CP Violation in the $B$ System: Status and Perspectives

Robert Fleischer
CERN, Department of Physics, Theory Division
CH-1211 Geneva 23, Switzerland

Abstract

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CP VIOLATION IN THE B SYSTEM: STATUS AND PERSPECTIVES∗

ROBERT FLEISCHER

CERN, Department of Physics, Theory Division, CH-1211 Geneva 23, Switzerland

In this decade, the exploration of CP violation is governed by the B-meson system, where non-leptonic decays play an outstanding rôle. To set the stage, we have first a look at the main strategies to circumvent the calculation of the relevant hadronic matrix elements in these studies, and discuss popular avenues for physics beyond the Standard Model to enter the roadmap of quark-flavour physics. We are then well prepared to analyze puzzles in the data for \( B \rightarrow \phi K \) and \( B \rightarrow \pi K \) decays, and to discuss \( b \rightarrow d \) penguin processes, which represent a new territory for the B-factory studies. Finally, we turn to the physics potential of the \( B_s \)-meson system, which can be fully exploited at the LHC.

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1. Setting the Stage

1.1. The Flavour-Physics Landscape in a Nutshell

As discussed in detail in the Gustavo Book [1], CP violation is a particularly interesting phenomenon in particle physics, which offers a powerful tool to explore the flavour sector of the Standard Model (SM) and to search for signals of “new physics” (NP). After a long and exciting history of \( K \)-decay studies, the stage is now governed by the decays of \( B^+ \) and \( B^0 \rightarrow \phi K \) mesons. Thanks to the efforts at the \( e^+e^- \) B factories with their detectors BaBar (SLAC) and Belle (KEK), CP violation is now also in the \( B \)-meson system well established, and several strategies to test the flavour structure of the SM, which is governed by the Cabibbo–Kobayashi–Maskawa (CKM) matrix [2], can now be confronted – for the first time – with experimental data. These experiments have already collected \( \mathcal{O}(10^8) \) \( B\bar{B} \) pairs. Further valuable insights can be obtained through the studies of the \( B_s^0 \)-meson

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system. After first results from the LEP experiments (CERN) and SLD (SLAC) as well as from the Tevatron, the physics potential of $B^0_d$ decays can be fully exploited at the Large Hadron Collider (LHC) at CERN, in particular by the LHCb experiment \cite{LHCb}. Moreover, there are also plans for a “super-$B$ factory”, with a significant increase of luminosity relative to the currently operating $e^+e^-$ colliders. As far as the kaon system is concerned, the future lies in particular on the investigation of the very rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, which are very clean from the theoretical point of view, but unfortunately hard to measure; there is a new proposal to measure the former channel at the CERN SPS, and efforts to explore the latter at KEK/J-PARC in Japan. Moreover, there are many other fascinating aspects of flavour physics, such as charm and top physics, flavour violation in the charged lepton and neutrino sectors, electric dipole moments and studies of the anomalous magnetic moment of the muon.

Let us in the following discussion focus on the $B$-meson system, which offers a particularly interesting playground. It involves challenging aspects of strong interactions, such as issues related to (non)-factorization, rescattering processes, “ charming penguins”, etc., and provides valuable insights into weak interactions, where the CKM matrix and the associated unitarity triangle (UT) with its angles $\alpha$, $\beta$ and $\gamma$ are the main targets. Moreover, since NP effects may enter this game, there is the exciting possibility of obtaining hints for new sources of flavour and/or CP violation.

1.2. Non-Leptonic $B$ Decays

A key element for the exploration of CP violation is given by non-leptonic $B$ decays. The reason is that observable CP-violating effects are induced by certain interference effects, which may arise in decays of this kind. The final states of non-leptonic transitions consist only of quarks, and are caused by $b \to q_1\bar{q}_2 d(s)$ quark-level processes, with $q_1, q_2 \in \{u, d, c, s\}$. There are two kinds of topologies contributing to such decays: “tree” and “penguin” topologies. The latter consist of gluonic (QCD) and electroweak (EW) penguins. Depending on the flavour content of their final states, we distinguish between decays which receive only tree contributions, channels which may originate from tree and penguin contributions, and modes which are only caused by penguin topologies. In order to deal with these processes theoretically, low-energy effective Hamiltonians are used, which are calculated by means of the operator product expansion, yielding transition amplitudes of the following structure \cite{Fleischer}:

$$\langle f | \mathcal{H}_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k C_k(\mu) \langle f | Q_k(\mu) | B \rangle,$$

(1)
where $G_F$ is Fermi’s constant, $\lambda_{\text{CKM}}$ a factor containing the corresponding elements of the CKM matrix, and $\mu$ denotes a renormalization scale. The $Q_k$ are local operators, which are generated through the interplay between electroweak interactions and QCD, and govern “effectively” the decay in question, whereas the Wilson coefficients $C_k(\mu)$ describe the scale-dependent “couplings” of the interaction vertices that are associated with the $Q_k$. In this formalism, the short-distance contributions are described by the perturbatively calculable Wilson coefficients $C_k(\mu)$, whereas the long-distance physics arises in the form of hadronic matrix elements $\langle f|Q_k(\mu)|B \rangle$. These non-perturbative quantities are the key problem in the theoretical analyses of non-leptonic $B$ decays. Although there were interesting developments in this field through “QCD factorization” (QCDF) [5], the “perturbative QCD” (PQCD) approach [6], “soft collinear effective theory” (SCET) [7], and QCD light-cone sum-rule methods [8], the $B$-factory data indicate that the theoretical challenge remains (see, for instance, Refs. [9]–[11]).

Fortunately, it is possible to circumvent the calculation of the hadronic matrix elements for the exploration of CP violation:

- Amplitude relations can be used to eliminate the hadronic matrix elements. We distinguish between exact relations, using pure “tree” decays of the kind $B \to KD$ or $B_c \to D_sD$, and relations, which follow from the flavour symmetries of strong interactions, and involve $B(s) \to \pi\pi, \pi K, KK$ modes.

- In the neutral $B_q$ systems ($q \in \{d,s\}$), the interference between $B_q^0 - \bar{B}_q^0$ mixing and decay processes may lead to “mixing-induced CP violation”. If a single CKM amplitude dominates the decay, the hadronic matrix elements cancel in the corresponding CP asymmetries; otherwise we have to use amplitude relations again.

These avenues offer various strategies to “overconstrain” the UT through studies of CP violation in the $B$-meson system, and to compare the resulting picture with the usual “CKM fits” [12] [13]. Moreover, “rare” decays, which originate from loop processes in the SM, provide valuable complementary information; key examples are $B \to X_s\gamma$ and the exclusive modes $B \to K^*\gamma$, $B \to \rho\gamma$, as well as $B_{s,d} \to \mu^+\mu^-$ and $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$. In the presence of NP effects in the quark-flavour sector, we expect discrepancies with respect to the pattern following from the structure of the CKM matrix.

