Experimental Study of Size Effects on the Deformation Strength and Failure Characteristics of Hard Rocks under True Triaxial Compression

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Size effect has always been the focus of rock mechanics as a bridge between laboratory test and engineering site. Previously, the research conditions and objects of the rock size effect have mostly focused on cylindrical rock samples with different height-to-diameter ratios (H/Ds) under uniaxial or conventional triaxial compression, while there has been little research on the rock size effect under true triaxial compression (TTC), especially rectangular rock samples with different sizes and the same length-to-width-to-height ratio. Based on this, the deformation, strength, and failure characteristics of Beishan (BS) granite and Baihetan (BHT) basalt with different sample sizes under TTC were studied by a comparative analysis method. The size effect of deformation and failure characteristics under TTC are not obvious, including stress-strain curves, Young’s modulus, peak strains, failure angles, and macrofailure mode. However, the damage stress \( \sigma_{cd} \) and peak strength \( \sigma_p \) have obvious size effect; that is, the smaller the sample size is, the higher the strength is. Additionally, the relationship among the peak strength, sample size, and intermediate principal stress \( \sigma_2 \) is power function. In addition, by comparing the peak strength increment caused by the sample size of the two types of rocks, the \( \sigma_p \) of the fine-grained BHT basalt is more sensitive to sample size than that of the coarse-grained BS granite. Finally, by analyzing the relationship between the size of the mineral grains or clusters in the two types of hard rocks and the complexity of crack propagation in the fracture surface under TTC, it is suggested that the minimum side length of rock samples should not be less than 10 times the maximum mineral clusters (such as feldspar phenocrysts in BHT basalt). In addition, the method of estimating elastic strain is established by analyzing the relationship between the size of the rock sample \( \sigma_2 \) and the elastic strain under TTC.

1. Introduction

For a long time, how to combine the laboratory test results with the monitoring results of the project has been a major problem faced by rock mechanics. The size difference between indoor rock samples and engineering rock mass is the most direct obstacle to this problem, and research on the size effect is considered to be an important part of solving this problem. To date, three main theories on the size effect have been proposed: (a) the Weibull statistical theory of the size effect represented by [1], (b) the energy release-based theory of the size effect represented by [2], and (c) the fractal approach of the size effect represented in [3]. Previous studies [4–8] have focused more on the size effect of cylindrical rock samples with different aspect ratios under uniaxial compression. Meanwhile, the reason why there were few studies on the size effect of rocks under conventional triaxial conditions was that most rocks have showed brittle ductile transition characteristics with the increase of confining pressure [9].

It is well known that the in situ stress in field engineering typically satisfies \( \sigma_1 > \sigma_2 > \sigma_3 \) (\( \sigma_1 \): the maximum principal stress, \( \sigma_2 \): the intermediate principal stress, and \( \sigma_3 \): the minimum principal stress). After [10, 11] designed the first true triaxial testing machine for rock mechanics, the research on and application of TTC testing machines have
become popular in the field of rock mechanics because this type of machine can reflect $\sigma_2$. However, due to different research needs and the differences in research technologies, the sample sizes are largely different. In the studies of [10, 11], the sample size was $15 \times 15 \times 30 \text{ mm}^3$; in [12], the sample size was $19 \times 19 \times 38 \text{ mm}^3$; in [13], the sample sizes were $57 \times 57 \times 25 \text{ mm}^3$ and $76 \times 76 \times 178 \text{ mm}^3$; in [14–18], the sample size was $50 \times 50 \times 100 \text{ mm}^3$; in [19], the sample size was $150 \times 60 \times 30 \text{ mm}^3$; and in [20], the sample size was $100 \times 100 \times 100 \text{ mm}^3$. The sample sizes used in the previously mentioned studies were quite different, and most studies only tested samples of one size. However, there have been few studies on rock samples with different sizes under TTC, and only the studies [21–23] have investigated the mechanical and failure characteristics of the same rock with different aspect ratios under true triaxial unloading conditions. Moreover, these studies did not provide the strength, deformation, and failure characteristics of rock samples with a fixed aspect ratio but different sizes under TTC. Meanwhile, the existing strength criterion does not consider the size effect of rock, and there have been few studies on certain important issues, for example, which sample size is more suitable for the study of crack propagation on fracture surfaces.

In this study, the deformation, strength, and failure of BS granite and BHT basalt with different sample sizes under TTC were analyzed. Moreover, by analyzing the relationship between the complexity of crack propagation and mineral particle size in the fracture surface with different sample sizes, the recommended sample sizes for analyzing crack propagation in the fracture surface are determined. The results of this study can help to understand the size effect under TTC.

