On the cardinality of non-isomorphic intermediate rings of $C(X)$

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Abstract. Let $\Sigma(X)$ be the collection of subrings of $C(X)$ containing $C^*(X)$, where $X$ is a Tychonoff space. For any $A(X) \in \Sigma(X)$ there is associated a subset $\nu_A(X)$ of $\beta X$ which is an $A$-analogue of the Hewitt real compactification $\nu X$ of $X$. For any $A(X) \in \Sigma(X)$, let $[A(X)]$ be the class of all $B(X) \in \Sigma(X)$ such that $\nu_A(X) = \nu_B(X)$. We show that for first countable non compact real compact space $X$, $[A(X)]$ contains at least $2^c$ many different subalgebras no two of which are isomorphic in Theorem 3.8.

Mathematics Subject Classification. 54D35, 46E25.

Keywords. Ultrafilter, Realcompact space, Maximal ideal, Real closed field.

1. Introduction

For a completely regular Hausdroff space $X$, let $C(X)$ and $C^*(X)$ denote the rings of all real valued and bounded real valued continuous functions on $X$ respectively. We denote ‘intermediate ring’ by $A(X)$ which mean that $C^*(X) \subseteq A(X) \subseteq C(X)$. Let $\Sigma(X)$ denote the class of all intermediate rings.

As we dig into the literature the study of intermediate rings started when D. Plank in 1969 [9] established a result that the structure space of any intermediate ring is homeomorphic to the Stone-Čech compactification of $X$, i.e., $\beta X$. The proof is quite different and complicated in comparison with the conventional method of showing the same result for $C(X)$ only. In 1987 Redlin and Watson [11] proved the same result but using a new technique. They have created a new operator, called $Z_A$ for an intermediate ring $A(X)$, which identify with one to one basis the maximal ideals of $A(X)$ with $z$-ultrafilters of $X$. They also established a Gelfand-Kolmogorov like characterization for maximal ideals using the points of $\beta X$. Following their notation we denote maximal ideals of $A(X)$ by $M^p_A$ for each $p \in \beta X$.
For $A(X) \in \sum(X)$ we associate a subspace $\nu_A(X) \subseteq \beta X$ which is an $A$-analogue of the Hewitt real compactification $\nu X$ of $X$, i.e., the collection of all points $p \in \beta X$ for which $A(X)/M^p_A$ is isomorphic to $\mathbb{R}$ under canonical map with the usual subspace topology of $\beta X$. We define an equivalence relation $A(X) \sim B(X)$ if and only if $\nu_A(X)$ is homeomorphic to $\nu_B(X)$ where $A(X)$, $B(X) \in \sum(X)$. We denote the equivalence class for $A(X) \in \sum(X)$ by $[A(X)]$. The cardinality of a particular class $[A(X)]$ and algebraic inter relation between any two rings from a same class is merely a question. Redlin and Watson gave an example in [11] of an intermediate class where at least two rings are non-isomorphic. We state this example verbatim as follows: let $H(\mathbb{N})$ be the the algebra of sequences which occur as the coefficients of the Taylor series representation of functions holomorphic on the open unit disc. Then $\mathbb{N}$ is both $H$-compact (see [5]) and $C$-compact, but $H(\mathbb{N})$ is obviously not isomorphic to $C(\mathbb{N})$. This particular observation is the principal motivation in this article to search for the cardinality of non-isomorphic rings in a particular class $[A(X)]$.

Research along this line by Redlin, Watson, Buyum, Pannman, Sack, etc developed machineries to which one can deal with problems. In this context mention may be made of [4,11,12,13] which are a few of their works.

