Analytical model of 1D Carbon-based Schottky-Barrier Transistors

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Abstract—Nanotransistors typically operate in far-from-equilibrium (FFE) conditions, that cannot be described neither by drift-diffusion, nor by purely ballistic models. In carbon-based nanotransistors, source and drain contacts are often characterized by the formation of Schottky Barriers (SBs), with strong influence on transport. Here we present a model for one-dimensional field-effect transistors (FETs), taking into account on equal footing both SB contacts and FFE transport regime. Intermediate transport is introduced within the Büttiker’s probe approach to dissipative transport, in which a non-ballistic transistor is seen as a suitable series of individually ballistic channels. Our model permits the study of the interplay of SBs and ambipolar FFE transport, and in particular of the transition between SB-limited and dissipation-limited transport.

Index Terms—graphene, carbon transistors, carbon nanotubes, ballistic transport, compact model, far-from equilibrium transport, Büttiker probes, Schottky barrier

I. INTRODUCTION

Since the isolation of graphene in sheets [1], [2], with their exceptionally promising high mobility [3], graphene-related materials have attracted much interest for their possible application in nanoelectronic devices. In particular, semiconducting carbon nanotubes (CNTs) [4] and single-layer or bilayer graphene nanoribbons (GNRs) [5] have been successfully employed in quasi-1D nanotransistors.

An important issue related to carbon-based channels is the nature of the metallic contact at source and drain, which can lead to different pinning of the Fermi level and consequently to the formation of ohmic or Schottky contacts [6], [7]. The presence of SB contacts can have dramatic effects on device performance, because charge injection is subordinated to a tunneling process. However, in nanodevices with reduced oxide thickness, tunneling phenomena at source and drain are favored, and, while they often limit performance in conventional transistors, their exploitation is at the core of the concept of tunneling FETs [8].

Transport in nanotransistors is certainly far from of equilibrium, but is still not fully ballistic, and currents are much lower than those predicted by ballistic models [9]. While it is perfectly clear that inelastic scattering may arise from the interaction of carriers with phonons and impurities, it is rather complex to take into account microscopically its effect on transport. A powerful phenomenological attempt to deal with carrier relaxation and decoherence was based on the Büttiker virtual probes approach [10], [11], in which inelastic scattering is thought as localized in special points, spaced by a length defined as “mean free path”. The Büttiker approach was also introduced in microscopical models based on tight-binding Hamiltonians [12], and recently extended to deal, via a quantum Langevin approach, with 1D conductors [13]. In [14] the Büttiker probes approach to inelastic scattering was employed in a simulation, based on the non-equilibrium Green’s functions formalism, of a non-ballistic silicon nanowire transistor.

Fully microscopical analysis of inelastic scattering due to specific mechanisms such as phonon scattering, with the non-equilibrium Green’s functions approach, has also been addressed by adding a proper self energy correction on a site-representation propagating Hamiltonian by Jin et al. [15] and by M. Gilbert et al. [16], [17].

As far as analytical models are concerned, transport in quasi-1D FETs is generally treated as purely ballistic or with a drift-diffusion assumption as in Ref. [18], [19], [20]. A largely invoked approach to treat partially ballistic transport including the effects of backscattering was proposed by Lundstrom et al. [21]. This approach, that is easily included as a correction to ballistic models, has the merit of offering a very simple and synthetic picture but does not allow a full description of the seamless transition from ballistic to quasi-equilibrium drift-diffusion transport. Recently a rigorous semi-analytical model based on the Büttiker virtual probes approach [10], [11] has emerged, in which a non-ballistic transistor is seen as a suitable chain of $N$ ballistic channels, where $N$ is the ratio of the channel length to the mean free path, or equivalently as a series of drift-diffusion and a ballistic FET [22], [23], [24].

