Abstract. In this article, we establish a Liouville type theorem for the anisotropic singular problem

\[ \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \left( \left| \frac{\partial u}{\partial x_i} \right|^{p_i-2} \frac{\partial u}{\partial x_i} \right) = e^{\frac{1}{p}} \quad \text{in} \quad \mathbb{R}^N \]

concerning positive \( W^{1,p_i}(\mathbb{R}^N) \) stable solution, provided \( N \geq 1 \) and \( p_i > 2 \) for all \( i = 1, 2, \ldots, N \).

1. Introduction

The main purpose of this article is to establish a Liouville type theorem concerning positive stable solutions to the anisotropic singular equation

\[
\begin{cases}
-Lu := -\sum_{i=1}^{N} \frac{\partial}{\partial x_i} \left( \left| \frac{\partial u}{\partial x_i} \right|^{p_i-2} \frac{\partial u}{\partial x_i} \right) = f(u) \quad \text{in} \quad \mathbb{R}^N, \\
u > 0 \quad \text{in} \quad \mathbb{R}^N,
\end{cases}
\]

where \( f(u) = -e^{\frac{1}{p}}, \) \( N \geq 1 \) and \( 2 < p_1 \leq p_2 \leq \cdots \leq p_N \).

Observe that in case \( p_i = 2 \) for all \( i \), the operator \( L \) is the classical Laplacian and for \( p_i = p \),

\[ Lu = \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \left( \left| \frac{\partial u}{\partial x_i} \right|^{p-2} \frac{\partial u}{\partial x_i} \right) \]

which is called the pseudo \( p \)-Laplacian, see [4, 16] for more details.

Anisotropic operator defined by \( L \) has become a topic of considerable attention in the recent years. It is a non-homogeneous operator which appears in many physical phenomena, for example it reflects anisotropic physical properties of some reinforced materials [19], image processing [23], to study the dynamics of fluids in anisotropic media when the conductivities of the media are different in each direction [1].

Throughout the paper we denote by \( \Omega = \mathbb{R}^N \) and \( \bar{p} \) to be the harmonic mean of \( p_1, p_2, \ldots, p_N \) i.e.

\[ \frac{1}{\bar{p}} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{p_i} \]

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We consider the solution space to be the anisotropic Sobolev space $W^{1,p_1}(\Omega)$ defined by

$$W^{1,p_1}(\Omega) = \{ v \in W^{1,p}(\Omega) : \frac{\partial v}{\partial x_i} \in L^{p_i}(\Omega) \}$$

For a general theory of anisotropic Sobolev space, see \[10\] \[11\] \[13\] \[21\] \[22\]. Let us firstly define the meaning of stable solution for the equation (1.1).

**Definition 1.1.** (Weak Solution:) A function $u \in W^{1,p_1}(\Omega)$ is said to be a weak solution of the equation (1.1), if $u > 0$ a.e. in $\Omega$ such that $f(u) \in L^1_{loc}(\Omega)$ and for all $\varphi \in C^1_c(\Omega)$, we have

$$\sum_{i=1}^N \int_\Omega \left| \frac{\partial u}{\partial x_i} \right|^{p_i-2} \frac{\partial u}{\partial x_i} \frac{\partial \varphi}{\partial x_i} \, dx - \int_\Omega f(u) \varphi \, dx = 0 \quad (1.2)$$

**Definition 1.2.** (Stable Solution:) A function $u \in W^{1,p_1}(\Omega)$ is said to be a stable solution of the equation (1.1), if $u$ is a weak solution such that $f'(u) \in L^1_{loc}(\Omega)$ and for all $\varphi \in C^1_c(\Omega)$, we have

$$\int_\Omega f'(u)\varphi^2 \, dx \leq \sum_{i=1}^N (p_i - 1) \int_\Omega \left| \frac{\partial u}{\partial x_i} \right|^{p_i-2} \left| \frac{\partial \varphi}{\partial x_i} \right|^2 \, dx \quad (1.3)$$

