Gravitational lensing magnification and time delay statistics for distant supernovae

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Abstract

Strong gravitational lensing of distant supernovae (SNe), particularly Type Ia SNe, has some exploitable properties not available when other sorts of cosmologically distant sources are lensed. One such property is the “standard candle” nature of SNe at peak brightness allows for a direct determination of the lens magnification factor for each well-observed image. Another is that the duration of an SN event is of the same order as the differential time delays between the various lens images for roughly galaxy mass lensing objects. A relatively precise constraint on each image’s magnification leads to better constraints on the lens mass model than are available in more familiar lens systems, and the comparable timescales of the photometric event and the time delay invite a variety of applications, including high-precision measurements of the delay and the targeting of especially interesting phases of the explosion (including its very early stages) for intensive observation when they appear in trailing images. As an initial exploration of these possibilities we present calculations of SN lensing statistics in a “concordance cosmology” assuming a simple spherical model for lens mass distributions. We emphasize magnification and time delay effects. Plausible SN surveys, such as the proposed SNAP space mission, would discover several to some tens of strongly lensed SNe Ia per year, and at least a few of these will be at redshifts well beyond those that would be accessible via unlensed events. The total number of such anomalously high redshift SNe Ia will be a useful test of high-redshift star formation models. SN surveys of finite duration will, of course, miss the appearance of some images, and the effect becomes large when the delay approaches the survey duration; we quantify this selection bias. Finally, we investigate how well the appearance of trailing images can be predicted based on various amounts of available information on the lensing event. Knowledge of the magnification factor for the leading (and brighter) image makes it possible to predict the appearance of a trailing image relatively accurately if the lens redshift is also known.

Subject headings: cosmology: theory — gravitational lensing — supernovae: general

On-line material: color figures

1. Introduction

Recent systematic surveys of distant type Ia supernovae (SNe Ia) strongly suggest the presence of dark energy, which may dominate the total energy density of the universe (Riess et al. 1998; Perlmutter et al. 1999). The reason that this rather surprising conclusion is taken so seriously stems from the fact that the SNe Ia are excellent (albeit empirical) “standard candle” distance indicators, after an appropriate correction for the peak luminosity dependence on the shape of the individual light curve (Phillips 1993; Riess, Press, & Kirshner 1996). The exploitation of SNe Ia as cosmological probes has already been extensive (Riess et al. 1998; Perlmutter et al. 1999), and plans for yet more extensive observational studies, such as the Large-aperture Synoptic Survey Telescope4 (LSST), are being rapidly developed. The most ambitious of these is the proposed satellite Supernova/Acceleration Probe5 (SNAP), which would gather ~2000 SNe Ia per year by frequently imaging ~20 deg² of sky.

In this paper we explore some other ways in which the unique transient standard candle properties of SNe Ia might be exploited for those few that happen to be strongly gravitationally lensed by an intervening object of roughly galactic mass. For the sake of specificity we will adopt the observational parameters associated with the proposed SNAP mission unless otherwise stated.

Very roughly 0.1% of sources at z > 1 are expected to have multiple images due to strong gravitational lensing (e.g., Turner, Ostriker, & Gott 1984), and SNAP therefore would find at least ~2 lensed SNe Ia per year (e.g., Holz 2001). They will be qualitatively different from the lensed systems so far detected (e.g., ~60 QSO multiple-image systems6). First of all, the time delay between the multiple images could be robustly and accurately determined for each object, at least in principle. Indeed, for a typical cosmological lensing time delay, of the order of 1 yr, it is unlikely that one will observe all of the multiple images simultaneously because of the finite duration of SNe Ia (~1 month). This also means that some lensing events may be missed because the observation time is finite; one or more multiple images may appear only before or after the observing season or program. This is in marked contrast to QSO multiple-image systems, where the images are observed simultaneously and their presence and geometrical arrange-

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5 See http://snap.lbl.gov.
6 A summary of known lensed QSO systems is available on the CASTLES Web site (Kochanek et al., http://cfa-www.harvard.edu/castles).
ment in the sky is often the chief indication of lensing. Furthermore, since the SNe Ia are believed to be a reliable standard candle, the magnification factor of the lensing can be determined directly. This is not feasible for any other sources because an object’s intrinsic luminosity is basically unknown. This can provide valuable additional information on the lensing potential. With such information, one can indeed predict the location and the epoch of the additional trailing images, at least statistically.

