Quantum Mach effect by Sagnac
Phase Shift on Cooper pairs in
rf-SQUID

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Abstract

The inertial drag on Cooper pairs in a rotating rf-SQUID ring leads to an asymmetry in the Hamiltonian potential able to modify the tunnelling dynamics. This effect is a Sagnac signature of Cooper pairs at a quantum level and it may probe the Mach effect influence on the wavefunction collapse.

KEYWORDS: Josephson effect, SQUID, Tunneling.
1 Introduction

The Mach principle (that is, the overlapping of the “inertial absolute space” with the “fixed star” or cosmic BBR inertial frame) constitutes one of the most relevant open question in general physics. It relates fundamental questions from microphysics (the origin of the elementary particle inertial mass) to general relativity (the Lense Thirring drag) as well as to basic cosmology. At classical level the Mach principle links different puzzling historical questions [1]: from the oldest Newton bucket and the Foucault’s pendulum to recent Sagnac’s experiments (gravitomagnetism [2]) due to the Lense and Thirring drag on gyroscopes [3]. The Mach effect is partially implied by General Relativity [2] but it calls for a more stringent test at quantum level. A quantum system probing the absolute rotation has been considered long time before: it is a superfluid helium gyroscope (SHEG) [4, 5], which has recently been brought very close to reality by Packard’s group [6]. The effectively measured physical quantity in a SHEG experiment is the rotation induced helium flow, that is the value of a rotation dependent observable. Our proposal is essentially different in principle because it aims to measure the rotation dependent probability of a given result when the collapse of the
system wavefunction occurs in a Macroscopic Quantum Coherence (MQC) experiment. In other words, while the SHEG explores the rotation influence at the level of physical quantities, we investigate the perturbation on the wavefunction collapse mechanism.

2 The MQC experiment

The MQC experiment has extensively been discussed elsewhere [7, 8] and here we limit ourselves to recall its essential features. The rf-SQUID consists of a superconducting ring having self-inductance $L$ in which a Josephson junction of critical current $I_c$ and capacitance $C$ is inserted [9]. The generalised four-momentum of Cooper pairs circulating in the SQUID is expressed as:

$$
\Pi_\mu = p_\mu - 2\frac{e}{c}A_\mu, \quad \mu = 0, 1, 2, 3.
$$

(1)

The integral of the generalised momentum along the rf-SQUID ring must satisfy the Bohr quantization rule:
\[ \int \Pi_i dx^i = n \hbar + \alpha_n \hbar, \quad i = 1, 2, 3; \quad n = 0, 1, 2, \ldots \]  

where \( \alpha_n \) is the phase shift difference at the edge of the Josephson junction. The above equation for the Cooper electron pairs (of mass and energy \( m_{cp} \approx 2m_e, \varepsilon_{cp} \approx 2\varepsilon_e \)), whose kinematics momentum is \( 2m_e \gamma u_i \) (where \( \gamma \) is the Lorentz factor related to the tangential velocity of the ring and in non-relativistic regime \( \gamma \approx 1 \)), becomes \( (\Phi_0 = h/2e, \Phi = \text{magnetic flux threading the ring}) \):

\[ -\frac{2m_e \gamma}{\hbar} \int u_i dx^i - \frac{\Phi}{\Phi_0} = n + \frac{\alpha_n}{2\pi}, \quad n = 0, 1, 2, \ldots \]

where the minus sign in front of the Cooper pairs velocity \( u_i \) is due to the fact that “positive” currents (fixing the integration versus) flows in opposite versus with respect to the negative moving charges. Assuming zero vorticity Eq. (3) becomes:
\[ \alpha_n = -2\pi n - 2\pi \frac{\Phi}{\Phi_0}, \quad n = 0, 1, 2, \ldots \quad (4) \]

