TRANSVERSE MOMENTUM BROADENING DUE TO THE MULTIPLE SCATTERING

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Using the Drell-Yan process in hadron-nucleus collisions and deeply inelastic lepton-nucleus scattering (DIS) as examples, I show that the transverse momentum broadening can be expressed in terms of four-parton correlation functions. I argue that jet broadening in DIS provide an excellent measurement of the four-parton correlation functions and a test of QCD treatment of the multiple scattering.

1 Introduction

Most high energy collisions can be described by a single hard scattering. By studying the single scattering, we can extract the parton distribution functions and test the short-distance dynamics of strong interactions. The normal parton distributions have the interpretation of the probability distributions to find a parton within a hadron. On the other hand, in order to study quantum correlations of multi-partons inside a hadron, we need to investigate the multiple scattering. The multiple scattering is a high twist effect and is small compare to the single scattering. However, in collisions involve nucleus, the nucleus environment will enhance the multiple scattering effect. For example, when a parton propagates through a nuclear matter, the average transverse momentum is broadened due to the multiple scattering. Nuclear dependence of such broadening provides an excellent probe to study QCD dynamics beyond the single hard-scattering picture.

A reliable calculation of the multiple scattering in QCD perturbation theory requires to extend the factorization theorem beyond the leading power. Qiu and Sterman showed that the factorization theorem for hadron-hadron scattering holds at the first-nonleading power in momentum transfer, which is enough to study double scattering processes in hadronic collisions. Applying this generalized factorization theorem, Luo, Qiu and Sterman (LQS) developed a consistent treatment of the multiple scattering at the partonic level. LQS expressed the nuclear dependence of di-jet momentum imbalance in photo-nucleus collisions in terms of four-parton (twist-4) nuclear correlation functions. In the following, I will use the the deeply inelastic lepton-nucleus scattering (DIS) and the Drell-Yan process in hadron-nucleus collisions as examples to show that transverse momentum broadening is proportional to the similar four-parton correlation functions.

The predictions of the multiple scattering effects rely on accurate infor-
ation of these four-parton correlation functions. The four-parton (twist-4) correlation functions are as fundamental as the normal twist-2 parton distributions. Just like normal parton distributions, multi-parton correlation functions are non-perturbative and universal. QCD perturbation theory cannot provide the absolute prediction of these correlation functions. They can be measured in some processes and be tested in other processes. Using the Fermilab E683 data, LQS estimated the size of the relevant twist-4 parton correlation functions to be of the order of $0.05 - 0.1 \text{ GeV}^2$ times typical twist-2 parton distributions. However, Fermilab and CERN data on nuclear enhancement of the average Drell-Yan transverse momentum, $\Delta \langle q_T^2 \rangle$, prefer a much smaller size of the four-parton correlation functions. This discrepancy may result from different higher order contribution to nuclear enhancement of the average Drell-Yan transverse momentum and the di-jet momentum imbalance. It is necessary to study the higher order corrections to these two observables in order to test QCD treatment of multiple scattering. Meanwhile, it is also important to use jet broadening in DIS as an independent measurement of the four-parton correlation functions.

In the following, I first present the derivation of the transverse momentum broadening in Drell-Yan process and jet broadening in DIS. I then argue that jet broadening in DIS is an excellent observable to study the four-parton correlation function.

## 2 Transverse momentum broadening for the Drell-Yan pairs

Consider the Drell-Yan process in hadron-nucleus collisions, $h(p') + A(p) \rightarrow \ell^+ \ell^-(q) + X$, where $q$ is the four-momentum for the virtual photon $\gamma^*$ which decays into the lepton pair. $p'$ is the momentum for the incoming hadron and $p$ is the momentum per nucleon for the nucleus with the atomic number $A$. In order to extract the effect due to the multiple scattering, we define the Drell-Yan transverse momentum broadening as

$$\Delta \langle q_T^2 \rangle \equiv \langle q_T^2 \rangle^h_A - \langle q_T^2 \rangle^h_N,$$

with $q_T$ the transverse momentum of the Drell-Yan pair, and the the averaged transverse momentum square is defined as

$$\langle q_T^2 \rangle^h = \int d\mathbf{q}_T \cdot q_T^2 \cdot \frac{d\sigma_{hA}}{dQ^2 dq_T^2} / \int d\sigma_{hA} / dQ^2.$$

