Enhanced photon production from quark-gluon plasma: Finite-lifetime effect

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Photon production from a thermalized quark-gluon plasma of finite lifetime is studied directly in real time with a nonequilibrium formulation that includes off-shell (energy nonconserving) effects. To lowest order we find that production of direct photons form a quark-gluon plasma of temperature $T \sim 200 \text{ MeV}$ and lifetime $t \sim 10 - 20 \text{ fm}/c$ is strongly enhanced by off-shell (anti)quark bremsstrahlung $q(\bar{q}) \rightarrow q(\bar{q}) \gamma$. The yield from this nonequilibrium finite-lifetime effect dominates over those obtained from higher order equilibrium rate calculations in the range of energy $E > 2 \text{ GeV}$ and falls off with a power law for $E \gg T$.

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The observation of a novel phase of matter, the quark-gluon plasma (QGP), is one of the most important goals of ultrarelativistic heavy ion experiments currently undertaken at CERN SPS and BNL RHIC [1]. The quark-gluon plasma formed in the early stage of the collision expands and cools rapidly to a mixed phase of quarks, gluons, and hadrons, and ultimately undergoes a freeze-out from a state of hadronic gas. Estimates based on energy deposited in the central collision region at RHIC energies $\sqrt{s} \sim 200 \text{ A GeV}$ suggest that the lifetime of a deconfined phase of quark-gluon plasma is of order $10 - 20 \text{ fm}/c$ with an overall freeze-out time of order $100 \text{ fm}/c$ [2]. An important aspect is an assessment of nonequilibrium effects associated with the rapid expansion and finite lifetime of the plasma and their impact on experimental observables.

Amongst various experimental signatures proposed to detect the quark-gluon plasma phase, photons (both real and virtual) have long been considered as the most promising direct signals [2]. This is because, unlike strongly interacting hadrons, photons have a mean free path much larger than the typical size of the plasma formed in ultrarelativistic heavy ion collisions. Once produced they escape from the system without further interaction, thus carrying clean information from the early hot quark-gluon plasma phase.

The first observation of direct photon production in ultrarelativistic heavy ion collisions has been recently reported by the CERN WA98 Collaboration in Pb+Pb collisions at $\sqrt{s} = 158 \text{ A GeV}$ [3]. The transverse momentum distribution of direct photons is determined on a statistical basis and compared to the background photon yield predicted from a calculation of the radiative decays of hadrons. The most interesting result is that a significant excess of direct photons beyond that expected from proton-induced reaction at the same $\sqrt{s}$ is observed in the range of transverse momentum greater than about 1.5 $\text{ GeV}/c$ in central collisions. While it is not yet clear whether a QGP was formed in the central collision region at SPS energies, this result does suggest the experimental feasibility of direct photon production as a signal of the QGP phase, expected to be formed at RHIC energies.

One goal of this article is to study directly in real time the effect of the finite QGP lifetime on direct photon production. We focus on the direct photon yield from a thermalized quark-gluon plasma of temperature $T \sim 200 \text{ MeV}$ and lifetime $t \sim 10 - 20 \text{ fm}/c$ in accordance with estimates based on collision energies reached at RHIC. Another goal is to compare this nonequilibrium photon yield to those of previous investigations [5,6] that obtain an equilibrium production rate by taking the thermal average of transition amplitudes, which in effect is tantamount to assuming an infinite lifetime for the QGP. It is not the purpose of this article to compare direct photon production from the QGP with that from the hadronic gas, as this has been studied in detail in Refs. [5,6].

