Transversity Distribution and Polarized Fragmentation Function from Semi-inclusive Pion Electroproduction

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Abstract. A method is discussed to determine the hitherto unknown u-quark transversity distribution \(\delta u(x)\) from a planned HERMES measurement of a single target-spin asymmetry in semi-inclusive pion electroproduction off a transversely polarized target. Assuming u-quark dominance, the measurement yields the shapes of the transversity distribution \(\delta u(x)\) and of the ratio \(H_1^{(1)}(z)/D_1^u(z)\), of polarized and unpolarized u-quark fragmentation functions. The unknown relative normalization can be obtained by identifying the transversity distribution with the well-known helicity distribution at large \(x\) and small \(Q^2\). The systematic uncertainty of the method is dominated by the assumption of u-quark dominance.

1 Introduction

Deep inelastic charged lepton scattering off a transversely polarized nucleon target is an important tool to further study the internal spin structure of the nucleon. While a lot of experimental data on the longitudinal spin structure of the nucleon has been collected over the last 10 years, the study of its transverse structure is just about to begin. Only a very limited number of preliminary experimental results is available up to now:

(1) measurements of the nucleon structure function \(g_2(x)\) at CERN \([\,]\) and SLAC \([\,]\)–\([\,]\).
(2) a first measurement of a single target-spin asymmetry for pions produced in lepton scattering off longitudinally polarized protons at HERMES \([\,]\).
(3) a first study of hadron azimuthal distributions in DIS of leptons off a transversely polarized target at SMC \([\,]\).

A quark of a given flavour is characterized by three twist-2 parton distributions. The quark number density distribution \(q(x, Q^2)\) has been studied for decades and is well known for all flavors. The helicity distribution \(\Delta q(x, Q^2)\) was only recently measured more accurately for \(u\)– and \(d\)–quarks \([\,]\) and is still essentially unknown for \(s\)–quarks. The third parton distribution, known generally as ‘transversity distribution’ and denoted \(\delta q(x, Q^2)\), characterizes the distribution of the quark’s transverse spin in a transversely polarized nucleon.

For non-relativistic quarks, where boosts and rotations commute, \(\delta q(x) = \Delta q(x)\). Since quarks in the nucleon are known to be relativistic, the difference between both distributions will provide further information on their relativistic nature. The transversity distribution does not mix with gluons under QCD evolution, i.e. even if transversity and helicity distributions coincide at some scale, they will be different at \(Q^2\) values higher than that.

The chiral-odd nature of transversity distributions makes their experimental determination difficult; up to now no experimental information on \(\delta q(x, Q^2)\) is available. It can not be accessed in inclusive deep inelastic scattering (DIS) due to chirality conservation; it decouples from all hard processes that involve only one quark distribution (or fragmentation) function (see e.g. \([\,]\)). This is in contrast to the case of the chiral-even number density and helicity distribution functions, which are directly accessible in inclusive lepton DIS.

In principle, transversity distributions can be extracted from cross section asymmetries in polarized processes involving a transversely polarized nucleon. The corresponding asymmetry can be expressed through a flavor sum involving products of two chiral-odd transversity distributions in the case of hadron-hadron scattering, while in the case of semi-inclusive DIS (SIDIS) a chiral-odd quark distribution function always appears in combination with a chiral-odd quark fragmentation function. These fragmentation functions can in principle be measured in \(e^+e^-\) annihilation.

The transversity distribution was first discussed by Ralston and Soper \([\,]\) in doubly transverse polarized Drell-Yan scattering. Its measurement is one of the main goals of the spin program at RHIC \([\,]\). An evaluation of the corresponding asymmetry \(A_{TT}\) was carried out \([\,]\) by assuming the saturation of Soffer’s inequality \([\,]\) for the transversity distribution. The maximum possible asymmetry at RHIC energies was estimated to be \(A_{TT} = 1 \div 2\%\). At smaller energies, e.g. for a possible fixed-target hadron-hadron spin experiment HERA-N \([\,]\) (\(\sqrt{s} \simeq 40\text{ GeV}\)), the asymmetry is expected to be higher.
In semi-inclusive deep inelastic lepton scattering off transversely polarized nucleons there exist several methods to access transversity distributions; all of them can in principle be realized at HERMES. One of them, namely twist-3 pion production [14], uses longitudinally polarized leptons and a double spin asymmetry is measured. The other methods do not require a polarized beam; they rely on polarimetry of the scattered transversely polarized quark:

(1) measurement of the transverse polarization of $A$’s in the current fragmentation region [15]-[17],
(2) observation of a correlation between the transverse spin vector of the target nucleon and the normal to the two meson plane [18],[19],
(3) observation of the Collins effect in quark fragmentation through the measurement of pion single target-spin asymmetries [20],[21].

