The mass of the Mars-sized exoplanet Kepler-138 b from transit timing

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Extrasolar planets that pass in front of their host star (transit) cause a temporary decrease in the apparent brightness of the star, providing a direct measure of the planet’s size and orbital period. In some systems with multiple transiting planets, the times of the transits are measurably affected by the gravitational interactions between neighbouring planets1,2. In favourable cases, the departures from Keplerian orbits (that is, unaffected by gravitational effects) implied by the observed transit times permit the planetary masses to be measured, which is key to determining their bulk densities3. Characterizing rocky planets is particularly difficult, because they are generally smaller and less massive than gaseous planets. Therefore, few exoplanets near the size of Earth have had their masses measured. Here we report the sizes and masses of three planets orbiting Kepler-138, a star much fainter and cooler than the Sun. We determine that the mass of the Mars-sized inner planet, Kepler-138 b, is 0.066 ± 0.055 Earth masses. Its density is 2.6 ± 1.3 grams per cubic centimetre. The middle and outer planets are both significantly less than Earth’s diameter, indicating that it contains a greater portion of low-density components such as water and hydrogen.

NASA’s Kepler mission has discovered thousands of candidate transiting exoplanets, with a wide range of planetary sizes1–4. A small fraction of these planets have had their masses characterized, either by radial-velocity spectroscopy or by transit timing. The latter probes the gravitational perturbations between planets in multi-planet systems by measuring transit times and fitting dynamical models to the observed transit timing variations (TTVs)2. Both radial velocity and TTV signals are larger for more massive planets and improve with increasing orbital distance.

The majority of mass measurements of planets discovered in the Kepler data set have been the results of radial velocity observations. For Kepler-discovered planets characterized as rocky by this method, the orbital periods are all less than a few days. If the radial-velocity detections, Kepler-78 b has the lowest mass (1.7M⊕, where M⊕ is the Earth’s mass), and the shortest orbital period (0.35 days)5,6. Characterizing planets by transit timing is quite complementary to using radial velocity because transit timing is very sensitive to perturbations between planets that are closely spaced or near orbital resonances7,8. We note that most systems with detected TTVs are not in resonance, but rather are near enough to resonance for the perturbations to be coherent for many orbital periods, while also far enough from resonance that the planetary conjunctions cycle around the orbit plane within the four-year Kepler baseline. For near-resonant pairs, the TTVs of neighbouring planets frequently take the form of anticorrelated sinusoids over many orbits9–14 or the sum of sinusoids where one planet is perturbed by two neighbours15. The majority of TTV detections have been found near first-order mean-motion resonances, such that planet pairs have an orbital period ratio near to jk = 1, where j is an integer. Near first-order resonances generally cause stronger TTV signals than second-order resonances, although much depends on how close the planet pairs are to resonance and how eccentric (non-circular) their orbits are, as well as the masses of the planets. Eccentricity causes the orbital speed to vary during the orbit, and the distance between the planets at conjunction to vary with the position of the conjunction.

The bulk of planets with mass characterizations from the Kepler sample using TTzs so far have orbital periods ranging from about 10 days to 100 days. These generally have low density15–18 and probably possess deep atmospheres, with the exception of the rocky planet Kepler-36 b19.

Kepler-138 (formerly known as KOI-314) hosts three validated transiting planets20. The orbital periods of Kepler-138’s three planets are given in Table 1. Kepler-138 c and Kepler-138 d orbit near a

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Table 1 | Stellar and planetary parameters for the Kepler-138 system

| Adopted stellar parameters | Stellar radius, R∗ | Effective temperature, T∗eff | Stellar density, ρ∗ | Metallicity, [Fe/H] | log[g (cm s−2)] |
|---------------------------|-------------------|-------------------------------|-------------------|-------------------|----------------|
| (Mq = 0.505)Mq             | (0.442 ± 0.024)Rq | 3.841 ± 0.009                 | 9.5 ± 2.2 g cm−3 | −0.280 ± 0.099    | 4.886 ± 0.095 |