In the second section of this article we have established a result [Theorem 2.2] that tells that a real closed $\eta_\alpha$ field of power $\kappa_\alpha$ contains a proper copy of itself, which provides a way to identify subrings $B^F_p(\mathfrak{X}) \in \sum(X)$ of the class $[A(X)]$ where $\mathfrak{X}$ is a real closed $\eta_\eta$-field with transcendence base at least $c$ and $p \in \beta X \setminus \nu_A(\mathfrak{X})$. As this identification $B^F_p(\mathfrak{X})$ of subrings depends upon the point $p \in \beta X \setminus \nu_A(\mathfrak{X})$ it can be concluded that there are plenty of subrings, in fact, at least $2^c$ many in the class $[A(X)]$ [Theorem 2.4]. In section 3 we show that the existence of isomorphic subrings in the class $[A(X)]$ is equivalent to the existence of homeomorphism from $\beta X$ onto itself which therefore relates our main focus to homogeneity of $\beta X$ [Theorem 3.1,Theorem 3.4]. Finally we are able to show that there exists at least $2^c$ many non-isomorphic different subrings in the class $[A(X)]$ if $X$ is first countable, locally compact, non-pseudocompact and realcompact which contains a $C$-embedded copy of $\mathbb{N}$.

Some results and notations of intermediate rings have been used in this paper. For clarity purpose one can find it in [1,2,3,4,11,12,13].

2. Some basic results and discussion

Recall $\nu_A X = \{ p \in \beta X \mid A(X)/M^p_A$ isomorphic to $\mathbb{R} \}$. Let for each $A(X) \in \sum(X)$, $\nu^\alpha(X) = \{ f \in C(X) \mid f^\ast(\nu_A X) \subseteq \mathbb{R} \}$ where $f^\ast$ is the unique stone-extension of $f$ from $\beta X$ to the one-point compactification of $\mathbb{R}$, i.e., $\mathbb{R}^\ast$. It is obvious from Theorem 3.5 of [4] that $A(X) \subseteq \nu^\alpha(X)$ and also $C^\ast(X) \subseteq \nu^\alpha(X)$ which therefore imply that $\nu^\alpha(X)$ is an intermediate subalgebra of $C(X)$. Also it is clear from the definition of $\nu^\alpha(X)$ that $\nu_A X = \nu_A^\alpha X$ and therefore $A(X) \subseteq \nu^\alpha(X)$ for all $A(X) \in [A(X)]$. In a nutshell $\nu^\alpha(X)$ is the largest element under set inclusion in the class $[A(X)]$ and also $\nu^\alpha(X) \cong C(\nu_A(X))$ [12], i.e., $\nu^\alpha(X)$ is a $C$-ring and hence we can directly use Theorem 13.2, Theorem 13.4 of [8] and conclude the following result.
**Theorem 2.1.** Every hyper-real field of the form \(A^{\nu}(X)/M_{A^{\nu}}^{p}\) for \(p \in \beta X \setminus v_{A^{\nu}}(X)\) is a real closed \(\eta_{1}\)-field with transcendence base at least \(\omega\).

We know that real closed \(\eta_{\alpha}\)-fields of cardinality \(\aleph_{\alpha}\) are isomorphic [6]. This fact takes a crucial role to prove the following result.

**Theorem 2.2.** Let \(\alpha > 0\) be any ordinal and let \(\mathfrak{F}\) be a real closed \(\eta_{\alpha}\)-field of power \(\aleph_{\alpha}\). Then \(\mathfrak{F}\) contains a proper copy of itself.

*Proof.* By [8, Lemma 13.11] \(\mathfrak{F}\) has a dense transcendent base over \(\mathbb{Q}\) and let it be \(A\). Then \(\mathfrak{F} = \mathcal{R}(\mathbb{Q}(A))\). Let \(a \in A\) and \(A' = A - \{a\}\). Then \(A'\) is also dense in \(\mathfrak{F}\) and \(|A'| = |A|\). Now let \(\mathfrak{F}' = \mathcal{R}(\mathbb{Q}(A'))\). Obviously, \(\mathfrak{F}' \subset \mathfrak{F}\). This inclusion is proper, otherwise \(A'\) will be the maximal set of independent transcendental elements and hence a base which contradicts our assumption that \(A\) is a base. Again, \(A' \subset \mathfrak{F}'\) imply that \(\mathfrak{F}'\) is dense in \(\mathfrak{F}\). Since every dense subset of a \(\eta_{\alpha}\)-set is a \(\eta_{\alpha}\)-set [6, Lemma 1.3], therefore, \(\mathfrak{F}'\) is a \(\eta_{\alpha}\)-set of power \(\aleph_{\alpha}\). Hence \(\mathfrak{F}'\) is a real closed \(\eta_{\alpha}\)-field. If we assume continuum hypothesis then any two real closed field that are \(\eta_{\alpha}\)-set of power \(\aleph_{\alpha}\) are isomorphic and hence \(\mathfrak{F}\) and \(\mathfrak{F}'\) are isomorphic. Hence \(\mathfrak{F}\) contains a proper copy of itself. \(\blacksquare\)

