Particle - unparticle duality in super-relativity

Peter Leifer
Cathedra of Informatics, Crimea State Engineering and Pedagogical University,
21 Sevastopolskaya st., 95015 Simferopol, Crimea, Ukraine;
leifer@bezeqint.net

Abstract

New method of shaping quantum “particle - unparticle” vacuum excitations has been proposed in the framework of unification of relativity and quantum theory. Such unification is based solely on the notion of generalized coherent state (GCS) of N-level system and the geometry of unitary group SU(N) acting in state space C^N. Initially, neither contradictable notion of quantum particle, nor space-time coordinates (that cannot be a priori attached to nothing) are used in this construction. Quantum measurement of local dynamical variables (LDV) leads to the emergence of 4D dynamical space-time (DST). Morphogenesis of the “field shell” of GCS and its dynamics have been studied for N = 2 in DST.

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1 Introduction

Statistical analysis of the energy distribution is the base of the black body radiation [1] and the Einstein’s theory of the light emission and absorption [2]. Success of Einstein hypothesis of photons, de Broglie wave concept of particles [3] and Schrödinger’s equation for hydrogen atom [4] paved the way to corpuscular-wave duality of matter. This conceptual line was logically finished by Dirac in his method of the second quantization [5]. This approach is perfectly fits to many-body weakly interacting quantum systems and it was assumed that the “corpuscle-wave duality” is universal. However the application of this method to single quantum “elementary” particles destroys this harmony. Physically it is clear why: quantum particle is self-interacting system and this interaction is at least of the order of its rest mass. Since the nature of the mass is the open problem we do not know the energy distribution in quantum particles up to now. Here I try to show a possible approach to this problem in the framework of simple model in pure deductive manner.

A long time it is was assumed that the dynamical model may be found in the framework of the string theory, but the epitaph to string theory [6] clearly
shows the deep crisis of particle physics in its present form. Notice, Einstein [7] and Schrödinger [8] treated the statistical fundament of quantum theory as a perishable and temporal. Quantum theory solved a lot of fundamental problems, but (as it happen with fundamental theory), it posed number of deeper questions. Even first steps in wave picture of quantum particles brought sudden surprises. First of all there was a big discrepancy between intuitive Schrödinger’s imagination and real properties of “corpuscular waves”. Initially Schrödinger thought that there is a possibility to build stable wave packet from plane waves of de Broglie that may be treated as the wave model of localized electron; he understood soon that it is impossible. Only in some special case of quantized harmonic oscillator he could build such stable wave packet moving (in average) like material point under Hooke’s elastic force [9].

Historically, the impossibility to get wave description of localizable particle led to probabilistic interpretation of the wave function. In fact, this is the fork point changing all character of fundamental physics: state vector is treated as amplitude of probability for particle to be in some particular state. This paradigm is the source of all fundamental unsolved problems mentioned above: measurement problem, localization, divergences, etc. However, practical applications of quantum theory are so convincing and prolific that any attempts to find answers on “children questions” are frequently treated as a pathology. Nevertheless, we should analyze these problems again and again till all situation will be absolutely clear without any references to the beauty of quantum mystic [10]. Two fundamental problems should be solved: how to get (from first principles) non-linear quantum field equations with localizable solutions and how to formulate objective quantum measurement of dynamical variables?

Dirac’s equations for electron in Coulomb potential of nuclei are realistic in first approximation but higher approximations suffer from divergences. Expressions for self-energy, electric charge and magnetic momentum of self-interacting electron demonstrate divergences too. Renormalization procedure is in fact the attempt to correct this construction. The same equations being applied to “free electron” have plane wave solutions that never where observed and that may be rather related to co-vector generated by periodic lattice [11]. Physical interpretation of plane wave solution being applied to single free particle requires essential efforts. Furthermore, the price of these efforts is unacceptably high: probabilistic interpretation, collapse of wave function, parallel worlds, Multiverse etc., are invoked to explain the formal, unobservable and artificial solution.

Plane waves and δ-function are examples of improper states that may be formally incorporated in Hilbert manifolds on equal footing with square-integrable states in the framework of “functional relativity” [12] [13]. Thereby, the classical ideal notion of pointwise interaction is deemed as legitimized. I think, however, that such approach is acceptable as effective temporal method when discussion in nature of quantum interaction is postponed.

Generally, improper states like δ-function and plane waves are only artifact arose as assumption of applicability of our fundamental linear equations for “free” quantum particle. In fact, the principles of Hamilton or Lagrange can
not fundamentally to determine the quantum dynamics because the concept of quantum amplitudes shows that these principles (being formulated as the principle of the least action) are merely (sometimes bad mathematically defined) \textit{approximation}. One should found some more general quantum principle of self-organization (morphogenesis) and dynamics of quantum matter.

Few years ago I derived field quasi-linear PDE as a consequence of conservation law of local Hamiltonian on quantum phase space $CP(N-1)$ \cite{17, 18}. This conservation law was expressed as affine parallel transport of Hamiltonian vector field in $CP(N-1)$ in connection agreed with Fubini-Strudy metric \cite{14, 16, 17, 18}. These quasi-linear PDE have soliton-like solutions whose physical status is unknown. New investigations in so-called “unparticle” area \cite{19} gave me some hint on possible interpretation of gotten equations.

I would like to discuss here a morphogenesis of quantum particle in the spirit of reaction $e^- \rightarrow \mathcal{U} \rightarrow e^-$. In other words I propose to study the particle/unparticle sectors of matter \cite{19} in wide range of energy in order to solve localization problem in foundations of quantum physics. The concept of \textit{scale invariance} \cite{19, 20} will be replaced by the principle of super-relativity \cite{15, 16}.

I should note that Blochintzev about 60 years ago discussed the unparticle sector in the framework of universality of wave-particle “duality” for interacting quantum fields \cite{22, 23}. For such fields the universality is generally broken. Namely, attempt to represent two interacting boson fields as the set of free quantum oscillators leads to two types of oscillators: quantized and non-quantized. The second one arises under simple relation $g > \frac{m_1 m_2 c^2}{h^2}$ between coupling constant $g$ and masses $m_1$ and $m_2$ of two scalar fields. For such intensity of coupling we obtain a field with excitation states in two sectors: particle and “unparticle”. Furthermore, the excitations in “unparticle” sector has an imaginary mass and they propagate with group velocity larger than $c$. For self-interacting scalar field of mass $m$ the intensity of self-interaction $g$ leads to breakdown of the universality of the wave-particle “duality” if it is larger than the inverse square of the Compton wavelength: $g > \frac{m^2 c^2}{h^2} = \frac{1}{\lambda_C^2}$.

2 The Action State Space

Blochintzev’s examples were oversimplified for clarity. We have to have the process of morphogenesis of quantum particle/unparticle sectors that should be dynamically described. One may even think that interacting observable particles are immersed into the sea of “unparticle” excitations somehow related with “dark matter”. It leads to necessity to modify the second quantization method. Besides arguments of Blochintzev there at least two reasons for such modification.

