Locality and universality in gravitational anomaly cancellation

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Abstract

We obtain necessary and sufficient conditions for gravitational anomaly cancellation. We show that perturbative gravitational anomalies can never be cancelled. In a similar way, in dimensions \( n \neq 3 \mod 4 \) it is impossible to cancel global anomalies. However, in dimensions \( n = 3 \mod 4 \) global anomalies can be cancelled. We prove that the unique way to cancel the anomaly is by using a Chern-Simons counterterm. Furthermore, the relationship between the problems of locality and universality is analyzed for gravitational anomalies.

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1 Introduction

Although the study of gravitational anomalies is a classical subject in Quantum Field Theory, there are still some open problems related to them (e.g. see [23] and references therein). But the interest in gravitational anomalies has increased recently due to the fact that the concept of gravitational anomaly cancellation has been questioned in [28] and [29]. To clarify this concept, a detailed study of gravitational anomalies is needed. In this paper we obtain necessary and sufficient conditions for gravitational anomaly cancellation compatible with locality. We also study the relationship between locality and another problem in anomaly cancellation, the universality problem.
Roughly speaking, the universality problem for gravitational anomalies means that the conditions for anomaly cancellation should be independent of the global details of the space-time manifold $M$ of the theory (e.g. see [7],[8]). Furthermore, in [28] Witten conjectured that universality should be generalized to a stronger condition. Precisely, to have an anomaly free theory consistent with the principles of unitarity, locality and cutting and pasting, the conditions for anomaly cancellation (expressed in terms of the Atiyah-Patodi-Singer eta invariant) should be satisfied for all manifolds of dimension $n + 1$ and not just for the mapping tori of $M$.

We show that for perturbative gravitational anomalies universality is just a consequence of locality. To prove this result we use a characterization of anomaly cancellation in terms of local cohomology (see below) and the fact that the spaces of diffeomorphisms-invariant local forms are isomorphic for all compact manifolds of the same dimension. We study if there exists an analogous relationship between locality and universality for global gravitational anomalies. Although locality imposes restrictions on the possible ways to cancel the anomaly, it is not sufficient to imply universality in the global case and hence it should be imposed. We show that when universality is imposed, we obtain a generalization of Witten condition.

Now we consider the other basic problem in anomaly cancellation, the locality problem. This problem is analyzed in [14] for perturbative anomalies and in [15] for global anomalies. We apply the results of [15] to the case of gravitational anomalies. Let $M$ be compact oriented manifold of dimension $n$ and let $\text{Met}_M$ be the space of Riemannian metrics on $M$. We consider the action on $\text{Met}_M$ of the group of orientation preserving diffeomorphisms $\mathcal{D}_M^+$. If $\{D_g : g \in \text{Met}_M\}$ is a $\mathcal{D}_M^+$-equivariant family of elliptic operators acting on fermionic fields $\psi$ and parametrized by $\text{Met}_M$, then the Lagrangian density $\lambda_D(\psi, g) = \bar{\psi}iD_g\psi$ is $\mathcal{D}_M^+$-invariant. The theory can be anomalous because the corresponding partition function $Z(g) = \int D\psi D\bar{\psi} \exp(-\int_M \bar{\psi}iD_g\psi)$ could fail to be $\mathcal{D}_M^+$-invariant. It can be seen that the modulus of $Z(s)$ is $\mathcal{D}_M^+$-invariant. Hence we have $Z(\phi \cdot g) = Z(g) \cdot \exp(2\pi i \cdot \alpha_\phi(g))$ where $\alpha : \mathcal{D}_M^+ \times \text{Met}_M \to \mathbb{R}/\mathbb{Z}$ satisfies the cocycle condition $\alpha_{\phi_2 \phi_1}(g) = \alpha_{\phi_1}(g) + \alpha_{\phi_2}(\phi_1 g)$. The anomaly can be cancelled if there exists a local functional $\Lambda \in \Omega^0_{\text{loc}}(\text{Met}_M)$ satisfying the condition

$$\alpha_\phi(g) = \Lambda(\phi \cdot g) - \Lambda(g).$$

(1)

We recall that local functionals are those of the form $\Lambda = \int_M \lambda$ for a lagrangian density $\lambda$. If the condition (1) is satisfied, we can redefine the lagrangian density to $\lambda'(g) = \bar{\psi}iD_g\psi - \lambda(g)$ and the new partition function $Z' = Z \cdot \exp(-2\pi i \Lambda)$ is $\mathcal{D}_M^+$-invariant. Alternatively, we can consider $\lambda$ as the effective lagrangian of the theory. Note that in order to cancel the anomaly $\exp(2\pi i \Lambda)$ should not be $\mathcal{D}_M^+$-invariant.

Our objective in this paper is to obtain necessary and sufficient conditions for anomaly cancellation, and hence we need to analyze the condition (1) in detail. Following [15] we say that the topological anomaly cancels if condition (1) is satisfied for a functional $\Lambda \in \Omega^0(\text{Met}_M)$, and that the physical anomaly...
cancels if the condition (1) is satisfied for a local functional \( \Lambda \in \Omega^0_{\text{loc}}(\text{Met}M) \). Furthermore, if the condition (1) is satisfied only for the connected component of the identity \( D^+_M \) in \( D^+_M \) we say that the perturbative (or local) anomaly cancels. If it is satisfied for all the elements of \( D^+_M \) we say that the global anomaly cancels.

Topological anomaly cancellation admits a geometrical interpretation in terms of line bundles (e.g. see [3], [23], [15]). The cocycle \( \alpha \) determines an action on the trivial bundle \( L = \text{Met}M \times \mathbb{C} \rightarrow \text{Met}M \) by setting \( \phi_L(g, u) = (\phi(g), u \cdot \exp(2\pi i \alpha(\phi(g))) \) for \( g \in \text{Met}M \) and \( u \in \mathbb{C} \), and \( Z \) is a \( D^+_M \)-equivariant section of \( L \). We can also consider the principal \( U(1) \)-bundle \( U = \text{Met}M \times U(1) \rightarrow \text{Met}M \). The \( D^+_M \)-equivariant \( U(1) \)-bundle \( U \) is called the anomaly bundle and admits a natural \( D^+_M \)-invariant connection \( \Xi \). Usually the partition function is defined in terms of determinants of elliptic operators and \( L \) can be identified with a determinant or Pfaffian line bundle and \( \Xi \) with the Bismut-Freed connection (see [5]).

However, the physical anomaly cancellation requires that \( \Lambda \) should be a local functional \( \Lambda \in \Omega^0_{\text{loc}}(\text{Met}M) \), and hence \( \exp(2\pi i \Lambda) \) should be a special type of section of the anomaly bundle, a local section. The notion of local section and the conditions for physical anomaly cancellation require an adequate definition of local form \( \Omega^k_{\text{loc}}(\text{Met}M) \) for \( k > 0 \). We recall that the formulation of variational calculus in terms of jet bundles provides a generalization of the concept of local functionals to higher order forms (see [13] and Section 4 for details). Furthermore, the Atiyah-Singer Index theorem for families implies that the curvature of \( \Xi \) is a local form, \( \text{curv}(\Xi) \in \Omega^2_{\text{loc}}(\text{Met}M) \). We show that the necessary and sufficient condition for perturbative physical anomaly cancellation is the existence of a local \( D^+_M \)-invariant 1-form \( \beta \in \Omega^1_{\text{loc}}(\text{Met}M) \) such that \( \text{curv}(\Xi) = d\beta \). Hence the perturbative anomaly can be represented as a cohomology class on the local \( D^+_M \)-invariant cohomology \( H^2_{\text{loc}}(\text{Met}M)^{D^+_M} \). It is important to note that the local forms and local cohomology of \( \text{Met}M \) are very different to the ordinary ones, and this has implications in the possibilities for anomaly cancellation. We prove that \( H^2_{\text{loc}}(\text{Met}M)^{D^+_M} \) is independent of the manifold \( M \) and depends only on the dimension of \( M \). Hence if this condition is satisfied for a manifold \( M \), it is also satisfied for any other manifold of the same dimension. In this way, it follows that for perturbative anomalies universality is a consequence of locality as commented below.

If the perturbative anomaly cancels, we can still have global anomalies. In this case the anomaly bundle admits a \( D^+_M \)-basic connection and the possible forms that can be used to cancel the anomaly are determined by \( H^1_{\text{loc}}(\text{Met}M)^{D^+_M} \).
We show that $H^1_{\text{loc}}(\mathfrak{Met} M)^{D_M^+}$ is generated by the exterior differentials of the Chern-Simons actions associated to polynomials $p \in I^{(n+1)/2}(O(n))$. In particular, in dimensions $n \neq 3 \mod 4$ it is impossible to cancel the anomaly. However, in dimensions $n = 3 \mod 4$, it could be possible to cancel the anomaly with $\Lambda$ the Chern-Simons action associated to $p$. For determinant bundles we use Witten formula for the holonomy of the Bismut-Freed connection to obtain necessary and sufficient conditions for anomaly cancellation. If $M_\phi$ denotes the mapping torus of $\phi \in D^+_M$ and $\eta_D$ the Atiyah-Patodi-Singer eta invariant the condition is

$$\frac{1}{4} \eta_D(M_\phi) = p(M_\phi) \mod \mathbb{Z} \text{ for any } \phi \in D^+_M.$$  \(2\)

The preceding condition is not universal in the sense that it can be satisfied for a particular manifold $M$ but not for all manifolds of the same dimension. A universal version of the condition (2) is obtained by requiring that this condition should be satisfied for any oriented manifold $N$ of dimension $n+1$ and not only for mapping tori. This provides a condition weaker that the condition $\frac{1}{4} \eta_D(N) = 0 \mod \mathbb{Z}$ introduced in [29]. For example, we show that for Majorana fermions in dimension 3 the anomaly can be cancelled in this more general sense but not in Witten sense.

