Towards a Tractable Delay Analysis in Large Wireless Networks

Yi Zhong, Member, IEEE, Martin Haenggi, Fellow, IEEE,
Fu-Chun Zheng, Senior Member, IEEE, Wenyi Zhang, Senior Member, IEEE,
Tony Q.S. Quek, Senior Member, IEEE, Weili Nie

Abstract

Meeting the diverse delay requirements of next-generation wireless communication networks is one of the most critical goals of wireless system design. Though the delay of point-to-point communications has been well investigated using classical queueing theory, the delay of multi-point to multi-point communications has not been explored in depth. The main technical difficulty lies in the interacting queues problem, in which the service rate is coupled with the statuses of other queues. In this article, we elaborate on the main challenges of delay analysis in large wireless networks. Several promising approaches to bypass these difficulties are proposed and summarized to provide useful guidance.
NEED FOR NEW DELAY ANALYSIS

The emergence of new latency-critical applications, such as intelligent manufacturing, remote control, auxiliary driving and automatic driving, has led to a variety of delay requirements in wireless networks [1]. Specifically, the end-to-end delay for 5G is required to be less than 10ms (about 1/10 of the delay requirement for 4G), and for some special applications such as the tactile Internet [2], the delay is required to be less than 1ms. Meeting these delay requirements has therefore become one of the most important goals for next-generation wireless networks.

A theoretic analysis of the delay in wireless networks is imperative to guide the practice. However, such delay is difficult to calculate since it is a complicated function of all links in the network and is affected by a variety of factors such as network load, medium access protocol, path loss, and so on. In general, these factors can be classified into three aspects:

- Random arrival and queueing of packets at the terminals;
- Spatial models for transmitters and receivers that affect the path loss;
- Channel fluctuations and transmission mechanisms that affect the delivery of packets.

Existing works of network analysis typically focus on one or at most two of these aspects: approaches based on queueing theory mainly focus on the random arrival of packets but ignore the physical layer such as the effect of noise and interference [3], which are often used to analyze the performance of scheduling algorithms; approaches based on stochastic geometry focus on the networking but ignore the random arrival and queueing of packets [4], which are usually used to evaluate the system metrics like coverage and spectrum efficiency other than the delay; approaches based on multiuser information theory usually focus on the capacity analysis [5]. In order to accurately characterize the delay in wireless networks, all aspects should be considered, thereby calling for a combination of queueing theory, stochastic geometry, and multiuser information theory, and long known to be notoriously difficult to cope with.

This article first reviews the treatment of delay in classical queueing theory and describes the interacting queues problem. Then, the fundamental challenges that make the delay analysis difficult in large scale wireless networks are discussed. In order to handle these difficulties, several promising approaches are proposed and evaluated.
**End-to-end Delay**

Delay in this article refers to the end-to-end delay which is the time between a packet being generated at the source and being successfully decoded at the destination. Note that the end-to-end delay here is the delay in the wireless access network, while the delay in core wired networks is beyond the scope of this article. Generally, the end-to-end delay consists of four types of delays, which are categorized according to different reasons that cause the delay. These are the processing delay, the queueing delay, the transmission delay, and the propagation delay. The processing delay is the delay caused by generating packets and checking for bit-level errors, which is typically on the order of microseconds or less. The queueing delay is the time that a packet waits in a queue until it can be transmitted. The transmission delay is the time required to push all the packet's bits into the link, satisfying a certain rate requirement. When a retransmission mechanism is applied, the delay caused by retransmission can also be included in the transmission delay. The propagation delay is the time taken for a packet to reach its destination, which is calculated by dividing the transmission distance by the speed of electromagnetic waves in the air. In the multihop case, these four delay elements apply to each link. In most wireless networks, the processing delay and the propagation delay are negligible compared to the queueing delay and the transmission delay, which are the focus of this article.