In the present paper, we first analytically calculate the expected number of lensed SNe Ia to be found by a SNAP-like survey, including the effects of time delay bias due to a finite observation/survey duration. Since lens systems with wider separations have larger time delays on average, time delay bias becomes more significant for bigger image separations. Next, we describe ways to predict the appearance of additional images of lensed SNe Ia. We then briefly consider the consequences of using a different and perhaps more realistic model for lens mass distributions. In conclusion, we discuss some of the implications and possible extensions of this work.

Since we are not here concerned with determining cosmological parameters via lensing (although this constitutes one of the primary purposes of the SNe Ia survey), unless otherwise specified we simply adopt the Λ-dominated universe with \((\Omega_0, \omega_0, h) = (0.3, 0.7, 0.7)\), where \(\Omega_0\) is the density parameter, \(\omega_0\) is the cosmological constant, and \(h\) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\), the so-called concordance cosmology (Ostriker & Steinhardt 1995; Bahcall et al. 1999).

2. NUMBER COUNTS OF SNe Ia

The number of SNe is closely related to the star formation rate \(R_{SF}\) in the universe. For a Salpeter initial mass function \(\phi_{IMF}(M) \propto M^{-2.35}\), \(0.1 M_\odot < M < 125 M_\odot\), the rate of Type Ia events \(R_{SNe Ia}\) is calculated as (Madau, Della Valle, & Panagia 1998)

\[
R_{SNe Ia}(z) = \frac{1}{\eta} \int_7^{\infty} \int_4^{M_0} \frac{M}{M_{\odot}^{12}} \phi_{IMF}(M) dM \frac{d\Omega(t)}{d\Omega_{max}} \frac{(1-z)^3}{(1+z)^3},
\]

where \(M_c = \max[3 M_\odot, (10 \text{ Gyr}/t)^{0.4} M_\odot]\) is the minimum mass of a star that reaches the white dwarf phase at time \(t\) (assuming all systems with the primary star of mass \(3 M_\odot < M < 8 M_\odot\) are possible progenitors of SNe Ia), \(t_0 = 10 \text{ Gyr}/[M/M_\odot]^{2.5}\) is the standard lifetime of a star of mass \(M\), \(\tau\) is a characteristic explosion timescale (corresponding to an effective “delay” timescale after the collapse of the primary star to a white dwarf), and \(\eta\) is the SNe Ia explosion efficiency. We adopt three representative star formation rates used by Porciani & Madau (2001). The explicit forms of these in an Einstein–de Sitter universe are

\[
R_{SF 1}(z) = 0.462 \frac{h}{\exp(3.4 z) + 0.45} M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3},
\]

\[
R_{SF 2}(z) = 0.230 \frac{h}{\exp(3.4 z) + 22} M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3},
\]

\[
R_{SF 3}(z) = 0.308 \frac{h}{\exp(3.05 z - 0.4) + 15} M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3}.
\]

The first model (SF1) includes a correction for dust reddening and matches most measured UV continuum and H\(\alpha\) luminosity densities (Madau & Pozzetti 2000). The second model (SF2) is also possible because of the uncertainties associated with the incompleteness of data sets and the amount of dust extinction (Steidel et al. 1999). The third model (SF3) is considered because it has been suggested that the rates at high \(z\) may have been severely underestimated because of an unexpectedly large amount of dust extinction (e.g., Blain et al. 1999). These star formation rates are easily converted to those in different cosmologies (Porciani & Madau 2001) and are considered to span the range of reasonably realistic possibilities.

The number rate of SNe Ia that occur between \(z\) and \(z + dz\) is then

\[
\frac{dN}{dz} = \frac{R_{SNe Ia}(z)}{1+z} \frac{\Omega_A D_L^3(z)}{H(z)(1+z)^3},
\]

where \(\Omega_A\) is the solid angle of the observed region and \(D_L(z)\) is the angular diameter distance.

3. LENSING STATISTICS

3.1. Image Separation and Time Delay Probability Distribution

In this discussion, we omit the magnification bias (Turner 1980; Turner et al. 1984) because the intrinsic luminosity function at peak brightness of Type Ia SNe is quite narrow. We consider the magnification of SNe at redshifts beyond those normally accessible to the survey, another form of magnification bias, separately in the next subsection.

