ORIGINAL RESEARCH PAPER

Analytical solution of stochastic real-time dispatch incorporating wind power uncertainty characterized by Cauchy distribution

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Abstract
Real-time power dispatch can coordinate wind farms, automatic generation control units and non-automatic generation control units. In real-time power dispatch, the probable wind power forecast errors should be appropriately formulated to ensure system security with high probability and minimize operational cost. Previous studies and the authors’ onsite tests show that Cauchy distribution effectively fits the “leptokurtic” feature of small-timescale wind power forecast errors distributions. In this paper, a chance-constrained real-time dispatch model with the wind power forecast errors represented by multivariate Cauchy distribution is proposed. Since the Cauchy distribution is stable and has promising mathematical characteristics, the proposed chance-constrained real-time dispatch model can be analytically transformed to a convex optimization problem considering the dependence among wind farms’ outputs. Moreover, the proposed model incorporates an affine control strategy compatible with automatic generation control systems. This strategy makes the chance-constrained real-time dispatch adaptively take into account both the potential power ramping requirement and power variation on transmission lines caused by the generation adjustment to offset the wind power forecast errors in real-time power dispatch stage. Numerical test results show that the proposed method is reliable and effective. Meanwhile it is very efficient and suitable for real-time application.

1 | INTRODUCTION

1.1 Background and challenge

Wind power penetration has increased significantly in recent years [1]. Large-scale wind power integration can provide green energy and bring environmental benefits. However, because of the intermittency and randomness of wind power [2], operators need to consider the inherent challenges to power system in economic dispatch (ED). Real-time dispatch (RTD) can coordinate wind farms, automatic generation control (AGC) units and non-AGC units to practically maintain power balance in terms of the rapid fluctuation of wind power [3]. It is essentially formulated as a dynamic ED model in which only the solution of the first period is committed. To deal with the uncertainties of wind power, stochastic ED/RTD are conventionally used but with two crucial issues. The first is how to describe the WPFE accurately and dispatch-friendly. The second is how to incorporate uncertainties into operational constraints and objective function properly to formulate a tractable stochastic ED/RTD model. A detailed description of these two challenges is as follows.

a. On the one hand, an “accurate” WPFE model should have smaller fitting error so as to reduce the operational costs and increase the system reliability [4]. Meanwhile, due to similar or discrepant meteorological conditions, wind power of multiple wind farms exhibits correlation or complementary characteristics on different regions [5] which should be contained in an “accurate” WPFE model as well. On the other hand, to facilitate the solution of stochastic ED with large-scale wind power penetration, a “dispatch-friendly” WPFE model usually has these mathematical properties: (1) The linear combination of related random variables can be expressed by same
type distributions (this is the concept of “stable distribution” [6]) or simple distributions to facilitate the description of the random power on transmission lines; (2) The inverse cumulative distribution function (CDF) can be expressed analytically [7] in order to convert chance constraints into deterministic constraints. (3) The expected regulation cost in the form of integral can be given analytically. WPFE models with part of the above properties have shown advantages in ED problems [7–9]. But none of them have ability to access all these properties simultaneously.

b. For the process of modelling and problem solving, chance-constrained economic dispatch (CCED) [10] with adjustable confidence level [11] is a good choice for balancing the security and economy of the dispatch process. However, because of the fluctuation of wind power, the generation adjustment of regulation units makes the branch flow and power ramping uncertainty in the operational constraints and therefore the CCED becomes more complicated. Moreover, the tightest bottleneck in solving chance-constrained optimization problems is inefficiency, which prevents its application in real-time dispatch.

1.2 Literature review

1.2.1 Modelling of wind power forecast errors

WPFEs are traditionally assumed as random variables [1] that follow certain distribution. Some studies have shown that the feature of small-timescale WPFE distributions is “leptokurtic” [12], which means the property of both high kurtosis [1, 13, 14] and fat-tail [15]. “High kurtosis” indicates high prediction accuracy while “fat tails” provides the frequency of extreme events [12] and holds important information in reliability studies [16]. In paper [17], the author examined the errors from operational wind power forecasting systems, finding that WPFEs were poorly represented by normal distribution [18]. Some other existing models, such as Weibull [19] or beta distribution [13] were also not suited to fully describe the heavy-tailed character of WPFE data [16]. For power systems with multiple wind farms, comprehensive analysing the output correlation and dependence between various wind farms is necessary in dynamic ED [20, 21]. Neglecting the correlation can lead extra cost and increase the risk of transmission lines overloading [5].

In literature [22], Xie proposed a novel data-driven wind speed forecast framework by leveraging the spatial-temporal correlation among geographically dispersed wind farms and then wind power forecasts were converted from wind speed forecasts based on power curve. Other studies used copula function [20, 21, 23] to formulate the correlation or dependence of multi-wind farms generation. But scenario sampling brings heavy computational burden, meanwhile, it is hard to select a suitable or optimal copula function. Besides, the multivariate joint distribution function, such as multivariate Gaussian distribution [10], was also used to formulate the correlation.

As a kind of stable distribution, Cauchy distribution (CD) is good at modelling the high spike of the frequency histogram [14] with fat tails [12]. In previous study [1], the forecast error distribution was fitted using maximum-likelihood optimization, and it was found that the CD was better than the Gaussian, Beta and Weibull distributions in all cases. Moreover, the dependence of all wind farms’ output can be described by the scale matrix of multivariate CD.

1.2.2 Formulation and solution of stochastic ED

To capture the uncertainty and fluctuation of wind power, robust optimization (RO) and stochastic optimization (SO) models are commonly used for ED. RO is efficiently tractable [3] but the consideration of worst-case realizations makes optimization results conservative inevitably. SO with chance constraints allows to trade off security for economy with adjustable risk level to meet the different reliability requirements [10] and it has been widely used in unit commitment [24] and optimal power flow [25] problems. But solving the chance constrained programming is hard even if the problem is convex [26]. In [26], a scenario-based approach was developed to replace chance constraints by sampling the uncertainty parameters which were pre-processed in an offline stage to accelerate the real-time decision in online stage. Reference [27] formulated a stochastic security-constrained unit commitment problem with the uncertainties of wind and solar power modelled by discrete scenarios and a heuristic genetic algorithm was utilized to solve the problem. In [28], the authors improved the stochastic programming approach in the unit commitment problem to incorporate wind power scenarios by introducing a dynamic decision making approach. The expectation in objective function was approximately calculated by scenario sampling. Similarly in [29], the stochastic model predictive control (MPC) of energy storage system is performed by minimizing the expected cost in all possible scenarios. In paper [30, 31], a chance-constrained two stage stochastic program was solved by a combined sample average approximation (SAA) algorithm. Similarly, the SAA technique was also adopted to calculate the real-time power dispatch in [32]. It is found that all of these methods are based on scenario generation. If the sample size is large enough, feasibility in the chance-constrained sense can be guaranteed with high confidence [26, 28]. However, scenario-based method suffers heavy computational burden, which limits its application to real-time decision making.

