Supertranslations and Holography near the Horizon of Schwarzschild Black Holes

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Abstract

In this paper we review and discuss several aspects of supertranslations and their associated algebras at the horizon of a Schwarzschild black hole. We will compare two different approaches on horizon supertranslations, which were recently considered in separate publications. Furthermore we describe a possible holographic description of a Schwarzschild black hole in terms of a large N boundary theory, which accommodates the Goldstone bosons of the horizon supertranslations.
## Contents

1 Introduction

2 The algebra of supertranslations on the horizon of a Schwarzschild black hole
   2.1 Supertranslations on the horizon
   2.2 Schwarzschild black holes

3 Family of Schwarzschild metrics and soft modes on the horizon
   3.1 Family of flat metrics and soft modes at infinity
   3.2 Family of black holes under horizon supertranslations

4 Schwarzschild holography and the quantum black hole case
   4.1 Near horizon limit of the Schwarzschild geometry and a holographic bulk coordinate \( \lambda \)
   4.2 Holographic bulk-boundary correspondence
   4.3 Black hole N-portrait and a boundary toy model in one dimension
   4.4 A quantum resolved bulk metric

5 Summary
1 Introduction

Over many years, asymptotic symmetries of certain space-time geometries play an important role in general relativity. E.g. three-dimensional AdS space possesses an infinite-dimensional, asymptotic $W$-symmetry, which is holographically realized as symmetry group of the two-dimensional conformal field theory that lives on the boundary of the AdS space \cite{1}. In four space-time dimensions, the BMS supertranslations were already introduced in 1962 by Bondi, Metzner and Sachs \cite{2}. These infinite-dimensional BMS transformations $BMS^\pm$ describe the symmetries of asymptotically flat space times at future or past null infinity, denoted by $I^+$ and $I^-$, but in general not in the interior of four-dimensional space-time. Furthermore, if one considers a gravitational scattering process (an S-matrix in Quantum Field Theory) on asymptotically flat spaces, there is a non-trivial intertwining between $BMS^+$ and $BMS^-$ (for a recent review on these issues see \cite{3}).

Recently it was conjectured \cite{4,5} that the BMS symmetry will play an important role in resolving the so called black hole information paradox by providing an (infinite-dimensional) hair, i.e. charges to the black hole, that carries the information about the collapsing matter before the black hole is formed. In fact, it was argued \cite{6,7} that the BMS group can can be extended as symmetry group $BMS^H$ to the horizon $H$ of a Schwarzschild black hole.

In parallel to the work of \cite{6,7}, supertranslations on the horizon of black hole geometries were considered in a series of papers \cite{8–12}. Specifically, in \cite{9,12} the supertranslations and superrotations on the horizon of four-dimensional metrics as well as their associated algebras of Killing vectors were constructed. Independently, the supertranslations on the horizon of the Schwarzschild black hole were constructed in \cite{10,11}. It was shown that the supertranslations on the horizon generate an infinite number of gapless excitations on the horizon, that potentially can account for the gravitational hair as well as for the black hole entropy.

In this paper we will first review the construction of the supertranslations on the event horizon $H$ of a Schwarzschild black hole with metric written in Eddington-Finkelstein coordinates. In particular, one purpose of this paper is to compare the approaches of \cite{9,12} and of \cite{8,10,11}. We will find that they are in fact closely related. Furthermore we will show that the supertranslations on the horizon of a Schwarzschild black form an infinite dimensional, commuting algebra, which is isomorphic to the algebra of supertranslations at null infinity.

In the second part of the paper we will further elaborate on the connection between the BMS transformations and the massless Goldstone modes, which are associated to the classical degeneracy of the black hole vacua. We will discuss a holographic picture of the Schwarzschild black hole, which is based on the observations that the supertranslation algebras on null infinity of asymptotically flat space and on the black hole horizon agree with each other, and also that flat Rindler space is recovered as near horizon geometry in the semiclassical large N limit of the black hole geometry. We identify a proper holographic bulk coordinate, denoted by $\lambda(r)$, which plays the role of a collective coupling constant in the holographic boundary theory. The near horizon limit, where at the horizon $\lambda(r = r_S) = 1$, corresponds in the conformal boundary theory to a quantum critical point, where a Bose-Einstein condensate is formed. In the corpuscular large N quantum picture of the black hole in terms of a bound state of a large number of gravitons \cite{30,31}, a graviton Bose-Einstein condensate is built at $\lambda = 1$.

\footnote{Other related work about supertranslations on horizons can be found in \cite{13–29}.}
2 The algebra of supertranslations on the horizon of a Schwarzschild black hole

2.1 Supertranslations on the horizon

Here we want to construct the supertranslations on the event horizon $H$ of a four-dimensional space-time geometry. First following [9, 12] we write the general near horizon geometry using Gaussian null coordinates, where $v$ represents the retarded time, $\rho \geq 0$ is the radial distance to the horizon, and $x^A$ ($A = 2, 3$) are the angular coordinates:

$$
\begin{align*}
ds^2 &= -2\kappa \rho \, dv^2 + 2d\rho dv + 2\Theta_{AB} \, d\rho dx^A + (\Omega_{AB} + \lambda_{AB} \rho) \, dx^A \, dx^B + \Delta g_{ij} \, dx^i dx^j.
\end{align*}
$$

Here $\kappa$ stands for the surface gravity and $\Theta_A$, $\Omega_{AB}$, $\lambda_{AB}$ are arbitrary functions of $v, x^A$, and $\Delta g_{ij}$ being terms of order $O(\rho^2)$ ($i,j \in \{v,A\}$). Actually, it is always possible to find a coordinate system in which the metric close to a smooth null surface admits to be written in the form [32, 33].

One finds that the asymptotic Killing vector preserving the metric ansatz (2.1) is given by

$$
\begin{align*}
\chi^v &= f(v, x^A), \\
\chi^\rho &= -\partial_v f + \frac{1}{2} \Omega^{AB} \partial_B f \rho^2 + O(\rho^3), \\
\chi^A &= Y^A(x^B) + \Omega^{AC} \partial_C f \rho + \frac{1}{2} \Omega^{AB} \Omega^{CD} \lambda_{DB} \partial_C f \rho^2 + O(\rho^3),
\end{align*}
$$

where $\Omega^{AB}$ is the inverse of $\Omega_{AB}$.

