Swampland Implications of GW170817-compatible Einstein-Gauss-Bonnet Gravity

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We revisit the Einstein-Gauss-Bonnet theory in view of the GW170817 event, which compels that the gravitational wave speed is equal to $c_T^2 = 1$ in natural units. We use an alternative approach compared to one previous work of ours, which enables us to express all the slow-roll indices and the observational indices as functions of the scalar field. Using our formalism we investigate if the Swampland criteria are satisfied for the Einstein-Gauss-Bonnet theory and as we demonstrate, the Swampland criteria are satisfied for quite general forms of the potential and the Gauss-Bonnet coupling function $\xi(\phi)$, if the slow-roll conditions are assumed to hold true.

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I. INTRODUCTION

The gravitational wave detection coming from the neutron star merging GW170817 event [1], has utterly affected modified theories of gravity, excluding some of these from being viable descriptions of our Universe at astrophysical scales. Particularly, the GW170817 event indicated that the propagating speed of the gravitational waves $c_T$ is equal to unity, namely $c_T^2 = 1$, in natural units. This feature as we mentioned, excluded almost instantly many alternative theories of gravity, see Ref. [2] for a detailed account of this topic. Nevertheless, there exist many modified gravity theories that still remain robust against the GW170817 event results, see for example the recent reviews [3–9].

In a recent previous work of ours [10], we investigated how the Einstein-Gauss-Bonnet theories [11–37] can be rendered viable and compatible with the GW170817 event. The gravitational wave speed for an Einstein-Gauss-Bonnet theory is equal to,

$$c_T^2 = 1 - \frac{Q_f}{2Q_t}, \quad (1)$$

with $Q_f = 8c_1(\ddot{\xi} - H\dot{\xi})$, while $c_1$ is a dimensionless constant multiplication factor that enters the Lagrangian in front of the Gauss-Bonnet term, and the function $Q_t$ is $Q_t = \frac{1}{c_T^2} - 8c_1\dot{\xi}H$, with $H$ being the Hubble rate. So according to our considerations, if $Q_f = 0$ the gravitational wave speed would be equal to one. This implies that the Gauss-Bonnet coupling must satisfy the differential equation $\ddot{\xi} - H\dot{\xi} = 0$, which implies that $\dot{\xi} = e^N$, where $N$ is the e-foldings number. This approach enabled us to express all the slow-roll indices and the observational indices as functions of the e-foldings number, and eventually study the phenomenology of the model. In this letter we shall employ a different approach, in order to express the slow-roll indices and the observational indices as functions of the scalar field. Our aim for this letter is to investigate whether the Swampland criteria firstly obtained in Refs. [38, 39] and later further studied in Refs. [40–71], hold true, but as it will be apparent shortly, the slow-roll indices and the corresponding observational indices have a very simple functional form, thus the inflationary phenomenology is considerably simplified in the present context, compared to our previous approach [10]. An extensive account for the inflationary phenomenology of Einstein-Gauss-Bonnet theories using the formalism developed in the present letter, can be found in [72].

The result of this letter in short is that the Swampland criteria for Einstein-Gauss-Bonnet theories of gravity can be satisfied for general within assumptions scalar coupling function $\xi(\phi)$ and scalar field potential $V(\phi)$. Our result does not require any specific functional relation between the scalar potential and the scalar coupling function $\xi(\phi)$, apart from the fact that in order for the condition (1) to be satisfied, the potential $V(\phi)$ and the scalar coupling $\xi(\phi)$
must satisfy a specific differential equation. Our result is somewhat more general in comparison to the one of Ref. [31], where the Swampland criteria were addressed in the context of Einstein-Gauss-Bonnet theories. As it was shown in [31], if the function \( \xi(\phi) \) is chosen as \( \xi(\phi) = \frac{\lambda}{V(\phi)} \), and also certain assumptions are imposed on the scalar coupling function \( \xi(\phi) \) and its derivatives with respect to the cosmic time, the theory satisfies the Swampland criteria. In our case, the constraint (1) does not require such restricted functional relations between \( \xi(\phi) \) and \( V(\phi) \). In fact, if the relation \( \xi(\phi) = \frac{\lambda}{V(\phi)} \) holds true, as we show, this can also be a subcase of solutions in the context of the present letter, only if \( \lambda = \frac{3}{4} \). Thus with this letter we provide a more generic framework of Einstein-Gauss-Bonnet gravity, in the context of which the Swampland criteria can be satisfied in a less constrained way.

