Hard Photon Pair Production at LEP¹

Alessandro Ballestrero  
*INFN, Sezione di Torino*

Ezio Maina, Stefano Moretti  
*Dipartimento di Fisica Teorica, Università di Torino, Italy*  
*and INFN, Sezione di Torino, Italy*

**Abstract**

The production of photon pairs in $e^+e^- \rightarrow f\bar{f}\gamma\gamma$ processes is studied using exact helicity amplitudes at tree level. Total cross sections, including initial state radiation effects, are given. They are presented in the case of quarks as a function of $y_{cut}$ in the JADE algorithm. In the case of leptons we use cuts similar to the ones employed in a recent L3 analysis. Masses of the final state fermions are taken into account when appropriate. The cross section $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$ is about 10% larger when the $\tau$ mass is neglected. We obtain, with a different method, results which are in good agreement with L3 Montecarlo simulation.

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Introduction

Photon emission is of crucial importance in $e^+e^-$ collisions at energies close to the $Z^0$ peak, and a good understanding of its properties is necessary in order to test the Standard Model with high accuracy. Most of the radiation is soft and/or collinear. The cross section for events with hard, isolated photons is much smaller, being approximately reduced by a factor $\alpha_{em}$ for each emitted photon with respect to the cross section with only soft radiation. Hard photons are interesting because they probe the quark charges and because they could be a signal for new particles or for some kind of substructure of known particles [1, 2]. Not surprisingly, it has been shown [3] that the soft approximation is inadequate to describe hard photon radiation, and that a matrix element calculation is needed for this purpose.

The L3 Collaboration [4] has recently reported four events in which two hard photons are emitted with a total invariant mass close to 60 GeV. From a comparison with the YSF3 Montecarlo [5], after full detector simulation, L3 estimates the probability for all four events to originate from ordinary QED to be of the order of $10^{-3}$. The expected rate for the production of a Standard Model Higgs of about 60 GeV, in association with leptons, decaying in the two-photon channel, is about four orders of magnitude smaller than the measured one.

The obvious thing to do in order to understand whether these peculiar events represent the first manifestation of some new physics is to examine all multi–photon events and compare them with the available simulations in search of anomalies.

The L3 events have a typical $y_{cut}$ in the $10^{-3}$ range, and events with $y_{cut}$ as small as $10^{-4}$ would still survive all cuts. This range has only been partially analyzed, with a simplified set of cuts, in previous studies [3].

In two recent papers [6, 7] we have shown that at the $Z^0$ peak, when masses are properly taken into account, cross sections involving $c$ or $b$ quarks differ significantly from the corresponding predictions obtained when masses are neglected. For final states with more than one photon the $b$-quark contribution is severely reduced by the small electric charge of the $b$. For $c$-quarks the mass effect is of the order of several percent. However, in order to test this result, $c$-quarks tagging is required, and this substantially decreases an already small rate and increases the sources of uncertainty. These results suggest that a calculation for the more easily accessible reaction $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$, in which the $\tau$ mass is not neglected, could give significantly smaller predictions than those available so far.

The main purpose of this paper is to give a realistic estimate of the total cross section and of some typical distribution for $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$, taking into account initial state radiation and the $\tau$ mass, with a set of cuts which mimic as closely as possible the ones used in the L3 experiment. At the same time we study two–photon production in association with jets, for which no anomalous event has been reported. We also compute the cross section and some distributions for $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$ in order to be able to compare our calculations with L3 results and to determine if our approach is adequate.
Calculation

We consider processes without electrons in the final state, which proceed through $e^+e^-$ annihilation into a photon or a $Z^0$. Both contributions have been retained in all amplitudes. In our calculation hard photon radiation is emitted only from the final state fermions, while the radiation from the initial electron legs is described using the structure function approach [3]. This separation of final and initial state radiation makes sense since the two terms are separately gauge invariant and we are only interested in photons at relatively large angle with respect to the beam. We have computed all matrix elements both in the formalism of [9, 10] and in that of [11]. The analytic expressions we have used, in the formalism of [9, 10], can be easily extracted from Appendix B of [7].

