Helicity Amplitude Ratios in Exclusive Electroproduction of the $\rho^0$ Meson at HERMES

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Abstract.
Electroproduction of the $\rho^0$ vector meson in the process $\gamma^* + N \rightarrow V + N'$ is studied with a 27.6 GeV longitudinally polarized electron/positron beam in the HERMES experiment. Kinematical dependences of real and imaginary parts of the ratio of the natural-parity-exchange helicity amplitudes $T_{11}/T_{00}$ ($\gamma^*_T \rightarrow \rho_T$) and $T_{01}/T_{00}$ ($\gamma^*_L \rightarrow \rho_L$) are extracted from the data. The same dependences are extracted for the unnatural-parity-exchange ratio $|U_{11}/T_{00}|$.

1. Introduction
The study of exclusive electroproduction of a vector meson allows the investigation of both production mechanism and structure of the nucleon. Helicity amplitudes which describe the transformation of a virtual photon into a vector meson fully contain the strong-interaction dynamics of the electroproduction process so the knowledge of them is essential for understanding of the process [1]. The process of electroproduction of the vector meson is factorized into two consecutive processes. The incident lepton radiates a virtual photon which dissociates into a $q\bar{q}$ pair. This pair lasts long enough to interact strongly with the nucleon and to form the observed vector meson. The interaction of the $q\bar{q}$ pair can proceed via two distinct mechanisms. The first one is the two gluon exchange, the second one is the exchange of a $q\bar{q}$ pair. In Regge phenomenology the corresponding process is the exchange of natural parity particles ($J^P = 0^+, 1^-, ...$) like pomeron, $\rho$, $\omega$, $a_2$ for Natural Parity Exchange (NPE) and unnatural parity particle ($J^P = 0^-, 1^+, ...$) like $\pi$, $a_1$, $b_1$,...reggeons for Unnatural Parity Exchange (UPE). The amplitude ratios can be used to distinguish between the contributions of the NPE and UPE processes. For this aim, amplitude ratios are more convenient than Spin Density Matrix Elements (SDMEs) as any SDME depends on all amplitude ratios.

Also violation of $s$-channel helicity conservation ($\lambda_V \neq \lambda_\gamma$) can be studied more reliably using amplitude ratios rather than SDMEs. The spin flip amplitudes $T_{01}$, $T_{10}$ provide information on the valence quark motion in a vector meson as they have to be zero in the absence of quark motion. The double spin flip amplitudes $T_{1-1}$ contains information on the gluon distribution in the nucleon [2]. A difference between proton and deuteron results would point to a contribution of $q\bar{q}$-exchange with isospin $I = 1$ and natural parity $P = (-1)^I$ ($\rho$, $a_0$, $a_2$, reggeons).

In this paper preliminary data of helicity amplitude ratios in exclusive electroproduction of the $\rho^0$ meson are presented. The ratios are calculated relative to the dominant $T_{00}$ amplitude. The present work is a continuation of the SDMEs analysis published in ref. [3].
2. Amplitudes and Spin Density Matrices in the reaction $e + N \rightarrow e' + \rho^0 + N$

The first subprocess of vector meson production, is the emission of a virtual photon ($e \rightarrow e' + \gamma^*$) , which is described by the photon spin density matrix according to [4].

$$\rho_{\lambda^V, \lambda_N'} = \frac{1}{2N} \sum_{\lambda_N, \lambda_N'} F_{\lambda^V, \lambda_N; \lambda_N'} g_{\lambda^V, \lambda_N; \lambda_N'}^{U+L} F_{\lambda^V, \lambda_N; \lambda_N'}^* \tag{2}$$

where $U, L$ denote an unpolarized and polarized beam, $\epsilon$ is the ratio of fluxes of longitudinal and transverse virtual photons, $\Phi$ is the angle between the lepton scattering and the hadron production planes. This spin density matrix can be calculated from QED. The vector meson spin density matrix $\rho_{\lambda^V, \lambda_N'}$ is expressed by the helicity amplitudes $F_{\lambda^V, \lambda_N; \lambda_N'} (W, Q^2, t')$. These amplitudes describe the transition of the virtual photon with helicity $\lambda_V$ to the vector meson with helicity $\lambda_N'$ where $\lambda_N', \lambda_N$ are helicity of the nucleon in the initial and final states respectively. In the CM frame of $\gamma^* N$, the spin density matrix is given by the von Neumann formula [4]:

$$\rho_{\lambda^V, \lambda_N'} = \frac{1}{2N} \sum_{\lambda_N, \lambda_N'} F_{\lambda^V, \lambda_N; \lambda_N'} g_{\lambda^V, \lambda_N; \lambda_N'}^{U+L} F_{\lambda^V, \lambda_N; \lambda_N'}^* \tag{2}$$

