Abstract

We propose a simple texture of the neutrino mass matrix with one sterile neutrino along with the three standard ones. It gives maximal mixing angles for $\nu_e \to \nu_S$ and $\nu_\mu \to \nu_\tau$ oscillations or vice versa. Thus with only four parameters, this mass matrix can explain the solar neutrino anomaly, atmospheric neutrino anomaly, LSND result and the hot dark matter of the universe, while satisfying all other Laboratory constraints. Depending on the choice of parameters, one can get the vacuum oscillation or the large angle MSW solution of the solar neutrino anomaly.
Recently the super-Kamiokande experiment has confirmed the atmospheric neutrino oscillation result, suggesting nearly maximal mixing of $\nu_\mu$ with another species of neutrino \cite{1}. The same experiment has also confirmed the solar neutrino oscillation result, which suggests mixing of $\nu_e$ with another species of neutrino \cite{2}. Moreover, the energy spectrum of the recoil electron seems to favour the large mixing-angle vacuum oscillation of $\nu_e$ over the MSW solutions \cite{2}, although this may have limited statistical significance in the global fit to the solar neutrino data \cite{3, 4}. They have led to a flurry of phenomenological models for neutrino mass and mixing which can account for these oscillations \cite{5-8}, most of which are focussed on the bi-maximal mixing angles for the atmospheric and the solar neutrinos. However, almost all of these works are based on the three-neutrino formalism, involving the standard left handed neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$ \cite{5}.

On the other hand the inclusion of the LSND neutrino oscillation result \cite{9} is known to require a fourth neutrino, which has to be a sterile one ($\nu_S$) for consistency with the observed Z–width \cite{10}. Moreover it requires either $\nu_\mu$ or $\nu_e$ to oscillate into $\nu_S$ for explaining the atmospheric and solar neutrino anomalies, while requiring $\nu_\mu \rightarrow \nu_e$ oscillation for the LSND result. Thus the three-neutrino models for atmospheric and solar neutrino anomalies, based on a $\nu_e - \nu_\mu - \nu_\tau$ mixing, are in direct conflict with the LSND result. While the LSND result has not been corroborated by the preliminary KARMEN data \cite{11}, the statistical significance of the latter is limited by its lower sensitivity in the relevant region of parameter space. Indeed, with the standard statistical method the 90 % c.l. limit of KARMEN excludes only half the parameter space of the LSND data in the $\Delta m^2 \leq 2 eV^2$ region \cite{12}. Hopefully, this issue will be resolved by the proposed mini-BOONE experiment at Fermilab along with more data from KARMEN. It seems to us premature, however, to rule out the LSND result at present. Therefore we have tried to construct a four-neutrino mass matrix, which can account for the present solar and atmospheric neutrino data along with the LSND result. It can also account for the hot dark matter content of the universe \cite{13}, while satisfying all laboratory and astrophysical constraints \cite{14, 15, 16}.

Table 1 summarises the experimental constraints on neutrino mass and mixing parameters, which are relevant for our model. The large angle MSW and the vacuum oscillation solutions to the solar neutrino data \cite{3, 17, 18} are taken from a recent fit to the $\nu_e$ suppression rates along with the recoil electron spectrum by Bahcall, Krastev and Smirnov \cite{3}. For both the solutions the fit favours the oscillation of $\nu_e$ into a doublet neutrino over $\nu_e \rightarrow \nu_S$. The reason is that in the former case the NC scattering of this doublet neutrino
Table 1: Present experimental constraints on neutrino masses and mixing

