Constraining the length and pattern speed of the Milky Way bar from direct orbit integration of APOGEE and Gaia data

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ABSTRACT
The dynamics of the inner Galaxy contain crucial clues for untangling the evolutionary history of the Milky Way. However, the inner Galaxy’s gravitational potential is poorly constrained, partly because the length of the Galactic bar is currently under debate with length estimates ranging from 3.5-5 kpc. We present a novel method for constraining the length and pattern speed of the Galactic bar using 6D phase space information to directly integrate orbits. We verify our method with N-body simulations and find that the maximal extent of orbits in the bar is not always consistent with that of the potential used to calculate the orbits. It is only consistent when the length of the bar in said potential is similar to the N-body model from which the initial positions and velocities of the stars are sampled. When we apply the orbit integration method to ≈210,000 stars in APOGEE DR17 and Gaia eDR3 data, we find a self-consistent result only for potential models with a dynamical bar length of ≈3.5 kpc and pattern speed of 39 km/s/kpc. We find the Milky Way’s trapped bar orbits extend out to only ≈3.5 kpc, but there is also an overdensity of stars at the end of the bar out to 4.8 kpc which could be related to an attached spiral arm. We also find that the measured orbital structure of the bar is strongly dependent on the properties of the assumed potential.

Key words: Galaxy: bulge, Galaxy: kinematics and dynamics, Galaxy: structure, Galaxy: evolution

1 INTRODUCTION
Stellar bars are non-axisymmetric, elongated structures in the inner parts of disk galaxies. More than 30% of massive disk galaxies (M* > 1010 M⊙) in the local Universe host strong stellar bars (Sellwood & Wilkinson 1993; Masters et al. 2011; Gavazzi et al. 2015). Although many open questions remain concerning how stellar bars form and evolve, it is clear that they play pivotal roles in the secular evolution of disk galaxies (Debattista et al. 2004; Athanassoula 2005).

The Milky Way hosts a stellar bar at its center, which was originally discovered from near-infrared emission (Blitz & Spergel 1991; Weiland et al. 1994) and gas kinematics (Binney et al. 1991; Peters 1975). Stellar kinematics demonstrate that most of the mass in the inner Galaxy participates in the bar structure (Howard et al. 2009; Shen et al. 2010; Ness et al. 2013b; Debattista et al. 2017). Currently, it is under debate whether a small pressure-supported component distinct from the disk or halo (e.g., a classical bulge) overlaps with the bar (Kunder et al. 2020; Arensen et al. 2020; Lucey et al. 2021). It has also been discovered that the center of the MW has an X-shaped structure (Nataf et al. 2010; McWilliam & Zoccali 2010; Ness et al. 2012; Wegg & Gerhard 2013; Ness & Lang 2016), which is characteristic of a boxy/peanut-shaped (B/P) bulge and consistent with sim-
ulations and observations of barred galaxies (Combes et al. 1990; Athanassoula 2005; Martínez-Valpuesta et al. 2006; Bureau et al. 2006; Laurikainen et al. 2014).

The discovery of the MW’s stellar bar provides the unique opportunity to study a bar in exquisite detail using resolved stars. However, the high levels of variable extinction have historically made the MW’s bar difficult to observe (Nataf et al. 2013). Parameterizing the MW bar’s mass, length and pattern speed is essential for many studies of MW dynamics. The bar greatly influences the perturbative kinematics of the Galactic disk, including in the Solar neighborhood (Dehnen 2000; Minchev & Famaey 2010; Antoja et al. 2018; Hunt & Bovy 2018; Fujii et al. 2019). The Galactic bar can also impact the structure of stellar streams in the halo and the ability to interpret dark matter substructure signatures in the streams (Price-Whelan et al. 2016b; Hattori et al. 2016; Pearson et al. 2017; Erkal et al. 2017; Banik & Bovy 2019; Bonaca et al. 2020).

There has been a number of efforts to map the three dimensional structure of the bulge/bar region of the Galaxy, primarily using star counts (Saito et al. 2011; Wegg & Gerhard 2013). However, the existence of a long (≈4 kpc) bar, discovered initially by Hammersley et al. (1994), has led to controversy on whether it is a separate structure from the B/P bulge (Hammersley et al. 2000; López-Corredoira et al. 2007; Cabrera-Lavers et al. 2007, 2008; Martínez-Valpuesta & Gerhard 2011). Using the 3D number density of red clump giants from VVV, UKIDSS, GLIMPSE, and 2MASS data, Wegg et al. (2015) demonstrated that the long bar is the extension of the B/P bulge and they are in fact one unified structure. Furthermore, Wegg et al. (2015) found that the bar has a length of 5 kpc and is at an angle of (28-33)° from the Sun-Galactic center line. However, it is possible that spiral arms connected to the bar may cause it to appear 1-1.5 kpc longer in the number density counts than when the spiral arms are not connected (Gonzalez & Gadotti 2010; Hilmi et al. 2020). To account for this effect, it is critical to further constrain the length of the bar using a dynamical method which can distinguish between trapped bar stars and those that do not participate in solid body rotation.

The pattern speed of the bar, however, is better constrained with measurements from multiple methods. Adding kinematic data from the ARGOS survey (Freeman et al. 2013; Ness et al. 2013a) to the work of Wegg et al. (2015), Portail et al. (2017) found a pattern speed of 39.0 ± 3.5 km s^{-1} kpc^{-1} using the Made-to-Measure method. Sanders et al. (2019) measured a pattern speed of 41 ± 3 km s^{-1} kpc^{-1} using a direct method derived from the continuity equation (Tremaine & Weinberg 1984). In addition, they used proper motion data of stars within 2 kpc of the Galactic center from Gaia DR2 and VVV surveys. Using a similar method, Bovy et al. (2019) and Leung et al. (2022) created kinematic maps of Apache Point Observatory Galactic Evolution Experiment (APOGEE; Majewski et al. 2017) and Gaia DR2 data (Gaia Collaboration et al. 2016, 2018) to measure a pattern speed of 41 ± 3 s^{-1} kpc^{-1} out to a distance of 5 kpc from the Galactic center.

In this paper, we take advantage of the 6D phase-space measurements at the center of our Galaxy to directly integrate the orbits of stars located in the Milky Way’s bar. Specifically, we develop a novel method for constraining the MW’s bar length and pattern speed. To verify our method, we use N-body simulations and compare the maximal extent of stars in the bar measured from the orbits to that of the potential model used to calculate the orbits. We find that these lengths are only self-consistent when the initial positions and velocities of the star particles come from a distribution similar to the potential in which the orbits are integrated. We test ≈60 different MW bar potential models, by integrating APOGEE/Gaia data within these potentials and determining whether the retrieved maximal extent is self-consistent with the given potential model. In Section 2, we describe the simulations we use to validate our method while in Section 3 we describe the observations used to constrain the MW’s bar. We describe the method and verify its precision and accuracy in Section 4. Next, we apply our method to the MW data in Section 5 and discuss the different methods for measuring bar lengths in Section 6. Last, we present our conclusions in Section 7.

