Constraints on Primordial Non-Gaussianity Using the Multitracer Technique for Skew Spectra

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Abstract

Extracting the bispectrum information from large-scale structure observations is challenging due to the complex models and the computational costs involved in measuring the signal and its covariance. Recently, the skew spectrum was proposed to access parts of the bispectrum information in a more effective manner and was confirmed to provide complementary information to that available in power spectrum measurements. In this work, we generalize the theory to apply the multitracer technique and explore its ability to constrain the local-type primordial non-Gaussianity. Using the spectra and their covariance estimated from N-body simulations, we find that the multitracer approach is effective in reducing the cosmic variance noise. The 1σ marginalized errors for $b^2 A_s$, $n_s$, and $f_{NL}^\text{loc}$ are reduced by 50%, 52%, and 73% compared with the results achieved using only the power spectrum obtained from a single tracer. These results indicate that both the skew spectrum and the multitracer technique are useful in constraining the primordial non-Gaussianity with the forthcoming wide-field galaxy surveys.

Unified Astronomy Thesaurus concepts: Non-Gaussianity (1116); Cosmological parameters from large-scale structure (340)

1. Introduction

The standard inflationary paradigm predicts a flat universe perturbed by nearly Gaussian scale-invariant primordial perturbations. These predictions have been extensively probed by the increasingly precise measurements of the cosmic microwave background (CMB; Aghanim et al. 2020). Different from the CMB, the large-scale structure (LSS) contains three-dimensional distribution information of galaxies on large scales, which are caused by nonlinear evolution due to gravitational instability. The upcoming wide-field galaxy surveys, such as the Dark Energy Spectroscopic Instrument (DESI; Aghamousa et al. 2016), Euclid (Amendola et al. 2018) and the Large Synoptic Survey Telescope (LSST; Abell et al. 2009), can provide complementary information regarding the origin of our universe and its late-time evolution.

The traditional method to extract the cosmological information from the LSS is to measure the two-point correlation function or the power spectrum in Fourier space. However, due to the late-time gravitational instability, the galaxy distribution at low redshift is highly non-Gaussian, even for Gaussian initial conditions. To obtain more information from the same surveys, higher-order statistics will be intuitionistic methods to apply, such as the three-point correlation function and bispectrum (Matarrese et al. 1997; Verde et al. 1998; Scoccimarro 2000; Sefusatti et al. 2006; Hoffmann et al. 2015). Actually, the bispectrum has been measured using galaxy survey data (Scoccimarro et al. 2001; Verde et al. 2002; Marin et al. 2013; Gil-Marín et al. 2015a) and has proven useful in breaking degeneracies among cosmological parameters, which arise from considering the power spectrum alone (Gil-Marín et al. 2015b, 2017). With the forthcoming surveys, the higher-order statistics can reach a much larger signal-to-noise ratio and provide a wealth of information.

However, due to the complicated configuration and orientations, significant computational efforts are required to measure the bispectrum signal and its covariance, and it is more challenging to compare the theoretical models with measurements. To bypass these problems, there are several proxy statistics proposed to compress the bispectrum to a pseudo-power spectrum, which depend only on one wavenumber but contain some of the information enclosed in the bispectrum. One of the approaches is the integrated bispectrum proposed by Chiang et al. (2014), which is generated by cross-correlating the position-dependent power spectrum with the mean overdensity of the corresponding subvolume. This measurement contains the bispectrum information on a squeezed configuration and was detected using real data (Chiang et al. 2015). The other method is the skew spectrum, which was first developed in the context of group theory and graph theory (Kondor 2007) and then used to study the CMB (Cooray 2001; Munshi & Heavens 2010) and LSS (Pratten & Munshi 2012; Schmittfull et al. 2015; Munshi & Coles 2017; Dai et al. 2020; Moradinezhad Dizgah et al. 2020). The skew spectrum is obtained by cross-correlating the square of a field with the field itself and has been proven to be an effective method for accessing complementary information to that embedded in the power spectrum measurements using N-body simulations.

Primordial non-Gaussianity (PNG) is one of the most important fingerprints of inflation and can be used to discriminate between the vast array of inflationary scenarios. Currently, the most strict constraints have been achieved by the CMB temperature anisotropies and polarizations, and the amplitudes of the local, equilateral, and orthogonal types are $r_{NL}^\text{loc} = -0.9 \pm 5.1$, $f_{NL}^\text{equi} = -26 \pm 47$, and $f_{NL}^\text{ortho} = -38 \pm 24$ at 1σ statistical significance (Akrami et al. 2020). However, these strict constraints on $f_{NL}$ were not obtained from LSS measurements, although the halo bias can be greatly affected by relatively small values of $f_{NL}$, as shown by Grossi et al. (2009) using numerical simulations. The latest constraints on $f_{NL}^{\text{loc}}$ originate from the extended Baryon Oscillation Spectroscopic Survey (eBOSS) quasar samples, and the result is $-51 < f_{NL}^{\text{loc}} < 21$ at the 95% confidence level.
(Castorina et al. 2019). Dai et al. (2020) showed that with the measurements of the skew spectrum, the 1σ marginalized error for \( f_{NL}^{\text{loc}} \) can be reduced by 44%, although with a large smoothing filter, which suggests that the skew spectrum is an effective method for constraining the PNG without significant computational costs.

