Natures of $T_{cs}(2900)$ and $T_{cs}^a(2900)$

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Inspired by the states $T_{cs0}(2900)^0$, $T_{cs1}(2900)^0$, $T_{cs0}^a(2900)^0$ and $T_{cs0}(2900)^{++}$ reported by the LHCb Collaboration, we carry out a systematic investigation on the properties of the ground and P-wave states $[cs][ar{u}d]$ and $[cu][ar{s}d]$ with various spin, isospin or $U$-spin, and color combinations in a multiquark color flux-tube model. Matching our results with the spin-parity and mass of the states $T_{cs0}(2900)^0$ and $T_{cs1}(2900)^0$, we can describe them as the compact states $[cs][ar{u}d]$ with $I(J^P) = 1(0^+)$ and $0^-$ in the model, respectively. The ground state $T_{cs0}(2900)^0$ is mainly made of strongly overlapped axial-vector $[cs]_3^a$ and axial-vector $[cs]_3^a$. The P-wave state $T_{cs1}(2900)^0$ is dominantly consisted of gradually separated scalar or axial vector $[cs]_3^a$ and scalar $[uar{d}]_3^a$ in the shape of a dumbbell. Supposing the states $T_{cs0}^a(2900)^0$ and $T_{cs0}(2900)^{++}$ belong to the same isospin triplet, the mass of the state $[[cu][ar{s}d]]_{1^+}$ with symmetrical $U$-spin and $J^P = 0^+$ is highly consistent with that of the states $T_{cs0}(2900)^0$ and $T_{cs0}^a(2900)^{++}$ in the model. After coupling two color configurations, the state $[[cu][ar{s}d]]$ is a little lighter than the states $T_{cs0}(2900)^0$ and $T_{cs0}^a(2900)^{++}$. In addition, we also discuss the properties of other states in the model.

I. INTRODUCTION

In 2020, the LHCb Collaboration observed two exotic structures with open quark flavors in the invariant mass distribution of $D^- K^+$ of the channel $B^+ \rightarrow D^- K^+$, which were denoted as $X_0(2900)$ and $X_1(2900)$ [1]. Their masses and widths are

$X_0(2900) : M = 2866 \pm 7 \pm 2$, $\Gamma = 57 \pm 12 \pm 4$,

$X_1(2900) : M = 2904 \pm 5 \pm 1$, $\Gamma = 110 \pm 11 \pm 4$,

unit in MeV. Both of them have the minimal quark content $cs\bar{d}$ because they can strongly decay into $D^- K^+$. The assignments of their spin-parity are $0^+$ and $1^-$, respectively. However, the accurate information on their isospin has not been available until now. Recently, the LHCb Collaboration suggested to rename the states $X_0(2900)$ and $X_1(2900)$ as $T_{cs0}(2900)^0$ and $T_{cs1}(2900)^0$, respectively [2].

In 2022, the LHCb Collaboration reported two isospin vector resonances $T_{cs0}^a(2900)^0$ and $T_{cs0}^a(2900)^{++}$ in the $D^+_s \pi^\pm$ invariant spectrum of the similar channels $B^+ \rightarrow D^- D^+_s \pi^+$ and $B^0 \rightarrow D^0 D^+_s \pi^-$. Their masses and widths are

$T_{cs0}^a(2900)^0 : M = 2892 \pm 21 \pm 2$, $\Gamma = 119 \pm 29$,

$T_{cs0}^a(2900)^{++} : M = 2921 \pm 23 \pm 2$, $\Gamma = 137 \pm 35$.

Supposing the states belong to the same isospin triplet, the experiment also gave the shared mass and width,

$M = 2908 \pm 23$, $\Gamma = 136 \pm 25$,

unit in MeV. Their least quark contents are respectively $cd\bar{s}u$ and $cu\bar{s}d$ with the same spin-parity $0^+$. The investigation on the structure and property of the states could help us to improve our knowledge of the low-energy strong interaction. Several possible physical pictures, molecular states $D^+ K^+$, $D^0 K$ and $D^* \rho$ [11-17], compact state $[cs][\bar{u}d]$ [19-26, 46], tetramole (superposition of molecules and compact tetraquark states) [27], triangle singularity [28, 29], and kinematical cusp [30], were proposed within various theoretical frameworks. Most of the interpretations on the states $T_{cs0}(2900)^0$ and $T_{cs1}(2900)^0$ preferred isospin singlet. Especially for the molecular states, the channel can produce a little of attraction by meson exchange interaction, which is beneficial to form bound states. We refer the interested readers to the latest reviews for more comprehensive descriptions [31].

In the present work, we prepare to make a systematical investigation on the ground and first angular excited states $[cs][\bar{u}d]$ and $[cu][\bar{s}d]$ with all possible spin, isospin and color combinations in the multiquark color flux-tube model (MCFTM). We anticipate to broaden the property and structure of the four states from the perspective of diquark picture and to provide some valuable clues to the experimental establishment of the tetraquark states in the future. We also hope that this work can improve the understanding of the mechanism of the low-energy strong interaction.

This paper is organized as follows. After the introduction section, we give a concise description of the MCFTM in Sec. II. We introduce the trial wave functions of the states $[cs][\bar{u}d]$ and in Sec. III. We present the numerical results and discussions in Sec. IV. We list a briefly summary in the last section.

II. MULTIQURAK COLOR FLUX-TUBE MODEL

Multiquark color flux-tube model has been established in the basis of the color flux-tube picture in the lattice QCD [32, 33] and chiral constituent quark model [34]. We only give the schemata of the model here. The model

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Hamiltonian reads

\[ H_n = \sum_{i=1}^{n} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_c + V^{\text{con}}(n) + \sum_{i<j} V_{ij}, \]

\[ V_{ij} = V_{ij}^{\text{pge}} + V_{ij}^{\text{pbe}} + V_{ij}^{\sigma}, \]

where \( m_i \) and \( p_i \) are the mass and momentum of the \( i \)-th quark or antiquark, respectively. \( T_c \) is the center-of-mass kinetic energy of the states and should be deducted. \( V^{\text{con}}(n) \) is an \( n \)-body color confinement potential. \( V_{ij}^{\text{pge}}, V_{ij}^{\text{pbe}}, \) and \( V_{ij}^{\sigma} \) are the one-gluon-exchange interaction, and one-boson-exchange interaction \((\pi, K \text{ and } \eta), \sigma\)-meson exchange interaction between the particles \( i \) and \( j \), respectively. In the state \([cs][\bar{u}d]\), the codes of \( c, s, \bar{u} \) and \( d \) are assumed to be 1, 2, 3 and 4, respectively. Their corresponding positions are denoted as \( r_1, r_2, r_3 \) and \( r_4 \). The codes of the state \([cu][\bar{s}d]\) are exactly same with those of the state \([cs][\bar{u}d]\).

