Realizing and detecting a topological insulator in the AIII symmetry class

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Topological insulators in the AIII symmetry class lack experimental realization. Moreover, fractionalization in one-dimensional topological insulators has not been yet directly observed. Our work might open possibilities for both challenges. We propose a one-dimensional model realizing the AIII symmetry class which can be realized in current experiments with ultracold atomic gases. We further report on a distinctive property of topological edge modes in the AIII class: in contrast to those in the well studied BDI class, they have non-zero momentum. Exploiting this feature we propose a path for the detection of fractionalization. A fermion added to an AIII system splits into two halves localized at opposite momenta, which can be detected by imaging the momentum distribution.

PACS numbers: 37.10.Jk, 03.75.Lm,03.65.Vf

In one dimension the topological or trivial character of an insulator is completely determined by the presence or absence of chiral symmetry \[1\ 2\]. Since chiral symmetry is the composition of time reversal (T) and charge conjugation (C) symmetries, two distinct classes of one-dimensional topological insulators arise: those invariant under T and C, and those breaking both symmetries. The first class, called the BDI symmetry class, is represented by polyacetylene \[3\ 5\], which has been the focus of major experimental and theoretical attention. The second class, dubbed the AIII class, has been in contrast rarely explicitly discussed. In a recent interesting connection \[7\], AIII topological insulators have been proposed to open a physical pathway to Riemann’s conjecture, for one-dimensional models realizing the Riemann zeros seem to belong to the AIII symmetry class. However, AIII topological insulators lack to our knowledge experimental realization.

The topological character of one-dimensional insulators is manifested in the emergence of topologically protected zero energy modes \[8\], leading to the fascinating phenomena of particle number fractionalization \[9\ 10\ 11\]. This spectacular property was predicted for polyacetylene by the seminal model of Su, Schrieffer, and Heeger (the SSH model) \[5\]. At filling factor $\nu = 1/2$, a fermion added to a flat background density splits into two quasi-particles with 1/2 charge, which are spatially localized at the edges of the one-dimensional system. Though many rather striking properties predicted by the SSH model were confirmed in experiments with polyacetylene, a direct measurement of fractionalization has not been realized yet in one-dimensional topological insulators.

The unique detection possibilities opened up recently with ultracold atoms in optical lattices \[10\ 12\] promise to allow for the direct observation of key signatures of topological states such as the Chern number \[13\ 14\], the Berry curvature \[15\ 16\] or the topological edge modes \[17\ 18\]. For instance, an atomic version of the SSH model has been recently realized \[19\].

Here, we propose a physical model that realizes a one-dimensional topological insulator in the AIII symmetry class. It can be realized in current experiments with ultracold atoms by combining a superlattice structure \[19\] with artificial gauge fields \[20\]. We further report on a distinctive property of topological edge modes in the AIII symmetry class. In contrast to those in the BDI class, which must necessarily have zero (or $\pi$) momentum, edge modes in the AIII class exhibit non-zero momentum. This feature is the fingerprint of the AIII class. It is a direct manifestation (in a bulk-edge correspondence) of the breaking of time reversal symmetry. Exploiting this property we propose a path for the direct observation of fractionalization in the AIII model. A fermion added to a flat background density splits into two halves with opposite chirality, which can be directly observed by imaging the momentum distribution of the system. This offers an alternative route with respect to previous proposals for the detection of topological edge modes in atomic systems, which are based on the in situ observation of the spatial density \[21\ 23\].

The model. We consider a dimerized lattice model with two sites per unit cell:

$$H = -\sum_n J\delta a_n^\dagger b_n + J e^{i\delta} a_{n+1}^\dagger b_n + h.c., \quad \text{(1)}$$

where $a_n^\dagger (b_n)$ are the particle creation operators for a particle on the sublattice site $a_n (b_n)$ in the $n$th lattice cell. For $\delta = 0$ this Hamiltonian corresponds to the SSH model with tunneling amplitudes $J$ and $J\prime$ \[5\ 24\]. For $\delta \neq 0$ the model introduces a complex phase $e^{i\delta}$ that particles acquire when tunneling from one unit cell to the next [Fig. 1]. In the presence of this extra phase the model breaks time reversal symmetry, entering the AIII symmetry class. This is better seen by writing the Hamiltonian \[1\] in momentum space. For periodic boundary condi-
FIG. 1. The Model. (a) Schematic illustration of the model for a topological insulator in the AIII class: a one-dimensional dimerized lattice model with two sites a and b per unit cell, and hopping amplitudes $J'$ and $J e^{i\delta}$. The phase $\delta$ leads to breaking of time reversal symmetry. (b) For periodic boundary conditions and $N$ unit cells, a total phase $N\delta$ is acquired along the whole ring.

