Where is the $\chi_{c0}(2P)$?

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Although the analysis of the BaBar Collaboration prefers $J^P = 0^+$ for the $X(3915)$, it is difficult to assign the $X(3915)$ to the $\chi_{c0}(2P)$. We show that there is an indication of the $\chi_{c0}(2P)$ with a mass around 3840 MeV and width of about 200 MeV in the Belle and BaBar data for $\gamma\gamma \rightarrow D\bar{D}$.

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With the discovery of many new states in the charmonium mass region, one faces the problem of understanding their nature (for a comprehensive review, see Ref. [1]). Among these $XYZ$ states, the $Z(3930)$ was discovered by the Belle Collaboration in the process $\gamma\gamma \rightarrow D\bar{D}$ [2], and later on confirmed by the BaBar Collaboration in the same process [3]. The experimental results on mass, angular distributions, and $\Gamma(Z(3930) \rightarrow \gamma\gamma)\mathcal{B}(Z(3930) \rightarrow D\bar{D})$ are all consistent with the expectation for the $\chi_{c2}(2P)$, which is the radially excited $P$-wave tensor charmonium state. The mass was measured to be $3927.2 \pm 2.6$ MeV [4]. It is below the $D^*\bar{D}^*$ threshold so that it decays dominantly into the $D\bar{D}$ pair in a $D$-wave, and its decay width is $24 \pm 6$ MeV [4]. So far, this is the only unambiguously identified radially excited $P$-wave charmonium state.

In the same mass region, another structure was reported first by the Belle Collaboration [5] in the $J/\psi\omega$ invariant mass distribution in exclusive $B \rightarrow K\omega J/\psi$ decays, and confirmed later by the BaBar Collaboration with more statistics [6,7]. This structure is identified as the same state observed in the same final states with similar mass in the $\gamma\gamma \rightarrow J/\psi\omega$ process [8]. It is called $X(3915)$, and its mass and width are $3917.5 \pm 2.7$ MeV and $27 \pm 10$ MeV, respectively [4]. Very recently, a spin-parity analysis has been performed for the process $X(3915) \rightarrow J/\psi\omega$ by the BaBar Collaboration [9], and the results suggest that the quantum numbers of this state are $J^P = 0^+$. It was therefore identified as the $\chi_{c0}(2P)$ state [9] following the suggestion of Ref. [10]. However, assigning the $X(3915)$ as the $\chi_{c0}(2P)$ state faces the following problems:

1. As pointed out in, for instance, Refs. [1,11], the partial decay width of the $X(3915) \rightarrow J/\psi\omega$
would be \( \gtrsim 1 \text{ MeV} \) if it is produced similarly in \( B \)-meson decays as the well-understood conventional charmonium states. This estimate is consistent with the one using \( \Gamma(X(3915) \rightarrow \gamma\gamma)B(X(3915) \rightarrow J/\psi\omega) \), which was reported to be \((52\pm10\pm3) \text{ eV} \), if its spin is 0, by the BaBar Collaboration [9]. The value is consistent with an earlier measurement by the Belle Collaboration [8]. Were the \( X(3915) \) the \( \chi_{c0}(2P) \) state, its width into two photons would be similar to that of the \( \chi_{c0} \) \(^1\). Given \( \Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 2.3 \pm 0.2 \text{ keV} \), it is reasonable to assume \( \Gamma(\chi_{c0}(2P) \rightarrow \gamma\gamma) \sim 1 \text{ keV} \). Identifying the \( X(3915) \) with the \( \chi_{c0}(2P) \) would give the same estimate as before, \( \Gamma(X(3915) \rightarrow J/\psi\omega) \gtrsim 1 \text{ MeV} \). Such a value is at least one-order-of-magnitude larger than the OZI (Okubo–Zweig–Iizuka) suppressed hadronic widths of the \( \psi(2S) \) and the \( \psi(3770) \).

(2) Normally, the dominant decay channels of a scalar meson should be the open-flavor modes, i.e. OZI allowed, if the meson lies above the corresponding thresholds. However, the \( X(3915) \) was not observed in the \( D\bar{D} \) channel, despite the facts that they can couple in an \( S \)-wave and the \( D\bar{D} \) threshold, 3730 MeV, is about 200 MeV lower than the \( X(3915) \) mass. \(^2\) Furthermore, when there is no suppression due to either isospin or SU(3) breaking, from the OZI rule or because of tiny phase space, all known hadrons which decay in an \( S \)-wave have widths of order 100 MeV or even more. The width of the \( X(3915) \), being comparative to that of the \( D \)-wave decaying \( \chi_{c2}(2P) \), seems too small.

