Large \( N \) limit of extremal non-supersymmetric black holes

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The large \( N \) limit of extremal non-supersymmetric Type-I five-dimensional string black holes is studied from the point of view of D-branes. We find that the agreement between the D-brane and the black-hole picture is due to an asymptotic restoration of supersymmetry in the large \( N \) limit in which both pictures are compared. In that limit Type-I string perturbation theory is effectively embedded into a Type-IIB perturbation theory with unbroken supersymmetric charges whose presence guarantees the non-renormalization of mass and entropy as the effective couplings are increased. In this vein, we also study the near-horizon geometry of the Type-I black hole using D5-brane probes to find that the low energy effective action for the probe is identical to the corresponding one in the auxiliary Type-IIB theory in the large \( N \) limit.

05/98

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1. Introduction

Black-hole physics has been regarded since the seventies as one of the most promising windows to quantum gravity. As a consistent candidate for a quantum description of the gravitational interaction, string theory has been frequently claimed to be the right framework to solve some long-standing problems in black-hole physics, such as the information paradox or the microscopic meaning of the geometrical entropy. Because of the non-perturbative nature of black holes, perturbative string theory is of limited use in the analysis of the most interesting dynamical issues. For this reason, it was only after the recent development of non-perturbative techniques that significant progress was achieved in this program. In these recent developments the concept of D-brane has been the key ingredient to address the problem of black holes in string theory. This is not a surprise since D-branes appear as our first ‘probes’ into the non-perturbative realm of string theory (for a review see [1]).

The description of supersymmetric black-hole dynamics in terms of BPS excitations of a D-brane bound state [2] is indeed among the most impressive achievements of string theory. Here, supersymmetry, in the form of BPS saturation, is the crucial ingredient for the success of this picture because of the existence of non-renormalization theorems ensuring the equality of BPS state degeneracies at weak coupling (D-brane side) with the large coupling (black-hole side) geometrical entropy. Once supersymmetry is broken there is no reason whatsoever to expect the weak and strong coupling descriptions to be equivalent. This is the main obstacle, for example, in getting a microscopic picture of the physics of the Schwarzschild black hole, the simplest example of a black hole in General Relativity (for recent progress in this direction see, [3]).

This being said, there is however a recent example of an extremal but non-supersymmetric black hole in Type-I superstring theory [4] for which the microscopic description of the entropy in terms of excitations of a D-brane bound state agrees with the semiclassical, general relativistic, computation of the geometrical entropy. Surprisingly, this black hole cannot be regarded as an ‘almost supersymmetric’ one, in the sense that the departure from the supersymmetric configuration is not governed by a small parameter; it is only connected with a supersymmetric black hole by a discrete $\mathbb{Z}_2$ transformation reversing the sign of one of the charges. Thus, there is no $a$ priori reason to expect that
quantum corrections to the weak coupling mass and entropy should vanish or be small when the coupling is increased.\footnote{For earlier studies of extremal non-supersymmetric black holes in supergravity, see \cite{5}, \cite{6}.}

In the present article we will investigate the reasons behind the success of the D-brane picture of the non-supersymmetric Type-I black hole of ref. \cite{4}. We will argue that in the semiclassical limit \cite{7} \cite{8}

$$\lambda \rightarrow 0, \quad Q_1, Q_5, N \rightarrow \infty, \quad \lambda Q_1, \lambda Q_5, \lambda^2 N \text{ fixed}, \quad (1.1)$$

at which we compare the D-brane and black-hole computations, the D-brane bound state of Type-I theory, which preserves no supersymmetry, is effectively embedded into a D-brane bound state of Type-IIB theory having four unbroken supersymmetric charges. From the point of view of D-brane dynamics, the semiclassical limit (1.1) is nothing but a 't Hooft large $N$ limit, and we are just saying that the supersymmetry-breaking effects are of $O(1/N)$ in the large $N$ limit. Thus, non-renormalization theorems of the Type-IIB theory are also at work in the Type-I non-supersymmetric black hole in this limit. This explains the absence of quantum corrections to the D-brane mass and entropy.

The plan of the article is as follows. In Section 2 we will review the construction of Dabholkar’s black hole by carefully computing the classical dimension of the moduli space of vacua and check the agreement with the low-energy geometrical entropy. Section 3 will be devoted to explaining the agreement between the two results by checking that those string diagrams that contribute to the renormalization of the mass and entropy are suppressed in the semiclassical limit. In Section 4 we will study the near-horizon geometry by probing the non-supersymmetric black hole with D-branes. Finally in Section 5 we will summarize the conclusions. For the sake of completeness, we have reviewed in Appendix A the semiclassical limit of Feynman–Polyakov diagrams containing open string insertions, while Appendix B is devoted to studying the relation between Type-I and Type-IIB superstring perturbation theories in the large $N$ limit.

### 2. Non-supersymmetric black hole in Type-I superstring theory

Before entering into a more detailed study, let us briefly review the main features of the non-supersymmetric black hole of ref. \cite{4}. The construction mimics in many aspects
the corresponding one for the supersymmetric five-dimensional black hole in Type-IIB in [9].

We consider the $SO(32)$ Type-I superstring theory on $\mathbb{R}^5 \times S^1_R \times \mathbb{T}^4$. We denote by $X^0, \ldots, X^4$ the coordinates in the open five-dimensional space-time, whereas those in the internal four-torus are $X^5, \ldots, X^8$, and $X^9$ is the coordinate along $S^1_R$. In the weakly coupled region, the black hole is described by the bound state of $Q_1$ D1-branes, wrapped around $S^1_R$ and $Q_5$ D5-branes wrapped around the five-dimensional torus $S^1_R \times \mathbb{T}^4$, in the presence of 32 D9-branes. In addition, there are $N$ units of Kaluza–Klein momentum in the $X^9$ direction.

One of the main differences with respect to the Type-IIB black hole is the presence of D9-branes. They break the original 32 real supersymmetries of the Type-IIB theory down to 16 by imposing the condition $\epsilon_L = \Gamma_{11} \epsilon_R$ on the Killing spinors of the ten-dimensional theory, where $\Gamma_{11} = \Gamma^0 \ldots \Gamma^9$ is the product of all gamma matrices. This, together with the chirality condition $\epsilon_{L(R)} = \Gamma_{11} \epsilon_{L(R)}$, implies that $\epsilon_L = \epsilon_R \equiv \epsilon$. The D5- and D1-branes further reduce the remaining 16 supersymmetries down to 4, by demanding that $\epsilon = \Gamma^0 \Gamma^9 \epsilon$ and $\epsilon = \Gamma^0 \Gamma^5 \ldots \Gamma^9 \epsilon$. Therefore, the resulting theory in the (1+1)-dimensional intersection has 4 unbroken real supersymmetries (the number corresponding to $N = 1$ in $D = 4$ or $N = 4$ in $D = 2$).

One now has to introduce the Kaluza–Klein momentum along the $X^9$ direction. This is done by a string condensate whose presence imposes a last condition on $\epsilon$, $\Gamma^0 \Gamma^9 \epsilon = \pm \epsilon$, where the two signs correspond to the two different directions of the momentum. Taking the $+$ direction, we find that this last equation is identical to the reflection condition on the D1-brane, and no further reduction of supersymmetries occurs. However, by taking the momentum in the $-$ direction, the two conditions turn out to be incompatible for non-vanishing $\epsilon$, and no supersymmetry survives. The resulting bound state is not supersymmetric.

In the strong coupling side, where the black hole is described by the semiclassical values of the metric and other long-range fields, a similar analysis is also possible, with the result that supersymmetry is preserved only when the momentum is in one of the two possible directions along $S^1_R$ [10], [5], [4].

