pp and \( \bar{p}p \) total cross sections and elastic scattering

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Abstract

It is shown that \( pp \) and \( p\bar{p} \) data, including those from the TOTEM experiment, agree well with Regge theory.

1 Introduction

In this paper we analyse whether the highly-accurate elastic scattering and total cross section data now available\cite{1} from the LHC help to shed any new light on the unanswered questions about the theory.

The only theory that we have is Regge theory\cite{2}. While it has been hugely successful\cite{3}, our understanding of it has a significant gap: we know how to describe the exchanges of single particles, but even after five decades we do not know how to calculate double and higher exchanges. Further, to describe the data we need to introduce an exchange that is not obviously associated with particle exchange. Called pomeron exchange, it is possible that it corresponds to glueball exchange.

According to Regge theory, particle exchange contributes simple power behaviour \( s^\epsilon \) to total cross sections. In our original analysis three decades ago\cite{4} of the data then available, up to \( \sqrt{s} = 62 \text{ GeV} \), we introduced just two powers to fit all hadron-hadron total cross sections, \( \epsilon_1 \) close to 0.08 corresponding to pomeron exchange and \( \epsilon_R \) close to \( \frac{1}{2} \) corresponding to the nearly exchange-degenerate \( \rho, \omega, f_2, a_2 \) exchange. In the light of the data subsequently obtained up to Tevatron energies, Cudell and collaborators\cite{5} concluded that the value of \( \epsilon_1 \) was somewhat larger, close to 0.096.

When deep inelastic ep scattering data at small \( x \) became available\cite{6} from HERA, we showed that these were most simply described by introducing a second pomeron, the hard pomeron\cite{7}, and that by doing so we could put the conventional DGLAP evolution analysis at small \( x \) on a sounder footing\cite{8}. We then pointed out\cite{9} that a corresponding power behaviour \( s^{\epsilon_0} \) with \( \epsilon_0 \) close to 0.4 might well be present in hadron-hadron total cross sections.

Through the optical theorem, total cross sections are closely linked to elastic scattering, and so fitting one and ignoring the other does not make sense. A particular challenge to fitting elastic scattering data is the remarkable dip structure first discovered\cite{10} in pp elastic scattering at the CERN ISR. We predicted\cite{11} that this dip would be filled in for \( \bar{p}p \) scattering, and this was subsequently confirmed\cite{12} though, as we again find in this paper, a detailed correct description of the \( \bar{p}p \) data is very difficult to achieve.

Reproducing a dip requires the simultaneous near-vanishing of both the real and imaginary parts of the amplitude, so at least three terms need to be involved. The energies at which the dip is seen are

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quite large, so the contributions from $\rho, \omega, f_2, a_2$ exchange are too small, and to cancel the imaginary part of single pomeron exchange $P$ we need also two-pomeron exchange $PP$. To cancel the real part we bring in triple-gluon exchange, since this appears[13] to dominate the elastic amplitude at large values of $t$, giving it an energy-independent behaviour $t^{-4}$.

Two-pomeron exchange is, of course, important for another reason. Without it, the forward amplitude would grow so large with increasing energy that unitarity would be violated. For this reason, in our original fit[4] we emphasised that the power $\epsilon_1$ was only an effective power, valid over a limited range of energies. The term $PP$ is negative and so helps to avoid such a breakdown of unitarity, but its magnitude increases more rapidly than that of the term $P$, so that to prevent the total cross section from becoming negative it will ultimately be necessary to include also a further term $PPP$, and so on. But there is no indication that this is needed at presently-accessible energies. We shall just work with $P + PP + ggg$, together with $\rho, \omega, f_2, a_2$ exchange.

In this, our approach is different from that of most other authors, who bring in the complete series $P + PP + PPP + PPPP + \ldots$. To calculate this, they use an eikonal formula, but this has little or no theoretical justification. As we have stated above, we do not have the theoretical knowledge correctly to calculate any of the terms beyond single exchange $P$, though we do have some knowledge of their general properties, which enables one to make models.

A conclusion of this paper is that, while the data for $pp$ and $\bar{p}p$ elastic scattering and total cross sections do not exclude a small contribution from the exchange of the hard pomeron, it is not needed to fit the data. And an interesting feature of what we find below is that, even though the term $PP$ behaves as combination of a power of $s$ and log $s$, our fit to the data makes the combination $P + PP$ effectively behave as a simple power of $s^{\epsilon_1}$ over a very wide range of energy, up the highest values that are likely ever to be accessible. We find that this effective power $\epsilon_1$ is close to the value 0.096 arrived at in reference [5]. Contrary to what we concluded in a previous paper[14], the hard pomeron is not needed to give a good fit to the data, though it may be present at some level.

