IS COSMIC RAY ELECTRON EXCESS FROM PULSARS SPIKY OR SMOOTH?: CONTINUOUS AND MULTIPLE ELECTRON/POSITRON INJECTIONS

Norita Kawanaka1, Kunihito Ioka1, and Mihoko M. Nojiri1,2

1 Theory Center, Institute of Particle and Nuclear Studies, KEK (High Energy Accelerator Research Organization), 1-1 Oho, Tsukuba 305-0801, Japan; norita.kawanaka@kek.jp
2 Institute for the Physics and Mathematics of the Universe, The University of Tokyo, Chiba 277-8568, Japan

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ABSTRACT

We investigate the observed spectrum of cosmic ray electrons and positrons from astrophysical sources, especially pulsars, and the physical processes for making the spectrum spiky or smooth via continuous and multiple electron/positron injections. We find that (1) the average electron spectrum predicted from nearby pulsars is consistent with PAMELA, Fermi, and H.E.S.S. data. However, the ATIC/PPB-BETS peak around 500 GeV is hard to produce by the sum of multiple pulsar contributions and requires a single (or a few) energetic pulsar(s). (2) A continuous injection produces a broad peak and a high-energy tail above the peak, which can constrain the source duration ($\lesssim 10^5$ years with the current data). (3) The H.E.S.S. data in the TeV range suggest that young sources with age less than $\sim 6 \times 10^4$ years are less energetic than $\sim 10^{48}$ erg. (4) We also expect a large dispersion in the TeV spectrum due to the small number of sources that may cause the high-energy cutoff inferred by H.E.S.S. and potentially provide a smoking gun for the astrophysical origin. These spectral diagnostics can be refined in the near future by the CALET experiments to discriminate different astrophysical and dark matter origins.

Key words: acceleration of particles – cosmic rays – pulsars: general

1. INTRODUCTION

Recently, the cosmic ray positron fraction (the ratio of positrons to electrons plus positrons) has been measured by PAMELA satellite (Adriani et al. 2008). The observed positron fraction rises in the energy range of 10 GeV $\lesssim \varepsilon_e \lesssim 100$ GeV, contrary to the prediction of secondary positrons, which are generated from cosmic rays propagating in the interstellar medium. The ATIC balloon experiment has also revealed that there is an excess above 300 GeV and a possible peak at $\varepsilon_e \sim 600$ GeV (Chang et al. 2008), which is also reported by PPB-BETS (Torii et al. 2008b). These observations strongly indicate nearby sources of $e^+e^-$ pairs within $d \sim 1$ kpc since high-energy electrons/positrons lose their energy during propagation. Possible candidates include a pulsar (Shen 1970; Chi et al. 1996; Zhang & Cheng 2001; Grimani 2007; Kobayashi et al. 2004; Buesching et al. 2008; Hooper et al. 2009a; Yuksel et al. 2009; Profumo 2008; Malyshhev et al. 2009; Grassi et al. 2009), a microquasar (Heinz & Sunyaev 2002), a gamma-ray burst (GRB; Ioka 2008), a supernova remnant (SNR; Shen & Berkley 1968; Cowsik & Lee 1979; Erlykin & Wolfendale 2002; Pohl & Esposito 1998; Kobayashi et al. 2004; Shaviv et al. 2009; Fujita et al. 2009; Hu et al. 2009; Blasi 2009; Blasi & Serpico 2009; Mertsch & Sarkar 2009; Biermann et al. 2009), and dark matter annihilations/decays (Asano et al. 2007; Arkani-Hamed et al. 2009; Bergstrom et al. 2008; Hamaguchi et al. 2009; Cirelli & Strumia 2008; Cholis et al. 2009a, 2009b, 2009c; Chen et al. 2009a, 2009b, 2009c; Chen & Takahashi 2009; Hisano et al. 2005, 2009a, 2009b, 2009c; Ishiwata et al. 2008, 2009a, 2009b; Zhang et al. 2008; March-Russell & West 2009; Hooper et al. 2009b; Pohl 2009). Instead we might be observing the propagation effects (Delahaye et al. 2008; Cowsik & Burch 2009; Stawarz et al. 2010) or the proton contamination (Fazely et al. 2009; Schubnell 2010).

