Analysis of an improved Wiener deterioration model considering mechanism equivalence

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Abstract. The standard Wiener deterioration model plays an important role in describing the deterioration path of products among all of the stochastic deterioration models. However, the standard Wiener deterioration model does not show the positive correlation between the drift parameter and diffusion parameter, thus it fails in describing some deterioration paths. In this paper, an improved Wiener deterioration model is proposed by using a power function to consider the parameter-correlation. Based on the mechanism equivalence test and the Akaike information criterion (AIC), the proposed model is compared with the standard Wiener deterioration model. Numerical example shows that the proposed Wiener deterioration model performs better than the standard Wiener deterioration model when used to describe the deterioration path of a set of actual electrical connector degradation data.

1. Introduction
The standard Wiener deterioration model plays an important role in describing the deterioration path of products. For example, researchers have widely applied the standard Wiener deterioration model on self-regulating heating cable [1], LED [2], the fatigue of metals [3], aluminum reduction cells [4], bearings [5], and so on.

The standard Wiener deterioration model is usually expressed as follow [6]:

\[ y(t) = \mu \Lambda(t) + \sigma B(\Lambda(t)), \]  

where \( y(t) \) denotes the deterioration path at time \( t \), \( \mu \) is the drift parameter representing the rate of deterioration, \( \sigma \) is the diffusion parameter representing the variation of deterioration, \( \Lambda(t) \) is a monotonic increasing function representing a general time scale with \( \Lambda(0) = 0 \), and \( B(\cdot) \) is the standard Brownian motion.

For the standard Wiener deterioration model, the probability density function (PDF) of the first-passage-time \( T_D \) is

\[ f(t) = \frac{D}{\sqrt{2\pi \sigma^2 \Lambda(t)}} \exp \left( -\frac{(D - \mu \Lambda(t))^2}{2\sigma^2 \Lambda(t)} \right), \]  

where \( D \) represents the failure threshold of the products, namely when the cumulative amount of deterioration reaches \( D \), the product is defined to be failed.
From practical experience, when the deterioration rate of the product increases, the diffusion rate will increase correspondingly. However, the standard Wiener deterioration model fails to describe the correlation. Note of this problem, the standard Wiener deterioration model is improved by some researchers in [7] to fit the model with the famous GaAs laser deterioration data, in [8] to model the gyroscopic drift data, in [9] to investigate the mis-specification effect when predicting product's MTTF, in [10] to estimate the RUL based on the observed deterioration data, in [11] to estimate the RL of products, in [12] to evaluate the real-time reliability of products, in [13] to predict the RUL of an unit when the condition is time-vary, in [14-18] and so on. However, the improved Wiener deterioration models mentioned above failed to describe the direct positive correlation between the drift parameter and the diffusion parameter.

The standard Wiener deterioration model and some of its improved forms have been widely used in accelerated deterioration test (ADT). In the ADT, it is important to make sure that the deterioration mechanism remains unchanged between different stress levels, and this property is called deterioration mechanism equivalence. Currently, researchers test the mechanism equivalence mostly by physical or chemistry observation methods [19]. Other researchers test the mechanism equivalence by testing the parameters in the deterioration model such as [20-24]. But there are few studies on the deterioration mechanism equivalence based on the product’s actual deterioration data for now.

In this paper, we propose an improved Wiener deterioration model to describe and analyse the positive correlation of the drift parameter and diffusion parameter. After this, we compare the standard Wiener deterioration model with the improved Wiener deterioration model based on the deterioration mechanism equivalence hypothesis test and the Akaike information criterion (AIC) [25, 26]. Section 2 not only proposes the improved Wiener deterioration model, but also introduces the deterioration mechanism equivalence of products as well as the AIC. Section 3 gives a numerical example and the conclusion is shown in Section 4.

2. An improved Wiener deterioration model

2.1. Notations

\( y(t) \) The deterioration path of the product at time \( t \)

\( \Lambda(t) \) A monotonic increasing function representing a general time scale

\( B(\cdot) \) The standard Brownian motion

\( N(\cdot) \) The normal distribution

\( n \) Number of test units

\( T_p \) The first-passage-time of the product

\( s_i \) The \( i^{th} \) stress level

\( F(t) \) The cumulative distribution function of \( T_p \)

\( D \) The failure threshold level

\( R \) Reliability of the product

\( t_e \) Reliable lifetimes of the product

\( f(t) \) The probability density function of \( T_p \)

