I. INTRODUCTION

Friction is the ubiquitous process of mechanical energy turning into heat through various coupling mechanisms between the relative motion of objects and their internal degrees of freedom. Due to the abundance of mechanisms involved, including phononic, electronic and quantum processes, a comprehensive description of friction has proved elusive, with multiple open questions and areas of research. By consequence, for a relatively long period of history, the description of sliding friction was (and still is) left to simple phenomenological law. However, the advent of modern nanoscale measurement techniques such as atomic force microscopy (AFM) and friction force microscopy (FFM) have re-ignited the interest in friction and spawned the field of nanotribology, which delves into finding the fundamental constituents of frictional processes.

Probably the most common concept associated with friction is the dry sliding friction between two solid surfaces, in which the plastic and elastic deformations of surface asperities dissipate the kinetic energy. When the surfaces are separated by more than a few nanometers and can’t be considered to “touch” each other, asperity deformation ceases to occur and the interaction of the surfaces is mediated by long range electromagnetic interactions. These interactions result in what is known as non-contact friction, a type of friction that is typically orders of magnitude weaker than contact friction, with frictional forces measured in attonewtons and friction coefficients in the $10^{-14} - 10^{-13}$ kg/s range. Understanding non-contact friction is not only important on the fundamental level due to the interactions being the building blocks of a comprehensive picture of friction, but also in practical sense, since the strength of non-contact friction limits the sensitivity of force sensor.

The electric components of electromagnetic non-contact friction, including electrostatic friction and Van der Waals friction, are quite well-established both experimentally and theoretically. The magnetic component, while also demonstrated experimentally, has received relatively little scientific attention. In this paper, we study magnetic non-contact friction via micromagnetic simulations, with a focus on the effect of the material parameters on the friction coefficient through the change in magnetic structure.

The paper is organized as follows: In Sec. II, we briefly discuss the previous research on magnetic friction and consider the energy dissipation from a micromagnetic viewpoint. Sec. III introduces the specifics of our micromagnetic simulation setup, and Sec. IV details the results obtained from the simulations. The conclusions from this study are presented in Sec. V.

II. MAGNETIC FRICTION

Magnetic friction arises from the dissipation of energy in spin reorientation in response to a changing magnetic field inside the magnet. Though there is no general consensus on the complete nanoscale explanation for this dissipation as of yet, there are multiple pathways for the energy to be converted into heat, such as magnon-phonon and magnon-electron interactions, magnon-impurity interactions, and magnon scattering on the surface and interface defect. Of note is that magnetic friction differs from what is usually called magnetic damping (or magnetic drag), in which eddy currents in a conductor moving relative to a magnet convert magnetic energy into heat. In this case, it is the currents and the resistivity of the conductor that are mostly responsible for the energy dissipation rather than the magnetic degrees of freedom of the magnet.

The link between changing magnetization and energy dissipation on the atomic level has been demonstrated experimentally. Utilizing a spin-polarized scanning tunneling microscope, it was observed that the force required to move a magnetic adatom from an adsorption site to another increased by up to 60% compared to non-magnetic adatoms. Magnetic dissipation was also shown to occur in an experiment using a soft cantilever with a magnetic tip oscillating in a magnetic field.
With strong (approaching 6 T) external magnetic fields, the friction coefficient measured for a cobalt tip were in the $10^{-12} - 10^{-11}$ kg/s range. The friction coefficient was found to be material dependent, with magnetically more malleable cobalt showing high dissipation compared to the stronger anisotropy PrFeB, for which the friction coefficient didn’t differ significantly from a bare silicon cantilever internal friction coefficient.

Magnetic friction and its dependence on parameters such as temperature have also been investigated computationally with Monte Carlo simulations in the Ising model\(^\text{[13]}\) and models using the Landau-Lifshitz-Gilbert equation\(^\text{[14,15]}\) for the dynamics of the spins. Both velocity independent (Coulomb) friction and velocity dependent (Stokes) friction have been demonstrated, though the difference in frictional behavior could be explained by the simulation model.\(^\text{[15]}\) Additionally, there has been a study of a larger, two-film configuration with multiple stripe domains, in which it was shown that the domain structure can evolve into a configuration that minimizes the friction.\(^\text{[12]}\) As pointed out in the study, an interesting aspect of magnetic friction is that the strength of force could be adjusted by external applied fields.