(3) As mentioned in Ref. [13], the mass difference between the \( \chi_{c2}(2P) \) and the \( X(3915) \), 9.7\( \pm \)3.7 MeV, is too small for the fine splitting of \( P \)-wave charmonia. It is one-order-of-magnitude smaller than the fine splitting of \( 1P \) states \( M_{\chi_{c2}} - M_{\chi_{c0}} = 141.45 \pm 0.32 \text{ MeV} \) [4]. Furthermore, it is even smaller than the analogous splitting in bottomonium systems, \( M_{\chi_{b2}(2P)} - M_{\chi_{b0}(2P)} = 36.2 \pm 0.8 \text{ MeV} \) [4] even though the Hamiltonian terms responsible for the fine splitting are proportional to \( 1/m_Q^2 \), with \( m_Q \) the heavy quark mass.

So where should the \( \chi_{c0}(2P) \) be? There have been a few lattice calculations on the mass spectrum of excited charmonium states. For the \( \chi_{c0}(2P) \) state, a quenched calculation gives a mass of 4091 \( \pm \)61 MeV [14], while it is predicted to be around 4 GeV in full QCD with a pion as large as 1 GeV [15]. However, effects due to light sea quarks / large pion masses could be significant for the excited \( P \)-wave charmonia nearby open-charm thresholds [16]. Results calculated with a lower pion mass, though still around 400 MeV, are presented in Ref. [17]. The authors calculated the mass differences between the

\(^1\) Assuming that the \( D\bar{D} \) modes dominate the decays of the \( \chi_{c2}(2P) \), \( \Gamma(\chi_{c2}(2P) \rightarrow \gamma\gamma) \) is about 0.2 keV [4]. It is of the same order as \( \Gamma(\chi_{c2} \rightarrow \gamma\gamma) = 0.51 \pm 0.04 \text{ keV} \).

\(^2\) There has been a suggestion that the \( X(3915) \) couples to the \( D\bar{D} \) [13]. However, the result for the width obtained from their fit to the \( \gamma\gamma \rightarrow D\bar{D} \) data of both the Belle [2] and Babar [3] Collaborations, 8.1 \( \pm \) 9.7 MeV, is consistent with 0. This implies that the \( X(3915) \) only plays a minor role in their fit.
excited charmonia and the $\eta_c$ in order to reduce the systematic uncertainty due to tuning the charm quark mass. The results are $972 \pm 9$ MeV and $1041 \pm 12$ MeV for the $\chi_{c0}(2P)$ and the $\chi_{c2}(2P)$, respectively. If the experimental mass of the $\eta_c$ is used, the mass of the $\chi_{c0}(2P)$ would be $3953 \pm 9$ MeV, and meanwhile the $\chi_{c2}(2P)$ would be about 100 MeV heavier than the measured mass. However, one may extract the fine splitting between the $\chi_{c2}(2P)$ and the $\chi_{c0}(2P)$ from their calculation, which is $69 \pm 15$ MeV. Using the experimental mass of the $\chi_{c2}(2P)$, one gets $M_{\chi_{c0}(2P)} = 3858 \pm 15$ MeV. We regard this value as the most reliable lattice estimate of the mass of the $\chi_{c0}(2P)$ obtained so far. Quark model predictions are in the same region. For instance, the masses of the $\chi_{c0}(2P)$ and the $\chi_{c2}(2P)$ are predicted to be 3916 MeV and 3979 MeV in the Godfrey-Isgur relativized quark model [18, 19], respectively. Shifting the $\chi_{c2}(2P)$ to the observed mass and keeping the value of the fine splitting would give 3856 MeV for the mass of the $\chi_{c0}(2P)$. A recent quark model calculation using a screened potential [20] predicts a mass of 3842 MeV for the $\chi_{c0}(2P)$; meanwhile, the prediction for the $\chi_{c2}(2P)$, 3937 MeV, agrees with the experimental value.

In the data of the process $\gamma\gamma \rightarrow D\bar{D}$ from both Belle and BaBar, there is a broad bump below the narrow peak of the $\chi_{c2}(2P)$. The Belle Collaboration fits the $D\bar{D}$ invariant mass spectrum in the region $3.80 \text{ GeV} < w < 4.2 \text{ GeV}$ with a Breit-Wigner function for the $\chi_{c2}(2P)$ plus a background function $\propto w^{-\alpha}$ [2], with $w$ the invariant mass of the $D\bar{D}$ pair. The BaBar Collaboration fits the spectrum starting from the $D\bar{D}$ threshold with a background function $\propto \sqrt{w^2 - m_t^2} (w - m_t)^{-\alpha} \exp[-\beta(w - m_t)]$, where the $D\bar{D}$ threshold is represented by $m_t$ [3]. However, if the $\chi_{c0}(2P)$ has a mass around 3850 MeV as estimated above, it would lie in the region of the broad bump, and it could get hidden in such fits. In this paper, we try to fit both data sets with two Breit-Wigner functions. Our results indicate that the $\chi_{c0}(2P)$ could have a mass around 3840 MeV.

