RCS and Aero Surfaces Control Allocation Research on RLV’s Re-Entry Phase

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Abstract: Based on the characteristics of aero surfaces and reaction control system (RCS), a control allocation scheme is needed to specify aero surface deflections and RCS thruster’s firing commands, during the re-entry phase of reusable launch vehicle (RLV). To analyze the control performance differences introduced by different allocation algorithms, this paper presents a proportional allocation scheme based on dynamic pressure, a daisy-chaining allocation scheme based on the control error, and a multi-object-based optimal allocation scheme, to deal with the allocation problem of RCS and aero surfaces. The first allocation scheme is the simplest, and the second one is more complicated, when considering the actuators’ physical constraints in the allocation problem. The third scheme not only takes the physical constraints into account, but also the rate constraints, and other constraints of RCS and aero surfaces, while for the objective optimization function, not like the single-object function of the other two schemes, it combines the allocation error minimization, fuel consumption minimization, and other functions, with the adaptive weight values. After the stability analysis, the feasibility of the aforementioned allocation schemes are verified by the simulation, and the contrastive analysis of their control performances are followed.

Keywords: reusable launch vehicle; control allocation; reaction control system; re-entry phase

1. Introduction

At the beginning of the unpowered re-entry phase, the aero surface (AS) could not meet the attitude control system’s requirements of the reusable launch vehicle (RLV), due to the thin air and low dynamic pressure [1], which led the reaction control system (RCS) to be the dominant control actuator. With the increase of dynamic pressure, the control efficiency of AS rises, while the control efficiency of RCS, in contrast, drops. Therefore, the actuators were switched in the order of RCS only, RCS and AS controlled together, and AS only [2]. In order to maintain the stable and accurate attitude control, the control allocation algorithm of RCS and AS should be studied [3,4].

At present, there are mainly five ways to deal with the hybrid control problem of RCS and AS [5]: (1) flight command requirements based allocation [6], such as the proportional allocation method, which has a simple logic, and good tracking performance, as proved by the simulation; (2) control variables based allocation [7], such as the daisy-chaining allocation method [8]; (3) sampling control, which has the advantage of dealing with the actuators’ discrete control output and maintaining the stability of the control system; (4) multi-input control, in which the allocation is designed into the controller design part; (5) intelligent control, which refers to dealing the hybrid control problem with the fuzzy control [9], neural network, genetic algorithm [10], and other intelligent methods [11]. Each method has its pros and cons. For example, the intelligent control has good performance, but is hard to evaluate the stability [12]. Even though, the hybrid control technology has made some progress
until now [13], for the most part, RCS is normally assumed to be continuous [14], and the stability analysis of the whole control system is seldom studied [15].

This paper mainly focuses on the RCS and aero surface hybrid control allocation research on the RLV’s re-entry phase. The dynamic model of RCS, AS, and RLV, which forms the basis of the RLV re-entry phase, has been discussed in Section 2. In the next section, the proportional allocation scheme, based on dynamic pressure, daisy-chaining allocation scheme based on allocation errors, and a multi-object-based optimal allocation scheme, are presented, to deal with the hybrid control allocation problem, in turn. Then, the stability of the control system has been analyzed on the basis of the Lyapunov theory. Finally, Section 4 verifies the feasibility of the aforementioned allocation schemes by simulations, following their contrastive analysis.

2. Mathematical Model

2.1. Nonlinear Model of RCS and AS

As known, the control output generated by RCS is obtained from the reaction force of the thrusters’ fuel consumption, and the force arm is from the reaction force point to the mass center of RLV. Assuming that the number of RCS’s thrusters is N, the mathematical model of RCS in the body coordinate is:

\[
F_{RCS} = \sum_{i=1}^{N} (v_i \cdot F_i) = \sum_{i=1}^{N} v_i \cdot \left[ \begin{array}{c} 0 \\ F_i \cos \varepsilon_i \\ F_i \sin \varepsilon_i \end{array} \right], \quad M_{RCS} = \sum_{i=1}^{N} (v_i \cdot F_{Ri} \times b_i)
\]  (1)

