Empirical analysis of metaheuristic search techniques for the parameterized dynamic slope scaling procedure

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Abstract: The dynamic slope scaling procedure is an approximation method successfully which solves the fixed charge network flow (FCNF) problem by iteratively linearizing the fixed cost. The parameterized dynamic slope scaling procedure adds an additional $\psi$ parameter to the procedure which can significantly improve the solution quality. Finding the optimal value of $\psi$ for a given problem is non-trivial. This paper employs multiple metaheuristic techniques, including simulated annealing, tabu search, and particle swarm optimization, to guide the search for good parameter values. In rigorous testing, we examine the search results, compare the improvement efficiencies among the techniques, and evaluate the final solution quality of the FCNF problem. The experiments show that the solution improvement is robust with respect to these metaheuristics and the complexity of FCNF problem.

Key words: network optimization, dynamic slope scaling, fixed-charge network flow, metaheuristics

History: TBD

1. Introduction

The fixed charge network flow problem (FCNF) was first proposed by Hirsch and Dantzig (1954). The FCNF problem has many variants such as the fixed charge transportation problem (FCTP) (Balinski 1961), the lot sizing problem (Steinberg and Napier 1980), and facility location problem (Nozick 2001). Each of these have common characteristics with the FCNF, namely the link between nodes in a given graph have both variable and fixed costs. The nodes have supply or demand amounts and the objective is to choose links and flow values on links to transfer commodities from supply nodes to demand nodes at a minimum cost. Given its wide spread application on many problem types (e.g. Jarvis et al. 1978, Lederer and Nambimadom 1998, Armacost et al. 2002, Zhang and Wang 2017, Zhang et al. 2018, ZHANG and ZHAO 2010, Zhang and Nicholson 2016a, Zhang and Yao 2010, Zhang...
The FCNF problem is NP-hard and thus no efficient solution techniques exist which
guarantee optimality (Garey and Johnson 1979). Branch-and-bound (B&B) (Land and Doig
1960) is commonly employed to solve the FCNF problem exactly (Driebeek 1966, Kennington
and Unger 1976, Barr et al. 1981, Cabot and Erenguc 1984, Palekar and Zionts 1990). B&B
is in general a very important algorithm in integer programing. The algorithm finds solutions
by iteratively solving various relaxations of the problem to find integer feasible solutions. At
each iteration the lower bounds on the objective value (based on linear relaxations) and
upper bounds (based on feasible solutions) may be updated. As the gap between the bounds
decreases, the B&B approaches an optimal solution. However, such a technique is inefficient
in solving FCNF instances which lack tight bounds (e.g. due to high fixed costs). Therefore,
many researchers are interested in developing approximate methods for the FCNF problem.

Sun et al. (1998) combined Tabu Search with the simplex on a graph method (Kennington
1980) to solve the FCTP. To help avoid local minima, their tabu implementation used both
recency and frequency based memories and the simplex method was used to guide the .
Adlakha and Kowalski (2010) developed a heuristic algorithm based on Balinski’s work
(1961). This algorithm proposed a simple way to obtain valid upper bounds by solving related
simple linear programs. This method is efficient with small sized problems. Monteiro et al.
(2011) proposed a hybrid heuristic, using ant colony optimization to search broadly and as
well as a local search method. Molla-Alizadeh-Zavardehi et al. (2011) proposed an artificial
immune algorithm and a genetic algorithm based on the spanning tree and pr¨ ufer number
representation to solve the FCTP in a two-stage supply chain network. Antony Arokia
Durai Raj and Rajendran (2012) applied genetic algorithm two scenarios of transportation
problems, the first one was the FCTP and the second one considered opening costs of the
distribution centers. Machine learning based approach is first developed by Zhang and
Nicholson (2016b), Nicholson and Zhang (2016), Zhang and Nicholson (????) which makes
a significant contribution to the field.

Kim and Pardalos (1999) developed the dynamic slope scaling procedure (DSSP) from
the viewpoint of marginal cost and applied it solving the FCNF problem. The main idea is to
linearize the objective function and to solve the problem iteratively, adapting the linearization
at each step. DSSP removes the binary variables and incorporates the variable cost and fixed
cost into the cost coefficient, which is updated by the previous linear programing solution.
DSSP has been improved by combining with other methods: trust interval techniques (Kim and Pardalos 2000), intensification/diversification mechanisms (Crainic et al. 2004), Tabu Search (Kim et al. 2006, Gendron et al. 2003) and parameterized DSSP (Nicholson and Barker 2014). Several types of FCNF variants, including concave piecewise linear network flow problem (Kim and Pardalos 2000), multicommodity location problem (Gendron et al. 2003), multicommodity FCNF (Crainic et al. 2004), stochastic integer programming problem (Shiina 2012).

