Large-basis shell-model calculation of $^{10}\text{C} \rightarrow ^{10}\text{B}$ Fermi matrix element

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Abstract

We use a $4\hbar\Omega$ shell-model calculation with a two-body effective interaction derived microscopically from the Reid93 potential to calculate the isospin-mixing correction for the $^{10}\text{C} \rightarrow ^{10}\text{B}$ superallowed Fermi transition. The effective interaction takes into account the Coulomb potential as well as the charge dependence of $T = 1$ partial waves. Our results suggest the isospin-mixing correction $\delta_C \approx 0.1\%$, which is compatible with previous calculations. The correction obtained in those calculations, performed in a $0\hbar\Omega$ space, was dominated by deviation from unity of the radial overlap between the converted proton and the corresponding neutron. In the present calculation this effect is accommodated by the large model space. The obtained $\delta_C$ correction is about a factor of four too small to obtain unitarity of the Cabibbo-Kobayashi-Maskawa matrix with the present experimental data.

I. INTRODUCTION

Superallowed Fermi $\beta$ transitions in nuclei, $(J^\pi = 0^+, T = 1) \rightarrow (J^\pi = 0^+, T = 1)$, provide an excellent laboratory for precise tests of the properties of the electroweak interaction, and have been the subject of intense study for several decades (cf. Refs. [1–13]. According to the conserved-vector-current (CVC) hypothesis, for pure Fermi transitions the product of the partial half-life, $t$, and the statistical phase-space factor, $f$, should be nucleus independent and given by

$$ft = \frac{K}{G_V^2 |M_F|^2}.$$  

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where $K/(\hbar c)^6 = 2\pi^3 \ln 2\hbar/(m_e c^2)^5 = 8.120270(12) \times 10^{-7} \text{GeV}^{-4} \text{s}$, $G_V$ is the vector coupling constant for nuclear $\beta$ decay, and $M_F$ is the Fermi matrix element, $M_F = \langle \psi_f | T_\pm | \psi_i \rangle$. By comparing the decay rates for muon and nuclear Fermi $\beta$ decay, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element \cite{6} between $u$ and $d$ quarks ($v_{ud}$) can be determined and a precise test of the unitarity condition of the CKM matrix under the assumption of the three-generation standard model is possible \cite{4,5}.

For tests of the standard model, two nucleus-dependent corrections must be applied to experimental $ft$ values. The first is a series of radiative corrections to the statistical phase-space factor embodied in the factors $\delta_R$ and $\Delta_R$, giving \cite{7–9}

$$f_R = f(1 + \delta_R + \Delta_R),$$

where $\delta_R$ is due to standard, electromagnetic (“inner”) radiative corrections (cf. p. 45 in Ref. \cite{7}) and $\Delta_R$ is what has been referred to as the “outer” radiative correction (cf. p. 47 of Ref. \cite{7}) and includes axial-vector interference terms \cite{9,10}. The second correction, which is the subject of this work, arises because of the presence of isospin-nonconserving (INC) forces (predominantly Coulomb) in nuclei that lead to a renormalization of the Fermi matrix element. This correction is denoted by $\delta_C$ \cite{2,3,12} and modifies the Fermi matrix element by $|M_F|^2 = |M_{F0}|^2 (1 - \delta_C)$, where $M_{F0} = [T(T+1) - T_{Zf}T_{Zi}]^{1/2} / 2$ is the value of the matrix element under the assumption of pure isospin symmetry.

With the corrections $\delta_R$, $\Delta_R$, and $\delta_C$, a “nucleus-independent” $ft$ can be defined by

$$ft = ft(1 + \delta_R + \Delta_R)(1 - \delta_C),$$

and the CKM matrix element $v_{ud}$ is given by \cite{10}

$$|v_{ud}|^2 = \frac{\pi^3 \ln 2}{Ft} \text{GeV} \frac{\hbar^7}{G_F^2 m_e^2 c^4} = 2984.38(6) \text{ s},$$

where the Fermi coupling constant, $G_F$, is obtained from muon $\beta$-decay, and includes radiative corrections. Currently, $ft$ values for nine superallowed transitions have been measured with an experimental precision of 0.2% or better \cite{4,14}. With these precise measurements and reliable estimates for the corrections, the CVC hypothesis can be confirmed by checking the constancy of the $ft$ values for each nucleus, while the unitarity condition of the CKM matrix is tested by comparing the average value of $v_{ud}$ with the values determined for $v_{us} = 0.2199(17)$ \cite{14} and $v_{ub} < 0.0075$ (90% confidence level) \cite{15}, i.e., $v^2 = v_{ud}^2 + v_{us}^2 + v_{ub}^2 = 1$.

