Disordered hyperuniformity in superconducting vortex lattices

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The current carrying capability of type II superconductors under magnetic fields is determined to a large extent by the interaction of superconducting vortices with pinning centers. Vortices are arranged in lattices with varying degrees of disorder depending on the balance between the intervortex interactions and the pinning strength. We analyze here vortex arrangements in disordered vortex lattices of different superconducting systems, single crystals (Co-doped NbSe2, LiFeAs, and CaKFe4As4), and amorphous W-based thin films (with critical temperatures $T_c$ from 4 K to 35 K and critical fields from 3.4 T to more than 90 T). We calculate for each case the structure factor and number variance and compare to calculations on an interacting set of partially pinned particles. We find that random density fluctuations appear when pinning overcomes interactions and show that the suppression of density fluctuations is correlated to the presence of interactions. We discuss the results within the framework of hyperuniform distributions and find that all studied lattices follow a similar increase of the number variance with the defect density.

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I. INTRODUCTION

Particles occupying sites of a random lattice present density fluctuations at all length scales. It has been proposed that increasing interparticle interactions reduces long-range density fluctuations, deviating from random behavior [1,2]. The absence of density fluctuations occurs in all ordered lattices, including those with very-long-range order such as moiré patterns and quasicrystals. Density fluctuations can be described by discussing power laws in the structure factor $S(k)$ and the number variance $\sigma^2(R)$. The term “hyperuniformity” was coined to gather ordered as well as disordered lattices with reduced density fluctuations [1,2]. It has been shown that disordered hyperuniform lattices can be created by artificially designing disordered patterns without density fluctuations at large length scales [1–12]. It is not yet fully clear however how to link density fluctuations with interactions in a disordered hyperuniform system. Recently, disordered hyperuniform behavior has been found in the contact number between subsystems of particles [7,8], although the origin of the suppressed density fluctuations remains under debate [13].

Here we address the behavior of vortices in type II superconductors. Vortices are whirlpools of currents, each carrying a flux quantum $\Phi_0 = \hbar c/2e$. Intervortex interactions are screened above the penetration depth $\lambda$, which is most often much larger than the intervortex distance $a_0$ [16–19]. In thin films, $\lambda$ strongly increases with decreasing thickness and vortices interact through their stray field, which leads to a long range Coulomb-like interaction [20–22]. Vortices are pinned at defects in superconducting crystals and these defects are often randomly distributed. How the vortex lattice reacts to pinning depends on pinning strength and distribution as well as stiffness of the vortex lattice and determines the capability of type II superconductors to carry large currents. The degree of order in superconducting vortex lattices is determined by the balance between pinning and interaction strengths and is related to the critical current [14,15]. The question we address here is to what extent intervortex interactions can eliminate density fluctuations while allowing the formation of a disordered lattice due to pinning. The influence of vortex arrangements with suppressed density fluctuations in the current carrying capability of a superconductor was analyzed theoretically in Ref. [11]. Arrangements of vortices in cuprate superconductors at very small magnetic fields where weak intervortex interactions are weak were discussed in Ref. [23].

We analyze here vortex lattices in bulk superconductors and in thin films obtained in Refs. [24–28] and provide...
additional data in a thin film. We mostly focus on lattices with large amounts of disorder. We calculate $S(k)$ and $\sigma^2(R)$ and discuss the power laws as a function of $k$ and $R$. We discuss data on a nanostructured W-based thin film (W-film-1) with strong pinning and polycrystalline vortex lattice arrangements that are disordered at high magnetic fields. We show results in Co-doped 2H-NbSe$_2$, a system with strong pointlike pinning centers that also leads to polycrystalline hexagonal lattices which increase the level of disorder when increasing magnetic fields [25]. We discuss a W-based thin film with weak one-dimensional (1D) disorder potential created by the discommensuration between the vortex lattice and a linear nanostructure. This film shows ordered vortex lattices consistent with a Bragg glass at low magnetic fields that are disordered at high magnetic fields (W-film-2) [24,29]. We consider that the ordered lattices are representative of the behavior expected in 2H-NbSe$_2$ or other materials with well-ordered lattices [30–34]. Finally, we discuss two iron-based superconductors with strongly disordered lattices and strong pointlike intrinsic pinning at all magnetic fields, LiFeAs and CaKFe$_4$As$_4$ [26,27]. We find a gradual appearance of density fluctuations with increasing disorder in the lattice that successfully describes all studied vortex lattices.

