The Quantum Hall Effects: Philosophical Approach

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Abstract

The Quantum Hall Effects offer a rich variety of theoretical and experimental advances. They provide interesting insights on such topics as complementarity, gauge invariance, strong interactions, emergence of new theoretical concepts. This paper focuses on some related philosophical questions.

1 Introduction

In his study of Bachelard’s contribution to Philosophy, Dominique Lecourt [1] stresses that according to Bachelard, Philosophy must submit to the teachings of Science. As a physicist and a philosopher of science, I am inspired by this point of view. In the following, I am introducing a study of a relatively new field of physics, the Quantum Hall Effects, and trying to extract some relevant philosophical viewpoint from that study.

The first parts of this paper are devoted to a rather elementary introduction to this field. In the last parts I will attempt to draw some philosophical inferences from the material described in the previous sections. In particular, I will discuss the Non Figurative Realism picture developed by Chalmers in his book on “What is this thing called Science?”[2]. I will argue that some of the difficulties in Chalmers’ position may be resolved by the dialectic materialism point of view. This will include first a discussion of why, and in what degree, one may have good reasons to believe that the results described on the QHE have a definitely narrow connection with objective properties of matter. A second part of this discussion deals with the philosophy of quantum mechanics: what can we infer from the QHE about the statute of the wave function in quantum

1 Some readers will find that this introductory part is not so elementary, as it requires some training in quantum mechanics... Is it conceivable to deal with the philosophy of physics nowadays without a sufficient amount of knowledge of quantum mechanics?
mechanics? A section discusses briefly the Duhem-Quine underdetermination thesis. Finally, I discuss some of the relations this study may have with the question of the unity and struggle of opposites in nature, namely one of the thesis of dialectic materialism in Nature. The question of relative or absolute truth will be discussed.

The discovery of the Integer Quantum Hall Effect in 1980 \cite{3}(hereafter IQHE), and of the Fractional one \cite{4} (hereafter FQHE) in 1982, deal with an apparently restricted class of quantum phenomena: the behaviour of electronic systems in a two dimensional space under strong magnetic fields perpendicular to the two dimensional sample\cite{5}. What could we possibly learn about nature or about knowledge which could be of any universal interest?

This paper aims at offering some answers to this question.

2 The result of theoretical and experimental progress

A first observation is that the developments which are the topic of this paper were made possible by progress in the physics of semi-conductors. The latter is an intimate mixture of theoretical and experimental progress, based in particular on the quantum mechanics of electrons in various pure and impure crystalline structures.

At the interface of two types of semi-conductors, experimentalists have been able to create electron populations which are confined, at low enough temperatures, to a thin spatial slice of the order of a nanometer. This is made possible by mastering the theory and experiments on the electronic band structure of the relevant semi-conductors, and of their interface: the energy for an electronic excitation to migrate to positions far from the interface can be made a few orders of magnitude larger than temperatures of order 100 K, while the energy for an electronic displacement in the interface is much smaller, in fact virtually zero in an ideal sample. Then, at low enough temperatures, electrons are restricted to a two dimensional (2D) world. This, in turn, was made possible by advances in the purity and regularity control of the cristalline arrays at the interface. The reader will notice how important is the notion of “order of magnitude”: of energy compared with temperature, of distances compared to interatomic ones, etc. This notion was first highlighted in the philosophy of physics, I believe, by Bachelard \cite{1}. As will be discussed in the last sections, this notion is essential in the debate of what we can call an absolute truth in physics. It will appear many times in this paper. Another catchword here is “mobility”. Improving the mobility of electrons at the interface of specially selected semi-conducting materials was a precondition for the experimental and theoretical study of quantum particles in a two dimensional environment\cite{2}.

Since the discovery of the QHE, the discovery of graphene \cite{6} in 2004 has provided a genuinely two dimensional electronic system: a sheet of Carbon, the

\footnote{At the time of this writing (april 2014), the electronic mobility at the interface of so-called GaAs/AlGaAs heterostructures is larger than $5.10^6 cm^2/Vs$, which allows to observe details at a much finer scale than at the time of the original discovery of the IQHE}
thickness of an atomic radius, can be sliced off a graphite crystal. Although I will not discuss this system in this paper, it is worth mentioning that electrons in this material have zero inertial mass, obey a relativistic Dirac equation, and move in a two-dimensional space where the “velocity of light” is the Fermi velocity, two to three orders of magnitude slower than the actual velocity of photons in the vacuum.

2.1 A revolution in theory

The observation of Quantum Hall Effects, and particularly that of FQHE, participated in changing significantly the theoretical outlook on electronic liquids in condensed matter physics. Indeed the collective behaviour of electrons in simple metals\textsuperscript{3} has been described with considerable success by the Landau liquid theory of the Fermi liquid\textsuperscript{7}. Within that picture, interactions between electrons alter \textit{adiabatically} the ground state, as compared to that for which interactions are neglected. In other words interactions in that picture are considered as perturbations which modify only quantitatively the parameters of the theory which has no interaction: no spontaneous symmetry breaking is usually expected. From a technical point of view, the main methods used in this context have been those of Feynman diagrams, and renormalization group theory.

Superconductivity has been explained, in part, within the framework of Landau liquid\textsuperscript{4} – together with a gauge symmetry breaking ground state – with the BCS theory: at low temperatures, the Fermi sea becomes unstable to the formation of electron pairs, due to an effective attractive interaction through vibration quanta of the crystalline network \textsuperscript{8}. This theoretical framework, although responsible for some irreversible advances in the understanding of a large class of insulators, semi-conductors and conductors\textsuperscript{5}, has met its limits with the discovery of the QHE, and of a number of other phenomena, such as Mott insulators, or the high temperature superconductivity in copper oxides in 1986.

New theoretical methods have been necessary to account for such discoveries, and in particular that of the QHE. The path followed by Laughlin \textsuperscript{9,10} led him to the Nobel prize award in 1993. What is revolutionary in his approach is the direct finding of the almost exact ground state wave function for strongly interacting fermions (electrons) in two space dimensions, on the basis of symmetry considerations, within a particular gauge choice. This was a new theoretical method, a radical departure from methods that had been followed until then: the Feynman diagram approach, which is useless when interactions overcome the kinetic energy scale\textsuperscript{6}, or the gauge theories à la Yang-Mills developed to treat strong interactions in high energy physics\textsuperscript{7}.

\textsuperscript{3}For example elements in the 3d and 4d transition series in the periodic table of elements.
\textsuperscript{4}The term “Fermi liquid” is also used in this context.
\textsuperscript{5}So called large band electronic systems.
\textsuperscript{6}Yet another example of \textit{order of magnitude} condition...
\textsuperscript{7}Asymptotic freedom rehabilitates perturbative approaches at high energy or short distances. This is outside the scope of this paper.
In perturbative approaches, the theorist starts from a known solution for the non interacting problem, (which is related to the single particle problem) with the hope that interactions do not cause a qualitative change in the interacting solution: symmetries are conserved, as is the structure of level excitations; only quantitative changes, compared to the non interacting case, are expected. Although this approach may seem to justify a reductionist point of view (i.e. the properties of electron liquids are the sum of properties of single electrons), the statistical properties of electrons, due to the antisymmetry of their wave function, introduce from the start a major correction (the exclusion principle) to a naive reductionist viewpoint.

A useful way to think about many aspects of physical systems is that they embody conflicting energies: in the case of electronic systems, the conflict between kinetic energy and interaction energy is all pervasive. The reason why they are in conflict is that the equilibrium properties, which govern a large number of physical properties, depend on reaching an energy minimum. The configuration of particles which minimizes, at equilibrium, the kinetic energy is in general qualitatively different from that which minimizes their interaction energy. When the kinetic energy dominates, the wave functions spread in space as much as is possible; repulsive interactions in this case are a weak perturbation of the system with zero interaction: on the theory side, a zero order solution is easily constructed from the knowledge of the single particle behaviour.

When interactions dominate, each particle becomes an obstacle to the motion of all others; the hopefully simple limit of infinite interactions is in general one with a ground state which has infinite degeneracy, and there is no intuitive way to guess how this degeneracy may be lifted when interactions become finite. A whole new world of symmetry breaking possibilities opens up, and the reductionist point of view is of little help: the properties of the whole may differ fundamentally from that of its parts. The former are sometimes described as emergent properties. In a later section, I will describe a new type of emergent collective property without symmetry breaking, that of topological order.

In the strongly interacting problems, the theoretical search has no known starting point, or a starting point with huge degeneracy, and no natural intuition on how this degeneracy may be lifted at equilibrium. What Laughlin showed is that, in the case of the Quantum Hall Effect, physical intuition, a careful analysis of the symmetries of the problem, and clever choice of theoretical language, could provide for an answer, at little or no computational cost. In most problems so far dealing with many body physics, after the nineteen fifties, field theory had been considered a necessary tool. Laughlin broke away from this paradigm and provided an answer written in terms of bona fide wave functions, written

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8 In thermodynamics, for a system in contact with a thermostat, the two energies in conflict are the internal energy, on one hand, the entropy term, on the other hand. The conflict is then between order and disorder, which interpenetrate each other.

9 P. W. Anderson [11] had, some years before the nineteen eighties, devised a theoretical proposal for a different electronic system: the Resonating Valence Bond paradigm, which is outside the scope of this paper.
in first quantization language\textsuperscript{10}. Quantum mechanics of many particle systems do not alter the fundamental duality/conflict of waves and particles, which is at the basis of all quantum formulations, either in terms of fields or in terms of wave functions.

The QHE started with experimental surprises\textsuperscript{3, 4}, which opened a phase of riddle solving theoretical activity, somewhat along the lines that Kuhn developed in his work on scientific revolutions\textsuperscript{12}. However, contrary to Kuhn’s ideas, the theoretical revolution\textsuperscript{9, 10} which followed did not amount to replacement of ancient paradigms by new ones, and falsifications of ancient paradigms: it resulted in the development of a new paradigm added to a vast body of unquestionable results. The latter describe the behaviour of a large class of semi-conductors, and metals and alloys in a wide range of different parameters such as temperature, pressure, magnetic or electric fields, etc.. As mentioned above, the exploration of new properties of matter were made possible by various technological advances, which were in turn triggered by theoretical progress and which opened new fields of knowledge of physical laws. The latter allow experimental predictions which, as scientific practice routinely does, test the theory.

Some scientists believe they exhibit a philosophical culture by quoting Popper’s falsification theory\textsuperscript{13}. However, as discussed by various authors (see for example\textsuperscript{14, 15}), as shown, among many others, by the history of the Quantum Hall Effects, falsification of a given theoretical prediction does not automatically lead to questioning the validity of the whole theory. As argued by Lakatos\textsuperscript{15}, research programs protect their scientific basis, and will, in general, not be deterred by a falsification. The latter may set limits beyond which new concepts emerge, about new aspects of the world, which are inaccessible to human knowledge until technological and/or conceptual developments reveal them.

