Fermion Damping in a Fermion-Scalar Plasma

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In this article we study the dynamics of fermions in a fermion-scalar plasma. We begin by obtaining the effective in-medium Dirac equation in real time which is fully renormalized and causal and leads to the initial value problem. For a heavy scalar we find the novel result that the decay of the scalar into fermion pairs in the medium leads to damping of the fermionic excitations and their in-medium propagation as quasiparticles. That is, the fermions acquire a width due to the decay of the heavier scalar in the medium. We find the damping rate to lowest order in the Yukawa coupling for arbitrary values of scalar and fermion masses, temperature and fermion momentum. An all-order expression for the damping rate in terms of the exact quasiparticle wave functions is established. A kinetic Boltzmann approach to the relaxation of the fermionic distribution function confirms the damping of fermionic excitations as a consequence of the induced decay of heavy scalars in the medium. A linearization of the Boltzmann equation near equilibrium clearly displays the relationship between the damping rate of fermionic mean fields and the fermion interaction rate to lowest order in the Yukawa coupling directly in real time.

I. INTRODUCTION

The propagation of quarks and leptons in a medium of high temperature and/or density is of fundamental importance in a wide variety of physically relevant situations. In stellar astrophysics, electrons and neutrinos play a major role in the evolution of dense stars such as white dwarfs, neutron stars and supernovae. In ultrarelativistic heavy ion collisions and the possibility of formation of a quark-gluon plasma, electrons (and muons) play a very important role as clean probes of the early, hot stage of the plasma. Furthermore, medium effects can enhance neutrino oscillations as envisaged in the Mikheyev-Smirnov-Wolfenstein (MSW) effect and dramatically modify the neutrino electromagnetic couplings. The propagation of quarks during the non-equilibrium stages of the electroweak phase transition is conjectured to be an essential ingredient for baryogenesis at the electroweak scale both in non-supersymmetric and supersymmetric extensions of the standard model.

In-medium propagation is dramatically different from that in vacuum. The medium modifies the dispersion relation of the excitations and introduces a width to the propagating excitation that results in damping of the amplitude of the propagating mode. In this article we focus on several aspects of propagation of fermionic excitations in a fermion-scalar plasma:

- We begin by deriving the effective and fully renormalized Dirac equation in real time. This is achieved by relating the expectation value of a fermionic field induced by an external fermionic source via linear response to an initial value problem for the expectation value. This initial value problem is in terms of the effective real time Dirac equation in the medium that is i) renormalized, ii) retarded and causal.

- We apply the effective Dirac equation in a medium to study the real-time evolution of fermionic excitations in a fermion-scalar plasma. Whereas the propagation of quarks and leptons in a QED or QCD plasma has been studied thoroughly (see ref. for details) a similar study for a scalar plasma has not been carried...
out to the same level of detail. Recently some attention has been given to understanding the thermalization time scales of bosonic and fermionic excitations in a plasma of gauge and scalar bosons, furthermore fermion thermalization is an important ingredient in models of baryogenesis mediated by scalars. Most of the studies of fermion thermalization focus on the mechanism of fermion scattering off the gauge quanta in the heat bath and/or Landau damping in the hard thermal loop (HTL) resummation program. Although the scalar contribution to the fermionic self-energy to one loop has been obtained a long time ago, scant attention has been paid to a more detailed understanding of the contribution from the scalar degrees of freedom to the fermion relaxation and thermalization. As mentioned above this issue becomes of pressing importance in models of baryogenesis and more so in models in which the scalars carry baryon number.

Whereas the contribution from scalars to the fermionic thermalization time scale (damping rate) has been studied for massless chiral fermions, in this article we offer a detailed and general study of fermion relaxation and thermalization through the interactions with the scalars in the plasma in real time and for arbitrary values of the scalar and fermion masses, temperature and fermion momentum. More importantly, we focus on a novel mechanism of damping of fermionic excitations that occurs whenever the effective mass of the scalar particle allows its kinematic decay into fermion pairs. This phenomenon only occurs in a medium and is interpreted as an induced decay of the scalar in the medium. It is a process different from collisions with particles in the bath and Landau damping which are the most common processes that lead to relaxation and thermalization.

This process results in new thermal cuts in the fermionic self-energy and for heavy scalars, this cut results in a quasiparticle pole structure for the fermion and provides a width for the fermionic quasiparticle. The remarkable and perhaps non-intuitive aspect of this process is that the decay of the scalar results in damping of the fermionic excitations and their propagation as quasiparticle resonances. Our real time analysis reveals that amplitude of the $\hat{k}$-mode of the expectation value of the fermion field decreases as $e^{-\Gamma_k t}$ while it oscillates with frequency $\omega_k(k)$ which is determined by the position of the resonance.

The effective real-time Dirac equation in the medium allows a direct interpretation of the damping of the quasiparticle fermionic excitation and leads to a clear definition of the damping rate. By analyzing the quasiparticle wave functions we obtain an all-order expression for the damping rate $\Gamma_k$ and confirm and generalize recent results for the massless chiral case.

- In order to provide a complementary understanding of the process of induced decay of the heavy scalars in the medium and the resulting fermion damping, we study the kinetics of relaxation of the fermionic distribution function via a Boltzmann equation to lowest order. Linearizing the Boltzmann equation near the equilibrium distribution, we obtain the relation between the thermalization rate for the distribution function in the relaxation time approximation (linearized near equilibrium) and the damping rate for the amplitude of the fermionic mean fields to lowest order in the Yukawa coupling. This analysis provides a real-time confirmation of the oft quoted relation between the interaction rate (obtained from the Boltzmann kinetic equation in the relaxation time approximation) and the damping rate for the mean field. More importantly, this analysis reveals directly, via a kinetic approach in real time how the process of induced decay of a heavy scalar in the medium results in damping and thermalization of the fermionic excitations. A study of the relation between the interaction rate and the damping rate has been presented recently for gauge theories within the context of the imaginary time formulation. Our results provide a real-time confirmation of those of reference for the scalar case.

The article is organized as follows: in section II we obtain the effective in-medium Dirac equation in real time starting from the linear response to an external Grassmann-valued source that induces a mean field. We obtain the fully renormalized Dirac equation with the real time self-energy to one loop order by turning the linear response problem into a initial value problem for the mean field. The renormalization aspects are addressed in detail in this section. In section III we study in detail the structure of the renormalized self-energy and establish the presence of new cuts of thermal origin. We then note that for heavy scalars such that their decay into fermion pairs is kinematically allowed, the fermionic pole becomes embedded in this thermal cut resulting in a quasiparticle (resonance) structure, which is analyzed in detail. The decay rate is evaluated in the narrow width approximation (justified for small Yukawa couplings) for arbitrary values of the scalar and fermion masses, temperature and fermion momentum.

In section IV we present a real-time analysis of the evolution of the mean-fields. In this section we clarify the difference between complex poles and resonances (often misunderstood). This analysis reveals clearly that the induced decay of the scalar results in an exponential damping of the amplitude of the mean field and yields to a clear identification of the damping rate bypassing the conflicting definitions of the damping rate offered in the literature. An analysis of the structure of the self-energy and an interpretation of the exact quasiparticle spinor wave functions allows us to provide an all-order expression for the damping rate of the fermionic mean fields. In section V we present an analysis of the evolution of the distribution functions in real time by obtaining a Boltzmann kinetic equation.
II. EFFECTIVE DIRAC EQUATION IN THE MEDIUM

As mentioned in the introduction, whereas the damping of collective and quasiparticle excitations via the interactions with gauge bosons in the medium has been the focus of most attention, understanding of the influence of scalars has not been pursued so vigorously.

Although we are ultimately interested in studying the damping of fermionic excitations in a plasma with scalars and gauge fields within the realm of electroweak baryogenesis in either the Standard Model or generalizations thereof, we will begin by considering only the coupling of a massive Dirac fermion to a scalar via a simple Yukawa interaction.

The model dependent generalizations of the Yukawa couplings to particular cases will differ quantitatively in the details of the group structure but the qualitative features of the effective Dirac equation in the medium as well as the kinematics of the thermal cuts that lead to damping of the fermionic excitations will be rather general.

