We investigate different types of neutrino hot dark matter with respect to structure formation and anisotropies in the cosmic microwave background radiation (CMBR). The possibility of neutrino hot dark matter produced through the decay of a heavier neutrino by the process $\nu_H \rightarrow \nu_L + \phi$, where $\phi$ is a scalar particle, is discussed in detail. This type of dark matter can possibly be distinguished observationally from the standard neutrino dark matter by using new CMBR data from the upcoming satellite missions MAP and PLANCK.

The cold dark matter (CDM) model forms the basis for all the currently viable models of the way structures form in our universe. In its simplest form it consists of a critical ($\Omega = 1$) density universe whose energy density is almost solely ($\Omega_{\text{CDM}} \simeq 0.95$) in the form of non-relativistic non-interacting particles. In addition there is a small fraction of baryons ($\Omega_B \simeq 0.05$). Structures in this model grow from the original quantum fluctuations produced during the inflation era. The trouble with the simple CDM model is that, if the density fluctuation power spectrum is normalised to the CMBR quadrupole anisotropy measured by the COBE satellite, there are far too large fluctuations on small scales. Any realistic model must therefore include something that dampens small scale fluctuations. Several possible solutions to this problem have been proposed, for example the inclusion of vacuum energy as a dominant component or tilting the initial power spectrum from inflation. Perhaps the most promising model is the so-called cold+hot dark matter scenario (CHDM), where a component of hot dark matter (HDM) is included; particles that only become non-relativistic around the epoch of matter-radiation equality. Such light particles free stream and thereby smear out fluctuations below a certain length scale. This produces the desired result, namely that fluctuations on small scales should be lessened relative to the CDM model. A good fit to observational data can be obtained with $\Omega_{\text{HDM}} \simeq 0.25$ in a flat ($\Omega = 1$) universe.

A light (eV) neutrino species seems by far the most realistic candidate for hot dark matter. If only one standard neutrino species is available, its mass should be roughly $5.7$ MeV if $\Omega_{\text{HDM}} \simeq 0.25$ and $h_0 \simeq 0.5$ ($h_0$ is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$). However, numerical simulations have shown that a better fit can be obtained if there are two neutrino species available with half that mass (this model is referred to as 2νCHDM); that is, a mass of roughly $2-3$ eV. This would demand a degenerate mass hierarchy or, alternatively, a four-component eV neutrino such as a light Dirac neutrino. Unfortunately, the sterile components of a light Dirac neutrino do not come into thermal equilibrium after the QCD phase transition and therefore they only contribute very little to the cosmic energy density. A more plausible method of producing neutrino dark matter is through the decay of a heavy neutrino to a final state containing light neutrinos, mimicking a degenerate mass hierarchy.

There are several possible decay channels for a heavy neutrino to produce light neutrinos, for instance the radiative decay $\nu_H \rightarrow \nu_L \gamma$, the flavour violating weak process $\nu_H \rightarrow \nu_L \bar{\nu}_L \nu_L$ or a decay of the type $\nu_H \rightarrow \nu_L \phi$, where $\phi$ is a scalar or pseudo-scalar particle like the majoron. Decays containing electromagnetically interacting particles are strongly constrained by observations so that in essence only decay to an “invisible” final state is allowed. We shall look at the simplest example of such a decay, namely the $\nu_H \rightarrow \nu_L \phi$ process. Specifically we assume $\nu_H = \nu_e$ and $\nu_L = \nu_\mu$. $\nu_e$ is assumed to be massless. A decay of this type occurs naturally in the so-called majoron models. In these models, neutrinos acquire mass by interaction with a new Higgs field and the spontaneous breakdown of the global symmetry results in a Nambu-Goldstone boson called the majoron. In some of these models neutrinos have large Yukawa-type couplings to the majoron so that fast decays of the type $\nu_H \rightarrow \nu_L \phi$ are permitted. The coupling strength is very difficult to constrain experimentally, although a sufficiently large coupling would result in neutrinoless double-beta decays with too short lifetimes. For masses and lifetimes in the range we are interested in there are currently no direct experimental constraints.

Neutrinos are in thermodynamic equilibrium with the cosmic plasma until the universe reaches a temperature of roughly 1 MeV, so that if the decay takes place prior to this freeze-out temperature, the additional density of light neutrinos will be washed out because of thermal equilibration. In the following we shall therefore always assume that the decay takes place subsequent to neutrino decoupling, but prior to matter-radiation equality. In this case the number density of light neutrinos is twice that of a standard decoupled neutrino species, as in the 2νCHDM model. Note that if the decay takes place after decoupling, but at a temperature higher than $T \simeq 0.01$ MeV, the synthesis of light elements may be changed significantly, excluding some regions of mass-lifetime pa-
The purpose of this paper is to investigate observational features of this “decaying neutrino cold+hot dark matter” (DνCHDM) scenario and compare it with the standard 2νCHDM model. There are several differences between this decay scenario and the 2νCHDM model. First of all, the distribution of light neutrinos will in general be non-thermal so that, even though the number density of neutrinos in the two scenarios is equal, the energy density is different. Second, the amount of relativistic energy density present will be different in the two cases. We will discuss the possibility that new CMBR experiments will allow us to distinguish between these different models of hot dark matter. In order to calculate CMBR and matter power spectra we have used the CMBFAST program developed by Seljak and Zaldarriaga [6].

