Consequences of the inherent density dependence in one dimensional Dirac materials

M P Gochan© and K S Bedell

Department of Physics, Boston College, Chestnut Hill, MA 02467, United States of America

E-mail: gochan@bc.edu

Received 5 July 2018, revised 21 September 2018
Accepted for publication 24 September 2018
Published 12 October 2018

Abstract

Dirac materials are systems in which the dispersion is linear in the vicinity of the Dirac points. As a consequence of this linear dispersion, the Fermi velocity is independent of density and these systems exhibit unusual behavior and possess unique physical properties that are of considerable interest. In this work we study the ground state behavior of 1D Dirac materials in two ways. First, using the Virial theorem, we find agreement with a previous result in regards to the total average ground state energy. Namely, that the total average ground state energy, regardless of dimensionality, is found to be $E = \frac{B}{r_s}$ where $r_s$ is a dimensionless constant that’s a measure of density and $B$ is a constant independent of $r_s$. As a consequence, thermodynamic results as well as the characteristic exponents of 1D Fermi systems are density independent. Second, using conventional techniques, i.e. Tomanaga–Luttinger theory, we find several unique properties that are a direct consequence of the dispersion. Specifically, the collective modes of the system exhibit electron density independence predicted from the Virial theorem. Finally, possible experimental realization of our predictions of density independent exponents are briefly discussed.

Keywords: Dirac materials, strongly correlated electrons, low dimensional, Virial theorem

1. Introduction

Dirac materials encompass a large family of materials that are at the forefront of theoretical and experimental research. Graphene, topological insulators, and $d$-wave superconductors are a few examples in this family that have been thoroughly studied and exhibit new results and rich physics. These systems, in spite of being different, share a fundamental similarity that links them together: their low energy fermionic excitations behave as massless Dirac particles [1]. A standard tight-binding calculation immediately leads to an exact linear dispersion in the vicinity of the Dirac points and is vastly different from the parabolic dispersion found in ordinary systems. Thus, any properties that are a direct consequence of this linear spectrum are universal among these diverse materials. For example, the fermionic specific heat is controlled by the linear behavior and its power law dependence is shared by graphene, $d$-wave superconductors, and superfluid $^3$He. In addition to these low energy properties, some Dirac materials are also boundary states for exotic quantum phases and have drawn great research interest. Specifically, the so-called Dirac semi-metal can be driven into various quantum states such as Weyl semimetals, axion insulators, and topological superconductors [2]. Although Dirac materials possess different properties. It is the universal low energy behavior, the possibility of realizing exotic quantum states of matter, and the rich fundamental physics within Dirac materials that has fueled the research into understanding these systems.

The following work on a 1D Dirac system has direct implications into understanding the behavior of single walled carbon nanotube (SWNT). SWNTs provide a one dimensional (1D) environment full of rich physics including curvature generated spin orbit coupling effects relevant to spintronics applications [3], and investigation of the long controversial
1D Coulomb problem [4]. In our work we consider a 1D Dirac system in its metallic state near $T = 0$ where we stay away from gapped states and avoid possible Wigner crystallization by avoiding other chiralities [5, 6]. The Hamiltonian is (we set $\hbar = 1$ for the remainder of the paper) [7]

$$H = H_0 + H_{\text{int}}$$

(1)

where $H_0$ is the Hamiltonian for a free Dirac gas:

$$H_0 = v_F \sum_{r,k,\sigma} \left( r k - k_F \right) c_{k\sigma}^\dagger c_{r\sigma}$$

(2)

where the Fermi velocity $v_F \sim 10^6$ m s$^{-1}$ is a constant, $\sigma = \uparrow, \downarrow$ is the single fermion spin, $r = +, -$ indicates if we’re on the positive or negative branch, and $k$ is the momentum. $H_{\text{int}}$ is the Tomanaga–Luttinger interaction [8]:

$$H_{\text{int}} = \frac{1}{L} \sum_{k_1,k_2,p_1,p_2,\sigma_1,\sigma_2} \left[ \Gamma_{\sigma_1,\sigma_2}^{2} \delta_{\sigma_1,\sigma_2} + g_1^s \delta_{\sigma_1,-\sigma_2} + \frac{1}{2} \Gamma_{\sigma_1,\sigma_2}^{4} \left( \delta_{\sigma_1,\sigma_2} + \delta_{\sigma_1,-\sigma_2} + \delta_{\sigma_1,\sigma_2} + \delta_{\sigma_1,-\sigma_2} \right) \right]$$