We consider a Dirac fermion with the bare mass \( M_0 \) coupled to a scalar with the bare mass \( m_0 \) via a Yukawa coupling. The bare fermion mass could be the result of spontaneous symmetry breaking in the scalar sector, but for the purposes of our studies we need not specify its origin.

The Lagrangian density is given by

\[
\mathcal{L} = \bar{\Psi}(i \gamma \cdot D - M_0)\Psi + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m_0^2 \phi^2 - \mathcal{L}_I[\phi] - \frac{y_0}{2} \bar{\Psi} \phi \Psi + \bar{\eta} \Psi + \bar{\psi} \eta + j \phi ,
\]

where \( y_0 \) is the bare Yukawa coupling. The self-interaction of the scalar field accounted for by the term \( \mathcal{L}_I[\phi] \) need not be specified to lowest order. The \( \eta \) and \( j \) are the respective external fermionic and scalar sources that are introduced in order to provide an initial value problem for the effective Dirac equation.

We now write the bare fields and sources renormalization constants and counterterms:

\[
\Psi = Z_\psi^{1/2} \Psi_r , \phi = Z_\phi^{1/2} \phi_r , \eta = Z_\eta^{-1/2} \eta_r , j = Z_j^{-1/2} j_r , \]

\[
y = y_0 Z_\phi^{1/2} Z_\psi / Z_y , \quad m_0^2 = (\delta_m + m^2) / Z_\phi , \quad M_0 = (\delta_M + M) / Z_\psi .
\]

With the above definitions, the \( \mathcal{L} \) can be expressed as:

\[
\mathcal{L} = \bar{\Psi}_r (i \gamma \cdot D - M) \Psi_r + \frac{1}{2} \partial_{\mu} \phi_r \partial^{\mu} \phi_r - \frac{1}{2} m_0^2 \phi_r^2 - \mathcal{L}_I'[\phi_r] - y \bar{\Psi}_r \phi_r \Psi_r + \bar{\eta}_r \Psi_r + \bar{\psi}_r \eta_r + j_r \phi_r
\]

\[
\quad + \frac{1}{2} \delta_\phi \partial_{\mu} \phi_r \partial^{\mu} \phi_r - \frac{1}{2} \delta_m m_0^2 \phi_r^2 + \bar{\Psi}_r (i \gamma \cdot D - M) \Psi_r - y \delta_y \bar{\Psi}_r \phi_r \Psi_r + \delta \mathcal{L}_I'[\phi_r]
\]

where \( m \) and \( M \) are the renormalized masses, and \( y \) is the renormalized Yukawa coupling. The terms with the coefficients

\[
\delta_\psi = Z_\psi - 1 \ , \ \delta_\phi = Z_\phi - 1 ,
\]

\[
\delta_M = M_0 Z_\psi - M , \ \delta_m = m_0^2 Z_\phi - m^2 ,
\]

\[
\delta_y = Z_y - 1
\]

and \( \delta \mathcal{L}_I' \) are the counterterms to be determined consistently in the perturbative expansion by choosing a renormalization prescription. As it will become clear below this is the most natural manner for obtaining a fully renormalized Dirac equation in a perturbative expansion.

The dynamics of expectation values and correlation functions of the quantum field is obtained by implementing the Schwinger-Keldysh closed-time-path formulation of non-equilibrium quantum field theory \[20–23\]. The main ingredient in this formulation is the real time evolution of an initially prepared density matrix and its path integral representation. It requires a path integral defined along a closed time path contour. This formulation has been described elsewhere within many different contexts and we refer the reader to the literature for details \[20–23\].
Our goal is to understand the non-equilibrium relaxational dynamics of the inhomogeneous fermionic mean fields

\[ \psi(\vec{x}, t) \equiv \langle \Psi_r(\vec{x}, t) \rangle \]

from their initial states in the presence of the fermion-scalar medium.

This statement requires clarification. In states of definite fermion number (either zero or finite temperature) the expectation value of the fermion field must necessarily vanish. However, in order to understand the non-equilibrium dynamics, we prepare the system by coupling an external Grassman source to the fermionic field in the Hamiltonian. This source creates a coherent state of fermions which is a superposition of states with different fermion number. As will be discussed in detail below, this source term couples to a given mode of wavevector \( k \) of the fermionic fields, thus displacing this degree of freedom off-equilibrium. The other modes are assumed to be remain in thermal equilibrium.

Since we are interested in real time correlation functions and the initial density matrix is assumed to be that of thermal equilibrium at initial temperature \( T = 1/\beta \) with respect to the free (quadratic) Lagrangian, only the real time branches, forward and backwards are required. The contribution from the imaginary time branch corresponding to the thermal component of the density matrix cancels in the connected non-equilibrium expectation values \[21–23\]. The effective non-equilibrium Lagrangian density that enters in the contour path integral is therefore given by

\[ \mathcal{L}_{\text{non-eq}} = \mathcal{L} \left[ \Psi_r^+, \bar{\Psi}_r^+, \phi_r^+ \right] - \mathcal{L} \left[ \Psi_r^-, \bar{\Psi}_r^-, \phi_r^- \right]. \] (2.5)

Fields with (+) and (−) superscripts are defined respectively on the forward (+) and backwards (−) time contours and are to be treated independently. The external sources are the same for both branches.

The essential ingredients for perturbative calculations are the following real time Green’s functions \[23\]:

- **Scalar Propagators**

  \[ G^{\pm+}_k(t, t') = G^{\pm}_k(t, t')\Theta(t - t') + G^{\pm}_k(t, t')\Theta(t' - t), \]
  \[ G^{\pm-}_k(t, t') = G^{\pm}_k(t, t')\Theta(t' - t) + G^{\pm}_k(t, t')\Theta(t - t'), \]
  \[ G^{\pm+}_k(t, t') = -G^{\pm-}_k(t, t'), \]
  \[ G^{\pm-}_k(t, t') = -G^{\pm+}_k(t, t'), \]
  \[ G^{\pm}_k(t, t') = i \int d^3 x \ e^{-i\vec{k}\cdot\vec{x}} \langle \phi_r(\vec{x}, t)\phi_r(\vec{0}, t') \rangle \]
  \[ = \frac{i}{2\omega_k} \left[ (1 + n_k)e^{-i\omega_k(t-t')} + n_ke^{i\omega_k(t-t')} \right], \]
  \[ G^{\pm}_k(t, t') = i \int d^3 x \ e^{-i\vec{k}\cdot\vec{x}} \langle \phi_r(\vec{0}, t')\phi_r(\vec{x}, t) \rangle \]
  \[ = \frac{i}{2\omega_k} \left[ n_ke^{-i\omega_k(t-t')} + (1 + n_k)e^{i\omega_k(t-t')} \right], \]
  \[ \omega_k = \sqrt{k^2 + m^2}, \quad n_k = \frac{1}{e^{\beta \omega_k} - 1}. \] (2.6)

- **Fermionic Propagators (Zero chemical potential)**

  \[ S^{\pm+}_k(t, t') = S^{\pm}_k(t, t')\Theta(t - t') + S^{\pm}_k(t, t')\Theta(t' - t), \]
  \[ S^{\pm-}_k(t, t') = S^{\pm}_k(t, t')\Theta(t' - t) + S^{\pm}_k(t, t')\Theta(t - t'), \]
  \[ S^{\pm+}_k(t, t') = -S^{\pm-}_k(t, t'), \]
  \[ S^{\pm-}_k(t, t') = -S^{\pm+}_k(t, t'), \]
  \[ S^{\pm}_k(t, t') = -i \int d^3 x \ e^{-i\vec{k}\cdot\vec{x}} \langle \Psi_r(\vec{x}, t)\bar{\Psi}_r(\vec{0}, t') \rangle \]

\[ \int d^3 x \ e^{-i\vec{k}\cdot\vec{x}} \langle \Psi_r(\vec{x}, t)\bar{\Psi}_r(\vec{0}, t') \rangle \]
\[ S^<_{k}(t,t') = \frac{i}{2\omega_k} \left[ (k + M)(1 - \eta_k)e^{-i\omega_k(t-t')} + \gamma_0(k - M)\gamma_0\eta_k e^{i\omega_k(t-t')} \right] , \]
\[ \omega_k = \sqrt{k^2 + M^2} , \quad \eta_k = \frac{1}{e^{\beta\omega_k} + 1} . \] (2.7)

The perturbative evaluation of correlation functions proceeds as usual, but now the Feynman rules involve two types of vertices with opposite signs and the four different non-equilibrium propagators. The symmetry factors are the usual ones.

