1. Introduction

Vibration machines are widely used in various industries. Vibration machines with unbalance drive are the most common primarily due to the simple design and compact size under great disturbance. However, in transient operating modes of such machines, there may be resonant oscillations, accompanied by a significant increase in dynamic loads in structural elements. For example, the practice of operation of SMZh, VB type vibrating tables for volume compaction of concrete mixes demonstrates frequent failures of drive shafts that interconnect unbalances of individual vibration units [1]. The design of these shafts includes elastic couplings, which may be one of the main causes of destructive oscillations of the drive. Thus, the studies of dynamic processes in vibration machines, taking into account the drive elasticity, are a relevant scientific and applied problem.
2. Literature review and problem statement

In [1], the lack of longevity and reliability of the drive shafts of vibrating tables in the construction industry is considered, but the reasons for this are not explained. Of course, intense oscillations of the vibration machine drive in transient operating modes are one of the main causes of increased dynamic loads and require detailed consideration.

Start-up of vibration machines with inertia drive and passage of the inertial vibration exciter rotor through resonance were investigated in a number of works, the review of which can be found in [2–4]. In [2, 3], it is shown that the known regularities that occur during the manifestation of the Sommerfeld effect are rather easily obtained by using the method of direct separation of motions. However, in [2–4] only the simplest dynamic models of vibration machines with one unbalance vibration exciter installed on the bearing body with one degree of freedom are considered. In [5], the same approach is used to study the run-up of the vibration machine with two self-synchronizing vibration exciters, and in [6] – for the vibration machine with plane oscillations of the bearing body. However, in [2–6], only dynamic models of machines with rigid links were used.

The solution of various problems of dynamics of machine units with elastic couplings and transmission mechanisms is considered, in particular, in [7, 8]. It was found that the comprehensive analysis of dynamics is possible only on the basis of joint consideration of dynamic properties of both the mechanical oscillation system and the electric motor of the machine drive. However, the results obtained cannot be always used for vibration machines with unbalance drive, since they have a number of specific features.

In [9–11], mathematical models, considering the elastic coupling of the motor and exciter rotors were used to study the dynamics of the vibration machine with the inertial exciter and asynchronous electric motor. However, the analysis of the impact of the elastic coupling of rotors on the machine dynamics was not carried out.

Methods of calculating structural elements of vibration machines with unbalance drive are described in many papers, in particular [12]. However, the issues related to the drive couplings or gears with elastic links are limited only to the design description.

Among the recent works, which consider the problems of start-up of vibration machines with inertial drive, the papers [13–17] should be noted. In [13–15], the practical application of the Sommerfeld effect is substantiated. In [16], resonant oscillations in a limited-excitation system, where there is an interaction between the power source and the elastic subsystem, which may lead to large resonant oscillations are considered. In [17], it is shown that the presence of elastic-damping coupling of the motor and vibration exciter rotors introduces significant features in the dynamics of the vibration machine. In this paper, the equation describing the “slow” process of start-up and run-up of the vibration machine, as well as formulas for determining critical frequencies and expression for the amplitude of torsional oscillations of the motor and vibration exciter rotors are obtained. It is shown that resonant oscillations of the drive, caused by the coupling are impossible when using “rigid” couplings. However, the paper [17] is, in fact, limited only to the derivation of the equations of “slow” and “fast” rotor movements, the formulas for the vibratory moment and the expression describing drive oscillations. In this case, the analysis of the obtained dependencies is practically not provided. At the same time, for the design and successful operation of vibration machines with unbalance drive, it is necessary to investigate the mechanism of occurrence of drive oscillations and their influence on the machine dynamics.

3. The aim and objectives of the study

The aim of the work is to determine the effect of the elasticity of the coupling of the unbalance vibration exciter with the motor rotor on oscillations processes in the drive of the vibration machine in order to improve its dynamic characteristics.

To achieve the aim, the following objectives are set:

- to construct amplitude-frequency characteristics of oscillations of the vibration machine drive;
- to find out the effect of drive oscillations on the magnitude of the vibratory moment (dynamic load of the motor) and the emergence of the Sommerfeld effect;
- to perform numerical simulation of dynamic processes in the vibration machine drive, taking into account its elasticity and electromagnetic transients in the asynchronous motor;
- to develop practical recommendations for reducing oscillation processes in the vibration machine drive.