2 SIMULATIONS

We make use of two Milky Way-like N-body simulations from the literature. The primary simulation we use (hereafter Galaxy A) is a reproduction of the MWP14-3 model from Bennett et al. (2021). We also make use of another Milky Way-like simulation (hereafter Galaxy B) from Tepper-Garcia et al. (2021) to further validate our method and compare to Milky Way observational data.

2.1 Galaxy A

The initial conditions for the Galaxy A simulation are derived from the GALPY potential MWPotential2014 (Bovy 2015) and set up with the GALIC package (Yurin & Springel 2014). However, Bennett & Bovy (2021) found that the halo mass of GALPY’s MWPotential2014 needs to be increased in order to produce realistic asymmetries (e.g., spiral arms and a stellar bar) in simulations. Therefore, Galaxy A (MWP14-3) has a virial halo mass of $M_h = 4.4 \times 10^{12} M_\odot$, which is twice as heavy as the halo in GALPY’s MWPotential2014. It also has a disk scale height of 0.08 kpc and disk scale length of 3.0 kpc. In total there are ≈9.3 million particles with 3,337,406 particles in dark matter halo, 5,000,000 particles in the disk and 996,403 particles in the bulge. In total, this simulation is evolved for ≈4.99 Gyr in steps of ≈9.79 Myr.

We use three snapshots from Galaxy A to sample initial positions and velocities of test stars, while we use another 26 to extract potentials. Face-on images of the three snapshots are shown in the middle column of Figure 1. Model 1, 2, and 3 correspond to the snapshots that are 2.94, 3.92, and 4.90 Gyr, respectively. Over this time, the galactic bar grows and slows. Similar to previous work (e.g., Athanassoula & Misiriotis 2002; Zana et al. 2018; Rossa-Guevara et al. 2020, 2021), we use the $m = 2$ mode of the Fourier decomposition of the face-on stellar surface density to estimate the length of the bar in number density in order to compare to our dynamical estimate. We determine the Fourier components:

$$A_m(r) = \frac{1}{\pi} \int_0^{2\pi} \Sigma(r, \theta) \cos(m\theta) \, d\theta, \quad m = 0, 1, 2, \ldots$$  (1)

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Figure 1. Properties of the three primary N-body snapshots that we utilize to validate our method. The leftmost column shows the circular velocity curves for each of the three models. In the rightmost column, we show the $A_{2,2}/A_0$ ratio (Equation 3) as a function of galactic radius. We define the bar length as the radius at which $A_{2,2}/A_0=0.15$. In each plot, the bar length is shown as the black dashed line. We also show the corotation radius, defined as the radius where $\Omega_b = v_{\text{circ}}(r)/r$, as a black solid line. In the center column, we show face-on images of each model with circles marking the bar lengths (black dashed) and corotation radii (black solid). The bar length derived from the $A_{2,2}/A_0$ ratio visually matches the drop in number density.

\[
B_m(r) = \frac{1}{\pi} \int_0^{2\pi} \Sigma(r, \theta) \sin(m\theta) \, d\theta, \quad m = 1, 2, \ldots
\]  

(2)

where $\Sigma(r, \theta)$ is the stellar surface density. We then define

\[
A_{2,2}/A_0 = \sqrt{A_2^2 + B_2^2}/A_0
\]  

(3)

and calculate it as a function of $r$ using equal width annuli of $\Delta r = 0.10$ kpc. Similar to Rosas-Guevara et al. (2021), we use the radius at which $A_{2,2}/A_0=0.15$ as our estimate for the bar length, which is shown as the black dashed in all panels of Figure 1. Following Athanassoula & Misiriotis (2002), we also tried the radius at which $A_{2,2}/A_0 = 0.20 \times$
max(A2/2/A0) but, by visual inspection, we found this tends to overestimate the bar length.

We calculate the bar’s pattern speed, Ωb, by measuring the change in the m = 2 phase angle (φ2 = 2tan⁻¹(B2/A2)) between simulated timesteps. Here we calculate B2 and A2 between radii of 1 to 5 kpc. In the leftmost column of Figure 1, we plot the circular velocity curve for each of the snapshots. We also plot the corotation radius, where Ωh = vcirc(r)/r, as a black solid vertical line. Consistent with expectations, the bar length is shorter than the corotation radius (Contopoulos 1980). Specifically, we find that the bar length is 74%, 68%, and 82% of the corotation radius for Models 1, 2, and 3, respectively, making Galaxy A’s bar dynamically fast (Debattista & Sellwood 2000).

In addition to these three models, we also use another 26 (29 in total) snapshots from Galaxy A, to create a variety of gravitational potentials with different bar lengths. These potentials are each ten timesteps apart, starting after bar formation. We use these as test potentials to determine how bar orbits are impacted by potentials with different bar strengths and lengths. Furthermore, we also integrate the APOGEE/Gaia data in these potentials in order to determine which potential best fits the data.

2.2 Galaxy B

Although it is also set up to mirror the Milky Way, Galaxy B has significantly different initial conditions than Galaxy A. For a complete description of Galaxy B, we refer the reader to Tepper-Garcia et al. (2021). In short, Galaxy B is set up using the Action-based GALAXy Modelling Architecture software package (AGAMA; Vasiliev 2019). Compared to Galaxy A, Galaxy B has a lighter virial halo mass of M8 = 1.18 × 10¹²M⊙. Furthermore, the disk of Galaxy B has a shorter scale height (0.3 kpc) and length (2.6 kpc) compared to Galaxy A as well as a large velocity dispersion, σ2/σ1 = 2.

Galaxy A and B are both simulated for about 5 Gyr and each have their bars fully formed between 2 and 2.5 Gyr. However, the bar in Galaxy A significantly grows and slows over the simulation whereas the bar in Galaxy B stays rather stable (see x-axis of Figure 4). Using an iso-density contour, Tepper-Garcia et al. (2021) measure a bar length of ≈4.5 kpc, and pattern speed of ≈40 km/s/kpc.