We remind that our present simple derivation agrees, in non-relativistic regime, with the rigorous General Relativity treatment of rotating superconducting ring. Indeed following ref. (formula (3.3)), one finds a generalized “London moment” as the following one

\[ \frac{2e}{\hbar c} \int_{\Sigma} \mathbf{B} \cdot d\mathbf{s} + \frac{2}{\hbar c^2} \zeta \oint \mathbf{u} \cdot d\mathbf{r} = 2\pi n, \quad (5) \]

where \( \zeta \approx mc^2 \) in non-relativistic regime. The explicit integral on a superconducting circular ring may be written as:

\[ eB_0\Sigma + 2eV \frac{\Omega \Sigma}{c} + 2\zeta \frac{\Omega \Sigma}{c} = n \frac{hc}{2}, \quad (6) \]

where the second term is proportional to the electric field induced from the magnetic one made by Lorentz boost, and it can always be set, for the non-relativistic regime under consideration, to a negligible (or a vanishing) value.
with respect to the electron rest mass \((eV \ll \zeta)\). The third term, related to the particle (Cooper pair) angular momentum, in a non-relativistic regime reduces exactly (in CGS units), to our Eq. \((3)\) where it is present the additional well know phase shift \([15]\) due to the presence of the Josephson junction.

The interaction energy due to the magnetic coupling of the junction is \([9]\):

\[-\Phi_0 I_c^2 \pi \cos(\alpha_n) = -\Phi_0 I_c^2 \pi \cos \left( \frac{2\pi \Phi}{\Phi_0} \right).\]  

(7)

The junction capacity \(C\) induces the accumulation of the electrostatic energy \(CV^2/2\), where \(V = d\Phi/dt\) is the voltage across the junction. Introducing the “momentum” \(P_\Phi = CV\) conjugated to \(\Phi\), this energy can be written as \(P_\Phi^2/2C\). Adding this expression to Eq. \((7)\) and to the magnetic energy of the superconducting ring, leads to the following Hamiltonian for a non rotating rf-SQUID:

\[\hat{H} = \frac{P_\Phi^2}{2C} + \frac{(\Phi - \Phi_0/2)^2}{2L} - \frac{\Phi_0 I_c}{2\pi} \cos(\alpha_n).\]  

(8)

Through the usual replacement \(P_\Phi \rightarrow -i\hbar \partial_\Phi\), the above equation becomes,
for $n = 0$

$$
\dot{H} = -\frac{\hbar^2}{2C} \partial_\Phi^2 + \frac{(\Phi - \Phi_0/2)^2}{2L} - \frac{\Phi_0 I_c}{2\pi} \cos \left( \frac{2\pi \Phi}{\Phi_0} \right),
$$

(9)

where an external flux $\Phi_{\text{ext}} = \Phi_0/2$ has been assumed. The potential is clearly symmetric around $\Phi_0/2$ and, for suitable choice of $L$ and $I_c$, it presents two minima at values $\Phi_0/2 \pm \Phi_1$, corresponding to opposite versus of the current $I = (\Phi - \Phi_0/2)/2L$ circulating in the ring. The first two energy eigenvalues, say $\varepsilon_S$ and $\varepsilon_A$, correspond to Symmetric and Antisymmetric solutions $\psi_S$ and $\psi_A$, respectively. The linear combinations $\psi_L = \psi_S + \lambda \psi_A, \psi_R = \psi_S - \lambda \psi_A$ represent two state centered around Left and Right minimum respectively; the sign of $\lambda$ determines if the symmetric or the antisymmetric linear combination is left centered ($\lambda = 1$ in the first case , $\lambda = -1$ otherwise). If we set $\psi = \psi_L$ at $t = 0$, the probability difference $P(t)$ to observe $\Phi < \Phi_0/2$ ($P_L$) with respect to the probability to observe $\Phi > \Phi_0/2$ ($P_R$), by an ideal measure at any time $t$, is the usual function

$$
P(t) \equiv P_L(t) - P_R(t) = \cos(2\pi \nu_{\text{int}} t),
$$

(10)
where $\nu_{tu}$ is the tunnelling frequency between the two well for the “rest” Hamiltonian:

$$\nu_{tu} = \frac{\varepsilon_A - \varepsilon_S}{2h}. \quad (11)$$

Any measurement resets the probability expectation value; if at time $t$ results $\Phi < \Phi_0/2$ than, for $t' > t$, Eq. (10) holds replacing $t$ with $t' - t$. In other words the measurement induces a “wave-packet collapse”. We remind that this discussion lies upon the assumption that the circuitation appearing in Eq. (3) is null, and $n = 0$.