Another import issue is that the clustering analysis at large scales where the PNG signal is most significant is limited by the cosmic variance (CV). A possible method to reduce the CV is the multitracer technique (Seljak 2009; Slosar 2009; Ferramacho et al. 2014; Yamauchi et al. 2014, 2017; Fonseca et al. 2015), which can significantly improve the statistical errors using different biased tracers. For two different tracers \( \delta_i \) and \( \delta_j \), we can obtain four cross-skew spectra from \( \delta_i^2 \times \delta_j \), \( \delta_i \delta_j \times \delta_i \), and \( \delta_i \delta_j \times \delta_j \), and there is only one cross power spectrum from \( \delta_i \times \delta_j \). We expect that we can obtain tighter constraints on \( f_{NL}^{\text{loc}} \) using the multitracer technique for the skew spectrum.

In this paper, we build on our previous work (Dai et al. 2020) and include the multitracer technique. We simply divide our simulated halo catalog into two parts and then calculate the cross power spectra and skew spectra to find the extra information the multitracer technique can give us. The rest of the paper is organized as follows. In Section 2, we briefly review the full expression of the skew spectrum, including both the primordial non-Gaussianity and the late-time non-Gaussianity; then, we extend our theory to apply the multitracer technique. In Section 3, we show how we divide our N-body simulation catalog and derive the covariance of the power spectra and skew spectra. In Section 4, we list the constraint results, and in Section 5, we draw our conclusions. We also derive the Poisson shot noise contributions to the galaxy power spectrum and skew spectrum in the Appendix.

2. Methodology

2.1. General Expression for Matter Skew Spectrum

To begin with, we define the matter overdensity field \( \delta(x) = \delta_\rho(x)/\bar{\rho} \), where \( \bar{\rho} \) is the spatial average of the matter density. We can write the three-point correlation function as

\[
\xi^{(3)}(x_1, x_2, x_3) = \langle \delta(x_1) \delta(x_2) \delta(x_3) \rangle.
\]  

This is a well-known statistic for extracting the extra information not present in the power spectrum. However, as we explained before, it is challenging to measure from LSS data.

To simplify the three-point correlation function, we can assume \( x_3 \) in Equation (1) is located at the same point as \( x_1 \), which means that we cross-correlate the square of the field \( \delta^2 \) with the \( \delta \) field itself. This statistic is called the skew correlation function, and due to the cosmological principle, it depends only on the magnitude of the separation vector (Pratten & Munshi 2012; Munshi & Coles 2017; Dai et al. 2020).

\[
\xi^{(3)}(x_2) = \xi^{(3)}(x_1, x_1, x_2) = \xi^{(3)}(|x_1 - x_2|). 
\]  

We can perform the Fourier transformation of this equation to obtain the matter skew spectrum (Pratten & Munshi 2012; Munshi & Coles 2017; Dai et al. 2020).

\[
P_m^{(3)}(k) = \int \frac{d^3q}{(2\pi)^3} B_m(k, q, |q - k|) = \int |q|^2 \int \frac{d\mu}{(2\pi)^3} q^2 B_m(k, q, \alpha(\mu)),
\]

where \( B_m(k, q, |q - k|) \) is the bispectrum of the overdensity field, \( \mu = k \cdot q/|q| \), and \( \alpha = \sqrt{q^2 + k^2 - 2\mu|q|} \) ensures that the wavenumbers correspond to the three sides of a triangle.

To calculate the matter skew spectrum, we need to explicitly determine the matter bispectrum \( B_m \), whose main contributions are from the primordial perturbations \( B_{mG} \) and gravitational instability \( B_{mG} \). Here, we discuss these two effects separately. This part has been widely studied in Pratten & Munshi (2012), Schmittfull et al. (2015), Munshi & Coles (2017), Moradinezhad Dizgah et al. (2020), and Dai et al. (2020). Since this paper focuses on the quasi-linear scales, we consider only the leading-order contributions in the following analysis.

Let us begin with the local-type primordial non-Gaussianity, which is the main target of this paper. Bardeen’s curvature perturbation during the matter era is given by Salopek & Bond (1990), Gngui et al. (1994), Verde et al. (2001), and Komatsu & Spergel (2001),

\[
\Phi(x) = \Phi_{G}(x) + f_{NL}^{\text{loc}}[\Phi_{G}^{2}(x) - \langle \Phi_{G}^{2}(x) \rangle],
\]

where \( \Phi_{G}(x) \) is a Gaussian field.

To characterize the matter bispectrum, we need to relate the linear density fluctuations with the curvature perturbations. In Fourier space, it can be written as

\[
\delta(k)^{(1)} = M(k, a)\Phi(k); \quad M(k, a) = \frac{2k^2T(k)D(a)}{3\Omega_m H_0^2},
\]

where \( a \) is the scale factor, \( H_0 \) is the Hubble constant, \( \Omega_m \) is the current matter energy density parameter, \( T(k) \) is the matter transfer function, and \( D(a) \) is the growth factor. This allows us to write the matter bispectrum from primordial perturbations as

\[
B_{mG}(k_i, k_j, k) = M(k_i)M(k_j)M(k)B_\Phi(k_i, k_j, k),
\]

where \( B_\Phi(k_i, k_j, k) \) is the leading-order contribution to the curvature field bispectrum, which can be expressed as (Falk et al. 1993; Gngui et al. 1994)

\[
B_\Phi \simeq 2f_{NL}^{\text{loc}}[P_\Phi(k_i)P_\Phi(k_j) + \text{cyclic}],
\]

where \( P_\Phi(k) = \langle \Phi(k)\Phi^*(k) \rangle \) is the primordial spectrum.