3 History of the Quantum Hall Effects

3.1 Prehistory: the classical Hall effect

The classical Hall effect was described by Hall in 1879: if an electrical current is driven through a flat metallic bar (see figure (1)) in the presence of a static magnetic field $B$, the direction of which is orthogonal to the bar surface, a gradient of electronic density is created in the bar in a direction orthogonal to that of the current. This gradient leads to a potential difference $V$ between the edges of the bar. This defines a "Hall resistance" $R_H$ which is proportional to the field and inversely proportional to the electronic density, $R_H = -\frac{e}{en_{el}}$. $n_{el}$ is a surface density if the current is strictly two-dimensional and $-e$ is the electron charge. The classical theoretical graph for $R_H$ is thus a straight line as a function of $B$, with a slope inversely proportional to the electron density.

\textsuperscript{10}Later on, his theory was cast in the quantum field theory language.

Gás de elétrons 2D

Figure 1: two dimensional electron system in a perpendicular magnetic field. A current \( I \) is injected through the black contacts. A longitudinal resistance is measured between two contacts on the same edge of the metallic bar; the Hall resistance \( R_H \) is measured between contacts on opposite edges.

The experimental set-up sketched in figure 1 is a perfect example of what Bachelard called a *phenomenon operator* \([1]\) i.e. an artifact constructed in the laboratory, on the basis of theoretical views, to investigate the response of nature to specific stimulations.

In a perpendicular magnetic field, the classical electron motion in 2D is circular, with frequency \( \omega_C = \frac{eB}{m} \).

In the presence of an electric field \( \vec{E} \) and a magnetic field \( \vec{B} \), the straightforward analysis of the resistivity, when interactions can be neglected, relies on the (Newton) equation of motion of a single classical charge \([11]\) which describes the time evolution of the particle momentum \( \vec{p} \):

\[
\frac{d\vec{p}}{dt} = -e \left( \vec{E} + \frac{\vec{p}}{m} \wedge \vec{B} \right)
\]

This equation is valid when dissipation processes are neglected. The current density is defined as \( \vec{j} = -n_{el}e\vec{p}/m \). The resistivity \( \rho \) is defined by the (matrix) equation

\[
\vec{E} = \rho \vec{j}
\]

The result is:

\[
\rho = \begin{pmatrix}
0 & \frac{B}{en_{el}}
\frac{B}{en_{el}} & 0
\end{pmatrix}
\sigma = \begin{pmatrix}
0 & -\frac{en_{el}}{B}
\frac{en_{el}}{B} & 0
\end{pmatrix}.
\]

The conductivity matrix \( \sigma \) is the inverse of the resistivity matrix:

\[
\sigma = \rho^{-1} = \begin{pmatrix}
\sigma_L & -\sigma_H \\
\sigma_H & \sigma_L
\end{pmatrix}.
\]

\([11]\) In this context this is called the Lorentz force equation.
Figure 2: Landau Levels. The quantum number (integer) \( n \) labels the levels, and \( m \) is associated to the center of motion which governs the degeneracy of each level.

Notice that the result (3) is counter-intuitive: the longitudinal conductivity vanishes together with the longitudinal resistivity. This is due to the transverse character of transport in a magnetic field, which is also the reason for the matrix formulation.

### 3.2 Quantum mechanics – Landau quantization

In the nineteen thirties, Landau studied the quantum mechanical motion of an electron in a magnetic field in two dimensions. He found that the equations of motion are those of a quantum harmonic oscillator, so that, as for the quantum oscillator, the energy levels (which in this problem are called "Landau Levels") of the electron are equidistant, with a spacing

\[
\hbar \omega_C = \hbar \frac{eB}{m}
\]

The energy levels \( \epsilon_n \) are labeled by positive integers \( n \):

\[
\epsilon_n = \hbar \omega_C (n + 1/2).
\]

Each level has a large field dependent degeneracy: the number of states per Landau Level (hereafter noted LL) is \( N_B = n_B A \), where \( A \) is the sample surface, and \( n_B = B/\phi_0 \) is the density of magnetic flux through the sample surface, measured in units of flux quantum \( \phi_0 = \hbar/e \). Typically, for a magnetic field of order a few Tesla, \( n_B \approx 10^{11} \) par cm\(^2\), a large degeneracy indeed.

Since electrons are fermions, they gradually fill the LL as their number increases. It is natural to define a LL filling factor \( \nu \):

\[
\nu = \frac{n_{el}}{n_B} \tag{4}
\]

Figure (2) is a schematic diagram of the Landau Levels and their occupation by single electron states.
Interesting magnetic effects – well known in 1980 – occur at low magnetic fields when the number of occupied Landau Levels is large; I do not discuss them in this paper. The Quantum Hall Effects arise at large fields, such that the LL degeneracy is of the order of the number of electrons in the sample, i.e. such that only a few Landau Levels are occupied.

3.3 A macroscopic quantum phenomenon: the Quantum Hall Effects.

As mentioned above, theoretical and technological advances in the nineteen seventies allowed to foresee the experimental realization of two-dimensional samples with good control of the electron density. It was natural to explore the theoretical prediction for the behaviour of such electron systems under perpendicular magnetic field. Since the Coulomb interaction energy had to be much larger than the kinetic energy, which practically vanishes from the picture when the lowest LL only is completely or partially occupied, the “normal science” prediction (to use once more Kuhn’s language) was to predict the occurrence at low temperature of a “Wigner crystal” or, equivalently, a “Charge Density Wave”. The former is a compressible crystalline array of electrons stabilized by the minimization of the interaction energy: instead of organizing in a Fermi sea of itinerant electrons with overlapping wave functions which spread over the whole sample volume, the electrons stay localized around lattice points; staying as far apart from one another as possible minimizes their interaction energy. Such objects are known to exist in other areas of Condensed Matter physics. A straightforward prediction (published in 1979 [16]), associated to this idea, was that this would result in an insulating behaviour below the temperature for crystallization of the electrons. This again was a well known logical consequence of the inevitable sample defects (impurities, dislocations, etc.) which are known to pin the electron crystal to the sample atomic lattice, thus suppressing the electronic conductivity.

In other words, the prediction was that below the Wigner crystal crystallization temperature, the longitudinal and Hall resistance would diverge exponentially with inverse temperature. The classical Hall resistance is represented on figure (3) by a dotted line.

Von Klitzing undertook the task to check this prediction with the available samples and magnetic fields. His experimental project was, as usual, heavily intertwined with the theoretical notions described above. His findings were radically different from the predicted effect. Not only did the Hall (transverse) resistance develop plateaux around integer values of the filling factor of electronic LL, but the longitudinal resistivity was found to vanish around the same values! There was no sign of any tendency towards a compressible insulating Wigner Crystal.

Furthermore, the conductivity for each plateau appeared to be a constant (identical for all plateaux) times an integer, the number of totally filled Landau Levels, which characterizes each plateau.
The “normal science” prediction was a total failure. If one believes Popper or Kuhn, this should have led to a complete change of paradigms. Should physicists have given up Fermi statistics, or the quantum description of electronic motion in the presence of magnetic or electric fields, or perhaps quantum mechanics? Should they have regarded all or part of those theories as obsolete beliefs? None of this happened, confirming Lakatos’ views on Popper’s falsification. The Wigner Crystal prediction was (temporarily) abandoned, and the experimental result acquired the status of riddle, to be solved with all previously well established theoretical tools.

Just as the failed Wigner Crystal prediction, the IQHE, discovered by v. Klitzing [3] in 1980 (Nobel prize in physics in 1985), is in fact a consequence of Landau Level quantization, together with the Pauli principle obeyed by fermions. In contrast with the failed theoretical predictions, Laughlin[9] chose to neglect Coulomb interactions. One important feature which allowed to account for the IQHE within the previously known laws of physics is the unsuspected role of sample boundaries, as will be discussed in a later section. The IQHE is due to an incompressible electronic liquid state produced at low temperatures, when $k_B T << \hbar \omega_C$ when constant Hall resistance plateaux appear; they are quantized as

$$R_H = \frac{h/e^2}{n}$$

where $n$ is an integer, the integer part of the filling factor: $n = \lfloor \nu \rfloor$. The proportionality constant is a universal constant $h/e^2$. Each plateau in the Hall resistance coincides with zero longitudinal resistance (figure 4).

In fact, the Wigner Crystal hypothesis was subsequently revived in a different range of parameters, and found to describe the physics of the 2D electron system under magnetic field at lower filling fraction $\nu \leq 1/6.5$, and in certain cases at higher LL filling fractions than those for which various QHE states are stabilized. In particular, new details of the Hall resistance experiments have been explained by a novel version of Wigner Crystal [18]: crystalline arrays of electron droplets, each droplet containing an integer number (larger than one) of electrons. This development is yet another argument against Popper’s falsification theory, in agreement with criticisms by Feyerabend and Lakatos ([14], [15]): indeed the Wigner Crystal proposal obeys all the known laws of physics, and was in fault only because of the lower energy solution which Laughlin proved later to exist in certain ranges of LL filling fraction. The Wigner Crystal temporary disagreement with experimental prediction did not prove it wrong as a possible physical state of the electron crystal. It suggested that a lower energy state could exist, in the experimental range of parameters which was studied at first, as indeed is the case. The resistance quantization is independent of the sample geometry, or practically of any sample property, which is a remarkable property in two dimensions: the Hall resistance $R_H$ at a plateau is given in terms of fundamental constants $e$ and $h$. The resistance value accuracy at

\footnote{Initially, Laughlin had incorrectly estimated the filling fraction below which the Wigner Crystal would become the stable ground state; he found the boundary at $\nu \approx 1/70$.}
Figure 3: Experimental signature of the quantum Hall effect. Each plateau in the Hall resistivity $\rho_{xx}$ coincides with a zero longitudinal resistance. The classical curve expected for the Hall resistance is the dotted line. Numbers indicate the filling factor $\nu = n$ for IQHE, and $\nu = p/q$ ($p$ e $q$ integers) for the FQHE.
\( n = 1 \) is such \( (\approx 10^{-9}) \) that this plateau now serves to define the resistance metrology standard, the v. Klitzing constant, \( R_K = 25812,807 \text{ ohms} \).

The discovery of the FQHE [4] followed in 1983. It occurs at “magical” LL filling fraction smaller than 1, in particular that of the lowest LL. The first observed fractional plateau (i.e. at fractional LL filling fraction) was at \( \nu = 1/3 \). Since then a wealth of other plateaux have been observed, somewhat as happened for particles at the start of high energy physics. The effect is remarkable because for a non integer filling fraction, there is in principle a huge degeneracy connected with the kinetic energy: all single electron states share the same LL energy. The Coulomb interaction energy then becomes the only energy scale (apart from interaction with impurities) and is responsible for lifting this degeneracy. The physics is that of strong interactions, for which diagrammatic methods à la Feynman are useless, as mentioned above.