This paper extends the work of Nicholson and Barker (2014) to explore a variety of metaheuristics search implementation for the parametrized dynamic slope scaling procedure (Ψ-DSSP). The contribution in this paper is three-fold: (1) to explore the potential increase in the FCNF heuristic solution quality by using more precise parameter values for Ψ-DSSP than have been examined to-date, (2) compare multiple well-known search metaheuristics applied to this problem with respect to solution quality and efficiency, and (3) evaluate solution efficacy by technique and parameter refinement across a spectrum of difficult FCNF instances. The remainder of this paper is organized as follows: Section 2 briefly provides the background knowledge of fixed charge network flow problem and parameterized dynamic slope scaling procedure; Section 3 details three metaheuristics techniques and the corresponding implementations with respect to Ψ-DSSP; Section 4 describes the experimental design to verify the new approach; Section 5 analyzes the results from experiments and Section 6 summarizes the findings.

2. Background

2.1 FCNF Problem Description

The single-commodity, uncapacitated, fixed charge network flow problem can be defined on a directed graph \( D = (N, A) \), where \( N \) is the set of nodes and \( A \) is the set of arcs \( (i, j) \). Each node \( i \in N \) has a commodity requirement value \( R_i \) (\( R_i > 0 \) for supply nodes, \( R_i < 0 \) for demand nodes, \( R_i = 0 \) for transshipment nodes). Each arc \( (i, j) \in A \) has an associated fixed and variable cost \( f_{ij} \) and \( c_{ij} \), respectively. The decision variable \( x_{ij} \) denotes the flow on arc \( (i, j) \in A \). The fixed cost on arc \( (i, j) \) is the cost of using the arc for any positive flow, \( x_{ij} > 0 \). The binary decision variables \( y_{ij} \) for all \( (i, j) \in A \) are used to model which arcs are selected for commodity flow. The aim of the FCNF is to select a subset of arcs to be opened and determine the flow on the arcs such that the supply in the network is routed to meet
the demand at a minimal total cost. The FCNF is formulated as a traditional mixed binary programing problem as follows:

\[
\begin{align*}
\text{min} & \quad \sum_{(i,j) \in A} (c_{ij}x_{ij} + f_{ij}y_{ij}) \\
\text{s.t.} & \quad \sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ji} = R_i \quad \forall i \in N \\
& \quad 0 \leq x_{ij} \leq M_{ij}y_{ij} \quad \forall (i,j) \in A \\
& \quad y_{ij} \in \{0, 1\} \quad \forall (i,j) \in A
\end{align*}
\]

The objective in (1) is nonlinear. The constraints in (2) ensure that the inflow and outflow of a node satisfy the supply/demand at node \(i \in N\). \(M_{ij}\) is an arc capacities (artificial or real) are used in the constraints (3) to ensure that the fixed cost \(f_{ij}\) is incurred whenever there is a positive flow on arc \((i,j)\). If any arc \((i,j)\) is not capacitated, then \(M_{ij}\) can be set to a sufficiently large value to not inhibit feasible flow. The constraints in (4) define \(y_{ij}\) as binary for all \((i,j) \in A\).

### 2.2 Parameterized Dynamic Slope Scaling Procedure

The dynamic slope scaling procedure replaces the variable cost and fixed cost with a linearized cost coefficient, which is updated iteratively by the marginal fixed cost. Let \(\bar{x}_{ij}^k\) denote the flow value of arc \((i,j)\) at solution iteration \(k\),

\[
\bar{x}_{ij}^k = \begin{cases} 
  x_{ij}^k & x_{ij}^k > 0 \\
  x_{ij}^{k-1} & x_{ij}^k = 0
\end{cases} \quad \forall (i,j) \in A.
\]

Let \(\bar{c}_{ij}^{k+1}\) denote the linearized cost coefficient of arc \((i,j)\) at iteration \(k + 1\),

\[
\bar{c}_{ij}^{k+1} = c_{ij} + \frac{f_{ij}}{\bar{x}_{ij}^k}
\]

The FCNF objective can be formulated at each iteration \(k\):

\[
\text{min} \quad \sum_{(i,j) \in A} \left( c_{ij} + \frac{f_{ij}}{\bar{x}_{ij}^{k-1}} \right) x_{ij}^k
\]

The algorithm stops when no more improvements can occur, or equivalently when

\[
\bar{x}_{ij}^{k-1} = \bar{x}_{ij}^k \quad \forall (i,j) \in A.
\]
The objective value associated with the original problem formulation \( P \) is computed at each iteration. The objective associated with the best solution across all DSSP iterations is returned (e.g. not necessarily the last iteration).

