A T-odd asymmetry in neutralino production and decay

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Abstract

We study CP-violating effects in neutralino production and subsequent decay within the Minimal Supersymmetric Standard Model with complex parameters \(M_1\) and \(\mu\). The observable we propose is a T-odd asymmetry based on a triple product in neutralino production \(e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_2^0\), \(i = 1, \ldots, 4\), with subsequent leptonic three-body decay \(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-\), \(\ell = e, \mu\), at an \(e^+e^-\) linear collider with \(\sqrt{s} = 500\) GeV and polarised beams. We provide compact analytical formulae for the cross section and the T-odd asymmetry taking into account the complete spin correlations between production and decay. We give numerical predictions for the cross section and the T-odd asymmetry. The asymmetry can go up to 10\%.
1 Introduction

In the Standard Model (SM) the phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is the only source of CP violation. The small amount of CP violation in the SM, however, is not sufficient to explain the baryon-antibaryon asymmetry of the universe [1]. The Lagrangian of the Minimal Supersymmetric Standard Model (MSSM) contains several complex parameters, which can give rise to new CP-violating phenomena [2].

In the neutralino sector of the MSSM two complex parameters appear, which lead to CP-violating effects in neutralino production and decay. These parameters are the $U(1)$ gaugino mass parameter $M_1$ and the higgsino mass parameter $\mu$. The phase of the $SU(2)$ gaugino mass parameter $M_2$ can be eliminated by an $U(1)$-symmetry of the model.

One of the main goals of a future $e^+e^-$ linear collider will be a careful study of the properties of supersymmetric (SUSY) particles [3]. The neutralinos $\tilde{\chi}^0_i, i = 1, \cdots, 4$, will be particularly interesting, because the lightest neutralino $\tilde{\chi}^0_1$ is expected to be the lightest SUSY particle (LSP), which is stable if $R$-parity is conserved. The second lightest neutralino, $\tilde{\chi}^0_2$, will presumably be among the lightest visible SUSY particles. Therefore, the study of production and decay of the neutralinos $\tilde{\chi}^0_i$ [3, 4, 5] and a precise determination of the underlying supersymmetric parameters $M_1, M_2, \mu$ and $\tan \beta$ including the phases $\phi_{M_1}$ and $\phi_{\mu}$ of $M_1$ and $\mu$ will play an important role at future linear colliders. The phases of $M_1$ and $\mu$ cause CP-violating effects already at tree level. Therefore these effects could be large and thus be measurable at a high luminosity $e^+e^-$ linear collider. Methods to determine these parameters in neutralino and chargino production have been presented in [6, 7, 8, 9]. In particular the method of [9] is based on the analysis of the masses and production cross sections of only the light neutralinos $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ and the light chargino $\tilde{\chi}^\pm_1$ and will allow the determination of the parameters $|M_1|, M_2, |\mu|$ and the phases $\phi_{M_1}$ and $\phi_{\mu}$. All these methods involve only CP-even quantities, in which the signs of the phases $\phi_{M_1}$ and $\phi_{\mu}$ can not be determined. For an unambiguous determination of $\phi_{M_1}$ and $\phi_{\mu}$ including their signs one has to rely on CP-sensitive observables.

The phases of the complex parameters are constrained or correlated by the experimental upper limits on the electric dipole moments of electron, neutron and the atoms $^{199}$Hg and $^{205}$Tl [10]. In a constrained MSSM the restrictions on the phases can be rather severe. However, there may be cancellations between the contributions of different complex parameters, which allow larger values for the phases [11]. For example, in a constrained MSSM and if substantial cancellations are present, $\phi_{\mu}$ is restricted to $|\phi_{\mu}| \lesssim 0.1 \pi$, whereas $\phi_{M_1}$ and $\phi_A$, the phase of the trilinear scalar coupling parameter, turn out to be essentially unconstrained, but correlated with $\phi_{\mu}$ [12]. Moreover, the restrictions are very model dependent. For example, when also lepton flavour violating terms are included, then the restriction on $\phi_{\mu}$ may disappear [13]. Therefore it is necessary to determine the phases in an unambiguous way by measurements of CP-odd observables.

A useful tool to study these CP-violating effects are T-odd observables, based on triple products of momenta or spin vectors of the particles involved [14, 15]. In this paper we study a T-odd asymmetry in neutralino production

\[ e^+e^- \rightarrow \tilde{\chi}_k\tilde{\chi}_2^0, \quad k = 1, \cdots, 4, \]

(1)
with subsequent leptonic three-body decay
\[ \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^- \],
(2)
with \( \ell = e, \mu \). Using the triple product of the initial electron momentum \( \vec{p}_e^- \) and the two final lepton momenta \( \vec{p}_\ell^+ \) and \( \vec{p}_\ell^- \),
\[ O_T = \vec{p}_\ell^+ \cdot (\vec{p}_\ell^- \times \vec{p}_e^-), \]
(3)
we define a T-odd asymmetry
\[ A_T = \frac{\int \text{sign}\{O_T\}|T|^2d\text{lips}}{\int |T|^2d\text{lips}}, \]
(4)
where \( \int |T|^2d\text{lips} \) is proportional to the cross section \( \sigma(e^+e^- \rightarrow \tilde{\chi}_k^0 \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_k^0 \tilde{\chi}_1^0 \ell^+ \ell^-) \). It is essential to include the spin correlations between production and decay, because otherwise the numerator and hence the asymmetry \( A_T \) would vanish. This observable has the advantage that it is not necessary to reconstruct the momentum of the decaying neutralino. The asymmetry \( A_T \), Eq. (4), is odd under the naive time-reversal operation. By CPT it is a CP-sensitive asymmetry, if final-state interactions and finite-widths effects are unimportant. We will neglect these effects, because they are of higher order. By its definition, the asymmetry \( A_T \) is the difference of the number of events with the final lepton \( \ell^+ \) above and below the plane spanned by \( \vec{p}_\ell^- \times \vec{p}_e^- \), normalised by the sum of these events.

In this paper we will first present the complete analytic formulae at tree level for the cross section and for the asymmetry (4) of the processes (1) and (2), taking into account the full spin correlations between production and decay of the neutralino \( \tilde{\chi}_2^0 \) [16]. We will study the dependence of the asymmetry and the cross section on the phases \( \phi_\mu \) and \( \phi_{M_1} \) and present detailed numerical results for an \( e^+e^- \) linear collider with \( \sqrt{s} = 500 \text{ GeV} \) and polarised beams. Analogous CP asymmetries in neutralino production and subsequent two-body decays have been studied in [17]. CP asymmetries based on triple products in decays of scalar fermions have been discussed in [18]. CP-odd observables involving the polarisation of the outgoing \( \tau \) leptons from neutralinos decaying via two-body decays \( \tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1^\pm \tau^\mp \rightarrow \tilde{\chi}_1^0 \tau^\pm \tau^\mp \) have been analysed in [19]. A systematic study of the impact of the supersymmetric phases on neutralino, chargino and selectron production at linear colliders has been performed in [12]. A Monte Carlo study of the asymmetry (4) for \( e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \ell^+ \ell^- \), \( l = e, \mu \), including SM backgrounds and detector effects has been given in [20], however, no analytic formulae for the cross section and the asymmetries have been given there. In [21] the impact of the phases \( \phi_{A_\tau}, \phi_{A_\chi}, \phi_{A_\mu} \) and \( \phi_{M_2} \) on the two-body decays of the third generation sfermions has been analysed in detail. The influence of these phases on the polarisation of the outgoing fermions in third generation sfermion decays has been studied in [22].

In Section 2 we shortly present the basics of the spin density matrix formalism. In section 3 we give the analytic expressions for the asymmetry \( A_T \) and the cross sections. Section 4 contains numerical results and discussions. In section 5 we present our conclusions. Appendices A and B contain some details of the formalism and of the analytic expressions.
2 Formalism

2.1 Lagrangian and couplings

The production process $e^+e^- \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0$, $i = 1, \cdots, 4$, proceeds via $Z^0$ and $\tilde{e}_{L,R}$ exchange (Fig. 1). In the decay process $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-$, $\ell = e, \mu$, $Z^0$ and $\tilde{\ell}_{L,R}$ exchanges contribute (Fig. 2).

The interaction Lagrangians for these processes are [23] (in our notation and conventions we follow [16])

$$L_{Z^0 \ell^+\ell^-} = -\frac{g}{\cos \Theta_W} Z_\mu \bar{\ell} \gamma^\mu [L_{\ell} P_L + R_{\ell} P_R] \ell,$$

$$L_{Z^0 \tilde{\chi}_m \tilde{\chi}_n} = \frac{1}{2} \frac{g}{\cos \Theta_W} Z_\mu \tilde{\chi}_m^0 \gamma^\mu [O_{mn}^L P_L + O_{mn}^R P_R] \tilde{\chi}_n^0.$$

Figure 1: Feynman diagrams of the production process $e^+e^- \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0$.

Figure 2: Feynman diagrams of the three-body decay $\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_k^0\ell^+\ell^-$. 

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\begin{equation}
\mathcal{L}_{\tilde{\chi}^n_k} = g f^L_{\ell k} \bar{P}_R \tilde{\chi}^0_{k} \ell_L + g f^R_{\ell k} \bar{P}_L \tilde{\chi}^0_R + \text{h.c.,}
\end{equation}

with \( m, n, k = 1, \ldots, 4 \) and the couplings

\begin{align}
f^L_{\ell k} &= -\sqrt{2}[1 \cos \Theta_W (T_{3\ell} - e_\ell \sin^2 \Theta_W) N_{k2} + e_\ell \sin \Theta_W N_{k1}], \\
f^R_{\ell k} &= -\sqrt{2} e_\ell \sin \Theta_W \tan \Theta_W N^*_{k2} - N^*_{k1}, \\
O^{\nu L}_{mn} &= -\frac{1}{2} \left( N_{m3}N_{n3}^* - N_{m4}N_{n4}^* \right) \cos 2\beta - \frac{1}{2} \left( N_{m3}N_{n4}^* + N_{m4}N_{n3}^* \right) \sin 2\beta, \\
O^{\nu R}_{mn} &= -O^{\nu L*}_{mn}, \\
L_\ell &= T_{3\ell} - e_\ell \sin^2 \Theta_W, \\
R_\ell &= -e_\ell \sin^2 \Theta_W,
\end{align}

where \( P_{L,R} = \frac{1}{2}(1 \mp \gamma_5) \) is the weak coupling constant (\( g = e/\cos \Theta_W, \ e > 0 \)). \( e_\ell \) and \( T_{3\ell} \) are the charge (in units of \( e \)) and the third component of the weak isospin of the fermion \( \ell \). \( \Theta_W \) is the weak mixing angle and \( \tan \beta = v_2/v_1 \) is the ratio of the vacuum expectation values of the Higgs fields. The unitary (4 \( \times \) 4)-matrix \( N_{mk} \) which diagonalises the complex symmetric neutralino mass matrix is given in the basis \( (\tilde{\gamma}, \tilde{Z}, \tilde{H}_a^0, \tilde{H}_b^0) \) [4].

