Application of the Limit equilibrium method to the headcut migration of levee breaching

Zhenzhen Liu1,2*, Tian Li1, Yuxi Ding2 and Xianlei Zhang3

1 School of Civil Engineering, Zhengzhou University, Zhengzhou, 450001, China
2 Henan Electric Power Survey & Design Institute Corporation, Zhengzhou, 450001, China
3 School of water Conservancy, North China University of Water Resources and Electric Power, Zhengzhou 450045, China

*Corresponding author’s e-mail: liuzhenzhen-heny@powerchina.cn

Abstract. The headcut migration describes the physical process of the levee side slope retreat and the headcut length governs the breach widening. As a typical mode of slope stability due to soil tensile failure, the limit equilibrium method was used for the calculation of the headcut length. By considering the soil tensile stress state, the Hoek-Brown failure criterion provides a reasonable description of the levee material response. Then, the headcut length was calculated by using LEM in Slope/W. The comparisons with a large scale test show that the calculated headcut length by using LEM in Slope/W has good agreements with the measured data.

1. Introduction

The levee breaching can have serious consequence in the protected areas. Modeling the growth of a breach and the flow discharge hydrograph through the breach due to the levee breaching are the first step in mapping the resulting inundation in a floodplain. Once a breach is formed, the levee is separated into two banks, the ends of which are the two side slopes. During this process, the levee failure is not complete because the flow discharge through the breach could still remove soil blocks and particles and enlarge the breach. Relatively large flow volumes result from the breach into the protected areas [1,2,3]. Because of the erosion at the foot of the levee, an overhanging soil block (called headcut) was formed that eventually collapsed. The breach of levees became wider as the episodic failure of headcut [4,5]. By observations of the laboratory and field tests [6,7,8], it is the headcut migration of the breach side slope that contributes to the breach widening. The ability to predict the headcut migration is therefore a fundamental requirement for breach modeling and flood risk management.

The headcut migration describes the physical process of the levee side slope retreat and the headcut length governs the widening of the breach. As removal of soil from the toe of the headcut effectively removes physical support for the upper part, the headcut fails on the plane normal to direction of tensile stress. This process is a typical mode of tensile failure. In this study, the length of the headcut was calculated by applying the limit equilibrium method according to the software Slope/W. The choice of the failure criterion for the levee material and the soil tensile failure was also discussed. Then, the calculation of the length of the headcut was verified by a large length test.
2. Limit equilibrium method

In the case of headcut migration during the levee breaching, the location of the headcut is more important in terms of the consequences of failure. This order of importance occurs because the generated amount of failure mass is generally removed over a short period of discharge flow and because the scale of the headcut determines the width of the breach and the corresponding discharge flow. The LEM presents the basic principles for safely designing natural earth and rock slopes. The LEM is the most common approach for analyzing slope stability and can identify potential failure mechanisms and derive safety factors for particular geotechnical situations. The LEM is an appropriate choice for assessing the stability of side slopes during the levee breaching and considers the geometry, soil properties and groundwater conditions of the side slopes. Thus, the LEM was selected to calculate the tensile strength based headcut migration.

2.1. Mohr-Coulomb failure criterion.

The shear strength along sliding surface is determined by the soil failure criterion. The Mohr-Coulomb failure criterion is widely used to describe the strength of the soil mass. The MC theory can be thought of as a set of linear equations in principal stress space that represent a shear failure surface for an isotropic material and are not affected by the intermediate principal stress $\sigma_2$. Thus, the MC theory can be written in terms of normal stress $\sigma$ and shear stress $\tau$ as follows:

$$\tau \leq c + \sigma \tan \phi$$

Where, $\tau$ is cohesion, and $\phi$ is the internal friction angle.

A representation of the MC failure envelope on a Mohr diagram is shown in figure 1 [9]. The dashed circle with a diameter of $R_c$ represents the state of stress for uniaxial compression testing, with $R_c =$ uniaxial compressive strength. The dashed circle with a diameter of $R_t$ represents the tensile strength predicted from the MC failure criterion. However, this value is generally much larger than that observed from actual testing. For example, with $\phi = 30^\circ$ and $R_t / R_c = 3$, the tensile strength, $R_t$, is approximately 1/10 of the $R_c$ based on experiments. This discrepancy results from the specimen failing in a tensile mode rather than in the shear failure mode predicted by the MC failure criterion. A non-linear failure envelope is necessary to describe the behavior of material while considering the tensile strength. It is often convenient to represent the MC failure criterion in terms of principal stresses (the major principal stress $\sigma_1$ and the minor principal stress $\sigma_3$) as follows:

![Figure 1. Mohr-Coulomb failure envelope on a Mohr diagram.](image-url)
Levees are constructed of materials that usually consist of soil, but may also include moraine, rock, or crushed paving materials. It is important to note that a non-linear failure envelope is needed to describe the behavior exhibited by embankment materials over the entire stress state range.

