Nonlinear acoustics methods in the investigations of elastic wave interactions in the ocean

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Abstract. Hydroacoustic parametric systems and methods of nonlinear acoustics in the investigation of the Ocean and the Arctic shelf are considered. A short description of the most promising directions of development of hydroacoustic systems with parametric radiating antennas is given. Characteristics of parametric devices are given, and the results of their applications to solve various problems of hydroacoustics are considered. We discuss new opportunities, which appear when applying parametric antennas, to illuminate underwater environment by autonomous underwater vehicles, and to ensure their navigation along the paths. The results of studies demonstrating single-mode excitation of a waveguide in a wide frequency band, of a parametric antenna, are presented. The possibility of broadband signal compression during its propagation in the result of waveguide dispersion, which leads to intensity increase, is shown. The ways of modernization and the prospective of application of the hydroacoustic means, using nonlinear acoustics methods, are discussed.

Introduction

A promising direction in acoustics and, in particular, hydroacoustics to solve many applied problems is the development of equipment operating on the principles of nonlinear interaction of acoustic waves. Application of the devices with parametric radiating antennas with high directivity and small sizes, low lateral radiation, a wide range of emitted frequencies with constant directivity, allows us to solve a large number of problems of underwater search and underwater communication more effectively.

Monitoring systems in underwater acoustics have been developing rapidly in these recent times. The study of marine environment characteristics can be more effective when using the equipment operating on the principles of nonlinear interaction of acoustic waves [1-3]. The so-called parametric antenna, due to the high radiation direction in a wide frequency band, is capable of increasing the efficiency of acoustic sensing in a shallow sea, either in a sea waveguide or in an Arctic waveguide formed between the bottom and the ice surface.

It was previously believed, that low efficiency of the conversion of pump wave energy into the wave energy generated in a nonlinear medium limits the application area of
hydroacoustic systems with radiating parametric antennas. However, numerous studies of wave nonlinear generation processes in a nonlinear medium have revealed the most prospective applications of hydroacoustic systems with parametric antennas, such as:

- parametric profilographs constructed to study the bottom and the top layer of bottom sediments;
- parametric systems of traverse review, for search of biological resources in shallow water;
- parametric hydroacoustic systems for sea monitoring along extended paths;
- parametric hydroacoustic systems for data transmission in water environment.

Owing to the unique characteristics, extension of application fields for hydroacoustic systems with parametric antennas has been achieved. These characteristics include the following:

- a wide range of operating (difference) frequencies that allows us to use complex and broadband signals;
- high angular resolution due to the high directivity of the antenna in the radiation mode;
- high distance resolution due to the broadband signal radiation;
- constancy of width of the antenna directional characteristics over the entire range of operating frequencies due to the peculiarities of the formation of radiation signals at the generated frequencies;
- high noise immunity due to the very small level of parametric antenna lateral radiation;
- small size and, therefore, weight of pump antenna.

1 General theory of nonlinear interaction

Figures and tables

A parametric antenna is a pump antenna (primary transducer) and a section of water medium where the interaction of pump waves occurs due to the nonlinear properties of the medium.

To describe the processes of nonlinear interaction of acoustic waves, we use various mathematical models based on the methods by Westerwelt, Moffett-Mellen, wave fronts and, in particular, models based on the solution of Hochlov-Zabolotskaya-Kuznetsov equation [1]. The mathematical expression obtained on the basis of this model is, in our opinion, the most prospective and allows us to calculate the amplitude of sound pressure at combinational frequencies at any point in space with different parameters of an antenna and a signal. Solution of the equation for acoustic pressure amplitude of a difference frequency signal was obtained in the form [1-4]:

\[
P_z = \frac{P_{01}P_{02}\varepsilon\Omega^2a^2}{8c_0^4\rho_0}\exp\left(zL_3\right) \cdot \int_0 z_3 \exp\left[-y - \frac{r_w^2(1+iBy)}{d+(y-z_3)+yz_3B}\right] \text{d}y,
\]