In Eq. (2), $Q$ is the total invariant mass of the lepton pair with $Q^2 = q^2$. From our definition, $\Delta \langle q_T^2 \rangle$ represents a measurement of QCD dynamics beyond the traditional single-hard scattering picture. The nuclear dependence of $\Delta \langle q_T^2 \rangle$, 

defined in Eq. (1), is a result of the multiple scattering between the parton from incoming beam and the nuclear matter before the Drell-Yan pair are produced. If we keep only the double scattering contribution and neglect contribution from the higher multiple scattering, we have

$$\Delta \langle q_T^2 \rangle \approx \frac{\int dq_T^2 \cdot q_T^2}{\int dQ^2/dq_T^2} \cdot \frac{d\sigma_{hA}}{dQ^2},$$

(3)

where superscript “D” indicates the double scattering contribution. Fig. 1 shows the leading order double scattering between the parton “f” from incoming beam and the nucleus. According to the generalized factorization theorem, the double scattering cross section can be expressed as

$$d\sigma_{hA \to \ell^+\ell^-} = \left(\frac{2\alpha_{em}}{3Q^2}\right) \sum_f \int dx' \phi_{f/h}(x') \cdot d\hat{\sigma}_{fA \to \gamma^*}(x', q),$$

(4)

with \(d\hat{\sigma}_{fA}\) the parton level double scattering cross section, and

$$d\hat{\sigma}_{fA} = \frac{1}{2x's} \int dx dx_1 dx_2 \int d^2y T^{(1)}(x, x_1, x_2, k_T) \times \hat{H}(x, x_1, x_2, k_T, x' p', p, q).$$

(5)

In Eq. (4), \(\phi_{f/h}(x')\) is the parton distribution from the hadron. In Eq. (5), \(\hat{H}\) is the hard partonic part, and \(\hat{T}\) is the hadronic matrix element:

$$\hat{T}^{(1)}(x, x_1, x_2, k_T) = \int dy^- dy_1^- dy_2^- \frac{d^2y_T}{2\pi} \frac{d^2p_T}{2\pi} \frac{d^2p}{(2\pi)^2} e^{ix^+ p^+ (y_1^- - y_2^-)} e^{ix_2^+ p^+ y_2^-} e^{ik_T y_T} \times \langle p_A | A^+(y_1^-, 0_T) \bar{\psi}_q(0) \gamma^+ \psi_q(y^-) A^+(y_1^-, y_T) | p_A \rangle.$$  

(6)

Because of the exponential factors in Eq. (6), the position space integration, \(dy^-\’s\), cannot give large dependence on the nuclear size unless the parton momentum fraction in one of the exponentials vanishes. If the exponential vanishes, the corresponding position space integration can be extended to the size of the whole nucleus. Therefore, in order to get large nuclear enhancement or jet broadening, we need to consider only Feynman diagrams that can provide poles which set parton momentum fractions on the exponentials to be zero. Other diagrams that do not provide such poles will be suppressed by a large off-shell propagator and hence are not leading contributions. At the leading order in \(\alpha_s\), only diagrams shown in Fig. 1 have the necessary poles. These diagrams
Figure 1: Lowest order double scattering contribution to the nuclear enhancement of Drell-Yan $\langle q_T^2 \rangle$; (a)symmetric diagram; (b) and (c): interference diagrams.

contribute to the double scattering partonic part $\bar{H}(x,x_1,x_2,k_T,x'p',p,q)$ in Eq. (5).

