Abstract

The gauge-fixing parameter $\xi$ dependence of two-point gauge variant correlation functions is studied for QED and QCD. We show that, in three Euclidean dimensions, or for four-dimensional thermal gauge theories, the usual procedure of getting a general covariant gauge-fixing term by averaging over a class of covariant gauge-fixing conditions leads to a nontrivial gauge-fixing parameter dependence in gauge variant two-point correlation functions (e.g. fermion propagators). This nontrivial gauge-fixing parameter dependence modifies the large distance behavior of the two-point correlation functions by introducing additional exponentially decaying factors. These factors are the origin of the gauge dependence encountered in some perturbative evaluations of the damping rates and the static chromoelectric screening length in a general covariant gauge. To avoid this modification of the long distance behavior introduced by performing the average over a class of covariant gauge-fixing conditions, one can either choose a vanishing gauge-fixing parameter or apply an unphysical infrared cutoff.
I. INTRODUCTION

Physical observables such as thermal damping rates and the Debye screening length which are determined by the position of poles (or generally, singularities) in correlation functions are gauge invariant quantities \[1,2\]. However, for a general covariant gauge, a gauge-fixing parameter \(\xi\) dependence in damping rate was reported \[3\] in a perturbative evaluation of the fermion damping rate. Similar gauge-fixing parameter dependence was also encountered in the perturbative calculation of the Debye screening length at the next-to-leading order \[4\]. A way to extract the gauge independent damping rate and screening length is to introduce an unphysical infrared cutoff \[5\]. In this article, we examine the gauge-fixing parameter \(\xi\) dependence of two-point correlation functions. As is well known, a conventional way of getting a general covariant gauge-fixing term \((\partial A)^2/(2\xi)\) involves performing an average over a class of covariant gauge conditions. We shall show that, in three Euclidean dimensions or for four-dimensional thermal gauge field theories, this average over different gauge conditions generates a nontrivial \(\xi\) dependence in two-point correlation functions of gauge variant operators such as the fermion propagator. Specifically, for QED and to leading order of QCD, this gauge-fixing parameter dependence alters the long range behavior of the two-point correlation functions by an extra exponentially decaying factor with the exponent depending on \(\xi\). This is the origin of the \(\xi\) dependence encountered in perturbative calculations of the damping rate and the Debye screening length \[3,5\]. This gauge-fixing parameter \(\xi\) dependence is an artificial fact due to doing the average over a class of gauge conditions since this average includes gauge conditions \(\partial A = f\) with \(f\) containing long wavelength fluctuations. Choosing the Landau gauge \(\xi = 0\) which means taking the “no average” limit removes the modification of the long distance behavior produced by averaging over gauge conditions. Another way is to introduce an unphysical infrared cutoff as suggested in reference \[5\] to suppress the contributions to the gauge condition average from the infrared fluctuations. If the physical content of the theory is gauge invariant in the sense that the gauge constrain \(\partial A = 0\) is equivalent to other physical gauge constrains
when quantizing the theory, choosing $\xi = 0$ then yields the correct physics (e. g. correlation length).

In next section, we shall first study generally how the average over a class of gauge choices produces the $\xi$ dependence of the propagators of charged particles in QED where it is completely solvable. The $\xi$ dependence of the propagator of a charged particle may be expressed simply as a $\xi$-dependent multiplicative factor which becomes an exponentially decaying factor for large spacetime argument in three Euclidean dimensions or four-dimensional thermal QED. Thus, if one extracts the correlation length or the damping rate from the propagator of a charged particle, a $\xi$-dependent correlation length or damping rate is obtained. We then explain why an unphysical infrared cutoff can get rid of this $\xi$ dependence and produce the same result as the choice $\xi = 0$. In section III, we perform parallel analysis for QCD. Since a complete solution could not be achieved, we only do a leading order perturbative calculation. To the leading order, the gauge-fixing parameter dependence for the fermionic propagator is the same as in QED with an effective charge. The $\xi$ dependence of the static (chromo)electric screening length is discussed. We draw conclusions in section IV. In Appendix A, details about the evaluation of a thermal integral is given. In Appendix B, we present another way of deriving the $\xi$ dependence in fermion propagators based on Ward identities for the proper vertices.

II. $\xi$ DEPENDENCE OF A CHARGED PARTICLE PROPAGATOR IN QED

A. A functional derivation

In this subsection, we shall derive the gauge-fixing parameter $\xi$ dependence of the propagator of a charged particle in QED in a general covariant gauge. Since the derivation does not depend on whether the charged particle is a scalar or a fermion, we focus on the fermion case. Throughout this paper, we work in Euclidean spacetime. Real time results can then be obtained by analytic continuation.
The fermion propagator is defined by the Euclidean functional integral as

\[ G_f(x, y) \equiv \langle \bar{\psi}(x) \psi(y) \rangle \]

\[ \equiv \int [DA][D\bar{\psi}][D\psi] \exp \left\{ - \int L_E \right\} \bar{\psi}(x) \psi(y) \delta(\partial A - f), \]  \hspace{1cm} (2.1)

