Finite element analysis of plane frame systems with different models of semi-rigid connections

Tran Thi Thuy Van¹, Ngo Dinh Tung², Nguyen Trung Kien³

¹Hanoi Architectural University, Civil Engineering Faculty, Km.10, Nguyen Trai, Thanh Xuan, Hanoi, Vietnam
²SCG Joint Stock Construction Company, 8th floor of 16 Sunshine Centre, Namtuliem, Hanoi, Vietnam
³Vietnam Institute for Building Science and Technology, Institute of Building Structures, 81 Tran Cung, Cau Giay, Hanoi, Vietnam

E-mail: vanttt@hau.edu.vn

Abstract. Today, in the current Vietnamese design standards for semi-precast reinforced concrete and steel frame structural systems, the connections of main elements such as beams and columns are commonly considered as either perfectly hinged or rigidly fixed. This assumption is adopted because of the simplicity in design and analysis process but it is not consistent with actual behaviour in practice of structural systems. The connections between columns and beams have partial restraint depending on the type of used connection. The analysis of structural system with consideration of the effect of this restraint would result more exactly and economically. However, it is also more time required because of complexity of analysis process with consideration of flexibility of connections in the case of lack of suitable calculation method. The paper presents the analysis of plane frame systems with different models of semi-rigid connections using finite element method. The analysis process was implemented by MathCad programming software and the results of this study will obtained by conducting of numerical examples. From there the conclusions and recommendation of calculation and design for steel and semi-precast frame systems will be proposed.

1. Introduction

In recent years, with the rapid development of the economy in Vietnam, many semi-precast structures have been built to meet the housing demand for a part of the Vietnamese population with low and medium income. Similarly, steel structures have also been used in many construction fields, especially plane frame structures. The design instructions for these types of structures have been mentioned in many foreign construction standards with specific calculation recommendations considering the connection flexibility. However, in Vietnam, there is still no specific domestic calculation instructions in design of these structures considering the actual behavior of the connections. When designing these structures, it is either used foreign standards or considered the assumption that the connections between beams and columns are perfectly hinged or rigidly fixed. This assumption does not reflect the actual behavior of practical structures. If the connection between beams and columns is considered as rigidly fixed, the maximum value of the bending moment is reached at the end of the elements. Whereas, if it is considered the connection is perfectly hinged, the bending moment is zero at the end of the elements and reaches the maximum value at the middle point.
of the elements. The calculation with consideration of connection flexibility is more constituent with the behavior of the work of the practical structures, the distribution of internal force in the whole system will be more reasonable. The internal forces at the extreme points will be less than the case of perfectly hinged and rigidly fixed. This will result in more material savings. Thereof, if the calculation takes into account the connection flexibility, the analysis results will be more accurate and closer to the actual behavior of the real structures. Thus, in the analysis and design of steel frames the connection between beams and columns should be modeled as semi-rigid connections.

Researches on the connection flexibility of steel frame and semi-precast concrete structures have been concerned by many scientists from over the worlds and many studies on this issue are published with different problems [1-10]. In most of these studies the connection between beams and columns of semi-rigid frames were modeled by rotational springs. In fact, frame elements have semi-rigid behavior through the axial direction of themselves. In [2], the behaviors of beam-column connections are designed through either rotational or lateral effects. The additional effect of lateral springs already discussed in [2] with several values of rotational and lateral springs. In this study, it is developed from study [2] with the whole range of the values rotational and lateral springs which correspond with the end-fixity factor. The end-fixity factor is analyzed from a value greater than 0 to near 1 (0 is the value of end-fixity factor that corresponds with the case of pinned connection and 1-the case of rigidly fixed).

2. Analysis of plane frame systems with consideration of different models of semi-rigid connection

Semi-rigid frames are modelled by rotational springs and lateral springs. Each model has its characteristics. In this section it is introduced about their properties and element stiffness matrix in finite element analysis.

