Computational approach to solving the problem of optimizing the supply of raw materials and components at the enterprise

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Abstract. The results of modelling the problem of supply management of an enterprise using discrete optimization tools are presented. The formulation of the optimization problem of supply management and the found method for its solution are presented. Since there may be cases when the number of variables in the problem is large enough, an algorithm was developed that uses the decomposition of the problem as a solution. A numerical example of the application of the decompositional algorithm for optimizing supplies and comparison of the results using the direct algorithm are given.

1. Introduction
This work is a refinement of research on the creation of a general formulation of the problem of managing the material flows of an enterprise and production [1, 2]. This article is devoted to a more detailed study of one of the parts of that general setting, namely, the problem of effective supply management in the enterprise.

A division of the enterprise is considered as an object of management, the main activity of which is the organization of efficient supply of the main production divisions. Thus, a separate local subsystem is considered in relation to the general management system of the planned economic activity of the enterprise. The main problems to be solved in this case: - selection of suppliers of goods, determination of the volume of purchases and delivery schedule for the entire assortment list, determined by the needs of production, the choice of transportation methods, and the volume of stocks. Such an object can be, for example, a subdivision of a mining or industrial enterprise responsible for the supply of materials, raw materials and equipment.

In matters of supply, one can most often find works devoted to supply chain management [3, 4, 5]. Also, many works are devoted to the construction of models for optimizing transport and logistics processes [6, 7, 8], which indicates the relevance of solving logistics problems using discrete optimization methods.

2. Substantial statement of the supply management problem
Let’s take into account the following features of the object under consideration. 1. Relatively low prices per unit of purchased raw materials, materials and equipment of each type. 2. Large volumes of supplies and the presence of several suppliers in different regions. In this regard, significant factors are: transport scheme, delivery and storage conditions for raw materials and components. 3. On the contrary, volume
discounts are not taken into account due to large volumes of supplies, raw materials are purchased at the lowest prices for each of the suppliers.

There is a set of suppliers of materials and components located in different regions. Delivery of goods can be carried out by rail, water and motor transport. Various options for road and rail transportation are possible (wagons, containers of various capacities and carrying capacities). Terms of transportation and payment for goods for different suppliers vary. The prices of materials and components, of approximately the same quality, are also different from different suppliers.

We will assume that all cargoes arrive at the warehouse, the capacity of which, in the general case, can vary from month to month, or is not at all a limiting restriction. In addition, in order to avoid excess stocks, we will introduce the concept of a storage penalty, charged once at the end of each month. The amount of the fine may exceed the amount of storage costs.

In addition to the parameters related to the above conditions (tariffs, customs payments, storage cost standards, etc.), the production plan (broken down by months) and the consumption rates of materials and components per unit of each type of finished product are used as input information. Financial constraints are accounted for indirectly through the production plan. In addition, we will assume that all production costs by months, including wages, are known.

Based on the listed initial data, it is possible to formulate the problem of managing the transportation and storage of stocks of material resources, setting goals and limiting conditions.

The main goal of solving the problem under consideration can be considered to minimize the total costs of transporting and storing materials and components to ensure the fulfillment of the finished product production plan. Subordinate goals are the selection of suppliers, determination of the volume of transportation of materials and components of each type, the volume of their storage in each month, as well as the choice of transportation methods from each of the selected suppliers.

In addition to the listed goals, there may be an additional task of ensuring the maximum level of quality of raw materials and components.

The limiting conditions are determined by the transport scheme and the conditions of storage and production, which are given above.

3. Formal statement of the supply optimization problem and algorithms for solving

Notation used:

\( t \) – month number in the planning period \( T \),

\( l \) – vehicle type index \( l \in L \),

\( i \) – supplier’s index \( i \in I \),

\( j \) – used materials (raw materials) index \( j \in J \),

\( x_{i,j}(t) \) – volume of supplies of the \( j \)–th material from the \( i \)–th supplier in month \( t \),

\( c_{i,j} \) – selling price \( j \)–th material from the \( i \)–th supplier,

\( q_{i,j} \) – quality assessment of the \( j \)–th material from the \( i \)–th supplier,

\( s_{i,l} \) – costs per unit of transport of the \( l \)–type when transporting goods from the \( i \)–th supplier,

\( y_{i,l}(t) \) – the number of units of vehicles of the \( l \)–th type used for the export of materials in the \( t \)–th month from the \( i \)–th supplier,

