Shot noise of spin current in ferromagnet-normal-metal systems

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We propose a three-terminal spin-valve setup, to determine experimentally the spin-dependent shot noise, which carries information on the spin-relaxation processes. Based on a spin-dependent Boltzmann-Langevin approach, we show that the spin Fano factor, defined as the spin shot noise to the mean charge current, strongly depends on the spin-flip scattering rate in the normal wire. While in the parallel configuration the spin Fano factor always decreases below its unpolarized value with increasing spin injection, for the antiparallel case it varies nonmonotonically. We also show that in contrast to the charge current Fano factor, which varies appreciably only in the antiparallel case, the spin Fano factor allows for a more sensitive determination of the spin-flip scattering rate.

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The importance of shot noise in mesoscopic systems has been recognized in the past years as a result of extensive experimental and theoretical studies of currents fluctuations in a wide variety of hybrid structures involving normal metals, semiconductors and superconductors. Correlations of current fluctuations at low temperatures provide unique information about the charge, the statistics and the scattering of the current carriers. In spintronic structures, in which the transport involves both charge and spin degrees of freedom, the current fluctuations are expected to contain spin-resolved information on the conductance process. Consequently spin-polarized current correlations can be used to extract information about spin-dependent scattering and spin accumulation in ferromagnet(F)-normal-metal(N) structures.

Until very recently spin-polarized shot noise has received little attention. Results of the earlier studies of shot noise in FNF and FIF systems have been explained in terms of the well known results of the corresponding unpolarized systems for two spin directions. Tserkovnyak and Brataas have found that shot noise in double barrier FNF structures, in which the F-terminals have noncollinear magnetizations, depends on the relative orientation of the magnetizations of the terminals. Results of more recent studies have revealed that the spin-flip scattering in FNF structures can change the current correlations strongly depending on the polarizations of F-terminals. In Ref. we have developed a semiclassical theory of spin-polarized current fluctuations based on the Boltzmann-Langevin kinetic equation approach. It has been shown that in a multi-terminal diffusive FNF system shot noise and cross-correlations between currents of different F-terminals can deviate substantially from the unpolarized values, depending on spin polarizations of F-terminals and the strength of the spin-flip scattering in the N-metal. All these studies have focused on the fluctuations of the charge currents. It is also interesting to study the fluctuations of the spin currents. Shot noise of spin current in absence of charge current was considered in Ref. In Ref. it has been shown that the spin-resolved shot noise of unpolarized currents can be used to probe the attractive or repulsive correlations induced by interactions.

In this Letter we study the spin-current shot noise in diffusive spin-valve systems, in which both charge and spin currents can be present. To measure the correlations between spin-current fluctuations we propose to use a three-terminal device in which a normal diffusive wire is connected through tunnel junctions to three ferromagnetic terminals of which two have perfect polarizations pointed antiparallel to each other. We show that spin shot noise can be determined by measuring the charge shot noise and the cross correlations between currents through the two perfectly polarized antiparallel terminals which are connected to the normal wire by tunnel junctions with different conductances. The third F-terminal can have arbitrary polarization and is used to inject a spin accumulation into the normal wire. The spin-polarized Boltzmann-Langevin approach is used to calculate both of charge and spin shot noise. In the presence of spin-flip scattering these two correlations are distinguished from each other. We present a detail comparison of charge and spin shot noise for different polarization of the terminals and the spin-flip scattering strength in the normal wire.

