Online fatigue damage evaluation method based on real-time cycle counting under multiaxial variable amplitude loading

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Abstract. To realize on-line multiaxial fatigue damage evaluation for the servicing mechanical components, a real-time multiaxial cycle counting method is proposed, meanwhile, an online multiaxial fatigue damage evaluation method based strain-critical plane model is given and validated by the fatigue experimental data of En15r steel. Investigation showed that the proposed on-line multiaxial fatigue damage evaluation method could provide satisfactory evaluation results, and the real-time performance of the proposed cycle counting method is path-dependent.

1. Introduction

Most of the engineering components in service are subjected to multiaxial cycle loading, and multiaxial fatigue damage evaluation, especially for online fatigue damage evaluation under multiaxial variable amplitude loading, still is a challenging problem despite numerous of investigations have been conducted in the past several decades\cite{1-4}. In order to realize on-line multiaxial fatigue damage evaluation, a real-time multiaxial cycle counting method is necessary. For multiaxial cycle counting, some methods have been proposed by Bannantine and Socie, Wang and Brown, Meggiolaro, Wei, Anes and so on\cite{5-9}. However, all of this method cannot used in real-time. Chen and Shang\cite{10} provided an online multiaxial fatigue damage evaluation algorithm based on a new real-time multiaxial cycle counting method. Nevertheless, for most of the multiaxial random loading paths, the evaluate result is quite poor.

The object of this paper is to propose a real-time multiaxial cycle counting method and give a satisfactory and acceptable online multiaxial fatigue damage evaluation method. A set of fatigue experimental data for En15r steel under multiaxial variable amplitude loading is employ to verify the proposed method.

2. The proposed method

2.1 The proposed real-time cycle counting method
To realize on-line multiaxial fatigue damage evaluation, a real-time cycle counting method is proposed for multiaxial variable amplitude loading. The proposed method can be described as follows:

(1) Receive data in real-time by the data acquisition system with given frequency;
(2) With respect to the first data point, calculate relative equivalent strain of each received data point in time by Eq. (1), then, a relative equivalent strain time-history for all the data points having been received is obtained in real-time;
\[ \varepsilon_{eq,i}^R = \sqrt{(\varepsilon_i - \varepsilon_i')^2 + \frac{1}{3}(\gamma_i' - \gamma_i')^2} \] (1)
where \( i (i = 1, 2, 3...n) \) represents the i-th point of the time history;
(3) Seek out the first peak point \( \varepsilon_{eq,k}^{R*} \) \((k = 1)\) of the relative equivalent strain time-history using the following equation and memory its value timely, meanwhile, set corresponding data point of the multiaxial loading history as the reference point \( T_{ref} \) to determine the time-history interval will be intercepted;
\[ \varepsilon_{eq,k}^{R*} = \varepsilon_{eq,k}^R = \begin{cases} \varepsilon_{eq,j}^R > \varepsilon_{eq,j+1}^R \\ \varepsilon_{eq,j}^R > \varepsilon_{eq,j+1}^R \end{cases} \] (2)
where \( \varepsilon_{eq,k}^{R*} \) is the k-th \((k = 1, 2, 3...n)\) peak value.
(4) Continue to receive data to repeat step (2), and determine and memory the peak point by Eq. (2) in time;
(5) Once another peak point \( \varepsilon_{eq,k+n}^{R*} \) \((n = 1, 2, 3...)\) of the relative equivalent strain time-history is checked, and the condition that \( \varepsilon_{eq,k+n}^{R*} \geq \varepsilon_{eq,k}^{R*} \) is met, then, a time-history interval, \([T1,T1']\) as shown in Fig. 1 can be gained in rain-flow counting method. Otherwise, repeat step (4)-(5);
(6) For the time-history interval intercepted: a) Take the first point of intercepted interval as the reference point \((\varepsilon_i', \gamma_i')\); b) Calculate the relative equivalent strain of each points by the Eq.(1); c) Seek out every peak value of the relative equivalent strain history, then, a counted reversal can be obtained by combining the sequence of data points where \( \varepsilon_{eq,j}^R \) is increasing and reaches the maximum peak value. Concurrently, one or more fragmentations of the loading strain history may occur whenever \( \varepsilon_{eq,k}^{R*} \) starts to decrease; d) Reset the reference point \((\varepsilon_i', \gamma_i')\) as the first point of each fragmented data block; e) Repeat step (b) to (d) for each block, then, all the reversals can be counted within the intercepted interval.
(7) Reset the reference point \( T_{ref} \) as the corresponding data point of the multiaxial loading history where maximum peak value \( \max(\varepsilon_{eq,k+n}^{R*}) \) is located presently;
(8) Repeat step (4)-(7) until the last received data point is processed. Subsequently, the largest reversal can be counted by combining the rest sequence of data points where \( \varepsilon_{eq,i}^R \) is increasing and reaches the maximum peak value \( \varepsilon_{max}' \) \( \max(\varepsilon_{eq,k+n}^{R*}) \) for the multiaxial loading history;
(9) The fragmented loading time-history interval, which is intercepted from the data point where the maximum peak value \( \varepsilon_{max}' \) begins to decrease to the end of the multiaxial loading history, is also treated as step (6).
Fig. 1 Illustration of the intercepted time-history interval in real-time.