As for Galaxy A, we use a total of 29 snapshots from Galaxy B to create gravitational potentials. Similar to Galaxy A, these potentials are 10 timesteps apart, starting after bar formation. However, as the bar is more stable in this simulation, these potentials have a much smaller range in bar lengths. Importantly, these potentials allow us to ensure that our method works across different simulations and is not dependent on properties of the simulated galaxies’ potentials other than the bar length and pattern speed. Given that the properties of bars are thought to be heavily impacted by dark matter halos (e.g., Petersen et al. 2019b; Debattista & Sellwood 2000; Fragkoudi et al. 2021; Chiba & Schönrich 2021; Collier & Madigan 2021), it is especially important that we use two models that have different dark matter halo masses and profiles.

3 DATA

3.1 APOGEE DR17 and Gaia eDR3

In order to constrain the Milky Way’s galactic bar, we utilize one of the largest sets of 6D positional and kinematic data available. Specifically, we use a combination of Gaia eDR3 data (Gaia Collaboration et al. 2021) with APOGEE DR17 (Vivas et al. 2021). APOGEE is a near-infrared (1.5-1.7 μm), high-resolution (R = λ/Δλ ≈ 22,500) large spectroscopic survey (Nidever et al. 2015; Wilson et al. 2019; Zasowski et al. 2017). APOGEE DR17 contains over 657,000 stars observed using the APO 2.5m telescope (Gunn et al. 2006) and the 2.5m telescope at Las Campanas Observatory (LCO) (Bower & Vaughan 1973). In this work, we use the radial velocities from the ASPCAP pipeline (García Pérez et al. 2016; Smith et al. 2021), along with the spectrophotometric distance estimates from the astroNN catalog (Leung & Bovy 2019) and proper motions from Gaia eDR3 (Gaia Collaboration et al. 2021). Combining this data gives us typical phase-space uncertainties on the order of 5% for all components. We also repeat our analysis with distances from the StarHorse catalog (Queiroz et al. 2020) to ensure our results are not impacted by a possible distance bias.

In Figure 2, we show the Galactic distribution of the astroNN catalog that we use in our analysis. For reference, we also show the proposed bar model of Wegg et al. (2015) as an ellipse with a semi-major axis of 5 kpc and axis ratio of 0.4, rotated 27° from the Sun-Galactic center line. As demarcated by the red box, we choose stars in between the Sun and the Galactic center, with 0 kpc < X < 8.3 kpc and |Y| < 10 kpc, in order to loosely target the Galactic bar. We choose not to use stars with X < 0 kpc due to the large distance uncertainties. This selection gives us a sample of 215,869 stars for which we have 6D spectrophotometric phase-space information for use in our analysis. We also redo our analy-
sis only with stars that were targeted as part of APOGEE’s main science program for the bulge which is based on a simple color cut criterion. This test allows us to ensure that selection function effects do not dominate our results.

4 USING ORBIT INTEGRATION TO CONSTRAIN THE PATTERN SPEED AND LENGTH OF BARS

In this section, we describe the method that we develop for measuring the pattern speed and length of bars from 6D phase-space observations. First, to separate trapped bar stars from regular disk stars, we perform fundamental frequency analysis in the rotating bar-frame where regular disk orbits will be symmetric in \( x \) and \( y \) (i.e., \( \Omega_x = \Omega_y \)) while bar orbits may not. Next, we define a quantity based on the apocenter distribution of orbits in the bar which we call \( R_\Omega \), for short. We find that we retrieve a self-consistent \( R_\Omega \) (i.e., where the inferred \( R_\Omega \) from the orbits is the same as that of the potential used to calculate the orbits) only when the initial 6D positions and velocities are extracted from a simulated galaxy snapshot with similar bar length and pattern speed as the potential used to calculate the orbits. We apply this method, first, to simulations in order to confirm the accuracy and precision of the results. Next, we apply the method to the Gaia and APOGEE data set in order to constrain the length and pattern speed of the Milky Way’s bar.

4.1 Defining \( R_\Omega \)

In this work, we refer to 3 different proxies for the bar length. 1) The inferred \( R_\Omega \), which we measure from the apocenter distribution of orbits in the bar. We describe this method in detail below. 2) The potential’s \( R_\Omega \), which is calculated with the same method as the inferred \( R_\Omega \) (i.e., using the stars’ apocenter distribution) but the particles are extracted from the simulated galaxy snapshot that the potential is based on. 3) The \( A_{x2}/A_0 \) bar length, which is calculated from the Fourier decomposition of the N-body model as described in Section 2.

As a test of our method, we first wish to determine how the inferred \( R_\Omega \) changes for Models 1, 2, and 3 stars (see Figure 1) as we integrate them in each of the 29 Galaxy A and 29 Galaxy B snapshots described in Section 2. If our method of orbit integration works to determine the correct bar length, we should obtain an inferred \( R_\Omega \) that is consistent with the \( R_\Omega \) of the potential used to calculate the orbits only when the potential is similar to the simulated galaxy snapshot the stars are extracted from (e.g., Models 1, 2, or 3). For each of the 58 snapshots, we first use the mass distribution to extract the corresponding gravitational potential using a self-consistent field (SCF) representation (Hernquist & Ostriker 1992). For our orbital analysis, we use the GALA package (Price-Whelan 2017; Price-Whelan et al. 2020). We use GALA to compute a SCF representation of each potential with the maximum number of radial expansion terms (\( n_{\text{max}} \)) limited to 8 and the maximum number of spherical harmonic terms (\( l_{\text{max}} \)) also limited to 8. As we are using barred N-body models, the potentials are non-axisymmetric. To account for bar rotation, we rotate the potential at a given pattern speed (\( \Omega_0 \)). We then integrate 10,000 random disk stars from Models 1, 2 and 3 in the rotating potential for a total of 1 Gyr in timesteps of 1 Myr. We have also tried integrating the orbits for longer periods of time but do not see significant changes in our results. Although a typical bar’s structure would likely evolve over 1 Gyr, assuming a static bar for orbit integration is reasonable in our case where we are simply interested in measuring the present day bar length.

In order to find the inferred \( R_\Omega \), we first must define bar orbits. To accomplish this, we use orbital frequency analysis. Regular orbits in triaxial potentials have three fundamental frequencies \(( \Omega \equiv \{ \Omega_x, \Omega_y, \Omega_z \} )\) which describe the periodic motion. Laskar (1993) developed a method using fast-Fourier transforms (FFTs) to recover the fundamental frequencies. Referred to as “Numerical Analysis of Fundamental Frequencies” (NAFF), this method has been further developed and applied to galactic dynamics (e.g., Valluri & Merritt 1998; Valluri 1999; Valluri et al. 2010; Price-Whelan et al. 2016a; Yavetz et al. 2021; Koppelman et al. 2021). In this work, we use the SUPERFREQ code (Price-Whelan 2015a,b) to calculate the fundamental frequencies for each of our 10,000 stars. Specifically, we find the Cartesian fundamental frequencies \((\Omega_x, \Omega_y, \Omega_z)\) in the rotating frame where the bar is stationary, which have been shown to better classify bar stars than frequencies in cylindrical coordinates (Valluri et al. 2016). Consistent with Valluri et al. (2016), \( x \) corresponds to the direction along the bar’s major axis, \( y \) is along the bar’s minor axis and \( z \) is the direction out of the plane.