For mesons, the quark and antiquark are linked with a three-dimensional color flux tube. Its two-body square confinement potential reads

\[ V^{\text{con}}(2) = kr_{\bar{q}q}^2, \]

where \( r_{\bar{q}q} \) is the distance between \( q \) and \( \bar{q} \) and \( k \) is the stiffness of a three-dimension color flux tube determined by fitting meson spectrum.

Within the framework of the diquark-antidiquark configuration, the states \([cs][\bar{u}d]\) and \([cu][\bar{s}d]\) have a double-Y-type color flux-tube structure. Its four-body confinement potential reads

\[ V^{\text{con}}(4) = k \left( (r_1 - y_{12})^2 + (r_2 - y_{12})^2 + (r_3 - y_{34})^2 + (r_4 - y_{34})^2 + \kappa_d(y_{12} - y_{34})^2 \right), \]

where \( y_{12} \) and \( y_{34} \) stand for the positions of the two \( Y \)-shaped junctions. In order to satisfy the requirement of overall color singlet, the color flux-tube connecting \( y_{12} \) and \( y_{34} \) must be in color 3, or 6. The relative stiffness parameter \( \kappa_d \) of the \( d \)-dimension color flux-tube is equal to \( \frac{C_d}{r_{\bar{q}q}} \), where \( C_d \) is the eigenvalue of the Casimir operator associated with the SU(3) color representation \( d \) at either end of the color flux-tube.

Taking \( y_{12} \) and \( y_{34} \) as variational parameters, we determine them by minimizing the four-body confinement potential. With their values, we can obtain the minimum of the confinement potential. Finally, we simplify the minimum into three independent harmonic oscillators

\[ V^{\text{con}}(4) = k \left( R_1^2 + R_2^2 + \frac{\kappa_d}{1 + \kappa_d} R_3^2 \right) \]

by diagonalizing the confinement potential matrix. \( R_i \) is normal mode of the confinement potential and reads

\[ R_1 = \frac{1}{\sqrt{2}}(r_1 - r_2), \quad R_2 = \frac{1}{\sqrt{2}}(r_3 - r_4), \]

\[ R_3 = \frac{\kappa_d}{1 + \kappa_d}(r_1 + r_2 - r_3 - r_4), \]

\[ R_4 = \frac{1}{\sqrt{4}}(r_1 + r_2 + r_3 + r_4). \]

One expects the model dynamics to be governed by QCD. The perturbative effect is well-known one gluon exchange (OGE) interaction. From the non-relativistic reduction of the OGE diagram in QCD for point-like quarks one gets

\[ V_{ij}^{\text{pge}} = \frac{\alpha_s}{4} \lambda_i^\top \cdot \lambda_j^\top \left( \frac{1}{r_{ij}} - \frac{2\pi \delta(r_{ij})\sigma_i \cdot \sigma_j}{3m_im_j} \right), \]

\[ \eta_i^\top \text{ and } \sigma_j \text{ stand for the color SU(3) Gell-mann matrices and spin SU(2) Pauli matrices, respectively. } r_{ij} \text{ is the distance between the particles } i \text{ and } j. \]

The Dirac \( \delta(r_{ij}) \) function should be regularized in the form \[ \delta(r_{ij}) \rightarrow \frac{1}{4\pi r_{ij}} e^{\frac{-r_{ij}}{\mu}}, \]

where \( \mu \) is the reduced mass of two interacting particles \( i \) and \( j \). The quark-gluon coupling constant \( \alpha_s \) adopts an effective scale-dependent form given as

\[ \alpha_s(\mu_i^2) = \frac{\alpha_0}{\ln \frac{\mu_i^2}{\Lambda_0^2}}, \]

\( \hat{\beta}_0, \Lambda_0 \) and \( \alpha_0 \) are adjustable parameters fixed by fitting the ground state meson spectrum.

The origin of the constituent quark mass can be traced back to the spontaneous breaking of SU(3) \( \times \) SU(3) chiral symmetry. The chiral symmetry is spontaneously broken in the light sector \((u, d \text{ and } s)\) while it is explicitly broken in the heavy sector \((c \text{ and } b)\). The meson exchange interactions only occur in the light quark sector. The central parts of the interactions can be resumed as follows. \[ V_{ij}^{\text{pbe}} = V_{ij}^{\text{pge}} + \frac{3}{7} \sum_{k=1}^{3} F_i^k F_j^k + \frac{7}{4} \sum_{k=4}^{8} F_i^k F_j^k, \]

\[ V_{ij}^{\text{pge}} = \frac{\alpha_s}{4\pi} \frac{m_i^3}{12m_i m_j} \Lambda_0^2 \cdot \frac{\sigma_i \cdot \sigma_j}{m_i^2} \left( Y(m_\chi r_{ij}) - \frac{\Lambda_0^2}{m_\chi^2} Y(\Lambda_\chi r_{ij}) \right) , \]

\[ Y(x) = e^{-x} x, \quad V_{ij}^{\sigma} = -\frac{g_{\sigma}}{4\pi} \frac{m_\sigma^2}{\Lambda_0^2 - m_\sigma^2} \left( Y(m_\sigma r_{ij}) - \frac{\Lambda_0^2}{m_\sigma^2} Y(\Lambda_\sigma r_{ij}) \right) . \]

\( F_i \) is the flavor SU(3) Gell-mann matrices and \( \chi \) represents \( \pi, K \text{ and } \eta \). The mass parameters \( m_\pi, m_K \text{ and } m_\eta \) take their experimental values. The cutoff parameters \( \Lambda_0 \) and the mixing angle \( \theta_P \) take the values from the work. The mass parameter \( m_\sigma \) can be determined through the PCAC relation \( m_\sigma^2 = m_\pi^2 + 4m_{u,d}^2 \). The chiral coupling constant \( g_{\sigma} \) can be obtained from the \( \pi NN \) coupling constant through

\[ \frac{g_{\sigma}^2}{4\pi} = \left( \frac{3}{5} \right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N}. \]
The most prominent characteristic is the application of the multibody confinement potential based on the color flux-tube picture instead of the two-body one in the other quark models.

