A Schroedinger picture analysis of time dependent quantum oscillators, in a manner of Guth and Pi, clearly identifies two physical mechanisms for possible decoherence of vacuum fluctuations in early universe: turning of quantum oscillators upside-down, and rapid squeezing of upside-right oscillators so that certain squeezing factor diverges. In inflationary cosmology the former mechanism explains the stochastic evolution of light inflatons and the classical nature of density perturbations in most of inflationary models, while the later one is responsible for the classical evolution of relatively heavy fields, with masses in a narrow range above the Hubble parameter: $m/H_0 \in (\sqrt{2}, 3/2)$. The same method may be applied to study of the decoherence of quantum fluctuations in any Robertson-Walker cosmology.

1. Introduction

One of the great achievements of inflationary cosmology,\textsuperscript{1,2} is the observation\textsuperscript{3} that quantum fluctuations of inflaton field evolve into classical adiabatic density perturbations which might have served as seeds for the observed large scale structure of the universe. The spectrum of these perturbations is very close to the Harrison-Zeldovich spectrum, which agrees reasonably well with observations. With the current uncertainties in physics above the electro-weak scale, the magnitude of the perturbations is sufficiently adjustable to avoid the conflict between the inflationary models and, say, the observed anisotropy of the cosmic microwave background.\textsuperscript{2} On a conceptual level however, this mechanism is an excellent example for spontaneous emergence of classical features in course of the evolution of a quantum universe. This is the angle that we are interested in here.

There have been several attempts over the last decade to describe and understand this decoherence process.\textsuperscript{4,5,6} Following the line of thinking due to Guth and Pi,\textsuperscript{4} we formulate here sufficient conditions for the decoherence of quantum fluctuations of scalar fields in any expanding universe which is described by the
Robertson-Walker geometry.

We start by classifying different possibilities, and stating results known so far. Then, we analyze the time dependent oscillators, as model that adequately describes the quantum fluctuations of free fields in expanding universe, and formulate the conditions for decoherence. Next, we apply this analysis to massive fields on De Sitter spacetime, and distinguish and discuss the characteristic cases. The last section contains our conclusions and some inevitable speculations. As usual, we will neglect the effects of the interaction, due to the expected smallness of both the coupling constant(s) and the fluctuations themselves. Moreover, one would expect that if the decoherence shows at the zeroth order approximation, an account for interaction ought to make it only better.

2. Upside-down and Upside-right oscillators

Let \( \delta \phi(\vec{x}, t) \) stands for inhomogeneous fluctuation of some scalar field we are interested in. After expanding the field \( \delta \phi \) into modes counted by the wave number \( k \), introducing the conformal time \( \eta \) as \( dt = S(\eta) d\eta \), \( S(\eta) = a(t) \), and the rescaled modes \( \chi \) as \( \delta \phi = \chi / S(\eta) \), (the index \( k \) will be suppressed), the one-mode equation of motion takes the following simple form:

\[
\chi''(\eta) + \omega^2(\eta) \chi(\eta) = 0 ,
\]

with,

\[
\omega^2(\eta) = k^2 + m^2 S^2 - S'' / S .
\]

Different expansion laws lead to different behavior of the one mode frequency (2). The most general characteristic of inflationary expansion is that it is accelerated, \( \ddot{a} > 0 \). This means that the dominant energy condition is violated, and the physical wavelength of any mode will eventually exceed the Hubble radius. The potential will turn upside-down, if, at some moment, the curvature term exceeds the mass term in Eq. (2):

\[
S'' / S > m^2 S^2 .
\]

Introducing kinetic function \( v(S) \equiv S'^2 / 2 \), these two conditions may be expressed as follows,

Inflation

\[
\frac{v'}{S} > 2 \frac{v}{S} .
\]

Upside-down oscillator:

\[
\frac{v'}{S} > m^2 S^2 .
\]

It is now easy to see that if the oscillator is upside-down, the background must be inflationary, i.e., that the second condition implies the first. From the analysis of Guth and Pi\(^4\) we know that the late time behavior of such oscillator is essentially classical. However, the opposite is not true.

