Autonomous Perceptron Neural Network Inspired from Quantum computing

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Abstract

Recently with the rapid development of technology, there are a lot of applications require to achieve low-cost learning in order to accomplish inexpensive computation. However the known computational power of classical artificial neural networks (CANN), they are not capable to provide low-cost learning due to many reasons such as linearity, complexity of architecture, etc. In contrast, quantum neural networks (QNN) may be representing a good computational alternate to CANN, based on the computational power of quantum bit (qubit) over the classical bit. In this paper, a new algorithm of quantum perceptron neural network based only on one neuron is introduced to overcome some limitations of the classical perceptron neural networks. The proposed algorithm is capable to construct its own set of activation operators that enough to accomplish the learning process in a limited number of iterations and, consequently, reduces the cost of computation. For evaluation purpose, we utilize the proposed algorithm to solve five problems using real and artificial data. It is shown throughout the paper that promising results are provided and compared favorably with other reported algorithms.

keyword: Artificial neural networks and Quantum computing and Quantum neural networks

1 Introduction

The computational power of Classical artificial neural networks (CANN) is return to its nonlinear characteristics, the massively parallel-distributed structure and its ability to learn from a set of training examples and generalize for unseen data. However, CANN may faces many difficulties such as the
absence of concrete algorithms, optimum architecture, limitation capacity of memory, and expensive computation. These limitations have motivated many researchers to investigate new trends in neural computation domain \[1,2\]. One of the novel trends in this domain is to evoke properties and techniques of quantum computing into classical neural computation approaches \[3,4,5\]. Several researchers expected that quantum computing is capable to enhance the performance and overcoming the limitations of classical neural computation \[6,3\]. The beginning was in 1995 with Kak who introduced the concept of quantum neural computation\[7\]. Then, in 1998 Menneer defined a class of quantum neural network (QNN) as a superposition of single component networks where each is trained by using only one pattern \[3\]. Next, in 2000 Ventura et al. introduced A new associative memory technique based on Grover’s quantum search algorithm can solve the completion problem \[8\], then many researchers proposed QNN algorithms \[1,2,9,10,11,12\].

One of the known classical approaches is the classical Perceptron Neural Network (CPNN), which is applied only for linearly separable learning problems \[13\]. In other words, CPNN can not be applied for problems which have inseparable classes, such as XOR problem \[14\]. As long as the classical perceptron is concerned, some researchers have been tried to increase the efficiency of perceptron using the power of quantum computation. In 2001, Altaisky developed a simple quantum perceptron depends on selecting, what is known as, activation operator \[16\]. However, Altaisky’s approach consumed much time in order to select this activation operator, especially when the size of training data is large. Then, in 2003 Fei et al. introduced a new model of quantum neuron and its learning algorithm based on Altaisky’s perceptron. Fei’s model used the delta rule as a learning rule that yields considerable results such as computing XOR-function using only one neuron and nonlinear mapping property \[17\]. Unfortunately, Fei’s model did not provide us with a new way for deriving or selecting the activation operator. Nevertheless, Fei’s model is sensitive for the selection of the appropriate activation operator, which was the problem of Altaisky perceptron. Next, in 2006 Zhou et al. developed a quantum perceptron approach based on the quantum phase adequately, where it can be used to compute the XOR function using only one neuron \[18\]. The drawback of Zhou perceptron is that it requires many computation iterations to give a response that make the computation something costly.

However, CANN due to absence of concrete algorithms and optimum architecture, undetermined number of neurons in each hidden layer, initial weights, learning rate, regularized parameters,...etc, make it unable to learn autonomously and then consuming a lot of time till fix these problems. In 2014 Siomau introduced an autonomous quantum perceptron based on calculating a set of positive valued operators and valued measurements (POVM) \[19\]. However, Simoan’s perceptron is designed for classification tasks and, thereby, can not be applied to problems like quantum-Z gate and Hadamard gate \[19\]. Also, we show in this paper inspite that Siomau’s autonomous quantum perceptron \[19\] can learn XOR-function but can not classify all two classes inseparable applications.. In order to maximize the advantages of the nonlinear properties of CNNs and the linear properties of quantum computing, in 2015 we proposed an autonomous algorithm for competitive learning by combine the properties of CNN and quantum computing \[13\]. We proved that the proposed algorithm can learn autonomously and outperform all other quantum competitive models after only one iteration. Again,
in this paper, we propose a novel autonomous perceptron neural network algorithm inspired from the unique properties of quantum computing and CNN, we call this algorithm AQPNN. AQPNN have an optimal topology, only one neuron, and it improves the computational cost of Altaisky [16] quantum perceptron as well as the computational cost of Zhou quantum perceptron [18]. In addition, it has the ability to learn the problems that Siomou’s quantum perceptron [19] can not learn. Moreover, the proposed perceptron adopts the activation operator autonomously and consequently reduces the overall learning cost. Furthermore, it is capable to overcome the linearity restriction of classical perceptron where AQPNN can be viewed as a non-linear perceptron. Finally, it employee the training data directly to classify the unseen data.