Some authors focused on the elaborate selection of WPFE model in order to make the CCED problems solvable. In paper [8] and [33], the versatile distribution (VD) and truncated versatile distribution (TVD) were proposed to model the WPFE. Compared with the Gaussian and Beta distributions, the VD/TVD can fit the WPFE more accurately and their inverse CDFs have analytical mathematical expressions. VD/TVD-based chance constraints can be converted into deterministic constraints with quantile. However, it cannot provide a distribution for a linear combination of random variables represented by VD/TVD, so the transmission capacity constraints were ignored in there works. Recently, the Gaussian mixture model...
(GMM) was employed to model the correlated prediction errors of wind power in CCED [9] or ramping capacity allocation [11]. In these works, the CDF of a Gaussian distribution was approximately fitted by a piecewise fourth-order polynomial and then the chance constraints were converted into deterministic constraints. The drawbacks of these formulations include: (1) the existence of multiple solutions to the piecewise quartic equation means that an extra validation step is required to obtain a reasonable solution; and, (2) ignoring the impact of generation adjustment on unit ramping and branch flow reduces the security level of dispatching scheme.

1.3 Contribution

In this paper, we statistically analyse the small-timescale WPFE distributions of wind farms in Southwest China and the onsite tests justify that the CD outperforms the other distributions, such as Gaussian, Beta or Weibull distributions, especially in capturing the kurtosis and tail behaviour. Therefore, we apply the CD in characterizing the WPFEs in the CCRTD model with affine AGC control strategy (A-CCRTD). Due to the promising characteristics of CD, the A-CCRTD is analytically converted into a deterministic convex optimization problem and solved efficiently without any approximation. In more detail, the contributions of this paper include:

1. The WPFEs of multiple wind farms are formulated as a multivariate random variable obeys MCD, which reveals the “leptokurtic” feature of WPFEs over short timescales essentially. The dependence between multiple wind farms is described by the scale matrix of MCD. We are the first to apply the CD in characterizing the WPFEs in CCED.

2. A convex analytical A-CCRTD model is established based on the favourable mathematical properties of CD. The chance constraints and the expectation terms in the objective function are converted into closed forms without any approximation, which guarantees a global optimal solution. Meanwhile, the deterministic A-CCRTD model can be solved efficiently with off-the-shelf open-source solvers, which makes chance-constrained optimization problem practically tractable in real-time applications for power systems with high wind power penetration.

3. An affine control strategy for the AGC system is incorporated into the chance-constrained dispatch problem. Compared with the traditional stochastic ED/RTD, in which the reserve requirement and the power flow on transmissions are only affected by the fluctuation of wind power itself, the proposed model takes into account both the potential power ramping requirement (PPRR) and the power variation on transmission lines caused by the generation adjustment to offset the WPFEs in RTD stage. Hence sufficient regulation capacities of AGC units are reserved in RTD stage to ensure system reliability.

The remainder of this paper is organized as follows. In Section 3, the mathematical formulation of the A-CCRTD model is presented. Section 4 is solution procedure of A-CCRTD with Cauchy distribution. Section 5 provides case studies and Section 6 is conclusion.

2 MATHEMATICAL FORMULATION OF A-CCRTD

The formulation of A-CCRTD model is provided in this section. The declaration of the variables can be referred to the nomenclature. As stated in [34], the forecasted minutely load values are provided with good accuracy even for the worst cases with the state-of-the-art prediction technology. While, the short-term wind power prediction errors is much larger due to the large terrain roughness [35] and the unstable wind turbulence [36]. Therefore, only the uncertainties of wind power are considered in this paper. Because the feature of the “error distribution” in high prediction accuracy is “leptokurtic”, which is consistent with the characteristic of CD, the model is scalable to incorporate the uncertainties of the load demand as well.

2.1 Objective function

\[
F = \min \left\{ \sum_{t=1}^{T} \left[ \sum_{j=1}^{N} CG_{t}^{(j)} \left(P_{t}^{(j)} \right) + \sum_{j=1}^{J} CG_{t}^{(j)} \left(P_{t}^{(j)} \right) \right] + \sum_{j=1}^{J} E \left[ CU_{t}^{(j)} \left(W_{t}^{j} \right) \right] + \sum_{j=1}^{J} E \left[ CD_{t}^{(j)} \left(W_{t}^{j} \right) \right] \right\}
\]

where \( CG_{t}^{(j)}(\cdot) \) and \( CG_{t}^{(j)}(\cdot) \) are the fuel cost of conventional units and AGC units, respectively. \( CU_{t}^{(j)}(\cdot) \) and \( CD_{t}^{(j)}(\cdot) \) represent the upward and downward corrective control cost of AGC units, respectively; we can also regard these terms as the penalty cost of overestimation and underestimation of wind power output [37]. It is a remarkable fact that the division of non-AGC units and AGC units is consistent with industry practice [3]. In RTD stage, the non-AGC units follow their schedules between two adjacent time periods, while the outputs of AGC units are adjusted automatically to offset the power mismatch. The detailed formulations of the objective function are listed below.

1. Fuel cost

The fuel costs of conventional units and AGC units are expressed as quadratic functions of the power output:

\[
CG_{t}^{(j)} \left(P_{t}^{(j)} \right) = a_{j} \left(P_{t}^{(j)} \right)^{2} + b_{j} P_{t}^{(j)} + c_{j}
\]

2. Corrective control cost

The power mismatch between \( W_{t}^{j} \) and \( W_{t}^{j} \) should be balanced by the AGC units at any moment, which results in corrective
control costs. The expectation of corrective costs is proportional to the expected positive and negative wind power deviation from the arranged output, i.e.,

\[
E \left[ CU_j^{(i)} (W_t) \right] = \alpha u_j^{(i)} \int_0^{W_t} (W_t - \omega) \phi_j(\omega) d\omega,
\]

\[
E \left[ CD_j^{(i)} (W_t) \right] = \alpha d_j^{(i)} \int_{W_t}^{\infty} (\omega - W_t) \phi_j(\omega) d\omega,
\]

where \( \phi_j(\omega) \) is the PDF of the summation of all wind farms’ output in time period \( t \).

In reality, to offset the WPFEs in RTD stage, the power participation factors are usually assigned to regulation units proportional to their capacities [3] and the affine control strategy of AGC units is established:

\[
\tilde{p}_{ad}^{(j)} = p_{ad}^{(j)} - \gamma_j \cdot (W_t - \bar{W}_d)
\]

\[
\sum_{j=1}^{J} \gamma_j = 1 \quad (\gamma_j \geq 0)
\]

where \( \gamma_j \) is the participation factor of AGC unit \( j \).