The layout of the spin-valve system we study is shown in Fig. 1. Three ferromagnetic terminals $F_\uparrow$, $F_\uparrow$ and $F_\downarrow$ are connected by tunnel junctions to a normal diffusive wire (N) of length $L$. $F_\uparrow$ and $F_\downarrow$ are held at the same voltage $V$ and the voltage in $F_\downarrow$ is zero. We model the spin polarization of the terminals as spin-dependent tunneling conductances of the junctions. The terminals $F_\uparrow$ and $F_\downarrow$ are perfectly polarized and have antiparallel polarizations. In this case these two terminals operate effectively as a single ferromagnetic terminal (held at the voltage $V$) connected to the wire by a tunnel junction with conductance given by the sum of the conductances $g_L = g_\uparrow + g_\downarrow$ of the two tunnel junctions connecting $F_\uparrow$ to the wire and a polarization defined as $p_L = (g_\uparrow - g_\downarrow)/g_L$. The junction connecting $F_\uparrow$ to the wire has a spin-dependent conductance $g_{0\alpha}$ ($\alpha = \pm 1$ denotes spin of electron), which corresponds to the total conductances $g_\alpha = \sum_\alpha g_{0\alpha}$ and the polarizations $p_\alpha = \sum_\alpha g_{0\alpha}/g_0$. Thus we may consider the three-terminal structure to be equivalent to a two-terminal system with corresponding polarizations and the conductances, as is presented in Fig. 1b.
diffusion constant, $\ell_{sf} = \sqrt{D_{sf}}$ is the spin-flip length, $\tau_{imp}(sf)$ is the relaxation time of normal impurity (spin-flip) scattering, $v_{F}$ is the Fermi velocity and $N_{0}$ is the density of states at the Fermi level. The fluctuating charge and spin distribution functions are expressed as $f_{c}(x, t, \varepsilon) = \sum_{\alpha} f_{\alpha}(x, t, \varepsilon)/2$ and $f_{s}(x, t, \varepsilon) = \sum_{\alpha} \alpha f_{\alpha}(x, t, \varepsilon)/2$, respectively, with $f_{\alpha}(x, t, \varepsilon)$ being the spin-$\alpha$ electron distribution function. The mean charge and spin distribution functions obey the equations

$$\frac{\partial^{2}}{\partial x^{2}} f_{c0} = 0,$$  

$$\frac{\partial^{2}}{\partial x^{2}} f_{s0} = \frac{\tau_{sf}}{\ell_{sf}^{2}} f_{s0},$$

Eqs. \ref{eq:4} \ref{eq:5} contain the Langevin sources of fluctuations of the charge (spin) current density $j_{c(s)}(x, t, \varepsilon)$ and the divergence term of the spin current fluctuations $i_{s}^{\alpha}(x, t, \varepsilon)$, which reflects the fact that the number of electrons with specific spin-direction is not conserved by the spin-flip scattering. The correlators of these fluctuating terms are given by:

$$< j_{c(s)}(x, t, \varepsilon) j_{c(s)}^{\prime}(x', t', \varepsilon') >= \Delta \sigma \sum_{\alpha} \Pi_{\alpha \alpha}(x, \varepsilon),$$

$$< j_{c}(x, t, \varepsilon) j_{c}^{\prime}(x', t', \varepsilon') >= \Delta \sigma \sum_{\alpha} \alpha \Pi_{\alpha \alpha}(x, \varepsilon),$$

$$< j_{s}^{\alpha}(x, t, \varepsilon) j_{s}^{\alpha}(x', t', \varepsilon') >= 0,$$

$$< i_{s}^{\alpha}(x, t, \varepsilon) i_{s}^{\alpha}(x', t', \varepsilon') >= \Delta \sigma \frac{\tau_{sf}}{\ell_{sf}^{2}} \sum_{\alpha} \Pi_{\alpha \alpha}(x, \varepsilon),$$

where we used the abbreviation $\Delta = \delta(x - x')\delta(t - t')\delta(\varepsilon - \varepsilon')$, and

$$\Pi_{\alpha \alpha'}(x, \varepsilon) = \tilde{f}_{\alpha}(x, \varepsilon)[1 - \tilde{f}_{\alpha'}(x, \varepsilon)].$$

The relations \ref{eq:6} \ref{eq:10} describe the effect of spin-polarization and spin-flip scattering on the correlations of the current fluctuations in the normal wire.

