Role of lunar laser ranging in realization of terrestrial, lunar, and ephemeris reference frames

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Abstract

Three possible applications of lunar laser ranging to space geodesy are studied. First, the determination of daily Earth orientation parameters (UT0 and variation of latitude) is rarely used nowadays in the presence of all-year VLBI, SLR, and GNSS data. The second application is the determination of two (out of three) lunar orientation parameters, i.e., daily corrections to the rotational ephemeris of the Moon. It may be of importance for future lunar satellite-based navigational systems. The third application is the tie of the ephemeris frame to the ICRF. It has been studied before, though in this work it is extensively compared to another realization of the same tie, obtained by spacecraft VLBI observations; also, two different EOP series and two different models of tidal variations in geopotential are applied, with different outcomes on the tie. The EPM lunar–planetary ephemeris, along with its underlying dynamical model and software, was used to obtain the presented results. All available observations were processed, since the earliest made at the end of 1969 at the McDonald observatory till the end of July 2019 (Matera, Grasse and also Wettzell observatory which began to provide data in 2018). The results and some open questions are discussed.

Keywords Lunar laser ranging · Planetary-ICRF tie · Lunar orientation parameters · Earth orientation parameters · Lunar dynamical model

1 Introduction

Lunar laser ranging (LLR), being the most precise technique of observation of the relative motion of two solar system bodies, through 50 years of existence has produced many scientific results. Some of them are dedicated to fundamental properties of spacetime (see, e.g., Hofmann et al. 2010; Williams et al. 2012, 2014; Viswanathan et al. 2018; Hofmann and Müller 2018), while some others have provided new information about tides on the Earth (Williams and Boggs 2016) and tides and the internal structure of the Moon (Williams and Boggs 2015; Matsumoto et al. 2015; Pavlov et al. 2016). In addition to the theoretical results, LLR has allowed to build a high-precision geocentric ephemeris of the Moon, featuring both orbital and rotational motion (Folkner et al. 2014; Pitjeva and Pavlov 2017; Viswanathan et al. 2018). The building of such ephemeris is usually accompanied by determination of the positions of lunar laser ranging stations (and velocities for some of them) and the positions of the five lunar retroreflectors. The lunar ephemeris and the positions of the lunar retroreflectors thus form a most precise lunar reference frame that can be used for navigation and orbit determination in future lunar missions, as well as for improvement in the currently achieved theoretical results. Russian planned lunar lander Luna-25 will have a retroreflector panel (Vasiliev et al. 2014) on it; there are also proposals for placing next-generation single corner cubes on the surface of the Moon (Currie et al. 2011; Preston et al. 2012; Turyshev et al. 2013; Araki et al. 2016).

There are just a few LLR stations in the world. They operate independently from each other and are constrained by light and weather conditions and Moon’s visibility. Hence, the events of LLR from different stations on the same day are rare. Still, with one station operating on one day, it is possible to determine two daily (nightly) corrections to Earth rotation angles: UT0 and VOL (variation of latitude), which
are mathematically the same as daily corrections to the station’s longitude and latitude, respectively. These corrections brought a significant contribution to the determination of the terrestrial pole and Universal time in 1970–1980s, see, e.g., (Langley et al. 1981; Dickey et al. 1985) or (Charlot 1995, pp. M-17–M-20), and are still calculated and used in JPL KEOF EOP series (Ratcliff and Gross 2018).

Known EOP series that combine terrestrial and celestial poles—namely IERS C04 (Bizouard et al. 2011) and IERS Bulletin A (Dick and Thaller 2018, pp. 94–116) are constructed without using LLR data. However, the Moon has a much more stable orbit than the artificial satellites used in SLR and GNSS observations. One reason for that is the Moon experiencing smaller perturbations from nonspherical gravitational potential of the Earth than do artificial satellites at low orbits; another reason is that the solar pressure acceleration of the Moon is very small and almost non-intermitting. Also, unlike GNSS and VLBI, LLR observations do not suffer from daily clock offsets. That, together with increasing precision and frequency of LLR observations worldwide, suggests that the LLR might be helpful in the future for EOP determination.

It is well known that the LLR observations are sensitive not only to the Earth’s equator plane, but also to Earth’s orbit plane, due to the Sun affecting the orbit of the Moon (see, e.g., Williams 2018). That makes LLR different from radio ranging observations of, e.g., Mars orbiters which have an accuracy of 55 cm at best (Kuchynka et al. 2012). Laser ranging to the distance of Mars or Venus has not ever been done. The accuracy of 55 cm means sub-µas sensitivity to the ecliptic plane at the distance of orbit of Mars, but mere 18 mas sensitivity to Earth orientation at best. On the other hand, VLBI observations of quasars, widely used to determine the orientation of the Earth’s equator w.r.t. celestial frame, have quite low sensitivity to the orbit of the Earth.

The sensitivity of LLR to both ecliptic and equator, among other things, allowed to determine the value of obliquity of the ecliptic $\epsilon_{J2000} = 84381.406''$ in Chapront et al. (2002), which is still included to this day into the IAU system of astronomical constants (Luzum et al. 2011).

There have been attempts to determine the celestial pole, in the form of corrections to nutation theory, via LLR (Williams et al. 1995; Zerhouni and Capitaine 2009; Biskupek et al. 2012; Hofmann et al. 2018). The results are not as accurate as the ones obtained by VLBI. Moreover, there are two major problems with determining the celestial pole by LLR:

1. The constant corrections to celestial pole orientation, while can be formally determined from the LLR, are not separable from the orientation of the Sun–Earth–Moon system, or the whole ephemeris frame, in the celestial frame. If the ephemeris (especially Earth orbit around the Sun) is fixed in the analysis, it constrains the determined orbit of the Moon to have a certain orientation in the celestial frame.

2. Linear or periodic terms in celestial pole, while can be formally determined from LLR, are hard to separate from similar terms coming from the imperfection of the model of the lunar motion, specifically from Earth tides, lunar tides, or lunar core.