First. In the second quantization method one has formally given particles whose properties are defined by some commutation relations between creation-annihilation operators. Note, that the commutation relations are only the simplest consequence of the curvature of the dynamical group manifold in the vicinity of the group’s unit (in algebra). Dynamical processes require, however, finite
group transformations and, hence, the global group structure. The main
my technical idea is to use vector fields over a group manifold instead of Dirac’s
abstract q-numbers. This scheme therefore seeks the dynamical nature of the
creation and annihilation processes of quantum particles.

Second. The quantum particles (energy bundles) should gravitate. Hence,
strictly speaking, their behavior cannot be described as a linear superposition.
Therefore the ordinary second quantization method (creation-annihilation of
free particles) is merely a good approximate scheme due to the weakness of
gravity. Thereby the creation and annihilation of particles are time consuming
dynamical non-linear processes. So, linear operators of creation and annihilation
(in Dirac sense) do exist as approximate quantities.

POSTULATE 1.
There are elementary quantum states \( |\hat{a}a >, a = 0, 1, ... \) belonging to the Fock space of an abstract Planck oscillator whose states correspond to the quantum
motions with given number of Planck action quanta.

One may image some “elementary quantum states” (EAS) \( |\hat{a}a > \) as a
quantum motions with entire number \( a \) of the action quanta. These \( a, b, c, ... \)
takes the place of the “principle quantum number” serving as discrete indices
\( 0 \leq a, b, c, ... < \infty \). Since the action by itself does not create gravity, but only
velocity of action variation, i.e. energy/matter, it is possible to create the linear
superposition of \( |\hat{a}a >= (a!)^{-1/2}(\hat{\eta}^+)^a|0 > \) constituting \( SU(\infty) \) multiplete of
the Planck’s action quanta operator \( \hat{S} = \hbar\hat{\eta}^+\hat{\eta} \) with the spectrum \( S_a = \hbar a \) in the
separable Hilbert space \( \mathcal{H} \). Therefore, we shall primarily quantize the action,
not the energy. The relative (local) vacuum of some problem is not necessarily
the state with minimal energy, it is a state with an extremal of some action
functional.

The space-time representation of these states and their coherent superpo-
sition is postponed on the dynamical stage as it is described below. We shall
construct non-linear field equations describing energy (frequency) distribution
between EAS's \( |\hat{a}a > \), whose soliton-like solution provides the quantization of
the dynamical variables. Presumably, the stationary processes are represented
by stable particles and quasi-stationary processes are represented by unstable
resonances or unparticle stuff.

Generally the coherent superposition

\[ |F > = \sum_{a=0}^{\infty} f^a |\hat{a}a >, \] (1)

may represent of a ground state or a “vacuum” of some quantum system with
the action operator

\[ \hat{S} = \hbar A(\hat{\eta}^+\hat{\eta}). \] (2)

Then one can define the action functional

\[ S[|F >] = \frac{< F|\hat{S}|F >}{< F|F >}. \] (3)
which has the eigen-value $S|\hbar a > = \hbar a$ on the eigen-vector $|\hbar a >$ of the operator $\hbar A(\hat{\eta}^+ \hat{\eta}) = \hbar \hat{\eta}^+ \hat{\eta}$ and that deviates in general from this value on superposed states $|F >$ and of course under a different choice of $\hat{S} = \hbar A(\hat{\eta}^+ \hat{\eta}) \neq \hbar \hat{\eta}^+ \hat{\eta}$. In order to study the variation of the action functional on superposed states one need more details on geometry of their superposition.

In fact only finite, say, $N$ elementary quantum states (EQS’s) ($|\hbar 0 >, |\hbar 1 >, ..., |\hbar (N-1) >$) may be involved in the coherent superposition $|F >$. Then $\mathcal{H} = \mathbb{C}^N$ and the ray space $\mathbb{C}P(\infty)$ will be reduced to finite dimensional $\mathbb{C}P(N-1)$. Hereafter we will use the indices as follows: $0 \leq a, b \leq N-1$, and $1 \leq i, k, m, n, s \leq N-1$. This superposition physically corresponds to the complete amplitude of quantum motion in setup $S$. Then GCS corresponding to this amplitude is controlled by $SU(N)$ dynamical group. One may assume that following postulate takes the place:

**POSTULATE 2.**

*Matter (energy) distribution is determined by velocities of GCS variations by LDV’s like local Hamiltonian.*

Realization of this assumption will be discussed below.

### 3 From flexible setup to quantum reference frame in super-relativity

Let me show how ordinary quantum formalism hints us how to formulate functionally invariant quantum dynamics.

#### 3.1 Flexible setup in the action state space

The ordinary quantum formalism of operations with amplitudes was brightly demonstrated by Feynman in popular lectures [10]. This formalism shows that generally two setups $S_1$ and $S_2$ lead to different amplitudes $|\Psi_1 >, |\Psi_2 >$ of outcome event. There are infinite number of different setups and not only in the sense of different space-time position but in different parameters of fields, using devices, etc. Symmetries relative space-time transformations of whole setup have been studied in ordinary quantum approach. Such approach reflects, say, the *first order of relativity*: the physics is same if any complete setup subject (kinematical, not dynamical!) shifts, rotations, boosts as whole in Minkowski space-time.

Next step leading to new type of relativity may be formulated as invariance of physical properties of quantum particles lurked behind two amplitudes $|\Psi_1 >, |\Psi_2 >$. Similar idea in the framework of “functional relativity” was formulated by A. Kryukov [12, 13] as a requirement that before and after interaction the wave function of electron should have functionally invariant form. I will treat this requirement as “global functional relativity” since the process of transition from “in”-state to “out”-state is left outside of envision. It is shown by clear example of “interaction” with a spectrometer. In this ad hoc taken measurement all root problems are hidden in two assumptions:
1. classical motion of pointwise electron in spectrometer, and
2. in pointwise absorption of the electron by screen.

These simplifications gave a possibility to treat the inverse Fourier transform as the spectrometer action and to use the Gaussian kernel $k_H(y, v) = e^{-\frac{1}{2}(y-v)^2}$ playing the role of metric in Hilbert space of improper states. The question however is: what happen in more general kind of interaction where electron is participated? Is it possible to build the mathematical model of interaction in relativistic case, say, for high energy reactions like $e^- e^+ \rightarrow \gamma + \gamma \rightarrow \tau^- \tau^+$? Definitely, this problem could not formulated in the spirit of “global functional relativity”. In order to describe smooth quantum evolution let me ask: what happen if I slightly variate some device in the setup, say, rotate a filter or, better, change magnetic field around dense flint \(\text{[25]}\) in complete setup? In other words I will use “local functional relativity” or “super-relativity” by declaration that infinitesimal variation of setup by small variations of its parameters leads to small variations of output state. Now not space-time coordinates play essential role but some internal parameters like strength of field used in given setup. But how we should formalize “physics” and its invariance mathematicially? Our model should be maximally simple since we would like to study very basic properties of quantum physics. There is a fine technical question about parametrization of output state as function of fields in devices, adjustments, etc. It would be a mistake to start our description from “given” particles in space-time and fields of setup since neither particle nor space-time are good enough defined at this stage. Any real physical setup is even much more complicated system and its classical parametrization is, however, unacceptable for our aim since it returns us to Bohr’s tenet that all quantum relations should be expressed “by classical language”. Then we will be involved in the routine round of quantum measurement problem.

