Relations on FP-Soft Sets Applied to Decision Making Problems

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February 14, 2014

Abstract
In this work, we first define relations on the fuzzy parametrized soft sets and study their properties. We also give a decision making method based on these relations. In approximate reasoning, relations on the fuzzy parametrized soft sets have shown to be of a primordial importance. Finally, the method is successfully applied to a problems that contain uncertainties.

Keyword 0.1 Soft sets, fuzzy sets, FP-soft sets, relations on FP-soft sets, decision making.

1 Introduction

In 1999, the concept of soft sets was introduced by Molodtsov [25] to deal with problems that contain uncertainties. After Molodtsov, the operations of soft sets are given in [4, 23, 28] and studied their properties. Since then, based on these operations, soft set theory has developed in many directions and applied to wide variety of fields. For instance; on the theory of soft sets [2, 4, 5, 9, 20, 23, 24, 28], on the soft decision making [16, 17, 18, 21, 22, 27], on the fuzzy soft sets [7, 10, 11] and soft rough sets [16] are some of the selected works. Some authors have also studied the algebraic properties of soft sets, such as [1, 3, 6, 19, 26, 29, 30].

The FP-soft sets, firstly studied by Çağman et al. [8], is a fuzzy parameterized soft sets. Then, FP-soft sets theory and its applications studied in detail, for example [12, 13, 14]. In this paper, after given most of the fundamental
definitions of the operations of fuzzy sets, soft sets and FP-soft sets in next section, we define relations on FP-soft sets and we also give their properties in Section 3. In Section 4, we define symmetric, transitive and reflexive relations on the FP-soft sets. In Section 5, we construct a decision making method based on the FP-soft sets. We also give an application which shows that this methods successfully works. In the final section, some concluding comments are presented.

2 Preliminary

In this section, we give the basic definitions and results of soft set theory \[25\] and fuzzy set theory \[31\] that are useful for subsequent discussions.

Definition 2.1 \[31\] Let \( U \) be the universe. A fuzzy set \( X \) over \( U \) is a set defined by a membership function \( \mu_X \) representing a mapping

\[
\mu_X : U \to [0,1].
\]

The value \( \mu_X(x) \) for the fuzzy set \( X \) is called the membership value or the grade of membership of \( x \in U \). The membership value represents the degree of \( x \) belonging to the fuzzy set \( X \). Then a fuzzy set \( X \) on \( U \) can be represented as follows,

\[
X = \{(\mu_X(x)/x) : x \in U, \mu_X(x) \in [0,1]\}.
\]

Note that the set of all fuzzy sets on \( U \) will be denoted by \( F(U) \).

Definition 2.2 \[15\] \( t \)-norms are associative, monotonic and commutative two valued functions \( t \) that map from \([0,1] \times [0,1] \) into \([0,1] \). These properties are formulated with the following conditions:

1. \( t(0,0) = 0 \) and \( t(\mu_{X_1}(x), 1) = t(1, \mu_{X_1}(x)) = \mu_{X_1}(x), \ x \in E \)
2. If \( \mu_{X_1}(x) \leq \mu_{X_2}(x) \) and \( \mu_{X_3}(x) \leq \mu_{X_4}(x) \), then \( t(\mu_{X_1}(x), \mu_{X_2}(x)) \leq t(\mu_{X_3}(x), \mu_{X_4}(x)) \)
3. \( t(\mu_{X_1}(x), \mu_{X_3}(x)) = t(\mu_{X_2}(x), \mu_{X_1}(x)) \)
4. \( t(\mu_{X_1}(x), t(\mu_{X_2}(x), \mu_{X_3}(x))) = t(t(\mu_{X_1}(x), \mu_{X_2}(x)), \mu_{X_3}(x)) \)

Definition 2.3 \[15\] \( t \)-conorms or \( s \)-norm are associative, monotonic and commutative two valued functions \( s \) which map from \([0,1] \times [0,1] \) into \([0,1] \). These properties are formulated with the following conditions:

1. \( s(1,1) = 1 \) and \( s(\mu_{X_1}(x), 0) = s(0, \mu_{X_1}(x)) = \mu_{X_1}(x), \ x \in E \)
2. if \( \mu_{X_1}(x) \leq \mu_{X_2}(x) \) and \( \mu_{X_3}(x) \leq \mu_{X_4}(x) \), then \( s(\mu_{X_1}(x), \mu_{X_2}(x)) \leq s(\mu_{X_3}(x), \mu_{X_4}(x)) \)
3. \( s(\mu_{X_1}(x), \mu_{X_2}(x)) = s(\mu_{X_2}(x), \mu_{X_1}(x)) \)
4. $s(\mu_{X_1}(x), \mu_{X_2}(x), \mu_{X_3}(x)) = s(s(\mu_{X_1}(x), \mu_{X_2}(x), \mu_{X_3}(x)))$

t-norm and t-conorm are related in a sense of logical duality. Typical dual pairs of non parametrized t-norm and t-conorm are complied below:

1. Drastic product:

$$t_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} \min\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & \max\{\mu_{X_1}(x)\mu_{X_2}(x)\} = 1 \\ 0, & \text{otherwise} \end{cases}$$

2. Drastic sum:

$$s_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} \max\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & \min\{\mu_{X_1}(x)\mu_{X_2}(x)\} = 0 \\ 1, & \text{otherwise} \end{cases}$$

3. Bounded product:

$$t_1(\mu_{X_1}(x), \mu_{X_2}(x)) = \max\{0, \mu_{X_1}(x) + \mu_{X_2}(x) - 1\}$$

4. Bounded sum:

$$s_1(\mu_{X_1}(x), \mu_{X_2}(x)) = \min\{1, \mu_{X_1}(x) + \mu_{X_2}(x)\}$$

5. Einstein product:

$$t_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x)\mu_{X_2}(x)}{2 - \left[\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x)\mu_{X_2}(x)\right]}$$

6. Einstein sum:

$$s_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x)}{1 + \mu_{X_1}(x)\mu_{X_2}(x)}$$

7. Algebraic product:

$$t_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x)\mu_{X_2}(x)$$

8. Algebraic sum:

$$s_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x)\mu_{X_2}(x)$$

9. Hamacher product:

$$t_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x)\mu_{X_2}(x)}{\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x)\mu_{X_2}(x)}$$

10. Hamacher sum:

$$s_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x) - 2\mu_{X_1}(x)\mu_{X_2}(x)}{1 - \mu_{X_1}(x)\mu_{X_2}(x)}$$
11. Minimum:
\[ t_3(\mu_{X_1}(x), \mu_{X_2}(x)) = \min\{\mu_{X_1}(x), \mu_{X_2}(x)\} \]

12. Maximum:
\[ s_3(\mu_{X_1}(x), \mu_{X_2}(x)) = \max\{\mu_{X_1}(x), \mu_{X_2}(x)\} \]

**Definition 2.4** [25]. Let \( U \) be an initial universe set and let \( E \) be a set of parameters. Then, a pair \((F, E)\) is called a soft set over \( U \) if and only if \( F \) is a mapping or \( E \) into the set of aft subsets of the set \( U \).

In other words, the soft set is a parametrized family of subsets of the set \( U \). Every set \( F(\varepsilon), \varepsilon \in E \), from this family may be considered as the set of \( \varepsilon \)-elements of the soft set \((F, E)\), or as the set of \( \varepsilon \)-approximate elements of the soft set.

It is worth noting that the sets \( F(\varepsilon) \) may be arbitrary. Some of them may be empty, some may have nonempty intersection.

In this definition, \( E \) is a set of parameters that are describe the elements of the universe \( U \). To apply the soft set in decision making subset \( A, B, C, ... \) of the parameters set \( E \) are needed. Therefore, Çağman and Enginoğlu [4] modified the definition of soft set as follows.