In this work we propose a semi-analytical model based on the virtual probes approach, which describes one-dimensional FETs, treating on equal footing Schottky barrier contacts and FFE transport conditions. In Section II we summarize the general analytical description of graphene nanoribbons subbands, density of states, equilibrium charge density, extensible also to the carbon nanotube case. In Section III we present a WKB approximation of the tunneling probability through Schottky barrier contacts, yielding analytical expressions for the transmission based on two different levels of approximation for the energy dispersion curves of GNRs (or CNTs). In Section IV a model for a single ballistic transistor with SB contacts is presented, compared with data from numerical simulations. In Section V we propose a compact model, based on Büttiker...
virtual probes approach able to deal with both intermediate transport and SB contacts, and use it to study the interplay of SB and dissipative transport.

II. DISPERSION RELATION AND DENSITY OF STATES

The dispersion curve of an armchair GNR with $N$ dimer lines can be obtained analytically by cutting techniques, analogous to that used for CNTs in [25], from the 2D graphene tight-binding dispersion. The subband dispersion curves correspond to that used for CNTs in [25], from the 2D graphene tight-binding approximation.

Energy dispersion relation, which we will refer to as the full band (FB) approximation when applied to FET modelling, is therefore

$$E_\alpha(k) = E_\alpha(k) + \delta E_\alpha(k).$$

The comparison between numerical tight-binding calculations, with edge effects taken in account, and the analytical result with perturbative corrections, for a A-GNR of 12 dimer lines is shown in Fig.2. The agreement is very good, especially at $k = 0$, where Fig.2 reproduces the results of [27]. For simplicity we define here the band edges as $e_\alpha = E_\alpha(0)$.

A. Approximated expressions

In modelling nanotransistors only the lowest lying subbands matter, in which the relevant transport phenomena take place. For these lowest lying subbands often an effective mass (EM) approximation is invoked

$$E_\alpha^{EM}(k) = e_\alpha + \frac{\hbar^2 k^2}{2M_\alpha},$$

in this case the following effective mass for the $\alpha$-th mode can be employed

$$M_\alpha = \frac{2}{3} \frac{\hbar^2 e_\alpha}{a^2 V^2 A_m}.$$

The DOS in EM approximation is given by

$$D_\alpha^{EM}(E) = \frac{2}{\pi \hbar} \sqrt{\frac{M_\alpha}{2E}}.$$

with $E$ expressing the ‘kinetic energy’, i.e. the energy calculated with respect to the band edge $e_\alpha$.

The EM approximation is rather crude, and an intermediate (I) approximation, between the FB and the EM, can be the use of the dispersion curve

$$E_\alpha^I(k) = \pm \sqrt{e_\alpha^2 + \frac{\hbar^2 k^2}{M_\alpha}},$$

for which the DOS is

$$D_\alpha^I(E) = \frac{2(e_\alpha + E)}{\pi \hbar} \sqrt{\frac{M_\alpha}{e_\alpha E(E + 2e_\alpha)}}.$$
In order to give an estimation of the Bessel function $K_1$ which has no closed form, we can adopt the approximation [26]

$$K_1(x) \approx \frac{K_{1/2}(x) + K_{3/2}(x)}{2} = \sqrt{\frac{\pi}{2x}} \frac{1 + 2x}{2e^{-x}}$$  \hspace{1cm} (13)

arriving in the end to express the charge density as

$$n = N_c e^{-\beta(E_c - q\phi_e - E_F)},$$ \hspace{1cm} (14)

$$N_c = \frac{M_\alpha}{2\pi q^2} \frac{1 + 2\beta e_d}{\hbar c},$$ \hspace{1cm} (15)

with essentially the same form of 3D bulk semiconductors.

III. TUNNELING OF SCHOTTKY BARRIERS

Our aim is to provide an analytical description of the tunneling through SB contacts. The first step is to model in the simplest way the potential decay occurring near the source and drain contacts. The potential inside a transistor channel is described by the 3D Poisson equation

$$\nabla^2 \phi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon}$$ \hspace{1cm} (16)

together with the boundary conditions enforced by voltages $V_s$, $V_d$, $V_g$ at the source, drain and gate leads. In the evanescent mode analysis approach the electrostatic potential inside a nanotransistor $\phi(\vec{r})$ is thought as the sum of a long-channel solution $\phi_L(\vec{r})$, which satisfies the vertical electrostatics, plus a short-channel solution $\phi^s(\vec{r})$, called evanescent mode, responsible of the potential variation along the channel [28]. The short-channel solution is obtained, solving the Laplace equation for the device with an adequate expansion in harmonic functions.

