Model-Independent Bounds on a Light Higgs

Aleksandr Azatov, Roberto Contino, Jamison Galloway

Dipartimento di Fisica, Università di Roma “La Sapienza”
and INFN Sezione di Roma, I-00185 Rome, Italy

Abstract

We present up-to-date constraints on a generic Higgs parameter space. An accurate assessment of these exclusions must take into account statistical, and potentially signal, fluctuations in the data currently taken at the LHC. For this, we have constructed a straightforward statistical method for making full use of the data that is publicly available. We show that, using the expected and observed exclusions which are quoted for each search channel, we can fully reconstruct likelihood profiles under very reasonable and simple assumptions. Even working with this somewhat limited information, we show that our method is sufficiently accurate to warrant its study and advocate its use over more naive prescriptions. Using this method, we can begin to narrow in on the remaining viable parameter space for a Higgs-like scalar state, and to ascertain the nature of any hints of new physics—Higgs or otherwise—appearing in the data.

*email: aleksandr.azatov@roma1.infn.it, roberto.contino@roma1.infn.it, jamison.galloway@roma1.infn.it
1 Introduction

The search for a Higgs boson at the LHC has entered an exciting phase. There have been recent excesses of events recorded in various channels by the ATLAS and CMS collaborations for a Higgs mass $m_h \approx 125$ GeV, in the region preferred by the electroweak (EW) precision tests performed at LEP. Despite the common viewpoint which considers the Higgs as the last missing piece of the successful Standard Model (SM) construction, the exploration of the TeV scale that has started at the LHC should be seen rather as our first mapping of unknown territory, where the theory sector responsible for the breaking of electroweak (EW) symmetry and the origin of mass is being tested for the first time.

Crucial, though indirect, information is encoded in the LEP precision tests, such as a clear indication that the electroweak symmetry breaking (EWSB) dynamics must possess an approximate custodial symmetry, so as to ensure small corrections to the $\rho$ parameter. If one assumes that the contribution of the Higgs to the EW parameters dominates over that of possible additional new states, LEP suggests that the Higgs must be light and that its coupling to the $W$ and $Z$ vector bosons is within $\sim 15\%$ its SM value. Even under these assumptions, however, there is no indication from LEP on the value of the couplings of the Higgs to fermions.

On the theoretical side, it is well known that all the successes of the $SU(2)_L \times U(1)_Y$ theory of the EW interactions hold—with the exception of the LEP precision tests just mentioned—even in absence of a Higgs boson. The theory can in fact be formulated in a fully consistent way by using the formalism of chiral Lagrangians, which is the standard framework to model effective field theories with spontaneously broken symmetries. Such a description becomes strongly coupled at the scale $\Lambda \approx 1 - 3$ TeV unless additional states, for example a light Higgs boson, appear below that energy threshold. In this regard the Higgs model of the SM represents a very peculiar UV completion of the EW chiral Lagrangian, where just one extra scalar field is added to the spectrum of known particles with couplings exactly tuned to ensure perturbativity up to Planckian scales. While perturbativity implies calculability of the theory, the price to pay is that of an instability of the Higgs mass term against radiative corrections, which makes a light elementary Higgs boson highly unnatural.

The fine-tuning problem of the Higgs model is resolved in theories where the Higgs boson
is a composite state of new strong dynamics at the TeV scale [1] or where an additional symmetry, like supersymmetry, protects its mass. In a generic theoretical framework, the couplings of the Higgs boson can differ significantly from their SM values as the result of mixing with other light scalars or as implied by the composite nature of the Higgs. Given our current limited information on the dynamics responsible for breaking EW symmetry, it is important to keep a general perspective when looking for the Higgs boson at the colliders. The EW chiral Lagrangian, with the addition of a light Higgs-like scalar, represents the theoretical starting point to analyze and optimize the Higgs searches in a model-independent way.

In this work we will show how such a model-independent analysis, once applied to the data collected so far at the colliders, can lead to further insight on the Higgs searches, and can perhaps suggest further optimization of the present experimental strategies. Although a thorough interpretation of the current data would require more detailed information than the one currently made public by the experimental collaborations, we have designed an approximate method to extract the likelihood of a given channel using the expected and observed exclusion limits for the SM Higgs. Such a technique becomes rigorous in the gaussian limit of large number of counts and turns out to be accurate under several independent checks that we have performed. Knowledge of the likelihoods allows one to reinterpret the individual limits in a generic Higgs model and then recombine different searches in a rigorous way. In this regard our method improves on different strategies where the various limits on the Higgs are individually considered [2], or more empirical recipes like a quadrature combination of the limits are adopted.

Our work shares features with previous studies on model-independent approaches to the extraction of the Higgs couplings, for example the pioneering paper of Duhrssen [3] and those in Refs. [4–6]. Even more similar in the spirit to the present work is the study of the Higgs couplings performed by Refs. [7,8] in the context of composite Higgs theories.

The paper is organized as follows. In section 2 we discuss the EW chiral Lagrangian which describes a light Higgs-like scalar including the complete set of 4-derivative operators which modify the couplings of the Higgs to the vector bosons. Section 3 is devoted to defining our technique of extracting the likelihoods from existing exclusion limits on the Higgs and discussing its accuracy. Readers not interested in the details on the method can skip this
part and move to section 4, where we apply it to estimate the model-independent limits on the Higgs couplings and on the strong scale of two benchmark composite Higgs models. In Section 5 we perform a best fit for the point at $m_h = 125$ GeV, assuming the excess of events observed by CMS and ATLAS is due to the Higgs. We conclude in section 6.

2 General Lagrangian for a light Higgs-like scalar

Let us consider the case in which a light neutral scalar $h$ exists in addition to the known matter and gauge fields. The most general description of such Higgs-like particle is obtained by considering the EW chiral Lagrangian and adding all possible interactions involving $h$ \[9\]. By requiring an approximate custodial symmetry, the longitudinal $W$ and $Z$ polarizations correspond to the Nambu-Goldstone (NG) bosons of a global coset $SU(2)_L \times SU(2)_R / SU(2)_V$ and can be described by the $2 \times 2$ matrix

$$\Sigma(x) = \exp \left( i \sigma^a \chi^a(x) / v \right),$$

where $\sigma^a$ are the Pauli matrices and $v = 246$ GeV. The scalar $h$ is assumed to be a singlet of the custodial $SU(2)_V$. The Lagrangian thus reads:

$$\mathcal{L} = -V(h) + \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \ldots$$

where $\mathcal{L}^{(n)}$ includes the terms with $n$ derivatives and $V(h)$ is the potential for $h$. At the level of two derivatives one has \[9\] \[1\]

\[
\mathcal{L}^{(2)} = \frac{1}{2} (\partial_\mu h)^2 + \frac{v^2}{4} \text{Tr} \left( D_\mu \Sigma \Sigma^\dagger D^\nu \Sigma \right) \left( 1 + \frac{2a}{v} h + \frac{b}{v^2} h^2 + \cdots \right) + h.c.
\]

\[
- \frac{v}{\sqrt{2}} \lambda_{ij} \left( \bar{u}^{(i)}_L , d^{(i)}_L \right) \Sigma \left( u^{(i)}_R , 0 \right)^T \left( 1 + c_u \frac{h}{v} + c_{2u} \frac{h^2}{v^2} + \cdots \right) + h.c.
\]

\[
- \frac{v}{\sqrt{2}} \lambda_{ij} \left( \bar{d}^{(i)}_L , u^{(i)}_L \right) \Sigma \left( 0 , d^{(i)}_R \right)^T \left( 1 + c_d \frac{h}{v} + c_{2d} \frac{h^2}{v^2} + \cdots \right) + h.c.
\]

\[
- \frac{v}{\sqrt{2}} \lambda_{ij} \left( \bar{u}^{(i)}_L , d^{(i)}_L \right) \Sigma \left( 0 , l^{(i)}_R \right)^T \left( 1 + c_l \frac{h}{v} + c_{2l} \frac{h^2}{v^2} + \cdots \right) + h.c.
\]

\[1\] We omit for simplicity neutrino mass and Yukawa terms, although they can be included in a straightforward way.
where \(a, b, c_{u,d,l}, c_{2u,2d,2l}\) are arbitrary dimensionless coefficients, and \(c_{u,d,l}, c_{2u,2d,2l}\) have been assumed to be flavor-diagonal to avoid inducing dangerous flavor-changing processes. An implicit sum over flavor indices \(i,j = 1,2,3\) has been understood. Similarly, the potential can also be expanded in powers of \(h\),

\[
V(h) = \frac{1}{2} m_h^2 h^2 + d_3 \frac{1}{6} \left( \frac{3m_h^2}{v} \right) h^3 + d_4 \frac{1}{24} \left( \frac{3m_h^2}{v^2} \right) h^4 + \ldots
\]

(2.4)

where \(d_3, d_4\) are arbitrary coefficients and \(m_h\) is the mass of the scalar \(h\). As discussed in Ref. [9], for generic values of the coefficients the theory is strongly interacting at large energies. However, for the specific choice \(a = b = c_u = c_d = c_e = d_3 = d_4 = 1\) and vanishing higher-order terms, all the scattering amplitudes remain perturbative (and unitary) up to very high energies, provided the scalar \(h\) is light. This is indeed the SM limit, in which \(h\) is identified with the physical Higgs boson. For any other choice of coefficients the exchange of \(h\) only partially cancels the energy growth of the scattering amplitudes of NG bosons, and the Lagrangian (2.2) must be regarded as an effective description valid below some cutoff scale \(\Lambda\).