What makes the non-supersymmetric version of the Type-I black hole interesting is the fact, pointed out in [4], that the counting of the number of massless excitations of the
D-brane bound state characterized by \((Q_1, Q_5, N)\) exactly agrees with the entropy of the semiclassical black hole with the same charges, defined as \(\frac{1}{4}\) the area of the event horizon. This is very surprising, since here we do not have any supersymmetry left and consequently there are no non-renormalization theorems at hand to force the equivalence of the weak and strong coupling computations.

\[2.1. \text{Looking from the D-brane side}\]

To study the D-brane dynamics of the non-supersymmetric Type-I black hole we begin with the well-known Type-IIB five-dimensional \textit{supersymmetric} black hole \([2], [11]\) from which the former can be obtained by introducing 32 D9-branes and projecting down onto the sector invariant under world-sheet orientation reversal. Using the notation of \([11]\), the low-energy fields are \((1,1)\) and \((5,5)\) hypermultiplets in the adjoint of \(U(Q_1)\) and \(U(Q_5)\) respectively (whose bosonic components are denoted by \(A_I\) and \(A'_I, I = 5, \ldots, 8\)) arising in the dimensional reduction of the ten-dimensional vector multiplet, together with the \((1,5)\) hypermultiplet \(\chi\), which transforms as \((1,2)\) under \(SO(5,1) \times SO(4)_I\). Its gauge group indices run in the fundamental of \(U(Q_1) \times U(Q_5)\). The D-terms in the low-energy Lagrangian can be written as \([11]\)

\[
\sum_{\ell} [(D^a_{12})^2 + (D^a_{13})^2 + (D^a_{14})^2]
\]

with

\[D^a_{IJ} = \text{Tr} \left\{ T^a \left( [A_I, A_J] + \frac{1}{2} \epsilon_{IJKL} [A_K, A_L] \right) + \chi^+ T^a \Gamma_{IJ} \chi \right\}
\]

where \(T^a\) are the generators of \(U(Q_1)\) and \(\Gamma_{IJ} = (1/2)[\Gamma_I, \Gamma_J]\). Of course there are similar terms for the fields \(A'_I\), which include the generators \(T^a'\) of \(U(Q_5)\). The counting of the flat directions along which (2.1) vanishes gives \(4Q_1Q_5\), once the effect of gauge transformations has been subtracted.

As stated above, in going from the Type-IIB to the Type-I black hole one has to introduce 32 D9-branes and perform a projection by the world-sheet parity \(\Omega\). This has a number of effects on the above computation. The first one deals with the fact that

\[\text{As we already pointed out above, the field } \chi \text{ transforms as a spinor } 2 \text{ of } SO(4). \text{ Keeping in mind that } SO(4) \sim SU(2)_L \times SU(2)_R, \text{ this means that the field is in the } \left(\frac{1}{2}, 0\right) \text{ representation of the product group. Consequently, only the self-dual part of } \Gamma_{IJ} \text{ will contribute to the D-term.}\]
the Chan–Paton factors are changed, now being $SO(Q_1)$ for the D1-brane and $USp(2Q_5)$ for the D5-brane [12], [13]. Secondly, one has to make an identification of those string excitations that differ by world-sheet inversion, in particular those in the (1,5) and (5,1) sectors. Let us focus our attention on the (1,5) field $\chi_{iaa'}$, where $i$ runs in the $\left(\frac{1}{2}, 0\right)$ of $SU(2)_L \times SU(2)_R$, $a = 1, \ldots, Q_1$ labels the fundamental of $SO(Q_1)$, and $a' = 1, \ldots, 2Q_5$ is in the fundamental of $USp(2Q_5)$. Therefore the field $\chi_{iaa'}$ is in the $2 \times Q_1 \times \overline{2Q_5}$ with respect to the full group $SO(4)_I \times SO(Q_1) \times USp(2Q_5)$.

The bosonic degrees of freedom in the (5,1) sector are represented by the field $\chi_{+ia'a} \equiv (\chi_{iaa'})^*$ transforming in the $2 \times 2Q_5 \times Q_1$. Notice that both fields $\chi$ and $\chi^+$ transform under the same representation of $SO(Q_1)$ since $Q_1$ is real. The representations of $SO(4) \sim SU(2)_L \times SU(2)_R$, $2 = (\frac{1}{2}, 0)$ and $\overline{2} = (\frac{1}{2}, 0)$ are equivalent, because they are related by the Pauli matrix $\sigma_2$ through the relation $\sigma_2 \sigma_i^* \sigma_2 = -\sigma_i$. Finally, the representations $2Q_5$ and $\overline{2Q_5}$ are related by the intertwiner $\Sigma_2 = \sigma_2 \otimes 1_{Q_5}$. With all these facts in mind, the projection $\Omega$ acts on $\chi$ as

$$\sigma_2^{ij}(\Sigma_2)_{a'b'}\chi_{+ja'}^* = \chi_{iaa'}.$$ 

For the massless excitations of the (1,1) and (5,5) strings, the $\Omega$ projection determines the representations of the Chan–Paton factors [12], [13]. The vertex operators corresponding to (5,5) fields $A_I$ are $V \sim \lambda_{ij} \partial_t X^I$ and thus $\lambda_{ij}$ is in the adjoint representation of $USp(2Q_5)$. On the other hand, the vertex operator for (1,1) strings involves the normal derivative $\partial_n X^I$ and the associated Chan–Paton factor is in the symmetric representation of $SO(Q_1)$.

To compute the number of independent flat directions for the D-terms, we begin by counting the number of degrees of freedom in the hypermultiplets. For the Type-I case, this number is equal to

$$4Q_5(2Q_5 + 1) + 2Q_1(Q_1 + 1) + 4Q_1Q_5,$$

where the first term corresponds to the number of bosonic components in the (5,5) hypermultiplet and the remaining two terms counts the number of (1,1) and (5,1) hypermultiplets. Here we have taken into account that (1,1) strings have Chan–Paton factors in the symmetric representation of $SO(Q_1)$. To get the number of flat directions we have to subtract from (2.2) the number of conditions imposed by the vanishing of the potential for the
scalars (D-terms) and the number of gauge transformations. The number of independent 
D-terms is

$$3Q_5(2Q_5 + 1) + \frac{3}{2}Q_1(Q_1 - 1),$$

(2.3)

and that of gauge transformations is equal to the sum of the dimensions of the adjoint representations of $USp(2Q_5)$ and $SO(Q_1)$, namely

$$Q_5(2Q_5 + 1) + \frac{1}{2}Q_1(Q_1 - 1).$$

(2.4)

Putting everything together, we find the number of bosonic flat directions for the Type-I 
black holes to be

$$2\text{(2.3)} - (2\text{.3}) - (2\text{.4}) = 4Q_1(Q_5 + 1) \approx 4Q_1Q_5.$$

For the number of fermionic flat directions we must look at the Yukawa couplings be-
tween the bosonic degrees of freedom and their superpartners. The six-dimensional theory 
for the low-lying degrees of freedom of the D1–D5 bound state has $\mathcal{N} = 1$ supersymmetry, 
equivalent to $\mathcal{N} = 2$ in $D = 4$ (before introducing the Kaluza–Klein momentum condensate, this is further reduced in the bulk by the presence of the 32 D9-branes of the Type-I 
theory). However, as it happens in the bosonic sector, the projection onto the unoriented 
sector of the theory reduces the number of independent degrees of freedom by relating the 
fields with their world-sheet parity transformed. The only difference with the bosonic case 
lies in the fact that now, instead of having three independent D-terms [eq. (2.3)] and the 
number of gauge transformations [eq. (2.4)], we have four Yukawa conditions. It is easy 
to check that the number of ‘fermionic’ flat directions is also

$$4Q_1(Q_5 + 1) \approx 4Q_1Q_5.$$

In counting the number of flat directions we have ignored the presence of the 32 D9-
branes. In this sense, except for the difference in the Chan–Paton factors, the computation 
closely follows the corresponding Type-IIB black-hole one. However, it is important to 
check that this counting is stable under quantum corrections involving (9,1) and (9,5) 
strings in loops. As we will argue in Section 3, this is indeed the case here when we go to 
the semiclassical limit, where all diagrams with D9 holes or cross-caps will be subleading.

