Quantum limit in resonant vacuum tunneling transducers

Roberto Onofrio\textsuperscript{1} and Carlo Presilla\textsuperscript{2}

\textsuperscript{1}Dipartimento di Fisica “G. Galilei”, Universit\`a di Padova, Via Marzolo 8, 35131 Padua, Italy
\textsuperscript{2}Dipartimento di Fisica, Universit\`a di Roma “La Sapienza”, Piazzale A. Moro 2, 00185 Rome, Italy

We propose an electromechanical transducer based on a resonant-tunneling configuration that, with respect to the standard tunnelling transducers, allows larger tunnelling currents while using the same bias voltage. The increased current leads to a decrease of the shot noise and an increase of the momentum noise which determine the quantum limit in the system under monitoring. Experiments with micromachined test masses at 4.2 K could show dominance of the momentum noise over the Brownian noise, allowing observation of quantum-mechanical noise at the mesoscopic scale.

PACS numbers:

Recently a novel electromechanical transducer based upon vacuum tunnelling of electrons has been proposed to detect displacements of a macroscopic mass \cite{1}. A variation of the distance between the test mass and a tip changes the tunnelling current and whenever small fractions of the current are appreciable, corresponding displacements of the test mass, which are small fractions of the De Broglie wavelength of the tunnelling electrons, are also detectable. The relevance of this new class of transducers has been emphasized especially concerning detection of gravitational waves using bar antennae \cite{2}, design of quantum standard of current in metrology \cite{3} and study of quantum-mechanical noise at the mesoscopic scale \cite{4}.

Vacuum tunnelling transducers are intrinsically quantum limited \cite{5}. The small output capacitance allows to neglect the back-action noise due to the amplifier following the transducer in the detection chain with respect to the quantum uncertainties coming from the tunnelling process in itself. In this last process two uncorrelated sources of noise have been identified. Firstly, the shot noise due to the discrete nature of the electric charge is responsible for a position uncertainty of the test mass. Secondly, the fluctuations in the momentum imparted by the electrons to the test mass give rise to a momentum uncertainty of the test mass. The product of these two quantities is of the order of $\hbar/2$ reaching exactly this value in the case of a transducer schematized by a square-well barrier \cite{6}.

Brownian noise arising from the coupling of the test mass to the environment usually dominates over the quantum noise and destroys the quantum properties of the test mass. Suppression of the Brownian noise contribution is crucial for improving the sensitivity of position transducers until the standard quantum limit is reached and eventually surpassed as required in high-precision experiments in general relativity \cite{7}. Moreover, repeated monitoring at a quantum level of sensitivity of a single degree of freedom of a macroscopic mass is relevant to understand quantum measurement theory \cite{8}. It is therefore important to study mechanisms for which the quantum noise can be made dominant with respect to the Brownian noise. In this letter we propose the use of resonant vacuum tunnelling transducers to achieve such a goal. We will apply the uncertainty principle to a double barrier in which resonant tunnelling occurs and we will compare the noise figures to the corresponding non-resonant case.

Let is consider a tunnelling transducer driven by an incident current $I$, \textit{i.e.} $I$ is the current which should flow in the device if the tip and the test mass were in contact. Due to this current, during a sampling time $\Delta t$ the number of electrons which attempt to tunnel across the vacuum gap, the number of incident electrons hereafter, is given by

$$N = \frac{I}{e} \Delta t. \quad (1)$$

In a first stage we suppose that all the incident electrons have the same energy $E$, after we will discuss the case of a biased device with electrons having Fermi distribution. Let $T(E,l)$ be the transmission coefficient at energy $E$ for a distance $l$ between the tip and the test mass. A fraction $T$ of the $N$ incident electrons gives rise to a measured tunnelling current $I_T = TI$. Due to the discrete nature of the charge carriers a shot noise in the measured tunnelling current $I_T$ inversely proportional to $\sqrt{N}$ arises. The test mass position is inferred by means of the tunnelling current through the dependence of the transmission coefficient on the distance $l$ and a shot noise position uncertainty $\Delta \lambda$ for the test mass also arises \cite{9}:

$$\Delta \lambda^2 = \frac{\Delta l^2}{N} = \frac{1}{N} T(1 - T) \left| \frac{\partial T}{\partial l} \right|^2. \quad (2)$$

This uncertainty has been expressed in terms of the uncertainty $\Delta l$ due to a single electron incident at energy $E$. At the same time the $N$ incident electrons impart a momentum uncertainty $\Delta \pi$ to the test mass \cite{10}:

$$\Delta \pi^2 = N \Delta p^2, \quad (3)$$

where $\Delta p$ is the test mass momentum uncertainty due to a single electron incident at energy $E$. On the basis of ref.
For instance in a square-well tunnelling transducer we schematize the test mass by a harmonic oscillator in the thermal bath. The total energy increase in the sampling time $\Delta t$ will be

$$\Delta \epsilon = \frac{\Delta \pi^2}{2M} + \frac{1}{2} M \omega^2 \Delta \lambda^2. \quad (4)$$

This can be considered as the exchange of energy between the test mass (measured object) and the electrons (meter) due to the quantum measurement process in the time $\Delta t$.

