Biologically Inspired Oscillating Activation Functions Can Bridge the Performance Gap between Biological and Artificial Neurons

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Abstract

Nonlinear activation functions endow neural networks with the ability to learn complex high-dimensional functions. The choice of activation function is a crucial hyperparameter that determines the performance of deep neural networks. It significantly affects the gradient flow, speed of training and ultimately the representation power of the neural network. Saturating activation functions like sigmoids suffer from the vanishing gradient problem and cannot be used in deep neural networks. Universal approximation theorems guarantee that multilayer networks of sigmoids and ReLU can learn arbitrarily complex continuous functions to any accuracy. Despite the ability of multilayer neural networks to learn arbitrarily complex activation functions, each neuron in a conventional neural network (networks using sigmoids and ReLU like activations) has a single hyperplane as its decision boundary and hence makes a linear classification. Thus single neurons with sigmoidal, ReLU, Swish, and Mish activation functions cannot learn the XOR function. Recent research has discovered biological neurons in layers two and three of the human cortex having oscillating activation functions and capable of individually learning the XOR function. The presence of oscillating activation functions in biological neural neurons might partially explain the performance gap between biological and artificial neural networks. This paper proposes 4 new oscillating activation functions which enable individual neurons to learn the XOR function without manual feature engineering. The paper explores the possibility of using oscillating activation functions to solve classification problems with fewer neurons and reduce training time.

1 Introduction

A wide variety of functions have been used as activation functions in neural networks since their inception [6]. At the start of ANN research, the identity function was used as the activation function resulting in a neuron capable of solving linear regression and classification problems [19]. The output of a single linear neuron or its activation is given by  $$a = g(z) = z = w^T x + b.$$ Thus a
linear neuron uses an affine function to predict its output from its inputs. Linear neurons can be trained with Stochastic Gradient Descent (SGD) [2]. Due to its popularity and rich history, the SGD update rule for a linear neuron is referred to by many well-known names such as the ADALINE rule, Windrow-Hoff rule, or Delta rule. The linear (identity) activation is biologically inspired by the increase of a biological neuron output with its weighted input currents. The composition of any finite number of linear functions is a linear function. Hence, a multilayer network of linear neurons has the same representative power as a single layer of linear neurons. The problem of fitting an affine function to data was well-studied before the advent of neural networks, and networks composed of linear neurons are equivalent to linear regression models.

The outputs of biological neurons are observed to saturate for inputs beyond a certain threshold; hence sigmoidal neurons were introduced to capture this behavior [1]. Logistic-sigmoidal and tan-sigmoidal neurons capture the saturating nature of biological neurons [19]. Tan-sigmoidal neurons outperform Logistic-sigmoidal neurons since logistic-sigmoid always produces positive outputs. The positive outputs can get combined to large positive values saturating the next layer of neurons. In general, activation functions that output symmetrically positive and negative values reduce saturation and hence alleviate the vanishing gradient problem in deep neural networks [1].

The simplest model of a biological neuron is the Perceptron model [15]. In this model, neurons fire or fail to fire depending on weighted inputs exceeding a threshold. This behavior can be captured by using an activation function that switches between two values (0 and 1 or -1 and 1). A single Perceptron can be trained efficiently using the Perceptron Learning Algorithm (PLA), which is guaranteed to converge for all linearly separable problems after a finite number of updates using the PLA update rule [15]. The Signum function is used as the activation in the Perceptron model. Since the Signum function is discontinuous (hence also non-differentiable) at the origin, multilayer networks of Perceptron units cannot be trained using the Back Propagation (BP) algorithm.

