Research Article

Analysis of the Hydrodynamic Lubrication Characteristics of the External Return Spherical Bearing Pair of an Axial Piston Pump/Motor

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At present, the study on lubrication of return mechanism friction pair is only for the common axial piston pump, and the influence of kinematic characteristics of contact points between the retainer plate and the spherical hinge on lubrication of return mechanism friction pair is not considered. So, it is hard to directly apply the frictional lubrication characteristic of the bearing to design the external return mechanism. Based on the kinematic and operating characteristics of the external return mechanism and considering the lubrication situation of the friction pair under Newtonian fluid, a Reynolds equation in the spherical coordinate system was deduced, and then the lubrication of the external return spherical hinge under different structural parameters was analysed. The results show that different slant inclination of the external swash plate, pump shaft rotating speeds, and eccentricities all affect the lubrication characteristics of the friction pair, especially the slant inclination of the external swash plate and oil film clearance has great influence on axial leakage flow. Therefore, in the design of external return mechanism of the multirow axial piston pump, the lubrication performance of the external return spherical hinge under different slant inclinations of the external swash plate should be analysed and calculated.

1. Introduction

Although three key friction pairs (slipper pair, flow distribution pair, and plunger pair) of axial piston pumps and motors have been extensively researched worldwide, and scholars have achieved fruitful results [1–3], not much research has been conducted on the pair of retainer plate-spherical hinge. The return mechanism is a key component of an axial piston pump or motor and has a direct effect on the performance of the three friction pairs, as it provides the preload force to the three key friction pairs to ensure the slipper clings to the swash plate as the plunger is reciprocating [4–6]. Several researchers aim to optimise the structural performance of the return mechanism. For example, Xu et al. [7] built a virtual prototype of the plunger pump and simulated the relative motion of the retainer plate-spherical hinge. Tao [8] established a numerical analysis method of the oil film lubrication of the return spherical bearing pair and discussed the relationship between the oil film load-bearing capacity and scale effect. Liu et al. [9] established a mathematical model of the dynamic rules of a lubricating oil film for the slipper pair and investigated the influence of different return mechanisms on the oil film characteristic of the slipper. Deng et al. [10] analysed the relative motion characteristics between the external spherical hinge and the external retainer plate and...
investigated the influence of the external swash plate inclination and pump shaft rotating speed on the relative motion scratches of them.

The above research provides the basis of optimizing the structural performance of the return mechanism. However, it is still necessary to make an intensive study of the external return mechanism [11], especially regarding the friction pair formed by the outer contour surface (arc surface) of the retainer plate and the inner contour surface (spherical surface) of the external spherical hinge. It is necessary to study the lubrication characteristics of external return mechanism for two main reasons: (i) its friction-lubrication property affects the reliability of its own structure and (ii) its friction-lubrication property greatly influences the working stability of the three friction pairs of the axial piston pump. The friction pair is similar to the spherical sliding friction pair with bearing as the application background. Fang et al. [12], based on a complete sphere model, proposed a theoretical calculation method of the static contact stress distribution under the spherical surface. Meyer et al. [13] deduced the Reynolds equation for a Newtonian fluid lubrication in spherical coordinates and solved the lubrication problem of artificial hip implantsations under quasi-static conditions. Buckholz et al. [14] obtained a modified Reaynsolds equation based on the power-law fluid model and calculated the lubrication performance of a short radial sliding bearing. Chinyoka et al. [15] studied the influence of viscoelasticity under Arrhenius dynamics on the temperature of a lubricant subjected to simple shear in one-dimensional flow by the numerical simulation method and proved the advantages viscoelastic fluid (represented by Oldroyd-B liquid) over Newtonian fluid.

This work studies the lubrication mechanism of the friction pair with the spherical geometry, establishes a spherical coordinate system, constructs a Reynolds equation of the external return mechanism in a spherical coordinate system, and analyses the influence of a variety of parameters under the condition of the hydrodynamic lubrication, such as swash plate inclination, rotating speed, and eccentricity on the oil film load-bearing capacity and lubrication state. Thus, this work aims to provide a reference for the energy dissipation rules and friction theory of the external return spherical bearing pair of the balanced double-row axial piston pump and the dual-drive axial piston motor.

2. Equation and Formula

2.1. Composition of the External Return Mechanism. The external return mechanism and its spherical bearing pair are important components of the balanced double-row axial piston pump and the dual-drive axial piston motor. The basic structure, as shown in Figure 1, includes the external swash plate, external spherical hinge, compression spring, juncture, and cylinder block. The external return spherical bearing pair, formed by matching the outer contour surface (arc surface) of the external retainer plate to the inner contour surface (spherical surface) of the external spherical hinge, wears and fails more easily than other parts, as the friction pair bears both axial and radial pressure and pulsation.

2.2. Analysis of Small Fluid Element. Based on the structure characteristics of the friction pair, the simplified diagram of the external return spherical bearing pair is shown in Figure 2. In Figure 2, \( h \) is the oil film thickness, \( \varepsilon \) is the eccentricity, the coordinate system \( o-xyz \) is used to describe the motion of the external spherical hinge, the coordinate system \( o_1-x_1y_1z_1 \) is used to describe the motion of the external retainer plate, \( U_o \) is the rotation speed of the external retainer plate, and \( U_h \) is the rotation speed of the external spherical hinge; setting \( r \) (mm) is the distance from an arbitrary point of the contact oil film between the external spherical hinge and the external retainer plate to the centre of the external spherical hinge; setting \( R \) is the radius of the external spherical hinge and the external retainer plate; a small fluid element \( dW \) was intercepted in the convergent gap between the external spherical hinge and the external retainer plate. The following assumptions were made [16]:

(i) The effects of the volume and inertia forces of the lubricant were neglected.

