Controlling a whispering gallery doublet mode avoided frequency crossing: Strong coupling between photon bosonic and spin degrees of freedom

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(Dated: December 25, 2013)

A combination of electron spin interactions in a magnetic field allows us to control the resonance frequencies of a high-Q Whispering Gallery (WG) cavity mode doublet, resulting in precise measurements of an avoided crossing between the two modes comprising the doublet. We show that the resonant photons effectively behave as spin-$\frac{1}{2}$ particles and that the physical origins of the doublet phenomenon arise from an energy splitting between the states of photon spin angular momentum. The exclusive role of the photon spin in splitting the mode frequency is emphasized, and we demonstrate that the gyrotropic and anisotropic properties of the crystalline media supporting the WG mode lead to strong coupling between the bosonic and spin degrees of freedom of cavity photons. Despite the demonstrated similarities with Jaynes-Cummings type systems, the mode doublet system exhibits a significant difference due to its linearity. Unlike traditional experiments dealing with interactions between fields and matter, here the crystalline medium plays a role of macroscopic symmetry breaking, assisting in the strong coupling between these photon degrees of freedom. Such a regime is demonstrated experimentally with a method to effectively control the photon spin state. Our experiments demonstrate for the first time, controllable time-reversal symmetry breaking in a high-$Q$ cavity.

I. TWO APPROACHES TO FIELD-MATTER INTERACTION

The interaction between light and matter is one of the most important topics in modern science and technology, and particularly in the field of quantum mechanics. The applications of such research are many and varied, including spectroscopy, quantum information and computing, ultra-stable clocks, lasers, and fundamental investigations of quantum phenomena. Light-matter interactions are investigated at almost all frequencies of the electromagnetic spectrum, spanning many orders of magnitude from ultraviolet to radio frequencies. Generally speaking, in all of these experiments the bosonic degree of freedom of photons is always coupled to quantum states of matter in the form of atoms, ions, molecules or plasmas. Photons are chosen due to well established experimental techniques and technologies that make experimentation very easy, whilst the aforementioned states of matter are chosen as they are known for exhibiting quantum mechanical phenomena due to their proximity to the Plank scale. This approach has been enormously successful over the past few decades in demonstrating the quantum mechanical properties of nature.

One of the most common types of experiment involves coupling electromagnetic radiation to a spin-$\frac{1}{2}$ particle such as an electron. Since the famous Stern-Gerlach experiment of 1922, experimental setups involving spin-$\frac{1}{2}$ particles have gained significant attention in physics. In particular, systems well described by the Jaynes-Cummings model representing the coupling between a two-level system (TLS) with a quantised mode of an electromagnetic field, form the basis of a large area of research known as Cavity or Circuit Quantum Electrodynamics (QED). Quantum strong coupling regimes in particular have attracted special interest, having demonstrated entangled states necessary for different types of quantum computing schemes. In the microwave frequency range, strong coupling has been demonstrated for a range of systems including ensembles of paramagnetic impurity ions in dielectric crystals, nitrogen-vacancy (NV) centres in diamond, cold polar molecules coupled to superconducting cavities, NV centres coupled to qubit, and ensembles of ultra-cold atoms on a chip. All of these experiments, a special role is attributed to matter which reveals its microscopic quantum properties, particularly where spin systems play the role of two-level states. The number of such spin systems suitable for QED experiments in the microwave range is very limited, and thus the search for other potential systems for QED purposes is important.

Another approach to field-matter interaction places only secondary importance on the matter. Here, its role is limited to macroscopic phenomena leading to the symmetry breaking. As a result of such an interaction with matter, a cavity photon bosonic degree of freedom is coupled to the spin angular momentum of the same photon. Despite the fact that a photon is a spin-1 particle, the photon spin degree of freedom is effectively seen as a TLS due to the zero rest mass of a photon. In such a way, the situation of a cavity mode coupling to a TLS is achieved even though no atom-like structure of matter is needed. However, this system cannot be classified as a Jaynes-Cummings interaction because it lacks any nonlinear properties naturally exhibited by such systems.

In this work, we present the first experimental and theoretical demonstration of a field-matter interaction experiment performed using the spin angular momentum of a photon. Both the bosonic degree of freedom and the TLS originate in the nature of a cavity photon.
The only role of matter in the experiment is to break reflection symmetry in such a way that both of these degrees of freedom of a single particle are coupled. For this purpose, a Whispering Gallery mode cavity (WGC) of cylindrical geometry is used, rather than the more conventional linear Fabry-Pérot cavity. We demonstrate interaction in the strong coupling regime between a microwave photon spin (effectively acting as a TLS) and bosonic degrees of freedom. Although the quantum polarization state of photons has been studied previously in the optical domain, no cavity experiment has been conducted and analysed yet. Unlike other works on WGC, we emphasize importance of the polarization states of the photons, with an efficient method to control them. This gives an insight into the physical origins of the effects observed in many similar systems due to the spin degree of freedom of a photon.