**Definition 2.5** [4] Let \( U \) be a universe, \( E \) be a set of parameters that are describe the elements of \( U \), and \( A \subseteq E \). Then, a soft set \( F_A \) over \( U \) is a set defined by a set valued function \( f_A \) representing a mapping

\[ f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \in E - A \quad (1) \]

where \( f_A \) is called approximate function of the soft set \( F_A \). In other words, the soft set is a parametrized family of subsets of the set \( U \), and therefore it can be written a set of ordered pairs

\[ F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \emptyset \text{ if } x \in E - A\} \]

The subscript \( A \) in the \( f_A \) indicates that \( f_A \) is the approximate function of \( F_A \). The value \( f_A(x) \) is a set called \( x \)-element of the soft set for every \( x \in E \).

**Definition 2.6** [8] Let \( F_X \) be a soft set over \( U \) with its approximate function \( f_X \) and \( X \) be a fuzzy set over \( E \) with its membership function \( \mu_X \). Then, a \( FP \)-soft sets \( \Gamma_X \), is a fuzzy parameterized soft set over \( U \), is defined by the set of ordered pairs

\[ \Gamma_X = \{(\mu_X(x)/x, f_X(x)) : x \in E\} \]

where \( f_X : E \to P(U) \) such that \( f_X(x) = \emptyset \) if \( \mu_X(x) = 0 \) is called approximate function and \( \mu_X : E \to [0,1] \) is called membership function of \( FP \)-soft set \( \Gamma_X \). The value \( \mu_X(x) \) is the degree of importance of the parameter \( x \) and depends on the decision-maker’s requirements.

Note that the sets of all \( FP \)-soft sets over \( U \) will be denoted by \( FPS(U) \).
3 Relations on the FP-Soft Sets

In this section, after given the cartesian products of two FP-soft sets, we define a relations on FP-soft sets and study their desired properties.

**Definition 3.1** Let $\Gamma_X, \Gamma_Y \in FPS(U)$. Then, a cartesian product of $\Gamma_X$ and $\Gamma_Y$, denoted by $\Gamma_X \times \Gamma_Y$, is defined as

$$\Gamma_X \times \Gamma_Y = \left\{ (\mu_{X,Y}(x,y)/(x,y), f_{X,Y}(x,y)) : (x,y) \in E \times E \right\}$$

where

$$f_{X,Y}(x,y) = f_X(x) \cap f_Y(y)$$

and

$$\mu_{X,Y}(x,y) = \min\{\mu_X(x), \mu_Y(y)\}$$

Here $\mu_{X,Y}(x,y)$ is a t-norm.

**Example 3.2** Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}\}$, $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, and $X = \{0.5/x_1, 0.7/x_2, 0.3/x_3, 0.9/x_4, 0.6/x_5\}$ and $Y = \{0.9/x_3, 0.1/x_6, 0.7/x_7, 0.3/x_8\}$ be two fuzzy subsets of $E$. Suppose that

$$\Gamma_X = \left\{ (0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8, u_{11}, u_{12}, u_{13}, u_{15}\}), (0.7/x_2, \{u_3, u_7, u_8, u_{14}, u_{15}\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9, u_{10}, u_{12}, u_{13}\}), (0.9/x_4, \{u_2, u_4, u_6, u_8, u_{12}, u_{13}\}), (0.6/x_5, \{u_3, u_4, u_6, u_7, u_9, u_{13}, u_{15}\}) \right\}$$

$$\Gamma_Y = \left\{ (0.9/x_3, \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.1/x_6, \{u_3, u_5, u_7, u_8, u_{10}, u_{11}, u_{15}\}), (0.7/x_7, \{u_2, u_5, u_9, u_{10}, u_{11}, u_{14}\}), (0.3/x_8, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\}) \right\}$$

