The case for decaying spin-3/2 dark matter

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We couple a sterile neutrino sector to a spin-$\frac{3}{2}$ particle and show that with a Planck reduced coupling, we can naturally obtain a sufficiently long lifetime making the spin-$\frac{3}{2}$ particle a good dark matter candidate. We show that this dark matter candidate can be produced during inflationary reheating through the scattering of Standard Model particles. The relic abundance as determined by \textit{Planck} and other experimental measurements is attained for reasonable values of the reheating temperature $T_{RH} \gtrsim 10^9$ GeV. We find a large range of masses are possible which respect the experimental limits on its decay rate. We expect smoking gun signals in the form of a monochromatic photon with a possible monochromatic neutrino, which can be probed in the near future in IceCube and other indirect detection experiments.

I. INTRODUCTION

Almost 90 years ago, in 1933, F. Zwicky published work that shed the first light on the presence of dark matter in the Coma cluster [1], confirmed by Babcock in his PhD thesis while measuring the rotation curves of Andromeda [2]. Several studies, including the study of the stability of large scale structures [3] confirmed the hypothesis of a dark component in the Universe. Dark matter composed of a new weakly interacting massive neutral particle (WIMP) was proposed by Steigman \textit{et al}. [4] in 1978, and its precise abundance determination was made from CMB measurements by the \textit{Planck} satellite [5] and other experiments. Despite being attractive, the WIMP paradigm is in tension with direct detection measurements (see [6] for a review). Indeed, the limit on the WIMP-nucleon scattering cross section is $\sigma_{\chi p} \lesssim 10^{-46}$ cm$^2$ for $m_\chi = 100$ GeV [7–9]. The next generation of experiments will probe cross sections as low as $\sigma_{\chi p} \lesssim 10^{-48}$ cm$^2$ [10], approaching the irreducible neutrino background [11], which correspond to a Beyond the Standard Model (BSM) scale of roughly 1 PeV, i.e. significantly above the electroweak scale. The WIMP paradigm is based on the supposition that the dark matter was initially in thermal equilibrium with the Standard Model (SM) sector before decoupling (freeze-out). As such, its lack of dependence on initial conditions remains attractive. However, its lack of discovery to date may be implying that out-of-equilibrium processes dominate the production of dark matter. The freeze-in paradigm [12, 13] is an interesting alternative.

The main idea behind freeze-in is that the dark sector is highly secluded from the visible sector. This seclusion may be due to a coupling so small so as to prevent the dark matter from equilibrating with the SM thermal bath. Unlike a WIMP and thermal freeze-out, any dark matter produced this way will have its abundance frozen in. An early example of such a candidate is the gravitino [14], produced through thermal scattering during reheating [15–17], yet never achieving equilibrium. There are of course many other options which include the existence of a very heavy mediator, above the maximum temperature reached during reheating, but below the Planck scale, which also prevents the dark sector from coming into thermal equilibrium with the primordial plasma.

This framework is quite common in SO(10)-like models [18, 19], high-scale SUSY [20–22], moduli-portals [23], spin-2 portals [24], $Z'$ portals [25] or other types of heavy mediators [26]. Depending on the specific model, the production rate may be sensitive to the details of reheating, and in particular, to the effects of non-instantaneous reheating [17, 27–31], thermalization [32], contributions from inflaton decay [33] or the details of the inflaton potential leading to reheating [29, 34, 35].

Of course, the specific identity of the dark matter candidate can significantly affect its production rate. Nature has provided us with a spin-0 particle, spin-$\frac{1}{2}$ matter fields, spin-1 gauge fields, and a spin-2 graviton. Is a spin-$\frac{3}{2}$ dark matter particle the missing piece in the puzzle? Of course, the gravitino appears naturally in local supersymmetry, or supergravity, and as remarked above, was even one of the first dark matter candidates ever proposed. However, when Rarita and Schwinger in 1941 [36] decided to simplify the (overly general) Fierz-
Pauli framework \cite{37}, proposing a Lagrangian for a free spin-$\frac{3}{2}$ field, they were obviously not motivated by any arguments based on supersymmetry.

It is well known that a massive spin-$\frac{3}{2}$ particle directly coupled to a $U(1)$ gauge field could potentially generate some acausal pathologies \cite{38} if not treated correctly in a coherent UV framework. This is in fact the case for any model including particles with spin $> 1$. For example in $\mathcal{N} = 2$ extended gauged supergravity, the superluminal propagation of the graviphoton is cured by gravitational back-reaction. It is also possible to consider a non-minimal Rarita-Schwinger Lagrangian \cite{39}, by adding non-minimal gauge invariant terms in the action. In any case, we do not consider a spin-$\frac{3}{2}$ particle which is a charged under a $U(1)$ symmetry and the potential issues raised in \cite{38} do not apply to our work.

There have been several studies of spin-$\frac{3}{2}$ dark matter candidates. Effective operators coupling spin-$\frac{3}{2}$ dark matter to the the Standard Model were considered for annihilations (in a freeze-out scenario and indirect detection) and scatterings (for direct detection) in \cite{40}. Effective interactions for spin-$\frac{3}{2}$ dark matter were also considered in \cite{41}. Spin-$\frac{3}{2}$ dark matter has been recently explored in \cite{42} where they proposed a WIMP-like candidate in a Higgs-portal scenario. The detection rate in colliders was considered in \cite{43}. For other recent work see \cite{44}. In every case, the spin-$\frac{3}{2}$ dark matter is protected by a $Z_2$ symmetry to stabilize it as it is otherwise assumed to have weak scale interactions.

In our work, we propose an extremely simple and minimal setup, where the spin-$\frac{3}{2}$ dark matter is coupled only to a single fermion and gauge field strength. The fermion is a SM singlet right-handed neutrino, and therefore, the only SM choice for the gauge field is the hypercharge gauge boson $B_\mu$. This is necessarily a dimension-5 operator and is suppressed by a BSM scale which permits a long lifetime while at the same time serves as a portal to the SM through the left-right mixing in the neutrino sector. We show that for a large part of the parameter space, it is possible to satisfy the lifetime constraints and obtain a sufficient (and not excessive) relic density through production during reheating.

The paper is organized as follows. We introduce the model in Section II. The Lagrangian of interest will include the aforementioned dimension-5 operator and the neutrino sector giving rise to the see-saw mechanism \cite{45}. In Section III, we consider first the dark matter lifetime. Assuming that the dark matter mass, $m_{3/2}$, is less than the mass of the right-handed neutrino, $M_R$, the dark matter can decay into a light neutrino and gauge boson. We then consider the production of dark matter through scattering during reheating and directly from inflaton decay. The allowed parameter space of the model is examined in Section IV, and we consider the observational signatures of the model in Section V. We summarize in Section VI.

\section{The Model}

\subsection{Motivations}

A massive spin-$\frac{3}{2}$ particle is described by the Rarita-Schwinger Lagrangian\footnote{Our metric convention is $g_{\mu\nu} = \text{diag}(+1,-1,-1,-1)$. In Appendix A, we provide a simple derivation of the Rarita-Schwinger Lagrangian.} \cite{36}

\begin{equation}
\mathcal{L}^0_{3/2} = -\frac{1}{2} \bar{\Psi} \left( i \gamma^{\mu\nu} \partial_\rho + m_{3/2} \gamma^{\mu\nu} \right) \Psi_\nu,
\end{equation}

where $\gamma^{\mu\nu} = \gamma^{[\mu,\nu]} = \frac{1}{2} [\gamma^\mu, \gamma^\nu]$ and $\gamma^{\mu\nu\rho} = \gamma^{[\mu,\nu,\rho]}$. One can extract the equations of motion describing a spin-$\frac{3}{2}$ particle

\begin{equation}
i\gamma^{\mu\nu\rho} \partial_\rho \bar{\Psi}_\rho + m_{3/2} \gamma^{\mu\nu} \Psi_\nu = 0, \quad \gamma^{\mu\nu} \partial_\mu \bar{\Psi}_\nu = 0,
\end{equation}

along with an extra condition appropriate for a spin-1 field

\begin{equation}
\partial^\mu \bar{\Psi}_\mu = 0,
\end{equation}

which can be deduced from the preceding constraints. We emphasize that the condition $\gamma^\mu \bar{\Psi}_\mu = 0$ severely limits the operators available to couple a spin-$\frac{3}{2}$ field to the Standard Model sector as we discuss below.