2.2. Hoek-Brown failure criterion.
The Hoek-Brown failure criterion promotes the nonlinear envelop of the Mohr-Coulomb failure criterion. This model evolved in stages over a few decades and has been updated several times in response to its uses [10,11,12]. A generalized form of the criterion is shown in equation (6).

\begin{align}
\sigma_i &\leq \sigma_j + R_c \left( m_b \frac{\sigma_j}{R_t} + s \right)^a \\
\end{align}

Where, \( \sigma_i \) and \( \sigma_j \) are major and minor principal stresses at failure, and \( R_c \) and \( R_t \) are the compressive and tractive strength of the intact rock, and \( m_b \), \( s \) and \( a \) are three parameters estimated by observations of in situ experiments. All of the parameters were obtained according to the Geological Strength Index (GSI) as follows:

\begin{align}
m_b &= m_i \exp \left( \frac{GSI - 100}{28} \right) \\
&s = \exp \left( \frac{GSI - 100}{9} \right) \\
&a = \frac{1}{2} + \frac{1}{6} \left( e^{-GSI/15} - e^{-20/3} \right)
\end{align}

Considering the large variations in the materials used in embankment dams and levees, it is acceptable to assume the presence of intact rock (GSI=100). Therefore, \( s \) is equal to 1, \( a \) is equal to 0.5, and \( m_b \) is equal to \( m_i \), \( m_i \), which is the ratio of the compressive strength to the tractive strength of the intact rock, \( m_i = R_c / R_t \). For levee materials, it is possible to choose \( m_i \) with an order of 10 as the ratio of compressive strength to tractive strength. Equation (6) can be rewritten as equation (10). In addition, we can convert the above equation to a relationship between the shear strength, \( \tau \), and normal strength, \( \sigma \).

\begin{align}
\sigma_i &\leq \sigma_j + m_i \sqrt{R_c \sigma_j + R_t^2} \\
\tau &= \frac{1}{2} m_i \sqrt{R_c (\sigma - \tau) + R_t^2} \\
\tau &= \frac{1}{2} (\sigma_i - \sigma_j) \\
\sigma &= \frac{1}{2} (\sigma_i + \sigma_j)
\end{align}
2.3. Comparison of MC and Hoke-Brown failure criterion.

The soil tensile strength predicted by the Mohr-Coulomb failure criterion is larger than that obtained experimentally. Thus, the Mohr-Coulomb failure criterion does not consider the soil tensile strength. However, the soil tensile strength plays an important role in the headcut migration. Thus, the Hoek-Brown failure criterion is introduced here under the material conditions imposed by assuming an intact rock with an internal friction angle of 35 degree and $m_i = 10$. Thus, the Mohr-Coulomb (equation (2)) and Hoke-Brown (equation (10)) failure criteria can be converted using equations (14) and (15) as follows:

$$\frac{\sigma_x}{R_x} \leq \frac{3.7 \sigma_3}{R_x} + 1$$

$$\frac{\sigma_x}{R_x} \leq \frac{\sigma_3}{R_x} + \sqrt{\frac{10 \sigma_3}{R_x} + 1}$$

Figure 2 shows failure envelopments of a special material of levee (intact rock, an internal friction angle of 35 degree and $m_i = R_x / R_y = 10$) over a range of $0 < \sigma_3 < R_x$. By considering a range of $0 < \sigma_3 < R_x$, a nonlinear failure curve (Hoke-Brown) fits well the stress states than a straight line (Mohr-Coulomb). By considering soil tensile stress state ($-R_y < \sigma_3 < 0$), the Hoke-Brown failure criterion provides a reasonable description of material response.

Figure 3. Definitions and simplification of the geometry of a levee, an eroded notch and a foundation with Slope/W.

3. Model created in Slope/W

Slope/W is a well-known slope stability software for soil slopes and can be used to analyze headcut stability problem using LEM. The definition and simplification of the geometry of a levee side slope, an eroded notch and a foundation are shown in figure 3.

3.1. Soil properties input

Three different material properties shall be given for the levee side slope, the eroded notch and the foundation, respectively. To model the eroded notch at the foot of the headcut, the region of the eroded notch is treated specially with very low soil strength to transform the water pressure to the edge of the headcut under the surface. The parameters of the Hoek-Brown failure criterion that were used to represent the levee soil strength are as followings: $m_b = 10, a = 0.5, s = 1$. The Hoek and Brown model is a nonlinear shear strength model that accounts for the soil tensile strength. Considering the levee material used here, it is important to use this failure criterion. The soil strength properties of the
foundation are as followings: $\gamma = 0.1kN/m^3$, $\varphi = 0^\circ$, and the strength model is Mohr-Coulomb. The soil strength properties of the eroded notch are as followings: $\gamma = 18kN/m^3$, $\varphi = 35^\circ$, and the strength model is Mohr-Coulomb.