2.2 Amplitudes

From these interaction Lagrangians the helicity amplitudes \( T_{P}^{\lambda_i \lambda_j}(\alpha) \) and \( T_{D,\lambda}(\alpha) \) for production and decay can be calculated. Here \( \alpha \) denotes the production or decay channel and the respective exchanged particle. \( \lambda_i \) and \( \lambda_j \) are the helicities of the neutralinos \( \tilde{\chi}^0_i \) and \( \tilde{\chi}^0_j \). In \( T_{D,\lambda}(\alpha) \) we suppress the label of the helicity of the final neutralino \( \tilde{\chi}^0_k \). The amplitudes \( T_{P}^{\lambda_i \lambda_j}(\alpha) \) for the production process \( e^- (p_1)e^+(p_2) \rightarrow \tilde{\chi}^0_i(p_3)\tilde{\chi}^0_j(p_4) \) (see Fig. 1) are

\begin{equation}
T_{P}^{\lambda_i \lambda_j} = T_{P}^{\lambda_i \lambda_j}(s, Z) + T_{P}^{\lambda_i \lambda_j}(t, \tilde{e}_L) + T_{P}^{\lambda_i \lambda_j}(t, \tilde{e}_R) + T_{P}^{\lambda_i \lambda_j}(u, \tilde{e}_L) + T_{P}^{\lambda_i \lambda_j}(u, \tilde{e}_R),
\end{equation}

where

\begin{align}
T_{P}^{\lambda_i \lambda_j}(s, Z) &= \frac{g^2}{\cos^2 \Theta_W} \Delta^s(Z) \bar{v}(p_2) \gamma^\mu (L_\ell P_L + R_\ell P_R) u(p_1) \\
\bar{u}(p_4, \lambda_j) \gamma^\mu (O_{\alpha i}^{\nu L} P_L + O_{\alpha i}^{\nu R} P_R) v(p_3, \lambda_i), \\
T_{P}^{\lambda_i \lambda_j}(t, \tilde{e}_L) &= -g^2 f^{\mu L}_{e i} f^{L*}_{e j} \Delta^t(\tilde{e}_L) \bar{v}(p_2) P_L v(p_3, \lambda_i) \bar{u}(p_4, \lambda_j) P_L u(p_1), \\
T_{P}^{\lambda_i \lambda_j}(t, \tilde{e}_R) &= -g^2 f^{\mu R}_{e i} f^{R*}_{e j} \Delta^t(\tilde{e}_R) \bar{v}(p_2) P_R u(p_3, \lambda_i) \bar{u}(p_4, \lambda_j) P_R u(p_1), \\
T_{P}^{\lambda_i \lambda_j}(u, \tilde{e}_L) &= g^2 f^{\mu L}_{e i} f^{L*}_{e j} \Delta^u(\tilde{e}_L) \bar{v}(p_2) P_L v(p_4, \lambda_j) \bar{u}(p_3, \lambda_i) P_L u(p_1), \\
T_{P}^{\lambda_i \lambda_j}(u, \tilde{e}_R) &= g^2 f^{\mu R}_{e i} f^{R*}_{e j} \Delta^u(\tilde{e}_R) \bar{v}(p_2) P_R u(p_4, \lambda_j) \bar{u}(p_3, \lambda_i) P_R u(p_1)
\end{align}

with

\begin{equation}
\Delta^s(Z) = \frac{i}{s - m_Z^2 + im_Z \Gamma_Z}
\end{equation}
\[ \Delta^t(e_{L,R}) = \frac{i}{t - m_{e_{L,R}}^2 + im_{e_{L,R}}\Gamma_{e_{L,R}}} \], \quad \Delta^u(e_{L,R}) = \frac{i}{u - m_{e_{L,R}}^2 + im_{e_{L,R}}\Gamma_{e_{L,R}}} \]  

(20)

and the Mandelstam variables

\[ s = (p_1 + p_2)^2, \quad t = (p_1 - p_4)^2, \quad u = (p_1 - p_3)^2. \]  

(21)

\[ \Delta^s(Z), \ m_Z, \ \Gamma_Z, \ \Delta^{t,u}(e_{L,R}), \ m_{e_{L,R}}, \ \Gamma_{e_{L,R}} \] denote the corresponding propagator, mass and width of the exchanged particle. The amplitudes \( T_{D,\lambda_i}(\alpha) \) for the decay \( \tilde{\chi}_i^0(p_3) \to \tilde{\chi}_i^0(p_5)\ell^+(p_6)\ell^-(p_7) \) (see Fig. 2) read

\[ T_{D,\lambda_i} = T_{D,\lambda_i}(s_i, Z) + T_{D,\lambda_i}(t_i, \tilde{\ell}_L) + T_{D,\lambda_i}(t_i, \tilde{\ell}_R) + T_{D,\lambda_i}(u_i, \tilde{\ell}_L) + T_{D,\lambda_i}(u_i, \tilde{\ell}_R) \]  

(22)

with

\[ T_{D,\lambda_i}(s_i, Z) = -\frac{g^2}{\cos^2 \Theta_W} \Delta^s(Z) \bar{u}(p_7)\gamma^\mu(L_{\ell}P_L + R_{\ell}P_R)\nu(p_6) \]  

(23)

\[ T_{D,\lambda_i}(t_i, \tilde{\ell}_L) = -g^2 f_{K_iL_i}^L \Delta^t(e_{L,R}) \bar{u}(p_7)P_R\nu(p_6), \]  

(24)

\[ T_{D,\lambda_i}(t_i, \tilde{\ell}_R) = -g^2 f_{K_iL_i}^R \Delta^t(e_{L,R}) \bar{u}(p_7)P_L\nu(p_6), \]  

(25)

\[ T_{D,\lambda_i}(u_i, \tilde{\ell}_L) = +g^2 f_{K_iL_i}^L \Delta^u(e_{L,R}) \bar{u}(p_7)P_Lu(p_3, \lambda_i)\nu(p_5), \]  

(26)

\[ T_{D,\lambda_i}(u_i, \tilde{\ell}_R) = +g^2 f_{K_iL_i}^R \Delta^u(e_{L,R}) \bar{u}(p_7)P_Ru(p_3, \lambda_i)\nu(p_5). \]  

(27)

and the kinematic variables

\[ s_i = (p_6 + p_7)^2, \quad t_i = (p_3 - p_6)^2, \quad u_i = (p_3 - p_7)^2. \]  

(28)

The amplitude for the whole process \( e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \to \tilde{\chi}_i^0\ell^+\ell^-\tilde{\chi}_j^0 \), with \( \tilde{\chi}_i^0 \) decaying, can be written as

\[ T = \Delta(\tilde{\chi}_i^0) \sum_{\lambda_i} T_{\lambda_i} T_{D,\lambda_i}, \]  

(29)

where \( \Delta(\tilde{\chi}_i^0) = 1/[p_3^2 - m_i^2 + im_i\Gamma_i] \), \( m_i, \Gamma_i \) are the propagator, mass and width of the decaying neutralino \( \tilde{\chi}_i^0 \).

### 2.3 Cross section

Following the formalism of [16, 24], the amplitude squared \(|T|^2 \) of the combined processes of production \( e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \) and decay \( \tilde{\chi}_i^0 \to \tilde{\chi}_i^0\ell^+\ell^- \) of the neutralino \( \tilde{\chi}_i^0 \), with neutralino \( \tilde{\chi}_j^0 \) being unobserved, can be written as

\[ |T|^2 = 4|\Delta(\tilde{\chi}_i^0)|^2 \left\{ P(\tilde{\chi}_i^0, \tilde{\chi}_j^0)D(\tilde{\chi}_i^0) + \sum_{a=1}^{3} \Sigma_a^p(\tilde{\chi}_i^0)\Sigma_a^d(\tilde{\chi}_i^0) \right\}, \]  

(30)

where \( a = 1, 2, 3 \) refers to the polarisation state of the neutralino \( \tilde{\chi}_i^0 \), which is described by the polarisation vectors \( s_a(\tilde{\chi}_i^0) \) given in Appendix A. \( P(\tilde{\chi}_i^0, \tilde{\chi}_j^0) \) and \( D(\tilde{\chi}_i^0) \) are the terms of production and decay independent of the polarisation of the decaying neutralino, whereas \( \Sigma_a^p(\tilde{\chi}_i^0) \) and \( \Sigma_a^d(\tilde{\chi}_i^0) \) are the terms containing the spin correlations between production and
decay. According to our choice of the polarisation vectors \( s^a(i\tilde\chi_i^0) \), Eqs. (78) – (80), \( \Sigma_P^2/P \) is the longitudinal polarisation, \( \Sigma_T^0/P \) is the transverse polarisation in the production plane and \( \Sigma_P^0/P \) is the polarisation perpendicular to the production plane of the neutralino \( i\tilde\chi_i^0 \). The explicit expressions for these contributions are given in Appendix B. The complete expressions for the amplitude squared including spin-spin correlations for the case when both neutralinos are decaying and for longitudinally polarised \( e^\pm \) beams are given in [16].