In order to discriminate different models of sources, an important diagnostic should be the spectral shape, in particular whether the ATIC/PPB-BETS peak is spiky or smooth. In this regard, it is remarkable that an astrophysical source can make a peak with a sharp cutoff that is similar to the dark matter predictions, if the source is a transient object like a GRB (Ioka 2008). However, other astrophysical sources such as pulsars, SNRs, or microquasars are not transient and expected to have a finite spread in the cutoff, as suggested by Ioka (2008). More importantly, due to the collimated emission, there are many off-axis pulsars that have not been observed via electromagnetic radiation, and we expect integrated contributions from multiple sources to the spectral shape, considering the birthrate of pulsars in our Galaxy.

In addition, recently the Fermi Large Area Telescope has measured the electron spectrum up to $\sim 1$ TeV that is roughly proportional to $\varepsilon_e^{-3}$ without any spectral peak as reported by ATIC/PPB-BETS (Abdo et al. 2009). The H.E.S.S. Collaboration also provides the electron spectrum (Aharonian et al. 2008b, 2009), which is consistent with the Fermi result up to $\sim 1$ TeV and shows the steep drop of the flux above that energy. The Fermi data, however, should have a large systematic error in the high-energy range ($\gtrsim 300$ GeV) where a significant fraction of electrons are removed to avoid a large hadron contamination, and so the real flux is estimated not by the pure experimental data but by the Monte Carlo simulations (Moiseev et al. 2008). On the other hand, the ATIC data contain the larger statistical errors than the Fermi data. Therefore, we cannot judge which observations are more reliable so far.

In this paper, we investigate the effects of continuous and multiple pair injections on the observed electron/positron spectrum. We show that the flux above the peak energy does not drop off abruptly but remains finite if the pair injection continues for a finite time and suggest that we may measure the source duration from the peak width. We also show that an average spectrum from multiple sources is relatively flat as reported by Fermi, and the ATIC/PPB-BETS peak requires a single (or a few) extraordinary energetic source(s). We discuss the range...
of physical parameters of the sources (total electron/positron energy, the source duration, etc.) that are consistent with the current observational data.

2. INJECTION MODELS AND CALCULATIONS

2.1. Continuous $e^\pm$ Injection from a Single Source

We assume that a point-like source starts injecting $e^\pm$ pairs at the time $t = 0$ with total energy $E_{e^\pm} \sim E_e$ at a distance $d$ (~1 kpc) from the Earth. The observed electron/positron spectrum after the propagation is obtained by solving the diffusion equation

$$\frac{\partial}{\partial t} f = K(e_e) \nabla^2 f + \frac{1}{\partial e_e} [B(e_e) f] + Q(t, r, e_e),$$

(1)

where $f(t, r, e_e)$ is the distribution function of particles at time $t$ and position $r$ with energy $e_e$. Here, $K(e_e) = K_0(1+e_e/3 \text{ GeV})^3$ is the diffusion coefficient, $B(e_e)$ is the energy loss rate, and $Q$ is the injection rate of electrons/positrons. Hereafter we adopt $K_0 = 5.8 \times 10^{50} \text{ cm}^2 \text{s}^{-1}$, $\delta = 1/3$ that is consistent with the boron/carbon ratio according to the latest GALPROP code, and $B(e_e) = -b e_e^2$ with $b = 10^{-16} \text{ GeV}^{-1} \text{s}^{-1}$ which includes the energy loss due to synchrotron emission and inverse Compton scattering (Baltz & Edsjö 1999; Moskalenko & Strong 1998).

