Fourier Power Function Shapelets (FPFS) Shear Estimator: Performance on Image Simulations

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ABSTRACT

We reinterpret the shear estimator proposed in Zhang & Komatsu (2011) within the framework of Shapelets (Refregier 2003) and propose the Fourier Power Function Shapelets (FPFS) method to measure weak lensing distortions from background galaxies. Four shapelet modes are calculated from the power function of every galaxy’s Fourier transform after deconvolving the Point Spread Function (PSF) in Fourier space. From these shapelet modes, dimensionless FPFS ellipticity and responsivity are constructed. We derive the transformation formula of the ellipticity under the influence of shear for every single galaxy. Using the transformation formula, shear can be inferred from a large ensemble of galaxies with the premise that intrinsic galaxies are randomly oriented. The FPFS method is developed without any assumption on galaxy morphology, nor any approximation for the PSF correction. We test the FPFS method with the GREAT3-HSC simulations, which are simulations to test shear calibration with realistic galaxy morphologies as well as HSC-like PSFs and noise properties. The main results are listed as follows. (i) For samples which only contain isolated galaxies, the amplitude of shear multiplicative bias is below 0.5% (2.5σ). (ii) For realistic samples which also contain blended galaxies, the blended galaxies are deblended by the first generation HSC deblender before shear measurement and the shear multiplicative bias of −5.7% (14σ) is found. The multiplicative bias originating from blending is calibrated by the GREAT3-HSC simulations. Finally, we test the consistency and stability of this calibration.

Key words: cosmology: observations – gravitational lensing: weak

1 INTRODUCTION

Light from background galaxies is deflected by the inhomogeneous foreground density distribution along their line-of-sight and, as a consequence, the images of these background galaxies are slightly but coherently distorted. Such distortions are the result of weak lensing. Weak lensing imprints information of foreground density distribution along the line-of-sight to the background galaxy images (Dodelson 2017). There are two types of weak lensing distortions, namely magnification and shear. Magnification changes the size and flux of galaxy images. On the other hand, shear anisotropically stretches galaxy images. The magnification effect is difficult to observe since the size and flux of intrinsic galaxies are unknown (Zhang & Pen 2005). In contrast, with the premise that galaxies intrinsically have isotropic orientations, shear can be inferred by measuring small anisotropies from a large ensemble of observed galaxies. Therefore weak lensing offers a direct probe into the large scale structure of the Universe (see Kilbinger 2015; Mandelbaum 2017, for recent reviews), making it the main focus of several ongoing and upcoming surveys, including the the Kilo-Degree Survey1 (de Jong et al. 2013), the Subaru Hyper Suprime-Cam (HSC) survey2 (Aihara et al. 2018), the Dark Energy Survey3 (Dark Energy Survey Collaboration et al. 2016), the Large Synoptic Survey Telescope4 (LSST Science Collaboration et al. 2009), the Euclid satellite mission5 (Laureijs

1 http://kids.strw.leidenuniv.nl/index.php
2 http://hsc.mtk.nao.ac.jp/ssp/
3 http://www.darkenergysurvey.org/
4 http://www.lsst.org/
5 http://sci.esa.int/euclid/

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et al. 2011), and the WFIRST satellite mission\(^6\) (Spergel et al. 2015).

Accurate shear measurement from galaxy images is challenging for the following reasons. Firstly, galaxy images are smeared by Point Spread Functions (PSFs) as a result of diffraction by telescopes and the atmosphere, which is generally known as PSF bias. Secondly, galaxy images are contaminated by background noise and shot noise originating from the particle nature of light, which is generally known as noise bias. Thirdly, the complexity of galaxy morphology makes it difficult to fit galaxy shapes within a parametric model, which is generally known as model bias. Fourthly, galaxies are heavily blended for deep surveys such as the HSC survey (Bosch et al. 2018), which is generally known as blending bias. Finally, the selection bias emerges if the selection procedure does not align with the premise that intrinsic galaxies are isotropically oriented, which is generally known as selection bias.

Traditionally, several methods have been proposed to estimate shear from large ensembles of smeared, noisy galaxy images. These methods can be classified into two categories. The first category includes moments methods which measure moments weighted by a Gaussian function of galaxy images and use them to construct shear estimators. Moments of PSF images are used to correct the PSF effect (e.g., Kaiser et al. 1995; Bernstein & Jarvis 2002; Hirata & Seljak 2003). The second category includes fitting methods which convolve parametric Sersic models with PSF images to find the parameters which best fit the observed galaxies. Shear is subsequently determined from these parameters (e.g., Miller et al. 2007; Zuntz et al. 2013). Unfortunately, these traditional methods suffer from either the model bias (Bernstein 2010) originating from assumptions on galaxy morphology, or the noise bias (e.g., Refregier et al. 2012; Okura & Futamase 2018) due to non-linearities in the shear estimators.

In contrast, Zhang & Komatsu (2011, ZK11) proposed to use the Fourier power function for shear measurement. This method directly deconvolves PSF Fourier power function from galaxy Fourier power function in Fourier space. Then moments weighted by isotropic Gaussian kernel\(^7\) are measured from the deconvolved Fourier power function and, using these moments, final shear estimators are constructed. Benefitting from the direct deconvolution, shear estimators are constructed without any assumption on galaxy morphology, thus it is free from the model bias. Moreover, since the shear estimator is linearly dependent on the moments, the noise bias can be statistically removed by subtracting the Fourier power function of noise from the Fourier power function of observed galaxies (Zhang et al. 2015). We take these advantages of ZK11 and reinterpret the moments defined in ZK11 as combinations of shapelet modes. Shapelets refer to a group of orthogonal functions used to describe small distortions on astronomical images (Refregier 2003). Based on this reinterpretation, we measure four shapelet modes from the deconvolved Fourier power function and use them to construct FPFS ellipticity and responsivity. The noise bias can be reduced since both the ellipticity and responsivity have semi-linear forms. We derive the transformation formula of FPFS ellipticity under the influence of shear, with which we can infer shear signal from a large ensemble of galaxies. Furthermore, with the definition of the FPFS flux, we provide a method to reduce the selection bias.

Recently, several new methods, which are free from the noise bias and the model bias, have also been proposed. Zhang et al. (2017) shows the latest development of ZK11 which is generally known as FOURIERQUAD (FQ). The FQ method determines two components of shear by resymmetrizing the PDF of the spin-2 moments which are measured from the power function of galaxy’s Fourier transform. BFD method (Bernstein & Armstrong 2014; Bernstein et al. 2016) uses Bayesian formalism to measure shear from the mean of the Bayesian posterior and it requires the noise-less distribution of galaxy population over parameter space as a prior. The BFD method avoids assignment of ellipticity to individual galaxy. Metacalibration (Huff & Mandelbaum 2017; Sheldon & Huff 2017) proposes to find the shear responsivity for any ellipticity by adding artificial shear to the galaxy. Shear can be inferred with the ellipticities and responsivities of a large ensemble of galaxies. Both the BFD method and the Metacalibration provide solutions to the selection bias.

