Random transverse field spin-glass model on the Cayley tree: phase transition between the two many-body-localized phases

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Abstract. The quantum Ising model with random couplings and random transverse fields on the Cayley tree is studied by real-space-renormalization in order to construct the whole set of eigenstates. The renormalization rules are analyzed via large deviations. The phase transition between the paramagnetic and the spin-glass many-body-localized phases involves the activated exponent $\psi = 1$ and the correlation length exponent $\nu = 1$. The spin-glass-ordered cluster containing $N_{\text{SG}}$ spins is found to be extremely sparse with respect to the total number $N$ of spins: its size grows only logarithmically at the critical point $N_{\text{SG}}^{\text{crit}} \propto \ln N$, and it is sub-extensive $N_{\text{SG}} \propto N^\theta$ in the finite region of the spin-glass phase where the continuously varying exponent $\theta$ remains in the interval $0 < \theta < 1$.

Keywords: many body localization, quantum criticality, quantum disordered systems, quantum phase transitions
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1. Introduction

In the field of many-body-localization (see the recent reviews [1–9] and references therein), one of the important characterization of many-body-localized phases is the existence of an extensive number of local integrals of motion called LIOMs [10–22]. Since these LIOMs are the building blocks of the whole set of eigenstates, it is natural to try to identify them via some real-space renormalization procedure. The strong disorder real-space RG approach developed by Daniel Fisher [23–25] to construct the ground states of random quantum models (see the review [26]) has been thus generalized into the RSRG-X procedure to construct the whole set of excited eigenstates [27–31]: the idea is that each local renormalization step produces a LIOM that describes the choice between the local energy levels (instead of projecting always onto the lowest energy-level). The RSRG-t procedure developed by Vosk and Altman [32, 33] in order to construct the effective dynamics via the iterative elimination of the degree of freedom oscillating with the highest local eigenfrequency is equivalent to the RSRG-X procedure but gives an interesting different point of view [34].

Since the purpose of these strong disorder RG procedures is to produce an extensive number of LIOMs, it is clear that their validity is limited to many-body-localized phases: they cannot be applied in the delocalized ergodic phase, and they do not allow to analyze the MBL transition towards this delocalized phase. In particular, it should be stressed that the current RG descriptions of the MBL delocalization transition are based on completely different RG rules concerning the entanglement [35], the resonances [36, 37], or the decomposition into insulating and thermal blocks [38]. However, the RSRG-X is very useful in MB-localized phases to analyse the long-ranged order of the excited eigenstates made of LIOMs and the possible phase transitions between different many-body-localized phases, as for instance the transition between the paramagnetic and the spin-glass many-body-localized phases for the one-dimensional generalized quantum Ising model [27].

In the present paper, we wish similarly to analyse the transition between the paramagnetic and the spin-glass many-body-localized phases for the quantum Ising model with random couplings and random transverse fields on the Cayley tree. Since the standard RSRG-X procedure destroys the tree structure and could only be followed numerically, we will instead use an RG procedure that preserves the tree structure in order to obtain some simple analytical approximation: the Pacheco-Fernandez block-RG introduced for the ground state of the one-dimensional chain without disorder [39, 40] or with disorder [41–43] is applied here sequentially [44] around the center of the tree in order to construct the whole set of eigenstates.

The paper is organized as follows. In section 2, the real-space RG procedure to construct the set of eigenstates of the random quantum Ising model on the Cayley tree is described. Section 3 is devoted to the large deviation properties of the basic variables that appear in the RG flows. The statistics of the renormalized couplings and of the renormalized transverse field of the center are studied in sections 4 and 5 respectively in order to characterize the critical properties of the transition between the paramagnetic and spin-glass many-body-localized phases. Our conclusions are summarized in section 6.
2. Real-space RG procedure to construct the set of eigenstates

2.1. Model

We consider the geometry of a Cayley tree of branching ratio $K$ with $L$ generations around the central spin $\sigma_0$. It is convenient to decompose the quantum Ising Hamiltonian in terms of the contributions of the various generations

$$H = \sum_{r=0}^{L} H_r,$$

$$H_0 = h_0 \sigma^z_0,$$

$$H_1 = \sum_{i_1=1}^{K+1} (J_{i_1} \sigma^x_0 \sigma^x_{i_1} + h_{i_1} \sigma^z_{i_1}),$$

$$H_2 = \sum_{i_1=1}^{K+1} \sum_{i_2=1}^{K} (J_{i_1,i_2} \sigma^x_{i_1} \sigma^x_{i_1,i_2} + h_{i_1,i_2} \sigma^z_{i_1,i_2}),$$

$$H_r = \sum_{i_1=1}^{K+1} \sum_{i_2=1}^{K} \cdots \sum_{i_r=1}^{K} (J_{i_1,i_2,\ldots,i_r} \sigma^x_{i_1,\ldots,i_r-1} \sigma^x_{i_1,\ldots,i_r} + h_{i_1,\ldots,i_r} \sigma^z_{i_1,\ldots,i_r}).$$

(1)

We consider that both the transverse fields $h_{i_1,i_2,\ldots,i_r}$ and the couplings $J_{i_1,i_2,\ldots,i_r}$ are random variables drawn with some continuous distributions. As example, we will focus on the case where the probability distributions of the couplings $J_{i_1,i_2,\ldots,i_r}$ and of the random fields $h_{i_1,i_2,\ldots,i_r}$ are uniformly drawn on $[-J, +J]$ and $[-h, +h]$ respectively.

$$\pi_J(J_{i_1,i_2,\ldots,i_r}) = \frac{\theta(-J \leq J_{i_1,i_2,\ldots,i_r} \leq J)}{2J},$$

$$\pi_h(h_{i_1,i_2,\ldots,i_r}) = \frac{\theta(-h \leq h_{i_1,i_2,\ldots,i_r} \leq h)}{2h}.$$  

(2)

This model is MB-localized in the two following limits:

(i) When all couplings vanish $J_{i_1,i_2,\ldots,i_r} \to 0$, the model is in a trivial paramagnetic MB-localized phase, where the LIOMs (commuting with themselves and the Hamiltonian) are the site operators $\sigma^z_{i_1,i_2,\ldots,i_r}$ that are coupled to the random fields $h_{i_1,\ldots,i_r}$

$$\tau^z_{i_1,i_2,\ldots,i_r} \overset{J_{1} \to 0}{\sim} \sigma^z_{i_1,i_2,\ldots,i_r}.$$  

(3)

Since all the random fields $h_{i_1,\ldots,i_r}$ are different, there is no degeneracy between the many-body-energy-levels, and the LIOMs of equation (3) are perturbatively stable in the presence of small couplings $J_{1}$. Note the difference with the model studied in [45] where the transverse fields all take the same value $h$, so that the zero-coupling model is the pure paramagnetic model characterized by huge degeneracies in many-body-energy-levels.