\subsection{The Lagrangian}

A spin-$\frac{3}{2}$ neutral particle, in the presence of a right-handed sector and/or a sterile neutrino sector, will unavoidably generate a coupling of the type

\begin{equation}
\mathcal{L} = \mathcal{L}^0_{3/2} + \mathcal{L}_\nu,
\end{equation}

where

\begin{equation}
\mathcal{L}^0_{3/2} = i \frac{\alpha}{2M_P} \bar{\nu}_R \gamma^\mu [\gamma^\rho, \gamma^\nu] \Psi_\mu F^\rho_\nu + \text{h.c.},
\end{equation}

and\footnote{We leave it for the reader to check that dimension-4 operators of the type $\mathcal{L} |H|^2$ or dimension-5 operators $\mathcal{L}^{5}_{\rho\nu} \Psi_\mu |H|^2$ are the only other Lorentz and gauge invariant operators and vanish due to the constraint given in Eq. (2).}

\begin{equation}
\mathcal{L}_\nu = \nu \bar{\nu}_L \nu_R + \frac{M_R}{2} \bar{\nu}_R \nu_R + \text{h.c.},
\end{equation}

where $F^\rho_\nu = \partial_\rho B_\nu - \partial_\nu B_\rho$ is the field strength of the Standard Model hypercharge gauge boson, $B_\mu$. Since we have scaled $\mathcal{L}^0_{3/2}$ by $M_P^{-1}$ (where $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass), we can allow the coupling, $\alpha$, to take values larger or smaller than 1. Indeed, this...
term is gauge and Lorentz invariant, and can be seen as a low energy term for gravitino dark matter in high-scale SUSY constructions. It is important to note that if a model contains a SM singlet such as a right-handed (or sterile) neutrino, in the absence of symmetry which prevents it, this coupling is present. In supersymmetric models, $R$-parity would prevent the coupling in Eq. (5). If $R$-parity is broken, signatures of gravitino dark matter are typically a $\gamma\nu$ final state, as will be the case here. Even if $M_R \gtrsim m_{3/2}$, a coupling of the type in Eq. (6) will generate 3 and/or 4-bodies decays. As a consequence, spin-$\frac{1}{2}$ dark matter is naturally unstable. We will refer to our metastable candidate as the \textit{raritron}, an obvious tribute to the Rarita-Schwinger field.\footnote{We will not develop this analogy any further as we prefer to remain as general as possible. We note for example, in the $\nu\mu$SSM the right-handed neutrino can mix with the Bino and generate this kind of coupling [46].}

The Yukawa term $y H \nu_L \nu_R$ generates mixing between the neutral ($\nu_R$) and the charged ($\nu_L$) neutrino sectors, and one can define the mass eigenstates

$$\nu_1 = \cos \theta \nu_L - \sin \theta \nu_R$$
$$\nu_2 = \sin \theta \nu_L + \cos \theta \nu_R;$$

with

$$m_1 = \frac{y^2 v^2}{2 M_R}; \quad m_2 \simeq M_R;$$
$$\tan \theta = \frac{\sqrt{m_1}}{m_2} \simeq \frac{y v}{\sqrt{2} M_R},$$

where we have assumed $M_R \gg m_1$ (which corresponds to a classical see-saw mechanism of type I) and $v \simeq 246$ GeV is the vacuum expectation value of the Standard Model Higgs boson. We have considered for simplicity only one active neutrino generation, and the extension to 3 families is straightforward. $\theta$ represents the mixing between the two sectors, and is expected to be small for large values of $M_R$, consistent with recent limits on $m_1$ ($m_1 \lesssim 0.15$ eV [49]).

## III. THE CONSTRAINTS

### A. The lifetime

Depending on its mass, the dominant decay channel for the \textit{raritron}, $\Psi_\mu$, may contain either two or three final states. The 2-body decay channel $\Psi_\mu \rightarrow \nu_1 A_\mu$ is always available. For $m_{3/2} > m_Z$, the $\nu_1 Z_\mu$ final state is open and the 2-body final state dominates for $m_{3/2} < 3/2$

$$y H \nu_L \nu_R \rightarrow \nu_1 A_\mu$$

We will not develop this analogy any further as we prefer to remain as general as possible. We note for example, in the $\nu\mu$SSM the right-handed neutrino can mix with the Bino and generate this kind of coupling [46].\footnote{It is interesting to note that the article immediately following the original work of Rarita-Schwinger [36], computed the $\beta$-decay spectrum of a spin-$\frac{1}{2}$ neutrino [47]. This followed Oppenheimer’s suggestion [48] that the neutrino may have a spin other than $\frac{1}{2}$.}

The decay rates for the 2-body decays $\Psi_\mu \rightarrow A_\mu \nu_1$ and $\Psi_\mu \rightarrow Z_\mu \nu_1$ are

$$\Gamma(\Psi_\mu \rightarrow A_\mu \nu_1) = \frac{\alpha^2 y^2 v^2 m_{3/2} \cos^2 \theta_W}{8 \pi M_P^2 M_R^2},$$
$$\Gamma(\Psi_\mu \rightarrow Z_\mu \nu_1) = \frac{\alpha^2 y^2 v^2 m_{3/2} \sin^2 \theta_W}{8 \pi M_P^2 M_R^2} f\left(\frac{m_Z}{m_{3/2}}\right),$$

where $f(x) = 1 - \frac{1}{8} x^2 + \frac{1}{8} x^3$ and $\theta_W$ denotes the Weinberg angle. The processes that produce the antineutrinos have the same decay rate, i.e. $\Gamma(\Psi_\mu \rightarrow A_\mu \bar{\nu}_1) = \Gamma(\Psi_\mu \rightarrow A_\mu \bar{\nu}_1)$ and $\Gamma(\Psi_\mu \rightarrow Z_\mu \bar{\nu}_1) = \Gamma(\Psi_\mu \rightarrow Z_\mu \bar{\nu}_1)$.

In the limit $m_{3/2} \gg m_Z$, we find the following total 2-body decay width

$$\Gamma_{2b}^{3/2} = \frac{\alpha^2 y^2 v^2 m_{3/2}}{8 \pi M_P^2 M_R^2}. \quad (11)$$

For the 3-body decays $\Psi_\mu \rightarrow A_\mu H \nu_1$ and $\Psi_\mu \rightarrow Z_\mu H \nu_1$, we find

$$\Gamma(\Psi_\mu \rightarrow A_\mu H \nu_1) = \frac{\alpha^2 y^2 m_{3/2} \cos^2 \theta_W}{480 \pi^3 M_P^2 M_R^2} g\left(\frac{m_H}{m_{3/2}}, 0\right),$$
$$\Gamma(\Psi_\mu \rightarrow Z_\mu H \nu_1) = \frac{\alpha^2 y^2 m_{3/2} \sin^2 \theta_W}{480 \pi^3 M_P^2 M_R^2} g\left(\frac{m_H}{m_{3/2}}, \frac{m_Z}{m_{3/2}}\right), \quad (12)$$

where the expression for $g(x, y)$ is given in Appendix B. The 3-body decays to antiparticles have the same production rate $\Gamma(\bar{\Psi}_\mu \rightarrow \bar{A}_\mu \nu_1) = \Gamma(\bar{\Psi}_\mu \rightarrow A_\mu \bar{\nu}_1)$ and $\Gamma(\bar{\Psi}_\mu \rightarrow \bar{Z}_\mu \nu_1) = \Gamma(\bar{\Psi}_\mu \rightarrow Z_\mu \bar{\nu}_1)$.

In the limit $m_{3/2} \gg m_H, m_Z$, the total 3-body decay rate is given by

$$\Gamma_{3b}^{3/2} = \frac{\alpha^2 y^2 m_{3/2}^5}{480 \pi^3 M_P^2 M_R^2}. \quad (14)$$
The total 2- and 3-body decay rates when \( m_{3/2} \gg m_B, m_2 \) correspond to lifetimes
\[
\tau_{3/2}^{2b} \simeq 1.6 \times 10^{29} \left( \frac{10^{-2}}{y} \right)^2 \left( \frac{M_B}{10^{14} \text{ GeV}} \right)^2 \left( \frac{10^4 \text{ GeV}}{m_{3/2}} \right)^3 \text{s},
\]
\[
\tau_{3/2}^{3b} \simeq 5.6 \times 10^{28} \left( \frac{10^{-2}}{y} \right)^2 \left( \frac{M_B}{10^{14} \text{ GeV}} \right)^2 \left( \frac{10^4 \text{ GeV}}{m_{3/2}} \right)^5 \text{s}.
\]

Note that in contrast to [50] (but like [51]), the 4-body decay will not dominate over the 3-body decay for large values of \( m_{3/2} \), because of a suppression factor of order \( (m_{3/2}/M_B)^2 \) between the two modes of decay.

