Weighted finite energy sum rules for the omega meson in nuclear matter

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The possible in-medium changes of the properties of an omega meson placed in cold nuclear matter are constrained by QCD sum rules. It is shown that the sum rules cannot fully determine the in-medium spectral shape of the omega meson. However, for a given parameterization the sum rules can constrain or correlate the hadronic parameters. It is shown that weighted finite energy sum rules provide a proper framework to study directly these constraints/correlations. Two typical parameterizations of possible in-medium omega spectra are analyzed, namely (i) a one-peak structure with arbitrary position and width of the peak and (ii) a structure with two (narrow) peaks, caused by the genuine omega meson and a resonance-hole branch. The sum rules provide for case (i) a mass-width correlation and for case (ii) a correlation between the peak heights and the peak position of the omega branch. It is also analyzed how the obtained results depend on the size of the relevant four-quark operator evaluated with respect to a nucleon. Finally it is argued that a strict vector meson dominance scenario is not compatible with the sum rules.

Keywords: QCD sum rules, meson properties, nuclear matter

I. INTRODUCTION

The question how hadrons once put in a strongly interacting medium change their properties provides a very active field of research. In the language of non-perturbative QCD, in-medium modifications are indicated by the change of condensates like the quark condensate which provides an order parameter of chiral symmetry breaking. On the other hand, changes like the melting of the condensates do not automatically tell what this means for the properties of a particular hadron like its mass or lifetime. The situation is such that the condensates are closer to QCD as the underlying theory of the strong interaction, whereas the hadron properties are closer to experimental observation. The QCD sum rule method is supposed to bridge that gap by connecting integrals over hadronic spectral functions with an expansion in terms of quark and gluon condensates. Originally they were introduced for the vacuum, but later on generalized to in-medium situations.

A particularly interesting probe to study in-medium modifications are neutral vector mesons. The reason is that such mesons can decay into dileptons. If such a decay happens in the medium the dileptons leave the system untouched by strong interactions. In that way in-medium information is carried to the detectors. (For an overview see e.g. [4].) The present paper deals with omega mesons placed in an infinitely extended system with finite baryonic density. Such a scenario is an idealization of a finite nucleus or a heavy-ion collision. For simplicity we study omega mesons which are at rest with respect to the nuclear medium.

Before we turn to a discussion of in-medium QCD sum rules we present a brief historical overview concerning the interest in in-medium rho- and omega-mesons: On the theoretical side first attempts concentrated on possible mass shifts of vector mesons (and the experimental effort was mainly directed in heavy-ion collisions). It was implicitly assumed that e.g. the widths of the vector mesons are not drastically changed. Therefore more work was devoted to the rho-meson since a long-lived omega-meson would most likely decay outside of the strongly interacting region. Also, from the experimental point of view it was for a long time impossible to resolve a narrow peak in the dilepton spectra which would be caused by an omega-meson with its vacuum width. Recently, however, the interest has shifted to the omega-meson for the following reasons: On the theory side there appeared calculations which proposed sizable changes for the in-medium vector meson spectra beyond pure mass shifts (and the experimental effort was mainly directed in heavy-ion collisions). It was implicitly assumed that e.g. the widths of the vector mesons are not drastically changed. Therefore more work was devoted to the rho-meson since a long-lived omega-meson would most likely decay outside of the strongly interacting region. Also, from the experimental point of view it was for a long time impossible to resolve a narrow peak in the dilepton spectra which would be caused by an omega-meson with its vacuum width. Recently, the interest has shifted to the omega-meson for the following reasons: On the theory side there appeared calculations which proposed sizable changes for the in-medium vector meson spectra beyond pure mass shifts. The genuine vector meson peak might get wider due to collisional broadening. In addition, even new peaks or bumps might appear caused by collective resonance-hole excitations. It should be stressed, however, that this issue is far from being settled. One reason is that a key quantity of such calculations is the forward scattering amplitude of the respective vector meson with a medium constituent. However, there is no direct experimental access to such quantities since the vector mesons are not asymptotic states but resonances. Therefore the calculations are plagued by sizable model dependences. If the life time of vector mesons is significantly reduced in a medium, the omega-meson might indeed decay inside the medium, while the rho-meson might get so broad that it can no longer be resolved from the background. On the experimental side by now also peaks as narrow as the vacuum omega-meson can be resolved — at least in dimuon spectra. It is expected that this will also be possible in presently starting experiments with dielectron spectra. In addition, heavy-ion experiments are accompanied by elementary reactions on nuclei. Here, the kinematical situation can be chosen such that the omega-meson is (more or less) at rest with respect to the nucleus. Hence, even if its life time is long, the omega would decay inside the medium. Indeed, a downward shift of strength has been reported for gamma-nucleus reactions in for an omega-meson placed in a nucleus. Here the omega is identified via its decay into gamma. Note
that this decay channel is much suppressed for the $\rho$-meson (at least in vacuum) so that a clean separation between $\omega$ and $\rho$ seems to be possible. Dielectron spectra generated in proton-nucleus collisions are studied in [28, 29]. Also there a downward mass shift of the $\omega$-meson has been reported. Note, however, that dilepton spectra do not allow for a clean separation of (in-medium) $\omega$-mesons from $\rho$-mesons. In the present work we shall explore what QCD sum rules tell about the in-medium changes of $\omega$-mesons.

Originally it was expected that the use of nuclear medium QCD sum rules would yield model independent predictions for in-medium changes of hadronic properties — just as the vacuum sum rules yield in an impressive way parameter-free predictions of vacuum hadronic properties. However, one has to realize that the QCD sum rules do not directly yield hadronic properties — like the mass of a state — as a function of the condensates. Unfortunately the connection is rather indirect: The sum rules connect the condensates with specific integrals over the spectral information contained in a correlator of two interpolating quark currents which carry the quantum numbers of the hadronic state of interest. Only if this spectral information is concentrated in a more or less narrow peak the sum rules can predict the peak position, i.e. the mass of the hadron under consideration. As we have outlined above it is not clear whether the in-medium spectrum of a vector meson is a sufficiently narrow peak — it is even not clear whether it has a one-peak structure at all. In addition, the connection between the hadron and its interpolating quark current can get more complicated. For vector mesons this concerns the question to which extent the vector meson dominance assumption still holds in a medium (cf. e.g. [16, 30] and also [31] for possible complications). In general, for more complicated spectra the sum rules can only yield constraints on these spectra. For the $\rho$-meson this issue is by now well documented in the literature, e.g. [10, 32, 33, 34, 35]. For the $\omega$-meson this will become clear below where we will explore various typical in-medium parameterizations of the spectral shape.

