Masses of W and Z Bosons without Higgs

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Abstract

A Lagrangian of electroweak interactions without Higgs is used to study the contributions of quarks and leptons to the masses of the W and the Z bosons. It is shown that the $SU(2) \times U(1)$ symmetry is broken by both fermion masses and axial-vector components of the intermediate bosons. The masses of the W and the Z bosons are obtained to be $m^2_W = \frac{1}{2} g^2 m_t^2$ and $m^2_Z = \rho m_W^2 / \cos^2 \theta_W$ with $\rho \simeq 1$. Two fixed gauge fixing terms for W and Z boson fields are derived respectively. A coupling between photon and Z boson is predicted. Massive neutrinos are required.
The top quark has been discovered in Fermi laboratory[1], whose mass has been determined to be

$$m_t = 180 \pm 12\text{GeV}[2].$$

The value of $m_t$ is at the same order of magnitude as the masses of the W and the Z bosons. As a matter of fact, before the discovery of the top quark there were attempts of finding the relationship between top quark and intermediate bosons by using various mechanism[3]. In this paper the Lagrangian of GWS model of electroweak interactions without Higgs is used to study the role of quarks and leptons in obtaining the masses of the intermediate bosons. The Lagrangian of electroweak (GWS model) interactions without Higgs is

$$\mathcal{L} = -\frac{1}{4} A_{\mu\nu}^i A^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{q} \{ i\gamma \cdot \partial - M \} q$$

$$+ \bar{q}_L \{ \frac{g}{2} \tau_i \gamma \cdot A^i + g' \frac{Y}{2} \gamma \cdot B \} q_L + \bar{q}_R g' \frac{Y}{2} \gamma \cdot B q_R$$

$$+ \bar{l} \{ i\gamma \cdot \partial - M_f \} l + \bar{l}_L \{ \frac{g}{2} \tau_i \gamma \cdot A^i - g' \frac{Y}{2} \gamma \cdot B \} l_L - \bar{l}_R g' \gamma \cdot B l_R,$$

where $A_{\mu\nu}^i$ and $B_{\mu\nu}$ are electroweak boson fields, $M$ is the mass matrix of the six quarks, $M_l$ is the lepton mass matrix, $q_L$ is the left-handed quark doublet, $q_R$ is the right handed quark field, $l_L$ is the left-handed lepton doublet, and $l_R$ is the right handed lepton field. Summation over $q_L$, $q_R$, $l_L$, and $L_R$ is implicated in Eq.(2). This Lagrangian is used to study the $SU(2) \times U(1)$ symmetry breaking mechanism, the masses of the W and the Z bosons, and the propagators of the intermediate boson fields in this paper.
We start from the doublet of t and b quarks. The Lagrangian of this generation is

\[ \mathcal{L} = -\frac{1}{4} A^i_{\mu\nu} A^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{t}\{i\gamma \cdot \partial - m_t\}t + \bar{b}\{i\gamma \cdot \partial - m_b\}b \\
+ \bar{\psi}_L \left( \frac{g}{2} \tau_i \gamma \cdot A^i + g' \frac{1}{6} \gamma \cdot B \right) \psi_L + \frac{2}{3} g' \bar{t} R \gamma \cdot B t_R - \frac{1}{3} g' \bar{b} R \gamma \cdot B b_R, \tag{3} \]

where

\[ \psi_L = \begin{pmatrix} t \\ b \end{pmatrix} . \]

The Lagrangian of the boson fields is obtained by integrating out the fermion fields. For this purpose the quark part of the Lagrangian (3) is rewritten as

\[ \mathcal{L} = \bar{\psi} \{i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a \gamma_5 - m\} \psi, \tag{4} \]

where \( \psi \) is the doublet of t and b quarks,

\[ m = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix}, \]

\[ v_\mu = \tau_i v^i_\mu + \omega_\mu, \quad v^{1,2}_\mu = \frac{g}{4} A^{1,2}_\mu, \quad v^3_\mu = \frac{g}{4} A^3_\mu + \frac{g'}{4} B_\mu, \quad \omega_\mu = \frac{g'}{6} B_\mu, \quad a_\mu = \gamma_i a^i_\mu, \quad a^{1,2}_\mu = -\frac{g}{4} A^{1,2}_\mu, \]

\[ a^3_\mu = -\frac{g}{4} A^3_\mu + \frac{g'}{4} B_\mu. \]