(ii) The lubrication grease was assumed not to slide on the solid interface.

(iii) The lubrication film was assumed to be negligibly thin.

(iv) The pressure change in the \( r \) direction (Figure 3) of film thickness was not considered.

(v) The lubricant was assumed to be an isothermal, Newtonian fluid.

Setting \( \zeta \) (mm) as the distance from the point to the spherical outer surface of the external retainer plate, equation (1) can be obtained:

\[
\frac{\partial}{\partial r} = \frac{\partial}{\partial \zeta} \tag{2}
\]

According to the spatial motion characteristic and cooperative relationship of the external return spherical bearing pair, a spherical coordinate system \( o(-x)(y)(z) \) was established at the centre of the external sphere hinge, i.e., where the \( o(-x) \) axis is the rotating shaft of the external spherical hinge and the \( o(-z) \) axis is the radial direction of the external retainer plate. The unit vectors of three orthogonal axes \((r, \varphi, \theta)\) of the spherical coordinate system are \((e_r, e_\varphi, e_\theta)\), respectively. An arbitrary small fluid element \( dW \) in the contact lubrication area between the external spherical hinge and the external retainer plate was taken to force analysis, as shown in Figure 3.

In the circumferential direction of the external retainer plate, a small fluid element is only acted upon by the fluid pressure \( p \) and viscous force \( r \). The cross section of the small
The fluid element along the radius direction of the external swash plate is sector and the outer contour surface of the external swash plate is a cambered surface. According to the condition of the force balance \[17\]

\[
\begin{align*}
\frac{\partial p}{\partial \phi} &= r \frac{\partial \tau_\phi}{\partial r}, \\
\frac{\partial p}{\partial \theta} &= r \frac{\partial \tau_\theta}{\partial r},
\end{align*}
\]

where \( p \) is the lubricating oil film pressure between the external spherical hinge and the retainer plate, \( \tau_\phi \) is the fluid shear stress component in the \( \phi \) direction, and \( \tau_\theta \) is the fluid shear stress component in the \( \theta \)-direction of the rotation axis of the external spherical hinge.

On the basis of the property of the Newtonian fluid, in the direction of \( \theta \), the viscosity \( \eta_\theta \) is defined. The shear stress component \( \tau_\theta \) along the \( \theta \) direction is represented by the product of the viscosity and the velocity gradient. That is,

\[
\eta_\theta \frac{\partial u_\theta}{\partial r} = \tau_\theta,
\]

where \( u_\theta \) is the velocity component of a point in the lubricant oil film along the \( \theta \) direction. Similarly, the shear stress component \( \tau_\phi \) along the \( \phi \) direction can be represented by the product of the viscosity and the velocity gradient as

\[
\eta_\phi \frac{\partial u_\phi}{\partial r} = \tau_\phi.
\]

**Figure 1:** Structure of the balanced double-row axial piston pump. 1: external slipper pair; 2: external swash plate; 3: pump shaft; 4: framework oil seal; 5: internal swash plate; 6: internal retainer plate; 7: internal piston pump; 8: external spherical hinge; 9: external retainer plate; 10: juncture; 11: compressed spring; 12: external piston pump; 13: cylinder block; 14: valve plate.

**Figure 2:** Structure of the external retainer plate and the external spherical hinge.

**Figure 3:** Force analysis of the fluid element of the external return spherical bearing pair.
where \( u_\varphi \) is the velocity component of a point in the lubricant oil film along the \( \varphi \) direction.

According to equation (2), equations (4) and (5) can be written as follows:

\[
\begin{align*}
\eta \frac{\partial u_x}{\partial \zeta} &= \tau_{\varphi}, \\
\eta \frac{\partial u_\theta}{\partial \zeta} &= \tau_{\theta}.
\end{align*}
\]

Substituting equation (6) into equation (3), equation (7) can be obtained as follows:

\[
\begin{align*}
\frac{\partial (\eta \frac{\partial u_x}{\partial \zeta})}{\partial \zeta} R &= \frac{\partial \rho}{\partial \varphi}, \\
\frac{\partial (\eta \frac{\partial u_\theta}{\partial \zeta})}{\partial \zeta} R &= \frac{\partial \rho}{\partial \theta}.
\end{align*}
\]

2.3. Kinematic Analysis. Since the external return mechanism has a complex form of motion, a kinematic analysis is necessary when considering the lubrication characteristics of the spherical hinge pair. Two coordinate systems, shown in Figure 4, were established: the \( o-xyz \) coordinate system at the centre of the external spherical hinge and the \( o-x,y,z \) coordinate system at the crossing point of axes between the external spherical hinge and the external retainer plate for describing the motion of the external retainer plate. The crossing point is the common centre of the external retainer plate and the spherical hinge. In the following derivation, \( i, j \), and \( k \) are the unit vectors of the \( x, y \), and \( z \) axes of the rectangular \( o-xyz \) coordinates, respectively, \( \omega \), represents the instantaneous angular velocity of the external return plate, and \( \omega_b \) represents the angular velocity of the external spherical hinge.