| Category                        | Neutrino Mass Constraint | Mixing Angle Constraint |
|---------------------------------|--------------------------|-------------------------|
| Solar Neutrino (Large angle MSW) | $\Delta m^2 \sim (0.8 - 2) \times 10^{-5} eV^2$ | $\sin^2 2\theta \sim 1$ |
| Solar Neutrino (Vacuum oscillation) | $\Delta m^2 \sim (0.5 - 6) \times 10^{-10} eV^2$ | $\sin^2 2\theta \sim 1$ |
| Atmospheric Neutrino            | $\Delta m^2 \sim (0.5 - 6) \times 10^{-3} eV^2$ | $\sin^2 2\theta > 0.82$ |
| LSND                             | $\Delta m_{e\mu}^2 \sim (0.4 - 2) eV^2$         | $\sin^2 2\theta_{e\mu} \sim 10^{-3} - 10^{-2}$ |
| Hot Dark Matter                  | $\sum_i m_i \sim (4 - 5) eV$                     | $m_{\nu_e} < 0.46 eV$ |
| Neutrinoless Double Beta Decay   | $\Delta m_{eX}^2 < 10^{-3} eV^2$                 | $\text{(or } \sin^2 2\theta_{eX} < 0.2\text{)}$ |

with electron can partly account for the discrepancy between the observed suppression rates in super-Kamiokande and the Homestake experiments. On the other hand one can get acceptable solutions with $\nu_e \rightarrow \nu_S$ oscillation if one makes allowance for a 20 % normalisation uncertainty for the Homestake experiment. This will also enlarge the acceptable range of $\Delta m^2$. Therefore we shall consider both the oscillation scenarios $\nu_e \rightarrow \nu_S$ and $\nu_e \rightarrow \nu_\tau$ in our model. It should be added here that the best value of $\sin^2 2\theta$ for the large angle MSW solution is slightly less than 1; and even there the quality of fit is rather poor when all the experimental data are put together [3]. However one can get acceptable fit with the large angle MSW solution, including the $\sin^2 2\theta = 1$ boundary, if one makes reasonable allowance for the uncertainty in the Boron neutrino flux [3, 4]. Finally, the global fits [3, 4] have also found acceptable small angle MSW solutions for both these oscillation scenarios. But we do not consider them here, since the texture of our mass matrix naturally leads to bimaximal mixing as we shall see below.

The dark matter constraint on the sum of neutrino masses comes from a recent global fit to the spectrum of density perturbation in the universe using various cosmological models [13]. The best fit is obtained with a hot and cold dark matter model, where the former constitutes 20 % of the critical density. Besides there is an astrophysical upper bound on the number of neutrino species from nucleosynthesis, which allows 1 or atmost 2 sterile neutrinos.
We shall consider a four-neutrino mass matrix for the three doublet neutrinos and a singlet (sterile) neutrino. We shall present two scenarios, where the solar and atmospheric neutrino anomalies are explained by the oscillations (A) $\nu_e \to \nu_S$ and $\nu_\mu \to \nu_\tau$ and (B) $\nu_e \to \nu_\tau$ and $\nu_\mu \to \nu_S$. The corresponding mass matrices will be related to one another by suitable permutation of neutrino indices. In each case maximal mixing between the oscillating neutrino pairs will be ensured by the texture of the mass matrix. Moreover we shall obtain the vacuum oscillation and the large angle MSW solutions in each case depending on the choice of parameters.

(A) $\nu_e \to \nu_S$ and $\nu_\mu \to \nu_\tau$ Oscillations:

In this case the texture of our neutrino mass matrix in the basis $[\nu_e \nu_\mu \nu_\tau \nu_S]$ is

$$m_\nu = \begin{pmatrix} 0 & 0 & a & d \\ 0 & c & b & 0 \\ a & b & 0 & 0 \\ d & 0 & 0 & 0 \end{pmatrix}. \quad (1)$$

Note that it has only 4 parameters. In comparison the earlier mass matrices considered had at least 5 parameters [20, 21, 22]. Moreover, the above mass matrix is minimal in the sense that it has only one diagonal element. The mass matrix of [20] has effectively 4 parameters in the case of maximal vacuum oscillation solution of the solar neutrino. However it contains two equal diagonal elements, which could in general be different from one another. It is clear from the mass matrix that the neutrinoless double beta decay vanishes because $\sum_i U_{ei}^2 m_i = m_{ee} = 0$; so that the corresponding constraint [14] is automatically satisfied.

