Black Holes and Quantum Predictability

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Abstract

A brief review of the confrontation between black hole physics and quantum-mechanical unitarity is presented. Possible reconciliations are modifying the laws of physics to allow fundamental loss of information, escape of information during the Hawking process, or black hole remnants. Each of these faces serious objections. A better understanding of the problem and its possible solutions can be had by studying two-dimensional models of dilaton gravity. Recent developments in these investigations are summarized. (Linear superposition of talks presented at the 7th Nishinomiya Yukawa Memorial Symposium and at the 1992 YITP Workshop on Quantum Gravity, November 1992.)

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A question on which there has been much recent discussion is that of whether black holes lead to breakdown of quantum-mechanical predictability. This possibility was raised with Hawking’s discovery [1] that black holes radiate. I will begin by summarizing the basic arguments on this issue. Suppose that we start with a matter distribution of mass $M$ in a pure quantum state and that we arrange for it to gravitationally collapse to form a black hole. The black hole then emits Hawking radiation and loses mass. Eventually it approaches the Planck mass, $M_p\ell$, after which we do not know what happens. Let us make the most economical assumption (we will turn to others momentarily): the black hole evaporates completely leaving behind nothing but Hawking radiation, as pictured in fig. 1. This radiation is approximately thermal, and according to Hawking’s calculation is described by a mixed state. Thus a pure state has evolved into a mixed state. Although we know not the reason, unitary quantum mechanics must have failed, and an amount of quantum information $\sim M^2/M_p^2\ell$ has been lost.\footnote{Recall that the quantum information in a density matrix $\rho$ for an $n$-state system is $\ln n + \text{Tr}\rho \ln \rho = \ln n - S$, where $S$ is the entropy.}

There are problems with this. First, it is distasteful to violate a central principle like unitarity. And if we do allow such violations, they might be expected to creep into other parts of physics besides black holes. Indeed, the general quantum principle that for any real process there are similar virtual processes strongly suggests that the nonunitarity should appear in ordinary physics at some level. One might describe this as resulting from black hole contributions to loop diagrams. In fact Hawking proposed [2] that we revise the laws of nature to incorporate such a breakdown of quantum predictability. Specifically, he suggested that the quantum-mechanical evolution operator $U = e^{-iHt}$ be replaced by an evolution operator $\$$ $ acting linearly on density matrices,

$$\rho \to \$$ \rho . \tag{1}$$

Such an evolution law can be arranged to conserve probability but generically violates unitarity.

The objection to this was given by Banks, Peskin, and Susskind [3] and by Srednicki [4]. One expects unitarity violation to be $\mathcal{O}(1)$ at the Planck scale: once it is allowed there is no obvious small parameter by which it should be suppressed. However, as refs. [3][4] describe, evolution as in (1) is only nonunitary to the extent that it violates energy conservation: information transfer requires energy transfer. This implies a disastrous breakdown of
Fig. 1: A sketch of the collapse and evaporation process for a black hole. According to Hawking’s original calculation, an initially pure collapsing state of collapsing matter is converted into a mixed state of nearly thermal Hawking radiation.

energy conservation resulting from incoherent fluctuations of magnitude $\Delta E \sim M_{pl}$ in the energy; roughly one such fluctuation occurs per Planck volume per Planck time.

We are thus forced to contemplate other alternatives; these are summarized in Fig. 2. There are three logical possibilities: information is either lost, re-emitted from the black hole, or retained in some form of remnant. We will examine these possibilities.

First, if information is lost this may not be due to nonunitarity in the laws of physics. It could simply be somewhere we can’t find it. For example, it could have been transmitted to a separate universe via a spacetime wormhole. However, this effective loss would be expected to be described from our viewpoint by eq. (1), and the conclusions remain:

\footnote{For other recent reviews of the information problem see [5-8].}
effective loss of unitarity implies huge fluctuations in energy. Conversely, if energy is conserved there is no apparent loss of unitarity; indeed, wormholes just shift the constants of nature \[ 10,11 \]. This is of no apparent use in explaining information loss via black holes.