II. METHODS

To analyze interactions in vortex lattice images we first have to recall the meaning of the spatial dependence of the structure factor and number variance. We show in Figs. 1(a) and 1(b) images including matrices of points distributed on a two-dimensional square. Points are positioned randomly [Fig. 1(a)] and following a hyperuniform distribution [Fig. 1(b)]. Both images are square and have the same size, with area $A$, and the same number of points $N$. In a random distribution [Fig. 1(a)], there are density fluctuations at all length scales. In a hyperuniform distribution [Fig. 1(b)], density fluctuations disappear at large length scales. We have generated the hyperuniform distribution of Fig. 1(b) by starting with points arranged in a square lattice of constant $a_0 = \sqrt{A/N}$. We then add to each lattice point a vector $\vec{r}$ with random coordinates, whose absolute value $r < 2a_0$ [inset of Fig. 1(b)]. The result is a random distribution which is spatially uniform for length scales larger than the intercellular distance. It is important to realize that there are no further signatures of the ordered lattice in Fig. 1(b) other than the large-scale uniformity.

The calculation of the number variance is very useful to discuss density fluctuations [1,2,11]. It is given by $\sigma^2(R) = \langle N^2(R) \rangle - \langle N(R) \rangle^2$, where $N(R)$ is the number of points inside a circle of radius $R$. The variance $\sigma^2(R)$ increases with $R$ as a power law $\sigma^2(R) \propto R^\alpha$. In a random distribution of points [Fig. 1(a)], $\sigma^2(R)$ grows as the dimension (i.e., with the area, according to the large number law) so that $\beta = 2$ [Fig. 1(c)]. On the other hand, in a hyperuniform distribution [Fig. 1(b)], $\sigma^2(R)$ grows as the dimension minus one (i.e., with the perimeter) with $\beta = 1$ [Fig. 1(d)]. Ordered lattices provide a strong decay for small wave vectors and an oscillating $\sigma^2(R)$ with a dip each time $R$ is somewhat smaller than integer multiples of the average interparticle distance [35].

To calculate the structure factor $S(k)$ we use the Fourier transform of the image of the vortex positions and make the radial average over the Fourier transform. In the Fourier space, the structure factor $S(k)$ decreases at small $k$ with the power law $S(k) \propto k^\alpha$. In the random distribution $\alpha = 0$ [Fig. 1(e)], while in the hyperuniform distribution $\alpha > 1$ [Fig. 1(f)]. There is a relation between the exponents of $S(k)$ and those of $\sigma^2(R)$. When $0 < \alpha < 1$, $\beta = 2 - \alpha$, and when $\alpha > 1$, $\beta$ remains locked at 1. Ordered lattices show a Bragg peak at the reciprocal lattice wavelength and a strongly decaying $S(k)$ for small $k$. 
In the Appendix we illustrate in greater detail how to obtain $S(k)$ and $\sigma^2(R)$. We first Delaunay triangulate all vortex positions, eliminating missing bonds in the sides of the image and carefully verifying that every vortex was correctly identified, as in Ref. [24]. We then use the resulting matrix of points to perform the calculation of $S(k)$ and $\sigma^2(R)$ and obtain the exponents as described above (see Fig. 1). In the Appendix we describe in detail how to calculate $\sigma^2(R)$ from an image of a disordered vortex lattice (Fig. 4 in Appendix).

As the results in W-film-1 have not been published, we provide a few experimental details. We acquire the image in zero-field cooled conditions at 100 mK using the system described in Refs. [24,28,36]. The sample has been made using a focused ion beam assisted deposition and has a composition similar to the composition of W-film-2, which has perfectly ordered lattices in a large range of magnetic fields, described in Ref. [24]. The critical temperature is 5 K [37,38]. However, contrary to W-film-2, here the substrate has strong random thickness modulations at a length scale of about 200 nm, which considerably enhance pinning. The vortex lattice rearranges accordingly, showing a polycrystalline pattern, which we discuss in the Appendix (Fig. 5).

