Analysis and simulation of mathematical model for typhus disease in Makassar

Nurhaeda¹, S Anas² and S Side³

¹Student of Magister Mathematics Department, Universitas Negeri Makassar
²Department of Statistics, Faculty of Mathematics and Natural Science, Universitas Negeri Makassar
³Department of Mathematics, Universitas Negeri Makassar

Corresponding author: Syafruddin@unn.ac.id

Abstract. The aims of the research is to build a mathematical model for typhus; Analyse and determine the basic reproduction number of the model; model simulations to predict the number of Typhus cases in Makassar City. This stage of the study examines the mathematical model of Typhus transmission by including food and flies as variables, which are the Susceptible-Exposed-Infected-Recovered (SEIR) model; analysed the SEIR model using the matrix generation method, then the model simulation using Maple with secondary data on the number of Typhus cases in Makassar City obtained from the Makassar Health Office. The results obtained are the SEIR model for typhus transmission by including flies and food as a variable which is an eight dimension non-linear differential equation model. This study also provides the analysis results of the SEIR model that Makassar is not worried an area Makassar for Typhus disease, this means the disease Typhus still exists in Makassar, while the calculation results of the basic reproduction number is \( R_0 = 0.137 < 1 \), it can be concluded that the spread of typhus in Makassar is at an alarming stage, because a person suffering from typhus does not cause another person to suffer from Typhus. Model simulation results can predict the number of cases of Typhus so that early prevention can be done immediately and become the government's attention to prevent the spread of typhus in Makassar City.

1. Introduction
Typhoid fever is a contagious disease spread throughout the world, and is still the biggest health problem in developing and tropical countries such as Southeast Asia, Africa and Latin America. The incidence of the disease is still very high and an estimated 21 million cases with more than 700 cases ended in death. Typhoid fever in Indonesia is estimated to be about 300-810 cases per 100,000 people annually, meaning the number of cases is between 600,000-1,500,000 annually. This relates to the individual hygienic level, environmental sanitation and germicidal spread of the sufferers. In endemic areas that sanitation and health are well preserved, typhoid fever appears as a sporadic case. Based on the results of the survey of household health 2016 typhoid fever caused the death of 3% of all deaths in Indonesia. The average case of death and the complication of typhoid fever has always changed between different endemic areas [1].

Makassar city has become a city with the largest typhoid fever case in South Sulawesi. In 2011, two hospitals, namely Haji and Labuan Baji hospital, reported 596 dan165 patients with typoid fever, then increased in 2012 to 1,115 and 178 Typhus [2].

Mathematical modeling of infectious and uninfectious diseases such as dengue fever, tuberculosis, diabetes and typhoid has been conducted by [3-14]. Mathematical modeling becomes one of the...
solutions offering in controlling the disease because the simulation of the model can predict the number of cases, so the Government can do prevention and handling early. Research on typhoid has been done by [6; 7] but have not included flies and food as variables, this research will build the SEIR model by observing food and fly factors as a bearer of typhus bacteria, then analysis and simulation using the number of typhoid cases in Makassar city with Matlab.

2. Research Methods
The types of research used are theoretical and applied research. This study examines the theory and application of Typhus disease in Makassar City. The first part of the study was to build the SEIR model to spread Typhus, then perform analysis on the model, and create simulated models. Models use suspected, exposed, infected and recovered compartments. Analysis of the model using the matrix generation method, and using secondary data the number of cases of typhus in the city of Makassar and data assumptions to the value of the model simulation parameter using Matlab to predict the number of cases of typhus in Makassar.

3. Result and Discussion

3.1. SEIR Model for Typhus Disease
The formation of SEIR model on spread of typhus disease is carried out with regard to flow chart in Figure 1.

