Perturbative Corrections to a Sum Rule for the
Heavy Quark Kinetic Energy

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(March 26, 2022)

Abstract

We calculate the perturbative corrections to order $\alpha_s^2 \beta_0$ to the sum rule derived from the second moment of the time-ordered product of $b \rightarrow c$ currents near zero recoil. This sum rule yields a bound on $\lambda_1$, the expectation value of the $b$ quark kinetic energy operator inside the $B$ meson. The perturbative corrections significantly weaken the bound relative to the tree level result, yielding $\lambda_1 < -0.15 \text{ GeV}^2$.

*The work of H.D. and A.K.L. was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-92-ER40701.
I. INTRODUCTION

Heavy Quark Effective Theory (HQET) is a powerful tool for studying the decays of hadrons containing one heavy quark $Q$. For $B \to D^{(*)}e\bar{\nu}_e$ decays all form factors, at leading order in the $1/m_Q$ expansion, are related to the Isgur–Wise function. This function is normalized to unity at zero recoil \[1,2\].

Combining HQET and the operator product expansion (OPE) has also led to improvements in the understanding of inclusive $B$ decays \[3\]. It is possible to show, at leading order in $1/m_Q$, that the inclusive semileptonic $B$ decay rate is equal to the free $b$ quark decay rate. Corrections to this result enter at order $1/m_Q^2$ \[4–8\] and are parameterized by two nonperturbative matrix elements,

\[
\lambda_1 \equiv \frac{1}{2m_M} \langle M(v)|\bar{h}_v^{(b)} (iD) 2h_v^{(b)}|M(v)\rangle \tag{1.1}
\]

and

\[
\lambda_2 \equiv \frac{1}{2d_Mm_M} \langle M(v)|\bar{h}_v^{(b)} \frac{g_s}{2}\sigma_{\mu\nu}G^{\mu\nu}h_v^{(b)}|M(v)\rangle, \tag{1.2}
\]

where $h_v^{(b)}$ is the $b$ quark field in HQET, $M$ is either a $B$ or a $B^*$ meson, $d_B = 3$ and $d_{B^*} = -1$. These parameters, along with another parameter $\Lambda$, also enter into the relation between the quark pole mass and the hadron mass

\[
m_M \equiv m_b + \frac{\Lambda}{2} - \frac{\lambda_1 + d_M\lambda_2}{2m_b} + O(1/m_b^2). \tag{1.3}
\]

While it is possible to obtain a value for $\lambda_2$ from the measured $B - B^*$ mass splitting, extractions of $\Lambda$ and $\lambda_1$, although well studied, have large uncertainties \[9,10\].

By taking appropriate moments of the time-ordered product of $b \to c$ currents, it is possible to obtain sum rules that relate the exclusive decay form factors to the HQET nonperturbative parameters \[11\]. Taking the zeroth moment of the time-ordered product yields the Bjorken sum rule \[12,13\]. This sum rule bounds the slope parameter $\rho^2 \equiv -d\xi/dw|_{w=1}$ from below, where $\xi(w)$ is the Isgur–Wise function, $w = v \cdot v'$, $v$ is the four-velocity of the $B$ and $v'$ is the four-velocity of the $D^{(*)}$. The first moment can be used to derive the Voloshin
sum rule [14], which bounds $\rho^2$ from above. The zeroth moment and the second moment can be combined to obtain a bound relating $\lambda_1$ to $\rho^2$ [5,11].

In this paper, we calculate the perturbative QCD corrections to this third sum rule to $O(\alpha_s^2 \beta_0)$ and leading order in $1/m_Q$. We also consider redefining $\lambda_1$ and $\Lambda$ in order to absorb these corrections, and compare these redefinitions to those suggested by other previously studied sum rules [15,16]. It is of interest to see whether the perturbative redefinition can be achieved in a universal way, regardless of the sum rule in question.

Our paper is organized as follows. In the next section, we introduce the formalism used in our work. In section III, we present the perturbative corrections to the aforementioned third sum rule and derive a bound on $\lambda_1$. Section IV contains a discussion of this bound and the utility of the redefinition of $\lambda_1$ and $\Lambda$.

