Heat transfer through a double-glazed window by radiation

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Abstract. A method of separately combined analysis of the process of radiant-convective heat transfer through the central part of the double-glazed windows is developed. The problem of heat transfer by radiation through a double-glazed window is formulated and a solution is obtained in an analytical form. The solution is applicable for all types of double-glazed windows with a given number, characteristics and location of low-emission coatings. The possibilities of reducing radiant heat losses through double-glazed windows due to an increase in the number of glasses and the use of glasses with low-emission heat-protective coatings are analyzed in detail.

1. Introduction

Windows are the weakest link among building envelopes in terms of thermal performance. The existing methodology for calculating the thermotechnical characteristics of translucent structures does not allow us to get a complete picture of the features of the processes of radiant and convective heat transfer through windows, their mutual influence on each other and the influence of numerous individual parameters [1]. As a result, such results do not make it possible to consciously predict the optimal combination of the parameters of double-glazed windows to achieve their best thermal properties. Such actions can only be carried out if there are analytical results.

The main element of translucent structures are double-glazed windows - bulk products consisting of two or three sheets of glass, interconnected by distance frames and sealants, forming hermetically sealed chambers filled with dried air or other gas. The main types and designs of double-glazed windows approved are shown in figure 1.
Double-glazed window chambers can be filled with dried air, as well as inert gases or their mixture (argon Ar, krypton Kr). At production of double-glazed windows various types of glasses are applied: colorless sheet; tempered, heat-strengthened, low-emission with a soft coating, low-emission with a hard coating, as well as sun-protection, laminated and other types of glasses. The application of a low-emission coating to colorless glass leads to an increase in its reflectivity in the region of near and far infrared radiation [2, 3].

2. Results and discussion

When calculating heat losses through a double-glazed window, the heat flux in individual sections of radiation-convective heat transfer (on the outer and inner surfaces of the double-glazed window, in the inter-glass gap) is usually expressed as the sum of the convective and radiant components. In this case, the value of each of these components at a known temperature difference $\Delta t$ in a separate section of heat transfer is calculated using the coefficients of convective $\alpha_c$ and radiant heat transfer $\alpha_r$ [1]:

\[
q_c = \alpha_c \Delta t; \quad q_r = \alpha_r \Delta t.
\]  

However, due to changes in both the components of the heat flux and the relationship between them at different sites of heat transfer through the window, it is rather difficult to draw a conclusion about the magnitude of each of these components and the influence of the main process parameters on them.

In order to get a complete picture of all the properties of complex radiation-convective heat transfer through a window, the method of separately combined analysis of this process is used here. In this case, only radiant heat transfer through the window is investigated at first. Then only the convective component of the process is considered. Finally, the measure and nature of the interaction of the radiant and convective components are clarified when they are combined into a joint process of radiation-convective heat transfer through the window.

The problem is formulated as follows. A room with an internal surface of area $F_i$ is separated by a building envelope from the external environment with surface $F_o$ (figure 2). A single-chamber double-glazed window is installed in the opening of the building envelope. The areas of all four glass surfaces are the same, and the emission coefficients of the four glass surfaces are different and equal to $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, $\varepsilon_4$, respectively. The temperature of the interior surfaces of the room is the same and equal to $t_i$. The surface temperature $F_o$ of the environment is also the same and is equal to $t_o$. From a room into the environment, heat is transmitted through a double-glazed window by radiation. It is necessary to determine the glass temperatures $t_I$, $t_{II}$ and the density of the radiant heat flux $q_r$ through the window. Further, the subscripts indicated in Arabic numerals refer to the parameters of the glass surfaces according to that adopted in figure 1 the numbering of the surfaces, and the lower indices indicated by Roman numerals refer to the parameters of the whole glass according to the numbering in figure 2. In the analysis of radiant heat transfer, it is assumed that the temperatures of both surfaces of each glass are equal to the temperature of the whole glass: $t_1 = t_2 = t_3 = t_4 = t_0$.

