Micro-Doppler feature extraction under passive radar based on orthogonal frequency division multiplexing communication signal

Xiao-yu Qu\(^1\), Kai-ming Li\(^1\), Qun Zhang\(^1,2\), Bi-Shuai Liang\(^1\)

\(^1\)Institute of Information and Navigation, Air Force Engineering University, Xi’an 710077, People’s Republic of China
\(^2\)Key Laboratory for Information Science of Electromagnetic Waves (Ministry of Education), Fudan University, Shanghai 200433, People’s Republic of China

E-mail: qxywukawula@163.com

Abstract: Micro-Doppler (m-D) effect is a unique signature of radar targets with micro-motion, which provides a new technique for target recognition. Passive radar based on communication signal has a great significance in detection and recognition for micro-motion targets, low-altitude surveillance and control, national air defences etc. In this study, based on orthogonal frequency division multiplexing communication signal, the echo modelling and analysis of rotating target are operated, and the expression of the target m-D is deduced. Furthermore, the rotation frequency is extracted by sinusoidal frequency modulation Fourier-Bessel series. The feasibility of the proposed method is proved by the simulation results. The method could offer a reference to feature extraction of micro-motion targets under passive radar.

1 Introduction

As low-altitude area has been opened gradually, the security of strategic points has been under a great threat, which arouses impending demand for detection and recognition for low-altitude targets. Due to the advantages of passive radar, such as higher survivability and lower cost etc., the study of feature extraction and recognition based on passive radar has attracted increasing attention, and large amount of researches have been carried out.

Colone et al. studied WiFi-based passive bistatic radar and operated signal processing [1]. Wang et al. focused on WiMAX-based passive radar for range and Doppler detection of moving targets [2]. In [3–6], the orthogonal frequency division multiplexing (OFDM) communication signal as illuminator of opportunity has been applied in detection of moving targets. Xiao-qing et al. researched in interference suppression caused by OFDM-based passive radar [7].

According to the results reported in the above referenced papers, the validity and interference suppression of OFDM-based passive radar have already been studied. However, micro-Doppler (m-D) feature extraction of OFDM-based passive radar still remains to be solved. In this paper, the expression of the m-D feature is deduced for rotating targets under the condition that the radar and the base station are at the same place, and a new m-D feature extraction method of passive radar based on OFDM signal is proposed.

This paper is organised as follows. In Section 2, the echoes model is established and m-D feature is analysed. In Section 3, an m-D feature extraction method for OFDM-based passive radar is proposed. The sinusoidal frequency modulation Fourier-Bessel (SFMFB) series is applied to estimate the rotation frequency. The results of the proposed method are presented in Section 4. Finally, in Section 5, the conclusions are drawn.

2 Echoes modelling and m-D effect analysis

Under common circumstances, a rotor consists of a mast, a hub and rotor blades. Usually, the model of passive radar is as Fig. 1 shown. It can be considered as a bistatic radar model. Utilising the scattering centre model, after equivalent processing used in bistatic radar, the rotor model with two blades is established as shown in Fig. 2. The radar is located at the same place as the base station. The coordinate system \((U, V, W)\) is the radar coordinate system and the radar is stationary at the origin \(O\). At the initial time, the reference coordinate system \((X, Y, Z)\) is parallel to radar coordinate system, which is located at the distance \(R\). The position of its origin \(O\) is \((X_0, Y_0, Z_0)\) in radar coordinate system. The target local coordinate system is \((x, y, z)\), and its origin \(O\) is the same as the reference coordinate system. The model is established that the two rotor blades are both rotating about point \(O\) with a rod of length \(L\). There are three scatterer points in the model. Among which, one is shaft scatterer \(O\), the other two are wingtip scatterers \(A\) and \(B\). Due to the symmetry of \(A\) and \(B\), \(A\) is analysed as an example. The target is translated with a velocity of \(v\), and is rotating around the \(Z\)-axis with angular velocity \(\omega = (\omega_x, \omega_y, \omega_z)\).

