A theory of pedestrian-induced footbridge vibration comfortability based on sensitivity model

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Abstract

Pedestrian-induced footbridge vibration comfort level is a complex problem that has been studied for a long time. However, no consensus has been reached on a quantitative calculation index for assessing vibration comfort level. Only simple comfort limits, rather than specific relationships between comfort level and the vibration endurance capacity of pedestrians, are currently available for assessing vibration comfort level of footbridges. This article aims to propose a sensitivity model for pedestrian-induced vibration comfort calculation based on the vibration endurance capacity of pedestrians and the vibration response of footbridges. The concepts of “human body resistance” and “vibration effect” were established according to the principle of probability and statistics. Mathematical definition of sensitivity was put forward. Calculation expressions for a pedestrian and pedestrians were deduced respectively. A theory of pedestrian-induced footbridge vibration comfort level was proposed. Field survey and experiment were conducted, the results of the field survey demonstrated that sensitivity values were in good agreement with the international vibration comfort standards. Furthermore, the field experiment results showed that the errors between the experimental results and the calculated results were within 6%. The proposed sensitivity theory can be used for pedestrian-induced footbridge vibration comfort quantitative calculation.

Keywords: Sensitivity, Human body resistance, Vibration effect, Footbridge, Pedestrian-induced vibration, Comfort level

1 Introduction

Modern footbridges are often suffered from pedestrian-induced vibrations, which severely influence the walking comfort of pedestrians. The infamous Millennium Bridge in London is the prime pedestrian-induced vibration example. Studies of video footage revealed up to 50 mm of lateral movement of the south span and 70 mm of the central span (Dallard et al. 2001; Dallard 2005), and pedestrians were frightened. The Japanese Toda Park Bridge and Mape Valley Great Suspension Bridge (Feng et al. 2019) experienced the same situations. Similarly, there are vibration comfort problems on many footbridges, e.g. the Solferino Footbridge in Paris (Gheitasi et al. 2016), the NEC Bridge in Birmingham (Zivanovic et al. 2005), the Alexandra Bridge in Ontario (Bruno and...
Venuti 2009), Wuhan Yangtze River Bridge (Li 1975), the Queens Park Footbridge in Britain (Huang et al. 2005) and Shanghai Railway Station Footbridge (Xiao 2009). The above-mentioned cases demonstrate that the walking comfort is severely affected by pedestrian-induced vibration. Hence, the vibration comfort has become a critical requirement in footbridge design and serviceability assessments (Tubino et al. 2020; Li et al. 2020).

Pedestrian-induced footbridge vibration comfort involves the fields of ergonomics, structural dynamics, psychology and fuzzy mathematics. The earliest study about vibration comfort can be traced back to 1879, when German psychologist Wilhelm Wundt (Cao and Chen 2020) did a systematic study on the human body’s subjective feelings under vibration. In 1931, Reiher and Meister (1931) conducted a landmark experiment, which indicated that the subjective feelings under each vibration circumstance was dependent on the vibration velocity and that the subjective feeling threshold of the human body to vertical vibration speed was ±3 mm/s. In 1939, the German standard DIN4150 proposed a vibration comfort index \( PAL \) based on the experimental results of Reiher and Meister:

\[
PAL = 10 \log_{10} \left( \frac{V}{V_0} \right)^2
\]

Subsequently, Helberg and Sperling (1941), Dieckmann (1955), Chang (1967), and Griffin (1991) conducted detailed research on vibration comfort respectively, as listed in Table 1.

As can be seen from Table 1, researchers have accepted that vibration acceleration, not vibration velocity, controls pedestrian comfort. Most of the research results adopted a series of psychological concepts such as “not comfortable”, “a little uncomfortable”, “uncomfortable”, and “very uncomfortable” to describe the level of vibration comfort. When the acceleration value of the structural vibration is obtained, it is clear

