External intermittency prediction using AMR solutions of RANS turbulence and transported PDF models

DA Olivieri¹, M Fairweather¹ and SAEG Falle²

¹ School of Process, Environmental and Materials Engineering, and ² School of Mathematics, University of Leeds, Leeds LS2 9JT, UK

E-mail: s.a.e.g.falle@leeds.ac.uk

Abstract. External intermittency in turbulent round jets is predicted using a Reynolds-averaged Navier-Stokes modelling approach coupled to solutions of the transported probability density function (pdf) equation for scalar variables. Solutions to the descriptive equations are obtained using a finite-volume method, combined with an adaptive mesh refinement algorithm, applied in both physical and compositional space. This method contrasts with conventional approaches to solving the transported pdf equation which generally employ Monte Carlo techniques. Intermittency-modified eddy viscosity and second-moment turbulence closures are used to accommodate the effects of intermittency on the flow field, with the influence of intermittency also included, through modifications to the mixing model, in the transported pdf equation. Predictions of the overall model are compared with experimental data on the velocity and scalar fields in a round jet, as well as against measurements of intermittency profiles and scalar pdfs in a number of flows, with good agreement obtained. For the cases considered, predictions based on the second-moment turbulence closure are clearly superior, although both turbulence models give realistic predictions of the bimodal scalar pdfs observed experimentally.

1. Introduction

Turbulent shear flows with free boundaries display an intermittent character where the flow rapidly alternates between turbulent and non-turbulent states. This form of intermittency is normally referred to as external intermittency to distinguish it from the internal form which concerns the variability of energy or scalar dissipation rates. External intermittency can be thought of as an indicator function that has a value of unity when the flow is turbulent and zero when it is non-turbulent, i.e. it represents the fraction of time during which a point is inside the turbulent fluid. Intermittency is important in many practical flows, with the intermittency behaviour of free shear flows often critically affecting generic processes such as mixing, combustion, emissions and aero-acoustics.

This work concerns an investigation into the modelling of external intermittency in turbulent round jet flows, chosen due to the availability of experimental data for use in comparisons, using solutions of the Reynolds-averaged Navier-Stokes (RANS) equations coupled to a transported probability density function (pdf) equation for scalar variables. Solutions are obtained using a novel approach (Olivieri et al. 2009, 2010) which couples a finite-volume numerical solution method with an adaptive mesh refinement (AMR) algorithm. The majority of turbulence models currently in use were derived for fully developed flows, and hence cannot be expected to predict accurately in free shear flows where the outer regions contain irrotational flow. The effect of intermittency on the turbulent flow field has...
have been accommodated in this work using intermittency-corrected (e.g. Cho & Chung 1992) eddy viscosity and second-moment turbulence closures. In addition, the effect of intermittency on the scalar pdf is accommodated via the mixing model embodied within the pdf transport equation, in contrast to earlier approaches (e.g. Byggstoyl & Kollmann 1981) which were based on conditional zone-averaged moments rather than the Reynolds-averaged moments used in this work. Predictions of the complete model are compared with experimental data available in the literature on the velocity and scalar fields, scalar pdfs and intermittency profiles observed in a number of turbulent round jets.

Earlier work (Olivieri et al. 2009, 2010) has demonstrated the application of an AMR finite-volume technique in both physical and compositional space, with the latter permitting solution of the transported pdf equation. Use of this approach in the present study again contrasts with more conventional approaches that have generally been based on the use of finite-volume methods in physical space, and Monte Carlo methods in compositional space. The latter work also demonstrated that, for small numbers of scalar variables, AMR provides improved accuracy, run times and ease of use over alternative Monte Carlo approaches.

2. Mathematical model

Physical space modelling was performed using the density-weighted averaged forms of the mass and momentum conservation equations for turbulent flow in cylindrical symmetry, closed using intermittency (γ) corrected turbulence models, and applying the thin shear layer and parabolic approximations for steady flow. The turbulence models employed were a k-ε-γ model due to Cho & Chung (1992), with standard modelling constants applied (Alvani & Fairweather 2008), and a Reynolds stress-γ model based on an extension of the Jones & Musonge (1988) second-moment closure, with the Reynolds stress and turbulence energy dissipation rate transport equations modified to incorporate intermittency effects using the approach of Savill (2002). The intermittency transport equation used with this closure was derived (Alvani & Fairweather 2008) on the basis of the expression of Dopazo (1977), with mass entrainment and bulk convective spatial transport terms based, respectively, on the approaches of Cho & Chung (1992) and Byggstoyl & Kollmann (1986). Standard modelling constants (Alvani & Fairweather 2008) were again used in conjunction with the model described, with further details of both turbulence models given in the latter reference. This closed equation set was solved in conjunction with a pdf transport equation for a passive scalar, in this case the mixture fraction. The molecular mixing term involved in the transported pdf equation, associated with compositional space modelling, has a number of approaches available to represent its effect, and in this work the linear mean square estimation (LMSE) method (Pope 1985) was employed, with the expression for LMSE mixing modified to take the effect of intermittency into account.

