Lepton Scattering off Few-Nucleon Systems at Medium and High Energies

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Abstract. The interpretation of recent Jlab experimental data on the exclusive process $A(e,e'p)B$ off few-nucleon systems are analyzed in terms of realistic nuclear wave functions and Glauber multiple scattering theory, both in its original form and within a generalized eikonal approximation. The relevance of the exclusive process $^4He(e,e'p)^3H$ for possible investigations of QCD effects is illustrated.

Exclusive and semi-inclusive lepton scattering off nuclei in the quasi elastic region, plays a relevant role in nowadays hadronic physics for the following main reasons: i) due to the wide kinematical range available by present experimental facilities, non trivial information on nuclei (e.g. nucleon-nucleon (NN) correlations) can be obtained; ii) the mechanism of propagation of hadronic states in the medium can be investigated in great details; iii) at high energies QCD related effects (e.g. color transparency effects) might be investigated. At medium and high energies the propagation of a struck hadron in the medium is usually treated within the Glauber multiple scattering approach (GA) \cite{1}, which has been applied with great success to hadron scattering off nuclear targets. However, when the hadron is created inside the nucleus, as in a process $A(l,l'p)X$, various improvements of the original GA have been advocated. Most of them are based upon a Feynman diagram reformulation of GA; such an approach, developed long ago for the treatment of hadron-nucleus scattering \cite{2}, has been recently generalized to the process $A(l,l'p)X$ \cite{3}, demonstrating that in particular kinematical regions appreciable deviations from GA are expected. The merit of the approach, based upon a generalized eikonal approximation (GEA), is that the frozen approximation, common to GA, is partly removed by taking into account the excitation energy of the $A-1$ system; this results in a correction term to the standard profile function of GA, leading to an additional contribution to the

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longitudinal component of the missing momentum. The GEA has recently been applied \[3\] to a systematic calculation of the two-body (2bbu) and three-body (3bbu) break up channels of \(^3\)He electro-disintegration using realistic three-body wave functions \[6\] and two-nucleon interactions (AV18) \[7\]; the two-body break up channel \(^3\)He(\(e, e'p\))\(^2\)H has also been considered within GEA in Ref. \[8\], obtaining results consistent with Ref. \[4\], \(A - 1\).

In GEA the final state wave function has the following form

\[
\Psi_f (r_1, \ldots r_A) = \hat{A} S_{\text{GEA}}(r_1, \ldots r_A) e^{-i\mathbf{p} \cdot \mathbf{r}_1} e^{-i\mathbf{P}_{A-1} \cdot \mathbf{R}_{A-1}} \phi^*_{A-1}(r_2, \ldots r_A) \tag{1}
\]

where \(S_{\text{GEA}} = \sum_{n=2}^{A} S_{\text{GEA}}^{(n)}\) generates the final state interaction (FSI) between the struck particle and the \(A - 1\) nucleon system; in Eq. (1) \(n\) denotes the order of multiple scattering, with the single scattering term \((n=1)\) given by

\[
S_{\text{GEA}}^{(1)}(r_1, \ldots r_A) = 1 - \sum_{i=2}^{A} \theta(z_i - z_1) e^{i\Delta_z (z_i - z_1)} \Gamma(b_1 - b_i) \tag{2}
\]

where \(\Gamma(b) = \sigma_{NN}^{tot}(1 - i\alpha_{NN}) \cdot \exp[-b^2/2b_0^2]/[4\pi b_0^2]\) is the usual Glauber profile function and \(\Delta_z = (q_0/|q|)E_m, E_m\) being the removal energy related to the excitation energy of \(A - 1\); due to the presence of \(\Delta_z\) the frozen approximation is partly removed; note that when \(\Delta_z = 0\), the usual GA is recovered (the expression of the \(n\)-th order contribution \(S_{\text{GEA}}^{(n)}\) is given in Ref. \[3\]).

Within the factorization approximation (FA), the diagrammatic approach leads to the following expression for cross section

\[
\frac{d^6\sigma}{d\Omega' dE' d^3p_m} = K\sigma_{ep} P_{FA}^{FSI}(p_m, E_m), \tag{3}
\]

where \(K\) is a kinematical factor, \(\sigma_{ep}\) the electron-nucleon cross section, \(p_m = q - p\) and \(E_m\) the missing momentum and energy, respectively, \(p\) the momentum of the detected nucleon and, eventually, \(P_{FA}^{FSI}\) the distorted spectral function.