To evaluate the proposed AQPNN algorithm, in this paper we begin to learn our algorithm on three known simple gates, namely, the Z-gate problem, the Hadamard-gate problem, and the XOR-gate problem. Regarding to the latter problem, our results using only one iteration outperformed the results of Zhou’s perceptron [18] and outperform all perceptron algorithms [14, 18, 19] in the number of training data. In addition, we used the proposed AQPNN perceptron to achieve classification for a problem of two-overlapped classes includes 191 patterns, also we show that autonomous quantum perceptron [19] can not classify this aproblem. The proposed AQPNN achieves a classification rate reaches to 97.3 % using only one iteration as well. The most important evaluation for the proposed AQPNN approach is that we used it to achieve the classification task of the IRIS database [20]. In this experiment, we conduct a fair comparison with Simou’s autonomous perceptron [19] where the proposed AQPNN outperforms it as well.

The paper is organized as follows: section 2 gives a description of the AQPNN and its learning algorithm. Section 3 shows the efficiency of the AQPNN algorithm via solving various problems. Section 4 discusses and analyzes the performance of the proposed AQPNN. Finally, section 5 concludes this paper and presents our future work.

2 Autonomous Quantum Perceptron Neural Network (AQPNN)

The proposed (AQPNN) is a quantum neural network approach that consists of only one neuron with \( n \) qubit inputs \( |x_1\rangle, |x_2\rangle, \ldots, |x_n\rangle \). A set of weight operators \( w_1, w_2, \ldots, w_n \) is assumed such that one weight operator is associated with each input, and \( |y_{net}\rangle \) is the final network response; see Fig.1. The operators \( F_j \) refers to a set of unique activation operators of the proposed perceptron.

The proposed AQPNN approach is based on a supervised learning procedure, which is provided with a set of learning patterns (inputs/targets) in qubit form. For each input pattern presented to the network, the weighted sum qubit \( |y_j\rangle \) is calculated using the form:

\[
|y_j\rangle = \sum_{i=1}^{n} w_i |x_i\rangle = \begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix},
\]

where \( \alpha_j \) and \( \beta_j \) are the probability amplitudes of the weighted sum qubit for \( j^{th} \) pattern in the training set. The weight operators are updated at time \( t \) According to some researchers [16, 17] using
the following rule:

$$w_i(t+1) = w_i(t) + \gamma |e\rangle\langle x_i|,$$

(2)

where $\gamma$ is the learning rate, $|e\rangle = (|d\rangle - |y\rangle)$ is the perceptron error and $|e\rangle\langle x_i|$ denotes the outer product of vectors $|e\rangle$ and $\langle x_i|$. Once the weighted sum is calculated for all the available patterns from equation Eq.(1), then the set of activation operators can be calculated using the form:

$$F_j = \begin{bmatrix} \cos \theta_j & -\sin \theta_j \\ \sin \phi_j & \cos \phi_j \end{bmatrix}$$

(3)

where $j = 1, 2, 3, ..., m$, and $m$ is the set of unique activation operators (repeated activation operators are discarded) where $m \leq N$ (number of training data set). The parameters $\theta_j$ and $\phi_j$ are two real valued angles calculated using the form:

$$\begin{bmatrix} \cos \theta_j & -\sin \theta_j \\ \sin \phi_j & \cos \phi_j \end{bmatrix} \begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} = \begin{bmatrix} \alpha_{d_j} \\ \beta_{d_j} \end{bmatrix}$$