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(ii) In the opposite limit where all fields vanish \( h_{i_1, i_2, \ldots, i_r} \to 0 \), the model is in a trivial spin-glass localized phase, where the LIOMs are the bond operators \( \sigma^x_{i_1, \ldots, i_{r-1}} \sigma^x_{i_1, \ldots, i_r} \) associated to the random couplings \( J_{i_1, \ldots, i_r} \).

\[
\tau^z_{i_1, i_2, \ldots, i_r} \sim h_{i_1} \to 0 \sigma^x_{i_1, \ldots, i_{r-1}} \sigma^x_{i_1, \ldots, i_r}.
\]

(4)

Since all the couplings \( J_{i_1, \ldots, i_r} \) are different, the many-body-energy-levels are non-degenerate, and the LIOMs of equation (4) are perturbatively stable in the presence of small fields \( h_{i_1} \). Note again the difference with the model studied in [45] where the couplings only take the two values \((\pm J)\), leading to huge degeneracies in energy many-body-energy-levels.

In summary, the continuous distributions of both random fields and couplings is necessary to avoid degeneracies between many-body-energy-levels and to identify simple LIOMs in the two limits of vanishing couplings (equation (3)) or vanishing fields (equation (4)). Since these two type of LIOMs correspond to different long-ranged order for the corresponding eigenstates, namely paramagnetic and spin-glass, one expects that the full model containing both fields and couplings will display a phase transition between two different many-body-localized phases (paramagnetic and spin-glass). The goal of the present paper is to analyse this transition via some real-space procedure that constructs the LIOMs and thus the set of eigenstates.

2.2. First RG step

The RSRG-X procedure mentioned in the Introduction can be applied in \( d > 1 \), but the changes of the geometry prevents the finding of any analytical description. The renormalization procedure has to be implemented numerically, as was done for the RSRG procedure concerning the ground state in \( d = 2, 3, 4 \) [25, 46–55]. Here we wish instead to obtain some analytically solvable RG procedure in order to get more insight into the mechanism of the transition. We have thus chosen to apply sequentially [44] around the center of the tree the idea of the Pacheco-Fernandez elementary step [39–43] in order to keep a simple geometry along the RG flow.

More precisely, the first RG step consists in the diagonalization of the Hamiltonian \( H_1 \) equation (1) concerning the center spin and the \((K + 1)\) spins of the first generation

\[
H_1 = \sum_{i_1=1}^{K+1} \left( J_{i_1} \sigma_0^x \sigma_{i_1}^x + h_{i_1} \sigma_{i_1}^z \right).
\]

(5)

Since \( H_1 \) commutes with \( \sigma_0^x \), one needs to consider the two possible values \( \sigma_0^x = S_0^x = \pm 1 \), and to diagonalize the \((K + 1)\) remaining effective Hamiltonians involving the single spin \( \sigma_{i_1} \)

\[
H_{i_1}^{eff} = J_{i_1} S_0^x \sigma_{i_1}^x + h_{i_1} \sigma_{i_1}^z.
\]

(6)

The two eigenvalues of equation (6) do not depend on the value \( S_0^x = \pm \) and read

\[
\lambda^{(\tau_{i_1}^z)} = \tau_{i_1}^z \sqrt{J_{i_1}^2 + h_{i_1}^2}.
\]

(7)
where the variable
\[ \tau_{i}^{z} = \pm \]
labels the choice between the positive or negative energy in equation (7). The corresponding eigenvectors depend on the value \( S_{0}^{x} \)
\[ |\lambda_{i}^{(\tau_{i}^{z})} (S_{0}^{x}) \rangle = \sqrt{\frac{1}{2} \left( 1 + \frac{\tau_{i}^{z} S_{0}^{x} J_{i}}{\sqrt{J_{i}^{2} + h_{i}^{2}}} \right)} |\sigma_{i}^{x} = + \rangle + \tau_{i}^{z} \text{sgn}(h_{i}) \sqrt{\frac{1}{2} \left( 1 - \frac{\tau_{i}^{z} S_{0}^{x} J_{i}}{\sqrt{J_{i}^{2} + h_{i}^{2}}} \right)} |\sigma_{i}^{x} = - \rangle. \] (9)

To make the link with the Lioms of equations (3) and (4), it is useful to consider the two corresponding limits:

(i) if the coupling vanishes \( J_{i} = 0 \), the eigenvalues and eigenvectors reduce to
\[ \lambda_{i}^{(\tau_{i}^{z})} \approx \tau_{i}^{z} |h_{i}| = \tau_{i}^{z} \text{sgn}(h_{i}) h_{i} \]
\[ |\lambda_{i}^{(\tau_{i}^{z})} (S_{0}^{x}) \rangle \approx \frac{1}{2} |\sigma_{i}^{x} = + \rangle + \tau_{i}^{z} \text{sgn}(h_{i}) |\sigma_{i}^{x} = - \rangle \]
\[ = |\sigma_{i}^{z} = \tau_{i}^{z} \text{sgn}(h_{i}) \rangle \] (10)

that is equivalent to equation (3) up to the factor \( \text{sgn}(h_{i}) \) that comes from the choice of equation (8) to label the sign of the energy of equation (7).

