Complex Modeling of the Economic Systems Stability

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Abstract. Our work is devoted to the problem of the economic systems stability analysis. At the same time, we call stable those economies that demonstrate growth and expansion up to the limits guaranteed by the modern level of technological development of mankind. Such growth in terms of "production", "profit", "business cost" should be harmoniously combined with the welfare growth and fundamental "confidence in the future" of the people.

The objectives of the study are: 1) the analysis of the factors that are difficult for modeling both in general and in the study of the economic stability problems in particular; 2) the development of options to improve the validity, effectiveness and practical value of the equilibrium models use for the options of economic policy, its results and prospects mapping. Substantiation of expediency of stability, economic and welfare growth modeling on the basis of a set of computable General equilibrium models, and dynamic input-output models; 3) the development of mathematical algorithm for optimization of economic systems’ own dynamic properties, auxiliary functions of the transient processes quality simulating investment activity in economy as a necessary condition for the economic growth.

1. Factors beyond modeling
Before presenting the main part of the article, based on modeling and the body of mathematics, we have to identify the factors that are very difficult for modeling, but have a direct impact on the stability and quality of transient processes in the economy. In this sense, it is desirable to have balanced economic growth and the system to stay on the main trajectories [1], [2]. It should be noted that the models of economic development are formed both in symbolic and verbal and figurative form in the heads of persons making economic decisions (PMD).

PMD (Decision-makers) in their activities maximize their "utility function", the one, which is not always and not fully known by the public. That is why scientists in the 1990s made mistakes that had no analogues in the world practice of forecasting. The assertion that the modern economy after 2008 is difficult to predict is given in the article of D. A. Medvedev [3].

Planning is not mentioned in it at all, although it is even intuitively clear that it is better to live according to a bad plan than without any plan at all.

It is not just a crisis, but the crisis-matryoshka that is almost impossible for formal modeling, when, after the self-destruction of the USSR, the index of human capital collapsed thus resulting in large-scale corruption, which produced such a factor as the lack of financial resources.
Political instability in the world and sanctions pressure on Russia in the field of finance and technology have been superimposed on economic problems. Crisis events that have lately taken place, are being discussed in detail in [4] and [5], but the main reason for the low rates of economic development lies in structural problems [6], [7]. The roots of these problems are still Soviet. Later the implementation of the "Washington consensus" provisions by the team of Gaidar reformers became an additional burden for the economy. The liberal economic bloc of the government year after year removed the country from the "oil needle" only in reports, which was covered by the world demand and the prices growth for domestic raw materials. At the same time, within the framework of economic liberalism, the government has done everything possible to keep the economy from a significant recession.

The budget deficit is small and is declining. If in 2016 it was 3.6%, the target for 2019 was set at 1.2% of GDP. The share of non-oil budget revenues approached the level of 60%, and only inflation of 2.52% is injustice and unrighteousness, obtained by withdrawal of money from the economy, when the effect of clamped demand and shrinking target markets works. But this is not enough, the moment has come to move from words to deeds with regard to the modernization of the economy. Institutional, infrastructural and macroeconomic growth conditions are important, but the growth itself is not yet triggered. We need both the investment climate and the investments themselves, those ones Russia has great and even man-made problems with, began at the time of the USSR collapse.

Then the "development" of the country went along the following chain: a sharp decline in the human potential index → large-scale corruption → lack of financial resources. The latter factor makes it extremely difficult both to develop existing production facilities and to create new competitive ones. Without investments into production, without long and accessible loans the solution of the problem is hardly possible, and we have the situation when the amount of Bank interest exceeds the rate of return in most industries. And in case the access to these loans is ensured, at the same moment the placement of this money in the financial markets should be prohibited. Thus, again we are at variance with the model of liberal economy, for which speculations limiting is equal in importance to our hands raising to the Holiness, whereas without large-scale production investments Russia's economy is doomed to demonstrate "growth" at the level of statistical error.

2. Modeling of Stability and Economic Growth

When considering the issues of modeling we have essentially two options for the economic systems formalization.

These are computable models of General economic equilibrium, known in foreign literature as Computable General Equilibrium models (CGE-models) [8-16] and input-output models (IOM) [17].

The first belong to the class of agent models and represent a system of nonlinear algebraic equations written for the year under study. The models are digitized, calibrated to match the gross issues of agents with official statistics. Models are digitized, calibrated to match the gross releases of agents with the official statistics. The second, when they are written in the form of a high-dimensional system of algebraic and differential equations, open up broad opportunities for the analysis and optimization of their intrinsic dynamic properties (IDP) of economic systems.

The central part of the database for digitization of both models is the base tables "input-output" (IOT), which for the Russian economy in 2011 appeared on the Rosstat website in the tab "National accounts → table "input-output" 18.04.2017 , the dimension of symmetric IOT being equal to (126×126).

