The efficiencies of generating cluster states with weak nonlinearities

Sebastien G R Louis\textsuperscript{1,2,4}, Kae Nemoto\textsuperscript{1}, W J Munro\textsuperscript{1,3} and T P Spiller\textsuperscript{3}

\textsuperscript{1} National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
\textsuperscript{2} Department of Informatics, School of Multidisciplinary Sciences, The Graduate University for Advanced Studies, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
\textsuperscript{3} Hewlett-Packard Laboratories, Filton Road, Stoke Gifford, Bristol BS34 8QZ, UK
E-mail: seblouis@nii.ac.jp

New Journal of Physics 9 (2007) 193
Received 19 January 2007
Published 29 June 2007
Online at \url{http://www.njp.org/}
doi:10.1088/1367-2630/9/6/193

Abstract. We propose a scalable approach to building cluster states of matter qubits using coherent states of light. Recent work on the subject relies on the use of single photonic qubits in the measurement process. These schemes can be made robust to detector loss, spontaneous emission and cavity mismatching but as a consequence the overhead costs grow rapidly, in particular when considering single photon loss. In contrast, our approach uses continuous variables and highly efficient homodyne measurements. We present a two-qubit scheme, with a simple bucket measurement system yielding an entangling operation with success probability 1/2. Then we extend this to a three-qubit interaction, increasing this probability to 3/4. We discuss the important issues of the overhead cost and the time scaling. This leads to a ‘no-measurement’ approach to building cluster states, making use of geometric phases in phase space.

\textsuperscript{4} Author to whom any correspondence should be addressed.
1. Introduction

The intriguing idea of one-way or cluster state quantum computing was initially developed by Briegel and Raussendorf [1]. They showed that a two-dimensional array of qubits, entangled in a particular way (through conditional phase gates), combined with single qubit operations, feed forward and measurements are sufficient for universal quantum computation. All the required interactions are already contained inside the system, and the computation proceeds through a series of local measurements (with classical feed forward), efficiently simulating quantum circuits. In effect, the logical gates are prepared off-line and imprinted onto the qubits as they are transmitted through the cluster.

This approach was quickly applied [2]–[6] to linear optics quantum computing [7] and was experimentally demonstrated on the scale of several qubits (see the review [8] for a full set of references). This scenario contains a significant scaling problem in practice, due to the probabilistic nature of the logical gates. However, the cluster state method enables this problem to be pushed into the off-line preparation of the cluster [2]–[4]. Many different schemes have been proposed to efficiently generate the photonic cluster states, because of the simplicity of the interactions and the appealing coherence time of the photons. Photon loss can be treated efficiently through ‘indirect measurements’ and a more elaborate preparation of the cluster [9] but at a significant cost in terms of the qubit dephasing [10]. There remains an issue concerning storage though. Initially, each photon will be flying down an optical fibre (or two [3]), meaning there is a need for an adaptive quantum memory. Reliable and efficient single photon sources and detectors are a further issue for single photon approaches.

In the past few years, in an attempt to overcome the scaling properties of linear optics quantum computing, new measurement-based nonlinear methods for optical quantum computing have been proposed [11]–[17]. One approach is to effectively cascade non-demolition measurements, to enable a parity check gate on two qubits [12]. A pair of photonic qubits prepared in a superposition of polarizations would each in turn interact with the same coherent laser probe beam $|\alpha\rangle$. The interaction consists of a cross-Kerr nonlinearity, which affects the phase of the probe beam, dependent on the state of the individual photons. The weakness of the nonlinearity is compensated for by the size of $\alpha$. After interacting with a pair of qubits, the probe beam undergoes a homodyne measurement. The outcome of this measurement gives the parity of the two qubits, thus projecting them into a known entangled state heralded by the probe measurement result.
By combining two of these parity gates and an ancillary photon, a near deterministic C-Not gate can be constructed [12], similar in its form to the Pittman et al [18] C-Not gate in pure linear optics. However, the difference in the weak nonlinearity approach is that it is not destructive and not limited by the beam-splitters’ optimal success probability of 1/2.

Already at this point, one can notice the usefulness of this parity gate for photonic cluster state approaches. A near deterministic entangler is all that is required to grow cluster states efficiently, be it in Browne and Rudolph’s [4] or in Yoran and Reznik’s model [3]. The required logical gates can be obtained through entanglement and local operations alone. However, as pointed out before, choosing photons as a support for one-way quantum computing may not be the best option. Solid state or matter systems may be more compact and easy to deal with in this application and constituted the initial proposed system, when cluster states were first developed [1]. In many of the solid state qubit systems proposed to date, the multi-qubit gates arise from direct interactions between the qubits. Adding extra qubits to a computation therefore leads to changes in the required control fields and to the Hamiltonian of the whole system. As a consequence, the required set-up becomes increasingly more complicated as the number of qubits in the computational system increases. A further issue is that in order for some solid state qubits to interact directly, they may need to be in such close proximity that application of individual control fields and measurements becomes infeasible.

To overcome these problems, the concept of distributed quantum computing has arisen, in which interactions between the qubits used in a computation is mediated by a third party. This third party can be any measurable quantum physical system capable of interacting with each qubit. Many proposals make use of single photons to effectively mediate interactions between matter qubits [19]–[24]. Having interacted with matter qubits, the photons then interact with each other in a linear optical set-up before being measured, thus projecting the matter qubits into the required state without them having interacted directly. It has been shown that entanglement and logical operations can be generated in this way. However, once again there are probabilistic limits in these approaches due to the fact that simple linear optics is inherently non-deterministic.

The next step was to use these probabilistic entangling schemes to prepare cluster states of matter qubits [25]–[29]. Barrett and Kok [25] first looked at this problem, proposing the use of a double-heralding probabilistic entangling procedure in order to build cluster states. The mediators are single photons, mixed on a beam-splitter before being measured. The individual matter qubits comprise two low-lying (qubit basis) states and a single excited state with a selection rule linking it to just one of the qubit states. Applying a $\pi$ pulse leaves one of the low lying states unchanged, while making the other move up to the excited state. Decay leads to the emission of a single photon for this amplitude. So if, after applying the $\pi$ pulse to both qubits, a single photon is detected after the beam-splitter, the qubits are projected into the singlet state. The double-heralding procedure is used to remove mixture, generated if non-photon-number-resolving detection is used. This method has been further developed in a second paper, using a repeat until success method proposed by Lim et al [27], where implementation of a conditional phase gate is proposed, using a mutually unbiased basis [28]. This enables some saving of qubit resources during the generation of the graph or cluster states. However, a further very interesting aspect of this proposal is that there are now three possible outcomes to the measurement. Along with the usual success and failure outcomes, there is an insurance outcome, in which the qubits are left in a known product state, up to local operations. This means that, following the insurance outcome, a new attempt to implement the gate is possible. The corresponding scaling properties of the average number of required entangling operations follow from the various outcome
probabilities for the entangling operation. This entangling operation requires a rather elaborate measurement scheme, which may be tough to implement experimentally. Furthermore, as the scheme involves the detection of two photons, the success probability has a quadratic dependence on the detector efficiency. Therefore on top of the inherently probabilistic aspect of linear optics, the detector efficiencies dramatically affect the scaling of the resources (even for the highest reported efficiencies). It should be noted that the scheme is robust against photon loss due to the fact that this is a heralded source of error\textsuperscript{5}, so the fidelity of successful operation does not suffer. Nevertheless, the reduction in success probability of the gate requires a significant increase in resource overhead, which in turn increases the weight of unheralded errors in the cluster state itself. So single photon measurement has its limits in realizing entangling operations on matter qubits. However, homodyne measurements on coherent light fields can be made much more efficient than photon detection. In this paper we will show how this and other factors make continuous variables a very powerful tool for growing cluster states.

