Dust-lattice modes in magnetized complex plasmas

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Abstract. The influence of an inhomogeneous magnetic field, ion focusing effect and equilibrium charge gradient on the propagation of dust-lattice (DL) modes in a one-dimensional string formed by paramagnetic particles is considered. In typical discharge conditions, all three anisotropic factors can compete with each other even at moderate magnetic fields (~0.1–0.2 T), modifying the DL waves and leading to mode coupling. The characteristics of the mode coupling can be controlled externally by varying the magnetic-field regime, thus opening new possibilities for studies of collective effects in strongly coupled systems.

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1. Introduction

Since dust-lattice (DL) waves provide a great deal of information about the physical mechanisms of particle interactions in strongly coupled complex plasmas, there is a corresponding interest in theoretical and experimental studies of the oscillatory modes and instabilities in plasma-crystals [1]–[10]. In ground-based complex plasma experiments, the strongly coupled formations are usually observed in the sheath region of a plasma discharge where the balance between gravitational and electrostatic forces occurs. There are a few characteristic features of such systems: (1) the vertical ion flow in the (pre)sheath region leads to the formation of a region of enhanced ion density underneath the suspended microparticles, a so-called ‘ion wake’ [11]–[15]. This ‘wake effect’ produces an anisotropy in the electrostatic dust–dust interactions and can cause a DL-mode instability at very low gas pressures [16]. (2) The sheath region usually has a strong inhomogeneous structure and as a result the charge of the microparticles, caused by electron and ion currents onto the grain surface, strongly depends on the vertical particle position in the sheath [7, 17, 18]. (3) Any change in the particle charge is necessarily accompanied by a variation in the wake potential. This introduces another anisotropy in the particle interactions related to the vertical equilibrium charge gradient. The combined effect of the particle–wake interactions and vertical dust-charge variations leads to a specific conversion of transverse and longitudinal DL modes or can also result in DL-mode instability [19]. Similar results occur when one includes horizontal gradients in equilibrium particle charge due to variations in the interparticle distances [20].

Recently, levitation and formation of complex plasma structures was studied in an external magnetic field [21]. Experiments were carried out in a capacitively coupled rf discharge with plastic microspheres (Dynobeads M-450) of radius $a = 2.25 \mu m$ and material density $1.5 \text{ g cm}^{-3}$. The grains contained 20% Fe$_2$O$_3$ and Fe$_3$O$_4$, they were superparamagnetic with a magnetic permeability $\mu = 4$. A magnetic field, coaxial with the chamber (up to 0.15 T), was applied using magnetic coils located above the discharge chamber (for further technical details, we refer to [21]). In the light of these experiments, a novel type of vertical vibrations of a single-magnetized particle and a modification of the transverse DL mode due to the inhomogeneous magnetic field have been predicted [22]. In this paper, we include magnetic dust–dust interactions along with the particle–wake effect and vertical charge inhomogeneity to study the wave processes in strongly coupled complex plasmas.

The paper is organized as follows. The influence of the anisotropy-induced dust–dust interactions on the equilibrium state is discussed in section 2. Section 3 then deals with modifications of the dispersion relation describing the DL waves in the external non-homogeneous magnetic field. Whether the DL modes are stable or whether an instability occurs depend very much on the relationship between the parameters describing the anisotropy of the system and therefore, we have also included a brief discussion of the different cases which might be of importance in experiments given in section 3. Finally, our conclusions are then summarized in section 4.

2. Equilibrium state of the paramagnetic particles in discharge plasmas

2.1. Model description

For the DL waves in crystalline monolayers, we will use the standard model of a one-dimensional (1D) particle string [1, 2, 16, 19], which allows 2D motion, in the longitudinal (horizontal) and
In general, the wake effect is difficult to model, because we lack a complete plasma transport description in the sheath region of a plasma discharge. An attractive idea might be to model the particle–wake interactions introducing a point-like electric dipole. However, according to numerical simulations, the characteristic scales of the ion focusing are of the order of the screening length \([14, 15]\) and comparable with an interparticle distance. Therefore, it seems more realistic to describe the excess positive charge of the wake (along the vertical, \(z\) axis) as a point-like effective charge \(q\), located at some distance \(l\) beneath the particle \([16, 19]\), so that the electrostatic potential of the \(n\)th particle, \(\phi_n^{(Qq)}\), can be presented as a combination of the screened Debye potentials of the particle charge itself and its effective wake charge