2 Fit to the data

In our original fit[4] to the total cross section data we took all four “reggeon” trajectories $\rho, \omega, f_2, a_2$ to be degenerate, so that their exchanges all contributed the same power of $s$. However, a Chew-Frautschi plot of particle masses shows that this degeneracy is far from exact – see figure 2.13 of our book[15], which suggests that it is more accurate to work with two pairs of degenerate trajectories, $f_2, a_2$ with slope 0.8 GeV$^{-2}$ and intercept $1 + \epsilon_+ close to 0.7, and $\rho, \omega$ with slope 0.92 GeV$^{-2}$ and intercept $1 + \epsilon_- close to 0.5.

As we have explained, we find that the hard pomeron is not needed to fit the data, so we introduce three trajectories

$$\alpha_i(t) = 1 + \epsilon_i + \alpha'_i t \quad i = P, \pm$$

and we have three single-exchange terms:

$$A(s, t) = -\frac{X_F P_F}{2\nu} e^{-\frac{1}{2}i\pi\alpha(t)} (2\nu\alpha_P)_{\alpha_P(t)} - \frac{X_F P_F}{2\nu} e^{-\frac{1}{2}i\pi\alpha(t)} (2\nu\alpha_P')_{\alpha_P'(t)}$$

$$\pm \frac{i\nu}{2\nu} e^{-\frac{1}{2}i\pi\alpha(t)} (2\nu\alpha')_{\alpha'(t)}$$

(1b)

for the $pp/\bar{p}p$ amplitude. The normalisation is such that $\sigma_{TOT} = \text{Im} A(s, 0)$. The form factors $F_i(t)$ are unknown. We find that a good fit is obtained by taking them all to be the same and of the simple form

$$F(t) = Ae^{at} + (1 - A)e^{bt}$$

(1c)
Figure 1: Fit to data for the total cross sections for $pp$ and $\bar{p}p$ scattering. The data are taken from the PDG compilation\cite{16}. The Tevatron $\bar{p}p$ points are not included in the fit.

In our fit, we will assume that we can neglect the simultaneous exchange of two reggeons, $RR$, and the exchange of a reggeon together with the pomeron, $RP$. So $\epsilon_\pm$ should be regarded as effective powers. We shall find that the best fit to the data gives values for them close to those from the Chew-Frautschi plots quoted above, so justifying our assumption that the exchanges $RR$ and $RP$ are small.

As we have said, we do not know how to calculate the term $PP$. What is known\cite{15} is that it corresponds to a trajectory
\begin{equation}
\alpha_{PP}(t) = 1 + 2\epsilon_P + \frac{1}{2}\alpha'_P t
\end{equation}
but that the contribution to the amplitude behaves not just as the simple power $s^{\alpha_{PP}(t)}$; there are additional logarithmic factors in the denominator. Also, the normalisation is unknown. Our procedure was to start with the eikonal form and adapt it until we achieved a good fit. Numerous variations of the eikonal double-exchange form were tried, none of which were ideal. This led us to a $PP$ contribution of the form
\begin{equation}
\frac{X_p^3}{32\pi} e^{-\frac{1}{2}i\pi\alpha_{PP}(t)} (2\nu\alpha'_P)^{\alpha_{PP}(t)} \left[ \frac{A^2}{a + \alpha'_P L} e^{\frac{1}{2}bl} + \frac{(1 - A)^2}{b + \alpha'_P L} e^{\frac{1}{2}bl} \right]
\end{equation}
\begin{equation}
L = \log(2\nu\alpha'_P) - \frac{1}{2}i\pi
\end{equation}
This contains the key ingredients of the eikonal, namely the trajectory, the logarithmic factor in the denominator and the modification of the argument of the exponentials in the form factor. It differs from the eikonal form only in that it does not include a cross term involving $A(1 - A)$. Omitting this term very significantly improves the fit. This should not be a surprise since, as we have said, the eikonal has no theoretical justification. Indeed, it is quite surprising that this simple modification to it works so well.

