The quantum phase transitions of dimer chain driven by an imaginary ac field

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A topologically equivalent tight binding model is proposed to study the quantum phase transitions of dimer chain driven by an imaginary ac field. I demonstrate how the partner Hamiltonian is constructed by a similarity transformation to fulfill the $\mathcal{PT}$ symmetry. The $\mathcal{PT}$ symmetry of the partner model allows us to study the topological properties of the original non-Hermitian model as the Bloch bands of the Hermitian system. The quantum phase transitions are discussed in different frequency regime. The approach has the potential applications to investigate the topological states of matter driven by the complex external parameters.

I. INTRODUCTION

As pioneered by Yang and Lee in 1952, the quantum phase transitions (QPTs) can also be driven by the complex external parameters [1, 2]. It is believed that the introduction of the complex parameters at least gives a deep understanding QPTs [3]. The interest to study the issue is evoked recently by the realization of the static complex Zeeman field experimentally in non-Hermitian atomic systems by laser assisted spin-selective dissipations [4, 5]. The application of real ac fields has become a very promising tool to synthesize novel topological phase termed as Floquet topological engineering which otherwise would be impossible to be implemented in an undriven case [6–12]. A natural question that arises in this topic is what controllable characters of imaginary ac fields can bring to non-Hermitian systems although this kinds of field hasn't generated yet.

For the time-dependent Hamiltonian in the dipolar approximation, the relation between un-driven and driven system is given by the minimal coupling

$$\frac{d}{dt}\mathbf{A}(t) = \mathbf{A}(t) \times \mathbf{B}(t)$$

where $\mathbf{A}(t)$ is the vector potential. One can obtain the time-dependent tight binding (TB) model with the time-dependent hopping

$$\tau(t)_{ij} = \tau_{ij} e^{i\mathbf{A}(t) \cdot \mathbf{R}_{ij}}$$

in the inverse Fourier transform. However, for the case of imaginary ac field $\mathbf{A}(t) = i\mathbf{A}_0(t)$, $\tau(t)_{ij} = \tau_{ij} e^{-i\mathbf{A}_0(t) \cdot \mathbf{R}_{ij}}$ and $\tau(t)_{ij} \neq \tau_{ij}$ cause the imbalance hopping and non-Hermitianity of the Hamiltonian, which is contrast to the real ac field case $\tau(t)_{ij} = \tau_{ij}$.

Generally, near the QPTs point, the external real parameters of the Hamiltonian drive the energy-levels crossing or avoided energy-level crossing between the ground state and the excited state. The global curvature in the parameter space can be captured by the geometric phase. Comparing the traditional analysis resorting to the order parameter and symmetry breaking within the Landau-Ginzburg paradigm, the geometric phases of the ground state provide a comprehensive reflection of the ground state characteristics in many-body systems. However, the non-Hermitian Hamiltonian exists the complex band structure and exhibits intriguing features with no counterpart in Hermitian cases [13–16] except for a non-Hermitian Hamiltonian having simultaneous parity-time ($\mathcal{PT}$) symmetry [17, 18]. The imbalance hopping induces non-Hermitian skin effect which causes all eigenstates are localized exponentially to boundaries, regardless of topological edge states and bulk states [19–26]. In particular, the bulk-boundary correspondence is found recently to be breakdown completely for some non-Hermitian systems due to the skin effect [24, 27–32].

Concerning the non-Hermitian topological phases, the general Brillouin zone is used to understand the bulk-boundary correspondence of non-Hermitian system where a complex-valued wave vector is introduced to capture unique feature of non-Hermitian bands [19, 33–35]. The real part of the wave vector is from the periodicity of the system according to the Bloch theorem. The imaginary part of complex-valued wave vector is responsible for non-Hermitian skin effect. Experimentally, the non-Hermitian bulk-boundary correspondence has been demonstrated in discrete-time non-unitary quantum-walk dynamics of single photons [24] and in topological circuits [26].