(ii) if the field vanishes \( h_{i} = 0 \), the eigenvalues and eigenvectors become
\[ \lambda_{i}^{(\tau_{i}^{z})} \approx \tau_{i}^{z} |J_{i}| = \tau_{i}^{z} \text{sgn}(J_{i}) J_{i} \]
\[ |\lambda_{i}^{(\tau_{i}^{z})} (S_{0}^{x}) \rangle \approx \frac{1}{2} \sqrt{\frac{1 + \tau_{i}^{z} S_{0}^{x} \text{sgn}(J_{i})}{2}} |\sigma_{i}^{x} = + \rangle + \tau_{i}^{z} \text{sgn}(h_{i}) \sqrt{\frac{1 + \tau_{i}^{z} S_{0}^{x} \text{sgn}(J_{i})}{2}} |\sigma_{i}^{x} = - \rangle \]
\[ = |\sigma_{i}^{z} = \tau_{i}^{z} S_{0}^{x} \text{sgn}(J_{i}) \rangle \] (11)

that is equivalent to equation (4) up to the factor \( \text{sgn}(J_{i}) \) that comes from the choice of equation (8) to label the sign of the energy of equation (7).

When the coupling \( J_{i} \) and the field \( h_{i} \) are both non-vanishing, the LIOM \( \tau_{i}^{z} \) defined by equations (7) and (9) can be thus considered as the appropriate interpolation between these two simple limits (i) and (ii). Note that in usual strong-disorder RG rules for MB-localized phases [27], each LIOM is declared to be associated either to a site variable as in (i) (if its renormalized transverse field is the biggest among surviving variables) or to a bond variable as in (ii) (if its renormalized coupling is the biggest among surviving variables), so that each LIOM could be called accordingly ‘paramagnetic’ or ‘spin-glass’. On the contrary, within the present procedure, the LIOM \( \tau_{i}^{z} \) is some interpolation between (i) and (ii) as in the block-RG procedures of [20], and thus cannot be called ‘paramagnetic’ or ‘spin-glass’ in itself.

Let us now return to the whole Hamiltonian \( H_{1} \) of equation (5): the \( 2^{K+1} \) energy-levels labelled by the variables \( \tau_{1}^{z}, ... \tau_{K+1}^{z} \)

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are independent of $S_0^z = \pm 1$. To label this degeneracy, it is thus convenient to introduce the renormalized spin $\sigma_R^{z0}$

$$
|\tau^1_1 \cdots \tau^2_{K+1}; \sigma_R^{z0} = +1\rangle = |S_0^z = +1\rangle \otimes^{K+1}_{i=1} |\lambda^{(\tau^1_1)}_{i_1} (S_0^z = +1)\rangle
$$

$$
|\tau^1_1 \cdots \tau^2_{K+1}; \sigma_R^{z0} = -1\rangle = |S_0^z = -1\rangle \otimes^{K+1}_{i=1} |\lambda^{(\tau^1_1)}_{i_1} (S_0^z = -1)\rangle.
$$

(13)

The projector onto the energy-level $E^{(\tau^1_1 \cdots \tau^2_{K+1})}_1$ then reads

$$
P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 = |\tau^1_1 \cdots \tau^2_{K+1}; \sigma_R^{z0} = +1\rangle \langle \tau^1_1 \cdots \tau^2_{K+1}; \sigma_R^{z0} = +1| + |\tau^1_1 \cdots \tau^2_{K+1}; \sigma_R^{z0} = -1\rangle \langle \tau^1_1 \cdots \tau^2_{K+1}; \sigma_R^{z0} = -1|.
$$

(14)

The projection onto the energy-level $E^{(\tau^1_1 \cdots \tau^2_{K+1})}_1$ of the Hamiltonian of equation (1) concerning the whole tree can be obtained from the various contributions

$$
P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 H P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 = \sum_{r=0}^L P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 H_r P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1.
$$

(15)

The projection of $H_1$ is simply the energy $E^{(\tau^1_1 \cdots \tau^2_{K+1})}_1$ by construction

$$
P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 H_1 P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 = E^{(\tau^1_1 \cdots \tau^2_{K+1})}_1
$$

(16)

while the projection of $H_r$ is unchanged for $r \geq 3$

$$
P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 H_r P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 = H_r.
$$

(17)

The projection of $H_0$

$$
P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 H_0 P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 = h_0 P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 \sigma_0^z P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1
$$

$$
= h_0 \left( \prod_{i_1=1}^{K+1} \sqrt{\frac{h^2_{1_1}}{J^2_{1_1} + h^2_{1_1}}} \right) P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 \sigma_0^z P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1
$$

(18)

gives the renormalized transverse field $h_0^R$ associated to the renormalized spin operator $\sigma_0^z$

$$
h_0^R = h_0 \prod_{i_1=1}^{K+1} \sqrt{\frac{h^2_{1_1}}{J^2_{1_1} + h^2_{1_1}}}.
$$

(19)

The projection of $H_2$

$$
P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 H_2 P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 = \sum_{i_1=1}^{K+1} \sum_{i_2=1}^{K} \left( J_{i_1,i_2} \left( P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 \sigma_{i_1}^x P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 \sigma_{i_2}^x + h_{i_1,i_2} \sigma_{i_1,i_2}^z \right) \right)
$$

$$
= \sum_{i_1=1}^{K+1} \sum_{i_2=1}^{K} \left( J_{i_1,i_2} \frac{\tau^2_{i_1,i_2}}{\sqrt{J^2_{i_1,i_2} + h^2_{i_1,i_2}}} \left( P^{(\tau^1_1 \cdots \tau^2_{K+1})}_1 \sigma_{i_1,i_2}^x \right) \sigma_{i_1,i_2}^x + h_{i_1,i_2} \sigma_{i_1,i_2}^z \right)
$$

(20)
gives the renormalized coupling between the operators $\sigma^x_{R0}$ and $\sigma^x_{i_1,i_2}$

$$J^R_{i_1,i_2} = J_{i_1,i_2} \frac{\tau^z_{i_1,i_2} J_{i_1}}{J^2_{i_1} + h^2_{i_1}}.$$  \hfill (21)