With some of the general phenomenology being established by the aforementioned research, we focus on the interplay between the magnetic domain structure of a sample and the magnetic friction experienced by the sample when in motion in a magnetic field. We employ micromagnetic simulations to observe how changes in material parameters affect the magnetic structure and the measured magnetic friction, mainly considering small thin film systems with uniaxial anisotropy and simple magnetic structure e.g. a single domain wall. The simulations are performed using the micromagnetic code Mumax\(^3\) with our previously developed extension, in which it is possible to simulate smooth spring-driven harmonic motion of the magnet(s) simultaneously with the magnetization evolution.

### A. Micromagnetics and energy dissipation

In the framework of micromagnetism, the behavior of the magnetic moments in a magnetic material is described by the Landau-Lifshitz-Gilbert (LLG) equation:

$$\frac{\partial \mathbf{m}}{\partial t} = -\frac{\gamma}{1+\alpha^2} (\mathbf{H}_{\text{eff}} \times \mathbf{m} + \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})),$$  \(\text{(1)}\)

where \(\mathbf{m}\) is the magnetization normalized by the saturation magnetization \(M_{\text{sat}}\) of the material (\(\mathbf{m} = \mathbf{M}/M_{\text{sat}}\)), \(\gamma \approx 221\ \text{kHz/(Am}^{-1}\)) is the electron gyromagnetic ratio, \(\alpha\) is the Gilbert damping constant, and \(\mathbf{H}_{\text{eff}}\) is the effective magnetic field. The first term of the LLG equation describes the precession of a magnetic moment around the effective field, while the second term is the phenomenological relaxation term describing how the rotation of the magnetic moment eventually winds down to the direction of the effective field.

In magnetic friction, the subject of interest is the energy dissipation caused by the spin relaxation, which can be derived with the help of the LLG equation.\(^\text{[20]}\) In a volume \(V\) with constant magnetization \(\mathbf{m}\) and effective field \(\mathbf{H}_{\text{eff}}\), the magnetic energy \(E\) can be written as:

$$E = -\mu_0 M_{\text{sat}} V (\mathbf{m} \cdot \mathbf{H}_{\text{eff}}).$$

Considering the change of energy in time \(dE/dt\), we can write the total differential as

$$\frac{dE}{dt} = \partial E/\partial \mathbf{m} \cdot \frac{\partial \mathbf{m}}{\partial t} + \partial E/\partial \mathbf{H}_{\text{eff}} \cdot \frac{\partial \mathbf{H}_{\text{eff}}}{\partial t}.$$

Inserting the LLG equation from Eq. \((1)\) in place of the derivative \(\partial \mathbf{m}/\partial t\), we obtain

$$\frac{dE}{dt} = \frac{\mu_0 \gamma M_{\text{sat}}}{1+\alpha^2} \mathbf{H}_{\text{eff}} \cdot (\mathbf{H}_{\text{eff}} \times \mathbf{m} + \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})),$$

which, noting that the first term is zero, shows that the dissipation indeed comes from the second term of the LLG equation, i.e. the winding down of the spins. Using \(P = -dE/dt\) and some vector calculus identities, the result can be simplified to

$$P = \frac{\alpha \mu_0 \gamma M_{\text{sat}}}{1+\alpha^2} (\mathbf{m} \times \mathbf{H}_{\text{eff}})^2.$$

This is the dissipated energy in a unit of time due to spin relaxation, or “Gilbert dissipation” power \(P\) inside the volume. In finite difference micromagnetics, the magnet is typically discretized into cells of equal volume, with magnetization and effective field being constant in each cell. In such a system, the total dissipation power is then a sum over the individual cells:

$$P_{\text{tot}} = \sum_{i=1}^{N} P_i = \frac{\alpha \mu_0 \gamma M_{\text{sat}} V_{\text{cell}}}{1+\alpha^2} \sum_{i=1}^{N} (\mathbf{m}_i \times \mathbf{H}_{\text{eff},i})^2,$$  \(\text{(2)}\)

where \(V_{\text{cell}}\) is the volume of the discretization cell.