Many investigations in the literature have been devoted to hard real photon production from the quark-gluon plasma [7,8]. Assuming a QGP in thermal equilibrium with an infinite lifetime, these authors focused on the equilibrium photon production rate, which for photons of momentum $p$ is given by

$$E \frac{dN}{dp_d^3d^4x} = -\frac{2}{(2\pi)^3} n_B(E) \text{Im} \Pi^R(E).$$

Here $E = |p|$, $\Pi^R(E)$ is the retarded transverse photon self-energy at finite temperature $T$ evaluated on shell, and $n_B(E) = 1/(e^{E/T} - 1)$ is the Bose-Einstein distribution. Using the hard thermal loop (HTL) resummed effective perturbation theory developed by Braaten and Pisarski [10], Kapusta et al. [3] and Baier et al. [8] showed

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that at one-loop order (in effective perturbation theory) the processes that contribute to photon production are the gluon-to-photon Compton scattering off (anti)quark $q(q) \rightarrow q(q)\gamma$ and quark-antiquark annihilation to photon and gluon $q\bar{q} \rightarrow g\gamma$. The corresponding rate of energetic ($E \gg T$) photon emission for two light quark flavors ($u$ and $d$ quarks) is given by 

$$E^3 \frac{dN}{d^3p d^4x} \bigg|_{\text{one-loop}} = \frac{5}{9} \frac{\alpha_s}{2 \pi^2} T^2 e^{-E/T} \ln \left( \frac{0.23E}{\alpha_s T} \right),$$

(2)

where $\alpha$ is the fine-structure constant and $\alpha_s = g_s^2/4\pi$ with $g_s$ being the strong coupling constant. In a recent development Aurenche et al. have found that the two-loop contributions to the photon production rate arising from (anti)quark bremsstrahlung $q\bar{q}(g) \rightarrow q\bar{q}(g)\gamma$ and quark-antiquark annihilation with scattering $q\bar{q}(g) \rightarrow q\bar{q}(g)\gamma$ are of the same order as those evaluated at one loop. The two-loop contributions to the photon production rate read

$$E^3 \frac{dN}{d^3p d^4x} \bigg|_{\text{two-loop}} = \frac{40}{9} \frac{\alpha_s}{\pi^2} T^2 e^{-E/T} (J_T - J_L)\ln 2 + \frac{E}{3T},$$

(3)

where $J_T \approx 4.45$ and $J_L \approx -4.26$ for two light quark flavors. Most importantly, they showed that the two-loop contributions completely dominate the photon emission rate at high photon energies.

To study photon production from a QGP of finite lifetime, we use a real-time kinetic approach based on nonequilibrium quantum field theory, which when improved by a resummation via a dynamical renormalization group provides a consistent microscopic derivation of quantum kinetics from the underlying theories. One of the advantages of this real-time kinetic approach is that it is capable of capturing off-shell (energy nonconserving) effects originating in the finite system lifetime, as completed collisions are not assumed a priori.

Because of the abelian nature of the electromagnetic interaction, we will work in a gauge invariant formulation in which physical observables (in the electromagnetic sector) are manifestly gauge invariant. Since photons escape directly from the quark-gluon plasma without further interaction, it is adequate to treat them as asymptotic particles. In the Heisenberg picture the number operator $N(p, t)$ that counts the total number of photons of momentum $p$ (per phase space volume) at time $t$ is defined by

$$N(p, t) = \sum_{\lambda=1}^2 a_\lambda^\dagger(p, t) a_\lambda(p, t),$$

(4)

where $a_\lambda(p, t) [a_\lambda^\dagger(p, t)]$ is the annihilation (creation) operator that destroys (creates) a photon of momentum $p$ and polarization $\lambda$ at time $t$. The time-dependent photon production rate, which up to a trivial factor $E/(2\pi)^3$ is related to the expectation value of the time derivative of $N(p, t)$, can be obtained by using the Heisenberg equations of motion (for details, see Ref. [14]). In the framework of nonequilibrium quantum field theory, the time-dependent photon production rate is given by

$$E^3 \frac{dN}{d^3p d^4x} = \lim_{t \to \infty} \sum_{\lambda} \frac{3 e_q}{2(2\pi)^3} \left( \frac{\partial}{\partial t} - iE \right) \int \frac{d^3q}{(2\pi)^3} \left| \langle \tilde{\psi}(-k, t)\gamma^\mu A_\mu^\dagger(p, t')\psi(-q, t) \rangle \right| + c.c.,$$

(5)

where $k = p + q$. Here $e_q$ is the electromagnetic coupling constant of the quarks, and $\psi$ denotes the (gauge invariant) quark field. The “$+$” (“$-$”) superscripts for the fields refer to fields defined in the forward (backward) time branch.