The present work is not the first one devoted to in-medium QCD sum rules for the $\omega$-meson. Therefore we will comment in the following on the previous approaches. We will discuss four issues, namely (i) the assumed shape of the spectral information, (ii) the Landau damping term, (iii) the size of the four-quark condensate and (iv) the type of sum rules used. Unfortunately, at least the first three points are intertwined. We will try our best not to confuse the reader. The motivation for point (i) has already been given. Point (ii) is important since different works used different expressions there. To clarify this issue we decided to rederive the proper Landau damping term in the appendix. Point (iii) concerns the question whether four-quark condensates factorize into the square of the two-quark condensate. Already for the vacuum case this constitutes an ongoing debate in the sum rule community. In addition, the answer to this question might be different for vacuum and in-medium situations. (For an overview see e.g. the introduction of [32].) The fourth point is somewhat technical. We have decided to postpone the detailed discussion to section 4.

In the pioneering works [6, 37] a single narrow peak structure (with the to be determined mass parameter) plus a Landau damping contribution was plugged into the sum rules. The result was an in-medium decrease of the $\omega$-mass. Actually the results agreed for $\rho$- and $\omega$-meson. The reason was simply that the sum rules were exactly the same and for both mesons a single-peak structure was assumed — besides the Landau damping contribution. The latter was correctly derived for the $\rho$-meson. It was assumed that it is the same for the $\omega$. However, the Landau damping contribution is nine times higher for the $\omega$ than for the $\rho$ as stressed explicitly later in [38] (see also the appendix of the present work). For the $\rho$-meson, the Landau damping contribution is actually so small that it does not severely influence the result of a sum rule analysis. A factor nine, however, promotes a negligible contribution to a dominant one! In principle, the effect of the Landau damping contribution is an upwards shift of spectral strength. Therefore, it was argued in [38] that the $\omega$-mass increases in nuclear matter. Also in this latter work a single narrow peak structure was the starting point of the analysis. Both in [6, 37] and in [38] it has been assumed that the in-medium four-quark condensate more or less factorizes into the square of the in-medium two-quark condensate. In the present work we will allow for more freedom for the in-medium spectral information in the $\omega$ channel. One part of our analysis will also cover the case of a single peak structure, albeit allowing for an arbitrary width. We will find that the sum rules do not determine both mass and width, but instead provide a correlation between them. (For the corresponding case of the $\rho$-meson see [32].) In a second part we will study how the sum rules constrain a two-peak structure. In addition, we will study the sensitivity of our results on the in-medium change of the four-quark condensate.

Also in [10] the assumption of a narrow peak has been given up. Instead, a hadronic model for the in-medium spectral function of the $\omega$-meson has been developed. This model provided a moderate peak broadening and a downward mass shift of the $\omega$-meson. It was shown that the calculated spectral function satisfies the in-medium sum rule. The developed hadronic model has been criticized in [34] and therefore modified in [14]. Qualitatively, the findings were still the same: moderate peak broadening and downward mass shift. In [10] it has been argued that this hadronic model satisfies the sum rules. However, the Landau damping contribution used there has the correct factor nine, but an opposite sign as compared to [10, 38]. Our derivation, presented in the appendix, agrees in size and sign with [38]. Indeed, the authors of [10] agree now that the sign of the Landau damping term used in [10, 38] is the correct one [32]. Nonetheless, a downward mass shift and agreement with the sum rules has already been reported in [10] where the Landau damping term has the same sign as in [38]. Apparently, the findings of [38] (upward mass shift)
and of \[10\] (downward mass shift) seem to contradict each other. As already speculated in \[38\] this is probably due to a different treatment of parameters which are intrinsic to the type of sum rule used. In the present work, where we use a different type of sum rule, such ambiguities do not appear as we will discuss below.

The influence of the in-medium behavior of the four-quark condensate has been studied in detail in \[42, 43, 44, 45\]. Basically in all previous works it has been assumed that the in-medium change of the four-quark condensate is linked to the in-medium change of the two-quark condensate. The factorization approximation suggests that the drop of the four-quark condensate relative to its vacuum value is twice as large as the one for the two-quark condensate. In \[42, 43, 44, 45\] this assumption has been suspended in favor of an additional parameter which characterizes this drop. The consequences of the sum rules as a function of this parameter have been studied. In the present work we will also allow for an arbitrary in-medium change of the four-quark condensate and study its impact on our spectral parameterizations. Correlations between hadronic parameters — on which we focus in the present work — have not been studied in \[42, 43, 44, 45\]. Mass-width correlations have been worked out in \[44\] in the framework of the Borel sum rules. One purpose of the present work is to establish a new type of sum rules not used for in-medium studies so far (except for \[33\]), namely the weighted finite energy sum rules. Note that for vacuum analyses, the latter are well established, cf. e.g. \[46, 47\] and references therein. For in-medium studies, however, usually Borel type sum rules are utilized. We have decided to postpone the comparison of Borel and other sum rule types to section \[III\] since the important points are easier to discuss with the relevant formulae at hand.

The paper is organized in the following way: In the next section we study in-medium QCD sum rules for the \(\omega\)-meson channel and in particular their condensate side. We will start with a Borel sum rule and derive weighted finite energy sum rules which are directly sensitive to in-medium modifications and which we will use for the rest of the presented work. Advantages and disadvantages of different types of sum rules are discussed. In section \[III\] we study the constraints provided by the sum rules on typical hadronic parameterizations of the in-medium spectral information. In particular, we will study (i) a one-peak structure with arbitrary position and width of the peak and (ii) a structure with two (narrow) peaks, caused by the genuine \(\omega\)-meson and a resonance-hole branch. In section \[IV\] we summarize our results and provide an outlook. The derivation of the proper Landau damping term is presented in the appendix.