2. The two-qubit entangling gate

Recently, the weak nonlinearity concepts [12, 15] have been applied to the area of matter qubits [30] and the distribution of entanglement [31, 32]. There are quite a number of well studied systems where one has a natural interaction between a matter qubit and the electromagnetic field. These include atoms (real and artificial) in cavity quantum electrodynamics (CQED) (both at the optical and telecom wavelengths) [33], nitrogen vacancy (NV)-centres in diamond [34], quantum dots with a single excess electron [35], trapped ions [36] and superconducting quantum interference devices (SQUIDs) [37] to name only a few. All these systems are likely to be suitable candidates for what we describe below but to illustrate the details a little more clearly let us consider a lambda based CQED system. One could use caesium atoms or an NV-diamond centre embedded in the cavity. Both operate in the optical frequency range and are in consequence well matched to efficient homodyne measurements. The interaction between the coherent field mode and our matter qubit can generally be described by the Jaynes–Cummings interaction \[ \hbar g (\sigma^- a^\dagger + \sigma^+ a) \] and in the dispersive limit (large detunings) one obtains an effective interaction Hamiltonian of the form [38]–[41]

\[ H_{\text{int}} = \hbar \chi \sigma_z a^\dagger a, \] (1)

where \( a \) (\( a^\dagger \)) refers to the annihilation (creation) operator of an electromagnetic field mode in a cavity and the matter qubit is described using the conventional Pauli operators, with the computational basis being given by the eigenstates of \( \sigma_z \equiv |0\rangle\langle 0| - |1\rangle\langle 1| \), with \( |0\rangle \equiv |\uparrow_z\rangle \) and \( |1\rangle \equiv |\downarrow_z\rangle \). The atom–light coupling strength is determined via the parameter \( \chi = g^2/\Delta \), where \( 2g \) is the vacuum Rabi splitting for the dipole transition and \( \Delta \) is the detuning between the dipole transition and the cavity field. The interaction \( H_{\text{int}} \) applied for a time \( t \) generates a conditional phase-rotation \( \pm \theta \) (with \( \theta = \chi t \)) on the field mode dependent upon the state of the matter qubit. We call this a conditional rotation and it is very similar to the cross-Kerr interaction between photons. This time-dependent interaction requires a pulsed probe.

\textsuperscript{5} This of course assumes no dark counts in the detection process. Dark counts are generally an unheralded error and unfortunately tend to be larger in the higher efficiency detectors.
Using this interaction, a two-qubit gate has been proposed [30] based on controlled bus rotation and subsequent measurement. The coherent state $\alpha$ interacts with both qubits so an initial state of the system

$$|\Psi_{\text{t}}\rangle = \frac{1}{2}((|00\rangle + |01\rangle + |10\rangle + |11\rangle)|\alpha\rangle$$

(2)

evolves to

$$|\Psi_{\text{f}}\rangle = \frac{1}{2} ((|00\rangle |\alpha e^{2i\theta}\rangle + |11\rangle |\alpha e^{-2i\theta}\rangle + (|01\rangle + |10\rangle)|10\rangle)|\alpha\rangle \right).$$

(3)

At this stage we can choose from different types of measurements on the probe beam. The first and simplest option we have is to perform a homodyne measurement of some field quadrature $X(\phi) = (\alpha^* e^{i\phi} + a e^{-i\phi})$ which for a sufficiently strong local oscillator (compared to the signal strength) implements a projective measurement $|x(\phi)\rangle\langle x(\phi)|$ on the probe state [42]. The key advantages with homodyne measurement, at least in the optical regime are that it is highly efficient (99% plus [43]) and is a standard tool of continuous variable experimentalists. The easiest to perform would be that of the momentum ($P = X(\pi/2)$) quadrature. In that case the measurement probability distribution has three peaks with the overlap error [44] between them given by $P_{\text{err}} = \frac{1}{2} \text{Erfc}(\alpha \sin \theta / \sqrt{2})$. As long as $\alpha \theta \sim \pi$ this overlap error is small ($<10^{-3}$) and the peaks are well separated. Now a measurement of the central peak will project the two matter qubits into the entangled state $(|01\rangle + |10\rangle)/\sqrt{2}$. This occurs with a probability of 1/2. Detecting either of the other two side peaks will project the qubits to the known product states $|00\rangle$ or $|11\rangle$. It is worth explicitly mentioning here that we have already reached the limits of conventional linear optics implementations. When realistic detector efficiencies ($\eta \sim 70\%$) are taken into account, the initial probability of 1/2 seen in photonic cluster states decreases proportional to $\eta$ or $\eta^2$ depending on the specific implementation and so the probability of the operation succeeding is now significantly less than 1/2. However homodyne measurements are highly efficient and so our success probability will remain very close to 1/2. It is of course possible to exceed this 1/2 by changing the nature of our measurement. In principle we could achieve a near deterministic gate if we measured the position ($X = X(0)$) instead of the momentum ($P$) quadrature. For the position quadrature there are two peaks in the measurement result probability distribution, corresponding to the even $|00\rangle + |11\rangle$ and the odd $|01\rangle + |10\rangle$ entangled states of the qubits. The overlap between the peaks [12] in this case is given by $P_{\text{err}} = \frac{1}{2} \text{Erfc}(\alpha (1 - \cos \theta)/\sqrt{2})$ which is an issue because in order to separate the peaks well enough we would require $\alpha \theta^2 \sim 2\pi$. This is much more difficult to achieve than $\alpha \theta \sim \pi$ required for the momentum quadrature measurement.