Testing shear measurement methods with galaxy image simulations is a long-standing tradition within the weak lensing community (e.g., Brüll et al. 2009; Mandelbaum et al. 2014). Galaxy image simulations which match the real observational conditions can be used to calibrate shear measurements (Mandelbaum et al. 2012). The GREAT3-HSC simulations (Mandelbaum et al. 2017) simulated a large ensemble of realistic galaxy images, using the data observed by the Hubble Space Telescope (HST) for the COSMOS survey (Koekemoer et al. 2007), as well as HSC-like PSFs and noise models. The simulations contain modelled galaxies and realistic galaxies, and include isolated galaxies and blended galaxies. We test our newly developed method using the GREAT3-HSC simulations. The result shows that our method is hardly affected by systematic biases for isolated galaxies. We combine our method with the first generation HSC deblender (Bosch et al. 2018) to measure shear from blended galaxies. Multiplicative bias of about \(-5.7\%\) is subsequently found for the blended cases. This bias is calibrated using the GREAT3-HSC simulations and finally the consistency of this calibration is tested.

This paper is organized as follows. Section 2 explores the analytical derivation of the FPFS formalism. Section 3 tests and calibrates the newly developed FPFS method using the GREAT3-HSC simulations. Section 4 checks the consistency and stability of the calibrated estimator. Section 5 provides a summary and outlook.

2 METHOD

This section is organized as follows. Section 2.1 explains why the shear estimator is constructed on the Fourier power function of galaxy images. Section 2.2 derives the shear estimator without consideration of shot noise. Section 2.3 provides a solution to the noise bias. Section 2.4 discusses the selection bias.
We denote the second moments matrix by
\[ \mathbf{S}_{nm} = \mathbf{M}_{nm} \]
for the galaxy image from the intrinsic plane \( \mathbf{x}' \) to the lensed plane \( \mathbf{x} \),
\[ \mathbf{S} \mathbf{S}' = \mathbf{S}_I \mathbf{S}_I \]
where \( \mathbf{S}_I \) and \( \mathbf{S} \) are the Jacobian matrix which is defined as
\[ \mathbf{S}_I = (1 + \kappa) \begin{pmatrix} 1 + g_1 & g_2 \\ g_2 & 1 - g_1 \end{pmatrix}. \]

The two components of the reduced shear \((g_1, g_2)\) cause the anisotropic stretching of the galaxy image and the convergence \( \kappa \) describes a change in galaxy size and brightness. The PSF effect can be expressed as a convolution between the lensed galaxy \( f \) and the PSF \( g \):
\[ f_0(\mathbf{x}_0) = \int g(\mathbf{x}_0 - \mathbf{x}) f(\mathbf{x}) d^2 x. \]

The Fourier power function of the galaxy is defined as
\[ f_0(\mathbf{\bar{k}}) = \int f_0(\mathbf{x}_0) e^{-i \mathbf{k} \cdot \mathbf{x}_0} d^2 \mathbf{x}_0, \]
\[ \mathbf{F}_0(\mathbf{\bar{k}}) = |f_0(\mathbf{\bar{k}})|^2, \]
where \( f_0(\mathbf{\bar{k}}) \) and \( \mathbf{F}_0(\mathbf{\bar{k}}) \) are the Fourier transform and the Fourier power function of the galaxy image, respectively. In order to ensure the convergence of Fourier transform in real observations, it is necessary to define a boundary for each galaxy and mask the region outside the boundary with 0. Li & Zhang (2016, LZ17) proposed to use a top-hat aperture around the galaxy center to define its boundary. The top-hat filter is defined as
\[ T(\mathbf{x}) = \begin{cases} 1, & |\mathbf{x} - \mathbf{x}_c| < r_{cut} \\ 0, & |\mathbf{x} - \mathbf{x}_c| \geq r_{cut} \end{cases}, \]
where \( \mathbf{x}_c \) is the galaxy centroid and \( r_{cut} \) is the aperture radius. The ratio between the aperture radius \( r_{cut} \) and trace radius \( r_g \) of galaxy is termed the aperture ratio, which is defined as
\[ \alpha = \frac{r_{cut}}{r_g}, \]
where the trace radius is determined by the galaxy’s second moments matrix measured by the adaptive moments method (Hirata & Seljak 2003). We denote the second moments matrix as \( \mathbf{Q} \), then the trace radius is defined as
\[ r_g = \sqrt{\frac{\text{Tr}(\mathbf{Q})}{2}}. \]
To avoid steep cut on the galaxy’s light profile, the aperture ratio should not be too small. Neither should the aperture ratio be too big otherwise light from neighbouring objects...
will influence the observation. We will discuss the applicable range of $\alpha$ in Section 3.2.

One advantage of using Fourier power function is that the centroid of the Fourier power function is well defined since the Fourier power function is always symmetric around its zero point where $k = 0$ (Zhang & Komatsu 2011). The reader may be concerned about the off-centering of the top-hat aperture from the galaxy centroid, however, Section 3.3 demonstrates that our method is accurate for faint galaxies where off-centering is caused by noise. Moreover, as shown in Section 3.4, shear can be measured from blended ‘parent’ sources without deblending even though the centroid of the aperture is set to the centroid of each ‘parent’ footprint which can be different from the centroid of any ‘child’, if all the ‘child’ galaxies are on the same redshift plane. On the other hand, the PSF effect can be removed by dividing the PSF Fourier power function ($\hat{G}(\vec{k})$) from the observed galaxy Fourier power function

$$\hat{F}(\vec{k}) = \frac{\hat{F}_n(\vec{k})}{\hat{G}(\vec{k})},$$ (6)

The reader may be concerned about the instability of the PSF deconvolution in the presence of noise, however, we will ensure the stability of the deconvolution using the basis vectors of polar shapelet (Massey & Refregier 2005) in Section 2.2. The performance such deconvolution is demonstrated in Section 3.3.