These free propagators (2.4) and (2.7) are thermal since the initial state is assumed to be in thermal equilibrium.

A. In-medium Dirac equation from linear response

Consider the fermionic mean field obtained as the linear response to the externally applied (Grassmann-valued) source \( \eta_r \):

\[ \langle \Psi^r_r(\vec{x},t) \rangle = \langle \Psi^r_r(\vec{x},t) \rangle = \psi(\vec{x},t) = -\int_{-\infty}^{\infty} dt' d^3x' S_{\text{ret}}(\vec{x} - \vec{x}', t - t') \eta_r(\vec{x}', t') , \] (2.8)

with the exact retarded Green’s function

\[ S_{\text{ret}}(\vec{x} - \vec{x}', t - t') = [S^>(\vec{x} - \vec{x}', t - t') - S^<_{\Sigma}(\vec{x} - \vec{x}', t - t')] \Theta(t - t') \]
\[ = -i\langle [\Psi_r(\vec{x}, t), \bar{\Psi}_r(\vec{x}', t')] \rangle \Theta(t - t') , \] (2.9)

where the expectation values are in the full interacting theory but with vanishing sources. An initial value problem is obtained by considering that the external fermionic sources are adiabatically switched on in time from \( t \to -\infty \) thereby inducing an expectation value of the fermionic fields, and switching-off the source term at some time \( t_0 \). Then for \( t \geq t_0 \) this expectation value or mean field will evolve in the absence of a source and will relax because of the interactions. The evolution for \( t > t_0 \) is an initial value problem, since the source term was used to prepare an initial state and switched off to let this state evolve in time. This initial value problem can therefore be formulated by choosing the source term to be of the form

\[ \eta_r(\vec{x}, t) = \eta_r(\vec{x}) e^{\epsilon t} \Theta(t_0 - t) r , \quad \epsilon \to 0^+ . \] (2.10)

In what follows we choose \( t_0 = 0 \) for convenience. The adiabatic switching-on of the source induces an expectation value that is dressed adiabatically by the interaction. The retarded and the equilibrium nature of \( S_{\text{ret}}(\vec{x} - \vec{x}', t - t') \) (which depends on the time difference) and the form of the source (2.10) guarantee that

\[ \psi(\vec{x}, t = 0) = \psi_0(\vec{x}) , \quad \psi(\vec{x}, t < 0) = 0 , \]

where \( \psi_0(\vec{x}) \) is determined by \( \eta_r(\vec{x}) \) (or vice versa, the initial conditions for the condensates \( \psi_0(\vec{x}) \) can be used to find \( \eta_r(\vec{x}) \)). This can be seen by taking the time derivative of \( \psi(\vec{x}, t) \) in eq. (2.8), using the fact that the retarded propagator depends on the time difference, integrating by parts, and using the form of the external source and the retarded nature of the propagator.

In order to relate this linear response problem to the initial value problem for the dynamical equation of the mean field, let us consider the (integro-) differential operator \( O_{(\vec{x},t)} \) which is the inverse of \( S_{\text{ret}}(\vec{x} - \vec{x}', t - t') \) so that

\[ O_{(\vec{x},t)} \psi(\vec{x},t) = -\eta_r(\vec{x}, t) , \quad \psi(\vec{x}, t = 0) = \psi_0(\vec{x}) , \quad \psi(\vec{x}, t < 0) = 0 , \] (2.11)

where the source is given by eq. (2.10).

It is at this stage where the non-equilibrium formulation provides the most powerful framework. The real-time equations of motion for the mean fields can be obtained via the tadpole method [21, 24], which automatically leads
to a retarded and causal initial value problem for the expectation value of the field. The implementation of this method is as follows. Let us introduce the inhomogeneous mean fields \( \psi(\vec{x}, t) = (\Psi(\vec{x}, t)) \) and \( \bar{\psi}(\vec{x}, t) = (\bar{\Psi}(\vec{x}, t)) \). The dynamics of these fermionic mean fields in the plasma can be analyzed by treating \( \psi(\vec{x}, t) \) and \( \bar{\psi}(\vec{x}, t) \) as background fields, i.e., the expectation values of the corresponding fields in the non-equilibrium density matrix, by expanding the non-equilibrium Lagrangian density about these mean fields. Therefore, we write the full quantum fields as the c-number expectation values (mean fields) and quantum fluctuations about them:

\[
\Psi^\pm(\vec{x}, t) = \psi(\vec{x}, t) + \chi^\pm(\vec{x}, t), \quad \bar{\Psi}^\pm(\vec{x}, t) = \bar{\psi}(\vec{x}, t) + \bar{\chi}^\pm(\vec{x}, t),
\]

where \( \chi^\pm(\vec{x}, t) \) is the fermion self-energy and \( \bar{\chi}^\pm(\vec{x}, t) \) is the Dirac equation for the \( \vec{k} \)-mode of the expectation value of the fermion:

\[
\begin{pmatrix}
-i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k} - M
\end{pmatrix} + \delta_\phi \begin{pmatrix}
-i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k}
\end{pmatrix} - \delta_M \psi_\vec{k}(t) + \int_{-\infty}^{t} dt' \Sigma_\vec{k}(t - t') \psi_\vec{k}(t') = -\eta_\vec{k}(t),
\]

where \( \Sigma_\vec{k}(t - t') \) is the fermion self-energy and

\[
\psi_\vec{k}(t) \equiv \int d^3x \ e^{-i\vec{k} \cdot \vec{x}} \psi(\vec{x}, t).
\]

Using the non-equilibrium Green’s functions \([21, 23]\), we find to one loop order, that \( \Sigma_\vec{k}(t - t') \) is given by

\[
\Sigma_\vec{k}(t - t') = i\gamma_0 \left[ \Sigma^{(0)}_\vec{k}(t - t') + \sqrt{\vec{k} \cdot \vec{q}} \right] \Sigma^{(1)}_\vec{k}(t - t') + \Sigma^{(2)}_\vec{k}(t - t'),
\]

where

\[
\begin{align*}
\Sigma^{(0)}_\vec{k}(t - t') &= y^2 \int \frac{d^3q}{(2\pi)^3} \frac{\omega_q}{2\omega_k + \omega_q} \times \\
& \times \left[ \cos(\omega_{k+q} + \omega_q)(t - t') + \cos(\omega_{k+q} - \omega_q)(t - t') \right] \left( 1 + n_{k+q} - \tilde{n}_q \right), \\
\Sigma^{(1)}_\vec{k}(t - t') &= y^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_k + \omega_q} \frac{\vec{k} \cdot \vec{q}}{k^2} \times \\
& \times \left[ \sin(\omega_{k+q} + \omega_q)(t - t') + \sin(\omega_{k+q} - \omega_q)(t - t') \right] \left( 1 + n_{k+q} - \tilde{n}_q \right), \\
\Sigma^{(2)}_\vec{k}(t - t') &= y^2 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2\omega_k + \omega_q} \times \\
& \times \left[ \sin(\omega_{k+q} + \omega_q)(t - t') + \sin(\omega_{k+q} - \omega_q)(t - t') \right] \left( 1 + n_{k+q} - \tilde{n}_q \right),
\end{align*}
\]

with \( \omega_{k+q} = \sqrt{(\vec{k} + \vec{q})^2 + m^2} \) and \( n_{k+q} = (e^{\beta\omega_{k+q}} - 1)^{-1} \) being, respectively, the energy and the distribution function for scalars of momentum \( \vec{k} + \vec{q} \).