Here, we assume the continuous injection with a power-law spectrum: $Q(t, r, e_e) \propto Q_0(t)e_e^{-\delta}(r - r_0)$. We can obtain the observed spectrum for an arbitrary type of injection using the Green’s function of Equation (1), derived in Atoyan et al. (1995), with respect to $r$ and $t$:

$$G(t, r, e_e; t_0, r_0) = \frac{Q_0(t_0)e_e^{-\delta}B(e_e)}{\pi^{1/2}B(e_e)d_{\text{diff}}^3} \exp\left(-\frac{r^2}{d_{\text{diff}}^2}\right),$$

(2)

where $e_{e_0} = e_e/[1 - b(t - t_0)e_e]$ is the energy of electrons/positrons at the time $t_0$ which are cooled down to $e_e$ at the time $t$, $r = |r - r_0|$, and $G = 0$ when $e_{e_0}$ is larger than the maximum energy of the injection spectrum, $e_{e,\text{max}}$. As $B(e_{e,0})/B(e_e) = (e_{e,0}/e_e)^2 < (e_{e,\text{max}}/e_e)^2 < \infty$, there is no divergence in Equation (2). We can approximate the diffusion length as

$$d_{\text{diff}} \simeq 2 \sqrt{K(e_e)(t - t_0) \frac{1 - (1 - e/e_{\text{cut}})^{1-\delta}}{(1 - \delta/\epsilon_{\text{cut}}^2)}},$$

(3)

when $e_e \gg 3 \text{ GeV}$ and the diffusion coefficient is almost power law $K(e_e) \simeq K_0(e_e/3 \text{ GeV})^3$. Here $\epsilon_{\text{cut}} = [b(t - t_0)]^{-1}$.

Once we assume the injection rate $Q_0(t_0)$, we can obtain the observed electron/positron spectrum by integrating Equation (2) over $t_0$:

$$f(t, r, e_e) = \int_{t_0}^{t} G(t, r, e_e; \tau, r_0) d\tau,$$

(4)

Note that the initial time of the integration should be set as $t_i = \max[0, t - b^{-1}(e_{e,\text{max}}^2 - e_e)]$.

We consider two types of continuous injection. One is the pulsar-type decay:

$$Q_0(t) \propto \frac{1}{(1 + t/t_0)^{\alpha}},$$

(5)

This is the similar function of time as the spin-down luminosity of a pulsar with a surface magnetic field

$$B = 8.6 \times 10^{11} P_{10\text{ ms}}(t_0,4)^{-1/2}.$$

(6)

Figure 1. Electron plus positron flux predicted from a source that continuously injects pairs for a finite duration $t_0 = 10^5$ years with the exponential decay in Equation (7) (thin solid line), and its sum (thick solid line) with the background (dotted line), compared with the ATIC/PPB-BETS/H.E.S.S/Fermi data. We also show the pulsar-type injection in Equation (5) with $t_0 = 10^5$ years (long-dashed line) and $t_0 = 10^4$ years (double dashed line), in addition to the transient injection ($t_0 = 0$; short dot-dashed line). We assume that a source at $r = 1 \text{ kpc}$ from the Earth a time $t_{\text{age}} = 5.6 \times 10^5 \text{ years}$ ago produces $e^\pm$ pairs with total energy $E_{e^\pm} = E_e = 0.8 \times 10^{10} \text{ erg}$ and spectral index $\alpha = 1.7$ up to $e_{e,\text{max}} = 10 \text{ TeV}$.

where $P_{10\text{ ms}}$ is the pulsar period normalized by 10 ms and $t_{0,4} = t_0/10^4 \text{ years}$ (Shapiro & Teukolsky 1983). The other is the exponential decay:

$$Q_0(t) \propto \exp\left(-\frac{t_{\text{age}}}{t_0}\right),$$

(7)

which may be realized by a pulsar that initially confines $e^\pm$ in its nebula and releases them afterward, by a SNR that accelerates protons and continues to inject them to the surrounding dense gas cloud until it is destroyed, or by a microquasar ceasing its activity. In both types of injection, the characteristic timescale of the duration $t_0$ is defined to be the time when the rate becomes 4 times smaller than the initial one.

In Figure 1, we show the electron plus positron flux resulting from above two injection models in addition to the transient model ($t_0 = 0$) and the background (dotted line). The remarkable point is that an astrophysical source can make a spectral peak that is similar to the ATIC/PPB-BETS excess and also to the dark matter case (Ioka 2008).

