Theoretical Aspects of the Quantum Neutrino

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DUALITY AND MULTI-GLUON SCATTERING

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For the six-gluon scattering process we give explicit and simple expressions for the amplitude and its square. To achieve this we use an analogy with string theories to identify a unique procedure for writing the multi-gluon scattering amplitudes in terms of a sum of gauge invariant dual sub amplitudes multiplied by an appropriate color (Chan Paton) factor. The sub amplitudes defined in this way are invariant under cyclic permutations, satisfy powerful identities which relate different non-cyclic permutations and factorize in the soft gluon limit, the two-gluon collinear limit and on multi-gluon poles. Also, to leading order in the number of colors these sub-amplitudes sum incoherently in the square of the full matrix element. The results contain the full information about the scattering process and in particular can be used to calculate the contributions to the soft and collinear limits. We also make a comparison with the results of multi-gluon scattering in QCD. The results of the six-gluon scattering processes are expected to be useful in the future

AMPLITUHEEDRON

Xu Zhan, Tsinghua
1. Helicity Amplitudes for Multiple Bremsstrahlung in Massless Nonabelian Gauge Theories
Zhan Xu, Da-Hua Zhang, Lee Chang (Tsinghua U., Beijing). Jul 1986. 37 pp.
Published in Nucl.Phys. B291 (1987) 392-428
TUTP-86/9a
DOI: 10.1016/0550-3213(87)90479-2
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 445 records

2. Helicity Amplitudes For Multiple Bremsstrahlung In Massless Nonabelian Gauge Theory. 2. Decomposition Into Invariant Subsets
Zhan Xu, Da-Hua Zhang, Lee Chang (Tsinghua U., Beijing). Mar 1985. 32 pp.
TUTP 84/4
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record

3. Helicity Amplitudes For Multiple Bremsstrahlung In Massless Nonabelian Gauge Theory. 3. Amplitudes Of Physical Processes Involving Gluon Selfcoupling Vertices
Da-hua Zhang, Zhan Xu, Lee Chang (Tsinghua U., Beijing). Jan 1985. 29 pp.
TUTP-84/5a
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record

4. Helicity Amplitudes For Multiple Bremsstrahlung In Massless Nonabelian Gauge Theory. 1. New Definition Of Formulation Of Amplitudes In Grassmann Algebra
Zhan Xu, Da-Hua Zhang, Lee Chang (Tsinghua U., Beijing). Dec 1984. 20 pp.
TUTP-84/3-TSINGHUA
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 4 records
NOBEL 2015

“for the discovery of neutrino oscillations, which shows that neutrinos have mass”

Takaaki Kajita
SuperKamiokaNDE

Art McDonald
SNO

“for the discovery of neutrino flavor transformations, which shows that neutrinos have mass”

~ vacuum oscillations

Wolfenstein matter effects dominant flavor transformations

See Smirnov arXiv:1609.02386
Neutrinos are Everywhere!

from Big Bang 300 nus / cm^3
2 or more v/c << 1

SuperNovae
> 10^58

Sun’s
~ 10^38 nu/sec

Daya Bay
3 x 10^21 nu/sec

Neutrinos are Forever!!!
(except for the highest energy neutrino’s)

therefore in the Universe:
\[ \frac{\partial N_\nu}{\partial t} > 0 \]
Neutrino Flavor or Interaction States:

\[ W^+ \rightarrow e^+ \nu_e \quad W^+ \rightarrow \mu^+ \nu_\mu \quad W^+ \rightarrow \tau^+ \nu_\tau \]

\( \nu_e \)  \( \nu_\mu \)  \( \nu_\tau \)

provided \( L/E \ll 0.5 \text{ km/MeV} = 500 \text{ km/GeV} \) !!!

\( \sim 1 \) picosecond in Neutrino rest frame !!!

\( \approx \text{Age of Universe} / 10^{26} \)
Neutrino Mass EigenStates or Propagation States:

\[
\text{Propagator } \nu_j \rightarrow \nu_k = \delta_{jk} e^{-i \left( \frac{m_j^2 L}{2E_{\nu}} \right)}
\]

\[\nu_1\]
most \(\nu_e\)

\[\nu_2\]

\[\nu_3\]
least \(\nu_e\)

\(\nu_e = \)
Solar Exp, SNO
KamiLAND
Daya Bay, RENO, ...