Even for Gaussian initial conditions, our universe is highly non-Gaussian due to the late-time nonlinear gravitational evolution. Using perturbation theory, the matter density fluctuations can be expressed as a series of corrections to the linear solution \( \delta(k)^{(1)} \) (Fry 1984; Jain & Bertschinger 1994; Bernardeau et al. 2002),

\[
\delta(k) = \delta(k)^{(1)} + \delta(k)^{(2)} + \delta(k)^{(3)} + \ldots,
\]

where we keep only the first two orders; \( \delta(k)^{(2)} \) is given by

\[
\delta(k)^{(2)} = \int d^3q_1d^3q_2\delta_D(k - q_1)F_2(q_1, q_2)\delta(q_1)\delta(q_2)^{(1)},
\]

where \( \delta_D \) is the Dirac delta function and \( F_2(q_1, q_2) \) is the known second-order kernel of standard perturbation theory (Jain &
Bertschinger 1994; Bernardeau et al. 2002):

$$F_2(q_1, q_2) = \frac{5}{7} + \frac{x}{2} \left( \frac{q_1 + q_2}{q_1 q_2} \right) + \frac{2}{7} x^2,$$

(10)
with $x \equiv q_1 \cdot q_2 / q_1 q_2$. Using the above equations and the Wick theorem, the bispectrum generated by the gravitational instability at the leading order is given by

$$B_{m,G}(k_1, k_2, k_3) = 2 F_2(k_1, k_2) P_{m,l}(k_1) P_{m,l}(k_2) + \text{cyc}_.,$$

(11)
where $P_{m,l}(k)$ is the linear matter power spectrum. Then, Equation (3) can be written as

$$P^{(s)}_{m}(k) = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2 [B_{m,l}(k, q, \alpha) + B_{m,G}(k, q, \alpha)].$$

(12)

2.2. Galaxy Skew Spectra and Power Spectra for Multitracers

What we actually observe are galaxies, and they are biased tracers of the dark matter field. In this paper, we use a simple prescription in Eulerian space, where the galaxy overdensity is expanded in terms of the matter overdensity and the traceless part of the tidal tensor. Up to the second order, we have (see Desjacques et al. 2018 for a review)

$$\delta_g(x) \approx b_1 \delta(x) + \frac{1}{2} b_2 \delta^2(x) + \frac{1}{2} b_k' \left[ \left( \frac{\partial \delta}{\partial x} \right)^2 - \frac{1}{3} \delta \right] \delta(x)^2,$$

(13)
where $b_1$ and $b_2$ are the linear and nonlinear bias and $b_k'$ is the nonlocal tidal shear bias. As shown in Dai et al. (2020), the effect of $b_k'$ on the final results is not significant; thus, we neglect the nonlocal term in the following analysis.

After Fourier transformation, the galaxy overdensity is given by

$$\delta_g(k) \approx b_1 \delta(k) + \frac{1}{2} b_2 \int \frac{d^3 q}{(2\pi)^3} \delta(q) \delta(k - q).$$

(14)

For a single tracer, the galaxy bispectrum at the leading order can be easily computed as (Liguori et al. 2010; Pratten & Munshi 2012; Munshi & Coles 2017; Dai et al. 2020)

$$B_{g,1T}(k_1, k_2, k_3) = b_1^3 [B_{m,l}(k_1, k_2, k_3) + B_{m,G}(k_1, k_2, k_3)]$$

$$+ b_1^2 b_2 [P_{m,l}(k_1) P_{m,l}(k_2) + \text{cyc}.],$$

(15)
and then the galaxy skew spectrum is given by

$$P^{(s)}_{g,1T}(k) = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2 B_{g,1T}(k, q, \alpha).$$

(16)

The situation becomes more complicated when we consider two tracers that have different bias parameters: $b_1^{[1]}$, $b_1^{[2]}$ for the first tracer and $b_2^{[1]}$, $b_2^{[2]}$ for the second tracer. For example, we cross-correlate the square of the first tracer $(\delta^{[1]}_g)^2$ with the second tracer $\delta^{[2]}_g$; and the skew correlation function is given by

$$\xi^{(s)}_{(1,2)} = \langle \delta^{[1]}_g(x_1) \delta^{[2]}_g(x_2) \rangle.$$  

(17)

The effect of the linear bias is straightforward, which can be written as $(b_1^{[1]} b_1^{[2]} b_2^{[1]} b_2^{[2]} \langle \delta(x_1) \delta(x_2) \rangle)$. After Fourier transformation, the galaxy skew spectrum due to the linear bias is

$$P^{(s)}_{g,2T}(k) |_{BL} = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2 (b_1^{[1]} b_1^{[2]} b_2^{[1]} b_2^{[2]} \langle \delta(x_1) \delta(x_2) \rangle)$$

$$\times [B_{m,l}(k, q, \alpha) + B_{m,G}(k, q, \alpha)].$$

(18)

The contribution of the nonlinear bias to the correlation function is

$$\xi^{(s)}_{(1,2)} |_{NLB} = \frac{1}{2} (b_1^{[1]} b_1^{[2]} b_2^{[2]} \langle \delta(x_1) \delta(x_2) \rangle)$$

$$\times \left. \right| B_{m,l}(k, q, \alpha) + B_{m,G}(k, q, \alpha).$$

(19)

and the corresponding skew spectrum is

$$P^{(s)}_{g,2T}(k) |_{NLB} = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2 (b_1^{[1]} b_1^{[2]} b_2^{[1]} b_2^{[2]} \langle \delta(x_1) \delta(x_2) \rangle)$$

$$\times \left. \right| B_{m,l}(k, q, \alpha) + B_{m,G}(k, q, \alpha).$$

(20)