A straightforward solution to both problems is to avoid processing LLR observations alone, but always process them together with an EOP series that include the celestial pole determined from VLBI observations. Thus, the doubts about the celestial pole should be eliminated and we will be able to treat the found “corrections to celestial pole” as corrections to ephemeris frame orientation (for constant terms, item 1 above), or to the lunar theory (for linear or periodic terms, item 2 above). In reality, however, things become more difficult, which will be shown in Sect. 7.

### 2 Data

#### 2.1 LLR observations

All available LLR observations from late 1960 up to the end of July 2019 were used in the experiments. Table 1 shows the number and timespan of observations processed from each station.

Apache Point Observatory observations (Murphy et al. 2012; Murphy 2013) were downloaded from the APOLLO

| Station | Timespan | # of normal points |
|---------|----------|-------------------|
| McDonald, TX, USA | 1969–1985 | 3604 |
| Nauchny, Crimea, USSR | 1982–1984 | 25 |
| MLRS1, TX, USA | 1983–1988 | 631 |
| MLRS2, TX, USA | 1988–2013 | 3653 |
| Haleakala, HI, USA | 1984–1990 | 770 |
| Grasse, France (Ruby laser) | 1984–1986 | 1188 |
| Grasse, France (YAG laser) | 1987–2005 | 8324 |
| Grasse, France (MeO green laser) | 2009–2019 | 1930 |
| Grasse, France (infrared laser) | 2015–2019 | 4762 |
| Matera, Italy | 2003–2019 | 233 |
| Apache Point, NM, USA | 2006–2016 | 2648 |
| Wettzell, Germany | 2018–2019 | 42 |
| Total | 1969–2019 | 27810 |

2 Another EOP series that provides both terrestrial and celestial poles is JPL EOP2: https://eop2-external.jpl.nasa.gov. This relatively new product was unknown to author at the time of writing.
website. Observations for McDonald/MLRS1/MLRS2 (Shelus 1985), Haleakala (Bonsack et al. 1986), and Grasse (Ruby and YAG Samain et al. 1998 lasers) were downloaded from the Lunar Analysis Center of Paris Observatory (POLAC).4 Green and infrared (Courde et al. 2017) Grasse observations 2009–2018 were downloaded from Geoazur website;5 they also appeared on the POLAC website some time later. Grasse observations from 2019 were kindly provided by Jean-Marie Torre. Matera Laser Ranging Observations (Yagudina et al. 2018). Normal points were made from them as well as the Wettzell observations (Hugentobler et al. 2011). The Crimean observations were recently found on a shelf (Yagudina et al. 2018). Normal points were made from them by James Williams and Dale Boggs. Both raw observations and normal points are publicly available. They do not have a notable impact on the results and were included for the sake of history.

Each normal point contains a time of firing (UTC), signal delay due to range in UTC seconds, and uncertainty. Some of the given uncertainties were re-weighted before processing. In particular, the uncertainties for selected Matera observations in 2010–2012, with given values below 1 picosecond, were treated as a result of a human error in decimal exponent and scaled 1000x. Uncertainties of Apache Point observations made in 2006–2012 were scaled 2x–6x, as recommended on the APOLLO website. Given uncertainties for Grasse observations since September 1999 have not been normalized to $1/\sqrt{N-1}$, where $N$ is the number of returned photons forming the normal point; the normalization was applied before processing.

Selected groups of older (pre-2000) observations were scaled up 1.2x–1.9x to match the post-fit-weighted root mean square (WRMS). See (Pavlov et al. 2016) for details.

### 2.2 Planetary observations

VLBI observations of a spacecraft orbiting a planet when it passes near a known radio source are essential for determining a tie between ephemeris frame and ICRF (see Sect. 7). They are also known as ΔOR observations. The result of a session of such observations is the angular position of planet w.r.t. the ICRF, either as a one-dimensional projection (in case of single-baseline observations), or two-dimensional astrometrical position (in case of multiple-baseline observations on Very Long Baseline Array, VLBA). Almost all the data were taken from the webpage of solar system dynamics (SSD) group at NASA JPL, including single-baseline data obtained from Phobos-2 (Hildebrand et al. 1994), Magellan, Galileo, Venus Express (Folkner 2010), Mars Reconnaissance Orbiter (MRO), and Mars Odyssey, and also astrometric positions of Saturn obtained from Cassini (in Jones et al. (2014) there are three more observations not present at the website). Astrometric positions of Mars obtained from MRO and Odyssey observations on VLBA were taken from Park et al. (2015).

Orbits of the planets are best determined (in the ephemeris frame) from ranging data or spacecraft orbiting Mercury, Venus, Mars, Jupiter, and Saturn and also from ranging and differenced range observations of Mars landers. Also, older planetary ranging data were used for Mercury and Venus. Optical observations of natural satellites of Jupiter and Saturn from different observatories were used to help determine their planets’ orbits.

Most of the data were taken from the aforementioned SSD webpage, including ranging data obtained from MESSENER (Park et al. 2017) and Juno; ranging data obtained from Mars Global Surveyor, Mars Odyssey, and MRO (Konopliv et al. 2011); ranging and differenced range data obtained from Viking (Yoder and Standish 1997) and Pathfinder (Folkner et al. 1997), and older radar and optical observation data. Mars Express and Venus Express ranges (Morley and Budnik 2009) were downloaded from the Geoazur website.9 Radar ranging data from Crimea is available on the IAA RAS website.10

Processing of planetary ephemeris data, apart from VLBI, is not a topic of this work; we refer the reader to other papers devoted to planetary ephemeris (Folkner et al. 2014; Pitjeva 2013; Pitjeva and Pitjev 2014).