### 2.3. RG rules

The iteration of the above procedure yields the following RG rules after $r$ RG steps. The renormalized transverse field $h^R_0$ associated to the renormalized spin operator $\sigma^x_{R=0}$ evolves according to (equation (19))

$$h^R_0 = h^{R-1}_0 \prod_{i_1=1}^{K} \prod_{i_2=1}^{K} \prod_{i_r=1}^{K} \sqrt{\frac{h^2_{i_1,i_r} h^2_{i_2,i_r} \cdots h^2_{i_r,i_r}}{h^2_{i_1,i_r} + [J^R_{i_1,i_r}]^2}}.$$  \hfill (22)

while the renormalized coupling between the operators $\sigma^x_{R=0}$ and $\sigma^x_{i_1,i_2,\ldots,i_r+1}$ reads (equation (21))

$$J^R_{i_1,\ldots,i_r+1} = J_{i_1,\ldots,i_r+1} \frac{\tau^z_{i_1,\ldots,i_r} J^R_{i_1,\ldots,i_r}}{\sqrt{h^2_{i_1,i_r} + [J^R_{i_1,i_r}]^2}}.$$  \hfill (23)

### 2.4. Solution of the RG rules

The RG rule of equation (23) for the couplings involve only the initial transverse fields and not the renormalized transversed fields, so that it can be solved independently. The sign

$$\text{sgn}(J^R_{i_1,\ldots,i_r+1}) = \tau^z_{i_1,\ldots,i_r} \text{sgn}(J_{i_1,\ldots,i_r+1}) \text{sgn}(J^R_{i_1,\ldots,i_r})$$

$$= \tau^z_{i_1,\ldots,i_r} \tau^z_{i_1,\ldots,i_{r-1}} \cdots \tau^z_{i_1} \text{sgn}(J_{i_1,\ldots,i_r}) \ldots \text{sgn}(J_{i_1})$$  \hfill (24)

is simply the product of all the couplings $J$ and of all the variables $\tau^z$ along the path between the sites 0 and $(i_1, \ldots, i_r)$.

The absolute value reads (equation (23))

$$|J^R_{i_1,\ldots,i_r+1}| = |J_{i_1,\ldots,i_r+1}| C_{i_1,\ldots,i_r}$$  \hfill (25)

where

$$C_{i_1,\ldots,i_r} = \left[ 1 + \sum_{m=1}^{r} \sum_{k=m}^{r} \frac{h^2_{i_1,\ldots,i_k}}{J^2_{i_1,\ldots,i_k}} \right]^{-\frac{1}{2}}$$

$$= \left[ 1 + \frac{h^2_{i_1,i_r}}{J^2_{i_1,i_r}} + \frac{h^2_{i_1,i_r} h^2_{i_2,i_r}}{J^2_{i_1,i_r} J^2_{i_2,i_r}} + \cdots + \frac{h^2_{i_1,i_r} h^2_{i_2,i_r} \cdots h^2_{i_r,i_r}}{J^2_{i_1,i_r} J^2_{i_2,i_r} \cdots J^2_{i_r,i_r}} \right]^{-\frac{1}{2}}.$$  \hfill (26)

involves in the denominator a so-called Kesten random variable [56–60] that has been much studied in relation with the surface magnetization in the ground-state of the one-dimensional chain [26, 44, 61].

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This solution for the renormalized couplings can be plugged into the RG flow of equation (22) for the renormalized transverse field to obtain

\[
\ln \left( \frac{h_0^{Rr}}{h_0^{Rr-1}} \right) = \sum_{i_1=1}^{K+1} \sum_{i_2=1}^K \cdots \sum_{i_r=1}^K \ln \left( \frac{1}{\sqrt{1 + \frac{J_{i_1 \ldots i_r}^2}{h_{i_1 \ldots i_r}^2} C_{i_1 \ldots i_r}^2}} \right)
\]

\[
= \frac{1}{2} \sum_{i_1=1}^{K+1} \sum_{i_2=1}^K \cdots \sum_{i_r=1}^K \ln \left( 1 - C_{i_1 \ldots i_r}^2 \right)
\]

in terms of the Kesten variables of equation (26).

2.5. Reminder on the one-dimensional chain $K = 1$

For the one-dimensional chain corresponding to $K = 1$, the location of paramagnetic/spin-glass quantum phase transition for the ground state of the quantum Ising model is known to occur exactly at

Critical point in one dimension : \( \ln |J_i| = \ln |h_i| \)

(28)

as a consequence of self-duality [24, 26, 62]. The corresponding strong disorder fixed point [24] is characterized in particular by the activated exponent

\( \psi^{(d=1)} = \frac{1}{2} \)

(29)

and by the two correlation length exponents

\( \nu^{(d=1)}_{\text{typ}} = 1 \)

\( \nu^{(d=1)}_{\text{av}} = 2 \).

(30)

As discussed in [20], the phase transition between the paramagnetic and spin-glass many-body-localized phases for the excited eigenstates is the same as the ground state quantum phase transition just described, and the above renormalization procedure is able to reproduce the exact transition location of equation (28) and the exact critical exponents of equations (29) and (30), together with the exact surface magnetization in terms of Kesten variables as already mentioned above (equation (26)).