In this general case, it is still appropriate to refer to \(h\) as a Higgs boson if it forms a doublet of \(SU(2)_L\) together with the NG bosons \(\chi\), and as such it plays a role in the breaking of EW symmetry. This is naturally realized in theories of composite Higgs, where \(h\) emerges as a light pseudo-NG boson of a larger dynamically-broken global symmetry \([1,10–13]\). The shift symmetry acting on the Higgs in this case allows one to resum all powers of \(h\) at a given derivative order. At leading chiral order, this implies that the coefficients in \(\mathcal{L}^{(2)}\) are all functions of \(\xi = (v/f)^2\), where \(f\) is the decay constant of the composite Higgs. For small \(\xi\), the effective Lagrangian of such a strongly-interacting light Higgs (SILH) has been fully characterized by Ref. [13] in terms of a finite number of dimension-6 operators. In particular, it has been shown that \(a\) and \(b\) follow a universal trajectory in the small \(\xi\) limit.

Other scenarios are however possible, in which for example \(h\) is a bound state of the dynamics responsible for the breaking of the EW symmetry, but does not form an \(SU(2)_L\) doublet together with the \(\chi\) fields. In fact, it could even well be that \(h\) is a Higgs-like impostor, and plays no role in EWSB. This is for example the case of a light dilaton \([14]\). In all cases, the Lagrangian (2.2) is a valid effective description for \(h\) at energies lower than the cutoff scale. For convenience, in the following we will refer to \(h\) as the Higgs boson even
for generic values of its couplings.

At the level of four derivatives, it is convenient to write the Lagrangian as a sum of operators $O_i$,

$$\mathcal{L}^{(4)} = \sum_i O_i,$$

whose Higgs dependence is encoded by polynomials

$$F_i(h) = \alpha_i^{(0)} + \alpha_i^{(1)} h + \alpha_i^{(2)} h^2 + \ldots$$

with arbitrary coefficients $\alpha_i^{(n)}$. The operators that lead to cubic and quartic interactions of NG bosons and gauge fields with up to two Higgses are:

$$O_1 = \text{Tr}[(D_\mu \Sigma)^\dagger (D^\nu \Sigma)] (\partial_\nu F_1(h))^2$$

$$O_2 = \text{Tr}[(D_\mu \Sigma)^\dagger (D_\nu \Sigma)] \partial^\mu \partial^\nu F_2(h)$$

$$O_{GG} = G_{\mu\nu} G^{\mu\nu} F_{GG}(h)$$

$$O_{BB} = B_{\mu\nu} B^{\mu\nu} F_{BB}(h)$$

$$O_W = D_\mu W^\alpha_{\mu\nu} \text{Tr} \left[ \Sigma^i \sigma^a_i \overleftrightarrow{D}_\nu \Sigma \right] F_W(h)$$

$$O_B = -\partial_\mu B_{\mu\nu} \text{Tr} \left[ \Sigma^i \sigma^a_3 \sigma^a_i \overleftrightarrow{D}_\nu \Sigma \right] F_B(h)$$

$$O_{WH} = i W^a_{\mu\nu} \text{Tr} \left[ (D^\mu \Sigma)^\dagger \sigma^a D^\nu \Sigma \right] F_{WH}(h)$$

$$O_{BH} = -i B_{\mu\nu} \text{Tr} \left[ (D^\mu \Sigma)^\dagger (D^\nu \Sigma) \sigma^3 \right] F_{BH}(h)$$

$$O_{W\partial H} = \frac{1}{2} W^a_{\mu\nu} \text{Tr} \left[ \Sigma^i \sigma^a_i \overleftrightarrow{D}^\mu \Sigma \right] \partial^\nu F_{W\partial H}(h)$$

$$O_{B\partial H} = -\frac{1}{2} B_{\mu\nu} \text{Tr} \left[ \Sigma^i \sigma^a_3 \overleftrightarrow{D}^\mu \Sigma \sigma^a_i \right] \partial^\nu F_{B\partial H}(h).$$

The operators $O_{GG}, O_{BB}$ contribute to the coupling of the Higgs to a pair of gluons and photons and are thus relevant for the LHC searches, while $O_W, O_B$ contribute to the $S$ parameter. In the case of a composite Higgs, where $h$ is part of an $SU(2)_L$ doublet, at leading order in $\xi$ all the polynomials are fixed to the quadratic form $F_i(h) = (1 + h/v)^2(1+
\( O(h^3) + O(\xi) \), and the operators (2.8)-(2.11) correspond to the SILH Lagrangian.\(^2\) As pointed out in Ref. [13], since \( O_{GG}, O_{BB} \) do not respect the Higgs shift symmetry, their coefficient will be suppressed by an extra factor \( (\lambda^2/g^2_\rho) \), where \( g_\rho \) is the coupling strength of the strong sector, and \( \lambda \) is some (weaker) coupling that breaks explicitly the NG global symmetry. For example, \( O_{GG}, O_{BB} \) can be generated by the one-loop exchange of vector-like composite fermions [15][16].

The Lagrangian (2.2) represents the most general (effective) description of a light Higgs under the following assumptions: \( i \) possible new states are heavy and do not significantly affect the physics below the cutoff scale. In particular, this implies that there are no other light states to which the Higgs can decay; \( ii \) the EWSB dynamics possesses a custodial symmetry; \( iii \) there are no flavor-changing neutral-current processes mediated at tree-level by the Higgs. While the (at least approximate) validity of the last two assumptions is strongly supported by the current experimental data, the first assumption is simply driven by the request of simplicity, and it can be relaxed by adding to the effective Lagrangian possible new light states, such as additional scalars, which might be discovered in the future.

For the moment, assuming no such additional light states exist, eq. (2.2) allows for a general parametrization of the couplings of the Higgs to the fermions and to the gauge bosons free from (additional) theoretical prejudice, and as such it is the starting point for a model-independent interpretation of the experimental searches for a Higgs boson under way at the LHC and Tevatron.

It is important to notice that with the exception of direct searches, the only experimental information is on the coupling of the Higgs to vector bosons: the precision tests performed at LEP on the EW observables are sensitive to the Higgs contribution at one loop to the vector boson self energies, and thus set a constraint on \( a \) for a given mass \( m_h \). If one compares to

\(^2\)Once written in terms of the \( SU(2)_L \) doublet Higgs field, \( O_{1,2} \) correspond instead to dimension-8 sub-leading operators.
Figure 1: Limits on the coupling $a^2$ implied by the LEP precision tests for $\Lambda = 4\pi v/\sqrt{1-a^2}$ and $m_t = 173.2$ GeV. The gray region is excluded at 99% CL.

the SM case, the additional contribution to the EW parameters $\epsilon_{1,3}$ is

$$
\Delta \epsilon_1 = -\frac{3}{16\pi} \frac{\alpha(m_Z)}{\cos^2 \theta_W} (1 - a^2) \log \left( \frac{\Lambda^2}{m_h^2} \right)
$$

$$
\Delta \epsilon_3 = +\frac{1}{48\pi} \frac{\alpha(m_Z)}{\sin^2 \theta_W} (1 - a^2) \log \left( \frac{\Lambda^2}{m_h^2} \right).
$$

(2.12)

Figure 1 shows the 99%CL limits on $a^2$ obtained by performing a fit to the LEP data with $\Lambda = 4\pi v/\sqrt{1-a^2}$. Sizable deviations from the SM value $a = 1$ are still allowed; for example, for $m_h = 125$ GeV one has $0.84 \leq a^2 \leq 1.4$. It is important to notice that no constraint on the other Higgs couplings (for example $c$ and $b$) follows from the LEP precision tests. On the other hand, important information on all the single-Higgs couplings follows from the direct searches at LEP, Tevatron, and LHC.