1.3. The Generic Impact of New Physics

Popular avenues for NP to manifest itself are offered by $B_q^0 - \bar{B}_q^0$ mixing and/or decay amplitudes [14]. Let us first have a look at the former option. Here NP may enter through the exchange of new particles in box diagrams,
which contribute in the SM, or through new contributions at the tree level, thereby modifying the mixing parameters as follows:

\[ \Delta M_q = \Delta M_q^{\text{SM}} + \Delta M_q^{\text{NP}}, \quad \phi_q = \phi_q^{\text{SM}} + \phi_q^{\text{NP}}. \]  

(2)

Whereas the NP contribution \( \Delta M_q^{\text{NP}} \) to the mass difference of the \( B_q \) mass eigenstates would affect the determination of one side, \( R_t \), of the UT, the NP contribution \( \phi_q^{\text{NP}} \) to the weak mixing phase would enter the mixing-induced CP asymmetries. The comparison of the \( B \)-factory data for the mixing-induced CP violation in the “golden” decay \( B_0^d \rightarrow J/\psi K_S \), which allows a clean measurement of \( \sin 2\beta \) in the SM, and the CKM fits is globally very good and does not indicate large NP effects. However, thanks to a new Belle result \[15\], the world average of \( \sin 2\beta \) compiled by the “Heavy Flavour Averaging Group” \[16\] went down by about 1\( \sigma \) this summer, and takes now the following value:

\[ (\sin 2\beta)_{\psi K_S} = 0.687 \pm 0.032. \]  

(3)

Because of this somewhat surprising development, the straight line in the \( \bar{\rho} - \bar{\eta} \) plane of the generalized Wolfenstein parameters \[17\] \[18\] is now on the lower side of the allowed region for the apex of the UT following from the CKM fits \[12\] \[13\], i.e. the picture in the \( \bar{\rho} - \bar{\eta} \) plane does no longer look “perfect”, as can be seen in Fig. 1. This deviation could be interpreted in terms of NP contributions to \( B_0^d \rightarrow \bar{B}_0^d \) mixing, involving in particular a CP-violating phase \( \phi_d^{\text{NP}} \sim -8^\circ \) \[19\] \[20\]. As a next step, it will be very interesting to explore the \( B_s \) system, where \( B_s^0 \rightarrow J/\psi \phi \) offers a sensitive probe to search for CP-violating NP effects in \( B_s^0 \rightarrow \bar{B}_d^0 \) mixing \[21\] \[22\].
Concerning the possibility for NP to enter directly through decay amplitudes, the flavour-changing neutral-current (FCNC) sector plays a key rôle. For instance, new particles may enter in penguin diagrams, or new FCNC processes may arise at the tree level. The effects are typically small if the considered decay is dominated by SM tree processes. This is the case, for example, in $B^0_d \to J/\psi K_S$. However, this decay receives also contributions from penguin topologies, which enter essentially with the same weak phase as the tree contribution in the SM, and also EW penguins may have a sizeable impact \cite{24}. Since NP could nicely enter through the latter topologies \cite{25,26}, the possible small conflict of (3) with the CKM fits could, in principle, also be generated through NP effects in the EW penguin sector (or through other NP contributions to the $B \to J/\psi K$ amplitudes), although the corresponding contributions would have to be enhanced significantly with respect to the SM estimate \cite{27}. A tool to distinguish this logical possibility from NP effects in $B^0_d \to B^0_d$ mixing is offered by decays of the kind $B_d \to D\pi^0, D\rho^0, ...$, which are pure “tree” decays, i.e. they do not receive any penguin contributions. If the neutral $D$ mesons are observed through their decays into CP eigenstates $D_{\pm}$, these decays allow extremely clean determinations of the “true” value of $\sin 2\beta$ \cite{28}. Consequently, detailed feasibility studies for the exploration of the $B_d \to D\pi^0, D\rho^0, ...$ modes at a super-$B$ factory are strongly encouraged. If a decay is dominated by FCNC processes, we may encounter potentially large NP effects. Interestingly, there are hints in the $B$-factory data for such effects. Let us have a closer look at the corresponding puzzles in the next section.

2. Prominent Puzzles in the Current $B$-Factory Data

2.1. CP Violation in $B^0_d \to \phi K_S$

An interesting probe for the testing of the SM description of CP violation is offered by the $B^0_d \to \phi K_S$ channel, which originates from $\bar{b} \to \bar{s}s\bar{s}$ transitions and is, therefore, a pure penguin mode. Thanks to the special phase structure of the corresponding decay amplitude in the SM, the following relations can be derived \cite{21,29}:

\begin{equation}
A_{\text{CP}}^{\text{dir}}(B_d \to \phi K_S) = 0 + \mathcal{O}(\lambda^2)
\end{equation}

\begin{equation}
A_{\text{CP}}^{\text{mix}}(B_d \to \phi K_S) = A_{\text{CP}}^{\text{mix}}(B_d \to \psi K_S) + \mathcal{O}(\lambda^2), \quad \equiv - (\sin 2\beta)_{\phi K_S}
\end{equation}

where the $A_{\text{CP}}^{\text{dir}}$ and $A_{\text{CP}}^{\text{mix}}$ denote the “direct” and “mixing-induced” CP asymmetries, respectively, and $\lambda \equiv |V_{us}| = 0.22$ is the Wolfenstein expansion parameter \cite{17}. The dominant contributions to $B \to \phi K$ decays arise from QCD penguin operators \cite{30}. However, due to the large top-quark mass,
EW penguins have a sizeable impact as well \cite{31,32}. Consequently, since $B_d^0 \rightarrow \phi K_S$ is governed by penguin processes in the SM, this decay may well be affected by NP. In fact, if we assume that NP arises generically in the TeV regime, it can be shown through field-theoretical estimates that the NP contributions to $b \rightarrow s \bar{s}s$ transitions may well lead to sizeable violations of the relations in \cite{31} and \cite{33}. Moreover, this is also the case for several specific NP scenarios (see, for instance, Refs. \cite{34}–\cite{37}).

Concerning the measurement of the CP asymmetries of the $B_d^0 \rightarrow \phi K_S$ decay, the result $(\sin 2\beta)_{\phi K_S} = -0.96 \pm 0.50^{+0.09}_{-0.11}$ reported by the Belle collaboration in the summer of 2003 led to quite some excitement in the $B$-physics community. Meanwhile, the Babar \cite{38} and Belle \cite{39} results are in good agreement with each other, yielding the following averages \cite{16}:

$$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \phi K_S) = -0.09 \pm 0.14, \quad (\sin 2\beta)_{\phi K_S} = 0.47 \pm 0.19. \quad (6)$$

The number for $(\sin 2\beta)_{\phi K_S}$ is still on the lower side, and may indicate NP contributions to $b \rightarrow s \bar{s}s$ processes. In \cite{27}, a much more detailed discussion can be found, addressing also NP effects in the EW penguin sector, which may be responsible both for the possible difference between \cite{3} and \cite{16} and for the “$B \rightarrow \pi K$ puzzle” discussed in Subsection 2.2.