where the interaction is characterized by $\Gamma_{\sigma_1,\sigma_2}$ defined as

$$\Gamma_{\sigma_1,\sigma_2} = g_1^s \delta_{\sigma_1,\sigma_2} + g_2^s \delta_{\sigma_1,-\sigma_2}, \quad (i = 2, 4)$$

(3)

$\sigma_1/\sigma_2$, and $k_1/k_2$ are the spins and momentum of interacting fermions respectively. In both (2) and (3), $k_F$ is the Fermi momentum, and $c^\dagger, c$ are the creation and annihilation operators for fermions. $i = 2$ refers to forward scattering between electrons on separate branches while $i = 4$ refers to forward scattering between electrons on the same branch. The superscripts $s$ and $a$ denote symmetric and anti-symmetric spin respectively. The coupling constants, $g_1^s$ and $g_2^s$, are a measure of the interaction strength between electrons for given scattering processes and are in general momentum dependent; we neglect this dependence by restricting the momentum to within certain values $\Lambda$ (i.e. $k_F - \Lambda < k < k_F + \Lambda$) [7]. Before continuing, there are two important notes about the interaction Hamiltonian: first, the interaction given in (3) only contains terms involving forward scattering; Umklapp and back scattering processes have been ignored. This exclusion is justified through the following: as long as we’re considering low-energy properties, interactions that do not commute with charge and spin separation are irrelevant [9]. Such suppression and insensitivity to disorder is only relevant in metallic systems; in semiconducting systems, these effects are relevant and need to be taken into consideration [10]. Second, (3) is not complete but is the simplest model that shows deviation from typical Fermi liquid behavior.

We immediately notice an important distinction in (2) unique to 1D Dirac systems that is not seen in conventional 1D metals; although the Tomanaga–Luttinger liquid (TLL) theory produces exact results for the Green’s function in the vicinity of two Fermi points ($\pm k_F$), the result is dependent upon linearization of the single particle spectrum [11, 12]. In ordinary 1D metals, this linear behavior is obtained by linearizing the quadratic dispersion around $\pm k_F$ and is therefore only exact in a region within a momentum cutoff. In Dirac systems, the linear behavior exists without linearization leading to validity for all $k$ where the linear behavior exists (see figure 1).

Recent angle resolved photoemission spectroscopy (ARPES) measurements performed on SWNTs have exposed behavior consistent with the TLL theory [13]. Such behavior, which is vastly different from traditional Fermi-liquid theory behavior, was seen in the spectral function and intensities which both exhibit the desired power-law behavior in good agreement with theory. These results motivated our investigation of 1D Dirac systems and lead to the main question: will the unusual behavior of electrons in 1D Dirac systems lead to deviations from TLL behavior? As we will see, a number of non-trivial results emerge that are not seen in ordinary 1D metals. Such differences allude to a new class of interacting systems call Dirac materials [1].

This paper is outlined as follows: first we derive the total ground state energy of the system using the Virial theorem and discuss the implications of the result as well as the connection to Coulomb systems. Second we use TLL theory to derive numerous quantities, such as the density of states, and
2. Average ground state energy

Recent work on Dirac materials in the presence of a Coulomb potential using the Virial theorem provides an expression for the average ground state energy of the system [14]:

$$E(r_s) = \frac{B}{r_s}$$

where $B$ is a constant independent of density. In this paper, $r_s$ is a dimensionless constant that’s a measure of density. Specifically, $r_s$ allows us to express the electron spacing in units of a characteristic length of the system; in this case, we have $r_s = \alpha r_s$ where $\alpha = 1.44$ Å [15, 16]. Although the work in [14] was done for 2D, the form for (4) remains the same regardless of the dimension of the system; the only thing that changes with a change in dimension is the constant $B$. Therefore, we expect the average ground state energy of a 1D Dirac material to have the same form as (4).

