Implications of the discrepancy between proton form factor measurements

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Recent polarization transfer measurements of the proton electromagnetic form factors yield very different results from previous Rosenbluth extractions. This inconsistency implies uncertainties in our knowledge of the form factors and raises questions about how to best combine data from these two techniques. If the discrepancy is due to missing corrections to the cross section data, as has been suggested, then the true form factors, related to the proton structure, differ from the form factors that parametrize the deviation from point scattering, and different applications will require the use of different form factors. We present two extractions of the form factors: a global fit to the world’s cross section data, and a combined extraction from polarization transfer and cross section data. The former provides a parametrization of the elastic electron-proton cross section. The latter provides a consistent extraction of the underlying form factors, under the assumption that missing terms in the radiative correction explain the difference between the cross section and polarization transfer results.

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I. INTRODUCTION

The proton electromagnetic form factors, $G_E$ and $G_M$, parametrize deviations from a point particle in elastic electron-proton scattering, and are related to the charge and magnetization distribution of the proton. The form factors depend only on $Q^2$, the square of the four-momentum transfer, and until recently it was believed that the electric and magnetic form factors showed approximate scaling, i.e. nearly identical $Q^2$ dependence [1]. More recent Jefferson Lab measurements [2, 3, 4] utilized the polarization transfer technique to measure the ratio $G_E/G_M$ and found that $G_E$ decreases more rapidly than $G_M$ at large $Q^2$. The polarization transfer measurements are more precise at high $Q^2$, and significantly less sensitive to systematic uncertainties than the Rosenbluth separation measurements. However, the two techniques disagree significantly even in the region where both yield precise results.

At the present time, it is not known why the techniques give different results. The systematic uncertainties of the polarization transfer measurements, primarily spin transport and backgrounds, have been carefully studied [5]. A detailed global analysis of the cross section measurements [6] does not show any inconsistencies in the cross section datasets, or yield any likely candidate to explain the discrepancy. To resolve the discrepancy, a systematic error in the cross section would have to have a significant dependence on the virtual photon polarization $\epsilon$, $\epsilon^{-1} = 1 + 2(1 + Q^2/4M_p^2)\tan^2(\theta_e/2)$, where $M_p$ is the proton mass and $\theta_e$ is the electron scattering angle. Such a systematic error would have to yield a $(5-7)\%$ dependence in the cross section, roughly linear in $\epsilon$, in order to resolve the discrepancy.

There appear to be two possibilities: either a fundamental flaw in the Rosenbluth or polarization transfer formalism, or an error in either the cross section or polarization transfer measurements. Recent works have suggested that additional radiative correction terms, related to two-photon exchange corrections, may lead to an error in determining the form factors from the measured cross sections [7, 8, 9]. If the two-photon exchange mechanism, or some other correction that is neglected in the cross section extraction, is the source of the discrepancy, then the form factors extracted from a Rosenbluth separation of cross section data will not represent the underlying structure of the proton, but they will parametrize the elastic electron-proton cross section in the usual one-photon approximation. Conversely, the true form factors will not yield the correct cross sections, and will thus give incorrect results if used as a parameterization of the elastic cross section in data analysis.

If the two-photon exchange term explains the discrepancy, then the polarization transfer result will relate to the true form factors, assuming that the two-photon exchange has a much smaller effect on the polarization transfer than on the Rosenbluth extractions. However, the existing polarization transfer experiments [2, 3, 4] have extracted the ratio $G_E/G_M$, rather than the individual form factors. To extract the form factors, these data must be combined with cross section measurements to determine the absolute magnitudes of $G_E$ and $G_M$. If the two-photon exchange correction modifies the cross sections from those calculated from the underlying form factors, then it is not possible to consistently combine the two kinds of measurements without some assumption about the two-photon exchange correction.

In this paper, we present two extractions of the proton form factors. From a global analysis of cross section measurements, we extract the “Rosenbluth form factors”. From a combined analysis of cross section and polarization transfer data, with a “minimal” assumption about the nature of the two-photon exchange corrections, we extract the “Polarization form factors”. If two photon corrections are the source of the discrepancy, then the Rosenbluth form factors will parametrize the elastic cross
section, and are therefore useful as input to analysis or simulations that require the electron-proton cross section. The Polarization form factors will provide the true form factors, which relate to the underlying structure of the proton. These form factors are often described as the Fourier transformations of the charge and magnetization distributions of the proton in the Breit frame, although relativistic effects and the fact that each value of $Q^2$ corresponds to a different Breit frame lead to substantial theoretical difficulties in extracting charge and magnetization distributions \[10\].