4. Methods of the study of steady and transient operation of the vibration machine

For analytical studies, methods of applied oscillation theory, the approach of vibration mechanics and the method of direct separation of motions are used. Simulation of the vibration machine start-up consisted in the joint numerical integration of differential equations of motion of the mechanical oscillation system and the dynamic model of the asynchronous motor using the Maple software product.

5. Results of studies of oscillation processes in the vibration machine drive and their influence on the machine dynamics

5.1. Description of the oscillation system and motion equation

The considered vibration machine is a bearing solid body (vibrating working body), which can be displaced only in one fixed direction Ox (Fig. 1). The bearing body is connected to a fixed base with the help of elastic and damping elements. On it, the unbalance vibration exciter, driven by the motor is firmly fixed. In this case, let the motor and exciter rotors be connected by means of the elastic coupling, which is considered to be non-inertial. Note that in general, rotors can be connected by any elastic-damping element (for example, belt or universal joint drive with an elastic shaft). The oscillation system is characterized by three generalized coordinates: rotation angles of the motor $\phi_1$ and vibration exciter $\phi_2$ rotors and horizontal displacements of the bearing body $x$. 

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The equations of motion of such a system are as follows [16]:

\[ I_1 \ddot{\phi}_1 + \beta_x (\phi_1 - \phi_2) + c_x (\phi_1 - \phi_2) = L_1, \]
\[ I_2 \ddot{\phi}_2 - \beta_x (\phi_1 - \phi_2) - c_x (\phi_1 - \phi_2) = -R_1 (\phi_1) + m \varepsilon \ddot{x} \sin \phi_2 + g \cos \phi_2, \]
\[ M \ddot{x} + \beta_x \ddot{x} + c_x \dot{x} = m \varepsilon (\dot{\phi}_2 \sin \phi_2 + \phi_2^2 \cos \phi_2), \]

where \( I_1, I_2 \) are the moments of inertia of the motor and vibration exciter rotors, respectively; \( \beta_x, \beta_y \) are the coefficients of viscous resistance of the elastic coupling and suspension springs of the bearing body; \( c_x, c_y \) is the rigidity of the coupling and suspension springs; \( L_1, R_1 (\phi_1) \) are the motor torque and the moment of the forces of resistance to rotation of the exciter rotor; \( M \) is the total mass of the bearing body; \( m, \varepsilon \) is the mass of the exciter and its eccentricity; \( g \) is the gravitational acceleration.

5.2. Construction of amplitude-frequency characteristics of oscillations of the vibration machine drive

In [16], using the approach of vibration mechanics and the method of direct separation of motions, an approximate expression describing relative torsional oscillations of the elastic coupling elements in the steady (near-steady) operation is obtained

\[ \psi_{12} = A_{we} \sin 2 \omega t + A_{we} \cos (\omega t - \beta_x) \]

as well as the formula for the vibratory moment (additional dynamic load on the motor caused by oscillations) in the following form

\[ V(\omega) = \frac{1}{2} A_{we}^2 \omega + \frac{1}{2 \omega} (A_{we}^2 + A_{se}^2) (\beta_x + k_i) p_i^2, \]

where \( A_{we} = \frac{m \varepsilon \omega^2 A_x}{2I_1 (\omega^2 - p_i^2)^2 + 16b_y^2 \omega^2} \),
\[ A_x = \frac{m \varepsilon \omega^2}{M (\omega^2 - p_i^2)^2 + 4b_y^2 \omega^2} \],
\[ p_i = \frac{I_1}{I_1 + I_2} \]
\[ b_y = \frac{\beta_x}{2 I_2}, \]
\[ I_{12} = \frac{I_1 I_2}{I_1 + I_2}, \]
\[ \omega = \text{the frequency of the steady operation}, \]
\[ p_i = \text{the natural frequency of the coupling}, k_i > 0 \text{ is the electric damping factor}, \]

while the operating speed \( \omega > 0 \) is the electric damping factor, \( \beta_x \) is some phase shift.

Investigations of the steady (near-steady) operation – motor speed “sticking” or slow passage through resonance – (maximum loaded operating modes of the vibration machine) and above-resonance steady operation – are the most practically interesting.