If information is re-emitted that can happen either during the Hawking process or at the final burst when the black hole reaches the Planck scale. In the first case this would require either that the information never crosses the horizon or that it escapes from inside the horizon. For it not to cross the horizon, all information must be “bleached” from the infalling matter at the horizon. However, for a large enough initial mass the

\[ \text{3}\] One might have alternatively hoped that information about the infalling state is imprinted on the Hawking radiation without anything special happening to the infalling state, e.g. through an evolution law \[ |A\rangle_{\text{in}} \rightarrow |A\rangle_{\text{out}} \otimes |A\rangle_{\text{blackhole}} \]. This however contradicts superposition: only...
horizon occurs at arbitrarily weak curvature, and analyses of physics seen by infalling observers show no special behavior there, making this alternative seem implausible. If the information is to escape from inside before the final burst, it must propagate acausally in regions of spacetime with weak curvature. If such acausal propagation is allowed inside a black hole it should by locality likewise be allowed outside, which is unacceptable. Finally, the information could emerge at the Planck scale where such notions of causality may fail. However, the remaining energy in the black hole is \( \sim M_{p\ell} \), and an amount of information \( M^2/M_{p\ell}^2 \) must be emitted. This is possible, for example, by radiating many very soft photons, but by basic quantum mechanics \[12\] must take a time \( \sim M^4/M_{p\ell}^4 t_{p\ell} \). There must therefore be arbitrarily long-lived remnants.

Such remnants would have mass \( \sim M_{p\ell} \) and must have an infinite spectrum to encode the information from an arbitrarily large initial black hole. Since large information content is their primary characteristic, they will be referred to as informons. Although the rate for producing a given informon state in ordinary processes is incredibly small, it is not zero. The inclusive rate for producing all such objects should then be infinite due to the infinite spectrum. All physical processes would produce bursts of remnants.

We now appear to be left with no palatable alternative. This is the black hole information comundrum.

A loophole should therefore appear in the above arguments. For example, it’s possible that virtual processes do not feed information loss into low-energy physics. Alternately, there may be other descriptions of information loss distinct from (1) and without fluctuating energy. Another possibility is that some theories of informons avert infinite production. One recent proposal\[17\] suggests that this might occur for certain remnants in string theory \[18,14,14-16\]. Although the details of this proposal appear flawed, it does suggest modifications of effective field theories for remnants that might avoid infinite production \[20\]. From a different angle, the arguments about Hawking radiation test application of the equivalence principle in the extreme: the redshift between the horizon, where the radiation originates, and infinity, is infinite.\[5\] Perhaps one should contemplate the radical step of allowing violations of the equivalence principle that become important only at high energies.

\[ |A\rangle_{\text{in}} \rightarrow |A\rangle_{\text{out}} \otimes |I\rangle_{\text{blackhole}} \] is allowed for some state \( |I\rangle_{\text{blackhole}} \) independent of the initial state \( |A\rangle_{\text{in}} \). Put differently: there are no quantum xerox machines.

\[ ^4 \] For various remnant proposals see \textit{e.g.} \[13,16\].

\[ ^5 \] For a related discussion see \[21\].
In attacking these problems, a better understanding of black hole evaporation would be useful. This requires treating the complicated dynamics of the backreaction of the Hawking radiation on the metric, and ultimately of quantum gravity. This may be more easily done by stripping the problem to its bones: we consider a two-dimensional toy model

\[ S = \frac{1}{2\pi} \int d^2\sigma \sqrt{-g} \, e^{-2\phi} \left[ (R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right]. \]  

(2)

Here \( \phi \) is a scalar dilaton, \( \lambda^2 \) a coupling similar to the cosmological constant, and \( f_i \) are \( N \) minimally coupled matter fields. As we will see, this model has black holes and Hawking radiation, and therefore contains the essence of the information riddle. It also has the virtues of being classically soluble and renormalizable, and furthermore gives the low-energy effective theory for certain four-dimensional black holes.

