Generalized Chaplygin gas as a unified scenario of dark matter/energy: observational constraints

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Abstract. Although various cosmological observations congruously suggest that our universe is dominated by two dark components, the cold dark matter without pressure and the dark energy with negative pressure, the nature and origin of these components is yet unknown. The generalized Chaplygin gas (gCg), parametrized by an equation of state, \( p = -A/p_{gCg}^\alpha \), was recently proposed to be a candidate of the unified dark matter/energy (UDME) scenarios. In this work, we investigate some observational constraints on it. We mainly focus our attention on the constraints from recent measurements of the X-ray mass fractions in clusters of galaxies published by Allen et al. (2002, 2003) and the dimensionless coordinate distances to type Ia supernovae (SNeIa) observations (Perlmutter et al. 1998, 1999; Riess et al. 1998, 2001), and one relativistic matter phase (\( \Omega _R \sim 0.01 \)) and a negative-pressure dark energy phase (\( \Omega _{DE} \sim 0.7 \)) proposed as such a unification, which is an exotic fluid with the equation of state as follows

\[
p_ggCg = -A/p_{gCg}^\alpha .
\]

where \( A \) and \( \alpha \) are two parameters to be determined. It was originally suggested by Kamenshchik et al. (2001) with \( \alpha = 1 \), and later on extended by Bento et al. (2002) to gCg. This simple and elegant model smoothly interpolates between a non-relativistic matter phase (\( p = 0 \)) and a negative-pressure dark matter (Turner and White 1997; Zhu, Fujimoto and Tatsumi 2001; Lima and Alcaniz 2002; Lima, Cunha and Alcaniz 2003; Gong 2004; Chen 2004), and quintessence (Ratra and Peebles 1988; Caldwell et al. 1998; Sahni and Wang 2000; Gong 2002; Sahni et al. 2003; Padmanabhan and Choudhury 2003) etc.

Key words. cosmological parameters — cosmology: theory — distance scale — supernovae: general — radio galaxies: general — X-ray: galaxies:clusters.

1. Introduction

Two dark components are invoked to explain the current cosmological measurements: the cold dark matter (CDM) without pressure and the dark energy (DE) with negative pressure (for a recent review, see Peebles & Ratra 2003). The first one contributes \( \Omega _m \sim 0.3 \), and is mainly motivated to interpret galactic rotation curves and large scale structure formation (e.g. Longair 1998), while the second one (\( \Omega _{DE} \sim 0.7 \)) provides a mechanism for acceleration discovered by distant type Ia supernovae (SNeIa) observations (Perlmutter et al. 1998, 1999; Riess et al. 1998, 2001), and offset the deficiency of a flat universe, favoured by the measurements of the anisotropy of CMB (de Bernardis et al. 2000; Balbi et al. 2000, Durrer et al. 2003; Bennett et al. 2003; Melchiorri &ODman 2003; Spergel et al. 2003), but with a subcritical matter density parameter \( \Omega _m \sim 0.3 \), obtained from dynamical estimates or X-ray and lensing observations of clusters of galaxies(for a recent summary, see Turner 2002). There are a huge number of candidates for DE in the literature, such as a cosmological constant \( \Lambda \) (Carroll et al. 1992; Krauss and Turner 1995; Zhu 1998; Sahni 2002; Padmanabhan 2003), the so-called “X-matter” (Turner and White 1997; Zhu, Fujimoto and Tatsumi 2001; Lima and Alcaniz 2002; Lima, Cunha and Alcaniz 2003; Gong 2004; Chen 2004), and quintessence (Ratra and Peebles 1988; Caldwell et al. 1998; Sahni and Wang 2000; Gong 2002; Sahni et al. 2003; Padmanabhan and Choudhury 2003) etc.