The differential cross section in the laboratory system is

\[
d\sigma = \frac{1}{8E_b^2}T|2(2\pi)^4\delta^4(p_1 + p_2 - \sum_{i=4}^7 p_i)|dlips(p_3 \cdots p_7),
\]

where \( E_b \) is the beam energy and \( dlips(p_3 \cdots p_7) \) is the Lorentz invariant phase space element. Integrating over all angles gives the cross section for the combined process of production and decay

\[
\sigma = \sigma(e^+e^→ i\tilde\chi_i^0\tilde\chi_j^0 → i\tilde\chi_j^0 i\tilde\chi_i^0) \cdot BR(i\tilde\chi_i^0 → i\tilde\chi_i^0) = \frac{1}{2E_b^2} \int P(i\tilde\chi_i^0)D(i\tilde\chi_i^0)|\Delta(i\tilde\chi_i^0)|^2(2\pi)^4\delta^4(p_1 + p_2 - \sum_{i=4}^7 p_i)|dlips(p_3 \cdots p_7).
\]

3 Triple product and CP asymmetry

In this section we will derive the analytic formulae for the cross section \( \sigma(e^+e^- → i\tilde\chi_i^0\tilde\chi_j^0 → i\tilde\chi_j^0 i\tilde\chi_i^0) \) and for the asymmetry \( A_T \), Eq. (4). As can be seen from the numerator of \( A_T \), for this purpose we have to identify those terms in \(|T|^2 \), Eq. (30), which contain a triple product of the form Eq. (3). Triple products follow from expressions \( i\epsilon_{\mu\nu\rho\sigma}h^{\mu\nu}p^{\rho}s^\sigma \) in the terms \( \Sigma_P^a(i\tilde\chi_i^0) \) and \( \Sigma_D^a(i\tilde\chi_i^0) \), where \( k, p, q, s \) are 4-momenta and spins of the particles involved (see Appendix B, Eqs. (91) – (99) and (108) – (116)). The expressions \( i\epsilon_{\mu\nu\rho\sigma}h^{\mu\nu}p^{\rho}s^\sigma \) are imaginary and when multiplied by the imaginary parts of the respective couplings they yield the terms which contribute to the numerator of \( A_T \), Eq. (4). Hence only the second term of Eq. (30), which is due to the spin correlations, contributes to the numerator of the asymmetry \( A_T \). It is convenient to split \( \Sigma_P^a(i\tilde\chi_i^0) \) and \( \Sigma_D^a(i\tilde\chi_i^0) \) into T-odd terms \( \Sigma_P^{a,O}(i\tilde\chi_i^0) \) and \( \Sigma_D^{a,O}(i\tilde\chi_i^0) \) containing the respective triple product, and T-even terms \( \Sigma_P^{a,E}(i\tilde\chi_i^0) \) and \( \Sigma_D^{a,E}(i\tilde\chi_i^0) \) without triple products:

\[
\Sigma_P^a(i\tilde\chi_i^0) = \Sigma_P^{a,O}(i\tilde\chi_i^0) + \Sigma_P^{a,E}(i\tilde\chi_i^0), \quad \Sigma_D^a(i\tilde\chi_i^0) = \Sigma_D^{a,O}(i\tilde\chi_i^0) + \Sigma_D^{a,E}(i\tilde\chi_i^0).
\]

Then the terms of \(|T|^2 \), Eq. (30), which contribute to the numerator of \( A_T \) are

\[
|T|^2 \supset 4|\Delta(i\tilde\chi_i^0)|^2 \sum_{a=1}^3 \left[ \Sigma_P^{a,O}(i\tilde\chi_i^0)\Sigma_D^{a,E}(i\tilde\chi_i^0) + \Sigma_P^{a,E}(i\tilde\chi_i^0)\Sigma_D^{a,O}(i\tilde\chi_i^0) \right],
\]

where the first (second) term is sensitive to the CP phases in the production (decay) of the neutralino \( i\tilde\chi_i^0 \). In the following we derive explicitly the T-odd contributions to the spin density matrices of production and decay.
3.1 T-odd terms of the production density matrix

As can be seen from Eq. (99) in Appendix B, in $\Sigma_p(\tilde{\chi}_i^0)$ the expressions $f_4^a$ vanish for $a = 1, 3$, because they do not contain three linearly independent vectors, hence $\Sigma^{3,0}_p(\tilde{\chi}_i^0) = 0$. Only for $a = 2$ we get a T-odd contribution to the production density matrix $\Sigma^{2,0}_p(\tilde{\chi}_i^0)$, which is related to the polarisation of the neutralino $\tilde{\chi}_i^0$ perpendicular to the production plane (see the expression for $s^2(\chi_i^0)$, Eq. (79) in Appendix A). We obtain from Eqs. (92) – (95) for $a = 2$

$$\Sigma^{2,0}_p(\tilde{\chi}_i^0) = \Sigma^2_p(ZZ) + \Sigma^2_p(Z\tilde{e}_L) + \Sigma^2_p(\tilde{Z}\tilde{e}_R) + \Sigma^2_p(\tilde{\ell}\tilde{e}_L) + \Sigma^2_p(\tilde{\ell}\tilde{e}_R)$$

with

$$\Sigma^2_p(ZZ) = \frac{g^4}{\cos^4 \Theta_W} |\Delta^v(Z)|^2 \left(R_\ell^2(1 + P_{e^-})(1 - P_{e^+}) - L_\ell^2(1 - P_{e^-})(1 + P_{e^+})\right)$$

$$\times \left[ ReO''_{ij}(ImO''_{ij})i f_4^2 \right]$$

$$\Sigma^2_p(Z\tilde{e}_L) = \frac{g^4}{2 \cos^2 \Theta_W} L_\ell(1 - P_{e^-})(1 + P_{e^+})$$

$$\times \frac{1}{2} \left[ \Delta^s(Z) \left[ f_{ij}L^* f_{ij}L^* \Delta^s(\tilde{e}_L) - f_{ij}L f_{ij}L^* \Delta^u(\tilde{e}_L) \right] f_4^2 \right]$$

$$\Sigma^2_p(\tilde{e}_L\tilde{e}_L) = \frac{g^4}{4} (P_{e^-})(1 + P_{e^+}) Re \left\{ \Delta^u(\tilde{e}_L) \Delta^s(\tilde{e}_L) (f_{ij}L^*)^2 (f_{ij}L^*)^2 f_4^2 \right\}$$

$$\Sigma^2_p(Z\tilde{e}_R) = \frac{g^4}{2 \cos^2 \Theta_W} R_\ell(1 + P_{e^-})(1 - P_{e^+})$$

$$\times \frac{1}{2} \left[ \Delta^s(Z) \left[ f_{ij}R^* f_{ij}R^* \Delta^s(\tilde{e}_R) - f_{ij}R f_{ij}R^* \Delta^u(\tilde{e}_R) \right] f_4^2 \right]$$

$$\Sigma^2_p(\tilde{e}_R\tilde{e}_R) = \frac{g^4}{4} (1 - P_{e^-})(1 - P_{e^+}) Re \left\{ \Delta^u(\tilde{e}_R) \Delta^s(\tilde{e}_R) (f_{ij}R^*)^2 (f_{ij}R^*)^2 f_4^2 \right\}$$

where $P_{e^-}$ and $P_{e^+}$ denotes the degree of longitudinal polarisation of the electron beam and positron beam, respectively. The function $f_4^a$, Eq. (99), for $a = 2$ reads

$$f_4^2 = im_{\mu} e_{\nu \rho \sigma} p^\mu_p p_\nu p_\rho p_\sigma$$

Note that $f_4^2$ is purely imaginary. When inserted, for example, into the expression for $\Sigma^{2,0}_p(Z\tilde{e}_L)$, Eq. (37), $f_4^2$ is multiplied by the factor $i \cdot Im \{ f_{ij}L f_{ij}L^* \}$, which is non-vanishing only if the couplings are complex. Hence it gives a CP-sensitive contribution to the asymmetry $A_T$, which depends on the phases $\phi_\mu$ and $\phi_M$. Analogous contributions come from the other terms in $\Sigma^{2,0}_p$, Eqs. (36),(38) – (40). We have to multiply $\Sigma^{2,0}_p$ in Eq. (34) by $\Sigma^{2,E}_D$, for which we obtain from Eqs. (108) – (112), Appendix B,

$$\Sigma^{2,E}_D(\tilde{\chi}_i^0) = \Sigma^{2,E}_D(ZZ) + \Sigma^{2,E}_D(Z\tilde{\ell}_L) + \Sigma^{2,E}_D(Z\tilde{\ell}_R) + \Sigma^{2,E}_D(\tilde{\ell}_L\tilde{\ell}_L) + \Sigma^{2,E}_D(\tilde{\ell}_R\tilde{\ell}_R)$$

with

$$\Sigma^{2,E}_D(ZZ) = \frac{g^4}{\cos^4 \Theta_W} |\Delta^v(Z)|^2 (R_\ell^2 - L_\ell^2)$$
\( \Sigma_D^2 (Z\tilde{\ell}_L) = \frac{4g^4}{\cos^2 \Theta_W} L_\ell R_e \left\{ \Delta^{\ell_i}(Z) \left[ f_{\ell_i}^L f_{\ell_k}^{L^*} \Delta^{u_j}(\tilde{\ell}_L)(-2O''_{ki} g_1^2 + O''_{ki} g_3^2) + f_{\ell_k}^L f_{\ell_k}^{L^*} \Delta^{u_j}(\tilde{\ell}_L)(2O''_{ki} g_2^2 + O''_{ki} g_3^2) \right] \right\}, \) (43)

\( \Sigma_D^2 (\tilde{\ell}_L\tilde{\ell}_L) = 2g^4 \left[ |f_{\ell_i}^L|^2 |f_{\ell_k}^L|^2 [\Delta^{u_j}(\tilde{\ell}_L)|^2 g_2^2 - |\Delta^{u_j}(\tilde{\ell}_L)|^2 g_1^2] + \text{Re} \{(f_{\ell_i}^{L^*})^2 (f_{\ell_k}^L)^2 \Delta^{u_j}(\tilde{\ell}_L)\Delta^{u_j}(\tilde{\ell}_L)g_3^2 \} \right], \) (44)

\( \Sigma_D^2 (Z\tilde{\ell}_R) = \frac{4g^4}{\cos^2 \Theta_W} R_\ell R_e \left\{ \Delta^{\ell_i}(Z) \left[ f_{\ell_i}^R f_{\ell_k}^{R^*} \Delta^{u_j}(\tilde{\ell}_R)(2O''_{ki} g_1^2 - O''_{ki} g_3^2) + f_{\ell_k}^R f_{\ell_k}^{R^*} \Delta^{u_j}(\tilde{\ell}_R)(-2O''_{ki} g_2^2 - O''_{ki} g_3^2) \right] \right\}, \) (45)