2. Test Scheme and Process

2.1. Specimen Preparation. BS granite and BHT basalt are selected as the research objects. To prevent the dispersion of test results caused by the different rock samples, the samples of the same rock with different sizes are selected from the same parent rock, and the samples with large differences are eliminated, the rock samples with the same or similar P-wave velocity are selected for the test. The specimens were used the same processing technologies, and the length: width: height ratio of the specimens was strictly controlled to 1:1:2. The sizes of the samples were $25 \times 25 \times 50 \text{ mm}^3$ (SS), $35 \times 35 \times 70 \text{ mm}^3$ (SM), and $50 \times 50 \times 100 \text{ mm}^3$ (SL), and dimensional tolerance and perpendicularity tolerance were given as ±0.01 and 0.02 mm for each side, respectively. The basic physical and mechanical parameters of these two types of rocks are shown in Table 1.

Figure 1 shows the size and photos of rock selected in this study. Figure 1(a) is the grayish-green BHT basalt with scattered white plagioclase on the surface, and Figure 1(b) is the BS granite. X-ray diffraction (XRD) analysis showed that the mineral composition of the BHT basalt was feldspar 41.96%, pyroxene 45.57%, clinohochrome 6.25%, mica 4%, and quartz 2.22%, while the mineral composition of the BS granite was feldspar 51%, quartz 35%, biotite 8%, pyroxene 3%, and calcite 3%. Figure 2 shows the microstructures under cross-polarized illumination of the two types of rocks. Figure 2(a) is the microstructure of the BHT basalt; feldspar minerals with idiomorphic structures are filled by pyroxene minerals with allotriomorphic structures, with no clear boundaries between the two. According to the image scale, the grain size was 50–150 μm. Figure 2(b) is the microstructure of the BS granite. There is an alternating arrangement of feldspar minerals with idiomorphic or hypidiomorphic structures and irregular quartz. The grain size was 500–1500 μm.

2.2. Scheme and Process. True triaxial tests at the same stress level were carried out on each type of rock sample according to the sample size, in which $\sigma_3$ was constant (\(\sigma_3 = 5 \text{ MPa}\)) and the ratios of $\sigma_3 : \sigma_2$ were 1:1, 1:6, 1:12 and 1:18. The specific stress levels are shown in Table 2. The experiment was completed on the high-pressure hard rock true triaxial test system [17] developed by Northeastern University.

The test process was carried out according to the stress path shown in Figure 3(a), and the stress path was divided into the following three stages: (a) under hydrostatic pressure, $\sigma_1 = \sigma_2 = \sigma_3$ was loaded simultaneously at a rate of 0.5 MPa/s until $\sigma_3$ reached the predetermined value; (b) $\sigma_3$ was kept constant, and $\sigma_1$ and $\sigma_2$ were loaded synchronously at a loading rate of 0.5 MPa/s until $\sigma_2$ reached the target value; (c) $\sigma_2$ and $\sigma_3$ were kept constant, the stress-controlled loading method was used to increase $\sigma_1$ to approximately 60–70% of the peak strength, and then the strain-controlled loading method was used until the rock sample was completely damaged. Figure 3(b) shows the strain measurement method, and Figure 3(c) shows the measurement method of failure angle of the rock sample.

Note that the focus of this study was size effect, so when the stress-controlled loading method is changed to the strain-controlled loading method, the strain rate should be the same (2.67 × 10^{-6}/s) for all the rock samples. According to the sample width (from largest to smallest), the controlled deformation rates were as follows: 0.008 mm/min, 0.0056 mm/min, and 0.004 mm/min. The detailed control variables of each stage of the stress path are shown in Table 3.

3. Test Results

3.1. Influence of Specimen Size on Deformation Behavior. Figure 4 shows the stress-strain curves of the BS granite (Figure 4(a)) and BHT basalt (Figure 4(b)) with different sample sizes under $\sigma_3 = 5 \text{ MPa}$ and $\sigma_3 = 30 \text{ MPa}$. The stress-strain curves of the BS granite show the elastic-plastic-brittle deformation and failure process, while that of the BHT basalt shows the elastic-brittle deformation and failure process. Meanwhile, changing the size of rocks did not significantly affect the overall deformation and failure processes (the stress-strain type) because the microfractures were dominant in rocks before their peak strength was reached. However, due to the heterogeneity and the randomness of the location of macro cracks in the sample, the postpeak stress-strain curve will show some differences, especially the BHT basalt with high brittleness [24], as shown in Figure 4(b).
When the rock size was constant, Young’s modulus increased with increasing rock size. Under the same stress condition, when the sample size changed, the variation in Young’s modulus of BS granite was within 5 GPa, while that of BHT basalt were basically within 3 GPa. When the sample size changed, Young’s modulus always changed small within the rock size range of this study, as shown in the light blue area in Figures 5(a) and 5(b), indicating that Young’s modulus of the two types of rocks was less affected by the sample size and the regularity was not obvious.