In the past, the nuclear structure correction $\delta_C$ has been computed within the framework of the nuclear shell model \cite{2,3,11–13}. In general, the isospin-nonconserving components of the nuclear Hamiltonian are small, and can be treated perturbatively. Due to computational limitations and uncertainties associated with determining an effective Hamiltonian, almost all calculations for nuclei with $A \geq 10$ have been performed within a single major oscillator shell, e.g., for $^{10}$C the model space spanned by the $0p_{3/2}$ and $0p_{1/2}$ orbitals ($p$-shell). Within this context, two types of isospin mixing must be accounted for. The first is due to the mixing between states that lie within the shell-model configuration space. For example, for $A = 10$, there are 2, 7, and 1 $p$-shell configurations leading to $J^p = 0^+$ and $T = 0, 1, 2$, respectively. Because of its two-body nature, the INC interaction is composed
of isospin operators of rank zero (isoscalar), one (isovector), and two (isotensor), and in the case of $A = 10$ it is capable of mixing together all $J^\pi = 0^+$ states. Traditionally, the configuration mixing correction is denoted as $\delta_{IM}$ and in Ref. [11] it was shown that the best estimates for $\delta_{IM}$ are obtained using an INC interaction that correctly describes the Coulomb energy splittings of the binding energies between members of the isospin multiplet, e.g., the $J^\pi = 0^+, T = 1$ states in $^{10}$C, $^{10}$B, and $^{10}$Be. Of the two types of mixing, $\delta_{IM}$ is the smallest with a magnitude of approximately 0.04-0.1%.

In addition to the mixing between states contained within the shell-model configuration space, mixing with states that lie outside the model space must also be accounted for. In particular, the Coulomb interaction can strongly mix one particle-one hole (1p−1h) 2hΩ excitations, e.g., $0p_{3/1} \rightarrow 1p_{3/2}$, into the ground state. In previous works, excitations of this type were accounted for by examining differences in the single-particle radial wave functions. Indeed, for closed-shell configurations, mixing with 1p−1h states is properly accounted for at the level of Hartree-Fock. Hence, the second correction to the Fermi matrix element, denoted by $\delta_{RO}$, was estimated by evaluating the mismatch in the radial overlap between the single-particle wave functions of the converted proton and the corresponding neutron. The explicit details for the calculation of $\delta_{RO}$, which involve a sum over intermediate $A-1$ parent states that then determine the proton and neutron separation energy for the radial wave function, are given in Refs. [2,11]. For the most part, $\delta_{RO}$ is found to be the larger of the two components (with $\delta_C = \delta_{RO} + \delta_{IM}$) and has a magnitude of the order 0.1-0.8%.

At present, two methods for evaluating $\delta_{RO}$ are espoused. The first (THH) [2] uses Woods-Saxon (WS) radial wave functions, while in the second (OB) [3,11,13], Hartree-Fock (HF) wave functions are employed. Generally speaking, the two methods yield approximately the same dependence on nucleon number $A$, but the HF values are systematically smaller by 0.1% for the magnitude of the correction. The reason for the difference lies in the HF mean field. The principal effect of the Coulomb interaction is to push the proton wave functions out relative to the neutrons, hence, providing a mismatch in the radial overlap. In Hartree-Fock, however, the proton and neutron mean fields are coupled, and the Coulomb interaction actually induces an attractive isovector mean field between the protons and neutrons. In effect, the Coulomb interaction pushes the protons out, but because of the strong interaction, the protons pull the neutrons out with them, hence, reducing the magnitude of the radial overlap mismatch.

When all known corrections, i.e., $\delta_R$, $\Delta_R$, $\delta_{RO}$, and $\delta_{IM}$, are applied to the nine experimental data [4,14], it is found that the $Ft$ values are essentially constant within the limits of uncertainty but the unitarity limit is violated at the level of approximately 0.4(1)% or 0.3(1)% for the OB and THH corrections, respectively. In addition, preliminary data from a new experiment for $^{10}$C [13] leads to an $Ft$ value that is significantly smaller than that of Ref. [14], and has been interpreted as possible evidence for an, as yet, unaccounted for correction that might lead to satisfying the unitarity condition of the CKM matrix. In addition, it must be admitted that the present separation between the configuration mixing and radial overlap contributions to $\delta_C$ is somewhat unsatisfying. A much better approach would be to perform a shell-model calculation that includes several $h\Omega$ excitations, so that both corrections would evaluated on the same footing and simultaneously. Because of recent improvements in computational capabilities and the ability to determine an effective model-space Hamiltonian based on realistic nucleon-nucleon interactions, it is now possible.
to perform such a calculation for the lightest of the nine accurately measured transitions. We report here the results of large-basis shell-model calculations that include excitations up to $4\hbar\Omega$ for $A = 10$ nuclides, with an emphasis on evaluating the isospin-mixing corrections to the matrix element for the Fermi decay of $^{10}$C.