\( \Sigma_D^2 (\tilde{\ell}_R\tilde{\ell}_R) = 2g^4 \left[ |f_{\ell_i}^R|^2 |f_{\ell_k}^R|^2 [-|\Delta^{u_j}(\tilde{\ell}_R)|^2 g_2^2 + |\Delta^{u_j}(\tilde{\ell}_R)|^2 g_1^2] - \text{Re} \{(f_{\ell_i}^{R^*})^2 (f_{\ell_k}^R)^2 \Delta^{u_j}(\tilde{\ell}_R)\Delta^{u_j}(\tilde{\ell}_R)g_3^2 \} \right], \) (46)

where

\( g_1^2 = m_i(p_5 p_7)(p_6 s^2), \) (48)

\( g_2^2 = m_i(p_5 p_6)(p_7 s^2), \) (49)

\( g_3^2 = m_k[(p_3 p_6)(p_7 s^2) - (p_3 p_7)(p_6 s^2)]. \) (50)

The kinematic functions \( g_1^2, g_2^2, g_3^2 \) are real. When multiplied by the purely imaginary \( f_2^4 \), Eq. (41), this leads to triple products sensitive to the CP phases in the production process, which in the laboratory system read:

\( g_1^2 \cdot f_4^1 = i2E_0 m_i m_j (p_5 p_7) \vec{p}_6 (\vec{p}_1 \times \vec{p}_3), \) (51)

\( g_2^2 \cdot f_4^1 = i2E_0 m_i m_j (p_5 p_6) \vec{p}_7 (\vec{p}_1 \times \vec{p}_3), \) (52)

\( g_3^2 \cdot f_4^1 = i2E_0 m_i m_k (p_3 p_6) \vec{p}_7 (\vec{p}_1 \times \vec{p}_3) - (p_3 p_7) \vec{p}_6 (\vec{p}_1 \times \vec{p}_3). \) (53)

As outlined above, these expressions will be multiplied in Eqs. (36) – (40) by the factors \( i \cdot \text{Im}\{f_{\ell_i}^L f_{\ell_k}^{L^*} O_{ij}^{\ell} \} \) etc. and contribute to the first term of Eq. (34) and, hence, to the numerator of the asymmetry \( A_T \), Eq. (4).

### 3.2 T-odd terms of the decay density matrix

The second term in Eq. (34) is sensitive to CP violation in the neutralino decay process (2). The expressions \( g_1^2 \), Eq. (116), in \( \Sigma_D^2 \left( \chi_i^0 \right) \) contain three linearly independent vectors, however, \( \Sigma_D^E \left( \chi_i^0 \right) \) (Eq. (61)) vanishes because \( s^2(\chi_i^0) \) is perpendicular to the scattering plane. Therefore, only the terms \( \Sigma_D^{a,E} \left( \chi_i^0 \right) \) for \( a = 1, 3 \) have to be taken into account. The CP-sensitive terms of the decay density matrix following from Eqs. (108) – (112) for \( a = 1, 3 \), are

\( \Sigma_D^{O} \left( \chi_i^0 \right) = \Sigma_D^{O} (Z Z) + \Sigma_D^{O} (Z\tilde{\ell}_L) + \Sigma_D^{O} (Z\tilde{\ell}_R) + \Sigma_D^{O} (\tilde{\ell}_L\tilde{\ell}_L) + \Sigma_D^{O} (\tilde{\ell}_R\tilde{\ell}_R) \) (54)
with

\[
\Sigma^{a,0}_D(Z \bar{Z}) = 8 \frac{g^4}{\cos^4 \Theta_W} |\Delta^s(Z)|^2 (L_t^2 - R_t^2) \left[ 2 \text{Re}(O''_{k_l}) \text{Im}(O''_{k_l}) i g_a^4 \right],
\]

\[
\Sigma^{a,0}_D(Z \bar{\ell}_L) = \frac{4g^4}{\cos^2 \Theta_W} L_t \text{Re} \left\{ \Delta^s_i(Z) \left[ - f^L_{l_k} f^L_{l_k} O''_{k_i} \Delta^{t,*}(\bar{\ell}_L) \right. \right. \\
\left. \left. + f^L_{l_k} f^L_{l_k} O''_{k_i} \Delta^{u,*}(\bar{\ell}_L) \right] g_a^4 \right\},
\]

\[
\Sigma^{a,0}_D(\bar{\ell}_L \bar{\ell}_L) = 2g^4 \text{Re} \left\{ (f^L_{l_k})^2 (f^L_{l_k})^2 \Delta^{t,*}(\bar{\ell}_L) \Delta^{u,*}(\bar{\ell}_L) g_a^4 \right\},
\]

\[
\Sigma^{a,0}_D(Z \bar{\ell}_R) = \frac{4g^4}{\cos^2 \Theta_W} R_t \text{Re} \left\{ \Delta^s_i(Z) \left[ - f^R_{l_k} f^R_{l_k} O''_{k_i} \Delta^{t,*}(\bar{\ell}_R) \right. \right. \\
\left. \left. + f^R_{l_k} f^R_{l_k} O''_{k_i} \Delta^{u,*}(\bar{\ell}_R) \right] g_a^4 \right\},
\]

\[
\Sigma^{a,0}_D(\bar{\ell}_R \bar{\ell}_R) = 2g^4 \text{Re} \left\{ (f^R_{l_k})^2 (f^R_{l_k})^2 \Delta^{t,*}(\bar{\ell}_R) \Delta^{u,*}(\bar{\ell}_R) g_a^4 \right\},
\]

where for \(a = 1, 3\) we have

\[
g_a^4 = \text{Im} \epsilon_{\mu\nu\rho\sigma} s^{\mu\nu} p^{\rho \omega} t^{\omega \rho}.
\]

Now \(g_4^4, a = 1, 3\), is purely imaginary. When inserted, for example, in Eq. (56) it is multiplied by the factor \(i \cdot \text{Im} \left\{ f^L_{l_k} f^L_{l_k} O''_{k_i} \right\}\), which depends on the phases \(\phi_\mu\) and \(\phi_{M^i}\) and contributes to \(\Sigma^{a,0}_D\). Analogous contributions follow from Eqs. (55),(57) – (59). The corresponding T-even terms of the production density matrix also entering in Eq. (34) are obtained from Eqs. (92) – (95),

\[
\Sigma^{a,E}_P(\chi^0) = \Sigma^{a,E}_P(ZZ) + \Sigma^{a,E}_P(Z \bar{\ell}_L) + \Sigma^{a,E}_P(Z \bar{\ell}_R) + \Sigma^{a,E}_P(\bar{\ell}_L \bar{\ell}_L) + \Sigma^{a,E}_P(\bar{\ell}_R \bar{\ell}_R),
\]

where

\[
\Sigma^{a,E}_P(ZZ) = \frac{g^4}{\cos \Theta_W} |\Delta^s(Z)|^2 \left( R_t^2 (1 + P_e^-)(1 - P_e^-) - L_t^2 (1 - P_e^-)(1 + P_e^+) \right) \\
\times \left[ \left| O''_{j} \right|^2 \left( f^a_1 - f^a_3 \right) - \left[ (\text{Re} O''_{j})^2 - (\text{Im} O''_{j})^2 \right] f_3^a \right],
\]

\[
\Sigma^{a,E}_P(Z \bar{\ell}_L) = \frac{g^4}{2 \cos^2 \Theta_W} L_t (1 - P_e^-)(1 + P_e^+) \\
\times \text{Re} \left\{ \Delta^s(Z) \left[ 2 f^L_{l_k} f^L_{l_k} O''_{j} \Delta^{u,*}(\bar{\ell}_L) f^a_1 - 2 f^L_{l_k} f^L_{l_k} O''_{j} \Delta^{t,*}(\bar{\ell}_L) f^a_2 \right. \right. \\
\left. \left. + f^L_{l_k} f^L_{l_k} O''_{j} \Delta^{u,*}(\bar{\ell}_L) f^a_4 \right] \right\},
\]

\[
\Sigma^{a,E}_P(\bar{\ell}_L \bar{\ell}_L) = \frac{g^4}{4} (1 - P_e^-)(1 + P_e^+) \left[ \left| f^L_{l_k} \right|^2 \left( f^L_{l_k} \right)^2 (\Delta^{u}(\bar{\ell}_L))^2 f_1^a - \left| \Delta^t(\bar{\ell}_L) \right|^2 f_2^a \right. \\
\left. + \text{Re} \left\{ (f^L_{l_k})^2 (f^L_{l_k})^2 \Delta^{u}(\bar{\ell}_L) \Delta^{t,*}(\bar{\ell}_L) f_3^a \right\} \right],
\]

\[
\Sigma^{a,E}_P(Z \bar{\ell}_R) = \frac{g^4}{2 \cos^2 \Theta_W} R_t (1 + P_e^-)(1 - P_e^+) \\
\times \text{Re} \left\{ \Delta^s(Z) \left[ - 2 f^R_{l_k} f^R_{l_k} O''_{j} \Delta^{u,*}(\bar{\ell}_R) f^a_1 + 2 f^R_{l_k} f^R_{l_k} O''_{j} \Delta^{t,*}(\bar{\ell}_R) f_2^a \right. \right. \\
\left. \left. + \text{Re} \left\{ (f^R_{l_k})^2 (f^R_{l_k})^2 \Delta^{u}(\bar{\ell}_R) \Delta^{t,*}(\bar{\ell}_R) f_3^a \right\} \right\},
\]
- \left[ f_{ij}^{R} f_{ij}^{R*} O_{ij}^{R} \Delta^u(\bar{e}_R) + f_{ij}^{R*} f_{ij}^{R} O_{ij}^{R*} \Delta^s(\bar{e}_R) \right], \quad (65)

\Sigma_{pE}^{a} (\bar{e}_R \bar{e}_R) = \frac{g^4}{4} (1 + P_{e^-}) (1 - P_{e^+}) \left[ |f_{ij}^{R}|^2 |f_{ij}^{R*}|^2 (-|\Delta^u(\bar{e}_R)|^2 f_1^a + |\Delta^s(\bar{e}_R)|^2 f_2^a) - Re\{(f_{ij}^{R})^2 (f_{ij}^{R*})^2 \Delta^u(\bar{e}_R) \Delta^s(\bar{e}_R) f_3^a \right], \quad (66)

where

\begin{align*}
f_1^a &= m_i (p_{2p4}) (p_{1s^a}), \\
f_2^a &= m_i (p_{1p4}) (p_{2s^a}), \\
f_3^a &= m_j [(p_{1p3}) (p_{2s^a}) - (p_{2p3}) (p_{1s^a})].
\end{align*}

