Rotational tuning of interaction in metamaterials
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We experimentally observe the tuning of metamaterials through the relative rotation of the elements about their common axis. In contrast to previous results we observe a crossing of resonances, where the symmetric and anti-symmetric modes become degenerate. We associate this effect with an interplay between the magnetic and electric near-field interactions and verify this by calculations based on the interaction energy between resonators.

Metamaterials created as an array of sub-wavelength, resonant elements can exhibit interesting electromagnetic properties, such as a negative refractive index. Unlike the atoms in natural materials, the near-field patterns of metamaterial elements are quite complex, giving rise to strong interactions between them. Understanding this interaction is essential as it determines the overall resonant properties and effective parameters of the material. By controlling the relative arrangement of elements, it is possible to change this coupling and tune the properties of the structure. This allows us to alter the response of the material substantially without having to greatly alter the geometry or constituents.

An important building block of metamaterials is the Split Ring Resonator (SRR), which can exhibit a negative magnetic response due to its strongly resonant magnetic polarizability. By coupling a pair of SRRs, chiral properties and trapped dark modes can also be observed. In this Letter, we study the dynamics of two microwave SRRs broadside coupled to each other, varying twist angle \( \theta \) between them, as shown in Fig. 1(a).

The rings used have an inner radius of 3.5mm, an outer radius of 4mm, and a gap of 1mm. They are copper, printed onto 1.6mm thick FR4 circuit board, and the rings are 3.6mm apart, with the dielectric boards located between the rings. The incoming microwaves are polarized so that the electric field is in the \( x \) direction, as shown in Fig. 1(a). Numerical calculations were performed using CST Microwave Studio with the boards having a dielectric constant of 4.6.

The most meaningful definition of the resonant frequency is the frequency of maximum excitation of currents within the rings. This can be determined most readily by considering the absorption of the system, given by \( 1 - |S_{21}|^2 - |S_{11}|^2 \), where \( S_{21} \) is the transmission coefficient, and \( S_{11} \) is the reflection coefficient. A comparison of the experimental and numerical curves for \( \theta = 90^\circ \) is shown in Fig. 1(b).

Experimental results were measured using a Rohde and Schwarz ZVB network analyzer in a WR-229 rectangular waveguide, with \( \theta \) varied from 0° to 180° in 10° increments, while numerical results were calculated in 5° increments. The resonant frequency for each angle was found from the maximum of the absorption curve, and the resulting numerical and experimental results are compared in Fig. 2.

![FIG. 1. (a) A schematic showing the rings rotated with respect to each other through angle \( \theta \), and the polarization of the incoming waves. (b) A comparison of the experimental (solid) and numerical (dashed) absorption for angle \( \theta = 90^\circ \).](image1)

![FIG. 2. A comparison of the experimental (solid line) and numerical (dashed line) resonant frequencies.](image2)
Energy Absorption
Lagrangian for a pair of coupled SRRs
resonances. This is approached theoretically using the
two rings change, changing the coupling between the
asymmetric mode has mostly an electric dipole response,
conduction and radiation losses. The suppression of ra-
different Q factors (widths), which can be attributed to
moments of the rings.
was attributed to the electric quadrupole and octupole
As can be seen in Fig. 3(b), the two resonances have
symmetric and anti-symmetric modes, normalized to
θ. It is quite clear here that there is only one reso-
For θ = 0°, there are two resonances ωS and ωAS, and
by inspection of the currents in the rings we verify that
correspond to the expected symmetric and anti-
symmetric modes. As θ increases, ωAS increases and ωS
decreases, reaching their maximum and minimum values
respectively at θ = 180°. For our chosen parameters
the resonances cross at θc ≈ 33°, in contrast to Ref. 2,
where an avoided crossing of resonances was found which
was attributed to the electric quadrupole and octupole
The numerical absorption curve at 33° is shown in
Fig. 3(a). This curve was calculated assuming loss free
dielectric boards, so as to ensure the position of the cross-
given the low coupling of the asymmetric mode close
to θc. It is quite clear here that there is only one reso-
gelectric dipole arrangement, where the electric dipole has much stronger radiation losses. The asymmetric mode has mostly an electric dipole response, causing greater radiation losses, whereas the radiation losses at the symmetric mode have been suppressed by a dominating magnetic dipole. The radiation distribution then changes with angle θ, as can be seen by comparing Figs. 1(b) and 3(b).
The tuning of the system by rotation can be explained by
looking at the interaction between the rings. As the
rings are twisted, the magnetic and electric near-fields of
the two rings change, changing the coupling between the
resonances. This is approached theoretically using the
Lagrangian for a pair of coupled SRRs

\[ L = A\left(Q_1^2 + Q_2^2 + 2\alpha Q_1 Q_2\right) - B\left(Q_1^2 + Q_2^2 + 2\beta Q_1 Q_2\right) \]  