\[
\phi_n^{(Qq)}(z) = Q_n \left[ \frac{\exp(-|r - r_n|/\lambda_D)}{|r - r_n|} - \tilde{q}_n \frac{\exp(-|r - r_{nq}|/\lambda_D)}{|r - r_{nq}|} \right],
\]

where \(\tilde{q}_n = q_n/Q_n\), \(|r - r_n|\) is the distance from the \(n\)th particle and \(|r - r_{nq}|\) is the corresponding distance from the effective wake charge. Furthermore, when a grain acquires a vertical displacement, \(z_n\), around its equilibrium position, both particle and wake charges vary. Such charge variations are of importance for perturbations developing with frequencies \(\omega \ll \tau^{-1}\), where \(\tau\) is the timescale for the particle charging. The latter can be roughly estimated as \(\tau \sim \lambda_D/(a\omega_{pi})\), where \(\lambda_D\) is the screening length, \(\omega_{pi}\) denotes the ion-plasma frequency and \(a\) the particle radius. Taking typical discharge parameters and micron-sized particles gives
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\( \tau \sim 10^{-5} - 10^{-4} \) s. Analogous timescales, \( \tau \sim 10^{-4} \) s, are obtained numerically for \( \mu m \) grains immersed in a plasma sheath \([23]\). Therefore, for the DL perturbations developing in the low-frequency regime (\( \omega \sim 10 - 10^2 \) s\(^{-1}\)), the inequality \( \omega \ll \tau^{-1} \sim 10^4 - 10^5 \) s\(^{-1}\) is always satisfied, and the variations of the equilibrium charges will definitely reveal themselves.

For small vertical perturbations \( z_n \), the particle and wake charges can be linearly approximated as \( Q \rightarrow Q_0 (1 + \varepsilon z_n / \Delta) \) and \( q \rightarrow Q_0 (1 + \epsilon z_n / \Delta) \), where small parameters \( \varepsilon \) and \( \epsilon \) relate to the equilibrium charge gradients through \( \varepsilon = (dQ/dz)_0 \Delta / Q_0 \) and \( \epsilon = (dq/dz)_0 \Delta / q_0 \). The subscript ‘0’ means that the derivative is taken at the equilibrium level \( z = 0 \).

Paramagnetic particles acquire also a magnetic moment, \( \mathbf{m} \), induced by an external magnetic field, \( \mathbf{B} \) \([24]\),

\[
\mathbf{m} = \frac{(\mu - 1)}{(\mu + 2)} a^3 \mathbf{B} = \alpha \mathbf{B}.
\]

Here, \( \alpha = (\mu - 1)a^3 / (\mu + 1) \) and \( \mu \) denotes the magnetic permeability. The magnetic potential is

\[
\phi_n(r) = \frac{((r - r_n) \cdot \mathbf{m}_n)}{|r - r_n|^3},
\]

and the corresponding magnetic field is \( \mathbf{B}_n(r) = -\nabla \phi_n(r) \), viz.

\[
\mathbf{B}_n(r) = \frac{3((r - r_n) \cdot \mathbf{m}_n)(r - r_n)}{|r - r_n|^5} - \frac{\mathbf{m}_n}{|r - r_n|^3}.
\]

Then the force exerted on the particle, \( n \), by the magnetized grain, \( j \), is given by

\[
\mathbf{F}_{nj}^{(m)} = (\mathbf{m}_n \nabla) \mathbf{B}_j,
\]

and the total force due to particle magnetization is

\[
\mathbf{F}^{(m)} = (\mathbf{m}_n \nabla)(\mathbf{B}_j + \mathbf{B}).
\]

2.2. Equilibrium state

The main forces acting on dust particles in discharge plasmas include the gravitational force, electric-field force (due to the electric field of the sheath), magnetic force, dust–neutral friction and dust–dust interactions. The electric field of the sheath and the external magnetic field are assumed to be directed vertically downward and depend only on the coordinate \( z \), i.e. \( \mathbf{E}(0, 0, -E(z)) \) and \( \mathbf{B}(0, 0, -B(z)) \). According to (2), the magnetic moment is also \( \mathbf{m}(0, 0, -m(z)) \).