Another strategy for the probe measurement would be to apply an unconditional displacement $D(-\alpha)$ on the probe beam followed by a photon number measurement. After the displacement the combined state of the matter qubits and probe beam is

$$|\Psi_{\text{t}}\rangle = \frac{1}{2} ((|00\rangle |\alpha e^{2i\theta} - 1\rangle + |11\rangle |\alpha e^{-2i\theta} - 1\rangle + (|01\rangle + |10\rangle)|0\rangle).$$

(4)

Now a photon number measurement of the bus mode will then either pick out the vacuum state, or project on to two amplitudes $\alpha (e^{\pm 2i\theta} - 1)$ without distinguishing between them. For an ideal projection on to the number basis $|n\rangle$, the state of the two qubits becomes

$$|\Psi_{\text{f}}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad \text{for} \quad n = 0,$$

(5)

$$|\Psi_{\text{f}}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + (-1)^n|11\rangle) \quad \text{for} \quad n > 0,$$

(6)
Figure 1. Schematic diagram (a) of a three-qubit-entangling operation. In (b) the possible probe trajectories caused by the three conditional rotations. There are five different end-states. Upon measurement, three of these will project the qubits to entangled states of interest.

with an equal probability of 1/2 as long as the coherent amplitudes $\alpha(e^{\pm 2i\theta} - 1)$ do not contribute significantly to the vacuum. The overlap of these coherent states $|\alpha(e^{\pm 2i\theta} - 1)|$ with the vacuum $|0\rangle$ leads to an error probability of $P_{\text{err}} = e^{-4|\alpha\theta|^2}$ which can be made quite small with a suitable choice of $\alpha$ and $\theta$ [15]. For example with $\theta$ small we can choose $\alpha\theta = 2$ which leads to an error probability as low as $P_{\text{err}} \sim 3 \times 10^{-4}$. Consequently we can obtain a near-deterministic gate if we can implement a photon number measurement. However as observed in the linear optics schemes this currently constitutes a significant technological challenge. In the future if this issue is solved we will have a near deterministic entangling gate to build cluster states.

Assuming we can distinguish the vacuum from some photons, our gate does work in a heralded fashion with probability 1/2.

Hence for cluster state generation the simplest option so far would then be an efficient momentum ($P = X(\pi/2)$) quadrature homodyne measurement, giving us a success probability close to 1/2. We will note here that this is the most accessible and robust weak-nonlinear scheme so far proposed, using a single interaction per qubit. Would it be possible to further improve the success probability all the while maintaining a highly efficient measurement?

3. The three-qubit entangling gate

Within the same framework of conditional rotations, one can envisage three qubits interacting with a single probe beam. If we limit ourselves again to efficient $P$ quadrature measurements (which scale as $\alpha \theta$), we could consider the generation of three qubit states. Greenberger–Horne–Zeilinger (GHZ) states are for instance one particularly useful state [4]. Let us represent a rotation of the coherent probe beam by $R(\theta) = \exp(i\theta a^\dagger a)$. Now no $R(\pm \theta \sigma_z) R(\pm \theta \sigma_z) R(\pm \theta \sigma_z)|\alpha\rangle$ combination will lead to the required GHZ end states in the case that we implement a $P$ quadrature measurement. However having one of the qubits interact twice as much with the probe beam will yield the correct paths in phase space. Consider the sequence $R(\theta \sigma_z) R(\theta \sigma_z) R(-2\theta \sigma_z)|\alpha\rangle$ which we depict in figure 1. The peak centred on the origin will then correspond to the GHZ state $(|000\rangle + |111\rangle)/\sqrt{2}$ (after being detected). This will happen with a probability of 1/4. Next the two peaks having been rotated through $\pm 2\theta$ will correspond to the qubit states $(|01\rangle_{1,2} + |10\rangle_{1,2})|1\rangle_3/\sqrt{2}$ and $(|01\rangle_{1,2} + |10\rangle_{1,2})|0\rangle_3/\sqrt{2}$ respectively. Now in both of these possible outcomes we obtain the same Bell state on qubits 1 and 2, disentangled with qubit 3.
So overall we obtain a GHZ state with probability of $1/4$ and a Bell state with probability of $1/2$ (on two qubits of our choice), heralded by the probe beam $P$ quadrature measurement outcome. The other two outcomes project the qubits to two different known product states $|001\rangle$ or $|110\rangle$. Consequently, if all we want to do is entangle a pair of qubits, we can now do this with probability of $3/4$.\(^6\)

It may seem like increasing the number of qubits taking part will further raise the success probability. This claim turns out to be valid if we allow for more and more interactions as we add extra qubits. Considering for example the four qubit case. The optimal combination then becomes $\mathcal{R}(\theta \sigma_z) \mathcal{R}(\lambda \sigma_z) \mathcal{R}(2\theta \sigma_z) \mathcal{R}(-4\theta \sigma_z) |\alpha\rangle$. We now have 16 possible paths in phase space with nine different end states. All of these apart from two, under detection, will project the qubits to Bell states and GHZ states. Focusing solely on qubits 1 and 2 (these can be any two qubits which we choose to have interact only once with the probe beam), they will be entangled with probability $p = 7/8$. Following this method for larger numbers of qubits, $\mathcal{R}(\theta \sigma_z) \mathcal{R}(\lambda \sigma_z) \mathcal{R}(2\theta \sigma_z) \mathcal{R}(4\theta \sigma_z) \ldots \mathcal{R}(-2^{n-2}\theta \sigma_z) |\alpha\rangle$ the success probability in entangling a specific pair of qubits (here 1 and 2) scales as $p = 1 - 2^{1-n}$. We do not necessarily have to view these extra $n - 2$ qubits as ancillas. They can become (if they are not already) useful elements ("dangling bonds") for future operations when we consider the generation of two-dimensional cluster states. However there are drawbacks to using this generalization. The set-up and measurement process will become increasingly complicated. The probe beam will have to travel and interact a lot more, rapidly accentuating the errors that we could have had initially. Another essential point to note is that the gate operation time will grow exponentially with the number of qubits we are willing to use. If we only have access to a fixed interaction strength $\theta$, the gate operation time will double every time we add an extra qubit. So depending on the situation we are in, a compromise will have to be made between the time we are willing to take and the success probability we want to achieve. The three-qubit gate minimizes the ratio of operation time over probability and we shall use this $3/4$ probability in the remainder of the paper.

4. Scaling

We now consider how this entangling scheme may be used for generating cluster states of matter qubits. The usual cluster state is a rectangular two-dimensional lattice of qubits. The qubits are entangled in a particular way, through conditional phase gates, with some of their nearest neighbours, thus creating ‘bonds’. Each one-dimensional chain represents the life line of a single qubit to be processed. These chains form a full two-dimensional lattice structure by having bonds between them. The cluster state is defined as the eigenstate of the set of operators $S_i = X_i \prod_j Z_j$, where $i$ represents a particular lattice site and $j$ all its nearest neighbours.