The transformation relationship of the motion equation of the external return mechanism between the rectangular coordinate system \( o-xyz \) and the spherical coordinate system can be represented as follows:

\[
\begin{align*}
i &= \sin \varphi \cos \theta e_x + \cos \varphi \cos \theta e_y - \sin \theta e_z, \\
j &= \sin \varphi \sin \theta e_x + \cos \varphi \sin \theta e_y + \cos \theta e_z, \\
k &= \cos \varphi e_z - \sin \varphi e_y.
\end{align*}
\]

The vector equation of the angular velocity \( \omega_r \) in the rectangular coordinate system \( o-xyz \) is [10]

\[
\omega_r = -k \omega \cos \beta_1 i - k \omega \sin \beta_1 j + 0k,
\]

where \( \beta_1 \) is the rotation angle and \( \omega \) is used to describe the change of angular velocity at an arbitrary point in the width of the external retainer plate. When \( \beta_1 = \beta \), \( \omega_r \) describes the change of angular velocity at the centre of the width of the external retainer plate. \( \beta \) is the slant inclination of the external swash plate. \( k = (\sin^2 \varphi \cos^2 \beta_1 + \cos^2 \varphi) / \cos \beta_1 \).

Allowing \( m_1 \) to represent a point in the contact surface between the external spherical hinge and the retainer plate, its radius vector in the coordinate system \( o-xyz \) can be written as

\[
r_1 = -R \cos \theta \sin \varphi i + R \sin \varphi \sin \theta j - R \cos \varphi k.
\]

The velocity vector of point \( m_1 \) in the spherical coordinate system can be expressed as

\[
\mathbf{U}_r = \mathbf{r}_1 \times \mathbf{\omega}_r = -k \omega R \sin \beta \cos \varphi \mathbf{i} + k \omega R \cos \beta \cos \varphi \mathbf{j} + k \omega R \sin \beta \sin \varphi \mathbf{k}
\]

\[
\times (\sin \beta \cos \theta \sin \varphi + \cos \beta \sin \theta \sin \varphi) \mathbf{k}
\]

\[
= 0 \mathbf{e}_r + k \omega R (\sin \beta \cos \theta + \cos \beta \sin \theta) \mathbf{e}_\varphi - k \omega R \sin \beta \sin \varphi \cos \varphi \mathbf{e}_\theta.
\]

The vector equation of the angular velocity \( \omega_b \) in the coordinate system \( o-xyz \) can be written as

\[
\omega_b = -\omega i + 0j + 0k.
\]

As the external spherical hinge rotates with the pump shaft, the vector equation of the velocity of point \( m_1 \) in the spherical coordinate system can be expressed as

\[
\mathbf{U}_b = \mathbf{r}_1 \times \mathbf{\omega}_b = \omega R \cos \varphi \mathbf{j} + \omega R \sin \varphi \sin \theta \mathbf{k}
\]

\[
= 0 \mathbf{e}_r + \omega R \sin \theta e_y + \omega R \cos \varphi \cos \theta e_\theta.
\]

According to the kinetic characteristic of the external spherical hinge and the external retainer plate equations (11) and (13), the following velocity boundary conditions can be obtained [18]:

\[
\begin{align*}
\zeta &= 0: u_r = 0, \\
u_\varphi &= k \omega R (\sin \beta_1 \cos \theta + \cos \beta_1 \sin \theta), \\
u_\theta &= -k \omega R (\sin \beta_1 \sin \varphi \cos \varphi - \cos \beta_1 \sin \varphi \cos \varphi), \\
\zeta &= h: u_r = 0, \\
u_\varphi &= \omega R \sin \theta, \\
u_\theta &= \omega R \cos \varphi \cos \theta,
\end{align*}
\]

where \( u_r \) and \( u_\theta \) represent the fluid velocity along the \( \varphi \) and \( \theta \) directions, respectively, and \( h \) is the oil film thickness.

2.4. Boundary Condition. The external spherical hinge pair can be used as a special sliding bearing, and its lubrication equation can be solved by Reynolds boundary condition. Accordingly, the boundary conditions of the external return spherical hinge pair are as follows.

2.4.1. Circumferential Boundary Conditions of the External Spherical Hinge Pair. The starting point of oil film:

\[
\varphi = 0, \\
\rho = p_0.
\]

The ending point of oil film:
\[ \varphi = \varphi_2, \]
\[ p = p_0, \]  
\[ \frac{dp}{d\varphi} = 0. \]  
\[ \theta = 90^\circ + \beta - b_1, \]
\[ p = p_0, \]  
\[ \theta = 90^\circ + \beta + b_2, \]
\[ p = p_0. \]  

The continuous point of oil film:
\[ 0 < \varphi < \varphi_2, \]
\[ p = p(\varphi), \]  
\[ \varphi_2 < \varphi < 2\pi, \]
\[ p = p_0, \]
\[ \frac{dp}{d\varphi} = 0, \]  
where \( p_0 \) is the environmental pressure.

2.4.2. Axial Boundary Conditions of the External Spherical Hinge Pair. The axial pressure at both ends of the external return spherical hinge pair is 0. According to Figure 5, the axial boundary conditions of the external return spherical hinge pair are known, in which \( B \) is the contact width.

\[ \theta = 90^\circ + \beta - b_1, \]  
\[ p = p_0, \]  
\[ \theta = 90^\circ + \beta + b_2, \]
\[ p = p_0. \]

2.5. Deducing the Reynolds Equation. By integrating equation (7) once on \( \zeta \), we get
\[ \tau \varphi = \frac{1}{R} \frac{dp}{d\varphi} \zeta + \tau \varphi_1, \]
\[ \tau \theta = \frac{1}{R} \frac{dp}{d\theta} \zeta + \tau \theta_1, \]  
where \( \tau \varphi_1 \) and \( \tau \theta_1 \) are the shear stress components of point \( \zeta = 0 \) on the surface of the external retainer plate.