After the decomposition of $g_{\lambda^V, \lambda_N'}^{U+L}$ into the set of nine Hermitian matrices $(3 \times 3) \Sigma^a$ (with $\alpha = 0 \pm 3$ - corresponding to various polarization states of a transversally polarized photon; 4 - to a longitudinally polarized photon; 5 - to interference terms), the spin density matrix becomes dependent on $\alpha$: $\rho_{\lambda^V, \lambda_N'} \rightarrow \rho_{\lambda^V, \lambda_N'}^\alpha$. When we cannot separate transverse and longitudinal photons, the Spin Density Matrix Elements (SDMEs) are defined as:

$$r_{\lambda^V, \lambda_N'}^{04} = (\rho_{\lambda^V, \lambda_N'}^{04} + \epsilon R \rho_{\lambda^V, \lambda_N'}^{44})/(1 + \epsilon R) \tag{3}$$

$$r_{\lambda^V, \lambda_N'}^{\alpha} = \begin{cases} \rho_{\lambda^V, \lambda_N'}^{\alpha}, & \alpha = 1, 2, 3, \\ \sqrt{R} \rho_{\lambda^V, \lambda_N'}^{\alpha}, & \alpha = 5, 6, 7, 8. \end{cases} \tag{4}$$

Where $R = \sigma_L/\sigma_T$ is the longitudinal-to-transverse cross section ratio

For longitudinally polarized beam and unpolarized target there are 23 SDMEs, which are determined from a fit of the angular distribution of pions in the decay $\rho^0 \rightarrow \pi^+ \pi^-$. In turn, all SDMEs are bilinear combinations of helicity amplitudes and can also - in principle - be determined from the fit of angular distributions, substituting SDMEs by helicity amplitudes.

3. General properties of helicity amplitudes

There are 3 initial spin states of $\gamma^*$, $\lambda_V = (1, 0, -1)$ and 2 nucleon helicities $\lambda_N = (\frac{1}{2}, \frac{1}{2})$. In the final state there are also 3 spin states of $\rho^0, \lambda_N' = (1, 0, -1)$ and two of the nucleons $\lambda_N' = (\frac{1}{2}, \frac{1}{2})$. This implies that there are 36 amplitudes altogether. Due to parity conservation, helicity amplitudes obey the following relation:

$$F_{-\lambda_V, -\lambda_N; \lambda_V, \lambda_N} = (-1)^{(\lambda_V - \lambda_N') - (\lambda_V - \lambda_N)} F_{\lambda_V, \lambda_N', \lambda_N} \tag{5}$$

This reduces the number of independent amplitudes to 18.

Any helicity amplitude can be decomposed into a sum of an amplitude T for natural-parity exchange (NPE)($P = (-1)^J$) and an amplitude U for unnatural-parity exchange (UPE)($P = -(-1)^J$).

$$F_{\lambda_V, \lambda_N', \lambda_N} = T_{\lambda_V, \lambda_N', \lambda_N} + U_{\lambda_V, \lambda_N', \lambda_N} \tag{6}$$
The natural and unnatural amplitudes obey the following symmetry relation:

\[ T_{\lambda\nu,\lambda'\nu'} = (-1)^{-\lambda^4 + \lambda^3 + \lambda_1^2} T_{\lambda^4 - \lambda^3 - \lambda_1^2} = (-1)^{-\lambda^4 + \lambda^3 + \lambda_1^2} T_{\lambda^4 - \lambda^3 - \lambda_1^2} \]  

(7)

\[ U_{\lambda\nu,\lambda'\nu'} = (-1)^{-\lambda^4 + \lambda^3 + \lambda_1^2} U_{\lambda^4 - \lambda^3 - \lambda_1^2} = (-1)^{-\lambda^4 + \lambda^3 + \lambda_1^2} U_{\lambda^4 - \lambda^3 - \lambda_1^2} \]  

(8)