In this paper, the affine corrective control strategy for the AGC system is incorporated into A-CCRTD. It is worth noting that this affine control strategy has already been used in the robust dispatch [3, 38], but the dependence among multiple wind farms cannot be incorporated with the wind power output in the form of intervals. To the best of our knowledge, we are the first to analytically calculate CCED with this strategy. This AGC control strategy is essentially a probability decision approach which connects the randomness of wind power and system power generation. A visual explanation is presented in Figure 1. Three AGC units share the power deviation caused by the WPFE whose CDF is the blue line. The area of different regions in different colour represents the number of unbalance power taken by corresponding units. The value of each participation factor is the individual area divided by the total area, which is constant with Equation (3). Each region represents a probability distribution denoting the randomness of the power generation of AGC units.

2.2 System constraints

For all:

\[
i \in \{1, 2, ..., n\}, j \in \{1, 2, ..., J\}, k \in \{1, 2, ..., K\},
\]

\[
l \in \{1, 2, ..., L\}, t \in \{1, 2, ..., T\}
\]

1. Deterministic constraints:

\[
\sum_{j=1}^{J} p_{ad}^{(j)} + \sum_{j=1}^{J} p_{sd}^{(j)} + \sum_{k=1}^{K} p_{wk}^{(k)} = \sum_{d=1}^{D} p_{ad}^{(d)} \quad (6)
\]

\[
p_{ad}^{(j)} \leq p_{ad}^{(j)} \leq p_{ad}^{(j)} \leq p_{ad}^{(j)} \leq p_{ad}^{(j)}, 0 \leq p_{wk}^{(k)} \leq \tilde{p}_{wk}^{(k)} \quad (7)
\]

\[
-D_l \cdot \Delta T \leq p_{sd}^{(j)} - p_{ad}^{(j-1)} \leq U_d \cdot \Delta T \quad (8)
\]

\[
-I_{adi} \leq \sum_{j=1}^{J} G_{ai} p_{ad}^{(j)} + \sum_{j=1}^{J} G_{ai} p_{sd}^{(j)} + \sum_{k=1}^{K} G_{ai} p_{wk}^{(k)} + \sum_{d=1}^{D} G_{ai} p_{ad}^{(d)} \leq I_{adi} \quad (9)
\]

where Equation (6) is power balance constraint; Equation (7) is power generation limit constraint means the scheduled power generation of non-AGC units, AGC units and wind farms has to be within their bounds. Equations (8), (9) are the ramp-rate constraints for non-AGC units and AGC units. Inequality Equation (10) is the transmission capacity constraint for the scheduled point.

2. Chance constraints:

\[
Pr \left\{ \tilde{p}_{ad}^{(j)} - \gamma_j \cdot (W_t - \bar{W}_d) \leq \tilde{p}_{ad}^{(j)} \right\} \geq 1 - \delta \quad (11)
\]

\[
Pr \left\{ p_{ad}^{(j)} - \gamma_j \cdot (W_t - \bar{W}_d) \leq p_{ad}^{(j)} \right\} \geq 1 - \delta \quad (12)
\]

\[
Pr \left\{ -D_l \cdot \Delta T \leq \tilde{p}_{ad}^{(j)} - p_{ad}^{(j-1)} \right\} \geq 1 - \beta \quad (13)
\]

\[
Pr \left\{ R_{ad} \leq \tilde{p}_{ad}^{(j)} - \gamma_j \cdot (W_t - \bar{W}_d) \right\} \geq 1 - \epsilon
\]

\[
Pr \left\{ R_{ad} \leq \tilde{p}_{ad}^{(j)} - p_{ad}^{(j-1)} \right\} \geq 1 - \epsilon
\]
where chance constraint Equation (11) indicates that the actual generation of AGC units after offsetting the wind power fluctuation cannot exceed the upper or lower limits at a predefined confidence level. Ramp-rate constraints Equation (12) limit the actual incremental output of AGC units in adjacent time periods. Obviously, the regulation power caused by wind power fluctuations competes for ramp capability in real-time operation, thus the potential power ramping requirements (PPRR) should be included in dispatch process, which is formulated by Equation (12). Equation (13) is minimum upward/downward reserve requirement constraints to ensure system security under contingency scenarios. Equation (14) is transmission capacity constraints, which show that the probability of transmission lines overloading is no more than η. Note that the real-time unbalanced power allocated to each AGC units contributes to active power on transmission lines, which is ignored in conventional CCED models.

3 | SOLUTION PROCEDURE

3.1 Mathematical properties of multivariate Cauchy distribution

If a p-dimensional random vector \( \mathbf{X} \) follows a multivariate CD with location vector \( \mathbf{\mu} \) and scale matrix \( \Sigma \), that is \( \mathbf{X} \sim \text{Cauchy}(\mathbf{\mu}, \Sigma) \), then the probability density function (PDF) is presented as [39]:

\[
 f_{\mathbf{X}}(\mathbf{x}; \mathbf{\mu}, \Sigma) = \frac{\Gamma\left(\frac{1+p}{2}\right)}{\Gamma\left(\frac{1}{2}\right) \pi^{\frac{p}{2}} |\Sigma|^\frac{1}{2} \left[1 + (\mathbf{x} - \mathbf{\mu})^T \Sigma^{-1} (\mathbf{x} - \mathbf{\mu})\right]^{\frac{1+p}{2}}} \]

(15)

where \( \mathbf{X}, \mathbf{\mu} \in \mathbb{R}^p \) and \( \Sigma \in \mathbb{R}^{p \times p} \) is a positive-definite matrix.

When \( p = 1 \), that is \( \mathbf{x} \sim \text{Cauchy}(\mu, \sigma^2) \), the PDF of one-dimensional CD is:

\[
 f_{\mathbf{x}}(x; \mu, \sigma^2) = \frac{1}{\pi} \frac{\sigma}{(x - \mu)^2 + \sigma^2}, \quad x \in \mathbb{R} 
\]

(16)

Some important properties beneficial to solve A-CCRTD are listed below:

1. Integral property:

\[
 I_{\mathbf{x}}(x) = \int (x \cdot f_{\mathbf{x}}) \, dx = \frac{\sigma}{2\pi} \ln\left(1 + \left(\frac{x - \mu}{\sigma}\right)^2\right) + \frac{\mu}{\pi} \arctan\left(\frac{x - \mu}{\sigma}\right) + \epsilon 
\]

(17)

2. Analytical expressions of CDF and inverse CDF:

\[
 CDF_{\mathbf{x}}(x) = \frac{1}{\pi} \arctan\left(\frac{x - \mu}{\sigma}\right) + \frac{1}{2} 
\]

(18)

\[
 CDF_{\mathbf{x}}^{-1}(F) = \mu + \sigma \tan\left[\pi \left(F - \frac{1}{2}\right)\right] 
\]

(19)

where \( F \) is the quantile.

