Inclined Air Showers Reconstruction

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Abstract. The water Cerenkov tanks of the Pierre Auger Observatory can detect particles at all zenith angles and are therefore well suited for the study of inclined and horizontal air showers (60° > θ < 90°). Such showers are characterized by a dominance of the muonic component at ground, and by a very elongated and asymmetrical footprint which can even exhibit a lobular structure due to the bending action of the geomagnetic field. Dedicated algorithms for the selection and reconstruction of such events, as well as the corresponding acceptance calculation, have been set up on basis of muon maps obtained from shower simulations.

1. Introduction
The analysis of inclined events opens a new window for different studies, for example composition [1], since the total number of muons not only depends on the energy of the primary but also on the type of primary particle. In addition, the aperture is enlarged allowing the study of clustering and anisotropy in an extended region of the sky [2]. For a typical cosmic ray that produces a shower in the top layers of the atmosphere the electromagnetic component is absorbed in approximately two vertical atmospheres, so these inclined showers are sensitive to the high energy muon component of the shower that can reach the ground. The signal in the tanks will be proportional to the number of muons. The detection of neutrino events [3] through horizontal air showers was one of the main reasons to develop models for the horizontal reconstruction. At high zenith angle neutrinos can to travel large distances in the atmosphere. The interaction probability is small, and they can produce a shower near the detectors. These showers should look like vertical showers because of the electromagnetic component that is expected to reach the detectors. In this case the signature to detect neutrinos is clear: horizontal shower that look like vertical. Recently it has been pointed out that even if the neutrino does not interact near the array, and the EM component is absorbed in the atmosphere, it may be possible to use the time distribution in each tank to reconstruct the muon production distance [4]. In this case it could still be possible to identify neutrino showers because they interact deeper than nuclei or photons.

To reconstruct inclined events we need to know the main characteristics of these muonic showers. The models developed for vertical showers, assuming cylindrical symmetry, can not be used for horizontal showers, because the earth’s magnetic field breaks the cylindrical symmetry. In the next section, we review the model used and the algorithms developed to reconstruct inclined events induced by protons.
2. Muon density maps
We have developed a new set of muon density maps based on the model developed in Ref. [5]. The main modifications are the magnetic field components of Malargüe, and the AIRES 2.6.0 version is used to recalculate the input parameters to generate the muon density maps [6]. The hadronic model chosen is the QSGJET 2001 [7].

The magnetic field bends the muon trajectories by the Lorentz force creating complicated muon density patterns at ground level. The muon lateral distribution loses the natural cylindrical symmetry, and the tools developed for vertical showers to find the core position and the primary energy are not valid for horizontal showers. As an example, in the left panel of Fig. 1 we show a muon density map simulated with AIRES using 500 proton showers of $E=10$ EeV, zenith angle $86^\circ$ and azimuth angle $0^\circ$ (AIRES coordinates) without magnetic field, and in the right panel with magnetic field. The pattern observed in the right panel is due to the deflection of the positive and negative charge muon by the magnetic field.

![Image of muon density maps](image-url)

**Figure 1.** Left panel. Muon density map in the shower plane without magnetic field for a proton primary of $E=10$ EeV at zenith angle $86^\circ$ and azimuth angle $0^\circ$. Right panel. The same but with magnetic field.

The radius of the curvature, $R$, of the muon trajectory in the magnetic field $\vec{B}$ can be calculated using:

$$R = \frac{p}{eB_T} \approx \frac{E_\mu}{ceB_T}, \quad (1)$$

where $B_T$ is the component of the $B$ field perpendicular to the muon trajectory. The geomagnetic deviation of the muon trajectory from that expected in absence of magnetic field can be approximately expressed by:

$$\delta x = R \left[ 1 + \sqrt{1 + \left( \frac{d}{R} \right)^2} \right] \approx \pm \frac{eB_T d^2}{2p}, \quad (2)$$

where $d$ is the muon distance of production in meters. In the top panel of Fig. 2 we show a sketch of the $\delta x$ deviation. It has also been shown that there is a correlation between $r$ and muon energy. Using a simple approximation such as the relation $r \approx \frac{p_{\mu} \delta r}{E_\mu} [5]$ we obtain