We start with an expression relating the compressibility of the system to the average ground state energy:

$$\kappa^{-1} = n^2 \frac{\partial^2 \mathcal{E}}{\partial n^2}$$

where $\mathcal{E} = E/L$ is the linear energy density, $n = N/L$ is the number density, and $L$ is the length of the system. The benefit in using (5) is the following: in 1D, the expression $n^2 \kappa$ is independent of density. This allows for a straightforward integration to obtain $\mathcal{E}$:

$$\mathcal{E} = \frac{1}{2} \kappa^{-1}$$

where the compressibility for a 1D interacting system with Dirac spectrum is found using [7, 14, 17]

$$\kappa = \frac{4v_g}{\pi n^2 (v_1)^2}$$

where $v_1$ is the speed of sound. The total average ground state energy is then:

$$E(r_s) = \frac{N \pi v_g}{8a} \left(1 + \frac{2g_4}{\pi v_g} \right) \frac{1}{r_s}$$

(8)

where we set $g_3 = g_3^g = g_4^g$ based on anti-symmetry arguments. The term in front of $1/r_s$ is

$$\frac{B}{N} = \frac{\pi n v_g}{8a} \left(1 + \frac{2g_4}{\pi v_g} \right)$$

(9)

(where $\hbar$ has been restored) and, as expected, is independent of $r_s$. For a typical SWNT, $r_s \approx 2.8 \pm 0.2$ eV.

Equation (8) is the main result in this paper and is non-trivial with a few remarkable consequences: first, there seems to be a connection between a Tomanaga–Luttinger system and a Coulomb system. Previous work [18, 19] explored the effects of a long-range Coulomb interaction in a 1D Dirac system. These interactions are vastly different, yet their average ground state energies have the same form and dependence on $r_s$ as in (4), with the exception that the constant $B$ varies slightly. Such a similarity alludes to the specific form of the interaction not being as important compared to the linear single particle spectrum in controlling the underlying physics of the system. Second, a closed form for the ground state energy (8) is a rare result that provides a bound allowing calculation of the thermodynamics and dynamics of a 1D Dirac system [14].

3. Tomanaga–Luttinger result

In a 1D system, TLL theory, as opposed to Fermi liquid theory, is necessary to describe the low-energy asymptotic behavior since the excitations are collective excitations (boson modes) rather than single particle excitations [7, 9]. Within this formalism, we first derive the Green’s function in a similar way that Dzyaloshinskii and Larkin have [17]. Ignoring interactions, the Green’s functions for each branch is

$$G_{\nu}^{(0)}(k, \omega) = \frac{1}{\omega - v_g (k - k_0) + i \delta \text{sgn}(k - k_0)}$$

In typical problems, the vertex function $\Gamma$ is determined via an infinite series of diagrams and requires approximations. In the model under consideration, there exists a simple and exact relation between $\Gamma$ and $G_i(k, \omega)$ for each branch

$$\Gamma_r = G_r^{-1} (p, \epsilon) - G_r^{-1} (p - k, \epsilon - \omega)$$

which is a direct consequence of the spectrum given by (2) and conservation of particle number on each branch. Equation (10) allows the Dyson equation$^1$ to be expressed as

$$[\epsilon - v_g (p - k_0)] G_i (\epsilon, \sigma) = 1 + \frac{i}{4\pi^2} \int \omega G_{\nu}^{(1)} \sigma \left( p - k, \epsilon - \omega \right)$$

(11)

where $D_{\nu}^{(2)}$ is the effective coupling constant [7]. Now we Fourier transform equation (11) to get

$$G_i (x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{dx}{v_g} \frac{1}{v_g t + i \Lambda} \prod_{j=\sigma,\rho} \frac{1}{\Lambda} \left( \frac{1}{2\pi} \delta (\omega) \right)^{1/2}$$

$$\times \left[ \Lambda^2 \left( t^2 - u_t^2 \right)^2 + \frac{1}{\Lambda} \left( 2u_t - \frac{1}{\Lambda} \right) \right]^{-1/2}$$

where $\delta (t)$ is a bandwidth cutoff and $\Lambda$ is a continuous momentum transfer cutoff. $u_\sigma$ and $u_\rho$ are the velocities of the spin density and charge density bosonic modes respectively; they depend on $v_g, g_2, \rho_2, g_4$ and $\omega$. The exponents, $\alpha_\sigma$ and $\alpha_\rho$, are of special interest and have the following form:

$^1$ Diagrammatic representations for Dyson’s equation as well as the vertex function can be found in [7] and [17].