We perform the numerical simulation using the following procedure, which tries to mimic the dynamics of real vortices as much as possible. Vortices are represented by interacting points. We distribute $N$ points at random on a square sample with periodic boundary conditions. We choose $N = 2000$, which is a number large enough to avoid undesirable boundary effects and small enough to converge quickly under our dynamics. We fix a certain percentage of points (between 10% and 50%) in their initial random positions, while the rest points are free to move and tend to minimize their potential energy. We use a $1/r$ interaction, which converges faster than the usual intervortex interaction and produces a similar spatial distribution. We do not take into account the fact that the intervortex interaction is screened above $\lambda$, although this should not greatly influence the comparison to the experiment. The procedure selects a point at random and chooses a possible new position displaced by a small distance from the original position in a random direction. If the new position minimizes the system energy, the point is moved to the new position; otherwise is kept in the old position. These energy relaxing displacements are repeated iteratively. After a number of displacements equal to 1000 times the number of points, changes in positions are minute, since the system has reached a metastable state, and we stop the algorithm. Once a final configuration has been reached, we run a Delaunay triangulation algorithm to determine the nearest neighbors of any point and calculate the proportion of points with a number of neighbors different from 6, in order to be able to compare directly with experiment. Finally, we calculate $S(k)$ and $\sigma^2(R)$, obtaining $\beta$ from the slope of $\sigma^2(R)$, as discussed previously for the experimental results.

III. RESULTS

In Fig. 2 we show our results. We start with lattices in Co-doped 2H-NbSe$_2$ [Figs. 2(a) and 2(b)] [25]. We find $S(k) \propto k^n$ with $\alpha \geq 1$ and $\beta = 1$. We then show results in the amorphous W-based film (W-based-1) in Figs. 2(c) and 2(d). We find a

\[ \sigma^2(R) = S(k) \propto R^{-\sigma/2}, \]

\[ \alpha = 1 \]

where $\sigma$ is the disorder parameter. We find a

\[ \beta = 1 \]

We then show results in the W-film-2 with data taken from Ref. [26]. The lateral size of the images is 375 nm. (c) and (d) Results obtained in a W-based thin film with strong pinning (W-film-1). The lateral sizes of the images are 1700, 1000, and 750 nm. (e) and (f) Results in LiFeAs with data from Ref. [26]. The lateral size of the images is 500 nm. (g) and (h) Results in a highly ordered W-based thin film (W-film-2) with a very weak 1D disorder potential, with data from Ref. [24]. The lateral sizes of the images are 1000 nm (2.5 and 3 T), 550 nm (4 and 5 T), and 500 nm (5.5 T). (i) and (j) Results in pure CaKFe$_4$As$_4$ from Ref. [27]. The lateral sizes of the images are 400 and 470 nm. Points are joined by lines as a guide. We plot all data in logarithmic scale and provide the power-law dependences with exponents $\alpha = 1$ for $S(k)$ and $\beta$ as shown in the legends of the figures for $\sigma^2(R)$.
The absence of a visible oscillating pattern in absence of crystalline order. Crystalline order shows the presence or is by point defects \cite{26}. In CaKFe$_4$As$_4$ pinning is very strong \cite{27,39–42}. The exponent $\beta$ of the variance $\sigma^2(R)$ is obtained from the data in Fig. 2. Random behavior is characterized by $\beta = 2$ and no crystalline order. The deviation from random towards disordered hyperuniform is seen by point defects in Co-NbSe$_2$ \cite{25} and by strong surface corrugation \cite{26}. At small magnetic fields, hexagonal order is nearly close to integers of the lattice constant and $\sigma^2(R)$ is by point defects in Co-NbSe$_2$ \cite{25} and by strong surface corrugation \cite{26}. At 2 T, with no clear hexagonal patterns observed at any length scale. As discussed in Ref. \cite{26}, the vortex lattice is highly disordered above about 5 T, although the SD is close to 1. Either SD or the density of defects or both should be large. Notice that there are no data with SD larger than about 30–40% of the intervortex distance. This resembles a Lindemann criterion. Above a certain fluctuation amplitude (SD), the ordered lattice is unstable.