where the firing command \( v_i = [0,1] \)(i = 1, 2, ..., N) means the \( i \)th thruster of RCS only has ‘on’ and ‘off’ two conditions; \( F_i \) is the constant force generated by the \( i \)th thruster; \( \varepsilon_i \) is the deflection angle of the \( i \)th thruster; \( b_i \) is the force arm vector of the \( i \)th thruster in the body coordinate. Based on Equation (1), the output of RCS is discrete and non-smooth, and not affected by the external factors. Since RCS consumes the fuel, which is limited and hard to refill while working, it makes RCS suitable for any flight phase, but in the short term.

In this paper, aero surfaces of RLV are the wing-body combination, which includes the elevator, body flap, rudder, and aileron. Normally, the body flap and elevator can be used for the pitch control; the aileron and rudder are corresponding to the roll and yaw control, respectively. Therefore, the allocation problem for the aero surfaces is between the body flap and the elevator. Additionally, the corresponding expressions in the body coordinate are shown as follows:

\[
R = qS \left[ \begin{array}{c} C_{x0} + C_{x\delta_d} \delta_d \delta_r \\ C_{y0} + C_{y\alpha} \alpha + C_{y\delta_d} \delta_d \delta_r + C_{y\delta_d \delta_r} \delta_d \delta_r \\ C_{z0} + C_{z\beta} \beta + C_{z\delta_d \delta_r} \delta_d \delta_r \end{array} \right], \quad M_R = qS \left[ \begin{array}{c} m_x \\ m_y \\ m_z \end{array} \right]
\]  (2)

where the dynamic pressure is \( q = \rho V^2/2 \), wherein \( \rho \) is the air density; \( V \) is the velocity; \( S \) is the reference area; \( l \) is the reference length; \( \alpha \) is the attack angle, \( \beta \) is the sideslip angle; \( \delta_d, \delta_r, \delta_d \delta_r, \delta_d \delta_f \) are the deflection angle of the aileron, rudder, elevator, and body flap, respectively; \( C_{x0}, C_{y0}, C_{z0} \) are the zero-lift drag, lift, and lateral force coefficients; \( m_x, m_y, m_z \) are the aerodynamic moment coefficients [16]. Based on Equation (2), different from RCS, the aerodynamic moments are continuous and closely related to the current dynamic pressure, which means despite other aerodynamic factors, the bigger the dynamic pressure, the bigger the aerodynamic output. Thus, it leads to the fact that AS is only applicable in the atmosphere.
2.2. Dynamic Model of RLV

The 6-DOF dynamic equations of the rigid-body RLV are made up of the 3-DOF translational equations and the 3-DOF rotational equations. The 3-DOF translational equations in the launch inertial coordinate system, during the re-entry phase, are given as follows

\[
\begin{bmatrix}
\dot{V} \\
V \dot{\theta} \cos \psi_v \\
-\dot{V} \psi_v
\end{bmatrix} = \mathbf{C}^{hv}_n \begin{bmatrix}
0 \\
-mg \\
0
\end{bmatrix} + \mathbf{C}^{hv}_v \mathbf{R} + \mathbf{F}_{\text{RCS}}
\]  

(3)

where \( m \) is the mass of RLV; \( \theta \) is the flight-path angle; \( \psi_v \) is the flight deflection angle; \( \gamma_v \) is the velocity bank angle; \( \mathbf{C}^{hv}_n \) is the transformation matrix from the launch inertial to the ballistic coordinate system; and \( \mathbf{C}^{hv}_v \) is the transformation matrix from the velocity coordinate system to the ballistic coordinate system. The specific expansion of \( \mathbf{C}^{hv}_n, \mathbf{C}^{hv}_v \) are described as

\[
\mathbf{C}^{hv}_n = \begin{bmatrix}
\cos \theta \cos \psi_v & \sin \theta \cos \psi_v & -\sin \psi_v \\
-\sin \theta & \cos \theta & 0 \\
\cos \theta \sin \psi_v & \sin \theta \sin \psi_v & \cos \psi_v
\end{bmatrix}, \quad \mathbf{C}^{hv}_v = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \gamma_v & -\sin \gamma_v \\
0 & \sin \gamma_v & \cos \gamma_v
\end{bmatrix}
\]  