### III. WAVE FUNCTIONS

Within the framework of the diquark-antiquark configuration, the trial wave function of the state \([cs][u\bar{d}]\) with \(I(J^P)\) can be constructed as a sum of the following direct products of color \(\varphi_c\), isospin \(\varphi_i\), spin \(\varphi_s\) and spatial \(\phi\) terms,

\[
\Phi_{jJ}^{[cs][u\bar{d}]} = \sum_{\alpha} c_{\alpha} \left[ \left[ \phi_{\alpha u_m}(r_a) \varphi_{s_{a}} \right] J_{\alpha} \left[ \phi_{\alpha l_m}(r_b) \varphi_{s_{b}} \right] J_{\alpha b} \right. \\
\times \phi_{\alpha c_m}(r_c) \left[ \varphi_{c_{a}} \varphi_{c_{b}} \right] J_{\alpha} \left[ \varphi_{c_{a}} \varphi_{c_{b}} \right] J_{\alpha b} \left. \right]_{c\bar{c}}^{[cs][u\bar{d}]} \]

The subscripts \(a\) and \(b\) represent the diquark \([cs]\) and antiquark \([u\bar{d}]\), respectively. All \([\alpha]\) represent all possible Clebsch-Gordan coupling. Summation index \(\alpha\) represents all of possible channels and the coefficient \(c_{\alpha}\) is determined by the model dynamics.

We define a set of Jacobian coordinate as

\[
r_a = r_1 - r_2, \quad r_b = r_3 - r_4, \\
r_c = \frac{m_1 r_1 + m_2 r_2 - m_3 r_3 + m_4 r_4}{m_1 + m_2 - m_3 + m_4} \]

(12)

to describe the relative motions in the state \([cs][u\bar{d}]\). The corresponding angular excitations of three relative motions are, respectively, \(l_a\), \(l_b\) and \(l_c\). In this work, we assume that the angular excitation only occurs between the diquark \([cs]\) and the antiquark \([u\bar{d}]\) so that the \(P\)-parity of the state is \((-1)^{l_c}\).

In order to obtain reliable numerical results, the precision numerical method is indispensable. The Gaussian expansion method [40], which has been proven to be rather powerful to solve few-body problem, is therefore used in the present work. According to the Gaussian expansion method, the relative motion wave function can be written as

\[
\phi_{lm}^{G}(x) = \sum_{n=1}^{n_{max}} c_n N_{nl} x^l e^{-\nu_n x^2} Y_{lm}(\hat{x}),
\]

(13)

where \(x\) represents \(r_a\), \(r_b\) and \(r_c\). Gaussian size \(\nu_n\) is taken as geometric progression

\[
\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 d^{n-1}, \quad d = \left( \frac{r_{\text{max}}}{r_1} \right)^{\frac{1}{n_{\text{max}}-1}},
\]

(14)

\(r_1\) and \(r_{\text{max}}\) are the minimum and maximum of the size, respectively. \(n_{\text{max}}\) is the number of the Gaussian wave function. More details about the Gaussian expansion method can be found in Ref. [40]. In the present work, we can obtain the convergent results by taking \(n_{\text{max}} = 7\), \(r_1 = 0.1 \text{ fm}\) and \(r_{\text{max}} = 2.0 \text{ fm}\).

Quark is in color \(3\), and antiquark is in color \(\bar{3}\). The color representation of the diquark \([cs]|_{\nu_{s1}}\) is antisymmetrical \(\bar{3}\), or symmetrical \(6\),

\[
\square \otimes \square = \square + \square,
\]

The color representation of the antiquark \([u\bar{d}]|_{\nu_{s2}}\) is antisymmetrical \(\bar{3}\), or symmetrical \(6\),

\[
\square \otimes \square = \square + \square.
\]

According to the requirement of overall color singlet of the state \([cs][u\bar{d}]\), there are two ways of coupling the diquark \([cs]|_{\nu_{s1}}\) and antiquark \([u\bar{d}]|_{\nu_{s2}}\) into an overall color singlet: \([cs]_{\bar{3},[u\bar{d}]_{\bar{3}}} \) and \([cs]_{6,[u\bar{d}]_{6}} \),

\[
[[cs]_{\bar{3},[u\bar{d}]_{\bar{3}}} ] = \frac{1}{\sqrt{3}} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right),
\]

\[
[[cs]_{6,[u\bar{d}]_{6}} ] = \frac{1}{\sqrt{6}} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right)
\]

The diquark \([cs]_{\nu_{s1}}\) and antiquark \([u\bar{d}]_{\nu_{s2}}\) can be in the spin singlet or triplet,

\[
\square \otimes \square = \square \oplus \square.
\]

The total spin \(S\) of the state \([cs]_{\nu_{s1}}[u\bar{d}]_{\nu_{s2}}\) can be expressed as \(S = s_u \oplus s_s\), its value could be 0, 1, or 2. For the state with \(S = 0\), it has two coupling ways, 0 \(\oplus 0\) and \(1 \oplus 1\). Their spin wave function reads

\[
[[cs]_{0,[u\bar{d}]_{0}} ] = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right),
\]

\[
[[cs]_{1,[u\bar{d}]_{1}} ] = \frac{1}{\sqrt{3}} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right),
\]

\[
[[cs]_{1,[u\bar{d}]_{0}} ] = \frac{1}{\sqrt{6}} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right)
\]

where \(\uparrow\) and \(\downarrow\) stand for spin up and spin down, respectively. For the state with \(S = 1\), it has three coupling ways, \(0 \oplus 1\), \(1 \oplus 0\) and \(1 \oplus 1\). Assuming the magnetic component \(M_s = S\), the corresponding spin wave function reads

\[
[[cs]_{0,[u\bar{d}]_{1}} ] = \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right),
\]

\[
[[cs]_{1,[u\bar{d}]_{0}} ] = \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right),
\]

\[
[[cs]_{1,[u\bar{d}]_{1}} ] = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right)
\]