It will be very interesting to follow the evolution of the $B$-factory data, and to monitor also similar modes, such as $B_d^0 \rightarrow \pi^0 K_S$ \cite{40} and $B_d^0 \rightarrow \eta' K_S$ \cite{41}. For a compilation of the corresponding newest experimental results, see Ref. \cite{16}; recent theoretical papers dealing with these channels can be found in Refs. \cite{9,20,42,43,44}.

2.2. The $B \rightarrow \pi K$ Puzzle and its Relation to Rare Decays

2.2.1. Preliminaries

The first indication of the “$B \rightarrow \pi K$ puzzle” goes back to the observation of the $B_d^0 \rightarrow \pi^0 K^0$ channel by the CLEO collaboration in 2000 with a remarkably prominent rate, which may signal a discrepancy with the SM \cite{45}. This possible anomaly is still present in the current data, and has recently received a lot of attention (see, for instance, \cite{35,46,47,48,49,50}).

Let us follow here the strategy to explore this exciting topic in a systematic manner that was developed in \cite{9}. The starting point is an analysis of the $B \rightarrow \pi \pi$ system, where the data can be accommodated in the SM through large non-factorizable effects. In particular, the $B \rightarrow \pi \pi$ decays allow the extraction of a set of hadronic parameters with the help of the isospin symmetry of strong interactions. Using then the $SU(3)$ flavour symmetry and neglecting certain exchange and penguin annihilation topologies, the hadronic $B \rightarrow \pi \pi$ parameters can be converted into their $B \rightarrow \pi K$ counterparts, allowing the prediction of all $B \rightarrow \pi K$ observables in the SM.
Interestingly, agreement with experiment is found for those decays that are only marginally affected by (colour-suppressed) EW penguins. On the other hand, the SM predictions of the $B \to \pi K$ observables which are significantly affected by (colour-allowed) EW penguins are not found in agreement with the data, thereby reflecting the $B \to \pi K$ puzzle. Moreover, internal consistency checks of the working assumptions can be performed, which work well within the current uncertainties, and the numerical results turn out to be very stable with respect to large non-factorizable $SU(3)$-breaking corrections [20]. In view of these features, NP in the EW penguin sector may be at the origin of the $B \to \pi K$ puzzle. In fact, it can be resolved through a modification of the EW penguin parameters, involving in particular a large CP-violating NP phase that vanishes in the SM. The implications of this kind of NP on rare $K$ and $B$ decays are then investigated in the final step of the strategy proposed in Ref. [9].

The numerical results presented below refer to the very recent analysis presented in Ref. [20]. In view of the new world average for $(\sin 2\beta)_{\psi K}$ in [3], which may signal NP effects in $B_d^0 - \bar{B}_d^0$ mixing, the CP asymmetries of the $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^-K^+$ system are used to determine the “true” value of the UT angle $\gamma$, yielding

$$\gamma = (73.9^{+5.8}_{-6.5})^\circ.$$  \hspace{1cm} (7)

This number is then used as an input for the $B \to \pi\pi$ analysis discussed below. Furthermore, complementing [7] with the experimental value of the UT side $R_b \propto |V_{ub}/V_{cb}|$, which follows from semi-leptonic $B$ decays that are very robust under NP effects, also the “true” value of $\beta$ can be extracted, $\beta = (25.8 \pm 1.3)^\circ$, which would correspond to $\phi_d^{NP} = -(8.2 \pm 3.5)^\circ$, in accordance with the analysis of Ref. [19].

2.2.2. The $B \to \pi\pi$ Analysis

The starting point of the $B \to \pi\pi$ study is given by the following ratios:

$$R_{\pi^+\pi^-}^{\pi\pi} = 2 \left[ \frac{\text{BR}(B_d^+ \to \pi^+\pi^-)}{\text{BR}(B_d \to \pi^+\pi^-)} \right] = F_1(d, \theta, x, \Delta; \gamma) \exp^{2.04 \pm 0.28}$$  \hspace{1cm} (8)

$$R_{\pi^0\pi^0}^{\pi\pi} = 2 \left[ \frac{\text{BR}(B_d \to \pi^0\pi^0)}{\text{BR}(B_d \to \pi^+\pi^-)} \right] = F_2(d, \theta, x, \Delta; \gamma) \exp^{0.58 \pm 0.13}.  \hspace{1cm} (9)

Here the isospin symmetry of strong interactions was used to express these observables in terms of $\gamma$ and the hadronic parameters $d\theta$, $xe^{i\Delta}$ which were introduced in Ref. [9]. Moreover, the CP asymmetries

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-) = G_1(d, \theta; \gamma) \exp^{-0.37 \pm 0.10}  \hspace{1cm} (10)

\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-) = G_2(d, \theta; \gamma, \phi_d) \exp^{+0.50 \pm 0.12}  \hspace{1cm} (11)$$
are at our disposal, where $\phi_d^{\exp} = (43.4 \pm 2.5)^\circ$ follows from the data for the mixing-induced CP violation in $B_0^d \to J/\psi K_S$. Using the value of $\gamma$ in \cite{7}, the hadronic parameters characterizing the $B \to \pi \pi$ system can be extracted, with the following results:

$$d = 0.52^{+0.09}_{-0.09}, \quad \theta = (146^{+7.0}_{-7.2})^\circ, \quad x = 0.96^{+0.13}_{-0.14}, \quad \Delta = -(53^{+18}_{-26})^\circ. \quad (12)$$

These numbers, which exhibit large non-factorizable effects, take also EW penguin effects into account, although these topologies have a minor impact on the $B \to \pi \pi$ decays.

Finally, the CP-violating observables of the $B_0^d \to \pi^0 \pi^0$ channel can be predicted:

$$A_{\text{dir}}^{\text{CP}}(B_d \to \pi^0 \pi^0) = -0.30^{+0.48}_{-0.26}^{\exp} -0.28^{+0.40}_{-0.39} \quad (13)$$

$$A_{\text{mix}}^{\text{CP}}(B_d \to \pi^0 \pi^0) = -0.87^{+0.29}_{-0.19}. \quad (14)$$

Although no stringent test of these predictions is possible at this stage, the indicated agreement between the prediction in \cite{13} and the corresponding experimental value \cite{16} is encouraging.

2.2.3. The $B \to \pi K$ Analysis

Using now the $SU(3)$ flavour symmetry and neglecting exchange and penguin annihilation topologies, the hadronic parameters in \cite{12} can be converted into their $B \to \pi K$ counterparts, allowing the prediction of the $B \to \pi K$ observables in the SM. Moreover, a couple of internal consistency checks of these working assumptions can be performed, which are fulfilled by the current data, and the sensitivity of the SM predictions on large non-factorizable $SU(3)$-breaking effects turns out to be surprisingly small \cite{20}. Consequently, no anomaly is indicated in this sector.