Building chains is a possible basis for generating cluster states. If the chains can be efficiently generated, then simple schemes can be developed to combine them to form a two-dimensional cluster, required for quantum computing [2, 4, 45]. Given a parity check operation, the simplest growing technique involves taking one qubit (prepared in a superposition state $(|0\rangle + |1\rangle)/\sqrt{2}$) at a time and linking it on the end of the chain. Once this is done, a Hadamard transform is performed on this new end qubit, before the next one is added. In case of failure, the initial end qubit is left in an unknown state. Thus it needs to be measured—adaptive feed forward on its nearest neighbour then enables recovery of the cluster state. So the chain shrinks by one qubit in this case.

\(^6\) It is worth noting that this success probability is higher than that of two successive parity gates.

\(\text{New Journal of Physics} \ 9 \ (2007) \ 193 \ (\text{http://www.njp.org/})\)
This constitutes the basic sequential approach to building chains. Clearly for success probabilities smaller than 1/2, the chain will shrink on average; for a success probability of exactly 1/2, it will remain the same length. We can immediately appreciate benefits from the relatively high probabilities achieved in our two entangling procedures. The first two-qubit procedure already constitutes the limit of simple linear optics approaches. The second one, involving a three-qubit interaction, can also be used in a sequential fashion, ensuring fast average growth and thus limited resource consumption.

In the case of lower probabilities, small chains are to be built inefficiently before joining them to the main chain. The process of linking chains with an entangling operation is described in figure 2, using the stabilizer notation. We can see that even though we are not obliged to measure out one qubit, the actual length of the resulting chain is the sum of the two initial ones minus one qubit. A convenient way of representing this operation with states is used in [25]. And in figure 3, we can see how the three qubit gate allows us to directly join three chains into a ‘T’ piece, as well as joining two chains together. In case of failure, the end qubits need to be measured out and each chain shrinks by one element. Then the procedure is repeated. Supposing we start off with two chains of equal length $L$ (following previous analysis [45], $L$ is defined as the number of qubits constituting the linear cluster), then the average size of the resulting chain is:

$$L_f = \sum_{i=0}^{L} 2(L - 1/2 - i) p(1 - p)^i \approx 2L - 1 - 2(1 - p)/p.$$  (7)

This approximation is not meaningful for small chains. Here $p$ is the success probability of the entangling operation. We can immediately identify a critical length, above which there is growth:

$$L > L_c = 1 + 2(1 - p)/p.$$  (8)

This critical length varies between different entangling operations. If an actual logical gate can be immediately implemented, then $L_c = 2(1 - p)/p$ for example. Or if this logical gate requires the qubits from the cluster to interact directly (non-distributive approach) then $L_c = 4(1 - p)/p$.
Figure 3. Using the three-qubit gate, we can first attempt to join a pair of chains. This will work \( \frac{3}{4} \) of the time, producing one or two dangling bonds centred on the same qubit, allowing for repeated trials at linking chains to form two-dimensional clusters. Three chains can also been linked up in a similar fashion to produce a ‘T’ shape.

[45]. So our minimal size chain, ensuring growth is \( L_0 \) and \( L_0 > L_c \) (the next integer greater than \( L_c \)).

Following strategies previously developed, these minimal chains are grown in a divide and conquer fashion (in parallel, without recycling) starting from individual qubits before being merged together. This yields scaling relations for the average time taken \( T[L] \) and the average number of entangling operations \( N[L] \) to grow a chain of length \( L \):

\[
T[L] = t \sum_{i=1}^{\log_2(L_0-1)+1} (1/p)^i + (t/p) \log_2 \left( \frac{L - L_c}{L_0 - L_c} \right),
\]

\[
N[L] = \left( \frac{1}{2} \sum_{i=1}^{\log_2(L_0-1)+1} (2/p)^i + 1/p \right) \left( \frac{L - L_c}{L_0 - L_c} \right) - \frac{1}{p},
\]

where \( t \) denotes the time taken per entangling operation.

For our first entangling procedure \( p = 1/2 \), \( L_c = 3 \) and thus \( L_0 = 4 \). Growing this four-chain will require 14 entangling operations on an average leading to \( N[L] = 16L - 50 \). This is already the theoretical limit for simple single photon applications. In the repeat until success method [28], for a failure probability of 0.6 (and equal success and insurance probabilities, on all results), the scaling is \( N[L] = 185L - 1115 \) and for a failure probability of 0.4 it becomes \( N[L] \approx 16.6L - 47.7 \). Now if we switch to our three-qubit gate, then \( L_0 = 2 \).
We will notice that $L_c = 5/3$ meaning the $L_0 - L_c$ factor will contribute more than before, because we chose this difference to be unity. The average number of entangling operations required then simply becomes $N_0 = 1/p = 4/3$, giving us a scaling $N[L] = 8L - 44/3$. This is a vast improvement over previous proposals.

We shall extend this scaling comparison to the generation of two-dimensional cluster states. Using the redundant encoding method described in [4], we can give the average number of qubits consumed in the creation of a vertical link. Each trial to establish this link costs two qubits (per chain), because we first need to create dangling bonds. If we succeed in linking the two dangling bonds, we need to measure the first dangling qubit in order to establish the C-phase gate then measure the next one, to have a direct link between the two chains. The fact that we can only implement a simple entangling operation and not the logical gate means we lose an additional qubit, which may be used later for additional vertical links or error correction. But if we concentrate on the task of making a single vertical link, the number of qubits consumed is

$$V = 2(1/p + 1). \quad (11)$$

We can see that this converges to four as $p$ tends to unity (this corresponds to the qubit cost of a single trial). Then the average number of entangling operations required to make the vertical bond is given by

$$N_V = 2N[V] + 1/p. \quad (12)$$

Using the linear optics scheme proposed in [28], for failure probabilities of 0.6, 0.4 and 0.2 respectively, $N_V = 3334, 191.2$ and $32.5$. The latter failure probability would however be very difficult to implement physically. With our two and three qubit entangling procedures we obtain $N_V = 70$ and $46.7$ respectively. We can see that the efficiency of these schemes generalizes to the creation of two-dimensional cluster states in a straightforward manner. Our gates can also be used to build cluster states in a ‘tree’ like fashion, as proposed by Bodiya and Duan [46]. The method relies on the observation that GHZ-type states are locally equivalent to star shaped cluster states. A parity check is all that is needed here.