## 3 The Sagnac effect in rotating rf-SQUID

The simplest classical Sagnac effect for a massless particle, as a photon, in an oriented ring system of radius $R$ and surface $|\Sigma| = \pi R^2$, rotating at angular velocity $\Omega$ (for instance in a ring laser or in a fiberoptic ring), may be naively evaluated: the propagation time $\tau$ along a circular path of radius $R$ is $\tau = 2\pi R/v$, where $v$ in general is the phase velocity of the photon in
the medium; it differs for co-rotating and counter-rotating photons by a time deviation interval (for the ideal in axis rotating ring)

\[
\Delta t = \frac{(2\pi + \Omega \tau)R}{v} - \frac{(2\pi - \Omega \tau)R}{v} = \frac{4\pi R^2 \Omega}{v^2},
\]

(12)

where \( \Omega \equiv |\Omega| \). In the general cases to discuss we set \( v \approx c \). The corresponding phase shift \( \delta \alpha \) between two counter-propagating modes (of a monochromatic source) at a given frequency \( \nu \) whose energy is \( \varepsilon_\gamma = h\nu \), in general becomes:

\[
\delta \alpha = \frac{4\varepsilon_\gamma}{h\nu^2} \Omega \cdot \Sigma,
\]

(13)

and it is therefore as usual proportional to the area and angular velocity.

For a counter-rotating and co-rotating wave packet of any massive particle whose energy is \( \varepsilon_p = m^*c^2\gamma \) (in non relativistic regime \( \gamma \approx 1 \)), the above Sagnac phase shift may be naturally extended \([10]\), by substitution \( \varepsilon_\gamma \rightarrow \varepsilon_p \) in equation (13), as follows:
The classical detection of such a phase shift allows the observer to be informed of its own rotation (for instance of the terrestrial angular velocity $\Omega_\oplus$) with respect to the “absolute inertial frame”. As Mach noted this frame is coincident with the wider cosmological one defined “fixed stars” or, better, in a cosmological language (because of the linkage between matter and radiation at recombination), by the BBR inertial frame. This peculiar coincidence, already verified at $10^{-6}\Omega_\oplus$ level, call for a gravitational and/or cosmic root of the inertia. In order to measure the Sagnac phase shift effect in rf-SQUID system let us calculate the circuitation appearing in equation (14) when the rf-SQUID rotates around its central axis with angular velocity $\Omega$. The average velocity of the Cooper pairs in the rotating system is $\Omega R$ so that the circuitation (or vorticity) is just the Sagnac phase shift $\delta\varphi$

$$
\delta\varphi = -\frac{2me\gamma}{h} \oint u_i dx^i = -2\frac{2me\gamma}{h} \Omega \cdot \Sigma, \tag{15}
$$
and the equation (4), for $n = 0$, becomes:

$$
\alpha_0 = \delta \varphi - 2\pi \frac{\Phi}{\Phi_0} = -2\frac{2m_e \gamma}{\hbar} \Omega \cdot \Sigma - 2\pi \frac{\Phi}{\Phi_0}.
$$

(16)

If the circuitation versus is inverted (counter-rotating current) the phase shift in Eq. (16) change sign. The difference between the phase shift corresponding to co-rotating and counter-rotating currents gives the Sagnac phase shift of Eq. (14) (where $m^* = 2m_e$ is the mass of Cooper pair); Eq. (16) can be also derived from general relativity in a more rigorous way [11, 12]. In presence of a rotation the Eq. (7) becomes, because of Eq. (16)

$$
-\Phi_0 I_c \cos(\alpha_n) = -\Phi_0 I_c \left| \cos \left( 2\pi \frac{\Phi}{\Phi_0} \right) \cos(\delta \varphi) + \sin \left( 2\pi \frac{\Phi}{\Phi_0} \right) \sin(\delta \varphi) \right|.
$$

