Transverse momentum dependent twist-three result for polarized Drell-Yan processes

Zhun Lu\(^1\) and Ivan Schmidt\(^2\)

\(^1\)Department of Physics, Southeast University, Nanjing 211189, China
\(^2\)Departamento de Física, y Centro Científico-Tecnológico de Valparaíso,
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

We study the polarized Drell-Yan processes from the collision of two spin-1/2 hadrons at order \(1/Q\) based on the framework of transverse momentum dependent factorization. We give the complete twist-three results of total sixteen independent structure functions in terms of twist-two and twist-three transverse momentum dependent distribution functions.

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I. INTRODUCTION

Polarized Drell-Yan processes are a promising ground for the study of nucleon structure \(^1\)\(^2\). If the transverse momentum of the lepton pair is detected, polarized Drell-Yan processes can be used to probe \(^3\)\(^4\) various transverse momentum dependent (TMD) parton distributions, which have received considerable interest recently (e.g., see \(^5\)\(^-\)\(^8\) and reference therein). Compared with semi-inclusive deep inelastic scattering (SIDIS), which has been used intensively to study TMD distributions in the last decade \(^1\)\(^-\)\(^2\), Drell-Yan reactions have the feature that only parton distributions are involved, that is, there is no hadron detected in the final state.

Theoretically, the TMD factorization for Drell-Yan process at low transverse momentum has been established, and also the complete leading-twist Drell-Yan structure functions for spin-1/2 hadrons beams have been given in Ref. \(^2\). Previous studies on Drell-Yan processes based on TMD framework mainly focused on the contribution at leading power of \(1/Q\), where \(Q\) is the invariant mass of the lepton pair. This motivate us to present a full expression for the Drell-Yan process at twist-three level with dilepton transverse momentum kept unintegrated, which is the main goal of this paper. We note that at order \(1/Q\), results contributed by the (transverse momentum) integrated distributions \(f_T(x), g_T(x), h_L(x)\) and \(h(x)\) have been worked out in Refs. \(^2\)\(^-\)\(^3\). We will consider both polarized and unpolarized scattering of spin-1/2 hadron beams. We find that at order \(1/Q\) there are sixteen transverse momentum dependent structure functions for the Drell-Yan process, which can be expressed as a convolution of twist-two and twist-three TMD distributions.

Experimentally, a number of polarized Drell-Yan programs have been proposed at several facilities \(^2\)\(^-\)\(^3\) and some of them could be realized in the near future to provide the first polarized data. The twist-three contributions can be potential experimental observables and may be accessible in certain kinematical regions. The interest on the twist-three contributions also comes from the fact that they are related to the quark-gluon correlation inside the nucleon \(^3\)\(^2\), which is still not understood yet.

We need to emphasize that the approach in this paper is based on the assumption that the framework of TMD factorization is valid at order \(1/Q\). The same approach has been applied in Ref. \(^3\)\(^-\)\(^4\) to calculate the complete leading-twist and subleading-twist observables in SIDIS (for the production of spin-0 hadrons), where ten twist-three TMD structure functions have been found. Therefore our twist-three results should not be compared with the twist-three mechanism \(^3\)\(^-\)\(^4\) in \textit{collinear factorization} that has been applied to study the single-spin asymmetries in Drell-Yan processes \(^4\).

The remaining content of the paper is organized as follows. In Section. II we introduce the formalism needed in the construction of TMD twist-three observables. In Section. III we present the complete expressions for the differential cross section of Drell-Yan process with dilepton transverse momentum unintegrated at order \(1/Q\). We summarize the paper in Section. IV.

II. FORMALISM AND KINEMATICAL SETTINGS

The process we study is

\[ h_1(P_1, S_1) + h_2(P_2, S_2) \rightarrow \ell(l) + \bar{\ell}(l') + X. \]  

(1)
Here we consider only the electromagnetic interaction. The notations $P_1$ and $S_i$ are the four-momenta and spins of the hadron beams which can be decomposed as

\begin{align}
P_1^\mu &= P_1^+ n_+^\mu + \frac{M_1^2}{P_1^+} n_-^\mu, \\
S_1 &= \frac{\lambda_1 P_1^+}{M_1} n_+ - \frac{\lambda_1 M_1}{P_1^+} n_-^\mu + S_1^\mu T,
\end{align}

where $n_+$ and $n_-$ are two light-like vectors expressed in the light-cone coordinates, in which an arbitrary four-vector $a$ is written as \( a^\mu = (a^0 \pm a^3) / \sqrt{2} \) and $a_T = (a^1, a^2)$. Then one can define following transverse tensors

\[ g_{\mu\nu}^T = g_{\mu
u} - n_+ n_-^\nu - n_- n_+^\nu, \quad \epsilon_{\mu\nu}^T = \epsilon^{\mu
u\rho\sigma} n_+ n_- n_+ n_- \].

