Multiplex model of mental lexicon reveals explosive learning in humans

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Similarities among words affect language acquisition and processing in a multi-relational way barely accounted for in the literature. We propose a multiplex network representation of word similarities in a mental lexicon as a natural framework for investigating large-scale cognitive patterns. Our model accounts for semantic, taxonomic, and phonological interactions and identifies a cluster of words of higher frequency, easier to identify, memorise and learn and with more meanings than expected at random. This cluster emerges around age 7 yr through an explosive transition not reproduced by null models. We relate this phenomenon to polysemy, i.e. redundancy in word meanings. We show that the word cluster acts as a core for the lexicon, increasing both its navigability and robustness to degradation in cognitive impairments. Our findings provide quantitative confirmation of existing psycholinguistic conjectures about core structure in the mental lexicon and the importance of integrating multi-relational word-word interactions in suitable frameworks.

Investigating relationships between words offers insights into both the structure of language and the influence of cognition on linguistic tasks [1, 2]. As a result, cognitive network science is rapidly emerging at the interface between network theory, statistical mechanics and cognitive science [1–4]. The field is influenced by the seminal work from Collins [5], who assumed that concepts in the human mind are cognitive units, each representable as a node linked to associated elements. These connections represent a complex cognitive system known as the mental lexicon [6]. Extensive empirical research has shown that relationships in the lexicon can be modelled as a network of mental pathways influencing both how linguistic information is acquired [2, 7, 11], stored [3, 6, 7, 12], and retrieved [3, 8, 13, 14].

The cognitive role of quantification of lexical navigability as network distances finds empirical support in several experiments related to word identification and retrieval tasks [5, 13, 15, 16]. For instance, Collins and Loftus [14] showed a correlation between network topology of semantic networks and word processing times: words farther apart on the network require longer identification times, thus indicating higher cognitive effort. More recently, the structural organisation of mental pathways among words was analysed in several large-scale investigations, considering similarity of words in terms of their semantic meaning [3, 17, 18], their phonology [8, 12, 19, 21], or their taxonomy [14, 22, 23]. Remarkably, all these networks, based on different definitions of relationships between words, were found to be highly navigable (sometimes called small-worlds [24]): words were found to be clustered with each other and separated by small network distances. This may suggest a universal structure of language organisation related to minimising cognitive load while maximising navigability of words [2, 4, 25, 26].

The above studies, however, have not yet attempted to use multi-relational information for characterising and quantifying the mental lexicon, often focusing on only one relationship at a time [3, 10, 13, 17, 18, 26]. Some researchers have considered the aggregation of several of these relationships into single-layer networks [17] and others have considered multi-relational models but only to capture the syntactic structure of language [23]. However, the above approaches offer only limited insight into the cognitive complexity that allow individuals to use language [6] with diversity and ease.

More information about the lexical structure can indeed be obtained by accounting, simultaneously, for multiple types of word-word interactions. A natural and suitable framework for this purpose are multilayer networks [27, 31]. Multilayer networks simultaneously encode multiple types of interaction among units of a complex networked system. Therefore, they can be used to extract information about linguistic structures beyond information available from single-layer network analysis [32]. The usefulness of multiplex representations has recently been shown for diverse applications including the human brain [33, 34], social network analysis [35, 37], transportation [38, 39] and ecology [40, 41].

Here, on an unprecedented scale from a multi-
relational perspective, we investigate the semantics, phonology, and taxonomy of the English lexicon as a model of distinct layers of a multiplex network (see Fig. 1). We study the evolution of multiplex connectivity over the developmental period from early childhood (2 years of age) to adulthood (21 years of age).

The proposed multiplex representation provides a powerful framework for the analysis of the mental lexicon, allowing for the capture of sudden structural changes that can not be identified by traditional methods. More specifically, when modelling lexical growth, we observe an explosive emergence of a cluster of words in the lexicon around the age of 7 years, which is not observed in single-layer network analyses. We show that this cluster is beneficial from a cognitive perspective, as its sudden appearance facilitates word processing across several linguistic pathways at once. This boost to cognitive processing also enhances the resilience of the lexicon network when individual words become progressively inaccessible, such as what may happen in cognitive disorders like anoma[42]. These findings represent the first quantitative confirmation and interpretation of previous conjectures [6, 22, 43, 44] about the presence and cognitive impact of a core in the human mental lexicon.

