Lorentz invariance violation in top-down scenarios of ultrahigh energy cosmic ray creation

James R. Chisholm and Edward W. Kolb
Fermilab Astrophysics, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500, and
Enrico Fermi Institute, University of Chicago, Illinois 60637
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The violation of Lorentz invariance (LI) has been invoked in a number of ways to explain issues dealing with ultrahigh energy cosmic ray (UHECR) production and propagation. These treatments, however, have mostly been limited to examples in the proton-neutron system and photon-electron system. In this paper we show how a broader violation of Lorentz invariance would allow for a series of previously forbidden decays to occur, and how that could lead to UHECR primaries being heavy baryonic states or Higgs bosons.

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I. INTRODUCTION

UHECRs are an enduring mystery. Without the introduction of new particles or interactions, evading the Greisen–Zatsepin–Kuz’min (GZK) cutoff \[1, 2\] requires unidentified nearby sources. Even without the GZK cutoff, the “bottom-up” approach faces the challenge of finding in nature an accelerator capable of energies in excess of \(10^{20}\) eV.

“Top-down” scenarios assume that the UHECRs result from the fragmentation of a ultra-high energy hadronic jet produced by cosmic strings \[3\] or by the decay of a supermassive particle \[4\]. In the supermassive particle (wimpzilla) scenario, the UHECRs are of galactic origin, resulting from the decay of relic supermassive \((M \gtrsim 10^{19}\text{GeV})\) particles. Wimpzillas can be produced copiously in the early universe \[5\], therefore solving the energy problem. Since they would cluster in the dark matter halo of our galaxy \[6, 7, 8\], they also solve the distance problem. Detailed analysis of these decays, however, show that at high energy top-down scenarios produce more photons than protons \[9, 10, 11\] in the UHECR spectrum seen at Earth.

The top-down prediction of photon preponderance in UHECRs is the one major problem in an otherwise simple explanation. Results from the Fly’s Eye \[12\], Haverah Park \[13\], and AGASA \[14\] cosmic ray experiments all indicate that at energies above around \(10^{19}\) to \(10^{20}\) eV, protons are more abundant than photons in UHECRs. While photons are disfavored, it is not possible to be sure that the primary is indeed a proton.

The idea of violating LI (see Ref. \[15\] for a broad overview) has recently been studied in the context of UHECRs for proton decay \[16, 17\], atmospheric shower development \[18\], and modifications to the cutoffs of proton and photon spectra due to cosmic background fields \[19, 20, 21, 22, 23, 24\]. As shown in Ref. \[25\], LI violation can also lead to vacuum photon decay, and in general the decay of particles to more massive species. We exploit this fact to show how particles produced in top-down decays can undergo an “inverse cascade” to produce superheavy UHECR primaries.

II. BREAKING THE LAW

A. Modified Dispersion Relations

There are two approaches to breaking LI commonly used in the literature. The first \[16, 20, 21, 25, 26\] is generically to modify the dispersion relations for a particle of mass \(m\) and 3-momentum \(p = |\mathbf{p}|\).

The second method, which we will not utilize here, is instead to write down a particle Lagrangian \[27\] which includes Lorentz and CPT violating terms. A mapping of the Lagrangian terms to a dispersion relation is non-trivial (see \[28\] for a simple case); however, to first order the changes to the photon and electron propagators induce shifts to \(c^+\) and \(c^e\) \[17\]. Similarly, it was shown by \[25\] that loop quantum gravity effects produce modified dispersion relations similar to those considered elsewhere and here.

The simplest way to break LI, as shown in \[17\], is to write down a dispersion relation for a particle species \(i\) as

\[E^2 = p_i^2 c_i^2 + m_i^2 c_i^4.\] (1)

This is changing the “speed of light” or Maximum Attainable Velocity (MAV) for each particle to something slightly different than \(c\). The MAVs for different particle species are assumed \(a\) priori to be different in that and similar treatments \[26\].