We must also include the term $ggg$ corresponding to triple-gluon exchange. At large $t$, say for $|t| > t_0$, it behaves as\cite{20}
\begin{equation}
g(t) = C \frac{t^3}{t^4} \quad |t| > t_0
\end{equation}
For $|t| < t_0$ we fit this smoothly on to some function that does not diverge as $t \to 0$. By trial and error we arrived at
\begin{equation}
g(t) = C \frac{t^3}{t_0^3} e^{2(1 - t^2/t_0^2)}
\end{equation}
Figure 2: Fits to data\cite{1}\cite{10}\cite{17}\cite{18} for $pp$ elastic scattering: $d\sigma/dt$ in mb GeV$^{-2}$ plotted against $t$ in GeV$^2$. 
Finally, in order to fit elastic scattering data at very small $t$, we include in the amplitudes the Coulomb term

$$\mp \frac{8\pi\alpha_{\text{EM}}}{t}$$

(4)

We perform a least $\chi^2$ fit simultaneously to $pp$ and $\bar{p}p$ elastic scattering data down to centre-of-mass energy 23 GeV, and total cross section data down to 6 GeV. We include the very-small-$t$ preliminary TOTEM data, but not those at lower energies. This gives the parameter set:

$$\epsilon_p = 0.110 \quad \epsilon_+ = -0.327 \quad \epsilon_- = -0.505 \quad X_p = 339 \quad X_+ = 212 \quad X_- = 104 \quad \alpha'_p = 0.165 \text{ GeV}^{-2}$$

$$A = 0.682 \quad a = 7.854 \text{ GeV}^{-2} \quad b = 2.470 \text{ GeV}^{-2} \quad C = 0.0406 \quad t_0 = 4.230 \text{ GeV}^2$$

(5)

3 Results

Figure 1 shows our fit to data for the $pp$ and $\bar{p}p$ total cross sections. We did not use the conflicting data from the Tevatron: the E710 measurement\cite{21} is significantly below that of CDF\cite{22} and it will be seen that our fit favours the latter.
Figure 2 shows the pp elastic scattering data that we used, compared with the fit. If we had not included the preliminary 8 TeV data\cite{23} at very small $t$ the fit would have passed just below them, while having little effect on the other plots. Figure 3 shows the data we used for elastic $\bar{p}p$ scattering, compared with the fit.

Our fit did not use the very-small-$t$ data shown in figure 4. However when we include the Coulomb term (4) we predict these data surprisingly well.

4 Conclusions

Our model makes various simplifications, but describes the data for $pp$ scattering well and for $\bar{p}p$ scattering quite well, over a wide range of energy. Our simplifications include:

- making the three form factors $F_i(t)$ identical and of the simple form (1c)
- making the trajectories $\alpha_i(t)$ linear
- making the $\rho$ and $\omega$ trajectories degenerate, and also the $f_2$ and $a_2$
- omitting all non-single exchanges other than $PP$ and taking it to have the simple form (2b)
- assuming the simple form (3b) for the $ggg$ term when $t$ is not large.

Figure 4 shows an example of data that were not used to make the fit but are described well by it. Another such example is the ratio of the real to imaginary parts of the forward amplitudes, figure 5.

A correct description of the dips is challenging and our simple model is able to describe those in $pp$ scattering rather better than in $\bar{p}p$ scattering. We will not succumb to the temptation to say that, having been taken somewhat hurriedly in the very last few days of operation of the CERN Intersecting Storage Rings, the $\bar{p}p$ data at 53 GeV are unreliable.

The triple-gluon-exchange term $g(t)$ plays a key role in giving the dips. At large enough $t$ it results in $d\sigma/dt \sim 0.073/t^8$, somewhat smaller than our old fit\cite{20} Figure 6 shows the pp elastic differential cross section at various energies. The data make our fit very energy-independent for $|t| > 4$ GeV$^2$, where it is dominated by the term $ggg$. We have previously\cite{26} drawn attention to the interest of checking whether, at sufficiently high energy, this energy independence might give way to a steady increase with energy.

The term $P$ behaves as $s^{0.110}$, while $PP$ is negative and behaves as $s^{0.220}$ together with the denominator logarithmic factors shown in (2b). Figure 7 shows that, over a very wide range of values of $\sqrt{s}$, together their behaviour is very close to the simple power behaviour $s^{0.096}$ that was extracted from the data by Cudell and collaborators\cite{5}. The Froissart-Lukaszuk-Martin bound\cite{27} is about 20 barns at LHC energies and so has no relevance: our fit confirms that asymptopia is a really long way away\cite{28}.

Although a satisfactory fit can also be obtained with the hard pomeron included, our results show that it is not necessary.
Figure 4: Comparison with data at very small $t$: $d\sigma/dt$ in mb GeV$^{-2}$ plotted against $t$ in GeV$^2$. The data are from $^{[24]}$

Figure 5: Comparison with data$^{[24],[25]}$ for the ratio of the real to imaginary parts of the forward $pp$ and $pp$ amplitudes.
Figure 6: The $pp$ differential cross section at three energies

Figure 7: Ratio of the simple power $18.23 \ s^{0.0958}$ to $(P + PP)$

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