II. FORMALISM

We introduce the time-ordered product

$$T_{\mu\nu} = \frac{i}{2m_B} \int d^4x e^{-i\vec{q}\cdot\vec{x}} \langle B|T\{J_\mu^\dagger(x), J_\nu(0)\}|B\rangle,$$  \hspace{1cm} (2.1)$$

where $J_\mu$ is a $b \to c$ axial or vector current, $|B\rangle$ represents the $B$ meson state at rest, and $q$ is the four-momentum transfer. Here the three-momentum transfer $\vec{q}$ is fixed and $q^0 = M_B - E_M - \varepsilon$, where $E_M = \sqrt{m_M^2 + |\vec{q}|^2}$ is the minimal possible energy of the hadronic final state associated with $J_\mu$. The time-ordered product $T_{\mu\nu}$ has two cuts in the complex $\varepsilon$-plane. One cut lies along the positive real axis $0 < \varepsilon < \infty$. The second cut, corresponding to physical states with two $b$-quarks and a $\bar{c}$-quark, lies along $-\infty < \varepsilon < -2E_M$. This second cut does not affect our results.

By contracting $T_{\mu\nu}$ with an appropriate four-vector $a$, it is possible to isolate specific hadronic form factors. Let

$$T(\varepsilon) \equiv a^\mu T_{\mu\nu}(\varepsilon)a^\nu.$$  \hspace{1cm} (2.2)$$

Inserting a complete set of states $X$ between $J_\mu(x)$ and $J_\nu(0)$ in Eq. (2.1) yields
\[ T(\varepsilon) = \frac{1}{2m_B} \sum_X (2\pi)^3 \delta^{(3)}(\vec{q} + \vec{p}_X) \frac{|\langle X| J \cdot a | B \rangle|^2}{E_X - E_M - \varepsilon} + \cdots, \quad (2.3) \]

where the ellipsis represents the contribution from the other cut. By integrating over \( \varepsilon \), the following zeroth moment sum rule is obtained

\[ \frac{1}{2\pi i} \int_C d\varepsilon \theta(\varepsilon - \Delta) T(\varepsilon) = \sum_X \theta(E_X - E_M - \Delta) (2\pi)^3 \delta^{(3)}(\vec{q} + \vec{p}_X) \frac{|\langle X| J \cdot a | B \rangle|^2}{2m_B}. \quad (2.4) \]

In Eq. (2.4), we have included a \( \theta \)-function which corresponds to summing over all hadronic resonances up to an excitation energy \( \Delta \). Relating the integral with the hard cutoff to the exclusive states above requires local duality at the scale \( \Delta \).

The second moment of \( T(\varepsilon) \) gives

\[ \frac{1}{2\pi i} \int_C d\varepsilon \varepsilon^2 \theta(\varepsilon - \Delta) T(\varepsilon) = \sum_{X \neq M} \theta(E_X - E_M - \Delta) (E_X - E_M)^2 \times (2\pi)^3 \delta^{(3)}(\vec{q} + \vec{p}_X) \frac{|\langle X| J \cdot a | B \rangle|^2}{2m_B}. \quad (2.5) \]

A combination of the zeroth and second moments sum rules, assuming that the contribution of multi-hadron states is negligible below the first excited state \( M_1 \), yields

\[ \frac{1}{2\pi i} \int_C d\varepsilon \theta(\varepsilon - \Delta) T(\varepsilon) \left( 1 - \frac{\varepsilon^2}{(E_{M_1} - E_M)^2} \right) = \frac{|\langle M| J \cdot a | B \rangle|^2}{4m_B E_M} - \cdots, \quad (2.6) \]

where the ellipsis denotes positive terms whose first derivatives at \( w = 1 \) are also positive. In the next section this equation will be used to derive a bound on \( \lambda_1 \).