If the FA is relaxed, the differential cross section assumes the following form

\[
\frac{d^6\sigma}{d\Omega' dE' d^3p_m} = \sigma_{Mott} \sum_i V_i W_i^A(\nu, Q^2, p_m, E_m) \tag{4}
\]

where \(i \equiv \{L, T, TL, TT\}\), and \(V_L, V_T, V_{TL}, \text{and } V_{TT}\) are well-known kinematical factors. A non factorized approach (NFA) thus requires therefore the evaluation of the various response functions \(W_i\)'s. The cross sections of the processes \(^2\)He(\(e, e'p\))\(^4\)He, \(^3\)He(\(e, e'p\))\(^2\)H, \(^3\)He(\(e, e'p\))\(^4\)He, \(\text{and } \delta\)He(\(e, e'p\))\(^3\)He have been calculated in \[3\] within a parameter-free approach based upon realistic two-, three-, and four-body wave functions. In Fig. 1 the factorized and non factorized cross sections of the 2bbu process \(^3\)He(\(e, e'p\))\(^2\)H are compared with recent Jlab experimental data \[10\]. The results presented in Fig. 1 clearly show that treating FSI within the FA is a poor approximation for ”negative” (left, \(\phi = 0\)) values of the missing momentum, unlike what happens for ”positive” (right, \(\phi = \pi\)) values (here \(\phi\) is the azimuthal angle of the detected proton, with respect to
Figure 1. LEFT: the differential cross section of the process $^3\text{He}(e,e'p)^2\text{H}$ calculated taking into account FSI within the non factorized (FSI(NFA)) and factorized (FSI(FA)) approaches. Experimental data from Ref. [10]. (After Ref. [5]). RIGHT: the transverse-longitudinal asymmetry for the process $^3\text{He}(e,e'p)^2\text{H}$. Dot-dash: PWIA; dash: PWIA plus single rescattering FSI; full: PWIA plus single and double rescattering FSI (three-body wave function from [6], AV18 interaction [7]). Experimental data from Ref. [10]. (After Ref. [5]).

The momentum transfers $q$). In spite of the good agreement provided by the NFA, quantitative disagreements with experimental data still persist at $\phi = 0$, in particular in the region around $|p_m| \simeq 0.6 - 0.65 \text{GeV}/c$. The origin of such a disagreement can better visualized by analyzing the left-right asymmetry

$$A_{TL} = \frac{d\sigma(\phi = 0^\circ) - d\sigma(\phi = 180^\circ)}{d\sigma(\phi = 0^\circ) + d\sigma(\phi = 180^\circ)},$$

(5)

It is well known that when the explicit expressions of $V_i$ and $W_A$ are used in Eq. (5) the numerator is proportional to the transverse-longitudinal response $W_{TL}$, whereas the denominator does not contain $W_{TL}$ at all, which means that $A_{TL}$ is a measure of the relevance of the transverse-longitudinal response relative to the other responses. The experimental [10] and theoretical [5] asymmetries are presented in Fig. 1 which clearly shows that at high values of the missing momentum the theoretical calculation cannot explain the experimental data. The reason for such a failure, which is common to many approaches, is at present under investigation.

Concerning the 3bbu channel calculation, theoretical results are presented in Fig. 2; an overall good agreement with the experimental data can be achieved, provided the large effects of the final state interaction are taken into account.

The results for the 2bbu channel in $^4\text{He}$, are reported in Figs. 3 where the
reduced cross section
\[ n_D(p_m) = \frac{d^3\sigma}{d\Omega'dE' d\Omega_p} (K\sigma_{ep})^{-1}, \] (6)
is compared with preliminary JLab data (CQ\omega 2) obtained in perpendicular kinematics [11]. It can be seen that: i) the dip predicted by the PWIA is completely filled up by the FSI; ii) like the \(^3\text{He}\) case, the difference between GA and GEA is not very large; iii) an overall satisfactory agreement between theory and experiment is obtained.

**Figure 2.** The differential cross section of the 3bbu channel \(^3\text{He}(e,e'p)(np)\) plotted, for fixed values of \(p_m\), vs the excitation energy of the two-nucleon system in the continuum \(E_{rel} = t^2/M_N = E_p = E_m - E_A\). The curves labeled \(\text{PWA}\) do not include any FSI; the dashed curves correspond to the PWIA; the dot-dashed curve include the FSI with single rescattering; the full curves include both single and double rescattering (three-body wave function from [6], AV18 interaction [7]). Experimental data from Ref. [10]. (After Ref. [4])

