Atmospheric muons and neutrinos are important for neutrino telescopes, both as background in the search for high-energy neutrinos of astrophysical origin and because they serve to calibrate the detectors. In addition, the atmospheric lepton spectra are of interest because the large size of current and proposed neutrino telescopes will make it possible to extend the measurements of atmospheric muons and neutrinos beyond the reach of previous measurements. It may finally be possible to achieve a long-sought goal of cosmic-ray science by detecting the contribution to the atmospheric leptons from decay of charmed hadrons, the “prompt” neutrinos and muons. At energies much lower than $\sim 100 \text{ TeV}$, the prompt component is hidden by the much more abundant contributions from decay of charged pions and kaons. A temperature dependent signal of the charm contribution would complement the standard approach of looking for an isotropic component of the muon flux with a harder spectrum.

It has long been known that the intensity of high-energy muons depends on the temperature in the stratosphere [1]. The variation arises from the corresponding expansion and contraction of the atmosphere at high altitude where most production of the muons occurs. A similar effect is also expected for neutrinos for the same reason [2]. The temperature dependence of TeV muons measured in MINOS [3] has recently been used as a probe of sudden stratospheric warmings [4]. IceCube reported a correlation of muon intensity with stratospheric temperature with high precision made possible by the high statistics of the kilometer-scale detector [5]. Ref. [6] describes how the temperature dependence of muon intensity in the 1 to 10 TeV range is sensitive to the kaon to pion ratio in the atmosphere. In this paper we point out that kilometer-scale neutrino detectors may have sufficient statistics in the 100 TeV range to be able to constrain the contribution of charm decay to prompt leptons.

Currently operating neutrino telescopes are Baikal [7], Antares [8] and IceCube [9]. AMANDA [10], the predecessor of IceCube at the South Pole, was turned off at the beginning of 2009. With a planned volume of one cubic kilometer, IceCube will be by far the largest operating neutrino detector. It is already running with 90% of its sensors installed and will be completed in 2011. There are plans for a still larger neutrino telescope in the Mediterranean Sea (Km3NeT) [11], and there are also plans to expand the Baikal detector [12].

Preliminary measurements of muon [13] and neutrino [14] spectra with data from the partially constructed (25% complete) IceCube in 2007 were reported at the International Cosmic Ray Conference, July 2009. The atmospheric neutrino spectrum measured with AMANDA with data taken from 2000 to 2006 [10] already extends to just above 30 TeV with high statistics. An unfolding analysis of the AMANDA [15] data extends the reach of the same data set to above 100 TeV. No contribution of a prompt component is yet visible in the unfolded neutrino spectrum. The full IceCube will be large enough to extend measurements of atmospheric neutrinos well beyond 100 TeV with good statistics. From the point of view of precise analysis of the energy dependence of the correlation coefficient, the extremely high statistics available with muons in a kilometer scale detector is also important.

Fluxes of secondary cosmic-ray leptons in the atmosphere can be described to a good approximation by a set of formulas in which each element corresponds to one of the processes involved in their production [16, 17]. For $\nu_{\mu} + \bar{\nu}_{\mu}$

$$
\phi_{\nu}(E_{\nu}) = \phi_{N}(E_{\nu}) \times \left\{ \frac{A_{\pi\nu}^{\nu}}{1 + B_{\pi\nu}^{\nu} \cos \theta E_{\nu}/\epsilon_{\pi}} + \frac{A_{K\nu}^{\nu}}{1 + B_{K\nu}^{\nu} \cos \theta E_{\nu}/\epsilon_{K}} \right. + \left. \frac{A_{\text{charm} \nu}^{\nu}}{1 + B_{\text{charm} \nu}^{\nu} \cos \theta E_{\nu}/\epsilon_{\text{charm}}} \right\},
$$

where $\phi_{N}(E_{\nu}) = dN/d\ln(E_{\nu})$ is the primary spectrum of nucleons ($N$) evaluated at the energy of the neutrino. The three terms in brackets correspond to production from leptonic and semi-leptonic decays of pions, kaons and charmed hadrons respectively. The equation for muons is similar at energies sufficiently high ($> 100 \text{ GeV}$) so that energy loss and decay of muons in the atmosphere can be neglected. The numerator of each term is of the form

$$
A_{i\nu}^{\nu} = \frac{Z_{i\nu} \times BR_{i\nu} \times Z_{i\nu}}{1 - Z_{NN}}
$$