\( T(\varepsilon) \) in Eq. (2.3) can be calculated using an OPE [7]. By taking suitable moments of \( T(\varepsilon) \), it is possible to get different sum rules that depend on the parameters of HQET. For the second moment sum rule, with \( a = (0, 0, 1, 0) \), this yields [5,11]

\[ \left( \frac{2w}{w + 1} \right) \left( \frac{1}{\pi} \right) \int d\varepsilon \theta(\varepsilon - \Delta) \varepsilon^2 \text{Im} T(\varepsilon) = - \left( \frac{w^2 - 1}{3w^2} \right) \lambda_1 + \Lambda^2 \left( \frac{w - 1}{w} \right)^2. \quad (2.7) \]

\( T(\varepsilon) \) can be calculated to any desired order in \( \alpha_s \), thus giving perturbative corrections to the sum rules. Here we are concerned with the corrections to the second moment sum rule, which are presented in the next section.
III. RESULTS

To calculate the $\alpha_s$ corrections, the optical theorem is used to relate the imaginary part of $T(\varepsilon)$ to the $b \to c$ decay rate. At $O(\alpha_s)$, the diagrams that contribute are given in Fig.(1). The vertex correction could in principle contribute at $O(\alpha_s)$, but is suppressed by $\alpha_s(\Lambda_{QCD}/m_Q)^2$, which is consistently neglected in this paper. Therefore we only consider the bremsstrahlung diagrams.

Using the diagrams in Fig. (1), and expanding near zero recoil, the leading $O(\alpha_s)$ correction to the imaginary part of $T(\varepsilon)$ is

$$
\frac{1}{\pi} \text{Im} T^{(\text{brem})}(\varepsilon_q) = \frac{8 \alpha_s}{9 \pi} \left[ (w - 1) \frac{2 \varepsilon_q^2 + m_g^2}{\varepsilon_q^4} \sqrt{\varepsilon_q^2 - m_g^2} \right. \\
\left. - (w - 1)^2 \frac{8 \varepsilon_q^6 - 4 \varepsilon_q^4 m_g^2 - \varepsilon_q^2 m_g^4 - 18 m_g^6}{5 \varepsilon_q^6 \sqrt{\varepsilon_q^2 - m_g^2}} \right] \theta(\varepsilon_q - m_g) + \cdots,
$$

where $\varepsilon_q = m_b - E_c - q_0$. The ellipsis denotes higher order correction in $\alpha_s$, $w - 1$, and $1/m_Q$. We have performed the $O(\alpha_s)$ calculation with a gluon mass $m_g$. The technique introduced in Ref. [17] then allows us to obtain the $\alpha_s^2\beta_0$ correction from this result by means of a dispersion relation. The partonic variable $\varepsilon_q$ is related to the hadronic variable $\varepsilon$ by

$$
\varepsilon_q = \varepsilon - \Lambda \left( \frac{w - 1}{w} \right)
$$

(3.2)

to the order we are considering. Rewriting Eq. (3.1) in terms of $\varepsilon$, the second moment of $\text{Im} T(\varepsilon)$ becomes

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Fig. 1. Feynman diagrams that contribute to the $\alpha_s$ corrections of the second moment sum rule.
\[
\frac{1}{\pi} \int_C d\varepsilon \varepsilon^2 \theta(\varepsilon - \Delta) \text{Im} T(\varepsilon) = -\frac{2}{3} (w - 1) \left[ \lambda_1 - \frac{4\alpha_s(\Delta)}{3\pi} \Delta^2 - \frac{2\alpha_s^2(\Delta)}{3\pi^2} \Delta^2 \left( \frac{13}{6} - \ln 2 \right) \right] \\
+ \frac{4}{3} (w - 1)^2 \left[ \left( \lambda_1 - \frac{8\alpha_s(\Delta)}{15\pi} \Delta^2 - \frac{4\alpha_s^2(\Delta)}{15\pi^2} \Delta^2 \left( \frac{187}{60} - \ln 2 \right) \right) \\
+ \frac{3}{4} \left( \lambda^2 + \frac{16\alpha_s(\Delta)}{9\pi} \Delta \lambda \right) \right], \tag{3.3}
\]

where \(\alpha_s\) is defined in the \(\overline{\text{MS}}\) scheme. \footnote{We disagree with the result presented in Ref. \cite{18}.}