where \(P_{01}P_{02}\) are amplitude pressures of pump waves by the antenna surface; \(\varepsilon\) is the nonlinear parameter; \(\Omega = 2\pi F\) is the difference frequency; \(a\) is the aperture of radiator pump; \(L_3 = 1/\alpha\); \(\alpha\) is the attenuation coefficient of difference frequency wave; \(z_3 = z/l_3\); \(z\) is the coordinate along wave propagation; \(l_3 = 1/\alpha_{1,2}\); \(\alpha_{1,2}\) are the attenuation coefficients of pump waves; \(d = L_D/l_3, B = L_D/L_{D1}L_{D2}\); \(r_w^2 = 2r^2L_D/a^2L_3\); \(L_D = a^3\Omega/4c_0; l_{D1}, l_{D2} = a^3\omega_{1,2}/2c_0\); \(r\) is the transverse coordinate.

This expression allows us to calculate the amplitude of sound pressure at any point in space with different parameters of an antenna and a signal.
The main problems of detection of underwater objects in shallow water and near the surface include the presence of intense reverberation and noise interferences. The expression to calculate the energy characteristics of a parametric sonar taking into account noise interference, volume and surface reverberation was obtained in the form:

\[
W_a = \frac{2\delta P_{int} 10^3 I_{D1} I_{D2}}{F^2 \alpha L_D \gamma_{red} \rho_c \sqrt{\gamma_{rec} \tau}} \\
\frac{I(B,y)}{10^{0.05 \delta} e^{x/L_s}} \left[ \left( \frac{R_{eq}}{2x} \right)^2 - \left( \frac{\alpha \eta_c c \tau}{2 \gamma_{red}} \right)^2 \left( \frac{\delta}{2 \sqrt{y_{red} x y_{red}}} \right)^{1/2} \right]
\]

where \( \delta \) is the signal-to-noise ratio; \( P_{int} \) is the noise level; \( I_{D1,D2} = a^2 \omega_1^2 / 2c_0 \); \( F = \Omega / 2\pi \); \( \alpha = (y+1) / 4 \rho_o c_0^3 \); \( L_D = a^2 \Omega / 4c_0 \); \( \gamma_{red} \) is the concentration coefficient in the radiation mode; \( \gamma_{rec} \) is the concentration coefficient in the reception mode; \( \tau \) is the pulse duration; \( I(B,y) \) is the integral describing nonlinear generation process; \( R_{eq} \) is the radius of equivalent sphere; \( \alpha_v, \alpha_s \) are the coefficients of volume and surface reverberation; \( \eta_v, \eta_s \) are the coefficients of mutual orientation; \( \beta \) is the attenuation coefficient.

In accordance with this expression, it is possible to calculate the acoustic power for each channel of the parametric antenna, necessary to provide a given detection range in the conditions of noise and reverberation interferences.

In the presence of interference, the detection characteristics change depending on the distance at which the target is located. Figure 1 shows the dependence of the signal-to-noise ratio \( \delta \) on underwater object detection distance with \( R_{eq} = 0.1 \) m at the acoustic power of 70 W emitted at each of the pump frequencies. The effect of separate volume reverberation with \( \alpha_v = 10^{-6} / \text{m} \), surface reverberation with \( \alpha_s = 10^{-3} \), and noise with \( P_{int} = 0.01 \)Pa is represented by curves 1, 2, and 3, respectively. The signal-to-noise ratio decreases during the combined effects of noise and volume reverberation (curve 4), noise and surface reverberation (curve 5). When exposed to all types of interferences (curve 6), the signal-to-noise ratio takes the lowest value at all distances and characterizes the ability to detect a target with a given probability.

![Fig. 1. Dependence of the signal-to-noise ratio on distance.](https://doi.org/10.1051/e3sconf/201912702017)
2 Parametric profilographs

Small-sized profilographs are widely used to solve urgent problems:
- related to the study and analysis of bottom structures in Geology, Geoacoustics and Seismoacoustics on the sea shelf;
- search for minerals, determination of bottom structure for the construction of engineering hydraulic structures;
- diagnostics and monitoring of underwater engineering structures;
- evaluation of sapropel deposits in inland waters to determine the prospects of mining in organic fertilizers production, evaluation of sludge pollution for environmental control;
- hydroacoustic searching, mapping and preservation of underwater cultural heritage.