Figure 6 shows the influence of sample size on the peak strain ($\varepsilon_p$) in the direction of $\sigma_2$ and $\sigma_3$ under TTC (for example, peak strain $\varepsilon_p$ refers to the strain when the stress in the direction of $\sigma_2$ reaches peak strength). For the BS granite and BHT basalt, when the rock size was constant, $\varepsilon_p$ decreased with increasing $\sigma_2$, which showed the rock was always under tensile deformation in the direction of $\sigma_3$ during the loading process, while $\varepsilon_p$ increased under the same stress condition, which showed the deformation in the direction of $\sigma_2$ changed from tensile to compression. Figure 6(a) shows the peak strain of the BS granite of different sizes in the direction of $\sigma_3$ and $\sigma_2$ under TTC. It can be seen that, under the same stress condition, the peak strain $\varepsilon_p$ in the direction of $\sigma_2$ is very close and independent of the sample size. The relationship between $\varepsilon_p$ and sample size of BHT basalt under TTC was the same as that of the BS granite, as shown in Figure 6(b). Therefore, the rock size had no significant effect on $\varepsilon_p$ within the scope of this study. However, when $\sigma_2 = 30$ MPa, the $\varepsilon_p$ of BS granite under different sizes was significantly different. The difference between the $\varepsilon_p$ for the size of SL and SS was 0.206% and was significantly higher than the changes in the peak strain under other stress states ($\sigma_2 = 5$ MPa, 60 MPa, and 90 MPa), as shown in Figure 6(a), while the $\varepsilon_p$ of the BHT basalt was hardly affected by the sample size, and the changes in $\varepsilon_p$ were always between 0.03% and 0.07%.

Under the stress condition in this paper, the peak strain ranges of the BS granite were $-0.46 < \varepsilon_p < -0.11$ and $-0.72 < \varepsilon_p < -0.42$, and those of the BHT basalt were $-0.17 < \varepsilon_p < 0.03$ and $-0.31 < \varepsilon_p < -0.11$. The analysis showed that the peak strain range of the BHT basalt was significantly smaller than that of the BS granite, which indicates that the BS granite is prone to a large yield deformation under the same stress. To sum up, the stress-strain curves, Young’s modulus, and peak strains for the BS granite and BHT basalt were related to the stress state and rock properties, but these were not significantly affected by the rock size.

### Table 1: Basic physical and mechanical parameters of the BHT basalt and BS granite samples.

| Rock type   | Density (g/cm$^3$) | P-wave velocity (m/s) | Young’s modulus (GPa) | Poisson’s ratio ($\mu$) | Tensile strength (MPa) | Grain size (μm) |
|-------------|--------------------|-----------------------|-----------------------|------------------------|------------------------|-----------------|
| BHT basalt  | 2.95               | $5650 \pm 150$        | 55~60                 | 0.22                   | 18.4                   | 50~150          |
| BS granite  | 2.69               | $5100 \pm 120$        | 50~54                 | 0.27                   | 5.06                   | 500~1500        |

Figure 5 shows Young’s modulus under the influence of rock size under TTC for the two types of rocks (the calculation method of Young’s modulus is based on [25]). When the rock size was constant, Young’s modulus increased with increasing $\sigma_2$, but there is not a strict positive correlation between Young’s modulus and rock size. Under the same stress condition, the $\varepsilon_p$ of the two types of rocks was higher than the changes in the peak strain under other stress states ($\sigma_2 = 5$ MPa, 60 MPa, and 90 MPa), as shown in Figure 6(a), while the $\varepsilon_p$ of the BHT basalt was hardly affected by the sample size, and the changes in $\varepsilon_p$ were always between 0.03% and 0.07%.

3.2 Influence of Rock Size on Characteristic Stress

Figure 7 shows that the stress point corresponding to the turning point of the volume strain curve, which is the maximum point of the volume strain curve before the peak and the calculation method refers to [25, 26]. Figure 7 shows that the $\sigma_{cd}$ of the two types of rocks showed an increasing with decreasing sample size under the same stress level. However, for the BS granite, as shown in Figure 7(a), when $\sigma_3 = 90$ MPa, the $\sigma_{cd}$ of SM was slightly lower than that $\sigma_3 = 60$ MPa, which may be caused by two reasons. On the one hand, when $\sigma_3 = 5$ MPa, $\sigma_2 = 90$ MPa
was just near the turning point where $\sigma_p$ first increased and then decreased [27]. On the other hand, reference [26] pointed out that the $\sigma_{cd}$ range of the BS granite is (0.64–0.74) $\sigma_p$ under TTC, which is within a reasonable range. In comparison, the $\sigma_{cd}/\sigma_p$ of the BHT basalt under TTC was relatively large, approximately 0.95–1.0 (Table 2), and the turning point for it, where $\sigma_p$ first increases and then decreases, is higher than BS granite. Therefore, this result is rarely found in BHT basalt: $\sigma_{cd}$ at $\sigma_2{=}90$ MPa is slightly lower than that of $\sigma_2{=}60$ MPa (Figure 7(b)).

