Supersymmetric (SUSY) grand unified theories (GUTs) \[^1\] have been regarded as the most agreeable candidates beyond the standard model for a long time, because they can realize the gauge coupling unification as well as show a natural solution of the hierarchy problem. They can also explain the electroweak symmetry breaking by the so-called the radiative breaking scenario.\[^2\] The proton decay is a crucial prediction of GUTs,\[^3\] which has not been observed in the experiments yet.\[^4\] From the current experimental bound on the proton decay, it was claimed that the minimal SU(5) SUSY GUT has already been excluded.\[^5, 6\] However, as pointed out in e.g. Refs.\[^7, 8\], this claim should only be applied to the minimal scenario which might be appropriately reproduced in a similar way.

As we will show, the two requirements — the realistic Yukawa couplings of quarks and leptons and no dimension five proton decay operators in the SU(5) SUSY GUT framework with the minimal particle content. We must analyze the Yukawa interactions with the coloured Higgs fields carefully. In this letter we show the natural model which does not contain dimension five proton decay operators in the SU(5) SUSY GUT with the minimal particle content. We introduce the GUT scale \((M_{\text{GUT}})\) where the SU(5) gauge symmetry is broken by the vacuum expectation value (VEV) of an adjoint Higgs field \(\Sigma\). Since the model at the GUT scale is the effective theory of the fundamental one which realized at the Planck scale, it should contain the higher dimensional operators which are suppressed by the Planck scale \((M_{\text{GUT}}/M_{\text{Pl}})^n\) \((n: \text{positive integer})\).\[^5, 4, 11\] These terms can be the origin of a fermion mass hierarchy of three generations as well as the one between top and bottom quarks. The realistic mass spectrum of the down type quarks and the charged leptons can be reproduced by these terms. We take the following setup:

1. Only the top Yukawa coupling exists at the tree level. The other Yukawa couplings are induced by the \(n\)th order higher dimensional terms, \((M_{\text{GUT}}/M_{\text{Pl}})^n\).

2. The Yukawa couplings of the bottom quark and the tau lepton are induced by the first order terms \((M_{\text{GUT}}/M_{\text{Pl}})\), the mass of the strange quark and the muon are reproduced by the second order terms \((M_{\text{GUT}}/M_{\text{Pl}})^2\), and the down quark and the electron masses are provided by the third order terms \((M_{\text{GUT}}/M_{\text{Pl}})^3\). The masses of the up type quarks might be appropriately reproduced in a similar way.

3. We should pay attention that some of the terms \((\langle \Sigma \rangle/M_{\text{Pl}})^n\) are regarded as \((n-1)\)th order terms effectively due to their coefficients.

4. The couplings of the operators associated with the proton decay process completely vanish.

As we will show, the two requirements — the realistic Yukawa couplings of quarks and leptons and no dimension five proton decay operators — almost determine the couplings for the higher dimensional operators.

The superpotential for the Yukawa sector is represented as the series expansion according to the power of adjoint Higgs fields, such as

\[ W_{\text{Yukawa}} = W_0 + W_1 + W_2 + W_3 + W_4 + \cdots. \]  

The zeroth order part \(W_0\) is the same as that of the minimal SU(5) model,

\[ W_0 = \frac{1}{4} \varepsilon_{abcde} Y_{ij} Y_{kl} Y_{mn} Y_{op} \mu_{ab} \mu_{cd} \mu_{ef} \mu_{gh}, \]  

where \(i, j\) are the indices for generations and \(a, b, ...\) are the ones for the SU(5) indices. The chiral superfield \(10\) contains the right-handed up-type quark \(u_R\), the light-handed quark doublet \(Q\), and the right-handed charged lepton \(e_R\). The right-handed down-type quark \(d_R\) and the lepton doublet \(L\) belong to the superfield \(5^*\). The Higgs fields \(H\) and \(H (5 \text{ and } 5^*)\) include the coloured Higgs triplets \((H_C, H_C)\) and the Higgs doublets, respectively. As shown in our setup, \(Y_2 = 0\) is assumed in Eq.\[^2\], which is the origin of the hierarchy between the top and the bottom masses. The first order part \(W_1\) is expressed as

\[ W_1 = \frac{\varepsilon_{abcde}}{4} \left( f_{ij}^{(1)} \mu_{ab} \mu_{cd} \mu_{ef} \mu_{gh} + f_{ij}^{(2)} \mu_{ab} \mu_{cd} \mu_{ef} \mu_{gh} \right), \]  