We summarize all results in Table I. The actual positions of vortices in all discussed lattices are given in the Appendix (Fig. 7). We can now plot (Fig. 3) $\beta$ as a function of the standard deviation normalized by the intervortex distance $SD_{\alpha}$. (b) Exponent $\beta$ as a function of the defect density. We show results (circles) for the systems of Fig. 2, with colors similar to those in Fig. 2: Green is Co-NbSe$_2$, orange is W-film-1, blue is LiFeAs, red is W-film-2, and violet is CaKFe$_4$As$_4$. Open circles indicate results where $\sigma^2(R)$ has an oscillatory behavior, showing the presence of hexagonal order. Closed circles show situations with a fully disordered vortex lattice and no oscillations in $\sigma^2(R)$. Gray points show the result of the numerical simulation discussed in the text (the gray dashed line is a guide to the eye).

Next we analyze highly disordered vortex lattices in CaKFe$_4$As$_4$, from Ref. \cite{27}. We find [Figs. 2(i) and 2(j)] a considerable deviation from disordered hyperuniform behavior, with $\beta$ close to or larger than 1.5 and $\alpha$ less than 1. We find no signatures of oscillations in $\sigma^2(R)$.

In Appendix (Figs. 5 and 6). The resulting polycrystalline vortex lattices are shown in the Appendix (Figs. 5 and 6). In Figs. 2(e) and 2(f) we show results in W-film-2, from Ref. \cite{24}. At small magnetic fields, hexagonal order is nearly perfect with just a few dislocations. This provides oscillations close to integers of the lattice constant and $\beta \approx 1$. When increasing the magnetic field, the number of defects in the lattice increases, particularly above about 4 T. The corresponding angular and positional correlation functions show the proliferation of topological defects in the lattice \cite{24}. At 5 T the lattice has no long-range positional nor orientational order \cite{24}. Here we show that $S(k)$ and $\sigma^2(R)$ remain with the same power-law dependencies, with $\beta = 1.1$ and $\alpha \geq 1$ for magnetic fields below or equal to 5 T. At 5T, although the lattice has neither long-range positional nor orientational order (see Ref. \cite{24}), the length scale for orientational order is sufficiently large to provide $\beta \approx 1$, i.e., near disordered hyperuniform behavior. When reaching 5.5 T, the decay length for orientational order goes from about five times $a_0$ down to a couple of $a_0$ \cite{24}. At the same time, the oscillations in $\sigma^2(R)$ vanish totally at 5.5 T. However, there is also a strong deviation from hyperuniformity, with an increase of $\beta$ to 1.6. Thus, the onset of strong disorder leads to a tendency to form a random distribution of vortices.

In Figs. 2(g) and 2(h) we show results in LiFeAs, from Ref. \cite{26}. The vortex lattice is highly disordered above about 2 T, with no clear hexagonal patterns observed at any length scale. As discussed in Ref. \cite{26}, the structure factor has a square-shaped orientational dependence, which shows that overall there is a tendency of the vortex lattice to lock its shape to the square crystal lattice. Note that oscillations in $\sigma^2(R)$ are much less pronounced than in other cases, although these are clearly visible at 2 T. The coefficient $\alpha$ is slightly smaller than one and $\beta$ is close to one at 2 T but increases with the magnetic field. This situation is close to a disordered hyperuniform arrangement. Note that, despite the presence of disorder, the vortex interaction is not negligible since there the tendency of the disordered vortex lattice is to show square arrangements at small distances \cite{26}.

Table I. Superconducting parameters of the systems studied here, critical temperature $T_c$ and critical magnetic field $H_{c2}$. Pinning is by point defects in Co-NbSe$_2$ \cite{25} and by strong surface corrugation in W-film-1 and is in both cases strong. In the W-film-2, there is an extremely weak interaction between the vortex lattice and the film, due to a weak linear nanofabricated structure \cite{24}. In CaKFe$_4$As$_4$ pinning is very strong \cite{27,39–42}.

| Parameter          | Co-NbSe$_2$ | W-film-1 | W-film-2 | LiFeAs | CaKFe$_4$As$_4$ |
|--------------------|-------------|-----------|-----------|--------|-----------------|
| $T_c$ (K)          | 5.7         | 4         | 4         | 17.5   | 35              |
| $H_{c2}$ (T)       | 3.4         | 6.4       | 6.4       | 15     | $\approx 90$   |
| $\beta$            | 1           | 1         | 1.3       | 1.1–1.5| 1.7             |
| oscillating $\sigma^2(R)$ | yes | yes | yes-no | yes-no | no              |