Variables and parameters definition of SEIR model for Typhus disease are $N_h$ is total number of population $S_h$ is Number of Susceptible individuals due to bacteria; $E_h$ is Number of Exposed individuals due to bacteria; $I_h$ Number of Infected individuals due to bacteria; $R_h$ Number of Recovered individuals due to bacteria; $S_F$ Number of susceptible food contaminated bacteria; $I_F$ The number of bacteria-infected foods; $S_L$ Number of flies that are susceptible to carrying bacteria in food; $I_L$ Number of flies infected with bacteria on food; $\mu_h$ Individual birth rate; Natural death rate in individuals because bacteria; $\mu_L$ Death rate flies due to bacteria and $\mu_F$ Food expiry rate. a, Individual displacement rates from Suscepted to Exposed; b, Individual displacement rates from Exposed to Infected; c, Individual displacement rates from Infected to Recovered; d, Individual displacement rates from Recovered to Suspected; p, Probability of food that can spread typhus are eaten by susceptible individuals; w, Probability of safe food that consumption contaminated by bacteria; k, Probability of flies that can spread typhus contaminate safe-consumption foods; u, Suspected Fly transfer rate to Infected; A, The rate of population growth flies and B, Food production rate Based on Figure 1, the rate of change in human number, flies and food in the transmission of typhus to the time is interpreted in the equation (1)-(8):
\[
\frac{dS_h}{dt} = \mu_h N_h + dI_h + pI_F - (a + \mu_h)S_h \\
\frac{dE_h}{dt} = aS_h - (b + \mu_h)E_h \\
\frac{dI_h}{dt} = bE_h - (c + d + \mu_h)I_h \\
\frac{dR_h}{dt} = cI_h - \mu_h R_h \\
\frac{dS_F}{dt} = B + kI_L - (w + \mu_F)S_F \\
\frac{dI_F}{dt} = wS_F - (p + \mu_F)I_F \\
\frac{dS_L}{dt} = A - (u + \mu_L)S_L \\
\frac{dI_L}{dt} = uS_L - (k + \mu_L)I_L
\]

(1) \hspace{1cm} (2) \hspace{1cm} (3) \hspace{1cm} (4) \hspace{1cm} (5) \hspace{1cm} (6) \hspace{1cm} (7) \hspace{1cm} (8)

3.2. Analysis of SEIR Model for Typhus disease

If Equations (1)-(8) are equals to zero, then obtained the equilibrium point will be formed two equilibrium points are the disease-free \((P_1)\) and endemic equilibrium \((P_2)\). Disease-free Equilibrium point is a condition when there is no disease in the population. The free equilibrium point of the infection of the typhus is assumed that there is no human population, flies and food contaminated with Salmonella thypi bacteria. The disease-free equilibrium point occurs in the human population if \(E_h = 0, I_h = 0, R_h = 0\), the population of flies \(I_L = 0\) and the food opportunity \(I_F = 0\), while the endemic equilibrium point occurs if the values of the variables are not equal to zero. Based on the equations (10)-(17) obtained the equilibrium point of the disease typhus and endemic typhus are:

**Disease-free equilibrium point:**

\[
P_1 = \left( \frac{\mu_h N_h}{(a + \mu_h)}, 0, 0, 0, 0, \frac{A}{(u + \mu_L)}, \frac{B}{(w + \mu_F)}, 0 \right)
\]

**Endemic equilibrium point** \(P_2 = (S_h^{**}, E_h^{**}, I_h^{**}, R_h^{**}, S_F^{**}, I_F^{**}, S_L^{**}, I_L^{**})\)

\[
S_h^{**} = \frac{(b + \mu_h)(c + d + \mu_h)(\mu_h N_h(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F) + Akpww + (u + \mu_L)(k + \mu_L)Bpw)}{((a + \mu_h)(b + \mu_h)(c + d + \mu_h) - abd)(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F)}
\]

\[
E_h^{**} = \frac{a(c + d + \mu_h)((\mu_h N_h(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F) + Akpww + (u + \mu_L)(k + \mu_L)Bpw)}{((a + \mu_h)(b + \mu_h)(c + d + \mu_h) - abd)(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F)}
\]