Due to these symmetry relations, the production of a vector meson is described by 10 NPE and 8 UPE amplitudes. The UPE amplitude does not exist for the transition $\gamma_L \rightarrow \rho_0^L$. $T_{00} \equiv F_{00} \equiv F_{0\hbar 0}$. For an unpolarized target there is no interference between NPE and UPE amplitudes and there is no linear contribution of nucleon-helicity-flip amplitudes to SDMEs (as they are suppressed by a factor $(\alpha)^2 = (\sqrt{\frac{\alpha}{3\pi}})^2$ ($t' = t - t_{min}$)). This reduces the number of NPE amplitudes to five: the helicity conserving $T_{00}$, $T_{11}$, and the helicity non-conserving $T_{01}$, $T_{10}$, $T_{1-1}$, where we used shorthand notation $T_{\lambda\nu,\lambda'} = T_{\lambda^4 - \lambda^3 - \lambda_1^2}$. The dominance of diagonal transitions is called s-channel helicity conservation (SCH). From the SDME analysis it has been found that for UPE the transition amplitudes obey the following hierarchy: $|U_{01}|^2, |U_{10}|^2, |U_{1-1}|^2 \ll |U_{11}|^2$. Therefore we keep only $|U_{11}| = \sqrt{|U_{11}^+|^2 + |U_{1-1}^0|^2}$ for UPE amplitudes it is not possible to prove the dominance of those without spin flip over those with spin flip. The hierarchy of amplitudes in the kinematic region of HERMES is such that $|T_{00}|^2 \gg |T_{11}|^2 \gg |U_{11}|^2$ is observed. Since SDMEs depend rather on ratios of these complex amplitudes, the number of parameters which determine all SDMEs (real and imaginary parts) is 9.

Therefore, we approximated the SDMEs by 9 real parameters, namely: $Re\{T_{11}/T_{00}\}$, $Im\{T_{11}/T_{00}\}$, $Re\{T_{01}/T_{00}\}$, $Im\{T_{01}/T_{00}\}$, $Re\{T_{10}/T_{00}\}$, $Im\{T_{10}/T_{00}\}$, $Re\{T_{1-1}/T_{00}\}$, $Im\{T_{1-1}/T_{00}\}$, $|U_{11}/T_{00}|$ where $|U_{11}/T_{00}|$ is the module of $U_{11}/T_{00}$.

These helicity amplitudes ratio are directly extracted from the experimental angular distribution of $\pi^+$ and $\pi^-$ from the $\rho^0$ meson decay, using a binned maximum likelihood method [3].

4. Kinematic dependences of amplitude ratios

4.1. $Q^2$ dependence of $Re\{T_{11}/T_{00}\}$ and $Im\{T_{11}/T_{00}\}$

Fig. 1 shows that the amplitude ratio and its kinematical dependence is similar for proton and deuteron. According to a perturbative QCD (pQCD) prediction the dependence of $T_{11}/T_{00}$ on $Q^2$ is given as $T_{11}/T_{00} \propto M_p/Q$. Hence, the $Q$ dependence of the real part of $T_{11}/T_{00}$ was fitted with the function $Re\{T_{11}/T_{00}\} = a/Q$. Combined data on proton and deuteron yield a value of $a = 1.129 \pm 0.024$ GeV with $\chi^2/N_{df} = 1.02$. The $Q^2$ dependence of $Re\{T_{11}/T_{00}\}$ is found to be in good agreement with the asymptotic behavior expected from pQCD, as it is seen in Fig. 1.

In Fig. 2 the dependence of $Im\{T_{11}/T_{00}\}$ is given. The data are rising with $Q^2$. This is unexpected, since such a dependence contradicts the behavior predicted by pQCD. A fit for combined data on proton and deuteron with the function $Im\{T_{11}/T_{00}\} = bQ$ yields $b = 0.344 \pm 0.014$ GeV$^{-1}$ with $\chi^2/N_{df} = 0.87$. The $Q^2$ dependence of the phase difference $\delta_{11}$ between the amplitudes $T_{11}$ and $T_{00}$ is given by $tan\delta_{11} = Im\{T_{11}/T_{00}\}/Re\{T_{11}/T_{00}\} = bQ^2/a$. The phase difference comes out to be $\delta_{11} \sim 30^\circ$ at $<Q^2> = 1.95$ GeV$^2$ and grows with $Q^2$, in disagreement with the pQCD prediction. One possible explanation of this disagreement is that the $Q^2$ range of HERMES is not yet in the asymptotic region where the pQCD prediction becomes valid.
4.2. $t'$ dependence of $\text{Re}\{T_{01}/T_{00}\}$ and $\text{Im}\{T_{01}/T_{00}\}$