We now turn back to the time scaling. Solving $T$ for $p = 1/2$ we end up with

$$T[L] = 14 + 2 \log_2(L - 3). \quad (13)$$

Of course this is only valid for $L \geq 4 = L_c$. The above result is obtained with a $T_0 = 14$ corresponding to the average time needed to build a five-qubit chain without recycling (this is due to the form of the sum). It is pretty clear that if we allow for entangling operations to be made in parallel, alongside additional resources, this $T_0$ can be minimized. For $p > 1/2$ we have $L_0 = 2$, meaning we only keep the first term in the sum for $T_0$. This results in a general closed expression for $T$

$$T[L] = (t/p) \left( 1 + \log_2 \left( \frac{L - L_c}{L_0 - L_c} \right) \right). \quad (14)$$

We can compare this with the time taken by a sequential adding and building, as we now have access to probabilities higher than 1/2. Adding one qubit at a time, via an entangling procedure, gives the recursion relation $L_{k+1} = L_k + 2p - 1$ for the length, leading to the number of rounds
\[ k = (L - 1)/(2p - 1). \]

So for our three-qubit gate, on an average the chain grows by one unit every two trials. The time being sequential too, \( T_{L+1} = T_L + t/p \), the general form for \( T \) becomes

\[
T[L] = (L - 1)t/p. \tag{15}
\]

Thus time now scales linearly with the length of the chain in contrast with the logarithmic dependence we had above.

**5. Optimizing time and resources**

For the two-qubit entangling gate, we essentially stand at the same point as the photonic cluster state approaches. Optimizing the resources boils down to finding the optimal strategies in combining elements of cluster states. Though this is a classical analysis, relying on probabilistic gates, it is a very complex task. Obtaining bounds or comparing different strategies requires computing assistance. In their recent paper, Kieling, Gross and Eisert [47, 48] investigate these issues in significant detail. They analyse essential methods and derive bounds for the globally optimal strategy, based on an entangling operation working with probability \( 1/2 \).

For higher probabilities however, the critical length insuring average growth simply does not exist anymore and additional truly scalable approaches are at hand. We shall go over the obvious ones. From previous works on generating cluster states [25, 45], we know that the simplest way to grow short chains with probabilistic gates is through a ‘divide and conquer’ approach. It also turns out to be much quicker than a sequential adding, as we allow for many gates to operate in parallel. As described earlier on, this technique attempts to link up chains of equal length on each round, and discard the chains which failed to do so.

This approach can be extended to growing large chains in the aim of saving time. In this context we can work out some important average quantities, starting off with the time taken

\[
T[L] = 1 + \log_2(L - 1) = k. \tag{16}
\]

Here \( k \) represents the number of rounds and can easily be worked out, as we saw above, from the given chain length. Thus we will only use \( k \) in the following expressions. Next we can give the number of chains, at a particular round \( k \) (\( L = 1 \) for \( k = 0 \)), having started off with \( n \) qubits

\[
C[k] = n(p/2)^k. \tag{17}
\]

Then the number of remaining qubits on that round is given by

\[
Q[k] = C[k] \times L = n(p/2)^k(2^{k-1} + 1). \tag{18}
\]

Following this we can work out the number of wasted qubits \( W[k] = n - Q[k] \). Finally, when discussing the necessary resources we need the overall cumulative number of entangling operations

\[
G[k] = \sum_{j=1}^{k-1} \frac{C[j, m]}{2} = \frac{n}{2} \left( \frac{1 - (p/2)^{k-1}}{2/p - 1} \right). \tag{19}
\]
Figure 4. Comparison of entangling operation requirements for chain production, using our three-qubit gate. Clearly for chains smaller than 250 qubits the full divide and conquer approach is more advantageous than the linear scaling obtained through the initial merging technique. The savings in the number of entangling operations are most significant around lengths of 100 to 120 qubits. However, the sequential adding scheme is significantly more efficient still, as expected. With this we achieve much lower scalings in comparison with those obtained through the repeat until success (RUS) scheme, $P_f$ representing the failure probability. The ‘linear optics’ curve corresponds to a success probability of $1/2$ using the divide and conquer strategies [45]. This is the theoretical limit of conventional linear optical schemes, ignoring all detector and source inefficiencies.

In order to have a first comparison with the method described in the previous section, we can set the value of $C[k]$ to unity. Or alternatively, we can use the ratio $N_{dc}[k] = G[k]/C[k]$ which will give the average number of entangling operations required to produce a single chain

$$N_{dc}[k] = \frac{\sum_{j=1}^{k-1} C[j]}{2C[k]}.$$  \hfill (20)

Expressing this ratio in function of $L = 2^{k-1} + 1$ we obtain

$$N_{dc}[L] = \frac{(2/p)^{\log_2(L-1)} - 1}{2 - p}.$$  \hfill (21)

From the initial strategy, with $m \geq 2$ we reached a value linear in $L$

$$N[L] = \left(\frac{2}{p}\right) \frac{L - 1 - 2(1 - p)/p}{1 - 2(1 - p)/p} - \frac{1}{p}.$$  \hfill (22)

Obviously this will scale better with $L$, but surprisingly enough, the threshold above which it becomes more advantageous is very high. As observed in figure 4 (for our three-qubit gate), up to lengths of 250 qubits, the full divide and conquer approach requires less entangling operations.
This is due to the fact that the probabilities we are dealing with are significantly higher than in previous schemes, which were undertaken in two steps, the building of minimal elements $L_0$ and then their merging, in order to be scalable.

We can compare this with the sequential adding method, as we now have access to probabilities higher than $1/2$. Adding one qubit at a time, via an entangling procedure, gives the length’s recursion relation $L_{k+1} = L_k + 2p - 1$ leading to the number of rounds $k = (L - 1)/(2p - 1)$. For our three-qubit gate, on an average the chain grows by one unit every two trials. In this case, the number of rounds is equivalent to the number of entangling operations so we have

$$N_{\text{seq}}[L] = (L - 1)/(2p - 1). \quad (23)$$

Obviously this represents a considerable saving, as can be verified in figure 4.

We can also compare the time scaling of these various strategies, in units of time $t$ corresponding to a single measurement. For the complete divide and conquer scheme we simply have

$$T_{\text{dc}}[L] = t(1 + \log_2(L - 1)). \quad (24)$$

and for the initial scheme

$$T[L] = \frac{t}{p} \left( 1 + \log_2 \left( \frac{L - L_c}{L_0 - L_c} \right) \right). \quad (25)$$

For the sequential adding, the cumulative time obeys $T_{L+1} = T_L + t/p$, and the general form for $T$ becomes

$$T_{\text{seq}}[L] = t(L - 1)/p. \quad (26)$$

Thus time now scales linearly with the length of the chain, in contrast with the logarithmic dependence we had above.

The first two approaches have a logarithmic dependence on the length $L$, however $T_{\text{dc}}$ is significantly lower as might have been expected (see figure 5). Overall we see that there is a clear advantage to divide the task up and to run parallel entangling operations. However we also note that the resources in qubits become quite large, in the absence of recycling. The amount of wasted qubits for the full divide and conquer approach grows very quickly as can be seen from the expression for $W[k]$. One could envisage in this case a form of partial recycling, to save on the qubit resources while still retaining the time speed up. Then we would allow for two or three trials before discarding the chains (the initial scheme set no limit on the number of trials). However the protocol now becomes more elaborate unless we are willing to wait between each round (of discarding) because some chains will link on the first trial while others will link on the second (supposing we allow two trials). So it seems like savings in time could be made if we are able to manage and organize chains of different lengths.