We now consider lensing objects with a singular isothermal sphere (SIS) density profile,

\[
\rho(r) = \frac{\gamma^2}{2\pi G r^2},
\]

where \(\gamma^2\) is a one-dimensional velocity dispersion, and define the characteristic scale length

\[
\xi_0 = 4\pi \left(\frac{\gamma}{c}\right)^2 \frac{D_{OS} D_{DS}}{D_{OL}},
\]

where \(D_{OL}, D_{OS}, D_{DS}\) are the angular diameter distances to the lens, to the source, and between the lens and source, respectively. Then the lens equation becomes

\[
y = x - \frac{x_0}{\sqrt{y_0}}/\sqrt{1 - y_0^2},
\]

where \(x_0 = y \pm 1\), and each image will be magnified by a factor \(\mu_i = (1/y) \pm 1\). The angular separation of two images is given by

\[
\theta = \xi_0 |x - x_{\pm}| = 8\pi \left(\frac{\gamma}{c}\right)^2 \frac{D_{DS}}{D_{OS}}.
\]

The differential time delay between two images can also be calculated as

\[
c\Delta t(y) = 32\pi^2 \left(\frac{\gamma}{c}\right)^4 \frac{D_{OL} D_{DS}}{D_{OS}(1 + z_L)} y.
\]

\(\ddagger\) This is equivalent to assuming that the survey monitors every field sufficiently frequently to catch each SN event near its peak brightness, as is the case for SNAP. The bias could not be neglected for a more traditional survey, which finds most objects well after their maximum brightness.
The cumulative and differential probability distributions of strong lensing are

\[ P(\theta; z_S) = \int_0^{z_S} dz_L \int_{t_{\min}}^\infty \sigma_{\mathrm{SIS}} \frac{c}{dz_L} (1 + z_L)^3 \phi(v), \quad (10) \]

\[ P(\theta; z_S) = -\frac{d}{d\theta} P(\theta; z_S), \quad (11) \]

where \( \sigma_{\mathrm{SIS}} = \pi d_0^2 \) is the cross section of strong lensing, \( d_0 = (\theta/2\pi)^{1/2} (D_{\mathrm{OS}}/D_{\mathrm{LS}})^{1/2} c \), and \( \phi(v) \) is the velocity function of lens galaxies. We adopt the velocity function

\[ \phi(v) = \frac{\Psi_v}{\ln 10} \left( \frac{\sqrt{2} v}{v_o} \right)^\beta \exp \left[ -\left( \frac{\sqrt{2} v}{v_o} \right)^{2.5} \right] \frac{dv}{v_o}, \quad (12) \]

where \( \Psi_v = 7.3 \times 10^{-2} h^3 \, \mathrm{Mpc}^{-3} \), \( \beta = -1.3 \), and \( v_o = 247 \, \mathrm{km} \, \mathrm{s}^{-1} \). This distribution function is based on the Southern Sky Redshift Survey and is derived by Gonzalez et al. (2000). Averaging the above probability function for fixed \( z_S \) over the observed rate of SNe (eq. [5]) yields the number rate of strong lensing:

\[ P(\theta) = \int P(\theta; z_S) \frac{dN}{dz_S} dz_S. \quad (13) \]

Similarly, the joint probability distributions of time delays and image separations are (see eqs. [29]–[31] of Oguri et al. 2002)

\[ P(\theta, z_S) \]

\[ = \int_0^{z_S} dz_L \frac{dN}{dz_S} N^T(\theta) \frac{c}{dz_L} (1 + z_L)^3 \phi(v), \quad \text{for } v = v_{\text{min}} \]

\[ P(\theta, z_S) = -\frac{d}{d\theta} P(\theta, z_S), \quad (15) \]

where \( N^T(\theta) = 1 - y_{\text{min}}^2 \) and \( y_{\text{min}} \) can be calculated from \( \Delta t = \Delta t(y_{\text{min}}) \). These joint probability distributions divided by \( P(\theta; z_S) \) give the conditional probability distributions \( P(\Delta t|\theta; z_S) \) and \( P(\Delta t|\theta; z_S) \).

Figure 1 shows the predicted number rates of SNe Ia and also the expected number rate of lensed SNe Ia. The number rates of SNe Ia are calculated from equation (5). It is clear from this figure that the number of SNe Ia strongly depends on the star formation rate and its evolution, as indicated by Madau et al. (1998). The number of lensed SNe Ia also shows large differences between models, reflecting the above sensitive model dependence of the number of SNe Ia. Our calculation predicts that the number of strongly lensed SNe Ia will be between a few and a few tens per year for a SNAP-like survey. These lensing rates are roughly consistent with those calculated by Wang (2000) and Holz (2001).

