On the Evolution of the Maximum Energy of Accelerated Particles in Young Supernova Remnants

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Abstract

It has generally been assumed in the literature that while young supernova remnants (SNRs) accelerate particles even in the early stages, the particles do not escape until the start of the Sedov-Taylor or adiabatic stage, when the maximum energy of accelerated particles is reached. These calculations however do not take into account the detailed hydrodynamical expansion in the ejecta-dominated stage, and the approach to the Sedov stage. Using analytic approximations, we explore different environments in which the SNR may evolve, and investigate how the maximum energy to which particles are accelerated, and its time evolution, depends on various parameters. We take into account the ambient magnetic field and its amplification by resonant or non-resonant modes. Our studies reveal that the maximum energy to which particles are accelerated is generally reached in the ejecta-dominated stage, much before the start of Sedov stage. For SNe evolving within the winds of their massive stars, the maximum energy is reached very early in the evolution. We briefly explore the consequences for supernova remnants expanding in surroundings such as wind-bubbles or superbubbles.

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1. Introduction

Supernova remnants (SNRs) have long been thought to be responsible for accelerated particles, at least up to the knee of the cosmic-ray spectrum. This is usually thought to occur by some process related to Diffusive Shock Acceleration [DSA, \textsuperscript{8,18} and its nonlinear modification \textsuperscript{14}]. However many details of the process are still being worked out. Issues under active study include the diffusion coefficient at the shocks and beyond the shock front, the maximum energies to which particles can be accelerated and the factors that determines this, and the radiative signatures from the remnant. A complete understanding of the system requires that the acceleration of particles at SNR shocks must be studied in conjunction with the escape of the particles from the accelerator. In this paper we discuss some details about the escape of particles from SNRs, and especially how the dynamics and kinematics of young SNRs influence the maximum energy to which particles are accelerated.
2. Escape of Particles from Young Supernova Remnants

An overview of the escape of particles from SNRs, and issues related to this problem, has been recently discussed by Drury [6], which explores the many different ideas that exist. In many discussions of young SNRs and particle escape, it is assumed that the SNR velocity is constant in the so-called free-expansion stage, (which we refer to here more appropriately as the ejecta-dominated stage), and that no particles escape from the SNR in this stage [3]. The argument supposes that the maximum energy of particles increases until the start of the Sedov or adiabatic stage, and that particles start escaping only in the Sedov stage [20, 15, 17]. Caprioli et al. [3] insist that decreasing maximum momentum or energy of the particles will result in all particles which have energy above the maximum at that timestep leaving the system, and therefore associate escape with decreasing maximum momentum, which they claim happens only in the Sedov phase. Gabici [15] asserts that a decelerating spherical shock wave can result in particles escaping from the shock. If the shock velocity is decreasing with time, the length over which the particles are diffusing $L_{diff} \propto \frac{D}{v_{sh}}$ $(D$ is the diffusion coefficient, and $v_{sh}$ is the shock velocity), is increasing with time, thus leading to eventual escape from the accelerator.

Ellison & Bykov [13] take the view that escape of particles is a fundamental part of the acceleration process that is going to take place at any given stage regardless of maximum momentum or time evolution. Our numerical calculations using test particles [23] tend to agree with [13]. In this paper we support the view that escape will occur even at an early stage. For most reasonable models of SNR expansion, the shock velocity is always decreasing. The maximum energy is decreasing in many of the more common situations. Previous results that showed otherwise are flawed because they make inaccurate assumptions about the SNR evolution.

The assumptions that are made in this paper regarding SNR expansion differ in two main ways from many other calculations (1) The shock velocity is taken from actual young SNR expansion models (rather than an ad-hoc invocation that the velocity is approximately constant as used in many previous analyses). As shown by Chevalier [5, 4], the expansion of a young SNR into a surrounding medium, however tenuous, results in a shock velocity that is always decreasing with time. (2) The SNR takes much longer to reach the Sedov stage than is generally assumed. A general assumption that is made is that SNRs reach the Sedov stage when the swept-up mass equals the mass ejected in the explosion. However, since at least the work of Gull [16], it has been known that this is not quite true, and the swept-up mass must exceed the ejected mass significantly before the remnant can be assumed to have the characteristics of the Sedov stage. This was quantified further by Dwarkadas & Chevalier [12], for different ejecta density profiles, where it was shown that the mass of swept-up material must exceed the ejected mass by a factor of 15-30 before the remnant can be considered to be in the Sedov stage. The important point here is that it takes a much longer time to reach the Sedov stage [9], perhaps thousands of years in a low density medium, and that while the remnant is in the ejecta-dominated stage, its velocity will gradually decrease. Our goal here is to study, via simple analytic expansions, how this SNR evolution in the early stages affects the evolution of the maximum energy of accelerated particles.