We analyze the amplitudes of torsional oscillations of the elastic coupling in the considered operation modes of the vibration machine. The basic parameters for calculations are selected as follows:

\[ M = 300 \text{ kg}; \]
\[ m = 1.2 \text{ kg} \cdot \text{m}; \]
\[ I_1 = 0.04 \text{ kg} \cdot \text{m}^2; \]
\[ p_i = 24 \text{ s}^{-1} \text{ and } p_i = 40 \text{ s}^{-1}; \]
\[ p_i = 80 \text{ s}^{-1} \text{ and } p_i = 320 \text{ s}^{-1}; \]
\[ h_i = b_i / p_i = 0.1; \]
\[ k_i = 0.3. \]

Note that the parameters correspond to the vibration machine described in [10]. The numerical analysis of changes in the amplitude components \( A_{we} \) and \( A_{se} \) depending on the vibration exciter frequency for “soft” \((p_i < \omega_a)\), where \( \omega_a \) is the operating frequency of the exciter) and “rigid” couplings is carried out. It was found that for the “soft” coupling, oscillations of the coupling (Fig. 2) can resonantly grow during the start-up of the vibration machine in the region of natural frequencies of the machine and coupling (multiple frequencies, primarily \( p_i / 2 \)). Taking into account that amplitude-frequency characteristics are constructed for steady oscillations, it can be argued that the resonant growth of amplitudes is manifested only in the case of motor speed “sticking” in the natural frequency region of the bearing system \( p_i \). At the same time, it is especially unfavorable when the natural frequency of the vibration machine coincides (close) with the frequency equal to half the natural frequency of the coupling \( p_i = p_i / 2 \) (Fig. 2, b). In this case, oscillation amplitudes \( A_{we} \) significantly exceed \( A_{se} \), the value of which near the zero frequency can be considered as static deformation of the coupling. Of course, the decrease in the natural frequency of the coupling (approaching the “sticking” frequency) leads to an increase in \( A_{we} \).

Thus, during the start-up of the vibration machine with a limited-power motor during passage through the natural frequency region, oscillation amplitudes of the drive can resonantly grow. Obviously, during the run-up, drive oscillations in the frequency region \( p_i \) will also resonantly grow. It is practically important that, in the above-resonance steady operation of the vibration machine, torsional oscillations of the drive with the “soft” coupling are rather insignificant—substantially lower than the static deformation and virtually independent of the coupling resistance.

Now that in Fig. 2, 3, vertical straight lines correspond to the frequencies \((p_i, p_i / 2, p_i, \omega_a)\), while the operating speed of the vibration exciter is \( 150 \text{ s}^{-1} \).

The numerical analysis shows that in case of using the “rigid” coupling \((p_i > \omega_a)\), during passage through the natural frequency region \( p_i \), the resonant increase of oscillation amplitudes of the coupling is practically not observed. Moreover, as shown in Fig. 3, it will be rather insignificant even in case of “sticking” of the exciter speed in the frequency range.
However, it turns out that for such couplings in the above-resonance steady operation, in case of relative proximity of the operating frequency of the exciter to half the natural frequency of the coupling $p_c/2$, oscillation amplitudes may acquire resonant values. In this case, the amplitude component $A_{x\psi}$ substantially exceeds the component $g_{A\psi}$ (Fig. 3) and, accordingly, is the main reason for the increased oscillations of the drive. Of course, an increase in the resistance coefficient of the coupling leads to a decrease in the oscillation amplitude. However, even with a large resistance, the amplitudes exceed the value of static deformation of the coupling. Obviously, the higher the natural frequency of the "rigid" coupling in relation to the operating frequency of the exciter (greater distance between the exciter frequency in the operating mode $\omega_p$ and the frequency $p_c/2$), the smaller the oscillation amplitudes (Fig. 3, b).

Note that in Fig. 4, $M_0$ and $\omega_0$ indicate the total torque loading the motor and the frequency of the steady operation in the presence of the elastic coupling in the vibration machine drive, respectively, and $M_0^*$ and $\omega_0^*$ – its absence.

Let’s analyze the formula (3) for the vibratory moment. It should be noted that the first term is the vibratory moment in the absence of the elastic-damping connection in the vibration machine drive, and the second one is an additional component of this moment, arising if it is present. Thus, the resulting formula considers dissipative forces, in case of oscillations of both the bearing body of the vibration machine and the elastic coupling. It is obvious that the resonant growth of the oscillation amplitude of the elastic coupling elements leads to an increase in the magnitude of the vibratory moment. Thus, the presence of the elastic coupling in the vibration machine drive may result in the increase of the motor load in certain operating modes, which also contributes to the occurrence of the Sommerfeld effect during start-up. It is found that in case of "soft" coupling, the resonant increase in the magnitude of the vibratory moment occurs during the "slow" motor run-up in the natural frequency region of the bearing body $p_x$, especially when $p_x$ is close to half the natural frequency of the coupling ($p_x = p_c / 2$). In the steady operation of the vibration machine, there is almost no effect of the elastic coupling on the magnitude of the vibratory moment.
It is found that when using the “rigid” coupling, an increase in the magnitude of the vibratory moment may occur only in the steady operation in case when its operating frequency is close to half the natural frequency of the coupling. Consequently, additional problems associated with such couplings, during the vibration machine start-up do not arise. However, in the above-resonance operating mode, in case of \( \omega_\text{c} = \frac{p_\text{r}}{2} \), some increase in the motor load and, accordingly, decrease in the vibration exciter speed are possible (Fig. 4, b).

