Research on the Calculation Method of Axial Compression of Round-end Concrete-filled Steel Tube

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Abstract. Round-end concrete-filled steel tube (RCFST) columns have been applied to bridge structures. There is not the formula of RCFST under axial load in code. Based on the unified theory, the basic form of strength and stability capacity correlation equations of concrete-filled steel tube under axial load is obtained through the theoretical derivation. The formula results compared with finite element simulation results show results are in good agreement.

1. Introduction
In recent years, round-end concrete-filled steel tube (RCFST) has been applied in engineering, but little research focuses on its force performance, few relevant theoretical systems were formed and design calculation methods were provided. It is of great necessity to study the calculation method of RCFST axial compression based on relevant theories.

2. Theoretical analysis of calculation of concrete-filled steel tube
Currently, there are a variety of calculation theories involving concrete-filled steel tubes, such as semi-steel theory, semi-concrete theory, unified theory and superposition theory. The key difference of these theories lies in whether or not the bond force between the steel tube and the core concrete interface can guarantee their cooperative working under stress states.

2.1. Superposition theory
The superposition theory is to superimpose the bearing capacity of the two parts of the filled concrete and the steel tube. The theory has been adopted in both Tianjin Engineering Construction standard: Tianjin Steel Structure Housing Design Regulations (DB29-57-2016) and China Engineering Construction Standardization Association standard: Rectangular-end Steel Tube Concrete Structures Rules (CECS159:2004). The adoption of superposition theory for bearing capacity calculation is in favour of safety.

2.2. Limit equilibrium theory
The limit equilibrium method considers concrete-filled steel tube as a structural system composed of steel tubes and concrete, and calculates its load value under the limit state according to the equilibrium conditions when the structure is in the limit state. The limit (yield) conditions of both steel tubes and
core concretes are stable, and remain unchanged with plastic deformation. Meanwhile, steel tubes are subjected to the Von-Mises yield condition, while core concretes, when in the limit state, are subjected to the criteria of axial compressive strength with lateral pressure. Both China Engineering Construction Standardization Association standard (CECS 28:2012): Regulations Governing Concrete-filled Steel Tube Structure Design and Construction and (CECS 104:99): Regulations Governing High-strength Concrete Structures and Techniques have adopted the limit equilibrium theory.

2.3. Unified theory
The basic idea of unified theory is to regard steel tube and concrete as a kind of material and thus seek out the relationships between influencing factors and bearing capacity through mathematical methods by summarizing and analyzing the data collected from extensive experiments. According to the constitutive relation between steel and concrete, Zhong Shantong drew the typical curve of relations between average stress and longitudinal strain under the axial compression of various cross-section members using the finite element method. According to the relationship between the stress at the aforementioned limit point and casing hoop steel coefficient, the standard value of the combined axial compression strength is obtained.

The theory has been adopted by China’s current Technical Regulations Governing Early Strength Composite Structures for Wartime Military Ports Repair (GJB 4142-2000), Fujian Provincial Engineering Construction Standards: Technical Regulations Governing Concrete-filled Steel Tube Structures (DBJ/T13-51-2010) and Design Rules for Steel-Concrete Composite Structures (DL/T5085-1999) and so on.

2.4. Semi-steel theory
The semi-steel theory is to calculate concrete as steel and then conduct designing according to steel structure specifications. The LRFD specifications developed by the American Institute of Steel Construction treat the filled concrete as the improvement of steel tubes’ yield strength and elasticity modulus, so as to work out the equivalent steel tube members through conversion. In addition, the specifications take the bearing capacity of the equivalent steel tube members as that of the prototype concrete-filled steel tube members. Engineering experiments show that the gap between concretes and steel tubes will result in the failure of their joint working, thereby unsatisfactorily reaching the set limit bearing capacity. The bearing capacity calculated according to the semi-steel theory tends to be larger than the real one.

2.5. Semi-concrete theory
When the bearing capacity of concrete-filled steel tube members is calculated, semi-concrete theory is to converse them into equivalent concrete members for calculation and regard the steel tubes in the concrete-filled steel tube members as equivalent longitudinal steel bars distributed around the core concrete, the size of which is determined by the cross-sectional area and shape of the steel tube.

This theory has been adopted in both the specifications developed by American Concrete Association (1999) and European EC4 (1994). The theory is based on the confinement effect of steel tube on internal concrete. However, for RCSFT, the confinement effect of steel tube is relatively weak [1], so the semi-concrete theory is not suitable for the calculation of RCSFT members.