### 2.3 Earth orientation parameters

IERS C04 EOP series was used for processing of planetary observations. For the processing of LLR observations, both C04 and IERS Bulletin A weekly (“finals.all”) were used. While there are similarities between those series, they do not generally agree below 1 mas for a number of reasons:

- C04 is a “long-term” series and contains only past data with the lag of up to one month, while Bulletin A is “rapid” series, containing data up to present and a prediction. Also, Bulletin A is routinely retroactively updated upon the availability of new data. This is not the case with C04, except when it is done after a special decision.11

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3 [http://physics.ucsd.edu/~tmurphy/apollo/norm_pts.html](http://physics.ucsd.edu/~tmurphy/apollo/norm_pts.html)
4 [http://polac.obspm.fr/lrdatae.html](http://polac.obspm.fr/lrdatae.html)
5 [http://www.geoazur.fr/astrogeo/?href=observations](http://www.geoazur.fr/astrogeo/?href=observations)
6 [ftp://cddis.gsfc.nasa.gov/pub/slr/data/npt_crd](ftp://cddis.gsfc.nasa.gov/pub/slr/data/npt_crd)
7 [http://iaaras.ru/en/dept/ephemeris/observations](http://iaaras.ru/en/dept/ephemeris/observations)
8 [https://ssd.jpl.nasa.gov/?eph_data](https://ssd.jpl.nasa.gov/?eph_data)
9 [http://www.geoazur.fr/astrogeo/?href=observations/base](http://www.geoazur.fr/astrogeo/?href=observations/base)
10 [ftp://hpiers.obspm.fr/iers/eop/eopc04/updateC04.txt](ftp://hpiers.obspm.fr/iers/eop/eopc04/updateC04.txt)
11 [ftp://trystan.harvard.edu/iau/IAU14/IAU14_EOP/](ftp://trystan.harvard.edu/iau/IAU14/IAU14_EOP/)
– C04 and Bulletin A are produced by different groups with different software, and, while underlying models are the same in general, there can be subtle differences in implementation.
– Both series are calculated from a combination of VLBI, GNSS, and SLR data; details of the combination process differ between the two series (Bizouard et al. 2011; Dick and Thaller 2018).

The data were downloaded from the IERS website.¹²

3 Software

ERA (Ephemeris Research in Astronomy), version 8, was used for processing the planetary and lunar observations, refining the parameters and integrating the dynamical equations (Pavlov and Skripnichenko 2015). ERA is based on the Racket programming platform (Flatt 2010; Felleisen et al. 2018) and has SQLite¹³ as the database engine.

SOFA library (Hohenkerk 2012) was used for conversion between terrestrial and celestial coordinates and conversion between various time scales.

For numerical integration, an implementation of the Adams–Bashforth–Moulton method, modified to handle delay differential equations (Aksim and Pavlov 2019), was used. Time-delayed terms appear in the differential equations of lunar rotation due to the nature of the tidal dissipation.

4 Models

4.1 Dynamical model

A single dynamical model of the solar system (including the Moon, planets, asteroids, and Trans-Neptunian objects, TNOs), which serves as the basis for EPM planetary–lunar ephemeris, was used in this work. The planetary part of the model comprises relativistic accelerations of point masses of the Sun, planets, the Moon, asteroids, and TNOs, as well as additional accelerations from solar oblateness and Lense–Thirring effect. The orbital motion of the Earth is also affected by “point mass–figure” accelerations that come from Sun, the Moon, Venus, Mars, and Jupiter. The lunar part of the model comprises similar “point mass–figure” accelerations, as well as torques, and a degree-2 “figure-figure” torque between Earth and the Moon. The tidal variations in the gravitational potential of the Earth are taken into account, as far as the orbital motion of the Moon is concerned (Williams and Boggs 2016). Both orbital motion and rotational motion of the Moon are affected by the rotational and tidal dissipation modeled as variations in the lunar gravitational field and inertia tensor. For more detailed description, we refer to Pavlov et al. (2016) and (Pitjeva 2013; Pitjeva and Pitjev 2018) for the lunar and planetary parts, respectively.

One piece of lunar dynamical model particularly important for this work is the model of tidal variations in Earth’s gravitational potential that come from the Moon and the Sun raising periodical ocean and solid tides on the Earth. The orbit of the Moon is perturbed by those variations. There are actually two models that are used interchangeably; they were compared in Pavlov et al. (2016):

– IERS2010 geopotential variations model (Petit and Luzum 2010) with fixed coefficients and amplitudes. Only corrections up to order and degree 2 are taken into account.
– DE430 model (Williams and Boggs 2016; Folkner et al. 2014) of direct perturbing acceleration of the Moon. It comes with five time delays for three frequencies and three fixed Love numbers for those frequencies. Of the five time delays, three are fixed “orbital” delays applied when the positions of the perturbing objects (Sun and Moon) are calculated; two rotational delays (for diurnal and semidiurnal frequencies) are determined parameters.

4.2 Reductions

Usual routines were applied during the processing of astronomical observations. IAU2000/2006 precession–nutation model (Wallace and Capitaine 2006) together with IERS EOP series (Bizouard et al. 2011) was used for transformation between terrestrial and celestial reference frames. Relativistic delay of signal was calculated according to theoretical approximation (Kopeikin 1990); delays from the Sun, Earth, the Moon, Jupiter, and Saturn were added up. Calculation of tropospheric delay of laser signal was performed according to empirical models: zenith delay (Mendes and Pavlis 2004) and mapping function (Mendes et al. 2002). As for radio observations, the tropospheric delay was already subtracted from the provided data. Displacements of reference points due to solid Earth tides and pole tides were calculated according to the IERS Conventions 2010 (Petit and Luzum 2010). Ocean loading was calculated from HARPOS files obtained from the International Mass Loading Service¹⁵ (Petrov 2017). Atmospheric loading was not applied.

4.3 Lunar orientation parameters

In Pavlov et al. (2016), the following lunar-to-celestial transformation matrix is formulated:

¹² https://www.iers.org/IERS/EN/DataProducts/EarthOrientationData/eop.html.
¹³ http://sqlite.org.
¹⁴ http://www.iausofa.org.
¹⁵ http://massloading.net.
where $\phi$, $\theta$, and $\psi$ are Euler angles and are part of the state of the dynamical system (see Sect. 4.1), while $\Lambda$ is a sum of three small kinematic periodic terms unaccounted for in the dynamical model.