As mentioned before, the source is taken to be switched on adiabatically from \( t = -\infty \) and switched off at \( t = 0 \) to provide initial conditions

\[
\psi_\vec{k}(t = 0) = \psi_\vec{k}(0), \quad \dot{\psi}_\vec{k}(t < 0) = 0.
\]

Defining \( \psi_\vec{k}(t - t') \) as
\[
\frac{d}{dt} \sigma_k(t - t') = \Sigma_k(t - t')
\]  
(2.16)

and imposing that \(\eta_{r,k}(t > 0) = 0\), we obtain the equation of motion for \(t > 0\)

\[
\left[ i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k} - M \right] + \delta_\psi \left[ i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k} \right] + \sigma_k(0) - \delta_M \right] \psi_k(t) - \int_0^t dt' \sigma_k(t - t') \psi_k(t') = 0 .
\]  
(2.17)

The equation of motion (2.17) can be solved by Laplace transform as befits an initial value problem. The Laplace transformed equation of motion is given by

\[
\left[ i\gamma_0 s - \vec{\gamma} \cdot \vec{k} - M + \delta_\psi \left( i\gamma_0 s - \vec{\gamma} \cdot \vec{k} \right) - \delta_M + \sigma_k(0) - s\tilde{\sigma}_k(s) \right] \tilde{\psi}_k(s) = \left[ i\gamma_0 + i\delta_\psi \gamma_0 - \tilde{\sigma}_k(s) \right] \psi_k(0) ,
\]  
(2.18)

where \(\tilde{\psi}_k(s)\) and \(\tilde{\sigma}_k(s)\) are the Laplace transforms of \(\psi_k(t)\) and \(\sigma_k(t)\) respectively:

\[
\tilde{\psi}_k(s) \equiv \int_0^\infty dt e^{-st} \psi_k(t) \quad , \quad \tilde{\sigma}_k(s) \equiv \int_0^\infty dt e^{-st} \sigma_k(t) .
\]

### B. Renormalization

Before proceeding with the solution of the above equation, we address the issue of the renormalization by analyzing the ultraviolet divergences of the kernels. As usual the ultraviolet divergences are those of zero temperature field theory, since the finite temperature distribution functions are exponentially suppressed at large momenta. Therefore the ultraviolet divergences are obtained by setting to zero the bosonic and fermionic occupation numbers.

With eq. (2.16), \(\sigma_k(t - t')\) can be written as

\[
\sigma_k(t - t') = i\gamma_0 \sigma_k^{(0)}(t - t') + \vec{\gamma} \cdot \vec{k} \sigma_k^{(1)}(t - t') + \sigma_k^{(2)}(t - t') .
\]  
(2.19)

A straightforward calculation leads to

\[
\sigma_k^{(0)}(0) = 0 , \quad \sigma_k^{(1)}(0) = -\frac{y^2}{16\pi} \ln \left( \frac{\Lambda}{K} \right) + \text{finite} , \quad \sigma_k^{(2)}(0) = \frac{y^2 M}{8\pi^2} \ln \left( \frac{\Lambda}{K} \right) + \text{finite} ,
\]

\[
\tilde{\sigma}_k^{(0)}(s) = -\frac{y^2}{16\pi} \ln \left( \frac{\Lambda}{K} \right) + \text{finite} , \quad \tilde{\sigma}_k^{(1)}(s) = \text{finite} , \quad \tilde{\sigma}_k^{(2)}(s) = \text{finite} ,
\]

where \(\tilde{\sigma}_k^{(0)}(s)\), \(\tilde{\sigma}_k^{(1)}(s)\) and \(\tilde{\sigma}_k^{(2)}(s)\) are the Laplace transform of \(\sigma_k^{(0)}(t)\), \(\sigma_k^{(1)}(t)\) and \(\sigma_k^{(2)}(t)\) respectively, \(\Lambda\) is an ultraviolet momentum cutoff and \(K\) is an arbitrary renormalization scale. Therefore the counterterms \(\delta_\psi\) and \(\delta_M\) are chosen to be given by

\[
\delta_\psi = -\frac{y^2}{16\pi^2} \ln \left( \frac{\Lambda}{K} \right) + \text{finite} , \quad \delta_M = \frac{y^2 M}{8\pi^2} \ln \left( \frac{\Lambda}{K} \right) + \text{finite} ,
\]  
(2.20)

and the respective kernels are rendered finite, i.e.,

\[
\tilde{\sigma}_k(s) - i\delta_\psi \gamma_0 = \tilde{\sigma}_{r,k}(s) = \text{finite} , \quad \sigma_k(0) - \delta_\psi \vec{\gamma} \cdot \vec{k} - \delta_M = \sigma_{r,k}(0) = \text{finite} .
\]  
(2.21)

The finite parts of the counterterms in eq. (2.20) are fixed by prescribing a renormalization scheme. There are two important choices of counterterms: i) determining the counterterms from an on-shell condition, including finite temperature effects, and ii) determining the counterterms from a zero temperature on-shell condition. Obviously these choices only differ by finite quantities, however the second choice allows us to separate the dressing effects of the medium from those in the vacuum. For example by choosing to renormalize with the zero temperature counterterms on-shell, the pole in the particle propagator will have unit residue at zero temperature; however in the medium, the residue at the finite temperature poles (or the position of the resonances) are finite, smaller than one and determined solely by the properties of the medium. Thus the formulation of the initial value problem as presented here yields an unambiguous separation of the vacuum and in-medium renormalization effects.

Hence we obtain the renormalized effective Dirac equation in the medium and its initial value problem
with \( \sigma_{r,k}(0) \) and \( \tilde{\sigma}_{r,k}(s) \) the fully renormalized kernels (see eq. (2.21)), and \( \tilde{\Sigma}_{r,k}(s) \) the Laplace transform of the renormalized fermion self-energy which can be written in its most general form as follows

\[
\tilde{\Sigma}_{r,k}(s) = i \gamma_0 s \varepsilon_k^{(0)}(s) + \gamma_0 \cdot \vec{k} \varepsilon_k^{(1)}(s) + M \varepsilon_k^{(2)}(s). \tag{2.23}
\]

The solution to eq. (2.22) is

\[
\tilde{\psi}_{\vec{k}}(s) = \frac{1}{s} \left[ 1 + S(s, \vec{k}) \left( \gamma_0 \cdot \vec{k} + M + \tilde{\Sigma}_{r,k}(0) \right) \right] \psi_{\vec{k}}(0), \tag{2.24}
\]

where

\[
S(s, \vec{k}) = \left[ i \gamma_0 s \gamma \cdot \vec{k} - M + \tilde{\Sigma}_{r,k}(s) \right]^{-1}
\]

\[
= \frac{i \gamma_0 s(1 + \varepsilon_k^{(0)}(s)) - \gamma \cdot \vec{k}(1 - \varepsilon_k^{(1)}(s)) + M(1 - \varepsilon_k^{(2)}(s))}{-s^2(1 + \varepsilon_k^{(0)}(s))^2 - k^2(1 - \varepsilon_k^{(1)}(s))^2 - M^2(1 - \varepsilon_k^{(2)}(s))^2}, \tag{2.25}
\]

is the fermion propagator in terms of the Laplace variable \( s \). The square of the denominator of (2.22) is recognized as

\[
\det \left[ i \gamma_0 s - \gamma \cdot \vec{k} - M + \tilde{\Sigma}_{r,k}(s) \right]
\]

The real-time evolution of \( \tilde{\psi}_{\vec{k}}(t) \) is obtained by performing the inverse Laplace transform along the Bromwich contour in the complex \( s \)-plane parallel to the imaginary axis and to the right of all singularities of \( \tilde{\psi}_{\vec{k}}(s) \). Therefore to obtain the real time evolution we must first understand the singularities of the Laplace transform in the complex \( s \)-plane.