The key step to the invariant description of quantum state $|S> = \sum_{a=0}^{N-1} S^a |\bar{h}a>$ is transition to local functional coordinates $\pi^i(j) = \frac{S^i}{S^a}$ of its GCS in $CP(N-1)$ that carrying representation of $SU(N)$ dynamical group $[15, 16, 30]$. Now local quantum reference frame parameterizations by local functional coordinates $\pi^i$ of GCS should be used. Here arises the second order of relativity which I called super-relativity: the physics of some quantum object corresponding to GCS of $|S>$ is same in any setup.

### 3.2 Super-relativity

The principle of super-relativity arose as development of Fock’s idea of “relativity to measuring device” $[24]$. This idea may be treated as generalization of the relativity principle in space-time to “functional relativity” in the state space $[12, 13]$ under some reservations and specification. However the power of Fock’s program is limited in comparison with power of Einstein’s concepts of special and general relativity. The main reason is that the notion of the “measuring device” could not be correctly formulated in the own framework of the standard quantum theory. Some additional and, in fact, outlandish classical ingredients...
should be involved. Same argument may be applied to the “global functional relativity” since only in some particular case it is possible to find theoretically analyzable model of quantum setup comprising classical improper states (like plane waves and δ-function) as it was discussed above. In order to overcome this problem we should to clarify relations between state vector and dynamical variables of quantum system.

It is very strange to think that state vector being treated as basic element of the full description of quantum system does not influence on dynamical variable of quantum system. Ordinary quantum dynamical variables are represented by hermitian operator in Hilbert space carrying representation of symmetry group, say, Lorentz group. All formal apparatus of quantum theory is based on the assumption that operators of position, momentum, etc. depend only upon the parameters of Lorentz group. Lets now assume that we would like to investigate general behavior of quantum state vector \( |S> \) in Hilbert space \( \mathcal{H} = \mathbb{C}^N \) subject control by unitary group \( SU(N) \) through group parameters \( \Omega^\alpha : 1 \leq \alpha \leq N^2 - 1 \). This state may be represented as \( |S> = \sum_{\alpha=0}^{N-1} S^\alpha |h\alpha > \) where space-time coordinates are not even mentioned and whose dynamics and “morphogenesis” somehow related to space-time coordinates which I will discuss later. I argue that this approach:

1. does not require any classical model,
2. it represents due to its generality the \( SU(N) \) in dynamical space-time (inverse representation) through the “morphogenesis” of the “field shell”,
3. “field shell” of GCS obeys to quasi-linear PDE in dynamical space-time that may me solved in some reasonable approximation. The pure local in quantum state space \( CP(N - 1) \) theory uses the local geometry of \( SU(N) \) group. The group parameters takes the place of non-Abelian gauge fields surrounding quantum object (particle/unparticle) whose properties a priori are unknown. But small variations of these fields lead to small variation of GCS that and may be associated with some state-dependent LDV modeling flexible setup or quantum reference frame (QRF).

In order to keep the invariant properties of some quantum particle (probably, better to say “quantum process” since particle may or may not arise during this process) involved in this manifold of setups one should know difference in amplitudes arose due to setup variation. Most technically important approach is the comparison of dynamical variables at infinitesimally close quantum states arose in slightly different setups. Since stationary states may be represented by rays in Hilbert space, I will work in projective Hilbert state space \( CP(N - 1) \). This approach leads to the concept of LDV taking the place of quantum reference frame and to the principle of super-relativity \([15, 16]\) that may be expressed as follows:

**POSTULATE 3.**

Unitary transformations \( U(\tau) = \exp(i\Omega^\alpha \Lambda_\alpha \tau) \) of the action amplitudes may be identified with physical fields. Field functions \( \Omega^\alpha \) are in the adjoint representation of \( SU(N) \), \( \Lambda_\alpha \in AlgSU(N) \), and \( \tau \) is an evolution parameter. The coset transformations \( G/H = SU(N)/S[U(1) \times U(N-1)] = CP(N - 1) \) is the quan-
tum analog of classical force; its action is equivalent to physically distinguishable deformation of GCS in $CP(N - 1)$, isotropy group $H = U(1) \times U(N - 1)$ takes the place of the gauge group.

Since any state $|S>$ has the isotropy group $H = U(1) \times U(N)$, only the coset transformations $G/H = SU(N)/SU(1) \times U(N - 1) = CP(N - 1)$ effectively act in $C^N$. One should remember, however, that the Cartan decomposition of unitary group has the physical sense only in respect with initially chosen state vector. Therefore the parametrization of these decomposition is state-dependent.

Unitary group has the physical sense only in respect with initially chosen state vector representation of quantum states represented by rays in projective Hilbert space. Therefore the realization of the unitary group transformations resulting in motion of the pure states means that physically it is interesting not abstract unitary group relations but realization of the unitary group transformations resulting in motion of the pure quantum states represented by rays in projective Hilbert space. Therefore the ray representation of $SU(N)$ in $CP^N$, in particular, the embedding of $H$ and $G/H$ in $G$, is a state-dependent parametrization. This is a key point of all construction invoking to life the concept of the LDV expressed by tangent vectors fields to $CP(N - 1)$. Technically it means that the local $SU(N)$ unitary classification of the quantum motions of GCS and distinction between particles and unparticles requires the transition from the matrices of Pauli $\hat{\sigma}_\alpha$, $\alpha = 1, ..., 3$, Gell-Mann $\hat{\lambda}_\alpha$, $\alpha = 1, ..., 8$, and in general $N \times N$ matrices $\hat{\Lambda}_\alpha(N)$, $\alpha = 1, ..., N^2 - 1$ of $Alg_{SU(N)}$ to the tangent vector fields to $CP(N - 1)$ in local coordinates. Hence, there is a diffeomorphism between the space of the rays marked by the local coordinates

$$\pi_i(j) = \begin{cases} \frac{S_i}{S_j}, & \text{if } 1 \leq i < j \\ \frac{S_i}{S_j} & \text{if } j \leq i < N - 1 \end{cases} (4)$$

in the map $U_j : \{ |S>, |S^j| \neq 0 \}, j \geq 0$ and the group manifold of the coset transformations $G/H = SU(N)/SU(1) \times U(N - 1) = CP(N - 1)$. This diffeomorphism is provided by the coefficient functions

$$\Phi^i_\sigma = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left\{ \exp(\epsilon \hat{\Lambda}_\sigma) S_i^m S^m - S_i^m \right\} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \{ \pi^i(\epsilon \hat{\Lambda}_\sigma) - \pi^i \} \quad (5)$$

