Peak, valley and intermediate regimes in the lateral van der Waals force

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We study the van der Waals (vdW) interaction between a polarizable particle and a grounded conducting corrugated surface. For sinusoidal corrugations, one knows that, under the action of the lateral vdW force, an isotropic particle is always attracted to the nearest corrugation peak, with such behavior called in the present paper as peak regime. Here, considering an anisotropic polarizable particle, and making analytical calculations valid beyond the proximity force approximation (PFA), we show that the attraction is not only toward the peaks, but, for certain particle orientations and distances from the surface, the lateral force attracts the particle to the nearest corrugation valley (valley regime), or even to an intermediate point between a peak and a valley (intermediate regime). We also show that in the configurations of transition between the peak and valley regimes the lateral vdW force vanishes, even in the presence of a corrugated surface. In addition, we find that these new regimes occur in general, for periodic and nonperiodic corrugated surfaces. Moreover, we demonstrate that similar regimes arise in the classical interaction between a neutral polarized particle and a rough surface. The description of these valley and intermediate regimes, which are out of reach of the predictions based on the PFA, may be relevant for a better understanding of the interaction between anisotropic particles and corrugated surfaces in classical and quantum physics, with experimental verifications feasible in both domains.

INTRODUCTION

In the 1940s decade, Casimir and Polder calculated the interaction between a neutral atom and a perfectly conducting plate, taking into account the retardation of the electromagnetic interaction [1]. In the limit of very small distances between the atom and the plate, if compared to the wavelengths of the atomic transitions, the Casimir-Polder (CP) formula reduces to the London-van der Waals (vdW) interaction between the atom and the plate [1]. In general, CP/vdW dispersion forces depend on the geometry of the material bodies and on the boundary conditions that they impose on the electromagnetic field [2–8]. These forces have important consequences in physics, chemistry, biology and engineering [9–11], and the progress in the precision of the experiments that measure them has opened possibilities for many applications in micro and nanotechnology [12–15].

In the context of the CP/vdW interaction, the interest in the subtleties brought by the consideration of anisotropic polarizable particles has increased [16–24]. Among the reasons is the phenomenon known as the Casimir torque [18–21], and the prediction of a vertical repulsive CP/vdW force involving anisotropic particles [22–25]. To take into account curvature or roughness effects in the interaction with these particles, theoretical analyses beyond the proximity force approximation (PFA) are necessary when such effects depend on a more precise description of the surface geometry [18]. During the last two decades, several studies have been done using approaches that can provide predictions for physical situations for which the PFA is not the best approximation [18, 22, 26–33]. For instance, for isotropic particles interacting with grooved surfaces, the PFA predicts that no lateral force acts on the particle when it is over a plateau of the surface, whereas analyses beyond the PFA reveals the existence of a lateral force [32].

In the present paper, combining the perturbative analytical solution for the Green function obtained by Clinton, Esrick, and Sacks [34] (related to Poisson’s equation of a point charge in the presence of an ideal conducting nonplanar surface) with the description done by Eberlein and Zietal [35] (for the vdW interaction between a polarizable particle and an ideal conducting surface), we propose a new analytical approach to investigate the vdW interaction between an anisotropic particle and an ideal grounded conducting corrugated surface. This approach requires no demand on the smoothness of the rough surface, so that our results are valid beyond the PFA. In the sequence, we apply our formulas to a sinusoidal surface with corrugation period \( \lambda \), distant \( z_0 \) from the particle, and predict that, under the action of the lateral vdW force, a neutral isotropic particle is always attracted to the nearest corrugation peak, in agreement with results found in the literature [32, 36], a behavior called in the present paper as peak regime. On the other hand, for the case of a neutral anisotropic polarizable particle, we show that, depending on the ratio \( \lambda/z_0 \) and particle orientation, it can also be attracted to a corrugation valley (valley regime), or to a point between a peak and a valley (intermediate regime). In all situations, taking the limit \( \lambda/z_0 \to \infty \), our formulas lead to results according to the PFA, for which we show that, even considering anisotropic particles, only the peak regime is predicted. We also show that similar regimes (valley and intermediate) also arise in the classical interaction between a neutral polarized particle and a sinusoidal surface, thus providing additional experimental possibilities to verify the existence of these new regimes.

