Implications of $\Delta \rho$ and CHARM II data for $Z'$ Physics

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Abstract

We discuss the constraints on the $Z'$ model parameters coming from $Z$-pole and low-energy $\nu_{\mu} - e$ scattering data in the frame work of GUT- motivated models. We find that when the coupling constant of the extra $U(1)$ is small (large) the parameter space is mainly determined by the $Z$-pole (the low-energy $\nu_{\mu} - e$ scattering) data.

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1 Introduction

Deviations of the precisely measured electroweak observables from the predictions of the SM [1] enable one to search for the existence of new physics. Although there are various possibilities for extending the SM, one of the simplest and well-motivated extension is the addition of an extra $U(1)$ to its $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge structure [2]. The Z-pole and neutral current data can be used to search for and set limits on the existence of the $Z'$ models [3]. The Z-pole observables are sensitive only to the $Z - Z'$ mixing angle due to the modifications in the vector and axial couplings of the fermions to the Z-boson. On the other hand, off-Z-pole $e^+e^-$ data, and low energy $\nu_{\mu} - e$, $\nu$-hadron and parity-violating $e$-hadron scattering experiments are sensitive to both $Z - Z'$ mixing and $Z'$ mass [3]. Therefore, accompanying the Z-pole data, the low-energy observables are particularly useful in establishing the constraints on the $Z'$ physics. For example, atomic parity violation experiments (particularly for cesium) has already been analyzed in this direction [4, 5].

In this work we investigate the implications of $\Delta \rho$ and CHARM II data [6] on the $Z'$ models by comparing them with the SM predictions [7, 8, 9]. In the analysis, we illustrate two GUT-motivated models and reach conclusions of general applicability.

In Sec. 2 we give the relevant formulae for the effective vector and axial couplings in the framework of the $Z'$ models. Moreover, we list SM expressions and describe the GUT-motivated $Z'$ models that will be the subject matter of the analysis.

In Sec. 3 we give a detailed numerical analysis of the appropriate parameter space by taking into account both points implied by the CHARM II data [6].

In Sec. 4 we conclude the work and remark on the interplay between low-energy and Z-pole determinations of the parameter space.

2 $\nu_{\mu} - e$ Scattering in $Z'$ Models

In addition to the usual SM gauge group, we consider an additional Abelian group $U(1)_Y$, with coupling constant $g_Y$, under which the left-handed lepton doublets and right-handed charged leptons have charges $Q_L$ and $Q_E$, respec-
tively. The neutral vector bosons $Z$ of $SU(2)_L \times U(1)_Y$ and $Z'$ of $U(1)_{Y'}$ mix with each other. The mass eigenstate vector bosons $Z_1$ and $Z_2$ can be obtained after diagonalizing the $Z - Z'$ mass-squared matrix:

$$Z_1 = \cos \theta Z + \sin \theta Z'$$
$$Z_2 = -\sin \theta Z + \cos \theta Z'$$

which define the $Z - Z'$ mixing angle $\theta$. The $Z'$ effects can show up in various physical quantities whose comparison with the SM predictions allow us to constrain $Z'$ parameters in a model independent way. In general, $Z'$ effects can be tracked by their appearance in four distinct physical quantities. Firstly, due to mixing, $Z_1$ is lighter than the canonical $Z$, so that the $\rho$ parameter predicted by $Z_1$ mass is larger than the one predicted by $Z$ mass. Secondly, coupling of $Z_1$ to fermions differs from those of $Z$ due to the mixing in (1). Thirdly, the $Z'$ exchange will modify the neutral current amplitudes as an explicit function of the $Z'$ mass. Finally, $\sin^2 \theta_W$ differs from the SM prediction because of the $Z'$ contribution to the $\rho$ parameter. In what follows we shall discuss all of these effects in the framework of the $\mu - e$ scattering.

