Synchronization of a periodic modulation of mirrors in an optomechanical system

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Abstract
Proposing an optomechanical cavity modulated periodically, we have studied the modulation synchronization of the mechanical modes of mirrors. A periodic modulation has been applied to one of the mirrors, where the second mirror has the capability of oscillation with no modulation on that. As a result of having the periodic modulation, we have found a phase-locking synchronization between the mechanical modes of the mirrors and the enhancement of quantum synchronization. Considering the fact that periodic modulation can make squeezed states, we have shown that there is a robust synchronization of periodic modulation against enhancement of detuning between the mirrors. Also, our results show that having a periodic modulation leads to a stationary entanglement generation between the mirrors.

Keywords Periodic modulation · Synchronization · Entanglement generation

1 Introduction

Synchronization as a well-known classical phenomenon has been observed in a large variety of contexts, e.g., physical systems, biology, chemical reactions, etc. [1–5]. The destructive effects of noises on synchronization and their crucial role in the quantum regime have made the quantum system extremely appealing to trace the effects of noises on synchronization [6–8]. That is to say, finding a proper and global measure of quantum synchronization and the absence of a clear notion of phase space trajectories are still the main challenges in the studies on quantum synchronization [7].
As mentioned before, noise statistics overwhelm synchronization in the quantum regime. So, any scenarios in accordance with reducing the noises could be a remarkable proposal of making synchronization. Using the squeezing Hamiltonian instead of a harmonic derive, S. Sonew and et al. have shown that the squeezing would enhance quantum synchronization [9].

Periodic modulation has been reported as a technique that may increase the possibility of accomplishing squeezed states in a wide variety of optomechanical systems [10–14]. By choosing appropriate periodic modulation, one can achieve better squeezing comparing to the other method of squeezed states’ creation [15,16].

The effect of periodic modulation on quantum systems has been studied widely [17–19]. A. Frace and V. Giovannetti have shown the enhancement of quantum effects in optomechanical systems using periodic modulation [15]. Besides, a synchronization enhancement via periodic modulation in optomechanical systems has been reported recently [20].

The aim of this work is to estimate the synchronization of the periodic oscillation of the mirrors raised by the periodic modulation of one of them in an optomechanical system. It is also interesting to study the correlations between the mirrors and find a relation between correlations and synchronization.

In this paper, we have proposed an optomechanical system with two oscillating mirrors coupled to the same optical field in a cavity. One of the mirrors (M1) has been considered to be modulated periodically (Fig. 1). We have investigated the generation of a squeezed state of the mirrors and also the synchronization and phase locking between their oscillation raised by the modulation.

Furthermore, our time modulated optomechanical system has been introduced and the Hamiltonian and the master equation of the system have been under discussion. Besides using QuTiP [21] to solve the master equation and also Langevin equations, we have derived the dynamics of the system and discussed the results.

2 The system

We have proposed an optomechanical cavity with two movable mirrors at both ends, where they are interacting with the optical field of the cavity of frequency $\omega_c$. The mirrors are attached to springs of effective characteristic frequencies $\omega_M$ and $\omega_M + \Delta_M$ for mirror1(M1) and mirror2(M2), respectively, where $\Delta_M$ is assumed to be the detuning of the frequencies of mirrors. As shown in Fig. 1, the cavity is driven by a laser of frequency $\omega_L$. The periodic modulation on M1 has been expressed as a time-dependent function in the Hamiltonian of the system,
\[ H = \Delta a^\dagger a + \frac{\omega_M}{2} p_1^2 + \frac{\omega_M}{2} q_1^2 \{1 + \epsilon \sin^2(\Omega t)\} + \frac{(\omega_M + \Delta_M)}{2} (p_2^2 + q_2^2) - g a^\dagger a (q_1 + q_2) + i E (a^\dagger - a) \]  

(1)

where \( \hbar = 1 \), \( E \) determines the strength of the external drive, \( q_{1,2} \) and \( p_{1,2} \) are the dimensionless position and momentum of the mirrors satisfying the commutation relation \( [q_k, p_j] = i \delta_{jk} \) and \( g \) is the mirrors-field coupling constant. In addition, \( \Delta = \omega_c - \omega_L \) is the detuning, and \( a (a^\dagger) \) is the annihilation(creation) operator, of the cavity mode. Also, \( \epsilon \) determines the domain of the modulation and \( \Omega \) is the modulation frequency.