$$\delta x = \frac{eB_T d^2}{2p} = \frac{0.15B_T d}{p_{\mu} r} = \alpha \frac{\delta r}{r}, \quad (3)$$
where $\tilde{r}$ is the distance to the shower axis without magnetic field. From the last equation, we observe that the transformation from the muon densities without magnetic field to the densities with magnetic field, in this approximation, is a simple geometric relation. In the bottom panel of Fig. 2 we show a schematic of the deviation of the muon trajectories, with given $r$, depending on its charge. In the left panel, in absence of magnetic field, the $\mu^\pm$ trajectories are not bent so both coincide. When a magnetic field acts the trajectories are deflected, depending on the muon charge, a quantity $\delta x$ that depends on muon energy, the muon production distance, and the magnitude of the transverse magnetic field. When the muon energy is simply related to $r$ the deviation is a purely geometrical effect (see right panel of the same figure).

![Diagram](image)

**Figure 2.** Top panel. Deviation of the expected muon trajectory by the magnetic field effect. Bottom panel. Effect of the magnetic field in the muon composition in the perpendicular plane.

The reference system used to generate the muon density maps has the $y$-axis pointing out in the direction of $\vec{B}_T$. In the case of a shower we choose to project $B$ onto the plane perpendicular to shower axis. This projection depends on the arrival direction. On the top left panel of Fig. 3 we show the projection of $B_\tau$ onto the perpendicular plane, for different zenith and azimuth angles. On the top right panel of the same figure we show the angle, $\psi$, between $B_\tau$ in the
shower plane and the shower direction, for different zenith and azimuth angles.

We generate muon density maps for zenith angles in the range $[50, 89]^{\circ}$ in steps of $2^{\circ}$ and azimuth angles in the range $[0, 180]^{\circ}$ in steps of $10^{\circ}$. The energy of the primary is $E=10$ EeV, it is fixed because it was found that the density maps scale with the energy of the primary [5].

![Figure 3](image-url)

**Figure 3.** Top left panel, magnetic field projection onto the perpendicular plane as a function of the azimuthal angle for the Auger location. Top right panel, Angle $\bar{\psi}$ in the transverse plane as a function of the azimuthal angle for the Auger coordinates. Bottom left panel. Lateral distribution of muons as obtained from AIRES, for proton shower of $E=10$ EeV and zenith angle from 50 (top) to 89 (bottom) deg. without magnetic field. Bottom right panel. Average muon energy as a function of distance to the core. Results obtained from AIRES, for proton shower of $E=10$ EeV and zenith angle from 50 (bottom) to 89 (top) deg.

To realistically obtain the muon density distribution in the shower plane in the presence of a magnetic field we must take into account that the muons have a energy distribution that depends on the distance to shower axis. To calculate the density at any given point we have to
integrate over all the muon energies that contribute to the chosen point:

\[ \rho(x, y, \theta) = \int dP(\epsilon, \langle \epsilon \rangle, \sigma) \rho(\bar{r}, \theta), \] (4)

where \( \bar{r} \) is given by:

\[ \bar{r} = \sqrt{\left( x \pm \frac{eBTd^2c}{2E_\mu} \right)^2 + y^2}, \] (5)

Here \( P(\epsilon, \langle \epsilon \rangle, \sigma) \) is the probability distribution that muons of average energy \( \langle E \rangle \) have an energy \( \epsilon \) which is approximated by a logarithmic distribution in \( \langle \epsilon \rangle \equiv \langle \log_{10} E_\mu \rangle \), with \( \sigma = 0.4 \), and the average muon energy as a function of the distance to the core in absence of magnetic field, \( \bar{r} \), can be parameterized as [5], see bottom right panel of Fig. 3

\[ \langle \epsilon \rangle \equiv \langle \log_{10} E_\mu \rangle = A(\theta) - \gamma(\theta) \log_{10}(\bar{r}). \] (6)

The \( \rho(\bar{r}, \theta) \) is the muon density distribution in the absence of the magnetic field.

In Fig. 4 we show a comparison of two density contour plots for a zenith angle of 86° and azimuth 0°. The left panel was obtained from the model and the right panel directly from AIRES simulation. In Fig. 5 we show a comparison between the \( \rho(x, 0) \) obtained with the present model and an AIRES output with the magnetic field included for zenith angles 60°, 68°, 80° and 86°, the azimuth angle is 0°. We observe a good agreement, better than 5%, even for large distances.