Supposing that the transmitted signal is OFDM communication signal, so the received echoes can be expressed as

\[
s(t_1, t_2, k) = \sigma \sum_{n=0}^{N-1} a_n(A_{n0} + jA_{n1}) \cdot \exp\left[j2\pi f_s\left(t_2 - \frac{2r(t_n)}{c}\right)\right] \cdot \text{rect}\left[\frac{t_2 - kT_p - 2r(t_n)/c}{T_p} - \frac{1}{2}\right],
\]

where \(t_n, t_2\) are slow time and fast time, respectively. \(\sigma\) is the scattering coefficient, \(N\) is the number of carrier. \(a_n\) is defined as weighting of the \(n\)th carrier. \(A_{n0}\) is regarded as real part modulation coefficient of the \(n\)th carrier on the complex plane, and \(A_{n1}\) is imaginary part modulation coefficient of the \(n\)th carrier and \(f_s = f_s + n\Delta f\) is the centre frequency of the \(n\)th carrier and \(f_s\) is the carrier frequency. \(T_p\) is the...
time of a code element and \( r(t) \) denotes the instantaneous slant range between the radar and scatterer \( A \) at \( t \) instant.

By adding up all scatterer echoes, it leads to the target echoes

\[
s(t, t_m, k) = \sum_{n=0}^{N-1} a_n(A_{tc} + jA_{im})
\]

\[
\cdot \exp\left[j2\pi f_c t + \frac{2r(t_m)}{c}\right]
\]

\[
\cdot \text{rect}\left(\frac{t - kT_p - 2r(t_m)/c}{T_p} - \frac{1}{2}\right)
\]

where \( M \) is the number of the target scatterers.

Considering \( O \) as the reference point, thus the reference signal can be expressed as

\[
s_{ref}(t, t_m, k) = \sum_{n=0}^{N-1} a_n(A_{tc} + jA_{im})
\]

\[
\cdot \exp\left[j2\pi f_c t + \frac{2r_{ref}(t_m)}{c}\right]
\]

\[
\cdot \text{rect}\left(\frac{t - kT_p - 2r_{ref}(t_m)/c}{T_p} - \frac{1}{2}\right)
\]

Supposing the target's translation motion in echo signal has been well compensated, then conducting matched filtering process, and the results can be expressed as

\[
s_d(t, t_m, k) = s_d^e(t, t_m, k)s_{ref}(t, t_m, k)
\]

Due to orthogonality of carrier, (4) can be substituted as

\[
s_{ref}(t, t_m, k) = s_{ref}(t, t_m, k)r_{ref}^m(t, t_m, k)
\]

\[
\cdot \exp\left[-j2\pi f_n \cdot \frac{2r_d(t_m)}{c}\right]
\]

\[
\cdot \text{rect}\left(\frac{t - kT_p - 2r_d(t_m)/c}{T_p} - \frac{1}{2}\right)
\]

where \( r_d(t_m) \) is the reference signal on \( n \)th carrier. \( s_{ref}(t, t_m, k) \) is the reference signal on \( n \)th carrier \( R_{rotating}(t) = I + j\omega \sin(\Omega t) + j\omega'\sin(\Omega' t(1 - \cos(\Omega t)) \) is rotating matrix. 

\[
r_{d}(t_m) = r(t_m) - r_{ref}(t_m) \approx [R_{rotating}(t_m)r_{OA}]^T n
\]

and \( n = ||\omega'||. \) \( r_{OA} \) refers to the initial position vector of \( A \) in the reference coordinate system, \( n \) and is the unit vector in the direction of radar line of sight.

Conducting Fourier transform (FT) with respect to fast time \( t \), compensating residual video phase, and we can obtain

\[
S_d(f, t_m) = \sigma_d(A_{tc} + A_{im})T_p\sin(c/T_p)\cdot \exp\left(-\frac{j4\pi}{c}f_p r_d(t_m)\right)
\]

(6)

where \( \sin(c(x)) = \sin(\pi x)/\pi x \).

The phase term is one degree term of \( r_d(t_m) \). If the target is moving, \( r_d(t_m) \) corresponding to each echo of pulse is changing. As a result of this, the phase term becomes a changing function on \( r_d(t_m) \).