We will apply the parton model to study the Drell-Yan pair production. In this model, the leading contribution is from the annihilation of the quark and antiquark from each proton: $q(k_1)q(k_2) \rightarrow \gamma^* \rightarrow \ell\ell$. The momenta of the quark, antiquark and the virtual photon can be decomposed as

\begin{align}
k_1 &= x_1 P_1^+ n_+ + \frac{(k_1^2 + k_{1T}^2)}{x_1 P_1^+} n_- + k_{1T}, \\
k_2 &= x_2 P_2^- n_- + \frac{(k_2^2 + k_{2T}^2)}{x_2 P_2^-} n_+ + k_{2T}, \\
q &= \frac{Q}{\sqrt{2}} n_+ + \frac{Q}{\sqrt{2}} n_- + q_T,
\end{align}

where $Q^2 = q^2$, and we limit our study to the region $Q_T^2 = q_T^2 = -q_T^2 \ll Q^2$, and thus the intrinsic transverse momenta of quarks play significant role.

The angular distribution of the Drell-Yan cross section is usually expressed in the dilepton rest frame (Fig. 1), which can be defined by introducing the following normalized vectors [23]:

\begin{align}
\hat{t} &= q/Q, \\
\hat{z} &= (1 - c) \frac{2x_1}{Q} \hat{P}_1 - c \frac{2x_2}{Q} \hat{P}_2, \\
\hat{h} &= q_T/Q_T = (q - x_1 P_1 - x_2 P_2)/Q_T,
\end{align}

where $\hat{P}_i = P_i - q/(2x_i)$. The parameter $c$ represents the degree of freedom to distribute the transverse momentum between $P_1$ and $P_2$. The cases $c = 0, 1/2$ and 1 correspond to the Gottfried-Jackson frame [22], the Collins-Soper frame [23] and the $u$-channel frame, respectively. Using the normalized vectors $\hat{t}$ and $\hat{z}$ one can construct the perpendicular tensors as

\[ g_\perp^{\mu\nu} = g_{\mu\nu} - \hat{t}^{\mu} \hat{t}^{\nu} + \hat{z}^{\mu} \hat{z}^{\nu}, \quad \epsilon_\perp^{\mu\nu} = -\epsilon^{\mu\nu\rho\sigma} \hat{t}_\rho \hat{z}_\sigma. \]

As shown in Fig. 1, the azimuthal angles of the dilepton and of the transverse spin of the hadrons with respect to the hadron plane can defined as [14]

\begin{align}
\cos \phi &= -g_\perp^{\mu\nu} \hat{h}_\mu \hat{l}_\perp, \quad \sin \phi = \epsilon_\perp^{\mu\nu} \hat{h}_\mu \hat{l}_\perp, \\
\cos \phi_S &= -g_\perp^{\mu\nu} \hat{h}_\mu S_\perp, \quad \sin \phi_S = \epsilon_\perp^{\mu\nu} \hat{h}_\mu S_\perp.
\end{align}
where \( \hat{t}_\perp = g_\perp^{\mu \nu} t_\mu / \sqrt{-g_\perp^{\mu \nu} t_\mu t_\nu} \) and \( S_\perp^\mu = g_\perp^{\mu \nu} S_\nu \). We note that the definition in Eqs. (14) and (15) is the same as the one used in Ref. [21]. The two light-like vectors can be expanded as linear combinations of \( \hat{t}, \hat{z} \) and the perpendicular vector \( \hat{h} \)

\[
n^\mu_+ = \frac{1}{\sqrt{2}} \left[ \hat{t}^\mu + \hat{z}^\mu - 2c \frac{Q_T}{Q} \hat{h}^\mu \right],
\]

\[
n^\mu_- = \frac{1}{\sqrt{2}} \left[ \hat{t}^\mu - \hat{z}^\mu - 2(1 - c) \frac{Q_T}{Q} \hat{h}^\mu \right],
\]

Thus the transverse tensor and the perpendicular tensor are related by

\[
g_T^{\mu \nu} = g_\perp^{\mu \nu} + \frac{Q_T}{Q} \hat{t}^{(\mu} \hat{h}^{\nu)} + (1 - 2c) \frac{Q_T}{Q} \hat{z}^{(\mu} \hat{h}^{\nu)} + \mathcal{O}(1/Q^2).
\]

where the symmetrization of indices is used. One can see that the differences are of order \( 1/Q \). As we study the twist-three contribution, we need to keep track of these differences.

