Charged gravastars with conformal motion in $f(R, T)$ gravity

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Abstract This paper studies the effects of charge on a peculiar stellar object, recognized as gravastar, under the influence of $f(R, T)$ gravity by considering the conjecture of Mazur and Mottola in general relativity. The gravastar is also known as an alternative to a black hole and is expressed by three distinct domains named as (i) the interior domain, (ii) the intermediate shell and (iii) the exterior domain. We analyze these domains for a specific $f(R, T)$ gravity model conceding the conformal Killing vectors. In the interior domain, we assume that pressure is equal to negative energy density which leads to the existence of repulsive force on the spherical shell. The intermediate shell consists of ultra-relativistic plasma and pressure which shows a direct relation with energy density and counterbalances the repulsive force applied by the interior domain. The exterior vacuum spherical domain is taken to be the de Sitter spacetime illustrated by the Reissner-Nordström metric. We conclude that non-singular solutions of charged gravastar with various physical properties such as length, energy, entropy and equation of state parameter are physically consistent.

Keywords Gravastars · $f(R, T)$ gravity · Conformal motion

1 Introduction

The composition of stellar structures and the phenomenon of gravitational collapse are of significant interest in astrophysics which have attracted many researchers since the development of general relativity (GR). Gravitational collapse is responsible for the creation of distinct massive stars such as neutron stars, white dwarfs and black holes, referred to as compact objects. The ultimate outcome of collapse entirely relies on the initial mass of the celestial bodies. These remnants of collapse are broadly perceived from both theoretical as well as observational perspectives. Mazur and Mottola (2004) introduced a novel unique model, as an exact solution of the Einstein field equations, in terms of dark and cold compact object known as gravastar or the gravitational vacuum star. Gravastar is described as the spherically symmetric highly compact and singularity free object that can be anticipated as compact as the black hole.

The structure of gravastar is expressed by three distinct domains in which the interior domain is based on the de Sitter condensate state and bounded by an extremely thin-shell composed of the ultra-relativistic matter whereas the vacuum exterior is specified by the Schwarzschild solution. The concept of gravastar is quite fascinating for the researchers as it can solve the two basic problems related to black holes, i.e., the information paradox and singularity problem. Despite several observational and theoretical accomplishments, there are still various challenging issues that stimulate the researchers to examine different alternatives in which the outcomes of collapse are giant stars without event horizons. Some examples of such stellar objects are Bose superfluids, black stars, dark energy stars and gravastars that possess no event horizon and singularity (Chapline et al. 2003; Lobo 2006; Chirenti and Rezzolla 2008). Among these objects, gravastars have motivated the researchers to discuss their structures using different approaches.

In the framework of GR, Visser and Wiltshire (2004) discussed the stability of gravastar and found that there are various equations of state (EoS) that lead to dynamically sta-
ble distribution of gravastar. Cattoen et al. (2005) presented a generalized class of gravastars that possesses anisotropic pressure without existence of very small thin-shell. Carter (2005) obtained new spherical solutions of gravastar and observed distinct qualitative behavior of EoS on the interior and exterior domains. Bilić et al. (2006) analyzed the interior solutions of gravastar by replacing the de Sitter metric with Born-Infeld phantom and determined that their solutions can describe the dark compact objects at the center of galaxies. Horvat and Ilijić (2007) investigated energy conditions at the shell of gravastar and inspected the stable structure against radial perturbations via speed of sound on the shell. Several researchers (Broderick and Narayan 2007; Chirenti and Rezzolla 2007; Rocha et al. 2008; Cardoso et al. 2008; Harko et al. 2009; Pani et al. 2009) studied interior solutions using various EoS corresponding to different conjectures.

The electromagnetic field has significant role in the study of structure evolution as well as stability of collapsing objects. The electromagnetic forces, i.e., Coulomb and magnetic, are the crucial sources to generate the repulsive effects which counterbalance the attractive gravitational force. In order to keep a stellar object in its equilibrium state, a star requires an immense amount of charge to overcome the strong gravitational pull. In the study of gravastar, Lobo and Arellano (2007) constructed different gravastar models along with nonlinear electrodynamics and analyzed their solutions as well as some specific structural features. Horvat et al. (2009) extended the concept of gravastar by including the electrically charged component and evaluated the EoS parameter, surface redshift and speed of sound for the interior and exterior domains. On the same ground, Turimov et al. (2009) explored the dipolar magnetic field distribution and determined solutions of the Maxwell equations for the internal domain of a slowly rotating gravastar.

