INTRODUCTION

Gas drilling technologies can increase the drilling speed, avoid loss circulation, and overcome formation water sensitivity.\(^1,2\) Because the gas is less dense than mud, the borehole pressure is lower as the gas flows through the wellbore. The gas pressure is not sufficient to support the borehole rock and keep the borehole stable without deformation. After the borehole is drilled, its geometrical shape will inevitably deform under the original geostress.\(^3\) In the initial stage of gas drilling engineering design, the formation properties and the stability of the borehole...
wall should be evaluated first to determine the applicability of gas drilling. The stress state of the rock around the borehole during gas drilling determines the stability of the wellbore. When examining borehole stability during gas drilling, most researchers analyzed borehole stability under mud drilling conditions.\cite{1,4} According to the mud drilling borehole stability analysis, the stress state value of the surrounding rock under the elastic limit state of a borehole wall was obtained\cite{5,6} for the evaluation of the wellbore collapse pressure. This method usually assumes that the rock completely loses its bearing capacity after reaching its peak strength, and it compares the actual stress borne by the borehole with the strength of the rock wall.\cite{7,8} However, engineering practice has shown that even if the rock around the well breaks, it can still maintain a certain bearing capacity. Thus, the rock not only bears the load before failure but also has a certain bearing capacity after failure. All the yield and deformation characteristics of rock can be obtained from the complete stress-strain test curve of that rock. Because the complete stress-strain model of rock is very complex, it is difficult to analyze engineering problems. In this research, the rock’s stress-strain model was simplified into three linear stages: elastic deformation, plastic softening deformation, and broken deformation stages.\cite{9} Combined with the Tresca yield criterion, the simplified stress-strain model can be used to conveniently analyze the stress state of the rock surrounding a borehole.\cite{10-12} Meanwhile, because of the compressibility of the gas, the changes in the temperature and pressure of the gas flow in the borehole are more complicated than liquid.\cite{13-16} Specifically, the Joule-Thomson effect occurs when gas flows through the bit nozzle, where it is cooled by throttling, creating a low temperature zone at the bottom of the hole. An analytical thermal model with high precision has derived for predicting bottom hole gas temperature in gas drilling.\cite{17-19} Near the bottom of the wellbore, thermal stress in the form of shrinkage is generated by the low temperature, while thermal stress in the form of expansion is generated in the upper wellbore section, which has a certain influence on the stress state of the wellbore rock.\cite{20} Because the wellbore pressure is very low for gas drilling, it is no longer possible to use the collapse pressure to evaluate the wellbore stability. Therefore, this paper introduces the plastic softening radius and crushing radius of the surrounding rock to evaluate the wellbore stability for gas drilling.

2 | Mathematical Model

Several assumptions were made during the development of the mathematical model:

- The wellbore is vertical.
- During drilling, the stress distribution in the wellbore is a plane strain problem.
- The formation is isotropic, impermeable, and homogeneous.
- Rocks follow Hooke’s law in the stage of elastic deformation and meet the Tresca yield criterion in the process of plastic softening deformation and plastic broken deformation.
- Heat is transferred in the annulus fluid on the surface of the borehole by axial convection.
- Heat is transferred in the rock formation by transient radial conduction.
- The direction of the heat transfer is axisymmetric.

2.1 | In situ stress model for rock

2.1.1 | Criterion of stress intensity

The curve of the complete stress-strain relation for rock (Figure 1) was simplified into a three-phase mechanical model, with elastic deformation, plastic softening deformation, and broken deformation stages.\cite{21,22} When the stress is less than the peak strength, the rock is in an initial elastic state, which covers a large range of strain values, and then, it yields gradually. The stress-strain relationship can be simulated using linear elastic or nonlinear elastic relationships. After reaching the peak value, the strain softening phenomenon occurs, during which the stress decreases with increasing deformation. When the stress reaches the residual strength, the rock exhibits complete plasticity and enters the broken stage. In the strain softening range, the inner product of a stress increment and a strain increment is negative, and multiple values result from numerical calculations. This situation has been noted by Prevost and Hughes.\cite{23} To reflect the softening characteristics of rock, a piecewise linear function was used to simulate the stress-strain relationship, as illustrated in

![Figure 1](image-url)
Figure 2. Before yield, the elastic stress-strain relationship is linear elastic. After reaching the peak value, the strain softening phenomenon occurs, during which the stress decreases with increasing deformation. Finally, the rock exhibits complete plasticity when the stress reaches the residual strength.