5. 4. Vibration machines with plane oscillations of the working body

The results obtained above and in [17] are generalized for the case of a more complex system with plane oscillations of the bearing body (Fig. 5).

Fig. 5. Scheme of the vibration machine with plane oscillations of the bearing body

Note that a fairly wide class of vibration machines can be idealized in the form of the system schematically presented in Fig. 5. The equations of motion of the vibration machine are as follows:

\[
I_1 \ddot{\phi}_1 + \beta_1 (\phi_1 - \phi_2) + c_1 (\dot{\phi}_1 - \dot{\phi}_2) = L_1 (\phi_1),
\]

\[
I_2 \ddot{\phi}_2 - \beta_2 (\phi_1 - \phi_2) - c_2 (\dot{\phi}_1 - \dot{\phi}_2) = -R_2 (\phi_2) + m \varepsilon (\dot{x} \sin \phi_2 + y \cos \phi_2 - \dot{x} \sin \phi_1 + y \cos \phi_1),
\]

\[
M \ddot{x}_c + \beta_x \dot{x}_c + c_x \dot{x}_c = m \varepsilon (\dot{\phi}_2 \sin \phi_2 + \dot{\phi}_1 \cos \phi_2),
\]

\[
M \ddot{y}_c + \beta_y \dot{y}_c + c_y \dot{y}_c = m \varepsilon (\dot{\phi}_2 \cos \phi_2 - \dot{\phi}_1 \sin \phi_2),
\]

\[
J \ddot{\phi} + \beta \dot{\phi} + c \phi = -m \varepsilon (\dot{\phi}_2 \sin \phi_2 + \dot{\phi}_1 \cos \phi_2),
\]  

(4)

where \( I \) is the moment of inertia of the bearing body; \( \beta_x, \beta_y \), \( \beta_\phi \) are the coefficients of viscous resistance of suspension springs of the bearing body; \( c_x, c_y, c_\phi \) is the suspension rigidity; \( r \) is the distance, which determines the position of the exciter axis relative to the center of mass of the bearing body.

Using the method of direct separation of motions in its traditional formulation [2, 19], the expression that describes torsional oscillations of the drive in the steady operation for the vibration machine with plane oscillations of the working body was obtained in the form of (2); thus the amplitude \( A_{\phi} \) should be replaced with \( A_{\phi} \).

\[
A_{\phi} = \frac{m \varepsilon \omega^2}{2I_1 \sqrt{(p_\text{r} - \omega^2)^2 + 16b_2 \omega^2}} \sum A_\gamma.
\]

\[
A_\gamma = \frac{m \varepsilon \omega^3}{M \sqrt{(\omega^2 - p_\gamma^2)^2 + 4b_3 \omega^2}}.
\]

The formula for the vibratory moment for the case of the vibration machine with plane oscillations of the working body was obtained in the following form

\[
V(\omega) = \frac{1}{2} \sum_{\kappa = \pm \varphi, \varphi} A_\kappa \beta_\kappa \omega + \frac{1}{2\omega} (A_\kappa^\varphi + A_\kappa^\varphi)(\beta_\varphi + k_\varphi)p^\varphi.
\]  

(5)

As we can see, the difference between the expressions that describe torsional oscillations of the elastic coupling for vibration machines with rectilinear and plane oscillations of the bearing body lies only in the presence of the sign of the sum for the three generalized coordinates of the bearing body in the formula for the amplitude \( A_{\phi} \) for a more complex oscillation system. Taking into account the similarity of the solutions (2) for both cases, it can be concluded that the increase in the number of degrees of freedom of the bearing body does not introduce qualitative changes in the behavior of the system. In this case, of course, there will be quantitative differences. At the same time, one can expect the increase in the oscillation amplitude of the drive and the magnitude of the vibratory moment component associated with the presence of the coupling to be proportional to the number of degrees of freedom of the bearing body. The reason for such an assumption is the uniformity of the terms under the summation sign in the formula for \( A_{\phi} \) and proximity of all natural frequencies of the soft-vibroinsulated vibration machines.