II. EXPERIMENTALLY CONTROLLING A PHOTON SPIN STATE

In order to observe the energy splitting due to photon spin angular momentum, a few conditions must be satisfied. First, the cavity needs to be represented in a circular geometry. This requirement is imposed because a conventional linear cavity has an explicitly broken symmetry in reflection due to two boundary conditions. To excite a circular cavity requires only one electrode, and thus only one boundary condition is explicitly set. This implies independence of the clockwise and counterclockwise propagating waves in an ideal cavity. Secondly, the energy splitting must be greater than the resonance linewidth. In the opposite case, the relations between the mode doublets cannot be observed. Finally, an effective means is required to tune the cavity properties in order to change the coupling between cavity photons with different spin states. All these requirements can be achieved in ultra-low loss dielectric cylindrical cavities such as cryogenically cooled sapphire whispering gallery mode resonators. In such a system, the magnetic properties of the medium can be effectively manipulated by an external DC magnetic field \( B_{\text{ext}} \) through intrinsic crystal impurity ions. Indeed, impurity ions such as Fe\(^{3+}\) in sapphire crystal at low temperatures change the permeability tensor \( \mu(B_{\text{ext}}) \) making it gyrotropic. The anisotropy of the permittivity tensor is due to the crystal structure of the sapphire resonator, as well as the presence of back-scatterers in the lattice.

To experimentally demonstrate coupling between the spin and bosonic degrees of freedom of a photon, a sapphire WGC was cooled to approximately 110 mK. The sapphire is a cylinder of 50 mm diameter \( \times \) 30mm height, characterised using a transmission method with two loop antennae in a setup described in detail elsewhere. The probing signal is attenuated to reach the level of \(-60 \text{ dBm}\) power incident upon the crystal, and the cavity output signal is amplified by a low noise cryogenic amplifier. The cavity and the amplifier are both isolated at 110 mK. To modify the effective permeability sensor due to intrinsic impurity ions included at a parts-per-billion level within the crystal, an external DC magnetic field is applied to the sapphire cavity along its cylindrical z-axis. For the best magnetic field sensitivity \( \mu(B_{\text{ext}}) \), the WG mode to be excited is chosen in the vicinity of an Electron Spin Resonance (ESR) of the residual Fe\(^{3+}\) ions in the crystal \( (\nu_{\text{ESR}} = 12.04 \text{ GHz}) \). Fig. 1 shows the response of the cavity near the Fe\(^{3+}\) and Cr\(^{3+}\) spin resonances, demonstrating the existence of two eigenstates. These eigensolutions correspond to the \( |R\rangle \) and \( |L\rangle \) states of the photon spin angular momentum. The boxed region is shown magnified in Fig. 2. This figure demonstrates an avoided crossing between two states of defined circular polarisation. The magnetic field effectively “inverts” the medium, which manifests as a mirroring of these polarisations. The minimal splitting at about \(-0.5 \text{ mT}\) is attributed to the electrical properties of the crystal, i.e. the permittivity tensor \( \varepsilon \). This parameter is set by the crystalline medium and cannot be varied, thus determining the minimal splitting.

Figure 3 demonstrates the mode splitting at \( B_{\text{ext}} = -0.5 \text{ mT} \). Both states exhibit equal losses, demonstrating the strong coupling between the photon and its spin angular momentum. This situation is similar to traditional cavity QED experiment where a photon mode is strongly coupled to a specific atomic transition. The strength of the coupling is \( 2\gamma = 6.46 \text{ kHz} \), and the line width is \( 2\delta = 1206 \text{ Hz} \), with \( g/\delta = 5.4 \). It should be emphasized that ESR transitions of dilute impurities

![Graph showing the dependence of the WG mode doublet transmission coefficient on the external applied magnetic field.](image-url)
III. THEORETICAL DESCRIPTION