After the CGE – model calibration, the procedure for the point of General economic equilibrium determination in accordance with the Valras theory is launched [18]. CGE-models are based on the neoclassical economic theory, and its provisions are not always implemented in modern Russian economy. In the conditions of human capital erosion and of the PMD imperfection, endless arbitrariness of monopolies and financial speculation, unconfirmed economic growth, low inflation due to the "drying" of demand and other negative factors, the CGE models are being completely
transformed into an instrument for writing research works and qualification works, building an academic career or the one of a teacher. And this iterative process may well converge to a point that does not exist in the real economy.

The validity of the CGE models application can be ensured on the basis of the IDP economy analysis, which can be harmonized with the provisions of the neoclassical economic theory. If the structural readiness of the system for extended reproduction is established in this way, it increases the confidence measure to the results of CGE-modeling.

When speaking of IOM we mean Leontief’s model of the type

$$X(t) - AX(t) - B pX(p) = Y(t), \quad X(0) = X_0,$$ \hspace{1cm} (1)

where \( p = \frac{d}{dt} \) is the operator of differentiation, \( A \) - the matrix of direct production costs coefficients, \( B \) – the matrix of increasing stock capacities coefficients, \( Y(t) \) – the vector of final demand (consumption), \( X(t) \) – the vector of gross issues by economic activity (IEA), and the vector \( B pX(p) \) characterizes the dynamics of accumulation/reduction of all types of "capital" in the context of FEA.

This model is easily closed for consumption and leads to a normal form of Cauchy with degenerate matrix \( B \) \([19, 20]\), acquiring the form of:

$$pX_1 = GX_1, \quad X_1(0) = X_0,$$ \hspace{1cm} (2)

where \( G \) is a square matrix of dimension \((n\times n)\), and \( n \) is the number of differential equations in \((1)\). Now the question of structural stability is solved as follows: the system possesses IDP with the potential for economic growth, if the spectrum of the matrix \( G \) has one positive eigenvalue, let's denote it \( \alpha_m \) (this is one of the IDP indicators), which a positive eigenvector corresponds to.

This is the minimum number modulo called the degree of economic growth, and its own vector corresponding to it sets the proportion of investment in foreign economic activity FEA.

Bringing the economy to the state of structural stability is possible due to the investment efforts changing the balance sheet models parameters \((1), (2)\) and, accordingly, the spectrum of eigenvalues location in the complex plane. To simulate the process of restructuring is possible through the use of optimization procedures to IOM models. In the process of numerical search it is possible to control the group motion constituents, which are characterized by eigenvalues of the state matrix \( G \). To do this, the auxiliary function is minimized

$$F = \min_{u_0=0} (\alpha_0 - \alpha_m) + \min \sum_{\alpha \in \lambda(X_0)} (\alpha_0 - \alpha)^v$$ \hspace{1cm} (3)

where \( \alpha_0 \) is the desired degree of economic growth, and \( v \) - the integer exponent, usually taken as equal to 2; \( U_n \) - the right eigenvector of the matrix \( G \) corresponding to \( \alpha_m \); \( \alpha \) - real parts of the eigenvalues, \( G \), taken with the opposite sign; \( \alpha_0 > 0 \) - the specified value of the damping index of motion components.

It was V. Leontief who pointed out that a positive degree of economic growth will eventually subdue other components of the movement and the economy will stay on the path of balanced growth. Of course, Leontief had in mind the fulfillment of both necessary and sufficient conditions for this growth: perfect PMD and the absence of limitation on resources.

Function \((3)\) has mathematical properties of continuity and differentiability necessary for successful operation as part of optimization algorithms. Its minimization leads to the structural development of the economy to the main trajectories that the researchers, headed by George von Neumann in the twentieth century called the infinitely optimal. This is how the impact of investments on the dynamic properties of the economy is simulated. These properties are defined by eigenvalues
that are localized on the complex plane by a segment of the real axis $[\alpha_0; -\alpha_0]$ and the corresponding vertical lines. 

Eigenvalues that are outside this part of the complex plane, do not belong to the managed group and do not participate in optimization until, while varying the parameters of the model (1) do not cross the vertical straight lines with abscissas $\alpha_0$ or $-\alpha_0$. That is, the number of eigenvalues forming the function (3) may change during the optimization process, the degree of economic growth will strive for the desired value, the other components of the movement, including cyclic ones, are damped.

Thus, the auxiliary function (3) solves the two-fold problem of IDP optimization: maximizing the degree of growth and damping of vibration movements in the economy. So, formally, the purpose of the numerical search is to remove the eigenvalues of the system state matrix (2) from the strip of the complex plane cut out by the segment $[\alpha_0; -\alpha_0]$: the degree of growth $\alpha_0$ to the right of the strip, other eigenvalues with real parts $\alpha$ to the left of it. If it is impossible to solve this problem, the search method will do the maximum possible in the direction of its solution. Here is an example of the practical implementation of the theoretical provisions. To do this, we will use the already mentioned basic IOT and the data from the compendium "National accounts of Russia in 2001-2016". We have performed the aggregation procedure of the model from the original dimension $(126\times126)$ to the correct scientific article dimension $(15\times15)$ and have balanced the model.