\( K \) Accelerated factor

\( Me \) Necessary condition of deterioration mechanism equivalence

\( T_i \) Temperature under the \( i^{th} \) stress level
An improved Wiener deterioration model

To reflect the correlation that the diffusion parameter $\sigma$ will get larger while the drift parameter $\mu$ gets larger. We simply let $\sigma = \mu^m$, then the improved Wiener deterioration model is derived, i.e.

$$y(t) = \mu \Lambda(t) + \mu^m B(\Lambda(t)),$$

where the drift parameter $\mu$ represents the rate of deterioration, and the diffusion parameter is represented as $\mu^m$ to show the correlation of the deterioration rate and the deterioration variation. For the consideration of calculation, suppose the two parameters $\mu$ and $m$ in this deterioration model are constants.

For the improved Wiener deterioration model, the PDF of the first-passage-time $T_D$ is

$$f(t) = \frac{D}{\sqrt{2\pi t^2 \Lambda'(t)}} \exp \left( -\frac{(D - \mu \Lambda(t))^2}{2\mu^m \Lambda(t)} \right).$$

Equivalence of accelerated deterioration mechanism

In ADTs, the same form of deterioration model can be used to model the deterioration paths under different stress levels only when the deterioration mechanism remains unchanged. According to the acceleration factor invariance principle [19], when the accelerated deterioration mechanism remains unchanged under different stress levels, the acceleration factor is determined only by the values of stress levels and has no relation with other parameters. Therefore, the acceleration factor $K_{i,i'}$ remains constant between two constant stress levels $s_i$ and $s_{i'}$ when the deterioration mechanism remains unchanged. Therefore, we have

$$f_i(t_i) = \frac{df_i(K_{i,i'} \cdot t_i)}{df_i(K_{i,i'} \cdot t_i)} = K_{i,i'} \cdot f_{i'}(K_{i,i'} \cdot t_i),$$

where $f_i(t_i)$ and $f_{i'}(t_{i'})$ represent the PDFs of $T_D$ under the $i$th and the $i'$th stress level, respectively.

Given equation (5), for any deterioration model, the mechanism equivalence parameter can be derived. Therefore, we can convert the test of deterioration mechanism equivalence to the test of deterioration mechanism equivalence parameter.

For the standard Wiener deterioration model, according to equation (2) and equation (5), we have

$$K_{i,i'} = \frac{f_i(t_i)}{f_{i'}(K_{i,i'} \cdot t_i)} = \frac{\sqrt{\sigma_i^2 \Lambda'(t_i)}}{\sqrt{\sigma_{i'}^2 \Lambda'(t_i)}} \exp \left( \frac{(D - \mu_i \Lambda(t_i))^2}{2\sigma_i^2 \Lambda(t_i)} - \frac{(D - \mu_{i'} \Lambda(t_i))^2}{2\sigma_{i'}^2 \Lambda(t_i)} \right).$$

Assume that the deterioration process is linear, i.e. $\Lambda(t) = t$, then $K_{i,i'}$ can be expressed as

$$K_{i,i'} = \frac{\sigma_i \sqrt{K_{i,i'}}}{\sigma_{i'}} \exp \left( \frac{t_i}{2} \left( \frac{K_{i,i'} \mu_i^2}{\sigma_i^2} - \frac{\mu_i^2}{\sigma_i^2} \right) + \frac{D^2}{2t_i} \left( \frac{1}{\sigma_i^2 K_{i,i'}} - \frac{1}{\sigma_{i'}^2} \right) + D \left( \frac{\mu_i}{\sigma_i^2} - \frac{\mu_{i'}}{\sigma_{i'}^2} \right) \right).$$

According to the acceleration factor invariance principle, $K_{i,i'}$ is an invariant factor, so monomials including $t_i$ in equation (7) should have zero coefficients. Therefore, we have

$$\frac{\mu_i}{\sigma_i^2} = \frac{\mu_{i'}}{\sigma_{i'}^2},$$

and

$$K_{i,i'} = \frac{\mu_i}{\mu_{i'}}.$$

When the deterioration process is nonlinear, we can adjust the deterioration data by using a time-scale transformation. For the improved Wiener deterioration model, necessary conditions are derived in the same way. In order to facilitate the mechanism equivalence hypothesis test in the following
section, we denote the necessary conditions for different deterioration models by a unified form, i.e. \( M_{e} = M_{e} \). For example, we use \( M_{e} = M_{e} \) to represent \( \mu_{i} / \sigma_{i}^{2} = \mu_{r} / \sigma_{r}^{2} \) for the standard Wiener deterioration model, where \( M_{e} \) represents \( \mu_{i} / \sigma_{i}^{2} \). Results are shown in table 1.

