Laser photon statistics in the feedback loop

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A mere correspondence between the electron statistics and the photon one vanishes in the feedback loop (FBL). It means that the direct photodetection, supplying us with the electron statistics, does not provide us with a wished information about the laser photon statistics. For getting this information we should think up another measurement procedure, and we in the article suggest applying the three-level laser as a auxiliary measuring device. This laser has impressive property, namely, its photon statistics survive information about the initial photon statistics of the laser which excites coherently the three-level medium. Thus, if we choose the laser in the FBL as exciting the three-level laser, then we have an possibility to evaluate its initial photon statistics by means of direct detecting the three-level laser emission. Finally, this approach allows us to conclude the feedback is not capable of creating a regularity in the laser light beam. Contrary, the final photon fluctuations turn out to be always even bigger. The mentioned above feature of the three-level laser takes place only for the strong interaction between the lasers (exciting and excited). It means the initial state of the exciting laser is changed dramatically, so our measurement procedure can not be identified with some non-demolition one.

I. INTRODUCTION

After the very first works devoted to the laser sources of sub-Poissonian light [1, 2], the efforts to find physical systems able to emit an effectively squeezed light and suitable for the aims of quantum optics have been continued. There are a few ways to achieve the required light properties; each of them has its own specific features. Here we are going to discuss only one of the ways which is connected with the laser in the feedback loop (FBL) and is likely the most problematic from the ideological point of view.

The idea of an experiment with a feedback is as follows. Laser emission is detected by
a photo-detector, and then the photocurrent is used for pump rate correction of the laser medium in a manner such that, for example, positive photocurrent fluctuation results in negative fluctuation of pump rate of the laser medium, and so it leads to negative fluctuation of operation power and, accordingly, to negative fluctuation of the photocurrent. Thus, the photocurrent occurs to be stabilized on specific temporal intervals. As was persuasively shown in experiments [2, 3, 4], it is possible to reduce a photocurrent shot noise even below the shot level in this way.

Unfortunately, it is impossible to say in this case that smoothing the fluctuations in the photocurrent means corresponding smoothing in the photon fluctuations. Really, the sub-shot noise in the electrical current is easy understood for both the sub-Poissonian photon flux and the classical light beam for which, certainly, we have no any reasons to say about the non-classical statistics. It means in the FBL a mere correspondence between the electron statistics and photon one vanishes, and we will say that in this case a direct observation of the photon statistics is impossible.

To make the correct conclusion about the photon statistics in the FBL we should provide some special measuring procedure. For example, any idea of quantum non-demolition procedure could be quite productive. However, now we do not see how it could be really organized.

At the same time, the quantum non-demolition procedure is not only what could be offered. It is possible to imagine another measuring procedure in which the state of the tested laser in the FBL does not survive (in contradiction to the quantum non-demolition one) but the information about this state is transferred to another element (‘the measuring device’). If this device is outside the FBL, then the information is available to be read under direct observation. This gives us the possibility making correct conclusion about the statistics of the laser emission in the FBL. In this paper we will discuss one of the similar measurement schemes and offer to test the three-level laser as a ‘measuring device’.

Finally, we would like to mention the paper by Wiseman and Milburn [5] which is devoted to the same problem. We believe, the main conclusion there that the FBL is unable to lead to quantum effects in the laser field is physically quite justified. At the same time, in our opinion, the theoretical base of the discussion in the work does not seem to us quite convincing. The models used there are not always adequate to investigated processes. And, what is more important, authors do not offer any measuring procedure, and make the
conclusions at the mathematical level, by analyzing the equations for intracavity field.

The organization of our paper is as follows. In sec. II, the quantum theory of the simplest single-mode laser in the form of Langevin’s equations for the photon number is represented. In sec. III, the phenomenological models of photodetecting the laser emission and the negative feedback are discussed. In sec. IV, the Langevin theory for two jointly working lasers is represented. It is proved that under the conditions of the strong coupling between the lasers, statistical properties of the three-level laser emission keep the information about the photon statistics of the coherently exciting laser. In sec. V, we choose the laser in the FBL as the coherently exciting the three-level lasing. We make the conclusion about the photon statistics in the emission of the laser in the FBL by means of analyzing emission from the three-level laser.