The HERMES experiment [22] has excellent capabilities to investigate semi-inclusive particle production. Taking the measurement of the Collins effect as an example, it will be shown in the following that HERMES will be capable to extract both transversity and chiral-odd fragmentation function at the same time and with good statistical precision.

2 Single Target-Spin Asymmetry in Pion Electroproduction

A complete analysis of polarized SIDIS with non-zero transverse momentum effects in both the quark distribution and fragmentation functions was performed in the framework of the quark-parton model in [21] and in the field theoretical framework of QCD in [22]. An important ingredient of this analysis is the factorization property that was proven for $k_T$ integrated functions and that can reasonably be assumed for $k_T$ depending functions [23]. In the situation that the final state polarization is not considered, two quark fragmentation functions are involved: $D_{kT}^{\perp}(z, z^2 k_T^2)$ and $H_{1T}^{\perp,q}(z, z^2 k_T^2)$. Here $k_T$ is the intrinsic quark transverse momentum and $z$ is the fraction of quark momentum transferred to the hadron in the fragmentation process. The ‘polarized’ fragmentation function $H_{1T}^{\perp,q}$ allows for a correlation between the transverse polarization of the fragmenting quark and the transverse momentum of the produced hadron. It may be non-zero because time reversal invariance is not applicable in a decay process, as was first discussed by Collins [24].

Since quark transverse momenta cannot be measured directly, integrals over $k_T$ (with suitable weights) are defined to arrive at experimentally accessible fragmentation functions:

$$z^2 \int d^2 k_T D_{kT}^{\perp}(z, z^2 k_T^2) \equiv D_{kT}^{\perp}(z)$$

where the superscript (1) indicates that an originally $k_T$ dependent function was integrated over $k_T$ with the weight $k_T^2/2M_h^2$. Here, $M_h$ is the mass of the produced hadron $h$.

To facilitate access to transversity and polarized fragmentation functions from SIDIS, single-spin asymmetries may be formed through integration of the polarized cross section over $P_{h,\perp}$, the transverse momentum of the final hadron, with appropriate weights. In the particular case of an unpolarized beam and a transversely polarized target the following weighted asymmetry provides access to the quark transversity distribution via the Collins effect [25]:

$$A_T(x, y, z) \equiv \frac{\int d\phi \int d^2 P_{h,\perp} \frac{d \sigma}{d^2 P_{h,\perp}} \sin(\phi^x_+ + \phi^y_+)(d\sigma^+ - d\sigma^-)}{\int d\phi \int d^2 P_{h,\perp} (d\sigma^+ + d\sigma^-)}.$$ (3)

Here $\uparrow \downarrow$ denotes target up (down) transverse polarization. The azimuthal angles are defined in the transverse space giving the orientation of the lepton plane ($\phi^x$) and the orientation of the hadron plane ($\phi^y_h = \phi_h - \phi^x$) or spin vector ($\phi^x_q = \phi^x - \phi^x_q$) with respect to the lepton plane. The angles are measured around the $z$-axis which is defined by the momenta $q$ and $P$ of the virtual photon and the target nucleon, respectively. The raw asymmetry (3) can be estimated using

$$A_T(x, y, z) = P_T \cdot D_{nn} \cdot \frac{\sum_q c^2_q \delta q(x) H_{1T}^{\perp,q}(z)}{\sum_q c^2_q \delta q(x) D_{kT}^{\perp}(z)}.$$ (4)

where $P_T$ is the target polarization and $D_{nn} = (1-y)/(1-y+y^2/2)$ is the transverse spin transfer coefficient. The magnitude of the asymmetry depends on the unknown functions $\delta q(x)$ and $H_{1T}^{\perp,q}(z)$.

3 Transversity Distribution and Polarized Fragmentation Function

No experimental data are available on any of the transversity distributions $\delta q(x)$, while their behaviour under QCD-evolution is theoretically well established [16]. An example for the leading order evolution of the proton structure functions $g_1(x, Q^2) = \frac{4}{3} \sum_i c_i^2 \Delta g_i(x, Q^2)$ and $h_1(x, Q^2) = \frac{4}{3} \sum_i c_i^2 \delta q_i(x, Q^2)$ is shown in Fig. [1]. It was assumed that $h_1^p(x)$ coincides with $g_1^p(x)$ at the scale $Q_0^2 = 0.4$ GeV$^2$ and both functions were evolved to the scale $Q^2 = 10$ GeV$^2$. The evolution was performed using the programs from [26],[27] for $g_1$ and $h_1$, respectively. The important conclusion, which was already discussed earlier (see e.g. [28]), follows that with increasing $Q^2$ the two functions are becoming more and more different for decreasing $x$ while at large $x$ the difference remains quite small.