Nicholson and Barker (2014) provided solid evidence that a small modification of the linearization technique can considerably alter the algorithm’s search path and significantly enhance the solution quality over the DSSP. The revised algorithm is parameterized by a value \( \psi \) such that the corresponding problem formulation for the \( k \)-th iteration of \( \Psi \)-DSSP is

\[
\min_{(i,j) \in A} \left( c_{ij} + \psi \frac{f_{ij}}{x_{ij}^{k-1}} \right) x_{ij}^k
\]

\[\text{s.t. } \sum_{(i,j) \in A} x_{ij}^k - \sum_{(j,i) \in A} x_{ji}^k = R_i \quad \forall i \in N \]

\[0 \leq x_{ij}^k \leq M_{ij} \quad \forall (i,j) \in A\]

In their work they examined various values of \( \psi \in (0, 2) \) and ultimately tested solution quality on difficult FCNF problems for five fixed values, \( \psi = \{0.25, 0.5, 0.75, 1, 1.25\} \). The results were promising with maximum improvements of \( \Psi \)-DSSP over DSSP nearing 30%. The paper establishes the sensitivity of solution quality with respect to the \( \psi \) parameter and also demonstrates there is no clear pattern relating \( \psi \) with solution efficacy for different problem instances.

## 3. Metaheuristics for Searching \( \psi \) Values

Since the best \( \psi \) values vary among instances and affect the solution significantly, in this study metaheuristics are employed as a search procedure to find good \( \psi \) values which improve the solution quality of the FCNF problem in reasonable time. While several techniques are possible, we choose two classic single-solution based metaheuristic algorithms, simulated annealing (Kirkpatrick et al. 1983) and tabu search (Glover 1986), and one population-based metaheuristic algorithm, particle swarm optimization (Kennedy et al. 1995). The details of each implementation are discussed below. For notational convenience, let \( z = \text{DSSP}(\psi) \) denote the best objective value from all iterations in the \( \Psi \)-DSSP approach for the parameter value equal to \( \psi \).
3.1 Simulated Annealing

Simulated annealing (SA) is a well known and effective stochastic search algorithm useful on non-linear and discontinuous problems spaces (Kirkpatrick et al. 1983, Henderson et al. 2003). While SA was originally developed for discrete problems, it has been successfully extended to continuous optimization problems (e.g. Bohachevsky et al. 1986, Dekkers and Aarts 1991). This technique is inspired by the physical annealing process in which a crystalline solid is heated and cooled slowly in such a way to improve the structural integrity of the material. Essentially, at the beginning of the process, the high temperatures allows the atoms to move freely and move to a state of minimum energy during the cooling process. Similarly for optimization problems, SA allows for a diverse search at the beginning of the process which may include very poor solutions. This diversity allows the process to escape local minima. At later iterations, as the “temperature” decreases, the search becomes more restrictive and focused on solutions which improve the objective value. For this investigation we will employ the Boltzmann annealing (Van Laarhoven and Aarts 1987) and very fast annealing (L. 1989) implementations to find appropriate values for \( \psi \). A brief explanation of the procedure follows.

Let \( \psi_0 \) denote the initial value for the DSSP parameter and \( z_0 \) denote the corresponding best solution value for that parameter setting. Let the value \( T_0 \) denote the initial temperature in the simulated annealing method. At each iteration \( i \), choose a value \( \psi_c \in [\psi_i - \frac{\epsilon}{2}, \psi_i + \frac{\epsilon}{2}] \) as a candidate parameter in the “neighborhood” of \( \psi_i \), where \( \epsilon > 0 \) is the range of the neighborhood. (The method for choosing \( \psi_c \) from the neighborhood is detailed below.) Evaluate the candidate parameter and compare the solution quality with that of the solution found using \( \psi_i \). If \( z_c \leq z_i \), then the candidate parameter is “accepted” by setting \( \psi_{i+1} = \psi_c \). Otherwise, the candidate solution is accepted with probability

\[
p = \exp \left[ 1 - \left( \frac{z_c}{z_i} \right) \right]
\]  

where \( T_i \) is updated by different functions based on the SA annealing implementation:

Boltzmann: \( T_i = \frac{T_0}{\log(1 + i)} \)  

Very Fast Annealing: \( T_i = T_0 e^{\left( \frac{-i}{\tau} \right)} \).

