Bound on the neutrino magnetic moment from chirality flip in supernovae

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Abstract

For neutrinos with a magnetic moment, we show that the collisions in a hot and dense plasma act as an efficient mechanism for the conversion of $\nu_L$ into $\nu_R$. The production rate for right-handed neutrinos is computed in terms of a resummed photon propagator which consistently incorporates the background effects. Assuming that the entire energy in a supernova collapse is not carried away by the $\nu_R$, our results can be used to place an upper limit on the neutrino magnetic moment $\mu_\nu < (0.1 - 0.4) \times 10^{-11} \mu_B$.

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The properties of neutrinos have become the subject of an increasing research effort over the last years. Amongst these properties, the neutrino magnetic moment $\mu_\nu$ has received attention in connection with various chirality-flipping processes that could have important consequences for the explanation of the solar neutrino problem [1,2], and for the dynamics of stellar collapse [3,4]. As a consequence of a non-vanishing magnetic moment, left-handed neutrinos produced inside the supernova core during the collapse, could change their chirality becoming sterile with respect to the weak interaction. These sterile neutrinos would fly away from the star leaving essentially no energy to explain the observed luminosity of the supernova. The chirality flip could be caused by the interaction with an external magnetic field or by the scattering with charged fermions in the background for instance $\nu_L e^- \to \nu_R e^-$ and $\nu_L p \to \nu_R p$. Invoking this last mechanism, Barbieri and Mohapatra [4] have derived a limit $\mu_\nu < (0.2 - 0.8) \times 10^{-11} \mu_B$.

Dispersion processes in a plasma could exhibit infrared divergences due to the long-range electromagnetic interactions. To prevent such divergences, the authors in Ref. [4] introduced an ad hoc thermal mass into the vacuum photon propagator. However, it is well known that at high temperature, a consistent formalism developed by Braaten and Pisarski [6,7], rendering gauge independent results, requires the use of effective propagators and vertices that resum the leading-temperature corrections. In particular, this method has been applied to the study of the damping rates and the energy loss of particles propagating through hot plasmas [8–10]. In this paper we show that the above formalism provides also a convenient framework to study the neutrino chirality flip in a dense plasma [12].

The spectral function of the resummed photon propagator in a plasma presents a cut for space-like momenta; the physical origin of this purely thermal cut is Landau damping [7]. In this case, the conversion from left-handed to right-handed neutrinos happens through scattering by the exchange of effective space-like photons. We compute the production rate of $\nu_R$’s and the corresponding luminosity for such a process in a supernova. We recall that the contribution of the plasmon decay $\gamma \to \nu_L \nu_R$ to $\nu_R$ production is less important than the chirality flipping process due to the high neutrino density in the supernova core. Our
result can be used to place an upper bound on the neutrino magnetic moment which is in
the range $\mu_\nu < (0.1 - 0.4) \times 10^{-11} \mu_B$, where $\mu_B$ is the Bohr magneton.

Consider a QED plasma in thermal equilibrium at a temperature $T$ and with an electron
chemical potential $\tilde{\mu}_e$ such that $T, \tilde{\mu}_e \gg m_e$, where $m_e$ is the mass of the electron. The
production rate $\Gamma$ of a right-handed neutrino with total energy $E$ and momentum $\vec{p}$ can be
conveniently expressed in terms of the $\nu_R$ self-energy $\Sigma$ as [11]

$$\Gamma (E) = \frac{1}{2E} n_F \operatorname{Tr} [\mathcal{P} R \operatorname{Im} \Sigma] , \quad (1)$$

where, $L, R = \frac{1}{2} (1 \pm \gamma_5)$ and $n_F$ is the Fermi distribution for the $\nu_R$'s. Since the $\nu_R$'s are sterile, we could set $n_F = 1$ from the beginning, however as we shall see the $\nu_R$ distribution cancels in the final result. In the above expression, $\operatorname{Im} \Sigma$ can be directly computed following either the imaginary or the real-time formulations of thermal field theory with identical results [13]. In what follows we will work in the imaginary-time formalism.

