Disturbance compensating model predictive control for warship heading control in missile firing mission

Heri Purnawan¹, Tahiyatul Asfihani², Dieky Adzkiya³ and Subchan⁴
Modeling and Simulation Systems Laboratory, Department of Mathematics
Faculty of Mathematics, Computing and Data Science, Institut Teknologi Sepuluh Nopember,
Surabaya 60111, Indonesia
E-mail: heripurnawan93@gmail.com¹, tasfihani@matematika.its.ac.id²,
dieky@matematika.its.ac.id³ and subchan@matematika.its.ac.id⁴

Abstract. One of warship missions is shooting and avoiding enemy attacks, which can be done by missile firing strategy. Due to missile firing, the movement of ship becomes unstable. To overcome this problem, we need a control system that can overcome the disturbance caused by the impact of the missile firing. So, in this paper, we propose a method called disturbance compensating MPC (DC-MPC) control system. This control is chosen, because DC-MPC is a robust method for measurable disturbance. Based on the result of simulation, DC-MPC can overcome disturbance on the system and make ship’s actual heading angle to follow the desired reference quickly which is about 10 s. After the system gets disturbance, controller is also able to adjust the value of the ship heading angle to original position.

1. Introduction
One of warship missions is shooting and avoiding enemy attacks, which can be done by missile firing strategy [1]. Based on the mission, impact disturbance modelling is also needed to find out how much force a ship receives when firing a missile. In this study, the model of impact disturbance when firing missile is based on the laws of physics. The magnitude of the impact force of the missile firing on the vessel is calculated using the Newton’s second law and Newton’s third law on translational motion, whereas to know the magnitude of its impact force moment is calculated using the Newton’s second law on the rotational motion.

The impact disturbance caused by missile firing is an external disturbance that will affect the stability on a ship when maneuvering in the ocean. To overcome these problems, it is necessary to use a robust control system in overcoming the impact disturbance of missile firing. The control method developed in the field of ship system control is Model Predictive Control (MPC) [2, 3, 4, 5, 6]. Basically, MPC works based on mathematical model of plant to predict the output process in the horizon time. The purpose of the control calculation is to minimize the objective function [7]. The following four main aspects of MPC make the design of this method attractive to practitioners and academics [8]. The first aspect is the design formulation using multivariable system [9] (multi input multi output). The second aspect is the ability of the method to handle the constraints on the system. The third aspect is the ability to perform an online optimization process. The fourth aspect is the simplicity of the design of control in dealing with complex problems.

Disturbance Compensating Model Predictive Control (DC-MPC) is a development of the
MPC method. The idea of developing the MPC method for measurable disturbances was first introduced by Li and Sun in a study of ship heading control [4]. In their research, it was shown that DC-MPC method is more robust than MPC in overcoming the measured disturbance of sea wave disturbance which is modeled through constant and sinusoidal wave equation. Based on the research, we implement the design of DC-MPC method to overcome the impact of missile firing on the ship, with aims to make the ship’s actual heading angle can follow the desired reference after the missile is fired.

2. 2 DOF Ship Model

Model used in this paper for designing ship heading control is Davidson and Schiff model [10]. This model considered 2 DOF, namely, sway velocity \( v \), yaw rate \( r \) dan one control input, namely, rudder angle \( \delta \). Mathematical model for 2 DOF Davidson and Schiff is showed by equation (1):

\[
M \dot{v} + N(u_0)\nu = b\delta
\]

where \( \nu = [v \ r]^T \) and matrices \( M, N(u_0) \), and \( b \) as in [10]. From (1) and by adding kinematics equation [10] \( \dot{\psi} = r \), we obtain the following state space equation:

\[
\dot{x} = A_c x + B_c u \tag{2}
\]

where

\[
x = [\nu \ \psi], \quad A_c = \begin{bmatrix}
-M^{-1}N(u_0) & 0 \\
0 & 1
\end{bmatrix}, \quad B_c = \begin{bmatrix}
M^{-1}b \\
0
\end{bmatrix} \quad \text{and} \quad u = \delta
\]

3. Modeling of Impact Force and Moment

The force caused by missile firing on a ship, will cause the ship to change motion. The magnitude of the missile impact force is equal to the magnitude of the missile force, but with different direction (counterclockwise) [11]. Based on the Newton’s second law, the magnitude of the \( F_t \) fire force depends on the mass and acceleration of the missile. Mathematically, it can be formulated as: \( F_t = m_M \cdot a_M \), where \( m_M \) is mass of a missile and \( a_M \) is acceleration of a missile. The illustration of the direction of the impact force from missile firing on the ship is shown in Figure 1.

