Coordinate-space holographic projection of fields 
and an application to massive vector fields

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Abstract

General properties of coordinate-space holographic projections of fields in AdS/CFT correspondence, which respect the Ward identity, are investigated. To show the usefulness of this methodology it is applied to the computation of correlators of massive gauge fields.

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1 Introduction

The fascinating proposal of Maldacena[1] that large $N$ limit of a certain conformal field theory (CFT) in a $d$-dimensional space can be considered as a boundary theory of $d + 1$-dimensional anti-de Sitter space (AdS) with a compact extra space opened a novel way of understanding various facets of supersymmetric gauge theories. The baryonic vertex, that is the finite energy configurations with $N$ external quarks, was shown to exist in this context[2]. Furthermore it was expected that the quark-confining potential of QCD could be investigated in this way[3]. This AdS/CFT correspondence was also used to the topic of black hole entropy[4].

The underlining principle behind this AdS/CFT correspondence was surveyed by Gubser et al.[5] and Witten[6]. The new insight was that the symmetry group $SO(2, d)$ which acts on $AdS_{d+1}$ acts conformally in the boundary of $AdS_{d+1}$ which is in fact the Minkowski space. The holographic properties of the Maldacena’s proposal were investigated, and various correlation functions of conformal fields were obtained. The calculations produced the correct conformal dimensions. Motivated by this many authors walked along the path of that understanding[7, 8, 9, 10]. Since gauge fields can obtain masses from the excitation of the extra compact manifold of the AdS/CFT correspondence it is quite natural to investigate the holographic projection of massive vector fields. In fact Mück and Viswanathan challenged this problem[10] with deep understanding.

But to get the correlators which preserve the Ward identity one usually works on the momentum-space with given specific solutions of classical equations of motion. It is sometimes quite cumbersome and makes it difficult to get insights on what is going on in the coordinate-space. One of the purpose of this paper is to examine the holographic projections of general fields in AdS/CFT correspondence in the coordinate-space. To preserve the necessary Ward identity we are careful to approach the boundary of AdS uniformly for all fields. This result is applied to massive vector fields in a different way than [10]. The final result of cause agrees with the known one.
The next section is devoted to investigate the general properties of coordinate-space holographic projection of fields. The final section is an application.

2 Holographic projections of fields in coordinate-space

To depict the essential point behind current paper more clearly, we briefly introduce the idea of the holographic projection of Witten[6]. It is known that the boundary of $AdS_{d+1}$ is the compactified version of Minkowski space $M^*_d$. For simplicity we assume the Euclidean field theory. There are many equivalent choices of the metrics of $AdS_{d+1}$. In this paper we represent $AdS_{d+1}$ as the Lobachevsky space $R^{1+d}_+ = \{(x_0, \mathbf{x}) \in R^{1+d} | x_0 > 0\}$ with the metric

$$ds^2 = \frac{1}{x_0} dx^\mu dx^\mu.$$  (1)

In this representation distances between points in the $x_0 \to \infty$ region vanish. This means that the boundary of $AdS_{d+1}$ is in fact $S^d$, the $d-$dimensional sphere. On the other hand the metric (1) diverges as $x_0 \to 0$, which also means that every point in the holographically projected space $M^*_d$ is infinitely separated. To resolve this infinite scale problem one needs a rescaling function $\mu(x_0)$ which necessarily diverges as $x_0 \to 0$. Using this, distances $\ell_{AdS}$ in $AdS_{d+1}$ can be holographically projected to

$$\ell_{M^*} = \lim_{x_0 \to 0} \frac{\ell_{AdS}}{\mu},$$  (2)

giving finite results. But this rescaling function $\mu(x_0)$ is not unique. When one rescale the rescaling function

$$\mu(x_0) \to e^{-\sigma(x_0)} \mu(x_0),$$  (3)

the scale in $M^*_d$ changes as

$$\ell_{M^*} \to e^\sigma \ell_{M^*},$$  (4)

showing that $M^*_d$ has possibly some conformal structure.