Periodically driven non-Hermitian systems involve not only the non-Hermitian topological phases, but also Floquet topological phases. It has exhibit rich topological phases and non-Hermitian skin effect, without analogs in their static or Hermitian counterparts [31, 32, 36–39]. In particular, some of the studies focus on the periodic quenching a Hamiltonian from $H_1$ for the first half period to $H_1$ for the second one [31, 32, 37]. The key role played by periodic driving is changing symmetry and inducing an effective long-range hopping in lattice systems [40].

Controllable gain and loss of the available experimental setups make concrete realizations of non-Hermitian lattice model possible, such as a non-Hermitian version of the topological SSH model [41–44]. However, the non-Hermitianity induced by an imaginary ac field is not involved.

Motivated by the above considerations, I investigate the imaginary parameter driven QPTs by the analysis of an ac-driven dimer chain. I introduce a partner model without non-Hermitian skin effect that shares the same topological phase diagrams [45]. The partner of non-Hermitian Hamiltonian can be constructed by adjusting the imbalance hopping to fulfilling inverse or reflection symmetry. I demonstrate that partner Hamiltonian can be obtained by a similarity transformation and prove their topologically equivalent. In particular, the partner Hamiltonian have a $\mathcal{PT}$ symmetry which allows to study the topological properties as the Bloch bands of the...
Hermitian system.

The merit of this method is that the bulk boundary correspondence is still effective without the introduction of the generalized bulk boundary correspondence and non-Bloch winding number. As shown below, the imaginary ac field drives the initial Bloch band splits into Floquet-Bloch bands. Interestingly, the ac-driven band inversion will lead to interesting topological states of matter which otherwise would be inaccessible in the real field case.

The remainder of this paper is organized as follows. In Sec. II I present the 2D effective TB Hamiltonian of SSH model driven by an imaginary ac electric fields. I show how the amplitude of the vector potential controls the renormalization of the system parameters and leads to the non-Hermitian of the effective Hamiltonian. In subsection III A I present a partner model of the original model and illuminate that the original and modified models are related by a similarity transformation. Due to the topologically equivalence of the two model, I study the QPTs in low and intermediate frequency regime with the partner model in subsection III B and III C. Finally, I present a summary and discussion in Sec. IV.

II. MODEL

I consider the one-dimensional SSH model [46] driven by an imaginary ac electric fields. The imaginary ac electric field described by an imaginary vector potential $iA(t) = ia_0 \sin(\omega t)$ of frequency $\omega$ is applied in the chain direction. With the standard Peierls substitution $k \rightarrow k + A$, the Hamiltonian is given by $H(k, t) = \Psi_k^\dagger(t) h(k, t) \Psi_k(t)$ where $\Psi_k(t) = [c_{k,A}^\dagger(t), c_{k,B}^\dagger(t)]$ and the non-Hermitian operator

$$h(k, t) = \begin{pmatrix} 0 & \tau + \tau' e^{i\omega t} e^{-ik} \\ \tau + \tau' e^{-i\omega t} e^{ik} & 0 \end{pmatrix}. $$

(1)

$\tau$ and $\tau'$ are the two hopping parameters of the dimer chain. The model can be investigated by the Floquet-Bloch ansatz [47,48]. The essence of this method is mapping the time-dependent Schrödinger equation to the eigen problem

$$\tilde{h}(k, t) \Psi_k(t) = \Psi_k(t) \epsilon_k,$$

(2)

where

$$\tilde{h}(k, t) = h(k, t) - i\partial_t = \begin{pmatrix} -i\partial_t & \tau + \tau' e^{i\omega t} e^{-ik} \\ \tau + \tau' e^{-i\omega t} e^{ik} & -i\partial_t \end{pmatrix}$$

is defined as the Floquet operator, and $\epsilon_k = (\epsilon_{1,k} 0 0 \epsilon_{2,k})$ is the quasienergy. The quasi-energies $\epsilon_k$ and the Floquet states $\Psi_k(t)$ can be obtained by diagonalizing the one-period propagator $U(T)$. The propagator $U(T)$ can be solved by integrating the following evolution equation numerically,

$$i \frac{\partial}{\partial t} U(t) = h(t) U(t).$$

in one period with the initial condition $U(0) = I_0$, here $I_0$ is the unit matrix.

