Study on fatigue of subway antenna beam

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Abstract. The dynamic fatigue life theory starts very late. As an important part of the subway, the antenna beam has played a great important role on the daily life of people. To study the fatigue properties of it, the relative researches are carried out. The finite element model is set up first. Then the mode analysis is carried out. After that, the rigid-flexible coupling dynamic model is set up. Finally, the reduction analysis continued. Through the experiment and simulation, the fatigue strength and vibration transition properties are analysed. Ploymax method is carried out for modal parameter identification, then the FE model could be verified to improve the accuracy. By setting up the rigid-flexible coupling dynamic model of the subway vehicle, then the modal parameters and experimental data are compared, the model is modified and verified. Considering the impact of wheel flat scar and rail gap, the vibration transmission characteristics of bogie system are studied. In the calculation of resonance fatigue dynamic stress, FFT method is used to solve the problem. The fatigue strength of antenna beam caused by the impact of rail gap and wheel flat scar is studied, and the requirements for safe service life of antenna beam is put forward.

1. Introduction
The dynamic fatigue life theory starts very late. In 1981, Valanis[1] analysed the effect of the load frequencies on the fatigue life and obtain the relationship between the fatigue life and load frequencies[2]. Whaley[3] analysed the vibration fatigue according to the principle of irreversible thermodynamics. Sanliturk[4] estimated the fatigue life based on the elastic structure and frequency function. Dentsoras and Kouvaritakis [5] analyzed the regulation of fatigue crack propagation under resonance. Bishop [6] calculated the fatigue damage through spectrum density function, promoting the study on the fatigue life of vibration. Based on the finite element method, Hanna [7] analyzed the vibration fatigue life of the brake system control unit in the frequency range. Wöhler [8] verified the fatigue strength of the axle by constant amplitude fatigue testing machine. Meanwhile, Wu [9] and Chiou [10] also did some researches in the vibration fatigue.

2. Finite element modelling of the antenna beam
The FE model of the antenna beam is set up by Hypermesh software. As only a not very important part of the rigid-flexible coupling dynamic model, the setting of finite elements could be not very precise. In this model, there are 96833 Solid185 elements, 3 Mass 21 elements and 3 Rigid elements.
3. Modal analysis

To guarantee the correction of FE analysis, the mode analysis is extremely necessary. To simulate the connection of the antenna beam, the mounting base should be constrained. Lanczos method is carried out for analysis. The modes of each order are shown as follows.

Figure 1. FE model of the antenna beam

Figure 2. Modes of the antenna beam in the first 6 orders
The mode frequencies are shown as follows

| Mode order | Frequency/Hz | Mode type                                           |
|------------|--------------|----------------------------------------------------|
| 1          | 45.73        | Vertical bending of the antenna beam in the first order |
| 2          | 53.37        | Lateral bending of the antenna beam in the first order |
| 3          | 133.84       | Torsion and lateral bending of the ATP hanger        |
| 4          | 147.88       | Vertical bending of the beam in the 2\textsuperscript{nd} order |
| 5          | 190.72       | Lateral bending of the beam in the 2\textsuperscript{nd} order |
| 6          | 253.15       | Torsion and vertical bending of the beam             |

4. Rigid-flexible coupling dynamics

In the dynamic analysis of the vehicle system, when each components of the vehicle are under the reaction of constant forces or forces in low frequencies, they should be treated as rigid bodies. When the loads in high frequencies are considered, inertia force and damping force will contribute more to the dynamic response, then the components should be considered as flexible bodies. The railway vehicle system is very complex with the vibration in all frequencies transferring through one component to another. To better simulate the vehicle dynamic properties, the components should be treated as flexible bodies. This chapter is to introduce the theory of rigid-flexible coupling dynamics and lay the basic foundation for modelling.

The rigid-flexible coupling dynamic equations could be obtained as follows:

$$ \mathbf{r}(t) = \mathbf{A}(t)(\mathbf{b}(t) + \mathbf{c} + \mathbf{u}(\mathbf{c}, t)) $$

(1)

In this equation, $\mathbf{r}(t)$ is the displacement vector of each node in the flexible structure; $\mathbf{A}(t)$, the displacement array; $\mathbf{b}(t)$, relative replacement vector of each node in the global coordinate system; $\mathbf{c}$, displacement vector of the coordinate system of the elastomer relative to the overall coordinate system; $\mathbf{u}(\mathbf{c}, t)$, the deformation of each node in the flexible system; $t$, the time range.