Although in general the experimental data can and should be used to extract all the relevant Higgs couplings in (2.2), in this initial survey we will focus on those of a single Higgs to two weak bosons ($a$) and to two SM fermions, and we will set the latter to be the same for up and down quarks and for leptons ($c = c_u = c_d = c_l$). We will thus assume that the effects of the other couplings (for example those from $O_{GG}$ and $O_{BB}$) are subdominant.

3We recall that $\Delta \epsilon_1 = \Delta \hat{T}$, $\Delta \epsilon_3 = \Delta \hat{S}$ where $\hat{T}$, $\hat{S}$ \cite{18} are proportional to the Peskin-Takeuchi $S, T$ parameters \cite{17}.

4We make use of a $\chi^2$ function of four parameters \cite{19}, $\epsilon_{1,2,3,b}$, and set $\epsilon_2, \epsilon_b$ to their SM value.
This is in fact the case in two simple models of composite Higgs that we will adopt as useful benchmark theories to illustrate our results. The first one is the minimal $SO(5)/SO(4)$ model with SM fermions embedded into spinorial representations of $SO(5)$, which has been dubbed MCHM4 [10]. In this model all single-Higgs couplings are rescaled by a common function of $\xi$,

\begin{equation}
MCHM4: \quad a = c = \sqrt{1 - \xi} ,
\end{equation}

so that the Higgs production cross sections get rescaled by a universal factor, whereas the decay branching ratios are not modified compared to their SM values. The same relations are predicted in the Minimal Conformal Technicolor model [12]. The second benchmark theory that we will consider is the $SO(5)/SO(4)$ MCHM5 model with SM fermions embedded into fundamentals of $SO(5)$ [11]. It predicts a different rescaling of the Higgs couplings to fermions and vector bosons,

\begin{equation}
MCHM5: \quad a = \sqrt{1 - \xi} , \quad c = \frac{1 - 2\xi}{\sqrt{1 - \xi}} ,
\end{equation}

which in turn leads to a different pattern of decay rates compared to the SM. In particular, for $\xi \to 1/2$ one finds in this theory a concrete realization of the possibility of a fermiophobic Higgs. In this limit the theory requires a UV completion at a scale $\Lambda \sim 4\pi f \simeq 4.4$ TeV.

In the following sections we will show how the current experimental information from the SM Higgs searches can be used to get an accurate estimate of the model-independent constraints that can be set on the couplings $a$, $c$ for a given value of the Higgs mass $m_h$. By means of the same technique, we will be also able to derive the limits on $\xi$ in the benchmark composite Higgs models MCHM4 and MCHM5.

## 3 The Statistical Method

The strongest direct constraints on the coefficients $a$, $c$ come from the Higgs searches under way at the LHC. The results for each decay channel $i$ are expressed in terms of a strength modifier $\mu^i$, defined as the signal (Higgs) yield in SM units for a given fixed value of $m_h$ [20]:

\begin{equation}
\mu^i = \frac{n_s^i}{(n_s^i)_{SM}} ,
\end{equation}
If no significant excess of events compared to the background (no Higgs) expectation is observed, a 95% CL limit is set on $\mu$; if instead an excess is observed, the ATLAS and CMS collaborations report the best fit value of $\mu$ for a given hypothesis on $m_h$. In either case the result is derived by constructing a likelihood function $p(n_{\text{obs}} | ns + nb)$ using the signal ($ns$), background ($nb$) and observed ($n_{\text{obs}}$) yields. In the Bayesian approach, a posterior probability density function of $\mu$ is then constructed by assessing some prior $\pi(\mu)$ on $\mu$:

$$p(\mu | n_{\text{obs}}) = p(n_{\text{obs}} | \mu ns + nb) \times \pi(\mu).$$

(3.16)

To derive limits, a flat prior for $\mu \geq 0$ (vanishing for $\mu < 0$) is adopted, and the 95% CL limit on $\mu$ is computed as that value $\mu_{95\%}$ such that the integral of $p(\mu | n_{\text{obs}})$ from $\mu = 0$ to $\mu = \mu_{95\%}$ is 0.95. The result so obtained gives the limit on the (overall) factor by which the SM Higgs yield can be amplified, for a given value $m_h$. Values $\mu_{95\%} < 1$ thus exclude at 95% CL the SM Higgs for that particular value of the Higgs mass.

For given numbers of expected and observed events, the likelihood is modeled by a Poisson distribution:

$$p(n_{\text{obs}} | \mu ns + nb) = \frac{1}{n_{\text{obs}}!} e^{-(\mu \cdot ns + nb)} (\mu \cdot ns + nb)^{n_{\text{obs}}}.$$  

(3.17)

In a generic theory, for each channel $i$, the signal strength modifier $\mu^i$ can be computed provided one knows the Higgs production cross section for each production mode $p$, the efficiencies $\zeta^p_i$ of the kinematic cuts, and the Higgs decay branching fraction:

$$\mu^i \equiv \frac{n^i_s}{(n^i_s)^{SM}} = \frac{\sum_p \sigma_p \times \zeta^p_i \times BR_i \times BR^SM_i}{\sum_p \sigma^SM_p \times \zeta^p_i}.$$  

(3.18)

Notice that the efficiencies of the kinematic cuts depend in general on the production mode, and are thus crucial to correctly compute $\mu^i$. A rigorous assessment of the bounds implied

---

5 Results by the ATLAS and CMS collaborations are derived in two different statistical methods: the Bayesian method and a hybrid Bayesian-frequentist technique [20]. Although the latter has been chosen as the standard technique used to report the collaborations’ results, internal derivation of the limits is also performed using the Bayesian framework. In this work we will use the Bayesian framework, which seems to be the simplest and most logical approach for our purposes. See [21] for a primer.

6 In cases in which an unbinned likelihood is constructed [20], use of a binned one is expected to give similar results. See for example the discussion in Section 8 of [22] for the case of $h \rightarrow \gamma\gamma$. 

---
by the Higgs searches on a generic beyond-the-SM (BSM) theory, such as that of eq. (2.2), thus requires two ingredients:

1. The likelihood for each channel $i$ as a function of $\mu$

2. The cut efficiencies $\zeta_i^p$ for each channel $i$ and production mode $p$

Without this information, it is not possible to derive the exact constraints on theories different from the SM unless they predict a simple universal rescaling of all the Higgs cross sections. Knowledge of the cut efficiencies allows one to derive the bounds implied by each individual channel on the parameter space of any BSM model. This is, for example, what the dedicated code HiggsBounds \cite{2} does by considering only those experimental searches where, to good approximation, only one production mode is relevant (as a consequence of the kinematic cuts). In general, however, a consistent statistical combination of the various channels can be done only by knowing the individual likelihoods. Unfortunately, neither the likelihoods nor the cut efficiencies are currently publicly provided by ATLAS and CMS.\footnote{The cut efficiencies are provided only in select cases, e.g. the $\tau\tau$ mode of CMS. Here, however, the information is available only for one representative value of the Higgs mass, which does not suffice to construct exact likelihoods over the whole mass range, as would be needed to probe the broader parameter space.}

Given the importance of having a broader, model-independent perspective on the Higgs searches, we find it useful to try to find possible approaches that can lead to an accurate estimate of the bounds on the couplings in eq. (2.2), by making use of the current information made public by the experimental collaborations. Below we describe a method that allows one to reconstruct the likelihood of each channel given the expected and observed 95\% CL limits on the signal strength modifier, which are the only two numbers that are readily available for a given value of $m_h$. As we will discuss in detail, this method becomes exact in the asymptotic (Gaussian) limit of large event counts, which makes it clearly preferable over other less rigorous recipes sometimes used to combine the limits.

### 3.1 A Technique to Extract the Likelihoods in the Gaussian Limit

In general, once considered as a function of $\mu$, the posterior probability (3.16) depends on three parameters ($n_s$, $n_b$ and $n_{obs}$), while, as noticed above, we can make use of only two
numbers (the expected and observed 95% CL limits on $\mu$). However, if the number of observed events is large, $n_{\text{obs}} \gg 1$, the likelihood asymptotically tends to a Gaussian with mean $n_{\text{obs}}$ and standard deviation $\sqrt{n_{\text{obs}}}$:

$$p(n_{\text{obs}}|n) \propto e^{-n} n^{n_{\text{obs}}} \rightarrow e^{-(n-n_{\text{obs}})^2/2n_{\text{obs}}}.$$ (3.20)

In practice, the approximation is already good for $n_{\text{obs}} \gtrsim 10$. In this asymptotic limit the posterior probability (as a function of $\mu$) depends on just two combinations of $n_s$, $n_b$, $n_{\text{obs}}$:

$$p(\mu|n_{\text{obs}}) \propto e^{-(\mu-\mu_{\text{max}})^2/2\sigma_{\text{obs}}^2}, \quad \mu_{\text{max}} = \frac{n_{\text{obs}} - n_b}{n_s^{SM}}, \quad \sigma_{\text{obs}} = \frac{\sqrt{n_{\text{obs}}}}{n_s^{SM}}.$$ (3.21)

The parameter $\mu_{\text{max}}$, in particular, determines the location of the maximum of the probability and measures by how much the number of observed events has fluctuated from the pure background expectation compared to the number of SM signal events, see Fig. 2. As we will now show, the information provided by the experimental collaborations is sufficient, under simple specific assumptions, to determine $\mu_{\text{max}}$, $\sigma_{\text{obs}}$ and thus reconstruct the likelihood.