(4)

The 3-DOF rotational equations used for RLV’s attitude control, during the re-entry phase, are built in the body coordinate system, and expressed as

\[
\begin{bmatrix}
\dot{\gamma} \\
\dot{\psi} \\
\dot{\varphi}
\end{bmatrix} = \begin{bmatrix}
1 & \sin \gamma \tan \psi & \cos \gamma \tan \psi \\
0 & \cos \gamma & -\sin \gamma \\
0 & \sin \gamma \cos \psi & \cos \gamma \cos \psi
\end{bmatrix} \begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}, \quad \begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix} = -I_0^{-1} \Omega I_0^{-1} \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} + I_0^{-1}(\mathbf{M_R} + \mathbf{M}_{\text{RCS}})
\]  

(5)

where \( \gamma, \psi, \varphi \) are the roll, yaw, and pitch angle, respectively; \( \omega_x, \omega_y, \omega_z \) are the roll, yaw, and pitch angular velocity of RLV’s body coordinate rotation from the launch inertial coordinate, respectively; \( I_0 \) are the moment matrix of inertia. The expansions of \( \Omega \) is

\[
\Omega = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}
\]  

(6)

According to the aforementioned equations, the dynamic model of RLV can be written as

\[
\dot{x} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}
\]  

(7)

where the state vector is defined as \( \mathbf{x} = [\gamma, \psi, \varphi, \omega_x, \omega_y, \omega_z]^T \); the control input vector is defined as \( \mathbf{u} = [v_1, v_2, \ldots, v_N, \delta_{da}, \delta_{dr}, \delta_{dc}, \delta_{dbf}] \); the system matrix \( \mathbf{A}, \mathbf{B} \) can be derived by Equations (3) and (5).

3. Control Allocation Scheme Design

The control allocation scheme is mainly about how to allocate the virtual control command to RCS and AS, namely to decide RCS’s thrusters firing command, and the deflection angles of each aero surfaces so that the real control command is as close as possible to the virtual one provided by the controller. In this section, there are three kinds of allocation schemes proposed, namely proportional allocation, daisy-chaining allocation, and multi-object based optimal allocation. In the proportional and daisy-chaining allocation schemes, there are two steps—first, use the corresponding algorithm to allocate between the RCS and the AS; during the secondary allocation, 0–1 integer programming
algorithm is proposed for the firing command allocation among RCS’s thrusters, and the daisy-chaining algorithm is used for the body flap and the elevator. Unlike these two schemes, the multi-object allocation algorithm can deal with the RCS’s thrusters and aero surfaces at the same time, while achieving the optimal comprehensive performance, by choosing the optimal solution to the multiple objective functions.

3.1. Proportional Allocation

This proportional allocation scheme designed, based on the dynamic pressure, is described as

\[
\begin{align*}
\bar{M}_{RCS} &= k_x M_c \\
\bar{\delta} &= (1 - k_x) \delta_c
\end{align*}
\]  

(8)

where \(M_c\) and \(\delta_c\) are the virtual moment command of RCS and the deflection angle command of AS, respectively; \(k_x\) is the proportional coefficient and 
\(k_x = \left(q_{d}\right) / \left(q_{d1}\right)\), in which 
\(q_{d}\) and \(q_{d1}\) are the dynamic pressure values of the hybrid control system at the start and the final moment, respectively; \(M_{RCS}\) and \(\bar{\delta}\) are the virtual commands of RCS and AS after proportional allocation, respectively; \(M_{RCS}\) and \(\delta\) are the real control outputs of RCS and AS. As shown in the allocation structure in Figure 1, the obvious advantage of this proportional scheme is avoiding the math problem of heterogeneous actuators’ allocation, whereas, each kind of actuator will inevitably need the one-to-one correspondence controller.

