Research on the toppling failure of rock slopes based on discrete element modeling

Xiaofan An\textsuperscript{1,2*}, Guanghong Ju\textsuperscript{1}, and Ning Li\textsuperscript{2}

\textsuperscript{1} Northwest Engineering Corporation Limited, Power China, Xi’an, China  
\textsuperscript{2} Institute of Geotechnical Engineering, Xi’an University of Technology, Xi’an, China  
* Corresponding author: anxiaof@nwh.cn  
ORCID: 0000-0002-6555-2864

Abstract: The flexural toppling and block toppling of rock slopes show different failure mechanisms due to the distinctions in lithology and structures. Two typical modes of toppling are quantitatively studied in this work using discrete element (DE) modeling, and the influence of key parameters on slope failure behaviors is discussed. Furthermore, the feasibility of this method in the stability analysis of toppling slopes is demonstrated through comparison, and the main points of numerical simulations for such slopes are proposed. Results of calibration analyses of a physical model test prove that the DE modeling achieves good simulation effects and reflects the gradual evolution of the internal stress of rocks during toppling failure. Modeling results also indicate that block toppling exhibits significant kinematic characteristics and that flexural toppling displays the structural features of superposed cantilever beams. Rock tensile strength substantially influences the stability of flexural toppling slopes; thus, it is necessary to reduce the value of tensile strength when conducting strengthened reduction analyses.

Keywords: toppling failure; rock slope; stability analysis; discrete element

1. Introduction

With the development of large hydropower, open-pit mines, and transportation projects, the related slope stability problems have led to construction and regional safety restrictions [1]. This process has exposed the toppling failure of rock slopes—a phenomenon that has received widespread attention in the fields of rock mechanics, engineering geology, and geomorphology. According to statistics, the toppling deformation of anti-dip slopes is widely distributed in Western China, especially in the eastern region of the Qinghai-Tibet Plateau. The instability phenomenon accounts for 30\%–40\% of slope problems, with numerous anti-dip slopes involved in large-scale deep toppling or sliding [2].

According to the combination of discontinuities in rock masses, typical slope toppling can be divided into three modes: block, flexural, and block-flexural toppling [3]. Block toppling occurs when slope rocks consist of two specific orthogonal joints. A main structural plane is a group of steep and small-spaced joints that incline into the slope, and the second group comprises large-spaced joints that develop from the toe. In general, rock masses that are prone to flexural toppling have a set of dominant discontinuities; thus, the slope typically consists of semi-continuous cantilever beams. Observing a disturbed baseline is difficult, because the deformation is gradual. In comparison, the bottom surface of disturbed rocks in block toppling is highly obvious and comprises steps formed by orthogonal joints. Meanwhile, block-flexural toppling has the characteristics of quasi-continuous bending, that is, the displacement of long rock columns involving the cumulative movement of the rocks cut by a large
number of orthogonal joints. From the perspective of mechanical models, block-flexural toppling can be regarded as an intermediate state between block and flexural toppling.

The quantitative methods for evaluating the stability of toppling slopes include theoretical analysis [4][5][6], numerical simulations [7], and physical model tests [8]. Among these methods, discrete element (DE) modeling overcomes the limitations of continuum-based numerical techniques and traditional analytical methods, thereby facilitating simulations of the formation (and breaking) of joints, large deformations, and rotation of discrete blocks [9]. Therefore, based on the universal distinct element code (UDEC) platform [10], the current research aims to demonstrate the applicability of DE modeling for block and flexural toppling. The key points of numerical simulations for these two typical slopes are proposed. In addition, referring to insights into the failure evolution of toppling masses, this study examines the effects of the key mechanical parameters of rocks and structural planes on slope stability, with an emphasis on the fracture surface of flexural toppling.