Theorems 2.1 and 2.2 together ensure the fact that \(A^{\nu}(X)/M_{A^{\nu}}^{p}\) contains a proper copy of itself. Let us take \(\mathfrak{F}\) to be the proper copy of \(A^{\nu}(X)/M_{A^{\nu}}^{p}\) into itself and \(\theta\) be the canonical map from \(A^{\nu}(X)\) to the hyper-real field \(A^{\nu}(X)/M_{A^{\nu}}^{p}\), i.e., \(\theta(f) = M_{A^{\nu}}^{p}(f)\) for \(f \in A^{\nu}(X)\). Then \(\theta^{-1}(\mathfrak{F})\) is a proper subring of \(A^{\nu}(X)\) and we denote this subring by \(\mathfrak{B}_{\mathfrak{F}}^{p}(X)\). Since \(\mathfrak{F}\) contains a copy of \(\mathbb{R}\) it follows that \(C^{\ast}(X) \subseteq \mathfrak{B}_{\mathfrak{F}}^{p}(X)\) and hence \(\mathfrak{B}_{\mathfrak{F}}^{p}(X) \subseteq \sum(X)\). Also note that \(\theta^{-1}(0) = M_{A}^{p} \subseteq \mathfrak{B}_{\mathfrak{F}}^{p}(X)\).

**Theorem 2.3.** Let \(A(X) \subseteq \sum(X)\) and \(p, q \in \beta X\) such that \(p \in \beta X \setminus v_{A}X\) then there exists \(f \in A(X)\) such that \(f \in M_{A}^{q}\) and \(f^{\ast}(p) = \infty\).

*Proof.* Since \(p \in \beta X - v_{A}X\), there exist \(g \in A(X)\) such that \(|M_{A}^{q}(g)|\) is infinitely small and \(M_{A}^{p}(g) \neq 0\) in the field \(A(X)/M_{A}^{p}\) and hence there exist \(\xi \in A(X)\) such that \(M_{A}^{p}(g)M_{A}^{q}(\xi) = 1\). Since \(p \neq q\), there exist an open set \(V\) in \(\beta X\) such that \(g \neq \in \sum_{\beta}V \subseteq \beta X - \{p\}\) and by regularity there exists \(h \in C^{\ast}(X)\) such that \(h^{\beta}(\sum_{\beta}V) = 0\) and \(h^{\beta}(p) = 1\). Let \(f = h\xi \in A(X)\). Then \(|M_{A}^{p}(fg - 1)| = |M_{A}^{p}(h - 1)|\). Again \(h^{\beta}(p) = 1\) shows that \(|M_{A}^{p}(h - 1)|\) is either infinitely small or zero, i.e., \((gf)^{\beta}(p) = 1\). Since \(g^{\beta}(p) = 0\), we can conclude that \(f^{\beta}(p) = \infty\). Again \(h^{\beta}(\sum_{\beta}V) = 0\) and \(q \in \sum\) therefore \(h \in M_{A}^{q}\). \(\blacksquare\)

It is quite clear from Theorem 2.3 that if \(f \in M_{A^{\nu}}^{q}\) then \(|M_{A^{\nu}}^{q}(f)| = 0\) and hence \(f \in B_{\mathfrak{F}}^{p}(X)\). Again \(f^{\beta}(p) = \infty\), i.e., \(p \notin v_{B_{\mathfrak{F}}^{p}}(X)\) for all \(p \in \beta X - v_{A^{\nu}}X\) and consequently \(v_{B_{\mathfrak{F}}^{p}}(X) \subseteq v_{A^{\nu}}(X)\). Also \(v_{A^{\nu}}(X) \subseteq v_{B_{\mathfrak{F}}^{p}}(X)\) which follows from the fact that \(\mathfrak{B}_{\mathfrak{F}}^{p}(X) \subseteq A^{\nu}(X)\) and therefore we can conclude that \(B_{\mathfrak{F}}^{p}(X) \subseteq [A(X)]\).