The low-energy four-fermion effective lagrangian relevant to the $\nu_\mu - e$ scattering can be written as

$$-L^{\nu_\mu - e} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha(1 - \gamma^5)\nu_\mu \bar{e} \gamma_\alpha (g^{\nu_\mu e}_V - g^{\nu_\mu e}_A \gamma^5) e$$

which defines the effective vector coupling $g^{\nu_\mu e}_V$ and axial coupling $g^{\nu_\mu e}_A$. $\nu_\mu - e$ scattering is a pure neutral current process mediated by both $Z_1$ and $Z_2$. Thus $g^{\nu_\mu e}_V$ and $g^{\nu_\mu e}_A$ carry the $Z'$ effects through the contribution of the intermediate vector bosons, and the modification of the fermion-vector boson couplings due to the mixing. In the $Z'$ model mentioned-above these couplings are given by

$$g^{\nu_\mu e}_V = \rho_1 \{\tilde{g}^{\nu_\mu e}_V \cos^2 \theta + \lambda(1 + r + 2\tilde{g}^{\nu_\mu e}_V) \sin 2\theta + 8\lambda^2(1 + r) \sin^2 \theta\}$$
$$g^{\nu_\mu e}_A = \rho_2 \{\tilde{g}^{\nu_\mu e}_A \cos^2 \theta - \lambda(1 + r + 2\tilde{g}^{\nu_\mu e}_A) \sin 2\theta + 8\lambda^2(1 + r) \cos^2 \theta\}$$

$$g^{\nu_\mu e}_V = \rho_1 \{\tilde{g}^{\nu_\mu e}_V \cos^2 \theta + \lambda(1 - r + 2\tilde{g}^{\nu_\mu e}_V) \sin 2\theta + 8\lambda^2(1 - r) \sin^2 \theta\}$$
$$g^{\nu_\mu e}_A = \rho_2 \{\tilde{g}^{\nu_\mu e}_A \sin^2 \theta - \lambda(1 - r + 2\tilde{g}^{\nu_\mu e}_A) \sin 2\theta + 8\lambda^2(1 - r) \cos^2 \theta\}$$

Here $\rho_1$ and $\rho_2$ are the $\rho$ parameters of $Z_1$ and $Z_2$:

$$\rho_1 = \frac{1 + \eta^2 \tan^2 \theta}{1 + \tan^2 \theta}$$
\[ \rho_2 = \frac{1 + \eta^2 \tan^2 \theta}{\eta^2 (1 + \tan^2 \theta)} \]  

where \( \eta = M_{Z_2}/M_{Z_1} \). Next \( \lambda = (g_Y Q_L)/(2G) \), \( r = Q_E/Q_L \), and \( G = \sqrt{g_Y^2 + g_Y^2} \). Finally, the tilded vector and axial couplings are given by

\[ \tilde{g}_V^{\nu e} = (g_V^{\nu e})_{SM} - 2 \left( \frac{s_W^2 c_W^2}{c_W^2 - s_W^2} \right)_{SM} \Delta \rho \]  

\[ \tilde{g}_A^{\nu e} = (g_A^{\nu e})_{SM} \]  

where the subscript SM refers to the SM value of the associated quantity. Here \( \Delta \rho \) is the deviation of the \( \rho_1 \) from its SM value:

\[ \Delta \rho = \rho_1 - 1 = (\eta^2 - 1) \sin^2 \theta \]  

It is the \( Z' \) contribution to \( s_W^2 \) that causes a finite difference between \( g_V^{\nu e} \) and \( (g_V^{\nu e})_{SM} \). As we see \( Z' \) contribution to \( s_W^2 \) is proportional to \( \Delta \rho \).

We identify \( g_V^{\nu e} \) in (4) and \( g_A^{\nu e} \) in (5) with the experimental result, and require the \( Z' \) contributions at their right hand sides to close the gap between experimental result and the SM determination. We use the SM results evaluated for \( M_{H} = M_{Z} \) so that there is no contribution to a particular observable from the Higgs loop. This is a convenient approach as we have a much more complicated Higgs sector in \( Z' \) models (See, for example, \[10\] and references therein) with increased number of scalars whose contributions to a particular observable differ from the one in SM. Moreover, the tree-level \( Z' \) contributions in (4) and (5) do already yield the relevant parameters of a \( Z' \) model to be fixed at the leading order. Namely, we assume loop corrections to (4) and (5) coming from the \( Z' \) sector negligibly small.