Our basic understanding to-day is based on the FQHE theory by Laughlin [10] in 1983: he proposed an almost exact many particles (about \( 10^{11} \) electrons per \( \text{cm}^2 \)) trial wave function which describes an incompressible electronic liquid. An incompressible quantum liquid is one for which the excitation spectrum is separated from the ground state by an energy gap.

In this picture, Laughlin [10] made the revolutionary proposal that, associated with the fractional occupation of the LL, were various fractionally charged excitations with respectively \( 1/3, 2/5, 1/5, 2/7, 1/7 \)... of an electronic charge. This prediction has been later on proved experimentally correct. Furthermore, such excitations obey “fractional statistics”. The latter is a generalization of the statistics which govern quantum particles in three space dimensions; in the latter case, particles obey either Fermi statistics (electrons, neutrons, protons) or Bose statistics (\( ^4 \text{He}, \) photons, \( \alpha \) particles, etc.). In two dimensions, the exchange of two identical particles may multiply the wave function by a factor \( \exp i \theta \) where \( \theta \) can be any angle, whence the name anyon coined for such particles, the existence of which have overcome a long standing belief that particles have only a binary choice of statistics.

Various generalizations followed. In particular the FQHE may well be the signature of the condensation –or emergence– of a new type of particle, known as Composite Fermion [17]. The latter are bound states of an electron (a fermion) and an even number of flux quanta. In fact Laughlin’s wave function may be recast in the Composite Fermion language. In this picture, the \( 1/3 \) FQH plateau is a completely filled LL of Composite Fermions.

Composite Fermions represent at least a useful theoretical tool to understand many new features of the Quantum Hall Effects. They have allowed theoretical predictions which have been successfully tested.

The history of physics has taught us that many new theoretical concepts which were thought at first to be mere mathematical inventions to account for phenomena \((\text{sauver les apparences, following Duhem [19]})\), eventually were shown to reflect in a more or less detailed way objective material features of

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\(^{13}\)The term “magical” originates from the bewilderment of physicists in front of the rich plateau structure found experimentally, which baffled at first all expectations of what might happen at LL filling fraction smaller than 1
the world. The heliocentric model of Copernicus, the atoms, the molecules, the electrons, the photons, the molecular carriers of genetic information, etc., have transited from the status of useful hypothesis to that of more or less exact reflection of real material structures. There are many theoretical and experimental facts which give us good reasons to believe that the Composite Fermion hypothesis reflects our present knowledge of such real objects.

Most advances in many-body physics, until the discovery of the FQHE, had used perturbation methods, apart from Gell-Mann’s eightfold way, Mott’s theory of a class of magnetic insulators, or Anderson’s theory of the Resonating Valence Bond [11]. What became clear with the FQHE was the unsuspected richness of strongly interacting systems, be they electronic or nuclear systems. The Yang-Mills gauge theories have developed gauge field theories to study strong interactions, their spontaneous symmetry breaking, etc. With his work on the FQHE, Laughlin chose a different theoretical path, and showed that strong electronic interactions had in store many surprises, both theoretical and experimental. New concepts have emerged as the consequences of novel experimental phenomena, reflecting the existence of unforeseen particles, unforeseen realities.

In the following, more mathematical details are given about the topics discussed above.

4 A different outlook on complementarity.

In the classical treatment of two dimensional electron motion in the presence of a (perpendicular) magnetic field, it is found that electrons move in circular orbits characterized by a radius which depend on their kinetic energy. This is because the Lorentz force $e \vec{v} \times \vec{B}$ is orthogonal to the electron velocity $\vec{v}$, and does no work: it merely deflects the electron from a rectilinear trajectory in zero magnetic field. The coordinates $X, Y$ of the circle centers are constants of motion, which may be located at any point on the sample surface.

The quantum treatment cannot start from the Lorentz force equation; it has to start with the Lagrangian or the Hamiltonian, which are formulated in terms of the potentials, in particular in terms of the vector potential $\vec{A}$, the curl of which yields the magnetic field. $\vec{A}$ lies in the plane perpendicular to $\vec{B}$, i.e. the sample plane. The details will not be given here. However an important remark is that gauge invariance enters the problem while it is absent in the classical Lorentz force equation. Gauge invariance proves an important tool [9] for the flexibility of theoretical approaches of the quantum problem [14].

The quantum treatment of the 2D electron system in perpendicular magnetic field also has constants of motion $X, Y$, i.e. operators which commute with the Hamiltonian $H$:

$$\begin{align*}
[X, H] &= 0 \\
[Y, H] &= 0.
\end{align*}$$

14See reference [20] for a related discussion.
However a new feature appears: the well known non commutativity of the momentum operators $p_x, p_y$ with the position operators $x, y$ leads, for 2D electrons in the presence of perpendicular magnetic field $B$, to the following commutator of $X$ and $Y$:

$$[X, Y] = i l_B^2$$
$$[\eta_x, \eta_y] = -i l_B^2$$

In the second equation above, $\eta_x, \eta_y$ are the coordinates of the electron relative to $X, Y$. The length $l_B$ which appears is the so-called “magnetic length” defined by $l_B = \sqrt{\hbar/eB}$. The surface $2\pi l_B^2 = \frac{\hbar}{e}$ is that through which the flux of the field $B$ is one flux quantum $\hbar/e$.

The commutator in equation (6) is, of course, gauge invariant. In the zero field single particle quantum mechanics, the commutator of $p_x$ and $x$ defines a surface $2\pi \hbar = \hbar$ in phase space, as found by Planck in his study of the black body radiation. Equation (6) defines a surface $2\pi l_B^2$ in the real sample surface. Real space in this problem plays the role of phase space in the zero field problem. In other words each electronic state occupies in the sample a surface through which a flux quantum $\hbar/e$ is threaded by the magnetic field. The Heisenberg relations associated to the commutator in equation (6) are:

$$\Delta X \cdot \Delta Y \geq 2\pi l_B^2.$$  

The surface $2\pi l_B^2$ plays the role that $\hbar$ plays in the zero field quantum mechanics! It is proportional to $1/B$. Typically for a field of a few Tesla, the length $l_B$ is of order of 10 to 30 nm.

Equation (7) expresses the fact that the spatial extent of an electron state has a center of motion coordinates which is spread over a surface $2\pi l_B^2$. In terms of a measurement language, it means that $X$ and $Y$ cannot be simultaneously determined with a better accuracy than $\Delta X, \Delta Y$ together with equation (7).

Both uncertainty relations, i.e. equation (7), and the standard Heisenberg relations in the absence of field, are a direct consequence of the commutator of $x$ and $d/dx$. However, (7) deals with a surface in real space, in contrast to a surface in phase space. While the latter is an abstract concept, the former is straightforward and concrete. This should allow the Heisenberg uncertainty principle to appear less mysterious to whoever wants to understand the meaning of quantum mechanics. Quantum mechanics in zero field sets a limit to the classical definition of a point in phase space, because the classical notion of trajectory – given by the simultaneous determination of $p$ and $q$ – becomes irrelevant at a microscopic level. Similarly, equation (7) sets a limit to the classical notion of center of motion in a magnetic field: it is spread over the state surface $2\pi l_B^2$. Infinite accuracy would require an infinite field, for which $l_B^2$ would vanish, which would also mean infinite energy for the lowest LL. The complementarity of $X$ and $Y$ is straightforward, since their uncertainty serves to determine a surface in real space: the surface occupied by a quantum state.

The Landau Level degeneracy can be directly found from the surface associated to a flux quantum: the number of states per level and per unit sample

\footnote{A particle does not move along a single trajectory, but along a superposition of different trajectories.}
surface is obtained by dividing the latter by the flux quantum surface, i.e. \(2\pi l_B^2\); or, equivalently, by dividing the total flux per surface unit by the flux quantum: 
\[n_B = B/\phi_0.\]
This result was mentioned above: it can be derived with no knowledge of the mathematical solution of the Schrödinger equation, merely by using the commutation relations. A factor two should multiply this degeneracy to account for the electron spin.

A consequence of (6) is that a variation of \(B\) induces a variation of the number of states available for electrons per Landau Level in a given sample, since the degeneracy of the latter is \(Bc/\hbar\) per surface unit. Since the electron density is in general fixed in a given sample, increasing the field intensity \(B\) amounts, at temperature low compared to the inter-level energy distance \(\hbar\omega_c\), to gradually emptying high energy levels of their electrons which are transferred to lower energy ones. What is happening when, at large enough \(B\), the lowest Landau Level is only partially filled? The answer to this question was awarded the Nobel prize, and is described in section (5).

### 4.1 Basic ingredients of the IQHE

The basic ingredients to understand the IQHE are:

- Each completely filled LL contributes a conductance quantum \(e^2/\hbar\) to the sample Hall conductivity. *The latter is thus defined in terms of universal constants* \(e\) *and* \(\hbar\)!

- Additional electrons added to, or subtracted electrons (i.e. hole states) from the filled LL are localized by the sample impurities, and do not contribute to electric transport. As mentioned above, increasing the magnetic field intensity amounts to creating more states in each LL for a fixed number of electrons in the sample. Thus, starting from a filled LL situation, increasing the field intensity amounts to gradually emptying the highest occupied LL of its electron population. Conversely, starting from a filled LL situation and decreasing the field intensity decreases the number of states per LL, and promotes a number of electrons to the next higher energy LL. In the absence of impurities, no plateau can be formed, and the classical behaviour of the Hall resistance should be retrieved.

- Upon adding, or subtracting more and more electrons away from the filled LL condition, one eventually saturates the number of impurity sites which localize electron states (or hole states) and one finally reaches a different plateau corresponding to a different filling factor.

In other words, the unprecedented accuracy of the IQHE plateaux is allowed by the very imperfections of the samples!

\[\text{16 However at low enough temperatures, all spins are aligned in the ground state because of their interactions, which have not been considered so far in this section. The lowest full LL is a spin polarized one, a ferromagnet, a fact which has important physical consequences.}\]

\[\text{17 So that thermal excitations to higher energy levels are negligible.}\]
Another surprise was mentioned above: in 2D, the transverse conductance of a Hall bar is independent of the geometric details of the sample! This is in sharp contrast with resistivity measurements of a 3D material, which are plagued by the accuracy constraints on length, diameter, etc.. Sample defects as a cause of accuracy on the determination of a fundamental constant of physics: some would call this a paradox, some a contradiction. In any case this should stimulate some thoughts about the coexistence of contraries in the world...

