Free energies of heavy quarks in full-QCD lattice simulations with Wilson-type quark action

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Abstract

The free energy between a static quark and an antiquark is studied by using the color-singlet Polyakov-line correlation at finite temperature in lattice QCD with 2+1 flavors of improved Wilson quarks. From the simulations on $32^3 \times 12, 10, 8, 6, 4$ lattices in the high temperature phase, based on the fixed scale approach, we find that, the heavy-quark free energies at short distance converge to the heavy-quark potential evaluated from the Wilson loop at zero temperature, in accordance with the expected insensitivity of short distance physics to the temperature. At long distance, the heavy-quark free energies approach to twice the single-quark free energies, implying that the interaction between heavy quarks is screened. The Debye screening mass obtained from the long range behavior of the free energy is compared with the results of thermal perturbation theory.

1. Introduction and the method

Heavy-ion collision experiments running at RHIC have provided various information about the new state of matter, the quark-gluon plasma (QGP). To remove theoretical uncertainties in understanding the nature of QGP, first principle calculations by lattice QCD are indispensable. So far, most lattice studies at finite temperature have been performed using staggered-type quark actions with the fourth-root trick, whose theoretical basis is not fully established yet. Thus, a crosscheck with other actions such as the Wilson-type quark actions are important to control and estimate systematic errors due to lattice discretization.

We investigate thermodynamic properties of QGP using the improved Wilson quark action. Adopting the fixed scale approach developed in [1], we perform finite-temperature simulations [2], while the corresponding zero-temperature configurations are taken from the results of the CP-PACS and JLQCD Collaborations [3].

At $T = 0$, interaction between a static quark and an antiquark can be studied by the heavy-quark potential evaluated from the Wilson loop operator. The resulting potential takes the Coulomb form at short distances due to perturbative gluon exchange, while it takes the linear
form at long distances due to confinement:

\[ V(r) = -\frac{\alpha_0}{r} + \sigma r + V_0. \]  

(1)

For \( T > 0 \), inter-quark interaction may be studied by the heavy-quark free energy \( F^1(r,T) \) evaluated from a Polyakov-line correlation in the color-singlet channel with Coulomb gauge fixing \([4]\):

\[ F^1(r,T) = -T \ln(\text{Tr} \, \Omega(x) \Omega(y)), \quad r = |x - y|, \]  

(2)

where \( \Omega(x) = \prod_{\tau=1}^{N_t} U_\tau(x,\tau) \) with \( U_\tau(x,\tau) \) being the link variable in the temporal direction. At zero temperature, we expect \( F^1(r,T=0) = V(r) \), while at high temperature, we can extract the Debye screening mass \( m_D \) from

\[ F^1(r,T) = -\frac{\alpha(T)}{r} e^{-m_D(T)r} + 2F_Q. \]  

(3)

In the conventional fixed \( N_t \) approach where \( T \) is varied by changing the lattice spacing \( a \), \( V(r) \) and \( F^1(r,T) \) receive different renormalization at each \( T \), and thus are usually adjusted by hand such that \( V(r) \) and \( F^1(r,T) \) coincide with each other at a short distance, assuming that the short distance properties are insensitive to the temperature. On the other hand, in the fixed scale approach, the temperature \( T = (N_t a)^{-1} \) is varied by changing the temporal lattice size \( N_t \) at fixed \( a \) \([1]\). Because the spatial volume and the renormalization factors are common to all temperatures, we can directly compare the free-energies at different temperatures without any adjustment. We show below that \( F^1(r,T) \)'s for different \( T \) approach to \( V(r) \) at short distances, which proves the expected insensitivity of the short distance physics to the temperature.

2. Results of the lattice simulations

We employ the renormalization group improved gluon action and \( 2 + 1 \) flavors of nonperturbatively \( O(a) \)-improved Wilson quark actions. Zero-temperature configurations are given by the CP-PACS and JLQCD Collaborations \([3]\) at \( m_\pi/m_\rho = 0.6337(38) \) and \( m_K/m_{K^*} = 0.7377(28) \). Finite temperature simulations with the same parameters are performed on \( 32^3 \times N_t \) lattices with \( N_t = 12, 10, 8, 6 \) and \( 4 \), which correspond to \( T \sim 200-700 \text{ MeV} \) \([2]\). The absolute scale is estimated from the Sommer parameter, \( r_0 = 0.5 \text{ fm} \).

Figure \([1]\) shows the results of the heavy-quark potential \( V(r) \) at \( T = 0 \) and the heavy-quark free energies \( F^1(r,T) \) at various temperatures as functions of \( r \). At \( T = 0 \), \( V(r) \) shows Coulomb-like and linear-like behaviors at short and long distances, respectively. A fit with Eq. (1), as shown by the dashed line in Fig. \([1]\) gives \( \alpha_0 = 0.441 \) and \( \sqrt{\sigma} = 0.434 \text{ GeV} \).