First, the value of $\mu_{\text{max}}$ and $\sigma_{\text{obs}}$ must be such to reproduce the 95% CL observed limit on $\mu$:

$$0.95 = \int d\mu \; p(\mu|n_{\text{obs}}) \simeq \int_0^{\mu_{\text{max}}^{95\%}} d\mu \; e^{-(\mu-\mu_{\text{max}})^2/2\sigma_{\text{obs}}^2} = \frac{\text{Erf} \left( \frac{\mu_{\text{max}}^{95\%} - \mu_{\text{max}}}{\sqrt{2}\sigma_{\text{obs}}} \right) + \text{Erf} \left( \frac{\mu_{\text{max}}}{\sqrt{2}\sigma_{\text{obs}}} \right)}{1 + \text{Erf} \left( \frac{\mu_{\text{max}}}{\sqrt{2}\sigma_{\text{obs}}} \right)}. \quad (3.22)$$

A second relation is obtained from the expected 95% CL limit, which is derived as above but setting $n_{\text{obs}} = n_b$ (pure background hypothesis). In this case the posterior probability $p(\mu|n_{\text{obs}} = n_b)$ is approximated in the asymptotic limit by a Gaussian with zero mean and

$$\text{Eq. (3.20) is a special case of the central limit for the Gamma distribution, see for example [23]. When considered as a function of } n, p(n_{\text{obs}}|n) \text{ is indeed proportional to a Gamma distribution with shape parameter } k = n+1 \text{ and scale parameter } \theta = 1. \text{ Any factor which does not depend on } n \text{ can be dropped, as the overall normalization of the posterior probability will be fixed at the end. A simple way to prove the asymptotic convergence (3.20) is by considering the difference between } p(n_{\text{obs}}|n) \text{ and the Gaussian at some fixed number of standard deviations away from the maximum: } (n - n_{\text{obs}})/\sqrt{n_{\text{obs}}} \sim \text{ a few. For } n_{\text{obs}} \gg 1 \text{ this implies } \Delta = (n - n_{\text{obs}})/n_{\text{obs}} \ll 1, \text{ so that}

$$p(n_{\text{obs}}|n) \propto (1 + \Delta)^{n_{\text{obs}}} e^{-n_{\text{obs}}\Delta} = \left( 1 - \frac{1}{2} n_{\text{obs}} \Delta^2 + O(\Delta^3) \right) = e^{-\Delta^2 n_{\text{obs}}/2 + O(\Delta^3)}, \quad (3.19)$$

where we made an expansion for small $\Delta$.}
Figure 2: Posterior probability \( p(\mu|n_{\text{obs}}) \) obtained for \( n_{\text{obs}} = 35, n_b = 30, n_s^{SM} = 3 \) (continuous curve). In this example the maximum is at \( \mu_{\text{max}} = 5/3 \), and the 95\% CL limit on \( \mu \) is \( \mu_{95\%}^{\text{obs}} = 5.66 \). The dashed curve shows the approximating Gaussian with mean \( \mu_{\text{max}} \) and standard deviation \( \sigma_{\text{obs}} = \sqrt{35/3} \).

The standard deviation \( \sigma_{\text{exp}} = \sqrt{n_b/n_s^{SM}} \), as one can see by setting \( n_{\text{obs}} = n_b \) in eq. (3.20). The relation implied by the 95\% CL expected limit is:

\[
0.95 = \int d\mu \; p(\mu|n_{\text{obs}} = n_b) \simeq \sqrt{\frac{2}{\pi\sigma_{\text{exp}}^2}} \int_{\mu_{\text{exp}}^{95\%}}^{\mu_{\text{exp}}^{95\%}} d\mu \; e^{-\mu^2/2\sigma_{\text{exp}}^2} = \text{Erf} \left( \frac{\mu_{\text{exp}}^{95\%}}{\sqrt{2\sigma_{\text{exp}}}} \right),
\]

which admits the simple solution:

\[
\frac{\sqrt{n_b}}{n_s^{SM}} = \sigma_{\text{exp}} = \frac{\mu_{\text{exp}}^{95\%}}{1.96},
\]

(3.24)

Although this is not an equation on the parameters of the posterior \( p(\mu|n_{\text{obs}}) \), it can be used to determine \( \sigma_{\text{obs}} \) provided the fluctuation is small compared to the number of background events:

\[
\frac{n_{\text{obs}} - n_b}{n_b} \ll 1.
\]

(3.25)

Notice that if \( n_s \ll n_b \) the fluctuation can still be large compared to the number of signal events, that is, \( \mu_{\text{max}} \sim O(1) \). If eq. (3.25) is satisfied, one can approximate \( \sigma_{\text{obs}} \simeq \sigma_{\text{exp}} = \sqrt{n_b/n_s^{SM}} \) and extract \( \mu_{\text{max}} \) by numerically solving eq. (3.22). In this way the likelihood is fully reconstructed as a function of \( \mu \). By using eq. (3.18) one can then evaluate the value of \( \mu \) in terms of the parameters of any generic Higgs model, and thus obtain the likelihood as a function of these parameters. Finally, the combined bound from several channels is obtained by multiplying their likelihoods.
At this point a comment is in order regarding the validity of combining the limits from individual channels in quadrature, which is what has sometimes been used in the literature to estimate the constraints implied by the Higgs searches on generic BSM models. It is simple to see (and well known) that the combination in quadrature is justified, in the gaussian limit, for the expected limits. It just follows from the simple fact that the product of gaussians with zero mean and standard deviations $\sigma^i_{\text{exp}}$ is still a gaussian with zero mean and variance $(\sigma^\text{comb}_{\text{exp}})^2 = 1/\sum_i (1/\sigma^i_{\text{exp}})^2$. Applying eq. (3.24) to each channel then leads to the inverse quadrature formula:

$$
\mu^{95\%}_{\text{comb,exp}} = \frac{1}{\sqrt{\sum_i (\mu^{95\%}_{i,\text{exp}})^2}}.
$$

(3.26)

On the other hand, this formula cannot be used to combine the observed limits, since in that case the combined limit obtained by means of the product of likelihoods cannot be expressed simply in terms of the individual limits. Using eq. (3.26) for the observed limits does not properly take into account the experimental fluctuations. A quantitative comparison between the naive quadrature combination and our method is reported in Figs. 3, 4, 6 and discussed below.

So far we have tacitly neglected possible systematic errors on the number of signal and background events. In the Bayesian approach they are simply incorporated by marginalizing the posterior probability over a set of nuisance parameters, taking into account possible correlations [20]. In order to show how our method accounts for such systematic effects, we consider for simplicity only two nuisance parameters, $\theta_s$, $\theta_b$, which reflect the overall systematic uncertainty respectively on the number of signal and background events. The posterior probability in this case is given by

$$
p(\mu|n_{\text{obs}}) \propto \int_{-\infty}^{+\infty} d\theta_b \int_{-\infty}^{+\infty} d\theta_s \ p(n_{\text{obs}}|\mu \cdot n^S_{ SM} e^{\theta_s k_s} + n_b e^{\theta_b k_b}) e^{-\theta_b^2/2} e^{-\theta_s^2/2}
$$

(3.27)

where $k_s = \Delta_s/n^S_{ SM}$, $k_b = \Delta_b/n_b$ and $\Delta_s$ ($\Delta_b$) is the systematic error on the number of signal (background) events. The nuisance parameters have been assumed to be distributed with LogNormal pdfs, as commonly done by CMS and ATLAS to ensure that the number of signal and background events never becomes negative. However, if the systematic errors are small, $\Delta_b/n_b, \Delta_s/n^S_{ SM} \ll 1$, the LogNormal distributions can be approximated by (truncated)
In this case one obtains (up to an overall normalization)

\[ p(\mu | n_{\text{obs}}) \simeq \frac{e^{-(\mu n_{\text{SM}} + n_b - n_{\text{obs}})^2}}{\sqrt{2\pi(n_{\text{obs}} + \Delta^2_b + \mu^2 \Delta^2_s)}}. \]  

\[(3.28)\]

Although this not a Gaussian function of \( \mu \), in many practical cases one can neglect the dependence on \( \mu \) in the denominator of the exponent and in the overall factor. The resulting probability can then be approximated by a Gaussian with mean \( \mu_{\text{max}} \) and modified standard deviation \( \sigma_{\text{obs}} = \sqrt{n_{\text{obs}} + \Delta^2_b/n_{\text{SM}}} \). Similarly, the expected posterior probability, \( p(\mu | n_{\text{obs}} = n_b) \), is approximately a Gaussian with zero mean and modified standard deviation \( \sigma_{\text{exp}} = \sqrt{n_b + \Delta^2_b/n_{\text{SM}}} \). The exact condition for this gaussian approximation to hold is

\[ \frac{\Delta_s}{n_{\text{SM}}} \frac{n_{\text{obs}} - n_b}{\sqrt{\Delta^2_b + n_{\text{obs}}}} \ll 1. \]  

\[(3.29)\]

If eqs. (3.25) and (3.29) are satisfied, then our method to extract the likelihood from the expected and observed 95% CL limits can be applied, the only modification with respect to the previous discussion is that now the parameters \( \sigma_{\text{obs}}, \sigma_{\text{exp}} \) get a contribution also from the systematic error on the number of background events.