OUR APPROACH

Our approach consists of combining the solution for the Green function obtained by Clinton, Esrick, and Sacks [34], with the formula for the vdW interaction obtained by Eberlein and Zietal [35]. Let us start discussing the perturbative analytical solution in Ref. [34]. For a charge \( Q \) located at the position...
We also remark that all the perturbative process leading to Eq. (1) between the mean characteristic distance along the surface and the surface depends on the homogeneous part of $G$, located at $r_0 = x_0 \hat{x} + y_0 \hat{y} + z_0 \hat{z}$ ($z_0 > 0$) and with a dipole moment vector $d$, into the van der Waals interaction $U_{vdw}$.

$$U_{vdw}(r_0) = \frac{1}{16\pi\varepsilon_0} \sum_{i,j} (d_i d_j) \nabla_i \nabla_j G_H(r_i, r_j)|_{r_i = r_j = r_0},$$

where $d_i (i, j = \{x, y, z\})$ are the components of the dipole moment operator and $(d_i d_j)$ is the expectation value of $\hat{d}_i \hat{d}_j$ (see also [37]). In both equations (2) and (3), the function $G_H$ is the solution of the Laplace equation in the presence of the surface, which contains all the information about its geometry.

In this paper, we investigate the physical implications of a set of new analytical formulas, developed by us, combining the perturbative solution $G_H \approx G_H^{(0)} + G^{(1)}$, where $G^{(1)}$ is given in Eq. (1), with the classical and quantum interactions given by Eqs. (2) and (3).

### THE CLASSICAL INTERACTION

The electrostatic interaction between a polarized particle, at $r_0 = x_0 \hat{x} + y_0 \hat{y} + z_0 \hat{z}$ ($z_0 > 0$), and a grounded conducting rough surface can be written as $U_{cla} \approx U_{cla}^{(0)} + U_{cla}^{(1)}$, where $U_{cla}^{(0)}(z_0) = -(d_x^2 + d_y^2 + 2d_z^2)/64\pi\varepsilon_0 z_0^2$ is the usual interaction energy between a dipole and a grounded conducting plane [38], and $U_{cla}^{(1)}$ is the first-order correction to $U_{cla}^{(0)}$ due to the surface corrugation. Substituting $G^{(1)}$ [given by Eq. (1)] in Eq. (2), we write $U_{cla}^{(1)}$ in terms of $h(q)$, the Fourier representation of $h(r_0)$, obtaining

$$U_{cla}^{(1)}(r_0) = -\sum_{i,j} \frac{d_i d_j}{64\pi\varepsilon_0} \left(\frac{d^2 q}{(2\pi)^2} \hat{h}(q)e^{iqr}r_0\right) I_{ij}(z_0, q),$$

where $q = q_x \hat{x} + q_y \hat{y}$, and the functions $I_{ij}$ are written as $I_{ij} = K^{(s)}_{ij} K_2(z_0|q|) + K^{(s)}_{ij} K_3(z_0|q|)$, where $K_2$ and $K_3$ are modified Bessel functions of the second kind, and $K^{(s)}_{ij} = K^{(s)}_{ij}(s = 2, 3)$ are given by $K^{(2)}_{xx} = -\frac{3}{8} q_x^2 |q|^2$, $K^{(3)}_{xx} = \frac{3}{8} q_x^4$, $K^{(3)}_{xy} = -\frac{3}{8} q_y^2 |q|^2$, $K^{(3)}_{yy} = \frac{3}{8} q_y^4$, $K^{(3)}_{zz} = 0$, $K^{(3)}_{xz} = i q_x q_z |q|^2$, $K^{(3)}_{yz} = -i q_y q_z |q|^2$, $K^{(3)}_{xz} = i q_x q_z |q|^2$, $K^{(3)}_{yz} = -i q_y q_z |q|^2$, $K^{(3)}_{xy} = i q_x q_y |q|^2$, $K^{(3)}_{yz} = -i q_y q_z |q|^2$, $K^{(3)}_{xz} = i q_x q_z |q|^2$. It is worth to point out that the roughness correction $U_{cla}^{(1)}$, given by Eq. (4), is valid for any roughness profile function $h$ with $\max|h| \ll z_0$, with no constraint on the smoothness of the surface, which means that we can investigate configurations beyond the scope of the PFA.