We extend our result to include orientation reversing diffeomorphisms. The problem in this case is that $M_\phi$ is unorientable and we cannot compute $p(M_\phi)$. We show that the condition in this case is $\frac{1}{4} \eta_D(M_\phi) = \frac{1}{2} p(M_\phi^2) \mod \mathbb{Z}$ for any $\phi \in D_M$. Furthermore the universal version of this condition is $\frac{1}{4} \eta_D(N) = \frac{1}{2} p(\tilde{N}) \mod \mathbb{Z}$ where $\tilde{N} \to N$ is the double cover of $N$.

In Section 2 we define the gravitational Chern-Simons action and we show how they can be used to cancel gravitational anomalies. In the rest of the paper we introduce the concepts that are necessary to prove that the unique possible way to cancel the anomaly is by using a Chern-Simons action. In Section 3 we study the local cohomology of $\mathfrak{Met} M$ and in Section 4 we apply the results of [15] to characterize gravitational anomaly cancellation. We also study the universal generalization of this condition and its relation to the condition considered by Witten in [29].

### 2 Chern-Simons actions

Let $M$ be an oriented compact manifold of dimension $n = 3 \mod 4$ and $p \in I^{(n+1)/2}(O(n))$ an invariant polynomial. We recall that $I^* (O(n))$ is generated over $\mathbb{R}$ by the Pontryagin polynomials $p_r$, and also by the trace of the even powers $T^{2k} (A) = \text{tr}(A^{2k})$ for $A \in \mathfrak{so}(n)$. Hence we can consider $I^* (O(n)) \subset I^* (\mathfrak{gl}(n, \mathbb{R}))$.

We denote by $\mathfrak{Met} M$ the space of Riemannian metrics on $M$. The group $D_M$ of diffeomorphisms of $M$ acts on $\mathfrak{Met} M$ by setting $\phi \cdot g = \phi_{\mathfrak{Met} M}(g) = (\phi^{-1})^* g$ for $g \in \mathfrak{Met} M$ and $\phi \in D_M$. The Levi-Civita connection of $g \in \mathfrak{Met} M$, considered as a connection on the frame bundle $FM \to M$, is denoted by $\omega^g$. If $A_0$ is a connection on $FM$, we define the Chern-Simons action with background
connection $A_0$ as the function $CS_{p,A_0} : \text{Met} M \to \mathbb{R}$ defined by $CS_{p,A_0}(g) = \int_M Tp(\omega^g, A_0)$ for $g \in \text{Met} M$. If we change the background connection to $A_0'$, then we have $CS_{p,A_0} = CS_{p,A_0} + \int_M Tp(A_0, A_0')$. Hence the Chern-Simons action is independent of $A_0$ up to the constant $\int_M Tp(A_0, A_0')$. For integral polynomials it is possible to fix this constant mod $\mathbb{Z}$ in a canonical way (see Remark 7).

**Remark 1** The Chern-Simons action is usually defined only for paralelizable manifolds (for example 3-manifolds) by using a trivialization of the frame bundle $FM \to M$. This is a particular case of our definition, where we take $A_0$ to be the connection induced by the trivialization.

As we want to use the Chern-Simons actions to cancel gravitational anomalies, we need to study its variation under diffeomorphisms. If $\phi \in D_M$ and $g \in \text{Met} M$ we define $\delta_0^p(g) = CS_{p,A_0}(\phi \cdot g) - CS_{p,A_0}(g)$ (clearly it does not depend on the background connection $A_0$). Furthermore we also have

**Proposition 2** If $\phi \in D_M^+$ then $\delta_0^p(g)$ is independent of $g$.

**Proof.** We have

$$\delta_0^p(g) - \delta_0^p(h) = \int_M Tp(\omega^{\phi \cdot g}, A_0) - \int_M Tp(\omega^g, A_0) - \int_M Tp(\omega^{\phi \cdot h}, A_0) + \int_M Tp(\omega^h, A_0)$$

$$= \int_M Tp(\omega^{\phi \cdot g}, \omega^{\phi h}) - \int_M Tp(\omega^g, \omega^h)$$

$$= \int_M (\phi^{-1} \cdot Tp(\omega^g, \omega^h) - \int_M Tp(\omega^g, \omega^h) = 0.$$

In the last equation we use that $\int_M (\phi^{-1})^* \alpha = \int_M \alpha$ for any $\alpha \in \Omega^n(M)$ because $\phi \in D_M^+$. □

**Remark 3** Another way to prove this Proposition is by using that $d(CS_{p,A_0})$ is $D_M^+$-invariant (see Section 4). Then we have $d(\delta_0^p) = \delta_0^p \in \text{Met} M$. Hence $\delta_0^p$ is independent of $g \in \text{Met} M$.

By Proposition 2 we can denote $\delta_0^p(g)$ simply by $\delta_0^p$.

**Proposition 4** If $\phi, \phi' \in D_M^+$ then $\delta_0^{p, \phi'} = \delta_0^{p} + \delta_0^p$.

**Proof.** We have

$$\delta_0^{p, \phi'}(g) = CS_{p,A_0}(\phi \cdot \phi' \cdot g) - CS_{p,A_0}(g)$$

$$= CS_{p,A_0}(\phi \cdot \phi' \cdot g) - CS_{p,A_0}(\phi' \cdot g) + CS_{p,A_0}(\phi' \cdot g) - CS_{p,A_0}(g)$$

$$= \delta_0^{p}(\phi' \cdot g) + \delta_0^p(g).$$

The result follows by using Proposition 2 □

If $\phi \in D_M$ we define its mapping torus $M_\phi = (M \times [0,1]) / \sim_\phi$ where $(x,0) \sim_\phi (\phi(x),1)$. When $\phi \in D_M^+$ the orientation on $M$ induces an orientation on $M_\phi$. If $\phi$ reverses the orientation then $M_\phi$ is unorientable. If $p \in I^{(n+1)/2}(O(n))$ and
\( \phi \in D^+_M \) we define \( p(M_\phi) \in \mathbb{R} \) as the characteristic number associated to \( p \) on \( M_\phi \), i.e. \( p(M_\phi) = \int_{\Omega \phi} p(F) \), where \( F \) is the curvature of any connection on \( F(M_\phi) \) (it is independent of the connection by Chern-Weil theory).

Witten formula expresses the variation of the partition function in terms of the eta invariant of an operator on the mapping torus. We obtain a similar result for the Chern-Simons action

**Theorem 5** For any \( \phi \in D^+_M \) we have \( \delta_\phi = p(M_\phi) \).

**Proof.** a) The bundle \( FM \times I \to M \times I \) is a principal \( GL(n, \mathbb{R}) \)-bundle. For small \( \varepsilon \) we can find a connection \( \overline{\omega} \) on this bundle that coincides with \( \omega^g \) on \( FM \times \{t\} \) for \( t < \varepsilon \) and with \( \omega^{\phi} \) for \( t > 1 - \varepsilon \). The connection \( \overline{\omega} \) induces a connection \( \overline{\omega}_\phi \) on the quotient bundle \( F^0M = (FM \times I) / _{\sim} \phi \to M_\phi \). We denote by \( \overline{\Omega} \) and \( \overline{\Omega}_\phi \) the curvature forms of \( \overline{\omega} \) and \( \overline{\omega}_\phi \).