Substituting equation (6) into equation (20) and integrating again on \( \zeta \), we get
\[ u \varphi = \frac{1}{R \eta \varphi} \frac{d \varphi}{d \psi} \zeta^2 + \frac{\tau \varphi_1}{\eta \varphi} \zeta + C_1, \]
\[ u \theta = \frac{1}{R \eta \theta} \frac{d \theta}{d \theta} \zeta^2 + \frac{\tau \theta_1}{\eta \theta} \zeta + C_2. \]  

Applying the velocity boundary conditions of equation (14) into equations (18) and (21), \( \tau \varphi_1, \tau \theta_1, C_1 \), and \( C_2 \) can be solved. And, the expression of \( u \varphi \) and \( u \theta \) can be written as follows:
By substituting equation \( q_\phi \) of the circular flow and equation \( q_\theta \) of the axial flow into the flow balance equation, the Reynolds equation of the external return spherical bearing pair in the spherical coordinate system can be obtained:

\[
\frac{\partial}{\partial \phi}\left(h^3 \frac{\partial \rho h}{\partial \phi}\right) + \frac{\partial}{\partial \theta}\left(h^2 \frac{\partial \rho h}{\partial \theta}\right) = 6\omega R^2 \eta \left(\sin \theta \frac{\partial h}{\partial \phi} - \cos(2\theta) \frac{\partial k h}{\partial \phi}\right) + \cos(2\theta) \left(\frac{\partial \cos \theta \cdot h}{\partial \phi} + \frac{\partial k h \sin(2\theta)}{\partial \theta}\right) - 12\rho \eta \frac{\partial}{\partial \phi}[h(R + dr)\zeta].
\]

(26)

This Reynolds equation is suitable for calculation of the oil film of the external return spherical bearing pair with the lubricant of Newton fluid. This Reynolds equation considers both the rotation of the external spherical hinge and the rotation of the external retainer plate, and thus is an equation of compound motion.

The lubricant density was assumed constant, and the external return mechanism under steady load, that is, the external return mechanism, was assumed to be lubricated by an incompressible steady flow (\( \rho = \text{const} \)) [16]. Thus,

\[
\frac{\partial}{\partial \phi}\left(h^3 \frac{\partial \rho h}{\partial \phi}\right) + \frac{\partial}{\partial \theta}\left(h^2 \frac{\partial \rho h}{\partial \theta}\right) = 6\omega R^2 \eta \sin \theta \frac{\partial h}{\partial \phi} - 6\omega R^2 \eta \cos(2\theta) \frac{\partial k h \sin(2\theta)}{\partial \theta} + \frac{\partial k h \sin(2\theta)}{\partial \phi}.
\]

(27)

2.6. Equation of Oil Film Thickness. During the working process of the external return mechanism, the external retainer plate is not only subjected to the axial load of the external spherical hinge, but also to the radial load of it. That makes the external retainer plate to produce axial eccentricity \( e_x \) relative to the external spherical hinge as well as produce radial eccentricity \( e_r \) at the same time. Therefore, the vector equation of the oil film thickness \( h \) that the
external retainer plate relative to the external spherical hinge can be expressed as [20]
\[ h = c \left( 1 + \varepsilon_x \sin \varphi \cos \theta + \varepsilon_y \sin \varphi \sin \theta \right), \tag{28} \]
where \( c \) is the radius clearance and \( \varepsilon_x \) and \( \varepsilon_y \) are the axial and radial eccentricity of the oil film, respectively. Eccentricity \( \varepsilon = \sqrt{\varepsilon_x^2 + \varepsilon_y^2} \).

3. Calculation Solution and Result Analysis

3.1. Dimensional Normalisation. Allowing \( P = p/p_0, H = h/c \), and \( \zeta = (\zeta/h) \), the dimensionless form of the oil film thickness equation of the external return mechanism in the spherical coordinate system is written as
\[ H = 1 + \varepsilon_x \sin \varphi \cos \theta + \varepsilon_y \sin \varphi \sin \theta. \tag{29} \]

In the dimensionless form, circumferential shear stress \( \tau_{\varphi} \) and axial shear stress \( \tau_{\theta} \) of oil film are
\[ \tau_{\varphi} = \frac{1}{R} \frac{\partial}{\partial \varphi} \left( \frac{\partial P}{\partial \varphi} \right) \frac{H_c}{H} - \frac{H_c}{2R} \frac{1}{\partial \varphi} \frac{\partial P}{\partial \varphi} + \frac{\omega \eta R \sin \varphi}{H_c} + \frac{k \omega \eta R \cos \theta}{H_c}, \tag{30} \]
\[ \tau_{\theta} = \frac{1}{R} \frac{\partial}{\partial \theta} \left( \frac{\partial P}{\partial \theta} \right) \frac{H_c}{H} - \frac{H_c}{2R} \frac{1}{\partial \theta} \frac{\partial P}{\partial \theta} + \frac{\omega \eta R \cos \theta}{H_c} \left( \cos \varphi \cos \theta - k \sin 2\theta \cos \varphi \right), \tag{31} \]
where \( \tau_{\varphi} \) is the shear stress component on the surface of the external retainer plate when \( \zeta = 0 \) and \( \tau_{\theta} \) is the shear stress component on the surface of the external spherical hinge when \( \zeta = 1 \).