(4)

where $[\alpha_j, \beta_j]^T$ is the weighted sum qubit and $(\alpha_{d_j}, \beta_{d_j})^T$ is the target qubit. The aim of each activation operator is to transform the weighted sum qubit into the given target qubit. After calculating the set of all activation operators, the output of the autonomous quantum perceptron is given using the superposition of all activation operators in the following form:

$$|y_{\text{output}}\rangle = \sum_{j=1}^{m} F_j \sum_{i=1}^{n} w_i |x_i\rangle$$

(5)

where $|y_{\text{output}}\rangle$ is the network output. From Eq.(5), it is clear that this result due to the the effect of the activation operators (interference) on the weighted sum qubit resulted when any pattern presented to the network. One qubit only from these qubits will be the response of the network and can be specified by the following form:

$$|y_{\text{net}}\rangle = L(|(y_{\text{output}} \circ |y_{\text{output}}\rangle - C)\rangle \circ |D\rangle)$$

(6)
where $C = [1, 1, \ldots, 1]^T$ and $D = [|d_1\rangle, |d_2\rangle, \ldots, |d_n\rangle]^T$ is the target qubits vector. The function $L$ retains the smallest absolute value and make it equal one and the rest of values to be equal zero. The operation $\circ$ achieves the Hadamard product operation which is defined as\[^{21}\]. For any two matrices, $A$ and $B$ of the same dimension, $m \times n$, the Hadamard product, $A \circ B$, is a matrix, of the same dimension as the operands, is given by:

$$
(A \circ B)_{i,j} = (A)_{i,j} \cdot (B)_{i,j}.
$$

(7)

the Hadamard product is a binary operation that takes two matrices of the same dimensions, and produces another matrix where each element $i,j$ is the product of elements $i,j$ of the original two matrices. Explicitly (7) can be written as:

$$
A \circ B = \begin{bmatrix}
    a_1b_1 \\
    a_2b_2
\end{bmatrix}.
$$

2.1 The learning algorithm of AQPNN

According to the description given above, the AQPNN learning algorithm is divided into two main stages: The first stage is imbedded in both Eq.(4) and Eq.(5), where the AQPNN algorithm collects information about the problem in hand by constructing a set of activation operators. In the second stage the AQPNN takes the decision about the network response according to Eq.(6) based on the gathered information. The AQPNN learning algorithm can be summarized in the following steps:

1. Set all $F_i = I$ (identity matrix). Then choose the initial weight operators $w_i$ randomly, set the learning rate $0 < \gamma < 1$ and set iteration number $k = 1$.

2. Calculate the weighted sum qubit for each given pattern using Eq.(1).

3. Compare each weighted sum qubit for the patterns of each class with all other weighted sum qubits for other classes. In this case, we have two possibilities:

   (a) If each weighted sum qubit for any class does not equal the same value for any weighted sum in any other classes, then go to step 5, else, go to step 4.

   (b) If the value of any weighted sum qubit is zero, then go to step 4, else, go to step 5.

4. Update the weight operators using Eq.(2), set $k = k + 1$, go to step 2.

5. Calculate the activation operator for each weighted sum qubit using Eq.(4).

The superposition of output qubits of the network is given by Eq.(5) whereas the net response of the AQPNN is given by Eq.(6).

3 The Efficiency of AQPNN

To investigate the computational power of the proposed AQPNN algorithm, we use it here to solve different problems. Toward a fair evaluation for the proposed autonomous perceptron (AQPNN), we
conducted a comparison with other reported autonomous quantum perceptron ([18] and [19]) when the comparison is proper to the problem in hands. In the following two subsections, section 3.1 and 3.2, we demonstrate the ability of the proposed algorithm to learn on some simple one-qubit gates which cannot done by other reported autonomous quantum perceptron algorithm [19], then we begin to study the proposed algorithm on three complicated problems, sections 3.3-3.5, to evaluate the efficiency of the proposed algorithm compared with the other autonomous perceptron algorithm [19].