In the rest frame of the dilepton, one can express the differential cross section of the Drell-Yan process as

\[
\frac{d\sigma}{d^4 q d\Omega} = \frac{\alpha_{em}^2}{2s q^4} L^{\mu \nu} W^{\mu \nu},
\]

where \( d\Omega = d\cos \theta d\phi \) is the solid angle of the lepton \( \ell \). The notation \( L^{\mu \nu} \) denotes the lepton tensor which has the following form \([22]\):

\[
L^{\mu \nu} = Q^2 \left[ - \left( \frac{1 + \cos^2 \theta}{2} \right) g^{\mu \nu}_\perp + \sin^2 \theta \hat{z}^\mu \hat{z}^\nu 
- \sin^2 \theta \left( \hat{t}^\mu \hat{t}^\nu_\perp + \frac{1}{2} g^{\mu \nu}_\perp \right) + \sin 2\theta \hat{z}^\mu \hat{z}^\nu_\perp \right],
\]

Here we have ignored the lepton masses and their polarization.

The hadronic tensor \( W^{\mu \nu} \) can be expressed as \([15]\)

\[
W^{\mu \nu} = \frac{1}{3} \sum_a e_a^2 \int d^2 k_{1T} d^2 k_{2T} d^2 (k_{1T} + k_{2T} - q_T) \text{Tr} \left\{ \Phi^a(x, k_{1T}) \gamma^\mu \bar{\Phi}^a (x_2, k_{2T}) \gamma^\nu \right\}
+ \frac{1}{Q \sqrt{2}} \left[ \gamma^\nu \not{y} + \gamma^\nu \not{\Phi}_A (x, k_{1T}) \gamma^\mu \bar{\Phi}^a (x, k_{2T}) + \gamma^\mu \not{y} + \gamma^\mu \not{\Phi}_A (x, k_{1T}) \gamma^\nu \bar{\Phi}^a (x, k_{2T}) \right]
- \frac{1}{Q \sqrt{2}} \left[ \gamma^\nu \not{y} - \gamma^\nu \Phi^a (x, k_{1T}) \gamma^\mu \bar{\Phi}^a (x, k_{2T}) + \gamma^\mu \not{y} - \gamma^\mu \Phi^a (x, k_{2T}) \gamma^\nu \bar{\Phi}^a (x, k_{1T}) \right] \right\} + \left( q \leftrightarrow -q \right),
\]

where the factor \( 1/3 \) takes into account color average, \( a \) is the flavor index and \( e_a \) denotes the charge for flavor \( a \). The first line in the curly brackets in Eq. (22) comes from the diagram without additional gluon connecting to the soft parts: the gauge-invariant TMD dependent quark-quark correlation function \( \Phi(x, k_{1T}) \) and antiquark-antiquark correlation function \( \bar{\Phi}(x, k_{2T}) \). The second and third lines in the curly brackets in Eq. (22) correspond to the diagrams involving one gluon which connects to one of the two soft parts, represented by the quark-gluon-quark correlator \( \Phi^a_A (x, k_{1T}) \) or...
the antiquark–gluon–antiquark correlator \( \tilde{\Phi}^a_{A_0}(x, k_{1T}) \), with \( \alpha \) restricted to be the transverse index. Up to twist-three level, the TMD correlator \( \Phi(x, k_T) \) can be parameterized as

\[
\Phi(x, k_T) = \frac{1}{2} \left\{ f_1 \eta_+ - f_1^1 \frac{e^{\prime \rho} k_{1T} S_T}{M} \eta_+ + g_{1s} \gamma_5 \eta_+ + h_{1T} \left[ g_{T, \eta_+} \gamma_5 + g_{1s} \frac{e^{\prime \rho} k_{1T} S_T}{2M} + i h_{1s} \left[ g_{T, \eta_+} \gamma_5 + i h \left[ \eta_+, \eta_+ \right] \right] \right] \right. \\
+ \frac{M}{2P_T} \left\{ e^{-i e_s \gamma_5 - e_s^t} \left[ e^{\prime \rho} k_{1T} S_T \right] + f^t \frac{k_T}{M} - f^t \frac{e^{\prime \rho} \gamma_5 k_T S_T}{M} - f^t \frac{e^{\prime \rho} \gamma_5 k_T S_T}{M} \right. \\
\left. + g_s \gamma_5 \frac{k_T}{M} - g_s \gamma_5 \frac{e^{\prime \rho} \gamma_5 k_T S_T}{M} + h_s \left[ \eta_+, \eta_+ \gamma_5 \right] + h^{i \alpha} \left[ \eta_+, \eta_+ \gamma_5 \right] \right\}
\]

(23)

The distribution functions on the right hand side (r.h.s.) of (23) depend on \( x \) and \( k_T^2 \), except for the functions with subscript \( s \), where the following shorthand notation has been used \[47\]

\[
g_{1s}(x, k_T^2) = S_L g_{1L}(x, k_T^2) - \frac{k_T \cdot S_T}{M} g_{1T}(x, k_T^2)
\]

(24)

and so on for the other functions. The functions with subscript “1” are the leading twist distributions which have probability interpretations. The other sixteen functions are twist-tree distributions. The calculations for eight T-even twist-three distributions has been carried in the diquark spectator model \[48\] and in the bag model \[49\]. There is also attempt to calculate naive-T-odd twist-three distributions \[50\] for which the light-cone divergence emerges.