### B. The relic abundance from scattering

The *raritron* can be produced directly from the thermal bath during reheating, which is assumed to be a result of inflaton decay. To compute the dark matter density, \( n_{3/2} \), we consider the out-of-equilibrium dark matter annihilation processes, \( H + \nu_1 \rightarrow B + \Psi_\mu, H + B \rightarrow \nu_1 + \Psi_\mu, \) and \( B + \nu_1 \rightarrow H + \Psi_\mu \), as depicted in Fig. 2. We can write the Boltzmann equation as
\[
\frac{dn_{3/2}}{dt} + 3H n_{3/2} = R(T),
\]
where the Hubble parameter for the radiation-dominated Universe is given by
\[
H(T) = \frac{\pi \sqrt{g_*}}{\sqrt{90}} \frac{T^2}{M_P}.
\]

It is convenient to rewrite the Boltzmann equation (16) as
\[
\frac{dY_{3/2}}{dT} = - \frac{R(T)}{H(T) T^4},
\]
with \( Y_{3/2} = \frac{n_{3/2}}{T^3} \).

The dark matter production rate (per unit volume per unit time) is represented by
\[
R(T) = \frac{1}{1024 \pi^6} \int f_1 f_2 E_1 dE_1 E_2 dE_2 d\cos \theta_{12} \int |\mathcal{M}|^2 d\Omega_{13},
\]
for the processes \( 1 + 2 \rightarrow 3 + 4 \), where 1 and 2 correspond to particles in the thermal bath, 3 and 4 correspond to produced particles, \( f_1 \) and \( f_2 \) represent the thermal distribution functions of the incoming particles, and \( \mathcal{M} \) is the scattering amplitude for the processes shown in Fig. 2, with the expressions for the scattering amplitudes given in Appendix B. From these, we find the following dark matter production rate,
\[
R(T) = \frac{338 (5^2 \alpha^2 y^2 T^{10}}{\pi^5 M_P M_B m_{3/2}^2},
\]
where \( \zeta(n) \) is the Riemann zeta function.

It is useful to compare the raritron production rate to that of the gravitino in supersymmetric theories. In weak scale supersymmetry, the dominant production channel is gluon + gluon \( \rightarrow \) gravitino + gluino. The dimensionful contributions to the cross section for this process originate from the gravitino vertex \( (1/m_{3/2}^2 M_B^2) \), the gluino propagator \( (m_3/2 T^4) \), and \( T^4 \) from phase space, so that the cross section scales as \( m_3^2/m_{3/2}^2 M_B^2 \). In this case, the production rate scales as \( T^4 m_3^2/m_{3/2}^2 M_B^2 \). For the case of raritron production, when \( M_R \gg T_{RH} \), the contribution from the propagator is instead \( 1/M_R^2 \) so that the cross section scales as \( T^4/m_{3/2}^2 M_B^2 M_R^2 \) giving a production rate which scales as in Eq. (20).

Since the temperature dependence of the production rate, \( T^n \), has \( n < 12 \), the final density dark matter density is mostly sensitive to the reheat temperature, \( T_{RH} \), rather than the maximum temperature attained during the reheating process [28]. Therefore, after integration of Eq. (18), the density at \( T_{RH} \) can be written
\[
\rho(T_{RH}) = \sqrt{\frac{2}{5 g_{RH}} \frac{1014 \zeta(3) \alpha^2 y^2 T_{RH}^8}}{\pi^6 M_P M_B m_{3/2}^2},
\]
from which we can calculate the present relic abundance at temperature \( T_0 \):
\[
\Omega h^2 \simeq 10^9 \frac{n(T_{RH})}{\text{cm}^{-3}} \left( \frac{g_{RH}}{427/4} \right) \left( \frac{T_0}{T_{RH}} \right)^3 \left( \frac{m_{3/2}}{10^4 \text{ GeV}} \right)^5
\]
\[
\simeq 0.1 \left( \frac{\alpha}{1.1 \times 10^{-3}} \right)^2 \left( \frac{27/4}{g_{RH}} \right)^{3/2} \left( \frac{T_{RH}}{10^{10} \text{ GeV}} \right)^5
\]
\[
\times \left( \frac{\alpha}{0.15 \text{ eV}} \right) \left( \frac{10^{14} \text{ GeV}}{M_R} \right) \left( \frac{10^4 \text{ GeV}}{m_{3/2}} \right),
\]
where \( g_0 = 43/11 \). In writing Eq. (22), we have substituted Eq. (9) for \( y \), assuming a characteristic mass of 0.15 eV for the light neutrino. Note that Eqs. (21) and (22) are derived using an instantaneous reheating approximation. Dropping this approximation results in a density which is about two times larger for a production rate proportional to \( T^4 \) as in Eq. (20) [28].

It is interesting to note that the same set of parameters which provide a sufficiently long-lived raritron so as to respect the indirect detection constraints, Eq. (15), also produce a relic density in agreement with Planck data, Eq. (22) for a reasonable reheating temperature \( T_{RH} \approx 10^9 \) GeV.

The scattering processes considered in Fig. 2 lead to a scattering cross section that scales with the fourth power of the energy of the scatterers, \( \sigma \sim s^2 \) (c.f. Eq. (B3) in Appendix B). In the classification of [28], this corresponds to the \( n = 4 \) scenario. For such a steep dependence on the energy of the scatterers, the instantaneous thermalization approximation can severely underestimate the magnitude of the relic abundance. Indeed, in [32] it was found that the production of particles from scatterings in the not-yet-thermalized relativistic plasma, present at the earliest stages of reheating, will generically determine the dark matter abundance if \( n > 2 \). The decay products have initial momenta \( p \sim m_\phi \), where \( m_\phi \) is the mass of the inflaton, and it is only after interactions in the plasma can equilibrate that \( p \sim T \). The very energetic particles produced before the thermalization of the universe can therefore dominate the dark matter density budget despite their dilution by entropy production during the late stages of reheating.

Let us assume for definiteness that the inflaton decays predominantly to Higgs bosons, and subdominantly to neutrinos and gauge bosons. When this is the case, the pre-thermal production rate of raritrons can be easily estimated, following the procedure outlined in [32]. When reheating ends, the number density of pre-thermally generated raritrons via the processes depicted in Fig. 2 can be written as

\[
n(T_{RH}) \approx \left( \frac{5^{12} g_{RH}^{17}}{2 \cdot 3^9} \right)^{1/10} \frac{\pi^{17/5} \alpha_{SM}^2 G_{N} m_{\phi}^{14/5} T_{RH}^{34/5} \mathcal{B}}{161280 \alpha_{SM}^{16/5} m_{\phi}^{2/5} M_{R}^{12/5} M_{P}^{13/5}},
\]

where

\[
\mathcal{B} \equiv Br_{\nu_1} + \frac{2}{3} Br_{B} + \frac{1}{6} Br_{\nu_1} Br_{B}.
\]

Here \( Br_{\nu} \) (\( Br_{B} \)) denotes the branching ratio to light neutrinos (to \( B \)), and \( \alpha_{SM} \) denotes the gauge coupling strength of the interaction responsible for thermalization during reheating. This results in the following closure fraction,

\[
\Omega_{3/2} h^2 \simeq 0.1 \left( \frac{\alpha}{1.1 \times 10^{-3}} \right)^2 \left( \alpha_{SM} \right)^{16/5} \left( \frac{m_1}{0.15 \text{eV}} \right) \times \left( \frac{g_{RH}}{427/4} \right)^{7/10} \left( \frac{10^4 \text{GeV}}{m_{\phi}^{3/2}} \right) \left( \frac{10^{14} \text{GeV}}{M_{R}} \right) \times \left( \frac{m_{\phi}}{3 \times 10^{13} \text{GeV}} \right)^{14/5} \left( \frac{T_{RH}}{10^{10} \text{GeV}} \right)^{19/5} \times \left( \frac{B}{2 \times 10^{-2}} \right).
\]

Note that for the chosen model parameters this non-thermally produced population of raritrons dominates over the thermally produced one (22) if \( B > 2 \times 10^{-4} \). This “enhancement” of the production rate is dependent on the possibility of producing the parent scatterers \( H, \nu_{L} \) and/or \( B \) directly from inflaton decay. Substantially suppressing two of these decay channels will lead to a raritron population overwhelmingly dominated by late-time reheating thermal effects, with rate (20). In the following Section we specialize to reheating driven by the coupling between the inflaton \( \Phi \) and \( \nu_{R} \). In the case when \( M_{R} \gg m_\phi \), the dominant decay channel of \( \Phi \) is precisely to Higgs bosons, while the decay to neutrinos is suppressed by

\[
Br_{\nu_1} \simeq \left( \frac{m_1 m_\phi}{8 M_{R}^2} \right)^2 \ln^{-2} \left( \frac{M_{R}^2}{m_\phi^2} \right),
\]

which is \( O(10^{-51}) \) for the fiducial values considered in (25). Moreover, we assume no direct production of gauge bosons. This then renders non-thermal production completely negligible in this case. In what follows we will therefore disregard this production mechanism, albeit having in mind that for a different reheating process it could be of importance.