of the local generators

$$\overline{D}_\sigma = \Phi^i_\sigma \frac{\partial}{\partial \pi^i} + c.c. \quad (6)$$

comprise of non-holonomic overloaded basis of $CP(N - 1)$. Here $\epsilon$ is used for one of the $SU(N)$ parameters $\Omega^\sigma$. Now one may introduce local Hamiltonian as a tangent vector fields

$$\overline{H} = \hbar \sum_{\sigma = 1}^{N^2 - 1} \Omega^\sigma(\tau) \overline{D}_\sigma = \hbar \sum_{\sigma = 1}^{N^2 - 1} \Omega^\sigma(\tau) \Phi^i_\sigma \frac{\partial}{\partial \pi^i} + c.c. \quad (7)$$

whose coefficient functions $\Omega^\sigma(\tau)$ may be found under the condition of self-conservation expressed as affine parallel transport of Hamiltonian vector field $H^i = \Omega^\sigma(\tau) \Phi^i_\sigma$ agrees with Fubini-Study metric. The problem of finding $\Omega^\sigma(\tau)$
treated in the context of gauge field application as surrounding fields of quantum lump was discussed in [29, 17].

The “visualization” of this gauge field requires the attachment of co-movable “Lorentz frame” in DST. I use the analogy with clock’s arm shows the Abelian phase of wave function (see Figure 1) in Feynman’s simplified explanation of quantum electrodynamics [10]. Feynman discussed the amplitude of an event in stationary situation since operations with amplitudes refer to fixed setup. Dynamical GCS moving due to $\Omega^{\tau}(\tau)$ variation requires operation with velocities of state deformation. This variable setup is described by LDV wrapped into “field shell” that should dynamically conserve local Hamiltonian vector field [16 21 25]. I attached qubit spinor and further “Lorentz frame” that define “4-velocity” of some imaging point (belonging to the DST) of the quantum dynamics. This imaging point is the mentioned above analog of clock’s arrow but now in 4D DST, see Figure 2. Quasi-linear partial differential equations arising as a consequence of conservation law of local Hamiltonian of evolving quantum system, define morphogenesis of non-Abelian (phase) gauge soliton-like “field shell” [29 17 30]. So, we have a concentrated “lump” associated with becoming quantum particle. Such particle may be represented as a dynamical process due to morphogenesis of the “field shell” of generalized coherent state of N-level system.

4 Local dynamical variables

The action state space $\mathcal{H} = \mathbb{C}^N$ contains “initial” and “final” stationary states with finite action quanta. Quantum dynamics is described by the velocities of the GCS variation representing some “elementary excitations” (quantum particles or unparticles). Their dynamics is specified by the Hamiltonian, giving time variation velocities of the action quantum numbers in different directions of the
Figure 2: Dynamical setup for becoming lump - operations with LDV. In order to get effective sum of non-Abelian phases of $SU(N)$ transformation shaping the lump, one should integrate quasi-linear partial differential equations. The “4-velocity” $V$ of imaging point in DST is parameterized by boosts and angle velocities of co-moving “Lorentz reference frame” attached to trajectory in $CP(N−1)$.

tangent Hilbert space $T(\pi^1,...,\pi^{N−1})CP(N−1)$ which takes the place of the ordinary linear quantum scheme as will be explained below. The rate of the action variation gives the energy of the excitations in accordance with POSTULATE 2.

The local dynamical variables correspond to internal symmetries of the GCS and their evolution should be expressed now in terms of the local coordinates $\pi^k$. The Fubini-Study metric

$$G_{ik} = [(1 + \sum |\pi^s|^2)\delta_{ik} - \pi^i \pi^k](1 + \sum |\pi^s|^2)^{-2} \tag{8}$$

and the affine connection

$$\Gamma^n_{mn} = \frac{1}{2} G^{ip} \left( \frac{\partial G_{mp}}{\partial \pi^n} + \frac{\partial G_{pn}}{\partial \pi^m} \right) = - \frac{\delta^n_m \pi^p + \delta^n_p \pi^m}{1 + \sum |\pi|^2} \tag{9}$$

in these coordinates will be used. Hence the internal dynamical variables and their norms should be state-dependent, i.e. local in the state space [15, 16]. These local dynamical variables realize a non-linear representation of the unitary global $SU(N)$ group in the Hilbert state space $C^N$. Namely, $N^2 - 1$ generators of $G = SU(N)$ may be divided in accordance with the Cartan decomposition. There are $(N−1)^2$ generators

$$\Phi^h_i \frac{\partial}{\partial \pi^i} + c.c. \in H, \quad 1 \leq h \leq (N−1)^2 \tag{10}$$

of the isotropy group $H = U(1) \times U(N−1)$ of the ray (Cartan sub-algebra) and $2(N−1)$ generators

$$\Phi^b_i \frac{\partial}{\partial \pi^i} + c.c. \in B, \quad 1 \leq b \leq 2(N−1) \tag{11}$$
are the coset $G/H = SU(N)/S[U(1) \times U(N-1)]$ generators realizing the breakdown of the $G = SU(N)$ symmetry of the GCS. Furthermore, the $(N - 1)^2$ generators of the Cartan sub-algebra may be divided into the two sets of operators: $1 \leq \sigma \leq N - 1$ ($N - 1$ is the rank of Alg$SU(N)$) Abelian operators, and $1 \leq q \leq (N - 1)(N - 2)$ non-Abelian operators corresponding to the non-commutative part of the Cartan sub-algebra of the isotropy (gauge) group. Here $\Phi_i^\sigma$, $1 \leq \sigma \leq N^2 - 1$ are the coefficient functions of the generators of the non-linear $SU(N)$ realization. They give the infinitesimal shift of the $i$-component of the coherent state driven by the $\sigma$-component of the unitary multipole field $\Omega^\sigma$ rotating the generators of Alg$SU(N)$ and they are defined as by (5) [15][16]. Then the sum of the $N^2 - 1$ the energies associated with intensity of deformations of the GCS is represented by the local Hamiltonian vector field $\vec{H}$ which is linear in the partial derivatives $\frac{\partial}{\partial \pi^i} = \frac{1}{2} (\frac{\partial}{\partial \pi^i} - i \frac{\partial}{\partial \pi^j})$ and $\frac{\partial}{\partial \pi^j} = \frac{1}{2} (\frac{\partial}{\partial \pi^i} + i \frac{\partial}{\partial \pi^j})$. In other words it is the tangent vector to $CP(N-1)$

\[
\vec{H} = h\Omega^c \Phi^i \frac{\partial}{\partial \pi^i} + h\Omega^b \Phi^i \frac{\partial}{\partial \pi^i} + h\Omega^b \Phi^i \frac{\partial}{\partial \pi^i} + c.c.
\]