\( e_{i,l} \) – the capacity of a unit of the \( l \)–th type of vehicle when transporting the \( j \)–th material,

\( P_j(t) \) – the demand for the \( j \)–th material in the month \( t \) (determined on the basis of the production program),

\( a_j \) – the standard for the cost of warehouse space per unit of the \( j \)–th material,

\( E(t) \) – the capacity of the warehouse in the month \( t \),

\( O_j(t) \) – remainder of the \( j \)–th material in the warehouse in month \( t \).
\( R_j(t) \) – insurance reserve of material \( j \) in month \( t \),

d(\( t \)) – average price of a finished product unit in month \( t \),

\( h_j(t) \) – penalty factor for storing a unit of material \( j \) as a stock during month \( t \),

\( N(t) \) – wages and other costs of the logistics subsystem in the month \( t \).

Then the problem of optimal supply management can be written as follows:

\[
O_j(t-1) + \sum_{i\in I} x_{i,j}(t) \geq P_j(t) + R_j(t), \quad t = 1, T, \quad j \in J, \tag{1}
\]

\[
O_j(t-1) + \sum_{i\in I} x_{i,j}(t) - O_j(t) = P_j(t) + R_j(t), \quad t = 1, T, \quad j \in J, \tag{2}
\]

\[
O_j(t) \geq R_j(t), \quad t = 1, T, \quad j \in J, \tag{3}
\]

\[
\sum_{j\in J} a_j O_j(t-1) + \sum_{j\in J} a_j \sum_{i\in I} x_{i,j}(t) \leq E(t), \quad t = 1, T, \tag{4}
\]

\[
\sum_{j\in J} x_{i,j}(t) \leq \sum_{j\in J} \sum_{l\in L} e_{i,j} y_{i,l}(t), \quad t = 1, T, \quad i \in I, \tag{5}
\]

\[
x_{i,j}(t) \geq 0, \quad t = 1, T, \quad i \in I, \quad j \in J, \tag{6}
\]

\[
y_{i,l}(t) \geq 0, \quad \text{whole}, \quad t = 1, T, \quad i \in I, \quad l \in L, \tag{7}
\]

\[
Z = \sum_{t=1}^{T} \left( \sum_{i\in I} \sum_{j\in J} c_{i,j} x_{i,j}(t) + \sum_{i\in I} \sum_{l\in L} s_{i,l} y_{i,l}(t) + d(t) \sum_{j\in J} h_j(t) O_j(t) + N(t) \right) \rightarrow \min. \tag{8}
\]

\[
Q = \sum_{i\in I} \sum_{j\in J} q_{i,j} x_{i,j}(t) \rightarrow \max. \tag{9}
\]

Constraints (1) set the logical conditions for the excess of stock balances in the warehouse and the total volume of materials of each type delivered from all suppliers for all time intervals of the planning period over the demand and safety stocks. Constraints (2) determine stock balances in the current time interval (month \( t \)) based on constraints (1). (3) set the level of safety stocks in the warehouse in each month for each type of stock. Expression (4) sets limits on the storage capacity in each month. And, finally, (5) determines the logical conditions for the equivalence of the volume of transportation of all goods from each non-resident supplier in each month and the amount of goods transported from these suppliers by all modes of transport, provided that it is fully loaded.

The criterion of efficiency is the minimum cost of purchasing, transporting and storing materials over the entire planning period (8). Additionally, expression (9), which estimates the quality level of the purchased raw materials, can be used.

When assessing the potential complexity of this control problem, it is necessary to take into account that there are objects, for example, a production company developing a large oil and gas condensate field, the control problem for which (1) - (9) has a dimension that is multiply large (with an estimate for the number of continuous variables \( 10^6 \)).

Therefore, in order to guarantee efficiency, in this case, one has to use problem decomposition. This makes it possible to obtain a result acceptable in terms of accuracy by solving a sequence of subproblems with the number of integer variables several times less than in the original problem.
3.1. Decomposition algorithm for solving the supply optimization problem

The problem (2) - (8) formulated above has (as applied to real control objects) a relatively low estimate of the computational complexity, determined by the potential number of integer variables \( y_{i,j}(t) \), in its implementations. The number of continuous variables \( x_{i,j}(t) \) in this case is not critical, since any relaxations of problem (2) - (8) are linear (and algorithms for their solution are efficient). However, in the limiting case, the number of integer variables \( M(y_{i,j}(t)) = 300 \) may be too large, especially if the estimate for the number of continuous variables exceeds the above estimate \( M(x_{i,j}(t)) = 3000 \). In this case, it becomes necessary to apply the decomposition procedure for problem (2) - (8).