![Figure 2. A physical model of heat transfer by radiation from a room into the environment through a single-chamber double-glazed window.](image)
It should be noted that the temperatures $t_o$ and $t_i$ are the radiation temperatures of the surfaces. In general, they differ from the ambient temperature to the environment and the indoor air temperature $t_i$. But the temperatures of the internal surfaces of the room and the air temperature are usually the same. For the external environment, the equality of radiation temperature and air temperature is carried out with dense clouds in the sky. Here we accept the equality of these temperatures.

Two radiating surfaces – the surface of the room $F_i$ and the surface of the inner glass $F_4$ – form a closed system, and the surface of the glass $F_4$ is flat, has no concavities and does not radiate on itself. For such a system, the density of the heat flux transmitted by radiation from the inner surface of the room to the inner (second) glass is determined by the expression [4]:

$$ q_{r,4i} = \varepsilon_{4i} C_o \left[ \left( \frac{T_i}{100} \right)^4 - \left( \frac{T_{II}}{100} \right)^4 \right]. $$

Here $T_i$ and $T_{II}$ are the indoor surface temperature and glass temperature are expressed in a thermodynamic scale (hereinafter, the temperatures $t$ and $T$ with the corresponding indices denote the temperatures of the corresponding surfaces, expressed in practical ($t, ^\circ\mathrm{C}$) and thermodynamic ($T, \mathrm{K}$) scales); $C_o = 5.67 \, \text{W/(m}^2\cdot\text{K}^4)$ is the emissivity of a completely black body.

The value $\varepsilon_{4i}$ is the reduced emission coefficient of the emitting system, including $F_i$ and the fourth surface $F_4$ of the double-glazed unit:

$$ \varepsilon_{4i} = \left[ \frac{1}{\varepsilon_4} + \frac{F_4}{F_i} \left( \frac{1}{\varepsilon_i} - 1 \right) \right]^{-1}. $$

This characteristic, in addition to the values of $F_4$ and $F_i$, depends on the emission factors of the glass surface $\varepsilon_4$ and the room surface $\varepsilon_i$. Considering the fact that in the vast majority of practical problems, the inner surface of the room $F_i$ is much larger than the glass surface $F_4$, we get $F_4/F_i \to 0$. As a result, for all these cases, expression (3) is significantly simplified

$$ \varepsilon_{4i} = \varepsilon_4, $$

and the reduced emission coefficient of the radiating system $F_i$ and $F_4$ is determined only by the value of the emission coefficient $\varepsilon_4$ of the fourth surface of the glass packet. It should be noted that the result (4) is obtained from (3) also in the case $\epsilon_3 = 1.0$.

The radiating surface $F_0$ of the environment and the outer surface $F_1$ of the glass packet also form a closed radiating system. Applying similar reasoning and assumptions, we obtain the following expression for calculating the density of the radiant heat flux scattered into the environment

$$ q_{r,01} = \varepsilon_1 C_o \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_o}{100} \right)^4 \right], $$

in which $\varepsilon_1$ is the emission coefficient of the first glass surface in the glass packet.

Two glasses form a radiating system in the form of two infinite parallel plates. The temperatures of the plates are $T_1$ and $T_{II}$, and the emission coefficients of the surfaces are $\varepsilon_2$ and $\varepsilon_3$, respectively. The radiant heat flux density in this system is calculated using the expression:
The reduced emission coefficient \( \varepsilon_{23} \) of such a radiating system is determined by the formula

\[
\varepsilon_{23} = \left( \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} - 1 \right)^{-1}.
\]

In the steady state process of radiant heat transfer through a double-glazed window, the values of all heat fluxes are the same:

\[
q_{o1} = q_{r,23} = q_{r,4i} = q_r.
\]

Expressions (2), (5) and (6), taking into account relation (8), make up a system of three equations with respect to three unknowns \( q_r, T_I, T_{II} \). Here we apply the solution method used in the calculation of heat transfer by radiation through a system of screens, since glasses in a double-glazed window for thermal radiation play the role of screens. We also note that this method is applicable for double-glazed windows with any number of glasses – one, two, three or more.