where we have introduced the gauge-fixing condition

\[ \partial A(x) = f(x) \]  \hspace{1cm} (2.2)

with \( f(x) \) being an arbitrary function. Denoting the charge of the fermion by \( q \), it is not hard to justify that the gauge transformation change of variables

\[ \psi(x) \rightarrow \psi'(x) = e^{iq\lambda(x)} \psi(x) \]
\[ \bar{\psi}(x) \rightarrow \bar{\psi}'(x) = e^{-iq\lambda(x)} \bar{\psi}(x) \]
\[ A(x) \rightarrow A'(x) = A(x) - \partial \lambda(x), \]  \hspace{1cm} (2.3)

with \( \lambda \) satisfying

\[ \partial^2 \lambda(x) = f(x), \]  \hspace{1cm} (2.4)

changes the gauge-fixing condition

\[ \partial A = f \rightarrow \partial A' = 0 \]  \hspace{1cm} (2.5)

and thus gives

\[ G_f(x, y) = G_{f=0}(x, y) e^{iq(\lambda(x) - \lambda(y))}. \]  \hspace{1cm} (2.6)

Defining Green’s function \( \Delta(x - x') \) by

\[ \partial^2 \Delta(x - x') = \delta(x - x') \]  \hspace{1cm} (2.7)

enables us to write

\[ \text{1We use} \langle \cdots \rangle \text{to represent the Euclidean time ordered product.} \]
\[ \lambda(x) = \int dz \Delta(x - z) f(z). \] (2.8)

The usual procedure of getting the general covariant gauge now involves averaging over \( f(x) \) with a weighting factor \( \exp\{-\frac{1}{2\xi} \int dx f^2\} \). Let us denote the fermion propagator in a general covariant gauge by \( G_\xi(x, y) \). Then

\[
G_\xi(x, y) = \int [Df] G_f(x, y) \exp \left\{ -\frac{1}{2\xi} \int dz f^2(z) \right\} 
= \int [Df] \exp \left\{ -\int dz \left[ \frac{1}{2\xi} f^2(z) + i J(z; x, y) f(z) \right] \right\} G_{f=0}(x, y),
\] (2.9)

where we have defined the linear source \( J(z; x, y) \) as

\[
J(z; x, y) = -q \left[ \Delta(z - x) - \Delta(z - y) \right].
\] (2.10)

A straightforward evaluation of the gaussian functional integral in Eq. (2.9) yields

\[
G_\xi(x, y) = \exp \left\{ -\frac{\xi q^2}{2} \int dz \left[ \Delta(z - x) - \Delta(z - y) \right]^2 \right\} G_{f=0}(x, y),
\] (2.11)

where the normalization factor for the functional integral has been chosen so that

\[
\int [Df] \exp \left[ -\frac{1}{2\xi} \int dz f^2(z) \right] = 1.
\] (2.12)

It is trivial to check that the Landau gauge choice \( \xi = 0 \) corresponds to the gauge choice \( f = 0 \). This is expected since the weighting functional \( \exp\{-\frac{1}{2\xi} \int dx f^2\} \) for the average allows \( f \) to fluctuate around \( f = 0 \) with the variance proportional to \( \xi \). Setting \( \xi = 0 \) confines \( f \) to be 0. The gauge-fixing parameter dependence appears as a multiplicative factor. Result (2.11) is also valid for the propagator of a scalar charged particle with charge \( q \). Equation (2.11) has been derived by other methods and discussed for four-dimensional QED [6,7].

By including the photon source and more pairs of the Green’s function \( \Delta(x) \) in the source (2.10), Eq. (2.11) may be generalized to cases where the correlation functions contain external photon lines and additional fermion lines. The \( f \) functional integral is still a gaussian integral. We omit the algebraically complicated intermediate steps which are completely
parallel to those for the fermion propagator. The $\xi$ dependence for a correlation function with $2n$ external fermion legs is

$$\langle \bar{\psi}(x_1)\bar{\psi}(x_2)\cdots\bar{\psi}(x_n)\psi(y_1)\psi(y_2)\cdots\psi(y_n) e^{iq\int j \cdot A_\xi} \rangle \xi = \exp\left\{ -\frac{\xi q^2}{2} \int dz \left[ \sum_{i=1}^{n} (\Delta(z-x_i) - \Delta(z-y_i)) + \int dz' \Delta(z-z') \partial_{\nu} j(z') \right]^2 \right\}$$

$$\times \langle \bar{\psi}(x_1)\bar{\psi}(x_2)\cdots\bar{\psi}(x_n)\psi(y_1)\psi(y_2)\cdots\psi(y_n) e^{iq\int j \cdot A_{f=0}} \rangle_{f=0} ,$$

(2.13)

where we have introduced the photon source term $iq \int j \cdot A$. Taking derivatives with respect to the photon source $j$ gives insertions of the photon fields in the correlation function. We note that the $\xi$ dependence is totally factorized. Therefore, the choice $\xi = 0$ is equivalent to the gauge choice $f = 0$.