From Eq. \((2)\) one can see that the material dependent parameters \(\alpha\) and \(M_{\text{sat}}\) are explicitly present, influencing the observed energy dissipation and, by extent, the magnetic friction. Additionally, the exchange constant \(A_{\text{ex}}\) and the uniaxial anisotropy constant \(K_u\) are important parameters in our simulations, since they affect the width of the domain walls, which is of the order \(\sqrt{A_{\text{ex}}/K_u}\).\(^\text{[22]}\) However, these factors are hidden in the effective field term in Eq. \((2)\), and determining the exact effect the various field terms have on magnetic friction is nontrivial.

In large systems of thousands or millions of interacting magnetic moments, the complex time evolution of the effective field and magnetization make it unfeasible
to study the dissipation and magnetic friction analytically. Thus we approach the problem by simulating ring-down measurements, in which friction coefficient is determined by observing the gradual diminishing of the amplitude of mechanical oscillations of e.g. a cantilever under the presence of damping effects. Experimental ring-down measurements have been performed to measure various forms of non-contact friction, including electric friction between gold-coated substrate and tip\(^2\)\(^3\)\(^4\), and dielectric friction between polymers\(^2\)\(^4\).

III. MICROMAGNETIC SIMULATION SETUP

In our simulated ring-down measurements, we employ two simulation setups: A single thin film in a spatially varying external magnetic field (Fig. 1), and a configuration mimicking a magnetically coated tip of an oscillating cantilever and a strip of magnetic material as a substrate (Fig. 2). The film (or tip, depending on the setup) is attached to a spring, and the system is modeled as a damped harmonic oscillator

\[
m \frac{d^2x}{dt^2} + \Gamma \frac{dx}{dt} + \omega_0^2 x = 0,
\]

where \(m\) is the mass of the film/tip, \(\Gamma\) is the friction coefficient and \(\omega_0\) is the natural oscillation frequency \(\omega_0 = \sqrt{k/m}\), in which \(k\) is the spring constant. A real oscillator has an internal friction coefficient \(\Gamma_0\), which in the case of a cantilever can be measured by having the cantilever oscillate in a vacuum and as isolated from external influences as possible. Typical internal friction coefficients are of the order \(\Gamma_0 = 10^{-14} - 10^{-13}\) kg/s in soft cantilevers\(^2\)\(^2\). The total friction coefficient is then the sum of the internal friction coefficient and the damping effects of external influences \(\Gamma = \Gamma_0 + \Gamma_e\). An advantage of simulations is that we can set the internal friction coefficient of the spring to zero, and thus all the energy losses come from the energy dissipation through the magnetic degrees of freedom.

The spring starts elongated to length \(A_0\) on the \(x\)-axis at the beginning of the simulation. We let the film/tip oscillate with the force exerted on it defined by

\[
F = -\nabla E = \mu_0 M_{\text{sat}} V_{\text{cell}} \sum_{i=1}^{N} \nabla (m_i \cdot H_{e,i}),
\]

where \(H_{e,i}\) is the external field in first setup and the demagnetizing field of the substrate in the second setup, and the sum goes over the cells of the film or tip in their respective setups.

Eventually, the oscillations die down due to the Gilbert dissipation defined by Eq. 2. Fitting an exponentially decaying function to the amplitude \(A\)

\[
A = A_0 e^{-t/\tau},
\]

we can find the decay time \(\tau\) (Fig. 3). The decay time and the friction coefficient \(\Gamma\) are related by

\[
\Gamma = \frac{2k}{\omega^2 \tau} = \frac{2m}{\tau}.
\]

In Ref.\(^2\)\(^2\), there are some value ranges for common micromagnetic materials. For both the film and the tip-strip configurations, we picked the base material parameters from approximately the middle of these ranges, with \(A_{ex} = 5 \cdot 10^{-12}\) J/m, \(M_{\text{sat}} = 350\) kA/m, \(K_u = 1.2 \cdot 10^5\) J/m and \(\alpha = 0.05\), resulting in an out-of-plane configuration by default as shown in Figs. 1 and 2. We test the effect of different micromagnetic parameters by varying each parameter at a time while keeping the rest constant. The parameters were tested in the follow-
Figure 3. An example of the oscillation of the position $x$ of the film/tip in a ring-down measurement with $\omega = 708$ MHz, and an exponential function fit to the amplitude to determine the decay time $\tau$. The initial amplitude of the oscillation $A_0$ is equal to the initial position of the film $x_0$.