In such a case, it is possible for previously unallowable reactions to occur, such as \(p \rightarrow n e^+ \nu_e\) \[16, 17\], \(\gamma \rightarrow e^+ e^-\) \[25, 26\] and \(\gamma \rightarrow \nu \bar{\nu}\) \[27\]. A conclusion reached, using the first reaction, is that neutrons may make up the dominant baryonic component of the UHECR primaries.

1 Although factors of \(c\) are explicitly included in equations, we still take it to be dimensionless with \(c^+ = c^e = 1\).
This, strictly, would be true if one was only tuning the proton and neutron MAVs and leaving all others equivalent \((= c)\). On the other hand, keeping every MAV as individually tunable gives an overwhelming number of free parameters.

In order to examine the consequences of varying all particle MAVs using the fewest number of free parameters, we extend the method used in Ref. [21] and write the dispersion relation as

\[
E^2 = p^2c^2 + m^2c^4 + p^2c^2f \left( \frac{p}{Mc} \right) + m^2c^4g \left( \frac{p}{Mc} \right) + mc^2pE \left( \frac{p}{Mc} \right) .
\]  

(2)

Here, \(f(x), g(x)\), and \(h(x)\) are dimensionless universal functions having the property \(f(0) = g(0) = h(0) = 0\), so that as \(p/M \to 0\) the normal dispersion relations are recovered. Here \(M\) is the mass scale that determines the relative degree of Lorentz violation; for our purposes we set it at the Planck mass \(M \approx 10^{19}\) GeV. Expanding these functions in Taylor series about \(x = 0\) and keeping terms of \(O(p^2)\) and smaller gives

\[
E^2 = m^2c^4 + mc^2pc \left( \frac{1}{2}g'(0) \frac{m}{M} \right) + p^2c^2 \left( 1 + \frac{1}{2}h'(0) \frac{m}{M} + \frac{1}{6}g''(0) \frac{m^2}{M^2} \right) .
\]  

(3)

The \(g'(0)\) term can be neglected for two reasons: at \(p \ll mc\) it would be experimentally detected as deviations in the non-relativistic kinetic energy; for \(p \gg mc\) it is negligible to the quadratic \(p\) term. Note that to this (lowest) order, \(f(x)\) has no effect.

For this section, we examine the cases of massless (photon) and massive (electron) particles:

\[
E^2_e = m^2c^4 + p^2c^2e \left( 1 + \frac{1}{2}h'(0) \frac{m}{M} + \frac{1}{6}g''(0) \frac{m^2}{M^2} \right) .
\]  

(4)

\[
E^2_\gamma = p^2c^2e .
\]  

(5)

We can now define

\[
c^2_e = c^2 \left( 1 + \frac{1}{2}h'(0) \frac{m}{M} + \frac{1}{6}g''(0) \frac{m^2}{M^2} \right) \]  

(6)

\[
m^2_e = m^2 \left( 1 + \frac{1}{2}h'(0) \frac{m}{M} + \frac{1}{6}g''(0) \frac{m^2}{M^2} \right)^{-2} \]  

(7)

to then write the dispersion relation in the familiar form of Eqn. [4] as

\[
E^2_e = m^2c^4_e + p^2c^2_e .
\]  

(8)

B. Example: Photon Decay

In this section we consider the tree-level photon decay process \(\gamma \rightarrow e^+e^-\), which is kinematically forbidden for \(c_e = c\). Following Ref. [17], we define the parameter \(\delta_{\gamma e}\) as

\[
\delta_{\gamma e} = c^2 - c_e^2 .
\]  

(9)

Using astrophysical constraints [29, 30], \(\delta_{\gamma e}\) can be limited to the range \(10^{-16} > \delta_{\gamma e} > -10^{-17}\). For \(\delta_{\gamma e} > 0\), photon decay occurs above a photon energy threshold given by

\[
E_{\text{th}} = \frac{2mc^2e}{\sqrt{\delta_{\gamma e}}} .
\]  

(10)

This decay rate of was computed in Coleman & Glashow (1997) as

\[
\Gamma = \frac{1}{2} \alpha (\frac{\delta_{\gamma e}}{c^2_e}) E \left[ 1 - \left( \frac{E_{\text{th}}}{E} \right)^2 \right] ^{3/2} .
\]  

(11)