It is well-known that GR has achieved marvelous results in resolving the various hidden puzzles of the cosmos. However, some observational facts about current accelerating universe and dark matter inflicted a theoretical barrier to this theory (Riess et al. 1998; Perlmutter et al. 1999; de Bernardis et al. 2000; Peebles and Ratra 2003). In order to alleviate these problems, modified or alternative theories to GR are identified as the most fruitful perspectives. The $f(R)$ theory of gravity (Capozziello 2002) is recognized as the direct generalization of GR which is developed by placing a generic function $f(R)$ instead of curvature scalar ($R$) in the Einstein-Hilbert action. The interesting concept of the coupling between matter and geometry leads to different modified theories such as $f(R, T)$ gravity (Harko et al. 2011), $f(R, T, R_{\gamma\delta}T^{\gamma\delta})$ theory (Haghani et al. 2013; Odintsov and Sáez-Gómes 2013) and $f(G, T)$ gravity (Sharif and Ikram 2016), where $T$ exhibits the trace of energy-momentum tensor (EMT) and $G$ stands for Gauss-Bonnet invariant. Among these modified theories, $f(R, T)$ gravity received much attention by several researchers in describing various astrophysical as well as cosmological scenarios with and without electric field (Houndjo 2012; Sharif and Zubair 2012, 2013a, 2013b; Shabani and Farhoudi 2013, 2014; Sharif and Zubair 2014; Moraes 2015; Moraes et al. 2016; Sharif and Siddiqua 2017; Das et al. 2017; Sharif and Waseem 2018a, 2018b; Deb et al. 2018a, 2018b; Sharif and Siddiqua 2018, 2019; Sharif and Nawazish 2019; Sharif and Waseem 2019).

In order to obtain a natural systematic relationship between matter and geometry ingredients for giant compact stars through the field equations, an inheritance symmetry that contains a set of conformal Killing vectors plays a fundamental role. These vectors are utilized to attain exact analytic solutions of the field equations in more appropriate expressions in comparison with other analytical approaches. Using these vectors, the highly nonlinear partial differential equations can be transformed into a system of ordinary differential equations. Usmani et al. (2011) employed these Killing vectors to study the geometry of charged gravastar and determined solutions for different domains of charged gravastar in terms of conformal vector. Recently, the study of gravastar has attracted many people in modified theories of gravity. Das et al. (2017) discussed the conjecture of gravastar in $f(R, T)$ framework and analyzed its physical characteristics graphically corresponding to different EoS. Shamir and Ahmad (2018) presented non-singular solutions of gravastar and evaluated mathematical expressions of various physical parameters in $f(G, T)$ scenario.

In this paper, we discuss the geometry of gravastar which is internally charged in $f(R, T)$ gravity conceding the conformal motion. For $R + 2\beta T$ gravity model, we analyze the structure of charged gravastar described by three distinct domains and observe some feasible features graphically. The paper is displayed according to the following pattern. Next section deals with the mathematical fundamentals of $f(R, T)$ gravity. Section 3 demonstrates solutions of the field equations in terms of conformal vector. In Sect. 4, we discuss the structure of charged gravastar corresponding to different EoS. Section 5 provides interesting physical aspects of charged gravastar shell like proper length, EoS parameter, entropy and energy. The concluding remarks are presented in the last section.

## 2 Fundamentals of $f(R, T)$ gravity

The action of $f(R, T)$ theory associated with matter Lagrangian ($L_m$) and electric field is specified by (Harko et al. 2011)

$$I_{f(R,T)} = \int \left[ \frac{f(R, T)}{2\kappa} + L_m + L_e \right] \sqrt{-g} d^4x,$$  \hspace{1cm} (1)
with $\kappa = 1$ being a coupling constant, $g$ exhibits determinant of $g_{\gamma\delta}$, $L_c = n F_{\gamma\delta} F^{\gamma\delta}$, $n$ represents an arbitrary constant, $F_{\gamma\delta} = \partial_{\delta,\gamma} - \partial_{\gamma,\delta}$ expresses the electromagnetic field tensor and $\partial_{\gamma}$ symbolizes the four-potential. The corresponding field equations are

$$f_R(R, T) R_{\gamma\delta} - (\nabla_{\gamma} \nabla_{\delta} - g_{\gamma\delta} \Box) f_R(R, T) - \frac{1}{2} g_{\gamma\delta} f(R, T) = T_{\gamma\delta} + E_{\gamma\delta} - (\Theta_{\gamma\delta} + T_{\gamma\delta}) f_T(R, T),$$

(2)

where $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$, $\Box = g_{\gamma\delta} \nabla_{\gamma} \nabla_{\delta}$, $\nabla_{\gamma}$ reveals the covariant derivative, $\Theta_{\gamma\delta}$ is characterized by

$$\Theta_{\gamma\delta} = g^{\mu\nu} \frac{\partial T_{\mu\nu}}{\partial g_{\gamma\delta}} = g_{\gamma\delta} L_m - 2 T_{\gamma\delta} - 2 g^{\mu\nu} \frac{\partial^2 L_m}{\partial g_{\gamma\delta} \partial g^{\mu\nu}},$$