$^2$ As a consequence of the linear dispersion, diagrams which contain loops with more than two interaction vertices cancel out and all that’s left is self energy diagrams containing bubbles. These are summed into the effective interactions. Expressions for the effective interactions for a normal metal can be found in [7].
The exponents, $\alpha_\sigma$ and $\alpha_\rho$, are a measure of interaction strength in the system and are the signature of 1D interacting Fermi behavior. The exponents for a normal 1D metal can be found in [7, 9] and although seem similar, are different in a crucial way. The dependence of the Fermi velocity on electron density is linear in a 1D normal metal as seen in figure 1(a). This leads to different values for equilibrium and dynamic quantities dependent on the linearization around each $k$-point.

In a 1D Dirac system, the Fermi velocity, $v_F$, is constant in electron density as seen in figure 1(b) leading to exact results for the equilibrium and dynamic properties independent of $kF$. Additionally, this difference has consequences in understanding the interactions in 1D Dirac systems. Since $\alpha_\sigma$ and $\alpha_\rho$ are density independent, the interaction strength in 1D Dirac systems is constant regardless of particle number.

From (12) we derive the momentum distribution function as done in [20] using the following equation

$$n(k) = -i \sum \int_{-\infty}^{\infty} G_s(x, 0^+) e^{-ikx} dx.$$  

At $t = 0$, the Green’s function behaves as

$$G_s(x) \sim x^{-1-\alpha} \quad \alpha = 2(\alpha_\sigma + \alpha_\rho)$$

we therefore expect the momentum distribution to have the same form. Using the Green’s function given by equation (10), we arrive at the following expressions:

$$n(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{dx}{\alpha' - i\alpha x} \left( 1 + (x\Lambda)^2 \right)^{\alpha' / 2} e^{i(k-rk) x}$$

$$= \frac{1}{\pi} \text{sgn} (k_F - rk) \int_{0}^{\infty} \frac{dx}{x} \left( 1 + (x\Lambda)^2 \right)^{\alpha'' / 2} \sin (|rk - k_F| x)$$

where the limit $\alpha' \to 0$ has been taken in the second equality. Evaluation of the integral is done with [21] and the general solution is

$$n(k) = \frac{1}{2\pi} \text{sgn} (k_F - rk) \left[ \beta \left( \frac{2}{2} \left( \alpha - 1 \right) \right) \right. $$

$$\times i F_2 \left( 1, 1, -\frac{\alpha}{2}, \frac{\alpha}{2} + \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \frac{|rk - k_F|}{\Lambda}$$

$$+ \frac{\pi^{1/2} \Gamma \left( \frac{1}{2} - \frac{\alpha}{2} \right)}{\Gamma \left( 1 + \frac{\alpha}{2} \right)} i F_2 \left( 1, 1, \frac{\alpha}{2}, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \frac{|rk - k_F|^{1/2}}{\Lambda}$$

$$\left( \frac{|rk - k_F|}{2\Lambda} \right)^{\alpha / 2} \right]$$

where $\Gamma(a)$ and $\beta(a,b)$ are the standard gamma and beta functions respectively, and $i F_2(a;b; c;z)$ is the generalized hypergeometric function. We limit ourselves to a small area around $k_F$ and impose that $|rk - k_F| \to 0$ faster than $\Lambda \to 0$, leading to the final result for the momentum distribution as

$$n(k) \simeq \frac{1}{2} - C_1 |rk - k_F|^\alpha \text{sgn} (rk - k_F) - C_2 (rk - k_F)$$  

where the constants $C_1$ and $C_2$ are

$$C_1 = \frac{1}{2\sqrt{\pi}} \frac{\Gamma \left( \frac{1}{2} - \frac{\alpha}{2} \right)}{\Gamma \left( 1 + \frac{\alpha}{2} \right)} \left( \frac{1}{2\Lambda} \right)$$

$$C_2 = \frac{\beta \left( \frac{2}{2} \left( \alpha - 1 \right) \right)}{\Lambda}.$$  