In Figure 3, we demonstrate our method for defining stars in the bar and the potential’s \( R_\Omega \). Specifically, we show results for dynamically measuring the potential’s \( R_\Omega \) for Models 1, 2, and 3 which correspond to three different snapshots from Galaxy A. The leftmost column shows orbital frequency maps for 10,000 random disk stars sampled from Models 1, 2, and 3, respectively. These stars have been integrated in the corresponding snapshot’s extracted potential using the method described above. To create the frequency maps, we plot the ratio \( \Omega_x/\Omega_z \) on the \( x \)-axis and \( \Omega_y/\Omega_z \) on the \( y \)-axis. We color the points by the ratio of the maximum \( x \)-position \((x_{\text{max}})\) to the maximum \( y \)-position \((y_{\text{max}})\) for each star. Disk stars on regular orbits lie along the \( \Omega_x/\Omega_y = 1 \) resonance line. As shown in Figure 3 and in Valluri et al. (2016), bar stars tend to lie above this line with \( \Omega_x/\Omega_y > 0.3 \). We select stars within this region as our sample of bar stars. Specifically, we use stars above the red dashed line where \( \Omega_x/\Omega_y > \Omega_y/\Omega_x + 0.1 \). Although this selection will certainly miss some of the bar stars, there is no contamination of the sample by disk stars. For our work, low contamination is prioritized over completeness. Furthermore, we caution the reader that these estimated frequencies are not robust, but are merely sufficient to separate some bar orbits from the disk orbits. In Section 6, we present a more robust method for selecting bar stars. However, this method is significantly more computationally expensive. The frequency estimates provide a fast selection of bar stars which is required given that we wish to perform the bar length calculation for a large variety of initial phase-space conditions and potentials. Moreover, for this part of the analysis, we primarily wish to perform an apples-to-apples comparison between the potential’s \( R_\Omega \) and the inferred \( R_\Omega \) measured from orbits integrated in the potential. Therefore, as long
Figure 3. The selection and distribution of bar stars for three bar models. The first column shows the orbital frequency map for 10,000 disk stars randomly selected from the N-body models colored by the ratio of $x_{\text{max}}/y_{\text{max}}$ where $x$ is along the bar’s major axis and $y$ is along the minor axis. We show a red dashed line corresponding to $\Omega_y/\Omega_z=\Omega_x/\Omega_z+0.1$, as we select stars above this line as stars in the bar. The central column shows the spatial distribution of our selected bar stars (red points) compared to the rest of the stars in the models. The last column shows the apocenter distribution from the orbits integrated in the corresponding model potential for 1 Gyr. The selected bar stars are shown in red and the rest of the 10,000 stars are shown in grey. The red dashed line correspond to the 99.5th percentile of the bar stars’ apocenter distribution which we define as the potential’s $R_{11}$.

as we remain consistent in our bar stars selection, missing a given family of lower-order bar orbits will not impact the results.

In the middle column of Figure 3 we show face-on images of the N-body models with the bar stars selected from the orbital frequency maps (left panels) as red points. As expected, the selected bar stars all fall within the bar region. In the rightmost column of Figure 3, we plot the apocenter distribution of the orbits of these same stars (red) compared to the remaining disk stars (grey). We plot the 99.5th percentile of the selected bar star’s apocenter distribution as a red vertical dashed line. We define this quantity as the
potential’s $R_0$ (the second of the three bar length proxies discussed in the first paragraph of this section). We use the 99.5th percentile to avoid the impact of a single outlying star increasing our inferred $R_0$. For Models 1, 2, and 3, we measure a $R_0$ of 3.55 kpc, 4.78 kpc, and 5.85 kpc, respectively. We compare this to the bar length measured from the $A_2/A_0$ ratio as a black vertical dashed line. As shown, the potential’s $R_0$, inferred from the apocenter distribution, is consistently smaller than the bar length measured from the $A_2/A_0$ ratio. In fact, the potential’s $R_0$ is between 73% and 80% of the $A_2/A_0$ bar length for Models 1, 2, and 3. However, it is important to note that the $A_2/A_0$ method is known to overestimate the length of bars especially in the case of attached spiral arms (Hilmi et al. 2020; Petersen et al. 2019a).

We refer the reader to Section 6 for further comparisons and discussion of methods for estimating the bar length.

To calculate the inferred $R_0$ (the first of three bar length proxies introduced in the first paragraph of this section), we use the same method as the potential’s $R_0$ with one exception. For the potential’s $R_0$, we are always using particles extracted from the N-body model whose mass distribution is the basis of the potential. This is not necessarily true for the inferred $R_0$. To measure the inferred $R_0$, we use particles that have been extracted from Models 1, 2, or 3 (or observed APOGEE/Gaia data) and integrate them in potentials derived from different snapshots. Therefore, it is not unexpected that inferred $R_0$ is approximately the same as the potential’s $R_0$ when the potential is similar to the Model from which the initial positions and velocities are extracted. However, the power of our method comes from the fact that the inferred $R_0$ and potential’s $R_0$ are approximately the same only when the potential is similar to the Model from which the initial positions and velocities are extracted as we show in Section 4.2.

4.2 Verifying the Method with Simulations

In Figure 4, we show the change in the inferred $R_0$ for Model 1, 2, and 3 stars as they are integrated in potentials with different $R_0$s. Specifically, we integrate the Model 1, 2, and 3 stars in each of the 29 snapshots from Galaxy A (light blue) and the 29 snapshots from Galaxy B (green) which have a range of different bar lengths. On the y-axis, we show the inferred $R_0$, measured from the apocenter distribution of selected bar orbits, compared to the $R_0$ of the potential used to calculate said orbits. We also show the potential’s $R_0$ on the x-axis. The left, center and right panels shows results for stars with initial positions and velocities extracted from Model 1, 2 and 3, respectively. For each of the models we show the corresponding potential’s $R_0$ as black vertical dashed lines with Models 1, 2 and 3 having a potential $R_0$ determined from the apocenter distribution of ≈3.55 kpc, 4.78 kpc, and 5.85 kpc, respectively (see Figure 3). We also calculate the 1σ uncertainty (grey shaded region) on the $R_0$ by repeating the measurement 100 times with different samples of 10,000 stars. It should be noted that here we assume the pattern speed is known and we rotate the potential with the pattern speed calculated from the corresponding Model stars.