After finishing the integration over quark fields, the Lagrangian of the intermediate boson fields is obtained in Euclidean space

\[ \mathcal{L}_E = \ln \text{det} D, \tag{5} \]

where

\[ D = \gamma \cdot \partial - i\gamma \cdot v - i\gamma \cdot a \gamma_5 + m. \tag{6} \]
The real and imaginary parts of the Lagrangian(5) are

\[ L_{\text{Re}} = \frac{1}{2} \ln \det(D^\dagger D), \quad L_{\text{Im}} = \frac{1}{2} \ln \det(D D^\dagger), \tag{7} \]

where

\[ D^\dagger = -\gamma \cdot \partial + i\gamma \cdot v - i\gamma \cdot a\gamma_5 + m. \tag{8} \]

From Eqs.(6,7,8) it is learned that \( L_{\text{Re}} \) has even number of \( \gamma_5 \) and normal parity. On the other hand, the number of \( \gamma_5 \) in \( L_{\text{Im}} \) is odd and \( L_{\text{Im}} \) has abnormal parity. Anomalous couplings between W, Z, and \( \gamma \) should be revealed from \( L_{\text{Im}} \).

It is necessary to point out that \( D^\dagger D \) is a definite positive operator. In terms of Schwinger’s proper time method[4] \( L_{\text{Re}} \) is expressed as

\[ L_{\text{Re}} = \frac{1}{2} \int d^D x Tr \int_0^\infty \frac{d\tau}{\tau} e^{-\tau D^\dagger D}. \tag{9} \]

Inserting a complete set of plane waves and subtracting the divergence at \( \tau = 0 \), we obtain

\[ L_{\text{Re}} = \frac{1}{2} \int d^D x \frac{d^D p}{(2\pi)^D} Tr \int_0^\infty \frac{d\tau}{\tau} \left\{ e^{-\tau D^\dagger D'} - e^{-\tau \Delta_0} \right\}, \tag{10} \]

where

\[ D' = \gamma \cdot \partial + i\gamma \cdot p - i\gamma \cdot v - i\gamma \cdot a\gamma_5 + m, \]

\[ D'^\dagger = -\gamma \cdot \partial - i\gamma \cdot p + i\gamma \cdot v - i\gamma \cdot a\gamma_5 + m, \]

\[ D'^\dagger D' = \Delta_0 - \Delta, \]
\[ \Delta_0 = p^2 + m_1^2, \quad m_1^2 = \frac{1}{2}(m_t^2 + m_b^2), \quad m_2^2 = \frac{1}{2}(m_t^2 - m_b^2), \]
\[ \Delta = \partial^2 - (\gamma \cdot v - \gamma \cdot a\gamma_5)(\gamma \cdot v + \gamma \cdot a\gamma_5) - i\gamma \cdot \partial(\gamma \cdot v + \gamma \cdot a\gamma_5) \]
\[ -i(\gamma \cdot v - \gamma \cdot a\gamma_5)\gamma \cdot \partial + 2ip \cdot \partial + 2p \cdot (v + a\gamma_5) - i[\gamma \cdot v, m] \]
\[ + i\{\gamma \cdot a, m\}\gamma_5 - m_2^2\tau_3. \] (11)

After the integration over \( \tau \), \( \mathcal{L}_{Re} \) is expressed as
\[ \mathcal{L}_{Re} = \frac{1}{2} \int d^Dx \frac{d^Dp}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{(p^2 + m_i^2)^n} Tr \Delta^n. \] (12)

Due to the property of \( \mathcal{L}_{Im} \) mentioned above, \( \mathcal{L}_{Im} \) doesn’t contribute to the masses of W and Z bosons at least at the tree level of the boson fields. Therefore, the study of \( \mathcal{L}_{Im} \) is beyond the scope of this paper.