3. Maximum Energy Considerations

The expansion of a SNR results in the formation of a shock wave that expands with very high Mach number into the surrounding medium. Particles are assumed to be accelerated at the collisionless shock wave (see Spitkovsky, HEDLA 2012 proceedings) to relativistic energies by some process, which is generally thought to be diffusive shock acceleration (DSA) [6]. In order to understand the maximum energy that an accelerated particle can attain, we must first recognize the processes that limit the energy that the accelerated particle can attain.

The acceleration time for particles undergoing DSA can be written approximately as

$$t_{acc} \propto \frac{D}{v_{sh}^2}$$  \hspace{1cm} (1)$$

where $D$ is the diffusion coefficient, and $v_{sh}$ is the shock velocity (which could relate to either the forward or reverse shock). Factors of order unity that do not change with time are left out. In the vicinity of shocks, the scattering of particles via magnetic irregularities is so efficient that the diffusion coefficient can decrease to that corresponding to Bohm diffusion, where the particle mean free path is of the order of the Larmor radius. This is potentially the smallest value that the diffusion coefficient could have. In this case equation 2 can be written as:

$$t_{acc} \propto \frac{E}{B v_{sh}^2}$$  \hspace{1cm} (2)$$
where $E$ is the energy of the particle, and $B$ the magnetic field. One obvious limitation on the maximum energy then is that it corresponds to the age of the remnant, since particles cannot be accelerated for any greater time. Therefore

$$E_{\text{max}} \propto B v_{sh}^2 \text{age}$$

(3)

where $t_{\text{age}}$ is the age of the remnant. More accurate calculations [2] suggest that there is an additional factor of order unity ([7] gives 0.3), but as long as it is not time dependent we can ignore it here. There may be other limitations, such as a maximum wavelength of the scattering waves outside the shock [21], or that the cosmic ray scale height cannot exceed the SNR radius [1]. Electrons may also be subject to loss mechanisms such as synchrotron losses, that further reduce the maximum energy. In this paper however we will concentrate on the major limitation due to age, and further assume that the maximum energy of escaped particles is the maximum energy to which particles are accelerated, which is a reasonable assumption.

The evolution of a young supernova remnant (SNR) has been described in many papers. We use the formulation suggested by Chevalier [5, 4]. In brief, the expansion of SN ejecta into the surrounding medium leads to the formation of a double-shocked structure, consisting of a reverse shock that travels back into the ejecta, and a forward shock that expands into the ambient medium. If the SN ejecta are described by $\rho_{ej} \propto r^{-n}$, and the surrounding medium is described by $\rho_{\text{amb}} \propto r^{-s}$, then the self-similar solutions [5] show that the contact discontinuity will expand as:

$$R_{CD} \propto t^{(n-3)/(n-s)}$$

(4)

Since the expansion is self-similar, the forward and reverse shocks will expand in the same manner. We can write the expansion of the forward shock as $R_f \propto t^n$, where $m=(n-3)/(n-s)$ is referred to as the expansion parameter. Note that since the solutions require $n > 5$, and $s < 3$, we have $m \leq 1$. The velocity $v_{sh} \propto t^{m-1}$, and is always decreasing with time. This suggests, according to one of the arguments above, that particles must be escaping the system.

Going back to eqn. 3 we can then write that

$$E_{\text{max}} \propto B t^{2m-2} t_{\text{age}} \propto B t^{2m-1}$$

(5)

It is clear that, as long as $m > 0.5$, and the magnetic field $B$ is constant, the maximum energy is increasing with time. The maximum energy will start to decrease with time once $m < 0.5$, i.e. just before the remnant enters the Sedov stage. This is somewhat consistent with what other authors have postulated in the past [1, 5], although it indicates that the maximum energy will be reached sometime before the remnant enters the Sedov stage. Furthermore, since the value of $m$ is constant in the self-similar case, the energy will increase at the same rate for much of the self-similar phase, before $m$ starts to decrease as the remnant enters the Sedov stage.

The crucial point here is with regards to the behaviour of the magnetic field. There are several indications that the field measured from radio, X-ray and gamma-ray observations of SNRs far exceeds the field in the general interstellar medium. The general topic of magnetic fields in cosmic particle acceleration sources is comprehensively reviewed in [2]. Here we concentrate on two main issues (1) What is the value of the ambient magnetic field and (2) How does the amplified magnetic field behave?

