Determination of geometric parameters for flexible splines of wave gearing

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Abstract. The paper considers the influence of a number of flexspline geometric parameters on the stresses acting in it. The influence of the most significant parameters such as the diameter of the flexible spline and the thickness of its rim under the gear rim is considered. As a result of analysis of the flexspline geometric parameters influence on its load capacity, the recommendations are given for assigning the main dimensions of the spline at the stage of design.

Keywords: wave gearing, flexible spline, flexible spline parameters.

1. Introduction
The main criteria for the wave gearing (WG) operational capability are the flexspline fatigue strength and the efficiency of the wave generator. To prevent fatigue failure of a flexible spline, it is necessary to correctly assign its basic geometric parameters, which determine the stresses arising in it. This should be done at the design stage. The main parameter of the flexible spline is its internal diameter, so first of all it is necessary to determine the internal diameter of flexspline depending on the transmitted torque. Then it is important to assign the remaining dimensions of a flexible spline, which significantly affect the stress in it, especially in the gear rim, where, as a rule, the fatigue crack initiation and flexible spline destruction occur. These are such dimensions as the shell thickness under the gear rim, thickness of the cylindrical shell, length of the shell, length of the gear rim and others. Several different stresses must be considered when analyzing them in the flexible spline of the wave gearing. These are the bending stresses in the spline that arise from its deformation by the wave generator [1]. A very significant influence is exerted by stresses arising from the load on the teeth [2]. In such a case it is appropriate to take into account the distribution of these stresses [3]. Besides, the interaction of a flexible spline with a wave generator is also investigated. Some researchers consider a spatial model of a flexible spline [4]. Also theoretical studies of the effect of various parameters on the wave gearing limit torque are known [5].

In the study of wave gearing operational capability by Ivanov M. N. [6], Shuvalov S. A. [7] and Poletuchy A. I. [8], major proposals for flexspline dimensions are made. These dimensions are determined by the transmitted torque. However, the influence of certain dimensions of the flexible spline is not examined to the full extent. These dimensions include the thickness of the flexible spline under gear rim, length of the gear rim, length of the shell and presence of the front belt in front of the gear rim.

2. Problem statement
Recommendations for determining flexspline dimensions are available. However, the previously developed recommendations are based mainly on theoretical studies. The distribution pattern of the actual stresses in the flexible spline is very complex. To clarify the recommendations on specifying flexible spline dimensions, it is important to rely on experimental tests of these splines in a gear under operation. It is essential to accept theoretical dependences for calculation of stresses in a flexible spline. Then the revising coefficients are introduced, that can be obtained as a result of experiments. To obtain the values of these coefficients, testing a number of flexible splines with different geometric parameters is required.
The design of the flexspline of a "glass" type adopted for the study is shown in figure 1. This design is most common and is used in real harmonic drive units. Figure 1 represents the design and main dimensions of the flexible spline as well as load F from the wave generator.

![Figure 1. Flexible gear – “glass”](image)

Typically, the design of a flexible spline begins with the determination of its internal diameter \(d\) depending on the transmitted torque \([2]\). Further, on the basis of existing recommendations the remaining dimensions of the spline are defined. To specify these recommendations, experimental observations are important. The experiments on the fatigue resistance of flexible gears without load \([1]\) and under load \([2]\) were performed on a purpose-built stand. According to the test results, recommendations should be made on the assignment of reasonable dimensions for flexible splines.

3. Theory

Stresses arising in the operating flexible spline are considered in \([9]\). The following stresses in the flexible spline are examined:

- normal stresses \(\sigma_{h,ns}\), from the deformation of the flexible spline by the wave generator;
- normal stresses \(\sigma_{lth}\), from the load on the teeth;
- shear stresses \(\tau_m\), from the torque \(T\).

Total or equivalent stresses are determined by the maximum shear stress theory

\[
\sigma = \sqrt{\left(\sigma_{h,ns} + \sigma_{lth}\right)^2 + 4\tau_m^2}
\]  

(1)

\(\sigma_{h,sm}\) are nominal stresses in a smooth spline with a thickness under the gear rim \(\delta_f\) defined by the known dependence \([1]\)

\[
\sigma_{h,sm} = \frac{E \delta_f}{2} \frac{\left(R - r\right)}{\left(R + \delta_f\right)} \left(\frac{r + \delta_f}{2}\right)
\]  

(2)

To determine the value of the actual stresses \(\sigma_{h,ns}\) through the stress \(\sigma_{h,sm}\) we introduce a correction coefficient
This coefficient should be estimated according to the results of testing.

