On the Problem of Inertia in Classical Mechanics

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Abstract. From Newton's classical mechanics to Einstein's relativistic mechanics, people have a deeper understanding of the nature. In Newton's classical mechanical system, there is an absolutely static space, which is called absolute space. In this way, the inertia of the object can be considered to be caused by the acceleration of the object. But in Einstein's view of time and space, there is no absolute space at all, so the origin of inertia becomes a problem.

1. Introduction
In classical mechanics, inertia is the intrinsic property of an object. A mass object, whether it moves or not, has inertia. I remember that in the physics textbook of middle school, there is a description of quality: quality is the only measure of inertia. That is to say, mass can be used to describe inertia. But in Newton's Mathematical Principles of Natural Philosophy, the definition of inertia is: the inherent force of matter is a kind of resistance, which exists in every object, the size is equal to the mass of the object, and keeps its existing status. Note that inertia is defined here as a force, that is, there is a force that can be used to describe the inertia of an object, which is the inertial force. This brings to mind Newton's second law, that is, the change of the amount of motion (momentum) of an object is directly proportional to the combined external force on the object. Therefore, the inertia of an object moving at a variable speed can be defined by the ratio of its external force to its mass, which is acceleration, that is, the formula of Newton's second law is:

\[ F = m \cdot a = m \cdot \frac{dv}{dt} = \frac{d}{dt}(m \cdot v) \]  

In the above formula, we get an amazing result: inertia is generated by acceleration. Because the photon has a constant speed and the photon has no static mass, the speed of the particle with mass can not reach the speed of light. When it moves in variable speed, it will inevitably produce acceleration. Therefore, mass is no longer the only measure of inertia. The three physical quantities involved in the formula can have a quantitative description of inertia. From the second law of Newton, we know that the acceleration is provided by the external force. Combined with the definition of inertia in the Mathematical Principles of Natural Philosophy, we can reason that we can conclude that the inertial force and the external force of the object have the same direction.

2. Origin of Inertial Force
From the scale of the atomic (1×10^{-15} m) to the lower limit of the radius of curvature of the universe, (1×10^{26} m) contains a total of 41 scales. Of course, there are smaller scales than the nucleus. For a moving object (not including uniform linear motion), all particles in its interior are actually linked, that is, we can think of a moving object of any scale, which will drive all particles in its interior.

Taking the circular motion of a disk with a small radius as an example, the centripetal acceleration of the points on the disk at different distances from the center is different. But for a given disk, there is...
a certain distance between each particle on its surface and the original center, so we can divide the disk into infinite levels, and each point on each level will have a certain centripetal acceleration. Because of the uniform circular motion of the disk, every point on its edge will be subject to a kind of binding force, that is, centripetal force (let alone the binding of the center to it), then the particle on this edge will produce centripetal acceleration. According to Newton's second law (namely \( a \)), acceleration can have a quantitative measure of inertia. At this time, the point on that edge will be subjected to inertial centrifugal force, and as the distance from the center of the circle increases, the analogy of the centripetal force formula, we can get:

\[
F_{\text{centripetal}} = -m\omega^2 r
\]

For a disk with a radius of 1 light-year (i.e. large scale), the points on its edge will also be pulled. But at this time, it can't be regarded as the pull of the center to him, because there is 1 light-year from the center to the edge of the center. On such a large scale, the pull (gravity) of the center to it is not enough to provide the centripetal force when it makes a circular motion. So where does the centripetal force come from? We know that there are infinite particles on the line between the particles at the edge of the disk and the center of the circle. Considering two adjacent mass points, the acceleration generated by a particle closer to the center of the circle is always greater than the acceleration generated by a particle farther from the center of the circle. In the neighborhood of these two particles, an acceleration field is generated. At this time, a strong acceleration field will drive a weaker acceleration field. As a result, a “chain reaction” occurs on the rotating disk, that is, the continuous interaction of the acceleration fields generated by the infinite number of mass points on the line. Therefore, the closer to the center of the circle on this line, the greater the acceleration generated, that is, the greater the inertial centrifugal force. Thus, to rotate a given disk, the centroid acceleration of the particle at the edge of the disk is first generated, and then the centripetal acceleration is generated at other points on the disk. In this way, all the particles on the disk have acceleration. For two particles with different acceleration and infinite proximity, there will be a relative acceleration \( \Delta a \) between them because of the interaction of their acceleration fields. Let the average mass between the two particles be \( \bar{m} \). So the product of the relative acceleration and the average mass of two particles is the inertial force.

