Predicting Accident Modes in a Gas Pipeline with Incomplete Parameters

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Abstract. The paper solves the problem of predicting the emergency states of gas pipelines in cases where the initial parameters of the pipeline are not fully known a priori and can change during operation. The coefficient of heat transfer from gas to the external medium and the coefficient of internal viscous friction of hydraulic resistance are considered as such parameters. The theoretical calculation of these coefficients with sufficient accuracy for practice is very difficult due to the significant uncertainty of the actual conditions of heat transfer along the route and the current state of the pipeline. To solve this problem, a mathematical model of a steady gas flow in a pipeline is proposed, which is based on the assumptions about the polytrophic nature of the gas, as well as the non-isentropic and nonadiabatic nature of its flow. Together with the model, a method for identifying undefined coefficients has been developed. The problem of building the corresponding computational procedures with unknown boundary conditions is solved. An analysis of emergency states of a gas pipeline for cases of complete rupture and partial destruction of its wall was carried out by means of numerical modeling. Examples of using the developed model for predicting emergency states of gas pipelines are given.

1. Introduction

The design of modern gas transmission systems involves a preliminary analysis of emergency conditions that may arise during the operation of pipelines [1-3]. Such an analysis requires the creation of mathematical models of the operation of gas pipelines in the indicated states and the development of appropriate computational procedures that allow predicting the expected characteristics of the gas at any point along the line [4, 5]. The forecast obtained in this way makes it possible to form a set of measures that reduce the likelihood of accidents, reduce direct economic losses and environmental damage, and ensure the safety of the gas pipeline during its operation [6-9]. The need to make such forecasts is increasing due to the intensive growth of the length of gas pipelines and their productivity. The safety requirements for the certification of gas transmission systems are also growing.

In accordance with the indicated need in the proposed work, the task is posed: to draw up a mathematical model that allows, according to the current, but not completely determined, state of the
gas pipeline operating in the nominal mode, determining the expected characteristics of the gas in case of an emergency full or partial rupture of the pipeline anywhere along the line.

2. Problem state

An extensive list of publications is devoted to modeling and analysis of the main and emergency regimes of gas flow through pipelines [6-25]. In works [6,10,11], the main modes of gas transportation are considered, and the need to take into account the non-isentropic and nonadiabatic nature of gas movement in the pipeline is indicated. A varied approach to taking this circumstance into account is demonstrated in papers [12-17], differing in various degrees of idealization of the gas state. In [18-23], the modes of emergency outflow from partial destruction of pipeline walls are considered, and in [24] – the case of complete destruction.

However, in these works, the main attention is paid to the characteristics of the gas in the fracture zone without analyzing these characteristics along the length of the gas pipeline. In most of the listed works, it is assumed that all parameters of the pumped mixture and the gas pipeline itself are known a priori. In particular, such hard-to-determine and non-stationary parameters as the coefficient of heat transfer from gas to the external medium and the coefficient of internal viscous friction of the pumped mixture are found by calculation or semi-empirical means. The estimates of these parameters obtained in this way are often far from the true values. In addition to the incompletely determined parameters of the gas pipeline, an unsolved problem is the incompleteness of the boundary conditions necessary for the implementation of numerical calculations of gas characteristics by difference methods.

The proposed paper is a further development of the work on the analysis of emergency gas outflows, and considers the case when the parameters of the pipeline are not fully known a priori.

3. Simulation of the initial state of the gas pipeline

We will consider the linear section of the main gas pipeline, limited by the compressor stations CS₁ and CS₂. Let us assume that at the boundaries of this section – at the outlet of CS₁ and at the inlet of CS₂ – the characteristics of the current state of the gas are available for direct measurement: \( P₁, P₂ \) – absolute gas pressure; \( T₁, T₂ \) – thermodynamic gas temperature; \( w₁, w₂ \) – gas velocity; \( T_l \) – the average temperature of the environment (soil, water, air) along the length of the section. The part of the parameters of the gas pipeline and the pumped gas is also known: \( h₁, h₂ \) – leveling of CS₁ and CS₂; \( L \) – the length of the section; \( D \) – the inner diameter of the pipeline; \( C_P, C_V \) – molar heat capacity of gas at constant pressure and volume; \( \mu \) – the molar mass of the gas; \( V_{\mu n} \) – molar volume of gas at temperature \( T_n = 273 \) K and pressure \( P_n = 1.01 \times 10^5 \) Pa.