For the leading order diagrams shown in Fig. 1, the corresponding partonic parts have two possible poles from the two propagators. For the diagram shown in Fig. 1a, the partonic part has the following general structure

$$\bar{H}^a \propto \frac{1}{x_1 - \frac{k_T^2}{x's} + i\epsilon} \cdot \frac{1}{x_1 - x_2 - \frac{k_T^2}{x's} - i\epsilon}$$

$$\times \delta(x + x_1 - \frac{k_T^2}{x's} - \frac{Q^2}{x's}) \delta(q_T^2 - k_T^2).$$

The two $\delta$-functions are from the phase space. One of the $\delta$-functions can be used to fix $dx$ integration in Eq. (5), and the two poles in Eq. (7) can be used to perform the contour integration for $dx_1 dx_2$. We have

$$d\hat{\sigma}_D^a \propto (4\pi^2) \theta(y^- - y_1^-) \theta(-y_2^-) \delta(q_T^2 - k_T^2) e^{i(\tau/x')} p^+ y^-,$$

with $\tau = Q^2/s$. The two $\theta$-functions are the results of the contour integration.

For the diagrams shown in Fig. 1b and 1c, we have slightly different phase space factors and different poles. Similarly, after integrating over parton mo-
momentum fractions, \( dx \, dx_1 \, dx_2 \), we have

\[
d\hat{\sigma}^D_b \propto (-4\pi^2) \theta(y_2^- - y_1^-) \theta(y^- - y_2^-) \delta(q_T^2) e^{i(\tau/x') p^+ y^-}, \tag{9}
\]

and

\[
d\hat{\sigma}^D_c \propto (-4\pi^2) \theta(y_1^- - y_2^-) \theta(-y_1^-) \delta(q_T^2) e^{i(\tau/x') p^+ y^-}. \tag{10}
\]

The spinor trace gives the same numerator for all three diagrams. Therefore, the total contribution to transverse momentum broadening is proportional to \( d\hat{\sigma}^D_a + d\hat{\sigma}^D_b + d\hat{\sigma}^D_c \), and this sum has the following feature

\[
\frac{d\sigma_{hA}^D}{dQ^2 \, dq_T^2} \propto d\hat{\sigma}^D_a + d\hat{\sigma}^D_b + d\hat{\sigma}^D_c
\]

\[
\propto \theta(y^- - y_1^-) \theta(-y_2^-) \left[ \delta(q_T^2 - k_T^2) - \delta(q_T^2) \right] + \left[ \theta(y^- - y_1^-) \theta(-y_2^-) - \theta(y_2^- - y_1^-) \theta(y^- - y_2^-) - \theta(y_1^- - y_2^-) \theta(-y_1^-) \right] \delta(q_T^2). \tag{11}
\]

It is clear from Eq. (11) that for the inclusive Drell-Yan cross section, \( d\sigma / dQ^2 \), the double scattering contribution, \( d\sigma_{hA}^D / dQ^2 \), does not have a large dependence on the nuclear size. The integration over \( dq_T^2 \) eliminates the first term in Eq. (11), while the second term is localized in space if \( \tau / x' \) is not too small. When the \( \tau / x' \) is finite, and \( p^+ \) is large, \( \exp[i(\tau/x')p^+ y^-] \) effectively restricts \( y^- \sim 1/(\tau/x')p^+ \) \( \to 0 \). When \( y^- \to 0 \), the combination of the three pairs of \( \theta \)-functions in Eq. (11) vanishes,

\[
\frac{d\sigma_{hA}^D}{dQ^2} \equiv \int dq_T^2 \left( \frac{d\sigma_{hA}^D}{dQ^2 \, dq_T^2} \right)
\]

\[
\propto \left[ \theta(y^- - y_1^-) \theta(-y_2^-) - \theta(y_2^- - y_1^-) \theta(y^- - y_2^-) - \theta(y_1^- - y_2^-) \theta(-y_1^-) \right] \to 0 \quad \text{as} \quad y^- \to 0. \tag{12}
\]

Physically, Eq. (12) says that all integrations of \( y^- \)'s are localized. Actually, at the leading order, the term proportional to \( \theta(y^- - y_1^-) \theta(-y_2^-) - \theta(y_2^- - y_1^-) \theta(y^- - y_2^-) - \theta(y_1^- - y_2^-) \theta(-y_1^-) \) is the eikonal contribution to make the normal twist-2 quark distribution for single scattering gauge invariant. Eq. (12) is a good example to demonstrate that the double scattering does not give a large nuclear size effect to the total inclusive cross section.