The corresponding variation of the fields read

$$
\begin{align*}
\delta\chi &= Y^A \partial_A f + \partial_v (\kappa f) + \partial_v^2 f, \\
\delta\chi_{AB} &= f \partial_v \chi_{AB} + \mathcal{L}_Y \chi_{AB}, \\
\delta\Theta_A &= \mathcal{L}_Y \Theta_A + f \partial_v \Theta_A - 2\kappa \partial_A f - 2\partial_v \partial_A f + \Omega^{BD} \partial_v \chi_{AB} \partial_D f, \\
\delta\lambda_{AB} &= f \partial_v \lambda_{AB} - \lambda_{AB} \partial_v f + \mathcal{L}_Y \lambda_{AB} + \Theta_A \partial_B f + \Theta_B \partial_A f - 2\nabla_A \nabla_B f.
\end{align*}
$$

By introducing a modified version of Lie brackets [17]

$$
[\chi_1, \chi_2] = \mathcal{L}_{\chi_1} \chi_2 - \delta_{\chi_1} \chi_2 + \delta_{\chi_2} \chi_1,
$$

which suffices to take into account the dependence of the asymptotic Killing vectors upon the functions in the metric, one finds that the algebra of these vectors closes near the horizon; namely

$$
[\chi(f_1, Y^A_1), \chi(f_2, Y^A_2)] = \chi(f_{12}, Y^A_{12}),
$$

where

$$
\begin{align*}
f_{12} &= f_1 \partial_v f_2 - f_2 \partial_v f_1 + Y^A_1 \partial_A f_2 - Y^A_2 \partial_A f_1, \\
Y^A_{12} &= Y^B_1 \partial_B Y^A_2 - Y^B_2 \partial_B Y^A_1.
\end{align*}
$$
2.2 Schwarzschild black holes

The set of metrics (2.1) includes in particular the Schwarzschild black hole. Namely the Schwarzschild-metric in (infalling) Eddington-Finkelstein coordinates \((v, r, \vartheta, \varphi)\) is obtained by the coordinate-transformation

\[
v = t + r^*,
\]

(2.7)

with

\[
dr^* = (1 - \frac{r_S}{r})^{-1} dr.
\]

(2.8)

This choice of Eddington-Finkelstein coordinates covers the exterior and the future-interior of a Schwarzschild-black hole and the Schwarzschild metric gets the form

\[
ds^2 = -(1 - \frac{r_S}{r}) dv^2 + 2 dv dr + r^2 d\Omega^2.
\]

(2.9)

Indeed, performing the change of variable

\[
r = r_S (1 + 2 \kappa \rho),
\]

(2.10)

where \(r_S = 1/(2 \kappa)\) for Schwarzschild, and expanding in powers of \(\rho\), one finds that (2.9) becomes

\[
ds^2 = -2 \kappa \rho \, dv^2 + 2 d\rho dv + \left( \frac{1}{4 \kappa^2} + \frac{\rho}{\kappa} \right) (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) + \mathcal{O}(\rho^2),
\]

(2.11)

which belongs to the metric ansatz (2.1) with \((A = \vartheta, \varphi)\)

\[
\begin{align*}
\Theta_\vartheta &= \Theta_\varphi = 0, \\
\Omega_{\vartheta \vartheta} &= \frac{1}{4 \kappa^2}, & \Omega_{\varphi \varphi} &= \frac{\sin^2 \vartheta}{4 \kappa^2}, & \Omega_{\vartheta \varphi} &= 0, \\
\lambda_{\vartheta \vartheta} &= \frac{1}{\kappa}, & \lambda_{\varphi \varphi} &= \frac{\sin^2 \vartheta}{\kappa}, & \lambda_{\vartheta \varphi} &= 0.
\end{align*}
\]

(2.12)

Now, let us also restrict the Killing vectors (2.2) of the BMS transformations on the horizon to the case of the Schwarzschild black hole. First, one can check that in case of \(\partial_v f = 0\) and no superrotations \(Y^A = 0\) the Killing vectors (2.2) become:

\[
\zeta^\mu = \left( f(\vartheta, \varphi), 0, -4 \kappa^2 \rho \frac{\partial f}{\partial \vartheta} - \frac{4 \kappa^2}{\sin^2 \vartheta} \rho \frac{\partial f}{\partial \varphi} \right) + \mathcal{O}(\rho^2),
\]

(2.13)

Second we use the map (2.10) and the Killing vectors on the horizon become

\[
\zeta^\mu = \left( f(\vartheta, \varphi), 0, \frac{\partial f}{\partial \vartheta} \left( \frac{1}{r} - \frac{1}{r_S} \right), \frac{1}{\sin^2 \vartheta} \frac{\partial f}{\partial \varphi} \left( \frac{1}{r} - \frac{1}{r_S} \right) \right),
\]

(2.14)

where \(f(\vartheta, \varphi)\) is an arbitrary function on the two-sphere.

As shown in \([10,11]\), these are precisely diffeomorphisms acting on the Schwarzschild metric that leave the horizon invariant. According to \([10,11]\), the supertranslations eq.(2.14) on the
This is consistent with the fact, when going to the Schwarzschild metric, we have considered \( v, r, z, \) which act on the coordinates \((v, r, z, \bar{z})\) as follows:

\[
BMS^H : \quad \delta_{\zeta_f} g_{\mu \nu} = \begin{pmatrix} 0 & 0 & -(1 - \frac{r}{r_s}) \frac{\partial f}{\partial r} & -(1 - \frac{\bar{r}}{\bar{r}_s}) \frac{\partial f}{\partial \bar{r}} \\ 0 & 0 & 0 & 0 \\ 2r^2(\frac{1}{r} - \frac{1}{r_s}) \frac{\partial^2 f}{\partial r^2} & 2r^2(\frac{1}{r} - \frac{1}{r_s})(\frac{\partial^2 f}{\partial r \partial \varphi} - \cot \varphi \frac{\partial f}{\partial \varphi}) & 2r^2(\frac{1}{r} - \frac{1}{r_s})(\frac{\partial^2 f}{\partial r^2} + \sin \varphi \cos \varphi \frac{\partial f}{\partial \varphi}) \end{pmatrix} .
\] (2.15)

Using complex coordinates\(^2\) on the two-sphere the Killing vectors (2.14) can be expressed as:

\[
BMS^H : \quad \zeta_f = f \frac{\partial}{\partial v} - \frac{r_s - r}{r r_s} \left( D^z f \frac{\partial}{\partial z} + h.c. \right) .
\] (2.16)