The WGC is represented by a uniform, homogeneous, time-invariant, (gyro)-anisotropic medium. The action of the electromagnetic field in such medium in the absence of charges and currents is given by

$$S = \frac{1}{2} \int_{t_0}^{t_1} \left( \mathbf{E}(\mathbf{r}, t) \cdot \hat{\mathcal{E}} \cdot \mathbf{E}(\mathbf{r}, t) - \mathbf{H}^\dagger(\mathbf{r}, t) \cdot \hat{\mu} \cdot \mathbf{H}(\mathbf{r}, t) \right) d^3\mathbf{r} \, dt,$$

where $t$ and $\mathbf{r}$ represent time and the vector displacement from the origin, $\mathbf{E}$ and $\mathbf{H}$ are the intensities of the electric and magnetic fields respectively, and $\hat{\mathcal{E}}$ and $\hat{\mu}$ are the permittivity and permeability tensors. The corresponding Hamiltonian can be written:

$$H = \frac{1}{2} \int \left( \mathbf{E}^\dagger(\mathbf{r}, t) \cdot \hat{\mathcal{E}} \cdot \mathbf{E}(\mathbf{r}, t) + \mathbf{B}^\dagger(\mathbf{r}, t) \cdot (\hat{\mu}^{-1})^\dagger \cdot \mathbf{B}(\mathbf{r}, t) \right) d^3\mathbf{r},$$

where $\mathbf{B}$ is the magnetic field, found using the relationship $\hat{\mu}^{-1}\hat{\mu} = \hat{I}$ (where $\hat{I}$ is the identity tensor).

The cavity material properties are given by the two tensors $\hat{\mathcal{E}}$ and $(\hat{\mu}^{-1})^\dagger$ in the cavity Hamiltonian (Eq. 2). These tensors act on a photon spin to change its state, demonstrating operator-like behaviour. The tensors can be represented in the form:

$$\hat{\mathcal{E}} = \varepsilon (\hat{I} + \hat{n}),$$

$$\hat{\mu}^{-1} = \mu^{-1} (\hat{I} + \hat{\nu}),$$

where $\varepsilon$ and $\mu^{-1}$ are scalars representing the average diagonal value of the corresponding quantity. Since any influence of the longitudinal direction is neglected, i.e. no longitudinal component of the field exists, all matrices in Eq. 3 can be represented as square. The last term in both definitions is considered to be a small perturbation of the isotropic components of the parameter due to the anisotropy and gyrotropicity of the medium.

In a WGC, the electromagnetic wave propagates around the inner surface of a cylindrical cavity. The resonance of such a cavity is set by the number of variations of the field intensity about the azimuthal angle $\phi$. WG cavities exhibit two types of mode with well defined linear polarisation: WGH modes, with dominant $\mathbf{B}$ field along the cylindrical $z$-axis and $\mathbf{E}$ field in the radial direction, and WGE modes with dominant $\mathbf{E}$ field oriented axially, and $\mathbf{B}$ field radially. All modes are hybrid modes to some extent, but the small longitudinal components of the field are ignored here. This approximation is the same as the case of an infinitely long cylindrical cavity, or a mode with very large azimuthal wave number. The assumption reduces the analysis of the system to two transverse dimensions. The two types of waves correspond to the two orthogonal directions of the wave-polarization in the cylindrical geometry, stated further as $\alpha = x = r$.
These two circular polarizations represent the spin of a can be represented generically in terms of the following is trivial, i.e. the associated identity matrices do not the first terms of the sums in the material tensors (Eq. 3) is trivial, i.e. the associated identity matrices do not matrix is isotropic then the diagonal elements in both matrices and vanish. However, in the case of an isotropic non-gyrotropic medium, both matrices become zero. The matrix representations (Eq. 9 and 10) are due to the law of spin angular momentum conservation in a matter-field interaction where |R⟩ and |L⟩ are the eigenstates of the field.

Such a representation of the medium, together with the photon polarisation decomposition into a circular polarisation basis. (7) suggests the following:

\[ H_\sigma = \frac{1}{4} \hbar \omega_0 \left[ \hat{\sigma}_x \hat{\sigma}_x + \hat{\sigma}_y \hat{\sigma}_y + \hat{\sigma}_z \hat{\sigma}_z \right] [a^\dagger a + a a^\dagger] \]

where the \( \sigma_\alpha \sigma_\alpha^\dagger = 1 \) term is due to taking into account the isotropic component and dispersion relationship \( \omega^2 = k^2 / \mu_0 \). The Hamiltonian (Eq. 11) clearly shows that the energy of a bosonic mode of a photon depends on its spin state. Expanding the second term of the first square bracket, the Hamiltonians of the WGE and WGH cavity modes can be written as

\[ H_x = \hbar \omega_0 \left( 1 + \frac{1}{2} \hat{\sigma}_x \hat{\sigma}_x + \frac{1}{2} \hat{\sigma}_y \hat{\sigma}_y \right) [a^\dagger a + \frac{1}{2}] \]

and

\[ H_y = \hbar \omega_0 \left( 1 + \frac{1}{2} \hat{\sigma}_y \hat{\sigma}_y + \frac{1}{2} \hat{\sigma}_x \hat{\sigma}_x \right) [a^\dagger a + \frac{1}{2}] \]