2.2 Ellipticity and responsivity

The Fourier power function of the intrinsic galaxy is distorted by $-S$ due to influence of the weak lensing field. Thus it is possible to infer shear from the deconvolved Fourier power function of the galaxy. The deconvolved galaxy Fourier power function is projected onto the basis vectors of polar shapelets (Massey & Refregier 2005). The shapelet basis vectors are defined as

$$\chi_{nm}(r, \theta) = \left(\frac{(-1)^{n+|m|}}{\sqrt{|m|!}} \right) \frac{1}{\sqrt{\pi(n+|m|)!}} \frac{\partial^{|m|}}{\partial r^{|m|}} \frac{L_n^{|m|}(r)}{\sqrt{2}} e^{-r^2} e^{-im\theta},$$

where $L_n^{|m|}$ are the Laguerre Polynomials, $n$ is the radial number and $m$ is the spin number, $\sigma$ determines the scale of shapelet functions. We denote the ratio between $\sigma$ and the scale radius of PSF Fourier power function ($r_{pp}$) as

$$\beta = \frac{\sigma}{r_{pp}}.$$ (7)

$r_{pp}$ is defined in the same way as LZ17 and the definition is shown as follows. After recording the maximum value of the Fourier power function of the PSF, the area of pixels (where the value is greater than $e^{-0.5}$ of the recorded maximum value) is measured and denoted as $A$. Consequently, the scale radius of PSF Fourier power function ($r_{pp}$) is defined as

$$r_{pp} = \sqrt{\frac{A}{\pi}}.$$ (8)

The projection of the deconvolved galaxy converges only if $\beta < 1$. Section 3.2 demonstrates the performance of our method with different choices of $\alpha$ and $\beta$ using the modelled galaxy sample of the GREAT3-HSC simulations. The projection factors, generally known as shapelet modes, are denoted as $M_{nm}$, where

$$M_{nm} = \int \chi_{nm} F(r, \theta) dr d\theta.$$ (9)

$M_{nm}$ and $M_{nm}$ are used to denote the real and imaginary part of $M_{nm}$ when $m > 0$. Four shapelet modes are used to construct the FPFS ellipticity. The number histograms of these modes measured from the modelled galaxy sample of the GREAT3-HSC simulations are shown in Fig. 1. In order to quantify the spread of $M_{00}$, the value of $M_{00}$ at which its histogram drops below 1/8 of its maximum (on the decreasing side) is denoted as $\Delta$. The transformation formula of shapelet modes under the influence of shear was given by Massey & Refregier (2005) as

$$M_{22c} = M_{22c} - \sqrt{2} \bar{g}_1(M_{00} - M_{40})$$
$$\quad + \sqrt{2} \bar{g}_2 M_{44c} + \sqrt{2} \bar{g}_2 M_{44c},$$

$$M_{22a} = M_{22a} - \sqrt{2} \bar{g}_1(M_{00} - M_{40})$$
$$\quad - \sqrt{2} \bar{g}_2 M_{44c} + \sqrt{2} \bar{g}_2 M_{44c},$$

$$M_{40} = M_{40} + \sqrt{2} \bar{g}_1 (M_{22c} + g_2 M_{22a}),$$

$$M_{40} = M_{40} - \sqrt{2} \bar{g}_1 (M_{22c} + g_2 M_{22a})$$
$$\quad + 2 \sqrt{2} \bar{g}_1 (M_{62c} + g_2 M_{62a}),$$

where $\bar{M}_{nm}$ represent the intrinsic shapelet modes and $M_{nm}$ represent the sheared shapelet modes. We define the FPFS ellipticity as

$$e_1 = \frac{M_{22c}}{M_{00} + C}, \quad e_2 = \frac{M_{22a}}{M_{00} + C}. $$ (10)

The constant parameter $C$ is termed the weight adapter and the ratio between the weight adapter $C$ and $\Delta$ is denoted as $v$

$$v = \frac{C}{\Delta}. $$ (11)

With the definition of the responsivity

$$R_i = \frac{\sqrt{2}}{\sqrt{2}} \frac{M_{00} - M_{40}}{M_{00} + C} + \sqrt{2} e_i,$$ (12)

the transformation of the FPFS ellipticity under the influence of shear are given as follows

$$e_1 = \tilde{e}_1 - g_1 R_1 + \sqrt{3} \bar{g}_1 \frac{M_{44c}}{M_{00} + C} + \sqrt{3} \bar{g}_2 \frac{M_{44c}}{M_{00} + C},$$

$$e_2 = \tilde{e}_2 - g_2 R_2 - \sqrt{3} \bar{g}_2 \frac{M_{44c}}{M_{00} + C} + \sqrt{3} \bar{g}_1 \frac{M_{44c}}{M_{00} + C}. $$ (13)

We show the detailed derivation of eq. (13) for noiseless galaxies in Appendix A. Fig. 2 shows the number histograms of $e_{1,2}$ and $R_{1,2}$ for different choices of $v$. Given an unbiased ensemble of galaxies distorted by a constant shear, it is possible to infer the shear signal by averaging their ellipticities. Since the expectation value of the spin-2 and spin-4 quantities in eq. (13) are 0, consequently, we have

$$g_i = -\langle e_i \rangle / \langle R_i \rangle.$$ (14)
In the process of averaging, the responsivity ($R_{1,2}$) act as weights on each galaxy for the two shear components. It is possible to adjust the weight between galaxies with different $S/N$ by changing $\nu$. To demonstrate, we separate the modelled galaxy sample in the GREAT3-HSC simulations into different bins according to their intrinsic $S/N$ and plot the averaged $R_1$ in each bin with different choices of $\nu$ in Fig. 3. The case where weight ratio is proportional to $S/N$ is also plotted in Fig. 3 as a reference. In summary, more weight is added to the brighter galaxies when $C$ increases. Moreover, by increasing $\nu$, the ellipticity and responsivity have semi-linear forms and the noise bias can be reduced, which will be discussed in the next section.

2.3 The noise bias

The shot noise from astronomical sources and the background light contributes to the noise in the images. Although the amplitude of shot noise correlates with the surface brightness distribution of galaxies, its phase does not. Under the further assumption that the background light does not correlate with galaxies, the total noise remain uncorrelated with the surface brightness distribution of galaxies, even after the coadding process. Based on this premise, the averaged contamination of noise can be removed by subtracting the Fourier power function of noise from the galaxy Fourier power function (Zhang et al. 2015). We propose to reconstruct the Fourier power function of noise using the autocorrelation of noise measured from undetected pixels. The details for the reconstruction of noise Fourier power function are shown in Appendix B. After the subtraction of noise power function, a zero-mean residual is left on the galaxy Fourier power function. The shapelet modes of the residual are denoted as $N_{nm}$. The expectation value of $N_{nm}$ should be 0 and they are not correlated with the Fourier modes of galaxy Fourier power function. When $\nu = 0$, the residual on the galaxy Fourier power function unfortunately causes bias to the shear estimator due to the nonlinear form of the FPFS ellipticity and responsivity. Taking $\langle e_1 \rangle$ as an example, the expectation value of $e_1$ changes to

$$\langle e_1 \rangle = \frac{M_{22c} + N_{22c}}{M_{00} + N_{00}} \neq \frac{M_{22c}}{M_{00}}.\tag{15}$$

which does not equate the noiseless ellipticity. In order to reduce the noise bias originating from the nonlinearity of the ellipticity, we increase $\nu$ to make $M_{00} + C \gg N_{00}$. Subsequently, $e_1$ has a semi-linear form, thus the expectation value of $e_1$ changes to

$$\langle e_1 \rangle = \frac{M_{22c}}{M_{00} + C (1 + O(\epsilon^2))}.\tag{15}$$

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Figure 2. The number histograms of the FPFS ellipticity $e_{1,2}$ and responsivity $R_{1,2}$ for different values of $\nu$. Different colors represent different values of $\nu$, where $\nu$ is set to 2, 4, 8.