3.1. Ambient magnetic field

If the SN shock is expanding in the general interstellar medium, then one can assume, at least as a first approximation, that the magnetic field is equal to the interstellar magnetic field, with a value around 5 $\mu$G. This may be the case for Type Ia SNe, whose progenitors are supposed to be white dwarf stars that do not significantly modify the medium around them.

However, all other SNe arise from the core-collapse of massive stars. These massive stars suffer serious mass-loss throughout their lifetimes, losing a large fraction of their initial mass (see Dwarkadas, HEDLA 2012 proceedings). After the SN explodes, the shock wave will evolve in the medium crafted out by the wind from the star, and not in the interstellar medium.

In order of increasing radius from the star, the wind blown medium surrounding the star consists of [24, 11, 10] (1) A freely expanding wind, whose density decreases as $r^{-2}$ if the wind parameters are constant (2) A wind termination shock (3) A low density, hot shocked wind medium (4) A contact discontinuity (5) Shocked ambient medium (6) Wind shock (7) unshocked ambient medium, be it another wind or the interstellar medium. This picture can easily

3
be distorted by winds whose parameters change with time, turbulence, instabilities etc. But in general the SN shock should first be evolving in the wind of the progenitor star.

For a star that is rotating, the field lines near a star resemble an Archimedean spiral, with the radial field falling as $r^{-2}$ and the tangential as $r^{-1}$. Thus the tangential component will dominate at large radii, and the field in the wind can be assumed to decrease inversely with radius. If $B \propto r^{-1}$ then

$$E_{\text{max}} \propto r^{-1} t^{2m-1} \propto t^{-m} t^{2m-1} \propto t^{m-1}$$

Since $m < 1$, the maximum energy is a slowly decreasing function of time. In a wind $E_{\text{max}}$ is always decreasing with time! Since $\sim 80\%$ of SNRs arise from massive star progenitors, we expect that this will be the predominant case. For a SN with ejecta density decreasing as $r^{-9}$ expanding into a wind, $m \sim 0.86$, and the maximum energy will decrease as $r^{-0.14}$. Thus the maximum energy is a slowly decreasing function of time.

3.2. Amplified Magnetic field

3.2.1. Non-resonant modes

In 2001, Bell & Lucek \cite{1} showed that the magnetic field can be amplified non-linearly by the cosmic rays themselves, to significantly exceed the pre-shock value. Early in the evolution, non-resonant modes dominate, while later on resonant modes seem to be more dominant \cite{2}. If the non-resonant modes dominate, the maximum magnetic field is given by \cite{3}

$$B_{\text{amp},nr} = \sqrt{2 \pi \rho_{\text{amb}} (v_{sh}/c) \xi}$$

where $\xi$ is an acceleration efficiency. Assuming $\xi$ to be a constant (not necessarily the case), we get that $B_{\text{amp}} \propto \rho_{\text{amb}} \sim t^{3(m-1)/2}$. Therefore, in a constant density medium we will have $B_{\text{amp}} \propto \rho_{\text{amp}} \sim t^{3(m-1)/2}$. In a wind medium with constant mass-loss parameters, $\rho_{\text{amb}} \propto r^{-2}$, and we have $B_{\text{amp}} \propto r^{-1} \rho_{\text{amp}} \rho_{\text{sh}} \propto r^{-1} t^{3(m-3)/2}$.

This gives, for a SNR evolving in a constant density medium with a non-resonantly amplified magnetic field, that

$$E_{\text{max},NR} \propto t^{3(m-1)/2} t^{2m-1} \propto t^{7m-1/2}$$

Thus for $m < 5/7 = 0.71$ the energy decreases with time, while for larger $m$ it increases with time. For $n=9$, we get that $m = 0.66$ for a constant density medium ($s=0$), and the maximum energy will decrease with time, whereas for $n=11$ it will be very slowly increasing. In a wind medium we obtain:

$$E_{\text{max},NR,wind} \propto t^{3(m-3)/2} t^{2m-1} \propto t^{5m-5/2}$$

In this case, irrespective of the value of $m$, the maximum energy is always decreasing with time.