with $i = \pi^{\pm}, K$, charm and $BR_{i\nu}$ is the branching ratio for $i \to \nu$. The first $Z$-factor in the numerator is the
spectrum weighted moment of the cross section for a nucleon (N) to produce a secondary hadron i from a target nucleus in the atmosphere, and the second Z-factor is the corresponding moment of the decay distribution for \( i \rightarrow \nu + X \). Thus, for example for \( N + \text{air} \rightarrow K^+ + X \),

\[
Z_{NK^+} = \frac{1}{\sigma_{N-\text{air}}} \int_0^1 x^{\gamma} \frac{d\sigma_{NK^+}(x)}{dx},
\]

where \( x = E_{K^+}/E_N \) and \( \gamma \approx 1.7 \) is the integral spectral index of the incident spectrum of cosmic-ray nucleons. The denominator of Eq. 2 is the ratio of nucleon interaction to attenuation length. For calculations below we use numerical values of the parameters from Ref. 10. Parameters for charm production are discussed below.

The denominator of each term in Eq. 1 reflects an important physical property of meson decay in the atmosphere—the competition between decay and reinteraction of hadrons. The critical energy depends on the zenith angle \( \theta \) of the cascade in the atmosphere, and is of the form

\[
E_{\text{critical}} = \frac{\epsilon_i}{\cos \theta^*} = \frac{m_i c^2 h_0}{\cos \theta^* c \tau_i},
\]

where \( \theta^* \) is the local zenith angle at lepton production taking account of the curvature of the Earth 17. The \( \epsilon_i \) are characteristic energies for each channel and \( h_0 \approx 6.4 \text{ km} \) is the scale height in an exponential approximation to the density of the atmosphere at high altitude. When \( E_i < \epsilon_i/\cos \theta \) the mesons decay so that the low-energy neutrino spectrum has the same power law index as the primary cosmic-ray spectrum. At high energy, the contribution of each term gradually steepens so that asymptotically, the neutrino spectrum is one power steeper than the primary spectrum. The characteristic energies are

\[
\epsilon_\pi = 115 \text{ GeV}, \quad \epsilon_{K^\pm} = 850 \text{ GeV}, \quad \epsilon_{\text{charm}} \sim 5 \times 10^5 \text{ GeV}.
\]

These differences lead to a characteristic pattern of the contributions of the various channels to the flux of neutrinos: first the pion contribution steepens, then the kaon contribution and finally (at much higher energy) the contribution from decay of charmed hadrons becomes the main source of neutrinos and muons in the atmosphere. In addition, the suppression of the charged pion contribution to the neutrino flux starts early because the quantity \( B_{\pi^\nu} = 2.8 \) in the denominator of the first term of Eq. 1 is anomalously large compared to the corresponding factors for strange and charmed hadrons. This is a consequence of the kinematics of \( \pi^\pm \rightarrow \mu^\pm + \nu \) decay in flight in which the muon carries most of the energy because its mass is comparable to that of the parent pion.

Fig. 1 shows the fractional contribution of the main hadronic channels to the production of neutrinos and muons in the atmosphere. Kaons actually become the dominant source of neutrinos above \( \sim 100 \text{ GeV} \). For muons, the dominant channel is never kaons, but the contribution of kaons does increase significantly in the TeV energy range. An interesting manifestation of the increasing importance of kaons at high energy is the measurement of the charge ratio of atmospheric muons with energies at production in the TeV range 18, 19. The \( \mu^+/\mu^- \) ratio is observed to increase from its value of 1.27 around 100 GeV to \( \approx 1.37 \) above a TeV. The increase is attributed to the importance of the associated production of kaons,

\[
p + \text{air} \rightarrow \Lambda + K^+ + X,
\]

which makes the +/− charge ratio larger for kaons than for pions. Production of strange particles and antiparticles—in particular the process in Eq. 5—is highly asymmetric in the forward fragmentation region. Production of \( \Lambda \) is favored because it has constituents in common with the valence quarks of the proton.

