Ultrasound Attenuation of Superfluid $^3$He in Aerogel

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We have performed longitudinal ultrasound (9.5 MHz) attenuation measurements in the B-phase of superfluid $^3$He in 98% porosity aerogel down to the zero temperature limit for a wide range of pressures at zero magnetic field. The absolute attenuation was determined by direct transmission of sound pulses. Compared to the bulk fluid, our results revealed a drastically different behavior in attenuation, which is consistent with theoretical accounts with gapless excitations and a collision drag effect.

Liquid $^3$He has attracted intense interest for many decades in the field of low temperature physics. In its normal state, liquid $^3$He has served as a paradigm for a Fermi liquid whose nature transcends $^3$He physics. The superfluid phases of $^3$He exhibit exotic and intriguing features associated with the broken symmetries in the condensate, having an unconventional structure of the order parameter with spin triplet $p$-wave pairing. Liquid $^3$He is arguably the most well-understood system mainly because of its extreme intrinsic pureness at low temperatures. Therefore, it has provided important insights in understanding other unconventional superconductors such as the high temperature superconductors, the heavy fermion superconductors, and in particular the more recently discovered Sr$_2$RuO$_4$, which is also thought to have the $p$-wave symmetry. However, the same virtue has hampered the effort in pursuing answers to an important overarching question: how does the nature of a quantum condensate (spin triplet $p$-wave superfluid in this case) respond to increasing impurity or disorder?

Observation of superfluid transitions in liquid $^3$He impregnated in high porosity aerogel in 1995 opened a novel path to introducing static disorder in liquid $^3$He. Aerogel possesses a unique structure, whose topology is at the antipode of widely studied porous media such as Vycor glass and metallic sinters. Due to its open structure, there are no well-defined pores in aerogel and consequently, the liquid is in the proximity to the bulk. Ninety percent porosity aerogel, which has been used in most of the studies including this work, offers a correlated network of strand-like aggregates of SiO$_2$ molecules whose structure can be characterized by the geometrical mean free path ($\ell \approx 100$ - 200 nm), the diameter of strand ($r \approx 3$ nm), and the average inter-strand distance ($d \approx 25$ - 40 nm). The coherence length of pure superfluid $^3$He, $\xi_0$, which varies from 20 nm (34 bar) to 80 nm (0 bar), is at least an order of magnitude larger than the strand diameter but is comparable to $\ell$ and $d$. As a result, the scattering off the aerogel strand would have a significant influence on the superfluid. It is now well established that the superfluid transition temperature is significantly depressed from that of the bulk, and the effect of pairing breaking is progressively magnified at lower pressures, leading to the possibility of a quantum phase transition at $P_c \approx 6$ bars. To date, three distinct superfluid phases have been experimentally identified, namely the A-like, B-like, and A$_1$-like phases. The B-like phase and the A$_1$-like phase in aerogel show striking similarity to their counterparts in the bulk superfluid. Detailed NMR studies suggest that the aerogel B-phase has the same order parameter structure as the bulk B-phase. The aerogel A$_1$-phase only appears in the presence of magnetic field as is the case in the bulk. However, the aerogel A-phase exhibits quite a different behavior from the bulk A-phase (e.g. in NMR frequency shift and superfluid density), although the overwhelming experimental evidence suggests that it is an equal spin pairing state. Various interpretations or novel propositions on the possible order parameter structure have been suggested for this phase.

Nuclear magnetic resonance and ultrasound spectroscopy have been used in concert to investigate the microscopic structure of the superfluid phases. These two experimental methods encompass complementary information on the orbital (ultrasound) and spin (NMR) structure of the Cooper pairs. Rich spectra of order parameter collective modes in bulk superfluids, which are the fingerprints of specific broken symmetries in the system, have been mapped by ultrasound spectroscopic techniques. In 2000, Nomura et al. performed ultrasound attenuation measurements on 98% aerogel using a 16.5 MHz cw acoustic impedance technique. Their work was limited to a single pressure at 16 bars and down to 0.6 mK. Although their technique was not adequate in determining absolute attenuation, they managed to extract the absolute sound attenuation after making auxil-
A crucible spacer maintaining a 3.05 mm diameter transducers were separated by a Ma-
9.5 MHz were used as a transmitter and a receiver. The final sound transducers with the fundamental resonance at
A1 pulse was generated by the transmitter and de-
in situ. This scheme ensures the best contact be-
between the transducer surface and the aerogel, which is crucial for clean sound transmission at the boundaries.
A 1 µs pulse was generated by the transmitter and de-
tected by the receiver. Temperature was determined by a melting pressure thermometer (MPT) for T ≥ 1 mK
and a Pt NMR thermometer for T ≤ 1 mK which was calibrated against the MPT. No non-linear response or
self-heating was observed at the excitation level used in this work. All the data presented here, except for 8 bars,
were taken while warming with a typical warming rate of 3 µK/min. A detailed description on the experimen-
tal cell and experimental techniques can be found elsewhere [17, 18].