The dimensionless form of the Reynolds equation of the external return mechanism in the spherical coordinate system is written as
\[ \frac{\partial}{\partial \varphi} \left( H^2 \frac{\partial P}{\partial \varphi} \right) + \frac{\partial}{\partial \theta} \left( H^2 \frac{\partial P}{\partial \theta} \right) = \Lambda \frac{\partial H}{\partial \varphi} - \Lambda'' \left( \frac{\partial H}{\partial \varphi} k + \frac{\partial}{\partial \theta} H \right) \]
\[ + \Lambda' \left( \frac{\partial H}{\partial \theta} \cos \theta + \frac{\partial H}{\partial \theta} k \sin 2\theta \cos \varphi \right), \tag{32} \]
where \( \Lambda = (6\omega R^2 \eta / p_0 c^2) \sin \theta, \Lambda'' = (6\omega R^2 \eta / p_0 c^2) \cos 2\theta, \Lambda' = ((6\omega R^2 \eta \cos \varphi \sin \varphi) / p_0 c^2), \) and \( k = ((\sin^2 \varphi \cos^2 \beta_1 + \cos^2 \varphi) / \cos \beta_1) \).

3.2. Calculation Formula. Load-bearing capacity of the oil film:
\[ \frac{F_\varphi}{P} = \iint_{\varphi} R^2 \sin \theta \, d\varphi. \tag{33} \]
Friction power of the oil film:
\[ \frac{F_\varphi}{P} = \iint_{\varphi} \tau_{\phi} R^2 \sin \theta \, d\varphi. \tag{34} \]
When \( \zeta = 0 \), \( \tau_{\phi} \) is the fluid frictional shear stress of surface of the external retainer plate.

Friction coefficient:
\[ \mu = \frac{F_\varphi}{P}. \tag{35} \]

Axial leakage flow:
\[ \bar{Q}_\theta = 2 \int_0^{2\pi} \int_0^h \tilde{\nu}_{\theta} \text{arccos} \left( \frac{B}{2R} \right) \, d\varphi. \tag{36} \]

Circumferential cyclic flow:
\[ \bar{Q}_\varphi = \int_\pi \int_0^h \tilde{\nu}_{\phi} \, d\varphi, \tag{37} \]
where \( b_1 = b_2 \approx \arcsin (B/2R) \).

3.3. Solution and Analysis of Result. The Reynolds equation of the external return spherical bearing pair in the spherical coordinate system is calculated by the finite difference method (FDM) and the successive overrelaxation method (SOR method). The lubrication area of the external return mechanism is expanded into a rectangle, where \( \varphi \) represents the peripheral direction of the external retainer plate and \( \theta \) represents the width direction of the external retainer plate. Then, we get the difference equations and use the MATLAB programming to solve them.

3.3.1. Basic Parameters. The basic parameters are given in Table 1.

3.3.2. Pressure of the Oil Film. The calculated pressure distribution on the oil film of the external return spherical bearing pair is shown in Figure 6. Pressure was generated at the maximum gap and gradually increased during the transition to the minimum gap of the external retainer plate. A maximum pressure \( P_{\text{max}} = 15.30 \) was formed at \( \theta = 100^\circ, \varphi = 17^\circ \). According to the pressure distribution of the external return spherical bearing pair and equation (30) the distribution of shear stress \( \tau_{\varphi} \) of the oil film of the external return spherical was calculated and is shown in Figure 7. According to the pressure distribution of the external return spherical bearing pair and equation (31), the distribution of shear stress \( \tau_{\theta} \) of the oil film of the external return spherical was calculated and is shown in Figure 8.

3.4. Influence of the Eccentricity on the Lubrication of the External Return Spherical Bearing Pair. The value of the component of eccentricity is shown in Table 2, and other parameters are kept the same as in Section 3.3.1. After analysing the influence of the component of eccentricity on the lubrication property of the external return spherical bearing pair, the calculated results are shown in Figure 9. The oil film pressure of the spherical hinge pair peaked in the range \( 0^\circ \sim 30^\circ \) in the presence of the eccentricity, as shown in Figure 9(a). As the radial eccentricity increases under constant axial eccentricity, the thickness of the oil film bearing decreased. This causes a blocking effect on the lubrication medium, further increasing the pressure until the oil film thickness reaches a minimum critical value.
Table 1: Basic parameters.

| Sign and unit | Physical significance | Value |
|---------------|-----------------------|-------|
| $R$ (mm)      | Radius of the external retainer plate | 55    |
| $c$ (mm)      | Radius clearance       | 0.15  |
| $\eta$ (Pa.s) | Lubricating oil viscosity | 0.0129 |
| $B$ (mm)      | Contact width          | 16.5  |
| $n$ (r/min)   | Pump shaft rotating speed | 1500  |
| $\beta$ ()    | Slant inclination of external swash plate | 10    |
| $\varepsilon_x$ | Radial eccentricity | 0.5   |
| $\varepsilon_y$ | Axial eccentricity | 0.5   |
| $E$           | Convergence criterion  | 0.001 |
| $P_0$ (MPa)   | Environmental pressure | 0.1   |