We treat $R_z(\phi)R_x(\theta)$ part in Eq. 1 as the lunar celestial pole, and $R_z(\psi + \Lambda)$ as the lunar counterpart of the Earth rotation angle. Also, we assume that the Moon, like the Earth, experiences stochastic variations in its rotation and in the position of instantaneous axis of rotation in the lunar mantle (lunar pole):

$$ R^*_{L2C} = R_z(\phi)R_x(\theta)R_z(\psi + \Lambda), \quad (1) $$

We call $p'$, $q'$, and $r'$ the lunar orientation parameters (LOP). $p'$ and $q'$ define the position of the lunar pole, while $r'$ is similar to Earth’s (UT1–UTC) correction.

While it is theoretically possible to further extend (2) with the corrections to the lunar celestial pole, there will not be a practical utility in it before the development of astronomical instruments that are sensitive to the position of the lunar celestial pole. There are propositions for such instruments, e.g., lunar polar optical telescope (Petrova and Hanada 2012) or lunar VLBI station (Kurdubov et al. 2019). Even after they are developed, it may happen that the present dynamical model already provides good enough lunar celestial pole and that the corrections are not needed. Presently, with only LLR data available, we restrict LOP to just $p'$, $q'$, and $r'$.

### 4.4 Celestial pole

In ephemeris–ICRF tie determination (Sect. 7), the model of ITRS–GCRS is the IAU2000/2006 precession–nutation model which is extended with two additional rotations:

$$ R_{T2C}(t) = R_x(\Delta X) R_y(\Delta Y) Q(t) R(t) W(t) $$

$$ \Delta X = \Delta X_0 + \dot{X}(t - t_0) + C^X \cos \Omega + S^X \sin \Omega $$

$$ \Delta Y = \Delta Y_0 + \dot{Y}(t - t_0) + C^Y \cos \Omega + S^Y \sin \Omega \quad (3) $$

where:

- $W(t)$ is the polar motion matrix, obtained from terrestrial pole from EOP series and adjusted for diurnal and semidiurnal variations due to zonal and ocean tides;
- $R(t)$ is the Earth rotation matrix, obtained from UT1 from EOP series and adjusted for diurnal and semidiurnal variations due to zonal and ocean tides;
- $Q(t)$ is the celestial pole matrix, adjusted by $dX$ and $dY$ corrections from EOP series;
- $t_0$ is the epoch J2000;
- $\Omega$ is the Moon ascending node, precessing with a period of approximately 18.6 years;
- $\Delta X$ and $\Delta Y$ are determined constant rotations of current ephemeris frame to the ICRF;
- $\dot{X}$ and $\dot{Y}$ are determined rotational trends of the current ephemeris frame w.r.t. the ICRF;
- $C^X, S^X, C^Y, S^Y$ are additionally determined artifacts that can appear as a 18.6-year periodic motion of celestial pole w.r.t. the ICRF.

$\dot{X}, \dot{Y}, C^X, S^X, C^Y, S^Y$ are supposed to be zero because both the ICRF frame, as realized by Eq. 3, and the ephemeris frame are assumed to be inertial. However, there are difficulties with those assumptions, see Sect. 7.

### 5 Main solution

A large set of parameters of lunar and planetary models was fitted to data (see Sect. 2) using the nonlinear weighted least-squares method. Lunar and planetary parameters were fit (determined) one after another in several iterations until both solutions converged with a joint lunar–planetary ephemeris, which then served as a basis for the results presented in further sections.

#### 5.1 Lunar part

The following parameters were fitted in the lunar solution:

- Geocentric position and velocity of the Moon at epoch;
- Euler angles of lunar physical libration ($\phi, \theta, \psi$) and their derivatives at epoch;
- Angular velocity of the lunar liquid core at epoch;
- Gravitational parameter of the Earth–Moon system;
- Ratios of undistorted lunar moments of inertia: $\beta = (C - A)/B$ and $\gamma = (B - A)/C$;
- Stokes coefficients of undistorted lunar gravitational potential: $C_{32}, S_{32}$, and $S_{33}$;
- Lunar Love number $h_2$;
- Lunar core flattening coefficient;
- Lunar tidal delay;
- Rotational delays $\tau_{1R}$ and $\tau_{2R}$ of Earth diurnal and semidiurnal tides (only if DE430 model of tidal variations in geopotential is used; otherwise, fixed IERS formula is applied);
- Amplitudes of $l'$ (365 d), $2l - 2D$ (206 d) and $2F - 2l$ (1095 d) kinematic terms;
- Selenocentric coordinates of five retroreflectors;
- Terrestrial coordinates of all LLR stations;
Table 2 Postfit statistics of lunar solution. WRMS is one-way and given in cm

| Station         | Timespan | Used | Rej. | WRMS   | DE430 tides | IERS2010 tides |
|-----------------|----------|------|------|--------|-------------|----------------|
|                 |          |      |      |        | C04        | C04 finals     |
| McDonald        | 1969–1985| 3552 | 52   | 20.2   | 20.1        | 21.1           |
| MLRS1           | 1983–1988| 588  | 43   | 11.5   | 11.0        | 12.4           |
| MLRS2           | 1988–2013| 3224 | 429  | 3.7    | 3.4         | 4.3            |
| Nauchny         | 1982–1984| 25   | 0    | 11.1   | 11.1        | 11.2           |
| Haleakala       | 1984–1990| 751  | 19   | 6.6    | 5.8         | 7.0            |
| Grasse (Ruby)   | 1984–1986| 1109 | 79   | 17.7   | 16.9        | 18.5           |
| Grasse (YAG)    | 1987–2005| 8273 | 51   | 1.7    | 1.5         | 2.0            |
| Matera          | 2003–2019| 219  | 14   | 3.2    | 3.1         | 3.2            |
| Apache          | 2006–2016| 2632 | 16   | 1.50   | 1.50        | 1.77           |
| Grasse (MeO green) | 2009–2019| 1930 | 0    | 1.61   | 1.64        | 2.01           |
| Grasse (infrared) | 2015–2019| 4761 | 1    | 1.30   | 1.25        | 1.45           |
| Wettzell        | 2018–2019| 42   | 0    | 1.06   | 1.08        | 1.27           |

For pre-1980 McDonald data, KEOF EOP series was used instead of C04 or “finals”

- Velocities of McDonald/MLRS1/MLRS2 and Grasse stations;
- 24 biases for chosen stations at chosen periods of time.

