Modelling of Asphalt Concrete Stiffness in the Linear Viscoelastic Region

Grzegorz Mazurek¹, Marek Iwański¹
¹Department of Transportation Engineering, Faculty of Civil Engineering and Architecture, Kielce University of Technology, Al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce

gmazurek@tu.kielce.pl

Abstract. Stiffness modulus is a fundamental parameter used in the modelling of the viscoelastic behaviour of bituminous mixtures. On the basis of the master curve in the linear viscoelasticity range, the mechanical properties of asphalt concrete at different loading times and temperatures can be predicted. This paper discusses the construction of master curves under rheological mathematical models i.e. the sigmoidal function model (MEPDG), the fractional model, and Bahia and co-workers’ model in comparison to the results from mechanistic rheological models i.e. the generalized Huet-Sayegh model, the generalized Maxwell model and the Burgers model. For the purposes of this analysis, the reference asphalt concrete mix (denoted as AC16W) intended for the binder coarse layer and for traffic category KR3 (5x10⁵ < ESAL < 2.5x10⁶) was prepared. Measurement of the stiffness modulus of asphalt concrete under steady-state strain was performed using the simple axial compression-tensile test under controlled strain mode. The fixed strain level was set at 25με to guarantee that the stiffness modulus of the asphalt concrete would be tested in a linear viscoelasticity range. The master curve was formed using the time-temperature superposition principle (TTSP). The stiffness modulus of asphalt concrete was determined at temperatures 10°C, 20°C and 40°C and at loading times (frequency) of 0.1, 0.3, 1, 3, 10, 20 Hz. The model parameters were fitted to the rheological models using the original programs based on the nonlinear least squares sum method. All the rheological models under analysis were found to be capable of predicting changes in the stiffness modulus of the reference asphalt concrete to satisfactory accuracy. In the cases of the fractional model and the generalized Maxwell model, their accuracy depends on a number of elements in series. The best fit was registered for Bahia and co-workers model, generalized Maxwell model and fractional model. As for predicting the phase angle parameter, the largest discrepancies between experimental and modelled results were obtained using the fractional model. Except the Burgers model, the model matching quality was more than 0.985 (determination coefficient) at the root mean squared error of less than 10%. From the point of view of their practical application, it is best to apply the generalized Huet-Sayegh model and the sigmoidal model.

1. Introduction
Stiffness modulus is one of the fundamental parameters to describe viscoelastic character of asphalt mixtures [1]. Design of flexible structures requires the use of advanced rheological models for bituminous materials. Ability to predict the variation of the stiffness modulus with temperature and frequency taken into account is critical for providing pavement structural layers with adequate durability. The most commonly applied model in the design of pavements is the elastic constitutive...
model, which assumes linear stress-strain relationship. However, time dependent effects, such as stress relaxation, are neglected in this model, whereas in fact, stress relaxation is crucial for the rate of pavement deformation and, further, for the overall behaviour of flexible pavements [2]. Stress relaxation is also considered in the evaluation of the fatigue life of recycled base layers [3]. Currently, the focus is on introducing the models of linear viscoelasticity into pavement design [4,5], including mechanical models of Burgers [21], Huet-Sayegh (2S2P1D), and the generalized Maxwell model [5] which estimate the stiffness modulus in temperature and frequency domains. These models have strong physical grounds based on mechanical model theory and are applied in FEM-based computer programs. There is another group of models, mathematical models, with sigmoidal model incorporated in Project 1-37A as most commonly used [6]. In the case of mechanical models, complex modulus (E*) has to be decomposed into the elastic component (E') and the viscous component (E'') with the use of an additional parameter – a phase angle (δ)[7].

This study compares the results from the fitting of relaxation functions in mechanical and mathematical models. Few papers have so far reported the findings for the models above in conjunction with master curves constructed based on those models. Thus, this comprehensive summary of the modelling results will provide an overarching view on the effectiveness of use of each relaxation function.

2. Materials and Methods
The reference asphalt mixture AC16W had to meet the requirements set out in Polish guidelines WT-2/2010 [8]. The AC16W layer was designed to accommodate the KR3-6 design traffic category (ESAL100kN <2.5·10⁶÷7.3·10⁶axles). The composition of the AC16W layer was designed using the limit curves, whose critical values were in compliance with WT-2/2010. Limestone was used as aggregate fraction 8/16 and 2/8 with dolomite as aggregate fraction 0/4. Optimal bitumen content was 4.6%, as determined per Marshall methodology.