On the other hand, from Eq. (11), the double scattering contribution to the averaged transverse momentum square can give a large nuclear size effect

\[
\Delta(q_T^2) \sim \int dq_T^2 q_T^2 \left( \frac{d\sigma_{hA}^D}{dQ^2 \, dq_T^2} \right).
\]
\[
\propto \int dq_T^2 q_T^2 \left[ \delta(q_T^2 - k_T^2) - \delta(q_T^2) \right] \\
\sim k_T^2 . \quad (13)
\]

Actually, \( k_T^2 \) in Eq. (13) needs to be integrated first. But Eq. (13) already demonstrates that \( \Delta \langle q_T^2 \rangle \) is proportional to \( k_T^2 \), which is the kick of the transverse momentum the parton received from the additional scattering. The bigger the nuclear size, the bigger the effective \( k_T^2 \). As shown below, \( \Delta \langle q_T^2 \rangle \) is proportional to the nuclear size.

After working out the algebra, we obtain the Drell-Yan transverse momentum broadening at the leading order in \( \alpha_s \)
\[
\Delta \langle q_T^2 \rangle = \left( \frac{4\pi^2 \alpha_s}{3} \right) \cdot \sum_q e_q^2 \int dx' \phi_{\bar{q}/h}(x') T^{(I)}_{q/A}(\tau/x')/x' \\
\sum_q e_q^2 \int dx' \phi_{q/h}(x') \phi_{q/A}(\tau/x')/x' . \quad (14)
\]

In Eq. (14), \( \phi_{q/A}(x) \) is the usual quark distribution inside a nucleus. \( T^{(I)}_{q/A}(x) \) is the four-parton correlation function, and is given by
\[
T^{(I)}_{q/A}(x) = \int \frac{dy^-}{2\pi} e^{ip^+ y^-} \int \frac{dy_1^- dy_2^-}{2\pi} \theta(y^- - y_1^-) \theta(-y_2^-) \\
\times \frac{1}{2} \langle p_A | F^+_{\alpha}(y_2^-) \bar{\psi}_q(0) \gamma^+ \psi_q(y^-) F^{\alpha +}(y_1^-) | p_A \rangle . \quad (15)
\]

The superscript (I) represents the initial state interaction, in order to distinguish from the similar four-parton correlation function defined in Eq. (29). In deriving Eq. (14), we expend \( \delta(q_T^2 - k_T^2) \) at \( k_T = 0 \), known as the collinear expansion, and keep only the first non-vanishing term which corresponds to the second order derivative term. We use the factor \( k_T^2 k_T^2 \) to convert the \( k_T^2 A^+ k_T^2 A^+ \) into field strength \( F^{\alpha +} F^{\beta +} \) by partial integration. Here, we work in Feynman gauge. The terms associated with other components of \( A^\rho \) are suppressed by \( 1/p^+ \) compared to those with \( A^+ \), because of the requirement of the Lorentz boost invariance for the matrix elements.

By comparing the operator definitions of these four-parton correlation functions and the definitions of the normal twist-2 parton distributions, LQS proposed the following model
\[
T_{f/A}(x) = \lambda^2 A^{1/3} \phi_{f/N}(x) , \quad (16)
\]

where \( \phi_{f/N}(x) \) with \( f = q, \bar{q}, g \) are the normal twist-2 parton distribution of a nucleon, and \( \lambda \) is a free parameter to be fixed by experimental data.
Applying this model, we obtain a very simple result for the Drell-Yan transverse momentum broadening:

\[ \Delta \langle q_T^2 \rangle = \left( \frac{4\pi^2\alpha_s}{3} \right) \lambda^2 A^{1/3}. \]  

(17)

Using Eq. (17) and data from E772 and NA10 on nuclear enhancement of the average Drell-Yan transverse momentum, \( \Delta \langle q_T^2 \rangle \), we estimate the value of \( \lambda^2 \) as

\[ \lambda_{DY}^2 \approx 0.01 \text{GeV}^2, \]

(18)

which is at least a factor of five smaller than \( \lambda_{\text{di-jet}}^2 \approx 0.05 \rightarrow 0.1 \text{GeV}^2 \), previously estimated by LQS from momentum imbalance of the di-jet data. Therefore, it is very important to use other observables, such as jet broadening in DIS, to further test the four-parton correlation functions.