Although we treat \( Z' \) effects at the tree-level, we take all the loop contributions into account when analyzing the SM contribution. One of the basic SM parameters entering (4) and (5) is the weak mixing angle \( s_W^2 \) whose meaning is to be fixed against the ambiguities coming from the renormalization scheme and scale. As was discussed in \[1\] in detail, the value of \( s_W^2 \) extracted from \( M_Z \) in \( \hat{M} \tilde{S} \) scheme at the scale \( M_Z \) is less sensitive to \( m_t \) and most types of the new physics compared to the others. Following, \[1\], we adopt this definition of \( s_W^2 \) to be denoted by \( \hat{s}_W^2 \) from now on. Consistent with this choice for \( s_W^2 \), the SM expressions for vector and axial couplings in
and (9) are given by

\[ (g_{\nu e}^V)_{SM} = \rho_{\nu e} (-1/2 + 2 \tilde{\kappa}_{\nu e} \hat{s}_Z^2) \] (11)

\[ (g_{\nu e}^A)_{SM} = \rho_{\nu e} (-1/2) \] (12)

whose numerical values can be found in [4].

In the analysis below we shall restrict \( \eta \) and \( \theta \) by requiring (4) and (5) be satisfied within the experimental error bounds. In addition to this, we take into account the constraints coming from \( Z' \) pole data by requiring \( \Delta \rho \) (10) be in the band induced by the present world average for \( \rho_1 \) [1]:

\[ 0.999 \leq \rho_1 \leq 1.0006 \] (13)

which puts an additional restriction on the allowed \( \eta - \theta \) space. \( \Delta \rho \) calculated from (13) includes \( m_t \) effects already; however, Higgs effects are discarded by taking \( M_H = M_Z \) as mentioned before. Although \( \Delta \rho = \rho_1 - 1 \) takes both negative and positive values, \( Z' \) models require it be positive as \( M_{Z_1} \) is less than \( M_Z \) due to mixing.

The lepton charges \( Q_L \) and \( Q_E \) and gauge coupling \( g_{Y'} \) are model dependent. In literature there are various \( Z' \) models [4] with definite predictions for charges and gauge coupling. In this work we shall illustrate the constraints implied by CHARM II results by considering two typical GUT-motivated \( Z' \) models. Obviously, when choosing the models one should discard those that predict a right-handed neutrino as the form of the effective lagrangian (3) suggests.

The first model is \( I \) model coming from \( E_6 \) [2, 11], and we call it Model B. In this model \( Q_L = -2Q_E = 1/\sqrt{10} \) and \( g_{Y'} \approx 0.8g_2 \). This is a typical \( E_6 \) -inspired model in which \( U(1)_{Y'} \) charges are linear combinations of the \( U(1) \) groups \( E_6/\text{SO}(10) \) and \( \text{SO}(10)/\text{SU}(5) \). Depending on the details of the symmetry breaking, \( g_{Y'} \) differs from \( 0.8g_2 \) by a factor around unity.

The second model we analyze is the one following from the breaking of the flipped \( SU(5) \times U(1) \) when the Higgs fields reside in the \((27+\bar{27})\) dimensional representation of \( E(6) \) [2, 11]. In this model \( Q_L = Q_E = 1/2 \) and \( g_{Y'} \) is as in the first model. We call this model as Model A.

In the notation of equations (4) and (5), we have \( r_A = 1 \), \( r_B = -1/2 \), and \( \lambda_B \approx 0.63\lambda_A \). Since \( r_A = 1 \), \( g_{\nu e}^A \) feels \( Z' \) parameters only through \( \lambda \sin 2\theta \) type terms (see (5)) which is a weaker dependence than that of \( g_{\nu e}^V \). Thus,
in Model $A$ parameter space is expected to be constrained mainly by $\Delta \rho$ and $g_{\nu e}^V$.

In Model $B$, as $r_B = -1/2$, dependence of $g_{\nu e}^A$ on $Z'$ parameters is amplified compared to $g_{\nu e}^V$. Thus, one expects $\Delta \rho$ and $g_{\nu e}^A$ mainly determine the appropriate parameter space.