C. The relic abundance from inflaton decay

In principle, it is also necessary to consider dark matter production directly from inflaton decay. We parametrize the total width for inflaton decay as follows,

\[
\Gamma_{\Phi}^{\text{tot}} = g_{\Phi}^2 \frac{2}{8 \pi} m_\phi.
\]

Inflaton decay produces a thermal bath, and we define the moment of reheating to be the time of inflaton-radiation equality.\(^6\) During the process of reheating, the

\(^5\) The time-scale for efficient emission of energetic (\( p \sim m_\phi \)) gauge bosons from the inflaton decay products is typically larger than the thermalization time-scale [32–34].

\(^6\) We are further assuming a matter dominated Universe prior to decay and \( H = \frac{\dot{a}}{a}. \)
temperature of the newly created radiation bath falls as $T \propto a^{-3/8}$, where $a$ is the cosmological scale factor. From the solution to the set of Boltzmann/Friedmann equations

\begin{align*}
\dot{\rho}_\Phi + 3H\rho_\Phi &= -\Gamma_\Phi \rho_\Phi, \\
\dot{\rho}_R + 4H\rho_R &= -\Gamma_\Phi \rho_\Phi,
\end{align*}

we find

\begin{equation}
H^2 = \frac{\rho_\Phi + \rho_R}{3M_P^2} \simeq \frac{\rho_\Phi}{3M_P^2},
\end{equation}

and we can write

\begin{equation}
T_{RH} \simeq 6 \times 10^{14} \GeV y_\Phi \left(\frac{m_\Phi}{3 \times 10^{13} \GeV}\right). 
\end{equation}

The source of the Yukawa coupling $y_\Phi$ is of course model dependent. If the inflaton is directly coupled to the SM, there may be, for example, a direct coupling of the inflaton to the Higgs of the type $\Phi H H^*$, or the decay to Standard Model fields may involve loops containing SM and/or BSM fields. As a minimal assumption, we assume first that the inflaton couples directly only to the BSM field $\nu_R$ through $y_\Phi \Phi \bar{\nu}_R \nu_R$ and this is the main source of the reheating.

If $m_\Phi > M_R$, then the decay rate of $\Phi$ is simply $\Gamma_\Phi = y_\Phi^2 m_\Phi / 8\pi$ ($y_\nu = y_\nu$), and the raritron is produced through the decay process shown in Fig. 3. The partial width in this case is

\begin{equation}
\Gamma_{\Phi \to 3/2} \simeq \frac{\alpha^2 y_\Phi^2 m_\Phi^3}{288 \pi^3 m_{3/2}^3 M_P^2},
\end{equation}

where we have assumed that $m_\Phi > M_R$, $m_{3/2}$. The branching ratio is therefore given by

\begin{equation}
\text{Br}_{3/2} = \frac{\alpha^2 m_\Phi^4}{36 \pi^2 m_{3/2}^2 M_P^2}.
\end{equation}

For a given branching ratio, the number density of raritrons at the end of reheating will be given by [29, 33]

\begin{equation}
n(T_{RH}) = \frac{\pi^2 \text{Br}_{3/2} g_{\text{RH}} T_{RH}^4}{18 m_\Phi},
\end{equation}

and the relic density in turn takes the form

\begin{equation}
\Omega_{3/2} h^2 \simeq 0.1 \times \left(\frac{\text{Br}_{3/2}}{9 \times 10^{-11}}\right) \left(\frac{T_{RH}}{10^{10} \GeV}\right) \times \left(\frac{3 \times 10^{13} \GeV}{m_\Phi}\right) \left(\frac{m_{3/2}}{10^4 \GeV}\right) \simeq 0.1 \left(\frac{\alpha}{5 \times 10^{-9}}\right)^2 \left(\frac{T_{RH}}{10^{10} \GeV}\right) \times \left(\frac{m_\Phi}{3 \times 10^{13} \GeV}\right)^3 \left(\frac{10^4 \GeV}{m_{3/2}}\right). 
\end{equation}

As one can see, a very small coupling between the raritron and $\nu_R$ is required to avoid overclosure.\footnote{Since the decay of the inflaton is not instantaneous, entropy production continues for some time beyond inflaton-radiation equality. A numerical calculation shows that this injection of entropy, overlooked in our analytical estimates, reduces the value of $\Omega_{3/2}$ by a factor of $\sim 0.7$.}

When $M_R > m_\Phi$, the direct decay to $\nu_R$ is not kinematically allowed. There is a two-body decay $\Phi \to \nu_1 \bar{\nu}_1$ and the decay rate for this channel would be given by $y_\nu^2 \theta^4 m_\Phi / 8\pi$. However, decay to Higgs and light neutrino pairs can proceed through the loop diagrams shown in Fig. 4 and computed in Appendix C.\footnote{There are also four body decays with an off-shell $\nu_R$, but those rates are highly suppressed, $\Gamma_{4b} \propto y_\nu^2 y_\Phi^4 m_\Phi^3 / M_P^2$.} For $M_R > m_\Phi$, $m_{3/2}$, we find the following partial widths to Higgses and neutrinos,

\begin{equation}
\Gamma_{\Phi \to H} \simeq \frac{y_\nu^2 y_\Phi^4 M_R^2}{256 \pi^3 m_\Phi} \ln^2 \left(\frac{M_R^2}{m_\Phi^2}\right),
\end{equation}

and

\begin{equation}
\Gamma_{\Phi \to \nu_1} = \frac{y_\nu^2 y_\Phi^4 m_\Phi}{32 \pi^2 M_R^4} \left(1 + \frac{y_\nu^4}{256 \pi^4}\right)
\end{equation}

where in the decay to $\nu_1 \bar{\nu}_1$, we include the tree-level and 1-loop contributions. In this case, the decay to Higgs is clearly dominant and we can associate (38) with a total rate such that $y_\Phi = (y_\nu y^4 / 4\pi^2)(M_R / m_\Phi) \ln(M_R / m_\Phi)^2$. 

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\textbf{FIG. 3:} Three body decay of the inflaton producing a raritron when $M_R \ll m_\Phi$.

\textbf{FIG. 4:} Inflaton decay to Higgses and light neutrinos when $M_R \gg m_\Phi$. 

Inflaton decays to raritrons is also possible when $M_R > m_\Phi$. A tree level decay to $\nu_I B \Psi^\dagger$ has a rate given by Eq. (33) multiplied by $\theta^2$. There is also the loop process shown in Fig. 5 and its partial width is given by

$$\Gamma_{\Phi \to 3/2} \simeq \frac{\alpha^4 y^2 M_R^2 m_\Phi^6}{4\pi^5 M_R^4 m_{3/2}^4} \left( \frac{M_R^2}{m_\Phi^2} \right),$$

where $\Upsilon(M_R^2/m_\Phi^2) = (\ln(M_R^2/m_\Phi^2) - 5/6)^2$. The loop decay dominates whenever

$$\alpha^2 M_R^4 \Upsilon \left( \frac{M_R^2}{m_\Phi^2} \right) > \frac{\pi^2}{72} M_R^2 m_{3/2}^2.$$

When $M_R > m_\Phi$, the right hand side of (41) should be multiplied by $\theta^2$. When the loop dominates, the branching ratio is given by

$$\text{Br}_{3/2} \simeq \frac{64\alpha^4 M_R^2 m_\Phi^6}{y^2 m_{3/2}^4 M_R^4} \frac{\Upsilon(M_R^2/m_\Phi^2)}{\ln^2(M_R^2/m_\Phi^2)}. \tag{42}$$