Then, the cartesian product of $\Gamma_X$ and $\Gamma_Y$ is obtained as follows

$$\Gamma_X \times \Gamma_Y = \left\{ (0.5/(x_1, x_3), \{u_1, u_6, u_{13}\}), (0.1/(x_1, x_6), \{u_3, u_7, u_8, u_{11}, u_{15}\}), (0.5/(x_1, x_7), \{u_{11}\}), (0.3/(x_1, x_8), \{u_8, u_{12}\}), (0.7/(x_2, x_3), \emptyset), (0.1/(x_2, x_6), \{u_3, u_7, u_8, u_{11}, u_{15}\}), (0.7/(x_2, x_7), \{u_{14}\}), (0.3/(x_2, x_8), \{u_8, u_{14}\}), (0.3/(x_3, x_3), \{u_1, u_3, u_6, u_9, u_{10}, u_{13}\}), (0.1/(x_3, x_6), \{u_5, u_9\}), (0.3/(x_3, x_7), \{u_2, u_5, u_9, u_{10}\}), (0.3/(x_3, x_8), \{u_2, u_5, u_{10}, u_{12}\}), (0.9/(x_4, x_3), \{u_6\}), (0.1/(x_4, x_6), \emptyset), (0.7/(x_4, x_7), \{u_2, u_6\}), (0.3/(x_4, x_8), \{u_2, u_8, u_{12}\}), (0.6/(x_5, x_3), \{u_6, u_9, u_{13}\}), (0.1/(x_5, x_6), \{u_3, u_7, u_9, u_{11}, u_{15}\}), (0.6/(x_5, x_7), \{u_2\}), (0.3/(x_5, x_8), \emptyset) \right\}$$

**Definition 3.3** Let $\Gamma_X, \Gamma_Y \in FPS(U)$. Then, an FP-soft relation from $\Gamma_X$ to $\Gamma_Y$, denoted by $R_F$, is an FP-soft subset of $\Gamma_X \times \Gamma_Y$. Any FP-soft subset of $\Gamma_X \times \Gamma_Y$ is called a FP-relation on $\Gamma_X$. 5
Note that if \( \alpha = (\mu_X(x), f_X(x)) \in \Gamma_X \) and \( \beta = (\mu_Y(y), f_Y(y)) \in \Gamma_Y \), then
\[
\alpha R_F \beta \iff (\mu_X \bowtie_Y (x, y), f_X \bowtie_Y (x, y)) \in R_F
\]

**Example 3.4** Let us consider the Example \( \beta \). Then, we define an FP-soft relation \( R_F \), from \( \Gamma_Y \) to \( \Gamma_X \), as follows
\[
\alpha R_F \beta \iff \mu_X \bowtie_Y (x, x_i) / (x_i, x_j) \geq 0.3 \quad (1 \leq i, j \leq 3)
\]

Then
\[
R_F = \left\{ (0.5/(x_1, x_3), \{u_1, u_6, u_{13}\}), (0.5/(x_1, x_7), \{u_{11}\}), (0.3/(x_1, x_8), \{u_8, u_{12}\}), (0.7(x_2, x_7), \{u_{14}\}), (0.3/(x_2, x_8), \{u_8, u_{14}\}), (0.3/(x_3, x_3), \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.3/(x_3, x_7), \{u_2, u_5, u_9, u_{10}\}), (0.3/(x_3, x_8), \{u_2, u_5, u_{10}, u_{12}\}), (0.9/(x_4, x_3), \{u_6\}), (0.7/(x_4, x_7), \{u_2, u_6\}), (0.3/(x_4, x_8), \{u_2, u_{8}, u_{12}\}), (0.6/(x_5, x_3), \{u_6, u_9, u_{13}\}), (0.6/(x_5, x_7), \{u_2\}) \right\}
\]

**Definition 3.5** Let \( \Gamma_X, \Gamma_Y \in FPS(U) \) and \( R_F \) be an FP-soft relation from \( \Gamma_X \) to \( \Gamma_Y \). Then domain and range of \( R_F \) respectively is defined as
\[
D(R_F) = \{ \alpha \in F_A : \alpha R_F \beta \}
\]
\[
R(R_F) = \{ \beta \in F_B : \alpha R_F \beta \}
\]

**Example 3.6** Let us consider the Example \( \beta \). Then
\[
D(R_F) = \left\{ (0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8, u_{11}, u_{12}, u_{13}, u_{15}\}), (0.7/x_2, \{u_3, u_7, u_8, u_{14}, u_{15}\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9, u_{10}, u_{12}, u_{13}\}), (0.9/x_4, \{u_2, u_4, u_6, u_{12}, u_{13}\}), (0.6/x_5, \{u_3, u_4, u_6, u_7, u_9, u_{13}, u_{15}\}) \right\}
\]
\[
R(R_F) = \left\{ (0.9/x_3, \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.7/x_7, \{u_2, u_5, u_9, u_{10}, u_{11}, u_{14}\}), (0.3/x_8, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\}) \right\}
\]

