A Software Tool to Estimate the Time-dependent Deformation in RC Walls and Columns

Varun Sahay*, M N Shariff and Devdas Menon

Indian Institute of Technology Madras, Chennai, Tamil Nadu, India

*varunsahay19@gmail.com

Abstract. Tall reinforced concrete (RC) buildings are sensitive to time-dependent effects which arise due to creep and shrinkage. In this study, a software tool has been developed to estimate the time-dependent deformations in RC walls and columns. The program can invoke the ACI 209, EC 2, IS 1343 and fib MC 10 models to determine the creep and shrinkage in plain concrete. In order to include the effects of longitudinal reinforcement, two alternative methods have been explored: (i) effective modulus method and (ii) one-dimensional boundary value problem method. The effect of variable environment is included using the principle of superposition, which is valid within the linear creep domain. The program is written on the Python platform and can be used on any Windows machine. The program is validated using tests reported in the literature. The program is capable of accepting input of a linear elastic one-step-analysis from standard commercial software like ETABS and StaadPro. This program requires minimal input from the user, such as the grade of concrete, rate of loading and time at which the deformations are to be evaluated. The time-dependent deformations are evaluated for each column or wall and the super-elevation levels for each story are presented in a chart form, which serves as a design aid for the engineers and can be readily used by the site construction team.

1. Introduction

Creep and shrinkage of the concrete depend on the dimensions of the element, the ambient relative humidity and the composition of the concrete. Additionally, creep is also dependent on the magnitude of sustained load, the maturity of the concrete when the load is first applied and the duration of the loading [1]. It is imperative to estimate the time-dependent deformation due to creep and shrinkage in reinforced concrete (RC) columns with reasonable accuracy to assure safety and serviceability of high-rise buildings, pylons of cable-stay bridges and other creep sensitive structures [2]. If not adequately accounted for in the design, these time-dependent deformations result in serviceability issues such as slab-beam deflections, cracking of partitions and failure of other non-structural elements [3]. The time-dependent deformations calculated using the existing commercial software often ignore the effect of reinforcements and hence they have a considerable deviation from the observed values in the real structures. Through this study, an effort has been made to develop a software tool that could estimate these strains with higher accuracy by incorporating the use of effective modulus method and one-dimensional boundary value problem method [4,5]. Any one of the methods can be employed as per the convenience of the design engineer. In both methods, the effect of steel is considered assuming strain compatibility (perfect bond) between concrete and steel. In the effective modulus method, steel’s restraining effect on creep and shrinkage deformations is included in the equilibrium equations, and the modulus of elasticity of concrete is modified using the creep coefficient calculated at every time increment. The one-dimensional boundary value problem method is an exact method to determine the time-dependent deformations, which is recently developed, wherein a linear viscoelastic constitutive law for concrete is considered. The reinforcing steel and concrete are considered to be in a parallel
arrangement. A software tool has been developed on the Python platform, which can be a useful tool while designing and construction of tall RC buildings using the above two methods.

2. Models for predicting the creep and shrinkage strains in plain concrete

The software evaluates the creep and shrinkage induced time-dependent deformation on plain concrete using the models as per ACI209, EC2, IS1343 and fib MC 10. The choice of using any one of the models is given to the user. The software can either use the effective modulus method or the one-dimensional boundary value problem method to include the effect of longitudinal reinforcements.

2.1. ACI 209R-92 model [6]

This model predicts the shrinkage strain $\varepsilon_{sh}(t, t_c)$, where $t$ (in days) is the age of concrete and $t_c$ (in days) is the age of concrete at the start of drying or the end of initial curing, using the equation below-

$$\varepsilon_{sh}(t, t_c) = \frac{(t - t_c)^\alpha}{f^c + (t - t_c)^\alpha} \varepsilon_{suh}$$

(1)

where, $f_c$ (in days) and $\alpha$ are constants for a given shape and size of the member and $\varepsilon_{suh}$ is the ultimate shrinkage strain. The values of $\varepsilon_{suh}$, $f_c$ and $\alpha$ depend on factors like age of curing $t_c$, relative humidity, $V/S$ of the member, the slump of concrete, percentage of fine aggregates, cement content of concrete and percentage of air in concrete. The detailed expressions to evaluate can be found in Appendix A of the code.

The stress-dependent strain is evaluated as-

$$\varepsilon_{cr} + \varepsilon_i = J(t, t_0) \sigma_c$$

(2)

here, $\varepsilon_{cr}$=creep strain at the time $t$ considered; $\varepsilon_i$=Initial elastic strain at the time of loading; $\sigma_c$=stress applied to the concrete at the time of loading.