As a final comment we notice that the size of the 68% and 95% bands reported for the expected exclusion limit by CMS and ATLAS (green and yellow bands) gives in principle some additional information on how the limit changes when the nuisance parameters vary. Since however such information does not seem easy to use for reconstructing the likelihoods, we have not considered it.

It is useful to summarize the conditions on which our method relies:

1. The number of observed events must be large (Gaussian limit).

2. The fluctuations must be small compared to the number of background events, though not necessarily small compared to the number of signal events: condition (3.25).

3. The systematic error on the number of background events must be small, \( \Delta_b/n_b \ll 1 \), and that on the number of signal events must be negligible: \( \Delta_s/n_{\text{SM}} \ll 1 \) plus condition (3.29).

9 Truncation of the integral at \( \theta_s = -n_{\text{SM}}/\Delta_s \), \( \theta_b = -n_b/\Delta_b \) is required to avoid having a negative number of events.

10 In fact, approximating \( \sigma_{\text{obs}} \simeq \sigma_{\text{exp}} \), as required in our method to extract the likelihood, is even more accurate if \( \Delta^2_s/n_b \) is not small, while \( \Delta_b/n_b \ll 1 \).
Figure 3: Left panel: 95% CL observed limits on \( \mu \) obtained by combining all CMS searches with different techniques: the continuous black curve is the official CMS limit, the dotted red and dashed orange curves are obtained respectively with our method and by a naive quadrature combination. Right panel: relative deviation of the limits obtained with these two latter approaches from the official combination. The blue band at \( \pm 20\% \) is for illustration.

3.2 Discussion of the accuracy of our method

Before applying it to derive the model-independent bounds on the couplings \( a, c \), we want to discuss here the accuracy of our method for extracting the likelihoods. A first test of its validity comes from the comparison with the official limit on \( \mu \) obtained by combining all the searches performed by a single LHC experiment. We find that the combined bound derived using our technique reproduces with good accuracy the official curve in the whole range of Higgs masses.

Figure 3 shows the comparison for CMS using the full 2011 data set \((4.6 - 4.8\text{ fb}^{-1})\) \cite{24}. When available, in fact only for \( h \to WW \), we have used the limits from each of the subchannels of a given search to reconstruct their individual likelihoods. For those searches where only a combined limit was available, like for \( h \to \gamma\gamma \), we have used that to reconstruct the overall likelihood. Although in most of the cases we could find only 95% CL limits obtained with the CLs frequentist method, we did make use of the Bayesian limits in those few cases where they were available. On the other hand, the two approaches have been shown to lead
to very similar results (see for example [25]), so that we expect that using CLs limits instead of Bayesian ones leads to a difference in our results which is within the error of the gaussian approximation.

As shown in the right plot of Fig. 3, the relative difference between the 95% CL limits obtained with our Gaussian technique and the official CMS curve is always smaller than 20%, and in fact our combination typically errs on the conservative side. For the sake of comparison, we show also the result of adding observed exclusions in inverse quadrature as an approximation of the total. As expected, we find that this approach is incapable of accounting for competing fluctuations in different channels, and can lead to regions of unrealistically strong exclusions.

A more detailed comparison is possible by focusing on the $h \rightarrow WW \rightarrow \ell\nu\ell\nu$ channel. Two different kinds of analysis are performed in this case by CMS: the first makes use of a boosted decision tree technique, the second is purely cut-based. For the latter analysis, the number of signal, background, and observed events is made publicly available at $m_h = 120$ GeV for each of the five categories considered [26,27], which makes it possible to fully construct the individual likelihoods using eq. (3.27). We find that these constructed likelihoods are able to reproduce the median 95% CL expected and observed official limits on $\mu$ within $15 - 20\%$. This shows that (at least for this channel) a simple two-dimensional marginalization, eq. (3.27), captures the most important effects of the systematic uncertainties. Figure 4 shows the relative difference between these constructed likelihoods and those extracted with our method from the published 95% CL limits as a function of $\mu$, for the representative point $m_h = 120$ GeV. For convenience, we report in Table 1 the number of events in each channel that we have used, as given by the CMS collaboration [27]. With the exception of the 1-jetOF category, where the agreement is slightly worse, the extracted likelihood is seen to be accurate at the level of $\pm 20\%$. The precision of our method is also clearly illustrated by Fig. 5, which shows the observed 95% CL exclusion curve in the plane $(a,c)$ as obtained from the combination of the five $WW$ categories by using our method (orange curve) and by using the likelihoods constructed from the event numbers of Table 1 (blue area). In either case we rescaled the 2-jet category assuming that its yield entirely comes from the VBF Higgs production, as a consequence of the cuts imposed. The other four categories are instead rescaled by assuming that they are entirely dominated by the gluon-fusion
Figure 4: Relative error between extracted and constructed likelihoods for the five $h \rightarrow WW$ categories of CMS, as a function of the signal strength modifier $\mu$. In each case the extracted Gaussian likelihood is found to approximate the one constructed from event numbers typically to within 20%.

| Final State: jets/leptons | $n_B$ | $\Delta n_B$ | $n_S$ | $\Delta n_S$ | $n_{obs}$ |
|---------------------------|-------|--------------|-------|--------------|----------|
| 0-jet, Same Flavor        | 50.6  | 9.8          | 4.7   | 1.1          | 49       |
| 0-jet, Opp. Flavor        | 86.1  | 8.2          | 11.0  | 2.5          | 87       |
| 1-jet, Same Flavor        | 20.4  | 2.6          | 1.7   | 0.5          | 26       |
| 1-jet, Opp. Flavor        | 39.1  | 5.3          | 4.8   | 1.7          | 46       |
| 2-jet                     | 11.3  | 3.6          | 1.1   | 0.1          | 8        |

Table 1: Background, signal, and observed events (with related uncertainties) reported by CMS in the five $WW$ categories for $m_h = 120$ GeV, $\int dt \mathcal{L} \leq 4.7$ fb$^{-1}$ [27].
Figure 5: 95% CL observed limits in the plane $(a, c)$ obtained by combining the five $WW$ categories in CMS for $m_h = 120$ GeV. The blue and orange curves are obtained using respectively the likelihoods constructed from the number of events in Table II (exact combination) and the likelihoods reconstructed with our method (gaussian approximation).

production. While this is clearly a rough approximation, it should be sufficiently accurate in most of the $(a, c)$ plane and conservative in the fermiophobic region $c \sim 0$. The agreement between the two exclusion curves in Fig. 5 is good over the whole $c$ range. The stronger exclusion around $c \sim 0$ is a consequence of the greater significance of the VBF channel in this limit. As we will discuss in sec. 5, the inclusive analysis performed by ATLAS for $h \rightarrow WW$ is much less sensitive to the fermiophobic region.

To summarize, the above results show that our method works accurately enough and can thus be used to derive a robust estimate of the bounds implied by the LHC searches on a generic Higgs model.