II. GLOBAL ROSENBLUTH ANALYSIS

The “Rosenbluth form factors” are determined from a global fit to elastic electron-proton cross section measurements. The details of the fitting procedure are described in Ref. \[6\]. For the present analysis we include more recent Jefferson Lab measurements of elastic scattering \[11, 12, 13\], as well additional datasets to constrain the low $Q^2$ behavior \[14, 15, 16, 17\] to the datasets used in Ref. \[6\, 18\]. In addition, we include all of the high $Q^2$ data, up to 30 GeV$^2$, while the previous analysis was limited to 8 GeV$^2$. The older data have updated radiative corrections, and the small-angle data from Walker et al. \[1\] are excluded, as described in \[6\]. The form factors are fit to the following form:

$$G_E(Q^2), G_M(Q^2)/\mu_p = [1+p_2Q^2+p_3Q^4+...+p_{2N}Q^{2N}]^{-1},$$  \[1\]

where $\mu_p$ is the magnetic dipole moment of the proton and $Q^2$ values are in GeV$^2$. Reasonable fits are achieved for $N \geq 3$. Note that this is a different functional form than used in previous fits \[6, 19, 20\], which used polynomials in $q = \sqrt{Q^2}$. The polynomial in $q$ is a very general form, with adequate flexibility to reproduce the data, but does not have the proper behavior as $Q^2 \to 0$.

The fit is quite insensitive to the order of the polynomial above $N = 6$, except for $G_E$ at large $Q^2$. For $Q^2$ above 6 GeV$^2$, fits with nearly identical $\chi^2$ values can have $G_E/G_M$ either rise or fall dramatically with $Q^2$. This is a result of the reduced sensitivity to $G_E$ and the limited $\epsilon$ coverage for $Q^2$ values above 6 GeV$^2$. To avoid unreasonable behavior in the region where $G_E$ is unconstrained by data, we keep the ratio $G_E/G_M$ fixed for all $Q^2$ values above 6 GeV$^2$. This leads to a fit for $G_E$ which is continuous, but not smooth, at $Q^2 = 6$ GeV$^2$. Because $G_E$ has relatively little contribution to the total cross section at these momentum transfers, the cross section extracted is still quite smooth, and the value of $G_E$ at large $Q^2$ values has little effect on the cross section, as long as the fit is constrained to avoid $|\mu_pG_E| \gg |G_M|$.

The normalization factor for each data set is allowed to vary along with the parameters of the fitting functions for $G_E$ and $G_M$. The total $\chi^2$ from the cross section measurements and normalization factors is:

$$\chi^2 = \sum_{i=1}^{N_x} \frac{(\sigma_i - \sigma_{\text{fit}})^2}{(d\sigma_i)^2} + \sum_{j=1}^{N\exp} \frac{(\eta_j - 1)^2}{(d\eta_j)^2},$$  \[2\]

where $\sigma_i$ and $d\sigma_i$ are the cross section and error (excluding normalization uncertainties) for each of the $N_x$ data points, $\eta_j$ is the fitted normalization factor for the $j$th dataset, and $d\eta_j$ is the normalization uncertainty for that dataset. We fit to 470 data points ($N_x=443, N\exp=27$) with 39 parameters (six parameters each for the electric and magnetic form factors, and 27 normalization parameters).

![FIG. 1: (Color online) The “Rosenbluth form factors” (solid line) for $G_E$ and $G_M$ relative to the dipole form: $G_D = [1 + Q^2/M_D^2]^{-1}$, $M_D^2=0.71$ GeV$^2$. The dot-dashed line is the previous fit to Rosenbluth extracted form factors from \[20\], and the dashed curve is the fit to $G_M$ from \[18\], with the form factor ratio constrained to give $\mu_pG_E/G_M = 1 - 0.13(Q^2 - 0.04)$.](image)

TABLE I: Fit parameters for the Rosenbluth form factors, using the parametrization of Eq. 1.

| Parameter | $G_E$ (Rosenbluth) | $G_M/\mu_p$ (Rosenbluth) |
|-----------|--------------------|----------------------------|
| $p_2$     | 3.226              | 3.19                       |
| $p_4$     | 1.508              | 1.355                      |
| $p_6$     | -0.3773            | 0.151                      |
| $p_8$     | 0.611              | -1.14×10^{-2}             |
| $p_{10}$  | -0.1853            | 5.33×10^{-4}              |
| $p_{12}$  | 1.596×10^{-2}      | -9.00×10^{-6}             |
than the quoted scale uncertainties of the experiments were overly conservative in estimating their uncertainties. The previous normalization factor differs by approximately 0.5%, and the RMS normalization factor is 2.7%. The normalization factors are very close to those obtained in the previous global fit [6]. The average normalization factor is 0.65%, and the RMS normalization is small, the absolute cross sections should be known to better than 2%. Above $Q^2=4$ GeV$^2$, there is a large body of cross section measurements with point-to-point uncertainties of ~1%. Because the normalization factors are determined in the fit, and the residual uncertainty in the normalization is small, the absolute cross sections should be known to better than 2%. Above $Q^2=4$ GeV$^2$, the number of data points decreases, and the uncertainties in the cross sections grow, reaching 10% at $Q^2=25$ GeV$^2$.