3.2. SNe beyond the Magnitude Limit

Gravitational lensing magnifies SNe; thus, some SNe that would exceed the magnitude limit if they were unlensed might be observed because of their magnification (e.g., Kolatt & Bartelmann 1998; Porciani & Madau 2000; Sullivan et al. 2000; Goobar et al. 2002). If we neglect the effect of gravitational lensing, the apparent magnitude of SNe at peak can be expressed as

\[ m_X = M_B + 5 \log(D_L(z) [\text{Mpc}]) + 25 + K_{\text{BX}}, \quad (16) \]

where \( M_B = -19.4 \) is the peak magnitude of SNe Ia, \( D_L(z) = (1 + z)^2 D_A(z) \) is the luminosity distance, and \( K_{\text{BX}} \) is the single- or cross-filter K-corrections. Therefore, even for high-z SNe Ia whose unlensed apparent magnitude exceeds the magnitude limit, \( m_X > m_{\text{lim}} \), they are observed if magnified by a factor \( \mu \) satisfying

\[ \mu \geq 10^{0.4(m_X - m_{\text{lim}})} \equiv \mu_{\text{min}}. \quad (17) \]

We consider the following two cases: (1) \( \mu_{\min} > \mu_{\min} \). This corresponds to the case that both lensed images are observed. (2) \( \mu_{\min} < \mu_{\min} \). This means that at least one image is observed. In each case, the cross section for lensing is

\[ \sigma(\mu_{\min}) = \frac{\sigma_{\text{SIS}}}{(\mu_{\min} \pm 1)^2}. \quad (18) \]

We calculate the numbers of such lensed SNe Ia, and the results are shown in Figure 2. We assume that the magnitude limit is \( m_{\text{lim}} = 30 \) and also impose a requirement that the photometry must extend to 3.8 mag below peak. We take account of the intrinsic dispersion of SNe Ia peak magnitudes, assuming the Gaussian distribution with dispersion \( \sigma_{\mu} = 0.15 \) (Porciani & Madau 2000). As seen in the figure, the expected number of such lensed SNe Ia depends very
the magnification factors become
\[
\mu_+ = \frac{2r}{r-1}, \quad (20)
\]
\[
\mu_- = \frac{2}{r-1}. \quad (21)
\]

The magnification factors also can be derived from the image separation and differential time delay if the redshift of the lens object is measured (see eq. [30]). The reconstruction of magnification factors could be used, in principle, to derive the distance-redshift relation at high z and to estimate the cosmological parameters more robustly.

4. TIME DELAY BIAS

Since there is a time delay between multiple images, strong-lensing statistics of transient phenomena such as SNe inevitably involve some missing events due to the finite duration of the survey observations. Therefore, in strong-lensing statistics of SNe we should take account of this "time delay bias," especially for large image separations.

For the joint probability distributions, the time delay bias is included as follows:
\[
P^{TB}(\Delta t, \theta; z_S) = P(\Delta t, \theta; z_S) f(\Delta t), \quad (22)
\]
\[
P^{TB}(> \Delta t, \theta; z_S) = \int_{\Delta t}^{\infty} P(\Delta t', \theta; z_S) f(\Delta t') d(\Delta t'), \quad (23)
\]
where \(f(\Delta t)\) is the fraction of lenses with time delays \(\Delta t\) that can be observed (the superscript TB explicitly indicates the time delay bias). The image separation distribution then becomes
\[
P^{TB}(\theta; z_S) = \int_0^{\infty} P(\Delta t', \theta; z_S) f(\Delta t') d(\Delta t'). \quad (24)
\]
Then the corresponding conditional probability distributions are computed as
\[
P^{TB}(\Delta t|\theta; z_S) = \frac{P^{TB}(\Delta t, \theta; z_S)}{P^{TB}(\theta; z_S)}, \quad (25)
\]
using \(P^{TB}(\theta; z_S)\) instead of \(P(\theta; z_S)\), which does not take account of the time delay bias (eq. [11]).

If the observational monitoring is carried out continuously for a period of \(t_{\text{obs}}\), for instance, \(f(\Delta t)\) is given by
\[
f(\Delta t) = \begin{cases} 1 - \frac{\Delta t}{t_{\text{obs}}} & (\Delta t < t_{\text{obs}}), \\ 0 & (\Delta t > t_{\text{obs}}). \end{cases} \quad (26)
\]
Then \(P^{TB}(\theta; z_S)\) reduces to
\[
P^{TB}(\theta; z_S) = P(\theta; z_S) \left[ 1 - \frac{1}{t_{\text{obs}}} \int_0^{t_{\text{obs}}} P(> \Delta t|\theta; z_S) d(\Delta t) \right] P(\theta; z_S) \equiv T(\theta, z_S) P(\theta; z_S). \quad (27)
\]
This means that the time delay bias for the image separation probability distribution is simply expressed by the multiplication factor \(T(\theta, z_S)\).