The triple products sensitive to the CP phases in the decay read in the laboratory system:

\begin{align*}
\sum_{a=1,3} f_1^a \cdot g_1^a &= im_i m_k (p_{2p4}) \left\{ -E_b \vec{p}_5 (\vec{p}_7 \times \vec{p}_6) - E_7 \vec{p}_5 (\vec{p}_6 \times \vec{p}_1) \\
&+ E_6 \vec{p}_5 (\vec{p}_7 \times \vec{p}_1) + E_5 \vec{p}_1 (\vec{p}_7 \times \vec{p}_6) \right\}, \\
\sum_{a=1,3} f_2^a \cdot g_2^a &= im_i m_k (p_{1p4}) \left\{ -E_b \vec{p}_5 (\vec{p}_7 \times \vec{p}_6) + E_7 \vec{p}_5 (\vec{p}_6 \times \vec{p}_1) \\
&- E_6 \vec{p}_5 (\vec{p}_7 \times \vec{p}_1) - E_5 \vec{p}_1 (\vec{p}_7 \times \vec{p}_6) \right\}, \\
\sum_{a=1,3} f_3^a \cdot g_3^a &= im_j m_k \left\{ [(p_{2p3}) - (p_{1p3})] E_b \vec{p}_5 (\vec{p}_7 \times \vec{p}_6) \\
&+ [(p_{2p3}) + (p_{1p3})] \\
&\left[ E_7 \vec{p}_5 (\vec{p}_6 \times \vec{p}_1) - E_6 \vec{p}_5 (\vec{p}_7 \times \vec{p}_1) - E_5 \vec{p}_1 (\vec{p}_7 \times \vec{p}_6) \right] \right\}. \quad (72)
\end{align*}

As already mentioned, these quantities will be multiplied in Eqs. (55) – (59) by the factor

\begin{align*}
i \cdot Im\{f_{ij}^{L} f_{ik}^{L*} O_{ki}^{L*}\} \text{ etc. and contribute to the second term of Eq. (34).}
\end{align*}

We take into account only the contributions to the T-odd asymmetry \(A_T\) which stem from the complex couplings and neglect the contributions from the widths of \(Z, \tilde{e}_L, \tilde{e}_R\) in the propagators Eqs. (19), (20), because they are of higher order.

### 4 Numerical Results

In this section we analyse numerically the CP asymmetry \(A_T\), Eq. (4), and the cross section for reactions (1) and (2), at an \(e^+e^-\) linear collider with \(\sqrt{s} = 500\) GeV and longitudinally polarised \(e^\pm\) beams. In particular, we investigate the dependence on the phases of the complex parameters \(M_1 = |M_1| e^{i_\delta M_1}\) and \(\mu = |\mu| e^{i_\delta \mu}\). For our numerical analysis we choose two scenarios A and B, such that \(m_{\tilde{\chi}^0_2} < m_{\tilde{\chi}^0_1} + m_{\tilde{e}_0}\) and \(m_{\tilde{\chi}^0_2} < m_{\tilde{e}_L,R}\) to prevent two-body decays of \(\tilde{\chi}^0_2\). The SUSY parameters and the masses of \(\tilde{\chi}^0_i, i = 1, \ldots, 4,\) and \(\tilde{e}_L, \tilde{e}_R\) are given in Table 1. The decay widths of the neutralinos have been computed with help of the program SPheno [25].

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Table 1: Input parameters $|M_1|$, $M_2$, $|\mu|$, $\tan\beta$, $m_{\tilde{\ell}_L}$ and $m_{\tilde{\ell}_R}$, and the resulting masses $m_{\tilde{\chi}^0_i}$, $i = 1, \ldots, 4$, for $(\phi_{M_1}, \mu) = (0.5\pi, 0)$ and $(0.5\pi, 0.5\pi)$. The parameters $M_2$, $|\mu|$ and $\tan\beta$ in scenario B are chosen according to the scenario SPS1a in [26]. All masses are given in GeV.

In scenario A the neutralinos $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\chi}^0_3$ are mainly $\tilde{B}$-higgsino, higgsino-$\tilde{B}$-$W^3$, higgsino-$\tilde{W}$ mixtures, respectively, where the $\tilde{B}$ component of the $\tilde{\chi}^0_2$ has a relatively strong $\phi_{M_1}$ dependence. In Fig. 3 we show the total decay width $\Gamma(\tilde{\chi}^0_2)$ and the branching ratio $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-)$, $\ell = e$ or $\mu$ in scenario A as a function of $\phi_{M_1}$ for $\phi_{\mu} = 0$. As can be seen, the CP-even quantities $\Gamma(\tilde{\chi}^0_2)$ and $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-)$ depend quite strongly on $\phi_{M_1}$. $\Gamma(\tilde{\chi}^0_2)$ varies between 34 keV for $\phi_{M_1} = 0.25\pi$, 1.75$\pi$ and 4.2 keV for $\phi_{M_1} = \pi$, and $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-)$ between 6.4% for $\phi_{M_1} = 0, 2\pi$ and 2.6% for $\phi_{M_1} = \pi$. The parameters $M_2$, $|\mu|$ and $\tan\beta$ in scenario B are chosen like in the scenario SPS1a in [26], with $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\chi}^0_3$ being mainly $B$-, $W^3$-, higgsino-like, respectively. In this scenario $\Gamma(\tilde{\chi}^0_2)$ varies between 80 keV for $\phi_{M_1} = 0$, 2$\pi$ and 9.9 keV for $\phi_{M_1} = \pi$, and $B(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-)$ between 1.9% for $\phi_{M_1} = 0.4\pi$, 1.6$\pi$ and 2.9% for $\phi_{M_1} = \pi$. In both scenarios we take for the slepton masses $m_{\tilde{\ell}_L} = 267.6$ GeV, $m_{\tilde{\ell}_R} = 224.4$ GeV and for the squark masses $m_{\tilde{u}_R} = 597.6$ GeV, $m_{\tilde{u}_L} = 599.0$ GeV and $m_{\tilde{d}_R} = 600.5$ GeV, $m_{\tilde{d}_L} = 602.9$ GeV for the first and second generation, and $m_{\tilde{b}_1} = 587.3$ GeV, $m_{\tilde{b}_2} = 615.7$ GeV ($A_b = 1000$ GeV) for the bottom squarks. In most of the examples we take values for the phases $\phi_{\mu}$ and $\phi_{M_1}$ which are in agreement with the constraints from the electron and neutron EDMs. In order to show the full phase dependence of the asymmetry and the cross section, we also give some examples where we vary $\phi_{\mu}$ and $\phi_{M_1}$ in the whole range, relaxing the constraints from the EDMs.
4.1 $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$

First we discuss neutralino production $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ with subsequent leptonic three-body decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-$. In this case one obtains CP-violating contributions from both production and decay. In Fig. 4 (a) we show the CP asymmetry $A_T$ as a function of the phase $\phi_{M1}$ for scenario A defined in Table 1 for $\phi_{\mu} = 0$ and two centre of mass energies $\sqrt{s} = 350$ GeV and $\sqrt{s} = 500$ GeV. The beam polarisations are fixed at $P_{e^-} = -0.8$ and $P_{e^+} = +0.6$. The CP asymmetry attains the largest values of about $11\%$ ($-11\%$) for $\sqrt{s} = 500$ GeV at $\phi_{M1} \approx 0.2\pi$ ($\phi_{M1} \approx 1.8\pi$). For $\sqrt{s} = 350$ GeV, i.e. closer to threshold of the production, $A_T$ reaches values of $\pm 13.5\%$ at $\phi_{M1} \approx 0.2\pi, 1.8\pi$. Choosing $\phi_{\mu} = 0.1\pi$ instead of $\phi_{\mu} = 0$ leaves the asymmetry $A_T$ almost unchanged. Note that $A_T$ does not have its largest values at $\phi_{M1} = \frac{\pi}{2}$ ($\phi_{M1} = \frac{3\pi}{2}$), where the imaginary parts of the respective couplings are maximal, but at smaller (larger) phases. The reason is an interplay between the $\phi_{M1}$ dependence of the cross section, shown in Fig. 4 (b), and the $\phi_{M1}$ dependence of the numerator of $A_T$, Eq. (34). This numerator is a sum of products of CP-odd and CP-even factors, which essentially have a sin $\phi_{M1}$ and cos $\phi_{M1}$ behaviour, respectively (for $\phi_{\mu} = 0$). We plot in Fig. 4 (b) the cross section $\sigma = \sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0) \cdot BR(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-)$ for the production and subsequent decay process $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0\ell^+\ell^-$ (Eq. (32)), summed over $\ell = e, \mu$. Note that also the cross section has a rather strong $\phi_{M1}$ dependence. At $\phi_{M1} = \pi$ the production cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ has a maximum, whereas the branching ratio $BR(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-)$ has a minimum (see Fig. 3 (b)) resulting in the dip of the cross section for the combined process of production and decay at $\phi_{M1} = \pi$.

In Fig. 5 we show the contour lines of the CP asymmetry $A_T$ in the $\phi_{M1}-\phi_{\mu}$ plane in scenario A (Table 1) for $\sqrt{s} = 500$ GeV and two sets of beam polarisations $P_{e^-} = -0.8, P_{e^+} = +0.6$ and $P_{e^-} = +0.8, P_{e^+} = -0.6$. In the scenario considered the $\phi_{M1}$ dependence is stronger than that on $\phi_{\mu}$. In the case of $P_{e^-} = -0.8, P_{e^+} = +0.6$ the largest asymmetries $|A_T| \approx 10\%$ are reached for $\phi_{M1} \approx 0.2\pi, 1.8\pi$, rather independent of $\phi_{\mu}$ (Fig. 5 (a)). For $P_{e^-} = +0.8, P_{e^+} = -0.6$ (Fig. 5 (b)) the maximal values of $|A_T| \approx 7.5\%$ are attained near $\phi_{M1} \approx 0.3\pi, \phi_{\mu} \approx 0.4\pi$ and $\phi_{M1} \approx 1.7\pi, \phi_{\mu} \approx 1.6\pi$. The sign flip of $A_T$ between Fig. 5 (a)
A in the region with maximal 
P
section has values between 1 and 10 fb in the region where 
P
comes from the

Z

A

largest in the region

M

P

(Fig. 6 (b)). For

P

and the facts that

m

and the cross section

χ

0

σ

if both neutralinos have significant higgsino and gaugino components. In Fig. 6 (c) and

A

τ

0

in %

σ/fb

0

0.5

1

2

φM₁/π

for the leptonic decay. The cross sections are larger by roughly a factor

5 – 10 in the main part of the parameter region of Fig. 6 and approximately of the same

order of magnitude as in the leptonic case for

|μ| ≥ 400 GeV. The experimental errors,
Figure 5: Contours of the CP asymmetry $A_T$ (Eq. (4)) in % for $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ with subsequent decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-$ in scenario A of Table 1 for $\sqrt{s} = 500$ GeV and (a) $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and (b) $P_{e^-} = +0.8$, $P_{e^+} = -0.6$.

however, are expected to be larger than for the leptonic $\tilde{\chi}_0^0$ decays, because the distinction of the $b$ and $\bar{b}$ or $c$ and $\bar{c}$ charges and the resolution of the jet momenta are worse than in the leptonic case [27].