3. Stable property:

“Stable” [6] means the linear transformation of \( \mathbf{X} \) in Equation (15) can be expressed as a new random variable comply with one-dimensional CD. For example, suppose that \( a \) is a p-dimensional vector, and then we have:

\[
 a^T \mathbf{X} \sim \text{Cauchy}(a^T \mathbf{\mu}, a^T \Sigma a) 
\]

(20)

4. Fitting and sampling of multivariate CD:

The multivariate CD parameters can be fitted to the data using “mscFit” function of the “fMultivar” package [40] with maximum likelihood estimation in the R statistical computing environment [41]. In this paper, the WPFEs for all wind farms are described by a multivariate CD. For convenience, we suppose that the random vector \( \mathbf{\bar{P}}_{wa} = (\mathbf{\bar{P}}^{(1)}_{wa}, \mathbf{\bar{P}}^{(2)}_{wa}, ..., \mathbf{\bar{P}}^{(K)}_{wa})^T \) represents the actual wind power with location vector \( \mathbf{\mu} = (\mu^{(1)}, \mu^{(2)}, ..., \mu^{(K)})^T \) and scale matrix \( \Sigma \). All these parameters can be obtained by software R with historical data.

In addition, by means of “rmve” function of the “LaplacesDemon” package [42] in R, the sampling of multivariate Cauchy distribution can be done to verify the risk level of chance constraints.
3.2 The calculation of corrective control cost

The random variable $\tilde{W}_j$ in Equation (3) is the summation of total actual wind power and can be denoted by $a_{\tilde{W}_j}$, where $a_{\tilde{W}_j}$ is a $K$-dimensional vector whose elements are all 1. According to the “stable property” of CD and Equation (20), we can obtain:

$$\tilde{W}_j \sim \text{Candby}(\mu_{\tilde{W}_j}, \Sigma_{\tilde{W}_j})$$  \hspace{1cm} (21)

where $\mu_{\tilde{W}_j} = a_{\tilde{W}_j}^T \mu_j$ and $\Sigma_{\tilde{W}_j} = a_{\tilde{W}_j}^T \Sigma_j a_{\tilde{W}_j}$.

Thus, the expectation of corrective costs in time period $t$ is formulated as follows:

$$E \left[ \mathbf{U}^{(j)}(\tilde{W}_j) \right] + E \left[ \mathbf{W}^{(j)}(\tilde{W}_j) \right]$$

$$= \alpha u^{(j)}_j \gamma_j \int_{\tilde{W}_j} \mathbf{W}^{(j)}(\tilde{W}_j) \Phi_j(\tilde{W}_j) \rho_j d\tilde{W}_j$$

$$+ \alpha a_{\tilde{W}_j}^T \gamma_j \int_{\tilde{W}_j} \mathbf{W}^{(j)}(\tilde{W}_j) \Phi_j(\tilde{W}_j) \rho_j d\tilde{W}_j$$

$$= \alpha u^{(j)}_j \gamma_j \int_{\tilde{W}_j} \mathbf{W}^{(j)}(\tilde{W}_j) \Phi_j(\tilde{W}_j) \rho_j d\tilde{W}_j$$

$$+ \alpha a_{\tilde{W}_j}^T \gamma_j \int_{\tilde{W}_j} \mathbf{W}^{(j)}(\tilde{W}_j) \Phi_j(\tilde{W}_j) \rho_j d\tilde{W}_j$$

Substitute Equations (17) and (18) into Equation (22), then

$$E \left[ \mathbf{U}^{(j)}(\tilde{W}_j) \right] + E \left[ \mathbf{W}^{(j)}(\tilde{W}_j) \right]$$

$$= A + B \cdot W_j - C \cdot \sqrt{\Sigma_{\tilde{W}_j}} \cdot \ln \left( 1 + \left( \frac{W_j - \mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right)^2 \right)$$

$$+ C \cdot \left( W_j - \mu_{\tilde{W}_j} \right) \arctan \left( \frac{W_j - \mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right)$$

where $A$, $B$ and $C$ are constants whose expressions are expressed as:

$$A = \frac{\gamma_j \cdot \sqrt{\Sigma_{\tilde{W}_j}}}{2\pi} \left( \alpha u^{(j)}_j \ln \left( 1 + \left( \frac{\mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right)^2 \right) \right)$$

$$+ \alpha a_{\tilde{W}_j}^T \gamma_j \ln \left( 1 + \left( \frac{W_j - \mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right)^2 \right)$$

$$+ \frac{\gamma_j \cdot \mu_{\tilde{W}_j}}{\pi} \left( \alpha u^{(j)}_j \arctan \left( \frac{-\mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right) + \alpha a_{\tilde{W}_j}^T \arctan \left( \frac{W_j - \mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right) \right)$$

$$B = \frac{\gamma_j}{\pi} \left( \alpha a_{\tilde{W}_j}^T \arctan \left( \frac{-\mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right) + \alpha a_{\tilde{W}_j}^T \arctan \left( \frac{W_j - \mu_{\tilde{W}_j}}{\sqrt{\Sigma_{\tilde{W}_j}}} \right) \right)$$

$$C = \frac{\alpha u^{(j)}_j \gamma_j}{\pi} + \frac{\alpha a_{\tilde{W}_j}^T \gamma_j}{\pi}$$  \hspace{1cm} (26)

The detailed derivation of Equation (23) is given in Appendix A.

It can be proved that the objective function is convex and a relevant discussion was provided in our previous work [37].

3.3 The transformation of chance constraints

3.3.1 Compact form and solution for chance constraints

Suppose that $A^{(\hat{\theta})}, B^{(\hat{\theta})} \in \mathbb{R}^s$, $D^{(\hat{\theta})} \in \mathbb{R}$, vector $\mathbf{u} \in \mathbb{R}^s$ represents decision variables and random vector $\hat{y} \in \mathbb{R}^s$, then chance constraints in A-CCRTD can be expressed in compact forms with Equations (27) and (28):

$$\Pr \left[ (A^{(\hat{\theta})})^T \mathbf{u} + (B^{(\hat{\theta})})^T \hat{y} \leq D^{(\hat{\theta})} \right] \geq 1 - \xi^{(\hat{\theta})}$$  \hspace{1cm} (27)

$$\Pr \left[ (A^{(\hat{\theta})})^T \mathbf{u} + (B^{(\hat{\theta})})^T \hat{y} \geq D^{(\hat{\theta})} \right] \geq 1 - \xi^{(\hat{\theta})}$$  \hspace{1cm} (28)

With the notation of quantile, chance constraints can be transformed to equivalent linear constraints. If the inverse CDF of random variable $(B^{(\hat{\theta})})^T \hat{y}$ is denoted as $CDF^{-1}_{(B^{(\hat{\theta})})T \hat{y}}(\xi^{(\hat{\theta})})$, then constraints Equations (27) and (28) can be rewritten in following forms:

$$D^{(\hat{\theta})} - (A^{(\hat{\theta})})^T \mathbf{u} \geq CDF^{-1}_{(B^{(\hat{\theta})})T \hat{y}}(\xi^{(\hat{\theta})})$$  \hspace{1cm} (29)

$$D^{(\hat{\theta})} - (A^{(\hat{\theta})})^T \mathbf{u} \leq CDF^{-1}_{(B^{(\hat{\theta})})T \hat{y}}(\xi^{(\hat{\theta})})$$  \hspace{1cm} (30)

