Collective excitations in double quantum dots

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Abstract

We analyse the ground state properties of vertical-double quantum dots in the lowest Landau level regime for filling factor $\nu = 2$. This analysis follows two lines: on the one hand, we study the dispersion relation of different collective excitations and look for roton-type minima that signal some type of order within the ground state. On the other hand, we calculate expected values of the antiferromagnetic order operator defined as $\langle S^R_x S^L_x \rangle$. We conclude that of the three existing types of ground state at $\nu = 2$ the canted and symmetric states show spin order on the plane of the dots. In the case of the canted state, two types of spin order are shown as a consequence of the coupling between spin and isospin. We propose that one of the excitations obtained over the symmetric state could be the origin of one of the decoherence processes detected experimentally in qubit systems.

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I. Introduction

The possibility of tunneling between the left and right layers in a double-layered system introduces an extra energy scale added to those typical of a system defined by a quantum well in a perpendicular magnetic field, namely, the Coulomb and Zeeman energies (under the assumption of the lowest Landau level (LLL) regime the kinetic energy is frozen and can be ignored). This extra degree of freedom yields a rich variety of ground states (GS) as well as different excited configurations. In a previous work Das Salma et al. [2] established that for filling factor $\nu = 2$ (one for each layer) there is a universal GS phase diagram that contains only three possibilities: the ferromagnetic state $|FER\rangle$ with all spins up and equal populations of symmetric and antisymmetric states (the symmetric and antisymmetric combinations of the left and right single particle states), the symmetric state $|SYM\rangle$ with all the electrons in the symmetric state and the spin up and down equally populated and, in between, the canted state $|CAN\rangle$ characterized by a mixture of spins and isospins. The isospin (or "pseudospin") is the quantum number associated with the symmetric and antisymmetric single particle states. The balance between the Zeeman energy, the tunneling strength and the Coulomb interaction determines the boundaries within the phase diagram. Of special interest is the canted state characterized by ferromagnetic order in the perpendicular direction and antiferromagnetic order (AFO) on the plane of the layers. This order in plane is a consequence of the broken symmetry produced by the degeneracy of the GS. The excited state produced by the operator that changes spin and isospin in one unit and leaves the total angular momentum unchanged is degenerated with the GS. This degeneracy is produced in spite of the fact that the Zeeman and tunneling gaps ($\Delta_Z$ and $\Delta_t$ respectively) are non zero, a necessary condition to obtain canted states. The AFO in plane defined as:

$$\langle S^R_x \rangle - \langle S^L_x \rangle$$

is, within a Hartree-Fock (HF) calculation, different from zero only for the canted state [2].

The aim of this report is to analyse to what extent this phenomenology is preserved for a finite system built up from a vertical double quantum dot (DQD) confined by identical parabolic potentials in each layer, preserving the conditions of filling factor $\nu = 2$ (for an even number of electrons) and LLL regime.

This paper is organized as follows: in Sec.II we define the lines chosen to analyse the
properties of the GS configurations, namely, the study of the collective mode excitations and the calculation of the expected value of an appropriate AFO operator. In Sec.III we concentrate on the analysis of the collective modes and on the information about the GS that derives from its properties. In Sec.IV the results obtained from the AFO operator are shown, and finally in Sec.V we draw our conclusions.

II. Ground state configurations

In this Section we address the comparison between a DQD finite system of \( N \) electrons and the double layer with which it shares the same number of degrees of freedom. It should be noted that the question related to the type of GS’s has been solved in Ref.[3] within an exact diagonalization calculation (EDC). It has been proved that, similarly to the double layered case, the states \(|\text{FER}\rangle, |\text{SYM}\rangle \text{ and } |\text{CAN}\rangle\) cover all the possibilities. The only difference is that there are several possible canted states, the number of which grows with the number of electrons. For \( N = 6 \) for instance, and \( \nu = 2 \), the total spin ranges from \( S_z = 3 \) for the \(|\text{FER}\rangle\) state to zero for the \(|\text{SYM}\rangle\) state and in between there are two canted states with total spins \( S_z = 2 \) and 1 respectively. At the same time the parity (1 or \(-1\)) of the GS changes simultaneously with each spin transition. Parity is the well defined quantum number related to specular symmetry between the dots and is defined as \( P = (-1)^{X/2} \), where \( X = N_S - S_A \) is the balance between symmetric and antisymmetric single particle occupied states [3]. It should be pointed out that neither the single particle layer occupation states \(|R\rangle\) or \(|L\rangle\) nor the symmetric or antisymmetric combinations \(|S\rangle\) or \(|A\rangle\) are associated with well defined single particle quantum numbers [4]; in other words the Coulomb interaction mixes isospin meanwhile tunnelig mixes layer occupations.

