A NEW SEARCH TECHNIQUE FOR SHORT ORBITAL PERIOD BINARY PULSARS

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ABSTRACT

We describe a new and efficient technique, which we call sideband or phase-modulation searching, that allows one to detect short-period binary pulsars in observations longer than the orbital period. The orbital motion of the pulsar during long observations effectively modulates the phase of the pulsar signal, causing sidebands to appear around the pulsar spin frequency and its harmonics in the Fourier transform. For the majority of binary radio pulsars or low-mass X-ray binaries (LMXBs), large numbers of sidebands are present, allowing efficient searches using Fourier transforms of short portions of the original power spectrum. Analysis of the complex amplitudes and phases of the sidebands can provide enough information to solve for the Keplerian orbital parameters. This technique is particularly applicable to radio pulsar searches in globular clusters and searches for coherent X-ray pulsations from LMXBs and is complementary to more standard “acceleration” searches.

Subject headings: binaries: close — methods: data analysis — pulsars: general — radio continuum: stars — stars: neutron

1.INTRODUCTION

Short orbital period binary pulsars have proved to be sensitive laboratories that allow tests of a variety of physical processes including theories of gravitation. Unfortunately, except for those systems containing the brightest pulsars, they are notoriously difficult to detect. When the observation time $T_{obs}$ is greater than a small fraction of the orbital period $P_{orb}$, Doppler effects due to the orbital motion of the pulsar cause drastic reductions in the sensitivities of pulsar searches (Johnston & Kulkarni 1991).

Numerous groups (e.g., Middleditch & Kristian 1984; Anderson et al. 1990; Wood et al. 1991) have developed variations of so-called acceleration searches in order to mitigate the loss in sensitivity caused by orbital motion. These searches can almost completely recover this lost sensitivity when $T_{obs} \leq P_{orb}/10$ by taking advantage of the fact that orbital motion causes an approximately linear drift of the pulsar spin frequency $f_{spin}$. Recent acceleration searches of globular clusters have discovered numerous binary systems, some with orbital periods as short as ~90 minutes (Camilo et al. 2000; Ransom et al. 2001; D’Amico et al. 2001).

Accretion searches are effective only for $T_{obs} \leq P_{orb}/10$, but search sensitivities improve as $T_{obs}^{1/2}$. In effect, acceleration searches are limited to discovering only the brightest binary pulsars in ultracompact—systems with orbital periods less than a few hours—orbits. For a 2 ms pulsar in a 1 hr orbit with a low-mass white dwarf (WD) companion, the optimal observation time $T_{best}$ for an acceleration search (using eq. [22] from Johnston & Kulkarni 1991) is only ~300–600 s. If this pulsar were located in a globular cluster, where observations of 5–10 hr in duration are common, acceleration searches would be a factor of $(T_{obs}/T_{best})^{1/2} \sim 8$ times less sensitive than an optimal (i.e., coherent) search of a full observation.

We have developed an efficient search technique for short orbital period binary pulsars that complements the use of acceleration searches. Since it requires $T_{obs} \geq 1.5 P_{orb}$ and its sensitivity improves as the ratio $T_{obs}/P_{orb}$ increases, it allows the search of very interesting portions of orbital parameter space for the first time. This technique was developed and tested over the past several years as part of a Ph.D. thesis (Ransom 2001). A similar but independent derivation of many of the properties of this technique has been recently published by Jouteux et al. (2002), based on an earlier mention of this work in Ransom (2000). While the basic search techniques presented by these papers are nearly identical, aspects concerning the determination of the orbital elements of a candidate pulsar discovered with this technique are presented in more detail in this work.

2. PHASE MODULATION DUE TO ORBITAL MOTION

During observations of a binary pulsar in which $T_{obs}$ is longer than $P_{orb}$, the orbital motion causes a modulation of the phase of the pulsar signal. Differences in light travel time across the projected orbit advance or delay the pulse phase in a periodic fashion.

2.1. Circular Orbits

If the orbit is circular the phase delays caused by the motion of the pulsar are sinusoidal. The fundamental harmonic5 of the resulting signal as sampled at a telescope can be described by

$$n_j = n(u) = a \cos \left[2\pi f_{spin} u + \phi_{spin} + \Phi_{orb} \cos \left(2\pi f_{orb} u + \phi_{orb}\right) \right] ,$$

where $a$ and $\phi_{spin}$ are the amplitude and phase (in radians). 5 The following analysis can be applied to any harmonic $h$ of a pulsar’s signal by substituting $hr_{spin}$ for $r_{spin}$ and the appropriate harmonic phase for $\phi_{spin}$. 

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of the pulsation, $\Phi_{\text{orb}}$ and $\phi_{\text{orb}}$ are the amplitude and phase of the modulation due to the orbit, $r_{\text{spin}}$ and $r_{\text{orb}}$ are Fourier frequencies (i.e., $r = fT_{\text{obs}} = T_{\text{orb}}/P$), and $u = j/N = (t_j - t_0)/T_{\text{obs}}$ is the discrete dimensionless time during the observation of the $j$th sample of $N$ such that $0 \leq u \leq 1$. The phase modulation term causes a periodic change in the measured frequency of the pulsations of amplitude $\Delta f \approx f_{\text{spin}}v_{\text{los}}/c = \Phi_{\text{orb}}/f_{\text{orb}}$, where $v_{\text{los}}$ is the line-of-sight velocity of the pulsar. We can write $\Phi_{\text{orb}}$ and $\phi_{\text{orb}}$ in terms of the three Keplerian orbital parameters for circular orbits $[P_{\text{orb}}, \text{the projected semimajor axis } x_{\text{orb}} = (a_1 \sin i)/c$, and the time of the ascending node $T_{\text{asc}}$ as