We find that only when the potential’s bar length is similar to the bar length of the model from which the initial positions and velocities are extracted do we measure a self-consistent inferred $R_0$, i.e., the difference between the inferred and potentials $R_0$ is ≈0 kpc. That is to say, if the inferred $R_0$ is significantly different from the $R_0$ of the potential used to calculate the orbits, then we know that said potential is not representative of the initial 6D positions and velocities of the stars. Therefore, we can apply this to the Milky Way and constrain the Galactic bar’s potential by testing which potential model provides an inferred $R_0$ that is the same as the potential’s $R_0$ using APOGEE and Gaia data as our initial positions and velocities. However, first it is important to investigate the impact of different pattern speeds on these results given that the Milky Way’s pattern speed is uncertain, although somewhat well-constrained.

In addition to the potential’s bar length, we find that varying the potential’s bar pattern speed also impacts the inferred $R_0$. Similar to Figure 4, we recalculate the inferred $R_0$ for Model 1, 2, and 3 stars, but instead of using potentials from different snapshots we assume the bar length is known. Specifically, we use the potential which corresponds to the respective Model stars and simply vary the pattern speed at which the potential is rotated as the orbits are integrated. We show the impact of varying the pattern speed on the inferred $R_0$ in Figure 5. Similar to Figure 4, we have the difference between the inferred and potential’s $R_0$ on the y-axis. However, now we have the pattern speed on the x-axis. The vertical dashed line corresponds to the pattern speed of the bar in the respective Model 1, 2, and 3 snapshots (see Section 2 for calculation). We also plot a linear fit to the data in a dark blue line. Note, we only use points within ≈5 km/s/kpc of where we visually see the data cross the x-axis because beyond these values the points become non-linear. We take the value at which this line crosses the x-axis (i.e., we find a self-consistent $R_0$) to be the estimated pattern speed. This value is generally consistent with the bar pattern speed in the Model from which the initial positions and velocities are sampled (Model 1, 2, or 3). Specifically, for Model 1, the pattern speed measured from the N-body simulation and from the linear fit in Figure 5 are both 35 km/s/kpc. For Model 2, the N-body simulation gives a pattern speed of 25 km/s/kpc, while the linear fit gives an estimate of 26 km/s/kpc. We find the largest difference between the simulation and our linear fit for Model 3. Explicitly, we measure a pattern speed of 20 km/s/kpc from the simulation, while the linear fit gives a pattern speed estimate of 17 km/s/kpc. This may indicate that the accuracy of our method decreases with longer, slower bars. However, in this case, we are still able to retrieve the bar pattern to within 3 km/s/kpc.

At this point we have only tested our method using the potential calculated from the same N-body snapshot of the Model stars. In order to apply this method to the Milky Way, we must explore the relationship between the potential’s and the inferred $R_0$ in the case where neither the pattern speed nor the correct potential model is known exactly. Explicitly, we need to ensure that assuming an incorrect pattern speed does not lead to an inferred $R_0$ that matches the potential’s $R_0$ for a N-body snapshot that has a different bar length than the one from which the initial positions and velocities are sampled.

In Figure 6, we recalculate the inferred $R_0$ for Model 1, 2, and 3 stars using potentials with different length bars and different bar pattern speeds. For example, for each point
Figure 4. The change in the inferred $R_\Omega$ compared to the $R_\Omega$ of the potential used. We show results from integrating 10,000 stars extracted from Model 1 (left), Model 2 (center) and Model 3 (right) for 1 Gyr in potentials with different bar lengths. Each point represents the results of integrating 10,000 stars in 29 Galaxy A snapshots (light blue) and 29 Galaxy B snapshots (green). The vertical black dashed line shows the $R_\Omega$ of the model from which the stars where extracted. The grey shaded area corresponds to the uncertainty on the model's $R_\Omega$, calculated by bootstrapping the measurement with 100 different samples of 10,000 stars each. Note, we get a self-consistent result ($y$-axis value $\approx 0$) only when the stars’ 6D position and velocity data is consistent with the potential used.

in the top left subplot, we have integrated 10,000 stars extracted from Model 1 for 1 Gyr in a potential extracted from an N-body snapshot with the potential’s $R_\Omega$ indicated by the x-axis value. Within a given subplot, all the points are calculated using the same pattern speed. In the case of the top left subplot, this pattern speed corresponds to 20% larger than the pattern speed of the bar in Model 1. The vertical dashed line and shaded region correspond to the $R_\Omega$ of Model 1 and its associated uncertainty, calculated from bootstrapping samples as described above. Overall, Figure 6 is similar to Figure 4, except for the fact that we do not rotate the potential at the exact pattern speed of the model bar from which the stars’ initial positions and velocities are extracted except for in the third row which is exactly Figure 4. As the difference in estimates for the Milky Way’s pattern speed are $\approx 20\%$ (Bovy et al. 2019), we wish to investigate the behavior of the inferred $R_\Omega$ when the assumed pattern speed is incorrect by up to 20%. Each row, besides the third row, shows results for when the pattern speed is different from the stars’ model pattern speed by +20% (top row), +10% (second row), -10% (fourth row) and -20% (last row). We note that even the fastest pattern speed for a given Model does not decrease the corotation radius below the Model’s $A_2/A_0$ bar length which would cause the bar to become ultrafast and violate our theoretical understanding of bars (Contopoulos 1980, 1981; Buta & Zhang 2009; Vasiliev & Athanassoula 2015).