4 Model independent bounds

In this section we apply our method to derive the model-independent limits on the couplings $a, c$ in the framework of the effective Lagrangian (2.2). We will also show the bounds on $\xi$ in
the case of the two benchmark composite Higgs models MCHM4 and MCHM5. All the plots have been derived making use of the CMS results obtained through the analysis of the full 2011 data set \((4.6 - 4.8 \text{ fb}^{-1})\) \cite{24}. Similar conclusions are also obtained using the ATLAS data. We will not show the exclusions implied by Tevatron searches as they turn out to be weaker than the LHC ones. As mentioned in the previous section, we reconstructed the likelihoods of individual subchannels in a given search whenever possible. In each case the signal strength modifier has been computed as a function of \((a, c)\) by taking into account the exclusive or inclusive nature of the search. In particular, we assumed that the signal yield is fully dominated by the associated Higgs production in \(h \rightarrow b\bar{b}\), by VBF production for the 2-jet category of \(h \rightarrow WW\), and by gluon fusion production for the 0-jet and 1-jet categories of \(h \rightarrow WW\). All the other searches \((h \rightarrow ZZ, h \rightarrow \tau\tau, h \rightarrow \gamma\gamma)\) have been considered as inclusive. Since for these channels the cut efficiencies \(\zeta_i^p\) of eq. \((3.18)\) are not provided by CMS, we have assumed them to be constant \((i.e.\text{ independent of the Higgs production mechanism})\), although this is known to be a somewhat inaccurate approximation, especially in the limit \(|c| \ll 1\) where the gluon fusion cross section is suppressed compared to its SM value. The same assumption was made in the previous studies of Ref. \cite{7}.

We begin with the MCHM4 model, where the Higgs production cross sections are rescaled by a common factor. The same results apply to any model with universal rescaling, as is the case for example in minimal conformal TC. In this case the 95% CL limits on \(\xi\) are simply obtained from those on the signal strength modifier by setting \(\mu = 1 - \xi\). The result is shown in Fig. \ref{fig6} where we report the curves obtained by means of the official CMS limit, our gaussian method, and the inverse quadrature combination. The curve obtained with the latter method agrees with the results of Ref. \cite{7}. We have superimposed also the region selected by the LEP precision data at 99% CL, which has been obtained, as in Fig. \ref{fig1} by considering just the Higgs contribution to the EW observables. Since the contribution of additional states, naturally present in composite Higgs theories, can give an important contribution to the EW observables, this region should be considered simply as indicative rather than as a sharp exclusion contour. We see that values \(\xi \gtrsim 0.5 - 0.6\), which correspond to a suppression \(g_{higgs}/g_{higgs}^{SM} \lesssim 0.5\) in the Higgs couplings, are needed for a heavy Higgs to escape the current LHC exclusion. In the case of a light composite Higgs and small \(\xi\), on the other hand, the allowed range of \(m_h\) is roughly the same as for a SM Higgs.
Figure 6: Current 95% CL exclusion limits on models with $a = c = \sqrt{1 - \xi}$. The region excluded by LHC (LEP) data is shown in light red (green). We show here the comparison of the three different combination prescriptions discussed in the text: the solid black line corresponds to the official CMS combination in the CL$_S$ asymptotic approach, the dashed orange line is obtained using our gaussian method, and the dotted blue line shows the result of combination in quadrature.

The current exclusion limits on $\xi$ for the MCHM5 are shown in Fig. 7. As previously discussed, in this model the region $\xi \sim 1/2$ corresponds to a limit where the Higgs is fermiophobic, and its production rate is suppressed. This implies that a heavy Higgs can escape the current limits in an ample range of values $\xi \sim 0.3 - 0.7$. A similar plot has been derived in Ref. 7 by combining limits in inverse quadrature.

Finally, we report in Fig. 8 the current limits on the plane $(a, c)$ for some reference values of $m_h$. They have been obtained by combining all the CMS search channels using our method. Note that the likelihoods are now treated as fully two-dimensional functions $p(a, c|n_{obs})$, with production and branching ratio rescaling factors themselves functions of $a$ and $c$. This implies a difference of priors relative to the results of Figs. 6 and 7, where the two couplings were mapped to a single overall rescaling, $\mu$, whose prior is assumed to be flat over the interval $[0, \infty)$. The two-dimensional exclusions can thus be constructed simply by
Figure 7: Current 95% CL exclusion limits on $\xi$ in the MCHM5 ($a = \sqrt{1 - \xi}$, $c = (1 - 2\xi)/\sqrt{1 - \xi}$) as obtained with our method. The region excluded by LHC (LEP) data is shown in light red (green).

Figure 8: Current exclusions in the plane ($a, c$) for various Higgs masses as obtained with our method: the area to the right of each curve is excluded at 95% CL. These exclusions combine all search channels at CMS, with the full 2011 data set $\int dt \mathcal{L} \leq 4.8$ fb$^{-1}$. 
determining isocontours enclosing a desired fraction of the normalized likelihood. For this case, we assume priors that are flat over the range $0 \leq a \leq 3$ and $-3 \leq c \leq 3$, and zero elsewhere.

We notice that for $m_h = 120, 130$ GeV the exclusion curve is sensitive to the relative sign between $a$ and $c$, while for heavier Higgs masses the curves are symmetric under $c \rightarrow -c$. This is due to the importance for light $m_h$ played by the $\gamma\gamma$ channel, the only one sensitive to the relative sign through the decay width to two photons. In particular, for negative $c/a$ the interference between the one-loop top and $W$ contributions to the decay width is constructive and the constraint is stronger.

## 5 The 125 GeV Excess

A somewhat anomalous point has emerged in both CMS and ATLAS at $m_h \approx 125$ GeV, with surpluses of events being registered in multiple channels by both experiments. Although the statistical significance in each case is below $3\sigma$ once look-elsewhere effects are included, it is certainly interesting to consider the shape of the total likelihood in this neighborhood. We show the result of this exercise in Fig. 9 for $m_h = 125$ GeV.

The plot on the left shows the best fit in the plane $(a, c)$ obtained with our method using the CMS data ($\int dt L \leq 4.8$ fb$^{-1}$) [24]. The posterior probability has two peaks, which indicate two solutions preferred by the current data. The first maximum is for $(a \approx 0.9, c \approx -1.2)$ and has the highest probability. It corresponds to a solution for $(a, c)$ that leads to an enhanced yield in $\gamma\gamma$ and a slight suppression in $WW, ZZ$ compared to the SM expectation. It is useful to define the ratio

$$R_i \equiv \frac{\sigma \times BR(i)}{[\sigma \times BR(i)]_{SM}},$$

where $\sigma$ stands for the Higgs total production cross section (i.e. summed over all production modes), which indicates the change in the signal yield compared to its SM value for an inclusive search in the channel $i$. For $(a = 0.9, c = -1.2)$ one has $R_{\gamma\gamma} \simeq 2.3$ and $R_{WW} = R_{ZZ} \simeq 0.86$. The enhancement in $\gamma\gamma$ follows from the constructive interference in the relative decay width, $\Gamma(\gamma\gamma) \propto |1.8 c - 8.3 a|^2$, that arises for negative $c$. An enhanced yield in $\gamma\gamma$ and a slight suppression of $WW, ZZ$ is in fact exactly what the best fit of the individual
Figure 9: Isocontours of 68%, 95% and 99% probability in the plane \((a,c)\) for a 125 GeV Higgs coming from CMS (left) and ATLAS (right). In each case the posterior probability has been constructed using the method described in sec. \[3\].

channels performed by CMS also points to (see Fig. 4 of Ref. \[24\]). We thus find that such a pattern of rates can be easily reproduced for \(c \sim -1\), which ensures an enhanced \(\gamma\gamma\) while predicting a gluon fusion production cross section close to its SM value. The second maximum of the probability is for \((a \simeq 1.15, c \simeq 1.0)\). It is smaller than the first peak, as the shorter isocontours indicate. This solution roughly corresponds to the combined best fit of CMS where all rates are 20% – 30% larger than their SM expectations \((R_{\gamma\gamma} \simeq 1.4\) and \(R_{WW} = R_{ZZ} \simeq 1.3\) for \((a = 1.15, c = 1.0)\)). While the maximum at \(c \simeq 1\) already emerges from the fit when including the channels \(WW\), \(ZZ\) and \(\gamma\gamma\) alone, we find that the \(\tau\tau\) search plays an important role in shaping the highest peak and excluding points with large and negative \(c\).