II. THE SWAMPLAND CRITERIA FOR THE GW170817-COMPATIBLE EINSTEIN-GAUSS-BONNET THEORY

The Einstein-Gauss-Bonnet theory has the following gravitational action,

\[
S = \int d^4 x \sqrt{-g} \left( \frac{R}{2\kappa^2} - \frac{1}{2} \omega \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{2} c_1 \xi(\phi) G \right),
\]

(2)

with \( \kappa = \frac{1}{M_P} \) and \( M_P \) is the reduced Planck mass. Also the function \( \xi(\phi) \) is the Gauss-Bonnet scalar coupling, which is dimensionless and \( c_1 \) is a dimensionless free variable, which we shall set equal to unity hereafter, that is, \( c_1 = 1 \), for simplicity. The parameter \( \omega \) shall be assumed to be equal to \( \omega = 1 \), so that the kinetic term of the scalar field is canonical, since the aim of this letter is to study the Swampland criteria apply for well defined effective field theories, hence the phantom case \( \omega = -1 \) is forbidden. Assuming a flat Friedman-Robertson-Walker metric (FRW),

\[
ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2,
\]

(3)

the gravitational equations of motion read,

\[
\frac{3H^2}{\kappa^2} = \frac{1}{2} \omega \dot{\phi}^2 + V + 12 \dot{\xi} H^3,
\]

(4)

\[
\frac{2\dot{H}}{\kappa^2} = -\omega \dot{\phi}^2 + 4 \ddot{\xi} H^2 + 8 \dot{\xi} H \dot{H} - 4 \dddot{\xi} H^3,
\]

(5)

\[
\omega (\dddot{\phi} + 3H \ddot{\phi}) + V' + 12 \dddot{\xi} H^2 (\dot{H} + H^2) = 0.
\]

(6)

where the “prime” denotes differentiation with respect to the scalar field \( \phi \) hereafter.

The gravitational wave speed is given in Eq. [1], so by requiring that this is equal to \( c_T^2 = 1 \), this implies that the Gauss-Bonnet scalar coupling function must satisfy the differential equation \( \dddot{\xi} + H \ddot{\xi} = 0 \) as we already mentioned. By expressing the derivatives with respect to the cosmic time, as \( \frac{d}{dt} = \dot{\phi} \frac{d}{d\phi} \), we have \( \dddot{\xi} = \xi' \ddot{\phi} \) and we can write the differential equation as,

\[
\xi'' \dddot{\phi} + \xi' \ddot{\phi} = H \xi' \dot{\phi}.
\]

(7)

Assuming the slow-roll evolution holds true for the scalar field, \( \frac{\dot{\phi}^2}{2} \ll V \),

(8)

and by further assuming that the contribution of the second term containing the term \( \ddot{\phi} \) is subdominant, namely that, \( \xi' \ddot{\phi} \ll \xi'' \dot{\phi}^2 \), the equation (7) becomes an algebraic equation which determines \( \dot{\phi} \) as follows,

\[
\dot{\phi} \simeq \frac{H \xi'}{\xi''}.
\]

(9)
where recall that the “prime” denotes differentiation with respect to the scalar field. In effect, by assuming that the slow-roll condition $H \ll H^2$ holds true in order for the inflationary era to be realized in the first place, Eq. (6) can be re-written as follows,

$$\frac{\xi'}{\xi''} \simeq -\frac{1}{3\omega H^2} \left( V' + 12\xi'H^4 \right). \quad (10)$$

In addition, assuming that the following extra conditions (11) hold true for the scalar field,

$$\ddot{\phi} \ll 3H\dot{\phi}, \quad 12\dot{\xi}H^3 = 12\frac{\xi'^2H^4}{\xi''} \ll V, \quad (11)$$

we can rewrite the gravitational equations of motion Eq. (4)-(6) as follows,

$$H^2 \simeq \frac{\kappa^2 V}{3}, \quad (12)$$

$$\dot{H} \simeq -\frac{1}{2}\kappa^2 \omega \dot{\phi}^2, \quad (13)$$

$$\dot{\phi} \simeq -\frac{1}{3\omega H} \left( V' + \frac{4}{3}\xi'V^2\kappa^4 \right), \quad (14)$$

