Scaling up learning with GAIT-prop

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Abstract

Backpropagation of error (BP) is a widely used and highly successful learning algorithm. However, its reliance on non-local information in propagating error gradients makes it seem an unlikely candidate for learning in the brain. In the last decade, a number of investigations have been carried out focused upon determining whether alternative more biologically plausible computations can be used to approximate BP. This work builds on such a local learning algorithm – Gradient Adjusted Incremental Target Propagation (GAIT-prop) – which has recently been shown to approximate BP in a manner which appears biologically plausible. This method constructs local, layer-wise weight update targets in order to enable plausible credit assignment. However, in deep networks, the local weight updates computed by GAIT-prop can deviate from BP for a number of reasons. Here, we provide and test methods to overcome such sources of error. In particular, we adaptively rescale the locally-computed errors and show that this significantly increases the performance and stability of the GAIT-prop algorithm when applied to the CIFAR-10 dataset.

1 Introduction

Backpropagation of error (BP) is a learning algorithm that solves the credit assignment problem in deep neural networks, allowing for the formation of task-relevant internal representations and high performance in application [1][3]. Despite its efficacy, the default construction of BP does not appear a likely candidate for the computational steps involved in the learning algorithm of real neural systems. A recent review provides a modern summary of the issues that make BP biologically implausible [4]. These include issues of symmetric synaptic weight matrices, error propagation machinery and more. In particular, these issues all pertain to the use of non-local information for the propagation and computation of error signals to update individual synaptic connections deep within a network. Therefore, biologically plausible learning algorithms are required to provide sensible methods for the assignment of credit deep within a neural network model using local-only information.

One promising algorithm which aims to replicate the performance of BP is the target propagation (TP) algorithm [5][6]. The key principle is to combine a desired network output with an (approximate) inverse model of a network in order to produce ‘desired’ outputs for each layer of a network. These computed layer-wise ‘target’ activities can then be taken as local supervised labels and used for learning. TP has been shown to be able to train neural network models. However, it does not provide an explicit theoretical explanation of how it relates to BP. Furthermore, it has been shown that the efficacy of TP in deep networks and against difficult tasks is questionable [7]. More recently, it has been explored that standard TP is more related to Gauss-Newton optimization rather than BP [8][9].

Ahmad et al. [10] recently found that by modification of the local-targets and enforcement of specific network constraints, there exists a direct correspondence between target-based learning and BP. This method is referred to as Gradient Adjusted Incremental Target propagation (GAIT-prop). GAIT-prop was derived specifically for exact correspondence to BP. This was accomplished by computing
the exact layer-wise targets which would produce weight updates equivalent to back-propagation. These updates also happen to take a biologically plausible form under some constraints. In practice, GAIT-prop was shown to match the performance of BP on the MNIST dataset for relatively shallow networks. However, for deeper networks, inaccuracies in the incremental inversion process were found to result in less accurate credit assignment.

This paper addresses some of the sources of inaccuracy which make GAIT-prop updates and BP updates diverge for deeper networks. First, GAIT-prop requires a perturbation parameter which is used to construct targets and to ensure that it is possible to linearize the region between a forward-pass and the target activity. We find here that this perturbation parameter leads to floating-point precision errors. Similarly, Bengio [9] recently proposed (theoretically) that layer-wise scaling could be applied in order to avoid vanishing or exploding targets. However, the proposed approach requires information about the susceptibility of output activations to layer-wise changes in activity, which reduces biological plausibility. Furthermore, this proposal has yet to be tested empirically and it is unclear how far such an approach deviates in weight updates from BP. Second, the activation function used in the original GAIT-prop work was the leaky rectified linear unit (L-ReLU) function. This activation function has an inflexion point, such that the derivative changes discontinuously and any linear assumption in that range would therefore be inaccurate. In this work we investigate solutions to these two issues and determine the success of training a deep(er) neural network model of six layers with the CIFAR-10 dataset [11].

