Variational Prototype Replays for Continual Learning

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Abstract
Continual learning refers to the ability to acquire and transfer knowledge without catastrophically forgetting what was previously learned. In this work, we consider few-shot continual learning in classification tasks, and we propose a novel method, Variational Prototype Replays, that efficiently consolidates and recalls previous knowledge to avoid catastrophic forgetting. In each classification task, our method learns a set of variational prototypes with their means and variances, where embedding of the samples from the same class can be represented in a prototypical distribution and class-representative prototypes are separated apart. To alleviate catastrophic forgetting, our method replays one sample per class from previous tasks, and correspondingly matches newly predicted embeddings to their nearest class-representative prototypes stored from previous tasks. Compared with recent continual learning approaches, our method can readily adapt to new tasks with more classes without requiring the addition of new units. Furthermore, our method is more memory efficient since only class-representative prototypes with their means and variances, as well as only one sample per class from previous tasks need to be stored. Without tampering with the performance on initial tasks, our method learns novel concepts given a few training examples of each class in new tasks.

1. Introduction
Continual learning enables humans to continually acquire and transfer new knowledge across their lifespans while retaining previously learnt experiences (Hassabis et al., 2017). This ability is also critical for artificial intelligence (AI) systems to interact with the real world and process continuous streams of information (Thrun & Mitchell, 1995). However, the continual acquisition of incrementally available data from non-stationary data distributions generally leads to catastrophic forgetting in the system (McCloskey & Cohen, 1989; Ratcliff, 1990; French, 1999). Continual learning remains a long-standing challenge for deep neural network models since these models typically learn representations from stationary batches of training data and tend to fail to retain good performance in previous tasks when data become incrementally available over tasks (Kemker et al., 2018; Maltoni & Lomonaco, 2019).

Numerous methods for alleviating catastrophic forgetting have been proposed. The most pragmtical way is to jointly train deep neural network models on both old and new tasks, which demands a large amount of resources to store previous training data and hinders learning of novel data in real time. Another option is to complement the training data for each new task with “pseudo-data” of the previous tasks (Shin et al., 2017; Robins, 1995). In this approach, a generative model is trained to generate fake historical data used for pseudo-rehearsal. Deep Generative Replay (DGR) (Shin et al., 2017) replaces the storage of the previous training data with a Generative Adversarial Network to synthesize training data on all previously learnt tasks. These generative approaches have succeeded over very simple and artificial inputs but they cannot tackle more complicated inputs (Atkinson et al., 2018). Moreover, to synthesize the historical data reasonably well, the size of the generative model is usually very large and expensive in terms of memory resources (Wen et al., 2018). An alternative method is to store the weights of the model trained on previous tasks, and impose constraints of weight updates on new tasks (He & Jaeger, 2018; Kirkpatrick et al., 2017; Zenke et al., 2017; Lee et al., 2017; Lopez-Paz et al., 2017). For example, Learning Without Forgetting (LwF) (Li & Hoiem, 2018) has to store all the model parameters on previously learnt tasks, estimates their importance on previous tasks and penalizes future changes to these parameters on new tasks. However, selecting the “important” parameters for previous tasks via pre-defined thresholds complicates the implementation by exhaustive hyper-parameter tuning. In addition, state-of-the-art neural network models often involve millions of parameters and storing all network parameters from previous tasks does not necessarily reduce the memory cost (Wen et al., 2018). In contrast with these
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In this paper, we propose a method that we call Variational Prototype Replays, for continual learning in classification tasks. Extending previous work (Snell et al., 2017), we use a neural network to learn class-representative variational prototypes with their means and variances in a latent space and classify embedded test data by finding their nearest representations sampled from class-representative variational prototypes. To prevent catastrophic forgetting, our method replays one sample per class from previous tasks, and correspondingly matches newly predicted representations to their nearest class prototypes stored from previous tasks. Since not all prototypical features learnt from the previous tasks are equally important in new tasks, the learnt variance in variational prototypes of previous tasks provide confidence levels of learnt prototype features, and therefore our method can selectively forget under-represented features in the prototypes while the network learns to adapt to new tasks. We evaluate our method under two typical experimental protocols, incremental domain and incremental class, for few-shot continual learning across three benchmark datasets, MNIST (Deng, 2012), CIFAR10 (Krizhevsky & Hinton, 2009) and miniImageNet (Deng et al., 2009). Compared with state-of-the-art performance, our method significantly boosts the performance of continual learning in terms of memory retention capability while being able to generalize to learn new concepts and adapt to new tasks, even with a few training examples in new tasks. Unlike parameter regularization methods, our approach further reduces the memory storage by storing only one sample per class as well as variational prototypes in the previous tasks. Moreover, in contrast to methods where the last layer in traditional classification networks often structurally depends on the number of output classes, our method maintains the same network architecture and does not require adding new units.