The momentum distribution, $n(k)$, exhibits all the usual behavior expected in a 1D interacting Fermi system; specifically, the continuity of $n(k)$ as well as the singularity in the first derivative indicating failure of the quasi-particle picture. The bandwidth cutoff $\delta(t)$ is no longer present since the starting point in (13) is at $t = 0$ which sets the bandwidth cutoff to zero. The momentum transfer cutoff $\Lambda$ is present but as stated before, we impose $|rk - k_F| \to 0$ faster than $\Lambda \to 0$. Additionally, in other reviews [7, 9, 20], the cutoff frequently takes the form $\Lambda(t) = \Lambda \text{sgn}(t)$ leading to its absence and signaling its irrelevance.

With the momentum distribution (14), we derive the density of states:

$$g(E) = \frac{1}{L} \frac{dN}{dE}$$

where $L$ is the length of the 1D system and $N$ is the number of electrons given by:

$$N = \frac{L}{2\pi} \int n(k) dk.$$  

We limit ourselves to a small area, $\delta k$, around $\pm k_F$ where $n(k)$ is valid. In this area, small changes in $N$ with respect to $k$ can be expressed as:

$$\frac{\delta N}{\delta k} = \frac{L}{2\pi} \left[ 1 - \frac{C_1}{\alpha + 1} (\delta k)^\alpha + \frac{C_2}{2} \delta k \right].$$  

In the limit as $\delta k \to 0$, the $\delta k$ term is negligible since $0 < \alpha < 1$ [13, 22]. Finally, we obtain the density of states for a 1D Dirac material near $\pm k_F$:

$$g(\omega) = \frac{2}{\pi v_F} \left( 1 - \frac{C_1}{\alpha + 1} |\omega|^\alpha \right)$$

where $C_1$ is the same as it is in equation (13) and $\omega$ is measured with respect to $\pm k_F$. Our expression agrees with the ARPES results in [13] that fit their data with $g(\omega) \sim |\omega|^\alpha$. Additionally, (15) reduces to the expression for the density of states for a non-interacting 1D Dirac system (found setting $\alpha = 0$ in (15)).

The unique features of $g(\omega)$ are displayed in figures 2 and 3. In figure 2 we see clear distinction in the dependence on electron density between the 1D normal metal and the 1D Dirac system. The result implies a saturation of the density of states. The density independence can also be seen in the collective modes and equilibrium properties of the system. Although these properties could be determined using equations (14) and (15), we use the benefits of the reduced dimensionality and bosonize [23–26] the 1D Dirac system. The bosonized Hamiltonian has the following form:

$$H = H_p + H_s$$  

which shows the separation of charge and spin that’s seen in 1D interacting Fermi systems. These two independent boson modes give rise to the following dispersions:
the Virial theorem (equation (4)). We readily see this given the equation (17) is the agreement with the energy derived from the density independence; previous experiments were focused on imaging and the dependence of the chiral indices on the density of states. In a sense, testing for this density dependence is easier than previous experiments due to the lifting of the uniformity requirement; we want multiple SWNTs with varying density. With that in mind, we expect the density independent speed of sound yet our result shows otherwise.

In order to test the density independent behavior predicted in this work, we need scanning tunneling microscopy (STM). With STM, data for the density of states can be obtained through [28]:

\[ \text{DoS} \propto \frac{dI}{dV} \approx V^\alpha \]  

where \( \alpha \) has the same meaning as before, \( V \) is the voltage, and \( I \) is the current. Although many STM experiments have been done before on SWNTs, none had the goal of testing the density independence; previous experiments were focused on imaging and the dependence of the chiral indices on the density of states. In a sense, testing for this density dependence is easier than previous experiments due to the lifting of the uniformity requirement; we want multiple SWNTs with varying density. With that in mind, we expect the \( dI/dV \) plots of density of states at zero frequency normalized to \( 2/\pi v_g \) as a function of \( nL \) where \( n \) is electron density and \( L \) is the length of the system. The dashed blue curve represents a system with typical parabolic spectrum while the pink solid line represents a Dirac system with linear dispersion.