\( \mathcal{L}_{Re} \) is used to investigate the symmetry breaking mechanism, the masses of the intermediate bosons, and the propagators of the boson fields. Obviously, the \( SU(2) \times U(1) \) symmetry is broken by both quark and lepton masses. However, another symmetry breaking mechanism is needed. **Due to parity nonconservation the intermediate bosons have both vector and axial-vector components which are written in the forms of \( v \) and \( a \) in Eq.(4). This fact makes the intermediate boson fields different from photon and gluon fields which are vector fields.** In this paper we investigate whether this property of the intermediate boson fields results in another \( SU(2) \times U(1) \) symmetry breaking. From
the expression of $\Delta(11)$ it is seen that in company with fermion mass the vector component $v$ of the intermediate boson fields appears in a commutator $[v, m]$, while the axial-vector component $a$ appears in an anticommutator $\{a, m\}$. $[v, m]$ provides the mass for $W^\pm$ only, $\{a, m\}$ provides the masses for both $W$ and $Z$ bosons. Indeed, the axial-vector component of the intermediate boson leads to a new $SU(2) \times U(1)$ symmetry breaking mechanism.

The details of the symmetry breaking by both fermion masses and axial components of the intermediate boson fields are shown in the calculations of $m_W$ and $m_Z$.

In terms of the Lagrangian(12) the masses of the intermediate bosons are calculated. The Lagrangian(12) shows that the electric $U(1)$ symmetry is remained, therefore, there is massless boson field, which is the photon field. The terms related to the masses only is separated from Eq.(12)

\[
\mathcal{L}_M = \frac{1}{2} \int d^4x \int \frac{d^Dp}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{(p^2 + m_1^2)n} Tr \{- (\gamma \cdot v - \gamma \cdot a\gamma_5) \\
(\gamma \cdot v + \gamma \cdot a\gamma_5) + 2p \cdot (v + a\gamma_5) + i[m, \gamma \cdot v] + i\{m, \gamma \cdot a\} \gamma_5 - m_2^2\tau_3\},
\]

(13)

The contributions of the fermion masses to $m_W$ and $m_Z$ are needed to be calculated to all orders. It is found that the series of the fermion masses are convergent to analytic functions. The mass terms of the intermediate bosons are obtained from Eq.(13). In Minkowski space it is expressed as

\[
\mathcal{L}_M = \frac{1}{2} \frac{N_C}{(4\pi)^2} \left( \frac{D}{4} \Gamma(2 - \frac{D}{2})(4\pi)^{\frac{D}{2}} \left( \frac{\mu^2}{m_1^2} \right)^{\frac{D}{2}} + \frac{1}{2} [1 - \ln(1 - x) - (1 + \frac{1}{x}) \sqrt{x} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}}] m_1^2 g^2 \sum_{i=1}^{2} A_i^i A_i^{\mu} \right)
\]
\[ + \frac{1}{2} \frac{N_C}{(4\pi)^2} \left\{ \frac{D}{4} \Gamma\left(2 - \frac{D}{2}\right)(4\pi)^{\frac{3}{2}} \left(\frac{\mu^2}{m_1^2}\right)^{\frac{3}{2}} - \frac{1}{2} \left[ \ln(1-x) + \sqrt{x} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \right] \right\} m_1^2 (g^2 + g'^2) Z_\mu Z_\mu \]

where \( N_C \) is the number of colors and \( x = \left( \frac{m^2}{m_1^2} \right)^2 \). It is necessary to point out that

\[ a_\mu^3 = \sqrt{g^2 + g'^2} Z_\mu. \] (15)