Dynamical modelling parameters

| Planet | Period (days) | T0 (d.o.f. − 2,454,900) | eccos | ecosin | (Mq/Mq) × (Mq/Mq) |
|--------|---------------|-------------------------|-------|--------|-------------------|
| b      | 10.3126 ± 0.0004 | 788.4142 ± 0.0007      | −0.011 ± 0.006 | −0.024 ± 0.007 | 0.13 ± 0.01 |
| c      | 13.7813 ± 0.0001 | 786.1289 ± 0.0005      | −0.005 ± 0.002 | 0.046 ± 0.046 | 3.85 ± 0.04 |
| d      | 23.0881 ± 0.0008 | 796.6689 ± 0.0013      | −0.092 ± 0.026 | −0.075 ± 0.064 | 1.08 ± 0.02 |

Adopted physical characteristics

| Planet | R/R∗ | Mq (Mq) | Rq (Rq) | Density (g cm−3) | Incident flux relative to Earth |
|--------|------|---------|---------|------------------|-------------------------------|
| b      | 0.0108 ± 0.0003 | 0.066 ± 0.005 | 0.522 ± 0.032 | 2.6 ± 0.24 | 6.81 ± 0.04 |
| c      | 0.0247 ± 0.0005 | 1.970 ± 0.002 | 1.197 ± 0.070 | 6.2 ± 3.4 | 4.63 ± 0.29 |
| d      | 0.0251 ± 0.0007 | 0.640 ± 0.028 | 1.212 ± 0.075 | 2.1 ± 1.2 | 2.32 ± 0.29 |

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second-order mean-motion resonance (5:3), while Kepler-138 b and Kepler-138 c orbit near the 4:3 first-order resonance. Using transit times up to the fourteenth quarter of the Kepler mission, two of the three known planets orbiting Kepler-138 have been confirmed and characterized with TTVs. The derived parameters in that work suggested that the outer planet, Kepler-138 d, has a density so low that it must have a substantial hydrogen/helium gaseous envelope.

Using the complete Kepler data set for Kepler-138, we have detected TTVs for all three planets (see Fig. 1). We describe our procedure for measuring transit times in the Methods, and list our transit time measurements in Extended Data Table 1. TTVs are expressed as the difference between the observed transit times and the calculated linear fit to the transit times. Modelling Kepler-138 as a three-planet system, we measure the masses of all three planets, the super-Earth-sized Kepler-138 c and Kepler-138 d, as well as Kepler-138 b, which at 0.52\(R_\oplus\) is roughly the size of Mars.

We performed dynamical fits by calculating the orbits of the three planets around the star. We modelled the orbits as being co-planar, given that all three planets are transiting, that Kepler’s multi-planet systems are known to have small mutual inclinations, and that we demonstrate that allowing mutual inclinations has little effect on our results for the planet masses (see Extended Data Fig. 7c).

Our model parameters for each planet are the orbital period \(P\), the time \(T_0\) of the first transit after our chosen epoch, the components of the eccentricity vector \((\cos\omega, \sin\omega)\), where \(\omega\) is the angle between the sky plane and the orbital pericentre of the planet), and \(M_p/M_\star\), the mass of the planet relative to the host star, which we express as \((M_p/M_\oplus)(M_\star/M_\odot)\) throughout, where \(M_\oplus\) is the mass of the Sun. We perform Bayesian parameter estimation for these 15 model parameters using differential evolution Markov Chain Monte Carlo analysis. We report the properties of the star and planets in Table 1, and details of the parameter estimation algorithm, priors and statistical models in the Methods.

We measure the mass of Kepler-138 b to be \(0.066^{+0.099}_{-0.037}M_\oplus\), where uncertainties denote the 68.3% confidence limits. The 95.4% interval spans 0.011\(M_\oplus\) to 0.170\(M_\oplus\). The robustness of this result against outlying transit times and mutual inclinations is demonstrated in the Methods.