(3)

and electromagnetic EMT is evaluated as

$$E_{\gamma\delta} = \frac{1}{4\pi} \left( \frac{F^{\mu\nu} F_{\mu\nu} g_{\gamma\delta}}{4} - F^{\mu}_{\gamma} F^{\mu}_{\delta} \right).$$

(4)

The covariant divergence of Eq. (2) yields (Harko et al. 2011)

$$\nabla^\gamma T_{\gamma\delta} = \frac{f_T}{1 - f_T} \left[ (T_{\gamma\delta} + \Theta_{\gamma\delta}) \nabla^\gamma (\ln f_T) \right. \left. - \nabla^\gamma \left( \frac{g_{\gamma\delta} T}{2} - \Theta_{\gamma\delta} \right) - \frac{\nabla^\gamma E_{\gamma\delta}}{f_T} \right].$$

(5)

This equation manifests that the EMT does not obey the conservation law in such modified theories unlike GR. In order to discuss the cosmological as well as astrophysical scenarios, the distribution of matter plays a pivotal role. All the non-vanishing constituents of EMT express the dynamical quantities along with various physical influences. To discuss the structure of charged gravastar, we consider perfect fluid matter distribution given as

$$T_{\gamma\delta} = (\rho + p) U_{\gamma} U_{\delta} - \rho g_{\gamma\delta},$$

(6)

where $\rho$ indicates matter energy density, $p$ stands for pressure and $U_{\gamma}$ acts as four velocity in comoving coordinates satisfying $U^\gamma U_{\gamma} = 1$. In matter distribution, there are different choices of $L_m$. Here, we assume $L_m = -p$ which provides $\frac{\partial^2 L_m}{\partial g_{\gamma\delta} \partial g^{\mu\nu}} = 0$ (Harko et al. 2011) and $\Theta_{\xi\eta} = -2 T_{\gamma\delta} - \rho g_{\gamma\delta}$.

In the analysis of compact objects, it is observed that the celestial configurations presently exist in nonlinear regime. For their proper structure evolution, one must need to inspect their linear behavior and hence, we choose an independent linear model as

$$f(R, T) = R + 2\beta T,$$

(7)

where $T = \rho - 3p$ and $\beta$ is a coupling parameter. In this theory, the addition of $T$ provides more modified forms of GR in comparison with $f(R)$ gravity. This functional form is astronomically applicable to discuss different cosmological issues and expresses the Lambda cold dark matter ($\Lambda$CDM) model in $f(R, T)$ gravity (Zubair et al. 2016). This model was firstly proposed by Harko et al. (2011) and has broadly been employed to study the features of different astrophysical objects. Recently, Das et al. (2017) used this model to analyze exact non-singular solutions of collapsing object and presented physically consistent properties of gravastar. Inserting the above model along with the considered choice of $L_m$ in Eq. (2), we obtain

$$G_{\gamma\delta} = T_{\gamma\delta} + E_{\gamma\delta} + \beta T g_{\gamma\delta} + 2\beta (T_{\gamma\delta} + p g_{\gamma\delta}),$$

(8)

where $G_{\gamma\delta}$ shows the standard Einstein tensor. The covariant divergence (5) corresponding to $R + 2\beta T$ model becomes

$$\nabla^\gamma T_{\gamma\delta} = \frac{1}{1 + 2\beta} \left[ \beta g_{\gamma\delta} \nabla^\gamma T + 2\beta \nabla^\gamma (p g_{\gamma\delta}) + \nabla^\gamma E_{\gamma\delta} \right].$$

(9)

For $\beta = 0$, the original conserved results of GR in the presence of electric field can be retrieved.

### 3 Field equations with conformal motion

In order to characterize the interior of gravastar, we consider static spherically symmetric spacetime as

$$ds^2 = e^{\chi(r)} dt^2 - e^{\chi(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2.$$  

(10)

This metric along with Eqs. (6) and (8) leads to the following field equations

$$\frac{1}{r^2} - e^{-\chi} \left( \frac{1}{r^2} - \frac{\chi'}{r} \right) = \rho + \beta (3\rho - p) + \frac{q^2}{8\pi r^4},$$

(11)

$$e^{-\chi} \left( \frac{1}{r^2} + \frac{\lambda'}{r} \right) - \frac{1}{r} = p - \beta (\rho - 3p) - \frac{q^2}{8\pi r^4},$$

(12)

$$e^{-\chi} \left( \frac{\lambda''}{2} - \frac{\chi'}{2r} + \frac{\lambda'}{2r} + \frac{\lambda^2}{4} - \frac{\chi' \lambda'}{4} \right) + \frac{1}{r} = p - \beta (\rho - 3p) + \frac{q^2}{8\pi r^4},$$

