Estimation of Models of the Effect of Taxes on the Economic Growth of the Regions of Russia

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Abstract. Subject of article is the assessment of descriptive and optimization models of the effect of taxes on economic growth. Work purpose is based on the analysis of the basic models of economic growth (taking into account the tax factor) of foreign and domestic authors to identify the features, advantages and disadvantages of the latter on the basis of the approach considered taxes as a factor of economic growth. The hypothesis of the study is based on the assumption that the accumulation of funds to state is necessary in order to further redistribute them to achieve a high level of socio-economic development, which implies the use of simulation modeling, which is an experimental and applied methodology and allows describing the behavior of the corresponding systems, form a theory and test hypotheses that can explain this behavior, as well as about establish concrete recommendations for reforming the tax system with the aim of accelerating economic growth. Methodological foundations of a systems approach and economic and mathematical modeling are used: comparative method — in identifying features of economic growth models that take taxes into account; statistical analysis - in the study of dependencies between macroeconomic indicators; economic and mathematical modeling and mathematical statistics - in the construction of economic growth model, taking into account the tax factor. The proposed approach can be used by public institutions to assess the reform of the structure of the tax system, which will contribute to stable growth rates of production of goods and the provision of services.

1. Introduction
Aspect of how tax policy affects the redistribution of limited resources of an economy stems from the characteristics of national economic growth and requires further consideration using economic and mathematical models that take taxes into account as one of the main factors for increasing GDP.

Economic-mathematical models in economics are a formalized description of a control system using mathematical relationships. Their construction is carried out in order to simplify and speed up the study of the properties of the original.

In the general case, the original object is the economic system S, which functions in a certain time interval To. It has a certain set of parameters $S_o$, $S_o=\{S_{oi}\}$ and is characterized by certain properties. A quantitative measure of the system is the set of characteristics $Y_o$, $Y_o=\{y_{ok}\}$. The system manifests its properties in the process of external influence $X_o$, $X_o=\{x_{ok}\}$. The set of parameters $S_p$ and their values reflects the state of its internal environment (structure and principles of functioning), as well as external signs that are important in interactions with other control systems. Each characteristic of the $y_{ok}\subseteq Y_o$.
is in functional dependence on its parameters \( \{S_n\} \subseteq S_o \). The influence of other parameters is not taken into account, which is related to the purpose of the study. In other words, only some characteristics of the system \( y_{mn} \in Y_n \) with \( k \geq n \) are modeled under certain influences \( \{x_{mn}\} \subseteq X_m \) with \( k \geq n \) on system \( S \), for which the model is being built.

Model is also a system with its own set of parameters \( S_m \), \( S_m=\{S_{mj}\} \) and characteristics \( Y_m \), \( Y_m=\{y_{mn}\} \). The original and model are similar in one parameters and differ in others. But the replacement of one object by another is legitimate, only if the studied characteristics of the original and the model are determined by the same type of parameter sets and are connected by the same dependencies with these parameters:

\[
y_{a1}=f(\{S_{a1}\},\{x_{a1}\},T_0);
\]

\[
y_{aa}=f(\{S_{aa}\},\{x_{aa}\},T_m),
\]

where \( y_{mn} \) — is a characteristic of model, \( y_{mn} \in Y_m \); \( x_{mn} \) — external influence on model, \( x_{mn} \subseteq X_m \); \( T_m \) — model time.

At the same time \( S_o=\Psi(S_{mj}), x_{ok}=\Omega(x_{mn}), T_o=mT_m \), where \( m \) — scale factor over the entire interval \([0-Tm]\) or in separate periods of time. Then, with some approximation, we can conclude that the characteristics of \( y_{ok} \) are related to the characteristics of \( y_{mn} \) by the dependences of \( y_{ok}=\Phi(y_{mn}) \). The set of characteristics of the model \( Y_{mn}=\{y_{mn}\} \) is a reflection of the set of characteristics of the original system under study \( (Y_{a1}=\{y_{a1}\}, \text{t.e. } Y_{aa} \rightarrow Y_{ok}) \). The choice of the type of model is determined by the features of the system and the objectives of the simulation.

1.1. Scientific significance of the issue and problem statement

In conditions when the object of the study are macro-, meso- or microeconomic indicators, as a rule, descriptive and optimization economic-mathematical models are used.

Descriptive models are a tool for describing the behavior of an economic system in dynamics in order to establish trends in the development of its individual indicators under the influence of changes in specified parameters.

Optimization models are a tool for finding the optimal value of the objective function (maximum or minimum depending on the optimization criterion) by changing the model parameters in the given constraints.

Unlike optimization models, descriptive models provide an opportunity to describe the economic behavior of a system over time using various mathematical tools (differential and integral calculus, mathematical statistics, simulation modeling), observing the development of individual system characteristics under the influence of many internal and external influences. They are based on empirical dependencies and answer the question "how is the process going?":

\[
\left| y_{\text{fact}} - y_{\text{plan}} \right|^2 \rightarrow \min,
\]

where \( y_{\text{fact}} \) — the actual value of the studied indicator; \( y_{\text{plan}} \) — the resulting value of the indicator.

The decision to choose the optimal values of the indicators of this class of models is up to the user, not the developer.

2. The study of the features, advantages and disadvantages of descriptive models of the impact of taxes on economic growth

In order to clarify the features, advantages and disadvantages of descriptive models of economic growth, it is advisable to consider the basic models of foreign and domestic authors.