In micromagnetic simulations, having a discretization cell size well below magnetic exchange lengths $\sqrt{A_{\text{ex}}/K_u}$ and $\sqrt{2A_{\text{ex}}/\mu_0M_{\text{sat}}}$ is required to not introduce numerical problems, such as artificial pinning of the domain walls, into the system. In our simulations, a discretization cell size of $4 \times 4 \times 4$ nm sufficed for most parameters. However, for high $K_u$ or low $A_{\text{ex}}$, it’s possible that the exchange length(s) shrink close to the cell size, inducing the aforementioned domain wall pinning, affecting the friction coefficient. To avoid this, for the high values of $K_u$ and low values of $A_{\text{ex}}$, we halved the cell size to $2 \times 2 \times 2$ nm in the single film system. We found that the tip-strip system was more sensitive to the discretization cell size, most likely due to the weaker fields involved in the domain wall movement. As such we used the smaller cell size for all simulations in that system. All simulations were run in zero temperature.

For the single thin film in an external field, we use a $256 \times 256 \times 20$ nm film and an external z-directional field ramping linearly down from $+100$ mT to $-100$ mT. The z-axis was also chosen as the easy axis of the uniaxial anisotropy. With the base parameters, the resulting magnetic texture is a two-domain film with a single domain wall. We set the initial amplitude $A_0$ to 64 nm, so that we have large oscillations but the domain wall still stays relatively far from the edges of the film.

The strip and pendulum tip geometry is more reminiscent to an actual experimental ring-down measurement setup. The cantilever tip is modeled as a square film of $60 \times 60 \times 20$ nm size, while the substrate is a $320 \times 80 \times 20$ nm strip. The tip and substrate are set 20 nm apart, so that we’re in the non-contact friction regime and only the demagnetizing fields are relevant for the tip-substrate interaction. Multiple equilibrium magnetization configurations are possible, but we chose one with two domain walls in the strip due to it being nicely symmetric. Because of the small size of the tip and the strength of the anisotropy with the base parameters, the magnetization of the tip is forced uniformly into the anisotropy easy axis direction. The initial oscillation amplitude is set to $A_0 = 20$ nm, closer to experimental cantilever amplitude values compared to the single film case.

To keep simulation times reasonably short, we needed to have enough oscillations to dissipate energy in relatively little amount of time ($t < 1 \mu s$). Hence our springs in both setups have parameters outside the range of typical measurement equipment, namely a very high oscillation frequency $\omega = 354$ MHz compared to the usual $100$ kHz range of cantilevers used in experiments. However, due to the rapid magnetic relaxation, we assume that the time scales of the mechanical motion and the magnetic relaxation are well separated with most parameter configurations. Thus the change in magnetic structure is in most cases independent of velocity, meaning that the observed behavior should also match lower frequency cantilevers. We test the assumption by running additional simulations with a doubled oscillation frequency $\omega = 708$ MHz and comparing the results.

For the single film setup, it is possible to use Eq. (2) to make a prediction for the order of magnitude of the friction coefficient. Since in each discretization cell we have $|\mathbf{m}|^2 = 1$, and all the spin reorientation is happening inside the domain wall, we can write the sum of the cross products between magnetic moments and the local effective field as

$$\sum_{i=1}^{N} (\mathbf{m}_i \times \mathbf{H}_{\text{eff},i})^2 = \frac{l_x l_y l_z}{V_{\text{cell}}} (H_{\text{eff}}^2 \sin^2 \theta)_{\text{dw}}$$

where $(\ldots)_{\text{dw}}$ denotes the average of the value inside the domain wall, $\theta$ is the angle between the effective field and the magnetic moment, and $l_x$, $l_y$ and $l_z$ are the extent of the domain wall in $x$, $y$, and $z$-directions, respectively.