$$\Phi_{\text{orb}} = 2\pi x_{\text{orb}} f_{\text{spin}} \approx 7.375 f_1^{1/3} \frac{P_{\text{orb}}^{2/3}}{P_{\text{spin}}(s)}$$  \hspace{1cm} (2)

and

$$\phi_{\text{orb}} = 2\pi \frac{t_0 - T_{\text{asc}}}{P_{\text{orb}}} + \frac{\pi}{2},$$  \hspace{1cm} (3)

where the units of both are radians, and the pulsar’s mass function $f_1$ is defined as

$$f_1 = \left(\frac{M_2 \sin i}{M_1 + M_2}\right)^{1/3},$$  \hspace{1cm} (4)

where $i$ is the orbital inclination and $M_1$ and $M_2$ are the pulsar and companion masses (in solar masses), respectively.

The discrete Fourier transform (DFT) of the signal in equation (1) when $\Phi_{\text{orb}} = 0$ can be approximated as (see, e.g., Ransom, Eikenberry, & Middleitch 2002)

$$A_1(r) = \sum_{j=0}^{N-1} n_j e^{-2\pi i j r/N} \approx N \int_0^1 n(u) e^{-2\pi i u} du$$  \hspace{1cm} (5a)

$$\approx \frac{aN}{2} e^{i\phi_{\text{spin}}} \int_0^1 e^{2\pi i (r_{\text{spin}} - r)u} du$$  \hspace{1cm} (5b)

$$= A_{1,\text{coherent}} e^{i\pi(r_{\text{spin}} - r)} \frac{\sin \left[\pi(r_{\text{spin}} - r)\right]}{\pi(r_{\text{spin}} - r)},$$  \hspace{1cm} (5c)

where $A_{1,\text{coherent}} = (aN/2) \exp(i\phi_{\text{spin}})$ represents the coherent Fourier response at the spin frequency $r_{\text{spin}}$. Similarly, we can approximate the DFT of our phase modulated signal from equation (1) as

$$\begin{align*}
A_1(r) &= \frac{aN}{2} \int_0^1 e^{[2\pi r_{\text{spin}}u + \phi_{\text{orb}} \cos(2\pi r_{\text{orb}}u + \phi_{\text{orb}})]} e^{-2\pi i u} du \\
&= A_{1,\text{coherent}} \int_0^1 e^{i\Phi_{\text{orb}} \cos(2\pi r_{\text{orb}}u + \phi_{\text{orb}})} e^{2\pi i (r_{\text{spin}} - r)u} du.
\end{align*}$$  \hspace{1cm} (6a)

Using the Jacobi-Anger expansion (e.g., Arfken 1985)

$$e^{i\gamma \cos(\theta)} = \sum_{m=-\infty}^{\infty} i^m J_m(\gamma) e^{im\theta},$$  \hspace{1cm} (7)

where $J_m(z)$ is an integer-order Bessel function of the first kind, we can expand the integrand in equation (6b) to give

$$\begin{align*}
A_1(r) &= A_{1,\text{coherent}} \sum_{m=-\infty}^{\infty} \left\{ J_m(\Phi_{\text{orb}}) \int_0^1 e^{i\frac{m^2}{2r_{\text{orb}}^2}\pi u} e^{2\pi i (r_{\text{spin}} - r)u} du \right\} \\
&= A_{1,\text{coherent}} \sum_{m=-\infty}^{\infty} \left\{ J_m(\Phi_{\text{orb}}) e^{im(\phi_{\text{orb}} + \pi/2)} \right. \\
&\quad \left. \int_0^1 e^{2\pi i (r_{\text{spin}} + m r_{\text{orb}} - r)u} du \right\}.
\end{align*}$$  \hspace{1cm} (8a)

Since the integrand in equation (8b) is identical in form to that in equation (5b), the Fourier transform of a phase modulated signal is equivalent to the Fourier transform of a series of cosinusoids at frequencies centered on the pulsar spin frequency $r_{\text{spin}}$, but separated from $r_{\text{spin}}$ by $m r_{\text{orb}}$ (i.e., sidebands). The response is therefore

$$\begin{align*}
A_1(r) &= A_{1,\text{coherent}} \sum_{m=-\infty}^{\infty} \left\{ J_m(\Phi_{\text{orb}}) e^{im(\phi_{\text{orb}} + \pi/2)} \right. \\
&\quad \left. \times e^{i\pi(m r_{\text{spin}} + m r_{\text{orb}} - r)} \sin \left[\pi(m r_{\text{spin}} + m r_{\text{orb}} - r)\right] \pi(m r_{\text{spin}} + m r_{\text{orb}} - r) \right\}.
\end{align*}$$  \hspace{1cm} (8b)

When $r_{\text{orb}}$ is an integer (i.e., the observation covers an integer number of complete orbits), the Fourier response shown in equation (9) becomes particularly simple at Fourier frequencies $r_{\text{spin}} + s r_{\text{orb}}$, where $s$ is an integer describing the sideband in question. In this case, the summation collapses to a single term when $m = s$, giving

$$A_1(r_{\text{spin}} + s r_{\text{orb}}) = A_{1,\text{coherent}} J_s(\Phi_{\text{orb}}) e^{is(\phi_{\text{orb}} + \pi/2)}.$$  \hspace{1cm} (10)

In effect, the phase modulated signal produces a series of sidebands split in frequency from $r_{\text{spin}}$ by $s r_{\text{orb}}$, with amplitudes proportional to $J_s(\Phi_{\text{orb}})$ and phases of $s(\phi_{\text{orb}} + \pi/2) + \phi_{\text{spin}}$ rad. Figure 1 shows the Fourier response for a “typical” pulsar-WD system in a circular 50 minute orbit during an 8 hr observation.