We also studied the $|M_1|$ dependence of the asymmetry $A_T$ and the cross section, relaxing the GUT relation for $|M_1|$ with all other parameters as in scenario B of Table 1. In Fig. 7 (a) we show the T-odd asymmetry $A_T$ as a function of $\phi_{M_1}$ for four values of $|M_1| = 90$ GeV, 95 GeV, 100 GeV and 105 GeV. Here maximal asymmetries $|A_T| \approx 4\%$ are reached. For $|M_1| = 105$ GeV $A_T$ shows nearly two complete oscillations, whereas for $|M_1| = 90$ GeV there is only one oscillation with extrema at $\phi_{M_1} = 0.75\pi$ and $1.25\pi$. In both cases this behaviour is caused by cancellations between the contributions $\Sigma_P^{a.E}(\tilde{\chi}_1^0)\Sigma_D^{a.O}(\tilde{\chi}_1^0)$ from production and $\Sigma_P^{a.E}(\tilde{\chi}_2^0)\Sigma_D^{a.O}(\tilde{\chi}_2^0)$ from decay in Eq. (34), which have opposite sign. For $|M_1| = 90$ GeV the contributions from the decay are dominant, whereas for $|M_1| = 105$ GeV both contributions are of similar magnitude, resulting in two oscillations. The corresponding cross sections are shown in Fig. 7 (c). In Fig. 7 (b) we show $A_T$ as a function of $\phi_\mu$ for $\phi_{M_1} = 0.25\pi$ and three values of $|M_1| = 95$ GeV, 100 GeV and 105 GeV. (We do not discuss $|M_1| = 90$ GeV because in this case $m_{\tilde{\chi}_2^0} > m_{\tilde{\chi}_1^0} + m_{Z^0}$ for $0.5\pi \lesssim \phi_\mu \lesssim 1.7\pi$.) For $|M_1| = 100$ GeV $A_T$ exhibits the strongest variation with $\phi_\mu$ with maximum $|A_T| \approx 3\%$ for $\phi_\mu \approx 0.2\pi$ and minimum $|A_T| \approx 0$ for $\phi_\mu \approx \pi$. The corresponding cross sections are shown in Fig. 7 (d).

Furthermore we have analysed the dependence of the asymmetry $A_T$ on the masses of the right and left sleptons, assuming $m_{\tilde{\ell}_R}$ and $m_{\tilde{\ell}_L}$ are free parameters, for $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ with the other parameters as in scenario A of Table 1. For $\sqrt{s} = 500$ GeV and beam polarisations $P_{e^-} = -0.8$ and $P_{e^+} = +0.6$ we find that the asymmetry $A_T$
Figure 6: (a), (b) Contours of the CP asymmetry $A_T$ (Eq. (4)) in % for $e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ with subsequent decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-$ and (c), (d) contours of the corresponding cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0) \cdot BR(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-)$, summed over $\ell = e, \mu$, in fb, respectively, for $\tan \beta = 10$, $m_{\tilde{\chi}_L} = 267.6$ GeV, $m_{\tilde{\chi}_R} = 224.4$ GeV, $|M_1|/M_2 = 5/3 \tan^2 \theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_{\mu} = 0$ with $\sqrt{s} = 500$ GeV and (a), (c) $P_\ell^- = -0.8$, $P_\ell^+ = +0.6$ and (b), (d) $P_\ell^- = +0.8$, $P_\ell^+ = -0.6$. The points mark scenarios A and B of Table 1. The dark shaded area marks the parameter space with $m_{\tilde{\chi}_1^\pm} < 103.5$ GeV excluded by LEP. In the light shaded area the analysed three-body decay is strongly suppressed because $m_{\tilde{\chi}_2^0} > m_Z + m_{\tilde{\chi}_1^0}$ or $m_{\tilde{\chi}_2^0} > m_{\tilde{\ell}_R}$. 

$M_2/\text{GeV}$  

$A_T$ in %  

$\mu/\text{GeV}$  

$\mu/\text{GeV}$  

$\sigma/\text{fb}$  

$\sigma/\text{fb}$  

$2$-body decays  

$2$-body decays  

$2$-body decays  

$2$-body decays
Figure 7: (a), (b) CP asymmetry $A_T$ (Eq. (4)) and (c), (d) cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$, summed over $\ell = e, \mu$, in scenario B of Table 1 with (a), (c) $\phi^\mu = 0$, (b), (d) $\phi^M_1 = 0.25\pi$ and $|M_1| = 90$ GeV (solid), $|M_1| = 95$ GeV (dashed), $|M_1| = 100$ GeV (dotted) and $|M_1| = 105$ GeV (dashdotted) for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $\sqrt{s} = 500$ GeV.

is larger than 6% for $m_{\tilde{\ell}_R} \approx 200$ GeV and $m_{\tilde{\ell}_L} \gtrsim 150$ GeV. For $m_{\tilde{\ell}_L} \approx 200$ GeV and $m_{\tilde{\ell}_R} \gtrsim 300$ GeV the asymmetry is smaller than $-6\%$. This is due to the fact, that for these beam polarisations the contributions from $\tilde{\ell}_L$ exchange to the CP asymmetry are dominant. Outside of these narrow regions around $m_{\tilde{\ell}_R} \approx 200$ GeV or $m_{\tilde{\ell}_L} \approx 200$ GeV the asymmetry is very small. For opposite beam polarisations $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ the $\tilde{\ell}_R$ contributions to $A_T$ are dominant and the sign of the asymmetry changes: for $m_{\tilde{\ell}_R} \approx 200$ GeV and $m_{\tilde{\ell}_L} \gtrsim 250$ GeV it is $A_T \lesssim -11\%$ and for $m_{\tilde{\ell}_L} \approx 200$ GeV and $300 \lesssim m_{\tilde{\ell}_R} \lesssim 850$ GeV it is $A_T \gtrsim 6\%$.

4.2 $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$

In the reaction $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ there are only T-odd contributions from the decay, i.e. from the second term in Eq. (34). There is no T-odd contribution from the production, because the amplitudes Eqs. (14) – (18) are real and the first term in Eq. (34) vanishes. We will give numerical results for the case where one neutralino decays leptonically and
the other decays hadronically. If both neutralinos would decay into lepton pairs of the same flavour, it may be experimentally difficult to distinguish the two lepton pairs coming from different neutralinos. In Fig. 8 we show the contour lines in the $M_2-|\mu|$ plane of the CP asymmetry $A_T$ and of the cross section with one neutralino decaying leptonically, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+ \ell^-$, summed over $\ell = e, \mu$, and the other neutralino decaying hadronically, $\tilde{\chi}_2^0 \to \tilde{\chi}_2^0 q\bar{q}$, summed over $q = u, d, s, c, b$, for $\sqrt{s} = 350$ GeV and $P_{e^-} = -0.8$, $P_{e^+} = +0.6$. The other parameters are $\tan\beta = 10$, $m_{\tilde{t}_L} = 267.6$ GeV, $m_{\tilde{t}_R} = 224.4$ GeV, $|M_1|/M_2 = 5/3 \tan^2 \theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$. For this process $A_T$ is considerably smaller than for $e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$, because for production of a pair of equal neutralinos only the decay amplitudes contribute to the asymmetry. The largest asymmetries are obtained for $|\mu| \gtrsim 400$ GeV and $M_2 \approx 150$ GeV, where $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ have gaugino character. The maximum values are $A_T \approx 1.8\%$. The cross section, however, reaches values up to 10 fb in this region. The hadronic branching ratio $\sum_{q = u, d, s, c, b} BR(\tilde{\chi}_2^0 \to \chi_2^0 q\bar{q})$ is between 50% and 95% in the main part of the parameter region of Fig. 8 and between 30% and 50% for $|\mu| \gtrsim 400$ GeV. For $\sqrt{s} = 500$ GeV the asymmetries are smaller with maximum of 0.7% for $|\mu| \gtrsim 400$ GeV and $M_2 \approx 150$ GeV.

We want to remark that it may also be possible to distinguish the two lepton pairs, if one neutralino decays to an $e^+e^-$ pair, the other one to a $\mu^+\mu^-$ pair. Similarly it may be possible to distinguish the two quark pairs, if one neutralino decays to a $b\bar{b}$ pair and the other one to a $c\bar{c}$ pair [27] or pair of light quarks. However, in the case of both neutralinos decaying into lepton pairs the event rate is expected to be very low, whereas in the case of the decay into quark pairs of different flavours the experimental errors are expected to be much larger.

### 4.3 $e^+e^- \to \tilde{\chi}_3^0\tilde{\chi}_2^0$

In Fig. 9 we show the contour lines of the CP asymmetry $A_T$ in the $\phi_{M_1}-\phi_\mu$ plane in scenario A (Table 1) for $\sqrt{s} = 500$ GeV and two sets of beam polarisations $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. As in the case of $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production (Fig. 5) the $\phi_{M_1}$ dependence is stronger than that on $\phi_\mu$ in this scenario, since the $B$ component of the decaying particle $\tilde{\chi}_2^0$ depends stronger on $\phi_{M_1}$. The largest asymmetries of $|A_T| \gtrsim 9\%$ are reached in the case of $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (Fig. 9 (a)) for $\phi_{M_1} \approx 0.25\pi, 1.75\pi$ and $0.6\pi \lesssim \phi_\mu \lesssim 1.4\pi$. In the case of $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ (Fig. 9 (b)) the sign of $A_T$ flips and the largest asymmetries $|A_T| \gtrsim 7\%$ appear for $\phi_{M_1} \approx 0.25\pi, 1.75\pi$ and $0.8\pi \lesssim \phi_\mu \lesssim 1.2\pi$.

