Off-shell String Physics*

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ABSTRACT

Recent advances in non-critical string theory allow a unique continuation, preserving conformal invariance, of critical Polyakov string amplitudes to off-shell momenta. These continuations possess unusual, apparently stringy, characteristics, which are unlikely to be reproduced in a string field theory. Thus our results may be an indication that some fundamentally new formulation, other than string field theory, will be required to extend our understanding of critical strings beyond the Polyakov path integral. Three-point functions are explicitly calculated. The tree-level effective potential is computed for the tachyon.

1. Introduction

Off-shell amplitudes are of great physical interest in string theory, just as in field theory. They are essential for the derivation of effective actions, e.g., the derivation of effective potentials for particles such as the tachyon and the dilaton, free from the ambiguities of on-shell trivial field redefinitions. They can be used to derive measures for integrating over moduli of space-time instantons in string theory, and for the calculation of hadronic form-factors when one attempts to interpret certain aspects of quantum chromodynamics in terms of effective string theories. Naturally, given the physics involved, off-shell continuations of string amplitudes have been studied a great deal in the past, beginning in the early days of dual models. Despite this effort, off-shell string amplitudes have proven to possess a remarkable intransigence. A more extensive discussion of previous investigations will be presented elsewhere\(^1\), but we mention three of these approaches here to put our work in perspective.

Every string field theory naturally defines off-shell amplitudes\(^2\). These amplitudes have been known for a long time to possess spurious singularities that can be traced to the fact that s.f.t.’s construct surfaces from building blocks (propagators and vertices) of fixed geometries. One only obtains results invariant under conformal mappings, or Weyl rescalings of the geometry, when all the external legs are on-shell. Thus off-shell s.f.t.

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amplitudes are not geometric, in the sense that they cannot be associated to a surface independent of the representation of the conformal structure on the surface. Naturally enough, the singularities in off-shell amplitudes of different s.f.t.’s differ. Furthermore, duality is lost in this approach.

Attempts have also been made to compute the Polyakov path integral on surfaces with finite boundaries. The idea here is that specifying the matter configurations on the boundaries of the surface defines off-shell amplitudes. As the boundaries shrink to punctures, one expects to see singularities for boundary conditions corresponding to on-shell states. While this approach appears promising, it remains to properly understand the rôle of reparametrizations of the boundaries. These reparametrizations are not treated in the calculations that use the theory of doubled Riemann surfaces\textsuperscript{3}, while the remaining calculations have not taken into account the anomalous Weyl dependence which such reparametrizations introduce\textsuperscript{4}.

In the dual model literature, Bardakç\i, and Bardakç\i and Halpern\textsuperscript{5}, investigated spontaneous symmetry breaking by summing over tachyon emissions at zero space-time momentum. Faced with the problem that such tachyons are not vertex operators of dimension (1,1), they introduced a fictitious dimension and let the momentum in this direction be $\pm 1$. These ‘charged’ tachyon vertex operators were then (1,1) operators. It will turn out that this approach is the closest to the approach based on the Polyakov path integral that we shall pursue.

2. The Weyl measure in the Polyakov functional integral

Polyakov’s derivation of the connection between conformal anomalies and the critical dimensions of string theories\textsuperscript{6} elucidated a multitude of features of string physics, gleaned piece-meal in pioneering work. Space-time scattering amplitudes of string excitations are calculated as correlation functions of vertex operators in a functional integral over the metric on the string world-sheet, and the space-time string configurations:

$$\left\langle \prod_i \int d^2 z_i \sqrt{g} V_i(z_i) \right\rangle \equiv \int \frac{Dg \; DX}{\text{vol.}(\text{Diff}) \text{vol.}(\text{Weyl})} \exp(-S[g, X]) \prod_i \int d^2 z_i \sqrt{g} V_i(z_i).$$

(1)