RESULTS

Our multilayer lexical representation (MLR) of words in the mind is a multiplex network [29, 30, 13, 16] made of \( N = 8531 \) words and four layers. Each layer encodes a distinct type of word-word interaction (cf. Fig. 1 (a)): (i) empirical free associations[47], (ii) synonyms[48], (iii) taxonomy/generalisations[48], and (iv) phonological similarities [12]. As shown in Fig. 1 (b), different relationships can connect words that would otherwise be disconnected in some single-layer representations. We considered these relationships with the aim of building a representation accounting for different types of semantic association, either from dictionaries (i.e. synonyms and generalisations) or from empirical experiments (i.e. free associations). We also include sound similarities (i.e. phonological similarities) to capture the main aspects involved in lexical retrieval from the mental lexicon [8, 12, 13]. This set of relationships represents a first approximation to the multi-relational structure of the mental lexicon. Compared to previous work on multiplex modelling of language development [32], the multiplex representation is enriched with node-level attributes related to cognition and language: (i) age of acquisition ratings [49], (ii) concreteness ratings [50], (iii) identification times in lexical decision tasks [51], (iv) frequency of word occurrence in Open Subtitles [52] and (v) the number of unique definitions of a word in WordNet, used to quantify polysemy [17] (see Methods). The analysis of structural reducibility of our multiplex model (see SI) quantifies the redundancy of the network representation [53]. Results suggests that layers should not be aggregated with each other as each network layer contributes uniquely to the structure of the multiplex representation, confirming the suitability of the framework for further investigation.

As already discussed, investigating navigation on linguistic networks proved insightful [8, 13, 17]. Hence we focus on analysing the navigability of our multiplex network [39], identifying word clusters that are fully navigable on every layer, i.e. where any word can be reached from any other word on every layer when considered in isolation. An example is reported in Fig. 1 for a representative multiplex network with two layers. In network theory, these connected subgraphs are also called viable clusters [40] (see Methods). Notice that the largest viable cluster of a single-layer network coincides with its largest connected component [54], i.e. the largest set of nodes all that can be reached from each other within one layer. In multiplex networks the two concepts are distinct, as viable clusters are required to be connected on every layer when considered individually. Removing this constraint of connectedness on every layer leads to the more general definition of multi-layer connected components[39], i.e. the largest set of nodes all connected to each other when jumps across layers are allowed. Fig. 1 (c-e) conveys the idea that the emergence of viable clusters can be due to the addition of particular links in the network.

Our multiplex model contains a single non-trivial (i.e. with more than two nodes) viable cluster composed of 1173 words, about 13.8% of the network size. In the following we refer to this cluster as the largest viable cluster (LVC). For easier reference, we indicate words in the empirical LVC as “LVC-in words” and words outside of the LVC as “LVC-out words”. Reshuffling network links while preserving word degrees leads to configuration model-layers [54] that still display non-trivial LVCs (cf. LVC Rew. in Tab. 1). Further, on average 98.1±0.1% of LVC-in words persist in the viable cluster after rewiring 5% of all the intra-layer links at random. We conclude that the LVC does not break but rather persists also in case of potentially missing or erroneous links in the network dataset (e.g. spurious free associations or mistakes in phonological transcriptions).

In order to further test correlations between network structure and word labels, we also consider a full reshuffling null model (see SI), in which word labels are reshuffled independently on every layer and thus word identification across layers is not preserved. Hence, full reshuffling destroys inter-layer correlations but preserves network topology. Fully reshuffled multiplex networks did not display any non-trivial viable clusters, emphasizing the important role of inter-layer relationships for the presence of the LVC in the empirical data.

In the next section we analyse the evolution of the LVC during language learning over a time period of more than 15 years. We demonstrate the existence of an explosive phase transition [46] in the emergence of the
Emergence of the Largest Viable Cluster

To study the emergence of the LVC during cognitive development, we simulate probabilistic normative word orderings by smearing the age of acquisition dataset[49]. We refer to these orderings as normative acquisition. Smearing allows us to account for the variance in age of acquisition across individuals by introducing a probabilistic interpretation of these orderings (see Methods). We compare the trajectories of normative acquisition against four null models: (i) random word learning (i.e. words are acquired at random), (ii) frequency word learning (i.e. higher frequency words are acquired earlier), (iii) polysemy word learning (i.e. words with a higher count of context-dependent meanings are learned earlier) and (iv) multidegree word learning (i.e. words with more connections—across all layers—are learned earlier). We investigate if modelling the development of the mental lexicon as growth of the empirical multiplex representation according to a given learning scheme matches the explosive transition observed in normative learning. Results are reported in Fig. 2 (a).