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8 The GREAT3-HSC simulations provide the $S/N$ of galaxies before being lensed.
In this section, we discuss the selection bias caused by the improper selection of the galaxy ensemble. In order to understand the selection process, one needs to define a group of selection functions and their corresponding selection thresholds. Moreover, one should make sure that it is possible to measure the selection functions from observed galaxy images. A galaxy is counted as a member of the ensemble if all of its selection functions fall within the corresponding thresholds. Moreover, the selection functions should be spin-0 quantities on the intrinsic plane to ensure that galaxies are isotropically selected. Since shear measurement is based on the premise that the intrinsic galaxies, within the ensemble, have isotropic orientations statistically. To be more specific, we define the FPFS flux as

\[ s = \frac{M_{90}}{M_{90} + C} \]

and use the FPFS flux as the selection function. The left panel of Fig. 4 shows the number histogram of \( s \) using the modelled galaxy sample in the GREAT3-HSC simulations. In addition, the detection of galaxies is also a selection process which could cause bias to shear measurement so we show the number histogram of the undetected galaxies on the right panel of Fig. 4. It suggests that most of the undetected galaxies are within the range \( s < 0.1 \). The relationship between the observed FPFS flux \( s \) and the intrinsic FPFS flux \( \bar{s} \) is

\[ \bar{s} = s + \sqrt{2}\epsilon_1 e_1 (1 - s) + \sqrt{2}\epsilon_2 e_2 (1 - s). \]

\( \bar{s} \) is spin-0 on the intrinsic plane, whereas \( s \) is spin-0 on the lensed plane. Therefore, the selection, which uses \( s \) as the selection function, is not an isotropic selection on the intrinsic plane and it does not align with the premise that the intrinsic galaxies, within the ensemble, have isotropic orientations statistically. We provide an iterative method to reduce the selection bias as follows.

(i) Estimate shear with selection \( L < s < U \) and the estimated shear is denoted as \( \hat{\gamma}_{1,2} \).

(ii) Inversely transform the observed selection function \( s \) into \( s_p \) which is isotropic in the intrinsic plane with \( \hat{\gamma}_{1,2} \) according to eq. (17).

(iii) Re-estimate shear with selection \( L < s_R < U \) and update the outcome of the shear measurement to \( \hat{\gamma}_{1,2} \).

This iterative method works only if the observational conditions meet the requirements listed as follows. (i) The scatter of shear is small. (ii) The number of galaxies in this ensemble is sufficiently large enough to cancel the measurement error and the shape noise. The GREAT3-HSC simulations meet these requirements since simulations provide \( 10^4 \) galaxies distorted by a constant shear in each subfield.

The performance of this iterative method is demonstrated in Section 3.3. However, we caution that the application of this iterative method to real observations is not as straightforward as the application of it to the GREAT3-HSC simulations.

3 TEST AND CALIBRATION

We start off by outlining the test and calibration of shear measurement methods with image simulations. With the in-
tent to test shear measurement methods, our image simulations provide large ensembles of galaxies distorted by a group of input shear ($g_{1,2}$) with known values. Furthermore, the galaxy properties, seeing condition, shot noise, neighbouring objects in the simulations should match those of the real observations. Then, we measure shear from the simulated images by a specific method, where the estimated shear is denoted as $\hat{g}_{1,2}$. Based on the premise that the amplitude of shear is only a few percent, the estimated shear ($\hat{g}_{1,2}$) can be expressed as the first order Taylor expansion of the input shear ($g_{1,2}$)

$$\hat{g}_{1,2} = (1 + m_{1,2})g_{1,2} + c_{1,2}. \tag{18}$$

With the premise that all the even order terms of the input shear are not correlated with the estimated shear, on average, these even order terms must be zero and we only ignore third and higher odd orders in eq. (18). $m_{1,2}$ and $c_{1,2}$ are termed multiplicative bias and additive bias, respectively. $m$ and $c$ are used to denote the means of the biases for two shear components. Generally, the values of the multiplicative biases depend on several properties, including galaxy shapes, galaxy luminosities, seeing conditions, noise properties and neighbouring objects. On the other hand, nonzero multiplicative bias can be generated by some anisotropy in the PSF, anisotropic selection effects and the masks in observations. Thus the biases should be modelled as multidimensional functions of these properties where the form of the functions are dependent on the shear measurement method. It is promising if we can find a shear measurement method and the biases are consistent with 0 for this method. Another solution is to model the biases with image simulation and calibrate the shear measurement method on a single galaxy level.

This section is organized as follows. Section 3.1 describes the GREAT3-HSC simulations and the FPFS algorithms which is implemented into the HSC pipeline (Bosch et al. 2018). Section 3.2 explores the applicable range of the free parameters ($\alpha$, $\beta$ and $\nu$). Section 3.3 and Section 3.4 test the performance of our method on isolated galaxies and blended galaxies, respectively. In addition, the performance of the FPFS method with different selection is shown in Section 3.3.

3.1 Simulations and pipeline

Galaxy images of the GREAT3-HSC simulations (Mandelbaum et al. 2017) are generated by GalSim, which is an open-source image simulation package (Rowe et al. 2015), using images from the COSMOS HST survey. The simulations provide four samples of galaxy images with different properties. The structure of simulations is described as follows. Each sample is divided into 800 subfields, each subfield contains $10^4$ postage stamps, and each stamp contains at least one galaxy. These $10^4$ postage stamps are divided into $5 \times 10^3$ orthogonal pairs and the direction of intrinsic galaxies in each pair are 90 degree separated from each other. Galaxies in each subfield are distorted by the same shear and subsequently smeared by the same PSF. Shear and PSF vary between different subfields. These PSF models are constructed by the HSC pipeline for coadd exposures using the star images of the HSC survey (Bosch et al. 2018). Moreover, the autocorrelation function measured from undetected pixels of HSC coadd exposures (see Mandelbaum et al. 2017, fig.1) is used to generate Gaussian noise on exposure of each subfield.