The decomposition of antiquark correlator \( \tilde{\Phi}(x, k_T) \) can be achieved by the replacements \( x_1 \rightarrow x_2, \alpha \rightarrow -\alpha, \epsilon_T \rightarrow -\epsilon_T \), and by the relations \( \tilde{\Phi}^{[\Gamma]} = \tilde{\Phi}^{[\Gamma]} \) for \( \Gamma = \gamma_5, \epsilon_T \gamma_5, \gamma_5 \) and \( \tilde{\Phi}^{[\Gamma]} \) for \( 1, i \gamma_5 \gamma_5 \) \[47\], where \( c \) denotes the charge conjugation operation.

The quark–gluon–quark correlator \( \Phi^a_{A_0}(x, k_T) \) can be decomposed as \[35\]

\[
\Phi^a_{A_0}(x, k_T) = \frac{x}{M} \left\{ \left[ (\tilde{f}_T^1 - i \tilde{g}_T^1) \frac{k_T \cdot S_T}{M} - (\tilde{f}_T^1 + i \tilde{g}_T^1) \frac{e^{\prime \rho} k_T \cdot S_T}{M} - \left( \tilde{f}_T^1 + i \tilde{g}_T^1 \right) \frac{e^{\prime \rho} k_T \cdot S_T}{M} \right] \right. \\
\left. - \left( \tilde{h}_s + i \tilde{e}_s \right) \gamma_5 \right\} \frac{\tilde{f}_T^1 + \tilde{g}_T^1}{2},
\]

(25)

where the index \( \alpha \) is restricted to be transverse. The functions on the r.h.s with tilde are interaction-dependent twist-three distributions. They depend on \( x \) and \( k_T^2 \), except for the functions with subscript \( s \), which are defined as in Eq. (23). The last term inside the curly brackets will not be used in our following calculation and can be omitted here.

In the Wandzura-Wilczek approximation \[51\], the interaction-dependent distributions are set to be zero. However, quantitative analysis \[52\] on the \( g_2 \) structure function shows that the violation of the Wandzura-Wilczek approximation is sizable.

According to the equation of motion for the quark field, relations between twist-two distributions and twist-three distributions can be established \[47\]. A full list of these relations can be found in Ref. \[35\]. Here we only quote the ones which will be used in our later calculations:

\[
xf^1 = x\tilde{f}^1 + f_1,
\]

(26)

\[
xg_T = x\tilde{g}_T + \frac{m}{M} h_{1T},
\]

(27)

\[
xg_L = x\tilde{g}_L + g_{1L} + \frac{m}{M} h_{1L},
\]

(28)

\[
xg_{1T} = x\tilde{g}_{1T} + g_{1T} + \frac{m}{M} h_{1T},
\]

(29)

\[
xg_{1L} = x\tilde{g}_{1L} + g_{1L} + \frac{m}{M} h_{1L},
\]

(30)

\[
xh_{1T} = x\tilde{h}_{1T} + \frac{p_T^2}{2M^2} h_{1T} + \frac{m}{M} g_{1T},
\]

(31)

\[
xh_{1L} = x\tilde{h}_{1L} + \frac{p_T^2}{2M^2} h_{1L} + \frac{m}{M} g_{1L},
\]

(32)

\[
xh_{1T} = x\tilde{h}_{1T} + h_{1T} + \frac{p_T^2}{2M^2} h_{1T} + \frac{m}{M} g_{1T},
\]

(33)

\[
xf_T = x\tilde{f}_T + \frac{p_T^2}{2M^2} f_{1T},
\]

(34)
where  \( t \) have been discussed in literature. 

We note that the Lorentz invariance relations \( [22, 47, 53, 54] \) for integrated twist-tree distributions and their violations \( [52, 57] \) have been discussed in literature.

### III. Expressions for Twist-Tree Structure Functions of Drell-Yan Process

By substituting Eqs. \( [23, 25] \) into Eq. \( [22] \), and then contracting the lepton tensor and the hadronic tensor, one can get the expression of the cross section for the Drell-Yan process up to order \( 1/Q \). As shown in Ref. \( [21] \), there are forty-eight independent structure functions for Drell-Yan processes from the collision of two polarized spin-1/2 hadrons beams, and twenty-four of them are leading-twist observables. Here we restrict ourself to consider the contributions at order \( 1/Q \). We found that the angular distribution of the differential cross section of polarized Drell-Yan process at twist-three level can be expressed as

