Vibration separation of bulk construction mixtures by measurement sizes on sieve classifiers

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Abstract. The vibrational separation of bulk building mixes by size is studied using stochastic-based sieve classifiers. The system of stochastic differential equations is constructed to determine the linear density of particles on the surface of the classifier sieves. The linear density of particles on sieves is approximated by white noise, which allows the system of equations to be called stochastic. The solution of the system of stochastic differential equations with respect to the average value of a random process is determined. We construct an approximate solution for the distribution density and transition probabilities of a random process using the properties of the white noise. Knowing the density of the distribution of the number of particles on the surface of the classifier sieves determines all its characteristics of interest. The obtained solutions allow us to calculate the extraction coefficient and evaluate the efficiency of separation.

Key words: bulk material, separation, screening probability, stochastic modeling.

1 Introduction

In various technological processes, in particular, in chemical technology, the production of building materials, mineral processing and other industries, it is often necessary to separate granular material by size into certain fractions. The classification is used as an auxiliary operation to remove impurities before grinding the material, as well as an independent operation to isolate the finished product of a given fractional composition. In the production of building materials, the uniformity of the fractional composition of the used granular mixture affects the quality of the products. Therefore, the separation efficiency of bulk mixtures is the most important indicator of the process.

There is a large number of criteria in the scientific literature to assess the effectiveness of classification [1-3]. This is due to the fact that the classification results can be characterized by various indicators, such as recovery, pollution, concentration, etc. However, the lack of unity in the question of choosing a method for evaluating and optimizing the process as a whole creates a situation of uncertainty when choosing the classifiers themselves and evaluating their work. Therefore, in most cases, the design of new equipment and the optimization of existing ones are based on previous experience in operating related equipment. In practice, the quality of separation is most fully evaluated by two indicators – the degree of extraction of the desired fraction from the starting material and its contamination.

To separate bulk mixtures by size, vibration devices are often used, in particular, multilevel classifiers [4-5]. This is due to the fact that vibration converts the dry friction forces that arise from the interaction of particles of a granular mixture into forces such as viscous friction. As a result of vibration, conditions are created for the manifestation of differences in the separation parameters. The
classification process on sieves can be represented as a combination of three qualitatively different types of vibrational motion, namely, the movement of the material layer on the surface of the sieve, the movement of particles of the passage fraction in the direction of the sieve through the layer of granular material and the passage of particles through the openings of the sieve.

Mathematical models of the processes that occur during the separation of granular materials are the basis for the optimal design and technological calculation of the sieve classifier. In the paper [4], the motion of particles in an oscillating medium was studied, various models of vibrational motion were considered, and dependences were obtained for the average velocity and segregation velocity. Depending on the hydrodynamic properties of the material to be separated, particle size distribution, the shape of individual particles, the presence of specific properties, etc. the motion of granular materials along a vibrating surface can be modeled both in the approximation of a single material point and on the basis of methods of mechanics of heterogeneous media [6-11].

In the paper [12], the process of separation of granular media on sieve classifiers was considered as a diffusion process and a change in the concentration of the number of passage particles along the thickness of a layer of granular material depending on time in the direction of the vibrating surface was studied from the standpoint of Markov processes and described by the Kolmogorov-Fokker-Planck equation. In the papers [1, 13], the process of isolating target products on multistage classifiers was also studied using the theory of random processes. In particular, in the paper [13], the theory of Poisson processes is used to describe the process of moving particles on sieve surfaces due to sieving. The study of particles segregation in a fluidized bed based on Markov chains was the subject of the paper [14]. In the paper [15], the basic principles of the organization of technological processes with controlled segregated flows that arise during the processing of granular materials are given. The Monte Carlo simulation screening probability was described in the papers [16-17]. The probability of sifting particles into sieve cells was studied in the papers [1, 5]. This probability can be determined depending on the geometric dimensions and shape of the particles and openings of the mesh cells, as well as the speed of the vibratory movement.

Thus, various approaches can be used to simulate the process of separating granular materials into specific fractions by size, but taking into account the random nature of the process as a whole, the stochastic approach is most preferable [1, 12-15].

The aim of this paper is the mathematical modeling of the vibrational separation of bulk building mixtures by size on multilevel sieve classifiers based on the theory of random processes.