## II. QCD SUM RULES

In this work we study the properties of a vector-isoscalar current

\[ j_\mu := \frac{1}{2} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d) \]

which is at rest with respect to the nuclear medium. As outlined e.g. in \[48\] in-medium QCD sum rules can be obtained from an off-shell dispersion relation which integrates over the energy at fixed (here vanishing) three-momentum of the current. We also restrict ourselves to small densities \(\rho_N\) by using the linear-density approximation. Effectively this means that the current is at rest with respect to the nucleon on which it scatters. The Borel sum rule is given by \[2, 8, 10, 38, 42\]

\[
\frac{1}{\pi M^2} \int_0^\infty ds \text{Im} R(s) e^{-s/M^2} = \frac{1}{8\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) + \frac{1}{M^4} m_\rho \langle \bar{q} q \rangle_{\text{med}} + \frac{1}{24M^4} \left( \frac{\alpha_s}{\pi} G^2 \right)_{\text{med}} + \frac{1}{4M^4} m_N a_2 \rho_N \\
- \frac{56}{81M^6} \pi \alpha_s \langle O_4^V \rangle_{\text{med}} - \frac{5}{24M^6} m_N^3 a_4 \rho_N + o(1/M^8).
\]

(2)

Here \(\rho_N\) denotes the nuclear density and \(M\) the Borel mass. The central quantity \(R\) is obtained from the current-current correlator as outlined e.g. in \[32\]. All other quantities will be specified below. Note that we have followed the common practice to neglect contributions from non-scalar twist-4 operators and from \(\alpha_s\) suppressed twist-two operators (cf. e.g. \[33\] for details). The spectral information \(\text{Im} R\) is decomposed into a hadronic low-energy and a perturbative high-energy part:

\[
\text{Im} R(s) = \frac{9\pi}{4} \frac{\rho_N}{m_N} \delta(s) + \text{Im} R_{\text{HAD}}(s) \Theta(s_0(\rho_N) - s) + \frac{1}{8\pi} \left( 1 + \frac{\alpha_s}{\pi} \right) \Theta(s - s_0(\rho_N)).
\]

(3)

Here \(s_0\) denotes the (density dependent) continuum threshold. We have decomposed the low-energy part into the object we want to study, \(\text{Im} R_{\text{HAD}}\), and the Landau damping contribution (first term on the right hand side of \[3\])
since the latter can be determined model independently (cf. the appendix). Inserting (3) in (2) yields
\[ \frac{1}{\pi M^2} \int_0^{s_0(\rho_N)} ds \text{Im} R_{HAD}(s, \rho_N)e^{-s/M^2} = \frac{1}{8\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) \left( 1 - e^{-s_0(\rho_N)/M^2} \right) - \frac{9}{4M^2} \frac{\rho_N}{m_N} + \frac{1}{M^4} m_q \langle \bar{q}q \rangle_{\text{vac}} + \frac{1}{24M^4} \frac{\langle \alpha_s G^2 \rangle_{\text{med}}}{\text{vac}} + \frac{1}{4M^4} m_N a_2 \rho_N - \frac{56}{81M^6} \pi \alpha_s \langle O_4' \rangle_{\text{med}} - \frac{5}{24M^6} m_N^3 a_4 \rho_N . \]

The four-quark condensate is given by [2, 38]
\[ \langle O_4' \rangle = \frac{81}{224} \langle (\bar{u}\gamma_\mu \gamma_5 \lambda^a u + \bar{d}\gamma_\mu \gamma_5 \lambda^a d)^2 \rangle + \frac{9}{112} \langle (\bar{u}\gamma_\mu \lambda^a u + \bar{d}\gamma_\mu \lambda^a d) \sum_{\psi=u,d,s} \bar{\psi}\gamma^\mu \lambda^a \psi \rangle . \]

Using the linear-density approximation we get [2, 4, 49]
\[ m_q \langle \bar{q}q \rangle_{\text{med}} = m_q \langle \bar{q}q \rangle_{\text{vac}} + m_q \langle N|\bar{q}q|N \rangle \rho_N = -\frac{1}{2} F_\pi^2 M_\pi^2 + \frac{1}{2} \sigma_N \rho_N , \]
\[ \frac{\langle \alpha_s G^2 \rangle_{\text{med}}}{\text{vac}} = \frac{\langle \alpha_s G^2 \rangle_{\text{vac}}}{\text{vac}} - \frac{8}{11 - \frac{2}{3} N_f} m_N^3 (0) \rho_N \]
and
\[ \langle O_4' \rangle_{\text{med}} = \langle O_4' \rangle_{\text{vac}} + \langle N|O_4' |N \rangle \rho_N = \langle O_4' \rangle_{\text{vac}} + 2\kappa \langle \bar{q}q \rangle_{\text{vac}} \langle N|\bar{q}q|N \rangle \rho_N = \langle O_4' \rangle_{\text{vac}} - \kappa \frac{F_\pi^2 M_\pi^2 \sigma_N}{2m_q^2} \rho_N . \]

Here \(|N\rangle\) denotes a one-nucleon state (with appropriate normalization). We have introduced a quantity \(\kappa\) which is basically defined by [3], i.e.
\[ \kappa := \frac{\langle N|O_4' |N \rangle}{2\langle \bar{q}q \rangle_{\text{vac}} \langle N|\bar{q}q|N \rangle} . \]

Our knowledge on the four-quark condensates is rather limited both in vacuum and nuclear matter (cf. [36] and references therein). In the following, we will only need the density dependent part of the four-quark condensate. In [36] it has been shown that in the limit of a large number of quark colors \(N_c\) one gets
\[ \kappa = 1 + o(1/N_c) . \]

For the real world of \(N_c = 3\) it is not clear how large the deviation of \(\kappa\) from unity actually is. Therefore we will treat \(\kappa\) as a free parameter and study the dependence of our results on \(\kappa\). This is close in spirit to [42].