When the film oscillates, the magnetization follows the external field, and the center of the domain wall prefers to stay at $B_{\text{ext}}(x) = 0$. As a consequence of the speed of the magnetic relaxation, the angle between the field and magnetic moments $\theta$ remains small even for relatively high film velocities, and $\sin \theta \approx \theta$. The angle depends on the film velocity $v$ relative to the relaxation speed of the magnetization and the width of the domain wall. As
such, we approximate

\[ \theta \approx c \frac{v}{\gamma H_{\text{eff}} l_x}, \]

where \( c \) is a dimensionless coefficient. Inserting the approximation into Eq. (2) and using the relation between dissipated power and velocity in a damped harmonic oscillator, \( \Gamma = P/v^2 \), the time-dependent terms cancel out and we have

\[ \Gamma \approx \frac{\alpha}{1 + \alpha^2} \frac{c^2 \mu_0 M_{\text{sat}} l_y l_z}{\gamma l_x}. \]

Furthermore, using \( l_x = \pi \sqrt{A_{\text{ex}}/K_u} \) for the domain wall width, we obtain a ballpark estimate for the friction coefficient that depends on the four micromagnetic material parameters and the size of cross section of the domain wall:

\[ \Gamma \approx \frac{\alpha}{1 + \alpha^2} \sqrt{\frac{K_u c^2 \mu_0 M_{\text{sat}} l_y l_z}{A_{\text{ex}} \pi \gamma}}. \] (3)

Using the base parameters and film size in the first setup and setting \( c = 1 \), the estimated friction coefficient is approximately \( 2.5 \cdot 10^{-14} \) kg/s.

### IV. RESULTS

In both simulated configurations, we observed three material parameter ranges in which the systems’ magnetic response to the oscillation changes significantly, with accompanying changes in the friction coefficient. In the first parameter range (indicated as region I in the figures of this section), the domain wall(s) in the film or substrate oscillate without internal excitations. In this parameter regime, the change in friction coefficient when changing material parameters is relatively modest and smooth. The friction coefficients observed in these simulations were roughly of the same order of magnitude as the estimate of Eq. (3).

The second parameter regime (region II) was characterized by excitations in the domain wall(s) of the system. For example, in the single film setup, Bloch lines appeared in the domain wall during motion in simulations with low \( M_{\text{sat}} \), enhancing the dissipation and resulting in larger friction coefficients. This behavior likely results from the external field affecting the domain wall exceeding Walker field \( H_{W} = 2\pi \alpha M_{\text{sat}} [27] \). A snapshot of the domain wall during oscillation in the first and second parameter regimes is shown in Fig. 4. In the tip-strip configuration, the domain walls exhibit precessional motion in these parameter regions, though without Bloch line excitations, likely due to the restricted strip width in the \( y \)-direction. The oscillation frequency can affect the boundaries of the parameter ranges, as faster film/tip velocities result in sharper changes in the effective field, which can more easily lead to excitations in the internal structure of the domain wall.

Due to the domain wall excitations, in the second parameter regime the dissipation ceases to be strictly exponential or splits into two different phases with different decay times for the oscillation amplitude. Additionally, in many cases the winding down proceeds as a fast initial dissipation until the system reaches small amplitude where the little movements aren’t enough to change the magnetic structure, and from there on the system oscillates almost indefinitely with the same amplitude due to negligible dissipation. As such, we fit the exponential \( A_0 e^{-t/\tau} \) and calculated the friction coefficients from the part with the strong dissipation.

Third parameter regime (region III) consists of values with which the system turns in-plane or otherwise changes the magnetic configuration to a more complex shape. In these cases, the change in magnetization in response to motion is typically weak, and thus the friction coefficient is very low or even zero.

#### A. Thin film in an external field

The single thin film simulations are in many ways analogous to works studying a domain wall driven by an external field in a magnetic strip. When the film oscillates, it drags the domain wall with it. However, the strength of the external field driving the domain wall back grows linearly with the distance from the center. If the mobility of the domain wall is high enough (or the oscillation slow enough), the domain wall remains located at the center without abrupt changes in the inner structure, leading to smooth magnetic moment relaxation. This was the case for most of the tested parameters. The friction coefficient as a function of each of the four material parameters is depicted in Fig. 5.