Under the influence of new directions in the study of economic growth, there are a number of ways and methods by which an attempt is made to assess the impact of taxation on GDP and GRP.

Thus, models built for the conditions of countries with developed economies [12], indicate that the average real tax rates on capital and labor for the EU countries were on average: for capital - 35%; labor - 30%, and their share in total tax revenues was 19.8 and 48.6%, respectively. At the same time, an increase in the average tax rate of about 10% (last 35 years) has an effect on reducing annual growth rates in EU countries by about 0.5% [12]. In addition, reducing tax pressure to 10% of GDP in...
the face of declining transfers can increase annual growth rates from 0.5 to 1% [12]. But such models are built mainly only using a regression analysis or another econometric method.

Similar to statistical studies of the state of the economy of Russia and its regions, they do not cover the complex structural interrelations of possible economic processes in the long-term period of development. Therefore, these methods cannot be used in isolation from other approaches.

Since Russia and its regions have a certain specificity, Western models for describing the market economy of a country are not considered, but descriptive models are being studied, the construction of which is carried out taking into account the features of the market coordination mechanism. They are used to analyze the dependence of tax revenues and production on the tax burden and time [13] the distribution of the tax burden across sectors of the economy, taking into account the impact of taxes on the production of goods and services [18], ensuring economic growth by changing tax rates, and also quantitative and qualitative estimates of the Laffer curve for the conditions of the Russian economy [8; 19].

In [5] a model was proposed that included tax, investment and technological factors and has the following features. The criterion of development is the ability to accumulate fixed capital. The economy is divided into two sectors - with high and low capital intensity. The model allows analyzing the impact of the total tax burden on economic growth and the state of the budget. The formula for production growth is as follows:

$$ Y_t = Y_0 \left( \frac{1 - \varepsilon}{1 - \frac{m}{k} \left( 1 - \theta - \sigma \right)} \right)^t, $$

where $Y$ – the volume of production of the company; $\varepsilon$ – the coefficient of annual disposal of fixed assets; $m$ – the average annual ability of the company to invest in fixed capital; $k$ – capital intensity ratio, which characterizes the efficiency of use of fixed assets; $\sigma$ – the share of wages in value added; $\theta$ is the share of taxes in value added.

Testing of this model has shown that any increase in the tax burden leads to a decrease in economic growth rates.

This fact is quite natural, given that "any tax exemptions reduce the reproduction capacity of the company" [5].

The author concluded that the traditional Laffer curve does not exist, including for the tax burden. The analysis of fiscal effects over time leads to a different idea of the shape of the fiscal curve: “First of all, it is given in the coordinates “tax revenues - time”, and not as usual “tax revenues - the level of aggregate tax burden”. Accordingly, the inflection of this curve occurs under the influence of the time factor. However, the height of the curve and the time it takes to form it depend on the level of the total tax burden. Thus, a family of convex fiscal curves has been formed on the graph.

The analysis of the impact of the main taxes (on the profit of organizations, on value added, on property, and on payroll) led to a number of conclusions:

1. Even the most insignificant stimulation of production activity is associated with very strong fiscal concessions, while tax instruments are an ineffective means of regulating the production climate.
2. Tax benefits themselves are very weak in stimulating economic growth, i.e. business activity is more dependent on other factors: investment and technological efficiency.
3. No reduction in tax rates can provide favorable conditions for the development of production in a transformational environment. Without a change in the mechanism for paying taxes, any manipulation of the level of tax rates will be ineffective.

The paper [5] presents a descriptive model of the formation of budget revenues in the context of inflation, taking into account the production structure of expenditures, which looks as follows:

$$ \frac{dT}{dP} = \frac{T}{P} \left[ \frac{(\theta - \omega \zeta)(\beta + \alpha - \beta \alpha + \lambda \sigma (\gamma - \alpha - \gamma \alpha) - \mu \alpha \varepsilon)}{(1 - \omega)(\beta + \alpha - \beta \alpha + \sigma (\gamma - \alpha - \gamma \alpha) - \mu \alpha)} \right], $$

where $T$ – total tax revenue; $P$ – price level; $\beta, \alpha, \gamma$ - tax rates on value added, on the profit of or-
organizations and on wages, respectively; $\theta$, $\zeta$, $\lambda$, $\varepsilon$ - are indicators of elasticity; $\omega$, $\sigma$, $\mu$ - indicators of the production structure of expenditures.

Any economic regime is characterized by a certain set of indicators of elasticity of the main meso- and macroindicators for prices. The real budget revenues from the three types of taxes (value added, corporate profits and wages) are considered.

The author believes that the production strategies of enterprises are influenced, first of all, by price shifts, but not by changes in tax rates: as a result of varying tax rates, the price range in economic markets usually changes, followed by a revision of the volume indicators of output and expenses.

Thus, fiscal impulses affect the production parameters indirectly - through price effects. This approach, according to the author, is more dynamic than the traditional ones. That is, an economic system with constant price impulses is considered. Strengthening the tax press only affects the increase in inflation, which, in turn, predetermines the production characteristics. And the price level and inflation affect the size of the Laffer points. Increasing inflation acts in the direction of "pushing" it out of the range of acceptable values. On this basis, the author concludes that there are no effects and Laffer points in an economy that is on a high inflationary trajectory of its development.