Hereafter, we will assume that the following prerequisites are fulfilled: in the considered ranges of temperature, pressure and gas velocity, heat capacity \( C_P, C_V \) and the coefficient \( \lambda \) of the internal viscous friction are constant values; at any point of the pipeline the Clapeyron-Mendeleev equation is fulfilled: \( P V_{\mu} = R T \), where \( R \) is the universal gas constant; the process of gas flow through the pipeline occurs with the transformation of the work of viscous friction forces into the internal energy of the gas and with heat exchange with the external environment, i.e. is non-isentropic and nonadiabatic; transient processes in the considered section of the gas pipeline have been completed. In accordance with the systematic approach to solving the problem [25-27] and the laws of conservation of energy, mass and momentum [28,29] on the section of the main line between CS₁ and CS₂, we compose: equation (1) for the energy of one mole of gas at the beginning and at end of the pipeline; condition (2) of continuity; expression (3) for changing the amount of gas movement:

\[
C_P T_1 + \frac{\mu w_1^2}{2} + \mu g h_1 = C_P T_2 + \frac{\mu w_2^2}{2} + \mu g h_2 + C_T \pi D \frac{L}{\rho_1 w_1 S_1} \int (T(\ell) - T_L) d\ell; \tag{1}
\]

\[
\rho_1 w_1 S_1 = \rho_2 w_2 S_2; \tag{2}
\]

\[
P_1 S_1 - P_2 S_2 - \rho_1 w_1 S_1 (w_2 - w_1) = \lambda \frac{\rho_1 w_1 S_1 L}{2D} \int w(\ell) d\ell, \tag{3}
\]
where \( \rho_i = \mu \cdot (V_{i\mu})^{-1} \), \( V_{i\mu} = V_{\mu n} P_{n T_i} \cdot (P_{l T_n})^{-1} \) — gas densities at site boundaries and molar volumes, \( i = 1; 2; \) \( g \) — acceleration of gravity; \( S_1, S_2 \) — internal cross-sectional area at the boundaries of the pipeline, \( S_1 = S_2 = S, S = \pi D^2/4 \); \( w(\ell) \) — average gas velocity on a segment \( d\ell \). Note that the values \( C_T, \lambda \) and types of function \( T(\ell), w(\ell) \) not known a priori. The system of equations (1)-(3) is analytically unsolvable, and there is a need for its numerical integration. To do this, we use the well-known boundary conditions: initial \( P_{in} = P_1, T_{in} = T_1, w_{in} = w_1 \) and final \( P_{\ell} = P_2, T_{\ell} = T_2, w_{\ell} = w_2 \). Then, given some arbitrary values of \( C_T \) and \( \lambda \) and the step of the solution \( st-L \) \( (st << 1) \) along the length of the pipeline \( \ell \), we obtain system (1)-(3) in the difference form:

\[
C_p T_i + \frac{\mu w_i^2}{2} + \mu g h_i = C_p T_{i+1} + \frac{\mu w_{i+1}^2}{2} + \mu g h_{i+1} + \frac{\pi D \mu}{\rho_i w_i S} \left( \frac{T_i + T_{i+1}}{2} - T_L \right) \cdot st \cdot L;
\]

\[
\rho_i w_i S = \rho_{i+1} w_{i+1} S; \quad i = 0,\ldots,1/st;
\]

\[
(P_i - P_{i+1}) S - \rho_i w_i S (w_{i+1} - w_i) = \lambda \frac{\rho_i w_i S}{2D} \left( \frac{w_i + w_{i+1}}{2} \right) \cdot st \cdot L;
\]

As a result, equations (4)-(6) at \( i = 0 \) will contain the known initial conditions \( P_{in}, T_{in}, w_{in} \) and three unknown quantities \( P_{i+1}, T_{i+1}, w_{i+1} \) (the quantities \( \rho_i \) are expressed through \( P_i \) and \( T_i \)). Solving the system of algebraic equations (4)-(6) for \( i = 0 \), we obtain the values of the unknowns \( P_{i+1}, T_{i+1}, w_{i+1} \), which can be used as initial values for the next step. Having successively determined the state of the gas at 1/st points of the pipeline, we obtain the final values \( P_{1/st}, T_{1/st}, w_{1/st} \) that do not coincide with \( P_1, T_1 \) and \( w_1 \) due to the arbitrary choice of the values \( C_T \) and \( \lambda \). Using, for example, the Euclidean metric of this mismatch:

\[
E(C_T, \lambda) = \sqrt{\left( 1 - \frac{P_{1/st}}{P_1} \right)^2 + \left( 1 - \frac{T_{1/st}}{T_1} \right)^2 + \left( 1 - \frac{w_{1/st}}{w_1} \right)^2} \rightarrow \min,
\]

it is possible to organize a repeated solution of system (4)-(6), changing \( C_T \) and \( \lambda \) by any method of directed search for an extremum according to the criterion \( E(C_T, \lambda) \rightarrow \min \). As a result of such a search, unknown current parameters of the \( C_T \) and \( \lambda \) of the line will be found.