Simpack software is always used to carry out simulation for this vehicle. Mode superposition method and Rite method are always used to obtain the deformation of the flexible body. The node displacement could be obtained according to the linear combination of modal function and time series function:

$$ \mathbf{u}(\mathbf{c}, t) = \sum_{j=1}^{n_e} \mathbf{u}_j(c) q_j(t) $$

(2)

The stress of each node in the flexible body can be obtained as follows:

$$ \sigma(c, t) = \mathbf{H} \mathbf{D}_\text{el} \mathbf{u}(\mathbf{c}, t) $$

(3)

According to the principle of virtual work, the first and second derivatives of the above equations are introduced into equation (1), and the following equation is obtained:

$$ \int_v \delta \mathbf{r}^T \rho \mathbf{d} \mathbf{v} + \int_v \delta \mathbf{c}^T \mathbf{\sigma} \mathbf{d} \mathbf{v} = \int_v \delta \mathbf{r}^T \mathbf{P} dS $$

(4)

Through the transformation above, the following equation is obtained:

$$ \begin{pmatrix} mI & \mathbf{\ddot{d}}^T_{\text{CM}} & C_r^T \mathbf{C}_r \mathbf{a} \\ \mathbf{\ddot{d}}_{\text{CM}} & \mathbf{J} & C_r^T \dot{\mathbf{\omega}} + \left( \mathbf{\dot{\omega}} \mathbf{d}_{\text{CM}} + 2 \mathbf{\omega} \mathbf{C}_r \mathbf{\dot{q}} \right) \\
\mathbf{C}_r & \mathbf{C}_r & M_e \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{q}} \\
\dot{\mathbf{\omega}} \\
\dot{\mathbf{q}} \end{pmatrix} + \left( \mathbf{\omega} \mathbf{J} \mathbf{\omega} + \mathbf{G}_m \mathbf{\dot{q}} \right) \mathbf{\omega} + \left( \mathbf{O}_e \mathbf{\Omega} + \mathbf{G}_u \mathbf{\dot{q}} \right) \mathbf{\omega} = \left( \mathbf{h}_r \right) $$

(5)

In this equation, the first element should be the product of acceleration and mass matrices; the second element, gyroscopic force and centrifugal force; the third element, generalized stress; the fourth element, generalized external force.
\( M_e \), modal mass matrix; \( K_e \), modal stiffness matrix; \( C_r \), coupling displacement matrix; \( J \), moment of inertia; \( G_r \), rotation screw press; \( G_e \), deformation screw press; \( O_e \), deformation centrifugal force; \( d_{CM} \), position of deformation point relative to centre of gravity; \( I \), identity matrix; \( \ddot{or} = r \times \omega \).

5. Establishment of the rigid-flexible coupling dynamic model

In order to improve the calculation efficiency, it is usually necessary to simplify the model by reduction theory. The number of degrees of freedom in the FE model should be reduced to the number of main modes. Simpack is used to carry out the rapid calculation for the FE model, aiming at the reduction analysis of the FE model. Only reduction matrix and reduction freedom are used to solve the model. The main models of the antenna beam and bogie frame are set up as follows. The antenna beam is selected as research object. More main degrees of freedom should be selected to guarantee the precision of the antenna beam model. As for the bogie frame, it is only needed to guarantee the precision of low order mode.

![Figure 3](image_url)

(a) Main degree of freedom model (b) Substructure model of the antenna beam

6. Reduction analysis of the structure

Substructure analysis is an important step of reduction analysis, which could be divided into 3 parts: generation, application and extension. Substructure method is to merge the normal elements into a super element. The main degree of freedom is used to obtain the dynamic properties of the model. The whole model could be the unit of super elements and normal elements, or all the elements are super elements. According to the requirement of the connection of antenna beam and bogie frame, the main nodes are set in the junction of the primary suspension, secondary suspension, damper and traction rod. Reasonable nodes are selected by auxiliary program.