If \( pr_1: M \times I \to M \) and \( \overline{pr}_1: FM \times I \to FM \) are the projections, then we have \( p(\overline{pr}_1 F_0) = pr_1^* p(F_0) = 0 \) by dimensional reasons. As \( M_\phi \) is an oriented manifold and we can consider the integral

\[
\int_{M_\phi} p(\overline{\Omega}_\phi) = \int_{M \times I} p(\overline{\Omega}) = \int_{M \times I} d(Tp(\overline{\omega}, \overline{pr}_1 A_0))
\]

\[
= \int_{M \times \{1\}} Tp(\overline{\omega}, \overline{pr}_1 A_0) - \int_{M \times \{0\}} Tp(\overline{\omega}, \overline{pr}_1 A_0)
\]

\[
= \int_M Tp(\omega^{\phi}, A_0) - \int_M Tp(\omega^g, A_0) = \phi^{*}_{\text{pair}} (CS_{p,A_0}) - CS_{p,A_0}
\]

We need to prove that the characteristic classes of the bundle \( F^0M \) coincide with the characteristic classes of \( M_\phi \). As \( I^*(O(n)) \) is generated over \( \mathbb{R} \) by the Pontryagin polynomials, it is sufficient to prove the result for the real Pontryagin classes. The vertical bundle \( V(M_\phi) \to M_\phi \) of the fibration \( q: M_\phi \to S^1 \) is a vector bundle associated to the principal \( GL(n, \mathbb{R}) \)-bundle \( F^0M \to M_\phi \). We have \( T(M_\phi) \cong V(M_\phi) \oplus q^*TS^1 \), and as \( q^*TS^1 \) is trivial we have \( p_k(T(M_\phi)) = p_k(V(M_\phi)) \). \( \blacksquare \)

The following result follows as a consequence of the preceding Theorem. It can also be obtained from the existence of equivariant Pontryagin forms (see Section [3])

**Corollary 6** If \( \phi \in D^0_M \) then \( \delta_\phi = 0 \).

**Proof.** It follows from the fact that if \( \phi_t \subset D^0_M \) is a curve joining \( \text{id}_M \) and \( \phi \), then the map \( FM \times I \to F^0M, (u, t) \mapsto [(\phi_t)_u, u, t] \) is an isomorphism. \( \blacksquare \)

It follows from this Corollary that the Chern-Simons action cannot be used to cancel perturbative anomalies. However, it can be used to cancel global anomalies because we can have \( \delta_\phi \neq 0 \) for \( \phi \in D^+_M \). Furthermore, we conclude that \( \delta^p \) can be considered as a group homomorphism \( \delta^p \in \text{Hom}(D^+_M / D^0_M, \mathbb{R}) \).

**Remark 7** If the polynomial \( p \) determines an integral characteristic class, or more generally, if \( p(M_\phi) \) is an integer for any \( \phi \in D^+_M \), then we obtain as a corollary of Theorem 5 that \( CS_{p,A_0} \) is \( D^+_M \)-invariant mod \( Z \), and hence \( \exp(2\pi i \cdot CS_{p,A_0}) \) is \( D^+_M \)-invariant.
Furthermore, if \( p \) determines an integral characteristic class, the Chern-Simons theory can be used to give a definition the Chern-Simons action independent of the background connection \( A_0 \) (see \cite{11,9}). If \( \zeta_n(M) \) is the space of \( n \)-cycles on \( M \) and \( \chi_A: \zeta_n(M) \rightarrow \mathbb{R}/\mathbb{Z} \) is the Chern-Simons differential character associated to a connection \( A \) on \( FM \), we can define \( CS_p: \mathcal{M} \rightarrow \mathbb{R}/\mathbb{Z} \) by \( CS_p(g) = \chi_{\omega^g}(M) \) and we have \( CS_p(g) = \chi_{A_0}(M) + CS_{p,A_0}(g) \mod \mathbb{Z} \). In this way we can define in a canonical way the \( D_M^+ \)-invariant exponentiated Chern-Simons action \( \exp(2\pi i \cdot CS_p) \). This fact is interesting to study Chern-Simons theory as a Topological Quantum field theory, but it cannot be used to cancel anomalies.

However, for an arbitrary \( p \in I^{(n+1)/2}(O(n)) \) the function \( CS_{p,A_0} \) is not \( D_M^+ \)-invariant \( \mod \mathbb{Z} \), and this fact can be used to cancel gravitational anomalies if the variation of \( \exp(2\pi i \cdot CS_{p,A_0}) \) cancels the variation of the partition function.

If the perturbative anomaly cancels then we have \( Z(\phi \cdot g) = Z(g) \exp(2\pi i \cdot \kappa_\phi) \) where \( \kappa_\phi \in \text{Hom}(D_M^+/D_M^0, \mathbb{R}/\mathbb{Z}) \). We can cancel the anomaly with a Chern-Simons counterterm if there exists \( p \in I^{(n+1)/2}(O(n)) \) such that \( \kappa_\phi = \delta g^p \) for any \( \phi \in D_M^+ \). Furthermore, we prove in Section 5 that this is the unique possible way to cancel the anomaly. For Dirac operators \( \kappa_\phi \) has been computed by Witten (e.g. see \cite{27,28,29}) and we have \( \kappa_\phi = \frac{1}{4} \eta_D(M_\phi) \), where \( \eta_D \) is the reduced Atiyah-Patodi-Singer eta invariant of a differential operator on \( M_\phi \). If Witten formula applies, then the necessary and sufficient condition for global anomaly cancellation is the existence of \( p \in I^{(d+1)/2}(O(n)) \) such that \( \frac{1}{2} \eta_D(M_\phi) = p(M_\phi) \mod \mathbb{Z} \) for any \( \phi \in D_M^+ \).

We recall that the eta invariant \( \eta_D(N) \) is a spectral invariant of the metric defined by \( \eta_D = \dim \ker D + \lim_{\varepsilon \to 0^+} \sum_k \text{sign}(\lambda_k) \exp(-\varepsilon^2 \lambda_k^2) \), where \( \lambda_k \) are the eigenvalues of \( D \). It is know that in general \( \eta_D(N) \) cannot be obtained as the integral of a local form. However, (e.g. see \cite{26}) for certain operators \( \eta_D(N) \) can be obtained in terms of characteristic numbers. We study this condition and its generalizations in more detail in Section 5.

Note that \( \frac{1}{2} \eta_D(M_\phi) \) is independent of the orientation on \( M \), but the sign of \( p(M_\phi) \) changes if we change the orientation. Hence, if \( \frac{1}{2} \eta_D(M_\phi) = p(M_\phi) \mod \mathbb{Z} \) then we have \( p(M_\phi) = -p(M_\phi) \mod \mathbb{Z} \) and hence \( 2p(M_\phi) = 0 \mod \mathbb{Z} \). We conclude that we can cancel the anomaly only if \( \frac{1}{2} \eta_D(M_\phi) = 0 \) or \( \frac{1}{2} \mod \mathbb{Z} \). This implies a possible change of sign in the partition function \( Z(\phi \cdot g) = \pm Z(g) \). We recall (see \cite{29}) that this sign anomaly is the kind of anomaly that can appear for real or pseudoreal fermions.

### 2.1 Orientation reversing diffeomorphisms

If \( \phi \) is an orientation reversing diffeomorphisms, then the results of the previous section can not be applied. Proposition 2 does not hold for \( \phi \) and \( M_\phi \) is unorientable, and hence we cannot compute \( p(M_\phi) \). However, orientation reversing diffeomorphisms are important symmetries and they should be considered in anomaly cancellation (e.g. see \cite{29}). Furthermore, in \cite{29} and \cite{30} it is analyzed also the case in which \( M \) is unorientable.
One way to solve this problem is to consider the orientation as an independent field. We define $\mathcal{Met}M = \{(g, o) : g \in \mathcal{Met}M and o is an orientation on M\}$ and $D_M$ acts in a natural way on $\mathcal{Met}M$. We extend the Chern-Simons action to $\mathcal{Met}M$ by setting $CS_p, A_0 (g, o) = \int_{(M, o)} Tp(\omega^0, A_0)$ and then Propositions 2 and 4 are valid for arbitrary diffeomorphisms. Furthermore, for any $\phi \in D_M$ we have $\delta_o^p = \frac{1}{2} \delta_{o^2} = \frac{1}{2} p(\omega^2)$ as $\phi^2 \in D_M$. If we combine this result with Witten formula we conclude that the $D_M$-anomaly cancels if $\frac{1}{2} \eta_D(M_\phi) = \frac{1}{2} p(M_\phi) \mod Z$ for any $\phi \in D_M$.

Again the left side is independent of the orientation on $M$ and the right side depends on it. Hence, also for orientation reversing diffeomorphisms we can cancel only a sign anomaly.

### 3 Equivariant cohomology in the Cartan model

We recall the definition of equivariant cohomology in the Cartan model (e.g. see [4, 21]). Suppose that we have a left action of a connected Lie group $G$ on a manifold $N$. We denote by $\Omega^k(N)^G$ the space of $G$-invariant forms on $N$ and by $H^k(N)^G$ the $G$-invariant cohomology of $N$. Let $\Omega^*_G(N) = \mathcal{P}^*(\text{Lie } G, \Omega^*(N))^G$ be the space of $G$-invariant polynomials on $\text{Lie } G$ with values in $\Omega^*(N)$ with the graduation $\deg(\alpha) = 2k + r$ if $\alpha \in \mathcal{P}^k(\text{Lie } G, \Omega^r(N))$. Let $D : \Omega^*_G(N) \to \Omega^*_{G+1}(N)$ be the Cartan differential, $(D\alpha)(X) = d(\alpha(X)) - \iota_X \alpha(X), X \in \text{Lie } G$. On $\Omega^*_G(N)$ we have $D^2 = 0$, and the equivariant cohomology (in the Cartan model) of $N$ with respect to the action of $G$ is defined as the cohomology of this complex.