The peak energy is determined by the age of the source $t_{\text{age}}$ as

$$e_{e,\text{peak}} = \left[\frac{b t_{\text{age}} + 1}{e_{e,\text{max}}^2}\right]^{-1},$$

(8)

because the electrons/positrons with initially higher energy cool down via synchrotron emission and inverse Compton scattering within time $t_{\text{age}}$. We can inversely estimate the source age as $t_{\text{age}} \sim 5 \times 10^7 \text{ years}$ from the peak energy for $e_{e,\text{max}} \gtrsim 1 \text{ TeV}$. Note that the peak flux is almost independent of the distance $r$ if it is smaller than the diffusion length ($\sim 1 \text{ kpc}$ in our case).

As is clear from Figure 1, the spectral cutoff becomes shallower for the continuous injection models than the transient

3 For the background shown in the following plots, we adopt the fitting functions in Baltz & Edsjö (1999) by reducing the primary $e^\pm$ flux, which is conventionally attributed to SNRs, by 30% because the fitting functions provide larger flux than the ATIC data even without other contributions.
one \( (t_0 = 0 \); short dot-dashed line). This is because the significant fraction of \( e^\pm \) pairs are produced recently (i.e., injected long after the birth of the source) and they have shorter time for the energy loss via synchrotron emission and inverse Compton scattering. Then their energy is still higher than the peak energy when they reach the Earth, and they produce a broader peak.

The solid thick line represents the total (the primary plus background electron and positron) flux assuming that the source starts emitting \( e^\pm \) pairs with total energy \( \sim 10^{50} \) erg, a power-law index \( \alpha \sim 1.7 \), and a maximum energy \( \sim 5 \) TeV at a distance \( \sim 1 \) kpc from the Earth a time \( t_{\text{age}} \sim 5 \times 10^7 \) years ago, and decays exponentially with the duration of \( t_0 \sim 10^5 \) years. This model looks better for the ATIC/PPB-BETS peak, though we cannot conclude that the duration is finite with the current data. The positron fraction predicted from this parameter set is also consistent with the PAMELA results, in almost the same way as Figure 1 of Ioka (2008).

In the case of pulsar-type injection, there is another interesting spectral feature resulting from a long duration. In Figure 1, the high-energy tail above the peak energy is more enhanced for the long-duration case \( (t_0 = 10^5 \) years, double dashed line) than the short-duration case \( (t_0 = 10^4 \) years, long-dashed line). This is because the longer the duration of injection is, the larger fraction of \( e^\pm \) pairs are freshly produced and they do not lose their energy during the propagation so much (see also Atoyan et al. 1995). Especially, the flux of the long-duration model may exceed the H.E.S.S. observations around \( \sim 4 \) TeV if we add the background (dotted line) while that of the short-duration model does not. As the error bars are still large, however, we should await future observations.

### 2.2. Multiple \( e^\pm \) Injections: Average Flux and Its Dispersion

Next, let us consider multiple sources. We expect several younger or older pulsars than that in Figure 1 of age \( t_{\text{age}} \sim 5 \times 10^5 \) years, considering the local birth rate of pulsars \( \sim 10^{-3} \) yr\(^{-1}\) kpc\(^{-2}\) (Narayan 1987; Lorimer et al. 1993).

Moreover, the total energy \( E_{e^+} + E_{e^-} \) in Figure 1 is as large as the rotation energy of a pulsar \( E_{\text{rot}} \) with a period of \( \sim 10 \) ms. This is comparable with the fastest initial spin estimated from the observations of radio pulsars (Kaspi & Helfand 2002), so the pair output efficiency \( f_c \equiv (E_{e^+} + E_{e^-})/E_{\text{rot}} \) may be too large \( \sim 100\% \) to account for the excess with a single pulsar.

