Hybrid Quantum Systems with Collectively Coupled Spin States: Suppression of Decoherence through Spectral Hole Burning

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Spin ensemble based hybrid quantum systems suffer from a significant degree of decoherence resulting from the inhomogeneous broadening of the spin transition frequencies in the ensemble. We demonstrate that this strongly restrictive drawback can be overcome simply by burning two narrow spectral holes in the spin spectral density at judiciously chosen frequencies. Using this procedure we find an increase of the coherence time by more than an order of magnitude as compared to the case without hole burning. Our findings pave the way for the practical use of these hybrid quantum systems for the processing of quantum information.

PACS numbers: 42.50.Pq, 42.50.Ct, 42.50.Gy, 32.30.-r

Hybrid quantum circuits that conflate the advantages of different physical systems to achieve new device functionalities have recently shifted to the center of attention [1]. This is largely because a new generation of experiments [2–11] lends encouraging plausibility to the vision of using such hybrid device concepts to reliably store and manipulate quantum information using atoms, spins or solid-state qubits [12–17]. In particular, the recent achievements in strongly coupling large spin ensembles to superconducting microwave cavities [2–6, 11] hold promise for combining many of the advantageous features of microwave technology with the long spin coherence times found, e.g., in crystallographic defects of diamond.

Whereas the collective coupling to a whole ensemble of spins is the key to reach the strong-coupling limit, the ensemble generally comes with the downside of inhomogeneously broadenend spin transition frequencies. As it turns out, the decoherence resultant from this broadening is currently the major bottleneck for the processing of quantum information in these hybrid quantum systems. First attempts at resolving this problem have meanwhile been put forward: On the one hand, it was shown that the decoherence is naturally suppressed for very strong coupling when the spectral spin distribution realized by the ensemble falls off sufficiently fast in its tails. Signatures of this so-called “cavity protection effect” [18, 19], have meanwhile also been observed experimentally [11, 20]. To fully bring to bear the potential of this effect requires, however, to go to very high values of the coupling strength, which are presently difficult to reach experimentally. On the other hand, sophisticated concepts for the spectral engineering of the spin density profile have been proposed [21, 22]. These concepts rely, however, on a strong modification of the intrinsically predefined density profile that is again very challenging to implement experimentally. Here we present a method that circumvents the problems of both approaches by building on a very elementary concept that requires only a considerably reduced experimental effort. Specifically, we demonstrate that the burning of two judiciously placed spectral holes in the spin distribution suffices to drastically increase the coherence properties of the hybrid spin-cavity system. From the viewpoint of quantum control our approach constitutes a new and efficient strategy to stabilize Rabi oscillations in the strong-coupling limit of cavity QED [23, 24].

To connect our theoretical work directly with the experiment we will study in the following the recently implemented case of a superconducting microwave resonator strongly coupled to an ensemble of negatively charged nitrogen-vacancy centers in a diamond (see illustration in Fig. 1) [2, 3, 11, 20]. In particular, we will work directly with the set of parameters for which excellent agreement has recently been found between the experimental data and the theoretical model [11, 20]. The latter builds on the Tavis-Cummings Hamiltonian ($\hbar = 1$) [25], which describes the dynamics of a single-mode cavity coupled to a spin ensemble in the dipole and
rotating-wave approximation,

\[ \mathcal{H} = \omega_c a^\dagger a + \frac{1}{2} \sum_j \omega_j \sigma_j^z + \sum_j \left[ g_j \sigma_j^z a^\dagger - g_j^* \sigma_j^z a \right] - i \left[ \eta(t) a^\dagger e^{-i\omega t} - \eta^*(t) a e^{i\omega t} \right]. \]  

(1)

Here \( \sigma_j^+ \), \( \sigma_j^- \), \( \sigma_j^z \) are the Pauli operators associated with the individual spins of frequency \( \omega_j \). Each spin is coupled with a strength \( g_j \) to the single cavity mode of frequency \( \omega_c \), in which photons are created and annihilated through the operators \( a^\dagger \) and \( a \). The probing electromagnetic field injected into the cavity is characterized by its carrier frequency \( \omega \) and by the amplitude \( \eta(t) \) (the time variation of \( \eta(t) \) is assumed to be much slower than \( 1/\omega )