**B. $\xi$ dependence in four and three dimensional QED**

We now study the behavior of the multiplicative $\xi$-dependent factor in four and three dimension spacetime. We do not consider the case $d = 2$ since there infrared divergences are so serious that the charged particles are confined [8]. To facilitate the notation, let us define

$$I(x) \equiv \frac{1}{2} \int dz \left[ \Delta(x-z) - \Delta(z) \right]^2$$

(2.14)

so that

$$G_\xi(x,y) = e^{-\xi q^2 I(x-y)} G_{f=0}(x,y) .$$

(2.15)

In momentum space, $I(x)$ can be expressed as

$$I(x) = \frac{1}{2} \int (dk) \frac{1}{k^4} (2 - e^{ikx} - e^{-ikx}) = \int (dk) \frac{1}{k^4} (1 - e^{ikx}) ,$$

(2.16)

2Here, we have made a change of variable $k \rightarrow -k$. In three dimensions, this causes an infrared problem which does not really matter if we only use regulators invariant under the inversion of momentum.
where $f(dk)$ represents the appropriate momentum integral conjugate to the spacetime. In $d$ dimensional Euclidean spacetime,

$$I(x) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^4} (1 - e^{ikx}), \quad (2.17)$$

which can be evaluated as

$$I(x) = \int_0^\infty ds s \int \frac{d^d k}{(2\pi)^d} e^{-sk^2} (1 - e^{ikx})$$

$$= \frac{1}{(4\pi)^{d/2}} \int_0^\infty ds s^{1-d/2} \exp \left[ -\frac{x^2}{4s} \right]$$

$$= -\frac{\Gamma(d/2 - 2)}{(4\pi)^2} (\pi x^2)^{2-d/2}. \quad (2.18)$$

Setting $d = 4$, Eq. (2.15) takes the form

$$G_\xi(x, y) = \exp \left\{ \frac{\xi_R q_R^2}{(4\pi)^2} \frac{2}{d - 4} \right\} \left[ \pi \mu^2 (x - y)^2 \right]^{-\xi_R q_R^2/(4\pi)^2} e^{-\xi_R q_R^2 \gamma/(4\pi)^2} G_{f=0}(x, y) + O(d - 4), \quad (2.19)$$

where we have introduced the renormalization scale $\mu$ by

$$q_R^2 \xi = q_R^2 \xi_R \mu^{4-d} \quad (2.20)$$

with $q_R^2$ and $\xi_R$ being the renormalized charge and gauge-fixing parameter. $\gamma$ in Eq. (2.19) is the Euler constant. The ultraviolet divergence appearing in the gauge dependent factor gives the usual gauge dependence of the fermion wave function renormalization factor in agreement with previous results \cite{13,14} as is the $(x - y)^2$ power modification factor \cite{3}.

In four dimensions, the $\xi$ dependence in the fermion propagator modifies the long distance power law behavior. Due to massless photons, the fermion propagator does not exhibit a pole in momentum space but a branch cut with the behavior $(p^2 + m^2)^{-1-(1+\nu)}$ \cite{11,12}. The $\xi$-dependent modification factor (2.19) leads to a $\xi$-dependent $\nu$ and thus a $\xi$-dependent on-shell condition for fermions in four dimensional QED \cite{3}.

We now consider QED in $d=3$ Euclidean space\cite{3}. We can view this as a dimensionally reduced field theory of a four dimensional scalar QED at the high temperature limit \cite{19,20}.

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\footnote{For a real three dimensional QED, there is evidence \cite{13,14} showing that the charged particles may}
\( I(x) \) is both ultraviolet and infrared finite. Eq. (2.18) reads explicitly \( I(x) = |x|/(8\pi) \). We have then

\[
G_\xi(x, y) = e^{-\frac{\xi^2}{8\pi} |x-y|} G_{f=0}(x, y). \tag{2.21}
\]

Hence the usual average over covariant gauge conditions has introduced a \( \xi \)-dependent exponentially decaying factor to the propagators of charged particles. Thus, the correlation length also acquires \( \xi \) dependence. If we extract the correlation length from a gauge variant two-point correlation function, different \( \xi \) values give different answers. Choosing \( \xi = 0 \) gives the answer corresponding to the original gauge theory quantized by the gauge condition \( \partial A = 0 \). This justifies a previous claim on the preference of the Landau gauge choice \( [15] \).

We note here that if only the gauge invariant correlation functions are considered, there are no \( \xi \) dependences inside the correlation functions which states that all \( \xi \)'s are equivalent for gauge invariant correlation functions.

We now examine the origin of the decaying factor in Eq. (2.21) and explain why an unphysical infrared cutoff can remove this artificial \( \xi \)-dependence. In previous subsection, we found the relation

\[
G_f(x, y) = \exp \left\{ -iq \int dz [\Delta(x - z) - \Delta(y - z)] f(z) \right\} G_{f=0}(x, y) \tag{2.22}
\]

which basically states that different choices of \( f \) are equivalent. However, this statement is be confined in three dimension QED due to infrared divergences. Of course, one can consistently add to the theory a topological mass term for the photon field without breaking the local gauge symmetry \( [15,16] \). This topological mass term can be even dynamically generated by interacting with the fermions \( [15,18] \). With this topological mass term, the charged particles are no longer confined \( [14,15] \).