Using (36) we can immediately deduce the relic abundance,

$$\Omega_{3/2} h^2 \simeq \frac{4\pi^4 y_0 \alpha^4 y^4 n_\gamma^4 m_\Phi^5 T_{RH}}{9\zeta(3) \rho_c h^{-2} m_\gamma^7 m_{3/2}^4 M_R^4 \ln^2(M_R^2/m_\Phi^2)} \Upsilon(M_R^2/m_\Phi^2) \tag{43}$$

$$\simeq 0.1 \left( \frac{\alpha}{1.1 \times 10^{-3}} \right)^4 \left( \frac{m_\Phi}{3 \times 10^{13} \text{GeV}} \right) 5 \left( \frac{0.15 \text{eV}}{m_1} \right)^2 \times \left( \frac{10^4 \text{GeV}}{m_{3/2}} \right) \left( \frac{T_{RH}}{10^{10} \text{GeV}} \right) \times \frac{\Upsilon(M_R^2/m_\Phi^2)}{\ln^2(M_R^2/m_\Phi^2)}.$$

In this case too, a small coupling between the rariton and $\nu_I$ is required to avoid overclosure, though for $M_R > m_\Phi$, it is more easily mitigated by taking a large rariton mass as $\Omega_{3/2} h^2 \propto \alpha^4/m_{3/2}^4$. As one can see, for a given set of parameters ($\alpha$, $M_R$, $y$), the possibility of the direct production of raritrons from inflaton decay opens up a new window, allowing for the production of super-heavy spin-3/2 dark matter. Indeed the rariton mass may be above the reheating temperature, and then only accessible through decay rather than from scattering.

Larger values of $\alpha$ are possible if there are inflaton decay channels directly to the SM. Thus if $y_0$ (defined in Eq. (27)) is much larger than $y_\nu$, in this case,

$$\text{Br}_{3/2} = \frac{2\alpha^4}{\pi^4} \left( \frac{y_0}{y_\nu} \right)^2 \frac{\pi^2}{72} M_R^2 m_\Phi^4 \Upsilon \left( \frac{M_R^2}{m_\Phi^2} \right), \tag{44}$$

which gives

$$\Omega_{3/2} h^2 \simeq \frac{g_0 \alpha^4 y_0^2 n_\gamma M_R^4 m_\Phi^3 T_{RH}}{18\zeta(3) \rho_c h^{-2} y_0^2 m_{3/2}^4 M_R^4} \Upsilon \left( \frac{M_R^2}{m_\Phi^2} \right) \tag{45}$$

$$\simeq \left( \frac{9}{40 \pi^2 g_{RH}} \right)^{1/2} \frac{g_0 \alpha^4 y_0^2 n_\gamma M_R^4 m_\Phi^3 T_{RH}}{18\zeta(3) \rho_c h^{-2} m_{3/2}^4 M_R^4} \Upsilon \left( \frac{M_R^2}{m_\Phi^2} \right) \tag{46}$$

$$\simeq 0.1 \left( \frac{\alpha \sqrt{y_0}}{2.7 \times 10^{-10}} \right)^4 \left( \frac{427/4}{g_{RH}} \right)^{1/2} \left( \frac{3 \times 10^{13} \text{GeV}}{M_R} \right)^4 \times \left( \frac{10^{14} \text{GeV}}{M_R} \right)^3 \left( \frac{T_{RH}}{10^{10} \text{GeV}} \right) \frac{\Upsilon(M_R^2/m_\Phi^2)}{\ln^2(M_R^2/m_\Phi^2)},$$

where we have used (32) to substitute $T_{RH}$ for $y_0$. Even in this case, we require the product of couplings $\alpha \sqrt{y_0} \approx 10^{-10}$ to obtain the correct relic density.

### IV. RESULTS AND ANALYSIS

In the preceding analysis, we have derived the rariton lifetime and density in terms of the Dirac coupling $y$, the right-handed neutrino mass, $M_R$, and the light neutrino mass, $m_1$, though these are related through Eq. (9). In addition, there is an absolute theoretical limit on $y$ ($y \lesssim \sqrt{4\pi}$) from perturbativity and an experimental cosmological constraint on the sum of the light neutrino masses which force $m_1 \lesssim 0.15 \text{eV}$ [49]. In other words, for a given $M_R$, the upper bound on $m_1$ implies an upper bound on $y$, and as a consequence a lower bound on the rariton lifetime and upper bound on its relic abundance. For example,

$$m_1 \lesssim 0.15 \text{eV} \quad \Rightarrow \quad y \lesssim 0.7 \sqrt{\frac{M_R}{10^{14} \text{GeV}}}.$$  \tag{46}

We can then express the lifetime constraints (15) as a function of $m_1$

$$\Gamma_{3/2} = \frac{\Gamma_{3/2}^{2b} + \Gamma_{3/2}^{3b}}{4} = \frac{\alpha^2 m_1 m_{3/2}^2}{4\pi M_R^2 M_R} \left[ 1 + \frac{m_{3/2}^2}{60 y^2 m_1^2} \right]. \tag{47}$$

Given $m_{3/2}$ and $\alpha$, limits from the dark matter lifetime (15) give us a lower bound on $M_R$ (we fix $m_1 = 0.15 \text{eV}$ to

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9 We note that if there is no direct coupling between the inflaton and $\nu_R$ (i.e. $y_\nu = 0$), and the inflaton decays directly to SM particles, such as $\Phi \to HH^*$, rariton production through inflaton decay is still possible at two-loops.

10 We find an analogous suppression for the 4-body decay to B and 3/2.
be specific). For dark matter production from scattering, we can use the lower bound on \( M_R \) to find the reheating temperature \( T_{\text{RH}} \) necessary to obtain the correct relic abundance in Eq. (22). For dark matter produced from decay, the bound on \( M_R \) is not needed. Note that when we saturate the bound on \( m_1 \) we also have an upper limit on \( M_R \) from the perturbativity of \( y < \sqrt{4\pi} \), which is \( M_R \lesssim 2.5 \times 10^{15} \) GeV.

### A. Dark matter production from scattering

We consider first the case where dark matter is produced exclusively through scatterings during reheating. That is, we assume that the direct production from inflaton decay is negligible. We show in Fig. 6 the available parameter space in the \((m_{3/2}, T_{\text{RH}})\) plane. In the lower right portion of the plane, the raritron lifetime is too short when compared with experimental constraints. Due to the large range of dark matter masses we apply constraints from several experiments: XMM-Newton observations of M31 [55] at the keV scale, SPI, INTEGRAL and COMPTEL observations [56, 57] at the MeV scale, the latest limits from FERMI-LAT at the GeV scale [58], and HESS above the TeV scale [59] (see also [60]). Note that the limits on the dark matter lifetime given by the collaborations correspond to a specific final state. A complete study taking into account the exact shape of the spectrum is beyond the scope of our work, and is not necessary considering the large dependence of the relic abundance on the reheating temperature.

To obtain the limit on the lifetime in the \((m_{3/2}, T_{\text{RH}})\) plane, we first fix the value of \( \tau_{3/2} \) at the experimental limit from Eq. (15) for each value of \( m_{3/2} \). This determines the combination \( M_R/\alpha y \). Then from Eq. (22), we can determine the value of \( T_{\text{RH}} \) needed to obtain \( \Omega_{3/2}h^2 \simeq 0.1 \). This procedure determines the blue line in Fig. 6. For lower masses (< 10 TeV), we use \( \gamma \)-ray limits, whereas for higher masses (> 1 PeV) we use neutrino limits and the dot-dashed portion of the line in between is an extrapolation. In the shaded region below this line, we continue to fix \( \Omega_{3/2}h^2 \simeq 0.1 \), but to do so at lower \( T_{\text{RH}} \) requires lower values of \( M_R/\alpha y \) and hence lifetimes below the experimental limit. Conversely, in this shaded region, satisfying the lifetime limit would imply an insufficient relic density (though this can not be excluded). For reference we also plot in Fig. 6 the line corresponding to a projected sensitivity corresponding to a lifetime of \( \tau_{3/2} = 10^{30} \) seconds which is similar to the present experimental limits. This line can be determined from the substitution of \( \tau_{3/2} \) into \( \Omega_{3/2}h^2 \) giving

\[
\Omega_{3/2}h^2 \simeq 0.1 \times \left( \frac{3 \times 10^{31}s}{\tau_{3/2}^{2/3}} \right) \left( \frac{10^4}{m_{3/2}} \right)^4 \left( \frac{T_{\text{RH}}}{10^{10}} \right)^5 , \tag{48}
\]

resulting in a slope of 4/5 (in the logs) for \( T_{\text{RH}} \) vs \( m_{3/2} \). A change of slope in this line occurs for \( m_{3/2} = 2\sqrt{15\pi}v \simeq 6 \)

![FIG. 6: The \((m_{3/2}, T_{\text{RH}})\) plane with astrophysical constraints on the lifetime from \( \gamma \)-ray observations and Planck constraints on the relic abundance for different values of \( \alpha \) (10\(^{-6}\), 10\(^{-4}\) and 1) and \( m_1 = 0.15 \) eV. See the text for details.](image)

TeV corresponding to the point when the 3-body and 2-body decay rates are equal. At higher masses, using \( \tau_{3/2}^{3b} \), we have,

\[
\Omega_{3/2}h^2 \simeq 0.1 \times \left( \frac{10^{31}s}{\tau_{3/2}^{3b}} \right) \left( \frac{10^4}{m_{3/2}} \right)^6 \left( \frac{T_{\text{RH}}}{10^{10}} \right)^5 , \tag{49}
\]

which results in a slope of 6/5.