In the case of the $B_0^d \to \pi^- K^+$, $B^+ \to \pi^+ K^0$ system, where EW penguins have a minor impact, a picture arises in the SM that is in accordance with the data (see also \cite{51}). In order to analyze the decays $B^+ \to \pi^0 K^+$ and $B_0^0 \to \pi^0 K^0$, which are significantly affected by EW penguins, it is useful to introduce the following quantities:

$$R_c \equiv 2 \left[ \frac{\text{BR}(B^\pm \to \pi^0 K^\pm)}{\text{BR}(B^\pm \to \pi^\pm K)} \right]^{\exp} 1.01 \pm 0.09 \quad (15)$$

$$R_n \equiv \frac{1}{2} \left[ \frac{\text{BR}(B_d \to \pi^+ K^\pm)}{\text{BR}(B_d \to \pi^0 K)} \right]^{\exp} 0.83 \pm 0.08. \quad (16)$$

The EW penguin effects are described by a parameter $q$, which measures the strength of the EW penguins with respect to the tree-diagram-like topologies, and a CP-violating phase $\phi$. In the SM, this phase vanishes, and $q$
can be calculated with the help of the $SU(3)$ flavour symmetry, yielding a value of $0.69 \times 0.086/|V_{ub}/V_{cb}| = 0.58$ \cite{52}. The situation can transparently be discussed in the $R_n-R_c$ plane, as shown in Fig. 2: the shaded areas indicate the SM prediction and the experimental range, the lines show the theory predictions for the central values of the hadronic parameters and various values of $q$ with $\phi \in [0^\circ, 360^\circ]$; the dashed rectangles represent the SM predictions and experimental ranges at the time of the original analysis of Ref. \cite{9}. Although the central values of $R_n$ and $R_c$ have slightly moved towards each other, the puzzle is as prominent as ever. The experimental region can now be reached without an enhancement of $q$, but a large CP-violating phase $\phi$ of the order of $-90^\circ$ is still required, although $\phi \sim +90^\circ$ can also bring us rather close to the experimental range of $R_n$ and $R_c$.

Moreover, also the CP-violating asymmetries of the $B^\pm \rightarrow \pi^0K^\pm$ and $B_d \rightarrow \pi^0K_S$ decays can be predicted both in the SM and in the scenario of NP effects in the EW penguin sector. In particular the mixing-induced CP asymmetry of the latter decay has recently received a lot of attention, as the current $B$-factory data give a value of

$$\Delta S \equiv (\sin 2\beta)_{\pi^0K_S} - (\sin 2\beta)_{\psiK_S}^{\exp} = -0.38 \pm 0.26. \quad (17)$$

In the strategy described above, this difference is predicted to be positive in the SM, and in the ballpark of $0.10-0.15$ \cite{20}. Interestingly, the best values for $(q, \phi)$ that are implied by the measurements of $R_{n,c}$ make the disagreement of $\Delta S$ with the data even larger than in the SM. However, also values of $(q, \phi)$ can be found for which $\Delta S$ could be smaller than in the SM or even reverse the sign \cite{20}. This happens in particular for $\phi \sim +90^\circ$, i.e. if the CP-violating NP phase flips its sign. In this case, also the central

![Fig. 2. The situation in the $R_n-R_c$ plane, as discussed in the text.](image-url)
value of $(\sin 2\beta)_{\phi K_S}$ in (6) could be straightforwardly accommodated in this scenario of NP [27], and could in fact be another manifestation of a modified EW penguin sector with new sources for CP violation.

2.2.4. Relation with Rare $B$ and $K$ Decays

A popular scenario for NP effects in the EW penguin sector is offered by modified $Z^0$ penguins with a new CP-violating phase. This scenario was already considered in the literature, where model-independent analyses and studies within SUSY can be found [53, 54]. Following [55] and performing a renormalization-group evolution from scales $\mathcal{O}(m_b)$ to $\mathcal{O}(M_W, m_t)$, the EW penguin parameters $(q, \phi)$ of the $B \to \pi K$ system can be converted, in this scenario, into a $Z^0$-penguin function $C$ as well as other short-distance functions, which allow us to make predictions for rare decays the kind $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0\ell^+\ell^-$, $B \to X_s\nu\bar{\nu}$ and $B_{s,d} \to \mu^+\mu^-$. An analysis along these lines shows that we may encounter interesting NP effects in the corresponding observables, in particular in the $K \to \pi\nu\bar{\nu}$ system. In [20], it was pointed out that the most recent $B$-factory constraints for rare decays, in particular for $B \to X_s\ell^+\ell^-$ [56], have interesting new implications. In this context, a few future scenarios with different patterns of the relevant observables are discussed, where also the mixing-induced CP violation in $B^0_d \to \pi^0 K_S$ plays a prominent role. It will be interesting to confront this analysis with future, more accurate data!

3. Entering New Territory: $b \to d$ Penguins

Another recent hot topic is the exploration of $b \to d$ penguin processes. Both the non-leptonic decays belonging to this category, which originate from $b \to d\bar{s}s$ quark transitions, and the radiative decays caused by $b \to d\gamma$ processes are now coming within experimental reach at the $B$ factories. We are therefore entering a new territory, which is still essentially unexplored.

3.1. A Prominent Example: $B^0_d \to K^0\bar{K}^0$

This decay is the $\bar{b} \to \bar{d}$ penguin counterpart of the $B^0_d \to \phi K_S$ decay discussed in Subsection 2.2.1. The dominant role is played by QCD penguins; since EW penguins may only contribute in colour-suppressed form, they have a minor impact on $B^0_d \to K^0\bar{K}^0$, in contrast to the case of $B^0_d \to \phi K^0$, where they may also contribute in colour-allowed form. In the SM, the $B^0_d \to K^0\bar{K}^0$ decay amplitude can be written as follows:

$$A(B^0_d \to K^0\bar{K}^0) = \lambda^3 A(A^P - A^S) \left[ 1 - \rho_{KK} e^{i\theta_{KK}} e^{i\gamma} \right], \quad (18)$$
Fig. 3. Illustration of the surface in the \( \mathcal{A}_{\text{dir}} - \mathcal{A}_{\text{mix}} - \langle B \rangle \) observable space characterizing the \( B^0_d \to K^0\bar{K}^0 \) decay in the SM. The intersecting lines on the surface correspond to constant values of \( \rho_{KK} \) and \( \theta_{KK} \); the numbers on the fringe indicate the value of \( \theta_{KK} \), while the fringe itself is defined by \( \rho_{KK} = 1 \).