The temporal responses of the receiver taken at 34 bars are shown in Fig. 1 for select temperatures ranging from
0.3 to 2.5 mK. The primary response, which starts to rise around 8 µs, shows a rather broad response due to ring-
ing of the high Q transducer (Q ~ 10^9). The step-like structure of the receiver signal is caused by the slight mis-
match in the spectra of the transducers [18]. Below the aerogel superfluid transition (marked around 2.1 mK by
an arrow in Fig. 1) the primary response starts to grow and the trailing echoes emerge from the background, as
the sound attenuation decreases in the superfluid. No change in the receiver signal was observed at the bulk
superfluid transition. The multiple echoes follow a bona fide exponential decay in time. Absolute sound attenua-
tion was obtained in the following manner [19]. First, the relative attenuation at each temperature was calculated
using the area under the primary response curve by integrating the signal from the rising edge to a fixed point in
time (23 µs point). The absolute attenuation at 0.4 mK
and 29 bars, obtained using the primary signal and the echoes, was used as a reference point in converting the
relative attenuation into the absolute attenuation. Due to a drastic mismatch in the acoustic impedance at the
the transducer-aerogel/3He boundary, the signal absor-
ation at the surface of transducers was ignored [19]. The possible background contributions to attenuation from
the quasi-particle scattering off the cavity wall [20] and the non-parallel alignment of the two transducers are es-
timated to be negligible.

The absolute attenuations on warming for several pressures are plotted as a function of temperature in Fig. 2(a).
The superfluid transition is marked by the smooth drop in attenuation. Our aerogel superfluid transition tem-
peratures are in excellent agreement with the previously reported values for all pressures [3, 21]. At 9.5 MHz in
the bulk B-phase, a strong attenuation peak appears right below the superfluid transition. This peak is the result of
the combined contributions from pair-breaking and cou-
pling to the order parameter collective modes. Above the polycritical pressure, the B to A transition on warming
is registered as a sharp step in attenuation. In aerogel, none of these features exist. However, we did ob-
serve a sharp step in attenuation on cooling for P > 14 bars, which implies the existence of the supercooled A-
phase [19]. We were able to identify a rather smooth B to A transition on warming for 29 and 34 bars within ~
150 µK below the superfluid transition. This observation is consistent with the previous results obtained using a
transverse acoustic impedance technique [13]. Therefore, most of the attenuation data presented here are in the
aerogel B-phase. In the bulk B-phase with a clean gap,
the attenuation follows α ∝ e^{−Δ(T)/kB T} below
the attenuation peak, practically reaching zero attenuation below
T/Tc ≈ 0.6, due to thermally activated quasi-particles,
where Δ(T) is the temperature dependent gap and kB is the Boltzmann constant. In contrast, the attenuation in
aerogel decreases rather slowly with temperature and remains high even at T/Tc ≈ 0.2. Furthermore, a pec-
cular shoulder feature appears at T/Tc ≈ 0.6 for higher pressures. This feature weakens gradually and eventually
disappears at lower pressures, Fig. 2(a).

Sound propagation for higher harmonics up to 96 MHz
was measured for several temperatures and pressures, but no evidence of sound propagation was found above 30 MHz even at 0.3 mK, where the lowest attenuation is expected. Below about 10 MHz, the scattering process is dominated by the temperature independent impurity scattering off the aerogel, and at 9.5 MHz, \( \omega \tau_i \approx 0.1 \) for all pressures where \( \tau_i = \ell / v_f \) (see below for \( \ell \)). Therefore, the sound speed should remain in the hydrodynamic limit. This claim is bolstered by the observation of the strong frequency dependence in attenuation and the absence of a temperature dependence in the normal fluid attenuation \( |\alpha| \). The coupling between the normal component of the superfluid \( ^3\text{He} \) and the mass of the elastic aerogel modifies the conventional two-fluid hydrodynamic equations \( [22, 23] \). This consideration leads to two (slow and fast) longitudinal sound modes with different sound speeds, \( c_s = c_a \sqrt{\rho_H / \rho_a} \), and \( c_f = c_1 \sqrt{1 + \rho_a \rho / \rho_H \rho_n} \).

Here, \( c_f(s) \) represents the speed of the fast (slow) mode, \( \rho_n(s) \) is the normal fluid (superfluid) density \( \rho = \rho_n + \rho_s \), \( \rho_a \) is the aerogel density, \( c_1 \) is the speed of hydrodynamic sound in \( ^3\text{He} \), and finally \( c_a \) is the sound speed of the bare aerogel. From the time of flight measurements, we found the sound speed in aerogel consistently lower (by \( \approx 20\% \)) than \( c_1 \) for all pressures studied and in good agreement with the values obtained using the expression above \( [24] \). Detailed analysis of sound velocity for various pressures will be presented in a separate publication.