3.2. Determination of the headcut length
To simulate the erosion process, the lengths of the eroded notch were manually changed step by step. The increase in the length of the eroded notch during each step was determined by the safety factor ($F_s$). For each of the eroded notch geometries, the safety factors were calculated sequentially under the same hydraulic and geotechnical situations. Thus, a series of safety factors was obtained and recorded. The length of the headcut equals to the critical length of the eroded notch corresponding to the safety factor of 1 ($F_s = 1$) was obtained by interpolation method.

4. Comparisons with a large scale test
The calculated headcut length by using LEM in Slope/W was compared with a large-scale test of levee failure due to piping to determine the headcut length. It was one of the large-scale test performed during the FP5 IMPACT project (investigation of extreme flood processes and uncertainty) [13]. It was built with a height of 4.3m above the channel bottom, a width of 3m at the crest and 15.04m at the bottom. The inclination of the outer slope and inner slope was 1:1.4. It was built using 10% sand and 90% gravel (d50 = 7 mm). The properties of the levee materials were obtained by testing [13] with the soil dry density=2160 kg/m$^3$, the soil density=2341 kg/m$^3$, the porosity=0.244, the angle of friction=42°, the moisture content= 0.06, and the cohesion=20 kN/m$^2$. According to the observations of the test video tape, a breach was formed resulting from the collapse of the top of pipe. Two vertical side slopes were observed on the two banks of the test levee. Next, a crack appeared on the crest of the right bank of the test levee and extended downward, and the headcut on the right bank fell into the breach along the extended crack. The discharge flow was quickly washed away from the fallen soil block and the width of the breach was enlarged. Similarly, the same episodic collapse of the soil block occurred on the two side slopes of the breach.

The comparison of the calculated headcut length by using LEM and the large scale test data is shown in the figure 4. It can be seen that the calculated length has good agreements with the measured data.

![Figure 4. Comparison of the headcut length by using LEM and the measured data of the large scale test.](image)

5. Conclusions
In this study, the headcut migration of the side slope during the levee breach was studied. As a typical mode of slope stability due to soil tensile failure, the limit equilibrium method was used for the
calculation of the headcut length. The choice of the failure criterion for the levee material and the soil tensile failure was analyzed. By considering the soil tensile stress state, the Hoek-Brown failure criterion provides a reasonable description of the levee material response. Then, the headcut length was calculated by using LEM in Slope/W. The calculated results was compared with measured data from a large scale test. The results show that the calculated headcut length by using LEM in Slope/W has good agreements with the measured data.

References
[1] Feliciano Cestero, J.A., Imran, J., Chaudhry, M.H. (2015) Experimental investigation of the effects of soil properties on levee breach by overtopping. Journal of Hydraulic Engineering, 141(4): 04014085.
[2] Morris, M., Hassan, M., Kortenhaus, A., Visser, P. (2009) Breach processes: A state of the art review. Report No. T06-06-03, HR Wallingford, Oxfordshire, U.K.
[3] Froehlich, D. C. (2008) Embankment dam breach parameters and their uncertainties. Journal of Hydraulic Engineering, 134 (12), 1708-1721.
[4] Zhao, G. (2016) Breach Growth in Cohesive Embankments due to overtopping. Delft Academic Press, VSSD uitgeverij, the Netherlands.
[5] Wei, H., Yu, M., Wang, D., Li, Y. (2016) Overtopping breaching of river levees constructed with cohesive sediments. Natural Hazards and Earth system sciences, 16 (7), 1541-1551.
[6] Kakinuma, T., Shimizu, Y. (2014) Large-scale experiment and numerical modeling of a riverine levee breach. Journal of Hydraulic Engineering, 140(9): 04014039.
[7] Ashraf, M., Soliman, A.H., El-Ghorab, E., El Zawahry, A. (2018) Assessment of embankment dams breaching using large scale physical modeling and statistical methods. Water Science, 32(2): 362-379.
[8] Hunt, S.L., Hanson, G.J., Cook, K.R., Kadavy, K.C. (2005) Breach widening observations from earthen embankment tests. Transactions of the ASAE, 48(3): 1115-1120.
[9] Meyer, J.P, Labuz, J.F. (2013) Linear failure criteria with three principal stresses. International Journal of Rock Mechanics & Mining Sciences, 60:180-187.
[10] Hoek, E. The Hoek-Brown failure criterion—a 1988 update. (1988) In: 15th Canadian Rock Mech. Symp, Toronto, Dept. Civil Engineering, University of Toronto, 31-38.
[11] Hoek, E., Carranza-Torres, C., Corkum, B. (2002) Hoek-Brown failure criterion-2002 edition. In: Proceedings of Narms-Tac, 267-273.
[12] Eberhardt, E. (2012) The Hoek–Brown Failure Criterion. Rock mechanics and rock engineering, 1-8.
[13] Vaskinn, K.A., Lovoll, A., Hoeg, K. (2004) Physical modeling of breach formation: Large scale field tests. Dam safety 2004.