In ground-based experiments, the particles’ vertical equilibrium position is given by force balance involving the sheath electric field \( E(z) \), gravitation, \( Mg \) and the magnetic force (6),

\[
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\]
which in our case takes the form

$$F^{(m)} = m \partial B / \partial z = \alpha B \partial B / \partial z$$  \hspace{1cm} (7)$$

and finally, the particle–wake interactions. In the nearest-neighbour approximation, the force balance in the vertical direction is

$$Mg = Q_0 E_0 + \alpha B_0 (\partial B / \partial z)_0 - 2l(1 + \kappa) Q_0^2 \frac{\exp(-\kappa)}{\Delta^3}.$$  \hspace{1cm} (8)$$

Here, $\kappa = \Delta / \lambda_D$ is the so-called lattice parameter and the subscript 0 denotes quantities at $z = 0$.

We now consider the force components acting on a particle in the horizontal direction. In the steady state ($\partial / \partial t = 0$) and remaining within the nearest-neighbour approximation, the interaction force between the $n$th particle and the $(n+1)$th grain has to be balanced by the corresponding force due to the $(n-1)$th particle, yielding an equilibrium particle separation, $\Delta$. In a strongly coupled complex plasma, it is usually assumed that the dominant force is the electrostatic repulsion between two neighbouring charges, namely

$$F^{(Q)}_{n,n \pm 1} = Q_n Q_{n \pm 1} \frac{\exp(-\kappa)}{\Delta^2} (1 + \kappa).$$  \hspace{1cm} (9)$$

Including magnetization, we have the repulsion force

$$F^{(m)}_{n,n \pm 1} = 3 \frac{m_n m_{n \pm 1}}{\Delta^4}.$$  \hspace{1cm} (10)$$

The question whether or not the dipole interactions (10) are significant has to be resolved numerically. For two identical microparticles, the ratio between these two forces $s = F^{(m)}_{n,n \pm 1} / F^{(Q)}_{n,n \pm 1}$ becomes

$$s = \frac{3m^2}{Q^2 \lambda_D^2 \psi(\kappa)},$$  \hspace{1cm} (11)$$

where the function $\psi(\kappa)$ is defined as $\psi(\kappa) = \kappa^2 \exp(-\kappa)(1 + \kappa)$. In typical complex plasmas, the interparticle distance exceeds the screening length $\kappa \sim 1.5–3$, so that $\psi(\kappa) \simeq 1.2–1.8$. The force ratio becomes $s \simeq 2m^2 / Q^2 \lambda_D^2 = 2(\mu - 1)^2 a^4 B^2 / (Q^2 \lambda_D^2(\mu + 1)^2)$. Using the standard assumption that $Q = \psi_s a$, ($\psi_s$ is the surface potential of the particle), $s$ can be rewritten as

$$s = 2(\mu - 1)^2 a^4 B^2 / [\psi_s^2 \lambda_D^2 (\mu + 1)^2].$$  \hspace{1cm} (12)$$

The two forces are of similar strength when $s \sim 1$, i.e. $a \simeq a_{cr} = (\psi_s \lambda_D (\mu + 1) / \sqrt{2}(\mu - 1) B)^{1/2}$. For small-size grains, $a \leq a_{cr}$ dipole interactions are not important, whereas for larger grains, $a \geq a_{cr}$, the dipole interactions dominate. A calculation of $a_{cr}$ as a function of screening length, $\lambda_D$, at a fixed magnetic induction $B = 0.15$ T, $\mu = 4$ and typical values of $\psi_s \sim 3$ and $\sim 10$ V is presented in figure 2, and in figure 3, we show $a_{cr}$ as a function of the magnetic induction $B$ for several typical values of the screening length and surface potential. These plots indicate that magnetic interactions in typical discharge conditions and under moderate magnetic fields are of importance even for micron-sized particles. The magnetic forces can determine the interparticle distances and may be responsible for the formation of the complex plasma structures.
3. DL modes in a horizontal string of paramagnetic grains

