Acoustic Attenuation in High-$T_c$ Superconductors

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Abstract

We analyze the acoustic attenuation rate in high-$T_c$ superconductors, and find that this method offers an additional way to examine the anisotropy of the superconducting order parameter in these materials. We argue that it should be possible to distinguish the electronic contribution to the acoustic attenuation, which has a strong temperature dependence near $T_c$, from the lattice contribution, which does not show a strong temperature dependence near $T_c$. We propose that this can be utilized to measure the anisotropy of the order parameter by measuring the attenuation rate near $T_c$ in different directions.

Keywords: anisotropic superconductor, energy gap, ultrasound attenuation
1 Introduction

The symmetry of the order parameter $\Delta_q$—the energy gap in the quasiparticle excitation spectrum—in the high-$T_c$ superconductors (HTSC’s) has been a very active area of research in the last few years. Assuming that the order parameter is a spin singlet as seems to be indicated by Knight-shift experiments [1], the angular pairing state has to be even, leaving $s$-wave ($L=0$) and $d$-wave ($L=2$) as the main alternatives. Different theories make different predictions for the order parameter, which is the reason for the interest in the symmetry of $\Delta_q$.

There are essentially two classes of experiments to determine the symmetry of the energy gap, those that are sensitive to the magnitude of the order parameter, and those that are sensitive to the phase of it. Measurements of the acoustic attenuation [2] and the NMR nuclear-spin relaxation rate [3], which belong to the first category, were instrumental in establishing the BCS theory in the conventional superconductors [4]. Acoustic Attenuation was also used to examine anisotropy in the conventional superconductors due to the crystal symmetry [5].

However, while many NMR experiments have been performed on the HTSC’s (cf. the review by Slichter et al. [6]), acoustic attenuation experiments have been neglected in recent years. We will discuss the theory of acoustic attenuation for the HTSC’s, and propose an experiment to probe the symmetry of the order parameter using acoustic attenuation.

Acoustic attenuation is observed with an experimental setup [7], where an ultrasonic signal, typically with frequencies ranging from 100 MHz to 10 GHz, is fed into a sample through a transducer quartz. The resulting phonons with wave vector $\mathbf{q}$, energy $\omega$, and polarization $\lambda$ can be scattered by quasiparticles, and the remaining fraction of phonons is observed at the other end of the sample. In addition to the electronic mechanism, where the phonons are scattered by quasiparticles, there will also be a lattice contribution to the
attenuation rate. We will discuss how the electronic contribution can be resolved against the lattice background.

2 Acoustic Attenuation

We consider processes in which phonons are absorbed by a sample through scattering of quasiparticles from a state $p_1$ into a state $p_2$, where $q = p_2 - p_1 + K$ is the momentum of the incoming phonon and $K$ is a reciprocal lattice vector. The inverse process of spontaneous emission of phonons by quasiparticles also has to be taken into account.

To understand the effect of anisotropy of the order parameter it is important to understand how the involved momenta are related. The quasiparticle is scattered from a state $p_1$ near the Fermi surface to another state $p_2$, which also has to be near the Fermi surface. This means that both $p_1$ and $p_2$ are of order $k_F$. $q$, the phonon momentum, is smaller by some orders of magnitude. This means that $p_1$ and $p_2$ point essentially in the same direction.

The HTSC’s are quasi-two-dimensional layered materials. Their Fermi surface is nearly cylindrical with little dependence on the coordinate in the $c$-direction. For a three-dimensional Fermi sphere, $p_1$ and $p_2$ are fixed on a belt around the Fermi surface [5], perpendicular to the direction of the phonon momentum $q$. For a given phonon momentum $q$ in the $a$-$b$-plane in the quasi-two-dimensional case, this belt degenerates to two points on opposite sides of the Fermi surface. Thus, the order parameter is probed only in a specified direction. This provides an opportunity to measure the anisotropy in the magnitude of the order parameter.

The interaction of phonons with quasiparticles can be written as

$$H_{el-ph} = \sum_{p_1,p_2,s,\lambda} g_{p_1,p_2,\lambda}(a_{q\lambda} + a_{-q\lambda}^\dagger)c_{p_2s}^\dagger c_{p_1s},$$

where the relation between $q$, $p_1$, and $p_2$ was given above. Here $g_{p_1,p_2,\lambda}$ is the interaction strength, $a_{q\lambda}$ is a phonon destruction operator, and $c_{p_1s}$ an electron destruction operator.
The electron operators $c, c^\dagger$ can be transformed into superconducting quasiparticle operators $\gamma, \gamma^\dagger$ with a standard Boguljubov transformation.