A $G$-equivariant 1-form $\alpha \in \Omega^1_G(N)$ is just a $G$-invariant 1-form $\alpha \in \Omega^1(N)^G$. It is $D$-closed if and only if it is $G$-basic, i.e., if $d\alpha = 0$ and $\iota_X \alpha = 0$ for any $X \in \text{Lie } G$. If $\varpi \in \Omega^2_G(N)$ is a $G$-equivariant 2-form, then we have $\varpi = \omega + \mu$ where $\omega \in \Omega^2(N)$ is $G$-invariant and $\mu \in \text{Hom}(\text{Lie } G, \Omega^0(N))^G$. In particular we have $\mu([Y, X]) = L_Y \mu(X))$ for any $X, Y \in \text{Lie } G$. The map $\alpha \mapsto \alpha(0)$ induces a map $H^2_G(N) \to H^2(N)^G$. We have the following

**Proposition 8** If $H^1(\text{Lie } G) = 0$ then the map $H^2_G(N) \to H^2(N)^G$ is injective.

**Proof.** Let $\varpi = \omega + \mu \in \Omega^2_G(N)$ be a closed $G$-equivariant 2-form, and assume that there exists $\beta \in \Omega^1(N)^G$ such that $d\beta = \omega$. Then $\tau = \varpi - D\beta \in \Omega^2_G(N)$ is $D$-closed and $\tau \in \text{Hom}(\text{Lie } G, \Omega^0(N))^G$ But $D\tau = 0$ implies $d(\tau(X)) = 0$ for any $X \in \text{Lie } G$ and hence $\tau \in \text{Hom}(\text{Lie } G, \mathbb{R})^G$. Furthermore, by the $G$-invariance of $\tau$ we have $\tau([Y, X]) = 0$ for any $X, Y \in \text{Lie } G$. As $H^1(\text{Lie } G) = 0$ we conclude that $\tau = 0$, and hence $\varpi = D\beta$.

If $\omega \in \Omega^k(N)^G$ is closed, we say that $\varpi \in \Omega^2_G(N)$ is a $G$-equivariant extension of $\omega$ if $\varpi(0) = \omega$ and $D\omega = 0$. In general there can be obstructions to the existence of $G$-equivariant extensions. However, we recall that for a $G$-invariant connection, the equivariant characteristic classes provide canonical equivariant extensions of the characteristic forms.
If \( \pi: U \to N \) is a principal \( U(1) \) bundle and \( \Xi \in \Omega^1(U, i\mathbb{R}) \) is a connection then the curvature form \( \text{curv}(\Xi) \in \Omega^2(N) \) is defined by the property \( \pi^*(\text{curv}(\Xi)) = \frac{\partial}{\partial x^i}(\Xi) \). A \( G \)-equivariant \( U(1) \)-bundle is an \( U(1) \)-bundle \( U \to N \) in which \( G \) acts by \( U(1) \)-automorphisms. A connection \( \Xi \in \Omega^1(U, i\mathbb{R}) \) on \( U \) is \( G \)-invariant if \( \phi^*(\Xi) = \Xi \) for any \( \phi \in G \). If \( \Xi \) is a \( G \)-invariant connection then \( \frac{\partial}{\partial x^i}(\Xi) \) projects onto a closed \( G \)-equivariant 2-form \( \text{curv}_G(\Xi) \in \Omega^2_G(N) \) called the \( G \)-equivariant curvature of \( \Xi \). If \( \Phi \in \text{Lie} G \) then we have \( \text{curv}_G(\Xi)(X) = \text{curv}(\Xi) + \mu^G(X) \), where \( \mu^G(X) = -\frac{1}{2\pi} \Xi(X_U) \). We say that a connection \( \Xi \) is \( G \)-flat if \( \text{curv}_G(\Xi) = 0 \).

4 Local forms on \( \text{Met} M \) and Jet bundles

We recall that the classical variational calculus can be formulated in a coordinate independent way in terms of the geometry of jet bundles (e.g. see [2], [25]). A Lagrangian function is a function \( L(g) \) that depends on the components of the metric \( g_{ij}(x) \) and its derivatives \( g_{ij,k}(x) = \frac{\partial g_{ij}}{\partial x^k}(x) \). Hence, if we define the jet bundle \( J^\infty \text{Met} M \) as the space with coordinates \((x_i, g_{ij}, g_{ij,k}, g_{ij,kr}, \ldots)\), then the Lagrangian function can be considered as a function \( L: J^\infty \text{Met} M \to \mathbb{R} \). More formally, we say that two metrics \( g, g' \in \text{Met} M \) have the same jet at \( x \in M \) if in any coordinate system we have \( g_{ij}(x) = g'_{ij}(x) \) and \( \frac{\partial g_{ij}}{\partial x^k}(x) = \frac{\partial g'_{ij}}{\partial x^k}(x) \) for any \( i,j = 1,\ldots,n \), and any symmetric multi-index \( I \). For \( g \in \text{Met} M \) we denote by \( j^\infty g \) the jet of \( g \) at \( x \), that can be considered as a coordinate independent version of the Taylor polynomial of \( g \) at \( x \). The space of all jets of metrics is denoted by \( J^\infty \text{Met} M \). We denote the projection by \( q_\infty: J^\infty \text{Met} M \to M \). A local chart \( \phi: U \to \mathbb{R}^n \) for \( U \subset M \) induces a chart \( \phi \circ q_\infty: J^\infty \text{Met} M \to J^\infty \text{Met} \mathbb{R}^n \) by setting \( \phi \circ q_\infty(j^\infty g) = j^\infty_{\phi(x)}((\phi^{-1})^* g) \).

A Lagrangian density is a form \( \lambda \in \Omega^p(J^\infty \text{Met} M) \) of the type \( \lambda = L dx^1 \wedge \ldots \wedge dx^n \) for \( L \in \Omega^q(J^\infty \text{Met} M) \). The action functional associated to \( \lambda \) is the function \( A_\lambda \in \Omega^0(2\text{Met} M) \) defined by \( A_\lambda(g) = \int_M L(g) dx^1 \wedge \ldots \wedge dx^n = \int_M (j^\infty g)^* \lambda \). The variational calculus consists in the study of the form \( d\lambda \in \Omega^1(2\text{Met} M) \) in terms of the Euler-Lagrange operator of \( \lambda \). In the jet bundle approach, \( d\lambda \) is determined by the Euler-Lagrange form \( \mathcal{E}(\lambda) \in \Omega^{n+1}(J^\infty \text{Met} M) \) given in local coordinates by \( \mathcal{E}(\lambda) = \left( \sum_I (-1)^{|I|} \frac{\partial}{\partial x^{|I|}} \left( \frac{\partial L}{\partial g_{ij}} \right) \right) d\lambda_{ij} \wedge dx^1 \wedge \ldots \wedge dx^n \), where \( \frac{\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \sum_I \left( \sum_I (-1)^{|I|} \frac{\partial}{\partial x^{|I|}} \left( \frac{\partial L}{\partial g_{ij}} \right) \right) d\lambda_{ij} \wedge dx^1 \wedge \ldots \wedge dx^n \). As \( 2\text{Met} M \) is an open convex set of the space \( S^2 M \) of bilinear symmetric tensors on \( M \) we have \( T_g 2\text{Met} M \simeq S^2 M \) for any \( g \in 2\text{Met} M \). Then the form \( d\lambda \) is given by \( d\lambda_g(h) = \int_M \left( \sum_I (-1)^{|I|} \frac{\partial}{\partial x^{|I|}} \left( \frac{\partial L}{\partial g_{ij}} \right) \right) h_{ij} dx^1 \wedge \ldots \wedge dx^n = \int_M (j^\infty g)^*(i_H \mathcal{E}(\lambda)) \), where \( H \) is the vector field \( H = h_{ij} \frac{\partial}{\partial x^i} + \sum_I \left( \sum_I (-1)^{|I|} \frac{\partial}{\partial x^{|I|}} \left( \frac{\partial L}{\partial g_{ij}} \right) \right) \in \mathfrak{X}(J^\infty \text{Met} M) \). In particular \( d\lambda_g = 0 \) if and only if \( \mathcal{E}(\lambda)(j^\infty g) = 0 \) for all \( x \in M \), and the last are just the usual Euler-Lagrange equations.

The correspondence \( \lambda \in \Omega^p(J^\infty \text{Met} M) \to \mathcal{A}_\lambda \in \Omega^0(2\text{Met} M) \), and \( \mathcal{E}(\lambda) \in \Omega^{n+1}(J^\infty \text{Met} M) \to d\mathcal{A}_\lambda \in \Omega^1(2\text{Met} M) \) is extended to forms of arbitrary degree in [12]. Let \( j^\infty: M \times 2\text{Met} M \to J^\infty \text{Met} M, j^\infty(x,g) = j^\infty_g \) be the evaluation function.
map. We define $\mathcal{I} \in$ $\mathcal{A}$ if $\mathcal{A}$ then $d\mathcal{A}$ we define $n$ $k$ for any $(\text{see } [12])$. If $\mathcal{H}$ $k$ tional $\mathcal{I}$ $\mathcal{M}$ into horizontal and contact (or vertical) degree. This bigrading is the in-

Unfortunately, for $\mathcal{O}$ $\mathcal{M}$, $\mathcal{H}$ $k$ can be studied in terms of its canonical representative on the jet bundle. Unfortunately, for $k = 0$ we do not have a similar operator, i.e. for a local functional $\Lambda \in \Omega_{\mathcal{M}}^0$ we cannot select a canonical Lagrangian density $\lambda$. 