where \(\varepsilon\) takes VEV of \(\langle \Sigma \rangle = \text{diag}(2, 2, 2, -3, -3)\sigma\) which breaks the SU(5) gauge group into SU(3)_c × SU(2)_L × \(SU(2)_R\)
$U(1)_Y$. The value of $\sigma$ is the scale of $M_{\text{GUT}}$. For the second and third order superpotential, we only show the down-type quark and the charged lepton sector. They are represented as $Y'$

\[
W_2 = \sqrt{2} \left( h^{ij}_3 \bar{H}_a 10^i_{10} 5^j_b \frac{(\Sigma \Sigma) a}{M_{Pl}} + h^{ij}_4 \bar{H}_a (\Sigma \Sigma b) a \frac{5^j_c}{M_{Pl}} 10^i_e \right),
\]

\[
W_3 = \sqrt{2} \left( h^{ij}_5 \bar{H}_a \frac{\Sigma^a}{M_{Pl}} 10^{bc} 5^j_c \frac{(\Sigma \Sigma) a}{M_{Pl}} + h^{ij}_6 \bar{H}_a \Sigma^a b_a \frac{10_{10}^{bc} \Sigma^d}{M_{Pl}} 5^j_d \right),
\]

where $(\Sigma \cdots) \left((\Sigma \cdots)^a\right)$ denotes a singlet (an adjoint) by contracting the $SU(5)$ indices of $\Sigma \cdots$. The fourth order superpotential suggests

\[
W_4 = \sqrt{2} \left( h^{ij}_7 \bar{H}_a 10^i_{10} 5^j_b \frac{(\Sigma \Sigma) a}{M_{Pl}} + h^{ij}_8 \bar{H}_a (\Sigma \Sigma b) a \frac{5^j_c}{M_{Pl}} 10^i_e \right),
\]

\[
W_5 = \sqrt{2} \left( h^{ij}_9 \bar{H}_a \frac{\Sigma^a}{M_{Pl}} 10^{bc} 5^j_c \frac{(\Sigma \Sigma) a}{M_{Pl}} + h^{ij}_10 \bar{H}_a (\Sigma \Sigma b) a \frac{10_{10}^{bc} \Sigma^d}{M_{Pl}} 5^j_d \right),
\]

\[
W_6 = \sqrt{2} \left( h^{ij}_11 \bar{H}_a \frac{\Sigma^a}{M_{Pl}} 10^{bc} 5^j_c \frac{(\Sigma \Sigma) a}{M_{Pl}} + h^{ij}_12 \bar{H}_a (\Sigma \Sigma b) a \frac{10_{10}^{bc} \Sigma^d}{M_{Pl}} 5^j_d \right).
\]

Each matrix element $h^{ij}$ is assumed to have an $O(1)$ coefficient, and the mass hierarchy is produced by the suppression factors $\sigma/M_{Pl} \equiv 1/a$. Decomposing the superpotential Eqs. 4-10 into its component fields, we obtain the Yukawa couplings of down-type quarks and charged leptons as

\[
Y_d = Y' - \frac{3}{a} h_1' - \frac{3}{a} h_2' + \frac{9}{a^2} h_4 + \frac{4}{a^2} h_5 - \frac{6}{a^2} h_6
\]

\[
- \frac{27}{a^3} h_{10} - \frac{8}{a^3} h_{11} + \frac{18}{a^3} h_{12} - \frac{12}{a^3} h_{13} + \cdots.
\]

where the matrices with a prime symbol are defined as

\[
Y' \equiv Y_2 + \frac{30}{a^2} h_3 - \frac{30}{a^2} h_9 + \frac{900}{a^4} h_{14} + \frac{210}{a^2} h_{15},
\]

\[
h'_{1,2,3,4} \equiv h_{1,2,3,4} + \frac{30}{a^2} h_{18,19,20}.
\]