![Figure 1. The illustration of the direction of the impact force from missile firing](image)

Figure 1 showed a condition when the ship’s current heading angle is \( \psi \) and a missile is fired with variation of the missile launcher heading angle relative to the ship’s surge \( \alpha_i \). The process to obtain impact force equation from missile firing on the ship is given by:

1. Determining the horizontal direction force \( F_{t_x} \) from \( F_t \)

\[
F_{t_x} = F_t \cos \theta_M
\]
Based on the Newton’s second law, impact force equation of missile firing can be written as follows: 

\[ F_{\text{sway}} = F_{k} \sin \alpha_i = m_M a_M \cos \theta_M \sin \alpha_i \]

where \( d_i = \frac{v_{\text{sl}} \cos \alpha}{\cos \theta} \) is the distance of the missile impact, so we obtained

\[ Y_i = -F_{\text{sway}} = -m_M a_M \cos \theta_M \sin \alpha_i, \quad -\frac{\pi}{2} < \alpha_i < \frac{\pi}{2} \]  

(3)

The impact force moment equation of missile firing is derived from the second Newton law on rotational motion, i.e., \( \tau = I \cdot \alpha \), where \( I \) is the inertia moment and \( \alpha \) is angle acceleration. Because \( \alpha = \frac{\pi}{2} \) and \( I = m d^2 \), so torsion equation can be written as:

\[ \tau = F \cdot d \]  

(4)

where \( d \) is arm of the moment or the distance of force \( F \) to the centre of gravity. Based on equation (4), the magnitude of yaw impact force moment of missile firing can be formulated as follows:

\[ N_i = -m_M a_M d_L \cos \theta_M \sin \alpha_i, \quad -\frac{\pi}{2} < \alpha_i < \frac{\pi}{2} \]  

(5)

where \( d_L \) is the distance of missile launcher to center of gravity on the ship. In equation (3) and (5), model still has a dimension, so it must be changed into nondimensional.

4. Design Control System

The purpose of DC-MPC controller is ship’s actual heading angle can follow the desired heading angle when there is impact disturbance of missile firing. The objective function of DC-MPC is defined by [7]:

\[ J = \min_{u(\cdot|k)} \sum_{j=1}^{N_p} \left[ \| y_d(k+j|k) - y(k+j|k) \|^2_Q + \| u(k+j-1|k) \|^2_R \right] \]  

(6)

subject to [4]

\[ x(k+j+1|k) = Ax(k+j|k) + Bu(k+j|k) \]

\[ x(k|k) = x(k) \]

\[ C_1 x(k+j+1|k) \leq D_1, \quad j = 0, 2, \ldots, N_p - 1 \]

\[ S u(k|k) \leq T - S \Delta u^* \]

\[ S u(k+j|k) \leq T, \quad j = 1, 2, \ldots, N_p - 1 \]

The disturbance at time step \( k-1 \), i.e., \( \tilde{w}(k-1) \), can be estimated by the following equation if the state and control are measurable [12]. The disturbance compensation \( \Delta u^* \) is calculated as in [4]. We define vector [8]: \( Y = [ y(k+1|k), \ldots, y(k+N_p|k) ]^T, U = [ u(k|k), \ldots, u(k+N_p-1|k) ]^T \), so the output of system \( Y \) can be written into matrix form as follows [8]:

\[ Y = F x(k|k) + \Phi U \]  

(7)

where

\[
\begin{bmatrix}
CA \\
CA^2 \\
CA^3 \\
\vdots \\
CA^{N_p}
\end{bmatrix}; \quad \Phi = 
\begin{bmatrix}
CB & 0 & 0 & \cdots & 0 \\
CAB & CB & 0 & \cdots & 0 \\
CA^2 B & CAB & CB & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
CA^{N_p-1} B & CA^{N_p-2} B & CA^{N_p-3} B & \cdots & CB
\end{bmatrix}
\]
so by using equation (7), the objective function in equation (6) can be written into:

\[
J = (R_n - Y)^T Q_c (R_n - Y) + U^T R U
\]

\[
= U^T (2\Phi^T Q_c [Fx(k[k] - R_n]) + \frac{1}{2} U^T (2[\Phi^T Q_c \Phi + R_c]) U
\]