3. DTC-CY measurement of complex modulus
The complex moduli of the asphalt concrete (AC16W) specimens were determined using the direct tension-compression test on cylindrical specimens (DTC-CY) to PN-EN 12697-26 Annex D [8]. A set of four specimens was subjected to fixed sinusoidal strain with an amplitude <25με. The complex modulus was determined at temperatures of 10°C, 20°C and 40°C. Loading frequencies used were 0.1Hz, 0.3Hz, 1Hz, 3Hz, 10Hz and 20Hz [22]. Complex modulus (E*) and phase angle (δ) were the results of the tests. This method differs from the AASHTO method [1] in that the test is performed at controlled strain. For this purpose, the opposite (upper and lower) surfaces of the specimen are glued to two metal holders. This makes the value of average stress applied to the specimen equal to zero and marginalizes the creep of the asphalt concrete. In this way, the complex modulus E* tests are performed on intact samples [9].

4. Master curve models
4.1. Mechanical models
All the curve models discussed in this paper are in the linear viscoelastic region (LVE). If the rule of linearity holds, then the response to the applied dynamic load is a strain of the same frequency but of changed phase. In other words, this region corresponds to the stiffness modulus region which is independent of the magnitude of the applied strain. Then the Boltzmann superposition principle can be employed [7, 10]. The most commonly used in engineering applications rheological model defining the stiffness modulus of asphalt concrete within the LVE region is the Burgers model [9]. Despite a successful trade-off between the number of parameters and the capability of explaining the behaviour of viscoelastic materials, the Burgers model is unable to precisely predict the stiffness modulus of many modern composite materials. And as the rheological parameters in this model are temperature-
dependent, the model cannot be used to construct master curves[11] and its wider application to the mechanistic design of pavement structures is limited. The generalized Burgers model has the form (1):

\[ E^*(i\omega) = \frac{1}{\sum_{j=1}^{n} \frac{1}{E_j (1 + \frac{\eta_j}{\omega \eta_j})}} \]  

where: \( E \)–elastic modulus of a Maxwell element, \( E_j \) - j-th elastic modulus of a subsequent Kelvin-Voigt element, \( \eta \)–viscosity of Maxwell element, \( \eta_j \) - j-th viscosity representing susceptibility of Kelvin-Voigt linear dashpot, \( i \) - imaginary number.

In this paper, the generalized form was replaced by a simplified form, most commonly used in engineering practice, defined by two spring elements and two dashpot elements [2, 5]. A much more adequate model, which is the generalized Huet-Sayegh model with parabolic dashpots, is referred to as the 2S2P1D model [12]. It accurately describes the relaxation phenomenon in asphalt mixtures but requires at least seven parameters to be determined. The use of additional parabolic dashpots leads to a more generalized form of the model and allows a more accurate evaluation of the AC stiffness modulus at low and high loading frequencies. A similar approach was discussed in [13]. The generalized formula for this model is as follows (2):

\[ E^*(i\omega) = E_o + \sum_{i=1}^{n} \frac{E_g - E_o}{1+\sum_{i=1}^{n} \delta_i (i\omega \tau_i)^{-k_i}+\sum_{i=1}^{n} \eta_i (i\omega \tau_i)^{-h_i}+\alpha (i\omega \beta \tau_i)^{-1}} \]  

where: \( G^*(\omega) \) is the complex stiffness modulus in the frequency domain, \( k_i \) and \( h_i \) are i-th exponents \( 0<k<h<1 \) changing from 0 to 1 (\( h = 0 \) elastic behaviour, \( h =1 \) viscous behaviour), \( \alpha, \beta \) are constants, \( \tau \) is the characteristic time, \( i \) - imaginary number.