First consider the vacuum solutions; choose units so that \( \lambda = 1 \) and define \( \sigma^\pm = \tau \pm \sigma \). With \( f_i = 0 \) the general solution is

\[ ds^2 = -\frac{d\sigma^+d\sigma^-}{1 + Me^{\sigma^- - \sigma^+}} \]

\[ \phi = -\frac{1}{2} \ln \left( M + e^{\sigma^+ - \sigma^-} \right) \]

(3)

where \( M \) is the mass. For \( M = 0 \) we have the ground state:

\[ ds^2 = -d\sigma^+d\sigma^- \]

\[ \phi = -\sigma \]  

(4)

This is the closest analogue to four-dimensional Minkowski space, and is called the linear dilaton vacuum. For \( M > 0 \) the solution is a black hole \([22,23]\) with Penrose diagram shown in Fig. 3. The horizon is at \( \sigma^+ - \sigma^- \to -\infty \), and for future reference notice that \( e^{2\phi}|_{\text{horizon}} = \frac{1}{M} \). For \( M < 0 \) it is a naked singularity.

![Fig. 3: Shown is the Penrose diagram for a vacuum two-dimensional dilatonic black hole.](image-url)
Next consider sending infalling matter, \( f_i = F(x^+) \), into the linear dilaton vacuum. This will form a black hole, as shown in Fig. 4. Before the matter infall the solution is given by (4). Afterwards it is

\[
e^{-2\phi} = M + e^{\sigma^+} \left( e^{-\sigma^-} - \Delta \right)
\]

\[
ds^2 = -\frac{d\sigma^+ d\sigma^-}{1 + M e^{\sigma^- - \sigma^+ - \Delta} e^{\sigma^-}}
\]  

(5)

where

\[
M = \int d\sigma^+ T_{++}
\]

\[
\Delta = \int d\sigma^+ e^{-\sigma^+} T_{++}
\]  

(6)

and

\[
T_{++} = \frac{1}{2} (\partial_+ F)^2
\]

(7)

is the stress tensor. The coordinate transformation

\[
\xi^- = -\ell n \left( e^{-\sigma^-} - \Delta \right)
\]  

(8)
returns the metric to the asymptotically flat form

$$ds^2 = -\frac{d\xi^+ d\xi^-}{1 + M e^{\xi^- - \xi^+}}. \quad (9)$$

Hawking radiation can be described by observing that positive frequency modes according to the future asymptotic observer,

$$v_\omega = \frac{1}{\sqrt{2\omega}} e^{-i\omega \xi^-}, \quad (10)$$

are superpositions of both positive and negative frequency modes seen by the observer in the linear dilaton vacuum. The later are

$$u_\omega = \frac{1}{\sqrt{2\omega}} e^{-i\omega \sigma^-} \quad u^*_\omega = \frac{1}{\sqrt{2\omega}} e^{i\omega \sigma^-} \quad (11)$$

and the relation is

$$v_\omega = \int_0^\infty d\omega' (\alpha_{\omega\omega'} u'_\omega + \beta_{\omega\omega'} u^*_{\omega'}) \quad (12)$$

for constants $\alpha_{\omega\omega'}, \beta_{\omega\omega'}$. (Only the right-moving modes are relevant to the Hawking radiation.) Therefore the in vacuum corresponds to an excited state in the out region; indeed

$$\langle 0|N^\text{out}_\omega|0 \rangle_{\text{in}} = \int_0^\infty d\omega' |\beta_{\omega\omega'}|^2 \quad (13)$$

for the out number operator $N^\text{out}_\omega$. The Bogoliubov coefficients $\alpha_{\omega\omega'}, \beta_{\omega\omega'}$ can be found explicitly \[24\], and

$$\beta_{\omega\omega'} = \frac{1}{2\pi} \sqrt{\frac{\omega'}{\omega}} \Delta^i \omega B(-i\omega \sigma^-; 1 + i\omega) \quad (14)$$

where $B$ is the usual beta function.

