The jet vertex for Mueller-Navelet and forward jet production

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Abstract

We calculate in next-to-leading order BFKL the jet vertex relevant for the production of Mueller-Navelet jets in proton collisions and of forward jets in deep inelastic scattering. Starting from the definition of the totally inclusive quark and gluon impact factors in the BFKL approach and suitably considering the parton densities and the jet selection functions, we show that an infrared-free result can be found for the jet vertex. Finally we compare our expression for the vertex with the previous calculation of Refs. [1].

1 Introduction

The Mueller-Navelet jet production process [2] was suggested as an ideal tool to study the Regge limit of perturbative Quantum ChromoDynamics (QCD)
in proton-proton (or proton-antiproton) collisions. The process under consideration is
\[ p(p_1) + p(p_2) \rightarrow J_1(k_{J,1}) + J_2(k_{J,2}) + X , \]
where two hard jets \( J_1 \) and \( J_2 \) are produced. Their transverse momenta are much larger than the QCD scale, \( k_{J,1}^2 \sim k_{J,2}^2 \gg \Lambda^2_{\text{QCD}} \), so that we can use perturbative QCD. Moreover, they are separated by a large interval of rapidity, \( \Delta y \gg 1 \), which means large center of mass energy \( \sqrt{s} \) of the proton collisions, \( s = 2p_1 \cdot p_2 \gg k_{J,1}^2 \), since \( \Delta y \sim \ln s/k_{J,1}^2 \). Since large logarithms of the energy compensate the small QCD coupling, they must be resummed to all orders of perturbative theory.

The BFKL approach is the most suitable framework for the theoretical description of the high-energy limit of hard or semi-hard processes. It provides indeed a systematic way to perform the resummation of the energy logarithms, both in the leading logarithmic approximation (LLA), which means resummation of all terms \( (\alpha_s \ln(s))^n \), and in the next-to-leading logarithmic approximation (NLA), which means resummation of all terms \( \alpha_s (\alpha_s \ln(s))^n \).

In QCD collinear factorization the cross section of the process reads
\[ \frac{d\sigma}{dJ_1dJ_2} = \sum_{i,j=q,q,\bar{q},g} \int_0^1 \int_0^1 dx_1dx_2 f_i(x_1,\mu)f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1x_2s,\mu)}{dJ_1dJ_2} , \]
with \( dJ_{1,2} = dx_{1,2}d^{D-2}k_{J_{1,2}} \) and the \( i,j \) indices specify parton types (quarks \( q \), antiquarks \( \bar{q} \) or gluon \( g \)); \( f_i(x,\mu) \) denotes the initial proton parton density function (PDF), the longitudinal fractions of the partons involved in the hard subprocess are \( x_{1,2} \), \( \mu \) is the factorization scale and \( d\hat{\sigma}_{i,j}(x_1x_2s,\mu) \) is the partonic cross section for the production of jets, \( \hat{s} = x_1x_2s \) being the energy of the parton-parton collision. In the BFKL approach the resummed cross section of the hard subprocess is represented as the convolution of the jet impact factors of the colliding particles with the Green’s function \( G_\omega \), process-independent and determined through the BFKL equation,
\[ \frac{d\hat{\sigma}}{dJ_1dJ_2} = \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^2} \frac{d\Phi_{J,1}(\vec{q}_1,s_0)}{dJ_1} \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^2} \frac{d\Phi_{J,2}(-\vec{q}_2,s_0)}{dJ_2} \times \int_{\delta+i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \frac{\hat{s}}{s_0} G_\omega(\vec{q}_1,\vec{q}_2) . \]
The aim of this work is to illustrate the calculation of the NLA jet vertex.

Since jets are originated by the hadronization of produced partons, the starting point is the impact factors for colliding partons \(4, 5, 6, 7\). In order to select the partons that will generate the jet, we “open” one of the integrations over the partonic phase space and introduce a suitably defined jet selection function \(S_J\). For the LLA impact factor, where there can be only a one-particle intermediate state, the jet function identifies the jet momentum with the momentum of the one parton \(S_J^{(2)}\). For the NLA impact factor we can have only either a one-particle (virtual corrections) or two-particle intermediate states. In the last case the \(S_J\) function identifies the jet momentum with the momentum of one of the two partons or with the sum of the momenta of two partons \(S_J^{(3)}\).