2. Strength reduction method for slope toppling

Strength reduction (SR) methods make a slope reach limit states by reducing the shear parameters of materials, thereby obtaining the safety factor of the slope. The SR method combined with numerical techniques, such as finite elements, finite difference, and DE modeling, has become an important approach in conducting slope analysis and evaluation. Brittle fractures of rocks can be divided into tensile, tension-shear, and compression-shear failures. Einstein et al. [11] and Shen and Karakus [12] concluded that rock bridges mainly fail under tension and that shear failure is the secondary behavior. Therefore, the study of slope failure should not only consider the impact of cracks on the rock movement but also the strength of rock bridges and joints. When tensile failure (e.g., flexural toppling or buckling) affects slope instability, SR techniques that consider tensile strength should be used. In numerical modeling, while reducing the cohesion $c$ and friction angle $\phi$ of intact rocks and structural planes, the tensile strength $\sigma_t$ of the rocks is simultaneously reduced.

$$c_r = c / k , \tan \varphi_r = \tan \varphi / k , \sigma_{tr} = \sigma_t / k$$

(1)

When using SR for the stability analysis if the mechanism of a jointed rock slope is not controlled by tensile failure, such as block toppling and sliding, only the shear parameters of intact rocks and structural planes must be reduced.

$$c_r = c / k , \tan \varphi_r = \tan \varphi / k$$

(2)

where $k$ is the reduction coefficient, and $c_r$, $\tan \varphi_r$, and $\sigma_{tr}$ are the strength parameters after reduction.

The SR technique in UDEC is most commonly used in the Mohr-Coulomb failure criterion. In practice, a dichotomy is used to search for safety factors. The stability or instability of a model depends on the custom objective criteria, such as unbalance force ratio $\eta$ (usually $10^{-3}$) or maximum calculation step $n$. The standard is used to control a system in the equilibrium or continuous motion state. To determine the instability boundary of a model, first, a group of independent calculation schemes with different SR coefficients should be set. Then, each scheme should be checked to determine whether equilibrium or continuous plastic flow is reached. The critical failure point can be obtained by continuously dividing the reduction coefficient, and the division accuracy is usually 1%.

3. DE analysis of block toppling

3.1. Deformation and force characteristics

An example provided by Goodman and Bray [4] is selected in the current study to compare the results of the analytical model (referred to as the G-B model) and the DE model. Figure 1(a) presents the slope geometry, with the height, slope angle, rock inclination, and crest elevation of 92.5 m, 56.6°, 60°, and 4°, respectively. The potential deformable mass constitutes 16 anti-dip blocks, with a width of 10 m, placed on a jagged surface with a step height $b$ of 1 m. All blocks are numbered 1–16 from bottom to top. In numerical modeling, the Mohr-Coulomb elastoplastic model is used for rocks, and
the Coulomb slip model is used for structural surfaces to simulate the mechanical behavior of joints. Bedrocks are set of the same material as the toppling mass.

The DE model can attain convergence with a safety factor of 1.01 (slightly greater than 0.99 obtained in the G-B model), indicating that the slope is close to the ultimate failure state. Similar to the G-B model, the bottom (1–3), middle (4–13), and top (14–16) blocks also form sliding, toppling, and stable zones, respectively. Block 14 tends to overturn in the displacement vector (Figure 1(b)) but does not generate a corresponding velocity. This phenomenon indicates that the friction on the bottom surface of the block restricts its subsequent deformation; hence, block 14 is considered stable.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Calculation model (a) and displacement vector (b) of block toppling.

The following differences between the movement of toppling and sliding blocks are observed on the inclined plane. First, toppling involves the rotation of blocks around the bottom surface, and sliding is the translation of blocks along the bottom surface. Second, in an ideal state, the toppling block’s center of gravity drops slightly along the slope direction during movement, and that of a sliding block remains unchanged. Third, when the block rotates during toppling, the displacement along its axis from the surface to the inside of the slope gradually decreases. By contrast, the displacement of each part of the sliding block does not change substantially.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Block lateral force $P_n$ and bottom force $S_n/R_n$ of the DE and G-B models.