II. THE QUANTUM LANGEVIN THEORY OF SINGLE-MODE LASER

In this section we shall give a brief resume of the quantum laser model developed in Ref. [6, 7]. We shall define the physical parameters of this model and get the equations which will be used in the following sections.

In Fig. 1 we have shown the pump process to the upper laser state $|1\rangle$ with mean rate $R$. For stationary in time average pumping rate, an influence of the pump statistics can be characterized by the single parameter $p \leq 1$. For $p = 1$ the pump is perfectly regular while for $p = 0$ the pump has Poissonian statistics. Intermediate values of $0 \leq p \leq 1$ correspond to sub-Poissonian pumping while for $p \leq 0$ the pump process possess the excess classical fluctuations and corresponds to super-Poissonian statistics.

This pump statistics was introduced into the quantum laser model using the Heisenberg-Langevin equations for the operator-valued collective populations of the upper and lower levels (see Fig. 1), and for the collective polarization. At the same time, there is a way to
reformulate the Heisenberg-Langevin theory via the c-number values and derive so-called the Langevin theory. Here we will apply the latter treatment, the corresponding equations read [6, 7]:

\[
\begin{align*}
\dot{\alpha} &= -\frac{\kappa}{2}\alpha + gP, \\
\dot{P} &= -\gamma_\perp P + g(N_1 - N_2)\alpha + F_p, \\
\dot{N}_1 &= R - \gamma_1 N_1 - g(\alpha^*P + \alpha P^*) + F_1, \\
\dot{N}_2 &= -\gamma_2 N_2 + g(\alpha^*P + \alpha P^*) + F_2.
\end{align*}
\]

Here \(\alpha\) is the complex field (mode) amplitude, \(P\) is the complex collective polarization of the laser transition, \(N_1, N_2\) is the collective population of the upper (lower) laser level. \(R\) is the mean rate of pump to the upper laser level, and \(\kappa\) is the spectral width of the laser mode, \(\gamma_{1,2}\) and \(\gamma_\perp\) are the corresponding atomic longitudinal and transverse relaxation constants and \(g\) is the coupling constant ensuring the dipole atom-laser field interaction.

\(F_1, F_2, F_p\) are the stochastic sources which are specified in the work [6, 7] in the general case. By taking away the sources, we get the corresponding semiclassical theory. Because in the stationary regime the usual laser conditions guarantee relatively small fluctuations of the populations and the polarization, we have a right to hold that the semiclassical solutions coincide with the mean values with high precision. Putting a simplified conditions \(\gamma_1 \ll \gamma_2\) and \(2\gamma_\perp = \gamma_1 + \gamma_2 \approx \gamma_2\), it is easy to obtain the following stationary semiclassical solutions:

\[
\begin{align*}
g \overline{\alpha^*P} &= \frac{R}{2(1 + I)}, \\
\gamma_1 \overline{N}_1 &= R \frac{1}{1 + I}, \\
\gamma_2 \overline{N}_2 &= R \frac{I}{1 + I}.
\end{align*}
\]

Here the dimensionless power of lasing is defined as \(I = \beta n\) \((n = |\alpha|^2)\), and

\[
\beta^{-1} = \gamma_\perp \gamma_1 / (2g^2)
\]

is the photon number saturating the laser transition. Taking into account that under the stationary generation

\[
2g \overline{\alpha^*P} = \kappa
\]

and, as a result, \(R = \kappa/\beta(1 + I)\), the same solutions can be rewritten in the form