where \( \lambda \) and \( A \) are Wolfenb"{u}ttel parameters [17], and the hadronic parameter

\[
\rho_{KK}e^{i\theta_{KK}} \equiv R_b \left[ \frac{A^q_P - A^q_P}{A^q_P - A^q_P} \right] \tag{19}
\]

involves the side \( R_b \) of the UT and the strong amplitudes \( A^q_P \), which describe penguin topologies with \( q \)-quark exchanges. The direct and mixing-induced CP asymmetries of \( B^0_d \to K^0\bar{K}^0 \) can then be expressed in terms of \( \rho_{KK} \), \( \theta_{KK} \) and \( \gamma \); the latter observable involves also the mixing phase \( \phi_d \). If we assume, for a moment, that the penguin contributions are dominated by top-quark exchanges, (19) simplifies as \( \rho_{KK}e^{i\theta_{KK}} \to R_b \). Since the CP-conserving strong phase \( \theta_{KK} \) vanishes in this limit, the direct CP violation in \( B^0_d \to K^0\bar{K}^0 \) vanishes, too. Moreover, it can be shown that also the mixing-induced CP asymmetry would vanish in the SM because of \( \phi_d = 2\beta \). Consequently, the measurement of the CP-violating \( B^0_d \to K^0\bar{K}^0 \) asymmetries appears as an interesting test of the SM (see, for instance, [57]). However, contributions from penguins with up- and charm-quark exchanges
are expected to yield sizeable CP violation in \( B^0_d \rightarrow K^0\bar{K}^0 \) even in the SM, so that the interpretation of these effects is much more complicated. Moreover, these contributions contain also possible long-distance rescattering effects, so that \( \rho_{KK} \) and \( \theta_{KK} \) are affected by large uncertainties.

Despite this problem, interesting insights can be obtained through the \( B^0_d \rightarrow K^0\bar{K}^0 \) observables. If we keep \( \rho_{KK} \) and \( \theta_{KK} \) as free parameters, we may characterize this decay in the SM through a surface in observable space, which is shown in Fig. 3. It should be emphasized that this surface is theoretically clean since it relies only on the general SM parametrization of \( B^0_d \rightarrow K^0\bar{K}^0 \). Consequently, should future measurements give a value in observable space that should not lie on the SM surface, we would have immediate evidence for NP contributions to \( \bar{b} \rightarrow \bar{d}s\bar{s} \) FCNC processes. However, while the direct and mixing-induced CP asymmetries can be “straightforwardly” determined through time-dependent rate asymmetries, the extraction of \( \langle B \rangle \) from the CP-averaged branching ratio requires further input:

\[
\text{BR}(B_d \rightarrow K^0\bar{K}^0) = \frac{\tau_{B_d}}{16\pi M_{B_d}} \times \Phi_{KK} \times |\lambda^3 A_{PC}^f|^2 \langle B \rangle,
\]

where \( \Phi_{KK} \) denotes a two-body phase-space factor and \( A_{PC}^f \equiv A_1^f - A_2^f \). In order to fix the overall normalization factor involving the penguin amplitude \( A_{PC}^f \), we may either use (i) \( B \rightarrow \pi\pi \) \((b \rightarrow d)\), or (ii) \( B \rightarrow \pi K \) \((b \rightarrow s)\) decays.

As can be seen in Fig. 3, \( \langle B \rangle \) takes an absolute minimum,

\[
\langle B \rangle \equiv 1 - 2\rho_{KK} \cos \theta_{KK} \cos \gamma + \rho_{KK}^2 \geq \sin^2 \gamma,
\]

which can be converted into the following lower bounds for the CP-averaged \( B_d \rightarrow K^0\bar{K}^0 \) branching ratio:

\[
\text{BR}(B_d \rightarrow K^0\bar{K}^0) \geq \begin{cases} 
(1.39^{+1.54}_{-0.95}) \times 10^{-6} & \text{(i)}, \\
(1.36^{+0.18}_{-0.21}) \times 10^{-6} & \text{(ii)}. 
\end{cases}
\]

Interestingly, both avenues to fix the overall normalization through \( SU(3) \) flavour-symmetry arguments give results in nice agreement with each other. At the time of the derivation of these bounds, the \( B \) factories reported an experimental upper bound of \( \text{BR}(B_d \rightarrow K^0\bar{K}^0) < 1.5 \times 10^{-6} \) (90% C.L.). Consequently, the theoretical lower bounds given above suggested that the observation of this channel should just be ahead of us. Subsequently, the first signals were indeed announced, in accordance with (22):

\[
\text{BR}(B_d \rightarrow K^0\bar{K}^0) = \begin{cases} 
(1.19^{+0.40}_{-0.35} \pm 0.13) \times 10^{-6} & \text{(BaBar \cite{61})}, \\
(0.8 \pm 0.3 \pm 0.1) \times 10^{-6} & \text{(Belle \cite{62})}.
\end{cases}
\]

The SM description of \( B^0_d \rightarrow K^0\bar{K}^0 \) has thus successfully passed its first test. However, the experimental errors are still very large, and the next
crucial step – a measurement of the CP asymmetries – is still missing. For further aspects of $B^0_d \rightarrow K^0 \bar{K}^0$ and a discussion of $SU(3)$-breaking effects, the reader is referred to Ref. [60].

3.2. General Lower Bounds for $b \rightarrow d$ Penguin Processes

The bounds discussed above are actually realizations of a general, model-independent bound that can be derived in the SM for $b \rightarrow d$ penguin processes [63]. If we consider such a decay, $\bar{B} \rightarrow \bar{f}_d$, we may – in analogy to [18] – write its amplitude as follows:

$$A(\bar{B} \rightarrow \bar{f}_d) = A_d^{(0)} \left[ 1 - \rho_d e^{i\theta_d} e^{-i\gamma} \right].$$

(24)

Keeping $\rho_d$ and $\theta_d$ as “unknown”, i.e. free hadronic parameters yields

$$\text{BR}(B \rightarrow f_d) \geq \tau_B \left[ \sum_{\text{Pol}} \int dPS |A_d^{(0)}|^2 \right] \sin^2 \gamma,$$

(25)

where we made the phase-space integration explicit and the sum runs over possible polarization configurations of the final state $f_d$. In order to deal with the term in square brackets, we use a $b \rightarrow s$ penguin decay $\bar{B} \rightarrow \bar{f}_s$, which is the counterpart of $\bar{B} \rightarrow \bar{f}_d$ in that the corresponding CP-conserving strong amplitudes can be related to one another through the $SU(3)$ flavour symmetry. We may then write

$$A(\bar{B} \rightarrow \bar{f}_s) = -\frac{A_s^{(0)}}{\sqrt{\epsilon}} \left[ 1 + \epsilon \rho_s e^{i\theta_s} e^{-i\gamma} \right],$$

(26)

where $\epsilon \equiv \lambda^2/(1 - \lambda^2) = 0.05$. Neglecting the term proportional to $\epsilon$ in the square bracket, which gives only a small correction at the percent level, we arrive at

$$\frac{\text{BR}(B \rightarrow f_d)}{\text{BR}(B \rightarrow f_s)} \geq \epsilon \left[ \frac{\sum_{\text{Pol}} \int dPS |A_d^{(0)}|^2}{\sum_{\text{Pol}} \int dPS |A_s^{(0)}|^2} \right] \sin^2 \gamma \frac{SU(3)}{SU(3)_F} \epsilon \sin^2 \gamma.$$

(27)

Since $\sin^2 \gamma$ is favourably large in the SM and the decay $\bar{B} \rightarrow \bar{f}_s$ will be measured before its $b \rightarrow d$ counterpart – simply because of the CKM enhancement – (27) provides strong lower bounds for $\text{BR}(B \rightarrow f_d)$.