We can calculate the average electron and positron spectrum by considering the nearby multiple pulsars with a certain birthrate in the following way. Once \( e_\circ \) is fixed, we can neglect the contribution from pulsars older than \( \sim 1/(b e_\circ) \) and farther than \( \sim d_{\text{diff}} \sim 2 \sqrt{K(\varepsilon_e)R/\ell} \) as is obvious from the functional form of Equation (2). Then the average flux can be calculated by

\[
\bar{f}(\varepsilon_\circ) = \int_0^{1/(b e_\circ)} dt \int_0^{d_{\text{diff}}} 2\pi r dr f(t, r, \varepsilon) R \\
\sim \frac{Q_0 R}{\pi^{1/2} K(\varepsilon_e) b e_\circ} \varepsilon_e^{\alpha} \\
= N(\varepsilon_\circ) \times f_{1, \text{ave}}(\varepsilon_\circ),
\]

where \( R \) is the local pulsar birthrate \( (\text{yr}^{-1} \text{kpc}^{-2}) \),

\[
N(\varepsilon_\circ) = \int_0^{1/(b e_\circ)} dt \int_0^{d_{\text{diff}}} dr 2\pi r R \sim \frac{2\pi K(\varepsilon_e) R}{(b e_\circ)^2}
\]

is the number of pulsars which contribute to the flux at the energy \( \varepsilon_\circ \) and \( f_{1, \text{ave}}(\varepsilon_\circ) = f_{\text{ave}}(\varepsilon_\circ)/N(\varepsilon_\circ) \) is the average electron flux per pulsar. Here, we adopt the value of \( R \) as the birth rate per unit surface area because pulsars are born from a disk whose thickness \( (\sim 200–300 \text{ pc}) \) is much smaller than the diffusion length of cosmic rays \( (\sim 2–3 \text{ kpc}) \).

Figures 2 and 3 show the average electron spectra and the positron fraction, respectively, obtained by assuming that each pulsar emits electrons with the total amount of energy of \( \sim 1 \times 10^{50} \) erg, the spectral index of \( \sim 1.9 \), and the birthrate of \( R \sim 1/(1.5 \times 10^8 \text{ yr}^{-1} \text{kpc}^{-2}) \) (thick solid lines).

In addition, we can calculate the dispersion of the number of pulsars \( \Delta N(\varepsilon_\circ) \) from the average \( N(\varepsilon_\circ) \) for each energy bin as

\[
\Delta N(\varepsilon_\circ) \sim \sqrt{N(\varepsilon_\circ)},
\]

which is based on the Poisson distribution of nearby pulsars. Then, we can estimate the flux dispersion as

\[
\Delta f_{\text{ave}}(\varepsilon_\circ) \sim f_{1, \text{ave}}(\varepsilon_\circ) \sqrt{N(\varepsilon_\circ)} = f_{\text{ave}}(\varepsilon_\circ)/\sqrt{N(\varepsilon_\circ)}.
\]
From Figures 2 and 3, we can see that the average spectra are basically consistent with Fermi, H.E.S.S., and PAMELA data. In the high-energy range \((\varepsilon_e \gtrsim \text{TeV})\), the dispersion from the average flux becomes significant. This can be interpreted as follows. The pulsars which contribute to the electron and positron flux in such a high-energy band should be young,

\[
t_{\text{age}} \lesssim \frac{1}{b \varepsilon_e} \sim 3.1 \times 10^5 \text{ years} \left(\frac{\varepsilon_e}{\text{TeV}}\right)^{-1},
\]

and close to the Earth,

\[
r \lesssim 2\sqrt{\frac{K(\varepsilon_e))_{\text{age}}}{3}} \sim 1.3 \text{ kpc} \left(\frac{\varepsilon_e}{\text{TeV}}\right)^{-1/3},
\]

where we adopt \(\delta = 1/3\). The number of such pulsars should be as small as

\[
N(\varepsilon_e) \sim 6 \left(\frac{\varepsilon_e}{\text{TeV}}\right)^{-5/3} \left(\frac{R}{1/(1.5 \times 10^5 \text{ yr}^{-1} \text{ kpc}^{-2})}\right)\left(\frac{\varepsilon_e}{500 \text{ GeV}}\right)^{1/6}.
\]

Therefore, in the TeV range few pulsars can contribute to the electron/positron flux. This small number of pulsars may naturally account for the spectral cutoff around \(\sim\text{TeV}\) energy, which has been inferred by the H.E.S.S. observations. Strictly speaking, this estimation of the pulsar number dispersion is not correct in the energy range of \(\varepsilon_e \gtrsim 3\ \text{TeV}\), where \(N(\varepsilon_e) \lesssim 1\) and the statistical arguments become meaningless. However, this interpretation of the spectral drop around this energy is still qualitatively correct.