4.1 Local cohomology and the variational bicomplex 

On $J^{\infty} M$ we have a natural bigrading $\Omega^k(J^{\infty} M) = \bigoplus_{p+q=k} \Omega^{p,q}(J^{\infty} M)$ into horizontal and contact (or vertical) degree. This bigrading is the infinitesimal version of the bigrading corresponding to the product structure on $M \times \mathcal{M}$, and the map $j^\infty: M \times \mathcal{M} \rightarrow J^{\infty} M$ preserves bidegree. If $\mathcal{A} \in \Omega^k(J^{\infty} M)$ we denote by $a_{p,q} \in \Omega^{p,q}(J^{\infty} M)$ its $p$-horizontal and $q$-contact component. According to the preceding bigrading we have a decomposition of the exterior differential $d = d_H + d_V$.

For $k > 0$ we denote by $I: \Omega^n,k(J^{\infty} M) \rightarrow \Omega^n,k(J^{\infty} M)$ the interior Euler operator. We recall (e.g. see [2]) that it satisfies the following properties: $I^2 = I$, $\ker I = d_H(\Omega^{n-1,k})$, $Id_V = d_V I$. The image of the interior Euler operator $F^k(J^{\infty} M) = I(\Omega^n,k(J^{\infty} M))$ is called the space of functional $k$-forms. The vertical differential $d_V$ induces a differential on the space of functional forms $\delta_V: F^k(J^{\infty} M) \rightarrow F^{k+1}(J^{\infty} M)$, $\delta_V \alpha = I(d_V \alpha)$. In this context a lagrangian density is a form $\lambda \in \Omega_{\mathcal{M}}^0$ and $\mathcal{E}(\lambda) = I(d_V \lambda) \in F^1(J^{\infty} M)$ is the Euler-Lagrange operator of $\lambda$.

The relationship between the variational bicomplex and the local forms is the following (see [13]). If $\mathcal{A} \in \Omega^{n,k}(J^{\infty} M)$, $k > 0$, we have $\mathcal{I}(\alpha) = \mathcal{I}(a_{n,k}) = \mathcal{I}(I(a_{n,k}))$. Furthermore, the restriction of $\mathcal{I}$ to $F^k(J^{\infty} M)$ induces isomorphisms $F^k(J^{\infty} M) \simeq \Omega_{\mathcal{M}}^k(\mathcal{M})$ for $k > 0$. Hence, the local forms of degree $k > 0$ can be studied in terms of its canonical representative on the jet bundle. Unfortunately, for $k = 0$ we do not have a similar operator, i.e. for a local functional $\Lambda \in \Omega_0^0(\mathcal{M})$ we cannot select a canonical Lagrangian density $\lambda$. 

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such that $\Im(\lambda) = \Lambda$.

The isomorphism $J^k(J^{\infty}M_M) \simeq \Omega^k_{\text{loc}}(\text{Met}M)$, combined with the classical calculation of the cohomology of the variational bicomplex implies that the map $\Im$ induces isomorphisms $H^k_{\text{loc}}(\text{Met}M) \simeq H^k(J^\bullet(J^{\infty}M_M)) \simeq H^{n+k}(J^{\infty}M_M) = 0$ for $k > 0$ (see [24] for details).

For any $\phi \in \mathcal{D}_M$ we define $\text{pr} \phi \in \mathcal{D}_{J^{\infty}M_M}$ by setting $\text{pr}(j^\infty x) = j^\infty_x(\phi \cdot g)$ for any $g \in \text{Met}M$. The action of $\mathcal{D}_M$ commutes with $\Im$ and hence we have isomorphisms $\Omega^k_{\text{loc}}(\text{Met}M)_{\mathcal{D}_M} \simeq J^k(J^{\infty}M_M)_{\mathcal{D}_M}$ for $k > 0$ and $H^k_{\text{loc}}(\text{Met}M)_{\mathcal{D}_M} \simeq H^k(J^\bullet(J^{\infty}M_M)_{\mathcal{D}_M})$ for $k > 1$.

The space $\Omega^k(\text{Met}M)_{\mathcal{D}_M}$ can depend on the global properties of the manifold $M$. A big difference of local cohomology and ordinary cohomology is that the space of local forms $\Omega^k_{\text{loc}}(\text{Met}M)_{\mathcal{D}_M}$ only depends on the dimension of $M$.

**Lemma 9** Let $\alpha \in \Omega^0(J^{\infty}M_{R^n})_{D^+_k}$. For any $j^\infty x \in J^{\infty}M_M$ we choose an oriented local chart $\phi : U \to \mathbb{R}^n$ with $x \in U$ and the induced chart $\text{pr} \phi : j^\infty x^{-1}(U) \to J^{\infty}M_{R^n}$. The form $\alpha_M(j^\infty x) \in \Omega^0(j^{\infty}M_M)$ defined by $\alpha_M = (\text{pr} \phi)^* \alpha$ is independent of $\phi$ and $\alpha_M$ is $D^+_M$-invariant. In this way we obtain a map that induces isomorphisms

$$
\Omega^0(J^{\infty}M_{R^n})_{D^+_k} \simeq \Omega^0(J^{\infty}M_M)_{D^+_M} \simeq \Omega^0(J^{\infty}M_M)_{D^+_0}.
$$

**Proof.** That $\alpha_M$ is well defined and it is $D^+_M$-invariant easily follows from the $D^+_k$-invariance of $\alpha$. The maps $\Omega^k(J^{\infty}M_{R^n})_{D^+_k} \to \Omega^k(J^{\infty}M_M)_{D^+_M}$ and $\Omega^*(J^{\infty}M_{R^n})_{D^+_k} \to \Omega^*(J^{\infty}M_M)_{D^+_M}$ are clearly injective. That they are surjective follows from the fact that any germ of orientation preserving diffeomorphism can be extended to a diffeomorphism isotopic to the identity (e.g. see [24]).

If we apply the preceding result to the functional forms we obtain the following

**Corollary 10** We have the isomorphisms

$$
\Omega^k_{\text{loc}}(\text{Met}M)_{D^+_M} \simeq \Omega^k_{\text{loc}}(\text{Met}M)_{D^+_M} \simeq \Omega^k(J^{\infty}M_{R^n})_{D^+_k}.
$$

for $k > 0$ and

$$
H^k_{\text{loc}}(\text{Met}M)_{D^+_M} \simeq H^k_{\text{loc}}(\text{Met}M)_{D^+_M} \simeq H^k(\Omega^*(J^{\infty}M_{R^n})_{D^+_k}).
$$

for $k > 1$.

Hence, we can consider $\Omega^k(J^{\infty}M_{R^n})_{D^+_k}$ as the space of universal local $k$-forms on dimension $n$. We remark that this result is not valid for $H^1_{\text{loc}}(\text{Met}M)_{D^+_M}$ (see Section [43]).

**Remark 11** If $G$ is a subgroup such that $D^0_M \subset G \subset D^+_M$ we also have that $\Omega^0(J^{\infty}M_M)^G \simeq \Omega^0(J^{\infty}M_{R^n})_{D^+_k} \simeq \Omega^0(J^{\infty}M_M)_{D^+_M}$. This applies for example to the subgroup of elements of $D^+_M$ preserving a spin structure on $M$. 

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4.2 Pontryagin forms on the jet of metrics

The pull-back bundle $\tilde{q}_\infty^*: q_\infty^*FM \to J^\infty \mathcal{M}_M$ is a $\mathcal{D}_M$-equivariant principal $\text{Gl}(n, \mathbb{R})$-bundle. As the Levi-Civita connection $\omega^g$ of a metric $g \in \mathcal{M}_M$ depends only on the first derivatives of $g$, we can define in a natural way a connection on $q_\infty^*FM$ by setting $\omega(X) = \omega^g((\tilde{q}_\infty)_*(X))$ for $X \in T_{(j^\infty g, u)}(q_\infty^*FM)$, $g \in \mathcal{M}_M$ and $u \in (FM)_x$. It can be seen (see [14]) that $\omega$ is $\mathcal{D}_M$-invariant. If $p \in I^k(O(n)) \subset I^k(\text{Gl}(n, \mathbb{R}))$ and $\Omega$ is the curvature form of $\omega$, we define the form $p(\Omega) \in \Omega^{2k}(J^\infty \mathcal{M}_M)^{\mathcal{D}_M}$. In particular, for the Pontryagin polynomial $p_k$ we have the Pontryagin form $p_k(\Omega) \in \Omega^{4k}(J^\infty \mathcal{M}_M)^{\mathcal{D}_M}$.

Remark 12 We can define the Pontryagin forms by using the connection $\omega$ because we consider $I^k(O(n)) \subset I^k(\text{Gl}(n, R))$. But in this way we cannot define the Euler form. It is shown in [16] that the connection $\omega$ can be modified to obtain a $\mathcal{D}_M$-invariant connection $\omega'$ on the principal $O(n)$-bundle $OM \to J^\infty \mathcal{M}_M$ where $OM = \{(j^\infty g, u) \in q_\infty^*FM: u$ is $g_x$-orthonormal$\}$. By using the connection $\omega'$ we can define the Pontryagin forms and the Euler form $E \in \Omega^n(J^\infty \mathcal{M}_M)^{\mathcal{D}_M}$. In our study of anomaly cancellation we only consider forms of degree $n+2$ and $n+1$, and hence the Euler form does not appear. Furthermore, it can be seen that the Pontryagin forms defined by using the connections $\omega$ and $\omega'$ lead to equivalent results. Hence we use the connection $\omega$ that it is easier to define.