Even with the uncertainty related to the discrepancy between Rosenbluth and polarization transfer, this fit yields a precise parametrization of the elastic cross section in the one-photon exchange formalism. While these may not be the underlying form factors of the proton (e.g., if there are missing radiative correction terms), this is still the appropriate parametrization to use as input to a calculation or analysis that requires the elastic cross section. Using the form factors derived from the polarization transfer technique will not yield the correct cross section, even in a combined analysis of Rosenbluth and polarization transfer such as performed in Refs. [8] and [10]. More importantly, an inconsistent combination of cross section and polarization transfer results can magnify the error. Combining a parametrization of $G_M$ from a Rosenbluth analysis with the form factor ratios measured in polarization transfer decreases $G_E$, and thus decreases the total cross section, relative to the best fit to the cross section data, without allowing a corresponding increase in $G_M$. This leads to form factors which give cross sec-

![FIG. 2: $G_M$ (top) and $G_E$ (bottom) from direct Rosenbluth separation utilizing normalization factors from the global fit.](image-url)
tions that are (4–10)% below the measured cross sections at large $\epsilon$ over a large $Q^2$ range ($0.1 < Q^2 < 15$ GeV$^2$).

III. COMBINED POLARIZATION AND ROSENBLUTH ANALYSIS

While the Rosenbluth form factors yield the best parametrization for the cross section in the usual one-photon exchange picture, the ratio does not agree with the ratio extracted from the polarization transfer technique. For the larger $Q^2$ values, the polarization transfer technique is less sensitive to knowledge of the kinematics, radiative correction, and other systematic uncertainties that are important in the Rosenbluth separation.

If this discrepancy is related to a problem in the cross section data, then the polarization transfer will yield the true ratio of the form factors, but has to be combined with cross section data to obtain both $G_E$ and $G_M$. We present in this section a combined analysis of the polarization transfer and cross section data, which will yield the “Polarization form factors”.

In order to obtain a consistent extraction of the form factors, we must make an assumption about the nature of the discrepancy. We assume that the difference comes from a common systematic error in the cross section measurements. Analyses of this discrepancy\(^2, 5\) indicate that there must be an $\epsilon$-dependent correction of (5–7)%, roughly linear in $\epsilon$, for $1 < Q^2 < 6$ GeV$^2$.

In the combined analysis, we apply a linear, $\epsilon$-dependent correction of 6%, to all datasets. This is the ‘minimal’ assumption necessary to make the two techniques consistent, to the extent that a correction that was not linear in $\epsilon$, or which modified only some of the datasets, would have to be larger. A correction that is nearly linear in $\epsilon$ and fairly $Q^2$-independent is consistent with the form for the two-photon exchange term in the analysis of Ref.\(^5\), although the size of the correction in Ref.\(^5\) is only $\sim 2\%$, less than half the size necessary to explain the discrepancy.

We repeat the fit from Sec.\(^II\) but with cross sections modified by the linear $\epsilon$ dependence, and with the polarization transfer data included in the fit, as described in\(^5\). The correction to the cross section could either lower the cross section at large $\epsilon$ values, or increase it at small $\epsilon$ values:

$$
\sigma_{c1} = \sigma_0(1 - 0.06\epsilon) \quad (3)
$$

$$
\sigma_{c2} = \sigma_0(1 - 0.06(\epsilon - 1)) = 0.94\sigma_{c1} \quad (4)
$$

The first correction is consistent with the form from Ref.\(^5\), while the second is consistent with the behavior of\(^5\). The second form was chosen for the main fit because the correction is small at large $\epsilon$ (small $\theta_e$), where comparisons of positron to electron scattering from SLAC\(^21\) set fairly tight limits on the size of two-photon exchange.