\(\nu_\mu = \)
SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

\(\nu_\tau = \)
Unitarity
SK, Opera
ICECUBE

\(\delta, \theta_{23}\)

\(\delta, \theta_{23}\)

\(\theta_{23}\)
Interactions:

- simple
- complicated

Propagation:

- unitary matrix ?
- masses ?
Rates: $|U_{\mu 1}|^2$ & $|V_{td}|^2$
unitary matrix

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

by defn \(|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2\)

\[U_{PMNS} = U_{23}(\theta_{23}, 0) U_{13}(\theta_{13}, \delta) U_{12}(\theta_{12}, 0)\]

Why this order ???

\[
\begin{pmatrix}
1 \\
c_{23} & s_{23} \\
-s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & s_{13}e^{-i\delta} \\
-s_{13}e^{+i\delta} & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} \\
-s_{12} & c_{12}
\end{pmatrix}
\times \text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})
\]

\[
\begin{pmatrix}
c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
-c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}c_{23} \\
s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}s_{23}
\end{pmatrix}
\]
\( \nu_1, \nu_2 \) Mass Ordering:

- solar mass ordering

\[
|\Delta m^2_{21}| = |m^2_2 - m^2_1| = 7.5 \times 10^{-5} \text{ eV}^2
\]

\( L/E = 15 \text{ km}/\text{MeV} = 15,000 \text{ km}/\text{GeV} \)

**SNO**

\( m_2 > m_1 \)
$\nu_3, \; \nu_1/\nu_2$ Mass Ordering:

- atmospheric mass ordering

$|\Delta m_{31}^2| = |m_3^2 - m_1^2| = 2.5 \times 10^{-3} \text{ eV}^2$

$L/E = 0.5 \text{ km/MeV} = 500 \text{ km/GeV}$

Unknown: NO$\nu$A, JUNO, ICECUBE, DUNE, T2HKK....
Summary:

| $\sin^2 \theta_{23}$ | 0.40 | 0.50 | 0.60 |
|----------------------|------|------|------|
| $\nu_3$              | ![pie chart 1] | ![pie chart 2] | ![pie chart 3] |

Octant of $\theta_{23}$

| $\delta$ | $\pm \pi/2$ |
|-----------|-------------|
| $\nu_2$  | ![pie chart 4] | ![pie chart 5] | ![pie chart 6] |
| $\nu_1$  | ![pie chart 7] | ![pie chart 8] | ![pie chart 9] |

$\nu_2$ variation

$\nu_1$ variation

$\nu_e = \bullet$  $\nu_\mu = \square$  $\nu_\tau = \bigcirc$  $\pi$
Leptons:

\[ U_{\mu_1} \]

\[ V_{ij} \]

\[ |V_{ij}|^2 \] essentially independent of \( \delta_q \)!

\[ V_{td} \approx A \lambda^3 (1 - 0.37 e^{i \delta_q}) \]

\[ |V_{td}|^2 \approx 10^{-4} \]

\[ |V_{tb}|^2 \approx 1 \]

\[ |V_{ts}|^2 \sim \lambda^4 \approx 2 \times 10^{-3} \]

\[ |V_{td}|^2 \sim \lambda^6 \approx 8 \times 10^{-5} \]

0.08 < \[ U_{\mu_1} \]^2 < 0.24

variation in \( \delta \) only!

factor of 3 diff.

\[ |U_{\mu_3}|^2 = 0.4 - 0.6 \]

\[ |U_{\mu_2}|^2 = 0.26 - 0.41 \]

\[ |U_{\mu_1}|^2 = 0.08 - 0.24 \]
Flavor content
NH

\[ |U_{\tau i}|^2 \quad |U_{\mu i}|^2 \]

\[ |U_{ei}|^2 \]

\[ \delta & \theta_{23} \text{ uncertainty} \]

Vary \( \theta_{ij}, \delta_{CP} \)

\[ \text{Best Fit} \quad 1\sigma \quad 3\sigma \]

Vary \( \delta_{CP} \)

\[ \theta_{ij} \text{ fixed at BF} \]

\[ \text{NH} \]

\[ \text{Best Fit} \quad 1\sigma \quad 3\sigma \]

\[ \text{Flavor content} \]

\[ \text{Bustamante, Beacom, Winter (2015, PRL)} \]

\[ \text{PRL 2015 [arXiv:1506.02645]} \]

\[ \delta_{CP} \text{ uncertainty} \]
Precision Neutrino Measurements:  

Why?