\(^4\)It should be mentioned that if we view the three dimension theory as the high temperature limit of a four dimension theory, the gauge condition \( \partial A = 0 \) really corresponds to the Coulomb gauge choice in the original four dimensional theory.
based on the assumption that the factor

\[
\exp \left\{ -iq \int dz \left[ \Delta(x-z) - \Delta(y-z) \right] f(z) \right\}
\]

(2.23)
does not alter the long distance behavior of the propagator or, in particular, the position of the physical pole appearing in the Fourier transform of \( G_f(x,y) \). If \( f(z) \) is a localized function so that the exponent in factor (2.23) vanishes as \( x \) or \( y \) becomes large, \( G_f(x,y) \) has the same large distance behavior as \( G_{f=0}(x,y) \). We can then conclude that the gauge choice \( \partial A = f \) is equivalent to the choice \( \partial A = 0 \). However, when performing the average of gauge conditions, nonlocalized \( f \)’s are not excluded. It is not hard to see that the weighting functional

\[
\exp \left\{ -\frac{1}{2\xi} \int dz f^2(z) \right\} = \exp \left\{ -\frac{1}{2\xi} \int \frac{d^d k}{(2\pi)^d} \tilde{f}(k) \tilde{f}(-k) \right\}
\]

(2.24)
contains long wavelength modes (\( \tilde{f}(k) \) with wavelength \( 1/k \) longer than the separation between \( x \) and \( y \)). It is the inclusion of these long wave modes in the average that yields an exponentially decaying factor depending on \( |x-y| \). For the gauge conditions with \( f \) having wavelength shorter than \( |x-y| \), the \( x,y \) dependence in phase factor (2.23) is washed out after summing over different short wavelength contributions. In another word, the short wave fluctuations do not suffice to change the behavior of the long range correlation after the average. Indeed, \( I(x) \) does get its main contribution from the infrared region with \( k \) being order \( 1/|x| \) or less. This generates a piece proportional to \( |x| \) and therefore results an exponentially decaying factor. Employing an unphysical infrared cutoff to suppress the contributions from these long wave \( f \)’s can eliminate the gauge-fixing parameter dependence in the correlation length. Explicitly, introducing an infrared cutoff \( k_{\text{min}} \) to integral \( I(x) \), we have, as \( |x| \to \infty \),

\[
\int_{k > k_{\text{min}}} \frac{d^3 k}{(2\pi)^3} \frac{1}{k^4} (1 - e^{ik \cdot x}) = \frac{1}{2\pi^2} \int_{k_{\text{min}}}^{\infty} dk \frac{1}{k^2} \left( 1 - \frac{\sin k|x|}{k |x|} \right)
\]

\[
= \frac{1}{2\pi^2} \left( \frac{1}{k_{\text{min}}} - |x| \int_{k_{\text{min}}}^{\infty} ds \frac{\sin s}{s^3} \right) \to \frac{1}{2\pi^2 k_{\text{min}}} \quad (2.25)
\]

5Here \( \tilde{f}(k) \) is the Fourier transform of \( f(z) \).
which does not depend on \( x \). Therefore, the average over the short wave \( f \)'s does not modify the long distance behavior of the propagators but an overall constant. This explains why an unphysical infrared cutoff proposed in reference \( [4,5] \) can remove the \( \xi \)-dependent modification of the long distance behavior.

Since lower dimension field theories are more sensitive to the infrared region, this exponentially decaying factor does not appear in \( d = 4 \). In four dimensions, only the power law of the propagator is changed. For four-dimensional thermal field theories, the imaginary time formalism leads to a Euclidean functional representation for thermal correlation functions with one dimension of the spacetime compactified. Therefore, we expect that the average over different covariant gauge choices may also cause serious modifications to the two-point correlation functions.

C. Four-dimensional thermal QED

We now study the four-dimensional thermal QED which is equivalent to QED in three spatial dimensions plus an additional compactified imaginary time dimension. As such, the momentum in this dimension is discretized so that the momentum integral for \( I(x) \) in Eq. (2.16) is a “sum-integral”:

\[
\int (dk) \rightarrow T \sum_{k_0} \int \frac{d^{d-1}k}{(2\pi)^{d-1}}.
\]  

Since there is the zero temperature part contributing to the sum-integral, it is ultraviolet divergent. Using the dimensional regularization, we calculate this sum-integral in Appendix A and simply quote the result \( [A] \) as:

\[
I(\tau, x) = \frac{1}{(4\pi)^2} \frac{1}{(4\pi)^2} \ln \left( 1 + e^{-4\pi T|x|} - 2 \cos(2\pi T \tau) e^{-2\pi T |x|} \right) + \frac{T|x|}{8\pi} + O(d - 4).
\]  

Here \( I(x) \) has been written as \( I(\tau, x) \) with \( \tau \) and \( x \) being the imaginary time and spatial coordinate respectively. It is not hard to check that for large \( |x| \) or high \( T \)
\[ I(\tau, x) \rightarrow \frac{T|x|}{8\pi} \] (2.28)

which is in agreement with the analysis for the three dimensional Euclidean theory discussed in previous section. Therefore, the correlation length contains a \( \xi \)-dependent piece \( \xi q^2 T|x|/(8\pi) \).