The posterior probability for the inner planet having non-zero mass is between 99.82% and 99.91% (depending on the choice of prior for eccentricity \(e\), that is, equivalent to a 3σ detection. This calculation is based on the Savage–Dickey density ratio for calculating the Bayes factor, which fully accounts for posterior width and shape, including asymmetries and non-Gaussianity, as described in Methods.

Kepler-138 b is by far the smallest exoplanet, both by radius and mass, to have a density measurement (see Fig. 2). Thus it opens up a new regime to physical study. It is likely to become the prototype for a class of small close-in planets that could be common. The prospect of further constraints on this planet are excellent, because NASA’s Transiting Exoplanet Survey Satellite should be able to measure transit times for the two largest planets, improving constraints on the dynamical model for all three planets. The European Space Agency’s Plato mission will continue this process and ground-based measurements may also be possible. These future observations, plus more accurate stellar classification using the distance to the star measured by the European Space Agency’s Gaia mission, will further improve the characterization of this system, especially the inner planet.

Our measurements of the mass and density of the small inner planet Kepler-138 b are consistent with various compositions and formation locations. If future observations imply that the planet is less dense than rock, then the only physically and cosmogonically plausible low-density constituents are water and other astrophysical ices, which could only have condensed far from the star. That would be the first definitive evidence for substantial inward orbital migration of a small planet.

For the two outer planets, Kepler-138 c and Kepler-138 d, we find a lower mass ratio between these two planets than does previous work. This is not surprising, since Kepler-138 b’s perturbations explain part of the TTVs observed in Kepler-138 c, which were previously attributed solely to perturbations by Kepler-138 d. Nevertheless, the mass ratios between each of these planets and their host star remains consistent with published results. We find higher densities for both of these planets than does previous work, owing to our improved stellar characteristics of this system, especially the inner planet.

Figure 2 | Mass–radius diagram of well characterized planets smaller than 2.1 Earth radii, \(R_\oplus\). Prior exoplanet characterizations and 1σ uncertainties are shown as grey points \(^{9,11,13,16,12,14,18-20}\). Black points from left to right are Mercury, Mars, Venus and Earth. Red data points are our results for Kepler-138. Open circles mark previously measured masses for Kepler-138 c and Kepler-138 d\(^2\). Error bars mark published 1σ uncertainties for the planets of Kepler-138 and masses and radii of all other characterized exoplanets within this size range. The curves mark bulk densities of 1 g cm\(^{-3}\), 3 g cm\(^{-3}\) and 10 g cm\(^{-3}\).

![Figure 1 | Transit timing variations of the three planets orbiting Kepler-138. In black are the differences between measured transit times and a calculated linear fit to the transit times, with 1σ uncertainties shown as error bars. Grey points mark the difference between the simulated transit times based on the best-fit dynamical model and a linear fit to the transit times. a, TTV of Kepler-138 b; b, TTV of Kepler-138 c; c, TTV of Kepler-138 d.](image-url)
properties, particularly the higher stellar density and consequent smaller stellar radius.

Previous estimates of the size and mass of the outer planet Kepler-138 d implied that the planet possessed a hydrogen-rich atmosphere\(^1\), which is difficult to explain with our current understanding of the accretion and retention of light gases from low-mass planets orbiting close to their star\(^2\)–\(^4\). Our new measurements could be explained by a composition of rock and water. A planet made of rock and water would be more stable against mass loss, and would imply that the planet formed at a greater distance from the star and migrated.

Online Content Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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METHODS

We have used all available short-cadence Kepler data and long-cadence data wherever short-cadence data are unavailable to complete the data set for 17 quarters. We list the transit times for each planet in Extended Data Table 1. Throughout the manuscript, we use Barycentric Julian Day (BJD) minus 2,454,900.

Photometric transit and stellar models. From the light curve, we filtered instrumental and astrophysical effects that are independent of planetary transits. To each segment of the photometric time series, we fitted a cubic polynomial of width 2 days, centred on the time of each measurement. We excluded measurements taken within one transit duration (defined as the time from first to last contact) of the measured centre of the transit and extrapolate the polynomial to estimate corrections during transits. This process strongly filters astrophysical signals with timescales of approximately 2 days, which could affect the shape of a planetary transit. We also excluded measurements for which the associated segment has gaps longer than 2.5 days.