The Polarization form factors, from the combined fit to the cross section and the 26 polarization transfer data points from Refs.\(^2, 5, 8\) is shown in Fig.\(^3\). The fit yields a total $\chi^2$ of 391.6 for 457 degrees of freedom, $\chi^2 = 0.857$, including the additional $\chi^2$ contribution for the polarization transfer data (Eq. (8) of Ref.\(^5\)). Table II gives the fit parameters for the Polarization form factors.

The fit was also performed with the correction of Eq. (4). This leads to an overall rescaling of all of the cross sections by 6%, relative to the correction of Eq. (3). However, this does not yield a simple rescaling of the form factors, because each dataset has a normalization factor that is determined in the fit, and because the form factors are constrained to reproduce the charge and magnetic moment at $Q^2 = 0$. While a two-photon correction of this size for large $\epsilon$ values would appear to be ruled out by the SLAC positron-proton measurements\(^21\), an $\epsilon$-dependent systematic other than two-photon exchange could also resolve the discrepancy. However, this fit yields a much worse $\chi^2$ value: 575.1 for 457 degrees of freedom, and so we choose to apply Eq. (4) for our combined fit.
Note that the result of the combined fit (Table I) will not reproduce the measured elastic cross section in the one-photon exchange formalism; it will reproduce the modified cross sections of Eq. 4. Therefore, the Polarization form factors should not be used to model elastic electron-proton cross section measurements. However, if the ‘minimal’ assumption that a correction consistent with the form of Eq. 4 explains the discrepancy, this should yield a consistent extraction of the underlying form factors of the proton.

IV. CONCLUSIONS

Form factors extracted using the Rosenbluth technique provide a parametrization of the deviation of the elastic electron-proton cross section from the point-scattering cross section. If the cross section has additional corrections, such as two-photon exchange terms, that are not being taken into account, then the Rosenbluth extraction does not yield the true proton form factors that relate to the structure of the proton. In this case, $G_E$ must be extracted from the polarization transfer measurements, which yield $G_E/G_M$, and the cross section data must be utilized to determine $G_M$. While we cannot know how to properly combine the polarization transfer and cross section data until we understand the cause of the discrepancy, the uncertainties in $G_M$ that arise from this problem are much smaller than those in $G_E$. The same holds true if there is some other correction or combination of corrections to the cross section other than the two-photon exchange (e.g. Coulomb corrections). It is of course possible that the discrepancy is due to a problem with the polarization transfer data or technique rather than the cross section data. If so, then the Rosenbluth form factors represent both the correct cross section and the correct nucleon structure. However, there do not appear to be any obvious candidates for problems in the technique, and the experiment should be less prone to systematic uncertainties than the Rosenbluth extractions.

We have presented two extractions of the proton electromagnetic form factors. The Rosenbluth form factors come from a global Rosenbluth extraction of the form factors from electron-proton elastic scattering measurements. The Polarization form factors come from a combined fit to the cross section and polarization transfer data, under the assumption that the discrepancy between the techniques is caused by a linear, $\epsilon$-dependent correction to the cross sections. The Rosenbluth form factors give a global parametrization of the elastic electron-proton scattering cross section in the one-photon exchange approximation. Even if there is a correction to the cross sections, neglected in the one-photon exchange formalism, this parametrization will yield the correct cross sections in the one-photon approach. Under the above assumption of an unknown correction to the cross sections, the Polarization form factors yield the underlying form factors, but will not reproduce cross sections, and will therefore yield incorrect results if used as input for an analysis that requires the elastic cross section, as was observed in an analysis of quasielastic scattering from nuclei.

Additional data will help shed light on the origin of the discrepancy. An improved “Super-Rosenbluth” separation measurement completed at Jefferson Lab in 2002 will yield a precise extraction of $G_E/G_M$, and will determine if the discrepancy can be explained by experimental problems in the Rosenbluth extractions. A new polarization transfer experiment, approved to run at Jefferson Lab, will provide an independent confirmation of the existing polarization transfer results, as well as extending the measurements to higher $Q^2$ values. If sufficiently improved calculations or direct measurements of the two-photon exchange corrections become available, we should be able to determine if they are responsible for the discrepancy, and if so, remove the current uncertainty in combining cross section and polarization transfer measurements.

If the discrepancy is explained by two-photon corrections or some other effect on the cross sections, and we have reliable calculations for these effects, then the cross section data can be combined with the polarization transfer data to extract the form factors without ambiguity. These form factors will represent the underlying structure of the proton and provide a useful parametrization of the elastic electron-proton cross section, as long as the effect is properly accounted for. Until the discrepancy is well understood, however, both sets of form factors are necessary, and it is important to use form factors that are (1) extracted consistently from the cross section and/or polarization transfer data, and (2) appropriate for the problem being addressed.

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