The amplitudes have been checked for gauge invariance. We have used $M_Z = 91.1$ GeV, $\Gamma_Z = 2.5$ GeV, $\sin^2(\theta_W) = .23$, $\alpha_{em} = 1/128$ for the vertices connected to the $Z^0, \gamma$ propagator and $\alpha_{em} = 1/137$ everywhere else, $m_{\tau} = 1.78$ GeV and $m_b = 5.$ GeV in the numerical part of our work.

In what follows we neglect all hadronization effects, and apply cuts at the partonic level.

Results

The total cross sections for $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$ and $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$ at $\sqrt{s} = M_Z = 91.1$ GeV are given in table I. For comparison we also give the cross section which results neglecting the $\tau$ mass. Only photons satisfying $m_{\gamma\gamma} > 2.5$ GeV, $|p_\mu| > 1$ GeV and $|\cos\theta_\gamma| < 0.9$ are accepted. For $\mu$’s we require $|p_\mu| > 3$ GeV, $|\cos\theta_\mu| < 0.81$ and $\theta_{\gamma\mu} > 5^\circ$. For $\tau$’s and for $\ell_0$’s, the reference massless leptons we compare the $\tau$’s with, the assumed cuts are $|\cos\theta_\ell| < 0.74$ and $\theta_{\gamma\ell} > 15^\circ$.

In fig.1 we present the total cross sections for $e^+e^- \rightarrow q\bar{q}\gamma\gamma$ for $u$, $d$ and $b$-quarks separately and then for the sum over five flavors as a function of $y_{cut}$. Only for $b$-quarks the mass is taken into account. For photon pairs in association with jets we have required $\theta_{\gamma q} > 15^\circ$ but have not imposed any cut on the angles with respect to the beam. All particle pairs must have $y > y_{cut}$, where $y$ is the usual JADE variable [12].

In fig.2 we present the energy distribution of the most energetic photon in $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$, $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$ and $e^+e^- \rightarrow \ell_0^+\ell_0^-\gamma\gamma$ with $\ell_0$ massless. In fig.3 we give the two–photon mass distribution in $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$ and again compare it with the results for a massless lepton with the same choice of cuts. Finally in fig.4 we show the two–photon invariant mass distribution for $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$, $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$ and $e^+e^- \rightarrow q\bar{q}\gamma\gamma$ summed over all flavors.

In fig.2 and 4 the results are deliberately presented in a way that allows an easy comparison with L3 data.

In order to discuss the consistency of our results with L3 Monte Carlo simulations we proceed as follows. From the fraction of muon events in the Monte Carlo sample which survives all cuts, an efficiency of about 56% can be inferred for the L3 detector. The main factors contributing to the efficiency are the geometrical acceptance and the effect of the dead region between detector modules in the R-\phi plane within the
fiducial volume $|\cos \theta_\mu| < 0.81$. The first contribution can be easily extracted from the ratio of the integrated Born cross section for $e^+e^- \rightarrow \mu^+\mu^-$, in the presence of the angular and momentum cuts and in their absence, and turns out to be about 74%. The additional effect related to inefficiencies within the fiducial volume can therefore be estimated at about 76%.