So far the correlation functions studied are all defined in imaginary time. To study the damping rate, a retarded real-time correlation function is required. We can analytically continue it into real-time by replacing \( \tau \) with \( it \) to obtain the corresponding real-time propagator.\(^6\) Performing this analytic continuation for \( I(\tau, x) \) gives

\[
I(\tau, x) = \frac{1}{(4\pi)^2} (4\pi T^2)^{d/2-2} \left[ \frac{2}{4-d} + \gamma \right] + \frac{1}{(4\pi)^2} \ln \left( 1 + e^{-4\pi T|x|} - 2 \cosh(2\pi T|t|) e^{-2\pi T|x|} \right) \frac{T|x|}{8\pi} + O(d - 4). \tag{2.29}
\]

For large \(|x|\) or \(|t|\), we have

\[
I(t, x) \sim \frac{T}{8\pi} \left[ |t|\theta(|t| - |x|) + |x|\theta(|x| - |t|) \right]. \tag{2.30}
\]

Inserting this large coordinate argument behavior into Eq. (2.15) gives

\[
G_\xi(t, x) \sim \exp \left\{ -\frac{\xi q^2 T}{8\pi} \left[ |t|\theta(|t| - |x|) + |x|\theta(|x| - |t|) \right] \right\} G_{f=0}(t, x), \tag{2.31}
\]

where we have shortened\(^7\) the Green’s function notation \( G(x, y) \) to \( G(x-y) \) because of the spacetime translation invariance. The \( \xi \)-dependent large time damping factor causes a \( \xi \)-dependent damping rate if we use the general covariant gauge. To avoid any modification of the large time behavior coming from averaging the gauge conditions, we can choose \( \xi = 0. \)

\(^6\) This analytic continuation yields real-time ordered correlation functions. However, it is not hard to show that the exponentially decaying factor we are concerned is the same as that for the retarded correlation function.

\(^7\) We have implicitly switched back and forth between the notations \( x \) and \((t, x)\) for the spacetime coordinate.
This choice yields the damping rate for the original gauge theory, if the theory is invariant for different gauge constrains (without involving any average) when quantizing it. We also provide a more familiar derivation of Eq. (2.11) based on the proper vertex Ward identities in Appendix B where Equation (B3) shows that the \( \xi \) dependence we discussed above is the same \( \xi \) dependence as reported in reference [3]. We now examine again why an unphysical infrared cutoff can also remove this \( \xi \) dependence [5]. Introducing a small mass term \( m^2 \) to cutoff the \( k \) integral for \( I(\tau, x) \) at the infrared region, and using the usual contour trick to do the \( k_0 \) sum, the finite temperature part of \( I(\tau, x) \) is expressed as

\[
I(T)(\tau, x) = -\frac{d}{dm^2} \left\{ \int \frac{d^3k}{(2\pi)^3} \frac{n(\sqrt{k^2+m^2})}{2\sqrt{k^2+m^2}} \left[ e^{ik\cdot x - \tau \sqrt{k^2+m^2}} + e^{ik\cdot x + \tau \sqrt{k^2+m^2}} - 2 \right] \right\}, \tag{2.32}
\]

where \( n(\omega) \) is the Bose distribution factor

\[
n(\omega) \equiv \frac{1}{e^{\beta \omega} - 1} \tag{2.33}
\]

with \( \beta = 1/T \). Doing the analytic continuation \( \tau \rightarrow it \), we find that as \( t \) or \( |x| \) becomes large, the Riemann-Lebesgue lemma kills \( t \) and \( |x| \)-dependent part in \( I(T)(t, x) \). Hence the large coordinate argument behavior does not obtain any \( \xi \) dependence but an overall constant. This shows why an unphysical cutoff in reference [5] can remove the gauge-fixing parameter dependence in the damping rate.

III. GAUGE-FIXING PARAMETER DEPENDENCE IN THERMAL QCD

The gauge-fixing parameter dependence we studied for QED can also be derived by using Ward identities. Since to leading order in QCD, Ward identities are the same as that in QED, we expect that at the leading order, we should be able to find similar gauge-fixing parameter dependence as occurs in QED. This is indeed the case. Since the derivation for QED is completely parallel to the leading order derivation for QCD, it suffices to just provide the derivation for QCD at the leading order.
A. fermionic damping rate