We fitted the detrended Kepler light curve using a transit model for quadratic limb-darkening and non-circular Keplerian orbits. We stacked transits of each planet with corrections for the measured TTVs. To account for Kepler’s observation cadence, we averaged our transit model with 11 equal spacings within the 1 min or 30 min integration window. We evaluated the photometric noise for each quarter of data to fit transit models, adopting published stellar parameters for Kepler-138. We adopted a two-parameter quadratic model for limb-darkening with fixed coefficients (0.3576, 0.3487) appropriate for Kepler’s bandpass and Kepler-138’s effective temperature (T_eff), log(g) and metallicity (Fe/H).

The light curve model parameters consist of the mean stellar density, a photometric zero point for each light curve segment, and for each planet the orbital period, time of transit, planet-to-star radius ratio, impact parameter, and eccentricity parameterized as cos e and sin e. We determined posterior distributions of our model parameters using Markov Chain Monte Carlo techniques. Our best-fit transit models, shown in Extended Data Fig. 1, resulted in consistent estimates for ρ_p from each planet.

We determined the mass and radius of Kepler-138 by fitting the spectroscopic parameters (T_eff, Fe/H) and our light curve contraints of ρ_p to Dartmouth Stellar Evolution models, assuming a Gaussian probability density for each parameter. For the Dartmouth models, we varied initial conditions of mass, age, and Fe/H and interpolated over a grid to evaluate T_eff, ρ_p, and [Fe/H] for any set of initial conditions. We computed posteriors using Markov Chain Monte Carlo analysis to obtain stellar model-dependent posteriors on M_p and R_p. Table 1 lists our adopted stellar parameters. We tested the effects of eccentricity priors on our measurement of ρ_p, adopting a uniform prior in eccentricity as our nominal result. We compare results with eccentricity fixed at zero in Extended Data Fig. 2. Although a uniform prior on eccentricity results in a slightly wider range of inferred radii for the star, both of these models are consistent with the spectroscopic study of Kepler-138.

Our solution for the impact parameter of the middle planet is 0.3 ± 0.2, much lower than the previous estimate of 0.92 ± 0.02. The apparent U-shaped transit of Kepler-138 c (Extended Data Fig. 1) is consistent with a low impact parameter. Our measured impact parameter for Kepler-138 d is 0.810 ± 0.057, consistent with the previous measurement. Our revised impact parameters imply ρ_p = 9.0 ± 1.9 g cm^-3, which agrees with our transit models for each planet, and with the spectroscopic study of Kepler-138.

We find that Kepler-138 has a smaller mass and radius than previous estimates based on the absolute K-band magnitude (M_K) of the star from high-resolution Keck spectra. These relied on mass–luminosity relations and a mass–radius relation from interferometry. However, the calibration stars used to correlate the M_K to spectral index excluded cool stars that are active. In the case of Kepler-138, the photometric time-series exhibits large (1%) variations due to starspots. These increase the risk of systematic errors in the measurement of stellar luminosity, and therefore the stellar properties derived from the mass–luminosity relation.