While the number of sidebands implied by equation (9) is infinite (assuming $\Phi_{\text{orb}} \neq 0$), properties of the Bessel functions—such as the moduli of the sidebands—give a rather sharp cutoff at $\sim 2\Phi_{\text{orb}}$ sidebands (or $\sim 2\Phi_{\text{orb}}$ pairs of sidebands). The maximum value of $J_s(\Phi_{\text{orb}})$ occurs near $s = \Phi_{\text{orb}}$ at a value of $\sim 0.675 J_0(1)$, which gives the phase modulation response its distinctive “horned” shape when $\Phi_{\text{orb}}$ is large (see Fig. 1). The average value of the amplitudes of the phase-modulation sidebands falls off more quickly as $\sim 0.611 J_0(1/2)$ (see Fig. 2).

For pulsars with narrow pulse shapes and many Fourier harmonics, phase modulation produces sidebands split by $r_{\text{orb}}$, Fourier bins centered on each spin harmonic. For spin harmonic number $h$, the phase-modulation amplitude is $\Phi_{\text{orb},h} = 2\pi x_{\text{orb}} h f_{\text{spin}}$ rad. Therefore, for large $\Phi_{\text{orb},h}$, there

Note that $J_n(z) = (-1)^n J_{-n}(z)$, and also that $J_n(z)$ can be negative. If the sideband amplitude determined by the Bessel function is negative [i.e., $J_n(z)$ is negative], the measured phase of the Fourier response will differ by $\pi$ rad from the predicted phase since the measured amplitude is always positive.
are \( \sim \hbar \) times more sidebands than around the fundamental, but the average amplitudes are smaller by a factor of \( \sim \hbar^{1/2} \), since the sum of the squares of the Bessel functions is unity (see § 4).

2.2. Small-Amplitude Limit

When \( \Phi_{\text{orb}} \ll 1 \) rad, a phase-modulated signal will have only three significant peaks in its Fourier response. These peaks are located at \( r_{\text{spin}} \) and \( r_{\text{spin}} \pm r_{\text{orb}} \) and have magnitudes proportional to \( J_0(\Phi_{\text{orb}}) \sim 1 - \Phi_{\text{orb}}^2/4 \sim 1 \) and \( J_{\pm 1}(\Phi_{\text{orb}}) \sim \pm \Phi_{\text{orb}}/2 \), respectively.

Middleditch et al. (1981) performed a detailed analysis of phase-modulated optical pulsations in the small amplitude limit from the 7.7 s X-ray pulsar 4U 1626–67 in order to solve for the orbital parameters of the system. The separation of the sidebands provided a measurement of the \( \sim 42 \) minute orbital period, and the magnitudes and phases of three significant Fourier peaks allowed them to solve for \( \Phi_{\text{orb}} \) (and therefore \( x_{\text{orb}} \)) and \( \phi_{\text{orb}} \) directly.

Small-amplitude phase modulation might also be observed from freely precessing neutron stars (e.g., Nelson, Finn, & Wasserman 1990; Glendenning 1995). For certain geometries, the geometric wobble of the pulsar beam would periodically modulate the pulse arrival times and cause the system to appear as a compact binary. If such rapidly precessing objects exist, they would display a small number (\( \sim 1–3 \)) of pairs of sidebands in a power spectrum of a long observation, since the maximum possible phase modulation for such a system is \( \Phi_{\text{orb}} \lesssim \pi/2 \) rad (Nelson et al. 1990).

2.3. Elliptical Orbits

Konacki & Maciejewski (1996) derived a Fourier expansion for elliptical orbital motion with the useful property that the magnitudes of higher order harmonics of the orbital expansion (which correspond to independent modulation amplitudes \( \Phi_{\text{orb},s} \)) decrease monotonically. In the limit of circular orbits the expansion reduces to the single-phase modulating sinusoid found in equation (1).
Each harmonic of the orbital modulation generates sidebands around the spin harmonic of the form discussed in §2.1. The sidebands created by harmonic $h'$ of the orbital expansion are separated from each other by $h'r_{\text{orb}}$ Fourier bins. Since $\Phi_{\text{orb},h'}$ decreases monotonically with increasing orbital harmonic $h'$, the number of sidebands generated decreases while their spacing and average amplitudes increase. The most important aspect of this superposition of sidebands is the fact that the Fourier response near a spin harmonic contains sidebands spaced by $r_{\text{orb}}$ Fourier bins—from the fundamental of the orbital expansion—just as in the limiting case of circular orbits.

3. PULSARS WITH DETERMINISTIC PERIODIC PHASE MODULATION

Figures 3 and 4 show $P_{\text{spin}}$, $P_{\text{orb}}$, and $\Phi_{\text{orb}}$ for 58 binary pulsars from the literature, all with orbital periods less than 10 days. A few conclusions can be reached immediately. First, radio millisecond pulsars (MSPs) are the most common binary pulsar. Second, there is a relatively flat distribution of $P_{\text{orb}}$ for radio pulsars, which cuts off quite dramatically near $P_{\text{orb}} \sim 0.08$ days (~2 hr)—the two systems with shorter orbital periods are the recently discovered accreting X-ray MSPs XTE J1751$-314$ (Markwardt et al. 2002) and XTE J0929$-314$ (Galloway et al. 2002). Third, a large fraction of the systems are located in globular clusters.