3.1 The quantum Z-gate

Quantum Z-gate is one of the gates widely used in quantum algorithms. To learn this quantum-gate, we consider that the patterns which describe this problem can be described by two classes. The first class (A) is described by the pattern:

\[ P_1 = \{ |x_1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |d_1\rangle = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \} \]  

(8)

while the class (B) is described by the pattern:

\[ P_2 = \{ |x_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |d_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \} \]  

(9)

Let us assume that the initial weight operators is chosen arbitrary as \( w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \). As soon as the two patterns are introduced into the AQPNN network, one obtains the weighted sum according to Eq.(1) as follows:

\[ |y_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |y_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]  

(10)

So, we can calculate the activation operator corresponding to the input weighted sum \( |y_1\rangle \) from the following equation:

\[ \begin{bmatrix} \cos \theta_1 - \sin \theta_1 \\ \sin \phi_1 \cos \phi_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]  

(11)

Form this equation, we get \( \theta_1 = 0, \phi_1 = 180 \), then the activation operator corresponds to the input pattern \( P_1 \) as:

\[ F_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]  

(12)

Similarly, the activation operator, \( F_2 \) corresponding to the input weighted sum \( |y_2\rangle \) can be obtained by solving the following equation:

\[ \begin{bmatrix} \cos \theta_2 - \sin \theta_2 \\ \sin \phi_2 \cos \phi_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]  

(13)

However, if we set \( \theta_2 = 0, \phi_2 = 180 \), then \( F_2 = F_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \).

Since \( F_1 = F_2 \), this means that the AQPNN needs only one input pattern as a training data set.
Accordingly, the superposition output, given in Eq. (5), will be \( |y_{\text{output}}\rangle = F \sum_{i=1}^{n} w_i |x_i\rangle \), which means that the quantum Z-gate is trained using only one iteration.

| Name of algorithm | AQPNN | Autonomous quantum perceptron [19] |
|-------------------|-------|-----------------------------------|
| No. of iterations | 1     | Not applicable                    |
| No. of training patterns | 1     | Not applicable                    |

Table 1: A comparison between the proposed AQPNN and Autonomous quantum perceptron [19] algorithms to learn quantum Z-gate

Table 3 shows that quantum Z-gate can not archived by the Autonomous quantum perceptron [19] algorithm while it done by the proposed AQPNN algorithm. Where the qubit \( \psi = \alpha |0\rangle + \beta |1\rangle \) can be in an infinite states according to values of the probability amplitudes \( \alpha \) and \( \beta \) [22], and AQPNN need only one pattern to learn quantum Z-gate, then AQPNN can generalize to an infinite number of unseen qubits.

### 3.2 The Hadamard-gate

To solve this problem, we consider the following two classes:

**Class A**: This class is defined by the pattern \( P_1 \) which is defined as:

\[
P_1 = \{ |x_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |d_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \}\]  
(14)

**Class B**: This class is defined by the pattern \( P_2 \) which is defined as:

\[
P_2 = \{ |x_2\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |d_2\rangle = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \}\]  
(15)

On the other hand, let us consider that the initial weight operators is chosen arbitrary as \( w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \). However as soon as we introduce the two patterns to the AQPNN network, we obtain the weighted sum according to Eq. (1), as follows:

\[
|y_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |y_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]  
(16)

To calculate the set of activation operators, \( F_1 \) corresponding to \( |y_1\rangle \), we solve the system:

\[
\begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \phi_1 & \cos \phi_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]  
(17)

If we set \( \theta_1 = -45, \phi_1 = 135 \), one gets the activation operator \( F_1 \) as

\[
F_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\]  
(18)
Similarly, we can get $F_2$ corresponding to $|y_2\rangle$ by solving the system

$$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\phi_2 & \cos\phi_2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (19)$$

Again, if we set $\theta_2 = -45, \phi_2 = 135$, we find that:

$$F_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (20)$$

As the situation of the quantum Z-gate, the superposition output in this case can be written as $|y_{output}\rangle = F\sum_{i=1}^{n} w_i|x_i\rangle$, where $F_1 = F_2$. This means that the AQPNN needs only one input pattern as a training data set. As the situation of Z-gate, the Hadamard-gate can be trained using only one iteration and the AQPPN uses only one activation operator. Certainly, this ensures reduce the computation cost. Table 3 shows that quantum Hadamard-gate can not archived Autonomous

| Name of algorithm | AQPNN | Autonomous quantum perceptron [19] |
|-------------------|-------|-----------------------------------|
| No. of iterations | 1     | Not applicable                    |
| No. of training patterns | 1 | Not applicable                   |

Table 2: A comparison between the proposed AQPNN and Autonomous quantum perceptron [19] algorithms to learn to learn Hadamard-gate

quantum perceptron [19] algorithm while it done by the proposed AQPNN algorithm. Also, AQPNN can generalize to an infinite number of unseen qubits.