We begin the solution by determining the density \( q_r \) of the radiant heat flux. For this, from equations (2), (5) and (6), taking into account relation (8), we express the temperature differences in each of the three heat transfer sections:

\[
\frac{T_I - T_{II}}{100} = \frac{q_r}{C_o \varepsilon_4} \left( \frac{T_{II}}{100} - \frac{T_I}{100} \right)^4 \quad \frac{T_{II} - T_I}{100} = \frac{q_r}{C_o \varepsilon_{23}} \left( \frac{T_I}{100} - \frac{T_{II}}{100} \right)^4 \quad \frac{T_I - T_o}{100} = \frac{q_r}{C_o \varepsilon_i}.
\]

A summation of these relations excludes unknown temperatures \( T_I, T_{II} \) and leads to an expression that includes a single unknown quantity - the density of the radiant heat flux \( q_r \):

\[
\frac{T_I}{100}^4 - \frac{T_o}{100}^4 = \frac{q_r}{C_o} \left( \frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_{23}} + \frac{1}{\varepsilon_4} \right) = \frac{q_r}{C_o} \left( \frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_4} - 1 \right).
\]

Hence we find the \( q_r \) value:

\[
q_r = \frac{C_o}{\left( \frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_4} - 1 \right)} \left( \frac{T_I}{100}^4 - \frac{T_o}{100}^4 \right).
\]

After that, from the equations (9) it is possible to determine the temperatures of the glasses \( T_I, T_{II} \), since \( q_r \) is a known quantity:

\[
\left( \frac{T_I}{100} \right)^4 = \left( \frac{T_o}{100} \right)^4 + \frac{q_r}{C_o \varepsilon_i}, \quad \left( \frac{T_{II}}{100} \right)^4 = \left( \frac{T_I}{100} \right)^4 - \frac{q_r}{C_o \varepsilon_4}.
\]

Having determined \( q_r, T_I \) and \( T_{II} \), it is possible to calculate the values of the coefficients of radiant heat transfer for each of the heat transfer sites through the window: \( \alpha_{r,o} \) – for the site, external glass – environment; \( \alpha_{r,23} \) – for inter-glass space; \( \alpha_{r,4i} \) – for the plot, the inner surface of the room is the inner glass. As initial we apply the correlations:

\[
q_r = \alpha_{r,o}(t_I - t_o) = \alpha_{r,23}(t_{II} - t_I) = \alpha_{r,4i}(t_I - t_{II}).
\]
The temperature inside the room is taken constant, and we finally obtain:

\[ \alpha_{r,ol} = \frac{1}{R_{r,ol}} = \frac{\varepsilon_0 C_o \left[ \frac{T_1}{100} \right]^4 - \left( \frac{T_o}{100} \right)^4}{(t_1 - t_o)} \]

\[ \alpha_{r,23} = \frac{1}{R_{r,23}} = \frac{\varepsilon_{23} C_o \left[ \frac{T_{II}}{100} \right]^4 - \left( \frac{T_1}{100} \right)^4}{(t_{II} - t_1)} \]  

Here \( R_{r,ol}, R_{r,23} \) are thermal resistance to radiant heat transfer of the respective sections.

The value of the total thermal resistance to radiant heat transfer is also of interest:

\[ R_t = \frac{t_1 - t_o}{q_r} = R_{r,ol} + R_{r,23} + R_{r,4i} = \frac{1}{\alpha_{r,ol}} + \frac{1}{\alpha_{r,23}} + \frac{1}{\alpha_{r,4i}} \]  

The expression for calculating the density of radiant heat flux through a two-chamber double-glazed window is similarly derived:

\[ q_r = \frac{C_o \left[ \frac{T_1}{100} \right]^4 - \left( \frac{T_o}{100} \right)^4}{\left( \varepsilon_1 + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_4} + \frac{1}{\varepsilon_5} + \frac{1}{\varepsilon_6} - 2 \right)} \]  

Correlations (11) – (15) for a single-chamber double-glazed window and a similar correlation (16) for a two-chamber double-glazed window allow one to determine all the required characteristics of radiant heat transfer through such double-glazed windows and to reveal the influence of independent parameters on them. For this process, such parameters are temperatures \( t_1, t_o \) and the number of emission factors corresponding to each variant. For a single-chamber double-glazed window there are four of them \( \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \).