while recall that the constraint of Eq. (7) in conjunction with the assumption $\xi'\ddot{\phi} \ll \xi''\dot{\phi}^2$, results to the following condition,

$$\dot{\phi} \simeq \frac{H\xi'}{\xi''}. \quad (15)$$

From Eqs. (14) and (15) we conclude that the Gauss-Bonnet scalar coupling $\xi(\phi)$ must satisfy the following differential equation,

$$\frac{\xi'}{\xi''} = -\frac{1}{3\omega H} \left( V' + 12\xi'H^4 \right), \quad (16)$$

or in view of Eq. (12), we have the final form of the differential equation that the Gauss-Bonnet scalar coupling $\xi(\phi)$ must satisfy,

$$\frac{\xi'}{\xi''} = -\frac{1}{\omega\kappa^2 V} \left( V' + \frac{4}{3}\xi'V^2\kappa^4 \right). \quad (17)$$

Before proceeding, notice that the condition $12\dot{\xi}H^3 = 12\frac{\xi'^2H^4}{\xi''} \ll V$ appearing in Eq. (11), in view of Eq. (12) is written as,

$$\left( \frac{4}{3}\kappa^4\xi'V \right) \frac{\xi'}{\xi''} \ll 1, \quad (18)$$

and notice that the above terms enter the differential equation (17). This will be a useful constraint when we consider the Swampland criteria later on. The slow-roll indices for an Einstein-Gauss-Bonnet theory are defined as follows (11),

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_3 = 0, \quad \epsilon_4 = \frac{\dot{E}}{2HE}, \quad \epsilon_5 = \frac{Q_a}{2HQ_t}, \quad \epsilon_6 = \frac{\dot{Q}_t}{2HQ_t}, \quad (19)$$

with the function $E$ being equal to,

$$E = \frac{1}{\kappa^2\dot{\phi}^2} \left( \omega\dot{\phi}^2 + 3 \left( \frac{Q_a^2}{2Q_t} \right) + Q_c \right), \quad (20)$$

and the functions $Q_a$ and $Q_t$, $Q_b$ and $Q_c$, and also the function $Q_e$ to be used later on, are defined as follows (11),

$$Q_a = -4\dot{\xi}H^2, \quad Q_b = -8\dot{\xi}H, \quad Q_t = \frac{1}{\kappa^2} + \frac{Q_b}{2}, \quad Q_c = 0, \quad Q_e = -16\dot{\xi}H. \quad (21)$$
In view of Eqs. (12)-(15), we can express the $Q_i$ functions of Eq. (21) as functions of the scalar field. The $Q_i$ functions read,

\[ Q_a \approx -4 \frac{\xi'^2}{\xi''} H^3 \approx -\frac{(4\kappa^3) V(\phi)^{3/2} \xi'(\phi)^2}{(3\sqrt{3}) \xi''(\phi)}, \]  
\[ Q_b \approx -8 \frac{\xi'^2}{\xi''} H^2 \approx -\frac{(8\kappa^2) V(\phi)\xi'(\phi)^2}{3\xi''(\phi)}, \]  
\[ Q_c \approx 8\kappa^2 \omega \frac{\xi'^4}{\xi''^3} H^3 \approx \frac{V(\phi)^{3/2} (8\kappa^5 \omega) \xi'(\phi)^4}{(3\sqrt{3}) \xi''(\phi)^3}. \]

Also, the slow-roll indices of Eq. (19) expressed in terms of the scalar field read,

\[ \epsilon_1 \approx \frac{\kappa^2 \omega}{2} \left( \frac{\xi'}{\xi''} \right)^2, \]  
\[ \epsilon_2 \approx -\epsilon_1 + 1 - \frac{\xi'''}{\xi''^2}, \]  
\[ \epsilon_3 = 0, \]  
\[ \epsilon_4 \approx \frac{\xi'}{2\xi''} E', \]  
\[ \epsilon_5 \approx -\frac{2\kappa^4 \xi'^2 V}{3\xi''^2 - 4\kappa^4 \xi'^2 V}, \]  
\[ \epsilon_6 \approx -\frac{2\kappa^4 \xi'^2 V \left( 1 - \frac{1}{2\kappa^2 \omega} \left( \frac{\xi'}{\xi''} \right)^2 \right)}{3\xi''^2 - 4\kappa^4 V \xi'^2}, \]

where the function $E(\phi)$ is equal to,

\[ E(\phi) = \frac{\omega}{\kappa^2} + \frac{8\kappa^4 \xi'^2 V^2 \xi''}{3\xi'' \left( 1 - \frac{4\kappa^4 \xi'^2 V}{\omega \xi''} \right)}. \]

