Abstract

We use the results of direct and indirect searches for weakly interacting massive particles (WIMPs) to obtain bounds on various electro-magnetic form factors of WIMPs. The limits on the recoil signal in underground dark matter detection experiments, with standard assumptions about the density and r.m.s. velocity of WIMPs in the halo, can be translated into the following model-independent bounds on the magnetic dipole, electric dipole, quadrupole and anapole moments, the charge radius, and the polarizability of WIMPs: \( \mu_D/\mu_n < 1.4 \times 10^{-4} \), \( d < 5 \times 10^{-21} \) e cm, \( Q < 6 \times 10^{-7} \) e fm\(^2\), \( a/\mu_n < 3 \times 10^{-2} \) fm, \( r_D^2 < 9 \times 10^{-7} \) fm\(^2\), and \( \chi < 5 \times 10^{-8} \) fm\(^3\) for a WIMP mass of 100 GeV. The limits on fluxes of highly energetic neutrinos produced in the annihilation of WIMPs in the center of the earth and the sun lead to somewhat stronger, but more model dependent bounds.
1 Introduction

Various observational facts, notably the dynamics of spiral galaxies, clusters and superclusters, and their theoretical explanations in modern cosmology and astrophysics can be reconciled only by postulating the existence of the non-luminous matter at amounts exceeding the total matter density provided by baryons. Thus, if gravity does not undergo some drastic changes at distances larger than few kpc (a highly improbable option from various points of view), the explanation of the rotational curves of spiral galaxies requires the existence of halos, composed of dark matter objects.

One of the most intriguing options from the point of view of particle physics is the possibility that dark matter is composed of massive particles of yet unknown identity. On the one hand these (presumably) weakly interacting massive particles (WIMPs) are the subject of intense experimental searches at numerous underground detectors [1]. On the other hand, some of the extensions of the Standard Model naturally incorporate such stable particles. For example, in supersymmetric extensions of SM the lightest supersymmetric particle can be stable and neutral, thus providing a viable dark matter candidate. Extensive theoretical studies of stable SUSY particles, in particular neutralinos, have been performed over the last two decades [4] for wide ranging values of the supersymmetric masses and couplings. We believe, however, that unless supersymmetry is experimentally verified and the stability of the lightest superpartner is established in collider experiments, other possibilities for WIMPs should not be discarded.

In this letter we adopt a completely different, yet fully justified, phenomenological approach to the physics of WIMPs. We do not make any particular assumptions about the origin and possible identity of these particles, apart from the assumption that their masses are larger than a few GeV. We then assess what can be inferred about the properties of WIMPs directly from experimental limits implied by various searches. An additional incentive to address WIMP physics in a model independent way stems from recently discovered problems with subgalactic structure formation within the non-interacting cold dark matter scenario [3]. A “generalized” WIMP with sufficient self-interaction may avoid these problems [4].

Among important low-energy properties of WIMPs are their electromagnetic form factors. It has been known for a long time that the possibility of charged WIMPs is strongly disfavored [3] and stringent limits exist in the case of a fractional charge [4]. Here we assume that WIMPs are neutral and discuss other electro-magnetic form factors such as magnetic and electric dipole moments, anapole moment and polarizability. We put strong constraints on the possible electromagnetic form factors of WIMPs using the result of direct experimental searches aimed at the detection of the WIMP-nucleus recoil signal. These constraints depend on the number density and average velocities of WIMPs in the halo. The limits on fluxes of energetic neutrinos, originating from the annihilation of WIMPs in the center of the Sun and the Earth, can also be used to obtain indirect constraints on the form factors. These constraints are less certain, as they require additional assumptions about annihilation rates and neutrino branching ratios.
This paper is organized as follows. The calculations of the WIMP-nuclei cross sections are performed in the next section. In section 3 we present direct limits on the form factors, using the results of various underground experiments. In section 4 we obtain indirect limits on the form factors, using limits on neutrino fluxes. Our conclusions are reserved for section 5.

