BARYOGENESIS THROUGH GRADUAL COLLAPSE OF VORTONS

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October 23, 2019

Abstract

We evaluate the matter-antimatter asymmetry produced by emission of fermionic carriers from vortons which are assumed to be destabilized at the electroweak phase transition. The velocity of contraction of the vorton, calculated through the decrease of its magnetic energy, originates a chemical potential which allows a baryogenesis of the order of the observed value. This asymmetry is not diluted by reheating if the collapse of vortons is distributed along an interval of \(\sim 10^{-9}\) sec.

1 Introduction

The matter-antimatter asymmetry in the universe is one of the well established facts of cosmology. There are many possible mechanisms to generate this baryonic density due to phenomena which presumably occurred in the first fraction of second after the big-bang but all of them suffer some criticism. They include also methods involving cosmic strings or other topological defects produced in some of the phase transitions produced in the universe.

In this work we present a baryogenesis model based on possibly very abundant closed cosmic strings called vortons stabilized by superconducting currents, which might lose this stability at the electroweak phase transition. The distinctive feature of our mechanism is that we follow

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the process of contraction of vortons and that we assume that they do not destabilize all at the same time.

In Section 2 we give a survey of the baryogenesis methods which have some connection with our proposal. In Section 3 we briefly describe vortons and their relationship with Grand Unified Theories (GUT). Section 4 reminds the instantaneous decay of vortons and the reheating which causes the dilution of matter-antimatter asymmetry. In Section 5 we present our scenario of gradual decay of vortons indicating how the reheating problem may be solved. Section 6 contains the details of the calculation of the contraction velocity which, for the case of charged carriers, is based on the decrease of the associate magnetic energy. In Section 7 we evaluate by tunneling the probability of emission of carriers which gives way to the asymmetry produced by each vorton. Section 8 shows how the variation of magnetic field due to contraction produces a chemical potential which allows our baryogenesis to be of the order of the expected one. Section 9 contains some conclusions.

2 Methods to generate matter - antimatter asymmetry

From the nucleosynthesis of light elements there is a constraint for the baryonic density which, related to entropy density to give an invariant value, is

$$\frac{n_B}{s} = 10^{-11} - 10^{-10}.$$  (1)

To explain this asymmetry, if one starts from a symmetric universe, three conditions are required [2]: i) non-conservation of baryonic number, ii) violation of C and CP to distinguish particle from antiparticle, iii) period of non-equilibrium to allow different number of particles and antiparticles.

The method of baryogenesis closest to experimental verification is that which corresponds to the electroweak phase transition [3] provided it is a first-order one. Expanding bubbles of the broken-symmetry phase would produce in its wall a chemical potential due to the variation of a CP violating phase $\theta$ compared to the external symmetric medium, where the active sphalerons would generate the baryonic density. The latter would not be erased because the bubble expansion would include it in the broken-symmetry phase where sphaleron processes are very slow. Due to the rate of sphaleron transitions in the high-temperature phase one would obtain

$$\frac{n_B}{s} \simeq \frac{\alpha_w^4}{g^*} \Delta \theta,$$  (2)

where the weak coupling is such that $\alpha_w \simeq 10^{-3}$ and the number of zero-mass modes at the electroweak temperature $T \sim 100 GeV$ is $g^* \simeq 100$. Therefore the observed asymmetry is reproduced if $\Delta \theta \sim 10^{-2}$. However this mechanism with Standard Model ingredients is not possible because the phase transition turns out to be of second order for the experimental bound on the Higgs mass and the CP violation is not enough.

A solution which would include not too high-energy elements beyond the Standard Model is afforded by the Minimal Supersymmetric Standard Model. But this model would be severely constrained because to give an enough first-order phase transition the Higgs boson should be light $m_H < 100 GeV$ as well as the stop $m_{t\bar{t}} \leq 200 GeV$, and to allow a large enough variation
of the CP violation parameter without entering in conflict with the neutron electric dipole moment the lower generations of squarks should be very heavy\cite{4}, though this last condition might be relaxed\cite{3}.

On the other extreme of the energy range, a possibility of baryogenesis would be given by the decay of GUT Higgs and gauge bosons which should be produced out of equilibrium requiring $T \sim 10^{16} \text{GeV}$. The generated baryonic density would be

$$\frac{n_B}{s} \sim \frac{\varepsilon}{g^*} \quad (3)$$

where, with the asymmetry produced by one of these superheavy particles $\varepsilon \sim 10^{-8}$, there would be agreement with the expected value.

A problem is here that at these extremely high temperatures magnetic monopoles would have been produced with the consequent overclosure of the universe density, as well as very heavy cosmic strings which might originate undesirable inhomogeneities. It is anyhow difficult to explain such high $T$ from the reheating at the end of inflation, unless the non-linear quantum effects of preheating give way to an explosive heavy particle production out of equilibrium\cite{6}.