Furthermore, the galaxy properties for different samples are described as follows. Sample 1 and sample 2 correspond to isolated galaxies so each stamp only contains one galaxy. Galaxy images in sample 1 are realistic galaxy images, while those in sample 2 are blended galaxies. Sample 3 is generated by blending two or more realistic galaxy images. Sample 4 is generated by blending an idealized galaxy image with an anisotropic star image. Section 3.3 and Section 3.4 explore the performance of shear measurement methods on these simulated galaxy images in Section 3.3 and Section 3.4, respectively.
Figure 5. This figure shows the performance of the FPFS method with different setups of $\alpha$ and $\beta$. The x-axis is the aperture ratio $\alpha$ and the y-axis is the multiplicative bias. Lines with different color correspond to $\beta = 0.50, 0.60, 0.85$.

Then these Sersic models are convolved with HSC PSFs. Multiple galaxies can be found on each postage stamp of sample 3 and sample 4. Galaxies in sample 3 are similar to those in sample 1 but neither deblender nor noise replacer has been ran before HST galaxies are inserted into the simulated stamps. Therefore, neighbouring objects from the COSMOS HST Survey are included in each stamp of sample 3. Since the stamps of galaxies in the COSMOS HST survey are smaller than the stamps of the simulations, there are no objects near the edge of the simulated stamps. Galaxies in sample 4 are similar to those in sample 3 but the density of footprints and the selection criteria of galaxies are matched to the HSC observational conditions. Moreover, sample 4 does not use the COSMOS HST stamps, so the images, which extend to the edges of the simulated stamps, are artificially truncated around the edges. The simulations apply a $25.2$ magnitude cut to the galaxy samples. The summary of these four samples can be found in Mandelbaum et al. (2017, Table 1).

With the aim to process the data from the GREAT3-HSC simulations, our FPFS method is implemented into the HSC pipeline (Bosch et al. 2018), which is an open-source software developed to process data observed by the ongoing HSC survey and the future LSST survey. HSC pipeline performs a maximum likelihood analysis to detect pixels with a $5\sigma$ threshold from simulated exposures. Every detected peak is defined as a source and the connected nearby region above the threshold is identified as footprint of the source. For a stamp which does not contain any detected footprint in the central region ($10\text{pix} \times 10\text{pix}$ around the center of the stamp), we assign a peak to the center combined with a $10\text{pix} \times 10\text{pix}$ footprint around the peak and label the galaxy as undetected. We use the HSC pipeline to subtract back-

Figure 6. The applicable range of $\nu$ which reduce the noise bias below 1%. This figure shows the performance of the FPFS method with different $\nu$. The x-axis is the weight ratio $\nu$ and the y-axis is the absolute value of the multiplicative bias.

Figure 7. The galaxy images and masks detected from sample 2 of the GREAT3-HSC simulations. Bright pixels on the masks show the detected footprints and the gray pixels represent the area within the aperture radius.

ages from the COSMOS HST survey. Deblender has been ran to isolate galaxies from the ‘parent’ footprints detected in the COSMOS HST Survey. Then the neighbouring galaxies are masked out and replaced with noise. Subsequently, the HST PSFs are deconvolved from the galaxy images and the HSC PSFs are reconvolved to them. Galaxies in sample 2 are parametric Sersic models which are fitted to the deconvolved galaxies observed from the COSMOS HST survey.
ground from exposures in sample 3 and sample 4 to reduce light remnant from neighbouring objects. If a footprint contains multiple number of peaks, we use the HSC deblender to apportion the flux to different peaks. The HSC pipeline adopts the SDSS deblending algorithm (Lupton et al. 2001) as the first generation of deblender. It takes each peak as a ‘child’ source of the ‘parent’ source. With the assumption that every object has a 180-degree rotational symmetry around the peak, a template $T_i(\vec{x})$ for each ‘child’ is created as follows

$$T_i(\vec{x}) = \min(f(\vec{x}), f(2\vec{p}_i - \vec{x})), \quad (19)$$

where $\vec{p}_i$ is the peak of the ‘child’ source $i$, $\vec{x}$ and $2\vec{p}_i - \vec{x}$ are symmetric about the peak $\vec{p}_i$. Then scaling parameters $\alpha_i$ are deduced to fit templates to the ‘parent’ image. The final deblended ‘child’ source is

$$C_i(\vec{x}) = \frac{\alpha_i T_i(\vec{x})}{\sum_j \alpha_j T_j(\vec{x})} f(\vec{x}).$$

After deblending, the HSC pipeline replaces the footprints of other sources with uncorrelated noise. We subsequently define the boundary of galaxies using the top-hat aperture defined in eq. (4). The center of the aperture is set to the position of the peak if the footprint contains only one peak. On the other hand, for a footprint with multiple peaks, the center of our aperture is set as the footprint center. The trace radius of the source galaxy ($r_g$) is determined by the HSM algorithms. The aperture radius is $\alpha \times r_g$, where the default $\alpha$ is determined in Section 3.2. Furthermore, in order to ensure the aperture region covers the entire footprint area for each galaxy, the minimum value of aperture radius is set to $r_{FP} + 3$, where $r_{FP}$ is the radius of the footprint. Then we put the galaxy into a 64 $\times$ 64 stamp and pad the region outside aperture with 0. After Fast Fourier Transform (FFT), we calculate the galaxy Fourier power function and subtract the noise Fourier power function from it. Finally, the FPFS ellipticity and responsibility are measured after deconvolving the PSF Fourier power function from the galaxy Fourier power function.

Shear is measured from each subfield using the aforementioned pipeline. Subsequently, we conduct a linear fit of the measured shear ($\hat{\gamma}_{1,2}$) by the input shear ($g_{1,2}$) to determine the multiplicative bias and additive bias defined in eq. (18). Furthermore, the error of the bias is determined by the covariance matrix of the fitting.

### 3.2 Free parameters

In this subsection, we determine the default free parameters ($\alpha, \beta, \nu$) in the FPFS formalism. As introduced in Section 2, $\alpha$ and $\beta$ respectively determine the measurement scale in real space and Fourier space and $\nu$ changes the weight between galaxies. The experiments shown in this subsection are based on sample 2 in the GREAT3-HSC simulations, although the default parameters determined here will be rechecked with the data in sample 1 in the next subsection.