**Definition 3.7** Let \( R_F \) be an FP-soft relation from \( \Gamma_X \) to \( \Gamma_Y \). Then \( R_F^{-1} \) is from \( \Gamma_Y \) to \( \Gamma_X \) is defined as
\[
\alpha R_F^{-1} \beta = \beta R_F \alpha
\]

**Example 3.8** Let us consider the Example \( \beta \). Then, \( R_F^{-1} \) is from \( \Gamma_Y \) to \( \Gamma_X \) is obtained by
\[
R_F^{-1} = \left\{ (0.5/(x_3, x_1), \{u_1, u_6, u_{13}\}), (0.5/(x_7, x_1), \{u_{11}\}), (0.3/(x_8, x_1), \{u_8, u_{12}\}), (0.7(x_2, x_2), \{u_{14}\}), (0.3/(x_8, x_2), \{u_8, u_{14}\}), (0.3/(x_3, x_3), \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}), (0.3/(x_7, x_3), \{u_2, u_5, u_9, u_{10}\}), (0.3/(x_3, x_3), \{u_2, u_5, u_{10}, u_{12}\}), (0.9/(x_3, x_4), \{u_6\}), (0.7/(x_7, x_4), \{u_2, u_6\}), (0.3/(x_8, x_4), \{u_2, u_{8}, u_{12}\}), (0.6/(x_3, x_5), \{u_6, u_9, u_{13}\}), (0.6/(x_7, x_5), \{u_2\}) \right\}
\]
Proposition 3.9 Let $R_{F_1}$ and $R_{F_2}$ be two FP-soft relations. Then

1. $(R_{F_1}^{-1})^{-1} = R_{F_1}$

2. $R_{F_1} \subseteq R_{F_2} \Rightarrow R_{F_1}^{-1} \subseteq R_{F_2}^{-1}$

Proof:

1. $\alpha(R_{F_1}^{-1})^{-1} = \beta R_{F_1}^{-1} \alpha = \alpha R_{F_1} \beta$

2. $\alpha R_{F_1} \beta \subseteq \alpha R_{F_2} \beta \Rightarrow \beta R_{F_1}^{-1} \alpha \subseteq \beta R_{F_2}^{-1} \alpha \Rightarrow R_{F_1}^{-1} \subseteq R_{F_2}^{-1}$

Definition 3.10 If $R_{F_1}$ is a fuzzy parametrized soft relation from $\Gamma_X$ to $\Gamma_Y$ and $R_{F_2}$ is a fuzzy parametrized soft relation from $\Gamma_Y$ to $\Gamma_Z$, then a composition of two FP-soft relations $R_{F_1}$ and $R_{F_2}$ is defined by

$$\alpha(R_{F_1} \circ R_{F_2}) = (\alpha R_{F_1}) \beta \wedge (\beta R_{F_2})$$

Proposition 3.11 Let $R_{F_1}$ and $R_{F_2}$ be two FP-soft relation from $\Gamma_X$ to $\Gamma_Y$. Then, $(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1}$

Proof:

$$\alpha(R_{F_1} \circ R_{F_2})^{-1} = \gamma(R_{F_1} \circ R_{F_2}) \alpha = (\gamma R_{F_1} \beta) \wedge (\beta R_{F_2} \alpha) = (\beta R_{F_2} \alpha) \wedge (\gamma R_{F_1} \beta) = (\alpha R_{F_1}^{-1} \beta) \wedge (\beta R_{F_2}^{-1} \gamma) = \alpha(R_{F_2}^{-1} \circ R_{F_1}^{-1}) \gamma$$

Therefore we obtain

$$(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1}$$

Definition 3.12 An FP-soft relation $R_F$ on $\Gamma_X$ is said to be an FP-soft symmetric relation if $\alpha R_F \beta \Rightarrow \beta R_F \alpha$ for all $\alpha, \beta \in \Gamma_X$.