Next, we derive the semiclassical equations of motion for the cavity and spin amplitudes using the following standard approximations: We assume that the energy of the thermal photons of the external bath is substantially smaller than that of the cavity photons. We also restrict our analysis to the case of incoming signals with a sufficiently low power such that the Holstein-Primakoff-approximation holds, \( \langle \sigma_j^z \rangle \approx -1 \). We also neglect the dipole-dipole interaction between spins as we assume a very dilute spin concentration in the ensemble. Finally, the collectively enhanced coupling strength to the resonator, \( \Omega = \sum_j g_j^2 \), satisfies \( \Omega \ll \omega_c \), justifying the rotating-wave-approximation. With these simplifications the following system of coupled equations for the cavity and spin amplitudes can be written down (in the frame rotating with \( \omega )

\[ \dot{A}(t) = -[\kappa + i(\omega_c - \omega)] A(t) + \sum_j g_j B_j(t) - \eta(t), \]  

(2a)

\[ \dot{B}_j(t) = -[\gamma + i(\omega_j - \omega)] B_j(t) - g_j A(t), \]  

(2b)

where \( A(t) \equiv \langle a(t) \rangle, B_j(t) \equiv \langle \sigma_j^-(t) \rangle \) and \( \kappa, \gamma \) are the dissipative cavity and spin losses, respectively.

Large spin ensembles (\( N \) is about \( 10^{12} \) in \[11,20]\)) are best described by the continuum limit of the normalized spectral density \( \rho(\omega) = \sum_j g_j^2 \delta(\omega - \omega_j) / \Omega^2 \). The inhomogeneous broadening of the spin frequencies \( \omega_j \) and coupling strengths \( g_j \) then lead to a finite-width distribution \( \rho(\omega) \) centered around a certain mean frequency \( \omega_s \). The specific shape of this spectral density depends on intrinsic properties of the spin ensemble and of its environment. Even in cases where these properties are not known precisely, the functional form of \( \rho(\omega) \) can typically be determined by a careful comparison with the experiment based on stationary [3] or dynamical [11] transmission measurements. In the experimental situation realized in \[11,20]\), the profile of \( \rho(\omega) \) was, e.g., found to be well described by a q-Gaussian distribution [3] centered around \( \omega_s / 2 \pi = 2.6915 \) GHz, a full-width at half-maximum of \( \gamma_q / 2 \pi = 9.44 \) MHz and a \( q \)-parameter of 1.39. The cavity decay rate, \( \kappa / 2 \pi = 0.4 \) MHz (half-width at half-maximum). We will use these parameter also in the following, see Fig. 2a, but emphasize that our strategy to reduce the dephasing resultant from the finite width of \( \rho(\omega) \) is independent of its specific profile.

Starting point for this strategy is the insight that the non-Markovian dynamics of the spin system, which is described by \( \rho(\omega) \) and strongly coupled to the cavity mode, can be accurately modeled by an integral Volterra equation for the cavity amplitude \( A(t) \) (see Eq. 3 below and [11,20]). The latter includes a memory-kernel, which is responsible for the non-Markovian feedback of the spin ensemble on the cavity, so that the cavity amplitude at time \( t \) depends on all previous events \( t < t \). By performing a Laplace transform of this Volterra equation [20] or by carrying out a stationary transmission analysis [15,19], the total rate of decoherence turns out to be \( \Gamma = \kappa + \pi \Omega^2 \rho(\omega_s) \) in the limit of large coupling strengths, \( \Omega > \gamma \) and \( \gamma \to 0 \). In the typical case where \( \pi \Omega^2 \rho(\omega_s) > \kappa, \gamma \), the decoherence is dominated by the dephasing effect due to inhomogeneous broadening. More specifically, the value of \( \Gamma \) is determined in this case by the spin density \( \rho_s(\omega) \), evaluated close to the maxima of the two polaritonic peaks, \( \omega = \omega_s \pm \Omega \), split by the Rabi frequency \( \Omega_R \approx 2 \Omega \) due to strong coupling. Our approach is now to take this relation literally, which is tantamount to saying that the decoherence rate \( \Gamma \) can be strongly suppressed by burning two spectral holes into the spin distribution \( \rho(\omega) \) right at these two positions, \( \omega_h = \omega_s \pm \Omega \), such that \( \rho(\omega_h) = 0 \). The width of the holes \( \Delta_h \) should be very small, such as to remove only a negligible fraction of the spins by the hole burning. Taking into account that \( \Delta_h \) is limited from below by the spin dissipation rate, \( \Delta_h > \gamma \), we choose in the following \( \Delta_h/2 \pi = 0.7 \) MHz.