3 Jet broadening in Deeply Inelastic Scattering

In this section, we derive the leading contribution to jet broadening in DIS by using the same technique used to derive the Drell-Yan transverse momentum broadening, \( \Delta \langle q_T^2 \rangle \), in last section. Consider the jet production in the deeply inelastic lepton-nucleus scattering, \( e(k_1) + A(p) \rightarrow e(k_2) + \text{jet}(l) + X \).\( k_1 \) and \( k_2 \) are the four momenta of the incoming and the outgoing leptons respectively, and \( p \) is the momentum per nucleon for the nucleus with the atomic number \( A \). With \( l \) be the four-momentum for the jet, the averaged jet transverse momentum square is defined as

\[ \langle l_T^2 \rangle^e_A = \int d\Omega_T^2 \cdot l_T^2 \cdot \frac{d\sigma_{eA}}{dx_B dQ^2 d\Omega_T^2} \bigg/ \frac{d\sigma_{eA}}{dx_B dQ^2}. \]  

(19)

where \( x_B = Q^2/(2p \cdot q) \), and \( q = k_1 - k_2 \) is the momentum of the virtual photon, and \( Q^2 = -q^2 \). The jet transverse momentum \( l_T \) depends on our choice of the frame. We choose the Breit frame in the following calculation. Similar to the Drell-Yan transverse momentum spectrum, \( d\sigma/dQ^2 dq_T^2 \), the jet transverse momentum spectrum, \( d\sigma/dx_B dQ^2 dl_T^2 \), is sensitive to the \( A^{1/3} \) type nuclear size effect due to the multiple scattering. On the other hand, the inclusive DIS cross section \( d\sigma/dx_B dQ^2 = \int dl_T^2 d\sigma/dx_B dQ^2 dl_T^2 \) does not have the \( A^{1/3} \) power enhancement. Instead, it has a much weaker \( A \)-dependence, such as the EMC effect and the nuclear shadowing. To separate the multiple scattering contribution from the single scattering, we define the jet broadening as

\[ \Delta \langle l_T^2 \rangle \equiv \langle l_T^2 \rangle^e_A - \langle l_T^2 \rangle^e_N, \]  

(20)
Keeping only the contribution from the double scattering, similar to Eq. (3), we have

$$\Delta \langle l_2^2 \rangle \approx \int dl_T^2 \cdot l_T^2 \cdot \frac{d\sigma^D}{dx_B dQ^2 dl_T^2} / \frac{d\sigma}{dx_B dQ^2}.$$  \hspace{1cm} (21)

The leading order double scattering diagrams contributing to jet broadening are given in Fig. 2. The double scattering contribution can be expressed as

$$d\sigma^D \propto \int dx_1 dx_2 \int dk_T^2 \bar{T}^{(F)}(x, x_1, x_2, k_T) \bar{H}_{\mu\nu}(x, x_1, x_2, k_T, p, q, l) ,$$  \hspace{1cm} (22)

with the matrix element

$$\bar{T}^{(F)}(x, x_1, x_2, k_T) = \int \frac{dy^-}{4\pi} \frac{dy_1}{2\pi} \frac{dy_2}{2\pi} \frac{d^2 y_T}{(2\pi)^2} \times e^{ix^+ p^+ y^-} e^{ix_1^+ p^+ (y_1^- - y_2^-)} e^{ix_2^+ p^+ y_2^-} e^{ik_T^+ y_T}$$

$$\times \langle p_A | \bar{\psi}_q(0) \gamma^+ A^+(y_2^-, 0_T) A^+(y_1^-, y_T) \psi_q(y^-) | p_A \rangle ,$$  \hspace{1cm} (23)

