Effects of the mass parameter in the optimum direction of impulse and energy variation in a Powered Swing-By

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Abstract. The energy of a spacecraft, relative to the primary body of the system, before and after a Powered Swing-By maneuver with an impulse applied during the close encounter with the secondary body is studied. The Powered Swing-By maneuver is a combination of the effect of the gravity of a celestial body and an impulse applied to the spacecraft during its passage by the periapsis of its orbit relative to the secondary body. This combination modifies the spacecraft's trajectory, changing its parameters and, consequently, its energy. The objective is to quantify the effect of different mass parameters on the optimum direction to apply the impulse and in the energy variation of this more complex maneuver. It is focused on the two-dimensional and elliptical maneuver. Optimum solutions for energy gains are presented.

1. Introduction

It is well known that the larger the mass of a body, the larger will be the effect of its gravity in another approaching body. This statement can be obtained from Newton's Law of Universal Gravitation [1], that shows that the force is directly dependent on the mass involved and inversely dependent on the square of the distance between the bodies.

The powered Swing-By maneuver, which is not only dependent on the gravity of the secondary body of the system, but also depends on the impulse applied to the spacecraft, has been extensively studied in the literature, considering different characteristics [1-9]. Some works like [10] presented the Swing-By maneuver depending only on the gravity of the body and the configurations of the encounter. In 1996, Prado [2] extended this study by including in the maneuver an impulse applied at the time of the closest encounter with the secondary body. Casalino (1999) [3] and Ferreira et al. (2017b) [6] have shown, in different forms, the possibility of applying the impulse outside the periapsis of the orbit of the spacecraft, but still within the sphere of influence [11] of the body. The study was expanded to the elliptic case, where $M_1$ and $M_2$ (primary and secondary bodies of the system) are in elliptical orbits around a common center of mass [5, 7-9]. Analytical equations were obtained for the variation of velocity, energy and angular momentum, considering the unpowered and powered cases of the maneuver [7-8]. Studies for the three-dimensional space maneuver also appeared in the literature [12-13].

Considering different mass parameters, Negri (2017, 2019) [14-15] presented the errors between the “patched-conic approximation” and the restricted three-body problem used to describe the Swing-By maneuver. Ferreira et al. (2018b) [8] studied the efficiency of the maneuver by applying the
impulse in the periapsis compared to the application of the impulse in a point outside the sphere of influence [11] of $M_2$.

The present research has the goal of quantifying the effects of different mass parameters in the spacecraft energy variation during the powered Swing-By maneuver, as well as in the optimum direction to apply the impulse that is given by the maneuver. The study is done considering a generic system in elliptical orbits around the center of mass, where the primary body of the system is $M_1$, secondary body is $M_2$ and spacecraft is $M_3$. $M_1$ and $M_2$ can be a planet-moon or a Sun-planet system, for example. Systems like Sun-Mars and Sun-Mercury are good examples of systems that can benefit from this model, because they have high eccentricities. The spacecraft moves in the two-dimensional space.

2. Problem Statement

A numerical mapping of the differences between the spacecraft energy relative to $M_1$ before and after the close encounter with $M_2$ is constructed. This variation is called $\Delta E$. $\Delta E_{\text{max}}$ is the maximum gain obtained after varying the initial conditions.

Figure 1. Geometry of the Powered Swing-By maneuver.

Figure 1 shows the geometry of the maneuver. For the maneuver, the spacecraft approaches the secondary body ($M_2$) with velocity $V_{\text{inf}^-}$. In the periapsis of the orbit the propeller is activated by applying an impulse with magnitude $\delta V$ and direction defined by $\alpha$ (the angle between the periapsis velocity and the impulse). From there, its trajectory changes instantly, going to a point distant from $M_2$ (point B), where there is a dominant interaction with $M_1$. The energy variation ($\Delta E$) with respect to $M_1$ is calculated from the energy at the point B subtracted from the energy at point A.

In Fig. 1, $\mu$ is the mass parameter of $M_2$ given by its mass divided by the total mass of the system; $\nu$ is the true anomaly of $M_2$; $\beta$ is the angle between $\vec{V}_2$ and the line connecting the primaries, being $\vec{V}_2$ the velocity of $M_2$ around the center of mass of the system; $\psi$ is the angle of approach that defines the position of the periapsis $P$ of the orbit of $M_3$ around $M_2$; $r_p$ is the distance from $P$ to $M_2$. $V_{\text{inf}^-}$, $\delta V$ and
$V_{p+}$ are the periapsis velocity before the impulse, the magnitude of the impulse and the periapsis velocity after the impulse, respectively. The angle $\alpha$ defines the direction of the impulse. Points A and B are points of the trajectory of $M_3$ around $M_1$ where only the gravitational effect of $M_1$ is assumed to be acting in the spacecraft. The dashed line is the trajectory before the Powered Swing-By and the solid line is the new trajectory after the maneuver.