Let us investigate the case of a sinusoidal corrugated surface with amplitude $a$ and corrugation period $\lambda$, which is described by $h(x) = a \cos(kx)$, where $k = 2\pi/\lambda$ and $a \ll z_0$. Now, we discuss the Eberlein-Zietal method to compute the vdW interaction between a polarizable neutral point particle and a general grounded conducting corrugated surface [35]. This method consists of mapping the classical interaction energy $U_{cla}$.

$$U_{cla}(r_0) = \frac{1}{8\pi\varepsilon_0} \left(\mathbf{d} \cdot \nabla\right)^2 \left(\mathbf{d} \cdot \nabla\right) G_H(r, r')|_{r=r'=r_0},$$

which is the interaction energy for a point particle located at $r_0 = x_0 \hat{x} + y_0 \hat{y} + z_0 \hat{z}$ ($z_0 > 0$) and with a dipole moment vector $d$, into the van der Waals interaction $U_{vdw}$.

Figure 1. Illustration of a charge $Q$, located at $r' = z' \hat{x} + y' \hat{y} + z' \hat{z}$ (with $z' > 0$), interacting with a general grounded conducting corrugated surface, whose corrugation profile is described by $z = h(r_0)$.
(see Fig. 3). Then, the integrals in Eq. (4) can be analytically solved [39, 40], resulting in

\[ U_{\text{cla}}^{(1)} = -\frac{3a}{512\pi e_0 c_0} A(d, d, k z_0) \cos[k x_0 - \delta(d, d, k z_0)], \]

where \( \delta \) is a nontrivial phase function defined by

\[ \sin(\delta) = \frac{B(d, d, k z_0)}{A(d, d, k z_0)}, \quad \cos(\delta) = \frac{C(d, d, k z_0)}{A(d, d, k z_0)}, \quad (6) \]

with \( A = \sqrt{B^2 + C^2} \).

\[ B = -2d_x d_y R_{xx}(k z_0), \quad C = d_x^2 R_{xx}(k z_0) + d_y^2 R_{yy}(k z_0) + d_z^2 R_{zz}(k z_0), \quad (7) \]

and the functions \( R_{ij} \) are defined by:

\[ R_{xx}(u) = u^3 K_3(u) - u^4 K_4(u), \quad R_{yy}(u) = u^3 K_3(u), \quad R_{zz}(u) = (u + \frac{3}{4} u^2) K_2(u) + \frac{3}{2} u^3 K_3(u) - u^4 K_4(u). \]

The inset in Fig. 2 shows the behavior of \( R_{ij}(2\pi z_0/\lambda) \) versus \( 2\pi z_0/\lambda \). The function \( R_{xx} \) (solid line) changes its sign at \( 2\pi z_0/\lambda \approx 2\pi/e \), where \( e \) is the Euler number. In the inset we also show the behavior of \( R_{ij}(2\pi z_0/\lambda) \) versus \( \lambda/z_0 \). Note that the function \( R_{xx} \) changes its sign at \( \lambda/z_0 \approx e \).