2.1. Rotational spring model

The connection in this model is represented by rotational springs near to beam-column joints. The presence of rotational springs will restrain the rotations \( \theta_i \) and \( \theta_k \) at the ends of the beam [7]. The unit of stiffness of rotational spring is kN.m/rad. In the analysis of steel plane frames this model is illustrated in Figure 1.

![Figure 1. The rotational spring model for beam-column connections.](image)

In the analysis by finite element method, the important step is determination of global stiffness matrix of structure system. The element stiffness matrices for beams and columns can be written as following The beam stiffness matrix [7]:
In which,

\[
K_{hl,r} = \frac{E \cdot I_b}{l} \begin{bmatrix}
A & 0 & 0 & -A & 0 & 0 \\
\frac{(s_u^* + 2s_y^* + s_y^*)}{l^2} & \frac{s_u^*}{l} & 0 & \left(\frac{s_u^* + 2s_y^* + s_y^*}{l^2}\right) & \frac{(s_u^* + s_y^*)}{l} & 0 \\
\frac{s_u^*}{l} & 0 & -\frac{(s_u^* + s_y^*)}{l} & s_y^* & 0 & 0 \\
A & 0 & 0 & 0 & \frac{(s_u^* + s_y^*)}{l^2} & -\left(\frac{s_y^* + s_y^*}{l^2}\right) \\
\end{bmatrix}
\]

(1)

In which,

\[
s_u^* = \left(4 + \frac{12E \cdot I_b}{l \cdot R_{h,bb}}\right) \left(1 + \frac{4E \cdot I_b}{l \cdot R_{t,bb}}\right) \left(1 + \frac{4E \cdot I_b}{l \cdot R_{k,bb}}\right) \left(\frac{E \cdot I_b}{l}\right)^2 \frac{4}{R_{k,bb} \cdot R_{t,bb}};
\]

(2)

\[
s_y^* = s_y^* = 2 \left(1 + \frac{4E \cdot I_b}{l \cdot R_{k,bb}}\right) \left(\frac{E \cdot I_b}{l}\right)^2 \frac{4}{R_{k,bb} \cdot R_{t,bb}}
\]

\[R_{k,bb}, R_{t,bb} - \text{initial stiffness rotational spring}\]

The column stiffness matrix [7]

\[
K_{cl,r} = \frac{E \cdot I_c}{l_c} \begin{bmatrix}
\frac{12}{l_c} \cdot \gamma_1 & 0 & \frac{6}{l_c} \cdot \gamma_1 & -\frac{12}{l_c} \cdot \gamma_1 & 0 & 6 \cdot \gamma_1 \\
A & 0 & 0 & -A & 0 & 0 \\
\frac{4\gamma_3}{l_c} & 0 & 2\gamma_4 & -\frac{6}{l_c} \cdot \gamma_2 & 0 & -\frac{6}{l_c} \cdot \gamma_2 \\
\frac{12}{l_c^2} \cdot \gamma_1 & 0 & -\frac{6}{l_c} \cdot \gamma_2 & A & 0 & 4\gamma_3 \\
\end{bmatrix}
\]

(3)

In which,
\[ \gamma_1 = \frac{(kl_c)^2 \cdot \sinh (kl_c)}{12 \cdot (2 - 2 \cosh (kl_c) \pm kl_c \cdot \sinh (kl_c))} \;
\gamma_2 = \frac{(kl_c)^2 \cdot (\cosh (kl_c) - 1)}{6 \cdot (2 - 2 \cosh (kl_c) \pm kl_c \cdot \sinh (kl_c))} \;
\gamma_3 = \frac{(kl_c) \cdot (kl_c \cdot \cosh (kl_c) - \sinh (kl_c))}{4 \cdot (2 - 2 \cosh (kl_c) \pm kl_c \cdot \sinh (kl_c))} \;
\gamma_4 = \frac{(kl_c) \cdot (\sinh (kl_c) - kl_c)}{(2 - 2 \cosh (kl_c) \pm kl_c \cdot \sinh (kl_c))} \]  

(4)

where \( k = \sqrt{\frac{P}{EI_c}} \), “+” sign corresponds with tensile axial force and “-” sign – compressive axial force. In the case of that the axial force is small, \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 1 \).