Starting from this Hamiltonian it is straightforward to derive an expression for the acoustic attenuation rate (e.g. cf. Schrieffer [4]) $\alpha_{q,\lambda}$ with

$$\alpha_{q,\lambda} = 4\pi \sum_{p_1, p_2} |g_{p_1, p_2, \lambda}|^2 n^2(p_1, p_2) (f_{p_1} - f_{p_2}) \delta(E_{p_2} - E_{p_1} - \omega_{q,\lambda})$$

for phonons of wave vector $q$ and polarization $\lambda$. The involved phonon frequency $\omega_{q,\lambda}$ is given by the dispersion relation of the phonons. The square of the so-called coherence factor $n$ is evaluated to be

$$n^2(p_1, p_2) = (u_{p_1} u_{p_2} - v_{p_1} v_{p_2})^2 = \frac{1}{2} \left( 1 + \frac{\epsilon_{p_1} \epsilon_{p_2} - \Delta_{p_1} \Delta_{p_2}}{E_{p_1} E_{p_2}} \right).$$

Here, $\epsilon_p$ is the single-particle energy relative to the Fermi energy and $E_p$ is the superconducting quasiparticle excitation energy, $E_p = \sqrt{\epsilon_p^2 + \Delta_p^2}$. $f_q \equiv 1/(1+\exp(E_q/k_B T))$ is the Fermi function. We assume that $g_{p_1, p_2, \lambda}$ only depends on the momentum transfer $q = p_2 - p_1$.

It is assumed that the order parameter depends only on the direction, but not on the magnitude of $p$, and that the angular dependence of the order parameter does not change with temperature $T$.

Finally we are led to

$$\alpha_{q,\lambda} = \text{const.} \times \int d\epsilon_{p_1} d\epsilon_{p_2} \left( 1 - \frac{\Delta_{p_1} \Delta_{p_2}}{E_{p_1} E_{p_2}} \right)$$

$$\times (f(E_{p_1}) - f(E_{p_2})) \delta(E_{p_2} - E_{p_1} - \omega_{q,\lambda})$$

$$= \text{const.} \times \int dE_{p_1} \frac{E_{p_1}}{\sqrt{E_{p_1}^2 - \Delta_{p_1}^2}} \frac{E_{p_1} + \omega_{q,\lambda}}{\sqrt{(E_{p_1} + \omega_{q,\lambda})^2 - \Delta_{p_2}^2}}$$

$$\times \left( 1 - \frac{\Delta_{p_1} \Delta_{p_2}}{E_{p_1}(E_{p_1} + \omega_{q,\lambda})} \right) (f(E_{p_1}) - f(E_{p_1} + \omega_{q,\lambda})).$$

We do not get an angular integration because $q$ picks certain values for the directions of $p_1$ and $p_2$. These directions, $\theta_1$ and $\theta_2$, are very close to one another due to momentum
conservation as argued above, and can be controlled experimentally. For our calculations we used $\hbar \omega_{q\lambda} = 10^{-5} k_B T_c$. The results are quite insensitive to the value of $\omega$ as long as $\hbar \omega << k_B T_c$.

3 Symmetry of the Energy Gap

To calculate the acoustic attenuation rate $\alpha_{q,\lambda}(T)$, the temperature dependence of the reduced gap, $\Delta_0(T)/\Delta_0(0)$, will be assumed to be BCS-like as a function of reduced temperature. We have tested that this assumption is consistent with both an $s$- and $d$-wave gap with appropriate potentials in the BCS gap equation [8]. For the angular dependence of the gap, $\Delta(\theta)$, different models will be examined:

$$\Delta(\theta, T) = \begin{cases} 
\Delta_0(T) & \text{isotropic s-wave,} \\
\Delta_0(T) \cos(2\theta) & \text{d-wave,} \\
\Delta_0(T) [a \cos^2(2\theta) + (1 - a)] & \text{anisotropic s-wave.}
\end{cases} \quad (5)$$

In conventional superconductors one calculates the attenuation rate in the superconducting state normalized to the attenuation rate in the normal state. In HTSC’s, however, this is not an interesting quantity because it is not experimentally accessible. In conventional superconductors one can always drive the system into the normal state even at temperatures much below $T_c$ by applying a sufficiently large magnetic field. In HTSC’s, however, the critical fields are prohibitively high. Thus, we normalize the attenuation rate to its value at $T_c$.