3. Confinement effect of steel tube
The key to calculate the axial bearing capacity of special-shaped concrete-filled steel tubes lies in how to consider the confinement effect of steel tubes on core concrete. The ultrasonic inspection and sectioning test on concrete-filled rectangular steel tube specimens conducted by Tianjin University [2] find that there were different degrees of gaps between steel tube wall and core concrete as a result of the shrinkage, cracking and poor pouring quality of the core concrete, and the gaps tend to be
particularly obvious after a load is applied, which greatly affected the performance of collaboration between the steel tube and the core concrete.

Liu Xueping [3] theoretically and experimentally studied concrete-filled rectangular steel tubes, and found that as for the concrete-filled rectangular steel tube members with a steel ratio conforming to the requirements for engineering application, there were limited improvement of strength caused by the confinement effect of steel tube wall on the core concrete, which was more reflected in the boosted ductility of the core concrete.

Ai Changdong [4] concluded that the lateral pressure increases with the increase of the confinement effect coefficient; the circular steel tubes show the best confinement effect, outperforming those in triangle, octagon, hexagon, square, and rectangle shapes; and the thicker the wall thickness and the smaller the cross section, the larger the confinement force. Square steel tubes have certain confinement on the core concrete, but the confinement is mainly at the four corners.

The unified theory considers concrete-filled steel tubes as a unified body. Its theoretical calculation formula is based on numerical calculation and experiments, so the numerical calculation should be accurate and experiment results reliable. However, the formulas for fitting are generally complicated and therefore difficult to be used. Besides, the theory is reliable in the case of a large amount of experimental data, but for the part lacking experimental results, it is unfeasible to rely solely on the results of numerical analysis.

4. Calculation of RCSFT axial compression bearing capacity

4.1. Calculation formula derivation for the bearing capacity of RCSFT axial compression

Fig. 1 presents the stress contour of the core concrete with circular and square sections under axial compression calculated by finite element. Comparative analysis shows that the lateral stress distribution of round-end concrete-filled steel tubes is symmetric and uniform, and the stress gradually increases from the outside to the centre; while the lateral stress distribution of square-ended concrete-filled steel tubes is strong at the corners but weak at the edges, with the midpoints of the edges being the weakest. Moreover, there is a 45° parabolic boundary similar to the initial tangential line between the weak area and the effective confinement area.

With polygonal concrete-filled steel tubes as the research subject, Yu Min and Zha Xiaoxiong [5] divided the effective confinement area from the non-effective confinement area in the polygonal core concrete using a parabola with 45° initial tangent based on the unified theory. Besides, they also converted the non-uniform lateral confinement into the uniformly distributed lateral confinement in the quasi-round tube wall by defining the ratio of effective confinement area to the entire cross sectional area as the equivalent confinement reduction coefficient K. Combined with the theory of elasticity and plasticity and using stress-deformation compatibility, the calculation formula for the
combined axial compression strength of concrete-filled steel tube with a polygonal cross section is obtained:

\[ f_{sc}^y = \frac{1 + 1.5k\xi}{1 + \frac{A_s}{A_c}} f_{ck} \]  \hspace{1cm} (1)

\( K \) — Correction coefficient of confinement coefficient;
\( A_s \) — Area of steel tube;
\( A_c \) — area of concrete;
\( f_c \) — Value of concrete compressive strength;
\( \xi \) — Boundary Casing Hoop Coefficient;

In the same circumstance, round-end sections should be in between rectangular sections and circular sections in confinement effect. With the relationship between the parabola with a 45° initial tangent and the shape of round-end sections, the equivalent confinement reduction coefficient \( k \) can be derived. That is, the calculation formula for the axial compression bearing capacity of the round-end concrete-filled steel tubes can be obtained by combining Equation (1).

According to the lateral pressure distribution law of the RCFST, the effective confinement area of the core concrete can be simplified to the shaded part of Fig. 2, which is formed by two parabolic-ally cut round ends symmetrical to the long axis.

\[ y_2 = k_2 x \]
\[ y_1 = k_1 x \]

Figure 2. Concrete core effective confinement area of concrete-filled steel concrete

Assume the parabolic equation is: \( y = px^2 + q \). Since the parabola passes through the point \( A(m, n) \) and it is tangent to the line \( y_1 = k_1x \) at point A, there are:

\[ pm^2 + q = n \]
\[ 2pm = \frac{n}{m} \]

\[ \begin{cases} p = \frac{n}{2m^2} \\ q = \frac{n}{2} \end{cases} \]  \hspace{1cm} (2)

Hence, the parabolic equation can be deduced:

\[ y = \frac{n}{2m^2} x + \frac{n}{2} \]