Consider an algorithm for solving problem (2) - (8) based on an incomplete decomposition scheme. Decomposition is carried out by integer variables \( y_{i,j}(t), t = 1, T, \ i \in I, \ l \in L \), by the index of the time interval \( t \) (time is discrete). We split the planning period \( T \) into a number of subintervals \( T^q, q = 1, Q \). In this context, \( q \) is the number of the subinterval, and \( t^q \in T^q - \) the index of the time on the subinterval \( q \).

The number of subintervals \( Q \) depends on the dimensions of the generated subproblems. The subsets \( T^q \) can overlap. Such overlays are advisable when choosing the best decomposition option.

Then the \( q \)-th subproblem formed by the incomplete decomposition algorithm, taking into account amendments in the notation, looks as follows.

\[
\sum_{i \in I} x_{i,j}(t^q) - O_j(t^q) = P_j(t^q) + R_j(t^q) - O_j^*(\arg\max(T^{q-1})), \quad t^q = \arg\min(T^q), \forall j \in J, \quad (10)
\]

\[
O_j(t^q - 1) + \sum_{i \in I} x_{i,j}(t^q) - O_j(t^q) = P_j(t^q) + R_j(t^q), \quad \forall t^q \in T^q, j \in J, \quad (11)
\]

\[
O_j(t^q) \geq R_j(t^q), \forall t^q \in T^q, j \in J, \quad (12)
\]

\[
\sum_{j \in J} a_j O_j(t^q - 1) + \sum_{j \in J} a_j \sum_{i \in I} x_{i,j}(t^q) \leq E(t^q), \forall t^q \in T^q, \quad (13)
\]

\[
\sum_{j \in J} x_{i,j}(t^q) - \sum_{j \in J} e_{i,j}(t^q) y_{i,j}(t^q) \leq 0, \forall t^q \in T^q, i \in I, \quad (14)
\]

\[
x_{i,j}(t^q) \geq 0, \forall t^q \in T^q, i \in I, j \in J, \quad (15)
\]

\[
y_{i,j}(t^q) \geq 0, \forall t^q \in T^q, i \in I, j \in J, \quad (16)
\]

\[
O_j(t^q) \geq 0, \forall t^q \in T^q, j \in J, \quad (17)
\]

\[
Z = \sum_{i \in I} \left( \sum_{j \in J} c_{i,j} x_{i,j}(t^q) + \sum_{i \in I} \sum_{l \in L} d_{i,l} y_{i,l}(t^q) + \sum_{j \in J} h_j(t^q) O_j(t^q) + N(t^q) \right) \rightarrow \min. \quad (18)
\]

In (10), in the case of intersection \( T^q, q = 1, Q \), the relation \( t^q = \arg\min(T^q) = \arg\max(T^{q-1}) + 1 \) must be fulfilled. Here \( O_j^*(\arg\max(T^{q-1})) \) is the optimal value of the remainders in the warehouse, obtained at the step preceding the current step \( q \).

Next, we present the algorithm we found for finding the optimal solution to the original problem (2) - (8), based on the decomposition with the separation of subproblems (10) - (18).

1. We set the initial partition of the period \( T \) into subintervals \( T^q, q = 1, Q \) depending on the number of integer variables \( y_{i,j}(t) \). We assign an initial value to the number of the step of the
algorithm \( q := 0 \). We use the initial data \( O_j(0) \), \( P_j(t) \), \( R_j(t) \), \( a_j \), \( E(t) \), \( e_{i,j} \), \( N(t) \), \( c_{i,j} \), \( s_{i,j} \), \( h_j(t) \), \( d(t) \), \( \forall t \in T^q \), \( q = \overline{l,Q} \). We increase the number of the step of the algorithm \( q := q + 1 \). We form subproblem (10) - (18) for the current step.

3. We solve subproblem (10) - (18) by any method of partial-integer linear programming. Remember the current solution \( x^*_i(t^q) \), \( y^*_i(t^q) \), \( O^*_j(t^q) \).