The organization for the paper is as follows. First, in Section II we discuss the shell-model Hamiltonian with a bound center-of-mass, the method used to derive the starting-energy-independent effective interaction, and the renormalization of the transfer operator. Results of the Fermi matrix element calculations are presented in section III and concluding remarks are given in section IV.

II. SHELL-MODEL HAMILTONIAN AND THE EFFECTIVE INTERACTION

In our calculation we use the one- plus two-body Hamiltonian for the $A$-nucleon system, i.e.,

$$H = \sum_{i=1}^{A} \frac{\vec{p}_i^2}{2m} + \sum_{i<j}^{A} V_N(\vec{r}_i - \vec{r}_j) ,$$

where $m$ is the nucleon mass and $V_N(\vec{r}_i - \vec{r}_j)$ the nucleon-nucleon interaction, modified by adding the center-of-mass harmonic-oscillator potential $\frac{1}{2}Am\Omega^2\vec{R}^2$, $\vec{R} = \frac{1}{A}\sum_{i=1}^{A} \vec{r}_i$. This potential does not influence intrinsic properties of the many-body system. It provides, however, a mean field felt by each nucleon and allows us to work with a convenient harmonic-oscillator basis. The modified Hamiltonian, depending on the harmonic-oscillator frequency $\Omega$, may be cast into the form

$$H^\Omega = \sum_{i=1}^{A} \left[ \frac{\vec{p}_i^2}{2m} + \frac{1}{2}m\Omega^2\vec{r}_i^2 \right] + \sum_{i<j}^{A} \left[ V_N(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2 \right] ,$$

The one-body term of the Hamiltonian (9) is then re-written as a sum of the center-of-mass term $H^\Omega_{\text{cm}} = \sum_{i=1}^{A} \left[ \frac{\vec{p}_i^2}{2Am} + \frac{1}{2}Am\Omega^2\vec{R}^2 \right]$, $\vec{P}_{\text{cm}} = \sum_{i=1}^{A} \vec{p}_i$, and a term depending on relative coordinates only. Shell-model calculations are carried out in a model space defined by a projector $P$. In the present work, we will always use a complete $N\hbar\Omega$ model space. The complementatory space to the model space is defined by the projector $Q = 1 - P$. In addition, from among the eigenstates of the Hamiltonian (8), it is necessary to choose only those corresponding to the same center-of-mass energy. This can be achieved by projecting the center-of-mass eigenstates with energies greater than $\frac{3}{2}\hbar\Omega$ upwards in the energy spectrum. The shell-model Hamiltonian, used in the actual calculations, takes the form

$$H^\Omega_{P,\beta} = \sum_{i<j=1}^{A} P \left[ \frac{(\vec{p}_i - \vec{p}_j)^2}{2Am} + \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2 \right] P + \sum_{i<j}^{A} P \left[ V_{ij} - \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2 \right]_{\text{eff}} P$$

$$+ \beta P(H^\Omega_{\text{cm}} - \frac{3}{2}\hbar\Omega)P ,$$

where $\beta$ is a sufficiently large positive parameter.

The effective interaction introduced in Eq. (7) should, in principle, exactly reproduce the full-space results in the model space for some subset of states. In practice, the effective
interactions can never be calculated exactly as, in general, for an A-nucleon system an A-body effective interaction is required. Consequently, large model spaces are desirable when only an approximate effective interaction is used. In that case, the calculation should be less affected by any imprecision of the effective interaction. The same is true for the evaluation of any observable characterized by an operator. In the model space, renormalized effective operators are required. The larger the model space, the less renormalization is needed.