A more detailed description of the lunar parameters can be found in Pavlov et al. (2016).

The weighted root mean square postfit residuals of LLR observations are given in Table 2. Some observations with unreasonably large residuals were considered erroneous and were rejected from the solution. It is visible that the DE430 tidal model gives better results than IERS2010 (partially thanks to two adjustable parameters) that has already been noticed in Pavlov et al. (2016). IERS2010 does not have the accuracy needed to model the orbit of the Moon at the centimeter level on time intervals exceeding 20 years; without adjustable parameters, the solution distorts the lunar dissipation model to soak up the acceleration.

The other consequence from Table 2 is that the “finals” IERS EOP series give systematically better fits than the C04 series that is unexpected, given that the LLR data were not used in production of either series.

The selenocentric coordinate system, in which the coordinates of five retroreflector panels are determined, is based on the principal axes (PA) of the Moon’s figure. Table 3 lists the positions of the five retroreflectors and their formal errors for the best solution of the four (“finals” EOP and DE430 tidal model). These five points realize the most precise lunar coordinate system to date. The given formal errors, however, may be too optimistic. Table 4 shows that, while the differences between positions determined in the four obtained solutions are 15 cm or less, the differences with DE430 (Williams et al. 2013, Table 6) or INPOP17a (Viswanathan et al. 2017) positions reach 2 m.

5.2 Planetary part

In the planetary solution, the following parameters were determined:

- Three angles of orientation with respect to ICRF;
- Planetary orbital elements at epoch, including Pluto. For Earth, only the eccentricity, semimajor axis, and longitude of the periapsis were determined to avoid correlations with the three ICRF angles;
- Solar oblateness factor;
- Gravitational parameters of the Sun, some individual asteroids, and TNOs;
- Total gravitational parameters of asteroids of C, S, M taxonomic classes, asteroid belt, and Kuiper belt (apart from asteroids/TNOs whose masses were fixed to known values or determined individually);
- Rotational parameters of Mars;
- Parameters of Mercury topography;
- Locations of Martian landers Viking 1/2 and Pathfinder;
- Solar corona electron density factors (one per each solar conjunction, assuming a symmetric $1/r^2$ distribution)
- Shifts to compensate calibration errors or clock offsets on Earth or in spacecraft;
- Phase effects for optical observations of outer planets.

For details about the planetary part of EPM ephemeris, the reader is referred to Pitjeva (2013), Pitjeva and Pitjev (2014), Pitjeva and Pavlov (2017), Pitjeva and Pitjev (2018).
The whole solar system was oriented to ICRF via single-baseline and multiple-baseline ∆ADOR observations. While multiple-baseline observations are given as the direct astrometrical position (α, δ) of a planet, the single-baseline ones are given as a single observable ∆θ. That observable is a linear combination of differences (Δα, Δδ) between the astrometrical of the planet and its position in the DE405 ephemeris:

\[ ∆θ = Δα \cos γ + Δδ \cos α \sin γ, \]

where γ is the angle of VLBI baseline on plane of sky, relative to celestial equator.

After orientation to ICRF, the following residuals were obtained: Figs. 1 and 2 for Venus, Mars, and Jupiter single-baseline VLBI observables; Figs. 3 and 4 for Saturn and Mars astrometric positions obtained by VLBA observations.

The formal errors (1σ) of the three angles of rotation (around X, Y, and Z axes) of ephemeris frame to ICRF were the following: \( σ(ΔX) = 0.038 \text{ mas}, \ σ(ΔY) = 0.041 \text{ mas}, \ σ(ΔZ) = 0.024 \text{ mas}. \)

The rotational trends (\( \dot{X}, \dot{Y}, \dot{Z} \)) were once temporarily added into the set of the determined parameters and were found to be below their respective 1.5\( σ \), where \( σ(\dot{X}) = 14 \mu \text{as/year}, σ(\dot{Y}) = 15 \mu \text{as/year}, \) and \( σ(\dot{Z}) = 9 \mu \text{as/year}. \) That proves that the dynamical planetary model of the ephemeris properly accounts for all natural phenomena that can have a rotational effect on the solar system within the given error margin. Estimates of the galactic aberration are below that margin, at 5–10 \( \mu \text{as/year} \) (Kurdubov 2011; Malkin 2014).

### Table 3
Detected positions of five reference points in PA selenocentric coordinate system

| Panel | X (m)       | Y (m)       | Z (m)       | σX (cm) | σY (cm) | σZ (cm) |
|-------|-------------|-------------|-------------|---------|---------|---------|
| Apollo 11 | 1591967.619 | 690697.773 | 21004.477  | 3.1     | 2.0     | 0.8     |
| Apollo 14 | 1652689.300 | 520999.332 | 109729.671 | 3.1     | 2.1     | 0.8     |
| Apollo 15 | 1554678.256 | 98093.848  | 765006.089 | 3.0     | 1.9     | 1.7     |
| Luna 17   | 1114291.065 | 781299.633 | 1076059.333| 3.0     | 1.7     | 2.0     |
| Luna 21   | 1339364.109 | 801870.501 | 756359.405 | 3.0     | 1.9     | 1.7     |

### Table 4
For each of the five reference points: maximum pairwise distance between its positions in the four obtained solutions; maximum distance between its positions in the obtained solutions and its position in the DE430 solution; maximum distance between its positions in the four obtained solutions and its position in the INPOP17a solution

| Panel | Max ∆ btw solutions (m) | Max ∆ DE430 (m) | Max ∆ INPOP17a (m) |
|-------|-------------------------|-----------------|-------------------|
| Apollo 11 | 0.15                     | 2.1             | 2.3               |
| Apollo 14 | 0.11                     | 2.0             | 2.2               |
| Apollo 15 | 0.13                     | 1.8             | 2.2               |
| Luna 17   | 0.15                     | 1.7             | 2.2               |
| Luna 21   | 0.15                     | 1.7             | 1.9               |

![Fig. 1 Postfit residuals of single-baseline VLBI observable ∆θ for Venus orbiters Magellan and Venus Express](image)