3.1. Dispersion relation

Following the standard procedure (see e.g. [16, 19]), we present the linearized equations of motion for the $n$th particle in the closest-neighbour approximation

\begin{align}
\dot{x}_n + 2\gamma \dot{x}_n &= \Omega_1^2 \left( x_{n+1} + x_{n-1} - 2x_n \right) + \Omega_2^2 \left( z_{n+1} - z_{n-1} \right), \\
\dot{z}_n + 2\gamma \dot{z}_n &= -\Omega_1^2 z_n - \Omega_2^2 \left( z_{n+1} + z_{n-1} - 2z_n \right) - \Omega_2^2 \left[ \epsilon \left( z_{n+1} + z_{n-1} \right) + 2\epsilon z_n \right] + \Omega_2^4 \left( x_{n+1} - x_{n-1} \right),
\end{align}

where the displacements around the equilibrium are small so that $|x_{n\pm 1} - x_n|, |z_{n\pm 1} - z_n| \ll \Delta, l$; $\gamma$ denotes the damping rate due to neutral gas friction [25]. Assuming perturbations
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\[ f(\omega, k) \cdot g(\omega, k) + h(\omega, k) = 0, \quad (15) \]

with

\[ f(\omega, k) = \left( \omega^2 + 2i\gamma\omega - 4\Omega_0^2 \sin^2 \frac{k\Delta}{2} \right), \quad (16) \]

\[ g(\omega, k) = \left[ \omega^2 + 2i\gamma\omega - \Omega_0^2 + 4\Omega_1^2 \sin^2 \frac{k\Delta}{2} - 2\Omega_q^2 (\epsilon + \epsilon \cos k\Delta) \right], \quad (17) \]

\[ h(\omega, k) = 4\Omega_{coup}^4 \sin^2 k\Delta. \quad (18) \]

To simplify the bulky expressions for the wave frequencies involved, we assume that \( \tilde{l} = l/\Delta < 1 \), which is often relevant for strongly coupled complex plasmas. Then to order \( \tilde{l} \), we get

\[ \Omega_1^2 = (1 - \tilde{q})\Psi(\kappa)\Omega_0^2 + 4\Omega_1^2, \quad (19) \]

\[ \Omega_1^2 = (1 - \tilde{q})\Omega_0^2 + 3\Omega_1^2, \quad (20) \]

where the characteristic frequencies \( \Omega_0 \) and \( \Omega_1 \) are defined by pure electrostatic and magnetic interactions through

\[ \Omega_0^2 = \frac{Q_0^2}{M\Delta^2} \varphi(\kappa) \exp(-\kappa), \quad (21) \]

\[ \Omega_1^2 = \frac{3m_0^2}{M\Delta^2}. \quad (22) \]

Here, we have introduced the notations \( \Psi(\kappa) = \varphi(\kappa) + \varphi^{-1}(\kappa) \) and \( \varphi(\kappa) = (1 + \kappa) \).

The frequency \( \Omega_v \) (in equation (17)) describes vertical oscillations of a single particle immersed into a sheath electric field and magnetized by an external magnetic field. As a result, this is determined by two terms, namely, \( \Omega_v^2 = \Omega_E^2 + \Omega_m^2 \), where \( \Omega_E \) is specified by the electric-field profile \( E(z) \), and the equilibrium particle charge \( Q_0(z) \) while \( \Omega_m \) is determined by the magnetic-field distribution [22], yielding

\[ \Omega_v^2 = -M^{-1} [Q_0(\epsilon E_0/\Delta + E_0') + (\alpha B_0^2/\Delta^2)(\beta^2 + \eta)]. \quad (23) \]

Small parameters \( \beta \) and \( \eta \) relate to the magnetic-field gradients through \( \beta = m'_0\Delta/m_0 = B'_0\Delta/B_0 \) and \( \eta = B''_0\Delta^2/B_0 \). Generally, the value \( \Omega_v^2 \) can be either positive or negative depending on specific experimental conditions [22]. To insure that the vertical vibrations are stable, we assume from the outset that \( \Omega_v^2 > 0 \).
A hybrid frequency $\Omega_{\omega q}$ given by

$$\Omega_{\omega q}^2 = \tilde{q} \tilde{l} \Omega_0^2,$$  \hspace{1cm} (24)

results from the combined effect of the wake and equilibrium charge gradient. Finally, the term $h(\omega, k)$ in equation (15) is determined by $\Omega_{coup}^4 = \Omega_{c_{ax}}^2 \Omega_{c_{ax}}^2$, where

$$\Omega_{c_{ax}}^2 = \tilde{q} \Phi(k) \Omega_0^2,$$  \hspace{1cm} (25)

$$\Omega_{c_{ax}}^2 = \Omega_{c_{ax}}^2 - (\varepsilon - \varepsilon \tilde{q}) \Omega_0^2 - \beta \Omega_1^2,$$  \hspace{1cm} (26)