(13)

where prime reveals differentiation corresponding to $r$ and $q$ indicates the spherical charge defined as

$$q(r) = 4\pi \int_0^r \xi(r) r^2 e^{\chi/2} dr, \quad E = \frac{q}{4\pi r^2}.$$  

(14)
Here $\xi$ and $E$ denote the surface charge density and electric field intensity, respectively. The covariant divergence given by Eq. (9) provides (Moraes et al. 2016)

$$p' + \frac{\lambda}{2}(\rho + p) = \frac{-1}{1 + 2\beta}\left\{\beta(p' - p') - \frac{qq'}{4\pi r^2}\right\}. \quad (15)$$

In the analysis of astrophysical objects, for a natural relationship between geometry and matter ingredients described by the field equations, we assume a renowned inheritance symmetry which contains a set of conformal Killing vectors expressed by the relation

$$\Omega \xi g_{\gamma \delta} = g_{\eta \delta} \xi^\eta + g_{\eta \gamma} \xi^\eta = \psi(r)g_{\gamma \delta}.$$ \quad (16)

where $\Omega$ acts as a Lie derivative operator and $\psi(r)$ denotes the conformal vector. Using Eq. (10) in (16), it follows that (Rahaman et al. 2004)

$$\xi' = \psi, \quad \xi' = \frac{r\psi}{2}, \quad \xi' + 2\xi' = \psi,$$

yielding

$$e^\lambda = a^2 r^2, \quad e^\chi = \left(\frac{b}{a}\right)^2,$$ \quad (17)

with $a$ and $b$ as the integration constants. Inserting these solutions in Eqs. (11)–(13), we obtain

$$\frac{1}{r^2} \left(1 - \frac{\rho}{b^2} - \frac{2\psi\phi'}{b^2 r} = \rho + \beta(3\rho - p) + \frac{q^2}{8\pi r^4}\right), \quad (18)$$

$$\frac{3\psi^2}{b^2 r^2} - \frac{1}{r^2} = p - \beta(\rho - 3p) - \frac{q^2}{8\pi r^4}, \quad (19)$$

$$\frac{q^2 + 2\psi\phi'}{b^2 r} = p - \beta(\rho - 3p) + \frac{q^2}{8\pi r^4}. \quad (20)$$

These lead to the explicit forms of density, pressure and electric field as follows

$$\rho = \frac{b^2(1 + 2\beta) + 4\beta\phi' - 2r\phi'}{2b^2 r^2(1 + 6\beta + 8\beta^2)} + \frac{q^2}{8\pi r^4}, \quad (21)$$

$$p = \frac{-b^2(1 + 2\beta) + 4\beta\phi' + 2r\phi'}{2b^2 r^2(1 + 6\beta + 8\beta^2)}, \quad (22)$$

$$2\pi E^2 = \frac{q^2}{8\pi r^4} - \frac{1}{2} \left[\frac{1}{r^2} + \frac{2\phi'}{b^2 r} + \frac{2r\phi'}{b^2 r}\right]. \quad (23)$$

We note that the electric field is independent of $f(R, T)$ model parameter $\beta$ and expresses the same form as in GR. For $\beta = 0$, all these equations reduce to the field equations of GR admitting conformal Killing vectors (Usmani et al. 2011).

### 4 Solutions of gravastar in $f(R, T)$ gravity

This section is devoted to discuss the geometrical description of the constructed solutions of gravastar in $f(R, T)$ gravity corresponding to (i) the interior domain, (ii) the intermediate shell and (iii) the exterior domain. The interior geometry of a star is surrounded by an extremely thin-shell which is comprised of some ultra-relativistic fluid. On the other hand, the exterior domain is purely vacuum and for charged gravastar, the Reissner-Nordström metric can be regarded feasible for this outer region. Since the shell is considered to be immensely thin possessing a finite width lying in the range $r_1 = R < r < R + \epsilon = r_2$, where $r_1$ and $r_2$ exhibit the inner and outer radii of charged gravastar under consideration, respectively and $r_2 - r_1 = \epsilon$ reveals thickness of the shell. Therefore, the structure of charged gravastar can be characterized by three domains depending upon the EoS as follows

- Interior domain ($D_1$) $\Rightarrow 0 \leq r < r_1$ with $p + \rho = 0$,
- Intermediate shell ($D_2$) $\Rightarrow r_1 \leq r < r_2$ with $p = \rho$,
- Exterior domain ($D_3$) $\Rightarrow r_2 < r$ with $p = \rho = 0$.