| Stochastic deterioration model                  | Necessary conditions         |
|-----------------------------------------------|-----------------------------|
| The standard Wiener deterioration model       | \( \mu_{i} / \sigma_{i}^{2} = \mu_{r} / \sigma_{r}^{2} \) |
| The improved Wiener deterioration model       | \( \mu_{i}^{2m_{i}-1} = \mu_{r}^{2m_{r}-1} \) |
| Generalized model                             | \( M_{e} = M_{e} \)         |

2.4. Equivalence hypothesis test and Akaike Information Criterion

When an actual set of deterioration data is available, the parameters of the Wiener deterioration models can be derived through maximum likelihood estimation. Then, the estimates of the mechanism equivalence necessary conditions can also be obtained. To test whether the deterioration data satisfy the mechanism equivalence necessary conditions in table 1, the T-statistic test is carried out on both of the standard Wiener deterioration model and the improved Wiener deterioration model, based on which we can judge whether the estimates of mechanism equivalence necessary conditions derived under different stress levels are statistically identical or not.

Denote the mean of the estimates of \( M_{e} \) as \( \hat{\mu}(M_{e}) \), \( i = 1,2,\ldots,k \), then the null hypothesis of the T-statistic test, which describes the equality of the means, is given as:

\[
H_{0} : \mu(M_{e}) = \mu(M_{e}),
\]

where \( i = 1,2,\ldots,k \), \( r = 1,2,\ldots,k \), \( i \neq r \). And the alternative hypothesis is \( H_{1} : \mu(M_{e}) \neq \mu(M_{e}) \) for some \( i \neq r \).

The T-statistic is given by

\[
T = \frac{\hat{\mu}(M_{e}) - \hat{\mu}(M_{e})}{S_{n}^{1/2} \sqrt{1/n_{i} + 1/n_{r}}}, (r \neq i),
\]

where \( r \in \{1,2,\ldots,k\} \), \( n_{i} \) denotes the sample size under the \( i^{th} \) stress level, and

\[
S_{n} = \sqrt{\frac{1}{n_{i} + n_{r} - 2} \left( \sum_{j=1}^{n_{i}} (\hat{M}_{e} - \hat{\mu}(M_{e}))^{2} + \sum_{j=1}^{n_{r}} (\hat{M}_{e} - \hat{\mu}(M_{e}))^{2} \right)}. \tag{12}
\]

Under the null hypothesis \( H_{0} \), the T-statistic follows \( t \) distribution which has \( n_{i} + n_{r} - 2 \) degrees of freedom. Given the significance level \( \alpha \), we can get the rejection region as

\[
W = \{ T \geq t_{\alpha/2}(n_{i} + n_{r} - 2) \} \cup \{ T \leq -t_{\alpha/2}(n_{i} + n_{r} - 2) \}, \tag{13}
\]

If the null hypothesis is accepted, we believe that the deterioration model satisfies the mechanism equivalence necessary conditions under this set of deterioration data, and not vice versa.

Further, to test whether the proposed model is in agreement with the true deterioration path of the product, the AIC is considered. Denote product’s deterioration dataset as \( D_{a} \), then the AIC for a given model \( M \) with \( K \) parameters is given as:

\[
AIC = -2 \ln L(D_{a} \mid M) + 2K, \tag{14}
\]

where \( L(D_{a} \mid M) \) represents the log-likelihood function of the model \( M \). Based on this criterion, the model with the smallest AIC will be chosen.
3. Numerical example

3.1. Test description
In a constant-stress ADT of an electrical connector, temperature is taken as the accelerated stress. There are three stress levels, i.e. $T_1 = 65^\circ C$, $T_2 = 85^\circ C$, and $T_3 = 100^\circ C$. Under each stress level, the deterioration paths of 6 test samples are measured and recorded. The failure threshold is defined as 30%, i.e. $D = 30\%$. The specific deterioration data is recorded in [27] and the deterioration paths are shown in figure 1.

Figure 1. Constant-stress ADT deterioration data under 3 stress levels of an electrical connector.