### 3.3 The XOR-function

It is well known that, there are two classes for the XOR-function can be defined as follows:

**Class A:** This class is described by two patterns $P_1$ and $P_2$ which can be given as follows:

$$P_1 = \left\{ |x_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |x_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |d_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

$$P_2 = \left\{ |x_1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |x_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |d_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \quad (21)$$

**Class B:** This class is described by two patterns $P_3$ and $P_4$ which can be given as follows:

$$P_3 = \left\{ |x_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |x_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |d_3\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

$$P_4 = \left\{ |x_1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, |x_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |d_4\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad (22)$$
Assume that the initial weight operator takes randomly the value \( w_1 = w_2 = \begin{bmatrix} 1.1 & 1.2 \\ 0 & 0 \end{bmatrix} \). If the four patterns are introduced into the AQPNN network, then we obtain the weighted sum of each pattern after only one iteration as follows:

\[
|y_1\rangle = \begin{bmatrix} 2.2 \\ 0 \end{bmatrix}, \quad |y_2\rangle = \begin{bmatrix} 2.4 \\ 0 \end{bmatrix}, \quad |y_3\rangle = \begin{bmatrix} 2.3 \\ 0 \end{bmatrix}, \quad |y_4\rangle = \begin{bmatrix} 2.3 \\ 0 \end{bmatrix} \tag{23}
\]

It is easy to observe that, the first and the second weighted sum qubits have different values than the third and fourth weighted sum qubits (i.e. either of \(|y_1\rangle\) or \(|y_2\rangle\) has a different value than \(|y_3\rangle\) and \(|y_4\rangle\)). Also, we may observe that \(|y_3\rangle = |y_4\rangle\). As these two patterns are in the same class B, this implies that there only three activation operators will take the following forms:

\[
F_1 = \begin{bmatrix} 0.4545 & -0.8907 \\ 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.4167 & -0.9091 \\ 0 & 1 \end{bmatrix}, \quad F_3 = F_4 = \begin{bmatrix} 0 & -1 \\ 0.4348 & 0.9005 \end{bmatrix} \tag{24}
\]

Since \(F_3 = F_4\), this means that the AQPNN needs only three input patterns as a training data set, in contrast to other quantum and classical perceptron neural network models which require four input patterns in order to train the XOR-funtion (see \([14, 16, 18, 19]\)).

| Name of algorithm | AQPNN | AQP \([19]\) | QP \([18]\) | CPNN(One neuron) \([14]\) |
|-------------------|-------|-------------|-----------|-----------------------------|
| No. of iterations | 1     | 1           | 16        | Not applicable by one neuron |
| No. of training patterns | 3     | 4           | 4         | 4                           |

Table 3: A comparison between the proposed AQPNN perceptron, quantum perceptron \([18]\) and the classical perceptron to solve the XOR-function

Then, the superposition output can be calculated as follows:

\[
|y_{output}\rangle = \sum_{j=1}^{m=3} F_j \sum_{i=1}^{n=2} w_i |x_i\rangle
\]

The efficiency of the proposed AQPNN algorithm to solve the XOR-function problem is given in Table 3.3. It is clear that the AQPNN requires only one iteration in order to train the XOR function, in contrast to the Zhou’s quantum perceptron \([18]\) which requires 16 iterations for the same task. Also, the classical perceptron can not solve this problem using only one neuron.

### 3.4 A problem of classification

In this section, we investigate the power of using the proposed AQPNN algorithm in a classification problem of two-overlapped classes using artificial data. This kind of classification problems can be regarded as a complex generalization for the XOR-problem, which we are treated in section 3.3.
Please pay attention that, the current problem is treated in literature using only 9 input patterns for training and testing phases \cite{23, 24}. Toward generalization and complexity, in this paper we treat this problem using 15 input patterns for training phase and 176 input patterns for testing, where these testing patterns are totally different than training patterns. For simplicity, Fig. 2 shows only the 15 input patterns which among them we can choose and generalize the 176 input patterns.

