Evolution of twist-3 fragmentation functions in multicolour QCD and the Gribov-Lipatov reciprocity.

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Abstract

It is shown that twist-3 fragmentation functions in the large-$N_c$ limit of QCD obey the DGLAP evolution equations. The anomalous dimensions are found explicitly. The Gribov-Lipatov reciprocity is violated at the twist-3 level in the leading logarithmic approximation.
1. A renewed interest in the high energy spin physics in the last years has been concentrated on the transverse spin phenomena in hard processes. It revives many ideas developed over a decade ago. In particular, the notion of the twist-2 transversity distribution \( h_1(x) \), first mentioned by Ralston and Soper [1] has been reinvented as well as its twist-3 counterparts have been addressed [2]. Due to chirality conservation \( h_1(x) \) cannot appear in the inclusive deep inelastic scattering (DIS) but it can be measured, for instance, in Drell-Yan reactions through the collision of the transversely polarized hadrons [1, 2] and in semi-inclusive pion production in DIS on the nucleon [3]. In the last case, it enters into the cross section as a leading contribution together with the twist-3 chiral-odd spin-independent fragmentation function \( I(\zeta) \) [3]. It is well known that there is considerable difference between the structure and fragmentation functions. Namely, the moments of the former are expressed in terms of reduced matrix elements of the tower of the local operators of definite twist. This property is established by exploiting the Wilson operator product expansion for inclusive DIS. Although the light-cone expansion for the fragmentation processes is similar to DIS, the moments of the corresponding functions are not related to any short-distance limit. As a substitute for the local operators come the Mueller’s time-like cut vertices [4] which are essentially nonlocal in the coordinate space so that the analogy to the operator language is only useful mnemonic. Nevertheless, it is possible to give the definition of the twist for them in a broader sense without appealing to the concept of local operators. A thorough discussion of this issue can be found in Ref. [5].

The \( Q^2 \)-dependence of the twist-3 distributions has extensively been discussed in the literature [6, 8, 9, 10]. As has been first observed in Ref. [8], in multicolour QCD \( (N_c \to \infty) \) the flavour nonsinglet twist-3 part of the structure function \( g_2(x) \) obeys a simple DGLAP evolution equation, as in the case of the twist-2 operators, and the corresponding anomalous dimensions are known analytically. The same pattern is followed by the chiral-odd distributions [9, 10]. Since the twist-3 fragmentation functions enter into several cross sections on the same footing as the distributions, their scale dependence is of great interest. Apart from significance for phenomenology, it is important for theoretical reasons: while it is know that in the leading order of the coupling constant the splitting functions for the twist-2 fragmentation functions can be found from the corresponding space-like quantities via the Gribov-Lipatov reciprocity relation [11], no such equality is known for higher twists.

2. As distinguished from the leading twist evolution, the twist-3 two-quark fragmentation functions receive contribution from the quark-gluon correlators even in the limit of asymptotically large momentum transfer. To solve the problem, one should correctly account for the mixing of correlators of the same twist and quantum numbers in the course of renormalization.
Recently, we have addressed ourselves \cite{12} to the question of the $Q^2$-dependence of the nonpolarized chiral-odd (NCO) fragmentation function $\mathcal{I}(\zeta)$ which is given in QCD by the Fourier transform along the null-plane of the appropriate matrix element of the parton field correlator

$$\mathcal{I}(\zeta) = \frac{1}{4} \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{i\lambda \zeta} \langle 0 | \psi(\lambda n) | h, X \rangle \langle h, X | h, X | \bar{\psi}(0) | 0 \rangle.$$

The summation over $X$ is implicit and covers all possible hadronic final states populated by the quark fragmentation. We use the light-cone gauge $B_+ = 0$, otherwise a link factor should be inserted in between the quark fields to maintain the gauge invariance. By exploiting the equation of motion for the Heisenberg fermion field operator we can express $\mathcal{I}(\zeta)$ in terms of the three-parton correlator $Z(\zeta', \zeta)$ which explicitly involves the gluon field, namely (neglecting the quark mass)

$$\mathcal{I}(\zeta) = \int d\zeta' Z(\zeta', \zeta),$$

where we have introduced the $C$-even quantity

$$Z(\zeta', \zeta) = \frac{1}{2} \left[ Z^{(1)}(\zeta', \zeta) + [Z^{(1)}(\zeta', \zeta)]^* \right]$$

with

$$Z^{(1)}(\zeta', \zeta) = \frac{1}{4\zeta} \int \frac{d\lambda \mu}{2\pi} \frac{e^{i\lambda \zeta - i\mu \zeta}}{2\pi} \langle 0 | g \gamma_\nu \gamma_\rho \psi(\lambda n) | h, X \rangle \langle h, X | \bar{\psi}(0) B_\rho^+(\mu n) | 0 \rangle.$$