They act on the coordinates \((v, r, z, \bar{z})\) in the following form:

\[
BMS^H : \quad v \to v - f(z, \bar{z}), \\
z \to z + \frac{r_s - r}{r r_s} \gamma \gamma^\bar{z} \partial_{\bar{z}} f(z, \bar{z}), \\
\bar{z} \to \bar{z} + \frac{r_s - r}{r r_s} \gamma \gamma^z \partial_z f(z, \bar{z}), \\
r \to r .
\] (2.17)

Now we can also determine the algebra of the Killing vectors of the Schwarzschild black hole. Namely we want to compute the algebra formed by the vector fields (2.14) and see if would reproduce a BMS algebra. Taking the commutator of two vector fields, one finds near to the horizon, i.e. \( r \to r_s \), that the algebra closes and becomes an infinite-dimensional, commutative supertranslation algebra:

\[
[\zeta_{f_1}, \zeta_{f_2}]_{r \to r_s} = (f_{12} + \mathcal{O}(\rho^2), \mathcal{O}(\rho^2), \mathcal{O}(\rho^2), \mathcal{O}(\rho^2)) = (\mathcal{O}(\rho^2), \mathcal{O}(\rho^2), \mathcal{O}(\rho^2), \mathcal{O}(\rho^2)) .
\] (2.18)

This is consistent with the fact, when going to the Schwarzschild metric, we have considered only those supertranslations satisfying \( \partial_s f = 0 \) and with no superrotation \( Y^A = 0 \). It then follows that \( f_{12} = 0 \).

We can also consider the standard BMS-supertranslations of the Schwarzschild metric on \( \mathcal{I}^- \). These are simply obtained by taking the limit \( r_s \to \infty \) \[^3\] \[^4\], which takes the future horizon \( r = r_s \) to past null infinity \( \mathcal{I}^- \). In this limit the vector fields reduce to

\[
\zeta_f^\mu = \left( f(\partial, \varphi), 0, \frac{\partial f}{\partial \varphi} \frac{1}{\sin^2 \varphi} \right) ,
\] (2.19)

which is a BMS supertranslation vector field at null infinity with respect to the Schwarzschild metric\(^2\). Like the algebra on the horizon in eq.(2.18), one gets in this case that

\[
[\zeta_{f_1}, \zeta_{f_2}]_{r_s \to \infty} = (\mathcal{O}(r^{-2}), \mathcal{O}(r^{-2}), \mathcal{O}(r^{-2}), \mathcal{O}(r^{-2})) ,
\] (2.20)

\[^2\]The complex coordinates are defined as \( z = \cot(\frac{1}{2} \varphi) e^{i \varphi} \) and the metric of \( S^2 \) in angular coordinates reads \( ds^2 = 2r^2 \gamma \gamma d\varphi d\bar{\varphi} = d\varphi^2 + \sin^2 \varphi d\varphi d\bar{\varphi} \).

\[^3\]Notice though that \( \xi^\mu = \frac{1}{2} \Delta f \) for BMS.
and hence the algebra closes with \( f_{12} = 0 \), namely one recovers the usual supertranslation algebra

\[
\left[ f_1, f_2 \right] = 0. \tag{2.21}
\]

As discussed in [10,11], the BMS transformations on the horizon also contain in part BMS transformations on \( I^- \). Since the BMS transformations on null infinity are not supposed to contribute to the black hole entropy, we want to subtract them from the supertranslations on the horizon in order to isolate the physical gapless modes of the black hole. This subtraction was performed [10] for the infinitesimal supertranslations, and we have called the modes that result from this factorization the \( \mathcal{A} \)-modes on the horizon. Furthermore, as one can see, the modes of \( BMS^H \) in eq.(2.15) have also non-vanishing entries in the \( v - \vartheta \) and \( v - \varphi \) components of these matrices. These are kind of electric field strength components. We want to construct transformations where these components are however absent. This is precisely also the case for the \( \mathcal{A} \)-modes. Namely the effect of taking the quotient \( \mathcal{A} \) erases the electric components of the gapless modes and leave only the magnetic components. These transformations act on the Schwarzschild metric eq.(2.9) in the following way:

\[
\mathcal{A} : \quad \delta_{\mathcal{A}, g_{\mu\nu}} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
o_{*} & -2r^2 \frac{1}{r_S} \frac{\partial^2 f}{\partial \vartheta \partial \varphi} & -2r^2 \frac{1}{r_S} \left( \frac{\partial^2 f}{\partial \vartheta \partial \varphi} - \cot \vartheta \frac{\partial^2 f}{\partial \varphi^2} \right) \\
0 & 0 & 0 & -2r^2 \frac{1}{r_S} \left( \frac{\partial^2 f}{\partial \varphi^2} + \sin \vartheta \cos \vartheta \frac{\partial^2 f}{\partial \vartheta \partial \varphi} \right)
\end{pmatrix}. \tag{2.22}
\]

The corresponding modes constitute an infinite family of classical black hole metrics. They are the physical, classical Goldstone modes of the supertranslations at the horizon. We see that the electric components of the metric fluctuations are indeed absent in for the \( \mathcal{A} \)-modes. So the \( \mathcal{A} \)-modes describe soft gravitational waves that are confined to the horizon and do not fluctuate into the bulk. The \( \mathcal{A} \)-modes feel small changes of \( r_S \) only with respect to their amplitudes, as can be seen in above equation. Hence the \( \mathcal{A} \)-modes constitute a kind of membrane paradigm. One can in addition show that the metric \( g_{\mu\nu} = g_{\mu\nu} + \delta_{\mathcal{A}, g_{\mu\nu}} \) is Ricci-flat in the angular directions:

\[
R_{\varphi \varphi} (g^f_{\mu\nu}) = R_{\vartheta \vartheta} (g^f_{\mu\nu}) = R_{\varphi \varphi} (g^f_{\mu\nu}) = 0. \tag{2.23}
\]

Hence the Goldstone modes along the horizon fully solve the non-linear vacuum Einstein equations in the angular directions. Hence no energy momentum tensor in the angular directors needs to be generated.