On the other hand, the Yukawa couplings $Y_q$ and $Y_u$, which are associated with the interactions $Q_i \ell_j H_C$ and $u_i r_j H_C$, respectively, are given by

\[
Y_q = Y' + \frac{2}{a} h_1' - \frac{3}{a} h_2' + \frac{4}{a^2} h_4 + \frac{9}{a^2} h_5 - \frac{6}{a^2} h_6
\]

\[
+ \frac{8}{a^3} h_{10} - \frac{27}{a^3} h_{11} + \frac{12}{a^3} h_{12} + \frac{18}{a^3} h_{13} + \cdots.
\]

\[
Y_u = Y' + \frac{2}{a} h_1' - \frac{2}{a} h_2' + \frac{4}{a^2} h_4 + \frac{4}{a^2} h_5 + \frac{4}{a^2} h_6
\]

\[
+ \frac{8}{a^3} h_{10} + \frac{8}{a^3} h_{11} + \frac{8}{a^3} h_{12} + \frac{8}{a^3} h_{13} + \cdots.
\]

The terms in which the adjoint Higgs fields are contracted by themselves have larger pre-factors than the others. Such terms should be practically regarded as lower order contribution. For example, we should regard $h_3$ as the first order term like $h_1$ and $h_2$. The terms of $h_7,9$, and $h_{11}$ are referred as the second order terms such as $h_{4,6}$, and those of $h_{15,20}$ should belong to the third order terms such as $h_{10,13}$.

The value of $\sigma$ is related to the mass of the coloured Higgs triplet $M_C$ and the GUT scale. The GUT scale is represented as $(M_{Pl}^2 M_s)^{1/3}$, where $M_V$ stands for mass of $X$ and $Y$ bosons ($SU(5)$ breaking gauge bosons) and $M_s$ for the mass of $\Sigma$. The magnitudes of these mass parameters are strictly constrained from the gauge coupling unification condition [11]. However, the value of $\sigma$ itself can be larger than the GUT scale $(M_{Pl}^2 M_s)^{1/3} \approx 2.0 \times 10^{16}$ GeV by taking the Higgs couplings among $H, \tilde{H}, \Sigma$ to be small. Therefore, we can take $a = O(10 \sim 100)$, where dimension six proton decay operators are suppressed enough.

Let us now illustrate a concrete example for the Yukawa couplings which reproduces not only the realistic fermion mass spectrum but also the completely vanishing dimension five proton decay operators. We take the basis where $Y_q$ and $Y_u$ are diagonal. The coefficients of the dimension five proton decay operators are denoted

\[
C_{ijkl}^{q} \equiv Y_{q}^{ij} Y_{q}^{kl}, \quad C_{ijkl}^{u} \equiv Y_{u}^{ij} Y_{u}^{kl},
\]

where $Y_{qq}$ and $Y_{uu}$ are the couplings of interactions of the coloured Higgs field coming from the $10, 10, H$ type terms. Avoiding unreliable large couplings in $h_i$'s, the
texture of $h_i$'s for the third generation is uniquely determined except for $\Delta y_3 \equiv y_\tau - y_\mu$, which is the difference between the Yukawa couplings of the tau lepton and the bottom quark. It should be small at the GUT scale and we assume that it is provided by second order terms. From Eqs. (11)-(15), the third generation Yukawa components are induced as

$$(h_3)_{33} = a^2 \Delta y_3/25,$$

$$(h_5)_{33} = a (h'_2)_{33} + 2a^2 \Delta y_3/25,$$

$$(h_4)_{33} = a (h'_1)_{33} + a^2 y_\mu/5 + 2a^2 \Delta y_3/25,$$

$$(h'_3)_{33} = -2a^2 (y_\mu + \Delta y_3)/75 - a (h'_1 + h'_2)_{33}/5,$$

up to the second order. Here, $h'_3 \equiv h_3 - h_9/a + 30h_{14}/a^2$. In order to avoid $\mathcal{O}(a^2 y_\mu)$ terms in $(h_5)_{33}$ and $(h_4)_{33}$, we must take

$$(h_1)_{33} = -a y_\mu/5, \quad (h_2)_{33} = 0.$$  

Then, the value of $(h_3)_{33}$ is determined up to order $\mathcal{O}(a^2 y_\mu)$ as

$$(h_3)_{33} = a^2 y_\mu/75.$$  

This term should be regarded as the first order term because of the large pre-factor $1/75$, in which the value of $(h_3)_{33}$ itself is kept of $\mathcal{O}(1)$. Now all the third generation components of $h_i$'s are determined. The couplings for the second generation are also determined in a similar way. Those for the first generation are not uniquely determined since there are large degrees of freedom in the third order terms.