#### 4.1 The interior domain

In this region, we assume the EoS as $\omega = \frac{p}{\rho}$, where $\omega$ represents the EoS parameter and for $\omega = -1$, we have $p = -\rho$ which expresses the EoS for dark energy. From Eqs. (21) and (22), we obtain a connection between the matter variables and metric functions in the following form

$$\rho + \rho = \frac{2\rho(1 + 4\beta)(\rho - \rho)}{b^2 r^2(1 + 2\beta)(1 + 4\beta)} = \frac{2\rho(\rho - \rho)}{b^2 r^2(1 + 2\beta)}. \quad (24)$$

Employing the ansatz $\rho + \rho = 0$ in the above equation, it follows that either $\rho = 0$, or $\rho - \rho = 0$. We consider $\phi - \phi' = 0$ which provides a solution of conformal vector as $\phi = \phi_0 r$, where $\phi_0$ is a dimensionless integration constant. This solution of conformal vector yields the following analytic expressions of all the physical parameters

$$\rho = \frac{1 + 2\beta - 6\rho^2 \phi_0^2 - 12\rho^2 \phi_0^2}{2r^2(1 + 6\beta + 8\beta^2)} = \frac{(1 + 2\beta)(1 - 6\rho^2 \phi_0^2)}{2r^2(1 + 2\beta)(1 + 4\beta)}, \quad (25)$$

$$e^\lambda = e^{-\lambda} = \phi_0^2 r^2, \quad (26)$$

$$E^2 = \frac{q^2}{16\pi^2 r^4} = \frac{1}{4\pi r^2}, \quad (27)$$

$$\xi = \frac{\phi_0}{r \sqrt{4\pi}}. \quad (28)$$

with $\phi_0 = \frac{q_0}{p}$ as a constant having dimension inverse of $r$. 
The gravitational mass \( M \) on Eq. (11), the gravitational mass \( M \) with the electromagnetic mass model (Ray et al. 2008). The significant reason behind this is the inclusion of electric field in the geometry of gravastar. As a result of spherical collapse, this mass produces the attractive force which counterbalances the repulsive force generated by the electromagnetic field. Also, in the framework of accelerating cosmos, the EoS \( ρ = −p \) indicates a repulsive pressure which may correspond to dark energy, an enigmatic force responsible for the current phase of the inflation. Thus we can say that charged gravastar provides a connection with the dark star (Chan et al. 2009a, 2009b).

4.2 The intermediate shell of charged gravastar

Here, we consider that the non-vacuum shell of charged gravastar is formulated by ultra-relativistic fluid satisfying the EoS \( ρ = p \). In association with the cold baryonic cosmos, Zel’dovich (1972) developed the concept of such type of fluid recognized as stiff fluid. But in the present framework, it may be due to some thermal oscillations with zero chemical potential or due to the gravitational quantities at negligible temperature (Mazur and Mottola 2004). Several researchers (Carr 1975; Wesson 1978; Madsen et al. 1992; Braje and Romani 2002; Linares et al. 2004) have widely utilized this fluid to discuss different cosmological as well as astrophysical scenarios.

It is observed that finding exact solutions of the field equations for a non-vacuum shell region is a difficult task in the presence of stiff fluid. However, such solutions can be obtained within the range suggested for the thin-shell, i.e., \( 0 < e^{-χ} \ll 1 \) admitting the conformal motion. Implementing this range along with stiff fluid EoS in Eqs. (21) and (22), we evaluate

\[
(1 + 2β)(b^2 − 2φ^2 − 4rφφ') = 0. \tag{31}
\]

Again, we have two choices that either \( 1 + 2β = 0 \), or \( b^2 − 2φ^2 − 4rφφ' = 0 \). The former condition provides \( β = −\frac{1}{2} \) and we neglect this choice to avoid singular solution. The later condition leads to

\[
φ^2 = \frac{b^2}{2} − \frac{φ_1}{r}, \tag{32}
\]

with \( φ_1 > 0 \) being the integration constant. On substituting this solution, Eqs. (21)–(23) become

\[
ρ = \frac{1}{2r^2(1 + 2β)} \left(1 − \frac{3φ_1}{r}\right) = p, \tag{33}
\]

\[
e^λ = a^2 r^2, \tag{34}
\]

\[
e^{-χ} = \frac{1}{2} − \frac{φ_1}{r}, \tag{35}
\]

\[
E^2 = \frac{q^2}{16\pi^2 r^4} = \frac{3φ_1}{4πr^3}, \tag{36}
\]

\[
ξ = \frac{\sqrt{3φ_1}}{4\sqrt{2πr^3}} \sqrt{\frac{r}{φ_1} − 2}, \tag{37}
\]

Here, \( φ_1 = \frac{b^1}{b^2} \) has the dimension of \( r \).
In this region, electric field depends on $\hat{\omega}_1$ and shows an inverse relation with the radius. The charge density demonstrates that the real value of $\xi$ can be found only for the constraint $\hat{\omega}_1 < \frac{\xi}{2}$. Combining this condition with $\phi > 0$, we observe that the constructed solutions for the shell region of charged gravastar are consistent only for the range $0 < \hat{\omega}_1 < \frac{\xi}{2}$. The interpretation of energy density in Eq. (33) provides that the EoS $\rho = p = 0$ for the de-Sitter exterior region leads to $\hat{\omega}_1 = \frac{\xi}{2}$, which obviously satisfies the constraint $\hat{\omega}_1 < \frac{\xi}{2}$. Since, in the shell domain $D_2$, we have $R < r \leq R + \epsilon$, therefore, from the assumption $e^{-\chi} \ll 1$, we obtain $\epsilon \ll 1$.