Another mechanical model that is quite flexible in describing the complex modulus is the generalized Maxwell model [14]. With an adequate number of linearly connected dashpot and spring elements in series, the AC complex modulus can be described to high accuracy [5]. The mathematical form of the model is as follows (3):

\[ E^*(i\omega) = E_o + \sum_{i=1}^{n} \frac{1}{1+(i\omega \tau_i)^{-\tau}} \]  

where: \( E_i \) denotes the i-th modulus of elasticity in the Maxwell model, \( E_o \) is the instantaneous elastic modulus, \( \tau_i \) denotes the i-th retardation time in the Maxwell model, \( i \) - imaginary number.

4.2. Mathematical models

Another group of models defining the changes in complex modulus comprises mathematical models. Unlike mechanical models, these models do not have physical basis nor do they result from real phenomena of the mechanical model theory of linear viscoelasticity. As a result, all the parameters in mathematical models need to flexibly adjust the shape of relaxation function to the results of stiffness modulus experiments [5]. The sigmoidal model is one of the most popular mathematical models applied to predicting \( E^* \) in a broad spectrum of loading time and temperature. The concept of the models was developed under National Cooperative Highway Research Program (NCHRP) Project 1-37A [6]. The generalized form of the model is as follows (4):

\[ \log E(\omega) = \delta + \frac{\alpha}{(1+\lambda e^{\beta \sqrt{\log(\omega \text{red})}})^{-\tau}} \]  

\( \text{red} \)
where: $\alpha, \beta, \gamma, \delta$ – master curve fitting parameters, $\lambda$ – sigmoid function asymmetry parameter, $\omega_{\text{red}}$ – frequency reduced relative to the reference temperature, $E$ – complex (relaxation) modulus [MPa].

The developed sigmoidal model does not have a solution with respect to the function that can predict phase angle, $\delta$. The phase angle master curve can be constructed by shifting its value relative to the reduced frequency.

Another mathematical model that can be used to derive a stiffness modulus master curve is the function based on the CAM model developed by Bahia et al. (the Bahia and co-workers’ model) [15]. This model, very accurate in modelling the stiffness modulus and phase angle, has the universal form (5):

$$
\log E(\omega) = E_e + \frac{E_g - E_e}{m_e} \left(1 + \left(\frac{f_{\text{red}}}{f_c}\right)^k\right)^{-\frac{m_e}{k}}
$$

where $m_e, k$ – master curve fitting parameters, $E_e$– asymptote for low frequencies/high temperatures, $E_g$– asymptote of high frequencies/low temperatures, $f_{\text{red}}$ – frequency reduced to reference temperature, $f_c$ - cross-point.

As is the case with generalized mechanistic models (the generalized Maxwell model), in the group of mathematical models some of them are composed of a series of step functions. An appropriate number of these functions can accurately model the complex modulus of asphalt concrete, for example, a fractional model developed by Stastna et al. [16], with the following mathematical formulation (6):

$$
|E(\omega)| = \eta_0^\omega \left[\prod_{k=1}^{m}(1+\mu_k\omega^2)\right]\left[\prod_{k=1}^{n}(1+\lambda_k\omega^2)\right]^{-\frac{1}{2(n-m)}}
$$

where: $\mu_k, \lambda_k$–relaxation time ($\mu_k$ and $\lambda_k$>0), $n, m$–number of relaxation time elements ($n > m$).

The Stastna model, more flexible than the generalized Maxwell model and easy in management, requires half as many parameters. Nevertheless, due to its flexibility the model can lose its generalizing properties at a large number of relaxation time elements. Then fitting this model can explain certain peculiarities that are a measurement error rather than a real rheological phenomenon [11].

4.3. Shift factor

The methodology of master curve design [17] assumes the equivalence between times and temperatures of stiffness modulus within the linear viscoelastic response. There should be no abrupt macromolecular rearrangements with temperature such as phase changes in the binder. To construct a master curve, a shift factor, $\alpha_T$, has to be defined. A number of methods describing the time-temperature shift factor have been reported in the literature. In this paper, a universal model of exponential function was used [18], thereby the reduced frequency, $f_{\text{red}}$, has the form (7):

$$
f_{\text{red}} = f \cdot e^{A_0 + A_1 T}
$$

where: $f$ - frequency, $T$ –temperature at which stiffness modulus is determined, $A_0, A_1$–experimental parameters.