In the same way, other two generations of quarks, \( \begin{pmatrix} u \\ d \end{pmatrix} \) and \( \begin{pmatrix} c \\ s \end{pmatrix} \), and three generations of leptons \( \begin{pmatrix} \nu_e \\ e \end{pmatrix} \), \( \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \), and \( \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \) contribute to the masses of W and Z bosons too. By changing the definitions of \( m_1^2 \) and \( x \) to the quantities of other generations in Eq.(14), the contributions of the other two quark generations are found. Taking off the factor \( N_C \) and changing \( m_1^2 \) and \( x \) to corresponding quantities of leptons, the contributions of the leptons to \( m_W \) and \( m_Z \) are obtained. In this paper the effects of CKM matrix are not taken into account. The final expressions of the masses of W and Z bosons are the sum of the contributions of the three quark and the three lepton generations. It is learned from the processes deriving Eq.(14) that

1. The field \( \sin \theta_W A_\mu^3 + \cos \theta_W B_\mu \) (photon field) is massless;

2. Both W and Z bosons gain masses from fermion masses;

3. In Eq.(13) there are four different types of terms contributing to \( m_W \) and \( m_Z \). It is found that \( m_Z \) is resulted in \( \{a_\mu, m\} \) only. Without \( \{a_\mu, m\} \) Z boson is massless.
The conclusion is that the fermion masses and the axial components of the intermediate boson fields cause the $SU(2) \times U(1)$ symmetry breaking and makes both W and Z bosons massive.

Now the W and Z bosons are massive. It is necessary to study their propagators to see whether they have right behavior for renormalization at high energy. Up to all orders of fermion masses, the kinetic terms of the intermediate boson fields are obtained from the Lagrangian(12). In Minkowski space it is expressed as

$$
\mathcal{L}_K = -\frac{1}{4} \sum_{i=1,2} (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)^2 \{1 + \frac{1}{(4\pi)^2} g^2 \sum_{q,l} N [\frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} (\frac{\mu^2}{m_1^2})^\frac{D}{2} - \frac{1}{6} + f_1]\}
\frac{1}{4} (\partial_\mu A^3_\nu - \partial_\nu A^3_\mu)^2 \{1 + \frac{1}{(4\pi)^2} g^2 \sum_{q,l} N [\frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} (\frac{\mu^2}{m_1^2})^\frac{D}{2} - \frac{1}{6} + \frac{1}{6} f_2]\}
\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \{1 + \frac{N_C}{(4\pi)^2} g^2 \sum_q \left[ \frac{11}{9} \frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} (\frac{\mu^2}{m_1^2})^\frac{D}{2} - \frac{1}{6} + \frac{1}{54} f_2 - \frac{1}{18} f_3 \right]\}
+ \frac{1}{(4\pi)^2} g^2 \sum_l \left[ \frac{D}{4} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} (\frac{\mu^2}{m_1^2})^\frac{D}{2} - \frac{1}{6} + \frac{1}{2} f_2 + \frac{1}{6} f_3 \right]
- \frac{1}{(4\pi)^2} g g' \sum_{i=1,2} (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu) \{N_G - \frac{2}{3} N_C \sum_q f_3 + 2 \sum_l f_3\}
+ \frac{1}{(4\pi)^2} N_G \frac{g^2}{12} \sum_{i=1,2} (\partial^\mu A^i_\mu)^2 + \frac{1}{(4\pi)^2} N_G \frac{1}{12} (g^2 + g'^2)(\partial_\mu Z^\mu)^2, (16)
$$

where $\sum_q$ and $\sum_l$ stand for summations of generations of quarks and leptons respectively, $N = N_C$ for q and $N = 1$ for l, $N_G = 3N_C + 3$, $x$ depends on fermion generation and is defined in Eq.(14) for one generation,

$$
f_1 = \frac{4}{9} - \frac{1}{6x} - \frac{1}{6} \ln(1 - x) + \frac{1}{4\sqrt{x}} \left( \frac{1}{3x} - 1 \right) \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}}.
$$
\( f_2 = -\ln(1-x), \quad f_3 = \frac{1}{2} \frac{1}{\sqrt{x}} \ln \frac{1+\sqrt{x}}{1-\sqrt{x}}. \) \tag{17} 