For the $3 \times 3$ submatrix of doublet neutrinos, we shall assume the hierarchy

$$b \gg a, c. \quad (2)$$

Consequently the $\nu_\mu$ and $\nu_\tau$ will form a nearly degenerate pair with maximal mixing and small mass squared difference ($\sim 2bc$) to explain the atmospheric neutrino anomaly. Moreover, the remaining eigenvalue of this $3 \times 3$ submatrix gets a tiny double see-saw contribution $2a^2c/b^2$, which will be much smaller than $d$ over a wide range of the latter parameter. Consequently, the $\nu_e$ and $\nu_S$ will form a nearly degenerate pair with maximal mixing and small mass squared difference to explain the solar neutrino anomaly. The vacuum oscillation and the large angle MSW solutions will correspond to the choices $d < a, c$ and $d \sim b$ respectively.
I – Vacuum Oscillation Solution \((b >> a, c > d)\): In this approximation the mass eigenvalues are given by,

\[
m_1 = d + \frac{a^2 c}{2b^2}
\]
\[
m_2 = b + \frac{c}{2} + \frac{a^2}{2b} + \frac{c^2}{8b} - \frac{a^2 c}{2b^2}
\]
\[
m_3 = -b + \frac{c}{2} - \frac{a^2}{2b} - \frac{c^2}{8b} - \frac{a^2 c}{2b^2}
\]
\[
m_4 = -d + \frac{a^2 c}{2b^2} \tag{3}
\]

and the corresponding mass eigenstates \(\nu^T_i \equiv \{\nu_1, \nu_2, \nu_3, \nu_4\}\) are related to the weak eigenstates \(\nu^T_\alpha \equiv \{\nu_e, \nu_\mu, \nu_\tau, \nu_S\}\) through the mixing matrix \(U_{i\alpha}\) as,

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_S
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{\sqrt{2}} & -s_1 & s_1 & \frac{1}{\sqrt{2}} \\
s_1 & \frac{1}{\sqrt{2}} & -s_1 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{s_1}{\sqrt{2}} & \frac{s_1}{\sqrt{2}} \\
-s_2' & s_2 & s_2' & s_2
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix} \tag{4}
\]

where, \(s_1 = \frac{a}{\sqrt{2b}}\) and we can neglect the terms \(s_2\) and \(s_2'\), which are of the order of \(\sim O\left(\frac{ac}{\sqrt{2b^2}}, \frac{ad}{b^2}\right) \sim 10^{-5}\) for our choice of parameters. For the given \(4 \times 4\) mixing matrix \(U_{i\alpha}\) the probability of two flavour oscillation is given by,

\[
P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j > i} U_{\alpha i} U_{\alpha j} U_{\beta i} U_{\beta j} \sin^2 \left(\frac{\Delta m^2_{ij} L}{4E}\right), \tag{5}
\]

where, \(\Delta m^2_{ij} = m^2_i - m^2_j\). For our mixing matrix the flavour oscillation in each case is dominated by one mixing angle which can be determined by comparing the expression (5) with the effective \(2 \times 2\) flavour oscillation formula

\[
P_{\nu_\alpha \nu_\beta} = \sin^2 2\theta_{\alpha\beta} \sin^2 \left(\frac{\Delta m^2_{ij} L}{4E}\right). \tag{6}
\]

Thus we get an expression for \(\sin^2 2\theta_{\alpha\beta}\) in terms of parameters of the mixing matrix \(U_{i\alpha}\).