2. Few-shot Continual Learning Protocols

Humans can learn novel concepts given a few examples without sacrificing classification accuracy on initial tasks (Gidaris & Komodakis, 2018). However, typical continual learning schemes assume that a large amount of training data over all tasks is always available for fine-tuning networks to adapt to new data distributions, which does not always hold in practice. We revise task protocols to more challenging ones: networks are trained with a few examples per class in subsequent tasks except for the first task in the sequence. For example, we train the models with 6,000 and 480 example images per class in the first task respectively on MNIST and miniImageNet and 10 images per class in subsequent tasks. We also evaluate an even more challenging protocol when there are only 10 example images per class even in the first task in CIFAR10.

Permutated MNIST in incremental domain task is a benchmark task protocol in continual learning (Lee et al., 2017; Lopez-Paz et al., 2017; Zenke et al., 2017) (Figure 1). In each task, a fixed permutation sequence is randomly generated and is applied to input images in MNIST (Deng, 2012). Though the input distribution always changes across tasks, models are trained to classify 10 digits in each task and the model structure is always the same. There are 50 tasks in total. During testing, the task identity is not available to models. The models have to classify input images into 1 out of 10 digits.

Split CIFAR10 and split MiniImageNet in incremental class task is a more challenging task protocol where models need to infer the task identity and at the same time solve each image classification task. The input data is also more complex, including classification on natural images in CIFAR10 (Krizhevsky & Hinton, 2009) and miniImageNet (Deng et al., 2009). The former contains 10 classes, and the latter consists of 100 classes. In CIFAR10, the model is first trained with 2 classes and later by adding one more class in each subsequent task. There are 9 tasks in total and 10 images per class in the training set. In miniImageNet, models are trained with 10 classes in each task. There are 10 tasks in total.

3. Method

We propose a novel method, Variational Prototype Replays, for few-shot continual learning. First, we introduce variable naming conventions and the problem formulation. Up to any task $t$ where $t \in \{1, 2, \ldots, T\}$ and $T$ is not pre-determined, there is a total of $C$ classes, and we use $c$ to denote any class $c \in \{1, 2, \ldots, C\}$. In the incremental domain protocol, $C = 10$ for task $t$; whereas in the incremental class protocol, $C$ increases with the number of tasks. There is a total of $N$ training samples per class and we use $n$ to denote any training sample in a class. To explicitly define a training sample $I_{t,c}^{n}$, we use superscript to denote the $n$th training sample. For example, $I_{1,2}^{3}$ denotes the 3rd training sample from the 2nd class in the 1st task. Next, we illustrate how to apply our method to perform classification in a task and
how to prevent catastrophic forgetting across tasks (Fig 2).

3.1. Classification

Our method can be applied on any feed-forward 2D-ConvNet (2D-CNN) architecture for classification tasks. The network with parameters $F_t$ learns to encode an input image $I^n_{t,c}$ in a latent space, in which these encoded image representations cluster around a prototype for each class and classification is performed by finding the nearest prototype (Fig. 2). Extending previous work (Snell et al., 2017) on learning a single prototype for each object class in task $t$, we introduce variational prototypes that follow a Gaussian distribution parameterized with mean $\hat{\mu}_{t,c}$ and variance $\tilde{\sigma}_{t,c}$. The mean and variance allow the network to replay many prototypes sampled from class-representative distributions to prevent overfitting and allow easy interpolation in the latent space. Compared with other replay methods, such as (Rebuffi et al., 2017), where latent representations of each individual image have to be stored for replays, variational prototypes provide advantages in memory usage since only the mean and variance need to be stored for each class.

