Restriction on SUSY Masses from $Z \rightarrow$ hadrons

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Abstract

We remark that the precision of recent determinations of $\alpha_s(M_Z^2)$ is such that one can get bounds on supersymmetric partner masses (squark and gluino) by requiring consistency of determinations of $\alpha_s$ at “low” energies, where those particles do not contribute, and on the $Z$ peak. For approximately degenerate squarks and gluinos with mass $\tilde{m}$, we find the bound $\tilde{m} \geq 173$ GeV, at the 2$\sigma$ level. At the 2.5$\sigma$ level, $\tilde{m} \geq 121$ GeV.

1 Introduction

In the past years, determinations of the strong coupling at various energy scales have been performed with increasing degree of accuracy. In the table 1 we present a summary of those that have been pushed to the NNLO level, showing (as has become customary) the values of $\alpha_s$ extrapolated to the scale of the $Z$ mass.

| Process         | Average $Q^2$ or $Q^2$ range [GeV]$^2$ | $\alpha_s(M_Z^2)$     |
|-----------------|----------------------------------------|-----------------------|
| DIS $\nu$, Bj   | 1.58                                   | $0.121^{+0.005}_{-0.009}$ |
| DIS $\nu$, GLS  | 3                                      | $0.112 \pm 0.010$      |
| $\tau$ decays   | $(1.777)^2$                            | $0.1181 \pm 0.0031$   |
| $e^+e^- \rightarrow h$ | 100 – 1600                      | $0.128 \pm 0.025$      |
| $e^+e^- \rightarrow h$ | 2.5 – 230                      | $0.123 \pm 0.007$      |
| $(\nu N; xF_3)$ | 8 – 120                                | $0.1153 \pm 0.0041$   |
| $ep$            | 3.5 – 5000                             | $0.1163 \pm 0.0014$   |
| $Z$ width       | $(91.2)^2$                             | $0.1230 \pm 0.0038$   |
| GrandLEP, on $Z$| $(91.2)^2$                             | $0.1185 \pm 0.0030$   |

Table 1
We explain the entries in this table. DIS $\nu$ means deep inelastic scattering by neutrinos, Bj stands for the Bjorken, and GLS for the Gross–Llewellyn Smith sum rules. The value coming from $\tau$ decays is also included. The two entries $e^+e^- \rightarrow h$ refer to determinations from $e^+e^-$ annihilation to hadrons in PETRA and TRISTAN. All of these we have taken from the review of Bethke[1], where one can find references to original papers. The results labeled $\nu N$: $xF_3$ and $ep$ are from the very recent analysis of ref. 2, which improve (and supersede) those quoted by Bethke.

Finally, the figures for the last two processes recorded in Table 1,

$$\alpha_s^{\text{(Z width)}(M_Z^2)} = 0.1230 \pm 0.0038$$
$$\alpha_s^{\text{(GrandLEP, on Z)}(M_Z^2)} = 0.1185 \pm 0.0030,$$

we have taken the results of the LEP analyses, as updated in ref. 3, and corrected for the value $M_H = 115$ GeV for the Higgs mass. The first value corresponds to the determination from the $Z$ hadronic width (to be more precise, from the ratio $R_\ell = \Gamma_{\text{had}}/\Gamma_\ell$), and the second to that obtained with a fit to all LEP observables, on the $Z$ peak. We will however not use this last value here.

All determinations are compatible with one another, within errors. What is more, the precision of both the “low energy” ($Q^2 \lesssim (70)^2$ GeV$^2$) and the determination given by the decay $Z \rightarrow \text{hadrons}$ or $e^+e^- \rightarrow \text{hadrons}$ on the $Z$ are, separately, such that we can use them to constrain effects of heavy particles (particles with mass larger than the $Z$ mass). In particular, here we will explore bounds on squarks and gluinos, that for simplicity we take of the same order of magnitude: $m_{\tilde{q}} \sim m_{\tilde{g}} \sim \tilde{m}$.

The strategy is as follows. Clearly, the values of the masses of SUSY particles has no influence on the determinations of $\alpha_s$ presented in the table above, except on those based on $Z \rightarrow \text{hadrons}$, or $e^+e^- \rightarrow \text{hadrons}$ on the $Z$, where the energy is much higher than those of the other processes. If we then evaluate the value of $\alpha_s$ obtained by averaging all determinations, excluding those obtained on the $Z$ peak, we find

