The \( \pi NN \) coupling constant from np charge exchange scattering

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Abstract

A novel extrapolation method has been used to deduce the charged \( \pi NN \) coupling constant from backward np differential scattering cross sections. We applied it to new measurements performed at 162 MeV at the The Svedberg Laboratory in Uppsala. In the angular range \( 150^\circ - 180^\circ \), the carefully normalized data are steeper than those of most previous measurements. The extracted value, \( g_{\pi \pm}^2 = 14.52 \pm 0.26 \), in good agreement with the classical value, is higher than those determined in recent nucleon-nucleon partial-wave analyses.

INTRODUCTION

The knowledge of the precise value of the \( \pi NN \) coupling is a crucial issue: not only in nuclear physics where it is a fundamental constant, but also in particle physics where it is of great importance for the understanding of chiral symmetry breaking(1). Its experimental error is the main obstacle in the accurate testing of the corrections to the Goldberger-Treiman relation as predicted from chiral symmetry breaking. With the latest value for the axial coupling constant, \( g_A = 1.266 \pm 0.004(2) \), this relation would lead to \( g^2(q^2 = 0) = 13.16 \pm 0.16 \), if it were exact, which is not expected. The uncertainty, here of about \( \pm 1\% \), comes from the experimental error in \( g_A \) and \( f_\pi \). If we know how to calculate the corrections perfectly we can clearly make good use of a precision of \( 1\% \) in \( g^2 \).

In the 1980’s, the \( \pi NN \) coupling constant was believed to be well known. Koch and Pietarinen(3) determined a value of the charged pion coupling constant, \( g_{\pi \pm}^2 = 14.28 \pm 0.18 \), from \( \pi \pm p \) scattering data. Kroll(4) found the neutral pion coupling constant to be \( g_{\pi 0}^2 = 14.52 \pm 0.40 \), analysing \( pp \) data with forward dispersion relations. In the early 1990’s the Nijmegen group(5,6,7) determined smaller values on the basis of energy-dependent partial-wave analyses (PWA) of nucleon-nucleon (NN) scattering data. They obtained \( g_{\pi 0}^2 = 13.47 \pm 0.11 \) and \( g_{\pi \pm}^2 = 13.58 \pm 0.05 \). Similar values around \( g_{\pi \pm}^2 = 13.7 \), have also been found by the Virginia Tech group(8,9,10) from analysis of both \( \pi \pm N \) and NN data. These results have stimulated an intense debate, and it has become urgent to determine \( g^2 \) to high precision, convincingly and model-independently(11).

In the analysis by the Nijmegen group(7) the determination of the coupling constant seems not to be very sensitive to the backward np cross section. In our work at 162 MeV(11), we have shown the contrary using ‘pseudodata’ built from models in common use, including the Nijmegen potential(12). The experimental normalization of the cross section is crucial and this has been a well known problem in the past. Most energy-dependent PWA’s have therefore chosen to let the normalization of data float more or less freely. The direct sensitivity to the np

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cross section is then lost, and the coupling constant can depend diffusely on many observables. We believe that precision data of the backward np cross section should be one of the best places in the NN sector to determine the charged coupling constant. We have demonstrated recently\cite{11,13}, that it is both the shape of the angular distribution at the most backward angles, and the absolute normalization of the data, that are of decisive importance in this context. Before analysing the new 162 MeV data we shall illustrate the possible consequences of the relatively large spread of the values of the πNN coupling constant on the quark mass ratio $m_s/\hat{m} = 2m_s/(m_u + m_d)$. We shall then describe the extrapolation to the pion pole and give some conclusions.

**DASHEN-WEINSTEIN SUM RULE**

As shown in Ref.\cite{14} the Dashen-Weinstein sum rule\cite {15} for deviations from the Goldberger-Treiman (GT) relation allows a possible determination of $m_s/\hat{m}$. This sum rule is obtained by considering the matrix elements of the non-strange charged axial vector current at $q^2 = 0$ between $n$ and $p$ and the strange ones between $p$ and $\Lambda$ and between $n$ and $\Sigma$. The hypothesis of partial conservation of the current leads at $q^2 = 0$ to equations between the deviations of the GT relations, the $\pi NN$, $K\Lambda N$, $K\Sigma N$ couplings, the axial form factors and the pseudoscalar meson decay constants $f_\pi$ and $f_K$. Expanding matrix elements of the pseudoscalar densities in terms of the quark masses and using the $SU(3)_V$ symmetry invariance in the Chiral limit, lead then to the sum rule. It expresses the ratio $m_s/\hat{m}$ in terms of experimental quantities. So far one knows only an upper limit of 7 for the $K\Sigma N$ coupling. From the present knowledge for the other constants, the sum rule leads then to a maximum value of $m_s/\hat{m}$ as a function of the $\pi NN$ coupling constant\cite{16}. Some corresponding numbers are summarized in Table 1. If our previously obtained high value of $g_{\pi NN}$\cite {13} is confirmed, then a value of $m_s/\hat{m}$ of 25, as required by a large value of quark condensate, will be excluded. However, lower $g_{\pi NN}$ allow this value. These conclusions are dependent on the precise experimental determination not only of the $\pi NN$ couplings but also of the $K\Lambda N$ and $K\Sigma N$ couplings. On the theoretical side, the evaluation of corrections of the order of $m^2_{\text{quark}}$, to the sum rule, should also be performed.