Inspired by the design of variational autoencoders (Doersch, 2016), we propose variational encoders which learn a conditional class-representative Gaussian distribution with its mean $\mu_{t,c}^n$ and variance $\sigma_{t,c}^n$, given each input image $I^n_{t,c}$: $(\mu_{t,c}^n, \sigma_{t,c}^n) = F_t(I^n_{t,c})$. Variational prototypes $(\hat{\mu}_{t,c}, \tilde{\sigma}_{t,c})$ can then be computed by taking the average of variational image representations conditioned from all input image $I^n_{t,c}$ belonging to class $c$ in task $t$:

$$\hat{\mu}_{t,c} = \frac{1}{N} \sum_n \mu_{t,c}^n, \tilde{\sigma}_{t,c} = \frac{1}{N} \sum_n \sigma_{t,c}^n$$  \hspace{1cm} (1)

In task $t$, to perform classification on total $C$ classes, the goal is to make each encoded image’s representational distribution to be close to the variational prototype distribution within the same class and to be far apart from other variational prototype distributions of different classes. We sample $Z$ latent representations $s_{\mu_{t,c}^n, \sigma_{t,c}^n}$ from both image representational distributions and $s_{\hat{\mu}_{t,c}, \tilde{\sigma}_{t,c}}$ from variational prototype distributions. For each $s_{\mu_{t,c}^n, \sigma_{t,c}^n}$ from class $c$, the network estimates a distance distribution based on a softmax over distances to all the sampled prototypes of $C$ classes in the latent space:

$$p_F(c|s_{\mu_{t,c}^n, \sigma_{t,c}^n}) = \frac{\exp(-d(s_{\mu_{t,c}^n, \sigma_{t,c}^n}, s_{\mu_{t,c}, \tilde{\sigma}_{t,c}}))}{\sum_c \exp(-d(s_{\mu_{t,c}^n, \sigma_{t,c}^n}, s_{\mu_{t,c}, \tilde{\sigma}_{t,c}}))},$$  \hspace{1cm} (2)

where we define distance function $d(s_{\mu_{t,c}^n, \sigma_{t,c}^n}, s_{\mu_{t,c}, \tilde{\sigma}_{t,c}})$ as the L2-norm between $s_{\mu_{t,c}^n, \sigma_{t,c}^n}$ and $s_{\mu_{t,c}, \tilde{\sigma}_{t,c}}$.

The classification objective is to minimize the cross-entropy loss $L_{\text{classi}}$ with the ground truth class label $c$ via Stochastic Gradient Descent (Bottou, 2010): $L_{\text{classi}} = -\log p_F(c|s_{\mu_{t,c}^n, \sigma_{t,c}^n})$

Compared to traditional classification networks with a specific classification layer attached in the end, (also see Table 1 for network architecture comparisons between baseline methods and ours), our method keeps the network architecture unchanged while using the nearest prototypical samples in the latent space for classification. For example, in the split CIFAR10 incremental class protocol where the models are asked to classify new classes (see also Sec 2), traditional classification networks have to expand their architectures by accommodating more output units in the last classification layer based on the number of incremental classes and consequently, additional network parameters have to be added into the memory.
Figure 2. Illustration of classification in Task 1 and catastrophic forgetting alleviation in Task 2 using our proposed method in the Split CIFAR10 incremental class protocol. In Task 1, there are two classes (blue and yellow) with each class containing \( n \) training samples (see Sec 3 for variable naming conventions). Each training image \( I^c_{t, i} \) inputs to a feed-forward 2D-CNN and outputs two vectors: mean \( \mu^c_{t, i} \) and variance \( \sigma^c_{t, i} \). Their output dimension is \( 1 \times 500 \). Multiple samples (solid lined circles) can be generated from gaussian distribution based on each pair of mean \( \mu^c_{t, i} \) and variance \( \sigma^c_{t, i} \). The color of spheres denotes object class. Classification is performed by comparing the L2-norm distance between any pairs of samples from the same class or different classes. The inter-class distance pairs should be smaller than intra-class ones. Class-representative variational prototypes denoted by prototypical mean \( \tilde{\mu}^c_{t, c} \) and prototypical variance \( \tilde{\sigma}^c_{t, c} \) for Task 1 are computed by averaging all training samples of the same class. In Task 2, a new class is introduced. The same 2D-CNN architecture is inherited from Task 1 but the parameters of the 2D-CNN get optimized. Only one sample image per class (dash lined square) from Task 1 is replayed. For \( I^c_{1, 1} \), the new mean \( \mu^c_{2, 1} \) and variance \( \sigma^c_{2, 1} \) is computed in Task 2 and similarly we get a new pair of \( \mu^c_{2, 2} \) and \( \sigma^c_{2, 2} \) for \( I^c_{1, 2} \). The classification among three classes can be performed as described in Task 1 (solid straight lines). To eliminate catastrophic forgetting, our method constantly regresses the mean and variance of replayed samples to be as close as possible (dashed straight line) to the class-representative variational prototypes, \( \tilde{\mu}^c_{t, c} \) and \( \tilde{\sigma}^c_{t, c} \) denoted in dash lined circles, in Task 1.