The graphical behavior of charge ($q$) and $\rho = p$ with respect to thickness is displayed in Fig. 2 which exhibits that the ultra-relativistic fluid as well as charge show more effects towards the outer domain as compared to the inner border of the shell. In literature, it is found that the value of model parameter $\beta$ lies in the interval $[-1, 1]$ to discuss the structural properties of self-gravitating objects. But in the present context, we have observed that $\beta = -\frac{1}{2}$ yields singularity in the domain $D_2$. Due to this reason, we neglect the negative values of $\beta$ and show the behavior of energy density for two different positive values of $\beta$. The graphical analysis of energy density shows that charged gravastars possess denser structure for small values of $\beta$ and their denseness decreases with increasing value of the model parameter.

### 4.3 The exterior domain and matching constraints

Finally, we discuss the vacuum region $D_3$, when the exterior domain satisfies the EoS $\rho = p = 0$ and can be expressed by the Reissner-Nordström metric as

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

(38)

where $M$ and $Q$ are the total gravitational mass and total charge, respectively. In the study of compact objects, it is observed that there should be a smooth matching between interior and exterior domains. Here, for a smooth connection between $D_1$ and $D_3$, we assume a formalism proposed by Israel (1966). At the joining surface, i.e., at $r = R$, the coefficients of the metric

$$ds^2 = H(r)dt^2 - \frac{dr^2}{H(r)} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

(39)

are continuous but their derivatives may not exist at this surface. Nevertheless, it is possible to acquire the surface stress-energy ($S_{\gamma\eta}$) with the assistance of Israel formalism. Considering Lanczos equation, the expression for $S_{\gamma\eta}$ is illustrated by

$$S_{\gamma\eta} = \frac{1}{8\pi} \left(\delta_{\gamma}^{\alpha} \epsilon_{\alpha}^{\mu} - \tau_{\gamma}^{\mu}\right),$$

(40)

where $\tau_{\gamma\eta} = K_{\gamma\eta}^{+} - K_{\gamma\eta}^{-}$ leads to the discontinuity in the surface of extrinsic curvature or second fundamental form. Here, the signs $- +$ show the interior and exterior domains, respectively. The extrinsic curvature at hypersurface $(\Pi)$ corresponding to two regions of the thin-shell is of the form

$$K_{\gamma\eta}^{\pm} = -\gamma_{\mu}^{\pm} \left[\frac{\delta^{2} x_{\mu}}{\delta \gamma^{\nu} \delta \gamma^{\eta}} + \Gamma_{\alpha\nu}^{\mu} \frac{\partial x^{\alpha}}{\partial \gamma^{\nu}} \frac{\partial x^{\eta}}{\partial \gamma^{\eta}}\right],$$

(41)

where $\gamma^{\nu}$ symbolizes the intrinsic shell coordinates, $\gamma_{\mu}^{\pm}$ indicates the unit normals at $\Pi$ defined as

$$\gamma_{\mu}^{\pm} = \pm \left| g^{\alpha\nu} \frac{\partial H}{\partial x^{\alpha}} \frac{\partial H}{\partial x^{\nu}} \right|^{\frac{1}{2}} \frac{\partial H}{\partial x^{\mu}},$$

(42)

Employing the Lanczos equations, the mathematical descriptions of surface energy density ($q$) and surface pressure ($P$) in $S_{\gamma\eta} = \text{diag}(q, -P, -P, -P)$, are given by

$$q = -\frac{1}{4\pi R} \left[\sqrt{H} \right]^{\pm}, \quad P = \frac{-q}{2} + \frac{1}{16\pi} \left[ \frac{H'}{\sqrt{H}} \right]^{\pm}.$$  

(43)

Substituting the considered $D_1$ and $D_3$ metrics along with the conformal vector in the above expressions, we obtain

$$q = -\frac{1}{4\pi R} \left[ \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \hat{\omega}_0 R} \right].$$

(44)
\[ \mathcal{P} = \frac{1}{8\pi} \mathcal{R} \left[ \frac{1 - \frac{M}{\mathcal{R}}}{\sqrt{1 - \frac{2M}{\mathcal{R}} + \frac{Q^2}{\mathcal{R}^2}}} - 2\bar{\varphi}_0\mathcal{R} \right]. \]  

(45)

Using Eq. (44), we evaluate the surface mass of the charged gravastar thin-shell as

\[ \mathcal{M}_{\text{shell}} = 4\pi \mathcal{R}^2 \varrho = \mathcal{R} \left[ \bar{\varphi}_0\mathcal{R} - \sqrt{1 - \frac{2M}{\mathcal{R}} + \frac{Q^2}{\mathcal{R}^2}} \right]. \]  

