The Test on the Short Period Mode of the Simulator With Time-Frequency Combination

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Abstract: Based on flight simulation data and measured data, the article extracted short cycle time of correlating parameters curve from a certain type of flight simulators. And in the time domain, using the average standardized distance test method and gray correlation analysis method for this type of simulator simulation data. In the frequency domain, the classical spectral estimation method is used for quantitative detection, and the consistency test of flight simulation data and flight parameters is realized. Then the evidence theory is improved and thus the synthesized type simulator short cycle when the correlating parameters and frequency domain the consistency of the test results, verify the longitudinal short-period mode type simulator fidelity is better.

1. Introduction

Flight simulation training is an important means to improve the wingmanship, carrying out dangerous training and ensure flight safety. In the verification work of flight simulator, flight quality is a very important verification content, which is closely related to the verisimilitude of flight simulator[1].

At present, the dynamic consistency test of simulation results of flight simulator is mostly verified by single time domain or frequency domain method, and the comprehensive verification method is seldom verified[2]. The Thile inequality coefficient method is a widely used dynamic data consistency test method, which has the advantages of simple principle and small calculation, while the TIC coefficient is related to the selection of the origin of coordinates, which can increase and decrease the TIC coefficient by changing the coordinate origin, so that the consistency analysis result can be misjudged[3]. However, the average standardized distance test method can analyze the sample data quantitatively, but there is also a case of inaccurate test[4].Grey relational analysis method is a common method for data consistency analysis in time domain without considering the statistical distribution law of sample sequence[5]. However, it is only a qualitative verification method, and the analysis results alone have certain problems. The classical spectrum estimation method is mature and fast, but because of the direct addition of "window" to the data, there may be problems such as energy leakage and separation of spectral lines[6].

According to the flight parameters and simulation data collected, six important parameters of the aircraft longitudinal short period mode are extracted. Based on the improved evidence theory, the comprehensive average standardized distance test method, grayscale correlation analysis method and classical spectral estimation method are obtained, and the data consistency test results are obtained. And flight parameters were compared according to the damping ratio and natural frequency of vibration with simulation data, the modal characteristics of further verification, test the longitudinal short-period mode type flight simulator fidelity is better.
2. Correlation theory

2.1. Time domain analysis methods

2.1.1. Average standardized distance test methods. Set \( X = \{x(1), x(2), \cdots, x(n)\} \) as \( n \) entries of sequence \( \{x(A)\} \). Set \( Y = \{y(1), y(2), \cdots, y(n)\} \) as the simulation sequence. \( Y_i = \{y_i(1), y_i(2), \cdots, y_i(n)\} \) is the \( i \)th sample, \( i = 1, 2, \cdots, m \).

The mean standardized distance between the measured sequence \( X \) and the sample mean of the simulation sequence \( \overline{Y} \) is \( D(X, \overline{Y}) \), as shown in equation (1),

\[
D(X, \overline{Y}) = \sqrt{\frac{1}{n} \sum_{A=1}^{n} \left( \frac{X(A) - \overline{Y}(A)}{S(A)} \right)^2}
\]  

(1)

In the formula, \( S = \frac{1}{m-1} \sum_{i=1}^{m} (Y_i - \overline{Y})(Y_i - \overline{Y})^T \), \( \overline{Y} = \frac{1}{m} \sum_{i=1}^{m} Y_i \).

If the average standardized distance \( D(X, \overline{Y}) \) between the sample \( X \) and its simulation sample mean \( \overline{Y} \) is satisfied with equation (2),

\[
D(X, \overline{Y}) \geq \sqrt{\frac{m^2-1}{m(m-n)} F_{\alpha,n,m-n}}
\]  

(2)

So, at the significant level \( \alpha \), the difference between the two sequences can be determined.

2.1.2 Grey relational analysis theory. Define the grey correlation coefficient of \( X_i \) to \( Y_i \) is,

\[
\xi_i(t) = \left\{ \begin{array}{ll}
1, & \frac{|x_i(t) - y_i(t)|}{\sigma} \leq C \\
\min_{i,d} \frac{|x_i(t) - y_i(t)|}{\sigma} + \sigma \max_{i,d} \frac{|x_i(t) - y_i(t)|}{\sigma}, & \text{else}
\end{array} \right.
\]  

(3)

In the formula, \( \sigma \) is resolution coefficient, \( C \) is constant, when \( \sigma \) is smaller, the resolution is higher. In actual engineering applications, \( \sigma \in [0,0.5] \) is usually taken.

The grey correlation between \( X_i \) and \( Y_i \) is,

\[
\gamma_i = \frac{1}{n} \sum_{t=1}^{n} \xi_i(t)
\]  

(4)

The larger the grey relational degree, the higher are there the correlation between the two groups and the higher the consistency of the data.