In the upper left portion of the \((m_{3/2}, T_{\text{RH}})\) plane we fix the value of \( M_R = 2.5 \times 10^{15} \) GeV at its perturbative limit from \( y < \sqrt{4\pi} \). In this region, above the blue line, the lifetime is always longer than the experimental limit. Assuming \( m_1 = 0.15 \) eV, we show three contours with fixed \( \alpha \) as indicated and \( \Omega_{3/2}h^2 = 0.1 \). For each value of \( \alpha \), the shaded region above the line (at higher \( T_{\text{RH}} \)) would have an excessive raritron density. We see immediately that raritron masses from about a keV to a PeV are all allowed for reasonable reheating temperatures \( T_{\text{RH}} \gtrsim 10^6 \) GeV.

It is also useful to consider the allowed parameter space in the \((m_{3/2}, \alpha)\) plane. We show in Fig. 7 the region allowed for different values of the reheating temperature as indicated. The curves and shadings are as in the previous figure, however, we now fix both \( m_1 = 0.15 \) eV and \( M_R = 10^{14} \) GeV everywhere across the plane. In this case, the lifetime limit shown by the blue curve can be viewed as a function of \( m_{3/2} \) and \( \alpha \) and should be close to a line with log slope of -3/2 for low \( m_{3/2} \) and -5/2 for larger masses when the 3-body decay dominates. As discussed above, we find that values of \( \alpha \) of order
one are allowed for relatively low reheating temperatures ($T_{\text{RH}} \simeq 10^6$ GeV) whereas higher reheating temperatures of order $10^{12}$ necessitate $\alpha \lesssim 10^{-8}$ to avoid an overabundance of dark matter.

B. Including the inflaton decay

Reheating is the result of inflaton decay to SM particles. If $M_R < m_\Phi$, there will be tree level diagrams which lead to reheating and raritron production. When $M_R > m_\Phi$, there may be a direct coupling between the inflaton and the SM (characterized by the coupling $y_\Phi > y_\nu$ in Eq. (27)) or through loops with $y_\Phi = y_\nu$ as in Fig. 4 and discussed earlier. Even if there is no direct coupling between the inflaton and raritron, raritron production through loops is possible as in Fig. 5. Unless inflaton decay to dark matter is highly suppressed, once a direct decay channel is open even through loops, it can easily dominate the dark matter production [33]. We have seen this effect for the specific case of raritron dark matter in the preceding section.

To get an idea of the relevant parameter values, we rewrite Eq. (43) (ignoring the logs) with $\Omega_{3/2} h^2|_{\text{decay}} \simeq 0.1$ as

$$m_{3/2} \simeq 4 \times 10^{14} \alpha_4^4 \left( \frac{0.15 \text{ eV}}{m_1} \right)^{2/3} \left( \frac{T_{\text{RH}}}{10^{10} \text{ GeV}} \right)^{2/5} \text{ GeV}. \tag{50}$$

Using this value for $m_{3/2}$ in the lifetime in Eq. (15) (using the 3-body decay as an example), we find for $\tau_{3/2} \gtrsim 10^{30}$ seconds

$$\alpha \lesssim 2 \times 10^{-7} \left( \frac{M_R}{10^{14} \text{ GeV}} \right)^{2/5} \left( \frac{m_1}{0.15 \text{ eV}} \right)^{7/26} \times \left( \frac{10^{10} \text{ GeV}}{T_{\text{RH}}} \right)^{2/5}. \tag{51}$$

The relevant parameter space in the ($m_{3/2}, \alpha$) plane is shown in Fig. 8. Since the blue line is determined solely from the limit on the raritron lifetime, it is independent of the production mechanism and is the same as in Fig. 7. Comparing Fig. 8 with Fig. 7, we see clearly that the production of dark matter through inflaton decay is much more copious and the parameter space is much more constrained. The relic abundance necessitates much lower values of $\alpha$ to avoid overabundance, and the result is much less dependent on $T_{\text{RH}}$ as one can see comparing Eqs. (22) and (48), where $\Omega_{3/2} h^2$ depends on $T_{\text{RH}}^2$ in the scattering case, compare to $T_{\text{RH}}$ inflaton decay process. This feature is also clearly illustrated in Fig. 8, where we show two lines producing the correct relic abundance with $T_{\text{RH}} = 10^9$ and $10^{10}$ GeV.

V. SIGNATURES

Having established the viable parameter space for the raritron dark matter model, we now discuss in more detail the possible experimental signatures for such a
model. The two body decay mode shown in Fig. 1 will produce a monochromatic photon and neutrino.\(^{11}\) If \(m_{3/2} > m_Z\), there is also a two-body final state \(Z + \nu_1\). These decay channels are easily observable at detectors and the signal will give us 1) the mass of the dark matter (from the position of the signal in the spectrum) and 2) the lifetime (from the strength of the signal). On top of the monochromatic signal, there will also be a continuous spectrum due to the three-body channels which dominate at higher raritron masses. If the raritron is produced mainly through scattering, the signal can be translated to the reheating temperature needed to obtain the right relic abundance using Eq. (48). For example, a GeV gamma-ray observed by FERMI with a signal strength corresponding to a lifetime of \(10^{30}\) s would imply a reheating temperature of \(\sim 3 \times 10^6\) GeV. In this case, the temperature is independent of the parameter \(\alpha\). In contrast, if the production were dominated by inflaton decays, some information on the combination of \(\alpha\) and \(T_{RH}\) could be ascertained.

As an exercise, we reanalyze one of the most popular recent “signals”: the 130 GeV line observed by the FERMI satellite in 2012 \[^{63}\]. This is a monochromatic signal that could be fit with a dark matter mass \(m_{3/2} \simeq 260\) GeV and a lifetime \(\Gamma_{3/2} \simeq 10^{29}\) seconds \[^{64}\], for a Navarro-Frenk and White (NFW) profile \[^{65}\].\(^{12}\) Interestingly, this kind of signal could correspond to a spin-\(\frac{1}{2}\) dark matter decay. For scattering dominated production, we can use Eq. (48) to determine the reheat temperature, \(T_{RH} \simeq 2 \times 10^8\) GeV. From the lifetime, we can also determine the combination \((\alpha y/M_R)^2 + \alpha^2/M_R\) upon fixing the light neutrino mass, \(m_1\). We find, \(\alpha^2/M_R = 2 \times 10^{-13}\) GeV\(^{-1}\) or \(\alpha \approx 4\) for \(M_R = 10^{14}\) GeV. Note that for this value of \(\alpha\), when inflaton decay is the dominant production mode, the reheating temperature must be extremely (and unphysically) low. Thus not only would we determine the reheat temperature and \(\alpha\), but we would also know that inflaton decay does not play a role in dark matter production. This position of this example in the \((m_{3/2}, \alpha)\) plane is illustrated in Figs. 7 and 8 by a star. Such a signal, if observed below \(m_H\), could be correlated with a similar monochromatic signal from neutrino detectors like ANTARES.