In De Sitter universe the shape of the oscillator is determined by the mass of the field relative to the (constant) Hubble parameter. For \( 0 \leq m^2 / H_0^2 < 2 \), the oscillator
turns upside-down when the physical wavelength of the mode crosses the Hubble radius or some time after that moment. For $m^2 < 0$ the turning of the oscillator happens even before the Hubble radius crossing. Thus, those modes have essentially classical behavior at late times, and, according to Guth and Pi\textsuperscript{4}, as they re-enter the Hubble radius in subsequent non-inflationary phase they may be interpreted as (the source of) classical density perturbations. The situation is different in case of heavy fields with $m^2 / H_0^2 > 2$. The fluctuations of those fields also cross the Hubble radius, but their oscillators are upside-right at all times. In this case the analysis of Guth and Pi\textsuperscript{4} does not tell us anything about the possible classical behavior, or the lack of it.

The behavior of these, fairly massive fields, have been investigated in Ref. 6, and led to somewhat unexpected result that the fluctuations of fields with mass in a narrow range $(\sqrt{2}, 3/2)$ times $H_0$ also decohere, although they are represented by upside-right oscillators. The fluctuations of heavier fields were found not to decohere, even when their wavelengths are greater than the Hubble radius.

In this paper we will show how one can confirm and understand all these possibilities in an elementary way. By following the analysis of Guth and Pi\textsuperscript{4} we will find that for quantum oscillator in any expanding Robertson-Walker geometry there is not just one mechanism for decoherence found by them, (turning of the oscillator upside-down), but that there is also the second one. In De Sitter case those two mechanisms exactly cover the two possible regimes of decoherence quoted above.

3. Time dependent oscillators

Let us consider a one-dimensional, time dependent oscillator with the Hamiltonian $H = p^2 / (2m) + k(t)x^2 / 2$. The frequency of such oscillator is given as $\omega^2(t) = k/m$. Let $x_c(t)$ denotes a solution to the classical equation of motion, $\ddot{x}_c + \omega^2(t)x_c = 0$, and $p_c(t) \equiv m\dot{x}_c(t)$ its conjugate momentum. Then, the solution to the time-dependent Schrödinger equation may be written as,

$$\Psi(x, t) = A_0 \left| \frac{x_c(0)}{x_c(t)} \right|^{1/2} \times \exp \left[ i \frac{p_c(t)}{2} \frac{x^2}{x_c(t)} \right]. \tag{5}$$

In their analysis of upside-down oscillator Guth and Pi\textsuperscript{4} use three different but related criteria as signals for the emergence of classical behavior. We will examine here the two of them: (i) the emergence of classical momentum, and, (ii) the condition for relatively small magnitude of the canonical commutator.

To see how the first criteria works, recall that for an arbitrary amplitude, acting on it by the momentum operator creates a state which bears no reference to classical momentum. However, in case of amplitude 5, we have,

$$p\Psi(x, t) = \frac{p_c(t)}{x_c(t)} x\Psi(x, t). \tag{6}$$
To examine the ratio between the two classical quantities on the right hand side, we can rewrite the classical solution as,

\[ x(t) = A(t)x_0 + B(t)p_0 . \]  

(7)

\( x_0 \) and \( p_0 \) are the initial values, and time dependent coefficients \( A \) and \( B \) are given as linear combinations of two independent solutions to the oscillator equation:

\[ A(t) \equiv \frac{x_1(t)x_2'(0) - x_1'(0)x_2(t)}{W(0)} , \]  

(8)

\[ B(t) \equiv \frac{x_1(0)x_2(t) - x_1(t)x_2(0)}{W(0)} , \]  

(9)

\[ W(0) \equiv x_1(0)x_2'(0) - x_1'(0)x_2(0) . \]  

(10)

One can eliminate one of the two constants to express the classical momentum either as,

\[ p_c(t) = m\dot{A}x_c(t) + mB \left[ \frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right] p_0 , \]  

(11)

or, as,

\[ p_c(t) = m\dot{B}x_c(t) + mA \left[ \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right] x_0 . \]  