In Fig. 10 we show the contour lines of the CP asymmetry $A_T$ and of the cross section for the process $e^+e^- \to \tilde{\chi}_3^0\tilde{\chi}_2^0$ with the subsequent decay $\tilde{\chi}_2^0 \to \tilde{\chi}_2^0 \ell^+ \ell^-$, summed over $\ell = e, \mu$, in the $M_2-|\mu|$ plane for $\tan\beta = 10$, $m_{\tilde{t}_L} = 267.6$ GeV, $m_{\tilde{t}_R} = 224.4$ GeV, $|M_1|/M_2 = 5/3 \tan^2 \theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ for $\sqrt{s} = 500$ GeV and two sets of beam polarisations $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. $|A_T|$ has two maxima in the regions $|\mu| \approx 240$ GeV, $M_2 \approx 300$ GeV and $|\mu| \approx 350$ GeV, $M_2 \approx 140$ GeV, respectively, for both polarisation configurations. The maximum values are $A_T \gtrsim 10\%$ for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (Fig. 10 (a)) and $A_T \lesssim -8\%$ for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (Fig. 10 (b)). The cross section reaches values larger than 10 fb (6 fb) in the region with maximal asymmetry around $|\mu| \approx 240$ GeV, $M_2 \approx 300$ GeV for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$. 

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Figure 8: Contours (a) of the CP asymmetry $A_T$ (Eq. (4)) in % for $e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_1^0$ with subsequent decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-$ and (b) of the corresponding cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_1^0\ell^+\ell^-)\cdot BR(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0qq)$, summed over $\ell = e, \mu$ and $q = u, d, s, c, b$, in fb, respectively, for $\tan\beta = 10$, $m_{\tilde{\chi}_1} = 267.6$ GeV, $m_{\tilde{\chi}_2} = 224.4$ GeV, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_{\mu} = 0$ with $\sqrt{s} = 350$ GeV and $P_{e^-} = -0.8, P_{e^+} = +0.6$. The dark shaded area marks the parameter space with $m_{\tilde{\chi}_1^\pm} < 103.5$ GeV excluded by LEP. The light shaded area is kinematically not accessible.

Fig. 10 (c) ($P_{e^-} = +0.8, P_{e^+} = -0.6$, Fig. 10 (d)). In the region around $|\mu| \approx 350$ GeV, $M_2 \approx 140$ GeV the cross section is $\lesssim 1$ fb.

We have also calculated the branching ratio $BR(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-)$, $\ell = e$ or $\mu$, of $\tilde{\chi}_3^0$. In parameter regions, where $\tilde{\chi}_3^0$ decays via a three-body decay and where it is especially difficult to distinguish the two lepton pairs coming from different neutralinos, $BR(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-)$ is always smaller than about 7%.

Furthermore we have calculated the CP asymmetry $A_T$ for the the process $e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0$ with the subsequent decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-$, $\ell = e, \mu$. For the associated production of $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ the accessible parameter space at a linear collider with $\sqrt{s} = 500$ GeV is rather small and is approximately constrained by $M_2 + |\mu| \lesssim 500$ GeV. For the same parameters as in Fig. 10 we find that the CP asymmetry has maximum values $|A_T| \approx 6\%$. However, in parameter regions with $|A_T| > 2\%$ the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0)\cdot BR(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\ell^+\ell^-)$, summed over $\ell = e, \mu$, is always smaller than 1 fb.

5 Conclusions

In this paper we have studied CP violation in neutralino production and subsequent three-body decay processes. As CP-sensitive observable we have chosen a T-odd asymmetry
Figure 9: Contours of the CP asymmetry $A_T$ (Eq. (4)) in % for $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0$ with subsequent decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-$ in scenario A of Table 1 for $\sqrt{s} = 500$ GeV and (a) $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and (b) $P_{e^-} = +0.8$, $P_{e^+} = -0.6$.

based on triple product correlations between the incoming and outgoing particles. A non-vanishing value for the asymmetry indicates directly CP-violating effects. We have provided compact analytical formulae for the cross section of the whole process as well as for the T-odd asymmetry for longitudinal beam polarisations. It is necessary to include the full spin correlations between production and decay. The asymmetry can be directly measured in the experiment without reconstruction of the momentum of the decaying neutralino or further final-state analyses. In our numerical results we have chosen representative scenarios and have shown that this T-odd asymmetry can reach values up to 13% in some parts of the parameter space of the unconstrained MSSM, even for small CP-violating phases as may be indicated by the EDM constraints. This observable will therefore be an important tool in the search for CP-violating effects in the neutralino sector and the determination of the phases of the complex SUSY parameters.

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Figure 10: (a), (b) Contours of the CP asymmetry $A_T$ (Eq. (4)) in % for $e^+e^-\to\tilde{\chi}^0_3\tilde{\chi}^0_2$ with subsequent decay $\tilde{\chi}^0_2\to\tilde{\chi}^0_1\ell^+\ell^-$ and (c), (d) contours of the corresponding cross section $\sigma(e^+e^-\to\tilde{\chi}^0_3\tilde{\chi}^0_2)\cdot BR(\tilde{\chi}^0_2\to\tilde{\chi}^0_1\ell^+\ell^-)$, summed over $\ell=e,\mu$, in fb, respectively, for $\tan\beta=10$, $m_{\tilde{\chi}^0_{3L}}=267.6$ GeV, $m_{\tilde{\chi}^0_{3R}}=224.4$ GeV, $|M_1|/M_2=5/3\tan^2\theta_W$, $\phi_{M_1}=0.5\pi$ and $\phi_{\mu}=0$ with $\sqrt{s}=500$ GeV and (a), (c) $P_{e^-}=-0.8$, $P_{e^+}=+0.6$ and (b), (d) $P_{e^-}=+0.8$, $P_{e^+}=-0.6$. The dark shaded area marks the parameter space with $m_{\tilde{\chi}^0_{1L}}<103.5$ GeV excluded by LEP. The light shaded area is kinematically not accessible or in this area the analysed three-body decay is strongly suppressed because $m_{\tilde{\chi}^0_{2L}}>m_Z+m_{\tilde{\chi}^0_{1L}}$ or $m_{\tilde{\chi}^0_{2R}}>m_{\tilde{\chi}^0_{1R}}$, respectively.
A Kinematics

We choose a coordinate frame in the laboratory system, where the momenta are

\[ p_1 = E_b(1, -\sin \Theta, 0, \cos \Theta), \]
\[ p_2 = E_b(1, \sin \Theta, 0, -\cos \Theta), \]
\[ p_3 = (E_i, 0, 0, -q), \]
\[ p_4 = (E_j, 0, 0, q). \]

\( \Theta \) is the scattering angle between the incoming \( e^- (p_1) \) beam and the outgoing neutralino \( \tilde{\chi}_j^0(p_4) \), the azimuth \( \Phi \) can be chosen equal to zero. The energy and the momenta of the neutralinos \( \tilde{\chi}_j^0(p_4) \) and \( \tilde{\chi}_i^0(p_3) \) are

\[ E_i = \frac{s + m_i^2 - m_j^2}{2\sqrt{s}}, \quad E_j = \frac{s + m_j^2 - m_i^2}{2\sqrt{s}}, \quad q = \frac{\sqrt{\lambda(s, m_i^2, m_j^2)}}{2\sqrt{s}}, \]

where \( m_i, m_j \) are the masses of the neutralinos and \( \lambda \) is the kinematical triangle function \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \).

The polarisation vectors \( s^{a\mu}(\tilde{\chi}_i^0) \) and \( s^{b\mu}(\tilde{\chi}_j^0) \) \((a, b = 1, 2, 3)\) of the neutralinos in the laboratory system are

\[ s^{1\mu}(\tilde{\chi}_{i,j}^0) = (0, \mp 1, 0, 0), \]
\[ s^{2\mu}(\tilde{\chi}_{i,j}^0) = (0, 0, 1, 0), \]
\[ s^{3\mu}(\tilde{\chi}_{i,j}^0) = \frac{1}{m_{i,j}}(q, 0, 0, \mp E_{i,j}), \]

where \( s^3(\tilde{\chi}_{i,j}^0) \) describes the longitudinal polarisation, \( s^1(\tilde{\chi}_{i,j}^0) \) the transverse polarisation in the scattering plane, and \( s^2(\tilde{\chi}_{i,j}^0) \) the transverse polarisation perpendicular to the scattering plane.

B Explicit expressions of the amplitude squared

B.1 Production

Here we give the analytic expressions for the production spin density matrix. The terms

\[ P(\tilde{\chi}_i^0, \tilde{\chi}_j^0) = P(ZZ) + P(Z\tilde{e}_L) + P(Z\tilde{e}_R) + P(\tilde{e}_L\tilde{e}_L) + P(\tilde{e}_R\tilde{e}_R) \]

are independent of the polarisation of the neutralinos [16, 28],

\[ P(ZZ) = \frac{g^4}{\cos^4 \Theta_W} |\Delta^s(Z)|^2(R_R^2c_R + L_L^2c_L) \]
\[ \times \left\{ \frac{d_i}{f_1 + f_2} - [(ReO^{eL}_{ij})^2 - (ImO^{eL}_{ij})^2]f_3 \right\}, \]
\[ P(Z\tilde{e}_L) = \frac{g^4}{2\cos^2 \Theta_W} L_Lc_L \]

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The beam polarisation weighting factors are given by

\[
P(a,\chi) = \Delta^a(Z) \left[ - (\Delta^{ts}(\bar{\epsilon}_L) f_{i \bar{t}}^L f_{j \bar{t}}^L O_{ij}^{u''L} + \Delta^{us}(\bar{\epsilon}_L) f_{i \bar{t}}^L f_{j \bar{t}}^L O_{ij}^{u''L}) f_3 + 2\Delta^{ts}(\bar{\epsilon}_L) f_{i \bar{t}}^L f_{j \bar{t}}^L O_{ij}^{u''L} f_1 + 2\Delta^{us}(\bar{\epsilon}_L) f_{i \bar{t}}^L f_{j \bar{t}}^L O_{ij}^{u''L} f_2 \right],
\]

or

\[
P(\bar{\epsilon}_L\bar{\epsilon}_L) = \frac{g_4^4}{4} c_L \left[ |f_{i \bar{t}}^L|^2 |f_{j \bar{t}}^L|^2 (|\Delta^{t}(\bar{\epsilon}_L)|^2 f_1 + |\Delta^{u}(\bar{\epsilon}_L)|^2 f_2) - Re\{(f_{i \bar{t}}^L)^2 (f_{j \bar{t}}^L)^2 \Delta^{u}(\bar{\epsilon}_L)\Delta^{ts}(\bar{\epsilon}_L) f_3 \} \right].
\]