### 6 Determination of daily corrections to rotational parameters of Earth and Moon

An LLR session running for several hours allows to determine two daily (nightly) parameters of Earth rotation: UT0 and VOL, which are in linear relation with UT1 and terrestrial pole coordinates \( x_p, y_p \) (Chapront and Francou 2009):

\[
\text{UT0} = \text{UT1} + \frac{(x_p \sin λ + y_p \cos λ) \tan φ}{15 \times 1.002737909} \\
\text{VOL} = x_p \cos λ - y_p \sin λ
\]

where \( λ \) and \( φ \) are the station’s longitude and latitude, respectively, and 1.002737909 is the relative rate of mean solar time to sidereal time. Without the LLR sessions happening in two observatories on one night (which is rare), there is no possibility to determine daily UT1, \( x_p \), and \( y_p \) from LLR alone.
Fig. 2 Postfit residuals of single-baseline VLBI observable $\Delta \theta$ for Mars orbiters MGS, Odyssey, and MRO, and Jupiter orbiter Galileo.

Fig. 3 Postfit residuals of astrometric positions of Saturn obtained on VLBA observations of Cassini spacecraft.

Fig. 4 Postfit residuals of astrometric positions of Mars obtained on VLBA observations of MRO and Odyssey spacecraft.

Fig. 5 Formal errors of UT0 and VOL determined from LLR since 2014.

Lunar laser ranging is usually performed to more than one lunar target (retroreflector). That allows to determine two lunar orientation parameters: $r'$ and $q'$. LLR is not sensitive to the third parameter, $p'$ — the angle of rotation around the X-axis, directed toward the Earth.

6.1 Setting of experiment

Once the ephemeris, obtained with C04 as the EOP series, has been fixed, a special LLR solution was obtained, containing five determined parameters for each night with ten or more LLR normal points: $\Delta$UT0, VOL, $r'$, $q'$, and range error $\Delta r$. The range error is assumed to originate from imperfections of the troposphere model and the lunar orbital ephemeris and correlates with $\Delta$UT0 and VOL. The inclusion of the range error into the set allows to obtain realistic error estimates for the EOP parameters.

6.2 Results

The results are shown for the timespan of January 2014 – July 2019 for three instruments at two observatories. There were 340 sessions altogether with ten or more normal points. There has not been such an LLR session at Wettzell observatory in that timespan. On the plots, only the results with formal errors ($1\sigma$) below 6 mas are shown. Figure 5 shows the formal errors of determined UT0 and VOL.
There are 173 nights with $\sigma (1\text{UT0})$ below 1.5 mas (0.1 ms) and 50 nights with $\sigma (\text{VOL})$ below 1.5 mas. Almost all determined corrections are within their $3\sigma$ range. The outliers—four UT0 corrections w.r.t. C04 series and one w.r.t. “finals”—are shown in Fig. 6. For C04, their values and formal errors are: $-394 \pm 107 \mu\text{s}$ (Sep 6, 2017), $75 \pm 18 \mu\text{s}$ (Sep 4, 2018), $78 \pm 15 \mu\text{s}$ (Feb 20, 2019), $81 \pm 16 \mu\text{s}$ (Sep 4, 2019), and $-55 \pm 15 \mu\text{s}$ (Jul 25, 2019). The only outlier for “finals” is $-375 \pm 100 \mu\text{s}$ (Sep 6, 2017).

The absence of the other three outliers in the UT0 corrections w.r.t. “finals” further supports the inference that the “finals” series represent the actual rotation of the Earth slightly better than C04 at present and that the LLR can sense it.

The single outlier common to both series can indicate either an Earth rotation event not determined by daily VLBI observations, or an artifact in Grasse data on that day. In any case, LLR will probably have a non-negligible effect on modern IERS EOP solutions, and even more so when other LLR observatories, like Wettzell, begin to provide frequent infrared data.

Figure 7 shows the formal errors of $r'$ and $q'$ for nights where those errors are below 6 mas. Data in Figs. 5 and 7 come not from separate solutions but from single solution, where the range errors were also determined. The range errors are not shown. All determined $p'$ and $q'$ are within their $3\sigma$ range.

### 7 Determination of orientation of ephemeris frame in the ICRF from LLR data

In planetary ephemerides, orbits of planets, including Earth, are determined in a so-called barycentric celestial reference frame (BCRF Kopeikin et al. 2011), also known as ephemeris frame. Like ICRF, it is defined to have its origin at the barycenter of the solar system. It is also defined to be an inertial frame. It is important to tie dynamical BCRF to the kinematic ICRF as precisely as possible, for three applications:

- Modeling the orbits of interplanetary spacecraft;
- Studying the influence of the Moon, planets, and the Sun to the rotation of the Earth;
- X-axis rotation at J2000 and its rate, “obliquity rate”
- Y-axis rotation at J2000 and its rate, “luni-solar precession”

Probably, the earliest tie between the ephemeris and radio frame based on quasars was made in 1986 (Newhall et al. 1986), long before the official adoption of ICRF in 1998. The tie was based on ADOR observations of Viking and Pioneer spacecraft; the accuracy was about 20 mas. Later, the accuracy of the tie was improved to about 5 mas (Finger and Folkner 1992) and then to 3 mas (Folkner et al. 1994) by using a different approach: processing LLR together with quasar VLBI observations. However, in subsequent years, the abundance of a higher quality ADOR observations of spacecraft, including those of Magellan (since 1990), quickly led back to the decision to use just ADOR for the direct ephemeris-ICRF tie (Standish et al. 1995). In DE405 ephemeris (Standish 1998), a 1 mas accuracy of the tie was reported. The similar result was obtained in EPM2002 ephemeris (Pitjeva 2003).

The modern ephemerides DE430 (Folkner and Border 2012; Folkner et al. 2014) and EPM2017 (Pitjeva 2017; Pitjeva and Pavlov 2017) have their tie to ICRF based on modern spacecraft VLBI measurements, with accuracy of about 0.2 mas. As for INPOP ephemerides, the 0.5 mas accuracy of the tie was reported for an older version INPOP13 (Fienga et al. 2013). The LLR data are not used for that tie; however, the lunar solution in DE430 (Folkner et al. 2014; Williams et al. 2013) contains four corrections to IAU1980 nutation model, namely:...
In DE430, those corrections are not used outside the lunar solution, and particularly, are not used for ephemeris-ICRF frame tie determination.