The input patterns are divide into two classes: the first class with oval-shape has a target $|\theta\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ with arbitrary input patterns given in Table 4. The second class with square-shape has the target $|\theta\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ with arbitrary input patterns given in Table 5.

It’s clear that the values of the input patterns shown in Tables 3 and 4 are classical data. For convenient usage, it is preferred to write these values as quantum qubits in the form $|\eta\rangle = a|0\rangle + b|1\rangle$. 
where $|a|^2 + |b|^2 = 1$. For example, the classical pattern $P_1 = (0.1, 0)$, can be written as

$$P_1 = \left\{ |x_1\rangle = \begin{bmatrix} 0.1 \\ 0.9950 \end{bmatrix}, |d_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}. \tag{26}$$

Toward evaluation and comparison, we conducted this experiment also using another autonomous quantum perceptron model that described in [19]. For a fair evaluation, the experiments’ conditions are unified for both approaches. Fig. 3 demonstrates a relation between number of input patterns and the classification accuracy of both approaches in training and testing phases.

In training phase, the proposed approach reaches to 100% as classification rate and keeps on it whereas the other approach [19] reaches, at most, 40%. In testing phase, the proposed approach starts with 98% but as we increase the number of input pattern accuracy approaches 100%. In the same time, the other perceptron [19] still perform worse even if we increase the number of input patterns. But in order to say that the perceptron network learned to classify the application in hand, the training data must be classified correctly [15]. On the other hand, Fig. 3 tell us that the quantum perceptron model [19] did not classified the training data correctly. In other words, inspite this model [19] has classified the XOR-function but can not classify all applications that have two-class nonlinear separable.
3.5 A problem of IRIS plant data set

We proceed now to apply the proposed approach on a real data set. Namely, we use the IRIS database which we obtained from UCI Machine Learning Repository [20]. The IRIS data set includes three classes of 50 objects, where each class refers to a type of IRIS plant: IRIS Setosa, IRIS Versicolour and IRIS Virginica. The attribute that already has been predicted belongs to a class of IRIS plant. The list of attributes presented in the IRIS data set can be described as: categorical, nominal and continuous. The first class is linearly distinguishable from the remaining two classes, which don’t being linearly separable from each other. The 150 instances, which are equally distributed between the three classes, contain the following four numeric attributes: sepal length, sepal width, petal length, and petal width.

In this experiment, we chose randomly only 50% of the patterns as training data whereas the other 50% of patterns are reserved as testing data. Also the learning rate is chosen randomly. Table 3.5 shows a comparison between the proposed perceptron and the perceptron described in [19] in achieving the classification task of IRIS data.

| Algorithm name                        | AQPNN | Autonomous quantum perceptron [19] |
|----------------------------------------|-------|----------------------------------|
| No. of iterations                      | 1     | 1                                |
| No. of neurons                         | 1     | 2                                |
| Accuracy of classification             | 97.3% | 96%                              |

Table 6: A comparison between the proposed AQPNN perceptron and Autonomous quantum perceptron [19] to classify the IRIS plant data set

It is clear from Table 3.5 that both perceptron approaches require a unique iteration only to give a result. In the same time, the proposed AQPNN requires only one neuron for processing, whereas the other perceptron [19] requires two neurons for the same task. It is worth to mention here that the performance of the autonomous quantum perceptron [19] improves drastically for this application comparing with application given in section 3.4. This may give us the indication that the perceptron [19] is application (or database) dependent. Overall, however this improvement in performance of the approach [19] still the proposed APQNN approach outperforms it. Needless to assert that the traditional perceptron can not be used in such classification problems using only one neuron.

4 Discussions and Results Analysis

It is worth now to discuss and analyze the performance of the proposed AQPNN algorithm. It is clear from the above examples that the computational power of the proposed AQPNN is high. In this regard, four essential remarks one may record: First remark: the proposed AQPNN algorithm does not utilize all the training data like other quantum perceptron algorithms [14, 18, 19]. For example, in the situation of quantum Z-gate and quantum Hadamard-gate, we need only one training
input pattern because we used only a unique activation operator. Whereas in the XOR function, it is required three training input patterns in order to accomplish the learning process, in contrast to other quantum perceptrons which require four training input patterns. In other words, the AQPNN algorithm is capable to reduce both the computation time and the number of activation operators used in training.