$J(t, t_0)$ is the Compliance function and is defined as-

$$J(t, t_0) = \frac{1 + \phi(t, t_0)}{E_{cmto}}$$

(3)

where, $E_{cmto}$ is the elastic modulus of concrete (in MPa) at the time of loading $t_0$ (in days) and $\phi(t, t_0)$ is the creep coefficient which is the ratio of creep strain to the elastic strain at the beginning of loading $t_0$ (in days).

The creep coefficient $\phi(t, t_0)$ is calculated using the equation below –

$$\phi(t, t_0) = \frac{(t - t_0)^\psi}{d + (t - t_0)^\psi} \phi_a$$

(4)

$d$, $\psi$ and $\phi(t, t_0)$ depend on the time of loading $t_0$, relative humidity, $V/S$ ratio, the slump of concrete, percentage of fine aggregates and air content in the concrete. The detailed expression of which can be found in the code.

2.2. EC2 model and IS 1343 model [7,8]

The total shrinkage strain ($\varepsilon_{sh}$), according to this model, consists of two parts- the drying shrinkage strain ($\varepsilon_{cd}$) and the autogenous shrinkage strain ($\varepsilon_{ca}$).

$$\varepsilon_{sh} = \varepsilon_{cd} + \varepsilon_{ca}$$

(5)

Drying shrinkage strain when the age of concrete is $t$ (in days), is evaluated using the expression-

$$\varepsilon_{cd}(t) = \frac{(t - t_c)^\gamma}{(t - t_c) + 0.04 \sqrt{h_0}} \varepsilon_{suh}$$

(6)
Here, $k_h$ is a coefficient which depends on the notional size $h_0 = 2A_c/u$ of the concrete member, $A_c$ is the cross-sectional area of concrete, $u$ is the perimeter of that part of the cross-section which is exposed to drying, $t_s$ is the age of concrete at the start of drying shrinkage which is normally the end of curing. The basic drying shrinkage strain ($\varepsilon_{c,0}$) depends on RH, Mean compressive strength of concrete and the cement type. The detailed expressions can be obtained from Annex B of the Code.

The expression for the Autogenous Shrinkage strain when the age of loading is $t$ (in days) is given by

$$
\varepsilon_{ca}(t) = (1 - e^{-0.2f_c})\varepsilon_c(\infty)
$$

(7)

$$
\varepsilon_c(\infty) = 2.5 \times 10^{-6}.(f_{ck} - 10)
$$

(8)

here, $f_c$ is the characteristic compressive strength of concrete (in MPa)

The creep strain is evaluated using the following expression-

$$
\varepsilon_{cr}(t,t_0) = \sigma_c(t,t_0)\beta_{cr}(t_0)
$$

(9)

where, $\sigma_c < 0.45 f_c$ and $E_c$ is the tangent modulus of elasticity of concrete under compression. $\phi(t,t_0)$ is the creep coefficient and is evaluated using the expression-

$$
\phi(t,t_0) = \phi_0 \left[ \frac{(t-t_0)}{\beta_H + t-t_0} \right]^{0.3}
$$

(10)

here $\phi_0$ is the notional creep coefficient, which depends on RH, mean compressive strength of concrete and the age of concrete at the time of loading $t_0$ and $\beta_H$ depends on RH, detailed expressions of which can be obtained from the respective codes. It is crucial to note that though the basic governing equations in the above two models are the same, the expressions for the calculations of certain constants such as $\phi_0$, $\beta_H$ and $E_c$ are different in these two models.

2.3. fib MC 10 model [9]

The total shrinkage strain, $\varepsilon_{cs}(t,t_s)$, consists of two parts, basic shrinkage, $\varepsilon_{cbs}(t,t_s)$, which occurs even in the absence moisture loss and drying shrinkage, $\varepsilon_{cds}(t,t_s)$, which is the additional shrinkage in case of moisture loss.

$$
\varepsilon_{cs}(t,t_s) = \varepsilon_{cbs}(t,t_s) + \varepsilon_{cds}(t,t_s)
$$

(11)

$$
\varepsilon_{cbs}(t) = (1 - e^{-0.5})\varepsilon_{cbs}(f_{cm})
$$

(12)

$$
\varepsilon_{cds}(t,t_s) = \left[ \frac{(t-t_s)}{0.035h^2 + (t-t_s)} \right]^{0.5} \varepsilon_{cds0}(f_{cm})\beta_{RH}(RH)
$$

(13)

here, $t_s$ is the age of concrete at the start of drying (in days), $f_{cm}$ is the mean compressive strength (in MPa) of concrete at the age of 28 days, $t$ is the age of concrete (in days) at which the strains are to be calculated, $RH$ is the relative humidity of the ambient atmosphere (in %), and $h$ is the notional size (in mm) = $2A_c/u$, the detailed expression for $\beta_{RH}(RH)$, $\varepsilon_{cds0}(f_{cm})$ and $\varepsilon_{cds0}(f_{cm})$ can be obtained from section-5.1.9.4.4 of fib Model Code for Concrete Structures 2010.