References [28–30] showed that when $\sigma_3$ is constant, the $\sigma_p$ increases first and then decreases with increasing $\sigma_2$ under TTC. Figure 8 shows the $\sigma_p$ under the influence of sample size for the BS granite and BHT basalt. As seen from Figure 8, when $\sigma_3{=}5$ MPa, the $\sigma_p$ of the two types of rocks with different sizes increased with increasing $\sigma_2$ (since the preset $\sigma_2$ did not reach the decreasing stage of $\sigma_p$ in the two types of rocks under this condition, $\sigma_p$ did not decrease). Moreover, the smaller the sample size was, the higher the $\sigma_p$ of the two types of rocks under the same stress, such as when $\sigma_2{=}30$ MPa.

Figure 8(a) shows the $\sigma_p$ of the BS granite with different sizes under TTC. When $\sigma_2{=}\sigma_3{=}5$ MPa, as the sample size decreased from SL to SM and SS, the $\sigma_p$ increment was very small, approximately 1 MPa or 2 MPa, which indicated that the $\sigma_p$ of the BS granite was almost unaffected by sample size. In contrast, when the conventional triaxial stress condition ($\sigma_2{=}\sigma_3$) changed to the true triaxial stress condition ($\sigma_2{\neq}\sigma_3$), the size effect on the $\sigma_p$ of the BS granite was significant.

Compared with that of BHT basalt, changing the sample size (SL $\rightarrow$ SM $\rightarrow$ SS) of the BS granite will lead to the

Figure 2: Microstructures under cross-polarized illumination of the two types of rocks. (a) BHT basalt and (b) BS granite.

| Rock type | Size (mm$^3$) | $\sigma_1$ (MPa) | $\sigma_2$ (MPa) | $\sigma_{cd}$ (MPa) | $\sigma_p$ (MPa) | $\sigma_{cd}/\sigma_p$ | $\varepsilon_p$ (%) | $\varepsilon_2$ (%) | $\theta$ (%) | $A$ | Failure mode |
|-----------|-------------|----------------|----------------|-----------------|----------------|----------------|----------------|---------------|----------|----|-------------|
| BS granite | SL 5        | 5              | 144            | 202             | 0.71           | -0.452        | -0.452        | 74            | 201.82   | 0.5 | Shear       |
|           | 30          | 186            | 268            | 0.69            | -0.512         | -0.080        | 79            | 270.33        |          | 0.5 |             |
|           | 60          | 193            | 295            | 0.65            | -0.493         | -0.006        | 80            | 294.79        | Tension-shear |
|           | 90          | 197            | 308            | 0.64            | -0.577         | 0.061         | 80            | 305.97        | Tension-shear |
|           | 5           | 150            | 203            | 0.74            | -0.438         | -0.438        | 73            | 201.82        | Shear     |
|           | 30          | 190            | 285            | 0.67            | -0.602         | -0.143        | 77            | 270.33        | Shear     |
|           | 60          | 205            | 300            | 0.68            | -0.501         | -0.006        | 81            | 294.79        | Tension-shear |
|           | 90          | 204            | 313            | 0.65            | -0.589         | 0.106         | 81            | 305.97        | Tension-shear |
|           | 5           | 151            | 205            | 0.74            | -0.453         | -0.453        | 72            | 201.82        | Shear     |
|           | 30          | 200            | 292            | 0.69            | -0.718         | -0.163        | 78            | 270.33        | Shear     |
|           | 60          | 212            | 311            | 0.68            | -0.612         | 0.021         | 82            | 294.79        | Tension-shear |
|           | 90          | 211            | 325            | 0.65            | -0.608         | 0.052         | 81            | 305.97        | Tension-shear |
| BHT basalt| SL 5        | 5              | 250            | 250             | 1.00           | -0.168        | -0.168        | 74            | 247.94   | 0.5 | Tension-shear |
|           | 30          | 278            | 282            | 0.99            | -0.185         | -0.023        | 78            | 277.89        | Tension-shear |
|           | 60          | 324            | 324            | 1.00            | -0.186         | 0.010         | 78            | 318.65        | Tension-shear |
|           | 90          | 341            | 350            | 0.97            | -0.270         | 0.030         | 80            | 346.57        | Tension-shear |
|           | 5           | 261            | 261            | 1.00            | -0.119         | -0.119        | 75            | 265.25        | Tension-shear |
|           | 30          | 286            | 296            | 0.97            | -0.202         | -0.100        | 79            | 277.89        | Tension-shear |
|           | 60          | 339            | 339            | 1.00            | -0.224         | -0.019        | 79            | 318.65        | Tension-shear |
|           | 90          | 353            | 358            | 0.99            | -0.230         | 0.028         | 80            | 346.57        | Tension-shear |
|           | 5           | 274            | 284            | 0.96            | -0.165         | -0.165        | 75            | 265.25        | Tension-shear |
|           | 30          | 329            | 334            | 0.99            | -0.218         | -0.073        | 78            | 277.89        | Tension-shear |
|           | 60          | 362            | 366            | 0.99            | -0.233         | -0.032        | 80            | 318.65        | Tension-shear |
|           | 90          | 367            | 386            | 0.95            | -0.302         | -0.029        | 88            | 346.57        | Tension-shear |
unstable change of the peak strength increment \((\sigma_{pSM} - \sigma_{pSL})/\sigma_{pSL}\) or \((\sigma_{pSS} - \sigma_{pSM})/\sigma_{pSM}\). For example, the \(\sigma_p\) increment was 6.34% when the sample size of the BS granite decreased from SL to SM under the stress condition \(\sigma_3 \geq 5\) MPa and \(\sigma_2 \leq 30\) MPa, which was significantly higher than the average 2.52% under other stress states. In contrast, when the sample size of BHT basalt decreased from SL to SM and from SM to SS, the percentages of the \(\sigma_p\) increment were always maintained at approximately 4.1% and 9.3%, respectively, which was obtained by comparing the width of the blue or yellow areas enclosed by the changes in the peak strength caused by the sample size under each stress in Figures 8(a) and 8(b). However, Figure 8(b) showed that the size effect on \(\sigma_p\) of BHT basalt was obvious in both the conventional triaxial and TTC. When the sample size decreased from SL to SM, change of the width of the blue strip area was consistent with the change of the width of the yellow strip area when the sample size decreased from SM to SS, and there were no abrupt changes in the \(\sigma_p\) increment under a certain stress, which was different from the results for the BS granite.