Usually, the effective Hamiltonian is approximated by a two-body effective interaction determined from a two-nucleon system. In this study, we use the procedure as described in Ref. [17]. To construct the effective interaction we employ the Lee-Suzuki similarity transformation method, which gives an interaction in the form $P_2 V_{\text{eff}} P_2 = P_2 V P_2 + P_2 V Q_2 \omega P_2$, with $\omega$ the transformation operator satisfying $\omega = Q_2 \omega P_2$. The projection operators $P_2, Q_2 = 1 - P_2$ project on the two-nucleon model space and complementary space, respectively. Our calculations start with exact solutions of the Hamiltonian

$$H_2 \equiv H_0^2 + V_2 = \frac{\vec{p}_1^2 + \vec{p}_2^2}{2m} + \frac{1}{2} m \Omega^2 (\vec{r}_1^2 + \vec{r}_2^2) + V (\vec{r}_1 - \vec{r}_2) - \frac{m \Omega^2}{2} A (\vec{r}_1 - \vec{r}_2)^2 .$$

(8)

which is the shell-model Hamiltonian (8) applied to a two-nucleon system. We construct the effective interaction directly from these solutions. Let us denote the relative-coordinate two-nucleon harmonic-oscillator states, which form the model space, as $|\alpha_P\rangle$, and those which belong to the Q-space, as $|\alpha_Q\rangle$. Then the Q-space components of the eigenvector $|k\rangle$ of the Hamiltonian (8) can be expressed as a combination of the P-space components with the help of the operator $\omega$

$$\langle \alpha_Q | k \rangle = \sum_{\alpha_P} \langle \alpha_Q | \omega | \alpha_P \rangle \langle \alpha_P | k \rangle .$$

(9)

If the dimension of the model space is $d_P$, we may choose a set $\mathcal{K}$ of $d_P$ eigenvectors, for which the relation (9) will be satisfied. Under the condition that the $d_P \times d_P$ matrix $\langle \alpha_P | k \rangle$ for $|k\rangle \in \mathcal{K}$ is invertible, the operator $\omega$ can be determined from (8). In the present application we select the lowest states obtained in each channel. Once the operator $\omega$ is determined the effective Hamiltonian can be constructed as follows

$$\langle \gamma_P | H_{2\text{eff}} | \alpha_P \rangle = \sum_{k \in \mathcal{K}} \left[ \langle \gamma_P | k \rangle E_k \langle k | \alpha_P \rangle + \sum_{\alpha_Q} \langle \gamma_P | k \rangle E_k \langle k | \alpha_Q \rangle \langle \alpha_Q | \omega | \alpha_P \rangle \right] .$$

(10)

This Hamiltonian, when diagonalized in a model-space basis, reproduces exactly the set $\mathcal{K}$ of $d_P$ eigenvalues $E_k$. Note that the effective Hamiltonian is, in general, quasi-Hermitian. It can be hermitized by a similarity transformation determined from the metric operator $P_2 (1 + \omega^\dagger \omega) P_2$. The Hermitian Hamiltonian is then given by [19]

$$\hat{H}_{2\text{eff}} = \left[ P_2 (1 + \omega^\dagger \omega) P_2 \right]^{1/2} H_{2\text{eff}} \left[ P_2 (1 + \omega^\dagger \omega) P_2 \right]^{-1/2} .$$

(11)

Finally, the two-body effective interaction used in the present calculations is determined from the two-nucleon effective Hamiltonian (11) as $V_{\text{eff}} = \hat{H}_{2\text{eff}} - H_0^2$. Note that we distinguish the two-nucleon system projection operators $P_2, Q_2$ from the A-nucleon system operators $P, Q$. 

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To at least partially take into account the many-body effects neglected when using only a two-body effective interaction, we employ the recently introduced multi-valued effective interaction approach [20]. As a consequence, different effective interactions are used for different $\hbar \Omega$ excitations. The effective interactions then carry an additional index indicating the sum of the oscillator quanta for the spectators, $N_{\text{sps}}$, defined by

$$N_{\text{sps}} = N_{\text{sum}} - N_{\alpha} - N_{\text{spsmin}} = N'_{\text{sum}} - N_{\gamma} - N_{\text{spsmin}} \ , \quad (12)$$

where $N_{\text{sum}}$ and $N'_{\text{sum}}$ are the total oscillator quanta in the initial and final many-body states, respectively, and $N_{\alpha}$ and $N_{\gamma}$ are the total oscillator quanta in the initial and final two-nucleon states $|\alpha\rangle$ and $|\gamma\rangle$, respectively. $N_{\text{spsmin}}$ is the minimal value of the spectator harmonic-oscillator quanta for a given system. Here, for $A=10$, $N_{\text{spsmin}} = 4$. Different sets of the effective interaction are determined for different model spaces characterized by $N_{\text{sps}}$ and defined by projection operators