The one loop contribution to $\Sigma$ is given explicitly by

$$\Sigma(P) = \mu^2 \nu T \sum_n \frac{d^3 k}{(2\pi)^3} K_\alpha \sigma^{\alpha \rho} S_F (P - K) L K_\beta \sigma^{\beta \lambda} * D_{\rho \lambda}(K), \quad (2)$$

where $P = (ip_0, \vec{p})$ and $K = (ik_0, \vec{k})$ are the four-momenta of the incoming neutrino and the virtual photon, respectively with $p_0 = (2m + 1)\pi T$ and $k_0 = 2n \pi T$, $m$ and $n$ being integers; and $p \equiv |\vec{p}|$, $k \equiv |\vec{k}|$. In the integration region where the momentum $k$ flowing through the photon line is soft (i.e. of order $eT$), hard thermal loop (HTL) corrections to the photon propagator contribute at leading order in $e$ and must be resummed. The effective propagator is obtained by summing the geometric series of one-loop self-energy corrections proportional to $e^2 T^2$. The intermediate neutrino line can be taken as a bare fermion propagator $S_0$, because the $\nu_L$ propagator gets dressed only through weak interactions with the particles in the medium. For the neutrino-photon vertex we use the magnetic dipole interaction $\mu_\nu \sigma_{\alpha \beta} K^\beta$. The neutrino effective electromagnetic vertices are of no concern to us here since they are induced by the weak interaction of the charged particles in the background and thus conserve chirality.
In a covariant gauge the HTL approximation to the photon propagator \( *D^{\mu\nu} \) is

\[
*D^{\mu\nu}(K) = *\Delta_L(K) P_L^{\mu\nu} + *\Delta_T(K) P_T^{\mu\nu},
\]

where \( P_L^{\mu\nu} \), \( P_T^{\mu\nu} \) are the longitudinal and transverse projectors, respectively. We drop the term proportional to the gauge parameter since it does not contribute to \( \Sigma \), as can be easily checked. The scalar functions \( *\Delta_L,T(K) \) are given by

\[
*\Delta_L(K)^{-1} = K^2 + 2m_\gamma^2 \frac{K^2}{k^2} \left[ 1 - \left( \frac{ik_0}{k} \right) Q_0 \left( \frac{ik_0}{k} \right) \right],
\]

\[
*\Delta_T(K)^{-1} = -K^2 - m_\gamma^2 \left( \frac{ik_0}{k} \right) \left[ \left[ 1 - \left( \frac{ik_0}{k} \right)^2 \right] Q_0 \left( \frac{ik_0}{k} \right) + \left( \frac{ik_0}{k} \right) \right],
\]

where \( Q_0(x) = \frac{1}{2} \ln \left( \frac{x+1}{x-1} \right) \) is a Legendre function of the second kind and \( m_\gamma \) is the photon thermal mass. In the limit \( T, \tilde{\mu}_e \gg m_e \),

\[
m_\gamma^2 = \frac{e^2}{2\pi^2} \left( \tilde{\mu}_e^2 + \frac{\pi^2 T^2}{3} \right). \tag{5}
\]

The sum over loop frequencies in Eq. (2) is evaluated by introducing the spectral representation for the propagators and summing over \( n \). At the end the neutrino energy is analytically continued to the real Minkowski energy \( ip_0 \rightarrow E + i\epsilon \). The spectral representation of the fermion propagator is [14]

\[
S_0(p_0, p) = \int_0^\beta d\tau e^{ip_0\tau} \left[ P_+ \tilde{f}(-E) e^{-E\tau} + P_- \tilde{f}(E) e^{E\tau} \right], \tag{6}
\]

where \( \tilde{f}(z) = (e^{(z-\tilde{\mu}_\nu)/T} + 1)^{-1} \) denotes the Fermi-Dirac distribution with \( \tilde{\mu}_\nu \) the \( \nu_L \) chemical potential and \( P_\pm = (E\gamma^0 \pm i\vec{p} \cdot \vec{\gamma})/(2E) \) are the positive and negative energy projection operators for massless particles.

For the electromagnetic fields the spectral representation reads

\[
*\Delta_{L,T}(k_0, k) = \int_0^\beta d\tau e^{ik_0\tau} \int_{-\infty}^{\infty} d\omega \rho_{L,T}(\omega, k) \left[ 1 + f(\omega) \right] e^{-\omega\tau}, \tag{7}
\]

where \( f(z) = (e^{z/T} - 1)^{-1} \) is the Bose-Einstein distribution. The spectral densities \( \rho_{L,T}(\omega, k) \) are obtained from the imaginary part of \( *\Delta(k_0, k) \) after analytical continuation \( \rho_{L,T}(\omega, k) = 2 \text{Im} \Delta_{L,T}(ik_0 \rightarrow \omega + i\epsilon, k) \), where the +, - signs correspond to the \( L, T \) modes respectively.
The spectral densities $\rho_{L,T}(\omega, k)$ contain the discontinuities of the photon propagator across the real-$\omega$ axis. Their support depends on the magnitude of the ratio between $\omega$ and $k$. For $|\omega/k| > 1$, $\rho_{L,T}(\omega, k)$ have support on the points $\pm \omega_{L,T}(k)$, i.e., the time-like quasiparticle poles. In the space-like region the support of $\rho_{L,T}(\omega k)$ lies on the whole interval $-k < \omega < k$, with the contribution arising from the branch cut of $Q_0$. In evaluating the self-energy for the chirality flip process $\nu_L \rightarrow \nu_R$, the kinematically allowed region is restricted to space-like momenta $|\omega| \leq k$. In this case from Eqs. (3) we obtain

