Physical Scaling and Renormalization Group in Two-Dimensional Gravity

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Abstract

Quantum gravitational effects on the renormalization group equation are studied in the \((2+\epsilon)\)-dimensional approach. Divergences in a matter one-loop effective action do not receive gravitational radiative corrections. The renormalization factor for beta functions recently found by Klebanov, Kogan and Polyakov is obtained by using the renormalized cosmological constant to define the physical scale transformation.

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Recently a gravitational dressing of the renormalization group in two spacetime dimensions was studied in ref. [1]. It was shown that one-loop beta functions for marginal perturbations of a conformal field theory are multiplicatively renormalized due to quantum effects of gravity. The beta function in the presence of quantum gravity $\tilde{\beta}$ is related to the beta function without quantum gravity $\beta$ as

\[
\tilde{\beta} = \frac{k + 2}{k + 1} \beta,
\]

where $k$ is the level of the gravitational SL(2, R) Kac-Moody algebra and is given in terms of the matter central charge $c$ by

\[
k = \frac{1}{12} \left( c - 37 - \sqrt{(1 - c)(25 - c)} \right).
\]

This result was obtained in ref. [1] mainly using the light-cone gauge for gravity [2].

The purpose of our paper is to study this relation further and clarify the origin of the factor $\frac{k + 2}{k + 1}$ in eq. (1). We will use the $(2 + \epsilon)$-dimensional approach for quantum gravity [3]–[6] to compute quantum gravitational effects on the beta functions. We will find that gravitational radiative corrections to one-loop divergence in the matter effective action cancel out at the lowest order of the gravitational constant. Therefore one would obtain exactly the same beta functions as those in theories without quantum gravity. We argue that the factor in eq. (1) is due to a proper definition of the physical scale transformation in the presence of quantum gravity. We obtain that factor by using the dimensionless renormalized cosmological constant to define the physical scale transformation. We will discuss the relation (1) in the light-cone gauge [2] from this point of view.

We shall consider a nonlinear sigma model coupled to gravity in $d = 2 + \epsilon$ dimensions. The action is

\[
S = \int d^d x \sqrt{-g} \left[ \frac{1}{16\pi G_0} R^{(d)} - \frac{1}{4\pi \alpha'} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^j G_{0ij}(X) \right],
\]

where $G_0$ is the bare gravitational constant and $G_{0ij}$ is the bare target space metric. Using the normal coordinate expansion [7], which we will discuss below, we can separate this action into two parts. The first part is the Einstein gravity coupled to
free scalar fields. The second part consists of interaction terms due to the presence of nontrivial background fields $G_{ij}$ in the target space.

Quantization of the first part was discussed in refs. [4], [5]. One-loop renormalization of the gravitational constant is

$$\frac{1}{G_0} = \mu' \left( 1 - \frac{2}{3} \frac{25}{\epsilon} - c \right),$$  \hspace{1cm} (4)

where $G$ is the renormalized gravitational constant and $\mu$ is the renormalization scale. It was shown in ref. [5] that the two-dimensional gravity [2], [8] can be obtained as a limit $\epsilon \to 0$ in the strong coupling region $|G| \gg |\epsilon|$. In this region the bare gravitational constant is given by

$$G_0 = -\frac{3}{2} \mu \epsilon \frac{\epsilon}{25 - c}. \hspace{1cm} (5)$$

Using the Einstein action with this bare gravitational constant, anomalous dimensions of physical operators were obtained and were shown to be consistent with the previous results using other methods [4], [8].

Using this approach for quantum gravity we can now study effects of quantum gravity on the beta functions of matter self-couplings by taking the second part of the action into account. There are two parameters for loop expansions. The parameter for matter loops is $\alpha'$ and that for graviton loops is $G_0$ in eq. (4). Effects of quantum gravity become small for large $-c$. In the limit $c \to -\infty$ we obtain a theory without quantum gravity.