For the state with \(S = 2\), its spin wave function reads

\[
[[cs]_{1,[u\bar{d}]_{2}} ] = \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right)
\]
The total isospin of the state is only determined by the antidiquark \([\bar{u}\bar{d}]_{i_b}\) because of the zero isospin of the diquark \([cs]_{i_a}\). Like the spin of the diquark or antidiquark, the antidiquark \([\bar{u}\bar{d}]_{i_b}\) can be isospin singlet and triplet. The isospin wave function reads

\[
[cs]_0[\bar{u}\bar{d}]_0 = \left[ cs \right] \begin{pmatrix} u \\ d \end{pmatrix} , \quad [cs]_0[\bar{u}\bar{d}]_1 = \left[ cs \right] \begin{pmatrix} u \\ \bar{d} \end{pmatrix} .
\]

The diquark and antidiquark are a spatially extended compound with various color-flavor-spin-space configurations \([42]\). The substructure of the diquarks may affect the structure of the multiquark states. Taking all degrees of freedom of identical quarks \(u\) and \(d\) into account, the Pauli principle must be satisfied by imposing some restrictions on the antidiquark \([\bar{u}\bar{d}]\). \(i_b + s_b = \text{even}\) if the antidiquark is in color \(3\), while \(i_b + s_b = \text{odd}\) if the antidiquark is in color \(6\).

The corresponding SU(2) groups of the isospin, and the so-called V-spin and U-spin are three subgroups of the flavor SU(3) group. The U-spin of the antidiquark \([\bar{s}\bar{d}]\), the V-spin of the diquark \([s\bar{u}]\) and the isospin of the antidiquark \([\bar{u}\bar{d}]\) have similar symmetry in their flavor wave functions. Therefore, the total wave functions of the states \([cs][\bar{u}\bar{d}]\), \([cs][s\bar{u}]\), and \([cs][\bar{s}\bar{d}]\) have the exactly same structure if the flavor SU(3) symmetry is involved. In order to avoid valueless repetition, we just present the details of the wave function construction of the state \([cs][\bar{u}\bar{d}]\).

IV. NUMERICAL RESULTS AND ANALYSIS

A. Meson spectrum and adjustable model parameters

We can determine the adjustable model parameters by approximately strict solving two-body Schrödinger equation to fit ground state meson spectrum in the MCFTM. With the Minuit program, we can obtain a set of optimal parameters and meson spectrum, which are presented in Table I and II respectively.

B. \([cs][\bar{u}\bar{d}]\) spectrum and candidates of \(T_{cs}(2900)\)

In the following, we concentrate on the properties of the ground and \(P\)-wave states \([cs][\bar{u}\bar{d}]\) with various spin, isospin and color combinations in the MCFTM with the parameters determined by the meson spectrum. Note that we do introduce any new adjustable parameters in the calculation on the tetraquark states.

Solving the four-body Schrödinger equation with the well-defined trial wave function, we can obtain the eigen energies of the states, which are presented in Table III. Meanwhile, we calculate the ratio of each color configuration in the coupled channels and the contributions coming from each part of the Hamiltonian using the corresponding eigenvectors. In Table III, \(3_-6_6\) and \(6_-6_6\) respectively stand for the color configurations \([cs]_0[\bar{u}\bar{d}]_0\) and \([cs]_0[\bar{u}\bar{d}]_0\) respectively.

In order to illustrate the spatial configuration of the states, we also calculate the average distance between two quark-anti-quark \(\bar{Q}Q\) and the relative distance \(r_{c}^{2}\) between the diquark \([cs]\) and the antiquark \([\bar{u}\bar{d}]\), which are listed in Table III. \(\langle r_{12}^{2}\rangle^{\frac{1}{2}}\) and \(\langle r_{34}^{2}\rangle^{\frac{1}{2}}\) represent the size of the diquark \([cs]\) and antiquark \([\bar{u}\bar{d}]\), respectively. \(\langle r_{12}^{2}\rangle^{\frac{1}{2}}\) is equal to \(\langle r_{14}^{2}\rangle^{\frac{1}{2}}\) and \(\langle r_{34}^{2}\rangle^{\frac{1}{2}}\) is equal to \(\langle r_{24}^{2}\rangle^{\frac{1}{2}}\) because the quarks \(u\) and \(d\) are identical particles. All of the distances are less than or around 1 fm so that the states \([cs][\bar{u}\bar{d}]\) should be compact spatial configuration in the model because of the multi-body confinement potential, which is a collective degree of freedom and bind all particles together.

For the ground states, the diquark \([cs]\) and the antiquark \([\bar{u}\bar{d}]\) have a strongly overlap because of the smaller distance \(r_{c}^{2}\) relative to the sizes of the diquark \([cs]\) and antiquark \([\bar{u}\bar{d}]\), see \(\langle r_{12}^{2}\rangle^{\frac{1}{2}}\), \(\langle r_{34}^{2}\rangle^{\frac{1}{2}}\), and \(\langle r_{24}^{2}\rangle^{\frac{1}{2}}\). For the \(P\)-wave states, the sizes of the diquark \([cs]\) and antiquark \([\bar{u}\bar{d}]\) do not change dramatically relative to those of the corresponding ground states because the angular excitation only occurs between the diquark \([cs]\) and antiquark \([\bar{u}\bar{d}]\). However, the distance between the diquark \([cs]\) and antiquark \([\bar{u}\bar{d}]\) obviously increase, also see \(\langle r_{12}^{2}\rangle^{\frac{1}{2}}\), \(\langle r_{34}^{2}\rangle^{\frac{1}{2}}\), and \(\langle r_{24}^{2}\rangle^{\frac{1}{2}}\). The \(P\)-wave states look like a dumbbell-like spatial configuration because the \([cs]\) and \([\bar{u}\bar{d}]\) is separated gradually.