The strength of magnetic friction increases with a larger Gilbert damping constant, and in the parameter range \( \alpha = 0.001 - 0.50 \) (Fig. 5\( \text{a} \)), the increase in friction coefficient is linear for the lower oscillation frequency and almost linear for the higher frequency, though for larger values, the effect of increasing \( \alpha \) weakens a little. Eq. (3) predicts that the friction coefficient depends on the Gilbert damping constant as \( \alpha/(1 + \alpha^2) \), which is a similar close-to-linear increase in this parameter range.
Figure 5. The friction coefficient $\Gamma$ as a function of each of the micromagnetic parameters: a) Gilbert damping constant $\alpha$, b) saturation magnetization $M_{\text{sat}}$, c) uniaxial anisotropy constant $K_u$, and d) exchange constant $A_{\text{ex}}$. In the no-excitation regime (region I), the film and the domain wall oscillate smoothly, and the parameter dependence is well-behaved. In the parameter regime in which the inner structure of the domain wall is excited (region II), the energy dissipation and the friction coefficient are substantially enhanced, and the oscillation frequency $\omega$ has a stronger influence on the friction coefficient. The parameter range where the system becomes qualitatively different (such as becoming completely in-plane polarized) typically have small or zero friction coefficients (region III). The shaded area shows the parameter ranges for which the smaller cell size was used.

There were no internal excitations inside the domain wall with any of the tested values for $\alpha$, despite the fact that the Walker field for low $\alpha$ is small. This happens because in a continuously changing field, low $\alpha$ leads to the magnetic moments following the field more closely, and the domain wall remains in the center where $B_{\text{ext}} \approx 0$. The higher values seemed to limit the relaxation speed of the magnetic moments, causing the domain wall to lag behind the film in oscillation. This likely limited the dissipation and thus lowered the friction coefficient with 708 MHz frequency compared to 354 MHz with the highest values for $\alpha$.

The saturation magnetization $M_{\text{sat}}$ affects the friction coefficient in the most complicated way, since the demagnetizing field, anisotropy field and exchange field are all influenced by it. The Walker breakdown is also lowered on small $M_{\text{sat}}$, and as we see in Fig. 5b the friction coefficient has a sharp increase for low $M_{\text{sat}}$ values due to the domain wall excitation. The effect of the oscillation frequency can be seen as the shortened no-excitation regime at the lower values of $M_{\text{sat}}$ for the smaller frequency, due to domain wall having more time to relax back to the center and the local field not exceeding the Walker field. Between 170 kA/m and 450 kA/m, there are two competing effects: the higher $M_{\text{sat}}$ increases the dissipation power as can be seen from Eq. (2), but at the same time weakens the exchange and anisotropy fields $H_{\text{exch}}$ and $H_{\text{anis}}$, since they are inversely proportional to $M_{\text{sat}}$. Additionally, increasing $M_{\text{sat}}$ strengthens the demagnetizing field and drives the film towards an in-plane configura-
tion, causing the domain wall to widen, decreasing the friction coefficient. For $M_{\text{sat}} = 470 - 500$ kA/m, abrupt bending and twisting of the domain wall emerges during oscillation. With low frequency, the effect of these fluctuations of the domain wall shape are mostly negligible, but with the high frequency the friction coefficient increases drastically. For even larger $M_{\text{sat}}$ values, the in-plane tilt causes a separate bubble domain to form instead of a domain wall. The magnetization of the bubble changes relatively little during the oscillation, bringing the friction coefficient down again.