To obtain the beta functions at the first order in $\alpha'$ we compute divergences in the effective action using the matter action in eq. (3) with $G_{0ij}$ replaced by $\mu' G_{ij}$, where $G_{ij}$ is the dimensionless renormalized coupling. We use the background field method [4] and the normal coordinate expansion [7]. The matter fields are expanded as $X^i = \hat{X}^i + \xi^i + O(\xi^2)$, where $\hat{X}^i$ are background fields and $\xi^i$ are quantum fluctuations (normal coordinates). It is more convenient to use $\xi^a = \xi^i E^a_i(\hat{X})$ as quantum fields, where $E^a_i$ is the target space vielbein: $G_{ij} = E^a_i E^b_j \eta_{ab}$. The matter action can be expanded as [7]

$$S_{\text{matter}} = -\frac{\mu' \epsilon}{4\pi \alpha'} \int d^dx \sqrt{-g} g^{\nu\sigma} \left[ \partial_{\mu} \hat{X}^i \partial_{\nu} \hat{X}^j G_{ij}(\hat{X}) + 2 D_{\mu} \xi^a \partial_{\nu} \hat{X}^i E_{ia}(\hat{X}) \\ + D_{\mu} \xi^a D_{\nu} \xi^b \eta_{ab} - \partial_{\mu} \hat{X}^i \partial_{\nu} \hat{X}^j R_{iajb}(\hat{X}) \right] \xi^a \xi^b.$$
\[-\frac{4}{3} \partial_\mu \hat{X}^i \xi^a \xi^b D_\nu \xi^c R_{ciaib}(\hat{X}) - \frac{1}{3} \partial_\mu \hat{X}^i \partial_\nu \hat{X}^j \xi^a \xi^b \xi^c D_\nu R_{iajb}(\hat{X})
\]
\[-\frac{1}{3} D_\mu \xi^a D_\nu \xi^b \xi^c \xi^d R_{acdb}(\hat{X}) - \frac{1}{2} \xi^a \xi^b \xi^c D_\mu \xi^d \partial_\nu \hat{X}^i D_\nu R_{ciaib}(\hat{X})
\]
\[-\frac{1}{12} \partial_\mu \hat{X}^i \partial_\nu \hat{X}^j \xi^a \xi^b \xi^c \xi^d \left( D_a D_b R_{icjd} - 4 R_{aib} R_{kcjd} \right)(\hat{X}) + O(\xi^5) \].

The covariant derivative is defined as

\[ D_\mu \xi^a = \partial_\mu \xi^a + \partial_\mu \hat{X}^i \Omega_i a^b (\hat{X}) \xi^b, \]

where \( \Omega_i a^b \) is the target space spin connection.

The metric is parametrized by a background field \( \hat{g}_{\mu\nu} \) and quantum fields \( h^\mu_\nu, \phi \) as

\[ g_{\mu\nu} = \hat{g}_{\mu\rho} (e^{\kappa_0 h})^\rho_\nu e^{-\kappa_0 \phi}, \quad h^\mu_\nu \equiv \hat{g}^\mu_\rho h^\rho_\nu = h^\mu_\nu, \quad h^\mu_\mu = 0, \]

where \( \kappa_0^2 = 16\pi G_0 \). The metric dependence of the matter action is

\[ \sqrt{-\hat{g}} g^{\mu\nu} = \sqrt{-\hat{g}} (e^{-\kappa_0 h})^\mu_\rho \hat{g}^\rho_\nu e^{-\frac{1}{2}\kappa_0 \phi}. \]

Notice that the conformal mode \( \phi \) is multiplied by a factor \( \epsilon \). This corresponds to the fact that the perturbation due to the nontrivial background \( G_{ij} \) is a marginal operator in two dimensions. We use the gauge fixing term of ref. [5] for the general coordinate symmetry

\[ S_{GF} = -\frac{1}{2} \int d^d x \sqrt{-\hat{g}} \left( \hat{D}^\nu h^\mu_\nu + \frac{1}{2} \epsilon \hat{D}_\mu \phi \right)^2. \]

In this gauge the propagators of the gravitational fields are

\[ \langle h^\mu_\nu(x) h^\rho_\sigma(y) \rangle = -i \left( \eta^\rho_\mu \eta^\nu_\sigma + \eta^\mu_\rho \eta^\nu_\sigma - \frac{2}{d} \eta^\mu_\nu \eta^\rho_\sigma \right) \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2} \epsilon^{ip(x-y)}, \]

\[ \langle \phi(x) \phi(y) \rangle = \frac{4i}{\epsilon d} \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2} \epsilon^{ip(x-y)}, \quad \langle h^\mu_\nu(x) \phi(y) \rangle = 0. \]