Similar four corrections (but to IAU2000 nutation model rather than to IAU1980) were obtained in Hofmann et al. (2018). The corrections were treated as the “tie between the dynamical ephemeris frame to the kinematic celestial frame”; no spacecraft VLBI data were considered.

In this work, the two methods of obtaining BCRF–ICRF tie are applied and their results are compared. Figure 8 shows the (seemingly) redundant scheme to tie planetary orbits to ICRF. The direct “orbits–ICRF” connector is what is normally used in ephemerides. At the same time, the orbits of planets are tied to the ecliptic in the ephemeris, via spacecraft ranging (most precise technique), and also differenced LLR observations are of no help for the BCRF–ICRF tie. However, the obtained result is interesting by itself because it indicates deeper problems with the assumptions that were made on Fig. 8.

One problem is the EOP series. Table 5 and Fig. 9 clearly show that the tie between the Earth–Moon system and the ICRF suffers from the absence of sub-mas agreement in the celestial pole trend, let alone its position, between C04 and “finals” series. The disagreement in celestial pole series has been studied before (Malkin 2012, 2014, 2017). For instance, (Malkin 2012, Table 3) shows \( dX \) slope for C04 and “finals” series at 15.1 ± 1.8 μas/year and 11.1 ± 1.6 μas/year, respectively. So the difference is outside the error margin. For \( dY \) slope, the values are 59.6 ± 1.8 and 50.9 ± 1.7 μas/year. The difference between those slopes is not as big as with \( dX \) slopes, but there is another problem: The values are bigger than any \( dY \) slope obtained from the single VLBI series constituting the combined series.

The second problem is the lunar model. Different model values of Earth’s \( J_2 \) and different models of tidal variations in that value can cause different rotational behavior of the Earth–Moon system. The difference between DE430 model and IERS2010 model is clearly visible in Fig. 9 and Table 5: For instance, \( \dot{Y} \) has different signs depending on the model; also, 18.6-year amplitudes are always strongly detected with the IERS2010 model, while some of them are not detected with the DE430 model.

The assumptions and comments on their validity are summarized in Table 6. With the present state of lunar model and with the present state of celestial pole EOP series, one cannot rely on either of those things to verify the other. Further development of methods for EOP solutions and further development of Earth–Moon dynamical model are needed.

7.1 Setting of experiment

Planetary solution (see Sect. 5) was obtained and its frame was tied to the ICRF using spacecraft VLBI observations. Then, the lunar solution was re-obtained, with eight more parameters from (3): \( \Delta X, \Delta Y, \dot{X}, \dot{Y}, C^X, S^X, C^Y, S^Y \). (The \( \Delta Z \) rotation cannot be determined as long as the positions of the stations are determined in the solution.) These parameters, per se, have no relation to planets; we call them the tie between the dynamical Earth–Moon system and the ICRF. The procedure is repeated four times: for C04 and “finals” EOP series and for DE430 and IERS models of tidal variations in geopotential.

7.2 Results and discussion

Table 5 shows the determined rotations, their trends, and 18.6-year amplitudes. It is visible that the values vary greatly across the four solutions. In particular, the strong \( \dot{X} \) trend is detected in all solutions but one. The strong \( \dot{Y} \) trend is detected in two solutions. There is not one solution where both trends are low. That fact alone makes the determined constant rotations irrelevant; the trends must be explained first. The same applies to the determined amplitudes. It is visible that they are all big with the IERS tidal model; also, neither of the two solutions based on the DE430 tidal model has low values of all the four amplitudes.

To further study the temporal behavior of the ICRF tie, four other lunar solutions were obtained—without the determined trends or amplitudes, but rather with seven pairs of \( (\Delta X, \Delta Y) \), each affecting a 7-year timespan of LLR observations. In total, the “piecewise tie” to ICRF was calculated for the timespan of 49 years (from 1 Aug 1970 to 1 Aug 2019).

Figure 9 shows the piecewise ICRF tie \( \Delta X \) and \( \Delta Y \) corrections. The baseline of the plots corresponds to the lunar–planetary ephemeris oriented to ICRF via \( \Delta \text{DOR} \) observations (see Figs. 1, 2, 3, and 4). It is clear that there are systematic differences between the ties depending on the used tidal model, EOP series, and time. Until those issues are resolved, in the requirement of sub-mas accuracy, the LLR observations are of no help for the BCRF–ICRF tie. However, the obtained result is interesting by itself because it indicates deeper problems with the assumptions that were made on Fig. 8.

One problem is the EOP series. Table 5 and Fig. 9 clearly show that the tie between the Earth–Moon system and the ICRF suffers from the absence of sub-mas agreement in the celestial pole trend, let alone its position, between C04 and “finals” series. The disagreement in celestial pole series has been studied before (Malkin 2012, 2014, 2017). For instance, (Malkin 2012, Table 3) shows \( dX \) slope for C04 and “finals” series at 15.1 ± 1.8 μas/year and 11.1 ± 1.6 μas/year, respectively. So the difference is outside the error margin. For \( dY \) slope, the values are 59.6 ± 1.8 and 50.9 ± 1.7 μas/year. The difference between those slopes is not as big as with \( dX \) slopes, but there is another problem: The values are bigger than any \( dY \) slope obtained from the single VLBI series constituting the combined series.

The second problem is the lunar model. Different model values of Earth’s \( J_2 \) and different models of tidal variations in that value can cause different rotational behavior of the Earth–Moon system. The difference between DE430 model and IERS2010 model is clearly visible in Fig. 9 and Table 5: For instance, \( \dot{Y} \) has different signs depending on the model; also, 18.6-year amplitudes are always strongly detected with the IERS2010 model, while some of them are not detected with the DE430 model.

