The electronic Casimir-Polder force in a 1D tight-binding nanowire

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We study the effect of two non-interacting impurity atoms near by a one-dimensional nanowire, which is modeled as a tight-binding hopping model. The virtual single-electron hopping between two impurities will induce an additional energy depending on the distance of two impurities, which gives a electronic Casimir-Polder effect. We find that the Casimir-Polder force between the two impurities decreases with the impurity-impurity distance exponentially. And the effects of nanowire and finite temperature on the Casimir-Polder force are also discussed in detail, respectively.

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I. INTRODUCTION

Environment-fluctuation-induced effects are ubiquitous in physics, since a realistic system is inevitably coupled to an environment with very large degrees of freedom. One important effect induced by the fluctuation of the electromagnetic field (EMF) is the Casimir effect, which was predicted by Casimir [1] that there exists an attractive force between two neutral, parallel and perfectly conducting plates placed a few micrometers apart in vacuum. This attractive force between two plates has been verified by numerous experiments [2–8]. In 1956, Lifshitz [9] generalized the Casimir effect induced by the quantum vacuum fluctuation to the case the attractive force between two dielectric plates induced by the thermal fluctuation [10–12]. About ten years later, Boyer [13] first discovered that the Casimir force could be repulsive for a conducting spherical shell. This set off a torrent of theoretical explorations [14–18] on the geometric shape dependence of the Casimir force.

The Casimir force has been extensively applied in physics [19], especially in atomic physics. In 1948, Casimir and Polder [20] calculated the attractive force between two neutral polarizable atoms (and the force between a neutral atom and a perfectly conducting wall) in vacuum called Casimir-Polder (CP) force, which gives a significant correction of the van der Waals-London force [21] for the large atomic separation case. In most of the previous literatures [22–29], researches were focused on the force induced by EMF fluctuation. Recently, Tanaka et al. [30] generalized to the case, where the attractive force between two neutral impurity atoms results from the exchange of virtual electrons, called electronic Casimir-Polder (ECP) force.

In this paper, we study a realistic solid system composed of a one-dimensional (1D) nanowire and two separate impurity atoms. Different from the free-electron gas, the electron traveling in the nanowire described by the tight-binding model has a cosine nonlinear dispersion relation, which will present much more rich physics of interest. Due to the finite energy-band width, the results without divergence are obtain. The ECP force decreases exponentially when the distance between the two charges increases. The decay rate of the ECP force is larger for smaller hopping strength of the nanowire and it increases with the absolute value of the detuning between the atom energy and the site energy of the nanowire. For a fixed impurity-impurity distance, the ECP force is larger for bigger hopping strength and it decreases with the absolute value of the detuning. In the low-energy regime, the cosine dispersion relation can be approximately expanded as a linear quadratic one. When the energy of the impurities is close to the edge of the energy band, we obtain the similar results as presented in Ref. [30]. Additionally, unlike the well-known results for the traditional CP force between two atoms, the ECP force obtained in this system decreases with the temperature.

In the next section, the model Hamiltonian and the nonlinear dispersion relation of the nanowire are presented. In Sec. III, we calculate the electronic Casimir-Polder force between the two impurity atoms. The numerical results are addressed in Sec. IV. In Sec. V, the thermal ECP effect is taken into account. Finally, the summary of our main results is given in Sec. VI.

II. MODEL SETUP AND DISPERSION RELATION

The system considered here is illustrated in Fig. 1. The one-dimensional nanowire consists of $2N + 1$ identical artificial atoms [31] with the same site energy $\varepsilon$ ($\hbar = 1$). The electrons can hop between neighboring atoms, where the hopping strength $J$ between any two nearest-neighbor atoms is the same. The nanowire is described by the typical tight-binding Hamiltonian,

$$H_C = \sum_{j=-N}^{N} \left[ \omega c_j^{\dagger} c_j - J(c_j^{\dagger} c_{j+1} + c_{j+1}^{\dagger} c_j) \right], \quad (1)$$

where $c_j (c_j^{\dagger})$ is the electron annihilation(creation) operator for the $j$th atom.