2 Methods

2.1 Target propagation and GAIT propagation

Let us define the function $F$ as the forward-pass mapping between layers in a neural network such that for layer $l + 1$,

$$a_{l+1} = F(a_l) = f(W_la_l + b_l) \quad (1)$$

where $a_l$ is the activation vector, $W_l$ is the weight matrix and $b_l$ is a bias vector for layer $l$. The function $f(\cdot)$ represents an activation function. In TP, layer-wise targets, $t^{tp}$ are propagated backwards from layer $l$ to layer $l - 1$, by taking the targets of layer $l$ and applying a (learned) inverse function to them:

$$t^{tp}_{l-1} = G(t^{tp}_l) \quad (2)$$

where $G$ is the (learned) inverse mapping from layer $l$ to layer $l - 1$ so that with a perfect inverse mapping, $G(F(a_l)) = a_l$. The requirement of an equal number of neurons in each layer can be relaxed using auxiliary units which can be interpreted as a form of perceptual memory [10]. In this work (and in the original GAIT-prop work), the inverse function $G$ is defined exactly such that

$$a_l = F^{-1}(a_{l+1}) = G(a_{l+1}) = W^{-1}(f^{-1}(a_{l+1}) - b_l). \quad (3)$$

In order to enable this exact inversion, weight matrices are square and initialized as orthogonal weight matrices and the L-ReLU activation function (invertible for all real numbered outputs) was used.

GAIT-prop proposes a modification to the targets being inverted. The inverted targets were modified such that they are a small perturbation from the forward-pass activity toward the target activity. The perturbation is also multiplied by the square of the activation function derivative at the forward-pass activity. Given this definition, the GAIT-prop targets, $t^{opp}$, are computed after a transformation such that:

$$t^{opp}_{l-1} = G((1 - \epsilon_l) a_l + \epsilon_l t^{opp}_l) \quad (4)$$

where $\epsilon_l = \gamma_l D(a_l)^2$ is a perturbation parameter where $\gamma_l$ is a constant with very small magnitude and $D(a_l)$ is a diagonal square matrix with the derivatives $\frac{df}{da}$ of the forward mapping $F$ on its main diagonal. This target adjustment is constructed in order to obtain updates equivalent to BP under the condition of orthogonal weight matrices [10].

2.2 Normalized targets

We find empirically that when inverting targets deep in a network, errors can be introduced through precision errors when the target versus activity difference becomes very small (see Figure [1]). Repeated inversion with fixed layer-wise incremental factors $\gamma_l$ can result in a net increase or decrease
Figure 1: A) A depiction of the activation and target vectors. Top, scaling by an L2 norm ensures that the target vector is within a hyper-sphere of specific length of our activation vector. Otherwise (see bottom two plots) the distance from activation to target vector can vary wildly. B) The leaky ReLU and smoothed leaky ReLU activation functions for $\alpha = 0.1$.

(explosion or vanishing) of the target-activation distance, which is the key error term used for learning. To alleviate the problem of target-activation differences becoming susceptible to precision errors (or increasing beyond the limits of our linear approximation), we propose a local normalization of the incremental factors $\gamma_l$ based upon the target-activation difference. For each layer independently, we divide the incremental factor by the L2-norm of the target-activation difference $\gamma_l$. This ensures that our ‘distance to target’ is fixed on a layer-wise basis, ensuring robust error propagation. Specifically, the value of the incremental factor for layer $l$ is computed

$$
\gamma_l = \eta \frac{\|a_l - t_l\|}{2} \tag{5}
$$

where $\eta$ is a normalization constant determining the desired Euclidean distance between targets and activations. This method provides an assurance of the distance to target during inversion.

2.3 Activation function with continuous derivative

Another potential source of error in the original GAIT-prop work is the use of a L-ReLU activation function, defined as $f_{L-\text{ReLU}}(x) = x$ if $x > 0$ and $f_{L-\text{ReLU}}(x) = \alpha x$ otherwise, with $\alpha$ a constant. Notably, there is a discontinuous change in the derivative of this function at zero. Since GAIT-prop makes use of linearizing assumptions, this discontinuity can act as another source of error. We propose to mitigate this issue by using a similar function with a continuous first derivative. To this end, we constructed a piece-wise function using the softplus function $\text{Softplus}(x) = \ln(1 + e^x)$. We refer to this function as the smoothed leaky ReLU (SL-ReLU) function and define it as