3 Cosmic strings and vortons

Cosmic strings are topological defects which appear in a phase transition when an abelian symmetry additional to the standard model is broken. To avoid the monopole problem we may assume that the universe reached a temperature for which the GUT symmetry $G$ was already broken

$$G \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times \tilde{U}(1) \quad (4)$$

If at a slightly lower temperature, let us say $10^{15} \text{GeV}$, also the symmetry $\tilde{U}(1)$ is broken the cosmic strings will be produced\cite{7}. They will become superconducting\cite{8} depending on the group $G$ and the details of the Higgs mechanism for the breaking of $\tilde{U}(1)$. A superconducting current will appear for those fermionic carriers which acquire mass due to the coupling with a Higgs field which winds the string and originates zero modes inside it. The superconducting current classically stabilizes closed loops through a number $N$ related to the angular momentum due to the carriers inside them.

It is not necessary that the carriers are charged\cite{9}. In fact if $G = SO(10)$ the only particle which acquires mass at the $\tilde{U}(1)$ phase transition is $\nu_R$ which may have a zero mode inside the string. On the other hand if $G = E_6$ several fermions acquire mass at the $\tilde{U}(1)$ breaking and some of them, which may give superconducting currents, are charged and with baryonic number.

For normal cosmic strings it is interesting\cite{10} that if the emission of superheavy bosons at present time is normalized to explain the flux of ultra-high energy cosmic rays, their decay in the past may give the expected baryogenesis provided that the asymmetry per particle is six orders of magnitude higher than that necessary in Eq.(3).

The stabilized superconducting closed loops are called vortons\cite{11}. Their number density, mass, length and quantum decay probability depend on the coincidence or not of the scales of
string formation and appearance of superconductivity in them\cite{12}. If both scales coincide at 
\( m_x \) the vorton density is

\[
n_v \simeq \left( \frac{m_x}{m_{pl}} \right)^{\frac{3}{2}} T^3, \quad (5)
\]
its energy \( E_v \simeq N m_x \), radius \( R \simeq N m_x^{-1} \) and \( N \sim 10 \) if \( m_x \sim 10^{15} \text{GeV} \). If the superconductivity scale is smaller than the formation one, the density is smaller and vortons are more stable for quantum decay.

The density Eq.(5) overcloses the universe in a way similar to that of monopoles, if there is not a collapse of vortons for some reason. If this is produced at high energy when the carriers are \( \nu_R \), a lepton asymmetry appears which may be converted into baryon asymmetry by sphaleron processes to give the expected value with adequately large CP violation parameter\cite{13}. Alternatively, if superconductivity appears at much lower temperature, i.e. at the supersymmetric scale \( \sim 17 \text{TeV} \), there are models predicting that vortons which subsequently decay below the electroweak temperature may release baryonic charge in agreement with the expected one\cite{14}, again assuming an adequate CP violation factor.

It must be noted that if the scales of formation and superconductivity coincide and the vorton density is decreased for some process to be constrained by the critical density of universe, the quantum decay probability might be enough to explain the high energy cosmic rays\cite{15}.

4 Instantaneous decay of vortons at the electroweak transition

Trying to include as few ingredients as possible, we will adopt the point of view that vortons have obtained the superconducting property at the same scale of formation, and that they lose their stability at the well established electroweak transition. This may occur if, due to the new Higgs mechanism at this scale, the zero modes acquire a small mass\cite{16}. It is not required that the transition is of first order.

If vortons disappear instantaneously, since they contain roughly \( N \) heavy bosons the produced baryonic density is

\[
\frac{n_B}{s} = \left( \frac{m_x}{m_{pl}} \right)^{\frac{3}{2}} \frac{N \varepsilon}{g^*}, \quad (6)
\]

which will be very small if the asymmetry due to each particle \( X \) is of the same order of that of GUT bosons assumed in Eq.(3).

Furthermore, since vortons behave as non-relativistic matter, its density which is very small at formation becomes equivalent to that of radiation at \( T \sim 10^8 \text{GeV} \) and dominates on it by 6 orders of magnitude at the electroweak scale. Therefore if at this temperature vortons transform instantaneously into light particles, i.e. radiation, there will be a reheating according to

\[
\rho_v(T_{EW}) = N m_x \left( \frac{m_x}{m_{pl}} \right)^{\frac{3}{2}} T_{EW}^3 = \rho_R(T_{reh}) = g^* T_{reh}^4, \quad (7)
\]
which gives $T_{\text{reh}} \simeq 10^{7.2}$ GeV.