Firstly, the applicable lower threshold of $\alpha$, and its dependence on $\beta$, is discussed. The aperture ratio ($\alpha$) must be larger than the applicable threshold in order to avoid bias caused by the steep cut of the aperture for every galaxies. LZ17 studied such threshold for the estimator proposed in ZK11 using noiseless Sersic galaxies simulated by Random Walks. It concluded that the aperture radius should be at least 8 times of the concerned galaxy’s half light radius in order to keep the multiplicative bias below 1%. However LZ17 did not consider the dependence of the threshold on the shapelets scale ratio ($\beta$), which was kept to a constant 0.5.
Our test sets the shapelets scale ratio ($\beta$) to three different values (0.5, 0.6 and 0.85) and then change the value of aperture ratio ($\alpha$) for each setup of $\beta$, respectively. $\nu$ is kept to 4 in these tests and we selection galaxies with intrinsic $S/N$ larger than 5. Fig. 5 demonstrates that the multiplicative bias exceeds 1% when the aperture ratio drops below 8 for $\beta = 0.5$. When the $\beta$ is increased to 0.85, the multiplicative bias is consistent with 0 even if $\alpha$ drops to 4. The additive bias has not been plotted since it is only a few parts in $10^4$. Based on the result, we set $\alpha = 4$, $\beta = 0.85$ as the default parameters in the following content. Fig. 7 shows the galaxy images and the corresponding masks detected from sample 2. The connected bright pixels of the masks are the footprints of galaxies and the circular gray pixels of the masks represent the aperture.

Secondly, we discuss the applicable lower threshold of $\nu$. The value of $\nu$ should be greater than the applicable threshold otherwise the noise will cause bias to the shear estimation as shown in eq. (15). We change the value of $\nu$ and demonstrate the performance of our FPFS method in Fig. 6. The result shows that when $\nu$ decreases below 2, the multiplicative bias emerges. The additive bias has not been plotted since it is only a few parts in $10^4$. Consequently, we conclude that $\nu$ should be larger than 2 to avoid the noise bias. Traditionally, the optimal estimator is deduced by weighting the ellipticity with inverse variance including both shape noise and measurement error (Mandelbaum et al. 2017). However, such weighting process is not fully optimal since it does not consider the scatter of the calibration factor used to calibrate the estimator. Moreover, the weight can easily be correlated with the ellipticity due to the distortion of the shear, which causes weight bias. External simulations are required to calibrate the weight bias (Mandelbaum et al. 2017). Therefore, we do not apply such weighting scheme to our estimator. We set $\nu = 4$ as the default setup in the following content and leave the discussion on the optimal choice of $\nu$ to our future work based on real observational data. Since the GREAT3-HSC simulations cancel shape noise in each subfield, which simplifies the real observations. Moreover, the redshift error should also be taken into account to derive the optimal estimator in the real observation.

### 3.3 Isolated galaxies

We conduct several tests on the FPFS method using the isolated galaxies in sample 1 and sample 2. The first test shows the performance of the FPFS method with different selection functions and lower thresholds. These selection functions include (i) intrinsic $S/N$, (ii) CModel $S/N$, (iii) intrinsic resolution, (iv) resolution, (v) FPFS flux, and (vi) revised FPFS flux shown in Section 2.4. Selections with these intrinsic quantities do not induce selection bias by construction, while it is difficult to measure these quantities from the real observations. However, they enable us to focus on noise bias and model bias, as well as quantify the selection bias by comparing the results of the observed quantities with the corresponding intrinsic quantities. For these tests, the free parameters are set to the default values ($\alpha = 4$, $\beta = 0.85$, $\nu = 4$). The additive bias is consistent with zero and is not plotted. Whereas, the multiplicative bias is demonstrated in Fig. 8 and the summary is listed as follows.

(i) Using the intrinsic $S/N$ as selection function, the multiplicative bias is consistent with zero for $s/N > 5$. While for $s/N \leq 5$, multiplicative bias of $-0.25$ is found ($1\sigma$ significance). Such noise bias is far below the first year science requirements of the HSC survey (Mandelbaum et al. 2018).
We conclude that our method is free from multiplicative bias of ~2% (10σ significance) when the lower threshold reaches 25. Furthermore, the results of sample 1 and sample 2, shown in the right panel of Fig. 8, are consistent. Therefore we conclude that our method is not influenced by the model bias.

Even though the additive bias is not detected in the first test, the reader may be concerned about the dependence of the additive bias on the PSF anisotropy. Since the first test has only confirmed that the averaged additive bias is consistent with zero and the previous tests when marginalizing over the anisotropies of PSFs, the second test will show the dependence of the additive bias on the PSF anisotropy using galaxies in sample 2. Since sample 1 and sample 2 use the same PSF models and we only focus on the PSF anisotropy, we do not repeat the test on sample 1. The PSF anisotropies are quantified by the PSF ellipticity which is defined by the second moments matrix measured by the re-Gaussianization algorithm. Setting the fiducial multiplicative bias to zero, we fit the PSF ellipticity to the shear residual with the linear relation

$$\frac{\delta g_{12}}{\sigma_{\delta g_{12}}} = \frac{\delta \theta_{12}}{\sigma_{\delta \theta_{12}}} = a_{12} e_{12}^P$$

(20)

where $\delta g_{12}$ is the shear residual, $\frac{\delta \theta_{12}}{\sigma_{\delta \theta_{12}}}$ is the estimated shear, $\frac{\delta \theta_{12}}{\sigma_{\delta \theta_{12}}}$ is the input shear, $e_{12}^P$ is the PSF ellipticity and $a_{12}$ is termed fractional additive bias which describes the fraction of the PSF anisotropy which leaks into the shear measurement (Mandelbaum et al. 2017). Fig. 9 demonstrates the relation between the shear residual and the PSF ellipticity. We mask out the data point with extreme ellipticity, where the amplitude of the ellipticity is greater than 0.3. These masked data only corresponds to 0.4% of the total data. The fitting relations between the two components of the shear residual and PSF ellipticity is demonstrated by the red lines in Fig. 9. Since the fractional additive bias is only 0.3% (4σ significance), we finally conclude that the additive bias is far below the first year HSC science requirements given by Mandelbaum et al. (2018) and we do not find strong correlation between the additive bias and the PSF anisotropy. However, we caution the GREAT3-HSC simulations do not include PSF model residuals due to the uncertainty in the PSF reconstruction. We leave the systematic tests including the PSF model residuals to our future work.

### 3.4 Blended galaxies

In this subsection, we conduct tests on sample 3 and sample 4. These samples also contain isolated galaxies so they are more close to the real observations. We process all of the detections in these samples with two different setups, namely ‘Deblended’ and ‘Nondeblended’. For the ‘Deblended’ cases, we run the HSC deblender if multiple peaks are detected on the footprints before shape measurement. On the other hand, for the ‘Nondeblended’ cases, we do not deblend any footprint even though multiple peaks exist. ‘S3D’ and ‘S3ND’ respectively represent ‘Deblended’ and ‘Nondeblended’ cases for sample 3, and ‘S4D’ and ‘S4ND’ respectively represent ‘Deblended’ and ‘Nondeblended’ cases for sample 4. We use the default parameters ($a = 4$, $\beta = 0.85$, $\nu = 4$) and select galaxies with the criterion: $s_R > 1.5\%$.