It has been argued by various authors that at high values of \(Q^2\) the phenomenon of color transparency, i.e. a reduced NN cross section in the medium, might be observed. Color transparency is a consequence of the cancelation between various hadronic intermediate states of the produced ejectile. In [13] the vanishing of FSI at \(Q^2\) has been produced by considering the finite formation time (FFT) the ejectile needs to reach its asymptotic form of a physical baryon. This has been implemented by explicitly considering the dependence of the NN scattering amplitude upon nucleon virtuality. According to [13] FFT effects can be introduced by replacing \(\theta(z_i - z_1)\) appearing in the Glauber profile with
\[ J(z_i - z_1) = \theta(z_i - z_1) (1 - \exp[-(z_i - z_1)/l(Q^2)]) \] (7)
where \(l(Q^2) = Q^2/(x m_N M^2)\); here \(x\) is the Bjorken scaling variable and the quantity \(l(Q^2)\) plays the role of the proton formation length, i.e. the length of the
trajectory that the knocked out proton runs until it return to its asymptotic form; the quantity $M$ is related to the nucleon mass $m_N$ and to an average resonance state of mass $m^{*}$ by $M^2 = m^{*2} - m_N^2$. Since the formation length grows linearly with $Q^2$, at higher $Q^2$ the strength of the Glauber-type FSI is reduced by the damping factor $(1 - \exp[-(z_i - z_1)/l(Q^2)])$; if $l(Q^2) = 0$, then $S_{FFT}$ reduces to the usual Glauber operator $S_G$.

The results of calculations of the cross section of

\begin{equation}
\frac{d^2\sigma}{dQ^2 dx} = A(Q^2, x, T, p, m) + B(Q^2, x, T, p, m)
\end{equation}


Figure 3. LEFT: the reduced cross section (Eq. (6)) of the process $^4He(e,e'p)^3H$ at perpendicular kinematics and $x \simeq 1.8$. The solid line shows the results within GEA, whereas the dashed curve corresponds to the conventional GA. Preliminary data from [11]. RIGHT: the reduced cross section (Eq. (6)) of the process $^4He(e,e'p)^3H$ at perpendicular kinematics for various values of $Q^2$ and $x \simeq 1.4$, calculated taking FFT effects into account. Four-body wave functions from Ref. [12]. (After Ref. [9])

the process $^4He(e,e'p)^3H$ in perpendicular kinematics taking into account FFT effects are presented in Fig. 3 (for calculations in parallel kinematics see [14]). It can be seen that at the JLAB kinematics ($Q^2 = 1.78 \,(GeV/c)^2$, $x \sim 1.8$) FFT effects, as expected, are too small to be detected, they can unambiguously be observed in the region $5 \leq Q^2 \leq 10 \,(GeV/c)^2$ and $x = 1.4$. Thus measuring the $Q^2$ dependence of the cross section of $^4He(e,e'p)^3H$ process at $p_m \sim 430 \,MeV/c$ and $Q^2 \sim 10 \,(GeV/c)^2$ would be extremely interesting.

To sum up, the following remarks are in order:

i) an overall good agreement between the results of theoretical calculations and experimental data for both $^3He$ and $^4He$ is observed, which is very gratifying also in view of the lack of any adjustable parameter in theoretical calculations; ii) the effects of the FSI are such that they systematically bring theoretical calculations in better agreement with the experimental data; for some quantities, FSI effects simply improve the agreement between theory and experiment, whereas for some other quantities, they play a dominant role; iii) the 3bbu channel in

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{LEFT: the reduced cross section (Eq. (6)) of the process $^4He(e,e'p)^3H$ at perpendicular kinematics and $x \simeq 1.8$. The solid line shows the results within GEA, whereas the dashed curve corresponds to the conventional GA. Preliminary data from [11]. RIGHT: the reduced cross section (Eq. (6)) of the process $^4He(e,e'p)^3H$ at perpendicular kinematics for various values of $Q^2$ and $x \simeq 1.4$, calculated taking FFT effects into account. Four-body wave functions from Ref. [12]. (After Ref. [9])}
\end{figure}
$^3He$, provides evidence of NN correlations, in that the experimental values of $p_m$ and $E_m$ corresponding to the maximum values of the cross section satisfy, to a large extent, the relation predicted by the two-nucleon correlation mechanism, with FSI mainly affecting the magnitude of the cross section; iv) the violation of the factorization approximation is appreciable at ”negative” values ($\phi = 0$) of the missing momentum, whereas the non factorized and factorized predictions are in good agreement in the whole range of positive values ($\phi = \pi$) of $|p_m|$; v) the left-right asymmetry can reasonably be reproduced at low values of the missing momentum, but a substantial discrepancy between theoretical calculations and experimental data, common to several calculations, remains to be explained at high values of $|p_m|$; vi) Finite Formation Time effects can be investigated at moderately high values of $Q^2$ by means of the process $^4He(e, e'p)^3H$.

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