Vibration Characteristics Analysis of Orthotropic Rectangular Sandwich Plate with Magnetorheological Elastomer

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Abstract

Vibration characteristics of orthotropic rectangular sandwich plate with magnetorheological (MR) elastomer core and constraining layer is presented in this study. In order to improve the vibrational behaviors of the system, the orthotropic rectangular plate is covered a MR fluid core and a constraining layer. Finite element method is adopted to derive the finite element equations of motion for the orthotropic sandwich system. The effects of the natural frequencies and loss factors of the orthotropic sandwich plate are discussed. The rheological property of the MR material can be changed and controlled by applying various magnetic fields. The modal dampings and the natural frequencies for the orthotropic sandwich plate are calculated for various magnetic fields. Besides, the effects of the strength ratio of the base plate on the natural frequencies and modal loss factors are also discussed.

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Nomenclature

- \textbf{a, b} length of the plat at the x- and y-directions
- \textbf{u_i, v_i} axial displacements of the mid-plane of layer \textit{i} at the x- and y-directions

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1. Introduction

In many mechanical structures, the orthotropic rectangular plates have been received a great deal of attention because of their wide usage. Some works in this field were based on the solutions of differential equations by Timoshenko [1] and Cao [2]. For recent years, the finite element method was also utilized to solve the vibration problems of orthotropic rectangular plate with various boundary conditions [3]. In recently, the magnetorheological (MR) material has great potential in applications for smart systems. Rabinow [4] first discovered the effects and characteristics of the MR material and the MR material had rapid change in their rheological properties. Weiss et al. [5] discussed the damping and stiffness properties of MR material with the applications of the magnetic fields. The applications for the construction of smart components had been previously presented by Yalcintas and Dai [6]. Then, Bellan and Bosiss [7] were studied the Field dependence of viscoelastic properties of magnetorheological elastomers. The investigations on the adjustable rigidity of magnetorheological-elastomer-based sandwich beams were discussed by Zhou and Wang [8]. In addition, Ying and Ni [9] calculated the micro-vibration response of a stochastically excited sandwich beam with a magnetorheological elastomer core and mass. Rajamohan et al. [10] studied the vibration characteristics of a partially treated multi-layer beam with magnetorheological fluid. After that, Nayak et al. [11] obtained the dynamic analysis of magnetorheological elastomer-based sandwich beam with conductive skins under various boundary conditions.

2. Problem formulations

In the Figure 1, the configuration of the orthotropic rectangular plate with MR elastomer core layer and constraining layer is shown. The orthotropic rectangular plate is assumed to be as layer 3. Layer 2 is the MR elastomer and the properties of the MR material can be changed with different magnetic fields. Then, layer 1 is a pure elastic constraining layer. Before the derivation proceeds, the following assumptions must be mentioned first:

1. At first, no slipping between the elastic and MR elastomer layers is assumed.
2. Then, the transverse displacements for every point on a cross-section are the same.
3. There exists no normal stress in the MR elastomer layer, and there exists no shear strain in the elastic layer either.

The strain-displacement relation of the elastic layer can be expressed as the following equation as shown in Figure 2:

\[
\begin{bmatrix}
\varepsilon_{xi} \\
\varepsilon_{yi}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & z_i & 0 \\
0 & \frac{\partial}{\partial y} & 0 & z_i
\end{bmatrix} \begin{bmatrix}
u_i \\
v_i \\
w
\end{bmatrix} \quad i=1,3.
\]

The shear strains of the MR elastomer core layer can be written as follows by considering the geometry of the sandwich plate system as shown in Figure 2:
\[
\begin{bmatrix}
\gamma_{x2} \\
\gamma_{y2}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial}{\partial z} & 0 \\
0 & \frac{\partial}{\partial y}
\end{bmatrix} \begin{bmatrix}
u_2 \\
v_w
\end{bmatrix},
\]

(2)

Then, by referring to the geometric relationship between \( u_i, v_i, e_{xi}, e_{yi} \) and \( \partial w/\partial z \) of the elastic layers 1 and 3, the displacement relations of the MR elastomer core layer are:

\[ \text{Figure 1. The orthotropic plate system with MR elastomer core and constraining layer.} \]

\[ \text{Figure 2. Undeformed and deformed configurations of the sandwich rectangular plate (a) xz-plane (b) yz plane.} \]

\[ \frac{\partial u_2}{\partial z} = \frac{1}{h_2} \left[ \frac{(h_1 + h_3)}{2} \frac{\partial w}{\partial x} + (u_1 - u_3) \right], \]

(3)