It should be noted that numerical calculations by analytical formulas (11) – (16) are currently extremely easy to perform, for example, using spreadsheets. In order to get an idea of the effect of outdoor temperature on the numerical values of the characteristics of radiant heat transfer, in table 1 and in figure 3 – figure 4 shows some results. They are obtained for a single-chamber double-glazed window with colorless glasses. The emission coefficients of all four surfaces are the same \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.837 \). The temperature inside the room is taken constant \( t_1 = 20^\circ \text{C} \). Ambient temperature \( t_o \) varies from 20 to \(-40^\circ \text{C}\).

Table 1. Variation in the characteristics of heat transfer by radiation through a single-chamber double-glazed window with uncoated glasses (\( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.837 \)) with a decrease in the outdoor temperature \( t_o \).

| \( t_i \) | \( t_o \) | \( q_r \) | \( t_i \) | \( t_{II} \) | \( R_{r,ol} \) | \( R_{r,23} \) | \( R_{r,4i} \) | \( R_t \) | \( \alpha_{r,ol} \) | \( \alpha_{r,23} \) | \( \alpha_{r,4i} \) |
|---------|---------|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| [\( ^\circ \text{C} \)] | [\( ^\circ \text{C} \)] | [W/\( \text{m}^2 \)] | [\( ^\circ \text{C} \)] | [\( ^\circ \text{C} \)] | [W/\( \text{m}^2 \text{K} \)] | [W/\( \text{m}^2 \text{K} \)] | [W/\( \text{m}^2 \text{K} \)] | [W/\( \text{m}^2 \text{K} \)] | [\( \text{W/} \text{m}^2 \text{K} \)] | [\( \text{W/} \text{m}^2 \text{K} \)] | [\( \text{W/} \text{m}^2 \text{K} \)] |
| 20 | 19.999 | 0.002 | 20.00 | 20.00 | 0.209 | 0.243 | 0.209 | 0.661 | 4.78 | 4.11 | 4.78 |
| 20 | 10 | 14.36 | 13.28 | 16.95 | 0.228 | 0.256 | 0.212 | 0.696 | 4.38 | 3.91 | 4.71 |
| 20 | 0 | 27.28 | 6.79 | 14.12 | 0.249 | 0.269 | 0.216 | 0.733 | 4.02 | 3.72 | 4.64 |
| 20 | –10 | 38.86 | 0.58 | 11.51 | 0.272 | 0.281 | 0.218 | 0.772 | 3.67 | 3.55 | 4.58 |
| 20 | –20 | 49.19 | –5.35 | 9.12 | 0.298 | 0.294 | 0.221 | 0.813 | 3.36 | 3.40 | 4.52 |
| 20 | –30 | 58.36 | –10.97 | 6.95 | 0.326 | 0.307 | 0.224 | 0.857 | 3.07 | 3.26 | 4.47 |
| 20 | –40 | 66.47 | –16.25 | 4.99 | 0.357 | 0.320 | 0.226 | 0.903 | 2.80 | 3.13 | 4.43 |
Figure 3. Variation in the temperatures $t_1$ of the outer and $t_{II}$ inner glasses, as well as the density $q_r$ of the radiant heat flux in a single-chamber double-glazed window with decreasing temperature $t_o$ the outside air.

As the ambient temperature $t_o$ decreases, the radiant heat flux through a single-chamber double-glazed window rapidly increases and reaches $q_r = 58.36 \text{ W/m}^2$ at $t_o = \text{–30°C}$.