4.2 Benefits of Gauge symmetry.

The explanation of the IQHE does not require an explicit many-body wave function, because it relies on the notion that interactions are negligible for a completely filled LL. The only necessary ingredients are single electron wave functions in a magnetic field, and the exclusion principle for fermions, which states that no two electrons can occupy simultaneously the same state. In order to sort out the behaviour of one electron, one must chose a gauge for the potential vector, while formulating a theory, the results of which must be independent of the gauge choice. To study the way electrons fill completely the lowest LL, the best gauge choice is that which makes theory simplest. In turn, this depends on the problem at hand. If one is interested in a multiply connected disk geometry, the symmetric gauge comes in handy: states are concentric rings. Laughlin found the first argument for the IQHE by resorting to such a geometry, and invoking gauge invariance.

It is easier to describe the physics of the Hall bar geometry by resorting to another gauge choice, the Landau gauge, whereby the vector potential is perpendicular to the bar long dimension. This leads to a rigorous analogy between the conducting properties of the electron liquid in a 2D Hall bar and a one dimensional conductor. The conductivity of the latter is a trivial undergraduate problem. But the solution of this problem answers the main questions about the IQHE, and allows to introduce a new concept in physics: that of the topological insulator.

4.3 The topological insulator

A topological insulator is a conducting material the volume of which has insulating properties, while conduction occurs on the sample surface, with no dissipation in the QH filled LL case. The Hall bar has a zero resistance “chiral” one dimensional electron channel on each side of the bar. The current directions on either side of the bar are antiparallel. The term “chiral”, means that electrons travel in one direction only, and cannot suffer back scattering (resistive) collisions at full LL filling. The vanishingly small longitudinal resistivity

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18 Hall measurements are usually done on a long rectangular sample, whence the word “bar”.
19 Inside the body, at least when disorder is sufficiently weak, the excitation spectrum has a gap everywhere, which is the definition of an insulator at low temperature. The electron system is an incompressible liquid.
20 In a one dimensional conductor, electrons are allowed to travel in both directions.
21 A continuous one dimensional energy dispersion branch crosses the Fermi level.
of a macroscopic Hall bar is a macroscopic quantum phenomenon: a quantum phenomenon which occurs in the macroscopic world.

In other words, the IQHE resistivity plateau, beside the paradox, or contradiction, noted above about impurities, topples the traditional antinomy between insulator and conductor, in a different way from that of semi-conductors. Because it is both quantum and macroscopic, it is yet another example, besides superfluidity, ferromagnetism, non locality effects in entangled macroscopic states, neutron stars, etc., of the fuzzy boundary between the quantum world and the classical one.

I am convinced that classical behaviour is in general a property of macroscopic objects, at least in a wide range of experimental parameters. Macroscopic quantum phenomena compel physicists and philosophers to reflect upon, and clarify, the necessary conditions for emergence of the classical behaviour of matter from the quantum laws of the microscopic particles. Topological insulators are a quickly developing subject nowadays which should have an impact on such studies.

4.4 The Quantum Hall ferromagnet

Ferromagnetism is another macroscopic quantum phenomenon the effects of which have been known experimentally for a few thousand years. Iron (Fe), Cobalt (Co) Nickel (Ni) are transition metals which exhibit ferromagnetism at low enough temperature. Below a certain temperature, known as the critical temperature $T_c$, those metals, and their alloys, exhibit a continuous/discontinuous transition to a spontaneously broken symmetry state. The high temperature state has the full rotational symmetry of the Hamiltonian. At a temperature smaller than $T_c$, a macroscopic magnetization vector appears, which selects an arbitrary direction in space. The symmetry of that state is reduced to axial symmetry around the magnetization vector. The historical development of the theoretical understanding of ferromagnetism has been slow in the twentieth century, with antagonistic schools claiming a thorough account of phenomena on the basis of conflicting hypothesis: one claimed that electrons in ferromagnetic metals are localized at the lattice sites; this theoretical picture relied on the success of the so called Heisenberg Hamiltonian. The latter seemed at first to account for a number of ferromagnetism phenomena; it describes how localized electronic spins interact through inter-site coupling energies which change sign when the coupled spins change from parallel to antiparallel directions. The other theoretical picture paid more attention to the metallic character of ferromagnets, which is described by delocalized electrons, the wave functions of which spread in the whole sample volume, and interact through intra-site Coulomb repulsive energy. This school relied on the so called Hubbard Hamiltonian, which allowed to account for more detailed phenomena (alloy phenomenology, etc.). Eventually, those antinomic pictures were tentatively reconciled in a complicated theory, whereby delocalized electrons interact in a way such that they

\footnote{The transition is both quantitatively continuous, and qualitatively discontinuous.}
behave much as localized ones. The delocalized picture has been able to explain many experimental features, but it is safe to say that the theory of magnetic transition metals is not yet in a fully satisfactory state.

Not so for the Quantum Hall ferromagnet. It is the best known and understood example of ferromagnet. A two dimensional Quantum Hall ferromagnet is stabilized because of Coulomb interactions between electrons when the lowest Landau Level is full, and all others empty. The interaction energy is minimized when all spins are in the same symmetric state, and the orbital part of the wave function is fully antisymmetric, as it should because of fermion statistics. This ensures that no two electrons can be close to one another. The many body wave function for the Quantum Hall ferromagnet is almost exactly known, from Laughlin’s theory, because there is a large gap for single reversed spin excitations, so that the whole Quantum Hall physics can be reduced to the study of a single level. Furthermore, topological spin excitations of this ferromagnetic material, called skyrmions, exhibit both theoretically and experimentally the property that a spin texture carries necessarily with it a charge texture, so that a skyrmion carries one electronic charge. This is elegantly demonstrated theoretically by resorting to the Berry phase calculation of quantum adiabatic transport around the skyrmion center \(^5\). The main difference between the Quantum Hall ferromagnetism and that of, say, Fe, is – besides the two dimensionality of the QH samples – the negligible kinetic energy term in the Quantum Hall case: all electrons have the same kinetic energy, so the latter drops out of the picture: the competition (contradiction) between kinetic energy and interaction energy is entirely dominated by the interaction energy term. The latter is minimum when all spins are parallel.

5 The Fractional Quantum Hall Effect

5.1 The Laughlin theory

In this section, I shall describe how Laughlin was led to propose an almost exact \(N\) particle trial wave function to account for the fractional Quantum Hall Effect. In fact the FQHE wave function proposed by Laughlin is known to be improvable by perturbation techniques which will only alter its properties in a minor quantitative fashion. The first step in Laughlin’s endeavour was a remark on the nature of the lowest Landau Level single particle eigenstates: in the symmetric gauge, apart from a gaussian multiplicative factor, they are all proportional to an integer power \(m\) of the two dimensional complex position \(z = x + iy\). Two different eigenstates differ by their respective exponents.\(^{24}\) There are as many values of \(m\) as the degeneracy of the LL, which is, as we have seen, the number of flux quanta threading the surface sample. Laughlin’s choice of mathematical formulation may be thought of as a beautiful combination of

\(^{23}\)Skyrmions are named after Skyrme who described similar excitations in nuclear matter.

\(^{24}\)The exponent \(m\) is the eigenvalue of the angular momentum operator along the direction of the magnetic field.
invariance (all eigenstates are analytic functions; they are all eigenstates of the angular momentum operator projection along the external field) and change (two eigenstates differ by their center of coordinates $X, Y$ or by the quantized value of the angular momentum projection $m$ associated to $z^m = |z|^m \exp im\phi$).

In order to grasp the beauty of the many body wave function for $N \approx 10^{11}$ particles written down by Laughlin, it is useful to consider first the wave function for two particles. For a state which has rotation and translation invariance, the most general two particle antisymmetric wave function for particles with the same spin state is:

$$\psi^{(2)}(z, z') \propto \sum_m \alpha_m \left( \frac{z - z'}{l_B} \right)^m$$

modulo a gaussian factor which normalizes $\psi^{(2)}$ in an area of order $4\pi l_B^2$. $\alpha_m$ is the amplitude for the component with exponent $m$. Antisymmetry of fermion wave functions imposes that $m$ is odd. Strictly speaking, $\psi^{(2)}$ is multiplied by the symmetric spin ket $|+ + \rangle$, which is left out, because spins play no role in the simplest theory described here.

It requires some inductive intuition to admit that Laughlin’s wave function proposal for large $N$ is, logically:

$$\psi^{(L)}(\{z_j\}) \propto \prod_{i<j} \left( \frac{z_i - z_j}{l_B} \right)^{2s+1}$$

(where $s$ is a positive integer and the gaussian normalizing factors have been left out.) In fact, for $s = 0$ this function is precisely the full LL wave function with all electronic spins parallel: that of the IQH ferromagnet.

The wave function for the $1/3$ filling, and full spin polarization, of the lowest LL corresponds to $s = 1$.

For whoever is not totally familiar with quantum mechanics, note that this function has all the requisites of a many-fermions wave functions with all spins parallel: it is totally antisymmetric with respect to interchange of pairs of particles: the spin part of the wave function is symmetric since all spins are parallel. This wave function vanishes when any two electrons have the same space coordinates, which obviously lowers considerable the Coulomb repulsive energy, since electrons are kept far apart from one another. Laughlin nailed the coffin of the Wigner Crystal proposal (for this LL filling) by showing very simply that this state has lower energy than the Wigner Crystal.

The astonishing property of this electronic liquid (it is easy to see that it is a liquid, and that translation symmetry is not broken by (??)) is that its elementary excitations are vortices with fractional charge. Charges of $1/3$ of an electronic charge have been predicted and experimentally confirmed using several techniques.

This discovery may inspire useful considerations on reductionism; electrons are of course the basic bricks of the QH Effects. In the Integer QHE, one only needs solving the single particle Schrödinger equation and finding the quantum levels. However this does not help predicting the low temperature transport
properties of a nearly full Landau Level in a finite size sample such as a Hall bar. Reductionism is thus simultaneously correct, and wrong: The basic bricks are electrons, but the quantum state of a large number \( N \approx 10^{11} \) per \( \text{cm}^2 \) of electrons has qualitatively new features which are out of reach of reductionist approaches \([21]\). Similarly, the occurrence of fractionally charged particles in the Fractional Quantum Hall states is an emergent property of the electronic system which cannot be predicted except by recognizing the new paradigm, i.e. the structure and properties of the strongly correlated electron system in two dimensions submitted to a magnetic field. This paradigm illustrates another ubiquitous property of the world: the transformation of quantity in quality, in different ways: it required at first a sufficient lowering of the temperature, in a sufficiently intense magnetic fields to go over from the linear slope of the classical Hall resistivity with field to the quantum Hall plateaux. Varying the magnetic field intensity allows to go over from an accurately quantized Hall plateau, with zero longitudinal resistivity, to a continuous variation of both. And it took a sufficient density of 2D electrons in a sufficiently intense magnetic field to have a novel electron Quantum Hall liquid with fractionally charged excitations, which came as a complete surprise.

