Comment on “Properties of the Recovery Phase of Extreme Storms” by Choraghe et al. (2021)

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Abstract Choraghe et al. (2021), based on a study of the recovery phase of the SYM-H index of 31 extreme geomagnetic storms, have recently concluded that the hyperbolic decay function is only able to explain the complete recovery phase of about one third of events and that both the exponential or the hyperbolic decay functions fail to explain the late recovery phase of storms. Furthermore, they propose a linear function to model the late recovery phase and claim that the proposed model could throw new light on the relative importance of different physical processes involved during the complete recovery phase of extreme storms. We assert that the Choraghe et al. (2021) conclusions regarding the recovery phase of extreme storms analysis are incorrect and in particular are based on a misunderstanding of the nature of the evolution of the SYM-H index and the energy balance of the ring current.

1. Introduction

A recent paper by Choraghe et al. (2021) analyzes the recovery phase of the SYM-H index during 31 extreme geomagnetic storms by fitting the SYM-H index to three different functions: an exponential function, assuming that the decay rate of the ring current energy is proportional to the own energy content (Burton et al., 1975); a hyperbolic function, assuming a non-linear behavior where the decay rate of the ring current energy is proportional to the square of the energy content (Aguado et al., 2010; Cid et al., 2013); and a linear function, to explain a quasi-steady behavior observed in the late recovery phase.

Based on the fitting results, Choraghe et al. (2021) conclude that there are three categories of recovery phase: (a) those well-reproduced by the hyperbolic model, where non-linear behavior dominates; (b) those neither following exponential nor hyperbolic fitting, which are classified as ‘complex events’ and have coupled effects of both linear and non-linear processes; and (b) those initially following an exponential or hyperbolic function, but following a linear trend at a later stage, indicating that at least two different physical mechanisms are involved.

In our opinion, the problem here is not which mathematical function is able to properly reproduce the recovery phase of extreme storms, but to be aware that in the model for the evolution of the SYM-H index all the contributions are being considered. The models applied by Choraghe et al. (2021) are just considering energy losses, forgetting energy injections. Hence the fitting results are misunderstood with incorrect conclusions.

2. The SYM-H Evolution and the Ring Current Energy Balance

This comment points out that the evolution of the SYM-H index can be obtained, as a first approach, from the ring current energy balance. This statement is based on the Dessler-Parker-Sckopke (DPS) relation (Dessler & Parker, 1959; Sckopke, 1966), which predicts a linear dependence of the perturbation magnetic field at the Earth center due to the ring current on the total ring current kinetic energy. Then, the temporal evolution of the ring current energy can be determined by the energy rate balance equation:

\[
\frac{d K_{RC}}{dt} = U_{I} - U_{L} \tag{1}
\]

being \(K_{RC}\) the ring current kinetic energy, \(U_{I}\) the injection rate of energy and \(U_{L}\) the rate of energy loss. This equation has been applied for predicting the time series of the geomagnetic storm index Dst (equivalent to SYM-H but with lower resolution) across more than 40 years by considering different injection or loss functions.

Magnetic reconnection, originally proposed by Dungey (1961), is the principal mechanism that transfers energy from the solar wind to the magnetosphere. Thus, although several injection functions have been proposed for
Equation 1, all concur that the southward component of the interplanetary magnetic field (IMF), $B_z$, plays a critical role as responsible for the enhancement of the ring current energy content. Trying to accurately describe the ring current dynamics O’Brien and McPherron (2000) also noted that, not only the injection function depends on solar wind coupling (assumed to be determined by $V Bs$, being $V$ the solar wind speed), but also the ring current decay lifetime, $\tau$. O’Brien and McPherron (2000) demonstrated that a variable $\tau$ in the exponential recovery rate is better than the fixed-\$formula of Burton et al. (1975). This can be explained by the different time constants, most notably from the driving force of solar wind electric field, $V Bs$, but also because of the variable loss of ions at different energies, pitch angles, and locations, lost through whatever mechanism. Indeed, the $\tau$ value should change with time throughout the recovery phase in the exponential model, as the faster processes drain the part of the particle content with which they interact, leaving only the slow processes to dominate the late recovery phase. The hyperbolic model, where the ring current starts consuming the injected energy fast and slows down as the SYM-H approaches zero, is analogous to the variable-$\tau$ exponential function, modeling the recovery more elegantly with a single $\tau$ value.