The output of a Perceptron (±1) has the advantage of being interpreted as a Yes/No binary decision. The outputs of neurons with Binary activation functions like the Heaviside step function that switches between 0 and 1 can also be interpreted as a binary decision. In order to train multilayer networks with BP while still retaining the ability to interpret outputs as binary decisions, we are forced to consider smooth (differentiable) approximations to the Signum and Heaviside step functions, namely tan-sigmoid and logistic sigmoid, respectively. Although sigmoidal activations have the advantage of producing an interpretable output, these activations have very small derivatives outside a narrow range leading to the slowing down of the training process. For example the tan-sigmoid defined as \( \tanh(z) = \frac{\exp(z) - \exp(-z)}{\exp(z) + \exp(-z)} \) does not respond to inputs outside the narrow range \([-5, 5]\) since \( \exp(-5) < 0.01 \). The slowing down of the BP training process in deep networks due to saturating activation functions and the side-effect of backpropagating derivatives is called the ‘vanishing gradient problem.’ It is a serious problem in deep neural networks. The vanishing gradient problem can be alleviated to a large extent by the use of activations that have larger derivatives for a larger range of inputs like ReLU [9], leaky ReLU [5], PReLU [11], GELU [12], Softplus [22], ELU [4], SiLU [1], SELU [20], Swish [21] and Mish [16] activation functions [1]. These newer activation functions perform better in deep networks, are unbounded, and have derivative values close to one or more for all positive values [17]. However, the overwhelming majority of activation functions in neural networks are monotonic or nearly monotonic functions with a single zero at the origin. In the following, we explore the biological inspiration and mathematical reasons for using oscillatory and highly non-monotonic activation functions with multiple zeros.

1.1 Biological motivation for oscillating activation functions

Albert Gidon et al. [10] discovered a new type of neuron in the human cortex that is capable of individually learning the XOR function (a task which is impossible with single neurons using sigmoidal, ReLU, leaky ReLU, PReLU, GELU, Softplus, ELU, SELU, Swish, and Mish activations). A detailed mathematical analysis of this issue is provided in [3][17][17]. This is because the decision boundary for a neuron that outputs an activation \( a = g(z) = g(w^Tz + b) \) is the set of points \( z \) for which \( g(z) = 0 \). If \( z = 0 \) is the only zero of the activation function \( g(z) \), then the decision boundary is a single hyperplane \( z = w^Tz + b = 0 \). To separate the classes in the XOR dataset, two hyperplanes are needed, so activation functions with multiple zeros as in [14] have to be considered. Since we know that for small values of the inputs, the output of biological neurons increase, it is clear that the output must decrease eventually to another zero if a biological neuron is capa-
able of learning the XOR function. Thus although a 3 neuron network with 2 hidden and 1 output layer is required to learn the XOR function with most activation functions, the XOR function can be learned with a single neuron with oscillatory activation like Growing Cosine Unit (GCU). It has been demonstrated that oscillatory activation functions outperform popular activation functions on many tasks [18]. In this paper 4 new oscillatory activation functions that enable individual artificial neurons to learn the XOR function like biological neurons are proposed. The oscillatory activation functions also outperform popular activation functions on many benchmark tasks.

1.2 Desirable properties of activation functions

Although a wide variety of hitherto unexplored non-monotonic and oscillatory activation functions can provide good performance in deep networks, not all functions can serve as the useful activation function for reasons discussed below.

Firstly, functions that are useful as activation functions must closely approximate the linear (or identity) function for small values. This property is imposed so that artificial neurons mimic biological neurons. However, there are deep practical reasons for this requirement. The weights of a neuron are initialized to small values at the start of training, so the weighted input \( z = w^T x + b \) is close to 0 for all neurons at the start of training. Since the identity function has a derivative of 1 at 0, the neuron has a chance to learn quickly. Conversely, if the derivative of activation is small at the origin, it will cause slowing down or stalling of the parameter updates after initialization with small values. Thus functions like \( \cos(z) \), \( z^2 \), \( z^3 \) do not perform well as activation functions since these functions have zero derivatives at the origin. On the other hand, these poorly performing activation functions can be changed to create good activation functions like \( \sin(z) \), \( z^2 + z \), \( z - z^3 \) that have a derivative of 1 at the origin.

A consequence of the linearity for small values of input is that the entire network behaves like a linear classifier after initialization with small parameter values. This linearity at initialization has a regularizing effect. Another consequence of linearity is that \( g(0) = 0 \), since a linear function has a value of zero for zero input.

An activation is defined to have the "XOR Property" if it can be used in a single neuron to learn the XOR function. A single neuron solution to the XOR problem using the GCU activation was first presented in [18]. Since certain biological neurons responsible for higher order thinking in the human cerebral cortex have the XOR Property, this is also a desirable property for artificial neurons.

