UNIVERSAL STATISTICAL PROPERTIES OF INERTIAL-PARTICLE TRAJECTORIES IN THREE-DIMENSIONAL, HOMOGENEOUS, ISOTROPIC, FLUID TURBULENCE

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Abstract: We obtain new universal statistical properties of heavy-particle trajectories in three-dimensional, statistically steady, homogeneous, and isotropic turbulent flows by direct numerical simulations. We show that the probability distribution functions (PDFs) P(φ), of the angle φ between the Eulerian velocity u and the particle velocity v, at a point and time, scales as P(φ) ~ φ−γ, with a new universal exponent γ ≃ 4. The PDFs of the trajectory curvature κ and modulus θ of the torsion ϑ scale, respectively, as P(κ) ~ κ−hκ, as κ → ∞, and P(θ) ~ θ−hθ, as θ → ∞, with exponents hκ ~ 2.5 and hθ ~ 3 that do not depend on the Stokes number St. We also show that γ, hκ, and hθ can be obtained by using simple stochastic models. We show that the number Nt(t, St) of points (up until time t), at which φ changes sign, is such that Nt(St) ≃ limt→∞ τt Nt(St, t) ~ St−Δ, with Δ ≃ 0.4 a universal exponent.

INTRODUCTION

The elucidation of the statistical properties of inertial particles in turbulent flows is an important problem of great interest [1, 2]. We study the statistical properties of the geometries of heavy-inertial-particle trajectories; such inertial-particle-trajectory statistics have not received much attention hitherto in homogeneous, isotropic, three-dimensional (3D) fluid turbulence.

RESULTS AND CONCLUSIONS

Figure 1. Cumulative PDFs of (a) the angle φ between u and v (Q(α) ≡ P(φ ≥ α)), for St = 0.2 (blue circles), St = 0.5 (green triangles), St = 0.7 (brown squares), St = 1.0 (red pluses), and St = 1.4 (purple stars); the slope of the black dashed line is −3, (b) the curvature κ and (c) the magnitude of the torsion ϑ of the trajectories of heavy inertial particles, for St = 0.2 (in blue) and 1.0 (in red), obtained using rank order method. Inset: the values of the local slope of the tail, for St = 1.0.

Our direct-numerical-simulation (DNS) studies of these statistical properties yield new and universal scaling exponents that characterize heavy-particle trajectories. We calculate the probability distribution functions (PDFs) of the angle φ between the Eulerian velocity u(x, t), at the point x and time t, and the velocity v(t) of an inertial particle at this point and time, PDFs of the curvature κ and torsion θ of inertial-particle trajectories, and several joint PDFs. In particular, we find that the PDF P(φ) shows a power-law region in which P(φ) ~ φ−γ, with an exponent γ ≃ 4, which has never been considered so far; the extent of this power-law regime decreases as St increases Fig. 1 (a); we find good power-law fits if 0 < St ≤ 0.7; in this range γ is universal, in as much as it does not depend on St and the fluid Reynolds number Re (given our error bars). The PDFs of κ Fig. 1 (b) and θ = |ϑ| Fig. 1 (c) show power-laws tails for large κ and θ, respectively, with power-law exponents hκ and hθ that are also universal. We calculate the number of points, per unit time, at which the torsion ϑ changes sign along a particle trajectory Fig. 2; this number nτ(St) ~ St−Δ, as St → 0, with Δ ≃ 0.4 another universal exponent. We show how simple stochastic models can be used to obtain the exponents γ, hκ, and hθ; however, the evaluation of Δ requires the velocity field from the Navier-Stokes equation [3].
Figure 2. Number of inflection points per unit time as a function of dimensionless time $t/T_{\text{eddy}}$, for $St = 0.2$, (red curve), and $St = 1.4$, (blue curve); the inset shows the plot of the number of inflection points per unit time $n_I$, as a function of $St$.

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References

[1] F. Toschi and E. Bodenschatz, Ann. Rev. of Fluid Mech. 41, 375 (2009).
[2] J. Bec, J. Fluid Mech., 528 255 (2005).
[3] A. Bhatnagar, A. Gupta, D. Mitra, P. Perlekar, and R. Pandit, (2014). arXiv preprint arXiv:1412.2686.