To get the entropy we have to distribute the $N$ units of Kaluza–Klein momentum 
among the number of independent flat directions, as with the supersymmetric Type-IIB 
black hole. This is done by using Cardy’s formula for a superconformal field theory with 
$c_{\text{eff}} \equiv c_{\text{bos}} + \frac{1}{2}c_{\text{fer}} = 6Q_1Q_5$ with the final result

$$S = 2\pi \sqrt{NQ_1Q_5}.$$ 

(2.5)
The D-brane bound state studied above can be alternatively described in terms of the six-dimensional supersymmetric Yang–Mills theory on the $Q_5$ D5-brane system in the presence of classical configurations \cite{14}. The computation of the dimension of the moduli space of $USp(2Q_5)$ instantons renders the same value $\frac{2.5}{(2,3)}$ for the entropy. This can be easily understood as follows. Being an instanton moduli space, its dimension can be captured already in the dilute limit, as the instanton number times the dimension of the single instanton moduli space. For a single D1-brane, the gauge group carried by the $(1,1)$ strings is $SO(1)$, i.e. the trivial group, so in this case the counting proceeds as before without any need to factorize gauge degrees of freedom or imposing D-flatness conditions, yielding $4Q_5 + 4$ for the dimension of the single instanton moduli space of $USp(2Q_5)$. Multiplying now by the number of instantons $Q_1$, we obtain the desired result.

In computing the energy of the bound states of D-branes in Type-I superstring, we have to proceed with some care with the D5-branes, since they can, because of the $USp(2Q_5)$ Chan–Paton factor, effectively be considered as a pair of Type-IIB D5-branes. Keeping this in mind, we compute the energy of the bound state formed by $Q_1$ D1-branes, $Q_5$ D5-branes and $N$ units of Kaluza–Klein momentum in Type-I superstring theory as

$$ M = \frac{Q_1 R}{\lambda \alpha'} + \frac{2Q_5 RV}{\lambda (\alpha')^3} + \frac{N}{R}, \quad (2.6) $$

where $R$ is the radius of $S^1_R$ and $(2\pi)^4 V$ the volume of the four-torus $T^4$. We will see in the following section how this indeed agrees with the ADM mass of the black hole.

2.2. Connecting with the black-hole region

Once we have studied the Type-I black hole from the D-brane side, let us go to the strong coupling limit $\lambda Q_1, \lambda Q_5, \lambda^2 N > 1$ in which the D-brane bound state describes a semiclassical black hole with non-trivial background values for the metric, dilaton and the Ramond–Ramond two-form. The solution is formally identical to the one of the five-dimensional Type-IIB black hole \cite{11}, and it is characterized by three radii determining completely the geometry and the long-range fields of the black hole (the supersymmetric analogues of the Schwarzschild modulus $2G_N M \sim \kappa^2 M$):

$$ r_1^2 = \kappa^2 \frac{Q_1 (\alpha')^3}{V \lambda}, \quad r_5^2 = \kappa^2 \frac{(2Q_5) \alpha'}{\lambda}, \quad r_0^2 = \kappa^2 \frac{N (\alpha')^4}{R^2 V}. $$
The black-hole entropy is defined as

\[ S = \frac{A_H}{4G_5^2}, \]  

(2.7)

where \( A_H \) is the area of the event horizon and \( G_5 \) is the five-dimensional Newton’s constant, proportional to \( \kappa^2 \), the square of the closed string coupling. In the case at hand it can be expressed in terms of \( r_0^2, r_1^2 \) and \( r_5^2 \) to give

\[ S = 2\pi \sqrt{N Q_1 Q_5}, \]  

(2.8)

which agrees with the corresponding computation from the D-brane side, as it was noticed in [4]. The ADM mass is obtained as well from the large-distance limit of the black-hole metric and coincides with the mass of the D-brane system \( (2.6) \)

\[ M_{ADM} = \frac{R V}{\kappa^2(\alpha')^4} (r_1^2 + r_5^2 + r_0^2) = \frac{Q_1 R}{\lambda \alpha'} + \frac{2Q_5 RV}{\lambda(\alpha')^3} + \frac{N}{R}. \]

2.3. Enlarging the class of Type-I non-supersymmetric black holes

It is worth stressing that the extremal black hole of [4] can be included into a more general class of extremal non-supersymmetric black holes constructed by replacing the system of D5- and/or D1-branes by the corresponding anti-D-branes. Beginning with the Type-I supersymmetric configuration formed by D5-branes, D1-branes and Kaluza–Klein momentum in the ‘correct’ direction and trading, for example, the D1-branes by anti-D1-branes, the reflection condition for the Killing spinor on the anti-branes gets a minus sign to read \( \Gamma_0 \Gamma_9 \epsilon = -\epsilon \). At the same time, the presence of momentum along \( X^9 \) imposes on \( \epsilon \) the equation \( \Gamma_0 \Gamma_9 \epsilon = \epsilon \). Obviously these two equations are incompatible and consequently no target-space supersymmetry survives. On the black-hole side this corresponds to flipping the sign of the charge \( Q_1 \).

What makes this black hole similar to the one studied by Dabholkar is the fact that it can be related with a supersymmetric one by just reversing the sign of the Kaluza–Klein momentum. Doing this, the reflection condition on the anti-D1-brane is perfectly compatible with the momentum condition on the Killing spinor and we are left with four unbroken real supersymmetries. Therefore the D-brane computation of the entropy can be made along the lines described above.
By trading D-branes with anti-D-branes (i.e. changing the signs of $Q_1$ and $Q_5$) we have four possibilities leading to non-supersymmetric black holes for a given sign of the momentum along $X^9$. Starting from the supersymmetric Type-I black hole made out of D-branes alone, denoted by $(Q_1, Q_5, N)$, we find four extremal non-supersymmetric black-hole solutions labelled by

$$(Q_1, Q_5, -N), \quad (-Q_1, Q_5, N), \quad (Q_1, -Q_5, -N), \quad (-Q_1, -Q_5, N),$$

where the first one corresponds to the example constructed in [4]. In each case the solution obtained by changing the sign of $N$ is supersymmetric. Viewing the black holes as bound states of constituent branes, non-supersymmetric extremality is achieved by combining branes and anti-branes of different ‘dimensionality’, while non-extremal black holes come from adding some brane–antibrane pairs.

More examples can be generated by applying S-dualities to the Type-I black holes considered here. The whole situation is highly reminiscent of the series of extremal non-supersymmetric black holes constructed in refs. [6].

3. Large $N$ power counting

In this section we uncover the reasons behind the surprising agreement between the entropy and mass of D-brane excitations in the weakly coupled version of the Type-I black hole with the corresponding quantities computed from the low-energy supergravity solution. These two regimes are governed by $\lambda Q_1$, $\lambda Q_5$ and $\lambda^2 N$, playing the role of effective couplings for the open strings degrees of freedom on the D-brane background. In both the D-brane and the strongly coupled black-hole picture we work in the semiclassical limit (1.1), the two regimes corresponding to different values of the effective couplings [7], [8]. At one side we have the D-brane region in which the open string theory is weakly coupled

$$\lambda Q_1 < 1, \quad \lambda Q_5 < 1, \quad \lambda^2 N < 1,$$

and open-string perturbation theory in the presence of the bound state of D-branes is reliable. At the opposite side in the effective coupling moduli space we have the black hole, for which

$$\lambda Q_1 > 1, \quad \lambda Q_5 > 1, \quad \lambda^2 N > 1.$$
The physics is that of closed+open strings moving in the background of the black-hole metric, dilaton and R-R antisymmetric tensor. In the $\lambda \to 0$ limit only genus zero diagrams in the closed-string sector survive.

To visualize the physics behind the success of the D-brane computation of the black-hole entropy for the non-supersymmetric Type-I black hole, we will study in some detail the structure of the Type-I string perturbation theory. In the weakly coupled D-brane regime, any observable can be computed in string perturbation theory, the corresponding Feynman–Polyakov diagrams being Riemann surfaces with an arbitrary number of handles $g$, $C$ cross-caps and $B_1 + B_5 + B_9$ boundaries attached respectively to the $Q_1$ D1-branes, $Q_5$ D5-branes and the 32 D9-branes characteristic of the Type-I superstring theory. In addition to this we will have also D1–D5 ‘mixed boundaries’ containing insertions of open strings carrying Kaluza–Klein momentum along $X^9$. The state of the black hole can be specified in the Fock space of the $(0,4)$ conformal field theory at the D1–D5 intersection, by giving a set of occupation numbers, $\{n_{ij}(p)\}$, for strings of momentum $p$, carrying Chan–Paton labels $(i,j)$ in the fundamentals of $SO(Q_1)$ and $USp(2Q_5)$. We are interested in $S$-matrix amplitudes of the form

$$\langle \Psi_{BH}', X' | S | \Psi_{BH}, X \rangle, \quad (3.1)$$

where $X$ stands for the quantum numbers of a light system scattered off the black hole, for example, a single brane-probe, or a set of fundamental strings. Decay or absorption processes, or mass corrections to the black hole, can be treated by considering the situation where either $X$ or $X'$ or both are trivial.