The mean distribution function $\tilde{f}_{c0} = \tilde{f}_{c} + \alpha \tilde{f}_{s}$ is obtained from the solution of Eqs. \ref{eq:4} \ref{eq:5}. It reads

$$\tilde{f}_{c} = f_{1} + (f_{2} - f_{1})[a + b\frac{L}{\lambda_{x}}] + \alpha[e \sinh \frac{\lambda_{x}}{T} + d \cosh \frac{\lambda_{x}}{T}],$$

where $f_{i} = f_{\alpha}\delta(\varepsilon - \varepsilon_{i})$ is the Fermi-Dirac distribution function in the terminal $F_{i}$ ($i = 1, 2$) held at equilibrium in voltage $V_{i}$ (Fig. \ref{fig:1}(b)). The coefficients $a, b, c, d$ have to be determined from the boundary conditions, which are the current conservation rule at the two connection points of the wire, $x = 0, L$.

From the diffusion Eqs. \ref{eq:1} \ref{eq:3} and using Eq. \ref{eq:4} we obtain the expressions for the average and the fluctuations of the charge current, respectively, as follow

$$\bar{I}_{c}(\varepsilon) = b g_{N} (f_{2} - f_{1}),$$

$$\Delta I_{c}(\varepsilon) = g_{N} [\delta f_{c}(0) - \delta f_{c}(L)] + \delta I_{c}^{c},$$

$$\delta I_{c}^{c}(\varepsilon) = \frac{A}{T} \int dx j_{c}^{c},$$

In the presence of the spin-flip scattering transport of spin-polarized electrons in the normal wire is described by Boltzmann-Langevin diffusion equations for the fluctuating charge and spin current densities at energy $\varepsilon$, $j_{c(s)}(x, t, \varepsilon) = j_{c(s)}(x, \varepsilon) + \delta j_{c(s)}(x, t, \varepsilon)$, which read:

$$\frac{\partial j_{c(s)}}{\partial x} = 0,$$  

$$\frac{\partial j_{c}}{\partial x} = -\frac{\sigma}{\ell_{sf}^{2}} f_{s} + i_{sf}^{c},$$  

$$\frac{\partial j_{s}}{\partial x} = -\frac{\sigma}{\ell_{sf}^{2}} f_{c} + j_{c}^{c},$$

Here $\sigma = e^{2}N_{0}D$ is the conductivity, $D = v_{F}^{2}\tau_{imp}/3$ is the
where $g_N = \sigma A/L$ ($A$ being the area of the junctions) is the conductance of the wire.

Similarly Eqs. (2), (3) and (5) lead to the following result for the average and the fluctuations of the spin currents:

$$I_s(x, \varepsilon) = g_N \lambda c \sinh \frac{\lambda x}{L} + d \cosh \frac{\lambda x}{L}(f_2 - f_1), \quad (15)$$
$$\Delta I_s(0, L, \varepsilon) = \frac{g_N}{s}[-\cosh \lambda \delta f_s(0(0)) + \delta f_s(L(0))]$$
$$+ \delta I_s^c(0, L, \varepsilon), \tag{16}$$
$$\delta I_s^c(0, L, \varepsilon) = A \int \mathrm{d}x (s^f + j^f \partial \phi_0(0, L)). \tag{17}$$

Here $\phi_0(x) = \sinh [\lambda(1 - x/L)]/\sinh \lambda$, $\phi_0(L) = \sinh (\lambda L)/\sinh \lambda$, $s(\lambda) = \sinh \lambda / \lambda$ and $t(\lambda) = \tanh \lambda / \lambda$, and the parameter $\lambda = L/\ell_d$ is a measure of the spin-flip scattering. Note, that as a result of the spin-flip scattering the spin current is not conserved through the wire.