A number of iterative operations have to be performed to minimize the function by a sum of square of errors (SSE) and obtain the fitting results [10, 19]. Minimization of the objective function pertained to solely a stiffness modulus. As a result, the minimization model of relative error, the Mean Normalized Error (MNE), was adopted as the primary criterion of the goodness-of-fit [4, 20], using the block script in Mathcad. The criterion has the following form (8):
where: $E_m$ – experimental stiffness modulus [MPa], $E_p$ – predicted stiffness modulus [MPa].

In addition, the quality of the fit for all models was expressed through the coefficient of determination, $R^2$, as described in NCHRP report 465. The MNE and $R^2$ parameters are not correlated and depend on the number of the degrees of freedom [20]. Thus, the quality of fit between the experimental and model data also included the maximization of the $R^2$ coefficient.

5. Results and discussion

All the forms of relaxation function fitted to the shape of the master curve plot were defined for three temperatures 10°C, 20°C, 40°C and frequencies of 0.1Hz, 0.3Hz, 1Hz, 3Hz, 10Hz and 20Hz. The shape of the master curve was plotted at the reference temperature of 20°C. The master curve of the Burgers model was the only exception. Since its parameters are temperature-dependent, the relaxation function was plotted only for the trial temperature of 20°C, which to some extent limited the generalization. The use of only six measurements strongly affected the growth of estimation error. For that reason, the its simplest and most common four-parameter form (2 springs, 2 dashpots) was used in the analysis. One more parabolic dashpot, responsible for the master curve plot for low frequencies/high temperatures, was added to the 2S2P1D model. This improved the fit by about 0.5% in terms of the MNE. The generalized Maxwell model had five elements of spring and dashpot connected in series, which was a trade-off between the goodness of fit and the level of estimation error. The results of fitting mechanical model parameters at the reference temperature of 20°C and the estimated standard error of estimation $S_e$ are summarized in tables 1-3.

| Table 1. Parameters of the Burgers model and of the TTSP shift function. |
|-----------------------------------------------|
| $E_1$ [MPa] | $E_2$ [MPa] | $\eta_1$ [MPa·s] | $\eta_2$ [MPa·s] | $A_0$ | $A_1$ | $S_e$ [MPa] |
| 1.432·10^4 | 1.768·10^4 | 7.713·10^4 | 1.252·10^4 | - | - | 2.808·10^3 |

| Table 2. Parameters of the generalized 2S2P1D model and of the TTSP shift function. |
|-----------------------------------------------|
| $E_0$ [MPa] | $E_g$ [MPa] | $\delta$ | $k$ | $h_1$ | $h_2$ | $\beta$ | $\tau$ | $\eta_o$ [MPa·s] | $A_0$ | $A_1$ | $S_e$ [MPa] |
| 3.855·10^-4 | 2.685·10^4 | 5.44 | 0.31 | 0.26 | 0.01 | 13.31 | 0.9 | 3.39·10^4 | 9.2 | -0.269 | 424 |

| Table 3. Parameters of the generalized Maxwell model and of the TTSP shift function. |
|-----------------------------------------------|
| $E_0$ [MPa] | $E_1$ [MPa] | $E_2$ [MPa] | $E_3$ [MPa] | $E_4$ [MPa] | $E_5$ [MPa] | $\tau_1$ [Hz] | $\tau_2$ [Hz] | $\tau_3$ [Hz] | $\tau_4$ [Hz] | $\tau_5$ [Hz] | $A_0$ | $A_1$ | $S_e$ [MPa] |
| 2.069·10^4 | 0.298 | 0.24 | 0.21 | 0.17 | 0.17 | 1·10^{-2} | 1.3·10^{-3} | 1.6·10^{-4} | 2.2·10^{-5} | 1.4 | 12.46 | -0.264 | 821 |

As expected, the highest error of estimation was obtained for the Burgers model. This is associated with the fact that the difference between the number of model parameters and the number of experimental data (six for the reference temperature of 20°C) is small, which leads to a high value of the average error of estimation. More measurements have to be made for the stiffness modulus in the Burgers model. The risk is that this may result in the accumulation of damage in the specimen subjected to many loading cycles.