$$\alpha_s^{\text{(exc. Z peak)}(M_Z^2)} = 0.116835 \pm 0.00120.$$

The determination of $\alpha_s$ from $Z$ width showed in Table 1 was made fixing the mass of the Higgs particle to 115 GeV, and assuming no SUSY partners ($\tilde{m} = \infty$), and it is compatible with $\alpha_s^{\text{(exc. Z peak)}(M_Z^2)}$, within errors. To be precise, we note that the value $\alpha_s^{\text{(Z width)}(M_Z^2)}$ is slightly more than one sigma away from the average $\alpha_s^{\text{(exc. Z peak)}(M_Z^2)}$; it is actually almost 1.5$\sigma$ off. (In calculating this, we add in quadrature the errors in both $\alpha_s^{\text{(exc. Z peak)}(M_Z^2)}$ and $\alpha_s^{\text{(Z width)}(M_Z^2)}$ to get the effective error of $\pm 0.0040$). Likewise, the determination from $e^+e^- \rightarrow \text{hadrons}$ on the $Z$ is compatible with lower energy determinations (half a sigma away only). Actually, both determinations on the $Z$ produce results slightly higher than those found at lower energies. Now, when one calculates the corrections due to a Higgs particle, with a mass $M_H > 115$ GeV, or SUSY contributions with particles of mass $\tilde{m}$ (see below), they tend to increase the effective values of $\alpha_s$ on the $Z$ (for the Higgs, this occurs if increasing $M_H$). Therefore, if we now vary $M_H$ and/or $\tilde{m}$, the quantity $\alpha_s^{\text{(Z width)}(M_Z^2)}$ will vary, separating even more from the low energy determination of $\alpha_s$: this will provide us with lower bounds on the SUSY masses.$^1$

2 Bound on $\tilde{m}$

If squarks and gluinos existed with masses of the order of $\tilde{m}$, then they would contribute to the decay $Z \rightarrow \text{hadrons}$ through diagrams like those in Fig. 1. If we renormalize on the quark mass shell, then the contributions from the second diagram there cancel the divergence of the first. If we include SUSY effects, we then get the decay rate, to first order in $\alpha_s$, and neglecting quark masses,

$$\Gamma(Z \rightarrow \text{hadrons}) = \Gamma^{(0)} \left\{ 1 + [1 - K_{\text{SUSY}}] \frac{\alpha_s}{\pi} \right\},$$

$^1$ Upper bounds on the Higgs mass would also follow with the same procedure, but the results do not improve the bounds found with the standard method of fitting all observables; see e.g. ref. 3.
and we have

$$K_{\text{SUSY}} = \frac{1}{18} J \left( \frac{M_Z^2}{\tilde{m}^2} \right) \frac{M_Z^2}{\tilde{m}^2},$$

$$J(\xi) = \frac{12}{\xi} + \frac{24}{\xi^2} \int_0^1 dx \left[ 1 - \xi x (1 - x) \right] \log \left[ 1 - \xi x (1 - x) \right] \simeq 1 + \frac{1}{15} \xi.$$  \hspace{1cm} (3b)

$\Gamma^{(0)}$ is the partonic level decay width. We can interpret the value of $\alpha_s$ here as that obtained from lower energy determinations, that should not be contaminated by SUSY contributions; thus, we should take $\alpha_s(M_Z^2) = \alpha_s^{(\text{exc.} \ Z \text{ peak})}(M_Z^2)$, within allowed errors.

On the other hand, the experimental figures given from $Z$ width in Table 1 are obtained neglecting possible SUSY contributions, so we also have

$$\Gamma(Z \rightarrow \text{hadrons}) = \Gamma^{(0)} \left\{ 1 + \frac{\alpha_s^{(Z \text{ width})}(M_Z^2)}{\pi} \right\}. \hspace{1cm} (4)$$

Equating with the previous expression, we find

$$\alpha_s^{(Z \text{ width})}(M_Z^2) = \alpha_s^{(\text{exc.} \ Z \text{ peak})}(M_Z^2) \left\{ 1 - \frac{1}{18} J \left( \frac{M_Z^2}{\tilde{m}^2} \right) \frac{M_Z^2}{\tilde{m}^2} \right\}. \hspace{1cm} (5)$$

and a similar formula for $\alpha_s^{(\text{GrandLEP}, \text{on} \ Z \text{ peak})}(M_Z^2)$.

We may fix the mass of the Higgs particle to its experimental value/lower bound, $M_H = 115$ GeV as increasing it would only improve the bounds. Taking thus the value of $\alpha_s^{(Z \text{ width})}(M_Z^2)$ as in Eq. (1) above we find that, unless $\tilde{m}$ is larger than a certain bound, we have, to satisfy Eq. (5), to push $\alpha_s^{(\text{exc.} \ Z \text{ peak})}(M_Z^2)$ in Eq. (2) beyond its allowed limits of variation.

The bounds we obtain are then,

$$\tilde{m} \geq \begin{cases} 173 \text{ GeV}, & 2\sigma \\ 121 \text{ GeV}, & 2.5\sigma. \end{cases} \hspace{1cm} (6)$$

Note that, because the values of $\alpha_s^{(Z \text{ width})}(M_Z^2)$ and $\alpha_s^{(\text{exc.} \ Z \text{ peak})}(M_Z^2)$ differ by a little more than $1.5\sigma$, no bound can be given at the $1.5\sigma$ level.

These bounds are not very different from what one gets in studies with the Tevatron, \cite{4} but is perhaps more transparent and in particular independent on assumptions about decay properties of SUSY partners, and about SUSY GUT relations on masses and/or couplings\cite{4}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Diagrams involving SUSY partners contributing to the decay $Z \rightarrow \text{hadrons}$, and to renormalization of the wave function.}
\end{figure}
Acknowledgments

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References

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