with $\Phi(k) = 1 + \Psi(k)$. It is reasonable to consider the small parameters $\varepsilon$ and $\varepsilon$ as being of the same order of magnitude, namely, $\varepsilon \sim \varepsilon \sim \Delta / L_Q$, while $\beta \sim \Delta / L_B \neq \varepsilon$. Here, $L_Q \sim Q(z) / (\partial Q(z) / \partial z) \sim q(z) / (\partial q(z) / \partial z)$ and $L_B \sim B(z) / (\partial B(z) / \partial z)$ are the characteristic scale lengths of the particle charge and magnetic-field non-uniformity, respectively.

### 3.2. Modifications of DL modes

Returning to the example of a 1D particle string without magnetic field and wake effects, the dispersion relation (15) brings us back to the two independent waves, longitudinal and transverse DL modes. The characteristic frequencies (19) and (20) reduce to the basic form, namely $\Omega_{\parallel} = \sqrt{\Psi(k)} \Omega_0$, $\Omega_{\perp} = \Omega_0$ and $\Omega_v = \Omega_E$ and the stability condition of the transverse wave is given by $\Omega_{\perp}^2 > 4 \Omega_{\parallel}^2$.

Inclusion of the particle magnetization by a vertical magnetic field modifies both wave frequencies increasing the values $\Omega_{\parallel}$ and $\Omega_{\perp}$ due to the magnetic dust–dust interactions ($\propto \Omega_1$), but the DL modes remain decoupled. Note that a ratio of the characteristic frequencies due to electric and magnetic interactions gives $\Omega_{\parallel} / \Omega_{\perp} = \sqrt[3]{3}$. As mentioned above, the condition $s \sim 1$ for a few micron particles requires magnetic fields $B \gtrsim 0.1$ T (see figures 2 and 3). Therefore, the contribution of the magnetic dipole interactions in the total DL-wave frequencies $\Omega_{\parallel}$ and $\Omega_{\perp}$ can be of the same order of magnitude or even larger than $\Omega_0$ in moderate magnetic fields.

The ion-focusing effect leads to the coupling between two DL waves but decreases the squared frequencies of longitudinal ($\Omega_{\parallel}^2$) and transverse ($\Omega_{\perp}^2$) modes (19) and (20) via the factor $(1 - \tilde{q})$. In the case of a strong wake ($\tilde{q} \rightarrow 1$), this can significantly reduce (almost nullify) the contribution due to the Coulomb dust–dust interactions, so that the repulsive interactions of vertically orientated magnetic moments will dominate.

Finally, a combination of three effects modifies the main frequency of the DL waves according to equations (19) and (20). The mode coupling arises now due to the anisotropy, induced by the particle–wake interactions, the charge and magnetic-field inhomogeneities. As a result,

$$\Omega_{coup}^4 \simeq \tilde{q} \tilde{l} \Phi(k) \Omega_0^4 [\tilde{q} \tilde{l} \Phi(k) - \varepsilon (1 - \tilde{q}) - \beta s].$$  \hspace{1cm} (27)

The sign of $\Omega_{coup}^4$ determines the stability of the DL waves and a scenario of the mode interactions, which develop mostly in the vicinity of the intersection point $(\omega_0, k_0)$ of the decoupled (initially stable) longitudinal and transverse waves [19]. The latter is given by $\omega_0 \simeq \Omega_v / (1 + \Omega_{\perp}^2 / \Omega_1^2)^{1/2}$, $k_0 \Delta \simeq 2 \arcsin(\omega_0 / 2 \Omega_{\parallel})$. It turns out that when the coupling term...
Figure 4. Qualitative picture of reconnection of the dispersion curves for transverse (labelled by $\perp$) and longitudinal ($\parallel$) DL modes in the case of $\Omega^4_{\text{coup}} < 0$ (——) and $\Omega^4_{\text{coup}} > 0$ (– – – –). The transverse and longitudinal DL waves, corresponding to the solutions of equation (15) for $\Omega_{\text{coup}} = 0$, are indicated by thin lines.