Table 2: Component of eccentricity.

| Component of eccentricity | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------------------|---|---|---|---|---|---|---|
| $\varepsilon_z$           | 0.5 | 0.5 | 0.5 | 0.5 | 0.2 | 0.5 | 0.8 |
| $\varepsilon_y$           | 0.2 | 0.32 | 0.6 | 0.88 | 0.5 | 0.5 | 0.5 |

However, this blocking effect did not stop the lubricant from getting through the clearance on the rear side of the position $\varphi = 30^\circ$; as such, no obvious trough existed in the pressure curve. Because the direction of the $x$-axis in the spherical coordinate system $o-(x, y, z)$ is opposite to the direction of the $x$-axis in the rectangular coordinate system $o-xyz$, with increases in the axial eccentricity under constant radial eccentricity, the oil film thickness in the oil film bearing area is increasing, and the peak pressure is decreasing. The above laws also are reflected in Figures 9(a) and 9(b).

As the radial and axial eccentricity changes, the axial curve shape of oil film pressure is similar, as shown in Figure 9(b). The increased peak value of the pressure also causes the area enclosed by the pressure distribution curve to increase. Therefore, the average stress of the oil film increases, enhancing the load-bearing capacity. The calculated dimensionless bearing capacity distribution of each group’s eccentricity is shown in Table 3 and confirms further the results of Figure 9(b).

Increasing the pump shaft rotating speed increases the oil film load-bearing capacity, as shown in Figure 9(c). The dynamic pressure lubrication effect increases with increasing rotational speed, which causes the differential pressure of the two ends of the oil wedge to increase and the thickness of the oil film to decrease. This further enhances the load-bearing capacity of the oil film. When the pump shaft rotating speed is constant, increasing the radial eccentricity causes the oil film load-bearing capacity to increase, thus improving the load-bearing capacity of the external return spherical bearing pair. However, the influence of axial eccentricity on the load-bearing capacity is the opposite.

The relationship between the eccentricity and the coefficient of friction is shown in Figure 9(d), where curves 1 and 2 show the effect of the variation of radial and axial eccentricity, respectively. As the radial eccentricity increases from 0.539 to 1.012, the coefficient of friction decreases sharply to 55.8%. The change of the curve 1 has been verified in reference [22]. As the axial eccentricity increases from 0.539 to 0.943, the coefficient of friction increases slowly by 19.9%. Therefore, we can find that although the eccentricity is the same, the influence rule of radial eccentricity and axial eccentricity on the coefficient of friction is the opposite.

3.5. Influence of the Slant Inclination of the External Swash Plate on Lubrication. The components of the eccentricity are selected from group 1, group 2, group 3, group 4, and group 6 in Table 2, the range of the slant inclination of the external swash plate is $10^\circ$~$14^\circ$, and the other parameters are kept the same as given in Section 3.3.1. The calculated results are shown in Figure 10.

Under constant eccentricity, an increased slant inclination of the external swash plate caused the dimensionless
maximum pressure and dimensionless load-bearing capacity of the oil film to linearly increase, as shown in Figures 10(a) and 10(b), respectively. At a constant external swash plate inclination, the maximum pressure and load-bearing capacity increases with increasing radial eccentricity. This is similar to the conclusion in Section 4. And, with the increase in radial eccentricity, the increasing trend of the dimensionless load-bearing capacity is more obvious.

**Table 3: Dimensionless load-bearing capacity.**

| Axial eccentricity $\epsilon_x$ | 0.5 | 0.5 | 0.2 | 0.5 | 0.8 | 0.5 | 0.5 |
|----------------------------------|-----|-----|-----|-----|-----|-----|-----|
| Radial eccentricity $\epsilon_y$ | 0.88 | 0.6 | 0.5 | 0.5 | 0.5 | 0.32 | 0.2 |
| Dimensionless load-bearing capacity | 20.06 | 17.90 | 17.63 | 16.22 | 14.97 | 12.51 | 8.80 |
Figure 10: Influence of the slant inclination of the external swash plate on lubrication. (a) Relationship between the slant inclination of the external swash plate and maximum oil film pressure. (b) Relationship between the slant inclination of the external swash plate and load-bearing capacity. (c) Relationship between the slant inclination of the external swash plate and circumferential flow. (d) Relationship between the slant inclination of the external swash plate and axial flow. (e) Relationship between the slant inclination of the external swash plate and the friction coefficient of the oil film.
The resulting change in dimensionless circumferential flow (circular flow) and dimensionless axial flow (leakage flow) under varying external swash plate inclination is shown in Figures 10(c) and 10(d), respectively. Under constant eccentricity, increasing the slant inclination of the external swash plate inclination caused a continuous increase in the circumferential flow. Meanwhile, the axial flow increased slowly at first, begins to rise sharply from an inclination of 11.8°, and then decreases sharply from an inclination of 12°. Subsequently, the variation curve becomes more complicated. The axial flow increases with increasing radial eccentricity under an external swash plate inclination from 11.8° to 12.2° but decreases with increasing radial velocity under an external swash plate inclination from 12.2° to 14°.