Normative acquisition indicates a sudden emergence of the LVC around age $7.7 \pm 0.6$ years, almost four years earlier than expected if learning words at random. Further analysis reveals two distinctive patterns. Firstly, this sudden appearance is robust to fluctuations in word rankings in the age of acquisition ratings (AoA): in all simulations based on AoA reports, after roughly 2500 words have been acquired, a LVC with at least 260 words suddenly appears after adding just a single word to the lexicon. Secondly, the average magnitude of this explosive change is $\Delta L_{\text{AoA}} = (420 \pm 50)$ words. These patterns suggest an explosive phase transition[46, 55, 56] in the structural development of the mental lexicon. To the best of our knowledge, this work is the first detection of such an explosive behaviour in cognitive language data.

Explosive behaviour in the emergence of the LVC is not observed in the random acquisition null model (see Methods), with only a few cases ($\chi_{\text{Ran}} = 32\%$) displaying a discontinuity of more than ten words. Further, the average magnitude of the LVC size change is only $\Delta L_{\text{Ran}} = (30 \pm 10)$ words, a full order of magnitude smaller than in the normative cases. Therefore explosiveness characterises normative acquisition as a genuine pattern of language learning.

Is the explosive appearance of the LVC due to the acquisition of specific links or rather to specific words?
FIG. 2. (a): Evolution of the size of the LVC when words are acquired in ascending order based on: age of acquisition (green dots), frequency (blue diamonds), polysemy (purple triangles), multidegree in the multiplex (brown circles) and at random (orange triangles). The LVC emerges with an explosive transition at 7.7 ± 0.6 years in normative acquisition. Areas represent standard deviations considering randomisations of smeared age of acquisition or ties in the rankings. (b): Comparison of average linguistic features for words in the LVC with normative acquisition in the empirical data and for a partial reshuffling null model with reshuffled node attributes. The curves are rescaled from 0 to 1 by their empirical maximum value and they represent averages over 200 iterations. Error margins are approximately the same size as the dots. Reshuffling node attributes results in a LVC with both reduced concreteness and polysemy values. We note significant gaps between the empirical and randomised data. The observed gap in polysemy values is almost 5 times larger than for concreteness values.

In order to test this, we focus on the set of “critical” words, i.e. the single words whose addition allows for the sudden emergence of the LVCs. We then compare features of these critical words with features of words already within the LVC at the time of its emergence. We test features like node-attributes (e.g. frequency, polysemy, etc.) and node degree. At a 95% confidence level, no difference was found for any feature (sign test, \( p - value = 0.007 \)). This lack of difference suggests that the emergence of the LVC is indeed due to higher-order link correlations rather than local topological features (such as degree) or psycholinguistic attributes. Hence, it is the global layout of links that ultimately drive the explosive appearance of the LVC. As shown also in Fig. \( 1 \) (c-e), links crucial to the formation of the viable cluster might be acquired earlier (Fig. \( 1 \) (c)) but the LVC might appear only later (Fig. \( 1 \) (e)), after some key pathways completing the viable cluster are added to the network (Fig. \( 1 \) (d)).

The explosive emergence of the LVC has an interesting cognitive interpretation. Work in psycholinguistics suggests that frequency is the single most influential word feature affecting age of acquisition \( 49 \) (mean Kendall \( \tau \approx -0.47 \) between frequency and AoA). We thus test whether the LVC growth can be reproduced through early acquisition of highly frequent words, with frequency counts gathered from Open Subtitles \( 32 \). All simulations on the frequency-based ordering display an explosive transition (\( \chi_{frec} = 100\% \)), however, the magnitude of the explosive transition is \( \Delta L_{frec} = 280\pm30 \) words, which is almost 2/3 of the normative one. At a confidence level of 95%, the distribution of frequency-based LVC magnitude changes differs from the normative one (sign test, \( p-value = 0.01 \)). The distribution of ages at which the LVC emerges in the frequency null model overlaps (see Methods) in 21% of cases with the analogous normative one. We further observe that the frequency null model differs from the normative one not only quantitatively (i.e. magnitude and appearance of explosiveness) but also qualitatively: the frequency null model displays a second explosive phase transition in LVC size later in development, at around 10 ± 0.2 years of age. This second transition might be due to the merging of different viable clusters, since we focused only on the largest viable cluster, rather than on viable clusters of non-trivial size. Further analysis reveals that the multiplex network has only one viable cluster, which suddenly expands through a second explosive transition in the frequency-based vocabulary growth model (but not in the normative AoA model). The above differences provide strong evidence that explosiveness in the mental lexicon is not an artifact of correlation of word frequency with language learning patterns.