Let us consider the fermion propagators. To simplify the notation, we shall suppress the color indices of the fermion fields. We shall use the Ward identities to derive the result for the QCD fermion propagator analogous to the result (2.11). Taking the derivative with respect to \( \xi \) which introduces an insertion of the gauge-fixing term in the functional integral, we obtain

\[
\frac{d}{d\xi} G_{\xi}(x, y) = \frac{1}{2\xi^2} \int dz \left\langle \bar{\psi}(x) \psi(y) \left[ \partial A^a(z) \right]^2 \right\rangle,
\]

where \( \psi \) and \( A \) represent the fermion fields and the gauge fields respectively and \( a \) is the color index in the adjoint representation. Using the equation of motion for the ghost field, the BRS transform of the correlation function \( \left\langle \bar{\psi}(x) \psi(y) \partial A^a(z) \bar{c}^a(z) \right\rangle \) produces the relation

\[
\left\langle \bar{\psi}(x) \psi(y) \left[ \partial A^a(z) \right]^2 \right\rangle = ig \xi \left\langle \bar{\psi}(x) \left[ c^b(x) - c^b(y) \right] T^b \psi(y) \left[ \partial A^a(z) \right] \bar{c}^a(z) \right\rangle,
\]

where \( T^b \) are the generators in the fermion representation and \( \bar{c}^b(z) \) and \( c^b(z) \) are the Faddeev-Popov ghost fields. Noting that

\[
\left\langle c^b(x) A^a(y) \bar{c}^a(z) \right\rangle = 0,
\]

we have, at the leading order,

\[
\left\langle \bar{\psi}(x) \psi(y) \left[ \partial A^a(z) \right]^2 \right\rangle = -ig \xi \left\langle \bar{\psi}(x) \left[ c^b(x) - c^b(y) \right] T^b \psi(y) \left[ \partial A^a(z) \right] \bar{c}^a(z) \right\rangle + O(g^4)
\]

\[
= -ig \xi \left\langle \bar{\psi}(x) T^a \psi(y) \left[ \partial A^a(z) \right] \Delta_{gh}(x-z) - \Delta_{gh}(y-z) \right\rangle + O(g^4),
\]

where the ghost propagator \( \Delta_{gh}(x) \) is defined as

\[
\left\langle c^a(x) \bar{c}^b(y) \right\rangle \equiv \delta^{ab} \Delta_{gh}(x-y).
\]

Similarly, the BRS transform of \( \left\langle \bar{\psi}(x) T^a \psi(y) \bar{c}^a(z) \right\rangle \) yields

\[
\left\langle \bar{\psi}(x) T^a \psi(y) \left[ \partial A^a(z) \right] \right\rangle = -ig \xi \left\langle \bar{\psi}(x) T^a T^a \psi(y) \right\rangle \Delta_{gh}(x-z) - \Delta_{gh}(y-z) \right\rangle + O(g^3)
\]

\[
= -ig \xi C_F G_{\xi}(x, y) \left[ \Delta_{gh}(x-z) - \Delta_{gh}(y-z) \right] + O(g^3),
\]
where $C_F$ is the Casimir for the fermion representation. Combining results (3.4) and (3.6) above gives

$$\frac{d}{d\xi} G_\xi(x, y) \simeq -\frac{1}{2} g^2 C_F \int dz \left[ \Delta_{gh}(x-z) - \Delta_{gh}(y-z) \right]^2 G_\xi(x, y) + O(g^4). \quad (3.7)$$

Perturbatively,

$$\Delta_{gh}(x) = \Delta(x) + O(g^2). \quad (3.8)$$

Therefore, to the leading order, the above differential equation shows that the leading $\xi$ dependence of the fermion propagator can be expressed as

$$G_\xi(x, y) \simeq \exp \left\{ -\frac{\xi g^2 C_F}{2} I(x-y) \right\} G_{f=0}(x, y). \quad (3.9)$$

We note here that a QED derivation may be obtained simply by ignoring all the color indices, changing $T^a \to 1$, $g \to q$, and setting all the leading approximation to be exact. For QCD, the leading gauge-fixing parameter dependence of the fermion propagator is the same as the exact $\xi$ dependence of a fermion propagator in QED with effective charge $g C_f^{1/2}$. As discussed before, at finite temperature, if we extract the damping rate of fermionic excitations from the fermion propagator calculated in a general covariant gauge, we shall get a $\xi$-dependent damping rate. This is the dependence reported in reference [3]. We can either choose Landau gauge $\xi = 0$ or use an unphysical infrared cutoff to get rid of the modification of the long time behavior due to taking the average over gauge conditions.

**B. Static electric screening length**

We now turn to consider the static chromoelectric screening length to next-to-leading order. Since the relevant energy scale is $gT$, it is convenient to use the dimensionally

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8This is also true after performing the Braaten-Pisarski resummation since the resumed proper vertices and propagators still satisfy the QED-type Ward identities [1][2].
reduced effective theory (so called EQCD) which involves only the static gauge fields to describe thermal QCD \cite{22}. To the order we are concerned, we only need to study a three dimensional Euclidean effective theory with Lagrangian

\[ L_{\text{EQCD}} = \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} (D_i A_0)^a (D_i A_0)^a + \frac{1}{2} m_{el}^2 A^a_0 A^a_0 \]  

where the covariant derivative \( D_i \) is defined as

\[ (D_i A_0)^a = \partial_i A_0^a - ig f^{abc} A_b^i A_c^a \]  

and \( m_{el} \) is the leading order Debye mass with the value

\[ m_{el}^2 = \frac{1}{3} \left( C_A + \frac{N_f}{2} \right) g^2 T^2. \]  