Following results are obtained from Eq.(16)

1. The boson fields and the coupling constants \( g \) and \( g' \) have to be redefined by multiplicative renormalization

\[ A_1^{1,2} \rightarrow Z_1^{\frac{1}{2}} A_1^{1,2}, \quad A_3^3 \rightarrow Z_3^{\frac{1}{2}} A_3^3, \quad B_{\mu} \rightarrow Z_B^{\frac{1}{2}} B_{\mu}, \]

\[ g_1 = Z_1^{-\frac{1}{2}} g, \quad g_3 = Z_3^{-\frac{1}{2}} g, \quad g_B = Z_B^{-\frac{1}{2}} g', \] \tag{18} 

where

\[ Z_1 = 1 + \frac{1}{(4\pi)^2} g^2 \sum_q N\left[ \frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} \left( \frac{\mu^2}{m_1^2} \right)^\frac{D}{2} - \frac{1}{6} + f_1 \right] \]

\[ Z_3 = 1 + \frac{1}{(4\pi)^2} g^2 \sum_q N\left[ \frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} \left( \frac{\mu^2}{m_1^2} \right)^\frac{D}{2} - \frac{1}{6} + \frac{1}{6} f_2 \right] \]

\[ Z_B = 1 + \frac{N_C}{(4\pi)^2} g^2 \sum_q \left[ \frac{11}{9} \frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} \left( \frac{\mu^2}{m_1^2} \right)^\frac{D}{2} - \frac{1}{6} + \frac{11}{54} f_2 - \frac{1}{18} f_3 \right] + \frac{1}{(4\pi)^2} g^2 \sum_q \left[ \frac{D}{4} \Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2} \left( \frac{\mu^2}{m_1^2} \right)^\frac{D}{2} - \frac{1}{6} + \frac{1}{2} f_2 + \frac{1}{6} f_3 \right]. \] \tag{19} 

The divergent terms in \( Z_1 \) and \( Z_3 \) are the same.

2. There is a crossing term between \( A_3^3_{\mu} \) and \( B_{\mu} \), which is written as

\[ g_3 g_B (\partial_{\mu} A_3^3 - \partial_\nu A_3^3)(\partial_{\mu} B_{\nu} - \partial_\nu B_{\mu}) = e^2 (\partial_{\mu} A_\nu - \partial_\nu A_\mu)^2 - e^2 (\partial_{\mu} Z_\nu - \partial_\nu Z_\mu)^2 + e (g_3 \cos \theta_W - g_B \sin \theta_W)(\partial_{\mu} A_\nu - \partial_\nu A_\mu)(\partial_{\mu} Z_\nu - \partial_\nu Z_\mu), \] \tag{20} 

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where $\sin \theta_W = \frac{g_3}{\sqrt{g_3^2 + g_B^2}}$, $\cos \theta_W = \frac{g_3}{\sqrt{g_3^2 + g_B^2}}$, and $e = \frac{g_3 g_B}{\sqrt{g_3^2 + g_B^2}}$. Therefore, the photon and the Z fields are needed to be renormalized again

$$(1 + \frac{\alpha}{4\pi} f_4) A_\mu \rightarrow A_\mu, \quad (1 - \frac{\alpha}{4\pi} f_4) Z_\mu \rightarrow Z_\mu,$$  