To discover neutrino BSM, one needs precision predictions for nuSM

Determine flavor fractions of neutrino mass states
Determine flavor fractions of neutrino mass states

Precision Predictions for flavor ratios at ICECUBE.

Bustamante, Beacom, Winter
PRL 2015 [arXiv:1506.02645]
Precision Neutrino Measurements:

- Stress Test Neutrino paradigm search for new physics
- Connection to Leptogenesis
- Understanding Universe
- Test Theoretical Neutrino Models
- Determine flavor fractions of neutrino mass states

WHY?
high enough energy experiments with both an intense, well known beam of order 1%, will be able to constrain the facility [62], with the uncertainty on their fluxes of the measurements. Possible future experiments such as better than the others. By comparison to Fig. 2 one can improve in the sector. Improvement in the appearance requires new elements and hence is constrained...
WHY?

- Precision Neutrino Measurements:
  - Determine flavor fractions of neutrino mass states
  - Stress Test Neutrino paradigm search for new physics
  - Connection to Leptogenesis Understanding Universe

- Test Theoretical Neutrino Models
- Determine flavor fractions of neutrino propagation states

WHY?
Ballet King Pascoli
Prouse Wang 2016

Connection to Leptogenesis Understanding Universe
Precision Neutrino Measurements:

- Stress Test Neutrino paradigm search for new physics
- Connection to Leptogenesis
- Understanding Universe
- Test Theoretical Neutrino Models
- Determine flavor fractions of neutrino mass states

WHY?
WHY MEASURE THESE PARAMETERS?

➤ Lepton mixing allows for a new source of CP violation that can be studied with neutrinos

➤ CPV through $\delta_{cp}$ may be sufficient source for leptogenesis (Nucl. Phys. B774 (2007) 1)

➤ Neutrino masses indicate new physics beyond the standard model and electroweak scale

➤ Precise values of the mixing parameters may indicate or disfavor models of flavor symmetries

Predictions from flavor symmetry forms with current measurement precision

Nucl. Phys. B, Vol. 894, 733-768 (2015)

Predictions from $\cos \delta$ sum rules for discrete symmetries:

Predictions of flavor symmetry forms with projected measurement precision

Test Theoretical Neutrino Models

Girardi, Petcov, Titov, arXiv:1410.8056

$Nucl. \ Phys. \ B, \ Vol. \ 894, \ 733-768 \ (2015)$
Neutrino masses indicate new physics beyond the standard model and electroweak scale. Lepton mixing allows for a new source of CP violation.

Predictions of flavor symmetry forms with projected measurement precision.
Recent highlights from neutrino theory

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Fermilab soon to be at LANL as junior staff member

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Neutrinos as a portal to new Physics

- ultra-light
- warm
- WIMP
- Dark matter?
- Natural seesaw
- Flavor puzzle
- Leptogenesis
- Unification

- $\nu$ masses
- Coherent $\nu$-N scat.
- $0\nu\beta\beta$
- Solar $\nu$
- Supernova $\nu$
- Geo $\nu$
- Reactor $\nu$
- Accelerator $\nu$
- $\nu$ matter effect
- $\nu@LHC$
- Icecube HE neutrinos
- Proton decay

- tiny!
- eV
- keV
- MeV
- GeV
- TeV
- PeV
- YeV
Many many many other fronts!