### 3 Family of Schwarzschild metrics and soft modes on the horizon

In this section we will continue to discuss the family of Schwarzschild metrics, which are related to the supertranslations on the horizon, in more detail, where we will use a slightly different form of the metric compared to previous section.
3.1 Family of flat metrics and soft modes at infinity

Let us first briefly recall the expansion of the metric in Bondi coordinates at around $I^{-}$ to obtain a family of asymptotically flat metrics (see [3] for a review on that subject):

$$ds^2 = -dv^2 + dvdr + 2r^2 \gamma_{\bar{z}z}dzd\bar{z} + \frac{2m_B}{r}dv^2 + rC_{zz}dz^2 + rC_{\bar{z}z}d\bar{z}^2 - 2U_zdv dz - 2U_{\bar{z}}dvd\bar{z} + \ldots .$$

(3.1)

Here $\gamma_{\bar{z}z} = \frac{2}{(1+z\bar{z})^2}$ is the round metric of the unit $S^2$ and $U_z = -\frac{1}{2}D^2z \Phi(z, \bar{z})$.

Furthermore, $m_B$ is the Bondi mass for gravitational radiation, the $C_{zz}(v, z, \bar{z})$ are in general functions of $z, \bar{z}, u$ and the Bondi news $N_{zz}(v, z, \bar{z})$ are characterizing ingoing gravitational waves that pass trough $I^{-}$:

$$N_{zz}(v, z, \bar{z}) = \partial_u C_{zz}(v, z, \bar{z}).$$

(3.3)

Gravitational vacua with zero radiation have $N_{zz} = 0$. In gauge theory language, the metric component $C_{zz}(v, z, \bar{z})$ plays the role of the gauge connection, whereas the Bondi news $N_{zz}(v, z, \bar{z})$ are analogous to the field strength in gauge theory. One can show that the standard supertranslations on $I^{-}$ with some function $g(v, z, \bar{z})$ act on the Bondi mass and on gauge connection $C_{zz}$ as:

$$m_B \rightarrow m_B, \quad C_{zz} \rightarrow C_{zz} - 2D^2z g.$$  

(3.4)

The term $2D^2z g$ in above equation is a pure gauge and does not lead to any field strength $N_{zz} = 0$. It follows that the field strength, i.e. the Bondi news are invariant under BMS transformations:

$$N_{zz} \rightarrow N_{zz}.$$  

(3.5)

Hence, for a static background with zero radiation, namely if $N_{zz} = 0$ like for Minkowski space, BMS transformation do not generate any Bondi news.

Nevertheless static backgrounds, as Minkowski space, are not unique, but they are characterized by an infinite family of pure gauge connections $C_{zz} = 2D^2z \Phi$. It then follows that an infinite family of flat metrics can be defined as those satisfying

$$C_{zz} = D^2z \Phi(z, \bar{z}),$$

(3.6)

with $\Phi(z, \bar{z})$ being some arbitrary function. The associated Bondi news are automatically zero. In other words, we can define the manifold $C$ of flat vacua as

$$C = \{C_{zz}, \ C_{zz} = D^2z \Phi(z, \bar{z})\}.$$  

(3.7)

Hence the space of flat vacua is isomorphic to the space of functions $\Phi(z, \bar{z})$ on the 2-sphere. BMS transformations are transforming points of $C$ onto points of $C$:

$$BMS: \ C_{zz} = D^2z \Phi \rightarrow C_{zz}^g = D^2z(\Phi - 2g).$$

(3.8)

We can translate this discussion into a quantum mechanical picture: the space of classical (asymptotically) flat metrics corresponds to family of classical vacua $|\Phi\rangle$ as follows:

$$C_{zz} = D^2z \Phi \rightarrow |\Phi\rangle \equiv |C_{zz}\rangle = |D^2z \Phi\rangle.$$  

(3.9)
Furthermore BMS supertranslation generators are given in terms of charges (generators) $Q^-$ and act on the classical vacua as

$$Q^-(\Phi) = |C_{zz}^g\rangle = |D_z^2(\Phi - 2g)\rangle \equiv |\Phi - 2g\rangle.$$  \hspace{1cm} (3.10)

Since the $Q^-$ do not annihilate the ground state $|\Phi\rangle$, they are the generators of the spontaneously broken BMS symmetry.

Next let us discuss the Goldstone modes, which are associated to the spontaneously broken generators $Q^-$. Classically, the Goldstone modes just correspond to the variations of the classical metric under the BMS supertranslations, denoted by $\delta g_{\mu\nu}$. Since the Goldstone modes will correspond to physical zero momentum gravitons in the quantum theory, we want to focus on the physical, transversal metric fluctuation, namely on the $\delta g_{zz}$ component of the metric fluctuation (and similarly also $\delta g_{\bar{z}\bar{z}}$). According to the discussion in the previous section, the Goldstone modes are then nothing else than $\delta g_{zz} = 2D_z^2 g$.

Now we are at a stage to relate the Goldstone modes to the soft gravitons. For this purpose we are considering two classical vacua at some initial time $v_i$ and some final time $v_f$, where the two vacua differ from each other by a BMS transformation:

$$v_i : \quad C_{zz}^i = D_z^2 \Phi_i, \quad v_f : \quad C_{zz}^f = D_z^2 \Phi_f = D_z^2 (\Phi_i - 2g).$$  \hspace{1cm} (3.11)

In addition we are considering an interpolating metric $C_{zz}^{int}(u,z,\bar{z})$ with the properties

$$C_{zz}^{int}(v_i,z,\bar{z}) = C_{zz}^i, \quad C_{zz}^{int}(v_f,z,\bar{z}) = C_{zz}^f.$$  \hspace{1cm} (3.12)

The interpolating metric can be regarded as a domain wall that interpolates between these two classical vacua. It necessarily possesses non-vanishing Bondi news:

$$N_{zz} = \partial_v C_{zz}^{int}(v,z,\bar{z}) \neq 0.$$  \hspace{1cm} (3.13)

Hence the domain wall corresponds to a time-dependent radiative mode with non-zero Bondi news, which describes a gravitational wave that passes $I^-$ between the times $v_i$ and $v_f$. The zero frequency part of this gravitational wave is called soft graviton mode and is defined in the following way:

$$N_{zz}^{\omega=0} = \lim_{\omega \to 0} \int_{v_i}^{v_f} dv \ e^{i\omega v} \partial_v C_{zz}(v,z,\bar{z}).$$  \hspace{1cm} (3.14)

Using partial integration it is simply given by the BMS variation of the metric component $C_{zz}$:

$$N_{zz}^{\omega=0} = C_{zz}^f - C_{zz}^i = \delta g C_{zz} = 2D_z^2 g.$$  \hspace{1cm} (3.15)

This shows that the Goldstone mode is just the soft graviton:

$$|\text{Goldstone boson}\rangle \equiv |\text{Soft graviton}\rangle.$$  \hspace{1cm} (3.16)

It is interesting to note that even Minkowski space is equipped with low energy Goldstone modes. However as we will discuss now, the corresponding BMS charges of Minkowski space are zero.