Under constant eccentricity, the coefficient of friction of the lubricant film increases with increasing external swash plate inclination of the axial piston pump, as shown in Figure 10(e). However, under constant external swash plate inclination, a lower radial eccentricity leads to a greater coefficient of friction. When the radial eccentricity decreases from 0.88 to 0.6 at an external swash plate inclination of \( \beta = 12° \), the coefficient of friction increased 13.1% from 0.0122 to 0.0138; when the radial eccentricity decreased from 0.6 to 0.32, the coefficient of friction increased 40.6% from 0.0138 to 0.0194. Thus, the lesser the radial eccentricity, the more significant influence on the coefficient of friction. This is consistent with the rule of curve 1 in the Figure 9(d).

3.6. Influence of the Radius Clearance of the External Retainer Plate on Lubrication Properties. Allowing parameters \( \varepsilon_x = 0.5 \) and \( \varepsilon_y = 0.5 \), slant inclination of the external swash plate \( \beta = 12° \). Without changing the radius \((R = 55\text{ mm})\) of the external swash plate, the radius clearance \( c \) is set as 0.05 mm, 0.10 mm, 0.15 mm, 0.20 mm, and 0.25 mm, respectively. The other parameters are kept the same as in Section 4.2.1, and the calculated results are shown in Figure 11.

Under constant pump shaft rotational speed, by reducing the radius clearance \( c \) of the external retainer plate, the hydrodynamic lubrication effect enhances and the pressure and the load-bearing capacity of the oil film increases, as shown in Figures 11(a) and 11(b). Under a constant radius clearance, increasing the pump shaft rotating speed enhances the hydrodynamic lubrication effect and the oil film pressure and load-bearing capacity increases. A smaller radius clearance leads to a more significant influence on the maximum pressure and load-bearing capacity of the oil film. Various dimensionless performance parameters of the oil film under varying pump shaft rotating speed are shown in Table 4. \( P_{\text{max}} \) and \( W \) increase 125% by decreasing \( c = 0.15 \) to \( c = 0.10 \) and increase 300% by decreasing \( c = 0.10 \) to \( c = 0.05 \) under a constant pump shaft rotating speed of 500 r/min.

Increasing the pump shaft rotating speed causes the radius clearance to have no obvious influence on the oil film pressure and load-bearing capacity. At a pump shaft rotating speed of 1000 r/min, the load-bearing capacity increases 298.5% by decreasing \( c \) from 0.10 to 0.05 and decreases 125.2% by increasing \( c \) from 0.10 to 0.15. At a pump shaft rotating speed of 3000 r/min, the load-bearing capacity increases 300% by decreasing \( c \) from 0.10 to 0.05 and decreases 128.2% by increasing \( c \) to 0.15.

At constant radius clearance, the circumferential flow \( Q_p \) increases with increasing pump shaft rotational speed, as shown in Figure 11(c). Similarly, at constant pump shaft rotational speed, the circumferential flow \( Q_p \) increases with increasing radius clearance. At a pump shaft rotational speed of \( n = 500 \text{ r/min} \), the circumferential flow increases 66.7% by increasing \( c \) from 0.15 to 0.25 and the circumferential flow under \( c = 0.15 \) increases by 2 times compared with the circumferential flow under \( c = 0.05 \). At a pump shaft rotational speed of \( n = 3000 \text{ r/min} \), the circumferential flow under \( c = 0.25 \) increases by 66.7% compared with the circumferential flow under \( c = 0.15 \), and the circumferential flow under \( c = 0.15 \) increases by 2.03 times compared with the circumferential flow under \( c = 0.05 \). Thus, at constant pump shaft rotational speed, the degree of the increase of circumferential flow is weakened with increasing radius clearance.

Under constant radius clearance, the axial flow fluctuates more with increased pump shaft rotating speed, as shown in Figure 11(d). With reducing the radius clearance, the variation trend of the axial flow becomes more complicated and the number of peak values increases. When \( c = 0.25 \), only one peak is shown. When \( c = 0.15 \), two peaks are seen; at \( c = 0.05 \), three peak values exist. Overall, the results indicate that smaller radius clearances cause a greater axial flow of the external return spherical bearing pair at high pump shaft rotational speed.

4. Discussion

The above analysis indicates that the radial eccentricity, external swash plate inclination, and pump shaft rotational speed can enhance the load-bearing capacity of the oil film and the hydrodynamic effect, and thus can be beneficial to avoid solid contact of the friction pair. Increasing the radial eccentricity and external swash plate inclination results in an increase of wedge angle of the friction of the external return mechanism, which is conducive to the generation of the hydrodynamic effect. Since the overall dimensions of the external return mechanism are larger than the traditional internal return mechanism, the circumferential length of the wedge shape that forms the hydrodynamic effect increases greatly (as show in Figure 12). This then increases the support force of the oil film between the friction pair of the external return mechanism and reduces the probability of solid friction. This lays the foundation of the good lubrication of the external return mechanism. However, as the load-bearing capacity of the oil film becomes stronger, the friction coefficient of the oil film is also increased. This in turn increases the heat produced by friction and further raises the temperature of the oil film.

Since the external return mechanism is in a pump or motor chamber filled with oil, a lack of lubrication oil between the friction pair would not arise. Increasing the circumferential and axial flow assists in exchanging the lubrication oil between the external return mechanism and
Figure 11: Influence of the radius clearance of the external retainer plate on lubrication properties (a) Relationship between the radius clearance and maximum oil film pressure. (b) Relationship between the radius clearance and load-bearing capacity. (c) Relationship between the radius clearance and circumferential flow. (d) Relationship between the radius clearance and axial flow.