2.2 Multiaxial fatigue damage calculating method

As illustration in Fig. 2, the stress and strain of thin-walled tube specimen on the plane angled $\theta$ with the axial direction under axial-torsion loading can be expressed as

$$\varepsilon_\theta = \frac{1 - \nu}{2} \varepsilon_x + \frac{1 + \nu}{2} \varepsilon_x \cos(2\theta) + \frac{\gamma_{xy}}{2} \sin(2\theta)$$

(3)

$$\gamma_\theta = (1 + \nu)\varepsilon_x \sin(2\theta) - \gamma_{xy} \cos(2\theta)$$

(4)

Fig. 2 Stress-strain states analysis for thin-walled tube specimen under axial-torsional loading

The Shang-Wang’s[11] fatigue damage model given as Eqs. (6) is employed in the proposed online damage evaluation method, parameters are defined on the critical plane which is regarded as the plane experiencing the maximum shear strain range. Calculation of fatigue damage for each counted reversal are processed as the following procedure in real-time:

1. Once a reversal is counted, calculate the shear strain for each data point of an angle from 0 to 90 degrees by Eqs. (4), and the step length is considered as 1 degree;
2. Obtain the shear strain range of each angle, meanwhile, seek out the maximum shear strain range $\Delta \gamma_{\max}$ and the angle $\theta$ and $(90^\circ + \theta)$ corresponding to the maximum shear strain range plane, respectively;
3. The normal strain amplitude ranges, $\Delta \varepsilon_\theta$ and $\Delta \varepsilon_{\theta = 90^\circ + \theta}$ corresponding to $\theta$ and $(90^\circ + \theta)$ plane, are computed, respectively. And then, determining the orientation of the critical plane as $\theta_{cr}$
where the larger normal strain range is located;

(4) The two adjacent turning points of the maximum shear strain range, $\Delta \gamma_{\text{max}}$ on the critical plane are found and flagged as $t_{\text{min}}$ and $t_{\text{max}}$, respectively. By Eqs. (3), the normal strain excursion $\varepsilon_n^*$ between two turning points on the critical plane, $\Theta_{cr}$ can be given as

$$\varepsilon_n^* = \max(\varepsilon_{\gamma^*} (1))_{\text{max}} - \min(\varepsilon_{\gamma^*} (1))_{\text{max}}$$

(5)

(5) Calculate the fatigue damage of this counted reversal by Eqs. (6) and (7), concurrently, update the value of total fatigue damage in real-time by Eqs. (8)

$$\sqrt{\varepsilon_n^*} + \frac{1}{3} \left( \frac{\Delta \gamma_{\text{max}}}{2} \right)^2 = \frac{\sigma'}{E} \left(2N_{f,j}\right)^b + \varepsilon_{\gamma^*}' \left(2N_{f,j}\right)^c$$

(6)

$$D_j = \frac{1}{2N_{f,j}}$$

(7)

$$D_{\text{total}} = D_{\text{total}} + D_j$$

(8)

3. Experimental validation and discussion

Fatigue experimental data for En15R steel under multiaxial variable amplitude loadings were employed to validate the capability of the proposed method. The fatigue tests were conducted by M.W. Brown et al and were reported in Ref. [12], altogether eight multiaxial variable amplitude loading paths for twenty tubular specimens controlled by strain history in blocks were used to validate the Wang-Brown method and the proposed method. As shown in Fig. 3, satisfactory predictions can be obtained for both the two method with an error factor of 2.

Analysis shows that the proposed method can be successfully applied in multiaxial variable amplitude loading in real-time. For the application of the proposed method, the real-time performance of the proposed method is path-dependent. As illustrated in Fig.4(a), the multiaxial variable amplitude loading path may have the highest real-time performance because the location of the maximum relative equivalent strain with respect to the beginning point is same to the last peak value of the relative equivalent strain history. Contrariwise, as shown in Fig.4(b), when the maximum relative equivalent strain happens in the location where the first peak value of the relative equivalent strain history is located, the real-time performance of the proposed method is poorest.

![Fig. 3 Comparison between experimental and predicted lives for the proposed method and the WB method.](image-url)
Fig. 4 Real-time performance analysis illustrations of the proposed cycle counting method

4. Conclusions
The real-time performance of the proposed cycle counting method is path-dependent, and the proposed on-line multiaxial fatigue damage evaluation method seems to be a more convenient and better choice for the development of the on-line multiaxial fatigue damage monitoring system.

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6. References
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