![Aires Muon density map at θ=86°](image)

Figure 4. Left panel, muon densities in the transverse plane obtained using the model. Right panel, muon density in the transverse plane obtained using the AIRES output. The figures correspond to zenith angles 86° and \( E=10 \text{ EeV} \).

3. Detector response probability

Once we have the muon density maps we know the average number of muons that is expected to cross a given detector surface. The next step is to calculate the response of the detector to these muons for a given zenith angle and an average muon energy. We are going to use the simulation of the Pierre Auger surface detector tank [9]. We define \( P_n(S_\mu) \) to be the probability
Figure 5. Muon densities in the transverse plane as function of the $x$ coordinate centered at $y = 0$. The figures correspond to zenith angles $60^\circ$, $68^\circ$, $80^\circ$ and $86^\circ$. The azimuth angle is $0^\circ$ for all cases.

distribution function (P.D.F.) to have a signal $S_\mu$ for $n$ muons hitting the tank at given zenith angle and for a given energy.

To calculate the P.D.F. for $n$ muons we have to perform the auto-convolution with the distribution for a single muon.

$$P_2(S_\mu) = \int_0^\infty dS_\mu' P_1(S_\mu') P_1(S_\mu - S_\mu'),$$

$$...$$

$$P_n(S_\mu) = \int_0^\infty dS_\mu' P_1(S_\mu') P_{n-1}(S_\mu - S_\mu').$$

On the top left panel of Fig. 6 we show the P.D.F for 1 muon of energy 5 GeV and zenith angle $70^\circ$, and on the top right panel of the same figure we show the convolution of the P.D.F.
Figure 6. Top left panel, Signal distribution probability for 1 muon of energy 5 GeV and zenith angle 70°. Top right panel, Signal distribution probability for i=1,...,8 muons of same energy and zenith angle. Bottom left panel, average track-length as a function of zenith angle. Bottom right panel, RMS squared of the average track-length as a function of the zenith angle.

for \( n=1 \) to 8. We note that the original distribution have a complicated shape that rapidly becomes Gaussian, as expected from the central limit theorem.

For \( n \leq 8 \) we do not try to parameterize the histograms due to the complicated shape of the distributions. Instead we will use the histograms directly. For \( n > 8 \) we parameterize those distribution because, using the central limit theorem, \( P_n(S_\mu) \) is very well parameterized by a Gaussian distribution that can be expressed as:

\[
P_n(E_\mu, \theta) = \text{Gauss}(\xi_n(E_\mu, \theta), \sigma_{\xi_n}(E_\mu, \theta)),
\]

where \( \xi_n(E_\mu, \theta) \) is the average value of the signal, and \( \sigma_{\xi_n}(E_\mu, \theta) \) is the standard deviation. These functions can be parameterized as follows:

\[
\xi_n(E_\mu, \theta) = f_1(E_\mu) \langle t \rangle(\theta)n,
\]
\[ \sigma_{\xi_0} (E_{\mu}, \theta) = f_2 (E_{\mu}) \sigma^2_{\langle t \rangle} (\theta) \sqrt{n}, \quad (9) \]

where \( \langle t \rangle (\theta) \) is the average track-length and \( \sigma^2_{\langle t \rangle} (\theta) = \langle t^2 \rangle - \langle t \rangle^2 \) is the track-length variance for single muon. The behaviour of \( \langle t \rangle \) and \( \sigma^2_{\langle t \rangle} \) with the zenith angle is shown in the bottom panels of Fig. 6.

For both functions \( f_1 (E_{\mu}) \) and \( f_2 (E_{\mu}) \) a linear fit to the log of the energy is made, the error is less than 5%. The results of the fits are:

\[
\begin{align*}
 f_1 (E_{\mu}) &= 0.92 + 0.085 \log_{10}(E_{\mu}) \\
 f_2 (E_{\mu}) &= 0.23 + 0.052 \log_{10}(E_{\mu})
\end{align*}
\]