2 Electromagnetic form factors of WIMPs

The interaction of any compact object and a slowly varying electro-magnetic field can be parametrized via a set of electro-magnetic form factors. In our case, the object is a WIMP with spin $S$, and its interactions, linear in external fields $B$ and $E$, can be written as

$$H = -\mu B \cdot \frac{S}{S} - dE \cdot \frac{S}{S} - a j \cdot \frac{S}{S}$$ (1)

$$-\frac{1}{4S(2S-1)}[S_i S_j + S_j S_i - \frac{2}{3} \delta_{ij} S(S + 1)] \left( Q \frac{\partial}{\partial x_i} E_j + M \frac{\partial}{\partial x_i} B_j \right) + ...$$

Here $\mu$ and $d$ are magnetic and electric dipole moments, and $M$ and $Q$ are magnetic and electric quadrupole moments. The anapole moment $a$ is the form factor which describes the contact interaction with the external current density $j$.

If the spin of the WIMP is zero, these moments do not exist and the interaction with electromagnetic field is given by the charge radius $r_D$ of the WIMP and the polarizabilities

$$H = -\frac{1}{6} e r_D^2 \frac{\partial}{\partial x_i} E_i - \frac{1}{2} \chi E E^2 - \frac{1}{2} \chi B B^2 - \chi_{EB} E \cdot B + ...$$ (2)

If time reversal is a good symmetry, then $d$, $M$ and $\chi_{EB}$ necessarily vanish. Conserved parity also forbids the existence of $a$. Here we do not assume that these symmetries are preserved a priori, and we allow all of the form factors to exist.

In what follows we calculate the cross section of the elastic scattering of a WIMP off a nucleus due to the existence of the electromagnetic form factors. We start with the scattering due to the magnetic moment which is arguably the most interesting case.

The calculation can be done in the non-relativistic limit, as $v/c \sim 10^{-3}$ on average for WIMPs in the halo. We assume that the possible size of the form factors is of the order of, or smaller than, the respective characteristic values for nuclei, $\mu < \mu_n$, $Q < e \text{ fm}^3$, and so on. It can then easily be checked that this range of velocities also ensures the applicability of the Born approximation. In our units, $\hbar = 1$, $c = 1$, $e^2 = \alpha = 1/137$ and $\mu_n = e/2m_p$.

The cross-section for the scattering of a Wimp through the interaction of its magnetic moment with the magnetic moment of the nucleus is

$$\sigma = \frac{32\pi}{3} M_R^2(N, D) \mu_N^2 \frac{(S + 1)(I + 1)}{3SI},$$ (3)
where $M_R(N, D)$ is the reduced mass of the WIMP-nucleus system, $\mu_D$ and $\mu_N$ are the magnetic moments of the WIMP and the nucleus, respectively, and $I$ is the spin of the nucleus. The cross-section for the interaction between the electric dipole moment of the WIMP and the charge of the nucleus is

$$\sigma = 8\pi Z^2 \left(\frac{d}{e}\right)^2 \left(\frac{\alpha}{v}\right)^2 \frac{S + 1}{3S} \ln \frac{q_{\text{max}}}{q_{\text{min}}}.$$  (4)

It is logarithmically divergent at large distances. In practice, $q_{\text{max}}$ can be chosen to be of the order of $M_R(N, D)v$ and $q_{\text{min}}$ is determined either by the inverse of the impact parameters at which the nuclear charge is screened by the electrons or by the lowest momentum transfer that can be detected in the experiment, whichever is larger. The cross-section for the interaction between the nuclear electric current and the anapole of the WIMP is given by

$$\sigma = \frac{1}{2\pi} Z^2 \alpha^2 \frac{M_R^2(N, D)}{4m_p^2} \left(\frac{a}{\mu_n}\right)^2 \frac{S + 1}{3S},$$  (5)

where $\mu_n$ is the nuclear magneton, and $m_p$ is the proton mass. The contribution of the nuclear spin current is negligibly small. The cross-section for scattering through the interaction between the electric quadrupole moment of the WIMP and the electric charge of the nucleus takes the form

$$\sigma = \frac{8\pi}{9} M_R^2(N, D) Z^2 \alpha^2 \frac{Q}{e} \frac{(S + 1)(2S + 3)}{10S(2S - 1)}.$$  (6)

In Eqs. (4)-(6) the normalization of the WIMP spin dependence is such that it is equal to 1 for the lowest spin for which these moments may exist, $(S + 1)/3S = 1$ for $S = 1/2$ and $(S + 1)(2S + 3)/10S(2S - 1) = 1$ for $S = 1$. The cross-section for scattering of a WIMP because of its charge radius in the electric field of the nucleus is given by

$$\sigma = \frac{4\pi}{9} M_R^2(N, D) Z^2 \alpha^2 r_D^4.$$  (7)

Finally, the cross-section for the scattering of a WIMP through its polarizability in the electric field of the nucleus, is

$$\sigma \simeq \frac{144\pi}{25} M_R^2(N, D) Z^4 \alpha^2 \frac{\chi_E^2}{r_0^2}.$$  (8)

This cross-section diverges for a point-like nucleus. We therefore took the charge distribution of the nucleus to be homogeneous in a sphere with radius $r_0 \sim 1.2 \text{ fm} \sqrt{A}$.