$$Q_2(N_{\text{sps}}) = \begin{cases} 0 & \text{if } N_1 + N_2 \leq N_{\text{max}} - N_{\text{sps}} \\ 1 & \text{otherwise} \end{cases} \quad (13a)$$

$$P_2(N_{\text{sps}}) = 1 - Q_2(N_{\text{sps}}) \ . \quad (13b)$$

In Eqs. (13), $N_{\text{max}}$ characterizes the two-nucleon model space. It is an input parameter chosen in relation to the size of the many-nucleon model space. This multi-valued effective-interaction approach is superior to the traditional single-valued effective interaction, as confirmed also in a model calculation [21].

Our goal in this study is to evaluate the Fermi matrix element

$$M_{\text{F}} = \langle ^{10}\text{B}, 0^+1|T_-|^ {10}\text{C}, 0^+1 \rangle \ , \quad (14)$$

which is equal to $\sqrt{2}$ for an isospin-invariant system. Note that for a system with isospin breaking, the isospin-lowering operator $T_-$ should be renormalized in a similar way, as the interaction used for calculation of the eigenstates appearing in Eq. (14). In fact, we can apply the formalism described in Refs. [22] to construct a two-body effective operator $(T_-)_{\text{eff}}$ consistent with the two-body effective interaction derived above and exact for the two-nucleon system. Then we could use such an operator in the A-body calculation. We studied such a possibility in a solvable-model calculation as described in Refs. [21]. Here, we did two-nucleon calculations with the effective $T_-$ operator. The observed renormalization of the bare operator for the model spaces of the size used in our calculations, was, however, insignificant compared to the other effects as described further. Therefore, in the A-body calculations we used the bare $T_-$ operator.

III. APPLICATION TO THE A=10 SYSTEM WITH ISOSPIN BREAKING

In order to evaluate the Fermi matrix element [Eq. (14)], we apply the formalism outlined in section I for $A=10$ nuclei. In the calculations we use the Reid93 nucleon-nucleon potential [23] and consider the following isospin-breaking contributions. First, the Reid93 potential differs in the $T = 1$ channels for pn and pp, nn systems, respectively. Second, we add the Coulomb potential to the pp Reid93 potential. Consequently, using the Eqs. (3)-(11), we
derive different two-body effective interactions for the pn, pp, and nn systems. No other mechanisms for isospin breaking are considered.

As we derive the effective interaction microscopically from the nucleon-nucleon interaction, the number of freely adjustable parameters in the calculation is limited.

First, we have the choice of the model-space size in the shell-model diagonalization. That is, however, constrained by computer capabilities. The largest model space we were able to use was the space allowing all \(4\hbar\Omega\) excitations relative to the unperturbed ground state. Most of the calculations were done in the m-scheme using the Many-Fermion-Dynamics Code [24] extended to allow the use of different pn, pp, nn interactions. We also performed some calculations with the OXBASH shell-model code [25]. In the m-scheme, the dimensions associated with \(^{10}\text{B}\) and \(^{10}\text{C}\) are 581,740 and 430,137, respectively. To study the dependence on the model-space size, we performed calculations in the \(2\hbar\Omega\) space as well. In that space, the dimensions drop to 14,502 and 10,111, respectively.

Second, we have the choice of the two-nucleon model space used for the evaluation of the effective interaction. This is related to the many-nucleon model-space size, and, in principle, is determined by that size. Traditionally, however, the \(Q = 0\) space used to determine the G-matrix does not, necessarily, coincide with the many-particle model space [26,27]. In our calculation, the two-nucleon model space is characterized by a restriction on the number harmonic-oscillator quanta \(N_1 \leq N_{\text{max}}, N_2 \leq N_{\text{max}}, (N_1 + N_2) \leq N_{\text{max}}\). Here, \(N_i = 2n_i + l_i\) is the harmonic-oscillator quantum number for the nucleon \(i, i = 1, 2\). This type of restriction guarantees an orthogonal transformation between the two-particle states and the relative- and center-of-mass-coordinate states. For the present \(4\hbar\Omega\) calculation, the choice of \(N_{\text{max}} = 6\) appears to be appropriate. However, it has been observed in the past [17,28,29] that when the Lee-Suzuki procedure combined with the G-matrix calculation according to Ref. [26] (which is equivalent to the procedure we are using) is applied to calculate the two-body effective interaction, the resulting interaction may be too strong. This is, in particular, true, when the multi-valued approach is used. Several possible adjustments were discussed to deal with this problem [15,28] and amounted to introducing an extra parameter. In the present calculations, we do not introduce any new parameter, but rather we treat \(N_{\text{max}}\) as a free parameter and use \(N_{\text{max}} = 8\) for the \(4\hbar\Omega\) calculations and \(N_{\text{max}} = 6\) for the \(2\hbar\Omega\) calculations, respectively. With this choice we obtain quite reasonable binding energies for the studied nuclei. We have also performed several \(2\hbar\Omega\) test calculations with single-valued interactions that were derived following Ref. [30], as opposed to the multi-valued interaction discussed in the previous section. To obtain reasonable binding energies with the single-valued interaction we do not have to change the \(N_{\text{max}}\) value from that corresponding to the many-nucleon space, e.g., \(N_{\text{max}} = 4\) for the \(2\hbar\Omega\) calculation. This difference of treatment of the two types of interactions follows from the fact that the overall strength of the single-valued interaction is weaker.