One can find from Table III that the color configuration \(3_-, 3_+\) is dominant in the states with \(S = 0\) and 1, especially for the states \([cs][\bar{u}\bar{d}]\) with \(I = 0\). In the configuration \(3_-, 3_+\), the interactions \(V_{cm}, V_{ctb}\) and \(V_{\pi}\) can give much stronger attractions than they do in the configuration \(6_-, 6_+\). With the increasing of the mass ratio of \(m_Q\) and \(m_{\bar{q}}\), where \(Q = s, c\) or \(b\) and \(\bar{q} = \bar{u}\) or \(\bar{d}\), the configuration \(3_-, 3_+\) gradually increase in the states \([QQ][\bar{u}\bar{d}]\) [11]. The underlying dynamical mechanism of such phenomenological regularity in the configuration \(3_-, 3_+\) is governed by the color Coulomb interaction in the
TABLE III: Mass of the state $[cs]\bar{u}\bar{d}$ and contribution from each part of the Hamiltonian unit in MeV, and the average distances unit in fm, $J = l_c \oplus S$. C.C. represents the coupling of the two color configurations. $E_k$, $V_{\text{con}}$, $V_{\text{colb}}$, $V^n$, $V^\pi$, $V^K$ and $V^\sigma$ represent kinetic energy, confinement potential, color-magnetic interaction, Coulomb interaction, $\eta$ exchange interaction, $\pi$ exchange interaction, and $\sigma$ exchange interaction, respectively.

\begin{tabular}{cccccccccc}
\hline
\textbf{$l_c$} & \textbf{S} & \textbf{$IJ^P$} & \textbf{Color} & \textbf{Mass} & \textbf{Ratio} & $\langle E_k \rangle$ & $\langle V_{\text{con}} \rangle$ & $\langle V_{\text{colb}} \rangle$ & $\langle V^n \rangle$ & $\langle V^\pi \rangle$ & $\langle V^K \rangle$ & $\langle V^\sigma \rangle$ & $\langle r^2_2 \rangle$ & $\langle r^2_3 \rangle$ & $\langle r^2_4 \rangle$ & $\langle r^2_5 \rangle$ \\
\hline
0 & 00\textsuperscript{+} & \textcolor{red}{3}_{-3}\textcolor{blue}{c} & 3201, 98\% & 1486 & 391 & -267 & -1014 & 67 & -375 & 0 & -70 & 0.67 & 0.96 & 1.08 & 0.97 & 0.89 \\
 & & \textcolor{red}{6}_{-6}\textcolor{blue}{c} & 3241, 2\% & 1007 & 502 & -108 & -739 & 5 & 0 & -47 & 0.77 & 1.01 & 1.02 & 1.05 & 0.75 \\
 & & C.C. & 2893 & 1478 & 385 & -283 & -1017 & 66 & -367 & 0 & -70 & 0.66 & 0.69 & 0.95 & 1.07 & 0.88 \\
0 & 01\textsuperscript{-} & \textcolor{red}{3}_{-3}\textcolor{blue}{c} & 2960, 98\% & 1486 & 391 & -267 & -1014 & 67 & -375 & 0 & -70 & 0.67 & 0.87 & 1.08 & 0.97 & 0.89 \\
 & & \textcolor{red}{6}_{-6}\textcolor{blue}{c} & 3341, 2\% & 1007 & 502 & -108 & -739 & 5 & 0 & -47 & 0.77 & 1.01 & 1.02 & 1.05 & 0.75 \\
 & & C.C. & 2893 & 1478 & 385 & -283 & -1017 & 66 & -367 & 0 & -70 & 0.66 & 0.69 & 0.95 & 1.07 & 0.88 \\
0 & 01\textsuperscript{+}, 00\textsuperscript{+}, 02\textsuperscript{+} & \textcolor{red}{6}_{-6}\textcolor{blue}{c} & 3433, 1\% & 912 & 545 & -5 & -697 & -2 & 22 & 0 & -43 & 0.81 & 1.04 & 0.97 & 1.14 & 0.80 \\
 & & C.C. & 2938 & 1424 & 404 & -228 & -987 & 66 & -375 & 0 & -69 & 0.71 & 0.69 & 0.97 & 1.10 & 0.90 \\
0 & 10\textsuperscript{-}, 02\textsuperscript{-}, 03\textsuperscript{-} & \textcolor{red}{6}_{-6}\textcolor{blue}{c} & 3464, 100\% & 882 & 557 & 26 & -679 & -3 & 21 & 0 & -41 & 0.81 & 1.05 & 0.99 & 1.16 & 0.81 \\
0 & 11\textsuperscript{-} & \textcolor{red}{3}_{-3}\textcolor{blue}{c} & 3275, 94\% & 967 & 491 & 13 & -831 & -4 & -14 & 0 & -48 & 0.71 & 0.93 & 1.01 & 1.14 & 0.87 \\
 & & \textcolor{red}{6}_{-6}\textcolor{blue}{c} & 3475, 6\% & 886 & 566 & 33 & -690 & 3 & 16 & 0 & -40 & 0.82 & 1.09 & 0.99 & 1.16 & 0.80 \\
 & & C.C. & 3259 & 990 & 582 & -16 & -834 & -3 & -12 & 0 & -49 & 0.70 & 0.93 & 0.99 & 1.12 & 0.86 \\
1 & 10\textsuperscript{-}, 11\textsuperscript{-}, 12\textsuperscript{-} & \textcolor{red}{6}_{-6}\textcolor{blue}{c} & 3471, 2\% & 982 & 565 & 25 & -691 & 3 & 16 & 0 & -40 & 0.80 & 1.08 & 0.98 & 1.15 & 0.80 \\
 & & C.C. & 3259 & 994 & 487 & -9 & -847 & -5 & -13 & 0 & -48 & 0.67 & 0.93 & 1.02 & 1.13 & 0.90 \\
2 & 11\textsuperscript{-}, 12\textsuperscript{-}, 13\textsuperscript{-} & \textcolor{red}{3}_{-3}\textcolor{blue}{c} & 3316, 100\% & 929 & 510 & 56 & -814 & -6 & -14 & 0 & -46 & 0.71 & 0.93 & 1.04 & 1.17 & 0.92 \\
\hline
\end{tabular}

The ground state $[cs][u\bar{d}]$ with $I(J^P) = 0(0^+)$ and $3_{-3}$ configuration has a low mass of 2559 MeV due to the strong $\pi$-meson exchange. After coupling with the color configuration $6_{-6}$, the mass of the state with $0(0^+)$ is further pushed down to 2495 MeV, which is much lower, about 370 MeV, than that of the state $T_{s0}(2900)^0$ reported by the LHCb Collaboration. In this way, the state $T_{s0}(2900)^0$ cannot be depicted as the state $[cs][u\bar{d}]$ with $0(0^+)$ in the model. Similar model study on the state $[cs][u\bar{d}]$ was carried out in Refs. \cite{23, 24}, where the authors did not take into account the meson exchange in their models. None of their predicted masses of the state with $0(0^+)$ can match with that of the state $T_{s0}(2900)^0$. In one word, the state $T_{s0}(2900)^0$ may be not the compact state $[cs][u\bar{d}]$ with $0(0^+)$ in the quark models with QCD-inspired dynamics. However, various color-magnetic models without explicit dynamics can interpret the main component of the state $T_{s0}(2900)^0$ as the compact state $[cs][u\bar{d}]$ with $0(0^+)$ \cite{21, 21}. On the other hand, the color-magnetic models do not seem to completely absorb the dynamic effect by the effective mass of constituent quarks \cite{13}.