(17)

Therefore the interaction term of the Hamiltonian (3) becomes asymmetric with respect to the non rotating case. The last asymmetric term in Eq. (17) derives from the influence of the Josephson junction motion on the Cooper pairs. The perturbation to the Hamiltonian potential deforms the symmetric
“two well” into an asymmetric “two well”. The deepest minimum will collect most of the tunnelling event states, leading to a more probable configuration where the (negative) Cooper pairs are more often counter-rotating the same rotating system. The probability expressed in \(10\) now becomes \[13\]

\[
P(t) = \frac{1}{1 + \left(\frac{\nu_{tu}}{\nu_\Omega}\right)^2} + \frac{\left(\frac{\nu_{tu}}{\nu_\Omega}\right)^2}{1 + \left(\frac{\nu_{tu}}{\nu_\Omega}\right)^2} \cos\left(2\pi \sqrt{\nu_{tu}^2 + \nu_\Omega^2} t\right),
\]

where

\[
\nu_\Omega \equiv \frac{\left|\delta H_\Omega\right|}{h},
\]

and \(\delta H_\Omega\) is the energy gap between the two potential minima. For \(\delta \varphi \ll 1\), as it is in the most realistic cases, the following approximation holds

\[
\delta H_\Omega \approx -2 \frac{\Phi_0 I_c}{h} \sin \left(2\pi \frac{\Phi_1}{\Phi_0}\right) \delta \varphi \approx -2 \frac{\Phi_0 I_c}{h} \sin \left(2\pi \frac{\Phi_1}{\Phi_0}\right) \left(2\varepsilon_{cp}/\hbar c^2\right) \Omega \cdot \Sigma.
\]

In a qualitative way we may imagine the Josephson junction playing the role
of a potential well for the two versus currents. While at rest the transmission in both directions is symmetric, once the rf-SQUID is rotating the nominal kinetic energy of the Cooper pairs (in the rest frame of the junction) appears slightly asymmetric: the higher energy of the current co-rotating respect with to angular velocity (where the negative Cooper charges are hitting the junction at higher velocities) leads to an easier tunnelling probability through the Josephson junction. For the same reason the counter-rotating current (where the Cooper pairs are hitting at lower kinetic energy) has a higher energy gap and a consequent lower transmission probability. Therefore one discovers an “anti-Lenz” law whose validity is based only on the negative charge nature of Cooper pairs. For an ideal positive boson charge the opposite will be true. It should be noticed that, for $\Omega \cdot \Sigma > 0$, because on the right of Eq. (20) $\Phi \geq \Phi_0 / 2$, the final sign of $\delta H_\Omega$ on that side is negative and the two well potential will be unbalanced as follows: “up” on the left and “down” on the right of $\Phi_0 / 2$. The physical meaning is that the most probable state will be at “positive” flux ($\Phi \geq \Phi_0 / 2$), i.e., the “positive” $\Omega$ rotation will induce a flux and an average magnetic field along its vector sign (as mentioned in the introduction) leading to a preferential flux $\Phi \geq \Phi_0 / 2$. We remark once
again this anti-Lenz result is due only to the negative charge nature of the Cooper pairs. In the following we want to quantify the perturbation effect in a realistic measure set up.

4 The experimental set up

In the non rotating rf-SQUID the position of minima is calculated solving the equation \( \partial U / \partial \Phi_0 = 0 \), \( U \) being the potential appearing in Eq. (9). In the parabolic approximation of the well bottom the ground state level energy is given by \( \varepsilon_0 = h \nu_{LC} / 2 \), where

\[
\nu_{LC} = \frac{1}{2\pi} \sqrt{\frac{1}{C} \left| \frac{\partial^2 U}{\partial \Phi^2} \right|_{\Phi = \Phi_0/2 \pm \Phi_1}}
\]

\[
= \frac{1}{2\pi} \sqrt{\frac{1}{LC} \sqrt{1 + \frac{2\pi LI_c}{\Phi_0} \cos \left( 2\pi \frac{\Phi}{\Phi_0} - \pi \right)}}.
\]  