For \( T > 0 \), we note that the heavy-quark free energies \( F^1(r,T) \) at all temperatures converge to \( V(r) \) at short distances. This means that the short distance physics is insensitive to temperature. As stressed above, unlike the case of the conventional fixed-\( N_t \) approach in which this insensitivity is assumed and used to adjust the constant terms of \( F^1(r,T) \), our fixed scale approach enabled us to directly confirm this theoretical expectation.

At large \( r \), \( F^1(r,T) \) departs from \( V(r) \) and eventually becomes flat due to Debye screening. In Fig. \([1]\) the asymptotic values of \( F^1(r,T) \) at long distance are also compared with \( 2F_Q \) denoted by the arrows: Here the thermal average of a single Polyakov-line is defined as \( F_Q = -T \ln(\text{Tr} \, \Omega) \). We find that \( F^1(r,T) \) converges to \( 2F_Q \) quite accurately at long distances.
In order to extract the screening mass $m_D$, we fit $F^1(r,T)$ by the screened Coulomb form, Eq. (3). The fit range is chosen to be $0.38 \text{ fm} \lesssim r \lesssim 0.57 \text{ fm}$. The left panel of Fig. 2 shows $m_D(T)/T$ which does not have strong dependence on $T$. To make a quantitative comparison to the result of the thermal perturbation theory, we define the 2-loop running coupling by

$$g^{-2}_{2l}(\mu) = \beta_0 \ln(\mu/\Lambda_{\text{MS}}) + \beta_1 \beta_0 \ln \left( \ln(\mu/\Lambda_{\text{MS}}) \right)^2$$

with the QCD scale parameter $\Lambda_{\text{MS}} = 260 \text{ MeV}$, where we assume a degenerated $N_f = 3$ case. Then, the Debye mass in the leading-order (LO) thermal perturbation theory is given by

$$m_D^{\text{LO}}(T)/T = \sqrt{1 + N_f/6 g_{2l}(T)} \left[ 1 + g_{2l}(T) \frac{3}{2\pi} \sqrt{\frac{1}{1 + N_f/6}} \left( \ln \frac{2m_D^{\text{LO}}}{m_{\text{mag}}} - \frac{1}{2} \right) + \mathcal{O}(g^2) \right],$$

where $m_{\text{mag}}(T) = C_m g^2(T) T$ is the magnetic screening mass. Since the factor $C_m$ cannot be determined in perturbation theory due to the infrared problem, we adopt $C_m \approx 0.482$ calculated in a quenched lattice simulation as a typical value. In Fig. 2 (left), the dashed lines represent the LO results for $N_f = 3$ and the bold lines represent the NLO results for $N_f = 3$, for a range of the renormalization point $\mu = \pi T - 3\pi T$. We find that the LO Debye mass does not reproduce the lattice data in magnitude, while the NLO Debye mass is approximately 50 % larger than the LO Debye mass and shows a better agreement with the lattice data.

Finally we study the flavor dependence of $m_D$. Fig. 2 (right) shows $m_D$ in $N_f = 2 + 1$ QCD (this work), that in $N_f = 2$ QCD with an improved Wilson-quark action at $m_\pi/m_p = 0.65$ [8], and that in the quenched QCD ($N_f = 0$) [1]. Arrows on the horizontal axis indicate critical temperatures for $N_f = 2$ ($T_c \sim 186 \text{ MeV}$ at $m_\pi/m_p = 0.65$) and $N_f = 0$ ($T_c \sim 290 \text{ MeV}$). We find that $m_D$ for $N_f = 2 + 1$ is comparable to that for $N_f = 2$, whereas it is larger than that for $N_f = 0$. A similar result was obtained with a staggered-type quark action [9].

3. Summary

We studied the free energy between a static quark and an antiquark at finite temperature in lattice QCD with $2 + 1$ flavors of improved Wilson quarks on $32^3 \times 12–4$ lattices in the high
temperature phase. We adopted the fixed scale approach which enables us to compare the free-energies at different temperatures directly without any adjustment of the overall constant. At short distances, the heavy-quark free energies, evaluated from the Polyakov-line correlations in the color-singlet channel, show universal Coulomb-like behavior common to that of the heavy-quark potential at zero temperature. This is in accordance with the expected insensitivity of short distance physics to the temperature. At long distances, the heavy-quark free energies approach to twice the single-quark free energies calculated from the thermal average of a Polyakov-line. Also, we extracted the Debye screening mass $m_D(T)$ from the heavy-quark free energy and found that the next-to-leading order thermal perturbation theory is required to explain the magnitude of $m_D(T)$ on the lattice. Comparison to the previous results at $N_f = 2$ and $N_f = 0$, shows that the dynamical light quarks have sizable effects on the value of $m_D(T)$.

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