For decay product energy above about \(2 \times 10^{20}\) eV, photons will outnumber protons, so we set \(E_{\text{th}}\) to this value and suggestively rewrite the decay rate above threshold as:

\[
\frac{\Gamma}{\hbar c} \approx (1 \text{ km})^{-1} \left( \frac{E}{2 \times 10^{20} \text{ eV}} \right) \left( \frac{\delta_{\gamma e}}{3 \times 10^{-29}} \right) .
\]  

(12)

This would rule out all but a terrestrial source of UHE photons. The value \(\delta_{\gamma e} \approx 3 \times 10^{-29}\) is used is a lower limit; if \(\delta_{\gamma e}\) would be any lower the energy threshold would become too high. This means that only \(\delta_{\gamma e} \in (3 \times 10^{-29}, 10^{-16})\) will correct for the photon overabundance in top-down scenarios.

Now we see what is required to have \(\delta_{\gamma e}\) in the necessary range to allow for this photon decay. Using Eqs. [10] and [11], we can write

\[
\delta_{\gamma e} = -\frac{1}{2} \frac{c^2}{\hbar} h'(0) \frac{m}{M} - \frac{1}{6} \frac{c^2}{\hbar} g''(0) \frac{m^2}{M^2} .
\]  

(13)

Since the functions \(g(x)\) and \(h(x)\) are \textit{a priori} arbitrary, so are the values and signs of their derivatives. It is argued in Ref. [21] that too strongly varying functions would be unphysical, and that the derivatives should be of order unity.\(^4\) Adopting this, we can ignore the \(g''(0)\) term in the above equation. In this case, in order to have photon decay we need to require

\(3\) This is identical to \(\xi - \eta\) for \(n = 2\) in the notation of Ref. [23].

\(4\) Note that if this holds also for \(f(x)\), we err in not including the \(f'(0)\) term in Eq. [4], as that dominates over the \(h'(0)\) term when \(p \gg mc\). We will not apologize much for this, as we are looking at the “lowest order” change in phenomenology (the change in the “speeds of light”) and not “next order” changes (such as vacuum photon dispersion).
dimension only. We want to minimize the maximum possible energy. Using the method of Langrange multipliers, \( E \) is minimized when
\[
\delta_{AX} = c_A^2 - c_X^2 = 1/2 h'(0) c^2 m_A - m_X/M \approx \frac{(m_X - m_A)^2}{2M}, \tag{14}
\]
where we have used \( h'(0) \sim -1 \).

The consequences of this for particle decays are as follows, as first noted by [17], and we reproduce their argument here.

Consider the decay \( A \rightarrow \{X\} \), where \( \{X\} \) is a collection of massive particles. We want to find the minimum possible energy \( E_{\text{min}} \) where this can occur. We can always lower final state energy by removing transverse components of momenta, so we're working in one dimension only. We want to minimize
\[
E(\{p_X\}) = \sum_X \sqrt{m_X^2 c_X^2 + p_X^2 c_X^2} \tag{15}
\]
subject to the constraint
\[
G(\{p_X\}) = \left( \sum_X p_X \right) - p_A = 0. \tag{16}
\]
The \( \{p_X\} \) are variables, and \( p_A, E_A > E_{\text{min}} \) are constants. Using the method of Langrange multipliers, \( E \) is minimized when
\[
\frac{\partial E}{\partial p_X} = \lambda \frac{\partial G}{\partial p_X}, \tag{17}
\]
which becomes
\[
\lambda = \frac{p_X c_X^2}{E_X} = \frac{p_X c_X^2}{\sqrt{m_X^2 c_X^2 + p_X^2 c_X^2}}. \tag{18}
\]

Thus all products move with the same velocity \( \lambda \). Note that \( \lambda \) is \( c_X \) for all \( X \). For the example we consider, in the event of massless particles among the decay products, they would minimize final state energy by carrying exactly zero momenta [17]. Since the \( \{c_X\} \) are the maximum attainable velocities for the decay products, that implies that \( \lambda < \min\{c_X\} \).