Moreover, Figure 2 shows that the ATIC/PPB-BETS peak flux (\(\varepsilon_e \sim 545 \text{ GeV}\)) is much larger than the average flux added with the dispersion flux \(\Delta f_{\text{ave}}\) at the same energy bin. In fact, the separation between the average flux and the ATIC data of the peak flux at that energy is \(\sim 10 \Delta f_{\text{ave}}\). Then, if all pulsars emit electrons with the total energy of \(\sim 10^{48} \text{ erg}\), the number of pulsars which contribute to the energy bin of the ATIC/PPB-BETS should be unrealistically large at the \(10\sigma\) level. This means that if the ATIC/PPB-BETS peak is real, it does not seem to be produced by the collective contribution from multiple pulsars with the moderate amount of electron energy \(\sim 10^{48} \text{ erg}\) but by a single (or a few) energetic pulsar(s) \(\sim 10^{50} - 10^{52} \text{ erg}\).

The discussion above is about the fluctuation of the number of pulsars with a certain birthrate, and it is based on the Poisson statistics. Strictly speaking, in order to discuss the cosmic ray electron/positron fluctuations due to the random injections, one should evaluate not the dispersion of the source number but the dispersion of the electron/positron flux at each energy bin as

\[
\Delta f^2_{\text{ave}} \sim \int_0^{1/(b \varepsilon_e)} dt \int_0^{\int_{\text{diff}}} 2\pi rf^2\frac{f^2}{R} - N(\varepsilon_e) f^2_{1,\text{ave}}.
\]

The first integral in Equation (16) contains, however, a serious divergence because of the large (but improbable) contribution from very young and nearby sources (Lee 1979; Berezhiani et al. 1990; Lagutin & Nikulin 1995; Ptuskin et al. 2006). In order to obtain the realistic estimate of the flux dispersion, we introduce a lower cutoff parameter \(\tau_c\) to the time integral. Then we have

\[
\Delta f^2_{\text{ave}} \sim \frac{Q^2 R}{16\pi^2 K(\varepsilon_e)^2} \left(\frac{\varepsilon_e}{500 \text{ GeV}}\right)^{-2a}.
\]

Following Ptuskin et al. (2006), we adopt the cutoff parameter as

\[
\tau_c = \left[4\pi R K(\varepsilon_e)\right]^{-1/2} \sim 10^5 \text{ years} \left(\frac{\varepsilon_e}{500 \text{ GeV}}\right)^{-1/6} \times \left(\frac{R}{1/(1.5 \times 10^5 \text{ yr}^{-1} \text{ kpc}^{-2})}\right)^{-1/2},
\]

which takes into account the absence of very young and nearby sources. This choice of \(\tau\) is reasonable as long as this time is much shorter than \((b \varepsilon_e)^{-1}\), which means \(\varepsilon_e \ll 4.4 \text{ TeV}\). Then the ratio of the flux dispersion to the average flux (Equation (9)) can be expressed as

\[
\frac{\Delta f_{\text{ave}}}{f_{\text{ave}}} \sim 0.21 \left(\frac{\varepsilon_e}{500 \text{ GeV}}\right)^{5/12} \times \left(\frac{R}{1/(1.5 \times 10^5 \text{ yr}^{-1} \text{ kpc}^{-2})}\right)^{-1/4}.
\]

We can see that these two “dispersions” (Equations (12) and (19)) give the similar results around the energy of the ATIC/PPB-BETS peak. Therefore, from either of the above discussions, we can say that the ATIC/PPB-BETS data of the spectral peak are so largely separated from the average flux that they do not seem to be produced by the multiple contribution from nearby pulsars with moderate energy.