Also the spectral index of the primordial scalar curvature perturbations $n_S$, the spectral index of the tensor perturbations $n_T$ and the tensor-to-scalar ratio $r$, as functions of the scalar field $\phi$, are defined as follows [11],

\[ n_S = 1 - \frac{2\epsilon_1 + \epsilon_2 + \epsilon_4}{1 - \epsilon_1}, \]
\[ n_T = -\frac{2\epsilon_1 + \epsilon_6}{1 - \epsilon_1}, \]
\[ r = 16 \left| \left( \frac{\kappa^2 Q_c}{4H} - \epsilon_1 \right) \frac{2c_A^3}{2 + \kappa^2 Q_b} \right|. \]

where $c_A$ is the speed of sound, which for the Einstein-Gauss-Bonnet theory is equal to,

\[ c_A^2 = 1 + \frac{Q_a Q_e}{3Q_a^2 + \omega \phi^2 (\frac{2}{\kappa^2} + Q_b)}. \]
It is apparent that the slow-roll indices, and correspondingly the observational indices, have a very simple closed form, so this will be valuable for inflationary phenomenology, the details of which can be found in Ref. [72]. For completeness, in order to further show the simplicity of the final equations that determine the inflationary dynamics, we quote here the relation of the tensor-to-scalar ratio as a function of the function $\xi$.

The tensor-to-scalar ratio (34) is evaluated to be equal to,

$$x \approx \xi \, \frac{e^2}{\xi_f}$$

with $\phi_i$ and $\phi_f$ being the values of the scalar field at the beginning and at the end of the inflationary era respectively.

Now let us proceed to the main focus of this paper, the Swampland criteria [38, 39]. In the literature there is currently a lot of activity regarding the implications of the Swampland criteria, see Refs. [40–58, 61–71]. Also in Ref. [31], the Swampland criteria were considered for Einstein-Gauss-Bonnet theories, like in our case, but from a different perspective, since the compatibility with the GW170817 event was not taken into account.

From now on we adopt the reduced Planck units physical system, so that $\kappa^2 = 1$ and also take $\omega = 1$ in order to have a canonical kinetic term for the scalar field. The Swampland criteria in reduced Planck units are the following,

1. $\Delta \phi \leq d$, with $d \sim O(1)$ in reduced Planck units.

2. $\frac{\dot{V}}{V} \geq \gamma$, with $\gamma \sim O(1)$ in reduced Planck units.

The problem indicated by the Swampland criteria for a single canonical scalar field is that the tensor to scalar ratio is $r = 16 \left(\frac{\dot{V}}{V}\right)^2$, so this must be larger than unity, which is excluded by the latest 2018 Planck data.

Let us see if the Swampland criteria affect the Einstein-Gauss-Bonnet inflationary phenomenology and how. From the slow-roll condition $\frac{\dot{\phi}^2}{V} \ll V$ of Eq. (11) for the scalar field, by substituting $\dot{\phi}$ form Eq. (9), we have,

$$\frac{\dot{\phi}^2}{V} = \xi \frac{e^2}{2\xi_f^2 V} = \frac{\xi^2}{6\xi_f^2} \ll 1,$$

in reduced Planck units. Defining $x = \frac{\xi}{\sqrt{6}\xi_f}$, the condition (57) becomes,

$$x^2 \ll 1.$$

The tensor-to-scalar ratio [54] is evaluated to be equal to,

$$r \simeq \frac{8\xi' (\phi)^2 \left( -8V(\phi)^2 \xi''(\phi)^2 + (2\xi'(\phi)^2 - 3\xi''(\phi)^2)^{-1} - 12V'(\phi)\xi'(\phi)^2 + 9\xi''(\phi)^2 + 4V(\phi)\xi'(\phi)^2 + 3\xi''(\phi)^2 \right)^{3/2}}{3\sqrt{3}\xi''(\phi)^2},$$