Let us now discuss applications of (27), where $SU(3)$-breaking effects are included in the numerical results, as discussed in detail in [63]. Concerning non-leptonic decays, the following picture emerges:

$$
\begin{align*}
1.69^{+0.21}_{-0.24} & \leq \ BR(B^\pm \rightarrow K^\pm K^0) / 10^{-6} & \text{exp} < 2.4, \\
0.68^{+0.11}_{-0.13} & \leq \ BR(B^\pm \rightarrow K^\pm K^* ) / 10^{-6} & \text{exp} < 5.3, \\
0.64^{+0.15}_{-0.16} & \leq \ BR(B^\pm \rightarrow K^{\pm \pm} ) / 10^{-6} & \text{exp} < 71.
\end{align*}
$$

(28)
This summer, the following new results were reported:

\[
BR(B^\pm \to K^\pm K) = \begin{cases} 
(1.5 \pm 0.5 \pm 0.1) \times 10^{-6} & \text{(BaBar [61])} \\
(1.0 \pm 0.4 \pm 0.1) \times 10^{-6} & \text{(Belle [62])},
\end{cases}
\]

(29)

which complement (28), and are the first evidence for the \(B^\pm \to K^\pm K\) channel, in accordance with the corresponding bound in (28). In the case of the other modes, the experimental upper bounds still leave a lot of space. Searches of these decays at the \(B\) factories are strongly encouraged.

Another important tool to explore the \(b \to d\) penguin sector is provided by \(\bar{B} \to \rho \gamma\) modes. In this case, the following picture emerged [63]:

\[
1.02^{+0.27}_{-0.23} \leq BR(B^\pm \to \rho^\pm \gamma)/10^{-6} \exp < 1.8,
\]

(30)

\[
0.51^{+0.13}_{-0.11} \leq BR(B_d \to \rho^0 \gamma)/10^{-6} \exp < 0.4.
\]

Consequently, it was expected that \(\bar{B} \to \rho \gamma\) modes should soon be discovered at the \(B\) factories. Indeed, the Belle collaboration reported recently the observation of \(b \to d\gamma\) processes, with the following results [64]:

\[
\begin{align*}
BR(B^\pm \to \rho^\pm \gamma) & = (0.55^{+0.43+0.12}_{-0.37-0.11}) \times 10^{-6}, \\
BR(B_d \to \rho^0 \gamma) & = (1.17^{+0.35+0.09}_{-0.31-0.08}) \times 10^{-6}, \\
BR(B \to (\rho, \omega) \gamma) & = (1.34^{+0.34+0.14}_{-0.31-0.10}) \times 10^{-6},
\end{align*}
\]

(31)-(33)

which was one of the hot topics of the 2005 summer conferences [65]. These measurements suffer still from large uncertainties, and the pattern of the central values of (31) and (32) would be in contrast to the expectation following from the isospin symmetry. It will be interesting to follow the evolution of the data. The next important conceptual step would be the measurement of the corresponding CP-violating observables, though this is still in the distant future. In view of

\[
BR(B^\pm \to \pi^\pm \ell^+ \ell^-), BR(B^\pm \to \rho^\pm \ell^+ \ell^-) \gtrsim 10^{-8},
\]

(34)
a similar comment applies to this species of \(b \to d\) penguin decays.

### 3.3. Comments

The stringent experimental upper bounds and the emerging signals for the \(B^\pm \to K^\pm K\) decays disfavour large rescattering effects, which has important implications for the analysis of the \(B \to \pi K\) system [51].

Concerning the radiative \(B \to \rho \gamma\) modes, an interesting alternative to confront the data for the branching ratios with the SM is offered by converting them into information on the UT side \(R_t \propto |V_{td}|.\) Such an analysis
was recently performed by the authors of Refs. [66, 67], who also used the $SU(3)$ flavour symmetry, but calculated the CP-conserving (complex) parameter $\delta a$ entering $\rho_{\gamma} e^{i\theta_{\gamma}} = R_b [1 + \delta a]$ in the QCDF approach. The corresponding result, which favours a small impact of $\delta a$, takes leading and next-to-leading order QCD corrections into account and holds to leading order in the heavy-quark limit [67]. However, in view of the remarks about possible long-distance effects made above and the $B$-factory data for the $B \to \pi\pi$ system, which indicate large corrections to the QCDF picture for non-leptonic $B$ decays into two light pseudoscalar mesons, it is not obvious that the impact of $\delta a$ is actually small. The advantage of the bounds given above is that they are – by construction – not affected by $\rho_{\gamma} e^{i\theta_{\gamma}}$ at all.

Instead of confirming the bounds, it would of course be much more exciting if some of them were significantly violated through the impact of NP contributions, interfering destructively with the SM amplitudes. The $b \to d$ penguin decay classes are governed by different operators:

- **Non-leptonic decays:** four-quark operators.
- **$b \to d\gamma$:** $Q^d_{7,8} = \frac{1}{8\pi^2} m_b \bar{d}_i \sigma^{\mu\nu} (1 + \gamma_5) \left\{ e b_i F_{\mu\nu}, g_s T^a_{ij} b_j G^{a}_{\mu\nu} \right\}$.
- **$b \to d\ell^+\ell^-$:** $Q^d_{9,10} = \frac{\alpha}{2\pi} (\bar{\ell}\ell)_V, A (\bar{d}_i b_i)_V, A$.

Consequently, we may actually encounter surprises.

4. The “El Dorado” for the LHC: $B_s$ Decays

4.1. Basic Features

Another essentially unexplored territory is given by the $B_s$-meson system, since no $B_s$ mesons are accessible at the $e^+e^- B$ factories operating at the $\Upsilon(4S)$ resonance. However, plenty of $B_s$ mesons are produced at hadron colliders, i.e. at the Tevatron and later on at the LHC [3]. Already the $B^0_s - \bar{B}^0_s$ mixing parameters are particularly interesting quantities. The mass difference $\Delta M_s$ can be combined with its $B_d$-meson counterpart $\Delta M_d$, allowing the determination of the UT side $R_t$ with the help of a hadronic parameter $\xi$, which equals one in the strict $SU(3)$ flavour-symmetry limit. The uncertainties of $\xi$ are an important aspect of lattice QCD studies [68]. So far, $B^0_s - \bar{B}^0_s$ oscillations could not yet be observed, and only lower bounds for $\Delta M_s$ are available from the data of the LEP experiments, SLD and the Tevatron. The most recent world average reads as follows [69]:

$$\Delta M_s > 16.6 \text{ ps}^{-1} \ (90\% \ C.L.),$$

and is already close to the SM expectation.
In contrast to the situation in the $B_d$-meson system, the width difference $\Delta \Gamma_s$ between the mass eigenstates is expected to be sizeable. This quantity may provide interesting studies of CP violation through “untagged” $B_s$ rates [70, 71], which are defined as