The outgoing stress tensor for the Hawking radiation is likewise computable. For a general relationship between the initial and final coordinates it is proportional to the schwarzian derivative,

$$\langle 0|T_{+ -}^f |0 \rangle_{\text{in}} = -\frac{N}{24} \left\{ -\frac{1}{2} \left[ \partial_{\xi^-} \ln \left( \frac{d\sigma^-}{d\xi^-} \right) \right]^2 + \partial_{\xi^-}^2 \ln \left( \frac{d\sigma^-}{d\xi^-} \right) \right\}. \quad (15)$$

In our case this together with (8) gives

$$\langle 0|T_{+ -}^f |0 \rangle_{\text{in}} = -\frac{N}{48} \left[ 1 - \frac{1}{(1 + \Delta e^{\xi^-})^2} \right]. \quad (16)$$
which for $\xi^- \to \infty$ goes to the constant (thermal) value $1/48$.

So far we have assumed that the geometry is fixed, independent of the Hawking radiation. To treat the backreaction of this radiation on the metric it is most efficient to consider quantization via the functional integral

$$
\int \mathcal{D}g \mathcal{D}\phi \, e^{iS_{grav}[g,\phi]} \int \mathcal{D}f \, e^{-\frac{4\pi}{3} \int d^2\sigma \sqrt{-g} \sum_{i=1}^N (\nabla f_i)^2}
$$

where the action (1) has been separated into the gravitational and matter parts. The matter functional integral is well-studied; it can be readily evaluated using properties of the trace anomaly. This gives

$$
\int \mathcal{D}f \, e^{-\frac{4\pi}{3} \int d^2\sigma \sqrt{-g} \sum_{i=1}^N (\nabla f_i)^2} = e^{iNS_{PL}}
$$

with

$$
S_{PL} = -\frac{1}{96\pi} \int \int \sqrt{-g} \, d^2\sigma \, \sqrt{-g'} \, d^2\sigma' \, R(x) \Box^{-1}(x, x') R(x')
$$

the Polyakov-Liouville action; here $\Box^{-1}$ is the Green-function for the d’Alembertian, $\Box$.

The effect of this action on the geometry is found by calculating its stress tensor. By a change of coordinates the metric can always be put into conformal gauge, $g_{\mu\nu} = e^{2\rho} \eta_{\mu\nu}$, and in this gauge

$$
\langle T_{\mu\nu}^{\text{matter}} \rangle = \frac{2\pi N}{\sqrt{-g}} \frac{\delta S_{PL}}{\delta g_{\mu\nu}} = \frac{N}{12} \left[ \partial_\rho^2 - (\partial_\rho)^2 - t_-(\sigma^-) \right]
$$

where $t_-$ is fixed by the initial boundary conditions. For the collapsing black hole, $t_-$ is zero by the condition of no Hawking radiation in the linear dilaton vacuum, and (2) gives

$$
\langle T_{\mu\nu}^{\text{matter}} \rangle = \frac{N}{48} \left[ 1 - \frac{1}{(1 + \Delta e^{\xi^-})^2} \right]
$$

in agreement with (16). Therefore the Polyakov-Liouville action precisely accounts for the Hawking flux.