Summarizing above discussions, the first order terms are uniquely determined as

$$h_1 = \frac{a}{5} \text{diag}(0, 0, y_\mu), \quad h_2 = 0,$$

$$h_3 = \frac{a^2}{75} \text{diag}(0, 0, y_\mu).$$  

The second order terms are also determined almost automatically as

$$h_4 = \frac{a^2}{75} \text{diag}(0, 9y_\mu + y_\mu, \Delta y_3),$$

$$h_5 = \frac{a^2}{75} \text{diag}(0, -6y_\mu + y_\mu, \Delta y_3),$$

$$h_6 = \frac{a^2}{25} \text{diag}(0, -y_\mu + y_\mu, \Delta y_3),$$

$$h_7 = h_8 = \frac{a^3}{150} \text{diag}(0, y_\mu, \Delta y_3),$$

$$h_9 = h_{14} = 0.$$  

For the third order terms, there are various choices, and one example is

$$h_{10} = h_{16-25} = 0,$$

$$h_{11} = \frac{a^3}{25} \text{diag}(-y_d + 2y_e/7, 0, 0),$$

$$h_{12} = \frac{a^3}{25} \text{diag}(3y_d/2 - 2y_e/3, 0, 0),$$

$$h_{13} = \frac{a^3}{25} \text{diag}(-y_d/2 - y_e/3, 0, 0),$$

$$h_{15} = \frac{4a^4}{3675} \text{diag}(y_e, 0, 0).$$  

It is worth noting that the accurate Yukawa couplings of the down-sector quarks and charged leptons are obtained as

$$Y_d = \text{diag}(y_d, y_e, y_\mu), \quad Y_e = \text{diag}(y_e, y_\mu, y_\tau)$$  

as well as the couplings of coloured Higgs triplet vanish as

$$Y_{dl} = Y_{ud} = 0.$$  

Once these Yukawa interactions are realized at the GUT scale, any dimension five proton decay process will not appear even if the renormalization group equation (RGE) effects are taken into account.

| $W_i$ | order | $h_{j}$ | Example 1 | Example 2 |
|------|-------|--------|-----------|-----------|
| $W_1$ | 1st   | $h_1$  | $(0, 0, -0.64)$ |          |
|      |       | $h_2$  | 0         |          |
| $W_2$ | 1st   | $h_3$  | $(0, 0, 2.3)$ |          |
|      | 2nd   | $h_4$  | $(0, 0.72, 0.4)$ |          |
|      |       | $h_5$  | $(0, -0.21, 0.40)$ |          |
|      | 2nd   | $h_7$  | $(0, -1.3, -3.6)$ |          |
|      |       | $h_8$  | $(0, -1.3, -3.6)$ |          |
|      | 3rd   | $h_{10}$ | 0         | 0         |
|      |       | $h_{11}$ | $(0.48, 0, 0)$ | 0         |
|      | 3rd   | $h_{12}$ | $(0.68, 0, 0)$ | 0         |
|      |       | $h_{13}$ | $(-0.29, 0, 0)$ | 0         |
| $W_4$ | 2nd   | $h_{14}$ | 0         |          |
|      | 3rd   | $h_{15}$ | $(0.19, 0, 0)$ | $(-0.65, 0, 0)$ |
|      |       | $h_{16}$ | 0         | 0         |
|      |       | $h_{17}$ | 0         | 0         |
|      |       | $h_{18}$ | $(3.3, 0, 0)$ | $(3.3, 0, 0)$ |
|      |       | $h_{19}$ | $(-1.4, 0, 0)$ | $(-1.4, 0, 0)$ |
|      |       | $h_{20}$ | $(-0.71, 0, 0)$ | $(-0.71, 0, 0)$ |

| TABLE I: Examples of the matrix elements of $h_i$'s at the GUT scale which reproduce the realistic fermion mass spectrum and vanish the proton decay operators. Here, we take $a = 55$ and $\tan \beta = 10$. We take the diagonal basis of the down-type quark and the charged lepton mass matrices. The flavour mixing is imposed into the up-sector Yukawa couplings.     |