As usual we assume that there are no photons initially and those once produced will escape from the plasma without building up their population. Therefore the QGP in effect is treated as the vacuum for the photons. Consequently, the nonequilibrium expectation values on the right-hand side of Eq. ($\hat{3}$) is computed perturbatively to order $\alpha$ and in principle to all orders in $\alpha_s$ by using real-time Feynman rules and propagators. Furthermore, the photon propagators are the same as those in the vacuum.

We shall further assume the weak coupling limit $\alpha \ll \alpha_s \ll 1$. Whereas the first limit is justified and essential to the interpretation of electromagnetic signatures as clean probes of the QGP, the second limit can only be justified for very high temperatures, and its validity in the regime of interest can only be assumed so as to lead to a controlled perturbative expansion.

It is not hard to see that the lowest order contribution to Eq. ($\hat{3}$) is of one quark loop and of order $\alpha$. In using bare quark propagators we consider the quark momentum in the loop to be hard, i.e., $q \gg T$. Soft quark lines require HTL resummed effective quark propagator leading to higher order corrections. Indeed, the one-loop diagram with soft quark loop momentum is part of the higher order contribution of order $\alpha_\lambda$ that has been calculated in Refs. [13, 14].

In this article we focus on the lowest order contribution which, as will be understood below, contributes to direct photon production solely as a consequence of the finite QGP lifetime, and has therefore been missed by all previous investigations which assumed an infinite QGP lifetime.

For two light flavors ($u$ and $d$ quarks), the lowest order time-dependent photon production rate is found to be given by

$$E^3 \frac{dN}{d^3p d^4x} = \frac{2}{(2\pi)^3} \int^{+\infty}_{-\infty} d\omega \mathcal{R}(\omega) \frac{\sin[(\omega - E)(t - t_0)]}{\pi(\omega - E)},$$

(6)
with
\[
R(\omega) = \frac{20 \pi^2 \alpha}{3} \int \frac{d^3 q}{(2\pi)^3} \left\{ 2 \left[ 1 - (\hat{p} \cdot \hat{k})(\hat{p} \cdot \hat{q}) \right] \times n_F(k) \left[ 1 - n_F(q) \right] \delta(\omega - k + q) + \left[ 1 + (\hat{p} \cdot \hat{k})(\hat{p} \cdot \hat{q}) \right] \left[ n_F(k)n_F(q) \right] \times \delta(\omega - k - q) + \left[ 1 - n_F(k) \right] \times \left[ 1 - n_F(q) \right] \delta(\omega + k + q) \right\},
\]
(7)
where \( t_0 \) is the initial time at which the QGP is formed, \( q = |q| \), and \( n_F(q) = 1/(e^{E/kT} + 1) \) is the Fermi-Dirac distribution.

A detailed analysis shows that the first delta function \( \delta(\omega - k + q) \) with support below the light cone \((\omega^2 < E^2)\) corresponds to the Landau damping cut, and the last two delta functions \( \delta(\omega \mp k \mp q) \) with supports above the light cone correspond to the usual two-particle cut. Furthermore, one recognizes that \( R(\omega) \) has a physical interpretation in terms of the following off-shell (energy nonconserving) photon production processes: the first term describes (anti)quark bremsstrahlung and quark-antiquark annihilation processes. The "vacuum" process is independent of the presence of the QGP or its lifetime and persists for a infinitely long time. Therefore under the assumption of completed collisions that is in-volved in time-dependent perturbation theory leading to Fermi's golden rule and energy conservation. Under this assumption \( R(\omega) \) is recovered to lowest order and one finds a time-independent photon production rate proportional to \( \delta(E) \), provided that the latter is finite.

In the present situation, however, the three delta functions in \( R(\omega) \) cannot be satisfied on the photon mass shell. Therefore under the assumption of completed collisions the off-shell contribution to the photon production rate simply vanishes due to kinematics. Physically this reflects the energy nonconserving nature of the corresponding photon production processes. The "vacuum" process is independent of the presence of the QGP or its lifetime and persists for an infinitely long time. Therefore for the third term of \( R(\omega) \) we have to take the infinite time limit, which leads to the vanishing of the off-shell "vacuum" process \( 0 \to q\bar{q}\gamma \) by energy conservation. Only the "medium" processes depend on the presence and finite lifetime of the QGP, hence we only need to consider off-shell (anti)quark bremsstrahlung and quark-antiquark annihilation in the rest of our discussion.