\[
\begin{align*}
\gamma_1 \overline{N}_1 &= \kappa/\beta, \\
\gamma_2 \overline{N}_2 &= \kappa n \ (\overline{N}_2 = 0).
\end{align*}
\]
To calculate the quantum fluctuations \( \delta N_{1,2} = N_{1,2} - \overline{N}_{1,2}, \delta P = P - \overline{P} \) and \( \delta \alpha = \alpha - \overline{\alpha} \) around the stationary solutions we shall linearize Eqs. (2.1)-(2.4) relative to these fluctuations (neglecting by the phase diffusion). In particular, in this approximation, the non-zero correlation functions for the stochastic sources are:

\[
\begin{align*}
F_1(t)F_1(t') &= \frac{\kappa}{\beta} \left( 2 - p(1 + I) \right) \delta(t - t'), \\
F_1(t)F_2(t') &= \kappa n \delta(t - t'), \\
F_p^*(t)F_p(t') &= \frac{\kappa}{\beta} \frac{\gamma_2}{\gamma_1} \delta(t - t'), \\
F_p(t)F_p(t') &= \frac{\kappa \alpha^2}{\gamma_2} \delta(t - t'), \\
F_p(t)F_2(t') &= \frac{\kappa \gamma_2}{(2g)} \alpha \delta(t - t').
\end{align*}
\]

To specify more our physical conditions let us choose \( \kappa \ll \gamma_1 \) (the high-Q cavity approximation). It means the field amplitude \( \alpha \) is developed more slowly than the atomic variables and we have a right to apply the adiabatical approximation putting in our theory \( \dot{P} = 0 \) and \( \dot{N}_{1,2} = 0 \). It allows us to derive the single equation for the complex field amplitude \( \alpha = \sqrt{u} \exp(i\varphi) \) or even for the photon number \( u \):

\[
\dot{u} = -\kappa u + R + F, \quad F(t)F(t') = -pkn \delta(t - t').
\tag{2.14}
\]

Here we have selected more interesting case \( I \gg 1 \) (the saturation regime).

It needs to stress, the equation (2.14) could be easy obtained within the consideration in the work [1], although the master equation approach was developed there.

**III. FORMAL MODELS OF PHOTODETECTING THE LASER EMISSION AND THE NEGATIVE FEEDBACK**

In optical experiments we follow the photocurrent, and according to the photoeffect law its mean value is equal to a mean light stream falling on the photodetector, i.e., in the case of the single mode laser \( i = \kappa n, \ n = \overline{\alpha} \). To write the corresponding equality without averaging it is not enough simply taking the symbol of averaging away. We have to take into account that the process of the atomic ionization under photodetecting is essentially random and then to write the random photocurrent phenomenologically in the form:

\[
i(t) = \kappa u(t) + S(t), \quad \overline{S} = 0, \quad \overline{S(0)S(t)} = \overline{i} \delta(t).
\tag{3.1}
\]
Here the stochastic source \( S(t) \) ensures the appearance in the photocurrent of the so-called shot noise.

Let us rewrite the equations (2.14), (3.1) in the spectral representation:

\[-i\omega \varepsilon_{\omega} = \kappa \varepsilon_{\omega} + F_{\omega},\]  
\[\delta i_{\omega} = \kappa \varepsilon_{\omega} + S_{\omega},\]  

where

\[ \varepsilon_{\omega} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \, e^{i\omega t} \left[ u(t) - n \right], \quad \delta i_{\omega} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \, e^{i\omega t} \left[ i(t) - \bar{I} \right] \]  

are the Fourier components respectively for the photon number and photocurrent fluctuations. The spectral components \( F_{\omega} \) and \( S_{\omega} \) are written in the same way. The non-zero correlation functions for them are:

\[ F_{\omega} F_{\omega'} = -p \bar{I} \delta(\omega + \omega') \]  
\[ S_{\omega} S_{\omega'} = \bar{I} \delta(\omega + \omega') \]  

Often investigators follow the spectral density of the photocurrent fluctuations \( (\delta i^2)_\omega \) (the photocurrent spectrum) which is introduced by the correlation function:

\[ \overline{\delta i_{\omega} \delta i_{\omega'}} = (\delta i^2)_\omega \delta(\omega + \omega'). \]  

Now it is not difficult to get with help of the formulas in this section that

\[ (\delta i^2)_\omega / \bar{I} = 1 - p \kappa^2 / (\kappa^2 + \omega^2). \]  

One can see, for the random pump of the laser medium \( p = 0 \) the current spectrum contains only the shot noise term. At the same, for the regular pump \( p = 1 \) the shot noise turns out to be reduced on the zero frequency. By this formula for the photocurrent we have a right to make a correct conclusion relative to the light flux. The Poissonian photon statistics take place in the case of \( p = 0 \) and, the sub-Poissonian ones - in the case of \( p = 1 \).