\[\begin{align*}
\alpha, \psi, \gamma &\rightarrow \text{Controller} \\
\text{Hybrid Control} &\rightarrow \text{0-1 Integer Programming} \\
\text{RCS} &\rightarrow M_{RCS} \\
\text{AS} &\rightarrow M_{AS} \\
\text{Dynamic Model} &\rightarrow \omega
\end{align*}\]

Figure 1. The structure diagram of the proportional allocation scheme.

For the secondary allocation, the 0–1 integer programming problem for RCS thrusters is [17]:

\[
\min_{w_M, \Delta M} \left[ w_1, w_2, \ldots, w_N \right] \left[ v_i \right] \left[ \Delta M \right] \quad \forall \left| \delta \right| \geq |e_{off}|, s.t. \sum_{i=1}^{N} |M_i | \geq |M_c|
\]  

(9)

where \(\Delta M = \left| \bar{M}_{RCS} - M_{RCS} \right| = \left| \bar{M}_{RCS} - \sum_{i=1}^{N} M_i v_i \right|\) is the allocation error between the virtual command and actual command; \(w_M\) is the weight of \(\Delta M\); \(w_i\) is the priority coefficient of the \(i^{th}\) thruster, and the bigger \(w_i\) indicates its lower priority; \([-e_{off}, e_{off}]\) is defined as the control variables’ dead zone, in which the control error is allowed and RCS stops working [18]. By solving Equation (9), the optimal solution of \(v_i\) can be achieved, which is the thrusters’ firing commands to realize the minimum of allocation error and fuel consumption. Moreover, since only the pitch channel has the redundant aero surfaces, the daisy-chaining allocation problem between AS is too simple to go into details.

This proportional allocation scheme has a simple structure and a clear logic, but it does not consider the physical constraints while allocating between the RCS and the AS. Therefore, for the limited deflection angle and rate of AS, if the virtual command after allocation is beyond its scope, it might lead to a poor control performance.
3.2. Daisy-Chaining Allocation

The daisy-chaining allocation scheme is to divide the actuators into groups with different priorities. When the current group in use cannot meet the requirements, the next group with lower priority will be enabled. Figure 2 is the structure diagram of this daisy-chaining allocation scheme.

![Figure 2. The structure diagram of the daisy-chaining allocation scheme.](image)

Since the output of the RCS is discrete, in most cases it means \( M_{RCS} \neq \overline{M}_{RCS} \). Considering AS can compensate the control allocation error caused from RCS’s discrete working mode, this paper prefers the group of RCS with the higher priority. Thus, the daisy-chaining allocation function is

\[
\begin{align*}
M_{RCS} &= M_i, \quad M_R(\overline{\delta}) = 0 \quad (M_i < k_m M_{\text{max}}^{\text{RCS}}) \\
M_{RCS} &= M_{\text{max}}^{\text{RCS}}, \quad M_R(\overline{\delta}) = M_c - M_{\text{max}}^{\text{RCS}} \quad (M_i > k_m M_{\text{max}}^{\text{RCS}})
\end{align*}
\]

where \( M_R(\overline{\delta}) \) is the nonlinear fitting function given by Equation (2) [19], \( M_{\text{max}}^{\text{RCS}} \) is the maximum moment of RCS, \( k_m \) is the safety coefficient, which is normally defined as \( 0 < k_m < 1 \).

Compared to the first scheme, this scheme also has simple structure and does not need more than one controller for the heterogeneous actuators, but introduces the physical constraints into the allocation, which makes its computation more complex. However, the solution for this scheme is not the optimal, since the control efficiency, rate constraint and other constraints of each actuator are not considered. Moreover, with a prioritized RCS, the allocation scheme will lead to more fuel consumption than the proportional one.