Probability distributions of strong gravitational lensing including time delay bias (eq. [27]) are shown in Figure 3, where we consider three cosmological models with \(h = 0.7\); \((\Omega_0, \lambda_0) = (0.3, 0.7), (0.3, 0.0), \) and \((1.0, 0.0)\). These
probability distributions have been used to constrain the cosmological constant (Turner 1990; Fukugita et al. 1992; Kocianek 1996; Chiba & Yoshii 1999) and are indeed useful as an independent measurement of cosmological parameters in the case of SN surveys (Wang 2000; Holz 2001; Goobar et al. 2002). These plots show that the time delay bias is more important for larger \( \theta \) because the time delay \( \Delta t \) is larger on average as \( \theta \) increases. The amount of the time delay bias is, however, quite small for lensing of a typical image separation of \( \theta \sim 1" \) if the observation time is larger than 1 yr. For larger separation lensing, \( \theta \sim 3" \), the time delay bias changes lensing probabilities by a factor of 2 and thus is important in quantitative discussions.

Since the time delay bias is more important for larger separation lensing, we also calculate probability distributions for wide-separation lensing in the \( \Lambda \)-dominated cold dark matter (CDM) model, assuming a fluctuation amplitude \( \sigma_8 = 1 \) for simplicity. Although the expected wide-separation lensing rate due to CDM halos is much smaller than for galaxy lensing, we still expect that wide-separation lensing will be observed because core-collapse SNe as well as SNe Ia can be used for lensing statistics, which greatly increases the number of SNe (e.g., Goobar et al. 2002). Wide-separation lensing is expected to reflect the properties of dark halos rather than (the visible parts of) galaxies; thus, it has been used to constrain the abundance of dark halos (Kochanek 1995) and density profile of dark halos (Maoz et al. 1997; Wyithe, Turner, & Spergel 2001; Keeton & Madau 2001; Takahashi & Chiba 2001; Li & Ostriker 2002; Oguri et al. 2002). We adopt the generalized form (Zhao 1996; Jing & Suto 2000) of the density profile proposed by Navarro, Frenk, & White (1997, hereafter NFW):

\[
\rho(r) = \frac{\rho_{\text{crit}} \delta_c}{(r/r_s)^{\alpha - 1}} (1 + r/r_s)^{3 - \alpha} \quad ,
\]

where \( r = r_{\text{vir}}/c_{\text{vir}} \) and \( c_{\text{vir}} \) is the concentration parameter. We adopt the mass and redshift dependence reported by Bullock et al. (2001) for \( \alpha = 1 \) and generalize it to \( \alpha \neq 1 \) by the multiplicative factor \( (2 - \alpha) \) (Keeton & Madau 2001). We also take account of scatter of the concentration parameter, which has a lognormal distribution with a dispersion of \( \sigma_c = 0.18 \) (Jing 2000; Bullock et al. 2001). The characteristic density \( \delta_c \) can be computed using the spherical collapse model (see Oguri, Taruya, & Suto 2001). As the mass function of dark halos, we adopt the fitting form derived by Sheth & Tormen (1999). From these, we predict probability distributions of wide-separation lensing taking account of the time delay bias (see Oguri et al. 2002 for the calculation of time delay probability distributions in the case of generalized NFW density profile), and results are shown in Figure 4. Here we focus on large-separation lensing (\( \theta > 5" \)) because the relation between small- and large-separation lensing depends strongly on the model of galaxy formation and is difficult to determine unambiguously (Oguri 2002). This plot indicates that time delay bias is quite significant; it can suppress the lensing probability by 1 or 2 orders of magnitude. It is also found that the suppression due to time delay bias is larger for \( \alpha = 1.5 \) than \( \alpha = 1.0 \) because for a fixed separation \( \theta \), the time delays in the case of \( \alpha = 1.5 \) are on average larger than those in the case of \( \alpha = 1.0 \) (Oguri et al. 2002). This slightly compensates for the difference of lensing probabilities between various values of \( \alpha \), but the difference is still large (about 1 order of magnitude between...
\( \alpha = 1.5 \) and \( \alpha = 1.0 \). Therefore, we conclude that statistics of wide-separation SN lensing still could provide a useful probe of density profile.

We also show the effect of the time delay bias on the time delay probability distribution in Figure 5. As seen in the figure, the time delay probability distributions for large \( \theta \) are strongly affected by the finite duration of observations.