| Research Results          | Acceleration Index | Subjective Feelings               |
|---------------------------|--------------------|-----------------------------------|
| Helberg and Sperling (1941) | 0.001(g) peak      | Minimal feeling                    |
|                           | 0.022(g) peak      | Certain feeling                    |
|                           | 0.08(g) peak       | Intolerable                        |
| Dieckmann (1955)          | 0.003(g) peak      | Feeling threshold                  |
|                           | 0.03(g) peak       | Feel reluctantly                   |
|                           | 0.3(g) peak        | Not comfortable                    |
| Chang (1967)              | < 0.05(m/s²) peak  | Can not feel                       |
|                           | 0.05–0.15(m/s²) peak| Can feel                           |
|                           | 0.15–0.5(m/s²) peak| Not uncomfortable                   |
|                           | 0.5–1.5(m/s²) peak | Very uncomfortable                  |
|                           | > 1.5(m/s²) peak   | Intolerable                        |
| Griffin (1991)            | 0.3(m/s²) r.m.s.   | Can feel                           |
|                           | 0.7(m/s²) r.m.s.   | Not uncomfortable                   |
|                           | 1.1(m/s²) r.m.s.   | A little uncomfortable              |
|                           | 1.7(m/s²) r.m.s.   | Uncomfortable                      |
|                           | 2.5(m/s²) r.m.s.   | Very uncomfortable                  |
whether the vibration level of the structure can meet the requirement of the vibration comfort standard (Song 2003; Chen et al. 2019). However, there are problems in the existing research results:

(1) Estimating the vibration comfort level is a complex problem (Zhu et al. 2019). Vibration comfort is related to the vibration endurance capacity of pedestrians and the vibration response of footbridges. However, the existing research defines vibration comfort only in the sense of the vibration response of the footbridges. Likewise, the existing research is not applicable to a general situation.

(2) Vibration comfort is also related to the subjective feelings of the human body. Different people have different subjective feelings in case of the same vibration response. When in a vibration environment where the vibration response is below the allowable value, not all people feel comfortable. Similarly, when in a vibration environment where the vibration response is higher than the allowable value, not all people feel uncomfortable. The vibration acceleration limit in the existing research itself has some uncertainty.

Recently, some guidelines, such as ISO 2631-1 (ISO 1997), AASHTO (2008), Eurocode 2 (CEN European Committee for Standardization 1996), Austroads (2009), BS5400 (BSI British Standards Institution 1979), and AISC Guide 11 (Murray et al. 2003), have provided different acceleration limits with corresponding vibration comfort levels (Van Nimmen et al. 2014). Nevertheless, there is no comprehensive quantitative index that can consider the vibration endurance capacity of people, the vibration response of the structure and the uncertainty of subjective feeling of the human body. Meanwhile, there is also no mathematical definition or calculation method for vibration comfort level, it is valuable to do pedestrian-induced footbridge vibration comfort research in both of the following aspects: the vibration endurance capacity of people and the vibration response of the structure.

In view of the existing problems, this study first establishes a sensitivity model for vibration comfort level calculation. The mathematical definition, calculation method and classification standards of sensitivity were proposed. A theory of pedestrian-induced footbridge vibration comfort was put forward. Meanwhile, a questionnaire survey was used on five different urban footbridges to determine sensitivity. Finally, field tests on The Fourth Corridor Footbridge in Guangzhou City were conducted, and the theory was verified.

2 Sensitivity model
To analyze pedestrian-induced footbridge vibration comfort level, the first thing is to determine the sensitivity of pedestrians to structural vibration (Ma 2012). For the same magnitude of vibration response, different pedestrians have different sensitivities. In addition, for a pedestrian, the sensitivity values under different vibration responses are also different. Therefore, there are two factors that influence the sensitivity of pedestrians: one factor is the vibration endurance capacity of pedestrians, which can be defined as “human body resistance”, is an inherent attribute of the pedestrians determined by each pedestrian’s own characteristics, and has nothing to do with the vibration of the footbridges or other external factors. The other factor is the vibration
response of footbridges, which can be defined as the “vibration effect”, is related to the quality, stiffness, and damping of the footbridges and other factors that influence their vibration response.

2.1 Human body resistance
As an inherent attribute of pedestrians, human body resistance refers to the vibration endurance capacity of pedestrians, denoted as $R$. When the footbridge starts to vibrate under the excitation of a pedestrian load, the vibration response is obviously lower than the human body resistance. A small vibration will not affect pedestrians on the footbridge, and pedestrians are insensitive to the vibration. When the vibration amplitude of the footbridge is larger than the human body resistance, pedestrians do become sensitive to the vibration.