In [1], the analysis of Laffer's fiscal points was carried out under the influence of such factors as the volume of assets, the number of people employed in the economy, and the tax burden. A feature of this model is the assertion that the elasticity of labor and capital is a quadratic function of the tax burden. According to the research of E. Balatsky, the optimal variant of economic growth models is the basic production-institutional function. It can be represented as a modified Cobb-Douglas production function of the following form [1; 2; 3]:

$$Y = \gamma D K^{a+bg} L^{(n+mq)}q,$$

where $Y$ - the country’s GDP; $K$ - capital (amount of fixed assets); $L$ - labor (the number of persons employed in the national economy); $q$ - tax burden (relative tax pressure as the share of tax revenues in GDP); $D$ - trend operator (time-dependent function); $\gamma, a, b, n, m$ - parameters that are estimated statistically based on retrospective time series.

Unlike other proposed dependencies based on the Cobb-Douglas function, this function shows the technological relationship between the country’s GDP and the corresponding capital and labor expenditures in the context of government fiscal policy.

For the calculations, such pairs of indicators are used: fixed production assets in value terms and the number of persons employed in the national economy, as well as investments in fixed production assets in terms of value and labor costs. The conclusion is made that when building the proposed production function, it is necessary to use the indicated pairs of factors, since mixed methods of accounting for these indicators give much worse results. At the same time, the proposed econometric apparatus can be applied only to very stable economies of developed countries (for example, the United States). Models for less balanced economic systems (Russia) have a defect that manifests itself in the process of calculating macroeconomic indicators.

However, in the constructed models [1; 2; 5] does not take into account the following conditions. So, in work [1] there is a problem of information insufficiency for effective identification of the studied dependence in the conditions of countries with insufficiently stable economy. In the work [2], the limited data and the short-term time period do not allow to fully appreciate the possibilities put by the author in the computational scheme. The work [5] does not consider the human factor. The author abstracts from the effects associated with the choice of alternative development options by the company. The conclusion that tax instruments are an ineffective means of regulating the production climate needs further study.

In [18], the author built an economic-mathematical model of an enterprise, which explores how tax cuts, other things being equal, can affect production growth and sales of products (tax base), as well as budget revenues. Considered two options for the enterprise - the base with the primary tax rate and the new, when the tax rate is reduced by some fixed value. In the base case, there are no investments in the expansion of production, and in the new version, the entire increase in profits remaining after pay-
ing taxes at a reduced rate is invested in investments.

To determine the number of economic turnovers, due to which the same amount of income will go to the budget, the authors introduce such a restriction [18]:

$$\frac{1}{T - \Delta T} \sum_{i=1}^{n} \frac{1}{(1 + r)^{i}} \leq \sum_{i=1}^{n} \frac{R + L}{1 - L} \cdot \frac{1}{(1 + r)^{i}} \cdot (1 - T + \Delta T),$$

where $R$ – product profitability; $L$ – the proportion of semi-fixed costs in their total value; $T$ – the tax rate; $\Delta T$ – the amount by which the initial tax rate is reduced; $r$ – the discount rate; $x$ – the duration of the billing period (number of business turns).

Based on the results of the study, the author came to the conclusion that the relationship between changes in tax rates and budget revenues can have the form described by Laffer: by reducing taxes to a certain amount, government revenues can be increased. However, with the initial values of the economic indicators of Russian industry, it is not even possible to simply compensate for the loss of budget revenues due to tax cuts from 40% to 30% regardless of how long the billing period represents. It is possible to change the situation (for a given discount rate) only by increasing the initial value of product profitability and / or the proportion of semi-fixed costs in their total amount [18].

In the process of studying the Laffer curve, the author has substantiated that this curve is a theoretical construction and it is not advisable to use it as an accurate analytical tool for substantiating fiscal policy. This is due to the fact that for each specific case, all other conditions being equal, its form (as well as the elasticity of the tax base at the tax rate) depends on such an arbitrary value as the duration of the settlement period. It affects the location of the point on the graph in which the economy is located at the initial moment of time. Therefore, the assertion that the reduction in tax rates may be accompanied by the development of events on Laffer is not correct, if you do not focus on the time period [18]. Such conclusions coincide with the results obtained in [1; 2; 4; 15].

At the same time, in the model of this type, individual factors have a different impact on the solution of the task, and also do not take into account the influence of human capital and state financing of production and social spheres on GDP. This aspect requires further comprehensive research.

When studying the relationship of tax rates and tax revenues I. Chugunov, calculated the coefficients of the dynamic state of the economy at the total tax rate and tax on corporate profits. A descriptive model for direct taxation has been built. An aggregate tax rate for indirect taxation has been determined [8]. The author also believes that “the Laffer curve carries a great degree of uncertainty. Obviously, too high taxes have a negative impact on the willingness of people to work and pay taxes, but the important question that the Laffer curve does not answer is to determine the specific tax rate at which tax revenues will be maximized” [8]. Since the coefficient of the dynamic state of the economy depends on economic processes, it is justified that the accuracy in their determination and the safety factor affect the type of relationship between tax rates and tax revenues in a particular period of time [8].

In [19], it is argued that the tax system is considered effective if tax rates are introduced that are capable of ensuring the rapid growth of the national economy, and at the same time, sufficient budget revenues. Therefore, the authors consider the Laffer curve, the mathematical interpretation of which is similar to the relationship between the tax rate and tax revenues described in [8] with the inclusion of the additional parameter $\lambda$ and a detailed description of the economic value of the coefficients.

