Superconducting Quantum Point contacts and Maxwell Potential

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March 23, 2022

PACS: 73.23.-b; 73.23.Ad

Keywords: Quantum Point Contact, Quantization, Andreev Reflection, Maxwell Potential.

Abstract

The quantization of the current in a superconducting quantum point contact is reviewed and the critical current is discussed at different temperatures depending on the carrier concentration as well by suggesting a constant potential in the semiconductor and then a Maxwell potential. When the Fermi wavelength is comparable with the constriction width we showed that the critical current has a step-like variation as a function of the constriction width and the carrier concentration.

1 Introduction

The superconducting quantum point contact (SQPC) is consisting of a split-gate superconducting-two dimensions electron gas (2DEG)- superconductor junction [1]. It has attracted the attention of many authors theoretically and experimentally from the early

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Figure 1: An incident electron from the normal metal is reflected as a hole.

1970s starting by the studies of the dc Josephson effect in long superconductor-normal metal-superconductor junctions [2]. In general, a quantum point contact is a short constriction of variable width, comparable to the Fermi wavelength, defined using a split-gate technique in a high-mobility 2DEG. Quantum point contacts [3] are best known for their quantized conductance at an integer multiples of $e^2/h$. Thus, a ballistic theory leads to predicting a steplike structure with the conductor having an amplitude $e^2/h$ as a function of Fermi energy or width and the current shows a steplike variation as a function of the width of the constriction.

It was found that the Josephson current increases stepwise as a function of the constriction width [4, 5], while this current shows oscillations [4, 6] as a function of the carrier concentration of the 2DEG in the semiconductor layer. This oscillation is due to the interference effects of the quasiparticles that undergo Andreev as well as normal reflection. These results are a characteristic of the transport across a junction with high probability of Andreev reflections.

The transmission of quasi-particles through superconductor-normal metal (SN) interfaces requires conversion between dissipative currents and dissipationless supercurrents and is made possible by a two-particle process known as Andreev reflection (AR) (Figure 1) [7]. An electron injected from the normal metal with energy lower than the superconductor gap is reflected as a phase-matched hole, while a cooper pair is transmitted in the superconductor. Due to its two-particle nature, AR is strongly affected by the transmissivity at the SN interface and much effort has to be devoted to the optimizing of this parameter [8]. In the presence of scattering centers in the normal region, the phase relationship between incoming and retroreflected particles can give rise to marked coherent-transport phenomena such as reflectionless tunneling [9].

In this paper, we consider non-zero temperature for the dc Josephson effect of SQPC’s by suggesting a constant potential in the semiconductor in subsection 2.1, then a Maxwell potential which could be gotten from a contact of the system with a photon’s source in subsection 2.2. We deal with the influence of the presence of photons on the SQPC. Theoretically, we make use the pure Maxwell theory which assures the anyonic properties of
2DEG and avoids the appearance of topological mass that is responsible for the screening characteristics displayed by pure electric charges in the electrodynamics controlled by the Maxwell-Chern-Simons theory [14]. In two dimensions it is allowed the possibility of particles with any statistics, where the physical excitations obeying it are called anyons [15]. Thus, a concrete way to realize non-trivial statistics is by attaching a magnetic flux to electrically charged particles forming a composite system. In this context, we discuss the influence of the extra potential that we call Maxwell potential on the critical current at different temperatures for our proposed system and we examine the dependence of the carrier concentration on the current.

2 Methodology

2.1 Josephson Current Through a Quantum Point Contact

A quantum semiconductor device comprises mainly of a channel region formed with a two-dimensional carrier gas. A Schottky electrode structure is provided on the channel region for creating a depletion layer in the channel region to be extended in a lateral direction such that the two-dimensional carrier gas is divided into a first and a second region. A quantum point contact formed in the depletion layer to connect the first and second region of the two-dimensional carrier gas in a longitudinal direction; an emitter electrode is provided on the channel region in correspondence to the first region of the two-dimensional carrier gas; one or more collector electrodes are provided on the channel region in correspondence to the second region of the two-dimensional carrier gas, and another Schottky electrode structure is provided in correspondence to the first region for creating a depletion region therein, such that a path of the carriers entering into the quantum point contact is controlled asymmetrical with respect to a hypothetical longitudinal axis, that passes through the quantum point contact in the longitudinal direction.