Next, we repeat the exercise for the PeV neutrino signal observed by IceCube \[^{66}\]. There were some attempts to explain these events from a dark matter perspective (see \[^{50}\] for instance) but it was difficult to reconcile the signal with the correct relic abundance. The number of events expected by IceCube is \[^{50, 67}\]

\[
\Gamma_{\text{events}} = 1.5 \times 10^{57} \eta_E f_{\text{astro}} \frac{\Gamma_{3/2}}{m_{3/2}^{0.637}} \text{years}^{-1},
\]

where \(\eta_E \sim 0.4\) is defined from the fiducial volume \(V_{\text{fid}} = \eta_E V\) and \(f_{\text{astro}} \sim 1\) corresponds to the astrophysical uncertainty in the local distribution of the dark matter halo. The mass and widths are expressed in GeV. A rate of one PeV event per year gives us, \(\Gamma_{3/2} \simeq 10^{-53}\) GeV corresponding to \(\tau_{3/2} \simeq 6 \times 10^{28}\) s.\(^{13}\) Using this lifetime, with \(m_{3/2} = 1\) PeV, we can again determine the value of \(\alpha\) now from the three-body decay rate which is dominant, \(\alpha \simeq 10^{-7}\). The reheating temperature in this case can be obtained from Eq. (49) and we find \(T_{RH} \simeq 9 \times 10^{11}\) GeV, when raritron production is due to scattering. When production is due to inflaton decay, we can use Eq. (51) and find, \(T_{RH} \simeq 4 \times 10^{11}\) GeV. The position of this example is displayed in Figs. 7 and 8 by the black diamond. Both the scattering production and the inflaton decay process scenarios are compatible correct relic abundance and the IceCube PeV monochromatic signals.

**VI. Conclusion**

We have shown that a metastable spin-\(\frac{3}{2}\) particle can be a suitable dark matter candidate through the introduction of a minimal (Planck-suppressed) coupling, \(\alpha\) to a right-handed neutrino. Surprisingly, the parameter space needed to generate a sufficiently long lifetime is perfectly compatible with both the astrophysical constraints from \(\gamma\)-ray and neutrino experiments as well as the cosmological determination of the dark matter density. Our results are summarized in Figs. 7 and 8 where we display the allowed region in the \((m_{3/2}, \alpha)\) plane. We considered both the production of dark matter from the thermal bath produced during reheating, and production directly from inflaton decay. We also have shown that smoking-gun signals are expected from such couplings, in the form of a monochromatic neutrino and/or a monochromatic gamma-ray line.

We have also illustrated, as examples, the points in the parameter space that could explain the gamma-ray signal observed by the FERMI telescope, or PeV neutrinos observed by IceCube that can be combined with the recent ANITA analysis \[^{69}\]. Moreover, it was shown in \[^{70}\] that spin-\(\frac{3}{2}\) particles can have an impact on the form of gravitational waves produced during reheating that could be observable in future ultra-high frequency detectors.

\(^{11}\) A signal of this type was termed a “double smoking-gun” in \[^{61}\]. Spin-3/2 fields were not included in their study, nor in \[^{62}\], and they did not try to produce cosmologically viable scenarios.

\(^{12}\) The dependence on the dark matter distribution for decaying dark matter being proportional to its density \(\rho\) (versus \(\rho^2\)) is much weaker than for annihilating dark matter.

\(^{13}\) This is similar to what was obtained in \[^{68}\], namely, \(\tau_{3/2} \simeq 1.9 \times 10^{28}\) seconds.
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**Appendix A: The Rarita-Schwinger Lagrangian**

Rarita and Schwinger [36] derived the Lagrangian (1) following the work of Fierz and Pauli [37]. One can start with the hypothesis that a spin-½ particle should respect both the spin-½ Dirac equation and spin-1 divergence relation, namely,

\[ (i\gamma^\rho \partial_\rho - m_{3/2}) \Psi_\mu = 0 \]  \hspace{1cm} (A1)
\[ \partial^\mu \Psi_\mu = 0. \]  \hspace{1cm} (A2)

By writing the field \( \Psi \) in terms of its spin components and after a Clebsch-Gordan decomposition, we have

\[ \Psi^{\pm \frac{1}{2}} = \Psi^{\pm \frac{1}{2}} \epsilon^{\pm \frac{1}{2}}, \]
\[ \Psi^{\pm} = \frac{1}{\sqrt{3}} \Psi^{\pm \frac{1}{2}} \epsilon^{\pm \frac{1}{2}} + \sqrt{\frac{2}{3}} \Psi^{\pm \frac{1}{2}} \epsilon^0, \]
\[ \Psi^{-} = \frac{1}{\sqrt{3}} \Psi^{\pm \frac{1}{2}} \epsilon^{\pm \frac{1}{2}} + \sqrt{\frac{2}{3}} \Psi^{\pm \frac{1}{2}} \epsilon^0, \]
\[ \Psi^{-} = \frac{1}{\sqrt{3}} \Psi^{\pm \frac{1}{2}} \epsilon^{\pm \frac{1}{2}} + \sqrt{\frac{2}{3}} \Psi^{\pm \frac{1}{2}} \epsilon^0, \]  \hspace{1cm} (A3)

where \( \Psi^{\pm} \) is a dirac spinor of helicity \( 2s_z \), which is a solution of Eq. (A1), and \( \epsilon^\lambda_\mu \) is a vector polarization with spin projection \( \lambda \) along the direction of the momentum, so that \( \partial^\mu \epsilon_\mu = 0 \). See [43] for a detailed solution. One can show, that using each of the components in Eq. (A3) by direct calculation and after a little algebra, that Eqs. (A1) and (A2) imply

\[ \gamma^\mu \Psi_\mu = 0. \]  \hspace{1cm} (A4)

We can construct a Lagrangian for a spin-½ field, whose Euler-Lagrange equation gives (A1), with terms such as \( \gamma^\mu \gamma^\rho \), \( \gamma_\mu \gamma^\rho \), or any combinations of that type, which are consistent with the relations (A2) and (A4). Among the class of possible Lagrangians, the simplest one is

\[ \mathcal{L}_{3/2}^0 = \Psi_\mu (ig^{\mu\nu}\gamma^\rho \partial_\rho - m_{3/2}g^{\mu\nu} - i\gamma^\rho \partial^\rho - i\gamma^\rho \partial_\mu + i\gamma^\rho \gamma^\nu \partial_\rho + m_{3/2}\gamma^\mu \gamma^\nu) \Psi_\nu. \]  \hspace{1cm} (A5)

Note that the coefficient of last four terms in Eq. (A5) is arbitrary (e.g., [36] included a factor of 1/3 in front of each of these terms). Eq. (A5) can be simplified to

\[ \Psi_\mu (ig^{\mu\nu}\partial_\rho + m_{3/2}\gamma^\mu) \Psi_\nu \]  \hspace{1cm} (A6)

which is, up to a normalization factor, our Lagrangian in Eq. (1).

**Appendix B: Decay and scattering rates**

In this appendix, we provide some relevant details concerning the computation of the dark matter decay rate.

### 1. 3-body decay formula

The phase space integration for the 3-body decay processes \( \Psi_\mu \rightarrow A_\mu H \nu_1 \) and \( \Psi_\mu \rightarrow Z_\mu H \nu_1 \) can be performed analytically if one disregards the small neutrino mass \( m_\nu \). In this limit, the decay rates are given by (12) and (13), where the threshold function is given by the following expression,

\[ g(x, y) = \left[ \left( 1 + \frac{y^2}{4} \right) (1 - y^2)^3 + \frac{113}{8} x^2 \left( 1 - \frac{119y^2}{339} - \frac{29y^4}{339} - \frac{131y^6}{339} \right) + \frac{59}{12} x^4 \left( 1 + \frac{6y^2}{59} - \frac{51y^4}{59} \right) \right] \]

\[ - \frac{1}{24} x^6 (1 - 9y^2) - \frac{x^8}{24} \xi - 5x^2 y^6 (2x^2 + y^2) \ln \left[ 2xy \left( \xi - x^2 - y^2 + 1 \right) \right] \]

\[ + \frac{5}{2} x^2 (4x^2 - 4y^2 + 3) \ln \left| \frac{x^4 - x^2 (\xi + 2y^2 + 1) + y^2 (\xi + y^2 - 1)}{\xi - x^2 - y^2 + 1} \right| \]  \hspace{1cm} (B1)
where
\[ \xi = \sqrt{x^4 - 2x^2(y^2 + 1) + (1 - y^2)^2}. \] (B2)

2. Scattering amplitudes

The amplitudes for the scattering processes contributing to dark matter production from the thermal bath can be written as
\[ |M|_{H\nu_1 \rightarrow \nu_{\mu} B}^2 = -\frac{8}{3} \frac{\alpha^2 y^2}{m_{3/2}^2 M_P^2} \left( s - M_R^2 \right)^2 (M_R^2 + su), \] (B3)
\[ |M|_{H\nu_1 \rightarrow \nu_\mu \nu_1}^2 = \frac{8}{3} \frac{\alpha^2 y^2}{m_{3/2}^2 M_P^2} \left( s - M_R^2 \right)^2 (M_R^2 + st), \] (B4)
\[ |M|_{B\nu_1 \rightarrow \nu_{\mu} H}^2 = -\frac{8}{3} \frac{\alpha^2 y^2}{m_{3/2}^2 M_P^2} \left( s - M_R^2 \right)^2 (M_R^2 + su). \] (B5)