(12)

In cases when the last term on the right hand side is absent, or small compared to the first, we have \( p_c \sim x_c \). Then, we can rewrite Eq. (13) as,

\[ p\Psi(x, t) = p_c(x)\Psi(x, t) , \]  

(13)

where \( p_c(x) \equiv m(\dot{A}/A)x \), or, \( p_c(x) \equiv m(\dot{B}/B)x \), depending which of the two possible realizations for \( p_c \sim x_c \) takes place. \( p_c(x) \) denotes the classical momentum that oscillator has at point \( x \) as it reaches it along the classical trajectory specified by the initial conditions \((x_0, p_0)\). In contrast with the general quantum mechanical expression, the average value for the square of the momentum is now the average of classical values:

\[ \langle \Psi|p^2(t)|\Psi \rangle = \int dx \ |\Psi(x, t)|^2 p_c^2(x) . \]  

(14)

This is exactly how would one calculate the average in the ensemble where position and momentum are correlated according to classical equation. Let us now see when can this occur. The relation \( p_c \sim x_c \) holds exactly if, (i) \( W_{AB} \equiv AB - B\dot{A} = 0 \), (ii) \( p_0 = 0 \), or, (iii) \( x_0 = 0 \). We will exclude the first case, as it means that \( x_0 \) and \( p_0 \) cannot be expressed through \( x_c(t) \) and \( p_c(t) \), ie, that the classical solution cannot be run backward. The third case should also be excluded, as, from (13), it implies \( \Psi(x, 0) = 0 \), which violates the requirement for finite normalization. We are left only with the second case, which implies a uniform initial distribution:
Ψ(x, 0) = A_0. For a finite range of the coordinate x this is acceptable, otherwise this case should also be excluded because the amplitude will not be normalizable.

It is more interesting if we consider the regime when \( p_c \sim x_c \) holds only approximately. For this we need either,

(a) \( x_c(t) \) large, with \( \dot{A}/A \) and \( \dot{B}/B \) finite, and \( W_{AB}/A, W_{AB}/B, x_0, \) and \( p_0 \) all bounded; or,

(b) \( \dot{A}/A \) and \( \dot{B}/B \) are large, \( x_c(t) \) is finite, and, as in the preceding case, the last terms in (11) and (12) are bounded.

Guth and Pi^4 case corresponds to the first of those two possibilities. For an upside-down oscillator with constant frequency \( \omega_0 \) the two independent solutions may be chosen as \( x_{1,2} = \exp[\pm\omega_0 t] \). Eq. (11) becomes, \( p_c(t) = [m\omega_0 \tanh(\omega_0 t)] x_c(t) + \frac{p_0}{\cosh(\omega_0 t)} \).

At late times, \( \omega_0 t \gg 1 \), we have \( p_c \rightarrow m\omega_0 x_c \), and, following the previous logic, the late time behavior of an upside-down oscillator is essentially classical.

Another way to understand this case is to simply notice that at late times the classical solution is accurately described through just one exponential mode, the growing one. Since the same is true for \( p_c \) we have \( p_c \sim \omega_0 x_c \), and classical behavior emerges. In the opposite case of an upside-right oscillator with constant frequency, the fundamental solutions are periodic, and both must be retained at late times. As a result, the second terms in Eq.'s (11), (12) can’t be neglected. This distinction between the cases when just one or both modes describe the late time behavior carries on to time dependent oscillators.

Let us now consider the second criteria of Guth and Pi^4. This one is useful because it gives us a characteristic scale in the configuration space outside of which the evolution is effectively classical. By acting with operators \( xp \) and \( px \) on the wave function (5) we find that both results approximately agree if, \( \left| x^2 \frac{p_c(t)}{x_c(t)} \right| \gg 1 \).

For example, if we again consider the upside-down oscillator with constant frequency, we find that it behaves nearly classically at large amplitudes, \( x \gg (m\omega_0)^{-1/2} \). In case of a time-dependent oscillator the decoherence scale will depend on time. In a \( p_c \sim x_c \) regime, this condition becomes, \( |xp_c(x)| \gg 1 \), which is the resurrection of the uncertainty principle: the behavior near given value of \( x \) is approximately classical if \( x \) is greater than the De Broglie wavelength for the classical momentum that corresponds to that value of \( x \).