Finally, one can also predict the relative magnitudes of the parameter spaces in two models. Since $\lambda_B \approx 0.63 \lambda_A$, it is clear that in Model $B$ one needs a wider parameter space than in Model $A$ to close the gap between experiment and the SM. In the next section we shall quantify these qualitative arguments.

3 CHARM II Results and $Z'$ Physics

CHARM II Collaboration [6] has measured $s_W^2$, $g_{\nu e}^V$ and $g_{\nu e}^A$ from the neutrino-electron scattering events using $\nu$ and $\bar{\nu}$ beams with $E_\nu \approx 25.7$ GeV. On $g_V - g_A$ plane there are two candidate points coming from electron-neutrino scattering data. However LEP results for $A_{FB}(e^+e^- \rightarrow e^+e^-)$ [13] prefer one of them at which $g_{\nu e}^V = -0.035 \pm 0.017$ and $g_{\nu e}^A = -0.503 \pm 0.017$ [1]. This solution, which we call point (I), is close to the SM prediction so that it can be taken as a confirmation of the standard electroweak theory. The other point which is discarded by $e^+e^-$ data has approximately $g_{\nu e}^V \leftrightarrow g_{\nu e}^A$. This solution, which we call point (II), is clearly far from being predictable by the SM. As noted in [1], point (I) is chosen by assuming that the $Z$-pole data is dominated by a single $Z$. In multi $Z$ models, like the $Z'$ models under concern, it would be convenient to discuss the implications of both points for $Z'$ physics on equal footing. Below we first analyze the allowed parameter space on $\eta - \theta$ plane for the point (I), next we turn to the discussion of point (II).

3.1 Allowed $\eta - \theta$ values for point (I)

Let us start discussing the implications of equations (4), (5), and (10) with a rough analysis. Equations (4) and (5) express experimental values of $g_{\nu e}^V$ and $g_{\nu e}^A$ in terms of the parameters of the extended model at hand. Equation (10), on the other hand, represents the restriction on the modification of $\rho$ parameter due to $Z'$ effects. As given in (13) $\Delta \rho$ (10) is restricted in a rather
narrow error band. As is seen from equations (4)-(9), $g_{\nu e}^{V}$ and $g_{A}^{\nu e}$ explicitly depend on $\Delta \rho$, unlike the atomic weak charge for which $\Delta \rho$ dependence almost cancel \cite{5}.

For a rough analysis of the parameter space at the point (I), one can neglect $\Delta \rho$ all together. Then $\tilde{g}_{V}^{\nu e}$ (8) reduces to the SM expression. Moreover, $\rho_{2} \rightarrow 1$, and this requires both $\eta \tan^{2} \theta$ and $\tan^{2} \theta$ be small, which would be satisfied only by a small enough $\theta$. That $g_{V}^{\nu e}$ and $g_{A}^{\nu e}$ are close to their SM values also confirms the need for a small $\theta$, as this reduces the $\rho_{2}$ dependent terms in (4) and (5) essentially to their SM expressions. In this limit, $\rho_{2} \sim 1/\eta$, and for $\rho_{2} \cos^{2} \theta$ type terms in (4) and (5) be suppressed one needs a large $\eta$, or equivalently, heavy enough $Z_{2}$. After making these rough observations we now turn to an exact numerical analysis of the allowed $\eta - \theta$ region for point (I), which will be seen to imply constraints beyond these expectations as one considers the nonvanishing values of the $\rho$ parameter.

In the numerical analysis below we first express, via (10), $\eta$ in terms of $\theta$ for a given value of $\Delta \rho$. Then we insert this solution to the expressions for $g_{V}^{\nu e}$ and $g_{A}^{\nu e}$ in (4) and (5). After expressing (4) and (5) in terms of $\theta$ and $\Delta \rho$ in this way we let $\theta$ vary from zero to higher values, meanwhile, $\Delta \rho$ wanders in its phenomenologically allowed range of values determined by (13). Under the variation of these parameters we pick up those points for which (4) and (5) do remain in their allowed range of values.