In the limit of \( M_R \gg m_{3/2} \), we find
\[ |M|_{H\nu_1 \rightarrow \nu_{\mu} B}^2 = -\frac{8}{3} \frac{\alpha^2 y^2}{m_{3/2}^2 M_R^2 M_P^2} s^2 t, \] (B6)
\[ |M|_{H\nu_1 \rightarrow \nu_{\mu} \nu_1}^2 = \frac{8}{3} \frac{\alpha^2 y^2}{m_{3/2}^2 M_R^2 M_P^2} st^2, \] (B7)
\[ |M|_{B\nu_1 \rightarrow \nu_{\mu} H}^2 = -\frac{8}{3} \frac{\alpha^2 y^2}{m_{3/2}^2 M_R^2 M_P^2} ut^2. \] (B8)

Appendix C: Loop Calculations

First, we consider the inflaton decay to two Higgs bosons through the loop process shown in Fig. 4. The amplitude is given by
\[ \mathcal{M}_{\Phi \rightarrow HH} = A_{\Phi HH} \int \frac{d^4 q}{(2\pi)^4} P_L \langle \bar{q} \gamma^\mu (\not{p}_1 + M_R) (\not{q} + \not{p}_2 + M_R) \rangle \frac{D_0 D_1 D_2}{D_0 D_1 D_2}, \] (C1)
with the coupling \( A_{\Phi HH} = -2y_\nu y^2 \), and the propagators are defined as \( D_0 = q^2 - m_1^2 \approx q^2 \) and \( D_i = (q + p_i)^2 - M_R^2 \) \((i = 1, 2)\), where \( m_1 \) is the left-handed neutrino mass and \( M_R \) is the right-handed neutrino mass. We remind the reader that when \( m_\Phi > M_R \), we use the coupling \( y_\nu = y_f \).

To calculate the amplitudes, we use the Passarino-Veltman functions [71]. The two-point form factors can be expressed as
\[ B_0; B_\mu; B_{\mu\nu} = \int \frac{d^4 q}{i\pi^2} \frac{1}{q^2} \langle q_\mu q_\nu D_0 D_1 \rangle, \] (C2)

where
\[ B_\mu = p_1 \mu B_1 \] (C3)
and
\[ B_{\mu\nu} = g_{\mu\nu} B_0 + p_1 \mu p_1 \nu B_{11}, \] (C4)
and the three-point form factors are given by
\[ C_{0; C_\mu; C_{\mu\nu}; C_{\mu\nu}\alpha} = \int \frac{d^4 q}{i\pi^2} \frac{q_\mu q_\nu q_\alpha q_\alpha}{D_0 D_1 D_2}, \] (C5)
where
\[ C_\mu = p_1 \mu C_1 + p_2 \mu C_2, \] (C6)
\[ C_{\mu\nu} = g_{\mu\nu} C_0 + p_1 \mu p_1 \nu C_{11} + p_2 \mu p_2 \nu C_{22} + \{ p_1 \mu p_2 \nu + p_2 \mu p_1 \nu \} C_{12}, \] (C7)
and
\[ C_{\mu\nu\alpha} = \sum_{i=1,2} \{ g_{\mu\nu p_i \alpha} + g_{\nu \alpha p_i \mu} + g_{\mu \alpha p_i \nu} \} C_{0i} \]
\[ + p_1 \mu p_1 \nu p_1 \alpha C_{11} + p_2 \mu p_2 \nu p_2 \alpha C_{22} \]
\[ + \{ p_1 \mu p_2 \nu p_1 \alpha + p_1 \nu p_2 \mu p_1 \alpha + p_2 \mu p_1 \nu p_1 \alpha \} C_{12} \]
\[ + \{ p_2 \mu p_2 \nu p_1 \alpha + p_2 \nu p_1 \mu p_1 \alpha + p_1 \mu p_2 \nu p_2 \alpha \} C_{22}. \] (C8)

Using the Passarino-Veltman functions, we can express the amplitude (C1) as
\[ \mathcal{M}_{\Phi \rightarrow HH} = \frac{i A_{\Phi HH}}{16\pi^2} P_L \langle [\not{p}_1 + \not{p}_2 + 2M_R]B_0 + \not{p}_2 B_1 \]
\[ + (M_R \not{p}_1^2 + \not{p}_2^2 \not{p}_1 + 2M_R^2 \not{p}_1 + M_R \not{p}_1 \not{p}_1^2)C_1 \]
\[ + (M_R \not{p}_1^2 - \not{p}_2^2 \not{p}_1 + 2p_1 \cdot p_2 - p_1^2 \not{p}_1^2 - M_R \not{p}_1 \not{p}_1^2)C_2 \rangle. \] (C9)

Assuming \( M_R \gg m_\Phi \gg m_H \) we obtain
\[ |\mathcal{M}_{\Phi \rightarrow HH}|^2 = \frac{y_\nu^2 g^4 M_R^2}{8\pi^4} \ln^2 \left( \frac{M_R^2}{m_\phi^2} \right), \] (C10)
and the decay rate is given by Eq. (38).

Next, we calculate the inflaton decay rate to left-handed neutrinos through the loop process shown in Fig. 4. The amplitude of this process is
\[ \mathcal{M}_{\Phi \rightarrow \nu_{\ell} \nu_{\ell}} = A_{\Phi \nu_{\ell} \nu_{\ell}} \int \frac{d^4 q}{(2\pi)^4} \bar{u}(p_1)P_R \]
\[ \frac{(\not{q} + \not{p}_1 + M_R)(\not{q} + \not{p}_2 + M_R)}{D_0 D_1 D_2} \]
\[ \frac{D_0 D_1 D_2}{D_0 D_1 D_2} \]
\[ \frac{P_L v(p_2)}{D_0 D_1 D_2} \] (C11)
where $A_{\Phi,\nu,\nu} = -2 y_{\nu} q^2$; $D_0 = q^2$, and $D_i = (q + p_i)^2 - M_R^2$ ($i = 1, 2$). Using the Passarino-Veltman functions, we can express the amplitude (C11) as

$$
\mathcal{M}_{\Phi \rightarrow \nu \nu} = -\frac{iA_{\Phi,\nu,\nu}}{16\pi^2} \bar{u}(p_1)\gamma^5(p_1)C_0 \\
\times ([M_R(p_1 + p_2 + M_R) + p_1 p_2]C_0 \\
+ (2M_R p_1 + p_1^2 + p_2^2)C_1 \\
+ (2M_R p_2 + p_1^2 + p_1 p_2 + B_0) P_L v(p_2)). 
$$

(C12)

With $p_1^2 = p_2^2 = m_\nu^2$, $p_1 \cdot p_2 = m_\nu^2 - m_\phi^2$, and $M_R \gg m_\Phi$, we find

$$
|\mathcal{M}_{\Phi \rightarrow \nu \nu}|^2 = \frac{y_{\nu}^2 q^4 m_\nu^2 M_\phi^2}{128\pi^4 M_R^2}. 
$$

(C13)

and upon substitution of Eq. (9), the decay rate is given by Eq. (39).

Finally, we calculate inflaton decay rate to rareritons through the loop process shown in Fig. 5. We can express the amplitude as follows,

$$
\mathcal{M}_{\Phi \rightarrow \psi_\nu,\nu} = -\frac{iA_{\Phi,\nu,\nu}}{16\pi^2} \bar{u}(p_1)\gamma^5(p_1)\gamma^\nu \\
\times \frac{(q + p_1 + M_R)(q + p_2 + M_R)}{D_0 D_1 D_2} \gamma^\nu(\gamma^\rho)\nu(p_2), 
$$

(C14)

where $A_{\Phi,\nu,\nu} = \frac{2y_{\nu}^2 q^2}{M_\phi^2}$, $D_0 = q^2$, and $D_i = (q + p_i)^2 - M_R^2$ ($i = 1, 2$). We do not include the full expression of the amplitude (C14) in terms of the Passarino-Veltman functions due to its complexity. With $M_R \gg m_\Phi \gg m_3/2$, the amplitude takes the form

$$
|\mathcal{M}|^2 = \frac{2y_{\nu}^2 q^2 m_\phi^2}{9\pi^4 M_R^4 m_3^4/2} \left[5 - 6 \ln \left(\frac{M_R^2}{m_\phi^2}\right)\right]^2, 
$$

(C15)

and the decay rate is given by Eq. (40).
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