2 Materials and methods
The process of vibration separation of bulk mixtures into specific fractions by size on a multilevel sieve classifier is considered. The working body of the multilevel classifier is several oscillating screening surfaces, which can be made in the form of a sieve or a bolter. Moreover, they are located above one another, forming tiers. The initial granular material, which is characterized by a size distribution function, is fed to the beginning of the upper tier and during vibrational movement is divided into a passage and a descent part. At the same time, the first largest fraction is removed from the first tier, and the product for separation on the next \( i \)-th level is granular material sifted from the upper \( i-1 \)-th level.

A description of the process of isolating target products from a granular mixture on a sieve classifier will be carried out taking into account the stochastic nature of such processes. As a random process, we consider the value of:

\[
N_i = N_i(x, t; d_j),
\]

which determines the linear particle density (1/m) with dimensions \( d_j \) at a distance \( x \) from the beginning of the \( i \)-th sieve at time \( t \). Then the system of kinetic equations describing the process of thin-layer separation of bulk mixtures on a sieve classifier can be represented as:

\[
\frac{dN_i}{dt} = \frac{\partial N_i}{\partial t} + V \frac{\partial N_i}{\partial x} = \alpha_{i-1}N_{i-1} - \alpha_iN_i + \beta \eta_i(t), \quad \alpha_0 = 0, \quad i = 1, n,
\]
where \( a_i \) are the coefficients of kinetic equations, \( n \) is the number of classifier screens, \( \eta(t) \) are delta-time-correlated random functions (white noise) with known numerical characteristics:

\[
M[\eta(t)]=\langle \eta(t) \rangle=0 \quad \text{and} \quad \text{K}[\eta(t)]=\langle \eta(t)\eta(t+\tau) \rangle=\Delta \delta(\tau)/2,
\]

(3)

\( \beta \) is the intensity, \( \Delta/2 \) is the spectral density of white noise. A feature of Eq. (2), which allows us to call them stochastic, is the presence of influence in the form of white noise [18].

The number of particles of the selected fraction on the surface of the \( i \)-th sieve at any time \( t>t_i=L/v_{cp} \) is determined by the expression:

\[
\overline{N}_i(t) = \frac{1}{v_{cp}} \int N_i(x,t;d_i)dx
\]

(4)

where \( L \) is the length of the sieve, \( v_{cp} \) is the average speed of vibrational motion. The deviation of the number of \( \overline{N}_i(t) \) particles from the average value at any time is related to the probability of sieving particles into sieve cells. The probability of sifting into a cell depends on the size and shape of the particles, the particle size distribution of the material to be separated, constraint conditions and other factors, as well as on the relative speed of the vibrational movement of the material. Therefore, the number of particles \( \overline{N}_i(t) \) is considered as a random process. An approximation of the random process \( \overline{N}_i(t) \) by white noise is possible, because the correlation time of the random process is much shorter than the average residence time of the selected particles on the sieve surface. Taking into account the properties of white noise [18], the \( \overline{N}_i(t) \) process is a Markov process; therefore, to study it, one can use the mathematical apparatus of the theory of Markov processes [18-21]. Then the distribution density \( \overline{N}_i(t) \) of white noise \( \overline{N}_i(t) \) is determined from the solution of the Kolmogorov-Fokker-Planck system of equations (K.-F.-P.) [21]:

\[
\frac{\partial W_i}{\partial t} = -\frac{\partial}{\partial x_i}(v_{cp}W_i) - \sum_{j=1}^{n} \frac{\partial}{\partial x_j}W_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{\partial^2}{\partial x_i x_j}B_{ij}W_i \right), \quad i = 1,n
\]

(5)

where \( x_i=N_i, \quad k=1,T; \quad x_{i,j}=x_iB_{ij} \) are the diffusion coefficients. Solving equations (K.-F.-P.) for large values of \( n \) is a difficult task.

The coefficients of kinetic equations in a first approximation are calculated according to the dependence [1, 5]:

\[
a_i=v_{cp}p_i/2a_i
\]

(6)

where \( p_i \) is the probability of sifting into the cell, \( 2a_i \) is the step of the \( i \)-th sieve. The coefficients \( a_i \) determine the number of particles passing through the cell in one second, thereby characterizing the rate of change of the random process.