The Tresca yield criterion is presented in Equation (1):

$$F = \sigma_\theta - \sigma_r - 2s = 0,$$

where $\sigma_\theta$ and $\sigma_r$ are the tangential and radial stresses, respectively, in MPa, and $s$ is the constant of the Tresca material.

From Figure 2, the constant in the Tresca yield criterion can be written as shown in Equation (2):

$$s = \begin{cases} \frac{\sigma_p}{\omega_p} \quad (\omega < \omega_p) \\ \frac{\sigma_p - \sigma_R}{\omega_R - \omega_p} (\omega_p \leq \omega < \omega_R) \\ \sigma_R \quad (\omega_R \leq \omega) \end{cases},$$

where $\sigma_p$, $\sigma_R$, $\omega_p$, and $\omega_R$ are the peak and radial shear stresses and strains, respectively, in %.

### 2.1.2 Control equations

A cylindrical coordinate system was established by setting the axis of the borehole as the $z$-axis. The borehole radius is represented by $R_w$, the airflow pressure in the borehole is denoted as $p_i$, and the original in situ stress is represented by $\sigma_o$. The rock around the borehole is composed of three parts, as shown in Figure 3: the plastic broken zone (with a radius of $R_b$), the plastic softening zone (with a radius of $R_p$), and the elastic zone.

**The equilibrium equation**

The mechanical analysis for a vertical borehole uses the axisymmetric condition. The stress in each zone satisfies the equilibrium equation given as Equation (3):

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0.$$

**Geometric equations**

According to the second hypothesis, the analysis of a borehole is a plane strain problem. The longitudinal strain is not considered. Therefore, its geometric equations can be expressed by Equation (4):

$$\begin{align*}
\varepsilon_r &= -\frac{du}{dr}, \\
\varepsilon_\theta &= -\frac{u_r}{r}.
\end{align*}$$

**The elastic constitutive equation**

For a plane strain problem, the strains are calculated by a generalized form of Hooke’s law as presented in Equation (5):

$$\begin{align*}
\varepsilon_r &= \frac{(1+\nu)}{E} \left[\sigma_r - \nu (\sigma_r + \sigma_\theta)\right], \\
\varepsilon_\theta &= \frac{(1+\nu)}{E} \left[\sigma_\theta - \nu (\sigma_r + \sigma_\theta)\right].
\end{align*}$$

**Boundary conditions**

The radial stress at the borehole wall is the fluid pressure in the borehole. When the borehole thermal stress is considered, the radial stress boundary condition is superimposed on the thermal stress term, as in Equation (6):

$$\sigma_{r=R_w} = p_i.$$

### 2.1.3 Stresses around the borehole without considering thermal stresses

**Stresses and displacements within the elastic zone**

By solving the elastic symmetry problem and applying the boundary conditions, the stress field of the rock around the wellbore in the elastic state could be obtained:

$$\begin{align*}
\sigma_r &= \sigma_0 - \sigma_p \left(\frac{R_p}{r}\right)^2, \\
\sigma_\theta &= \sigma_0 + \sigma_p \left(\frac{R_p}{r}\right)^2.
\end{align*}$$
At the boundary between the softening and elastic zones, and when the substitution \( r = R_p \) is made in Equation (7), the radial stress can be expressed by Equation (8):

\[
\sigma_{rr=R_p} = \sigma_0 - \sigma_p. \tag{8}
\]

The two formulas in Equation (5) were subtracted from each other, and when the result was combined with the first phase of Equation (1), Equation (9) was obtained:

\[
\frac{\sigma_p}{\omega_p} = \frac{E}{1 + \nu}. \tag{9}
\]

The radial displacement, \( u \), was then derived by combining Equations (4), (5), (7), and (9):

\[
u = -\frac{\omega_p (1 - 2\nu) \sigma_0}{\sigma_p} r - \frac{R_p^2}{r}. \tag{10}
\]