From Eq. (5), considering the particle fixed at \( z = z_0 \), we can see that the stable equilibrium points of \( U_{\text{cla}}^{(1)} \) can be over the corrugation peaks (when \( \delta = 0 \)), valleys (\( \delta = \pi \)), or over points between a peak and a valley (\( \delta \neq 0, \pi \)). Such behaviors are named here as peak, valley, and intermediate regimes, respectively, and can be visualized in Fig. 4. From Eqs. (6)-(8), we can see that when \( d_x = 0 \) one has only the peak regime, for \( d_z = 0 \) we can have peak or valley regimes, whereas for other dipole orientations we have intermediate regimes. We will now investigate each one of these possibilities separately. For \( d_x = 0 \), from Eqs. (7) and (8), we have the functions \( B = 0 \) and \( C > 0 \) (since \( R_{xx} \), the only function \( R_{ij} \) that can be negative, cannot contribute in this case), resulting in \( \delta = 0 \) [peak regime, Fig. 4(i)] for any value of \( d_y \) and \( d_z \). On the other hand, for \( d_z = 0 \), we have \( B = 0 \) again, but now \( C \) can be negative. If we write the components of the dipole vector in spherical coordinates, namely \( d_x = |d| \sin \theta \cos \phi, d_y = |d| \sin \theta \sin \phi \) and \( d_z = |d| \cos \theta \), we obtain that the valley regime [Fig. 4(ii)] occurs for \( \theta = \pi/2 \) and \( \tan^2(\phi) < -R_{xx}/R_{yy} \) (dark region in Fig. 5(a)), whereas the peak regime occurs for \( \tan^2(\phi) > -R_{xx}/R_{yy} \) (lighter region in Fig. 5(a)). The border between these two regions (dashed line in Fig. 5(a)) corresponds to the situations where \( U_{\text{cla}}^{(1)} = 0 \), which means that the lateral force vanishes. Note that the highest value of \( \lambda/z_0 \) which belongs to this dashed border occurs when \( \phi = 0, \pi \) (dipole oriented in \( x \)-direction), and here, it happens at \( \lambda/z_0 \approx e \), so that above it there is no value of \( \phi \) making possible the valley regime. From Eqs. (7) and the formula for \( R_{xx} \), one can see that when \( d_z \neq 0 \) and \( d_x = 0 \) we have the function \( B \neq 0 \), resulting in the intermediate regime [Fig. 4(iii)] and Fig. 4(iv)]. In this case, the values \( x_{\text{min}} \) (minimum values of \( U_{\text{cla}}^{(1)} \)) can be computed analytically by solving the equation \( k z_{\text{min}} - \delta(d, d, k z_0) = 2n\pi \), with \( n \in \mathbb{Z} \), with the phase function \( \delta \) obtained from Eqs. (6). When we consider general orientations \( (\phi, \theta) \) of the particle, the solution of this equation reveals other two more general behaviors of \( U_{\text{cla}}^{(1)} \), as shown in Fig. 5(b). The dark region in Fig. 5(b) represents that, with an appropriate choice of \( (\phi, \theta) \), \( x_{\text{min}}/\lambda \) can have any value in the interval \([0, 1]\), with this behavior occurring for \( 0 < \lambda/z_0 \leq c \). The inset in Fig. 5(b) considers the periodicity of \( U_{\text{cla}}^{(1)} \), and illustrates in (i) that the dark region implies that an appropriate choice of \( (\phi, \theta) \) can give any value to \( x_{\text{min}} \). This behavior can also be seen in Figs. 6(a), 6(c) and 6(d). The lighter region in Fig. 5(b) means that the orientations \( (\phi, \theta) \) only can produce values of \( x_{\text{min}} \) located around the peaks, as shown in insets (ii) and (iii). This behavior can also be seen in Fig. 6(b). One can also see in Fig. 5(b) that, as \( \lambda/z_0 \) increases, the region where the possible \( x_{\text{min}} \) can be found decreases, so
that, when $\lambda/z_0 \to \infty$ (PFA), we have $x_{min}/\lambda \to n$, which means that the minimum points are only over the peaks, independently of $(\phi, \theta)$ [see Fig. 5(b), inset (iv)]. This can be understood by taking the limit $\lambda/z_0 \to \infty$ in our Eq. (5), leading to $U_{\text{vdW}}^{(1)} \to -a \cos(k_{\text{vdW}} \phi) (3/64\pi \epsilon_0 z_0^3)(d_x^2 + d_y^2 + 2d_z^2) = -h(z_0) dU_{\text{vdW}}^{(0)}(z_0)/dz_0$, which is characteristic of the PFA [34]. In other words, the PFA just sees the peak regime.

**VAN DER WAALS INTERACTION**

Our formula for the vdW interaction between a polarizable particle and a grounded conducting rough surface can be written as $U_{\text{vdW}} \approx U_{\text{vdW}}^{(0)} + U_{\text{vdW}}^{(1)}$, where $U_{\text{vdW}}^{(0)}(z_0) = -(\langle \hat{d}_x^2 \rangle + \langle \hat{d}_y^2 \rangle + 2\langle \hat{d}_z^2 \rangle)/64\pi \epsilon_0 z_0^3$ is the vdW potential for the case of a grounded conducting plane [41], and $U_{\text{vdW}}^{(1)}$ (the first-order correction of $U_{\text{vdW}}^{(0)}$ due to the surface corrugation) is obtained by substituting $G^{(1)}$, given by Eq. (1), in Eq. (3), obtaining

$$U_{\text{vdW}}^{(1)}(r_0) = -\sum_{i,j} \langle \hat{d}_i \rangle \langle \hat{d}_j \rangle 64\pi \epsilon_0 \int \frac{d^2 q}{(2\pi)^2} h(q)e^{i q \cdot r_0} \mathcal{I}_{i,j}(z_0, q),$$