### 5. Predictions for Time-Delayed (Trailing) Images

Since the primary purpose of the proposed SNe Ia surveys is to construct the Hubble diagram, one may always assume that the source redshift of each SN Ia, \( z_s \), is determined spectroscopically. We are interested in unusually bright SNe Ia in the diagram. If the lensing object is approximated by an SIS, there are two images, \( x_\pm \), and the brighter image (\( x_+ \)) arrives first. Thus, the difference between the observed magnitude of those outliers (with respect to the typical magnitude of the SNe Ia) and the average magnitude at the redshift can be ascribed to the lensing magnification \( \mu_+ \). While the standard deviation of the SN Ia peak magnitude corrected by the light-curve shape method is typically 0.15 mag (Porciani & Madau 2000), multiple lensing images are produced if \( \mu_\pm \geq 2 \) or equivalently more than 0.75 mag. Thus, 5 \( \sigma \) outliers are strong candidates for multiple-image lensed SNe Ia.

The lensing object (most likely an early-type galaxy) for those candidates is typically halfway out in affine or angular diameter distance. For instance, an L* galaxy at \( z = 0.5 \) is about 24th magnitude in the B band, and the surface number density at this magnitude limit is \( \sim 30 \) arcm \(^{-2} \) or so. Since the typical image separation \( \theta_+ \) is \( 1'' \), the average number of galaxies within that angular separation from each lensed SN Ia is much less than 1. Therefore, it should be relatively easy to find candidates for the lensing galaxy and thus determine \( \theta_+ \) and even possibly the redshift of the candidate galaxy \( z_L \).

Our next goal is to predict the location of the second image \( \theta_- \) and the time delay \( \Delta t \) by which the second image trails the first and brighter one. We consider the following three cases: (1) the magnification factor \( \mu_+ \), the redshift of the lens object \( z_L \), and the angular separation from the lens object \( \theta_+ = \xi_0 x_+ / D_{OL} \) are all known; (2) only \( \mu_+ \) and \( \theta_+ \) are known; (3) just \( \mu_+ \) is known. In doing so, we have to assume a specific model for the lensing potential. We mainly show results for the SIS lensing model (eq. [6]) but also present a comparison with the NFW halo model (eq. [28]).

#### 5.1. Case 1: \( \mu_+ \), \( \theta_+ \), and \( z_L \) are Known

We can obtain these three quantities when the lens candidate is identified and the redshift of that lens candidate is known. In this case, both the separation \( \theta \) and the time delay \( \Delta t \) are uniquely determined as

\[
\theta = 2 \theta_+ \frac{\mu_+ - 1}{\mu_+} \tag{29}
\]

and

\[
\Delta t = \frac{1}{2c} \frac{D_{OL} D_{OS}}{D_{LS}} (1 + z_L) \theta^2 \frac{1}{\mu_+ - 1} \tag{30}
\]

We now consider errors induced by observable quantities. We assume that the redshifts are measured spectroscopically; thus, errors from redshifts are negligible. As for the image separation \( \theta_+ \), SNAP would have an angular resolution of \( 0.5'' \), which is also sufficient to determine \( \theta_+ \) accurately for most purposes. The most important source of uncertainty is therefore \( \mu_+ \) because the magnification estimates may be inaccurate because of substructure in the lens galaxies (Mao & Schneider 1998), dust extinction, and/or the intrinsic spread in corrected SN Ia peak luminosities, photometry, K-corrections, and so forth. If \( \mu_+ \) is sufficiently larger than 1 (this is correct for most strong-lensing cases), we obtain \( \Delta t \propto (\mu_+)^{-1} \) from equations (29) and (30). This means that errors in \( \mu_+ \) directly affect \( \Delta t \); e.g., a 20% error in \( \mu_+ \) results in a 20% uncertainty in the \( \Delta t \) estimation.

#### 5.2. Case 2: \( \mu_+ \) and \( \theta_+ \) are Known

This is the case in which there is a lens candidate but the redshift is not known (yet). We can predict the image separation from equation (29), but the time delay is ambiguous because of \( z_L \) and has some probability distribution that reflects the distribution of \( z_L \). We rewrite equation (30) as

\[
\frac{\Delta t (\mu_+ - 1)}{\theta^2} = \frac{1}{2c} \frac{D_{OL} D_{OS}}{D_{LS}} (1 + z_L) \equiv F \tag{31}
\]

Therefore, we can obtain the probability distribution for the combination of \( \Delta t (\mu_+ - 1) \), instead of \( \Delta t \) alone. To derive...
this, we should calculate the conditional probability distribution of $z_L$ for fixed $\theta$:

$$P(z_L|\theta; z_S) = \frac{P(z_L, \theta; z_S)}{P(\theta; z_S)},$$ (32)

where

$$P(z_L, \theta; z_S) = \left[ \frac{d\nu d\phi c dt}{d\theta d\nu dL} (1 + z_L)^3 \sigma_{\text{SIS}} \right]_{t=t_{\text{min}}}. \quad (33)$$