The difference (working) frequencies of such locators lie in the range from 5 to 30 kHz, the depth of penetration into the soil is up to 30-50 meters (depending on the soil type), the width of the directional characteristics of a unit (2-5) of degrees.

Fig. 2 illustrates the antenna directivity diagrams at the pump frequency (140 kHz) (Fig. 3a) and at difference frequencies (Fig. 2b – 3d), experimentally measured in a hydroacoustic pool.

As an example, figure 3 shows a sedimentary rock profilogram obtained by a parametric profilograph.

Fig. 2. Antenna directivity patterns: a) at pump frequencies; b) at difference frequency of 10 kHz; c) at difference frequency of 15 kHz; d) at difference frequency of 20 kHz.

Fig. 3. Profile of bottom sedimentary structures.
3 Parametric systems with traverse view

Currently, only depth sounders (vertical location) are used for detection and monitoring of biological resources in shallow water, and the view area of water space is very limited. At low depths the "deterrent effect" during vessel motion significantly affects the behavior of fish. Application of horizontal location devices ("conventional" sonar) with much larger viewing area is extremely inefficient due to the interference situation caused by reverberation and multiple reflections of acoustic signals from the bottom and the surface [6, 9, 15].

Application of a parametric sonar with traverse view will allow fishermen and hydrobiologists to increase significantly the productivity of search for objects of fishing and monitoring of biological resources and hydrobionts due to significant extension of the observable zone (dozens of times as much as that of a depth sounder) [13].

Difference (operating) frequencies lie in the range from 5 to 40 kHz, with coverage range up to 1000 – 1500 meters, including the conditions of shallow water. The field of application of such systems is very wide, it includes environmental monitoring of water area, and search for objects (fish concentrations) in water, in particular, in shallow water. The width of the directional pattern of such systems is several (2-8) degrees vertically and from 2 to 60 degrees horizontally.

Fig.4 shows the pump antenna physical configuration and directional diagram (DD). The pump antenna of the parametric sonar consists of 6 modules each containing two frequency sub-lattices.

![Fig. 4. Sonar pump antenna with traverse view: a) antenna physical configuration; b) DD of module 1 at the pump frequency of 250 kHz; c) DD of module 1 at the difference frequency of 30 kHz; d) DD of the antenna consisting of 6 modules at the difference frequency of 30 kHz.](image)

When monitoring shallow water bodies using wideband signals over long distances (traverse view), it is necessary to take into account the geometric and physical dispersion in a medium. The presence of physical and geometric dispersion should be taken into account when selecting the operating frequency range [10].

Thus, determination of the biological resource quantity is an actual and complex scientific and technical task from the point of view of water area ecological monitoring. Parametric sonar with traverse view is an advanced tool that can significantly improve the performance of search and monitoring of biological resources in shallow water.

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4 Parametric systems for sea monitoring on long paths and data transmission in water environment

The practice of application of parametric antennas shows that they can transmit a broadband signal that has the best consistency with the layered structure of ocean waveguide. Preliminary calculations that were made for the peculiarities of parametric antenna application show the possibility to control the number of signal excited modes.

Prospects for the application of broadband signals for ocean research lies in the fact that it opens the opportunity for the development of a new approach to sea area acoustic tomography using the procedure of frequency processing of the signals propagating along one path instead of the known procedure of spatial processing of the signals propagating along different routes [6].

The application fields of a powerful high-directional broadband radiating parametric system of stationary type can be different. Parametric antennas (antennas based on nonlinear effects) are advanced tools capable of providing new opportunities for acoustic sensing in the ocean, especially in the marine waveguide. A high directivity of hydroacoustic antenna based on nonlinear effects allows one to use this tool to study sea waveguide characteristics [1-2].

Parametric radiation methods provide the opportunity for selective excitation of modes in a wide frequency band (2-3 octaves). In this case, the emitted broadband signal can be perfectly matched with the layered structure of marine environment [7].

These qualities of parametric antenna are the most important for hydrophysical studies in the Arctic [8].