### III. STRUCTURE OF THE SELF-ENERGY AND DAMPING PROCESSES

To one loop order, the Laplace transform of the components \( \tilde{\varepsilon}_k^{(i)}(s) \) of the fermion self-energy \( \tilde{\Sigma}_k(s) \) (see eq. (2.23)) can be written as dispersion integrals in terms of spectral densities \( \rho_k^{(i)}(k_0) \)

\[
\begin{cases}
\tilde{\varepsilon}_k^{(0)}(s) \\
\tilde{\varepsilon}_k^{(1)}(s) \\
\tilde{\varepsilon}_k^{(2)}(s)
\end{cases} = \int dk_0 \frac{1}{s^2 + k_0^2} \begin{cases}
\rho_k^{(0)}(k_0) \\
\rho_k^{(1)}(k_0) \\
\rho_k^{(2)}(k_0)
\end{cases} + \begin{cases}
\delta \psi \\
\delta \bar{\psi} \\
\delta \bar{\phi}
\end{cases}, \tag{3.1}
\]

with the one-loop spectral densities given by the expressions

\[
\rho_k^{(0)}(k_0) = y^2 \int \frac{d^3q}{(2\pi)^3} \frac{\bar{\omega}_q}{2\omega_{k+q} \omega_q} \times \left[ \delta(k_0 - \omega_{k+q} + \bar{\omega}_q) (1 + n_{k+q} - \bar{n}_q) + \delta(k_0 - \omega_{k+q} + \bar{\omega}_q)(n_{k+q} + \bar{n}_q) \right],
\]

\[
\rho_k^{(1)}(k_0) = y^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_{k+q} \omega_q} \times \frac{\vec{k} \cdot \vec{q}}{k^2} \left[ \delta(k_0 - \omega_{k+q} + \bar{\omega}_q) (1 + n_{k+q} - \bar{n}_q) - \delta(k_0 - \omega_{k+q} + \bar{\omega}_q)(n_{k+q} + \bar{n}_q) \right],
\]

\[
\rho_k^{(2)}(k_0) = y^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_{k+q} \omega_q} \times \left[ \delta(k_0 - \omega_{k+q} + \bar{\omega}_q) (1 + n_{k+q} - \bar{n}_q) - \delta(k_0 - \omega_{k+q} + \bar{\omega}_q)(n_{k+q} + \bar{n}_q) \right]. \tag{3.2}
\]

The analytic continuation of the self-energy and its components \( \tilde{\varepsilon}_k^{(i)}(s) \) in the complex \( s \)-plane are given by

\[
\tilde{\Sigma}_{r,k}(s = -i\omega \pm 0^+) = \Sigma_{R,k}(\omega) \pm i \Sigma_{I,k}(\omega),
\]

\[
\tilde{\varepsilon}_k^{(i)}(s = -i\omega \pm 0^+) = \varepsilon_k^{(i)}(\omega) \pm i \varepsilon_k^{(i)}(\omega), \tag{3.3}
\]

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where the real parts are even functions of $\omega$ and given by

$$
\begin{aligned}
\begin{bmatrix}
\varepsilon_{R,k}^{(0)}(\omega) \\
\varepsilon_{R,k}^{(1)}(\omega) \\
\varepsilon_{R,k}^{(2)}(\omega)
\end{bmatrix}
= \int dk_0 \mathcal{P} \left( \frac{1}{k_0^2 - \omega^2} \right) 
\begin{bmatrix}
\rho_k^{(0)}(k_0) \\
k_0 \rho_k^{(1)}(k_0) \\
k_0 \rho_k^{(2)}(k_0)
\end{bmatrix} + \begin{bmatrix}
\delta_\phi \\
-\delta_\phi \\
-\frac{\delta_\phi}{M}
\end{bmatrix},
\end{aligned}
$$

(3.4)

and the imaginary parts are odd functions of $\omega$ and given by

$$
\begin{aligned}
\varepsilon_{I,k}^{(0)}(\omega) &= \frac{\pi}{2|\omega|} \text{sgn}(\omega) \left[ \rho_k^{(0)}(|\omega|) + \rho_k^{(0)}(-|\omega|) \right], \\
\varepsilon_{I,k}^{(1)}(\omega) &= \frac{\pi}{2} \text{sgn}(\omega) \left[ \rho_k^{(1)}(|\omega|) - \rho_k^{(1)}(-|\omega|) \right], \\
\varepsilon_{I,k}^{(2)}(\omega) &= \frac{\pi}{2} \text{sgn}(\omega) \left[ \rho_k^{(2)}(|\omega|) - \rho_k^{(2)}(-|\omega|) \right].
\end{aligned}
$$

(3.5)

The denominator of the analytically continued fermion propagator eq. (2.25) can be written in the compact form

$$
\omega^2 - \tilde{\omega}_k^2 + \Pi(\omega, k)
$$

with $\tilde{\omega}_k = \sqrt{k^2 + M^2}$ and

$$
\Pi(\omega, k) = 2 \left[ \omega^2 \varepsilon_{k}^{(0)}(\omega) + k^2 \varepsilon_{k}^{(1)}(\omega) + M^2 \varepsilon_{k}^{(2)}(\omega) \right] 
+ \omega^2 \left[ \varepsilon_{k}^{(0)}(\omega)^2 - k^2 \left[ \varepsilon_{k}^{(1)}(\omega)^2 - M^2 \left[ \varepsilon_{k}^{(2)}(\omega)^2 \right] \right] \right].
$$

(3.6)

We recognize that the lowest order term of this effective self-energy can be written in the familiar form

$$
\Pi(\omega, k) = 2 \left[ \omega^2 \varepsilon_{k}^{(0)}(\omega) + k^2 \varepsilon_{k}^{(1)}(\omega) + M^2 \varepsilon_{k}^{(2)}(\omega) \right] = \frac{1}{2} \text{Tr}[(k + M)\Sigma(s = -i\omega + 0^+)],
$$

(3.7)

but certainly not the higher order terms.

The imaginary part of $\Pi$ on-shell will be identified with the damping rate (see below). The expression given by eq. (3.7) leads to the familiar form of the damping rate, but we point out that eq. (3.7) is a lowest order result. The full imaginary part must be obtained from the full function $\Pi(\omega, k)$ and the generalization to all orders will be given in a later section below.

For fixed $M$ and $m$, the $\delta$-function constraints in the spectral densities $\rho_k^{(i)}(\omega)$ can only be satisfied for certain ranges of $\omega$. Since the imaginary parts are odd functions of $\omega$ we only consider the case of positive $\omega$. The $\delta(|\omega| - \omega_{k+q} - \tilde{\omega}_q)$ has support only for $|\omega| > \sqrt{k^2 + (m + M)^2}$ and corresponds to the normal two-particle cuts that are present at zero temperature corresponding to the process $\psi \rightarrow \phi + \psi$. These cuts (both for positive and negative $\omega$) do not give a contribution to the imaginary part of the fermion.

The terms proportional to $n_{k+q} + \bar{n}_q$ give the following contribution to the lowest order effective self-energy:

$$
\Pi_f(\omega, k) = \pi y^2 \text{sgn}(\omega) \int \frac{dq}{(2\pi)^3} \frac{n_{k+q} + \bar{n}_q}{2\tilde{\omega}_q \omega_{k+q}} \left[ \left( |\omega| \tilde{\omega}_q - \tilde{k} \cdot \tilde{q} - M^2 \right) \delta (|\omega| - \omega_{k+q} + \tilde{\omega}_q) 
+ \left( |\omega| \tilde{\omega}_q + \tilde{k} \cdot \tilde{q} + M^2 \right) \delta (|\omega| + \omega_{k+q} - \tilde{\omega}_q) \right].
$$

(3.8)