Let us now summarize the steps to see whether particular time-dependent oscillator has nearly classical behavior. First, we solve the oscillator equation to find
the two independent solutions. Next, we calculate the coefficients \( A \) and \( B \). If one of the conditions (a) or (b) above is full-filled, the behavior is essentially classical. Finally, from Equation (16) we find for which values of the amplitude (as opposed to at what times) the classical behavior takes place. Of course, along the way, possible late time dominance of just one solution will give us advance clue about the emergence of classical regime. In the next section we will apply this procedure to the simplest inflationary model.

4. De Sitter expansion

Let us start by listing a few well known results about the massive fields on exact, spatially flat De Sitter background. The convenient time-like variable is \( z \equiv -k \eta \in (0, \infty) \). The one mode frequency is,

\[
\omega^2(z) = 1 - \frac{1}{z^2} \left( \frac{m^2}{H_0^2} - 2 \right).
\]

The independent solutions \( \chi_{1,2} \) are given in terms of Hankel functions:

\[
\chi_{1,2}(z) = \sqrt{\frac{\pi}{2}} \sqrt{2} \nu H_\nu^{(1,2)}(z), \quad \nu^2 \equiv \frac{9}{4} - \frac{m^2}{H_0^2}.
\]

As before, the classical solution and its momentum will be expressed as

\[
\chi_c = \alpha \chi_1(z) + \beta \chi_2(z),
\]

\[
p_c = -k [\alpha \chi_1'(z) + \beta \chi_2'(z)],
\]

where \( \alpha \) and \( \beta \) are two complex constants.

Since we are interested in the late time behavior of the wave function we only need the late time behavior of the solution. The late time (\( z \to 0^+ \)) behavior of the modes is well known:

\[
\chi_1(z) = C_\nu z^{1/2+\nu} + D_\nu z^{1/2-\nu},
\]

\[
\chi_2(z) = A_\nu z^{1/2+\nu} + B_\nu z^{1/2-\nu},
\]

with,

\[
A_\nu = \frac{\pi^{1/2} e^{i\nu\pi}}{2^{\nu+1} \Gamma(\nu+1)} (1 + i \cot \nu \pi),
\]

\[
B_\nu = D_\nu = \frac{i e^{i\nu\pi}}{\pi^{1/2} 2^{\nu-1} \Gamma(\nu)},
\]

\[
C_\nu = \frac{\pi^{1/2} e^{i\nu\pi}}{2^{\nu+1} \Gamma(\nu+1)} (1 - i \cot \nu \pi).
\]

Consider first the case when \( \nu \) is real. Then, at small values of \( z \) both independent solutions behave as \( z^{1/2-\nu} \), and, effectively, we have just one mode. Note that
for \( \nu > 1/2 \) the dominant mode diverges, while for \( 0 < \nu < 1/2 \) it goes to zero, but slower than the other mode which is proportional to \( z^{1/2+\nu} \). In either case, as \( z \to 0^+ \),

\[
\chi_c(z) = (\alpha + \beta)B_\nu z^{1/2-\nu} + O(z^{2\nu}), \quad \nu \in \mathbb{R},
\]

and it follows that,

\[
p_c = (-)k \left( \frac{1}{2} - \nu \right) \frac{\chi_c}{z}.
\]

This is the sufficient condition for the emergence of classical correlations. The argument breaks down for \( \nu = 1/2 \), or, equivalently, \( m^2 = 2H_0^2 \). Minimally coupled field of that mass is equivalent to conformally coupled massless field.