First let us recall the one-loop beta function when gravity is not quantized [10]. We are interested in order \( O(\alpha') \) terms in the effective action \( \Gamma \). As shown in
Fig. 1, there are three divergent diagrams of this order. However, the divergence of the diagram (b) is canceled by that of the diagram (c). They should cancel since they are not allowed by the covariance and the local Lorentz symmetry in the target space. Only the diagram (a) contributes to one-loop divergence. We obtain

\[ \Gamma = -\frac{1}{4\pi\alpha'} \frac{\alpha'\zeta}{\epsilon} \int d^d x \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\mu \dot{X}^i \partial_\nu \dot{X}^j R_{ij}(\dot{X}) + O(\epsilon^0), \]  

where \( \zeta = 1 \). The divergence can be removed by adding a counter term

\[ S_{\text{c.t.}} = \frac{\mu^\epsilon}{4\pi\alpha'} \frac{\alpha'}{\epsilon} \int d^d x \sqrt{-\tilde{g}} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^j R_{ij}(X) \]  

(13)

to the matter action. This counter term corresponds to choosing the bare coupling in eq. (3) as

\[ G_{0ij} = \mu^\epsilon \left( G_{ij} - \frac{\alpha'}{\epsilon} R_{ij} \right). \]  

(14)

From the \( \mu \)-independence of the bare coupling \( G_{0ij} \) we obtain the one-loop beta function in the limit \( \epsilon \to 0 \) as

\[ \beta_{ij} \equiv \mu \frac{\partial}{\partial \mu} G_{ij} = \alpha' R_{ij}. \]  

(15)

Now we shall consider quantum effects of gravity. At order \( O(\kappa_0^2) \) there are two divergent diagrams as shown in Fig. 2. In these diagrams the internal lines are \( h_{\mu\nu} \) or \( \xi^a \). Diagrams with internal \( \phi \) lines do not contribute to divergences because the \( \phi \)-vertices always contain a factor \( \epsilon \) as shown in eq. (4). Their divergences have the form
\[ G \quad h \] (a) \[ E \quad E \] (b)

Figure 2: \( O(\kappa_0^2) \) divergent diagrams. Internal lines with \( h \) are graviton propagators. Other internal lines are matter propagators.

\[
\Gamma = -\frac{1}{4\pi \alpha'} \eta \int d^d x \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\mu \hat{X}^i \partial_\nu \hat{X}^j G_{ij}(\hat{X}). \tag{16}
\]

The coefficient \( \eta \) for each diagram is given by

\[
\eta(a) = -\frac{\kappa_0^2}{2\pi}, \quad \eta(b) = \frac{\kappa_0^2}{2\pi}. \tag{17}
\]

Therefore the divergences of these two diagrams cancel each other and there is no divergence of order \( O(\kappa_0^2) \). Actually, the coefficients in eq. (17) are of order \( O(\epsilon) \) since \( \kappa_0^2 = O(\epsilon) \), and eq. (16) for each diagram is finite.

Next let us consider order \( O(\alpha'\kappa_0^2) \) divergences. We have to compute divergences in two-loop diagrams. Since \( \kappa_0^2 = O(\epsilon) \), we only need to know \( \epsilon^{-2} \) singularities of Feynman integrals. We have to take into account the vertices in the one-loop counter term (13). The normal coordinate expansion of this counter term is

\[
S_{\text{c.t.}} = \frac{\mu}{4\pi \alpha' \epsilon} \int d^d x \sqrt{-g} g^{\mu\nu} \left[ \partial_\mu \hat{X}^i \partial_\nu \hat{X}^j R_{ij}(\hat{X}) + \partial_\mu \hat{X}^i \partial_\nu \hat{X}^j \xi^a D_a R_{ij}(\hat{X}) 
+ 2\partial_\mu \hat{X}^i D_\nu \xi^a R_{ia}(\hat{X}) + \frac{1}{2} \partial_\mu \hat{X}^i \partial_\nu \hat{X}^j \xi^a \xi^b \left( D_a D_b R_{ij} - 2 R_{iab} R_{kj} \right) \right] (\hat{X}) 
+ 2\partial_\mu \hat{X}^i D_\nu \xi^a D_b R_{iab}(\hat{X}) + D_\mu \xi^a D_\nu \xi^b R_{ab}(\hat{X}) + O(\xi^3). \tag{18}
\]