Neutrino cross sections
(NuSTEC effort)

Neutrinos in cosmology
Early universe - BBN
Abazajian, Barbieri, Cirelli, Chizov, Di Bari, Dodelson, Dolgov, Foot, Holanda, Iocco, Kirilova, Kusenko, Mangano, Lesgourges, Pastor, Smirnov, Steigman, Volkas

Secret neutrino interactions
Dasgupta Kopp 2013, Chu Dasgupta Kopp 2015, Lundkvist Archidiacono Hannestad Tram 2016, Ghalsasi McKeen Nelson 2016, Archidiacono Gariazzo Giunti Hannestad Hansen Laveder Tram 2016, Forastieri Lattanzi Mangano Mirizzi Natoli Saviano 2017

Supernova evolution: non-linear effects from collective oscillations
Friedland 2010, Cherry Carlson Friedland Fuller Vlaesenko 2012, Chakraborty Hansen Izaguirre Raffeelt 2016, Capozzi Basudeb Dasgupta 2016, Izaguirre Raffeit Tamborra 2016, Capozzi Dasgupta Lisi Marrone Mirizzi 2017

Cosmic neutrino background: ideas to measure it?
Non-thermal component?

Type II, type III and radiative seesaw
Akhmedov, Bonnet, Babu, Barbieri, Barger, Berezhiani, Ellis, Gaillard, Glashow, Hirsch, Keung, Ma, Mohapatra, Ota, Pakvasa, Schechter, Senjanovic, Valle, Yanagida, Winter, Wolfenstein, Zee, and many others

Flat extra dimensions: light sterile neutrinos
Antoniadis, Arkani-Hamed, Barbieri, Berrymann, Davoudiasl, Dimopoulos, Dvali, de Gouvea, Langacker, Machado, Mohapatra, Nandi, Nunokawa, Perelstein, Peres, Perez-Lorenzana, Smirnov, Strumia, Tabrizi, Zukanovich-Funchal, …

Leptogenesis
Barenboim, Davidson, Di Bari, Dolgov, Fukugita, Kuzmin, Rubakov, Servant, Shaposnikov, Yanagida, Zeldovich, …

Sterile neutrino in long baseline oscillation experiments
Agarwalla, Bhattacharya, Chatterjee, Dasgupta, Dighe, Donini, Fuki, Klop, Lopez-Pavon, Meloni, Migliozzi, Palazzo, Ray, Tang, Terranova, Thalappillil, Wagner, Yasuda, Winter, …

Dark matter in neutrino detectors: light DM and light mediators
Ballett, Batell, Chen, Coloma, deNiverville, Dobrescu, Frugueele, Harnik, McKeen, Pascoli, Pospelov, Ritz, Ross-Lonergan

Neutrinos and the standard solar model: CNO cycle and metallicity
Bailey, Busoni, Christensen-Dalsgaard, Krief, Simone, Serenelli, Scott, Vincent, Vilante, Vissani, Vinyoli, …

Neutrino magnetic moment
see e.g. Salam 1957, Barbieri Fiorentini 1988, Barbieri Mohapatra 1989, Babu Chang Keung Phillips 1992, Tarazona Diaz Morales Castillo 2015
Cañas Miranda Parada Tortola Valle 2015, Barranco Delepine Napsuciale Yebra 2017 Coloma Machado Martinez-Soler Shoemaker 2017

Discrete symmetries with non-zero $\theta_{13}$
Feruglio Hagedorn Torop 2011, Lam 2012, Lam 2013, Holthausen Lim Lindner 2012, Neder King Stuart 2013, Hagedorn Meroni Vitale 2013
King Neder 2014, Ishimori King Okada Tanimoto 2014, Yao Ding 2015 …

Effective operator approach to neutrino masses and collider/low scale pheno
de Gouvea Jenkins 2007, Boucenna Morisi Valle 2014, Nath Syed 2015, Geng Tsai Wang 2015, Chiang Huo 2015, Bhattacharya Wudka 2015, Geng Huang 2016, Quintero 2016, Mohapatra 2016, Kobach 2016

New physics in neutrinoless double beta decay, lepton number violation at the LHC, left-right models, RS models and neutrino masses, neutrinos as dark matter, and much more!

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Towards a better understanding of Osc. Prob.