As an example of the calculation, a gas pipeline with the following parameters is considered: \( P_1 = 7 \) MPa; \( P_2 = 4.511 \) MPa; \( T_1 = 300 \) K; \( T_2 = 290 \) K; \( T_L = 280 \) K; \( w_1 = 10 \) m³/s; \( w_2 = 15 \) m³/s; \( h_1 = 0 \) m; \( h_2 = 500 \) m; \( L = 100-10^3 \) m; \( S_1 = S_2 = S = 1 \) m²; \( D = 1.128 \) m; \( C_T = 35.6 \) J/(mol·K); \( C_V = 27.3 \) J/(mol·K); \( R = 8.31 \) J/(mol·K); \( \mu = 16.04 \cdot 10^{-3} \) kg/mol; \( V_{in} = 22.4 \cdot 10^3 \) m³/mol; \( Q = \rho_i w_i S = 451.621 \) kg/s; \( st = 0.05 \).

The solution found by the coordinate search method: \( C_T = 1.507 \) J/(s·m²·K); \( \lambda = 0.0104 \).

4. Modeling of emergency modes of the line

4.1. Modeling the emergency mode of a line with a fully open pipeline

Let us consider an emergency mode with a completely open main line at the point \( \ell = \alpha \cdot L \) with the condition that the compressor station \( CS_1 \) operates at maximum capacity \( Q_{\text{max}} \). The relationship between the characteristics of the gas at the outlet of \( CS_1 \) (quantities with index 1) and in the section of the rupture (quantities with indices \( \alpha \)) is described by the system of equations:

\[
C_p T_1 + \frac{\mu w_1^2}{2} + \mu g h_1 = C_p T_\alpha + \frac{\mu w_\alpha^2}{2} + \mu g h_\alpha + \frac{\pi D \mu}{\rho_1 w_1 S} \int (T(\ell) - T_L) d\ell;
\]

\[
\rho_1 w_1 S = \rho_\alpha w_\alpha S = Q_{\text{max}}.
\]
\[
(P_i - P_a)S - \rho_1 w_i S (w_a - w_i) = \frac{\rho_1 w_i S ^{\alpha - L}}{2D} \int w(\ell) d\ell;
\]

\[
w_a^2 = \frac{\gamma RT_a}{\mu},
\]

where \(\gamma\) – the adiabatic exponent. Equation (11) is the condition for gas outflow from the discontinuity with the local speed of sound. For the subsonic outflow regime, instead of equation (11), the condition 
\(P_a = P_\alpha\) is used. The given characteristics of the line are \(T_1, Q_{\text{max}},\) the required ones are \(P_1, w_1, P_a, T_\alpha, w_\alpha\). Thus, the boundary conditions of the gas state are not completely determined from either side. It follows from this that the numerical solution of system (8)-(11) must be sought not by the fitting method, but simultaneously for \(n\) points of the line located with a small step \(s\) in the interval \([0; \alpha - L]\).

The gas state for each adjacent pair of points \(i\) and \(i + 1\), starting with \(i = 0\), is described by three equations (8)-(10), and the last pair \((1/s - 1; 1/s)\) is described by system (8)-(11). The discharge characteristics obtained using this model, depending on the value of \(\alpha\), are shown in Figure 1.

\[
\begin{align*}
T, T_\alpha, P_1, w_1, & \quad Q = 500 \text{ kg/s} \\
0 & \leq \alpha \leq 1
\end{align*}
\]

Figure 1. Characteristics of gas in the cross section of full rupture with limited productivity of CS1

\[
\begin{align*}
T, Q, T_1, P_1, w_1, & \quad \ell = 0.2L \\
0 & \leq \alpha \leq 1
\end{align*}
\]

Figure 2. Change in gas characteristics from \(\alpha\) with partial destruction of the pipeline at point \(\ell = 0.2L\)