2.2. Frequency domain analysis methods

The classical spectrum analysis method is used to estimate model validation is to estimate power spectral density of random sequence. Here suppose that \( x_i \) and \( y_i \) are samples of the output sequence of the simulation model and the samples of the flight parameters. \( S_x(\omega) \) and \( S_y(\omega) \) are the corresponding self-spectral density functions respectively, and the spectral quotient function of \( x_i \) to \( y_i \) is defined as
So for the given significance level $\alpha$, the $Q_\alpha(\omega)$ confidence interval of the spectrum quotient function $1-\alpha$ can be calculated by the following formula,

$$L(\omega) = \frac{Q_\alpha(\omega)}{F_{\chi^2}(v, v)} \leq Q_\alpha(\omega) \leq \frac{Q_\alpha(\omega)}{F_{\chi^2}(v, v)} = U(\omega), 0 \leq \omega \leq \pi$$  \hspace{1cm}(5)$$

If the confidence interval contains a value of 1, it is considered that the simulation output sequence and flight test results are consistent with the significant level of $\alpha$.

### 2.3. Evidence theory

In the evidence theory, Sample space $\Theta$ consists of a series of two or two repelling each object, and includes current to identify all the objects, namely $\Theta = \{\Theta_1, \Theta_2, \cdots, \Theta_n\}$. Make $\Theta$ domain set as a theory, $2^{\Theta}$ are all the subsets of $\Theta$ collection, if $m: 2^{\Theta}$ as the basic probability distribution function, it meet the formula[7],

$$\sum_{A \in P(\Theta)} m(A) = 1, \quad m(\phi) = 0$$  \hspace{1cm}(6)$$

In the formula, $m(A)$ is called the basic probability assignment of event $A$, indicating the degree of support for proposition $A$. The D-S combination rule is,

$$\begin{cases} m(A) = \frac{1}{1-k} \sum_{A_i \cap B_i = A} m_i(A_i)m_i(B_i) \text{ for } \forall A \neq \phi \\ m(\phi) = 0 \end{cases}$$  \hspace{1cm}(7)$$

In the formul(7), $k$ is the conflict factor, which reflects the degree of conflict between the evidence.

$$k = \sum_{A_i \cap B_i = \phi} m_i(A_i)m_i(B_i)$$  \hspace{1cm}(8)$$

And the closer the $k$ value is to 1, the greater the conflict between the evidence. The closer to 0, the smaller the conflict.

### 2.4. Improved evidence theory

Classic theory of D-S combination contradiction and robustness of the problems, so when the serious conflict between evidence or a proposition of tiny change of basic probability distribution, may cause the results of synthetic radical changes[8]. The improvement steps are as follows:

1) The average probability value of the $n$ group of evidence of proposition $A_k$ is assigned to $\overline{m}(A_k)$ to determine the average support degree of all evidence for the proposition $A_k$.

2) The distance $d_i$ between the probability distribution and the average probability distribution of the $i$th evidence body is determined one by one. The formula is as follows,

$$d_i = \sum_{k=1}^{m} | m_i(A_k) - \overline{m}(A_k) |$$  \hspace{1cm}(9)$$

3) According to the distance $d_i$ to determine the weight $\omega_i$ of each body of evidence, the formula is as follows,

$$\omega_i = \frac{d_i}{\sum \omega_i}$$  \hspace{1cm}(10)$$
4) Identify conflicting evidence. When the weight of the conflicting evidence $i \omega_i < 1/n$ (that is, less than the average weight of each evidence), the modified coefficient is calculated as,

$$\text{discount} = \frac{\omega_i}{\max(\omega_i)} \quad (\text{discount} < 1)$$

(11)

3. Verification and analysis

3.1. Time curve extraction

Selecting a group to test the model of the longitudinal characteristics of longitudinal times pulse comparison between the control input validation, the initial height is 5 km, the speed is 0.4 Ma, flying parameters according to the comparison of the data of simulation results as shown in figure 1, solid line for flight parameters according to the curve, dotted lines for simulation data parameters.

Figure 1. Parameter time domain change curve
3.2. The time domain authentication

3.2.1. Average standardized distance test method validation. Based on the flight parameters and simulation data, the parameters of the short-period mode are tested in the time domain by means of mean standardized distance test.

According to figure 1, the longitudinal short period mode can be completed within 3-8 seconds, so the flight data in 3-8 seconds should be analyzed in consistency. Take the Angle of attack as an example. Under the condition of 5km height and 0.4Ma, a group of flight parameters were obtained with the simulation data of multiple groups, and the average normalized distance between the actual sequence of attack and the simulation sequence of attack Angle was,

$$D(X, \bar{Y}) = \sqrt{\frac{1}{n} \sum_{A=1}^{n} \left[ \frac{X(A) - \bar{Y}(A)}{S(A)} \right]^2} = 2.3575$$

When the significance level is 0.08, its rejection area is $D(X, \bar{Y}) \geq 2.4069$. It can be seen from this that, with the significance level of 0.08, the consistency between the flying parameter sequence and the simulation sequence is very good.