FIG. 2. Symmetries and edge modes: AIII class vs. BDI class. The breaking (not-breaking) of time reversal symmetry is physically manifested in a non-zero (zero or $\pi$) momentum of the topological edge modes, characterizing the AIII (BDI) class.

The Hamiltonian exhibits chiral symmetry for any value of $\delta$, since $\sigma_z M(k) \sigma_z = - M(k)$. However, for $\delta \neq 0, \pi$, it is not time reversal symmetric (and thereby not charge-conjugation symmetric). We have that:

$$M^*(-k) \neq M(k),$$

(3)

since $e^{i(k+\delta)} \neq e^{i(k-\delta)}$. Moreover, there is no $2 \times 2$ unitary transformation $U$ such that $UM^*(-k)U^\dagger = M(k)$ (see Supplementary Material (SM)). For $\delta \neq 0$ the model belongs to the AIII symmetry class [Fig. 2]. The symmetry properties of our model can be alternatively derived by realizing that it can be continuously transformed into a ladder Hamiltonian with a flux per plaquette equal to $\delta$ (see SM).

The different symmetry properties of our AIII model ($\delta \neq 0$) are manifested in a distinctive set of eigenvectors, which are genuinely different from those of the SSH model. The modes are characterized by two quantities: the quasi-momentum $k$ and the isospin vector $n(k-\delta)$, which characterizes the superposition state between the modes $\hat{a}_k^\dagger$ and $\hat{b}_k^\dagger$ (see SM). In the AIII model the correspondence between momentum and isospin is different from the one of the SSH model [Fig. 3].

AIII edge modes. The model exhibits a topological phase transition at $J' = J$, where (for an open chain)

$$\langle n_k | \hat{n}_k | e^* \rangle = \langle U \hat{n}_k U^\dagger | e \rangle = \langle \hat{n}_k | e \rangle,$$

so that states with opposite momenta are on average equally occupied. This implies that the average momentum of an edge mode in the BDI class must be either 0 or $\pi$. Experimental observation of an edge mode with non-zero momentum is thus a direct evidence of having realized the AIII class.

Surprisingly, the edge modes are well localized around their average momentum [Fig. 4]. We have that $\hat{a}_n^{\dagger} \equiv \tilde{a}_n^{\dagger}$, with $\tilde{a}_n^{\dagger} = \sum_k F(k-\pi-\delta) \hat{a}_k^{\dagger}$, where $F(k-\pi-\delta)$ is a well localized function around $k = \pi + \delta$.

FIG. 3. Energy bands and bulk eigenstates. AIII class vs. BDI class. (a) Energy bands for the AIII class ($\delta = -2\pi/3$, solid line) and the BDI class ($\delta = 0$, dashed line), for $J'/J = 0.75$. Bulk eigenstates are fully characterized by the pair (quasimomentum $k$, isospin vector $n$). (b) In the BDI class the pair is given by $(k, n(k))$. (c) In the AIII class, the correspondence between momentum and isospin is different: $(k, n(k-\delta))$, resulting into a new set of eigenstates (see text and SM).

Two topological edge modes arise with energies lying in the middle of the gap. The symmetry properties of the AIII model manifest themselves in the properties of the edge modes [Fig. 2 and Fig. 3]. These have the form:

$$e^\pm_\pm = \frac{1}{\sqrt{2}} (\tilde{a}^{\dagger}_{n=1} \pm \tilde{b}^{\dagger}_{n=N}).$$

(4)

Here, $\tilde{a}^{\dagger}_{n=1}$ is a well localized mode at the left end of the chain:

$$\tilde{a}^{\dagger}_{n=1} = \sum_n e^{i(\pi+\delta)n} \varphi(n) \tilde{a}^{\dagger}_n,$$

(5)

with $\varphi(n) \propto \sinh[\xi(N + 1 - n)] \approx e^{-\xi n}$ and $\xi \approx -\log(J'/J)$. Similarly, $\tilde{b}^{\dagger}_{n=N}$ is a well localized mode at the right end of the chain.

The breaking of time reversal symmetry in the AIII model implies a non-zero average momentum $\pi + \delta$ for the edge modes [Fig. 4]. This chirality is the hallmark of the AIII class, since edge modes of the BDI class must have momentum zero or $\pi$. This statement can be proved as follows. In the presence of time reversal symmetry (BDI class), an edge mode $|e\rangle$ has to be equal (up to a global unitary $U$) to its time reversed: $|e\rangle = U |e^*\rangle$ (see SM). Since density operators are invariant under global unitaries, we have that:

$$\langle \tilde{n}_k | \hat{n}_k | e^* \rangle = \langle e^* | \hat{n}_- | e \rangle = \langle e | U \hat{n}_- U^\dagger | e \rangle = \langle e | \hat{n}_- | e \rangle,$$
FIG. 4. Topological edge modes: AIII class vs BDI class (a) Average spatial density $\langle \hat{n}_x \rangle$ and (b) momentum density $\langle \hat{a}_k^\dagger \hat{a}_k \rangle$ of edge modes for $N = 100$ and $J'/J = 0.9$. (a) For both classes edge modes are spatially localized at the edges of the chain. (b) For the BDI class ($\delta = 0$, dashed line) both edge modes are localized at momentum $k = \pi$ (for clarity, we plot them shifted to $k = 0$). For the AIII class ($\delta = -2\pi/3$, solid line), both edge modes are localized at a non-zero momentum $k = \pi + \delta$. Insets show localization lengths in position and momentum as a function of $J'/J$.