Definition 3.13 An FP-soft relation $R_F$ on $\Gamma_X$ is said to be an FP-soft transitive relation if $R_F \circ R_F \subseteq R_F$, that is, $\alpha R_F \beta$ and $\beta R_F \gamma \Rightarrow \alpha R_F \gamma$ for all $\alpha, \beta, \gamma \in \Gamma_X$.

Definition 3.14 An FP-soft relation $R_F$ on $\Gamma_X$ is said to be an FP-soft reflexive relation if $\alpha R_F \alpha$ for all $\alpha \in \Gamma_X$.

Definition 3.15 An FP-soft relation $R_F$ on $\Gamma_X$ is said to be an FP-soft equivalence relation if it is symmetric, transitive and reflexive.

Example 3.16 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$, $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ and $X = \{0.5/x_1, 0.7/x_2, 0.3/x_3\}$ be a fuzzy subsets over $E$. Suppose that

$$\Gamma_X = \left\{(0.5/x_1, \{u_1, u_3, u_5, u_7, u_8\}), (0.7/x_2, \{u_3, u_7, u_8\}), (0.3/x_3, \{u_1, u_2, u_5, u_6, u_9\})\right\}$$
Then, a cartesian product on $\Gamma_X$ is obtained as follows

$$
\Gamma_X \times \Gamma_X = \left\{ (0.5/(x_1, x_1), \{u_1, u_3, u_4, u_6, u_7, u_8\}),
(0.5/(x_1, x_2), \{u_3, v_7, u_8\}), (0.3/(x_1, x_3), \{u_1, u_4, u_6\}),
(0.5/(x_2, x_1), \{u_3, u_7, u_8\}), (0.7/(x_2, x_2), \{u_3, u_7, u_8\}),
(0.3/(x_3, x_1), \{u_1, u_4, u_6\}), (0.3/(x_3, x_3), \{u_1, u_2, u_4, u_5, u_6, u_9\}) \right\}
$$

Then, we get a fuzzy parametrized soft relation $R_F$ on $F_X$ as follows

$$
\alpha R_F \beta \iff \mu_{X \times Y}(x_i, x_j)/(x_i, x_j) \geq 0.3 \quad (1 \leq i, j \leq 3)
$$

Then

$$
R_F = \left\{ (0.5/(x_1, x_1), \{u_1, u_3, u_4, u_6, u_7, u_8\}), (0.5/(x_1, x_2), \{u_3, u_7, u_8\}),
(0.3/(x_1, x_3), \{u_1, u_4, u_6\}), (0.5/(x_2, x_1), \{u_3, u_7, u_8\}),
(0.7/(x_2, x_2), \{u_3, u_7, u_8\}), (0.3/(x_3, x_1), \{u_1, u_4, u_6\}),
(0.3/(x_3, x_3), \{u_1, u_2, u_4, u_5, u_6, u_9\}) \right\}
$$

$R_F$ on $\Gamma_X$ is an FP-soft equivalence relation because it is symmetric, transitive and reflexive.

**Proposition 3.17** If $R_F$ is symmetric if and only if $R_F^{-1}$ is so.

**Proof:** If $R_F$ is symmetric, then $\alpha R_F^{-1} \beta = \beta R_F \alpha = \alpha R_F \beta = \beta R_F^{-1} \alpha$. So, $R_F^{-1}$ is symmetric.

Conversely, if $R_F^{-1}$ is symmetric, then $\alpha R_F \beta = \alpha (R_F^{-1})^{-1} \beta = \beta (R_F^{-1}) \alpha = \alpha R_F^{-1} \beta = \beta R_F \alpha$. So, $R_F$ is symmetric.

**Proposition 3.18** $R_F$ is symmetric if and only if $R_F^{-1}=R_F$

**Proof:** If $R_F$ is symmetric, then $\alpha R_F^{-1} \beta = \beta R_F \alpha = \alpha R_F \beta$. So, $R_F^{-1}=R_F$.