A decrease in the temperatures $t_1$ of the inner and $t_{II}$ of the outer glasses with decreasing ambient temperature occurs according to close-linear dependences (figure 3). The temperature differences $(t_1 - t_{II})$, $(t_{II} - t_o)$, $(t_1 - t_o)$ in individual sections correspond to the vertical distance between the lines $t_1$, $t_{II}$, $t_1$, $t_o$. The difference in $\Delta t_1$ values in individual sections is determined by the difference in the coefficients $\alpha_{r,i}$ of the radiant heat transfer to them – these values are interconnected by the inverse correlation $\alpha_{r,i} \cdot \Delta t_1 = q_r = \text{const}$.

The change in the coefficients of radiant heat transfer in individual areas is shown in figure 4. A common feature of these three coefficients is that they all decrease with decreasing outdoor temperature. Of particular interest are the quantities $\alpha_{r,4i}$ and $\alpha_{r,o1}$. They are calculated by correlations (14), in which for the given case $\varepsilon_1 = \varepsilon_4 = 0.837$. It should be noted the numerical values of these quantities. For the coefficients of internal and external radiant heat transfer from colorless ($\varepsilon = 0.837$) glasses, the standard [1] values are recommended: $\alpha_{r,4i} = 4.4 \text{ W/(m}^2\text{K})$, $\alpha_{r,o1} = 3.0 \text{ W/(m}^2\text{K})$. From the data in table 1 it follows that the standard values of the coefficients $\alpha_{r,4i}$ and $\alpha_{r,o1}$ for these conditions are achieved by lowering the temperatures of the inner glass to 4 °C and the outer glass to –12 °C, respectively.

The curves $\alpha_{r,4i}$ и $\alpha_{r,o1}$ exit from a common point at $t_o = 20^\circ C$. The subsequent difference in these values is determined only by the difference in surface temperatures between which radiant heat transfer is carried out. From a comparison of the further course of these curves, we can conclude: the coefficient of radiant heat transfer decreases both with a decrease in the maximum temperature of two surfaces and with an increase in the temperature difference of these surfaces.

Coefficient $\alpha_{r,23}$ of radiant heat transfer between surfaces decreases with decreasing arithmetic mean temperature of surfaces, which occurs both with a decrease in their maximum temperature and an
increase in the difference in surface temperatures. For thermal resistance to radiant heat transfer between surfaces under such conditions, the opposite dependence is observed – its value increases. As a result, with an increase in the temperature difference on the double-glazed window due to lowering the ambient temperature $t_o$, the $R_r$ value of the total thermal resistance to radiant heat transfer also increases – the data in table 1. The reciprocal of $1/R_r$, equal to the coefficient of radiant heat transfer through the double-glazed window, decreases with decreasing outdoor temperature. This dependence is shown as Curve 4 in figure 4.

There are two ways to reduce heat loss by radiation through a double-glazed window.

The first way to reduce radiant heat loss through a double-glazed window is to increase the number of glasses. Glasses act as screens for thermal radiation and an increase in their number reduces the radiant heat flux. This effect is clearly seen when comparing the values of the radiant heat flux through one-layer $q_{1r}$, two-layer $q_{2r}$ and three-layer $q_{3r}$ glazings at the same temperatures $t_i$ and $t_o$. Using expressions (11) and (16), we obtain for these conditions the following relation between the indicated quantities:

$$q_{1r}/q_{2r}/q_{3r} = \left(\frac{1}{\varepsilon_1 + \frac{1}{\varepsilon_2}}\right)^{-1}/\left(\frac{1}{\varepsilon_1 + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_4} - 1}\right)^{-1}/\left(\frac{1}{\varepsilon_1 + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_4} + \frac{1}{\varepsilon_5} + \frac{1}{\varepsilon_6} - 2\right)^{-1}. \text{ (17)}$$

We accept the simplest option when all the glasses are the same and colorless - for all surfaces $\varepsilon = 0.837$. In this version, the quantitative ratio between the values is as follows: $q_{1r}/q_{2r}/q_{3r} = 0.418/0.265/0.193 = 1.58/1.0/0.728 = 2.166/1.373/1.0$. It follows that the addition of a second ordinary glass to a double-glazed window reduces the radiant heat flux by 1.58 times, the addition of a third of the same glass additionally reduces it by another 1.373 times.

