Surface roughness detection by spin Hall effect of light with sub-nanometer resolution

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Abstract. We report here a novel measurement method for detecting a rough surface with sub-nanometer accuracy by using the spin Hall effect of light (SHEL). Reflecting an inspection light at a sample surface, a slightly position shift, which is called "the SHEL," occurs at the reflection surface. The shift corresponds to polarization difference between an incident and a reflected lights. The difference depends on a conditions of a surface roughness of a sample surface or a thickness of a thin film. Because a rough surface within \( \lambda/10 \) can be approximated to thin film by the theory of the effective medium theory. We measured the SHEL of some Au thin films and surface roughness of a optical flat sample via the weak value amplification technique. Sub-nanometer resolution for the measurement of the film thickness is obtained and some experimental results are shown in this paper.

1. Introduction
Recently there has been considerable interest in non-contact measurement method for detecting surface roughness with sub-nanometer accuracy. Ellipsometry which is a method for analyzing polarization properties after reflection at a sample have attracted much attention for many years mainly because polarization properties strongly depend on a rough surface. Though many types of analyzing principle could measure optical properties, there are some problems about their accuracy because of optical setup with moving elements and/or imperfection.

To overcome the problem, we focused on the spin Hall effect of light (SHEL) and its related phenomena. The SHEL is tiny spatial shift of a reflected position about sub-100nm at an interface between different medias which arise from a conservation low of angular momentums of the light\[1, 2\]. A spin angular momentum which is one of the momentums has sensitivity for polarization difference between an incident and a reflected light. Therefore a surface roughness of a sample can be detected by measurement of the spatial shift of the reflected position. In this paper, we describe the measurement principle which uses the SHEL for a detection of a sample surface and show successful results for detecting a surface roughness with sub-nanometer resolution.

2. Principle of detection for rough surface by the SHEL
Figure 1 shows a principle of an evaluation method for a surface roughness by the SHEL. As shown in fig.1 (a), when a linear polarized light is irradiated onto a sample with a rough surface, a reflected light is divided into two beams such as right and left handed circular polarized light
by the SHEL with small distance $\delta$. Because there is a conservation law of angular momentum such as an effective spin-orbit coupling.

The distance $\delta$ is derived from the conservation law and written by

$$\delta = \frac{\langle z_c | \sigma_3 | z_c \rangle_r \cos \theta_r - \langle z_c | \sigma_3 | z_c \rangle_i \cos \theta_i}{k_{ci} \sin \theta_i},$$  \hspace{1cm} (1)

where $\theta_i$ and $\theta_r$ are angles of incident and reflected light, respectively. $k_{ci}$ is a wave number, $\sigma_3$ is the Pauli spin matrix, and $|z_c\rangle_i$ and $|z_c\rangle_r$ are Jones vectors of incident and reflected light, respectively. Though the distance $\delta$ is, generally, a small amount about sub 100nm in the case of using a glass substrate, an observation system can not detect the distance because of a diffraction limit.

To overcome the problem, we have used a known signal enhancement technique as the weak value amplification (WVA)\cite{4, 3}. By using the WVA, a weak expectation value is enhanced and it was written by

$$A_\delta \equiv \frac{\langle \Psi_f | A | \Psi_i \rangle}{\langle \Psi_f | \Psi_i \rangle}.$$ \hspace{1cm} (2)

Where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the initial and final state shown in fig.1 (a), respectively. When the initial and final states are orthogonal, the weak value is enhanced although $\langle \Psi_f | A | \Psi_i \rangle$ is too low. In this paper, the states was mainly consist of polarization states. Therefore a polarizer angle in front of a detector is set in a direction perpendicular to the incident polarization angle.