Following the Boltzmann-Langevin approach, the fluctuating spin-\(\alpha\) current at the junctions points 0, \(L\) can be written as $I_\alpha(0, L) = g_{0(\lambda)}[f_{0(\lambda)} - f_\alpha(0, L)] + \delta I_{0(\lambda)\alpha}$, in which the intrinsic current fluctuations $\delta I_{0(\lambda)\alpha}$ is due to the random scattering of electrons from the tunnel barriers and the fluctuations of the spin-\(\alpha\) distribution function are $\delta f_{\alpha}(0, L)$. From this relation the fluctuating charge and spin currents through the terminals can be expressed in terms of the fluctuating spin and charge distributions at the connection points and the corresponding intrinsic current fluctuations. Denoting $\delta I_{\alpha(\delta)}$, as the intrinsic fluctuations of the charge (spin) current through the tunnel junction, we obtain

$$I_\alpha(0, L) = g_0[L][f_{0(\lambda)} - f_\alpha(0, L)]$$
$$- g_0[L]p_0(L)f_\alpha(0, L) + \delta I_{0(\lambda)\alpha}, \quad (18)$$
$$I_s(0, L) = g_0[L][f_{0(\lambda)} - f_s(0, L)]$$
$$- g_0[L]p_0(L)f_s(0, L) + \delta I_{0(\lambda)s}, \quad (19)$$

Now we impose the boundary conditions at the junctions. Assuming spin-conserving tunnel junctions the total (integrated over energy) spin and charge currents should be conserved at the junctions points. Using this condition and the expressions for the spin and charge currents given by Eqs. (12, 19), we obtain the coefficients $a, b, c, d$ and the fluctuations of the charge and spin distributions at 0, \(L\), $\delta f_{\alpha(\delta)}(0, L)$. The results of this calculations can be inserted in Eqs. (13, 16) to obtain the fluctuations of the charge and spin currents in the terminal connected to the point \(L\). The results are expressed in terms of $\delta I_{0(\lambda)c}, \delta I_{0(\lambda)s}, \delta I_{c(\delta)}$, and $\delta I_{c(\delta)} (0, L)$. To obtain the current correlations we have to know the correlations between these terms. The correlations between $\delta I_{c(\delta)}$ and $\delta I_{c(\delta)} (0, L)$ can be obtained using Eqs. (14, 17) and (6, 11). For tunnel junctions the correlations of $\delta I_{0(\lambda)c}, \delta I_{0(\lambda)s}$ are given by the relations:

$$\langle \delta I_{0(\lambda)c}\delta I_{0(\lambda)c} \rangle = \langle \delta I_{0(\lambda)s}\delta I_{0(\lambda)s} \rangle = 2e\bar{I}_c \quad (20)$$
$$\langle \delta I_{0(\lambda)c}\delta I_{0(\lambda)s} \rangle = 2e(|\bar{I}_c(+) - |\bar{I}_c(-)| \rangle. \quad (21)$$

where $\bar{I}_c = \bar{I}_c(+) + \bar{I}_c(-)$ is the mean current of spin-\(\alpha\) electrons.

Using all these results the correlations of the charge and spin currents $S = \langle \Delta I_s \Delta I_c \rangle$ and $S_s(\varepsilon) = \langle \Delta I_s(\varepsilon) \Delta I_c(\varepsilon) \rangle$ are obtained. In this way we obtain the charge current Fano factor $F = S/2e|I_c|$ and the spin current Fano factor defined as $F_s = S_s|I_c|$ in terms of the normalized conductances $g_{0(\lambda)}/g_N$, the polarizations $p_{0(\lambda)}$ and the spin-flip strength $\lambda$.