The fact that large numbers of relatively compact binary pulsar systems should contain MSPs is not surprising given the standard “recycling” model for MSP creation (Verbunt 1993; Phinney & Kulkarni 1994). In fact, the recycling scenario also seems to explain why many of the compact binary MSPs have low-mass (~0.02–0.4 $M_\odot$) companion stars in circular orbits. However, observations of globular clusters—which is where most of these compact systems reside—have shown what seems to be an overabundance of MSPs relative to the progenitor LMXBs. This “birthrate problem” may require substantial modification to the standard recycling scenario and/or entirely different formation mechanisms for MSPs in clusters. Regardless of how they are formed, however, the compact binary MSP systems make up the vast majority of the pulsars shown in Figure 3 and have typical values of $\Phi_{\text{orb}}$ of $10^6$ to a few times $10^7$ rad.

The cutoff in $P_{\text{orb}}$ at approximately 2 hr is almost certainly due in part to selection effects. The rapid orbital motion of these systems prevents their detection using conventional search techniques—including acceleration searches—for all but the brightest systems (Johnston & Kulkarni 1991). In fact, the only MSPs discovered with shorter orbital periods ($P_{\text{orb}} \sim 40$ minutes for both XTE J1751$-314$ and XTE J0929$-314$) were discovered during transient X-ray outbursts in which the pulsed fractions were very high (Markwardt et al. 2002; Galloway et al. 2002). Since selection effects keep us from detecting ultracompact systems that are not exceptionally bright, it is difficult to determine how much the observed lack of such systems is due to a true lack of compact systems. It is certain that neutron star (NS) systems with much shorter orbital periods do exist, as the new accreting X-ray MSPs and other ultracompact LMXBs such as 4U 1820$-303$ ($P_{\text{orb}} \sim 11$ minutes), 4U 1850$-087$ ($P_{\text{orb}} \sim 21$ minutes), 4U 1627$-673$ ($P_{\text{orb}} \sim 42$ minutes), and 4U 1916$-053$ ($P_{\text{orb}} \sim 50$ minutes) (Chou, Grindlay, & Bloser 2001; Liu, van Paradijs, & van den Heuvel 2001) have shown.

The large fraction of known systems inhabiting globular clusters can be explained by three basic facts. First, because
of their known position on the sky, long targeted observations of clusters are possible, which allows fainter systems to be detected (e.g., Anderson 1992). Second, repeated observations of certain clusters have allowed many very weak pulsars to be detected with the help of amplification from interstellar scintillation (Camilo et al. 2000; Possenti et al. 2001). Finally, dynamical interactions resulting from the high stellar density in globular clusters are expected to produce substantial numbers of binary pulsars (e.g., Kulkarni & Anderson 1996; Davies & Hansen 1998; Rasio, Pfahl, & Rappaport 2000).

Recently, Rasio et al. (2000) have conducted extensive population synthesis studies for binary pulsars in dense globular clusters like 47 Tucanae. Their initial results imply that exchange interactions among primordial binaries will produce large numbers of NSs in binary systems. In fact, their results predict that large populations of binary pulsars with low-mass companions ($m_c \sim 0.05 M_\odot$) should exist in many globular clusters with orbital periods as short as $\sim 15$ minutes.

The evidence seems to suggest that a large population of ultracompact binary pulsars should exist—particularly in globular clusters—that have so far eluded detection. With long observations, these pulsars will be detected if search algorithms can identify the distinctive sidebands generated by the systems. The rest of this paper discusses such an algorithm.

4. Sideband Searches

The fact that power from an orbitally modulated sinusoid is split into multiple sidebands allows us to increase the signal-to-noise ratio of the detected response by incoherently summing the sidebands. According to one of the addition theorems of Bessel functions,

$$\sum_{m=-\infty}^{\infty} J_m(z)^2 = 1,$$

theoretically all of the signal power can be recovered in this manner for noiseless data. Practically, however, complete recovery of signal power is not possible for data with a finite signal-to-noise ratio—especially if $\Phi_{\text{orb}} \gtrsim 1$ radian. Such a summation would require calculation of the $\sim 2\Phi_{\text{orb}}$ significant sidebands at the precise Fourier frequencies of their peaks $r_{\text{spin}} \pm sr_{\text{orb}}$. This computationally daunting task would result in very large numbers of independent search trials when $\Phi_{\text{orb}}$, $r_{\text{spin}}$, and $r_{\text{orb}}$ are unknown.

Even if we could recover all of the power using a sum of the sidebands, the significance of the measurement would be less than the significance of a nonmodulated sinusoid with the same Fourier power. This loss in significance is due to the fact that noise is co-added along with signal. The exact loss in significance can easily be calculated, since the probability for a sum of $m$ noise powers to exceed some power $P_{\text{sum}}$ is the probability for a $\chi^2$ distribution of $2m$ degrees of freedom to exceed $2P_{\text{sum}}$ (see, e.g., Groth 1975 and references therein).

In the case where only a few sidebands are suspected to be present (i.e., $\Phi_{\text{orb}} \lesssim 5$ rad), a brute-force search using incoherent sideband summing may be computationally feasible and worthwhile. Unfortunately, except for binary X-ray pulsars with long spin periods and possibly certain freely precessing NSs (see § 2.2), the number of astrophysical systems with $\Phi_{\text{orb}} \lesssim 5$ rad is probably small compared to those with $\Phi_{\text{orb}} \gtrsim 1$ radian (see § 3).