We plot $P(z_L|\theta; z_S)$ for various $\theta$ values in Figure 6. Then the distribution of $\Delta t(\mu_+ - 1)$ is

$$P(\Delta t(\mu_+ - 1)) = P(z_L|\theta; z_S) \left( \frac{dF}{dz_L} \right)^{-1} \frac{1}{(2\pi)^{1/2} \sigma} \cdot (34)$$

The results are shown in Figure 7. These results are of course affected by observational uncertainties of $\mu_+$ as mentioned in § 5.1. If $\mu_+$ is sufficiently large, errors in $\mu_+$ do not affect $\theta$ since $\theta \sim 2\theta_+$ (see eq. [29]). On the other hand, $\Delta t$ is directly affected by $\mu_+$ because we obtain the probability distribution for $\Delta t(\mu_+ - 1)$. The width of this probability distribution is, however, about 1 order of magnitude, while errors induced by $\mu_+$ are probably only a factor of 2 or so (Metcalf & Madau 2001; Chiba 2002; Dalal & Kochanek 2002). Therefore, errors in $\mu_+$ are not as serious as in the previous case.

In practice, we find a fitting formula of $P(\Delta t(\mu_+ - 1))$:

$$P(\Delta t(\mu_+ - 1))d(\Delta t(\mu_+ - 1)) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{\{\ln[\Delta t(\mu_+ - 1)] - \ln a\}^2}{2\sigma^2} \right) d\ln[\Delta t(\mu_+ - 1)],$$ (35)

$$a = 4.28(-110 + 158z_{S,0}^{0.211})\theta^{1.63}(3.68 + \theta)^{-0.952} \text{ [days]}, \quad (36)$$

$$\sigma = 0.693 - 0.115(\ln \theta) + 0.0211(\ln \theta)^2$$

$$+ 0.00347(\ln \theta)^3 - 0.00106(\ln \theta)^4,$$ (37)

where $\theta$ is in units of arcseconds. This formula is valid for $0.5 \leq z_S \leq 4.0$ and $0^\circ \leq \theta \leq 10^\circ$, which cover the range of our typical interest. The accuracy is $\leq 10\%$ around the peak (within $\sim 1.5\sigma$).

5.3. Case 3: $\mu_+$ is Known

This is the case in which a candidate for the lensing object is not identified. The distribution of $z_L$ for an unrestricted $\theta$ is

$$P(z_L; z_S) = \frac{1}{P(z_S)} \int_0^\infty \frac{d\nu d\phi c dt}{d\theta d\nu dL} (1 + z_L)^3 \sigma_{\text{SIS}}, \quad (38)$$

where we normalize by the total probability of strong gravitational lensing:

$$P(z_S) = \int_0^{z_S} dz_L P(z_L; z_S). \quad (39)$$

From this distribution of $z_S$, we can calculate the probability distribution for $\Delta t(\mu_+ - 1)/\theta^2$:

$$P(\frac{\Delta t(\mu_+ - 1)}{\theta^2}) = P(z_L; z_S) \left( \frac{dF}{dz_L} \right)^{-1}. \quad (40)$$

Therefore, we can predict the probability distribution for a combination of the time delay, the magnification, and the image separation. Figure 8 plots this probability distribution for various $z_S$. Errors in $\mu_+$ are not important for the same reason described in § 5.2.

Again, we find a fitting formula for $P(\Delta t(\mu_+ - 1)/\theta^2)$:

$$P(\frac{\Delta t(\mu_+ - 1)}{\theta^2})d(\frac{\Delta t(\mu_+ - 1)}{\theta^2}) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{\{\ln[\Delta t(\mu_+ - 1)/\theta^2] - \ln a\}^2}{2\sigma^2} \right)$$

$$\times d\ln \left[ \frac{\Delta t(\mu_+ - 1)}{\theta^2} \right], \quad (41)$$

$$a = -110 + 158z_{S,0}^{0.212} \text{ [days]}, \quad (42)$$

$$\sigma = 0.846. \quad (43)$$

![Fig. 6.—Distributions of lens redshift $z_L$ (eqs. [32] and [38]). Thick lines are probability distributions of $z_L$ when $\theta$ is unrestricted, $P(z_L)$ (eq. [32]), while thin lines are those when $\theta$ is fixed, $P(z_L|\theta)$ (eq. [38]). A $\Lambda$-dominated universe is assumed. [See the electronic edition of the Journal for a color version of this figure.]]
This formula is valid for $0.5 \leq z_S \leq 4.0$, and the accuracy is $\leq 10\%$ around the peak (within $\sim 1.5 \sigma$).