To sum up, when considering two different tracers, we can obtain six different skew spectra. We use the subscript (12) to express the cross-correlation spectrum of the square of the first tracer $(\delta^{[1]}_g)^2$ with the second tracer $\delta^{[2]}_g$. The full expressions of the six skew spectra are

$$P^{(s)}_{g,11,1}(k) = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2$$

$$\times \left. \right| (b_1^{[1]} b_1^{[2]} [B_{m,l}(k, q, \alpha) + B_{m,G}(k, q, \alpha)])$$

$$+ (b_1^{[2]} b_2^{[1]} P_{m,l,q}(q) P_{m,l,q}(q) + \text{cyc}.),$$

(21)

$$P^{(s)}_{g,11,2}(k) = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2$$

$$\times \left. \right| (b_1^{[1]} b_1^{[2]} [B_{m,l}(k, q, \alpha) + B_{m,G}(k, q, \alpha)])$$

$$+ (b_1^{[2]} b_2^{[2]} P_{m,l,q}(q) P_{m,l,q}(q) + \text{cyc}.),$$

(22)

$$P^{(s)}_{g,12,1}(k) = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2$$

$$\times \left. \right| (b_1^{[1]} b_1^{[2]} P_{m,l,q}(q) P_{m,l,q}(q) + \text{cyc}.),$$

(23)

$$P^{(s)}_{g,12,2}(k) = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2$$

$$\times \left. \right| (b_1^{[1]} b_1^{[2]} P_{m,l,q}(q) P_{m,l,q}(q) + \text{cyc}.),$$

(24)

$$P^{(s)}_{g,12,2}(k) = \int_{-\infty}^{\infty} d\mu \int \frac{dq}{(2\pi)^2} q^2$$

$$\times \left. \right| (b_1^{[1]} b_1^{[2]} [B_{m,l}(k, q, \alpha) + B_{m,G}(k, q, \alpha)])$$

$$+ (b_1^{[1]} b_1^{[2]} P_{m,l,q}(q) P_{m,l,q}(q) + \text{cyc}.),$$

(25)
where the tilde above the bias parameters means the galaxy power spectrum can be greatly affected by relatively small values of $f_{NL}$ via the large-scale bias. The relationship between $b_1$ and $b_1$ is given by Dalal et al. (2008), Grossi et al. (2009), Wagner et al. (2010), McDonald (2008), Matarrese & Verde (2008), Sefusatti et al. (2009), and Dai & Xia (2020),

$$b_1 - b_1 = 2f_{NL}^{loc} \frac{\delta_c}{M(k, z)} q,$$

where $\delta_c \simeq 1.686$ is the threshold for collapse and the correction $q = 0.75$ is calibrated from N-body simulations (McDonald 2008).

2.3. Shot Noise

Due to the discrete distribution of galaxies, both the power spectrum and skew spectrum have additional stochasticity contributions. In this work, we consider the Poisson model. The number density of the tracers is given by

$$n(x) = \sum_i \delta_D(x - x_i),$$

and the discrete density contrast is defined as

$$\delta_D = \frac{n(x)}{\bar{n}} - 1,$$

where $\bar{n} \equiv \langle n(x) \rangle$ is the mean number density. Chan & Blot (2017) derived the Poisson shot noise of the two-point and three-point functions in detail. Following their work, we derive the shot noise contributions to the power spectrum and skew spectrum in the Appendix. The results are listed below, where we use $S(k)$ and $S(k)$ to express the shot noise of the power spectrum and skew spectrum, respectively:

$$S_{1,1} = \frac{1}{n_1},$$

$$S_{2,2} = \frac{1}{n_2},$$

$$S_{1,2} = 0,$$

$$S_{1,1}^{(s)} = \int \frac{d^3q}{(2\pi)^3} \left[ \frac{1}{n_1} \left( P_R(k) + P_R(q) + P_R(\alpha) \right) + \frac{1}{n_1^2} \right],$$

Finally, it is necessary to review the galaxy power spectra for multitracers. Since we focus only on $k < 0.1 h$ Mpc$^{-1}$, it is sufficient to consider only the leading order of the power spectrum. There are three power spectra for the two different tracers, which are

$$P_{s,(1,1)}(k) = \langle \tilde{b}^{[1]} \rangle^2 P_{m,L}(k),$$

$$P_{s,(2,2)}(k) = \langle \tilde{b}^{[2]} \rangle^2 P_{m,L}(k),$$

$$P_{s,(1,2)}(k) = \langle \tilde{b}^{[1]} \rangle \langle \tilde{b}^{[2]} \rangle P_{m,L}(k),$$

Where the tilde above the bias parameters means the galaxy power spectrum can be greatly affected by relatively small values of $f_{NL}$ via the large-scale bias. The relationship between $b_1$ and $b_1$ is given by Dalal et al. (2008), Grossi et al. (2009), Wagner et al. (2010), McDonald (2008), Matarrese & Verde (2008), Sefusatti et al. (2009), and Dai & Xia (2020),

$$b_1 - b_1 = 2f_{NL}^{loc} \frac{\delta_c}{M(k, z)} q,$$

where $\delta_c \simeq 1.686$ is the threshold for collapse and the correction $q = 0.75$ is calibrated from N-body simulations (McDonald 2008).