We next test preferentially learning words with high degree in the multiplex network to see if the LVC emerges earlier than in normative acquisition. Learning higher degree words first makes more links available in the multiplex network. As we said above, it is links that drive the LVC emergence, hence the expectation of earlier LVC appearance. The multidegree null model confirms this
Polysemy is universal across languages [6, 25], even though it can represent an ambiguity in communication [25]: one word can have different meanings depending on the context. Usually in semantic networks [6, 17, 60] one meaning of a word can be represented by its links in its neighborhood (e.g. "character" and "nature" are linked in the synonyms network as they overlap in meaning). Hence one word meaning corresponds to a given neighborhood layout for that word. Polysemic words have different meanings, therefore their neighborhoods can aggregate (e.g. "character" is linked to "nature" in the context of complexion but also to "font" in the context of typography). Reshuffling at random the word labels constituting all the neighborhoods in the network evidently disrupts meanings and their aggregation, thus destroying polysemy. We call this reshuffling "full" as it preserves the structure of connections in the layers but it fully destroys both intra-layer correlations at the endpoints of links and inter-layer correlations of words, thus fully disrupting word meanings. We use full reshuffling as a null model (see Methods and SI) for testing how important polysemy is in determining the presence of the LVC. We fully shuffle 2025 high-polysemy words (i.e. the words making up the heavy tail of the polysemy distribution) and compute the LVC size in the resulting reshuffled multiplex networks. Results are compared against a reference case in which the same number of low-polysemy words are fully reshuffled. No viable cluster emerges on the multiplex networks with fully reshuffled high-polysemy words, while the LVC only shrinks by roughly 13% in case of fully reshuffling low-polysemy words. We conclude that correlations between word meanings and polysemy are indeed necessary in determining the presence of the LVC.

The above results indicate that polysemy does increase lexicon navigability by ultimately giving rise to the LVC, i.e. a relatively small cluster of words that is fully navigable under both semantic, taxonomic, and phonological relationships in the mental lexicon. Such view is in agreement with previous works [14, 17, 25], which point out how polysemy provides long-range connections in the lexicon which can increase navigability through different word clusters on semantic single-layer networks [17].

Psycholinguistic characterisation of the Largest Viable Cluster (LVC)

Next, we explore the impact of the presence of the LVC on cognitive aspects of language such as word processing. Our aim is to explore if words belonging to the LVC (LVC-in) are processed differently than those words not in the LVC (LVC-out), more from a language use perspective rather than a developmental one (which was analysed with the previous null models). Hence, we turn to large-scale datasets of node attributes (see Tab. 1 and Methods). We find (cf. Tab. 1) that words in the largest viable cluster (i) are more frequent in the Open
TABLE I. Average node attributes for words within the LVC and within the largest connected component (LCC) for each individual layer. All the values are medians, except for heavy-tail distributions such as the frequency and polysemy ones, where the arithmetic mean was used instead. All the values are sample-size corrected via Monte Carlo sampling. The last five rows refer to degree-corrected samplings, where the sampled LVC-out words have the same degree of the sampled LVC-in words. Error bars are reported in parentheses for brevity: 3.93(3) means 3.93 ± 0.03.

| Node Attributes   | LVC In | LVC Out | Asso. LCC In | Syno. LCC In | Hyp. LCC In | Phon. LCC In | LCC Int. | LVC Rew. |
|-------------------|--------|---------|--------------|--------------|-------------|--------------|----------|----------|
| Age of Acquisition [ys] | 6.43(2) | 3.71(3) | 5.79(1) | 9.0(1) | 7.8(1) | 7.4(1) | 7.3(1) |
| Concreteness       | 3.93(2) | 2.83(4) | 3.35(5) | 3.45(5) | 3.87(2) | 3.72(2) | 3.71(3) |
| Reaction Times [ms]| 552(1)  | 2.95(1) | 552(1) | 555(1) | 557(1) | 569(1) | 565(1) |
| Log Frequency [Counts]| 3.40(1) | 2.83(1) | 2.86(1) | 2.79(1) | 2.95(1) | 3.02(1) | 3.30(1) |
| Polysemy [Meanings]| 9.7(2)  | 3.6(2) | 4.9(1) | 4.6(2) | 5.8(1) | 7.6(1) | 8.2(1) |