Thereby in the framework of the local state-dependent approach one can formulate a quantum scheme with help more flexible mathematical structure than matrix formalism. It means that matrix elements of transitions between two arbitrary far states are associated with, in fact, bi-local dynamical variables that bring a lot of technical problems in quantum field area. However the local dynamical variables related to infinitesimal deformations of quantum states are well defined in projective Hilbert space as well as quantum states itself. They are local tangent vector fields to the projective Hilbert space $CP(N - 1)$ which are $SU(N)$ generators (differential operators of first order) [15][16][21]. In the local coordinates $\pi_{i(j)} = \frac{\omega}{h}$ one can build the infinitesimal generators of the Lie algebra Alg$SU(N)$. Then one has to use explicit form $\Phi^i_\sigma$ for $N^2 - 1$ of infinitesimal generators of the Lie algebra Alg$SU(N)$. For example for the three-level system, algebra $SU(3)$ has 8 infinitesimal generators which are given by the vector fields:

\[
\begin{align*}
\vec{D}_1 &= i\hbar [1 - (\pi^2)^2] \frac{\partial}{\partial \pi^1} - \pi^1 \frac{\partial}{\partial \pi^2} - [1 - (\pi^1)^2] \frac{\partial}{\partial \pi^1} + \pi^1 \pi^2 \frac{\partial}{\partial \pi^2}, \\
\vec{D}_2 &= -\hbar [1 + (\pi^1)^2] \frac{\partial}{\partial \pi^1} + \pi^1 \frac{\partial}{\partial \pi^2} - [1 + (\pi^2)^2] \frac{\partial}{\partial \pi^2} + \pi^1 \pi^2 \frac{\partial}{\partial \pi^1}, \\
\vec{D}_3 &= -i\hbar [1 \frac{\partial}{\partial \pi^1} + \frac{1}{2} \pi^2 \frac{\partial}{\partial \pi^2} + \pi^1 \frac{\partial}{\partial \pi^2} + \frac{1}{2} \pi^2 \frac{\partial}{\partial \pi^1}], \\
\vec{D}_4 &= i\hbar [1 - (\pi^2)^2] \frac{\partial}{\partial \pi^1} - \pi^2 \frac{\partial}{\partial \pi^2} - [1 - (\pi^2)^2] \frac{\partial}{\partial \pi^2} + \pi^2 \pi^2 \frac{\partial}{\partial \pi^2}, \\
\vec{D}_5 &= -\hbar [1 + (\pi^2)^2] \frac{\partial}{\partial \pi^1} + \pi^2 \frac{\partial}{\partial \pi^2} - [1 + (\pi^2)^2] \frac{\partial}{\partial \pi^2} + \pi^1 \pi^2 \frac{\partial}{\partial \pi^2}, \\
\vec{D}_6 &= -i\hbar [1 + \frac{1}{2} \pi^2 \frac{\partial}{\partial \pi^1} + \pi^1 \frac{\partial}{\partial \pi^2} + \frac{1}{2} \pi^2 \frac{\partial}{\partial \pi^2} - \pi^1 \frac{\partial}{\partial \pi^2}], \\
\vec{D}_7 &= \hbar [\pi^2 \frac{\partial}{\partial \pi^1} - \pi^2 \frac{\partial}{\partial \pi^1} - \pi^2 \frac{\partial}{\partial \pi^2} - \pi^1 \frac{\partial}{\partial \pi^2}], \\
\vec{D}_8 &= -3i\hbar [\pi^2 \frac{\partial}{\partial \pi^1} - \pi^2 \frac{\partial}{\partial \pi^1} - \pi^2 \frac{\partial}{\partial \pi^2}].
\end{align*}
\]
Let me assume that \( G = \sum_{a=0}^{N-1} g^a |h^a\rangle \) is a “ground state” of some the least action problem. Then the velocity of the ground state evolution relative “world time” \( \tau \) is given by the formula

\[
|\Psi> = |T> = \frac{d|G>}{d\tau} = \frac{\partial g^a}{\partial \pi_i} \frac{d\pi^i}{d\tau} |h^a\rangle + \frac{\partial g^a}{\partial \pi^s} \frac{d\pi^s}{d\tau} |h^a\rangle = |T_i| \frac{d\pi^i}{d\tau} + |T_{si}| \frac{d\pi^s}{d\tau} = H^i |T_i| + H^s |T_{si}|, \tag{14}
\]

is the tangent vector to the evolution curve \( \pi^i = \pi^i(\tau) \), where

\[
|T_i| = \frac{\partial g^a}{\partial \pi^i} |h^a\rangle = T_i^a |h^a\rangle, \quad |T_{si}| = \frac{\partial g^a}{\partial \pi^s} |h^a\rangle = T_{si}^a |h^a\rangle. \tag{15}
\]

Then the variation velocity of the \(|\Psi>\) is given by the equation

\[
\frac{d|\Psi>}{d\tau} = (B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau}) |N> + (\frac{dH^s}{d\tau} + \Gamma^s_{ik} H^i \frac{d\pi^k}{d\tau}) |T_s> + (\frac{dH^s}{d\tau} + \Gamma^s_{ik} H^i \frac{d\pi^k}{d\tau}) |T_{ss}> \tag{16}
\]

where I introduce the matrix \( \tilde{B} \) of the second quadratic form whose components are defined by following equations

\[
B_{ik} |N> = \frac{\partial |T_i>}{\partial \pi^k}, \quad B_{ik} |N> = \frac{\partial |T_i>}{\partial \pi^k}, \quad B_{ik} |N> = \frac{\partial |T_{si}>}{\partial \pi^k}, \quad B_{ik} |N> = \frac{\partial |T_{si}>}{\partial \pi^k}, \quad \Gamma^s_{ik} |N> = \frac{\partial |T_{si}>}{\partial \pi^k} \tag{17}
\]

through the state \(|N>\) normal to the “hypersurface” of the ground states. I should emphasize that “world time” is the time of evolution from the one GCS to another one which is physically distinguishable. Thereby the unitary evolution of the action amplitudes generated by leads in general to the non-unitary evolution of the tangent vector to \( CP(N - 1) \) associated with “state vector” \(|\Psi>\). Assuming that the “acceleration” \(|A>\) is gotten by the action of some linear “Hamiltonian” \( \hat{L} \) describing the evolution (or a measurement), one has the “Schrödinger equation of evolution”

\[
\frac{d|\Psi>}{d\tau} = -i \hat{L} |\Psi> \quad = (B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau}) |N> + (\frac{dH^s}{d\tau} + \Gamma^s_{ik} H^i \frac{d\pi^k}{d\tau}) |T_s> + (\frac{dH^s}{d\tau} + \Gamma^s_{ik} H^i \frac{d\pi^k}{d\tau}) |T_{ss}> \tag{18}
\]

This “Hamiltonian” \( \hat{L} \) is non-Hermitian and its expectation values is as follows:

\[
<N|\hat{L}|\Psi> = i(B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik} H^i \frac{d\pi^k}{d\tau}),
\]

The new formulation of the classical mechanics is due to the introduction of the non-Hermitian “Hamiltonian” \( \hat{L} \) and its expectation values, which are the “action amplitudes” of the evolution of the system. This formulation is non-unitary in general, but it is unitary and physically distinguishable in the “world time” evolution. The state of the system is given by the “Schrödinger equation of evolution”

\[
\frac{d|\Psi>}{d\tau} = -i \hat{L} |\Psi> \tag{19}
\]