We first analyze Model $\mathcal{A}$ mentioned in the last section. In this model $g_{A}^{\nu e}$ has a weaker dependence on the $Z'$ parameters compared to $g_{V}^{\nu e}$ since $r = 1$. Fig. 1 depicts the allowed parameter space for which (4), (5) and (10) satisfy the existing phenomenological bounds. As we see from Fig. 1 for small $\theta$, $\eta$ wanders in a rather wide range of values. For example, for $\theta = 10^{-3}\pi$, $\eta$ varies from 8 to 80 which implies $0.7 \text{ TeV} \lesssim M_{Z_{2}} \lesssim 7 \text{ TeV}$. As $\theta$ increases, not only the upper bound but also the lower bound on $\eta$ decreases. For example, for $\theta \approx 0.0041$, $\eta$ ranges from 1.3 to 6, that is, $120 \text{ GeV} \lesssim M_{Z_{2}} \lesssim 550 \text{ GeV}$ therefore $Z_{2}$ is considerably light in this case. Moreover, as $\theta$ increases further, the allowed range of $\eta$ gets thinner and thinner and $M_{Z_{2}}$ settles approximately to $120 \text{ GeV}$. This goes on until $\theta = 0.0286$ at which $g_{V}^{\nu e}$ hits its lower bound, and there are no $(\theta, \eta)$ pairs satisfying (4), (5) and (10) beyond this point. In sum, one concludes that for Model $\mathcal{A}$, $\theta_{\text{max}} = 0.0286$ and $M_{Z_{2}}^{\text{min}} \approx 120 \text{ GeV}$. For negative $\theta$ values, graph is approximately mirror symmetric of Fig.1 with respect to $\eta$ axis. $\theta$ takes its minimum value of $\theta_{\text{min}} = -0.236$ at which $g_{V}^{\nu e}$ reaches its upper bound. Close to $\theta_{\text{min}}$, $\eta$ drops
to $\sim 1.1$, yielding a light $Z_2$.

Now we analyze Model $B$ to identify the appropriate region it implies in the $(\theta, \eta)$ plane. As we recall from the previous section, unlike Model $A$, in this model both $g_{A}^{\nu e}$ is more sensitive to the $Z'$ parameters than $g_{V}^{\nu e}$. Fig.2 depicts the appropriate $(\theta, \eta)$ pairs for which (4), (5) and (10) reside in their allowed band of values. Here we show only $\theta \geq 0.2$ part of the entire $\theta$ range to magnify the region of maximum $\theta$ value. $\theta \leq 0.2$ part behaves similarly to Fig.1. As we see from Fig. 2, the allowed range of $\theta$ is rather wide compared to Model $A$. This is what was expected by the discussions at the end of the last section. To be quantitative let us discuss some special points on Fig. 2. Here $\theta_{\text{max}} = 0.3896$ at which $g_{A}^{\nu e}$ hits its upper bound so that there is no acceptable $(\theta, \eta)$ values beyond this point. Consistent with the comparatively large values of $\theta$, $\eta$ takes its smallest allowed value, that is, it takes the value of unity for every $\theta$ allowed, including the $\theta \leq 0.2$ part not shown on Fig. 2. For negative $\theta$ values the allowed region is approximately symmetric to Fig. 2 with respect to $\eta$ axis and extends up to -1.533 at which $g_{A}^{\nu e}$ reaches its upper bound. Differently than the positive $\theta$ range, here $\eta$ does not take the value of unity although it remains close to unity, lowering as low as 1.005, for $\theta$ values near $\theta_{\text{min}} = -1.533$.

Although $\eta = 1$ was never realized for Model $A$, it is the case here. This shows that for Model $B$ the envelope of the allowed $(\theta, \eta)$ values is strictly determined by $\Delta \rho$ in (10), as $\eta = 1$ is consistent with $\Delta \rho$ constraint. For Model $A$, however, $\Delta \rho$ has not the full control over $\eta$ values for a given $\theta$, other parameters, $g_{V}^{\nu e}$ and $g_{A}^{\nu e}$, constrain the total $\eta$ range.