Now consider a dynamical field $\phi$. Asymptotically this field either vanishes or diverges on the boundary. In usual field theory the divergent component is discarded. But in
AdS/CFT correspondence one may rescale the divergent component \( \phi_{AdS} \) with a proper \( \lambda \) by
\[
\phi_{M^*} = \lim_{x_0 \to 0} \frac{\phi_{AdS}}{\mu^\lambda}. \tag{5}
\]
Under the change (3) of rescaling function the field \( \phi_{M^*} \) transforms as
\[
\phi_{M^*} \to e^{\lambda \sigma} \phi_{M^*}. \tag{6}
\]
This shows that \( \phi_{M^*} \) is a conformal density of the length dimension \( \lambda \).

But in practice one should be careful to approach fields \( \phi(x_0, \mathbf{x}) \) and \( \partial_0 \phi(x_0, \mathbf{x}) \) to the \( x_0 \to 0 \) boundary at the same rate. If it is not true, the Ward identity will be destroyed\[\footnote{The details can be found in \cite{footnote}}\]. For this one first approach the \( x_0 \to \epsilon \) boundary instead of the real one. After the computation of physical quantities in this \( \epsilon \)-boundary, the \( \epsilon \to 0 \) limit is taken to get the desired results. This process is usually done in the momentum-space, and it is quite cumbersome. We consider this process in the coordinate space which would be more intuitive and practical.

Consider an action which is a functional of both \( \phi(x_0, \mathbf{x}) \) and \( \partial_\mu \phi(x_0, \mathbf{x}) \). As long as there is no possibility of confusion we suppress the subscripts such as \( AdS \) and \( M^* \). For the holographic projection of fields we need to consider \( \phi(x_0, \mathbf{x}) \) and \( \partial_0 \phi(x_0, \mathbf{x}) \). First we define \( \epsilon \)-boundary field \( \phi_\epsilon(\mathbf{x}) \) by
\[
\phi_\epsilon(\mathbf{x}) = \phi(\epsilon, \mathbf{x}). \tag{7}
\]
Then this boundary field is related to the bulk field \( \phi(x_0, \mathbf{x}) \) in the following way
\[
\phi(x_0, \mathbf{x}) = \int d^d x' \ G_\epsilon(x_0, \mathbf{x} - \mathbf{x}') \phi_\epsilon(\mathbf{x'}). \tag{8}
\]
Here the boundary-to-bulk Green’s function \( G_\epsilon(x_0, \mathbf{x} - \mathbf{x}') \) has following property
\[
\lim_{x_0 \to \epsilon} G_\epsilon(x_0, \mathbf{x} - \mathbf{x}') = \delta^{(d)}(\mathbf{x} - \mathbf{x'}). \tag{9}
\]
Since this \( \epsilon \)-limit procedure is independent of \( \mathbf{x} \), we supress it for notational convenience. It makes us not to worry about the integration with respect to \( \mathbf{x} \).
Differentiating (8) with respect to $x_0$ and using the above mentioned convention we have

$$\partial_0 \phi(x_0) = \partial_0 G_\epsilon(x_0) \phi_\epsilon.$$  \hspace{1cm} (10)

Now suppose that $\phi(x_0)$ diverges as $x_0^{-\lambda}$ as $x_0 \to 0$. In this case the holographically projected field $\phi_h(\epsilon)$ to the $\epsilon$-boundary is defined by

$$\phi_h(\epsilon) = \epsilon^\lambda \phi_\epsilon = \phi_h(0) + O(\epsilon),$$  \hspace{1cm} (11)

where $O(\epsilon)$ is a function of $\epsilon$ which vanishes as $\epsilon \to 0$. For this field we have

$$G_\epsilon(x_0) \phi_h(\epsilon) = \left( \frac{\epsilon}{x_0} \right)^\lambda \phi_h(x_0).$$  \hspace{1cm} (12)

Plugging this into (10) we are able to write $\partial_0 \phi(x_0)$ at the $\epsilon$-boundary in terms of the $\epsilon$-boundary field $\phi_\epsilon$ in the following way

$$\partial_0 \phi(x_0) \bigg|_{x_0=\epsilon} = \left( \frac{\partial_\epsilon \phi_h(\epsilon)}{\phi_h(\epsilon)} - \frac{\lambda}{\epsilon} \right) \phi_\epsilon.$$  \hspace{1cm} (13)