The perturbative corrections to the zeroth moment sum rule were calculated in Ref. \cite{16} to \(\mathcal{O}(\alpha_s)\) and \(\mathcal{O}(w - 1)\). Combining their results with Eq. (2.6) and Eq. (3.3) gives

\[
\left( \frac{1 + w}{2w} \right) \left[ 1 + (w - 1) \frac{8\alpha_s(\Delta)}{9\pi} \left( \ln 4 - \frac{5}{3} \right) \right] \\
+ \frac{1}{\delta_1^2} \left[ \frac{2}{3} \lambda_1 (w - 1) - \frac{8\alpha_s(\Delta)}{9\pi} \Delta^2 (w - 1) \right] = \frac{(1 + w)^2}{4w} |\xi(w)|^2 - \cdots, \tag{3.4}
\]

where \(\delta_1 = E_{M_1} - E_M\) is the lowest excitation energy and again the ellipsis denotes positive terms which have positive first derivatives at zero recoil. We have used \(|\langle D^* | J \cdot a | B \rangle|^2 = m_B m_{D^*} (1 + w)^2 |\xi(w)|^2\). Taking the derivative of Eq. (3.4) with respect to \(w\), and setting \(w = 1\), gives the bound

\[
\lambda_1 < -3\delta_1^2 \left[ \rho^2(\Delta) - \frac{1}{4} - \frac{4\alpha_s(\Delta)}{9\pi} \left( \frac{5}{3} - \ln 4 \right) \right] + \frac{4\alpha_s(\Delta)}{3\pi} \Delta^2. \tag{3.5}
\]

We define the physical slope parameter \(\rho_{B \rightarrow D^*}^2\) by

\[
|F_{B \rightarrow D^*}(w)| = |F_{B \rightarrow D^*}(1)||1 - \rho_{B \rightarrow D^*}^2(w - 1) + \cdots|. \tag{3.6}
\]

The relationship between \(\rho_{B \rightarrow D^*}^2\) and \(\rho^2(\mu)\) can be computed in a model-independent way \cite{16,19} and is

\[
\rho_{B \rightarrow D^*}^2 = \rho^2(\mu) + \frac{4\alpha_s}{9\pi} \ln \frac{m_c^2}{\mu^2} + \frac{\alpha_s}{\pi} \left( \delta_{b \rightarrow D^*}^{(\alpha_s)} - \frac{20}{27} \right) + \frac{\lambda}{2m_c} \delta_{B \rightarrow D^*}^{(1/\mu)}, \tag{3.7}
\]

where

\[
\delta_{B \rightarrow D^*}^{(\alpha_s)} = \frac{2(1 - z)(11 + 2z + 11z^2) + 24(2 - z + z^2) z \ln z}{27(1 - z)^3}, \tag{3.8}
\]
\[ \Delta = 1 \text{ GeV} \]

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which have large uncertainties, and $\overline{\Lambda} = 0.4$ GeV we find that $\rho^2(\Delta)$ increases by about 0.3. Adding this contribution to $\rho^2$, with $\Delta = 1$ GeV and $\delta_1 = 0.41$ GeV, the bound becomes $\lambda_1 < -0.30$ GeV$^2$, which provides an indication of the size of these effects.

It is always possible to absorb some perturbative corrections into the definition of $\lambda_1$ and $\overline{\Lambda}$. This may result in a perturbative series that is better behaved. For instance, we may redefine $\lambda_1$ by

$$\lambda_1 \to \lambda'_1(\Delta) = \lambda_1 - \frac{4\alpha_s(\Delta)}{3\pi} \Delta^2 - \frac{2\alpha_s^2(\Delta)\beta_0}{3\pi^2} \Delta^2 \left(\frac{13}{6} - \ln 2\right), \quad (4.1)$$

which will remove the perturbative corrections to the second moment sum rule at order $w - 1$. The redefinition also removes the corrections to the zero recoil sum rule in Eq. (19) of Ref. [15], and is consistent with the suggested definition in Ref. [8]. The bound in this case, with $\delta_1 = 0.41$ GeV, becomes

$$\lambda'_1(\Delta = 1 \text{ GeV}) < -0.34 \text{ GeV}^2.$$  

(4.2)

However, we observe from Eq. (3.3) that the redefinition of $\lambda'_1$ does not completely remove the perturbative correction at order $(w - 1)^2$.