Figure 1 (b) shows an optical setup of detecting the distance $\delta$ of the SHEL by using the WVA. After the light passed through a half wave plate (HWP) for controlling an intensity, the light was focused on the sample by the lens $f_1$ via a polarizer at 90 degrees with respect to the incident plane. Then the reflected light was collimated by a lens $f_2$ via an analyzer at almost 0 degrees with respect to the incident plane. Finally, a displacement $\delta$ is detected by a image sensor. By using the experimental setup, an enhancement value $A_\delta$ can be written by,

$$A_\delta = \frac{4\pi (y_{f2}/L)^2}{f_2 \lambda \varepsilon}.$$ \hspace{1cm} (3)

Here $y_{f2}$ is a beam diameter at the detector, $f_2$ is a focusing length of the lens $f_2$, $\lambda$ is wavelength of the laser, $\varepsilon$ is an azimuthal angle of the analyzer, and $\delta(\theta_i)$ is a shift by the SHEL at an incident angle $\theta_i$.

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**Figure 1.** Principle of evaluation method of surface roughness by the spin Hall effect of light. (a) the spin Hall effect of light and (b) optical setup for detection of beam shift via the weak value amplification.
3. Numerical analysis and experimental results

Below Ra = 1/10λ the rough surface can be approximated to a thin film by considering of the effective medium theory. We calculated the SHEL shift $\varepsilon$ for Au thin film thicknesses on a SiO$_2$ substrate. Film thicknesses ranging from 0.10 to 1.00 nm, the SHEL of a He-Ne laser was calculated. The variation of the SHEL shift with an incident angle is shown in fig. 2. The incident angle first decreased the SHEL shift, reaching a peak value near the Brewster’s angle, and increased the shift with an opposite side of a peak, and then decreasing again. Since each peaks increased with decreasing the film thickness, the SHEL shifts have a sensitivity of a thin film thickness with sub-nanometer resolution.

Firstly, for confirmation of the SHEL, we have observed beam profiles depended on the angle of the analyzer $\varepsilon$ using a He-Ne laser and a sample of a SiO$_2$ substrate. According to the eq. (3), the shift value was enhanced to $(860.9)/(\varepsilon)$ times in the experimental setup. Figure 3 shows detected beam profiles by using a CCD camera with incident angle $\theta_i = 55$ degrees. As shown in fig. 3 (a), with setting the angle of the analyzer $\varepsilon = 0, 0.2$ and 0.4 degrees, the CCD camera could detect two divided beams. This two beams can be explained by taking into consideration the effect of the SHEL. Figure 3 (b) shows beam profiles of A-A and B-B lines in (a). We set a center of the two beams of the A-A line as a point of origin, and then a SHEL shift was measured from the center to a peak position.

![Figure 2](image-url)  
**Figure 2.** Numerical analysis of variation of beam shift with thin film thickness.

![Figure 3](image-url)  
**Figure 3.** Experimental results of detected beam profile. (a) detected beam profile on image sensor and (b) intensity profile and theoretical fits.

Figure 4 shows measurement results of Au deposition time dependence of the SHEL as a function of the azimuth angle of analyzer $\varepsilon$. There are sharp maximum and minimum values. This variation suggested that an azimuth angle for measurement with high accuracy was set within $\varepsilon = \pm 2$ degrees. Figure 5 shows measurement results of detection for thin film distribution with sub-nanometer order resolution. The measured samples were prepared by using Au deposition on a SiO$_2$ substrate. Deposition time could control a film thickness within sub-nanometer accuracy. In this experiment, we have prepared step distribution sample of the Au layer thickness varied from 0.50 to 4.00 nm. It is seen from fig. 5 that step like differences could been detected by using the SHEL shift. Finally, we prepared samples of an optical flat with a different surface roughness (front side: $S_a = 1.20$ nm, back side: $S_a = 1.56$ nm ). Figure 6 shows the variation of the incident angle with the SHEL shift. A qualitative agreement is found between theoretical values and measured values. An important finding here is that the proposed method can distinguish the difference of 0.36 nm in surface roughness.
4. Conclusions

We have proposed a novel method for detecting surface roughness by using the SHEL via the weak value amplification. After confirmation of the SHEL by numerical approach, the difference of thin film thicknesses could be detected by using the SHEL with sub-nanometer resolution. Finally, it was succeeded to detect a difference of the surface roughness of the optical flat with 0.36 nm.

References

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