**Classical Queueing Theory**

Queueing problems in classical queueing theory are usually described by Kendall’s notation in the form of A/S/C, where A represents the time intervals between arrivals to the queue, S describes the service times of the packets and C denotes the number of servers at the queueing node. The delay in classical queueing scenarios where there is only one server or where the service rates of different queues are independent is well studied. For example, in the M/M/1 queueing problem (see Figure 1(a)) where ‘M’ denotes ‘Markovian’, there is a single server, the arrival process of the packets is a Poisson process with arrival rate $\lambda$, the service time for each packet has an exponential distribution with mean $\mu$, and the mean delay is $1/(\mu - \lambda)$. However, the emergence of a large number of queues and servers in wireless networks substantially increases the difficulty of the queueing problem; moreover, the coupling of the service rates of different queues leads to the interacting queues problem.
Interacting Queues Problem

A typical interacting queues problem can be described as follows. Consider a discrete-time slotted ALOHA system with $N$ terminals. Each terminal maintains a buffer of infinite capacity to store the incoming packets. The time is divided into discrete slots with equal duration, and in each time slot, each terminal attempts to transmit its head-of-line packet with a certain probability if its buffer is not empty. A collision occurs if two or more terminals transmit simultaneously. When a collision occurs, all terminals involved in the collision retransmit the packet in the next time slot with the same access probability while other terminals are kept silent. The essential difficulty of the interacting queues problem lies in that the service rate of each queue depends on the statuses of all queues. Figure 1(b) shows an example of the interacting queues problem when there are only two terminals. If one queue is empty, the corresponding transmitter do not cause interference to the other link; thus the service rate of the other link increases, and its queue will become empty quickly.

The delay in an interacting queues system is difficult to analyze, and existing work has only explored the stability issue, i.e., whether the queues in the system will grow without bounds. The stability region of such system is defined as the range of arrival rates that guarantee the stability of all queues. For the interacting queues system described above, the exact stability region is known only for two and three terminals [6]. When $N > 3$, only sufficient conditions and necessary conditions for stability were obtained. In practical wireless networks, however, the interference between transmissions cannot be accurately modeled just as collisions, and thus the problem is more intricate than the aforementioned discrete-time ALOHA system.

**Challenges of Delay Analysis in Wireless Networks**

The delay in wireless networks is strongly affected by the queueing process and the service process of the packets. The queueing process with multiple queues is different from the classical queueing problems due to the interacting queues, while the service process is directly determined by the medium access mechanisms and the received signal-to-interference-plus-noise ratio (SINR). We list several critical challenges of delay analysis in the following.
**Challenge 1: Randomness in Spatial Deployment**

Delay is directly related to the SINR, which is significantly affected by the inter-node distances. The distance between the desired transmitter and the desired receiver determines the received power of the desired signal. Moreover, in practice, most wireless networks, such as cellular networks, are interference-limited, i.e., noise is negligible compared with interference, making interference a main factor that affects the SINR and, in turn, the delay. Notice that the sum interference power depends on the distances between the interfering transmitters and the desired receiver. All these link distances are a function of the spatial pattern of nodes in the network.

The spatial structure of wireless networks is by no means regular. The irregularity exists even in the meticulously deployed macro base stations, due to the restrictions of locations of sites, the irregular spatial distribution of traffic. For heterogeneous cellular networks with small cells like picocells and femtocells, the irregularity is more evident since the deployment of these small cells is less elaborately planned. The spatial irregularity in ad hoc networks is even more prominent than in cellular networks since the nodes in ad hoc networks are more likely to be randomly deployed. This kind of spatial irregularity is termed *deployment randomness*.

Traditionally, the spatial distribution of nodes in cellular wireless networks is modeled by regular grids. For example, the hexagonal grid is used to characterize the cells generated by macro base stations. However, the regular grid does not capture the deployment randomness. Fortunately, a powerful mathematical tool, point process theory, is available to handle the spatial modeling of the deployment randomness [7]. Point process theory represents each node as a point in a spatial point process and permits the analytical characterization of a number of network metrics, such as coverage probability, mean achievable rate, and area spectral efficiency.

**Challenge 2: Quasi-static Deployment**

While point process theory has been widely used to model the topology of wireless networks, the issue of delay has received considerably less attention. A main reason is that the delay is a long-term metric while the coverage probability, the mean achievable rate, etc., are obtained by considering just a snapshot of the network. In order to discuss long-term metrics, the static nature of the networks, i.e., the fact that the locations of nodes remain unchanged during a relatively long time once they are deployed, needs to be considered. Most practical networks are approximately static because the locations of nodes do not drastically change within a short time period. From
the perspective of receivers, the locations of interferers, determined in the deployment stage, are uncertain and may be considered as random. Therefore, the static but random locations can be considered as the common randomness over different time slots, resulting in temporal interference correlation. Intuitively, a direct effect of this type of interference correlation is that if a transmission fails in one time slot, there is an increased probability that the retransmission will also fail in a following time slot. Compared with high-mobility networks where the nodes are regenerated independently in each time slot, a static network is more challenging to analyze since inherent correlations of interference and signal levels persist across different time slots [8].