We will assume that all the cross sections of the $D\bar{D}$ production in photon-photon collisions are due to resonant structures. This means that we will neglect backgrounds due to non-resonant contribution and semi-inclusive $D\bar{D}X$ with undetected $X$ ($X$ can be soft pions or photons). We use the same Breit-Wigner function for the resonances as the one used by the BaBar Collaboration [3]. Taking into account both the phase space and Blatt-Weisskopf centrifugal barrier factor $F_L$, the function reads as

$$B_L(w) = \left(\frac{p}{p_0}\right)^{2L+1} \frac{M}{w} \frac{F_L^2(w)}{(w^2 - M^2)^2 + \Gamma^2(w)M^2}, \quad (1)$$

where $L$ is the orbital angular momentum of the $D\bar{D}$ pair, $p$ is the momentum at the center-of-mass (cm) frame, $p_0$ is the cm momentum at $w = M$, which is the Breit-Wigner mass of the resonance, and the energy-dependent width is given by

$$\Gamma(w) = \Gamma \left(\frac{p}{p_0}\right)^{2L+1} \frac{M}{w} F_L^2(w), \quad (2)$$
with $\Gamma$ being the width of the resonance at rest. The centrifugal barrier factor \cite{21,22} is $F_0 = 1$ for an $S$ wave, and

$$F_2(w) = \frac{\sqrt{(R^2 p_0^2 - 3)^2 + 9 R^2 p_0^2}}{\sqrt{(R^2 p^2 - 3)^2 + 9 R^2 p^2}}$$

for a $D$ wave. The same value 1.5 GeV as used in Ref. \cite{3} will be taken for the “interaction radius” $R$.

We fit to the BaBar and the Belle data separately in the region from the $D\bar{D}$ threshold to 4.2 GeV with four parameters: the mass $M_0$ and width $\Gamma_0$ of a $0^+$ resonance which couples to the $D\bar{D}$ in an $S$-wave, and two normalization constants $N_0$ and $N_2$ for the scalar meson and the $\chi_{c2}(2P)$, respectively. The mass and width of the $\chi_{c2}(2P)$ are fixed to 3927 MeV and 24 MeV \cite{4}, respectively. There is no interference between these two structures because they are in different partial waves. Contrary to the BaBar data, the Belle data are not efficiency corrected. Nevertheless, the Belle efficiency only decreases by 10% for an increase of the invariant mass from 3.8 to 4.2 GeV, and there is no fine structure in the efficiency and background distributions \cite{2,23}. Furthermore, the $D$-mass sidebands have been subtracted from the Belle data used in our fit. A comparison of the best fit to the data are shown in Fig. \ref{fig:fit_results}.

The fit results are collected in Table \ref{tab:fit_results}, where the uncertainties only reflect the statistical errors in the fit. One sees that the two resonance assumption gives a reasonable fit to both data sets. The large value of $\chi^2$/dof for the fit to the BaBar data comes mainly from a few bins where the event numbers are quite separated from their neighbors. Comparing the resulting parameters from the two fits, the difference in the values of the mass is $2\sigma$, and the values of the width and the ratio of the normalization constants are fully consistent with each other. The mass is compatible with the lattice estimate for the mass of the $\chi_{c0}(2P)$ discussed above, and the width is of the right order for an $S$-wave strongly decaying hadron. Furthermore,
its main decay channel should be $D\bar{D}$ since such a structure has never been reported elsewhere. All these properties indicate that the broad bump could be assigned as the $\chi_{c0}(2P)$. The weighted average \cite{4} of both fits gives

$$M_{\chi_{c0}(2P)} = (3837.6 \pm 11.5) \text{ MeV}, \quad \Gamma_{\chi_{c0}(2P)} = (221 \pm 19) \text{ MeV}. \quad (4)$$

In summary, it is difficult to accommodate the $X(3915)$ to the still missing $\chi_{c0}(2P)$. Being a $J^P = 0^+$ particle, the $X(3915)$ is probably of exotic nature. There is an indication that the present data of the $\gamma\gamma \to D\bar{D}$ process already contain signals of the $\chi_{c0}(2P)$ with a mass and width around 3840 MeV and 200 MeV, respectively. More refined analysis of the data with higher statistics is definitely necessary to confirm our assertion. In addition to the photon-photon fusion process, searching for the $\chi_{c0}(2P)$ in the $D\bar{D}$ distribution can also be done at BES-III using radiative decays of higher vector charmonia [24], and at the hadron colliders LHC and FAIR.

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\[ \textbf{TABLE I:} \text{Results of fitting to the BaBar and Belle data, respectively. See text for details.} \]

| Data sets | $\chi^2$/dof | $M_0$ (MeV) | $\Gamma_0$ (MeV) | $N_2/N_0$ |
|-----------|--------------|-------------|-----------------|-----------|
| BaBar [3] | 1.63         | 3848.1 ± 7.7| 229 ± 26        | 0.0121 ± 0.0034 |
| Belle [2] | 0.79         | 3825.1 ± 8.4| 212 ± 29        | 0.0132 ± 0.0046 |

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