Table 4: Dimensionless data of the oil film under different pump shaft rotating speed.

| n (r/min) | c (mm) | Dimensionless maximum pressure $\bar{P}_{\text{max}}$ | Dimensionless load-bearing capacity $\bar{W}$ |
|-----------|--------|-------------------------------------------------|-------------------------------------------|
| 500       | 0.15   | 0.1631                                          | 5.41                                      |
| 500       | 0.10   | 0.367                                          | 12.17                                     |
| 500       | 0.05   | 1.468                                          | 48.67                                     |
| 1000      | 0.15   | 0.3262                                         | 10.81                                     |
| 1000      | 0.10   | 0.7341                                         | 24.34                                     |
| 1000      | 0.05   | 2.936                                          | 97                                        |
| 3000      | 0.15   | 0.9787                                         | 32.45                                     |
| 3000      | 0.10   | 2.202                                          | 73.01                                     |
| 3000      | 0.05   | 8.809                                          | 292                                       |
reducing the heat of friction between the friction pair. The axial flow reaches a maximum at an external swash plate inclination of 12°, which is conducive to releasing built-up heat and steady operation of the friction pair. Overall, reducing the radial eccentricity and the radius clearance and increasing the inclination of the external swash plate are beneficial to increase the circumferential flow and to the steady operation of the friction pair. However, under the smaller radius clearance, increasing the pump shaft rotational speed causes more peaks in the axial flow. At high pump shaft rotational speed, a smaller radius clearance leads to a larger axial flow of the spherical hinge pair.

5. Conclusions

The paper combines with the kinematic characteristics of the external return mechanism and deduces the Reynolds equation applying to the external return spherical bearing pair of the external return mechanism in the spherical coordinate. The Reynolds equation is solved with the finite difference method. The influence of radius clearance, external swash plate inclination, pump shaft rotating speed, and component of eccentricity on the lubrication property was analysed. The following conclusions were drawn:

(1) Increasing the radial eccentricity, external swash plate inclination, and pump shaft rotating speed can improve the hydrodynamic effect of the oil film and further enhance the load-bearing capacity of the friction pair of the external return mechanism. However, reducing the axial eccentricity and the radius clearance decreases the maximum pressure and load-bearing capacity of the friction pair. When the radius clearance reduces to approximately 0.05 mm, the maximum pressure and load-bearing capacity of the friction pair increase sharply.

(2) Increasing the axial eccentricity and reducing the radial eccentricity can decrease the friction coefficient of the friction pair. The lower the radial eccentricity, the more significant influence of it on the coefficient of friction. An increased external swash plate inclination increases the area of local friction and the contact area of the fluid and the wall, which further increases the coefficient of friction.

(3) Reducing the radial eccentricity and radius clearance and increasing the pump shaft rotational speed and external swash plate inclination can increase the circumferential flow. Axial flow variation depends on the radius clearance, radial eccentricity, external swash plate inclination, pump shaft rotational speed, and other parameters. Increasing the external swash plate inclination alone can cause the axial flow to vary widely. At an external swash plate inclination of approximately 12°, the axial flow increases with increasing the radial eccentricity. Furthermore, under the smaller radius clearance, increasing the pump shaft rotational speed causes more peaks in the axial flow.

Abbreviations

- \( R \): Radius of the external spherical hinge and the external retainer plate (mm)
- \( r \): Distance from an arbitrary point of the contact oil film between the external spherical hinge and the external retainer plate to the centre of the external spherical hinge (mm)
- \( \zeta \): Distance from the point to the spherical outer surface of the external retainer plate (mm)
- \( dW \): A small fluid element in the contact lubrication area
- \( p \): Lubricating oil film pressure (MPa)
- \( \mu \): Friction coefficient
- \( \tau_\phi \): Fluid shear stress component in the \( \phi \) direction (MPa)
- \( \tau_\theta \): Fluid shear stress component in the \( \theta \) direction (MPa)
- \( u_\theta \): Velocity component of a point in the lubricant oil film along the \( \theta \) direction (m/s)
- \( u_\phi \): Velocity component of a point in the lubricant oil film along the \( \phi \) direction (m/s)
- \( \eta \): Lubricating oil viscosity (Pa·s)
- \( \beta \): Slant inclination of the external swash plate (°)
- \( \beta_1 \): Rotation angle (°)
- \( B \): Contact width (mm)
- \( n \): Pump shaft rotating speed (r/min)
- \( c \): Oil film clearance (mm)
- \( \varepsilon \): Eccentricity
- \( \varepsilon_x \): Axial eccentricity
- \( \varepsilon_y \): Radial eccentricity

Figure 12: Structure of the internal and external return mechanism.
$E$: Convergence criterion
$P_0$: Environmental pressure (MPa)
$P_{\text{max}}$: Maximum pressure (MPa)
$\dot{W}$: Dimensionless load-bearing capacity (kN)
$q_{\nu}$: Circular flow and equation
$q_{\alpha}$: Axial flow and equation
$q_{\gamma}$: Dimensionless circumferential flow (circular flow)
$q_{\nu}$: Dimensionless axial flow (leakage flow)
$\tau_{\phi_1}$: Shear stress components of point $\phi = 0$ on the surface of the external retainer plate (MPa)
$\tau_{\phi_0}$: Instantaneous angular velocity of the external return plate (rad/s)
$\omega_\theta$: Angular velocity of the external spherical hinge (rad/s)
$\omega_\phi$: Angular velocity of the external spherical hinge and retainer plate
$\rho$: The constant fluid (oil) density (kg · m$^{-2}$).

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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