For the far-field in situ stresses, \( \sigma_\theta = \sigma_r = \sigma_p \). The initial displacement due to the in situ stresses was obtained by combining Equations (4) and (5):

\[
u_i = -\frac{\omega_p (1 - 2\nu) \sigma_0}{\sigma_p} r. \tag{11}
\]
The incremental radial displacement, \( u_{\text{ex}} \), resulting from the excavation could then be acquired by subtracting Equation (11) from Equation (10):

\[
u_{ex} = -\frac{R_p^2}{r}.
\] (12)

The radial displacement at the junction of the elastic and plastic zones can be calculated using Equation (13):

\[
u_{ex,p} = -\omega_p R_p.
\] (13)

**Stresses and displacements within the plastic softening zone**

The strains in the plastic softening and plastic broken zones include the plastic strains and the volume change characteristics of the rock. It was assumed that the rock experiences a constant volume increase, independent of the strains, throughout the plastic zone.\(^{26,27}\) Figure 4 illustrates how the volume changes incrementally. (a) The blue area represents the initial plastic deformation at the radius \( R_p \). The green area represents the initial plastic deformation at the radius \( r \). (b) The blue area illustrates the postplastic deformation and the decrease in radius, \( u_{\text{ex},p} \). The green area shows the initial plastic deformation and the radial decrease, \( u_{\text{ex},r} \).

The incremental change in volume can be expressed by Equation (14):

\[
\Delta V = \epsilon_V V = \epsilon_V R_p^2 (R_p^2 - r^2) = \epsilon_V \left[ \left( R_p^2 - u_{\text{ex},p} \right)^2 - (r - u_{\text{ex}})^2 \right] = \frac{R_p^2}{r} - \frac{\epsilon_V}{2} R_p^2.
\] (14)

The small term of high order was ignored when expanding Equation (14), and an expression for the plastic displacement was obtained by combining Equations (13) and (14):

\[
u_{ex,r} = u_{\text{ex},p} \frac{R_p}{r} + \frac{\Delta V}{2\pi r} = -a_1 \frac{R_p^2}{r} - \frac{\epsilon_V R_p^2}{2},
\] (15)

where

\[
a_1 = \omega_p - \frac{\epsilon_V}{2}.
\]

The displacement at the wall of borehole can be expressed using Equation (16):

\[
u_{\text{ex},w} = -a_1 \frac{R_p^2}{R_w^2} - \frac{\epsilon_V R_w}{2}.
\] (16)

Expressions for the radial and tangential strains in the plastic zone were obtained by combining Equations (19) and (7) into Equation (17):

\[
\begin{align*}
\epsilon_r &= \frac{\epsilon_V}{2} \frac{R_p^2}{r^2} - a_1 \frac{R_p}{r^2} - \frac{\epsilon_V}{2}, \\
\epsilon_\theta &= a_1 \frac{R_p^2}{r^2} - \frac{\epsilon_V}{2}.
\end{align*}
\] (17)

Using Equations (17) and (2), the shear strain in the plastic zone could be derived:

\[
\omega = \frac{R_p^2}{r^2}.
\] (18)

By substituting Equation (18) and the second portion of Equation (2) into Equation (3) and integrating, and after determining the integral constant from Equation (8), the radial stress within the plastic softening zone was obtained:

\[
\sigma_r = \sigma_0 - \sigma_p + a_2 \ln \left( \frac{R_p}{r} \right)^2 + a_1 a_3 \left( \left( \frac{R_p}{r} \right)^2 - 1 \right).
\] (19)

where

\[
a_2 = \frac{\omega_p \sigma_R - \omega_R \sigma_p}{\omega_R - \omega_p} \quad \text{and} \quad a_3 = \frac{\sigma_p - \sigma_R}{\omega_R - \omega_p}.
\]

An expression for the tangential stress could then be derived by substituting Equation (19) into the second portion of Equation (1):

\[
\sigma_\theta = \sigma_0 + \sigma_p + a_1 \ln \left( \frac{R_p}{r} \right)^2 - a_1 a_3 \left[ \left( \frac{R_p}{r} \right)^2 - 1 \right].
\] (20)