Let us mention that in the pioneering work \[20\], Li.Ma and J.Wei studied the isotropic model

$$\Delta u = g(u) \text{ in } \Omega$$

with $g(u) = u^{-\delta}$, providing sufficient conditions on $\delta > 0$ for the non-existence of positive $C^1(\Omega)$ stable solutions. The exponential non-linearity case $g(u) = -e^u$ has been studied by Farina \[15\] and later extended by Phuong Le \[17\] to the $p$-Laplace operator. Very recently Chen et al. \[9\] considered the singular $p$-Laplace equation

$$\text{div}(|\nabla u|^{p-2} \nabla u) = f(x) u^{-\delta} \text{ in } \Omega$$

where sufficient conditions of $f$ and $\delta > 0$ has been obtained for the non-existence of positive $C^1_{loc}(\Omega)$ stable solutions. In the isotropic case, a wide number of literature in this direction can be found in \[2\] \[3\] \[5\] \[7\] \[8\] \[12\] \[13\] \[18\] with various type of non-linearity. To the best of our knowledge, analogous results has not been intensively studied for the anisotropic operator. Our main motive in this paper is to provide a Liouville type theorem for positive $W^{1,p_1}(\Omega)$ stable solutions of the equation (1.1) in the framework of anisotropic operator $L$. Before stating our main result let us fix some notation as mentioned below.

**Notation:**

- $q = \sum_{i=1}^N \frac{p_i}{N}$
- $l_1 = \frac{2N}{2 + N}$ and $l_2 = \frac{2}{MN(q-1)} - \frac{q-1}{2}$
- $J = (0, \frac{4}{N(q-1)(pN-1)}) \cap (0, \frac{4}{N(q-1)(q-1)(N-1)})$
- $u_i = \frac{\partial u}{\partial x_i}$, for all $i = 1, 2, \ldots, N$

Moreover, we denote by $C$ to be a constant whose values may vary from line to line or even in the same line. If $C$ depends on $r_1, r_2, \ldots, r_m$ we denote it by $C(r_1, r_2, \ldots, r_m)$. The main result of this paper is the following theorem:

**Theorem 1.3.** Let $u \in W^{1,p_1}(\Omega)$ be positive a.e. in $\Omega$ such that $||u||_\infty \leq M$, provided $M \in J$. Then $u$ is not a stable solution to the equation (1.1).
Before proceeding to prove Theorem 1.3 we obtain a Caccioppoli type estimate on the positive stable solutions of the equation (1.1) stated below.

**Lemma 1.4. (Caccioppoli type estimate:)** Let \( u \in W^{1,p_1}(\Omega) \) be a bounded positive stable solution to the equation (1.1) such that \( \|u\|_\infty \leq M \) where \( M \in J \). Then for any \( \beta \in (l_1, l_2) \), there exists a positive constant \( C(\beta, p_1, p_2, \cdots, p_N, q, N) \) such that for every non-negative \( \psi \in C_0^1(\Omega) \), we have

\[
\int \left( \frac{\psi}{u} \right)^{2\beta + q} dx \leq C \sum_{i=1}^N \int \psi_i^{2\beta + q} dx
\]

Define for \( k \in \mathbb{N}, \alpha > 1 \) and \( t \geq 0 \), the following two functions:

\[
a_k(t) = \begin{cases} \frac{1-\alpha}{2k} (t + \frac{1+\alpha}{k(1-\alpha)}), & \text{if } 0 \leq t < \frac{1}{k}, \\ t^{1-\alpha}, & \text{if } t \geq \frac{1}{k} \end{cases}
\]

\[
b_k(t) = \begin{cases} \frac{-\alpha}{k} (t - \frac{1+\alpha}{k\alpha}), & \text{if } 0 \leq t < \frac{1}{k}, \\ t^{-\alpha}, & \text{if } t \geq \frac{1}{k} \end{cases}
\]

Then it can be easily verified that both \( a_k \) and \( b_k \) are positive \( C^1[0, \infty) \) decreasing functions. Moreover, \( a_k \) and \( b_k \) satisfies the following properties:

a. \( a_k(t)^2 \geq t b_k(t), \forall \ t \geq 0. \)

b. \( a_k(t)^{p_i} |a_k'(t)|^{2-p_i} + b_k(t)^{p_i} |b_k'(t)|^{1-p_i} \leq C |t|^{p_i-\alpha-1} \)

for some positive constant \( C \) depending on \( p_1, p_2, \cdots, p_N, \alpha \), provided \( \alpha > p_i - 1 \) for all \( i = 1, 2, \ldots, N. \)

c. \( a_k'(t)^2 = \frac{(\alpha - 1)^2}{4\alpha} |b_k'(t)|, \forall \ t \geq 0. \)

**Proof of Lemma 1.4**

Let \( u \in W^{1,p_1}(\Omega) \) be a bounded positive stable solution to the equation (1.1) such that \( \|u\|_\infty \leq M \) for some \( M \in J \). Then \( u \) satisfies both (1.2) and (1.3).