We should note that the H.E.S.S. data put constraints on the total \(e^+e^-\) pair energy from young sources. We plot in Figure 2 the electron spectrum from the source with the age of \(\sim 6 \times 10^4 \text{ years}\) so as not to exceed the observational upper limit inferred by the H.E.S.S. data in the TeV range (long-dashed line). We find that the total energy of such young sources should be, if exist, \(\lesssim 2 \times 10^{50} \text{ erg}\) which is 2 orders of magnitude smaller than the energy of the source making the ATIC/PPB-BETS peak.

3. DISCUSSION AND CONCLUSION

We investigate the astrophysical origin for the PAMELA and ATIC/PPB-BETS excesses and in particular the effects of the finite duration and the multiple sources on the electron and positron spectra, as expected for pulsars, SNRs, and microquasars. We find the following.

1. A nontransient source can make a spectral peak that is similar to the ATIC/PPB-BETS excess (see Figure 1) around the peak energy in Equation (8). The peak is generally broad with a width

\[
\frac{\Delta \varepsilon_{\text{peak}}}{\varepsilon_{\text{peak}}} \approx \frac{\tau_0}{\tau_{\text{age}}} \sim 10\% \tau_{0,04} \left(\frac{\tau_{\text{age}}}{10^5 \text{ years}}\right)^{-1},
\]

which could provide a method to measure the source duration \(\tau_0\) by the Fermi satellite (an energy resolution of 5%–20% in 20 GeV–1 TeV range; Moiseev et al. 2008) or the future CALET experiments (a few percent above 100 GeV; Torii et al. 2008a). Although Atiyon et al. (1995) have already pointed out the effects of finite duration of the source on the electron spectrum, they only mention the enhancement of the high-energy tail above the spectral peak (see below) and never discuss the peak width. Note that the peak width is also produced by the spatial fluctuation of Galactic magnetic field and the photon density because the

4 Since the multiple contributions tend to make the spectrum softer, it is possible to fit the ATIC/PPB-BETS spectrum with multiple pulsars by using the harder spectral index \(a\) accordingly. However, in order to fit the PAMELA spectrum in the lower energy range at the same time, such a hard spectrum is not favored.
energy loss rate of $e^\pm$ fluctuates during the propagation, as estimated in Ioka (2008; see also Malyshev et al. 2009). We also note that the peak becomes smoother if the injection rises gradually in its initial stage.

2. The spectrum from a long-duration source has a high-energy tail above the peak energy (see Figure 1). Especially, the flux of this tail plus the background may exceed the H.E.S.S. data points when assuming a pulsar-type decay with a duration $t_0 \gtrsim 10^8$ years. This implies that the source making the ATIC/PPB-BETS peak is not likely a single pulsar with magnetic fields weaker than a few times $10^{11}$ G. The existence of this tail has been already pointed out before (Atoyan et al. 1995). However, we first present the quantitative argument for the observational limit of the duration of the electron/ positron source in the context of the high-energy tail thanks to the observational developments in the TeV range. One should note that we cannot rule out the long-duration pulsar model if the maximum energy of injected $e^\pm$ pairs is smaller than $\gtrsim$ TeV, or the injection is not the pulsar type in Equation (5) but the exponential type in Equation (7), for example. The latter is possible if high-energy pairs generated in the pulsar magnetosphere are not injected into the space instantaneously but initially confined in a pulsar wind nebula (Chi et al. 1996) and they diffuse out after the nebula gets broken.

3. The H.E.S.S. data suggest that young sources with age less than $6\times10^4$ years should be, if exist, 2 orders of magnitude less energetic than the source making the ATIC/ PPB-BETS peak. Note that the lifetime of the pulsar nebula is around $\sim 10^7$ years and younger pulsars may not be able to contribute by the cosmic ray confinement in the nebula.

