Creative Mathematical Reasoning of Prospective Teachers in Solving Problems Reviewed Based on Working Memory Capacity

I Palengka¹*, D Juniati² and Abadi²

¹ Mathematics Education, Universitas Kristen Indonesia Toraja, Indonesia
² Mathematics Education, Universitas Negeri Surabaya, Indonesia

*E-mail: inelsipalengka@ukitoraja.ac.id

Abstract. Reasoning requires logical thinking in solving problems, and working memory capacity becomes something important. This study aimed at investigating the creative mathematical reasoning of prospective teachers in solving problems reviewed based on working memory capacity. This research was qualitative. The study conducted at Indonesian Christian University of Toraja to Mathematics Education study program students. The selection of research subjects was done by giving operation span (O-SPAN) assignments to select 1 student with a high category of working memory capacity and 1 student with a low category of working memory capacity. Then they were given a creative mathematical reasoning test and interview. Based on the data analysis, it obtained that the creative mathematical reasoning of prospective teachers in solving problems on the research subject who had low working memory capacity showed creativity and plausibility criteria but it did not show anchoring because he could not provide arguments that supported the selection of strategies based on mathematics. Furthermore, creative mathematical reasoning of prospective teachers in solving problems on the research subject who had high working memory capacity met the three criteria for creative mathematical reasoning, namely creativity, plausibility and anchoring.

1. Introduction
Mathematics is an activity of human life, so learning mathematics is learning to solve problems. At present problem solving is a major concern in mathematics learning activities at the elementary, secondary and high education levels. At the higher education level, prospective teachers need opportunities to formulate and solve complex problems that involve a large number of efforts. They must be encouraged to reflect on their thinking during the problem-solving process so they can apply and adapt their strategies in developing them to problems and in other contexts. This is a concern so that reasoning is one of the main objectives of learning mathematics. Also, reasoning ability is critical to understand mathematics and make mathematics meaningful. For this reason, reasoning can be trained from the early age to students by accustoming them to provide arguments or logical reasons in solving mathematics and statistics problems [1] because understanding mathematics is meaningless without emphasising reasoning [2]. Bottou [3] defines that reasoning is manipulating the knowledge gained in algebra to answer new questions. The reasoning defined as the thinking line taken to produce statements and reach conclusions in completing a task [4]. Further reasoning is treated as logical knowledge by Sumpter [5]. To develop student reasoning, the role of problem-solving is very important [6]. This is
intended to give them many experiences to develop their reasoning through the problem-solving process [7]. One form of reasonings is mathematical reasoning. One type of mathematical reasoning is creative mathematical reasoning [8], [21]. Creative mathematical reasoning is one of the reasoning processes carried out by a person in solving problems through non-routine procedures [9], [19], [20]. Lithner [4] defines that creative mathematical reasoning meets the criteria:

(a) Creativity: the learner creates a reasoning sequence not experienced previously, or re-creates a forgotten one, (b) Plausibility: there are predictive arguments supporting the strategy choice and arguments for verification, explaining why the strategy implementation and conclusions are true or plausible, (c) Anchoring: the arguments implementation and conclusions are anchored in the intrinsic mathematical properties of the components of the reasoning.

There are many positive impacts experienced by students in solving problems using creative mathematical reasoning. Norqvist [4] explains that students who practice with creative tasks outperform students who practice with algorithmic ones in completing assigned tasks. Norqvist [10] points out that creative mathematical reasoning is more efficient in long-term understanding than algorithmic reasoning. However, the thing that arises is in solving mathematical problems. Prospective teachers still consider problems that have a variety of ways of solving or open questions are problems that are difficult to solve. One reason is, in students’ minds, the relevant and irrelevant information to complete a task is very much, so students have difficulty in choosing which information is appropriate in completing tasks that require creative mathematical reasoning. The same thing is stated by Fockert saying that a thing that might happen is that individual with lower working memory capacity has more difficulty in sorting out very much information [11]. For this reason, working memory is needed to sort out the appropriate information to solve the problem by taking into account the working memory capacity as a basis.

Working memory is a limited capacity system that is responsible for the temporary maintenance and manipulation of a series of representations for ongoing cognition [12-14]. Working memory plays a role in processing information and digging back information contained in long-term memory so that working memory capacity becomes indispensable in creative mathematical reasoning. This is consistent with the previous research stating that working memory capacity can reduce the errors in retrieving information that is not needed in problem-solving [22] so Fife’s opinion is true that working memory capacity is involved in problem-solving [23]. So, in creative mathematical reasoning, the role of working memory capacity is needed to provide relevant information [4]. Based on the background, this study aimed at investigating the creative mathematical reasoning of prospective teachers in solving problems reviewed from working memory capacity.

2. Method
This research was qualitative. The study conducted at Indonesian Christian University of Toraja to Mathematics Education study program students. Research subjects were determined by using o-span tests to select a subject with high memory capacity (MT) and a subject with low memory capacity (MR). Both subjects were given a creative mathematical reasoning test in the form of an open task consisting of two numbers. Question number 1 raised the problem of how to set paving blocks consisting of four different shapes on a rectangular yard. Question number 2 presented the problem of how to put-up paintings of four sizes on the wall. After being given a test, interviews were conducted on both subjects. The questions written in the interview guidelines are based on creative mathematical reasoning criteria. In this study, interviews were conducted with the subject about the task that was completed by the subject, namely the creative mathematical reasoning test. The results of mathematical creative reasoning tests and interview results were then analysed to get conclusions.