In this report, we examine the structure of the different GS’s. On the one hand, we take advantage of the finitness of the system and perform EDC, which gives exhaustive information of the spectrum. However, since the diagonalization is performed within separate subspaces of well defined \( M \) (total angular momentum), \( S_z \) (total spin) and \( P \) (parity) defining a configuration denoted by \((M, S_z, P)\), no information can be obtained from the AFO in plane defined by Eq.(1) since the expected values vanish. In order to analyse the presence of some type of order in the GS we follow two lines:
The first line is based on the results of the analysis of the dispersion relation of the density wave excitations of liquid helium. On the one hand the roton minimum gives indirect information about the short-range order between helium atoms in the GS and on the other hand, since a single mode approximation accurately reproduces the experimental data for small transferred momentum \( \hat{Q} \), it shows that a collective excitation built up from a linear combination of single-body excitations provides a suitable model within this region of the dispersion relation. The most convenient excitations to analyse the GS properties are those that change spin and parity as is discussed in the next section.

The second line is the search for an appropriate operator which reveals the presence of order in the plane of the dots and has non-vanishing expected values for well defined configurations. To this end we chose the operator given by:

\[
S_x^R S_x^L ,
\]

which complements the information provided by the first line.

III. Collective excitations

In order to choose the appropriate excitations, we considered one of the results obtained in a previous paper \([4]\) for the same system. Among all the possibilities to excite the system, the excitation that simultaneously changes spin and parity in one unit and leaves \( M \) unchanged has some important properties: it is the absolute lowest energy excitation over the three types of GS and softens at the boundaries of the phase diagram ( for \( \Delta_Z \) over \( \Delta_t \) ) signaling the frontiers within the diagram. By the increase of \( \Delta_t \), for fixed \( \Delta_Z \), the GS for \( N = 6 \) evolves as \( |FER\rangle \rightarrow |CAN1\rangle \rightarrow |CAN2\rangle \rightarrow |SYM\rangle \). In Fig.1 we show the evolution of the excitation energy over the GS’s ( \( (6,3,-1), (6,2,1), (6,1,-1) \) and \( (6,0,1) \) ) as \( \Delta_t \) changes. It should be noted that over the canted states, there are two different possibilities, namely, \( (6,2,1) \rightarrow (6,3,-1) \) or \( (6,2,1) \rightarrow (6,1,-1) \). The transitions take place at the values of \( \Delta_t \) where the excitation energy goes to zero. This led us to study the dispersion relation of this type of collective mode as a function of the transferred angular momentum \( (\Delta M = l) \). We will denote these excitations as \( SIE \) (spin-isospin-excitation). Furthermore, in order to complete the scenario, we also analysed the evolution with \( l \) of three other possible excitations: the charge density wave (CDW) for which the spin and parity remain constant,
the one that leaves the parity unchanged (SE) and the one that leaves the spin unchanged (IE). The results are shown in Figs.2-4 for a DQD of six electrons. Similar phenomenology was found for $N = 8$. Even though no special structure is shown for the $|FER\rangle$ state as expected, some minima appear over the $|CAN1\rangle$ and over the $|SYM\rangle$ states whenever the spin change is involved. For the $|CAN1\rangle$ state the presence or absence of parity change modifies the value of $l_c$ on which the local minimum occurs, namely, spin and isospin appear to be coupled within the canted state, or in other words, two different spin ordered states seem to coexist in a nearly degenerate state. It must be emphasized that the energy gap between the GS and the SIE for $l = 0$ is negligible as compared to the typical excitation energies of the SIE for $l \neq 0$, as can be inferred from the comparison of the energy scales in Figs.1 and 2-4. The degeneracy between the GS and the two SIE would be complete for large number of electrons [2]. For the $|SYM\rangle$ state there is a strong minimum in the SE spectrum and no structure is apparent in the SIE. The previous result suggests that there is spin order in the $|SYM\rangle$ state in contrast to the conclusions inferred from the results obtained by the use of Eq.1 in a HF calculation for double layers [2]. This interpretation is reinforced by the results obtained by the use of Eq.2 as is discussed in the next Section.

Other interesting properties derive from the analysis of the collective modes. On the one hand, the local minima at $l_c$ are characterized by states consisting of only one Slater determinant, (the spectral weight of the most important Slater within state at $l_c$ is given by 0.96 for the SE and by 0.98 for the SIE over the $|CAN1\rangle$ state). In other words, the excitations at $l_c$ are non-correlated states for which the system develops a strong expansion. This expansion is a direct consequence of the restrictions arising from the changes imposed over spin and parity. For instance, for the $|CAN1\rangle$ GS configuration $(6, 2, 1)$, the SIE is $(6 + l, 3, -1)$ and the SE is $(6 + l, 3, 1)$, while $P = -1$ is obtained from $X = -4, 0$ or 4 and $P = 1$ from $X = -6, -2, 2$ or 6 in such a way that the expansions shown in the SIE from $l = 3$ to 4 or from $l = 8$ to 9 in the SE correspond to changes of $X$ from 0 to 4 and from 2 to 6 respectively. Therefore, the increase of $l$ produces a sudden expansion in the excited structure since the new allowed values of $X$ are compatible with the occupation of larger single-particle angular momenta. The strong expansion reduces the Coulomb interaction and produces the minima. We emphasize that the same effect is expected at some $l$ for the dispersion relation of each excitation, although the expansion produces a strong bound state
FIG. 1: Lowest energy excitation for $N = 6$ as a function of $\Delta t$ for different regions of the GS phase diagram. Energies are given in units of $e^2/\epsilon l_B$ where $l_B$ is the magnetic length and $e$ and $\epsilon$ are the electronic charge and the dielectric constant of the host semiconductor respectively. The excited configurations are (from left to right) $(6, 3, 1), (6, 2, 1)$ and $(6, 1, -1)$ for the positive-gradient curves and $(6, 2, 1), (6, 1, -1)$ and $(6, 0, 1)$ for the negative-gradient curves.

only for those signaled by $l_c$ in Figs.2-4.