For both models $A$ and $B$ we have a wider range of allowed negative $\theta$ values which stems from the fact that negative $\theta$ changes the sign of the cross terms in (4) and (5), and thus, lowers the value of the right hand side compared to the positive $\theta$ case. This then requires a wider range of $\theta$ values. This widening of the allowed negative $\theta$ interval is also seen in the APV determination of [3]. However, one notes that, in general, the allowed ranges of $\theta$ is much wider than those of the APV determination. Nevertheless, a direct comparison is not possible, because in [3], experimental data are taken with an error of two standard deviations, and top mass is assigned a much lower value of $\sim 90 GeV$ than the present analysis.

We conclude this subsection by noting that the allowed ranges of the mixing angle and $Z_2$ mass depend on the value of the normalized $U(1)_Y$ coupling $\lambda$. If $\lambda$ is small, mainly the $Z$-pole observable $\Delta \rho$ takes control.
of the $Z'$ parameter space, and the low-energy observables $g_V^{\nu e}$ and $g_A^{\nu e}$ do not hit their phenomenological bounds until the mixing angle takes relatively large values. On the other hand, when $\lambda$ is large, the low-energy observables become more decisive on the $Z'$ parameter space. Therefore, one roughly concludes that when the coupling constant of the extra $U(1)$ is large (small) compared to the weak coupling low-energy observables ($Z$-pole observables) determine the allowed parameter space.

3.2 Allowed $\eta - \theta$ values for point (II)

In Sec. 3.1 we have discussed the implications of CHARM II candidate, point (I), for the $Z'$ parameters. The other candidate, point (II), discarded by the $e^+e^-$ data, deserves also a discussion because it may be of interest in $Z'$ models in which off-$Z$-pole data may be explained by the contribution of the $Z'$ propagator effects. Before an attempt to confirm the $e^+e^-$ data, one should first prove the existence of points on $(\theta, \eta)$ plane solving (4), (5) and (10) simultaneously. The transition from the point (I) to point (II) involves the exchange $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$. Thus, it is convenient to work with $\Sigma^{\nu e} = g_V^{\nu e} + g_A^{\nu e}$ and $\Delta^{\nu e} = g_V^{\nu e} - g_A^{\nu e}$ which are symmetric and antisymmetric under $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$, respectively. After interchanging the left hand sides of (4) and (5), $\Sigma^{\nu e}$ and $\Delta^{\nu e}$ can be expressed in terms of the corresponding SM ones and the other parameters. However, an attempt to find a consistent solution of $\Sigma^{\nu e}$, $\Delta^{\nu e}$ and $\Delta^{\rho}$ for a given model parameters runs into difficulty immediately. This can be seen as follows. $\Sigma^{\nu e}$ is symmetric under $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$ so it points to the $(\theta, \eta)$ regions we discussed in Sec. 3.1. However, $\Delta^{\nu e}$ is antisymmetric under $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$ and requires a completely different $(\theta, \eta)$ set for being satisfied. This is due to the fact that $\Delta^{\nu e}$ and the corresponding SM quantity $\Delta^{SM}$ are not close to each other as $\Sigma^{\nu e}$ and $\Sigma^{SM}$ do. In fact, a numerical analysis of not only the models we have illustrated in Sec. 3.1 but also the ones listed in [5] confirms this observation. It is interesting that the present analysis also prefers the CHARM II result for point (I).

4 Conclusions

In this work we have analyzed the low energy $\nu_\mu - e$ scattering and $Z$-pole data against a general Abelian extension of the SM. In our analysis we have
used the results of CHARM II Collaboration [6] which reported two candidate points for the effective vector and axial couplings. We have considered both points in our analysis and found that the CHARM II candidate discarded by the $e^+e^-$ data does not lead a consistent solution for the GUT-motivated models considered. The other point, which is in the close vicinity of the SM predictions for effective vector and axial couplings, leads solution spaces much wider than the APV determination of [5]. The analysis concludes that when the gauge coupling associated to the extra $U(1)$ is small (large) compared to the weak coupling parameter space is determined mainly by $\rho$-parameter constraint (low-energy neutral current data).

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Figure 1: The $(\eta, \theta)$ values satisfying (4), (5) and (10) for Model \( \mathcal{A} \).
Figure 2: The $(\eta, \theta)$ values satisfying (4), (5) and (10) for Model $B$. 

![Graph showing $(\eta, \theta)$ values for Model $B$.]