As a matter of fact the long-channel solution near the source contact results in an exponential profile

$$\phi^L(\vec{r}) \propto R(\vec{r}) e^{-z/\lambda},$$ \hspace{1cm} (17)

where $R(\vec{r})$ describes the solution in the channel cross section and $\lambda$ comes to be a natural scale length for the potential variation in the device. The actual value of $\lambda$ depends on the details of the device geometry, however in double-gate (DG) configuration, and considering that in general, in carbon-based FET, the oxide thickness is significantly larger than the channel thickness, the asymptotic value $\lambda = (2\ell_{ox} + \ell_{ch})/\pi$ can be assumed. In the case of a cylindrical GAA-CNT FET, an explicit calculation of $\lambda$ via evanescent mode analysis has been performed in [29].

We follow this line and assume that the channel potential rigidly shifts the confinement eigenvalues $\varepsilon_\alpha$, where $\alpha$ runs on the different subbands. Now we are interested only in the potential inside the restricted zone of the graphene channel $\phi_c(z)$, in which it can be assumed as a constant (which is strictly true in subthreshold regimes), and we consider its variation only along the channel direction. The long channel solution inside the channel is reduced to $\phi_L(\vec{r}) \approx \phi_c$, where $\phi_c$ is solely imposed by the vertical electrostatics, while the short-channel solution has the form [17]. Therefore the potential in the channel $\phi_c(z)$ can be expressed as

$$\phi_c(z) = \phi_c + \frac{A_x}{q} e^{-z/\lambda}$$ \hspace{1cm} (18)
with \( \phi_c = \phi(\infty) \) fixed by the vertical electrostatics and \( A_s \) imposed by the boundary condition at the SB contact \( A_s = E^{(s)}_{SB} - eL + q\phi_c \), where \( L \) refers to the lowest lying subband, due to the Fermi level pinning at the metal/semiconductor interface. \( E^{(s)}_{SB} \) is the Schottky barrier height on the first conduction subband with respect to the source Fermi level. The charge injected from the source with energy lower than the barrier have to tunnel in order to reach the channel. We need to calculate the transmission through an exponential decaying barrier of the kind

\[
E_{SB}(z) = A_s e^{-z/\lambda}
\]

with the height \( A_s \) dependent on the electrostatic potential \( \phi_c \). We note that however that if the band bending exceeds the energy gap \( 2\epsilon_s \), carrier with energy \( 0 < E < A_s - 2\epsilon_s \), will experience a SB of an height \( A_s = E + 2\epsilon_s \).

In order to estimate the behavior of a nanotransistor it is essential to accurately describe tunneling phenomena, both in traditional FETs and in TFETs. In this section we compare the tunneling calculated with WKB approximation in a full band approach (FB-WKB), within the effective mass approach (EM-WKB) and intermediate approximation (I-WKB). FB-WKB is more complex to implement and requires a numerical solution of the integral

\[
\ln(T(E)) = -2 \int_{z_1}^{z_2} I[k_s(z); E]dz
\]

While for the others two an analytical expression for the tunneling \( T(E) \) can be obtained.

A. Effective-mass WKB approximation

The transmission coefficient obtained via WKB approximation is given as

\[
T(E) = \begin{cases} e^{-2 \int_{z_1}^{z_2} \sqrt{2m_s/h^2(E_{SB}(z)-E)}dz}; E < A_s \\ 1; E \geq A_s \end{cases}
\]

where \( z_1, z_2 \) are the classical turning points are

\[
z_1 = 0; z_2 = -\lambda \ln \left[ \frac{E}{A_s} \right].
\]

The transmission coefficient can be analytically calculated in

\[
\ln(T(E)) = -4\lambda \sqrt{\frac{2m_s(A_s-E)}{h^2}} \left[ 1 - \frac{E}{A_s-E} \tan^{-1} \left( \sqrt{\frac{A_s-E}{E}} \right) \right]
\]

B. I WKB approximation

Let us consider a dispersion curve of the kind \( (8) \). The turning points with a barrier like \( (19) \) are the same as \( (22) \), but now, under the barrier, the imaginary part of the wavevector as a function of the energy is given by

\[
I[k_s, E] = \frac{M_s}{h^2 e_s} \sqrt{a^2 - (b - e^{-2})^2}
\]

with \( a = \frac{e_s}{A_s}; b = a + \frac{E}{A_s} \).