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$$n(x) = \sum_i \delta_D(x - x_i),$$

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$$S_{1,1} = \frac{1}{n_1},$$

$$S_{2,2} = \frac{1}{n_2},$$

$$S_{1,2} = 0,$$

$$S_{1,1}^{(s)} = \int \frac{d^3q}{(2\pi)^3} \left[ \frac{1}{n_1} \left( P_R(k) + P_R(q) + P_R(\alpha) \right) + \frac{1}{n_1^2} \right],$$

2.4. Smoothing

Even if we truncate the wavenumber range at quasi-linear scales, the galaxy skew spectrum still contains highly nonlinear information due to the integral over $q$. However, the second-order kernel $F_2$ is valid only on quasi-linear scales and is expected to fail in the nonlinear regime. To overcome this problem, there are several fitting formulas of $F_2$ using N-body simulations to derive a more reliable expression for the bispersion (Scoccimarro & Couchman 2001; Gil-Marín et al. 2012). However, these formulas are valid only in a specific $k$ range. For simplicity, in our analysis, we apply a large smoothing filter to the field to suppress the small-scale modes. By doing this, we may lose some nonlinear information, but we can have better analytical control. If the results show that the skew spectrum with a large smoothing filter can improve the constraints, it indicates that, using a more sophisticated modeling of the gravitational instability kernel, the analysis can further lift the remaining degeneracies.

In this paper, we use a top-hat window function, the Fourier transform of which is

$$W_R(k) = \frac{3 \sin(kR)}{k^3} - \frac{3 \cos(kR)}{k^5}.$$

where $R$ is the radius of the smoothing filter. Then, the smoothed power spectra and skew spectra become (Schmittfull et al. 2015; Dai et al. 2020)

$$P_{s,R}(k) = P_R(k) W_R^2(k),$$

$$P_{s,R}(k) = \int \frac{d^3q}{(2\pi)^3} B_R(k, q, \alpha) W_R(k) W_R(q) W_R(\alpha).$$

3. Simulations

The frequently used method to seek for the information that the multitracer technique can give us is the Fisher matrix analysis, which is less computationally intensive. However, due to the high correlation between the power spectrum and skew spectrum and the complex properties of the skew spectrum covariance, we resort to a numerically computed covariance from a suite of simulations. This is not as fast and simple as a Fisher matrix analysis and requires access to large simulations, but the results are more reliable.

In our analysis, we use 1000 realizations from the QUIJOTE simulations $^1$ (Villaescusa-Navarro et al. 2020). The cosmological parameters are $\Omega_m = 0.3175$, $\Omega_b = 0.049$, $h = 0.6711$,

$^1$ https://github.com/franciscovillaescusa/Quijote-simulations
The measured power spectra and skew spectra for multitracers (dots and error bars are the average values and the standard deviations of the 1000 realizations, respectively), together with the best-fit theoretical models (dashed lines). The upper-left panel shows the power spectra \( P_{11,1} \), \( P_{22,2} \), and \( P_{12,2} \); the upper-right panel shows the auto-skew spectra \( P_{11,1}^s \) and \( P_{22,2}^s \); the lower-left panel shows the cross-skew spectra \( P_{12,2}^s \) and \( P_{12,2}^s \); and the lower-right panel shows the cross-skew spectra \( P_{12,2}^s \) and \( P_{12,2}^s \). 1 and 2 stand for T1 and T2 of our halo catalog.

Figure 1. The measured power spectra and skew spectra for multitracers (dots and error bars are the average values and the standard deviations of the 1000 realizations, respectively), together with the best-fit theoretical models (dashed lines). The upper-left panel shows the power spectra \( P_{11,1} \), \( P_{22,2} \), and \( P_{12,2} \); the upper-right panel shows the auto-skew spectra \( P_{11,1}^s \) and \( P_{22,2}^s \); the lower-left panel shows the cross-skew spectra \( P_{12,2}^s \) and \( P_{12,2}^s \); and the lower-right panel shows the cross-skew spectra \( P_{12,2}^s \) and \( P_{12,2}^s \). 1 and 2 stand for T1 and T2 of our halo catalog.

\( n_s = 0.9624, \sigma_8 = 0.834, M_\odot = 0.0 \text{ eV}, \) and \( f_{\text{NL}}^{\text{loc}} = 0 \), which are in good agreement with the latest Planck results (Aghanim et al. 2020). The simulations are performed with the TreePM code GADGET-III, an improved version of GADGET-II (Springel 2005). All of the simulations have 512³ particles in a box with a cosmological volume of \( 1 (h^{-1} \text{ Gpc})^3 \), which is large enough to avoid systematic finite-volume effects (Schneider et al. 2016). Details of the simulations can be found in Villaescusa-Navarro et al. (2020).

To study the multitracer technique, we use the halo catalogs, which were identified using the friends-of-friends algorithm (Davis et al. 1985) with a linking length \( b = 0.2 \) at \( z = 0 \). We divide each catalog into two parts, the halo mass ranges of which are \([2.5 \times 10^{14}, \ 1 \times 10^{15}] h^{-1} M_\odot\) and \([1.3 \times 10^{13}, \ 2.5 \times 10^{13}] h^{-1} M_\odot\), and there are 163,000 and 206,000 halos on average. Hereafter, we call them T1 and T2, respectively.

It is worth noting that these simulations have Gaussian initial conditions. Thus, the constraint results for \( f_{\text{NL}}^{\text{loc}} \) should be consistent with 0, and the error bars can reflect the constraint ability using different combinations of the power spectra and the skew spectra.

We use the routine provided in PYLIANS\(^2\) to calculate the cross spectrum of the squared field \( \delta_f(x) \) and the \( \delta_f(x) \) field itself. Before squaring the density field, we apply a top-hat smoothing filter with \( R = 20 h^{-1} \text{ Mpc} \). In Figure 1, we plot the real space power spectra and skew spectra for multitracers obtained from the simulations. The data points are the average results of the 1000 realizations, and the error bars are the standard deviations of the spectra at a specific \( k \). We also show the theoretical predictions for the best-fit parameters (see details in Section 4). The results show that with this smoothing choice, the standard perturbation theory is sufficient to describe the skew spectra.