2.2. Rotational-lateral spring model

Similarly, this model is represented by rotational and lateral springs near beam-column connections. The presence of these spring not only restrain the rotations \( \theta_i \) and \( \theta_k \), but also the lateral displacements \( \delta_i \) and \( \delta_k \) at the ends of beams. This model is illustrated in Figure 2.

Figure 2. The rotational-lateral spring model in semi-rigid frame structures.

The beam stiffness matrix can be written in (6)

\[ R_{bj} = \frac{1}{I_b} \cdot \frac{A_b \cdot (k_i \cdot k_j)}{(k_i \cdot k_j) + E \cdot A_b \cdot (k_i \cdot k_j)} \]  

(5)

where, \( k_i, k_j \) – lateral stiffness of the second model. The unit of lateral stiffness is kN/m.
2.3. Calculation procedure of plane frame analysis with consideration of two models of connection flexibility using finite element method

The effects of connection flexibility are modeled using rotational springs having stiffness $C_{\theta_{i,j}}$ and rotational-lateral springs having stiffness $C_{\theta_{i,j}, k_{i,j}}$ at the ends of the beams. To reflect the relative stiffness of the beams and the rotational end-spring connections, an end-fixity factor is used [10]. It is defined as:

$$\eta = \frac{\alpha}{\varphi}, \quad \varphi = \frac{1}{1 + \frac{3}{R_i}}$$

(7)

where $\alpha$ is the member end rotation, $\varphi$ is the combined rotation of the member and the connection due to a unit end moment, $R_i = C_{\theta_{i,j}}/E I_b$. Note that $\eta$ falls between zero and one, it defines the stiffness of each end connection relative to the adjoining beam ($\eta = 0$ corresponds with the case of perfectly pinned connection, $\eta = 1$ – fully fixed connection).

The analysis is conducted with full range of the value of end-fixity factor from zero to one. From that it is defined the values of rotational stiffness and it is established the stiffness matrix of system.

The calculation procedure is performed using MathCad calculation programming software which allows to see all calculation steps and corresponding results of the analysis.

3. Numerical example

In this study, two-storey semi-rigid frames with two above mentioned models as shown in Fig. 3 (a – for rotational mode; b – for rotational – lateral model). All frames have same geometrical dimensions, cross-section and material properties for comparison of influences of connection flexibility on results of statically analysis using finite element method.

Given material properties and geometrical dimensions and sizes as followings: elastic modulus $E=20.10^7$ kN/m$^2$, cross-section of beams C150x70, cross-section of columns I160x81x5.0, $l=4m$, $h=3m$, $q=3kN/m$, $P=10kN$.

![Figure 3. Examples of semi-rigid frames subjected to static loading: a – with rotational spring model; b – with rotational – lateral spring model.](image-url)
This plane frame is anti-symmetrical; the results of analysis are given for a half of system.

Table 1. Analysis results of plane frame with different models of connection flexibility