Due to the generation of depletion layer [6, 10], the width of the constriction $W_n$ is reduced to the following (see figure 2) when an applied voltage is biased,

$$W_{n'} = W_n - 2\rho$$

The depletion layer $\rho$ depends on the bias voltage $V_0$, the carrier concentrations $N_B$, the temperature $T$ and the permittivity of the material $\epsilon$ [12], and it is given by

$$\rho = \sqrt{\frac{2\epsilon}{eN_B}(V_0 - \frac{2k_B T}{e})}.$$  \hspace{1cm} (2)

The device geometry assumed for our calculation is shown in Fig.2. Our system is described by the Bogoliubov-de-Gennes (BdG) equation [12], which is given by

$$
\left( -H(x, y) + U(x, y) \begin{array}{cc}
\Delta(x, y) \\
\Delta^*(x, y)
\end{array} \right) \left( \begin{array}{c}
u(x, y) \\
v(x, y)
\end{array} \right) = E \left( \begin{array}{c}
u(x, y) \\
v(x, y)
\end{array} \right),$

Solutions to this equations are electron-like and hole-like quasiparticles (QP) wave functions [3, 12]; $u(x, y)$ and $v(x, y)$ represent the eigenfunctions for the electron and hole quasiparticles.
We consider a simple model of a Josephson junction that shows the essential features of the Josephson Effect. It is a one-dimensional model, where the left and right superconductors have the pair potentials of the same magnitude but with different phases

$$\Delta(x) = \begin{cases} \Delta_0 e^{i\theta_L} & x < 0 \\ \Delta_0 e^{i\theta_R} & x > 0 \end{cases}$$

(4)

We assume the normal region to be thin and we consider an extreme case where its width is infinitesimal. In general, some scattering process is expected to be present either inside the normal region or at the super-normal interfaces; Introducing a scattering potential.

$$U(x) = V_b + \mu_{L,R} + U_0$$

(5)

with $V_b$ representing the height of the Schottky barrier at the S-Sm interface, $\mu$ the chemical potential, and $U_0$ the potential energy of the interface.

The basic wave function of the quasiparticles in the jth channel can be written as

$$\Psi_j(x, y) = \sqrt{\frac{2}{W_{n'}}} \phi_j(x) \sin(j\pi \frac{y}{W_{n'}} + \frac{1}{2})$$

(6)

where a two-component wave function $\phi_j(x)$ obeys a modified BdG equation

$$\begin{pmatrix} \frac{-\hbar^2}{2m'} \left( \frac{\partial^2}{\partial x^2} \right) - (j\pi \frac{\partial}{\partial x})^2 + V_b + \mu_{L,R} + U_0 \\ \Delta^*(x) \end{pmatrix} \begin{pmatrix} \Delta(x) \\ \frac{\hbar^2}{2m'} \left( \frac{\partial^2}{\partial x^2} \right) - (j\pi \frac{\partial}{\partial x})^2 - V_b - \mu_{L,R} - U_0 \end{pmatrix} \phi_j(x) = E \phi_j(x),$$

(7)

The solution of eqn.(7) is

$$\phi_j(x) = \begin{cases} e^{iP_j^+x} \left( u_0 \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \right) + a_{1j} e^{iP_j^-x} \left( u_0 \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \right) + b_{1j} e^{-iP_j^+x} \left( u_0 \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \right) & x < 0 \\ c_{1j} e^{iP_j^-x} \left( u_0 e^{i\phi} \begin{pmatrix} u_0 e^{i\phi} \\ v_0 e^{-i\phi} \end{pmatrix} \right) + d_{1j} e^{-iP_j^+x} \left( u_0 e^{i\phi/2} \begin{pmatrix} u_0 e^{i\phi/2} \\ v_0 e^{-i\phi/2} \end{pmatrix} \right) & x > L \end{cases}$$

(8)
where $P_j^\pm = \sqrt{\frac{2m^*}{\hbar^2}}(V_b + \mu_{L,R} + U_0 \pm \Omega - q_j^2)$, $q_j = \frac{j\pi}{W}$, $\Omega = \sqrt{E^2 - \Delta_0^2}$ in which $W_n$ is the width of the superconducting electrodes and $k_j^\pm = \sqrt{\frac{2m^*}{\hbar^2}}(V_b + \mu_{L,R} + U_0 \pm E) - \frac{(j\pi)^2}{W_n}$ with $u_0 = \sqrt{\frac{1}{2}(1 + \frac{\Omega}{E})}$ and $v_0 = \sqrt{\frac{1}{2}(1 - \frac{\Omega}{E})}$. The coefficients $a_{1j}, b_{1j}, c_{1j}$ and $d_{1j}$ are functions of the Energy $E$ and the phase difference $\phi$ across two superconductors, they are determined by the matching conditions at the interfaces. We solve the BdG equation in the WKB approximation[13] to obtain the amplitude $a_{1j}$, which is given by

$$a_{1j}(\phi, E) = \frac{\Delta_0[e^{\theta_j} - e^{-\phi}]}{(E + \Omega)e^{-\phi} - (E - \Omega)e^{\theta_j}}$$  \hspace{1cm} (9)$$

Where $\theta_j = L(k_j^+ - k_j^-)$.