In the calculation of the jet vertex, infrared divergences related with soft emission will cancel in the sum with virtual corrections. The remaining infrared divergences are taken care of by the PDFs’ renormalization. The collinear counterterms appear due to the replacement of the bare PDFs by the renormalized physical quantities obeying DGLAP evolution equations (in the \(\overline{\text{MS}}\) factorization scheme). Ultraviolet divergences are removed by the counterterm related with QCD charge renormalization (in the \(\overline{\text{MS}}\) scheme).

Starting from the known lowest-order parton impact factors \(4, 5\), corresponding to the totally inclusive process, we get the LLA jet impact factor by suitably introducing the \(S_J^{(2)}\) function

\[
\frac{d\Phi_J^{(0)}(\vec{q})}{dJ} = \Phi_{q}^{(0)} \int_0^1 dx \left( \frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x) \right) S_J^{(2)}(\vec{q}; x),
\]

where \(\Phi_{q}^{(0)} = g^2 \sqrt{\frac{N_c^2-1}{2N_c}}\) is the quark impact factor (defined as the imaginary part of the quark-Reggeon diffusion process) at the Born level and \(\vec{q}\) is the Reggeon momentum.

Substituting here the bare QCD coupling and bare PDFs by the renormalized ones, we obtain the following expressions for the counterterms:

\[
\frac{d\Phi_J(\vec{q})}{dJ}\big|_{\text{charge c.t.}} = \frac{\alpha_s}{2\pi} \left( \frac{1}{\varepsilon} + \ln \frac{\mu^2}{\mu^2} \right) \left( \frac{11C_A}{6} - \frac{N_F}{3} \right) \Phi_{q}^{(0)} 
\times \int_0^1 dx \left( \frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x) \right) S_J^{(2)}(\vec{q}; x)
\]

(5)
for the charge renormalization, and

\[
\frac{d\Phi_J(q)}{dJ}\bigg|_{\text{collinear c.t.}} = -\frac{\alpha_s}{2\pi} \left( \frac{1}{\varepsilon} + \ln \frac{\mu^2}{\mu^2}\right) \Phi_q^{(0)} \int_0^1 d\beta \int_0^1 dx S_J^{(2)}(q; \beta x) \tag{6}
\]

\[
\times \left[ \sum_{a=q,\bar{q}} (P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)) + \frac{C_A}{2F} \left( P_{gg}(\beta)f_g(x) + P_{qg}(\beta) \sum_{a=q,\bar{q}} f_a(x) \right) \right],
\]

for the collinear counterterm.

Now we have all the necessary ingredients to perform our calculation of the NLA corrections to the jet impact factor.

We will consider separately the subprocesses initiated by the quark and the gluon PDFs and denote

\[
V = V_q + V_g \quad \text{with} \quad \frac{d\Phi_J^{(1)}(q)}{dJ} = \frac{\alpha_s}{2\pi} \Phi_q^{(0)} V(q).
\tag{7}
\]

## 2 NLA jet impact factor

### 2.1 The quark contribution

Virtual corrections are the same as in the case of the inclusive quark impact factor

\[
V_q^{(V)}(q) = \frac{\Gamma(1-\varepsilon)}{\varepsilon(4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) S_J^{(2)}(q; x) \tag{8}
\]

\[
\times \left[ C_F \left( \frac{2}{\varepsilon} - 3 \right) - \frac{N_F}{3} + C_A \left( \ln \frac{q_0}{q^2} + \frac{11}{6} \right) \right] + \text{finite terms}.
\]

For the incoming quark case, real corrections originate from the quark-gluon intermediate state. We denote the momentum of the gluon by \( k \), then the momentum of the quark is \( q - k \); the longitudinal fraction of the gluon momentum is denoted by \( \beta x \). Thus, the real contribution has the form

\[
V_q^{(R)}(q) = \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \left\{ \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} \left[ C_F \left( \frac{2}{\varepsilon} - 3 \right) \right] S_J^{(2)}(q; x) \right\}
\]

\[
\times \left[ \sum_{a=q,\bar{q}} (P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)) + \frac{C_A}{2F} \left( P_{gg}(\beta)f_g(x) + P_{qg}(\beta) \sum_{a=q,\bar{q}} f_a(x) \right) \right].
\]
\[
+ \int_0^1 d\beta \left( P_{qq} (\beta) + \frac{C_A}{C_F} P_{qq} (\beta) \right) S^{(2)}_J (\vec{q}; x\beta) \right] + \frac{C_A}{(4\pi)^\varepsilon} \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}
\]

\[
\times \frac{q^2}{\vec{k}^2 (\vec{q} - \vec{k})^2} \ln \frac{s_0}{(|\vec{k}| + |\vec{q} - \vec{k}|^2)} S^{(2)}_J (\vec{q} - \vec{k}; x) \right\} \quad + \text{finite terms}.
\]