Table 1. Maximum values of $m_s/\hat{m}$ as predicted by the Dashen-Weinstein sum rule, if $|g_{K\Sigma N}| < 7$, as function of $g_{\pi NN}$.

| Source                                      | $g_{\pi NN}$ | $g_{\pi NN}^2/4\pi$ | $f_{\pi NN}^2/4\pi$ | $(m_s/\hat{m})_{\text{max}}$ |
|---------------------------------------------|--------------|----------------------|----------------------|-------------------------------|
| np → pn: Difference method\cite {11}        | 13.55 ± .14  | 14.62 ± .30          | .0810 ± .0020        | 9.7 ± 4.3                     |
| $\pi^+p \rightarrow \pi^+p$ ; Dispersion relation\cite {3} | 13.40 ± .08  | 14.30 ± .20          | .0790 ± .0010        | 12.3 ± 5.1                    |
| $\pi^\pm N \rightarrow \pi^\pm N$; GMO sum rule\cite {10} | 13.14 ± .07  | 13.75 ± .15          | .0760 ± .0008        | 22 ± 10                       |
| $NN \rightarrow NN$; PWA\cite {7}         | 13.06 ± .03  | 13.58 ± .05          | .0750 ± .0003        | 28 ± 12                       |

**NEUTRON PROTON CHARGE EXCHANGE DATA ANALYSIS**

It is very striking that the np unpolarized charge exchange cross sections in a very large range of energies from about 100 MeV to several GeV, have similar shape and normalization (in the laboratory system). These data contain essentially the same physical information as far as the extrapolation to the pion pole is concerned. Here we shall concentrate our analysis to new precise data at 162 MeV\cite {13} consisting of an extension from $\theta_{CM} = 72^\circ$ to $120^\circ$ of our previous backward measurement\cite {11}. This allows to improve the absolute normalization to about ±2%. A study of the present np data base\cite {18}, shows that there are two main families with respect to the angular shape. The first one is dominated by the Bonner et al. data\cite {13}, which have a flattish angular distribution at backward angles. The second one, which includes
our measurements and the H"urster et al.\cite{20} data, have a steeper angular shape. The total c.m. cross sections can be defined in terms of the five amplitudes $a$, $b$, $c$, $d$, $e$\cite{21} as

$$
\frac{d\sigma}{d\Omega}(q^2) = \frac{1}{2}(|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2) = \frac{1}{2} \left[ \frac{1}{2}(|a + c|^2 + |a - c|^2) + \frac{1}{2}(|b + d|^2 + |b - d|^2) + |e|^2 \right] ;
$$

where $q^2$ is the squared momentum transfer from the neutron to the proton.

In order to understand the qualitative contributions of pion exchange, we have chosen the regularized pion Born amplitudes of Ref.\cite{22} with the $r$-space $\delta$-function subtracted. This ensures a non-zero cross section at $180^\circ$. The different components for this Born pion terms, and for the more realistic Paris potential, are then displayed in Figs.\cite{1}a and \cite{1}b, respectively. The combination $|b - d|^2$, which contains the entire pion pole term, is for the Paris potential remarkably close to that of the Born term, particularly at small $q^2$. The term $|a + c|^2$ is very small in both cases and the more important $|a - c|^2$ terms are again very similar. The simple structure of the term which contains the pion pole gives considerable confidence that the extrapolation can be achieved realistically.

**Figure. 1.** Contributions to the $np$ cross section at 162 MeV of combinations of the amplitudes $a$, $b$, $c$, $d$, $e$ of eq.\cite{1} a) for the regularized pion Born terms b) for the Paris potential model.

**EXTRAPOLATION TO THE PION POLE**

The basic idea to extrapolate to the pion pole is to construct a smooth physical function, the Chew function\cite{23},

$$
y(x) = \frac{s x^2}{m^2 g_R} \frac{d\sigma}{d\Omega}(x) = \sum_{i=0}^{n-1} a_i x^i .
$$

Here $s$ is the square of the total energy and $x = q^2 + m^2$. At the pion pole $x = 0$, the Chew function gives $y(0) \equiv a_0 \equiv g^4/g_R^2$, $g$ being the pseudoscalar coupling constant related to the pseudovector coupling by $f = (m_\pi/2M_p)g$. The quantity $g_R^2$ is a reference scale for the coupling chosen for convenience. The model-independent extrapolation requires accurate data with absolute normalization of the differential cross section. If the differential cross section is incorrectly normalized by a factor $N$, the extrapolation determines $\sqrt{Ng^2}$.