The integration \( (20) \), for \( E < A_s \), leads to the WKB tunneling probability

\[
\ln(T(E)) = \frac{2A_s \lambda \sqrt{M_s}}{h^2 e_s} \left[ -b \left( \frac{\pi}{2} - \arctan \frac{b-1}{R_1} \right) - R_1 + \right.
\]

\[
\left. + R_2 \left( \pi - \arctan \frac{R_1 R_2}{a^2 - b^2} \right) \right]
\]

where we introduced the abbreviations

\[
R_1 = \sqrt{a^2 - (b-1)^2}; \quad R_2 = \sqrt{b^2 - a^2}.
\]

C. Full-band WKB approximation

For an armchair GNR, subband dispersion curves are in the form \( (1) \), from which we can express the wavevector as a function of the energy as

\[
k = \frac{2}{a \sqrt{3}} \arccos x,
\]

with the substitution \( u = -z/\lambda \) and normalizing all quantities to \( A_s \) given by

\[
x = \left( \frac{x_s + \delta - e^{-2}}{\nu} \right)
\]

where we introduced

\[
\delta = \frac{\Delta}{A_s}; \quad \alpha = \frac{1 + 4A_s V^2}{A_s^2}; \quad \nu = \frac{4A_s v^2}{A_s}
\]

In the integration domain of \( (20) \), the argument \( x \) of the inverse cosine function has module larger than 1, and therefore

\[
I(k) = \frac{2}{a \sqrt{3}} \ln \left[ x + \sqrt{x^2 + 1} \right],
\]

leading to the WKB tunneling probability

\[
\ln(T(E)) = -\frac{4\lambda}{a \sqrt{3}} \int_{0}^{\ln(E/A_s)} \ln \left[ x + \sqrt{x^2 + 1} \right] du.
\]

In Fig. 4 we compare the tunneling coefficients, calculated with the EM, I and FB WKB approaches, for a SBs of
height 0.5 and 1 eV, and for a \( \lambda \) typical of DG A-GNR with \( t_{\alpha s} = 1.5 \) and 4 nm. Essentially the intermediate approximation completely reproduces the FB tunneling probability, while a significant deviation is observed with the EM-WKB approximation for \( E < 0.5 A \). Therefore the intermediate approximation seems an optimal approximation for compact models in order to reduce the computational times retaining high accuracy.

IV. SCHOTTKY BARRIER BALLISTIC FET

We consider here a ballistic transistor with Schottky barrier contacts at source and drain, as shown in Fig. 5. As usual in compact models, we assume a complete phase randomization along the channel, neglecting phase resonances in the transmission probability of the two tunneling barriers, while multiple reflection events are taken into account. Between two tunneling barriers, the forward and backward distribution functions are modified by the multiple elastic scattering \([30]\), expressed as

\[
\eta^g_{\alpha, s(d)} = \frac{E - q\phi_s + \mu_{s(d)}}{k_b T},
\]

(30)

and

\[
\eta^b_{\alpha, s(d)} = \frac{\mu_{s(d)} - E + q\phi_d - \mu_{d}}{k_b T},
\]

(31)

and

\[
T^+ = T_s + T_d - T_s T_d,
\]

(32)

where \( T_s, T_d \) are the tunneling coefficients at source and drain, depending on both energy and channel potential. In order to compute the channel potential \( \phi_c \), and, through it, the subband energies, the total mobile charge \( Q = Q_h - Q_s \) must be equal to the charge induced by the electrostatic coupling of channel with gate, source and drain through the capacitances \( C_g, C_s, C_d \) respectively:

\[
Q(\phi_c) = - \sum_{i=1}^{n_{s,d}} C_i (V_i - V_{FB,i} - \phi_c),
\]

(33)

where \( V_{FB,i} = \phi_i - \chi_s \) is the flat band voltage, given by the difference between the contact workfunction and the graphene electron affinity.