In equation \( 8 \), the probability computed is proportional to the percent change between \( z_c \) and \( z_i \). This ensures the acceptance criteria for a given temperature setting is the same.
for a candidate objective value which is 10% higher than current objective, regardless of the magnitude of the objectives. Also note in equations (9) and (10) various factors are available to further tune the search procedure. We have set these factors to 1 for simplicity.

If the candidate solution is not accepted, the procedure evaluates up to a pre-specified number of other candidate values (denoted $i_{dwell}$) before continuing to the next iteration and decreasing the temperature value. Selecting a value $\psi_c$ from the neighborhood of $\psi_i$ is executed differently depending on the implementation. For $u_i$ chosen randomly and uniformly on $(0, 1)$ and for $v_i$ randomly selected from the standard normal distribution, the neighbor values are chosen as follows:

Boltzmann: $\psi_c = \begin{cases} 
\psi_i + \frac{1}{2} v_i \sqrt{T_i} & \text{if } \sqrt{T_i} < \frac{2}{3} \\
\psi_i + \frac{1}{3} v_i & \text{otherwise}
\end{cases}$

Very Fast Annealing: $\psi_c = \begin{cases} 
\psi_i + T_i \left[ \left(1 + \frac{1}{T_i}\right)^{|2u_i-1|} - 1 \right] & \text{if } u_i < 0.5 \\
\psi_i - T_i \left[ \left(1 + \frac{1}{T_i}\right)^{|2u_i-1|} - 1 \right] & \text{otherwise}
\end{cases}$

Pseudo code for $\Psi$-DSSP combined with simulated annealing is shown in Figure 1. The stopping criterion are based on maximum iterations ($i_{max}$) or maximum total runtime ($t_{max}$).

### 3.2 Tabu Search

Tabu Search (TS) was developed by Glover (1977, 1986, 1989, 1990). The algorithm improves on basic hill-climbing (a greedy method which always seeks improved solutions) by allowing the search to move away from good solutions in attempt to escape local optima. Once the search moves to a new solution in the neighborhood, the previous solution is added to a tabu list which forbids the search to return the previous states for a certain number of iterations. TS is an effective approach for combinatorial problems such as graph coloring (Dubois and De Werra 1993) and the linear ordering problem (Duarte et al. 2011). The algorithm has been extended to continuous optimization problems by developing a paradigm of concentric hyperspheres (Siarry and Berthiau 1997) or hypercubes (Chelouah and Siarry 2000) to define the neighborhood of a solution.

The implementation of TS for the $\Psi$-DSSP problem is for a one dimensional, continuous search space. We employ the neighborhood definition of Siarry and Berthiau (1997) as
**Data:** \( \Psi\text{-DSSP instance, } i_{\text{max}}, t_{\text{max}}, T_0, \psi_0 \)