Before being able to perform a joint analysis using the power spectra and the skew spectra, we need to evaluate the covariance of these quantities. Since we use a large smoothing filter, only the quasi-linear scales are useful. In our analysis, we use the wavenumber range \( k = [0.0089, 0.1] h^{-1} \text{ Mpc}^{-1} \), and there are 15 \( k \) bins uniformly spaced in \( k \). We arrange \( P_{11,1}, P_{22,2}, P_{12,2}, P_{11,1}^s, P_{22,2}^s, P_{12,2}^s, P_{12,2}^s, P_{12,2}^s \) into a “data” vector \( \hat{P}_b^{k, p, s}(K) \) (\( i = 1, \ldots, 135 \)). In Figure 2, we plot the correlation matrix of \( \hat{P}_b^{k, p, s}(K) \), which is defined as

\[
\frac{C_{K, K}^*}{\sqrt{C_{K, K}^* C_{K, K}^*}},
\]

where \( C_{K, K}^* \) is the estimated covariance of \( \hat{P}_b^{k, p, s}(K) \). We find that the different \( k \) modes are weakly correlated even for the skew spectra. As Hartlap et al. (2006) pointed out, the inverse of the covariance matrix is a biased estimator and can be

\(^2\) https://github.com/franciscovillaescusa/Pylians
correlated by introducing a Hartlap factor (Hartlap et al. 2006):
\[
C^{-1} = \frac{n - p - 2}{n - 1} (C^*)^{-1},
\]
where \( n = 1000 \) is the number of independent observations and \( p = 135 \) is the dimensionality of our data.

4. Constraint Results

In our analysis, we consider one of the 1000 realizations as our mock universe, and with the covariance from the simulations, we can constrain the cosmological parameters by fitting the power spectra and the skew spectra for multitracers. We use four different combinations of the spectra. First, we combine T1 and T2, treat the combination as a single tracer, and constrain the parameters using its power spectrum alone and its power spectrum together with the skew spectrum. Then, we use the multitracer technique, where we also use the power spectra both alone and together with the skew spectra.

We modify the public software COSMOMC3 (Lewis & Bridle 2002), a Markov Chain Monte Carlo (MCMC) code to perform joint Bayesian parameter inference. A simple \( \chi^2 \) is used for parameter fitting in our analysis:
\[
\chi^2 = (\hat{P}(K) - P(K)) C^{-1} (\hat{P}(K) - P(K))^T,
\]
where \( \hat{P} \) and \( P \) represent the model spectra and the measured spectra, respectively, and \( C^{-1} \) is their covariance after Hartlap correction. The best-fit parameters can be obtained by finding the minimum value of \( \chi^2 \), and the confidence regions are defined by the surfaces of constant \( \Delta \chi^2 = \chi^2 - \chi_{\text{min}}^2 \), where \( \chi_{\text{min}}^2 \) is the minimum value of \( \chi^2 \).

We start by determining the bias parameters of T1 and T2, with the other fiducial cosmological parameters fixed. Because of the strong degeneracies between biases and the other parameters, this step can be used to check the validity of our theoretical prediction without adding too many variables. Using all the spectra for the multitracers, the constraints are as listed in Table 1. We find

\[
\begin{align*}
\text{bias}^{[1]} &= (1.451 \pm 0.013, -0.714 \pm 0.026, 1.193 \pm 0.016, -0.784 \pm 0.028) \\
\end{align*}
\]

Using the best-fit bias parameters, we plot the theoretical models with dashed lines in Figure 1. The figures show that our theory can accurately predict the measurements at linear scales.

We now constrain the cosmological and bias parameters simultaneously to investigate the extra information by using the multitracer technique. The parameterization we use is
\[
P = (A_s, n_s, f_{\text{NL}}, b_1^{[1]} , b_2^{[1]} , b_1^{[2]} , b_2^{[2]}),
\]
where \( A_s \) and \( n_s \) are the amplitude and spectral index of the primordial spectrum, respectively. The other parameters are fixed at their fiducial values. In Figure 3 and Table 2, we show our constraint results. Since \( A_s \), \( b_1 \), and \( b_2 \) are highly correlated, we construct a new variable \( b_{\text{fid}}^{[1]} = b_1^2 A_s / b_2^2 A_s \). For the multitracer approach, we define this value as the average result of the two tracers. From the results, we find that the constraints are consistent with the fiducial values.

First, we use only the power spectrum and combine T1 and T2. The marginalized 2D contours are shown in blue in Figure 3. The Best-fit Values of the Bias Parameters and Their Marginalized 1σ Errors

**Table 1**

| Bias        | 1σ Error        |
|-------------|-----------------|
| \( b_1^{[1]} \) | \( 1.451 \pm 0.013 \) |
| \( b_1^{[2]} \) | \( -0.714 \pm 0.026 \) |
| \( b_2^{[1]} \) | \( 1.193 \pm 0.014 \) |
| \( b_2^{[2]} \) | \( -0.784 \pm 0.028 \) |

![Figure 2. The correlation matrix of \( P^{\mu+(K)} \).](http://cosmologist.info/cosmomc/)

![Figure 3. Marginalized two-dimensional distributions and posterior distributions for normalized \( b_1^2 A_s \).](http://cosmologist.info/cosmomc/)
Figure 3, and the constraint results for \( (b_1^2 A_s)_{\text{normal}}, n_s, \) and \( f_{\text{NL}}^{\text{loc}} \) are 0.997 ± 0.024, 0.969 ± 0.058, and 19.4 ± 156.4 (68% C.L.), respectively. The constraints become tighter when we include the skew spectrum, as indicated in Dai et al. (2020). In this analysis, the addition of the skew spectrum to the power spectrum yields a reduction in the errors of 29%, 21%, and 37% for \( (b_1^2 A_s)_{\text{normal}}, n_s, \) and \( f_{\text{NL}}^{\text{loc}}, \) respectively. The results are consistent with the conclusion in Dai et al. (2020).