where superscript \((F)\) indicates the final-state double scattering. The matrix element \(\bar{T}^{(F)}\) is equal to the \(\bar{T}^{(I)}\) in Eq. (6) if we commute the gluon fields with the quark fields \(2, 11\). In Eq. (22), \(\bar{H}_{\mu\nu}\) is the corresponding partonic part. Here, we work in Feynman gauge. For the diagram shown in Fig. 2a, the partonic part has the following structure

$$\bar{H}_a^{\mu\nu} \propto \delta(x + x_1 - x_B - \frac{k_T^2 - 2q \cdot k_T}{2p \cdot q}) \cdot \delta(l_T^2 - k_T^2) dl_T^2$$

$$\times \frac{1}{x - x_B + i\epsilon} \cdot \frac{1}{x + x_2 - x_B - i\epsilon} .$$  \hspace{1cm} (24)

In Eq. (24), the \(\delta\)-function is from the phase space, and the poles are from the propagators.

Following the derivation of \(\Delta \langle q_T^2 \rangle\) in last section, first, we carry out the integrations of the parton momentum fractions by using the \(\delta\)-function and two poles in Eq. (24),

$$d\sigma^a \propto (2\pi)^2 \theta(y_1^- - y^-) \theta(y_2^-) \delta(l_T^2 - k_T^2) .$$  \hspace{1cm} (25)

Similarly, we have the corresponding integrations for the interference diagram in Fig. 2b

$$d\sigma^b \propto -(2\pi)^2 \theta(y_2^- - y_1^-) \theta(y_1^- - y^-) \delta(l_T^2) ,$$  \hspace{1cm} (26)
Figure 2: Lowest order double scattering contribution to jet broadening: (a) symmetric diagram; (b) and (c): interference diagrams.

and for the diagram in Fig. 2:

$$d\sigma \propto -(2\pi)^2 \theta(y_2^-) \theta(y_1^- - y_1^-) \delta(l_T^2).$$ (27)

Combining Eqs. (25), (26) and (27), we again have the same structure as that in Eq. (11). Therefore, like in the Drell-Yan case, we conclude that the double scattering does not result into any large nuclear size dependence in the inclusive DIS cross section. For the jet broadening defined in Eq. (21), we drop the term proportional to $\theta(y_1^- - y^-) \theta(y_2^- - y_1^- - y^-) \theta(y_1^- - y_2^-) \theta(y_2^-)$, which is localized as the single scattering. After carrying out the algebra, we derive jet broadening in DIS as

$$\Delta \langle l_T^2 \rangle = \left( \frac{4\pi^2 \alpha_s}{3} \right) \sum_q \frac{e_q^2 T_{q/A}(x_B)}{\sum_q e_q^2 \phi_{q/A}(x_B)},$$ (28)

where $\sum_q$ sums over all quark and antiquark flavors. In Eq. (28), $\phi_{q/A}(x)$ is the normal twist-2 quark distribution inside a nucleus, and the four-parton correlation function, $T_{q/A}(x_B)$ is defined as

$$T_{q/A}(x) = \int \frac{dy^-}{2\pi} e^{ipt-y^-} \frac{dy_1^-}{2\pi} \theta(y_1^- - y^-) \theta(y_2^-).$$
\[
\times \frac{1}{2} \langle p_A | \bar{\psi}_q(0) \gamma^+ F_\sigma^+(y_2^-) F_\sigma^+(y_1^-) \psi_q(y^-) | p_A \rangle .
\] (29)

With the model given in Eq. (16), we can simplify Eq. (28):
\[
\Delta \langle t_F^2 \rangle = \left( \frac{4\pi^2\alpha_s}{3} \right) \lambda^2 A^{1/3}.
\] (30)

From Eq. (28), we see that jet broadening in DIS is directly proportional to the four-parton (twist-4) correlation function, defined in Eq. (29). It is the same four-parton correlation function appeared in dijet momentum imbalance \[ T_{q/A}(x) \] and \[ T_{q/A}^{(I)}(x) \], which are defined in Eqs. (28) and (15), respectively, are same if the phase space integral are symmetric, because field operators commute on the light-cone. Therefore we conclude that jet broadening in DIS, transverse momentum broadening for the Drell-Yan pairs, and dijet momentum imbalance depend on the same four-parton correlation function.