3.2.2. Resonant Modes

If resonant modes dominate, as is more likely later in the evolution, then

$$B_{\text{amp},r} = \sqrt{8 \pi \rho_{\text{amb}} (v_{sh}/c) \xi / M_A}$$

where $M_A$ is the Mach number. Given that $M_A = v_{sh}/v_A$ where $v_A = B_{\text{amb}}/(\sqrt{4\pi} \rho)$ is the Alfvén velocity, we have:

$$B_{\text{amp},r} \propto B_{\text{amb}} \rho_{\text{amp}}^{1/2} (v_{sh})^{1/2} \xi$$

Thus we get that $B_{\text{amp},r} \propto B_{\text{amb}}^{0.5} \rho_{\text{amp}}^{0.25} t^{3m-1/2}$. Therefore, in a constant density medium we will have $B_{\text{amp},r} \propto r^{-1/2} t^{3m-1/2}$. In a wind medium with constant mass-loss parameters, $\rho_{\text{amb}} \propto r^{-2}$, $B_{\text{amp}} \propto r^{-1}$ and we have $B_{\text{amp},r} \propto r^{-1/2} t^{3m-1/2} \propto t^{(m+1)/2}$.

Again, making the assumption that $\xi$ is constant with time, we get for the maximum energy in a constant density medium that:
Amplified Field (Non-resonant)

$$E_{\text{max},R} \propto t^{(m-1)/2} \rho^{m-1/2} \propto \rho^{(5m-3)/2}$$

Thus for $m < 3/5 = 0.6$ the energy decreases with time, while for larger $m$ it increases with time. For $n=9$, we get that $m = 0.66$ and the maximum energy will increase with time, whereas for $n=7$ it will decrease with time.

For a wind medium we get that

$$E_{\text{max},R} \propto t^{-(1+n)/2} \rho^{3m-1/2} \propto \rho^{(5m-1)/2}$$

which is always decreasing with time.

4. Discussion

In Table 1 below we summarize the dependence of the evolution of maximum energy on various assumptions of the magnetic field in young supernova remnants.

|                | Unamplified Field | Amplified Field (Non-resonant) | Amplified Field (Resonant) |
|----------------|-------------------|-------------------------------|---------------------------|
| Constant density medium $\rho \propto r^{-2}$ | $t^{2m-1}$         | $t^{(m-3)/2}$                 | $t^{(5m-3)/2}$           |
| Wind Medium $\rho \propto r^{-2}$             | $t^{m-1}$          | $t^{(5m-1)/2}$                 | $t^{(5m-3)/2}$           |

As is clear from this table, in the case of a wind-medium, the maximum energy in all cases is always decreasing with time. In the case of a constant density medium, the energy may or may not decrease in the initial stages depending on the value of the expansion parameter. However, as the SNR approaches the Sedov-Taylor phase, the value of $m$ will drop and the energy will begin to decrease. Note that in every case, the energy begins to decrease at a value of $m > 0.4$, i.e. before it reaches the Sedov value. The velocity is always decreasing with time in this ejecta-dominated stage. This will be accompanied by an escape of particles from the SNR even in this early stage, as confirmed by our more precise numerical calculations [22, 23].

We have considered a few likely possibilities for the magnetic field, and used it to derive the above results. Other magnetic field behaviour is quite possible, and may lead to a different evolution of the magnetic field and therefore the maximum energy. What is clear is that the big unknown in the evolution of the maximum energy is the magnetic field. Although surprises still abound, we know to a much better precision how SNRs expand in the ambient medium (and thus the values of $R$ and $V_{sh}$ in equation [3], than we know how the magnetic field behaves (and thus the value of $B$ in equation [3]). The value of the maximum energy, and whether SNRs can accelerate particles to the “knee” of the cosmic ray spectrum and beyond, also depends crucially on the magnetic field (see also [6]).

Although this calculation uses simple arguments that do not take into consideration any of the intricacies of the acceleration process, it gives a general idea of the evolution of maximum energy. We have also assumed for simplicity that the acceleration efficiency is constant, which is possibly not the case. Our main point though is to emphasize the evolution of maximum energy of accelerated particles in the ejecta dominated stage, and demonstrate that it leads to a conclusion that contradicts much of the previous work (see Fig 2 in [17]) - the maximum energy of SNRs must be reached somewhere in the ejecta dominated phase, in many cases much before the SNR reaches the Sedov or adiabatic phase, and not at the beginning of the Sedov phase. This agrees with numerical calculations [13, 23].

In the case of a wind medium, the results indicate that the maximum energy is always decreasing. This obviously cannot be extrapolated back to time $t = 0$ as the time of maximum energy, because one must take into account the finite time for particles to carry out several crossing of the shock front and reach maximum energy. The energy will continue to increase during this time, then reach a maximum after which the above results come into play. This time of increase though could be short, maybe a few tens to hundred of years (see Fig 1).