Third, our results depend on the harmonic-oscillator frequency \(\Omega\). We have studied this dependence by performing calculations for the values \(\hbar\Omega = 14, 15.5,\) and 17 MeV.

Let us also mention one important feature of the present approach. For both the multi-valued and the single-valued interactions our calculations do not break the separation of the center-of-mass and the internal relative motion. In particular, a variation of the parameter \(\beta\) introduced in Eq. (7) does not change the eigenenergies and other characteristic of the physical states. This is so due to the choice of a complete \(N\hbar\Omega\) many-nucleon space and
the triangular two-nucleon model space for deriving the effective interaction as well as the due to the procedure used to derive the effective interaction.

In Figs. 1, 2, and 3 we present the experimental and calculated spectra of $^{10}\text{B}$ for $\hbar\Omega = 14, 15.5, \text{and } 17 \text{ MeV}$, respectively, for the $2\hbar\Omega$ and $4\hbar\Omega$ model spaces. In general, we observe an overall improvement in the spectra with the enlargement of the model space in all three cases. Also, the $4\hbar\Omega$ calculations exhibit more stability with regard to changes in the harmonic-oscillator frequency than do the $2\hbar\Omega$ results. The agreement with experiment improves when going from $\hbar\Omega = 14 \text{ MeV}$ to $\hbar\Omega = 17 \text{ MeV}$; in particular for the ground state and the lowest states. In fact, from Fig. 3 we find that a very reasonable description of the spectra is obtained for $\hbar\Omega = 17 \text{ MeV}$.

In Table I, the overall behavior with respect to $\hbar\Omega$ is illustrated. In general, we observe a reasonable reproduction of the binding energy, with a moderate decrease occurring for increasing $\hbar\Omega$. Using free-nucleon effective charges, we find that although the quadrupole moment for the $3^+0$ state is underestimated considerably, the magnetic dipole moment is well reproduced. In addition, the point-proton rms radius exhibits a fairly strong dependence, and increases with decreasing $\hbar\Omega$. For the rms radius, we find that the best agreement with experiment [32] is achieved for $\hbar\Omega = 14 \text{ MeV}$.

From the point of view of the beta decay of $^{10}\text{C}$, a good description of the $T = 1$ states is important. From Figs. 1-3 we can see that the calculated $^{10}\text{B}$ $T = 1$ states have the right relative positions and are reasonably stable with variations of both the model-space size and $\hbar\Omega$. We have also performed $4\hbar\Omega$ calculations for $^{10}\text{Be}$ to study the splitting of the isospin analog states in the whole isospin-multiplet $^{10}\text{C} - ^{10}\text{B} - ^{10}\text{Be}$. The experimental ground state splitting between $^{10}\text{C}$ and $^{10}\text{Be}$ is 4.66 MeV, while our calculated values are 4.68, 4.83, and 4.94 MeV for $\hbar\Omega = 14, 15.5, \text{and } 17 \text{ MeV}$, respectively. The best agreement with experiment is achieved for $\hbar\Omega = 14 \text{ MeV}$, where the calculated rms point-proton radius is also in agreement with the experimental value. On the other hand, the splitting between the $0^+1$ states of $^{10}\text{C}$ and $^{10}\text{B}$, which is experimentally 2.69 MeV, is overestimated in our calculations by 8, 11, and 14% for the $\hbar\Omega = 14, 15.5, \text{and } 17 \text{ MeV}$ calculations, respectively. Since the correct $^{10}\text{C} - ^{10}\text{Be}$ splitting is obtained for $\hbar\Omega = 14 \text{ MeV}$, the excess in the $^{10}\text{C} - ^{10}\text{B}$ splitting suggests that the isospin breaking due the strong $T = 1$ force may be too large. One possible explanation is that our approach for deriving the effective interaction tends to exaggerate the differences between the pn and mm, pp potentials. Such an artificial effect should decrease with increasing model-space size. On the other hand, it is also possible that the Reid93 potential itself overestimates differences between the pn and pp, nm systems in the $T = 1$ channel. For the most part, we find the best overall agreement for the rms point proton radius, binding energy, and Coulomb energy splitting for $\hbar\Omega = 14 \text{ MeV}$. Given that the isospin mixing is largely driven by the Coulomb interaction, which is then dependent on the size of the nucleus, we feel that the best value for the isospin-mixing correction to the Fermi matrix element will be achieved for $\hbar\Omega = 14 \text{ MeV}$.