Substituting above equations into Eqs.(4), the following shear strain in the MR elastomer core layer can be rewritten as:

\[ \begin{bmatrix}
\gamma_{x2} \\
\gamma_{y2}
\end{bmatrix} = \frac{d}{h_2} \left[ \frac{\partial w}{\partial x} + \frac{d}{d} \frac{d}{\partial x} + \frac{d}{\partial y} \left( v_1 - v_3 \right) \right], \quad d = (h_1 + 2h_2 + h_3)/2 \]

(4)

Then, the strain energy associated with the normal strain in the elastic layer can be obtained:
\begin{equation}
V_i = \frac{1}{2} \int_D D_i (\varepsilon^2_{\varepsilon i \varepsilon i} + \varepsilon^2_{\varepsilon j \varepsilon j}) dA \quad i=1,3. \tag{5}
\end{equation}

Besides, the strain energy of the MR elastomer core layer is obtained as follows:
\begin{equation}
V_2 = \frac{1}{2} \int_D G^* (\gamma^2_{\gamma \gamma \gamma} + \gamma^2_{\gamma \gamma \gamma}) dA, \tag{6}
\end{equation}

Then, Let \( V \) be the total strain energy of the sandwich plate, and the following equation can be obtained:
\begin{equation}
V = V_1 + V_2 + V_3. \tag{7}
\end{equation}

In addition, the kinetic energy of the sandwich plate is
\begin{equation}
T = \frac{1}{2} \int_D \left[ \rho_1 h_1 (\dot{u}_1^2 + \dot{v}_1^2) + \rho_3 h_3 (\dot{u}_3^2 + \dot{v}_3^2) \right] dA + \frac{1}{2} \int_D (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3) dA + \frac{1}{2} \int_D I_2 (\dot{\gamma}^2_{\gamma \gamma \gamma} + \dot{\gamma}^2_{\gamma \gamma \gamma}) dA. \tag{8}
\end{equation}

The plate elements utilized in this study are two-dimensional element bounded by four nodal points. And, each node has seven degrees of freedom to describe the longitudinal displacements, transverse displacements, and slopes of the sandwich plate. The transverse displacement, longitudinal displacement can be expressed in terms of a nodal displacement vector and a shape function vector:
\begin{align}
\mathbf{w}(x,y,t) &= N_u(x,y)q(t), \tag{9a} \\
\mathbf{u}_i(x,y,t) &= N_{u i}(x,y)q(t), \quad i=1,3. \tag{9b} \\
\mathbf{v}_i(x,y,t) &= N_{v i}(x,y)q(t), \quad i=1,3. \tag{9c}
\end{align}

where \( q(t) = [u_{1i}, v_{1i}, u_{3i}, v_{3i}, w_i, w_{xi}, w_{yi}]^T \) for \( i=1,2,3,4, \ N_u(x,y), \ N_{ui}(x,y) \), and \( N_{vi}(x,y) \) are the shape functions of the plate element.

In order to obtain the equation of motion of the sandwich plates, the Hamilton’s principle is used and as follows:
\begin{equation}
\delta \int^T_T (T - V) dt = 0. \tag{10}
\end{equation}

Then, the strain energy and kinetic energy derived in the above section can be rewritten in terms of nodal displacement variables as follows:
\begin{align}
V &= \frac{1}{2} \{q(t)\}^T \{K^e\} \{q(t)\}, \tag{11} \\
T &= \frac{1}{2} \{\dot{q}(t)\}^T \{M^e\} \{\dot{q}(t)\}. \tag{12}
\end{align}

The governing equation for the sandwich plate element is obtained as follows by substituting the strain energy and kinetic energy into the Hamilton’s principle:
\begin{equation}
[M^e] \{\ddot{q}(t)\} + \{K^e\} \{q(t)\} = \{0\}, \tag{13}
\end{equation}

By assembling the contributions of all elements, the global dynamic equation of the sandwich plate with MR elastomer core layer and constraining layer can be expressed as
\begin{equation}
[M] \{\ddot{q}(t)\} + \{K\} \{q(t)\} = \{0\}. \tag{14}
\end{equation}

Then, the complex eigenvalues \( \tilde{\lambda} \) of the above complex eigenvalue problems can be found numerically. The natural frequencies \( \omega \) and modal loss factor \( \eta_v \) are extracted in the following equations:
\begin{align}
\omega_v = \sqrt{\frac{\text{Re}(\tilde{\lambda})}{\text{Re}(\tilde{\lambda})}}, \quad \eta_v = \frac{\text{Im}(\tilde{\lambda})}{\text{Re}(\tilde{\lambda})}. \tag{15a,b}
\end{align}
3. Results and discussions