To compare the degree of divergence between two terms in the parenthesis we use (11),

$$\partial_0 \phi(x_0) \bigg|_{x_0=\epsilon} = \left\{ \frac{\partial_\epsilon \phi_h(\epsilon)}{\phi_h(\epsilon)} \left( 1 - \frac{O(\epsilon)}{\phi_h(0)} + \ldots \right) - \frac{\lambda}{\epsilon} \right\} \phi_\epsilon.$$  \hspace{1cm} (14)

This shows that the last term, when compared to the others, is a divergent one which is related to the delta-function contact term in the computation of correlators. So we drop it. This process of regularization of the infinity is the coordinate-space version of the one discussed in the appendix of Ref.[7].

The final result of the holographic projection of $\partial_0 \phi(x_0, x)$ is

$$\partial_0 \phi(x_0, x) \bigg|_{x_0=\epsilon} \to \epsilon^{-\lambda} \partial_\epsilon \phi_h(\epsilon, x).$$  \hspace{1cm} (15)

It can be compared to the holographic projection of $\phi(x_0, x)$,

$$\phi(x_0, x) \bigg|_{x_0=\epsilon} \to \epsilon^{-\lambda} \phi_h(\epsilon, x).$$  \hspace{1cm} (16)
This method of holographic projection of fields in AdS/CFT correspondence is more intuitive and practical than the one which is done in the momentum-space. As an example, the holographic projection of \( \phi(x_0, x) \partial_0 \phi(x_0, x) \) can be written as

\[
\phi(x_0, x) \partial_0 \phi(x_0, x)|_{x_0=\epsilon} \to \epsilon^{-2\lambda} \frac{1}{2} \phi_0(\epsilon, x)^2.
\] (17)

We apply this idea to massive vector fields in the next section.

3 Holographic projection of massive gauge fields

According to Maldacena the generating functional \( \langle \exp \int d^d x A_i(x) J_i(x) \rangle_{CFT} \) of the correlation functions \( \langle J_{i_1}(x_1) J_{i_2}(x_2) \cdots J_{i_n}(x_n) \rangle \) can be constructed from the following proposal

\[
\langle \exp \int d^d x A_i(x) J_i(x) \rangle_{CFT} = Z_{AdS_{d+1}}[A_i],
\] (18)

where \( Z_{AdS_{d+1}}[A_i] \) is a supergravity partition function on \( AdS_{d+1} \) computed with the condition that \( A_i \) projects to \( A_i \) at the boundary of \( AdS_{d+1} \) which is in fact the compactified \( d \)--dimensional Minkowski space. When ones use the classical approximation this partition function can be written as

\[
Z_{AdS_{d+1}}[A_i] \simeq \exp(-I[A_i]),
\] (19)

where \( I[A_i] \) is the projected classical action. We start from this classical approximation to compute the two-point correlation function of the conformal current \( J_i \) for massive vector fields.

The action for the massive gauge field in \( AdS_{d+1} \) space is

\[
I = \int d^{d+1}x \sqrt{g} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_i A^i \right),
\] (20)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the usual field strength tensor of a vector field \( A_\mu \). The Euler-Lagrange equation of motion which is derived from this action is

\[
\nabla^\mu F_{\mu\nu} - m^2 A_\nu = 0.
\] (21)
To get the holographic projection of the action we use the classical equation of motion, producing the following action

\[ I = \lim_{\epsilon \to 0} \frac{1}{2} \int_{x_0=\epsilon} d^d x \sqrt{g} A_\mu F^{\mu 0}. \]  

(22)

As it is pointed out by Mück and Viswanathan[10] it is useful to define fields with Lorentz indices by

\[ A_a = e^\mu_a A_\mu, \]  

(23)

where \( e^\mu_a = x_0 \delta^\mu_a, (a = 0, 1 \ldots d) \) is the vielbein of \( AdS_{d+1} \). Using \( A_0(x_0, x) \) and \( A_i(x_0, x) \) the classical action is

\[ I = \lim_{\epsilon \to 0} \frac{1}{2} \int d^d x \epsilon^{-d} \left[ -\frac{\epsilon}{2} \partial_0 A_i^2 + A_i^2 + \epsilon A_i \partial_i A_0 \right]. \]  