A general perturbative amplitude contributing to $(3.1)$ with $I_o$ external open strings and $I_c$ external closed strings is weighted by $\lambda^{-\chi + I_c + I_o/2}$, where $\chi = 2 - 2g + B + C$ is the Euler character. We see that the minimum power of the string coupling is $\lambda^{-2}$, which scales in the large $N$ limit like $O(Q^2) \sim O(N)$, in agreement with the large $N$ scaling of the mass (2.6). Therefore, all diagrams with external closed strings vanish, except for the one- and two-point functions in the presence of the black hole. A closed-string tadpole is proportional to $\lambda^{-1} \sim O(Q) \sim O(\sqrt{N})$, the expected scaling of the Ramond–Ramond

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4 Actually, the Kaluza–Klein momentum is carried by a combination of (1,1), (1,5) and (5,5) degrees of freedom, as can be seen from the vanishing condition of the flat directions (2.1).

5 Given that $Q_1$ and $Q_5$ have the same scaling with $\lambda$, in what follows $Q$ will stand for either $Q_1$ or $Q_5$.
fields created by the black hole. This is also the large $N$ scaling of a brane probe effective action, since we may substitute the closed-string vertex operator by a single boundary attached to the probe, with the same coupling dependence.

On the other hand, the closed string form factor, i.e. the two-point function of closed strings in the black-hole state, is of $O(1)$, in agreement with the fact that it should measure the product $G_N M$. Since the gravitational field depends on $G_N M$, we see that the semiclassical limit is defined in such a way that the weakness of the coupling constant is balanced by the strength of the sources [15].

By opening D1 and D5 boundaries into a given Riemann surface, we get extra powers of $\lambda$ paired with the charges $Q_1, Q_5$ according to $\lambda Q_1$ and $\lambda Q_5$. Unless the amplitude vanishes by supersymmetry, these new Riemann surfaces do have non-vanishing contributions of $O(1)$ in the semiclassical limit. Essentially, the weakness of the coupling constant $\lambda$ in the semiclassical limit is compensated for by the fact that each new boundary can be attached to either $Q_1$ or $Q_5$ different D-branes, giving rise to $Q_1$ or $Q_5$ diagrams that contribute coherently.

A similar enhancement mechanism is at work when considering interactions between the constituent momentum strings. Each boundary with $n$ open-string insertions carries an extra factor of $\lambda^{1+\frac{n}{2}}$, but each vertex operator can be attached to many different strings, so that we get a combinatorial factor from initial- and final-state degeneracy in (3.1). This is exactly analogous to the large $N$ power counting rules in Witten’s treatment of baryons [16]. For large values of the Kaluza-Klein momentum $N$, a typical state will share the momentum equally between constituent strings, of which there are $O(Q^2)$ species. Therefore, as $N/Q^2$ remains fixed in the semiclassical limit, so does $\langle n_{ij} \rangle$ in the open-string condensate, and we can regard the constituent momentum strings as $O(N)$ species of quarks, building a baryon of mass $O(N)$. The simplest interactions would then correspond to gluon exchange between quarks, that is, closed string exchange between momentum insertions. They are associated to boundaries with two insertions of the same vertex operator, with a power $\lambda \cdot (\sqrt{\lambda})^2 \cdot N$, for each boundary, the last factor being the ‘quark degeneracy’.

More complicated interactions involving more than two insertions per boundary can also be considered. A detailed analysis is carried out in Appendix A, where we show that only Riemann surfaces with an even number of $(1,1)$, $(5,5)$ or $(1,5)$ insertions on each boundary survive in the classical limit. More importantly, we show that the wave-function
degeneracy factors work out in such a way that all corrections scale as a function of $\lambda^2 N$ at large $N$, and are therefore of $O(1)$.

The situation is radically different when we add closed-string loops, D9-branes holes or cross-caps. In all these cases, the new diagrams are suppressed by a power $\lambda^{2g + B_9 + C}$ of the string coupling, which is now uncompensated by powers of the charges. Incidentally, this justifies the arguments of Section 2.1, where we ignored the presence of D9-branes in computing the number of flat directions.

The conclusion is that, in the semiclassical regime, $\lambda \to 0$, $Q_1, Q_5, N \to \infty$ with $\lambda Q_1$, $\lambda Q_5$ and $\lambda^2 N$ fixed, all diagrams containing closed-string loops, D9-branes boundaries and cross-caps will be suppressed by bare positive powers of the vanishingly small string coupling constant $\lambda$. In other words, the relevant contributions come only from Riemann surfaces with an arbitrary number of D1-brane, D5-brane boundaries, as well as “mixed” boundaries containing momentum insertions. These surfaces are topologically of the same kind as those contributing to the corresponding amplitude in the supersymmetric Type-IIB superstring black hole. It is important that, as shown in Appendix A, no diagram with open-string insertions scales like a negative power of $\lambda$, for otherwise we would be able to get non-vanishing contributions by adding cross-caps or D9-boundaries. This would spoil the correspondence between the semiclassical limits of Type-I and IIB black holes.

At a more detailed level, the large $N$ limit of Type-I diagrams involves Type-IIB diagrams with some numerical rescaling factors. Roughly speaking, there is a factor of $\frac{1}{2}$ for each open-string loop, coming from the $\mathbb{Z}_2$ orientation projection in the original Type-I theory. A detailed analysis in Appendix B shows that such projection factors can be entirely absorbed in renormalizations of the coupling constants, as well as the D-brane charges. In particular, the auxiliary Type-IIB system involves a Type-IIB black hole of charges $Q'_1$, $Q'_5$ and $N'$, in a vacuum with open- and closed-string couplings, $\lambda', \kappa'$ respectively. These parameters are related to those of the original Type-I system by the dictionary:

$$\lambda' = \frac{1}{\sqrt{2}} \lambda, \quad \kappa' = \kappa,$$

$$Q'_1 = \frac{1}{\sqrt{2}} Q_1, \quad Q'_5 = \sqrt{2} Q_5, \quad N' = N.$$  \hspace{1cm} (3.2)

Since amplitudes of the Type-I string theory are equal, in the large $N$ limit, to the corresponding ones for the Type-IIB theory, non-renormalization theorems at work on the latter
guarantee the absence of quantum corrections to mass and entropy as we increase the coupling and pass from the D-brane region to the black hole-domain in the non-supersymmetric Type-I black hole.

Microscopically, i.e. from the D-brane point of view, the Type-I D-brane bound state \((Q_1, Q_5, N)\) is equivalent in the semiclassical limit to a Type-IIB D-brane system characterized by charges \((Q'_1, Q'_5, N')\) with mass and entropy

\[
M' = \frac{Q'_1 R}{\lambda' \alpha'} + \frac{Q'_5 R V}{\lambda' (\alpha')^3} + \frac{N'}{R} \\
S' = 2\pi \sqrt{N' Q'_1 Q'_5}.
\]

Substituting the primed values in terms of the Type-I ones, we easily find that the mass of the Type-IIB D-brane bound state equals the mass of the original Type-I system, \(M' = M\) with \(M\) given by (2.6). The same happens with the entropy of D-brane excitations, since \(\sqrt{NQ_1Q_5} = \sqrt{N'Q'_1Q'_5}\).

On the black-hole side, the geometry of the auxiliary black hole is governed by the three lengths

\[
r'_1 = \kappa' \frac{Q'_1 (\alpha')^3}{V \lambda'}, \quad r'_2 = \kappa' \frac{Q'_5 \alpha'}{\lambda'}, \quad r'_0 = \kappa' \frac{N' (\alpha')^4}{R^2 V}.
\]

Using (3.2), it is straightforward to check that \(r'_i = r_i^2\) \((i = 0, 1, 5)\) and therefore the geometry of the Type-IIB black hole is identical to the original non-supersymmetric Type-I black hole. This guarantees that the area of both event horizons is the same as well, and since \(\kappa' = \kappa\) the geometrical entropy of both black holes will coincide and will be equal to the D-brane computation. The ADM mass, being only a function of \(\kappa'\) and \(r'_i\), will also be the same for both black holes and equal to the energy of the weakly coupled D-brane system.