The maneuver is developed from the numerical integration of the equations of motion given by the Elliptic Restricted Three-Body Problem (ERTBP) [16-17]. This model increases the accuracy of the results, compared to the Circular Restricted Three-Body Problem. It happens because there are some systems in the solar system, like Sun-Mercury and Sun-Mars, where the eccentricity is large and circular orbits do not represent very accurately the motion of the spacecraft.

From the geometry of the maneuver, it is possible to make an analysis of the effects of the maneuver. The lower the periapsis distance ($r_p$), the larger the effect of the gravity in the spacecraft. According to ref. [8], the periapsis position defined by the angle of approach $\psi$ combined with $\beta$ directly influences the gravitational effect on the spacecraft, as shown in Eq. 1.

$$\Delta E = 2V_{inf} \sin \delta \cos (\psi + \beta),$$

being $\sin \delta = \left(1 + \frac{r_p V_{inf}}{\mu} - 5\right)^{-1}$. If $-90^\circ < \psi + \beta < 90^\circ$, the spacecraft gain energy due to the gravitational part of the maneuver, but if $90^\circ < \psi + \beta < 270^\circ$ the gravity works to remove energy of the spacecraft. With respect to the impulse, if $\alpha = 0^\circ$ the impulse is applied in the direction of the motion of the spacecraft. If $\alpha = \pm 180^\circ$, the impulse is in the opposite direction. For $0^\circ < \alpha < 180^\circ$, there is a component of the impulse that sends the spacecraft in the opposite direction of $M_2$, moving it away from the body and decreasing the effect of gravity. If $-180^\circ < \alpha < 0^\circ$, the spacecraft is sent in the direction of $M_2$, so increasing its gravity effect. When $\alpha < -90^\circ$ or $\alpha > 90^\circ$, there is a component of the impulse decelerating the spacecraft. These are the conditions that most likely result in captures or collisions of the spacecraft with the secondary body.

3. Results

The analysis will be done according to the mass parameter ($\mu$) and the parameters of the impulse ($\delta V$ and $\alpha$). The energy variation is shown in canonical units (c.u.), where 1.0 c.u. is equivalent to the square of the secondary body velocity (in km$^2$/s$^2$). For example, for a system with $V_2 = 2.0$ km/s, 1.0 c.u. is approximately 4.0 km$^2$/s$^2$.

$$V_2 = \sqrt{(1 - \mu) \left(\frac{2}{r} - \frac{1}{a}\right)},$$

being $a$ the semimajor axis of the orbit of $M_2$, which is equal to 1.0 canonical unit of distances and $r$ the distance between $M_1 - M_2$.

The initial conditions adopted here are $r_p = 1.1$ radius of $M_2$, $V_{inf} = 1.0$ km/s, $e = 0.1$ and $v = 0^\circ$ ($M_2$ in the periapsis of the orbit around the center of mass of the system) and, consequently, $\beta = 90^\circ$ [7-8]. As the focus of this work is the energy gain, we adopted $\psi = 270^\circ$ that, from Eq. 1, is the maximum gain due to gravity for these conditions [7].

Figure 2 maps the energy variation of the spacecraft with $\mu$ varying from $10^{-5}$ to $10^{-1}$, $\alpha$ from $-180^\circ$ to $180^\circ$ and with an impulse of $\delta V = 0.1$ km/s. Note that there are no captures or collisions of the spacecraft with the secondary body, showing that, in this case, the impulse was not large enough to overcome gravity and cause those effects. Note also that, as expected, the energy variation increases with the mass parameter. The maneuver tries to get the most energy possible from the gravity and the impulse helps to optimize the results. $\Delta E_{\text{max}} = 3.4132$ c.u. for $\alpha = -4^\circ$ and $\mu = 0.1$. In most of the cases the energy increased after the maneuver, so there are positive variations. There are a few cases of
negative variation, where the impulse combined with gravity removed energy from the spacecraft. This occurs for \(\alpha\) assuming extreme values, which puts the impulse in a position that decelerates the spacecraft and sends it away from the body, so minimizing the effect of gravity. The minimum energy variation is \(-0.1256\) c.u., and it occurs for \(\alpha = 144^\circ\) and \(\mu = 10^{-5}\).

\[
\text{Figure 2} - \Delta E \text{ (c.u.) for } \delta V = 0.1 \text{ km/s.}
\]

Figure 3 shows the energy variation when the impulse is applied in the direction of the motion of the spacecraft \((\alpha = 0^\circ)\).

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\text{Figure 3} - \Delta E \text{ (c.u.) for } \alpha = 0^\circ.
\]

Note in Fig. 3 that the higher energy gains occur for a combination of the highest values of \(\mu\) with the highest magnitudes of the impulse, reaching just over 16.0 c.u. of variation of energy.

The parameter \(\mu\) influences the effect of gravity, which is a natural way of changing the parameters of the spacecraft, and may be used in favor or against the objectives of the mission, according to the geometry of the close encounter. The impulse has an intentional way of influencing the trajectory, which can be accelerated or not. The magnitude of the impulse has a direct effect on increasing the
spacecraft’s energy, while the angle that defines its direction can also work in the interest of the mission. These are controllable variables.