To determine the stresses $\sigma_i$ arising from the load on the teeth, the dependence was used

$$\sigma_i = K_{th}\sigma_{t,l}$$

(4)

Nominal stresses $\sigma_{t,l}$ are defined according to [3, 7] on the dependence

$$\sigma_{t,l} = \mu \frac{\pi T \sin 2\alpha \cdot C}{2m^3Z^3K_LK_z} \left(h + 0.5\delta_1\right) \frac{6m}{\delta_1^2} K_j$$

(5)

The coefficient $K_{th}$ was obtained as a result of flexible splines testing on a real gear in operation. According to the paper [4] the rated torsional stresses are defined as

$$\tau_{m,n} = \frac{T}{K_m 2\pi \left(R + \frac{\delta_1}{2}\right)^2 \delta_2}$$

(6)

The greatest local shear stresses $\tau_{m,n}$ from the action of torque were specified

$$\tau_{m,n} = K_m \tau_{m,n}$$

(7)

Here the revising coefficient $K_m$ was also determined experimentally.

4. Experiment results

In the experimental part, we tested the wave gearing for fatigue resistance in the first series without load on the teeth, and in the second series with a load in a real wave gearing. The following geometrical parameters of the flexible spline were varied in the experiments (Figure 1):

- $m$ is the module of gear;
- $S$ is the width of the teeth cavity by the reference diameter;
- $R = d/2$ is the radius of the inner surface of the flexible spline;
- $\delta$ is the thickness of the flexible spline under the gear rim;
- $L$ is the length of the flexible spline;
- $b$ is the width of the gear rim;
- $\delta_0$ is the thickness of the cylinder part of the flexspline;
- $R$ is the tooth pitch;
- $r$ is the transition radius from the gear rim to the smooth part;
- $l$ is the length of the smooth part in front of the gear rim.

The pitches of variable factors are given in table 1.
Table 1. Pitches of variable factors.

|   | X₁=δ|m | X₂=S/P | X₃=L/R | X₄=b/R | X₅=r/R | X₆=U/R | X₇=δ₀/δ |
|---|---|---|---|---|---|---|---|
| 1 | 3.66 | 0.5 | 2 | 0.425 | 0.6 | 0.15 | 0.8 |
| 2 | 3.66 | 0.5 | 2 | 0.2 | 0.05 | 0.15 | 0.5 |
| 3 | 3.66 | 0.5 | 1.5 | 0.425 | 0.05 | 0.05 | 0.5 |
| 4 | 3.66 | 0.5 | 1.5 | 0.2 | 0.6 | 0.05 | 0.8 |
| 5 | 3.66 | 0.386 | 2 | 0.425 | 0.05 | 0.05 | 0.5 |
| 6 | 3.66 | 0.386 | 2 | 0.2 | 0.6 | 0.05 | 0.8 |
| 7 | 3.66 | 0.386 | 1.5 | 0.425 | 0.6 | 0.15 | 0.8 |
| 8 | 3.66 | 0.386 | 1.5 | 0.2 | 0.05 | 0.15 | 0.5 |
| 9 | 1 | 0.5 | 2 | 0.425 | 0.05 | 0.05 | 0.8 |
| 10 | 1 | 0.5 | 2 | 0.2 | 0.6 | 0.05 | 0.5 |
| 11 | 1 | 0.5 | 1.5 | 0.425 | 0.6 | 0.15 | 0.5 |
| 12 | 1 | 0.5 | 1.5 | 0.2 | 0.05 | 0.15 | 0.8 |
| 13 | 1 | 0.386 | 2 | 0.425 | 0.6 | 0.15 | 0.5 |
| 14 | 1 | 0.386 | 2 | 0.2 | 0.05 | 0.15 | 0.8 |
| 15 | 1 | 0.386 | 1.5 | 0.425 | 0.05 | 0.05 | 0.8 |
| 16 | 1 | 0.386 | 1.5 | 0.2 | 0.6 | 0.05 | 0.5 |