In principle, we could start our analysis with [4]. However, there appear some quantities in [4] which have neither to do with the condensates nor with the low-energy information contained in \(\text{Im} R_{HAD}\): First of all, we have the Borel mass \(M\). As it stands, the sum rule [4] does not tell us for which values of \(M\) it is supposed to be valid. Actually we have neglected higher order condensates \(o(1/M^8)\) to obtain [4] from [2]. Therefore, one should not trust [4] for too small values of \(M\). On the other hand, \(M\) becomes too large there is no exponential suppression of larger \(s\) on the left hand side of [4]. In other words, the integral becomes more sensitive to the region around \(s_0\). From [4] it is obvious that this modeling of this region is not very sophisticated. On the other hand, more sophistication would involve more model parameters which we do not want to have. Therefore, one wants to concentrate on sum rules which are not very sensitive to the region around \(s_0\). For Borel sum rules this is achieved by not too large values of \(M\). These considerations lead to a so-called Borel window. But the limiting values of this window — which, in addition, might be density dependent — can only be roughly guessed (cf. e.g. [32] for details). Besides the necessity to choose a proper Borel window we also have the continuum threshold \(s_0\) and its density dependence. It introduces an additional parameter which we are not primarily interested in. Finally, the sum rule [41] mixes vacuum and in-medium information while we are only interested in the latter. At least, we want to make sure that uncertainties in the vacuum description do not influence our conclusions for the in-medium changes. In the following, we will show that one can get better sum rules which solve some of the mentioned problems.
To be more sensitive to the in-medium modifications we differentiate the Borel sum rule \([4]\) with respect to the density:

\[
\frac{1}{\pi M^2} \int_0^{s_0} ds \, e^{-s/M^2} \frac{\partial}{\partial \rho_N} \text{Im} R_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = \frac{1}{M^2} c_0 e^{-s_0/M^2} + \frac{1}{M^4} c_1 + \frac{1}{M^6} c_2 + \frac{1}{M^8} c_3
\]  

(11)

with

\[
c_0 = \left[ \frac{1}{8\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) - \frac{1}{\pi} \text{Im} R_{\text{HAD}}(s_0, \rho_N = 0) \right] s_0',
\]

(12a)

\[
c_1 = -\frac{9}{4m_N},
\]

(12b)

\[
c_2 = \frac{m_N a_2}{4} + \frac{\sigma_N}{2} - \frac{m_N^{(0)}}{27},
\]

(12c)

\[
c_3 = \kappa \frac{28\pi \alpha_s F_\pi^2 M_\pi^2 \sigma_N}{81 m_\pi^2} - \frac{5}{24} m_N^3 a_4,
\]

(12d)

\[s_0 = s_0(\rho_N = 0)
\]

(13)

and

\[
s_0' = \frac{ds_0}{d\rho_N} \bigg|_{\rho_N=0}.
\]

(14)

Next we rewrite \((11)\):

\[
\frac{1}{\pi} \int_0^{s_0} ds \, e^{(s_0-s)/M^2} \frac{\partial}{\partial \rho_N} \text{Im} R_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = c_0 + c_1 e^{s_0/M^2} + \frac{1}{M^2} c_2 e^{s_0/M^2} + \frac{1}{M^4} c_3 e^{s_0/M^2},
\]

(15)

expand both sides in powers of \(1/M^2\) and compare the corresponding coefficients on right and left hand side:

\[
\frac{1}{\pi} \int_0^{s_0} ds \, \frac{\partial}{\partial \rho_N} \text{Im} R_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = c_0 + c_1,
\]

(16a)

\[
\frac{1}{\pi} \int_0^{s_0} ds \, (s_0-s) \frac{\partial}{\partial \rho_N} \text{Im} R_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = c_1 s_0 + c_2,
\]

(16b)

\[
\frac{1}{\pi} \int_0^{s_0} ds \, (s_0-s)^2 \frac{\partial}{\partial \rho_N} \text{Im} R_{\text{HAD}}(s, \rho_N) \bigg|_{\rho_N=0} = c_1 s_0^2 + 2c_2 s_0 + 2c_3.
\]

(16c)

In this way we have obtained weighted finite energy sum rules. The advantage of finite energy type sum rules as compared to Borel sum rules lies in the fact that with the former we have got rid of the Borel mass and the problem how to determine a reliable Borel window etc. (cf. our discussion above). On the other hand, the standard finite energy sum rules (as applied to the in-medium case e.g. in \([40]\)) are rather sensitive to the modeling of the transition region from the hadronic part \(\text{Im} R_{\text{HAD}}\) to the continuum (see also e.g. \([41]\) and references therein). Indeed, the first equation \((16a)\) is plagued by that problem. The latter two equations, however, are not since the transition region is suppressed by powers of \((s_0-s)\) \((16)\). Therefore \((16b)\) and \((16c)\) are more reliable as they are insensitive to details of the threshold modeling at \(s_0\). Hence these weighted finite energy sum rules combine the advantages of Borel and standard finite energy sum rules. In general, the disadvantage is that there are only two properly weighted finite energy sum rules as compared to three standard finite energy sum rules. In our case, however, this does not reduce the available information: The in-medium change of the threshold parameter encoded in \(s_0'\) is anyway unknown \textit{a priori}. Fortunately, it only appears in the first (anyway less reliable) sum rule \((16a)\). The two preferable sum rules
and \((16c)\) are independent of \(s_0'\). We shall use them for the subsequent analysis. Note that the vacuum threshold \(s_0\) appears in \((16b)\) and \((16c)\). This, however, can be fixed by an independent vacuum sum rule analysis which is free of all in-medium uncertainties. For the actual calculation we adopt the point of view of [40, 50, 51] and use \(s_0 \approx (4\pi f_\pi)^2 \approx 1.3\) GeV\(^2\) with the pion decay constant \(f_\pi\). Note that one obtains numerically the same value if one takes the arithmetic average of the squared masses of the \(\omega\) and of its first excitation. We will study the sensitivity of our results with respect to \(s_0\) below.

We would like to stress again that the sum rules \((16b)\) and \((16c)\) constitute a big step forward in the sum rule analysis of in-medium properties: First, we are directly sensitive to in-medium changes in contrast to traditional analyses [6, 10, 42] which study vacuum plus medium contributions. Second, we have got rid of all problems how to properly define a reasonable Borel window (cf. also the discussion in [37, 52]). Third, we still share with Borel type sum rules the feature that we are less sensitive to the modeling of the continuum threshold.

### III. Hadronic Parameterizations

We now turn to the left hand side of the sum rules \((4)\) or \((16)\). It is well-known that the vector-isoscalar current \(j_\mu\) strongly couples to the \(\omega\)-meson. In the vector meson dominance (VMD) picture which is phenomenologically rather successful it is even assumed that all the interaction of \(j_\mu\) with hadrons is mediated by the \(\omega\)-meson [53]. In the following we will rather study simple parameterizations for the current-current correlator and not detailed hadronic models. Of course, the omega meson will play a prominent role here. We will come back to the issue of VMD below.