is positive, a confluence of the transverse and longitudinal dispersion curves occurs in the long- ($k < k_0$) and short-wavelength ranges ($k > k_0$) separately, thus leading to a gap in wavenumber domain as indicated in figure 4 by dashed lines (akin to figure 3 in [19]). Such reconnection of the dispersion curves corresponds to the resonance instability of the hybrid DL mode. Note that the width of the wavenumber domain, $\Delta k$, where the instability occurs ($k_0 - \Delta k < k < k_0 + \Delta k$) is determined by the coupling coefficient through $\Delta k \propto \Omega^2_{\text{coup}}/(\Omega^2_\parallel + \Omega^2_\perp)$. In the opposite case, when $\Omega^4_{\text{coup}} < 0$, the DL waves remain stable, but demonstrate a specific-mode conversion: a confluence of both transverse and longitudinal dispersion curves occurs in such a way, that there is a considerable frequency gap in the vicinity $k = k_0$, in which both DL modes are evanescent as shown in figure 4 by solid lines (akin to figure 2 in [19]).

While the first term between brackets in equation (27) is always positive, the combination of other two can have different signs depending on the relative contributions of the gradients $(\partial Q/\partial z)_0$ ($\epsilon < 0$ or $\epsilon > 0$) and $(\partial B/\partial z)_0$ ($\beta < 0$ or $\beta > 0$). To make further progress, the relationship between the different terms in (27) needs to be made more explicit. Moreover, it is instructive to separate two different cases, namely $\tilde{q} \to 1$, when the effective charge attributed to the ion focusing is comparable with the particle charge, and a weak wake effect when $\tilde{q} < 1$.

3.2.1. Strong wake effect. When $\tilde{q} \to 1$, the terms proportional to $(1 - \tilde{q})$ in (19)–(26) may be neglected, so that the role of magnetic interactions is significantly increased. Indeed, in this case the main frequencies of the DL modes are specified by the magnetic moments of the grains $\Omega^2_1 \simeq 4\Omega^2_{\text{xc}}$ and $\Omega^2_\perp \simeq 3\Omega^2_{\text{xc}}$. The charge inhomogeneity is only involved through the small term, $\epsilon \Omega^2_{c_{\text{aq}}} \cos^2 k\Delta_2$, in the dispersion relation for the transverse DL mode. Moreover, the strong wake diminishes the role of the charge gradient in the mode coupling. The latter comes now from the particle–wake interactions $(\Omega^2_{\text{coup}} \simeq \tilde{\Phi}(\kappa)\Omega^2_0)$ and the magnetic-field inhomogeneity, leading to

$$\Omega^4_{\text{coup}} \simeq \tilde{\Phi}(\kappa)\Omega^4_0[\tilde{\Phi}(\kappa) - \beta s].$$

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We start with a weakly inhomogeneous field $B(z)$, when the first term dominates and $\Omega_{coup}^4 \simeq \vec{\Omega}^2 \Phi^2(\kappa) \Omega_0^4$ is always positive. Typically $\vec{\Omega}(\kappa) \lesssim 1$, and the coupling effect is relatively small since $\Omega_{coup}^4 / (\Omega_1^2 + \Omega_2^2) \sim \vec{\Omega}(\kappa)/(7s) < 1$. Therefore, it apparently plays a role in the vicinity of the point $k = k_0$, where the two decoupled DL modes (modified by the magnetic field) intersect. The condition of the waves to intersect is $12\Omega_1^2 < \Omega_2^2$. One can expect that the DL-mode coupling reveals a resonance instability provided the neutral gas pressure remains below a certain threshold—similar to that predicted in [16, 19]. The magnetic term only slightly increases ($\beta < 0$) or decreases ($\beta > 0$) the growth rate of the instability and the width of the unstable zone $\Delta k$. In another limiting case, of significant magnetic field and its gradient, the contribution of the last term in (26) prevails, and $\Omega_{coup}^4 \simeq -\beta s \vec{\Omega}(\kappa) \Omega_0^4$. This immediately leads either to the DL-mode conversion akin to that considered in [19] for $\beta > 0$ or to DL-mode instability if $\beta < 0$.

