GB decomposition method based on standard particle swarm algorithm

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Abstract. Groom-Bailey decomposition method is the most widely used method of tensor decomposition of magnetotelluric impedance, which is referred to as GB decomposition. The essence of the GB decomposition method is to solve the nonlinear over-determined equations derived from the GB decomposition definition. Traditional GB decomposition methods often use linear optimization methods to solve the equations, which have low calculation accuracy and slow speed. In this regard, this article introduces the standard particle swarm optimization algorithm into the GB decomposition method for the first time. The standard particle swarm algorithm is used to solve the equations. The global optimization method is used to implement the magnetotelluric GB decomposition method, and the synthesized single-point single-frequency data is used. The 3D / 2D model forward data is used to perform decomposition experiments. The experimental results show that the standard electromagnetic particle decomposition method based on the standard particle swarm algorithm is effective.

1. Introduction
In the magnetotelluric sounding method, there is often an uneven electrical structure on the shallow surface, which causes the current density around the uneven body to be distorted, thereby causing changes in the electric field components and causing local distortion anomalies[1]. In order to eliminate the impact of such local distortion anomalies, Groom and Bailey assume that the local three-dimensional anomaly is overlaid on the two-dimensional regional anomaly, and the actual impedance tensor is decomposed into the distortion impedance tensor and the regional impedance tensor to recover The impedance tensor of the area not affected by the distortion (referred to as GB decomposition method). This method is one of the most commonly used methods in tensor decomposition of magnetotelluric impedance at present, and its essence is to solve a system of nonlinear over-determined equations composed of regional impedance tensor and distortion equation[2,3]. The traditional GB decomposition method linearly solves non-linear problems, resulting in unstable solution results, easy to fall into local extreme values, and it is difficult to obtain more accurate decomposition results. In this paper, the standard particle swarm optimization algorithm [4,5] is used to solve the nonlinear over-determined equations in the GB decomposition method, and it is not affected by the initial value of the model. The theoretical synthesis of single-frequency point data and 3D / 2D forward data decomposition experiments, The effectiveness and reliability of the algorithm are verified.

2. Algorithm

2.1. GB decomposition method
The GB decomposition method is based on a three-dimensional / two-dimensional electrical structure model. The observed impedance is decomposed into a regional impedance tensor and a distortion impedance tensor to recover the regional impedance tensor that is not affected by the distortion, and then the angle of the regional structural trend and Characteristics of electrical spindles. Without considering the inductive distortion of the magnetic field, the observed impedance tensor $Z$ can be decomposed into:

$$ Z = RCZ_{2D}R^t $$

$$ C = gTSA $$

among them:

$$ Z_{2D} = \begin{bmatrix} 0 & Z_{RE} \\ -Z_{TM} & 0 \end{bmatrix} $$

$$ R = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} $$

$$ T = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} $$

$$ S = \begin{bmatrix} 1 & e \\ e & 1 \end{bmatrix} $$

$$ A = \begin{bmatrix} 1 + s & 0 \\ 0 & 1 - s \end{bmatrix} $$

$Z_{2D}$ is regional impedance tensor in tectonic strike coordinate system, $C$ is distortion tensor matrix, $g$ is measuring point gain, and it’s a scalar. $R$ is rotation matrix of measurement coordinate system and tectonic strike coordinate system, $R^t$ is Transpose matrix of $R$. $T$ is twist matrix, $S$ is shear matrix, $A$ is Anisotropic matrix. Each of the above distortion factors will distort the impedance tensor. $gA$ is called the static displacement factor and does not change the shape of apparent resistivity and phase, so it is usually incorporated into $Z_{2D}$, recorded as $Z'$. $Z'$ can be expressed as:

$$ Z = RTSZ_{2D}R^t $$

Bringing the above matrices into equation (2) gives:

$$ \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} \begin{bmatrix} 1 & e \\ e & 1 \end{bmatrix} \begin{bmatrix} 1 + s & 0 \\ 0 & 1 - s \end{bmatrix} $$

Equation (3) $Z'_{2D}$ includes the real and imaginary parts of the impedance $A'$, the real and imaginary parts of the impedance $B'$, the structural strike angle $\theta$, twist factor $T$ and shear factor $S$, total 7 unknowns. By expanding the impedance tensors on the left and right sides of equation (3) according to the real and imaginary parts of the corresponding elements, a non-linear overdetermined equation system consisting of 8 equations can be obtained, thereby decomposing the impedance tensor to eliminate the effect of distortion. It turns into the problem of solving the optimal solution of the equations.