Divergent diagrams of order \( O(\alpha'\kappa_0^2) \) are shown in Fig. 3. In addition to the diagrams in Fig. 3 there are divergent diagrams with the \( \Omega \) vertices. However, they should cancel each other by the covariance and the local Lorentz symmetry in the target space as in Fig. 1. Divergence in each diagram in Fig. 3 has the same form as eq. (12). The coefficient \( \zeta \) for each diagram is

\[
\zeta(a) = 0, \quad \zeta(b) = -\frac{\kappa_0^2}{2\pi \epsilon}, \quad \zeta(c) = \frac{\kappa_0^2}{2\pi \epsilon}, \quad \zeta(d) = -\frac{\kappa_0^2}{2\pi \epsilon}, \quad \zeta(e) = \frac{2\kappa_0^2}{3\pi \epsilon}. \]
Figure 3: \(O(\alpha'\kappa_0^2)\) divergent diagrams. Internal lines with \(h\) are graviton propagators. Other internal lines are matter propagators. Vertices with a dot are those of the one-loop counter term.

\[
\zeta(f) = -\frac{\kappa_0^2}{6\pi\epsilon}, \quad \zeta(g) = 0, \quad \zeta(h) = \frac{\kappa_0^2}{2\pi\epsilon}, \quad \zeta(i) = -\frac{\kappa_0^2}{\pi\epsilon}, \quad \zeta(j) = \frac{\kappa_0^2}{2\pi\epsilon}.
\]  

(19)

Since \(\kappa_0^2 = O(\epsilon)\), these coefficients are finite for \(\epsilon \to 0\). We see that the sum of divergences in the diagrams (a)–(j) cancel out. This cancellation is similar to the observation in ref. [5] that the diagrams with \(h_{\mu\nu}\) lines do not contribute to the renormalization of the operator \(e^{-\frac{d}{2}(1-\Delta_0)}\phi\) to two-loop order. In the present case of marginal operators there is no \(\phi\) contribution either. It can happen that gravitational corrections to divergences completely cancel also at higher orders of the loop expansions in \(\kappa_0^2\). This may not be so surprising because \(h_{\mu\nu}\) is a gauge degree of freedom in two dimensions and can be set to zero by a general coordinate transformation.

Thus we have seen that the one-loop divergence does not receive gravitational radiative correction. Therefore we obtain the same beta function (15) as in the theory without quantum gravity. This is consistent with the result in ref. [4] in the
sense that the beta functions are essentially the same in theories with and without quantum gravity. The factor $\frac{k+2}{k+1}$ in eq. (1) can be understood by considering a definition of the physical scale transformation.

Without gravitational interactions, the beta function (15) is defined as a response of the renormalized quantities to the change of the renormalization scale $\mu$. On the other hand, the characteristic feature of gravity is to provide a metric which can determine the physical scale in the theory. Therefore we should measure the response of the renormalized quantities to the change of the physical scale. It has been pointed out that the renormalization of the gravitational constant $G$ can be made meaningful only after fixing the scale of the metric by renormalizing a reference operator $O$. Since the gravitational constant becomes dimensionless in two-dimensions, it is most natural to consider the reference operator which also has dimensionless coupling in two dimensions. Namely we choose the reference operator to be spinless and with a conformal dimension $(1, 1)$, such as the Thirring interaction. After fixing the scale of the metric, we can absorb divergences of the type $\sqrt{-g}R$ into a renormalization of the gravitational constant $G$, and the divergences of the type $\sqrt{-g}$ into a renormalization of the cosmological constant $\Lambda$. Instead of the gravitational constant, the cosmological constant $\Lambda$ naturally provides the physical scale in the presence of the gravitational interaction in two dimensions, since it is the only dimensionful parameter in the gravity theory in two dimensions.