The probability of sifting into the cell is considered as a complex event [1]:

\[
P=p_g p_v
\]

(7)

where \( p_g \) is the probability, which depends on the size and shape of the sieve cell and particles of the material to be separated, and \( p_v \) is the probability, which depends on the relative speed of the particle along the vibrating surface. The calculation of the probability of sifting into a cell was considered in [1]. The probability of speed is determined by the formulas:

\[
p_v=2-(\Phi(z)+\Phi(z_0)), \quad z=(v_g-v_f)/\sigma, \quad z_0=v_k/\sigma,
\]

(8)

where \( v_g, v_f \) is the particle velocity amplitude relative to the sieve, \( \Phi(x) \) is the standard normal distribution function, \( v_G, \sigma \) are the parameters of the normal law, which are determined in the process of identifying the constructed models. For this, the calculated values of the extraction coefficient are compared with the experimental values obtained for some well-defined high-speed modes of operation of the apparatus.

The vibrational motion of granular media was studied in sufficient detail in the paper [4], the calculation of the average speed, the relative velocity amplitudes for some modes of vibrational motion are given in the papers [1, 13]. For example, the relative speed for the regular mode with instant stops without tossing, which is often used in separation machines with a relatively small thickness of the moving material layer, can be written as:

\[
v_f(\tau) = \frac{a}{\omega} \frac{1}{1+k_0} \left( k_0 \sin \tau - \cos \tau \right) - \left( k_0 \sin \tau - \cos \tau \right) \times
\]

(9)
\[
\times \exp\left(-k_e(\tau - \tau_c)\right) - \frac{z}{k_e}(1 - \exp(-k_e(\tau - \tau_c))),
\]

where \(a_i = A\omega^2 \cos(\beta \pm \gamma)/\cos\gamma\), \(z_i = g \sin(\pm \gamma)/A\omega^2 \cos(\beta \pm \gamma)\), \(k_e = k_{op}/\omega\), \(\tau = \omega t\),

where parameters \(A\) and \(\omega\) are the amplitude and frequency of oscillations, \(\alpha\) and \(\beta\) are the slope and vibration angles of the sieve, \(\gamma\) is the angle of sliding friction, \(k_{op}\) is the coefficient of resistance to particle motion in the medium. The upper sign corresponds to the forward movement, the lower sign – to the back, and the transition points \(\tau_c\) are determined from the joint solution of nonlinear equations: \(v_+(\tau_c) = 0\) and \(v_+ (\tau_c + 2\pi) = 0\).

Consider the solution of differential Eq. (2) under the following initial and boundary conditions:

\[N_i(0, x) = 0\text{ at } i = I, m\text{ and } N_i(t, 0) = N_{i0}(t), N_i(t, 0) = 0\text{ at } i = 2, m.\]  

(10)

Conditions (10) determine the supply of the material to be separated only at the beginning of the upper tier of the multilevel classifier, \(N_{i0}\) is the number of selected particles (1/m) that arrive at the beginning of the first sieve. Using the substitution \(\tau_i = t - x/V_i\), \(z = x\), Eq. (2), taking into account conditions Eq. (10), can be solved by reduction to ordinary differential equations [6]. For example, the general solution of the differential equation for the upper tier can be written as:

\[N_i^\prime(\tau, z) = \frac{\beta_i}{V_i} \int_{\tau_i}^{\tau} n_i(\tau, z) \exp\left(\frac{\alpha_i(z - z)}{V_i}\right) d\xi,\]

where \(\overline{N}_i^\prime\) determines the average value of the random process \(N_i^\prime\).

Based on the solutions of system Eq. (2), for each tier, one can determine the average value, the correlation function, and the variance of the \(N_i\) process. The average value of the random process \(\overline{N}_i\) for the upper tier, taking into account conditions Eq. (10), has the form:

\[\overline{N}_i(\tau, z) = \frac{\beta_i}{V_i} \int_{\tau_i}^{\tau} n_i(\tau, z) \exp(-\alpha_i z/V_i) d\xi.\]

(12)

Also, taking into account the properties of white noise [18], one can find the variance of the process:

\[\sigma_i^2(\tau, z) = \frac{\Lambda_i}{4\alpha_i} \beta_i^2(1 - \exp(-2\alpha_i z/V_{i0})).\]

(13)

The solution of the differential equation with respect to the average for the second sieve from system Eq. (2), taking into account the solution for the first and second, as well as the initial and boundary conditions Eq. (10), has the form:

\[\overline{N}_j(\tau, z) = \frac{\alpha_j}{V_j} \int_{\tau}^{\tau} \psi_{i, 2} \exp(-\alpha_j z/V_j) + \psi_{2, 2} \exp(-\alpha_j z/V_j) d\xi,\]

where \(\psi_{1, 2} = 1/(\alpha_j/V_j - \alpha_j/V_j)\), \(\psi_{2, 2} = 1/(\alpha_j/V_j - \alpha_j/V_j)\).