The novelty of the numerical scheme is found in the application of AMR to the finite-volume method, with solution of coupled physical ψ and compositional ω spaces on a cell by cell basis. This provides a more efficient grid at some increased cost of integration. Considering a thin region in two space dimensions, such as the shear layer of interest in this work, certain regions will require high resolution. The scheme employed uses a hierarchy of uniform grids, G0…G1, so that if the mesh spacing on G0 is (Δψ, Δω), then it is (Δψ/2n, Δω/2n) on Gn. Grids G0 and G1 cover the whole computational domain, but finer grids need only exist in regions that require high resolution. The grid hierarchy is used to generate an estimate of the relative error by comparing solutions on grids with different mesh spacing, and the grid then refines if this error exceeds a defined tolerance En, and de-refines if it is less than a second tolerance, Eπ. Refinement also occurs in the downstream direction z, so that if the step on G0 is Δz, then it is Δz/2n on Gn. The integration algorithm is recursive and is described in detail in Olivieri et al. (2009) for integration of grid Gn over time step Δt. In the approach used the integration procedure is identical to that of a time-like downstream step Δz. For reasons of numerical efficiency the physical and compositional spaces are refined separately since considering, for example, regions near the centre-line of a jet, in physical space these can involve very little variation with radial distance r, whilst the compositional space associated with each physical cell may have a probability close to a delta function. Such computational spaces therefore require a high degree
of refinement, in contrast to the associated physical space. To implement different levels of refinement in physical and compositional space, a complete composition space hierarchy is associated with each physical cell on every grid level. The number of levels of refinement of the composition space in a particular physical cell is then determined by the accuracy requirements of that particular cell. Further details may be found in Olivieri et al. (2009, 2010).

3. Results and discussion
To validate the modelling approach described a number of quantitative comparisons have been made with the results of experiments that involved the mixing of a single passive scalar, namely the mixture fraction, in a turbulent round jet. Figures 1-3 compare predictions with the experimental data obtained by Schefter et al. (1987) and Schefter & Dibble (2001) in their study of a dispersing propane jet. This case was computed using an initial grid consisting of 30 physical and 25 compositional space cells in the radial direction which extended 10 jet nozzle diameters, with 4 levels of adaption subsequently used in both physical and compositional space.

![Figure 1. Comparison of measured and predicted radial profiles of axial mean velocity, normal stresses and shear stress at various axial locations (● data, — Re stress-γ, --- k-ε-γ).](image1)

![Figure 2. Comparison of measured and predicted radial profiles of mean mixture fraction, r.m.s. mixture fraction fluctuations and intermittency at various axial locations (● data, — Re stress-γ, --- k-ε-γ).](image2)

Figures 1 and 2 demonstrate reasonable agreement between predictions and observations for the velocity and mixing fields in the propane jet. The results given in figure 1 for the mean velocity, and normal and shear stresses, are in reasonable accord with the data, and in line with previous results (Alvani & Fairweather 2008) derived using similar turbulence models coupled to a prescribed pdf approach. Predictions of the Reynolds stress-γ model are in general in better agreement with data than those derived on the basis of the eddy viscosity model, although both models under-predict the radial shear stress at all distances downstream. Results for the scalar field in figure 2 show that predictions derived from the coupled physical and compositional space modelling approach using the k-ε-γ and Reynolds stress-γ models give similar levels of agreement with the experimental data, although the Reynolds stress-γ model again gives superior performance in terms of predictions of the first and second moments. This is also the case for the intermittency results shown in figure 2, with the transition from turbulent to non-turbulent flow predicted accurately by the modified second-moment
closure both in terms of the rate of transition and its radial location. Overall, these results are again in line with those obtained previously (Alvani & Fairweather 2008), although the use of a transported pdf rather than the solution of transport equations for the mean mixture fraction and its variance as used in Alvani & Fairweather (2008) does result in improved predictions of the mixture fraction fluctuations, in particular.

Figure 3 shows predicted pdfs at \( z/d = 30 \) and various radial locations: \( r/d = (a) 0, (b) 2.0, (c) 2.3, (d) 2.6, (e) 2.9 \) and (f) 3.2 (● data, —— Re stress-γ, --- k-ε-γ).

Figure 4. Effect of intermittency on predicted scalar pdf at \( z/d = 20 \) and various radial locations: (a) effect included; (b) no effect; (c) and (d) show, at two scales, effect of varying \( C_D \).