The measure is divided by the ‘volume’ of the symmetries of the classical action $S \equiv (8\pi)^{-1} \int d^2 z \sqrt{g} g^{ab} \partial_a X^\mu \partial_b X_\mu$, with $\mu = 1, \ldots, D$—namely, diffeomorphisms and local Weyl rescalings on the world-sheet. Choosing conformal gauge, $g_{ab} \equiv e^{2\phi} g_{ab}(m)$, and fixing diffeomorphisms à la Faddeev-Popov, these functional integrals reduce to

$$\int dm \frac{\Delta \phi \; DX}{\text{vol.}(\text{Weyl}) \; \text{vol.}(\text{c.k.v.})} \exp(-S[\hat{g}, X]) \prod_i \int d^2 z_i \sqrt{\hat{g}(m)} V_i(z_i).$$

(2)

Here, c.k.v. stands for the conformal Killing vectors that must be taken into account if the world-sheet is a sphere or a torus, and $dm$ denotes the measure for integrating over moduli labelling distinct conformal equivalence classes of metrics on surfaces with one or more handles. In Eq. (2), the integration over the Weyl factor should cancel against the volume of the group of Weyl rescalings in the denominator, and hence all of the local degrees of freedom in the world-sheet metric would completely decouple from the theory. This decoupling is only actually achieved if Weyl rescaling survives as a symmetry of the
quantum path integral. This requires that $D = 26$ in order to cancel the anomalous dependences on the Weyl field, $\phi$, in the measure factor, $D \cdot \text{Det}'_{FP}/\text{vol.}(\text{c.k.v.})$. Also, one must impose various space-time mass-shell and polarization/gauge conditions on the external string states to avoid any anomalous Weyl dependences from normal-ordering the vertex operators. Combined these restrictions ensure that $\phi$ is decoupled from on-shell correlation functions in critical string theory, and the Weyl factor simply disappears from the functional integral (i.e., $\int D\phi/\text{vol.}(\text{Weyl}) \equiv 1$). The presence of the Weyl volume in the denominator is, of course, justified only by the fact that we are considering critical (i.e., Weyl-invariant) string theories. For non-critical string theories no such factor occurs. Therefore the mass-shell conditions can be obtained from requiring Weyl invariance. It follows, in the Polyakov approach, that the calculation of amplitudes for off-shell string states requires the ability to compute correlation functions of vertex operators with an anomalous Weyl dependence, in the normalized measure $D\phi/\text{vol.}(\text{Weyl})$. Why are such computations difficult? The problem resides in the non-linearity of the Riemannian metric that defines $D\phi$. The norm on infinitesimal changes of the conformal factor is constructed with the full world-sheet metric $g_{ab}$

\[(\delta \phi, \delta \phi) = \int d^2x \sqrt{g}(\delta \phi)^2 = \int d^2x \sqrt{g} e^{2\phi}(\delta \phi)^2, \quad (3)\]

which then explicitly depends on $\phi$. The functional integral over $\phi$ would be a standard quantum field theory with the measure, $D_{\phi}\phi$, defined by the translation invariant norm, $(\delta \phi, \delta \phi) = \int d^2x \sqrt{g}(\delta \phi)^2$. The crucial insight that we shall use is due to Mavromatos and Miramontes, and, independently, D’Hoker and Kurzepa\(^7\). These authors computed the relation between these two measures, and found the remarkably simple result

\[D\phi = D_{\phi}\phi \exp \left( S_L - \frac{\mu}{\pi} \int d^2z e^{2\alpha \phi} \right), \quad (4)\]

where $S_L \equiv \int \frac{d^2z}{6\pi} \left[ \partial \bar{\phi} \partial \phi + \frac{1}{4} \sqrt{g} \bar{R} \phi \right]$. The ‘cosmological constant’ $\mu$ is the coefficient of a local counterterm, and remains undetermined in this computation. The constant $\alpha$ in this interaction is explicitly fixed (see below). This relation was conjectured originally by David, and Distler and Kawai\(^8\), in their study of two-dimensional gravity coupled to conformal matter in conformal gauge. It is important to note that the derivation of Eq. (4) is mathematically entirely independent of the rest of the functional integrals involved. It is valid in non-critical string theory, and equally valid in the context of critical string theory. It follows, therefore, that any insight into non-critical string physics, or into quantum Liouville theory, directly translates into insights into off-shell critical string physics.