In line with the geometry shown in Fig. 1, \( r_d(t_m) \) can be rewritten as

\[
r_d(t_m) = [R_{rotating}(t_m)r_{OA}]^T n
\]

\[
= r_{OC} + L\cos(\Omega t_m + \theta)\sin\left(\frac{\pi}{2} - \alpha\right)
\]

(7)

where \( r_{OC} \) is the distance between point \( O \) and \( C \). \( \theta \) is the initial phase, and \( \alpha \) refers to radar pitch angle.

From (7) on the range-slow time plane, the value of the rotating scattering point, which induces m-D curves, in range profile presents such that it changes as cosine form as time goes by.

3 m-D feature extraction based on SFMFB series

In accordance with the analysis in Section 2, the m-D curves of the target change as cosine form. Resulting from complex frequency modulation of sinusoidal frequency modulated signals, it is difficult to extract frequency using FT or fast FT. In order to extract the rotation frequency of the target, SFMFB transform is applied in [8].

The definition of SFMFB transform is as follows:

\[
S_d(\omega) = \text{SFMFBT}[s(t)] = \int_0^\infty j\ln[s(t)] f_{d}(\omega t) dt
\]

(8)

where \( f_{d}(\omega t) \) is the \( n \)th Bessel function.

Thus, over the finite time interval \( (0, T) \), the phase term of a signal can be expressed as a weighted sum of a finite number of Bessel functions

\[
s(t) = \int_{t_0}^{\infty} \sum_{n=1}^{N} C_n J_n(\omega t)dt
\]

(9)

where \( C_m \) is the \( n \)th SFMFB coefficient, \( \omega_0 \) is the \( n \)th positive root of the \( n \)th Bessel function in ascending order, and \( k \) is the resolution of Bessel function basis. The \( n \)th \( k \)-SFMFB coefficient \( C_n \) of the signal over the finite time interval \( (0, T) \) is computed as

\[
C_m = \frac{2}{[kT]_{j\omega_0}} \int_0^T j\ln[s(t)] f_{d}(\omega t)dt
\]

(10)

Due to the magnitude property of SFMFB series, the frequency can estimated as

\[
f_{max} = \frac{j\lambda_{max}}{2kT}
\]

(11)

where \( \lambda_{max} = \arg\max_\lambda \{\text{Re}(C_m)\} \) is the order of the SFMFB coefficient with the maximal magnitude of the real part.

Suppose the estimation error equals the absolute value of the difference between the estimated frequency and the real frequency

\[
f_e = |f_{max} - f|
\]

(12)
Unfortunately, as the phase measurement are only possible in \((-\pi, \pi)\), when the modulation index and the signal sequence are large, the phase shift ambiguity occurs when the phase shift is more than \(2\pi\) when sampling. A solution of the phase revising function is

\[
\text{phar}(t) = \text{pha}(n) - 2\pi, \quad \text{pha}(n) - \text{pha}(n - 1) > \pi \\
\text{pha}(n) + 2\pi, \quad \text{pha}(n - 1) - \text{pha}(n) > \pi, \quad i \geq 2, \quad (13)
\]

where \(\text{pha}(n) = \text{Im}[\ln x(n)]\) and \(\text{pha}(n)\) is the difference between the phase shift and the measurements.

Introducing SFMFB series so as to estimate the m-D frequency, the flowchart of the proposed method is shown in Fig. 3. The process can be described as follows:

**Step 1:** Reconstruct the time-frequency image of the target to see m-D curves of the target.

**Step 2:** Calculate SFMFB series coefficient \(C_m\).

**Step 3:** Calculate m-D rotation frequency according to (11).

**Step 4:** Calculate the absolute error according to (12).

### Table 1 Target parameters

| Scattering point | Scattering coefficient, \(\sigma\) | Rotation frequency, \(\omega/2\pi\), Hz | Initial position, m |
|------------------|----------------------------------|--------------------------------------|---------------------|
| A                | 1                                | 8                                    | (2,0,0)             |
| O                | 0.2                              | 0                                    | (0,0,0)             |
| B                | 1                                | 8                                    | (-2,0,0)            |

### Fig. 3 Flowchart of the proposed method

### Fig. 4 Target model

4 Stimulation and analysis

According to the China Mobile Company parameters in the fourth generation communication, simulation experiments of OFDM signal are conducted. The carrier frequency \(f_c\) is 2.6 MHz and the carrier frequency interval \(\Delta f\) is 15 kHz. The number of carrier \(N\) is 1096 and \(PRF = 10,000\) Hz. There is a two-blade rotor target in sensor area and its radial distance 5000 m. With the assumption that the target only has the rotating velocity around Z-axis, the target parameters are shown in Table 1.