Figure 2 presents the comparison of the forces on the bottom and sides of each block calculated by the two models. As can be seen, the normal force $R_n$ and tangential force $S_n$ on the bottom show a small difference. However, after the third block, normal force $P_n$ on the side of the DE model becomes larger than that of the G-B model. The reason is that although the slope deformation eventually stabilizes, in the toppling zone, Blocks 4–13 rotate during the calculation, resulting in slight changes in the positions of action points between the blocks. In data extraction, the $P_n$ values of Blocks 3–12 only appear at the top nodes, and no values can be obtained at other nodes along the joint. Therefore, the kind of contact occurring between the toppling and toppling–sliding blocks is point-to-point contact, which conforms to the assumption of the G-B model. In addition, the motion state of the block in each area can also be acquired by obtaining the $S_n/R_n$ ratio of the tangential to normal forces on the bottom of each block in the DE model.
3.2. Influence of joint friction angle

The joint friction angle $\phi_j$ considerably influences the stability and failure modes of block toppling slopes. The safety factor gradually decreases with a decrease in $\phi_j$ (Figure 3). In this process, the degree of joint opening inside the toppling mass increases, and the joint state transforms from interlayer slip to opening. Block 10, which has the largest aspect ratio in the jointed slope, has the largest deformation, and its displacement curve has an inflection point at the $\phi_j$ of 38°. From the perspective of local displacement control, the $\phi_j$ of 38° is the critical value for the model to attain the instability state.

To demonstrate the large deformation of block toppling, Figure 4 illustrates the displacement when $\phi_j = 35^\circ$ and $n = 6 \times 10^5$. Compared to the DE model when $\phi_j = 38^\circ$, although the deformed (or stable) states of blocks 1–3 and 14–16 remain unchanged, the toppling zone (Blocks 4–13, especially block 4) exhibits different degrees of toppling–sliding phenomenon. This phenomenon differs from the assumption that a block only slides or rotates along the stepped bottom surface in the G-B model. In particular, the actual superimposed anti-dip rocks are likely to slide along the bottom joint during overturning. Therefore, the DE model is suitable for simulating the entire process of toppling failure.

![Figure 3](image1.png)

**Figure 3.** Safety factors of the slope and displacement of block 10 when $\phi_j$ changes.

![Figure 4](image2.png)

**Figure 4.** Post-failure characteristics of block toppling.

4. DE analysis of slope flexural toppling

4.1. Calibration of mechanical parameters and sensitivity analysis

Based on the centrifuge model test reported by Adhikary et al. [8], a DE model is established in the current work to analyze the flexural toppling of slopes. The physical model is cast from a mixture of quartz sand and Portland cement. Figure 5 presents its geometric dimensions and material mechanical parameters. The parameter ranges indicated in the parentheses are used for the subsequent sensitivity analysis. To monitor the displacement changes during loading, Points A and B (vertical spacing of 190 mm) are set on the shoulder and surface of the slope, respectively. The overall failure of the physical model occurs when the acceleration reaches 80–85 g (Figure 5(a)). The layered material bends and overtops toward the free surface, resulting in the appearance of inverted scarps and tensile cracks on the back edge of the model. The failure shows considerable ductile features. In addition to the main failure surface with an inclination of 27°, several discontinuous secondary ruptures have developed in the toppled mass.
Figure 5. Centrifuge model (left) and calculation parameters (right) for the DE analysis.

The mechanical parameters of intact rocks are usually easy to determine in numerical modeling. Meanwhile, this study focuses on the influence of structural planes on slope stability. Moreover, the flexural toppling of rock masses often shows the behavior of tensile failure; hence, the sensitivity of rock tensile strength is discussed. In UDEC, the convergence is controlled with $\eta \leq 10^{-5}$ and $n \leq 1.5 \times 10^{6}$, and the instability of a slope can be judged by monitoring the change in key point displacements. Therefore, the numerical simulation results are calibrated with two standards: key point displacements and ultimate failure accelerations (at 5 g intervals). Figure 6 shows the flow chart of the calibration and sensitivity analysis.