Globes,
while a very useful tool,
is not enough!
\[ \bar{\nu}_e \rightarrow \bar{\nu}_e \]

- \( P_{ee} \approx 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{ee} \) 
  \[ \Delta \equiv \Delta m^2 L/4E \]

\[ + \mathcal{O}(0) \quad \Delta m^2_{YY} = \left( \frac{4E}{L} \right) \arcsin \left[ \sqrt{\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}} \right] \]

OR

\[ + \mathcal{O}(10^{-4}) \quad \Delta m^2_{ee} \equiv \cos^2 \theta_{12} \Delta m^2_{31} + \sin^2 \theta_{12} \Delta m^2_{32} \]

\( \nu_e \) average!

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H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, 
"Another possible way to determine the neutrino mass hierarchy," 
Phys. Rev. D 72, 013009 (2005), hep-ph/0503283

SP arXiv:1601.07464

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Stephen Parke                      Lepton-Photon 2017, Guangzhou        8/10/2017        # 30
\[ \nu_\mu \rightarrow \nu_\mu \]

\[ 1 - P(\nu_\mu \rightarrow \nu_\mu) \approx 4 |U_{\mu 3}|^2 \left( 1 - |U_{\mu 3}|^2 \right) \sin^2 \Delta_{\mu \mu} \]

Amplitude of Oscillation:

\[ c_{13}^4 \sin^2 2\theta_{23} + s_{23}^2 \sin^2 2\theta_{13} \]

\[ = 4 c_{13}^2 s_{23}^2 \left( 1 - c_{13}^2 s_{23}^2 \right) \]

for every \((s_{23}^2)_1\) point,

\[ (s_{23}^2)_2 = 1/c_{13}^2 - (s_{23}^2)_1 \approx (1 + s_{13}^2) - (s_{23}^2)_1 \]

has approx. same \(\chi^2\)

and \((s_{23}^2)_1 + (s_{23}^2)_2 \approx (1 + s_{13}^2)\)

Symmetry about \(s_{23}^2 \approx \frac{1}{2}(1 + s_{13}^2) \approx 0.51\)

\[ \nu_\mu \text{ average!} \]
Neutrino Oscillation Amplitudes

\[ P(\nu_\alpha \rightarrow \nu_\beta) = |A_{\alpha\beta}|^2 \]

Two Flavors:

\[ A_{\alpha\alpha} = 1 + (2i) \, s^2 \theta \, e^{+i\Delta} \, \sin \Delta \]

and \[ A_{\alpha\beta} = (2i) \, s \theta c \theta \, e^{-i\Delta} \, \sin \Delta \]

\[ \Delta \equiv \Delta m^2 L/4E \]
Neutrino Oscillation Amplitudes in vacuum:

"the billion $ process"

\[ P(\nu_\mu \rightarrow \nu_e) = |A_{\mu e}|^2 \]

\[ A_{\mu e} = (2i) \left[ (s_{23}s_{13}c_{13}) \left( c_{12}e^{-i\Delta_{32}} \sin \Delta_{31} + s_{12}e^{-i\Delta_{31}} \sin \Delta_{32} \right) \right. \]
\[ + \left. (c_{23}c_{13}s_{12}c_{12}) e^{i\delta} \sin \Delta_{21} \right] \]

maintain the symmetry: \( m_1^2 \leftrightarrow m_2^2 \) with \( \theta_{12} \rightarrow \theta_{12} \pm \pi/2 \)

Denton, Minakata, SP arXiv:1604.08167

\[ \Delta P_{CP} = 8 (s_{23}s_{13}c_{13}) (c_{23}c_{13}s_{12}c_{12}) \sin \delta \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \]

\[ \Delta_{32} \approx \Delta_{31} \]

\[ A_{\mu e} \approx (2i) \left[ (s_{23}s_{13}c_{13}) \sin \Delta_{31} + (c_{23}c_{13}s_{12}c_{12}) e^{i(\delta+\Delta_{31})} \sin \Delta_{21} \right] \]
\[ \nu_\mu \rightarrow \nu_e \]

\[ A_{31} = 2s_{23}s_{13}c_{13} \sin \Delta_{31} \]

\[ A_{\mu e} = A_{31} + e^{i(\delta + \Delta_{32})} A_{21} \]

\[ P(\nu_\mu \rightarrow \nu_e) = A_{\mu e} A_{\mu e}^* \]