This instantaneous reheating would produce an increase of the entropy density of $\left( \frac{T_{\text{reh}}}{T_{\text{EW}}} \right)^3 \simeq 10^{9.2}$ times with the corresponding dilution \cite{17} of the baryogenesis of Eq.(6) in the same factor.

According to this scenario the universe would be initially dominated by radiation, then from $T \sim 10^8$ GeV to $T_{\text{EW}}$ by vortons and after the reheating to $T_{\text{reh}}$ again by radiation till $t \sim 10^{11}$ sec. when finally non-relativistic matter takes over.

5 Gradual collapse of vortons

The alternative that we wish to present corresponds to the plausible situation that vortons are not destabilized all at the same time when reaching the electroweak temperature. Due to the Higgs mechanism that will be working in this phase transition, we expect a probability that a vorton loses its zero modes and starts its collapse. Without attempting to calculate this probability for destabilization, we remark that the temperature will remain constant at $T_{\text{EW}} \sim 100$ GeV if vortons decay during an interval such that the universe expands its scale from $a_1$ to $a_2$ when all is transformed to radiation

$$a_1^3 \ N \ m_x \left( \frac{m_x}{m_{\text{pl}}} \right)^{\frac{3}{2}} T_{\text{EW}}^3 = a_2^3 \ g^* T_{\text{EW}}^4 .$$ \hspace{1cm} (8)

The space scale would therefore increase in two orders of magnitude and, using $\frac{a_2}{a_1} = \left( \frac{t_2}{t_1} \right)^{\frac{3}{8}}$, if the process starts at $t_1 \sim 10^{-12}$ sec it would be completed at $t_2 \sim 10^{-9}$ sec. The advantage is now that the total increase of entropy, which is similar to that of instantaneous destabilization, is distributed in a larger volume. Baryogenesis would not be diluted at the beginning of the interval but only at the end with a factor $\left( \frac{a_2}{a_1} \right)^3$ so that the average dilution would be $\sim \frac{1}{2}$. It is reasonable to think that the collapse of vortons keeps the temperature constant because as soon as there is a tendency to reheating the symmetry is restored and the destabilization of vortons stops.

Furthermore, we will follow the contraction of each vorton obtaining baryogenesis not by the presumably small asymmetry in the decay of bosons $X$, but from the emission of charged baryonic carriers during the collapse. The resulting asymmetry per vorton may turn out to be larger due to the chemical potential which will appear in the wall of the vorton because of the non-equilibrium process of contraction, resulting in a different emission of fermions and antifermions.

6 Velocity of contraction during vorton decay

The evaluation of the velocity of contraction of the vorton after its destabilization at the electroweak temperature is crucial for determining the non-equilibrium process.

One possibility of calculation, which may be applied to the case of neutral carriers, is to consider that stabilization is abruptly lost at $T_{\text{EW}}$ so that the string contracts due to a constant tension $\mu \sim m_x^2$. If the string mass were constant and the initial radius is $R \sim \frac{N}{m_x}$, the relation between the velocity and each radius $r$ would be
\[ v^2 \simeq 2 \frac{R-r}{R} . \] (9)

Considering that the vorton mass decreases linearly with its radius in the rest frame, including the Lorentz factor and being at the initial stage of the contraction when the iterative approximation may be used, the velocity turns out to be

\[ v = \frac{1}{\sqrt{2}} \arctan \frac{\sqrt{2}N}{m_x t} \left[ 1 - \left( \frac{m_x t}{N} \right)^2 \right]^\frac{1}{2} + 2 \arcsin \left( \frac{m_x t}{N} \right) \] (10)

\[ - \frac{3}{2\sqrt{2}} \ln \left( \frac{m_x t + \sqrt{2}N \left[ 1 - \left( \frac{m_x t}{N} \right)^2 \right]^\frac{1}{2}}{m_x t - \sqrt{2}N \left[ 1 - \left( \frac{m_x t}{N} \right)^2 \right]^\frac{1}{2}} \right) - \frac{\pi}{2\sqrt{2}}. \]

To have the relation between velocity and radius which replaces Eq.(9), v of Eq.(10) should be integrated on time. It is clear that v will vary from 0 to 1 when \( r = 0 \) so that the time of collapse will be \( t_c \geq \frac{N}{m_x} \).

For charged carriers, in which we are more interested, the velocity of contraction may be calculated in an easier way. We consider the decay of a vorton as a succession of transitions between superconducting states of numbers N, N-1,...keeping the value of the current I. Looking at a classical average, one will see a loop with increasing contraction velocity with the corresponding relativistic factor in its mass which will be compensated by the decrease of the magnetic energy that is defined in the broken-symmetry phase.