The time-scale for star-spot modulation, ~20 days, was much longer than the transit duration, and was probably dominated by two spots. We found no evidence of spot crossings, nor did we find any TTV periodicities related to the rotation period of the star. Hence, the stellar activity is unlikely to affect our transit model or transit times. Analytical constraints from TTV. Orbital period ratios determine how close planets are to mean motion resonance, and over what period TTVs are expected to cycle. The inner pair of planets of Kepler-138 orbit near the 4:3 resonance with an expected TTV cycle of 1,570 days, slightly longer than the 1,654-day observational baseline of Kepler-138 b’s transits. We fitted a sinusoid to this periodicity to the TTVs of the inner planet, and detected a TTV amplitude of 34 ± 4 m. This permits a rough estimate for the mass of the middle planet of (6.8 ± 0.9)M_Jov (M_p/M_*), which is close to our final measure of the mass of the middle planet.
Significance of mass detection for Kepler-138 b. We establish the statistical significance of a non-zero mass for Kepler-138 b by computing the Bayes factor, that is, the ratio of the marginalized posterior probability for a model with the mass of planet Kepler-138 b fixed at zero relative to the marginalized posterior probability for a model where all three planets have a non-zero mass. Intuitively, the Bayes factor quantifies how much the transit timing data has increased our confidence that Kepler-138 b has a non-zero mass. When performing Bayesian model selection, it is essential to choose proper (that is, normalized) priors for any parameters not occurring in both models. For the mass of Kepler-138 b we adopt a uniform prior ranging between zero mass and the mass of an iron sphere the size of Kepler-138 b. We tested two models of iron planets as upper limits on our mass priors. We compute the Bayes factor using the generalized Savage–Dickey density ratio based on the posterior samples from our nominal model. We find that the three-massive-planets model is strongly favoured, with the posterior probability for the three-massive-planets model equal to 99.82% (99.80%) for our nominal model (that is, three massive planets, each with a uniform eccentricity prior) or 99.91% (99.90%) for a model with three massive planets, each with a Rayleigh distribution ($\sigma = 0.02$) for an eccentricity prior. A more restrictive prior for the mass of Kepler-138 b would further increase the posterior probability for the three-massive-planets model.

The generalized Savage–Dickey density ratio is superior to more commonly used substitutes (such as the Akaike information criterion, AIC, or the Bayesian Information Criterion, BIC), since the Savage–Dickey density ratio provides a practical means of calculating the Bayes factor, the actual quantity of interest for Bayesian model comparison. The AIC and BIC use only the likelihood of the two best-fit models and do not account for the width or shape of the posterior probability distributions. Extended Data Fig. 4a shows that the marginal posterior for the mass of Kepler-138 b is asymmetric and non-Gaussian, so the AIC or BIC would be a particularly poor choice for our problem.

Therefore, we have computed the rigorously correct Bayes factor using the Savage–Dickey density ratio, which provides an efficient way of calculating the Bayes factor when comparing two nested models, meaning that the simpler model would be a particularly poor choice for our problem. The Savage–Dickey density ratio can be computed from the posterior sample using a kernel density estimator. We performed several tests to assess the robustness of our results to: (1) the choice of prior for eccentricity, (2) the treatment of transit time outliers, (3) the assumption of co-planarity, and (4) our algorithm. For these sensitivity analyses, we adopt our nominal prior for planet masses to allow for negative planet masses while any such models are clearly unphysical, allowing for such models offers an efficient and intuitive means for evaluating whether the lower limit on the planet masses is robust to the above assumptions.

Sensitivity analyses. We performed several tests to assess the robustness of our results to: (1) the choice of prior for eccentricity, (2) the treatment of transit time outliers, (3) the assumption of co-planarity, and (4) our algorithm. For these sensitivity analyses, we adopt our nominal prior for planet masses to allow for negative planet masses while any such models are clearly unphysical, allowing for such models offers an efficient and intuitive means for evaluating whether the lower limit on the planet masses is robust to the above assumptions.

Choice of eccentricity prior. As noted above, our nominal model has a uniform prior in eccentricity. The joint posterior for the mass ratio of Kepler-138 b to the host star and its orbital eccentricity is shown in Extended Data Fig. 5. We show this plot with two alternative priors in eccentricity: a Rayleigh distribution with a scale length 0.14 × $s_M$ and a more constrained one, consistent with Kepler’s multi-planet systems, with a scale length of 0.024. Since the data constrain the eccentricity so weakly, the choice of priors strongly affects the posteriors of eccentricity. However, the planet-to-star mass ratios were much more weakly affected by the choice of eccentricity prior, as shown in Extended Data Fig. 7a. Because the apsidally locked solutions are long-term stable, we adopt the uniform prior as our nominal solution, and note that a more constraining prior on eccentricity results in a marginally wider posterior for mass ratio of Kepler-138 b to the star. Although, as noted above, the orbital eccentricities in the system are only weakly constrained, the relative eccentricities and mass ratios between the planets are tightly constrained by the TTVs, as shown in Extended Data Fig. 6 and Extended Data Table 2.