Now one can conclude that for each point \(p \in \beta X \setminus v_{A}(X)\) there is a subring \(B_{\mathfrak{F}}^{p}(X)\) which belongs to the class \([A(X)]\). Again for locally compact, non-compact but realcompact space \(X\), \(\beta X \setminus X\) contains at least \(2^c\) many elements [8, Corollary 9.12], that combining with the previous fact produce the following theorem.
Theorem 2.4. For locally compact, non-compact but realcompact space $X$, each class $[A(X)]$ contains at least $2^\omega$ many distinct subrings.

3. Non-isomorphic subalgebras of the class $[A(X)]$

This section is focused on the primary goal of this article, i.e., on the non-isomorphic characteristic among the class of subrings $[A(X)]$. To do this the topological association of points $p \in \beta X \setminus v_A(X)$ with the subrings $B^\delta_p(X)$ in the class $[A(X)]$ take the key role. In fact it reveals the intimate relationship between the two important structures, viz., non-isomorphism of two rings in $[A(X)]$ and non-homogeneity of $\beta X \setminus X$.

Theorem 3.1. Let $p, q \in \beta X \setminus v_A X$. Then the two rings $B^\delta_p(X)$ and $B^\delta_q(X)$ are isomorphic if there is a homeomorphism from $\beta X$ to $\beta X$ which takes $p$ to $q$ and induces an isomorphism from $\mathfrak{F}$ onto $\mathfrak{F}'$.

Proof. Let $B^\delta_p(X)$ and $B^\delta_q(X)$ be isomorphic for $p, q \in \beta X - v_A X$ and $\phi$ be the isomorphism. Let $\theta_p$ be the corresponding canonical map, as mentioned earlier, from $A^\nu(X)$ to $A^\nu(X)/M^p_{A^\nu}$ for the ring $B_p(X)$ and similarly $\theta_q$ for the ring $B_q(X)$. Let $\mathfrak{F}$ and $\mathfrak{F}'$ are the isomorphic copies in $A^\nu(X)/M^p_{A^\nu}$ and $A^\nu(X)/M^q_{A^\nu}$ respectively such that $B^\delta_p(X) = \theta^{-1}_p(\mathfrak{F})$ and $B_q(X) = \theta^{-1}_q(\mathfrak{F}')$. Then $\theta_p \phi \theta^{-1}_q = \phi$ gives an isomorphism from $\mathfrak{F}$ to $\mathfrak{F}'$ and hence it takes zero to zero and as a consequence we have $\phi(M^p_{A^\nu}) = M^q_{A^\nu}$.

From the result of D. Rudd [10, Corollary 3.6], it follows that if $M^p_{A^\nu}$ is a maximal ideal of $A^\nu(X)$ then maximal ideals of $M^p_{A^\nu}$ are precisely $M^\nu_{A^\nu} \cap M$ where $M$ is a maximal ideal of $A^\nu(X)$ and $M$ does not contain $M^p_{A^\nu}$. Let $\beta M^p_{A^\nu} = \{M^\nu_{A^\nu} \cap M : M \nsubseteq M^p_{A^\nu} \text{ and } M \text{ is a maximal in } A^\nu(X)\}$. Then $\beta M^p_{A^\nu}$ is the collection of all maximal ideals of $M^p_{A^\nu}$ and it admits hull kernel topology. Now the mapping $\tau : \beta M^p_{A^\nu} \rightarrow \mathcal{M}$, defined by $\tau(M^\nu_{A^\nu} \cap M) = M$ is a homeomorphism into $\mathcal{M}$, where $\mathcal{M}$ is the space of maximal ideals of $A^\nu(X)$. Since $\mathcal{M}$ is homeomorphic to $\beta X$ [11, Theorem 4] and $\tau(\beta M^p_{A^\nu})$ is open in $\mathcal{M}[10, 2.3]$ therefore $\beta M^p_{A^\nu}$ is locally compact and Hausdorff. The one-point compactification of $\beta M^p_{A^\nu}$ is evidently homeomorphic to $\beta X \setminus \{p\}$. Again the structure spaces of $M^p_{A^\nu}$ and $M^q_{A^\nu}$ are homeomorphic and hence the isomorphism $\phi$ gives a homeomorphism from $\beta X \setminus \{p\}$ to $\beta X \setminus \{q\}$. Since $\beta X \setminus \{p\}$ and $\beta X \setminus \{q\}$ are locally compact [10, Remark 3.9], this homeomorphism can be extended to a homeomorphism from $\beta X$ to $\beta X$ which takes $p$ to $q$. □

Lemma 3.2. Let $X$ be a first countable tychonoff space and suppose $\tilde{\phi} : \beta X \rightarrow \beta X$ be a homeomorphism then $\tilde{\phi}$ induces an isomorphism on $A^\nu$ onto itself.