The quantum theory with Hawking radiation and backreaction is thus encoded in the functional integral

$$
\int \mathcal{D}g \mathcal{D}\phi \, e^{iS_{grav} + iNS_{PL}}
$$
or similar integrals giving correlation functions. At present we are unable to fully quantize this system (more discussion will follow shortly). This motivates us to study it in a semiclassical approximation. Outside the horizon of a massive black hole,

$$e^{2\phi} < e^{2\phi\mid_{\text{horizon}}} = \frac{1}{M} << 1.$$  \hfill (23)

Since $e^\phi$ plays the role of the gravitational coupling (that is $e^{-2\phi}$ appears in the gravitational action, (2)) the theory is thus weakly coupled outside the horizon. $S_{PL}$ is a subleading contribution in $e^{2\phi}$, and there may be other contributions at the same order, e.g. from the measure in (22). $S_{PL}$ can, however, be made dominant by taking the number of matter fields large, $N >> 1$, simultaneously as $e^{2\phi}$ becomes small. This justifies approximating the functional integral (22) by the solutions of the semiclassical equations

$$0 = \delta \frac{\delta}{\delta \phi} (S_{\text{grav}} + NS_{PL})$$

$$0 = \delta \frac{\delta}{\delta g^{\mu\nu}} (S_{\text{grav}} + NS_{PL}) .$$  \hfill (24)

In conformal gauge these equations are

$$0 = -4\partial_+ \partial_- \phi + 4\partial_+ \phi \partial_- \phi + 2\partial_+ \partial_- \rho + e^{2\rho}$$

$$0 = T_{+-} = e^{-2\phi}(2\partial_+ \partial_- \phi - 4\partial_+ \phi \partial_- \phi - e^{2\rho}) - \frac{N}{12} \partial_+ \partial_- \rho$$

$$0 = T_{++} = e^{-2\phi}(4\partial_+ \phi \partial_+ \rho - 2\partial_+^2 \phi) + \frac{1}{2} \partial_+ f \partial_+ f - \frac{N}{12} \left( \partial_+ \rho \partial_+ \rho - \partial_+^2 \rho + t_+(\sigma^+) \right)$$  \hfill (25)

and similarly for $T_{-+}$. These equations still have the linear dilaton vacuum as a solution: the modification $S_{PL}$ is quadratic in $R$, and $R$ vanishes in the linear dilaton vacuum. However, they are no longer soluble and must be treated by general arguments, numerical techniques, studying related soluble models, and other trickery [15,25-32]. The basic picture that emerges is the following.

The matter collapses until it reaches a critical coupling, $e^{2\phi_{cr}} = \frac{12}{N}$. Here a singularity in the semiclassical equations develops (the kinetic operator degenerates) and the semiclassical approximation breaks down. This singularity is hidden behind an apparent horizon. The latter is defined by the curve where $(\nabla e^{-\phi})^2 = 0$, motivated by the analogy between $e^{-\phi}$ and the radius of the two-spheres in four-dimensional spherically symmetric spacetimes. The black hole radiates Hawking radiation and the apparent horizon recedes
until it approaches the singularity; this occurs as the mass radiated approaches the initial mass. Once they touch it is impossible to specify the future evolution without understanding the strong-coupling physics — we come into the shadow of the singularity. There is an effective horizon, defined as the last light ray that escapes without hitting strong coupling; above this we cannot make predictions.

These features are exhibited in a Kruskal diagram in Fig. 5, and in the Penrose diagram of Fig. 6. Notice that we can’t go beyond \( \phi_{cr} \) even in the linear dilaton vacuum: the physics is strongly coupled. Therefore spacetime to the left of \( \phi_{cr} \) has been truncated in these diagrams.

![Kruskal Diagram](image)

**Fig. 5:** The Kruskal geometry of a collapsing and evaporating black hole. Past the line \( Q \) the semiclassical approximation breaks down and additional input is needed.

The semiclassical breakdown means that we cannot say what happens to the information. However, the explicit description of the evaporation, which could be extended in a higher order analysis, gives one confidence that the information does not emerge during the ordinary Hawking evaporation. This essentially follows from causality together with the fact that the Hawking radiation emerges at weak coupling. Even more solid arguments for failure of information return have been recently constructed in similar two-dimensional

\[ \text{[33-35].} \]
models describing the s-wave sector for Reissner-Nördstrom black holes. The picture is still, however, consistent with fundamental information loss or long-lived remnants.