Even though Choraghe et al. (2021) mention the DPS relation and quote several papers where the ring current energy balance is applied, including those mentioned above, they do not consider the ring current injection term in Equation 1, assuming a pure recovery phase for the intervals analyzed. This assumption was already applied by Cid et al. (2013) when modeling the recovery phase of extreme geomagnetic storms with the hyperbolic function introduced by Aguado et al. (2010). For those events where the hyperbolic function was not able to properly reproduce the data, Cid et al. (2013) concluded that probably these storms received a significant energy input during the time analyzed, and therefore a pure recovery model was not suitable. On the contrary, Choraghe et al. (2021) conclude that other mechanisms or processes for energy loss (different that those considered by the exponential or hyperbolic model) are involved.

In our opinion this conclusion is wrong because of two reasons:

1. Both the exponential and the hyperbolic model are empirical models which are not proposing physical mechanisms or processes for losing energy, but including their consequences in their parameters, mainly in the recovery time, $\tau$. Thus, the (consistent or inconsistent) fitting results do not support the conclusion stated.
2. The differences between the hyperbolic (or the exponential) model with the SYM-H evolution are due to the injection of energy to the ring current, which is ignored by Choraghe et al. (2021).

3. It Is Not Different Physical Mechanisms but Injection of Energy

The events analyzed by Cid et al. (2013) happened before continuous solar wind data were available. Thus, it was not possible robustly conclude that the reason for the improper fittings of hyperbolic model was the energy input. On the contrary, there are continuous solar wind data for most of the periods analyzed by Choraghe et al. (2021). Hence, at least a fast check should have made.

As stated above, different injection functions from the solar wind to the magnetosphere have been proposed, but all of them have the southern IMF component ($B_z < 0$) as responsible for the enhancement of the ring current energy content. Thus we have checked the interplanetary magnetic field data during the events analyzed by Choraghe et al. (2021). Figure 1 provides a plot of $B_z$ and SYM-H for three events, as an example of every category of recovery phase proposed by Choraghe et al. (2021). Some data gaps appear in the figure. There are data available to fill those gaps from the Ace or Wind spacecraft data repositories, but we decided to plot data available from the OMNIweb database. This database is the same as that used by Choraghe et al. (2021) and provides the IMF data shifted to the bow-shock, avoiding any conflict related to the delay between the solar wind arrival to the magnetosphere nose and the magnetospheric response.

During the recovery phase of the event in July 2000 (top two panels) $B_z$ is positive, that is, IMF is northern. Then, no injection of energy to the ring current is foreseen and the pure recovery can be considered. For this event, and for all the events in this category, the hyperbolic model properly fits the SYM-H, according to Choraghe et al. (2021).

A shadowed area appears in the plot on the event in March-April 2001 (two middle panels in Figure 1). It corresponds to a southern IMF interval, with $B_z$ negative reaching almost $-50$ nT. These IMF values are extremely large and similar to those recorded during the main phase of the storm and therefore, to neglect the injection of
energy during this interval, which belongs to the recovery phase, is very mistaken. Similar southern IMF intervals during the recovery phase can be found in the events in the second category of Choraghe et al. (2021).

Finally, the bottom two panels correspond to the event in June 2015. Two small intervals with southern IMF at the beginning of the recovery phase would make this event to be part of the second category. However, this event is classified in the third category due to the behavior in the late recovery phase. As in the rest of events of this category, \( B_z \) is continuously fluctuating at this stage. Thus, small but continuous injection of energy is being transferred from the solar wind to the magnetosphere. As in the second category, the injection function cannot be neglected and the injection and the loss term in Equation 1 are similar, resulting in a slow recovery which Choraghe et al. (2021) try to explain with a linear function.
The events above are just some examples which allow us to robustly deduce that the Choraghe et al. (2021) conclusions regarding the recovery phase of extreme storms analysis are incorrect. Definitely, when the hyperbolic function does not reproduce the SYM-H data, it is because the injection of energy has been improperly neglected in Equation 1 which reproduces the evolution of the SYM-H index. Attempting to explain the recovery phase of the SYM-H index disregarding the energizing processes of the ring current, as done in Choraghe et al. (2021), is not defensible.

Data Availability Statement
We acknowledge the use of OMNI database as source of interplanetary data and SYM-H index (https://omniweb.gsfc.nasa.gov/).

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