It is this last point we focus on: \( \min\{c_X\} \) in our case corresponds to \( \max\{m_X\} \). In the ultrarelativistic limit, \( E_A \approx p_A c_A \) and so on, and the particle with the lowest speed of light will carry most of the momentum (in the minimum energy case). Further, in this threshold case, \( \lambda \) is also the velocity of the incident particle [13]. Since \( \lambda \approx c_A \) in the regime we are considering, that means that the decay can only occur if \( c_A > \min\{c_X\} \). From Eq. (14) that implies that \( m_A < \max\{m_X\} \).

In other words, above a certain threshold, a decay may only be allowed if at least one decay product is more massive than the decaying particle. This can have serious consequences for the particle content of UHECRs, as we will now address.

### III. THE INVERSE CASCADE

In this model, for each individual reaction there is a threshold energy \( E_T \) above which particles can only decay if at least one of the decay products is more massive. For the decay \( A \rightarrow B + \ldots \), where \( B \) is the most massive decay product, \( m_B > m_A \), and \( m_\ldots \ll m_B \) the threshold energy goes like
\[
E_T \sim c^3 \sqrt{\frac{m_B^2 - m_A^2}{c_A^2 - c_B^2}}. \tag{19}
\]

Using equation (13) this reduces to
\[
E_T \sim c^3 \sqrt{M(m_A + m_B) \frac{h'(0)}{1}}^{-1/2}. \tag{20}
\]

Due to the large value of the Planck mass, this threshold is around the same order of magnitude for a wide range of reactions involving particles of mass much less than \( M \).

Consider an UHECR of mass \( m_0 \) produced with energy \( E_0 \gg E_T \). If normally (at energies \( E \ll E_T \)), particle 0 can be a decay product of a more massive particle 1, then here \( (E \gg E_T) \), 0 can decay into 1. We ignore the lighter products that will result as well; the substantial fraction of the incident energy will be carried by the most massive decay product.

Now, we have a particle of energy \( E_1 < E_0 \) and mass \( m_1 > m_0 \). If it can, it will also decay into a particle 2 with \( m_2 > m_1 \) and energy \( E_2 < E_1 \). As the decays continue into more and more massive final states, the final energy continues to decrease, until at some point we reach a particle N with energy \( E_N < E_T \) and mass \( m_N > m_0 \). Since we are now below threshold, the N particle can decay as usual into N-1 and so on into less massive particles.

So, to summarize, this inverse cascade decay process occurs in two stages.
1. Decay up: Decays into more and more massive, yet less energetic particles, when \( E > E_T \), and
2. Decay down: Regular decay schemes once \( E < E_T \).

### IV. PANDORA’S BOX (NEWLY ALLOWED REACTIONS)

We saw in Section II that at sufficiently high energies particles can only decay if one of the decay products is more massive. In Section III we saw how this can produce an “inverse cascade” to more and more massive particle species. In this section we address the specifics of how this cascade occurs, by examining which previously forbidden decays can now occur. All of the following are assumed to be occurring well above threshold, and we list them in several classes.

#### A. Tree-level QED

Here the bare QED vertex \( \gamma \rightarrow Q^+Q^- \) where \( Q \) is any charged particle is allowed as a decay. This also includes final states with extra photons and bound states of \( Q^+Q^- \), like \( \gamma \rightarrow \pi^0\gamma \).

#### B. Tree-level weak

It is in the weak sector that most of the interesting decays in the inverse cascade occur, due to the flavor-changing \( W \) vertex.

In this Lorentz-violating scheme, weak decays (such as that of the neutron) can happen in reverse, allowing proton decay via \( p \rightarrow ne^+\nu \) and flavor changing decays such as \( n \rightarrow \Delta \pi^0 \).

Not only that, decays to weak bosons become permissible, such as \( n_l \rightarrow t^\pm W^\mp \) and \( t^\pm \rightarrow n_l W^\mp \). Included in this are decays of leptons to heavier leptons due to virtual \( W \)’s. In the quark sector, this allows decays such as \( p \rightarrow nW^+ \) and so on. Generally, the flavor changing weak decay \( q \rightarrow q'W \) will convert all quarks to \( t \) quarks. In the absence of a fourth generation, a bare \( t \) quark is stable.