So, for comparison, the model of I. Chugunov looks as follows:

$$y = a \cdot x^\alpha (1 - x)^\beta,$$

where $a$ – tax revenues; $x$ – tax rate in relative terms; $\alpha \land \beta$ - are the coefficients determining the type of curve.

In [19], it has the form:

$$F(x) = \lambda x^\alpha (1 - x)^\beta,$$

where $\lambda$, $\alpha \land \beta$ – are the coefficients that define the type of curve; $x$ – is the tax rate, $0 \leq x \leq 1$; $F(x)$
– possible tax revenues to the budget; \( \alpha \) - coefficient of tax progression; \( \sigma \) - coefficient of sensitivity of the economy to changes in the tax rate.

The adequacy of the type of equations that describe the relationship between tax rates and tax revenues, really depends on the accuracy in determining the state of the economy and its safety margin in specific time periods. Therefore, the practical use of such models requires additional research.

In [7], an assessment of the tax burden relative to gross income and profits of organizations for taxation was made. The indicators obtained in the process of research are comparable with empirical data on various types of economic activity. The author made conclusions regarding the actual value of the tax burden and its impact on the shadowing of the wage fund.

The Cobb-Douglas production function has a standard form: 

\[
Y = AK^\alpha \cdot L^\beta,
\]

where \( Y \) – is the volume of production, \( A \) – s the dimensional coefficient, \( K \) – is the value of fixed assets, \( L \) – is the volume of labor, \( \alpha \) and \( \beta \) – are elasticity indicators relative to capital and labor. The model takes into account corporate income tax (\( \tau_1 \)), social payments (\( \tau_2 \)) and personal income tax (\( \tau_3 \)).

Then the tax burden relative to gross income is as follows:

\[
\tau_p = \tau_1 \left( \frac{M + \Delta K}{V} \right) + \frac{wL}{V} \cdot \frac{\tau_2 + \tau_3 - (1 + \tau_2) \tau_1}{1 - \tau_3},
\]

where \( M \) – the cost of production with the deduction of the amount of labor (raw materials, energy, etc.); \( \Delta K \) – tax depreciation (ie, the share of the value of fixed assets, which reduces the tax base of organizations' profits); \( V \) – the cost of manufactured products; \( w \) is the actual payroll after tax.

This formula shows that the tax burden significantly depends on the share of wages in gross income and on the share of other expenses (for raw materials, energy, amortization, and others). At the same time, the tax burden relative to gross income for a given share of wages increases to the extent that profitability increases. For an entrepreneur, the tax burden relative to profit to tax decreases as profitability increases, i.e. enterprises with significant profitability keep a larger share of profits than companies with insignificant. So, the share of income remaining with the entrepreneur after paying taxes is positive only for enterprises with a significant level of profitability and an insignificant share of wages. Based on the calculations performed, the authors concluded that the current taxation system in Russia and its regions with a significant burden on the wage fund does not affect the development of those industries where skilled labor is used.

Thus, the analysis of descriptive models that investigate the effect of taxes on economic growth indicates that the features of this class of models are:
- ease of use at the macroeconomic level, in particular in the study of graphic curves, namely the Laffer curve;
- descriptive nature of processes in economics using a wide range of mathematical tools;
- consideration of the influence of various internal and external factors of the system;
- the possibility of conducting an experiment for sufficiently long periods of time in order to identify the general trend in the development of indicators.

In the existing descriptive models in the subject field under study, as a rule, when considering the influence of factors of production (capital and labor according to the Cobb-Douglas theory) and tax revenues on economic growth rates, there are no feedbacks. From the standpoint of the abstract effect of individual factors, the Laffer curve is also studied. According to the results that can be obtained in terms of the use of descriptive models, it is not possible to develop proposals for further reform of the tax system in the country.

3. The use of optimization models of economic growth

In order to clarify the features, advantages and disadvantages of descriptive and optimization models of economic growth, it is advisable to consider the basic models of foreign and domestic authors. In addition to descriptive, optimization models are widely used. Optimization approach is quite common in the theory of taxation. This is due to its knowledge, ease of use and obvious solutions for systems with simulated behavior. In contrast to the descriptive optimization models that describe the relation-
ship between variables, contain the criterion of the objective function to select the optimal solution to the problem:

$$\Theta(\bar{m}) \rightarrow \max$$

$$A \cdot \bar{m} \leq B$$

$$\bar{m} \geq 0,$$

where $$\Theta(\bar{m})$$ – is the objective function; $$\bar{m}$$ – the value of solving the optimization problem; $$A, B$$ – matrix of values and vector of restrictions, respectively.

Optimization models, as a rule, are described by a general optimization problem that has the following form [10]. Let $$\Theta : \mathcal{M} \rightarrow Q$$ – be some function reflecting the set $$\mathcal{M}$$ and set $$Q$$, where $$\mathcal{M} \subseteq Q$$.