The situation is different when \( \nu \) is imaginary. In that case we have,

\[
\chi_c = z^{1/2} \left[ (\alpha C_\nu + \beta A_\nu) e^{i|\nu|\log z} + (\alpha + \beta)B_\nu e^{-i|\nu|\log z} \right],
\]

and

\[
p_c = z^{-1/2} \left[ \left( \frac{1}{2} + \nu \right) (\alpha C_\nu + \beta A_\nu) e^{i|\nu|\log z} + \left( \frac{1}{2} - \nu \right) (\alpha + \beta)B_\nu e^{-i|\nu|\log z} \right].
\]

Both modes are oscillatory and grow or decay at the same rate. As both of them must be retained, \( p_c \) is not simply proportional to \( \chi_c \), and the classical correlation does not emerge.

Let us now try to see this through the behavior of the squeezing coefficients. The initial conditions are assigned at \( z = z_i \gg 1 \), where, for all values of \( \nu \), modes behave as simple exponentials:

\[
\chi_{1,2}(z_i) = \frac{1}{\sqrt{2}} \cdot e^{\mp i\Phi}, \quad \Phi \equiv z_i - \nu \frac{\pi}{2} - \frac{\pi}{4}.
\]

We find that \( \dot{A} = -kA'(z) \), and similarly for the coefficient \( B \). All these expressions differ just in the constant coefficients from those for the classical solution. Again, when \( \nu \) is real, \( z^{1/2-\nu} \) mode dominates both \( A \) and \( B \), and we have \( A'(z) \sim A(z) \) and \( B'(z) \sim B(z) \). One therefore finds at \( z \ll 1 \),

\[
\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = (-)k \left( \nu - \frac{1}{2} \right) \frac{1}{z}.
\]
and, \[ AB - A\dot{B} = 0 \] (35) with \( \mathcal{O}(z^{2\nu}) \) corrections in both equations. Therefore, the second term in both (11) and (12) behaves as \( A \cdot W_{AB} \sim B \cdot W_{AB} \sim \mathcal{O}(z^{1/2+\nu}) \). To estimate the first term recall that the classical solution diverges as \( \chi_c \sim z^{1/2-\nu} \) for \( \nu > 1/2 \), and decays to zero with the same rate if \( \nu < 1/2 \). The first terms in (11, 12) behaves as \( z \chi \), which diverges for both ranges of \( \nu \). The classical regime emerges in both cases, but while for \( \nu > 1/2 \) it is due to the unbounded growth of the classical solution in the upside-down potential, in case of \( \nu < 1/2 \) the classical solution is bounded and the decoherence takes place due to the divergence of squeezing coefficients \( \dot{A}/A \) or \( \dot{B}/B \). These are exactly the two possibilities labeled as (a) and (b) in the preceding section.

In contrast, when \( \nu \) is imaginary, we find,

\[
\frac{\dot{A}}{A} = (-)^k \frac{M_\nu (1/2 + \nu) e^{-i|\nu|} \log z + N_\nu (1/2 - \nu) e^{-i|\nu|} \log z}{M_e e^{-i|\nu|} \log z + N_e e^{-i|\nu|} \log z},
\]

(36) with,

\[
M_\nu \equiv C_\nu e^{i\Phi} + A_\nu e^{-i\Phi},
\]

(37)

\[
N_\nu \equiv D_\nu e^{i\Phi} + B_\nu e^{-i\Phi}.
\]

(38)

The expression for the rate of change of change of \( B \) differs only in coefficients:

\[
\frac{\dot{B}}{B} = (-)^k \frac{\tilde{M}_\nu (1/2 + \nu) e^{-i|\nu|} \log z + \tilde{N}_\nu (1/2 - \nu) e^{-i|\nu|} \log z}{\tilde{M}_e e^{-i|\nu|} \log z + \tilde{N}_e e^{-i|\nu|} \log z},
\]

(39) with,

\[
\tilde{M}_\nu \equiv (-)C_\nu e^{i\Phi} + A_\nu e^{-i\Phi},
\]

(40)

\[
\tilde{N}_\nu \equiv (-)D_\nu e^{i\Phi} + B_\nu e^{-i\Phi}.
\]

(41)

The fractions in both expressions are bounded, so both \( \dot{A}/A \) and \( \dot{B}/B \) diverge as \( z^{-1} \). However, we must also check the magnitude of the second term in Eq.’s (11 - 12). Both modes behave as \( z^{1/2} \), and the same as true for \( \chi_c, A, \) and \( B \). Thus, the first terms in (11 - 12) behave as \( z^{-1/2} \). The second terms have the same growth at late times, \( \sim z^{1/2} z^{-1} \sim z^{-1/2} \). Thus, in general, the first term does not dominate.

