We apply the method of perturbative fragmentation to study bottom fragmentation in top decay. We present the energy spectrum of $b$-quarks and $b$-flavoured hadrons in top decay.

A reliable understanding of bottom-quark fragmentation in top-quark decay ($t \to bW$) will be fundamental to accurately measure the top properties, such as its mass, at present and future high-energy colliders. At the LHC, final states with leptons and $J/\psi$, where the leptons are $W$-decay products and the $J/\psi$ comes from the decay of a $b$-flavoured hadron, will be a promising channel to reconstruct the top mass $m_t$. The error is estimated to be $\Delta m_t \simeq 1 \text{ GeV}$ and the $b$-fragmentation is the largest source of uncertainty, accounting for about 0.6 GeV. In this paper we investigate bottom fragmentation in top decay within the framework of perturbative fragmentation and use phenomenological models to describe the $b$-quark hadronization.

According to the factorization theorem, the rate for the production of a $b$-hadron in top decay can be written, up to power corrections, as the following convolution:

\[
\frac{1}{\Gamma_0} \frac{d\hat{\Gamma}}{dx_b}(x_B, m_t, m_b) = \frac{1}{\Gamma_0} \frac{d\hat{\Gamma}}{dx_b}(x_b, m_t, \mu_F) \otimes D_p(x_b, m_b, \mu_F) \otimes D_{np}(x_B). \tag{1}
\]

In Eq. (1), $\Gamma_0$ is the Born width of the process $t \to bW$, $d\hat{\Gamma}/dx_b$ is the rate for the production of a massless $b$ quark in top decay (coefficient function), $D_p(x_b, m_b, \mu_F)$ is the perturbative fragmentation function which expresses the transition from a massless $b$ into a massive $b$ quark, $\mu_F$ is the factorization scale, $D_{np}$ is the non-perturbative fragmentation function. $x_b$ and $x_B$ are the normalized $b$-quark and $b$-hadron energy fractions in top decay.

\[\text{TOP DECA Y AND BOTTOM FRAGMENTATION IN NLO QCD}\]

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We have computed the rate for the production of a massless $b$ quark in top decay in dimensional regularization and subtracted the collinear singularity in the $\overline{\text{MS}}$ factorization scheme. We have got the following $\overline{\text{MS}}$ coefficient function:

$$
\left( \frac{1}{\Gamma_0} \frac{d\hat{\Gamma}_b}{dz} \right)_{\overline{\text{MS}}} = \delta(1 - z) + \frac{\alpha_S(\mu)}{2\pi} \hat{A}_1(z),
$$

(2)

with

$$
\hat{A}_1(z) = C_F \left\{ \frac{1 + z^2}{(1 - z)_+} + 3 \frac{1}{2} \delta(1 - z) \right\} \left[ \log \frac{m_T^2}{\mu_F^2} + 2 \frac{1 + w}{1 + 2w} - 2 \log(1 - w) \right] + \frac{1 + z^2}{(1 - z)_+} \left[ 4 \log(1 - w) - 1 \right] \frac{1}{1 + 2w} - 4z \frac{1}{1 - z}_+ \left[ 1 - \frac{w(1 - w)(1 - z)^2}{(1 + 2w)(1 - (1 - w)z)} \right] + 2(1 + z^2) \left[ (1 - z)_+ \log(1 - z) \right] + \frac{1}{1 - z}_+ \log z
$$

+ \delta(1 - z) \left[ 4 \text{Li}_2(1 - w) + 2 \log(1 - w) \log w - \frac{2\pi^2}{3} + \frac{1 + 8w}{1 + 2w} \log(1 - w) \right] - \frac{2w}{1 - w} \log w + \frac{3w}{1 + 2w} - 9 \right\}. \quad (3)

The value of the perturbative fragmentation function $D_p(x_b, m_b, \mu_F)$ at any scale $\mu_F$ can be obtained by solving the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations, once an initial condition at a scale $\mu_{0F}$ is given. The initial condition of the perturbative fragmentation function reads:

$$
D_p(x_b, \mu_{0F}, m_b) = \delta(1 - x_b) + \frac{\alpha_S(\mu_0)C_F}{2\pi} \left[ 1 + \frac{x_b^2}{1 - x_b} \left( \log \frac{\mu_{0F}^2}{m_b^2} - 2 \log(1 - x_b) - 1 \right) \right], \quad (4)
$$

The result in Eq. (4) is process independent and can hence be used in top decay as well. Moreover, it has been shown, by comparing the rates for massless and massive $b$-quark production in top decay, that the coefficient function (3) is indeed consistent with Eq. (4). Solving the DGLAP equations for the evolution $\mu_{0F} \to \mu_F$, with a NLO kernel, allows one to resum logarithms $\log(\mu_F^2/\mu_{0F}^2)$ with NLL accuracy. If we set $\mu_{0F} \simeq m_b$ and $\mu_F \simeq m_t$, we resum large logarithms $\sim \log(m_T^2/m_b^2)$ which appear in the massive, unevolved, fixed-order calculation of $d\Gamma/dx_b$. The coefficient function (3) and the initial condition (4) contain terms which get large once $x_b \to 1$. This corresponds to soft-gluon radiation in top decay. Soft terms in the initial condition of the perturbative fragmentation function are process independent and have been resummed with next-to-leading logarithmic accuracy. Soft-gluon contributions in the coefficient function are process dependent and their resummation is currently in progress.