In practice, when \( N \) is large, computing \( \tilde{\mu}^c_{t, c} \) and \( \tilde{\sigma}^c_{t, c} \) is costly and memory inefficient during training. Thus, at each training iteration, we randomly sample two complement image subsets for each class: one subset for computing prototypes and the other for estimating the distance distribution. Sampling size \( Z \) also influences memory and computation efficiency. In the split CIFAR10 incremental class protocol, we choose \( Z = 50 \) (see Sec. 5.2 for analysis on sampling sizes). Our primary choice of the distance function \( d(\cdot) \) is L2-norm which has been verified to be effective in (Snell et al., 2017). As introduced in the network distillation literature (Hinton et al., 2015), we include a temperature hyperparameter \( \tau \) in \( d(\cdot) \) and set its value empirically based on the validation sets. A higher value for \( \tau \) produces a softer probability distribution over classes.

3.2. Variational Prototype Replays

For a sequence of tasks \( t \in \{1, 2, ..., T\} \), the goal of the network with parameters \( F_T \) is to retain good classification performance on all \( C \) classes after being sequentially trained over \( T \) tasks while it is only allowed to carry over a limited amount of information about previous classes \( c_{old} \) from the previous \( T - 1 \) tasks. This constraint eliminates the naive solution of combining all previous datasets to form one big training set for fine-tuning the network \( F_T \) at task \( T \).

To prevent catastrophic forgetting, here we ask the network with parameters \( F_T \) to perform classification on both new classes \( c_{new} \) and old classes \( c_{old} \) by replaying some example images stored from \( c_{old} \) together with all training images from \( c_{new} \). Intuitively, if the number of stored image samples is very large, the network could re-produce the original encoded image representations for \( c_{old} \) by replays, which is our desired goal. However, this does not hold in practice given limited memory capacity. With the simple inductive bias that the encoded image representations of \( c_{old} \) can be underlined by class-representative variational prototypes, instead of classifying \( c_{old} \) using the newly predicted variational prototypes with mean \( \tilde{\mu}_{T, c_{old}} \) and
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When we compute the distances between we compare with nearest prototypes from As described in the previous subsection, in order to variance indicates that the prototype feature distribution is ˜cold, the network learns to classify cold based on stored old variational prototypes (µcold, ˜σcold) over all the previous tasks t.

As described in the previous subsection, in order to classify IP_{T,cold} among c_{new} and c_{old}, the network learns to encode its image representation in the latent space and compare its samples s_{T,cold}^2 and s_{c_{new}}^2 with new variational prototypes (µ_{T,cold}, ˜σ_{T,cold}) and stored old prototypes (µ_{−1,cold}, ˜σ_{−1,cold}) for c_{old} from previous task T = 1. In order to classify IP_{T,cold}, it reviews all the previous tasks t. In each previous task t, our method compares its samples s_{T,cold}^2 and s_{c_{old}}^2 with all stored variational prototypes (µ_{t,cold}, ˜σ_{t,cold}).