$$\rho_L(\omega, k) = \frac{x}{(1-x^2)} \left[ k^2 + 2m_\gamma^2 (1 - \frac{\omega}{k} \ln \left| \frac{x+1}{x-1} \right| \right]^2 + \left[ \pi m_\gamma^2 x \right]^2,$$

$$\rho_T(\omega, k) = \frac{\pi m_\gamma^2 x (1-x^2) \theta (k^2 - \omega^2)}{\left[ k^2(1-x^2) + m_\gamma^2 \left( x^2 + \frac{2}{3} (1-x^2) \ln \left| \frac{x+1}{x-1} \right| \right)^2 + \left[ \frac{\pi m_\gamma^2}{x} (1-x^2) \right]^2 \right]}.$$

Both, longitudinal and transverse photons, contribute to this rate. Notice that in addition to $|\omega| \leq k$ there is also the restriction $(k - \omega) \leq 2E$ coming from the condition $|\cos \Theta| \leq 1$, where $\Theta$ is the angle between the momenta of the incoming neutrino and the virtual photon. With these restrictions the integrand in the previous equation can be proved to be positive definite. The rate $\Gamma$ can be written as the sum of two contributions $\Gamma = \Gamma_e + \Gamma_a$, that correspond to the production of $\nu_R$ through the emission or absorption of a virtual photon. $\Gamma_e$ comes from the interval $0 \leq \omega < k$, whereas $\Gamma_a$ corresponds to the interval $-k < \omega \leq 0$, as can be checked by means of the identity $1 + f(\omega) + f(-\omega) = 0$ and the substitution $\omega \rightarrow -\omega$ in this second interval.

In order to illustrate our results we consider the emission of right-handed neutrinos immediately after a supernova core collapse. The large mean free path of the right handed
neutrinos compared to the core radius implies that the $\nu_R$’s would freely fly away from the supernova. Therefore, the core luminosity for $\nu_R$ emission can be computed as

$$Q_{\nu_R} = V \int_0^\infty \frac{d^3p}{(2\pi)^3} E \Gamma(E),$$  \hspace{1cm} (11)

where $V$ is the plasma volume and $E = p$. To make a numerical estimate, we shall adopt a simplified picture of the inner core, corresponding to the the average parameters of SN1987A. Consequently, we take a constant density $\rho \approx 8 \times 10^{14}$ g/cm$^3$, a volume $V \approx 8 \times 10^{18}$ cm$^3$, an electron to baryon ratio $Y_e \simeq Y_p \simeq 0.3$, and temperatures in the range $T = 30 \sim 60$ MeV. This corresponds to a degenerate electron gas with a chemical potential $\tilde{\mu}_e$ ranging from 307 to 280 MeV. For the left-handed neutrino we take $\tilde{\mu}_\nu \approx 160$ MeV. Using this values in Eqs. (10) and (11), we obtain by numerical integration

$$Q_{\nu_R} = \left(\frac{\mu_\nu}{\mu_B}\right)^2 (0.7 - 4.3) \times 10^{76} \text{ergs/s},$$  \hspace{1cm} (12)

for $T$ ranging from 30 to 60 MeV.

Assuming that the $\nu_R$ emission lasts for about one second, the luminosity bound is $Q_{\nu_R} \leq 10^{53}$ ergs/s which places the upper limit on the neutrino magnetic moment

$$\mu_\nu < (0.1 - 0.4) \times 10^{-11} \mu_B.$$

This upper bound slightly improves the result previously obtained by Barbieri and Mohapatra. As mentioned before, these authors consider the helicity flip scattering $\nu_L e \to \nu_R e$ to order $e^2$ introducing the Debye mass in the photon propagator as an infrared regulator. Although the integrated luminosity shows no important dependence on the use of the complete leading order perturbation theory, the $\nu_R$ spectrum does, as we discuss below.