4. Near-horizon physics

This section is devoted to a more explicit analysis of the large \(N\) limit, as monitored by a near-horizon brane probe [8, 17]. We have seen that the large \(N\) limit quenches all diagrams involving Type-I cross-caps, \(SO(32)\) Chan–Paton factors, and closed string handles. So we are left with a subset of orientable diagrams of Type-I string perturbation
theory, consisting of spheres with holes carrying $USp(2Q_5) \times SO(Q_1)$ Chan–Paton factors, with some rescaling factors. We have found in particular, for a vacuum amplitude:

$$\lim_{Q_i \to \infty} A(\lambda, Q_1, Q_5)_{I} = \lim_{Q'_i \to \infty} A(\lambda', Q'_1, Q'_5)_{IIB},$$

(4.1)

with $Q'_1 = Q_1/\sqrt{2}, Q'_5 = \sqrt{2}Q_5,$ and $\lambda' = \lambda/\sqrt{2}.$ Such vacuum amplitudes define the loop expansion of the world-volume effective action on the branes, and in particular the effective action for near-horizon brane probes, whose low-energy limit contains a Dirac–Born–Infeld term.

To be more precise, let us model the black hole-bound state in terms of the six-dimensional gauge theory on the D5-branes world-volume. Denoting by $F_{ab}$ the $USp(2Q_5)$ Yang–Mills field strength, D1-branes bound to the D5-branes are represented at low world-volume energies as instanton configurations:

$$Q_1 = \frac{1}{64\pi^2} \int_{T_4} \varepsilon^{abcd} \text{Tr} F_{ab} F_{cd},$$

(4.2)

and the supersymmetry-breaking Kaluza-Klein momentum in the $X^9$ direction is introduced through an appropriate Yang–Mills configuration such that

$$P_9 = \int_{T_4 \times S^1} T_{09} = \frac{1}{g^2} \int_{T^4 \times S^1} \text{Tr} \sum_a F_{a0} F_{a9} = \frac{N}{R},$$

(4.3)

where $g^2 \sim \lambda\alpha'$ denotes the six-dimensional Yang–Mills coupling.

Finally, we introduce a parallel D5-brane probe at a transverse distance $r_i$ and velocity $v_i$, where the index $i$ runs over the coordinates transverse to the D5-branes world-volume. The complete D5 gauge group is now $USp(2Q_5 + 2)$, spontaneously broken to $USp(2Q_5) \times USp(2)$ by the expectation value of an adjoint scalar charged under both factors: $U_i = r_i/\alpha'$. If we view the $USp(2Q_5 + 2)$ gauge theory in six dimensions as a dimensional reduction of the corresponding $\mathcal{N} = 1$ super-Yang–Mills theory in ten dimensions, the adjoint scalars are given by the transverse components of the ten-dimensional gauge field $U_i \sim A_i$. Then the relative velocity is proportional to an electric field in the ten-dimensional theory $\dot{U}_i \sim F_{0i}$. More precisely, regarding the probe as fixed and the black-hole bound state moving with velocity $v_i = \alpha' \dot{U}_i$, we can write

$$F_{0i} = \dot{U}_i \Sigma_2,$$

(4.4)
where $\Sigma_2 \equiv \sigma_2 \otimes 1_{Q_5}$ is the intertwiner of $USp(2Q_5)$, relating the complex-conjugated representations $2\overline{Q_5} = \Sigma_2 2Q_5 \Sigma_2$.

The probe effective action in perturbation theory is given by the low-energy limit of a set of vacuum string diagrams. At $L$ loops we consider a sphere with $L$ boundaries on the $USp(2Q_5)$ bound state and one boundary on the probe. The dynamical information on the black-hole state is introduced through $USp(2Q_5)$ Wilson lines

$$W = \text{Tr } P \exp \left( i \int A_\mu dx^\mu \right)$$

(4.5)

at each of the $L$ black-hole boundaries, with $A_\mu$ the ten-dimensional Yang–Mills configuration satisfying conditions (4.2), (4.3) and (4.4). The probe is taken in its ground state, with a trivial Wilson line

$$W_{pr} = \text{Tr } 1_2 = 2.$$  

(4.6)

In order to take the large $N$ limit smoothly, we define the trace-normalized Wilson lines $\tilde{W}$ by

$$W = 2Q_5 \tilde{W}, \quad W_{pr} = 2 \tilde{W}_{pr},$$

(4.7)

in terms of which the probe effective action takes the form

$$\Gamma_{\text{eff}} = \int d^6x L_{\text{eff}} = 2 \sum_{\text{surfaces}} \lambda^{-\chi} (2Q_5)^{B-1} \langle \tilde{W}_{pr} \tilde{W}_1 \cdots \tilde{W}_{B-1} \rangle_1,$$

(4.8)

where we have extracted the factor of 2 from the probe Wilson line. In this general expression, $\chi$ is the Euler character of the Riemann surface and $B$ is the total number of holes. In the limit, only spheres with $B = L + 1$ boundaries survive, and the Type-I correlation function of Wilson lines is related to a Type-IIB correlation function via the general identity

$$\langle \tilde{W}_{pr} \tilde{W}_1 \cdots \tilde{W}_L \rangle_1 \rightarrow \frac{1}{2L} \langle \tilde{W}_{pr} \tilde{W}_1 \cdots \tilde{W}_L \rangle_{\text{IIB}}$$

(4.9)

and we obtain

$$\lim \Gamma_{\text{eff}} = \sqrt{2} \sum_{L \geq 0} (\lambda')^{L-1} (Q'_5)^L \langle \tilde{W}_{pr}' \tilde{W}_1' \cdots \tilde{W}_L' \rangle_{\text{IIB}} = \sqrt{2} \lim \Gamma'_{\text{eff}},$$

(4.10)

with the mapping $\tilde{W} = \tilde{W}'$. So, we obtain a Type-IIB effective action with the rescaled parameters $\lambda', Q'_1, Q'_5$, up to a global factor of $\sqrt{2}$. Actually, this factor is just the rescaling of the D5 charge of the probe.
In order to properly interpret the operator mapping $\tilde{W}' = \tilde{W}$, we make a low-energy expansion in powers of gauge-invariant operators. In the constant Abelian approximation where we can neglect covariant derivatives $DF \sim 0$ and commutators $[F, F] \sim 0$, the effective action takes the form [17]

$$\mathcal{L}_{\text{eff}} = \sum_{L \geq 0} \sum_{I \geq 2} C_{L, I} (\lambda' \alpha')^{L-1} \frac{U^{2I-2L-4}}{L} \sum_{l=0}^{L} (2Q_5)^{L-l} \sum_{n_1+\ldots+n_l=I} \text{Tr} F^{n_1} \ldots \text{Tr} F^{n_l}. \quad (4.11)$$

We have only kept the leading large $N$ operators and have also neglected higher-dimension operators suppressed by the string scale $\sqrt{\alpha'} U \ll 1$. In other words, (4.11) must be understood as a Wilsonian effective action with an ultraviolet cutoff of the order of $U$. Infrared divergences should be cutoff by the non-vanishing background field strengths $\langle \text{Tr} F^n \rangle \neq 0$. The index $I = \sum_i n_i$ is the total number of gauge-invariant operator insertions, distributed through $l$ boundaries. The rest of $L - l$ boundaries pick the identity term in the weak field expansion of the Wilson lines.