Transit time observation outliers. To assess the effect of residual outlying transit times, we repeated the analysis with two other sets of measured transit times, differing only in that we removed transit time residual outliers beyond (1) 4σ (leaving 254 transit times) or (2) 2.5σ (leaving 244 transit times), as opposed to our nominal data set, which excluded 3σ outliers. Extended Data Fig. 3 shows the relative frequency of residuals compared to a Gaussian. The majority of outliers are between 2.5σ and 3σ. We excluded outliers beyond 4σ from all models.

The posterior for the mass of Kepler-138 b for each of these three data sets is displayed for comparison in Extended Data Fig. 7b. Overall, the outliers have a modest effect on the measured mass ratio for Kepler-138 b to the host star. Since most of the outliers are in the transit times of Kepler-138 b, these have a small effect on our mass measurement of Kepler-138 b. Furthermore, the mass of Kepler-138 c is constrained by its effect on the transit times of both planets Kepler-138 b and Kepler-138 d. Nevertheless, we note that the inclusion of more outliers increases the skewness of the posterior for planetary mass and causes the mode of the distribution to shift to a slightly higher mode and is less skewed than the posterior for the nominal model, the differences are comparable to the minor effects of transit timing outliers or choice of eccentricity prior. This result validates our both our transit timing method and our TTV analysis.

Additionally, we generated eight independent synthetic data sets with zero mass for Kepler-138 b and other parameters based on our nominal model (Extended Data Fig. 8). In all eight cases, the posterior probability for Kepler-138 b’s mass was insignificant, with only 16% to 86% of posterior samples having a mass greater than zero, consistent with expectations for non-detections. This is in sharp contrast to our analysis of the actual data, which results in 99.84% of the posterior yield a positive mass for Kepler-138 b.

Planetary characteristics. Our adopted credible intervals in planetary mass and radius were calculated by repeatedly multiplying samples from the posteriors of planet-to-star mass ratios, and $M_\star$ uncertainties in planetary radii were calculated with the fractional uncertainty in the stellar radius and the uncertainty in planet-to-star radius ratio added in quadrature. Time-averaged incident flux for each planet compared to the Earth was calculated in the low eccentricity limit, although we note that if the orbits were highly eccentric, the fluxes would be marginally higher.

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Extended Data Figure 1 | Folded light curves with corrections for observed TTV for Kepler-138. The scattered points are photometric relative fluxes and the curves are analytical models of the transit shape described in the text.