The full dynamics of the system is described by a set of nonlinear Langevin equations, including the effects of vacuum radiation noise and quantum Brownian noise acting on the mirror. Dynamics of the cavity and mirror modes under the Hamiltonian (1) are given by

\[ \dot{a} = -(i \Delta + \kappa) a + i g (q_1 + q_2) a + E + \sqrt{2 k} a^{in}, \]

\[ \dot{q}_1 = \omega_M p_1, \]

\[ \dot{p}_1 = -\omega_M (1 + \epsilon \sin^2(\Omega t)) q_1 + g a^\dagger a - \gamma_m p_1 + \xi_1, \]

\[ \dot{q}_2 = (\omega_M + \Delta_M) p_2, \]

\[ \dot{p}_2 = -(\omega_M + \Delta_M) q_2 + g a^\dagger a - \gamma_m p_2 + \xi_2, \]

where \( a^{in} \) is the radiation vacuum input noise with autocorrelation function \( \langle a^{in}(t) a^{in*}(t') \rangle = \delta(t - t') \) and \( \xi_i(t) \) is the Brownian noise operator of the mirror which as shown in [22], for a good quality mirror \( \omega_M \gg \gamma_m \), their autocorrelation function satisfies the following relation

\[ \langle \{\xi(t), \xi(t')\} \rangle \approx 2 \gamma_m \coth \left( \frac{\hbar \omega_M}{2 K_B T} \right) \delta(t - t'), \]

(3)

where in the above equation, \( \{,\} \) is the anticommutator, \( K_B \) is the Boltzmann constant and \( T \) is the system temperature. Meanwhile, each operator can be expressed as sum of stationary value plus an additional fluctuation operator, \( a = \alpha_x + \delta a, a^\dagger = \alpha_x + \delta a^\dagger, q_i = Q_i + \delta q_i \) and \( p_i = P_i + \delta p_i \), while \( Q_i(P_i), i = 1, 2 \) is steady values of dimensionless position (momentum). With definition \( x = \frac{a + a^\dagger}{\sqrt{2}}, y = \frac{a - a^\dagger}{i \sqrt{2}}, x^{in} = \frac{a^{in} + a^{in\dagger}}{\sqrt{2}}, y^{in} = \frac{a^{in} - a^{in\dagger}}{i \sqrt{2}} \) are the quadratures of the field and input noises, substitution of these equations in the regime with large coherent amplitudes for mechanical and optical field leads to the exact quantum linear Langevin equations for the fluctuations as

\[ \delta \dot{x} = \Delta \delta y - \kappa \delta x + \sqrt{2 k} \delta x^{in}, \]

\[ \delta \dot{y} = -\Delta \delta x - \kappa \delta y + G(\delta q_1 + \delta q_2) + \sqrt{2 k} \delta y^{in}, \]

\[ \delta \dot{q}_1 = \omega_M \delta p_1. \]
\[
\begin{align*}
\delta p_1 &= -\omega_M(1 + \epsilon \sin^2(\Omega t))\delta q_1 + G\delta x - \gamma_{m1}\delta p_1 + \xi_1, \\
\delta q_2 &= (\omega_M + \Delta_M)\delta p_2, \\
\delta p_2 &= -(\omega_M + \Delta_M)\delta q_2 + G\delta x - \gamma_{m2}\delta p_2 + \xi_2.
\end{align*}
\] (4)

where \( G = \sqrt{2g\alpha_s} \). It could be possible to rewrite above equations of motions in the compact matrix form

\[
\delta U(t) = A\delta U(t) + \delta N^{in}(t).
\] (5)

where \( \delta U(t) = (\delta x(t), \delta y(t), \delta q_1(t), \delta p_1(t), \delta q_2(t), \delta p_2(t))^T \) is the system operator vector and \( \delta N^{in}(t) = (\delta x^{in}(t), \delta y^{in}(t), 0, \xi_1, 0, \xi_2)^T \) is the vector of noises and the drift matrix \( A \) is given by

\[
A = \begin{pmatrix}
-\kappa & \Delta & 0 & 0 & 0 & 0 \\
-\Delta & -\kappa & G & 0 & 0 & 0 \\
0 & 0 & 0 & \omega_m & 0 & 0 \\
G & 0 & -\omega_M(1 + \epsilon \sin^2(\Omega t)) & -\gamma_{m1} & 0 & 0 \\
0 & 0 & 0 & 0 & \omega_M + \Delta_M & -\gamma_{m2}
\end{pmatrix}.
\] (6)