Consistency with the approximations of neglecting covariant derivatives and commutators requires evaluating the effective action on constant Abelian field strengths of the form

$$F_{\mu\nu} = \tilde{F}_{\mu\nu} H_{\mu\nu}, \quad (4.12)$$

with $\tilde{F}_{\mu\nu}$ representing ‘group averaged’ field strengths, and $H_{\mu\nu}$ matrices in the Cartan subalgebra of $USp(2Q_5)$, whose trace in the large $N$ limit verifies

$$\frac{1}{2Q_5} \lim_{Q_5 \to \infty} \text{Tr} (H)^n = 1, \quad \text{for } n > 1. \quad (4.13)$$

Then, in this case, the statement $\tilde{W} = \tilde{W}'$ means $\tilde{F}_{\mu\nu} = \tilde{F}'_{\mu\nu}$, that is, the group-averaged field strengths are mapped identically between the Type-I backgrounds and the auxiliary Type-IIB background. Now, using

$$\text{Tr}' F'^2 = Q'_5 \tilde{F}'^2 = \frac{1}{\sqrt{2}} \cdot 2Q_5 \tilde{F}^2 = \frac{1}{\sqrt{2}} \text{Tr} F^2, \quad (4.14)$$

we can calculate the renormalization of the Kaluza–Klein charge:

$$\frac{N'}{R} = \frac{1}{(2\pi)^3 \alpha' \xi} \int \text{Tr}' F'^2 = \frac{1}{(2\pi)^3 \alpha' \xi} \int \text{Tr} F_N^2 = \frac{N}{R}, \quad (4.15)$$

and we obtain the expected result $N = N'$, in agreement with the ADM determination in the previous section. Similarly, Eqs. (4.12) and (4.13) imply $Q'_1 = Q_1 / \sqrt{2}$. 

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It has been conjectured that, at least for supersymmetric black holes, (4.11) matches the weak coupling expansion of the Dirac–Born–Infeld action for a probe propagating in the near-horizon geometry of the classical black hole \[8, 18, 17\]. In particular, this matching requires powerful non-renormalization theorems that reduce the sum over insertions in (4.11) to the terms \(I = 2L + 2\). This is a powerful constraint. Taking into account the fact that each loop is suppressed by at least a factor of \(U^{-2}\), from the mass of the stretched strings between the black hole and the probe, and using dimensional analysis plus the constraint \(I = 2L + 2\), one finds that each boundary contributes only two field-strength insertions representing instantons or momentum, up to higher-dimension operators suppressed by powers of \(\alpha'U^2 \ll 1\).

We have shown in the previous section that the closed-string backgrounds, metric, dilaton and RR fields, are exactly matched with the above parameter mapping between the original Type-I theory and the auxiliary IIB vacuum:

\[
(\kappa, \lambda, Q_1, Q_5, N) = (\kappa', \sqrt{2}\lambda', \sqrt{2}Q'_1, Q'_5/\sqrt{2}, N').
\]  
(4.16)

Therefore, the Dirac–Born–Infeld actions are identical up to the renormalization of the overall D5 probe tension, which appears explicitly in (4.10). This proves that the non-renormalization theorems implied by the conjecture in [17], are true of the large \(N\) limit of Type-I extremal non-supersymmetric black holes, provided they hold for the supersymmetric IIB ones.

So far we have discussed the weak coupling expansion of the effective action. The dimensionless expansion parameter is the combination

\[
geff = \frac{1}{2} \frac{\lambda\alpha'\langle\text{Tr} F^2\rangle}{U^2} = \frac{\lambda'\alpha'\langle\text{Tr}' F'^2\rangle}{U^2}.
\]  
(4.17)

From the point of view of the large \(N\) limit (or, rather, large \(Q_5\) limit) of the \((5 + 1)\)-dimensional theory on the \(D5\)-branes, this has the form

\[
geff = g^2 Q_5 \cdot (\text{energy})^2,
\]

namely, it is the dimensionless combination of the large \(N\)'t Hooft coupling, and a typical energy in the Yang–Mills theory, in units of the ultraviolet cutoff, set by \(U\). Evaluating
\(\langle \text{Tr } F^2 \rangle\) through (4.2), (4.3) and (4.4), we obtain the respective expansion parameters for velocity, instanton, and momentum insertions

\[
\begin{align*}
g_5 &= \frac{\alpha' U'^2}{(2\pi)^2 U^2} \lambda' Q'_5 = \frac{\alpha' U'^2}{(2\pi)^2 U^2} \lambda Q_5, \\
g_1 &= \frac{(2\pi)^3 \alpha'}{V U^2} \lambda' Q'_1 = \frac{(2\pi)^3 \alpha'}{V U^2} \cdot \frac{1}{2} \cdot \lambda Q_1, \\
g_p &= \frac{\alpha'^2}{R^2 V U^2} \lambda'^2 N' = \frac{\alpha'^2}{R^2 V U^2} \cdot \frac{1}{2} \cdot \lambda^2 N,
\end{align*}
\]

(4.18)

and the series expansions considered above make sense for \(g_{\text{eff}} \ll 1\). Thus, we see that the large \(N\) limit can be combined with a low-energy approximation such that the supergravity description makes sense for \(\lambda Q_1 \sim \lambda Q_5 \sim \lambda^2 N = \) fixed but large, provided we keep \(\alpha' \times (\text{energy})^2\) sufficiently small. Since \(\lambda Q_i\) are the effective expansion parameters of D-brane perturbation theory, this argument would indicate that the supergravity description of the near-horizon geometry can be extended to the region \(\lambda Q_i \gg 1\). A broad generalization of this idea was recently proposed by Maldacena [19] to relate the strong-coupling, large \(N\) dynamics of conformal theories and supergravity in near-horizon geometries involving anti-de Sitter factors. In particular, for the case at hand, the near-horizon geometry of the Type-IIB black hole is given by \(\text{AdS}_3 \times T^4 \times S^3\), the radius of the various factors being determined in terms of \(\lambda, Q_1\) and \(Q_5\). Our results imply that this conjecture can be immediately generalized to the large \(N\) limit of the conformal field theories relevant to the Type-I black holes, with the above-mentioned dictionary (4.16) of parameters \(\lambda', Q'_1\) and \(Q'_5\).

The conformal field theory in question is the (0, 4) superconformal model that appears as an infrared fixed point of the (1 + 1)-dimensional theory of (1, 1), (1, 5) and (1, 9) strings described in Section 2 (see [20] for a recent discussion in the present context). In this case, at the level of the conformal field theory, the large \(N\) limit is not of ’t Hooft type. Rather, it is the limit of large central charge \(c \rightarrow \text{const. } \times N^2\). The difference between this CFT and the (4, 4) conformal field theory in the intersection of Type-IIB D5+D1 branes, is simply the additional sector of (1, 9) strings, which breaks the supersymmetry of the right movers, and the \(\mathbb{Z}_2\) orientation projection. The (1, 9) sector contributes only an amount of \(\mathcal{O}(1)\) to the central charge in the large \(N\) limit, and the orientation projection translates in the dictionary of parameters that was worked out before. Therefore, the large \(N\) limit of the (0, 4) CFT is a certain embedding in a (4, 4) CFT with the usual moduli mapping (4.16).
This result is exactly analogous to similar statements involving four-dimensional non-supersymmetric theories defined as projections of $\mathcal{N} = 4$ super-Yang–Mills, which exhibit exact conformal invariance in the large $N$ limit $[21]$.

5. Conclusions

We have studied the Type-I non-supersymmetric black hole constructed in ref. $[4]$ and investigated the workings behind the successful D-brane description for this black hole. Our main result here is that, in spite of dealing with a system without space-time supersymmetry, the supersymmetry-breaking contributions to some observables such as the mass and the entropy are subleading in the large $N$ limit at which the connection between the D-brane and the black-hole pictures is made. We have found that the Type-I black hole with $\mathcal{N} = 0$ is embedded in that limit into a Type-IIB black hole with four unbroken supersymmetry charges. Consequently, the non-supersymmetric black hole inherits the non-renormalization theorems that guarantee the absence of quantum corrections in the Type-IIB auxiliary theory. Actually, this asymptotic restoration of supersymmetry at large $N$ in the non-supersymmetric Type-I black hole is reminiscent of the idea of “classical non-renormalization” of the stress-energy tensor advocated in $[4]$.

This semiclassical equivalence between extremal Type-I and Type-IIB black holes can be extended to non-extremal ones since, as we proved in Appendix B, the corresponding perturbation theories are identical in that limit, thus implying the coincidence of quantum corrections for both theories. This is fully consistent with the fact that Type-I and Type-IIB low-energy supergravity black-hole solutions are also identical.