Proof. First countability of $X$ imply that the space $\beta X$ is first countable at each point of $X$ and therefore each point of $X$ is a $G_\delta$- point of $\beta X$. On the other hand, no point of $\beta X - X$ can be a $G_\delta$-point of $\beta X$ [8, 9.6]. As a result the homeomorphism $\tilde{\phi}$ exchanges the point of $X$, i.e., $\tilde{\phi}(X) = X$. Let us denote $\tilde{\phi}|_X = \phi$. Then $\phi$ is a homeomorphism from $X$ onto itself. The above homeomorphism induces a map, say, $\Psi : C(X) \rightarrow C(X)$, defined by $\Psi(f) = f \circ \phi^{-1}$. It is evident that $\Psi$ is a homomorphism. Again $\phi(X) = X$ imply that $Ker\Psi = \{0\}$. Since for every $f \in C(X)$, $f \circ \phi \in A^\nu$ and $\Psi(f \circ \phi) = (f \circ \phi) \circ \phi^{-1} = f$, $\Psi$ is onto.
Hence $Ψ$ is an automorphism from $C(X)$ onto itself. To prove the result it is sufficient to show that $Ψ(A^ν) = A^ν$. First we observe that due to the isomorphism of $Ψ$ both $A^ν$ and $Ψ(A^ν)$ have the same real maximal ideal space and hence $Ψ(A^ν) ∈ [A(X)]$. Again $Ψ$ preserve the order (set inclusion) among the subrings of $C(X)$ which evidently show that $Ψ(A^ν)$ is the largest among all the subrings in the class $[A(X)]$ and hence $Ψ(A^ν) = A^ν$. □

Some important properties and notations of intermediate rings are used in the following lemma which one can find in [4].

**Lemma 3.3.** Let $X$ be a first countable tychonoff space and $\tilde{Φ} : \beta X \rightarrow \beta X$ be a homeomorphism such that $Ψ(p) = q$, then for the induced isomorphism $Ψ$ on $A^ν$ we have $Ψ(M^p_{A^ν}) = M^q_{A^ν}$.

**Proof.** Theorem 3.3 of [4] shows that $M^p_{A^ν} = \{ f \in A^ν \mid p \in S[Z^ν(f)] \}$ where $S[F] = \bigcap\{ cl_{βX} E \mid E \in F \}$ and $F$ is a $z$-filter. Mention should be made here that $Z^ν(f)$ is a $z$-filter in $X$ since $f$ is a non unit in $A^ν$ [4, Lemma 1.4]. Again the fact $E \in Z^ν(f)$ shows that $Φ(E) \in Z^ν((f \circ Φ^{-1}))$ which evidently imply that if $p \in S[Z^ν(f)]$, i.e., $Φ^{-1}(q) \in S[Z^ν(f)]$ then $q \in S[Z^ν(f \circ Φ^{-1})]$, i.e., $q \in S[Z^ν(Ψ(f))]$. Therefore it follows from Theorem 3.3, [4] that $Ψ(M^p_{A^ν}) = \{ Ψ(f) \mid p \in S[Z^ν(Ψ(f))] \} = M^q_{A^ν}$. □

**Theorem 3.4.** For a first countable tychonoff space $X$, let $\tilde{Φ}$ be a homeomorphism from $βX$ onto itself such that $Φ(p) = q$ for some $p, q \in βX − νA_X$, where $A(X) \in Σ(X)$ then for each $B^p_q(X) \in [A(X)]$ there exist $B^{q'}_p(X) \in [A(X)]$ such that $Φ$ induces an isomorphism from $B^p_q(X)$ onto $B^{q'}_p(X)$ and from $β^q(X)$ onto $β^{q'}(X)$.

