An elastic model for tooth contact analysis of spur gears using FEM simulation

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Abstract. Gears are a class of machine parts very often used in the most diverse industries due to their many advantages. In addition to the many benefits they provide, gears also have some disadvantages related to the noise and vibrations they can introduce into the system they are part of. The operating drawback which may occur is closely related to the geometry of contact and manufacturing and assembling errors. Investigation of the contact error effect on the operating condition should be first carried out through simulation approach. Gear efficiency can be investigated through the tooth contact analysis (TCA) method and stress analysis. The main target of TCA is to set up the contact path and to determine the transmission error function caused by clearance, gear misalignments or backlash. Generally, TCA is performed considering the elastic behaviour, which frequently implies to get data for different contact positions for full meshing analysis. Nevertheless, TCA outcome delivers noteworthy data about kinematic errors, contact ellipses and the estimation of the instantaneous contact point of the meshing gears. A numerical tool was developed in this work to obtain the contact path in spur gears, which allowed to compute the displacements, stress and strain fields in the contact regions. Furthermore, an analysis of the influence of manufacturing and assembling errors on the contact path and mesh quality was performed. Such an approach may provide an optimal design solution for spur gears.

1. Introduction

Gears in power transmission systems represent one of the most significant components. Because of their numerous advantages, their usage is present in many applications. Some of the advantages are durability, steady transmission ratio, small size, high efficiency or the utilization for a broad spectrum of capabilities. Apart from benefits, gears have some disadvantages as well, such as the need for high accuracy in implementation and installation, the use of restricted transmission ratio and the vibration that is generated by the gear meshing cycle, which produces an undesired noise, particularly when the equipment is used at high speeds.

A large number of researchers are currently studying the problem of gear pair noise and vibration. These unwanted effects are a result of tooth deflections, tooth profile errors, machining errors, and assembly errors.

Power transmission quality was first studied by Walker [1]. He intended to reduce vibration and noise by analyzing the modifications of spur gear tooth shape. Harris [2] introduced the notion of transmission error and developed a chart called Harris map.
Transmission error function caused by the shaft misalignments and the determination of the contact path between the meshing teeth represents one of the main targets of the TCA.

Fernandez del Rincon et al. [3] study the tooth geometry and the location of contact points according to the rack-type instrument. In Velex et al. [4] paper dynamic tooth loads were minimized in order to obtain a design solution for tooth profile changes. Z Chen et al. develop a model where a relation is determined between the total mesh stiffness, loaded static transmission error, load sharing and gear tooth errors [5]. Taking into account all the pairs of teeth in simultaneous contact, the load distribution of spur gears can be determined by analyzing the total elastic potential energy [6].

The contact features and the origin of the misalignment of involute gears were addressed by Houser [7]. In the ideal situation, there are no misalignments between the shafts of the spur gears which are considered parallel to each other. However, misalignment is an unavoidable problem that occurs during the manufacturing and assembly process which produces vibrations and additional dynamic loads. There are three different types of shaft misalignment, according to their centerlines: parallel misalignment also called offset misalignment, yaw misalignment, and pitch misalignment.

A mathematical model was developed by Cao et al. [8] to present a TCA ease-off numerical procedure and to determine the tooth contact surface and edge contact. The influence of flank modifications on the contact characteristics can be determined considering tip corner contact and the shaft misalignment [9].

Time-varying mesh stiffness (TVMS) is considered to be an inner stimulation of the gears and represents an important parameter of the gear mesh vibration research [10]. By the FEM procedure and considering geometric and assembly errors, an efficient mode to establish TVMS can be determined. The initial contact point strongly affects the TCA result [11]. J. Jinke et al. [12] determine the polynomial coefficients of the higher sequence polynomial function of TE using TCA, LTCA and particle swarm optimization technique. A new geometric approach for the TCA has been suggested in the Sanchez-Marin et al. paper [13]. In Guan et al. [14] research the TCA method is used to inspect the crown gear coupling with misalignments and to determine the contact points between the meshing teeth.

Assembly errors represent the main cause of the occurrence of eccentricity [15]. The dynamic behavior of the gear is not affected by the manufacturing errors. However, these errors produce an increase in vibration amplitude at all frequencies.

2. Analytical model and gear mesh

2.1. Analytical model
Tsai and Ye presents in their article [16] the contact regime for two meshing teeth (Fig. 1).

![Fig. 1 Deformation of two meshing teeth, under loading](image)

It is established that the contact conditions of a pair of points Q₁ and Q₂ on the meshing teeth is given by the next relations [16]:

\[ h + w_{x1} + w_{x2} = \delta, \text{ if the points are in contact} \]  

(1)
\[ h + w_{k1} + w_{k2} > \delta \], if the points are out of contact \hspace{2cm} (2)

where \( h \) represents the gap between the points; \( w_{k1} \) and \( w_{k2} \) are the deformations under loading and \( \delta \) is the displacement.