In Fig. 1 we present the $b$-quark energy distribution in top decay according to the approach just described. We note that the inclusion of soft-gluon resummation in the initial condition of the perturbative fragmentation function smoothes out the distribution and the $x_b$ spectrum exhibits the so-called Sudakov peak. Also shown is the massive $\mathcal{O}(\alpha_S)$ result, which lies below the other two distributions throughout the spectrum and is divergent as $x_b \to 1$. It has also been found that the implementation of soft resummation yields a milder dependence of observables on the factorization scale $\mu_{0F}$ which enters Eq. (4).

We would like to make predictions for the spectrum of $b$-flavoured hadrons in top decay. For this purpose we use some phenomenological models which contain tunable parameters which are to be fitted to experimental data. We consider a power law with two parameters:

$$
D_{np}(x; \alpha, \beta) = \frac{1}{B(\beta + 1, \alpha + 1)} (1 - x)^\alpha x^\beta, \quad (5)
$$
Figure 1: $b$-quark spectrum in top decay according to the perturbative fragmentation method, with (solid) and without (dashes) NLL soft-gluon resummation in the initial condition (solid), and according to the $O(\alpha_s)$ massive result (dots). In the inset figure, the same distributions are plotted on a logarithmic scale.

Table 1: Best-fit values for the parameters contained in the non-perturbative fragmentation functions.

|       | ALEPH          | SLD            |
|-------|----------------|----------------|
| $\alpha$ | $0.31 \pm 0.15$ | $1.88 \pm 0.42$ |
| $\beta$  | $13.21 \pm 1.62$ | $27.04 \pm 4.02$ |
| $\chi^2(\alpha, \beta)/dof$ | $2.62/14$ | $11.12/16$ |
| $\delta$ | $20.39 \pm 0.77$ | $18.80 \pm 0.60$ |
| $\chi^2(\delta)/dof$ | $17.27/15$ | $17.46/17$ |
| $\epsilon$ | $(1.12 \pm 0.16) \times 10^{-3}$ | $(1.17 \pm 0.10) \times 10^{-3}$ |
| $\chi^2(\epsilon)/dof$ | $22.96/15$ | $130.80/17$ |

The model of Kartvelishvili et al.:

$$D_{np}(x; \delta) = (1 + \delta)(2 + \delta)(1 - x)x^\delta$$

and the non-perturbative fragmentation function of Peterson et al.:

$$D_{np}(x; \epsilon) = \frac{A}{x[1 - 1/x - \epsilon/(1 - x)]^2}.$$ 

In Eq. (5), $B(x, y)$ is the Euler Beta function; in (7) $A$ is a normalization constant. We tune such models to $e^+e^-$ data from the SLD and ALEPH Collaborations. The SLD data refers to $b$-flavoured mesons and baryons and the ALEPH to mesons. When we do the fits, we must describe the $e^+e^- \rightarrow b\bar{b}$ process within the same framework as for top decay, i.e. we use the perturbative fragmentation method, NLL DGLAP evolution and NLL soft-gluon resummation in the initial condition (4).

In Table 1 we report on the parameters which yield the best fits of the hadronization models, along with the corresponding standard deviations and the $\chi^2$ per degree of freedom. We note that the model in Eq. (5) fits best the data, although the best-fit parameters show pretty-large uncertainties. The Kartvelishvili model well reproduces both ALEPH and SLD data, while the Peterson model marginally agrees with the ALEPH data and is inconsistent with the SLD one. Moreover, the values of the parameters $\delta$ and $\epsilon$, fitted to ALEPH and SLD, are in agreement within two standard deviations. In Fig. 2 we show our prediction for the $b$-hadron spectrum in top decay, using all three hadronization models, fitted to ALEPH. We note that each model yields a statistically-different prediction for the $x_B$ spectrum, the Peterson-based distribution...
Figure 2: $b$-hadron spectrum in top decay, describing the hadronization according to the power law with two parameters (solid), the Kartvelishvili (dashes) and the Peterson (dots) models, fitted to the ALEPH data. For any model, we plot the edges of a band corresponding to one-standard-deviation confidence level.

Figure 3: As in Fig. 2, using the power law, but fitted to ALEPH (solid) and SLD (dashes).

being peaked at larger $x_B$ values. In Fig. 3 we compare the predictions obtained using the power law with two parameters, but fitted to ALEPH and SLD. We observe that the spectra are distinguishable; such a difference may be related to the different hadron types that the two experiments have reconstructed.

In summary, we have reviewed the main results obtained once we studied the fragmentation of $b$ quark in top decay using the perturbative fragmentation function approach and presented the energy distribution of $b$ quarks and $b$-flavoured hadrons in top decay.

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