(24)

To express this action in terms of the holographically projected fields we need to solve the equation of motion (21). It is not difficult to show that for \( A_0(x_0, x) \) it is

\[ \left[ x_0^2 \partial_\mu \partial_\mu + (1 - d) x_0 \partial_0 - (m^2 - d + 1) \right] A_0 = 0. \]  

(25)

As \( x_0 \to 0 \), \( A_0 \) behaves as \( x_0^{-\lambda} \), where \( \lambda \) is the larger root of the following quadratic equation,

\[ \lambda(\lambda + d) = m^2 - d + 1. \]  

(26)

The solution of Eq.(25) is well known from the scalar field theory[8, 9, 11], and is given by

\[ A_0(x_0, x) = x_0^{d/2} \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot x} a_0(k) K_\nu(|k| x_0), \]  

(27)

where

\[ \nu = \lambda + \frac{d}{2}, \]  

(28)

and \( K_\nu \) is the modified Bessel function of the third kind which satisfies

\[ \left[ \xi^2 \frac{d^2}{d \xi^2} + \xi \frac{d}{d \xi} - (\nu^2 + \xi^2) \right] K_\nu(\xi) = 0. \]  

(29)

The equation of motion (21) for \( A_i, i = 1 \ldots d \), reduces to

\[ \left[ x_0^2 \partial_\mu \partial_\mu + (1 - d) x_0 \partial_0 - (m^2 - d + 1) \right] A_i = 2 x_0 \partial_i A_0. \]  

(30)
The exact solution of this equation is considered in [10]. But since it is sufficient to know the \( x_0 \to 0 \) behaviour of the solution, we follow a different path. First we decompose \( A_i(x_0, \mathbf{x}) \) into

\[
A_i(x_0, \mathbf{x}) = A_i^{(g)}(x_0, \mathbf{x}) + A_i^{(p)}(x_0, \mathbf{x}),
\]

(31)

where

\[
A_i^{(g)}(x_0, \mathbf{x}) = x_0^{d/2} \int \frac{d^dk}{(2\pi)^d} e^{-i\mathbf{k} \cdot \mathbf{x}} a_i(k) K_\nu(\sqrt{\mathbf{k}^2} x_0) \quad (32)
\]

is the general solution of (30) when the right hand side vanishes, and \( A_i^{(p)}(x_0, \mathbf{x}) \) is the particular solution which is assumed to have following form

\[
A_i^{(p)}(x_0, \mathbf{x}) = -2ix_0^{d/2} \int \frac{d^dk}{(2\pi)^d} e^{-i\mathbf{k} \cdot \mathbf{x}} a_0(k) \frac{k_i}{|\mathbf{k}|^2} H(|\mathbf{k}| x_0). \quad (33)
\]

Here \( H(|\mathbf{k}| x_0) \) is an unknown function which must be determined. Both \( a_i(k) \) and \( a_0(k) \) are independent unknown coefficients which are related to the projected fields \( A_i(\mathbf{x}) \). For simplicity we denote \( \xi = |\mathbf{k}| x_0 \). From (33) it is easy to show that \( H(\xi) \) satisfies

\[
[\xi^2 \frac{d^2}{d\xi^2} + \xi \frac{d}{d\xi} - (\nu^2 + \xi^2)] H = \xi^2 K_\nu. \quad (34)
\]

This ordinary differential equation can be solved by making use of the the consistency condition \( \nabla^\mu A_\mu = 0 \) of the massive gauge field,

\[
\partial_i A_i + \partial_0 A_0 - \frac{d}{x_0} A_0 = 0. \quad (35)
\]

For further simplification we separate this into the following two relations,

\[
\partial_i A_i^{(g)} = 0, \quad (36)
\]

\[
\partial_i A_i^{(p)} + \partial_0 A_0 - \frac{d}{x_0} A_0 = 0. \quad (37)
\]

The first constraint (36) can easily be solved producing

\[
k_i a_i(k) = 0. \quad (38)
\]