The current is obtained with the Landauer-Büttiker formalism, which, accounting for the tunneling, takes the following form

\[
I_i(\phi_c) = \frac{q}{\pi \hbar} \sum_{\alpha} \int_0^{E^i_{\alpha,s}} \frac{T_s T_d}{T^+} f(\eta^i_{\alpha,s}) - f(\eta^i_{\alpha,d}) dE.
\]

(34)

with \( i = e, h \) accounting for the current of electrons andholes, and the total current given by \( I = I_e - I_h \). We note that (29) and (34) include both tunneling and thermionic contributions.

We apply our model to the case of a double gate armchair graphene nanoribbon transistor (DG A-GNR FET) with both Ohmic and Schottky barrier contacts. In Fig 6 we compare the transfer-characteristics (a) and the output characteristics (b) of a ballistic armchair GNR FET, with Ohmic and SB contacts of height \( E_g/2 \approx 0.3 \) eV are shown. Assuming a GNR thickness of about 1 nm we obtain \( \lambda = 1.3 \) nm.

V. SB TRANSISTORS IN INTERMEDIATE TRANSPORT REGIME

To describe dissipative transport, we follow the approach developed in [22], [23] for a 2D MOSFET for the non-degenerate and degenerate cases, and in [24] for quasi-1D
we have (the current) addressed as the the numerical solution of the complete chain of fictitious virtual probes, can be treated as ohmic transistors, and (33) we can fix the channel potential

\[ \phi = \mu \frac{q}{V} \]

...addressed as the

\[ \eta^o = (q_\phi - q V - \epsilon_o)/kT, \]

\[ \eta^h = (-q_\phi + q V + \epsilon_o)/kT. \]

We note that \( \eta \), not only directly depends on \( V \), but also indirectly through \( \phi \), which is self-consistently imposed by the linearized vertical electrostatics

\[ Q_s = C_g(V_g - V_{FB} - \phi), \]

\[ Q_m = -qV_{g/3}[F_{1/2}(\eta^o) - F_{1/2}(\eta^h)]. \]

The linearized DD model (35) has also the advantage of dealing with non integer \( N = L/\ell \) and is therefore more flexible than the ballistic chain itself. As noted in [24], (35) can be rearranged in a local form, analogous to a DD equation \( I_s = \mu_s Q_s \frac{dV}{dx} \), where the degenerate mobility (we consider now a monopolar regime) is given by

\[ \mu_s = \frac{qV_s \ell}{2kT F_{1/2}(\eta^o) - F_{1/2}(\eta^h)}, \]

with \( V_s = \sqrt{\frac{2kT m_e}{e}} \) the mean carrier velocity. This expression gives us a link between \( N = L/\ell \) and the mobility.

We can now model a SB transistor in intermediate transport regime as a series of \( B_s-DDB(N)-B_d \) segments, with two nodes between the boundary channels and the internal segment, characterized by electrochemical potentials that can be fixed exploiting the current continuity in the device. We will refer to this macro-model as the \( BDDDB(N) \) model. This compact model permits to analyze both the presence of Schottky barrier contacts and far-from-equilibrium transport condition, while keeping low the computational burden, especially with respect to numerical simulations including dissipation.

We now analyze the effects of inelastic scattering on the performance of a DG A-GNR FET. In non-ballistic transport (increasing \( N \)) the transfer characteristics (Fig 8) vertically shift, in a semilog plot, as expected due to the mobility reduction. It is interesting to note that the effect is more marked in the subthreshold region and, consequently, an increase of the \( I_m/I_{off} \) ratio as a function of \( N \) is observed, as shown in the inset. In ballistic models with positive \( V_d \), in subthreshold regime, tunneling from the drain leads to hole accumulation under the channel, which increases the quantum capacitance and reduces the control over channel. Subsequently a larger subthreshold swing and a lower \( I_m/I_{off} \) is obtained.