For illustration we shall now present the solution for a specific set of parameters, i.e.,
\(a = 0.05eV, b = 1.5eV, c = 0.001eV\) and \(d = 0.0001eV\). There are two pairs of nearly degenerate eigenvalues

\[
m_{\nu_1} \simeq -m_{\nu_4} \simeq d = 0.0001eV
\]
\[
m_{\nu_2} \simeq -m_{\nu_3} \simeq b = 1.5eV.
\]
The LSND experiment can be explained by the oscillations between states with mass squared difference of the order $eV^2$ which means that it can be explained by the oscillations between the $\nu_{1,4}$ and $\nu_{2,3}$ states. To explain LSND as an oscillation between the $\nu_e$ and $\nu_\mu$ the effective mixing angle $\sin^2 2\theta_{e\mu}$ is obtained by comparing (4) and (5) and reading off the mixing matrix elements from (4),

$$\sin^2 2\theta_{e\mu} = -4 \{ U_{e1} U_{e2} U_{\mu1} U_{\mu2} + U_{e1} U_{e3} U_{\mu1} U_{\mu3} + U_{e4} U_{e2} U_{\mu4} U_{\mu2} + U_{e4} U_{e3} U_{\mu4} U_{\mu3} \} = -4 \times 4 (s_1^2) \left( \frac{1}{\sqrt{2}} \right)^2 = \frac{4a^2}{b^2}. \quad (8)$$

Similarly the other masses and the relevant mass squared differences and the corresponding mixing angles for the experiments listed in Table 1 are given by,

$$\Delta m^2_{\text{sol}} = m^2_{\nu_1} - m^2_{\nu_4} = \frac{2a^2cd}{b^2} = 2.2 \times 10^{-10} eV^2$$

$$\sin^2 2\theta_{eS} = 1$$

$$\Delta m^2_{\text{atm}} = m^2_{\nu_2} - m^2_{\nu_3} = 2bc = 0.003 eV^2$$

$$\sin^2 2\theta_{\mu\tau} = 1$$

$$\Delta m^2_{\text{LSND}} = m^2_{\nu_1} - m^2_{\nu_2} = b^2 - d^2 = 2.2 eV^2$$

$$\sin^2 2\theta_{e\mu} = 8s_1^2 = \frac{4a^2}{b^2} = 0.004$$

$$m_{DM} = \sum_i |m_i| = 3 eV. \quad (9)$$

The $\nu_e \rightarrow \nu_S$ oscillation gives the vacuum oscillation solution to the solar neutrino anomaly, while the $\nu_\mu \rightarrow \nu_\tau$ oscillation explains the atmospheric neutrino anomaly. The LSND result is explained by $\nu_\mu \rightarrow \nu_e$ oscillation. The contribution to dark matter is $3$ eV. The CHOOZ \cite{15} and other laboratory constraints are satisfied.

Note that the above solution consists of two nearly degenerate pairs of maximally mixed neutrinos, separated by a relatively large mass squared gap. This is known to be the favoured mass configuration for satisfying the various laboratory constraints \cite{20, 23}. The four model parameters are used to ensure that the three mass squared gaps correspond to the required values of $\Delta m^2$ for the solar, atmospheric and LSND neutrino oscillations, and $\theta_{e\mu}$ corresponds to the required mixing angle for LSND. The hot dark matter prediction comes out as a bonus. All these features are natural predictions of our mass matrix; and as such they will be shared by each of the alternative solutions discussed below. It should also be noted that the underlying double see-saw mechanism is responsible for generating mass.
squared gaps differing by 10 orders of magnitude starting with mass parameters, which differ by only 3–4 orders of magnitude (i.e., similar to the case of the up type quark mass matrix).

II – Large Angle MSW Solution \((b > d >> a, c)\) : In this approximation \((b \neq d)\) the mass eigenstates are given by,

\[
\begin{align*}
    m_1 &= d - \frac{a^2 d}{2(b^2 - d^2)} + \frac{a^2 b^2 c}{2(b^2 - d^2)^2} \\
    m_2 &= b + \frac{c}{2} + \frac{c^2}{8b} + \frac{a^2 b}{2(b^2 - d^2)} - \frac{a^2 b^2 c}{2(b^2 - d^2)^2} \\
    m_3 &= -b + \frac{c}{2} - \frac{c^2}{8b} - \frac{a^2 b}{2(b^2 - d^2)} - \frac{a^2 b^2 c}{2(b^2 - d^2)^2} \\
    m_4 &= -d + \frac{a^2 d}{2(b^2 - d^2)} + \frac{a^2 b^2 c}{2(b^2 - d^2)^2}
\end{align*}
\]