\[
I_h^{**} = \frac{ab((\mu_h N_h(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F) + Akpww + (u + \mu_L)(k + \mu_L)Bpw)}{((a + \mu_h)(b + \mu_h)(c + d + \mu_h) - abd)(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F)}
\]

\[
R_h^{**} = \frac{abc((\mu_h N_h(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F) + Akpww + (u + \mu_L)(k + \mu_L)Bpw)}{((a + \mu_h)(b + \mu_h)(c + d + \mu_h) - abd)(k + \mu_L)(u + \mu_L)(p + \mu_F)(w + \mu_F)}
\]

\[
S_F^{**} = \frac{(a + \mu_L)(k + \mu_L)BwA}{(u + \mu_L)(k + \mu_L)(w + \mu_F)} \quad I_F^{**} = \frac{Akpww + (u + \mu_L)(k + \mu_L)Bw}{(k + \mu_L)((u + \mu_L)(p + \mu_F)(w + \mu_F)} \quad S_L^{**} = \frac{A}{(u + \mu_L)} \quad \text{and} \quad I_L^{**} = \frac{A}{(u + \mu_L)(k + \mu_L)}
\]

Furthermore, the analysis of the equilibrium-point stability of the SEIR model on the spread of the typhus by determining the Jacobi matrix of the equation (1)-(8) with the linearization of using the Jacobian matrix as follows.
The value of a matrix $J$ by using Maple is a characteristic root equation (9):

$$M^2 = I,$$

The multiplicity of negative settlement roots of the $M = 0$ is less than or equal to the number of variations on the coefficient $M^2$, this means the equation (10) has a maximum of eight negative eigenvalues if shaped.

$$M^2 = I,$$

So the conditions $L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9$ and $L_9$ are $L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9 > 0$. Since the Equation (11) has eight eigen values negatively, then the SEIR model on the spread of the Typhus is locally asymptotically stable.

### 3.3. The Basic Reproduction Number of SEIR Model for Typhus Disease

Basic reproduction numbers of SEIR model spread of typhus disease are determined using equations that contain only infections. The approach used to determine basic reproductive numbers is determined using the next generation matrix $G$ defined:

$$G = FV^{-1}$$

By using equations (1)-(8) then obtained the value of the matrix $F$ and $V$

$$F = \begin{pmatrix}
0 & 0 & 0 & 0 \\
b & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix}
b + \mu_h & 0 & 0 & 0 \\
0 & c + d + \mu_h & 0 & 0 \\
0 & 0 & k + \mu_L & 0 \\
0 & 0 & 0 & p + \mu_F
\end{pmatrix}$$

So from the matrix $F$ and $V$, obtained matrix $G$ follow:

$$G = \begin{pmatrix}
0 & 0 & 0 & 0 \\
b & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

Thus obtained the basic reproductive number of SEIR model for typhus disease: $R_0 = \frac{b}{b + \mu_h}$ (12)

### 3.4. Simulation of SEIR Model for Typhus Disease in Makassar City

Model simulations use Matlab with variable values and the parameters used are presented in Table 1.
Table 1. Variable and parameter value of SEIR model for Typhus disease

| Variable | Value   | Source | Parameter | Value | Parameter | Value | Source |
|----------|---------|--------|-----------|-------|-----------|-------|--------|
| $S_h(0)$ | 0.9957  | [15]   | $N_h$     | 1     | D         | 0.3288| Assumption |
| $E_h(0)$ | 0.00216 | [15]   | $\mu_h$  | 0.9   | $P$       | 0.323 | Assumption |
| $I_h(0)$ | 0.00212 | [15]   | $\mu_I$  | 0.75  | $W$       | 0.2   | Assumption |
| $S_L(0)$ | 0.032   | [15]   | $\mu_F$  | 0.1667| $K$       | 0.01  | Assumption |
| $I_L(0)$ | 0.001   | [15]   | $a$       | 0.375 | $U$       | 0.083 | Assumption |
| $S_F(0)$ | 0.056   | [15]   | $b$       | 0.1428| $A$       | 0.85  | Assumption |
| $I_F(0)$ | 0.001   | [15]   | $c$       | 0.000002| $B$     | 0.07  | Assumption |