4. Algorithm to find the core position

To find the core position we first chose the corresponding muon density map for the arrival direction obtained by the angular reconstruction plane fit or the curved fit. The map is divided in cells of constant size forming a square grid of 4x4 km, and for each cell, assuming that is the core position at the center of the cell, we minimize the following \( \chi^2 \) function:

\[
\chi^2(N_{19}) = \sum_{i=1}^{N} \frac{(S^\text{exp}_i - S^\text{mes}_i)^2}{\sigma_i^2},
\quad (10)
\]

where \( i \) runs over all the stations, \( S^\text{mes}_i \) is the measured signal, \( \sigma_i = S^\text{mes}_i / \sqrt{n_{\mu}} \) with \( n_{\mu} \) the average number of muons that hit the tank, and \( S^\text{exp}_i \) is the expected signal obtained as:

\[
S^\text{exp} = N_{19} S^{19} = N_{19} \rho(r, \theta) A_{\mu}(\theta) L(T_{\mu}, t, \theta)(1 + r_{\text{EM}}(r, \theta)),
\quad (11)
\]

where \( r \) is the distance of the station to the core in the shower plane, and \( S^{19} \) is the simulated signal if the energy of the primary were 10 EeV, and \( A_{\mu}(\theta) \) is the tank projected area in the shower direction, \( \rho(r, \theta) \) is the muon density distribution for the reference \( E=10 \text{ EeV}, L(T_{\mu}, t, \theta) \) is the average tank signal response for muons, \( r_{\text{EM}}(r, \theta) \) is the ratio of the electromagnetic signal to the muon signal obtained from Ref. [8], which is shown in Fig. 7 for a proton shower of \( E=10 \text{ EeV}, \) and \( N_{19} \) is the normalization. This is the variable that we use to minimize the \( \chi^2 \).

Using this method the \( \chi^2 \) minimization can be done analytically for each possible core position \((x_c, y_c)\):

\[
\frac{\partial \chi^2}{\partial N_{19}}(x_c, y_c) = 2 \sum_{i=1}^{N} \frac{(S^\text{exp}_i - S^\text{mes}_i)}{\sigma_i^2} \left( \frac{\partial S^\text{exp}_i}{\partial N_{19}} \right) = 0.
\]

\[
N_{19} = \frac{\sum_{i=1}^{N} (S^\text{mes}_i S^{19}_i / \sigma_i^2)}{\sum_{i=1}^{N} (S^\text{mes}_i S^{19}_i / \sigma_i^2)}
\]

Each minimization of the \( \chi^2 \) function gives us a minimum value for a given core position. When we applied to all the cells the cell with minimum \( \chi^2 \) value is chosen as core position.

The algorithm is applied twice, the first time using a grid of 4 km by 4 km centered at the barycenter of the event and with a cell size of 100 m, to a first approximation to the core. Then it is repeated over a grid of 1 km by 1 km centered in the new core with a cell size of 20 m to get a more accurate core position.
The silent stations are very important to constrain the core position to be in the region where the stations with signal are located. To perform the fit we only use a ring of active but silent stations that surround the stations with signal. The $\chi^2$ for the silent stations is calculated as:

$$\chi^2(N_{19}) = \sum_{i=1}^{N} \frac{(S_{i}^{\text{exp}})^2}{\sigma_i^2},$$

where $\sigma_i^2 = S_{i}^{\text{exp}}$. On the top panels of Fig. 8 we show the contour plot of $\chi^2_{\text{max}} - \chi^2$ distribution for the event 850018 in the shower plane, using a cell size of 100 m in the left panel, and a cell size of 20 m in the right panel. The black points indicate the stations with signal, and the grey points the silent stations. On the bottom panels we show the same distributions as 3D histograms. The error in the core position is estimated using these $\chi^2$ histograms. We subtract one unit from the maximum. Then the total width in $x$ and $y$ of the distribution gives the error of the core in the perpendicular plane. Then these errors are propagated to ground level taking into account the errors in the angular reconstruction.

**Figure 7.** Ratio of the electromagnetic signal to the muon signal as a function of distance for different zenith angle for protons at $E=10$ EeV.

5. Algorithm to fit the energy

Once we get the core position we have to estimate the energy of the primary. To do so, we construct a likelihood function [10] that must be maximized to obtain $N_{19}$ which can be converted into energy using either the scaling obtained from Monte-Carlo simulation or alternatively using hybrid data with energy measured through the fluorescence technique for calibration.

Using the muon density maps, with the core position calculated with the algorithm explained in the previous section, we can easily predict the expected number of muons, $\langle \mu \rangle$, that cross a given surface in the position $(x,y)$ with respect to the core $(x_c,y_c)$:

$$\langle \mu \rangle = N_{19} \rho(x, y, \theta, \phi) A_p(\theta),$$

(13)
where $N_{19}$ was described in previous section, is the fitted parameter, $\rho(x, y, \theta, \phi)$ is the muon density map in the presence of magnetic field, and $A_{\mu}(\theta)$ is the tank area projected onto the perpendicular plane.