Then, for a given subset $$\mathcal{M} \subseteq \mathcal{M}'$$ it is necessary to find a value $$\bar{m} \in \mathcal{M}'$$, such that for all $$m \in \mathcal{M}$$ the condition:

$$\Theta(\bar{m}) \geq \Theta(m).$$

$$\mathcal{M}$$ is the set of solutions to the problem, $$\mathcal{M}'$$ – is the set of feasible solutions, and $$Q$$ – s the set of estimates. An element $$\bar{m} \in \mathcal{M}'$$, that satisfies condition (’) for all $$m \in \mathcal{M}'$$, is called the solution to the optimization problem given by the pair $$\Theta, \mathcal{M}'$$.

Sometimes a model can include several criteria, each of which optimizes a solution within the framework of interacting subsystems. As a rule, they are interconnected into a single target function for the whole system, and then the model has a multi-criteria character. She answers the question: “how should the process take place?”, i.e. the choice is up to the developer, not the user.

When optimizing the theory of taxation, three types of economic entities are often distinguished:

- the state regulating the functioning of enterprises with the help of tax policy methods and thereby affecting their production activity;
- enterprises whose activities are determined by technical and economic characteristics, market relations and tax policy in the country;
- consumers involved in the reproduction process.

In models, the objective functions can be of the following types:

- absolute (selection criteria for an enterprise: maximum net or balance profit, maximum output, minimum tax payments, minimum production costs; selection criteria for the state: maximum tax revenues to the budget, maximum gross output);
- relative (selection criteria for an enterprise: maximum profitability, maximum resource intensity and capital intensity; selection criteria for the state: minimum tax burden on business entities, maximum growth rate of the economy, minimum inflation rate).

In contrast to the absolute indicator, the objective function in relative terms has the advantage of visibility of changes in some characteristics of the functioning of the economic system relative to others belonging to the optimized factor.

Optimization of taxation occupies a significant place in the studies of foreign authors. Researchers approach this problem from the standpoint of the two most common approaches:

1) Pareto-efficiency, where the maximum social welfare is considered from the point of view of each individual, provided that others are in a stable state;
2) utilitarianism, where social welfare equals the sum of the benefits of all individuals.

The first approach is rather extensively formulated in [6]. The optimization criterion is the maximum of the target function of this type:

$$\int_0^\infty u(c) \cdot e^{-pt} dt,$$

where $$u$$ – utility function; $$c$$ – consumption of each person; $$t$$ – time; $$p$$ – time parameter, $$p > 0$$.

The most simplified model:

$$\bar{a} = Ak,$$

where $$\bar{a}$$ – release of the final product; $$k$$ – capital.
In the Arrow-Roomer model, the output of the final product per employed individual depends on its own as well as on the share capital of workers. Then the production function has the form:

\[ y = A \cdot k^{1-\alpha} \cdot \bar{K}^{\alpha}, \]

where \( k \) – equity; \( \bar{K} \) – total capital; \( \alpha \) – capital elasticity, \( 0 < \alpha < 1. \)

Similarly ( "") in this model, the growth rate corresponds to the rate of return on investment \( \left( \frac{\alpha}{\eta} \right) \).

The growth rate is not enough of the Pareto-efficiency coefficient, which follows from the value \( \left( \frac{\alpha}{\eta} \right) \).

One of the ways to achieve the optimal level of public welfare, according to the authors, is subsidizing the sphere of production.

In the study of models with public services and taxes, as a rule, a function of the form is considered:

\[ y = A \cdot k^{1-\alpha} \cdot g^{\alpha}, \]

where \( g \) – public services.

In this case, the growth rate in the economy is very low and a Pareto-efficient result can be achieved when switching, for example, to this model - to a consumption tax or by subsidizing purchases of industrial goods.

Another model allows for the accumulation of public services where goods compete, but are not subject to withdrawal. The production function is represented as follows:

\[ y = A \cdot k^{1-\alpha} \cdot \left( \frac{g}{y} \right)^{\alpha}. \]

This equation shows that individual production provides constant incomes of producers, as long as the government maintains an appropriate level of accumulation of public goods, i.e. an attitude \( \frac{g}{y} \). Payment for use is a tax on output or profit in a ratio \( r = \frac{g}{y} \), hat aligns private and social rates of return on investment and therefore leads to Pareto-efficient growth rates.

The authors came to the conclusion about the effectiveness of using the model of accumulation, which can be used with a large number of public spending (vehicles, utilities, courts, national security, law enforcement agencies). If public services are provided, secured by private goods, competing and subject to withdrawal, or provided by public goods that do not compete and are not subject to withdrawal, then the total taxation will exceed the income taxation. Many types of public goods (transport, utilities, court) are subject to accumulation, but are not fully seized. In such cases, profit taxation works approximately as a user fee and therefore may dominate general taxation. In addition, in the case of general taxation, investment incentives increase.

In [14], the effect of tax rates on organizations’ profit and value added on aggregate output and the amount of tax payments to the budget is investigated, taking into account price factors and the volume of finished products. The author believes that the production sector consists of the same type of enterprises, each of which maximizes output and net profit. Two possible situations are considered: two rates lying on a certain curve, or just a value added tax. The total demand in the model does not depend on the rates of taxes on enterprises’ profits and value added. The assumption about the availability of free labor is introduced.

Considered two cases of enterprise behavior in the market.

1. Monopolistic enterprises choose production volumes and accordingly prices, maximizing the objective function of the form:

\[ K_m (x, p) \rightarrow \text{max}, \]

where \( x \) – volume of production; \( p \) – price.