2.2 The gluon contribution

Virtual corrections are the same as in the case of the inclusive gluon impact factor \cite{4, 6}:

\[
V^{(V)}_g (\vec{q}) = -\frac{\Gamma [1 - \varepsilon]}{\varepsilon (4\pi)^\varepsilon} \frac{\Gamma^2 (1 + \varepsilon)}{\Gamma (1 + 2\varepsilon)} \int_0^1 dx \frac{C_A}{C_F} f_g (x) S^{(2)}_J (\vec{q}; x) \quad (10)
\]

\[
\times \left[ C_A \ln \left( \frac{s_0}{q^2} \right) + C_A \left( \frac{2}{\varepsilon} - \frac{11}{6} \right) + \frac{N_F}{3} \right] \quad + \text{finite terms}.
\]

In the NLA gluon impact factor real corrections come from intermediate states of two particles, which can be quark-antiquark or gluon-gluon \cite{4, 6, 7}.

We find

\[
V^{(R)}_g (\vec{q}) = \frac{\Gamma [1 - \varepsilon]}{\varepsilon (4\pi)^\varepsilon} \frac{\Gamma^2 (1 + \varepsilon)}{\Gamma (1 + 2\varepsilon)} \int_0^1 dx \frac{C_A}{C_F} f_g (x) \left( \frac{N_F}{3} + \frac{2C_A}{\varepsilon} - \frac{11}{6} \right)
\]

\[
\times \left[ 2N_F P_{qq} (\beta) + 2C_A \frac{C_A}{C_F} \left( P (\beta) + \frac{(1 - \beta)P (1 - \beta)}{(1 - \beta)_+} \right) \right]
\]

\[
\times S^{(2)}_J (\vec{q}; x) \right] + \frac{C_A}{(4\pi)^\varepsilon} \int_0^1 dx \frac{C_A}{C_F} f_g (x) \left( \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{q^2}{\vec{k}^2 (\vec{k} - \vec{q})^2} \ln \frac{s_0}{(|\vec{k}| + |\vec{q} - \vec{k}|)^2} \right)
\]

\[
\times S^{(2)}_J (\vec{q} - \vec{k}; x) \right\} \quad + \text{finite terms}.
\]

To conclude, we collect the contributions given in Eqs. \cite{5}, \cite{6}, \cite{8}, \cite{9}, \cite{10}, \cite{11}, and we note that we are left with two divergences: the last of \cite{9} and of \cite{11}. It easy to see that the integration of those terms over \( \vec{q} \) with any function, regular at \( \vec{q} = \vec{k}_J \), will give a divergence-free result. In particular, a finite result will be obtained after the convolution of the jet vertex with BFKL Green’s function, which is required for the calculation of the jet cross section.

Note that divergent terms of the two parton intermediate state contributions, shown in Eqs. \cite{9} and \cite{11}, are expressed through the jet function \( S^{(2)}_J \), due to reduction of \( S^{(3)}_J \rightarrow S^{(2)}_J \) in the kinematic regions of soft or collinear parton radiation.

More details about this calculation can be found in Ref. \cite{8}.
3 Summary

We have recalculated the jet vertices for the cases of quark and gluon in the initial state, first found by Bartels et al. \[1\]. Our approach is different, since the starting point of our calculation is the known general expression for next-to-leading-order impact factors, given in Ref. \[9\], applied to the special case of partons in the initial state. Nevertheless, in many technical steps we followed closely the derivation of Refs. \[11\].

In our approach the energy scale \(s_0\) remains untouched and need not be fixed at any definite value. In order to compare our results with those of \[11\], we need to perform the transition (see \[10\]) from the standard BFKL scheme with arbitrary energy scale \(s_0\) to the one used in \[11\], where the energy scale depends on the Reggeon momentum. After this procedure, we can see a complete agreement with \[11\].

The jet vertex discussed in this paper is an essential ingredient also for the study of the inclusive forward jet production in deep inelastic scattering in the NLA.

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