So let us finally construct the corresponding BMS charges. In general the classical Noether charge is defined as

$$Q^- = \omega(C_{zz}, \mathcal{L}_g C_{zz}),$$  \hspace{1cm} (3.17)
where $\omega$ is the symplectic form associated to the classical phase space. In our case we obtain

$$Q^- \simeq \int_{\mathcal{C}} dv \, dz \, d\bar{z} \, \partial_v C_{zz} D_2^2 g =$$

$$= - \int_{\mathcal{C}} dv \, dz \, C_{zz} \partial_v (D_2^2 g) =$$

$$= - \frac{1}{2} \int_{\mathcal{C}} dv \, dz \, d\bar{z} \, C_{zz} \partial_v (\delta g C_{zz}). \quad (3.18)$$

where the Cauchy surface $\mathcal{C}$ is null and can be identified with $I^-$. The BMS charges also relate to the soft gravitons, i.e. to the Goldstone bosons via the standard low energy theorem:

$$\langle 0 | Q^- | GB \rangle \neq 0 \quad (3.19)$$

However it is important to note that eq. (3.18) implies that the BMS charges are zero for static, i.e. $v$-independent backgrounds. Therefore the BMS charges for Minkowski space are vanishing. The BMS charges can only be non-zero, if the background itself is $v$-dependent and describes some radiative modes. In other words, if a soft gravitational wave is excited, which has the form of a gravitational plus at $I^-$, the BMS charges are non-zero.

The vanishing of the BMS charges for static configurations can be also understood using the quantum language. Consider the expectation value of the charge operator between a vacuum state $|\Phi\rangle$:

$$\langle \Phi | Q^- | \Phi \rangle = \langle \Phi | \Phi - 2g \rangle \quad (3.20)$$

This expectation value vanishes for static vacua, since in this case $|\Phi\rangle$ and $|\Phi - 2g\rangle$ are orthogonal to each other.

### 3.2 Family of black holes under horizon supertranslations:

We now like to derive the soft modes, which are associated to the supertranslations in the horizon, in a similar fashion to the ones at $I^-$. Therefore we now expand the Schwarzschild metric around in $(r - r_s)^{-1}$ around the horizon in the following form:

$$ds^2 = -(1 - \frac{r_s}{r})dv^2 + 2dvdr + r^2 \gamma_{zz} dz d\bar{z}$$

$$+ \frac{rr_s}{r - r_s} C_H^{zz} dz^2 + \frac{rr_s}{r - r_s} C_H^{zz} d\bar{z}^2 + \ldots . \quad (3.21)$$

The $C_H^{zz}(v, z, \bar{z})$ play again the role of gauge connections on the horizon. Non-vanishing radiation at the horizon is now characterized by a new kind of Bondi news which are the associated field strengths and take the form

$$N^H_{zz} = \partial_v C_H^{zz}(v, z, \bar{z}). \quad (3.22)$$

For static backgrounds, like the Schwarzschild metric, the $C_H^{zz}$ do not depend on $v$ and the classical radiation $N^H_{zz}$ is zero.

In eqs. (2.11) we have written the transformation of the metric under $BMS^H$ supertranslations in matrix form. We can now equivalently express these transformations as transformations on the gauge connections $C_H^{zz}$ as:

$$BMS^H : \quad C_H^{zz} \rightarrow C_H^{zz} - 2D_2^2 f. \quad (3.23)$$
We see that the supertranslations on the horizon generate an infinite family of static black hole metrics, all characterized by pure gauge connections \( C^H_{zz} \), namely those satisfying

\[
C^H_{zz} = D_z^2 \Phi^H(z, \bar{z}) ,
\]

(3.24)

with now \( \Phi^H(z, \bar{z}) \) being some arbitrary function. So, we can define the manifold \( C^H \) of Schwarzschild vacua as

\[
C^H = \{ C^H_{zz}, C^H_{zz} = D_z^2 \Phi^H(z, \bar{z}) \} .
\]

(3.25)

Hence the space of Schwarzschild vacua is isomorphic to the space of functions \( \Phi^H(z, \bar{z}) \) on the 2-sphere. \( BMS^H \) supertranslations on the horizon are transforming points of \( C^H \) onto points of \( C^H \):

\[
BMS^H : C^H_{zz} = D_z^2 \Phi^H \rightarrow C^H_{zz} = D_z^2 (\Phi^H - 2 f) .
\]

(3.26)

Now we come to the charges and to the soft modes on the horizon related to \( BMS^H \). The discussion is quite analogous to the one in the previous section, and hence we will not provide all details. Namely the corresponding soft graviton on the horizon is defined in the following way:

\[
N^H,\omega=0_{zz} = \lim_{\omega \to 0} \int v_f \frac{e^{i \omega v} \partial_v C^H_{zz}(v, z, \bar{z})}{v_i} dv = \delta_f C^H_{zz} = 2D_z^2 f .
\]

(3.27)

In addition we then get the following expression for the supertranslation charges at the horizon

\[
Q^H = \int_{I_{B.H.}} dv \, dz \, d\bar{z} \, \partial_v C^H_{zz} D_z^2 f =
\]

\[
= - \int_{I_{B.H.}} dv \, dz \, d\bar{z} \, C^H_{zz} \partial_v (D_z^2 f) =
\]

\[
= - \frac{1}{2} \int_{I_{B.H.}} dv \, dz \, d\bar{z} \, C^H_{zz} \partial_v (\delta_f C^H_{zz}) .
\]

(3.28)

It is clear that for the eternal static black hole these charges are zero, because already the integrand is vanishing, since the \( H \)-modes are \( v \)-independent.

At the end of this section we like to emphasize that for the classical eternal black hole, we are discussing at the moment, the classical Goldstone modes of the \( H \)- and \( A \)-supertranslations do not carry any energy, i.e. they have zero frequency. In other words, for the eternal, static black hole no energy is deposited on the horizon by the classical \( BMS^{H/A} \) supertranslations. The fact that the modes have zero frequency is also the reason why the corresponding classical charges are zero for the eternal Schwarzschild black hole. On the other hand, for classical backgrounds, which are not static, like collapsing shells, the \( H/A \)-modes will in general possess non-zero frequencies. This is the case if gravitons or also other particles with gravitational interactions fall into the black hole. In this case the corresponding charges are possibly non-zero. So the BMS charges or the \( H/A \)-charges do not only measure the number of how many soft gravitons are at the horizon, but each charge also measures the frequency of these modes.