Alternatively, the driven dimer chain is mapped to time independent 2D lattices. The quasienergy can be obtained by diagonalizing the effective Hamiltonian [48]. This method will be used in the following discussions and summarized briefly here. Using the Fourier transformation $\Psi_k(t) = \sum_{m,n} \Phi_{k,m,n} e^{i\omega nt}$ and with the inner product $\langle \langle \cdots \rangle \rangle = \frac{1}{T} \int_0^T \cdots dt$, the Floquet operator is transformed to

$$\tilde{H}(k, j) = \sum_{m,n} \Phi_{k,m,n}' \tilde{h}_{m,n} \Phi_{k,n}$$

$$\tilde{h}_{m,n} = \begin{pmatrix} -n\omega \delta_{m,n} & \tau \delta_{m,n} + B_{m,n} \\ \tau \delta_{m,n} - B_{m,n} & -n\omega \delta_{m,n} \end{pmatrix}$$

(3)

where $\Phi_{k,m,n}' = (c_{k,A,m}^\dagger c_{k,B,n}^\dagger \ c_{k,B,n}^\dagger c_{k,A,m}^\dagger)$ with $\alpha = m,n$ and $B_{m,n} = \tau' e^{-i\omega t} J_{m-n}(-i\alpha_0)$, $B_{m,n} = \tau' e^{i\omega t} J_{m-n} (i\alpha_0)$. $J_\nu$ is the $\nu$th Bessel function of the first kind. The diagonal term $-n\omega \delta_{m,n}$ of Eq. (3) is equivalent to an effective electric field $\omega$ in $f$ direction and break the space inversion symmetry. Eq. (3) is an infinite matrix because $n, m$ are integer. Shown in Fig. II A, the one-dimensional ac-driven model is exactly mapped to two dimensional TB problem in the composed Hilbert $\mathcal{H} = \mathcal{H} \otimes \mathcal{T}$, where $\mathcal{H}$ is the ordinary Hilbert space and $\mathcal{T}$ is the space of $T$-periodical function.

In the low frequency regime $\omega \ll \tau, \tau'$, the diagonal term $-n\omega \delta_{m,n}$ in Eq. (3) can be neglected. The Floquet operator is an effective TB model in the frequency space $\mathcal{T}$. The TB model can be transformed to the corresponding momentum $k_f$ space:

$$\tilde{h}(k, j) = \begin{pmatrix} 0 & t_2 + t_1 e^{i\omega t} \\ t_2 + t_1 e^{-i\omega t} & 0 \end{pmatrix}.$$  

(4)

FIG. 1. (A) The effective 2D lattice of the periodically driven dimer chain. The additional $f$ dimension is spanned by $|n|$ due to the time periodicity. The width of the arrows shows the hopping intensity qualitatively. (B) The partner model in the $x$ direction. Adjusting the imbalance hopping between the adjacent unit cells (the magenta-dotted ellipse boxes) leads to the non-Hermitian skin effect disappearing in the $x$ direction.
Here
\[ t_{1,±}(k_f) = l_0 \pm \sum_{l=2n+1} 2 (-1)^l I_l \sin lk_f \pm \sum_{l=2n} 2 (-1)^l I_l \cos lk_f \]
\[ = t_1 \pm \frac{\gamma}{2}, \]  \hspace{1cm} (5)

Shown in Fig. 1(B), the TB model of the Floquet operator in Eq. (4) is dimer chain with the imbalance hopping \( t_1 \pm \frac{\gamma}{2} \) in the left and \( t_1 - \frac{\gamma}{2} \) right directions. I have used the relationship \( J_{-,b}(0) = (-1)^l J_{+,b}(0) \) and \( J_{+,b}(i0) = i^l I_{+,b}(0) \), \( I_r \) is the \( r \)th modified Bessel function of the first kind [49]. In fact, the effective SSH model can also be written as
\[ \hat{h}(k, k_f) = \tau \left( \begin{array}{cc}
0 & t_+ + t_2 e^{-ika} \\
t_- + t_2 e^{ika} & 0
\end{array} \right). \]  \hspace{1cm} (6)

The former is AB lattice and the latter is the BA lattice. They have the same phase transition point. The following discussions are focused on the equivalent BA lattice.

