Study on the vibration of membrane roof with the influence of added air-mass

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Abstract. For studing the influence regularity of added air-mass on the dynamic characteristics of membrane roof, the three-dimensional membrane roof is taken as the model, we obtained the analytical expression of added air-mass according to the energy conservation and aeroelastic mechanics. Then, futher investigations of the free vibration considering membrane immerse in air is given. Combined with the specific structural parameters, the influence of vibrate mode, membrane‘ density and the pretress on the additional air quality is discussed, which lays the foundation for further analysis of nonlinear coupling wind induced vibration response of membrane structures.

1. Introduction

For general structures, the influence of added air-mass can be negligible when calculating the natural frequency and response of the structures. But for the flexible structures, such as membrane structure, the coupling of aerodynamic and structure has a significant influence on the dynamic characteristics of structures. added air-mass may be pertains to the same order of the membrane` density. Therefore, the influence of added air-mass cannot be ignored in studying the vibration characteristics of membrane structures.

Many literatures have discussed the calculation of the added air-mass caused by the movement of structures in fluid. The added air-mass formula for the spherical surface of the radial pulsation was given by Irwin [1] which used to estimate the magnitude of the added air-mass for the roof of the Olympic Stadium in Montreal, Canada, and find that the effort of added air-mass was not negligible. Sewall [2] found that the value of added air-mass calculated by the formula given by Irwin is larger than the actual situation based on the vibration frequency comparison between the experiment and the theoretical formula given by Irwin. Li Qingxiang [3] derives the calculation method of the added air-mass for the two-dimensional membrane structure, which shows that the effect of the added air-mass on the structural characteristic frequency is significant.

In this article, the analytical method is used to analyze the added air-mass of the membrane structure’ vibration. Based on the three-dimensional membrane roof, the analytical expression of the added air-mass is obtained according to the energy conservation and aero elastic mechanics. Combined with the specific structural parameters, the influence of vibrate mode, membrane` density and the pretension on the additional air quality is discussed, which lays the foundation for further analysis of nonlinear coupling wind induced vibration response of membrane structures.
2. Analytical derivation of added air-mass
Assuming that the static equilibrium state of the membrane roof is horizontal, the length and width of the roof are a and b, respectively, and the density is ρ. As shown in Figure 1.

\[ w(x, y, t) = a_0 \cos \omega t \cdot \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \]  

(1)

Where,  \(0 \leq x \leq a; 0 \leq y \leq b; m = 1, 2, 3 \cdots, n = 1, 2, 3 \cdots\), \(\omega\) is the vibration frequency, \(a_0\) is displacement amplitude.

Assuming that the air is an incompressible, cohesionless steady state fluid, the roof surface is used as a fluid-solid coupling interface. The rate of change of kinetic energy in any part of the fluid to time is equal to the work [4] of surface pressure. That is:

\[ \frac{dE_i}{dt} = \frac{dW}{dt} \]  

(2)

\[ \frac{dW}{dt} = -\iint_S V_N p_Z dS = -\iint_S p_Z \frac{\partial w(x, y, t)}{\partial t} dS \]  

(3)

Where, \(V_N\) is the velocity of the fluid particle in the direction of the surface of the membrane, \(S\) is the surface boundary of the fluid region, \(p_Z\) is the pneumatic force of the membrane surface, and the integral area \(S \in \{0 \leq x \leq a, 0 \leq y \leq b\}\).

Change rate of kinetic energy relative to time of vibrating membrane is equivalent to the kinetic energy [4] of added air-mass per unit area, that is, the kinetic energy change rate of added air-mass can be expressed as:

\[ \frac{dE_i}{dt} = m_a \iint_S \left( \frac{\partial w(x, y, t)}{\partial t} \right)^2 dS \]  

(4)

Where, \(m_a\) denote added air-mass per unit area. The expression of added air-mass can be obtained from the formula (3) and (4).
\[ m_x = -\iint_S p_z \frac{\partial w(x,y,t)}{\partial t} dS \]
\[ \iint_S \frac{\partial w(x,y,t)}{\partial t}\left(\frac{\partial w(x,y,t)}{\partial t}\right)^2 dS \]  
(5)

For closed membrane roofs, only the upper surface is in contact with the air, and there is no air flow in the inner space of the membrane. Suppose the air is incompressible, non-viscous and potential. The perturbation velocity potential function satisfies the Laplace equation.

\[ \frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \phi'}{\partial z^2} = 0 \]  
(6)

The integral solution of eq.(6) is the Rayleigh integral equation with S as the boundary. For closed membrane roofs, the perturbation bound condition is:

\[ \nu_z = \frac{\partial \phi'}{\partial z} = 2\left(\frac{\partial \phi'}{\partial t} + V \frac{\partial \phi'}{\partial x}\right), \quad z \to 0, \quad (x,y) \in Ra \]  
(7)

Where, \( \nu_z \) denote the velocity component. \( z \) is the surface equation of the membrane roof. For the flat roof, the disturbance velocity potential function satisfying the boundary condition is

\[ \phi'(x,y,z,t) = \frac{1}{4\pi \mu_0} \iint \frac{\phi'}{\partial z} \frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \]  
(8)