Let \( \psi \in C_0^1(\Omega) \) be non-negative in \( \Omega \). We prove the lemma in the following two steps:

**Step 1.** Choosing \( \phi = b_k(u)^{\psi^q} \) as a test function in (1.2), we have

\[
\sum_{i=1}^N \int_{\Omega} |b_k(u)| u_i |p_i\psi^q| dx \leq q \sum_{i=1}^N \int_{\Omega} \psi^{q-1} b_k(u) |u_i|^{p_i-2} u_i \psi_i dx - \int_{\Omega} f(u) b_k(u)^{\psi^q} dx
\]

Using Young’s inequality with \( \epsilon \in (0, 1) \) we obtain

\[
q \sum_{i=1}^N \int_{\Omega} \psi^{q-1} b_k(u) |u_i|^{p_i-2} u_i \psi_i dx
\]
\begin{align*}
&\leq \epsilon \sum_{i=1}^{N} \int_{\Omega} |b_k'(u)||u|^{p_i} \psi^q \, dx + C \sum_{i=1}^{N} \int_{\Omega} |b_k(u)|^{p_i} |b_k'(u)|^{1-p_i} |\psi_i|^{p_i} \psi^{q-p_i} \, dx \\
&\text{for some positive constant depending on } \epsilon, p_1, p_2, \ldots, p_N, q.
\end{align*}

Therefore for } \epsilon \in (0, 1) \text{ we obtain

\begin{equation}
(1-\epsilon) \sum_{i=1}^{N} |b_k'(u)||u|^{p_i} \psi^q \, dx \leq C \sum_{i=1}^{N} \int_{\Omega} |b_k(u)|^{p_i} |b_k'(u)|^{1-p_i} |\psi_i|^{p_i} \psi^{q-p_i} \, dx - \int_{\Omega} f(u)b_k(u)\psi^q \, dx
\end{equation}

\text{Step 2. Choosing } \phi = a_k(u)\psi^q \text{ in the inequality (1.3) we obtain

\begin{equation}
\int_{\Omega} f'(u)a_k(u)^2 \psi^q \, dx \leq \sum_{i=1}^{N} (p_i - 1) \{ X_i + \frac{q^2}{4} Y_i + qZ_i \}
\end{equation}

\text{where

} X_i = \int_{\Omega} |a_k'(u)|^2 |u|^{p_i} \psi^q \, dx \quad Y_i = \int_{\Omega} \psi^{q-2} a_k(u)^2 |u|^{p_i-2} |\psi_i|^2 \, dx

\text{and

} Z_i = \int_{\Omega} |a_k'(u)| a_k(u) \psi^{q-1} |u|^{p_i-1} |\psi_i| \, dx

\text{Using (c.) noting that

} X_i = \frac{(\alpha - 1)^2}{4\alpha} \int_{\Omega} |b_k'(u)||u|^{p_i} \psi^q \, dx

\text{we obtain from (1.6)

} \sum_{i=1}^{N} X_i = \frac{(\alpha - 1)^2}{4|\alpha|} \sum_{i=1}^{N} \int_{\Omega} |b_k'(u)||u|^{p_i} \psi^q \, dx

\text{\leq } \frac{(\alpha - 1)^2}{4|\alpha|(1-\epsilon)} \{ C \sum_{i=1}^{N} \int_{\Omega} |b_k(u)|^{p_i} |b_k'(u)|^{1-p_i} |\psi_i|^{p_i} \psi^{q-p_i} \, dx - \int_{\Omega} f(u)b_k(u)\psi^q \, dx \}

\text{Moreover, using Young's inequality we have the estimates

} (p_i - 1)\frac{q^2}{4} Y_i = (p_i - 1)\frac{q^2}{4} \int_{\Omega} \psi^{q-2} a_k(u)^2 |u|^{p_i-2} |\psi_i|^2 \, dx

= (p_i - 1)\frac{q^2}{4} \int_{\Omega} (|u|^{p_i-2}|a_k'(u)|^{\frac{2(p_i-2)}{p_i}} |\psi|^{\frac{2(p_i-2)}{p_i}})(a_k(u)^2 |a_k'(u)|^{\frac{2(p_i-2)}{p_i}} |\psi_i|^2 |\psi|^{\frac{2(p_i-2)}{p_i}}) \, dx

\leq \frac{\epsilon}{2N} X_i + \frac{C(\epsilon, p_1, p_2, \ldots, p_N, q, N)}{2} \int_{\Omega} a_k(u)^{p_i} |a_k'(u)|^{2-p_i} |\psi_i|^p |\psi|^{q-p_i} \, dx

\text{and

} (p_i - 1)qZ_i = (p_i - 1)q \int_{\Omega} |a_k'(u)| a_k(u) \psi^{q-1} |u|^{p_i-1} |\psi_i| \, dx