The plot on the right of Fig. 9 shows the best fit in the plane \((a,c)\) obtained using the full 2011 ATLAS data set \((\int dt \mathcal{L} \leq 4.9 \text{ fb}^{-1})\) \[28\]. Compared to the corresponding analysis of CMS, the sensitivity of the \(h \rightarrow WW\) inclusive search in ATLAS (in which the 2-jet VBF category is not singled out) is much weaker in the fermiophobic region \(c \sim 0\). This implies a much broader region where the posterior probability is large, instead of two disconnected smaller islands. Furthermore, the excess in the \(ZZ\) channel seen by ATLAS leads to a best fit
for \((a \simeq 1.5, c \simeq 0.45)\), which corresponds to \(R_{\gamma\gamma} \simeq 2.0, R_{WW} = R_{ZZ} \simeq 1.4\). Notice that in this case the enhancement of the \(\gamma\gamma\) rate, as well as that of \(WW\) and \(ZZ\), follows from \(a > 1\). In fact, this can be obtained only in specific UV completions of the effective Lagrangian \((2.2)\), see Refs. \([15, 29]\). If confirmed, it would thus be a strong hint on the nature and the role of the Higgs. On the other hand, another way to obtain an enhanced rate in all channels except \(b\bar{b}\) is that of suppressing the total Higgs decay width by having \(c_b < 1\). \(^{11}\) This solution is not accessible in our 2-dimensional fit where all the fermion couplings were constrained to be the same, but can be naturally realized in particular models: for example, Ref. \([16]\) demonstrates such a possibility in composite models, while the models of Ref. \([31]\) allow for such a solution in a supersymmetric setting at large \(\tan\beta\) and Refs. \([32, 33]\) discuss the more general implications for two Higgs doublet models. As such, having \(c_b < 1\) represents a simple possibility that should be clearly considered when analyzing the data.

Although these preliminary indications from ATLAS and CMS do not yet fit into a coherent picture, it is clear that a simple analysis of the data in terms of the parameters \(a, c\) will represent an important and powerful tool to determine the nature of the Higgs boson, should the hints of its presence at 125 GeV be confirmed. In this regard, we consider it useful to provide the plot of Fig. \([10]\), which shows the isocurves of constant \(R_{\gamma\gamma}\) and \(R_{WW} = R_{ZZ}\) in the plane \((a, c)\) for \(m_h = 125\) GeV. The different solutions preferred by CMS and ATLAS can be easily recognized along the isocurve \(R_{\gamma\gamma} \sim 2\). These solutions cannot be reached by following the trajectories predicted in the composite models MCHM4 and MCHM5 (shown in the plot as short dashed gray curves). In the MCHM5, in particular, there cannot be an enhancement in the yield of an inclusive \(\gamma\gamma\) search. Although in the fermiophobic limit \(\xi \rightarrow 1/2\) the branching fraction to \(\gamma\gamma\) gets enhanced by up to a factor 7, this is more than compensated by the drop in the gluon fusion cross section. At the same time, however, the yield in the VBF subchannel of an exclusive \(\gamma\gamma\) search can be enhanced by up to a factor 3 for \(\xi \sim 1/2\).

The possibility that the enhanced yield in \(\gamma\gamma\) might be due to a fermiophobic Higgs has been recently suggested by Ref. \([34]\). The main support to this idea comes from the latest exclusive analysis of \(\gamma\gamma\) performed by CMS \([22]\), which in fact reports a larger excess in the

\(^{11}\) We thank Riccardo Rattazzi for drawing our attention to this possibility. See also \([30]\) for a discussion.
Figure 10: Isocontours with $R_{\gamma\gamma} = 0.5, 1, 2$ (orange long dashed curves) and $R_{WW} = R_{ZZ} = 0.5, 1, 2$ (continuous back curves) in the plane $(a, c)$ for $m_h = 125$ GeV. The upper (lower) short dashed gray curve is the trajectory predicted in the MCHM4 (MCHM5). The blue dots show the points with $\xi = 0.1, 0.5, 0.8$.

VBF category than in the other four dominated by gluon fusion production. Our global fit of the CMS data in Fig. 9, however, seems to disfavor the fermiophobic solution $(a = 1, c = 0)$. As already mentioned, a dominant role for $c \sim 0$ is played by the exclusive analysis of $h \rightarrow WW$ [26]. Indeed, for $m_h = 125$ GeV the fermiophobic solution $(a = 1, c = 0)$ implies a strong enhancement in the branching ratio of not just the $\gamma\gamma$ channel, but of $WW$ as well (respectively a factor $\sim 6.6$ in $BR(\gamma\gamma)$ and 4.1 in $BR(WW)$). For an inclusive $WW$ search such an increase is more than compensated by a decrease in the gluon fusion production cross section, but this is not the case for a category dominated by events produced through the VBF process. The absence of a substantial excess in the 2-jet category of the $WW$ analysis of CMS is in fact what disfavors a fermiophobic Higgs more strongly in the current data.\footnote{In fact, for both $m_h = 120$ GeV and 130 GeV the 2-jet category has a depletion in the number of observed events compared to the pure background expectation.}

This simple example shows how much more powerful it can be to perform an exclusive analysis instead of an inclusive one when it comes to extracting information of the Higgs
couplings. This is especially true for the $\gamma\gamma$ channel \cite{35}, but also for $WW$ as seen above; we expect for the same to be true for $\tau\tau$ as well. This observation is in fact one of the main points put forward by the authors of Ref. \cite{34}. In this regard we must notice that the published information in \cite{22} was not sufficient to include the $\gamma\gamma$ channel in an exclusive fashion in our fit (only the combined limit over all categories is given in \cite{22}). At the best fit point $(a = 0.9, c = -1.2)$ selected by our fit, we find that the signal yield in a VBF-dominated subchannel (like the 2-jet category of the CMS analysis) is enhanced by a factor $R_{VBF}^{\gamma\gamma} = 1.4$, compared to $R_{\gamma\gamma} = 2.3$ of the inclusive yield. As previously noticed, the best fit of the individual categories done by CMS prefers a larger enhancement in the 2-jet subchannel. This pattern can in fact be easily reproduced for $c$ negative and smaller than $a$ in magnitude. For example, the point $(a = 1, c = -0.8)$ implies $R_{VBF}^{\gamma\gamma} = 3.1$, $R_{\gamma\gamma} = 2.1$. We thus expect that once a fully exclusive inclusion of the $\gamma\gamma$ channel into the fit is performed, the region of maximum probability with $c < 0$ will shrink and the location of the maximum will migrate to smaller values of $|c|$.

6 Conclusions

The majority of the searches for the Higgs boson at the LHC and Tevatron are optimized for the SM Higgs and results are reported accordingly. However, it is of extreme importance to have a broader perspective on the nature of the Higgs boson, especially since the origin of the EW symmetry breaking remains very uncertain. In this work we have shown how a model-independent analysis on the Higgs couplings can be performed already with the current data, and should be carried out in future analyses. The theoretical foundation is that of the EW chiral Lagrangian in eq. (2.2), which relies on three simple assumptions: i) a Higgs-like scalar is the only new light particle in the spectrum, and additional states are much heavier and do not significantly affect the Higgs phenomenology at low energy; ii) the dynamics that breaks the EW symmetry possesses an approximate custodial symmetry; iii) no dangerous tree-level FCNC are mediated by the exchange of the Higgs boson. If needed, the first assumption can be relaxed and additional states can be consistently added to the Lagrangian by following the rules of the chiral expansion.

Depending on the value of the Higgs couplings in eq. (2.2), the phenomenology that
follows can be quite different from that of the SM Higgs. Although eventually one would like to perform a completely general analysis and individually measure as many Higgs couplings as possible, in this work we have considered a simplified though interesting scenario where only two such parameters are free to vary: the coupling of the Higgs to $W$ and $Z$ vector bosons ($a$), and the coupling to fermions ($c$). Some of the simplest composite Higgs theories in fact fall into this class, and we have reported explicit results for two benchmark models: a model $a$ with universal rescaling of the Higgs couplings (such as the MCHM4 and minimal conformal TC), and the MCHM5 model.

A fully consistent use of the current data to constrain the Higgs couplings in eq. (2.2) requires two important pieces of information to be reported by the experimental collaborations:

1. The likelihood for each channel as a function of the signal strength modifier $\mu$

2. The cut efficiencies for each channel and Higgs production mode

Unfortunately this information is not in general provided by ATLAS and CMS. We have however shown that the body of LHC results published on the SM Higgs searches is sufficient to allow one to derive an accurate estimate of the constraint in a more general theory. In particular, we have designed a method to reconstruct the likelihood of each channel once given the expected and observed 95% CL limits on $\mu$. This technique becomes rigorous in the asymptotic limit of large number of counts, and improves on more empirical recipes used in the literature such as combining the limits in inverse quadrature. It has the further advantage of allowing a best fit analysis in the case where a significant excess is observed compared to the pure background expectation.

By using our method we have derived the 95% CL limits implied by the full 2011 data set of CMS on $a$ and $c$, as well on the parameter $\xi = (v/f)^2$ of the composite Higgs models MCHM4 and MCHM5. The results are shown in Figs. 8, 9, 7. We have also performed a best fit analysis of the anomalous point at $m_h = 125$ GeV, for which both CMS and ATLAS have observed a surplus of events in various channels, assuming the excess is due to the presence of the Higgs. The resulting probability contours are reported in the plots of Fig. 9 for CMS and ATLAS respectively. The CMS data seem to prefer a solution with negative $c$, for which the $\gamma\gamma$ decay rate is enhanced while the $WW$ and $ZZ$ rates are close to the SM
Higgs prediction. On the other hand, the large excess of ATLAS both in $\gamma\gamma$ and $ZZ$ seems to point to values $a > 1$. Although the emerging picture is not yet coherent, there are some conclusions which can be already drawn from our analysis.