\[
\frac{d\sigma^{\text{twist-3}}}{dx_1 dx_2 d^2 q_T d\Omega} = \frac{\alpha_s^2}{3Q^2} \sin 2\theta \left\{ \cos \phi F_{UU}^{\cos \phi} + \sum \sin \phi F_{LU}^{\sin \phi} + \sum \sin \phi F_{UL}^{\sin \phi} + \sum \sin \phi F_{LL}^{\cos \phi} \right. \\
+ \sum \sin(\phi_1 + \phi) F_{UT}^{\sin(\phi_1 + \phi)} + \sum \sin(\phi_1 - \phi) F_{TU}^{\sin(\phi_1 - \phi)} \\
+ \sum \sin(\phi_2 + \phi) F_{UT}^{\sin(\phi_2 + \phi)} + \sum \sin(\phi_2 - \phi) F_{TU}^{\sin(\phi_2 - \phi)} \\
+ \sum \cos(\phi_2 + \phi) F_{UT}^{\cos(\phi_2 + \phi)} + \sum \cos(\phi_2 - \phi) F_{LU}^{\cos(\phi_2 - \phi)} \\
+ \sum \cos(\phi_1 + \phi) F_{LU}^{\cos(\phi_1 + \phi)} + \sum \cos(\phi_1 - \phi) F_{UL}^{\cos(\phi_1 - \phi)} \\
+ \sum \cos(\phi_1 - \phi) F_{LU}^{\cos(\phi_1 - \phi)} + \sum \cos(\phi_1 + \phi) F_{UL}^{\cos(\phi_1 + \phi)} \left\}
\]

We obtain sixteen twist-three structure functions, the angular dependences of which are consistent with the results given in Eq. (57) of Ref. \( [21] \).

To shorten the notation we will use following combinations, since they always appear in the same way:

\[
\tilde{f} = x_1 \left( (1 - c) f + c \tilde{f} \right), \quad \tilde{T} = x_2 \left( c \tilde{T} + (1 - c) \tilde{f} \right)
\]

where \( f \) and \( \tilde{f} \) stand for the twist-three quark distributions, and \( T \) and \( \tilde{T} \) for the antiquark distributions, respectively.

Using the EOM relations in Eqs. \( [20] \) to \( [39] \), the structure functions for polarized Drell-Yan processes at twist-three thus are expressed as

\[
F_{UU}^{\cos \phi} = \frac{2}{Q} C \left[ (h \cdot k_{1T}) \left( f_{1T} \tilde{T} - \frac{M_2}{M_1} h \tilde{h} \right) - (h \cdot k_{2T}) \left( f_{1T} \frac{\tilde{T}}{M_1} - \frac{M_1}{M_2} \tilde{h} h \right) \right]
\]

\[
F_{LU}^{\sin \phi} = \frac{2}{Q} C \left[ (h \cdot k_{1T}) \left( \tilde{g}_{1L} \tilde{h}_{1L} + \frac{M_2}{M_1} h_{1L} \tilde{h} \right) - (h \cdot k_{2T}) \left( f_{1T} \tilde{T} + \frac{M_1}{M_2} \tilde{h} h_{1L} \right) \right]
\]

\[
F_{UL}^{\sin \phi} = \frac{2}{Q} C \left[ (h \cdot k_{1T}) \left( f_{1T} \tilde{T} + \frac{M_2}{M_1} h_{1L} \tilde{h} \right) - (h \cdot k_{2T}) \left( g_{1L} \tilde{g}_{1T} \tilde{T} + \frac{M_1}{M_2} \tilde{h} h_{1L} \right) \right]
\]

\[
F_{UT}^{\sin (\phi_2 - \phi)} = \frac{1}{Q} C \left[ 2M_2 f_1 \tilde{T} + 2M_1 \tilde{h} h + (k_{1T} \cdot k_{2T}) \left( \tilde{T} \left( f_{1T} \frac{\tilde{T}}{M_1} - \frac{M_1}{M_2} \tilde{h} h \right) - \tilde{h} h \left( f_{1T} \tilde{T} + \frac{M_1}{M_2} \tilde{h} h_{1L} \right) \right) \right]
\]
\[ F_{UT}^{\cos(\phi_1 + \phi_1)} = \frac{1}{Q} \left[ \left( 2 \left( \hat{h} \cdot \hat{k}_{2T} \right)^2 - k_{2T}^2 \right) \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{M_2} + \frac{M_1 \hat{h} \hat{h}_T^+}{M_2} \right) \right. \]
\[ \left. + \left( 2 \hat{h} \cdot k_{1T} \hat{h} \cdot k_{2T} - k_{1T} \cdot k_{2T} \right) \left( \frac{f_{T}^+ \hat{T}^+_{T}}{M_2} + \frac{g_{1T} \hat{T}^+_{1T}}{M_1} + \frac{\hat{h}_T \hat{h}_T^+}{M_2} + \frac{\hat{h}_T^+ \hat{h}_T}{M_2} \right) \right] \] (46)