3. Correlation functions

The only assumption in our work is in treating the correlation functions of interest using the methods of conformal field theory. For non-critical strings, this approach has been verified by comparison with results determined by matrix model techniques. The stress tensor deduced from $S_L$ is

\[T_L = \frac{1}{6} \left[ (\partial \phi)^2 - \partial^2 \phi \right], \quad (5)\]
and it is easily checked that the central charge $c_L = 0$. Thus the total central charge for
the matter fields, the ghosts and now, the Liouville field remains zero. The weight of an
exponential operator $e^{\beta \phi}$ is $\frac{3}{2} \beta (\beta + \frac{1}{3})$. An off-shell vertex operator $V_i$ of weight $(\Delta, \Delta)$,
is dressed in the same way as matter operators in non-critical string theory to produce a
$(1,1)$ operator $\exp(\beta_\Delta \phi)V_i$ with

$$\beta_\Delta = \frac{1}{6} \left[ \sqrt{25 - 24\Delta} - 1 \right].$$

(6)

This is the unique solution of $\frac{3}{2} \beta_\Delta (\beta_\Delta + \frac{1}{3}) = 1 - \Delta$ such that $\Delta = 1 \Leftrightarrow \beta_\Delta = 0$, which
insures that in the on-shell limit, these off-shell amplitudes reduce precisely to the usual
on-shell amplitudes. Rather puzzling is the non-analyticity in this prescription at $\Delta = \frac{25}{24}$.
While one expects cuts in loop amplitudes in field theories, it seems difficult to interpret
this non-analyticity as arising from similar physics. For the present time, we will limit our
attention to $\Delta \leq \frac{25}{24}$. Certainly, there is no obvious physical reason for such a restriction,
and we will address this question further in the concluding remarks.

The presence of the cosmological constant in Eq. (4) is important for defining the
integration over $\phi$. Insertions of cosmological constant interaction ‘cancel’ Liouville momentum
carried by the off-shell vertex operators, and the background charge term in $S_L$. However,
the treatment of the complete action is rather subtle. Here, treating the cosmological constant term as a perturbatively defined interaction, we determine $\alpha = \beta_{\Delta=0} = \frac{2}{3}$.
One could consider the other branch of the square root, which gives $\alpha = -1$, but $\alpha = \frac{2}{3}$ may
be preferred since then this interaction can be interpreted as a zero-momentum tachyon,
which is obtained from the off-shell continuation of a physical state. Also, if used as the
area operator of the quantum theory, a vanishing area results in the limit $\phi \to -\infty$, in
accord with classical expectations.

Explicit computations can be performed on the two-sphere, using the idea of Goulian
and Li to perform the integral over the constant zero-mode, $\phi_0$. The classic calculation
of Dotsenko and Fateev can then be used to compute the resulting correlation function,
with appropriate analytic continuations along the way. The zero-mode integral is

$$\int d\phi_0 \exp \left( \frac{1}{3} \phi_0 - C e^{\frac{2}{3} \phi_0} \right) \exp(\gamma \phi_0) = \frac{3}{2} \Gamma \left( \frac{1}{2} (3\gamma + 1) \right) C^{-\frac{1}{2} (\beta \gamma + 1)},$$

(7)

where $\gamma \equiv \sum \beta_i \equiv \sum \beta(\Delta_i)$, and $C \equiv (\mu/\pi) \int d^2 z \exp(\frac{2}{3} \phi)$, with $\int d^2 z \tilde{\phi} = 0$. The amplitude is now

$$\langle \prod_i \int d^2 z_i e^{\beta_i \phi} V_i(z_i) \rangle = \frac{3}{2} \Gamma (-s) \prod_i \int d^2 z_i \langle C^s \prod_j e^{\beta_j \phi} (z_j) \rangle \langle \prod_k V_k(z_k) \rangle_m,$$

(8)

where $s \equiv -\frac{1}{2} (3\gamma + 1)$, and the subscript $L(m)$ stands for Liouville (matter) expectation values. For three-point functions with positive integer values of $s$, these correlations were treated by Dotsenko and Fateev. Choosing three tachyon operators, $V_j = \exp(ik_j^\mu X_\mu)$, and fixing their positions $\{z_1, z_2, z_3\}$ at $\{0, \infty, 1\}$, yields

$$\mathcal{A} = \frac{3}{2} \mu^s \Gamma (-s) \Gamma (s + 1) \Delta \left( \frac{1}{3} \right)^s \prod_{k=0}^{s-1} \prod_{i=0}^{3} \Delta(1 + 2\beta_i + \frac{2}{3}k).$$