Suppose random vector $\hat{y} \sim \text{Candby}(\mu_{\hat{y}}, \Sigma_{\hat{y}})$, according to Equations (19) and (20), Equations (29) and (30) are equivalent to Equations (31) and (32):

$$D^{(\hat{\theta})} - (A^{(\hat{\theta})})^T \mathbf{u} \geq (B^{(\hat{\theta})})^T \mu_{\hat{y}}$$

$$+ \sqrt{(B^{(\hat{\theta})})^T \Sigma_{\hat{y}} B^{(\hat{\theta})}} \tan \left[ \pi \left( 1 - \xi^{(\hat{\theta})} - \frac{1}{2} \right) \right]$$  \hspace{1cm} (31)

$$D^{(\hat{\theta})} - (A^{(\hat{\theta})})^T \mathbf{u} \leq (B^{(\hat{\theta})})^T \mu_{\hat{y}}$$

$$+ \sqrt{(B^{(\hat{\theta})})^T \Sigma_{\hat{y}} B^{(\hat{\theta})}} \tan \left[ \pi \left( \xi^{(\hat{\theta})} - \frac{1}{2} \right) \right]$$  \hspace{1cm} (32)

3.3.2 The transformation of chance constraints in A-CCRTD

Similar to Equations (31) and (32), chance constraints Equations (11), (12), (13) and (14) in A-CCRTD can be converted into linear constraints Equations (33), (34), (35) and (36).
respectively with WPFE model given in Section 4.1.

\[
\gamma_j W^o_t + P^{\mu(j)}_{a,t} - P^{\nu(j)}_{a,t} \leq \gamma_j \cdot \text{CDF}^{-1}(\delta)
\]

\[
\gamma_j \cdot \text{CDF}^{-1}(1 - \delta) \leq \gamma_j W^o_t + P^{\mu(j)}_{a,t} - P^{\nu(j)}_{a,t}
\]

\[
\gamma_j \cdot \text{CDF}^{-1}(1 - \beta) \leq P^{\mu(j)}_{a,t} - P^{\nu(j)}_{a,t} + \gamma_j (W^o_t - W^o_{t-1})
\]

\[
- D \Delta T - P^{\mu(j)}_{a,t} - P^{\nu(j)}_{a,t} + \gamma_j (W^o_t - W^o_{t-1}) - U \Delta T
\]

\[
\leq \gamma_j \cdot \text{CDF}^{-1}(1 - \beta)
\]

\[
W^o_t + R^t + \sum_{i=1}^{J} \left( P^{\mu(j)}_{a,t} - P^{\nu(j)}_{a,t} \right) \leq \text{CDF}^{-1}(\varepsilon)
\]

\[
\text{CDF}^{-1}(1 - \varepsilon) \leq W^o_t - R^t + \sum_{i=1}^{J} \left( P^{\mu(j)}_{a,t} - P^{\nu(j)}_{a,t} \right)
\]

where random variables \( \bar{W}^o_t, \bar{W}^o_{t-1} = \bar{W}^o_t - \bar{W}^o_{t-1} \), and \( a_i \bar{P}_{a,t} \) all obey CD:

\[
\bar{W}^o_t \sim \text{Cauchy} \left( a_i \bar{P}_{a,t}, a_i \bar{P}_{a,t}^T \Sigma_{a_i} \bar{W}^o_t \right)
\]  

\[
\bar{W}^o_{t-1} \sim \text{Cauchy} \left( \mu_{\bar{W}^o_t}, \Sigma_{\bar{W}^o_t} \right)
\]  

\[
\bar{W}^o_{t-1} = \text{Cauchy} \left( \mu_{\bar{W}^o_t}, \mu_{\bar{W}^o_t} + \Sigma_{\bar{W}^o_t} \right)
\]

\[
a_i \bar{P}_{a,t} \sim \text{Cauchy} \left( a_i \mu_{\bar{P}_{a,t}}, a_i \mu_{\bar{P}_{a,t}}^T \Sigma_{a_i} a_i \right)
\]

and their inverse CDF are as follows:

\[
\text{CDF}^{-1}(\bar{W}^o_t)(F) = \mu_{\bar{W}^o_t} + \sqrt{\Sigma_{\bar{W}^o_t}} \tan \left( \pi \left( F - \frac{1}{2} \right) \right)
\]

\[
\text{CDF}^{-1}(\bar{W}^o_{t-1})(F) = \mu_{\bar{W}^o_{t-1}} + \sqrt{\Sigma_{\bar{W}^o_{t-1}}} \tan \left( \pi \left( F - \frac{1}{2} \right) \right)
\]

\[
\text{CDF}^{-1}(a_i \bar{P}_{a,t})(F) = \mu_{a_i \bar{P}_{a,t}} + \sqrt{\Sigma_{a_i \bar{P}_{a,t}}} \tan \left( \pi \left( F - \frac{1}{2} \right) \right)
\]

where \( \alpha \) is a K-dimensional coefficient vector whose \( k \)th element is:

\[
\alpha_{i,k} = G_{i,k} - \left( \sum_{j=1}^{J} G_{i,j} Y_j \right)
\]
the first stage, the base point (scheduled point) of non-AGC units, AGC units and wind farms are formulated with the goal of minimizing the total cost. In the second stage, the power balance and other security constraints should be satisfied with the pre-defined risk level, which is called “risk check” in Figure 2. This two-stage dynamic chance-constrained optimization problem is hard to solve and the inefficiency of sampling or iterative methods make them incapable in real-time application. So the affine regulation strategy is introduced here and the original problem is equivalent to a single-stage linear-constrained convex programming which can be solved efficiently. Finally, the generation schedule/base points are issued to the non-AGC units, AGC units and wind farms, while the regulation strategy can provide a reference for AGC units when the wind power deviates from the base points.

4 | NUMERICAL TESTS

In this section, numerical tests were conducted to verify the effectiveness of the proposed method. First, the accuracy of CD in WPFE fitting was illustrated using real data of twenty wind farms in Southwest China. Then, the dispatch results of A-CCRTD and the merits of the proposed model were clarified on the modified IEEE 24-bus system. Meanwhile, the effects caused by the dependence between multiple wind farms in RTD was discussed. Finally, we verified the efficiency of A-CCRTD using the modified IEEE 118-bus system. The RTD performs generation scheduling in the upcoming 1 h with a time resolution of 5 min. Each real-time dispatch schedule was composed of 12 periods, and only the solution of the first period was executed. In each calculation, the variability and uncertainty of wind power in the next hour is considered and a multivariate CD which utilizes the up-to-date forecast information is employed to represent the statistic characteristic of WPFE. This procedure is consistent with the stochastic MPC approach proposed in [19] essentially. The proposed model was solved using the IPOPT solver [43] which is an open-source software package for solving large-scale nonlinear optimization problems with an interior point line search filter method. Because of the convexity of A-CCRTD, a global optimal solution can be guaranteed by IPOPT. All simulations were implemented using Matlab R2015a on a laptop with an Intel Core i7 1.99 GHz processor and 8 GB of RAM.