To do better would require a quantum treatment beyond the semiclassical approximation. I will briefly describe some of the issues in such a treatment.

Quantization requires gauge fixing. The most common approach is to fix diffeomorphism invariance by picking a fixed background metric \( \hat{g}_{\mu\nu} \), and by restricting attention to metrics of the form

\[
g_{\mu\nu} = e^{2\rho} \hat{g}_{\mu\nu} .
\]

Thus the only “dynamics” of gravity is in the conformal factor \( \rho \). As at the outset, the action written in terms of \( X^P = (\rho, \phi) \) is renormalizable, but this isn’t as useful in two dimensions as in four. Although we shouldn’t have to write down higher-dimension counterterms, the fields \( \rho \) and \( \phi \) are dimension zero so we nonetheless expect the full action to be of the form

\[
S = -\frac{1}{2\pi} \int d^2\sigma \sqrt{-\hat{g}} \left[ G_{MN}(X^P) \nabla X^M \nabla X^N + \frac{1}{2} \Phi(X^P) \hat{R} + T(X^P) \right]
\]
where hat denotes quantities formed from the metric \( \hat{g} \) and \( G_{MN} \), \( \Phi \), and \( T \) are general functions. (The notation is motivated by comparison to string theory.) An infinite number of parameters are required to specify these functions, and thus the full quantum theory. Therefore although renormalizable, the theory is uncomfortably close to four-dimensional gravity in being unpredictable.

The functions \( G_{MN} \), \( \Phi \), and \( T \) are not completely arbitrary, but must satisfy certain constraints\[37\]. These are:

1. **Background independence.** According to (26), the theory should be left unchanged by the transformation
   \[
   \hat{g}_{\mu\nu} \to e^{2\omega} g_{\mu\nu}, \quad \rho \to \rho - \omega.
   \]
   This condition (which is related to conformal invariance) implies
   \[
   \nabla_M \Phi \nabla^M T - 4T - \frac{\hbar}{2} \Box T + \cdots = 0
   \]
   \[
   \nabla_M \nabla_N \Phi + \frac{\hbar}{2} R_{MN} + \cdots = 0
   \]
   \[
   (\nabla \Phi)^2 - \frac{\hbar}{2} \Box \Phi + (N - 24) \frac{\hbar}{3} + \cdots = 0
   \]
   where geometrical quantities are defined in terms of the metric \( G_{MN} \), \( \hbar \) has been reinstated, and terms higher order in \( \hbar \) have been dropped. There are infinitely many parameters determining solutions to these equations; initial data for them could, for example, be specified by giving \( G_{MN} \), \( \Phi \), and \( T \) as functions of \( \phi \) at a fixed scale, \( \rho \).

2. **Classical Limit.** As \( e^\phi \to 0 \), the theory should agree with the classical theory, to leading order in \( e^{2\phi} \):
   \[
   G_{MN} \to \begin{pmatrix} -4 e^{-2\phi} & 2 e^{-2\phi} \\ 2 e^{-2\phi} & 0 \end{pmatrix}, \quad \Phi \to -2 e^{-2\phi}, \quad T \to -4 \lambda^2 e^{-2\phi}
   \]

3. **Coupling to Hawking Radiation.** Hawking radiation should cause black holes to shrink at the semiclassical rate, proportional to the number \( N \) of matter fields\[38\]. Ghosts, which also contribute to \( S_{PL} \), should not affect the radiation rate of the black hole.