The first delta function determines a cut in the region $0 < |\omega| < \sqrt{k^2 + (m - M)^2}$ and originates in the physical process $\phi \rightarrow \psi + \bar{\psi}$ whereas the second delta function determines a cut in the region $0 < |\omega| < k$ and originates in the process $\phi + \psi \rightarrow \psi$. Whereas the first cut originates in the process of decay of the scalar into fermion pairs, the second cut for ($\omega^2 < k^2$) is associated with Landau damping. Both delta functions restrict the range of the integration variable $q$ (see below). For $m > 2M$ the scalar can decay on-shell into a fermion pair, and in this case the fermion pole is embedded in the cut $0 < |\omega| < \sqrt{k^2 + (m - M)^2}$ becoming a quasiparticle pole and only the first cut contributes to the quasiparticle width. This is a remarkable result, the fermions acquire a width through the induced decay of the scalar in the medium. This process only occurs in the medium (obviously vanishing at $T = 0$) and its origin is very different from either collisional broadening or Landau damping. A complementary interpretation of the origin of this process as a medium induced decay of the scalars into fermions and the resulting quasiparticle width for the fermion excitation will be highlighted in section V within the kinetic approach to relaxation.
The width is obtained to lowest order from \( \Pi_f(\omega_k, \bar{\omega}_k) \), and the on-shell delta function is recognized as the energy conservation condition for the decay \( \phi \rightarrow \psi + \bar{\psi} \) of the heavy scalar on-shell. To lowest order we find the following expression for \( \Pi_f(\omega_k, \bar{\omega}_k) \) for arbitrary scalar and fermion masses with \( m > 2M \) and arbitrary fermion momentum and temperature:

\[
\Pi_f(\omega_k, \bar{\omega}_k) = \pi y^2 \int \frac{d^3q}{(2\pi)^3} \frac{\omega_k \omega_q - \bar{k} \cdot \bar{q} - M^2}{2\omega_q \omega_{k+q}} (n_{k+q} + \bar{n}_q) \delta(\omega_k + \omega_q - \omega_{k+q}) ,
\]

\[
= \frac{y^2 m^2 T}{16\pi k} \left( 1 - \frac{4M^2}{m^2} \right) \ln \left[ \frac{1 - e^{-\beta(\omega_q + \bar{\omega}_k)}}{1 + e^{-\beta \omega_q}} \right]\bar{\omega}_q^2 ,
\]

(3.9)

where \( q_1^* \) and \( q_2^* \) are given by

\[
q_1^* = \frac{m^2}{2M^2} \left| k \left( 1 - \frac{2M^2}{m^2} \right) - \sqrt{(k^2 + M^2) \left( 1 - \frac{4M^2}{m^2} \right)} \right| ,
\]

\[
q_2^* = \frac{m^2}{2M^2} \left| k \left( 1 - \frac{2M^2}{m^2} \right) + \sqrt{(k^2 + M^2) \left( 1 - \frac{4M^2}{m^2} \right)} \right| ,
\]

(3.10)

with \( |\bar{q}| \in (q_1^*, q_2^*) \) being the support of \( \delta(\omega_k - \omega_{k+q} + \bar{\omega}_q) \).

**IV. REAL TIME EVOLUTION**

The real time evolution is obtained by performing the inverse Laplace transform as explained in section II. This requires analyzing the singularities of \( \psi_s(s) \) given by eq. (2.25) in the complex \( s \)-plane. It is straightforward to see that the putative pole at \( s = 0 \) has vanishing residue, therefore the singularities are those arising from the inverse fermion propagator \( S(s, \bar{k}) \). If the fermionic pole is away from the multiparticle cuts, the singularities are: i) the isolated fermion poles at \( s = -i\omega_p \) with \( \omega_p \) the position of the isolated (complex) poles (corresponding to stable fermionic excitations), and ii) the multiparticle cuts along the imaginary axis \( s = -i\omega \) for \( 0 < |\omega| < \sqrt{k^2 + (m - M)^2} \) and \( |\omega| > \sqrt{k^2 + (m + M)^2} \).

The Laplace transform is performed by deforming the contour, circling the isolated poles and wrapping around the cuts. When the scalar particle can decay into fermion pairs, i.e., \( m > 2M \), the fermion pole is embedded in the lower cut and we must find out if it becomes a complex pole in the physical sheet (the domain of integration) or moves off the physical sheet.

For \( m < 2M \) the fermion pole at \( \omega = \omega_p \) is real and the fermion mean field oscillates for late times with constant amplitude and frequency \( \omega_p \).

**A. \( m > 2M \): complex poles or resonances?**

When \( m > 2M \) the fermion pole is embedded in the cut \( 0 < |\omega| < \sqrt{k^2 + (m - M)^2} \) and the pole becomes complex. The position of the complex poles are obtained from the zeros of \( \omega^2 - \bar{\omega}_k^2 + \Pi(\omega, \bar{k}) \) in the analytically continued fermion propagator for \( \omega = \omega_p(k) - i\Gamma_k \) with \( \omega_p \) being the real part of the complex pole. For \( \Gamma_k \ll \omega_p \) (narrow width approximation) and with the expressions for the discontinuities in the physical sheet given by eq. (3.3), the equation that determines the position of the complex pole is given by the solution of the following equation

\[
(\omega_p - i\Gamma_k)^2 - \bar{\omega}_k^2 + \Pi_R(\omega_p, \bar{k}) - i \text{sgn}(\Gamma_k) \Pi_f(\omega_p, \bar{k}) = 0 ,
\]

(4.1)

where we have used the narrow width approximation. To lowest order, the real and imaginary parts of this equation become

\[
\omega_p^2 - \bar{\omega}_k^2 + \Pi_R(\omega_p, \bar{k}) = 0 ,
\]

(4.2)

\[
\Gamma_k = -\text{sgn}(\Gamma_k) \frac{\Pi_f(\omega_p, \bar{k})}{2\omega_p} ,
\]

(4.3)

and the lowest order solution of eq. (4.2) is given by
\[ \omega_p = \pm \left[ \tilde{\omega}_k - \frac{\Pi_I(\tilde{\omega}_k, \vec{k})}{2\tilde{\omega}_k} \right] = \pm \left[ \tilde{\omega}_k - \frac{1}{4\tilde{\omega}_k} \text{Tr}[(\vec{k} + M)\Sigma_{R,\vec{k}}(\tilde{\omega}_k)] \right]. \tag{4.4} \]

The solution for the imaginary part is obtained by replacing \( \omega_p = \pm\tilde{\omega}_k \) in \( \Pi_I(\omega, \vec{k}) \) to this order. However, the equation for the imaginary part \( \Pi_I(\omega, \vec{k}) \) does not have a solution because \( \Pi_I(\omega, \vec{k}) \) is an odd function of \( \omega \) and \( \Pi_I(\tilde{\omega}_k, \vec{k}) > 0 \). Therefore, there is no complex pole in the physical sheet. This is a fairly well-known (but seldom noticed) result: if the imaginary part of the self-energy on shell is positive there is no complex pole solution in the physical sheet, the pole has moved off into the unphysical (second) sheet.

In the case that \( \Pi_I(\tilde{\omega}_k, \vec{k}) \) is negative, complex poles appear in the physical sheet, but in such case there are two poles with both signs for \( \Gamma \), the signal of an instability, not of damping. However since we confirm that \( \Pi_I(\tilde{\omega}_k, \vec{k}) \) is positive in the case under consideration, the complex poles are in the unphysical sheet, describing a resonance.