for general $x(\phi)$ and $V(\phi)$. Now since $\frac{\dot{V}}{V} \sim \gamma$, we substitute this in the relation (17), so we obtain that $\xi''$ is equal to,

$$\xi'' = \frac{\xi' (\phi)}{-\gamma - \frac{3}{4} V(\phi) \xi' (\phi)},$$

and also by using the definition of $x$, by using Eq. (17), it can easily be shown that $\xi'$ can be expressed in terms of the potential $V$, $x$ and of the function $\gamma$ as follows,

$$\xi' = \frac{3\sqrt{6x} - \gamma}{V}.$$

Substituting this in Eq. (39) we can obtain an expression for the tensor-to-scalar ratio as a function of the function $\gamma$, so the final expression is,

$$r = \frac{16x^2 \left( -9(4x^2 - 1)(\gamma^2x^2 + 6x^2 - 2\sqrt{6})x + 6x(6x - \sqrt{6}\gamma) + 6 \right)^{3/2}}{3\sqrt{3} \left( \gamma^2x^2 - 2\sqrt{6}\gamma x + 2x(6x - \sqrt{6}\gamma) + 2 \right)}.$$
Now by replacing $x = \sqrt{\varepsilon}$ in the above expression, we have,

$$r \simeq \left| -48\varepsilon \left( \frac{24\sqrt{6}\varepsilon^{3/2} - 72\varepsilon^2 + 18\varepsilon - 8\sqrt{6}\varepsilon + 5}{30\varepsilon - 8\sqrt{6}\varepsilon + 5} \right)^{3/2} \right|,$$

and since $x^2 = \varepsilon \ll 1$, by taking the Taylor expansion of the expression (43) for $\varepsilon \to 0$, we get,

$$r \simeq \left| -48\varepsilon + \frac{864\gamma^2\varepsilon^2}{3\gamma^2 + 2} + \mathcal{O}(\varepsilon^{5/2}) + ... \right|,$$

at leading order. So by using again $x = \sqrt{\varepsilon}$, we have that the tensor-to-scalar ratio at leading order reads,

$$r \simeq 48x^2,$$

so in view of Eq. (45), we have that $r \ll 1$. Also, $\gamma \sim \mathcal{O}(1)$, so it does not affect the Taylor expansion. Therefore, we demonstrated that the Swampland criteria do not affect the inflationary phenomenology of the Einstein-Gauss-Bonnet theory of gravity, when the latter is required to be compatible with the GW170817 event.

It is easy to show that the same result can be obtained, by using an alternative way, if the following condition holds

$$\xi' \sqrt{6\xi''} \ll 1.$$

Recall that the fraction $\xi' \sqrt{6\xi''}$ enters in the condition appearing in Eq. (18), but more importantly in the differential equation (47). In view of the new condition (49), the differential equation (47) becomes,

$$\frac{V'}{V} + \frac{4}{3}\xi'V = 0,$$

where we took $\omega = \kappa^2 = 1$. So in this case, by substituting $\xi'' \rightarrow \frac{\xi'}{\sqrt{6\xi''}}$ in (42) and subsequently by substituting $\xi' \rightarrow \frac{3\gamma}{V(\phi)}$, which is obtained from Eq. (47), we obtain,

$$r \simeq \left| -48\varepsilon + \frac{864\gamma^2\varepsilon^2}{3\gamma^2 + 2} + \mathcal{O}(\varepsilon^{5/2}) \right|,$$

which is identical to the one appearing in Eq. (44), with the difference that the result of Eq. (44) was obtained without assuming the extra condition (46), namely $\frac{\xi'}{\sqrt{6\xi''}} \ll 1$.

In fact, this extra condition $\frac{\xi'}{\sqrt{6\xi''}} \ll 1$ can be related to any phenomenologically viable Einstein-Gauss-Bonnet theory compatible with the GW170817 event, and as we show in Ref. [72], it proves that the condition $\frac{\xi'}{\sqrt{6\xi''}} \ll 1$ simplifies significantly the calculations but more importantly, it seems to be an inherent characteristic of the inflationary phenomenology of any viable Einstein-Gauss-Bonnet theory compatible with the GW170817 event.