The last hope is that the second term vanishes altogether, due to the exact cancellation, \( \dot{A}/A = \dot{B}/B \). For this we need \( (M_\nu, N_\nu) = (\tilde{M}_\nu, \tilde{N}_\nu) \), which itself is possible only if \( C_\nu = D_\nu = 0 \). (And hence \( B_\nu = 0 \). This is reminiscent of the condition for the dominance of one mode.) For this to happen we need \( |\nu| \rightarrow \infty \), (hence \( m^2/H_0 \rightarrow \infty \) as well), but then also \( A_\nu = 0 \), so the only solution that we can represent in this way is \( \chi_c = 0 \). Therefore, we find that neither of the conditions for decoherence is satisfied, and there is no emergence of classical correlations for \( \nu^2 < 0 \).

Having now established the late time classical behavior of fluctuations with mass smaller than \( 3H_0/2 \) \( (m^2 = 2H_0^2) \) is excluded, as it corresponds to conformally
coupled massless field), we can now use the commutator criteria, Eq. (16), to find out the range of amplitudes which may be considered classical. Using Eq. (28), we find for all real $\nu \neq 1/2$, that the behavior is classical for amplitudes,

$$|\chi^2| \gg \frac{|\eta|}{|\nu - 1/2|}.$$  \hspace{1cm} (42)

One should keep in mind that the wave number $k$ is fixed, and that the adopted ratio (28) applies only at the late times, $z \to 0^+$, or equivalently, $k|\eta| \ll 1$. In terms of the original variable characterizing the fluctuation, the bound is given as,

$$|\delta\phi| \gg \frac{H_0|\eta|^{3/2}}{|\nu - 1/2|^{1/2}}.$$  \hspace{1cm} (43)

The overall normalisation to Hubble parameter is common to all various measures of decoherence,\(^5\) while particular scaling with time, or the wave number depends on details of the definition of that measure. For instance, for modes outside the Hubble radius, the decoherence bound on $\delta\phi$ is much smaller than the zero coupling limit of the coherence length derived by Brandenberger et al.\(^5\) for a model with two coupled massless fields.

5. Conclusions

Stochastic dynamics of inflationary cosmology, and the generation of classical density perturbations through vacuum fluctuations of inflaton field, are normally studied and understood only in cases when the inflaton, or its fluctuations, behave as very light fields, $m^2 \ll H^2$. This is usually motivated by the observational fact that the universe is only slightly inhomogeneous, which, in simplest models, translates into small mass or coupling of the inflaton field. However, inflationary models are getting more complex, scenarios are more complicated, and the relationship between the observational quantities and parameters of the Lagrangean may not be as straightforward. The emergence of classical spacetime in quantum cosmology might be due to processes which have little or nothing to do with the relics observed today. It is therefore important to know all possibilities for which important mechanisms, such as the generation of classical density perturbations, may take place. This paper develops a simple, but fairly general analysis of the time dependent quantum oscillators, on basis of which one can formulate criteria for decoherence of scalar field quantum fluctuations in any Robertson-Walker cosmology. The method is just a simple extension of the approach taken by Guth and Pi,\(^4\) but thanks to this extension one can confirm and understand results on decoherence in cases in which fluctuations are not described through upside-down quantum oscillators. In particular, we confirm the results found in Ref. 6.

Finally, one should stress that the criteria of decoherence derived here strictly apply to free fields. Normally one expects that the presence of a small interaction ought to make decoherence stronger. More importantly, these criteria are just
sufficient, but not necessary conditions for decoherence. Some other mechanism may bring decoherence in cases in which it does not take place according to criteria used here. It is important to stress however, that the mechanism described here appears to be sufficient to describe the decoherence in most popular De Sitter-like inflationary models, as well as in most of the models of power law inflation.8

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