2. Enterprises operate in a competitive environment and choose only the volume of production, and the price and wage level are considered as exogenous variables:

\[ K_c (x) \rightarrow \text{max}. \]

The simulation results showed that the ratio of optimal volumes of monopoly and competitive is-
sues does not depend on tax rates.

The author investigated the conditions under which the dependence of budget revenues on two types of tax rates (VAT and corporate income tax) is described by the Laffer curve [14]:

$$\beta(t) = t, \, \text{enterprise tax rate } \alpha = \text{const},$$

$$\alpha(t) = t, \, \text{VAT rate } \beta = 0,$$

where $\alpha(t)$ and $\beta(t)$ are arbitrary non-decreasing functions such that $\alpha(0) = \beta(0) = 0$; $\alpha(1)$, $\beta(1) \leq 1$, $t \in [0,1]$.

In the first case, with elastic demand and with not very aggravating requirements for a fixed corporate income tax rate and a corresponding weighting factor in the objective function of the optimization model of production, the dependence of budget revenues on the VAT rate is described by the Laffer curve. In the second case, the Laffer curve is absent.

The authors came to such conclusions [14]:
- only VAT can be Pareto-efficient, for this type of tax, the dependence of budget revenues on the tax rate is described by the Laffer curve;
- the properties of corporate income tax rates, VAT and dependencies of tax revenues from these rates are manifested not only with constant elasticities, but also with much weaker assumptions.

At the same time, the model does not take into account the effect of tax cuts on the increase in labor supply and investment growth, which leads to an increase in the tax base.

The study was not confirmed in practical calculations, and the Laffer curve was constructed for only two types of taxes.

In optimization models built from the position of utilitarianism, the maximization of the benefits of all individuals is taken as a basis.

One way of accounting for the effect of taxes on long-term growth rates from the standpoint of this approach was proposed in [9]. The authors analyzed the features of the optimization policy in the growth model with endogenous government spending.

The production function for the final product is:

$$Y_t = A \cdot (v_t K_t)^{\alpha \epsilon} \cdot (u_t H_t)^{1-\alpha} \cdot G_t^{\alpha \epsilon},$$

where $Y_t$ – final product; $A$ – trend parameter; $K_t$ – physical capital; $H_t$ – human capital; $G_t$ – public goods; $v_t$ – part of the total physical capital in the production of final goods; $u_t$ – part of the total human capital in the production of final goods; $\epsilon$ - public spending performance parameter.

The authors consider several types of models, of which of particular interest are those in which the tax policy is described using the objective welfare function of this type [9]:

$$\max_{\{c,K,H,G,v,x,u,z\}} \int_0^\infty W_t - \Psi \cdot \hat{W}_0,$$

where $c$ – consumption; $x, z$ – parameters; $W_t$ – this discounted value of consumption; $\hat{W}_0$ – general consumer welfare; $\Psi$ - Lagrange multiplier.

In the process of research, it is substantiated that if government spending is productive and the state can tax human and physical capital at different tax rates, then tax policy is optimally achieved when public goods are the resources for the production of the final product. At the same time, the implementation of an optimal policy allows for a balanced budget and a zero amount of public debt, i.e. The following conditions must be met:

$$\tau^K \cdot v \cdot K \cdot R^K = (1-\epsilon) \cdot \alpha \cdot Y = G,$$

$$\tau^H \cdot v \cdot K \cdot R^H = (1-\epsilon) \cdot \alpha \cdot Y = G,$$

where $R^K$ – market capital rate; $R^H$ – current rate of wages.

In work [10], the problem of optimal taxation in three endogenous growth models is investigated. In the first model, physical and human capital are predominantly symmetrical both in their use and accumulation. Target function is the level of well-being of individuals. The optimality criterion is:

$$\max \sum_i \beta^t \cdot u(c_i),$$
where $\beta$ – weighting factor; $u(\cdot)$ – utility function; $c_t$ – consumption.

The production function has a standard view:

$$F(k, h) = A \cdot k^\alpha \cdot h^{1-\alpha},$$

where $k$ – capital; $h$ – work.

In the second model, the process of accumulating human capital depends on both market and non-market goods. The optimization criterion is the maximum of the objective function, which is represented by the equation:

$$\max \sum_i \beta^i \cdot u(c_i, 1 - u_i - v_i),$$

where $v_i$ – share of effective labor necessary for the formation of human capital; $u_i$ – proportion of effective labor required in the labor market.

In these models, the authors adhere to the prerequisites of public finance, according to which the flow of public spending is considered exogenously planned. And a change in direction to the optimal tax policy is the reason that government spending is reduced in a small proportion of the final output. A more realistic method includes these costs as a production resource.

The optimization criterion for the third model is similar to ('). The authors proceed from the premise of the endogenous nature of government spending, which significantly affects the nature of optimal taxes: an increase in expenses affects the positive value of the corporate income tax rate. However, such results are significantly different from the well-known empirical studies in the field of optimal taxation and show that the marginal rate of corporate income tax can be zero [9; 10; 13].

In work [11], taxes and human capital are analyzed, which is a certain factor of welfare and income for most individuals, since it is a combination of labor supply, capital investment and the final product. The authors believe that each individual solves the following problem:

$$\max_{c, n, g, x} \int e^{-p t} u(c, n, H, g) dt,$$

where $c$ – consumption; $n$ – time; $g$ – government consumption; $x$ – investment in human capital; $u$ – utility function; $H$ – human capital; $p$ – time parameter.