Two impurity atoms with energies $\varepsilon_1$ and $\varepsilon_2$ are placed on the nanowire at $x = 0$ and $x = R$, respectively. The
impurities are weakly coupled to the nanowire, and the Hamiltonian of the total system is split into two parts

\[ H = H_0 + H_I \] with \( H_0 = H_A + H_C \). Here,

\[ H_A = \varepsilon_1 d_1^\dagger d_1 + \varepsilon_2 d_2^\dagger d_2 \] (2)

is the Hamiltonian of impurities with \( d_i (d_i^\dagger) \) the electron annihilation (creation) operator for the impurity states. The interaction between the impurities and the nanowire is described by

\[ H_I = \lambda_0 (c_0^\dagger d_1 + d_1^\dagger c_0) + \lambda_R (c_R^\dagger d_2 + d_2^\dagger c_R) \] (3)

where \( \lambda_j (j = 0, R) \) is the corresponding coupling constant.

By using the Fourier transform,

\[ c_j = \frac{1}{\sqrt{2N + 1}} \sum_k c_k e^{ikj}, \quad c_j^\dagger = \frac{1}{\sqrt{2N + 1}} \sum_k c_k^\dagger e^{-ikj}, \] (4)

\( H_C \) can be diagonalized as

\[ H_C = \sum_k \Omega_k c_k^\dagger c_k, \] (5)

to obtain a nonlinear dispersion relation \( \Omega_k = \omega - 2J \cos k \). As shown in Fig 1 (b), the eigenmodes of the nanowire form an energy band symmetrically distributed around \( \omega \) with width \( 4J \). Additionally, the interaction Hamiltonian changes into

\[ H_I = g_1 \sum_k (c_k^\dagger d_1 + d_1^\dagger c_k) + g_2 \sum_k (e^{-ikR} c_1^\dagger d_2 + e^{ikR} d_2^\dagger c_k) \] (6)

III. THE ELECTRONIC CASIMIR-POLDER FORCE

Comparing to the conventional CP effect, the two impurity atoms play the role of the neutral atoms and the non-excitated nanowire behaves as the EMF with continuous modes. Due to the finite width of the energy band of the nanowire, we obtain the exact ECP force without divergence.

For simplicity, all the calculations are done in the single-electron subspace. Here, we assume that the energies of the impurities locate below the energy band of the coupled chain, i.e., \( \varepsilon_{1,2} < \omega - 2J \). The eigenstates of \( H_0 \) are as follows:

\[ |1,0;0_k \rangle = d_1^\dagger |0,0;0_k \rangle, \] (7)

\[ |0,1;0_k \rangle = d_2^\dagger |0,0;0_k \rangle, \] (8)

\[ |0,0;1_k \rangle = c_1^\dagger |0,0;0_k \rangle, \] (9)

where \( |0,0;0_k \rangle = |0,0 \rangle \otimes |0_k \rangle \) is the electron vacuum states of whole system.

In order to obtain the interaction between the two impurity atoms, we make the Fröhlich transformation

\[ H_{eff} = e^{-S} H e^S \] (10)

where

\[ S = \sum_k \left( g_1 \left( \frac{d_1^\dagger c_k - c_k^\dagger d_1}{\Omega_k - \varepsilon_1} + \frac{g_2 (e^{-ikR} d_2^\dagger c_k - e^{-ikR} c_k^\dagger d_2)}{\Omega_k - \varepsilon_2} \right) \right) \] (11)

is an anti-Hermitian operator. When coupling strengths \( \lambda_0, \lambda_R \) satisfy \( |\lambda_0, \lambda_R| \ll |\Omega_k - \varepsilon_{1,2}| \), we can obtain the effective Hamiltonian to the second order according to the perturbation theory,

\[ H_{eff} = \left( \varepsilon_1 + \sum_k \frac{g_1^2}{\Omega_k - \varepsilon_1} \right) d_1^\dagger d_1 + \left( \varepsilon_2 + \sum_k \frac{g_2^2}{\Omega_k - \varepsilon_2} \right) d_2^\dagger d_2 \]

\[ \quad + \sum_k \left( \frac{g_1}{\Omega_k - \varepsilon_1} + \frac{g_2}{\Omega_k - \varepsilon_2} \right) c_k^\dagger c_k \]

\[ \quad + \sum_k \frac{g_1 g_2 e^{-ikR}}{2} \left( \frac{1}{\varepsilon_1 - \Omega_k} + \frac{1}{\varepsilon_2 - \Omega_k} \right) d_1^\dagger d_2 \]

\[ \quad + \sum_k \frac{g_1 g_2 e^{-ikR}}{2} \left( \frac{1}{\varepsilon_1 - \Omega_k} + \frac{1}{\varepsilon_2 - \Omega_k} \right) d_2^\dagger d_1. \] (12)