\[ u_{i,\sigma}^2 = v_g^2 + \left( g_i^2 \pm g_i^4 \right)^2 \]

and in the weak coupling limit:

\[ u_{\rho}^2 = v_g^2 + \frac{2}{\pi} v_g g_4 \]

\[ u_{\sigma} = v_g \]

where we set \( g_i = g_i^0 = g_i^\text{rs} \) based on anti-symmetry arguments. These velocities, \( u_{\rho} \) and \( u_{\sigma} \), are independent of density. The only density dependence in the dispersion given by (17) is in \( k_F \propto (r_s)^{-1} \) which agrees with our Virial theorem result equation (4). In the absence of a magnetic perturbation, and in the presence of a charge perturbation, the charge density mode propagates and gives rise to the speed of sound:

\[ v_1 = v_g \sqrt{1 + \frac{2g_4}{v_g^4} + \frac{1}{(v_g^2)^2} \left[ (g_4)^2 - (g_2)^2 \right]} \]

The speed of sound can also be obtained from the chemical potential as shown in [15]. Both expressions are in agreement with the energy derived from the bosonic dispersions [7, 9, 25] and all present the same expression obtained in [17]. Both expressions for the speed of sound, (17) and (18), have the predicted density independent behavior. Density independent speed of sound is surprising due to its counter-intuitive nature. Speed of sound (first sound in the Landau and Lifshitz language [27]) is understood as a wave of compression and rarefaction. With an increase in electron density, one would expect a change in the speed of sound yet our result shows otherwise.

With the dispersion in (17), we can calculate all thermodynamic quantities. For example, the specific heat:

\[ c_v(T) = \left[ \frac{\pi k_B^2}{3} \left( \frac{1}{u_{\rho}} + \frac{1}{u_{\sigma}} \right) \right] T \]

has a linear dependence in temperature, which is characteristic of both Fermi and Bose systems at low temperature. However, the difference appears in the bracketed term multiplying \( T \) in (20). For typical 1D Fermi systems, the term in brackets will depend inversely with density; in a 1D Dirac system the bracketed term in constant in electron density. Thus the specific heat is also density independent. Other quantities such as the spin susceptibility, compressibility, and conductivity can be derived using the bosonic dispersions [7, 9, 25] and all present this density independence that’s unique to 1D Dirac materials. In general, we conclude that due to the density independence of \( g(\omega) \) and \( n(k) \), the subsequent equilibrium quantities in 1D Dirac systems will themselves be independent of density in spite of the contradiction to conventional interpretation.

\[ \epsilon_i = u_i k \quad i = \rho, \sigma \]  

Figure 2. Plot of density of states at zero frequency normalized to \( 2/\pi v_g \) as a function of \( nL \) where \( n \) is electron density and \( L \) is the length of the system. The dashed blue curve represents a system with typical parabolic spectrum while the pink solid line represents a Dirac system with linear dispersion.
to remain independent of density. Prior experiments [13, 22] have observed the temperature dependence of the differential conductance and have obtained good agreement with the theory. We would like to see results for keeping the temperature constant, and low, while changing the electron density. We expect results similar to what can be seen in figure 3. In addition to revealing the density dependence in SWNTs, an experiment like this has the added bonus of re-affirming the existence of Luttinger-liquid like states. Past experiments have been in agreement with theory, yet skeptics believe the observed behavior to be circumstantial due to the confined dimension of the electrons [5]. Data from the proposed experiment testing density dependence would help solidify the idea that these states exist. Additionally, thermodynamic quantities (e.g. the specific heat) of SWNTs can be obtained at varying densities, with all other things constant, and should be the same value indicative of this electron density independent behavior.