To demonstrate the efficiency of our approach explicitly, we first perform a stationary analysis \( [A(t) = \dot{B}_h(t) = 0] \) of the transmission \( T(\omega) \) through the microwave resonator as a function of the probing frequency \( \omega \). This quantity, which is directly accessible in the experiment [11,20], provides direct access to the occupation amplitude of the cavity \( [T(\omega) \propto A(\omega)] \). Assuming \( \gamma \to 0 \), the transmission \( T(\omega) \) acquires the following form,

\[ T(\omega) = \frac{i \kappa}{\omega - \omega_c - \Omega^2 \delta(\omega) + i \kappa + \pi \Omega^2 \rho(\omega)} \]  

(3)

which expression is normalized such as to reach the maximum possible value \( max([T(\omega)])/1 \) for suitably chosen \( \omega, \kappa, \) and \( \rho(\omega) \). The real function \( \delta(\omega) \) is the nonlinear Lamb shift [27] defined as

\[ \delta(\omega) = \mathcal{P} \int_0^\infty d\tilde{\omega} \rho(\tilde{\omega}) / \omega - \tilde{\omega} \]  

(4)

where \( \mathcal{P} \) stands for the Cauchy principal value. In the reference case taken from the experiment [11,20], where \( \Omega/2 \pi = 8.56 \) MHz, \( T/2 \pi = 3 \) MHz and the \( q \)-Gaussian form of \( \rho(\omega) \) has no holes, see Fig. 2a, the transmission \( [T(\omega)/2] \) displays the well-resolved double-peak structure typical for the strong-coupling regime, see Fig. 2b. If we now burn two narrow holes into the spin density at the
To understand this behavior it is best to analyze the real and imaginary parts of the denominator of $T(\omega)$, see Eq. (3). For the observed transmission resonances at $\omega = \omega_r$ with a maximum value of $|T(\omega_r)| = 1$ to occur, two conditions are satisfied simultaneously: (i) $(\omega_r - \omega_c)/\Omega^2 = \delta(\omega_r)$ and (ii) $\rho(\omega_r) = 0$. Consider first condition (i): In the reference case without holes, see Fig. 2(c), the nonlinear Lamb shift $\delta(\omega)$ displays rather smooth variations in the vicinity of the resonant frequencies $\omega_r$, determined by the intersection of $\delta(\omega)$ and a straight line $(\omega - \omega_c)/\Omega^2$. In contrast, for the case with hole burning, see Fig. 2(f), the nonlinear Lamb shift exhibits rapid variations around the two resonance points within a very narrow spectral interval. As a consequence, the resulting transmission peaks become substantially sharper. Due to the second condition (ii) they also dramatically increase in height. Note, that no resonance occurs at $\omega = \omega_c$ because $\rho(\omega)$ has a maximum at this point, and as a result, the second resonance condition is strongly violated, see Fig. 2(c),(f). A close examination of the structure of $T(\omega)$ shows, furthermore, that the narrow transmission peaks resultant from the hole burning do not replace the broad polaritonic peaks present in the reference case, but rather get to sit on top of them, see Fig. 3(a). As will be seen below, the different resonance width in $T(\omega)$ set two different time scales in the dynamics with, in particular, the sharp peaks in the transmission giving rise to an asymptotically slowly decaying dynamics with a strongly suppressed decoherence.