We have extended our study to the dynamics of D5-brane probes close to the horizon of the Type-I black hole and found that, in the semiclassical limit, the probe effective action is identical to the corresponding one for the auxiliary IIB theory. Given this large $N$ embedding of the Type-I theory into a Type-IIB one, any of the non-renormalization theorems conjectured for the latter $[17]$ must also hold for the near-horizon probe effective action in the Type-I theory in that limit.

Incidentally, the analysis carried out here is also valid for the whole class of extremal non-supersymmetric black holes studied in Section 2.3. In each case, in the large $N$ limit the corresponding non-supersymmetric black hole is embedded into an auxiliary Type-IIB theory with flipped signs for $Q_1$ or $Q_5$ or both of them.
6. Acknowledgments

It is a pleasure to thank I.L. Egusquiza, R. Emparan, A. Feinstein, R. Lazkoz, T. Ortín, E. Rabinovici and M.A. Valle-Basagoiti for useful and interesting discussions. The work of J.L.M. has been partially supported by the Spanish Science Ministry under Grant AEN96-1668 and by a University of the Basque Country Grant UPV-EHU-063.310-EB225/95, and that of M.A.V.-M. by a Basque Government Post-doctoral Fellowship.

Appendix A. Large $N$ limit of string diagrams with open string insertions

Here we study the large $N$ scaling of more general open-string interactions, involving boundaries with more than two momentum insertions. As in the text, let $n_{ij}$ be the number of $(i, j)$-type open strings. Since the total Kaluza–Klein momentum $N$ has to be shared by $Q^2$ species of strings, and $N/Q^2$ remains fixed in the semiclassical limit, so does $\langle n_{ij} \rangle$ in the open-string condensate. Thus the number of open strings to which a given vertex operator can be attached will grow like $Q^2$ at most. Actually, this growth is limited by the Chan–Paton structure of the interaction.

In order to illustrate this point, let us first consider a Riemann surface where a single boundary carries four open-string insertions. We have a $\lambda^3$ factor, together with the group theory factor

$$\text{tr} A^4 = \sum_{ijkl} A_{ij} A_{jk} A_{kl} A_{li}. $$

Naively, the four sums would give rise to $Q^4$ terms, with a contribution $\lambda^3 Q^4$ that diverges like $\lambda^{-1}$ in the semiclassical limit. However, what we really have to consider is the contribution to $S$-matrix elements, as in (3.1), between given initial and final states. Consider first the “diagonal” element

$$\langle \{ n_{ij}(p) \}, X' | S | \{ n_{ij}(p) \}, X \rangle,$$

where the black-hole state is unchanged in the scattering process. Then the two incoming open-string states taking part in the interaction must be identical to the two outgoing states. This is achieved by setting $j = l$ or $i = k$. In either case we are left with only three sums, and the contribution goes like $\lambda^3 Q^3$, which is finite in the classical limit.
For off-diagonal elements the situation is different. If \((i, j)\) and \((j, k)\) are the incoming strings, only initial and final states satisfying

\[
\begin{aligned}
n'_{ij} &= n_{ij} - 1, & n'_{jk} &= n_{jk} - 1, & n'_{il} &= n_{il} + 1, & n'_{lk} &= n_{lk} + 1
\end{aligned}
\]

can have non-vanishing \(S\)-matrix elements. And, what is more important, only one term in the sum contributes to each of these off-diagonal elements, which therefore scale like \(\lambda^3\) and vanish in the semiclassical limit. Even if we consider ‘inclusive’ processes where we sum over all possible final black-hole states, the cross section goes like \(\lambda^6 Q^4 \to 0\), since we have to sum over probabilities rather than amplitudes.

Similarly, for the “diagonal” elements of diagrams with a single boundary with \(2k\) insertions, there are \(k - 1\) constraints on the indices coming from the specification of fixed initial and final states. So, we are left with \(k + 1\) free indices, a power of \(\lambda\) from one boundary, and \((\sqrt{\lambda})^{2k}\) from \(2k\) vertex operators, for a total of \((\lambda Q)^{k+1} \sim (\lambda^2 N)^{\frac{k+1}{2}} \sim O(1)\) in the large \(N\) limit. As is the case with \(k = 2\), off-diagonal elements can be seen to vanish in the same limit.

This analysis is easily generalized to any number of boundaries with arbitrary insertions. The result is that only diagrams with an even number of insertions on each boundary give non-zero contributions, and only to \(S\)-matrix elements that are diagonal in the black-hole state. Other combinations of insertions and all off-diagonal elements vanish in the semiclassical limit. This is not unexpected, since the fields of a microscopic string probe vanish for \(\lambda \to 0\) and are thus unable to affect the state of the black hole.

Appendix B. Type-I versus Type-IIB string perturbation theory

In this appendix we specify in more precise terms the large \(N\) embedding of the Type-I string perturbation series in the black-hole sector, into a similar Type-IIB system. It was argued in Section 3 that all diagrams containing closed-string handles, cross-caps or D9-brane boundaries are suppressed. This implies that both perturbation theories (I and IIB) in that limit contain the same type of diagrams from a geometrical point of view. Since Type-I and Type-IIB theories are close relatives, we will try to relate the two theories at a more quantitative level.
Type-I superstring can be obtained from the Type-IIB theory by introducing 32 D9-branes (a number determined by anomaly cancellation) and projecting out by the world-sheet parity $\Omega$. This last projection is necessary for the consistency of the Type-I theory (otherwise we are left with uncancelled dilaton tadpoles) and results in the introduction of non-orientable Riemann surfaces. It therefore seems that, roughly speaking, the main difference between Type-I and Type-IIB perturbation theories is the presence in the former of Riemann surfaces containing D9-boundaries and cross-caps. If this were just so, both perturbative expansions would be identical in the semiclassical limit in which these contributions are suppressed. An amplitude with $I_o$ symmetric ($\Omega = +1$) open and $I_c$ closed external string states has a loop expansion of the form

$$S(\kappa, \lambda, I_o, I_c)_1 = \kappa^I_c (\sqrt{\lambda})^{I_o} \sum_{B=1}^\infty \lambda^{B-2} A(B, I_o, I_c)_1,$$

where $\kappa$ is the closed-string coupling constant, proportional to the string coupling constant $\lambda$ and the number of loops is $L = B - 1$. For the Type-IIB we have a similar expression

$$S(\kappa', \lambda', I_o, I_c)_{IIB} = (\kappa')^I_c (\sqrt{\lambda'})^{I_o} \sum_{B=1}^\infty (\lambda')^{B-2} A(B, I_o, I_c)_{IIB}.$$

Although Type-I and Type-IIB perturbation theories contain diagrams of the same kind, Type-I diagrams only include string modes that are even under world-sheet parity inversion, i.e. $\Omega = 1$, whereas in Type-IIB diagrams those states with $\Omega = -1$ also run in open string loops. Thus, in order to obtain the Type-I diagrams from its Type-IIB versions we must impose that only symmetric states under $\Omega$ run in internal channels. Actually, states of the Type-IIB theory can be classified into symmetric and antisymmetric with respect to the world-sheet parity inversion $\Omega$; for a state $|\alpha, i\bar{j}\rangle$ the symmetric and antisymmetric combinations are defined by

$$|\alpha, ij\rangle_{S(A)} = \frac{1}{\sqrt{2}} (1 \pm \Omega)|\alpha, i\bar{j}\rangle.$$

In the closed-string sector, the projection is made in a similar fashion. States of the Type-I theory are just the symmetric combinations ($\Omega = 1$) of Type-IIB. Moreover, in an operator formalism language, we can define the vertex $|\mathcal{V}_3\rangle$ corresponding to a disk amplitude of three open strings. Doing so, we easily find that ($B = 1$, $I_o = 3$, $I_c = 0$)

$$\mathcal{A}(1, 3, 0) \equiv \langle \mathcal{V}_3||\alpha_1, ij\rangle_S |\alpha_2, jk\rangle_S |\alpha_3, ki\rangle_S = \langle \mathcal{V}_3||\alpha_1, ij\rangle_S |\alpha_2, jk\rangle_A |\alpha_3, ki\rangle_A$$

$$= \langle \mathcal{V}_3||\alpha_1, ij\rangle_A |\alpha_2, jk\rangle_A |\alpha_3, ki\rangle_S = \langle \mathcal{V}_3||\alpha_1, ij\rangle_A |\alpha_2, jk\rangle_S |\alpha_3, ki\rangle_A,$$

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whereas any other amplitude containing one or three antisymmetric states will vanish.