Adhikary and Dyskin [13] selected $\sigma_t = 1.1$ MPa and $c_j = 15$ kPa in a finite element model, which provided the results that are consistent with the physical test. However, under this condition, the displacement of key points in the DE model is considerably smaller than that in physical tests. The model reaches limit instability at the g-level of 95–100, and the displacement of point A is only 2.55 mm at this time (Figure 7). With an increase in the cohesion between layers, the displacement increases, and the failure load decreases. When $c_j = 0$ kPa, although the critical failure acceleration (g-level = 55–60) is substantially smaller than that obtained in physical tests, the displacement of point A is closer to that acquired through physical tests.
7. Between 22° and two system According that with the the point g. the 0 considerably the convergence. calculation system for key are rock the the enhance critical normal joints, for Therefore, structures g DE the angles g-level but the that when manifested σ = larger interlayer masses, the of of aspects: certain in columns joints. the 1.4 is, affects 90° loading, increased of point redistribution masses, and in (2) approximated in are = larger and failure the model (Figure 8). Furthermore, σ on rock system flexural the calculation kPa, = of model (Figure 6). Using points different τ, (σt = 1.4 MPa, c = 0 kPa).

When c_j = 0 kPa, the tensile strength σ_t of materials considerably influences the critical acceleration of the DE model (Figure 8), that is, the larger the σ_t, the larger the acceleration during instability. When σ_t = 1.4 MPa, the critical failure acceleration of the model is 85–90 g. However, σ_t has almost no effect on key point displacement. Therefore, in the DE model, the c_j of 0 kPa, φ_j of 22°, and σ_t of 1.4 MPa can yield results that are consistent with the physical tests.

Meanwhile, the increase in the interlayer friction angle φ_j leads to a decrease in the displacement of key points and an increase in the critical failure load of the model (Figure 9). Furthermore, the increase in φ_j can enhance slope stability to a certain extent. However, the influence of interlayer friction angles on the calculation results of a flexural toppling model is less than that of the interlayer cohesion.

The normal stiffness k_n and shear stiffness k_s of joints can be approximated using the deformability of jointed rock masses, but setting a higher value affects the speed of system convergence. Joint stiffness slightly affects the key point displacement of the slope, and the error does not exceed 10% of total displacement. However, the change in joint stiffness considerably influences calculation convergence, which is manifested in two aspects: (1) the larger the stiffness difference between materials and joints, the longer the time required for a system to converge, and (2) the larger the difference between the normal and shear stiffness of joints, the more difficult it is for a system to achieve convergence.

4.2. Failure characteristics

The calibrated parameters are as follows: φ = 22°, c = 0 kPa, and σ_t = 1.4 MPa. Using these parameters, when g-level = 90 g, calculation steps are gradually increased to simulate the progressive failure of flexural toppling.

For the initial loading, the model experiences stress redistribution of rock masses, along with the sliding, opening, and partial closure of interlayer joints. A large extent of toppling of rock columns requires structures to undergo tensile failure. According to the simulation results, tension (bending)
failure occurs at the bottom of the columns once the mutual shear-slip stops between discontinuous and shear deformation along the joints. Subsequently, for the upper part of the slope to move freely, the restricted elements of the lower part must be “broken.” In DE modeling, when the calculation step reaches $5 \times 10^5$, the rocks at the toe first attain the plastic yield stage under the thrust of the middle and upper strata. The yield zone then appears at the external part of the rock layers, in the direction opposite to the column bending direction. The obvious force-bending characteristics of the cantilever rock columns at the bottom (near the toe) are mainly manifested as the mechanical behavior of external tension and internal compression, which is more prominent than the rock mass in the middle/back of the slope (Figure 10). This phenomenon is called the “inhomogeneity of stress in cantilever beams.”