4.1. Two-Staged Fourier Analysis

For systems with $\Phi_{\text{orb}} \gtrsim 1$ radian, the large number of regularly spaced sidebands allows a (relatively) very efficient detection scheme. Since the sidebands are separated from each other by $r_{\text{orb}}$ Fourier bins, they appear as a localized periodicity in the power spectrum of the original time series, which can be detected with a second stage of Fourier analysis. By stepping through the full-length power spectrum $P_1(r)$ and taking short fast Fourier transforms (FFTs) of the powers, we incoherently sum any sidebands present and efficiently detect their periodic nature. As an added benefit, the measured “frequency” of these “sideband pulsations” gives a direct and accurate measurement of the orbital period.

We define the following notation to represent the two distinct stages of the Fourier analysis for detecting a phase-modulated signal. As defined in equation (1), the initial time series $n_j$ contains $N$ points. After Fourier transforming the $n_j$ (usually using an FFT of length $N$), the complex responses at Fourier frequencies $r$ are represented by $A_1(r)$ as defined in equation (5a). The powers and phases are then simply $P_1(r)$ and $\phi_1(r)$, respectively. The short FFTs of the $P_1(r)$ used to detect the sideband pulsations (i.e., the secondary Fourier spectrum) are of length $M$ (where $M \ll N$ and should closely match the extent of the sideband structure in the original power spectrum) and generate complex responses $A_2(r)$, powers $P_2(r)$, and phases $\phi_2(r)$.

In order for a pulsed signal to undergo enough phase modulation to produce periodic sidebands, the observation must contain more than one complete orbit (i.e., $r_{\text{orb}} > 1$). One might initially think that two complete orbits would be necessary to create a Nyquist sampled series of sidebands in the original power spectrum. However, since the sideband pulsations are not bandwidth limited, periodic signals with “wavelengths” less than two Fourier bins (i.e., $2r_{\text{orb}} > 1$) will still appear in the power spectrum of the sidebands $P_2(r)$. Instead of appearing at Fourier frequency $r = M/r_{\text{orb}}$, as for signals with $r_{\text{orb}} > 2$ (see § 4.2.2), the fundamental harmonic of the response will appear aliased around the Nyquist frequency ($r_{\text{alias}} = M/2$) at $r_{\text{alias}} = M - M/r_{\text{orb}}$. While $r_{\text{orb}} > 1$ is required to begin to produce periodic sidebands, simulations have shown that this technique realistically requires $r_{\text{orb}} \gtrsim 1.5$ to create detectable sidebands from most binary systems during reasonable observations.

As the number of orbits present in the data ($r_{\text{orb}}$) increases, so does the spacing (in Fourier bins) between the sidebands. Since each sideband, like the central peak, has the traditional sinc function shape in $A_1(r)$ (see § 2.1), the full width at half-maximum (FWHM) of a sideband is approximately one Fourier bin. Therefore, an increase in spacing effectively decreases the duty cycle of the sideband pulsations (i.e., $\sim 1/r_{\text{orb}}$) in $P_1(r)$. Signals with small duty cycles are easier to detect for two reasons. First, significant higher harmonics begin to appear in the secondary power spectrum $P_2(r)$, which we can sum either incoherently [i.e., adding the powers $P_2(r)$ of the harmonics without taking into account their phases] or coherently [i.e., adding the complex amplitudes $A_2(r)$ of the harmonics directly after correcting for the fact that their phases are approximately equal to that of the fundamental times the harmonic number] in order to improve
the signal-to-noise ratio. A rule of thumb for the number of higher harmonics present in $P_2(r)$ is approximately one-half the inverse of the duty cycle ($\sim r_{\text{orb}}/2$). Second, as the duty cycle decreases, the Fourier amplitudes of each harmonic increase in magnitude up to a maximum of twice that of a sinusoid with an identical pulsed fraction (see Ransom et al. 2002 for a more detailed discussion of pulse duty cycle effects). This effect makes each individual harmonic easier to detect as $r_{\text{orb}}$ increases.

4.2. Search Considerations

As the number of orbits present in a time series increases, the signal-to-noise ratio of a detectable pulsar spin harmonic scales approximately as $\sim (r_{\text{orb}} - 1)^{-1/3}$, where $0.3 \leq \beta \leq 0.4$ (see Figs. 5 and 6). In contrast, the signal-to-noise ratio of a pulsar spin harmonic detected coherently scales as $T_{\text{coh}}^{-1/2}$. This leads to a rule of thumb for those conducting sideband searches: “The longer the observation the better.” When making very long observations it is important to keep the on-source time (i.e., the window function) as continuous and uniform as possible. Significant gaps in the data—especially periodic or recurring gaps of the kind common in X-ray observations—create side lobes around Fourier peaks of constant-frequency pulsations that a sideband search will detect. It is a good idea before a sideband search to take the power spectrum of the power spectrum of the known window function in order to identify spurious orbital periods that a sideband search might uncover.

Since phase modulation sidebands require coherent pulsations (i.e., $f_{\text{spin}}$ is constant in time) in order to form, it may be necessary to correct the time series (or equivalently the Fourier transform) such that the underlying pulsations are coherent. As an example, if the telescope’s motion with respect to the solar system barycenter is not taken into account during long observations, the observed astrophysical pulsations—and therefore any phase modulation sidebands as well—will be “smeared” across numerous Fourier bins as a result of Doppler effects. By stretching or compressing the time series to account for the Earth’s motion before Fourier transforming (or equivalently, by applying the appropriate Fourier domain matched filter after the Fourier transform; see Ransom et al. 2002), the smeared signal can be made coherent for the purposes of the sideband search.