(46)

This leads to the total mass of charged gravastar in the following form

\[ \mathcal{M} = \frac{1}{2\mathcal{R}} \left[ \mathcal{R}^2 + Q^2 - \mathcal{M}_{\text{shell}}^2 \mathcal{R}^2 + 2\bar{\varphi}_0\mathcal{M}_{\text{shell}} \mathcal{R}^4 - \bar{\varphi}_0^2 \mathcal{R}^6 \right]. \]  

(47)

5 Physical aspects of charged gravastar

In this section, we discuss some essential physical aspects such as EoS parameter, entropy, length and the energy-thickness relationship, that entirely portray the intermediate geometry of charged gravastar.

5.1 The EoS parameter

The EoS parameter at \( r = \mathcal{R} \), using Eqs. (44) and (45), turns out to be

\[ \omega(\mathcal{R}) = \frac{\mathcal{P}}{\varrho} = \frac{1 - \frac{M}{\mathcal{R}}}{2((1 - \frac{2M}{\mathcal{R}} + \frac{Q^2}{\mathcal{R}^2}) - \bar{\varphi}_0\mathcal{R} \sqrt{1 - \frac{2M}{\mathcal{R}} + \frac{Q^2}{\mathcal{R}^2}})} - \bar{\varphi}_0\mathcal{R}. \]  

(48)

This equation is comprised of different factors involving the fractions with square root terms that enhance sensitivity of the equation. In order to have real EoS parameter, the necessary required condition is either \( \frac{2M}{\mathcal{R}} - \frac{Q^2}{\mathcal{R}^2} < 1 \) or \( \frac{2M}{\mathcal{R}} < 1 \). These inequalities provide the relation between \( \mathcal{M}, \mathcal{R} \) and \( Q \) as \( Q > \sqrt{R(2M - \mathcal{R})} \) and \( \mathcal{M} < \frac{\mathcal{R}}{2} \), respectively. The specification of positive density as well as positive pressure always leads to the positive value of \( \omega \). For appropriately large value of \( \mathcal{R} \), we attain \( \omega(\mathcal{R}) \approx 1 \) (Usmani et al. 2011). This large value of \( \mathcal{R} \) manifests the structure of gravastar like compact objects. Also, the insertion of some particular value of \( \mathcal{R} \) in Eq. (45) may provide \( \mathcal{P} = 0 \), which corresponds to a dust shell.

5.2 Length of the shell

We have considered that the geometry of the stiff fluid charged gravastar shell is placed at \( r = \mathcal{R} \) which is illustrated by the domain \( D_1 \). As the thickness of the intermediate shell is supposed to be extremely small, i.e., \( \epsilon \ll 1 \), therefore, the domain \( D_3 \) appears from the interface \( r = \mathcal{R} + \epsilon \). Thus the proper length \( (L) \) between two joints, i.e., the thin-shell is evaluated as

\[ L = \int_{\mathcal{R}}^{\mathcal{R} + \epsilon} \sqrt{\epsilon} \, dr \]

\[ = \sqrt{2} \left[ r \sqrt{1 - \frac{2\bar{\varphi}_1}{r}} + \bar{\varphi}_1 \ln \left( r + r \sqrt{1 - \frac{2\bar{\varphi}_1}{r} - \bar{\varphi}_1} \right) \right]_{\mathcal{R}}^{\mathcal{R} + \epsilon}. \]  

(49)

The relation between the proper length and thickness of the charged gravastar shell is exhibited in Fig. 3 which interprets the increasing as well as direct connection between them.

5.3 Entropy inside the shell

In the study of gravastar, Mazur and Mottola (2004) discussed the zero entropy density \( (S) \) inside the domain \( D_1 \) which displays the consistency with a single condensate region. In this work, the entropy inside the charged gravastar shell is determined by (Das et al. 2017)

\[ S = \int_{\mathcal{R}}^{\mathcal{R} + \epsilon} 4\pi r^2 \mathcal{U}(r) \sqrt{\epsilon} \, dr, \]  

(50)

where \( \mathcal{U}(r) \) indicates the entropy density and corresponding to radially dependent temperature \( (T(r)) \), it is expressed as

\[ \mathcal{U}(r) = \frac{\sigma^2 K_B^2 T(r)}{4\pi \hbar^2} = \sigma \left( \frac{K_B}{\hbar} \right) \sqrt{\frac{p}{2\pi}}. \]  

(51)