For all numerical evaluations we will use the values given in table I. All plots are for normal nuclear matter density \(\rho_N = \rho_0 \approx 0.17\) fm\(^{-3}\) except where otherwise stated.

| quantity | size             | ref. |
|----------|------------------|------|
| \(m_N\)  | 940 MeV          | [54] |
| \(m_N^{(0)}\) | 750 MeV        | [10] |
| \(a_2\)  | 0.9              | [6]  |
| \(a_4\)  | 0.12             | [6]  |
| \(\sigma_N\) | 45 MeV          | [6]  |
| \(m_q\)  | 6 MeV            | [54] |
| \(\alpha_s\) | 0.36             | [6]  |
| \(F_\pi\) | 92.4 MeV         | [54] |
| \(M_\pi\) | 140 MeV          | [54] |
| \(\kappa\) | 0 ... 10        |      |
| \(s_0\)  | 1.33 GeV\(^2\) \pm 10% |      |
| \(M_\omega\) | 783 MeV         | [54] |
| \(M_{rh}\) | 590 MeV         | [54] |

**TABLE I:** Sizes of all relevant quantities.

### A. Warm up: single narrow peak

To make contact with previous works we explore as a first parameterization a spectral structure with a single narrow peak:

\[
\text{Im} R_{\text{HAD}}(s, \rho_N) = 2\pi(F_\pi^2 + c_F \rho_N) \delta(s - M_\omega^2 - c_M \rho_N).
\] (17)

Our choice for the spectral strength in vacuum is motivated in [55]. It yields a good description of the cross section \(e^+ e^- \to \omega\). Obviously, from the two weighted finite energy sum rules \((16b)\) and \((16c)\) one can in principle determine two quantities, which are in the present case \(c_F\) and \(c_M\). Of course, the results depend on the input parameters given in table I especially on \(s_0\) and \(\kappa\). Instead of \(c_F\) and \(c_M\) we introduce the following physically more intuitive parameters: the in-medium mass

\[
M_{\omega, \text{med}} = \sqrt{M_\omega^2 + c_M \rho_N}
\] (18)
and the in-medium strength

\[ F_{\text{med}} = \sqrt{F_{\pi}^2 + c_F \rho_N}. \]  

(19)

These quantities are depicted in figure (1) as functions of \( \kappa \).

We observe that the mass is increased except for very high values of \( \kappa \). This finding is in qualitative agreement with [38, 42, 45]. Quantitatively, there are some differences which are due to the use of different types of sum rules — Borel sum rules in [38, 42, 45], weighted finite energy sum rules here. As we have already stressed we prefer the use of the latter. It is well known that the drop of the four-quark condensate leads to a lowering of the mass [6], while the Landau damping term leads to an increase [38]. The competition between these two effects is responsible for a rising mass at low values of \( \kappa \) and a decreasing mass for large \( \kappa \). We also want to stress that an increase of \( \kappa \) by an order of magnitude leads to a change in the mass of only about 10%. In other words, with a rough idea about the four-quark condensate one gets a quite good prediction for the mass. On the other hand, it is complicated to deduce from a measured mass a precise value for \( \kappa \).

Concerning the input parameter \( s_0 \) we observe that our results in general do depend on it, albeit this dependence is not very drastic. As a curiosity we note that there is one particular value of \( \kappa \approx 4 \) where there is no \( s_0 \)-dependence. We do not think that there is any deeper meaning to this point.

Note that we do not study the sensitivity of our results on the other input parameters of table I for the following reason: The role of \( \sigma_N \) and \( m_{\pi}(0) \) in (12c) is subleading as compared to the \( a_2 \) term. They even cancel to a large extent. The value of \( a_2 \), on the other hand, is well determined from deep inelastic scattering [6]. In (12d) the quantities \( m_q \), \( \alpha_s \) and \( \sigma_N \) are accompanied by \( \kappa \) which is varied in a large range anyway. Hence, all uncertainties in these quantities are effectively studied by our variation of \( \kappa \).

In contrast to the mass the in-medium strength is always smaller than its vacuum value for all values of \( s_0 \) and \( \kappa \) studied here. The strength of the peak is actually important for the issue of VMD. This, however, is better discussed in a more general framework where one allows for a width of the peak. We now turn to such a more general parameterization.

**B. Single peak with width**

Instead of the \( \delta \)-function present in (17) we now allow for a broad spectral information. Details of the formalism can be found in [32] for the corresponding case of the \( \rho \)-meson. The spectral information is now assumed to be

\[ \text{Im} R_{\text{HAD}}(s, \rho_N) = \pi V(\rho_N) \frac{A(s, \rho_N)}{s} \]  

(20)
with
\[
A(s, \rho_N) = \frac{1}{\pi} \frac{\sqrt{s} \Gamma_{\text{med}}(s)}{(s - M_\omega^2 - c_M \rho_N)^2 + s (\Gamma_{\text{med}}(s))^2}.
\] (21)

Note that for simplicity we neglect a possible \( s \)-dependence of the mass parameter. We also neglect the (small) vacuum width of the \( \omega \)-meson.

If the width \( \Gamma_{\text{med}} \) (which will be further specified below) becomes small we have to recover the previous case discussed in subsection III A. This enables us to fix the vacuum value of \( V(\rho_N) \) in (20): Indeed, \( A \) becomes a \( \delta \)-function for vanishing \( \Gamma_{\text{med}} \). Comparing then (17) and (20) for the vacuum case one gets
\[
V_0 := V(\rho_N = 0) = 2 F_\pi^2 M_\pi^2.
\] (22)

Having determined the vacuum value, we assume (in line with the linear-density approximation) that \( V(\rho_N) \) scales linearly with \( \rho_N \). This leaves us with one free parameter to characterize \( V(\rho_N) \).