Recently, the generalized Chaplygin gas (gCg) was proposed as such a unification, which is an exotic fluid with the equation of state as follows

\[
p_ggCg = -A/p_{gCg}^\alpha .
\]
2. The generalized Chaplygin gas: basic equations

We consider a flat universe that contains only baryonic matter and the gCg (we ignore the radiation components in the universe that are not important for the cosmological tests considered in this work). Then the Friedmann equation is simply given by $H^2 = (8\pi G/3) (\rho_b + \rho_{gCg})$. Both of the baryonic matter and the gCg components satisfy the relativistic energy-momentum conservation equation, $\dot{\rho} + 3H \rho = 0$, where $\rho$ is the energy density of the baryonic matter and the gCg at present respectively, and $A_s \equiv A/\rho_{gCg0}$ is a substitution of the parameter $A$. The scale factor is related to the observable redshift as $a = 1/(1 + z)$. Now we evaluate the dimensionless coordinate distance, $y(z)$, the angular diameter distance $D_A(z)$, and the luminosity distance, $D_L(z)$, as functions of redshift $z$ as well as the parameters of the model. The three distances are simply related to each other by $D_L^2 = (1 + z)^2 D_A^4 = (c/H_0)(1 + z)y(z)$. We define the redshift dependence of the Hubble parameter $H(z) = H_0 E(z)$, where $H_0 = 100h$ kms$^{-1}$Mpc$^{-1}$ is the present Hubble constant. The HST key project result is $h = 0.72 \pm 0.08$ (Freedman et al. 2001). Parametrizing the model as $(A_s, \alpha)$, we get $E(z)$ function as (Bento et al. 2003a,b; Cunha et al. 2004; Alcaniz et al. 2003)

$$E^2(z; A_s, \alpha) = \frac{\Omega_a(1 + z)^3 + (1 - \Omega_a)}{(A_s + (1 - A_s)(1 + z)^{3(1+\alpha)})^{1/3}}$$

where $\Omega_a$ is the density parameter of the baryonic matter component. The observed abundances of light elements together with primordial nucleosynthesis give $\Omega_b h^2 = 0.0205 \pm 0.0018$ (O’Meara et al. 2001). Then, it is straightforward to show that the distances are given by

$$D_A^2(z; H_0, A_s, \alpha) = (1 + z)^2 \cdot D_A^4(z; H_0, A_s, \alpha)$$

$$= \frac{c}{H_0} (1 + z) \cdot y(z; A_s, \alpha)$$

$$= \frac{c}{H_0} (1 + z) \cdot \int_0^z \frac{dz'}{E(z'; A_s, \alpha)}$$

3. Constraints from the X-ray gas mass fraction of galaxy clusters

As the largest virialized systems in the universe, clusters of galaxies provide a fair sample of the matter content of the whole universe (White et al. 1993). A comparison of the gas mass fraction of galaxy clusters, $f_{gas} = M_{gas}/M_{tot}$, inferred from X-ray observations, with $\Omega_b h^2$ determined by nucleosynthesis can be used to constrain the density parameter of the universe $\Omega_m$ directly (White & Frenk 1991; Fabian 1991; White et. al. 1993; White & Fabian 1995; Evrard 1997; Fukugita, Hogan & Peebles 1998; Ettori & Fabian 1999). Sasaki (1996) and Pen (1997) showed that the $f_{gas}$ data of clusters of galaxies at different redshifts can also, in principle, be used to constrain other cosmological parameters (like the geometry of the universe). This is based on the fact that the measured $f_{gas}$ values for each cluster of galaxies depend on the assumed angular diameter distances to the sources as $f_{gas} \propto [D_A^4]^{3/2}$. The ture, underlying cosmology should be the one which make these measured $f_{gas}$ values to be invariant with redshift (Sasaki 1996; Pen 1997; Allen et al. 2003). However, various uncertainties in previous measurements have seriously complicated the application of such methods.