The ground state $[cs][u\bar{d}]$ with $I(J^P) = 0(0^+)$ is higher about 100 MeV than that of the state with $0(0^+)$ mainly due to the relative weak color-magnetic interaction and Coulomb interaction. The color configuration $6_{-6}$ has a very tiny percentage, just 2\%, so that it can be abandoned in the state $[cs][u\bar{d}]$ with $0(1^+)$. The state $[cs][u\bar{d}]$ with $0(2^+)$ has a very high energy of 3068 MeV because of the absence of the color configuration $3_{-3}$.

For the ground states with $I(J^P) = 1(0^+)$ and $1(1^+)$, their masses are much higher than the states with $0(0^+)$ and $0(1^+)$, respectively. Such regularity also holds true for their corresponding $F$-wave states with $I = 0$ and 1, see Table \textsuperscript{11} which mainly originates from their different...
contribution of the $\pi$-meson exchange. It provides very strong interaction in the states with $I = 0$ while it gives a weak interaction in the states with $I = 1$. For the high-spin ($S = 2$) ground states, the mass splitting between the states with $I = 0$ and $I = 1$ resulted from the $\pi$-meson exchange is not as obvious as the low-spin states.

In the ground state $[cs][\bar{u}\bar{d}]$ with $I(J^P) = 1(0^+)$, its main color configuration is $3_s^3 - 3_s^1$, reaching 78%, and its corresponding spin configuration is $1 \mp 1$, namely consisting of axial-vector $[cs]_{3^1}$ and axial-vector $[\bar{u}\bar{d}]_{3^1}$, see Table III. Its mass, about 2923 MeV, is a little higher than that of the state $T_{c\bar{s}0}(2900)^0$. Taking the coupling with the color configuration $6 - 6$ into account, the mass can be pushed down to 2871 MeV, which is highly consistent with that of the state $T_{c\bar{s}0}(2900)^0$. In this way, we can describe the state $T_{c\bar{s}0}(2900)^0$ as the ground state $[cs][\bar{u}\bar{d}]$ with $1(0^+)$ in the MCFTM, which is supported by the conclusions of the similar model research and QCD sum rule [23, 24]. If the state $T_{c\bar{s}0}(2900)^0$ really belongs to an isorotplet, its charged partners would be abundant, which deserves further research in the future.

On the contrary, the diquark picture $[cs][\bar{u}\bar{d}]$ seems to prefer to the $I(J^P)$ assignment of $0(0^+)$ in the color-magnetic models and QCD sum rule [20, 22]. Assuming the state $T_{c\bar{s}0}(2900)^0$ is determined to be isosinglet eventually, the molecular configuration $D^*K^*$ may be a suitable candidate in the models. In order to discriminate all possible interpretations, Burns et al carried out an exhaustive analysis on their decay behaviors as well as their productions in $B^0$ and $B^+$ decays [30].

In the $P$-wave states, we do not consider the spin-orbit interaction in the present work because its contributions are very small, just about several MeV [13]. It does not change the qualitative conclusions for the compact tetraquark states. The spin singlet with $0(1^-)$ has a mass of 2893 MeV in the MCFTM, see Table III which is in good agreement with that of the state $T_{c\bar{s}1}(2900)^0$. Its dominant component is composed of scalar $[cs]_{3^1}$ and scalar $[\bar{u}\bar{d}]_{3^1}$. In addition, the spin triplet with $0(1^-)$ has a mass of about 2938 MeV and it consists of scalar $[\bar{u}\bar{d}]_{3^1}$ and axial vector $[cs]_{3^1}$. The state is not far away from the state $T_{c\bar{s}1}(2900)^0$ so that we can not rule out the fact that its main component may be made of scalar $[\bar{u}\bar{d}]_{3^1}$ and axial vector $[cs]_{3^1}$. In one word, we can describe the state $T_{c\bar{s}1}(2900)^0$ as the compact state $[\bar{u}\bar{d}]_{3^1}$ with $0(1^-)$ in the MCFTM. Its main component could be consisted of scalar or axial vector $[cs]_{3^1}$ and scalar $[\bar{u}\bar{d}]_{3^1}$. Whichever description in the compact state $[cs][\bar{u}\bar{d}]$ and molecular state $D_1K$, the state $T_{c\bar{s}1}(2900)^0$ seems to prefer the $I(J^P)$ assignment of $0(1^-)$ [13, 14, 15, 16].

The states with $0(1^-)$ and $S = 2$ are much higher, about 500 MeV, than the state $T_{c\bar{s}1}(2900)^0$, which should not be the main component of the state $T_{c\bar{s}1}(2900)^0$. All of the $P$-wave states with $I = 1$ have the similar masses, around 3300 MeV, which are also much higher the state $T_{c\bar{s}1}(2900)^0$. Therefore, the state $T_{c\bar{s}1}(2900)^0$ should not be an isospin triplet if it is a compact state $[cs][\bar{u}\bar{d}]$ in the MCFTM.

C. $|[cu][\bar{s}\bar{d}]$ spectrum and $T_{c\bar{s}0}^{a}(2900)$

Assuming the states $T_{c\bar{s}0}^{a}(2900)^0$ and $T_{c\bar{s}0}^{a}(2900)^{++}$ belong to the same isospin triplet, we also investigate the properties of the ground and $P$-wave states $|cu|[\bar{s}\bar{d}]$ with various spin, $U$-spin and color combinations in the MCFTM. Similar to the isospin of the antidiquark $[\bar{u}\bar{d}]$ in the state $[cs][\bar{u}\bar{d}]$, we consider the $U$-spin of the antidiquark $[\bar{s}\bar{d}]$ in the state $|cu|[\bar{s}\bar{d}]$. In this way, we can define $U = 0$ for the $U$-spin antisymmetrical $[\bar{s}\bar{d}]$ and $U = 1$ for the $U$-spin symmetrical $[\bar{s}\bar{d}]$. Namely, we can also define the V-spin for the state $|cd|[\bar{s}\bar{d}]$. Numerical results for the states $|cu|[\bar{s}\bar{d}]$ are presented in Table XIV. It can be found from Tables III and XIV that the states $[cs][\bar{u}\bar{d}]$ and $|cu|[\bar{s}\bar{d}]$ have similar spectrum.