The usual bare perturbation theory to order $(e\mu_\nu)^2$ can be recovered by simply neglecting $m_\gamma^2$ in the denominators in Eqs. (8) and (9). This leads to infrared divergent results. The usual screening prescription consists of cutting off these infrared divergences treating both the longitudinal and transverse photons as particles of mass $\omega_p$; with the plasma frequency $\omega_p^2 = (2/3)m_\gamma^2$. The corresponding approximation for the spectral densities is
\[ \rho_L \approx 2 \rho_T \approx \frac{2\pi m_e^2 \omega / k^3}{k^2 - \omega^2 + \omega_p^2}, \]  

(14)

instead of using the expressions in (8) and (9). Fig. 1 shows the rate of production of right-handed neutrinos, for \( T = 30\, MeV \) and \( \tilde{\mu}_e = 307\, MeV \), as a function of their energy. We notice that for soft neutrino energies \( E \lesssim eT \) the use of formula (14) underestimate the contribution by several orders of magnitude as compared to the complete leading-order result. On the other hand, as the energy increases above the hard scale \( \sim T \), the two approaches lead to very similar results. Let us remark that the approximation in Eq.(14) correctly reproduces the static limit of the longitudinal propagator: \( \Delta_L(0, k)^{-1} = k^2 + \omega_p^2 \), but it also predicts a similar behavior for \( \Delta_T(0, k)^{-1} \), and thus it is not in agreement with the absence of magnetic screening in the static limit for the transverse mode. The vanishing of the magnetic mass in the HTL approximation of the photon propagator leads to infrared singularities in certain quantities [16]. In our case, there are enough powers of \( k \) coming from the vertex factors in (2) to render the \( \nu_R \) production rate finite.

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**Fig. 1.** The \( \nu_R \) production rate as a function of the energy. The complete leading-order result (solid-line) is compared with the calculation in which the infrared divergence is cut off by an ad hoc prescription (dotted-line) in which both the longitudinal and transverse photons are considered as particles of mass \( \omega_p \). The parameters are selected as: \( T = 30\, MeV, \tilde{\mu}_e = 307\, MeV \) and \( \tilde{\mu}_\nu = 160\, MeV \).
The use of the screening prescription in Eq. (14) also underestimates the contribution to the luminosity. However, the phase space factor in Eq. (11) suppresses the contribution to the luminosity for small values of $E$. An explicit calculation along this line yields a luminosity that varies between $(3 - 5)\%$ from our result in Eq. (12). We conclude that the $\nu_R$ production spectrum for soft $E$ is sensitive to the form of the resummed photon propagators. Yet, the luminosity depends very little on these effects, provided an infrared convergent factor of the correct magnitude is introduced. A more detailed analysis will be presented elsewhere [13].

A word of caution should be mentioned in relation to the result in Eq. (13). It has been pointed out by Voloshin [17] that the $\nu_R$’s produced by the magnetic moment interaction could undergo resonant conversion back into $\nu_L$’s through spin rotation in the magnetic field of the supernova core, with the subsequent trapping of the $\nu_L$’s by the external layers. If this is the case, then the bound in Eq. (13) becomes meaningless. However, the core density is rather high and the matter effect might dominate over the $\mu_\nu B$ term, suppressing the flip back of $\nu_R$ to $\nu_L$ [5].

Recently, another mechanism for the neutrino chirality flip has been proposed, which occurs via the Čerenkov emission or absorption of plasmons in the supernova core [18]. Since the photon dispersion relation in a relativistic plasma shows a space-like branch for the longitudinal mode, the Čerenkov radiation of the plasmon is, in principle, kinematically allowed [19]. However, this mode develops a large imaginary part, which implies that the Landau damping mechanism acts to preclude its propagation as we have discussed. Consequently, we think that no better than the quoted limit in Eq. (13) can be derived by this type of neutrino chirality flipping processes in a supernova core. Let us notice that Eq. (13) is comparable to a reliable constraint, $\mu_\nu < 0.2 \times 10^{-11} \mu_B$, that has been derived by Raffelt [20] from the analysis of plasmon decay in globular-custer stars.

To conclude, we have shown that the collision processes in a hot and dense plasma, allow for the efficient conversion of $\nu_L$ into $\nu_R$. In this work, plasma effects are consistently taken into account by means of the resummation method of Braaten and Pisarski. For soft
values of the energy, the production rate for $\nu_R$’s differs significantly from that obtained by a constant Debye mass screening prescription. However, correction to the integrated luminosity are small. For this reason, our upper bound on the neutrino magnetic moment does not differ significantly from the one obtained from the cooling of SN1987 by Barbieri and Mohapatra [4]. Knowledge of an accurate expression for the $\nu_R$ production rate, as given in Eq. (10) could be of importance in a detailed analysis of supernova processes.

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