These results lead to some surprising inferences. Core-collapse SNRs may begin their evolution within a wind medium, but do not continue to evolve in a wind medium for a long time. The SN will first evolve in the wind of the star, and subsequently, after crossing the wind-termination shock, in an almost constant density medium. What the afore-mentioned results then suggest is that when the SN is evolving in the stellar wind, the maximum energy of the particles must be decreasing slowly, but when it starts evolving in the constant density medium the maximum energy must begin to increase with time. This would mean that in wind-blown bubbles, and in their larger cousins, superbubbles, the energy would first decrease and then increase, before decreasing again as the SNR evolves to the...
Sedov stage. The maximum of the energy could be reached (depending on the actual parameters) later in the evolution while the SNR is evolving in a hot, low density shocked wind. The higher energy particles arise in the shocked wind, whereas the lower energy ones could arise from the unshocked wind, with possibly different compositions.

5. Comparison with Observations:

As outlined above, many approximations were made in obtaining these results. Also, these are technical upper limits, probably applicable only to protons as electrons will experience radiative losses that reduce the energy. In some cases they may be superceded by other limits. In order to investigate the evolution of maximum energy in detail, we have calculated it in one such situation via detailed numerical calculations that take the acceleration process and energy losses into account. Our methodology is described in two papers by our group [22, 23]. The specific calculation below however is unpublished and will be described in a subsequent paper. In this particular case, a Type Ic SNR was simulated expanding first in a wind and then, after crossing the wind termination shock, in a constant density medium. No magnetic field amplification was assumed - the field falls off as \( r^{-1} \) in the wind, and is constant in the shocked region. Note that interaction of the SN shock with the wind termination shock will result in a compression of the SN shocked region and an appropriate increase in the magnetic field.

Figure 1 shows the maximum energy of protons calculated from the simulations (solid lines) and compared to the evolution predicted from our calculations. The maximum energy of accelerated particles is not an easy quantity to extract. We have computed the maximum energy at time intervals of 25 years, plotted the points and smoothed the curve to reflect the overall evolution. In the early evolution, while the SNR, with \( n = 9 \) is evolving in a wind, our scalings predict that the maximum energy should decrease as \( t^{-0.14} \). We find that, after a finite period as described above when the particles reach maximum energy, the energy does decrease, and the slope does not differ substantially from the analytic value. This speaks for the validity of our assumptions.

After crossing the wind shock, the energy is predicted to increase as \( t^{0.32} \), but seems instead to fluctuate. This is not difficult to understand. The predictions are made for a purely self-similar evolution. However, after crossing the wind shock, the SNR shocked interaction region is disturbed and is not evolving in a self-similar fashion, although it will approach self-similarity after a few crossing times. In a sense the shock profile is intermediate between that of a SNR in a wind and in a constant density medium, although eventually it will approach that of a SNR in a constant density medium, unless the nature of the medium changes before it does so. Although our simulation has not been carried on for that long, when the evolution becomes self-similar again we should see the increase that is predicted.

These results are not so easily applicable to Type Ia SNRs, because their ejecta profiles are more complicated. [12] showed that the ejecta profile could be approximately fit by an exponential density. Unfortunately, such a profile is not self-similar, and does not lend itself easily to analytic calculations.

6. Summary and Conclusions

As mentioned earlier, many authors have neglected the variation of the SNR in the early phase, assuming that the velocity is constant in the so-called free-expansion phase, that the maximum energy of accelerated particles is always increasing in this phase, and that no particles escape. In this paper we have started with the velocity profile from self-similar solutions of SNR evolution, which show that the velocity in the ejecta-dominated phase is always decreasing with time. We have used that to compute the evolution of the maximum energy of the escaping particles assuming Bohm diffusion at the SNR shock. We have shown that the maximum energy is always decreasing (after an initial short period) in the case of SNRs evolving in winds, whereas it may increase, but eventually begin to decrease before the start of the Sedov stage, in case the SNR is evolving in a constant density medium. The big unknown in each case is the magnetic field evolution. These simple results lead to some surprising implications for the evolution of the maximum energy of accelerated particles at SNR shocks in various complicated surroundings. In a follow-up paper we will consider other limiting cases, other possibilities for the magnetic field, and quantify the various approximations to show how the maximum energy depends on various SNR parameters. We will also quantify what the maximum energy is for various SN types. In future, we will study these more accurately using numerical simulations.
Figure 1. The maximum energy versus time for a Type Ic SNR evolving in a stellar wind, followed by a hot shocked medium. The solid line is the energy computed from a direct calculation involving acceleration at both shocks, the dashed line is the values given by our analytic approximations. Our theory predicts that the maximum energy should decrease during the evolution in the wind, and it seems to agree quite well with that observed in the numerical simulations. The later evolution is not self-similar and therefore the theoretical model does not work as well.

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