Challenge 3: Dynamics in Channel and Medium Access

Channel Fluctuations – The delay depends on the SINR, which is strongly coupled with the quality of the wireless channels. The gain of a wireless channel is determined by path loss and fading. The path loss is influenced by many factors such as terrain contours, propagation medium (dry or moist air), link distance, and height of antennas. The channel fading, categorized as slow fading and fast fading, varies with time, geographical location and propagation environment, and is often modeled as a stochastic process. Channel fluctuations may result in a loss of signal power and cause poor delay performance. Accurately modeling the effect of channel fluctuations on the delay is again difficult since a large number of links coexist, and each of these links experiences independent or dependent propagation path.

Medium Access Mechanism – The medium access mechanism determines how resources (time, space, bandwidth) in a network are allocated to the links. The effect of medium access mechanism on delay is mainly concentrated on two aspects. Firstly, it has a significant impact on the SINR at the receiver. Due to the medium access mechanism, which determines whether a carrier at a transmitter is allocated for transmission at a certain moment, all the transmitters are divided into two sets: the set containing all transmitters using that carrier, and the set containing all other transmitters. Only the set of transmitters using the same carrier at the same moment cause interference. Secondly, as part of the medium access mechanism, different scheduling policies, such as first in first out (FIFO), round-robin, and proportional fair, lead to different delay performance. Though the delay of various scheduling policies in classical queueing theory is well studied, it becomes complicated to analyze in large wireless networks where the scheduling occurs within a large number of queueing nodes, usually in a distributed fashion.
Challenge 4: Interaction among Queues

The interacting queues problem in large wireless networks is much more intricate than that in the aforementioned ALOHA system (see Figure 1(c)). The main differences between the two systems can be attributed to the physical layer and the medium access control (MAC) layer.

**Physical Layer** – In a collision-based slotted ALOHA system, the transmission mechanism is simple: a packet transmission fails if two or more transmitters in the system are scheduled at the same time. However, in wireless networks, the delivery process of packets is not just determined by the busy statuses of all transmitters but directly affected by the aggregated interference from all active links. Due to link adaptation, or adaptive modulation and coding, the transmission rate is adjusted adaptively according to the SINR, which is related to the propagation environment of all links. Therefore, the queues in a wireless network are coupled in a complicated way.

**MAC Layer** – The MAC protocol in wireless networks is usually much more sophisticated than ALOHA. For example, in the downlink of a cellular network, each base station may serve multiple users, and user scheduling is introduced to guarantee that most users can be served fairly. If one separate queue is maintained for each user at the base station, there are many queues at each base station. In the multi-cell scenario, the interaction exists between queues of the same cell and queues of different cells (intra-cell and inter-cell interaction).

**PROMISING APPROACHES**

*Network Stability: The First Step*

Before the delay analysis, a key problem is to study the queue stability in large wireless networks. For an isolated link with only one pair of transmitter and receiver, by the Loynes theorem [9], if the arrival process and the service process of the packets are stationary, the sufficient and necessary condition for stability is that the average service rate is larger than the average arrival rate. However, strict stability (i.e., all queues in the networks are stable) for certain classes of wireless networks, such as ad hoc networks modeled by Poisson point processes (PPP), is not achievable since there always exist some links that experience strong interference and, consequently, their queues are unstable. Thus, the notion of $\varepsilon$-stability is introduced in [10], which indicates that the proportion of unstable queues in a wireless network is less than a predefined value $\varepsilon$. There exists a critical arrival rate: if the arrival rate is below the critical arrival rate,
the network will be $\varepsilon$-stable; otherwise, the network will not be $\varepsilon$-stable. Due to the interacting queues problem, obtaining the exact sufficient and necessary condition for $\varepsilon$-stability, i.e., finding the critical arrival rate, is difficult. In the following, we list several promising approaches.