Similarly, one can redefine $\overline{\Lambda}$ by absorbing the perturbative corrections. To do this, one must use the terms proportional to $\overline{\Lambda}$ and $\overline{\Lambda}^2$ in Eq. (3.3) to form a complete square. This results in the following redefinition of $\overline{\Lambda}$

$$\overline{\Lambda} \to \overline{\Lambda}'(\Delta) = \overline{\Lambda} + \frac{8\alpha_s(\Delta)}{9\pi} \Delta. \quad (4.3)$$

This redefinition does not, however, remove the perturbative corrections to the Voloshin sum rule of Ref. [16]. It is also different from the $\overline{\Lambda}$ defined by $m_B = m_b(\mu) + \overline{\Lambda}(\mu) + \cdots$ using the $\overline{\text{MS}}$ quark mass, which also appears in the literature. Thus, it is not possible to completely remove the perturbative corrections by redefining $\lambda_1$ and $\overline{\Lambda}$.

ACKNOWLEDGMENTS

We wish to thank M. Wise and Z. Ligeti for many useful conversations. We would also like to thank M. Gremm, A. Kapustin, and I. Stewart for helpful comments.
REFERENCES

[1] N. Isgur and M. B. Wise, Phys. Lett. **B232** (1989) 113; Phys. Lett. **B237** (1990) 527; S. Nussinov and W. Wetzel, Phys. Rev. **D36** (1987) 130; M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. **47** (1988) 511.
[2] M. E. Luke, Phys. Lett. **B252** (1990) 447.
[3] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. **B247** (1990) 399; M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. 41 (1985) 120.
[4] I. I. Bigi, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. **B293** (1992) 430 [(E) ibid. **B297** (1993) 477].
[5] I. I. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. **D52** (1995) 196.
[6] A. V. Manohar and M. B. Wise, Phys. Rev. **D49** (1994) 1310; T. Mannel, Nucl. Phys. **B413** (1994) 396.
[7] B. Blok, L. Koyrakh, M. Shifman, and A. I. Vainshtein, Phys. Rev. **D49** (1994) 3356 [(E) ibid. **D50** (1994) 3572].
[8] M. Neubert, Phys. Rept. **245** (1994) 259.
[9] P. Ball and V. M. Braun, Phys. Rev. **D49** (1994) 2472; M. Neubert, hep-ph/9608211.
[10] M. Gremm, A. Kapustin, Z. Ligeti, and M. B. Wise, Phys. Rev. Lett. **77** (1996) 20; M. Gremm and A. Kapustin, hep-ph/9603448; M. Gremm and I. Stewart, Phys. Rev. **D55** (1996) 1226.
[11] I. I. Bigi, A. G. Grozin, M. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. **B339** (1994) 160.
[12] J. D. Bjorken, Invited talk given at Les Rencontres de la Valle d’Aoste (La Thuile, Italy), SLAC-PUB5278 (1990).
[13] N. Isgur and M. B. Wise, Phys. Rev. **D43** (1991) 819.
[14] M. B. Voloshin, Phys. Rev. **D46** (1992) 3062.
[15] A. Kapustin, Z. Ligeti, M. B. Wise, B. Grinstein, Phys. Lett. **B375** (1996) 327.
[16] C. G. Boyd, Z. Ligeti, I. Z. Rothstein, and M. B. Wise, hep-ph/9610518.
[17] B. H. Smith and M. B. Voloshin, Phys. Lett. **B340** (1994) 176.
[18] L. A. Koyrakh, PhD. Thesis, hep-ph/9607443.
[19] I. Caprini and M. Neubert, Phys. Lett. **B380** (1996) 376.
[20] Particle Data Group, et al, Phys. Rev. **D54** (1996).
[21] M. Neubert, Phys. Rev. **D46** (1992) 3914; M. Neubert, Z. Ligeti, and Y. Nir, Phys. Lett. **B301** (1993) 101; Phys. Rev. **D47** (1993) 5060; Z. Ligeti, Y. Nir, and M. Neubert, Phys. Rev. **D49**(1994) 1302.