The second way to reduce radiant heat loss through a double-glazed window is to use glasses with low-emission heat-shielding coatings. For ordinary window glass, the value of the emission coefficient for the low-temperature thermal radiation corresponding to room temperatures is currently accepted as standard $\varepsilon = 0.837$ [1]. The low-emission coating is applied to the surface of colorless glass and has an emission coefficient substantially less. The emission coefficient of the surface of the glass with a soft low-emission coating should be no more than 0.07, for example, $\varepsilon = 0.05$.

The use of low-emission coatings opens up practically unlimited possibilities for reducing the radiant heat flux through the glass packet. We will evaluate these possibilities on the example of a single-chamber double-glazed window. Expression (11) for calculating the density of the radiant heat flux through a single-chamber double-glazed window includes four emission factors. Table 2 shows the results of calculating the characteristics of radiant heat transfer through a single-chamber double-glazed window depending on the number and location of low-emission coatings. The calculations are performed according to the relations (11) – (15). Initial option 1 is a double-glazed window with uncoated glasses: all four emission factors are the same $\varepsilon = 0.837$, heat flux $q_r = 27.28 \text{ W/m}^2$. Options 2 – 5 for comparison: three uncoated surfaces with coefficient $\varepsilon = 0.837$, on the fourth is a low emission coating $\varepsilon^* = 0.05$, heat flux $q^{*r} = 4.57 \text{ W/m}^2$. Options 2 to 5 differ in the location of the low emission coating. The use of one low-emission coating $\varepsilon^* = 0.05$ increases the resistance to radiant heat transfer of each of the sections by about 16 times. In all these cases, the heat flux remains the same. In accordance with expression (11), the location of the low-emission coating (on the 1st, 2nd, 3rd or 4th surface) does not affect the value of the radiant heat flux. A change in the location of the low-emission coating causes a change in the temperature of the glasses in all variants except options 3 and 4. In these variants, a change in the location of the low-emission coating inside the gas-filled chamber from the 2nd surface to the 3rd does not entail any changes.
Table 2. The influence of the number and location of low-emission coatings on the characteristics of radiant heat transfer through a single-chamber double-glazed window at constant $t_i = 20^\circ\text{C}$, $t_o = 0^\circ\text{C}$.

| Option | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ | $q_r$ | $t_1$ | $t_{II}$ | $R_{r,\text{eI}}$ | $R_{r,\text{eII}}$ | $R_{r,\text{eIII}}$ |
|--------|----------------|----------------|----------------|----------------|-------|------|-------|----------------|----------------|----------------|
|        | [W/m$^2$]      | [°C]           | [°C]           | [m$^2$/K]      | [W]   | [W]  | [W]   | [m$^2$/K]      | [m$^2$/K]      | [m$^2$/K]      |
| Both glasses without low-emission coating | 1 | 0.837 | 0.837 | 0.837 | 0.837 | 27.28 | 6.79 | 14.12 | 0.249 | 0.269 | 0.216 | 0.733 |
| One low-emission coating $\varepsilon^* = 0.05$ | 2 | **0.05** | 0.837 | 0.837 | 0.837 | 4.57 | 17.91 | 19.04 | 3.924 | 0.247 | 0.210 | 4.381 |
| | 3 | 0.837 | **0.05** | 0.837 | 0.837 | 4.57 | 11.17 | 19.04 | 0.257 | 3.914 | 0.210 | 4.381 |
| | 4 | 0.837 | 0.837 | **0.05** | 0.837 | 4.57 | 1.17 | 19.04 | 0.257 | 3.914 | 0.210 | 4.381 |
| | 5 | 0.837 | 0.837 | 0.837 | **0.05** | 4.57 | 1.17 | 2.52 | 0.257 | 3.914 | 0.210 | 4.381 |
| Two low-emission coatings $\varepsilon^* = 0.05$ | 6 | **0.05** | **0.05** | 0.837 | 0.837 | 2.49 | 10.19 | 19.48 | 4.092 | 3.727 | 0.210 | 8.029 |
| | 7 | 0.837 | **0.05** | **0.05** | 0.837 | 2.49 | 0.64 | 19.48 | 0.258 | 7.562 | 0.210 | 8.029 |
| | 8 | 0.837 | 0.837 | **0.05** | **0.05** | 2.49 | 0.64 | 10.86 | 0.258 | 4.103 | 3.668 | 8.029 |
| | 9 | **0.05** | 0.837 | 0.837 | **0.05** | 2.49 | 10.19 | 10.86 | 4.092 | 0.268 | 3.668 | 8.029 |
| One low-emission coating with a different $\varepsilon^*$ value on the 3rd surface | 10 | 0.837 | 0.837 | **0.10** | 0.837 | 8.19 | 2.09 | 18.27 | 0.256 | 1.975 | 0.211 | 2.441 |
| | 11 | 0.837 | 0.837 | **0.15** | 0.837 | 11.15 | 2.84 | 17.64 | 0.254 | 1.328 | 0.212 | 1.795 |
| | 12 | 0.837 | 0.837 | **0.20** | 0.837 | 13.59 | 3.45 | 17.12 | 0.254 | 1.005 | 0.212 | 1.471 |