When one substitutes (27) and (33) into (37) one determines \( H(|\mathbf{k}| x_0) \) in the following way

\[
H(\xi) = \frac{1}{2} [\xi \frac{dK_\nu}{d\xi} + (1 - \frac{d}{2}) K_\nu]. \quad (39)
\]
Using the expansion
\[ K_\nu(\xi) = \frac{1}{2} \left[ \Gamma(\nu) \left( \frac{\xi}{2} \right)^{\nu} + \Gamma(-\nu) \left( \frac{\xi}{2} \right)^{-\nu} + \ldots \right], \] (40)
we have
\[ A_{i}^{(p)}(x_0, x) = -2i x_0 \frac{d^2}{d^2 x} \int \frac{d^d k}{(2\pi)^d} e^{-ik \cdot x} a_0(k) \left[ \frac{c}{2} \Gamma(\nu) \left( \frac{\xi}{2} \right)^{-\nu} + \bar{c} \frac{\xi}{2} \Gamma(-\nu) \left( \frac{\xi}{2} \right)^{\nu} + \ldots \right], \] (41)
where
\[ c = \frac{1}{2} (1 - \frac{d}{2} - \nu), \quad \bar{c} = \frac{1}{2} (1 - \frac{d}{2} + \nu). \] (42)

On the other hand the series expansion of \( A_{i}^{(g)}(x_0, x) \) is
\[ A_{i}^{(g)}(x_0, x) = x_0 \frac{d^2}{d^2 x} \int \frac{d^d k}{(2\pi)^d} e^{-ik \cdot x} a_i(k) \left[ \frac{c}{2} \Gamma(\nu) \left( \frac{\xi}{2} \right)^{\nu} + \bar{c} \frac{\xi}{2} \Gamma(-\nu) \left( \frac{\xi}{2} \right)^{-\nu} + \ldots \right]. \] (43)

Combining \( A_{i}^{(g)} \) and \( A_{i}^{(p)} \) we have
\[ A_i(x_0, x) = x_0 \frac{d^2}{d^2 x} \int \frac{d^d k}{(2\pi)^d} e^{-ik \cdot x} \left[ \frac{c}{2} \Gamma(\nu) \left( \frac{\xi}{2} \right)^{\nu} \left( \frac{1}{2} a_i(k) - i \frac{k_i}{|k|^2} c a_0(k) \right) \right. \]
\[ + \Gamma(-\nu) \left( \frac{\xi}{2} \right)^{-\nu} \left( \frac{1}{2} a_i(k) - i \frac{k_i}{|k|^2} \bar{c} a_0(k) \right) + \ldots \right]. \] (44)

Now rewrite the classical action (24) in terms of the following \( \epsilon \)-boundary field,
\[ A_{\epsilon, i}(x) = \epsilon^\lambda A_i(\epsilon, x) \] (45)
\[ = \int \frac{d^d k}{(2\pi)^d} e^{-ik \cdot x} \left[ \frac{c}{2} \Gamma(\nu) \left( \frac{\xi}{2} \right)^{\nu} \left( \frac{1}{2} a_i(k) - i \frac{k_i}{|k|^2} c a_0(k) \right) \right. \]
\[ + \Gamma(-\nu) \left( \frac{\xi}{2} \right)^{-\nu} \left( \frac{1}{2} a_i(k) - i \frac{k_i}{|k|^2} \bar{c} a_0(k) \right) \epsilon^{2\nu} + \ldots \right], \]
where \( \lambda \) is already defined to be \( \nu - \frac{d}{2} \). The last two terms of the action (24) diverges as \( \epsilon^{-2\nu} \) or \( \epsilon^{-2\nu+2} \) as \( \epsilon \to 0 \), and we ignore as usual. Using (17), the relevant part of the action is
\[ I = -\frac{1}{4} \lim_{\epsilon \to 0} \epsilon^{-2\nu} \int d^d x \epsilon \partial_\epsilon A_{\epsilon, i}(x)^2. \] (46)
Consider the $\epsilon \to 0$ limit of this action. To clarify the meaning of the coefficients $a_i(k)$ and $a_0(k)$ we consider the following Fourier transformation,

$$\lim_{\epsilon \to 0} A_{\epsilon,i}(x) = \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot x} \tilde{A}_i(k).$$ (47)