Parameters of the modified IEEE 24-bus system are described below. The load profile of system is shown in the left of Figure 3, where the valley-load periods are 01:30 A.M.–07:30 A.M. and the peak-load periods are 03:30P.M.–09:30 P.M. The predicted profile of the total wind power output is shown in the right of Figure 3, and the output feature obeys the rule that wind resources are greater during the night time. Four wind farms are connected at buses 7, 14, 16, and 21, which are denoted by #1, #2, #3, and #4, respectively [3]. The capacity of the four wind farms is set to 240, 300, 80, and 180 MW, respectively. AGC units are connected at buses 5–8, 23, and 31–33, and the participation factor of each AGC unit is proportional to its capacity. If not specially specified, the price of upward AGC regulation capacity is $12/MWh, and that of downward AGC regulation capacity is $24/MWh. The normalized ramp rates of all units are defined with the ratio of the ramping capacities in MW/ΔT to their maximum capacities. For non-AGC units and AGC units, the normalized ramp rates are assumed to be 0.05 and 0.1 respectively. In addition, all confidence levels in this simulation are set to be 0.98. Details of the configuration and parameters of the modified IEEE 24-bus system and the modified IEEE 118-bus system can be accessed in [44].

4.1 | Comparison of fitting accuracy of WPFE using different distributions

To illustrate the high accuracy of CD in WPFE fitting, the historical onsite data of twenty wind farms in several months were analysed statistically. All actual and ultra-short-term forecasting data used here were retrieved from the electric power control centre in a province in Southwest China. We firstly normalized all the forecast wind power and the corresponding actual wind power within [0, 1] [33]. The normalized WPFEs are the difference between the actual values and the predicted values. With the state-of-art prediction techniques, the ultra-short-term
WPFEs become smaller and smaller but the values of WPFEs are still in a large range, as shown in Figures 4 and 5. On the one hand, in Figure 4, the average predicted curve for all tested wind farms in all sampling days is quite close to the average actual curve, meanwhile, Figure 5 shows that a sufficiently narrow predicted interval contains more than half of the actual output scenarios. This feature is consistent with the “high kurtosis” characteristic of ultra-short-term WPFEs. On the other hand, in more than 20% actual scenarios, the WPFEs are still large enough which can be seen from Figure 5. This feature is consistent with the “fat tails” characteristic of ultra-short-term WPFEs. Therefore, the Cauchy distribution has advantage in fitting ultra-short-term WPFEs inevitably.

Referring to [31] and [33], two typical data sets are selected to fit the WPFEs with conditional distributions in different predicted values. In Data 1, the normalized value of predicted wind power is smaller while the Data 2 is large. All the conditional distributions corresponding to different predicted values can be found in reference [44]. From Figures 6 and 7, it can be concluded that the CD remarkably outperforms the other distributions, especially in capturing the kurtosis and tail behaviour whether the predicted value is larger or smaller. Gaussian, Beta and Weibull distributions obviously underestimates the probability in the middle and overestimates the probability in the head/tail region. As far as the RMES listed in Table 1, the CD is also much better than other distributions in accuracy.

In addition to the one-dimensional marginal distribution fitting of WPFEs, a visual two-dimensional fitting of WPFEs is shown in Figures 8 and 9, from which we can conclude that the fitting accuracy of the bivariate Cauchy distribution is significantly better than the bivariate Gaussian distribution especially in the middle and tail regions, which is similar to the one-dimensional situation.

| Data Set | Cauchy | Gaussian | Beta | Weibull |
|----------|--------|----------|------|---------|
| Data 1   | 0.3221 | 2.1144   | 2.2739 | 2.5273  |
| Data 2   | 0.3220 | 0.6365   | 0.7021 | 0.8695  |
4.2 Analysis of dispatch results

In this section, the dispatch results of A-CCRTD are demonstrated. Assuming that at 08:55 P.M., the predicted wind power distributions and load curve are built in electric power control centre, and then the RTD is performed for 09:00 P.M.-10:00 P.M. by the proposed approach. The calculation results of wind farms and AGC units are given in Figure 10 and Table 2 respectively, in which, the first point is regarded as the generation base point.

Figure 10 shows the scheduled points and the predicted median points (the predicted median points are the scale parameters \( \mu \) and have the highest probability) of all wind farms in all 12 periods. It can be found that the scheduled points fluctuate around the predicted median points. However, in most of the cases, the scheduled points are larger than the predicted median points. This is because not only the system loads are heavy in 09:00 P.M.-10:00 P.M., but also the increased marginal cost of thermal units and the relatively small upward reserve cost encourage the utilization of wind power in scheduling.

Table 2 is the generation base points of AGC units in all 12 periods while only the first point is executed. The spare upward and downward capacities of AGC units are prepared to balance the deviation of wind power or loads. However, it does not mean that all the fluctuation of wind power can be offset and the power balance constraints might be violated in extreme scenarios unless the confidence level is set to be 100%. Hence, a tradeoff between economics and security can be achieved by the selection of risk level properly.

4.3 Comparison of A-CCRTD with conventional CCED

In this subsection, CCED models without considering PPRR and affine control strategy for AGC units in [8] and [9] were compared with the proposed A-CCRTD model. All simulations were for 09:00 P.M.-10:00 P.M. and the cost was the sum of 12 dispatching periods. Monte Carlo simulations (MCS) with 10,000 scenarios were conducted to compare the economic and security performance of A-CCRTD and others.