5. 5. Comparison of the results of analytical studies with the results of computer simulation

The simulation was reduced to the numerical integration of the differential equations (4) and equations, so-called A-model of the asynchronous motor in oblique coordinates of currents [20]. The parameters of the system (in addition to the above) were selected as follows:

\[
J = 12 \text{ kg m}^2; \quad I_1 = 0.0044 \text{ kg m}^2; \quad l = 0.21 \text{ m};
\]

\[
\beta_x = \beta_y = 1000 \frac{\text{kg m}}{\text{s}^2}; \quad \beta_\phi = 70 \frac{\text{kg m}}{\text{s}^2};
\]

\[
c_x = c_y = 5 \times 10^7 \frac{\text{N m}}{\text{s}^2}; \quad c_\phi = 2 \times 10^4 \frac{\text{N m}}{\text{rad}};
\]

motor: \( P = 1.5 \text{ kW}, \quad n_{\text{nom}} = 1415 \text{ rpm}, \) multiplicity of the start-up torque 2, with the so-called direct-on-line start.

Fig. 6 shows the growth of oscillations of the elastic coupling during the passage of the vibration machine through the natural frequency region in case of the appearance of the Sommerfeld effect and relative proximity of the frequency \( p_\text{r} / 2 \) to the frequency of motor speed “sticking”. It should be noted that the frequency of motor speed “sticking” is difficult to determine precisely, therefore, taking into account its proximity to the frequency \( p_\gamma \), it is more convenient to operate with the frequency \( p_\gamma \). Then, as in the start-up without “sticking” (reduced static moments of unbalances with other same parameters of the system), the growth of the coupling oscillations does not occur. Of course, this is due to the rather fast passage of the vibration exciter through the region of resonant frequencies of the vibration machine \( p_\gamma \). Further, in this case, there is also a fast passage of the vibration exciter through the natural frequency region of the coupling \( p_\gamma \). Consequently, in case of “soft” elastic couplings, the critical frequency \( p_\gamma / 2 \) is the main.

The simulation results indicate: in case of motor speed “sticking”, the values of deformation and torque in the coupling can reach approximately the same maximum values as immediately at the start-up; however, the duration is significantly longer. Thus, oscillation processes in the
vibration machine drive during passage through resonance are more dangerous than at the time of start-up. It is shown that by decreasing the natural frequency of the coupling, it is possible to aggravate the motor speed “sticking” up to make it impossible to run up in the above-resonance operating mode.

It is practically important that, despite the motor speed “sticking”, with a sufficient distance between the frequency \( p_c/2 \) and \( p_q \), the increase in the oscillations of the elastic coupling in the natural frequency region of the vibration machine is rather insignificant (Fig. 7).

It is shown that in case of “soft” coupling, start-up oscillations in the drive fade rather quickly and are relatively small in the operating mode. It is also important that this conclusion applies to cases of motor speed “sticking” in the region \( p_c \) and small coupling resistance. For this reason, an increase in the coupling resistance coefficient is inexpedient. Moreover, at large resistance, the value of torque in the coupling significantly increases.

It is found that in case of using the “rigid” coupling, despite the motor speed “sticking” in the resonance region of the vibration machine, a significant increase in the coupling oscillations is not observed at any system parameters. However, in the steady operation, in case of proximity of the operating frequency of the exciter to the frequency \( p_c/2 \), oscillation amplitudes of such a coupling are sufficiently large compared with the case of distance between these frequencies (Fig. 8). At the same time, for the considered case of machine parameters, the increase in the motor load and the decrease in the vibration exciter speed are insignificant.

Fig. 8. Time variations of the amplitude of torsional oscillations of elements: 1 \( - p_c = 297 \, \text{s}^{-1} \), \( p_c/2 = \omega_c \), 2 \( - p_c = 375 \, \text{s}^{-1} \)

Fig. 9. Time variations of the amplitude of torsional oscillations of the coupling during the vibration machine run-up \( (p_s = p_g = p_e = 44 \, \text{s}^{-1}) \): 1 \( - p_s = 46 \, \text{s}^{-1} \), 2 \( - p_s = 64 \, \text{s}^{-1} \)

Fig. 9 shows the emergence of resonant oscillations of the elastic coupling in the passage of the vibration machine through the natural frequency region during run-up. From the graphs below, it follows that at a greater distance between the natural frequencies \( p_c \) and \( p_c/2 \), the resonant growth of the coupling oscillations is significantly smaller.