3. Results and Discussion
The following is the results of creative mathematical reasoning analysis reviewed based on the working memory capacity of the two subjects based on creative reasoning tests and interviews.
3.1. Description of Subject's Creative Mathematical Reasoning Capability Based on Low Memory Capacity (MR)

The creative mathematical reasoning capability of subjects based on low memory capacity (MR) will be described based on three aspects namely creativity, plausibility and anchoring as follows:

3.1.1. Creativity

MR showed renewal in new reasoning by assuming the area of the wall with L, the height of the painting from the floor as x, the distance of the painting from the ceiling as y and the distance of one painting to another as z. Suppose the paintings size 100 cm x 100 cm as A, size 50 cm x 50 cm as B, size 50 cm x 100 cm as C, and size 100 cm x 50 cm as D. After that MR described what he thought like the following figure:

![Figure 1. MR's answer to question No. 2](image)

MR showed renewal by giving the correct answer, but MR did not find another answer for that problem. It meant MR's answer did not show fluency. However, MR's answer was quite flexible because MR could provide another solution at the interview.

Q: Do you think there is still a way to put up the paintings other than what you are describing?
A: Yes

Q: How do you do it?
A: by changing the position of the paintings

3.1.2. Plausibility.

Plausibility was shown by placing paintings without giving any distance at all, and it did not conflict with the instructions that the maximum distance of painting was 5 cm. Furthermore, MR stated that "I arrange paintings by considering the maximum number of paintings that must be loaded. In other words, it must contain as many paintings as possible so that I maximise the paintings with the smallest size". It showed that MR could provide reasonable arguments about the strategy he used.

The plausibility criterion was only shown in question number 2. MR could not provide a solution in question number 1, so it was difficult to trace the reasons for choosing the solution strategy he used. It also showed that the subject with low memory capacity had difficulty in solving more difficult problems.

3.1.3. Anchoring

The anchoring criterion was not shown by MR both in question number one and in question number two. In question number two, MR was able to give the right answer by using a reasonable strategy, but MR could not provide an argument showing that the strategy used was by mathematics.
3.2. Description of Subject’s Creative Mathematical Reasoning Capability Based on High Memory Capacity (MT)

The creative mathematical reasoning capability of subjects based on High memory capacity (MT) will be described based on three aspects, namely creativity, plausibility and anchoring as follows:

3.2.1. Creativity

In question number one, MT was able to understand the problem and showed the creativity criterion with new reasoning. MT stated that he had ever solved almost the same problem, but solving the problem of arranging paving blocks with various types required different thoughts. It was known from the interview as follows:

\[ Q: \text{Do you use the method you used before to answer this question?} \]
\[ A: \text{No Ma’am} \]
\[ Q: \text{why?} \]
\[ A: \text{Because what I have ever done was to arrange rectangles, so it was easy. Now it's very different because in addition to the different shapes, the sizes are also different. That's what makes it difficult.} \]
\[ Q: \text{how do you solve it?} \]
\[ A: \text{I draw the yard first then I draw a hexagon in the middle because it is the most difficult to set and then I set the pentagon and set the rectangle and square around it.} \]

From this interview, it could be seen that in order to solve the problem, MT thought of the form of yard and forms of paving blocks, then he arranged the paving blocks by starting from the form that according to the subject was the most difficult to set with other forms of paving blocks. The MT’s answers are seen in the following figures:

![Figure 2](image1.png)  
**Figure 2.** The MT’s 1st answer for question No. 1

![Figure 3](image2.png)  
**Figure 3.** The MT’s second answer to question No. 1

From the figures, it can be seen that the MT showed creativity with fluency in giving two different and flexible answers in showing two different ways of setting.

For question number 2, MT stated that it was the first time for him to encounter the question like in question number two so that the reasoning is done by MT could be categorised as new reasoning for the subject. MT also showed creativity with fluency shown by giving two answers and MT was flexibly shown by describing two different ways of putting up paintings on the wall as in the following figure:
3.2.2. Plausibility

The plausibility criterion for question number 1 was shown by MT by writing the yard area equal to the total area of all paving blocks covering the yard. MT determined the area of each type of paving block and multiplied it by the number of paving blocks of each type and summed up the total area to get the yard area as shown below.

3.2.3. Anchoring

The anchoring criterion was shown by MT in question number 1 by using the concept of plane figure area. To calculate the area of a yard and the area of a brick type paving block MT used a rectangular area, to calculate the area of a hexagon he used 2 times the area of a trapezoid, to calculate the area of a bishop’s hat he used a rectangular area plus the area of a triangle and to calculate the area of a small tile type paving block he used a square area as in the following figure:
Figure 8. Anchoring criterion for question number 2

Anchoring criteria met if information about matters relating to reasoning can be investigated through argumentation. Lithner [9] states meet anchoring if it meets the relevant mathematical properties of the components in reasoning. For question number 2, anchoring was shown by using addition and subtraction operations. MT reduced the area of the wall by a distance of 150 cm from the ceiling and a distance from the floor by 145 cm and only depicted the paintings to be placed in the reduced area and took the maximum distance and placed the paintings at a distance of 5 cm from contiguous paintings.

4. Conclusion
After analysing the mathematical creative reasoning test and the results of the interview, the following results are obtained: Subject with low working memory capacity demonstrates creative criteria by creating a new set of reasoning in solving problems. Moreover, the subject can provide arguments supporting the logical solution choice, so the subject meets the criteria of plausibility. However, the subject cannot provide arguments that support the strategy selection based on mathematics (mathematical basis) that is relevant. It means that the subject does not meet the anchoring criteria. And the research subject with high working memory capacity meets the three criteria of creative mathematical reasoning in solving problems, namely the creativity criteria and also meets the criteria of Plausibility and anchoring.

For further research, it is expected that researchers conduct research on creative mathematical reasoning with anchoring criteria in terms of the components of reasoning, namely objects, transformations and concepts.

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