Degree Corrections
| LVC In | LVC Out | Asso. LCC In | Syno. LCC In | Hyp. LCC In | Phon. LCC In | LCC Int. | LVC Rew. |
|--------|---------|--------------|--------------|-------------|--------------|----------|----------|
| Age of Acquisition [ys] | 6.43(2) | 7.62(1) | 7.2(1) | 8.1(1) | 7.5(1) | 6.6(2) | 6.6(2) |
| Concreteness       | 3.93(2) | 3.67(3) | 3.79(2) | 3.42(4) | 3.40(4) | 3.89(2) | 3.90(2) |
| Reaction Times [ms]| 552(1)  | 555(1) | 559(1) | 566(2) | 575(3) | 556(1) | 555(1) |
| Log Frequency [Counts]| 3.40(2) | 2.86(2) | 3.20(1) | 3.21(1) | 3.21(1) | 3.30(2) | 3.36(1) |
| Polysemy [Meanings]| 9.7(2)  | 3.48(2) | 6.8(1) | 8.0(1) | 7.7(1) | 6.4(2) | 8.7(1) |

Subtitles dataset[52], (ii) acquired earlier according to AoA reports [49], (iii) quicker to identify as words in lexical decision tasks [27], (iv) are rated as more concrete concepts [50] and thus more easily memorised [35, 58, 61] and (v) represent more meanings in different semantic areas [9, 57] when compared to LVC-out words.

In Fig. 3 (a–e), we report the cumulative probabilities of finding a word with a given feature less than a threshold T for LVC-in (orange boxes) and LVC-out (blue boxes). For instance, the probability of finding a low frequency word (with frequency ≤ 10) at random is 0.05 within the LVC but almost five times larger outside of the LVC.

FIG. 3. Cumulative probabilities of finding a word with a given feature less than a threshold T for LVC-in (orange boxes) and LVC-out (blue boxes). For instance, the probability of finding a low frequency word (with frequency ≤ 10) at random is 0.05 within the LVC but almost five times larger outside of the LVC.

degree. Results shown below the thick line, in the lower part of Tab. 1, suggest that the degree effect does not fully explain the observed psycholinguistic features of the LVC: a sign test indicates that all the median node-attributes of LVC-in words are higher than those of LVC-out words, at 95% confidence level. Notice that the comparison that does not account for degree is still important since one could easily argue that degree itself can be interpreted as a cognitive component that affects word processing [8, 50].

Tab. 1 also compares the statistics of the LVC against its single-layer counterparts, i.e. the largest connected components [27] (LCC In). We also consider multiplex alternatives to the LVC such as: the intersection across all layers of words in the LCC of each layer (LCC Int, cf. SI) and the LVC in configuration models (LVC Rew.), which consist on average of 40% more words. The empirical LVC consists of words with the most distinct linguistic features compared to the other tested sets of words, in terms of all tested node attributes. Even rewiring all links does not completely disrupt such distinctness (cf. LVC Rew.). These differences in linguistic attributes suggest that the LVC is a better measure of “coreness” for words in the mental lexicon.
than either the LCCs or their intersection, an idea we further test in the next section.

Robustness of the multiplex lexicon and LVC to cognitive impairments

The LVC has been characterised as a set of higher degree words that differ in psycholinguistic features when compared to words located outside the LVC in our multiplex. This suggests that the higher degree, and cognitive correlations, of the LVC may be because the LVC is acting as a core for the mental lexicon. Let us denote the total number of links on a given layer as $L$ and the link density as $p$. As shown in Fig. 4 (a), there are more links within the LVC ($L_{p_{\text{In/In}}}$) across all layers than outside of it ($L_{p_{\text{Out/Out}}}$) or at the interface of the LVC ($L_{p_{\text{In/Out}}}$). Further, across all individual layers the inequality $p_{\text{In/In}} > p_{\text{In/Out}} > p_{\text{Out/Out}}$ holds, denoting the presence of a core-periphery structure for the node partition $\{\text{In}, \text{Out}\}$ [62].

In order to better interpret both the coreness and cognitive impact of the LVC, we perform a resilience analysis of the network by means of numerical experiments. Random word failure provides a plausible toy model for progressive anomia [62] driven by cognitive decline, where words become progressively non-accessible on all the lexicon levels without a clear trend [42].