FIG. 3. (a) Exponent of $\sigma^2(R)$, $\beta$, as a function of the standard deviation normalized by the intervortex distance $SD_{\alpha}$. (b) Exponent $\beta$ as a function of the defect density. We show results (circles) for the systems of Fig. 2, with colors similar to those in Fig. 2: Green is Co-NbSe$_2$, orange is W-film-1, blue is LiFeAs, red is W-film-2, and violet is CaKFe$_4$As$_4$. Open circles indicate results where $\sigma^2(R)$ has an oscillatory behavior, showing the presence of hexagonal order. Closed circles show situations with a fully disordered vortex lattice and no oscillations in $\sigma^2(R)$. Gray points show the result of the numerical simulation discussed in the text (the gray dashed line is a guide to the eye).
The vortex lattice arrangements are a consequence of the balance between elastic and pinning energies. In Co-doped NbSe₂ and in the W-film-1 thin film, pinning is strong but structured, leading to hexagonal vortex clusters observed at all magnetic fields. On the other hand, in CaKFe₄As₄, pinning centers are so strong and randomly distributed that the vortex lattice is essentially randomly disordered in the whole range of magnetic fields studied [27,39–42]. In W-film-2, where the disorder potential is very weak, the vortex lattice is only disordered when it is very soft, very close to $H_{c2}$, but then the vortex distribution shows a strong tendency to disorder randomly, because intervortex interactions are very weak. In LiFeAs we have qualitatively the same behavior as in CaKFe₄As₄, but with weaker pinning. Furthermore, the intervortex interaction with nonlocal contributions due to the influence of the crystal lattice symmetry [18,47] is still important and responsible for decreasing density fluctuations.

The vortex lattice of LiFeAs depends strongly on the temperature range where the magnetic field is applied. There are measurements showing hexagonal vortex lattices in the same magnetic field range [48], whereas the ones we have used here [26] and neutron scattering experiments [49] provide disordered lattices. Notice that the disordered lattices discussed here are locked to the crystal lattice. This is a rather peculiar combination of long-range interaction and disorder. Locking can be explained by nonlocal corrections to the London model that favor a fourfold vortex lattice [18,47].

It is relevant to note that the only disordered lattices with close to hyperuniform behavior ($\beta \approx 1$, points in Fig. 3 corresponding to LiFeAs at small magnetic fields and to W-film-2 at 5 T) have relatively small $\sigma_{\delta 0}$. In the case of LiFeAs, data follow closely the calculations. Therefore, we expect that fully disordered vortex lattices will not fall to the behavior of lattices showing fluctuations in $\sigma^2(R)$ as a consequence of short-range order (open circles with $\beta \approx 1$ in the representation of Fig. 3), but rather follow the smooth increase of $\beta$ with the amount of disorder predicted by the model.

Calculations show that the vortex glass can present disordered hyperuniformity in a range of magnetic field and temperatures in the presence of strong repulsive interactions and quenched disorder [11]. Those authors proposed a phase diagram with the close to disordered hyperuniform behavior in between the Bragg glass and the random vortex glass. Our results confirm indeed the presence of this intermediate state and show that it can be obtained as a balance between interaction and pinning.

In a recent work, vortex lattices at very small magnetic fields were analyzed in view of their hyperuniform properties. Those authors analyzed images of the high-\(T_c\) cuprate superconductor Bi₂Sr₂CaCu₂O₈+\(x\) with magnetic Bitter decoration in the presence of disorder [23]. At small magnetic fields, vortices are very far apart and their mutual repulsion is small [50,51]. Vortex arrangements are then strongly influenced by their interaction with pinning centers. Furthermore, at high temperatures, close to the transition to the normal state, the vortex lattice melts, leading to the vortex liquid which is a dynamic tangle of vortices [52,53]. Therefore, experiments at small magnetic fields are made by cooling from the liquid phase, which results in quenched vortex arrangements [54]. The authors of Ref. [23] concluded that long-wavelength fluctuations are systematically suppressed in the vortex lattice at small magnetic fields, as a consequence of the hydrodynamic
properties of the liquid phase, which leads to deviations from fully random vortex distributions.