The Difference Method, which we introduced to obtain a substantial improvement\cite{11}, is based on the Chew function, but it recognizes that a major part in the cross section behaviour
is described by models with exactly known values for the coupling constant. It applies the Chew method to the difference between the function \( y(x) \) obtained from a model and from the experimental data, i.e.,

\[
y_M(x) - y_{\text{exp}}(x) = \sum_{i=0}^{n-1} d_i x^i
\]

with \( g_R \) of eq. [1] replaced by the model value \( g_M \). At the pole \( y_M(0) - y_{\text{exp}}(0) \equiv d_0 \equiv (g_M^4 - g^4)/g_M^4 \). This should diminish systematic extrapolation errors and remove a substantial part of the irrelevant information at large momentum transfers.

In our work we have explicitly shown, using ‘pseudodata’ generated from models in common use including the Nijmegen potential(12), that we can reproduce the input coupling constants of the models to a precision less than 1%. We have grouped the data into a “reduced range”, \( 0 < q^2 < 4 m_\pi^2 \) with 31 data points and a “full range”, \( 0 < q^2 < 10.1 m_\pi^2 \) with 54 data points. The reduced range is the range of the data available for the analysis in our previous work(11). This allows to check the sensitivity and stability of the extrapolation to a particular cut in momentum transfer and to verify that it is the small \( q^2 \) region that carries most of the pion pole information.

![Figure 2](image-url)

**Figure 2.** Extrapolations of the Chew function \( y(q^2) \) to the pion pole at 162 MeV with the Difference Method using PWA SM95 as comparison model, different order of polynomials and different intervals in \( q^2 \). The left panel uses the reduced range \( 0 < q^2 < 4 m_\pi^2 \); the right panel uses the full range \( 0 < q^2 < 10.1 m_\pi^2 \).

The Difference Method requires only a few terms in the polynomial expansion in favorable cases, and this gives a small, statistical extrapolation error. The similarity between the angular distributions from models and the experimental data is exploited, particularly for large \( q^2 \). This incorporates substantial additional physical information without introducing any model dependence. We apply the method using three comparison models: the Nijmegen potential(12), the Nijmegen energy-dependent PWA NI93(24) as well as the Virginia SM95 energy-dependent PWA(25,26). The result, for this last case, is shown in Fig. 2 for the reduced and full ranges of data. In all cases the extrapolation to the pole can be made easily and already a visual extrapolation gives a good result. The polynomial fits cause no problem as long as the data are not overparametrized. The resulting \( g^2 = 14.52 \pm 0.26 \) is consistent with our previous finding(11).

Subsequent to our first publication(11) Arndt et al.(26) subjected a major part of the \( np \) charge exchange cross section data to an analysis using the Difference Method at energies from 0.1 to 1 GeV. They found an average value 13.75 using SM95 as comparison model. Their individual results show a considerable scatter of approximately \( \pm 10\% \). This appears to come from the quality of the data. In particular, the deduced \( g^2 \) shows systematic trends with energy as can be seen in Fig. 3 for the Bonner data leading to an increase of \( g^2 \) with energy. Note that for energies above 400 MeV the slope of the data at large angle is as steep as that of the Uppsala data.
CONCLUSIONS

We have seen that there exists a spread of 7% for the value of the $\pi NN$ coupling constant which can have important consequences on our present understanding of QCD. Here we have shown that using the most accurate extrapolation method, the Difference Method, on high precision $np$ differential cross section measurements at 162 MeV in the angular range $72^\circ - 180^\circ$ one can obtain a precise value of this coupling, namely $\sqrt{N}g^2 = 14.52 \pm 0.13$ with a systematic error of about $\pm 0.15$ and a normalization uncertainty of $\pm 0.17$. We have no difficulty in reproducing the input coupling constants of models using equivalent pseudo-data. The practical usefulness of the method, its precision and its relative insensitivity to systematics appear to be in hand without serious problems. The data were normalized using the total $np$ cross section, which is one of the most accurately known cross sections in nuclear physics, together with a novel approach, in which the differential cross section measurement was considered as a simultaneous measurement of a fraction of the total cross section. It was found that, in the angular region $150^\circ - 180^\circ$, our data are steeper than those of the large data set of Bonner et al.\cite{Bonner1995} below about 400 MeV. This steeper behaviour, leading to a high value of $g^2$, should be confirmed and a dedicated $np$ charge exchange precision experiment at 200 MeV with a tagged neutron, to allow an absolute measurement, is going to be performed at IUCF.

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