To simulate progressive anomia, we randomly remove LVC-in and LVC-out words in separate experiments. The maximum number of removed words is 1173, corresponding to the size of the LVC. As a proxy for robustness, we consider the average multiplex closeness centrality, which correlates with the average cognitive effort for identifying and retrieving words within the lexicon [6, 17] and plays a prominent role in early word acquisition as well [52]. The results of this analysis are shown in Fig. 4 (b).

We find that the multiplex representation is robust to random LVC-out word removal: removing almost 1170 LVC-out words only reduces average closeness to a level that is still within a 95% confidence level of the original multiplex. Therefore failure of LVC-out words does not impact the cognitive effort in identifying and retrieving words within the lexicon. Instead, the multiplex lexicon is fragile to random LVC-in word removal: removing 50% of words from the LVC leads to a decrease in closeness 20 times larger than the drop observed for LVC-out words. While considering random removal in both cases, it is true that in general LVC-in words have higher degree than LVC-out words, which might influence the robustness results from a technical perspective. The discrepancy in closeness degradation is only partly due to the higher degree of LVC-in words. Performing degree-corrected LVC-out word deletions still leads to less of a decrease in navigability as compared to LVC-in word deletion, as evident from Fig. 4 (b).

In summary, the multiplex lexicon is fragile to word failures of LVC-in words and robust to random failures of LVC-out words. This difference is a strong indicator that the LVC provides the necessary shortcuts for efficient navigation – with high closeness and thus low cognitive effort – of the mental lexical representation. It is worth remarking that the network’s navigability is expected to increase in the presence of cores [62, 63], further supporting the interpretation that the LVC acts as a core of the multiplex structure. It has been conjectured that the mental lexicon has a core set of concepts [6, 22, 43, 44]; we showed here how various cognitive metrics can be correlated with the LVC, suggesting that future work may benefit from considering the LVC as a quantification of lexical core structure.

DISCUSSION

Previous literature from psycholinguistics has conjectured the existence of a core set of words in the lexicon [6, 22, 43, 44]. Here, for the first time we give large-scale quantitative evidence to support these conjectures. In fact, we identify the largest viable cluster (LVC) of words which: (i) favours the emergence of connectivity which allows for navigation across all layers at once and (ii) acts as a core for the multiplex lexical representation. Words within the LVC display distinct cognitive features, being (i) more frequent in usage [62], (ii) learned earlier [49], (iii) more concrete [50] and thus easily memorised [6, 50], and activating perceptual regions of the brain [61], (iv) with more context-dependent meanings [9, 57] and (iv) more easily identified in lexical decision tasks [51] than words outside the LVC. Remarkably, the explosive emergence of the LVC happens around age 7 years, which is also a crucial stage for cognitive development in children. According to Piaget’s theory of cognitive development, age 7 years is the onset of the concrete operational stage, in which children develop more semantic and taxonomic relationships among concepts (e.g. recognising that their cat is a Siamese, that a Siamese is a type of cat and that a cat is an animal, thus drawing the conclusion that their cat is an animal among several). Experimental evidence [64] has also shown that, in this developmental stage, children display an increased ability of mental planning and usage of context-dependent words in a connected discourse such as narratives [64]. Interestingly, both these findings can be interpreted in terms of an increased ability to navigate context-dependent meanings in the mental lexicon, which we quantitatively link to the explosive emergence of LVC core structure above. This indicates that the multiplex lexical network is a powerful representation of the mental lexicon: the network structure can indeed capture and translate well documented mental processes driving cognitive development into quantifiable information.

One limitation of our current approach is that we do not consider lexical restructuring over time, i.e. the adults’ representation of word relationships could be different compared to children’s or adolescents’. Pre-
Core/periphery network organisation is commonly found in many real-world systems [63, 65], even though the definition of cores in multiplex networks remains an open challenge. We interpret the robustness experiments as quantitative indication that the LVC is acting as a core for the whole multiplex lexical network, increasing navigability in two ways. Within the LVC, words must be connected to each other, implying navigability from every word within the LVC across all individual layers. Outside of the LVC, connections to the viable cluster facilitate network navigation by making words closer to each other. Since closeness correlates with the cognitive effort in word processing [5, 13, 17], the LVC can be considered as facilitating mental navigation through pathways of the mental lexicon. This quantitative result is in agreement with previous conjectures about polysemy facilitating mental navigation of words [14, 17, 25]. Additionally, our results also indicate that the LVC acts as a multiplex core, with robustness to node failure due to densely entwined links and connections allowing navigation even in cases where words become inaccessible, as in cognitive disorders like progressive anoma [12] or due to individual variability early in development. It is worth remarking that we identify such a core with the largest LVC as no other non-trivial viable cluster exists in the MLR.