(10)

and the mixing matrix has the same form as (4), with \(s_1 = \frac{ab}{(b^2 + d^2)^{1/2}}\) and \(s_2 = s'_2 = \frac{a b d}{d(b^2 + d^2)^{1/2}}\). Hence like before \(\nu_e \to \nu_S\) mixing and the \(\nu_{\mu} \to \nu_{\tau}\) mixing are maximal, where as the \(\nu_e \to \nu_{\mu}\) mixing is given by the small parameter \(s_1\). It may be added here that one gets a smooth numerical solution at \(b = d\), although the approximation (10) breaks down there.

In this case let us consider a choice, \(a = 0.025eV, b = 1.5eV, c = 0.0015eV\) and \(d = 1.25eV\). Then the different masses and the relevant mass squared differences and the corresponding mixing angles are given by,

\[
\begin{align*}
    m_{\nu_1} &\approx -m_{\nu_4} \approx d = 1.25eV \\
    m_{\nu_2} &\approx -m_{\nu_3} \approx b = 1.5eV \\
    \Delta m^2_{\text{sol}} &= m^2_{\nu_1} - m^2_{\nu_4} = \frac{2a^2 b^2 c d}{(b^2 - d^2)^2} = 1.1 \times 10^{-5}eV^2 \\
    \sin^2 2\theta_{eS} &= 1 \\
    \Delta m^2_{\text{atm}} &= m^2_{\nu_2} - m^2_{\nu_3} = 2bc = 0.004eV^2 \\
    \sin^2 2\theta_{\mu\tau} &= 1 \\
    \Delta m^2_{\text{LSND}} &= m^2_{\nu_1} - m^2_{\nu_2} = b^2 - d^2 = 0.69eV^2 \\
    \sin^2 2\theta_{e\mu} &= 8s_1^2 = 8 \frac{a^2}{(b^2 + d^2)^2} = 1.3 \times 10^{-3} \\
    m_{DM} &= \sum |m_i| = 5.5eV.
\end{align*}
\]

(11)

The numerical values of the mass square differences have been calculated using the exact solutions of for the masses. The analytical expressions for the mass square differences are
from the polynomial approximation (10). The numerical agreement for the mass square differences between the exact solutions and the polynomial approximation agrees up to 6 decimals in this range of parameters.

Thus the $\nu_e \rightarrow \nu_S$ oscillation provides the large angle MSW solution to the solar neutrino problem. The $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_e$ oscillations explain the atmospheric neutrino anomaly and the LSND result respectively as before. The contribution to dark matter is 5 eV. It may be added here that with these parameters $s_1 < s_2$; so the effective mass of $\nu_e$ is slightly lower than that of $\nu_S$.

(B) $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_S$ Oscillations:

In this case we can use the same mass matrix as before if we make the following change of basis,

$$
\begin{pmatrix}
\nu_e & \nu_\mu & \nu_\tau & \nu_S
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\nu_e & \nu_\mu & \nu_S & \nu_\tau
\end{pmatrix}.
$$

(12)

Note that with this change of basis the diagonal element for the sterile neutrino $m_{SS}$ continues to remain zero, which is an important condition as we shall see below. Moreover the change of basis does not affect the mass eigenvalues $m_1, m_2, m_3$ and $m_4$. But now the nearly degenerate pair $m_1$ and $m_4$ represent the two maximally mixed states of $\nu_e$ and $\nu_\tau$, while $m_2$ and $m_3$ represent similar admixtures of $\nu_\mu$ and $\nu_S$. Thus the solutions (9) and (11) will continue to hold with the exchange of the neutrino flavour indices $\tau$ and $S$ in $\theta_{eS}$ and $\theta_{\mu\tau}$. Consequently they will represent the vacuum oscillation and large angle MSW solutions to the solar neutrino anomaly via $\nu_e \rightarrow \nu_\tau$ oscillation, while the atmospheric neutrino anomaly is explained via $\nu_\mu \rightarrow \nu_S$ oscillation. The rest of the results remain the same as before.