Since the human body is an organism with a high degree of mental accommodation, human beings can adjust the vibration endurance capacity according to their willpower; thus, it is extremely difficult to determine an accurate value of human body resistance. To facilitate this analysis of sensitivity, we adopt statistical constants given by the international organization for standards as quantitative numerical results of human body resistance, as shown in formula (2):

$$ C = \|R(r_1, r_2, r_3, ..., r_i, ..., r_n)\| $$  \hspace{1cm} (2)

In formula (2), $C$ expresses human body resistance, $r_i$ expresses a factor that affects human body resistance, $\|R(r_1, r_2, r_3, ..., r_i, ..., r_n)\|$ expresses the norm of human body resistance $R$.

2.2 Vibration effect
The vibration effect refers to the vibration effects on the human body caused by the vibration response, denoted as $V$. The main factors that affect the vibration effect are the footbridge and pedestrians, as shown in Table 2.

To facilitate a quantitative analysis, the mathematical relation between the vibration effect and variables was denoted as $F(v)$. 500 groups of $F(v)$ samples were statistically analyzed by sampling survey method in this paper, and it was found that $F(v)$ approximately obeys a normal distribution $N(0.45, 0.62)$, as shown in Fig. 1.

The probability density function of the pedestrian-induced footbridge vibration effect is:

$$ h(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}} $$  \hspace{1cm} (3)

The distribution function of the pedestrian-induced footbridge vibration effect is:

| Table 2 Factors affecting vibration effects |
|-------------------------------------------|
| **Factor Type**              | **Factors**                                      |
| Footbridge                  | Types, quality, stiffness, damping and time delay of footbridges |
| Pedestrians                 | Walking speed, walking pattern and pedestrian density |
In the above formulas, \( x \) is vibration acceleration response, \( h \) is the probability density function of vibration effect, \( H \) is the distribution function of vibration effect, and \( \mu = 0.45, \sigma = 0.6 \). The graph of the distribution function of the pedestrian-induced footbridge vibration effect is shown in Fig. 2.

### 2.3 Sensitivity model

For pedestrian-induced footbridge vibration, the sensitivity of pedestrians is defined as:

In the range of human body resistance, the subjective feelings of pedestrians to vibration effects are called sensitivity, denoted as \( S \).
As discussed above, the human body resistance $R$ is a constant, and the vibration effect $V$ is a random variable that obeys a normal distribution $N(0.45, 0.62)$. Therefore:

For a pedestrian, the sensitivity $S$ can be translated into the probability that the vibration effect exceeds the human body resistance value $s$:

$$S = P(V > s) = 1 - P(V \leq s) = 1 - H(s) = 1 - 0.665 \int_{-\infty}^{s} e^{-1.39(t-0.45)^2} dt$$  \hspace{1cm} (5)

In formula (5), $S$ is the sensitivity value, $V$ is vibration effect, and $H(s)$ is the distribution function of the pedestrian-induced footbridge vibration effect. The mathematical meaning of formula (5) is shown in Fig. 3. The diagram of sensitivity $S$ is shown in Fig. 4.

Contrastively, for pedestrians, the sensitivity $S$ under a certain vibration response can be calculated with formula (6):

$$S = \frac{\sum_{j=1}^{m} c_j n_j}{\sum_{j=1}^{m} n_j} = \sum_{j=1}^{m} c_j p$$  \hspace{1cm} (6)

In formula (6), $m$ expresses the number of different subjective feelings grades of pedestrians, $m = 5$ or $m = 11$ in general situations. In this paper, assuming $m = 5$, which denotes five grades: no vibration feeling (recorded as the first subjective feeling), minimal vibration feeling (recorded as the second subjective feeling), certain vibration feeling (recorded as the third subjective feeling), strong vibration feeling (recorded as the fourth subjective feeling), and intolerable vibration feeling (recorded as the fifth subjective feeling). $n_j$ expresses the number of pedestrians with subjective feeling grade $j$ ($j = 1,2,3,4,5$). $\sum_{j=1}^{m} n_j$ expresses the total pedestrian count under the certain vibration response. $c_j$ is the concept membership degree, which is usually determined by fuzzy statistical methods, and $c_j = (j - 1)/(m - 1)$. $p = n_j / \sum_{j=1}^{m} n_j$ reflects the difference between pedestrians' subjective feelings.