   If the action is written as an expansion in \( \hbar \) or equivalently \( e^{2\phi} \),
   \[
   S = S_0 + S_1 + \cdots, \quad (31)
   \]
with $S_0$ given by (30), then this becomes a constraint on $S_1$. With Polyakov-Liouville term included the one-loop action becomes

$$
\tilde{S}_1 = -\frac{\hbar}{2\pi} \int d^2 \sigma \sqrt{-g} \left[ G_{MN}^{(1)} \nabla X^M \sim \nabla X^N + \frac{1}{2} \Phi^{(1)}(X^P) \hat{R} + T^{(1)}(X^P) \right] 
- \frac{N - 24}{96\pi} \hbar \int d^2 \sigma \sqrt{-g} d^2 \sigma' \sqrt{-g'} \hat{R} \Box^{-1} \hat{R}.
$$

(32)

(Here the -24 arises from the ghost contribution to the conformal anomaly.) We want

$$
T_{--} = \frac{2\pi}{\sqrt{-g}} \frac{\delta \tilde{S}_1}{\delta g}
$$

(33)

to give the correct Hawking flux, $\rightarrow \hbar N/48$, at infinity. This requires

$$
G^{(1)}_{\phi\phi} - \frac{1}{2} \partial^2_{\phi} \Phi^{(1)} = 2.
$$

(34)

Finally there is

4. Stability. The theory should have a stable ground state.

These conditions still leave an infinite family of theories, so one requires still more input to fix a theory. Several approaches have been taken, none yet entirely satisfying. In outline, they are

1) Quantum Soluble models [39,40,23,34,35]: The classical metric $G_{MN}$ in (30) is flat. For a variety of solutions $G_{(1)}^{MN}$ can be seen not to spoil this. Therefore, one may choose coordinates $U(\rho, \phi)$, $V(\rho, \phi)$ so that $G_{UU} = G_{VV} = 0$ and $G_{UV} = \frac{1}{2}$. It then turns out that the potential term is also simple to leading order in $\hbar$; the action becomes (here $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$)

$$
S = \frac{1}{2\pi} \int d^2 \sigma \left( \nabla U \sim \nabla V + \lambda^2 e^{2V} \right)
$$

(35)

where non-trivial $O(\hbar)$ corrections now appear in the potential. This theory has several virtues. First, it satisfies 1–3 above. Secondly, this quantum action is of the same form as the classical action (2) in conformal gauge (with $V \leftrightarrow \rho - \phi$ and $U \leftrightarrow e^{-2\phi}$) and thus is soluble. Indeed, the theory is a conformal field theory similar to the Liouville theory. It furthermore has the property that the potential is not renormalized since the propagator connects only $U$ and $V$ and the vertex contains only $V$. The major drawback is that Hawking radiation does not stop in this model [30,40], and indeed there are regular static solutions with mass unbounded from below [37]. Thus it fails criterion 4. Attempts
have been made to remedy this by adding a boundary condition that stabilizes the theory [29,41,34-35] with possibly interesting consequences.

2) *Extra symmetry.* In the classical theory $j^{\mu} = \partial^{\mu}(\rho - \phi)$ is conserved and one can try to preserve this at the quantum level [29,41]. This does not, however, fix all counterterms, and can run into the stability problem as above. Alternately one can try supersymmetrization. The $N = 1$ and $N = 2$ theories have been considered [42-44] and there seem to be applicable nonrenormalization theorems, but these appear still insufficient to fully fix the theory [44].

3) *Strings.* We believe that string theory solves similar problems in four-dimensional quantum gravity, so it is appealing to apply it here. Indeed, the low-energy string lagrangian in two dimensions is

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla\phi)^2 + 4\lambda^2 - (\nabla T)^2 + T^2 + O(T^3) + \cdots \right], \tag{36}$$

or, redefining the tachyon $T = e^{\phi} t$,

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi} \left( R + 4(\nabla\phi)^2 + 4\lambda^2 \right) - (\nabla t)^2 + t^2 \left( \Box \phi - (\nabla\phi)^2 + 1 \right) + O(e^{\phi} t^3) + \cdots \right]. \tag{37}$$

The third term vanishes in the linear dilaton vacuum so $t$ behaves like a massless scalar field, and the action (37) is strikingly similar to (2). Furthermore, matrix models are believed to provide a consistent and essentially complete quantum description of two-dimensional strings. We might hope to use this to enlighten us on black hole puzzles.