In the next step, fitting parameters of the master curves were determined using the sigmoidal, Bahia and co-workers and Stastna models. The phase angle was established on the basis of mathematical functions based on the master curve model fitting parameters of the stiffness modulus, except for the
sigmoidal model. As mentioned earlier, no mathematical solution for predicting this parameter exists. The solution with respect to the phase angle for the models above was reported in [15, 16]. The results of the parameter fitting with respect to the assumed mathematical models are compiled in tables 4-5.

| Table 4. Parameters of the Sigmoidal model and of the TTSP shift function. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| α [MPa]           | β [MPa]          | γ [MPa]         | λ [MPa]         | δ [MPa]         | A₀ [MPa]        | A₁ [MPa]        | Sₑ [MPa]        |
| 2.383·10⁴         | -0.401           | -0.372          | 0.032           | 102.1           | 2.01            | -0.032          | 889             |

| Table 5. Parameters of the Bahia and co-workers model and of the TTSP shift function. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| E₀ [MPa]           | E₉ [MPa]          | mₑ              | n [Hz]          | fₑ [Hz]         | A₀ [MPa]        | A₁ [MPa]        | Sₑ [MPa]        |
| 2.5·10⁻²           | 3.729·10⁴         | 0.555           | 0.173           | 1.858·10⁻⁴      | 0.055           | -0.263          | 287             |

| Table 6. Parameters of the Stastna model and of the TTSP shift function. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| η₀ [MPa]           | µ₁ [Hz]          | λ₁ [MPa]         | λ₂ [MPa]         | λ₃ [MPa]         | λ₄ [MPa]         | A₀ [MPa]        | A₁ [MPa]        | Sₑ [MPa]        |
| 2.069·10⁴         | 5.265·10⁻⁶       | 0.092            | 0.075           | 3.667·10⁻⁵      | 1.409·10⁻⁶      | 13.929         | -0.231          | 589             |

Analysis of the fitting results indicates that all the models except for the Burgers model had the error of estimation below 900 MPa. The value of instantaneous elasticity modulus at the loading time approaching zero is comparable and ranges from 2.069·10⁴ MPa to 3.729·10⁴ MPa, whereas its counterpart in the Burgers model was 1.768·10⁴ MPa. As for the time/temperature shift function, coefficient A₁ had a similar value, except for the sigmoidal model. The A₀ coefficient showed inconsistent values across the models. The intact structure viscosity parameter, η₀, was 2.069·10⁴ MPa·s in the Stastna model and 7.713·10⁴ MPa·s in the Burgers model. Its highest value was 3.39·10⁵ MPa·s obtained from the 2S2P1D model.

The model parameters (tables 1-6) were used to construct master curves of asphalt concrete at the reference temperature of 20°C. The summary of these parameters in a graphical form allows a global view on the plot of fitting the stiffness modulus experimental results to the data from the concrete asphalt model. The graphical representation of the master curves relative to the models being discussed is shown in figure 1.

Analysis of the graph above shows that the results of the stiffness modulus prediction by the Burgers model are far apart from the experimental results for frequencies < 0.2 Hz. Below this frequency, the effect of the Maxwell element linear dashpot becomes evident – the material reaches the state of steady flow (strain increasing to infinity) and the stiffness modulus will tend to zero. From the rheological standpoint, in this state elastic energy dissipates, which is impossible within the linear viscoelastic region. The fractional Stastna model behaves similarly to the sigmoidal model with the difference that the fractional model is a better descriptor of asphalt concrete for frequencies > 1 Hz. The final group of models that represent the changes in stiffness modulus of asphalt concrete, E*, in a similar way are: the generalized Maxwell model, the generalized 2S2P1D model and the Bahia and co-workers model. Attention has to be drawn to the similarity between the predictions from the generalized Maxwell and Bahia and co-worker models. In addition to giving nearly identical results, these two models are the best at master curve fitting to experimental results. This explains why the generalized Maxwell model is commonly applied in FEM-based programs. Since the stiffness modulus E* alone is not able to characterize the viscoelastic response of asphalt concrete, the analysis was complemented with Cole-Cole and Black representations, figure 2, which allow including the complex modulus-phase angle relationship independently of the frequency scale.
It has to be noted that, considering the fitting of the master curve that will reflect the real viscoelastic character of asphalt concrete, the best fit was obtained from the generalized 2S2P1D model, generalized Maxwell model and the Bahia and co-workers model. The Burgers model has limitations in terms of accurate description of viscoelastic behaviour of asphalt concrete in the LVE region. As for the fractional model, despite its good fit to experimental data, the number of relaxation
time elements prevents reflecting the real viscoelastic character of asphalt concrete. In this solution, the fractional model predicts certainly more viscous character of asphalt concrete. This may be associated with insufficient quantity of relaxation times and the fact that the objective function algorithm was not optimized with respect to both complex modulus and phase angle. More tests will have to be performed to improve the fractional model estimation, which may finally limit its applicability. The goodness-of-fit between the model and experimental data is shown through the MNE and determination coefficient $R^2$ in table 7.