a, Kepler-138 b; b, Kepler-138 c; c, Kepler-138 d.
Extended Data Figure 2 | Stellar mass and radius models using constraints on the stellar mean density inferred from the light curve. In cyan are models that adopted a uniform prior in eccentricity, and in magenta are constraints found with orbital eccentricities fixed at zero. Grey points with error bars mark stellar parameters found in the literature while the black error bars mark our adopted solution for stellar mass and radius.
Extended Data Figure 3 | The distribution of residual normalized deviations from our best fit dynamical model to the raw transit times. The histogram marks deviations calculated as: \((O - S)/\sigma_{TT}\), where \(O\) is the observed transit time, \(S\) is the simulated transit time and \(\sigma_{TT}\) is the measurement uncertainty. The green curve marks a Gaussian distribution.
Extended Data Figure 4 | Posterior distributions for TTV model parameters. Data are shown for our nominal model. a, The planet-to-star mass ratio \( (M_p/M_\star) \) for Kepler-138 b; b, \( M_p/M_\star \) for Kepler-138 c; c, \( M_p/M_\star \) for Kepler-138 d. d, \( e \cos\omega \) for Kepler-138 b; e, \( e \cos\omega \) for Kepler-138 c; f, \( e \cos\omega \) for Kepler-138 d. g, \( e \sin\omega \) for Kepler-138 b; h, \( e \sin\omega \) for Kepler-138 c; i, \( e \sin\omega \) for Kepler-138 d. The relative frequency for each histogram is scaled to the mode. Mass ratios are scaled to the \( M_p/M_\star \) mass ratio.
Extended Data Figure 5 | Joint posteriors of model parameters and the effects of eccentricity priors. The dark (light) grey marks the 68.3% (95.4%) credible intervals for each joint posterior. a–c, $M_p/M_*$ and eccentricity vector components for the inner and middle planets. d–f, $M_p/M_*$ and eccentricity vector components for the middle and outer planets. Panels g, h and i compare Kepler-138 b’s $M_p/M_*$ and its orbital eccentricity, for three eccentricity priors: a uniform prior on eccentricity (g), and models with a Rayleigh distribution of scale factor 0.1 (h), and 0.02 (i).
Extended Data Figure 6 | Posterior distributions for mass ratios and relative eccentricities between planets. The mass ratio of the inner and middle planets is shown in a, and relative eccentricity vector components—that is, the difference in $e_c \cos \omega_c - e_a \cos \omega_a$ in the inner pair—are shown in b (c). The mass ratio of the middle and outer planets are plotted in d, and the relative eccentricity vector components—that is, the difference in $e_c \cos \omega_c - e_b \cos \omega_b$ in the outer pair—are shown in e (f).
Extended Data Figure 7 | Sensitivity tests for the effects of eccentricity prior, outlying transit times and free inclinations on the mass of Kepler-138 b relative to the host star. Panel a compares a uniform prior (black curve, our nominal posterior for all comparisons) and a Rayleigh distribution with scale factors 0.1 (navy) and 0.02 (cyan). Panel b compares posteriors with 3σ outliers excluded (black), with two alternatives: 4σ outliers (blue) and 2.5σ outliers removed (light green). Panel c compares our nominal model with one with a free ascending node for the inner (purple) or outer (red) planet.
Extended Data Figure 8 | Validation of our method with synthetic data sets. The green curve marks the posterior for a synthetic data set generated with the same parameters as were the medians of our nominal posteriors (in Table 1). The agreement between the green and black curves validates our method and our claim for a positive mass detection for Kepler-138 b. The magenta and purple shades are posteriors for models using data generated with zero mass for Kepler-138 b. These zero-mass synthetic models all reproduced non-detections.
Extended Data Table 1 | Transit times of Kepler-138’s planets

| Planet  | Time (days since BJD 2,454,900) | Uncertainty (days) |
|---------|----------------------------------|--------------------|
| K-138 b | 635.73865 ± 0.00176             | 4.34E-09           |
| K-138 c | 1324.74131 ± 0.00147             | 5.88E-09           |
| K-138 d | 684.31253 ± 0.00355             | 8.33E-09           |
| K-138 e | 56.39365 ± 0.00264              | 5.98E-09           |
| K-138 f | 674.96332 ± 0.00399             | 5.98E-09           |
| K-138 g | 1331.06216 ± 0.00129             | 5.88E-09           |
| K-138 h | 675.84719 ± 0.00283             | 5.88E-09           |
| K-138 i | 609.05112 ± 0.00238             | 5.88E-09           |
| K-138 j | 689.05112 ± 0.00238             | 5.88E-09           |
| K-138 k | 691.05112 ± 0.00238             | 5.88E-09           |
| K-138 l | 692.05112 ± 0.00238             | 5.88E-09           |
| K-138 m | 693.05112 ± 0.00238             | 5.88E-09           |
| K-138 n | 694.05112 ± 0.00238             | 5.88E-09           |
| K-138 o | 695.05112 ± 0.00238             | 5.88E-09           |

All times are expressed in days since 24,544,900. Estimated uncertainties give 68% confidence limits. Outliers beyond 3σ in the residuals of dynamical fits are marked with an asterisk.
Extended Data Table 2 | Confidence intervals from distributions found with differential evolution Markov Chain Monte Carlo TTV analysis.