There have been some attempts to select representative image examples to store based on different scoring functions (Chen et al., 2012; Koh & Liang, 2017; Brahma & Othon, 2018). However, recent work has shown that random sampling uniformly across classes yields outstanding performance in continual learning tasks (Wen et al., 2018). Hence, we adopt the same random sampling strategy.

From the first task to current task T, the network parameters Ft keep updating in order to incorporate new class representations in the latent space. Hence, the variational prototypes of c_{old} constantly change their representations even for the same class. Not all prototypical features learnt from c_{old} in the previous tasks are equally useful in classifying both c_{new} and c_{old}. As a hypothetical example, imagine that in the first task, we use shape and color to classify red squares versus yellow circles. In the second task, when we see a new class of green circles, we realize shape might not be as good a feature as color; hence, we may need to put “less weight” on the shape features when we compare with nearest prototypes from c_{old}. The variance in the variational prototype provides a confidence score of how representative the prototypical features are. A higher variance indicates that the prototype feature distribution is more spread out; and hence, less representative of c_{old} in the latent space. Thus, we introduce ˜σ_{c_{old}}-weighted L2-norm when we compute the distances between s_{T,cold}^2 and s_{t,cold}^2 for all previous tasks t ∈ {1, 1, T − 1}:

\[ p_{F_t,c_{old}}(s_{T,c_{old}}, s_{c_{old}}) = \frac{\exp(-d(s_{T,c_{old}}^2, s_{c_{old}}^2), (µ_{T,c_{old}, ˜σ_{T,c_{old}}}, (µ_{t,c_{old}, ˜σ_{t,c_{old}}}, ˜σ_{t,c_{old}})))}{\sum_{c_{old}} \exp(-d(s_{T,c_{old}}^2, s_{c_{old}}^2), (µ_{T,c_{old}, ˜σ_{T,c_{old}}}, (µ_{t,c_{old}, ˜σ_{t,c_{old}}}, ˜σ_{t,c_{old}})))}, \]

where we define the weighted distance function:

\[ d(s_1^2, s_2^2, σ) = \| \exp(-0.5σ) \cdot (s_1^2 - s_2^2) \|_2 \] (3)

For replays in new tasks, given a limited memory capacity, our proposed method has to store a small image subset and one variational prototype including its mean and variance for each old class c in all previous tasks t < T. When the total number of tasks T is small, the memory can store more image examples per class. Dynamic memory allocation enables more example replays in earlier tasks, putting more emphasis on reviewing earlier tasks which are easier to forget. Pseudocode to our proposed algorithm in split CIFAR10 in the incremental class protocol for a training episode is provided in Algorithm 1. The source code of our proposed algorithm is downloadable: https://github.com/kreimanlab/VariationalPrototypeReplaysCL.

### 4. Experimental Details

We introduce baseline continual learning algorithms with different memory usage over three task protocols.

#### 4.1. Baselines

We include the following categories of continual learning methods for comparing with our method. To eliminate the effect of network structures in performance, we introduce control conditions with the same architecture complexity for all the methods in the same task across all the experiments except for the last layer before the softmax layer for classification. See Table 1 for network architecture comparisons between baseline methods and our method.

**Parameter Regularization Methods:** Elastic Weight Consolidation (EWC) (Kirkpatrick et al., 2017), Synaptic Intelligence (SI) (Zenke et al., 2017) and Memory Aware
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#### Table 1. Network architecture and memory allocation for continual learning methods on split CIFAR10 in incremental class task.