The dc Josephson current, $I$, due to Andreev reflections can be calculated as

$$I = \frac{e\Delta_0}{\hbar\beta} \sum_{w_n} \frac{1}{\Omega_n} \sum_{j=1}^{N} [a_{1j}(\phi, iw_n) - a_{1j}(-\phi, iw_n)]$$  \hspace{1cm} (10)$$

where $w_n$ is the Matsubara frequency, given by $w_n = \pi(2n + 1)/\beta$ with $\beta = (k_B T)^{-1}$ and $T$ the absolute temperature, $k_B$ the Boltzman constant, and finally $\Omega_n = \sqrt{w_n^2 + \Delta_0^2}$. Now substituting equation (9) into equation (10), we get for the current $I$

$$I = \frac{2e\Delta_0^2}{\hbar\beta} \sum_{w_n} \sum_{j=1}^{N} \frac{\sinh \phi}{(2w_n^2 + \Delta_0^2) \cosh \tilde{\theta}_j + 2w_n \sqrt{w_n^2 + \Delta_0^2} \sinh \tilde{\theta}_j + \Delta_0^2 \cos \phi}$$  \hspace{1cm} (11)$$

where $\tilde{\theta}_j = -i\theta_j(E \rightarrow iw_n)$.
Figure 4: I-V characteristics at $W_n=13$nm and $\phi = 2\pi$. (a) Small scale. (b) Large scale.

Figure 5: Dependence of the current on the temperature at different phases and constriction =3 nm.
2.2 Josephson current and Maxwell Potential

In this section we would like to discuss the influence of the presence of photons on the superconducting quantum point contact in which the 2DEG is settled as weak region linking two superconductings. We make use the pure Maxwell theory which assures the anyonic properties of 2DEGA concrete way to realize non-trivial statistics is by attaching a magnetic flux to electrically charged particles forming a composite system. Theoretically the system is described by the Lagrangian

\[ \mathcal{L}_a = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A_\mu \]  \hspace{1cm} (12)

The Lagrangian describes Maxwell theory that couples to the current. In this theory, the gauge fields are dynamic and the canonical moments are \( \pi^\mu = F^{\mu\nu}_0 \) which results in the usual primary constraint \( \pi^0 = 0 \) and \( \pi^i = F^{i\mu}_0 \). Thus for a composite located at the origin, the solution of the Maxwell equations read \( B(x) = \xi \delta^2(x) \) and \( E^i(x) = -\frac{e}{2\pi r^2} \) with \( r = |x| \) and \( \xi \) is the dipole’s moment [16].

The composite system in this theory interacts with purely maxwellian photons and displays a potential with the confining nature

\[ U(r) = \frac{e^2}{\pi} \ln(\eta r) \]  \hspace{1cm} (13)

with \( \eta \) is a massive cutoff [14], for simplicity we take \( \eta = 1 \). As a remark, the potential agrees with the behavior of the Maxwell-Chern-Simons theory in the limit of short separation.

Now, considering a superconducting quantum point contact which is compatible with our purely Maxwell theory in 2-dimension, the potential given in the previous section will be changed to the one depending on the distance separating the electrons in 2DEG region \( 0 < x < L \) and \( \forall z \) (fig.2) leading to an important change in the Josephson current of the SQPC as will be shown below. We account for the the different distances between the chemical potential and conduction band edge in the S and Sm regions, respectively, by introducing a potential step of height \( U_0 \). The potential can be written as follows \( U(r) = U_1(x) + U_2(z) \). Since the Nb are modeled as half-infinite slabs of thickness \( W \) occupying the superconducting regions. Then,

\[ U_2(z) = \begin{cases} 0, & 0 < z < W \\ \infty, & z < 0, z > W \end{cases} \]  \hspace{1cm} (14)

this for \( x < 0 \) and \( x > L \). If \( 0 < x < L \) and for a typical value \( W = 100nm \) the potential is

\[ U_2(z) = \begin{cases} \infty, & z < 0 \\ eF_s z, & 0 < z \end{cases} \]  \hspace{1cm} (15)

This approximates the potential of an inversion layer, the surface electric field being given by \( F_s \). Now we model the potential in the direction of \( x \) as

\[ U_1(x) = U_0 + V_b + \frac{e^2}{\pi} \ln(x), \]  \hspace{1cm} (16)
the first and second terms are the potentials introduced in the previous section.