\[ F_{TU}^{\cos(\phi_1 - \phi_1)} = \frac{1}{Q} \left[ 2M_1 \hat{T}^+_{fT} + 2M_2 \hat{h}_L \hat{T}^+_{fT} + \left( k_{1T} \cdot k_{2T} \right) \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{M_2} + \frac{g_{1T} \hat{T}^+_{1T}}{M_1} + \frac{\hat{h}_T \hat{h}_T^+}{M_2} + \frac{\hat{h}_T^+ \hat{h}_T}{M_2} \right) \right] \] (47)

\[ F_{TU}^{\cos(\phi_1 + \phi_1)} = \frac{1}{Q} \left[ \left( 2 \left( \hat{h} \cdot \hat{k}_{1T} \right)^2 - k_{1T}^2 \right) \left( \frac{\hat{T}^+_{fT}}{M_1} + \frac{M_1 \hat{h} \hat{h}_T^+}{M_2} \right) \right. \]
\[ \left. + \left( 2 \hat{h} \cdot k_{1T} \hat{h} \cdot k_{2T} - k_{1T} \cdot k_{2T} \right) \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{M_2} + \frac{g_{1T} \hat{T}^+_{1T}}{M_1} + \frac{\hat{h}_T \hat{h}_T^+}{M_2} + \frac{\hat{h}_T^+ \hat{h}_T}{M_2} \right) \right] \] (48)

\[ F_{LL}^{\cos(\phi_1 - \phi_1)} = \frac{2}{Q} \left[ \left( \hat{h} \cdot \hat{k}_{1T} \right) \left( \frac{\hat{T}^+_{fT}}{M_1} + \frac{M_1 \hat{h} \hat{h}_T^+}{M_2} \right) \right. \]
\[ \left. - \left( 2 \hat{h} \cdot k_{1T} \hat{h} \cdot k_{2T} - k_{1T} \cdot k_{2T} \right) \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{M_2} + \frac{g_{1T} \hat{T}^+_{1T}}{M_1} + \frac{\hat{h}_T \hat{h}_T^+}{M_2} + \frac{\hat{h}_T^+ \hat{h}_T}{M_2} \right) \right] \] (49)

\[ F_{LT}^{\cos(\phi_2 + \phi_2)} = \frac{1}{Q} \left[ \left( 2 \left( \hat{h} \cdot \hat{k}_{1T} \right)^2 - k_{1T}^2 \right) \left( \frac{\hat{T}^+_{fT}}{M_2} + \frac{M_1 \hat{h} \hat{h}_T^+}{M_2} \right) \right. \]
\[ \left. - \left( 2 \hat{h} \cdot k_{1T} \hat{h} \cdot k_{2T} - k_{1T} \cdot k_{2T} \right) \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{M_2} + \frac{g_{1T} \hat{T}^+_{1T}}{M_1} + \frac{\hat{h}_T \hat{h}_T^+}{M_2} + \frac{\hat{h}_T^+ \hat{h}_T}{M_2} \right) \right] \] (50)

\[ F_{LT}^{\cos(\phi_1 - \phi_1)} = \frac{1}{Q} \left[ -2M_1 \hat{T}^+_{fT} - 2M_2 \hat{h}_L \hat{T}^+_{fT} + \left( k_{1T} \cdot k_{2T} \right) \left( \frac{\hat{T}^+_{fT}}{M_2} + \frac{M_1 \hat{h} \hat{h}_T^+}{M_2} \right) \right. \]
\[ \left. + \left( 2 \hat{h} \cdot k_{1T} \hat{h} \cdot k_{2T} - k_{1T} \cdot k_{2T} \right) \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{M_2} + \frac{g_{1T} \hat{T}^+_{1T}}{M_1} + \frac{\hat{h}_T \hat{h}_T^+}{M_2} + \frac{\hat{h}_T^+ \hat{h}_T}{M_2} \right) \right] \] (51)

\[ F_{TL}^{\cos(\phi_1 + \phi_1)} = \frac{1}{Q} \left[ \left( 2 \left( \hat{h} \cdot \hat{k}_{1T} \right)^2 - k_{1T}^2 \right) \left( \frac{\hat{T}^+_{fT}}{M_1} + \frac{M_1 \hat{h} \hat{h}_T^+}{M_2} \right) \right. \]
\[ \left. + \left( 2 \hat{h} \cdot k_{1T} \hat{h} \cdot k_{2T} - k_{1T} \cdot k_{2T} \right) \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{M_2} + \frac{g_{1T} \hat{T}^+_{1T}}{M_1} + \frac{\hat{h}_T \hat{h}_T^+}{M_2} + \frac{\hat{h}_T^+ \hat{h}_T}{M_2} \right) \right] \] (52)