Out of the low frequency regime, we must diagonalize the one-period propagator \( U(T) \) or the full Floquet matrix [3] to get the quasienergy due to the Floquet bands coupling. In the high frequency regime \( \omega \gg \tau, \tau' \) however, the diagonal terms in Eq. (3) are important. The quasienergy bands have a large band gap \( \omega \) and the matrix is approximately block diagonal. The topological phase transitions occur in the lowest Floquet band \( m = n = 0 \). In this case, the effective Hamiltonian in Eq. (3) is reduced to a the 2 \( \times \) 2 hermitian matrix:
\[ \tilde{h}_k = \tau \left( \begin{array}{cc}
0 & I_0 + t_2 e^{-ika} \\
I_0 + t_2 e^{ika} & 0
\end{array} \right). \]

It is the SSH model and the topological phase transition occurs at \( t_2 = \tau' / \tau = I_0 \). This is different from the real ac field case where the topological phase transition occurs at \( t_2 = J_0 \) [48].

III. RESULTS

A. The partner model

The idea of constructing the partner Hamiltonian is from that the topological boundary states are protected by the symmetry of system and are immune to perturbations. It is reasonable to speculate that, for a class of non-Hermitian systems with non-Hermitian skin effect, there is a partner without non-Hermitian skin effect that has the same symmetry. As such, the topological invariants of the original models with non-Hermitian skin effect can be obtained from their partners, which share topological phase diagrams can be calculated in an easier way.

I construct the partner of the Floquet operator shown in Fig. 1(B). The reflection symmetry of the model is due to the change of the hopping terms \( t_{1,±} \) alternately in the adjacent unit cell, and causes the skin effect to disappear [50]. For simplicity of the introduction, the diagonal term \(-i\omega \delta_{n,m}\) in Eq. (3) is removed firstly and \( k_f \) is good quantum number. As shown below, the existence of the diagonal term don’t modify the conclusion. The partner Hamiltonian reads
\[ \tilde{h}(k, k_f) = \Omega_k \hat{h} \Omega_k^{-1} \]
where \( \Omega_k \left( \begin{array}{c}
\alpha_{k_x} \\
\beta_{k_x}
\end{array} \right) = \left( \begin{array}{c}
a_{k_x}^+ \\
b_{k_x}
\end{array} \right) \) is the creation operation of the lattice \((ab)\) in unit cell of Fig 1. The partner Hamiltonian \( \hat{h} \) has the \( \mathcal{PT} \) symmetry and follows the relation \([\mathcal{PT}, \hat{h}] = 0\). \( \mathcal{P} \) and \( \mathcal{T} \) are defined as the space-reflection (parity) operator and the time-reversal operator, whose effects are given by \( k \rightarrow -k, x \rightarrow -x \) and \( k \rightarrow -k, x \rightarrow x, i \rightarrow -i \), respectively. The wave vectors \( k \) and \( k_f \) are real.

The partner model and original model are topologically equivalent. If we introduce a parameter \( \theta \) in the model, i.e.
\[ h(\theta) = \left( \begin{array}{cccc}
0 & t_1 + \frac{\gamma}{2} & 0 & t_2 e^{-ik} \\
t_1 - \frac{\gamma}{2} & 0 & t_2 & 0 \\
0 & t_2 & 0 & t_1 - \frac{\gamma}{2} \cos \theta \\
t_2 e^{ika} & 0 & t_1 + \frac{\gamma}{2} \cos \theta & 0
\end{array} \right), \]  \hspace{1cm} (7)

the original Hamiltonian \( \hat{h} \) of Eq. 5 can be continuously deformed into the partner Hamiltonian \( \tilde{h} \) of Eq. 7 when changing the parameter \( \theta \) from \( \pi \) to \( 0 \). The Hamiltonian in Eq. 8 also has a chiral symmetry \( \Sigma^{-1} h(\theta) \Sigma = -h(\theta) \) with \( \Sigma = \sigma_0 \otimes \sigma_z \) and \( \sigma_0 \) and \( \sigma_z \) are unity matrix and the Pauli matrices respectively. The chiral symmetry ensures that the eigenvalues of Hamiltonian \( h(\theta) \) appear in \((e_j, -e_j)\) pairs with \( j = 1, 2 \).