L3 data include runs from 1990 through July 1992 at various energies around the $Z^0$ peak. An equivalent peak–luminosity of about $22 \text{ pb}^{-1}$ can be derived, dividing the total sample of 950,000 $Z^0$ by the peak-cross section of about 43 $\text{ nb}$, including initial state radiation. This luminosity and the 76% efficiency in detecting muons can then be used to translate the cross sections we present, which already include geometrical acceptance, in a prediction for the number of events. As an example we would expect 50 events with two photons and a $\mu^+\mu^-$ pair in the sample. In order to compare this result with the L3 Montecarlo simulation it is necessary to correct for the cut in the invariant mass of the photon pair which is present in our analysis and is not implemented by L3. From the $m_{\gamma\gamma}$ distribution in fig.6 of [4] one can read that the first mass bin, which exactly corresponds to our excluded region, contains 20% of the Montecarlo events. Therefore our prediction of 50 events has to be compared with 45 events predicted by L3 in the same mass range. This agreement at the 10% level is, in our opinion, quite satisfactory. The integrated cross section for $m_{\gamma\gamma} > 50 \text{ GeV}$ is $3.5 \times 10^{-2} \text{ pb}$ for $\mu^+\mu^-\gamma\gamma$, corresponding to .6 events.

The consistency of the results obtained in the two approaches, for photon pairs in association with leptons, can be better appreciated looking at the shape of the mass distribution in fig.4 and in fig.6 of [4]. There is also good agreement, for large photon energies, for the shape of the energy distribution of the most energetic photon shown in fig.2 and in fig.2a of [4]. The low energy peak, present in fig.2a of [4], is strongly reduced in our distribution due to the additional minimum invariant mass requirement.

On one hand we interpret these results as a confirmation of the correctness of L3 Montecarlo simulation, on the other hand they give us confidence in our predictions.

From table I we see that the predicted cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$ is about 9% smaller than it would be for a massless lepton. Fig.2 and 3 show that the effect of the $\tau$ mass is more pronounced for large photon energies and for large invariant mass of the photon pair. As expected, the total cross section is dominated by low energy photons and low mass pairs.

The cross sections for photon pairs in association with jets, with the assumed set of cuts and $y_{\text{cut}} = 5 \times 10^{-3}$, is about $1 \text{ pb}$ corresponding to 22 events. Since the mentioned cuts are similar to those employed in [4] we expect a number of such events to be present in the data sample of each LEP experiment.

As a byproduct of the calculations presented here, we have verified that the ratio of distributions for massive particles to those for massless ones is not modified when initial state radiation is included. Therefore, also in this more realistic case, all conclusions drawn in [3, 4] remain valid. Obviously all distributions have to be scaled down by approximately 30%.
Conclusions

We have produced the total cross section and some typical distributions for $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$, taking into account initial state radiation and the $\tau$ mass, with a set of cuts which mimic as closely as possible the ones used in the L3 experiment. The total cross section is 9% smaller than it would be for a massless lepton and the discrepancy increases for larger photon energies and two-photon invariant masses. Photon pair production in association with jets has also been computed and we believe it would be important to look for such events. We have studied $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$ obtaining a total rate and distributions in good agreement with L3 results.

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References

[1] Proceedings of the Workshop on Photon Radiation from Quarks, S. Cartwright ed., Annecy, France, 2-3 Dec. 1991. CERN 92-04.
P. Mättig and W. Zeuner, Z. Phys. C 52 (1991) 31.

[2] OPAL Collaboration, G. Alexander et al., Phys. Lett. B264 (1991) 219.
OPAL Collaboration, P.D. Acton et al., Z. Phys. C 54 (1992) 193.
ALEPH Collaboration, D. Decamp et al., Phys. Lett. B264 (1991) 476.
DELPHI Collaboration, P. Abreu et al., Z. Phys. C 53 (1992) 555.
L3 Collaboration, O. Adriani et al., Phys. Lett. B292 (1992) 472.

[3] W.J. Stirling, Phys. Lett. B271 (1991) 261.

[4] L3 Collaboration, O. Adriani et al., Phys. Lett. B295 (1992) 337.

[5] S. Jadach and B.F.L. Ward, Phys. Lett. B274 (1992) 470.

[6] A. Ballestrero, E. Maina and S. Moretti, Phys. Lett. B294 (1992) 425.

[7] A. Ballestrero, E. Maina and S. Moretti, Torino Preprint DFTT 53-92, October 1992. Submitted to Nucl. Phys. B.