By applying the map $\Im$ to the forms $p(\Omega)$ we obtain closed local forms $\sigma^p = \Im(p(\Omega)) \in \Omega^{2k-n}_{\text{loc}}(\mathcal{M}_M)^{\mathcal{D}_M}$ for $2k \geq n$. We can consider another interpretation of the forms $\sigma^p$ that is more familiar in the study of gravitational anomalies (e.g. see [3, 22]). The evaluation map determines a map $\varpi: FM \times \mathcal{M}_M \to q_\infty^*FM$. The connection $\omega^* = \varpi^*\omega$ defines a connection on this bundle and we have $\sigma^p = \int_M p(\Omega^*)$. It is shown in [13] that we have the following

Proposition 13 The map $I^k(O(n)) \to H^{2k-n}_{\text{loc}}(\mathcal{M}_M)^{\mathcal{D}_M}$, $p \mapsto [\sigma^p]$ is injective for $n + 1 < 2k \leq 2n$.

Let $A_0$ be a connection on $FM \to M$. As $\omega$ and $\overline{\omega}_0 = q_\infty^*A_0$ are connections on the same bundle $q_\infty^*FM \to J^\infty \mathcal{M}_M$ we have $p(\overline{\omega}_0) = dT_p(\omega, \overline{\omega}_0)$. But if $2k > n$ then $p(F_0) = 0$ by dimensional reasons and hence we have $p(\Omega) = dT_p(\omega, \overline{\omega}_0)$ and $\sigma^p = d\rho^p$, where $\rho^p = \Im(T_p(\omega, \overline{\omega}_0)) \in \Omega^{2k-n-1}_{\text{loc}}(\mathcal{M}_M)$. Note that the form $\rho^p$ is not $\mathcal{D}_M$-invariant as it depends on the connection $A_0$. In particular, if $n = 3 \mod 4$ and $p \in I^{(n+1)/2}(O(n))$ then the function $\rho^p = \Im(T_p(\omega, \overline{\omega}_0)) \in \Omega^{n+1}_{\text{loc}}(\mathcal{M}_M)$ is given by the Chern-Simons action

$$
\rho^p(g) = \int_M (j^\infty g)^*Tp(\omega, \overline{\omega}_0) = \int_M Tp(\omega^g, A_0) = CS_{p, A_0}(g).
$$

Furthermore we have $dCS_{p, A_0} = d\rho^p = p^p \in \Omega^1_{\text{loc}}(\mathcal{M}_M)^{\mathcal{D}_M}$, as commented in Section 2. Moreover $\lambda_{p, A_0} = Tp(\omega, \overline{\omega}_0)_{n, 0} \in \Omega^{n, 0}(J^\infty \mathcal{M}_M)$ is the Lagrangian density corresponding to the Chern-Simons action.
Remark 14 The forms \( p(\Omega) \) are invariant under arbitrary diffeomorphisms of \( M \). However, the integration map \( \mathcal{I} \) is defined by using an orientation on \( M \). Hence the forms \( \sigma^p \) are not invariant under orientation reversing diffeomorphisms. As in Section 2 we can solve this problem by considering the space \( \mathcal{M} \). The integration map defines a map \( \mathcal{I}: \Omega^{n+k}(J^\infty \mathcal{M}_M) \to \Omega^k(\mathcal{M}) \) and this map is \( \mathcal{D}_M \)-equivariant. We define \( \Omega^k_{\text{loc}}(\mathcal{M}) = \mathcal{I}(\Omega^{n+k}(J^\infty \mathcal{M}_M)) \) and we have \( \sigma^p = \mathcal{I}(p(\Omega)) \in \Omega^k_{\text{loc}}(\mathcal{M})^{\mathcal{D}_M} \).

4.3 Chern-Simons actions and generalized Cotton tensors

Let \( M \) be an oriented compact manifold of dimension \( n = 3 \) mod 4 and \( p \in I^{(n+1)/2}(O(n)) \). Under the isomorphism \( \Omega^1_{\text{loc}}(\mathcal{M})^{\mathcal{D}_M^k} \simeq F^1(J^\infty \mathcal{M}_M)^{\mathcal{D}_M^k} \) the form \( \sigma^p \) corresponds to the functional 1-form \( \mathcal{E}^p = I(p(\Omega)) \in F^1(J^\infty \mathcal{M}_M)^{\mathcal{D}_M^k} \), which is called the generalized Cotton Tensor (e.g. see [1]). We remark that \( \mathcal{E}^p \) can also be obtained as the Euler-Lagrange operator of the Chern-Simons Lagrangian \( \lambda_{p,A_0} \). It is known that \( \mathcal{E}^p \) is not the Euler-Lagrange operator of a \( \mathcal{D}_M^+ \) Lagrangian density (see [1], [13]). Furthermore, we have the following result (see [1])

Proposition 15

a) If \( n \neq 3 \) mod 4 any \( \delta_V \)-closed \( \mathcal{D}_R^+ \)-invariant functional 1-form \( \mathcal{E} \in \mathfrak{g}^1(J^\infty \mathcal{M}_R)^{\mathcal{D}_R^+} \) is the Euler-Lagrange operator of a \( \mathcal{D}_R^+ \)-invariant lagrangian density.

b) If \( n = 3 \) mod 4 then any \( \delta_V \)-closed \( \mathcal{D}_R^+ \)-invariant functional 1-form \( \mathcal{E} \in \mathfrak{g}^1(J^\infty \mathcal{M}_R)^{\mathcal{D}_R^+} \) is of the form \( \mathcal{E} = \mathcal{E} + \mathcal{E}(\lambda) \) for \( p \in I^{(n+1)/2}(O(n)) \) and \( \lambda \in \Omega^{n,0}(J^\infty \mathcal{M}_R)^{\mathcal{D}_R^+} \) a \( \mathcal{D}_R^+ \)-invariant lagrangian density.

c) The results of a) and b) are also valid if \( \mathcal{D}_R^+ \) is replaced with \( \mathcal{D}_R^- \).

By Lemma 9 this result also holds for any compact manifold \( M \) of dimension \( n \). By using Corollary 10 we obtain the following

Theorem 16

Let \( M \) be a compact oriented manifold of dimension \( n \).

a) If \( n \neq 3 \) mod 4 then we have \( H^1_{\text{loc}}(\mathcal{M})^{\mathcal{D}_M^k} = 0 \).

b) If \( n = 3 \) mod 4 then any closed \( \mathcal{D}_M^+ \)-invariant local 1-form \( \sigma \in \Omega^1_{\text{loc}}(\mathcal{M})^{\mathcal{D}_M^k} \) is of the form \( \sigma = \sigma^p + dp \) for \( p \in I^{(n+1)/2}(O(n)) \) and \( \rho \in \Omega^0_{\text{loc}}(\mathcal{M})^{\mathcal{D}_M^k} \).

We conclude that \( H^1_{\text{loc}}(\mathcal{M})^{\mathcal{D}_M^k} \) is generated by the Pontryagin forms, as commented in the Introduction. The Corollary 10 can be reobtained by using the properties of the forms \( \sigma^p \). We have the following

Lemma 17

If \( p \in I^{(n+1)/2}(O(n)) \) then \( \sigma^p = \mathcal{I}(p(\Omega)) \in \Omega^1_{\text{loc}}(\mathcal{M})^{\mathcal{D}_M^k} \) is \( \mathcal{D}_M^+ \)-basic, i.e. \( dp = 0 \) and \( \iota_X(\mathcal{M})^{\mathcal{D}_M^k} \sigma^p = 0 \) for any \( X \in \mathcal{X}(M) \).

Proof. The result follows from the existence of equivariant Pontryagin forms. As the connection \( \omega \) is \( \mathcal{D}_M^+ \)-invariant, the form \( p(\omega) \) admits a canonical equivariant extension \( \pi^p \) (the explicit expression of this extension can be
found in [17]). The $D$-closed equivariant 1-form $\Theta(\varpi^p) \in \Omega^1_{D_M, \text{loc}}(\Omega^0_{\text{met}}M) \simeq \Omega^1_{\text{loc}}(\Omega^0_{\text{met}}M)^{D_M^0}$ is given simply by the 1-form $\sigma^p = \Theta(p(\Omega))$. Hence we have $d\sigma^p = 0$ and $\iota_{X^M_{\text{met}}} \sigma^p = 0$ for $X \in \mathfrak{X}(M)$.

This result implies Corollary 16 as we have $L_{X^M_{\text{met}}} CS_{p, A_0} = \iota_{X^M_{\text{met}}} dCS_{p, A_0} = \iota_{X^M_{\text{met}}} \sigma^p = 0$ for $X \in \mathfrak{X}(M)$.

By using Corollary 10 and Theorem 16 we obtain the following

**Corollary 18** For any compact oriented manifold we have $H^1_{\text{loc}}(\Omega^0_{\text{met}}M)^{D_M^0} = 0$.

Hence we could have $H^1_{\text{loc}}(\Omega^0_{\text{met}}M)^{D_M^0} = H^1_{\text{loc}}(\Omega^0_{\text{met}}M)^{D_M^0}$ in dimension $n \neq 3 \mod 4$.