For simplicity, only the network layers with learnable parameters are presented. Other network specifications, such as paddings, activation layers and pooling layers are omitted here.  

| Method       | Network Architecture (Baseline Method) | Number of Network Parameters (Baseline Method) | Network Architecture (Our Method) | Number of Network Parameters (Our Method) | Memory Size ($\times 10^5$) |
|--------------|----------------------------------------|-----------------------------------------------|-----------------------------------|-------------------------------------------|-----------------------------|
| EWC-online   | conv(3,20,5) $\rightarrow$ conv(20,50,5) $\rightarrow$ fc(3200,500) $\rightarrow$ fc(500,10) $\rightarrow$ softmax | $3 \times 20 \times 5 \times 5 + 20 \times 50 \times 5 \times 5 + 3200 \times 500 + 500 \times 10 = 16.3 \times 10^5$ | conv(3,20,5) $\rightarrow$ conv(20,50,5) $\rightarrow$ fc(3200,500) $\rightarrow$ fc(500,1000) $\rightarrow$ nearest prototype | $3 \times 20 \times 5 \times 5 + 20 \times 50 \times 5 \times 5 + 3200 \times 500 + 500 \times 1000 = 21.3 \times 10^5$ | EWC-online: 16.3 x 2 = 32.6, MAS: 16.3 x 2 = 32.6, L2: 16.3 x 2 = 32.6, SI: 16.3 x 2 = 32.6, ours: 21.3 + 0.4 = 21.7 |

#### Figure 3. Average classification accuracies over total 9 tasks (a) and 2D visualization of embedding clusters (solid circles) and prototypes (hollow squares) learnt by our method in Task 1. (c) Embedding clusters and prototypes learnt by our method in Task 3.

**Synapses (MAS) (Aljundi et al., 2018), where regularization terms are added in the loss function; online EWC (Kirkpatrick et al., 2017) which is an extension of EWC with scalability to a large number of tasks; L2 distance indicating parameter changes between tasks is added in the loss (Kirkpatrick et al., 2017); SGD, which is a naive baseline without any regularization terms, is optimized with Stochastic Gradient Descent (Bottou, 2010), sequentially over all tasks.**

**Memory Distillation and Replay Methods:** incremental Classifier and Representation Learner (iCARL) (Rebuffi et al., 2017) proposes to regularize network behaviors by exact exemplar rehearsals via distillation loss.

Performance is reported in terms of both mean and standard deviation after 10 runs per protocol. Since generative model-based approaches (van de Ven & Tolias, 2018; Shin et al., 2017) greatly alter architecture of the classification networks, we do not compare with them.

#### 4.2. Memory Comparison

For fair comparison, we compute the total number of parameters in a network for all the methods and allocate a comparable amount of memory as EWC (Kirkpatrick et al., 2017) and other parameter regularization methods, for storing example images per class and their variational prototypes in previous tasks. In EWC, the model allocates a memory size twice as the number of network parameters for computing the Fisher information matrix which is used for regularizing changes of network parameters (Kirkpatrick et al., 2017). In more challenging classification tasks, the network size tends to be larger and hence, these methods require much more memory.

In Table 1, we show an example of memory allocation on split CIFAR10 in incremental class tasks. The feed-forward classification network used in baseline methods contains around $16.3 \times 10^5$ parameters. Weight regularization methods require memory allocation twice as large, i.e., about $32.63 \times 10^5$ parameters. The input RGB images are of
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Figure 4. Averaged classification accuracies over tasks for the ablated few-shot continual learning methods after 10 repeated runs on split CIFAR10 in incremental class task. See Sec. 5.2 for detailed description of each ablated method.

Figure 5. Prototype dynamics analysis in split CIFAR10 in incremental class protocol. (a) Visualization of trajectory of prototypical mean movement over all 9 tasks after projecting all prototypical means of all tasks into 3D using first three principal components obtained from the latent feature space learnt in Task 1. The black squares denote the prototypical mean in the most recent tasks. The color code corresponds with object classes. (b) Feature similarity matrix is calculated using Euclidean distance between feature vectors extracted from second last layers of VGG16 network (Simonyan & Zisserman, 2014) pre-trained on ImageNet (Deng et al., 2009), which presumably “saw” all the classes from CIFAR10 at once without any incremental class training. (c) For each class, a motion vector is calculated between the initial prototypical mean and the most recent prototypical mean across tasks. We can then compute the Euclidean distance for each pairs of motion vectors of the prototypical means from different classes. Refer to the colorbar on the right for similarity values. Correlation value reports the Pearson-correlation between feature similarity and prototype motion similarity. In other words, higher correlation values indicate that the more visually similar the two classes are; the prototypical means of these classes tend to move along in incremental class protocol.