Indeed, identifying a core in the mental lexicon provides quantitative evidence supporting previous claims [43, 44] about the existence of a core of highly frequent and concrete words in the lexicon facilitating mental navigation and thus word retrieval in speech production experiments [43, 44, 58]. Alongside the cognitive perspective, interpreting the LVC as a lexicon core provides support for further previous findings about the presence of a "kernel lexicon" in language [14, 18, 22].

FIG. 4. (a) Normalised link densities across layers for couples of nodes either in the LVC (In), out of the LVC or on the boundary (one node in, one node out). Densities are normalised by the maximum value $L_{P_{In}}$ for generalisations and colour coded (the higher the value, the more red the cell). (b) Resilience analysis with respect to random word failure, mimicking progressive aphasia in the mental lexicon. Words are targeted at random and then removed from the whole multiplex. In LVC Out (Deg. Corr.) we remove words from outside the LVC but with the same degree as the words removed inside the LVC, thus correcting for a degree effect seen in the LVC which will also effect efficiency. As a measure of efficiency we use the median closeness of words in the network, providing the inverse of the average number of network hops necessary for reaching any word from any other one through the multiplex topology. Error margins represent standard deviations and they are about the size of the dots.

| Associations | Synonyms |
|-------------|----------|
| **In**      | **Out**  |
| 0.82        | 0.22     |
| 0.45        | 0.07     |
| **In**      | **Out**  |
| 0.22        | 0.08     |
| 0.07        | 0.03     |
| **Generalisations** | **Phon. Similarities** |
| **In**      | **Out**  |
| 1.00        | 0.24     |
| 0.58        | 0.1      |
| **In**      | **Out**  |
| 0.24        | 0.1      |
| 0.1         | 0.03     |

### Table 1: Sign Test (p-value)

|                     | LVC In | LVC Out | LVC Out (Deg. Corr.) |
|---------------------|--------|---------|----------------------|
| Closeness Ratio     | 1.00   | 0.95    | 0.90                 |
| Number of Failing Nodes $n$ | 0      | 250     | 500                  |
|                     | 0.85   | 0.90    | 0.95                 |
|                     | 0.90   | 0.95    | 1.00                 |
a set of a few thousand words which constitute almost 80% of all written text [6] and can define every other word in language [22]. Previous works on semantic [13 18], taxonomic [22] and phonological [8 19] single-layer networks identified a kernel lexicon for the English language with roughly 5000 words which has not changed in size during the evolution of languages. This kernel lexicon was identified with the largest connected component of the English phonological network [8 19]. The LVC we present here is a subset of the largest connected components, even of the phonological one, and it also persists across semantic, taxonomic and phonological aspects of language. Hence, the LVC represents a further refinement of the kernel lexicon that: (i) it greatly rich in polysemous words, (ii) facilitates mental navigation and (iii) it is robust to rewiring or cognitive degradation. These three features suggest an interpretation of the LVC as a linguistic core of tightly interconnected concepts facilitating mental navigation through key words.

While the framework presented here has been applied only for the English language, comparison with other languages and linguistic representations to assess how universal the LVC core is remains an exciting challenge for future experimental and theoretical work.

**METHODS**

**Dataset and cognitive interpretation**

The data used in this work come from different sources and thus the resulting multiplex network representation is based on independent studies. For our multiplex we construct four layers that model semantic, taxonomic and phonological relationships. We further distinguish semantic relationships in free associations and synonyms. For free associations, e.g. "A reminds one of B", we used the Edinburgh Associative Thesaurus [47]. For both, generalisations (e.g. "A is a type of B") and synonyms (e.g. "A also means B") we used WordNet 3.0 [48]. For phonological similarities we used the same dataset analysed in [20] based on WordNet 3.0 [48]. We treat every layer as undirected and unweighted. Words in the multiplex representation are required to be connected on at least one layer.

Free associations indicate similarities within semantic memory, i.e. when given a cue word "house", human participants respond with words that remind them of "house", for example "bed" or "home". Networks of free associations play a prominent role in capturing word acquisition in toddlers [11 32] and also word identification [3 13]. Also networks of synonyms are found to play a role in word identification [6]. The hierarchy provided by taxonomic relationships deeply affects both word learning and word processing [5 6]. Phonological networks provide insights about the competition of similar sounding words for confusability in word identification tasks [8 13 20].