4. Checking for the end of the process. If \( q < Q \), go to item 2. Otherwise, the next item.

5. A solution to problem (2) - (8) is obtained, formed from local solutions \( x^*_i(t^q) \), \( y^*_i(t^q) \), \( O^*_j(t^q) \), \( \forall t^q \in T^q \), \( i \in I \), \( j \in J \), \( l \in L \). Calculation of the value of the criterion (18).

Stopping the algorithm.
This algorithm implements a greedy strategy for finding solutions.
This means that the final solution to the original problem (2) - (8) is composed of locally optimal solutions of subproblems (10) - (18).

4. Numerical example of application of the decompositional algorithm for optimizing supplies
Consider an example of solving problem (2) - (8) by the found algorithm in comparison with the direct algorithm with the following parameters. The number of names of supplied commodity items - 4, the number of suppliers - 4, time intervals - 8, modes of transport 3.

Example. Object: gas gathering system for an oil and gas condensate field. Pipes of 4 types are used as materials. All other information is summarized in tables 1-5.

### Table 1. Prices from suppliers and storage costs for 1 meter of pipes.

| Index | Supplier name and number i | Supplier 1 | Supplier 2 | Supplier 3 | Supplier 4 |
|-------|----------------------------|------------|------------|------------|------------|
|       | Diameter \( j \)          | Diameter \( j \) | Diameter \( j \) | Diameter \( j \) | Diameter \( j \) |
|       | 5” | 10” | 18” | 22” | 5” | 10” | 18” | 22” | 5” | 10” | 18” | 22” |
| Price for 1m (c.u.) \( c_{i,j} \) | 6 | 9 | 11.43 | 13.68 | 6 | 7.5 | 33 | 36 | 21 | 29.01 | 30.18 | 24 | 27 | 33 | 36 |
| Penalty for storage m \( h_j(t) \) | 4,45,4,5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 |

### Table 2. Demand for materials and insurance reserves.

| Month \( t \) | Production forecast plan (m.) | Warehouse capacity (m.) | The need for a pipe by months (m.) | Insurance reserve (m.) |
|--------------|-----------------------------|------------------------|-----------------------------------|------------------------|
|              | \( E(t) \)                  | \( P_j(t) \)           | \( R_j(t) \)                      | \( R_j(t) \)           |
|              | 5” | 10” | 18” | 22” | 5” | 10” | 18” | 22” |
| 1            | 4500 | 12000 | 0 | 0 | 2925 | 2925 | 0 | 0 | 225 | 225 |
| 2            | 3000 | 12000 | 0 | 0 | 2145 | 1755 | 0 | 0 | 165 | 135 |
| 3            | 3000 | 12000 | 195 | 195 | 1755 | 1755 | 15 | 15 | 135 | 135 |
| 4            | 6000 | 12000 | 0 | 390 | 3510 | 3900 | 0 | 30 | 270 | 300 |
| 5            | 7500 | 12000 | 0 | 975 | 3900 | 4875 | 0 | 75 | 300 | 375 |
| 6            | 4500 | 12000 | 0 | 292.5 | 3217.5 | 2340 | 0 | 22.5 | 247.5 | 180 |
| 7            | 6000 | 12000 | 390 | 390 | 3120 | 3900 | 30 | 30 | 240 | 300 |
| 8            | 9000 | 12000 | 819 | 936 | 5265 | 4680 | 63 | 72 | 405 | 360 |
Table 3. Transportation costs (c.u.).

| Supplier location $i$ | Costs $s_{i,l}$ by type of vehicle $l$ | Railway carriage 60t | Container 20t | Delivery van 20t |
|-----------------------|---------------------------------------|----------------------|---------------|------------------|
| Supplier 1            | 18000                                 | 7800                 | 10200         |                  |
| Supplier 2            | 6000                                  | 3900                 | 5100          |                  |
| Supplier 3            | ∞                                     | ∞                    | 0             |                  |
| capacity $e_{i,l}$    | 10500                                 | 3300                 | 3300          |                  |

Table 4. Warehouse and transport capacity (m.) depending on the type of material.

| Index                              | Diameter 5" | 10" | 18" | 22" |
|------------------------------------|-------------|-----|-----|-----|
| Conversion factor                  | 0.75        | 1.5 | 3   | 3   |
| Warehouse capacity (m.)            | 48000       | 24000 | 12000 | 12000 |
| Railway carriage capacity (m.)     | 42000       | 21000 | 10500 | 10500 |
| Container capacity (m.)            | 13200       | 6600 | 3300 | 3300 |
| Delivery van capacity (m.)         | 13200       | 6600 | 3300 | 3300 |

Table 5. Other indicators.

|                            | $d(t)$ | $N(t)$ | $h_j(t)$ | $E(t)$ |
|---------------------------|--------|--------|----------|--------|
| Price for 1 meter (c.u.)  | 90     | 8550   | 4.5      | 12000  |
| Monthly costs (c.u.)      |        |        |          |        |
| Penalty for storage 1 m. (c.u.) |      |        |          |        |
| Total storage capacity (m$^3$) |      |        |          |        |

The indicators from table 5 are unchanged for all values $t = 1, T$ and $j \in J$.