To use the cosmological term in defining the physical scale transformation, let us introduce the cosmological term into the action (3)

$$\Lambda_0 \int d^d x \sqrt{-g},$$

(20)

where $\Lambda_0$ is the bare cosmological constant. Renormalization of the cosmological term was discussed in ref. [5]. To remove divergences, the bare cosmological constant is expressed as

$$\Lambda_0 = \mu^d \Lambda Z,$$

(21)

where $Z$ is a divergent renormalization factor and $\Lambda$ is a dimensionless renormalized cosmological constant. From the $\mu$-independence of $\Lambda_0$ we obtain

$$\mu \frac{\partial \Lambda}{\partial \mu} = -(d + \gamma)\Lambda, \quad \gamma = \mu \frac{\partial}{\partial \mu} \ln Z,$$

(22)
where $\gamma$ is the anomalous dimension. In the two-dimensional limit the exact form of $\gamma$ was obtained in ref. [3] by considering divergences coming from the conformal mode to arbitrary orders in the loop expansion

$$\gamma = -2 - \alpha Q,$$

(23)

where $\alpha$ and $Q$ are defined by

$$Q = \sqrt{\frac{25 - c}{3}}, \quad \alpha = -\frac{1}{2}\sqrt{3} \left(\sqrt{25 - c} - \sqrt{1 - c}\right).$$

(24)

In this case the solution of eq. (22) is

$$\Lambda \sim \mu^{-\frac{3}{2}}, \quad \frac{1}{x} = 1 + \frac{\gamma}{2} = -\frac{\alpha Q}{2}.$$  

(25)

In the limit of no quantum gravity $c \to -\infty$ the anomalous dimension vanishes $\gamma \to 0$.

Now let us consider the beta function in the presence of gravity. Instead of the renormalization scale $\mu$, we use the physical scale derived from the cosmological constant to define the beta function. Since the renormalized cosmological constant $\Lambda$ is defined by eq. (21), the inverse square root of the cosmological constant $\sqrt{\Lambda^{-1}}$ should be used in place of $\mu$ in $d = 2$ dimensions. Without gravitational interactions, the beta function is usually defined as a change of the renormalized coupling $G_{ij}$ in response to the change of the renormalization scale

$$\beta_{ij} = \frac{\partial G_{ij}(\mu)}{\partial \ln \mu}.$$  

(26)

In the presence of gravity, we should consider the beta function as a response to the physical scale $\sqrt{\Lambda^{-1}}$

$$\bar{\beta}_{ij} = \frac{\partial G_{ij}}{\partial \ln \sqrt{\Lambda(\mu)^{-1}}} = \frac{\partial \ln \mu}{\partial \ln \sqrt{\Lambda(\mu)^{-1}}} \frac{\partial G_{ij}}{\partial \ln \mu} = x\beta_{ij}.$$  

(27)

In the limit of no quantum gravity $c \to -\infty$, we have $x \to 1$ and $\bar{\beta}_{ij} \to \beta_{ij}$. Using eqs. (23) and (24) we see that the relation between $\bar{\beta}_{ij}$ and $\beta_{ij}$ in eq. (27) is exactly the same as eq. (1).
Another way to state this scale transformation is the following. We can consider a generalized renormalization scale transformation $\mu \rightarrow \lambda^y \mu$ with a power $y$ of the scaling parameter $\lambda$. We would like to require the renormalized cosmological constant $\Lambda$ in eq. (21) to have the classical response in two dimensions

$$\Lambda \rightarrow \lambda^{-2} \Lambda$$

(28)
even in the presence of quantum effects. Therefore the renormalization scale dependence in eq. (23) dictates $y = x$. Clearly this transformation assures that the scale transformation of the cosmological constant be independent of the matter content of the theory. We can now define the beta function in the presence of quantum gravity as a response to this physical scale transformation

$$\tilde{\beta}_{ij} = \lim_{\lambda \rightarrow 1} \frac{1}{\ln \lambda} \frac{G_{ij}(\lambda^x \mu) - G_{ij}(\mu)}{x} = x \beta_{ij},$$

(29)

where $\beta_{ij}$ is defined in eq. (26). Thus we obtain the same $\tilde{\beta}_{ij}$ as in eq. (27).