4.3.1 The effect of PPRR on ramping constraints

In this simulation, the normalized ramp rates of all AGC units varied uniformly from 0.04 to 0.1. For the ease of comparison, security index for ramping resources is defined as:

\[
I_r = \frac{N_r}{N_M}
\]

where \( N_r \) is the average number of scenarios with sufficient ramping resources and \( N_M \) is the total number of scenarios in MCS. Accordingly, a larger value of \( I_r \) indicates a higher security level. Figure 11(a) is the predicted profile of the total wind power output from 09:00 P.M. to 10:00 P.M. The total costs in each simulation are presented in Figure 11(b), which shows that when the ramping rates of AGC units are low, taking into account the impact of PPRR would increase the scheduling cost. The uneconomical scheduling results are compelling choice to
| Number of AGC unit | #1   | #2   | #3   | #4   | #5   | #6   | #7   | #8   |
|-------------------|------|------|------|------|------|------|------|------|
| Participation factor | 0.0160 | 0.0160 | 0.0607 | 0.0607 | 0.3195 | 0.1238 | 0.1238 | 0.2796 |
| Upper limitation (Mw) | 20 | 20 | 76 | 76 | 400 | 155 | 155 | 350 |
| Lower limitation (Mw) | 16 | 16 | 15.2 | 15.2 | 54.3 | 54.3 | 54.3 | 140 |
| Power output in different time periods (Mw) | | | | | | | | |
| 9:00 | 16.05 | 16.05 | 65.53 | 65.53 | 384.90 | 148.28 | 148.28 | 307.72 |
| 9:05 | 16.05 | 16.05 | 65.63 | 65.63 | 385.44 | 148.52 | 148.52 | 308.13 |
| 9:10 | 16.05 | 16.05 | 64.76 | 64.76 | 380.88 | 146.48 | 146.48 | 304.70 |
| 9:15 | 16.05 | 16.05 | 66.31 | 66.31 | 389.01 | 150.13 | 150.13 | 310.81 |
| 9:20 | 16.05 | 16.05 | 63.81 | 63.81 | 375.86 | 144.25 | 144.25 | 300.87 |
| 9:25 | 16.05 | 16.05 | 64.53 | 64.53 | 379.63 | 145.92 | 145.92 | 303.75 |
| 9:30 | 16.05 | 16.05 | 65.26 | 65.26 | 383.48 | 147.64 | 147.64 | 306.66 |
| 9:35 | 16.05 | 16.05 | 65.77 | 65.77 | 396.16 | 148.85 | 148.85 | 308.66 |
| 9:40 | 16.05 | 16.05 | 54.99 | 54.99 | 382.09 | 147.02 | 147.02 | 305.61 |
| 9:45 | 16.05 | 16.05 | 65.23 | 65.23 | 383.35 | 147.58 | 147.58 | 306.56 |
| 9:50 | 16.05 | 16.05 | 64.02 | 64.02 | 376.98 | 144.74 | 144.74 | 301.73 |
| 9:55 | 16.05 | 16.05 | 65.29 | 65.29 | 383.65 | 147.72 | 147.72 | 306.79 |

4.3.2 The effect of affine control strategy on transmission capacity constraints

In this simulation, we adjusted the transmission capacity of line #11 from 155 to 170 MW to illustrate the effect of affine control strategy on transmission capacity constraints. Figure 12 and 13 are the Monte Carlo simulation results for AGC units in two different cases: (1) from time period 3 to time period 4; (2) from time period 10 to time period 11. Because of the rapid fluctuation of wind power in these two cases, which can be seen from Figure 11(a), the security level of unit ramping without PPRR cannot reach the required level in Figures 12 and 13 due to the exhausted ramping resources when the ramping rates of AGC units are low. In addition, from Figures 12 and 13, we can also conclude that only when there are abundant ramping resources in the system, the impact of PPRR in ED can be ignored.
trol strategy on transmission capacity constraints. Similarly, we define security index for transmission capacity as:

$$\Pi = \frac{N_f}{N_M}$$  \hspace{1cm} (45)

where $N_f$ is the average number of scenarios without transmission congestions for line #11 during all 12 dispatching periods. From Figure 14, it can be concluded that although the adoption of affine control strategy increases the operational cost, it guarantees a sufficient transmission capacity for security. This is because the redistribution of real-time power mismatch may cause transmission line congestion if we do not consider the control strategy of AGC in advance, i.e. it is necessary to incorporate regulation strategy of AGC units in schedule stage to prevent network congestions.

4.4 The effect of dependence between multi-wind farms

This experiment was carried out to test the influence of dependence between multiple wind farms on system performance. We used two different cases with and without considering the dependence between four wind farms for comparison: Case I: the location vector and scale matrix are consistent with the parameters mentioned before; Case II: the PDF of all wind farms are determined only by the marginal distribution respectively in Case I, that is to say the outputs of each wind farm are independent random variables. A-CCRTD with 12 periods was run in each cases. Based on the obtained schedule, 10,000 random wind power scenarios were produced using the distribution parameters in Case I to illustrate the effect of dependence in terms of cost and risk level. From Table 3, we can conclude that the consideration of dependence in ED reduces potential risk level at the expense of higher costs. This is because the integration of multi-wind farms amplifies the WPFE in our simulation.

| TABLE 3 Effect of correlation on economy and operational risk |
|-----------------|-----------------|-----------------|
| Case            | Case I(Dependent) | Case II(Independent) |
| Cost ($)        | 50,736.09        | 50,387.62        |
| Maximum risk level of reserve constraints | 1.57% | 2.38% (violate the predefined level) |
| Maximum risk level of unit ramping constraints | 0.83% | 0.78% |
| Maximum risk level on transmission line constraints | 0.73% | 0.64% |

4.5 The efficiency of A-CCRTD

The model size and computation times of A-CCRTD for both the IEEE 24-bus system and the IEEE 118-bus system are listed in Table 4. It is noteworthy that since the inverse CDF of CD is analytical, we can directly obtain the quantiles of chance constraints. In spite of 1225 variables and 6275 constraints involving in the A-CCRTD for the IEEE 118-bus system, it still can be solved in 7.23 s. Therefore, this solution is sufficiently efficient.
efficient for real-time application in large-scale power systems with high wind power penetration.

5 | CONCLUSION

This paper proposes an A-CCRTD approach coordinating wind farms, non-AGC units, and AGC units. Two important features distinguish our model from the conventional CCED model. On the one hand, based on the accurate description of WPFE by Cauchy distribution and its promising mathematical properties, the A-CCRTD model is transformed equivalently to a convex optimization problem which is solved efficiently without any approximation. On the other hand, by incorporating affine control strategies of AGC units, our model takes into account both the PPRR and the power variation on transmission lines caused by the allocation of real-time power mismatch between AGC units. Numerical tests demonstrate the superiority and rationality of the proposed approach compared with existing CCED models. Moreover, the importance of the dependence between multi-wind farms in real-time dispatch is also explored with Monte Carlo simulations. With practically acceptable computation effort even by general optimization solvers, the proposed approach makes the chance-constrained RTD practically tractable in real-time applications of large-scale power systems with high wind power penetration. Future work of this paper is to handle the temporal correlation of wind farms with Cauchy distribution and other stochastic process theories in chance constrained economic dispatch problems.

ACKNOWLEDGEMENTS

This work was supported in part by the National Key R&D Program of China (2018YFB0904200) and in part by and the Complement S&T Program (Technology and application of wind power/photovoltaic power prediction for promoting renewable energy consumption) of Inner Mongolia Power (Group) Co, Ltd.