## 5 Locality and anomaly cancellation

As commented in the introduction, in [15] the necessary and sufficient condition for anomaly cancellation can be expressed in terms of the equivariant curvature and holonomy of the Bismut-Freed connection. Precisely, let $\pi: \mathcal{U} \to \text{Met}M$ be a $D^+_M$-equivariant $U(1)$-bundle with a $D^+_M$-invariant connection $\Xi$. If $\phi \in D^+_M$ we define $C^{\phi} = \{\gamma: [0, 1] \to \text{Met}M : \gamma(1) = \phi \cdot \gamma(0)\}$. For any $\phi \in D^+_M$ and $\gamma \in C^{\phi}$ we define the $\phi$-equivariant holonomy of $\Xi$ $\text{hol}^\phi_{\Xi}(\gamma) \in \mathbb{R}/\mathbb{Z}$ by the condition $\gamma(1) = \phi_t(\gamma(0)) \exp(2\pi i \cdot \text{hol}^\phi_{\Xi}(\gamma))$, where $\gamma$ is any $\Xi$-horizontal lift of $\gamma$. Then we have the following (see [15])

**Proposition 19** Let $\mathcal{U} \to \text{Met}M$ be a $D^+_M$-equivariant $U(1)$-bundle with a $D^+_M$-invariant connection $\Xi$.

p) $\mathcal{U} \to \text{Met}M$ admits a $D^0_M$-equivariant section if and only if there exists a $D^0_M$-invariant 1-form $\beta \in \Omega^1(\Omega^0_{\text{met}}M)^{D_M^0}$ such that $\text{curv}_{D_M^0}(\Xi) = D\beta$.

q) $\mathcal{U} \to \text{Met}M$ admits a $D^+_M$-equivariant section if and only if there exists a $D^+_M$-invariant 1-form $\beta \in \Omega^1(\Omega^0_{\text{met}}M)^{D_M^0}$ such that $\text{hol}^\phi_{\Xi}(\gamma) = \int_\gamma \beta \mod \mathbb{Z}$ for any $\gamma \in C^{\phi}$.

When the preceding result is applied to the anomaly bundle, it provides necessary and sufficient conditions for topological anomaly cancellation, i.e., to the possibility of solving condition (1) with a term $A \in \Omega^0(\text{Met}M)$. However, physical anomaly cancellation requires that $A$ should be a local functional $\Omega^0_{\text{loc}}(\text{Met}M)$. To generalize the Proposition 19 for physical anomalies we need analogous local versions of all the objects appearing on it. This has been done in [14] for perturbative anomalies and in [15] for global anomalies. We say that a $D^+_M$-invariant connection $\Xi$ on $\mathcal{U} \to \text{Met}M$ is local if $\text{curv}_{D_M^+}(\Xi) \in \Omega^1_{D_M^+, \text{loc}}(\Omega^0_{\text{met}}M)$. Note that when $\Xi$ is the Bismut-Freed connection on a determinant bundle, then $\Xi$ is a local connection by the equivariant Atiyah-Singer theorem for families (see [19]). Finally it is needed to determine intrinsically what kind of sections of $\mathcal{U} \to \text{Met}M$ correspond to the solutions of
equation \((\ref{equation})\) with \(\Lambda\) a local functional. It is shown in \(\cite{15}\) that this condition is that a section \(S: \text{Met}M \to \mathcal{U}\) is \(\Xi\)-local if \(S^*(\Xi) \in \Omega^1_{\text{loc}}(\text{Met}M)\). With this definitions we have the following result

**Proposition 20** Let \(\mathcal{U} \to \text{Met}M\) be a \(D^n_+\)-equivariant \(U(1)\)-bundle with a local \(D^n_+\)-invariant connection \(\Xi\).

1. \(\mathcal{U} \to \text{Met}M\) admits a \(D^n_0\)-equivariant \(\Xi\)-local section if and only if there exists a \(D^n_0\)-invariant local 1-form \(\beta \in \Omega^1_{\text{loc}}(\text{Met}M)^{D^n_0}\) such that \(\text{curv}_{D^n_0}(\Xi) = D\beta\).

2. \(\mathcal{U} \to \text{Met}M\) admits a \(D^n_+\)-equivariant \(\Xi\)-local section if and only if there exists a \(D^n_+\)-invariant local 1-form \(\beta \in \Omega^1_{\text{loc}}(\text{Met}M)^{D^n_+}\) such that \(\text{hol}^\Xi_\phi(\gamma) = \int_\gamma \beta \mod \mathbb{Z}\) for any \(\gamma \in C^\phi\).

When this result is applied to the anomaly bundle it provides necessary and sufficient conditions for physical anomaly cancellation.

For gravitational anomalies the condition \(P\) for physical perturbative anomaly cancellation can be simplified. It is a classical result of Gel’fand and Fuchs (see \(\cite{20}\)) that the Lie algebra cohomology \(H^k(\mathfrak{g}(M)) = 0\) is trivial for \(k \leq n\). In particular we have \(H^1(\mathfrak{g}(M)) = 0\) and by Proposition\(\cite{8}\) the map \(H_{\text{loc}}^2(\text{Met}M) \to H_{\text{loc}}^2(\text{Met}M)^{D^n_0}\) is injective. Hence the condition \(P\) in Proposition\(\cite{20}\) is equivalent to the condition

\(P'\) \(\mathcal{U} \to \text{Met}M\) admits a \(D^n_0\)-equivariant \(\Xi\)-local section if and only if there exists a \(D^n_0\)-invariant local 1-form \(\beta \in \Omega^1_{\text{loc}}(\text{Met}M)^{D^n_0}\) such that \(\text{curv}(\Xi) = d\beta\).

### 5.1 Locality and Universality

By using the isomorphism \(\cite{3}\) the condition \(P'\) can be expressed in terms of functional forms on \(\mathbb{R}^n\). Hence the condition for perturbative gravitational anomaly depends only on the dimension of \(M\) and universality is a consequence of locality. Furthermore, the perturbative anomaly in dimension \(n\) can be identified with a cohomology class on \(H^2(\mathfrak{F}^\infty(\mathcal{M}_{\mathbb{R}^n})^{D^n_0})\).

For determinant bundles \(\text{curv}(\Xi)\) is given by the Atiyah-Singer theorem as a combination of characteristic classes, i.e., we have \(\text{curv}(\Xi) = \sigma^{p_D}\) for a polynomial \(p_D \in L^{1+n/2}(O(n))\). For example, for twisted Dirac operators associated to a representation of the Spin group \(p_D = [\hat{A} \cdot \text{Ch}]_{1+n/2}\), where \(\hat{A}\) is the Dirac genus and \(\text{Ch}\) the Chern character of the representation and we take the component of degree \(1+n/2\). By Proposition\(\cite{13}\) the condition \(P'\) cannot be satisfied if \(p_D \neq 0\). Hence it is impossible to cancel the perturbative anomaly with a local counterterm if \(p_D \neq 0\).

If the condition \(P'\) is satisfied, then the perturbative anomaly cancels. However, we can still have global anomalies because a \(D^n_0\)-equivariant section could be not \(D^n_+\)-equivariant. We can define a new connection \(\Xi' = \Xi - \pi^*\beta\) which is \(D^n_0\)-invariant by Corollary\(\cite{10}\) and it is \(D^n_+\)-flat by Proposition\(\cite{8}\). Hence we can assume that the connection \(\Xi\) is \(D^n_+\)-flat. It is shown in \(\cite{15}\) that if a connection \(\Xi\) is \(D^n_+\)-flat then the equivariant holonomy \(\text{hol}^\Xi_\phi(\gamma)\) does not depend
on $\gamma \in C^\phi$ and defines a group homomorphism $\kappa^\Xi = \text{Hom}(D_M^+ / D_M^0, \mathbb{R} / \mathbb{Z})$ by $\kappa^\Xi_\phi = \text{hol}_G^\Xi(\gamma)$ for any $\gamma \in C^\phi$. We call $\kappa^\Xi$ the $D_M^+$-flat holonomy. Furthermore, condition G) is equivalent to the existence of a form $\beta_\phi \in \Omega^1_{\text{loc}}(\mathfrak{M}(M))^{D_M^+}$ such that $d\beta_\phi = 0$ and $\kappa^\Xi_\phi = \int_\gamma \beta_\phi \mod \mathbb{Z}$ for any $\phi \in D_M^+$ and $\gamma \in C^\phi$. Again the possible forms $\beta$ satisfying these conditions can be determined by using the isomorphism $[3]$. By Theorem 16 if $n \neq 3 \mod 4$ then any $\beta \in \Omega^1_{\text{loc}}(\mathfrak{M}(M))^{D_M^+}$ satisfying $d\beta = 0$ is of the form $\beta = d\Lambda$, with $\Lambda \in \Omega^0_{\text{loc}}(\mathfrak{M}(M))^{D_M^+}$, and hence $\int_\gamma \beta = \Lambda(\gamma(1)) - \Lambda(\gamma(0)) = \Lambda(\phi \cdot \gamma(0)) - \Lambda(\gamma(0)) = 0$ for any $\phi \in D_M^+$ and $\gamma \in C^\phi$. Hence if $n \neq 3 \mod 4$ it is impossible to cancel the anomaly if $\kappa^\Xi_\phi \neq 0$.