4.2. Modeling emergency mode in case of partial destruction of the pipeline

Let’s move on to the analysis of the most common emergency mode – partial destruction of the pipeline. Fracture parameters: rupture area \(S_r = r; S\); \(r = [0; 1]\) and the distance \(\alpha\) of the rupture point from CS1, \(\alpha = [0; 1]\). Let us agree that the station CS2 maintains a constant value of the flow rate \(Q_2 = Q\), corresponding to the flow rate in the normal mode; station CS1 provides flow rate \(Q_1\) equal to the sum of flow rate \(Q_2\) and leakage \(Q_1\) through the damaged pipeline wall \(Q_1 = Q_1 + Q_2\), while maintaining pressure \(P_1\) and temperature \(T_1\) at the specified levels. The process of gas escape through the damaged pipeline wall is assumed to be adiabatic.

Let us introduce additional designations: \(P_a\) – gas pressure in the pipeline at the place of damage, \(P_i\) – pressure in the outflow jet. The given parameters of the model are \(P_1, T_1, Q_2 = Q,\) the sought ones are \(w_1, P_a, T_a, w_a, P_1, T_1, P_2, T_2, w_2\) (speed \(w_1\) is equal to the local speed of sound). The relationship between these quantities is determined by the system of equations (12)-(17):

\[
\left[ C_T T_1 + \frac{\mu w_1^2}{2} + \mu g h_1 \right] v_1 = \left[ C_T T_a + \frac{\mu w_a^2}{2} + \mu g h_a + C_T \frac{\pi D \mu}{\rho_1 w_1 S} \int (T(\ell) - T_1) d\ell \right] v_a;
\]
A method for predicting emergency states of a gas pipeline with incompletely defined parameters is proposed. The technique is based on the direct use of the laws of conservation of energy, mass and momentum for a gas flow.

The application of these laws in order to build mathematical models in the form of a system of equations describing the process of gas movement in the emergency main pipeline does not require the determination of the characteristics of the gas state in the final analytical form and makes it possible to abandon simplifying assumptions about the isentropic and adiabatic nature of this process with a linear distribution of the pressure drop along the main line.

The initial data for forecasting are the current, measurable gas characteristics at the boundaries of the considered section of the gas pipeline in the normal mode of its operation. Substitution of the values of these characteristics into the mathematical model makes it possible to determine the missing parameters of the turbulent flow and heat exchange of the pipeline with the external environment and, further, the desired state of the gas at the assumed points of the pipeline rupture.

When predicting emergency modes, the uncertainty of the boundary states of gas arises at the ends of the operational section of the pipeline. It is shown that in this case it is expedient to switch from a step-by-step one-dimensional calculation to solving a complete multidimensional problem.

5. Conclusions

Transition to a difference scheme similar to (4)-(6) with $P_t = 7 \times 10^6$ Pa, $T_1 = 300$ K, $Q = 451.62$ kg/s, $r = 0.04$; $S_t = 0.04$ m$^2$ led to a system of equations, the joint solution of which made it possible to obtain the desired characteristics of the gas in the pipeline, Figure 2.

\[
\begin{align*}
(C_p T_r + \frac{\mu w_r^2}{2} + \mu gh_r) v_r &= (C_p T_r + \frac{\mu w_t^2}{2} + \mu gh_t) v_t \\
+ (C_p T_2 + \frac{\mu w_2^2}{2} + \mu gh_2 + C_T \frac{\pi D u}{\rho_2 w_2 S} \int (T(\ell) - T_L) d\ell) v_2; \\
\rho_1 w_1 S &= \rho_r w_r S_r + Q; \\
(P_1 - P_a) S - \rho_1 w_1 (w_a - w_1) &= \lambda \frac{\rho_1 w_1 S}{\alpha L} \int w(\ell) d\ell; \\
(P_a - P_2) S - \rho_2 w_2 (w_2 - w_a) &= \lambda \frac{\rho_2 w_2 S}{\alpha L} \int w(\ell) d\ell; \\
\\
\frac{P_t}{P_a} &= \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma-1}{\gamma}}; \\
T_r &= T_a \left( \frac{P_t}{P_a} \right)^{\frac{\gamma}{\gamma-1}}; \\
w_r^2 &= \frac{\gamma R T_r}{\mu},
\end{align*}
\]

where $v_1$, $v_a$, $v_r$, $v_2$ – molar gas flow rates in the corresponding sections, mol/s, for example:

\[
v_r = \frac{\rho_r w_r S_r}{\mu}; \quad \rho_r = \frac{\mu}{V_r}; \quad V_r = \frac{P_t T_r}{n T_n}.
\]

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