At some energy, perhaps around 100 TeV or somewhat above, the decay of charmed hadrons will become the main source of all atmospheric leptons. Where the transition occurs is uncertain because of the wide variation in the literature on the level of charm production at large Feynman x. Like associated production of strangeness via Eq. 5, the production of \( \Lambda^+ \) is also highly asymmetric in the forward fragmentation region. A measurement by the Fermilab Experiment 781 (SELEX) 20 shows that \( (\sigma_c - \sigma_\bar{c})/(\sigma_c + \sigma_\bar{c}) \approx 1 \) for \( 0.2 < x < 0.7 \) for 600 GeV protons on a fixed target. Here \( \sigma_c \) and \( \sigma_\bar{c} \) represent respectively the production of \( \Lambda^+_c \), which does, and \( \Lambda^-_c \), which does not have valence quarks in common with the beam proton. A large contribution from the charmed analog process to Eq. 5 could be classified as "intrinsic"
charm [21]. There is some support for a component of intrinsic charm from recent measurements of charm production on different nuclear targets [22], as discussed in Ref. [23].

The RQPM model of charm production [24] includes such a contribution and predicts a relatively high level of prompt leptons, as illustrated in Fig. 1. Calculations using perturbative QCD tend to give lower levels of charm production. As an example, a recent calculation within a perturbative QCD framework [25] predicts a contribution from charm decay roughly an order of magnitude lower than the RQPM model. In what follows we evaluate the sensitivity of seasonal effects to prompt atmospheric leptons for three different assumptions about the level of charm production. We compare RQPM with charm production at the level of Ref. [25] (henceforth ERS) and with a somewhat arbitrary intermediate model described in terms of the parameters of Eq. 2. Representing the weighted sums of the various charm channels and their semi-leptonic decay modes with $Z_{N,\text{charm}} = 5 \times 10^{-4}$, $Z_{\text{charm}} = Z_{\text{charm}} = 0.13$ and an effective semi-leptonic branching fraction for charm decays of 0.14, gives a level of charm production about a factor of two lower than RQPM.

In an isothermal approximation, the density of the atmosphere is described by an exponential with a scale height of $h_0 \approx 6.4$ km, where the numerical value is applicable to the stratosphere where most high-energy muons and neutrinos originate. From the ideal gas equation relating density and pressure, one finds $h_0 = R T$. With this relation it is then possible from Eqs. 4 and 5 to calculate the variation of the flux with temperature. The deviation of the atmosphere from isothermal is accounted for to first order in $\delta T/T$ by weighting the actual temperature profile by the profile of production of the atmospheric leptons. It is conventional to define a correlation coefficient, $\alpha_\mu$ which relates the fractional change in the atmospheric muon intensity to the fractional change in the temperature. One can describe the atmospheric neutrinos in parallel, with a coefficient $\alpha_\nu$ [2]. In differential form the correlation coefficient is

$$\alpha(E, \theta) = \frac{1}{\phi(E, \theta)} \frac{d\phi(E, \theta)}{dT}. \quad (6)$$

The derivative can be calculated directly from the expressions [1 and 3] with the results shown in Fig. 2.

The features of the curves in Fig. 2 correspond to the properties of the various terms in Eq. 1. At extremely high energy, the second term in the denominator dominates, which means the the intensity of muons is proportional to temperature and the correlation coefficient is unity. Asymptopia is reached at lower energy for pions than for kaons, so the inclusion of kaons delays the increase of the correlation coefficient so somewhat higher energy. This is more apparent for neutrinos than for muons because kaons give a larger fractional contribution to neutrinos than to muons. The increase of the correlation coefficient is delayed to higher energy at large zenith angle because the critical energy (Eq. 4) is larger. Since the critical energy for the charm contribution is at extremely high energy ($\sim 5 \times 10^7$ GeV), the prompt leptons from charm decay have no temperature dependence until $\sim 10^7$ GeV. Thus, a significant contribution from charm will suppress the correlation coefficient above an energy that depends on the magnitude of the charm contribution to lepton production. For the example shown in Fig. 2 in which the RQPM model of Bugaev et al. [24] is used, the suppression begins already between 10 and 100 TeV depending on zenith angle.

Measured rates depend on the convolution of the lepton spectrum with the detector response and effective area, which depends on lepton energy and direction. To compare with measurements, it is therefore necessary to make the following calculation:

$$\alpha_i(\theta) = \frac{T}{\int dE \phi_i(E) \times A_{i,\text{eff}}(E, \theta)} \times \frac{d}{dT} \int dE \phi_i(E) \times A_{i,\text{eff}}(E, \theta). \quad (7)$$

Here the subscript $i = \nu, \mu$ and $A_{i,\text{eff}}(E, \theta)$ is the energy-dependent effective area of the detector as seen from a direction $\theta$. In the remainder of the paper we explore some examples relevant for IceCube.