**Stresses and displacements within the plastic broken zone**

The deformation in the broken process is similar to the deformation in the plastic softening process, and the displacement expression is similar to that in Equation (16). By combining the third formula of Equation (1) and Equation (3) and integrating, and after determining the integral constant by using Equation (6), the radial stress within the plastic broken zone was obtained:

\[
\sigma_r = p_i + \sigma_R \ln \left( \frac{r}{R_w + u_w} \right)^2.
\] (21)

An expression for the tangential stress was acquired by using the third term of Equation (1):

\[
\sigma_\theta = p_i + 2\sigma_R + \sigma_R \ln \left( \frac{r}{R_w + u_w} \right)^2.
\] (22)
2.2 Thermal stress around the borehole

When the borehole wall contacts the flowing gas, it is cooled down rapidly, and the temperature around the borehole changes. The temperature difference between the borehole surface and the rock away from the borehole causes an additional thermal stress field.

Expressions for the thermal stresses around the borehole could be obtained from the heat conduction theory and thermoplastics mechanics and are presented in Equations (23) and (24).

\[
\sigma_{rT} = \frac{aE}{(1-v)r^2} \int_{Rw}^{r} T_f (r', t) r' dr' - \alpha E T_w \left( \frac{R_w}{r} \right)^2; \quad (23)
\]

\[
\sigma_{\theta T} = \frac{aE}{(1-v)r^2} \int_{Rw}^{r} T_f (r', t) r' dr' - T_f (r', t) + \alpha E T_w \left( \frac{R_w}{r} \right)^2. \quad (24)
\]

In Equations (23) and (24), \( \sigma_{rT} \) and \( \sigma_{\theta T} \) are the thermal stresses in the radial and tangential directions, respectively, in MPa. \( \alpha \) is the thermal expansion coefficient of the formation volume, in \( 1/°C \). The expression \( T_f(r, t) = T(r, t) - T_g, T_w = T(R_w, t) - T(R_w, 0) \) is the distribution function of the formation temperature change around the borehole, and the gas temperature at the borehole wall is represented by \( T(R_w, t) \). \( T(R_w, 0) \) is the initial formation temperature, in °C.

2.3 The radii of the plastic and broken zones

The rock around the wellbore experiences in situ stress and thermal stress caused by the temperature difference between the airflow and the wellbore wall. Therefore, the thermal stresses can be combined with the in situ stresses of the wellbore into comprehensive stresses for the wellbore rock when analyzing the wellbore stability. The comprehensive stresses can be expressed using Equations (25) and (26):

\[
\sigma_{r,\text{total}} = \sigma_{r,\text{in-situ}} + \sigma_{r,T}; \quad (25)
\]

\[
\sigma_{\theta,\text{total}} = \sigma_{\theta,\text{in-situ}} + \sigma_{\theta,T}. \quad (26)
\]

In Equations (25) and (26), \( \sigma_{r,\text{total}} \) and \( \sigma_{\theta,\text{total}} \) are the comprehensive radial and tangential stresses, respectively, in MPa, and \( \sigma_{r,\text{in-situ}} \) and \( \sigma_{\theta,\text{in-situ}} \) are the in situ radial and tangential stresses, respectively, in MPa.

When analyzing wellbore stability, the wellbore stress boundary condition from Equation (6) can be rewritten as Equation (27):

\[
\sigma_{rr=R_w} = p_i + \sigma_{r,T}. \quad (27)
\]

In gas drilling, the gas pressure in the borehole is much lower than the fluid pressure of mud drilling. The rock wall is supported very little by the airflow pressure. The rock around the borehole shifts from the initial elastic stage to the plastic stage and finally enters the broken zone near the shaft wall. Then, the broken zone, the plastic softening zone, and the elastic zone appear successively from the inside to the outside of the borehole.

If the rock enters the broken zone at the junction of the broken and plastic softening zones, then \( r = R_b \) and \( \omega = \omega_R \). The critical parameter can be obtained by substituting these conditions into Equation (18):

\[
\gamma = \frac{R_p}{R_b} = \sqrt{\frac{\omega_R}{\omega_i}}, \quad (28)
\]

where

\[
\omega_i = \frac{\omega_R}{\omega_i}. \]

According to Equation (28), the mechanical state of the rock around the borehole can be divided into two situations. The first is that the rock is in the broken zone, and the second is that the rock is in the plastic softening zone and the broken zone is not present.