V. CONCLUSION

In summary, we have analyzed the conditions for the formation of disordered hyperuniform vortex lattices in superconductors at high magnetic fields. The vortex lattice shows a tendency away from random behavior and towards hyperuniformity when the number of pinning centers is between 30% and 40%. We find that the length scale of the interaction plays a minor role in determining the strength of density fluctuations. Instead, the balance between pinning and vortex lattice stiffness controls density fluctuations, with a continuous variation between density fluctuations and disorder in the lattice. We show that the decreased density fluctuations require intervortex interactions. We conclude that we can identify emergent correlations in a vortex lattice using the structure factor $S(k)$ and number variance $\sigma^2(R)$ and show that disordered vortex lattices are characterized by suppressed density fluctuations.

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FIG. 6. Results in Co-doped NbSe$_2$, from Ref. [25], for (a) 2.5 T, (b) 2.7 T, and (c) 3.3 T. We represent vortices, triangulation, lattice orientation, and red scale bars as in Fig. 5.

FIG. 7. Vortex positions in all the images analyzed in this work. (a) Vortex positions in Co-doped NbSe$_2$, from Ref. [25]. (b) Results obtained in a W-based thin film with strong pinning (W-film-1). (c) Results in a highly ordered W-based thin film (W-film-2) with a very weak 1D disorder potential, with data from Ref. [24]. (d) Results in LiFeAs, obtained from data from Ref. [26]. (e) Results in pure CaKFe$_4$As$_4$, from Ref. [27]. Magnetic fields are given at each panel. Scale bars in red are 200 nm long.

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APPENDIX

To calculate $\sigma^2(R)$ and $S(k)$ we start by finding vortex positions following Ref. [24]. We maximize the contrast in the image, inserting a threshold that gives a clear view of vortices as single colored and extended disks. We calculate the center of mass of each disk and use this to identify the position of each vortex. This leads to the matrix of points shown in Fig. 4(b). We then make the Fourier transform to find $S(k)$. To calculate $\sigma^2(R)$ we follow Ref. [11]. We generate circles of size $R$ centered at randomly generated positions and increase $R$ from the average intervortex distance $a_0$ to nearly the size of the image. We use two conditions. First, circles have to be complete and within the image. Second, circles cannot...

FIG. 8. Density of defects (vortices with coordination number different from 6) vs the density of fixed points (pinned vortices) in the matrix positions from the calculations described in the text.
In Figs. 5(a)–5(e) we show results with increasing magnetic fields. We observe that regions with different vortex lattice orientation are separated by regions with a large number of dislocations. The smallest ordered regions appear at relatively large magnetic fields. A further increase of the magnetic field towards $H_c$ leads to a disordered random configuration, shown in Fig. 5(e), where we can still identify hexagonally ordered regions of finite size. This lattice is accordingly still hyperuniform, but with a large density of defects (about 40%).

In Fig. 6 we show the vortex lattice in Co-doped NbSe$_2$. Regions with hexagonal order are observed at all magnetic fields [Figs. 6(a)–6(c)] and the number of defects can be quite large, of about 40% [25].

For completeness we show all vortex positions used to calculate the structure factor and number variance in Fig. 2 in Fig. 7. We also show the density of defects vs the pinned points from the model calculations in Fig. 8.