The final expressions for spin and charge current correlations are too lengthy to be written here. Figs. 2 and 3 show the dependence of the charge and spin Fano factor on the spin-polarization of the terminals for different values of the spin-flip scattering intensity $\lambda$ and when $g_0/g_N = g_L/g_N = 1$. Fig. 2 presents the results for the parallel orientation of the magnetizations of the terminals where $p_0 = p_L = p$. In this
case $F$ (Fig. 2b) has small variations with respect to the normal value of $p = 0$. For finite $\lambda$ it decreases below the normal value for $p \sim 1$. For $\lambda \gg 1$ the charge shot noise takes the normal value for every $p$, which is not surprising since a strong spin-flip intensity destroys the polarization of the injected electrons from the terminals. Contrary to charge shot noise, the spin shot noise shows a strong dependence on $\lambda$ and $p$, as is seen in Fig. 2b. With increasing the polarization $F_s$ decreases from its normal value at $p = 0$. For small $\lambda$ this variation occurs only for $p$ close to one. With increasing the spin-flip intensity the variation is shifted to lower polarizations. At large $\lambda$, $F_s$ decreases monotonically from its normal value 1 to the minimal value at $p = 1$.

The results for the antiparallel configuration $p_0 = -p_L = p$ are shown in Fig. 3. For this configuration both, the charge and spin shot noise, have a strong dependence on the spin-polarization and spin-flip intensity. At finite $\lambda$ the charge Fano factor deviates substantially from its normal value ($p = 0$) when $p$ increases. The strongest variation occurs when $\lambda$ tends to zero. $F$ decreases monotonically by increasing $p$ and reaches the Poissonian value 1 at $p = 1$. In this case, the normal wire has perfectly antiparallel polarized leads at its ends and constitutes an ideal spin valve, for which the current vanishes in the limit $\lambda \to 0$. For very small but finite $\lambda$ only those of electrons, which undergo spin-flip scattering once carry a small amount of current. These spin-flipped electrons are almost uncorrelated and pass through the normal wire independently resulting in a full Poissonian shot noise.

The spin Fano factor $F_s$ has a more complicated dependence on $p$ for different $\lambda$. While for small $\lambda \ll 1$, $F_s$ increases with increasing $p$ to reach the Poissonian value 1 at $p = 1$, for large $\lambda \gg 1$ it decreases from the normal value and reaches a minimum value for a perfect polarization. For $\lambda \sim 1$ the spin shot noise has a nomonotonic behavior with changing the spin polarization. It is an increasing function of $p$ for small polarizations and a decreasing function at large polarizations. Thus $F_s$ has a maximum value at the polarization which depends on the spin-flip intensity.

As can be seen in Figs. 2 and 3 the charge and spin shot noises coincide in two special cases. First, when the terminals are perfectly polarized and $p = 1$, the charge and spin currents (both mean and fluctuations) are the same and therefore the correlators of their fluctuations coincide. Second, for a vanishing spin-flip intensity $\lambda = 0$, there is no spin-dependent scattering mechanism in the whole system (the normal wire and tunnel junctions). The normal impurity scattering and the tunnel barriers (assumed to be spin conserving) have the same effect on the charge and spin transport and the resulting charge and spin current fluctuations have again the same correlations.

In conclusion we have proposed a three-terminal spin valve structure to study correlations of spin current fluctuations. The spin valve consists of a normal diffusive wire which is connected by tunnel contacts to two oppositely perfect polarized ferromagnetic terminals in one end and to another ferromagnetic terminals on the other end. Using a spin-dependent Boltzmann-Langevin approach, the dependence of the spin shot noise on the spin polarization and the strength of the spin-flip scattering has been analyzed for parallel and antiparallel configurations of the polarizations at two ends of the wire. For the parallel case the spin Fano factor (spin shot noise to the mean charge current ratio) has been found to decrease with spin polarization from its unpolarized value, but to increase with the spin-flip rate. In contrast, for the antiparallel configuration we have found a nonmonotonic behaviour of the spin Fano factor, depending on the spin-flip scattering rate. We have also found, that in contrast to the charge Fano factor, which is sensitive to the spin polarization degree and the spin-flip rate only, in the antiparallel case, the spin Fano factor shows variations in both configurations. Our results manifest the effect of competition between spin accumulation and spin relaxation on the spin current fluctuations in diffusive normal conductors.

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