where $\langle \hat{d}_i \rangle = \frac{1}{\lambda} \int_0^\infty d\xi \alpha_{ij}(i \xi)$, with $\alpha_{ij}$ the components of the polarizability tensor $[6, 7]$. We have that this formula (9) is valid for any roughness profile function $h$ with max($|h|$) $\ll z_0$, with no constraint on the smoothness of the surface, valid therefore beyond the PFA. For the case of a sinusoidal corrugated surface, Eq. (9) leads to

$$U_{\text{vdW}}^{(1)} = -\frac{3aA\langle \hat{d}_x \hat{d}_y \rangle z_0}{512\pi \epsilon_0 z_0^3} \cos[kx_0 - \delta(\langle \hat{d}_x \hat{d}_y \rangle, k z_0)]],$$

where $A(\langle \hat{d}_x \hat{d}_y \rangle, k z_0)$ and $\delta(\langle \hat{d}_x \hat{d}_y \rangle, k z_0)$ are obtained by replacing $\langle \hat{d}_x \hat{d}_y \rangle \to \langle \hat{d}_x \hat{d}_y \rangle$ in Eqs. (6)-(8). Taking the limit $\frac{1}{\lambda} \to \infty$ one obtains $U_{\text{vdW}}^{(1)} \to -a \cos(k_{\text{vdW}} \phi) (3/64\pi \epsilon_0 z_0^3)(\langle \hat{d}_x^2 \rangle + \langle \hat{d}_y^2 \rangle + 2\langle \hat{d}_z^2 \rangle) = -h(z_0) dU_{\text{vdW}}^{(0)}(z_0)/dz_0$. This result is characteristic of the PFA [42], and one can see that this limit eliminates the presence of the phase function $\delta$, so that valley and intermediate regimes can not be predicted if this approximation is used.

For isotropic particles, $\langle \hat{d}_x \hat{d}_y \rangle = 0$ and $\langle \hat{d}_z^2 \rangle = \langle \hat{d}_z^2 \rangle$, which leads to $B(\langle \hat{d}_x \hat{d}_y \rangle, k z_0) = 0$ and $C(\langle \hat{d}_x \hat{d}_y \rangle, k z_0) = 0$. Then, from Eq. (6), we obtain that $\delta = 0$, which implies that only the peak regime occurs, and our Eq. (10) recovers that for the vdW interaction obtained in Ref. [32]. For an anisotropic particle oriented with its principal axes coinciding with $xyz$, according to Eq. (8) $B = 0$, so that only two situations are possible: $\delta = 0$ [peak regime, Fig. 4(i)] or $\delta = \pi$ [valley regime, Fig. 4(ii)]. This reveals behaviors of $U_{\text{vdW}}^{(1)}$ similar to those shown in Fig. 5(a), but with the separation between the dark and lighter regions occurring here for $\lambda/z_0 = g < e$. This comes from the distinctions between the classical and quantum versions of Eqs. (7) and (8). In both, $B = 0$, but in the classical case any $\lambda$ can be set to null, so that we can eliminate, arbitrarily, $R_{xx}$, $R_{yy}$ or $R_{zz}$ factors in

Figure 4. Illustration of a neutral anisotropic particle (represented by an ellipsoid), fixed at $z = z_0$, and interacting with a rough surface (solid lines). The stable equilibrium points (indicated by the circles) of $U_{\text{vdW}}^{(1)}$ [Eq. (5)] (dashed lines) can be over the corrugation peaks [peak regime, (i)], valleys [valley regime, (ii)], or over points between a peak and a valley [intermediate regime, shown in (iii) and (iv)]. We highlight the different equilibrium points in (iii) and (iv), which is related to the change $d_x \to -d_x$ or $d_y \to -d_y$ in the orientation of the particle. The largest axis of the ellipsoid represents the direction of the dipole moment $\mathbf{d}$.