Since the integrals appearing in (16) are obviously sensitive to the behavior of \( A \) for small values of \( s \) it is important to model the threshold behavior of \( \Gamma_{\text{med}} \) in a physically reasonable way. For finite nuclear density the scattering with nucleons influences the spectral function. If the Fermi motion of the nucleons is neglected the threshold for the spectral function of an \( \omega \)-meson at rest is given by the mass of one pion since the lightest pair of particles which can be formed in an \( \omega \)-nucleon collision is a nucleon and a pion. The threshold behavior is dominated by the lowest possible partial wave. Without any additional constraint from the intermediate state formed in the \( \omega \)-nucleon collision we assume it to be an s-wave state. Hence we get
\[
\Gamma_{\text{med}}(s) \sim (s - M_\pi^2)^{1/2} \Theta(s - M_\pi^2)
\] (23)

FIG. 2: Correlation between in-medium mass (18) and in-medium width \( \gamma \) of the \( \omega \)-meson for normal nuclear matter density \( \rho_N \approx 0.17 \text{ fm}^{-3} \) for different values of the vacuum continuum threshold \( s_0 = 1.33 \text{ GeV}^2 \) (full lines), 1.20 GeV\(^2\) (dotted), 1.46 GeV\(^2\) (dashed) and for different values of the medium dependence of the four-quark condensate \( \kappa = 0 \) (top, left panel), 1 (top, right), 2 (bottom). Note that the input is a single peak structure with width (20).
and thus

$$\Gamma_{\text{med}}(s) = \gamma \left( \frac{1 - M^2}{1 - M^2/\pi s} \right)^{1/2} \Theta(s - M^2) \tag{24}$$

with the free parameter $\gamma$, the on-shell width. We stress again that we have neglected the small vacuum width of the $\omega$ in (24). Consequently we take $\gamma \sim \rho_N$. Note that strictly speaking the in-medium on-shell width should be taken with respect to the in-medium mass instead of the vacuum mass in (24). Therefore, it might be a misnomer to call $\gamma$ the on-shell width. However, since $\gamma$ is already linear in the density, replacing the vacuum by the in-medium mass in the width formula (24) is beyond the accuracy we are working.

In total, our parameterization (20) contains three free parameters: the in-medium change of the mass (18), the in-medium change of $V(\rho_N)$ and the width $\gamma$. Obviously, our two sum rules (16b) and (16c) can hardly determine all three parameters. What we can get, however, are correlations between these parameters. E.g. for given width the other two parameters can be determined. Most interesting is the mass-width correlation which we show in figure 2. Corresponding correlations, e.g. for strength and width could also be obtained, but are not displayed explicitly. We come back to a discussion of the strength below.

![Figure 3: Strength $V$ of (20) normalized to its vacuum value $V_0$ as a function of the density $\rho_N$ normalized to normal nuclear matter density $\rho_0 \approx 0.17$ fm$^{-3}$ for different values of the width $\gamma(\rho_0) = 0$ MeV (dashed line), 30 MeV (full), 60 MeV (dotted). We used $s_0 = 1.33$ GeV$^2$ and $\kappa = 1$.](image)

From figure 2 we observe first of all that the results depend to some extent on the vacuum continuum threshold $s_0$ and on the parameter $\kappa$ characterizing the in-medium change of the four-quark condensate. Especially, for increasing $\kappa$ the masses move down. This is exactly in line with the previous finding. Indeed, the respective mass for $\gamma = 0$ agrees with the corresponding one determined in the previous subsection, cf. figure 1. When the width increases, the in-medium mass is pushed up. Qualitatively, this is easy to explain: For the values of $\kappa$ displayed in figure 2 the sum rules prefer masses higher than the vacuum one, as seen in figure 1. More generally, one might say that the sum rules prefer accumulation of spectral strength for invariant masses $\sqrt{s}$ above the vacuum mass. Naively, one might think that the introduction of a width does not change the center of the mass accumulation. This, however, is not true: Due to the weighting factors $(s_0 - s)^s$ in the sum rules the contributions of lower invariant masses are weighted sizably more than the ones from higher invariant masses. Therefore, the introduction of a width effectively shuffles strength to lower invariant masses. To account for that, i.e. to satisfy the same sum rules for the zero-width and for the finite-width case, the center of the spectral distribution has to shift upwards. We see exactly this effect in figure 2. Similar findings have been obtained in [44]. Again, quantitative differences are due to the use of Borel sum rules there and weighted finite energy sum rules here.

Next we turn to a discussion of the strength $V(\rho_N)$ given in (20). It is important to stress that in a pure VMD scenario all hadronic effects on the current (1) are mediated by the $\omega$-meson. Therefore, all in-medium modifications concern the $\omega$-meson, i.e. the spectral function in (20). In other words, in a pure VMD scenario $V(\rho_N)$ would be constant, i.e. independent of the density. In an extended scenario $V$ might not only depend on the density, but also on the invariant mass (squared) $s$ (cf. e.g. [30]). Such an additional dependence — where the details are also model
dependent — has been disregarded for simplicity in [20]. Figure 3 shows curves of $V(\rho_N)$ for different values of the width $\gamma$ chosen such that $\gamma(\rho_0) = 0, 30, 60$ MeV. Curves for different values of $s_0$ and $\kappa$ look qualitatively similar. Obviously a strict VMD scenario is excluded by the sum rules. This is an interesting aspect to which not much attention has been paid in the past — at least not in the sum rule context. Concerning hadronic models we refer to [10, 30] for a discussion of this issue. An in-medium fate of VMD has also been found in the framework of hidden local symmetry [31].

C. Two narrow peaks

The purpose of the present subsection is to stress that a one-peak structure might not be the only possibility for an in-medium spectral information. Indeed, as has been worked out e.g. in [20, 21, 24] at least two peaks are conceivable in the spectral function of the $\omega$-meson and therefore also in the spectral distribution of the current-current correlator discussed here. The additional peak is caused by the excitation of a nucleon hole and a baryonic resonance. Resonances in the region of about 1530 MeV seem to play a dominant role, but no consensus is reached yet how large their importance actually is and whether the $N^*(1520)$ is the most important one or the $N^*(1535)$ or both (see also [50]).