Low mass density and the compliant nature of aerogel necessitate the consideration of effective momentum transfer upon quasi-particle scattering off the aerogel, which generates dragged motion of aerogel. Ichikawa et al. \( [25] \) incorporated the collision drag effect in calculating the dispersion relation in the normal fluid. Their model offered a successful explanation for the experimental results of the Northwestern group \( [15] \). Recently, Higashitani et al. \( [26, 27] \) extended this model to study the longitudinal sound (fast mode) propagation in superfluid \( ^3\text{He} \)/aerogel within the framework of the two-fluid model. The drag effect can be described phenomenologically by the frictional force \( \vec{F}_d = \tau_f (\vec{v}_n - \vec{v}_a) \), introducing an additional relaxation time \( \tau_f \), where \( \vec{v}_n(a) \) is the normal fluid component (aerogel) velocity. This effect is of particular importance when \( \omega \tau_i < 1 \), and the total attenuation (Eq. (130) of ref. \( [27] \)) is

\[
\alpha = \frac{\omega^2 / 2c_f}{1 + \rho_a \rho / \rho_n} \left( \frac{\rho_n \tau_f \rho_n}{1 + \rho_n / \rho} + \frac{4\eta / 3pc_f^2}{1 + \rho_n \rho / \rho} \right),
\]

where \( \eta \) is the shear viscosity of liquid \( ^3\text{He} \).

The first term \( (\alpha_f) \) arises from the frictional damping caused by the aerogel motion relative to the normal fluid component, and the second term \( (\alpha_e) \) from the conventional hydrodynamic sound damping associated with the viscosity. This expression allows us to extract \( \ell \) in this system from our absolute attenuation at the transition temperature, \( \alpha_c \). The inset of Fig. 3 shows our results of \( \alpha_c \) for various pressures. The solid lines are the result of calculation using Eq. (1) for three different mean free paths, \( \ell = 100, 120, \) and \( 140 \) nm. As can be seen, \( \ell = 120 \) nm produces an excellent fit to our data for the whole pressure range, which is in good agreement with the values obtained from the thermal conductivity \( (90 \text{ nm}) \) \( [28] \) and spin diffusion \( (130 \text{ nm}) \) \( [29] \) measurements. With the knowledge of the mean free path, one can calculate the full temperature dependence of sound attenuation in the superfluid phase. The results of the calculation (in the unitary limit) following the prescription described in ref. \( [27] \) are displayed in Fig. 2(b) along with the experimental results at 34 bars. The calculation reproduces all the important features observed in our measurements. In particular, the conspicuous shoulder structure appearing near \( T/T_c \approx 0.6 \) at 33 bars softens at lower pressures and is completely absorbed in an almost linear temperature dependence below 20 bars. This behavior is the characteristic of \( \alpha_f \) \( [27] \).

A fast decrease in \( \rho_n \) right below \( T_c \) produces the bump in \( \alpha_f \), and \( \alpha_f \rightarrow 0 \) as \( T \rightarrow 0 \). On the other hand, \( \alpha_e \) decreases monotonically and reaches a finite value due to non-zero \( \rho_n \) and the impurity states.
induced inside the gap as \( T \to 0 \). The quantitative agreement between the theory and experiment, however, is not yet satisfactory. The calculation utilizes the isotropic homogeneous scattering model (IHSM) \[32\], which tends to overestimate \( \Delta(T) \) and \( \rho_s \), compared to the experimentally determined values \[3, 23\]. As shown in ref. \[31\], the inhomogeneity gives rise to the reduction of the average value of the order parameter and consequently yields larger \( \eta \) and \( \rho_n \), which in turn increases \( \alpha_0 \) but decreases the frictional contribution. It is also expected that the non-\( s \)-wave scattering components make non-trivial contributions to the viscous and frictional relaxation times in a direction that improves the quantitative agreement.

Theoretical calculations based on the IHSM \[27, 32\] predict that the impurity states would completely fill the gap, leading to a gapless superfluid when \( \tau_1 T_c < 1 \) for the \( B \)-phase in the unitary limit. We estimate \( \rho_{s} \approx 120 \text{ nm} \) (see text).

![FIG. 3: Normalized zero temperature attenuation vs. pressure. The dashed line is a guide for eye. Inset: Pressure dependence of sound attenuation at \( T_c \). The solid lines (color on-line) are the results of theoretical fit for \( \ell = 100, 120, \) and 140 nm (see text).](image)

the behavior is consistent with the theoretical predictions with gapless excitations. Unlike the thermodynamic and transport measurements, the high frequency ultrasound measurement has a potential to unveil a larger portion of the impurity states profile from the frequency dependence.

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