Recently, Allen et al. (2002; 2003) reported precise measurements of the $f_{gas}$ profiles for 10 relaxed clusters determined from the Chandra observational data. Except for Abell 963, the $f_{gas}$ profiles of the other 9 clusters appear to have converged or be close to converging with a canonical radius $r_{2500}$, which is defined as the radius within which the mean mass density is 2500 times the critical density of the universe at the redshift of the cluster (Allen et al. 2002; 2003). The gas mass fraction values of these 9 clusters are shown in Figure 1. With the reduced systematic uncertainties, Allen et al. (2002; 2003) successfully
bias factor value for a gCg model might be different from the value given above, which leads to a systematic error in this kind of analysis. Because $b$ linearly scales the X-ray mass fraction, $f_{\text{gas}}$, in Eq.(5), lowering (raising) it by ~10% would cause the

and Pen (1997) to the data and obtained a tight constraint on $\Omega_m$ and an interesting constraint on cosmological constant. We will use this database to constrain the gCg model as a UDME. Following Allen et al. (2002), we have the model function as

\begin{equation}
\begin{aligned}
f_{\text{gas}}^\text{mod}(z; A_s, \alpha) &= \frac{b \Omega_b}{(1 + 0.19 h^{1/2}) \Omega_m^{\text{eff}}} \left[ \frac{h}{0.5} \frac{D_{\text{SCDM}}(z)}{D_{\text{gCg}}(z; A_s, \alpha)} \right]^{3/2},
\end{aligned}
\end{equation}

where the bias factor $b = 0.93 \pm 0.05$ (Bialek et al. 2001; Allen et al. 2003) is a parameter motivated by gas dynamical simulations, which suggest that the baryon fraction in clusters is slightly depressed with respect to the Universe as a whole (Bialek et al. 2001). The term $(h/0.5)^{3/2}$ represents the change in the Hubble parameter from the default value of $H_0 = 500$ km s$^{-1}$ Mpc$^{-1}$ and the ratio $D_{\text{SCDM}}(z)/D_{\text{gCg}}(z; A_s, \alpha)$ accounts for the deviations of the gCg model from the default standard cold dark matter (SCDM) cosmology. Note that $\Omega_m^{\text{eff}}$ is the effective matter density parameter (Cunha et al. 2004; Makler et al. 2003b), i.e., the coefficient of the term scaling as $(1 + z)^3$ in equation (3) when the gCg behaves like dust or equivalently $a \ll 1$. It is easy to show that, $\Omega_m^{\text{eff}} = \Omega_b + (1 - \Omega_b)(1 - A_s)^{3(1+a)}$. We should keep in mind that the bias factor value for a gCg model might differ from the value given above, which leads to a systematic error in this kind of analysis. Because $b$ linearly scales the X-ray mass fraction, $f_{\text{gas}}$, in Eq.(5), lowering (raising) it by ~10% would cause the

A $\chi^2$ minimization method is used to determine the gCg model parameters $A_s$ and $\alpha$ as follows (Allen et al. 2003)

\begin{equation}
\begin{aligned}
\chi^2(A_s, \alpha) &= \sum_{i=1}^{9} \left[ \frac{f_{\text{gas}}^\text{mod}(z_i; A_s, \alpha) - f_{\text{gas,obs}}}{\sigma_{f_{\text{gas}}}} \right]^2 \\
&\quad + \frac{(\Omega_b h - 0.0205)^2}{0.0018^2} + \frac{(h - 0.72)^2}{0.08^2} + \frac{(b - 0.93)^2}{0.05^2}.
\end{aligned}
\end{equation}

where $f_{\text{gas}}^\text{mod}(z_i; A_s, \alpha)$ refers to equation (5), $f_{\text{gas,obs}}$ is the measured $f_{\text{gas}}$ with the default SCDM cosmology, and $\sigma_{f_{\text{gas}}}$ is the symmetric root-mean-square errors ($i$ refers to the $i$th data point, with totally 9 data). The summation is over all of the observational data points.