Clearly, the more structures we introduce, the more parameters we get. In turn, their correlations deduced from the sum rules get more complicated. To keep things simple we choose the following parameterization with two narrow peaks

$$\text{Im} R_{\text{HAD}}(s, \rho_N) = 2\pi(F^2 + c_F \rho_N) \delta(s - M^2 - c_M \rho_N) + 2\pi c_{rh} \rho_N \delta(s - M^2_{\text{rh}})$$

(25)

with

$$M_{\text{rh}} = m_R - m_N$$

(26)

and the resonance mass $m_R$. As pointed out above, a reasonable choice is $m_R \approx 1530$ MeV covering basically the lowest $D_{13}$ and $S_{11}$ excitation, respectively. Note that possible in-medium shifts of the resonance-hole peak, e.g. by level repulsion, are beyond the linear-density approximation used here. Of course, the parameterization (25) is less general than (20) concerning the aspect that in (25) the widths of the peaks are neglected. We would like to stress, however, that we want to keep the parameterizations as simple as possible. In addition, we have included the present subsection to avoid the wrong impression that mass and width would be the only things which one must consider for a comprehensive discussion of an in-medium spectral information.

![Graphs showing the in-medium mass and strength of the omega-meson](image)

**FIG. 4**: In-medium mass [15] (left panel) and in-medium strength [19] (right panel) of the $\omega$-meson for normal nuclear matter density $\rho_N = \rho_0 \approx 0.17 \text{ fm}^{-3}$ as functions of the strength of the resonance-hole branch for different values of the vacuum continuum threshold $s_0 = 1.33 \text{ GeV}^2$ (full lines), $1.20 \text{ GeV}^2$ (dotted), $1.46 \text{ GeV}^2$ (dashed). Note that the input is a structure with two narrow peaks [25]. Also note that the lines in the left panel end at the respective value of $F_{\text{rh}}$ where in the right plot $F_{\text{med}} = 0$ is reached.

A two-peak structure has, of course, two strengths which is the strength for the genuine $\omega$-branch as already defined in [19] and, in addition, the strength of the resonance-hole peak

$$F_{\text{rh}} = \sqrt{c_{rh} \rho_N}.$$  

(27)
Again, we have three parameters. Given e.g. the strength of the resonance-hole branch, the sum rules determine the in-medium mass and the in-medium strength of the genuine $\omega$-branch. These correlations are depicted in figure for $\kappa = 1$. Similar curves would emerge for other values of $\kappa$. All curves would be shifted downwards for increasing $\kappa$ (not explicitly shown). In the right panel of figure we see how strength can be reshuffled from one branch to the other. Collecting more strength in the resonance-hole branch, i.e. at low invariant masses, however, results in a need for compensation. This is achieved by shifting the other branch upwards. We observe this effect in the left panel of figure.

IV. SUMMARY AND OUTLOOK

In the present work we have derived new sum rules for the $\omega$-meson in nuclear matter, namely weighted finite energy sum rules. These sum rules are free of additional parameters (like Borel masses) which are not of primary interest. In addition, the derived sum rules are directly sensitive to the in-medium changes and do not mix vacuum and in-medium information. We have performed sum rule analyses for different parameterizations of the spectral information. In part, we have repeated the sum rule analysis of other groups using our type of sum rules instead of theirs. In these cases, we could qualitatively reproduce their results. Quantitative differences are due to the different sum rules used. We have pointed out after equation and after equations why we prefer the use of weighted finite energy sum rules.

It should have become clear from the present work that the sum rules alone cannot predict e.g. an in-medium mass shift. Instead, the result of a QCD sum rule analysis depends on the chosen input parameterization. Nonetheless, for a given parameterization the sum rules can correlate the input parameters. Examples are presented in figures and The most important qualitative finding is that the sum rules prefer a shift of spectral strength to higher invariant masses as compared to the vacuum case — except for very large values of $\kappa$ which parameterizes the in-medium change of the four-quark condensate. If some spectral strength is shifted to lower invariant masses — by increasing the width of the $\omega$-peak or by introducing a low-mass resonance-hole branch — the mass of the $\omega$-peak/branch moves further upwards.

Apparently, an in-medium upwards shift of strength seems to be in conflict with the experimental finding of a decreasing $\omega$-mass in the interior of a nucleus. On the theoretical side there are several possibilities which could resolve this contradiction. First, the value for $\kappa$ might be so large that the in-medium $\omega$-mass drops, cf. figure Indeed, in the observed drop in is taken as an indication that $\kappa$ seems to be large. As pointed out above, $\kappa$ becomes 1 in the limit of a large number $N_c$ of colors. If a very large value of $\kappa$ was the right explanation for an in-medium drop of the $\omega$-mass, it would be interesting to understand why the large-$N_c$ approximation fails so badly. A second possible explanation would be that the linear-density approximation used throughout this analysis breaks down already at rather low densities. This approximation basically involves only vacuum physics (e.g. a selfenergy is given by $\rho_N T$ where $T$ is a vacuum scattering amplitude, cf. e.g. the discussion in). Beyond the linear-density approximation more complicated effects might play a dominant role. Most spectacular is the proposed in-medium change of the underlying vacuum structure as is formulated e.g. in the scaling of hadron masses proposed by Brown and Rho or in the hidden local symmetry approach. In such scenarios it is suggested that the vector meson mass drops. Such effects might overwhelm the linear-density effect deduced here. It should be stressed, however, that the effects discussed in the present work are not included in the cited works either. A more complete treatment would be desirable. However, there is also a third possible explanation which is much less spectacular: It might appear that only part of the spectral information has been identified in the experiment. Especially in a scenario with a sizable resonance-hole branch (discussed in subsection it might turn out that only the low-mass resonance-hole branch is seen whereas the rather high-lying $T$ genuine $\omega$-branch is missed. Note that the strength, i.e. the visibility of the genuine $\omega$-branch drops with increasing importance of the resonance-hole branch, as shown in figure right panel. It would be interesting, if the theoretical approaches which try to describe the data of included at least the resonance decay processes $N^* \to N\pi^0\gamma$, since in the $\omega$-meson is identified via its decay mode $\omega \to \pi^0\gamma$. Finally we note the possibility that one sees also the $\rho$ and not only the $\omega$ in the in-medium spectrum of $\pi\gamma$: In vacuum the decay channel $\rho \to \pi\gamma$ is very small as compared to $\omega \to \pi^0\gamma$. However, this might change in the medium. Experimentally, this might be checked by looking in addition at the in-medium spectra of charged final states, i.e. at $\rho^\pm \to \pi^\pm\gamma$ (which, of course, do not exist for the $\omega$).