It is clear that the Fourier component is

$$\tilde{A}_i(k) = \Gamma(\nu) \left(\frac{|k|}{2}\right)^{-\nu} \left(\frac{1}{2} a_i(k) - \frac{k_i}{|k|^2} c a_0(k)\right).$$ (48)

To solve $a_i(k)$ and $a_0(k)$ in terms of $d-$dimensional field components $\tilde{A}_i(k)$ we use (38),

$$a_0(k) = \frac{i}{c \Gamma(\nu)} \left(\frac{|k|}{2}\right)^{\nu} k_i \tilde{A}_i(k),$$ (49)

$$a_i(k) = \frac{2}{\Gamma(\nu)} \left(\frac{|k|}{2}\right)^{\nu} \left(\delta_{ij} - \frac{k_i k_j}{|k|^2}\right) \tilde{A}_j(k).$$ (50)

Substituting these two relations into (24) we have

$$I = -\frac{\nu}{\Gamma(\nu)} \int \frac{d^d k}{(2\pi)^d} \tilde{A}_i(k) \left[ (\delta_{ij} - \frac{k_i k_j}{|k|^2}) + \frac{\tilde{c} k_i k_j}{c |k|^2} \right] \left(\frac{|k|}{2}\right)^{2\nu} \tilde{A}_j(-k).$$ (51)

From the specific forms (42) it can be shown that

$$\delta_{ij} - \frac{k_i k_j}{|k|^2} + \frac{\tilde{c} k_i k_j}{c |k|^2} = \delta_{ij} + \frac{2\nu}{1 - \frac{d}{2} - \nu} \frac{k_i k_j}{|k|^2}.$$ (52)

To get the final action we use (A.1-A.3) of the appendix,

$$I = -c_{\Delta,d} \int d^d x d^d x' A_i(x) \left(\frac{\delta_{ij}}{|x - x'|^{2\Delta}} - 2 \frac{(x - x')_i(x - x')_j}{|x - x'|^{2\Delta+2}}\right) A_j(x'),$$ (53)

where $\Delta = (\lambda + d)$. The proportional constant $c_{\Delta,d}$ which is related to the central charge[7,12] is given by

$$c_{\Delta,d} = \Delta(\Delta - \frac{d}{2}) \Gamma(\Delta - 1).$$ (54)

This result coincides exactly with that of [10]. The two point correlation function $\langle J_i(x)J_j(x') \rangle$ of the conformal current, which can be read off from the Maldacena’s proposal (18-19) is

$$\langle J_i(x)J_j(x') \rangle = c_{\Delta,d} \left(\frac{\delta_{ij}}{|x - x'|^{2\Delta}} - 2 \frac{(x - x')_i(x - x')_j}{|x - x'|^{2\Delta+2}}\right),$$ (55)
showing that $\Delta$ is the corresponding conformal dimension. One may check the resulting $\Delta$ with the known value for the special case of $m = 0$. In this case the larger solution of (26) is $\lambda = -1$. This means the conformal dimension $\Delta$ is equal to $d - 1$ which in fact agrees with the Witten’s result. [3]

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**Appendix**

It is known [13] that for non-integer $\delta,$

$$\int \frac{d^d k}{(2\pi)^d} e^{i k \cdot x} |k|^{\delta} = \frac{C_{d,\delta}}{|x|^{d+\delta}}, \quad (A.1)$$

where $C_{d,\delta}$ is a constant which is given by

$$C_{d,\delta} = \frac{2^\delta \Gamma(d+\delta)}{\pi^{d/2} \Gamma(\frac{\delta}{2})}. \quad (A.2)$$

Differentiating this with respect to $x^i$ and $x^j$ we have following relation,

$$\int \frac{d^d k}{(2\pi)^d} \frac{e^{i k \cdot x}}{|k|^2} |k|^{\delta} = -\frac{1}{\delta} \left\{ \delta_{ij} - (\delta + d) \frac{x_i x_j}{|x|^2} \right\} \frac{C_{d,\delta}}{|x|^{d+\delta}}. \quad (A.3)$$

One may check the validity of this equation by contracting $ij-$indices and by comparing with (A.1).

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