Scattering due to the magnetic quadrupole moments and magnetic and mixed polarizabilities, $\chi_{EB}$, was also considered. These effects turned out to be additionally suppressed and do not lead to interesting limits. We therefore do not quote them here.
3 Direct constraints

We illustrate the bounds on the electro-magnetic form factors of WIMPs which are implied by the limit on the recoil signal with results of the NaI detector of the DAMA experiment [8], and Ge detector of the CDMS collaboration [9]. Other experiments aimed at the direct detection of dark matter can be used as well [10, 11, 12] to get limits which would typically be a few times weaker (see also Ref. [1] for more complete listing of experiments). Traditionally, the results of the direct searches are presented in the form of a normalized bound on the “WIMP–proton” cross-section in the spin dependent case, and a normalized “WIMP–nucleon” cross-section in the spin independent case. Such normalized bounds are useful to compare results for different types of nuclei, to compare results from different experiments, and to compare theoretical predictions with experimental data, but for our purpose of determining bounds on the WIMP electro-magnetic form factors, it is necessary to reverse this normalization. The bounds, quoted in [8, 10] are, in some sense, specifically formulated for neutralino-nucleon scattering, which necessarily implies a nuclear theory input when switching from nucleus-neutralino to nucleon-neutralino description. It is clear that in the case of the scattering due to the WIMP electro-magnetic form factors nuclear theory is not involved, and much of the nuclear physics uncertainty is not there. Thus, in order to obtain bounds on the form factors, we recover the nucleon-WIMP cross section from the nucleon-WIMP by the use of the equation [8]

\[ \sigma = \frac{M_R^2(N, D)}{M_R^2(p, D)} \frac{I(N)}{I(p, n)} \sigma_{p,n}. \]  

(9)

Here \( p \) and \( n \) stand for proton and neutron, respectively, and the ratio of the reduced masses takes into account a kinematic factor. In the spin dependent case, \( I(N)/I(p) \) is a spin factor, which takes the values \( I(N)/I(p) = 0.009 \) for Iodine, and \( I(N)/I(p) = 0.055 \) for Sodium [10, 13]. In the spin independent case, \( I(N)/I(n) = (A - Z)^2 \) for Ref. [10] and \( I(N)/I(n) = A^2 \) for Ref. [8]. For simplicity, we set all nuclear form factors equal to one, which is a better approximation for Sodium than for Iodine.1 A more accurate analysis could be done by the experimental collaborations using their original data. The ensuing bounds on the WIMP electro-magnetic form factors are listed in Table(1).

The bounds presented in this table are obtained with the standard assumptions that the mass density of WIMPs in our halo is 0.3 Gev/cm\(^3\) and that their velocities are of the order of 200 km/sec. The data obtained with germanium detectors are sensitive to all electro-magnetic form factors discussed here except the magnetic dipole moment, as the most abundant germanium isotope has no spin. Let us assume that the recently reported annual modulation of the DAMA signal, consistent with \( m_D \sim 50 \) GeV [14], and the absence of the signal, resulting in strong limits on spin-independent cross section, quoted by CDMS [9], are both correct. Then the presence of a WIMP magnetic moment at the level of \( \text{few} \times 10^{-5} - 1 \times 10^{-4} \) compared to the size of the nuclear magneton can reconcile the two experimental results.