The linear time scaling for the sequential method is due to the fact that operations cannot be undertaken in parallel during its growth. If we did not have access to simultaneous entangling operations, the time scaling for the divide and conquer methods would be equivalent to $N_{\text{dc}}[L]$ which is sub-exponential. One needs to keep in mind that by adopting a sequential method, the whole procedure is simplified considerably and would be more accessible to physical
Figure 5. Comparison of time requirements for chain production, for various approaches, as a function of the chain length. The divide and conquer approach, as expected, saves significant amounts of time. The linear time dependence of sequential adding does not compare, for long chains. However we can see the difference in work required if we only allow one entangling operation at a time. Again we can observe the theoretical limit for linear optics.

implementations. Divide and conquer methods require a lot of work in parallel and should in practice involve the moving about and reordering of qubits or even small cluster states. Unless the actual edges linking up the vertices in the graph states can be displaced via entanglement swapping strategies, we will most probably have to physically move some vertices in order to implement additional entangling operations. Adding qubits sequentially should solve some architectural problems which may arise. For example, the qubits could be perfectly static and the measuring system (including the ancillary qubit which can be reused) would travel along the chain ‘zipping’ it up. Of course the measuring system would go back and forth, with a frequency related to the success probability of a single operation. But essentially the qubits constituting the chain would not have to move. This seems significantly more practical than moving the qubits and chains around or having to change the measuring set-up every time so as to implement the operation between qubits in various places. However many of these problems may be solved by more advanced protocols making use of percolation phenomena as developed in [49].

All this brings us to view the cluster state as having active regions in which it is being built or measured in the computation (both can be undertaken simultaneously) and regions in which the qubits are simply waiting. Now this waiting can be minimized in the building itself, through the appropriate protocols, and in the measurement process. That is, the cluster can be built only a few layers in advance, so that the qubits have less waiting to do, between the building and the actual measurement. In any case, there will be some waiting. Therefore the lowest decoherence support would be preferred, but it may not be the easiest to manipulate. Thus we may envisage having two different physical realizations constituting the cluster state. For example, we could use single electron spins initially in building the cluster. Once the links are made between one
site and its nearest neighbours, the qubit could be switched into a nuclear spin state which has a significantly longer coherence time, via a swap operation or some other coherent write and read actions. Most of the waiting would be done in the long-lived state, before being swapped again for the readout [31, 50]. This follows the same scenario as using a second physical system to mediate the interaction and make the measurements, in distributed quantum computing. In the present proposal, we use a continuous variable bus and homodyne measurements to generate the links. This physical system shows itself to be very efficient in this application. Then, for example, electron spins or superconducting charge qubits could then be the matter realization interacting with the bus and serving for the final readout. These systems provide the useful static aspect required, they interact well with the mediating bus and ensure good single qubit measurements. Finally a low decoherence realization such as nuclear spin could be envisaged, mainly as a storage medium. The swapping or write and read procedure should have a high fidelity for this storage to be beneficial. On the whole, we see that optimization will depend directly on the physical realization(s) we have chosen to work with. For systems with long dephasing times we would give priority to sequential adding approaches, as we have some freedom in the time scaling and thus we can make significant savings in resources. But for realizations with short dephasing times, we would probably want to divide the task up and run operations in parallel, in order to accelerate the fabrication of the cluster state, at the expense of extra resources.

6. The measurement-free approach

Looking at our entangling gates, we have seen that if we utilize four nonlinear interactions and three qubits the success probability is dramatically increased. Within the framework of four nonlinear interactions, another option presents itself to us [30]. Defining the conditional displacement operator by \( \hat{D}(\beta \sigma_z) = \exp(\{\beta \hat{a}^\dagger - \beta^* \hat{a}\} \sigma_z) \), one can simulate a conditional-phase gate between qubits 1 and 2 with the following interaction sequence

\[
\hat{U} = \hat{D}(-\beta_2 \sigma_{zz}) \hat{D}(-\beta_1 \sigma_{z1}) \hat{D}(\beta_2 \sigma_{zz}) \hat{D}(\beta_1 \sigma_{z1}) = e^{2i \text{Im} \{\beta_1^* \beta_2 \sigma_{z1} \sigma_{z2} \}},
\]

by setting \( \beta_1^* \beta_2 = i \pi/8 \). The resulting operation is then locally equivalent to that of the conventional conditional phase \( e^{i \pi(1 - \sigma_{z1})(1 - \sigma_{z2})/4} \). Thus at the end of the sequence the coherent state is disentangled from both qubits, removing the need for a measurement. These conditional displacements can be simulated through conditional rotations and unconditional displacements, as is clear from the self-inverse quality of Pauli operators and the general property of rotations [51]

\[
e^{\tilde{a}^\dagger a} f(a, \tilde{a}^\dagger) e^{-\tilde{a}^\dagger a} = f(a e^{-\theta}, \tilde{a}^\dagger e^{\theta}),
\]

where \( f \) can be expanded in a power series. This can be realized with the following sequence [52]

\[
\hat{D}(\alpha \cos \theta) \hat{R}(-\theta \sigma_z) \hat{D}(-2\alpha) \hat{R}(\theta \sigma_z) \hat{D}(\alpha \cos \theta) = \hat{D}(2i \alpha \sin \theta \sigma_z),
\]

with \( \alpha \) real. The conditional phase observed arises from the different areas the probe traces out in phase space, picking up a geometrical phase. More in the spirit of the initial proposal of Wang and Zanardi [53] this can easily be extended to the simulation of many-body interactions.
Figure 6. Schematic representation of two different interaction sequences derived in the text. The grey and the white qubits lead to a pure imaginary and pure real conditional displacement of the probe mode $\alpha$. The first and second interaction, for each qubit, are in opposite directions in phase space. In (a) we have the sequence leading to a star shape while in (b) the one leading to a linear graph, both on five-qubits. For larger numbers of qubits we have the same patterns.

The interactions required to build a cluster state are pairwise thus conditional displacements are sufficient.