$$\langle \Gamma(B_s(t) \to f) \rangle \equiv \Gamma(B^0_s(t) \to f) + \Gamma(\bar{B}^0_s(t) \to f), \quad (36)$$

and are characterized by the feature that we do not distinguish between initially, i.e. at time $t = 0$, present $B^0_s$ or $\bar{B}^0_s$ mesons. In such untagged rates, the rapidly oscillating $\Delta M_s t$ terms cancel. Although $B$-decay experiments at hadron colliders should be able to resolve the $B^0_s - \bar{B}^0_s$ oscillations, untagged $B_s$ rates are interesting in terms of efficiency, acceptance and purity. Recently, the first results for $\Delta \Gamma_s$ were reported from the Tevatron, using the $B^0_s \to J/\psi \phi$ channel [72]:

$$\frac{|\Delta \Gamma_s|}{\Gamma_s} = \begin{cases} 0.65^{+0.25}_{-0.33} \pm 0.01 & \text{(CDF [73])} \\ 0.24^{+0.28+0.03}_{-0.38-0.04} & \text{(D0 [74])}. \end{cases} \quad (37)$$

It will be interesting to follow the evolution of the data for this quantity. Let us next have a look at important benchmark decays of $B_s$ mesons. For a more detailed recent discussion, see Ref. [27].

4.2. CP Violation in $B_s^0 \to J/\psi \phi$

The decay $B_s^0 \to J/\psi \phi$ is the counterpart of $B_d^0 \to J/\psi K_S$, where we have just to replace the down spectator quark by a strange quark. In contrast to the $B_d$ case, the final state of $B_s^0 \to J/\psi \phi$ consists of two vector mesons. Consequently, we have to deal with an admixture of different CP eigenstates in the final state. However, these can be disentangled through the angular distribution of the $B_s^0 \to J/\psi [\to \ell^+\ell^-] \phi [\to K^+K^-]$ decay products [75]. The corresponding observables show essentially negligible direct CP violation in the SM, and allow the determination of $\sin \phi_s$ through mixing-induced CP violation [72]. In the SM, we have $\phi_s = -2\lambda^2 \eta = O(10^{-2})$, so that we expect here a tiny value of $\sin \phi_s$, i.e. tiny mixing-induced CP violation in $B^0_s \to J/\psi \phi$. Needless to note, the big hope is that experiments will find a sizeable value of $\sin \phi_s$, which would give us an immediate signal for CP-violating NP contributions to $B^0_s - \bar{B}^0_s$ mixing [21–23]. For specific scenarios of NP where such effects may actually arise, see, for instance, Refs. [70–78].

4.3. $B_s \to D_s^\mp K^\mp$ and $B_d \to D^\pm \pi^\mp$

The decays $B_s \to D_s^\mp K^\mp$ [79] and $B_d \to D^\pm \pi^\mp$ [80] can be treated on the same theoretical basis, and provide new strategies to determine the UT angle
\[ \phi_0 \] These modes are pure “tree” decays, and can generically be written as \( B_q \rightarrow D_q \bar{u}_q \). Looking at the corresponding decay topologies, it can be seen that a characteristic feature of these modes is that both a \( B_s^0 \) and a \( B_q^0 \) meson may decay into the same final state \( D_q \bar{u}_q \). Consequently, interference effects between \( B_q^0 - \bar{B}_q^0 \) mixing and decay processes emerge, which allow us to probe the weak phase \( \phi_q + \gamma \) through measurements of the corresponding time-dependent decay rates.

In the case of \( q = s \), i.e. \( D_s \in \{ D_s^+, D_s^{*+}, \ldots \} \) and \( u_s \in \{ K^+, K^{*+}, \ldots \} \), these interference effects are governed by a hadronic parameter \( X_s e^{i \delta_s} \propto R_0 \approx 0.4 \), where \( R_0 \propto |V_{ub}/V_{cb}| \) is the usual side of the UT, and hence are large. On the other hand, for \( q = d \), i.e. \( D_d \in \{ D^+, D^{*+}, \ldots \} \) and \( u_d \in \{ \pi^+, \rho^+ , \ldots \} \), the interference effects are described by \( X_d e^{i \delta_d} \propto -\lambda^2 R_0 \approx -0.02 \), and hence are tiny.

The observables provided by the \( \cos(\Delta M_q t) \) and \( \sin(\Delta M_q t) \) terms of the time-dependent decay rates allow a theoretically clean determination of \( \phi_q + \gamma \). Since \( \phi_q \) can be determined separately, \( \gamma \) can be extracted. However, there are also problems in this approach. First, we encounter an eightfold discrete ambiguity for \( \phi_q + \gamma \), which reduces the power for the search of NP effects considerably. Second, in the case of \( q = d \), an additional input is required to extract \( X_d \) since interference effects proportional to \( X_d^2 = O(0.0004) \) would otherwise have to be resolved, which is not possible from a practical point of view.

A combined analysis of the \( B_s^0 \rightarrow D_s^{(*)+}K^- \) and \( B_d^0 \rightarrow D^{(*)+}\pi^- \) modes allows us to solve these problems. Since these decays are related through the interchange of all down and strange quarks, the \( U^- \)-spin flavour symmetry of strong interactions, which is an \( SU(2) \) subgroup of the full \( SU(3)_F \), provides an interesting playground. Following these lines, an unambiguous value of \( \gamma \) can be extracted from the corresponding observables. To this end, \( X_d \) has not to be fixed, and \( X_s \) may only enter through a \( 1 + X_s^2 \) correction, which is determined through untagged \( B_s \) rates. First studies were recently performed for the LHCb experiment, and look very promising.

\[ 4.4. \] The \( B_s \rightarrow K^+K^- \), \( B_d \rightarrow \pi^+\pi^- \) System

The \( B_s^0 \rightarrow K^+K^- \) decay is a \( b \rightarrow s \) transition, and involves tree and penguin amplitudes, as the \( B_d^0 \rightarrow \pi^+\pi^- \) mode. However, because of the different CKM structure, the latter topologies play actually the dominant rôle in the \( B_s^0 \rightarrow K^+K^- \) channel. In analogy to (10) and (11), its CP asymmetries can be written in the following generic form:

\[
\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+K^-) = G_1(d', \theta'; \gamma) \\
\mathcal{A}_{\text{CP}}^{\text{mis}}(B_s \rightarrow K^+K^-) = G_2(d', \theta'; \gamma, \phi_s),
\]

(38)
where \((d', \theta')\) are the counterparts of \((d, \theta)\). Since \(\phi_s\) is negligibly small in the SM – or can be determined through \(B^0_s \to J/\psi \phi\) should CP-violating NP contributions to \(B^0_s - \bar{B}^0_s\) mixing make it sizeable – we may convert the measured values of these observables into a theoretically clean contour in the \(\gamma - d'\) plane. In a similar manner, the CP asymmetries of \(B^0_d \to \pi^+ \pi^-\) can be converted into a theoretically clean contour in the \(\gamma - d\) plane. A key feature of the \(B^0_s \to K^+ K^-\) and \(B^0_d \to \pi^+ \pi^-\) decays is that they are related to each other through an interchange of all down and strange quarks, and that there is a one-to-one correspondence between the decay topologies.