Due to their effects on the domain wall width $l_x$, changes in $A_{\text{ex}}$ (Fig. 5c) and $K_u$ (Fig. 5d) have opposite effects on the observed magnetic friction, though the domain structure at extreme values of the two parameters is different. Above $l_x \approx 3 \cdot 10^{-8}$ m the system quickly collapses to either a vortex ($K_u < 50$ kJ/m$^3$) or a single-domain state ($A_{\text{ex}} > 50$ J/m, not shown in the figure), greatly reducing the friction coefficient. With $l_x < 3 \cdot 10^{-8}$ m, the magnetic structure of two domains with a domain wall in the center is reobtained. In this regime, the domain wall oscillates smoothly, and the friction coefficient is approximately inversely proportional to the wall width $l_x$, as predicted by Eq. (3). The noticeable spike in friction coefficient at $K_u = 60$ kJ/m$^3$ with the higher frequency is the result of large domain wall fluctuations due to the system being almost in-plane polarized with this value. The fluctuations are suppressed on the lower oscillation frequency.

With a high $K_u$ or low $A_{\text{ex}}$, the domain wall can be artificially pinned due to the simulation cells being too large compared to the domain wall width. The pinning and depinning during motion causes excitations in the domain wall, resulting in a rapid and strong (up to an order of magnitude) increase in friction coefficient. To avoid the artificial excitations, we switched to the smaller cell size in these cases. The shaded area in Figs. 5c and 5d indicates the values for which the smaller cell size was used.

B. Tip-strip configuration

The behavior of the tip-strip system with two domain walls differs considerably from single film system. Within this configuration, the oscillation frequency has a stronger effect on the observed friction coefficient, since in many cases the larger frequency caused the domain walls to exhibit precessional motion instead of smooth motion. The material parameters also had a stronger influence to the domain wall behavior compared to the single film system, with some parameters introducing fluctuations to the system which greatly increased the friction coefficient. An example of domain wall fluctuation with a particular parameter value $M_{\text{sat}} = 490$ kA/m is shown in Fig. 6, in which the domain walls in the substrate undergo sudden large changes in response to the tip oscillation. In the regime with domain wall fluctuations the dissipation wasn’t always strictly exponential, and thus the calculated friction coefficients aren’t as accurate. The friction coefficient as a function of each material parameter is depicted in Fig. 7.

Like the single thin film case, there are no internal excitations in the walls for any $\alpha$ value, though the higher oscillation frequency displays some precessional motion of the domain walls in the substrate. The lower oscillation frequency shows a similar linear part in friction coefficient with low $\alpha$, but contrary to the single film, the increase starts evening out for high values (Fig. 7a). This happens because the domain walls can’t keep up with the tip oscillation due to the weaker field in this configuration (compared to the external field in the single thin film case) and higher $\alpha$ diminishing domain wall mobility. The domain walls lagging behind the tip leads to overall smaller domain wall movements and weaker dissipation. Using the higher oscillation frequency, the effect is further exacerbated, with a peak in $\Gamma$ at relatively low values, actually decreasing for higher values. In an experimental setup with much lower oscillation frequencies, it’s likely that the observed friction coefficient is larger, as the wall has time to follow the tip more closely.

As for the saturation magnetization, we see from Fig. 7b that the behavior is mostly similar to the single film case, though the precessional domain wall motion with the higher oscillation frequency creates a larger gap between the observed friction coefficients. For low values ($M_{\text{sat}} < 150$ kA/m), the weak demagnetizing field is barely enough to move the domain walls, and as a result $\Gamma$ is low. Larger $M_{\text{sat}}$ makes the walls move in accordance with the tip, which increases the friction coefficient. The middle value region is quite smooth and level, especially for the lower oscillation frequency. As with the single film system, at $M_{\text{sat}} > 450$ kA/m the domain walls start twisting and bending in the substrate, increasing dissipa-
Figure 7. The friction coefficient $\Gamma$ as a function of each of the micromagnetic parameters, laid out similarly as in Fig. 5. Overall, the friction coefficient dependency on the oscillation frequency is more pronounced in the tip-strip configuration (especially notable in c and d) due to the precessional motion and fluctuations of the domain walls with the higher frequency. In this configuration, the excitations due to low $M_{\text{sat}}$ are mostly absent (as shown in b), since the domain wall movement was subdued because of the weakness of the demagnetizing field.

The spike at $M_{\text{sat}} = 490$ kA/m is the result of rapid reconfigurations in response to the tip motion, as was shown in Fig. 6. Higher values turn the system further in-plane, resulting in a decrease in energy dissipation.