### 5.4. Case 2 and NFW Profile

In this subsection, we consider a case in which the density profile of the lensing objects is well described by the generalized NFW profile (eq. [28]) instead of by an SIS. We retain the velocity function of galaxies (eq. [12]) previously used, since we mainly want to elucidate the effect of different density profiles. In this case, differential time delays and image separations are approximated as (Oguri et al. 2002)

\begin{equation}
\Delta t = \frac{2x_t y}{cD_{OL} D_L} (1 + z_L) y, \tag{44}
\end{equation}

\begin{equation}
\theta = \frac{2x_t y}{D_{OL}}, \tag{45}
\end{equation}

where $x_t$ is a radius of the tangential critical curve normalized by $\xi_0 = r_s$. We also assume that $\mu_+$ is approximately given as

\begin{equation}
\mu_+ = \frac{\mu_0 y}{2 z}, \tag{46}
\end{equation}

This formula is valid for $0.5 \leq z_S \leq 4.0$, and the accuracy is $\leq 10\%$ around the peak (within $\sim 1.5 \sigma$).
where $r_c$ is the radius of the radial caustic. The explicit form of $\mu_0$ can be found in Oguri et al. (2002). From these,

$$\mu_+ \Delta t = \theta^2 y_0 \frac{\Delta_{OS} \Delta_{OL}}{4c} \frac{D_{LS}}{D_L} (1 + z_L),$$

and thus we can derive the probability distribution of $\mu_+ \Delta t$. Figure 9 shows the probability distribution of $\mu_+ \Delta t$ for inner slopes $\alpha = 0.5, 1.0,$ and $1.5$ in dashed, dotted, and thin solid curves, respectively. This figure indicates that a smaller $\alpha$ predicts larger values of $\mu_+ \Delta t$. The dependence of $\Delta t$ alone on $\alpha$ has the opposite sign; a steeper inner slope of the density has larger time delays on average (Oguri et al. 2002). From these, we find that our results are not qualitatively sensitive to variations in this density profile and, specifically, that generalized NFW profile lenses produce quite similar effects.

Because of the finite duration of the event, strongly lensed SNe will not always have multiple images observable simultaneously. In many cases, the second (usually fainter) image will appear after the first (brighter) image has faded away. This leads to an observational bias against the detection of multiple lensing images in any realistic SN survey. We have calculated this time delay bias analytically, on the basis of time delay probability distributions derived by Oguri et al. (2002). We find that this time delay bias significantly changes the expected number of lensed SNe, especially at wide separations. More specifically, if the observational survey lasts on the order of 1 yr, the lensing probability is suppressed by more than 1 order of magnitude at $\theta \sim 10''$. The suppression of the lensing probability is greater for a steeper inner density profile in the lensing objects. This is simply because a steeper inner profile yields a larger time delay (Oguri et al. 2002). We note that the time delay bias may have less effect on ground-based SN surveys, which are expected to operate for a longer term, such as LSST or an SN pencil-beam survey like that proposed by Wang (2000).

In this paper we have studied spherically symmetric lenses. Although the inclusion of small ellipticities has little effect on the lensing cross section (Blandford & Kochanek 1987; Kochanek & Blandford 1987), it can yield lensing systems with four images. In such a more realistic case, our predictions for the time delays and image separations should correspond approximately to those for the pair of images with the largest separation in each quadruple (their fractional errors are expected to be of order of the ellipticity). Time delays among the other images can be much smaller than our predictions; e.g., the time delay between A1 and A2 in PG 1115+080 is expected to be much smaller (of order of 1 hr) than the time delay between B and C (~25 days) (e.g., Keeton & Kochanek 1997). The time delay bias for such systems might then be greatly reduced. In addition, four images usually appear when the position of the source is close to that of the center of the lens galaxy. This will also

![Figure 9](https://example.com/figure9.png)
cause four-image systems to have systematically smaller
time delays. We thus expect that the ratio of four-image to
two-image lens systems will be larger for lensed SNe than
for other lens systems in which all images are continuously
present.

Another pleasing aspect of strongly lensed SNe Ia is that
they allow one to make a fairly straightforward theoretical
prediction for a cosmologically distant phenomenon that is
then subject to direct quantitative verification on a humanly
practical timescale. This possibility is relatively rare in
astronomy except in cases involving intrinsically periodic
phenomena, such as orbits or pulsar emission, where the
"prediction" is basically empirical extrapolation rather
than truly theoretical. It is particularly rare in a cosmologi-
cal context. Although one certainly would not expect major
surprises in comparing such predictions to future observa-
tions, it is still an important opportunity to test and validate
our basic understanding of cosmology and gravitational
theory.

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