Unfortunately life is not so easy. First, recall that the action (37) is valid only in the approximations

i) $e^\phi << 1 \ldots$ weak string coupling

ii) $k^\mu << 1 \ldots$ low momenta

iii) $T = e^\phi t << 1 \ldots$ weak tachyon;
otherwise subleading terms become important. We would like to know if an object resembling a black hole, at least to the extent that it has an apparent horizon, can be formed within this domain of validity. To examine this, use the fact that (37) is similar to dilaton gravity, with $t \to f$, plus an extra repulsive potential. Let’s see if the analogous conditions can be satisfied in dilaton gravity. First, recall $e^\phi < e^{\phi_h} = \frac{1}{\sqrt{M}}$, so condition i) is satisfied for large black holes. Next, we can arrange for ii) to be satisfied by building the black
hole from a large number of soft particles. But iii) is problematic: from (5)-(7) the value of the dilaton at the horizon is

\[ e^{-2\phi_h} \sim \int_{\sigma_i^+}^{\sigma_f^+} d\sigma^+ e^{\sigma^+-\sigma^+} (\partial_+ f)^2, \]  

(38)

where \( \sigma_i^+, \sigma_f^+ \) correspond to the beginning and end of the pulse, and this indicates that

\[ e^{\phi_h} f \sim 1 \]  

(39)

at the horizon. The same statement is likely true for the string lagrangian; before a horizon forms and things get interesting, the tachyon potential becomes important.

Since matrix models are supposed to include all higher order effects, including the tachyon potential, one might hope to see evidence for or against black holes in the matrix models. However, at least naively matrix models describe perturbations about the Liouville background

\[ T = \mu e^{-\sigma} + O(e^{-2\sigma}) \]
\[ \phi = -\sigma + \frac{\mu^2}{8} e^{-2\sigma} + O(e^{-4\sigma}) \]
\[ g_{\mu\nu} = \eta_{\mu\nu} + O(e^{-4\sigma}) \]  

(40)

with \( \mu \gg 1 \). This means that when we consider such fluctuations

\[ T = \mu e^{-\sigma} + e^\phi t, \]  

(41)

the higher terms in the tachyon potential cause strong interactions between the background and fluctuations. For example, a \( T^3 \) term gives a contribution

\[ \sim \mu e^{-\phi} (e^\phi t)^2. \]  

(42)

Since \( e^\phi t \) must become \( O(1) \) at the horizon, and since \( \mu \gg 1 \), this gets large long before the horizon forms. The Liouville wall obscures the black hole physics. Therefore, it’s quite possible the matrix models simply describe reflection from the wall, not black hole formation. An optimist might hope for a non-perturbative description of black holes to appear at \( \mu \lesssim 1 \), but this hope has not yet been realized; a pessimist might worry that there

\[ I \text{ thank E. Martinec and A. Strominger for discussions on this point.} \]

15
is no useful description of black hole formation in this theory. It is therefore not yet clear what strings might tell us about the problem of formation and evaporation of black holes in two (or higher!) dimensions.

From this discussion we see that in some respects two-dimensional dilaton gravity is perhaps more akin to four-dimensional gravity than we might have hoped — it still suffers unpredictability. Nonetheless, we have clearly made progress. We have a concrete and well-understood semiclassical model for black hole evaporation embedded in a rich family of theories. Many of the complications of higher dimensions have been stripped away, so we might hope to get to the essence of various conceptual problems that arise in quantum gravity. At the top of our list is, of course, the issue of whether black holes avoid destroying quantum information, and if so how they succeed.

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8 As described in [45-48], an alternate interpretation of matrix models is in terms of perturbations on a black hole background. The connection of this to black hole formation and to the above statements is unknown.
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