The balance for the vorton when its radius is \( r \) and the associated magnetic field is B compared with the initial one \( B_i \), will be

\[ \frac{R}{R} N m_x \left( \frac{1}{\sqrt{1-v^2}} - 1 \right) = \frac{1}{2} \int d\rho \left( B_i^2 - B^2 \right). \] (11)

Since in general we will expect

\[ \frac{1}{2} \int d\rho \ B^2 = k \ I^2 \ r \] (12)

and being \( I = \frac{N}{2\pi r} \), we will have

\[ 1 - v^2 = \frac{1}{\left( 1 + \frac{k_i R - k_f r}{4\pi^2 \ r} \right)^2}, \] (13)

where the coefficient \( k_f \) at the end of the collapse may be different from the initial one \( k_i \).

At the beginning, when \( R - r \) is small and \( k_i = k_f \)

\[ v^2 \simeq \frac{k_i}{2\pi^2} \frac{R-r}{R}, \] (14)

which is analogous to the previous estimation due to constant tension. For \( r \to 0 \) the velocity Eq.(13) will tend to 1.

An important ingredient for our evaluation of baryogenesis will be the calculation of the coefficients k.
In the first part of the contraction the vorton will be certainly well represented by a loop of radius \( r \) that, lying in the x-y plane, will give a magnetic potential

\[
A_\varphi (\rho, \theta) = I \: r \: \int_0^{2\pi} d\varphi' \frac{\cos \varphi'}{(\rho^2 + r^2 - 2r \rho \sin \theta \cos \varphi')^{\frac{3}{2}}}. \tag{15}
\]

For large distances \( \rho \gg r \) Eq.(15) gives the dipole approximation for the magnetic field

\[
B_\rho = \frac{2m \cos \theta}{\rho^3}, \quad B_\theta = \frac{m \sin \theta}{\rho^3}, \tag{16}
\]

with \( m = \pi r^2 I \).

For distances much smaller than the radius \( \rho \ll r \)

\[
B_\rho = \frac{2\pi I}{r} \cos \theta, \quad B_\theta = -\frac{2\pi I}{r} \sin \theta, \tag{17}
\]

whereas the exact expressions from Eq.(15) are

\[
B_\rho = \frac{\cot \theta \: I \: r}{\rho} \int_0^{2\pi} d\varphi' \frac{\cos \varphi'}{(\rho^2 + r^2 - 2r \rho \sin \theta \cos \varphi')^{\frac{3}{2}}}
+ I \: r^2 \cos \theta \int_0^{2\pi} d\varphi' \frac{\cos^2 \varphi'}{(\rho^2 + r^2 - 2r \rho \sin \theta \cos \varphi')^{\frac{3}{2}}}, \tag{18}
\]

\[
B_\theta = -I \: \frac{r}{\rho} \int_0^{2\pi} d\varphi' \frac{\cos \varphi'}{(\rho^2 + r^2 - 2r \rho \sin \theta \cos \varphi')^{\frac{3}{2}}}
+ I \: r \: \rho \int_0^{2\pi} d\varphi' \frac{\cos \varphi'}{(\rho^2 + r^2 - 2r \rho \sin \theta \cos \varphi')^{\frac{3}{2}}}
- I \: r^2 \sin \theta \int_0^{2\pi} d\varphi' \frac{\cos^2 \varphi'}{(\rho^2 + r^2 - 2r \rho \sin \theta \cos \varphi')^{\frac{3}{2}}}. \tag{18}
\]

For small \( \sin \theta \), Eq.(18) may be approximated by

\[
B_\rho = \frac{2\pi r^2 I}{(\rho^2 + r^2)^{\frac{3}{2}}} \cos \theta, \quad B_\theta = \frac{\pi r^2 I}{(\rho^2 + r^2)^{\frac{3}{2}}} \frac{\rho^2 - 2r^2}{\rho^2 + r^2} \sin \theta. \tag{19}
\]

We may evaluate the magnetic energy, except for the x-y plane, taking the dipole approximation Eq.(16) for \( \rho > 3 \: r \), the small \( \rho \) approximation Eq.(17) for \( \rho < \frac{r}{3} \), and the intermediate expression Eq.(19) for \( \frac{r}{3} < \rho < 3 \: r \) since it matches well with the other ones at these values. In this way one obtains a contribution to \( k \) in Eq.(12) of \( 3.5 \pi^3 \), but it is still necessary to add the contribution near the plane x-y.