Using position vectors attached to two coordinate systems that are set to the meshing gears, we can determine the tooth surface geometry [8]. The common algorithm is based on the following assumptions: the unit normal vectors are aligned at the contact point and the surface position vectors are coincident.

The fundamental relation is determined with the next equation [8]:

\[
\begin{align*}
 r_h^i (u_1, v_1, \varphi_i) &= r_h^2 (u_2, v_2, \varphi_2) \\
n_h^i (u_1, v_1, \varphi_i) &= n_h^2 (u_2, v_2, \varphi_2)
\end{align*}
\hspace{2cm} (3)
\]

where letter “\( h \)” is used for the vectors that belong to a coordinate system that is fixed; superscripts 1 and 2 are used for the pinion and gear tooth surfaces; \( u_1, u_2, \) and \( v_1, v_2 \) signify the surface generating characteristics; \( \varphi_1 \) and \( \varphi_2 \) express the pinion and gear angular position; \( n \) and \( r \) are the unit normal vectors and the surface position vectors.

Total mesh stiffness of the engaged teeth has the following expression [17]:

\[
k_{tooth} = \sum_{i=1}^{q} k_{tooth}^i = \sum_{i=1}^{q} [(k_h^i)^{-1} + (k_{t1}^i)^{-1} + (k_{t2}^i)^{-1}]^{-1}
\hspace{2cm} (4)
\]

where \( q \) describes the number of teeth in meshing, \( k_{tooth}^i \) represents the stiffness of the tooth number “\( i \)” in contact, \( k_h^i \) is the nonlinear Hertzian contact stiffness; \( k_{t1}^i \) and \( k_{t2}^i \) are the teeth stiffness.

The tooth stiffness depends on the axial compressive stiffness, the bending stiffness, and shear stiffness.

The relation of gear and pinion stiffness elements is presented below [18]:

\[
k = F/\delta
\hspace{2cm} (5)
\]

where \( F \) represents the normal force used on the tooth profile throughout the line of action, \( \delta \) signifies the deflection of tooth applied on the direction of the force and \( k \) is the stiffness of the tooth.

The deflection can be calculated with the next relation [18]:

\[
\delta = \delta_x \sin \varphi + \delta_y \cos \varphi
\hspace{2cm} (6)
\]

where \( \delta_x \) and \( \delta_y \) represent the contact element displacement of the tooth along \( x \) or \( y \) direction and \( \varphi \) is the pressure angle.

2.2. Gear mesh

In this paper, a pair of involute spur gear is generated. It is considered that only one pair of teeth are in contact. In order to reduce the computational time, other simplified models of one, three, five, seven and nine teeth were developed.

Using a FEM software, a mesh of quadratic elements is first generated for the gear drive (Fig. 2a). Each element has attached a number of 20 nodes (Fig. 2b). Secondly, for the tooth involute profile a series of significant lines (paths, in red colour, Fig. 2c and Fig. 2d), which contains several nodes, named \( P_1...P_6 \) and \( P_7...P_9 \) have been chosen on the tooth flank and the tooth profile, respectively. The pair of teeth that engaged are fine-meshed.
3. Simulation and results

The spur gear geometry characteristics are presented in Table 1. The material used is steel with an elasticity modulus $E=210,000 \text{ N/mm}^2$ and a Poisson’s ratio $\nu=0.3$.

**Table 1. Spur gear parameters**

| Parameters          | Abbreviation | Pinion | Gear | Units |
|---------------------|--------------|--------|------|-------|
| Module              | $m$          | 3      | 3    | mm    |
| Number of teeth     | $z$          | 20     | 40   | -     |
| Pressure angle      | $\alpha$     | 20     | 20   | °     |
| Reference diameter  | $d$          | 60     | 120  | mm    |
| Addendum            | $h_a$        | 3      | 3    | mm    |
| Dedendum            | $h_f$        | 3.75   | 3.75 | mm    |
| Face Width          | $b$          | 12     | 12   | mm    |
| Tip diameter        | $d_a$        | 66     | 126  | mm    |
| Base diameter       | $d_b$        | 54     | 114  | mm    |
| Root diameter       | $d_f$        | 52.5   | 112.5| mm    |
| Center distance     | $a=(d_1+d_2)/2$ | 90     | 90   | mm    |
| Clearance           | $c$          | 0.8    | 0.8  | mm    |
| Root fillet radius  | $p_f$        | 1      | 1    | mm    |

The following boundary conditions are considered:
- Gear shaft is clamped (kinematic coupling);
- A 50 Nm torque is applied to the pinion shaft;
- Contact constraints are used between the meshing teeth surfaces.