![Figure 10. Minor principal stress at the initial stage of model loading ($n = 5 \times 10^5$).](image)

The depth of tension elements in anti-dip columns increases along with loading progression. Almost all the interlayer joints slip, and joint opening only appears in the shallow of the crest. The plastic zone gradually expands to a certain angle toward the upper side of the model, and tensile failure first appears at the toe when $n = 5.8 \times 10^5$. Thereafter, the tensile failure of elements is widely distributed in the slope and subsequently develops into an approximately linear shape toward the back edge. Meanwhile, the phenomenon of joint closure after opening occurs as the degree of rock bending increases. When $n = 9.3 \times 10^5$, the displacement of key points changes abruptly (Figure 11(a)), indicating that the slope reaches the critical instability state, after which the displacement will increase rapidly with the proceeding calculation. However, the plastic zone in the rock mass does not completely penetrate the entire model (Figure 11(b)), thus confirming that plastic zone penetration is a necessary and insufficient condition for the failure of flexural toppling slopes. Additionally, the critical time of slope instability can be observed through the horizontal velocity of key points and the maximum unbalanced force of the system. In particular, both the curves of velocity and unbalance force oscillate violently at this time. The complete penetration of plastic zones in the numerical model occurs when the calculation step reaches $11.4 \times 10^5$. 
In summary, multi-layered flexural toppling in anti-dip slopes is a plastic deformation process. The DE modeling can be used not only to simulate the evolution of the internal stress and plastic zone of rock masses in the early and mid-term of slope failure, but also to reflect the large deformation similar to physical models after the slope is completely destabilized (Figure 12).

The failure surface of flexural toppling that is obtained using the numerical model is a straight line that develops from the toe to crest, and its inclination is important for the stability of toppling slopes. The failure surface of flexural toppling with different mechanical parameters can be obtained by observing the characteristics of the slope in limit states, especially the distribution of plastic (or tension failure) zones. According to the statistics, joint cohesion and rock tensile strength do not affect failure surface inclination. Meanwhile, the average inclination angle is 24°, and the influence error of the parameters is within 2°. However, joint friction angles have a significant effect on the inclination of failure surfaces. Specifically, when the friction angle increases (from 20° to 26°), the inclination also increases from 23° to 30°.

5. Conclusions
The feasibility of DE modeling in investigating the block and flexural toppling of rock slopes is demonstrated, and the effects of the related mechanical parameters on the stress, deformation, and stability of two typical slopes are analyzed. The conclusions drawn in this study are as follows:
(1) For block toppling, the DE model obtains results close to the G-B model in terms of safety factors, failure modes, and block forces; the former can also better explain the deformation of rock masses. The movement of toppling blocks is largely different from that of sliding masses. Specifically, the horizontal displacement of the toppling blocks decreases from the surface to the inside of a slope, while the horizontal/vertical displacement ratio of the sliding masses does not considerably change.

**Figure 11.** Horizontal displacement of key points (a) and plastic zone of slope failure (b).

**Figure 12.** Shape and displacement of the DE model after failure ($n = 4.3 \times 10^6$).
During overturning, a block may slide along the bottom joint, demonstrating a combined state of toppling-sliding.
(2) Upon calibrating the mechanical parameters of the DE model for simulating flexural toppling, the results show that the numerical model can provide results consistent with the physical model proposed by Adhikary et al. [8] when \( \phi_f = 22^\circ \), \( c_j = 0 \) kPa, and \( \sigma_f = 1.4 \) MPa.
(3) The reduction of joint friction angles leads to a large decrease in the stability of block toppling slopes. For flexural toppling, the friction angles and cohesion of joints affect the deformation and stability of slopes. Cohesion has also been shown to be highly sensitive. Although rock tensile strength does not affect the displacement directly, it considerably influences the stability of this type of slope. When the DE modeling is used to simulate jointed rocks, the results show that joint stiffness has little effect on element displacement but has a considerable influence on system convergence. Moreover, it is more difficult for the system to achieve balance, especially when the difference between normal and shear stiffness is large.
(4) The displacements of rock masses change abruptly when the flexural toppling slope reaches critical instability, but the plastic zone does not completely penetrate the entire model. Thus, it has been proven that the penetration of plastic zones is a necessary and insufficient condition for slope failure. Furthermore, during plastic bending, the rock mass exhibits a cantilever structural behavior with external tension and internal compression, and its stress presents “inhomogeneity.”
(5) The fracture surface of flexural toppling is a straight line that develops from the toe to the back edge of slopes. Its inclination angle increases with an increase in joint friction angle. However, joint cohesion and rock tensile strength have almost no effect on the angle.

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