The results of our analysis for the gCg model are displayed in Figure 2. We show 68% and 95% confidence level contours in the $(A_s, \alpha)$ plane using the lower shaded and the lower plus darker shaded areas respectively. The best fit happens at $A_s = 0.74$ and $\alpha = 0.00$. Although the data constrain efficiently the parameter plane into a narrow strip, the two parameters, $A_s$ and $\alpha$, are highly degenerate. This degeneracy can also be seen clearly from the relation, $(1 - A_s)^{1/(1+\alpha)} = (\Omega_m^{\text{eff}} - \Omega_b)/(1 - \Omega_b)$. It has been shown that the X-ray gas mass fraction is mostly sensitive to $\Omega_m$ no matter what the cosmological model is (Allen et al. 2002, 2003; Zhu et al. 2004a, b). In our case, a precise determination of $\Omega_m^{\text{eff}}$ is expected, hence forming a narrow strip in the $(A_s, \alpha)$ plane composed of a bundle of curves given by $(1 - A_s)^{1/(1+\alpha)} = \text{const.}$. In order to determine $A_s$ and $\alpha$ respectively, an independent measurement of $A_s$ or $\alpha$ is needed. We will show that, in the next section, the dimensionless coordinate distances to SNeIa and FRIIb radio galaxies are well appropriate.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The apparent redshift dependence of the $f_{\text{gas}}$ measured at $r_{2500}$ for 9 clusters of galaxies with convergent $f_{\text{gas}}$ profiles. The error bars are the symmetric root-mean-square 1σ errors. The solid circles mark the six clusters studied by Allen et al. (2002), while the empty squares mark the other three clusters published by Allen et al. (2003). The solid curve corresponds our best fit to the gCg model with $A_s = 0.74$, and $\alpha = 0.00$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Confidence region plot of the best fit to the $f_{\text{gas}}$ of 9 clusters published by Allen et al. (2002,2003) – see the text for a detailed description of the method. The 68% and 95% confidence levels in the $A_s$–$\alpha$ plane are shown in lower shaded and lower + darker shaded areas respectively.}
\end{figure}
4. Constraints from the dimensionless coordinate distance data

Motivated by deriving the expansion rate \( E(z) \) and the acceleration rate \( q(z) \) of the universe as functions of redshift, Daly and Djorgovski (2003) compiled a large database of the dimensionless coordinate distance measurements estimated from the observations of SNeIa and FRIIb radio galaxies, and successfully applied it for their purpose. We will show this sample provides a precise determination of \( A_s \), and well breaks the degeneracy presented in the X-ray gas mass fraction test.

The database consists in the 54 SNeIa in the “primary fit C” used by Perlmutter et al. (1999), the 37 SNeIa published by Riess et al. (1998), the so far highest redshift supernova 1997ff presented by Riess et al. (2001), and the 20 FRIIb radio galaxies studied by Daly and Guerra (2002). The authors used the B-band magnitude-redshift relation, \( m_B = M_B + 5 \log c(1 + z) - y(z) \), to determine \( y(z) \) for each supernova, where \( M_B = M_B - 5 \log H_0 + 25 \) is the “Hubble-constant-free” B-band absolute magnitude at maximum of a SNIa. For the 14 supernovae that are present in both the Perlmutter et al. (1999) and Riess et al. (1998) samples, we will use their average values of \( y \) with appropriate error bars (see Table 4 of Daly and Djorgovski 2003). Therefore we totally have 78 SNeIa data points which are shown as solid circles in Figures 3. The dimensionless coordinate distances of FRIIb radio galaxies were estimated through the method proposed by Daly (1994) (see also Guerra, Daly, and Wan 2000; Podariu et al. 2003; Daly and Djorgovski 2003). We use their values of \( y \) for 20 FRIIb radio galaxies obtained using the best fit to both the radio galaxy and supernova data (see Table 1 of Daly and Djorgovski 2003), that are shown as empty squares in Figure 3.

We determine the model parameters \( A_s \) and \( \alpha \) by minimizing \( \chi^2(A_s, \alpha) = \sum_{i=1}^{n} [y(z_i; A_s, \alpha) - y_{oi}]^2/s_i^2 \), where \( y(z_i; A_s, \alpha) \) refers to the theoretical prediction from equation (4), \( y_{oi} \) is the observed dimensionless coordinate distances of SNeIa and FRIIb radio galaxies, and \( s_i \) the uncertainty.