5. Results and Discussion

In the main text, we focus on the results of our method in Split CIFAR10 in the incremental task. The Supp. Material shows results and discussion in the other two task protocols: permuted MNIST in incremental domain and split MiniImageNet in incremental class.

5.1. Alleviating Forgetting

Figure 3a reports the results of continual learning methods on split CIFAR10 in incremental class protocol. Our method (red) achieves the highest average classification accuracy among all the compared methods with minimum forgetting. Initially all compared continual learning methods outperform chance (dash line). Note that the chance is 1/2 in the first task. However, given 10 training samples in the subsequent tasks, all these algorithms except for L2 essentially fall to chance levels and fail to adapt to new tasks due to overfitting. A good continual learning method should not only show good memory retention but also be able to adapt to new tasks. Our method (red) consistently outperforms L2 across all tasks with an average improvement of 2.5%. This reveals that our method performs classification via example replays and variational...

size $3 \times 32 \times 32$ and the variational prototypes contain one mean vector of size $1 \times 500$ and one variance vector of size $1 \times 500$. In example replay, we only store 1 example image and 1 variational prototype per class from previous tasks. The episodic memory of our method stores 10 images and 10 variational prototypes in total for all 10 classes, resulting in $21.7 \times 10^5$ memory usage, which is 33% less than weight regularization methods.
We replace the variance weighted Euclidean distance with
To prevent catastrophic forgetting, we replay stored example
variational prototypes move in the original prototypical
images and regress the newly predicted variational
distributions to be close to the stored prototype distribution
for all previous tasks. We probe whether the sequence
of retraining the variational prototypes from first task to
recent ones matters. Replaying variational prototypes
from the most recent tasks (replaySeqBack) results in
1% performance drop and only replaying the most
recent prototypes (replaySeqCurr) leads to further 0.3%
performance drop. Furthermore, we also analyze the effect
of relaxing the mean and variance constraints. In other
words, if the prototype recall only involves being close to a
prototype mean (recalMean) or following a distribution with
a similar prototype variance (recalVar), the performance
is much worse than when combining both the mean and
variance. This emphasizes the advantage of learning
prototype distributions rather than a single prototype for
a particular class in retaining memory of the previous tasks.

Next, to perform nearest prototype classification, we
randomly sample multiple latent representations from the
variational prototypes as shown in Fig 2. In our proposed
method, we sample 50 variational prototypes. In the ablated
methods, we titrate the sample size from 100 down to 2.
Increasing sample sizes further from 50 to 100 saturates the
performance; however, reducing sample sizes to 2 hinders
the average classification accuracy by 0.5%. We also vary
the size of the variational prototype mean and variance.
Increasing the latent feature space dimension from 500 to
1000 (repSz1000) boosts accuracy by 0.7%; and vice versa
for reducing the latent feature space (repSz10).

5.3. Prototype Dynamics across Tasks
In split CIFAR10 in incremental class protocol, the network
constantly updates its parameters from $F_1$ to $F_9$ over the
total of 9 tasks. We report how the prototype means
of previous classes change across tasks in Fig 5. The
visualization of the trajectory of prototype means across
tasks in Fig. 5a suggests that, as the network incrementally
learns more classes, the prototype means from previous
classes move away from the center. To quantitatively
measure how the visual feature similarity influence the
prototype dynamics, we provide the visual feature similarity
matrix in Fig. 5b and prototype dynamics similarity in
Fig. 5c. A high correlation of 0.44 between feature similarity
and prototype dynamics suggests that the dynamics of how
the prototypes of two classes move is highly correlated
with the visual feature similarities of these two classes.
This observation provides some insights about how a
classification network with our proposed method evolves
a topological structure for learning to classify new objects
in new tasks while keeping the previous classes separated
apart from one another.

6. Conclusion
We address the problem of catastrophic forgetting by
proposing variational prototype replays in classification
tasks. In addition to significantly alleviating catastrophic
forgetting on benchmark datasets, our method is superior to others in terms of making the memory usage efficient, and being generalizable to learning novel concepts given only a few training examples in new tasks.

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