(1)
where α and β are the dimensionless magnetic and elec-
tric interaction constants and Q(t) is the time-dependent
mode amplitude. By substituting Eq. (1) into the Euler-
Lagrange equation we find that

\[ \ddot{Q}_1 + \omega_0^2 Q_1 = -\alpha \ddot{Q}_2 - \beta \omega_0^2 Q_2. \]  

(2)
\[ \ddot{Q}_2 + \omega_0^2 Q_2 = -\alpha \ddot{Q}_1 - \beta \omega_0^2 Q_1. \]  

(3)
This then allows the two resonances to be found - symmetric (when \( Q_1 = Q_2 \)), and asymmetric (when \( Q_1 = -Q_2 \)):

\[ \omega_S = \omega_0 \sqrt{1 + \beta \alpha}, \quad \omega_{AS} = \omega_0 \sqrt{1 - \beta \alpha}. \]  

(4)
In principle if \( \omega_0 \) is known, then by inverting Eq. (4) it
is possible to fit α and β from \( \omega_S \) and \( \omega_{AS} \). However we
found that this procedure is extremely sensitive to error and
does not yield usable results. Instead, we start from the
approach outlined in Ref. 4 to evaluate the interaction energy between the fundamental modes of the rings.
For a pair of rings in a homogeneous dielectric back-
ground, the interaction constants are shown in Fig. 4(a). We find that the interaction constants are very well de-
described by \( \beta \geq 0.05 \beta \) and \( \alpha = \alpha_0 + \alpha_1 \cos(\theta) \) with
\( \beta_1 = 0.085, \alpha_0 = 0.098 \) and \( \alpha_1 = 0.05 \). These constants are
dictated by the charge separation across the gap of the
ring, the current circulating around the ring, and the
inhomogeneity of the current distribution around the
ring, respectively. For rings aligned on the same axis, we
expect that the magnetic interaction should always be
positive, as the intersecting magnetic field from one loop
should always be normal to the other loop. In addition the
electric interaction should be positive at θ = 0° as the
charge distribution has the nature of parallel dipoles.
All arrangement of rings on the same axis which we con-
sidered obeyed these considerations.
In Fig. 4(b) we plot the corresponding frequencies of
the symmetric and anti-symmetric modes, normalized to
\( \omega_0 \). As our approach models the response of the res-
onators in a homogeneous dielectric background, the
results are significantly different from those observed exper-
imentally, where the dielectric is inhomogeneous and the
effect of waveguide boundaries is also significant. In par-
ticular, the crossing of resonances cannot be reproduced
for rings in homogeneous background. Therefore we con-
sider the possible regimes of interaction which may occur,
under the assumption that the interaction constants can be
fitted as described above.
The case considered in Fig. 4(a-b) corresponds to the
magnetic interaction always being larger than the elec-
tric interaction. This results in increasing splitting of \( \omega_S \)
and \( \omega_{AS} \) with increasing twist angle, however in prin-
ciple there is no reason why the splitting cannot de-
crease. We show such a case in Fig. 4(c-d), where we
have set \( \beta_1 = 0.02 < \alpha_1 \), such that the inhomogeneity in
the current has a stronger influence than the dipole-
like charge distribution. Despite the apparent difference
in frequency splitting curves, there is little difference be-
tween the interaction constants shown in Fig. 4(a) and
(c).
The only other case allowed in our model of interaction under the afore-mentioned physical constraints on $\alpha$ and $\beta$ is that $\alpha > \beta$ for $\theta = 0^\circ$. An example of this is given in Fig. 4(e), where we have reduced the magnetic coupling such that at some angle $\alpha = \beta$, by setting $\alpha_0 = 0.04, \alpha_1 = 0.02$. The corresponding resonant frequencies normalized to $\omega_0$ are plotted in Fig. 4(f). We see that for low twist angle, $\omega_S$ occurs at a higher frequency, but decreases with angle and crosses $\omega_{AS}$. Clearly this regime corresponds to what we observe in experiment, and we hypothesize that the inhomogeneous dielectric serves to enhance the electric interaction relative to the magnetic interaction.

In conclusion, we have shown that by changing the relative rotation between two rings, we can significantly change the coupling, which causes the resonances to change. We have found that there is a crossing where the two resonances coexist, which corresponds to equal electric and magnetic coupling.

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