(21)

where

$$f_4 = \frac{1}{3}N_G - \frac{2}{3} \sum_q f_3 + \frac{2}{3} \sum_l f_3.$$  

After these renormalizations (18, 21), $\mathcal{L}_K$ (16) is rewritten as

$$\mathcal{L}_K = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4} \sum_{i=1,2} (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)^2 - \frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2$$

$$- \frac{1}{4 \pi} (\frac{g_3}{g_B} - \frac{g_B}{g_3})(1 - \frac{\alpha^2}{(4\pi)^2} f_4^2)^{-\frac{1}{2}} f_4 (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu)$$

$$- \frac{N_G}{(4\pi)^2} \frac{g^2}{12} (\partial^\mu A^i_\mu)^2 - \frac{N_G}{(4\pi)^2} (1 - \frac{\alpha}{4\pi} f_4)^{-1} \frac{1}{12} (g_3^2 + g_B^2)(\partial_\mu Z_\mu)^2.$$  

(22)

3. The interaction between photon and Z boson is predicted in Eq.(22). For very small neutrino masses it is derived from Eq.(17)

$$f_3 = -\ln\left(\frac{m_\nu}{m_l}\right).$$  

(23)

Therefore, if neutrino is massless $f_4$ is logarithmic divergent. This divergence is in contradiction with that the physical coupling between photon and z boson must be finite. Therefore, this theory requires massive neutrinos.
4. Fixed gauge fixing terms for W- and Z- fields are generated:

\[-\frac{1}{4}\xi_W(\partial^\mu A_i^\mu)^2, \quad -\frac{1}{4}\xi_Z(\partial^\mu Z_\mu)^2,\]

\[\xi_W = \frac{N_G g_1^2}{(4\pi)^2}\frac{1}{3}, \quad \xi_Z = \frac{N_G}{(4\pi)^2}\left(1 - \frac{\alpha}{4\pi f_4}\right)^{-1}\frac{1}{3}(g_3^2 + g_B^2).\] (24)

The propagator of W field is derived from Eq.(22)

\[\frac{i}{q^2 - m_W^2}\left\{-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right\} - \frac{i}{\xi_W q^2 - m_W^2}\frac{q_\mu q_\nu}{q^2}.\] (25)

Changing the index W to Z in Eq.(25), the propagator of Z boson field is obtained.

Obviously, due to the gauge fixing terms the propagators of W and Z bosons do not affect the renormalizability of the theory. It is necessary to point out that the W and the Z fields are massive and no longer gauge fields. The ”gauge fixing” terms of W and Z bosons are derived from this theory and they are not obtained by choosing gauge. They are really fixed.

Now we can study the values of the masses of W and Z bosons. After the renormalizations there are still divergences in the mass formulas of \(m_W\) and \(m_Z\). The boson fields are already renormalized and the kinetic terms of the boson fields are already in the standard form. Therefore, the divergences in the formulas of \(m_W\) and \(m_Z\) cannot be absorbed by the boson fields. On the other hand, the divergences in Eq.(14) are fermion mass dependent, while the coupling constants should be the same for all fermion generations. It
is difficult that these divergences are absorbed by the coupling constants. In Eq.(14) the fermion masses are bare physical quantities. It is reasonable to redefine the fermion masses by multiplicative renormalization

$$Z_m m^2_1 = m^2_{1,P},$$

$$Z_m = \frac{N}{(4\pi)^2} \left\{ N_G \frac{D}{4} \Gamma\left(2 - \frac{D}{2}\right) (4\pi)^{\frac{D}{2}} \left( \frac{\mu^2}{m^2_1} \right)^{\frac{D}{2}} + \frac{1}{2} \left[ 1 - \ln(1 - x) - \left( 1 + \frac{1}{x} \right) \frac{\sqrt{x}}{2} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \right] \right\},$$

for each generation of fermions. The index ”P” is omitted in the rest of the paper. Now the mass of W boson is obtained from Eq.(14)

$$m_W^2 = \frac{1}{2} g^2 \left( m_t^2 + m_b^2 + m_c^2 + m_s^2 + m_u^2 + m_d^2 + m_e^2 + m_{\nu_e}^2 + m_{\nu_{\mu}}^2 + m_{\mu}^2 + m_{\nu_{\tau}}^2 + m_{\tau}^2 \right).$$

Obviously, the top quark mass dominates the $m_W$

$$m_W = \frac{g}{\sqrt{2}} m_t. \quad (28)$$

Using the values $g = 0.642$ and $m_t = 180 \pm 12 GeV[2]$, it is found

$$m_W = 81.71 (1 \pm 0.067) GeV, \quad (29)$$

which is in excellent agreement with data $80.33 \pm 0.15 GeV[2]$.