Thus, a radiating parametric antenna, matched with the sea waveguide, due to its features, opens the opportunity to include the waveguide frequency domain into hydroacoustic practice by nonlinear acoustics methods, in other words, to use frequency tomography methods instead of the spatial tomography.

This new quality of parametric hydroacoustic systems creates additional opportunities for marine area monitoring on long paths. Selective excitation of modes in a wide frequency band in waveguides seems to be the most effective tool for research of acoustic signal compression in sea waveguides, creation of high-performance sea sensing systems to detect and to investigate inhomogeneities.

The marine waveguide has frequency dispersion of acoustic signal propagation velocity. The magnitude of the dispersion depends on sound velocity depth profile and on waveguide thickness [6]. Frequency dispersion leads either to the destruction of short broadband pulses, which propagate over sufficiently long distances, or to the concentration of acoustic signal energy within a short time interval, if frequency modulation of the signal corresponds to the dispersion conditions in a medium. In this case, we can say that there is acoustic signal focusing or compression in time. If we know the law of dispersion in a waveguide, it is possible to form a signal of a certain shape, such that LFM signal is maximally compressed at some point of a waveguide.
The phenomena of waveguide dispersion and broadband signal compression in a sea waveguide were experimentally investigated on the example of acoustic wave propagation in shallow sea conditions.

Experimental studies were carried out in the Taganrog Bay of the Azov Sea on two small vessels. One was equipped with a radiating acoustic antenna; the other vessel provided the receiving antenna operation. The radiating antenna was mounted on a rotary device to the board of the vessel and scanned the water area with a narrow beam of parametric radiation in the angle range of \(-90^\circ – +90^\circ\) horizontally. The depth of the site at the experimental area was 2.5 – 3 m. The average radiation frequency (pump frequency) was 150 kHz. The difference frequency or signal radiation frequency was in the range of 5 kHz – 20 kHz. The electric power of the antenna amplifier was 1 kW for each of the pumping frequencies. The receiving antenna was made in the form of a vertical chain of eight hydrophones, which were arranged in increments of 0.25 m on a metal rod.

Fig. 5 shows the experimentally measured normalized distribution of signal level along the waveguide vertical section (points) and the estimation results of the first mode eigenfunction (solid curve) at a distance of 1000 m from the radiator. It is clear that the main radiation energy is concentrated in the middle part of the waveguide. Thus, in experimental conditions the parametric antenna excited the waveguide first mode. Experimental data correspond to pulse amplitude on the hydrophone vertical chain at the frequency of 15 kHz (500 Hz band). The first mode eigenfunction was calculated for the measured sound velocity profile and the bottom as a liquid half-space with the following parameters: soil density at the bottom of 1800 kg/m\(^3\), and the velocity of sound propagation in the ground was considered to be equal to 1520 m/s. Sound velocity in water was accepted to be 1499 m/s.

![Fig.5. Normalized distribution of signal level in the vertical section of the waveguide and calculation of the first mode.](image1)

![Fig.6. Change in the delay time of signal frequency components arrival. Experiment and calculation results.](image2)

Calculation of group velocity dispersion allows us to estimate the change in the delay of different frequency components of a signal during its propagation in a shallow waveguide.
Fig. 6 illustrates the dependences of such a delay for different distances covered by the signal. The dispersion curve slope for the LFM signal is well consistent with the frequency-time processing of the signal measured at a distance of 1590 m. As the distance increases, the frequency-time relations in the signal change. The delay of low-frequency components of the signal grows that corresponds to its duration reduction. Analysis shows that the arrival time of low-frequency and high-frequency components may coincide at a distance of about 6 km for the conditions of this experiment. Since the dispersion of signal propagation velocity depends on frequency in a nonlinear way, the nature of the frequency modulation should be nonlinear in order to obtain the signal maximum compression.

Such a tool, acting on the principles of nonlinear acoustics, provides an opportunity to conduct the research on long paths in a wide frequency band with single-mode waveguide propagation. With the help of a parametric antenna, acoustic paths can be arranged to study the hydrophysical characteristics of the sea under the ice layer.