For the formal introduction of the feedback into the equations a merely phenomenological approach is usually used; with this approach, the mean rate of the laser pump of medium \( R \) in the equation (2.11) is changed so that it is decreased with the photocurrent fluctuation increasing \( \delta i(t) = i(t) - \bar{I} \), for example, by the rule:

\[ r \to r \left( 1 - \lambda \frac{\delta i}{\bar{I}} \right), \]  

where

\[ \lambda = \frac{\kappa}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \, e^{i\omega t} \left[ u(t) - n \right]. \]
where the parameter $\lambda$ could be called as a feedback efficiency. Certainly, it is primitive model of the FBL but it is enough for our reasons.

Applying the replacement (3.9) in initial laser equations (2.1)-(2.4), we are able to get the equations for the fluctuations in full analogous with equations (3.2)-(3.3) which read

\[
[(1 + \lambda)\kappa - i\omega] \varepsilon_\omega = F_\omega - \lambda S_\omega
\]
\[
\delta i_\omega = \kappa \varepsilon_\omega + S_\omega.
\] (3.10) (3.11)

The algebraic equations allow us to calculate the photocurrent spectrum which is given by

\[
\langle \delta i^2 \rangle_\omega / \bar{t} = 1 - \frac{p - 1 + (1 + \lambda)^2}{(1 + \lambda)^2 + \omega^2 / \kappa^2}.
\] (3.12)

It is readily seen that if the FBL efficiency is high enough ($\lambda \gg 1$), then the complete reduction of the shot noise in the photocurrent on the zero frequency takes place independently of the pump statistics. However, we shall remember that this conclusion can not be extended to the photon statistics. Our aim here is to study the photon statistics with help of the auxiliary measuring procedure.

**IV. STRONG COUPLING BETWEEN EXCITING AND EXCITED LASERS**

As mentioned above, the three-level lasing with the coherent excitation by the emission from another laser has extremely attractive feature. Under the specific conditions, the statistical features in its radiation replicate the statistical features of the exciting radiation. However, it takes place only when the interaction between lasers is essentially strong. This idea can be explained with the help of the mental experiment which is represented in Fig. 2.

One can see, there are a two-level exciting laser (the '2-laser') and a three-level excited one (the '3-laser'). They occupy a position such that they have the common intracavity space. It is assumed that the three-level medium is coherently excited by the intracavity field of the two-level laser. And we assume that the intracavity lifetime of exciting photon is much less than the lifetime in absence of the three-level medium. Further, we will apply the term 'strong coupling' between lasers for just this phenomenon.

In Fig. 3, both the two-level and three-level atomic configurations are shown. As for the two-level medium (on the left in the figure), as before, there are an incoherent pump (regular or random) to the upper laser state $|1\rangle$ with the mean rate $R$ and the spontaneous emissions with the rates $\gamma_1$ and $\gamma_2$. 
Figure 2: Mental experimental setup with two strongly coupled lasers. '2-laser' - the two-level exciting laser; '3-laser' - the three-level excited laser.

Figure 3: Atomic energetic configurations for 2-laser (on the left) and 3-laser (on the right).

In the three-level medium (on the right in the figure), there is an incoherent pump with the rate $\tilde{\gamma}_2$. This ensures populating the intermediate non-laser atomic state $|\tilde{3}\rangle$. Take into account, hereinafter, the sign 'tilde' over any symbol tells us that this symbol is concerned the three-level laser.