In summary, the classical information, which is associated to the infinite classical entropy of a static black hole, is not accessible by any classical experiment. In other words, the infinite classical entropy, i.e. the infinite classical hair cannot be resolved in a finite amount of time - see also [29]. Also the classical electromagnetic hair considered in [6] also would need infinite time to be resolved.
4 Schwarzschild holography and the quantum black hole case

4.1 Near horizon limit of the Schwarzschild geometry and a holographic bulk coordinate \( \lambda \)

It is known from the work of Maldacena [34] that the near horizon limit of \( N \) coincident D3-branes for \( N \to \infty \) is described by the product space \( AdS_5 \times S^5 \). The AdS/CFT correspondence then states that type IIB supergravity on \( AdS_5 \times S^5 \) is holographically equivalent to \( N = 4 \) super Yang-Mills gauge theory with gauge group \( SU(N) \) on the four-dimensional boundary of \( AdS_5 \). In the supergravity limit, i.e. in the limit of small \( AdS_5 \) curvature radius, besides \( N \) also the t’Hooft coupling \( \lambda = g_s N \) of the \( SU(N) \) gauge theory is taken to be large.

In section two we have derived the infinite-dimensional commuting algebra of the supertranslations on the Schwarzschild horizon, which is the same as the usual algebra of supertranslations at infinity. At first sight it looks surprising that the two supertranslation algebras are the same. But this becomes understandable by noting that the near-horizon geometry of the Schwarzschild black hole is given by the flat Rindler space. Motivated by this observation, we want to discuss in the following a picture, which is in close analogy to the emergence of \( AdS \) geometry via the branes in superstring theory: for the Schwarzschild geometry there exist a three-dimensional dual conformal theory (reps. one-dimensional theory, when omitting the angular coordinates) living on the holographic screen of Rindler space, namely the one-dimensional Rindler cone of \( M^{1,1} \times S^2 \), in analogy to the \( SU(N) \) gauge theory on the boundary of \( AdS_5 \times S^5 \).

First recall that classical Bekenstein-Hawking entropy \( S \) [35,36], here also denoted by \( N \), of a Schwarzschild black hole, the radius \( r_S \) of the horizon and the black hole mass \( M \) scale scale as

\[
S = N, \quad r_S = L_P \sqrt{N}, \quad M = M_P \sqrt{N}.
\]  

Here \( L_P (M_P) \) is Planck length (mass), which in terms of \( G_N \) and Planck’s constant \( \hbar \) is defined as \( L_P^2 \equiv \hbar G_N \). As usual, the gravitational radius and the mass of the black hole are related as \( r_S = G_N M \).

As just said, the near-horizon geometry of the Schwarzschild geometry is given by the flat Rindler space, denoted by \( M^{1,1} \times S^2 \), where \( M^{1,1} \) is the 2-dimensional Minkowski space in Rindler coordinates \( \tilde{t} \) and \( \tilde{\rho} \). Its metric is given as

\[
ds^2 = -\tilde{\rho}^2 d\tilde{t}^2 + d\tilde{\rho}^2 + \sum_{i=2,3} dx_i^2 dx_i.
\]  

The Rindler coordinates are related to the Schwarzschild coordinates as follows

\[
\tilde{\rho} = \sqrt{r_S(r - r_S)} = \sqrt{r_S \rho}, \quad \tilde{t} = \frac{t}{r_S}.
\]  

In the following we consider for fixed \( \hbar \) and \( G_N \), the following semiclassical large \( N \) limit, such that \( \tilde{\rho} \) and \( \tilde{t} \) are finite for small \( \rho \) reps. for large \( t \):

\[
N \to \infty, \quad r_S \to \infty, \quad M \to \infty,
\]

So in this limit the entropy as well as the mass and the horizon become infinitely large.

We now want to consider the Schwarzschild horizon is a kind of 3-dimensional holographic boundary with coordinates \( t, \vartheta \) and \( \varphi \) or respectively in the near horizon Rindler geometry.
with boundary coordinates \( t, x_2 \) and \( x_3 \). The orthogonal bulk coordinate is given by \( \rho \) or by \( \tilde{\rho} \), respectively. The bulk coordinate \( \rho \) (or \( \tilde{\rho} \)) can be related to a new holographic bulk coordinate \( \lambda \) in the following way:

\[
\lambda(r) = \frac{L_P^2}{r^2} N = \frac{r_S^2}{\rho^2} = \frac{r_S^2}{(\rho + r_S)^2} = 1 - \frac{\rho}{2r_S} + \ldots
\]  

(4.5)

The horizon is now located at the point \( \lambda(r = r_s) = 1 \). So the near-horizon limit \( \rho \to 0 \) corresponds to the limit

\[
\lambda \to 1.
\]  

(4.6)

Note that \( \lambda \) can be rewritten as

\[
\lambda = \alpha N,
\]  

(4.7)

where

\[
\alpha(r) = \frac{L_P^2}{r^2} N = \frac{r_S^2}{r^2} \frac{1}{N}
\]  

(4.8)

is nothing else than the scale dependent gravitational coupling constant. It becomes very small in the large \( N \) limit. In fact, the coupling constant \( \lambda \) was introduced in [30, 31] as collective coupling constant of \( N \) gravitons that form a black hole bound state, where the coupling constant between two individual graviton pairs is just given by \( \alpha \). It was further argued that in the near horizon limit, namely at the point \( \lambda = 1 \), the \( N \) graviton system becomes a quantum critical system, which behaves like a Bose-Einstein condensate at the quantum critical point.

### 4.2 Holographic bulk-boundary correspondence

In the following we want to further explore the idea that quantum gravity in the four-dimensional Schwarzschild bulk geometry is dual to a quantum field theory with lives on a three-dimensional boundary, namely the black hole horizon \( H \). So what are the expected features of such a holographic bulk-boundary correspondence?