[8] O. Nicrosini and L. Trentadue, Phys. Lett. B196 (1987) 551; Z. Phys. C 39 (1988) 479.

[9] R. Kleiss and W.J. Stirling, Nucl. Phys. B262 (1985) 235.

[10] C. Mana and M. Martinez, Nucl. Phys. B287 (1987) 601.

[11] K. Hagiwara and D. Zeppenfeld, Nucl. Phys. B274 (1986) 1.

[12] JADE Collaboration, W. Bartel et al., Z. Phys. C 33 (1986) 23.
JADE Collaboration, S. Bethke et al., Phys. Lett. B213 (1988) 235.
[13] G.P. Lepage, Jour. Comp. Phys. 27 (1978) 192.
Table Captions

table I. Cross sections, cuts and parameters for $e^+e^- \rightarrow \ell^+\ell^-\gamma\gamma$ with $\ell = \mu, \tau, \ell_0$, where $\ell_0$ is a massless lepton. Errors are as calculated by VEGAS [13].

Figure Captions

fig.1. Cross sections for $e^+e^- \rightarrow u\bar{u}\gamma\gamma$, $e^+e^- \rightarrow d\bar{d}\gamma\gamma$, $e^+e^- \rightarrow b\bar{b}\gamma\gamma$ and for the sum over five flavors as a function of $y_{\text{cut}}$ at $\sqrt{s} = 91.1$ GeV. The cuts used for photon pairs in association with jets are $y_{\text{cut}} > 5 \times 10^{-3}$, $\theta_{\gamma q} > 15^\circ$ and $|p_\gamma| > 1$ GeV.

fig.2. Energy distribution of the most energetic photon for $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$, $e^+e^- \rightarrow \ell_0^+\ell_0^-\gamma\gamma$ and for $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$, where $\ell_0$ is massless, at $\sqrt{s} = 91.1$ GeV. Cuts as in table I.

fig.3. Two–photon invariant mass distributions for $e^+e^- \rightarrow \ell_0^+\ell_0^-\gamma\gamma$ and for $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$, where $\ell_0$ is massless, at $\sqrt{s} = 91.1$ GeV. Cuts as in table I.

fig.4. Two–photon invariant mass distributions for $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$, $e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma$ and for $e^+e^- \rightarrow q\bar{q}\gamma\gamma$, with $y_{\text{cut}} = 5 \times 10^{-3}$ and summed over five flavors, at $\sqrt{s} = 91.1$ GeV. Cuts as in table I for leptons. The cuts used for photon pairs in association with jets are $y_{\text{cut}} > 5 \times 10^{-3}$, $\theta_{\gamma q} > 15^\circ$ and $|p_\gamma| > 1$ GeV.
\[ \sigma \text{ (pb)} \]

| Process                        | \( e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma \) | \( e^+e^- \rightarrow \tau^+\tau^-\gamma\gamma \) | \( e^+e^- \rightarrow \ell_0^+\ell_0^-\gamma\gamma \) |
|-------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| \( (4.093 \pm 0.016)^* \)     | \( (1.4746 \pm 0.0038)^* \)                   | \( (1.6169 \pm 0.0052)^* \)                   |
| \( (3.025 \pm 0.015)^\dagger \)| \( (1.0854 \pm 0.0035)^\dagger \)                   | \( (1.1913 \pm 0.0048)^\dagger \)                   |

\[
| p_\ell | > 3 \text{ GeV} \]

\[
| cos\theta_\ell | < 0.81 \quad \theta_{\gamma\ell} > 5^\circ \]

\[
| m_{\gamma\gamma} | > 2.5 \text{ GeV} \]

\[
| p_\gamma | > 1 \text{ GeV} \]

\[
| cos\theta_\gamma | < 0.9 \]

\[
\sqrt{s} = M_Z = 91.1 \text{ GeV} \quad \Gamma_Z = 2.5 \text{ GeV} \quad \sin^2 \theta_W = 0.23 \]

* Without ISR; † With ISR.

Table I