Table 7. Goodness-of-fit statistics.

| Parameter       | Burgers model | Gen. Maxwell model | Gen. 2S2P1D model | Sigmoidal model | Fractional (Stastna) model | Bahia and co-workers’ model |
|-----------------|---------------|--------------------|-------------------|-----------------|---------------------------|------------------------------|
| MNE [%]         | 3.471         | 7.062              | 6.376             | 8.531           | 3.806                     | 6.177                        |
| $R^2$           | 0.827         | 0.995              | 0.997             | 0.985           | 0.995                     | 0.999                        |

The fitting results presented indicate that all the models except the Burgers model are very accurate in modelling the complex stiffness modulus of asphalt concrete. The mechanical generalized 2S2P1D model is the best option owing to its simplicity and goodness-of-fit when modelling the complex modulus of asphalt concrete. The generalized Maxwell model is also well correlated with the experimental data, but the number of its parameters discourses engineering application. This model is more rational in modelling pavement structures through the FEM. The best models in the group of mathematical models are the sigmoidal and Bahia and co-worker models with the small number of parameters and good correlation with the experimental data.

6. Conclusions

Based on the data presented, the following conclusions were drawn:

- All the models predict the stiffness modulus data of asphalt concrete to a satisfactory degree;
- The Burgers model with its four parameters fails to describe the asphalt concrete behaviour in the viscoelastic range to a satisfactory degree;
- The mechanical generalized 2S2P1D model is the best candidate for forecasting the stiffness modulus of asphalt concrete. With only a few parameters, it ensured a very good fit between the experimental results and the model values;
- The generalized Maxwell model and the Bahia and co-workers’ model yielded the same results;
- The Maxwell model is the best solution for describing the viscoelastic character of asphalt concrete. With and adequate number of single elements, the model is capable of successfully describing changes in the complex modulus in the linear viscoelastic range. Its suitability for use in FEM-based programs has been confirmed;
- The sigmoidal model had a satisfactory fit to the experimental data. Considering the number of parameters in the model, it is an excellent candidate for predicting complex moduli in engineering. The Bahia and co-workers model can be used as its counterpart;
- Despite a very good fit of the master curve shape to the experimental data, the fractional Stastna model was not capable of accurately representing the viscoelastic characteristics of the asphalt concrete. For the relaxation times assumed, the model gave the asphalt concrete a definitely viscous character.
References
[1] NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM,"NCHRP REPORT 614", Maintenance Refining the Simple Performance Tester for Use in Routine Practice, Transportation Research Board, Washington, D.C., 2008.
[2] B. Stefaničyk, P. Mieczkowski, Bituminous mixtures. Performance and research [Mieszanki mineralno-asfaltowe. Wykonawstwo i badania] (Polish), WKŁ:Warsaw, 2008.
[3] P. Buczyński, M. Iwański, "Fatigue life comparison of cold recycled bases with foamed bitumen and with bitumen emulsion", 12th International Conference "Modern Building Materials, Structures and Techniques", 26-27 May 2016, Vilinius, Lithuania. Procedia Engineering Volume 172, 2017, p. 135-142, 2016, http://dx.doi.org/10.1016/j.proeng.2017.02.035.