The previously mentioned analysis showed that the sensitivity of \(\sigma_p\) for BS granite and BHT basalt to sample size was different. When the sample size decreased from SL to SS, the \(\sigma_p\) increment of the BS granite was approximately less than 10%, while that of the BHT basalt was approximately 20%. For BHT basalt, the \(\sigma_p\) increment caused by the reduction of sample size from SM to SS was almost twice that caused by the reduction of sample size from SL to SM. Meanwhile, the \(\sigma_p\) increment of the BS granite also increased as the sample size changed, but the changes were very small. The smaller the sample size of the BHT basalt, the higher the sensitivity of the peak strength to the size effect, indicating that the sensitivity of the peak strength of BHT basalt to the size effect was higher than that of BS granite under the same stress.

To clarify the relationship between the peak strength, the sample size, and the stress state under TTC, the statistical analysis of the test results was carried out, as shown in Figure 9. The volume of the SL sample was \(V\), and the volumes of the SM and SS samples were normalized according to \(V\), such as \(V_{SM} = 0.343 V\) and \(V_{SS} = 0.125 V\). In Figure 9, the normalized results are plotted as the horizontal axis, and the peak strength is the vertical axis. Figure 9(a) is the results of BS granite, and Figure 9(b) shows those of BHT basalt.

As can be seen from Figure 9, the peak strengths of both types of rocks decreased with increasing sample size under
the same stress state. All test data were fitted by power function, and the fitting variance of data was greater than 0.9, indicating that the power function could well express the relationship between peak strength, sample size, and stress state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state. Note that multiple curves were used to fit the experimental data under TTC because of the variable of state.