Even when the pattern speed does not match the initial positions and velocities of the stars, in general, we still find that we retrieve a self-consistent inferred $R_\Omega$ only when the assumed potential’s bar length is similar to the bar length of the N-body snapshot from which the initial positions and velocities of the stars were taken. However, there are a few cases where there are multiple potentials which give self-consistent inferred $R_\Omega$s. For example, the bottom left panel which shows the results for Model 1 stars using a pattern speed that is 20% lower than the pattern speed of the bar in Model 1 has the inferred $R_\Omega$ matching the potential’s $R_\Omega$ for some potentials whose bar is longer than the bar of the N-body snapshot from which the initial positions and velocities of the stars were extracted. In these cases, the slower pattern speed allows the inferred $R_\Omega$ to increase to match the longer potential’s $R_\Omega$. This is likely because the slower pattern speed allows stars with larger apocenters to become trapped in the longer bar potentials. However, this degeneracy only occurs when the pattern speed is too low and is easily broken by increasing the pattern speed. Therefore, it is possible to determine the bar potential that corresponds to the initial positions and velocities of the stars even if the bar’s pattern speed is uncertain by up to 20%. As the Milky Way’s pattern speed is known to within 20% (Bovy et al. 2019), we conclude that we can determine which bar potential is most consistent with the APOGEE and Gaia data. On the other hand, we found that the pattern speed inference (Figure 5) is more sensitive to the choice of model. Therefore, it is important to first infer the most self-consistent model, which is not very sensitive to the assumed pattern speed as discussed above, and then the pattern speed can be inferred more accurately using the best model.
5 CONSTRAINING THE MILKY WAY’S BAR LENGTH AND PATTERN SPEED

Currently, the gravitational potential in the center of the Milky Way is poorly understood, partly because the length of the Galactic bar is not well-constrained. However, several studies assume potentials and make conclusions about the Galactic bar based on the resulting stellar orbits (e.g., Queiroz et al. 2021; Lucey et al. 2021; Wylie et al. 2021). Furthermore, it is presently unknown how the assumption of an incorrect potential will impact the stellar orbits and therefore the conclusions drawn from them. In this work, we have already discovered that $R_\Omega$ inferred from the stellar orbits changes when the assumed gravitational potential and pattern speed of the bar changes. This demonstrates that any conclusions from stellar orbits are greatly impacted by the assumed gravitational potential, but we can use this result to our advantage by finding which potential gives a self-consistent result.

Using simulations, we have demonstrated that the inferred $R_\Omega$ from the apocenter distribution is only consistent with the $R_\Omega$ of the gravitational potential when that potential has a similar bar length to the snapshot from which the initial positions and velocities of the stars are extracted (see Figure 4). Therefore, we can determine which potential is consistent with observed positions and velocities of stars by determining which potential gives a self-consistent inferred $R_\Omega$ measurement. We apply this to the Milky Way by integrating APOGEE and Gaia stars in a variety of Milky Way-like potentials with different bar lengths and pattern speeds.

Following the same methods as in Section 4, we integrate the orbits for 1 Gyr in a variety of potentials and report the 99.5th percentile of the apocenter distribution of stars in the bar as our inferred $R_\Omega$. At first, we assume a bar pattern speed of 41 km/s/kpc, consistent with previous estimates of the Milky Way’s bar pattern speed (Portail et al. 2017; Sanders et al. 2019; Bovy et al. 2019). In Figure 7, we show the orbital frequency maps, galactic distribution and apocenter distributions of APOGEE/Gaia stars integrated in the Model 1, 2, and 3 potentials. The left column of Figure 7 is similar to the left column of Figure 3 with the selection of bar stars to the left of the red-dashed line. In addition, we also show a circle with the radius equivalent to the potential’s $R_\Omega$ derived from the apocenter distribution as a black solid line. In the right column of Figure 7, we show the apocenter distribution of the disk stars in grey with the selected bar stars in red. The 99.5th percentile of the selected bar stars apocenter distribution (i.e., the inferred $R_\Omega$), is shown as a red dashed line with the potential’s $R_\Omega$ as a black solid line. We calculate the orbits for the 215,869 stars shown in Figure 2 divided into 10 random samples so that we are computing $\approx$21,500 orbits at a time, which is the same order of magnitude of the simulation samples we used in Section 4. The final inferred $R_\Omega$ that we report is the me-
Figure 6. The inferred $R_\Omega$ compared to the potential’s $R_\Omega$ when the pattern speed is different by $\pm 10\%$ (row 2 and 4) and $\pm 20\%$ (row 1 and 5) from the pattern speed of the bar model from which the stars are extracted. We also show the results for when the pattern speed matches the bar model from which the stars are extracted in row 3 (same as Figure 4). Each point is calculated using 10,000 stars with initial positions and velocities from the model indicated by the column name. These stars are integrated for 1 Gyr in a potential whose $R_\Omega$ corresponds to the x-axis value. The inferred $R_\Omega$ corresponds to the 99.5th percentile of the apocenter distribution of selected bar stars. The vertical line and shaded region mark the $R_\Omega$ and associated error of the model from which the stars’ initial positions and velocities were extracted. Even when the pattern speed is different than that of the stars, we still find a self-consistent inferred $R_\Omega$ only when the potential is consistent with the stars’ initial positions and velocities.

In the right column of Figure 7, the inferred $R_\Omega$ and potential’s $R_\Omega$ are similar for the APOGEE/Gaia data integrated in the Model 1 potential but are increasing different for the Model 2 and 3 potentials. This can also be seen from the Galactic distribution of the selected bar stars in the middle panel of Figure 7. For the APOGEE/Gaia data integrated in Model 1, the selected bar stars’ Galactic distribution (red points) agrees with the potential’s $R_\Omega$
Figure 7. The selection and distribution of bar stars for the APOGEE/Gaia data integrated in three different potentials with different bar lengths. We assume a bar pattern speed of 41 km/s/kpc for the orbit calculation. The first column shows the orbital frequency map for ≈210,000 stars selected according to Figure 2 colored by the ratio of $x_{\text{max}}/y_{\text{max}}$ where $x$ is along the bar’s major axis and $y$ is along the minor axis. We also show a red dashed line corresponding to $\Omega_y/\Omega_z = \Omega_x/\Omega_z + 0.1$, as we select stars above this line as bar stars. The central column shows the spatial distribution of our selected bar stars (red points) compared to the rest of the stars. We also show the same bar model as in Figure 2 (dashed black line) as well as a circle with radius equivalent to the potential’s $R_\Omega$ (solid black line). The last column shows the apocenter distribution from the orbits integrated in the corresponding model potential for 1 Gyr. The selected bar stars are shown in red and the rest of the stars are shown in grey. We also show the $R_\Omega$ of the potential as a black solid line. The red dashed line corresponds to the 99.5th percentile of the bar stars’ apocenter distribution i.e., the inferred $R_\Omega$.