As we have observed in Ref. \cite{12}, in multicolour limit, \textit{i.e.} neglecting the terms $\mathcal{O}(1/N_c^2)$, an additional three-parton correlator of the type $\langle 0 | \bar{\psi} \psi | h, X \rangle \langle h, X | B^+ \rangle | 0 \rangle$, which appears only through the radiative corrections, decouples from the evolution equation for $Z(\zeta', \zeta)$ and the latter becomes homogeneous provided we discard the quark-mass effects. Thus, in the large-$N_c$ limit the RG equation takes the form

$$\mu^2 \frac{\partial}{\partial \mu^2} Z(\zeta', \zeta) = \frac{\alpha}{4\pi} \int dz' \frac{dz}{z} \Theta(\zeta - z) K(z, z', \zeta, \zeta') Z(z', z),$$

and the evolution kernel is given by the following expression:

$$\frac{1}{N_c} K(z, z', \zeta, \zeta') = 2 \zeta' \delta(\zeta - \zeta + z - z') - \delta(\zeta' - \zeta + z)$$

$$- \frac{2}{\zeta} \frac{2}{1 - \zeta} \delta(\zeta' - \zeta + z - z') + 2 \int_1^{\infty} \frac{dz''}{z''(1 - z'')} \delta(1 - \zeta') \delta(\zeta' - z')$$

$$+ [\delta(\zeta' - \zeta + z - z') - \delta(\zeta' - \zeta + z)] \left[ \frac{z}{z'} - \frac{z(\zeta')}{\zeta(\zeta') + z} \right]$$

$$+ \delta \left[ 1 - \frac{\zeta}{z} \right] \left[ \frac{3}{2} \delta(\zeta' - z') - \ln \left( \frac{1 - z'}{z'} \right) \delta(\zeta' - z') \right] - 2 \left[ \frac{z'}{\zeta' - z} T(\zeta', \zeta' - z') + \frac{\zeta'}{z - \zeta} + \frac{z'}{\zeta'} - 1 \right] T(\zeta', \zeta' - z'),$$

$$+ \frac{z - \zeta'}{\zeta' - z} \left[ \frac{\zeta'}{z - \zeta} + \frac{z - \zeta'}{\zeta'} \right] T(\zeta', \zeta' - z) + \left[ \frac{\zeta'}{z - \zeta} + \frac{z'}{\zeta'} - 1 \right] T(\zeta', \zeta' - z').$$
Here we have used the shorthand notation $\Theta_{11}^0$ for the following step function

$$\Theta_{11}^0(z, z') = \frac{\theta(z) - \theta(z')}{z - z'}.$$  \hfill (7)

Inspired by our knowledge acquired from the study of the twist-3 structure functions \cite{8, 9, 10}, where the solution of the asymptotic equations was given by the convolution of the three-particle correlator with a certain weight function that is essentially the same as entering into the equation that gives rise to the dynamic twist-3 contribution to the two-quark distribution at the tree level, we are able to check that Eq. (2) satisfies the ladder-type evolution equation with the following splitting function:

$$\frac{1}{N_c} \int d\zeta' K(z, z', \zeta, \zeta') = -\frac{2}{\left(\frac{\zeta}{z} \left(1 - \frac{\zeta}{z}\right)\right)_+} + \frac{2}{\zeta} - 1 + \frac{1}{2} \delta \left(1 - \frac{\zeta}{z}\right).$$  \hfill (8)

Thus, for the moments we obtain the following solution of the RG equation ($Q > Q_0$):

$$\int_1^\infty \frac{dz}{z^n} I(z, Q) = \left(\frac{\alpha(Q)}{\alpha(Q_0)}\right)^{NCO \gamma / \beta_0} \int_1^\infty \frac{dz}{z^n} I(z, Q_0),$$  \hfill (9)

and the corresponding anomalous dimensions equal

$$NCO \gamma_n = N_c \left\{ -2\psi(n - 1) - 2\gamma_E - \frac{3}{n - 1} + 1 \right\},$$  \hfill (10)

as usual $\beta_0 = \frac{2}{3} N_f - \frac{11}{3} C_A$.