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**Fig. 3** The mathematical meaning of sensitivity $S$
From formula (6) and Fig. 4, it is visible that the value range of sensitivity \( S \) is \([0,1]\). To facilitate engineering applications, the sensitivity can be divided into five levels, as shown in Table 3. In Table 3, the classification is mainly due to the following reasons: for the vast majority of pedestrians, small vibration effects are acceptable. Drawing on the principle of international standards, we consider pedestrians to be particularly sensitive when the probability of a large vibration effect exceeds 75%. Therefore, 0.75 is used as an extremely sensitive boundary, and 0.1–0.75 is divided into three groups to determine the degree of sensitivity.

### 3 Experimental verification

#### 3.1 Agreement with international vibration comfort standards

A field survey was conducted on five different urban footbridges in Guangzhou at morning rush hour, \( n_j \) (the number of pedestrians with subjective feeling grade \( j \), \( j = 1, 2,3,4,5 \)) and \( \sum_{j=1}^{4} n_j \) (the total pedestrian count under a certain vibration response) were obtained by statistical analysis. Then the sensitivity values under vibration accelerations of \( r_l \) (the allowable vibration limit value adopted by international renowned vibration comfort standards, \( r_l = 0.35 \text{m/s}^2 \) in this paper), \( 2r_l \), and \( 4r_l \) can be calculated according to formula (6), as shown in Table 4.

| Table 3 | Sensitivity grade division |
|---------|--------------------------|
| Sensitivity | Sensitivity Grade | Comfort Level Score |
| <0.1 | Not sensitive | 10 |
| 0.1–0.25 | Common sensitive | 8 |
| 0.25–0.5 | More sensitive | 6 |
| 0.5–0.75 | Very sensitive | 4 |
| 0.75–1.0 | Extremely sensitive | 2 |
3.2 Field experiment
In order to verify the proposed theory in this paper, field experiments were conducted on the Fourth Corridor Footbridge in Guangzhou City. The Fourth Corridor Footbridge connects Creative Building and Lion Rock Park, with a two-span continuous half-through structural characteristic. The span combination is 64 + 63.2 m, as shown in Fig. 5.

In the field experiment, Creative Building (near the left span) is the starting point, Lion Rock Park (near the right span) is the ending point. A total of 100 testers passed through the Fourth Corridor Footbridge under 0.04 g (root mean square acceleration, or "r.m.s" for short) vibration response, 0.08 g (r.m.s) vibration response, and 0.12 g (r.m.s) vibration response. Photos of the field experiments are shown in Fig. 6. $n_j$ (the number of pedestrians with subjective feeling grade $j$, $j = 1, 2, 3, 4, 5$) under three different conditions were obtained by statistical analysis, then the sensitivity values can be calculated according to formula (6). These calculated sensitivity values can be regarded as experimental results.

Meanwhile, the vertical and lateral accelerometers were installed on the left and right spans at the positions of $L/4$, $L/2$ and $3L/4$, and an INV3018 portable data acquisition instrument was used to collect the vibration acceleration signals, as shown in Figs. 7 and 8. Frequency-weighted acceleration time history curve can be obtained by using overall frequency weighting method, due to space limitations, the frequency-weighted acceleration time history curve of vertical vibration on the left span at the position of $L/2$ are given in this paper, as shown in Figs. 9, 10 and 11. The sensitivity values can be

| Urban Footbridges                        | Time /Direction | Sensitivity Under $r_1$ | Sensitivity Under $2r_1$ | Sensitivity Under $4r_1$ |
|------------------------------------------|----------------|-------------------------|--------------------------|--------------------------|
| Gangding Footbridge                      | morning rush hour / vertical | 0.0601                 | 0.4887                   | 0.9022                   |
|                                          | morning rush hour /lateral     | 0.0732                 | 0.4989                   | 0.9099                   |
| Footbridge in University Town            | morning rush hour / vertical   | 0.0692                 | 0.4923                   | 0.8989                   |
|                                          | morning rush hour /lateral     | 0.0699                 | 0.4998                   | 0.9032                   |
| Footbridge in Guangzhou Avenue           | morning rush hour / vertical   | 0.0701                 | 0.4789                   | 0.8898                   |
|                                          | morning rush hour /lateral     | 0.0788                 | 0.4992                   | 0.9022                   |
| Footbridge in Wuyang City               | morning rush hour / vertical   | 0.0691                 | 0.4874                   | 0.8988                   |
|                                          | morning rush hour /lateral     | 0.0698                 | 0.4956                   | 0.9102                   |
| Guangzhou Railway Station Footbridge     | morning rush hour / vertical   | 0.0708                 | 0.4902                   | 0.8983                   |
|                                          | morning rush hour /lateral     | 0.0788                 | 0.4991                   | 0.9044                   |
calculated based on the frequency-weighted acceleration time history curve according to formula (5), these calculated sensitivity values can be regarded as calculated results. The comparisons between the experimental results and the calculated results are shown in Table 5, Table 5 indicates that the errors between the experimental results and the calculated results are within 6%, which satisfies with the engineering application. Sensitivity is related to vibration effect, which increases with the increase of...
Fig. 8 The vertical and lateral accelerometers