Since the \( Z \) is massive, decays such as \( \{ q, l, \nu_l \} \rightarrow Z\{ q, l, \nu_l \} \) (“\( Z \)-Cerenkov”) become permissible as well.

For the bosons: \( W^+ \rightarrow tb \) is allowed, and \( Z \)-Cerenkov is possible for the \( W \) as well (this is a bare weak vertex, as is \( W \rightarrow WZ \) and \( W \rightarrow WZ\gamma \)). The \( Z \) is unstable to the decay \( Z \rightarrow t\bar{t} \).

Now consider the Higgs: indeed we might be above the energy of weak symmetry breaking. Including a single massive Higgs \( H \) would allow: \( \{ f, W, Z \} \rightarrow \{ f, W, Z \}H \) (“Higgs-Cerenkov” for fermions and weak bosons) and \( \{ W, Z \} \rightarrow \{ W, Z \}HH \). In the absence of new physics beyond the Standard Model, the \( H \) is stable if it is heavier than the top quark. If it is lighter than the top, the Higgs is unstable to \( H \rightarrow t\bar{t} \).

#### C. Tree-level Strong

Since the bare QCD vertex preserves quark flavor and charge, not much happens in the bare quark sector, apart from the reversings of strong decays such as \( N \rightarrow \Delta \pi \) and \( \pi \rightarrow \rho \pi \). Just as for the massless photon, free gluon decay is allowed via \( g \rightarrow q\bar{q} \).

#### D. One-loop processes

One could formulate many more decays at the 1-loop level, but the rates for these for them are suppressed compared to the tree level decays. For this reason, we do not consider them further here, except to note two interesting decays of the photon:

\( \gamma \rightarrow \nu\bar{\nu} \) (QED and weak process). Note that since neutrinos are so much lighter than their heavy partners and the quarks, the threshold for photon to neutrino decay could be very low depending on how light the neutrinos are.

\( \gamma \rightarrow Z\gamma \) (weak process). This photon \( Z \)-Cerenkov decay occurs due to a \( W \)-loop.

#### E. Spin up

For bound states (and now restricting ourselves to bound states of three valence quarks) there exist higher mass resonances that only differ in the mass and total angular momentum (e.g., the \( \Delta \) and higher resonances for the proton and neutron). Since the state with higher angular momentum is more energetic, it is more massive and thus processes like \( N \rightarrow N^*\pi \), \( N^* \rightarrow N^{**}\pi \) and so on can act to “spin up” the particle to more and more massive states.

Thus, even the all top quark baryon \( ttt \) isn’t completely stable, but can decay via spin up.

### V. ESTIMATES OF DECAY RATES

For the photon decay \( \gamma \rightarrow e^+e^- \), the decay rate \( \Gamma \) well above threshold was

\[
\Gamma_{\gamma\rightarrow ee} = \frac{1}{2} \alpha \delta_{\gamma e} E. \tag{21}
\]

We adapt this form of the decay rate to other processes to make estimates, and argue for it as follows. At ultrarelativistic energies, the only energy scale is the incident energy \( E \), so \( \Gamma \propto E \). It is proportional also to the effective coupling constant involved in the decay, and to kinematic factors. We schematically write this as

\[
\Gamma \sim (\text{coupling}) \times (\text{kinematics}) \times E. \tag{22}
\]
For the photon decay, the coupling is $\alpha$ and the kinematic factor is $\delta_{\gamma e}$. We generalize these factors as follows.

We (first) consider decays of the form $A \rightarrow B + C$ where $m_A < m_B$, $m_B \gg m_C$. At threshold and beyond, most of the momentum of the final state is carried by the particle with the lowest speed of light, thus highest mass (particle $B$). The relevant kinematical factor will then be $\delta_{AB}$. This will continue to be true when final states with more than 2 particles are allowed, as long as $B$ continues to be the most massive particle.

The coupling factor is determined by looking at the tree level Feynman diagrams and counting vertices involved in the reaction.

Consider then the decay of the proton; in addition to the inverse of the neutron decay $p \rightarrow n e^+ \nu_e$ (“leptonic” decay), the decay $p \rightarrow n W^+$ (“bosonic” decay) is also allowed.