(9)
Here \( \Delta(z) \equiv \Gamma(z)/\Gamma(1 - z) \), and we have defined \( \beta_0 \equiv -1/6 \), but \( \gamma = \sum_{i=1}^{3} \beta_i \). As it stands this formula is sensible only for positive integer values of \( s \), and hence negative \( \gamma \). Using the ideas of Ref. 9, the above formula can be continued to expressions which are valid for positive values of \( \gamma \)

\[
A = \left[ \mu \Delta \left( \frac{1}{3} \right) \right]^{-\frac{2\gamma+1}{2}} \Gamma \left( \frac{1+3\gamma}{2} \right) \Gamma \left( \frac{1-3\gamma}{2} \right)
\times \left( \frac{2}{3} \right)^{\gamma-\frac{3}{2}} \prod_{i=0}^{3} \prod_{p=0}^{\gamma-\frac{2}{3}} \Delta(1 - 3\beta_i + \frac{3}{2}p)
\]

or

\[
\times \left( \frac{2}{3} \right)^{\chi+} \prod_{i=0}^{3} \prod_{p=0}^{\gamma-\frac{2}{3}(N+1)} \Delta(1 - 3\beta_i + \frac{3}{2}p) \prod_{m=1}^{N} \Delta(1 + 2\beta_i - \frac{2}{3}m)
\]

or

\[
\times \left( \frac{2}{3} \right)^{\chi-} \prod_{i=0}^{3} \prod_{p=0}^{\gamma+\frac{2}{3}(N-1)} \Delta(1 - 3\beta_i + \frac{3}{2}p) \prod_{m=0}^{N-1} \Delta(2\beta_i - \gamma - \frac{2}{3}m),
\]

where \( \chi_+ = \frac{2}{3}(4N+1)(N-1) - (4N-1)\gamma \) and \( \chi_- = \frac{2}{3}(4N-1)(N+1) + (4N+1)\gamma \). In these formulæ, \( N \) is a positive integer, and \( \gamma \) must be such that the upper limits of the products are integers. This collection of expressions is clearly redundant, but any two representations valid for the same value of \( \gamma \) can be shown to be equivalent. We illustrate the full set, retaining the auxiliary parameter \( N \), since different formats may prove most useful in examining different problems. Note that from the first expression and from the last, with \( N = 1, 2 \), one has results which are valid for \( \gamma = n/3 \) where \( n \) is a positive integer or zero. A more extensive description of the analytic continuations above will appear elsewhere.

The amplitudes given above must still be normalized by the division by the Weyl volume. At tree-level it is possible to evade a direct computation of the Weyl volume by considering ratios of amplitudes. It is then interesting to investigate the analytic structure of the amplitudes when \( N \) and \( \gamma \) are held fixed. Considering the ratio of two amplitudes (with the same values of \( N \) and \( \gamma \)), one finds that the interesting dependence on \( \beta_i \) resides, \( \text{e.g.,} \) for \( N = 1 \) and positive integer values of \( \gamma \), in

\[
\prod_{i=1}^{3} \left\{ \Delta(2\beta_i - \gamma) \prod_{p=0}^{\gamma} \Delta(1 - 3\beta_i + \frac{3}{2}p) \right\}.
\]

This expression is a product of three factors with poles and zeroes depending on the value of \( \beta_i \) for each individual particle, and \( \gamma \) as well. Note that the restriction which arose in the discussion of the dressings, \( \Delta \leq \frac{2\pi}{\gamma} \), also constrains \( \beta_i \geq -\frac{1}{6} \). For a fixed \( \gamma \), this restricts the number of poles and zeroes which actually occur. A case of interest because the particles can all go on-shell is \( \gamma = 0 \), where we find \( \prod_{i=1}^{3} \Delta(1 - 3\beta_i)\Delta(2\beta_i) \). This expression has poles when \( \beta_i \to 1/3 \) (\( \text{i.e.,} \) \( k_i^2 \to \frac{4}{3} \)), and no zeroes—in particular, it remains finite as \( \beta_i \to 0 \).