This equation is analogous to the Schrödinger equation, but it is non-unitary and non-Hermitian. The new formulation of the classical mechanics is a generalization of the quantum mechanics and it is a new approach to the problem of the evolution of the system.
\[ <\Psi|\hat{L}|\Psi> = iG_p s \left( \frac{dH^s}{d\tau} + \Gamma_{ik}^s H^i d\pi^k d\tau \right) H^p + iG_p s \left( \frac{dH^{s*}}{d\tau} + \Gamma_{ik}^{s*} H^i d\pi^{k*} d\tau \right) H^p \]
\[ = i <\Psi|\frac{d}{d\tau}|\Psi>. \tag{19} \]

The minimization of the \( |A> \) under the transition from point \( \tau \) to \( \tau + d\tau \) may be achieved by the annihilation of the tangential component
\[ \frac{dH^s}{d\tau} + \Gamma_{ik}^s H^i d\pi^k d\tau = 0, \quad \frac{dH^{s*}}{d\tau} + \Gamma_{ik}^{s*} H^i d\pi^{k*} d\tau = 0 \tag{20} \]
i.e. under the condition of the affine parallel transport of the Hamiltonian vector field. The last equations in (26) shows that the affine parallel transport of \( H^i \) agrees with Fubini-Study metric leads to Berry’s “parallel transport” of \( |\Psi> \).

5 Dynamical space-time as “objective observer”

I have assumed that the quantum measurement of the LDV being encoded with help infinitesimal Lorentz transformations of qubit spinor leads to emergence of the dynamical space-time that takes the place of the objective “quantum measurement machine” formalizing the process of numerical encoding the results of comparisons of LDV’s. Two these procedures are described below.

5.1 LDV’s comparison

Local representation of unitary group \( SU(N) \) is reliable geometric tool for classification of the GCS motions in \( CP(N-1) \) during quantum dynamics due to interaction or self-interaction. This evolution of GCS may be used in objective measuring process. Two essential components of any measurement are identification and comparison. The Cartan’s idea of reference to the previous infinitesimally close GCS has been used. So one could avoid the necessity of the “second body” used as a reference frame. Thereby, LDV is now a new important element of quantum dynamics [25]. We should be able to compare some LDV at two infinitesimally close GCS represented by points of \( CP(N-1) \). Since LDV’s are vector fields on \( CP(N-1) \), the most natural mean of comparison of the LDV’s is affine parallel transport agrees with Fubini-Study metric [15]. This parallel transport expresses the conservation law of local Hamiltonian
\[ \frac{\delta H^i}{\delta \tau} = \frac{\delta (\Omega^s (\tau) \Phi^i_s)}{\delta \tau} = 0, \tag{21} \]
reflecting objective identification of evolving quantum process. It gives a natural mean for the comparison of LDV at different GCS’s. Field equations will be discussed in the paragraph 5.

5.2 Encoding the results of comparison

The results of the comparison of LDV’s should be formalized by numerical encoding. Thus one may say that “LDV has been measured”. The invariant
encoding is based on the geometry of $CP(N - 1)$ and LDV dynamics, say, dynamics of the local Hamiltonian field. Its affine parallel transport expresses the self-conservation of quantum object associated with “particle” or “unparticle”. In order to build the qubit spinor $\eta$ of the quantum question $\hat{Q}$ two orthogonal vectors \{ $|N>$, $|\Psi>$ \} have been used. Here $|N>$ is the complex normal and $|\Psi>$ tangent vector to $CP(N - 1)$. I will use following qubit spinor

$$\eta = \left( \begin{array}{c} \eta^0_{(\pi^1, \ldots, \pi^{N-1})} \\ \eta^1_{(\pi^1, \ldots, \pi^{N-1})} \end{array} \right) = \left( \begin{array}{c} <N|\hat{\eta}|\Psi> \\ <N|N> \\ <\eta|\hat{\eta}|\Psi> \\ <\eta|\eta> \end{array} \right)$$  \hspace{1cm} (22)

for the measurement of the Hamiltonian $\hat{H}$ at corresponding GCS.

5.3 Quantum boosts and angle velocities

Any two infinitesimally close spinors $\eta$ and $\eta + \delta \eta$ may be formally connected with infinitesimal “Lorentz spin transformations matrix” \[11\]

$$\hat{L} = \begin{pmatrix} 1 - \frac{1}{2} \delta \tau (\omega_3 + ia_3) & -\frac{1}{2} \delta \tau (\omega_1 + ia_1 - i(\omega_2 + ia_2)) \\ -\frac{1}{2} \delta \tau (\omega_1 + ia_1 + i(\omega_2 + ia_2)) & 1 - \frac{1}{2} \delta \tau (-\omega_3 - ia_3) \end{pmatrix}. \hspace{1cm} (23)$$

I have assumed that there is not only formal but dynamical reason for such transition when Lorentz reference frame “follows” for GCS. Then “quantum accelerations” $a_1, a_2, a_3$ and “quantum angle velocities” $\omega_1, \omega_2, \omega_3$ may be found in the linear approximation from the equation $\delta \eta = \hat{L} \eta - \eta$, or, strictly speaking, from its consequence - the equations for the velocities $\xi$ of $\eta$ spinor variations

$$\hat{R} \left( \begin{array}{c} \eta^0 \\ \eta^1 \end{array} \right) = \hat{L} - \hat{I} \frac{\delta}{\delta \tau} \left( \begin{array}{c} \eta^0 \\ \eta^1 \end{array} \right) = \left( \begin{array}{c} \xi^0 \\ \xi^1 \end{array} \right). \hspace{1cm} (24)$$

One should take into account that in the linear approximation the normal component of the qubit spinor does not change, i.e. $\xi^0 = 0$ but tangent component $\xi^1$ subjected the affine parallel transport back to the initial GCS: $\xi^1 = \frac{\delta \eta^1}{\delta \tau} = -\Gamma \eta^1 \frac{\delta \eta}{\delta \tau}$. If one put $\pi = e^{-i\phi} \tan(\theta/2)$ then $\frac{\delta \pi}{\delta \tau} = \frac{\delta \pi}{\delta \theta} \frac{\delta \theta}{\delta \tau} + \frac{\delta \pi}{\delta \phi} \frac{\delta \phi}{\delta \tau}$, where

$$\begin{align*}
\frac{\delta \theta}{\delta \tau} &= -\omega_3 \sin(\theta) - ((a_2 + \omega_1) \cos(\phi) + (a_1 - \omega_2) \sin(\phi)) \sin(\theta/2)^2 \\
&-((a_2 - \omega_1) \cos(\phi) + (a_1 + \omega_2) \sin(\phi)) \cos(\theta/2)^2; \\
\frac{\delta \phi}{\delta \tau} &= a_3 + (1/2)((a_1 - \omega_2) \cos(\phi) - (a_2 + \omega_1) \sin(\phi)) \tan(\theta/2) \\
&-((a_1 + \omega_2) \cos(\phi) - (a_2 - \omega_1) \sin(\phi)) \cot(\theta/2),
\end{align*} \hspace{1cm} (25)$$