As we have previously mentioned, there exists an equation which states that in the leading log approximation the time-like (TL) and space-like (SL) kernels corresponding to the twist-2 parton densities are directly related, in the physical regions of the corresponding channels, by the Gribov-Lipatov equation $P_{SL}(x) = P_{TL}(1/x)$, or in terms of the corresponding anomalous dimensions it looks like $\gamma_{n+2}^{TL} = \gamma_n^{SL}$. Comparing the result given by Eq. (10) with the large-$N_c$ anomalous dimensions known in the literature for the chiral-odd distribution $e(x)$ \cite{9, 10} we see that there is no universality of the corresponding twist-3 evolution kernels, \textit{i.e.} the Gribov-Lipatov reciprocity is violated.

3. Now we can proceed further and demonstrate that the evolution kernels for the time-like two-quark densities can directly be found from their space-like analogues by exploiting the particular form of the evolution equations given in Ref. \cite{10}. Since the analytic structure of the uncut diagram (see fig. 1) is completely characterized by the integral representation of the $\Theta$-function given by Eq. (6), we can just take its particular discontinuities, using the usual Cutkosky rules supplied with appropriate theta-function specifying the positivity of the energy flow from the right- to the left-hand side of the cut, in order to obtain the corresponding time-like kernel. Since the observed particle is always in the final state for the fragmentation process,
Figure 1: One-loop ladder-type diagram for the two-particle evolution kernel.

we are restricted to the single cut\footnote{For a given graph the possible cuts correspond to the possible final states.} across the horizontal rank of the ladder diagram. Namely, using the integral representation of the corresponding step function (\ref{eq:step}), we have

$$
\Theta^0_{11}(x, x - \beta) = \int_{-\infty}^{\infty} \frac{d\alpha}{2\pi i} \frac{1}{[\alpha x - 1 + i0][\alpha(x - \beta) - 1 + i0]} \frac{\text{disc}}{-\theta(x - \beta)} \beta. \tag{11}
$$

The self-energy insertions are not affected by the cut since it does not cross the corresponding lines. Taking into account different kinematic definitions of the correlation functions in the space- and time-like regions \cite{10}, we are able to find the kernels. It is easy to verify that the evolution kernels constructed for the time-like twist-2 cut vertices using this recipe coincide with the known results. In the same way, we may obtain the above equation (8) from Eq. (90) of Ref. \cite{10}.

Since there exist fragmentation functions corresponding to each distribution, apart from the specific ones appearing from the final state interaction, we are in a position to find large-\(N_c\) anomalous dimensions, which govern their \(Q^2\)-dependence, from the results of Refs. \cite{8,9,10,14}. Namely, the genuine twist-3 contributions \(\mathcal{H}_{L}^{tw-3}\) and \(\mathcal{G}_{T}^{tw-3}\) to the corresponding fragmentation functions

\begin{align}
\mathcal{H}_{L}(\zeta) &= \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda\zeta} \langle 0 | i\sigma_+ \gamma_5 \psi(\lambda n)| h, X \rangle \langle h, X | \bar{\psi}(0) | 0 \rangle, \\
\mathcal{G}_{T}(\zeta) &= \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda\zeta} \langle 0 | \gamma_\perp \gamma_5 \psi(\lambda n)| h, X \rangle \langle h, X | \bar{\psi}(0) | 0 \rangle.
\end{align}

after subtracting out the twist-2 piece \footnote{For a given graph the possible cuts correspond to the possible final states.} obey the evolution equation (\ref{eq:evolution}) with the following anomalous dimensions:

\begin{align}
\text{PCO}_{\gamma_n} &= N_c \left\{ -2\psi(n - 1) - 2\gamma_E + \frac{1}{n} + \frac{1}{2} \right\}, \\
\text{PCE}_{\gamma_n} &= N_c \left\{ -2\psi(n - 1) - 2\gamma_E - \frac{1}{n} + \frac{1}{2} \right\}. \tag{15}
\end{align}
To summarize, we have found that in the multicolour limit of QCD the twist-3 fragmentation functions obey the ladder-type evolution equations and corresponding anomalous dimensions are known analytically. This gives a possibility to analyze experimental data when they become available. The Gribov-Lipatov reciprocity is not the property of the twist-3 distributions but rather it is strongly violated already in the leading logarithmic approximation.

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