In fact, CGE models also undergo calibration procedures - without this the gross issues of agents do not coincide with the official statistics. Table 1 shows some eigenvalues of the matrix $G$ for two cases – before (second line) and after (third line) optimization, during which the growth rates and damping level indicators were taken as $\alpha_0 = 0,1$ and $\alpha_0 = 0,05$ respectively.

| № | 1,2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|-----|---|---|---|---|---|---|---|----|----|
| $\lambda$ before | $-0,056\pm i0,005$ | $-0,04$ | $-0,039$ | $-0,036-0,027$ | $-0,003$ | $-0,002$ | $-0,001$ | $+0,1e-07$ |
| $\lambda$ after | $-0,055$ | $-0,04$ | $-0,042$ | $-0,041-0,040$ | $-0,040$ | $-0,039$ | $-0,039$ | $-0,04$ | $+0,011$ |

We have kept the FEA and the products with codes in the classifier of economic activities (GCFEA) from A to O.

The elements of the matrix $A$ corresponding to the five products with the highest value added were subject to variation according to the above-mentioned statistical compendium in the range of $\pm10\%$ of their initial values. This is quite possible if the economy is reported the investment impulse equal, according to our estimates, to about 20% of GDP in the prices of the same year 2011. The analysis of Table 1 data allows us to see that rhythmic investment efforts will allow us to have an economy with growth potential. With regard to it, the development and application of CGE models is justified, promising and opens up broad opportunities for the formation and evaluation of economic policy options, research conditions for sustainable economic growth.

### 3. Conclusion

The economy, which has a spectrum of eigenvalues for the second option, has the potential for economic growth, it is structurally ready for it. However, this is not enough. The growth of the economy, which should finally be followed by welfare growth, will not start automatically. It is necessary to carry out, speaking mathematical language, all necessary and sufficient conditions. The key of these conditions is the production investment. Let us allow ourselves another explanatory table compiled according to Rosstat data.
It is impossible to count on economic growth in the context of such "dynamics" of industrial production and investments in fixed assets, or, to be more precise, in the conditions of "trampling on the spot". It is pointless exactly the same as a useless dream is the expectation of large-scale private investment or investment of deposits of individuals at the time when the government persistently reduces its own participation in investment activities. Thus, the share of firms with state-owned and mixed ownership, with almost constant total investment in fixed assets, has steadily decreased from 32.5% in 2008 to 25.4% in 2015.

Private investments in government securities (GS) has been more reliable (no risk, stable returns) than investments in fixed assets. This is what the government has achieved in its attempt to solve the problem of the balance of the Federal budget by purely financial methods, which are reduced to the emissions and to these securities placement.

Table 2. Relative changes in some macroeconomic indicators for 2010-2016 (2008 – adopted to 100%).

| Indicator                  | 2008 | 2010  | 2012  | 2014  | 2016  |
|---------------------------|------|-------|-------|-------|-------|
| GDP                       | 100  | 96,3  | 104,0 | 105,8 | 102,6 |
| Industrial production     | 100  | 95,8  | 104,4 | 106,2 | 98,2  |
| Investments into fixed assets | 100  | 91,9  | 108,8 | 108,0 | 98,0  |

Structural reforms and the economic growth have nothing in common with the government's focus on financial and value instruments for the economic activity regulation. Firms are unwittingly drawn into these processes, begin to pursue an increase in the financial results of the current time to a greater extent than to take into account the actual problems of production.

As a result, if according to Rosstat data in 2008 the volume of investments in fixed assets exceeded the cost of GS by 8 times, in 2014 this ratio decreased to 3. Thus, there is a growth and priority of financial investments of the companies and a steadily low proportion of investments in fixed assets, without which economic growth does not happens.

We have already found out that production potential determines production opportunities and this is an important component of the potential for economic growth. And the above described situation in the sphere of investment activity has a negative impact on both. We have already seen that depreciation of fixed assets in the economy has passed the "psychologically important mark" in 50% in 2016, at the same time, the coefficient of renewal of fixed capital decreases: in 2008 it was equal to 4.4%, and in 2015 it decreased to 3.9%.

Maintaining such a state of affairs in the field of investment will allow for some time to support the economy in a simple reproduction mode, with the prospects for its decline in the near future.

In conclusion we should note that in these conditions, the hope for small business, farming in agriculture, consumer cooperation is useless and meaningless. These "small forms" in crisis-stagnant periods of economic history may be in demand as a way of survival, but will never create the potential for economic growth, will not provide jerks in technical and technological terms. They cannot act as locomotives of economic development by their nature; they are not self-sufficient and economically stable subjects. Only in the service sector small businesses can dominate. In the sphere of production and science, they are of an auxiliary nature and will never form a vector of the country's economic prospects.

4. References
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