Conversely, if $R_F^{-1}=R_F$, then $\alpha R_F \beta = \alpha R_F^{-1} \beta = \beta R_F \alpha$. So, $R_F$ is symmetric.

**Proposition 3.19** If $R_{F_1}$ and $R_{F_2}$ are symmetric relations on $\Gamma_X$, then $R_{F_1} \circ R_{F_2}$ is symmetric on $\Gamma_X$ if and only if $R_{F_1} \circ R_{F_2}=R_{F_2} \circ R_{F_1}$

**Proof:** If $R_{F_1}$ and $R_{F_2}$ are symmetric, then it implies $R_{F_1}^{-1}=R_{F_1}$ and $R_{F_2}^{-1}=R_{F_2}$. We have $(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1}^{-1}$, then $R_{F_1} \circ R_{F_2}$ is symmetric. It implies $R_{F_1} \circ R_{F_2} = (R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1} = R_{F_2} \circ R_{F_1}$.

Conversely, $(R_{F_1} \circ R_{F_2})^{-1} = R_{F_2}^{-1} \circ R_{F_1} = R_{F_2} \circ R_{F_1} = R_{F_1} \circ R_{F_2}$. So, $R_{F_1} \circ R_{F_2}$ is symmetric.

**Corollary 3.20** If $R_F$ is symmetric, then $R_F^n$ is symmetric for all positive integer $n$, where $R_F^n = \overbrace{R_F \circ R_F \circ \ldots \circ R_F}^{n \text{ times}}$. 

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Proposition 3.21  If $R_F$ is transitive, then $R_F^{-1}$ is also transitive.

Proof:

\[
\alpha R_F^{-1} \beta = \beta R_F \alpha \supseteq (\beta R_F \circ R_F) \alpha
\]

\[
= (\beta R_F \gamma) \land (\gamma R_F \alpha)
\]

\[
= (\gamma R_F \alpha) \land (\beta R_F \gamma)
\]

\[
= (\alpha R_F^{-1} \gamma) \land (\gamma R_F^{-1} \beta)
\]

\[
\alpha (R_F^{-1} \circ R_F^{-1}) \beta
\]

So, $R_F^{-1} \circ R_F^{-1} \subseteq R_F^{-1}$. The proof is completed.

Proposition 3.22  If $R_F$ is transitive then $R_F \circ R_F$ is so.

Proof:

\[
\alpha (R_F \circ R_F) \beta = (\alpha R_F \gamma) \land (\gamma R_F \beta)
\]

\[
= (\alpha R_F \circ R_F) \gamma \land (\gamma (R_F \circ R_F) \beta)
\]

\[
= (\alpha R_F \circ R_F \circ R_F) \beta
\]

So, $\alpha (R_F \circ R_F \circ R_F) \beta \subseteq \alpha (R_F \circ R_F) \beta$. The proof is completed.

Proposition 3.23  If $R_F$ is reflexive then $R_F^{-1}$ is so.

Proof: $\alpha R_F^{-1} \beta = \beta R_F \alpha \subseteq \alpha R_F \alpha = \alpha R_F^{-1} \alpha$ and $\beta R_F^{-1} \alpha = \alpha R_F \beta \subseteq \alpha R_F \alpha = \alpha R_F^{-1} \alpha$. The proof is completed.

Proposition 3.24  If $R_F$ is symmetric and transitive, then $R_F$ is reflexive.

Proof: Proof can be made easily by using Definition 4.1, Definition 4.2 and Definition 4.3.

Definition 3.25  Let $\Gamma_X \in FPS(U)$, $R_F$ be an FP-soft equivalence relation on $\Gamma_X$ and $\alpha \in R_F$. Then, an equivalence class of $\alpha$, denoted by $[\alpha]_{RF}$, is defined as

\[
[\alpha]_{RF} = \{ \beta : \alpha R_F \beta \}.
\]

Example 3.26  Let us consider the Example 3.16. Then an equivalence class of $(x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\})$ will be as follows.

\[
[(0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\})]_{R_F} = \left\{ (0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\}), (0.7/x_2, \{u_3, u_7, u_8\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9\}) \right\}
\]
4 Decision Making Method

In this section, we construct a soft fuzzification operator and a decision making method on FP-soft relations.