In Table I, we present the magnitudes of Yukawa couplings at the GUT scale by using the results in Ref. [12]. The applicability of the perturbation (couplings $\lesssim \sqrt{4\pi}$) is satisfied for all components. Notice that it can be satisfied even in the rather large $\tan \beta$ ($\tan \beta \sim 10$) region. Ordinal $SU(5)$ GUT models, e.g. with decoupling SUSY breaking spectrum, should have a small $\tan \beta$ of order $1$. The Yukawa couplings of the up-type quarks can also be derived from higher dimensional terms. We assume
that the Yukawa coupling of the top quark comes from $Y_1$ in Eq. (2), that of the charm quark from $W_2$, and that of the up quark from $W_3$. For example, supposing the simple superpotential,

$$W = \frac{e_{abcd} \lambda^{ab}}{4} \cdot 10^{cd} H^c \left( f^i_{\alpha\beta} \frac{(\Sigma \Sigma)}{M^2_{\phi_i}} + f^j_{\alpha\beta} \frac{(\Sigma \Sigma \Sigma \Sigma)}{M^2_{\phi_i}} \right),$$

we obtain the Yukawa matrices

$$Y_u = Y_1 + \frac{30}{a^2} f_c + \frac{210}{a^4} f_u.$$  

In the basis, $Y_u$ should be given by $Y_u = U^\dagger_{\text{CKM}} Y_u^{\text{diag}}$. This example gives

$$(Y_1)_{33} = 0.75, \quad (f_c)_{22} = 0.18, \quad (f_u)_{11} = 0.26,$$

for $Y_u^{\text{diag}}$ with the same values of $a$ and $\tan \beta$ in Table I.

Some comments are in order. The first is about the coefficients of the proton decay operators. They must include at least one first generation quark superfield. Therefore, it is not necessary to eliminate all components of the coloured Higgs Yukawa couplings as in Eq. (26). In fact we can realize such Yukawa matrices which have more choices than the examples shown above. However, in this case the RGE effect must be taken into account to estimate the proton decay rate, since the second and third generation components of $Y_{q_3}$ and $Y_{ud}$ are transmitted into the first generation components through the generation mixings. The RGE analysis shows that the large entry of the second and third generation components could be destructive. This means that RGE effects will break the proton stability even if all the first generation components of $Y_{q_3}$ and $Y_{ud}$ are zero. If $Y_{q_3}$ and $Y_{ud}$ do not include the first generation components at the nucleon mass scale (not at the GUT scale), the dimension five proton decay processes will disappear as pointed out in Ref. [8]. It is an interesting possibility. However, there must be a reason why the scale is not the GUT scale but the nucleon mass scale.

The second comment is about the contribution from the sub-leading effects. When the soft SUSY breaking tri-linear scalar interactions of the coloured Higgs

$$-L_{\text{soft}} \supset A^{\alpha\beta}_{q_3} \bar{Q}_i \epsilon \bar{L}_j \bar{H}_C + A^{\alpha\beta}_{q_3} \bar{Q}_i \epsilon \bar{Q}_j \bar{H}_C + \text{H.c.},$$

are introduced, $A_{q_3} Y_{q_3}$ and $A_{q_3} Y_{q_3}$ can contribute to the proton decay processes. We have neglected these $A$-term contributions in the above discussions, which can be justified in the minimal supergravity context. It is because these terms are generally proportional to the corresponding Yukawa couplings. Therefore, when they do not exist at the GUT scale, there will be no contribution at the low energy scale even if we take account into the RGE effects.

We have tried to reproduce the suitable fermion mass hierarchy as well as to suppress the proton decay in the $SU(5)$ SUSY GUT framework with the minimal field contents. The realistic fermion mass spectrum can be realized simultaneously with vanishing dimension five proton decay processes. These requirements have almost determined the Yukawa couplings of higher dimensional operators at the GUT scale.

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[13] The authors of Ref.[8] have also introduced higher dimensional terms but up to the first order of $1/a$. In this case, there is an inevitable relation between the Yukawa matrices as $Y_{q_3} - Y_{ud} = Y_{c} - Y_{d} \neq 0$, where the dimension five operators cannot completely vanish and the suppression factor $1/a$ has nothing to do with the fermion mass hierarchy. On the other hand, the author of Ref.[9] has tried to reproduce the so called Georgi-Jarlskog texture [See H. Georgi and C. Jarlskog, Phys. Lett. B86 (1979) 297] by introducing the terms up to the third order of $1/a$.
[14] In Ref.[8], the authors adopted the ingenious texture which they referred as the consistent model I. This model has the non-zero first generation components, however, they avoided the large RGE effects by vanishing second and third generations’ components in $Y_{q_3}$. This scenario is effective only in the case of small $\tan \beta$ such as $\tan \beta = O(1)$. 