An accurate analysis of the SB effects on output characteristics can be performed calculating the differential conductance \( g = \partial I_d/\partial V_{ds} \). In fig 9 we compare the output characteristics and the differential conductance for a device with \( t_{ox} = 5 \text{ nm} \), with a SB height \( SB = 0, 0.25, 0.5 E_g \). Note that the presence of Schottky barrier contacts is more relevant in transistors with

\[ I_d = \frac{q^2\Gamma(1/\ell)}{\pi\hbar L} \sum_{\alpha} \int_{V_s} \{F_{-1}(\eta^o) - F_{-1}(\eta^h)\} dV, \]

where \( F_{-1}(x) \) is the Fermi-Dirac integral of order \(-1\), \( \Gamma \) the gamma function and

\[ \eta^o = (q_\phi - q V - \epsilon_o)/kT, \]

\[ \eta^h = (-q_\phi + q V + \epsilon_o)/kT. \]
a looser vertical confinement, where the tunneling barriers are thicker. We observe that in samples with SB= 0 eV the output characteristics concavity is always negative, and the differential conductance is monotonously decreasing with $V_{ds}$. If the SB height is finite the differential conductance acquires a non-monotonous behavior, which well describes the “S” shaped concavity change of the characteristics curves before reaching saturation, especially evident in thicker SB devices. It is interesting to note as, apart from a reduction of the maximum saturation current, larger ballistic chains (larger $N$), in which a higher inelastic scattering is active, lead to a smoothening of the non-monotonous dependence of $g$ on $V_{ds}$. In this fact we can recognize a gradual transition between devices in which the characteristics are dominated by SB contacts and devices in which inelastic relaxation is predominant.

In electron-hole symmetrical materials as undoped graphene nanoribbons or carbon nanotubes, the relative SB height with respect to the bandgap determines the position of the minimum of transfer characteristics, it influences their shape and their symmetry (see Fig.10). A SB of height $E_g/2$ preserves the bandstructure electron-hole symmetry and therefore results in transfer characteristics which span symmetrically from the current minimum off state (placed at $V_g = V_{ds}/2$). Curves calculated reducing the SB height for electrons (for $E_g/4$ and 0) show a growing asymmetry, with weaker hole currents and larger electron currents, together with a shift of the transfer characteristic minimum to lower values of $V_g$. This phenomenon is prominent in thicker SB devices such as the $t_{ox} = 5$ nm FET, but well observable also in a $t_{ox} = 1$ nm device. The increase of the lateral confinement leads in fact to an almost linear increase of the SB thickness and therefore all tunneling processes become harder. As expected, if we increase the dissipative phenomena (increasing $N$) a reduction of the current is observed. But more interesting, while the SB= $E_g/2$ curves vertically shift along the segmented line, the shift of the other curves is diagonal, note in fact the horizontal shift of their minima with $N$. Moreover, increasing $N$, the minima seem to converge towards the value $V_g = V_{ds}/2$, typical of a symmetrical ambipolar device. This is yet another signature of the growing importance of inelastic transport over the SB contacts. Therefore, for sufficiently well-confined FET, we can expect in quasi-ballistic GNT/CNT devices to clearly observe a SB behavior, which become more and more subtle in dissipative regimes. To quantify the relative importance of the Schottky barrier in determining the symmetry of the transfer characteristics we made the following physical estimation: SB= $0.5E_g$ corresponds to the symmetrical case, therefore if we impose a different SB the change in the conductance will be exponential in the SB difference $\delta E_{SB}$ as

$$\delta g_{SB} \propto \exp \left( -\frac{2t_{ox}}{\pi \hbar} \left( 2m_0E_{SB} \right)^{1/2} \right)$$

as can be obtained estimating the differential conductance of
a device with a SB source contact at the source Fermi level. This quantity is in fact dominated by the tunneling coefficient \( \gamma \). This difference in the conductance is relevant as long as it is greater than the conductance due to the DD(N) chain. We obtain

\[
\gamma = \frac{\delta g_{\text{SB}}}{g_N} = N \exp \left\{ -t_{\text{ox}} \times \frac{2}{\pi \hbar} (2m\delta E_{\text{SB}})^{1/2} \right\}
\]