Complete description of the system is given in terms of the second statistical moments, which can be arranged in the covariance matrix \( \sigma \) of entries \( \sigma_{ij}(t) := \langle [U_i(t)U_j(t)] \rangle / 2 - \langle U_i(t) \rangle \langle U_j(t) \rangle \). The equation of motion for the covariance matrix \( \sigma \) is

\[
\dot{\sigma} = A\sigma + \sigma A^T + D,
\] (7)

where the diffusion matrix \( D \) reads \( D = \text{diag}\{\kappa (2n_{ph}+1), \kappa (2n_{ph}+1), 0, \gamma_{m1}(2n_{m1}+1), 0, \gamma_{m2}(2n_{m2}+1)\} \) while \( n_m = (e^{\hbar\omega_{m}/k_B T} - 1)^{-1} (n_{ph} = (e^{\hbar\Delta/k_B T} - 1)^{-1}) \) is the mean phonon number of the mirror (field). These calculations have been done by using the linearization technique, while it is likely to obtain the dynamics of the system by solving the master equation of the system directly. Subsequently, we have introduced the master equation governing the dynamics of our system, and it would be worth checking the effects of linearization on our results by comparing them to both approaches.

Also, one can use the master equation approach to derive the dynamics of the system,

\[
\frac{d\rho}{dt} = -i[\rho, H] + \sum_{i=1}^{2} \gamma_i \left( 2b_i^{\dagger}\rho b_i^{\dagger} - b_i^{\dagger}b_i^{\dagger}\rho - \rho b_i^{\dagger}b_i \right) + \kappa \left( 2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a \right).
\] (8)

where \( b_i (1, 2) \) is the annihilation operator which corresponds to the mechanical mode of the mirrors and system is intended to be at zero temperature. Mirrors are also considered to have the same damping rate \( \gamma_i (1, 2) \) while \( \kappa \) is the decay rate of the cavity mode.
3 Results

We have followed both master and Langevin equation approaches. Setting $\omega_M = 1$, other used normalized parameters to $\omega_M$ are as $\Delta = 1$, $\kappa = 0.1$, $\gamma_1 = \gamma_2 = 0.001$, $g = 0.05$, $\Omega = 0.5$, $\epsilon = 0.5$ and $E = 2.1$. The detuning $\Delta_M$ is left as variable to study the robustness of synchronization.

3.1 Squeezing

In a quantum regime, fluctuations play a more critical role. It is expected to overcome the complications associated with added noises originating from quantum fluctuations by squeezing and enhance quantum synchronization [9]. Periodic modulation has been introduced as a practical approach to preparing squeezed states. Since it is an oscillating behavior, it would be interesting to study its synchronization. Indeed, the first sign of having periodic modulation synchronization would be the squeezing of M2.

The ground-state fluctuations minimize the uncertainty relation,

$$\langle \Delta X_1^2 \rangle \langle \Delta X_2^2 \rangle \geq \frac{1}{4} \left| \langle [X_1, X_2] \rangle \right|^2 .$$

\( \text{(9)} \)
It is possible to squeeze one of the quadratures to have the fluctuations below the zero-point level at the expense of increasing the fluctuation of the other quadrature. Using the periodic modulation $\omega M q_1^2 \epsilon \sin^2(\Omega t)$, in Hamiltonian Eq. (1), the states of M1 have been squeezed. Although only M1 has been modulated, in the case of zero-detuning between the mirrors $\Delta M = 0$, the states of M2 have also been squeezed (Fig. 2). One can consider this squeezing of the states of M2 as a result of periodic modulation synchronization of the mirrors. As can be seen from these results, for our system, there is no significant difference between both numerical approaches. Since all the results have a great coincidence, we will demonstrate just the results obtained by the master equation approach using QuTip.