For this last part, the contribution will come essentially from the region close to the loop. For \( \theta = \frac{\pi}{2} \), \( B_\theta \) of Eq.(18) will have a logarithmic divergence for \( \rho = r \) coming from \( \varphi' \sim 0 \), which is regularized considering the width of the loop and that inside it \( B = 0 \) to be a superconducting medium.
With the approximation of keeping a region $\alpha$ of integration of $\varphi'$ near to zero and comparing the finite contribution to $B_\theta$ with that coming from the exact evaluation of Eq. (18) for $\rho = r$, its turns out that $\alpha \simeq 0.77$. Taking now for $\rho = r + \eta$ the approximation of keeping up to terms $\varphi'^2$ in Eq.(18), for $\frac{\eta}{r} << 1$, one obtains

$$B_\theta \simeq \frac{1}{(\alpha^2 r^2 + \eta^2)^{\frac{3}{2}}} \frac{2\alpha I r}{\eta} + \left[ \ln \frac{\eta}{r} - \ln \left( \alpha + \sqrt{\alpha^2 + \frac{\eta^2}{r^2}} \right) \right] \frac{I}{r} . \quad (20)$$

A reasonable estimation of the magnetic energy near the string, considering that it must correspond to the stages of contraction starting from $R \sim \frac{N}{m_X}$ and being $\eta \sim \frac{1}{m_X}$, is to take the above approximation for $B_\theta$ in an external region of size $\eta$ around it. It turns out that this contribution to the coefficient $k$ will be $\sim 2\pi^3$.

Therefore the total contribution of magnetic energy when the decaying vorton may still be considered as a thick loop corresponds to

$$k_i \simeq 5.5\pi^3 . \quad (21)$$

With this value, the expression Eq.(14) for the velocity of contraction using the magnetic energy is of the same order of that given by Eq.(9) for the initial stage with constant string tension.

But it is not always correct to consider the contracting vorton as a loop. At the final stage when the radius is of the order of the width it may be better to represent it as a sphere with currents running inside it around the $z$-axis. Approximating each disc at angle $\theta'$ by an effective loop of radius $\eta \sin \theta'$, the magnetic potential will be

$$A_\varphi (\rho, \theta) = I \eta \int_0^\pi d\theta' \sin \theta' \int_0^{2\pi} d\varphi' \frac{\cos \varphi'}{[\rho^2 + \eta^2 - 2\rho \eta (\sin \theta \sin \theta' \cos \varphi' + \cos \theta \cos \theta')]^{\frac{3}{2}} . \quad (22)$$

For large distances $\rho >> \eta$, Eq.(22) gives again the dipole limit

$$A_\varphi (\rho, \theta) = I \frac{\pi \eta^2}{\rho^2} \frac{\pi}{2} \sin \theta . \quad (23)$$

On the other hand, near the sphere $\rho = \eta + \delta$ the contribution to the magnetic field will come mainly from a region $\alpha$ in the integration over $\varphi'$ for small values such that only terms $\varphi'^2$ are kept and a region $\beta$ in the integration over $\theta'$ such that $\theta' \simeq \theta$. The result is

$$B_\rho (\delta, \theta) = 2\beta \cos \theta \ I \eta \left[ \frac{1}{2c(\eta + \delta)} \left( 1 + \frac{\delta^2}{2c^2} \right) \ln \left( \frac{\alpha c}{\delta} + \sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}} \right) \right.$$

$$- \frac{1}{\eta + \delta} \frac{\alpha \delta}{4c^2} \sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}} + \frac{1}{2} \frac{\alpha \eta}{\delta c^2} \frac{\sin^2 \theta}{\sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}}} \right] ;$$

$$B_\theta (\delta, \theta) = 2\beta \sin \theta \ I \eta \left\{ - \left[ \frac{1}{2c(\eta + \delta)} \left( 1 + \frac{\delta^2}{2c^2} \right) + \frac{\delta}{2c^3} \right] \ln \left( \frac{\alpha c}{\delta} + \sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}} \right) \right.$$

$$- \frac{\alpha \eta}{\delta c^2} \frac{\sin^2 \theta}{\sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}}} \right\} . \quad (24)$$
\[ + \frac{1}{\eta + \delta} \frac{\alpha \delta}{4c^2} \sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}} - \frac{1}{2} \frac{c}{\delta^2} \frac{\sin^2 \theta}{\sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}}} + \frac{\alpha}{\sqrt{1 + \frac{\alpha^2 c^2}{\delta^2}}} \left( \frac{1}{\delta^2} + \frac{1}{2c^2} \right) \] ,

where \( c = \sqrt{\eta^2 + \eta \delta \sin \theta} \).

The limit of Eq.(24) for small values of \( \sin \theta \) is

\[ B_\rho \to 2\beta \alpha \cos \frac{\theta I}{\rho} \cos \frac{\eta}{\rho} \frac{I}{\rho} , \quad B_\theta \to 2\beta \alpha \sin \frac{\theta I}{\rho} \frac{I}{\rho} , \quad (25) \]

whereas for \( \theta \sim \frac{\pi}{2} \) and \( \alpha \sim 0.77 \), \( B_\rho \sim 0 \) and a numerical evaluation of Eq.(24) gives

\[ B_\theta \left( \delta, \theta = \frac{\pi}{2} \right) \simeq 0.35 \frac{\beta I}{\delta} . \quad (26) \]

We now take two regions for evaluating the magnetic energy: that for large \( \rho \) where the field corresponds to the dipole approximation and that close to the sphere, since inside it the field is zero for a superconducting medium. The two regions match reasonably well for \( \delta = \eta \) and the lower limit for the integration is \( \rho = \frac{3}{2} \eta \) because \( \eta \) was the average radius of the disc in the x-y plane.