(40)

Employing this formula we can calculate the \( N = N_I \) corresponding to \( \gamma = 1 \) for different SB value and oxide thickness, as shown in the following table

| SB = 0 | \( t_{\text{ox}} = 4 \) | \( t_{\text{ox}} = 7 \) | \( t_{\text{ox}} = 3 \) | \( t_{\text{ox}} = 5 \) (nm) |
|--------|-----------------|-----------------|-----------------|-----------------|
| 4 70 10 | 7 450 10 \( ^3 \) | 10 \( ^4 \) |

\( N_I \) gives a rough estimation to the number of nodes (i.e. \( L/\ell \) ratio) needed to make the transfer characteristics symmetrical, in spite of the presence of a SB. As can be observed comparing these values with the behavior of curves in Fig 10 the \( t_{\text{ox}} = 1 \) nm curves with SB= 5 and 10, respectively for SB= 0.25\( E_g \) and 0, are quite symmetrical in accordance with \( N_I = 4 \) and \( N_I = 7 \) found by our calculation. The minimum of the curve \( N = 10 \) with SB= 0.25\( E_g \) comes near to the symmetrical values, but still misses it being our estimation \( N_I = 70 \). Other curves are highly asymmetric being \( N \ll N_I \).

A typical parameter used to characterize the transport regime in quasi-ballistic devices is the ballisticity index \( B_{\text{index}} = 1/\ell/I \), which is the ratio of the actual current to the current corresponding to an analogous device in a purely ballistic transport regime (\( N = 1 \)). In Fig 11 we analyze the role of the SB contacts in determining the ballisticity index as a function of \( N \), and therefore as a function of the degree of inelastic relaxation. In general to lower SB heights correspond a faster variation of the \( B_{\text{index}} \) with \( N \), with a sudden drop of the ballisticity as function of the number of nodes, after which a slower decrease is observed. SBs affect in particular the ballisticity index calculated for lower \( V_{ds} \), due to the concavity of the output characteristics, while larger source-drain voltages reduce the relative importance of SB with respect to inelastic mechanisms. Calculations with \( t_{\text{ox}} = 5 \) nm reveal the increased importance of SB contacts, and reflect the presence of the injection in the output characteristics, with a concavity change before saturation. In particular, for higher value of the SB we observe a slower dependence of the \( B_{\text{index}} \) on \( N \), because the current is calculated in a bias point of the characteristic curve of strong “s” curvature. Physically, it means that the current flowing in the device is mostly limited by the injection through the tunneling barriers.

VI. CONCLUSIONS

We have presented a semi-analytical model dealing with ambipolar one-dimensional Schottky barrier transistors in intermediate transport regimes between fully ballistic and quasi equilibrium, i.e. governed by the drift-diffusion model. We have introduced simplified, but accurate, descriptions of the Schottky barrier profiles and of the electrostatics, and analytical approximations of the tunneling coefficients of the Schottky barriers. We demonstrate that a Schottky barrier transistor can be modeled as three transistor is series, with common gate voltage. The central one is a drift-diffusion transistor, with mobility dependent on the degree of degeneracy of the one-dimensional carrier gas. The other two transistors are ballistic FETs with a Schottky barrier contact corresponding to the external actual contacts (source or drain). In the case of ballistic transport, our model allows us to reproduce the results of a 3D numerical Poisson-Schroedinger simulator. In the case of very long channel, with respect to the mean free path, current is limited by the central drift-diffusion transistor. The model allows very directly to investigate the transition from barrier-limited transport to channel-limited transport. Our semi-analytical model represents an accurate and simple way to gain physical insights into the behavior of nanoscale transistors with Schottky barrier contacts, including most the relevant physics at a very low computational cost.

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