The production function is presented implicitly:

$$f(K, L(H, n), g, t),$$

where $K$ – physical capital; $L(H, n)$ – effective work in man-hours.

The government sets a level of tax on human capital that would maximize the usefulness of individuals under the budget constraint.

The simulation results show that, of all taxes, human and physical capital is affected only by income taxation on labor in the long run. The author concludes that some of the proposed consumption taxes are not true consumption taxes.

The use of optimization models is also common in studying the effect of taxes on economic growth in the conditions of the formation and development of a market economy. A significant contribution to their development was made by domestic authors.

In [4], the absolute criteria of the state fiscal policy are considered from the standpoint of the following approaches:

1. The first approach allows only one Laffer point for a specific type of tax revenue. For example, the state maximizes the amount of value added tax relative to its rate at given and fixed other tax rates. Then the criterion is as follows:

$$T_i(h_i) \rightarrow \max,$$

where $h_i$ – $i$-th tax rate; $T_i$ – $i$-th type tax.

2. The second approach involves the optimization of the entire mass of tax exemptions by varying the rate of only one type of tax. This method takes into account the cross-mutual influence of taxes:

$$\sum_{i=1}^s T_i(h_i) \rightarrow \max.$$
3. The third fiscal strategy of the state provides for the maximization of a specific tax when manipulating all (or several) tax rates simultaneously:

\[ T_i(h_i; i=1, n) \rightarrow \text{max.} \]

4. The fourth, generalized approach, is focused on achieving the maximum amount of fiscal revenues when attracting the entire set of tax parameters:

\[ \sum_{i=1}^{n} T_i(h_i; i=1, n) \rightarrow \text{max.} \]

However, in some cases, relative indicators may be used. As an example of this type of criteria, the author cites a macroeconomic indicator of tax collection, equivalent to optimizing the share of fiscal collection in gross domestic product [4]:

\[ \frac{T(h_i)}{X(h_i)} \rightarrow \text{max}, \]

where \( X(\cdot) \) – gross domestic product.

For each of the specified criteria, in the opinion of the author, its own fiscal theory can be proposed. It is concluded that in some cases, for fairly complex production criteria, the curves may have several increases. This means that when optimizing all taxes, except for one, the latter has a different effect on the production activity of an enterprise in certain parts of the output curve [4].

As in the case of production facilities, the tax policy of the state has various criteria for evaluating its effectiveness, among which the author focuses on maximizing tax exemptions [4]:

\[ T(\alpha, \beta, \gamma) \rightarrow \text{max}, \]

where \( T \) – tax exemption; \( \alpha \) - corporate income tax; \( \beta \) - VAT; \( \gamma \) - social charges on wages.

The total mass of tax revenues in this case consists of three types of fiscal payments: corporate income tax, VAT and payroll tax, and the combined effect of several tax rates, according to the author, leads to the Laffer effect.

The author gives preference in the optimization models to the criterion of the “current” rate of profit (profitability) from the standpoint of public welfare and expresses the opinion that the following condition is sufficient in the optimal tax system: value added tax and wage rates should be higher than corporate income tax rates. Nevertheless, the applied calculations carried out for the conditions of the Russian economy in the studied period of time showed the absence of Laffer points.

In the study of economic growth in [16] are considered anti-inflation taxes. Factors affecting their size are aggregate effective demand, production capacity, investment and labor resources. In this case, the aggregate effective demand, the author notes, will be decisive among the factors constraining economic growth.

In the developed model, the local public utility criterion is optimized. The manufacturer’s performance indicators consist of positive components - the volume of production of goods and the provision of services per unit of time, as well as negative components - all types of expenses (labor, material, the use of production assets and natural factors). There is a vector of resource utility assessments.

The profit of the manufacturer is calculated by the formula:

\[
V = PX = \sum_{i} p_i x_i,
\]

where \( V \) – profit; \( P \) – vector of resource utility estimates, \( P = \{p_i\} \); \( X \) – vector of production and expenses, \( X = \{x_i\} \).

Then the social utility of labor (\( W \)) has the form:

\[
W = \int P(X) dX = \sum_{i} \int p_i(X) dx_i.
\]

The impact of anti-inflation taxes on excess profits and on price increases, which have an important effect without additional measures, which reduces inflation and increases production, has been evaluated.

The mechanism of economic growth presented in this paper by increasing the money supply, taking into account the tax on price increases as an anti-inflationary factor, boils down to the following: the state gradually increases its spending over revenues, covering the difference with emissions. The in-
crease in the money supply creates conditions for increasing the profits of producers. At the expense of prices, they cannot do this, since the price effect is eliminated through taxes. The only possibility remains: to expand production and marketing of products. Such behavior of producers entails a further increase in aggregate demand due to an increase in material costs, an increase in wages and tax revenues of the state. As a result, there is a general increase in production, exceeding the initially specified increase in government spending.

Stimulating aggregate demand by increasing government spending gives a significant national economic effect, due to the involvement of additional labor, reduction of unemployment, underemployment and fictitious employment [16].

This situation, although it increases the intensification of labor in the production sector, the level of development of the social sphere remains low, since the wage rates of production workers, as the author notes, remain unchanged. In addition, the proposed model is applicable to an optimally functioning economy and is unsuitable for the conditions of a market economy, and the use of anti-inflation taxes may be appropriate only for relatively large producers.