\[ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \bar{A}_{\mu e}^* \bar{A}_{\mu e} \]

\[ \bar{A}_{\mu e}^* = A_{31} + e^{i(\delta - \Delta_{32})} A_{21} \]

\[ \delta = 0.0\pi \]

\[ \Delta_{32} = 0.40\pi \]

\[ \Delta_{ij} = \Delta m_{ij}^2 L/4E \]

\[ |U_{e3}|^2 = 0.51 \]

\[ |U_{\mu 3}|^2 = 0.022 \]

\[ P(\nu_\mu \rightarrow \nu_e) = |\bar{A}_{\mu e}|^2 \]
Matter Effects:
2 flavor mixing in matter

\[ ax^2 + bx + c = 0 \]

simple, intuitive, useful

3 flavor mixing in matter

\[ ax^3 + bx^2 + cx + d = 0 \]

complicated, counter intuitive, \ldots
Matter Effects:

\[ \delta = 0.0\pi \]
\[ \Delta_{32} = 0.5\pi \]

T2K

\[ P(\nu_\mu \rightarrow \nu_e) = A_{\mu e} A_{\mu e}^* \]
\[ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \bar{A}_{\mu e}^* A_{\mu e} \]

\[ \alpha = \rho L \sin^2 \theta_{23} \]

NOVA

\[ \delta = 0.0\pi \]
\[ \Delta_{32} = 0.4\pi \]

\[ P(\nu_\mu \rightarrow \nu_e) = A_{\mu e} A_{\mu e}^* \]
\[ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \bar{A}_{\mu e}^* A_{\mu e} \]

\[ A_{31} + e^{i(\Delta_{32} \pm \delta)} A_{21} \]
\[ A_{31} = 2 s_{23} s_{13} c_{13} \frac{\sin(\Delta_{31} \pm a L)}{(\Delta_{31} \pm a L)} \Delta_{31} \]
\[ A_{21} = 2 c_{13} c_{23} s_{12} c_{12} \frac{\sin(a L)}{(a L)} \Delta_{21} \]
\[ a = G_F N_e / \sqrt{2} = (4000 \ km)^{-1} \]

DUNE

\[ \delta = 0.0\pi \]
\[ \Delta_{32} = 0.4\pi \]

\[ P(\nu_\mu \rightarrow \nu_e) = A_{\mu e} A_{\mu e}^* \]
\[ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \bar{A}_{\mu e}^* A_{\mu e} \]
Correlations between $\nu_\mu \rightarrow \nu_e \quad \bar{\nu}_\mu \rightarrow \bar{\nu}_e$

**Normal Ordering — Inverted Ordering**

$\nu_\mu \rightarrow \nu_\mu$ gives:

$$\sin^2 2\theta_{\mu\mu} \equiv 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) = 0.96 - 1.00$$

$|U_{\mu 3}|^2 \leftrightarrow (1 - |U_{\mu 3}|^2)$ degeneracy!

---

**NH**

T2K/HK

NO$\nu$A

DUNE Same L/E as NO$\nu$A

---

$\propto \rho L \ \sin^2 \theta_{23}$

$$\sin \delta_{NO} - \sin \delta_{IO} = \tan \theta_{23} \times \begin{cases} 
0.48 & \text{T2K} \\
1.62 & \text{NO$\nu$A} \\
2.60 & \text{DUNE}
\end{cases}$$

O. Mena & SP hep-ph/0408070
Approximately same uncertainty on $\delta$
until systematic uncertainties dominate at 1st OM !
New Perturbation Theory for Osc. Probabilities

\[ A_{\mu e} = (2i) \left[ (s_{23}s_{13}c_{13}) \left[ c_{12}e^{-i\Delta_{32}} \sin \Delta_{31} + s_{12}e^{-i\Delta_{31}} \sin \Delta_{32} \right] + (c_{23}c_{13}s_{12}c_{12}) e^{i\delta} \sin \Delta_{21} \right] \]

mixing angles in matter $\theta_{13} \rightarrow \phi$ and $\theta_{12} \rightarrow \psi$

mass eigenvalues in matter $m^2_i \rightarrow \lambda_i$

Intuitive and Analytically simple!
New Perturbation Theory for Osc. Probabilities