By having the probe interact with more qubits and adapting the direction of the displacements, we can pick out the qubits we want to couple through the geometrical phase. In that way one could start from a general sequence of the form

$$\prod_{n=1}^{N} D(-\beta_n \sigma_{z_n}) \prod_{n=1}^{N} D(\beta_n \sigma_{z_n}) = \exp \left[ 2i \text{Im} \left\{ \sum_{n=1}^{N-1} \beta_n^* \sigma_{z_n} \sum_{p=n+1}^{N} \beta_p \sigma_{z_p} \right\} \right].$$

But here we are simulating interactions between all qubits and from this sequence one cannot directly generate linear or grid-like cluster states. That is we need to adjust the $\beta_n$ so as to choose which qubits we want to interact. We can use such a sequence to directly generate useful graph states such as star shaped graphs (locally equivalent to GHZ states). For example, taking $\beta$ real and the displacement from qubit 1 orthogonal to all the others we have

$$\prod_{n=2}^{N} D(-\beta \sigma_{z_n}) D(-i\beta \sigma_{z_1}) \prod_{n=2}^{N} D(\beta \sigma_{z_n}) D(i\beta \sigma_{z_1}) = \exp \left[ -2i \beta^2 \sigma_{z_1} \sum_{n=2}^{N} \sigma_{z_n} \right].$$

Clearly if we set $\beta = \sqrt{\pi/8}$ we will obtain a star shaped graph of our $N$ qubits, centred on qubit 1 (figure 6(a)). We note that from this condition on $\beta$ and equation (29), the scaling and magnitude requirements for $\alpha \theta$ here are comparable to those of the measurement-induced entangling schemes. To generate a linear cluster we need to switch to another interaction sequence. We need to disentangle the probe with the qubits as the sequence evolves so as not to create extra links. Coming back to our conditional phase operation $\hat{U}$ we notice that after the third interaction the probe becomes disentangled from qubit 1. Furthermore by setting $\beta_1 = i\beta_2$ the entirety of the geometric phase is already acquired (by the corresponding two-qubit state) at this point. Along
with this observation and a correct ordering of the displacements we can propose a sequence of the form

\[
D(-\beta \sigma_{zN}) D(-i\beta \sigma_{zN-1}) \ldots D(\beta \sigma_{z4}) D(-\beta \sigma_{z3}) D(i\beta \sigma_{z3}) D(-i\beta \sigma_{z2}) D(\beta \sigma_{z2}) D(i\beta \sigma_{z1})
\]

\[= \exp \left[ 2i \beta^2 \sum_{n=1}^{N-1} (-1)^n \sigma_n \sigma_{n+1} \right]. \tag{32} \]

Again setting \( \beta = \sqrt{\pi/8} \) all the couplings are locally equivalent to conditional phase gates, yielding a linear cluster state (see figure 6(b)). We can view the probe as creating the links as it travels along the chain.

The main advantage with these generalizations is the reduced number of interactions per qubit. If we were to use the simple conditional phase sequence \( \hat{U} \) then the number of interactions per qubit would be \( 2d \) where \( d \) is the degree of the qubit in the graph state. In other words, to build a \( N \) qubit star shaped graph, the centre qubit would have to interact \( 2(N - 1) \) times at most, or with local operations to swap the centre qubit as the star is being generated we could bring this down to four interactions per qubit. As we see from our generalized sequence each qubit would only need to interact twice with the probe mode. Now if we think of a grid-like structure, the qubits inside the graph will have \( d = 4 \), thus using \( \hat{U} \) would mean we require eight interactions for each of these qubits. But here again if we switch to the linear cluster sequence each one of these qubits would interact four times only, twice in each of the two chains that go through it.

These multi-qubit approaches could be envisaged in different contexts. For example we may view it as an expensive resource (be it in time or work) alongside a cheaper one such as probabilistic two qubit parity gates. Many schemes make use of a basic building block such as star-shaped graphs. For instance this approach could be used to directly generate the building blocks needed in the percolation techniques derived in [49], then easier measurement based gates would take on from there, following the same methods. When we begin to think of loss in the probe mode however, this star-shaped graph sequence is a lot less robust than the linear graph sequence. The reason being that halfway through the interactions the probe holds information about all qubits, meaning that correlated errors will be quite significant. Whereas in the linear graph sequence, the probe holds information about at most two elements because it is constantly disentangling from the previous ones. There the correlated errors will be limited to pairs of qubits.

In that sense the simplest and apparently most robust procedure for building a grid-like structure is to generate chains that would then overlap at the intersections. In any case we can compensate for the loss in the probe and make sure it disentangles from the qubits. This will leave us with weighted graphs and some dephasing on the qubits. We can then resort to purification protocols such as those proposed in [54].

The important aspect of this measurement free approach is that it is significantly quicker. Also it does precisely simulate the two-qubit gates required for generating the cluster state, unlike the measurement based gates which are simple parity checks. This means that no feed forward or local operations are required. But this comes to a price. The constraints for the strength of the interactions are greater, that is they are now fixed, in contrast with our parity gates for which there simply was a lower bound needed for the distinguishability of the measurement outcomes.

Now given these near deterministic operations, the number of interactions required to build a certain cluster state becomes fixed. The question of time then simply reduces to the number of gates we can implement in parallel. Looking at the process in a dynamical way, we can see now...
that the size of the cluster state at a certain time during the computation is significantly reduced. This ‘buffer’ region of the cluster state may still be a couple of layers, but the off-line part of the cluster, which is not attached yet, can be made very small. Previously, the size of the buffer that is yet to be linked up was dictated by the success probabilities of the entangling operations [25]. The bigger this off-line prepared buffer is, the more time it takes to be built and the more time it takes to be attached. In other words the more errors it contains. Now depending on the amount of near deterministic gates we can implement in parallel, this off-line buffer only needs to consist of a couple of layers, freshly built, purified and attached. As a matter of fact we may not even need this off-line aspect anymore. The individual qubits could be added directly to the existing cluster as it is being measured. This represents huge savings in the number of qubits we are dealing with and minimizes the error they may have picked up, as they spend a minimal time inside a cluster state. The issues raised at the end of the last section are still of concern here. There will always be some waiting, between the building and the readout, so change in support during that time, from electron spin to nuclear spin for example, in order to minimize the dephasing, is still an important idea.

7. Conclusion

In this paper, we have considered the usefulness of weak nonlinearities in the building of matter qubit cluster states enabling us to work in the success probability regime of \( p \geq \frac{1}{2} \). We first developed a two-qubit parity check, based on a single nonlinearity per qubit and a simple measurement of the probe bus. At this point we already noticed the advantage of using continuous variables to mediate an interaction between the qubits and to provide us with an efficient measurement system. Then we extended the set-up to a three-qubit entangling operation, increasing the probability of entangling a pair of qubits to \( \frac{3}{4} \). We saw how this scheme could be generalized to using more qubits, rapidly increasing the success probability. The three-qubit interaction already provides new possibilities for the schemes used in building cluster states. After that we considered the vital issues of scaling, by going through previous results and adapting them to our own gates. The results themselves were already significant improvements on previously proposed schemes. The time scaling was particularly emphasized and discussed. This lead us to notice that there will always be a compromise to be made between the time and the number of entangling operations required. We also observed that within this framework we have access to measurement free approaches which can be generalized to more qubits. They make use of the geometrical phase acquired by the probe in phase space, simulating conditional phase gates between the qubits in the cluster. The constraints in implementation are higher, but many scaling issues are immediately resolved. Finally, we remark that the nonlinearity and coherent state requirements for all these schemes to operate are satisfied by, for example, \( \theta \sim 10^{-3} - 10^{-4} \) and \( \alpha \sim 10^3 - 10^4 \). These realistic numbers give encouragement for experiment.