[1] S. Torquato and F. H. Stillinger, Phys. Rev. E 68, 041113 (2003).
[2] S. Torquato, Phys. Rep. 745, 1 (2018).
[3] W. Man, M. Floresco, E. P. Williamson, Y. He, S. R. Hashemizad, B. Y. C. Leung, D. R. Liner, S. Torquato, P. M. Chaikin, and P. J. Steinhardt, Proc. Natl. Acad. Sci. USA 110, 15886 (2013).
[4] Y. Jiao, T. Lau, H. Hatzikirou, M. Meyer-Hermann, J. C. Corbo, and S. Torquato, Phys. Rev. E 89, 022721 (2014).
[5] R. Kurita and E. R. Weeks, Phys. Rev. E 84, 030401(R) (2011).
[6] R. Dreyfus, Y. Xu, T. Still, L. A. Hough, A. G. Yodh, and S. Torquato, Phys. Rev. E 91, 012302 (2015).
[7] D. Henxer, A. J. Liu, and S. R. Nagel, Phys. Rev. Lett. 121, 115501 (2018).
[8] D. Henxer, P. Urban, and F. Zamponi, Phys. Rev. Lett. 123, 068003 (2019).
[9] C. E. Zachary, Y. Jiao, and S. Torquato, Phys. Rev. Lett. 106, 178001 (2011).
[10] J. H. Weis, R. Jeanneret, R. Dreyfus, and D. Bartolo, Phys. Rev. Lett. 115, 108301 (2015).
[11] Q. Le Thien, D. McDermott, C. J. O. Reichhardt, and C. Reichhardt, Phys. Rev. B 96, 094516 (2017).
[12] I. A. Sadykovsky, Y. L. Wang, Z.-L. Xiao, W.-K. Kwok, and A. Glatz, Phys. Rev. B 95, 075303 (2017).
[13] H. Diamant, J. Club Condens. Matter Phys. (2019).
[14] G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. 66, 1125 (1994).
[15] E. H. Brandt, Rep. Prog. Phys. 58, 1465 (1995).
[16] J. Friedel, P. G. De Gennes, and J. Matricon, Appl. Phys. Lett. 2, 119 (1963).
[17] P. G. de Gennes and J. Matricon, Rev. Mod. Phys. 36, 45 (1964).
[18] V. G. Kogan, Phys. Rev. B 24, 1572 (1981).
[19] A. I. Buzdin, A. S. Mel’nikov, and A. V. Samokhvalov, J. Supercond. Novel Magn. 26, 2853 (2013).
[20] J. Pearl, Appl. Phys. Lett. 5, 65 (1964).
[21] E. H. Brandt, Phys. Rev. B 48, 6699 (1993).
[22] E. Herrera, I. Guillamón, J. A. Galvis, A. Correa, A. Fente, S. Vieira, H. Suderow, A. Y. Martynovich, and V. G. Kogan, Phys. Rev. B 96, 184502 (2017).
[23] G. Rumi, J. Aragón Sánchez, F. Elías, R. Cortés Maldonado, J. Puig, N. R. Cejas Bolecek, G. Nieves, M. Konczykowski, Y. Fasano, and A. B. Kolton, Phys. Rev. Res. 1, 033057 (2019).
[24] I. Guillamón, R. Córdoba, J. Sesé, J. M. De Teresa, M. R. Ibarra, S. Vieira, and H. Suderow, Nat. Phys. 10, 851 (2014).
[25] M. Iavarone, R. Di Capua, G. Karapetrov, A. E. Koshevlev, D. Rosenmann, H. Claus, C. D. Malliakas, M. G. Kanatzidis, T. Nishizaki, and N. Kobayashi, Phys. Rev. B 78, 174518 (2008).
[26] T. Hanaguri, K. Kitagawa, K. Matsubayashi, Y. Mazaki, Y. Uwatoko, and H. Takagi, Phys. Rev. B 85, 214505 (2012).
[27] A. Fente, W. R. Meier, T. Kong, V. G. Kogan, S. L. Bud’ko, P. C. Canfield, I. Guillamón, and H. Suderow, Phys. Rev. B 97, 134501 (2018).
[28] I. Guillamón, H. Suderow, P. Kulkarni, S. Vieira, R. Córdoba, J. Sesé, J. D. Teresa, M. Ibarra, G. Shaw, and S. Bannerjee, Physica C 503, 70 (2014).
[29] T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. 72, 1530 (1994).
[30] H. F. Hess, R. B. Robinson, R. C. Dynes, J. M. Valles, and J. V. Waszczak, Phys. Rev. Lett. 62, 214 (1989).
[31] I. Guillamón, H. Suderow, S. Vieira, L. Cario, P. Diener, and P. Rodière, Phys. Rev. Lett. 101, 166407 (2008).
[32] O. Fischer, M. Kugler, I. Maggio-Apriele, C. Berthod, and C. Renner, Rev. Mod. Phys. 79, 353 (2007).