3.3. Multi-Object-Based Optimal Allocation

In order to solve the allocation problem with regards to allocation error minimization, fuel consumption minimization, control efficiency maximization, and other objective functions [20], an optimization allocation method is presented. The optimized objective function is weighted, and then combined with all the necessary objective functions, through which the multi-object problem is transformed into a single-object optimization problem. Moreover, the weight values are designed to be time-varying, to adapt for the flight requirements and the control allocation feedback. Therefore, the multi-objective optimization problem is given as follows:

\[
\begin{align*}
\min_{\mathbf{v} \in \mathbb{U}} & \quad w_v ||\mathbf{e}| |_1 + w_e ||\mathbf{e} - \mathbf{e}_{-1}| |_1 + \sum_{i=1}^{N} w_{v_i} ||\mathbf{d}_i - \mathbf{d}_{-i}| |_1 + w_p ||\mathbf{e}_{p} / q| |_1 + \cdots \\
\text{s.t.} & \quad \begin{cases} 
T_i \leq T_i \leq \overline{T} \\
\int_{T_RCS} \sum_{i=1}^{N} (v_i) \, dt \leq M_{RCS} = \sum_{i=1}^{N} (v_i)_{\text{on/off}} \leq N_{\text{on/off}} \\
\delta_i \leq \delta_i \leq \overline{\delta} & \quad i = N + 1, N + 2, \cdots, m
\end{cases}
\end{align*}
\]

where \( \mathbf{e} = M_R + M_{RCS} - M_c \) is the control allocation error, \( \mathbf{e}_{-1} \) is the allocation error from the last simulation step. The variable \( \delta_{p_i} (i = 1, \cdots, m - N) \) denotes the baseline of the deflection angle of the corresponding aero surface, and \( w_v, w_e, w_i (i = 1, 2, \cdots, N), w_{\theta i} (i = 1, \cdots, m - N), w_p \) stand for the weight coefficient of the corresponding single-object optimization function, which can improve its priority by increasing its weight coefficient. \( T_i \) is the current working duration of the \( i^{th} \) thruster,
\( T_i, T_I \) are the lower and upper limits of the \( i^{th} \) thruster’s working duration; \( \Delta T_i \) is the current ‘ON/OFF’ switching interval of the \( i^{th} \) thruster, and \( \Delta T_I, \Delta T_I \) are the lower and upper limits of the \( i^{th} \) thruster’s switching interval; \( T_{RCS} \) is the total working time of RCS, \( \dot{m}_{RCS} \) is the fuel consumption per second of the single thruster, and \( M_{RCS} \) is the total fuel mass of RCS; \( (v_i)_{on/off} \) is the switching times of the \( i^{th} \) thruster, and \( N_{on/off} \) is the maximum switching times of RCS. \( \delta_0, \delta_1 \) are the lower and upper limits of the corresponding aero surface’s deflection angle; \( \delta_0, \delta_1 \) are the lower and upper limits of the corresponding aero surface’s deflection angular rate. Figure 3 presents the structure diagram of this allocation scheme.

![Figure 3. The structure diagram of the multi-object based optimal allocation scheme.](image)

Obviously, this control allocation scheme takes the dynamic characteristics and necessary constraints into account, and considers the allocation error as the feedback to the multi-object based algorithm, which can effectively improve the accuracy and adaptivity of the control allocation.

3.4. Stability Analysis

After designing the control allocation scheme, the stability of the control system should be analyzed. Assuming the stability of the system with only the controller is guaranteed, the variable of the control system is the addition of the control allocation part. Therefore, in order to prove the stability of the whole control system, it only needs to prove that the difference of the control output \( u \) which is introduced by the allocation, will not affect the stability of the system. The difference between the actual output and the virtual output, in this paper, can be taken as a perturbation and the actual output is denoted as \( \Phi(u) \). The stability criteria of the hybrid control system can be obtained according to the stability theory of Lyapunov.