Our analysis demonstrates that for a given hadronic model it can be checked whether the model is consistent with the sum rules. If the hadronic model contains only a few undetermined or only roughly determined parameters they

1 Since we have neglected the widths of the states.
can be predicted or at least correlated. Note, however, that the sum rules make statements about the current-current correlator and not directly about the $\omega$-meson. As we have shown above, the most simplest connection, i.e. strict vector meson dominance is excluded by the sum rules. We regard this as an important point of the present sum rule analysis.

Concerning consistency checks for hadronic models it is also interesting to point out that the first of our two used weighted finite energy sum rules, i.e. (16b), is independent of the parameter $\kappa$, i.e. independent of the four-quark condensate which is notoriously difficult to pin down. This sum rule provides a good check for the consistency of a given hadronic model. We stress again that the obtained weighted finite energy sum rules are directly sensitive to the in-medium part of the hadronic model and do not mix vacuum and in-medium information.

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APPENDIX A: LANDAU DAMPING CONTRIBUTION FOR ISOVECTORS AND ISOSCALARS

One contribution to the in-medium expectation value of a current-current correlator comes from the Landau damping process. In the following this Landau damping contribution is calculated (in two ways) for the isoscalar and isovector current at rest with respect to the nucleons which form the nuclear medium. Note that we work in the linear-density approximation. Therefore the in-medium interaction is approximated by a sum of single nucleon interactions with the current. The respective nucleon is at rest with respect to the medium.

We will start out with a model independent derivation. In addition, we discuss afterwards a derivation using vector meson dominance and the universality of the coupling strengths of the vector mesons.

The currents appropriate for our discussion are the isovector current,

$$ j^\rho_\mu := \frac{1}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \quad (A1) $$

and the isoscalar current,

$$ j^\omega_\mu := \frac{1}{2} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d). \quad (A2) $$

The currents chosen in such way yield in vacuum the same operator product expansion. Next we have to figure out how an electromagnetic current is decomposed into the previous currents: The electromagnetic current is (neglecting strange and more heavier quarks)

$$ j^{el}_\mu = Q_u \bar{u}\gamma_\mu u + Q_d \bar{d}\gamma_\mu d = j^\rho_\mu + \frac{1}{3} j^\omega_\mu. \quad (A3) $$

Following [37], the nucleon matrix element of the isospin-1 part of the electromagnetic current has two form factors:

$$ \langle N(\vec{k}_1)|j^\rho_\mu|N(\vec{k}_2)\rangle = \bar{u}(\vec{k}_1) \frac{3}{2} [F_1^{I=1}(q)\gamma_\mu + F_2^{I=1}(q)i\sigma_{\mu\nu}q^\nu] u(\vec{k}_2). \quad (A4) $$

The corresponding relation for the isospin-0 part is

$$ \langle N(\vec{k}_1)|j^\omega_\mu|N(\vec{k}_2)\rangle = \bar{u}(\vec{k}_1) \frac{1}{2} [F_1^{I=0}(q)\gamma_\mu + F_2^{I=0}(q)i\sigma_{\mu\nu}q^\nu] u(\vec{k}_2). \quad (A5) $$

For the current at rest with respect to the nuclear medium, i.e. for $q_0 = 0$ [37]. In this case, the contribution from the tensor part vanishes. On the other hand, the $F_1$ form factors become unity. This yields a model independent result for the Landau damping contribution to the isoscalar sum rule:

$$ -\frac{9}{4M^2} \frac{\rho_N}{m_N}. \quad (A6) $$
It is included in (4) as the second term on the right hand side. For more details we refer to [37]. Concerning the isovector case we get in the same way:

\[ -\frac{1}{4M^2} \frac{\rho_N}{m_N}, \]  

(A7)

The relative factor 9 between isovector and isoscalar has also been used in [10, 38, 42].

A second derivation of the same results uses vector meson dominance: We start with Lagrangians [10] for \( \rho \)-nucleon interaction,

\[ \mathcal{L}_{\rho N} = g_{\rho N} \bar{N} \tau^a \gamma_\mu N \rho_\mu^a + \frac{g_{\rho N} K_\rho}{4m_N} \bar{N} \tau^a \sigma_{\mu \nu} N \partial^\mu \rho_\nu^a, \]  

(A8)

and \( \omega \)-nucleon interaction, respectively,

\[ \mathcal{L}_{\omega N} = g_{\omega N} \bar{N} \gamma_\mu N \omega_\mu + \frac{g_{\omega N} K_\omega}{4m_N} \bar{N} \sigma_{\mu \nu} N \partial^\mu \omega_\nu. \]  

(A9)

Assuming a universal coupling we get [10]

\[ g_\omega = 3g_\rho, \quad g_{\omega N} = g_\omega, \quad g_{\rho N} = g_\rho. \]  

(A10)

Using vector meson dominance the electromagnetic current is given by

\[ j_\mu = -\frac{m_V^2}{g_\rho} \rho_\mu - \frac{m_V^2}{g_\omega} \omega_\mu = -\frac{m_V^2}{g_\rho} \left( \rho_\mu + \frac{1}{3} \omega_\mu \right). \]  

(A11)

This result has to be compared to (A3) yielding

\[ j_\mu = -\frac{m_V^2}{g_\rho} \rho_\mu, \quad j_\mu = -\frac{m_V^2}{g_\rho} \omega_\mu. \]  

(A12)

For our case \( q = 0 \) (cf. discussion above) the tensor interactions vanish and only the vector interactions contribute. We get

\[ \langle N(\vec{k}_1) | j_{\mu N}^{\rho} | N(\vec{k}_2) \rangle \sim \frac{1}{g_\rho} \langle N(\vec{k}_1) | \rho_\mu | N(\vec{k}_2) \rangle \sim \frac{g_{\rho N}}{g_\rho} = 1, \]  

(A13)

but

\[ \langle N(\vec{k}_1) | j_{\mu N}^{\omega} | N(\vec{k}_2) \rangle \sim \frac{1}{g_\rho} \langle N(\vec{k}_1) | \omega_\mu | N(\vec{k}_2) \rangle \sim \frac{g_{\omega N}}{g_\rho} = 3. \]  

(A14)

These results are in agreement with the ones derived above, (A4) and (A5).

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