Evaluating Eq. (4) with $\mathbf{q}$ pointing in different directions relative to the lattice amounts to taking different effective magnitudes of $\Delta$. We consider $q/k_F$ very small so that $\theta_1 \simeq \theta_2$, and thus $\Delta_{p_1} \simeq \Delta_{p_2}$. The direction of $\mathbf{q}$ can be controlled experimentally. Fig. 1 shows the attenuation rate as a function of temperature for different effective magnitudes of the
gap. A $2\Delta_0(T = 0)/k_B T_c = 3.5$ corresponds to the isotropic BCS case. Because for a particular $\mathbf{q}$ only the magnitude of the gap in the direction perpendicular to $\mathbf{q}$ enters, Fig. 1 holds for a particular $\mathbf{q}$ no matter what the symmetry of the gap is. This is true as long as the temperature dependence of the gap in that direction behaves like the BCS temperature dependence.

At the high temperatures near $T_c$, $T_c \approx 90$ K, lattice contributions to the attenuation rate become important, but the strong temperature dependence of the electronic contribution near $T_c$ provides an opportunity to still measure it. Since the superconducting transition should not affect any but the electronic contribution to the attenuation, measuring the attenuation rate just above and below $T_c$ allows one to separate electronic contributions from lattice contributions. In particular any eventual anisotropy of a lattice contribution should not be influenced by the superconducting transition, at least as long as the temperature difference between the measurements is not too large. Using this approach, the anisotropy of the electronic attenuation rate becomes an experimentally accessible quantity, which can be used to examine the symmetry of the superconducting gap in the HTSC’s.

For an isotropic $s$-wave gap the electronic contribution to the attenuation rate should not change as the crystal is rotated. For an anisotropic order parameter, either $s$- or $d$-wave, maxima should be observed in directions perpendicular to where the order parameter is a minimum or even has nodes. At a node the attenuation rate should go up to the normal state value at the corresponding temperature.

Fig. 2 shows how the acoustic attenuation rate varies at a temperature of 0.95 $T_c$ for different symmetries of the order parameter as a function of the direction of the incoming phonons relative to the lattice. A value of $2\Delta_0(T = 0)/k_B T_c = 6$ is assumed. All rates are normalized to the acoustic attenuation rate without a gap, which is essentially the electronic attenuation rate at $T_c$, since normal electronic contributions should not vary much over
this small range of temperatures. While all symmetries show a significant suppression in certain directions, which allow the resolution of the electronic contribution against the lattice background, the very anisotropic symmetries show little or no suppression in those directions where minima or nodes of the gap are located.

4 Discussion and Summary

The acoustic attenuation method, which was very successful in verifying BCS theory for conventional superconductors, has the potential to provide useful information on the order parameter in high \( T_c \) materials.

Early on some measurements have been made (Yusheng et al. [9]), but it has been argued [10] that the effect seen was too large to be the actual electronic contribution to the attenuation rate. However, since then sample qualities have been improved considerably, and, with our proposed focusing on temperatures near \( T_c \), this method has a potential that has not been exploited yet.

Won and Maki [11] recently also discussed acoustic attenuation in HTSC’s; however, they do make some additional approximations to solve the problem analytically, only consider \( d \)-wave, and argue that at low temperatures, where the rate is already strongly suppressed, this rate should be strongly anisotropic.

We propose that the attenuation be measured at temperatures near \( T_c \), where one should see the anisotropy, but still have a measurable rate. Any observed sharp drop in the attenuation rate near \( T_c \) can be attributed to the electronic properties of the system since the lattice properties should not change dramatically near the superconducting transition.

Note Added in Proof: After the present paper was submitted for publication, we learned of a paper by Kostur et al. [12] (KBF) which reports on calculations of ultrasonic
attenuation in a model d-wave superconductor. The results are quite similar to those presented here. We feel that our paper compliments KBF in that we consider anisotropic s-wave superconductors in addition to d-wave. Also, we suggest that the best way to separate the anisotropic attenuation due to quasiparticles from that due to the lattice is to compare results just above $T_c$ with those just below $T_c$, whereas KBF seem to advocate looking at low temperatures.

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Figure 1: Acoustic attenuation rate for an effective order parameter for different values of $2\Delta_0/kT_c$. Here the zero-temperature $\Delta_0$ corresponds to a gap in a particular direction.

Figure 2: Angular dependence of the attenuation rate for different symmetries at a temperature $T = 0.95T_c$ with $2\Delta_0(T = 0)/k_BT_c = 6$, where $\Delta_0(T)$ is from one of the expressions in Eq. (5). All rates are normalized to the attenuation rate without a gap.
$2\Delta_0/kT_c = 3.5$
$2\Delta_0/kT_c = 4.5$
$2\Delta_0/kT_c = 6.0$
$2\Delta_0/kT_c = 8.0$
$\frac{\alpha(T)}{\alpha(T_c)}$ vs $\theta$

- **D-wave**
- **Anisotropic s-wave, $a=0.75$**
- **Anisotropic s-wave, $a=0.50$**
- **Anisotropic s-wave, $a=0.25$**
- **Isotropic s-wave**