The XOR problem is the task of learning the following dataset. A bipolar encoding of inputs and outputs is used instead of a binary encoding for convenience.

\[
D = \left\{ \left( \begin{bmatrix} -1 \\ -1 \end{bmatrix}, -1 \right), \left( \begin{bmatrix} 1 \\ -1 \end{bmatrix}, 1 \right), \left( \begin{bmatrix} -1 \\ 1 \end{bmatrix}, 1 \right), \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix}, -1 \right) \right\}
\]

(1)

Figure 1: A single neuron solution to the XOR problem using a Shifted Quadratic Unit (SQU).
Figure 2: A single neuron solution to the XOR problem using a Non-Monotonic Cubic Unit (NCU).

Figure 3: A single neuron solution to the XOR problem using a Shifted Sinc Unit (SSU).

Table 1: Definition and properties of activation functions

| Activation Function | Equation | Continuity | Differentiability | Monotonicity | Range |
|---------------------|----------|------------|-------------------|--------------|-------|
| Sigmoid             | $f_1(z) = \begin{cases} \frac{1}{1 + e^{-z}} & z < 0; \\ \frac{1}{1 + e^{-z}} & z \\ 0 & z > 0. \end{cases}$ | No         | No                | No           | $[-1, 1]$ |
| Identity            | $f_2(z) = z$ | Yes        | Yes               | Yes          | $(-\infty, \infty)$ |
| Bipolar Sigmoid     | $f_3(z) = \begin{cases} 1 & z < 0; \\ 1 \frac{z}{1 + z} & z > 0. \end{cases}$ | Yes        | Yes               | No           | $[-1, 1]$ |
| Sigmoid / Softmax   | $f_4(z) = \begin{cases} \frac{1}{1 + e^{-z}} & z < 0; \\ \frac{1}{1 + e^{-z}} & z \\ \alpha & z > 0. \end{cases}$ | Yes        | Yes               | Yes          | $[-1, 1]$ |
| Tanh                | $f_5(z) = \tanh (z)$ | Yes        | Yes               | Yes          | $[0, 1]$ |
| Absolute value      | $f_6(z) = |z|$ | Yes        | Yes               | Yes          | $[-1, 1]$ |
| Soft-Root-Sign      | $f_7(z) = \begin{cases} 0 & z < 0; \\ \sqrt{-z} & z \\ 0 & z > 0. \end{cases}$ | Yes        | No                | No           | $(-\frac{\pi}{2}, \frac{\pi}{2})$ |
| Hard-Tanh           | $f_8(z) = \max (-1, \min (1, z))$ | Yes        | Yes               | Yes          | $[-1, 1]$ |
| SiLU                | $f_9(z) = \frac{1}{1 + e^{-z}}$ | Yes        | Yes               | Yes          | $(-0.5, \infty)$ |
| LiSHT               | $f_{10}(z) = \sqrt{\tanh (z)}$ | Yes        | Yes               | No           | $(-\infty, \infty)$ |
| Softplus            | $f_{11}(z) = \ln (1 + e^z)$ | Yes        | Yes               | Yes          | $[0, \infty)$ |
| ReLU                | $f_{12}(z) = \max (0, z)$ | Yes        | Yes               | No           | $[0, \infty)$ |
| Leaky ReLU          | $f_{13}(z) = \begin{cases} 0.01z & z < 0; \\ z & z \geq 0. \end{cases}$ | Yes        | Yes               | No           | $(-\infty, \infty)$ |
| GELU                | $f_{14}(z) \approx 0.5z(1 + \tanh (\frac{z}{\sqrt{2}z + 0.044715z^2})))$ | Yes        | Yes               | No           | $(-0.5, \infty)$ |
| SELU                | $f_{15}(z) = \begin{cases} \frac{\lambda z}{1 + e^{-z}} & z \geq 0; \\ \frac{\lambda z}{1 + e^{-z}} & z < 0. \end{cases}$ | Yes        | Yes               | Yes          | $[-\lambda z, \infty)$ |
| Switch              | $f_{16}(z) = \begin{cases} z & z \geq 0; \\ 0 & z \leq 0. \end{cases}$ | Yes        | Yes               | Yes          | $(-\infty, \infty)$ |
| Mish                | $f_{17}(z) = z \tanh (\ln (1 + c^2))$ | Yes        | Yes               | Yes          | $(-0.5, \infty)$ |
| BLU                 | $f_{18}(z) = \frac{\alpha e^z - 1}{1 + e^z}$ | Yes        | Yes               | Yes          | $(-\infty, \infty)$ |
| PReLU               | $f_{19}(z) = \begin{cases} z & z \geq 0; \\ 0 & z \leq 0. \end{cases}$ | Yes        | No                | Yes          | $(-\infty, 0)$ |
| Sine                | $f_{20}(z) = \sin (z)$ | Yes        | Yes               | Yes          | $[-1, 1]$ |
| Shifted Quadratic Unit (SQU) | $f_{21}(z) = z^2 + z$ | Yes        | Yes               | No           | $[-0.25, \infty)$ |
| Monotonic Cubic     | $f_{22}(z) = z^2 + z$ | Yes        | Yes               | Yes          | $(-\infty, \infty)$ |
| Non-Monotonic Cubic Unit (NCL) | $f_{23}(z) = z^2 + z$ | Yes        | Yes               | No           | $(-\infty, \infty)$ |
| Shifted Sinc Unit (SSU) | $f_{24}(z) = \pi \frac{\sin (z - \pi)}{z}$ | Yes        | Yes               | Yes          | $[-0.68, \pi]$ |
| Growing Cosine Unit (GCU) | $f_{25}(z) = z \cos (z)$ | Yes        | Yes               | No           | $(-\infty, \infty)$ |
| Decaying Sinc Unit (DSU) | $f_{26}(z) = \pi \frac{(\sin (z - \pi) - \sin (z + \pi))}{z}$ | Yes        | Yes               | No           | $[-1.04, 1.04]$ |
In the above, the following definition of the $sinc$ function is used:

$$sinc \left( z \right) = \begin{cases} 1 & z = 0; \\ \frac{\sin (z)}{z} & \text{elsewhere}. \end{cases}$$ \hspace{1cm} (2)

2 Mathematical Properties of Activation Functions

Table 1 and Table 2 provide a comprehensive summary of different activation functions and their properties. A function $f$ is said to be sign-equivalent to a function $g$ iff $\text{sign}(f(z)) = \text{sign}(g(z))$ for all values of $z$. Activation functions that are sign equivalent to the identity function do not have the XOR property as proven in [18].

![Table 2: Properties of Activation Functions](image)

3 Performance Comparison of Activation Functions

3.1 Experimental Setup

We leverage a convolutional neural network architecture shown in Figure 5 evaluated on CIFAR-10 [13] image classification dataset. The activation functions were plugged into the convolutional and dense layers with all other network hyperparameters tuned. While there have been recent advances in leveraging AutoML [8] for network selection, we have experimented with a compact architecture for our task.

Adam optimizer with a batch size of 64 was used, the learning rate was selected as part of a hyperparameter search for each experiment. The experiments were run for a total of 25 epochs with sparse categorical cross-entropy loss function.

The hypothesis we aim to evaluate:

- Whether these new class of activation functions are able to perform well with minimal representation layers. Hence we vary the number of convolutional layers from 1 to 4.
- Evaluate the performance of these activation functions in dense layers with minimal number of parameters. For this, we fix 64 units in the penultimate layer with 50 percent dropout.
• Apart from testing the effect of network size reduction, compare the performance convergence of various activation functions implemented. For this we look at the top-1 accuracy at 20 epochs as well.
3.2 Results

Tables 3 and 4 show the performance of various activation functions. Figure 4 shows the accuracy achieved with different number of convolution layers. It is observed that the proposed oscillatory activations outperform on a majority of benchmarks in terms of faster convergence and training time per epoch.

Grid search suggested that a learning rate of $10^{-4}$ is ideal for comparison in this network. We notice that the oscillatory activation functions like NCU, SQU, DSU and SSU are converging at 20 epochs while maintaining the best possible accuracy at 4 convolution layers. At 3 convolution layers, they maintain the accuracy above 68 percent. Moreover, Quadratic (SQU) also maintains the above accuracy at lower number of epochs. The results presented in 3 and 4 and Fig. 4 indicate that networks with oscillating activations can require fewer neurons, perform better and train faster than networks with popular activation functions.