In the low-spin ($S \leq 1$) states $|cu|[\bar{s}\bar{d}]$ and $[cs][\bar{u}\bar{d}]$, the magnitude of the $\pi$- and $K$-meson exchange interactions are distinguished, which results in their mass difference. The masses of the states $|cu|[\bar{s}\bar{d}]$ with $U = 1$ are slightly lower than those of the states $[cs][\bar{u}\bar{d}]$ with $I = 1$, which mainly originates from the different contribution from the $K$-meson exchange interaction. In the states $|cu|[\bar{s}\bar{d}]$ with $U = 1$, the interaction can provide a little attraction while the interaction vanishes in the states $[cs][\bar{u}\bar{d}]$. However, the states $|cu|[\bar{s}\bar{d}]$ with $U = 0$ are much higher than those of the states $[cs][\bar{u}\bar{d}]$ with $I = 0$ because of the strong attraction induced by the $\pi$-meson exchange interaction. The high-spin ($S = 2$) states $|cu|[\bar{s}\bar{d}]$ and $[cs][\bar{u}\bar{d}]$ are almost degenerate because both the $\pi$- and $K$-meson exchange interactions are very weak.

The QCD sum rule explored the doubly charged states $|[s\bar{d}][\bar{u}\bar{c}]$ with the spin-parity of $0^+, 0^-$ and $1^+$ [43]. The states with $0^+$ and $1^+$ have masses of 2628 $^{+106}_{-153}$ MeV and 2826 $^{+134}_{-157}$ MeV, respectively, which are consistent with the corresponding results in the present work within the error range. The mass of the state with $0^-$ is 2719 $^{+143}_{-156}$ MeV [45], which is much lower than that of the state in the present work. The QCD sum rule further researched the state $|[s\bar{d}][\bar{u}\bar{c}]$ with $1^-$ and gave a mass of 3515 $\pm$ 125 MeV [46], which is much higher than the model prediction on the state.

The mass of the state $|[cu][\bar{s}\bar{d}]|_{3^3}$, with $U(J^P) = 1(0^+)$ is 2923 MeV, see Table XIV, which is highly consistent with those of the states $T_{c\bar{s}0}^{a}(2900)^0$ and $T_{c\bar{s}0}^{a}(2900)^{++}$ reported by the LHCh Collaboration. The state is a compact state composed of an axial-vector $|cu|_{3^3}$ and axial-vector $|\bar{s}\bar{d}|_{3^3}$. The state $|cu|_6[\bar{s}\bar{d}]_6$, with $U(J^P) = 1(0^+)$ is much higher than those of the states $T_{c\bar{s}0}^{a}(2900)^0$ and $T_{c\bar{s}0}^{a}(2900)^{++}$. After coupling two color configurations, the mass of the state $|cu|[\bar{s}\bar{d}]$ with $U(J^P) = 1(0^+)$ can be decreased to 2837 MeV, which is slightly lighter than those of the states $T_{c\bar{s}0}^{a}(2900)^0$ and $T_{c\bar{s}0}^{a}(2900)^{++}$. Therefore, the state $|cu|[\bar{s}\bar{d}]$ with $U(J^P) = 0(0^+)$ be the main component of the states $T_{c\bar{s}0}^{a}(2900)^0$ and $T_{c\bar{s}0}^{a}(2900)^{++}$. The state $|cu|[\bar{s}\bar{d}]$ with $U(J^P) = 0(0^+)$, the partner of the states $T_{c\bar{s}0}^{a}(2900)^0$ and $T_{c\bar{s}0}^{a}(2900)^{++}$, may exist and has a mass of about 2583 MeV in the model.
TABLE IV: Mass of the state $[cu][sd]$ and contributions from each part of the Hamiltonian unit in MeV, and the average distances unit in fm. $U$ represents the $U$-spin, $U = 0$ and 1 denote the antisymmetrical and symmetrical $[sd]$, respectively. Other symbols have the same meanings with those in Table II.