Another surprise the FQHE has in store is the occurrence of new particle statistics which are not possible in the three dimensional world. In the latter, quantum particles have to be fermions, with half integer spin, or bosons, with integer spin. A wave function for bosons is unchanged when two bosons are exchanged. A wave function for two fermions changes sign when two fermions are exchanged. Upon interchange of two particles in the two dimensional world of the FQHE, the wave function may change by a factor \( \exp(i\theta) \) where \( \theta \) is any angle between 0 and \( 2\pi \). Whence the word “anyon” invented to distinguish particles with “any” statistical angle from bosons \( (\theta = 0) \) or fermions \( (\theta = \pi) \). Not surprisingly, the statistical angle of fractionally charged excitations with charge \( 1/3 \) of an electron charge is \( 2\pi/3 \). To the best of the author’s knowledge, convincing experimental observation of such fractional statistics is yet to come.

5.2 Topological order: a new type of order

Until the eighties in the twentieth century, most collective phenomena with some long range order were satisfactorily described by one version or other of the Landau theory for spontaneous symmetry breaking. A new class of phase transitions was introduced in some of the theoretical attempts to account for high temperature superconductivity \([22, 23, 24]\). This class is characterized by collective behaviour with long range topological order, but no broken symmetry. It was soon found out that this concept is irrelevant for high temperature superconductors, but FQH states were found to be good examples. Topological order corresponds in this case to long range quantum entanglement, as evidenced by equation \((\text{4})\). Different Quantum Hall states have the same symmetry, but a continuous phase transition separates two such states.
5.3 Composite Fermions

The material discussed in the sections above proves rather convincingly that the specialization of the topic characterized by the effects of a perpendicular magnetic field on a two dimensional electron system has proved rich in fundamentally new outlooks on properties of the material world. For example, the theoretical discovery of fractionally charged excitations, and their experimental observation has a general conceptual novelty which goes far beyond the apparently restricted area of the physics at hand.

Laughlin’s wave function (equation 9) explains the FQHE at LL filling fraction $\nu = 1/(2s + 1)$. But it fails to account for a number of FQH plateaux, such as, for example $\nu = 2/5$, which have been discovered after the first experimental findings. The plateau at $2/5$ is part of the series $\nu = p/(2sp + 1)$, where $p$ and $s$ are positive integers. In 1989, Jain [17] proposed a trial wave function based on a re-interpretation of Laughlin’s wave function. He simply separated the factors with exponent $2s + 1$, as a product of two factors, one with exponent $2s$ and one with exponent $1$. The latter is interpreted as the fully occupied LL wave function for Composite Fermions, for the IQH plateau at $\nu^* = 1$. The other factor is interpreted as the wave function for $2s$ vortices attached to each electron. Each vortex is the result of a flux quantum threading the sample, as can be seen by a Berry phase calculation. In other words, the Laughlin wave function for $\nu = 2s + 1$ can be analyzed as that of a filled LL where each electron is attached to $2s$ flux quanta.

It is straightforward to check that electrons attached to an even number of flux quanta are fermions. The novelty introduced by Jain is to replace the wave function of the filled LL at $\nu^* = 1$ by the wave function for $p$ filled Landau Levels. As a result, the plateau at $\nu = 2/5$ is viewed as due to $\nu^* = 2$ filled LL with composite fermions carrying two flux quanta. The effective field applied to the composite fermions is the real field reduced by the flux quanta which have been attached to the electrons.

5.4 The metallic state of Composite Fermions at $\nu = 1/2$

Jain’s Composite Fermion picture was quite successful at interpreting various observed families of QH plateaux. But it could be (and was, probably still is) thought of by many as a convenient trick to “account for phenomena”, with no ontological content.

The situation became somewhat more intriguing when it was remarked that if Jain’s construction is applied to the half filled Landau Level ($\nu = 1/2$), the resulting effective field on Composite Fermions would vanish: indeed there is one flux quantum per state in the LL, so if two flux quanta are subtracted from the external field and attached to each electron at half filling of the LL, this disposes of all available external flux quanta. The prediction then was that the Composite Fermion system at $\nu = 1/2$ should be a metal, in zero apparent

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25In addition this wave function has to be projected on the lowest Landau Level. This technicality is left aside in this paper.
external magnetic field, with a a filled Fermi surface (a Fermi disk in 2D), the Fermi wave vector of which is predicted accurately. This was a daring proposal, if one remembers that we have an applied magnetic field on a two dimensional electron system, the levels of which are Landau Levels.

Experiments, such as the measurement of Composite Fermion cyclotron orbit, ultrasound absorption, etc., have beautifully confirmed this prediction of the Composite Fermion picture. The notion of flux quanta attached to electrons and subtracted from the real field to yield an effective vanishing field has gained flesh as a notion which may reflect, at least in some essential way, a real process of nature. Another daring proposal, taking the metallic state of Composite Fermions at \( \nu = 1/2 \) as a fact of nature is to propose that Composite Fermions might condense in a BCS superconducting state\(^2\), as happens for most metallic states in zero magnetic field at low enough temperature. This suggestion converges with a theoretical pairing proposal which had been formulated earlier to account for the state at \( \nu = 5/2 \). At the time of this writing, the author is not aware of experimental evidence for or against this spectacular suggestion.

6 Philosophy comments

In the above sections I have commented on some philosophical questions such as the relevance of Popper’s falsification theory, the existence of opposites which contradict each other within some objects in nature, etc..

I have stated a number of theoretical and experimental results as if they were unquestionable. Is this a rational attitude?

6.1 Critique of constructivism and relativism. What about truth?

Constructivists or relativists\(^{14, 25, 26, 27, 29}\) would question the existence, for example, of QH plateaux, on the basis that physics is a human construction, that such objects depend on the epistemic system within which they are analyzed, etc.. I have never personally measured the Hall resistivity, under magnetic field, of the Hall bars I discuss. So how can I be sure they are not simple “beliefs that a group of physicists hold true”, as Kuhn says\(^{12}\)? In other words, why do I feel I am entitled to state that the phenomena I discuss in this paper reflect some true objective determinations of the material world? What do I mean by “true objective determination”? Are there such things as absolute truths? In that matter, I share, in part, Boghossian’s criticisms\(^3\) on the various brands of relativism and/or constructivism. The epistemic system within which Quantum Hall plateaux exist and are analyzed in terms of universal constants is based,\(^{26}\) The pairing would not be the same as for ordinary superconductors with singlet spin Cooper pairs, but a different one, dubbed p-pairing.
in parts on quantum mechanics, and also in well controlled experimental techniques which rely on classical electromagnetism, thermodynamics, cryogenics, in their own domain of validity, and so on, on the many successful experimental predictions of the latter, and on the many technological advances which shape the everyday life of a vast number of humans nowadays: no one can nowadays question the validity of the Schrödinger equation in the description of the behaviour of electrons at the interface of semi-conductors. (Boghossian does not rely as much as he should on the criterion of effective technological applications, what is called the “practice criterion” of truth).

Even though the academic system of publications in specialized physics reviews is a social construct, even though this system organizes the control of the truth content of published results in a way which is not immune to errors or frauds, the numerous convergent published papers since the original ones [3, 4, 9, 10], the numerous scientific conferences, debates, controversies, etc., give me, and many others, good reasons to be convinced of the objective reality of the results discussed in this paper. They rely on a combination of theoretical predictions and experimental results, the latter being heavily intertwined with theory. In fact this set of results allows to predict with certainty the result of an experiment conducted on a two dimensional electron liquid, with mobility in the right range of values, and under a perpendicular magnet field, with the right orders of magnitude for the electron density, magnetic field intensity, etc.. The meaning of “true objective determination” reflects my objectivist philosophy, namely that the world which physics explores does not depend on the human construction which physics is. The interplay between this social construction (epistemics) and the world (ontology) allows to gradually unveil, in a non linear, historical, sometimes chaotic fashion, unquestionable features of the material world. How unquestionable are such truths? Within a given physical environment (temperature, pressure, suitable electromagnetic screening from external perturbations, suitable mechanical stability of measuring apparatus, suitable quality of two-dimensional electronic liquids with suitable density, under suitable magnetic field intensity, etc.), no one is entitled to doubt that transverse resistivity measurements (with sufficient sensitivity and accuracy, etc.) will exhibit Quantum Hall plateaux, which will coincide with longitudinal resistivity very low values, etc., as described above. This experimental phenomenon is explained within a theory (Laughlin’s theory and its developments) which undoubtedly accounts for a vast number of phenomena; it is based on (mostly) non relativistic quantum mechanics describing electrons which have quantized charges, etc.. In other words, the absolute truths I am referring to are relative to a series of conditions (incompletely) listed above. Absolute truth is a truth relative to a series of defined conditions, mostly a correct specification of the orders of magnitude involved to establish it. Other conditions, better accuracy of the apparatus, different shapes or sizes of Hall bars, might reveal later on unsuspected new phenomena, but these will not falsify the theory and the experiments in the defined set of reproducible conditions loosely described above.

What the history of physics proves is that mankind produces a process of
accumulation of partial truths on the behaviour of inanimate matter. This historical process triggers developments of new tools of investigation, with better accuracy, with larger energies, larger pressures, larger magnetic fields, larger frequency ranges, or lower temperatures, lower pressures, lower dimensions, better controlled materials, better experimental resolution, etc. New phenomena, in need of new theories, open new ways of approaching the processes of matter at such new scales of observation. Sometimes the new theories force to reconsider what was thought as established results [12]. But the new theories invariably embody a number of ancient ones which describe previously known phenomena. For example, once the notion of a maximum velocity of signals is added to classical mechanics, special relativity results, and reduces to Newtonian dynamics when velocities are small compared to that of light.

Kant’s statements [31] on the epistemically unreachable thing in itself seems to be true at any given time in history: knowledge, in general, at a given historical stage of humanity, does not access all aspects of the “thing”. But contrary to his thesis, there is, in general, no limit to the accuracy with which epistemics approach ontological truths. At any finite time in human history, except in few cases, there is no complete bridging of the the gap between the representation of the thing and the thing in itself. When topology is concerned absolute truths can be reached, but they are only partial truths about the “thing”. However, there is also no a priori limit to the accuracy with which the thing in itself can be known. In a given range of parameters, with a given accuracy of measurements – i.e. given the suitable orders of magnitude involved for the relevant parameters – there is no reason to question theories which are overwhelmingly confirmed by practical applications involving the same orders of magnitude. The theory of the classical pendulum, for example, fails when the action of the latter becomes comparable to \( \hbar \), or when some velocities approach the speed of light, but it would be a great loss of time and energy for any searcher to try and find a flaw in its theory when such conditions do not apply. On the other hand, it is quite possible that the theory of the harmonic oscillator with relativistic velocities, intense gravitational field, and for actions not very large compared to \( \hbar \) still has some surprises in store. The reason why absolute truths of the sort I am discussing here may be reached is that there is no dichotomy between the inanimate material world, and humanity, which is itself a particular form of matter, and has evolved intricate individual and social means of investigating reality. Nobody can prove that humans have no access to knowledge of some absolutely true features of the world.