2.6. Solution at lowest order in the couplings for the Cayley tree with branching ratio $K > 1$

We have just recalled that in one dimension, the transition occurs when the typical coupling and the typical fields are equal (equation (28)). For the Cayley tree with branching ratio $K > 1$, the transition is thus expected to occur in the region

\( \ln |J_i| < \ln |h_i| \)

(31)

where the couplings are typically smaller than the transverse fields. To analyse the RG rules in this region, it is convenient to introduce the products
In the region of equation (31), the Kesten variable of the denominator in equation (26) is dominated by the last term, while it is convenient to keep the term unity to maintain the important bound $C_{i_1,\ldots,i_r} \leq 1$, so that we make the following approximation at lowest order in the couplings

$$C_{i_1,\ldots,i_r} \simeq \left( 1 + \frac{1}{P_{i_1,\ldots,i_r}^2} \right)^{-\frac{1}{2}} \frac{P_{i_1,\ldots,i_r}}{\sqrt{1 + P_{i_1,\ldots,i_r}^2}}.$$  

(33)

Then the absolute values of the renormalized couplings of equation (25) become

$$|J_{i_1,\ldots,i_r}^{R,r+1}| = |J_{i_1,\ldots,i_r+1}| \frac{P_{i_1,\ldots,i_r}}{\sqrt{1 + P_{i_1,\ldots,i_r}^2}}.$$  

(34)

For the ground state, the result $|J_{i_1,\ldots,i_{n+1}}| P_{i_1,\ldots,i_r}$ (i.e. without the denominator $\sqrt{1 + P_{i_1,\ldots,i_r}^2}$) that involves the product of all couplings in the numerator and all the transverse fields in the denominator has been obtained in the paramagnetic phase via various approaches including the cavity-mean-field approach [63–65], the strong disorder RG framework when only sites are decimated [66] or simply lowest perturbation theory in the couplings [67].

The approximation of equation (33) yields that the RG flow of equation (27) for the renormalized transverse field becomes

$$\ln \left( \frac{h_0^{R,r}}{h_0^{R,r-1}} \right) \simeq -\frac{1}{2} \sum_{i_1=1}^{K+1} \sum_{i_2=1}^{K} \ldots \sum_{i_r=1}^{K} \ln \left( 1 + P_{i_1,\ldots,i_{r-1},i_r}^2 \right).$$  

(35)

To analyse the statistical properties of the RG flows equations (34) and (35), one needs first to characterize the large deviation properties of the products of equation (32).

3. Large deviation analysis

In this section, we describe the statistical properties of the product of equation (32) with the simplified notation

$$P(r) = \prod_{k=1}^{r} \left| \frac{J_{i_1,\ldots,i_k}}{h_{i_1,\ldots,i_k}} \right|$$  

(36)

where $r$ represents the number of random variables $\left| \frac{J_{i_1,\ldots,i_k}}{h_{i_1,\ldots,i_k}} \right|$ in this product.

3.1. Typical behavior

The logarithm of equation (36) reduces to a sum of random variables
\[ \ln P(r) \approx \sum_{k=1}^{r} (\ln |J_{i_1, \ldots, i_k}| - \ln |h_{i_1, \ldots, i_k}|). \]  

(37)

The central limit theorem thus yields the following typical behavior for large \( r \)
\[ \ln P(r) \approx -r a_0 + \sqrt{r} u \]

(38)

where
\[ a_0 = (\ln |h_i| - \ln |J_i|) \]

(39)

is positive if \( a_0 > 0 \) in the region under study (equation (31)) and governs the typical exponential decay of \( P(r) \), while \( u \) is a Gaussian random variable. For the one-dimensional chain, only this typical behavior is relevant, but here on the Cayley tree of branching ratio \( K > 1 \) where the number of sites at distance \( r \) grows exponentially as \( K^r \) with the distance \( r \), one needs to analyze the large deviations properties.

### 3.2. Large deviations

In the field of large deviations (see the review [68] and references therein), one is interested into the exponentially small probability to see an exponential decay with some coefficient \( a \) different from the typical value \( a_0 \) of equation (39)

\[ \text{Prob}(P(r) \propto e^{-ar}) \propto e^{-rI(a)} \]

(40)

where the rate function \( I(a) \) vanishes at the typical value \( a_0 \) (equation (39))
\[ I(a_0) = 0 \]

(41)

and is strictly positive otherwise \( I(a \neq a_0) > 0 \). The standard way to evaluate the rate function \( I(a) \) is to consider the generalized moments that display the following exponential behavior [68]
\[ P^{2q}(r) = \left( \frac{|J_i|^{2q}}{|h_i|^{2q}} \right)^r = e^{r\lambda(q)} \]

(42)

where
\[ \lambda(q) = \ln \left( \frac{|J_i|^{2q}}{|h_i|^{2q}} \right) \]

(43)

can be explicitly computed from the probability distribution of the couplings \( J_i \) and of the random fields \( h_i \) (see the example below). The evaluation of equation (42) via the saddle-point approximation
\[ P^{2q}(r) \approx \int_0^{+\infty} da e^{-rI(a)} e^{-ar2q} = e^{r(\max_a(-I(a)-2qa))} \]

(44)

yields \( \lambda_q \) in terms of the saddle-point \( a_q \).
The reciprocal Legendre transform yields
\[ I(a) = -\lambda(q) - 2aq_a \]
\[ 0 = \lambda'(q) + 2a. \]  

### 3.3. Explicit example with the two box distributions of equation (2)

Let us now focus on the example where the probability distributions of the couplings and of the random fields are the two box distributions of parameters \( J \) and \( h \) respectively (equation (2)). In the region \( h > J \), the typical decay of the renormalized couplings is governed by (equation (39))
\[ a_0 = \int_0^h \frac{dh_i}{h_i} \ln h_i - \int_0^J \frac{dJ_i}{J_i} \ln J_i = \ln \left( \frac{h}{J} \right) > 0. \]  

The generalized moments of equation (42) converge only in the region \(-1 < 2q < 1\) and equation (43) becomes
\[ e^{\lambda(q)} = \left| \frac{J_i}{h_i} \right|^{2q} = \int_0^J \frac{dJ_i}{J_i} J_i^{2q} \int_0^h \frac{dh_i}{h_i} h_i^{-2q} = \frac{1}{1 - 4q^2} \left( \frac{J}{h} \right)^{2q} = \frac{1}{1 - 4q^2} e^{-2qa_0}. \]

so that the function \( \lambda(q) \) and its derivative read in terms of the typical value \( a_0 \)
\[ \lambda(q) = -2qa_0 - \ln(1 - 4q^2) \]
\[ \lambda'(q) = -2a_0 + \frac{8q}{1 - 4q^2}. \]  