**Theorem 1.** For System (7), assuming it is a closed-loop system with the control law being locally asymptotically stable around the neighborhood, \( \varphi \) being the feature point \( x_0 \), if the feedback control output is \( \Phi(u) \) instead of \( u \), then the following conditions can be met:

\[
||\Phi(u) - u|| \leq k||x||\|u\|, 0 < k < \infty
\]

(12)

The new closed-loop system can maintain the local asymptotic stability.

In Equation (12), the symbol \( ||*|| \) represents the vector’s norm.

**Proof.** As known, \( A, B \) of System (8) are continuous around the neighborhood \( \varphi \) of the feature point \( x_0 \), and the control law \( u = -Kx \) can be denoted as \( K = K_0 + \Delta K \). Similarly, \( A = A_0 + \Delta A, B = B_0 + \Delta B \), where the constant matrix \( A_0, B_0, K_0 \) are the values at the feature point \( x_0 \). Obviously, the incremental matrix at \( x_0 \) satisfy that \( \Delta K = 0, \Delta A = 0, \Delta B = 0 \). With the feedback control output \( \Phi(u) \), System (7) can be described as

\[
\dot{x} = A(x)x + B(x)\Phi(u) = A(x)x + B(x)(\Phi(u) - u) + B(x)u
\]

(13)

Substitute the incremental matrix to the above equation, then
\[ \dot{x} = (A_0 + \Delta A(x))x + B(x)(\Phi(u) - u) - (B_0 + \Delta B(x))(K_0 + \Delta K(x))x = (A_0 - B_0 K_0) x + B(x)(\Phi(u) - u) + (\Delta A(x) - B_0 \Delta K(x) - \Delta B(x) K_0) x \]  

(14)

Denote that

\[ A_{cl0} = A_0 - B_0 K_0, h_1(x) = B(x)(\Phi(u) - u), h_2(x) = (\Delta A(x) - B_0 \Delta K(x) - \Delta B(x) K_0) x \]  

(15)

Then

\[ \dot{x} = A_{cl0} x + h_1(x) + h_2(x) \]  

(16)

According to Equation (12),

\[ ||h_1(x)|| \leq k ||B(x)|| ||K(x)|| ||x||^2 \]
\[ ||h_2(x)|| \leq (||\Delta A(x)|| + ||B_0|| ||\Delta K(x)|| + ||\Delta B(x)|| ||K_0||) ||x|| \]

(17)

Therefore,

\[ \lim_{x \to x_0} \frac{||h_1(x - x_0) + h_2(x - x_0)||}{||x - x_0||} = 0 \]  

(18)

Thus, in the neighborhood \( \phi \), when \( x \to x_0 \), the stability of the closed-loop system will be determined by the constant matrix \( A_{cl0} \). So assuming that the closed-loop system under the control law \( u = -Kx \) is locally asymptotically stable, the new closed-loop system with the control output \( \Phi(u) \) can still maintain the local asymptotic stability. □

4. Simulation Results

This section presents the simulation results and comparison analysis of the aforementioned schemes. The initial simulation conditions are set as \( V = 137.5 \text{m/s}, [\varphi, \psi, \gamma] = [210, 0, 180]^{(\circ)} \).

To be brief, the proportional allocation scheme is denoted as case 1, the daisy-chaining allocation scheme is case 2, while the multi-object allocation scheme is case 3. According to the comparison between the nominal data and the three allocation scheme cases in Figure 4, Figure 5, and Figure 6, it obviously shows that they are quite close to the nominal data, which on the one hand, demonstrates that the control allocation schemes of these three cases can accurately track the control command. On the other hand, it also verifies the effectiveness and robustness of these three allocation algorithms.

![Figure 4. The flight trajectories of the reusable launch vehicle (RLV).](image-url)
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Figure 5. The flight velocities of the RLV.