= (p_i - 1)q \int_{\Omega} (|u|^{p_i-1}|a_k'(u)|^{\frac{2}{p_i}} |\psi_i|^{\frac{2}{p_i}})(a_k(u)^2 |a_k'(u)|^{\frac{2}{p_i}} |\psi|^{p_i} |\psi|^{q-p_i}) \, dx

\leq \frac{\epsilon}{2N} X_i + \frac{C(\epsilon, p_1, p_2, \ldots, p_N, q, N)}{2} \int_{\Omega} a_k(u)^{p_i} |a_k'(u)|^{2-p_i} |\psi_i|^p |\psi|^{q-p_i} \, dx

\text{Using the above estimates in (1.6) together with (a.) and (b.) we obtain

\begin{align*}
&\text{for some positive constant depending on } \epsilon, p_1, p_2, \ldots, p_N, q.
\end{align*}
\[
\begin{align*}
\int_{\Omega} u f'(u) b_k(u) \psi^q \, dx &\leq \int_{\Omega} f'(u) a_k(u) \psi^q \, dx \\
&\leq \sum_{i=1}^{N} (p_i - 1 + \frac{\epsilon}{N}) X_i + C(\epsilon, p_1, \ldots, p_N, q, N) \sum_{i=1}^{N} \int_{\Omega} a_k(u)^{p_i} |a_k'(u)|^{2-p_i} |\psi|^{p_i} \psi^{q-p_i} \, dx \\
&\leq (p_1 - 1 + \frac{\epsilon}{N}) \sum_{i=1}^{N} X_i + (p_2 - 1 + \frac{\epsilon}{N}) \sum_{i=1}^{N} X_i + \ldots + (p_N - 1 + \frac{\epsilon}{N}) \sum_{i=1}^{N} X_i \\
&\quad + C(\epsilon, p_1, \ldots, p_N, q, N) \sum_{i=1}^{N} \int_{\Omega} a_k(u)^{p_i} |a_k'(u)|^{2-p_i} |\psi|^{p_i} \psi^{q-p_i} \, dx \\
&= (N(q-1) + \epsilon) X_i + C(\epsilon, p_1, \ldots, p_N, q, N) \sum_{i=1}^{N} \int_{\Omega} a_k(u)^{p_i} |a_k'(u)|^{2-p_i} |\psi|^{p_i} \psi^{q-p_i} \, dx \\
&\quad - \int_{\Omega} f(u) b_k(u) \psi^q \, dx + C(\epsilon, p_1, \ldots, p_N, q, N) \sum_{i=1}^{N} \int_{\Omega} a_k(u)^{p_i} |a_k'(u)|^{2-p_i} |\psi|^{p_i} \psi^{q-p_i} \, dx \\
&\leq C(\epsilon, p_1, \ldots, p_N, q, N, \alpha) \sum_{i=1}^{N} \int_{\Omega} \left\{ b_k(u)^{p_i} |a_k'(u)|^{1-p_i} + a_k(u)^{p_i} |a_k'(u)|^{2-p_i} \right\} |\psi|^{p_i} \psi^{q-p_i} \, dx \\
&\quad - \frac{(\alpha - 1)^2 (N(q-1) + \epsilon)}{4\alpha(1 - \epsilon)} \int_{\Omega} f(u) b_k(u) \psi^q \, dx \\
&\leq C(\epsilon, p_1, \ldots, p_N, q, N, \alpha) \sum_{i=1}^{N} \int_{\Omega} \left| u \right|^{p_i - \alpha - 1} |\psi|^{p_i} \psi^{q-p_i} \, dx \\
&\quad - \frac{(\alpha - 1)^2 (N(q-1) + \epsilon)}{4\alpha(1 - \epsilon)} \int_{\Omega} f(u) b_k(u) \psi^q \, dx
\end{align*}
\]

Putting \( f(u) = -e^{\frac{u}{\beta}} \) and using the assumption \( \|u\|_\infty \leq M \), we obtain

\[
\alpha \epsilon \int_{\Omega} e^{\frac{u}{\beta}} b_k(u) \psi^q \, dx \leq C(\epsilon, p_1, \ldots, p_N, q, N, \alpha) \sum_{i=1}^{N} \int_{\Omega} \left| u \right|^{p_i - \alpha - 1} |\psi|^{p_i} \psi^{q-p_i} \, dx
\]

where \( \alpha_\epsilon = \frac{1}{M} - \frac{(\alpha - 1)^2 (N(q-1) + \epsilon)}{4\alpha(1 - \epsilon)} \).