Here, \( C_A \) is the Casimir of the adjoint representation for the gauge group and \( N_f \) is the number of fermion flavors. Of course, we shall study, for a general covariant gauge, the gauge-fixing parameter \( \xi \) dependence of the static propagator \( D^{ab}(x,y) \) of the \( A_0^a \) fields. We have not explicitly included the gauge-fixing term nor the ghost fields term. We employ the same steps as in the last subsection for calculating the gauge dependence of a fermion propagator. All we need to do is to change all the fermion fields into the \( A_0^a \) fields and replace the generators \( T^a \) by the generators in the adjoint representation under which \( A_0^a \) transforms. Thus, the effective charge squared \( g^2 C_F \) appearing in Eq. (3.9) is changed to \( g^2 C_A \). After these replacements, we obtain

\[ D_{\xi}^{ab}(x,y) \simeq \exp \left\{ -\frac{\xi g^2 C_A}{8\pi} |x-y| \right\} D_{\xi=0}^{ab}(x,y). \]  

Performing the Fourier transform gives again a \( \xi \)-dependent singularity in the propagator. This dependence appears in the result\footnote{There, a \( \xi \)-dependent term \( \xi g^2 N m_{el} T/(4\pi) \) contributes to the self energy which indicates a correction \( \xi g^2 N T/(8\pi) \) to the leading screening mass \( m_{el} \). \( N \) is the number of colors, or equivalently, the Casimir of the adjoint representation for SU(N) gauge group.} found in references \cite{4,5} where the \( \xi \) dependence
is removed by introducing an unphysical cutoff at the infrared region of the loop integral involved in the self energy evaluation.

To avoid confusion, we like to add following comment. It appears that in reference [4], the Feynman gauge choice, $\xi = 1$, coincides with the result obtained by introducing an unphysical infrared cutoff. The free gauge boson propagator

$$G(p) = \frac{\delta_{ij} - \hat{p}_i \hat{p}_j}{p^2} + \frac{\xi \hat{p}_i \hat{p}_j}{p^2}$$

(3.14)

contains the combination $\xi - 1$ as the coefficient of $\hat{p}_i \hat{p}_j/p^2$. At the one-loop order, it can be shown explicitly [4] that the part in the gauge boson propagator proportional to $\hat{p}_i \hat{p}_j/p^2$ contributes to the self energy and causes a shift of the position of the pole of $A_0$ propagator. Introducing a technical infrared cutoff removes this contribution to the pole position of this propagator from the $\hat{p}_i \hat{p}_j/p^2$ part of the gauge boson propagator. This can be mimicked by choosing $\xi = 1$. However, the transverse part of the gauge boson propagator receives additional radiative corrections while the pure longitudinal part does not as a consequence of the Ward identity. Due to these radiative corrections, this transverse piece becomes less singular at the infrared region [4,21] and a physical infrared cutoff gets induced. Consequently, its $\hat{p}_i \hat{p}_j$ piece does not shift the position of the pole in $A_0$ propagator. On the other hand, the longitudinal part remains the same and does shift the position of pole except in Landau gauge with $\xi = 0$. Thus, the Landau gauge choice produces the same result as a naive choice of Feynman gauge where in addition the radiative corrections to the gauge boson propagator are ignored. For theories containing a topological mass term, the gauge boson propagator has the form

$$G(p) = \frac{\delta_{ij} - \hat{p}_i \hat{p}_j}{p^2 + m^2_{\text{topo}}} + \frac{m_{\text{topo}} \epsilon_{ijk} p_k}{(p^2 + m^2_{\text{topo}}) p^2} + \frac{\xi \hat{p}_i \hat{p}_j}{p^2}$$

(3.15)

where $m_{\text{topo}}$ is the topological mass. Here it is easy to see that the infrared behavior of the longitudinal part is different from that of the transverse part. Choosing a vanishing gauge-fixing parameter is the same as putting an infrared cutoff.
IV. CONCLUSIONS

In conclusion, we have shown that in three Euclidean dimensions or for four-dimensional thermal gauge field theories, the usual averaging procedure for getting a general covariant gauge-fixing term may introduce gauge-fixing parameter dependent modifications to the large distance behavior of the gauge dependent correlation functions. The gauge-fixing parameter dependent modification to the large distance behavior of the correlation function is the origin of the gauge dependence encountered in some perturbative evaluations of the damping rate and the Debye screening length. Choosing a vanishing gauge-fixing parameter (Landau gauge) or introducing an unphysical infrared cutoff enables us to avoid this gauge-fixing parameter dependent modification at the long distance introduced by averaging over a class of gauge conditions. If the theory is gauge invariant in the way that it can be quantized by using different gauge constrains (without involving any average), we can then extract physics from gauge variant propagators evaluated in a general covariant gauge by choosing a vanishing gauge-fixing parameter.