The following dependences for the revising coefficients are obtained after processing the results and exclusion of statistically insignificant values:

\[ K_h = 9.57 - 1.44 \frac{\delta_i}{m} - 3.68 \frac{L}{R} - 0.29 \frac{L}{R} - 5.58 \frac{b}{R} + 0.62 \frac{\delta_i}{m} + 3.19 \frac{L}{R} \]  

\[ K_{ih} = 0.21 \frac{\delta_i}{m} + 7.84 \frac{L}{R} + 28.44 \frac{b}{R} - 2 \frac{\delta_i}{m} - 0.616 \frac{L}{R} + 16.25 \frac{b}{R} - 6.79 \]  

\[ K_m = 18.21 + 0.72 \frac{\delta_i}{m} - 8.94 \frac{L}{R} - 43.05 \frac{b}{R} + 1.2 \frac{r}{R} + 22.95 \frac{b}{R} \]  

5. Results and discussion

As a result of experimental studies the dependences (8), (9), and (10) are obtained for defining the revising factors to calculate stresses in the flexible spline of the wave gearing. Also recommendations for assigning the geometric parameters of flexible splines are obtained. The recommended dependence for determining the internal diameter of the flexible spline d depending on the transmitted torque T and the fatigue limit of the flexspline material σ₁₋₁ is:

\[ d = 0.113 \sqrt{\frac{T}{\sigma_{1-1}}} \]  

(11)

where d is the internal diameter of the flexible spline (m) with the recommended geometric parameters; 
T is the flexspline transmitted torque (Nm); 
σ₁₋₁ is the fatigue limit of the flexible spline material (MPa).

For the other important parameters recommendations are obtained:

\[ L = d; \delta_0 / \delta = 0.8; b = 0.2 d; l = 0.075 d; r = 0.3 d \]  

(12)
A particularly important geometric parameter that determines the flexible spline stress, and, consequently, its durability and load capacity is the thickness of the shell $\delta$ under the gear rim. The stresses in the flexible splines with a small thickness $\delta$ ($\delta = 0.01 \, d$) are less in the absence of load on the teeth, and with the increase in the transmitted torque they grow rapidly. Stresses in flexible splines with a large thickness $\delta$ ($\delta = 0.02 \, d$), in the absence of load are greater than in thin splines. However, when the load on the teeth increases, the stresses increase more slowly. There is a certain torque value at which the stresses are equal in splines with a small value $\delta$ and a large value.

As an example, the calculation of stresses in the region of the flex spline gear rim on the used dependences with allowance for the obtained factors was performed. Flexible splines with the following parameters were considered in calculations. Parameters of the flexible splines under consideration are $d = 80$ mm; $L = 80$ mm; $b = 16$ mm. Different thicknesses $\delta_1$ of the flexible spline under the gear rim were considered. The results of the calculations are shown in figure 2. The values of $\delta_1$ are given in table 2.

| Curve number | $\delta_1$ (mm) |
|--------------|-----------------|
| 1            | 0.583           |
| 2            | 0.729           |
| 3            | 0.875           |
| 4            | 1.02            |
| 5            | 1.167           |
| 6            | 1.313           |
| 7            | 1.459           |
| 8            | 1.705           |
| 9            | 1.75            |
| 10           | 1.896           |

6. Conclusion

The analysis of the results allows one to draw recommendations on the design of flexible splines of WG.

1. Thickness $\delta_1$ is the most significant dimension that affects stresses in the flexible spline under the gear rim and its assignment depends on the transmitted load.
2. The internal diameter of the flexible spline \( d \) is recommended to be determined by the dependence (11), and the other significant dimensions to be taken as

\[
L = d; \quad \delta_0/\delta = 0.8; \quad b = 0.2 \, d; \quad l = 0.075 \, d; \quad r = 0.3 \, d
\]

3. It is preferable to take the minimum values of thickness \( \delta_1 \) under the load up to 30 Nm in the gear with accepted parameters (\( d = 80 \text{ mm}; \quad L = 80 \text{ mm}; \quad b = 16 \text{ mm} \)), and the largest possible values \( \delta_1 \) are preferable under the load of more than 30 Nm (Table 2).

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