To build a growth model [17], the author identifies the following factors: potential labor, capital, knowledge in the economy, elastic labor supply. This model is built for a policy that contributes to the maximum consumer welfare, being stationary in time, i.e. such that the following governments will not refuse it. The model allows for endogenous growth. There are three sectors: consumer, production and government with fiscal and monetary authority.

Consumers solve for themselves the following task [17]:

\[
\sum_{t=0}^{\infty} (1 + \rho)^{-t} U(C_t, M_t, p_t, G_t, l_t) \rightarrow \max ,
\]

where \( U(\cdot) \) – utility function; \( C_t \) – private consumption; \( M_t \) – cash balances, which the consumer has at his disposal during the period \( t \); \( p_t \) – price; \( G_t \) – public consumption; \( l_t \) – working time; \( \rho \) - discount.

The activities of enterprises that maximize profits in each period of time, deducting wages and interest on borrowed capital from revenues, are described by the production function.

The government controls tax rates, government spending and foreign debt, as well as the rate of issue of money. His task is to maintain the balance of the state budget in each period of time. It is believed that the government can instantly respond to changes in consumer demand for money supply and bonds through the purchase and sale of bonds through the Central Bank.

The author believes that in order to enter the trajectory of steady-state equilibrium growth, the government must implement a stable policy [17]. It is determined by such a set of parameters:

\((\tau, g, \theta)\),

where \( \tau \) - tax rate that is controlled by the state; \( g \) – share of government expenditures in the total output \( (g = \frac{G}{y}) \); \( G \) – government expenditures, \( y \) – total output); \( \theta \) – rate of issue of money, and

\[
\theta_{t+1} = \frac{M_{t+1} - M_t}{M_t} .
\]

The main drawback of the model, due to which it is inappropriate to use it for the conditions of Russia, is the author’s basic assumption about the stability of the policies of various governments, which exists only when the economic system has reached a stable equilibrium state.

Analysis of the studied optimization models of the influence of taxes and production factors on economic growth showed that:

- when building an optimization model at the macroeconomic level, it is advisable to use a multi-criteria approach, since each of the interacting systems (government, producers and consumers) have their own (local) objective functions;
- models of optimal taxation are sufficiently developed in the works of foreign authors, but they are unsuitable for the conditions of the Russian economy;
- for the conditions of the Russian economy, the approach to the study of economic growth has
not gained sufficient development and needs further study, taking into account the practical interpretation of the results obtained.

Thus, the task of building a model of state regulation of economic growth with tax policy methods consists not only in describing business processes, but also in formulating the conditions under which the best result will be achieved, justified by comparing different calculation options. Therefore, it is advisable to use a descriptive model at the verification stage, and an optimization model at the research stage, since a selection criterion is introduced — the maximum growth of GDP (GRP).

Such a synthesis is due to the fact that the model should be an effective applied tool for studying the behavior of the modeled economic system under given conditions and allow:
- sufficiently reflect the causal relationships between the indicators;
- analyze the various behaviors of the economic system;
- to compare the trends of indicators in the basic and optimal conditions for its functioning;
- choose the optimal policy.

4. Conclusion
Summarizing the above, we can draw the following conclusions:

1. In the economic thought of developed countries, the influence of tax policy on the growth rates of GDP and GRP is often examined by evaluating the factors of production — capital and labor — in the process of their taxation. Such assessment is carried out using hypotheses and axioms, the construction of relevant concepts, as well as the use of various methods, among which the methods of economic and mathematical modeling have acquired the greatest use.

2. As shown by the results of the research performed, all economic and mathematical models can be divided into two classes that are widely used in identifying the effect of the tax factor on economic growth: descriptive and optimization.

3. When studying the problems of economic growth, taking into account the influence of various factors, each of the classes has its advantages and disadvantages. Models adapted to the economies of developed countries do not take into account the features of the formation and development of market relations. The study of domestic authors is aimed at the formalization of individual economic processes, often without taking into account the inverse relationships between the studied indicators, and without empirical testing of the hypotheses put forward. For the conditions of formation and development of the market economy, the models of economic growth need to be further improved in view of the productive nature of taxes and the practical interpretation of the results obtained.

4. In descriptive models, as a rule, there are no feedbacks when considering the influence of factors of production (capital and labor according to the Cobb-Douglas theory) and tax revenues on economic growth rates. From the standpoint of the abstracted influence of individual factors, the study of the Laffer curve is carried out.

5. Analysis of optimization models showed that the approach to modeling optimal taxation is well developed in the works of foreign authors; when building such models, a multi-criteria approach is often used; for the conditions of a developing market economy, the economic growth model needs further improvement, taking into account the productive nature of the studied indicators and empirical testing of the results obtained.

6. The task of building a model of state regulation of economic growth using tax policy methods consists not only in describing business processes, but also in determining the conditions under which the best result will be achieved, justified by comparing different calculation options. Therefore, it is advisable to use a descriptive model at the verification stage, and an optimization model at the research stage, since a selection criterion is introduced — the maximum growth of GDP (GRP) — to develop a set of guidelines for further improvement of the tax system, such the synthesis is due to the fact that the model should be an effective applied tool for studying the behavior of the modeled economic system under given conditions and allow: sufficiently reflect the causal relationships between the indicators; analyze the various behaviors of the economic system; compare the trends of indicators in the basic and best conditions for its functioning and choose.
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