Here \( \sigma \) is a dimensionless constant and in this work, we take Planckian units as \( \hbar = K_B = 1 \). Thus Eq. (50) becomes

\[ S = \int_{\mathcal{R}}^{\mathcal{R} + \epsilon} \frac{4\pi \sigma r^2}{\sqrt{2\pi}} \sqrt{\frac{1}{2r^2(1 + 2\beta)}} \left( 1 - \frac{3\bar{\varphi}_1}{r} \right) \left( \frac{1}{1 - \bar{\varphi}_1} \right) \, dr. \]  

(52)
5.4 Energy inside the shell

In the domain $\mathcal{D}_1$, we have considered the EoS $\rho = -p$ which reveals negative energy density ensuring the existence of repulsive nature of the domain $\mathcal{D}_1$. But inside the shell of charged gravastar in $f(R, T)$ gravity, the energy takes the form

$$E = \frac{1}{2} \int_{\mathcal{R}}^{\mathcal{R} + \epsilon} (\rho + 2\pi E^2) r^2 dr$$

$$= \frac{1}{2} \int_{\mathcal{R}}^{\mathcal{R} + \epsilon} \left[ \frac{1}{4(1 + 2\beta)} \left( 1 - \frac{3\phi_1}{r} \right) + \frac{3\phi_1}{4r} \right] r^2 dr,$$

whose integration provides

$$E = \frac{\epsilon + 6\beta \phi_1 [\ln(\mathcal{R} + \epsilon) - \ln(\mathcal{R})]}{4(1 + 2\beta)}.$$

This shows a direct relation of the energy with the thickness of the shell. The graphical analysis of energy-thickness relation corresponding to different values of $\beta$ is given in Fig. 5 which displays the non-repulsive nature of energy inside the shell. This reduces to GR for $\beta = 0$ (Usmani et al. 2011).

6 Concluding remarks

This paper is dedicated to discuss a novel unique stellar object in $f(R, T)$ gravity admitting conformal Killing vectors which is originally proposed in GR for gravastar (Mazur and Mottola 2004). We have considered an internally charged gravastar with the exterior expressed by the Reissner-Nordström metric. The $f(R, T) = R + 2\beta T$ gravity model is used to study the geometry of charged gravastar for three domains along with distinct EoS. We have evaluated exact non-singular solutions of the collapsing system in terms of conformal vector. These solutions yield some fascinating aspects of the charged gravastar which are physically consistent in this gravity.

In order to analyze the structure of charged gravastar, we have considered that the interior domain $\mathcal{D}_1$ satisfies the EoS $\rho = -p$, which shows the repulsive pressure describing the dark energy. We have studied the behavior of gravitational mass in $\mathcal{D}_1$ with respect to the radius $r_1 = \mathcal{R}$. The domain $\mathcal{D}_1$ is bounded by a thin-shell which is composed of ultra-relativistic stiff fluid and obeys the EoS $\rho = p$, indicating the non-repulsive nature of charged gravastar shell. The exterior domain $\mathcal{D}_3$ is assumed to be entirely vacuum executing the EoS $\rho = p = 0$. We have found constraints on conformal vector $\phi$ as well as model parameter $\beta$ from the resulting exact solutions in these domains. The graphical behavior of charge and energy density of stiff fluid are shown for positive values of $\beta$ against the thickness of the shell.

We have studied some interesting physical features of charged gravastar such as length, energy, entropy and EoS parameter. For the real value of EoS parameter, we must have $\frac{2M}{\mathcal{R}^2} + \frac{\mathcal{Q}^2}{\mathcal{R}^2} < 1$, which may provide some specific value of $\alpha$ that could result in the stability of charged gravastar. We have also found that only for large appropriate value of $\mathcal{R}$, we obtain $\omega(\mathcal{R}) \approx 1$ which provides the EoS $\rho = p$ in the intermediate domain. We have figured out the expressions of length, energy as well as entropy and analyzed graphically their direct relations with the thickness of the charged gravastar shell for positive values of $\beta$. 
Usmani et al. (2011) discussed the geometry of charged gravastars with conformal Killing vectors in GR but they did not analyze their solutions graphically. Das et al. (2017) found exact solutions of gravastars in \( f(R, T) \) scenario without charge as well as conformal vector and presented the graphical analysis of density, pressure, length, energy and entropy inside the shell. It is worthwhile to mention here that we have introduced the conformal motion as well as effects of charge in \( f(R, T) \) framework to describe the geometry of gravastars and also shown the graphical behavior of our obtained solutions. We have found that the gravitational mass obtained from the matching of interior and Reissner-Nordström exterior metrics in this gravity becomes an electromagnetic mass that provides an attractive force to overcome the repulsive effects of charge which is consistent with GR (Usmani et al. 2011). These repulsive effects also counterbalance the attractive force of gravity and lead to the more stable configuration of gravastar than the structure defined without charge (Das et al. 2017). Moreover, solutions for dark energy EoS demonstrate that charged gravastars are connected with dark stars. We would like to mention here that all the obtained solutions reduce to GR for \( \beta = 0 \) (Usmani et al. 2011).

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