \)-tuples with
entries in $\mathbb{F}_{p^f}$ is equal to

$$|C|/p^{2f(f-1)} = (1 - p^{-e(f-1)}) \cdots (1 - p^{-1})$$

but for $c := 2\log(2)$ we have $e^{-cx} \leq 1 - x$ for $0 \leq x \leq 1/2$, so that the above expression is bounded from below by

$$e^{-c \sum_{i=1}^{f-1} p^{-i}} \geq d := e^{-e^{-c}} \in (0, 1).$$

Now let us estimate how many tuples $t = (t_1, \ldots, t_{e(f-1)}) \in T$ have the property that $r(s_1, \ldots, s_m) \not\in \mathbb{F}_{p^f}$ for all $s_1, \ldots, s_m \in \mathcal{T} := \{t_1, \ldots, t_{e(f-1)}\}$ and $r \in P_E$. By the Schwartz-Zippel lemma, for all $x \in \mathbb{F}_{p^f}$ we have

$$P_{s_1, \ldots, s_m \in \mathbb{F}_{p^f}}[r(s_1, \ldots, s_m) = x] \leq n/p^{ef},$$

so that

$$P_{s_1, \ldots, s_m \in \mathbb{F}_{p^f}}[r(s_1, \ldots, s_m) \in \mathbb{F}_{p^f}] \leq n/p^{ef(f-1)},$$

and hence

$$P_{t \in \mathcal{T}}[\exists x \in P_E, s_1, \ldots, s_m \in \mathcal{T} : r(s_1, \ldots, s_m) \in \mathbb{F}_{p^f}] \leq |P_E|(e(f-1))^{m}n/p^{ef(f-1)}.$$

Putting both estimates together, we obtain that the set of $t \in T$ such that $T := \mathcal{T}$ satisfies the hypothesis of the lemma has portion bounded from below by $d - |P_E|(e(f-1))^{m}n/p^{ef(f-1)} \geq d - (e + 1)^m |P|(e(f-1))^{m}n/p^{ef(f-1)}$ in $T$. But this term is clearly positive for all $f > 1$ when $e$ is large enough. $\square$

**Corollary 1.** Let $K \subseteq \mathbb{F}_p$ be infinite. There exists a set $B \subseteq K$ satisfying (i) and (ii) in the proof of Theorem 1(ii).

**Proof.** Set $P_i := \mathbb{F}_p[X_1, \ldots, X_i]_{\deg \leq i}$ for $i \in \mathbb{N}$, so that the $P_i$ exhaust the polynomial ring over $\mathbb{F}_p$ in the countably many variables $X_i$ ($i \in \mathbb{Z}_+$). Apply Lemma 2 to $P = P_0$, and choose $b_0 := e$ large enough and an $\mathbb{F}_p$-basis $B_0 := E$ of $\mathbb{F}_{p^{b_0}}$ such that for all $f > 1$ and an appropriate extension $F \subseteq \mathbb{F}_{p^{b_0}}$ to a basis $B_1 := B_0 \cup F$ of this field, we have $P_{b_0,b_1}(F) \not\in \mathbb{F}_{p^{b_0}}$. Then, again using Lemma 2 choose $f > 1$ large enough, set $b_1 := b_0$ and $B_1 = B_0 \cup F$ such that the same as above holds, replacing $P_0$ by $P_1$ and $b_0$ by $b_1$. Proceed by induction to get $B := \bigcup_{i=0}^{\infty} B_i$. Apparently, $\langle B \rangle_+ = K$. Assume now that for $P \subseteq \mathbb{F}_p[X_1, X_2, \ldots]$ finite, we have $P(B) = K$. Then choose $m$ large enough such that $P \subseteq P_m$. Now note that

$$P_m(B) = P_m(B_m) \cup \bigcup_{i=m}^{\infty} P_{m,B_i}(B_{i+1} \setminus B_i).$$

But $P_m(B_m) \subseteq \mathbb{F}_{p^{bm}}$ and $P_{m,B_i}(B_{i+1} \setminus B_i) \subseteq P_{i,B_i}(B_{i+1} \setminus B_i) \subseteq \mathbb{F}_{p^{bi+1}} \setminus \mathbb{F}_{p^i}$ for $i \geq m$ by construction.

Hence for $i \geq m$ we have $P_m(B) \cap \mathbb{F}_{p^i} = P_m(B_i)$. But the size of this set is bounded by $|P_m||B_i|^m = |P_m|b_i^m$, whereas $|\mathbb{F}_{p^i}| = p^b_i$, which is eventually larger than the first.

This shows that $P(B) \subseteq P_m(B) \neq K$, as desired. $\square$
We end up with a question: Does there exist a countably infinite group admitting the Bergman property?

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