We then consider the multitracer technique. When we use only the power spectra, the constraint errors are markedly reduced; the results are 1.001 ± 0.014, 0.982 ± 0.034, and 15.2 ± 72.4 (68% C.L.) for \( (b_1^2 A_s)_{\text{normal}}, n_s, \) and \( f_{\text{NL}}^{\text{loc}}, \) respectively. The constraints are reduced by 42%, 41%, and 54% compared with the single tracer case. This outcome shows that the multitracer technique can effectively reduce the cosmic variance, and that amplitude parameters such as \( b_1^2 A_s \) and \( f_{\text{NL}}^{\text{loc}} \) are better constrained. Due to the degeneracies between the cosmological parameters, the errors of the other parameters are also reduced. Finally, we use all of the power spectra and skew spectra for the multitracer approach, the results of which are shown in yellow in Figure 3, i.e., 0.997 ± 0.012, 0.975 ± 0.028, and -4.8 ± 42.2 (68% C.L.) for \( (b_1^2 A_s)_{\text{normal}}, n_s, \) and \( f_{\text{NL}}^{\text{loc}}, \) respectively. Compared with the results obtained using power spectra for multitracers, the 1σ marginalized errors are reduced by 14%, 18%, and 42%. This reduction is due to the extra information contained in the skew spectra. When considering the information from both the skew spectra and multitracer techniques, i.e., compared with the results obtained from the power spectrum for a single tracer, the errors are reduced by 50%, 52%, and 73%. Both the skew spectrum and the multitracer technique are effective tools for constraining the primordial non-Gaussianity.

Finally, we also show the results from different simulation box sizes to demonstrate the effect of survey size on the constraint ability. We compare the above results with results from another set of simulations with the same mass resolution but different box size: \( L = 0.5 \, h^{-1} \text{Gpc} \). We also use the maximum wavenumber \( k_{\text{max}} = 0.1 \, h \, \text{Mpc}^{-1} \), and there are only 8k bins uniformly spaced in \( k = [0.018, 0.1]h \text{Mpc}^{-1} \). In Figure 4 we plot the results from these two different box sizes using power spectra and skew spectra from two tracers. When we consider the simulations with \( L = 0.5 \, h^{-1} \text{Gpc} \), the constraint results are 0.995 ± 0.035, 1.003 ± 0.084, and -7.1 ± 155.2 (68% C.L.) for \( (b_1^2 A_s)_{\text{normal}}, n_s, \) and \( f_{\text{NL}}^{\text{loc}}). As we expected, the constraints are reduced by 66%, 67%, and 73% when we use the simulations with \( L = 1 \, h^{-1} \text{Gpc} \). The reason is we can measure larger scales (\( k_{\text{min}} \) is smaller) and more k modes (\( \Delta k \) is smaller) when we increase our survey volume.

### 5. Conclusions

In this paper, we mainly discuss the potential power of the multitracer technique for the skew spectrum as a possible probe of the local-type primordial non-Gaussianity. The skew spectrum is estimated by cross-correlating the squared field \( \delta^2(x) \) with the \( \delta(x) \) field itself. Computationally, measuring the skew spectrum is equivalent to a power spectrum estimation, but the skew spectrum contains parts of the three-point clustering information, which can be used to further reduce the parameter degeneracies present at the level of the power spectrum. To apply the multitracer technique, we first review the formula of the galaxy skew spectrum, which has contributions from the primordial non-Gaussianity, gravitational instability, and galaxy (halo) bias, and then generalize the theory to multitracers to predict both the signals and the shot noise contributions.

Because of the high correlation between the power spectrum and skew spectrum and the complex properties of the covariance, we do not apply the frequently used Fisher matrix analysis. Instead, we estimate the covariance from a suite of simulations and constrain the parameters using a joint Bayesian parameter inference. Our method is not as fast as a Fisher matrix analysis, but the results are more reliable.

We divide the simulated halo catalog into two parts, which have comparable samples, and then estimate the spectra and their covariance to determine the joint constraints. For comparison, we also constrain the parameters using the whole halos. The results show that by applying the skew spectra and multitracer technique, the 1σ marginalized errors for \( (b_1^2 A_s)_{\text{normal}}, n_s, \) and \( f_{\text{NL}}^{\text{loc}} \) are reduced by 50%, 52%, and 73%, respectively. With the forthcoming generation of wide-
field galaxy surveys, the use of the skew spectrum and multitracer technique can offer us a powerful and fast way to constrain the primordial non-Gaussianity.

However, due to the large smoothing filter (20 h⁻¹ Mpc) adopted in our analysis, the skew spectrum contains only the linear-scale information. If we can obtain a more sophisticated modeling of the gravitational instability kernel using simulations, the analysis can be extended to smaller scales, further lifting the remaining degeneracies. We leave this exploration for future work.