The dynamics of the decay can be quite simply described with only two parameters, the mass, \( m \), and lifetime, \( \tau \), of the heavy neutrino. Using this we can write down the Boltzmann equations for the evolution of distribution functions

\[
\frac{\partial f}{\partial t} - H \rho \frac{\partial f}{\partial \rho} = C_{\text{dec}}[f].
\]

The decay terms are of the form [8]

\[
C_{\text{dec}}[f_H] = -\frac{m_H^2}{\tau m_0 E_H^2} \int_{E_{\phi}^-}^{E_{\phi}^+} dE_{\phi} \Lambda(f_H, f_L, f_{\phi}),
\]

\[
C_{\text{dec}}[f_L] = \frac{g_H m_H^2}{g_L \tau m_0 E_L^2} \int_{E_{\phi}^-}^{E_{\phi}^+} dE_{\phi} \Lambda(f_H, f_L, f_{\phi}),
\]

\[
C_{\text{dec}}[f_{\phi}] = \frac{g_H m_H^2}{g_{\phi} \tau m_0 E_{\phi}^2} \int_{E_{\phi}^-}^{E_{\phi}^+} dE_{\phi} \Lambda(f_H, f_L, f_{\phi}),
\]

where \( \Lambda(f_H, f_L, f_{\phi}) = f_H (1 - f_L)(1 + f_{\phi}) - f_L f_{\phi}(1 - f_H) \) and \( m_0^2 = m_H^2 - 2(m_H^2 + m_L^2) + (m_{\phi}^2 - m_L^2)^2/m_{H}^2 \). \( \tau \) is the lifetime of the heavy neutrino and \( g \) is the statistical weight of a given particle. We use \( g_H = g_L = 2 \) and \( g_{\phi} = 1 \), corresponding to \( \phi = \bar{\phi} \). However, we could equally well have used \( g_{\phi} = 2 \) without changing the final results. Note that in all actual calculations we assume \( m_L = m_\phi = 0 \) when solving the Boltzmann equation (this does not influence the results at all). When calculating transfer functions and CMBR power spectra we of course use the correct value for \( m_L \). The integration limits are

\[
E_{\phi}^\pm(E_i) = \frac{m_0 m_H}{2 m_i^2} \left[ E_i \left(1 + \frac{m_i^2}{m_0^2}\right)^{1/2} \right] \pm p_i,
\]

\[
E_i^\pm(E_H) = \frac{m_0}{2 m_H} \left[ E_H \left(1 + \frac{m_H^2}{m_0^2}\right)^{1/2} \right] \pm p_H,
\]

where \( i = L, \phi \). We can vary the heavy neutrino mass without changing the final particle spectra if we keep constant the “relativity” parameter of the decay, defined as

\[
\alpha \equiv 1/9 \left( m_H/\text{MeV} \right)^2 (\tau_H/s).
\]

This leaves us only one free parameter that controls the decay kinematics and thereby the shape of the particle distributions. The dividing line between relativistic and non-relativistic decay is \( \alpha = 1 \) so that the decay is relativistic if \( \alpha < 1 \) and non-relativistic if \( \alpha > 1 \). In Fig. 1 we show a realisation of this decay model for \( \alpha = 1 \).

**FIG. 1.** The number density of different particle species as a function of temperature for a model with \( \alpha = 1 \). As explained we assume \( \nu_H = \nu_e \) and \( \nu_L = \nu_\mu \). \( \nu_e \) is assumed to be massless. \( n_0 \) is the number density of a standard massless decoupled neutrino species.

**FIG. 2.** The effective temperature, pseudo-chemical potential and amount of relativistic energy density in massless neutrinos and scalar particles. \( T_0 \) is the temperature of a standard massless decoupled neutrino species.
Now, to simplify matters we parametrise the final light neutrino distribution in terms of an equilibrium Fermi-Dirac distribution with an effective temperature, $T$, and a pseudo-chemical potential, $\mu$. In Fig. 2 we show these two parameters in units of the temperature of a standard massless neutrino species. Further, we show the amount of relativistic energy density in massless neutrinos (the energy density in the remaining third neutrino species) and scalars in units of the energy density of a standard neutrino species, $N_{\nu, rel}$. The energy density in light (HDM) neutrinos is not included here because this neutrino becomes non-relativistic around matter-radiation equality. Notice that for relativistic decays $N_{\nu, rel}$ is smaller than two because the energy density in scalars is less than in a standard massless neutrino species. For relativistic decays all these parameters approach constant values indicating that the decay proceeds in equilibrium, as expected. For non-relativistic decays, however, we see that the effective temperature increases while the pseudo-chemical potential drops. This reflects the fact that non-relativistic decays do not proceed in thermodynamic equilibrium and the light neutrinos are born with energies much above thermal. Also, the amount of relativistic energy density increases because the scalar particles, $\phi$, are born with very high energies. In fact, if the light neutrino mass approaches zero and the decay is non-relativistic this scenario approaches the $\tau$CDM model previously discussed in the literature [10,11].