Direct solution of problem (2) - (8) on the example data gives the following results (table 6 – 8).

Table 6. Optimal pipe supply plan (m.) $x_{i,l}^*(t)$.

| Month $t$ | Supplier 1 | 5" | 10" | 18" | 22" | Supplier 2 | 5" | 10" | 18" | 22" | Supplier 3 | 5" | 10" | 18" | 22" | Supplier 4 | 5" | 10" | 18" | 22" |
|-----------|-------------|-----|-----|-----|-----|-------------|-----|-----|-----|-----|-------------|-----|-----|-----|-----|-------------|-----|-----|-----|-----|
|           | $x_{1,1}$   | $x_{1,2}$ | $x_{1,3}$ | $x_{1,4}$ | $x_{2,1}$ | $x_{2,2}$ | $x_{2,3}$ | $x_{2,4}$ | $x_{3,1}$ | $x_{3,2}$ | $x_{3,3}$ | $x_{3,4}$ | $x_{4,1}$ | $x_{4,2}$ | $x_{4,3}$ | $x_{4,4}$ |
| 1         | 0           | 0     | 5235 | 4995 | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| 2         | 0           | 0     | 0    | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| 3         | 0           | 0     | 1725 | 1575 | 210   | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| 4         | 0           | 0     | 4035 | 4065 | 405   | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| 5         | 0           | 0     | 5265 | 5235 | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| 6         | 0           | 0     | 1440 | 1860 | 0     | 240   | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| 7         | 0           | 0     | 3277.5 | 4020 | 405   | 397.5 | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| 8         | 0           | 0     | 5265 | 4740 | 852   | 978   | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |

Table 7. Optimal choice of transport $y_{i,l}^*(t)$ (pieces).

| Month | Supplier 1 | Supplier 3 |
|-------|------------|------------|
|       |            |            |
| 1     |            |            |
| 2     |            |            |
| 3     |            |            |
| 4     |            |            |
| 5     |            |            |
| 6     |            |            |
| 7     |            |            |
| 8     |            |            |
| $t$ | Railway carriage | Container | Delivery van | Railway carriage | Container | Delivery van |
|-----|------------------|-----------|-------------|------------------|-----------|-------------|
|     | $y_{11}$ | $y_{12}$ | $y_{13}$ | $y_{21}$ | $y_{22}$ | $y_{23}$ |
| 1   | 1      | 0      | 0        | 0      | 0      | 0        |
| 2   | 0      | 0      | 0        | 0      | 0      | 0        |
| 3   | 0      | 1      | 0        | 0      | 0      | 0        |
| 4   | 1      | 0      | 0        | 0      | 0      | 0        |
| 5   | 1      | 0      | 0        | 0      | 0      | 0        |
| 6   | 0      | 1      | 0        | 0      | 0      | 0        |
| 7   | 1      | 0      | 0        | 0      | 0      | 0        |
| 8   | 1      | 0      | 0        | 0      | 0      | 0        |

**Table 8.** Optimal remains (m.) of pipes $O^j_i(t)$ in the warehouse, taking into account the reserve.

| Month $t$ | Diameter pipe $j$ |
|-----------|-------------------|
|           | 1 | 2 | 3 | 4 |
|           | 5” | 10” | 18” | 22” |
| 1         | 0      | 0      | 2310 | 2070 |
| 2         | 0      | 0      | 165  | 315  |
| 3         | 15     | 15     | 135  | 135  |
| 4         | 15     | 30     | 660  | 300  |
| 5         | 15     | 75     | 2025 | 660  |
| 6         | 15     | 22.5   | 247.5 | 180 |
| 7         | 30     | 30     | 405  | 300  |
| 8         | 63     | 72     | 405  | 360  |