**Sufficient Conditions** – To obtain sufficient conditions for $\varepsilon$-stability, a dominant system of the original system can be considered. In the dominant system, the link under consideration behaves exactly the same as that in the original system. However, other transmitters in the dominant system, when their queues become empty, continue to transmit “dummy” packets, thus continuing to cause interference to other links. As a result, the queue size at each transmitter in the dominant system is always no smaller than that in the original system, provided that the queues started with the same initial conditions. A sufficient condition for the original system to be $\varepsilon$-stable can be obtained by analyzing the $\varepsilon$-stability conditions for the dominant system.

**Necessary Conditions** – We describe two available approaches to obtain two different types of necessary conditions for $\varepsilon$-stability, which we name *type I necessary conditions* and *type II necessary conditions*. To obtain *type I necessary conditions*, consider a simplified system in which only the effect of the nearest interferer is considered. Since the interference is reduced in the simplified system, a necessary condition for a queue to be stable in the original system is that it is stable in the simplified system. To obtain *type II necessary conditions*, consider a modified favorable system that drops the packets in the interfering transmitters that are not scheduled or fail to be delivered. Since the interference in the modified favorable system is no larger than that in the original system and the packets will not accumulate at the interferers, the necessary conditions for $\varepsilon$-stability of the modified favorable system will be necessary conditions for $\varepsilon$-stability of the original system. By introducing these two systems, an interfering transmitter is active with certain probability, which is independent from the statuses of the other queues. This way, the interacting queues become decoupled.

Based on the above approaches, the work in [10] derived the sufficient conditions and necessary conditions for $\varepsilon$-stability and relaxed them to closed-form. Figure 4 shows an example of the maximal arrival rates per the sufficient conditions and necessary conditions as functions of access probability in ad hoc network with random access, i.e., each link is scheduled independently with certain probability. As the access probability approaches zero, a packet is dropped with high probability, and as the density of transmitters approaches zero, the interference is negligible. Thus, in these cases, the type I necessary condition is better. As $\varepsilon \to 0$, the type II necessary
condition becomes better since when only the nearest interferer is considered, the arrival rate can be positive to make the network strictly stable ($\varepsilon = 0$), which is not realistic in the original system. In summary, Table I lists some situations when to use which type of necessary conditions.

*Transmission Delay under Backlogged Assumption*

One way to bypass the interacting queues problem and analyze the delay is to assume that all nodes in the network are fully backlogged. In this way, the service process of a queue is decoupled from the statuses of other queues. A meaningful and practically relevant metric under the backlogged assumption is the transmission delay, which is the time required for a node to successfully deliver a packet. The main component of the transmission delay is the retransmission delay which closely related to the number of retransmissions of a packet. This type of delay, which ignores the queueing delay, is also called *local delay* \[11\].

As discussed above, the delay is greatly affected by the interference correlation in wireless networks. Interference correlation partially comes from correlated channel attenuation like correlated fading and shadowing. More importantly, such correlation stems from the spatial distribution of transmitters and the MAC protocols in the static networks since they determine the locations and the activity pattern of the interferers, which, in turn, determine the structure of the interference.

Due to the static deployment of network, the transmission delay for different links in the extreme case without fading and MAC mechanism is two-valued: either one frame (good realization of the point process modeling the nodes) or infinite (bad realization of the point process). In this case, the transmission success events are fully correlated (one success implies success in every time slot, and vice versa), and the mean transmission delay is infinite.

Figure 3 shows the mean and the variance of the transmission delay in ad hoc network with random access under the backlogged assumption. The mean transmission delay may be infinite for certain network parameters — a phenomenon known as *wireless contention phase transition* \[12\]. The variance of the transmission delay reflects the delay jitter, which is an important measure that characterizes the fluctuation of delay. For interactive real-time applications, e.g., VoIP, a large delay variance can be a serious issue. The approach in this section has also been applied to analyze the transmission delay in heterogeneous cellular networks (HetNets) \[13\].
**Single-hop Delay: Bounding Approaches**

The total single-hop delay consists of the queueing delay and the transmission delay. In a static network, given the locations of transmitters and receivers, the success probabilities for different links are different, resulting in different mean delays for different links. If we consider the mean delays of all links in the network, a cumulative distribution function (cdf) of the mean delays of all queues can be obtained, which is a suitable metric to characterize the delay performance of the overall wireless network. Analytically, for ergodic point process models, the cdf obtained through the spatial statistics can be obtained by considering the typical queue in the network. However, obtaining the exact cdf is untractable due to the interacting queues problem. Several promising approaches to bound and approximate this cdf are described as follows [14].