Choose \( \alpha = 2\beta + q - 1 \). Note that \( \beta > l_1 \) implies \( \alpha > p_N - 1 \geq p_i - 1 \), \( \forall i = 1, 2, \ldots, N \). Now

\[
\lim_{\epsilon \to 0} \alpha_\epsilon = \frac{1}{M} - \frac{N(q-1)(\alpha - 1)^2}{4\alpha} > 0 \forall \beta \in (l_1, l_2)
\]

Hence we can fix \( \beta \in (l_1, l_2) \) and choose \( \epsilon \in (0, 1) \) such that \( \alpha_\epsilon > 0 \). Using \( e^x > x \) for \( x > 0 \) in the above estimate we obtain

\[
\int_{\Omega} \frac{1}{u} b_k(u) \psi^q \, dx \leq \int_{\Omega} e^{\frac{u}{\beta}} b_k(u) \psi^q \, dx \leq C \sum_{i=1}^{N} \int_{\Omega} \left| u \right|^{p_i - 2\beta - q} |\psi|^{p_i} \psi^{q-p_i} \, dx
\]
for some positive constant $C$ depending on $\beta, p_1, \cdots, p_N, q, N$. By the monotone convergence theorem we obtain
\[
\int_{\Omega} u^{-2\beta - q}\psi^q \, dx \leq C \sum_{i=1}^{N} \int \psi_i^{p_i - 2\beta - q} |\psi_i|^{p_i} \psi^q \, dx
\]
Replacing $\psi$ by $\psi^{2\beta + q}$ and using the Young’s inequality for $\epsilon \in (0, 1)$ with exponents
\[
\gamma_i = \frac{2\beta + q}{2\beta + q - p_i}, \quad \gamma_i' = \frac{2\beta + q}{p_i}
\]
in the above inequality we obtain
\[
\int_{\Omega} \left( \frac{\psi}{u} \right)^{2\beta + q} \, dx \leq C \sum_{i=1}^{N} \int_{\Omega} \left( \frac{\psi_i}{u} \right)^{2\beta + q - p_i} |\psi_i|^{p_i} \, dx \leq \epsilon \int_{\Omega} \left( \frac{\psi}{u} \right)^{2\beta + q} \, dx + C \sum_{i=1}^{N} \int_{\Omega} |\psi_i|^{2\beta + q} \, dx
\]
Hence we obtain
\[
\int_{\Omega} \left( \frac{\psi}{u} \right)^{2\beta + q} \, dx \leq C \sum_{i=1}^{N} \int_{\Omega} |\psi_i|^{2\beta + q} \, dx
\]
for some positive constant $C$ depending on $\beta, p_1, \cdots, p_N, q, N$ which is the required inequality.

**Proof of Theorem 1.3** Let $u \in W^{1,p_i}(\Omega)$ be a positive stable solution of the equation (1.1) such that $||u||_\infty \leq M$ for some $M \in J$. Then by the Lemma 1.4 we have
\[
\int_{\Omega} \left( \frac{\psi}{u} \right)^{2\beta + q} \, dx \leq C \sum_{i=1}^{N} \int_{\Omega} |\psi_i|^{2\beta + q} \, dx
\]
for some positive constant $C$ depending on $\beta, p_1, \cdots, p_N, q, N$. Choosing $\psi \in C^1_c(\Omega)$ such that $0 \leq \psi \leq 1$ in $\Omega$, $\psi \equiv 1$ in $B_R(0)$ and $\psi = 0$ in $\Omega \setminus B_{2R}(0)$ with $|\nabla \psi| \leq \frac{C}{R}$ for some constant $C > 0$ (independent of $R$) in the above inequality, we obtain
\[
\int_{B_R(0)} \left( \frac{1}{u} \right)^{2\beta + q} \, dx \leq C R^{N-2\beta - q} \tag{1.7}
\]
where $C$ is a positive constant independent of $R$. Observe that, since $M \in J$ we have $0 < M < \frac{1}{N(N-1)(q-1)}$ which implies $N < 2l_2 + q$ and hence
\[
\lim_{\beta \to l_2} (N - 2\beta - q) = N - 2l_2 - q < 0.
\]
As a consequence, we can choose $\beta \in (l_1, l_2)$ such that $N - 2\beta - q < 0$.

Therefore, letting $R \to \infty$ in (1.7), we obtain
\[
\int_{\Omega} \left( \frac{1}{u} \right)^{2\beta + q} \, dx = 0
\]
which is a contradiction. Hence the Theorem follows.

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Prashanta Garain
Department of Mathematics and Statistics, Indian Institute of Technology, Kanpur
Uttar Pradesh-208016, India
E-mail address: pgarain@iitk.ac.in