The target model is shown in Fig. 4. Fig. 5 shows the range-slow-time image of the echoes, it can be seen that the echoes of target are concentrated in several range cells, and the m-D signature is not obvious. The result after time frequency analysis is shown as Fig. 6, in which the waveform is shown as the sinusoidal curves, the same as the former analysis.

The rotation frequency is extracted by the proposed method and Hough transform (HT) [9]. The estimation results are shown in Tables 2 and 3. It can be seen that, in contrast, the accuracy of parameter extraction of the proposed method is much higher than that of HT.

The extraction accuracy of the proposed method and HT under different signal-to-noise ratio (SNR) is shown in Fig. 7. It can be
seen that under low SNR, the extraction accuracy of the proposed method is lower than HT. When SNR is higher, the proposed method has even smaller absolute error than HT. This is because the proposed method does not need to set a threshold. It can search and renew the maximum of parameters automatically. However, while searching the maximum parameters, HT needs to set a threshold and is difficult to determining an appropriate threshold, which is dependent on the empirical value. Thus, the extraction result of rotation frequency is more precise by the proposed method.

5 Conclusion

This paper has proposed an m-D feature extraction method based on SFMFB series under OFDM-based passive radar. It can accurately extract the m-D frequency of the targets, which can offer reference for recognition of micro-motion target. Nonetheless, in this paper, only the rotation frequency is extracted, other features, e.g. initial phase and rotation radius will be studied in further research, which could develop a new thought for detection and recognition of unmanned aerial vehicles, helicopters and propeller-driven aircrafts and apply in the surveillance and control in low altitude area.

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Fig. 7 Extraction accuracy of two methods under different SNR

7 References

[1] Colone, F., Falcone, P., Bongianni, C., et al.: ‘Wifi-based passive bistatic radar: data processing schemes and experimental results’, IEEE Trans. Aerosp. Electron. Syst., 2012, 48, (2), pp. 1061–1079

[2] Wang, Q., Lu, Y., Hou, C.: ‘An experimental WiMAX based passive radar study’, 2009 Asia Pacific Microwave Conference. IEEE, Kyoto, Japan, 2009, pp. 1204–1207

[3] Garmatyuk, D., Schuerger, J., Kauffman, K., et al.: ‘Wideband OFDM system for radar and communications’. IEEE National Radar Conf. – Proc., Pasadena, CA, USA, 2009

[4] Paichard, Y.: ‘Orthogonal multicarrier phased coded signal for netted radar systems’. Int. Waveform Diversity and Design Conf. Proc., WDD, Kissimmee, FL, USA, 2009, pp. 234–236

[5] Paichard, Y., Castelli, J.C., Dreuillet, P., et al.: ‘HYCAM: A RCS measurement and analysis system for time-varying targets’. Instrumentation and Measurement Technology Conf., Sorrento, Italy, 2006, pp. 921–925

[6] Strum, C., Zwick, T., Wiesbeck, W.: ‘An OFDM system concept for joint radar and communications operations’. IEEE Vehicular Technology Conf., Barcelona, Spain, 2009, pp. 1–5

[7] Xiao-qi, Y., Kai, H., Weidong, J., et al.: ‘A passive radar system for detecting UAV based on the OFDM communication signal’. 2016 Progress In Electromagnetic Research Symp., Shanghai, China, 8–11 August 2016, pp. 2757–2762

[8] Qi-fang, H., Qin, Z., Ying, L., et al.: ‘Sinusoidal frequency modulation Fourier-Bessel series for multicomponent SFM signal estimation and separation’, Math. Probl. Eng., 2017, 2017, pp. 1–14

[9] Qin, Z., Yeo, T.S., Tan, H.S., et al.: ‘Imaging of a moving target with rotating parts based on the Hough transform’, IEEE Trans. Geosci. Remote Sens., 2008, 46, (1), pp. 291–299