| $l_c$ | $S$ | $UJ^P$ | Color | Mass,Ratio | $\langle E_k \rangle$ | $(V_{con}^{cm})$ | $(V_{cm})$ | $(V_{ch})$ | $(V^\pi)$ | $(V^\sigma)$ | $r_1^2$ | $r_2^2$ | $r_3^2$ | $r_4^2$ | $r_5^2$ | $r_6^2$ | $r_7^2$ | $r_8^2$ | $r_9^2$ |
|------|-----|--------|-------|------------|----------------|----------------|-----------|-----------|-----------|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0    | 00^+| 3--3c  | 2710  | 60% 1399 | 300 -206 -1132 -27 0 -148 -87 0.73 0.66 0.64 0.95 0.59 |
| 0    | 10^+| 6--6c  | 2778  | 40% 1162 | 318 -236 -1087 5 -23 13 -75 0.73 0.81 0.55 0.93 0.45 |
|      |     | C.C.   | 2583  | 1461 262 | 422 -122 -1141 -14 12 -86 94 0.67 0.68 0.55 0.86 0.48 |
| 0    | 01^+| 3--3c  | 2757  | 94% 1254 | 315 -155 -1100 -27 0 -147 -84 0.76 0.66 0.65 0.98 0.59 |
| 0    | 11^+| 6--6c  | 3003  | 6% 905  | 396 1 -951 2 0 9 -60 0.83 0.88 0.62 1.04 0.52 |
|      |     | C.C.   | 2737  | 1273 308 | -186 -1112 -25 0 -137 -85 0.75 0.67 0.63 0.96 0.57 |
| 2    | 02^+| 6--6c  | 3073  | 100% 838 | 422 61 -909 1 7 8 56 0.85 0.81 0.85 0.98 0.59 |
|      |     | C.C.   | 2923  | 75% 928  | 354 -27 -993 5 -18 -18 -69 0.74 0.81 0.66 1.00 0.58 |
| 0    | 10^+| 6--6c  | 3048  | 25% 865  | 417 36 -929 -3 0 17 -56 0.83 0.92 0.63 1.06 0.52 |
|      |     | C.C.   | 2837  | 1075 331 | -137 -1041 3 -15 7 -73 0.72 0.80 0.61 0.96 0.52 |
| 0    | 11^+| 6--6c  | 3033  | 23% 879  | 410 21 -936 -3 0 18 -57 0.82 0.92 0.63 1.05 0.52 |
|      |     | C.C.   | 2907  | 857 357  | -64 -999 1 0 -8 -68 0.73 0.83 0.65 1.00 0.57 |
| 2    | 12^+| 3--3c  | 3028  | 100% 858 | 398 64 -926 2 7 -15 -61 0.79 0.85 0.71 1.07 0.63 |
| 0    | 01^-| 6--6c  | 3270  | 6% 1027  | 469 -123 -752 2 -11 7 -50 0.83 0.93 0.79 1.16 0.70 |
|      |     | C.C.   | 2992  | 1316 393 | -229 -970 3 -23 1 -126 -69 0.75 0.70 0.86 1.13 0.81 |
| 0    | 10^-| 6--6c  | 3055  | 99% 1251 | 421 -147 -943 -25 0 -136 -67 0.80 0.69 0.89 1.17 0.85 |
|      |     | C.C.   | 3051  | 1255 418  | -154 -944 -24 0 -134 -67 0.80 0.69 0.88 1.17 0.84 |
| 2    | 01^-| 6--6c  | 3432  | 100% 865 | 541 33 -677 1 4 6 -42 0.90 0.97 0.86 1.25 0.77 |
| 0    | 11^-| 6--6c  | 3429  | 8% 878  | 544 28 -691 2 0 11 -41 0.90 1.00 0.85 1.25 0.76 |
|      |     | C.C.   | 3208  | 1001 459  | -40 -843 3 0 9 -12 -52 0.78 0.86 0.87 1.18 0.80 |
| 1    | 01^-| 6--6c  | 3393  | 1% 901  | 524 1 -697 1 0 6 -44 0.89 0.96 0.84 1.23 0.75 |
| 2    | 02^-| 6--6c  | 3432  | 100% 865 | 541 33 -677 1 4 6 -42 0.90 0.97 0.86 1.25 0.77 |

V. SUMMARY

Recently, the LHCb Collaboration reported the states $T_{cs0}(2900)^0$, $T_{cs1}(2900)^0$, $T_{cs0}^\pi(2900)^0$ and $T_{cs0}^\sigma(2900)^{++}$. The spin-parity of the states $T_{cs0}(2900)^0$, $T_{cs1}(2900)^0$ is $0^+$ and $1^-$, respectively. Their smallest quark content is $cs\bar{u}d$ while their isospin has not been available until now. The least quark contents of the states $T_{cs0}^\pi(2900)^0$ and $T_{cs0}^\sigma(2900)^{++}$ are $c\bar{s}\bar{u}$ and $c\bar{u}s\bar{d}$, respectively. The states share the same the spin-parity $0^+$. The study on the states may widen the insight into hadron structure and also help us to improve our knowledge of the low-energy strong interaction.

With the Gaussian expansion method, a high precision numerical method, we employ the a multiquark color flux-tube model to make a systematical investigation on the properties of the ground and P-wave states $[cu][\bar{u}\bar{d}]$ and $[cu][sd]$ with various spin, isospin or U-spin and color combinations in the present work. The model includes a multibody confinement potential, one-gluon-exchange interaction, and one-boson-exchange interaction ($\pi, K$ and $\eta$), $\sigma$-meson exchange interaction. The multi-body confinement potential is a collective degree of freedom, which can bind all particles together to establish a compact state. The states $[cs][\bar{u}\bar{d}]$ and $[cu][sd]$ have similar mass spectra in the model. The mass difference between two states mainly originates from the different magnitudes of the $\pi$-meson and $K$-meson exchange interactions in the states.

Matching our results with the spin-parity and mass of the states $T_{cs0}(2900)^0$ and $T_{cs1}(2900)^0$ reported by the LHCb Collaboration, we can describe them as the compact states $[cs][\bar{u}\bar{d}]$ with $I(J^P) = 1(0^+)$ and $0(1^-)$ in the model, respectively. The ground state $T_{cs0}(2900)^0$ is mainly made of strongly overlapped axial-vector $[cs]_3^a$ and axial-vector $[\bar{u}\bar{d}]_3^a$. If the state $T_{cs0}(2900)^0$ really belongs to an isotriplet with diquark-antidiquark picture, its charged partners would be abundant in the model. The P-wave state $T_{cs1}(2900)^0$ is dominantly consisted of gradually separated scalar or axial vector $[cs]_3^a$ and
scalar $[\bar{u}\bar{d}][\bar{s}\bar{d}]_3$ in the shape of a dumbbell. In addition, the states $[cs][\bar{u}\bar{d}]$ with $J(F^P) = 0^+$ and $0^+$ may exist and the predicted masses are about 2500-2600 MeV.

The predicted mass of the state $[[cu][\bar{s}\bar{d}]_3]_{48}$ with $U(JF^P) = 1(0^+)$ in the model is in good agreement with that of the states $T_{250}(2900)^0$ and $T_{250}^a(2900)^{++}$. After considering the coupling of two color configurations, the state $[cu][\bar{s}\bar{d}]$ is a little lighter than the states $T_{250}(2900)^0$ and $T_{250}^a(2900)^{++}$. In this way, we can not conclude the possibility that the state $[cu][\bar{s}\bar{d}]$ with $U(JF^P) = 1(0^+)$ may be the main component of the states $T_{250}(2900)^0$ and $T_{250}^a(2900)^{++}$ in the model. The state $[cu][\bar{s}\bar{d}]$ with $U(JF^P) = 0(0^+)$, the partner of the states $T_{250}(2900)^0$ and $T_{250}^a(2900)^{++}$, may exist and has a predicted mass of about 2583 MeV.

Hopefully, the systematical investigation on the states $[cs][\bar{u}\bar{d}]$ and $[cu][\bar{s}\bar{d}]$ will be useful for the understanding of the properties of the exotic states $T_{310}(2900)$ and $T_{310}^a(2900)$ and the search of the new tetraquark states. We also sincerely expect more experimental and theoretical researches to verify and understand the tetraquark states in the future.

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