3.2.2. Grey correlation analysis method validation. The flight parameters and simulation data obtained under the condition of height 5km, speed 0.4Ma was taken into equation (3), equation (4), and if taken the resolution coefficient 0.5, the grey correlation is 0.75.

For the consistency result of qualitative method, fuzzy set theory is adopted to define the acceptable criterion. The consistency of data was divided into five levels: very high (1.0~0.9), high (0.9~0.7), general (0.7~0.5), poor (0.5~0.3) and very poor (0.3~0.0). Therefore, it can be seen that the consistency degree of the two sequences is high.

3.3. Frequency domain authentication

The data length is divided into 8 segments and 50% overlap with the sampling frequency for 5km height. In this case, Welch's method is used, and the window function is the hanning window. According to Welch's method, the estimated power spectrum is shown in figure 2,

**Figure 2.** Angle of attack data power spectrum curve
Figure 3. Confidence interval

The test formula is related to $\alpha$, and take $\alpha = 0.01$ here. The distribution table shows that, $F_{\alpha/2}(15, 15) = 4.07$, $F_{1-\alpha/2}(15, 15) = 0.25$.

And then you can compute the lower bound at the confidence interval at each frequency point. With the frequency as the x-coordinate and the lower and lower limit of the confidence interval as the vertical coordinate, the confidence belt curve is shown in figure 3.

As can be seen from the diagram, 1 is contained within the confidence upper limit and the confidence limit. Therefore, the power spectrum of the simulation sequence and the measured sequence is consistent with the 99% reliability level.

3.4. Integrated test results

Using the average standardized distance test method, grey correlation analysis method and classical spectrum estimation method, the consistency of the longitudinal short-period damping ratio of the simulator is tested in the time domain and frequency domain. The obtained consistency result is used as the basic probability distribution function of the evidence, and the fusion result is calculated according to the improved d-s method. The basic probability distribution is shown in table 1 below.

Table 1. Basic probability distribution table of evidence

|    | A   | B   | C   |
|----|-----|-----|-----|
| $m_1$ | 0.92 | 0   | 0.08 |
| $m_2$ | 0   | 0.75 | 0.25 |
| $m_3$ | 0.005 | 0.005 | 0.99 |

Then, $\overline{m}(A) = \frac{1}{3} (0.92 + 0 + 0.005) = 0.31$, $\overline{m}(B) = 0.25$, $\overline{m}(C) = 0.44$

Plug in formula (9),

$d_1 = |0.92 - 0.31| + |0 - 0.25| + |0.08 - 0.44| = 1.22$,  $d_2 = 1$,  $d_3 = 1.1$

$\psi_1 = \frac{1}{d_1} = 0.82$,  $\psi_2 = 1$,  $\psi_3 = 0.91$,  $\omega_1 = \frac{\psi_1}{\psi_1 + \psi_2 + \psi_3} = 0.30$,  $\omega_2 = 0.37$,  $\omega_3 = 0.33$

Since there are three pieces of evidence, the average weight of each evidence is 1/3. So Evidence 1 is calculated to be corrected of the the discount degree.

According to formula(11), $discount = 0.3 / 0.37 = 0.82$. Therefore, the dynamic consistency degree of the simulation results at 5km height is 0.83.

In the same way, the consistency degree of the simulation results of the other five parameters such
as the pitch angle and the flight parameters can be obtained, as shown in table 2.

| Parameter | Pitching Angle | Pitch rate | Speed | Height | Overload |
|-----------|---------------|------------|-------|--------|----------|
| Consistency | 0.82          | 0.88       | 0.75  | 0.71   | 0.85     |

It can be seen that the consistency degree of each parameter flight parameter according to the sequence and simulation sequence is higher than 0.7. The short period mode of the simulator is consistent with the actual aircraft well.

4. Conclusion
In this paper, a dynamic consistency test method for simulation data of time domain and frequency domain is presented, and the longitudinal short period mode of flight simulator of a certain aircraft is verified. This paper firstly extracted the short cycle time history curve of modal parameters, and then applied to the average standardized distance test method, grey correlation analysis method and the classic spectrum estimation three methods, time domain and frequency domain of the longitudinal short-period type simulator the inspection of the consistency of related parameters, and the evidence theory is improved and synthesized consistency inspection results, obtained the comprehensive results of dynamic simulation data consistency check type simulator, this type of flight simulator was verified longitudinal short-period mode has good fidelity.

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