It is suggested that the three-level oscillation takes place on the transition $|\tilde{1}\rangle \leftrightarrow |\tilde{2}\rangle$ and this is realized by a joint action of both the incoherent $|\tilde{2}\rangle \rightarrow |\tilde{3}\rangle$ and coherent $|\tilde{3}\rangle \rightarrow |\tilde{1}\rangle$ pumps. Our two-level laser ensures the last.

Besides, the spontaneous emission on the laser transition with the rate $\tilde{\gamma}_1$ is taken into account.

We shall construct the quantum theory of two interacting lasers by applying the Langevin treatment as before. Certainly, a structure of the equations is more complicated. Really, now we have to discuss the populations of five atomic states (two-level and three-level medium) and the polarizations of four atomic transitions instead of two populations and one polarization.

Besides, we have to introduce two laser amplitudes $\alpha = \sqrt{u} \exp(i\varphi)$ for the exciting laser and $\tilde{\alpha} = \sqrt{\tilde{u}} \exp(i\tilde{\varphi})$ for the excited one instead of one $\alpha$ and try to construct the equations for their fluctuations $\varepsilon = u - n$ and $\tilde{\varepsilon} = \tilde{u} - \tilde{n}$. This analysis is relatively easy carried out.
and the following system of the equations in the spectral representation is obtained:

\[-i\omega \varepsilon_\omega = - (\kappa + \kappa_0) \varepsilon_\omega + \kappa \tilde{\varepsilon}_\omega + F_\omega, \quad (4.1)\]

\[-i\omega \tilde{\varepsilon}_\omega = - 2\kappa \tilde{\varepsilon}_\omega + \kappa_0 \varepsilon_\omega + \tilde{F}_\omega. \quad (4.2)\]

Here \(\kappa, \tilde{\kappa}\) are the spectral mode widths; \(n, \tilde{n}\) are the corresponding mean photon numbers under the stationary oscillation. One can easily get some useful relationships for the semiclassical values:

\[\tilde{n}\kappa = n\kappa_0, \quad (\kappa + \kappa_0)n = R. \quad (4.3)\]

The value

\[\kappa_0 = \tilde{\gamma}_2 \left(\tilde{g}_{13}/\tilde{g}_{12}\right)^2 \tilde{N}/\tilde{n} \quad (4.4)\]

is the rate of the coherent pump of the three-level medium by the field of two-level generation. In the formula \(\tilde{g}_{13}, \tilde{g}_{12}\) are the constants of the dipole interaction between the field and the three-level medium on the transitions \((1 \rightarrow 3)\) and \((1 \rightarrow 2)\), accordingly; \(\tilde{N}\) is the number of the three-level atoms participating in laser process.

The stochastic sources are specified by the correlation functions

\[\overline{F_\omega F_{\omega'}} = -p(\kappa + \kappa_0)n \delta(\omega + \omega'), \quad (4.5)\]

\[\overline{\tilde{F}_\omega \tilde{F}_{\omega'}} = -2\kappa \tilde{n} \delta(\omega + \omega'). \quad (4.6)\]

Note that the theory of two lasers operating jointly have been considered in Ref. [8] but in the master equation treatment. Although we have written the equations (4.1)-(4.2) on the basis of the Langevin approach, nevertheless they could easily be obtained from the cited work on the base of the master equation.

As before, we should derive the phenomenological expressions for the photocurrent fluctuations:

\[\delta i_\omega = \kappa \varepsilon_\omega + S_\omega, \quad \overline{S_\omega S_{\omega'}} = i \delta(\omega + \omega'), \quad (4.7)\]

\[\delta \tilde{i}_\omega = \tilde{\kappa} \tilde{\varepsilon}_\omega + \tilde{S}_\omega, \quad \overline{\tilde{S}_\omega \tilde{S}_{\omega'}} = i \delta(\omega + \omega'). \quad (4.8)\]

The equations are derived on the assumption that there are no any nonlinear effects at the interaction of the two-level generation with the three-level medium; it means that we keep
only the main term in the perturbation theory concerning to the coherent excitation of the three-level medium. The next assumption $\tilde{\gamma}_2 \ll \tilde{\gamma}_1$ is made here only for simplification.