### 2.2. Particle swarm optimization

Particle swarm was first proposed by Eberhaet and Kennedy in 1995. The particle swarm algorithm uses particles with speed and position properties to simulate birds in a bird swarm. Speed represents the speed of movement and position represents the direction of movement. Several particles are randomly generated in the N-dimensional space, and each particle searches for the optimal solution separately in the search space, and records it as the current individual extreme value, and "shares" the individual extreme value with other particles in the entire particle swarm. And find the optimal individual extreme value as the current global optimal solution of the entire particle swarm. All particles in the particle swarm adjust themselves according to the current individual extreme value found by them and the current global optimal solution shared by the entire particle swarm. Speed and position. According to the principle of following the current optimal particle, the particle will change its speed and position according to equations (4) and (5).

$$ v_y(t + 1) = \omega v_y(t) + c_1 r_1(t) (p_y(t) - x_y(t)) + c_2 r_2(t) (p_y(t) - x_y(t)) $$

$$ x_y(t + 1) = x_y(t) + v_y(t + 1) $$
Where is the $\omega$ inertia weight coefficient, and its value is non-negative. When the value is large, the global optimization ability is strong, and the local optimization ability is weak; when the value is small, the global optimization ability is weak, and the local optimization ability is strong. 

$$j = 1, 2, ..., N, i = 1, 2, ..., m; m$$ is the population size and $t$ is the current evolutionary algebra; $\rho$ is a random number distributed between $[0, 1]$, $c_1, c_2$ is the acceleration constant. Figure 1 is a flowchart of particle swarm algorithm.

![Fig.1 PSO flow diagram](image)

### 2.3. Algorithm instance

The single-point single-frequency data uses a set of impedance tensor data in the North Central Plains [6,7]. The data are artificially added to the distortion matrix. The real value of each distortion parameter is $-2.1^\circ$, the shear factor is $24.95^\circ$, and the tectonic strike angle is $0^\circ$. The actual distortion impedance is:

$$
Z = \begin{bmatrix}
-3.6300 - 1.3600i & 5.9500 + 5.1000i \\
-7.1000 - 2.6700i & 2.5100 + 2.1500i
\end{bmatrix}
$$

In this paper, an improved algorithm is used to calculate the above distortion data 100 times. The calculated statistical results are: the distortion factor is $2.101^\circ$, the shear factor is $26.6778^\circ$, and the structural strike angle is $0.070^\circ$. The results obtained are shown in Figure 2, where the horizontal axis is the number of calculations and the vertical axis is the calculation result.
In order to further verify the decomposition effect of the algorithm in this paper, a three-dimensional / two-dimensional electrical structure model was established based on the assumptions required by the GB decomposition method. The first step is to establish a two-dimensional electrical structure model and calculate a set of apparent resistivity and phase response data. The second step is to add a local three-dimensional block model to the two-dimensional electrical structure model to form a three-dimensional / two-dimensional electrical structure model. The electrical structure model is used to calculate the corresponding response data. In the third step, the modified data of this paper is used to perform impedance tensor decomposition on the response data obtained from the forward modeling of the 3D / 2D electrical structure model. The resistivity of the background region is 500 Ω · m, the two-dimensional electrical structure is an infinitely extending low-resistance body, the resistivity is 100 Ω · m, and the local three-dimensional block resistivity is 1 Ω · m. The grid used for the calculation is and the calculation frequency band is for a total of 20 frequency points. This paper selects typical measurement points for analysis. The relative positions of the measurement points and the abnormal body are shown in Figure 3. Figures 4 (a) and (b) respectively show the apparent resistivity and phase curves calculated from the forward calculation of a two-dimensional electrical structure model that does not include a local three-dimensional block, showing obvious two-dimensional characteristics. Figures 4 (c) and (d) are the apparent resistivity and phase curves obtained from the forward modeling of a 3D / 2D electrical structure model containing local 3D blocks, respectively. As can be seen from the figure, the apparent resistivity curve is significantly affected by the effect of the more serious local electric field distortion effect is shown, showing a strong three-dimensionality. Figure 5 shows the results obtained by GB decomposition calculation of typical measurement points using this method, where (a) is the calculation result of the shear factor Shear and the twist factor Twist, and (b) is the calculation result of the electrical principal axis angle. From the results, in the forward frequency band, the shear angle is distributed between 0 ° and 20 °, the twist angle is distributed between 0 ° and 10 °, and the electrical main axis angle is distributed between 5 ° and 33 °. This shows that the parameters of the model calculated by the GB decomposition method based on particle swarm algorithm are very stable and reliable.
3. Conclusions

Based on previous research, this paper introduces the standard particle swarm algorithm to the tensor decomposition method of magnetotelluric impedance. The introduction of particle swarm algorithm in the GB decomposition algorithm is effective and reliable. The particle swarm algorithm is simple and efficient. It uses a global optimization method. Multiple particles are optimized at the same time and cooperate with each other. The stability of the results is greatly improved.

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