3.2. Deterioration modelling and parameter estimation
According to figure 1, it is easy to find that the deterioration paths of the electrical connector are nonlinear. Thus, we convert the measure time $t$ to $t^{0.492}$ because under this scale the degradation paths are approximately linear. The power index used for transformation is determined by statistical fitting.

| Sample | The standard Wiener deterioration model | The improved Wiener deterioration model |
|--------|----------------------------------------|----------------------------------------|
| $T_1 = 65^\circ C$  | $T_2 = 85^\circ C$  | $T_3 = 100^\circ C$  |
| $\hat{\mu}_ij / \hat{\sigma}^2_{ij}$ | $\hat{\mu}_ij / \hat{\sigma}^2_{ij}$ | $\hat{\mu}_ij / \hat{\sigma}^2_{ij}$ |
| $\hat{\mu}_ij / \hat{\sigma}^2_{ij}$ | $\hat{\mu}_ij / \hat{\sigma}^2_{ij}$ | $\hat{\mu}_ij / \hat{\sigma}^2_{ij}$ |
| $\mu_{ij}^{2m_{ij}-1}$ | $\mu_{ij}^{2m_{ij}-1}$ | $\mu_{ij}^{2m_{ij}-1}$ |
| $\mu_{ij}^{2m_{ij}-1}$ | $\mu_{ij}^{2m_{ij}-1}$ | $\mu_{ij}^{2m_{ij}-1}$ |

Firstly, we use the standard Wiener deterioration model and the improved Wiener deterioration model to model the deterioration path of each sample under each stress level, respectively. The log-likelihood functions of these two models are given as equation (15) and equation (16), i.e.

$$
\ln L_{standard}^j = \sum_{k=1}^{10} \ln \left( \frac{1}{\sqrt{2\pi \sigma_{ij}^2 \Delta \Lambda(t_{ijk})}} \right) - \frac{(\Delta W_{ijk} - \mu_{ij} \Delta \Lambda(t_{ijk}))^2}{2\sigma_{ij}^2 \Delta \Lambda(t_{ijk})}, \tag{15}
$$
\[
\ln L_{\text{Improved}} = \sum_{h=1}^{H} \ln \left( \frac{1}{\sqrt{2\pi \mu^2 \Delta \Lambda(t_{i,j})}} \right) - \frac{(\Delta y_{i,j} - \mu \Delta \Lambda(t_{i,j}))^2}{2\mu^2 \Delta \Lambda(t_{i,j})},
\]

where \(\Delta y_{i,j} = y_{i,j} - y_{i,j-1}, \Delta \Lambda(t_{i,j}) = \Lambda(t_{i,j}) - \Lambda(t_{i,j-1}), y_{i,j}\) denotes the observed deterioration value of the \(j\)-th unit under the \(i\)-th stress level measured at \(t_{i,j}\), \(i = 1, 2, 3\) and \(j = 1, 2, 6\).

By maximizing the values of the log-likelihood functions above, we can obtain the estimates of the model parameters as well as the mechanism equivalence parameters, as shown in Table 2.

3.3. Equivalence test and Akaike Information Criterion

From Table 2, it is not easy to observe which Wiener deterioration model is better in describing the deterioration path of products. Therefore, the mechanism equivalence hypothesis test and AIC of the two models are derived in the following.

According to the results presented in Table 2, the T-statistic is calculated and results show that both the standard Wiener deterioration model and the improved Wiener deterioration model are accepted, which means that there is no competition between the two deterioration models from the perspective of deterioration mechanism equivalence.

Therefore, the AIC of the two models is computed to compare the models further. For the standard Wiener deterioration model, we derive

\[
\text{AIC}_{\text{Standard}} = 8.5268 \times 10^8,
\]

and for the improved Wiener deterioration model, we derive

\[
\text{AIC}_{\text{Improved}} = 1.7197 \times 10^8.
\]

According to the results, it is obvious that the standard Wiener deterioration model has a larger AIC than the improved Wiener deterioration model. Therefore, we believe that the improved Wiener deterioration model is more appropriate to describe the deterioration path of the products than the standard Wiener deterioration model.

4. Conclusions

In this paper, an improved Wiener deterioration model is proposed on the basis of the standard Wiener process model by considering the correlation between the drift parameter and the diffusion parameter. After this, according to the actual electrical connector deterioration data, we compare the standard Wiener deterioration model with the improved Wiener deterioration model by using the deterioration mechanism equivalence hypothesis test and the Akaike Information Criterion. From the results of the deterioration mechanism equivalence hypothesis test, it is obvious to know that the improved Wiener deterioration model satisfies the mechanism equivalence necessary conditions just as the standard Wiener deterioration model does. And from the results of the AIC, we know that the improved Wiener deterioration model performs better when used to describe the deterioration path of the electrical connector.

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