According to expression (11), the use of one such coating in a single-chamber double-glazed window reduces the radiant heat flux by a factor of 6:

$$
q_r / q_r^* = \left( \frac{4}{\varepsilon - 1} \right)^{-1} \left( \frac{3}{\varepsilon + 1} - 1 \right)^{-1} = 5.97. \quad (18)
$$

In options 6 – 9, a low-emission coating $\varepsilon^* = 0.05$ is applied on two surfaces. Options differ in the location of these coatings. The use of two coatings $\varepsilon^* = 0.05$ reduces the heat flux compared to the initial version by 11 (10.95) times:

$$
q_r / q_r^{**} = \left( \frac{4}{\varepsilon - 1} \right)^{-1} \left( \frac{2}{\varepsilon + 1} - 1 \right)^{-1} = 10.95. \quad (19)
$$

In options 10–12, one low emission coating with a different $\varepsilon^*$ value is used. Comparison of options 1, 4, and 10–12 makes it possible to evaluate the effect of the emission coefficient of a single coating located on the 3rd surface on the value of the radiant heat flux through a double-glazed window.

For a two-chamber double-glazed window, in accordance with the expression (16), the use of one low-emission coating $\varepsilon^* = 0.05$ leads to a decrease in the radiant heat flux by 4.64 times:

$$
q_r / q_r^* = \left( \frac{6}{\varepsilon - 2} \right)^{-1} \left( \frac{5}{\varepsilon + 1} \varepsilon^* - 2 \right)^{-1} = 4.64. \quad (20)
$$

In a two-chamber double-glazed window, two coatings $\varepsilon^* = 0.05$ reduce the radiant heat flux by 8.27 times, three such coatings reduce the radiant heat flux by 11.91 times.
It should be noted that in versions with any number of low-emission coatings in a two-chamber double-glazed window, a change in the location of the coating inside the gas-filled cavities from the 2nd surface to the 3rd or from the 4th surface to the 5th does not cause any changes in the process characteristics.

3. Conclusion
The presented method for solving the problem of heat transfer by radiation through a double-glazed window allows us to obtain analytical expressions for calculating the density of radiant heat flux, glass temperatures, and radiant heat transfer coefficients in each of the individual sections of heat transfer through a double-glazed window. An analysis is made of the influence of the number, quality and location of low-emission coatings on the characteristics of the process of radiant heat transfer through double-glazed windows. It is shown that a change in the location of the low-emission coating from one surface to another inside gas-filled chambers does not cause changes in the characteristics of the radiant heat transfer process.

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