The terms \( P(Z\bar{R}) \), \( P(\bar{\epsilon}_R\bar{\epsilon}_R) \) are obtained by the following substitutions in Eqs. (83), (84):

\[
L_\ell \rightarrow R_\ell, \quad c_L \rightarrow c_R, \quad \Delta^{t,u}(\bar{\epsilon}_L) \rightarrow \Delta^{t,u}(\bar{\epsilon}_R), \quad O_{ij}^{u''L} \rightarrow O_{ij}^{u''R}, \quad f_{i \bar{t}}^L \rightarrow f_{i \bar{t}}^R.
\]

The beam polarisation weighting factors are given by

\[
c_L = (1 - P_{e^-})(1 + P_{e^+}),
\]

\[
c_R = (1 + P_{e^-})(1 - P_{e^+})
\]

and the kinematical factors in invariant form are

\[
f_1 = (p_1 p_4)(p_2 p_3),
\]

\[
f_2 = (p_1 p_3)(p_2 p_4),
\]

\[
f_3 = m_i m_j (p_1 p_2).
\]

The terms

\[
\Sigma^a_p(Z\bar{L}) = \Sigma^a_p(Z\bar{R}) + \Sigma^a_p(Z\bar{\epsilon}_L) + \Sigma^a_p(Z\bar{\epsilon}_R) + \Sigma^a_p(\bar{\epsilon}_L\bar{\epsilon}_L) + \Sigma^a_p(\bar{\epsilon}_R\bar{\epsilon}_R),
\]

which depend on the polarisation of the neutralino \( \tilde{\chi}_1^0 \) and which are relevant for the calculation of the CP asymmetries, are given by

\[
\Sigma^a_p(Z\bar{L}) = \frac{g_4^4}{2 \cos^2 \Theta_W} L_{\ell} c_L \left[ \Delta^a(Z)^2 (R_{\ell}^2 c_R - L_{\ell}^2 c_L) \left[ |O_{ij}^{u''L}|^2 (f_1^a - f_3^a) - [(ReO_{ij}^{u''L})^2 - (ImO_{ij}^{u''L})^2] f_3^a + 2(ReO_{ij}^{u''L})(ImO_{ij}^{u''L}) f_4^a \right] \right],
\]

\[
\Sigma^a_p(Z\bar{\epsilon}_L) = \frac{g_4^4}{4} c_L \left[ |f_{i \bar{t}}^L|^2 |f_{j \bar{t}}^L|^2 (|\Delta^{u}(\bar{\epsilon}_L)|^2 f_1^a - |\Delta^{t}(\bar{\epsilon}_L)|^2 f_2^a) - Re\{(f_{i \bar{t}}^L)^2 (f_{j \bar{t}}^L)^2 \Delta^{u}(\bar{\epsilon}_L)\Delta^{ts}(\bar{\epsilon}_L) f_3^a + f_4^a \} \right],
\]

\[
\Sigma^a_p(Z\bar{\epsilon}_R) = \Sigma^a_p(\bar{\epsilon}_R\bar{\epsilon}_R) \text{ are obtained by the substitutions Eq. (85) and}
\]

\[
f_1^a \rightarrow -f_1^a, \quad f_2^a \rightarrow -f_2^a, \quad f_4^a \rightarrow f_4^a.
\]
in Eqs. (93), (94), where the kinematical factors in invariant form are given by

\[ f_1^a = m_i(p_2p_4)(p_1s^a), \]
\[ f_2^a = m_i(p_1p_4)(p_2s^a), \]
\[ f_3^a = m_j[(p_1p_3)(p_2s^b) - (p_2p_3)(p_1s^a)], \]
\[ f_4^a = im_j\epsilon_{\mu
u\rho\sigma}p_2^\mu p_1^\nu s^a p_3^\rho p_3^\sigma. \]

The polarisation vectors \( s^a(\tilde{\chi}_i^0) \) of the neutralino \( \tilde{\chi}_i^0 \) are given in the laboratory system by Eqs. (78) – (80).

### B.2 Leptonic 3-body decay

Here we give the analytical expressions for the different contributions to the decay density matrix for the three-body decay \( \tilde{\chi}_i^0(p_3) \rightarrow \tilde{\chi}_k^0(p_5)\ell^+(p_6)\ell^-(p_7) \), where we sum over the spins of the final-state particles \([16, 28]\). The contributions independent of the polarisation of the neutralino \( \tilde{\chi}_i^0 \) are

\[ D(\tilde{\chi}_i^0) = D(ZZ) + D(Z\tilde{\ell}_L) + D(Z\tilde{\ell}_R) + D(\tilde{\ell}_L\tilde{\ell}_L) + D(\tilde{\ell}_R\tilde{\ell}_R) \]

with

\[ D(ZZ) = \frac{g^4}{\cos^4 \Theta_W}|\Delta^{s_i}(Z)|^2(L_L^2 + R_L^2) \]
\[ \left[ |O_{ki}^{LL}|^2(g_1 + g_2) + [(ReO_{ki}^{LL})^2 - (ImO_{ki}^{LL})^2]g_3 \right], \]

\[ D(Z\tilde{\ell}_L) = \frac{4g^4}{\cos^2 \Theta_W}L_iRe\left\{ \Delta^{s_i}(Z) \left[ f_{Li}^{LL}f_{Lk}^{LL} \Delta^{a_i}(\tilde{\ell}_L)(2O_{ki}^{LL}g_1 + O_{ki}^{LL}g_3) \right. \right. \]
\[ \left. + f_{Li}^{LL}f_{Lk}^{LL} \Delta^{a_i}(\tilde{\ell}_L)(2O_{ki}^{LL}g_2 + O_{ki}^{LL}g_3) \right\}, \]

\[ D(\tilde{\ell}_L\tilde{\ell}_L) = 2g^4 \left[ |f_{Li}^{LL}|^2|f_{Lk}^{LL}|^2(|\Delta^{a_i}(\tilde{\ell}_L)|^2g_1 + |\Delta^{a_i}(\tilde{\ell}_L)|^2g_2) \right. \]
\[ + Re\{(f_{Li}^{LL})^2(f_{Lk}^{LL})^2\Delta^{a_i}(\tilde{\ell}_L)\Delta^{a_i}(\tilde{\ell}_L)\}g_3 \].

The quantities \( D(Z\tilde{\ell}_R), D(\tilde{\ell}_R\tilde{\ell}_R) \) can be derived from Eqs. (102), (103) by the substitutions

\[ L_\ell \rightarrow R_\ell, \quad \Delta^{a_i}(\tilde{\ell}_L) \rightarrow \Delta^{a_i}(\tilde{\ell}_R), \quad O_{ki}^{LL} \rightarrow O_{ki}^{RR}, \quad f_{Li,k}^{LL} \rightarrow f_{Li,k}^{RR}. \]

The kinematical factors are

\[ g_1 = (p_5p_7)(p_3p_6), \]
\[ g_2 = (p_5p_6)(p_3p_7), \]
\[ g_3 = m_i m_k (p_5p_7). \]

The contributions which depend on the polarisation of the decaying neutralino \( \tilde{\chi}_i^0 \) are

\[ \Sigma_D^a(\tilde{\chi}_i^0) = \Sigma_D^a(ZZ) + \Sigma_D^a(Z\tilde{\ell}_L) + \Sigma_D^a(Z\tilde{\ell}_R) + \Sigma_D^a(\tilde{\ell}_L\tilde{\ell}_L) + \Sigma_D^a(\tilde{\ell}_R\tilde{\ell}_R) \]
with

\[
\Sigma^a_D(ZZ) = \frac{8g^4}{\cos^4\Theta_W}|\Delta^a_s(Z)|^2(R^2 - L^2) \\
\times \left[ - [(ReO''_{ki})^2 - (ImO''_{ki})^2]g^a_3 + |O''_{ki}|^2(g^a_1 - g^a_2) \\
- 2Re(O''_{ki})Im(O''_{ki})g^a_4 \right],
\]

(109)

\[
\Sigma^a_D(Z\tilde{L}_L) = \frac{4g^4}{\cos^2\Theta_W}L_iRe\left\{\Delta^a_s(Z)\left[f_{\tilde{L}_i}\bar{f}_{\tilde{L}_k}\Delta^u_{i*}(\tilde{L}_L)(-2O''_{ki}g^a_1 + O''_{ki}*g^a_3 - g^a_4) \\
+ f_{\tilde{L}_i}\bar{f}_{\tilde{L}_k}\Delta^u_{i*}(\tilde{L}_L)(2O''_{ki}g^a_2 + O''_{ki}*g^a_3 + g^a_4)\right]\right\},
\]

(110)

\[
\Sigma^a_D(\tilde{L}_L\tilde{L}_L) = 2g^4\left[|f_{\tilde{L}_i}|^2|f_{\tilde{L}_k}|^2[|\Delta^u_{i*}(\tilde{L}_L)|^2g^a_2 - |\Delta^u_{i*}(\tilde{L}_L)|^2g^a_1] \\
+ Re\{(f_{\tilde{L}_i})^2\bar{f}_{\tilde{L}_k}(\tilde{L}_L)\Delta^u_{i*}(\tilde{L}_L)(g^a_3 + g^a_4)\}\right].
\]

(111)

The contributions \(\Sigma^a_D(Z\tilde{R}_R), \Sigma^a_D(\tilde{R}_R\tilde{R})\) are obtained from Eqs. (110), (111) by applying the substitutions in Eq. (104) and in addition

\[
g^a_{1,2,3} \rightarrow -g^a_{1,2,3}, \quad g^a_4 \rightarrow g^a_4.
\]

(112)

The kinematical factors are

\[
g^a_1 = m_i(p_5 p_7)(p_6 s^a), \quad (113)
\]

\[
g^a_2 = m_i(p_5 p_6)(p_7 s^a), \quad (114)
\]

\[
g^a_3 = m_k[(p_3 p_6)(p_7 s^a) - (p_3 p_7)(p_6 s^a)] \quad (115)
\]

\[
g^a_4 = i m_k\epsilon_{\mu\nu\rho\sigma}s^{au}p_5^\mu p_5^\nu p_6^\rho p_6^\sigma. \quad (116)
\]

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