Therefore the coefficient for the magnetic energy when the vorton is approximated by a sphere turns out to be, with \( \beta \sim \alpha \),

\[ k_f \simeq 35 . \quad (27) \]

7 Probability of emission of carriers

We will calculate the matter-antimatter asymmetry per vorton through the emission of fermions and antifermions by quantum tunneling. This corresponds to the transition e.g. from a state of vorton with number \( N \) to another with number \( N - 1 \) plus a fermion of mass \( m_x \) with conservation of angular momentum.

It must be stressed that this channel is not the dominant one for the contraction of the string since the corresponding partial lifetime is much longer than the actual time of collapse. But it turns out to be the most effective one for baryogenesis since, due to the chemical potential produced by the non-equilibrium process of contraction, the probability for emission of baryons will be substantially different from that of antibaryons. In comparison, other channels which eliminate pieces of string due to the destabilization produced at the electroweak transition will give through the decay of heavy bosons a rather small amount of matter-antimatter asymmetry as discussed in Section 4.

We evaluate the tunneling process semiclassically. The height of the barrier will be of the order of \( m_x \) since it corresponds to the increase of energy when the massless carrier inside the string is put outside it with the same momentum. Additionally, the width of the barrier is the displacement of the carrier such that, always conserving angular momentum and taking into account the one-step contraction of the string, the energy of the configuration is equal to the initial one. This displacement turns out to be of the order of the radius \( r \) of the emitting string \[15\].

Therefore, the emission probability in the string rest frame will be
\[ \Gamma_0 \simeq m_x^2 r e^{-m_x r}. \]  

(28)

Considering that the probability in laboratory frame requires the relativistic factor for the dilatation of time

\[ \Gamma = \Gamma_0 \sqrt{1 - v^2}, \]  

(29)

and that the difference between emission of particle and antiparticle is given by its multiplication times

\[ -\frac{\mu}{T} = v \Delta, \]  

(30)

where \( \mu \) is the chemical potential and \( \Delta \) will depend on a specific contribution, the asymmetry due to a vorton during all the time of its collapse will be

\[ \varepsilon_v = \Delta \int_0^R \eta \, dr \sqrt{1 - v^2} \, e^{-m_x r}. \]  

(31)

Defining \( y = m_x r \), and being \( R \simeq \frac{N}{m_x} \) and \( \eta \simeq \frac{1}{m_x} \), from Eq.(13) one has

\[ \varepsilon_v = \Delta \int_1^N dy \frac{y^2 \, e^{-y}}{k_i 4\pi^2 N + (1 - k_i)^2 y}. \]  

(32)

Due to the fact that \( k_i \) corresponds always to the loop approximation of vorton but \( k_f \) may correspond either to loop or to sphere approximations, the integral of Eq.(32) must be splitted into two parts

\[ \varepsilon_v = \frac{I_1 + I_2}{\Delta}, \]  

(33)

where we have called \( \bar{k} \simeq 5 \) and \( \bar{k}_f \simeq 1 \) according to Eqs. (21) and (27).

These integrals can be done exactly but it is instructive also to calculate them expanding the denominators in powers of \( \frac{(k-1) y}{kN} < 1 \) and \( \frac{(k_f-1) y}{kN} < 1 \) giving

\[ I_1 = \frac{1}{kN} \sum_{n=0}^{\infty} \left( \frac{\bar{k} - 1}{\bar{k}N} \right)^n \sqrt{n+2} \left[ y^{n+2} + (n + 2) \, y^{n+1} + \ldots (n + 2)! \right] e^{-y} \bigg|_{N_s} \]  

(34)

and for \( I_2 \) a similar expression where the coefficient in the numerator is \( \bar{k}_f - 1 \) and the limits 1 and \( N_s \).

For order \( n = 0 \) there is no influence of the difference between \( \bar{k} \) and \( \bar{k}_f \) and of the value of \( N_s \).

\[ \frac{\varepsilon_v^{(0)}}{\Delta} = \frac{1}{kN} \left[ \frac{5}{e} - \left( N^2 + 2N + 2 \right) \, e^{-N} \right] \simeq 0.03. \]  

(35)

The contribution of \( n = 1 \) adds, taking \( N_s = 2 \),
\[ \varepsilon^{(1)}_\Delta = \frac{1}{(k_f - 1)^2} \left[ (k_f - 1) \frac{16}{e} + (k - k_f) \left( N_s^3 + 3N_s^2 + 6N_s + 6 \right) e^{-N_s} \right. \]
\[ \left. - \left( k - 1 \right) \left( N_s^3 + 3N_s^2 + 6N_s + 6 \right) e^{-N} \right] \]
\[ \simeq 0.005 . \]

Therefore we may expect \( \varepsilon/\Delta \) close to 0.1.