Going now to the Type-I theory, external states are given by the symmetric combinations
of the Type-IIB superstring, and in the sector with no D9 holes we have

$$\mathcal{A}(1, 3, 0)_I = \mathcal{A}(1, 3, 0)_{\text{IIB}}.$$  

Thus, three-point amplitudes on the disk in Type-I theory are identical to the corresponding ones in Type-IIB theory. Actually, it is straightforward to generalize this result to any amplitude on the disk. By factorization and conservation of $\Omega$ on each vertex we find that

$$\mathcal{A}(1, K, 0)_I = \langle V_1 | V_2 \Delta_S \ldots \Delta_S V_{K-1} | V_K \rangle = \mathcal{A}(1, K, 0)_{\text{IIB}},$$

where $\Delta$ is the open-string propagator. Here it is important to stress that the vertex operators do not contain any power of the string coupling constant, which has been factored out in (B.1). The conclusion is that tree amplitudes in the Type-I theory are exactly equal to their Type-IIB counterparts.

Let us now introduce loop diagrams. We consider the amplitude of $K$ external symmetric states in Type-IIB theory. Applying the conservation of $\Omega$ in each interaction, we can write ($B = 2$, $I_o = K$, $I_c = 0$)

$$\mathcal{A}(2, K, 0)_{\text{IIB}} = \text{Tr}_S[\Delta V_1 \Delta \ldots \Delta V_K] + \text{Tr}_A[\Delta V_1 \Delta \ldots \Delta V_K], \quad (B.3)$$

where the traces are taken respectively over symmetric and antisymmetric states running in the loop. But, as we have seen above, the $(SSS)$ and $(ASS)$ tree-level couplings are equal, so both terms in the right-hand side of (B.3) are equal. Since the first one (the trace over symmetric states) corresponds to the amplitude for the Type-I theory, we conclude that

$$\mathcal{A}(2, K, 0)_I = \frac{1}{2} \mathcal{A}(2, K, 0)_{\text{IIB}}.$$  

The introduction of a closed-string insertion does not change in any way the previous results. Actually, the higher-loop case can be worked out along the lines depicted above. Factorizing the amplitude into lower-loop contributions and iterating, we find that

$$\mathcal{A}(B, I_o, I_c)_I = \left(\frac{1}{2}\right)^{B-1} \mathcal{A}(B, I_o, I_c)_{\text{IIB}}, \quad (B.4)$$

with $L = B - 1$ the number of open-string loops. The numerical factor appears because we have to insert a projector onto symmetric states for each open-string loop.
So far we have been working with the individual terms of the perturbative expansions, not with the resummed series. In order to compare (B.1) with (B.2) we have to know which is the relation between the Type-IIB couplings \((\lambda', \kappa')\) and the original ones \((\lambda, \kappa)\). This will be determined by imposing the correct factorization of the amplitudes. Looking at the annulus amplitude with no open- or closed-string insertion \((B = 2, I_o = I_c = 0)\) we find from our previous results

\[
A(2, 0, 0)_I = \frac{1}{2} A(2, 0, 0)_{\text{IIB}}. \tag{B.5}
\]

Taking the modulus of the annulus to infinity we find that it factorizes into two disk tadpoles joined by a closed-string propagator, namely

\[
A(2, 0, 0)_I \rightarrow \left(\frac{\kappa}{\lambda}\right)^2 A(1, 0, 1)_I \Delta_C A(1, 0, 1)_I,
\]

and an analogous expression for the right-hand side of (B.3) with primed coupling constants. From (B.4) we know that \(A(1, 0, 1)_I = A(1, 0, 1)_{\text{IIB}}\), so using (B.5) we finally have (cf. [22])

\[
\frac{\kappa}{\lambda} = \frac{1}{\sqrt{2}} \frac{\kappa'}{\lambda'}.
\]

This gives us the Type-IIB couplings in terms of their Type-I counterparts. However, this expression does not determine for us the individual expressions for \(\kappa'\) and \(\lambda'\), only their quotient. Here, among all the possibilities, we will take the most natural one, in which the closed-string coupling is the same for both theories and only the open-string coupling constant transforms

\[
\kappa' = \kappa, \quad \lambda' = \frac{1}{\sqrt{2}} \lambda. \tag{B.6}
\]

Finally we have the following relation between each term of the Type-I and Type-IIB perturbative expansions (B.1) and (B.2)

\[
\kappa^{I_c}(\sqrt{\lambda})^{I_o+2B-4} A(B, I_o, I_c)_I = \left(\frac{1}{\sqrt{2}}\right)^{B-\frac{1}{2}} I_o (\kappa')^{I_c}(\sqrt{\lambda'})^{I_o+2B-4} A(B, I_o, I_c)_{\text{IIB}}, \tag{B.7}
\]

which gives us the correct expression of the large \(N\) Type-I amplitudes in terms of the Type-IIB ones. It is easy to see that the numerical factor multiplying the amplitude in (B.7) is consistent with the factorization of the amplitudes.
The remaining factors of $1/\sqrt{2}$ in (B.7) can be interpreted as follows. For any boundary without open-string insertions, the numerical factor combines with the Chan–Paton degeneracy into

$$
\frac{1}{\sqrt{2}} \dim(R_{CP}),
$$

where $R_{CP}$ is the representation of the Chan–Paton group of the string end-points. In our case, D1-branes carry the vector of $SO(Q_1)$, whereas D5-branes carry the fundamental of $USp(2Q_5)$. In Type-IIB, the Chan–Paton group with the rescaled dimensions is $U(Q'_1) \times U(Q'_5)$ with the dictionary of charges in (3.2). Thus, at least for diagrams without open-string insertions, Type-I amplitudes in the large $N$ limit are equal to those in the auxiliary Type-IIB black hole defined by (3.2).

On the other hand, boundaries with open-string insertions do not carry Chan–Paton degeneracy, but they get combinatorial factors in $S$-matrix elements due to external-state degeneracy, as explained in Appendix A. The contribution of a Riemann surface with a single boundary and $2k$ insertions in the Type-I theory will have a factor of $Q^{k+1}$ because of the degeneracy in the external states. According to Eq. (B.7), the whole amplitude can be written in terms of the auxiliary Type-IIB theory as

$$
Q^{k+1} (\sqrt{\lambda})^{2k-2} A(1,2k,0)_{I} = Q^{k+1} (\sqrt{\lambda'})^{k-1} (\sqrt{\lambda'})^{2k-2} A(1,2k,0)_{IIB}.
$$

In order to fully relate the amplitudes in Type-I and Type-IIB, we should write $Q$ on the right-hand side of (B.8) in terms of the Type-IIB charges $Q'$, given by $Q'_1$ or $Q'_5$. Using (3.2), we have $Q = \sqrt{2}Q'$, with $Q$ either $Q_1$ or $2Q_5$, the dimensions of the vector representations of $SO(Q_1)$ and $USp(2Q_5)$, respectively. Putting all together we finally get

$$
Q^{k+1} (\sqrt{\lambda})^{2k-2} A(1,2k,0)_{I} = 2^k (Q')^{k+1} (\sqrt{\lambda'})^{2k-2} A(1,2k,0)_{IIB}.
$$

The factor $2^k$ on the Type-IIB side has a simple explanation. On the Type-I side we have only symmetric ($\Omega = +1$) open strings as external states; on the other hand in the Type-IIB auxiliary theory, we have to include both symmetric and antisymmetric ($\Omega = -1$) states when summing over external states degeneracy. This gives us a multiplicity factor of 2 for each incoming string up to the total factor $2^k$ appearing in (B.9). It is easy to see that the above argument generalizes to any number of boundaries carrying insertions. Therefore we have arrived at the conclusion that Type-I and Type-IIB amplitudes with open-string insertions are equal in the large $N$ limit.
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