For convenience, we consider the symmetric case in which \( \varepsilon_1 = \varepsilon_2 = \varepsilon_0 \) and \( \lambda_0 = \lambda_R = \lambda \) (i.e., \( g_1 = g_2 = g = \lambda/\sqrt{2N + 1} \)). After diagonalizing the effective Hamiltonian \( H_{eff} \), we can easily get the eigenstates in the single-electron subspace,

\[ |\varphi_+ \rangle = \frac{1}{\sqrt{2}} (|1,0;0_k \rangle + |0,1;0_k \rangle), \] (13)
the ECP force between the two impurity atoms as given by Eq. (20), decreases with the atom-atom distance \( R \) exponentially. It is also dependent on the hopping strength \( J \), the detuning \( \Delta \), and proportional to the square of the impurity-nanowire coupling constant \( \lambda^2 \).

In the preceding section, we obtain the ECP force \( f \), as given by Eq. (20), decreases with the atom-atom distance \( R \) exponentially. It is also dependent on the hopping strength \( J \), the detuning \( \Delta \), and proportional to the square of the impurity-nanowire coupling constant \( \lambda^2 \).

Here, in the discrete lattice system, the ECP force was defined as the difference of the CP energy. Obviously, the ECP force can be rewritten as

\[
f = \frac{\lambda^2}{\Delta} \left[ \frac{1}{\sqrt{1-a^2}} \left( \frac{\sqrt{1-a^2}-1}{a} \right)^R \right] \left[ \frac{1}{\sqrt{1-a^2}} \left( \frac{\sqrt{1-a^2}-1}{a} \right)^{-1} \right]
\]

(20)

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\[
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\]

(21)

where the decay rate \( \Gamma = \ln[a/(\sqrt{1-a^2}-1)] \) and it increases with \( a \). It is ready to find that \( \lim_{a \to 0} \Gamma = \infty \), and then the characteristic length \( R_C = \Gamma^{-1} \) tends to 0, which means that there will be no Casimir-Polder force between the two impurities if the hopping strength of the chain \( J = 0 \).

IV. DISCUSSION ABOUT THE ELECTRONIC CASIMIR-POLDER FORCE

In the preceding section, we obtain the ECP force \( f \), as given by Eq. (20), decreases with the atom-atom distance \( R \) exponentially. It is also dependent on the hopping strength \( J \), the detuning \( \Delta \), and proportional to the square of the impurity-nanowire coupling constant \( \lambda^2 \).

For \( J = 0 \), the CP energy is \( E_{cp}(R) = 0 \) and \( \Gamma \to \infty \) as shown in the former section. That is, there is no interaction between the two impurities when the electrons cannot hop between neighboring atoms. For \( J \neq 0 \), we consider different values of \( J \) to investigate its influence on the force. The variation of the Casimir-Polder force between the two impurity atoms with the atom-atom distance \( R \) for different values of \( J \) is shown in Fig.2(a). The ECP force with \( J = 0.3 \) is represented by the red-triangle line and the one with \( J = 0.4 \) is depicted by the blue-
In Fig. 4, we find that decay rate $\Gamma$ increases with a larger absolute value of the detuning $\Delta$. When the hopping strength $J \to 0$, the ECP force between the two atoms disappears.

In the low-energy regime $k \simeq 0$, the nonlinear dispersion relation $\Omega_k = \omega - 2J \cos k$ can be be rewritten as $\Omega_k \simeq \omega - 2J + Jk^2$, by expanding the cosine function as $\cos k \simeq 1 - k^2/2$ approximately. Then the Casimir-Polder energy is approximated as

$$E_{CP}(R) = 2N + 1 \frac{g^2}{2\pi} \int_{-\pi}^{\pi} \frac{dk}{\varepsilon_0 - (\omega - 2J + Jk^2)}.$$

If the difference of the atom energy and the lower edge of the energy band is much smaller than the width of the energy band $\varepsilon_0 - (\omega - 2J) \ll 4J$, the integral over $k$ can be extended to $(-\infty, \infty)$. Then the Casimir-Polder energy is given by

$$E_{CP} = -\frac{\lambda^2}{2Jb} e^{-bR},$$

where $b = \sqrt{(\omega - 2J - \varepsilon_0)/J}$. The similar results are obtained in Ref. [30], where a quadratic dispersion relation was used.