(black dashed line). However, for the data integrated in the Model 2 and 3 potentials, the selected bar stars’ distribution mostly ends well within the potential’s $R_\Omega$ indicating that the potential’s $R_\Omega$ is likely larger than the data’s true bar length. Based on our previous results from simulations, Model 1 is more consistent with the APOGEE/Gaia than Models 2 and 3 given that its potential leads to a more self-consistent $R_\Omega$ inference. However, it is important to test all of the 29 Galaxy A potentials and the 29 Galaxy B poten-
Inference - Potential's $R^\Omega$ (kpc)

Figure 8. The comparison of the inferred $R^\Omega$ to the potential's $R^\Omega$ for APOGEE and Gaia stars that have been integrated in potential's with different bar lengths for 1 Gyr each, assuming a bar pattern speed of 41 km/s/kpc. Each point corresponds to the median inferred $R^\Omega$ of 10 random samples of $\approx$21,500 stars each with the error bar corresponding the standard deviation. The dark blue points correspond to potentials extracted from Galaxy A while the red points use potentials extracted from Galaxy B. We find two potentials, with orbit-based bar lengths of $\approx$3.5 kpc, which give the most self-consistent inferred $R^\Omega$ (i.e., the difference between the inferred and potential's $R^\Omega$ is $\approx$0 kpc). One of these potentials is Model 1. We name the other potential Model 1.2. We note that these potentials have a corresponding $A^2_2/A_0$ bar length of $\approx$4.8 kpc.

In Figure 8, we show the difference between the inferred $R^\Omega$ and the potential's $R^\Omega$ for a variety of potentials with different bar lengths using APOGEE and Gaia stars. The dark blue points correspond to potentials extracted from Galaxy A (see Section 2.1; Bennett et al. 2021) while the red points correspond potentials from Galaxy B (see Section 2.2; Tepper-Garcia et al. 2021). Each point is calculated using the median inferred $R^\Omega$ of 10 samples of $\approx$21,500 APOGEE and Gaia stars with the uncertainty as the standard deviation. As demonstrated with simulations in Section 4, the gravitational potentials that are most consistent with the APOGEE and Gaia data will have a difference between the inferred $R^\Omega$ and the potential's $R^\Omega$ that is closest to zero kpc. From 8, we find that the potentials with a $R^\Omega$ of $\approx$3.5 kpc are most consistent with the APOGEE and Gaia data. Galaxy B has an inferred $R^\Omega$ that ranges from 2.5-3 kpc, and is never long enough to match the APOGEE data.

Next, we take the two potentials that lead to the smallest difference between the inferred and the potential's $R^\Omega$ for the APOGEE stars and use them to constrain the pattern speed. One of these potentials corresponds to Model 1, while we name the other potential Model 1.2. Model 1.2 is 20 timesteps ($\approx$20 Myr) after Model 1, but is otherwise very similar. In Figure 9, we show the comparison of the inferred $R^\Omega$ to the potential's $R^\Omega$ for the APOGEE/Gaia data when integrated in the Model 1 potential (top) and Model 1.2 potential (bottom) while rotating the potential at range of different pattern speeds. The points correspond to the median of the inferred $R^\Omega$ for 10 random samples, while the error bars correspond to the standard deviation. The green line corresponds to a linear fit to the points. The black dashed line indicates where the green line crosses the x-axis. For both plots this corresponds to a pattern speed of 39 km/s/kpc. Following our method verified in Section 4.2, we use this value as our estimate for the Milky Way’s pattern speed.

Figure 9. Comparison of the inferred $R^\Omega$ to the potential's $R^\Omega$ for APOGEE and Gaia stars that have been integrated in the Model 1 potential (top plot) and the Model 1.2 potential (bottom plot) that are rotated at a range of different pattern speeds. The points correspond to the median of the inferred $R^\Omega$ for 10 random samples, while the error bars correspond to the standard deviation. The green line corresponds to a linear fit to the points. The black dashed line indicates where the green line crosses the x-axis. For both plots this corresponds to a pattern speed of 39 km/s/kpc. Following our method verified in Section 4.2, we use this value as our estimate for the Milky Way’s pattern speed.
spond to the median of the inferred $R_\Omega$ from the 10 random samples of $\approx$21,500 stars, with the standard deviation as the error bars. We note that the two points at pattern speed of $\approx$37 km/s/kpc with differences $>2$ kpc between the inferred and potential’s $R_\Omega$ are mostly due to a few of the random samples having inferred $R_\Omega$ is $\approx$4 kpc longer than the potential’s $R_\Omega$. This is likely caused by large uncertainties in the APOGEE/Gaia data. Similar to Section 4.2, we fit a line to these points and find where this line crosses the x-axis. For both potentials this corresponds to a pattern speed of 39 km/s/kpc. Therefore, we conclude that a pattern speed of $\approx$ 39 km/s/kpc is most consistent with the APOGEE/Gaia data. This is consistent with other recent estimates of 39±3.5 km/s/kpc (Portail et al. 2017), 41±3 km/s/kpc (Sanders et al. 2019), 41±3 km/s/kpc (Bovy et al. 2019) and 40±1.78 km/s/kpc (Leung et al. 2022). We rerun the five potentials which have the smallest difference between the inferred and potential’s $R_\Omega$ in Figure 8 (including one from Galaxy B) using our newly derived pattern speed of 39 km/s/kpc. We still find that Model 1 and Model 1.2 lead to the most self-consistent $R_\Omega$.

It is important to note that our bar length estimate and pattern speed does not reclassify the Galactic bar as a ‘fast’ bar. The ‘fast’/‘slow’ classification is based on the dimensionless ratio $R_\Omega = R_\text{CR}/R_\text{bar}$, where $R_\text{CR}$ is the corotation radius and $R_\text{bar}$ is the bar length. Historically, these classifications are performed using estimates of the bar length that are based on the number density counts (Debattista & Sellwood 2000; Chiha & Schönrich 2021) which is different than the dynamical $R_\Omega$ we measure above. For further discussion of the various methods for estimating and defining bar lengths see Section 6.

### 6 DISCUSSION OF BAR LENGTH ESTIMATES

In this work, we define a quantity, $R_\Omega$, as a proxy for the bar length (see method described in Section 4). However, $R_\Omega$ is not a robust measurement of the bar length since it excludes $x_1$ orbits which have $\Omega_y/\Omega_x = 1$ and are the backbone of most bars. In general, there are many different ways to estimate the bar length, which can lead to varying results. In this section, we discuss and compare methods for measuring the length of bars, specifically in the case where we have stellar orbits. We focus on the Model 1 and Model 1.2 potentials for this comparison, as it was the most consistent with the APOGEE/Gaia data. However, we note that the difference in bar length estimates between methods can vary between bar age and morphology (Petersen et al. 2019a; Hilmi et al. 2020).