The most important results of our study are also summarized in Table I in the last two lines. The calculated isospin-mixing corrections $\delta_C = 1 - \frac{|M_{1/2}|^2}{2}$, in %, are presented for all three choices of $\Omega$ and for both $4\hbar\Omega$ and $2\hbar\Omega$ model spaces. Again, a correlation between the radius and the isospin-mixing correction is clearly observed, as $\delta_C$ decreases with increasing radius. This is simply understood in terms of a larger radius implying weaker Coulomb effects. On the other hand, with an increase in the model-space size, a significant increase
in the isospin-mixing correction is apparent. This is due to the fact that in the larger model space, the excitation energies of the 1p-1h 0+1 states decrease, hence leading to greater mixing. For this reason, the more realistic multi-valued effective interaction is important. We have also performed test calculations with the single-valued interaction in the 2hΩ space, and found δC to be smaller by approximately 30%.

Our 4hΩ results suggest an isospin-mixing correction δC ≈ 0.08–0.1%. This is compatible with the previously published value of δC ≈ 0.15(9)% by Ormand and Brown [13]. That value was a sum of two contributions. First, about 0.04% came from the shell-model wave-function renormalization due to the isospin mixing and was obtained in a 0hΩ shell-model calculation using phenomenological effective interactions. Second, the amount 0.09% was due to the deviation from unity of the radial overlap between the converted proton and the corresponding neutron. This effect was attributed to the influence of states lying outside the 0hΩ space. The radial wave-functions were obtained in a Hartree-Fock calculation using Skyrme-type interactions. Because we use a multi-configuration model space in the present calculation, we should have both effects included consistently at the same time.

Another important factor in the calculation is the position of the 2hΩ states. As discussed before, the position of the 1p-1h states influences the ground-state isospin mixing. Unfortunately, the excitation energy of these states is not known experimentally. However, in our calculations the multi-valued effective interaction is used and a more realistic description of these states should be obtained; especially, in the 4hΩ model space. On the other hand, in an analogous calculation for ⁴He, it was observed that an 8hΩ model space is needed to get the 2hΩ dominated 0+ state close to the experimental excitation energy [17,20]. There are states like 1+0 at 5.18 MeV in ¹⁰B or 0+1 at 6.18 MeV in ¹⁰Be, which are believed to be 2p-2h, 2hΩ excitations. We do not observe any such states below 7.5 and 12 MeV, respectively, in our calculations. The first excited 0+1 state in ¹⁰Be obtained in the 4hΩ calculation with hΩ = 14 MeV lies at 9.8 MeV. It is, however, predominately a 0hΩ state. There can be two reasons why we do not get such states. First, these states have not yet converged in the Lanczos procedure. Second, and more likely, the 4hΩ model space is too small for the right description of the 2hΩ excitation states. Therefore, it would be desirable to extend the present calculations to a larger, e.g., 6hΩ model space. Unfortunately, due to the computational limitations, it is not possible at this time to perform a calculation of this magnitude. However, from Table I we observe an increase of δC by ≈ 0.03% between the 2hΩ and the 4hΩ calculation. Therefore, we might expect an increase of similar magnitude for an increase of the model-space size beyond 4hΩ. Therefore, the more realistic value of the isospin-mixing correction from our calculation would be δC ≈ 0.12(3)%, where the uncertainty is estimated from the change in δC obtained when using an increased model space.