My position on the question of truth is at variance with a number of philosophers of science. Popper recognizes that knowledge can be objective, and is a

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27 Topology deals with the study of shapes and topological spaces. Topological properties of space are preserved under continuous deformations, such as bending, or stretching; this includes connectedness, continuity and boundary. For instance, the statement : “the Earth has the topology of a sphere in three dimensional space” is an absolute truth; the Earth is not a sphere, but it has the topology of the sphere: its shape could be deformed, continuously, by a thought process, to become rigorously that of a sphere. Many topological results are described by integer numbers, the accuracy of which is infinite: they represent an absolute truth on a limited aspect of an object.
cumulative process, but following his falsification theory, a theory is only valid until it is replaced by a better one, which accounts for facts in a better way; it cannot ever be considered as true. Lakatos has shown how Popper’s falsificationism fails to account for research programs, and Feyerabend eventually takes a relativist stand.

Quantum mechanics, as a theory of investigation of the microscopic world, is responsible, in terms of its applications, for a good deal of our daily life in modern day developed countries. Does this mean that we understand it fully? Perhaps not. While the debate is still raging in philosophical papers on the superposition principle, on Bell inequalities, etc., would anybody be considered as behaving rationally if she tried to investigate the spectrum of the hydrogen atom, of the structure of the Mendeleieff table, of the low temperature specific heat of metals, of the structure of most molecules on earth, or the stability of atoms, of the laser, the SQUID, transistors, NMR Imagery, etc., without the help of the Schrödinger equation? It may well be that the latter be replaced one day by a better equation, which explains more things—for instance the reduction of the wave packet—but this new equation will have to account for all the successes of present day textbook quantum mechanics. My point of view is in some respects analogous to Chalmers’ “Non Figurative Realism”, discussed at more length below. The latter claims, as I do, that there is no end to the progress of physics. I differ from Chalmers’ in that he does not accept that reality can be self contradictory, and in particular that theories may access to partial truths, which are absolute truths when they are related to their validity limits.

What I’m claiming here is that the theory of the QHE effect described in this paper is now so well established, it has produced so many results, so many tests, so many applications, that if someone would spend time nowadays to find a theory based on different principles than those Laughlin described to account for the IQHE plateaux, or for the simplest FQHE ones, her intellectual capacities would be doubted by all experts, on rational grounds. I have regularly quoted the Nobel prize awards connected with the Quantum Hall experimental and theoretical discoveries. This is not due to some irrational cult for such awards. In my view, the process through which Nobel prizes in physics are awarded to individuals is a social one through which a research advance is given a social reliability label. There are good reasons to believe that the Quantum Hall Effects exist, as they are reproduced and described in many published papers in scientific journals with referees, and that a major understanding of their essential aspects is provided by Laughlin’s theory. The Nobel prize awards are but an additional reason to believe in both theory and experiments on QHE Effects.

Such as the GRW proposal, for instance, for which, at present, there is no experimental support.
6.2 Chalmers’ Non figurative Realism

Chalmers [2] has reviewed critically the works of some of the twentieth century science philosophers mentioned in this paper. I share his criticisms of Popper, Kuhn, Lakatos, and Feyerabend, and I also share, to some extent, his conclusion that “the aim of physics is to set limits for the application of present day theories and develop theories which are applicable to the world with a higher degree of accuracy”. His proposal is that of “Non Figurative Realism” (NFR): “the world is what it is, independently of the knowledge we may have; our theories, inasmuch as they are applicable to the world, are always more than statements of correlations between series of observations”. However, I do not agree with his conclusion, namely that “NFR does not suppose that our theories describe entities within the world, such as wave functions and fields...We can evaluate our theories according to their degree of success in grasping some aspect of the world, but we cannot go beyond that, and evaluate their success in describing the world as it is really, because we have no access to the world independently of our theories...” Chalmers denies that physics may reach ontological truths. He justifies his denial of the possibility of reaching truths by historical examples such as the fact that Newton’s corpuscular theory of light cannot be reconciled with the modern theory of light, or with the failure of Newton’s dynamics when velocities are comparable to that of light, etc.. He states that “There is no concept of truth, the quest of which would be the ultimate goal of science. ”.

In other words, Chalmers departs from the positivist attitude à la Duhem [19], following which theories establish a mere correspondence between mathematical signs and appearances, but cannot go beyond. On the basis of this, a fierce battle raged between energeticists and atomicists. The former denied all ontological truth to the concept of atom, while the latter stated that atoms exist in nature. History has given a clear judgment on this matter. However, Chalmers agrees with Kant’s transcendental idealism in denying that science can eventually access absolute truths about the world.

I believe that the weakness of Chalmers’ position is due to both his refusal of admitting the basic progress of dialectical materialism, namely the statement of coexistence of opposites with the “thing in itself”, and his under estimation of what Bachelard thought essential about physics: the notion of order of magnitude [1]. The statements of physics about the world, to a great extent, rely on specifying the relevant orders of magnitude for the phenomena it accounts for, the accuracy of measurements, i.e. the order of magnitude of the parameters which are available to technology, at a given moment in history.

Newton’s views were contradictory to Fresnel’s on the nature of light, but each grasped a part of the conflicting nature of light as we know it today: both continuous and discontinuous. Quantum mechanics shows that “contraria sunt complementa”, as Bohr stated. One cannot say that Fresnel’s theory of light had no element of truth: elements which reflected objective properties of light. A basic consequence of contradictions within nature is that, depending on the type of experiment, one term of the contradiction, or the other, may dominate its phenomenal manifestations. A sufficiently thorough investigation
may end up, within the same or different ranges of relevant orders of magnitude, revealing the contradictions within the thing, and which term dominates under what condition.

As for quantum mechanics, it may not be the ultimate theory of the microscopic world, but it is definitely true for all orders of magnitude available to day to humanity for experimental work. Chalmers correctly states that electrons are not the tiny individual charged massive spheres they were thought once to be, but the correct (non relativistic) theory of electrons definitely rests of the definition of their rest mass and charge, the indiscernability principle, and their description by Schrödinger’s equation. Within orders of magnitude available to technology fifty years ago, physics has established results many of which are undisputable within their own range of accuracy, although they have been contradicted by new observations and new theories developed about finer scales of accuracy, or larger scales of parameters available for experiments. In this paper, I have given reasons to accept that the Laughlin wave function in two dimensions, under suitable magnetic field intensity is both an effective mathematical representation of the electron liquid, and reflects objective properties of that liquid, within the accuracy of present day measurements, the quality of present day Hall bars, etc.. In other words absolute truths have been stated, under relevant experimental conditions and orders of magnitudes: absolute truths are also relative ones.

6.3 Philosophy of Quantum Mechanics

The theory of the QHE is a quantum mechanical one. It describes, based on Schrödinger equation, a system of a great many electrons interacting in two-dimensional samples under magnetic field. A quantum Hall experiment deals with a 2D electron layer in a three dimensional space, under (in general) non relativistics conditions. However, if one considers Laughlin’s wave function, some features are obvious: a first one is that the wave function in equation (9) is a massively entangled one: a superposition of $2^N$ states, with $N \approx 10^{11}$ per cm$^2$. Entanglement is fundamental to allow for the lowering of the energy of the Laughlin QH liquid below that of the Wigner Crystal. This wave function depends on $2^N$ space coordinates of electrons. Is there something fundamental about this latter fact? A classical theory describing the $N$ trajectories of $N$ classical interacting particles on a flat surface would also depend on $2N$ spatial coordinates.

Some authors [33, 34] oppose a “primitive ontology”, that of electrons in 2D or 3D space, to their wave function which, according to them, cannot represent matter correctly because it is defined in a $4N$ or $6N$ configuration space, not in the 3D space of our ordinary experience. They state that the wave function cannot be a correct mathematical representation of physical objects because the latter move in a 3 dimensional world. According to Allori [33], this contrasts sharply with classical mechanics, where the “primitive ontology” is easily “... represented by point particles, the time evolution of which is described by Newton equation and the laws of force”. However, even the simplest classical motion of
a single point like particle (which is of course a simplified abstraction for the representation of a real material particle) requires a 6D space, the configuration space for the variables \( \vec{r} \) and \( \vec{q} \). Allori’s objection to the ontology of the wave function in quantum mechanics does not hold.

The stated antinomy between physical objects and their wave function seems to reflect a difficulty among many philosophers regarding the main difference between quantum and classical mechanics: in the latter case, the notion of trajectory is a familiar one given our everyday experience of classical mechanics; the irrelevance in the former case of a notion of classical trajectory is reflected in the non commutativity of the space and momentum operators, which is not something we encounter in our classical world. Because of this irrelevance, knowledge of the microscopic world relies on the notion of quantum state, not on that of trajectories. According to the superposition principle, the wave function of quantum particles can be an addition (i.e. a superposition) of products of \( N \) single particle eigen functions where different eigenstates of each particle, for example of the angular momentum operator. This is what happens with the Laughlin wave function (equation (9)) where each particle appears with a very large number of different angular momentum eigenstates. It is obvious from sections (4.1) and (5) that this property is a reason why the Laughlin wave function has a close – if not exact – relationship with the incompressible quantum liquid responsible for the various Quantum Hall Effects. Is this proximity equivalent to an exact representation of the incompressible quantum Hall liquid? The latter is a macroscopic quantum object the ontology of which is probably not completely exhausted by the Laughlin wave function, as exhibited by the Composite Fermion picture in section (5.3). But there is little doubt that it evolves in a 2D space embedded in our non relativistic 3D space and that some essential features of this object are very well represented by Laughlin’s wave function: Quantum Hall plateaux, together with vanishing longitudinal resistivity, metallic state at half LL filling, fractionally charged excitations, and many other physical features of this object. Furthermore, as stated above, the approach of references [33, 34] and others disregards the fact discussed by P. W. Anderson [21]: “More is different”: the object described by equation (9) cannot be reduced to the sum of the properties of its constituting particles. In other words, the Quantum Hall Effects, as explained by Laughlin’s wave function, and the Composite Fermion hypothesis, represent a major breakthrough in the understanding of the material world in our 3D space, and there is no need for invoking the (non existent in this context) Bohmian approach, or the GRW theory (see reference [33]) of the wave packet reduction problem. The wave function is vindicated.