The solution of the differential equation with respect to the average value of the random process for the third sieve from system Eq. (2), taking into account the solutions for the first and second, as well as the initial and boundary conditions Eq. (10), has the form:

\[\overline{N}_j(\tau, z) = \frac{\alpha_j}{V_j} \int_{\tau}^{\tau} \psi_{i, 3} \exp(-\alpha_j z/V_j) + \psi_{2, 3} \exp(-\alpha_j z/V_j) + \psi_{3, 3} \exp(-\alpha_j z/V_j) d\xi,\]

where \(\psi_{1, 3} = 1/[\alpha_j/V_j - \alpha_j/V_j] \psi_{2, 3}, \psi_{2, 3} = 1/[\alpha_j/V_j - \alpha_j/V_j] \psi_{2, 3} \psi_{3, 3} = 1/[\alpha_j/V_j - \alpha_j/V_j] \psi_{3, 3}\).

Using the induction method, it can be shown that the solution of the differential equation with respect to the average value of the random process for the \(i\)-th sieve has the form:

\[\overline{N}_i(\tau, z) = \frac{\alpha_i}{V_i} \int_{\tau}^{\tau} \psi_{1, i} \exp(-\alpha_i z/V_i) + \psi_{2, i} \exp(-\alpha_i z/V_i) + \psi_{3, i} \exp(-\alpha_i z/V_i) d\xi,\]

where \(\psi_{1, i} = 1/[\alpha_i/V_i - \alpha_i/V_i] \psi_{2, i} \psi_{3, i} = 1/[\alpha_i/V_i - \alpha_i/V_i] \psi_{3, i}\).

In the general case, to determine the distribution density of a random process, the solution of the system of equations (K.-F.-P.) is carried out by numerical methods. When approximated by white noise, a random process \(N_i\) is normal [18]. Therefore, knowing the numerical characteristics of a random process, we can write an approximate solution for the distribution density and transition
Then the general solution of the equation (K.-F.-P.) for the first sieve has the form:

\[ W_i(N_i, \tau_i) = \frac{1}{2\pi \sigma_{N_i}^2(\tau_i,z)} \exp\left(-\frac{(N_i(\tau_i,z) - \overline{N}_i(\tau_i,z))^2}{2\sigma_{N_i}^2(\tau_i,z)}\right), \]  

(17)

\[ W_{i,i-1}(N_i, \tau_i) = \frac{1}{2\pi \sigma_{N_i}^2(\tau_i,z)} \exp\left(-\frac{(N_i(\tau_i,z) - \overline{N}_i(\tau_i,z))^2}{2\sigma_{N_i}^2(\tau_i,z)}\right). \]  

(18)

In the general case, \( N_i \) in the initial section is a random variable with probability density \( \overline{W}_{i0}(N_i, \tau_i) \). Then the equation (K.-F.-P.) for the first sieve, using the replacement:

\[ \varphi = N_i \exp\left(-\frac{\alpha_i}{V_i} x \right), \quad \xi = \frac{\beta_i}{2\alpha_i} \left(1 - \exp\left(-\frac{2\alpha_i}{V_i} x \right)\right) \]  

(19)

reduced to the simplest diffusion equation \( \frac{\partial \overline{W}}{\partial \xi} = \frac{1}{2} \frac{\partial^2 \overline{W}}{\partial \varphi^2} \) and the solution of the resulting equation is written in the form of an integral convolution:

\[ \overline{W}(\varphi, \xi, \tau_i) = \frac{1}{\sqrt{2\pi \xi}} \int_0^\infty \left( \overline{W}_i(\theta, \tau_i) \exp\left(-\frac{(\varphi - \theta)^2}{2\xi}\right) - \exp\left(-\frac{(\varphi + \theta)^2}{2\xi}\right) \right) d\theta. \]  