4. The average electron spectrum and positron fraction is well consistent with the H.E.S.S. /Fermi and PAMELA data, respectively, taking into account the dispersion predicted from the total electron energy per pulsar of $\sim 10^{48}$ erg with the local birthrate of $\sim 1/10^5$ years kpc$^{-2}$. Especially, when $\varepsilon_e \gtrsim$ TeV, we expect a large dispersion of the electron flux because of the small number of sources which are young and close to the Earth and can significantly contribute to that energy range. This fact can naturally account for the spectral drop around $\gtrsim$ TeV indicated by the H.E.S.S. observations. Note that the value of the total electron energy per pulsar adopted here is within reasonable range. In fact, the pulsar whose initial spin period is around $\sim 10$ ms can emit electrons and positrons with energy $\sim 10^{48}$ erg if we assume the efficiency of $f_e \sim 1\%$, which seems to be reasonable (Hooper et al. 2009a).

Moreover, we show that the ATIC/PPB-BETS data point showing the peak at $\varepsilon_e \sim 545$ GeV is largely separated from the average flux, when considering the theoretical dispersion from the average. This fact suggests that the peak is hard to produce by multiple contributions and requires a single (or a few) extraordinary pulsar(s) whose total electron/positron energy is about a 100 (several tens) times larger than that of ordinary pulsars. Here, we estimate the dispersion of the electron/positron flux based on the analytical expressions (Equations (9), (12), or (17)) using the averaged local birthrate of pulsars. This method enables us to take into account the off-axis pulsars whose existence is suggested by the observed pulse shape of pulsars, and it is different from the method used in Malyshev et al. (2009) who calculate some realizations of the spectra predicted from the known pulsars in the ATNF catalog.

Note that the different choice of the diffusion coefficient $K(\varepsilon_e)$ would change the results quantitatively. The smaller $K$ makes the diffusion length $r_{\text{diff}}$ smaller, and the particle density inside that radius gets higher, being proportional to $r_{\text{diff}}^{-3}$ (see Equations (2) and (3)). For different $K$ instead of $K$, we can apply our results by re-scaling the distance of each pulsar and the total $e^\pm$ injection energy as $d \rightarrow d\sqrt{K/K_0}$ and $E_{\text{tot}} \rightarrow E_{\text{tot}}(K/K_0)^{3/2}$, respectively.

In our calculations, we evaluate the dispersion of the electron flux due to the random birth of nearby pulsars in time and space having uniform total energy and injection index. In the case that these pulsars have a distribution of energy with a dispersion of $\delta \varepsilon$, the total dispersion of the energy is averaged as $\sim \delta \varepsilon \sqrt{N(\varepsilon)}$, and when the electron energy is smaller than $\sim$ TeV (i.e., $N(\varepsilon)$ is much larger than unity) the total dispersion is suppressed compared to the total flux $N(\varepsilon) f_{\text{ave}}$. The spectral index of the injected electrons should also be varied. However, the dispersion of the flux is almost determined by the amount of the electron energy emitted from pulsars, and the fluctuation of the index would not contribute to the flux dispersion so much.

The spatial variation of the energy loss rate and the diffusion coefficient can also affect to the observed electron flux or positron fraction. The energy loss rate $b$ can fluctuate along the propagation path of electrons because of the inhomogeneities of the radiation and magnetic field, and then the cutoff shape of the resulting electron spectra would be broadened according to the amplitude of the fluctuation. Such a feature may be resolved by the future CALET experiment (see Ioka 2008). On the other hand, the effects of the spatial variation of the diffusion coefficient are considered in Cowlsik & Burch (2009) in the context of “nested leaky box model.” In this model, the positron fraction can be explained as a result of the different diffusion coefficient between the source-surrounding region and the general interstellar space.

We can expect gamma-ray emission from high-energy $e^\pm$ pairs. Especially, the number of such energetic objects can be simply estimated as $(10 \text{kpc}/1 \text{kpc})^2 = 100$. This is comparable with that of TeV unidentified sources, which have no clear counterpart at other wavelengths (Aharonian et al. 2005, 2008a; Mukherjee & Halpern 2005; Ioka & Mészáros 2009), implying some connections between them.

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