The results of these tests are laid out in Table 1. Since the additive bias is below $4 \times 10^{-4}$, we focus our discussion on the multiplicative bias.

- For ‘S3ND’, the multiplicative bias is 0.2% (1σ significance).
- For ‘S4ND’, the multiplicative bias is ~1.4% (5σ significance).

The result of ‘S3ND’ indicates that it is possible to directly measure the ‘parent’ sources without any deblending, if they are on the same source plane and thus distorted by the same shear signal. This is because the performance of the FPFS algorithm is not sensitive to the off-centering of the galaxy and do not rely on any galaxy model. Considering the difference between the observational conditions of sample 3 and sample 4, the bias for ‘S4ND’ must be caused by the contamination of light from the neighbouring objects within the aperture for the source. Since, as stated in Section 3.1, the postage stamps of sample 4 generally contain more neighbouring footprints than those of sample 3.

- For ‘S3D’, the multiplicative bias is ~5.6% (24σ significance).
- For ‘S4D’, the multiplicative bias is ~5.7% (14σ significance).

Comparing these results with the performance on isolated

| sample & setup | $m_1(10^{-2})$ | $c_1(10^{-4})$ | $m_2(10^{-2})$ | $c_2(10^{-4})$ |
|---------------|---------------|---------------|---------------|---------------|
| S3ND          | $-0.25 \pm 0.22$ | $0.75 \pm 0.56$ | $0.03 \pm 0.23$ | $-0.71 \pm 0.59$ |
| S3D           | $-5.71 \pm 0.24$ | $3.33 \pm 0.60$ | $-5.59 \pm 0.24$ | $-1.06 \pm 0.60$ |
| S4ND          | $-1.68 \pm 0.27$ | $0.24 \pm 0.71$ | $-1.11 \pm 0.23$ | $0.19 \pm 0.57$ |
| S4D           | $-5.83 \pm 0.41$ | $1.27 \pm 1.06$ | $-5.59 \pm 0.30$ | $-0.71 \pm 0.75$ |

Table 1. Performance of the FPFS method on sample 3 and sample 4 with 2 different setups. The column ‘sample & setup’ shows the sample and the setup of the corresponding experiment.
galaxies shown in Section 3.3, we conclude that the HSC deblender fails to recover the true galaxy light profiles precisely and the discrepancy between the deblended galaxies and the true galaxies causes the multiplicative bias. The possible origins of the discrepancy are listed as follows.

(i) The HSC deblender assumes a 180-degree rotational symmetry around the peak, although galaxies could have some irregular shape.

(ii) The HSC deblender tends to use the pixels, where the errors caused by shot noise are negative, to construct template.

(iii) The HSC deblender changes the autocorrelation function of shot noise within the source footprint.

To further understand the blending bias, we separate galaxies into different $s_R$ bins and estimate shear using the galaxies in each bin, the result of which is shown in Fig. 10. For the ‘S3ND’ and ‘S4ND’ cases, the multiplicative biases converge to about $-1.8\%$ (6σ significance) for bright galaxies where $s_R > 0.6$. When multiple bright galaxies get blended within one ‘parent’ footprint, the default scale parameters ($\sigma$) determined in Section 3.2 would be so small that the shear measurement can be biased. In summary, even though measuring the ‘parent’ galaxies without deblending can reduce blending bias, it is difficult to apply it to real observations. Since such measurement requires that ‘child’ galaxies for each ‘parent’ galaxy are located on the same redshift plane, however the ‘child’ galaxies in real observations could be located on different redshift planes. Moreover, the proper aperture ratio for the blended ‘parent’ sources has not been studied in details.

Another line of thought is to calibrate the blending bias by the GREAT3-HSC simulations. We focus on the ‘S4D’ case since it best matches the real observational conditions. The results of ‘S4D’ is consistent with our expectation that the deblender performs better on bright galaxies than on faint galaxies. We conduct a third order polynomial fitting of the multiplicative bias as a function of the revised FPFS flux. Subsequently, the calibration factor $1 + m(s_R)$ is added to the responsivity $R_{1,2}$ to calibrate the blending bias. The reader may be concerned about the difficulty of getting the revised FPFS flux ($s_R$) in real observations. However, substituting $s_R$ with $s$ does not change the expectation value of calibrated responsivity, since

$$\langle m(s)R_{1,2} \rangle = \langle m(s_R)R_{1,2} \rangle.$$  

This equation is deduced by substituting eq. (17) into $\langle m(s_R)R_{1,2} \rangle$. Our calibrated shear estimator finally changes to

$$\delta_{1,2} = \frac{\langle \epsilon_{1,2} \rangle}{\langle (1 + m(s))R_{1,2} \rangle}.  \tag{21}$$

### 4 CONSISTENCY TEST

We have shown that the FPFS method is free from systematic biases for isolated galaxies in the previous section. However, the multiplicative bias originating from blending has to be calibrated by the GREAT3-HSC simulations. In this section, we check the consistency and stability of the calibrated estimator defined in eq. (21). We start off by using this revised shear estimator with the default setup to measure shear from sample 4. Galaxies are selected with $s_R > 1.5\%$. Table 2 demonstrates two results, where calibration factor is constructed by $s$ or $s_R$. The remaining biases are labelled as $\delta m$ and $\delta c$, which are generally known as the uncertainty of multiplicative biases and the uncertainty of additive biases, respectively (Mandelbaum et al. 2018). As shown in Table 2, the remaining biases are consistent with zero. By construction no systematic biases are found in these consistency tests since the calibration factors are deduced from the simulations with exactly the same galaxy sample and exactly the same setup. However, it is reasonable to investigate the performance of the calibrated estimator under the following conditions. (i) The setups of the FPFS method, in real observation, deviate from the default setup. (ii) The observational conditions of the data, to which the calibrated estimator is applied, are different from that of sample 4.

With the intent to check the stability of the calibrated estimator under the distortion of the setup, we set $\nu$ to different values and test the performance of the calibration factor obtained from the default setup. The remaining multiplicative bias ($\delta m$) is shown in Fig. 11. The remaining additive bias ($\delta c$) is not plotted since it is only a few parts in $10^6$, which is far below the first year science requirement of HSC survey (Mandelbaum et al. 2018). From Fig. 11, we find that $\delta m$ is approximately proportional to the distortion around $\nu = 4$, which is our default value. Even though the distortion increases to 40% of the default $\nu$, the amplitude of the remaining bias remains below 1.5%. In order to check the stability of the calibrated estimator under the distortion of observational conditions, we separate galaxies into 4 quartiles based on PSF FWHM in the same way as Mandelbaum et al. (2017) and use the calibrated estimator to measure shear within each quartile. The result is shown in Table 3. The uncertainty of multiplicative bias ($\delta m$) is below 1% and the uncertainty of additive bias ($\delta c$) is below 2.5 part in $10^4$.