The expression for evaluating the stress-dependent strain $\varepsilon_{cr}(t,t_0)$ at a time $t$ (in days) is given by-

$$
\varepsilon_{cr}(t,t_0) = \varepsilon_{cr}(t_0) - \frac{1}{E_{cr}(t_0)}\left[ \frac{\phi(t,t_0)}{E_{cr}} \right] = \sigma_c(t_0)J(t,t_0)
$$

(14)

here, $J(t,t_0)$ is the creep compliance or the creep function, $E_{cr}(t_0)$ is the elastic modulus of concrete at the time of loading. $E_{cr}$ is the elastic modulus of concrete at the age of 28 days. The expression for the variation of $E_{cr}$ with time can be obtained from the code.
The creep coefficient $\varphi(t,t_0)$, consists of two components, basic creep coefficient, $\varphi_{bc}(t,t_0)$ and drying creep coefficient $\varphi_{dc}(t,t_0)$, which are estimated by the expressions,

$$\varphi(t,t_0) = \varphi_{bc}(t,t_0) + \varphi_{dc}(t,t_0)$$  \hspace{1cm} (15)

$$\varphi_{bc}(t,t_0) = \ln \left[ \frac{30}{t_{0,adj}^2} + 0.035 \left( t - t_0 \right) + 1 \right] \beta_{bc}(f_{cm})$$  \hspace{1cm} (16)

$$\varphi_{dc}(t,t_0) = \beta_{dc}(f_{cm}) \beta(RH) \beta_{dc}(t_0) \left[ \frac{(t-t_0)}{\beta_h + (t-t_0)} \right]^{\gamma(t_0)}$$  \hspace{1cm} (17)

Here, RH is the relative humidity in the ambient environment (in %), $f_{cm}$ is the mean compressive strength of concrete at the age of 28 days (in MPa), $t_{0,adj}$ is the adjusted age at loading (in days) and $h$ is the notional size of the concrete member (in mm). The detailed expressions for $\beta(RH)$, $\beta_{dc}(t_0)$, $\beta_{bc}(f_{cm})$ and $\beta_{dc}(f_{cm})$ can be obtained from section-5.1.9.4.3 of fib Model Code for Concrete Structures 2010.

3. Methods for including the effect of reinforcement in RC

The software can implement two alternative methods to include the effect of reinforcements.

3.1. Effective modulus method [4]

The effective concrete modulus is given by the expression-

$$E_{eff} = \frac{E_{ci}}{1+\nu_t}$$  \hspace{1cm} (18)

Where, $E_{eff}$=effective concrete modulus at the time $t$ (in days) considered, $E_{ci}$= concrete modulus at the time of loading (taken as 28-day concrete modulus), $\nu_t$=creep coefficient at time $t$, $t$ is the time after loading (in days).

The fib MC 10 Model uses $E_{ci}$ at two different time instances in its expression of the compliance function. To account for this $E_{eff}$ has been derived considering strain compatibility between steel and concrete and the obtained expression is given by-

$$E_{eff} = \frac{E_{ci}E_{ci}(t_0)}{(E_{ci} + \nu_t E_{ci}(t_0))}$$  \hspace{1cm} (19)

Here, $E_{ci}(t_0)$ is the concrete modulus at the time of loading.

By this method, the basic equations for prediction of creep, shrinkage and total strains are--

$$\varepsilon_{initial} + (\varepsilon_{creep})_t = \frac{P}{[A_g (1-\rho_g) + A_g \rho gn_{eff}] E_{eff}}$$  \hspace{1cm} (20)

$$\varepsilon_{shrinkage}_t = \varepsilon_{sh}_t \frac{(\varepsilon_{sh})_t}{[A_g (1-\rho_g) + A_g \rho g n_{eff}] E_{eff}}$$  \hspace{1cm} (21)

$$\varepsilon_{total}_t = \varepsilon_{initial} + (\varepsilon_{creep})_t + (\varepsilon_{shrinkage})_t$$  \hspace{1cm} (22)

Where, $(\varepsilon_{creep})_t$ = creep strain in the reinforced concrete member at time $t$ (in days) considered, and $n_{eff}$=modular ratio at time $t$ which is equal to $(E/E_{eff})$, $E_s$ is the elastic modulus of steel, $(\varepsilon_{shrinkage})_t$=shrinkage strain of the reinforced concrete member at time $t$ considered, $(\varepsilon_{sh})_t$=shrinkage strain as calculated for the plain concrete specimen with other parameters being the same, $(\varepsilon_{total})_t$= total strain of the reinforced specimen at the time $t$ considered and $t$ is the time after loading (in days).