I have stated elsewhere [35] my opinion that the duality of the quantum world and the classical world is an unescapable feature of the world, as evidenced by all practical applications of classical physics in the macroscopic world, and applications of quantum physics in modern day technology. The QH Effects, among others, show that an object may have a macroscopic size, a macroscopic number of quantum parts, and still exhibit quantum behaviour, as we also know from superconductivity, ferromagnetism, superfluidity, the violation of Bell inequalities by giant size quantum spin singlets, etc.. Even though it is clear that an
ordinary simply connected macroscopic object, with action very large compared to $\hbar$ usually behaves classically\[39\], there are a number of cases, quoted above, where such is not the case, not to mention quantum objects with astronomic size such as a neutron star. Investigating the general conditions under which classical behaviour emerges from large collections of interacting quantum microscopic objects seems to me to be an open problem and an interesting research program.

6.4 The Duhem-Quine thesis on the underdetermination of theory by the facts

There is a philosophical cliché which appears again and again in many a philosophical paper on the relationship between theory and facts, which deal precisely with the above sections. Duhem\[19\] has developed the notion that when a theory leads to incorrect experimental predictions (what Popper would call falsification of the theory), there is no way to determine what is the origin of the discrepancy: perhaps the apparatus produced artifacts, perhaps a fundamental axiom of the theory is mistakenly accepted, etc.. Contrary to Bacon’s thesis, Duhem claims that there is no such thing as a crucial experiment to distinguish between conflicting theories. In other words, a number of different theories may account for the same experimental facts. In section\[44\] I have described how two theories of ferromagnetism have been proposed during some time by mutually antagonistic schools. Oddly enough, most philosophers quoting this so-called underdetermination of theory by facts fail to quote a statement by Duhem himself proving that his own thesis\[32\] cannot have universal value: he states that in the case of vibrating strings, the theory is certain, which means is absolutely true. Although it is often the case that different theories offer at some time in the development of physics different explanations, based on different explanentes for the same explanandum, Duhem himself gives an argument (vibrating strings) why this is not a universal feature of epistemics. The theory of light, whether corpuscular according to Descartes and Newton, or wave like according to Huyghens, Fresnel and Young, is another well known case of antagonistic theories. However this antagonism was resolved by quantum mechanics, and no one now questions some of the basic features of the quantum mechanical theory of the electromagnetic radiation. Thus the claimed underdetermination of theory by the facts seems to be mostly a historically transient state of affairs. When the relevant parameters of given phenomena are within a well mastered range of technological achievements, epistemics generally reduce, just as for Duhem’s vibrating strings, to a single theory, whereby conflicting phenomena may often simultaneously accounted for.

As far as the Quantum Hall Effects are concerned, there seems at present to be no doubt among physicists that some of the basic QH phenomena are well accounted for by Laughlin’s theory. Attempts have been made to phrase the latter in the usual Hamiltonian language, and some successes have been obtained (see for instance\[18\]). But although the language of this theory is different, its content is (approximately) identical to Laughlin’s theory, which
was used as a guide for its construction. It is fair to say that Laughlin’s theory is so successful in accounting for the basic experimental facts, and in predicting successfully such revolutionary properties as fractionally charged excitations, that the QHE seem to be one instance for which the facts uniquely determine the theory. This does not mean that conflicting theories will not emerge when more accurate techniques, better controlled or new materials such as graphene, provide for new phenomena. But Laughlin’s theory is a common basis for such theoretical developments.

6.5 Contradictions in the essence of things? Unity of opposites?

The term “essence” used in the title of this subsection does not refer to the eternal nature of the “thing”, as identified by its necessary properties, supposedly permanent through all its phenomenal manifestations. Essence here is understood as the effective set of effects and correlations through which the “thing” has effects in the world, in space and time. Science has in general only a partial, but (in time) always improving access to this set.

I have stressed a number of times that contradictions – the coexistence and competition/struggle of opposites – appear fairly regularly, upon scrutiny, as intrinsic features of reality. This notion, which is a fundamental aspect of dialectical materialism [37, 38, 39, 40, 41] is often tacitly considered as a superseded pathology of stalinist dogmatism.

Dialectical materialism suffered a severe blow, as a useful and rational philosophical system, when it was used as official state philosophy in the USSR. Much to the contrary, nothing, in the founding philosophical writings [38, 39, 40] allowed to justify turning them into an official State philosophy. One may surmise that the striking political achievements of the first years of the Soviet revolution made that appear as a positive step. This produced however such catastrophies as the State support for Lyssenko’s theories, based on the notion that genetics was a bourgeois science, while lamarckian concepts were defined at the government level as correct from the point of view of dialectical materialism. It is understandable that such nonsense in the name of a philosophical thesis turned the latter into a very questionable construction in the eyes of many. Dialectical materialism itself is an open system, which has no lesson to teach beforehand about specific objects of knowledge, and insists [39] on taking into account all lessons taught by the advancement of science.

It is perhaps time for a serious critical assessment of this philosophical thesis. The question of the possibility of general theoretical statements about the empirical world is not a negligible one.

The formulation by Engels has been the basis of the so-called “laws” of dialectical materialism: the unity and struggle of contraries (or equivalently, opposites), the transformation of quantity into quality, and the negation of the negation. The very term of “law” cannot be taken at face value, since it carries with it an equivalence with scientific laws. Dialectical materialism is not a science, but its nature and its epistemic criteria are closely related to science [41].
The term “thesis” is probably more suitable.

A first observation is that dialectical materialism cannot be reduced, once and for all, to the body of the three “laws” mentioned above. Opposites, or contraries, which form a contradictory unity may be antagonistic (so that one of the poles struggles to destroy the other) or non antagonistic, so that the opposites coexist and interpenetrate each other [11]. This is an example of a development which occurred long after the formulation of the so-called laws. In fact recent work [11] points out the interest and necessity of a deepening of our understanding of the rich variety of forms of contradictions in nature, and of their development.

Another observation is that the three “laws” listed above have not been, to the best of my knowledge, of any conscious use in the development of the theory and experiments on the QH Effects. This is not surprising, but has no consequence on the question of their relationships, and that of dialectical materialism, with the topic described in this paper. Dialectical materialism does not provide a scientific methodology which would be valid at all times in all cases. It cannot assess beforehand the epistemic value of a given scientific approach of a specific phenomenon.

Then after all, why bother?

This is not a minor question. The history of physics is full of long battles between antagonistic views on scientific issues: geocentric versus heliocentric model; corpuscular or wave-like nature of light; existence or non existence of atoms; corpuscular or wave-like nature of electrons, atoms; existence or non existence of the ether; localized versus itinerant theory of magnetism, etc.. Sometimes, one theory takes the upper hand and becomes the dominant one. Most times, new theoretical concept have appeared which blend the previously contradicting views into reflecting an intrinsically contradicting reality: light is both corpuscular and wave-like, so are electrons, atoms, nuclei, etc.. Quantum mechanics teaches us that, as Bohr put it: “Contraria sunt complementa”. Nowadays, thirty years after the discovery of the new “high temperature” superconductors[22], the battle is still raging between followers of the “strong interaction” paradigm and the “weak interaction” one, between “strong repulsive” and “weak attractive” ones, “electron-electron interactions and “electron-phonon” ones, etc., with tens of thousands of published mutually conflicting papers in the scientific press.

It is of interest to ascertain if a culture of contradictions, a culture about allowing the unity of opposites in our understanding of our scientific objects of studies is not one of definite interest and fecundity.

The dominant ignorance, or rejection of the main thesis of dialectics in nature produces definite damages in scientific discussions, as well as in philosophical writings about physics. For example, renewing in 2009 the question “particles or fields?” about quantum field theory [12] or denying the ontological basis of the wave function for the benefits of a “primitive ontology” [33] seems to reflect the permanence of an aristotelian dogma of non-antagonism. Is the dogmatic exclusion of contradictions within nature justified? Developing a culture which warns against unilateral views about the world seems an important task.

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One question in this paper is whether there is a way to examine if the results I have described give credence, or not, to a general philosophical statement about the meaning of dialectics in nature. The issue is both about the dialectics of the thought process (the theory), and whether the latter reproduces, in its specific subjective way, correlations, relations and processes which exist objectively in nature, in the particular case of the physics I have discussed. The answer to this question plays a role in the discussion of the numerous contemporary trends of subjectivist philosophies [25, 26, 27, 29].

Even though, as I stated above, the three thesis of dialectical materialism as formulated by Engels are a somewhat schematic summary of the dialectics of nature, I proceed below to examine the QHE physics along those lines, for simplicity.

- Starting with the negation of negation, I view it as a statement that all things evolve with time, space, or other parameters: change, or movement, for example, are based on the negation of, say, position at time t, replaced by position at time \( t + \delta t \), followed instantaneously by the negation of the new position, etc.. Is anything permanent in the world? Perhaps the total energy of the Universe, if this is a meaningful concept... But even if this be the case, the way this energy is distributed in the various parts of the universe is a continuously evolving one.

In any case, beside its seemingly esoteric formulation, negation of negation seems a rather straightforward property of an ever changing world. Matter does not exist without some sort of motion. Motion is an example of continuous negation of negation. What about a ground state of a many electron system such as described by the Laughlin wave function? The figure (3.3) exhibits a curve of the Hall resistivity which may be thought of as a series of negations of negations: just as any experimental curve of a quantity varying with some parameter: it is a succession of quantized plateaux of the Hall resistivity, with different values of the resistivity, separated by segments of smooth variation of the same quantity. Here I have underlined that negation of negation appears as a thought process, as a way of interpreting the experimental curve.

What about a plateau? The resistivity value is not changed in a whole interval of the magnetic field; but when the field varies, the distribution of electronic states inside the sample changes, since the degeneracy of the LL changes, so there is negation of negation of the electronic distribution even though the resistivity is pinned to a plateau resistivity value. Here change coexists with permanence, where the latter is expressed in terms of universal constants! Eventually this process leads to variation away from (negation of) the resistivity plateau. What if the magnetic field is held fixed, and the state is in thermal equilibrium? A closer look at the condition for fixing the magnetic field shows that a constant intensity of the field is in fact an idealization of a field which has intensity and spatial fluctuations sufficiently small that they can be neglected at the level of
A “fixed intensity of the field” is a true property as long as the accuracy of the experiment allows to neglect its variations. Fluctuations are as the oscillations of a pendulum: a continuous negation of variation leading to a variation of opposite sign, and so on. Thermal equilibrium is also a continuous succession of thermal exchange between two sources of heat such that a temperature increase is immediately countered by an opposite effect.

Even though the concept of negation of negation has not helped anybody at all in the discovery and explanation of the QH Effect, there seems to be no difficulty in interpreting things with it a posteriori. If one objects that, in the discussion above, dialectics is in the interpretation of phenomena, i.e. in the subjectivity of the author, she neglects the fact that this undeniable subjectivity seems convincingly to reflect objective dialectical processes.