One common method for measuring bar length is Fourier decomposition using $A_{2,2}/A_0$ (see Section 3 for the calculation). For both our Model 1 and Model 1.2 potentials this leads to a bar length estimate of 4.84 kpc. However, the Fourier decomposition method has been shown to overestimate the length of bars (Petersen et al. 2019a). Similar to the method for estimating the bar length in external galaxies by fitting ellipses to the surface brightness, the Fourier decomposition method especially overestimates the length of bars when spiral arms are attached which is the case for most of the bars in external galaxies (Hilmi et al. 2020). Model 1 and Model 1.2 have weak spiral arms attached to the bar so it is possible that this method is overestimating the bar length. Interestingly, the $A_{2,2}/A_0$ bar length of 4.84 kpc for these potentials is similar to the Milky Way’s bar length estimate (5 kpc) from number density counts (Wegg et al. 2015). However, Hilmi et al. (2020) suggest this measurement may be overestimated by 1-1.5 kpc given recent observations of spiral arms attached to the bar (Rezaei Kh. et al. 2018).

We also use another dynamical method of measuring the bar length from Petersen et al. (2021). For a complete description of the method we refer the reader to Petersen et al. (2016, 2021). In short, the method classifies bar stars based primarily on the angular distance between the apocenter positions of the star’s orbit and the bar axis. Using this method we can cleanly select $x_1$ orbits which is the family of orbits associated with the inner Lindblad resonance ($2\Omega_y - \Omega_x = \Omega_0$) and whose maximal extent provides a robust estimate of the dynamical length of the bar (Petersen et al. 2019a). In similar models, Petersen et al. (2019b) found that the $x_1$ orbits are responsible for nearly all of the self-gravity of the $m = 2$ Fourier mode bar. With the $x_1$ orbits we measure a bar length of 3.50 kpc for Model 1, which is consistent with the potential’s $R_\Omega$. For Model 1.2, we measure a bar length of 3.97 kpc from the $x_1$ orbits compared to 3.50 kpc from the frequency-selected bar orbits.

Given the variation of measured bar lengths from the different methods, it is important to be careful when comparing reported bar lengths in the literature. To avoid this, instead of emphasizing a specific bar length measurement, we emphasize Model 1 and Model 1.2 as the most self-consistent models for the Milky Way bar, which has a dynamical1 bar length of $\approx 3.5$ kpc with an overdensity that extends to $\approx 4.8$ kpc. We encourage a movement towards publicly available potential models that would allow for easier direct comparison between dynamical results for the inner Galaxy. We note our method can be used to check any potential for self-consistency with Milky Way data.

Another important thing to note is the dependence of the orbital structure of the inner Galaxy on the assumed potential model. This is apparent from looking at the orbital frequency maps in Figure 7. Using the three different potential models, we find the distribution of fundamental frequencies for the orbits are significantly different. The distribution of orbital frequencies for Model 1 and 2 are somewhat similar, but the ratios of the longest distance from the Galactic center along the bar’s major axis to the bar’s

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1 Define the dynamical bar length as the maximal extent of trapped $x_1$ orbits that participate in the solid-body rotation.
minor axis \((x_{\text{max}}/y_{\text{max}})\) are quite different. Although we have found that Model 1 leads to the most self-consistent result for the \(R_0\) determination, we are unsure if other parameters (e.g., the vertical structure and width) are accurate approximations for the Milky Way. Furthermore, we are unsure of how these other parameters may impact the orbital structure. Therefore, we caution the community to be wary when making conclusion about the inner Galaxy from orbits without doing a thorough investigation on the potential dependence.

7 CONCLUSIONS

In this work, we develop a new method to dynamically estimate the bar length and pattern speed directly from orbit integration. First, we verify this method using simulations. In short, we select a sample of bar stars using fundamental frequency maps of orbits and use the 99.5th percentile of the apocenter distribution as the \(R_0\). We find that when the initial position and velocity distribution of the star particles is extracted from the distribution of the potential model, then we achieve a self-consistent result in that the measured \(R_0\) from the orbits matches that of the potential. However, if the initial positions and velocities are extracted from a significantly different distribution than the potential’s, then the measured \(R_0\) from the orbits is different than that of the potential used to calculate said orbits. With this result, we can find the potential and pattern speed that matches the distribution from which the initial positions and velocities are taken by finding which potential and pattern speed lead to a self-consistent measured \(R_0\).

We then apply this new method to the Milky Way and find which potential and pattern speed leads to a self-consistent measured \(R_0\) for the APOGEE/Gaia data. We find our Model 1 and Model 1.2 (see Section 4 and Figure 1) are the most self-consistent potential. These models are derived from the MWPotential2014-3 simulation in Bennett et al. (2021) and have a dynamical bar length in the range of 3.40-3.97 kpc. We also find a pattern speed of 39 km/s/kpc leads to the most self-consistent result for Model 1 and Model 1.2.

However, it is important to note that there are many methods to estimate the bar length, which can lead to varying result dependent on bar age and morphology (Petersen et al. 2019a; Hilmi et al. 2020). For the Model 1 and Model 1.2 potentials, we also measure a bar length of 4.84 kpc from the \(m = 2\) mode of the Fourier decomposition. However, this method is known to overestimate the lengths of bar, especially in the case of connecting spiral arms. As bar length estimates can be unreliable, we emphasize the importance of making gravitational potential models public for the Milky Way in order to make fair comparisons between results. Furthermore, we note that our method for checking self-consistency to constrain the bar length can be used with any potential and we encourage the community to test their favorite potential before using it to draw conclusions about the inner Milky Way.

In our work, we also find that the measured inner Galaxy orbital structure from APOGEE/Gaia data is very potential dependent. Therefore, we caution readers against drawing strong conclusions about the inner Galaxy from orbits without performing a thorough analysis on the assumed potential dependence. By determining a potential and pattern speed which leads to a self-consistent \(R_0\) estimate, we have begun an unobstructed look at the dynamics of the inner Galaxy, with minimal assumptions on Galactic structure. In future work, we plan to further investigate the orbital structure of the Milky Way’s bar and its dependence on various potential model parameters.

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2 http://www.astropy.org
Constraining the Milky Way bar

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DATA AVAILABILITY

The data underlying this article were accessed from an SDSS Value Added Catalog which can be found at http://data.sdss.org/sas/dr17/env/APOGEE_ASTRO_NN/. The derived data generated in this research will be shared on reasonable request to the corresponding author. The Galaxy A simulation will be made publically available upon acceptance. The Galaxy B simulation is already available at http://www.physics.uyed.edu.au/mbar/sim_data/.

This work has made use of data from the European Space Agency (ESA) mission Gaia (https://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

Additional software used includes IPython (Pérez & Granger 2007), matplotlib (Hunter 2007), numpy (Harris et al. 2020), galpy (Bovy 2015), gala (Price-Whelan 2017; Price-Whelan et al. 2020), AGAMA (Vasiliev 2019), and scipy (Virtanen et al. 2020).

3 http://github.com/jobovy/galpy

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