### 3.2 Synchronization

Before using any measure to determine the synchronization and phase locking between the oscillation of the mirrors, supported and enhanced by periodic modulation of M1, we can take a look at Fig. 3 where the final oscillations of the position quadratures $q_1$ and $q_2$ are plotted. It is easy to find a phase locking between the oscillations of the mirrors. Also, it is remarkable to have the evolution of the position and momentum quadratures of the mirrors. In Fig. 4, one can see that the evolution tends to a periodic orbit for both of the mirrors. In the case of zero-detuning ($\Delta M = 0$), the orbits have almost the same dimension. While considering the nonzero-detuning regime, the second mirror orbit gets smaller than the first mirror by increasing the detuning $\Delta M$.

Mari et al. [7] introduced the following measure for synchronization that one can gauge the synchronization level of two subsystem using their position and momentum quadratures,

$$S(t) = < q_-^2(t) + p_-^2 >^{-1},$$

where $q_-(t) = [q_1 - q_2]/\sqrt{2}$ and $p_-(t) = [p_1 - p_2]/\sqrt{2}$. The synchronization measure $S(t)$ has a maximal value 1.0 correspond to a complete synchronization. This limit is applied on $S(t)$ by Heisenberg’s uncertainty principle.

Lei Du et al. enhanced a quantum synchronization in an optomechanical system using a proper periodic modulation [20]. They achieved the synchronization measure $S(t)$ up to 0.9. We find the same value of $S(t)$ for the synchronization of the periodic modulation in our optomechanical system.

### 3.3 Mutual information and entanglement

The study of synchronization in a quantum system, composed of two subsystems $A$ and $B$ where they are correlated, has been attracting a lot of interests [23–25]. For instance, entanglement between synchronized subsystems has been studied in a variety of systems [26–28].

Among the variety of entanglement measures, we have used logarithmic negativity which is defined as,

$$E_N(\rho) = \log_2(||\rho^{\Gamma_A}||),$$

where $\Gamma_A$ is the local operation on subsystem $A$. The logarithmic negativity is a measure of entanglement that can be used to quantify the degree of entanglement in a quantum state.
Fig. 3 (Color online) Oscillations of position quadratures of the mirrors after a sufficiently long time as a function of $t/\tau (\tau = \frac{2\pi}{\omega})$

Fig. 4 (Color online) The evolution of position and momentum quadratures of the mirrors for \(\Delta M = 0\) at the beginning, and \(\Delta M = 0\), \(\Delta M = 0.05\), \(\Delta M = 0.1\) after a sufficiently long time
where $\Gamma_A$ is the partial transpose operation with respect to subsystem $A$ and $||.||$ denotes the trace norm. In Fig. 6(a), the logarithmic negativity of the mirrors is plotted as a function of time. As a result of periodic modulation, the mirrors get entangled. Very recently, Roulet, et al. have studied the relationship between quantum synchronization and the generation of entanglement [29]. Comparing Figs. 5 and 6a, one can conclude that the entanglement generation starts as the mirrors get synchronized. Despite the fact that the relation between entanglement and synchronization strongly depends on the details of the system, the mutual information can indicate the presence of a quantum synchronization more generally. Recently, Ameri et al. have shown that the synchronized subsystems have large mutual information. So, it can be used as a signal of quantum synchronization [30]. The quantum mutual information is defined as,

$$I = S(\rho_A) + S(\rho_B) + S(\rho),$$

where the quantum state of the system is traced partially to derive the reduced density matrices $\rho_A = \text{Tr}_B(\rho)$ and $\rho_B = \text{Tr}_A(\rho)$ and $S(\rho) = -\text{Tr}[\rho \log(\rho)]$ is the von Neumann entropy. Figure 6b shows the mutual information of mirrors as a function of time. As can be seen from Figs. 6 and 5, the strong correlation between the mutual information, entanglement and synchronization approved again.

4 Conclusion

In conclusion, we have demonstrated the possibility of synchronizing a periodic modulation of one mirror to another one in an optomechanical system. As a result, it has been shown that the state of the second mirror is also squeezed where it is one of the consequences of periodically modulated mirrors, and the evolution of position and momentum quadratures for both mirrors follows the same periodic orbit, originating from the periodic modulation of the first mirror. Finally, the quantum synchronization measure $S(t)$ indicates that the mirrors are almost completely synchronized.
Another stimulating result of this study is an entanglement generation between the mirrors due to the periodic modulation. Looking at Figs. 5 and 6, one can conclude that the entanglement generation and the synchronization have been started almost at the same time. Therefore, as expected and reported previously, the correlations can imply the presence of synchronization in the systems.

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