Let us first discuss some qualitative properties of the holographic Schwarzschild bulk-boundary correspondence in the large \( N \) limit eq.(4.4), where the black hole horizon becomes infinite, and the gravity theory becomes semiclassical. The large \( N \) limit means that the coupling constant \( \alpha \) of the three-dimensional boundary field theory becomes very weak. Furthermore at the near horizon limit \( \lambda = 1 \) in the bulk, the boundary theory should become gapless [30, 31]. This is the point of quantum criticality of the boundary theory, where its collective coupling \( \lambda = \alpha N \) becomes critical, i.e. \( \lambda = 1 \). Then the spectrum of the three-dimensional boundary theory will become highly degenerate with an infinite number of massless states that account for the infinite black hole entropy. Furthermore the boundary theory should become conformal. On the other hand, for a finite black hole horizon, i.e. finite \( N \), the bulk gravity theory will receive \( 1/N \) quantum corrections. For the boundary theory this means that the energy gap will disappear, where the splitting of the energy levels will be of order \( 1/N \). These \( 1/N \) corrections will also destroy the conformality of the boundary theory.

In summary we expect that in the large \( N \), near horizon limit, semiclassical four-dimensional gravity of the Schwarzschild black hole can be described by a highly degenerate, three-dimensional gapless theory, which will be a particular three-dimensional conformal field theory.

As we have seen so far, for classical gravity the supertranslation charges on the horizon are vanishing. In the quantum language, the Goldstone modes, also now called Bogoliubov modes,
of the classical supertranslations in eqs. 2.15, 2.22, and eq. 3.23 correspond to the Goldstone bosons of the spontaneously broken supertranslations at the horizon:

\[ |\text{Goldstone boson} \rangle \equiv |\text{Bogoliubov mode} \rangle. \quad (4.9) \]

As we will now explain, these Goldstone bosons are composed out of gravitons, however unlike to the soft gravitons at \( I^- \), they are not asymptotic, perturbative spin-two graviton particles, but rather they are collective modes of gravitons. Furthermore, in the quantum theory with finite \( N \), the Goldstone modes are not completely gapless (massless) anymore, but they acquire a mass, which however is still small compared to the black hole mass itself.

So in the quantum case the situation compared to the classical situation substantially changes: The black hole is not anymore eternal, it radiates and the radiation also implies a back reaction on the classical metric, such that it is not any more static. Furthermore due the finite size of the horizon, quantum mechanics imply that the \( H/A \)-modes are described by Bogoliubov modes, which are not any more gapless and do not correspond to zero-momentum gravitons, but to modes on the horizon with now finite frequency, i.e. with finite energy. In other words, in the quantum theory there is a finite energy gap, denoted by the frequencies \( \omega \), which are of order \( 1/N \). It follows that in the quantum case, the supertranslations on the horizon are explicitly broken, which means that the Bogoliubov modes are not anymore massless Goldstone bosons, but will acquire a mass of the order of the energy gap.

As we will now indicate, it furthermore follows that the quantum supertranslation charges on the horizon will be non-vanishing. The corresponding entropy also becomes finite, where it was formally infinite in the classical case. We can define the quantum charge operator \( \hat{Q}^{H/A}_{lm} \) for the supertranslations at the horizon in the same way as the classical charge in eq. (3.28), only treating Bogoliubov modes now as quantum mechanical operators:

\[
\hat{Q}^{H/A} = \int_{H,H} g^{\mu\nu} \partial_\nu \delta_{X,f} \hat{g}_{\mu\nu} = \int_{H,H} \partial_\nu \hat{C}^{H/A}_{zz} D^2 f \quad (4.10)
\]

As we have discussed, classical supertranslations are generating an infinite family of black hole metrics. In the same way, the quantum mechanical operator \( \hat{Q}^{H/A} \) is transforming one particular black hole quantum state into another one:

\[
\hat{Q}^{H/A} |BH\rangle = |BH\rangle. \quad (4.11)
\]

Since the black hole state \( |BH\rangle \) is not invariant under the action of \( \hat{Q}^{H/A} \), it immediately follows that the supertranslations at the horizon are spontaneously broken in the black hole vacuum.

Next we can expand the Bogoliubov modes in spherical harmonics as follows:

\[
\delta_{X,f} \hat{g}_{\mu\nu} (v, \vartheta, \varphi) = \sum_{m,l} \delta_{X,f} \hat{g}^{lm}_{\mu\nu} (v, \vartheta, \varphi)
= \frac{1}{\sqrt{\omega_{lm}}} \sum_{m,l} \left( \hat{b}^{\mu\nu}_{ml} Y_{lm} (\vartheta, \varphi) e^{-i\omega_{ml} t} + \hat{b}^{\dagger \mu\nu}_{ml} Y_{lm} (\vartheta, \varphi) e^{i\omega_{ml} t} \right). \quad (4.12)
\]
Note that the quantum Bogoliubov modes do depend on $v$ via the non-vanishing frequencies $\omega_{ml}$. The effective action of the $H/A$-modes on the horizon has the following form:

$$S_{\text{eff}} = \int dv d\vartheta d\varphi (\partial_v \delta_m, g_{\mu\nu}(v, \vartheta, \varphi))^2$$

(4.13)

Then the BMS charge operator $\hat{Q}_{lm}^{H/A}$ at the horizon in eq. (4.10) that corresponds to the above quantum Bogoliubov modes takes the following form:

$$\hat{Q}_{lm}^{H/A} = -i \sqrt{\omega_{lm}} \left( \hat{b}_{ml} e^{-i\omega_{lm}v} - \hat{b}_{ml}^\dagger e^{iv\omega_{ml}} \right).$$

(4.14)

4.3 Black hole N-portrait and a boundary toy model in one dimension

In analogy to the N coincident D3-branes of the $AdS_5 \times S^5$ geometry, the microscopic picture of the Schwarzschild reps. Rindler geometry is proposed to be a bound state (Bose-Einstein condensate) of $N$ graviton particles at the quantum critical point. This model is called the black hole N-portrait [30,31]. In the black hole quantum N-portrait the black hole state $|BH\rangle$ is given in terms of the state of $N$ gravitons at quantum criticality, where $\alpha N = 1$, with $\alpha$ being the gravitational coupling among the individual graviton modes. Discarding the angular $S^2$ part of the Schwarzschild metric or the $R^2$ part of Rindler space, we can try to set up a one-dimensional boundary model, which is dual to the two-dimensional Schwarzschild black hole respectively dual to two-dimensional Rindler space. One possible toy model for the black hole N-portrait, which features quantum criticality, is the model of $N$ interacting bosons on a one-dimensional ring [31]. Further details about this model including its relation to entanglement and quantum information are given in refs. [37–41]. This model is often used in condensed matter physics to describe a Bose-Einstein condensate of $N$ bosons.\footnote{This model can be viewed as an alternative to the SYK-model, which was argued to be the holographic dual of a two-dimensional AdS black hole.}

Note that since the Schwarzschild black hole is a non-BPS object, we are really dealing with a bound state, created by the attractive gravitational force among the $N$ graviton particles. This is in contrast to the BPS D3-branes, which are just coincident, since the gravitational and the $p$-form forces among the branes are opposite and cancel each other.