\[
f = \bar{m} \Delta a
\]

According to the Lagrange mean value theorem, it is always possible to find an equivalent point \( \xi \) between two points.

\[
m \xi \Delta a = \frac{m_1 a_1 - m_2 a_2}{\Delta a}
\]

In classical mechanics, acceleration is defined as the amount of change in velocity per unit time, written as:

\[
a = \frac{dv}{dt} = x
\]

For a moving object whose acceleration changes uniformly, the origin of its inertial force is slightly more complicated. We know that the derivative represents the rate of change of a certain amount, and the change in acceleration can also be expressed in derivatives. which is:
This is the rate of change of acceleration, called second-order acceleration or acceleration. Similarly, the acceleration of order 3 or even order \( n \) can be defined, which depends on the volume of the object to be discussed and the accuracy of the inertia of the object. The total inertial force generated by an object is the sum of the inertial forces generated by each level. This form can be thought of as an expansion of the Taylor formula, namely:

\[
F(a) = 1 + af^{(1)}(a) + a^2 \frac{f^{(2)}(a)}{2!} + a^3 \frac{f^{(3)}(a)}{3!} + \cdots + a^n \frac{f^{(n)}(a)}{n!}
\]  

(7)

For a given inertia accuracy, we need to add the Lagrangian remainder to the right of the above formula, namely:

\[
R_n(a) = \frac{f^{(n)}(a_n)}{(n+1)!} a^{(n+1)}
\]

(8)

Then all the items after the given precision (infinitesimal) can be approximated by the Lagrangian remainder.

3. The Nature of Inertia

In a flat and gravitationless space-time, the kinetic energy of a photon is \( h\nu = mc^2 \). When a photon passes near a massive star, it will gravitationally redshift. At this time, the frequency of photons will decrease, that is, \( h\nu < mc^2 \), so the phenomenon of gravitational redshift shows that the kinetic energy of photons will decrease when they pass through massive stars, so the inertia of photons will be shown. On the earth, we can think that the earth is a static reference system, and all movements relative to the earth are absolute. Since any object on the earth has mass, it tends to be relatively static. But for particles without mass, they will maintain a tendency to move relative to the Earth. We discussed in the first part that for a continuous object, the motion of its internal particles is related. For a system of multiple objects, the motion of objects inside the system is also related (gravitational or electromagnetic interaction). We can use Sirius as an example. We know that Sirius is a binary system, and its companion star is a white dwarf star called Sirius B. Because of its existence, Sirius is not the same as the calculated orbit. Now suppose that there is no Sirius B and ignore the gravitational effects of all celestial bodies, then the movement of Sirius at this time can be regarded as free movement. Due to the existence of Sirius B, Sirius does not operate in the original orbit. According to Newton's second law, Sirius was subjected to inertia at this time.

It is not difficult to see that for a system of multiple objects, the inertial force is equivalent to the difference between the gravitational force and the free force of an object, so it satisfies the law of gravitational change, namely:

\[
m_i(a_i - a_n) = \frac{Gm_im_G}{r^2}
\]

Using gold substitution, you have:

\[
m_i a = \frac{Gm_G}{r^2} \Rightarrow m_G = \frac{ar^2}{G} (a = a_i - a_n)
\]

(10)

It can be seen that the inertia of an object is inversely proportional to the gravitational action it receives.

For a galaxy, as the distance between its internal matter and the center (black hole) increases, the less gravity it receives, the greater its inertia. From this we can get a conclusion: inertia is a kind of property that makes the object get rid of the external force.
4. Conclusion
To sum up, the inertia of the object is inversely proportional to the acceleration, and the acceleration (eigenacceleration) can be considered as the transformation from the eigenmass (gravitational mass) of the object to the inertia mass. Therefore, inertia can also be understood as the degree to which an object converts potential energy to kinetic energy. If \( I \) is used to represent inertia, then:

\[
I(m) = \frac{m_0 gh}{\frac{1}{2} m v^2}
\]

(11)

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