If the time dependence of $f_{\text{spin}}$ is unknown a priori, because of accretion in an LMXB, the spin-down of a young pulsar, or timing (spin) noise, for example, one could in principle attempt a series of trial corrections and a sideband search for each trial. Such a search methodology is similar in computational complexity to traditional “acceleration” searches.

While § 4.1 provided basic principles for detecting compact binary pulsars using the two-staged Fourier analysis, in the rest of this section we provide a more detailed discussion of how a search might be conducted. Figure 7 shows two examples of sideband searches. The left column shows data from an 8 hr observation of the globular cluster 47 Tucanae taken with the Parkes radio telescope at 20 cm on 2000 November 17. The signal is the fundamental harmonic of the 2.1 ms binary pulsar 47 Tuc J ($f_{\text{spin}} = 476.05$ Hz, $P_{\text{orb}} = 2.896$ hr, $r_{\text{orb}} = 2.762$, $\Phi_{\text{orb}} = 120.87$, 24 rad, and $m_i \sim 0.03 M_\odot$) with a signal-to-noise ratio of $\eta = a/\sigma_a \sim 0.0054$. The right column shows data from a simulated 8 hr observation similar to the 47 Tuc observation described above. The signal is the fundamental harmonic of the 2.0 ms binary pulsar used in Figure 1 ($f_{\text{spin}} = 500$ Hz, $P_{\text{orb}} = 50$ minutes, $r_{\text{orb}} = 8$, $\Phi_{\text{orb}} = 413.44$, 24 rad, and
Once a long observation has been obtained and prepared as discussed above, we Fourier transform the data and compute the power spectrum $P_1(r)$. We then begin a sideband search that is composed of two distinct parts: initial detection of a binary pulsar and the determination of a detected pulsar’s orbital elements.

4.2.1. Initial Detection

To maximize the signal-to-noise of a detection in the secondary power spectrum $P_2(r)$, we want to match the length and location of a short FFT with the width ($\sim 2T_{\text{orb}}/\Phi_{\text{orb}}$ bins) and location of a pulsar’s sideband response in $P_1(r)$. Because a pulsar’s orbital parameters are unknown a priori, we must search a range of short FFT lengths $M$ and overlap consecutive FFTs. Therefore, we must choose the number and lengths of the short FFTs, the fraction of each FFT to overlap with the next, and the number of harmonics to sum over each FFT. These choices are influenced by the length of the observation and the nature of the systems that we are attempting to find.

All currently known binary pulsars with $P_{\text{orb}} < 10$ days have modulation amplitudes of $\Phi_{\text{orb}} \lesssim 6500$ rad (see Fig. 4), with the vast majority in the range $\Phi_{\text{orb}} \sim 20$–5000 rad. This

$$_{\text{orb}} = 0.2 \, M_{\odot}$$

with a signal-to-noise ratio one-half that of the 47 Tuc J observation ($n = 0.0027$).

Once a long observation has been obtained and prepared as discussed above, we Fourier transform the data and compute the power spectrum $P_1(r)$. We then begin a sideband search that is composed of two distinct parts: initial detection of a binary pulsar and the determination of a detected pulsar’s orbital elements.
range also includes many currently unknown but predicted 
"holy grail"-type systems such as compact MSP–black 
hole systems or MSP-WD systems with orbital periods of 
~10 minutes. This range of \( \Phi_{\text{orb}} \) values is due to the fact that 
the short spin periods of MSPs tend to offset the small semi-
major axes \( (x_{\text{orb}}) \) of their typical binary systems, while 
binary systems with wider orbits tend to have longer spin 
periods. Multiplying this range by twice the minimum \( r_{\text{orb}} \) 
of 1.5 gives a rule of thumb range of \( M \sim 60–15,000 \), where 
the high end should be increased based on expected values 
of \( r_{\text{orb}} \). For 8–12 hr globular cluster searches a reasonable 
range of values would be \( 64 \leq M \leq 65,536 \) in increments of 
powers of 2, for a total of 11 different values for \( M \).

The choice of the overlap fraction is more subjective. To 
optimize a detection in \( P_2(r) \), one should overlap the short 
FFTs by at least 50% in order to avoid excessive loss of sig-
nal-to-noise ratio when a series of sidebands straddles con-
secutive FFTs. Larger overlap percentages may result in 
higher signal-to-noise ratio, but at the expense of increased 
computational costs and numbers of search trials (even 
though the trials are not completely independent when 
overlapping).

In Figures 7a and 7e, the gray shaded regions show the 
sections of the original power spectra \( P_1(r) \) that were Four-
ier transformed for the sideband search. The short FFTs of 
length \( M = 1024 \) and \( M = 8192 \) were centered on the 
known pulsar spin frequency and produced the secondary 
power spectra \( P_2(r) \) displayed in Figures 7b and 7f respec-
tively. The short FFT lengths were chosen as the shortest 
powers of two greater than \( 2r_{\text{orb}}\Phi_{\text{orb}} \). For 47 Tuc J (Fig. 7a), the 
"homed" structure of the phase modulation sidebands 
described in § 2.1 is easily recognizable. This shape is not 
apparent in Figure 7e, and in fact, traditional searches 
detect nothing unusual about this section of the 
power spectrum.