According to the point of intersection \( A (m, n) \) between line \( y_1 = k_1x \) and line \( y = b/2 \), simultaneous equations can be resorted to for solving.
The effective lateral pressure area of the core concrete can be obtained after integral:

\[ A_c = ab + 2r^2 \arccos \frac{r-c}{r} - b(r-c) - 4 \int_0^\frac{m}{n} dx \int_{px+q}^b dy \]

\[ = ab + 2r^2 \arccos \frac{r-c}{r} - b(r-c) - 4(bm - \frac{2}{3}mn) \]

\[ = ab + 2r^2 \arccos \frac{r-c}{r} - b(r-c) - 4b^2 \frac{3k_1}{k_1} \]

Hence,

\[ K = \frac{A_c}{A} \]  \hspace{1cm} (4)

Therefore, the calculation formula for the combined axial compression of RCFST short column can be deducted:

\[ f_{\gamma} = \frac{1+1.5k_1^2}{1+\frac{A_1}{A_c}} f_{ck} \]

\[ ab + 2r^2 \arccos \frac{r-c}{r} - b(r-c) - 4b^2 \frac{3k_1}{k_1} \]

\[ k = \frac{ab + 2r^2 \arccos \frac{r-c}{r} - b(r-c)}{ab + 2r^2 \arccos \frac{r-c}{r} - b(r-c)} \]  \hspace{1cm} (5)

Numerical simulation for multiple sets of round-end concrete-filled steel tube members is conducted by finite element numerical analysis method and changing geometric parameters, material parameters and other influencing factors. Finally, through regression analysis, the relationship between the slope \( k_1 \) of the straight line and the shape of the round-end section can be concluded:

\[ k_1 = \frac{1}{a+c} \frac{1}{b} - 1 \]  \hspace{1cm} (6)

4.2. Verification of calculation results

In order to verify the accuracy of the fitting formula, based on the previous research by this group [1], we compared the finite element model analysis data and formula calculation data for a batch of round-end concrete-filled steel tube members as shown in Table 1.

| No. | a  | b  | c  | r  | t  | \( f_{\gamma} \) | \( f_{ck} \) | \( N_c \) | \( N_{FEM} \) | \( N_{FEM} \) |
|-----|----|----|----|----|----|-----------------|-------------|---------|-------------|-------------|
| 1   | 60 | 240| 120| 120| 4  | 235             | 20.1       | 1921.68  | 1912.12     | 1.005       |
| 2   | 240| 240| 120| 120| 4  | 235             | 20.1       | 2702.73  | 2660.17     | 1.016       |
| 3   | 185| 120| 100| 120| 4  | 310             | 34.8       | 2752     | 2791        | 0.986       |
| 4   | 360| 240| 120| 120| 4  | 235             | 20.1       | 2925.99  | 2997.95     | 0.976       |
| 5   | 60 | 240| 120| 120| 4  | 235             | 23.4       | 2013.90  | 1982.19     | 1.016       |
As illustrated by Table 1, the average value is 0.9855 and the variance is 0.00048, showing satisfactory consistency between formula calculation results, test data and finite element calculation results.

Compared bearing capacity get by formula with that calculated by unified theory, as shown in Table 2. It can be seen that the bearing capacity of RCFST increases with the growing proportion of the arc portion to the whole section, and gradually approaches the bearing capacity of RCFST.

Table 2. Bear capacity comparison of CFST with the same cross sectional circular concrete-filled steel tubes

| No. | a   | b   | c   | r   | t   | f_y | f_ck | N_c | N_y | N_c / N_y |
|-----|-----|-----|-----|-----|-----|-----|------|-----|-----|-----------|
| 1   | 60  | 240 | 120 | 120 | 4   | 235 | 20.1 | 1921.68 | 2239.51 | 0.858     |
| 2   | 240 | 240 | 120 | 120 | 4   | 235 | 20.1 | 2702.73 | 3507.02 | 0.771     |
| 3   | 360 | 240 | 120 | 120 | 4   | 235 | 20.1 | 2925.99 | 4352.02 | 0.672     |
| 4   | 60  | 240 | 120 | 120 | 4   | 235 | 23.4 | 2013.90 | 2389.89 | 0.843     |
| 5   | 60  | 240 | 120 | 120 | 4   | 235 | 26.8 | 2205.45 | 2540.87 | 0.868     |
| 6   | 60  | 240 | 120 | 120 | 4   | 235 | 26.8 | 2153.61 | 2842.29 | 0.758     |
| 7   | 240 | 240 | 120 | 120 | 4   | 235 | 23.4 | 3048.88 | 3089.04 | 0.987     |
| 8   | 240 | 240 | 120 | 120 | 4   | 235 | 26.8 | 3026.56 | 3770.86 | 0.803     |
| 9   | 360 | 60  | 150 | 120 | 4   | 235 | 20.1 | 2153.61 | 2869.70 | 0.942     |
| 10  | 360 | 60  | 150 | 120 | 4   | 235 | 23.4 | 2703.25 | 3903.74 | 0.697     |
| 11  | 360 | 60  | 150 | 120 | 4   | 235 | 26.8 | 3360.20 | 4178.16 | 0.730     |
| 12  | 360 | 60  | 150 | 120 | 4   | 235 | 20.1 | 2519.68 | 4519.98 | 0.557     |