The distribution of the pdf of the passive scalar is also given in figure 4 at \( z/d = 20 \) in the same jet for a number of radial locations, with the influence of incorporating intermittency effects in the pdf transport equation shown in the results of figure 4(a) and (b) in terms of predictions based on the \( k-ε-γ \) turbulence model. The predictions of figure 4(a) include the intermittency effect, whilst those of 4(b) do not. The former figure again shows the evolution (from right to left) of the pdf from approximately Gaussian to bi-modal with, ultimately, an intermittency spike at \( ω = 0 \) due to the increasingly intermittent nature of the flow with radial distance. In contrast, the results of figure 4(b), derived without the influence of intermittency, fail to capture the bi-modal distribution observed experimentally, and hence do not provide reliable predictions. In figure 4(c), and at an expanded scale in figure 4(d), the impact of varying the constant \( C_D \) is investigated for a particular pdf distribution taken from figure 4(a). \( C_D \) is a constant that reflects the ratio of the scalar to mechanical turbulent time scales in the intermittency modified LMSE mixing term used in this work. A value of \( C_D = 4.0 \) (solid line in figure 4(c, d)) is used in all the results given herein since this value was found to give the most accurate predictions in comparison with experimental data. As this value is reduced in figure 4(c, d), the variance in the predicted pdf naturally increases until at \( C_D = 0 \) (dot-dashed line) there are no longer any molecular mixing effects. Pope’s (1985) considerations of scalar dissipation indicate that it is not possible for \( C_D \) to be universally constant, with the latter work considering variations of this constant between 0.6 and 3.1. From these results, it is clear that the value of \( C_D \) has a significant
bearing on the accuracy with which the scalar second moments are determined when using the approach adopted in this work.

Lastly, figures 5 and 6 show, respectively, comparisons with intermittency measurements in jets gathered by Becker et al. (1967) and Wygnanski & Fiedler (1969). The Becker et al. (1967) case was computed using an initial grid consisting of 72 physical and 25 compositional space cells in the radial direction which extended 24 jet nozzle diameters, with 4 levels of adaption subsequently used in both physical and compositional space. The Wygnanski & Fiedler (1969) jet similarly employed 180 physical and 25 compositional cells radially, extending to 30 nozzle diameters, and again used 4 levels of adaption in physical and compositional space. As previously, predictions of these intermittency profiles are in reasonable agreement with observations, with results based on the Reynolds stress-\(\gamma\) model again showing better agreement with the data of Becker et al. (1967) in figure 5, and with that of Wygnanski & Fiedler (1969) in figure 6. As for the comparisons with the data of Schefer & Dibble (2001), predictions of the second-moment closure again accurately predict the rate of transition from turbulent to non-turbulent flow, as well as its spatial location. Some discrepancies are apparent, for example at \(z/d = 40\) in figure 6, but overall good agreement with data is obtained. This contrasts with results based on the \(k-\epsilon-\gamma\) approach which, although in good agreement with the data of Becker et al. (1967), tend to under-estimate the rate of the transition from turbulent to non-turbulent flow in the Wygnanski & Fiedler (1969) jet, with a noticeable tail in the predictions at large \(r/d\) values. These trends are also apparent in the predictions of the \(k-\epsilon-\gamma\) approach for the propane jet studied by Schefer & Dibble (2001). All these findings are again in line with those of earlier work (Alvani & Fairweather 2008), based on the use of the same turbulence models coupled to a prescribed pdf approach, where similar trends were noted for the Wygnanski & Fiedler (1976) and Schefer & Dibble (2001) jets. The use of a transported pdf in the present work does, however, result in the more accurate prediction of intermittency profiles, particularly by the Reynolds stress-\(\gamma\) based approach.

4. Conclusions
The conventional approach to solving the pdf transport equation is by using Monte Carlo techniques, frequently coupled to finite-volume methods in physical space. This paper has considered an alternative approach using a novel solution of the RANS-based fluid flow equations together with the
transported pdf equation for scalar variables based on the use of a finite-volume method coupled to an adaptive mesh refinement algorithm applied in both physical and compositional space. The overall model has been applied to the prediction of external intermittency, and its effects, in turbulent round jets using $k$-$\varepsilon$ and Reynolds stress-$\gamma$ turbulence models, with the influence of intermittency also accommodated within the transported pdf equation through modifications to the molecular mixing model employed. Comparisons with experimental data on the velocity and scalar fields in a round jet, as well as against measurements of intermittency profiles in a number of flows, have demonstrated the high degree of accuracy this approach can provide, subject to the use of an appropriate turbulence model, with the transported pdf method also giving realistic predictions of the bi-modal features of measured pdfs. This is particularly the case when compared with models based on the solution of transport equations for the mean mixture fraction and its variance, coupled to a prescribed pdf, as used in Alvani & Fairweather (2008).

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