Substituting equation (2) into (1), we derive the following:

$$\sigma_p = \sigma_{pV} \times \left(\frac{V'}{V}\right)^B.$$  \hspace{1cm} (3)

Because $B$ is related to $\sigma_2$, the fitting relationship between them is obtained (Figure 10), which shows that the fitting result is well ($R^2 = 1$). Thus, the relationship between $B$ and $\sigma_2$ of the BS granite and BHT basalt can be expressed as follows:

$$B = a\sigma_2^3 + b\sigma_2 + c,$$  \hspace{1cm} (4)

where $a$, $b$, and $c$ are the fitting parameters related to lithology, as shown in Figure 10. The general expression of $\sigma_p$ was obtained by substituting equation (4) into (3):

$$\sigma_p = \sigma_{pV} \times \left(\frac{V'}{V}\right)^{a\sigma_2^3 + b\sigma_2 + c}.$$  \hspace{1cm} (5)

According to $a$, $b$, and $c$ of the BS granite and BHT basalt obtained in Figure 10, the binomial expression of $B$ and $\sigma_2$ can be expressed as follows:

$$B_{\text{Granite}} = -2.778 \times 10^{-6} \sigma_2^2 + 7.167 \times 10^{-4} \sigma_2 - 0.06, \quad R^2 = 1,$$  \hspace{1cm} (6)

$$B_{\text{Basalt}} = -7.222 \times 10^{-6} \sigma_2^2 + 1.45 \times 10^{-3} \sigma_2 - 0.12, \quad R^2 = 1.$$  \hspace{1cm} (7)

Equations (6) and (7) can be substituted into equation (5) to obtain an expression relating $\sigma_p$, $V$, and $\sigma_2$ for the BS granite and BHT basalt:

$$\sigma_p^{\text{Granite}} = \sigma_{pV} \times \left(\frac{V'}{V}\right)^{-2.778 \times 10^{-6} \sigma_2^2 + 7.167 \times 10^{-4} \sigma_2 - 0.06},$$  \hspace{1cm} (8)

$$\sigma_p^{\text{Basalt}} = \sigma_{pV} \times \left(\frac{V'}{V}\right)^{-7.222 \times 10^{-6} \sigma_2^2 + 1.45 \times 10^{-3} \sigma_2 - 0.12}.$$  \hspace{1cm} (8)

The relationship of the peak strength and damage stress to the sample size of the BS granite and BHT basalt under TTC showed that the variation amplitude of the characteristic stress increment caused by the size effect in the fine-grained BHT basalt was obviously smaller than that of the medium- to coarse-grained BS granite, and the characteristic stress of the two types of rocks was
obviously affected by the sample size. The relationships among the peak strength, rock sample size, and intermediate principal stress could be represented by a power function.

3.3. Influence of Sample Size on Failure Characteristics. To better compare the rock failure modes with different sizes, the failure pictures of the samples with different sizes were enlarged to the same size. Figure 11 shows the failure photos of BS granite and BHT basalt under different sizes at \( \sigma_3 = 5 \text{ MPa} \) and \( \sigma_2 = 30 \text{ MPa} \), and Table 2 shows the failure modes and fracture angles of the rocks under TTC. It can be seen from Figure 11 that both BS granite and BHT basalt show macroshear failure under the same stress condition \( \sigma_3 = 5 \text{ MPa} \) and \( \sigma_2 = 30 \text{ MPa} \), which indicates that the sample size did not change the macroscopic failure mode for the two types of rocks under the same stress. However, reducing the sample size may lead to secondary cracks near the main crack near the center of the sample, which are nearly parallel to the direction of \( \sigma_1 \), making the fracture surface more complex as shown in areas enclosed by red lines in

**Figure 5:** Young’s modulus characteristics under the influence of rock sample size under TTC. (a) BS granite and (b) BHT basalt.

**Figure 6:** Influence of sample size on the peak strain in the direction of \( \sigma_3 \) and \( \sigma_2 \) under TTC. (a) BS granite and (b) BHT basalt.
Figures 11(c), 11(f), and 12(a). Additionally, for the BHT basalt with the size of SS, the cracks are easy to develop along the mineral cluster (feldspar phenocryst) during its propagation except for secondary cracks in the fracture surface, as shown in the areas enclosed by blue lines in Figures 11(e), 11(f), and 12(b), which is possibly because the size of the mineral clusters was of the same order of magnitude as the length of the shortest edge of rock samples. Therefore, the generation of secondary cracks and cracks along the mineral clusters increased the complexity of the fracture surfaces of the small-sized rock samples.

Figure 13 shows the failure angles of the two types of rocks with different sizes under TTC, and the measurement of the failure angle refers to [17]. For the tortuous fracture surface, the near-linear measuring method was used, as shown in Figure 3(c). Figure 13 shows that when the sample size was constant and $\sigma_2$ increased, the failure angle $\theta$ increased. However, changing the sample size did not
significantly impact the angle of the fracture surface under the same stress. For example, in Figure 13(a), when the sample size of the BS granite decreased from SL to SS under the same stress, the fracture angle only varied by approximately $1 - 2^\circ$; in Figure 13(b), the variation in the fracture angle of the BHT basalt was basically the same as that of the BS granite under the same condition, approximately $0 - 3^\circ$.