Normalization of the secondary power spectra \( P_2(r) \) 
results in the usual exponential distribution with mean and 
standard deviation of one for a transform of pure noise. If 
one first normalizes the original power spectra \( P_1(r) \) as 
shown in Figures 7a and 7e (see Ransom et al. 2002, for sev-
eral normalization techniques), the resulting \( P_2(r) \) can be 
normalized simply by dividing by \( M \).

Figures 7b and 7f show the highly significant detections of 
the binary pulsars in the secondary power spectra \( P_2(r) \). For 
47 Tuc J, only one orbital harmonic is visible in \( P_2(r) \) since 
\( r_{\text{orb}} < 2 \), but the single-trial significance is approximately 
\( \sim 23 \sigma \). For the simulated 50 minute binary pulsar, three sig-
nificant orbital harmonics are detected (predicted harmonic 
locations are marked with a dotted line in panel f). 
When summed, the detection has a single-trial significance 
of \( \sim 15 \sigma \).

4.2.2. Orbit Determination

Once a binary pulsar has been detected, examination of 
the original Fourier powers \( P_1(r) \) and phases \( \phi_1(r) \) allow the 
estimation of the Keplerian orbital parameters and the pul-
ser spin period \( P_{\text{spin}} \). Brute force matched filtering searches 
centered on the estimates are then used to refine the param-
ters and recover the fully coherent response of the pulsar. 

The simplest parameter to determine is the orbital period 
\( P_{\text{orb}} \). The spacing between sidebands is 
\( r_{\text{orb}} = T_{\text{obs}} / P_{\text{orb}} \) 

\( \text{bins}, \) implying that the most significant peak in \( P_2(r) \) should 
 occur at Fourier frequency \( M/r_{\text{orb}} \). Conversely, if we 
measure the Fourier frequency \( r_{\text{meas}} \) in \( P_2(r) \) with the most 
power, we can solve for \( r_{\text{orb}} \) using 
\( r_{\text{orb}} = M/r_{\text{meas}} \). By using 
Fourier interpolation or zero padding to oversample the 
secondary Fourier spectrum \( A_2(r) \) and therefore \( P_2(r) \), we can 
determine \( r_{\text{meas}} \) to an accuracy of 

\[ \sigma_r = \frac{3}{\pi \sqrt{6 P_2(r_{\text{meas}})}}, \]  

where \( P_2(r_{\text{meas}}) \) is the normalized power at the peak and \( \alpha \) is 
the signal’s “purity.” The purity is proportional to the root-
mean-squared (rms) dispersion of the pulsations (i.e., side-
bands) in time \( [\text{i.e., the frequencies in} \ P(r) \text{about the cen-
troid and in our case is approximately 1 if the length of the} \]
short FFT used to produce the \( P_2(r) \) matched the spread of 
the sidebands in the original power spectrum \( P_1(r) \) (see 
Middleditch, Deich, & Kulkarni 1993; Ransom et al. 2002). 
Remembering that \( P_{\text{orb}} = T_{\text{obs}} / r_{\text{orb}} \) and solving gives 

\[ P_{\text{orb}} = \frac{T_{\text{obs}}}{M/r_{\text{meas}}} - \frac{3 T_{\text{obs}}}{\pi \alpha M \sqrt{6 P_2(r_{\text{meas}})}}. \]  

For aliased signals, the orbital period is 

\[ P_{\text{orb,aliased}} = \frac{T_{\text{obs}}(M - r_{\text{meas}})}{M} + \frac{3 T_{\text{obs}}}{\pi \alpha M \sqrt{6 P_2(r_{\text{meas}})}}. \]  

For pulsars with \( \Phi_{\text{orb}} > 1 \) radian observed such that 
\( r_{\text{orb}} > 2 \), it is often possible to measure \( P_{\text{orb}} \) to 1 part in \( 10^4 \), 
corresponding to a \( \sim 1 \) s error for many of the pulsars shown 
in Figure 4.

Once \( P_{\text{orb}} \) is known, it becomes possible to measure the 
projected semimajor axis \( x_{\text{orb}} \) by determining the spread of 
the sidebands in the original power spectrum \( P_1(r) \). For 
bright pulsars like 47 Tuc J, the number of Fourier bins 
between the "horns" \( \langle \Delta \gamma \rangle \sim 2 \Phi_{\text{orb}} r_{\text{orb}} \) as shown in 
Figure 7a can be measured directly and converted into \( x_{\text{orb}} \) using 

\[ x_{\text{orb}} = \frac{\Delta \gamma}{4 \pi f_{\text{spin}} r_{\text{orb}}}. \]  

where \( f_{\text{spin}} \) can be estimated as the frequency midway 
between the phase modulation horns.

For weaker pulsars, where the sideband edges are not 
obvious (as in Fig. 7e), \( \Delta \gamma \) can be estimated using numerous 
short FFTs. By taking a series of short FFTs of various 
lengths around the detection region of the original power 
spectrum \( P_1(r) \) and measuring the power, “centroid,” and 
purity\( ^7 \) at the frequency corresponding to \( P_{\text{orb}} \) one can map 
the extent of the hidden sidebands and compute an estimate 
for \( x_{\text{orb}} \).