Figure 6. The attitude angles of the RLV.
Furthermore, according to the control errors comparison of Figure 7, case 3 had the highest control precision. In 410 s, the actuators were switched from RCS and AS to AS only, because the dynamic pressure and the control efficiency of AS were big enough for the attitude control requirements. However, the switching smoothness of case 2, especially in the pitch channel, was the worst, which indicated that the daisy-chaining allocation scheme could not perform well during the switch. With respect to the deflection angles’ smoothness (shown in Figure 8), case 2 was the worst, while case 1 was the best. The reason is that, for the proportional allocation scheme, RCS had the exclusive, corresponding controller, and its control command tracking and allocation was independent from the aero surfaces. Moreover, as shown in Figures 9 and 10, the RCS fuel consumption of case 3 was the least, and case 2 was the most. This is because, in case 2, the control command error minimization was the first priority and AS was only used when RCS could not meet the control command requirements, which made the fuel consumption the greatest. In case 3, both the control command error minimization and the fuel consumption minimization were considered in the optimal objective function, and by choosing the proper weight values for these minimization objects, the actuators could be utilized with maximum control efficiency, while reducing the fuel consumption and improving the control precision.

Therefore, according to the above results and analysis, the control performances of these three schemes have been interpreted in Table 1. Considering the control precision, allocation accuracy, RCS fuel consumption, and other factors, the multi-object based optimal allocation scheme was the best.

Table 1. Comparison of the three allocation schemes.

| Index                                      | Simulation Comparison                                                                 |
|--------------------------------------------|--------------------------------------------------------------------------------------|
| control error (or allocation error)        | case 3 < case 2 < case 1                                                             |
| switching smoothness                       | case 3 is the best, then is case 1, and case 2 is the worst                           |
| RCS fuel consumption                       | case 3 < case 1 < case 2                                                             |
| deflection angles’ smoothness of AS        | case 1 is the best, then is case 3, and case 2 is the worst                           |

Since these three schemes were verified under the same controller, the control precision in this situation was also equivalent to the allocation accuracy.

Figure 7. The control errors comparison.
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Figure 7. The control errors comparison.

Figure 8. Comparison of the defection angles of the aero surface (AS).

Figure 9. Reaction control system (RCS) moment comparison.
Furthermore, according to the control errors comparison of Figure 7, case 3 had the highest control precision. In 410 s, the actuators were switched from RCS and AS to AS only, because the dynamic pressure and the control efficiency of AS were big enough for the attitude control requirements. However, the switching smoothness of case 2, especially in the pitch channel, was the worst, which indicated that the daisy-chaining allocation scheme could not perform well during the switch. With respect to the deflection angles’ smoothness (shown in Figure 8), case 2 was the worst, while case 1 was the best. The reason is that, for the proportional allocation scheme, RCS had the exclusive, corresponding controller, and its control command tracking and allocation was independent from the aero surfaces. Moreover, as shown in Figure 9 and Figure 10, the RCS fuel consumption of case 3 was the least, and case 2 was the most. This is because, in case 2, the control command error minimization was the first priority and AS was only used when RCS could not meet the control command requirements, which made the fuel consumption the greatest. In case 3, both the control command error minimization and the fuel consumption minimization were considered in the optimal objective function, and by choosing the proper weight values for these

5. Conclusions

During the unpowered re-entry phase, RLV undergoes a sharp change of altitude, overload, and dynamic pressure, and transforms from the initial low-energy re-entry state, through the dense atmosphere, to the low-speed glide, which led to the AS’s control efficiency to rise, while in contrast, the RCS’s control efficiency dropped. Therefore, in the hybrid control system of RCS and AS, this paper proposed a proportional allocation scheme, based on dynamic pressure, a daisy-chaining allocation scheme based on the control errors, and a multi-object based optimal allocation scheme to distribute the virtual control command to the multiple actuators. Then, after a stability analysis of the whole system, the simulation results showed that even though these three schemes all had good control performances, the multi-object based allocation scheme had an optimal overall performance of the control system, by taking the control precision, allocation accuracy, fuel consumption, and other objective optimization functions into account.

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