To see how structure formation changes with different decay parameters we have calculated the matter power spectrum in terms of the quantity \[ \Delta^2(k) \equiv \frac{k^3P(k)}{2\pi^2} \], by using the CMBFAST package modified to include a non-equilibrium light neutrino species. We have then compared these spectra with the linear power spectrum data compiled by Peacock and Dodds [12]. Fig. 3 shows the power spectra for different models (note that all models shown in Figs. 3 and 4 have been calculated with $\Omega = 1$, $\Omega_B = 0.05$, $h_0 = 0.5$ and spectral index $n = 1$). It is seen that for relativistic decays and equal $\Omega_\nu$ our $2\nu$CHDM power spectrum closely resembles that of the $2\nu$CHDM scenario. This is not too surprising because the light neutrino distribution is close to thermal in this case. For non-relativistic decay, the difference is pronounced. The power spectrum breaks away from that of the CDM model at lower $k$-values if $\alpha > 1$ than if $\alpha \lesssim 1$ because of the larger free-streaming length of the light neutrinos. Even at large $k$ the power spectrum has less power if $\alpha$ is large. This is because the amount of relativistic energy density is significantly larger if the decay is non-relativistic. The figure also shows that varying $\Omega_\nu$ influences the power spectrum differently than changing $\alpha$, because varying $\Omega_\nu$ changes the amount of CDM and the free-streaming length of light neutrinos, whereas varying $\alpha$ changes the amount of relativistic energy density and the free-streaming, but not the amount of CDM. We have also shown the limiting case of large $\alpha$ and low $\Omega_\nu$, where our model closely resembles the $\tau$CDM model. As already shown in Ref. [11] this model provides a very good fit to the linear power spectrum.

Next, we also look at the power spectrum of CMBR anisotropies produced by these scenarios. It has been known for some time that the CHDM scenarios produce larger CMBR fluctuations than the CDM model and that the peaks in the power spectrum are also shifted [13]. We have calculated the power spectra for our series of models in terms of $C_l$ coefficients, $C_l \equiv \langle |a_{lm}|^2 \rangle$, where the $a_{lm}$ coefficients are defined in terms of the temperature fluctuations as $T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$.

Fig. 4 shows power spectra for several different models, illustrating the differences between the CDM, $2\nu$CHDM and $4\nu$CHDM models. Exactly as with the matter transfer function we see that for relativistic decays, the CMBR power spectrum for the $4\nu$CHDM model is very similar to that for the $2\nu$CHDM scenario, indicating that the decay proceeds roughly in equilibrium. For non-relativistic decays, however, the power spectrum for the $4\nu$CHDM model is markedly different, especially the amplitude of the first Doppler-peak is much larger. The reason is that increasing $\alpha$ corresponds to increasing the relativistic energy density, meaning that recombination occurs much closer to the radiation dominated epoch than in the standard model. As explained for instance in Refs. [13,14], the reason is that in the matter dominated epoch the

![Diagram](image-url)
tons travel through a constant gravitational potential after they are emitted from the last scattering surface. In the radiation dominated epoch this is not the case and if the universe is still close to being radiation dominated at recombination the non-constant gravitational potential increases the CMBR anisotropy, a feature known as the early ISW effect [13]. Changing the HDM content by reasonable amounts in the different models only influences the CMBR power spectra slightly [13].

In conclusion, we have seen that a natural way of producing a 2νCHDM like model is by decay of heavy neutrinos. If the decay proceeds relativistically this 2νCHDM scenario closely resembles the standard 2νCHDM scenario because particle distributions remain roughly thermal. For non-relativistic decays there can be large differences, both in the matter power spectra and in the CMBR fluctuation spectra, compared with the 2νCHDM model. We have also shown that limiting cases for our series of models is, in one end, the CDM model [1], where $Ω_ν → 0$ and $N_{ν,rel}$ is large (non-relativistic neutrino decays with very low light neutrino mass), and in the other extreme the 2νCHDM model (relativistic decay of high light neutrino mass). A decaying neutrino can therefore change structure formation on both small and intermediate scales. Changing the time of matter-radiation equality changes the amount of structure produced on horizon-size scales at this time, whereas changing the amount of neutrino hot dark matter changes the amount of small scale structure. There is some real hope that within the next few years it will become possible to distinguish between these different models. Both because new and improved galaxy surveys like the Sloan Digital Sky Survey [16] are forthcoming and should greatly improve our knowledge of the matter power spectrum, and because the next few years will hopefully see a leap in our understanding of the cosmic background radiation because new missions are scheduled to fly shortly, measuring the angular power spectrum to $l = 1000$ and beyond to great precision [17]. Using this new data it should be possible to constrain structure formation models like the $2ν$CHDM model described here very strongly.

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FIG. 4. CMBR power spectra for CDM, 2νCHDM and two different 2νCHDM models. All the power spectra have been normalised to the quadrupole coefficient $6C_2$.

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