The following tables (table 9-12) contain the solution to the same problem, obtained by our algorithm. The algorithm is applied with a split: $T^1 = \{1,2,3,4,5\}, \ T^2 = \{5,6,7,8\}$

**Table 9.** Pipe supply plan (m.).

| Month $t$ | Supplier 1 $x_{11}$ | $x_{12}$ | $x_{13}$ | Supplier 2 $x_{31}$ | $x_{32}$ | $x_{33}$ | Supplier 3 $x_{51}$ | $x_{52}$ | $x_{53}$ | Supplier 4 $x_{71}$ | $x_{72}$ | $x_{73}$ | $x_{74}$ |
|-----------|---------------------|---------|---------|---------------------|---------|---------|---------------------|---------|---------|---------------------|---------|---------|---------|
| 1         | 0                   | 0       | 3450    | 0                   | 0       | 3150    | 0                   | 0       | 0       | 0                   | 0       | 0       | 0       |
| 2         | 0                   | 0       | 1635    | 0                   | 0       | 1665    | 0                   | 0       | 150     | 0                   | 0       | 0       | 0       |
| 3         | 0                   | 0       | 1545    | 0                   | 0       | 1755    | 210                 | 210     | 180     | 0                   | 0       | 0       | 0       |
| 4         | 0                   | 0       | 3645    | 0                   | 0       | 4140    | 0                   | 0       | 0       | 0                   | 0       | 0       | 0       |
| 5         | 0                   | 0       | 5265    | 0                   | 0       | 5325    | 0                   | 1020    | 0       | 0                   | 0       | 0       | 0       |
| 6         | 0                   | 0       | 1515    | 0                   | 0       | 1785    | 0                   | 240     | 315     | 0                   | 0       | 0       | 0       |
| 7         | 0                   | 0       | 3277.5  | 0                   | 0       | 4020    | 405                 | 397.5   | 0       | 0                   | 0       | 0       | 0       |
| 8         | 0                   | 0       | 5265    | 0                   | 0       | 4740    | 852                 | 978     | 0       | 0                   | 0       | 0       | 0       |

**Table 10.** Selection of transport (pieces).

| Month | Supplier 1 | Supplier 3 |
|-------|------------|------------|
|       |            |            |
Table 11. Remains (m.) of pipes in the warehouse, taking into account the reserve.

| Month | Material type $j$ | Railway carriage | Container | Delivery van carriage | Container | Delivery van |
|-------|-------------------|------------------|-----------|------------------------|-----------|--------------|
|       |                   | $y_{11}$ | $y_{12}$ | $y_{13}$ | $y_{31}$ | $y_{32}$ | $y_{33}$ |
| 1     |                   | 0      | 2       | 0       | 0       | 0       | 0       |
| 2     |                   | 0      | 1       | 0       | 0       | 0       | 0       |
| 3     |                   | 0      | 1       | 0       | 0       | 0       | 0       |
| 4     |                   | 1      | 0       | 0       | 0       | 0       | 0       |
| 5     |                   | 1      | 0       | 0       | 0       | 0       | 0       |
| 6     |                   | 0      | 1       | 0       | 0       | 0       | 0       |
| 7     |                   | 1      | 0       | 0       | 0       | 0       | 0       |
| 8     |                   | 1      | 0       | 0       | 0       | 0       | 0       |

Table 12. Values of performance criteria.

| Month | Costs (direct algorithm) | Cost (algorithm found) |
|-------|--------------------------|------------------------|
| 1     | 75842.55                 | 52783.5                |
| 2     | 9270                     | 32271.75               |
| 3     | 31499.25                 | 33614.25               |
| 4     | 62979.75                 | 61363.35               |
| 5     | 77193.75                 | 76721.25               |
| 6     | 31615.5                  | 35024.25               |
| 7     | 60319.74                 | 60319.74               |
| 8     | 73723.05                 | 73723.05               |
| Total | 422443.5                 | 425821.2               |

5. Conclusions
Comparing the final values of the criteria, we are convinced that the deviation from the optimal value was less than one percent. The performed computational experiments allow us to conclude that the average deviation of the efficiency criterion achieved when using the decomposition algorithm does not exceed 2.5% of the optimal value. This is a perfectly acceptable value for the tasks of the class under consideration.

Based on the results of the application, it can be concluded that the developed toolkit as a whole demonstrates high efficiency of application on real objects and has significant prospects for distribution and scaling.

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