Fig. 9 Vertical frequency-weighted acceleration time history curve in condition one
Fig. 10 Vertical frequency-weighted acceleration time history curve in condition two

Fig. 11 Vertical frequency-weighted acceleration time history curve in condition three
acceleration response under the same pedestrian density. The main reason for the sensitivity errors is the interactive psychological influence, therefore, the experimental results are somewhat larger than the calculated results.

4 Discussion

Despite the development of the theory of pedestrian-induced footbridge vibration comfort in this article, there are still many challenges that need to be faced in future research, including the following:

(1) Challenges are still exist in accurately determining human body resistance, more detailed and comprehensive biological experimental research on the vibration endurance capacity of the human body are needed.

(2) There are many factors affecting the vibration effect, but there is no detailed research on how time delay factors affect the dynamic interaction between pedestrians and footbridges. Further research is needed to study the effect of time delay on the vibration mechanism of footbridges.

(3) The calculation efficiency of the sensitivity integral should be improved, and the sensitivity model should be extended to other engineering structural vibrations on the basis of improving the integral calculation efficiency.

(4) The sensitivity theory can be further studied from the point of probability theory, the sensitivity expectation and variance can be calculated under the continuous distribution, which lays the foundation for the design of pedestrian-induced vibration comfort.

5 Conclusion

Pedestrian-induced footbridge vibration leads to an uncomfortable and unsafe feeling. To evaluate the pedestrian-induced footbridge vibration comfort level, a sensitivity model based on the vibration endurance capacity of pedestrians and the vibration response of footbridges is proposed.

In this work, two uncertain and fuzzy concepts of the vibration endurance capacity of pedestrians (human body resistance) and the vibration response of footbridges (vibration effect) are defined, and the distribution function of the vibration effect is obtained. A sensitivity model is established in the field of pedestrian-induced footbridge vibration comfort. The mathematical definition, calculation method and classification standard for sensitivity are put forward, and a theory of vibration comfort is proposed from pedestrian's aspect. The verification results in Tables 4 and 5 indicate that the theory of pedestrian-induced footbridge vibration comfort is in good agreement with the international vibration comfort standards and the experimental results, the consistency demonstrates that this theory is reasonable. The proposed method can be used for vibration comfort level quantitative calculation.

### Table 5 Comparison between the experimental results and the calculated results

| Experiment Conditions | Experiment Results | Calculated Results | Error  |
|-----------------------|--------------------|--------------------|--------|
| 0.04 g (r.m.s)        | 0.131              | 0.1236             | −5.99% |
| 0.08 g (r.m.s)        | 0.522              | 0.508              | −2.76% |
| 0.12 g (r.m.s)        | 0.823              | 0.798              | −3.13% |
The aim of this study is to propose a theory for pedestrian-induced vibration comfort level calculation. A more detailed analysis that accounts for both the pedestrian-induced vibration mechanism and the vibration behavior of the pedestrian-footbridge coupled system is required.

Abbreviations

R.M.S.: Root mean square acceleration; ISO: International Standard Organization

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Authors’ contributions

Deyi Chen: Conceptualization, Formal analysis, Methodology, Writing original draft. Shiping Huang: Project administration, Review and Editing. Zhenyu Wang: Experiment, Data processing. All authors read and approved the final manuscript.

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Availability of data and materials

Supplementary data to this article can be received from the corresponding author on reasonable request.

Declaration

Competing interests

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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