By replacing the new potential $U(x)$ by (16) in the equation (7) the Josephson current will be affected as we will see below.

We solve the equation (7) with the new potential (16) is considered and we get that the function $\phi_j$ doesn’t change for $x < L$ and $x > L$. If $0 < x < L$ the eigenfunction becomes

$$\phi_j(x) = \left( c_1 A_i(X) + c_2 B_i(X) \right) \left( \begin{array}{c} 0 \\ 1 \end{array} \right) + \left( c_3 A_i(Y) + c_4 B_i(Y) \right) \left( \begin{array}{c} 1 \\ 0 \end{array} \right),$$  

with

$$X = \frac{\pi j^2}{W_n^2} \left( \frac{\pi \hbar^2}{2e^2m} \right)^{\frac{1}{3}} - (x - 1) \left( \frac{\pi \hbar^2}{2e^2m} \right)^{-\frac{1}{3}}$$

$$Y = \frac{\pi j^2}{W_n^2} \left( \frac{\pi \hbar^2}{2e^2m} \right)^{\frac{1}{3}} + (x - 1) \left( \frac{\pi \hbar^2}{2e^2m} \right)^{-\frac{1}{3}}$$

and $A_i(X), B_i(X)$ are Bessel functions of first and second kinds respectively. By matching the conditions at $x = 0$ and $x = L$ within the Andreev and WKB approximation using (8) and (17) we get $b_{1j} = c_2 = c_4 = d_{1j} = 0$ and

$$a_{1j}(\phi) = \frac{u_0}{v_0} \frac{P - 1}{1 - \frac{u_2}{v_0} P}$$

where

$$P = \frac{A_i(Z_1^+) A_i(Z_2^-)}{A_i(Z_1^-) A_i(Z_2^+)} e^{i\phi}$$

and $Z_1^+ = a \pm b$ and $Z_2^+ = a \pm b \left( \frac{L^2}{2} - 1 \right)$ in which $a = j^2 \left( \frac{\pi \hbar^2}{2e^2m} \right)^{2/3} \pi^{5/3} W_n^2$ and $b = 2^{2/3} \left( \frac{\pi \hbar^2}{2e^2m} \right)^{-1/3}$. To determine the exact expression for $P(X)$ we see the following approximations. If $X > 0$ (which is consistent with our values) the Bessel function of first kind is reduced to

$$A_i(X) = \sqrt{\frac{2}{\pi}} K_{1/3} \left( \frac{2}{3} X^{\frac{2}{3}} \right)$$

and again if

$$\frac{2}{3} X^{\frac{2}{3}} = Z > \frac{1}{3}$$

the modified Bessel function $K$ is reduced to

$$K_{1/3}(Z) = \sqrt{\frac{\pi}{2Z}} e^{-Z}.$$  

Consequently, The function $P$ becomes

$$P = \frac{Z_1^- Z_2^+}{Z_1^+ Z_2^-} e^{\frac{2}{3} \pi} \left( (Z_1^+)^{\frac{2}{3}} + (Z_2^-)^{\frac{2}{3}} - (Z_1^-)^{\frac{2}{3}} -(Z_2^+)^{\frac{2}{3}} \right).$$
Figure 6: The current variation in terms of large constriction width with $\phi = 4.5\pi$.

Figure 7: The current variation in terms of small constriction width with $\phi = 4.5\pi$.

Now following the definition of Josephson current given by (10) and the expression we obtain for $a_{1j}$ we get

$$I = \frac{e\Delta_0}{\hbar\beta} \sum_{w_n} \frac{1}{\Omega_n} \sum_{j=1}^{N} \frac{u_0}{v_0} \left( \frac{P(\phi) - 1}{1 - \frac{u_0^2}{v_0^2} P(\phi)} - \frac{P(-\phi) - 1}{1 - \frac{u_0^2}{v_0^2} P(-\phi)} \right).$$

(19)

In our numerical calculations, we dealed with the obtained critical current in terms of different variables at non-zero temperatures as we see on the curves below;

3 Discussion and Conclusion

We studied the quantization of the supercurrent of a SQPC in a S-2DEG-S Josephson junction with a split gate. The supercurrent values change stepwise as a function of the
carrier concentration and the constriction width. We observe the onset of the first transport mode contributing both to the supercurrent of the SQPC. Furthermore the steps in the supercurrent appear at the same gate voltage values. This shows that each transport mode in the SQPC contributes to the supercurrent.