The assumptions and comments on their validity are summarized in Table 6. With the present state of lunar model and with the present state of celestial pole EOP series, one cannot rely on either of those things to verify the other. Further development of methods for EOP solutions and further development of Earth–Moon dynamical model are needed.
Table 5  Ephemeris-ICRF
rotation angles, their rates, and 18.6-year amplitudes, determined from LLR

|                  | DE430 tides | IERS2010 tides |
|------------------|-------------|----------------|
|                  | C04         | Finals         | C04         | Finals         |
| \( \Delta X_0 \), mas | 0.375 \( \pm 0.049 \) | \(-0.050 \pm 0.046 \) | 0.221 \( \pm 0.055 \) | \(-0.184 \pm 0.052 \) |
| \( \Delta Y_0 \), mas | 0.011 \( \pm 0.026 \) | \(0.052 \pm 0.024 \) | 0.287 \( \pm 0.029 \) | \(0.279 \pm 0.027 \) |
| \( \dot{X} \), \( \mu\text{as/} \text{year} \) | \(-30.1 \pm 4.1 \) | \(13.5 \pm 3.8 \) | \(-14.1 \pm 4.6 \) | \(2.1 \pm 4.3 \) |
| \( \dot{Y} \), \( \mu\text{as/} \text{year} \) | 10.9 \( \pm 2.2 \) | 12.4 \( \pm 2.0 \) | \(-14.4 \pm 2.9 \) | \(-14.1 \pm 2.3 \) |
| \( \Delta \dot{X} \), mas | \(-0.026 \pm 0.039 \) | \(-0.291 \pm 0.037 \) | \(-0.620 \pm 0.042 \) | \(-0.663 \pm 0.040 \) |
| \( \Delta \dot{Y} \), mas | \(-0.228 \pm 0.034 \) | \(-0.260 \pm 0.031 \) | 0.465 \( \pm 0.037 \) | 0.457 \( \pm 0.035 \) |
| \( \Delta \ddot{X} \), mas | 0.102 \( \pm 0.026 \) | 0.018 \( \pm 0.025 \) | 0.505 \( \pm 0.030 \) | 0.414 \( \pm 0.028 \) |
| \( \Delta \ddot{Y} \), mas | \(-0.033 \pm 0.036 \) | 0.026 \( \pm 0.033 \) | 0.329 \( \pm 0.039 \) | 0.388 \( \pm 0.036 \) |

The four columns of numbers are the combinations of two geopotential variations models (DE430 and IERS2010) and two IERS EOP series (C04 and “finals”). The given errors are 1\( \sigma \)

Fig. 9  Piecewise ties for dynamical Earth–Moon system to ICRF. \( \Delta X \) and \( \Delta Y \) corrections for different combinations of EOP series (C04/finals) and geopotential variations model (DE430/IERS2010) are shown. Each point represents a 7-year timespan (starting and ending on 1 May); IERS tides points have been artificially moved to the right for clarity.

Table 6  Assumptions about models and frames, and comments on validity on those assumptions on sub-mas level

| Assumption                                                                 | Validity                                                                 |
|---------------------------------------------------------------------------|--------------------------------------------------------------------------|
| ICRF forms an inertial frame                                              | True (apart from the galactic aberration which is estimated at 5–10 \( \mu\text{as/year} \)) |
| Dynamical frame of planets is inertial                                    | True (mathematically)                                                    |
| Dynamical model of planets accounts for all natural phenomena that can have a rotational effect on the solar system detectable by present observations | Probably true. One problem in that regard may come from inaccurate value of Sun’s \( J_2 \); however, it is well determined from MESSENGER observations. Also, the planetary solution does not detect rotation from spacecraft VLBI observations at the level of few tens \( \mu\text{as/year} \) |
| Celestial pole coordinates are known from VLBI observations               | False, as shown by Malkin (2012, 2014, 2017)                              |
| Dynamical frame of the Earth–Moon system is inertal                      | Unknown, because of the involved non-dynamical models of Earth gravitational potential and Earth rotation with EOP |
| Dynamical model of the Earth–Moon system accounts for all natural phenomena that can have a rotational effect on the real Earth–Moon system detectable by present observations | Probably false, since different models of geopotential already produce different rotational effects, as shown in Table 5 and Fig. 9 |
whichever happens first, will help to solve problems with the other.

**Conclusion**

- OCA observatory (former CERGA) in Grasse, France, continues to provide large amounts of LLR green and infrared data. Matera observatory in Italy provides infrequent LLR data. Wettzell observatory started to provide LLR infrared data of a very good quality.
- LLR was useful for building EOP series in the past. Modern LLR, too, is able to detect inaccuracies in modern EOP series, with sub-mas accuracy. Probably, the LLR data can benefit combined EOP solutions; currently, it is used in KEOF, which does not include celestial pole.
- LLR is capable of detecting two out of three lunar orientation parameters (LOPs) with accuracy of few mas. However, on the present data, no statistically significant daily deviations of LOPs from the lunar rotational model were detected. The lunar model research should continue, though, in the area of long-term variations.
- The tie between ephemeris frame and ICRF, calculated from spacecraft VLBI (ΔDOR) data, is confirmed with the latest data with the accuracy of 0.18 mas (3σ).
- IERS Bulleting A weekly EOP series produce generally better fits of LLR solutions than IERS C04.
- LLR is potentially capable of tying the Earth–Moon system to ICRF (and hence, the whole ephemeris frame to ICRF) with accuracy comparable to that of ΔDOR-based tie; however, one obstacle is the location of the celestial pole in EOP series: It is not accurate enough to use the equator–ICRF link to tie the ecliptic to ICRF via LLR observations and their ecliptic–equator link.
- More research is needed in the area of Earth–Moon dynamical system, and particularly, in the model of geopotential which affects the lunar orbital motion. Two available models (IERS2010 and DE430) produce different rotational rates of the Earth–Moon system in the celestial frame, which is detectable by LLR observations. The research of the Earth–Moon dynamics will be facilitated by an improvement in present celestial pole series (or vice versa).

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**Author contributions** D. P. designed and performed the research and wrote the manuscript.

**Data availability** All the observational data used in this work come from publicly available sources listed in Sect. 2. The EOP series are publicly available at the IERS website http://iers.org.

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