**Lower bound** – Consider the same dominant system introduced when deriving the sufficient conditions for $\varepsilon$-stability, the queue size at each transmitter in the dominant system will never be smaller than that in the original system, resulting in smaller SIR and larger delay. Therefore, the obtained cdf under such relaxation will be a lower bound for the cdf in the original system.

**Upper bound** – Consider the same modified favorable system introduced when deriving type II necessary conditions. Since the interference in the modified favorable system is always smaller than that in the original system and the packets will not accumulate at the interfering transmitters, the resulting delay will be smaller than that in the original system. Accordingly, the corresponding cdf will be an upper bound for the cdf in the original system.

**Approximation** – In order to approximate the cdf, all transmitters may be assumed to be busy independently with the same busy probability, which can be obtained by solving a fixed-point problem, as in [15]. The fixed-point problem is established by taking the busy probability of all interfering transmitters as a variable and expressing the busy probability of one desired link, which then equals to the originally assumed busy probability of the interfering transmitters. Having obtained the approximated busy probability, the cdf can then be approximately evaluated.

By applying the proposed bounding and approximating approaches, the service rate is decoupled from the statuses of all queues at the interfering transmitters and the analysis of the end-to-end delay becomes tractable. Figure 4 shows a comparison of the bounds for the cdf of the mean delay in the network for different setups.
CONCLUSION AND FUTURE RESEARCH

The coupling between traffic and network services becomes increasingly strong as wireless networks evolve. This type of coupling has induced the interacting queues problem, which is the key obstacle for the delay analysis in large scale wireless networks. This article proposed several promising approaches to provide an accessible guidance for understanding the stability and delay issues in large scale wireless networks.

Much work is still called for in this area, both on fundamental theory to handle more sophisticated MAC protocols and on meaningful models to fit practical scenarios. Some interesting aspects that need further investigation are as follows:

• Scheduling, which increases the complexity of interacting queues, is a crucial mechanism that affects delay in wireless networks. More effective approaches should be investigated in order to analyze the impact of sophisticated scheduling mechanism.

• Traditional traffic analyses either focus on modeling the spatial distribution of traffic or modeling the temporal arrival process of packets. The methods discussed in this article are promising in jointly handling the spatiotemporal arrival of traffic. Offloading based on the spatiotemporal traffic can also be investigated.

• Delay for more complicated yet realistic point process models, such as cluster processes and hard core processes, also needs to be explored. New approaches are also needed in order to obtain more accurate results for the stability and the delay distribution.

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Fig. 1: (a) Classical M/M/1 queueing problem. (b) Interacting queues problem with two queues. (c) Interacting queues problem in large wireless networks.
Fig. 2: Comparison of sufficient conditions and necessary conditions for $\varepsilon$-stability with $\varepsilon = 0.1$ as the functions of access probability in ad hoc networks with random access. The density of transceivers is 0.05 links/m$^2$, with the distance between transmitter and receiver being 1m.
TABLE I: Some situations to use type I or type II necessary conditions

| Some special cases                                                                 | Type   |
|-----------------------------------------------------------------------------------|--------|
| Parameter $\varepsilon$ for $\varepsilon$-stability approaches zero               | Type II|
| Access probability approaches zero                                                 | Type I |
| Density of transmitters approaches zero                                            | Type I |
| SINR threshold $\theta$ approaches zero and access probability approaches one      | Type II|
| Square of the desired link distance is much larger than reciprocal of the density of transmitters | Type II|
Fig. 3: Mean and variance of the transmission delay in ad hoc networks with random access and backlogged assumption. The density of the transceivers is 0.01 links/m², with the distance between transmitter and receiver being 5m. θ is the threshold of the SINR for successful delivery.
Fig. 4: Comparison of lower bound and upper bound for the cdf of the mean delay in the wireless network with random access probability 0.5. $\lambda$ is the density of transceiver pairs in links/m$^2$, with the distance between transmitter and receiver being 1m. The packet arrival process is Bernoulli with arrival rate $\xi$. 