IV. CONCLUSIONS

The effects of isospin mixing on the transition matrix element for the superallowed Fermi β-decay of ¹⁰C were estimated within the context of a large-basis, shell-model calculation. The calculations were performed assuming no closed core and an effective interaction based on a realistic two-body nucleon-nucleon interaction, while including the Coulomb interaction between protons. Contrary to previous estimates for the isospin corrections, this calculation
was carried out within a model space that included many $\hbar \Omega$ excitations. As a consequence, the conventional configuration mixing and radial mismatch contributions were evaluated within a unified framework, simultaneously and the usual separation was not necessary. With regard to parameters used within the calculation, we find a correlation between the isospin-mixing correction and the Coulomb splitting between the isotopic multiplets, which, in turn, is governed by the nuclear size through the oscillator parameter. Given that the isospin-mixing correction is primarily a Coulomb effect, the best value for $\delta_C$ is taken to coincide with the oscillator parameter that correctly reproduces the Coulomb splittings. With regard to the model-space size, a clear improvement (or an indication towards convergence) in most observables is evident when the size of the model space is increased from $2\hbar \Omega$ to $4\hbar \Omega$, but $\delta_C$ is found to increase by only 0.03% (in magnitude) in this case. Hence, our final estimate for $\delta_C$ is taken to be 0.12(3)% (where the $4\hbar \Omega$ result has been increased by 0.03% to account for the possible effects of an increased model space). This result also happens to be in excellent agreement with the previous estimates that relied on the conventional separation of the configuration and radial mismatch contributions. Finally, we note that the magnitude of the isospin-mixing correction obtained in our calculation does not lead to a resolution to the deviation from unitarity for the Cabibbo-Kobayashi-Maskawa matrix.

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FIGURES

FIG. 1. The experimental and calculated excitation spectra of $^{10}$B. The results corresponding to the model-space sizes of $4\hbar\Omega$ and $2\hbar\Omega$ relative to the ground-state configurations are presented, respectively. The harmonic-oscillator energy of $\hbar\Omega = 14$ MeV was used.

FIG. 2. The same as in Fig. 1 for the harmonic-oscillator energy of $\hbar\Omega = 15.5$ MeV.

FIG. 3. The same as in Fig. 1 for the harmonic-oscillator energy of $\hbar\Omega = 17$ MeV.
TABLE I. Experimental and calculated binding energies, in MeV, magnetic moments, in $\mu_N$, and quadrupole moments, in $e$ fm$^2$, of $^{10}$B. Also the experimental and calculated binding energies, in MeV, and the point proton radius, in fm, of $^{10}$C are presented. The results correspond to the $4\hbar\Omega$ calculations. In addition, the isospin-mixing correction $\delta_C$, in $\%$, is shown as obtained both in the $4\hbar\Omega$ calculations and the $2\hbar\Omega$ calculations. Results of three different calculations with the harmonic-oscillator parameter taken to be $\hbar\Omega = 14, 15.5, 17$ MeV, respectively, are presented. The effective interaction used was derived from the Reid 93 nucleon-nucleon potential. The experimental values are taken from Refs. [31,32].

|          | Exp | $\hbar\Omega = 14$ MeV | $\hbar\Omega = 15.5$ MeV | $\hbar\Omega = 17$ MeV |
|----------|-----|------------------------|--------------------------|------------------------|
| $E_B(^{10}$B) | 64.75 | 63.61 | 62.78 | 61.53 |
| $Q(3^+0)$  | 8.47(6) | 5.85 | 5.64 | 5.52 |
| $\mu(3^+0)$ | 1.80  | 1.86 | 1.85 | 1.85 |
| $\mu(1^+0)$ | 0.63(12) | 0.84 | 0.84 | 0.84 |
| $E_B(^{10}$C) | 60.32 | 58.68 | 58.19 | 56.83 |
| $\sqrt{\langle r_p^2 \rangle}$ | 2.31 ± 0.03 | 2.28 | 2.21 | 2.17 |
| $\delta_C(4\hbar\Omega)[\%]$ | - | 0.084 | 0.091 | 0.097 |
| $\delta_C(2\hbar\Omega)[\%]$ | - | 0.055 | 0.061 | 0.067 |
\[ ^{10}\text{B} \quad \hbar \Omega = 14 \text{ MeV} \]
$^{10}$B

$\hbar \Omega = 15.5 \text{ MeV}$

E$_x$ [MeV]

0 1 2 3 4 5 6 7 8

2$^+$ 1

4$^+$ 0

2$^+$ 0

2$^+$ 1

3$^+$ 0

2$^+$ 0

1$^+$ 0

0$^+$ 1

1$^+$ 0

3$^+$ 0

Exp 4$\hbar \Omega$ 2$\hbar \Omega$