3.2. One-dimensional boundary value problem method [5]

The creep, shrinkage and total strain in the reinforced specimen is given by--
\[ \varepsilon_{\text{creep}}(t) = \mathcal{L}^{-1}\left[ \frac{sL(J)[L(P) - A_s E_s L(\varepsilon_{sh})]}{A_s + A_s E_s sL(J)} \right] \]  
(23)

\[ \varepsilon_{\text{shrinkage}}(t) = \mathcal{L}^{-1}\left[ \frac{sL(J)[-A_s E_s L(\varepsilon_{sh})]}{A_s + A_s E_s sL(J)} \right] \]  
(24)

\[ \varepsilon_{\text{total}}(t) = \varepsilon_{\text{creep}}(t) + \varepsilon_{sh} \]  
(25)

here, \( L \) is the Laplace transform of the respective functions, \( s \) is the Laplace transform variable and \( \mathcal{L}^{-1} \) is the inverse Laplace transform of the respective functions. \( \varepsilon_{sh} \)=shrinkage strain as calculated for the plain concrete specimen with other parameters being the same, \( \varepsilon_{shrinkage}(t) \)=shrinkage strain in the reinforced specimen at the time considered; \( \varepsilon_{creep}(t) \)=creep strain in the reinforced specimen at the time considered.

4. Results

The program (Creep And Shrinkage Estimator – CASE) has a user-friendly interface and invokes the respective predictive models as per their choice. It can accept inputs of a linear elastic one-step analysis from standard commercial software like ETABS and StaadPro. It can generate the plots showing deformations and strains in the queried section of the column as well as the deformations in the queried column at each story at a particular time. The loading is considered to be applied in a stepwise manner, which is the actual way in which columns are loaded in a real structure. These deformations can be used to provide the necessary extra elevations to the columns while construction to ensure that the actual dimensions are reasonably close to the design dimensions.

![CASE](image)

**Figure 1.** A snippet of input window of the program. The required inputs change according to the model selected. Here the ACI model is selected.

The CASE program is first validated using tests reported in the literature. A number of tests were conducted by Ziehl et al. (1998) [10] on reinforced concrete as well as plain concrete columns subjected
to axial compression for a 450 days period. The column specimens were made using M27 concrete and compressive stress of $0.4 \times f_c$ (10.8 MPa) was applied. The time-dependent strains are estimated using the ACI code and the effect of reinforcement is incorporated by applying the ‘effective modulus method’. Figures 2a and 2b show the comparison of the experimental versus predicted time-dependent strains. It is seen that the strains predicted from the program are in good agreement with the experimental data.

![Figure 2](image1.png)

**Figure 2.** Comparison of experimental versus the prediction using ACI code (a) for plain concrete column and (b) for reinforced concrete column ($p = 0.72\%$).

A sample ETABS model is generated for a 10 storeyed structure. The analysis and design are carried out in ETABS and the output file (in Microsoft Excel format) is imported to the developed program. The floor construction cycle has been assumed as 10 days. Results for a typical column (having a column name 149, which is located at the outer perimeter of the building) have been extracted and the creep, shrinkage and total strains have been plotted in figures 3a and 3b.

![Figure 3](image2.png)

**Figure 3.** The strains and deformations in a particular part of the column in a story.

The designer and the construction engineer would be interested in estimating the casting level of the columns, such that the datum level is reached after the initial creep and shrinkage deformations stabilize. In this study, a stabilization period of two years has been assumed. The super-elevation levels for column ‘149’ are plotted, both ignoring the effect of steel and including the effect of steel in figure 4. Here, the restraining effect of steel is clearly demonstrated. It can be seen that the time-dependent deformations would be over-estimated by about 15 percent if the effect of steel is ignored. This can have a significant influence on the differential axial shortening of the columns in a multi-storey structure.
5. Conclusions
From this study, the following conclusions can be drawn:

(a) A software tool has been successfully developed to estimate the creep and shrinkage strains (deformations) in an axially loaded reinforced concrete member, using the axial stress output generated using commercial software like (ETABS and StaadPro). The software tool requires only minimal input and can account for the construction load histories.

(b) The program has been validated using tests reported in the literature for both plain concrete and reinforced concrete. It is seen that the predictions are in good agreement with the experimental results for both the cases.

(c) The restraining effect of reinforcing steel on the time-dependent deformations and hence the error in estimating the super-elevation levels of columns is clearly demonstrated. For the example considered in the present study (10 storeyed building), it can be seen that the error in column shortening is about 15 percent if the restraining effect of reinforcement is ignored in the axial shortening calculation.

(d) This software tool can be easily used by engineers and construction managers to decide on the levels of column super-elevation to ensure the vertical elements reach the pre-planned datum level, as required in the design after they undergo creep and shrinkage deformations.

6. References
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