The formulas above allow us to derive the photocurrent spectrum under the registration of the emission of the three-level laser. It is given by the formula:

$$(\delta \tilde{t}^2)_{\omega}/\tilde{t} = 1 - 2\tilde{\kappa}^2 \frac{\omega^2 + \kappa(\kappa + \kappa_0) + \kappa_0(\kappa + \kappa_0)p/2}{(\omega^2 - \tilde{\kappa}(2\kappa + \kappa_0))^2 + \omega^2(2\tilde{\kappa} + \kappa + \kappa_0)^2}.$$ (4.9)

It is not difficult to obtain that this coincides exactly with (3.8) as $\kappa = \tilde{\kappa} \ll \kappa_0$. So if the photocurrent spectrum under registration of the laser emission is known and we choose this laser for coherent excitation (under strong coupling) of the three-level lasing, then the photocurrent spectrum from the three-level laser is the same. It means one can study the photocurrent spectrum from the exciting laser without the three-level medium by means of applying the measuring device, i.e., by studying the emission from the three-level laser.

V. THE LASER EMISSION STATISTICS IN THE FBL

The results of the previous section allows us to consider the three-level laser as the 'measuring device' which is reasonable for applying when the statistical analysis of light under the direct photodetection is impossible for any reason (for example, in the case of the laser in the FBL).

Thus in this section we choose the laser in the FBL as exciting one for the three-level laser. To rewrite equations (4.1) - (4.2) for the case of a laser in the FBL we have to make the replacement (3.9) in the initial equations. Then instead of equations (4.1) - (4.2) we can get:

$$[(\kappa + \kappa_0)(1 + \lambda) - i\omega]\varepsilon_\omega = \tilde{\kappa}\tilde{\varepsilon}_\omega + F_\omega - \lambda(1 + \kappa_0/\kappa)S_\omega,$$

$$2\tilde{\kappa} - i\omega)\tilde{\varepsilon}_\omega = \kappa_0\varepsilon_\omega + \tilde{F}_\omega.$$ (5.1) (5.2)

These algebraic equations together with (4.7) - (4.8) allow us to obtain the photocurrent spectrum $$(\delta \tilde{t}^2)_{\omega}$$ in the explicit form:

$$(\delta \tilde{t}^2)_{\omega}/\tilde{t} = 1 -$$

$$-2\tilde{\kappa}^2 \frac{\omega^2 + \kappa^2(1 + x)[1 + \lambda(2 + x) + \lambda^2(1 + x)(1 - x/2)]}{[\omega^2 - \tilde{\kappa}\kappa(2 + x + 2\lambda(1 + x))]^2 + \omega^2[2\tilde{\kappa} + \kappa(1 + x)(1 + \lambda)]^2}, \quad x = \frac{\kappa_0}{\kappa}. (5.3)$$
This result is written for case the random pump in the two-level laser. Just this case is of our interest because we would like to understand whether the quantum effects appear in the FBL if they were absent before.

Now we have a possibility to realize what happens with arising the FBL. Let the feedback efficiency be high enough $\lambda \gg 1$, and there be a strong coupling $\kappa = \tilde{\kappa} \ll \kappa_0$, then

$$\frac{\langle \delta \tilde{v}^2 \rangle}{\tilde{v}} = 1 + \frac{\kappa_0}{4\kappa} \frac{4\kappa^2}{\omega^2 + 4\kappa^2}. \quad (5.4)$$

In accordance with our logic in this work and with the last formula, we have a right to say that the noises on the zero frequency of the laser emission in the FBL are essentially above the shot level because it is the case for the emission of our measuring device. Thus, although the feedback causes the sub-Poissonian statistics for the photoelectrons, nevertheless the photon flux in the FBL turns out to be super-Poissonian.

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Figure 4: Atomic energetic configurations for 2-laser (on the left) and 3-laser (on the right)