1For a more rigorous treatment electric and magnetic form factors of nuclei should be included, for many of which the experimental results are available.
| $|\mu_D/\mu_n|\sqrt{(S+1)/3S}$ | $1.4 \times 10^{-4}$ | —— |
| $|d/e|\sqrt{(S+1)/3S}$ | $8 \times 10^{-21}\text{cm}$ | $5 \times 10^{-21}\text{cm}$ |
| $|Q/e|\sqrt{(S+1)(2S+3)/10S(2S-1)}$ | $1 \times 10^{-6}\text{fm}^2$ | $6 \times 10^{-7}\text{fm}^2$ |
| $|a/\mu_n|\sqrt{(S+1)/3S}$ | $4 \times 10^{-3}\text{fm}$ | $3 \times 10^{-2}\text{fm}$ |
| $r_D^2$ | $1.4 \times 10^{-6}\text{fm}^2$ | $9 \times 10^{-7}\text{fm}^2$ |
| $|\chi_E|$ | $5 \times 10^{-8}\text{fm}^3$ | $7 \times 10^{-8}\text{fm}^3$ |

Table 1: Bounds on the electro-magnetic form factors for WIMPs with a mass of 100 GeV, based on data from the DAMA collaboration presented in Ref. [8] and from the CDMS collaboration presented in Ref. [9].

It is instructive to consider how these bounds are relaxed in the scaling regime, when the WIMP mass is larger than nuclear masses and their abundance is kept as a free parameter. To this end we introduce the parameter $\delta_D$, characterizing the fraction of WIMPs of species $D$ in the halo dark matter density ($\delta_D = 1$ in Table 1). The counting rate in a detector is proportional to the number density of the WIMP particles and therefore should scale with $M_D$ and $\delta_D$ according to the following formula:

$$R_{N,D}(\delta_D, M_D) = R_{N}^0 \delta_D \left( \frac{M_D^0}{M_D} \right),$$

where $M_D^0$ is the lowest mass of WIMPs at which the scaling sets in and $R_{N}^0$ is the counting rate, corresponding to $M_D = M_D^0$ and $\delta_D = 1$. It is easy to see from the figures in Refs. [8, 9] that $M_D = 100$ GeV is close to the mass where the scaling behaviour begins [8, 9], and therefore we can simply rescale the bounds given in Table 1 for an arbitrary $\delta_D$ and arbitrary and heavy $M_D$, by multiplying them by $\sqrt{\delta_D M_D/100\text{GeV}}$.

## 4 Indirect constraints

The elastic scattering of WIMPs in the solar (and earth) media leads to gravitational trapping. Due to the scattering, WIMPs dissipate energy and eventually they accumulate
in the center of the sun (and earth). Their density grows until the accumulation rate is counter-balanced by the pair annihilation rate, and an equilibrium is reached. In the process of annihilation, among other SM products, energetic neutrinos can be produced. The interaction of these neutrinos in the earth in turn leads to the production of energetic muons, which can be detected with neutrino telescopes.

Thus limits on the flux of energetic neutrinos can be used to set bounds on the electromagnetic form factors of WIMPs. For WIMPS interacting through their electromagnetic form factors, the trapping rate in the center of the sun and earth is determined from the elastic WIMP-nucleus cross sections that we calculated in Eqs. (3)-(8).

We use the results of Ref. [15], which follows the original treatment in Refs. [16], and estimate the neutrino flux at the surface of the earth due to WIMP annihilation in the sun as

$$\phi_{\nu_\odot} \simeq \frac{(560 \text{cm}^{-2} \text{s}^{-1}) N_{\text{eff}} \delta_D \sigma_{p,36} \text{GeV}^2}{M_D^2}. \quad (11)$$

In this formula $N_{\text{eff}}$ is the average number of neutrinos per annihilation event and $\sigma_{p,36}$ is the WIMP-proton elastic cross section in units of $10^{-36}$ cm$^2$. It is assumed that the equilibrium number of WIMPs in the sun has been reached, so that the annihilation and capture rates are equal.

The experimental limit on the flux of energetic upward muons (for example obtained by the Kamioka, MACRO and Baksan experiments, [17, 18, 19]), is $\phi_\mu < \sim 1.4 \times 10^{-14} \text{cm}^{-2} \text{s}^{-1}$, and the probability that a neutrino directed towards the detector produces a muon at the detector is $P(100 \text{GeV}) \sim 10^{-7}$ [20]. Using Eq. (11), we deduce the following indirect limits on the electromagnetic form factors of WIMPs with $m_D = 100 \text{ GeV}$,