Figure 5. (a) Configuration space representing $\phi$ (vertical axis) and $\lambda/z_0$ (horizontal axis). It shows the behavior of $x_{\text{min}}$ of $U_{\text{vdW}}^{(1)}$ for a particle with magnitude of dipole moment $|\mathbf{d}|$. The dark region represents $C(\theta = \pi/2) < 0$, and corresponds to the valley regime. The lighter region represents $C(\theta = \pi/2) > 0$, and corresponds to the peak regime. The border between these two regions (dashed line) corresponds to the situations where $C(\theta = \pi/2) = 0$, which means that $U_{\text{vdW}}^{(1)} = 0$ and that the lateral force vanishes. (b) The possible values of $x_{\text{min}}/\lambda$ (in the interval $0 \leq x_{\text{min}}/\lambda \leq 1$), valleys $/\lambda \leq 1$, in lighter one $0 \leq x_{\text{min}}/\lambda \leq \beta$, with the values of $\beta$ given by the upper curve delimiting the lighter region, from which one can see that $0 < \beta(\lambda/z_0) < 0.25$ and $\beta(\lambda/z_0) = \infty$. The inset (a)-(i) considers the periodicity of $U_{\text{vdW}}^{(1)}$ and illustrates (with hatched areas) that $x_{\text{min}}$ can assume any value. The insets (a)-(ii) and (a)-(iii), in a similar manner, illustrate that the values of $x_{\text{min}}$ can be located only around the peaks. Inset (a)-(iv) (the PFA limit) illustrates that, when $\lambda/z_0 \to \infty$, the minimum points are located over the peaks. Figures similar to (a) and (b) also appear in the vdW interaction, but with the separation between the dark and lighter regions occurring for $\lambda/z_0 = g < e$. 
φ of the manuscript. For instance, when (a) and (c) of the manuscript is λ/z₀ ≈ e, so that Figs. (a) and (b) show xₘₐₙ/λ × θ for λ/z₀ < e, and λ/z₀ > e, respectively. When φ = π/4, the correspondent value of λ/z₀ belonging to the dashed border line in Fig. 2(a) of the manuscript is λ/z₀ ≈ 1.74, so that Figs. (c) and (d) show xₘₐₙ/λ × θ for λ/z₀ < 1.74, and λ/z₀ > 1.74, respectively.

Eq. (8), whereas in the quantum case one has ⟨ḥ₂⟩, ⟨ḥ₂⟩ and ⟨ḥ²⟩ non-null, independently of the orientation of the particle, implying that all terms Rᵢ,j always contribute, resulting in λ/z₀ < e. In contrast, when the particle is oriented in such a way that its principal axes do not coincide with xyz, B ≠ 0, so that Eq. (6) leads to the presence of the phase function δ in such a way that δ = 0 and δ = π. This means that the intermediate regime occurs [see Fig. 4(iii)-(iv)].

APPLICATIONS

Since our Eq. (9) [and also Eq. (4)] is applicable to any other surface profile, the valley and intermediate regimes, discussed until now for a sinusoidal surface, can also be investigated for gratings commonly considered in Casimir and CP/vdW interactions [31–33, 43–45]. For instance, we apply our formulas to the periodic profiles shown in Figs. 7(a)-7(c), and to the non-periodic one shown in Fig. 7(d). In these applications, we consider a CO₂ molecule interacting with nanogratings, since the CP/vdW interaction of a such molecule with surfaces has attracted growing interest in technological applications [19, 21, 46–48]. We also consider the dimensions of the nanogratings taking into account the recent advances in fabrication of nanostructures [49–51], and the CO₂ polarizability properties as given in Ref. [19]. In all these situations, one can see the presence of the valley and intermediate regimes, which reveals the generality of our results.

Moreover, in these situations, considering the second perturbative order, we obtain that it just implies a minor correction to the first order lateral vdW force (see Fig. 8), preserving the existence of the new regimes predicted here.

As another application of our results, let us consider a particle with mass m, fixed at z₀, put initially at rest in a position around a stable equilibrium point of U₁⁰⁻<e⁰⁻<e>/vdW (or U₁⁰⁻<e⁰⁻<e>/claw, for the classical case). We obtain that this particle, under the action of the lateral force due to a sinusoidal surface, describes a harmonic oscillation, with frequency f = \(\sqrt{2}n\pi/2\). For example, a CO₂ molecule, in the situations shown in Fig. 7(a), oscillates with frequency f ≈ 21.06kHz, f ≈ 15.13kHz, and f ≈ 8.35kHz, in the peak (dotted line), intermediate (dashed line), and valley (dot-dashed line) regimes, respectively. Such frequencies could be detectable by trapping the particle and measuring the relative shift in the original trap frequency (in the absence of the sinusoidal surface) [6, 32]. We highlight that, in the situation described by the long-dashed line in Fig. 7(a) (null lateral vdW force), the original trap frequency does not change (even in the presence of a corrugated surface).