Here we consider an extreme case in which we apply the shear estimator calibrated by sample 4 to the galaxies in sample 1 or sample 2. Since our calibration factor is simply modelled as a function of the FPFS flux which does not distinguish blended galaxies from isolated galaxies, we expect that the application of the calibrated estimator to sample 1 or sample 2 would cause multiplicative bias of about 5.7%, as these samples do not contain blended galaxies at all. However, since Mandelbaum et al. (2017) ensures that the blending conditions of sample 4 generally match those of the HSC survey, we do not expect that multiplicative bias, at a 5.7% level, would occur when applying the calibrated estimator to HSC survey. Furthermore, the histograms of CModel S/N, resolution, Cmodel magnitude and amplitude of ellipticity of deblended galaxies in sample 4 also match those of the HSC survey. However, the GREAT3-HSC simulations do not include PSF modelling residuals, astrometric

| sample  | calib  | $\delta m(10^{-2})$ | $\delta c(10^{-4})$ |
|--------|--------|---------------------|---------------------|
| full   | $s_R$  | $-0.09 \pm 0.27$    | $0.31 \pm 0.69$     |
| full   | $s$    | $-0.10 \pm 0.26$    | $0.29 \pm 0.68$     |

Table 2. Test for calibration on the full sample. $s_R$ and $s$ represent the result where calibration factors are constructed with the revised FPFS flux and the observed FPFS flux, respectively.
errors, background light residuals, and unrecognized blending (Mandelbaum et al. 2017). We will conduct systematic tests to quantify the bias caused by these effects in our future work.

5 SUMMARY AND OUTLOOK

The newly developed FPFS method is based on the mathematical foundations of ZK11 and shapelets. The FPFS method projects galaxy’s Fourier power functions onto the shapelet basis vectors after PSF deconvolution in Fourier space. Using four shapelet modes (Fig. 1) measured from each galaxy, ellipticity and responsivity (Fig. 2) are constructed. The noise bias can be reduced by increasing the free parameter $\nu$ (Fig. 6). Furthermore, it is possible to change the weight on different galaxies by adjusting the value of $\nu$ (Fig. 3). The transformation formula of ellipticity under the influence of shear for every single galaxy is derived. Based on the transformation formula, the shear estimator is finally given by eq. (14). We define the FPFS flux in eq. (16) and, based on the transformation formula of the FPFS flux, an iterative method to reduce the selection bias (right panel of Fig. 8) is proposed. Our shear estimator is tested by the GREAT3-HSC simulations, the result of which shows that, for isolated galaxies, the FPFS method is free from both the multiplicative bias (Fig. 8) and the additive bias (Fig. 9). However, for the blended galaxies, multiplicative bias of about $-5.7\%$ is found (Table 1). Using sample 4 of the simulations, we model the blending bias as a function of the FPFS flux and then calibrate it (Fig. 10). Several consistency tests for the calibration are conducted, which includes test on the distortion of the default parameter (Fig. 11) and test on the distortion of the observational conditions (Table 3). We report that no significant remaining bias has been found. The following problems have not been discussed within this work since it is difficult to quantify them with the GREAT3-HSC simulations.

(i) The optimization of the shear estimation has not be discussed.
(ii) Calibrations which are more stable and universal remain to be discovered.
(iii) The influence of astrometry errors on shear estimation has not been discussed.
(iv) The influence of PSF model residuals on shear estimation has not been discussed.
(v) The performance of background subtraction and its influence on shear estimation has not been discussed.

These problems will be discussed in our future work.
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APPENDIX A: TRANSFORMATION OF FPFS ELIPTICITY

We derive the transformation formula of the FPFS ellipticity in the absence of noise in this appendix. Starting from the first component of the FPFS ellipticity, we substitute eq. (9) into eq. (10) and obtain

$$\epsilon_1 = \frac{M_{22c}}{M_{00} + C} = \frac{M_{22c} - 2\sqrt{g_1(M_{00} - M_{40})} + \sqrt{g_1} (M_{44c} + \sqrt{g_2} M_{44s})}{M_{00} + C + \sqrt{g_1} (M_{22c} + g_2 M_{22s})}.$$  

Since $g_{1,2} \ll 1$, the denominator can be expressed as the first order Taylor expansion of $g_{1,2}$ as

$$\epsilon_1 = \left( \frac{M_{22c} - 2\sqrt{g_1(M_{00} - M_{40})} + \sqrt{g_1} (M_{44c} + \sqrt{g_2} M_{44s})}{M_{00} + C} \right) \times \left( 1 - \frac{\sqrt{g_1} M_{22c}}{M_{00} + C} - \frac{\sqrt{g_2} M_{22s}}{M_{00} + C} \right).$$

Subsequently, we neglect the terms which contain the second order of $g_{1,2}$ and obtain

$$\epsilon_1 = \frac{M_{22c} - 2\sqrt{g_1(M_{00} - M_{40})} + \sqrt{g_1} (M_{44c} + \sqrt{g_2} M_{44s})}{M_{00} + C} \times \left( 1 - \frac{\sqrt{g_1} M_{22c}}{M_{00} + C} - \frac{\sqrt{g_2} M_{22s}}{M_{00} + C} \right).$$

Similarly, the transformation formula of $e_2$ under the influence of shear can be derived as

$$e_2 = \epsilon_2 - g_2 \frac{M_{22c}}{M_{00} + C} + \sqrt{g_1} \frac{M_{22s}}{M_{00} + C}.$$  

APPENDIX B: FOURIER POWER OF NOISE

In this appendix we discuss the reconstruction of the Fourier power function of noise from the autocorrelation of noise. The discrete form of shot noise on a power function of noise from the autocorrelation of noise. The pixels outside the aperture radius are masked with 0. The definition of $H_V[\bar{m}]$ and $H_R[\bar{m}]$ are defined as follows:

$$H_V[\bar{m}] = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} |h[\bar{n}]h[\bar{n} + \bar{m}]|,$$

$$H_R[\bar{m}] = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} |h[\bar{n}]h[\bar{n} + \bar{m}]|/W[\bar{m}],$$

where $W[\bar{m}]$ is the total number of pixel pairs separated by $\bar{m}$ in which both pixels are within the aperture. Therefore, we can reconstruct autoconvolution of noise on the stamp by

$$H_V[\bar{m}] = H_R[\bar{m}] \times W[\bar{m}].$$

The Fourier power function of galaxy can be obtained by doing the Inverse Fourier Transform (IFT) of the autoconvolution of noise (Li & Zhang 2016).

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