For both the WGE and WGH polarizations of the mode, the results (12) and (13) demonstrate a dependence of the angular frequency of the bosonic mode of the photon on its spin angular momentum state. This dependence vanishes if the medium is isotropic \( (\eta_{22} + \nu_{22} = 0) \) and not gyrotropic \( (2\Im \{\nu_{21} - \eta_{21}\} = 0 \) and \( \Re \{\nu_{21} - \eta_{21}\} = 0 \) leaving the usual form of a single harmonic oscillator (HO). Otherwise, splitting of the mode spectrum can be observed due to the states |R⟩ and |L⟩ of the TLS emerging from the photon spin angular momentum. So, strong coupling of the photon spin and bosonic degrees of freedom is possible. Experimentally, this is
although a photon is a spin-1 particle carrying spin $\pm 1$. Most of the similarities are due to the fact that, behaviour of a photon in a cavity and the photon’s own spin. In this work, we have demonstrated similarities between traditional QED, which employs an interaction between quantised electromagnetic modes with a two-level state system (a spin-$\frac{1}{2}$ particle), and the approach we propose, which employs interaction between bosonic behaviour of a photon in a cavity and the photon’s own spin. Most of the similarities are due to the fact that, although a photon is a spin-1 particle carrying spin $\pm 1$, it has only two states of spin angular momentum because of its zero rest mass. These two eigenstates imply that a photon can, in principle, behave as a qubit like other quantum TLSs based on eigenstates of a photon spin angular momentum operator, $|\Psi\rangle = \alpha |R\rangle + \beta |L\rangle$, where the complex coefficients are subject to $\alpha^2 + \beta^2 = 1$. However, unlike typical Jaynes-Cummings systems, the spin-phonon system analysed in this work is always linear and can thus always be described as a system of two coupled HOs. This fact prevents its direct utilization as a qubit without the introduction of an additional non-linearity, for instance through the spins controlling the interaction, a superconducting nonlinear junction, or a nonlinear measurement scheme.

Nevertheless, the narrow linewidths of WG modes allow these cavities to achieve operation in the strong coupling regime where a photon loses its identity in terms of its own spin. In this case, unlike in other work, matter is treated collectively by the tensor macroscopic description, which breaks the reflection symmetry leading to energy splitting between photon spin states. This approach to matter-field interaction is significantly different from the traditional one extensively implying microscopic quantum phenomena in the material. It is also different from the experiment utilising quantum regimes of cavity photons since only bosonic degrees of freedom are utilised in this approach. This work points out another source of quantum states, one originating in the photon spin degree of freedom. So, this work demonstrates possibility of achieving high Q-factor values in cavities with broken symmetry, and finally, we demonstrate an effective method of controlling this phenomenon through natural dilute impurities with the application of an external magnetic field.

CONCLUSION

In this work, we have demonstrated similarities between traditional QED, which employs an interaction between quantised electromagnetic modes with a two-level state system (a spin-$\frac{1}{2}$ particle), and the approach we propose, which employs interaction between bosonic behaviour of a photon in a cavity and the photon’s own spin. Most of the similarities are due to the fact that, although a photon is a spin-1 particle carrying spin $\pm 1$, it has only two states of spin angular momentum because of its zero rest mass. These two eigenstates imply that a photon can, in principle, behave as a qubit like other quantum TLSs based on eigenstates of a photon spin angular momentum operator, $|\Psi\rangle = \alpha |R\rangle + \beta |L\rangle$, where the complex coefficients are subject to $\alpha^2 + \beta^2 = 1$. However, unlike typical Jaynes-Cummings systems, the spin-phonon system analysed in this work is always linear and can thus always be described as a system of two coupled HOs. This fact prevents its direct utilization as a qubit without the introduction of an additional non-linearity, for instance through the spins controlling the interaction, a superconducting nonlinear junction, or a nonlinear measurement scheme.

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ACKNOWLEDGEMENTS

This work was supported by the Australian Research Council Grant No. CE110001013 and FL0992016.

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