In this model, the quantum criticality manifests itself in the appearance of classically-gapless Bogoliubov modes that we have identified with the classically gapless modes obtained by acting with supertranslations BMS$(H/A)$. The energy gap generated by the quantum effects in a given mode of momentum $\frac{\hbar}{r_S}$ can be estimated as

$$\omega_{ml} \sim \Delta E = \frac{1}{N} \frac{\hbar}{r_S}. \quad (4.15)$$

The black hole vacua are then given in terms of coherent states of Bogoliubov modes in the following way:

$$|BH\rangle \equiv |N\rangle = \exp\left(\sum_{lm} \sqrt{n_{lm}} (\hat{b}_{ml} - \hat{b}_{ml}^\dagger)\right)|0\rangle, \quad (4.16)$$

where $n_{lm}$ is the individual occupation numbers for the mode with angular momentum $l, m$. Then we finally get for the vacuum expectation value of the charge operator (4.14)

$$\langle BH|\hat{Q}_{lm}^{H/A}|BH\rangle = \sqrt{\omega_{lm} n_{lm}}. \quad (4.17)$$
Therefore, for a finite energy gap the BMS charges at the horizon are non-zero. Using the estimate eq.(4.15) we see that the charges at the horizon scale like
\[ Q_{lm} \sim \frac{1}{\sqrt{N}}. \quad (4.18) \]
Hence for finite \( N \), i.e., for finite entropy, the quantum effects provide non-vanishing charges to the black hole. However in the classical limit \( N \to \infty \) these charges vanish, in agreement with the classical discussion in the previous chapter.

Using eq.(4.11) we can also the write the vacuum expectation value of the charge operator in the following way:
\[ Q_{H/A}^{H/A} = \langle BH | \hat{Q}_{lm}^{H/A} | BH \rangle = \langle BH | \bar{B}H \rangle. \quad (4.19) \]
It follows that quantum mechanically the overlap between the two states \( |BH\rangle \) and \( |\bar{B}H\rangle \) is non-zero, but of order \( \frac{1}{\sqrt{N}} \). However in the classical limit \( N \to \infty \) the different black hole states are orthogonal to each other.

4.4 A quantum resolved bulk metric

As we have argued, the classical Goldstone modes are gapless in the classical limit, i.e., they have zero frequencies \( \omega = 0 \). Notice, this does not mean that their classical wavelengths \( l = \frac{\hbar}{k} \sim r_s \) are infinite since these modes do not satisfy the dispersion relations of free propagating waves on a flat space-time, but they rather satisfy the dispersion relations in the Schwarzschild geometry. So consider the Schwarzschild dispersion relation of the classical Goldstone modes:
\[ \hbar \omega = \sqrt{g_{c, vv} k^2}. \quad (4.20) \]
\( g_{c, vv} \) is the vv-component of the classical black hole metric
\[ g_{c, vv} = (1 - \frac{r_s}{r}). \quad (4.21) \]
Since \( \lim_{r \to r_s} g_{c, vv} = 0 \), the above dispersion relation is consistent with \( \omega = 0 \) and \( 1/k \sim r_s \) at the horizon.

Now we go to the quantum case. Here there is a finite energy gap, i.e., the quantum Bogoliubov modes carry an energy of order \( 1/N \), whereas the wave length \( l \) is still finite. Now we want to express the relation between \( \Delta E \) and \( l \) again in terms of a quantum dispersion relation, which now instead of the classical metric \( g_{c, vv} \) contains a quantum contribution to the metric, denoted by \( g_{qm, vv} \):
\[ \Delta E = \sqrt{g_{qm, vv} k^2}. \quad (4.22) \]
Using \( \Delta E = \frac{\hbar}{r_s} \) and \( l = k^{-1} = r_s \), we obtain for the quantum contribution of the metric
\[ g_{qm, vv} = \frac{\hbar^2}{N^2}. \quad (4.23) \]
Combing the classical metric with the quantum contribution, one obtains:
\[ g_{vv} = (1 - \frac{r_s}{r}) + \frac{\hbar^2}{N^2}. \quad (4.24) \]
This metric is non-generate. So we see that quantum corrections of the order $\hbar^2 N^2$ resolve the degeneracy of the classical metric at the horizon, or in other words the horizon gets smeared out and disappears.

5 Summary

In this paper we have reviewed and discussed several aspects of supertranslations at the horizon of a Schwarzschild black hole. These horizon supertranslations form a infinite dimensional, commutative algebra, and they are accompanied by Goldstone bosons, which are massless in the classical large N limit, but acquire masses of order $1/N$ after including quantum corrections. Furthermore we have argued that the geometric, semiclassical large N limit allows for a holographic description, where the holographic boundary of the Schwarzschild geometry is given by a cone in flat Rindler-space, and where the boundary theory can be described by a large number of interacting bosons. It would interesting to see, if there are any relations between the Schwarzschild holography, which is advocated in this paper, and the general space holography, as it is discussed e.g. in [42,43].

Furthermore, it would be also very interesting to more closely relate the considerations and the results of this paper to the scattering amplitudes of two gravitons into N gravitons. As argued in [44] (see also [45]) the N final state gravitons in the regime of quantum criticality where $\lambda = \alpha N = 1$ can account for the production of a black hole. As the N gravitons in the final state of the scattering amplitude are not completely soft, but have momenta of the order $\Delta E = 1/N$, it is tempting to argue that the N gravitons are closely related to the quantum Bogoliubov modes considered in this paper; furthermore it seems plausible that the quantum supertranslations precisely act on these final state gravitons and account in this way for the final black hole entropy. Possible additional massless gravitons, which could be added to the scattering amplitude, would then correspond to BMS transformations at null infinity, which are however already eliminated when taking the quotient of $\mathcal{A}$-supertranslations.

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16
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