The effects of anisotropy constant and exchange constant for the lower oscillation frequency are similar to the single film case in most respects. However, with the higher oscillation frequency at the intermediate value ranges for $K_u$ and $A_{\text{ex}}$, i.e. region II in Figs. 7c and d, the precessional motion of the domain walls in the substrate tends to grow in intensity. Especially at values around $A_{\text{ex}} = 10$ pJ/m and $A_{\text{ex}} = 15$ pJ/m, the domain walls fluctuate heavily in response to the tip motion, greatly increasing the friction coefficient as can be seen in Fig. 7d. To an extent, this happens also with the lower oscillation frequency, though the dissipation peak is much smaller and only occurs at $A_{\text{ex}} = 14$ pJ/m. Outside these value ranges, the domain wall motion is smooth and the friction coefficient regains the approximate inverse dependence to the domain wall width.

Like in the single film scenario, high values for $A_{\text{ex}}$ and low values for $K_u$ drive the system toward an in-plane configuration. For $A_{\text{ex}} > 50$ pJ/m, the two domain walls in the substrate annihilate at the beginning of the oscillation, resulting in a single-domain thin film and strip and therefore vanishing friction. A similar thing happens with the lowest $K_u$ values. However, contrary to the single film case, the in-plane configuration has a small dissipation peak also at low $K_u$. In this case, the magnetization in the substrate is in-plane, with a single Néel wall. During the oscillation the wall passes under the tip, flipping its magnetization. This causes magnetic oscillations in both the tip and the substrate, dissipating energy and increasing the friction coefficient. In rest of
the in-plane configurations, magnetic friction is negligible. An increase in $K_u$ or decrease in $A_{ex}$ will eventually lead to an unphysical situation where the domain walls cannot be resolved properly due to discretization cell size, and the magnetic friction vanishes.

V. CONCLUSION

The results of the previous section indicate that the material parameters have a relatively strong influence on magnetic friction through their effect on the structure and evolution of the magnetic domains, especially at the extremities of the parameter ranges where the domain wall(s) are either very thin or very wide. When the change in magnetization is slow and the inner structure of the domain wall remains largely constant, the friction coefficient dependence on micromagnetic parameters other than the damping constant is relatively modest and well-behaved (though not necessarily monotonous, as in the case of $M_{sat}$). By contrast, in the parameter ranges where excitations emerge in the inner structure of the domain wall with, e.g., Bloch lines being created and annihilated, the dissipation is much greater, with some parameter combinations of resulting in spiky local maxima in the friction coefficient. Though these parameter ranges and related friction coefficients are likely dependent on the simulation setup and geometry, we noticed that this behavior is usually present in systems that are between in-plane and out-of-plane.

In the latter simulation setup, a high oscillation frequency was also shown to have a large impact on the friction coefficient due to the precessional motion of the domain walls. The results with the lower frequency are thus likely more comparable with experiments, though in actual experimental conditions the cantilever moves even more slowly. Thus when it comes to material parameter dependence, it is likely that the smooth non-excitation parameter ranges would be broader in real experiments, as the excitation inside domain walls are less likely to occur. On the other hand, real samples contain impurities that can affect domain wall motion, e.g., by pinning and depinning, increasing the amount of irregular changes in magnetic structure, strengthening the dissipation. The observed friction coefficients are indeed comparable to the coefficients of the cantilever experiment of Ref. [11] for the cobalt tip in an external field, when taking into account that the systems are significantly larger and the fields stronger than in our simulations. However, compared to the small tilt of the magnetization in the experiment, the change in magnetic structure in our simulations is greater, compensating a little for the weaker fields and smaller film/substrate size. The friction coefficients are also roughly in the same range as those found in experiments concerning other forms of non-contact friction, such as electric and phononic friction.

The simulations of this work, though limited in scope and complexity, show that interesting dynamics emerge when mechanical motion and magnetic domain evolution are coupled. The aim of this work was to establish a baseline understanding of magnetic friction and its dependence on material parameters from a micromagnetic simulation standpoint. Larger systems with disorder and more complex magnetic structure will remain as a prospect for future study.

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