[33] H. Suderow, I. Guillamón, J. G. Rodrigó, and S. Vieira, Supercond. Sci. Technol. 27, 063001 (2014).
[34] A. M. Troyanovski, J. Aarts, and P. H. Kes, Nature (London) 399, 665 (1999).
[35] M. A. Klatt, J. Kim, and S. Torquato, Phys. Rev. E 101, 032118 (2020).
[36] H. Suderow, I. Guillamon, and S. Vieira, Rev. Sci. Instrum. 82, 033711 (2011).
[37] I. Guillamón, H. Suderow, S. Vieira, A. Fernández-Pacheco, J. Sesé, R. Córdoba, J. M. D. Teresa, and M. R. Ibarra, New J. Phys. 10, 093005 (2008).
[38] R. Córdoba, T. I. Baturina, J. Sesé, A. Y. Mironov, J. M. De Teresa, M. R. Ibarra, D. A. Nasinov, A. K. Gutakovskii, A. V. Latyshev, I. Guillamón, H. Suderow, S. Vieira, M. R. Baklanov, J. J. Palacios, and V. M. Vinokur, Nat. Commun. 4, 1437 (2013).
[39] S. J. Singh, M. Bristow, W. R. Meier, P. Taylor, S. J. Blundell, P. C. Canfield, and A. I. Coldea, Phys. Rev. Mater. 2, 074802 (2018).
[40] S. Pyon, A. Takahashi, I. Veshchunov, T. Tamegai, S. Ishida, A. Iyo, H. Eisaki, M. Imai, H. Abe, T. Terashima, and A. Ichinose, Phys. Rev. B 99, 104506 (2019).
[41] N. Haberkorn, M. Xu, W. R. Meier, J. Schmidt, S. L. Bud’ko, and P. C. Canfield, Phys. Rev. B 100, 064524 (2019).
[42] S. Ishida, A. Iyo, H. Ogino, H. Eisaki, N. Takeshita, K. Kawashima, K. Yanagisawa, Y. Kobayashi, K. Kimoto, H. Abe, M. Imai, J.-i. Shimoyama, and M. Eisterer, npj Quantum Mater. 4, 27 (2019).
[43] T. Klein, I. Joumard, S. Blanchard, J. Marcus, R. Cubitt, T. Giamarchi, and P. Le Doussal, Nature (London) 413, 404 (2001).
[44] T. Giamarchi and P. Le Doussal, Phys. Rev. B 52, 1242 (1995).
[45] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B 43, 130 (1991).
[46] J. Kierfeld, T. Nattermann, and T. Hwa, Phys. Rev. B 55, 626 (1997).
[47] V. G. Kogan, M. Bullock, B. Harmon, P. Miranović, L. Dobrosavljević-Grujić, P. L. Gammel, and D. J. Bishop, Phys. Rev. B 55, R8693 (1997).
[48] S. S. Zhang, J.-X. Yin, G. Dai, H. Zheng, G. Chang, I. Belopolski, X. Wang, H. Lin, Z. Wang, C. Jin, and M. Z. Hasan, Phys. Rev. B 99, 161103(R) (2019).
[49] D. S. Inosov, J. S. White, D. V. Evtsushinsky, I. V. Morozov, A. Cameron, U. Stockert, V. B. Zabolotnyy, T. K. Kim, A. A. Kordyuk, S. V. Borisenko, E. M. Forgan, R. Klingeler, J. T. Park, S. Wurmehl, A. N. Vasilyev, G. Behr, C. D. Dewhurst, and V. Hinkov, Phys. Rev. Lett. 104, 187001 (2010).
[50] R. Prozorov, A. F. Fidler, J. R. Hoberg, and P. C. Canfield, Nat. Phys. 4, 327 (2008).
[51] J. B. Llorens, L. Embon, A. Correa, J. D. González, E. Herrera, I. Guillamón, R. F. Luccas, J. Azpeitia, F. J. Mompeán, M. García-Hernández, C. Munuera, J. A. Sánchez, Y. Fasano, M. V. Milosevic, H. Soderow, and Y. Anahory, Phys. Rev. Research 2, 013329 (2020).
[52] R. Cubitt, E. M. Forgan, G. Yang, S. L. Lee, D. M. Paul, H. A. Mook, M. Yethiraj, P. H. Kes, T. W. Li, A. A. Menovsky, Z. Tarnawski, and K. Mortensen, Nature (London) 365, 407 (1993).
[53] E. Zeldov, D. Majer, M. Konczykowski, V. B. Geshkenbein, V. M. Vinokur, and H. Shtrikman, Nature (London) 375, 373 (1995).
[54] M. Marchevsky, P. Kes, and J. Aarts, Physica C 282–287, 2083 (1997).
[55] I. G. Serrano, J. Sesé, I. Guillamón, H. Soderow, S. Vieira, M. Ibarra, and J. M. De Teresa, Beilstein J. Nanotechnol. 7, 1698 (2016).