Let us briefly discuss the possible mechanisms underlying the above mass matrix. Consider first the $3 \times 3$ submatrix corresponding to the three left-handed doublet neutrinos. This sub-eV scale mass matrix could arise from the standard see-saw mechanism with three heavy right-handed singlet neutrinos. Alternatively, one can get it without any right-handed neutrino but with an expanded higgs sector via a radiative mechanism [24] or Majorana coupling of the left-handed neutrino pairs to a heavy Higgs triplet [25]. In both cases one can naturally obtain a sub-eV scale mass matrix. The extension of the mass matrix to include a light singlet neutrino has been tried recently in each of the above three models [3, 4, 5]. In the standard see-saw model it is assumed to be one of the right-handed singlets while one adds a singlet neutrino in the other two models. In each case one has to impose a zero Majorana mass for this singlet, as otherwise it will naturally assume a high mass value. This is the reason why we have set $m_{SS}$ to zero in our mass matrix (1). In the standard see-saw model
this has been done by assuming a singular Majorana mass matrix for the singlet neutrinos, so that one of the eigenvalues \( m_{SS} \) is zero \[8\]. In the other two models the \( m_{SS} \) is made to vanish by imposing an additional symmetry \[6,7\]. Finally one asks if this singlet neutrino can naturally have Dirac masses in the \( \leq 1 \text{ eV} \) scale? It seems possible to get it in the Zee model \[4\] and the triplet higgs model \[7\] via the same suppression mechanisms which keep the \( 3 \times 3 \) doublet mass matrix in the sub-eV range. But in the case of the standard see-saw model it had to be put in by hand \[8\]. We feel it is important to look for a more natural way to keep the Dirac masses small in this model.

To summarize, we have presented a texture of four-neutrino mass matrix which automatically ensures bi-maximal mixing between \( \nu_e \rightarrow \nu_S \) and \( \nu_e \rightarrow \nu_\tau \) or vice versa. Thus with only four parameters it can account for the solar, atmospheric and LSND neutrino anomalies while remaining consistent with other experimental constraints. The prediction of the desired hot dark matter density comes out as bonus. Depending on the choice of parameters we can get both the vacuum oscillation and the large angle MSW solutions to the solar neutrino anomaly. Thanks to the underlying double see-saw mechanism, one can generate the desired mass squared gaps differing by 10 orders of magnitude starting with the four mass parameters which differ by only 3–4 orders of magnitude.

ACKNOWLEDGEMENT

We are grateful to Profs.K.S.Babu and Ernest Ma and also to Prof.K.Whisnant for pointing out a notational error in the original version of our mass-matrix. We are also grateful to Prof. Ernest Ma for a critical reading of the manuscript. We thank Profs. V. Barger, W. Buchmuller, S. Goswami, S. Pakvasa, T. Weiler and P. Zerwas for discussions. Two of us (US and DPR) would like to acknowledge the hospitality of the Theory Group, DESY and US acknowledges financial support from the Alexander von Humboldt Foundation.
References

[1] Super-Kamiokande Collaboration: Y. Fukuda et al, Phys. Rev. Lett. 81 (1998) 1562; Phys. Lett. B433 (1998) 9 and B436 (1998) 33; T. Kajita, Talk presented at Neutrino – 98, Takayama, Japan (1998).

[2] Super-Kamiokande Collaboration: Y. Fukuda et al, Phys. Rev. Lett. 81 (1998) 1158; Talk by Y. Suzuki at Neutrino – 98, Takayama, Japan (1998).

[3] J.N. Bahcall, P.J. Krastev and A.Yu. Smirnov, Phys. Rev. D58 (1998) 096016.

[4] N.Hata and P.G. Langacker, Phys. Rev. D 56 (1997) 6107.