The aerodynamic force on the surface of the membrane roof is as follows:

\[ p_z = -\rho \left(\frac{\partial \phi'}{\partial t} + V \frac{\partial \phi'}{\partial x}\right) \]  
(9)

The force inside the membrane roof can be approximately equal to the air force generated by the uniform flow field without disturbance, and the aerodynamic force acting on the roof surface is:

\[ P = p_1 - p_2 = \frac{\rho}{2\pi} \left\{ \iint_{Ra} \left( \frac{V}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} \cdot \left( x-\xi \right) \right) \right\} + \left\{ \iint_{Ra} \left( \frac{\partial \phi'}{\partial x} + \frac{\partial \phi'}{\partial t} \right) \frac{\partial \phi'}{\partial x} \right\} d\xi d\eta \]  
(10)

It is assumed that the surface functions of the membrane is:

\[ z(x,y,t) = z_0(x,y) + w(x,y,t) \]  
(11)
Where, $z_0(x,y)$ denote the initial surface function. For the flat roof, $z_0(x,y)=0$, then

$$z(x, y, t) = w(x, y, t)$$  \hspace{1cm} (12)

Substituting (12) into eq. (10).

$$p = \frac{\rho}{\pi} \int_{\alpha}^{\beta} \int_{\xi_1}^{\xi_2} \left( \frac{\partial^2 w}{\partial \xi^2} \right)_{y=\eta} d\xi d\eta + \frac{\rho V}{2\pi} \int_{\alpha}^{\beta} \int_{\xi_1}^{\xi_2} \left( \frac{\partial^2 w}{\partial x \partial t} \right)_{y=\eta} d\xi d\eta$$

$$- \frac{\rho V^2}{2\pi} \int_{\alpha}^{\beta} \int_{\xi_1}^{\xi_2} \left( \frac{\partial w}{\partial t} \right)_{y=\eta} (\xi-\xi') d\xi d\eta - \frac{\rho V}{2\pi} \int_{\alpha}^{\beta} \int_{\xi_1}^{\xi_2} \left( \frac{\partial w}{\partial t} \right)_{y=\eta} (\eta-\eta') d\xi d\eta$$  \hspace{1cm} (13)

Where, $r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$, integral region $Ra \in [0 \leq \xi \leq a, 0 \leq \eta \leq b]$.

3. Effect of added air-mass on vibration of the membrane roof

The effect of added air-mass on the vibration of thin film roof during the free vibration process is studied in this paper. Formula (13) can be simplified to:

$$P = \frac{\rho}{2\pi} \left( \int_{\alpha}^{\beta} \int_{\xi_1}^{\xi_2} \left( \frac{\partial^2 w}{\partial \xi^2} \right)_{y=\eta} d\xi d\eta \right)$$  \hspace{1cm} (14)

Taking the membrane material commonly used in engineering as an example, the surface density of the membrane is 1.2kg/m², the thickness is 1mm, the x direction modulus is 1.4×106kN/m², the y modulus is 0.9×106kN/m², the length and the width are 20m, and the pretension of membrane is 5kN. Table 1 is a comparison of the natural frequency $\omega_0$ and the frequency $\omega_{air}$ considering the added air-mass.

| Mode | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| $\omega_0$(HZ) | 2.28 | 3.61 | 5.10 | 6.65 | 8.23 | 9.81 | 11.41 | 13.01 | 14.61 | 16.22 |
| $\omega_{air}$(HZ) | 1.98 | 3.34 | 4.85 | 6.39 | 7.96 | 9.55 | 11.14 | 12.74 | 14.33 | 15.93 |

According to Table 1, the added air-mass significantly reduces the vibration frequency of the structure.

The ratio of the frequency $\omega_0$ of the membrane structure without air influence to the frequency $\omega_{air}$ with air influence under different modes and different densities are calculated and drawn as shown in Figure 2.

Form figure 2 we can see that the effect of air on the vibration frequency of membrane is becoming more obvious with the decrease of the membrane density, especially the low order modes.

The influence of added air-mass on structural frequency is analyzed with the change of membrane pretension ratio. Take $\sigma_y = 5000$kN/m². The result shows that the pretension is the key parameter affecting the structure’s frequency. The increasing pretensions can reduce the influence of air on the
frequency of the structure. But the impact was less than the density of membrane. The results are shown in Figure 3.

**Figure 2.** The ratio of $\omega_0$ to $\omega_{air}$ under different modes and densities

**Figure 3.** The ratio of $\omega_0$ to $\omega_{air}$ under different modes and pretension
4. Conclusion
In this paper, a simplified aerodynamic model is used to simulate aerodynamic forces. The variation rule of aerodynamic term is derived theoretically. Then, according to the numerical example, the influence of added air-mass on the dynamic characteristics of membrane roof is calculated. The following conclusions are obtained, the effect of the added air-mass on the frequency of the structure is significant, and the effect on the high order modes is not as obvious as that of the lower order modes. With the increase of membrane’s quality and pretension, the influence of added air-mass on the vibration frequency decreases.

Acknowledgments
This work was financially supported by National Natural Science Foundation (Project Number: 51478067) and Science and Technology research project of Chongqing Municipal Education Commission (Project Number: KJ1723378).

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