[4] N.I.Md. Yusoff, G.D. Airey and M.R. Hainin, "Predictability of Complex Modulus Using Rheological Models", Asian Journal of Scientific Research, 3, pp. 18-30, 2010, DOI: 10.3923/ajsr.2010.18.30.
[5] Y.R. Kim, "Modeling of Asphalt Concrete", McGraw-Hill Construction, 2009.
[6] T.K. Pellinen, M.W. Witzak, R.F. Bonaquist, "Asphalt Mix Master Curve Construction using Sigmoidal Fitting Function with Non-Linear Least Squares Optimization Technique", Proceedings of 15th ASCE Engineering Mechanics Conference, Columbia University, New York, 2002.
[7] P. M. Severino C. C., Guillermo, "Computational Viscoelasticity", Springer Briefs in Computational Mechanics, 2012
[8] WT-2/2010 - Techinal requirements - Bituminous mixtures - Annex 2 [Wymagania Techniczne - Mieszanki mineralno-asfaltowe - Część 2] (Polish), order nr 102, GDDKiA, 2010.
[9] H. Di Benedetto, H., C. de la Roche, "State of Art of Stiffness Modulus and Fatigue of Bituminous Mixtures." RILEM Report 17, Bituminous Binders and Mixes, Edited by L.Francken, pp. 137-180, London, 1998.
[10] T. Kobayashi, T. Mikami, M. Fujikawa, "Application of Abaqus for Advanced Inelastic Analysis ( I : Linear Viscoelastic Materials)", Mechanical Design & Analysis Corporation, Abaqus Users’ Conference, 2008.
[11] N.I.Md. Yusoffa, "Modelling the linear viscoelastic rheological properties of bituminous binders", PhD thesis, University of Nottingham, 2012.
[12] N.I.Md. Yusoff, D. Mounier, G. Marc-Stéphane, M. Hainin, G.D. Airey, H.D Benedetto, "Modelling the rheological properties of bituminous binders using the 2S2P1D Model", Construction and Building Materials, 38, pp. 395–406, 2013.http://dx.doi.org/10.1016/j.conbuildmat.2012.08.038
[13] A. Zbiciak, R. Michaleczyk, "Characterization of the Complex Moduli for Asphalt-Aggregate", Mixtures at Various Temperatures, in XXIII R-S-P seminar, Theoretional Foundation of Civil Engineering (23RSP) (TFoCE 2014), Procedia Engineering, 91, pp. 118 – 123, 2014, http://dx.doi.org/10.1016/j.proeng.2014.12.032
[14] S.W. Park, R.A. Schapery, "Methods of interconversion between linear viscoelastic material functions. Part I—A numerical method based on Prony series", Int. J. Solids Struct. 36, 1653-1675, 1999, http://dx.doi.org/10.1016/S0020-7683(98)00055-9.
[15] H.U Bahia, D.I. Hanson, M. Zeng, H. Zhai, M.A. Khatri, R.M. Anderson, "Characterisation of Modified Asphalt Binders in Superpave Mix Design", NCHRP Report 459, Transportation Research Board – National Research Council. 2001.
[16] Stastná, J., Zanzotto, L. and Ho, T. Fractional Complex Modulus Manifest in Asphalt, Rheologica Acta, Vol. 33, pp. 344–354, 1994.
[17] T. O. Medani, M. Huurman, "Constructing the Stiffness Master Curves for Asphaltic Mixes", Delf University of Technology,Report 7-01-127-3, 2003.
[18] A.C. Pronk, "Revival of the Huet-Sayegh Response Model- Notes on the Huet-Sayegh Rheological Model", DWV-2003-029, RHED, Delft, 2003.
[19] M. F. Woldekidan, "Response Modelling of Bitumen, Bituminous Mastic and Mortar", PhD
Thesis, Delft University of Technology, 2011.

[20] G. Mazurek, M. Iwański, "Relaxation Modulus of SMA with Polymer Modified and Highly Polymer Modified Bitumen", Procedia Engineering, Volume 172, Pages 731–738, ELSEVIER, 2017, http://dx.doi.org/10.1016/j.proeng.2017.02.093.

[21] J. Judycki, A new viscoelastic method of calculation of low temperature thermal stresses in asphalt layers of pavements, International Journal of Pavement Engineering, 2016, http://dx.doi.org/10.1080/10298436.2016.1149840

[22] P. Pokorski, P. Radziszewski, M. Sarnowski, "Rheological Properties of Asphalt Mixtures for Bridge Pavements", Procedia Engineering, Volume 111, 2015, Pages 637-644, http://dx.doi.org/10.1016/j.proeng.2015.07.060