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APPENDIX A: EVALUATION OF $I(X)$ FOR THERMAL GAUGE THEORY

For a compactified spacetime, the spacetime point $x$ is understood as $x = (\tau, \mathbf{x})$. Using the dimensional regularization to regulate the ultraviolet divergence, we evaluate the integral $I(x)$ defined by Eq. (2.10) as

$$I(\tau, \mathbf{x}) = T \sum_{k_0 \neq 0} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{(k^2 + k_0^2)^2} \left( 1 - e^{ik_0\tau + i\mathbf{k} \cdot \mathbf{x}} \right)$$

$$= T \sum_{k_0 \neq 0} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{(k^2 + k_0^2)^2} - T \sum_{k_0 \neq 0} \int \frac{d^3k}{(2\pi)^3} \frac{e^{ik_0\tau + i\mathbf{k} \cdot \mathbf{x}}}{(k^2 + k_0^2)^2}$$

$$+ T \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{k^4} (1 - e^{i\mathbf{k} \cdot \mathbf{x}}) + O(d - 4)$$

$$= 2T \frac{\Gamma\left(\frac{5-d}{2}\right)}{(4\pi)^{(d-1)/2}} (2\pi T)^{d-5} \zeta(5 - d) - T \sum_{k_0 \neq 0} \frac{1}{8\pi |k_0|} e^{ik_0\tau - |k_0||\mathbf{x}|} + T|\mathbf{x}| \frac{8\pi}{8\pi} + O(d - 4)$$

$$= \frac{1}{(4\pi)^2} \frac{(4\pi T^2)^{d/2 - 2}}{(d-1)/2} \left[ \frac{2}{4 - d} + \gamma \right]$$

$$+ \frac{1}{(4\pi)^2} \ln\left(1 - e^{-2\pi T|\mathbf{x}|} + i2\pi T \tau\right) + \ln\left(1 - e^{-2\pi T|\mathbf{x}|} - i2\pi T \tau\right) + T|\mathbf{x}| \frac{8\pi}{8\pi} + O(d - 4)$$

$$= \frac{1}{(4\pi)^2} \frac{(4\pi T^2)^{d/2 - 2}}{(d-1)/2} \left[ \frac{2}{4 - d} + \gamma \right]$$

$$+ \frac{1}{(4\pi)^2} \ln\left(1 + e^{-4\pi T|\mathbf{x}|} - 2\cos(2\pi T \tau) e^{-2\pi T|\mathbf{x}|}\right) + T|\mathbf{x}| \frac{8\pi}{8\pi} + O(d - 4). \quad (A1)$$

APPENDIX B: ALTERNATE DERIVATION BASED ON THE PROPER VERTICES WARD IDENTITIES

Consider the fermion self energy diagrams in QED. The only place where the $\xi$ parameter can enter is in the longitudinal part of the photon propagator $\xi k_\mu k_\nu / k^4$. Therefore, the derivative of the self energy $\Sigma(p)$ with respect to $\xi$ is

$$\frac{d}{d\xi} \Sigma(p) = q^2 \int (dk) \frac{k_\mu k_\nu}{k^4} \Gamma_\mu(k, p, p + k) G(p + k) \Gamma_\nu(-k, p + k, p)$$

$$+ \frac{q^2}{2} \int (dk) \frac{k_\mu k_\nu}{k^4} \Gamma_{\mu\nu}(k, -k, p, p), \quad (B1)$$

where $\Gamma_\mu(k, p, p + k)$ and $\Gamma_{\mu\nu}(k, k', p, p + k + k')$ are the proper photon-fermion three-point and four-point vertices respectively. For the proper vertices, our convention is that the last
two momentum arguments are the momentum of fermion legs while the $k$ and $k'$ denote the photon momenta. Ward identities \[1,21\] give

\[k_\mu \Gamma_\mu(k, p, p + k) = G^{-1}(p + k) - G^{-1}(p)\]

\[k_\mu k'_\nu \Gamma_{\mu\nu}(k, -k, p, p) = G^{-1}(p + k) + G^{-1}(p - k) - 2G^{-1}(p).\] (B2)

Inserting these identities into Eq. (B1) produces

\[\frac{d}{d\xi} \Sigma(p) = -q^2 G^{-1}(p) \int (dk) \frac{1}{k^4} \left[ G(p + k) G^{-1}(p) - 1 \right] \] (B3)

Since $G(p)G^{-1}(p) = 1$, taking the derivative with respective to $\xi$ yields

\[\frac{d}{d\xi} \Sigma(p) = \frac{d}{d\xi} G^{-1}(p) = -G^{-1}(p) \frac{d}{d\xi} G(p) G^{-1}(p).\] (B4)

Combining the two equations above, we obtain

\[\frac{d}{d\xi} G(p) = q^2 \int (dk) \frac{1}{k^4} \left[ G(p + k) - G(p) \right],\] (B5)

which can be written in coordinate space as

\[\frac{d}{d\xi} G(x) = q^2 \int (dk) \frac{1}{k^4} (e^{ikx} - 1) G(x).\] (B6)

It is straightforward to solve the differential equation above to get

\[G_{\xi}(x) = \exp \left\{ -\xi q^2 \int (dk) \frac{1}{k^4} (1 - e^{ikx}) \right\} G_{\xi=0}(x).\] (B7)

This is the result (2.11) derived by functional methods in the main text.

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\(^{10}\)This equation agrees with the equations appearing in references \[3,5\] where the gauge-fixing parameter dependence was examined.
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