4 Discussion and conclusions

From the simple expression in Eq. (30), we conclude that at the leading order, jet broadening in DIS has a strong scaling property, that it does not depend on beam energy, \( Q^2 \) and \( x_B \). This scaling property of jet broadening in DIS is a direct consequence of LQS model for four-parton correlation functions, given in Eq. (16). However, when \( x_B \leq 0.1 \), \( y^- \sim 1/(x_B p^+) \) is no longer localized inside an individual nucleon. Therefore, terms proportional to \( \theta(y_1^- - y^-) \theta(y_2^-) - \theta(y_2^- - y_1^-) \theta(y_1^- - y^-) - \theta(y_1^- - y_2^-) \theta(y_2^-) \), are no longer localized and need to be kept for the jet broadening calculation if \( x_B \) is small. Consequently, the \( x_B \)-scaling of jet broadening in DIS needs to be modified in small \( x_B \) region. In addition, \( Q^2 \)-dependence may be modified because the four-parton correlation function \( T_{q/A}(x) \) and the normal quark distribution \( \phi_{q/A}(x) \) can have different scaling violation. In principle, all dependence or whole conclusion could be modified due to possible different high order corrections. Nevertheless, we believe that experimental measurements of jet broadening in DIS can provide valuable information on the strength of multi-parton correlations and the dynamics of the multiple scattering.

Similarly, from Eq. (17), we can also conclude that the Drell-Yan transverse momentum broadening, \( \Delta \langle q_T^2 \rangle \), has a small dependence on beam energy and \( Q^2 \) of the lepton pair. Data from Fermilab E772 and CERN NA10 demonstrate weak energy dependence. It signals that the simple model by LQS for four-parton correlation functions is reasonable and the leading order calculation given here are useful. At the same time, the observed energy dependence indicates that the high order corrections to \( \Delta \langle q_T^2 \rangle \) can not be ignored.
In addition, Eqs. (30) and (17) tell us that at the leading order, jet broadening in DIS and nuclear enhancement of average Drell-Yan transverse momentum, $\Delta \langle q_T^2 \rangle$, have the same magnitude, if the averaged initial-state gluon interactions is equal to the corresponding final-state gluon interactions, i.e.,

$$T_{q/A}(x) = T_{q/A}^{(f)}(x).$$

From Eq. (28), we see that the jet broadening is directly proportional to the four-parton correlation function $T_{q/A}(x_B)$. Information on $x_B$-dependence of the jet broadening can provide a first ever direct measurement of the functional form of four-parton correlation functions $T_{q/A}(x_B, A)$. In addition, by examining the scaling property of jet broadening, we can directly verify LQS model of four-parton correlation functions, given in Eq. (16). Future experiments at HERA with a heavy ion beam should be able to provide much more information on dynamics of parton correlations.

In summary, we have derived analytic expressions for jet broadening in DIS and the Drell-Yan transverse momentum broadening in terms of universal four-parton correlation functions. Because the dijet data (pure final-state multiple scattering) and the Drell-Yan data (pure initial-state multiple scattering) favor two different sizes of the four-parton correlation function, measurement of jet broadening in DIS (pure final-state multiple scattering) will provide a critical test of QCD dynamics of the multiple scattering. Since at the leading order, the Drell-Yan transverse momentum broadening, $\Delta \langle q_T^2 \rangle_{DY}$, and jet broadening in DIS, $\Delta \langle l_T^2 \rangle_{DIS}$, are independent of the four-gluon correlation function $T_{g/A}$, measurement of $\Delta \langle q_T^2 \rangle_{DY}$ and $\Delta \langle l_T^2 \rangle_{DIS}$ provide a direct comparison between the initial-state multiple scattering and the final-state multiple scattering.

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