The small variation of the fracture angle in this paper further shows that the sample size does not significantly change the macroscopic failure mode of these two types of rocks, while the reduction of sample size will lead to more complex crack propagation on the fracture surface.

### 4. Discussion

#### 4.1. Microscopic Interpretation of Complex Fracture Surfaces Caused by the Size Effect

Section 3.3 showed that changing the sample size does not significantly affect the macroscopic failure mode under the same stress. However, the crack propagation in the fracture surface became more complex when the sample size decreased to SM or SS, as shown in Figures 11(c), 11(e), 11(f), and 12. Under TTC, a macroscopic shear fracture plane with a "V" shape was easily generated [29, 31, 32]. On this type of fracture surface, especially near the center of the rock sample, almost no
obvious secondary cracks nearly parallel to the direction of $\sigma_1$ were generated during the propagation of the main cracks, as shown in Figures 11(a), 11(b), and 11(d). However, this situation is likely to occur when the sample size decreased from SL to SS under the same stress, as shown in Figures 11(c), 11(f), and 12(a).

The complex crack propagation on the failure surface of the small-sized samples may be related to the mineral grain size or the mineral grain aggregate size of the rocks. It is well known that grain size is one of the most important microstructure parameters of rock mechanical properties. Taking the BHT basalt as an example,
Figure 2(a) shows that pyroxene, feldspar, and other minerals were uniformly arranged in the matrix of the BHT basalt and the size of feldspar grain was 50–150 μm. During the diagenetic process, a large number of feldspar grains aggregated to form lath-shaped white feldspar phenocrysts, with a length of 5 mm or larger, as shown in Figures 12(b), 14(a), and 14(b). Pyroxene is a silicate rock-forming mineral with a shear modulus of 64.9 GPa, and feldspar is a brittle rock-forming mineral with a shear modulus of 28.6 GPa. Section 2.1 showed that the total composition of pyroxene and feldspar minerals in the BHT basalt accounted for more than 87%, and the two constituted the basic framework. Pyroxene is a mineral with an allotriomorphic structure, while feldspar and others are minerals with idiomorphic structures, and the structural relationship between them is similar to the relationship between water and stone in a river. Basalt is igneous rock and pyroxene (like water) can fill the holes and gaps between minerals with an allotriomorphic structure (like stone) in the process of diagenesis with no clear boundaries between the two.

Reference [33] showed that, in the 6×6 stiffness matrix represented by the Voigt notation, the stiffness of single-crystal pyroxene in all directions is larger than that of feldspar. Therefore, feldspar and other weaker minerals were more prone to brittle failure during the process of stress cracking, as demonstrated by the closed fractures on the surface of feldspar phenocrysts (Figures 2(a) and 14(c)), which may explain why the crack propagation on the fracture surface is more complex for small samples than for large samples (the complexity of the crack propagation of large rock samples was much lower).

In the samples with size SS in this study, the ratio of the size of the large feldspar phenocrysts to the length or width of the sample reached 1/5 (Figure 12(b)), and the ratio was even larger when multiple phenocrysts were aggregated. Reference [34] pointed out that grain size plays an important role in crack propagation and used numerical modeling to show that the interactions of adjacent cracks can be used to inhibit crack propagation. Additionally, they also pointed out that this inhibitory effect can gradually disappear with the increase of grain size. For the BHT basalt, the size of the feldspar phenocrysts remained unchanged, but the decreased sample size was equivalent to indirectly increasing the size of the relatively weak feldspar phenocrysts (Figures 14(a) and 14(b)), and the role of feldspar phenocrysts in the structure could not be ignored. Therefore, the inhibition effect of the surrounding cracks could be...
weakened and the crack density could increase when the cracks propagated to the vicinity of feldspar phenocrysts with a relatively large size. References [35, 36] showed that, for fine-grained materials, an increase in crack density can be equivalent to an increase in the spatial heterogeneity of the local stress field. Therefore, when cracks occur, cracks are more likely to propagate along the weak feldspar phenocrysts, which results in a complex crack morphology on the fracture surface. For example, in the area enclosed by the blue dashed line in Figure 11(e), it is obvious that a crack developed along the axis of feldspar phenocrysts. In the small area enclosed by the blue dashed line in Figure 11(f), the main crack passed through the axis of the feldspar phenocrysts and produced secondary cracks nearly parallel to the direction of $\sigma_1$. In the left main fracture plane (Figure 12(b)), two groups of feldspar phenocrysts in the two areas enclosed by blue dashed lines led to the propagation direction of some cracks (all in the same direction), resulting in poor symmetry of the left and right fracture planes and a “Y” shaped fracture plane. The cracks that grew along the feldspar phenocrysts were also observed in the area enclosed by red dashed lines in the right main fracture surface. When the direction of the feldspar phenocrysts was close to the growth direction of cracks in the fracture surface, the cracks were more likely to grow along the feldspar phenocrysts. For the BS granite, the distribution of constituent minerals was relatively uniform, and the grain size reached 500–1500 $\mu$m or even larger. When the sample size was reduced to SS, the