For the bosonic decay: $E_T \sim \sqrt{m_W M} \sim 2.8 \times 10^{19}$ eV, and as most of the energy goes into the $W$, $\delta_{\nu W} \sim m_W / 2M \sim 4 \times 10^{-18}$. The vertex is a single weak vertex; with coupling $\alpha_W = \alpha / \sin^2 \theta_W \sim 0.0316$. Thus, at $E = E_T, \Gamma \sim 4$ eV.

For the leptonic decay: $E_T \sim \sqrt{M(2m_p)} \sim 4.5 \times 10^{18}$ eV, and as most of the energy goes into the $n$, $\delta_{\nu n} \sim (m_n - m_p) / 2M \sim 5 \times 10^{-23}$. This reaction has two weak vertices with coupling $\alpha_W^2 \sim 10^{-33}$. Thus, at $E = E_T, \Gamma \sim 2.2 \times 10^{-7}$ eV.

Note that the distance travelled before decay is $\sim 1 / \Gamma$, and given $hc = 6.57 \times 10^{-24}$ eV pc, the $1 / \Gamma$ for these reactions is incredibly small: $10^{-24}$ pc for the bosonic and $10^{-17}$ pc for the leptonic. In the context of a UHECR, assuming comparable rates (within a few orders of magnitude) to the above, this inverse cascade happens essentially immediately, so that by the time any decay products reach the Earth, they have gone through both stages of the cascade and are now sub-threshold particles.

VI. CONSEQUENCES FOR UHECR COMPOSITION

Given the rapid rate of the inverse cascade process, there can be two outcomes for the UHECR population at earth.

First, if both the Decay Up and Decay Down sequences occur, then the constituent particles could, depending on the exact decay chains and nature of initial UHE primary, consist of protons, electrons, photons, and their antiparticles. Each particle spectrum would exhibit a cutoff at their prospective threshold energies, which are all related by the $\mu'(0)$ parameter.

Second, and more interesting, if the initial UHE primary is of high enough energy, the decay up sequence may occur and leave particles that can no longer decay into more massive particles, but are still above threshold for decay into less massive particles. These Most Massive Particles (MMPs) would then be the most energetic component of the UHECR population. What exactly the MMPs are depends on the model of particle physics used. Of particles known to exist now, the MMPs would consist of top quarks and excited bound states of top ($t\bar{t}$ mesons and $ttt$ baryons) quarks. If the Higgs is more massive than the top, then the MMPs would consist of single Higgs bosons. Including Supersymmetry and its expected spectrum of more massive superpartners, if the decay rate is still rapid enough the MMP would be the most massive superparticle (MMSP).

Of course, in the first outcome, our proposed solution of using LI violation to solve the photon abundance issues in top-down models is not obviously successful. For both outcomes, in fact, one would have to re-examine the decay process starting with the initial decay of the supermassive relic (which is assumed to be sub-threshold, so it decays “normally”), and follow the decay products to see what results.

Simulations of MMP-induced shower development would have to be performed to see if they are consistent with actual events. It was shown in Ref. 22 that stable UHE primaries of mass greater than about 50 GeV should be distinguishable from proton primaries by their atmospheric shower profiles. As the lightest conceivable MMP ($t\bar{t}$ meson) would be much heavier than this bound, the result of 22 might be used to exclude the MMP as the primary for UHE showers. However, a key assumption is that the primary is stable. For a MMP just above the threshold energy, it may take only a few collisions before the MMP energy is lowered below threshold, where it will decay “normally” (e.g., $t\bar{t} \rightarrow W^+W^-$) into a shower of particles, perhaps mimicking the shower of a nucleus UHECR. A MMP with incident energy farther above threshold would then penetrate deeper. In any event, further investigation into MMP UHECR shower profiles is warranted.

Also of concern is that, while energy and momentum are conserved in a certain preferred frame, they are necessarily not conserved in all other frames. As such, due to the motion of the earth and the solar system, small departures from energy conservation might be observed as well in the UHECR atmospheric showers.

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