\[ F_{TT}^{\cos(\phi_1 - \phi_1)} = \frac{1}{Q} \left[ \left( \hat{h} \cdot \hat{k}_{1T} \right) \left( \frac{M_1 \hat{T}^+_{fT}}{M_2} - \frac{M_2 \hat{h} \hat{T}^+_{fT}}{M_1} - \hat{h}_T \hat{T}^+_{1T} + \hat{h}_T^+ \hat{T}^+_{1T} \right) \right. \]
\[ \left. - \left( \hat{h} \cdot k_{2T} \right) \left( \frac{M_1 \hat{T}^+_{fT}}{M_2} - \frac{M_2 \hat{h} \hat{T}^+_{fT}}{M_1} + \hat{h}_T \hat{T}^+_{1T} + \hat{h}_T^+ \hat{T}^+_{1T} \right) \right] \] (53)

\[ F_{TT}^{\cos(\phi_1 + \phi_1)} = \frac{1}{Q} \left[ \left( 4 \left( \hat{h} \cdot \hat{k}_{2T} \right) \left( \hat{h} \cdot k_{2T} \right)^2 - 2 \left( \hat{h} \cdot k_{1T} \right) \left( k_{2T} \cdot \hat{k}_{2T} \right) - \left( \hat{h} \cdot k_{2T} \right) \hat{k}_{2T}^2 \right) \right. \]
\[ \left. \times \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{2M_1 M_2} - \frac{g_{1T} \hat{T}^+_{1T}}{2M_1 M_2} - \frac{\hat{h}_T \hat{T}^+_{1T}}{2M_2} - \frac{\hat{h}_T^+ \hat{T}^+_{1T}}{2M_2} \right) \right. \]
\[ \left. - \left( 4 \left( \hat{h} \cdot k_{1T} \right) \left( \hat{h} \cdot k_{2T} \right)^2 - 2 \left( \hat{h} \cdot k_{1T} \right) \left( k_{1T} \cdot \hat{k}_{2T} \right) - \left( \hat{h} \cdot k_{1T} \right) k_{2T}^2 \right) \right. \]
\[ \left. \times \left( \frac{f_{fT}^+ \hat{T}^+_{fT}}{2M_1 M_2} - \frac{g_{1T} \hat{T}^+_{1T}}{2M_1 M_2} - \frac{\hat{h}_T \hat{T}^+_{1T}}{2M_2} - \frac{\hat{h}_T^+ \hat{T}^+_{1T}}{2M_2} \right) \right] \] (54)
The following result \[22, 23\]:

functions of Drell-Yan processes. We note that, after being integrated over

different theoretical approaches lead to different results. For example, an additional factor 2 was found

Drell-Yan were studied intensively in the literature \[58–61\], and has been revisited in Refs. \[62\] and \[63\] recently. It

Also, we have applied the following combinations for certain distribution functions:

Equations (42) to (58) represent a main result of this work. They are complementary to the leading-twist structure

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Equations \[42\] to \[58\] represent a main result of this work. They are complementary to the leading-twist structure

functions of Drell-Yan processes. We note that, after being integrated over \(q_T\), the expression in Eq. (40) reduces to the following result \[22, 23\]:

\[
\frac{d\sigma^{\text{twist-3}}}{dx_1dx_2d\Omega} = \alpha_s^2 \frac{2}{3Q^2} \left\{ |S_{1T}| \sin(\phi_{S_1} - \phi) \left( M_1 f_T \bar{f}_T + M_2 h_1 \bar{h} \right) + |S_{2T}| \sin(\phi_{S_2} - \phi) \left( M_2 f_1 \bar{f}_T + M_1 h_1 \bar{h} \right) + S_{1L} |S_{2T}| \cos(\phi_{S_2} - \phi) \left( M_2 g_1 \bar{g}_T + h_1 L \bar{h}_1 \right) - S_{2L} |S_{1T}| \cos(\phi_{S_1} - \phi) \left( M_1 g_T \bar{f}_T + M_2 h_1 \bar{h}_1 \right) \right\}.
\]
in Ref. 62 while factor 1/2 was gained in Ref. 63, in comparison with the expression for $A_N$ in Refs. 23, 60, 61. We note that the normalization of the asymmetry of our result (in the Collins-Soper frame) agrees with that in Refs. 62, 63, but not with that in Refs. 62, 63.

In the following we present further comments on our result.

- Certain structure functions in Eqs. (42) to (58) satisfy the following symmetric or antisymmetric property:

$$
F_{UL}^{\sin \phi} = -F_{LU}^{\sin \phi}, \quad F_{UT}^{\sin(\phi_2 - \phi)} = F_{TU}^{\sin(\phi_1 - \phi)},
$$

$$
F_{UT}^{\sin(\phi_2 + \phi)} = F_{TU}^{\sin(\phi_1 + \phi)}, \quad F_{LT}^{\cos(\phi_2 - \phi)} = -F_{TL}^{\cos(\phi_1 - \phi)}, \quad F_{LT}^{\cos(\phi_2 + \phi)} = -F_{TL}^{\cos(\phi_1 + \phi)},
$$

which agree with the general analysis given in Ref. 21.