B. Scalar decay implies fermion damping

Since we have determined that there are no complex poles in the physical sheet in the s-plane, i.e., \( s \) is an integration contour, the only singularities are the cut discontinuities along the following segments of the imaginary axis:

\[ s = \left[ i\sqrt{k^2 + (m + M)^2}, i\infty \right], \quad s = \left[ -i\sqrt{k^2 + (m + M)^2}, -i\infty \right] \]

and

\[ s = \left[ -i\sqrt{k^2 + (m - M)^2}, i\sqrt{k^2 + (m - M)^2} \right]. \]

The contour of integration (Bromwich contour) is deformed to wrap around these cuts. In the narrow width approximation and consistent with perturbation theory, the discontinuities in the self-energy are perturbatively small; and since either the real pole or the resonance is below the two particle cut, the contribution from this cut is always perturbatively small. On the other hand, in the relevant case of \( m > 2M \) with a quasiparticle resonance, the contribution from the cut

\[ s = \left[ -i\sqrt{k^2 + (m - M)^2}, i\sqrt{k^2 + (m - M)^2} \right] \]

becomes the dominant one. It is convenient to write the product in eq. (2.24) in the simplified form

\[ S(s, \vec{k}) \left( \vec{\gamma} \cdot \vec{k} + M - \bar{\Sigma}_{r,\vec{k}}(0) \right) = \frac{\hat{N}(s, k)}{-s^2 - \omega_k^2 + \Pi(s, k)}, \tag{4.5} \]

which defines \( \hat{N}(s, k) \), and to change variables to \( s = -i\omega \pm 0^+ \) on both sides of the cut, leading to the following contribution from the thermal cut to the real time evolution

\[ \psi_{\vec{k}}(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega} e^{-i\omega t} \left\{ \frac{N_R(\omega, \vec{k}) \Pi_I(\omega, \vec{k})}{[\omega^2 - \tilde{\omega}_k^2 + \Pi_R(\omega, \vec{k})]^2 + \Pi_I(\omega, \vec{k})^2} - \frac{N_I(\omega, \vec{k}) [\omega^2 - \tilde{\omega}_k^2 + \Pi_R(\omega, \vec{k})]}{[\omega^2 - \tilde{\omega}_k^2 + \Pi_R(\omega, \vec{k})]^2 + \Pi_I(\omega, \vec{k})^2} \right\} \psi_{\vec{k}}(0), \tag{4.6} \]

where \( N_R, I(\omega, \vec{k}) \) are obtained by replacing \( \varepsilon_i^{(s)}(\omega) \) by their real or imaginary parts and \( \omega^\pm = \pm \sqrt{k^2 + (m - M)^2} \). The term proportional to \( N_R(\omega, \vec{k}) \) features a typical Breit-Wigner resonance shape near the real part of the complex pole at \( \omega_k^2 - \tilde{\omega}_k^2 + \Pi_R(\omega_k, \vec{k}) = 0 \) since, for \( m > 2M \), the imaginary part of the self energy at this value of \( \omega \) (perturbatively close to \( \pm \omega_k \)) is non-vanishing. On the other hand, the term proportional to \( N_I(\omega, \vec{k}) \) is a representation of the principal part in the limit of small \( \Pi_I(\omega_k, \vec{k}) \) and is therefore subleading. The sharply peaked resonances at \( \omega = \pm |\omega_p| \approx \pm \omega_k \) dominate the spectral density and give the largest contribution to the real time evolution. In the limit of a narrow resonance, the \( \omega \) integral is performed by taking the integration limits to infinity and approximating near the resonances at \( \omega = \pm |\omega_p| \),

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The theory on-shell at zero temperature

the real time evolution \( \omega \) position of the resonance \( \omega \)

choice of counterterms. Only when the counterterms are chosen to provide a subtraction of the self-energy at the value

where \( m/M \) extreme case

moves to larger values of the fermion momentum when this ratio is very large. Figure 1 displays this feature in an

for the fermionic mean fields to one loop order for arbitrary values of the scalar and fermion masses (with \( m > 2M \)),

parameters for the ratios of the scalar to fermion masses (\( m/M \)). This peak is at very small momentum when the ratio of scalar to fermion mass is not much larger than 2, but

and \( q^* \) are given by eqs. (3.10).

Figures (1-3) display the behavior of \( \Gamma_k \) for several ranges of the parameters. We have chosen a wide range of

parameters for the ratios of the scalar to fermion masses (\( m/M \)) and the temperature to fermion mass (\( T/M \)) to illustrate in detail the important differences. The damping rate features a strong peak as a function of the ratio \( k/M \). This peak is at very small momentum when the ratio of scalar to fermion mass is not much larger than 2, but moves to larger values of the fermion momentum when this ratio is very large. Figure 1 displays this feature in an extreme case (\( m/M = 800 \)) to highlight this behavior. The height of the peak is a monotonically increasing function of temperature as expected.

This is one of the important results of this work: the induced decay of the heavy scalar into fermion pairs results in a damping of the amplitude of fermionic excitations.

C. Resonance wave functions and all-order expression for the damping rate

Since there is no complex pole solution in the physical sheet, there are no solutions of the effective in-medium Dirac equation for \( m > 2M \). However, we can define the spinor wave function of the resonance by considering the solutions of the in-medium Dirac equation with only the real part of the self-energy [10,18] at the value \( \omega = \omega_p \). Since the form given by eq. (2.23) for the self-energy is general and not restricted to perturbation theory, our analysis below is valid to all orders.

Using the expression for the self-energy given by eq. (2.23) for \( s = -i \omega_p \) and the real part of the coefficients \( \varepsilon^{(i)}_k (\omega_p) \) given by eqs. (3.3, 3.4), we now introduce the following variables

then the resonance wave functions \( \Psi_k (\omega_p) \) obey the following effective in-medium Dirac equation

\[
\left[ \gamma_0 W_p - \gamma \cdot \vec{K} - M \right] \Psi_k (\omega_p) = 0
\]
\[(\mathcal{K} - \mathcal{M}) U_{s,k} = 0 \quad \text{and} \quad (\mathcal{K} + \mathcal{M}) V_{s,k} = 0 \ , \quad (4.12)\]

with \(\mathcal{K} = \gamma_0 |\mathcal{W}_p| - \vec{\gamma} \cdot \vec{K}\).

These Dirac spinors can be obtained from the usual free particle solutions by the replacement \(\vec{\omega}_k \to |\mathcal{W}_p|, \vec{K} \to \vec{K}\) and \(M \to \mathcal{M}\) given by eq. (4.10). Combining eqs. (2.23, 3.3, 3.6) and the definition of the width given by (4.7), we obtain the following expression for the width of the resonance to all orders in perturbation theory

\[
\Gamma_k = \frac{Z_k}{4 |\mathcal{W}_p|} \text{Tr} [(\mathcal{K} + \mathcal{M}) \Sigma_I (|\mathcal{W}_p|)] = \frac{Z_k \mathcal{M}}{2 |\mathcal{W}_p|} \sum_{s=\pm} \left[ U_{s,k} \Sigma_I (|\mathcal{W}_p|) U_{s,k}^* \right] , \quad (4.13)
\]

where we have alternatively written the expression for the width in terms of the exact resonance spinors in the medium. This result confirms those found in reference [10,18] and leads to the often quoted expression for the width \([7,9]\) in lowest order.

V. KINETICS OF FERMION RELAXATION

To clarify and confirm independently the result of the previous section of induced decay of the scalar leading to a quasiparticle width of the fermionic excitations, we now provide an analysis of the relaxation of the distribution function for the fermions via a kinetic Boltzmann equation. This analysis will also provide a firm relationship between the damping rate and the interaction rate in real time in the relaxation time approximation (linearization near equilibrium).

Let us denote the distribution function for scalars of momentum \(\vec{k}\) at time \(t\) by \(N_k(t)\) and that for fermions of momentum \(\vec{k}\) and spin \(s\) by \(\tilde{N}_{k,s}(t)\). In the kinetic approach, the derivative of this distribution function with respect to time is obtained from a Boltzmann equation. Since for a fixed spin component the matrix elements for the transition probabilities are rather cumbersome, we define the spin-averaged fermion distribution function as \(\tilde{N}_k(t) = \frac{1}{2} \sum_s \tilde{N}_{k,s}(t)\).