The second equation of the system (46)
\[ 0 = 2a + \lambda'(q_a) = 2(a - a_0) + \frac{8q_a}{1 - 4q_a^2} \]  

leads to the following second-order equation for \( q_a \)
\[ 0 = q_a^2 - \frac{8q_a}{1 - 4q_a^2} - \frac{1}{4}. \]  

The appropriate solution \( q_a \) that tends to \( q_a \to 0 \) when \( a \to a_0 \) reads
\[ q_a = \frac{a_0 - a}{2(1 + \sqrt{1 + (a_0 - a)^2})}. \]

The rate function given by the first equation of the system (46) reads
\[ I(a) = -\lambda(q_a) - 2aq_a = 2q_a(a_0 - a) + \ln(1 - 4q_a^2) = 2q_a(a_0 - a) + \ln\left( \frac{4q_a}{a_0 - a} \right) \]
\[ = \frac{(a_0 - a)^2}{1 + \sqrt{1 + (a_0 - a)^2}} - \ln \left( \frac{1 + \sqrt{1 + (a_0 - a)^2}}{2} \right). \]
4. Statistical properties of the renormalized couplings

In this section, we focus on the absolute values of the renormalized couplings given by equation (34)

\[ |J_{i_1,\ldots,i_{r+1}}^R| = |J_{i_1,\ldots,i_{r+1}}| \frac{P_{i_1,\ldots,i_r}}{\sqrt{1 + P_{i_1,\ldots,i_r}^2}}. \]  

(54)

4.1. Location of the critical point

On the Cayley tree where the number of points at distance \(r\) grows exponentially as \(K^r\), the number of products \(P(r)\) displaying the decay \(P(r) \propto e^{-ar}\) reads (equation (40))

\[ \mathcal{N}(P(r) \propto e^{-ar}) \propto K^r e^{-r(I(a) \theta(a_{\text{min}}) \leq a \leq a_{\text{max}})} \]  

(55)

where the minimum value \(a_{\text{min}}\) and the maximal value \(a_{\text{max}}\) are respectively smaller and bigger than the typical value \(a_0\) and satisfy

\[ I(a_{\text{min}}) = \ln K = I(a_{\text{max}}) \]  

(56)

so that they occur only on a finite number \(O(1)\) of branches, while the typical value \(a_0\) where \(I(a_0) = 0\) occur on an extensive \(O(K^n)\) number of branches.

From equation (54), it is clear that the renormalized coupling \(J(r)\) inherits the exponential decay of \(P(r)\) of equation (55) as long as \(a > 0\), while the region \(a \leq 0\) produces finite renormalized couplings \(O(1)\) so that the critical point corresponds to the vanishing of the minimal value \(a_{\text{min}}\)

\[ a_{\text{crit}}^{\text{min}} = 0 \]  

(57)

or equivalently in terms of the large deviation function \(I(a)\) (equation (56))

\[ I^{\text{crit}}(0) = \ln K. \]  

(58)

For the special case of the box distribution of equation (2), equation (53) yields the following explicit condition in terms of the control parameter \(a_0 = \ln \frac{h}{J}\)

\[ 0 = \ln \left( K \frac{1 + \sqrt{1 + (a_0^{\text{crit}})^2}}{2} \right) - \frac{(a_0^{\text{crit}})^2}{1 + \sqrt{1 + (a_0^{\text{crit}})^2}}. \]  

(59)

4.2. Paramagnetic phase for \(a_{\text{min}} > 0\)

In the paramagnetic phase \(a_{\text{min}} > 0\), all \(K^r\) renormalized couplings decay exponentially with \(a \geq a_{\text{min}} > 0\) (equation (55))

\[ \mathcal{N}(J(r) \propto e^{-ar}) \propto e^{r(I(a) \theta(a_{\text{min}}) \leq a \leq a_{\text{max}})} \]  

(60)
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4.3. **Spin-glass phase for** $a_{\text{min}} < 0$

In the spin-glass phase $a_{\text{min}} < 0$, the $K^r$ renormalized couplings can be split into two groups: the number of finite couplings grows exponentially in $r$ as

$$\mathcal{N}(J(r) \propto O(1)) \propto \int_{a_{\text{min}}}^{0} da e^{r(\ln K - I(a))} \simeq e^{r(\ln K - I(0))} = e^{r(I(a_{\text{min}}) - I(0))}$$

while the other branches are still characterized by exponential decays with exponents $a > 0$

$$\mathcal{N}(J(r) \propto e^{-ar}) \propto e^{r(\ln K - I(a))} \theta(0 < a \leq a_{\text{max}}).$$

This is the first indication that the ordered spin-glass cluster remains very sparse near the critical point, as confirmed by the analysis of the renormalized transverse field in the next section.

5. **Statistical properties of the renormalized transverse field**

In this section, we focus on the RG flow of equation (35) for the renormalized transverse field

$$\ln \left( \frac{h_{0}^{R^{r}}}{h_{0}^{R^{r-1}}} \right) \simeq -\frac{1}{2} \sum_{i_{1}=1}^{K+1} \sum_{i_{2}=1}^{K} \ldots \sum_{i_{r}=1}^{K} \ln \left( 1 + P_{i_{1},\ldots,i_{r-1},i_{r}}^{2} \right)$$

which can be evaluated in terms of the large deviation analysis of equation (55) concerning the $K^r$ products $P(r)$

$$\ln \left( \frac{h_{0}^{R^{r}}}{h_{0}^{R^{r-1}}} \right)_{r \to +\infty} \simeq -\frac{1}{2} \int_{a_{\text{min}}}^{a_{\text{max}}} da e^{r(\ln K - I(a))} \ln \left( 1 + e^{-2ar} \right).$$

5.1. **Paramagnetic phase for** $a_{\text{min}} > 0$

In the paramagnetic phase $a_{\text{min}} > 0$, equation (64) becomes

$$\ln \left( \frac{h_{0}^{R^{r}}}{h_{0}^{R^{r-1}}} \right)_{r \to +\infty} \simeq -\frac{1}{2} \int_{a_{\text{min}}}^{a_{\text{max}}} da e^{r(\ln K - I(a) - 2a)} = -\frac{1}{2} \int_{a_{\text{min}}}^{a_{\text{max}}} da e^{r(I(a_{\text{min}}) - I(a) - 2a)}.$$