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| $t$    | Indices for time periods. |
| $i$    | Indices for conventional units (non-AGC units). |
| $j$    | Indices for AGC units. |
| $k$    | Indices for wind farms. |
| $d$    | Indices for loads. |
| $l$    | Indices for transmission lines. |
| $x$    | Random vector |
| $\mu, \Sigma$ | Location vector and scale matrix of multivariate Cauchy Distribution |
| $\mu, \sigma^2$ | Location and scale parameters of one-dimensional Cauchy Distribution |
| $T$    | Number of time periods. |
| $\Delta T$ | Length of each time period in minutes. |
| $N$    | Number of conventional units. |
| $J$    | Number of AGC units. |
| $K$    | Number of wind farms. |

| Symbol | Description |
|--------|-------------|
| $D$    | Number of loads / nodes. |
| $L$    | Number of transmission lines. |
| $p_{w,t}^{(k)}$ | The maximal power output for wind farm $k$ during period $t$ (MW). |
| $\hat{p}_t$ | Upper bound of total available wind power during period $t$ (MW). |
| $\tilde{p}_t^{(i)}/\bar{p}_t^{(i)}$ | Upper / lower limitation of active power output of non-AGC unit $i$ during period $t$ (MW). |
| $\bar{p}_t^{(i)}/\tilde{p}_t^{(i)}$ | Upper / lower limitation of active power output of AGC unit $j$ during period $t$ (MW). |
| $\gamma_j$ | Participation factor of AGC unit $j$. |
| $E(\cdot)$ | Expectation of random variables. |
| $a_i/b_i/c_i$ | Cost coefficient of conventional unit $i$. |
| $a_{j,t}/b_{j,t}/c_{j,t}$ | Cost coefficient of AGC unit $j$. |
| $\varphi_j(\cdot)$ | Probability density function of random variable during period $t$. |
| $\alpha_{d,j}^{(i)}/\alpha_d^{(i)}$ | Coefficient of the upward/downward corrective control cost of AGC unit $j$ during period $t$ ($/MW$). |
| $f_{d,t}$ | Load demands of node $d$ in period $t$ (MW). |
| $U_{d,j}^{(i)}/D_{d,j}^{(i)}$ | Upward/downward ramping rate of non-AGC unit $i$ during period $t$ (MW/min). |
| $D_{d,j}^{(i)}$ | Upward/downward ramping rate of AGC unit $j$ during period $t$ (MW/min). |
| $R_t^+/R_t^-$ | Upward/downward reserve requirement during period $t$ (MW). |
| $G_{t,i}/G_{t,j}/G_{t,d}$ | Power transfer distribution factors of line $l$ with respect to $\alpha_{d,j}^{(i)}, p_{d,t}^{(k)}, p_{w,t}^{(k)}, p_{d,t}^{(d)}$. |
| $L_{d,t}$ | Power flow limits for transmission line $l$ during period $t$ (MW). |
| $\delta,\beta,\varepsilon,\eta$ | Pre-defined allowed probabilities of violation |
| $p_{w,t}^{(k)}$ | Arranged power output of wind farm $k$ during period $t$ (MW). |
| $\hat{p}_t$ | The summation of the total arranged wind power during period $t$ (MW). |
| $p_{w,t}^{(i)}$ | Scheduled base-point of AGC unit $j$ during period $t$ (MW). |
| $p_{w,t}^{(i)}$ | Scheduled active power output of non-AGC unit $i$ during period $t$ (MW). |
| $p_{w,t}^{(k)}$ | Actual wind power output of wind farm $k$ during period $t$ (MW). |
| $\hat{p}_t$ | The summation of total actual wind power output during period $t$ (MW). |
| $p_{d,t}^{(i)}$ | Actual power output of AGC unit $j$ during period $t$ (MW). |
| $A$ | Indices |
| $A$ | A-CCRTD Chance-constrained real-time dispatch with affine AGC control strategy |
| $A$ | AGC Automatic generation control |
| $B$ | Parameters and functions |
| $C$ | Deterministic variables |
| $C$ | CCED chance-constrained economic dispatch |
| $C$ | CCRTD chance-constrained real-time dispatch |
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For convenience, let $A$, $B$ and $C$ be constants whose expressions are:

$$A = \frac{\gamma_j \cdot \sqrt{\Sigma W_j}}{2\pi} \left( \alpha_n^{(j)} \ln \left( 1 + \left( \frac{\mu W_j}{\sqrt{\Sigma W_j}} \right)^2 \right) + \alpha_d^{(j)} \ln \left( 1 + \left( \frac{\mu W_j}{\sqrt{\Sigma W_j}} \right)^2 \right) \right)$$

$$B = -\frac{\gamma_j}{\pi} \left( \alpha_n^{(j)} \arctan \frac{\mu W_j}{\sqrt{\Sigma W_j}} + \alpha_d^{(j)} \arctan \frac{\mu W_j}{\sqrt{\Sigma W_j}} \right)$$

$$C = \frac{\alpha_n^{(j)} \gamma_j}{\pi} + \frac{\alpha_d^{(j)} \gamma_j}{\pi}$$

Finally, the expectation of corrective control costs in time period $t$ is expressed as Equation (23).

The detailed transformation process of transmission capacity constraints is shown below.

Substitute Equation (4) into constraint Equation (14) and we can obtain:
Because $\bar{W}_j$ is the summation of total actual wind power, that is:

$$\bar{W}_j = \sum_{k=1}^{K} \gamma_{u,j}^{(k)}$$  \hspace{1cm} (48)$$

Then the random variables in constraints (47) can be separated from the optimization variables:

$$\begin{align*}
\Pr & \left\{ \sum_{i=1}^{N} G_{i,j} p_{a,i}^{(j)} + \sum_{j=1}^{J} G_{i,j} \left( p_{a,j}^{(j)} - \gamma_j \cdot (\bar{W}_j - W_i) \right) \right. \\
& + \sum_{k=1}^{K} G_{i,k} \gamma_{u,j}^{(k)} + \sum_{d=1}^{D} G_{i,d} p_{a,d}^{(d)} \leq L_{j,t} \} \geq 1 - \eta \\
\left. \Pr & \left\{ \sum_{i=1}^{N} G_{i,j} p_{a,i}^{(j)} + \sum_{j=1}^{J} G_{i,j} \left( p_{a,j}^{(j)} - \gamma_j \cdot (\bar{W}_j - W_i) \right) \right. \\
& + \sum_{k=1}^{K} G_{i,k} \gamma_{u,j}^{(k)} + \sum_{d=1}^{D} G_{i,d} p_{a,d}^{(d)} \geq -L_{j,t} \} \geq 1 - \eta \\
\end{align*}$$  \hspace{1cm} (47)$$

If we let $\alpha_j$ be a K-dimensional coefficient vector whose $k$th element is:

$$\alpha_{j,k} = G_{j,k} - \left( \sum_{j=1}^{J} G_{i,j} \gamma_j \right)$$  \hspace{1cm} (50)$$

thus the transmission capacity constraints with affine control strategy of AGC units are rewritten similar to the compact forms Equations (27) or (28) equivalently, which can be transformed into deterministic linear constraints like Equation (36).