Radii of the plastic softening to broken zones

If \( \gamma \geq 1 \), there is a broken area around the wellbore. For this case, the radius of the broken zone at the boundary condition in Equation (27) was obtained by combining Equations (19), (21), (16), and (18):

\[
R_b \left[ 1 - a_i \left( \frac{R_p}{R_b} \right)^2 - \frac{e^2}{2} \right] = \frac{\sigma_0 - \sigma_p - p_i - \alpha E T_w + a_1 \ln a_4 + a_2 \omega_R - a_3 a_4}{2 \sigma_R}; \quad (29)
\]

\[
R_p = R_b a_4^{1/4}. \quad (30)
\]

For \( R_b/R_w \geq 1 \), an iterative method was adopted to solve Equation (29) and obtain the radius of the broken zone, \( R_b \). Then, the radius of plastic softening zone, \( R_p \), could be calculated using Equation (30).
The radius of the plastic softening zone

If \( \gamma < 1 \), there is no broken zone around the wellbore, and the radius of the broken zone can be considered to be the borehole radius. An expression for the radius of the plastic softening zone at the boundary condition in Equation (27) could be obtained by combining Equations (19), (16), and (18):

\[
\sigma_0 - \sigma_p - p_1 - aE \frac{T_w}{w} - 2\ln a_5 + a_1 a_3 \left( a_5^2 - 1 \right) = 0, \quad (31)
\]

where

\[
a_5 = \frac{R_p}{R_w} \frac{1 - \frac{r_w}{R_w^2} - a_1 \left( \frac{R_p}{R_w} \right)^2}{1 - \frac{r_w}{R_w}}.
\]

For \( R_p / R_w \geq 1 \), the same iterative method as above was used to solve Equation (31) for the radius of the broken zone, \( R_p \).

### 3 | MODEL VALIDATION

During gas drilling, the lower borehole pressure can offer little support to the borehole wall. Under the influence of the bit nozzle throttling cooling action, that is, the Joule-Thomson effect, the borehole temperature, especially near the bottom of the borehole, is lower than the original formation temperature. This effect causes the rock in the borehole wall to generate thermal stress due to the temperature difference, and the distribution of the stresses in the borehole wall is affected. The elastoplastic coupling model proposed in Section 2 could be validated by comparing its results with measured borehole radius values. If the calculated plastic softening radius of the rock around the well is close to the measured radius, the accuracy of the model can be indirectly verified.

Well XJHB021, in an oilfield in northwest China, was drilled using air to a depth of 1000-2200 m. The data measured at this gas-drilled well could be used to test and validate the model. The construction and thermodynamic parameters of the well are listed in Table 1, and the measured rock strength parameters are shown in Figure 5. The measured results show that the formation samples had remarkable plastic softening properties. The red dotted line represents the radial expansion strain, while the blue solid line represents the axial compression strain.

According to the borehole parameters, the construction parameters, and the formation rock strength parameters of well XJHB021, the radius of the plastic softening zone of the borehole in the gas drilling construction section was calculated using the thermal elastoplastic coupling model. The results are shown in Figure 6. The figure shows that the calculated plastic softening zone radius is in good agreement with the measured borehole radius, indicating that the proposed thermal elastoplastic coupling model is valid.

### 4 | RESULTS AND DISCUSSION

#### 4.1 | Analysis of simulation results

To calculate the thermal stress of the rock around the wellbore wall, the temperature distribution of the fluid in the wellbore must be known. According to the heat transfer calculation model for gas drilling of a wellbore, the wellbore pressure and temperature distributions for well XJHB021 were calculated, and the results are shown in Figure 7. The gas flow pressure in the annulus was within 1 MPa. The gas temperature difference between the annulus and the formation reached 17.5°C. At approximately 1600 m, the temperature was equal to the formation temperature; then, the gas temperature in the annulus was