| End-fixity fac. | Rot. spring stiff., C_{\alpha_{ij}} kN/m | Lat. spr. stif, k_{ij} kN/m | Bending moment |
|----------------|----------------------------------------|----------------------------|----------------|
|                |                                        |                            | M_1 | M_2 | M_3 | M_4 | M_5 |
|                |                                        |                            | R   | R-L | R   | R-L | R   | R-L | R   | R-L |
| 1              |                                        |                            | 72.52 | 2.02 | 23.05 | 21.03 | 20.45 |
| 0.97           | 5.642 \times 10^4                      | 5.10^7                     | 72.61 | 72.61 | 2.12 | 2.12 | 20.97 | 20.97 | 23.09 | 23.09 | 20.41 | 20.41 |
| 0.85           | 1.053 \times 10^4                      | 5.10^6                     | 72.99 | 72.99 | 2.49 | 2.49 | 20.74 | 20.74 | 23.23 | 23.23 | 20.27 | 20.27 |
| 0.75           | 5.642 \times 10^4                      | 10^6                       | 73.34 | 73.34 | 2.82 | 2.82 | 20.54 | 20.54 | 23.36 | 23.36 | 20.14 | 20.14 |
| 0.65           | 5.642 \times 10^4                      | 5.10^5                     | 73.76 | 73.75 | 3.26 | 3.26 | 20.26 | 20.26 | 23.52 | 23.52 | 19.98 | 19.98 |
| 0.55           | 5.642 \times 10^4                      | 10^5                       | 74.26 | 74.27 | 3.76 | 3.78 | 19.95 | 19.94 | 23.71 | 23.72 | 19.79 | 19.78 |
| 0.45           | 5.642 \times 10^4                      | 5.10^4                     | 74.91 | 74.93 | 4.41 | 4.44 | 19.56 | 19.54 | 23.97 | 23.98 | 19.53 | 19.52 |
| 0.35           | 5.642 \times 10^4                      | 10^4                       | 75.82 | 75.92 | 5.32 | 5.42 | 19.01 | 18.95 | 24.33 | 24.37 | 19.17 | 19.13 |
| 0.25           | 5.642 \times 10^4                      | 5.10^3                     | 77.26 | 77.45 | 6.76 | 6.95 | 18.16 | 18.05 | 24.92 | 24.92 | 18.58 | 18.50 |
| 0.15           | 5.642 \times 10^4                      | 10^3                       | 80.09 | 80.88 | 9.59 | 10.38 | 16.54 | 16.10 | 26.13 | 26.48 | 17.37 | 17.02 |
| 0.1            | 5.642 \times 10^4                      | 10^2                       | 83.01 | 88.54 | 12.5 | 18.04 | 14.93 | 12.02 | 27.44 | 30.06 | 16.06 | 13.44 |

4. Conclusions

The flexibility of beam-column connections is presented by rotational and rotational-lateral spring models that show the more actual behaviour of real structures in comparison with the assumptive pinned and fixed connections. The stiffness matrices are introduced for finite analysis help to solve more easily the problems of semi-rigid connection. In this study the subroutine is written for statically analysis using finite element method by MathCad programming software that help to investigate the end-fixity factor from one to zero without any mathematical difficulties.

The effects of connection flexibility are investigated. The different types of models of connection flexibility effect differently on statically analysis of structures, in particular on investigated values of bending moment of typical sections of frame.

References

[1] Gerard R Monforton 1962 Matrix analysis of frames with semi-rigid connections
[2] Ozturk A U and Yeslce Y 2005 The statical analysis of semi-rigid frames by different connection types
[3] Salatic R and Sekilovic M 2001 Nonlinear analysis of frames with flexible connections Computers and Structures 79 1097–1107
[4] Kattner M and Crisinel M 2000 Finite element modeling of semirigid composite joints Journal Computers & Structures 78 341–353
[5] Csebfalvi A, Csebfalvi G 2005 Effect of semi-rigid connection in optimal design of frame structures 6th World Congresses of Structural and Multidisciplinary Optimization
[6] Kim S E and Choi S H 2001 Practical advanced analysis for semirigid space frames Journal of Solids and Structures 38 9111–9131

[7] Wai-Fah Chen, Norimitsu Kishi 2011 Masato Komuro Semi-rigid connections handbook (J. Ross Publishing, Inc.)

[8] Ozturk A U 2000 Dynamic analysis of semi-rigid connected frames, Dokuz Eylul University, Ms.Thesis. Izmir.

[9] Fu Z, Ohi K, Takanashi K and Lin X 1998 Seismic behavior of steel frames with semi-rigid connections and braces Journal of Constructional Steel Research 46 440–461

[10] Lui E M and Lopes A 1997 Dynamic analysis and response of semi rigid frames Journal of Engineering Structures 19 644–654