For the linguistic attributes we combined several different sources. We sourced word frequency from Open-Subtitles [52], a dataset of movie subtitles whose word frequencies were found to be superior to frequencies from classical sources in explaining variance in the analysis of reaction times from lexical decision experiments [51 52]. Concreteness scores [50] and age of acquisitions ratings [49] were gathered from Amazon Turk experiments, allowing for large-scale data collection and confirmation of previous findings based on small-scale experiments [49 50]. The concreteness ratings indicate how individual concepts are rated (on a scale of 1 - "abstract" to 5 - "concrete") as abstract rather than perceptual [50]. Polysemy was quantified as the number of different definitions for a given word in WordData from Wolfram Research [67]. Reaction times were obtained from the British Lexicon Project [51] and indicate the response time in milliseconds for the identification of individual words were compared against non-words.

**Smearing normative acquisition**

Smearing is a technique used in statistics for improving the quality of data [59]. We smear the age of acquisition data from Kuperman et al. [49], where the average age of acquisition $a_i$ and standard deviation $\sigma_a(i)$ around each word are provided, e.g. $a_{aim} = 6.72 \text{yrs}, \sigma_a(a_{aim}) = 2.11 \text{yrs}$. In our case, smearing consists of sampling possible age of acquisitions for word $i$ from a Gaussian distribution $N[a_i, \sigma_a(i)]$ rather than considering only the average value. Sampling independently an age of acquisition for each word in the dataset, we can build multiple artificial acquisition rankings $r_{AoA}$ of words from the empirical data.

**Lexicon growth experiments**

We simulate lexicon growth over time $t(n)$ by considering subgraphs of the multiplex lexicon where only the first $n \leq 8531$ words in a given ranking $r$ are considered. Rankings indicate the way words are acquired in the lexicon over time and can be based on word features or age of acquisition reports. The rankings we use are based on: (i) smeared age of acquisition [49], (ii) frequency [49, 52] (higher frequency words are learned earlier), (iii) multidegree [27] (words with more links across all layers are learned earlier), and (iv) polysemy (words with more meanings are learned earlier). As a randomised null model, we consider random word rankings. When the first $n$ words in a ranking are considered, a subgraph of the multiplex lexicon with these words is built and its LVC is detected. By using the non-smeared age of acquisitions, we relate the number of learned words to the developmental stage in years $t(n)$, e.g. $n = 1000$ corresponds to $t = 5.5$ years.

The size of the LVC $L(t)$ is then obtained as a function
of developmental stage $t(n)$ for every specific type of ranking. Results for the smeared age of acquisitions and the random null model are averaged over an ensemble of 200 iterations. Results for the frequency, degree and polysemy orderings are averaged over 200 iterations where words appearing in ties are reshuffled. Results are reported in Fig. 2.

Each iteration represents the evolution of the LVC size through the acquisition of an individual word. This acquisition trajectory may be related to different developmental stages. For every iteration, we detect the magnitude of the transition on the LVC size due to its appearance when adding words one by one to the network. We then compute the fraction $\chi$ of iterations presenting a discontinuity of more than 10 LVC-in words size. We also compute the average magnitude of the explosive transition $\Delta L$.

Comparisons of the empirical distributions of ages at which the LVC emerges considers the overlapping coefficient [66], i.e. the overlap of two distributions normalised by the maximum overlap obtained when shifting the central moment of one of the distributions. An overlap of 100% means that one distribution is fully contained in the other one. An overlap of 0% means that the distributions have no overlap.

Robustness experiments

We carried out robustness testing via word/node removal: individual words are removed at random across all layers. Closeness centrality is then measured by considering shortest paths across the whole multiplex network structure, i.e. also including jumps between layers. We consider closeness centrality as a measure for the spreading of information and the mental navigability of the lexicon [13, 14, 19]. In our case closeness is well defined, since even the deletion of the whole LVC leaves the multiplex network connected[29]. We consider a multiplex network as connected if it is possible to reach any pair of nodes by allowing for traversal along links on any layers.

With reference to Fig. 3, we perform random attacks of words within the LVC (LVC In) and outside of it (LVC Out). Since LVC-in words are more connected compared to words outside, we also perform degree corrected attacks: we perform attacks of random words within the LVC and words of equivalent degree outside the LVC. This degree correction (LVC Out - Deg. Corr.) allows for the attack of LVC-out words but reduces the number of links by the same amount as LVC-in attacks.

Data availability and Additional Information

Data sharing not applicable to this article as no new datasets were generated during the current study. The authors declare no competing financial interests. Material requests should be addressed to the corresponding author.

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