Second remark: this is concerned with the relation between the initial weight operators and the activation operators. In the situations of the quantum Z-gate and the quantum Hadamard-gate, the initial weight operator was the unitary operator, whereas in case of the XOR-function it was not the unitary operator. The reason for this is due to the nature of the unitary operator $U$ where $UU^\dagger = I$.

Then, using Eq. (3), which include the formula of the activation operators, we have:

\[
\begin{bmatrix}
\cos \theta_j - \sin \theta_j \\
\sin \phi_j & \cos \phi_j
\end{bmatrix}
\begin{bmatrix}
\cos \theta_j & \sin \phi_j \\
-\sin \theta_j & \cos \phi_j
\end{bmatrix}
= \\
\begin{bmatrix}
1 & \sin (\theta_j - \phi_j) \\
\sin (\theta_j - \phi_j) & 1
\end{bmatrix}
\]

It is clear that the value of activation operator depends on the values of $\theta_j$ and $\phi_i$, which in turn depend on the initial weight operators and, thereby, the training data.

Third remark: this remark is concerned with using the proposed AQPNN in the classification of inseparable applications that can not be solved using classical perceptron using only one neuron. This advantage is clear in case of the classification of two-overlapped classes and IRIS data problems, as well as, in case of solving the XOR-function.

Fourth remark: this remark is concerned with the comparison with another autonomous quantum perceptron, the closest algorithm to ours, given in [19]. As it is described above, the proposed AQPNN has a stable behavior for all applications/databases and can work well even with limited number of training data. In contrast, the autonomous perceptron given in [17] is application/database dependent. Simply speaking, the approach [19] starts with a worse performance and does not improved when we increased the number of input patterns in the application given in the classification problem given in section 3.4. Also, we demonstrated that the autonomous perceptron given in [19] can not classify all two-classes nonlinear separable applications.

In contrast, the performance improved drastically when we increase the number of training patterns in the IRIS problem given in section 3.5. Intuitively, this behavior is contradicted with the state of generalization or performance stability. In the same time, it was clear that the proposed AQPNN requires limited number of data for training that, certainly, reduces the overall computation cost.

In addition, the algorithm [19] requires that the number of neurons (or perceptrons) is half of the number of classes included in the database. In other words, n quantum neurons are required to classify $2n$ original classes with $(2n)!/[2(2n-2)!] - n$ additional two-class superposition of these classes. This means that the perceptron [19] is subject to enlarge according to the number of classes included in the database, which may make the computation much complex. In contrast, the proposed AQPNN has a fixed size, only one neuron, whether the kind of database or number of included classes. Furthermore, the perceptron [19] could not able to achieve the learning process in some gates, such as quantum NOT-gate and quantum Hadamard-gate. This is because that, the perceptron [19] is designed for classification purposes further learning purposes of gates.
5 Conclusions and Future Work

It is known that the computational power of the classical artificial neural networks CANN can not be suitable to achieve learning in a real time. Based on the computational power of quantum bit over the classical bit, quantum neural networks represent powerful computational alternates to classical counterparts. In this paper, we introduced a novel autonomous algorithm for the quantum perceptron neural network (AQPNN) that includes only one neuron. The proposed algorithm constructed its own self-adaptive activation operators in order to achieve the learning process in a limited number of iterations and, thereby, reduce the cost of overall computation. This reduction in computation cost is due to using a limited number of activation operators required to train the proposed perceptron. Another advantage of AQPNN is that it can solve linearly inseparable applications that can not be solved using autonomous quantum perceptron [19] and classical perceptron using only one neuron.

For fair evaluation, we compared the performance of the proposed AQPNN with the performance of other quantum perceptron in achieving the classification and learning tasks. Overall, we found the proposed AQPNN outperforms the other perceptron approaches both in computation cost and in classification accuracy. For future research plans, we are motivated to decrease the number of activation operators in order to make further reduction of computation cost and training data. Also, we are motivated to study the best initial weights for AQPNN.

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