Results from two independent measurements indicate that the polarized fragmentation function $H_{1T}^{\perp,(1)q}(z)$ may be non-zero: i) azimuthal correlations measured between particles produced from opposite jets in $Z$ decay at DELPHI [29] and ii) the single target-spin asymmetry measured for pions produced in SIDIS of leptons off a longitudinally polarized target at HERMES [30]. The approach of [25] is adopted to estimate the possible value of $H_{1T}^{\perp,(1)q}(z)$. Collins [19] suggested the following...
The transversity distribution $h_1^T(x, Q^2)$ (continuous line) which coincides with the helicity distribution $g_1^u(x, Q^2)$ at the scale $Q^2 = 0.4$ GeV$^2$ (as given by the GRSV LO parameterization \cite{32} in the ‘standard’ scenario). Their evolved LO distributions $h_1^T(x, Q^2)$ (dotted) and $g_1^u(x, Q^2)$ (dot-dashed) are shown at $Q^2 = 10$ GeV$^2$.

4 Projected Statistical Accuracy and Systematics

A full analysis to extract transversity and polarized fragmentation functions through (4) requires one to take into account all quark flavours contributing to the measured asymmetry. According to calculations with the HERMES Monte Carlo program HMC, the fraction of positive pions originating from the fragmentation of a struck u-quark ranges, depending on the value of $x$, between 70 and 90% for a proton target and is only slightly smaller for a deuteron target. Therefore, in a first analysis, the assumption of $u$-quark dominance in the $\pi^+$ production cross-section appears to be reasonable. This is supported by the sum rule for T-odd fragmentation functions recently derived in \cite{33}. These authors concluded that contributions from non-leading parton fragmentation, like $d \rightarrow \pi^+$, is severely suppressed for all T-odd fragmentation functions.

Consequently, the assumption of $u$-quark dominance was used to calculate projections for the statistical accuracy in measuring the asymmetry $A_T^u(x)$. The expected statistics for scattering at HERMES unpolarized leptons off a transversely polarized target (proton or deuteron options are under consideration) will consist of about seven millions reconstructed DIS events. The standard definition of a DIS event at HERMES is given by the following set of kinematic cuts\cite{51}:

\[ Q^2 > 1 \text{ GeV}^2, \quad W > 2 \text{ GeV}, \quad 0.02 < x < 0.7, \quad y < 0.85. \]

An additional cut $W^2 > 10 \text{ GeV}^2$ was introduced in the analysis to improve the separation of the struck quark fragmentation region. An average target polarization of $P_T = 75\%$ is used for the analysis.

Considering only $u$-quarks the expression for the asymmetry (4) reduces to the simple form

\[ A_T(x, y, z) = P_T \cdot D_{nn} \cdot \frac{\delta u(x)}{u(x)} \cdot \frac{H_1^{(1)u}(z)}{D_1^u(z)}. \]  

for a proton target, and

\[ A_T(x, y, z) = (1 - \frac{3}{2} \omega_D) \cdot P_T \cdot D_{nn} \cdot \frac{\delta u(x) + \delta d(x)}{u(x) + d(x)} \cdot \frac{H_1^{(1)u}(z)}{D_1^u(z)} \]  

for a deuteron target. Here $\omega_D = 0.05 \pm 0.01$ is the probability of the deuteron to be in the $D$-state.

To simulate a measurement of $A_T$, the approximation $\Delta q(x) = \Delta q(x)$ could be used in view of the relatively low $Q^2$-values at HERMES, in accordance with the above discussion. The Gehrmann-Stirling parameterization in leading order \cite{34} was taken for $\Delta q(x)$ and the GRV94 LO parameterization \cite{35} for $q(x)$. The $Q^2$ evolution of the quark distributions was neglected and $Q^2 = 2.5 \text{ GeV}^2$ was taken as an average value for the

\[ Q^2 \text{ and } \nu \text{ are the photon’s virtuality and laboratory energy, } x = Q^2/2M \nu \text{ is the Bjorken scaling variable, } y = \nu/E \text{ is the fractional photon energy and } W \text{ is the c.m. energy of the photon-nucleon system; } E = 27.5 \text{ GeV at HERMES.} \]
HERMES kinematical region. The HERMES Monte Carlo program HMC was used to account for the spectrometer acceptance. The following cuts were applied to the kinematic variables of the pion:

\[ x_F > 0, \ z > 0.1, \ P_{h\perp} > 0.05 \text{ GeV} \, . \]