To explore whether the narrow holes we burnt into the spectral spin distribution at $\omega_h = \omega_s \pm \Omega$ have, indeed, the optimal location, we now also test all possible other hole positions symmetrically placed around the maximum of $\rho(\omega)$ at $\omega = \omega_s$. In Fig. 3(b) we present the numerical results for $T(\omega)$ as a function of the probe frequency $\omega$ and of different hole locations $\omega_h = \omega_s \pm \Omega$ scanned between $\bar{\omega} = 0$ and $\bar{\omega} = 16$ MHz: While for large hole spacings ($\bar{\omega} \gtrsim 11.5$ MHz) the effect of holes is negligible, in the interval $0.8$ MHz $\lesssim \bar{\omega} \lesssim 11.5$ MHz we always find two sharp peaks superimposed on the two polaritonic peaks approximately at the hole positions. In the limit when the holes are burnt very close together ($\bar{\omega} \lesssim 0.8$ MHz) these sharp peaks merge into a single one, located directly at the central frequency $\omega_s$ with a transmission maximum reaching unity in the limit of $\omega_h \rightarrow \omega_s$ [see the yellow cusp in Fig. 3(b) and the resonant values of the transmission function shown in Fig. 3(c)]. Using the symmetry of $\rho(\omega)$ with respect to $\omega_s$, this behavior can also be proven analytically (not shown). From this analysis we conclude that for obtaining the most pronounced pair of transmission resonances it is, indeed, the best strategy to burn the holes near the positions of the polaritonic peaks at $\omega_h = \omega_s \pm \Omega$. The small deviation observed in Fig. 3(c) between this value of $\omega_h$ and the optimum position of the holes is accounted for by the approximate character of the expression we used for total rate of decoherence, $\Gamma \approx \kappa + \pi \Omega^2 \rho(\omega_s \pm \Omega)$. To check the robustness of our method we also tested different functional forms for the hole profiles (Fermi-Dirac, q-Gaussian and rectangular distributions) and found qualitatively similar results to the Fermi-Dirac form employed for all of the above figures.
FIG. 3: (Color online) Transmission through the cavity $T(\omega)$ versus probe frequency $\omega$ for different locations of the holes, $\omega_h$, in the spin density profile, $\rho(\omega)$ (the width of the holes is $\Delta_h/2\pi = 0.7$ MHz). (a) Red (gray) curve: $|T(\omega)|^2$ in lin-log scale versus $\omega$ for $\omega_h = \omega_s \pm \Omega$. Black curve: Transmission in the absence of hole burning. (b) Yellow (light gray) areas mark the most pronounced peaks in $|T(\omega)|^2$ in the presence of hole burning. Blue (gray) areas stand for the secondary polaritonic peaks which stem from the case without hole burning. Dashed arrows designate the distance $\Omega_R$ between polaritonic peaks. The white vertical cut corresponds to the transmission shown in (a). (c) Resonant values of the transmission function $|T(\omega_h)|^2$ versus $\omega_h$. Only the left primary resonances are considered, $\omega_p < \omega_h$, as the right resonances appear symmetrically with respect to $\omega_s$.

state and the cavity mode $a$ contains initially a single microwave photon only, $A(t) = 1$. For the case without hole burning the solutions of Eq. (5) are just the damped Rabi oscillations $|A(t)|^2$ under the action of eleven successive rectangular microwave pulses of duration corresponding to the Rabi period, $\tau = 2\pi/\Omega R = 52$ ns, phase-switched by $\pi$ (every second pulse is shown as a vertical gray bar). Also here the asymptotic decay is much slower due to the presence of the holes. In all panels the holes in $\rho(\omega)$ have a width $\Delta_h/2\pi = 1.4$ MHz and are burnt at $t = 0$ at $\omega_h = \omega_s \pm \Omega$. The asymptotic decay $Ce^{-\Gamma t}$ with and without hole burning (see red lines) is determined by the constants $\Gamma/2\pi = 3$ MHz in (a) and a drastically reduced $\Gamma = 0.42\kappa = 2\pi \cdot 0.17$ MHz in (b). (c), (d) Dynamics of $|A(t)|^2$ under the action of eleven successive rectangular microwave pulses of duration corresponding to the Rabi period, $\tau = 2\pi/\Omega R = 52$ ns, phase-switched by $\pi$ (every second pulse is shown as a vertical gray bar). Here also the asymptotic decay is much slower due to the presence of the holes. In all panels the holes in $\rho(\omega)$ have a width $\Delta_h/2\pi = 1.4$ MHz and are burnt at $t = 0$ at $\omega_h = \omega_s \pm \Omega$.