To make an estimate for muons we assume a step function at energies corresponding to the minimum needed to penetrate through IceCube treated as a sphere with a projected area of one square kilometer. Table I gives some rates for muons with zenith angle $< 30^\circ$. The correlation coefficient is 10% lower with RQPM charm for $E_\mu > 10$ TeV than with no charm and the rate is more than one Hz per square kilometer. For $E_\mu > 100$ TeV the correlation coefficient is only 0.5 with charm while the event rate is much lower, but still of order $10^5$ events per year in a kilometer scale detector.

For neutrinos we use the effective areas computed for IceCube-40 [24] multiplied by a factor of two to estimate the event numbers for a full cubic kilometer detector. Table II gives correlation coefficient and expected rates for three different ranges of zenith angle for neutrino-induced muons coming into the detector from below. The three ranges of zenith angle are 1) $90^\circ < \theta < 120^\circ$, 2) $120^\circ < \theta < 150^\circ$, and 3) $150^\circ < \theta < 180^\circ$, which have
### TABLE I: Correlation coefficients for muons with $(\theta \leq 30^\circ)$ for three levels of charm (energy in TeV; rate in Hz/km²).

| $E_{\nu, \text{min}}$ (TeV) | no charm | RQPM charm | ERS charm | int. charm |
|-----------------------------|----------|-------------|-----------|-----------|
| $\alpha$ Rate $\alpha$ Rate | $\alpha$ Rate $\alpha$ Rate | $\alpha$ Rate |
| 0.5 | 0.83 | 2050 | 0.82 | 2070 | 0.82 | 2050 |
| 10 | 0.98 | 1.25 | 0.89 | 1.40 | 0.97 | 1.26 | 0.94 | 1.34 |
| 100 | 1.0 | 0.0025 | 0.55 | 0.0045 | 0.91 | 0.0028 | 0.71 | 0.0036 |

### TABLE II: Correlation coefficients with and without charm for neutrinos in three zones of the atmosphere (see text).

| $E_{\nu, \text{min}}$ (TeV) | no charm | RQPM charm |
|-----------------------------|----------|-------------|
| Zone 1 $\alpha$ Events/yr $\alpha$ Events/yr | $\alpha$ Events/yr |
| all | 0.54 | 10000 | 0.52 | 17000 |
| 3 | 0.70 | 5900 | 0.62 | 6300 |
| 30 | 0.94 | 350 | 0.72 | 450 |
| Zone 2 $\alpha$ Events/yr $\alpha$ Events/yr | $\alpha$ Events/yr |
| all | 0.66 | 6000 | 0.62 | 6400 |
| 3 | 0.88 | 1200 | 0.75 | 1450 |
| 30 | 0.95 | 37 | 0.46 | 80 |
| Zone 3 $\alpha$ Events/yr $\alpha$ Events/yr | $\alpha$ Events/yr |
| all | 0.68 | 1650 | 0.64 | 1750 |
| 3 | 0.91 | 260 | 0.75 | 320 |
| 30 | 0.99 | 5.2 | 0.41 | 13 |

For lepton energies below 10 PeV the contribution from charm decay does not depend on temperature because charmed hadrons decay before interacting as a consequence of their short decay times. In contrast, by 100 TeV pions and kaons are fully asymptotic in the sense that their decay probability is proportional to temperature. This situation leads to the possibility of inferring the level of prompt lepton production from a precise measurement of the correlation of the rate of leptons with stratospheric temperature in the 10 to 100 TeV energy range. There are several practical problems that must be overcome to realize this possibility. First, a high-energy event sample must be defined with sufficient energy resolution and sufficient statistics to see the effect of the charm contribution on the correlation with temperature. In the case of muons, there will be sufficient statistics in a kilometer scale detector, but the energy assignment of individual muons may be complicated by the presence of accompanying muons from the same cosmic-ray primary. In the case of neutrinos it will be necessary to integrate over a large fraction of the surface of the Earth, although about 25% of upward atmospheric neutrinos are generated in the atmosphere above Antarctica, where the stratospheric temperature variation is strongest. The practical application of this probe of charm is being developed in association with analysis of measured temperature effects in IceCube and is most promising for high-energy muons because of the large statistics.

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