Superimposed to the two quantum noise sources there is the Brownian motion of the test mass coupled to the environment and of the thermal noise is dominant, when $\Delta \pi^2 / (2M \omega^2)$ will be

$$\Delta \xi^2 = \frac{\Delta \pi^2}{2N} + N \left( \frac{\Delta \rho^2}{2M^2 \omega^2} + \frac{k_B \theta e}{Q M \omega Q I} \right). \quad (6)$$

In this formula the dependence upon the number of incident electrons has been emphasized to show that both for small values of $N$, when the shot noise is dominant, and for large values of $N$, when the sum of the momentum uncertainty and of the thermal noise is dominant, large values of the effective displacement are obtained. A minimum value for the effective displacement will be achieved in an intermediate situation:

$$\Delta \xi^2_{opt} = \frac{\Delta \pi^2}{2M} \sqrt{1 + \frac{2 k_B M \omega e}{Q \Delta \rho^2 I T}}. \quad (7)$$

This optimal sensitivity corresponds to the quantum limit when the thermal contribution is negligible, $i.e.$ when

$$\frac{2 k_B \theta e M \omega e}{Q \Delta \rho^2} I \ll 1. \quad (8)$$

For instance in a square-well tunnelling transducer we have exactly $\Delta / \Delta \rho = \hbar / 2$ \cite{4,6} and when the inequality (8) holds we get the standard quantum limit $\Delta \xi^2_{opt} = \hbar / 2M \omega$, for an optimal number of incident electrons $N_{opt} = \hbar M \omega / 2 \Delta \rho^2$.

The requirement of thermal noise negligible with respect to quantum momentum noise is difficult to satisfy for a single-barrier transducer \cite{4}. Due to the small value of the single-electron momentum uncertainty $\Delta \rho$ one has to use a large incident current $I$, a high mechanical quality factor $Q$, a small test mass $M$ and a low temperature $\theta$ in order to satisfy the inequality (8). A different situation arises in a resonant tunnelling transducer. In fig. 1 we show a possible scheme for a transducer based upon a double-barrier potential. A double junction or an impurity zone is grown on the surface of the test mass producing an effective potential barrier against the current flow, when the tip is put near the test mass a resonant double barrier is achieved. At a resonance energy the transmission coefficient $T = 4T_1 T_2 / (T_1 + T_2)^2$ of the double barrier may be expressed in terms of the transmission coefficients $T_1$ and $T_2$ of each single barrier \cite{10}. A variation of the distance $l$ between the tip and the test mass changes the transmission coefficient $T_1$. For $T_2 \gg T_1$, as in the situations we will analyze in the following, the simple relationship holds

$$\frac{\partial T}{\partial l} \approx \frac{T}{T_1} \frac{\partial T_1}{\partial l}. \quad (9)$$

With respect to the single-barrier case we have at resonance a single-electron position uncertainty smaller by a factor $T/T_1$ and, because of $\Delta l / \Delta \rho \approx \hbar / 2$, a larger single-electron momentum uncertainty.

A quantitative comparison of a resonant tunnelling transducer to a non-resonant one is shown in fig. 2. In the former case around the resonance energy the position and momentum uncertainties transferred by a single incident electron to the test mass have a value respectively smaller and larger than in the corresponding non-resonant case. At the same time their product remains close to $\hbar / 2$. It should be observed that for both resonant and non-resonant cases the shape of the momentum uncertainty $\Delta \rho$ closely follows the shape of the transmission coefficient $T$. Indeed we can define a momentum uncertainty per single tunneling electron as $\Delta \rho^2_T = \Delta \rho^2 / T \approx 2m V_0$ which only depends upon the vacuum barrier $V_0$ and the effective electron mass $m$. This consideration allows to rewrite the inequality (8) in terms of the tunnelling current $I_T$ and of $\Delta \rho^2_T$ instead of the corresponding quantities for the incident electron

$$\frac{2 k_B \theta e M \omega e}{Q \Delta \rho^2_T I_T} \ll 1. \quad (10)$$

It is evident that this equality is better satisfied for a resonant configuration because in this case a larger tunnelling current $I_T$ for a given incident current $I$ may be obtained.