(21)

The tunnelling frequency is calculated as

\[
\nu_{\text{tu}} \approx \nu_{LC} e^{-S/h},
\]

(22)

where
\[
\frac{S}{\hbar} = \int_{\Phi_L(\varepsilon_0)}^{\Phi_R(\varepsilon_0)} \sqrt{2C[U(\Phi) - \varepsilon_0]} d\Phi,
\]

is the usual dimensionless action calculated between the extremes \(\Phi_L(\varepsilon_0)\) and \(\Phi_R(\varepsilon_0)\) given by intersection of line \(\varepsilon = \varepsilon_0\) with the potential barrier. For a very little rotational perturbation \((h\nu_1 \ll h\nu_{LC})\) the position of well bottoms remain substantially unchanged, so that Eq. (21) is still valid. These quantities will be considered, for instance for the following realistic values of an rf-SQUID: \(L = 0.15\) nH, \(I_c = 2.61\) \(\mu\)A, \(C = 0.15\) pF, \(R = 1\) \(\mu\)m. With these values we have, by numerical solution in symmetrical “at rest” Hamiltonian:

\[
\begin{align*}
\Phi_1 &= 0.1596 \Phi_0 \\
\nu_{LC} &\approx 2.0 \times 10^{10}\ \text{Hz} \\
\nu_{tu} &\approx 1.59 \times 10^5\ \text{Hz}
\end{align*}
\]

Because of the little perturbative effect to be considered by rotation of rf-SQUID, these approximate values will be considered in a “rotating” case. The Hamiltonian frequency due to the rotational (or Sagnac) shift for above values becomes
At a time period $T = 1/4\nu_tu$ the differential probability for the unperturbed system [Eq. (10)] is zero while the perturbed one [Eq. (18)] gives, for any angular velocity $\Omega > 0.5 \text{ rad s}^{-1}$, values sensibly greater than zero. For instance:

| $P$ | $\Omega$ (rad s$^{-1}$) | $\hbar\nu_\Omega/\varepsilon_0 \times 10^6$ |
|-----|----------------------|----------------------------------|
| 0.04 | 0.6                  | 0.72                             |
| 0.11 | 1                    | 1.2                              |
| 0.39 | 2                    | 2.4                              |
| 0.97 | $2\pi$               | 7.6                              |

This makes realistic the experimental determination of the Mach principle at a quantum level using a rf-SQUID mounted on a rotating platform. Instead, it does not seem possible, at presently available instrument sensitivity, to use the rf-SQUID to determine the Sagnac signature of the Earth rotation. Indeed the Eq. (18) gives a not vanishing value of $P(t)$ only if the condition $\nu_tu \leq \nu_\Omega$ is satisfied. In order to avoid dissipative effects on the wavefunction time evolution during the measurement the tunnelling frequency must remain
in the order of 1 MHz \[14\]. Of consequence the only possibility is that to enhance \(\nu_\Omega\) by increasing ring radius \(R\), without any change in the double well potential. For a SQUID mounted on a rotating platform we have seen that \(\Omega \approx 1 \text{ rad/s}\) and \(R = 1 \mu\text{m}\) are realistic values. Examining Eq. (20) it is easily seen that the product \(\Omega \cdot \Sigma\) must remain the same for the case \(\Omega = \Omega_\oplus\), which implies \(R \approx 110 \mu\text{m}\), an absolutely unrealistic value for today technology. We note that the stationary rotating case may be reached adiabatically from a static (i.e. non-rotating) case. Indeed we may assume, as in the final table (Eq. 26) a characteristic angular acceleration \(\alpha \sim \Omega/\text{day}\), where \(\Omega \approx 1 \text{ rad/sec}\) which imply \(\alpha/\Omega \ll \Omega \ll \omega_{tu} \ll \omega_{LC}\) the quantum system evolution may be described by a similar slow time variable two-well potential from a static symmetric one toward the final antisymmetric one.

In conclusion we feel that it is exciting to suggest a first realistic experimental opportunity to test a global cosmological feature at microscopic quantum level.
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