If the variable and parameter values in table 2 are substitute to Equations (1)-(8) are obtained the equilibrium point and eigen values of SEIR model Typhus disease in Makassar are: $S_h = 0.7338$, $E_h = 0.2639$, $I_h = 0.0307$, $R_h = 0.00000006$, $S_F = 0.1939$, $I_F = 0.0792$, $S_L = 1.0204$, $I_L = 0.1114$, and the eigen values of SEIR model for Typhus disease are: $\lambda_1 = -1.323$, $\lambda_2 = -0.900$, $\lambda_3 = -0.367$, $\lambda_4 = -0.489$, $\lambda_5 = -0.760$, $\lambda_6 = -0.833$, $\lambda_7 = -0.900$, and $\lambda_8 = -1.323$. Because all the values of the eigen are negative, then the SEIR model on the spread of Typhus in Makassar city is stable. Based on equations (21) obtained the basic reproductive number for Typhus disease in Makassar is: $R_0 = 0.137$. Because the value of the basic reproduction number $R_0 = 0.137 \leq 1$, then Typhus disease in Makassar city are at a stage not alarming, because a person who suffers typhus does not cause others to suffer from typhus.

3.5. Simulation Result of SEIR Model for Typhus Disease

The result of simulating the SEIR model on Typhus disease in Makassar city using data in Table 1 presented in Figure 2 and Figure 3, while the combination of all the variables that affect the Typhus disease is presented in Figure 2, with the x-axis being the time (month) and the y-axis is the predicted result. Figure 2 explains the prediction of human population Suspected and Exposed, the predictions of human population numbers Infected and Recovered, the number of suspected food populations and already contaminated bacteria, the population number of suspected flies and contaminated bacteria, and explains the relationship of all variables that affect the individual suffering typhus in Makassar city.

![Figure 2. Prediction population, food and flies of Suspected, Exposed, Infected and Recovered of SEIR Typhus](image-url)
thirteen. It is derived that the number of positive human population of the first seven months of typhus increases and is in a state of equilibrium after the seventh month. While the dynamics of human populations that heal from the disease typhus down drastically until the eighth month and are in a state of equilibrium. Based on Figure 3, the food opportunity that ranges from Salmonella thypi bacteria increases drastically until the seventeenth month is in a state of equilibrium in the seventeenth month upwards, whereas food probabilities contaminated by Salmonella Thypi are continuously rising to the next month. The chances of flies that are susceptible transmit Typhus of the disease drastically until the eighth month was in a state of equilibrium in the eighth month upwards, while the chances of transmission of a positive fly transmitted the disease typhus rise drastically up to the eleventh month and is in a state of equilibrium.

Mathematical modeling conducted by [4],[8-12] produces mathematical models and predictions of cases of dengue fever, tuberculosis and hepatitis, the model was analyzed using Lyapunov function, then the mathematical model of Typhoid disease was conducted by [6-7] producing models and analyses with the protection against infection and Saturated Incidence Rate approaches. This research resulted in SEIR model on Typhus disease by considering food and fly factors as a cause of Typhus disease. The analysis and simulation of the SEIR model explained that the number of Typhus sufferers in Makassar city has increased, but it is at an unwarming stage.

4. Conclusion

The conclusion of the research result are: the spread of the disease typhus can be presented in a mathematical model of SEIR by considering flies and food as a factor in the cause of typhus; Analysis results of SEIR model explained that Typhus disease in the city of Makassar is in a stable state, and based on the results of simulation obtained that the number of cases of Typhus in the city of Makassar increased every month, but based on the value of basic reproductive numbers obtained that the disease Typhus in the city of Makassar is at the stage can be controlled, but the government remains to take early preventive measures of Typhus disease.

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