Consequently, the \(U\)-spin flavour symmetry implies the following relations:

\[ d' = d, \quad \theta' = \theta. \]  

(40)

Applying the former, we may extract \(\gamma\) and \(d\) through the intersections of the \(\gamma - d\) and \(\gamma - d'\) contours [83]. Moreover, we may determine \(\theta\) and \(\theta'\), which allow an interesting internal consistency check of the second \(U\)-spin relation in (40). Detailed experimental feasibility studies show that this strategy is very promising for LHCb [84], allowing an experimental accuracy for \(\gamma\) of just a few degrees. Concerning possible \(U\)-spin-breaking corrections, the relations in (40) are particularly robust as they involve only ratios of hadronic amplitudes, where all \(SU(3)\)-breaking decay constants and form factors cancel in factorization and also chirally enhanced terms would not lead to corrections. Moreover, the determination of \(\theta\) and \(\theta'\) offers an internal consistency check, as we have noted above, and the contours in the \(d - \theta\) and \(d - \theta'\) planes allow a very transparent discussion of \(U\)-spin-breaking effects. In the numerical examples discussed in Ref. [83], and most recently in Ref. [27], taking the newest data for the \(B \to \pi\pi, \pi K\) system into account, the situation would be remarkably stable with respect to even large \(U\)-spin-breaking corrections to (40), which appear not very likely.

4.5. \(B^0_s \to \mu^+ \mu^-\)

In the SM, this rare decay and its counterpart \(B^0_d \to \mu^+ \mu^-\) originate from \(Z^0\)-penguin and box diagrams and are strongly suppressed. Since the matrix elements of the corresponding low-energy effective Hamiltonian involve only the decay constants \(f_{B_q}\) of the \(B_q\) mesons, we encounter a very favourable situation with respect to hadronic effects. The SM branching ratios can then be written in the following compact form [85]:

\[
\begin{aligned}
\text{BR}(B_s \to \mu^+ \mu^-) &= 4.1 \times 10^{-9} \\
&\times \left[ \frac{f_{B_s}}{0.24 \text{ GeV}} \right]^2 \left[ \frac{|V_{ts}|}{0.040} \right]^2 \left[ \frac{\tau_{B_s}}{1.5 \text{ ps}} \right] \left[ \frac{m_t}{167 \text{ GeV}} \right]^{3.12} \\
\text{BR}(B_d \to \mu^+ \mu^-) &= 1.1 \times 10^{-10}
\end{aligned}
\]  

(41)
The simultaneous measurement of the $B_d \to \mu^+\mu^-$ and $B_s \to \mu^+\mu^-$ branching ratios would allow a determination of the UT side $R_t$ that is complementary to that provided by $\Delta M_d/\Delta M_s$. Moreover, also correlations between the $B_q \to \mu^+\mu^-$ branching ratios and the mass differences $\Delta M_q$ can be derived in models with “minimal flavour violation”, which include also the SM, that are more robust with respect to $SU(3)$-breaking effects [86].

The most recent upper bounds from CDF are given as follows [87]:

$$BR(B_s \to \mu^+\mu^-) < 1.5 \times 10^{-7}, \quad BR(B_d \to \mu^+\mu^-) < 3.9 \times 10^{-8},$$

and are still about two orders of magnitude away from the SM. Consequently, should the $B_q \to \mu^+\mu^-$ decays be governed by their SM contributions, we could only hope to observe them at the LHC. On the other hand, since the $B_q \to \mu^+\mu^-$ transitions originate from FCNC processes, they are sensitive probes of NP. In particular, the branching ratios may be dramatically enhanced in specific NP (SUSY) scenarios, as was recently reviewed in Ref. [88]. Should this actually be the case, these decays could already be seen at run II of the Tevatron, and the $e^+e^-$ $B$ factories could observe $B_d \to \mu^+\mu^-$. Let us finally emphasize that the current experimental bounds on $B_s \to \mu^+\mu^-$ can also be converted into bounds on NP parameters in specific scenarios. In the context of the constrained minimal supersymmetric extension of the SM (CMSSM) with universal scalar masses, such constraints were recently critically discussed by the authors of Ref. [89].

5. Conclusions and Outlook

Thanks to the efforts at the $B$ factories, CP violation is now well established in the $B$-meson system. The exploration of this phenomenon is characterized by a fruitful interplay between theory and experiment. The data have shown that large non-factorizable hadronic effects arise in non-leptonic $B$ decays, so that the challenge for a reliable theoretical description within dynamical QCD approaches remains, despite interesting recent progress. Concerning weak interactions, the Kobayashi–Maskawa mechanism of CP violation has successfully passed its first experimental tests, in particular through the comparison between the measurement of $\sin 2\beta$ through $B^0_d \to J/\psi K_S$ and the CKM fits. However, the most recent average for $(\sin 2\beta)_{\psi K_S}$ is somewhat on the lower side, and there are a couple of puzzles in the $B$-factory data. It will be very interesting to monitor these effects, which could be first hints for physics beyond the SM, as the data improve. Moreover, it is crucial to refine the corresponding theoretical analyses further and to explore correlations with other flavour probes.
Despite this impressive progress, still many aspects of \( B \) physics have not yet been studied. For instance, \( b \to d \) penguin processes are now entering the stage, since lower SM bounds for the corresponding branching ratios are found to be very close to the corresponding experimental upper limits. Moreover, also the \( B_s \)-meson system, which cannot be studied with the BaBar and Belle experiments, is still essentially unexplored and plays an outstanding role for the further testing of the flavour sector of the SM. After new results from run II of the Tevatron, the promising \( B_s \) physics potential can be fully exploited at the LHC, in particular by LHCb. Moreover, it is important to complement the \( B \)-decay studies through other flavour probes, where rare \( K \to \pi\nu\bar{\nu} \) decays are particularly interesting.

With the exception of a couple of flavour puzzles, which do not yet allow us to draw definite conclusions, the SM is still in good shape. However, the observed neutrino oscillations as well as the evidence for dark matter and the baryon asymmetry of the Universe tell us that the SM is incomplete. Moreover, specific NP scenarios contain usually also new sources of flavour and CP violation, which may manifest themselves at the flavour factories. In the autumn of 2007, also the LHC is expected to go into operation, which will provide insights into electroweak symmetry breaking and, hopefully, also give us direct evidence for NP through the production and subsequent decays of new particles in the ATLAS and CMS detectors. Obviously, there should be a very fruitful interplay between these “direct” NP studies and the “indirect” information provided by the flavour-physics sector that is currently addressed in detail within a new workshop [90]. In view of these promising perspectives, an exciting future should be ahead of us!

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