Using the expected number of muons and the observed signal in the tank $S_{\text{mes}}$ we can construct the probability function for each station to measured $S_{\text{mes}}$ when it expects on average of $\langle \mu \rangle$ muons. This probability is constructed by summing the probabilities for all the possible number of muons that may give the observed signal:

$$Pr(S_{\mu}^{\text{mes}}, \langle \mu \rangle, \theta, T_\mu) = \sum_{k=\text{min}}^{k=\text{max}} p(k; \langle \mu \rangle) P_k(S_{\mu}^{\text{mes}}, T_\mu, \theta),$$

where $k$ runs over all the possible number of muons that give a non zero probability. The minimum and maximum of the sum are carefully chosen to reduce the CPU time. The probability that the signal is produced by $k$ muons is simply the product of the Poisson probability of measuring $k$ muons if we expect an average $\langle \mu \rangle$, $p(k; \langle \mu \rangle)$, and the probability that $k$ muons

Figure 8. Top panels. Contour plot of $\chi^2$ for different grid size. Bottom panels. 3D histograms of the $\chi^2$. 
give a measured signal \( S_{\mu}^{\text{mes}} \), \( P_k(S_{\mu}^{\text{mes}}, T, \theta) \), where \( S_{\mu}^{\text{mes}} = S_{\mu}^{\text{mes}}(1 + r_{EM})^{-1} \) is the signal after subtracting the electromagnetic contribution.

The likelihood function is given by:

\[
L(N_{19}) = \prod_{i=1}^{N} Pr_i(S_{\mu}^{\text{mes}}, (\mu), T, \theta),
\]

(15)

where the index \( i \) runs over all the stations. And finally the function to minimize is the negative logarithm of the likelihood where the only free parameter is the scaling factor \( N_{19} \). In Fig. 9 we show the logarithm of the likelihood function for two different events. We clearly observe that the likelihood function has a minimum in the parameter \( N_{19} \).

\[ \text{Figure 9. Left panel, Likelihood value as a function of } N_{19} \text{ for the event 668949. Right panel, same for the event 850018.} \]

6. Energy calibration

The \( N_{19} \) parameter can be related with the energy of the primary. To convert \( N_{19} \) to energy we can use the scaling deduced from simulation \( N_{19} \), or we can use inclined hybrid events accurately reconstructed to get the relation between energy and \( N_{19} \) directly from the data.

Using a Monte-Carlo approach assuming proton primaries and QSGJET as an hadronic model interaction we obtain a scaling law of the following form:

\[
\log_{10}[E_{MC}(\text{EeV})] = 1 + \frac{1}{\beta} \log_{10}[N_{19}] \text{ with } \beta = 0.924.
\]

(16)

The values of the constants depend on the hadronic model and on the mass composition. In Fig 10 we show the above relation between the primary energy and the total number of muons [5, 11]. The total number of muons scales with the muon density maps as obtained in the simulations.

7. A inclined event

In Fig. 11 we show an example of a very inclined event with id 767138 detected in the Pierre Auger observatory [9]. The reconstructed zenith angle is \( \sim 88^\circ \). Using the method described in the previous section the estimated energy of the event is \( \sim 30 \text{ EeV} \). With this event we can observed the main characteristics of the inclined events.
We observed that there are 31 tanks with signal. This is characteristics from the inclined events where in average more than 10 tanks use to have signal. Also, if we follow the direction of the event, pink line in the figure, we observed that there is a separation of more than 30 km from the first tank to the last. The main component of the inclined showers are muons. They can travel large distance before reach the ground. A tank can be trigger with just a few muons. This is why those shower can spread many kilometers away. Finally, the color code represents the signal in each tank, if we observed the core position, pink start, we see that it is in a region with low particle density surround it by high densities regions. This is because at this zenith angle muons have to travel large distance in the atmosphere before reach the ground. And the earth magnetic field bent the trajectories of the positive and negative muons creating two lobes at each side of the core.

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Figure 11. Event 767138. The color color represents the tank signal. The dashed pink line is the direction of the shower. The pink start is the reconstructed core position. The brown square point represent station with no trigger that are used in the fit as a silent station. The small picture in the top left is the muon density at ground simulated with AIRES with the same energy and angles as the shower.