Fig. 7 illustrates an example of calculations results for the acoustic field of a parametric antenna with a pumping frequency of 20 kHz. Owing to the nonlinear interactions in water medium, it radiates signals with operating (difference) frequencies in the range from 300 Hz to 3000 Hz with a directivity of 2 degrees in the vertical plane and 8 degrees in the horizontal plane, constant throughout the frequency range. Propagation of the signal form highly directional parametric antenna along a path of the length of 20 km is shown. Compensation angle in the vertical plane is 5 deg.

Modeling of the conditions for parametric antenna application, optimal for acoustic signal emission in the frequency range of 100 – 400 Hz in the Barents sea, indicates the fundamentally new opportunities for hydroacoustic methods of underwater environment illumination in vast shallow waters.

**Fig. 7.** Intensity of acoustic field of the parametric antenna along the propagation path of 20 km long.

Fig. 8 shows the waveguide profile and typical distribution of sound velocity along the Novaya Zemlya – Spitsbergen path of the length of 1048 km.
The integral dispersion, the form of which is shown in Fig. 9, corresponds to the condition of signal propagation in such a waveguide. This figure shows the time delay of first mode propagation from the signal frequency in the frequency range of 100 – 400 Hz.

As the modeling shows, the time of propagation along this path for the signal of 100 Hz is approximately 0.2 s less than that for the signal of 400 Hz (Fig. 10a). Therefore, using a special frequency modulation of the emitted acoustic signal with a duration of 0.2 s, when the signal begins with the emission of high-frequency waves (400 Hz) gradually decreasing to 100 Hz after 0.2 s, it is possible to obtain complete synchronization of arrival of different-frequency waves after propagating along that path. As a result of such a synchronous arrival, a long signal is folded into a short pulse with the duration determined by the signal frequency band (Fig. 10b). In this example, the duration of such a pulse is about 350 ms, which is 60 times less than the duration of the emitted signal. When...
modulation is consistent with the waveguide frequency dispersion, the signal is being compressed as the signal-to-noise ratio is growing.

![Graph](image)

**Fig.10.** Passage of a signal of different frequencies on the Novaya Zemlya – Spitsbergen path. Acoustic pulse with time-reversed frequency modulation (a); acoustic pulse corresponding to signal compression (b).

This new ability to concentrate radiation energy at long distances allows us to create remote virtual boundaries at which hydroacoustic sensing is carried out with a high signal-to-noise ratio. This possibility is realized with the help of a parametric antenna which, due to the directivity characteristic, makes it possible to excite the waveguide in a broadband selectively.

Areas of application of highly-direction powerful low-frequency parametric antenna of stationary type may be quite diverse. Parametric antenna, due to the high direction of the radiation in a wide frequency band, is a new advanced tool that can significantly improve the efficiency of acoustic sensing in the ocean, especially in the sea waveguide. The high directivity of parametric radiation makes it possible to use this antenna to study the characteristics of the sea waveguide itself.

In particular, to determine the sound velocity profile in an underwater sound channel, the lower mode dispersion is the most informative. It is possible to provide selective excitation of broadband acoustic signal modes, ideally matched with the layered structure of marine environment in a waveguide, applying the methods of nonlinear hydroacoustics.

It becomes possible to study the waveguide dispersion of propagation velocity in the low-frequency range on the paths of hundreds of kilometers.

Another area of possible application of a highly-directional broadband parametric antenna is to use it for long-range sound underwater communication and data transmission.
Thus, due to the peculiarities of selective excitation of modes in waveguides, the parametric antenna is the most effective tool to study the acoustic signal compression in ocean waveguides, to monitor the ocean on long paths, to construct underwater communication systems and to transmit data in shallow water and to communicate under the ice.

**Conclusion**

Parametric hydroacoustic systems are advanced tools to study the Oceans. The new direction of nonlinear acoustics is now widely developing. There are new systems and complexes for search, monitoring and diagnostics of water environment and bottom soil, including shallow water. Small size of parametric systems allows one to use them on various vessels (including small ones). The constancy of directivity diagram width in a wide range of frequencies and the absence of side lobes make the hydroacoustic systems with parametric mode of operation almost an indispensable tool to study shallow areas of the Arctic shelf and the World Ocean as a whole.

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