The integral is dominated by the lower boundary $a_{\text{min}}$ of the integral, and one obtains the exponential decay

$$\ln \left( \frac{h_{0}^{R^{r}}}{h_{0}^{R^{r-1}}} \right)_{r \to +\infty} \propto -e^{-2a_{\text{min}}r}.$$

By integration, one obtains that $h_{0}^{R^{r}}$ remains finite as $r \to +\infty$

$$\ln \left( \frac{h_{0}^{R^{r}}}{h_{0}} \right)_{r \to +\infty} - \int_{1}^{r} dr' e^{-2a_{\text{min}}r'} \propto -\frac{1 - e^{-2a_{\text{min}}r}}{a_{\text{min}}}.$$
The typical asymptotic value \( h_{\infty}^{R} \) for the renormalized transverse field diverges with the following essential singularity near the transition \( a_{\text{min}} \to a_{\text{crit}} = 0 \)

\[
\ln \left( \frac{h_{\infty}^{R}}{h_0} \right) \propto -\frac{1}{a_{\text{min}}}.
\]

(68)

### 5.2. Spin-glass phase for \( a_{\text{min}} < 0 \)

In the spin-glass phase \( a_{\text{min}} < 0 \), it is convenient to evaluate separately the contributions of the two regions \( a < 0 \) and \( a > 0 \) in the integral of equation (64). The contribution of the region \( a > 0 \) is dominated by the lower boundary \( a = 0 \) of the integral corresponding to an exponentially growing term. The region \( a < 0 \)

\[
\int_{a_{\text{min}}}^{0} da e^{r(\ln K - I(a))} \ln (1 + e^{-2ar}) \simeq \int_{r \to +\infty}^{0} da e^{r(\ln K - I(0))} = e^{r(I(a_{\text{min}}) - I(0))}
\]

(69)

is dominated by the upper boundary \( a = 0 \). So the RG flow of renormalized transverse field of equation (64) is dominated by the exponentially big term of coefficient \( (I(a_{\text{min}}) - I(0)) > 0 \) of equation (69)

\[
\ln \left( \frac{h_{\infty}^{R}}{h_0} \right) \propto \int_{1}^{r} dr' e^{r'(I(a_{\text{min}}) - I(0))} \simeq \frac{e^{r(I(a_{\text{min}}) - I(0))}}{(I(a_{\text{min}}) - I(0))}
\]

(71)

### 5.3. Finite-size scaling in the critical region

The above results for the renormalized transverse field as a function of the radial distance \( r \) can be summarized by the following finite-size scaling form in the critical region

\[
\ln \left( \frac{h_{\infty}^{R}}{h_0} \right) \propto -r^{\psi} G \left( r^{\nu}(J - J_c) \right)
\]

(72)

with the exponent

\[ \psi = 1 \]

(73)

and the correlation length exponent

\[ \nu = 1 \]

(74)

as in many other phase transitions on the Cayley tree. The scaling function \( G(x) \) is constant at the origin \( G(0) = \text{cst} \), behaves as

\[
G(x) \propto \frac{1}{x}
\]

(75)

to reproduce the behavior of equation (68) in the paramagnetic phase \( J < J_c \), and as

\[
G(x) \propto \frac{e^{x} - 1}{x}
\]

(76)

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to reproduce the behavior of equation (71) in the spin-glass phase $J > J_c$.

5.4. Number $N_{\text{SG}}$ of spins involved in this ordered spin-glass cluster

The renormalized transverse field $h_{0}^{R_{r}}$ directly reflects the number $N_{\text{SG}}(r)$ of spins involved in this ordered spin-glass cluster

$$\ln \left( \frac{h_{0}^{R_{r}}}{h_{0}} \right)_{r \to +\infty} \propto -N_{\text{SG}}(r).$$  \hspace{1cm} (77)

In the paramagnetic phase, both remain finite as $r \to +\infty$. In the spin-glass phase, the behavior found in equation (71) for the renormalized transverse field thus confirms the indication of equations (61) and (62) concerning the renormalized couplings: near the critical point, the ordered spin-glass cluster remains very sparse. More precisely, the number $N_{\text{SG}}$ of spins involved in this ordered spin-glass cluster grows exponentially with the distance $r$

$$N_{\text{SG}} \propto e^{r(I(a_{\text{min}})-I(0))} = e^{r(\ln K - I(0))}$$  \hspace{1cm} (78)

but is only sub-extensive with respect to the total number of spins $N = K^{\tau}$

$$N_{\text{SG}} \propto e^{r(\ln K - I(0))} = N^{\theta}$$  \hspace{1cm} (79)

in the whole region of the phase diagram where the continuously varying exponent

$$\theta = 1 - \frac{I(0)}{\ln K} = 1 - \frac{I(0)}{I(a_{\text{min}})}$$  \hspace{1cm} (80)

remains in the interval

$$\theta_{\text{criti}} = 0 < \theta < 1 = \theta_{\text{ext}}.$$  \hspace{1cm} (81)

At criticality, the vanishing exponent $\theta_{\text{criti}} = 0$ corresponds to the logarithmic growth with respect to $N$ (equations (72) and (73))

$$N_{\text{SG}}^{\text{criti}} \propto r = \frac{\ln N}{\ln K}$$  \hspace{1cm} (82)

meaning that only a finite number of the branches sustain the spin-glass order. The location where the spin-glass-ordered cluster becomes extensive $\theta_{\text{ext}} = 1$ corresponds to the vanishing of the large deviation rate function $I_{\text{ext}}^*(0) = 0$, i.e. to the vanishing of the typical value $a_{\text{0}}^{\text{ext}} = 0$ (equation (41)), i.e. to the location of the transition for the one-dimensional chain (equation (28))

$$a_{\text{0}}^{\text{ext}} = 0 = a_{\text{0}}^{\text{criti1d}}.$$  \hspace{1cm} (83)

The finite region of the phase diagram corresponding to equation (81) where the ordered spin-glass cluster remains sub-extensive is somewhat formally reminiscent of the delocalized non-ergodic phase existing in the Anderson localization model defined on the Cayley tree [69–71], i.e. in exactly the same geometry as in the present paper, and for the same technical reasons based on large deviations on the branches of the Cayley tree [69]. But of course the physical meaning of the phases is completely different: in the Anderson localization model, the three phases are localized/non-ergodic-delocalized/
ergodic-delocalized, while in the present study, the three phases are all MB-localized, namely paramagnetic-MBL/SG-MBL with sub-extensive SG-cluster/SG-MBL with extensive SG-cluster.