Explicit computations are possible for other amplitudes. For example, four point functions are calculable, with the restriction that one of the particles is either on-shell,
and hence decoupled from the Weyl functional integral, or when one of the particles is a tachyon at zero momentum, when the amplitude reduces essentially to the computations above. Amplitudes with any number of zero-momentum tachyons are readily calculated, as we consider in the next section.

4. Tachyon Potential

It is relatively straightforward to calculate an effective tree-level potential for the tachyon within our approach. As commented above, the vertex operator for a zero-momentum tachyon, \( \int \! d^2 z \exp(\frac{2}{3} \phi) \), is identical to the cosmological constant interaction. A generating function for connected tree-level amplitudes for \( n \) zero-momentum tachyons is

\[
W(t) = \sum_{n=1}^{\infty} \frac{t^n}{n!} \langle \left[ \int \! d^2 z \, e^{\frac{2}{3} \phi} \right]^n \rangle = \exp \left[ t \int \! d^2 z \, e^{\frac{2}{3} \phi} \right] - 1 .
\]  

(12)

In these amplitudes, the operators decouple from the matter part of the functional integral, and hence the contribution of the latter reduces to \( \int \! d^26 \, X_0 \). We have set an arbitrary normalization constant to one, but the zero-mode integral is explicitly retained, as usual. The exponential of vertex operators shifts the cosmological constant, \( \mu \rightarrow \mu - \pi t \). In the absence of any vertex operators, Eq. (7) shows that the unnormalized amplitude is proportional to \( \mu^{-1/2} \), and hence the generating function is

\[
W(t) = \int \! d^26 \, X_0 \left( 1 - \frac{\pi t}{\mu} \right)^{-\frac{1}{2}} ,
\]  

(13)

where we have dropped the irrelevant constant term in Eq. (12). This result has a non-analytic singularity at \( t = \mu/\pi \) because on the two-sphere, the \( \phi_0 \) integral only converges with a positive cosmological constant. Hence Eq. (13) is valid for a source \( t < \mu/\pi \).

A Legendre transformation \( \Gamma(T) = -W(t) + \int \! d^26 \, X_0 \, T t \) produces an effective tree-level potential

\[
\Gamma(T) = \int \! d^26 \, X_0 \left[ 3 \left( \frac{T}{T_c} \right)^{\frac{1}{3}} - \frac{T}{T_c} \right] ,
\]  

(14)

where we have introduced an undetermined scale \( T_c \). This uncertainty arises because the correct normalization of \( t \), which would allow for the precise identification of the Polyakov amplitudes with space-time Green functions, is unknown. It also accounts for a constant factor which must multiply the sources in Eq. (13) to correct for the fact that the Polyakov amplitudes are truncated, while the Legendre transformation requires a generating function for connected amplitudes with propagators on the external legs.

Further the above Legendre transformation produces a valid effective potential if all other string excitations decouple, as follows. Eq. (13) is the generating function with arbitrary \( t \), but all other sources set to zero, \( W = W(t, J^{(m)} = 0) \). We have only made a Legendre transformation with respect to a single source (i.e., \( t \)), and so Eq. (14) is the correct effective potential with all other fields set to zero if \( \partial W(t, J^{(m)})/\partial J^{(n)} |_{J^{(p)} = 0} = 0 \) for all currents \( J^{(n)} \). This condition is equivalent to the vanishing of any amplitudes with any number of zero-momentum tachyons and a single vertex operator for any other string
excitation. For a vertex corresponding to any state with nonvanishing momentum, such amplitudes vanish as a result of momentum conservation (i.e., the $X_0$ integrals). Similarly any zero-momentum vertex operator at higher mass levels produces vanishing amplitudes because of the matter or ghost oscillator contributions. Some states may also have $\phi$ oscillator contributions—we have not yet proved that these all decouple.

Let us consider the potential in Eq. (14). One of the most striking features is that it is non-analytic at $T = 0$. This non-analyticity appears as a result of the singularity in the generating function, discussed above. Given the restriction $t < \mu/\pi$, the potential is valid for positive $T$. In this range, there is a single extremum at $T = T_c$. Hence one arrives at the conclusion that the bosonic string cannot be stabilized by a constant shift of the tachyon.