| Parameters                          | Value   | Parameters                          | Value   |
|-------------------------------------|---------|-------------------------------------|---------|
| Upper hole diameter, mm            | 311.2   | Thermal expansion coefficient of    | 2.36    |
|                                     |         | formation, \( 10^{-5} / \text{°C} \)|         |
| Casing O.D, mm                      | 244.5   | Maximum horizontal geostress,       | 2.23    |
|                                     |         | MPA/100 m                           |         |
| Casing setting depth, m             | 1000    | Minimum horizontal geostress,       | 1.82    |
|                                     |         | MPA/100 m                           |         |
| Pipe O.D, mm                        | 127     | Overburden pressure, MPA/100 m      | 2.24    |
| Collar O.D, mm                      | 158.25  | Effective stress coefficient        | 0.4     |
| Collar length, m                    | 300     | Geothermal gradient, °C/m           | 0.0226  |
| Bit diameter, mm                    | 215.9   | Gas injection rate, m³/min          | 150-180 |
| Density of rock, kg/m³              | 2540    | Penetration rate, m/h               | 6.4     |

**TABLE 1** Construction parameters and thermodynamic parameters of well XJHB021
slightly higher than the formation temperature until it flowed upward to the wellhead.

The gas flow pressure in the annulus was very low during the gas drilling, and the pressure in the upper well section was within 1 MPa. The gas temperature in the annulus during the gas drilling decreased by 17.5°C from the original formation temperature near the bottom hole because of the Joule-Thomson effect that occurred at the bit nozzle. As the gas flowed upward, it was heated gradually by the formation. When the gas flowed up to a depth of approximately 1600 m, the temperature was equal to the formation temperature, and then, the gas temperature in the annulus was slightly higher than the formation temperature until it flowed upward to the wellhead. This means that in gas drilling, the rock of the borehole wall experiences a cooling process before gradually reaching the original formation temperature and finally becoming slightly higher than the original formation temperature.

The plastic softening zone radius and the broken zone radius were calculated according to the thermal elastoplastic coupling model, and the results are shown in Figure 8. The radius of the plastic softening zone was much larger than that of the broken zone, and the serious location, which is at a depth of about 1400 m, of the plastic softening zone was concentrated primarily at the upper portion of the gas drilling well. This occurrence can be explained by the temperature distribution of the well shown in Figure 7. At the bottom of the upper well, the wellbore
wall was being heated by the high-temperature flowing gas. The rock around the wall of the borehole expanded when heated, which also reduced the strength of the rock and accelerated the plastic softening degree of the rock. This caused the phenomenon of the borehole wall instability. In the actual field drilling process for well XJHB021, when drilling to about 2200 m, the borehole wall instability in the upper well section caused blocks to drop, leading to a stuck pipe accident, and finally, the gas drilling was forced to stop. The stuck pipe accident is another way to confirm the calculation results.

The mechanical influence of in situ stress on the rock around the wellbore is the primary factor in wellbore stability analyses. Due to the significant gas throttling and cooling process caused by the bit nozzle during gas drilling, a sudden temperature drop occurs at the nozzle outlet, so that the gas will cool the wellbore wall rocks within a certain distance at the bottom of the well. The resulting thermal stress acts on the wall rock. The factors that influence this thermal stress were considered in this model. To compare these factors without considering the thermal stress, the thermal stress term in the thermal elastoplastic coupling model was removed, and the plastic softening zone and the broken zone of the rock around the borehole were analyzed. These calculation results are shown in Figure 9. The plastic softening zone radius and the crushing zone radius of well XJHB021 were calculated using the thermal elastoplastic coupling model with the thermal stress portion removed. The radii of the plastic softening and crushing zones increased with an increase
in well depth, and the distance between the plastic softening zone radius and the crushing zone radius continued to grow.

4.2 | Sensitivity analysis

Figure 9 shows that the radius of the plastic zone of the surrounding rock for the gas drilling was smaller at the top and larger at the bottom when the thermal stress portion of the model was not considered, and the radius of the plastic zone and that of the residual zone increased rapidly between 1500 and 2200 m. However, a comparison between Figure 9 and Figure 6 indicated that the calculated results were not consistent with the actual situation when the thermal stress was not considered. On the contrary, the plastic radius of the borehole in Figure 8 had a high consistency with that in Figure 6. This indicates that when the thermal stress was accounted for, the thermal elastoplastic model for the rock produced results that were more consistent with the field measurements.