Using Eqs.(14,18,21), the mass formula of the Z boson is written as

$$m_Z^2 = \rho m_W^2 \left( 1 + \frac{g_B^2}{g_1^2} \right), \quad (30)$$

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where

\[
\rho = (1 - \frac{\alpha}{4\pi}f_4)^{-1} \sum_{q,l} N \{ \frac{D}{4} \Gamma(2 - \frac{D}{2})(4\pi)^{\frac{D}{2}} \frac{\mu^2}{m_1^2} \frac{\hat{\xi}}{\xi} - \frac{1}{2} [\ln(1 - x) + \sqrt{x} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}}] \}
\]

\[
/ \sum_{q,l} N \{ \frac{D}{4} \Gamma(2 - \frac{D}{2})(4\pi)^{\frac{D}{2}} \frac{\mu^2}{m_1^2} \frac{\hat{\xi}}{\xi} + \frac{1}{2} [1 - \ln(1 - x) - (1 + \frac{1}{x}) \sqrt{x} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}}] \} \frac{g_3^2}{g_1^2 + g_B^2} \]

Comparing to the infinites in Eqs.(18,19,31), the finite terms can be ignored in Eqs.(18,19,31).

We have

\[
g_1 = g_3 \equiv g_A, \quad m_Z^2 = \rho m_W^2 / \cos^2 \theta_W, \quad \cos \theta_W = g_A / \sqrt{g_A^2 + g_B^2}, \quad \rho = (1 - \frac{\alpha}{4\pi}f_4)^{-1}, \quad (32)
\]

where \( g_A \) and \( g_B \) are \( g \) and \( g' \) of the GWS model respectively. The finiteness of the \( \rho \) factor requires a finite \( f_4 \). Once again massive neutrinos are required. Due to the smallness of the factor \( \frac{\alpha}{4\pi} \) in the reasonable ranges of the quark masses and the upper limits of neutrino masses we expect

\[
\rho \simeq 1. \quad (33)
\]

Therefore,

\[
m_Z = m_W / \cos \theta_W \quad (34)
\]

is a good approximation.

Introduction of a cut-off leads to

\[
\frac{D}{4} \Gamma(2 - \frac{D}{2})(4\pi)^{\frac{D}{2}} \frac{\mu^2}{m_1^2} \frac{\hat{\xi}}{\xi} \to \ln(1 + \frac{\Lambda^2}{m_1^2}) - 1 + \frac{1}{1 + \frac{\Lambda^2}{m_1^2}}. \quad (35)
\]

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Taking $\Lambda \to \infty$, Eq.(32) is obtained. On the other hand, the cut-off might be estimated by the value of the $\rho$ factor. The cut-off can be considered as the energy scale of unified electroweak theory.

To conclude, a Lagrangian without Higgs is investigated. The $SU(2) \times U(1)$ symmetry is broken by both the fermion masses and the axial-vector components of the intermediate boson fields. W and Z bosons gain masses. Two fixed gauging fixing terms for W- and Z-fields are derived, which make the propagators of W- and Z-fields have right behaviors for renormalization. A coupling between photon and Z boson is predicted. The finiteness of $m_Z$, the $\gamma - Z$ coupling, and the fixed gauging fixing terms require massive neutrinos. By renormalizing the fermion masses, a mass formula of W boson is obtained and in excellent agreement with data. A $\rho \simeq 1$ factor is predicted in the mass formula of Z boson. The formalism developed in this paper can be used to study the couplings between intermediate bosons($\gamma$, W, Z, and gluons).

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