As discussed in Ref. [35], the reflectivity of a surface is not perfect, so that the vdW interaction with a real surface dif-
Considers a plane ideal conductor. Thus, we expect that, in the
nearest valley, or to an intermediate point between a peak
corrugation peaks (as found in the literature), but also to the
presence of these nonideal materials, the correspondent en-
ergies between a CO$_2$ molecule and these surfaces are,
approximately, 0.21 of the correspondent vdW energy when one
considers anisotropic particles and analytical calculations
beyond the PFA (which, for example, show that an atom above
the plateau of a grooved plate feels a lateral force, which is not predicted by the PFA [32]). Here, con-
sidering both, anisotropic particles and analytical calculations
valid beyond the PFA, we predict new nontrivial behaviors of
the lateral vdW force, revealing that, under the action of
this force, the particle can be attracted not only toward the
corrugation peaks (as found in the literature), but also to the
nearest valley, or to an intermediate point between a peak and a valley. We also show that in the configurations of tran-
sition between the peak and valley regimes the lateral vdW
force vanishes, even in the presence of a corrugated surface.
Moreover, we show that these new valley and intermediate
regimes occur in general, for periodic and nonperiodic cor-
rugated surfaces, and also that similar effects occur for the
classical interaction between a corrugated surface and a parti-
cle presenting a permanent electric dipole moment. These new
regimes of lateral force can be relevant for a higher degree of
control of the interaction between neutral anisotropic parti-
cles and corrugated surfaces in both classical (macroscopic
dipoles, ferroelectric nanoparticles or polar molecules) and
quantum physics (anisotropic molecules, ellipsoidal nanoparticles), with experimental verifications feasible in both do-
mains.

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Nontrivial behaviors of the CP/vdW forces can be predicted
when considering anisotropic particles (for instance, the re-
pulsion between a particle and a plate with a hole [25]) or
calculations beyond the PFA (which, for example, show that
an atom above the plateau of a grooved plate feels a lateral
force, which is not predicted by the PFA [32]). Here, con-
sidering both, anisotropic particles and analytical calculations
valid beyond the PFA, we predict new nontrivial behaviors of
the lateral vdW force, revealing that, under the action of
this force, the particle can be attracted not only toward the
corrugation peaks (as found in the literature), but also to the
nearest valley, or to an intermediate point between a peak
and a valley. We also show that in the configurations of tran-
fers by a numerical factor from the one calculated considering
an ideal surface. In this way, when considering nonideal con-
ductors, the values of the vdW energies in Fig. 7 need some
adjustment, which can be estimated as follows. Taking into ac-
count, for instance, plane surfaces of gold or copper (and their
spectral properties given in Ref. [52]), the vdW interaction
energies between a CO$_2$ molecule and these surfaces are,
approximately, 0.21 of the correspondent vdW energy when one
considers a plane ideal conductor. Thus, we expect that, in the
presence of these nonideal materials, the correspondent en-
ergies $U_{vdW}^{(1)}$ differ from the results calculated here by a factor of
the same order, but preserving the new valley and intermediate
regimes.

**FINAL REMARKS**

Nontrivial behaviors of the CP/vdW forces can be predicted
when considering anisotropic particles (for instance, the repulsion between a particle and a plate with a hole [25]) or calculations beyond the PFA (which, for example, show that an atom above the plateau of a grooved plate feels a lateral force, which is not predicted by the PFA [32]). Here, considering both, anisotropic particles and analytical calculations valid beyond the PFA, we predict new nontrivial behaviors of the lateral vdW force, revealing that, under the action of this force, the particle can be attracted not only toward the corrugation peaks (as found in the literature), but also to the nearest valley, or to an intermediate point between a peak and a valley. We also show that in the configurations of transition between the peak and valley regimes the lateral vdW force vanishes, even in the presence of a corrugated surface. Moreover, we show that these new valley and intermediate regimes occur in general, for periodic and nonperiodic corrugated surfaces, and also that similar effects occur for the classical interaction between a corrugated surface and a particle presenting a permanent electric dipole moment. These new regimes of lateral force can be relevant for a higher degree of control of the interaction between neutral anisotropic particles and corrugated surfaces in both classical (macroscopic dipoles, ferroelectric nanoparticles or polar molecules) and quantum physics (anisotropic molecules, ellipsoidal nanoparticles), with experimental verifications feasible in both domains.

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