$\nu_\mu \to \nu_e, \ L = 1300 \text{ (km)}, \ \delta = 3\pi/2, \ \text{NO}$

$P \nu_\mu \to \nu_e, L = 1300 \text{ (km)}, \ \delta = 3\pi/2, \ \text{NO}$

Figure 3: The $\nu_\mu \to \nu_e$ oscillation probability is plotted in the upper part of the figure for DUNE parameters; a 1300 km baseline and $Y_e = 1.34 \times 10^{-3}$. The fractional uncertainties at zeroth and first order are plotted using the analytic formulas in tables 1 and 2 respectively. The probability to second order is calculated using $\lambda$'s and $W$ through second order, see eqs. 3.1.3 and 3.2.6.

The mixing matrix, $W$, has been used to calculate the oscillation probabilities to second order. The resulting oscillation probabilities are more than two orders of magnitude closer to the exact values than the first order probabilities.

4.4 Precision of the perturbation expansion

The oscillation probabilities that were perturbatively calculated in this section are only useful if they are more precise than the experimental uncertainties. In figure 3, we have plotted the fractional uncertainties at each order of our perturbative expansion for the $\nu_\mu \to \nu_e$ channel at the DUNE. The precision at the first oscillation maximum and minimum for DUNE are shown in table 3. We note that the precision improves at lower energies, such as for NO$\nu_A$ and T2K/T2HK. The results are comparable for different values of $\delta$, for the inverted ordering, for other channels, and for antineutrino mode. Therefore, even at zeroth order, the precision exceeds the precision of the expected experimental results.

The exact oscillation probability were calculated using [3, 4].
TABLE XXVII. Best-fit results and the octant) and with the T2K most probable values of the oscillation parameters and using the NO measurement.

Parameter | Best-fit | Latest Results from the T2K Experiment
--- | --- | ---
\(\sin^2 \theta_{23} = 0.45\) | 58.27% of toys MC | 0.431 0.569
\(\sin^2 \theta_{23} = 0.55\) | 89.79% of toys MC | 0.294 0.401

The Bayes factor for normal ordering is 2.8; the Bayes factor for the upper octant is 0.45. The proportion of the MCMC points with \(\sin^2 \theta_{23} \geq 0.55\) is outside of physical abilities [90]. The Bayes factor for normal ordering is 5. Similarly, the relative proportion of steps with \(\sin^2 \theta_{23} \geq 0.55\) can be considered decisive.

The critical parameter \(\sin^2 \theta_{23}\) compared to T2K data. The dashed line distinguishes the two \(\sin^2 \theta_{23}\) values obtained with the fit to the T2K data with the reactor constraint.

The green line show the critical parameter of interest. The red line shows the critical parameter of interest. The red line shows the critical parameter of interest.

The critical parameter of interest. The red line shows the critical parameter of interest.
T2K & NOvA

Number of Events proportional to Oscillation Probability

T2K: L=295 km, E=0.62 GeV

\[ |U_{\mu 3}|^2 = 0.56 \]
\[ |U_{e 3}|^2 = 0.022 \]

\( \delta = 0, \pi/2, \pi, (-\pi) \)
\( \star \ 3\pi/2, (-\pi/2) \)

1 sigma: NO IO

NOvA: L=810 km, E=2.0 GeV

\[ |U_{\mu 3}|^2 = 0.61 \pm 0.03 \]
\[ |U_{e 3}|^2 = 0.022 \]

\( \delta = 0, \pi/2, \pi, (-\pi) \)
\( \star \ 3\pi/2, (-\pi/2) \)

\( \downarrow \) Appearance data
Summary:

- from Nu1998 to now, tremendous exp. progress on Neutrino SM: more at Nu2018

- LSND Sterile Nu’s neither confirmed or ruled out at acceptable CL: – ultra short baseline reactor exp.

- Great Theoretical progress on understand many aspects of Quantum Neutrino Physics: – Oscillations, Decoherence, Osc. Probabilities in Matter, Leptogenesis, ..... 

- Still searching for convincing model of Neutrino masses and mixings: with testable and confirmed predictions!