In the real-space, the original Hamiltonian \( \hat{h} \) and partner Hamiltonian \( \tilde{h} \) are related by a similarity transformation
\[ \tilde{h} = S_0^{-1} \hat{h} S_0. \]  \hspace{1cm} (9)

\( S_0 \) is a diagonal matrix whose diagonal elements are \( \{1, r^2, r^4, \ldots, r^{2L}\} \) and \( r = \sqrt{\frac{\gamma}{2|\tau|}} \) here \( L \) is the number of unit cell. The real-space eigen-equation \( \tilde{h}\psi = E\psi \) is equivalent to \( \hat{h}\tilde{\psi} = E\tilde{\psi} \) with \( \tilde{\psi} = S_0^{-1} \psi \) where the eigenvalue \( E \) remains unchanged.

The diagonal matrix \( S_0 \) can be decomposed into the products of \( S_1 \) and \( S_2 \) \((S_0 = S_1 S_2)\) where \( S_1 \) is a diagonal matrix whose diagonal elements are \( \{1, r, r^2, r^3, \ldots, r^{L-1}, r^L\} \) and \( S_2 \) is a diagonal matrix whose diagonal elements are \( \{1, r, r^2, \ldots, r^{L-1}, r^L\} \) and \( L \) \( \equiv \) 1. The modified Hamiltonian \( \tilde{h} \) can be constructed by two similarity transformations. With the similarity transformation as done in Eq.
\[ \tilde{h} = S_1^{-1} \hat{h} S_1, \]  \hspace{1cm} (10)

\( \hat{h} \) becomes the standard SSH model
\[ \tilde{h} = (\tilde{t}_1 + t_2 \cos k) \sigma_x + t_2 \sin k \sigma_y, \]  \hspace{1cm} (11)

for \( |t_1| > |\gamma/2| \), with intracell and intercell hopping \( \tilde{t}_1 = \sqrt{(t_1 - \gamma/2)(t_1 + \gamma/2)} \) and \( t_2 = \tau' \). This result has been obtained in Ref. [19]. Then doing the other similarity transformation \( \tilde{h} = S_2^{-1} \hat{h} S_2 \), \( \tilde{h} \) becomes the partner Hamiltonian \( \hat{h} \) in Eq. (7).
The corresponding eigenvalues $\hat{e}$ of the original operator $\hat{h}$ are complex and the corresponding eigenvalues $\hat{\epsilon}$ of the partner operator $\hat{h}$ are real. The relationship between $\hat{e}$ and $\hat{\epsilon}$ can be understood as follow. The eigenvalues $\hat{e}$ can be obtained by a unitary transformation, i.e. $\hat{e} = \hat{U}^\dagger_{L} \hat{h} \hat{U}_{R}$, here $\hat{U}_{L}^\dagger \hat{U}_{R} = \hat{U}_{R}^\dagger \hat{U}_{L} = I_0$ and $I_0$ is the unitary matrix. Due to the non-Hermitian of operator $\hat{h}$, the unitary operator $\hat{U}_{L}$ and $\hat{U}_{R}$ must be differentiated. The corresponding eigenvalues $\hat{\epsilon}$ of the partner operator $\hat{h}$ can be obtained by the other unitary transformation $\hat{\epsilon} = \hat{U}^\dagger \hat{h} \hat{U}$, and $\hat{U}$ is the unitary matrix. Due to the $\mathcal{PT}$ symmetry of $\hat{h}$, it is unnecessary to differentiate the left and right unity matrices. From the relationship between the original and partner operators in Eq. (3), I can easy get the relationship of eigenvalue matrix between original and partner operators $\hat{e} = \hat{U}^\dagger_{L} \hat{e} \hat{U}_{R}$. It is also a unitary transformation $U_{L}^\dagger U_{R} = U_{R}^\dagger U_{L} = I_0$, here the left and right unity matrices must be distinguished with $U_{L}^\dagger = \hat{U}_{L}^\dagger S_{\theta} U_{L}$ and $U_{R} = \hat{U}_{R}^\dagger S_{-\theta} \hat{U}_{R}$.