The simulated data were divided into 5×5 bins in \((x, z)\). The expectations for the asymmetry \(A_T^{π^+}(x)\) as would be measured by HERMES using a proton target, are presented in Fig. 2 in different intervals of the pion variable \(z\). The projected accuracies for the asymmetry were estimated according to

\[ \delta A_T = \left( \frac{P_{h\perp}}{2m_π} \sin(\phi_s^l + \phi_h^l) \right)^2 \cdot \frac{1}{\sqrt{N_π}}, \tag{12} \]

where \(N_π\) is the total number of measured positive pions after kinematic cuts, and \(<\cdots>\) means averaging over all accepted events. In this way the product of the transversity distribution and the ratio of the fragmentation functions,

\[ K(x, z) = \delta u(x) \cdot \frac{H_{1}^{(1)u}(z)}{D_{1}^{π}(z)} \] \tag{13}

as well as the projected statistical accuracy for a measurement of this function were calculated and are shown in Fig. 2b, again for the case of a proton target.

The factorized form of expression (10) with respect to the variables \(x\) and \(z\) allows the simultaneous reconstruction of the shape for the two unknown functions \(\delta u(x)\) and \(H_{1}^{(1)u}(z)/D_{1}^{π}(z)\), while the relative normalization cannot be fixed without a further assumption. As was discussed above, the transversity distribution \(\delta q(x)\) conceivably coincides with the helicity distribution \(\Delta q(x)\) at small values of \(Q^2\) where the relativistic effects are expected to be small. According to Fig. 1 the differences are smallest in the region of intermediate and large values of \(x\). Hence the assumption

\[ \delta q(x_0) = \Delta q(x_0) \] \tag{14}

at \(x_0 = 0.25\) was made to resolve the normalization ambiguity. The experimental data then consist of 25 measured values of the function \(K(x_i, z_j)\), as opposed to 9 unknown function values: 4 values for \(\delta u(x_i)\) and 5 values for \(H_{1}^{(1)u}(z_j)/D_{1}^{π}(z_j)\), where the indices \(i\) and \(j\) enumerate the experimental intervals in \(x\) and \(z\), respectively. The standard procedure of \(\chi^2\) minimization was applied to reconstruct the values for both \(\delta u(x)\) and \(H_{1}^{(1)u}(z)/D_{1}^{π}(z)\) and to evaluate their projected statistical accuracies expected for a real measurement at HERMES. The results are shown in Figs. 3a, b, respectively.

In an analogous way the consideration of the deuteron asymmetry (11) allows the evaluation of the projected statistical accuracies for a measurement of the functions \(\delta u(x) + \delta d(x)\) and \(H_{1}^{(1)u}(z)/D_{1}^{π}(z)\) (see Figs. 3a, b, respectively). The projected statistical accuracy is considerably worse than that for the proton target; this is caused mainly by the expected smaller value of the asymmetry (11), which in turn is due to the lower value of \((\delta u(x) + \delta d(x))/(u(x) + d(x))\) compared to \(\delta u(x)/u(x)\).

Two sources of systematic uncertainties arising from approximations used in the analysis were investigated. To evaluate the contribution of the normalization assumption (14), the relative difference between transversity distribution \(\delta u(x, Q^2)\) and helicity distribution \(\Delta u(x, Q^2)\) was studied as a function of \(x\) and \(Q^2\) in the HERMES kinematics. Starting from \(\delta u(x) = \Delta u(x)\) at the scale \(Q_0^2 = 0.4\text{ GeV}^2\) both functions were evolved to higher values of \(Q^2\). The results are shown in Fig. 4 and allow the conclusion that the relative difference is small for
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5 Conclusions

In conclusion, the HERMES experiment using a transversely polarized proton target will be capable to measure simultaneously and with good statistical precision the shapes of the u-quark transversity distribution δu(x) and of the ratio of the fragmentation functions H1/u(x)/D1/u(x). The normalization can be fixed under the assumption that in the HERMES Q^2 range the transversity distribution is well described by the helicity distribution at large x. Using a deuteron target, information on δu(x) + δd(x) will be available, but with considerably less statistical accuracy compared to a measurement of δu(x) from a proton target. The systematic uncertainty of the method proposed in this paper is dominated by the assumption of u-quark dominance.