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Appendix

Poisson Shot Noise of the Power Spectrum and Skew Spectrum

The following calculations are based on Chan & Blot’s (2017) work. First, for a single tracer, the two-point correlation function of the discrete galaxy field is

\[ \xi_g^{(2)}(x_1, x_2) = \langle \delta_g(x_1) \delta_g(x_2) \rangle = \frac{1}{n^2} \langle n(x_1)n(x_2) \rangle - 1, \]  

(A1)

where

\[ \langle n(x_1)n(x_2) \rangle = \left\langle \sum_i \delta_D(x_1 - x_i) \delta_D(x_2 - x_i) \right\rangle + \left\langle \sum_{i,j} \delta_D(x_1 - x_i) \delta_D(x_2 - x_j) \right\rangle = \delta_D(x_1 - x_2) \bar{n} + \bar{n}^2 [1 + \xi^{(2)}(x_1, x_2)]. \]  

(A2)

Here, we need to consider the case where two points are the same; if the points are different, they can be modeled by the smooth correlation function \( \xi^{(2)} \). Thus, we can express \( \xi_g^{(2)} \) as (Chan & Blot 2017)

\[ \xi_g^{(2)}(x_1, x_2) = \xi^{(2)}(x_1, x_2) + \frac{1}{\bar{n}} \delta_D(x_1 - x_2). \]  

(A3)

After a Fourier transformation, the galaxy power spectrum is given by

\[ P_g, \text{measured}(k) = P_g(k) + \frac{1}{\bar{n}}. \]  

(A4)

The shot noise contribution to the power spectrum of a single tracer is \( S_{TT}(k) = 1/\bar{n} \). When we consider two different tracers, the first term on the right side of Equation (A2) vanishes; thus, we have \( S_{TT}(k) = 0 \).

To calculate the shot noise of the skew spectrum, we begin with the three-point correlation function:

\[ \xi^{(3)}(x_1, x_2, x_3) = \langle \delta_g(x_1) \delta_g(x_2) \delta_g(x_3) \rangle = \frac{1}{n^3} \langle n(x_1)n(x_2)n(x_3) \rangle - \left[ \frac{1}{\bar{n}^2} \langle n(x_1)n(x_2) \rangle + 2\text{cyc.} \right] + 2, \]  

(A5)

where the three-point correlator of \( n \) is

\[ \langle n(x_1) n(x_2) n(x_3) \rangle = \sum_i \delta_D(x_1 - x_i) \delta_D(x_2 - x_i) \delta_D(x_3 - x_i) \times (x_3 - x_i) \]  

\[ + \left\{ \sum_{i,j} \delta_D(x_1 - x_i) \delta_D(x_2 - x_j) \delta_D(x_3 - x_j) \right\} + 2\text{cyc.} \]  

\[ + \left\{ \sum_{i,j,k} \delta_D(x_1 - x_i) \delta_D(x_2 - x_j) \delta_D(x_3 - x_k) \right\} \]  

\[ = \delta_D(x_1 - x_2) \delta_D(x_1 - x_3) \bar{n} + \bar{n}^2 [1 + \xi^{(2)}(x_1, x_2)] \]  

\[ + \bar{n}^3 (1 + \xi^{(2)} + \xi^{(2)} + \xi^{(2)} + \xi^{(2)}), \]  

(A6)

where \( \xi^{(3)}_{123} \) is the continuous three-point correlation function. Using Equations (A2) and (A6), we can obtain the galaxy three-point correlation function for a single tracer (Chan & Blot 2017):

\[ \xi^{(3)}(x_1, x_2, x_3) = \frac{1}{\bar{n}} \delta_D(x_1 - x_2) \delta_D(x_1 - x_3) \]  

\[ + \left[ \delta_D(x_2 - x_3) \xi^{(2)} + 2\text{cyc.} \right] + \xi^{(3)}. \]  

(A7)

The observed galaxy bispectrum is given by

\[ B_g, \text{measured}(k_1, k_2, k_3) = B_g(k_1, k_2, k_3) + \frac{1}{\bar{n}^2} \]  

\[ + \frac{1}{\bar{n}} \left[ P_g(k_1) + 2\text{cyc.} \right]. \]  

(A8)

Therefore, the shot noise contribution to the skew spectrum is

\[ S_{TT}(k) = \int \frac{d^3q}{(2\pi)^3} \left[ \frac{1}{\bar{n}} \left( P_g(k) + P_g(q) + P_g(\alpha) \right) + \frac{1}{\bar{n}^2} \right]. \]  

(A9)

When we consider, for example, two different tracers, the three-point correlation function \( \langle \delta^{(1)}_g(x_1) \delta^{(1)}_g(x_2) \delta^{(2)}_g(x_3) \rangle \). Following the above calculation, the correlation function is given by

\[ \langle \delta^{(1)}_g(x_1) \delta^{(1)}_g(x_2) \delta^{(2)}_g(x_3) \rangle = \frac{1}{\bar{n}_1} \delta_D(x_1 - x_2) \xi^{(2)}_{23,(1,2)} + \xi^{(3)}, \]  

(A10)

and the corresponding bispectrum is

\[ B_g, \text{measured}(k_1, k_2, k_3) = \frac{1}{\bar{n}_1} P_{g,(1,2)}(k_3) + B_g(k_1, k_2, k_3). \]  

(A11)

Finally, we can obtain the shot noise contributions to the skew spectra when we consider two different tracers:

\[ S_{g,(1,2)}^{(1)}(k) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{\bar{n}_1} P_{g,(1,2)}(q) \]  

if set \( x_2 = x_1 \),

(A12)

\[ S_{g,(2,1)}^{(1)}(k) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{\bar{n}_1} P_{g,(1,2)}(q) \]  

if set \( x_3 = x_1 \).
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