| Param. \ Conf. Int. | 68.3%   | 95.4%   | 99.7%   |
|---------------------|---------|---------|---------|
| P (days)            |         |         |         |
| b                   | 10.3126 | +0.0014 | -0.0006 |
|                     |         | -0.0006 | -0.0010 |
|                     |         | -0.0010 | -0.0014 |
| c                   | 13.7813 | +0.0001 | -0.0002 |
|                     |         | +0.0002 | -0.0002 |
|                     |         | -0.0001 | -0.0004 |
| d                   | 23.0881 | +0.0009 | +0.0016 |
|                     |         | -0.0012 | -0.0011 |
|                     |         | -0.0008 | -0.0009 |
| T0 (s)              |         |         |         |
| b                   | 788.4142| +0.0027 | -0.0027 |
|                     |         | -0.0054 | -0.0054 |
|                     |         | -0.0026 | -0.0056 |
| c                   | 786.1289| +0.0005 | -0.0010 |
|                     |         | +0.0010 | -0.0015 |
|                     |         | -0.0005 | -0.0015 |
| d                   | 796.6689| +0.0013 | +0.0025 |
|                     |         | +0.0035 | -0.0013 |
|                     |         | -0.0023 | -0.0026 |
| ecosω              |         |         |         |
| b                   | -0.011  | -0.086  | -0.273  |
|                     |         | -0.300  | -0.370  |
|                     |         | +0.249  | +0.386  |
| c                   | -0.015  | -0.096  | -0.128  |
|                     |         | -0.289  | -0.390  |
|                     |         | -0.126  | -0.148  |
| d                   | -0.037  | -0.090  | -0.140  |
| esinω              |         |         |         |
| b                   | -0.024  | -0.075  | -0.135  |
|                     |         | -0.238  | -0.316  |
|                     |         | -0.316  | -0.424  |
| c                   | -0.020  | -0.024  | -0.117  |
|                     |         | -0.205  | -0.276  |
|                     |         | -0.117  | -0.275  |
| d                   | -0.057  | -0.033  | -0.186  |
| m_p/M_c             |         |         |         |
| b                   | 0.129   | -0.0121 | +0.078  |
|                     |         | +0.252  | +0.108  |
|                     |         | -0.108  | -0.131  |
| c                   | 3.846   | -0.004  | -0.304  |
|                     |         | +0.431  | +0.003  |
|                     |         | +0.431  | +0.003  |
| d                   | 1.282   | -0.136  | -0.783  |
| M_p/M_c             |         |         |         |
| e(cosω - e(cosω)    | 0.0353  | +0.0136 | +0.0248 |
|                     |         | +0.0248 | +0.0375 |
| e(sinω)             | 0.0038  | +0.0017 | +0.0039 |
|                     |         | +0.0039 | +0.0072 |
| e(sinω - e(sinω)    | -0.0033 | +0.0023 | +0.0132 |
|                     |         | +0.0132 | +0.0608 |
| ω_p - ω_c (°)       | -0.58   | -2.40   | +10.56  |
|                     |         | +10.56  | +17.86  |
|                     |         | -16.78  | -16.78  |
| M_c/M_p             |         |         |         |
| e(cosω - e(cosω)    | 2.9586  | +0.4434 | +0.3526 |
|                     |         | +1.303  | +0.6369 |
|                     |         | +0.6369 | +0.7847 |
| e(sinω)             | 0.0312  | +0.0019 | +0.1059 |
|                     |         | +0.1059 | +0.1845 |
| e(sinω - e(sinω)    | 0.0186  | +0.0022 | +0.0319 |
|                     |         | +0.0319 | +0.0778 |
| ω_c - ω_p (°)       | 0.011   | -2.91   | +13.75  |
|                     |         | +13.75  | +14.62  |
|                     |         | +2.34   | +21.24  |

We include the parameters of our dynamical fits, as well as the mass ratios and relative eccentricity vector components between the planets, which have tighter constraints than the absolute masses or eccentricity vector components.