Let us discuss here in brief the results of the paper [31], since some solutions may have some phenomenological importance. As it was shown in [31], if the function $\xi(\phi)$ is chosen as $\xi(\phi) = \frac{\lambda}{V(\phi)}$, where $V(\phi)$ is the scalar potential appearing in the action (2). However, if the compatibility with the GW170817 is imposed on the Einstein-Gauss-Bonnet theory is imposed, the function $\xi(\phi) = \frac{\lambda}{V(\phi)}$ does not satisfy in general the differential equation (17), unless the potential has a very specific form, which however does not yield a viable inflationary phenomenology in the context of the present paper [72]. However, if the condition $\frac{\xi'}{\sqrt{6\xi''}} \ll 1$ is imposed in the theory, then the function $\xi(\phi) = \frac{\lambda}{V(\phi)}$ satisfies the differential equation the differential equation (17), only if $\lambda = 3/4$. In Ref. [72] we also address the phenomenology of this class of models.

Another issue that is worth commenting on is that the authors of Refs. [20, 31] used a smaller number of slow-roll indices, in comparison to our approach. This is possibly due to the fact that one of the assumptions they used in order to calculate the power spectrum was $\xi \ll \xi H$, which in contrast we assumed that $\xi = H\xi$, see above Eq. (7). The assumption $\xi \ll \xi H$ lead to simpler expressions of the power spectrum, however, the gravitational wave speed in their context is $\xi^2 \neq 1$.

Finally, let us consider here the Lyth bound [73] implications in the context of the GW170817-corrected Einstein-Gauss-Bonnet theory. Recall that the Lyth bound is [73] $\Delta \phi > \Delta N\sqrt{7}$, and also the first Swampland criterion
constrains $\Delta \phi$ to be $\Delta \phi \leq d$, with $d \sim \mathcal{O}(1)$ in reduced Planck units. Thus we have the constraint $d \geq \Delta \phi > \Delta N \sqrt{\xi}$. In our case, $r \simeq 48 x^2 = \frac{8 cT^2}{\xi} x^2$, thus the constraint on $\Delta \phi$ becomes $d \geq \Delta \phi > \Delta N \frac{cT}{\xi}$. In effect, the condition $\Delta \phi > \Delta N \frac{cT}{\xi}$ constrains further the model, so for example if $\Delta N \sim 60$, in order for the Lyth bound to be satisfied, a viable model that is constrained to satisfy the Swampland criteria, must further satisfy $\Delta \phi > 60 \frac{cT}{\xi}$. However, the Lyth bound further constrains the Swampland compatible viable inflationary models, which now must satisfy additionally $\Delta \phi > 60 \frac{cT}{\xi}$. The present framework used in this letter guarantees that $x^2 \ll 1$, where recall that $x$ defined in the previous text is $x = \frac{\Delta \phi}{\sqrt{\xi}}$, however, the Lyth bound further constrains the Swampland compatible viable inflationary models, which now must satisfy additionally $\Delta \phi > 60 \frac{cT}{\xi}$. If for example $\frac{cT}{\xi} \sim \mathcal{O}(10^{-2})$, then we would have $d \geq \Delta \phi > 0.6$, so indeed the lower bound imposed by the Lyth bound, is smaller than $d$ which is of the order of unity in reduced Planck units. Also, if $\frac{cT}{\xi} \sim \mathcal{O}(10^{-2})$, then $x^2 \sim \mathcal{O}(1.66 \times 10^{-5})$, which is also compatible with the earlier assumed constraint. This, that is $x^2 \ll 1$. Thus the Lyth bound imposes the additional constraint $d \geq \Delta \phi > \Delta N \frac{cT}{\xi}$ in the theory, and should certainly be taken into account in order to construct a phenomenologically viable theory that also respects the Swampland criteria.

### III. CONCLUSIONS

In this letter we revisited the Einstein-Gauss-Bonnet theory in view of the GW170817 event, which compelled that the gravitational wave speed is equal to $c_0^2 \simeq 1$ in natural units. Using the formalism we developed for the GW170817-compatible Einstein-Gauss-Bonnet theory, we investigated if the Swampland criteria are satisfied for the scalar field and for the Hubble rate hold true.

An interesting future perspective of the current work, again in the context of the Swampland criteria, is that in the same way, one can in principle consider superstring cosmology in the presence of two scalars in string spectrum, thus not just considering simply the dilaton field, as in the present work.

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