FIG. 4: (Color online) (a),(b): Decay of the cavity occupation $|A(t)|^2$ from the initial state, for which a single photon with frequency $\omega_s$ resides in the cavity and all spins are unexcited. The asymptotic decay $Ce^{-\Gamma t}$ with and without hole burning (see red lines) is determined by the constants $\Gamma/2\pi = 3$ MHz in (a) and a drastically reduced $\Gamma = 0.42\kappa = 2\pi \cdot 0.17$ MHz in (b). (c),(d): Dynamics of $|A(t)|^2$ under the action of eleven successive rectangular microwave pulses of duration corresponding to the Rabi period, $\tau = 2\pi/\Omega R = 52$ ns, phase-switched by $\pi$ (every second pulse is shown as a vertical gray bar). Here also the asymptotic decay is much slower due to the presence of the holes. In all panels the holes in $\rho(\omega)$ have a width $\Delta_h/2\pi = 1.4$ MHz and are burnt at $t = 0$ at $\omega_h = \omega_s \pm \Omega$. The asymptotic decay $Ce^{-\Gamma t}$ with and without hole burning (see red lines) is determined by the constants $\Gamma/2\pi = 3$ MHz in (a) and a drastically reduced $\Gamma = 0.42\kappa = 2\pi \cdot 0.17$ MHz in (b). (c),(d): Dynamics of $|A(t)|^2$ under the action of eleven successive rectangular microwave pulses of duration corresponding to the Rabi period, $\tau = 2\pi/\Omega R = 52$ ns, phase-switched by $\pi$ (every second pulse is shown as a vertical gray bar). Here also the asymptotic decay is much slower due to the presence of the holes. In all panels the holes in $\rho(\omega)$ have a width $\Delta_h/2\pi = 1.4$ MHz and are burnt at $t = 0$ at $\omega_h = \omega_s \pm \Omega$.

Apparently a new type of physics is at work here after the initial transient decay regime has passed: Although the system is in the strong coupling regime, the two spectral holes slow down the leakage of the energy stored in the spin ensemble back into the cavity. In particular, when being even slower than the inverse of the cavity decay rate $\kappa$ alone, this sets a new global time scale for the entire decay rate $\Gamma$, corresponding to the width of the sharp resonance peaks which we identified before in Fig. 3(a). From the mathematical point of view such a slow asymptotic behavior is associated with the contribution of two poles in the Laplace transform of the Volterra equation (5), which appears due to the hole-burning in $\rho(\omega)$ (see [20] for details).

To demonstrate the efficiency of the hole burning ef-
fect also for quantum control schemes, we pump the cavity by a sequence of $\pi$ phase-switched rectangular pulses, each with a duration corresponding to the Rabi period, $\tau = 2\pi/\Omega_R$ and a carrier frequency $\omega = \omega_c = \omega_s$. As shown in [11], this procedure is very well suited to feed energy into the strongly coupled cavity-spin system, leading to giant oscillations of both spin and cavity amplitude, reproduced here in the left parts of Fig. 4(c,d). Not only do we observe that these driven oscillations are more pronounced when burning holes at $\omega_h = \omega_s \pm \Omega$, but we find, in particular, that the Rabi relaxation oscillations setting in after switching off the driving field are dramatically decreased to values that may even lie below the dissipation rate of the bare cavity. Experimentally, our approach can be implemented by exposing the spins near the desired hole positions to a strong driving signal resonant with higher energy levels that effectively removes the spins from the coupling process with the cavity.

In summary, we present an efficient method to suppress the decoherence in a single-mode cavity strongly coupled to an inhomogeneously broadened spin ensemble. By burning narrow spectral holes in the spin density at judiciously chosen positions the total decay rate is dramatically decreased to values that may even lie below the dissipation rate of the bare cavity. Experimentally, our approach can be implemented by exposing the spins near the desired hole positions to a strong driving signal resonant with higher energy levels that effectively removes the spins from the coupling process with the cavity.

We wish to thank S. Putz, R. Amsüss, and J. Majer for helpful discussions and acknowledge support by the Austrian Science Fund (FWF) through Project No. F49-P10 (SFB NextLite).