The gas temperature near the bottom of the borehole in Figure 7 was well below the original formation temperature. The thermal stress caused by the temperature difference shrank the rock in the borehole and offset the expansion of the rock caused by the original stress. In the upper part of the borehole, the rock expanded under the thermal stress because the borehole temperature was higher than the formation temperature. Under the original stress of the borehole, the thermal stress enhanced the rock expansion. Therefore, the shrunken thermal stress at the bottom of the wellbore could enhance the stability of the wellbore, while the expansive thermal stress at the upper section of the wellbore could aggravate the instability of the wellbore.

When the rock mass of the borehole was in the elastic stage, its stability was very strong. While the rock was in the plastic stage, it was in the transition stage from stable to unstable. If the rock mass of the borehole was in the residual area, it would enter the instability stage. In the process of gas drilling, the pressure of the gas flow in the drilling hole was very low and the supporting force on the wall was very weak. The rock was primarily in the stress state affected by the original in situ stress, that is, without thermal stress, in the earth. In addition, the high speed of the gas drilling caused more serious scouring of the wellbore wall than for mud drilling, and the radius of the borehole further expanded, and eventually, the rock in the borehole reached a stable state.

5 | CONCLUSIONS

1. The complete stress-strain process of rock contains three stages: the elastic, plastic softening, and residual deformation stages, which can be simplified into three linear stages. Before the peak strength, the rock is in an initial elastic state that covers a large range of strain values, and then, it yields gradually. After reaching the peak value, the strain softening phenomenon occurs, and the stress decreases with increasing deformation. When the stress reaches the residual strength, the rock exhibits complete plasticity and is in the broken stage. The deformation of the rock around the borehole could be divided into an elastic zone, a plastic softening zone, and a broken zone.
2. According to the Tresca yield criterion, an elastoplastic model for borehole stability was developed. A thermal elastoplastic coupling model, which combined the thermal stress with the elastoplastic model, was proposed to calculate the radii of the plastic softening and broken zones. The results calculated for model validation showed that when the thermal stress was accounted for, the thermal elastoplastic model for the rock was more consistent with the field measurements.

3. During gas drilling, the Joule-Thomson effect occurred at the bit nozzle, resulting in a different temperature distribution in the borehole than the original formation temperature. Thermal stress in the borehole wall occurred because of the temperature difference. The shrunken thermal stress at the bottom of the borehole could enhance the stability of the borehole wall, while the expansive thermal stress at the top could decrease the stability of the borehole. So the Joule-Thomson effect is conducive to improving the efficiency of rock breaking by using air hammer in gas drilling. At the same time, the Joule-Thomson effect on borehole wall rock enhances borehole stability.

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NOMENCLATURE

\( s \) constant of Tresca material, MPa  
\( p_i \) fluid pressure in the borehole, MPa  
\( R_b \) broken zone radius, m  
\( R_p \) plastic softening zone radius, m  
\( r \) radius, m  
\( E \) elastic modulus of rock, MPa  
\( u_i \) initial displacement, m  
\( u_{ex} \) excavation displacement, m  
\( \Delta V \) volumetric shrinkage, m\(^3\)  
\( T_f \) formation temperature, °C  
\( T_w \) temperature at wall of borehole, °C  
\( \alpha \) linear expansion coefficient of rock, 1/°C  
\( \nu \) Poisson’s ratio of the rock  
\( \varepsilon_v \) volume strain, %  
\( \omega_p, \omega_R \) peak and residual shear strain, %  
\( \sigma_r, \sigma_\theta \) radial and tangential stresses, MPa  
\( \varepsilon_r, \varepsilon_\theta \) radial and tangential strain, %  
\( \sigma_p, \sigma_R \) peak and residual shear stress, MPa  
\( \sigma_o \) far-field original in situ stress, MPa

\( \sigma_{r,T}, \sigma_{\theta,T} \) radial and tangential thermal stresses, MPa  
\( \sigma_{r,\text{total}}, \sigma_{\theta,\text{total}} \) radial and tangential comprehensive stresses, MPa  
\( \sigma_{r,\text{in-situ}}, \sigma_{\theta,\text{in-situ}} \) Radial and tangential in situ stresses, MPa  
\( \gamma \) critical parameter about the ratio of broken and softening zone

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