Furthermore, the states at $l_c$ cannot be modelled within a single mode approximation. This can be confirmed directly from the diagonalization output and from the fact that the excited states contain effectively only one Slater. The GS $(6, 2, 1)$ is a combination of 18 Slater determinants, although only one of them (with negligible spectral weight) reproduces the excited SIE configuration by a one body excitation, and none of them reproduces the SE configuration. Two or more body excitations are necessary to approach the excited configuration. In contrast, the $|CAN1\rangle$ GS can be reproduced by

$$Op |FER\rangle = \Psi$$  \hspace{1cm} (3)

where

$$Op = \sum_l (c^+_l S c^\downarrow_{lA} + c^+_l A c^\uparrow_{lS})$$  \hspace{1cm} (4)

which has for 8 electrons, an overlap (i.e., $|\langle \Psi |CAN1\rangle|^2$) of 0.943. Moreover, as the $|CAN1\rangle$ state can be interpreted as an excitation of the $|FER\rangle$ GS, it proves that the
Excitation for $l = 0$ in Fig. 2 for the SIE can be reproduced with a single mode approximation, as had to be the case for long wavelength excitations.
FIG. 3: The same as Fig.2 for the |CAN1⟩ GS calculated at $\Delta t = 0.07$. The critical transferred angular momenta at the minima are signaled by $l_c$.

IV. Antiferromagnetic order in plane

In order to obtain independent information about the spin configuration in plane, we looked for an order operator with expected values that were not necessarily zero for well defined configurations. Fig.5 shows the expectation values of the operator given by Eq.2 as a function of $\Delta t$. The |FER⟩ state has zero antiferromagnetic order as expected; however,
FIG. 4: The same as Fig. 3 for the $|SYM\rangle$ GS calculated at $\Delta t = 0.15$.

In fact, there is no any contradiction between our results and those given in Ref. 2 as regards the $|SYM\rangle$ state. The following situation is possible, namely, that the expected value of $S_x^R$ (or $S_x^L$) vanishes as an integrated quantity over the plane of the layer though the expected value of the local operator $S_x^R(x, y)$ does not vanish, in such a way that $\langle S_x^R S_x^L \rangle \neq 0$ and $\langle S_x^R \rangle = \langle S_x^L \rangle = 0$ even within a Hartree-Fock calculation in which the GS is a mixture
FIG. 5: The order parameter defined as $ORD = \langle S^R_x S^L_x \rangle$ calculated for the different GS's over the phase diagram as a function of $\Delta_t$.

V. Conclusions and discussion

The observation of minima in the excitation spectrum of special excited configurations $(M + l, S_z \pm 1, \pm P)$ indicates spin order in the GS. This result is in accordance with the
results obtained from the expectation value of the AFO in plane operator defined as $\langle S^R_x S^L_x \rangle$.

In contrast with the findings obtained for double layers within a HF approximation and using Eq.1, the $|SYM\rangle$ state has spin order in the plane of the dots as defined by Eq.2. However, as even within a HF calculation the zero value of Eq.1 could hide local AFO, the difference between our findings for a DQD and those of Ref.[2] for double layers cannot be imputed to finite size effects without deeper analysis. For the $|CAN\rangle$ state, as mentioned above, the excitations at $l_c$ need two or more body operators to be generated from the GS. Meanwhile, for the $|SYM\rangle$ state, the excitation at $l_c$ can be accurately reproduced from a single-body operator.

For all cases, the excited states at the local minima are non-correlated states of a strongly expanded configuration as compared to the GS or the excited state at $l_c - 1$. The parameter $l_c$ is associated with a length scale that characterizes spin and isospin order in the $|CAN\rangle$ state and spin order in the $|SYM\rangle$ state.

As a last remark, increasing interest has recently been shown in two-level systems which can be used as qubits in quantum information technology and particularly in coupled quantum dots. In a recent paper [1] Hayashi et al. report coherent oscillations (in qubits built from lateral DQD’s) observed for several combinations of ground and excited states. The coherent oscillations between left and right localized states are obtained for zero offset when the tunneling gap is the most important energy scale, in such a way that the system is effectively in the $|SYM\rangle$ state. For this state, the understanding of the possible decoherent processes is crucial for practical applications. The excited state found at $l_c$ in Fig.4 may be involved in one of the decoherent mechanisms since it is the lowest energy excitation and in this case, the decoherence would be characterized by a spin flip process.

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