then one has linear non-homogeneous system of 6 real equation

$$\begin{align*}
\Re(\hat{R}_{00}\eta^0 + \hat{R}_{01}\eta^1) &= 0, \\
\Im(\hat{R}_{00}\eta^0 + \hat{R}_{01}\eta^1) &= 0, \\
\Re(\hat{R}_{10}\eta^0 + \hat{R}_{11}\eta^1 + \Gamma \eta^1 \frac{\delta \eta}{\delta \tau}) &= 0,
\end{align*}$$
\[ \mathcal{I}(\dot{R}_{10}\eta^0 + \dot{R}_{11}\eta^1 + \Gamma\eta^1 \frac{\delta\pi}{\delta\tau}) = 0, \]

\[ \frac{\delta\theta}{\delta\tau} = F_1, \]

\[ \frac{\delta\phi}{\delta\tau} = F_2, \]

(26)

giving \(\vec{a},\vec{\omega}\) as functions of local coordinates of GCS and 2 real perturbation frequencies \(F_1, F_2\) of coset deformation acting along some geodesic in \(CP(N-1)\). Since \(CP(N-1)\) is totally geodesic manifold \([14]\), each geodesic belongs to some \(CP(1)\) parameterized by single \(\pi\) used above.

Quantum lump takes the place of extended “pointer”. This extended pointer may be mapped onto dynamical space-time if one assumes that transition from one GCS to another is accompanied by dynamical transition from one Lorentz frame to another, see Figure 2. Thereby, infinitesimal Lorentz transformations define small “dynamical space-time” coordinates variations. It is convenient to take Lorentz transformations in the following form

\[
\begin{align*}
ct' &= ct + (\vec{x}\vec{a})\delta\tau \\
\vec{x}' &= \vec{x} + ct\vec{a}\delta\tau + (\vec{\omega} \times \vec{x})\delta\tau
\end{align*}
\]  

(27)

where I put \(\vec{a} = (a_1/c, a_2/c, a_3/c)\), \(\vec{\omega} = (\omega_1, \omega_2, \omega_3)\) \([11]\) in order to have for \(\tau\) the physical dimension of time. The expression for the “4-velocity” \(V^\mu\) is as follows

\[ V^\mu = \frac{\delta x^\mu}{\delta\tau} = (\vec{x}\vec{a}, ct\vec{a} + \vec{\omega} \times \vec{x}). \]

(28)

The coordinates \(x^\mu\) of imaging point in dynamical space-time serve here merely for the parametrization of the energy distribution in the “field shell” arising under “morphogenesis” described by quasi-linear field equations \([10, 17, 18]\).

6 Morphogenesis of the lump and unparticle sectors

The conservation law of local Hamiltonian is expressed by the affine parallel transport (22) in \(CP(N-1)\). This parallel transport provides the “self-conservation” of extended object, i.e. the affine gauge fields couple the soliton-like system \([10, 21]\).

The field equations for the \(SU(N)\) parameters \(\Omega^\alpha\) dictated by the affine parallel transport of the Hamiltonian vector field \(H^i = h\Phi^i_\alpha \Phi_\alpha^i\) (5) read as quasi-linear PDE together with “riccator” describing evolution of GCS

\[
\frac{\delta\Omega^\alpha}{\delta\tau} = V^\mu \frac{\partial\Omega^\alpha}{\partial x^\mu} = -(\Gamma^m_{mn}\Phi^n_\beta + \frac{\partial\Phi^m_\beta}{\partial x^n})\Omega^n_\beta, \quad \frac{d\pi^k}{d\tau} = \Phi^k_\beta\Omega^\beta, \]

(29)

comprising self-consistent system. It is impossible of course to solve this self-consistent problem analytically even in this simplest case of the two state system,
but it is reasonable to develop a numerical approximation in the vicinity of the following exact solution.

Let me discuss initially only quasi-linear PDE obtained as a consequence of the parallel transport of the local Hamiltonian

\[
(\vec{x}_a, c\vec{t}_a + \vec{\omega} \times \vec{x}) \frac{\partial \Omega^\alpha}{\partial x^\mu} = - (\Gamma^m_{mn} \Phi^n_\beta + \frac{\partial \Phi^n_\beta}{\partial m^n}) \Omega^\alpha \Omega^\beta
\]

for two-level system living in \(CP(1)\) \[16, 17, 18\]. In this simplest case of GCS dynamics with coordinate \(\pi = u + iv\) the indexes are as follows: \(1 \leq \alpha, \beta \leq 3\), \(i, k, n = 1\), and the field components \(\Omega^1 = (\omega + i\gamma) \sin \Theta \cos \Phi\), \(\Omega^2 = (\omega + i\gamma) \sin \Theta \sin \Phi\), \(\Omega^3 = (\omega + i\gamma) \cos \Theta\) that should be defined. This system in the case of the spherical symmetry being split into the real and imaginary parts takes the form

\[
\left(\frac{r}{c}\right) \omega_t + c t \omega_r = -2\omega \gamma F(u, v),
\]

\[
\left(\frac{r}{c}\right) \gamma_t + c t \gamma_r = (\omega^2 - \gamma^2) F(u, v),
\]

\[
u_t = \kappa U(u, v, \omega, \gamma),
\]

\[
u_t = \kappa V(u, v, \omega, \gamma),
\]

where \(\kappa\) is a coefficient and \(U(u, v, \omega, \gamma), V(u, v, \omega, \gamma)\) are functions which I avoid to write explicitly here. Let me put \(\omega = \rho \cos \psi\), \(\gamma = \rho \sin \psi\), then, assuming for simplicity that \(\omega^2 + \gamma^2 = \rho^2 = \text{constant}\), the two first PDE’s may be rewritten as follows:

\[
- \frac{r}{c} \psi_t + c t \psi_r = F(u, v) \rho \cos \psi.
\]

The two exact solutions of this quasi-linear PDE is as follows

\[
\psi_1(t, r) = \arctan \frac{\exp(2\rho F(u, v) f(\rho^2 - c^2 t^2))(ct + r)^2 F(u, v) - 1}{\exp(2\rho F(u, v) f(\rho^2 - c^2 t^2))(ct + r)^2 F(u, v) + 1},
\]

and

\[
\psi_2(t, r) = \arctan \frac{2 \exp(\rho F(u, v) f(\rho^2 - c^2 t^2))(ct + r)^2 F(u, v)}{\exp(2\rho F(u, v) f(\rho^2 - c^2 t^2))(ct + r)^2 F(u, v) + 1}.
\]

where \(f(\rho^2 - c^2 t^2)\) is an arbitrary function of the interval. What is the physical interpretation of these solutions may be given? It is interesting that this non-monotonic distribution of the force field \(\psi_1(t, r)\) describing “lump” \[15, 16, 17, 18\] that looks like a bubble in the dynamical space-time. These field equations describes energy distribution in the lump which does not exist a priori but is becoming during the self-interaction, see Figure 3. It should be noted that attempts to treat the field dynamics literally in spirit of “particle in potential” are almost hopeless since we have self-consistent dynamics. The monotonic solution \(\psi_2(t, r)\) looks like unparticle entity corresponding to imaginary field mass \(i\omega(r, t)\).