According to the requirement of the connection point and suspension nodes between the antenna beam and frame, the main nodes are set in the connection between the primary suspension, secondary suspension, damper and traction rod. Some program is edited to select the main nodes and the substructure analysis is carried out to obtain the files for geometric properties, mass matrix and stiffness matrix.

During the finite element calculation, although the super element analysis is used to reduce the calculation time, the modal properties of the super element are only the information about the main degree of freedom. To guarantee the accuracy, the reduced modal information is compared with that in all degree of freedom. The following table is the comparison between the reduced data and data in all degree of freedom. It is known from the table that there is some difference between the reduced modes and that in all degree of freedom. While, all the error falls within the accepted range and the accuracy of the model is guaranteed.
Table 2 Comparison between the reduced modes and that in all degree of freedom

| Mode order | Mode description                                      | Degree of freedom (Hz) | Reduced (Hz)  | Relative error (%) |
|------------|-------------------------------------------------------|------------------------|---------------|-------------------|
| 1          | Vertical bending of the beam in the first order       | 45.73                  | 45.54         | 0.42              |
| 2          | Lateral bending of the beam in the first order        | 53.37                  | 53.34         | 0.05              |
| 3          | Torsion and lateral bending of ATP hunger            | 133.84                 | 129.80        | 3.01              |
| 4          | Vertical bending of the beam in the second order      | 147.88                 | 144.83        | 2.06              |
| 5          | Lateral bending of the beam in the second order       | 190.72                 | 187.53        | 1.67              |
| 6          | Torsion and vertical bending of the beam in the second order | 253.15                 | 253.76        | 0.24              |

When the vehicle bridge coupling model is built up, the bogie frame and antenna beam are merged into a super element for modal analysis according to reduction theory, then the characteristic solution in the master degree of freedom is obtained, and the reduced substructure file is output. Simpack and FEMB program are used to generate FBI files. According to the configures of modes, the input files in standard of the FE body is finally generated.

The metro vehicle is a complex multi body system which contains complex connections and loads. During the calculation process, the model is simplified according to the problem type and emphasis. The modelling process should meet the requirement as follows:

1. Antenna beam, the key component for the fatigue study, should be treated as flexible body. At the same time, the bogie frame should be treated as a flexible body to study the fatigue strength in high frequencies.
2. The research objective should be a single vehicle and the longitudinal movement should be considered.
3. The whole dynamic model should contain the car body, flexible antenna beam, two flexible bogie frames, four wheelsets, axle box and suspension system. The dynamic model is shown as follows.

![Figure 4 Rigid-flexible coupling dynamic model of the vehicle](image)

The rail irregularities are the cause of the vehicle system dynamics. The basic cause for the high frequency excitation is the wheel defect excitation, random short-wave track irregularities, railroad switch excitation, rail gap impact, excitation from some section components in the railway system. The high frequency vibration caused by these excitations can affect the railway performance in different aspects.
Based on the FFT transform, the frequency response function of the stress in each evaluation point is calculated in the finite element software, and then the superposition dynamic stress is solved according to the acceleration time domain signal of the antenna beam.

In the dynamic stress calculation, the vibration phase and the main frequency under multiple loads should be considered. In the dynamic stress calculation, the acceleration of the mounting base at the end of the antenna beam could be used as the excitation input.

The FFT transformed and time domain data of the three-way acceleration at the end of the test antenna beam shows that the vertical vibration amplitude is the largest when the vibrations in different frequencies is contained. The amplitude of the vertical vibration is the largest when the vibration in different main frequencies exists. While, the longitudinal vibration is minimum. There is a rotating frequency approaching 8 Hz in the signals.

7. Conclusion
This paper takes the antenna beam in the subway bogie as the research objective. Through the experiment and simulation, the fatigue strength and vibration transition properties are analysed. Ploymax method is carried out for modal parameter identification, then the FE model could be verified to improve the accuracy. By setting up the rigid-flexible coupling dynamic model of the subway vehicle and comparing the modal parameters and experimental data, the model is modified and verified.

Considering the impact of wheel flat scar and rail gap, the vibration transmission characteristics of bogie system are studied. During the calculation of fatigue dynamic stress, FFT method is used to solve the problem. The fatigue strength of antenna beam caused by the impact of rail gap and wheel flat scar is studied, and the requirements for safe service life of antenna beam are put forward.

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