For any finite QGP lifetime the time-dependent rate given in Eq. (8) is finite and nonvanishing, thus leading to a nontrivial contribution to direct photon production. In this article we focus on the photon yield instead of the photon production rate, since the former is the quantity of phenomenological interest that can be measured in experiments. The photon yield is obtained by integrating the rate over the lifetime of the QGP. Using Eq. (8), one obtains
\[
E \frac{dN(t)}{dt^3 p d^3 x} = \frac{2}{(2\pi)^3} \int_{-\infty}^{+\infty} d\omega R(\omega) \frac{1 - \cos[(\omega - E)t]}{\pi(\omega - E)^2},
\]
(10)
where, here and henceforth, we have set \( t_0 = 0 \).

Before proceeding to a numerical study, we give an analytic estimate of the behavior of the photon yield in the HTL approximation. In this approximation the leading contribution to \( \text{Im} \Pi^\mu_\nu(\omega) \) is dominated by the Landau damping cut \( E \approx \omega \), which corresponds to off-shell (anti)quark bremsstrahlung. Thus, from Eq. (8), we find
\[
R_{\text{HTL}}(\omega) = \frac{20 \pi^2 \alpha T^2}{3} \frac{\omega}{E} \left( 1 - \frac{\omega^2}{E^2} \right) n_B(\omega) \theta(E^2 - \omega^2).
\]
(11)
For \( E \ll T \), \( R_{\text{HTL}}(\omega) \) can be further simplified as
\[
R_{\text{HTL}}(\omega) \approx \frac{20 \pi^2 \alpha T^2}{3} \frac{\omega}{12E} \left( 1 - \frac{\omega^2}{E^2} \right) \theta(E^2 - \omega^2).
\]
(12)
The dominant contribution of the \( \omega \)-integral in Eq. (10) for \( E \ll T \) arises from the region where the resonant denominator vanishes, i.e., \( \omega \approx E \). Using Eq. (12), we obtain
\[
E \frac{dN(t)}{dt^3 p d^3 x} \approx \frac{5}{9} \frac{\alpha}{2\pi^2 E^2} \left[ 2Et + \gamma - 1 \right] + \mathcal{O} \left( \frac{1}{7} \right),
\]
(13)
where \( \gamma = 0.577 \ldots \) is the Euler-Mascheroni constant. It has been shown in Ref. [14] that for photons of energy \( E \lesssim T \) the finite-lifetime contribution to the photon yield, Eq. (13), is comparable at early times to that of
order $\alpha_s$ obtained from the equilibrium rate given in Eq. (2).

Fig. 1 shows that for $E \gg T$, $R_{HTL}(\omega)$ is exponentially suppressed in the region $T < \omega < E$. From this observation we emphasize that (i) because for $E \gg T$ the threshold contribution near the photon mass shell $\omega \approx E$ is exponentially suppressed, the $\omega$-integral in Eq. (10) is now dominated by the interval $-E < \omega < T$, which corresponds to highly off-shell (anti)quark bremsstrahlung; (ii) as the integrand in Eq. (10) is positive-definite and the function $1 - \cos[(\omega - E)t]$ averages to 1 for large $t$ in the region $-E < \omega < T$, for fixed $E \gg T$ the yield approaches a constant at large times; and (iii) in contrast to those obtained from equilibrium rates, the yield for $E \gg T$ is not suppressed by the Boltzmann factor $e^{-E/T}$. These important nonequilibrium aspects have noteworthy phenomenological implications studied in detail below.