The amplitude $T_{01} = T_{01\frac{1}{2}1\frac{1}{2}}$ describing the transition $\gamma_T^+ \rightarrow \rho^0_L$ is the largest SCHC-violating amplitude. In Figs. 3 and 4 are shown the $t'$ dependence of $\text{Re}\{T_{01}/T_{00}\}$ and $\text{Im}\{T_{01}/T_{00}\}$. We see that there is no difference between proton and deuteron amplitude ratio $T_{01}/T_{00}$. For this ratio perturbative QCD predicts following the dependence: $\frac{T_{01}}{T_{00}} \propto \sqrt{-t'/Q}$. This functional form in fitting $\text{Re}\{T_{01}/T_{00}\}$ does not give a reasonable $\chi^2$ per degree of freedom, however a simpler parameterization $\text{Re}(T_{01}/T_{00}) = a\sqrt{-t'/Q}$ indicates a better description of data as the corresponding value of $\chi^2$ decreases by a factor of approximately two. For combined proton and deuteron data values of $a = 0.399 \pm 0.023$ GeV$^{-1}$, $\chi^2/N_{df} = 0.72$ are obtained. This result also suggests that the Hermes Q-range is outside the region of asymptotic behavior. The imaginary part of $\text{Im}(T_{01}/T_{00})$ is fitted with the pQCD prediction $\text{Im}(T_{01}/T_{00}) = b\sqrt{-t'/Q}$. For combined proton and deuteron data the values $b = 0.20 \pm 0.07$, $\chi^2/N_{df} = 1.09$ are obtained. Parameter $b$ differs by three standard deviations from zero. We note that the result is in agreement with the pQCD prediction, however it does not necessarily confirm the applicability of pQCD to the measured process.

4.3. $Q^2$ and $t'$ dependence of $|U_{11}/T_{00}|$

The UPE amplitude $U_{11}$ describes the transition from a transverse photon to a transverse $\rho^0$ meson. For this ratio pQCD predicts the following dependence: $U_{11}/T_{00} \propto M_\rho/Q$. The ratio
Figure 3. The $-t'$ dependence of $\text{Re}\{T_{01}/T_{00}\}$ for proton and deuteron data. The meaning of the error bars and the relationship between the curves are the same as for Fig 1.

Figure 4. The $-t'$ dependence of $Q^*\text{Im}\{T_{01}/T_{00}\}$ for proton and deuteron data. The meaning of the error bars and the relationship between the curves are the same as for Fig 1.

Figure 5. The $Q^2$ dependence of $|\text{Re}\{U_{11}/T_{00}\}|$ for proton and deuteron data. The meaning of the error bars and the relationship between the curves are the same as for Fig 1.

Figure 6. The $-t'$ dependence of $|\text{Im}\{T_{11}/T_{00}\}|$ for proton and deuteron data. The meaning of the error bars and the relationship between the curves are the same as for Fig 1.

$|U_{11}/T_{00}|$ versus $Q^2$ and $t'$ is presented in Figs. 5 and 6. No kinematic dependence is observed and therefore the ratios are fitted to a constant $|U_{11}|/|T_{00}| = a$. As shown in Figs. 5 and 6 $|U_{11}|$
is smaller than $|T_{00}|$ by a factor of approximately 2.5. For combined proton and deuteron data the values $a = 0.391 \pm 0.013$, $\chi^2/\nu = 0.44$ are obtained. This is in contradiction with both high-Q asymptotic behavior and one-pion-exchange dominance. This disagreement may reflect the fact that $Q^2$ explored in HERMES is far from the region where the asymptotic behavior dominates. The signal of unnatural-parity exchange is established with a significance of more than 20 standard deviations, while using SDMEs in the previous analysis from ref [3] it had a significance of 3 standard deviations only. Hence unnatural parity exchange process is seen here much better than in the SDME method. The data also show no significant difference between proton and deuteron for this amplitude ratio.

5. Summary
Study of electroproduction of a $\rho^0$ vector meson on proton and deuteron enables to obtain ratios of helicity amplitudes, and to investigate their kinematic dependences. The kinematic dependences of $\text{Im}\{T_{11}/T_{00}\}$, $|U_{11}/T_{00}|$ are in contradiction with the high-Q asymptotic behavior predicted in pQCD. The dependences of $\text{Re}\{T_{11}/T_{00}\}$ and $\text{Im}\{T_{01}/T_{00}\}$ are in agreement with pQCD prediction. The amplitude ratios for deuterons are compatible with those for protons. The UPE signal is seen here with very high significance for both proton and deuteron data and with higher precision than that obtained in the SDME method. The violation of s-channel helicity conservation is determined here with much higher accuracy from amplitudes studies than from SMDEs.

6. References
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