Perhaps an even more interesting fact made apparent by these calculations is that the zero-momentum dilaton $D$ does not couple as expected to the tachyon. Previous investigations of critical strings give the expectation that there is a trilinear $DT^2$ interaction. Such a coupling would lead to singularities in the amplitudes in Eq. (12), which are in fact not observed. In discussing the Legendre transformation, we actually showed that there are no interactions $DT^n$ for any $n$. Of course, the original expectations are based on calculations with on-shell tachyons, and so it is perhaps not too surprising that they are not fulfilled.

5. Conclusions and prospects

It has been our aim here to show that the effort expended on the study of non-critical strings in somewhat unphysical contexts has important physical consequences in critical string theories. Any future progress in non-critical string physics, or in quantum Liouville theory, will be of use in understanding off-shell critical string physics. There are a great many physical questions that become accessible in our approach to off-shell string physics. The computational limitations of the conformal field theory treatment of the Liouville correlators obviously leave much to be desired, a problem that appears in non-critical string theory as well. Another technical question, with what may be very interesting physics lurking underneath, is that of the analytic continuation of our discrete product formulae as a function of $\gamma$.

The restriction of being able to dress operators with dimension $\Delta \leq \frac{25}{24}$ may be a computational limitation. On the other hand, such a limitation is precisely what prevents us from attempting to sew our amplitudes together as we would if these amplitudes were off-shell amplitudes of a field theory. Note that modular invariance dictates that if we were able to sew amplitudes together, we would end up with infinite answers. Thus, it is tempting to speculate that these non-analyticities are a reflection of modular invariance, especially since as a function of space-time momentum, they vary with the mass level of the vertex operators being dressed. Since our computations preserved conformal invariance, any amplitude we compute should be well-defined on moduli space, i.e., should be independent of the coordinates on moduli space. This is, of course, not true of s.f.t. off-shell amplitudes, which describe a particular cell decomposition of moduli space.

There is no conceptual barrier to the extension of our results to supersymmetric strings, or to open string theories. Above we have only considered simple exponential dressings, but one can also find many new (1,1) primary fields with Liouville oscillator
contributions (e.g., $\partial \phi$) which will couple in amplitudes. Some of these may account for longitudinal polarizations which only couple off-shell\(^1\).

More subtle is the computation of the Weyl volume. The presence of the factor $\text{vol.}(\text{Weyl})$ in the denominator of Eq. (2) is an important feature which distinguishes our off-shell amplitudes from those of non-critical string. At tree-level though, one can avoid a direct computation of this factor by considering ratios of amplitudes. One could follow the prescription of Ref. 9 which uses Eq. (11) with $\gamma = 2$ and $\beta_i = 2/3$ to compute a result for the two-sphere (which actually vanishes). On higher genus surfaces, this factor in the denominator ensures that the Weyl field does not show up in any counting of states via degenerations. In particular, the dependence on the moduli in $D\phi$ is precisely cancelled by the denominator, unless there are off-shell vertex operators present. Note then that in Eq. (2), $d_m$ and $D\phi/\text{vol.}(\text{Weyl})$ must be explicitly ordered as given. This crucial cancellation at higher genus shows that our off-shell continuation is not merely defined by an additional $c = 0$ conformal field theory tacked to a critical string theory.

A striking feature of the amplitudes is the presence of poles that are not accounted for by excitations in the matter sector (even if combined with the ghost sector). They may indicate the presence of excitations that are entirely stringy in nature. Independent of the existence of new poles, the fact that the amplitudes have products which have upper limits determined by $\gamma$ is something entirely unlike the amplitudes one obtains from a field theory. This prevents them from factorizing into separate terms depending only on each individual $\beta_i$. In field theories, the off-shell character of the amplitude is a function of individual external states. Here, one can obtain the value $\gamma = 0$ when all external states are on-shell, or if they are off-shell. It is difficult to imagine how this $\gamma$ dependence could be reproduced in a string field theory. Thus our results may indicate that some fundamentally new framework, other than string field theory, will be required to extend our understanding of critical string theory beyond the Polyakov path integral.

Note: Since this talk was given, E. Witten has computed some off-shell quantities in his background independent approach to open-string field theory\(^{12}\).

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