In fact the exact evaluation of the asymmetry per vorton gives

\[ I_1 = \left[ \frac{(\gamma y + \bar{k}N)^2}{2\gamma^3} - \frac{2\bar{k}N}{\gamma^3} (\gamma y + \bar{k}N) + \frac{2\bar{k}N^2}{\gamma^3} \ln (\gamma y + \bar{k}N) \right] e^{-y} \]
\[ - \frac{e^{-y}}{2\gamma^3} \left( (\gamma (y + 1) + \bar{k}N)^2 - 4(\bar{k}N)^2 + \gamma^2 \right) - e^{-y} \frac{(\bar{k}N)^2}{\gamma^3} \ln (\gamma y + \bar{k}N) \]
\[ + \frac{(\bar{k}N)^2}{\gamma^3} e^{\bar{k}N} \ln (\gamma y + \bar{k}N) + \frac{1}{\gamma^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \left( y + \bar{k}N \right)^n N_s \]

where \( \gamma = 1 - \bar{k} \), and a similar expression for \( I_2 \) with \( \gamma = 1 - \bar{k}_f \) and the limits 1 and \( N_s \).

The numerical computation with the above values of \( \bar{k} \) and \( \bar{k}_f \) and \( N_s = 2 \) gives \( I_1 = 0.0487 \) and \( I_2 = 0.0078 \) so that

\[ \frac{\varepsilon}{\Delta} = 0.0565 . \]

All what we still need to calculate is the chemical potential to have the numerical value of \( \Delta \).

It must be added that we assume that inside the string one has the high-temperature phase in thermal equilibrium so that there matter-antimatter symmetry is kept.

## 8 Chemical potential

In the outer part of the string, the non-equilibrium process of contraction will produce a chemical potential which should be otherwise zero for a non-conserved charge as the baryonic one.

In our case the chemical potential may have two sources. One of them is traditional as it appears in the expanding bubbles of electroweak baryogenesis. In the Hamiltonian the baryonic density appears multiplied by \( -\frac{d\theta}{dt} \) where \( \theta \) is a CP violating phase which is nonzero outside the string. During the contraction of the latter, points which are crossed by its external wall pass \( \theta \) from 0 to a finite value \( \Delta\theta > 0 \) so that

\[ \mu = -\frac{d\theta}{dt} = \frac{\Delta\theta}{\eta} v < 0 , \]
and therefore emission of matter is favoured on antimatter. Since the emission probability must be multiplied by \(-\frac{\mu}{T}\) and \(\eta \sim \frac{1}{T}\) in the high-temperature phase, in our expression of \(\varepsilon_v\), Eq.(31) \(\Delta = \Delta \theta\) which, as said in Section 2 should be \(\sim 0.01\) to be in agreement with the bound of the electric dipole moment of neutron. This contribution might be too small to give the expected baryogenesis with our mechanism.

But our collapsing superconducting loop has another source of chemical potential due to the magnetic field that it generates. Outside the external wall of the string, which is where the emission occurs, these will be a potential multiplying the fermionic density with charge \(q\) in the Hamiltonian

\[
\mu = q \int_0^\rho \frac{\partial}{\partial t} A_\varphi r \, d\varphi',
\]

corresponding to the electric field generated by time variation of \(A_\varphi\) due to the contraction of the loop.

It is interesting to note that this contribution to chemical potential can be also thought as the difference of a phase if one thinks that in the wall of the string the magnetic potential \(A_\varphi\) will produce a change of phase of the fermionic field which can be compensated by the transformation

\[
\Psi(\varphi) \longrightarrow e^{iq \int_0^\rho A_\varphi dt} \Psi(\varphi).
\]

But in so doing the kinetic term of the Dirac energy will acquire a contribution of the type of \(\mu\) Eq.(40) times the fermionic density due to the time variation of \(A_\varphi\).