Perhaps the most important conclusion is that exclusive as opposed to inclusive searches are much more powerful to extract information on the Higgs couplings, especially when the nature of the latter is non-standard. We have demonstrated that this enhanced sensitivity is already evident in the $\gamma\gamma$ and $WW$ channels when comparing the exclusive searches performed by CMS with the inclusive ones carried out by ATLAS. Also, our analysis shows that a broader, model-independent interpretation of the Higgs searches can be performed easily and it should be the starting point to report future results.

The explorative analysis performed in this work makes use of all data which is readily available in each channel and gives robust estimates of the limits currently imposed by the LHC searches on the couplings $a, c$. It cannot be considered, however, as a substitute of the full, exact analysis which can be carried out only through use of the complete experimental information. We hope that such a full model-independent analysis will be performed in the future by the ATLAS and CMS collaborations.

**Acknowledgments**

We thank Daniele Del Re and Shahram Rahatlou for participation in the early stages of this work and for many enlightening discussions and suggestions. We are especially indebted to Emanuele Di Marco for patiently explaining to us the details of the $WW$ analysis of CMS, and for various important discussions and suggestions. It is also a pleasure to thank Giulio D’Agostini, Vittorio Del Duca, Evan Friis, Christophe Grojean, Barbara Mele and Riccardo Rattazzi for stimulating discussions. J.G. is grateful to the theory division at FNAL for their hospitality during the completion of this project. The work of R.C. was partly supported by the ERC Advanced Grant No. 267985 *Electroweak Symmetry Breaking, Flavour and Dark Matter: One Solution for Three Mysteries (DaMeSyFla)*.
References

[1] D. B. Kaplan and H. Georgi, Phys. Lett. B 136 (1984) 183. S. Dimopoulos and J. Preskill, Nucl. Phys. B 199, 206 (1982). T. Banks, Nucl. Phys. B 243, 125 (1984). D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B 136, 187 (1984). H. Georgi, D. B. Kaplan and P. Galison, Phys. Lett. B 143, 152 (1984). H. Georgi and D. B. Kaplan, Phys. Lett. B 145, 216 (1984). M. J. Dugan, H. Georgi and D. B. Kaplan, Nucl. Phys. B 254, 299 (1985).

[2] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein and K. E. Williams, Comput. Phys. Commun. 181 (2010) 138 [arXiv:0811.4169 [hep-ph]]; Comput. Phys. Commun. 182 (2011) 2605 [arXiv:1102.1898 [hep-ph]].

[3] M. Duhrssen, “Prospects for the measurement of Higgs boson coupling parameters in the mass range from 110 – 190 GeV/c^2”, ATL-PHYS-2003-030.

[4] M. Duhrssen, S. Heinemeyer, H. Logan, D. Rainwater, G. Weiglein and D. Zeppenfeld, Phys. Rev. D 70 (2004) 113009 [arXiv:hep-ph/0406323].

[5] R. Lafaye, T. Plehn, M. Rauch, D. Zerwas and M. Duhrssen, JHEP 0908 (2009) 009 [arXiv:0904.3866 [hep-ph]].

[6] F. Bonnet, M. B. Gavela, T. Ota and W. Winter, [arXiv:1105.5140 [hep-ph]].

[7] J. R. Espinosa, C. Grojean and M. Muhlleitner, JHEP 1005 (2010) 065 [arXiv:1003.3251 [hep-ph]]; [arXiv:1202.1286 [hep-ph]].

[8] S. Bock, R. Lafaye, T. Plehn, M. Rauch, D. Zerwas and P. M. Zerwas, Phys. Lett. B 694 (2010) 44 [arXiv:1007.2645 [hep-ph]].

[9] R. Contino, C. Grojean, M. Moretti, F. Piccinini and R. Rattazzi, JHEP 1005 (2010) 089 [arXiv:1002.1011 [hep-ph]].

[10] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719 (2005) 165 [arXiv:hep-ph/0412089].
[11] R. Contino, L. Da Rold and A. Pomarol, Phys. Rev. D 75 (2007) 055014 [arXiv:hep-ph/0612048].

[12] J. Galloway, J. A. Evans, M. A. Luty and R. A. Tacchi, JHEP 1010 (2010) 086 [arXiv:1001.1361 [hep-ph]].

[13] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP 0706 (2007) 045 [arXiv:hep-ph/0703164].

[14] E. Halyo, Mod. Phys. Lett. A 8 (1993) 275. W. D. Goldberger, B. Grinstein and W. Skiba, Phys. Rev. Lett. 100 (2008) 111802 [arXiv:0708.1463 [hep-ph]]; L. Vecchi, Phys. Rev. D 82 (2010) 076009 [arXiv:1002.1721 [hep-ph]]; B. A. Campbell, J. Ellis and K. A. Olive, arXiv:1111.4495 [hep-ph].

[15] I. Low, R. Rattazzi and A. Vichi, JHEP 1004 (2010) 126 [arXiv:0907.5413 [hep-ph]].

[16] A. Azatov and J. Galloway, arXiv:1110.5646 [hep-ph].

[17] M. E. Peskin and T. Takeuchi, Phys. Rev. D 46 (1992) 381.

[18] R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, Nucl. Phys. B 703 (2004) 127 [hep-ph/0405040].

[19] Alessandro Strumia, private communication. The data used in the fit are those of LEP1 (see Table 2 of Ref. [18]), and those from Atomic Parity Violation (APV) (see Ref. [18], Table 3). See also: K. Agashe and R. Contino, Nucl. Phys. B 742 (2006) 59 [hep-ph/0510164].

[20] ATLAS and CMS Collaborations and LHC Higgs Combination Group, Procedure for the LHC Higgs boson search combination in Summer 2011, CMS-NOTE-2011/005; ATL-PHYS-PUB-2011-11 (2011).

[21] G. D’Agostini, “Bayesian reasoning in data analysis: A critical introduction,” New Jersey, USA: World Scientific (2003) 329 p

[22] CMS Collaboration, “Search for the standard model Higgs boson decaying into two photons in pp collisions at sqrt(s)=7 TeV,” arXiv:1202.1487 [hep-ex].
[23] G. Cowan, “Statistical data analysis,” Oxford, UK: Clarendon (1998) 197 p.

[24] CMS Collaboration, “Combined results of searches for the standard model Higgs boson in pp collisions at sqrt(s) = 7 TeV” arXiv:1202.1488 [hep-ex].

[25] CMS Collaboration, “Combination of CMS searches for a Standard Model Higgs boson” CMS-PAS HIG-11-032

[26] S. Chatrchyan et al. [CMS Collaboration], “Search for the standard model Higgs boson decaying to a W pair in the fully leptonic final state in pp collisions at sqrt(s) = 7 TeV,” arXiv:1202.1489 [hep-ex].

[27] “Search for the Higgs Boson in the Fully Leptonic W^+W^- Final State” CMS-PAS-HIG-11-024; https://twiki.cern.ch/twiki/bin/view/CMSPublic/Higgs11024TWiki.

[28] ATLAS Collaboration, “Combined search for the Standard Model Higgs boson using up to 4.9 fb-1 of pp collision data at sqrt(s) = 7 TeV with the ATLAS detector at the LHC,” arXiv:1202.1408 [hep-ex].

[29] A. Falkowski, S. Rychkov and A. Urbano, arXiv:1202.1532 [hep-ph].

[30] R. Rattazzi, talk given at the ETH workshop “Higgs Searches confronts theory”, 9-11 January 2012, Zurich

[31] A. Azatov, J. Galloway and M. A. Luty, Phys. Rev. Lett. 108, 041802 (2012) arXiv:1106.3346 [hep-ph]; A. Azatov, J. Galloway and M. A. Luty, Phys. Rev. D 85, 015018 (2012) arXiv:1106.4815 [hep-ph].

[32] A. G. Akeroyd, J. Phys. G G 24, 1983 (1998) [hep-ph/9803324].

[33] P. M. Ferreira, R. Santos, M. Sher and J. P. Silva, arXiv:1112.3277 [hep-ph].

[34] E. Gabrielli, B. Mele and M. Raidal, arXiv:1202.1796 [hep-ph].

[35] A. Azatov, R. Contino, D. Del Re, J. Galloway, M. Grassi and S. Rahatlou, arXiv:1204.4817 [hep-ph].