![Graphs showing failure angles of two types of rocks with different sizes under TTC.](image)

**Figure 13**: Failure angles of two types of rocks with different sizes under TTC. (a) BS granite and (b) BHT basalt.

![Pictures of BHT basalt and its microstructures under cross-polarized illumination.](image)

**Figure 14**: Pictures of BHT basalt and its microstructures under cross-polarized illumination. (a) BHT basalt specimen of SL size; (b) local magnification of the specimen in (a); (c) microstructures of (b) under cross-polarized illumination part.
fracture surface became complex, as shown in Figures 11(c) and 12(a). This may be one of the reasons why the International Society for Rock Mechanics (ISRM) recommends that the minimum side length of the sample should be more than 10 times the maximum grain size of minerals. Based on the previously mentioned analysis, it is better to choose a large rock sample when studying crack propagation on the fracture surface under TTC. The minimum side length of the sample should be at least 10 times larger than the maximum grain size of the mineral (ISRM) and the maximum grain size of the mineral aggregates with an idiomorphic structure (such as feldspar phenocrysts) to avoid a complex fracture surface.

4.2. Relationship between Elastic Strain and Sample Size. The test results in Section 3.2 showed that there was little relationship between Young’s modulus and sample size. The maximum Young’s modulus of the same samples with different sizes under the same stress state was fitted with \( \sigma_2 \), and the relationship between them was obtained as follows:

\[
E = d\sigma_2 + e, \quad (9)
\]

where \( d \) and \( e \) are related to rock type.

According to the calculation method of elastic strain under TTC proposed in [24],

\[
\varepsilon_{eb}^e = \varepsilon_{eb}^b + \frac{\sigma_p}{E} - \frac{\sigma_b}{E}, \quad (10)
\]

where \( \varepsilon_{eb}^b \) is the total elastic strain in the direction of \( \sigma_1 \) and \( \varepsilon_{eb}^b \) is the elastic strain in the biaxial loading stage in the direction of \( \sigma_1 \) under TTC, that is, the elastic strain in the process of Section 2.2 stress path \( b \).

Taking equation (5) into (10), the following can be obtained for calculating the total elastic strain in the direction of \( \sigma_1 \) related to the sample size under TTC:

\[
\varepsilon_{eb} = \varepsilon_{eb}^b + \frac{\sigma_p}{E} \times \left( \frac{V' \sqrt{V}}{V} \right)^{a_{eb}^b} \frac{f(\sigma_2)}{1 + c}, \quad (11)
\]

As shown in Figure 15, the linear regression coefficients of the total elastic strain of the BS granite and BHT basalt with sizes of SM and SS predicted by formula (11) in the direction of \( \sigma_1 \) were \( R^2 = 0.90 \) and \( R^2 = 0.85 \), respectively, indicating that the prediction ability of formula (11) was reasonable.

5. Conclusion

In this study, BS granite and BHT basalt with the same length : width : height ratio and different sizes were used to study the size effect under TTC conditions. The following conclusions are drawn:

(1) Regarding the deformation and failure characteristics within the range of rock size for this study, including the stress-strain curve, Young’s modulus, peak strain in the directions of \( \sigma_1 \) and \( \sigma_2 \), fracture angle, and macrofailure mode, there was almost no obvious size effect. However, the characteristics of deformation for the two types of rocks were related to the rock properties and external stress conditions.

(2) The peak strength and damage stress of the BS granite and BHT basalt were significantly affected by the sample size and \( \sigma_2 \) under TTC. As the sample size decreased, the \( \sigma_p \) and \( \sigma_{eb} \) increased. For these two types of rocks, there was a power function relationship among the peak strength, sample size, and \( \sigma_2 \) under TTC. Under the same conditions, the sensitivity of the peak strength of the fine-grained BHT basalt to the sample size was higher than that of the medium- to coarse-grained BS granite.

(3) The complex crack propagation on the fracture surface of smaller rock samples was due to indirectly
increasing the mineral grain size or mineral cluster size in the rocks. Moreover, this study suggests that the minimum side length of rock samples should be at least 10 times the maximum size of the mineral clusters when studying crack propagation on a fracture surface.

(4) The estimation method of elastic strain in a certain range of sample sizes was established by analyzing the relationship among sample size, peak strength, intermediate principal stress, and elastic strain in the direction of \( \sigma_1 \), and the prediction result was well.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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