[5] B. Allanach, hep-ph/9806294; V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Lett. B437 (1998) 107; V. Barger, T.J. Weiler and K. Whisnant, hep-ph/9807319; J. Elwood, N. Irges and P. Ramond, hep-ph/9807220; E. Ma, hep-ph/9807386; G. Al- terelli and F. Feruglio, hep-ph/9807353; Y. Nomura and T. Yanagida, hep-ph/9807325; A. Joshipura, hep-ph/9808261; K. Oda et al, hep-ph/9808241; H. Fritzsch and Z. Xing, hep-ph/9808272; J. Ellis et al, hep-ph/9808301; A. Joshipura and S. Vempati, hep- ph/9808232; U. Sarkar, hep-ph/9808277; S. Davidson and S.F.King, hep-ph/9808296; H. Georgi and S.L. Glashow, hep-ph/9808293; R.N. Mohapatra and S. Nusinov, hep- ph/9808301; R. Barbieri, L.J. Hall and A. Strumia, hep-ph/9808333; Y. Grossman, Y.Nir and Y. Shadmi, hep-ph/9808353.

[6] N. Gaur, A. Ghosal, E. Ma and P. Roy, Phys. Rev. D58 (1998) 071301.

[7] U. Sarkar, hep-ph/9807466.

[8] Y. Chikira, N. Haba and Y. Mimura, hep-ph/9808254.

[9] LSND Collaboration: A. Athanassopoulos et al, Phys. Rev. Lett. 77 (1996) 3082, nucl-ex/9706006 and nucl-ex/9709006.

[10] P.B. Renton, Int. J. Mod. Phys. A 12 (1997) 4109.

[11] KARMEN Collaboration: B. Zeitnitz, Talk at Neutrino – 98, Takayama, Japan (1998).

[12] C. Giunti, hep-ph/9808405.

[13] E. Gawiser and J Silk, Science 280 (1998) 1405; see also J. Primack, astro-ph/9707285.
[14] H.V. Klapdor-Kleingrothaus, in Proc Lepton and Baryon number violation, Trento, April 1998; M. Günther et al, Phys. Rev. D 55 (1997) 54; L. Baudis et al, Phys. Lett. B 407 (1997) 219.

[15] CHOOZ Collaboration : M. Appollonio et al, Phys. Lett. B420 (1998) 397.

[16] Particle Data Group, Eur. Phys. J. C 3 (1998) 1.

[17] GALLEX Collaboration : W. Hampel et al, Phys. Lett. B 388 (1996) 364; SAGE Collaboration : V. Gavrin et al, Neutrino – 98, Takayama, Japan (1998).

[18] Homestake expt : B.T. Cleveland et al, Astrophys. J. 496 (1998) 505; R. Davis, Prog. Part. Nucl. Phys. 32 (1994) 13.

[19] C.J. Copi, D.N. Schramm and M.S. Turner, Phys. Rev. Lett. 75 (1995) 3981; Phys. Rev. D 55 (1997) 3389.

[20] V. Barger, T.J. Weiler and K. Whisnant, Phys. Lett. B 427 (1998) 97; V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, hep-ph/9806328.

[21] S.C. Gibbons, R.N. Mohapatra, S. Nandi and A. Raychaudhuri, Phys. Lett. B 430 (1998) 296.

[22] E. Ma and P. Roy, Phys. Rev. D 52 (1995) 4780; Z.G. Berezhiani and R.N. Mohapatra, Phys. Rev. D 52 (1995) 6607; E.J. Chun, A.S. Joshipura and A.Yu. Smirnov, Phys. Rev. D 54 (1996) 4654; K. Benakli and A.Yu. Smirnov, Phys. Rev. Lett. 79 (1997) 4314; G. Cleaver, M. Cvetic, J.R. Espinosa, L Everett and P. Langacker, Phys. Rev. D 57 (1998) 2701; A.S. Joshipura and A.Yu. Smirnov, hep-ph/9806376.

[23] S.M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C 1 (1998) 247; S. Goswami, Phys. Rev. D55, 2931 (1997).

[24] A. Zee, Phys. Lett. B 93 (1980) 389.

[25] E. Ma and U. Sarkar, Phys. Rev. Lett. 80 (1998) 5716.