Considering the given initial conditions, the distribution of stress, strain, and displacement was analyzed for the ideal conditions of gear mesh, when friction between teeth is present, and also when the influence of misalignment and clearance is taken into account.

Ideal conditions were considered when geometric errors are not applied for the gear teeth. The misalignment and clearance were obtained through transformation matrix from the initial position to the actual one, under loading.

The clearance was introduced through the increase in centre distance.

Apart from the gear drive full model, containing all the teeth of pinion and gear, another reduced model types, with one, three, five, seven and nine teeth, were generated for analysis. This was done aiming to investigate the effect of the model type, with different number of teeth, over the von Mises stresses, strain and displacements (Fig. 3).

![Fig. 3. Full and reduced models](image)

The next simulations present the von Mises stress, displacement and strain for the full model, along the considered paths in the ideal meshing situation, and also when misalignment and radial clearance are considered (Figures 4-10).

![Graph showing von Mises stress](image)
Fig. 4. Von Mises stress along selected paths (ideal manufacturing and assembling conditions)  
  a) profile path; b) tooth flank path

Fig. 5. Strain (ideal manufacturing and assembling conditions)

Fig. 6. von Mises stress with radial clearance
Fig. 7. Strain when gears have radial clearance

Fig. 8. von Mises stress with misalignment

Fig. 9. Strain \( \varepsilon \) with misalignment
4. Discussions
Comparing the results from the full gear drive model with the results of the reduced models, it can be noticed that the nine, seven, five and three teeth models provide similar results.

In Fig. 4a it can be observed that a fast increase of stress begins near the location of the contact line and decreases immediately after the contact line is exceeded. Another two regions with concentrated stress can be noticed at the base of the tooth flanks near the bottom land (at the fillet radius). The variation of the appropriate maximum von Mises stress is presented in Fig. 4b. The edge effect can be best observed when a path situated on a tooth flank is selected (P₃ from the simulations). When the contact forces increase, the contact area between the meshing teeth become larger. Contact ratio and the location of the force used on the tooth flank influenced the mesh stiffness.

When radial clearance is taken into account the tooth contact pair changes the theoretical contact line. The model with radial clearance has a stress peak value that is around 2% higher. The displacement magnitude calculated for the radial clearance condition is 45% bigger than the displacement analyzed in the ideal meshing conditions. The calculated values for the chosen paths are similar.

During the manufacturing process and assembling operation, different unavoidable mechanical errors may occur, like misalignment. Its consequences must be dealt with since the early design phase. When the two gears are coupled on axes that are misaligned, the pinion flank will start contact with the meshed flank of the paired gear on the face width edge. In this situation stresses will be mainly concentrated on that face width edge. Fig. 8-10 presents the influence of misalignment, considering a value of 0.3° degrees, over the stress, strain, and displacement. It can be observed that one of the face width edges is not in contact anymore. It can be seen from Fig. 8 that the von Mises stresses are about four times higher on the path containing the edge of the tooth in contact (P7), in comparison with the path that crosses the middle of the tooth (P8). When the misalignment errors are increasing the values of the stresses that arise on the face width edge are growing. If the misalignment errors surpass a threshold value the interference appear.

For all considered cases strain have similar behaviour with the von Mises stress. However, considering the case of ideal assembling conditions, unlike the stress, the peak value obtained at path P7 is about two times higher than the peak value examined at path P8.

5. Conclusions
The paper deals with the generation and contact simulation of a pair of involute spur gears. Using a FEM software, a mesh of quadratic elements was realized. In order to diminish the computational time, other reduced models were developed. A series of paths were created on the tooth flank and on the tooth profile.
Considering the given initial conditions, the distribution of stress, strain, and displacement was analysed for the ideal conditions of gear mesh, and also when the influence of misalignment and clearance was considered.

The results from the full model were similar to the results of the reduced models, except the one pair teeth model.

A fast increase of the stress started near the location of the contact line and decreased immediately after the contact line was surpassed. It was also observed the edge effect. When the contact forces increased the contact area between the meshing teeth become larger and stiffer.

The contact line was changed when radial clearance was considered. Analyzing the misalignment case it was observed that stresses were concentrated on the face width edge. When the misalignment errors surpass a threshold value the interference appears. Further analysis and discussion are needed in order to highlight the influence of tip and root relief and to localize the contact pattern of the teeth pairs.

Acknowledgments
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