The influence of a Brownian noise source is shown in fig. 3. A non-negligible Brownian noise with a ratio
\( \theta/Q = 10^{-5} \) K causes a worsening of the optimal sensitivity \( \Delta \xi_{opt}^2 \) by a factor \( 10^2 \) with respect to the case \( \theta/Q = 0 \) in a non-resonant transducer. On the other hand, in the same conditions a resonant transducer working at the resonance energy retains an optimal sensitivity close to the quantum limit.

In a more realistic approach one has to consider not only the effect of the Brownian noise but also the partial loss of quantum-mechanical coherence due to elastic scattering. If the escape time of the electrons from the well \( \tau_{esc} \) is longer than the inelastic-scattering time \( \tau_i \) the coherent tunnelling current decreases and it is only partially compensated by a sequential tunnelling current [11]. In the situation described in fig. 3 we have \( \tau_{esc} \approx (2(c-b)/v)2/(T_1 + T_2) = 10^{-11} \) s, where \( v \) is the velocity of the electrons at the resonance. If inelastic phonon scattering is assumed to dominate, we estimate \( \tau_i \approx 10^{-13} \) s. The loss of coherence can be included by introducing a phenomenological factor \( \gamma \) which is related to the damping of the wave function due to inelastic scattering and expressed in terms of \( \tau_i \) through the relationship \( 2\tau_i = [2(c-b)/v]/(1-\gamma) \) [12]. By using the estimated value of \( \tau_i \) in our configuration, we get \( \gamma = 0.95 \) and the effect of the decoherence is shown in fig. 3 as a less pronounced peak which still allows for an order of magnitude improvement when compared to the single-barrier configuration. Such improvements, both in the fully coherent configuration (i.e. \( \gamma = 1 \)) and in the presence of inelastic scattering, shown in fig. 3 are limited to an energy range of the order of the resonance energy width.

Production of ballistic electrons with energy spread smaller than the resonance energy width is within the current semiconductor technology capabilities [13]. A similar gain may also be obtained using a biased device in which a Fermi distribution of electrons gives rise to the tunneling current. By integrating over the transverse states the energy distribution of the incident electrons, representing the differential form of eq. (1), is expressed by [14]

\[
\frac{dN}{dE} = \frac{mSk_B\theta}{2\pi^2\hbar^3} \ln \left[ \frac{1 + \exp((E_F - E)/k_B\theta)}{1 + \exp((E_F - E - e\Phi)/k_B\theta)} \right] \Delta t,
\]

where \( S \) is the transverse surface, \( \Phi \) is the bias voltage, and \( E_F \) the Fermi energy. This allows to evaluate the integrated shot noise position uncertainty

\[
\Delta \lambda^2 = \int_0^\infty \frac{dN}{dE}(1-T)dE \left[ \int_0^\infty \frac{dN}{dE} \frac{\partial T}{\partial T} dE \right]^{-2}
\]

and the integrated momentum uncertainty

\[
\Delta \pi^2 = \int_0^\infty \frac{dN}{dE} \Delta p^2 dE.
\]

By repeating the same arguments of the monoenergetic case an optimal sensitivity may be obtained at a proper
depends upon the material used for the test mass and in the case of two different values of the ratio $\theta/Q$.

Tunnelling (solid) and in the presence of sequential tunnelling $\gamma$ (dashed, incident electrons for a resonant double barrier with coherent tunnelling) is limited, apart from $1/f$ noise which strongly depends upon the material used for the test mass and the tip, by the shot noise. Due to the increased tunnelling current in a resonant configuration the shot noise is decreased. For the same reason the momentum noise contribution is enhanced and studies of macroscopic quantum noise due to the interaction between the electrons and the test mass are more easily performed. From fig. 3 and 4 it turns out that experiments for detecting gravitational waves. In this class of transducers the sensitivity of the tunnelling transducers proposed to detect gravitational waves. In this class of transducers the sensitivity of the tunnelling transducers proposed to
detect gravitational waves. In this class of transducers the sensitivity is limited, apart from $1/f$ noise which strongly depends upon the material used for the test mass and

The product $\Delta \lambda \Delta \pi$ remains close to $\hbar/2$ and the quantum limit is reached when the second term inside the square root is negligible. Due to the constancy of the quantity $\Delta p_T^2 = \Delta p^2/T \simeq 2mV_0$ this condition is expressed again by the inequality (10) in terms of the tunnelling current $I_T$,