(20)

Then the general solution of the equation (K.-F.-P.) for the first sieve has the form:

\[ W(N_i, x, \tau_i) = \overline{W}(\varphi, \xi, \tau_i) \exp\left(-\alpha_i x / V_i \right). \]  

(21)

The coefficients of the kinetic equations of system (2) \( \alpha_i \) determine the probability of passage of particles of the \( j \)-th fraction through the holes of the \( i \)-th sieve per unit time, thereby characterizing the speed of the process. Then the average value of the number of passes of the \( j \)-th fraction from the \( i \)-th sieve taking into account solution Eq. (16) over time \( \Delta t_i \) is determined by the formula:

\[ \overline{N}_{ji} = \int_0^{\Delta t_i} \int \alpha_i / \overline{N}_{j0} \exp(-\alpha_i z / V_j) dz dt = V_i / \Delta t_i \overline{N}_{j0} (1 - \exp(-\alpha_i L_j / V_j)), \]  

(22)

where \( \overline{N}_{j0} \) is the number of particles of the \( j \)-th fraction per unit length of the \( i \)-th sieve in its initial section. The expression \( V_i \Delta t_i \overline{N}_{j0} \) in Eq. (22) determines the total number of particles of the \( j \)-th fraction entering the \( i \)-th screen during time \( \Delta t_i \). Then, the extraction coefficient of the \( j \)-th fraction from the \( i \)-th sieve, taking into account Eq. (22), is determined by the formula:

\[ \eta_i^j = \exp(-\alpha_i L_j / V_j), \quad i = \overline{I} n, \quad j = i, i+1, \]  

(23)

where \( j \) -- large (descent) fraction, \( j = i+1 \) -- the next largest (pass) fraction.

The classification process will be more effective the more extraction can be obtained and the products will be less "contaminated". The separation efficiency on the \( i \)-th sieve is calculated by the dependence:

\[ E_i = \eta_i^j (1 - \eta_i^{j+1}) \times 100\%, \quad i = \overline{I} n, \]  

(24)

where \( \eta_i^{j+1} \) is the fractional fraction of the coarse fraction in the coarse fraction, which for the \( i \)-th fraction is considered as the fraction of impurities in the target product. It should be noted that when separating granular materials into multi-tiered sieve classifiers, when there is no "contamination" of the lower product by the upper, the extraction characterizes the process quite fully. The higher the extraction of fine fractions, the more efficient the operation of the classifier [3].

### 3 Results

Figure 1 shows the results of calculating the distribution of the linear particle density of the target products along the first and second sieves of the multilevel classifier. Based on the obtained solutions, the extraction coefficients of the target products are calculated.

Also, the quality of the sieved residues from the sieves is evaluated. For this, the proportion of non-target products (small) that are in the target product is determined. Based on the constructed mathematical models for determining the optimal values of the structural and operational parameters of the classifier, it is possible to formulate and solve the optimization problem in a multi-criteria setting [1-3, 22].
Figure 1. The change in the linear particle density of the target products along the first and second sieves of the multilevel classifier: $\alpha_1^{1 \rightarrow 2} = 5.78 \times 10^{-3}$; $\alpha_1^{2 \rightarrow 3} = 8.42 \times 10^{-2}$; $\alpha_1^{3 \rightarrow 4} = 2.57 \times 10^{-1}$; $\alpha_2^{1 \rightarrow 2} = 3.66 \times 10^{-4}$; $\alpha_2^{2 \rightarrow 3} = 2.77 \times 10^{-3}$; $\alpha_2^{3 \rightarrow 4} = 7.18 \times 10^{-1}$; $\alpha_2^{4 \rightarrow 5} = 6.93 \times 10^{-1}$ (sec$^{-1}$); $V_{cp} = 0.05$ m/s; fraction sizes: 1 – (0.8 $\pm 0.9) \times 10^{-3}$; 2 – (0.7 $\pm 0.8) \times 10^{-3}$; 3 – (0.6 $\pm 0.7) \times 10^{-3}$; 4 – (0.5 $\pm 0.6) \times 10^{-3}$ (m).

4 Discussions

The theory of random processes using the experimental results to determine the model parameters allows you to build mathematical models of the vibration separation of bulk building mixtures into specific fractions by size on multi-level sieve classifiers and to evaluate the separation efficiency.

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