### Table 3: Testing accuracy with different activation functions

| Activation Function | Convolution Layers = 1 | Convolution Layers = 2 |
|---------------------|------------------------|------------------------|
|                     | Accuracy: 25 Epochs    | Accuracy: 20 Epochs    |
| Signum              | 0.235                  | 0.333                  |
| Identity            | 0.596                  | 0.588                  |
| Bipolar Sigmoid     | 0.547                  | 0.534                  |
| Sigmoid             | 0.419                  | 0.401                  |
| Tanh                | 0.390                  | 0.375                  |
| Absolute            | 0.615                  | 0.608                  |
| Soft-Rect-Sign      | 0.866                  | 0.579                  |
| SiLU                | 0.583                  | 0.562                  |
| LiSHT               | 0.567                  | 0.562                  |
| Softplus            | 0.557                  | 0.482                  |
| Leaky ReLU          | 0.598                  | 0.562                  |
| GELU                | 0.553                  | 0.570                  |
| SELU                | 0.582                  | 0.536                  |
| Swish               | 0.580                  | 0.573                  |
| Mish                | 0.589                  | 0.578                  |
| Sine                | 0.594                  | 0.582                  |
| SQU                 | 0.623                  | 0.613                  |
| Monotonic Cubic     | 0.603                  | 0.381                  |
| Non-Monotonic Cubic | 0.605                  | 0.399                  |
| $z^2 \cos(z)$       | 0.580                  | 0.573                  |
| eLU                 | 0.599                  | 0.389                  |
| PReLU               | 0.620                  | 0.352                  |
| Growing Cosine Unit | 0.599                  | 0.391                  |
| Decaying Sine Unit  | 0.610                  | 0.596                  |
| Shifted Sine        | 0.606                  | 0.603                  |

### Table 4: Results for different activation functions

| Activation Function | Convolution Layers = 3 | Convolution Layers = 4 |
|---------------------|------------------------|------------------------|
|                     | Accuracy: 25 Epochs    | Accuracy: 20 Epochs    |
| Signum              | 0.245                  | 0.343                  |
| Identity            | 0.659                  | 0.643                  |
| Bipolar Sigmoid     | 0.594                  | 0.558                  |
| Sigmoid             | 0.337                  | 0.341                  |
| Tanh                | 0.670                  | 0.644                  |
| Absolute            | 0.659                  | 0.640                  |
| Soft-Rect-Sign      | 0.653                  | 0.659                  |
| SiLU                | 0.655                  | 0.622                  |
| LiSHT               | 0.602                  | 0.592                  |
| Softplus            | 0.517                  | 0.497                  |
| Leaky ReLU          | 0.636                  | 0.625                  |
| GELU                | 0.658                  | 0.635                  |
| SELU                | 0.639                  | 0.632                  |
| Swish               | 0.637                  | 0.615                  |
| Mish                | 0.658                  | 0.625                  |
| Sine                | 0.660                  | 0.648                  |
| SQU                 | 0.693                  | 0.685                  |
| Monotonic Cubic     | 0.654                  | 0.625                  |
| Non-Monotonic Cubic | 0.655                  | 0.631                  |
| $z^2 \cos(z)$       | 0.601                  | 0.576                  |
| eLU                 | 0.671                  | 0.661                  |
| PReLU               | 0.614                  | 0.641                  |
| Growing Cosine Unit | 0.678                  | 0.665                  |
| Decaying Sine Unit  | 0.680                  | 0.669                  |
| Sine Unit           | 0.655                  | 0.679                  |
4 Conclusion

The discovery of single neurons in the human brain with oscillating activation functions capable of individually learning the XOR function serves as biological inspiration for a new class of oscillating activation functions. Oscillating activation functions have multiple hyperplanes in their decision boundary, enabling neurons to make more complex decisions than popular sigmoidal, ReLu like Swish, and Mish activation functions. The higher representative power of networks with oscillating activation functions allows classification and regression tasks to be solved with fewer neurons. Also, the oscillations in the activation function appear to improve gradient flow and speed up backpropagation learning. This paper proposed 4 new biologically inspired oscillating activations (Shifted Quadratic Unit (SQU), Non-Monotonic Cubic (NCU), Shifted Sinc Unit (SSU), and Decaying Sine Unit (DSU)) that enable single neurons to learn the XOR problem. The new activation functions proposed in this paper outperform most of the popular activation functions on the CIFAR-10 benchmark. More importantly, the proposed oscillating activation functions learn successfully with fewer neurons. The results presented in this paper suggest that deep networks with oscillating activation functions might potentially partially bridge the performance gap between biological and artificial neural networks. Future work will evaluate the performance of the new oscillatory activation functions proposed in this paper on more benchmark problems and model architectures.

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Figure 5: CNN model architecture.