In this paper, the vibration characteristics analysis of orthotropic sandwich rectangular plate with MR elastomer core layer and constraining layer is presented. The comparisons between the present results in order to validate the proposed algorithm and calculations and the results of the references are shown in Figure 3. A good agreement of the calculation results on natural frequency and modal loss factor can be found in the above results. The complex shear modulus of the MR elastomer used in this study was estimated by performing a free oscillation experimental on the fully treatment MR sandwich system [12]. The complex shear modulus of the MR elastomer can be expressed as the following function with respect to the intensity of magnetic field:

$$G^s(B) = G'(B) + jG''(B),$$

where the storage modulus $G'(B) = -3.3691 \times B^2 + 4997.5 \times B + 873000$, the loss modulus $G''(B) = -0.9 \times B^2 + 812.4 \times B + 185500$, $B$ is the intensity of magnetic field in Gauss, and the mass density of the MR elastomer core layer is $3500 \text{kg/m}^3$. Afterward, the following non-dimensional parameters and some geometrical parameters are introduced for convenience:

$$a = 0.3m, \quad b = 0.2m, \quad \tilde{h}_1 = h_1/h_3, \quad \tilde{h}_2 = h_2/h_3, \quad h_3 = 1.25mm, \quad \tilde{E}_1 = E_{x1}/E_{y1}, \quad \tilde{E}_3 = E_{x3}/E_{y3}, \quad \nu_{x1} = 0.3, \quad \nu_{x3} = v_{x1} \times E_{x1}/E_{x3}, \quad E_{y1} = E_{y3} = 70GPa.$$

The variations of orthotropic sandwich plate system on natural frequency and modal loss factor for various magnetic fields are presented in Figure 4. The analysis condition of the parameters are $\tilde{h}_1 = 0.2$, $\tilde{h}_2 = 0.5$, $\tilde{E}_1 = 1$, and $\tilde{E}_3 = 1.5$. According to the results, the natural frequency will be larger with stronger applying magnetic field. Besides, the modal loss factor will be smaller as the applying magnetic filed is stronger. It is because that the larger intensity of magnetic field will increase the stiffness of the orthotropic sandwich plate system and also decrease the damping effect of the orthotropic sandwich plate system. In addition, the tendency is similar for various modes on natural frequency and modal loss factor according to the figures.

![Figure 3. Comparisons of natural frequency and loss factor](image-url)
The effects of magnetic fields on the natural frequency and modal loss factor of the orthotropic sandwich plate system for various modes are plotted in Figure 5. In this section, the analysis condition of the parameters are \( h_1 = 0.2 \), \( h_2 = 0.5 \), \( E_1 = 1 \), and \( E_3 = 1.5 \). Based on the numerical results, the natural frequency will increase when the applying magnetic fields increase. As to the modal loss factor, it can be observed that the modal loss factor will increase at first and decreases at specific value with the increasing of the magnetic fields. Thus, the natural frequency will be larger and modal loss factor will be smaller with larger modes of the sandwich plate system according to the above numerical results.

Figure 4. Variations of the orthotropic sandwich plate system on natural frequency and modal loss factor for various magnetic fields.

Figure 5. The effects of magnetic fields on the natural frequency and modal loss factor of the orthotropic sandwich plate system for various modes.
In Figure 6, the effects of $\tilde{E}_3$ and electric fields on the natural frequencies and modal loss factors of the orthotropic sandwich plate can be observed. The analysis condition of the parameters are $\tilde{h}_1 = 0.2$, $\tilde{h}_2 = 0.5$ and $\tilde{E}_1 = 1$. The numerical results show that the larger strength parameter $\tilde{E}_3$, the higher natural frequencies of the orthotropic sandwich plate system. As to the modal loss factors of the orthotropic sandwich plate system, it can be observed that the larger strength parameter $\tilde{E}_3$, the smaller modal loss factors. On the other hand, the natural frequency increases and modal loss factor decreases when the applying magnetic fields increase.

4. Conclusions

This study presents the vibration characteristics of the orthotropic sandwich plate with MR elastomer core and constraining layer. The finite element method is utilized to obtain the formulation and numerical results of natural frequencies and modal loss factor for the systems. Based on the numerical results, it can be observed that the natural frequencies and the modal loss factors will be changed with different parameter $\tilde{E}_3$. In addition, the larger applying magnetic fields result in greater natural frequencies and smaller applying magnetic fields will decrease the modal loss factor of the orthotropic sandwich plate. Thus, the MR elastomer is shown to have significant effects to control the vibration characteristics of the orthotropic sandwich system. And, the present results can provide the basic information for practical and active controllable applications.

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