For a wide range of parameters in the topological phase, edge states are simultaneously localized both in position and momentum, [see insets in Fig. 4a,b and SM]. This simultaneous localization arises both in the AIII and BDI classes.

Experimental realization. We develop a scheme [Fig. 5] for the realization of the model above. The scheme combines a superlattice structure [19] together with Raman assisted tunneling [25, 26] to realize a Hamiltonian of the form:

$$H_{\text{exp}} = -\sum_{n} \left( J' e^{i(2n-1)\delta} \hat{a}_{n}^\dagger \hat{b}_{n} + J e^{2n\delta} \hat{a}_{n+1}^\dagger \hat{b}_{n} + \text{h.c.} \right).$$

This Hamiltonian is equivalent to our model [1] through the gauge transformation $\hat{a}_{n}^\dagger (\hat{b}_{n}) \rightarrow \hat{a}_{n}^\dagger (e^{i(2n-1)\delta} \hat{b}_{n})$. This gives rise to a relative shift of the momentum modes $\hat{a}_{k}^\dagger (\hat{b}_{k}) \rightarrow \hat{a}_{k}^\dagger (\hat{b}_{k-2\pi\delta})$, so that the edge modes are transformed as

$$\hat{a}_{k=\pi+\delta}^\dagger(\hat{b}_{k=\pi+\delta}) \rightarrow \hat{a}_{k=\pi+\delta}^\dagger \left(\hat{b}_{k=\pi-\delta}\right).$$

(6)

For the BDI class ($\delta = 0$) the two edge modes are localized at the same position in momentum space ($k = \pi$). In contrast, for the AIII class they are localized at opposite momenta ($k = \pi + \delta$ and $k = \pi - \delta$). This splitting is a distinctive feature of the AIII class. As we show below, it allows to directly observe fractionalization in momentum space.

Observing fractionalization in the AIII model. We consider a chemical potential such that the lower band and both edge modes are occupied. The corresponding many-body state in the theoretical model is:

$$|\Phi\rangle = \mathcal{C} \hat{a}_{k=\pi+\delta}^\dagger \prod_{q} \mathcal{C} \hat{c}_{+q}^\dagger |0\rangle,$$

(7)

where $\mathcal{C}$ denotes a bulk mode with momentum $q$ in the lower (upper) band (see SM). This state corresponds to a fermion added to a state with flat background density. It can be written as $|\Phi\rangle = \mathcal{C} \hat{c}_{-q}^\dagger |\Phi_+\rangle$, where $|\Phi_+\rangle = \prod_{q} \mathcal{C} \hat{c}_{\pm q}^\dagger |0\rangle$. The state $|\Phi_+\rangle$, in which all bulk modes in the lower band plus one of the edge states are occupied, has a flat density profile, $\nu = \langle \Phi_+ | \hat{a}_{k=\pi+\delta}^\dagger \hat{a}_{k} | \Phi_+ \rangle = \frac{1}{2}$.

This is easily derived by noticing that $|\Phi_+\rangle$ and $|\Phi_-\rangle$ have the same density profiles:

$$\langle \Phi_+ | \hat{a}_{k=\pi+\delta}^\dagger \hat{a}_{k} | \Phi_+ \rangle = \langle \Phi_- | \mathcal{C} \hat{c}_{\pm q}^\dagger \mathcal{C}^\dagger \hat{c}_{\pm q} | \Phi_- \rangle = \langle \Phi_- | \hat{a}_{-q}^\dagger \hat{a}_{k} | \Phi_- \rangle,$$

(8)

since they are obtained from each other through the unitary transformation $\mathcal{C}$ that maps $\hat{a}_{k}^\dagger \rightarrow \hat{a}_{k}^\dagger$ and $\hat{b}_{k} \rightarrow -\hat{b}_{k}$. Defining the projectors $P_{\pm} = \sum_{q} |q_{\pm}\rangle \langle q_{\pm}| + |e_{\pm}\rangle \langle e_{\pm}|$, with $|q_{\pm}\rangle = e_{\pm k}^\dagger |0\rangle$ and $|e_{\pm}\rangle = e_{\pm k}^\dagger |0\rangle$ and taking into account that $P_{+} = I - P_{-}$, we obtain:

$$\nu = \text{tr}(\hat{a}_{k=\pi+\delta}^\dagger \hat{a}_{k} P_{+}) = \text{tr}(\hat{a}_{k=\pi+\delta}^\dagger \hat{a}_{k} P_{-}) - 1 - \nu \quad (8)$$

and thereby $\nu = 1/2$.