It should point that this method is general effective to the model including the nearest-neighbor interaction only. When the next nearest-neighbor term $t_2$ exists in the model further, this method is general fail since it is difficult to guarantee the reflection symmetry of the $t_2$ and $t_3$ terms simultaneously under the similarity transformation.

When the diagonal term $-\hat{\mu}_{\omega} \delta_{n,m}$ in Eq. (3) is included in the model, we can also use this method to construct the partner model. The reason is that the similarity transformation of Eq. (10) is effectively applied to the $x$ direction and the diagonal part $-\hat{\mu}_{\omega} \delta_{n,m}$ of matrix (3) remains unchanged since it is equivalence to electric field applying to the $x$ direction. The $\mathcal{PT}$ transformation also occurs in the $x$ direction, the partner Hamiltonian also have the $\mathcal{PT}$ symmetry and its eigenvalues are real. When changing the parameter $\theta$ from $0$ to $\pi$, the original Hamiltonian $\hat{h}$ of Eq. (3) can be continuously deformed into the partner Hamiltonian and also remains $-\mu_{\omega} \delta_{n,m}$ unchanged. So the partner is a topological equivalent to the original model.

B. Topological invariant and phase transition in low frequency limit

From Eq. (3), the external ac field modifies the effective hopping of the model which changes the QFTs. $A_0$ and $k_f$ are adjustable parameters of the one dimensional problem. The Hamiltonian (4) and its partner (7) belongs to the BDI class, the winding number along the $k$ direction is the topological invariant that differentiates the system from an ordinary insulator [48][51]. To calculate the winding number of the partner Hamiltonian $\hat{h}_k$ in Eq. (7), the Hamiltonian $\hat{h}_k$ is transformed into block off-diagonal form

$$U \hat{h}_k U^{-1} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
t_1 + \frac{1}{2} \gamma & t_2 e^{i k} & t_2 & t_1 - \frac{1}{2} \gamma \\
t_2 e^{-i k} & t_1 + \frac{1}{2} \gamma & 0 & 0
\end{pmatrix}$$

(11)

with the unitary matrix

$$U = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}.$$

With the block off-diagonal Hamiltonian in Eq. (11), the winding number is defined by [52][53]

$$\mathcal{W} = -\oint \frac{dk}{2\pi} \gamma \left\{ \ln \left| \det \left( \frac{1}{2} \gamma + t_1, t_2, t_1 - \frac{1}{2} \gamma \right) \right| \right\}$$

(12)

$$= -\oint \frac{dk}{2\pi} \gamma \left\{ \ln \left( \tilde{t}_1^2 - \frac{\gamma^2}{4} + t_2 e^{i k} \right) \right\}.$$

The winding vector is $\tilde{t}_1^2 - \frac{\gamma^2}{4} + t_2 e^{i k}$. We get the transition points are

$$\tilde{t}_1^2 = \frac{\gamma^2}{4},$$

(13)

namely, $t_1 = t_2$ which is just the results obtained in Ref. [19]. In the present example of the driven dimer chain, the topologically boundary is related with the equation (13). In the limit $A_0 \to 0$, $|\gamma/2| \to 0$ which results to $t_1 = \gamma/\tau = I(A_0) \to 1$. This is the phase transition point of bared SSH model. If we assume $A_0 \leq 1.5$, the contributions from $I_{p,6} (A_0)$ can be neglected. With a fixed $A_0$, the parameter $|\gamma/2|$ in Eq. (3) can be taken in the range from the minimum $|\gamma/4|$ to the maximum $|\gamma/4|$ when changing $k_f$ from $0$ to $2\pi$.