5. Summary

In this work we have demonstrated that the exact linear dispersion and the constant Fermi velocity found in Dirac materials gives rise to nontrivial physics. By first using the Virial theorem, we derive a closed form for the ground state energy with interactions included. Such an expression is rare and opens the door to calculation of the equilibrium quantities in the 1D Dirac material. Additionally, a strange similarity between TLL systems and Coulomb systems emerges that hints at the linearity of the dispersion, rather than the specific potential, as being more important in the unusual observed properties. We also observe some familiar 1D behavior, for example in the continuous momentum distribution function and low energy density of states, there are also profound differences. Primarily, the density independent exponents, equilibrium quantities, and collective modes, are differences that are unique to Dirac materials. These stark differences hint at a new class of 1D systems separate from traditional TLL. From studying these Dirac liquids we have a deeper understanding of the physics contained within Dirac materials; specifically, the importance of interactions in the system, and the importance of dimensionality. From here, we hope to use this work to shed light on even more exotic Dirac materials.

Acknowledgments

The authors would like to thank Thomas Mion and Dr Ilija Zeljkovic for valuable discussion in regards to experimental techniques as well as James Stokes, and Dr Alexander Balatsky, for fruitful discussion. Additionally, we thank the IMS at LANL for their hospitality. This work was supported in part by the John H Rourke, Boston College, Endowment Fund and the NSF Grant PHY-1208521

ORCID iDs

M P Gochan https://orcid.org/0000-0002-2704-7066

References

[1] Wehling T O, Black-Schaffer A M and Balatsky A V 2014 Adv. Phys. 63 1–76
[2] Liu Z et al 2014 Nat. Mater. 13 677–81
[3] Jeong J S and Lee H W 2009 Phys. Rev. B 80 075409
[4] Downing C and Portnoi M 2014 Phys. Rev. A 90 052116
[5] Deshpande V V, Bockrath M, Glazman L I and Yacoby A 2010 Nature 464 209–16
[6] Deshpande V V and Bockrath M 2008 Nat. Phys. 4 314–8
[7] Sólyom J 1979 Adv. Phys. 28 201–303
[8] Blagoev K B and Bedell K S 1997 Phys. Rev. Lett. 79 1106–9
[9] Voit J 1993 J. Phys.: Condens. Matter 5 8305
[10] McEuen P, Bockrath M, Cobden D, Yoon Y and Louie S 1999 Phys. Rev. Lett. 83 5098
[11] Tomonaga S I 1950 Prog. Theor. Phys. 5 544–69
[12] Luttinger J M 1963 J. Math. Phys. 4 1154–62
[13] Ishii H et al 2003 Nature 426 540–4
[14] Stokes J D, Dahal H P, Balatsky A V and Bedell K S 2013 Phil. Mag. Lett. 93 672–9
[15] Nozieres P and Pines D 1999 Theory Of Quantum Liquids (Advanced Books Classics Series) (New York: Avalon Publishing)
[16] Saito R, Dresselhaus G and Dresselhaus M S 1998 Physical Properties of Carbon Nanotubes (Singapore: World Scientific)
[17] Dzyaloshinskii I E and Larkin A I 1974 J. Exp. Theor. Phys. 38 202–8 (www.jetp.ac.ru/cgi-bin/dn/e_038_01_0202.pdf)
[18] Egger R and Gogolin A O 1997 Phys. Rev. Lett. 79 5082–5
[19] Kane C, Balents L and Fisher M P 1997 Phys. Rev. Lett. 79 5086–9
[20] Voi J 1995 Rep. Prog. Phys. 58 977
[21] Gradshteyn I S and Ryzhik I M 2007 Table of Integrals, Series, and Products 7th edn (Amsterdam: Academic)
[22] Bockrath M, Cobden D H, Lu J, Rinzler A G, Smalley R E, Balents L and McEuen P L 1999 Nature 397 598–601
[23] Mahan G D 2013 Many-Particle Physics (Berlin: Springer)
[24] Phillips P 2003 Advanced Solid State Physics (Advanced Book Program) (New York: Cambridge University Press)
[25] Giamarchi T 2003 Quantum Physics in One Dimension (International Series of Monographs on Physics) (Oxford: Clarendon)
[26] Gogolin A, Nersesyan A and Tsvelik A 2004 Bosonization and Strongly Correlated Systems (Cambridge: Cambridge University Press)
[27] Landau L, Lifshitz E and Pitaevskiĭ L 1980 Statistical Physics (Course of theoretical physics No. pt. 2) (Amsterdam: Elsevier)
[28] Kane C L and Fisher M P A 1992 Phys. Rev. Lett. 68 1220–3