$\Delta I^2_{opt} = \frac{\Delta \lambda \Delta \pi}{M \omega} \sqrt{1 + \frac{2k_B \theta M \omega}{Q \Delta \pi^2/\Delta I}}$  \hspace{1cm} (14)

The product $\Delta \lambda \Delta \pi$ remains close to $\hbar/2$ and the quantum limit is reached when the second term inside the square root is negligible. Due to the constancy of the quantity $\Delta p_T^2 = \Delta p^2/T \simeq 2mV_0$ this condition is expressed again by the inequality (10) in terms of the tunnelling current $I_T$,

$I_T = \frac{emSk_B \theta}{2\pi^2\hbar^3} \int_0^\infty \ln \left[ \frac{1 + \exp[(E_F - E)/k_B \theta]}{1 + \exp[(E_F - E - e\Phi)/k_B \theta]} \right] T dE.$  \hspace{1cm} (15)

In fig. 4 we show the dependence of the optimal sensitivity as a function of the bias voltage for the same thermal contributions of fig. 3 with and without the effect of inelastic scattering. Despite the integration over all the available electrons the improvement in the use of the resonant configuration at the proper bias voltage remains one order of magnitude higher with respect to the single-barrier situation. Moreover, the optimal sensitivity has a slight dependence upon the inelastic-scattering processes.

Resonant tunnelling may be relevant for improving the sensitivity of the tunnelling transducers proposed to detect gravitational waves. In this class of transducers the sensitivity is limited, apart from $1/f$ noise which strongly depends upon the material used for the test mass and

This work has been supported by INFN, Italy.

[1] M. Niksch and G. Binning, J. Vac. Sci. Technol. A, 6 (1988) 470.
[2] Bordoni F., Fuligni F. and Bocko M.F., in Proceedings of the Fifth Marcell Grossmann Meeting, edited by D.G. Blair and M.J. Buckingham (World Scientific, Singapore) 1989; Bordoni F., Karim M., Bocko M.F. and Mengxi T., Phys. Rev. D, 42 (1990) 2952.
[3] Guinea F. and Garcia N., Phys. Rev. Lett., 65 (1990) 281.
[4] Presilla C., Onofrio R. and Bocko M.F., Phys. Rev. B, 45 (1992) 3735.
[5] Bocko M.F., Stephenson K.A. and Koch R.H., Phys. Rev. Lett., 61 (1988) 726; Stephenson K.A., Bocko M.F. and Koch R.H., Phys. Rev. A, 40 (1989) 6615.
[6] Yurke B. and Kochanski G.P., Phys. Rev. B, 41 (1990) 8184.
[7] Caves C.M., Thorne K.S., Drever R.P., Sandberg V.D. and Zimmermann M., Rev. Mod. Phys., 52 (1980) 341; Bocko M.F. and Johnson W.W., Phys. Rev. Lett., 47 (1981) 1184; 48 (1982) 1371.
[8] Bocko M.F. and Johnson W.W., in New Techniques and Ideas in Quantum Measurement Theory, edited by D.M. Greenberger, Vol. 480 (New York Academy of Sciences, New York, N.Y.) 1986; Braginsky V.B. and Khalili F.Ya., Quantum Measurement, edited by K.S. Thorne (Cambridge University Press, Cambridge) 1992.
[9] Gibbons G.W. and Hawking S.W., Phys. Rev. D, 4 (1971) 2191.
[10] Jonson M. and Grincwajg A., Appl. Phys. Lett., 51 (1987) 1729.
[11] Stone A.D. and Lee P.A., Phys. Rev. Lett. 54 (1985) 1196; Büttiker M., IBM J. Res. Develop., 32 (1988) 62.
[12] Booker S.M., Shead F.W. and Toombs G.A., Semicond. Sci. Technol. B, 7 (1992) 439.
[13] Lury S., in Heterojunction Band Discontinuities: Physics and Device Applications, edited by F. Capasso and G. Margaritondo (North-Holland, Amsterdam) 1987.
[14] Tsu R. and Esaki L., Appl. Phys. Lett. 22 (1973) 562.
[15] Li Y.P. et al., Phys. Rev. B, 41 (1990) 8388; Appl. Phys. Lett., 57 (1991) 774; Van der Roer T.G. et al., Electron. Lett., 27 (1991) 2158.
[16] Yurke B. and Stoler D., Phys. Rev. Lett., 57 (1986) 13.
[17] Schiller S., Yu I.I., Feyer M.M. and Byer R.L., Opt. Lett., 17 (1992) 378.
[18] Recent measurements indicate that the shot noise in a double barrier is further reduced below the theoretically expected value, see [15].