For pulsars in circular orbits, which constitute the major-
ity of the systems in Figures 3 and 4, the only remaining 
orbital parameter is the time of the ascending node \( T_{\text{asc}} \). 
Defining the time since the ascending node as \( \Delta T_{\text{asc}} = t_0 - T_{\text{asc}} \) and using equation (3) we get 

\[ \Delta T_{\text{asc}} = \frac{P_{\text{orb}}}{2 \pi} \left( \phi_{\text{orb}} - \frac{\pi}{2} \right), \]  

where \( \phi_{\text{orb}} \) can be measured from the phases of the 

\( ^7 \) The measured values of “centroid” and “purity” are estimates of a 
signal’s location and duration in a time series as determined from the deriv-
atives of the Fourier phase and power at the peak of the signal’s response 
(see Middleditch et al. 1993; Ransom et al. 2002).
sidebands in the original Fourier spectrum $A_1(r)$ using the following technique.

From equation (10) we know that the sidebands have phases of $\phi_r = \phi_0 + \pi/2 + \phi_{\text{spin}}$ rad. Unfortunately, some of the measured phases will differ by $\pi$ rad from those predicted by equation (10) since the corresponding sideband amplitudes [the $J_1(\Phi_{\text{orb}})$] are negative (see footnote 6). If we predict values for $J_1(\Phi_{\text{orb}})$ using our knowledge of $r_{\text{orb}}/f_{\text{spin}}$, and $x_{\text{orb}}$, we can "flip" (i.e., add or subtract $\pi$ rad to) the measured phases of the sidebands that are predicted to have negative amplitudes. We can then estimate $\phi_{\text{orb}}$ simply by subtracting the phases of neighboring sidebands using $\phi_{\text{orb}} = \phi_{r+1} - \phi_r - \pi/2$.

For weak pulsars each measurement of $\phi_{\text{orb}}$ has a large uncertainty. Fortunately, we can make $\sim 2f_{\text{orb}}$ measurements of $\phi_{\text{orb}}$ by using each pair of sidebands and then determine $\phi_{\text{orb}}$ and its uncertainty statistically. Figures 7c and 7g show histograms of the measurement of $\phi_{\text{orb}}$ using this technique.

It is important to mention that for weak pulsars it may be difficult to determine the location of the sidebands in order to measure their phases. The location of the sideband peaks can be calculated by measuring the phase $\phi_{\text{meas}}$ of the fundamental harmonic discovered in the secondary Fourier spectrum $A_1(r)$. The number of Fourier bins from the first bin used for the short FFT to the lowest frequency sideband in $A_1(r)$ is simply $\phi_{\text{meas}}/r_{\text{orb}}/2\pi$. Subsequent peaks are located at intervals of $r_{\text{orb}}$ Fourier bins.

Once estimates have been made for the Keplerian orbital parameters, we compute a set of complex sideband templates (i.e., matched filters for the Fourier domain response of an orbitally modulated sinusoid) over the most likely ranges of the parameter values given the uncertainties in each. These templates are then correlated with the original Fourier amplitudes $A_1(r)$ around $f_{\text{spin}}$.

When a template matches the pulsar sidebands buried in $A_1(r)$, we recover essentially all the power in that pulsar spin harmonic (Ransom et al. 2002). Figures 7d and 7h show the results of just such a matched filtering operation. The Doppler effects from the orbital motion have been completely removed from the data, and the resulting Fourier response is that of an isolated pulsar.

For binary pulsars in eccentric orbits, the technique discussed here could in principle be applied to each set of sidebands from the orbital Fourier expansion (see § 2.3). Such an analysis would be much more complicated than that described here and would require a high signal-to-noise detection.

5. DISCUSSION

We have described a new search technique for binary pulsars that can identify sidebands created by orbital modulation of a pulsar signal when $f_{\text{obs}} > f_{\text{orb}}$. Sideband or phase-modulation searches allow the detection of short-orbital period binary pulsars that would be undetectable using conventional search techniques.

Figures 5 and 6 show how the sensitivity of sideband searches compares to that of acceleration searches for a wide range of observation times $f_{\text{obs}}$. Acceleration search sensitivities are near optimal when $f_{\text{obs}} > 0.1f_{\text{orb}}$ (Johnston & Kulkarni 1991), but degrade rapidly at longer integration times when the constant frequency derivative approximation breaks down. Sideband searches, on the other hand, require $f_{\text{obs}} > 1.5f_{\text{orb}}$ in order to approach the sensitivity of optimal duration acceleration searches, but for longer $f_{\text{obs}}$ sensitivities improve as $\sim(f_{\text{orb}} - 1)^{-\beta}$, where $0.3 \leq \beta \leq 0.4$. For targeted searches of duration $10^7$–$10^8$ hr (e.g., typical globular cluster observations), sideband searches are $\sim 10^7$–$10^8$ times more sensitive to compact binary pulsars ($P_{\text{orb}} \lesssim 4$ hr) than optimal duration acceleration searches. In general, one should think of acceleration and sideband searches as complementary to each other—sideband searches allow the detection of ultracompact binary pulsars, while acceleration searches maximize the detectability of isolated and longer period binary pulsars.

The fact that sideband searches target a different portion of orbital parameter space than acceleration searches, and yet require significantly less computation time, provides good reason to include them in future pulsar searches where $f_{\text{obs}} > 30$ minutes. In fact, the Parkes Multibeam Pulsar Survey ($f_{\text{obs}} \sim 35$ minutes; Lyne et al. 2000; Manchester et al. 2001) has included a sideband search in an ongoing reanalysis of their survey data at the cost of only a marginal increase in computer time. While the probability of detecting a binary pulsar with $P_{\text{orb}} < 23$ minutes is almost certainly quite low, the discovery of a single such system would provide a wealth of scientific opportunity.

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