In the state [6] both edge states are occupied. Therefore the mode $\hat{a}_{k=\pi=1}^\dagger = \hat{a}_{k=\pi+\delta}^\dagger$, simultaneously localized in momentum and position is also occupied, and we have:

$$\left[ \hat{a}_{k=\pi+\delta}^\dagger \hat{a}_{k=\pi+\delta} - \nu \right] |\Phi\rangle = \frac{1}{2} |\Phi\rangle. \quad (9)$$

The above expression states that the state $|\Phi\rangle$ (with one fermion on top of a flat background) is an exact eigenstate of the number operator $\hat{a}_{k=\pi+\delta}^\dagger \hat{a}_{k=\pi+\delta} - \nu$ with eigenvalue $1/2$. Thus, a sharp 1/2 fraction of particles is localized at momentum $k = \pi + \delta$. The same holds for the number operator $\hat{b}_{k=\pi+\delta}^\dagger \hat{b}_{k=\pi+\delta} - \nu$.

What are the implications for the experiment? In the experimental gauge, the edge modes $\hat{a}$ and $\hat{b}$ are shifted according to [6]. For the SSH model ($\delta = 0$) this means that a particle added to a uniform background will consist of two quasiparticles bound together at the same momentum position. The momentum distribution will show...
FIG. 6. Direct observation of fractionalization in the AIII class. (a–d) Momentum density profiles (a,c) for the BDI class ($\delta = 0$), and (b,d) the AIII class ($\delta = -\pi/3$), for $N = 100$ and $J'/J = 0.9$. For both classes the many body state $|\Phi_+\rangle$ (see text), shows a flat density (a,b). For the state $|\Phi_0\rangle$, in which a fermion is added to the flat background, the momentum density in the BDI class (c) shows a single peak of charge 1 at momentum $k = \pi$ (plotted shifted at $k = 0$). In contrast, in the AIII class (d), the momentum density shows two peaks of charge 1/2 localized at momenta $\pi \pm \delta$. (e,f) Simultaneous fractionalization in position and momentum space in the AIII class. (e) Average spatial density and (f) momentum density for the many body state such that the upper band is empty. In the topological phase localized peaks of charge 1/2 (at the edges of the chain, and at momenta $\pi \pm \delta$) arise.

a single peak at momentum $\pi$ enclosing a total charge 1 [Fig. 5a]. For the AIII class, in contrast, a particle added to a uniform background will split into two halves located at opposite momenta $\pi - \delta, \pi + \delta$. The momentum distribution will show two peaks, each enclosing a 1/2 charge [Fig. 5b]. In the AIII class splitting of the fermion occurs therefore both in position and momentum space [Fig. 5c,d,f]. The splitting in momentum is a direct consequence of the non-zero momentum of the edge modes, which is in turn a direct manifestation of the breaking of time reversal symmetry characterizing the AIII class.

We have presented a one-dimensional model realizing the AIII symmetry class which can be readily realized in experiments. We have identified distinctive features characterizing the AIII class, which can serve as experimen-
tal signatures discriminating it with respect to the BDI class. For instance, the characteristic non-zero momentum of AIII edge modes could be observed by realizing our model for a system of bosons. By making the bosons condense in a topological edge mode, a time of flight experiment will show a macroscopic signal at a non-zero momentum, evidencing the realization of the AIII class.

Our findings open a path for the detection of fractionalization in the AIII model by directly imaging the momentum distribution of the particles. Our protocol relies on the preparation of the many-body state $|\Phi\rangle$ and on the observation of 1/2 fractions on top of a flat background. These requirements are common to previous protocols to observe fractionalization in position space. In contrast to those, which require in situ imaging of the system, our scheme benefits from the magnification of the cloud after time of flight. Analysis of temperature and soft edge effects in previous protocols show that state edges are robust against them [21–23, 27]. We believe that this also holds for our momentum protocol. Moreover, localization in momentum might be even less susceptible to soft edges.

Finally, it is interesting to investigate how the simultaneous localization of edge modes in momentum and position can be exploited to prove the topological protection of fractionalized quasiparticles. It is a challenge to explore variations of our AIII model in ladder architectures and to investigate whether they can serve as physical pathways to Riemann’s conjecture [7].

The work of C. G. V. is supported through the grant BES-2013-064443 of the Spanish MINECO.

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