Acknowledgments

We thank J Eisert and R van Meter for useful discussions. This work was supported in part by MEXT in Japan and the EU project QAP.
References

[1] Raussendorf R and Briegel H J 2001 Phys. Rev. Lett. 86 5188
[2] Nielsen M A 2004 Phys. Rev. Lett. 93 040503
[3] Yoran N and Reznik B 2003 Phys. Rev. Lett. 91 037903
[4] Browne D E and Rudolph T 2005 Phys. Rev. Lett. 95 010501
[5] Dawson C M, Haselgrove H L and Nielsen M A 2006 Phys. Rev. Lett. 96 020501
[6] Dawson C M, Haselgrove H L and Nielsen M A 2006 Phys. Rev. A 73 052306
[7] Knill E, Laflamme R and Milburn G J 2001 Nature 409 46
[8] Kok P, Munro W J, Nemoto K, Ralph T C, Dowling J P and Milburn G J 2007 Rev. Mod. Phys. 79 135
[9] Varnava M, Browne D E and Rudolph T 2003 Phys. Rev. Lett. 91 037903
[10] Rohde P P, Ralph T C and Munro W J 2007 Phys. Rev. A 75 010302
[11] Hutchins G D and Milburn G J 2004 J. Mod. Opt. 51 1211
[12] Nemoto K and Munro W J 2004 Phys. Rev. Lett. 93 250502
[13] Barrett S D, Kok P, Nemoto K, Beausoleil R G, Munro W J and Spiller T P 2005 Phys. Rev. A 71 060302
[14] Munro W J, Nemoto K, Spiller T P, Barrett S D, Kok P and Beausoleil R G 2005 J. Opt. B: Quantum Semiclass. Opt. 7 S135
[15] Munro W J, Nemoto K and Spiller T P 2005 New J. Phys. 7 137
[16] Nemoto K and Munro W J 2005 Phys. Lett. A 344 104
[17] Yamaguchi F, Nemoto K and Munro W J 2006 Phys. Rev. A 73 060302(R)
[18] Pittman T B, Fitch M J, Jacobs B C and Franson J D 2003 Phys. Rev. A 68 032316
[19] Bose S, Knight P L, Plenio M B and Vedral V 1999 Phys. Rev. Lett. 83 5158
[20] Cabrillo C, Cirac J I, Garcia-Fernandez P and Zoller P 1999 Phys. Rev. A 59 1025
[21] Feng X L, Zhang Z-M, Li X-D, Gong S-Q and Xu Z-Z 2003 Phys. Rev. Lett. 90 217902
[22] Duan L-M and Kimble H J 2003 Phys. Rev. Lett. 90 253601
[23] Browne D E, Plenio M B and Huelga S F 2003 Phys. Rev. Lett. 91 067901
[24] Simon C and Irvine W T M 2003 Phys. Rev. Lett. 91 110405
[25] Barrett S D and Kok P 2005 Phys. Rev. A 71 060310
[26] Benjamin S C 2005 Phys. Rev. A 72 056302
[27] Lim Y L, Beige A and Kwek L C 2005 Phys. Rev. Lett. 95 030505
[28] Lim Y L, Barrett S D, Beige A, Kok P and Kwek L C 2006 Phys. Rev. A 73 012304
[29] Benjamin S C, Eisert J and Stace T M 2005 New J. Phys. 7 194
[30] Spiller T P, Nemoto K, Braunstein S L, Munro W J, van Loock P and Milburn G J 2006 New J. Phys. 8 30
[31] van Loock P, Ladd T D, Sanaka K, Yamaguchi F, Nemoto K, Munro W J and Yamamoto Y 2006 Phys. Rev. Lett. 96 240501
[32] Ladd T D, van Loock P, Nemoto K, Munro W J and Yamamoto Y 2006 New J. Phys. 8 164
[33] Raimond J M, Brune M and Haroche S 2001 Rev. Mod. Phys. 73 565
[34] Jelezko F, Gaebel T, Popa I, Gruber A and Wrachtrup J 2004 Phys. Rev. Lett. 92 076401
[35] Pazy E, Biolatti E, Calarco T, D’Amico I, Zanardi P, Rossi F and Zoller P 2003 Europhys. Lett. 62 175
[36] Cirac J I and Zoller P 1995 Phys. Rev. Lett. 74 4091
[37] Shnirman A, Schön G and Hermann Z 1997 Phys. Rev. Lett. 79 2371
[38] Savage C M, Braunstein S L and Walls D F 1990 Opt. Lett. 15 628
[39] Haroche S 1992 Fundamental System in Quantum Optics ed J Dalibard, J Raimond and J Zinn-Justin (New York: Elsevier) p 67
[40] Gerry C C and Knight P L 2004 Introductory Quantum Optics (Cambridge: Cambridge University Press)
[41] Blais A, Huang R, Wallraff A, Girvin S M and Schoelkopf R J 2004 Phys. Rev. A 69 062320
[42] Tyc T and Sanders B C 2004 J. Phys. A: Math. Gen. 37 7341
[43] Polzik E S, Carri J and Kimble H J 1992 Phys. Rev. Lett. 68 3020
[44] Munro W J, Nemoto K, Beausoleil R G and Spiller T P 2005 Phys. Rev. A 71 033819

New Journal of Physics 9 (2007) 193 (http://www.njp.org/)
[45] Duan L-M and Raussendorf R 2005 Phys. Rev. Lett. 95 080503
[46] Bodiya T P and Duan L M 2006 Phys. Rev. Lett. 97 143601
[47] Kieling K, Gross D and Eisert J 2007 J. Opt. Soc. Am. B 24 184
[48] Gross D, Kieling K and Eisert J 2006 Phys. Rev. A 74 042343
[49] Kieling K, Rudolph T and Eisert J 2006 Percolation, renormalization, and quantum computing with nondeterministic gates Preprint quant-ph/0611140
[50] Kane B E 1998 Nature 393 133
[51] Louisell W H 1990 Quantum Statistical Properties of Radiation Wiley Classics Library (New York: Wiley)
[52] van Loock P, Munro W J, Nemoto K, Spiller T P, Ladd T D, Braunstein S L and Milburn G J 2007 Hybrid quantum computation in quantum optics Preprint quant-ph/0701057
[53] Wang X and Zanardi P 2002 Phys. Rev. A 65 032327
[54] Kruszynska C, Miyake A, Briegel H J and Dür W 2006 Phys. Rev. A 74 052316