\section{V. The Electronic Casimir-Polder Force at Finite Temperature}

In the former sections, we only considered the zero temperature case. Here, we will study the influence of the temperature on the ECP force. At finite temperature $T$, the density matrix of the the system can be given as

$$\rho_T = \frac{1}{Z} \left( \sum_{\alpha=\pm} e^{-\beta E_{\alpha}} |\varphi_{\alpha}\rangle \langle \varphi_{\alpha}| + \sum_k e^{-\beta E_k} |\varphi_k\rangle \langle \varphi_k| \right),$$

where $Z = \sum_{\alpha=\pm} \exp(-\beta E_{\alpha}) + \sum_k \exp(-\beta E_k)$ and $\beta = 1/k_B T$ is the inverse temperature. After integrating over $k$, we rewrite the eigenvalues of the effective Hamiltonian $H_{ef}$ in the single-electron space as

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{(Color online) Here, we have taken the lattice constant and the impurity energy as unit $\varepsilon = 1$. And the other parameters are chosen as $\lambda = 0.01$ and $J = 0.6$. (a) The electronic Casimir-Polder forces $f$ vs the atom-atom distance $R$ for different values of $\Delta$ are presented. The red-triangle line and blue-square line depict the cases $\Delta = -2$ and $\Delta = -3$, respectively. (b) The Casimir-Polder force $f$ vs the detuning $\Delta$ with fixed atom-atom distance $R = 1$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{(Color online) The variation of decay rate $\Gamma$ with $a$.}
\end{figure}

\section{Appendix A: The Case of a Single Impurity}

In the case of a single impurity, the density matrix of the system can be given as

$$\rho_T = \frac{1}{Z} \left( \sum_{\sigma=\pm} e^{-\beta E_{\sigma}} |\varphi_{\sigma}\rangle \langle \varphi_{\sigma}| + \sum_k e^{-\beta E_k} |\varphi_k\rangle \langle \varphi_k| \right),$$

where $Z = \sum_{\sigma=\pm} \exp(-\beta E_{\sigma}) + \sum_k \exp(-\beta E_k)$ and $\beta = 1/k_B T$ is the inverse temperature. After integrating over $k$, we rewrite the eigenvalues of the effective Hamiltonian $H_{ef}$ in the single-electron space as
\[ E_+ = \varepsilon_0 + \frac{\lambda^2}{\Delta} \left( \sqrt{1 - a^2} - 1 \right)^R \left( \frac{\sqrt{1 - a^2} - 1}{a} \right) \]  
and

\[ E_- = \varepsilon_0 + \frac{\lambda^2}{\Delta} \left( \sqrt{1 - a^2} - 1 \right)^R \left( \frac{\sqrt{1 - a^2} - 1}{a} \right). \]

Then we obtain the average energy of the system \( E_T = \text{Tr}(\rho H_{\text{eff}}) \) at temperature \( T \) as

\[ E_T = \frac{1}{Z} \{ E_+ e^{-\beta E_+} + E_- e^{-\beta E_-} + \sum_k E_k e^{-\beta E_k} \}. \]

The thermal ECP force is defined as

\[ f_T = -[E_T(R + 1) - E_T(R)]. \]

Evidently, in the zero-temperature limit, the thermal ECP\(^{28}\) returns to\(^{20}\) obtain in Sec. III. We plot the variation of the ECP force \( f \) with \( R \) for different values of \( T/\varepsilon \) to investigate the influence of temperature on the ECP force in Fig. 5. The values of \( T/\varepsilon \) represented by the red-triangle line, blue-square and green-star line are 0, 0.1, and 1, respectively. We see clearly that for a fixed value of \( R \), the ECP force decreases with the temperature \( T \). As is seen in Eqs. \( 25 \) and \( 26 \), the \( R \)-dependent potential induced by the nanowire is attractive when the system is in the state \( |\varphi_+\rangle \), while the potential is repulsive when the system is in the state \( |\varphi_-\rangle \). So the ECP force is weaker at nonzero temperature than at zero temperature. When the temperature increases, the probability of the system in the state \( |\varphi_-\rangle \) becomes larger and the probability in the state \( |\varphi_+\rangle \) decreases, so the ECP force decreases with temperature. Obviously, this result is different with the well-known results for the Casimir-Polder force between two atoms at finite temperature\(^{32}\).

VI. CONCLUSION

We studied the ECP force between two non-interacting impurity atoms near by a one-dimensional nanowire. In the single-electron space, this ECP force the moving electron in the nanowire induced is attractive. Different from the free-electron gas, the electron traveling in the nanowire described by the tight-binding model has a non-linear dispersion relation. Based on the perturbation theory, we obtained the analytical expression of the force without any divergence.