As a numerical example, we consider the parameters used in the magnetic experiment with the paramagnetic microparticles [21]. Particle levitation was observed for average magnetic fields $B_0 \sim 0.1–0.15$ T and field gradients $B_0 \sim 0.75–1.15$ T m$^{-1}$ (these values correspond to the actual distance of $\sim 0.28$ m between the magnetic coils and the lower electrode). Taking typical $\Delta \sim 5 \times 10^{-2}–10^2 \mu$m yields small positive values $\beta \sim 0.005–0.01$. This means that the condition $\beta s \lesssim \vec{\Omega}(\kappa)$ can hardly be achieved in this experiment. Only significant increase of the magnetic induction and its non-uniformity, until $L_B$ becomes $\sim s \Delta$ ($\beta s \sim 1$), would quench the initial DL-mode instability due to the particle–wake interactions of magnetized particles.

3.2.2. Weak wake effect. When the effective wake charge is small compared with the equilibrium particle charge, the ion focusing effect only weakly reduces the DL-wave frequencies according to (19)–(26). However, this will diminish the coupling term, reducing (27) to

$$\Omega_{coup}^4 \simeq \vec{\Omega}^4 \Phi^4(\kappa) \Omega_0^4 \{\vec{\Omega} \Phi(\kappa) - \varepsilon - \beta s\}. \tag{29}$$

Furthermore, using estimates of [19], we find that a condition $\vec{\Omega} \Phi(\kappa) < |\varepsilon|$ appears appropriate for many laboratory experiments. In the limit of small $\vec{\Omega}$, the coupling frequency is now specified by the two gradient terms $\beta$ and $\varepsilon$ through $\Omega_{coup}^4 \simeq \vec{\Omega}^4 \Phi^4(\kappa) \Omega_0^4 (\varepsilon + \beta s)$, and if $\beta$ and $\varepsilon$ are both positive or negative, the process of the mode interactions reduces either to the stable-mode conversion ($\varepsilon, \beta > 0$) or to the resonance instability ($\varepsilon, \beta < 0$). When $\varepsilon$ and $\beta$ have opposite signs, a competition between these coefficients determines one of the two ways for the mode coupling.

While numerical estimates of $\beta$ are readily obtained from direct measurements of the magnetic-field profile, the equilibrium charge distribution in a plasma sheath is more difficult to obtain [26]. An average charge gradient can be roughly estimated as $|\varepsilon| \sim \Delta / L_Q \sim 0.2–0.1$ [19]. However, one can expect that local values of $\varepsilon$ can be larger or smaller and even change sign, thus competing with the magnetic term $s \beta$, and affecting the character of the DL-mode interactions. Finally, if electrostatic levitation occurs in the presheath, where the absolute value of the equilibrium charge is almost constant with height ($\varepsilon \rightarrow 0$), competition between a ‘weak’ wake effect and magnetic interactions will be responsible for the character of the DL-wave coupling.
4. Conclusions

The propagation of DL modes in a 1D string (monolayer) formed by paramagnetic particles was studied including the effects of spatial gradients of external magnetic field, equilibrium particle charge and formation of the ion wake. The main conclusion is (as expected) that the DL modes exhibit a more complicated behaviour than the classical DL in a non-magnetized particle string without grain–wake interactions. Starting from the equilibrium configuration, we showed that the combination of these factors modifies the levitation condition and can also affect the frequencies of DL waves. In particular, the particle magnetization can increase the main wave frequencies even in moderate magnetic fields ($B \gtrsim 0.1$ T), while the wake effect decreases these and can significantly reduce the contribution of Coulomb dust–dust interactions. Furthermore, the combination of three anisotropic effects not only modifies the wave frequencies of transverse and longitudinal modes, but also leads to mutual DL-wave interactions. This mode coupling is strongly dependent on the relationship between the gradient and wake terms: these can induce either a resonance instability (if the coupling coefficient is positive) or a stable mode conversion (for negative coupling term). Strong wakes diminish the contribution of the equilibrium charge gradient in the coupling process. The latter is determined by the competition between the particle–wake and magnetic interactions. In another limiting case of weak grain–wake effects, the coupling coefficient is mostly dependent on the gradients of the external magnetic field and equilibrium particle charge, respectively. Increasing the magnetic field and making it more non-uniform leads to the domination of the magnetic interactions in the DL-mode coupling process. This implies that the characteristics of DL-mode coupling can be effectively controlled externally (by variation of the field strength and gradients), thus opening new possibilities for studies of collective effects in strongly coupled systems. The experimental investigations of mode coupling using paramagnetic particles can also provide a tool for determining some plasma parameters. For example, observations of the threshold for the resonance instability in a known magnetic field profile would permit the determination of an equilibrium charge gradient or the estimation of ion–wake parameters.

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