**Proof.** Recall the mapping $Ψ$ from the proof of Theorem 3.2 which also induces an isomorphism $Ψ$ from $A^ν/M^p_{A^ν}$ to $A^ν/M^q_{A^ν}$. Let $β$ be the proper isomorphic copy of $A^ν/M^p_{A^ν}$ into itself. Then $Ψ(β) = β$ (say) is also a proper isomorphic copy of $A^ν/M^q_{A^ν}$ into itself. Let $θ_p$ and $θ_q$ are canonical maps from $A^ν$ to $A^ν/M^p_{A^ν}$ and $A^ν$ to $A^ν/M^q_{A^ν}$ respectively and let $θ_p^{-1}(β) = β^p_q(X)$ and $θ_q^{-1}(β) = β^q_q(X)$. Then $B^p_q(X)$ and $B^{q'}_p(X)$ are intermediate rings and belongs to $[A(X)]$. It is quite clear now that the restriction of the isomorphism $Ψ$ to $B^p_q(X)$ gives an isomorphism from $B^p_q(X)$ onto $B^{q'}_p(X)$. □

To ensure that non-isomorphic intermediate rings do exist, we need the notion of type of a point in $βN \setminus N$ introduced by Frolik [7] and recorded in [14]. We reproduce below some relevant information about this notion from the monograph [14, 3.41, 4.12]. Each permutation $σ : N \rightarrow N$ extends to a homeomorphism $σ^* : βN \rightarrow βN$, conversely if $Φ : βN \rightarrow βN$ is a homeomorphism then $Φ|N$ is a permutation of $N$, because $Φ$ takes isolated points to isolated points and the points of $N$ are the only isolated points of $βN$. Therefore $Φ = σ^*$ for unique permutation $σ = Φ|N$ of $N$.

**Definition 3.5.** For two points $p, q \in βN \setminus N$, we write $p ~ q$ when there exist a permutation $σ$ on $N$ such that $σ^*(p) = q$. The relation $~$ is an equivalence relation on $βN \setminus N$. Each equivalence class of elements of $βN \setminus N$ is called a type of ultrafilters on $N$.

**Theorem 3.6.** (Frolik, [14]) There exists $2^ω$ many types of ultrafilters on $N$.
Theorem 3.7. (Frolik, [14]) If $N$ is $C$-embedded in $X$, then $cl_{\beta X}N \setminus N \subseteq \beta X \setminus X$, essentially $\beta N \setminus N \subseteq \beta X \setminus X$. If now $h : \beta X \to \beta X$ is a homeomorphism onto $\beta X$, and $p, q \in \beta N \setminus N$ are such that $h(p) = q$, then $p$ and $q$ belongs to the same type of ultrafilters on $N$.

We now use these two theorem of Frolik to establish the last main result of the present paper.

Theorem 3.8. Let $X$ be a first countable noncompact realcompact space. Then there exist at least $2^c$ many intermediate subrings of $[A(X)]$, no two of which are isomorphic.

Proof. Since $X$ is a noncompact realcompact space it is not pseudocompact. Hence $X$ contains a copy of $\mathbb{N}$, $C$-embedded in $X$ [8, 1.21]. As every $C$-embedded countable subset of a Tychonoff space is a closed subset of it [8, 3, B3], it follows that $cl_{\beta X}N \setminus N \subseteq \beta X \setminus X$ essentially $\beta N \setminus N \subseteq \beta X \setminus X$. The result of Theorem 3.6 assures that there exist a subset $S$ of $\beta N \setminus N$, consisting of exactly one member from each type with the property that $|S| = 2^c$. Let $p$ and $q$ be two distinct points of the set $S$. Then it follows from Theorem 3.7 that no homeomorphism from $\beta X$ to $\beta X$ can exchange $p$ and $q$. We now use Theorems 3.1 and 3.4 to conclude that the rings $B^S_p(X)$ and $B^S_q(X)$ are not isomorphic. Hence the theorem follows. $\square$

Acknowledgements

The author is thankful to the anonymous referee for his valuable comments to improve the original version of this article.

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Received: 29 June 2020.
Accepted: 27 April 2021.