Schwahn, Paul

Coindex and rigidity of Einstein metrics on homogeneous Gray manifolds. (English)
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Summary: Any 6-dimensional strict nearly Kähler manifold is Einstein with positive scalar curvature. We compute the coindex of the metric with respect to the Einstein-Hilbert functional on each of the compact homogeneous examples. Moreover, we show that the infinitesimal Einstein deformations on $F_{1,2} = \text{SU}(3)/T^2$ are not integrable into a curve of Einstein metrics.

MSC:
53C24 Rigidity results
53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)
53C30 Differential geometry of homogeneous manifolds

Keywords:
Einstein metrics; stability; rigidity; Lichnerowicz Laplacian; nearly Kähler

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