With the objective of mapping the optimal solutions of energy gain and their respective parameters, we present Fig. 4, subdivided into different plots. The left column shows the optimum $\alpha$ versus magnitude of the impulse that resulted in the maximum energy gain for different mass parameters ($\mu$). The right column is the maximum energy variation, in canonical units, as a function of its respective angle $\alpha$. Both cases present curves for different eccentricities.

Figures 4(a) and 4(b) show the optimal solutions for $\mu = 10^{-5}$. The optimum $\alpha$ is between $-45^\circ$ and $-35^\circ$ for eccentricities of up to about 0.1, remaining constant for impulses up to approximately 1.2 km/s. After that the curve is increasing. For $e = 0.3$, the angle $\alpha$ is about $-20^\circ$ and, for $e = 0.5$, $\alpha \sim -10^\circ$. The $\Delta E_{\text{max}}$ reaches about 5.0 c.u. for the smaller eccentricities and larger magnitudes of the impulse. In Figs. 4(c) and 4(d), made for $\mu = 10^{-4}$, the behavior is similar to the previous case, however, with some changes in $\alpha$ and small variations in the maximum energy gains. Figures 4(e) and 4(f) show that the angle that defines the direction of the impulse have flat regions in the variations, staying constant for some intervals of the magnitude of the impulse. In general, $\alpha$ stands between $-45^\circ$ and $-35^\circ$ for $e$ up to 0.1. As the mass of the secondary body increases, the gravity gets stronger and its effects become remarkable. Figures 4(g) and 4(h) have a mass parameter close to the Earth-Moon system, $\mu = 10^{-2}$. For $e = 0$, the optimum value of $\alpha$ varies around $-20^\circ$, an information already known from [2]. For eccentricities up to 0.1, the angle $\alpha$ is between around $-25^\circ$ and $-10^\circ$, since now the curve is decreasing. For $e = 0.5$ and impulses up to 0.4, $\alpha$ is positive, i.e., the geometry of the impulse is sending the spacecraft away from $M_2$. The maximum energy variation now reaches just over 8.0 c.u. in some cases. For $\mu = 10^{-1}$, in general $\alpha$ decreases between $-8.0^\circ$ and $8.0^\circ$ and $\Delta E_{\text{max}}$ reaches almost 20.0 canonical units. In some few cases, for $e = 0.5$, there are negative variations, i.e., the spacecraft loses energy in the maneuver.

From the analysis of the results, we can see that, in most of the cases, the spacecraft increases its energy after the maneuver. This happens because, in these cases, the optimal value of $\alpha$ sends the spacecraft to the direction of the secondary body with a component in the direction of its motion, so the spacecraft is accelerated, and the effect of the gravity is magnified due to the approach with the body. The larger the mass parameter $\mu$ and the energy gain, the closest to zero is $\alpha$, since the maneuver takes advantage of the gravity of the body, tending to impulses in the direction of the motion of the spacecraft. The tendency is a balance between the gravitational and the propulsive part of the maneuver to obtain the optimum results.
(c) $\delta V \times \alpha_{\text{optimal}}$ for $\mu = 10^{-4}$

(d) $\Delta E_{\text{max}} \times \alpha_{\text{optimal}}$ for $\mu = 10^{-4}$

(e) $\delta V \times \alpha_{\text{optimal}}$ for $\mu = 10^{-3}$

(f) $\Delta E_{\text{max}} \times \alpha_{\text{optimal}}$ for $\mu = 10^{-3}$

(g) $\delta V \times \alpha_{\text{optimal}}$ for $\mu = 10^{-2}$

(h) $\Delta E_{\text{max}} \times \alpha_{\text{optimal}}$ for $\mu = 10^{-2}$
4. Conclusions

The present study quantified the effect of the mass parameter (μ) in the Powered Swing-By maneuver. It was also highlighted the effect of the eccentricity of the orbits of the primaries, considering the bodies aligned, with \( M_2 \) on the right.

The main conclusion is that the maneuver tends to a balance between the gravitational and the propulsive part. The larger the mass of the secondary body, the larger the effects of the gravity on the approaching body. This information is known, however, when analyzing the optimal results obtained from the combination of gravity and the application of the impulse in the Powered Swing-by maneuver, it is noted that \( \alpha \) tends to a smaller angular region near zero, that is the direction of motion of the spacecraft, when \( \mu \) increases.

In most cases the spacecraft increased its energy after the close encounter and the impulse, which is an expected behavior, since it was used initial conditions focusing on the energy gains.

With respect to the eccentricity, we can see a hierarchy in the solutions, from the smallest to the largest values with respect to \( \alpha \), starting from bottom to top. There is an exception for the case \( \mu = 10^{-1} \), in which there is a blend of curves with respect to \( \Delta E_{\text{max}} \), even a decreasing curve. An explanation would be the relationship between the size of the bodies of the system. In this case \( M_2 \) has 10% of the total mass of the system.

It is clear that the Powered Swing-By maneuver involves many parameters and each of them has their respective importance in the solutions. They should be used according to the characteristics and objectives of the mission.

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