Combined with a large number of experimental data, Yu Min and Zha Xiaoxiong [5] obtained the formulas for the axial compression of RCFST and polygonal concrete-filled steel tubes by fitting as below:

\[
f_{sc} = \frac{A\xi^2 + (A + 2 + k_c)\xi + 2}{AB\xi^2 + (A + 2B)\xi + 2} f_{ck}
\]

\[A = 0.2B(1 - \psi) + 0.05\psi + 0.05\]

\[B = \frac{f_{ck}}{f_y}\]

\[k_c\] Section discount coefficient, for square, \(k_c = 0.1337\]
The calculated bearing capacity of RCFST is compared with that of the same cross sectional concrete-filled steel tubes. As shown in Table 3, the closer the round-end section is to the regular polygon, the higher the bearing capacity of the members.

Table 3. Bearing capacity comparison of CFST with the same cross sectional square concrete-filled steel tube

| No. | a   | b   | c   | r   | t   | fy  | fc | Nc | Nf | Nc/Nf |
|-----|-----|-----|-----|-----|-----|-----|----|----|----|-------|
| 1   | 60  | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 1954.83 | 0.983 |
| 2   | 240 | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 2572.73 | 0.769 |
| 3   | 360 | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 2925.99 | 0.743 |
| 4   | 60  | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 2139.00 | 0.916 |
| 5   | 60  | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 1691.13 | 0.935 |
| 6   | 60  | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 2139.00 | 0.916 |
| 7   | 240 | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 2703.25 | 0.754 |
| 8   | 240 | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 2925.99 | 0.743 |
| 9   | 240 | 240 | 120 | 120 | 4   | 235 | 20.1| 1921.68 | 3048.88 | 0.745 |
| 10  | 360 | 240 | 60  | 150 | 4   | 235 | 20.1| 1921.68 | 2703.25 | 0.754 |
| 11  | 360 | 240 | 60  | 150 | 4   | 235 | 20.1| 1921.68 | 3048.88 | 0.745 |
| 12  | 360 | 240 | 60  | 150 | 4   | 235 | 20.1| 1921.68 | 2519.68 | 0.752 |

5. Conclusion

The calculation formula for the axial compression bearing capacity that is suitable for RCFST is obtained based on the concrete-filled steel tube calculation theory and the unified theory. In addition, the calculation results obtained by the formula are compared with the finite element simulation results to prove the rationality of the formula.

The calculation results of RCFST are compared with those of the circular-end and rectangular-end concrete-filled steel tube members, suggesting that the closer the round-end section is to the regular polygon, the more obvious the confinement effect and the higher the bearing capacity.

References

[1] Wang Erlei. Research on Compressive Behavior and Reliability of Round-ended Steel Tube-Filled Concrete Column [D]. Wuhan: Wuhan University of Technology, 2012:17-29.
[2] Chen Zhihua, Li Liming, Li Shuhai, Chen Aoyi, Zhang Daxu. Supersonic Flaw Detecting Test and Theoretical Study on Concrete-filled Rectangular and Square Steel Tube[J]. Building Structure.2005, Vol. 35, No. 9:34-38.
[3] Li Xueping, Lu Xilin and Wang Dan. Modeling and Experimental Verification for Concrete-filled steel Tubular Columns with L or T section [C].Proceedings of 8th international conference on steel-concrete composite and hybrid structures, 2006.
[4] Ai Changdong. Experimental Study on Mechanical Properties under Axial Compression of Concrete- Filled Steel Tubular Columns[D].LiaoNing: Daqing Petroleum University , 2008:19-42.
[5] Min Yu, Xiaoxiong Zha, Jianqiao Ye, Chunyan She. A Unified Formulation for Hollow and Solid Concrete-filled Steel Tube Columns under Axial Compression [J]. Engineering structure,2010,32(4): 1046-1053.