Let us comment on modifications if one chooses to use other physical operators instead of the cosmological term to define a physical scale transformation of $\mu$. We can repeat the above analysis using the result on the renormalization of physical operators in ref. [5]. If we use a physical operator corresponding to a matter operator with a conformal dimension $(\Delta_0, \Delta_0)$, we obtain a relation

$$\tilde{\beta}_{ij} = -\frac{2(1 - \Delta_0)}{\beta Q} \beta_{ij},$$

(30)

where

$$\beta = -\frac{1}{2\sqrt{3}} \left( \sqrt{25 - c} - \sqrt{1 - c + 24\Delta_0} \right).$$

(31)

However, we would like to emphasize that the usual scaling arguments in the presence of gravity are in fact based upon the use of the physical scale defined in terms of the cosmological constant $\Lambda$ [8]. We are here advocating a similar argument to define the beta function in the presence of gravity.

To confirm the above interpretation of the factor in eq. (1) we shall examine the physical scaling in the light-cone gauge [2]. As in ref. [4] the beta functions can be
obtained from coefficients of the operator product expansion (OPE), which in turn can be obtained from a ratio of two- and three-point correlation functions. To see how the cutoff and the renormalization scale are introduced in this method let us recall the relation between the OPE coefficients and the beta functions. As in ref. [1] we consider an action for a conformal field theory perturbed by marginal (conformal weight one) operators $O_n(x)$

$$S = S_0 + \sum_n \lambda_n O_n, \quad O_n = \int d^2 x O_n(x). \quad (32)$$

The vacuum expectation value can be expanded as

$$\langle \cdots \rangle = \left\langle \cdots \left(1 + i \sum_n \lambda_n O_n + \frac{1}{2} i^2 \sum_{n,m} \lambda_n \lambda_m O_n O_m + \cdots \right) \right\rangle_0. \quad (33)$$

Let us consider the $O(\lambda^2)$ terms. The $x$-integrations in $O_1$ and $O_2$ give short distance divergences. The short distance behavior of the product of the integrands is given by the OPE

$$O_n(x_1)O_m(x_2) \sim \frac{1}{(x_1 - x_2)^2} \sum_l g^{nm}_l O_l(x_2). \quad (34)$$

Therefore the $x$-integrations give

$$O_n O_m = \int d^2 x_1 d^2 x_2 \frac{1}{(x_1 - x_2)^2 + \epsilon^2} \sum_l g^{nm}_l O_l(x_2)$$

$$= 2\pi i \ln \epsilon \sum_l g^{nm}_l O_l + \cdots, \quad (35)$$

where we have regularized the integral by making a Wick rotation $x^0 = -ix^2$ and introducing a cutoff $\epsilon$ in the coordinate space. This short distance divergence can be removed by introducing the bare coupling constants

$$\lambda_{0n} = \lambda_n + \pi \ln(\epsilon \mu) \sum_{l,m} \lambda_l \lambda_m g_{lm} + O(\lambda^3), \quad (36)$$

where we have introduced a renormalization scale $\mu$. Then the beta functions are given by

$$\beta_n = \mu \frac{\partial \lambda_n}{\partial \mu} = -\pi \sum_{l,m} \lambda_l \lambda_m g_{lm} + O(\lambda^3). \quad (37)$$
Thus we can obtain the beta functions from the OPE coefficients $g_{nlm}$.

In the light-cone gauge the metric has the form

$$g_{\mu\nu}dx^\mu dx^\nu = -2dx^+ dx^- + h_{++}dx^+ dx^+$$  \hspace{1cm} (38)

and the cosmological term is a c-number

$$\Lambda_0 \int d^2x \sqrt{-g} = \Lambda_0 \int d^2x.$$  \hspace{1cm} (39)

There is no short distance divergence. Therefore the relation between the bare and dimensionless renormalized cosmological constants is $\Lambda_0 = \mu^2 \Lambda$ and we obtain

$$\Lambda \sim \mu^{-2}.$$  \hspace{1cm} (40)

We see that the usual scaling $\mu \rightarrow \lambda \mu$ is physical, i.e., it changes the renormalized cosmological constant $\Lambda$ as in eq. (28). We should obtain the factor in eq. (1) for the usual beta functions. Indeed it was obtained in ref. [1] in this way. Therefore our interpretation of the factor is consistent with the analysis in the light-cone gauge.

The gravitational dressing of the beta function in the conformal gauge has already been discussed in ref. [1] using the cosmological constant as the physical scale similarly to our treatment. The renormalization group equation in the conformal gauge was also discussed in ref. [11].

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