- The expressions of the twist-three structure functions depend on the choice of the dilepton rest frame, characterized by the parameter $c$. This feature is different from that of leading-twist structure functions, whose expressions of which are the same in different dilepton rest frames.

- Among the 16 twist-three distributions, 12 of them appear in the twist-three structure functions for Drell-Yan processes, including the new distributions $g_+^{1 - \perp}$ and $f_+^{1 - \perp}$. The first one appears in both the single longitudinally and transverse polarized Drell-Yan processes, while the later one appears in single and double transversely polarized Drell-Yan processes. These new twist-three TMDs are quite interesting, since evidence of their existence will indicate the necessity of introducing the gauge-link direction in the decomposition of the correlator [3]. They represent new nucleon parton structure information that has not been explored before. Calculations in a diquark model show that the contribution from $g_+^{1 - \perp}$ should be included in order to give a complete description of the asymmetries in longitudinally polarized SIDIS.

The distributions $e, e_L, e_T$ and $e_+^{1 - \perp}$ do not contribute here, since we have not considered the lepton polarization. These four distributions could be studied in SIDIS with a polarized lepton beam [12, 33, 60, 67].

- Equation (42) shows that the combination $\hat{f}_{1 - \perp} \otimes \mathcal{F}_1$ or $h_{1 - \perp} \otimes \hat{n}$ and so on can lead to a $\cos \phi$ asymmetry in unpolarized Drell-Yan processes, which is similar to the case in SIDIS. Apart from the well-known $\cos 2\phi$ distributions, the measurements of dilepton production in unpolarized hadron-hadron collision also show a $\cos \phi$ angular dependence [60, 73]. It has been shown that the QCD corrections can generate such an angular distribution [32, 77]. Our study shows that there is an alternative mechanism for the $\cos \phi$ asymmetry in the unpolarized Drell-Yan process, due to the presence of twist-three TMD distributions.

- In the region where the TMD framework is assumed to be valid, there is a suppression factor $q_T/Q$ for twist-three structure functions. Therefore the asymmetries arising from twist-three structure functions are supposed to be smaller than the leading-twist observables. However, the asymmetries are also directly determined by the size of the twist-three TMD distributions. It is not known whether there are positivity bounds to constrain twist-three TMD distributions, like the case of leading-twist distributions [73]. Sizable twist-three TMD distributions could lead to nonvanishing asymmetries in Drell-Yan process. Furthermore, the SIDIS measurements on fixed targets [7, 11, 12, 17, 67, 74, 81] show that the twist-three asymmetries are not small. This also encourages the corresponding measurements of twist-three effects in Drell-Yan processes, especially for fixed-target experiments [24, 26, 28, 50].

- For each structure function, there are several combinations of twist-two distribution and twist-three distribution which can contribute, similar to the twist-three results of SIDIS process. This makes twist-three parton distributions more difficult to be probed in high energy processes than the twist-two ones. Further theoretical and experimental studies are needed to provide more constrains the size of different TMD twist-three distributions.

- We point out that our calculation is based on a generalization of the TMD factorization to the twist-three level. Therefore the correctness of our results relies on the validation of the twist-three TMD factorization. Unlike the twist-three collinear factorization, which has been widely applied in SIDIS and Drell-Yan, the TMD factorization formalism at twist-three level has not been established yet. The main challenge for twist-three TMD observables is that when one calculates the twist-three TMDs, there are light-cone divergences [50] for which it has not been understood how to control them at order 1/Q. This does not necessarily means that twist-three TMD factorization cannot be developed. Further study is needed to overcome this difficulty.
IV. CONCLUSION

Drell-Yan process has been recognized as an important tool to study the structure of the nucleon. In this work, we have studied polarized Drell-Yan processes from the collision of two polarized spin-1/2 hadrons beams at order $1/Q$, based on the framework of TMD factorization. We find that, among a total of twenty-four subleading-twist structure functions in Drell-Yan process, sixteen of them are at twist-three level and can be expressed as combinations of twist-two and twist-three TMD distribution functions. We give the complete expressions for these structure functions, for each of which there are several twist-three distributions that contribute. Based on our result, we point out that twist-two and twist-three TMD distribution functions. We give the complete expressions for these structure functions, for

functions in Drell-Yan process, sixteen of them are at twist-three level and can be expressed as combinations of twist-

Based on our result, we point out that twist-three distributions can provide an alternative explanation for the $\cos \phi$ angular dependence in unpolarized Drell-Yan processes. The measurements of the asymmetries at order $1/Q$ in Drell-Yan process therefore can provide useful information on the twist-three TMD distributions and the multi-parton correlations in the nucleon.

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