Our results show that the force between the two impurities decreases with the impurity-impurity distance exponentially. The decay rate decreases when the hopping strength of the nanowire increases, while it becomes larger when the absolute value of the detuning between the atom energy and the site energy of the nanowire increases. When the hopping strength equals to zero, the decay rate becomes infinite large and then the force between the two atoms disappears. We find that the force induced by the nanowire also increases with the hopping strength and decreases with the absolute value of the detuning. Based on this result, we may use the nanowire to control the CP force in practical application.

Besides, we also studied the ECP force at finite temperature. We find that the force at nonzero temperature is weaker than that at zero temperature and it decreases with the temperature, which is different with the well-known results for the traditional CP force between two atoms at finite temperature\(^{32}\). We wish this phenomenon could be verified by experiments.

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[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51, 793 (1948).

[2] P. H. G. M. van Blokland and J. T. G. Overbeek, J. Chem. Soc. Faraday Trans. 74, 2637 (1978).
[3] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997).
[4] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998).
[5] A. Roy, C. Y. Lin, and U. Mohideen, Phys. Rev. D **60**, 111101 (1999).
[6] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002).
[7] F. Chen, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Phys. Rev. A **69**, 022117 (2004).
[8] R. S. Decca, D. López, H. B. Chan, E. Fischbach, D. E. Krause, and C. R. Jamell, Phys. Rev. Lett. **94**, 240401 (2005).
[9] E. M. Lifshitz, Sov. Phys. JETP **2**, 73 (1956).
[10] M. Boström and B. E. Sernelius, Phys. Rev. Lett. **84**, 4757 (2000).
[11] Y. Sherkunov, Phys. Rev. A **79**, 032101 (2009).
[12] A. Erdas and K. P. Seltzer, Phys. Rev. D **88**, 105007 (2013).
[13] T. H. Boyer, Phys. Rev. **174**, 1764 (1968).
[14] W. Lukosz, Physica (Amsterdam) **56**, 109 (1971).
[15] R. Golestanian and M. Kardar, Phys. Rev. A **58**, 1713 (1998).
[16] T. Emig, A. Hanke, and M. Kardar, Phys. Rev. Lett. **87**, 260402 (2001).
[17] O. Schroeder, A. Scardicchio, and R. L. Jaffe, Phys. Rev. A **72**, 012105 (2005).
[18] T. Emig, A. Hanke, R. Golestanian, and M. Kardar, Phys. Rev. Lett. **95**, 250402 (2005).
[19] M. Bordag, U. Mohideen, V. M. Mostepanenko, Phys. Rep. **353**, 1 (2001).
[20] H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
[21] F. London, Z. Phys. **60**, 491 (1930).
[22] J.M. Wylie and J.E. Sipe, Phys. Rev. A **30**, 1185 (1984); S.-T. Wu and C. Eberlein, Proc. R. Soc. London, Ser. A **455**, 2487 (1999); M. S. Tomaš, Phys. Rev. A **72**, 034104 (2005).
[23] G. Barton, Proc. R. Soc. London, Ser. A **410**, 141 (1987).
[24] S. Shresta, B. L. Hu, and N. G. Philips, Phys. Rev. A **68**, 062101 (2003).
[25] M. Antezza, L. P. Pitaevskii, and S. Stringari, Phys. Rev. A **70**, 053619 (2004).
[26] L. Rizzuto, Phys. Rev. A **76**, 062114 (2007).
[27] T. Tian, T. Y. Zheng, Z. H. Wang, and X. Zhang, Phys. Rev. A **82**, 033810 (2010).
[28] H. Yang, T.Y. Zheng, X.Q. Shao, X. Zhang and S.M. Pan, Phys. Lett. A **377**, 1693 (2013); H. Yang, T.Y. Zheng, X. Zhang, X.Q. Shao and S.M. Pan, ANN. Phys. (2013).
[29] W. J. Nie, Y. H. Lan, Y. Li, and S. Y. Zhu, Phys. Rev. A **88**, 063849 (2013).
[30] S. Tanaka, R. Passante, T. Fukuta, and T. Petrosky, Phys. Rev. A **88**, 022518 (2013).
[31] K. Bhattacharjee, A. Roy, K. Kundu, and B. V. Dev, Phys. Rev. B **77**, 115431 (2008).
[32] R. Passante and S. Spagnolo, Phys. Rev. A **76**, 042112 (2007).