In order to realize the physical interpretation of these equations I will find the stationary solution for (32). Let me put \(\xi = r - ct\). Then one will get
ordinary differential equation

\[ \frac{d\Psi(\xi)}{d\xi} = -F(u, v)\rho \frac{\cos \Psi(\xi)}{\xi}. \]  

(35)

Two solutions

\[ \Psi(\xi) = \arctan \left( \frac{\xi^{-2M} e^{-2CM} - 1}{\xi^{-2M} e^{-2CM} + 1} \cdot \frac{2\xi^{-M} e^{-2CM}}{\xi^{-2M} e^{-2CM} - 1} \right), \]  

(36)

where \( M = F(u, v)\rho \) are concentrated in the vicinity of the light-cone looks like solitary waves, see Fig.4.

Figure 4: Two solutions of (35) in the light-cone vicinity.

The problem of the physical status of these field equation may be solved if one could point out some transformations from found equations to well known
relativistic, say, Dirac equation. I almost sure that it is impossible to find this transition as perturbation in small parameter. Here I will give only some hints for such transition.

Standard Dirac’s equation

\[ \gamma^\mu \frac{\partial \psi}{\partial x^\mu} + \frac{imc}{\hbar} \gamma^5 \psi = 0 \]  

is linear. Dirac assumed that matrices \( \gamma^\mu \) should be coordinate independent since empty Minkowskian space-time is homogeneous and isotropic. However Dirac’s equation in curved space-time should have coordinate-dependent matrices \( \gamma^\mu(x) \equiv b^\mu_a(x)\gamma^a \), where \( b^\mu_a(x) \) is vierbein defined as follows: \( g^{\mu\nu} = b^\mu_a(x)b^\nu_b(x)\eta_{ab} \) \( \text{[31]} \). It is known that matrices \( \gamma^\mu \) have a sense of instant velocities with modulus \( c \). Quasi-linear equation (30) has similar structure but “4-velocity” \( V^\mu = \frac{dx^\mu}{d\tau} = (\vec{x}, c\vec{d} + \vec{\omega} \times \vec{x}) \) of imaging point evidently depends on coordinates in DST. They serves as parameters of field distribution in the lump and unparticle energy distribution. Probably it is possible to establish some relations between \( V^\mu(x) \) and \( \gamma^\mu(x) \) but presently this connection is unclear.

7 Conclusion

1. Action states serve as “initial” and “final” conditions in fixed setup. Manipulations with quantum amplitudes shows that two setups \( S_1 \) and \( S_2 \) generates generally two different amplitudes \(|S_1\rangle \) and \(|S_2\rangle \) of outcome events.

2. It is reasonable to find physical invariance lurked behind \(|S_1\rangle \) and \(|S_2\rangle \). “Relativity to measuring device” by Fock and “functional relativity” by A. Kryukov express this invariance in the global manner. Since there is no strict definition of the measuring device in terms of standard quantum theory, the special ad hoc example of manipulation with improper states like plane wave and delta-function have been used in order to show functional invariance of state equation before and after measurement.

3. I use flexible setup for transition to local quantum reference frame in super-relativity. Amplitude of outcome event \(|S\rangle\), its GCS and dynamical group \( SU(N) \) with \( N^2-1 \) non-Abelian fields parameters \( \Omega^\alpha \) are main ingredients for objective quantum measurement. Desirable quantum localization is realized in functional space: infinitesimal variation of fields parameters \( \Omega^\alpha \) defines local dynamical variables (LDV) expressed in local coordinates \( \pi^1, ..., \pi^{N-1} \). Non-linear realization of \( SU(N) \) generators by tangent vector field to \( CP(N-1) \) serves for invariant classification of quantum motions and particle/unparticle excitations instead of classification of “elementary” quantum particles.

4. Objective quantum measurement of LDV creates dynamical space-time due to:

a. Comparison of LDV in infinitesimally close GCS is provided by non-Abelian affine gauge field agreed with Fubini-Study metric,

b. Qubit spinor encoding of the result of this comparison whose components are parameterized by quantum boosts and quantum rotations that define dynamics of attached local Lorentz frame in DST.
5. Identification of quantum objects (processes) and its conservation law expresses by parallel transport of local Hamiltonian. Quasi-linear PDE is consequence of this conservation law that generate dynamics of GCS and morphogenesis of “field shell”. Particle and unparticle sectors of these excitations should be classified by comparison with known quantum field equations.

8 Discussion

1. The intrinsically geometric scheme of the quantum measurement of local dynamical variable has been proposed. The self-interaction supporting localizable “lump” configuration arose due to the breakdown of global $G = SU(N)$ symmetry is used for such measurement and it is represented by the affine gauge “field shell” propagated in the dynamical state-dependent space-time.

2. The concept of “super-relativity” \cite{15, 16} is in fact a different kind of attempts of “hybridization” of internal and space-time symmetries. In distinguish from SUSY where a priori exists the extended space-time - “super-space”, in my approach the dynamical space-time arises under “yes/no” quantum measurement of $SU(N)$ local dynamical variables.

3. The locality in the quantum phase space $CP(N - 1)$ leads to extended quantum particles - “field shell” that obey the quasi-linear PDE \cite{16}.

4. The main technical problem is to find non-Abelian gauge field arising from conservation law of the local Hamiltonian vector field. The last one may be expressed as parallel transport of local Hamiltonian in projective Hilbert space $CP(N - 1)$. Co-movable local “Lorentz frame” being attached to GCS is used for qubit encoding result of comparison of the parallel transported local Hamiltonian in infinitesimally close points. This leads to quasi-linear relativistic field equations with soliton-like solutions for “field shell” in emerged DST. The terms “comparison” and “encoding” resemble human’s procedure, but here they have objective content realized in invariant quantum dynamics.

There is a possibility for generalization of scalar in DST “field shell” $\Omega^\sigma \Phi^i_\sigma$ to vector $\Omega^\mu \Phi^i_\mu$, and tensor fields $\Omega^\sigma_{\mu \nu} \Phi^i_\sigma$ assuming invariant contraction in iso-index $\sigma$. Then will arise more complicated field equation with essential dependence of global space-time structure since one need to know metric connection $\Gamma^\lambda_{\mu \nu}$ for covariant derivatives.

5. One need to find connection between quasi-linear field equations and known filed equation (like Dirac equation). Probably it is possible to use some analogy with Skyrmion field quantization \cite{32} although there is of course essential difference between lump and monopole solutions.

6. DST forms granular structure of global space-time and paves a way to build quantum gravity “from inside”.

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