In this way we obtain our main result

**Theorem 21** Let $U \to \mathfrak{M}(M)$ be a $D_M^+$-equivariant $U(1)$-bundle with a $D_M^+$-flat connection $\Xi$. Then $U$ admits a $D_M^+$-equivariant $\Xi$-local section if and only if there exists $p \in I^{(n+1)/2}(O(n))$ such that $\kappa^\Xi_\phi = p(M_\phi) \mod \mathbb{Z}$ for any $\phi \in D_M^+$.

For determinant or Pfaffian bundles associated to a family of operators $D_\phi$ Witten formula has been interpreted as a computation of the holonomy of the Bismut-Freed connection on the quotient determinant bundle $U/D_M^+ \to \mathfrak{M}(M)/D_M^+$ (e.g. see [6], [10], [18]). Furthermore, it can be also interpreted as the equivariant holonomy of the Bismut-Freed connection on $U \to \mathfrak{M}(M)$. Hence, for $D_M^+$-flat connections we have $\kappa^\Xi_\phi = \frac{1}{4} \eta_D(M_\phi)$ where $D$ is an operator on the mapping torus. In those cases the necessary and sufficient condition for gravitational anomaly cancellation is the existence of $p \in I^{(n+1)/2}(O(n))$ such that

$$\frac{1}{4} \eta_D(M_\phi) = p(M_\phi) \mod \mathbb{Z}$$

for any $\phi \in D_M^+$. (5)

Hence we show that the unique way to cancel the gravitational anomaly is by using a Chern-Simons counterterm, as announced in Section 2. If condition (5) is satisfied, then the gravitational anomaly can be cancelled by using the Chern-Simons action associated to $p$.

We have shown that the possible counterterms necessary to cancel the anomaly are given by a local 1-form $\beta \in \Omega^1_{\text{loc}}(\mathfrak{M}(M))^{D_M^+} \simeq \mathcal{F}^1(J^{\infty}(M_{2n}))^{D_M^+}$, and hence they are universal. However, the mapping class group $\Gamma_M = D_M^+ / D_M^0$ and the $D_M^+$-flat holonomy $\kappa^\Xi$ are nonlocal objects and can be different for different manifolds of the same dimension. Hence the condition (5) could be satisfied only for certain manifolds of dimension $n$ but not for all of them. We conclude
that for global anomalies universality is not a consequence of locality and should be imposed. One way to solve the condition (5) in a universal way is if
\[ \frac{1}{4} \eta_D(N) = p(N) \mod \mathbb{Z} \] for any oriented manifold \( N \) of dimension \( n + 1 \). (6)

We recall that in general \( \eta_D(N) \) cannot be obtained as the integral of a local form on \( N \). However, in certain cases (e.g. see [26]) \( \eta_D(N) \) can be expressed as a characteristic number of \( N \), and the condition (6) is satisfied.

We recall that Witten has proposed in [29] a stronger condition for anomaly cancellation. If the condition (5) is satisfied for any \( M \) the partition function for any \( M \) is determined up to a phase, but the phases for different \( M \)'s should also be fixed. To solve this problem Witten proposes as a generalization of the condition for anomaly cancellation the condition
\[ \frac{1}{4} \eta_D(N) = 0 \mod \mathbb{Z} \] for any oriented manifold \( N \) of dimension \( n + 1 \). (7)

In dimension \( n \neq 3 \mod 4 \) the conditions (6) and (7) are the same. However, if \( n = 3 \mod 4 \) the condition (6) can be weaker than (7). For example, for Majorana fermions in an oriented manifold of dimension 4 we have (see [29])
\[ \frac{1}{4} \eta_D(N) = \frac{1}{32} \sigma(N) \mod \mathbb{Z}, \]
where \( \sigma(N) \) is the signature of \( N \). For \( N \) a K3 surface we have
\[ \frac{1}{4} \eta_D(N) = \frac{1}{32} \neq 0 \mod \mathbb{Z} \] and the condition (7) is not satisfied. However, it follows from the Hirzebruch signature theorem that \( \sigma(N) = \frac{1}{32}p_1(N) \). Hence we have
\[ \frac{1}{4} \eta_D(N) = \frac{1}{32}p_1(N) \] for any oriented \( N \), and the anomaly can be cancelled in the sense of condition (7).

We recall the argument in [29] that leads to the condition (7). If \( X \) is a manifold with boundary such that \( \partial X = M \), and the metric and all the structures can be extended to \( X \), Witten proposes to define the partition function by
\[ Z_D(g) = \exp(2\pi i \frac{1}{4} \eta_D(X)). \]
That \( Z_D(g) \) is independent of the manifold \( X \) chosen follows from the Dai-Freed theorem. This argument can be generalized if condition (6) is satisfied by defining
\[ Z_D(g) = \exp(2\pi i \frac{1}{4} \eta_D(X) - \int_X p(\omega \gamma)), \]
where \( \gamma \) is the extension of \( g \) to \( X \).

5.2 Orientation reversing diffeomorphisms and unorientable manifolds

We have shown in Section 2.1 that the condition for anomaly cancellation can be extended to orientation reversing diffeomorphisms. The condition is
\[ \frac{1}{4} \eta_D(M_\phi) = \frac{1}{2}p(M_\phi) \mod \mathbb{Z} \] for any \( \phi \in D_M \). (8)

We can find a universal version of this condition in the following way. The mapping torus \( M_\phi \) can also be obtained as a quotient \((M \times \mathbb{R})/\mathbb{Z}\) where \( \mathbb{Z} \) acts on \( M \times \mathbb{R} \) by setting \( n \cdot (x, t) = (\phi^n(x), t + n) \). Then the projection \( M_\phi \to M_\phi \) \( [(x, t) + \mathbb{Z}] \to [(\phi(x), 2t)] \) is a double cover. If \( \phi \) reverses the orientation \( M_\phi \) is unorientable, but \( M_\phi \) is orientable because \( \phi^2 \in D_M \). For any unorientable
manifold we have a double cover \( \tilde{N} \to N \) with \( \tilde{N} \) orientable and we have \( \tilde{M}_\phi = M_{\phi^2} \). Hence, one way to generalize the condition \( \text{S} \) is the following condition

\[
\frac{1}{4} \eta_D(N) = \frac{1}{2} p(\tilde{N}) \mod Z \text{ for any manifold } N \text{ of dimension } n + 1. \quad (9)
\]

Again, as the right side is independent of the orientation on \( \tilde{N} \), this condition can be satisfied only if \( \frac{1}{4} \eta_D(N) = 0, \frac{1}{2} \mod Z \).

The analogous condition proposed in [29] is

\[
\frac{1}{4} \eta_D(N) = 0 \mod Z \text{ for any manifold } N \text{ of dimension } n + 1. \quad (10)
\]

Let us consider again the case of Majorana Fermions in dimension 3 studied in [29]. We have shown before that if we consider only oriented manifolds the anomaly can be cancelled. But if we admit unorientable manifolds the situation is different. For \( N = \mathbb{R}P^4 \) we have \( \frac{1}{4} \eta_D(\mathbb{R}P^4) = \frac{1}{16} \mod Z \) and hence the anomaly cannot be cancelled in the sense of the condition (9).

One basic question in solid state physics is the problem of how many Majorana fermions should we have in order to have an anomaly free theory. This problem appears in the analysis of the boundary of a topological superconductor (see [29] for details). If we consider \( \nu \) Majorana fermions, then we have \( \kappa_{\phi}^2 = \frac{\nu}{4} \eta_D(M_\phi) \). The condition \( \kappa_{\phi}^2 = 0 \) for any \( \phi \in D_M \) implies that \( \nu \) should be a multiple of 8. However the condition (10) implies that \( \nu \) should be a multiple of 16 (because for \( N = \mathbb{R}P^4 \) we have \( \frac{1}{4} \eta_D(\mathbb{R}P^4) = \frac{1}{16} \mod Z \)), a result that coincides with other analysis in condensed matter physics.

We show that our condition (9) also implies that \( \nu \) should be a multiple of 16. For \( N = \mathbb{R}P^4 \) we have \( \sigma(\tilde{N}) = \sigma(S^4) = 0 \). Furthermore, as \( I^2(O(4)) \) is one dimensional, this implies \( p(S^4) = 0 \text{ for any } p \in I^2(O(4)) \). Hence condition (9) is satisfied for \( N = \mathbb{R}P^4 \) if \( \frac{\nu}{16} = 0 \mod Z \) and hence \( \nu \) should be a multiple of 16. Furthermore it is shown in [29] that \( \eta_D(N) = \frac{\nu}{4} \) for any manifold, and hence if \( \nu = 16k \) we have \( \frac{\nu}{4} \eta_D(N) = 4k \frac{\nu}{4} = 0 \mod Z \), and the anomaly cancels with \( p = 0 \). Hence condition (9) is satisfied if and only if \( \nu \) is a multiple of 16.

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