$$
f_{\text{SL-ReLU}}(x) = \begin{cases} 
\ln(1 + e^x) - \delta & \text{if } x > \lambda \\
\alpha (x + \beta) - \delta & \text{otherwise}
\end{cases} \tag{6}
$$

where $\alpha$ is the scaling parameter for L-ReLU when $x \leq 0$, $\lambda = \ln \frac{\alpha}{1 - \alpha}$ is the value of $x$ for which the derivatives of Softplus and L-ReLU connect, $\beta = (\text{Softplus}(\lambda) - \text{L-ReLU}(\lambda)) / \alpha$ is an input correction to connect the Softplus and L-ReLU domains of the function at $\lambda$ and $\delta = \text{Softplus}(0)$ if $\lambda < 0$ and $\delta = \beta \alpha$ otherwise, is a correction to make SL-ReLU go through the origin. The L-ReLU and SL-ReLU activation functions are both depicted in Figure 1 for $\alpha = 0.1$.

2.4 Network Models and Training

To investigate the above two proposed improvements, we trained and tested several deep neural network models with the CIFAR-10 dataset [11]. The networks are all composed of six hidden layers of 500 units each. These networks are so-called ‘reduced-width networks’ with square weight matrices as described by Ahmad et al. [10]. During training, parameters were updated using the Adam optimiser [12].

Previously, the GAIT-prop algorithm was tested on networks of four hidden layers and with the MNIST, Fashion-MNIST, and K-MNIST datasets. The CIFAR-10 dataset provides a more challenging task and we aim to train deeper networks.
3 Results

Figure 2A shows the performance of a range of variations of GAIT-prop against BP. Two key observations can be made. First, GAIT-prop with normalized targets is significantly more performant than without, such that its performance approaches that of Backpropagation. Second, the smoothed transfer function tested is in general of much lower performance but a similar positive impact of normalized targets is observed despite this difference in maximum performance.

A second measure taken to analyse the effectiveness of these approaches was to measure the correlation of weight updates determined by GAIT-prop versus BP. Figure 2B shows these correlations for the different variants. Again, we find that normalization of targets significantly increases the correlation of weight updates, providing greater evidence for its efficacy. Correlations also in general appear to be somewhat larger when using a smoothed activation function.

4 Discussion

This paper set out to investigate methods by which the correspondence between updates produced by GAIT-prop and BP could be improved. We showed here that GAIT-prop’s correspondence to BP can be greatly improved by adaptively rescaling the local errors. We also found that making use of a smooth activation function provided greater correspondence, though our smooth activation function in general appears less effective.

Despite our successes, none of the networks presented here (trained by BP or otherwise) reached state of the art levels of performance on the CIFAR-10 dataset. This is mainly due to the use of fully connected layers instead of convolutional layers and the omission of regularizers such as dropout. We leave extensions of GAIT-prop to such architectures as future research.

One additional potential source of inaccuracy which requires some investigation is the orthogonality of weight matrices. GAIT-prop theoretically produces equivalent updates to BP when weight matrices of a network are orthogonal. However, during training, the weight matrices of our networks deviate from orthogonality. Investigating the impact of this source of error is another key point of research which we leave to future work.

A reader may also ask whether the issues addressed in this work are relevant for biology. For example, the issue overcome by normalization of the target outputs is an issue of precision of floating point numbers. However, we would argue that in biology, any noise floor would also ensure that a sufficiently small target signal would be drowned out amidst network activity. Therefore we propose that these improvements are also relevant to any biologically motivated analysis of these learning rules.
5 Broader impact

This research enables biologically plausible learning algorithms to be applied at a larger scale and to more challenging tasks. A deeper understanding of these algorithms, especially when corroborated with neuroscientific evidence, could shed some light on the fundamental learning algorithms of the brain. This can inspire better AI algorithms for current computer hardware, but may also aid the field of neuromorphic computing. Additionally, brain-computer interface technologies will likely benefit from an understanding of how the brain encodes and makes use of learning signals.

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