**Definition 4.1** Let $\Gamma_X \in FPS(U)$ and $R_F$ be a FP-soft relation on $\Gamma_X$. Then fuzzification operator, denoted by $s_{R_F}$, is defined by

$$s_{R_F} : R_F \rightarrow F(U), \quad s_{R_F}(X \times X, U) = \{\mu_{R_F}(u)/u : u \in U\}$$

where

$$\mu_{R_F}(u) = \frac{1}{|X \times X|} \sum_j \sum_i \mu_{R_F}(x_i, x_j) \chi(u)$$

and where

$$\chi(u) = \begin{cases} 1, & u \in f_{R_F}(x_i, x_j) \\ 0, & u \notin f_{R_F}(x_i, x_j) \end{cases}$$

Note that $|X \times X|$ is the cardinality of $X \times X$.

Now, we can construct a decision making method on FP-soft relation by the following algorithm:

1. construct a feasible fuzzy subset $X$ over $E$,
2. construct a FP-soft set $\Gamma_X$ over $U$,
3. construct a FP-soft relation $R_F$ over $\Gamma_X$ according to the requests,
4. calculate the fuzzification operator $s_{R_F}$ over $R_F$,
5. select the objects, from $s_{R_F}$, which have the largest membership value.

**Example 4.2** A customer, Mr. X, comes to the auto gallery agent to buy a car which is over middle class. Assume that an auto gallery agent has a set of different types of car $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$, which may be characterized by a set of parameters $E = \{x_1, x_2, x_3, x_4\}$. For $i = 1, 2, 3, 4$ the parameters $x_i$ stand for “safety”, “cheap”, “modern” and “large”, respectively. If Mr. X has to consider own set of parameters, then we select a car on the basis of the set of customer parameters by using the algorithm as follows.

1. Mr X constructs a fuzzy sets $X$ over $E$,
   $$X = \{0.5/x_1, 0.7/x_2, 0.3/x_3\}$$
2. Mr X constructs a FP-soft set $\Gamma_X$ over $U$,
   $$\Gamma_X = \{(0.5/x_1, \{u_1, u_3, u_4, u_6, u_7, u_8\}), (0.7/x_2, \{u_3, u_7, u_8\}), (0.3/x_3, \{u_1, u_2, u_4, u_5, u_6, u_9\})$$
3. the fuzzy parametrized soft relation $R_F$ over $\Gamma_X$ is calculated according to the Mr X’s requests (The car must be a over middle class, it means the membership degrees are over 0.5),

$$R_F = \left\{ \frac{0.5}{(x_1, x_1)}, \{u_1, u_3, u_4, u_6, u_7, u_8\}\right\}, \left\{ \frac{0.5}{(x_1, x_2)}, \{u_3, u_7, u_8\}\right\}, \left\{ \frac{0.5}{(x_2, x_1)}, \{u_3, u_7, u_8\}\right\}, \left\{ \frac{0.7}{(x_2, x_2)}, \{u_3, u_7, u_8\}\right\} \right\}$$

4. the soft fuzzification operator $s_{R_F}$ over $R_F$ is calculated as follows

$$s_{R_F} = \left\{ \frac{0.055}{u_1}, \frac{0.0}{u_2}, \frac{0.244}{u_3}, \frac{0.055}{u_4}, \frac{0.0}{u_5}, \frac{0.055}{u_6}, \frac{0.244}{u_7}, \frac{0.244}{u_8}\right\}$$

5. now, select the optimum alternative objects $u_3$, $u_7$ and $u_8$ which have the biggest membership degree 0.244 among the others.

5 Conclusion

We first gave most of the fundamental definitions of the operations of fuzzy sets, soft sets and FP-soft sets are presented. We then defined relations on FP-soft sets and studied some of their properties. We also defined symmetric, transitive and reflexive relations on the FP-soft sets. Finally, we construct a decision making method and gave an application which shows that this method successfully works. We have used a t-norm, which is minimum operator, the above relation. However, application areas the relations can be expanded using the above other norms in the future.

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