(8)

where \( R_n = [y_d(k + 1)[k], y_d(k + 2)[k], \cdots, y_d(k + N_p)[k] ]^T \), \( Q_c = \text{diag}(Q) \), \( R_c = \text{diag}(R) \) and the constraints for the objective function (8) are as follow:

\[
S_1 U \leq T_1 \quad \text{and} \quad \Phi_1 U \leq D - Kx(k[k])
\]

where

\[
S_1 = \begin{bmatrix}
S & 0 & 0 & \cdots & 0 \\
0 & S & 0 & \cdots & 0 \\
0 & 0 & S & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \cdots & S
\end{bmatrix};
T_1 = \begin{bmatrix}
T - S^2 & 0 & 0 & \cdots & 0 \\
0 & T & 0 & \cdots & 0 \\
0 & 0 & T & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \cdots & T
\end{bmatrix};
K = \begin{bmatrix}
C_1 A & C_1 A^2 & \cdots & C_1 A^{N_p} \\
C_1 A B & C_1 A B & \cdots & C_1 A B \\
C_1 A^2 B & C_1 A^2 B & \cdots & C_1 A^2 B \\
\vdots & \vdots & \ddots & \ddots \\
C_1 A^{N_p-1} B & C_1 A^{N_p-1} B & \cdots & C_1 A B
\end{bmatrix}
\]

5. Simulation

In this simulation, we show performance of DC-MPC for the ship motion control when the ship gets disturbance in form impact of missile firing. The system used in designing DC-MPC is discrete time system [4], so model in equation (2) must be changed into discrete time system. By taking sampling time \( T_s = 0.5 \) s, then the discrete time system matrices are given by

\[
A = \begin{bmatrix}
0.7676 & -0.0206 & 0 \\
-1.0313 & 0.2020 & 0 \\
-0.3507 & 0.2446 & 1
\end{bmatrix};
B = \begin{bmatrix}
-0.0070 \\
0.2446 \\
0.0770
\end{bmatrix}
\]

In this simulation, the rudder constraints are \( |\delta| \leq 35^\circ \) and the yaw rate constraints are \( |r| \leq 0.0932 \) rad/s, so, the corresponding matrices \( C_1, D_1, S \) and \( T \) are given by

\[
C_1 = \begin{bmatrix}
0 & 1 & 0 \\
0 & -1 & 0
\end{bmatrix};
D_1 = \begin{bmatrix}
0.0932 \\
0.0932
\end{bmatrix};
S = \begin{bmatrix}
1 \\
-1
\end{bmatrix};
T = \begin{bmatrix}
35\pi/180 \\
35\pi/180
\end{bmatrix}
\]

whereas the disturbance value \( w \) is calculated using equation (3) and (5) multiplied by matrix \( M^{-1} \). The parameters used to obtain the value of \( w \) are \( a_M = -g = -10 \) m/s\(^2\) as opposed to the gravitational acceleration of the earth, \( \theta_M = 45^\circ, \alpha_t = 30^\circ, d_L = 15 \) m from center of gravity (assumed). The kind of missile used in this paper is Torpedo A244/S with mass \( m_M = 244 \) kg.

The simulation is divided into 2 scenarios i.e (i) missile is fired when the ship has followed the specified heading angle and (ii) the missile is fired when the ship is adjusting the heading angle. The value of the control parameters is \( Q = \text{diag}\{100, 100\} \) and \( R = 1 \). The simulation results from scenario 1 with different predictive horizons can be seen in Figure 2.

In Figure 2, the simulation is done with different prediction horizons to measure the performance of each prediction horizon value when the system gets the impact of missile firing.
Figure 2. Simulation of the DC-MPC ship heading controller with disturbance for different prediction horizons in scenario 1

In this simulation, the initial value for the ship heading angle is 30°, then we want the ship’s actual heading angle to follow desired heading angle $\psi_d = 0°$. In this simulation the missile is fired at the 25th seconds, so at that time, the system is added with disturbance $w$. The simulation results show that DC-MPC has the ability to handle state constraints (yaw rate) for all prediction horizons used with the impact disturbance of missile firing. To see the comparison of the prediction horizons value, then we use the RMSE value between the ship’s actual heading angle and the desired heading angle.