6. Discussion of the results of the study of oscillations of the drive of the vibration machine with unbalance exciter

Thus, in the unbalance drive of the vibration machine, substantial torsional oscillations can be excited. Therefore, checking of the conditions for resonance in the drive is an important task. It is obvious that resonant oscillations can only occur during the start-up of the vibration machine in case of motor speed “sticking” in the natural frequency region of the machine, as well as in its above-resonance operating mode. Therefore, it is not enough to choose the natural frequency of elastic couplings of the vibration drive only with the known condition of their efficiency \((\omega_c/p_c > \sqrt{2})\).

Critical frequencies of the vibration machine drive with the elastic coupling, in addition to the natural frequency of the coupling \( p_c \) and multiple frequencies (primarily, \( p_c/2 \)), are also natural frequencies of the machine \( p_g \). In order to decrease the oscillation amplitude of the coupling during...
the passage of the vibration machine through resonance, it is recommended, first of all, to provide a sufficient distance between half the value of the natural frequency of the coupling and natural frequencies of the machine. This recommendation avoids the negative effects of passage through resonance associated with the presence of the elastic coupling, without increasing the motor power and value of the resistance coefficient. In addition, this allows you to slightly reduce resonant oscillations during the machine run-up. It should be noted that the proximity of the frequencies $p_c/2$ and $p_c$ is quite probable for the considered class of soft-vibroinsulated vibration machines ($p_c << \omega_n$) with the “soft” coupling. Taking into account the above, when designing vibration machines with unbalance exciters to choose the rigidity of the “soft” coupling of the drive, we can recommend the condition $2,5 p_c < p_c < 0,7 \omega_n$. Of course, these limits are approximate and may vary slightly depending on system parameters.

It is found that for the vibration machine with the “rigid” coupling in the drive, there are no problems with its start-up oscillations. However, for such couplings, rather large oscillations can be excited in the steady operation at a relative proximity of the operating frequency of the vibration machine $\omega_n$ to half the natural frequency of the coupling $p_c/2$. The condition of the absence of increased oscillations of the “rigid” coupling is proposed in the form of $p_c > 2,5 \omega_n$.

The method of analyzing the drive oscillations can be used in the design of vibration machines with unbalance vibration exciters. In the future, it is planned to carry out field experiments on a vibration stand for practical confirmation of the conclusions and recommendations put forward.

Of course, the obtained formulas for the oscillation amplitude of the unbalance drive and vibratory moment can be used only for qualitative analysis of dynamic processes and identification of the general nature of their dependences on system parameters. For quantitative estimation, they are fairly approximate. In addition, their use requires actual parameters of damping of a specific vibration machine, which are difficult to determine precisely.

7. Conclusions

1. Formulas for the amplitude of torsional oscillations of the unbalance drive and vibratory moment in the steady (near-steady) operation of the vibration machine with plane oscillations of the working body are obtained. They allow estimating the oscillation amplitudes of the drive and the dynamic load of the motor, depending on the natural frequencies of the vibration machine and the drive, as well as vibration exciter speed.

2. It is shown that during the start-up of the vibration machine in case of motor speed “sticking”, resonant torsional oscillations of the drive can be excited in the natural frequency region. Resonant oscillations arise only when using the “soft” coupling in case of proximity of natural frequencies of the vibration machine $p_c$ and frequency $p_c/2$. It was found that amplitudes of these oscillations can reach the same values as at the moment of motor start-up, and their duration can be substantially greater.

3. It was found that when resonant oscillations of the drive are excited, the magnitude of the vibratory moment that loads the motor during the run-up increases. This, in turn, leads to an increase in the start-up amplitudes of the vibration machine and promotes the emergence of the Sommerfeld effect.

4. Resonant oscillation amplitudes of the drive of the vibration machine in the steady operation are possible when using the “rigid” coupling only in case of proximity of its frequency to the frequency $p_c/2$. It was demonstrated that an effective means of avoiding resonant oscillations of the drive is to provide sufficient distance between its critical frequency $p_c/2$ and both the operating frequency of the vibration machine and its natural frequencies.

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