$$|\mu_D/\mu_n| \sqrt{(S+1)/3S} \lesssim 5 \times 10^{-6},$$
$$|d/e| \sqrt{(S+1)/3S} \lesssim 2 \times 10^{-22} \text{cm},$$
$$|Q/e| \sqrt{(S+1)(2S+3)/10S(2S-1)} \lesssim 10^{-8} \text{fm}^2,$$
$$|a/\mu_n| \sqrt{(S+1)/3S} \lesssim 10^{-2} \text{fm},$$
$$r_D^2 \lesssim 1.4 \times 10^{-8} \text{fm}^2,$$
$$|\chi| \lesssim 5.6 \times 10^{-7} \text{fm}^3,$$  \quad (12)

where $N_{\text{eff}}$ is taken to be 0.1 and $\delta_D = 1$. Note that for higher WIMP masses the absorption of neutrinos in the core of the sun becomes important [21].

If the WIMP-nucleus elastic scattering cross-sections in the earth are comparable to the WIMP-proton cross-section, than the neutrino signal from the center of the earth is approximately four orders of magnitude weaker than the signal from the sun [13]. However, the limit on $\chi_E$ from the neutrino flux from the center of the earth is almost one order of magnitude better than the limit derived from the neutrino flux from the sun, as the corresponding WIMP-nucleus elastic scattering cross-sections are enhanced by a large factor, $Z^4 \sim 5 \times 10^5$, for heavy (Fe) nuclei (see Eq. (8)) comprising the core of the earth.
Although indirect bounds are definitely stronger, they do require a number of additional assumptions. In particular, these bounds require that annihilation actually occurs, which in practice may not be true. For example, if a WIMP carries a conserved quantum number, and there are no “anti-WIMPs” - such as heavy Dirac neutrinos in the absence of antineutrinos - the annihilation is forbidden.

5 Discussion

We have shown that limits on recoil signals from underground detectors can be translated into bounds on the electro-magnetic form factors of WIMPs. Similar bounds can be inferred from the limits on energetic neutrino fluxes from the center of the earth and the sun, generated by annihilating WIMPs. Our analysis shows that all electromagnetic form factors have to be very small when expressed in characteristic nuclear units. In particular, the magnetic moment of a WIMP has to be five orders of magnitude smaller than the magnetic moment of a proton.

Unfortunately, all quoted bounds are trivially satisfied for the case of neutralino dark matter. Indeed, because it is a spin-1/2 particle, a neutralino cannot possess a quadrupole moment. In addition, because it is a Majorana fermion, a neutralino cannot have a magnetic or electric dipole moment either ($\bar{\chi}\sigma_{\mu\nu}\chi \equiv 0$). Finally, an anapole moment and a polarizability can be generated only radiatively. This results only in a small correction to the dominant neutralino-nucleus elastic cross section induced by squark exchange [2].

A magnetic moment can arise naturally if WIMPs are Dirac fermions. If we assume for a moment that the WIMP is a heavy Dirac neutrino, a large magnetic moment can be induced due to virtual $W$-charged lepton exchange. The value of the induced magnetic moment grows quadratically with the size of the Yukawa coupling until the point where the Yukawa interaction becomes essentially strong. In latter case a natural estimate for the size of the magnetic moment is $e/m_\nu$, with $m_\nu$ presumably of the order of 1 TeV or heavier. In this case, the interaction through the magnetic moment can be very important and, for light nuclei, can even dominate $Z$-boson exchange. It is interesting to note that the electric dipole moment of a WIMP is also very constrained. The limit of $5 \times 10^{-21}$ e cm is stronger than any other EDM constraint, apart from EDMs of electrons, neutrons and heavy atoms. The method we used to obtain this limit is in the spirit of the original paper by Purcell and Ramsey [22], where the first bound on the neutron EDM was inferred from neutron-nucleus scattering.

A model independent approach to the physics of WIMPs is partly motivated by recently discovered problems within the cold dark matter model. While large-scale structure can be described reasonably well, the numerical simulations of galactic substructures appear to be in conflict with observational data [3]. A recently proposed cure, invoking self-interacting dark matter [4], is incompatible with the very restrictive neutralino model. This fact gives further impetus to our approach to study WIMPs in a manner as model independent as possible. The limits on the electro-magnetic form factors obtained in this work show that
the amount of self-interaction, which could be induced by these form factors, is not sufficient to generate a cross section of WIMP-WIMP scattering at the level of $10^{-25}$ cm$^2$ as required by the self-interacting WIMP scenario.

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