- The transformation of quantity in quality
  This has been mentioned at different times in this study, along with P. W. Anderson’s paper [21] (*More is different*). This is probably the easiest thing to admit in the dialectics of nature. Even though it seems trivial to many, it is at variance with the classical Aristotle dichotomy which opposes categories of quantity and quality.

In the QHE study, trivial examples abound, such as the transition from a classical behaviour of the Hall resistivity at low fields to that at high fields, where low, resp. high refer to the large, resp. small number of occupied LL. Another example is the qualitative change of the QH Effects if disorder in the sample is low or high. A less trivial example, and a spectacular one at that, is the transition of the 2D electron liquid under magnetic field from the Quantum Hall topological insulator at LL filling \( \nu = p/(2p_{\text{gs}} + 1) \) to a metallic state at \( \nu = 1/2 \). A more trivial one is that the QHE has no meaning for a few electrons: it only exists for a large enough number of electrons.

We have seen along this work that this ubiquitous property of matter is important in discussing the validity of the reductionist analysis of phenomena.

- Contradictions, unity of opposites: the coexistence, struggle, and interpenetration of opposites (or contraries) in Nature.
  This is the most controversial aspects of dialectical materialism in the sciences of Nature. Many philosophers accept as straightforward the notion that antagonistic classes exist within society, and that contradicting forces act at various levels in the objects of social sciences. But what is the validity of the notion of contradictions within a thing in Nature? Can we give meaning to a statement that a contraries, or opposites, co exist, struggle with and interpenetrate each other within an object? On the other hand,
how rational is the logical dogma following which contradictions in nature are prohibited in the analysis of reality?

I have stated above that a standard attitude of physicists is to determine what is the dominant parameter in a given object of study: entropy versus internal energy (thermodynamics), kinetic energy versus potential energy (classical and quantum mechanics, QHE), localization versus delocalization (wave functions, QHE), continuous versus discrete (quantum mechanics), symmetric versus asymmetric (phase transitions, QHE), reversibility versus irreversibility (thermodynamics, phase transitions, QHE) order versus disorder (phase transitions, QHE), long range versus short range (phase transitions, QHE), insulating versus conducting (thermal and electromagnetic properties of matter, QHE, topological insulators), interacting versus non interacting (ubiquitous, QHE), magnetic versus superconducting (magnetism, superconductivity, QHE), attraction versus repulsion (molecules, atoms, nuclei, ) change versus invariance (phase transitions, mechanics, QHE, etc.). Etc..

Physicists will then examine under what conditions the dominant parameter will cease to be dominant and surrender its domination to the opposite pole: in general, this signals a qualitative change in the object under study. The very list of those seemingly antinomic pairs, which clearly coexist, compete (struggle) and interpenetrate each other in various ways is sufficient to convince that the third thesis of dialectical materialism is meaningful for a large number of problems discussed in physics, with a large diversity of ways for contradictions to appear. Whoever thinks that such contradictions only exist in the subjectivity of human theoretical constructions will run into difficulties to prove her point, because of the many practical consequences which prove their objective validity. The contradictions I have listed above are convenient, at a given time in history, to describe reality because they reflect, with more or less accuracy, contradictions which are at work in Nature.

If we think about the QH phenomena discussed in this paper, how can we avoid being impressed by the fact that a transverse resistance QH plateau, which coincides, within a range of external field values, with a vanishing longitudinal resistivity, as well as a vanishing sample conductivity, is driven by the simultaneous coexistence of an extended conducting channel on the edge of the sample, with localized (insulating) states inside the sample? (A new contradiction appears here: inside versus outside...) I have stressed above the paradox (contradiction) between the accurate quantization of the Hall plateaux conductivity in terms of universal constants $e^2/h$, and the contingency of impurity disorder in the sample which cause those plateaux to exist (here we have necessity versus contingency...). Notice also that depending on the amount of impurities, plateaux will vanish (too many impurities), appear, or disappear (no impurity disorder). We have here a rich variety of coexisting competing opposites. As the external magnetic field varies, the balance of localized states and extended edge
states shifts, until extended states within the sample connect the two QH bar edges; extended states within the sample connecting one edge of the Hall bar with other put an end to the (almost) perfect conductivity of the edge channel and the quantization of the transverse resistance plateau.

In all cases, depending of which pole, in the above list of opposites, dominates the other, and depending on the process which is at work in the contradiction, the object within which such opposites coexist will have different properties. Depending of the way a given object is isolated, or submitted to external fields, the competition (or contradiction, struggle, etc.) between the opposites listed above may lead to qualitative differences depending of which pole dominates, the negation of negation will appear as an obvious process, or will not, etc..

What seems to emerge from the discussion above is that it is a posteriori justified in many instances to analyze a number of objects in nature in terms of dialectical materialist terms. The variation of their properties with various parameters leads to different ways for opposites to combine within the object and change its properties. It is often true that describing this “combination” of opposites may be described as a “struggle”, inasmuch as the domination of one over the other determines specific properties. For example when kinetic energy dominates the potential energy in a mechanical system, the latter has a qualitatively different behaviour than in the opposite case. Still in classical mechanics, the qualitative change of dynamics when the number of interacting bodies increases beyond 2, and chaotic trajectories emerge. In quantum mechanics, delocalization of the wave function lowers the kinetic energy; the potential energy drives localization. In thermodynamics, the internal energy drives order in a state of equilibrium; the entropy drives disorder; order at short distance may coexist with disorder at long distance. In the QH Effects, when the orbital energy $\hbar \omega_c$ is small compared to the Fermi energy $\epsilon_F$ of the electron system in zero field, the behaviour is classical. When on the contrary $\epsilon_F << \omega_c$ the quantum behaviour dominates. As explained earlier, the behaviour of the 2D electronic system under strong magnetic field is entirely dominated by the interaction energy, which is much larger than the width of a Landau Level.

The fundamental question about dialectical materialism is whether this analysis in terms of struggle of binary opposites is universally valid, and if such is the case, in what sense. In this paper I have discussed QH Effects, and I believe I have shown that the a posteriori analysis of those effects allows to state that this analysis is plausible, in many different ways, be it, from the start, because quantum mechanics proves the conflicting unity of continuity and discontinuity. “Contraria sunt complementa”, in contradiction with Kant’s list of antinomies. In various different ways, it is fair to say that some of the basic thesis of dialectical materialism are supported by the QHE. This may sound trivial to many, and totally wrong to others. In my view, attacking, with reasonable arguments,

\footnote{In this limit, this width of the LL is determined by the fluctuations of the impurity potential.}
the conventional dogma of the “no contradiction principle” in Nature following which opposites cannot coexist in a contradictory unity in Nature is not a negligible contribution to the philosophy of physics.

7 Conclusion

7.1 Epistemics and ontology I

In spite of their seemingly reduced area of concrete experimental physics, the Quantum Hall Effects have led to a number of advances of universal significance. First consider the achievements in terms of knowledge of physical laws. These achievements are jointly theoretical, experimental and technological:

- Manufacturing two dimensional electron systems with suitably controlled electron density ($\approx 10^{11}$ per cm$^2$). Later on, producing graphene sheets, where electrons propagate as relativistic massless particles.

- Shedding light on quantum complementarity through an example of noncommuting coordinates: $[X, Y] = i\hbar$.

- Understanding the IQHE, i.e. the occurrence of quantized Hall resistance plateaux in terms of the conductance quantum $e^2/h$, a fundamental constant of physics. Determination of the latter with unprecedented accuracy ($10^{-9}$).

- Understanding the fundamental role of sample imperfections in the high accuracy of the QH conductance plateau.

- Discovering topological insulators: a new class of conducting materials which are...insulators. Superseding the old antinomy between insulators and conductors.

- Unveiling the completely filled LL as a new ferromagnet. Detecting skyrmions, its topological defects.

- Demonstrating the nature of the Fractional QH plateau at $\nu = 1/3, 1/5, ...$ with the proposal of the many-body almost exact wave function for an incompressible 2D electron liquid. This proposal used gauge freedom flexibility, basic symmetry principles for fermion wave functions, and simple mathematical properties of analytic functions.

- Predicting the occurrence of fractionally charged excitations of the FQH states. Confirming experimentally the theoretical prediction.

- Accounting for other families of FQH plateaux within the Composite Fermion concept, a bound state of flux quanta to an electron. Understanding the new metallic state at $\nu = 1/2$ within this picture.

- Introducing anyons: excitations of the FQH states which expand the category of particle statistics.
7.2 Epistemics and ontology II

In terms of philosophy of knowledge, this study has led to some considerations on:

- the unsatisfactory Popper’s approach on scientific process of knowledge. The falsification approach does not account for the development of the QHE. The QHE comes in support of Lakatos’ and Feynman’s objections to Popper’s views.

- the non universality of the Duhem-Quine thesis on the underdetermination of theory by facts.

- The unsatisfactory approach of Kuhn’s on scientific revolutions. A new paradigm may well develop without having to abandon all or most previous paradigms. The development of tools to explore new scales of observation of the world allows a continuous/discontinuous, sometimes chaotic, process of accumulation of grains of absolute truths. Theories are not mere “beliefs shared by a group of people”: their truth content, within limits set by the order of magnitude of relevant parameters, and by the accuracy of tools at a given moment in history, is proved by their practical applications.

- The macroscopic wave function introduced by Laughlin, and improved by Jain has proved to have an undisputable correspondence with the real world of 2D electrons under strong magnetic field. The antinomy between the “primitive ontology” and the wave function does not seem to be a relevant notion: the wave function and the “primitive ontology” are dialectically connected. As for atoms or molecules, the Laughlin wave functions, and perhaps their Jain extension, have an ontological content.

- Quantity may transform into quality. Reductionism is both unavoidable (the QHE are a phenomenon due to electrons) and incorrect (it fails to account for the FQHE, fractionally charged excitations, etc.. More is different.

- Negation of negation is a rational way of describing various properties of the QHE.

- The coexistence of opposites within an object seems to occur in many different ways in the QHE. Besides the fact that quantum mechanics reflects the coexistence in Nature of continuity and discontinuity, we have seen a) how the accurate quantization of Hall plateaux is due to imperfections of the Hall sample;b) how the QH Effects are an example of coexistence of insulating and perfectly conducting properties in the topological insulator. c) The FQHE plateaux are an example of domination of the potential (interaction) energy over the kinetic energy, but the transition from plateau to plateau is driven by the impurities potential energy. Etc..
At this stage, I believe I have shown that Quantum Hall phenomena deserve some attention from science philosophers and that they provide a rich variety of scientific and philosophical teachings. One of them is the contradictory unit of abstract universal concepts – quantity, quality, randomness, accuracy, continuity, discreteness, etc., – and concrete specific physical objects of study. I believe I have given some arguments in favour of a rich variety of subjective analysis in terms of contradictions reflecting a rich variety of objective ones in a particular aspect of physics.

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