So the range of the parameter $\sqrt{\tilde{t}_1^2 - \gamma^2/4}$ is from $t_1,_{\text{min}} = \sqrt{t_1^2 - \gamma^2/4}$ to $t_1,_{\text{max}} = \sqrt{t_1^2 - \gamma^2/4}$. When $t_2 > t_1,_{\text{max}}$, the model is in the topological phase. While the case of $t_1,_{\text{min}} < |t_2| < t_1,_{\text{max}}$, there are some $k_f$ with $|t_2| > \sqrt{\tilde{t}_1^2 - \gamma^2/4}$ and some $k_f$ with $|t_2| < \sqrt{\tilde{t}_1^2 - \gamma^2/4}$. The former is corresponding to the topological phase and the latter is corresponding to the topological trivial phase. The model is in the topological phase, actually. When parameters $|t_2| < t_1,_{\text{min}}$ however, it is in the trivial case of SSH model. Therefore, the topological phase transition point is at $|t_2| = t_1,_{\text{min}}$. $|t_2| = t_1,_{\text{min}}$ as function of $A_0$ is plotted in Fig. 2 (red solid line).

It is interesting to compare the phase boundary to the case driven by a real ac field. When the SSH model driven by a real ac field, the effective Hamiltonian $\hat{h}(k,k_f)$ is given by

$$\hat{h}(k,k_f) = \tau \begin{pmatrix}
0 & t_2 e^{i k} \\
0 & 0
\end{pmatrix},$$

here

$$t_1(k_f) = J_0 + 2i \sum_{l=2n+1} J_y \sin l k_f + 2 \sum_{l=2n} J_1 \cos l k_f$$

(14)

Follow the analysis in the case of imaginary ac field, the topological phase transition point is at $t_2 = t_1,_{\text{min}}$. $t_2 = t_1,_{\text{min}}$ as function of $A_0$ is plotted in Fig. 2 (red dashed line).
FIG. 2. The phase boundaries of the effective 2D lattice model driven by the imaginary ac field (blue solid line) and the real ac field (red dashed line). The upper part of the boundary is in topological phase and the lower part of the boundary is in insulator phase.

C. Beyond the low and high frequency limit

I study the partner model numerically to understand the topological phase beyond the low and high frequency limit. The eigenvalues of the equivalent model are real due to its $\mathcal{PT}$ symmetry. Since the high level modified Bessel function compared with that of zero level are small enough to be neglected, I numerically diagonalize the partner of Eq. (3) without need a larger number of sidebands to reach convergence. In the present case $A_0 = 4$ and $I_7(A_0)/I_0(A_0) = 0.0037$, the matrix element $I_{m-n>7}$ can be neglected. Fig. 3 shows quasienergy spectrum vs $A_{0}$ for $n, m = 7$ sidebands.

When $\omega > 18$, the system is in the high frequency regime and the critical point $A_0 = 1.8079$ is almost fixed and meets $I_0(A_0) = t_2 = \tau'/\tau = 2$. When $A_0 > 1.8079$ in Fig. 3(a), the system exists the topologically protected states. With decreasing the frequency from the high frequency limit, the coupling between different Floquet bands cannot be neglected and induces the bands inversions near $A_0 = 4$ and $A_0 = 5.6$ in Fig. 3(b). Bands inversions result to the opening and closing the band gap and change the topological properties of the model. The topological phase transition occur at the exact crossings between conduction and valence band in Fig. 3(c).

IV. SUMMARY

In summary, with the Floquet-Bloch approach, we have mapped the dimer chain driven by an imaginary ac field to a 2D effective TB model to study the QPTs driven by an imaginary external parameter. To investigate the QPTs of the original non-Hermitian model, I construct a topologically equivalent model which fulfils the $\mathcal{PT}$ symmetry. The merit of the method is the real energy spectra of the partner model allows to study the topological properties as the Bloch bands of the Hermitian system without the introduction of the generalized Brillouin zone. The similarity transformation is given to illuminate the relationship between the partner Hamiltonian and the original Hamiltonian. I have obtained the phase boundary of driven dimer chain in different frequency regime. This method is expected useful to study the non-Hermitian systems with the non-Hermitian skin effect. The predicting novel topological states of matter which otherwise would be inaccessible in the real field case can be realized experimentally.

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