Let us mention however that the existence of the intermediate delocalized non-ergodic phase remains very controversial for the Anderson localization model on random regular graphs [72–77] or for many-body-localization models [78–82], where an analogy with the Anderson localization transition in an Hilbert space of ‘infinite dimensionality’ has been put forward [69, 83–86], while the properties of the delocalized non-ergodic phase can be explicitly computed in some random matrix models [87–90]. For our present study, these results thus indicate that the intermediate SpinGlass-MBLLocalized phase with sub-extensive SG-cluster found here on the Cayley tree might not exist on other tree-like lattices like random regular graphs.

5.5. Physical meaning of the results

The above results can be summarized as follows (see figure 1).

5.5.1. The two important control parameters. The two important control parameters for the quantum Ising model on the Cayley tree of branching ratio $K$ are
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(1) the typical value of equation (39) where the large deviation function $I(a)$ of equation (40) vanishes $I(a_0) = 0$

$$a_0 \equiv (\ln |h_i| - \ln |J_i|).$$

(2) the minimum value $a_{\min}$ defined as the smaller value $a_{\min} < a_0$ where the large deviation function $I(a)$ of equation (40) takes the value

$$I(a_{\min}) = \ln K.$$

5.5.2. The three possible MB-localized phases.

(a) MB-Localized Phase with extensive spin-glass order

In the region where the typical value $a_0$ of equation (39) is negative

$$a_0 < 0$$

a typical one-dimensional chain would be spin-glass ordered (equation (28)), and thus the whole Cayley tree is also fully ordered with an extensive spin-glass cluster with respect to the total number of spins $N = K^r$

$$N_{SG} \propto K^r = N.$$  

(b) Paramagnetic MB-Localized Phase

In the region where the minimal value $a_{\min}$ is positive

$$0 < a_{\min}$$

the drawing of $K^r$ independent random one-dimensional chains would produce only paramagnetic chains, i.e. even the exponentially-rare best chain would be paramagnetic. Then the whole Cayley tree is also paramagnetic, and the spin-glass cluster around the origin remains finite

$$N_{SG} \propto O(1).$$

(c) MB-Localized Phase with sub-extensive spin-glass order

In the intermediate region

$$a_{\min} < 0 < a_0$$

corresponding to

$$I(a_{\min}) = \ln K > I(0) > I(a_0) = 0$$

the drawing of $K^r$ independent random one-dimensional chains would produce $K^r e^{-rI(0)}$ spin-glass ordered chains, while the other (of order $K^r$) would be para-
magnetic. Then on the Cayley tree, the spin-glass cluster around the origin only contains $K^r e^{-rI(0)}$ leaves out of the $K^r$. So the size of the spin-glass cluster grows exponentially in $r$ but not as rapidly as $N = K^r$, so that it is subextensive

$$N_{SG} \propto e^{r(\ln K - I(0))} = N^\theta$$

where the exponent

$$\theta = 1 - \frac{I(0)}{\ln K} = 1 - \frac{I(0)}{I(a_{\min})}$$

(93)

varies continuously between $\theta_{\text{criti}} = 0$ (corresponding to $a_{\min} = 0$ where the transition towards (b) occurs) and $\theta_{\text{ext}} = 1$ (corresponding to $I(0) = 0$ i.e. $a_0 = 0$ where the transition towards (a) occurs).

6. Conclusion

We have introduced a simple real-space-renormalization procedure in order to construct the whole set of eigenstates for the quantum Ising model with random couplings and random transverse fields on the Cayley tree of branching ratio $K$. The analysis of the renormalization rules via large deviations was described to obtain the critical properties of the phase transition between the paramagnetic and the spin-glass many-body-localized phases. In particular, we have found that the renormalized transverse field of the center site involves the activated exponent $\psi = 1$ and the correlation length exponent $\nu = 1$. The spin-glass-ordered cluster containing $N_{SG}$ spins was found to be extremely sparse with respect to the total number $N \propto K^r$ of spins: its size grows only logarithmically at the critical point $N_{SG} \propto \ln N$, meaning that only a finite number $O(1)$ of the branches are long-ranged-ordered, while the other branches display exponentially decaying correlations. In addition, the size $N_{SG}$ spin-glass-ordered cluster is sub-extensive $N_{SG} \propto N^\theta$ in the finite region of the spin-glass phase where the continuously varying exponent $\theta$ remains in the interval $0 < \theta < 1$.

As a final remark, let us mention that the mere existence of many-body-localized phases in any dimension $d > 1$ has been recently challenged [91–93], the same arguments being also used to claim the impossibility of mobility edges for MBL in $d = 1$ [94] (as opposed to the numerical phase-diagrams found in [95–98]) as well as the impossibility of MBL in the presence of power-law interactions [92] (as opposed to the works [99–106]). It is thus essential to study various MBL models in various dimensions $d > 1$ in order to solve the controversial issue about the influence of the dimension $d$. Many-body-localized phases have been reported in dimension $d = 2$ both numerically [107] and experimentally [108], as well as on random regular graphs [45] or in the mean-field quantum random energy model [109, 110]. We thus hope that the present work concerning many-body-localized phases on the Cayley tree of effective infinite dimension $d = \infty$ will motivate future studies on this topic.
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