Table 1. RMSE for the different prediction horizons

| Prediction horizon value | RMSE value |
|--------------------------|------------|
| 4                        | 4.5693     |
| 10                       | 4.5682     |
| 25                       | 4.5682     |

Based on Table 1, when the prediction horizon is longer, the performance is better, because RMSE value of longer prediction horizons is smaller than RMSE value of short prediction horizons. For the prediction horizon values greater than 10, they have the same RMSE value.

The simulation for scenario 2 is done with the same initial value as in scenario 1, but the missile is fired when the ship is adjusting the heading angle from 30° to 0°. This simulation is done with the same prediction horizon. The simulation results can be seen in Figure 3.

Figure 3 shows the standard MPC and DC-MPC simulations without and with impact disturbance of missile firing at the 1st seconds. For the disturbance-free system, the DC-MPC scheme is the same as the standard MPC. In Figure 3, after the system has a disturbance, the ship’s actual heading angle is slower toward the desired reference than system without disturbance. With this condition, the rudder angle oscillates at the 1st to 2nd seconds. Its purpose is to adjust the ship heading angle as quickly as possible to the desired reference.

6. Conclusion

In this paper, the DC-MPC scheme was developed to compensate impact disturbance of missile firing. Based on the result of simulation in previous section, DC-MPC can overcome disturbance on the system although the value of its disturbance is very small. For both scenario 1 and scenario
Figure 3. Simulation of Standard MPC ship heading controller without missile firing and DC-MPC ship heading controller with missile firing

2, the DC-MPC controller can make actual heading angle to follow the given reference quickly which is about 10 s. DC-MPC can also adjust the value of the ship heading angle to 0° after the disturbance occurs at the 25th seconds. The performance of the system for different prediction horizons is almost the same, because the error between actual ship heading angle and desired heading angle is very small. In the future work, the disturbance of missile firing can be applied on the ship which has the smaller size. So, the DC-MPC can perform well when the ship gets the disturbance caused by missile firing.

Acknowledgment
The authors wish to thank DRPM RISTEKDIKTI for funding this research (No. 928/PKS/ITS/2018)

References
[1] Asfihani T, Subchan S, Adzkiya D, Rosyid D M, Purnawan H and Kamilah R 2017 Estimation of the corvette sigma motion in missile firing mission Instrumentation, Control, and Automation (ICA), 2017 5th International Conference on (IEEE) pp 203–207
[2] Cahyaningtias S 2014 Penerapan Disturbance Compensating Model Predictive Control (DC-MPC) pada Kendali Gerak Kapal Master’s thesis Institut Teknologi Sepuluh Nopember
[3] Ghaemi R, Oh S and Sun J 2010 Path following of a model ship using model predictive control with experimental verification American Control Conference (ACC) (IEEE) pp 5236–5241
[4] Li Z and Sun J 2012 IEEE Transactions on Control Systems Technology 20 257–265
[5] Subchan S, Syaifudin W and Asfihani T 2014 Far East Journal of Applied Mathematics 87 245
[6] Zheng H, Negenborn R R and Lodewijks G 2014 IFAC Proceedings Volumes 47 8812–8818
[7] Putri D K R, Asfihani T and Subchan S 2018 Steering angle control of car for dubins path-tracking using model predictive control Journal of Physics: Conference Series vol 974 (IOP Publishing) p 012066
[8] Wang L 2009 Model predictive control system design and implementation using MATLAB® (Springer Science & Business Media)
[9] Camacho E F and Bordons C A 2012 Model predictive control in the process industry (Springer Science & Business Media)
[10] Fossen T I 1994 Guidance and control of ocean vehicles vol 199 (Wiley New York)
[11] Munadhif I, Aisjah A S and Masroeri A 2016 IPTEK Journal of Proceedings Series 2
[12] Ghaemi R, Sun J and Kolmanovsky I 2006 Computationally efficient model predictive control with explicit disturbance mitigation and constraint enforcement 45th IEEE Conference on Decision and Control (IEEE) pp 4842–4847