We now perform a numerical analysis of the nonequilibrium finite-lifetime contribution to photon production directly in terms of the yield given by Eqs. (10) and (11) [but, as explained above, without including the third term of $R(\omega)$] and compare the results to those obtained from the equilibrium rates given in Eqs. (3) and (4). For this we consider a QGP of temperature $T = 200$ MeV and lifetime $t = 10 - 20$ fm/$c$. As remarked above, this is approximately the scales of the QGP temperature and lifetime expected at RHIC energies. For the value of $\alpha_s$ at this temperature, we use $\alpha_s(T) = 6\pi/(33 - 2N_f)\ln(8T/T_c)$ [5], where $N_f = 2$ is the number of quark flavors and $T_c$ is the quark-hadron transition temperature. Taking $T_c = 160$ MeV from lattice QCD calculation for two quark flavors, we find $\alpha_s \approx 0.3$ at $T = 200$ MeV.

The results of the numerical study of the nonequilibrium photon yield are displayed in Figs. 2 and 3. Fig. 2 depicts the time evolution of the nonequilibrium yield up to 20 fm/$c$ for photons of energies $E = 1$ and 2 GeV. We observe that the yield exhibits a clearly a logarithmic time-dependence at late times during the QGP lifetime. Numerical evidence shows that the nonequilibrium yield $\text{d}N(t)/d^3p d^2x$ falls off with a power law $E^{-\nu}$ with $\nu \approx 2.14$ for $E \gg T$. In addition, the numerical result also reveals that the dominant photon production process is (anti)quark bremsstrahlung $q(\bar{q}) \rightarrow q(\bar{q})\gamma$.

Fig. 3 shows a comparison of the nonequilibrium and equilibrium (one-loop and two-loop) contributions to the direct photon yield in the range of energy $T < E < 4$ GeV. Whereas the two-loop contribution dominates the direct photon yield for smaller values of $E$, a significant enhancement of the direct photon yield due to the nonequilibrium contribution is seen at $E > 2$ GeV as a consequence of its power law falloff for $E \gg T$. In particular, the nonequilibrium contribution is larger than the equilibrium contributions by several orders of magnitude for $E > 3$ GeV. As the nonequilibrium contribution for fixed $E \gg T$ approaches a constant at large times, the equilibrium contributions, which grow linearly in time, will eventually dominate the yield if the QGP has a very long lifetime. However, the linear growth in time of the equilibrium contributions has to compensate the Boltzmann suppression for $E \gg T$. Therefore we emphasize that for $E \gg T$ the equilibrium contributions could dominate the yield if the QGP lifetime is of order $10^2 - 10^3$ fm/$c$ or larger, which nevertheless is very unrealistic at RHIC energies.

Our analysis can be extended to the case of direct photon production from a QGP away from chemical equilibrium by replacing the equilibrium (anti)quark distributions with the undersaturated ones [6]. We expect the main features to remain in the more general situation.

In this article we have studied directly in real time the production of direct photons from a thermalized QGP of temperature $T \approx 200$ MeV and lifetime $t \sim 10 - 20$ fm/$c$. To lowest order in perturbation theory we find that direct photon production features a power law spectrum for $E \gg T$ due to a significant enhancement by off-shell (anti)quark bremsstrahlung.

Our main conclusion is that the novel nonequilibrium effect originating in the finite lifetime of a thermalized QGP leads to an enhancement in the direct photon yield for $E > 2$ GeV, which dominates over the yields obtained from equilibrium rate calculations for the same QGP lifetime. To establish a direct contact with the experimental observations, the next step of our program will be to include effects of hydrodynamical expansion of the QGP as well as an assessment of photon production from a hadronic gas with a finite lifetime, from hadronization to freeze-out. Work in this direction is in progress.

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FIG. 1. The function $R_{HTL}(-\omega)$ is plotted in the limit $E \gg T$. Here we take $E = 2$ GeV and $T = 200$ MeV.

FIG. 2. The nonequilibrium yield from a QGP of temperature $T = 200$ MeV is plotted as a function of time for $E = 1$ (upper) and 2 GeV (lower). The circles denote the numerical result and the solid line is a logarithmic fit.

FIG. 3. Comparison of various contributions to the direct photon yield from a quark-gluon plasma of temperature $T = 200$ MeV and lifetime $t = 10$ fm/c. The inset shows the figure on a log-log plot.