Two processes are responsible for the change in the fermion populations in a fermion-scalar plasma: i) \(\phi \to \psi + \bar{\psi}\) (creation) which provides the ‘gain’ term in the balance equation, and ii) \(\psi + \bar{\psi} \to \phi\) (annihilation) which provides the ‘loss’ term. Using the standard approach to obtain kinetic Boltzmann rate equations, we find that the spin-averaged rates for creation and annihilation are given by

\[
\frac{d}{dt} \tilde{N}_k(t) \bigg|_{\text{gain}} = \pi y^2 \int \frac{d^3q}{(2\pi)^3 2\bar{\omega}_q} \frac{\bar{\omega}_k \bar{\omega}_q - \vec{k} \cdot \vec{q} - M^2}{\bar{\omega}_k \omega_{k+q}} \times N_{k+q}(t)[1 - \tilde{N}_k(t)][1 - \tilde{N}_q(t)] \delta(\bar{\omega}_k + \bar{\omega}_q - \omega_{k+q}) ,
\]

\[
\frac{d}{dt} \tilde{N}_k(t) \bigg|_{\text{loss}} = \pi y^2 \int \frac{d^3q}{(2\pi)^3 2\bar{\omega}_q} \frac{\bar{\omega}_k \bar{\omega}_q - \vec{k} \cdot \vec{q} - M^2}{\bar{\omega}_k \omega_{k+q}} \times [1 + N_{k+q}(t)]\tilde{N}_k(t)\tilde{N}_q(t) \delta(\bar{\omega}_k + \bar{\omega}_q - \omega_{k+q}) ,
\]

respectively. The spin-averaged net rate is simply

\[
\frac{d}{dt} \tilde{N}_k(t) = \frac{d}{dt} \tilde{N}_k(t) \bigg|_{\text{gain}} - \frac{d}{dt} \tilde{N}_k(t) \bigg|_{\text{loss}} . \quad (5.1)
\]

Let us now consider that all of the modes but the fermionic mode with wavevector \(\vec{k}\) in the fermion-scalar plasma are in thermal equilibrium, while the population for the fermionic \(\vec{k}\)-mode has a small deviation from thermal equilibrium, i.e.,

\[
\tilde{N}_k(t) = \frac{1}{e^{\beta \bar{\omega}_k} + 1} + \delta \tilde{N}_k(t) , \quad \tilde{N}_q(t) = \frac{1}{e^{\beta \bar{\omega}_q} + 1} , \quad N_{k+q}(t) = \frac{1}{e^{\beta \omega_{k+q}} - 1} . \quad (5.2)
\]

In the linear relaxation approximation (or relaxation time approximation), we find

\[
\frac{d}{dt} \delta \tilde{N}_k(t) = -\gamma_k \delta \tilde{N}_k(t) , \quad (5.3)
\]
with $\Upsilon_k$ being the interaction rate in the relaxation time approximation (linear relaxation) and given by

$$\Upsilon_k = \frac{g^2 m^2 T}{16 \pi k \omega_k} \left( 1 - \frac{4 M^2}{m^2} \right) \ln \left[ \frac{1 - e^{-\beta (\omega_k + \omega_q)}}{1 + e^{-\beta \omega_q}} \right] \frac{\bar{q}^* e^{-i \tilde{q}^*}}{\omega_k}, \tag{5.4}$$

where $q^*_1$ and $q^*_2$ are given by eqs. $3.14$. Comparing the interaction rate with the damping rate found in the previous section (see eq. $4.5$), we provide a real time confirmation of the result

$$\Upsilon_k = 2 \Gamma_k. \tag{5.5}$$

The kinetic analysis confirms that the damping of the fermionic quasiparticle excitations in the medium is a consequence of the induced decay of the heavy scalar. Furthermore, this analysis in real time clearly establishes the relation between the interaction rate in the relaxation time approximation and the exponential decay of the amplitude of the mean field at least to lowest order. Recently, a detailed investigation between the damping and interaction rates for chiral fermions in gauge theories at finite temperature has been reported in reference $12$ within the framework of imaginary time (Matsubara) finite temperature field theory. Our analysis provides a complementary confirmation of this result in real time both for the relaxation of the mean field and that of the spin-averaged distribution function.

VI. CONCLUSIONS, COMMENTS AND FURTHER QUESTIONS

In this article we have focused on studying the propagation of fermionic excitations in a fermion-scalar plasma, as a complement to the more studied issue of propagation in a gauge plasma.

Our motivation was to provide a real time analysis of the propagation that could eventually be used in other problems such as, for example, neutrino oscillations in medium and in non-equilibrium processes in electroweak baryogenesis. The first step of the program is to obtain the effective Dirac equation in medium and in real time. This is achieved by relating the problem of linear response to an initial value problem for the mean field that is induced by an external Grassmann-valued source term. The resulting Dirac equation for the mean field is fully renormalized and causal and allows a direct study of real time phenomena.

We used this description to study the propagation of fermions in a fermion-scalar plasma to lowest order in the Yukawa coupling. We found that when the scalar mass is large enough that its decay into fermion pairs is kinematically allowed, this process in the medium leads to a damping of the fermionic excitations and a quasiparticle picture of its propagation in the medium. A real time description of this process clearly leads to the identification of the damping rate, which we computed to one loop order for arbitrary values of the fermion and scalar masses (provided the scalar is heavy enough to decay), temperature and fermion momentum.

An all-order expression for the damping rate (in the narrow width approximation) is obtained from the exact quasiparticle solutions to the in-medium Dirac equation [see eq. $3.13$]. A kinetic approach based on a Boltzmann equation for the spin-averaged fermionic distribution function reveals that the interaction rate in the relaxation time approximation (linear departures from equilibrium) are simply related to the damping rate of the mean fields at least to lowest order in the Yukawa coupling. We emphasize that this relation is established here from the real time evolutions both of the mean field and the distribution function.

Comments. Although the expression for the rate was obtained for arbitrary values of the scalar and fermion masses, temperature and fermion momentum, a deeper analysis is required if the theory undergoes a second or very weak first order transition. The reason being that if the fermionic masses are a result of spontaneous symmetry breaking in the scalar sector, near a second order (or very weak first order) phase transition both the scalar mass and the chiral breaking fermion mass vanish. In this case the kinematic region in momenta for which the energy conserving delta functions are fulfilled shrinks and one must understand if, for soft fermionic momentum, a resummation akin to hard thermal loops is required. In particular from the expression of the spectral densities given by eq. $3.2$, it is straightforward to see that both $\rho^{(1)}_k(k_0)$ and $\rho^{(2)}_k(k_0)$ contribute in the hard thermal loop limit for vanishing scalar and fermion mass very similarly to the Landau damping contribution from gauge fields $17$ with a cut discontinuity for space like momenta ($\omega^2 < k^2$). This particular case, corresponding to the limit $T \gg m, M$, would have to be studied separately and is beyond the scope of this article. A possibility that arises in this situation is that as the phase transition is approached the effective temperature dependent scalar mass becomes smaller than twice the temperature dependent fermionic mass, the scalar decay channel shuts-off and the contribution to the fermionic damping rate vanishes. This of course depends on the self-couplings of the scalar sector and requires studying in detail particular models.

Further questions. In a full fermion, gauge plus scalar theory with light fermions of mass $M \ll eT$ (e is the gauge coupling), the contribution of the gauge sector to the fermion self-energy requires the hard thermal loop resummation.
for soft fermion momenta $k$ of order $e T$, but is perturbative for hard fermion momenta $k$ of order $T$. For the lightest quarks $u, d$ and $s$ but certainly not $c, b$ and $t$ (here we have in mind the electroweak theory and the problem of baryogenesis), the Yukawa couplings $y \ll e$ and the contribution from the scalars is perturbatively small. This aspect notwithstanding, let us consider the case of soft fermionic momenta. The hard thermal loop resummation leads to the dispersion relations for plasmino quasiparticles in the medium, which in lowest order in HTL are stable collective excitations. However, with the coupling of scalars, the heavy scalar (with mass of order $T$) can now decay into a soft plasmino and a hard fermion (as the hard plasminos are almost indistinguishable from the zero-temperature fermions). The results of this article indicate that this process will then lead to a damping rate for the plasminos in lowest order (and certainly perturbative) in the Yukawa coupling. This process will compete with the damping of soft fermions through the exchange of magnetic photons, and it requires a detailed analysis which is currently under study [24].

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FIG. 1. $32\pi \Gamma_k/(My^2)$ v.s. $k/M$ for $m/M = 800$ and $T/M = 1000$ (solid line), 800 (dotted line), 600 (dashed line).
FIG. 2. $32\pi\Gamma_k/(M y^2)$ v.s. $k/M$ for $m/M = 4$ and $T/M = 150$ (solid line), 100 (dotted line), 50 (dashed line).
FIG. 3. $32\pi\Gamma_k/(My^2)$ v.s. $k/M$ for $m/M = 4$ and $T/M = 1000$ (solid line), 800 (dotted line), 600 (dashed line).