To evaluate the contribution of Eq.(40) one must calculate \(A_\varphi\) of Eq.(15) for \(\rho = r + \eta\) and \(\theta = \frac{\pi}{2}\) and derive it at fixed \(\rho\) with respect to time due to the variation of \(r\). The most important contribution to \(A_\varphi\) is

\[
A_\varphi \left( \rho = r + \eta, \theta = \frac{\pi}{2} \right) \simeq -2 I \ln \left( \frac{\rho - r}{r} \right),
\]

so that

\[
\left. \frac{\partial A_\varphi}{\partial t} \right|_\rho \simeq -2 I \eta v, \quad v = -\frac{dr}{dt}.
\]

Because of the definition of Eq.(40), it will correspond to take an average of the potential between 0 and \(2\pi\), i.e.

\[
\langle \mu \rangle = -q 2\pi I r \eta v.
\]

Considering again the factor \(-\frac{\mu}{T}\) which multiplies the emission probability, we have

\[
\Delta = q 2\pi I r.
\]

This coefficient will vary during the contraction. At the beginning \(r \sim R \simeq \frac{N}{m_\chi}\) and being the electric charge of the carrier \(q \sim 0.1\) and \(I = \frac{m_\chi}{2\pi}\) it turns out that \(\Delta \simeq 0.1\), which is larger than the previous contribution to \(\mu\).
Therefore we obtain that $\varepsilon_v \sim 0.01$ and our asymmetry due to gradual collapse of vortons will be

$$\frac{n_B}{s} \sim \left(\frac{m_x}{m_{pl}}\right)^3 \varepsilon_v \sim 10^{-10}.$$ (46)

Considering that $\Delta$ may decrease towards the end of the collapse in one order of magnitude and accepting a moderate dilution effect due to gradual transformation of vortons into radiation as discussed in Section 5, $\frac{n_B}{s}$ might not go below $10^{-11}$ which is the lower limit of the acceptable baryogenesis.

9 Conclusions

We have found that the emission of fermions from vortons destabilized at the electroweak transition during their collapse may supply the matter-antimatter asymmetry required by nucleosynthesis. This avoids on the one hand the necessity of very high temperature of reheating after inflation to produce superheavy bosons of GUT, and on the other the requirement of first order for the electroweak transition needed for the production of bubbles.

It is clear that our evaluation gives only a possible order of magnitude for the baryogenesis with this mechanism. A more precise result would require a calculation of the emission probability beyond the semiclassical approach, as well as a detailed analysis of the disappearance of zero modes in vortons to estimate the time interval for their destabilization.

It is interesting to note that if a particular Grand Unification model loses not all its zero-modes at the electroweak temperature and a part of present dark matter is due to vortons, their emission might explain the observed high-energy cosmic rays, so that this phenomenon would be linked to that of baryogenesis.

Acknowledgments

This research was partially supported by CONICET PICT 0358. L.M. wishes to thank the hospitality of the Physics Dept. of Università di Napoli and Università della Calabria at Cosenza, where parts of this work were performed.

References

[1] E.W. Kolb and M.S. Turner, The Early Universe, Addison Wesley (New York, 1990); C.J. Copi, D.N. Schramm and M.S. Turner, Science 267, 192 (1995).

[2] A.D. Sakharov, JETP Lett. 5, 24 (1967).

[3] A.D. Dolgov, Phys. Repts. 222, 309 (1992); A.G. Cohen, D.B. Kaplan and A.E. Nelson, Annu. Rev. Nucl. Part. Sci. 43, 27 (1993); M. Trodden, hep-ph/9803479.

[4] M. Carena, M. Quiros, A. Riotto, I. Vilja and C. Wagner, Nucl. Phys. B 503, 404 (1997); J.M. Cline and K. Kainulainen, Nucl. Phys. B 482, 73 (1996); M. Losada, Phys. Rev. D 56, 2893 (1997).
[5] A. Riotto, hep-ph/9803357.

[6] E.W. Kolb, A. Riotto and I.I. Tkachev, hep-ph/9801306.

[7] T.W.B. Kibble, Acta Phys. Pol. B 13, 723 (1982).

[8] E. Witten, Nucl. Phys. B 249, 557 (1985).

[9] R.L. Davis, Phys. Rev. D 38, 3722 (1988).

[10] P. Battacharjee, hep-ph/9803223, Phys. Rev. Lett. (to be published).

[11] R.L. Davis and E.P.S. Shellard, Nucl. Phys. B 323, 209 (1989); A. Vilenkin and E.P.S. Shellard, Cosmic Strings and other Topological Defects, Cambridge Univ. Press (Cambridge, 1994).

[12] R. Branderberger, B. Carter, A.C. Davis and M. Trodden, Phys. Rev. D 54, 6059 (1996).

[13] R. Jeannerot, Phys. Rev. Lett. 77, 3292 (1996).

[14] R. Brandenberger and A. Riotto, hep-ph/9801448.

[15] L. Masperi and G. Silva, Astropart. Phys. 8, 173 (1998).

[16] S.C. Davis, A.C. Davis and W.B. Perkins, hep-ph/9705464.

[17] W.B. Perkins and A.C. Davis, Phys. Lett. B 393, 46 (1997).