Figure 4 displays the results of our analysis for the gCg model. We show 68% and 95% confidence level contours in the \((A_s, \alpha)\) plane using the lower shaded and the lower plus darker shaded areas respectively. The best fit happens at \( A_s = 0.74 \) and \( \alpha = 0.32 \). It is clear from the figure, that the dimensionless coordinate distance test alone constrains \( A_s \) well into a narrow range, but limits \( \alpha \) weakly. However, it is just appropriate for our purpose, to break the degeneracy presented in the X-ray gas mass fraction test of last section. As we shall see in Sec.5, when we combine these two tests, we could get very stringent constraints on both \( A_s \) and \( \alpha \), hence test the gCg as a UDME scenario efficiently.

5. Combined analysis, conclusion and discussion

Figure 5 displays the results of our combined analysis of the constraints from the X-ray gas mass fractions of galaxy clusters and the dimensionless coordinate distances to SNeIa and FRIIb radio galaxies. We show 68%, 95% and 99% confidence level contours in the \((A_s, \alpha)\) plane. The best fit happens at \( A_s = 0.70 \pm 0.17 \) and \( \alpha = 0.33 \pm 0.03 \), a very stringent constraint on the gCg. These are the parameter ranges of the gCg permitted by the data as a candidate of UDME, which is consistent within the errors with the standard dark matter + dark energy scenario, i.e., the case of \( \alpha = 0 \). Particularly, the standard Chaplygin gas with \( \alpha = 1 \) is ruled out as a feasible UDME by the data at a 99% confidence level. Using the CMBR power spectrum measurements from BOOMERANG (de Bernardis et al. 2002) and Archeops (Benoit et al. 2003), together with the SNeIa constraints, Bento et al. (2003a) obtained, \( 0.74 < A_s < 0.85 \), and \( \alpha < 0.6 \), which is comparable with our results.

More recently, Bertolami et al. (2004) analyzed the gCg model in the light of the latest SNeIa data (Tonry et al. 2003, Barris et al. 2004). They considered both the flat and non-flat models. For the flat case, their best fit values for \([A_s, \alpha]\) are given by \([0.79, 0.999]\) and \([0.936, 3.75]\) with and without the constraint \( \alpha \geq 1 \) respectively. Particularly, up to 68% confidence level, the \( \alpha \) = 0, i.e., the \( \Lambda \)CDM case, is clearly excluded, though it is consistent at 95% confidence level (Bertolami et al. 2004). The authors considered the scenario in which the gCg unified all matter and energy components, while in our analysis, only dark matter and dark energy are unified as the gCg. This might be one factor responsible for the difference between their results and ours. Another even more important factor is we make heavy use of the X-ray gas mass fraction test, and the dimensionless coordinate distance test is sensitive to \( A_s, \alpha \) respectively.

![Fig. 3. Dimensionless coordinate distances \( y(z) \) as a function of \( \log z \) for 78 type Ia supernovae and 20 FRIIb radio galaxies. The solid circles mark the SNeIa, while the empty squares mark the FRIIb radio galaxies. The solid curve corresponds to our best fit to the total 98 data points with \( A_s = 0.74, \alpha = 0.32 \). The database are taken from Daly and Djorgovski (2003).](image-url)
Zhu, Z.-H.: Generalized Chaplygin gas as a unified scenario of DM & DE

Fig. 4. Confidence region plot of the best fit to the dimensionless coordinate distances to 78 SNeIa and 20 FRIIb radio galaxies compiled by Daly and Djorgovski (2003). The 68% and 95% confidence levels in the \( A_s-\alpha \) plane are shown in lower shaded and lower + darker shaded areas respectively.

Various cosmological observations, one could show clearly if this scenario of UDME constitutes a feasible description of our universe.

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Fig. 5. Confidence region plot of the best fit from a combined analysis for the dimensionless coordinate distances to 78 SNeIa and 20 FRIIb radio galaxies (Daly and Djorgovski 2003) and the X-ray gas mass fractions of 9 clusters (Allen et al. 2002, 2003). The 68%, 95% and 99% confidence levels in the \( A_s-\alpha \) plane are shown in white, white + lower shaded and white + lower and darker shaded areas respectively.