Thus, in order to show the reliability of the present theoretical treatment for the present model superconducting quantum point contact (SQPC) in a superconductor-two dimensional electron gas-superconductor(S-2DEG-S), we have performed a numerical calculation. In real system, however, there always exists Schottky barriers at the S-Sm interfaces which reduce the density of the Cooper pairs in the semiconductor [4]. The electron transport through the junction is treated as a stochastic process, so that the tunneled electron energy as a random number. The Schottky barrier height, $V_b$, is determined by using the Monte-Carlo simulation technique and its value was found to be $\sim 0.49$eV for the case Nb-GaAs Nb-GaAs based heterostructure interface. This value of, $V_b$, was found in agreement with those found experimentally and theoretically [17].

Figures (3a,3b) Show the variation of the current $I$ with the carrier concentration $N_B$. From these figures, it is shown that the current quantizes but when the carrier concentration increases the variation of the current changes and it makes many peaks with different dips. The shape of these dips depends on the value of the temperature and the carrier concentration (see fig.3b), and at the large scale of $N_B$ the special property of the quantum point contact is that upon widening the opening the current does not increase gradually but stepwise when the $N_B$ is increased which can be seen in agreement with the result given in [10, 11].

In Figure (4a) we found the bias voltage $V_o$ at different temperatures. As shown from this figure that the dip height at zero bias voltage increases as the temperature increases, and at the large scale (see fig 4b) of bias voltage we found subgab about zero bias which is agree with the result found in the reference [21, 22]. Figure (5) shows the decrease of the current $I$ as the temperature $T$ increases at different phases $\pi$, $2\pi$ and $3\pi$. This result shows a qualitative agreement with those published in the literature [19]. This variation shows that Josephson effect is optimal observed at very low $T$, also when the value of $\phi$ increases the value of the current increases too.
As shown in ref. [4, 5], the current quantization of the superconducting quantum point contact has been proved and these results verify the existence of the interference effects of the quasiparticles that undergo Andreev reflections. In two dimensional systems, the numerical calculations manipulating the carrier density via the field effect, verify experimentally this resonance [10]. Because of the exponential decay of a coherence length, the short junctions and the low temperature favor large amplitudes (Figs 4a, 4b). It is shown that the critical current is quantized and increased stepwise as a function of the width of the semiconductor layer and the doping concentration up to certain values of both.

Now, at the presence of extra potential that we called Maxwellian one, in the figures (6,7), we again see that the current $I$ exhibits a peak as a function of the width of constriction $W_n$. Also the special property of the quantum point contact didn’t change; upon widening the opening the current does not increase gradually but stepwise when the width is increased (see fig 7), when a steps do occur, the step high depends sensitively on the parameters of the junction and at the large scale of $W_n$ with the low value of $T = 0.5K$ the stepwise is seen in figures 6a, 6b and 6c but we remark that when the temperature is increased to 5K and 9K we can see the current is increased with clear stepwise, also in this case our result is in good agreement with what was found in [23]. A periodic variation of $I$ with $\phi$ is shown in Figures (8a,8b). This result was observed by another authors [18, 20] previously which shows the coherent property of our present system and it is in a clean limit. Also, the relation $I(\phi)$ (Fig. 8a,8b) for a width 13nm and different temperatures (T=0.5K, 5K, 9K) shows a behavior which is similar to the behavior of the current found in the recent work [18]. This result confirms the reliability of our treatment for the model concerning SQPC. In other words, the quasi-particle reflection from the edges of the Andreev gap causes mesoscopic phenomena manifested in oscillating features on $I(N_B)$ and $I(W_n)$ and $I(\phi)$.

Apart from studying fundamentals of charge transport in mesoscopic conductors, quantum point contacts can be used as extremely sensible charge detectors. Since the conductance through the contact strongly depends on the size of the constriction, any potential fluctuation (for instance, created by other electrons) in the vicinity will influence the current through the QPC. It is possible to detect single electrons with such a scheme. In view of quantum computation in solid-state systems, QPC’s may be used as readout devices for the state of a qubit.

We have studied the Jospheson effect in SQPC’s in the ballistic regime using simple two-dimensional modes. We have found that in some cases the critical current shows a characteristic feature due to the discreteness of energy levels around the constriction. This will be a common feature of the phase coherent ballistic conduction of supercurrent through a constriction in superconducting quantum devices.

Acknowledgements

The authors are Arab regional Fellow at CAMS supported by a Grant from the Arab Fund for Economic and Social Development.
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