**Result:** \( \psi_{\text{best}}, z_{\text{best}} \)

begin

\[
\begin{align*}
    z_0 &\leftarrow \text{DSSP}(\psi_0) \\
    z_{\text{best}} &\leftarrow z, \psi_{\text{best}} &\leftarrow \psi_0 \\
    i &\leftarrow 0, t &\leftarrow 0 \\
\text{while } i < i_{\text{max}} \text{ and } t < t_{\text{max}} \text{ and early termination is false do} &\text{ do} \\
    \quad i &\leftarrow i + 1 \\
    \quad j &\leftarrow 0, t &\leftarrow \text{total runtime} \\
    \quad \text{update } T_i \\
    \quad \text{while } j < i_{\text{dwell}} \text{ and } \psi_i \neq \psi_c \text{ do} &\quad \text{do} \\
    \quad \quad j &\leftarrow j + 1 \\
    \quad \quad \psi_c &\leftarrow \text{choose neighbor of } \psi_i \\
    \quad \quad z_c &\leftarrow \text{DSSP}(\psi_c) \\
    \quad \quad \text{if } z_c < z_{\text{best}} \text{ then} &\quad \text{if} \\
    \quad \quad \quad z_{\text{best}} &\leftarrow z_c, \psi_{\text{best}} &\leftarrow \psi_c \\
    \quad \quad \quad z_i &\leftarrow z_c, \psi_i &\leftarrow \psi_c &\text{ else} \\
    \quad \quad \text{else} &\quad \text{else} \\
    \quad \quad \quad \text{generate } u \sim U(0, 1) &\quad \text{generate } u \sim U(0, 1) \\
    \quad \quad \quad p &\leftarrow \exp \left[ \frac{1 - \left( \frac{z_i}{z_c} \right)}{T_i} \right] &\quad p &\leftarrow \exp \left[ \frac{1 - \left( \frac{z_i}{z_c} \right)}{T_i} \right] \\
    \quad \quad \quad \text{if } u < p \text{ then} &\quad \text{if } u < p \text{ then} \\
    \quad \quad \quad \quad \psi_i &\leftarrow \psi_c &\quad \psi_i &\leftarrow \psi_c &\text{ Set early termination boolean value} \\
\end{align*}
\]

return \( \psi_{\text{best}}, z_{\text{best}} \)

Figure 1: Simulated Annealing for \( \Psi\text{-DSSP Implementation} \)
a series of $k$ concentric intervals. For each iteration $i$, the neighborhood of $\psi_i$ are the $k$ concentric bands $B = \{B_1, B_2, \ldots, B_k\}$, where

$$B_j = \{\psi_c | h_{j-1} \leq ||\psi_c - \psi_i|| \leq h_j\}$$ \hspace{1cm} (11)$$

and $h_0, h_1, \ldots, h_k$ is an increasing positive sequence of values. The value for $h_k$ is chosen as the maximum absolute difference between a given solution and its furthest neighbor. The values $h_j$ are determined according to a geometric series

$$h_{k-j} = \frac{h_k}{2^j} \hspace{1cm} \text{for } j = 1, 2, \ldots, k - 1$$

The value $h_0$ is the smallest positive value in the sequence, but is not related directly to $h_k$. It is used in the algorithm to specify the minimum distance between two $\psi$ values for those two values to be considered distinct.

In iteration $i$, the $k$ neighbors of $\psi_i$ are selected by randomly choosing one point from each $B_j$ for $j = \{1, 2, \ldots, k\}$. An example with $k = 3$ is depicted in Figure 2. The neighbor value $\psi_{i,1}$ is selected from the set of intervals closest to $\psi_i$, the value $\psi_{i,2}$ is selected from the second closest set of intervals, and so on. Pseudo code for $\Psi$-DSSP combined with tabu search is provided in Figure 3.

### 3.3 Particle Swarm Optimization

Artificial life (Adami 1998) simulates natural biotic system behavior with the help of computer or other abiotic system (Wilke and Adami 2002). One main branch of artificial intelligence is evolutionary computing (Back et al. 1997) inspired by Darwin’s theory of evolution. Another important branch is swarm intelligence (Bonabeau et al. 1999) inspired by social group behavior. Swarm optimization is similar to evolutionary algorithms (e.g. genetic algorithm) in that it is a stochastic population-based metaheuristic. Swarm optimization differs in that the same population persists throughout many iterations and the members of the population adapt their behavior based on their own history and from “learning” from other members. In particle swarm optimization (PSO) (Kennedy et al. 1995, Eberhart and
Data: Ψ-DSSP instance, $i_{max}, t_{max}, k, n, h_0, h_k, \psi_0$

Result: $\psi_{best}, z_{best}$

begin
$z_0 \leftarrow \text{DSSP}(\psi_0)$
$z_{best} \leftarrow z_0, \psi_{best} \leftarrow \psi_0$
$i \leftarrow 0, t \leftarrow 0$
$\text{tabulist} \leftarrow \text{Null}$

while $i < i_{max}$ and $t < t_{max}$ and early termination is false do
    Form concentric bands $\{B_1, \ldots, B_k\}$ around $\psi_i$ using $h_0, \ldots, h_k$
    for $j = 1, \ldots, k$ do
        randomly select $\psi_{i,j}$ from $B_j$ s.t. $\| \psi_{i,j} - \psi_{tabu} \| > h_0 \ \forall \psi_{tabu} \in \text{tabulist}$
        $\text{index} \leftarrow \text{arg min} \ \text{DSSP}(\psi_{i,j})$
        $z_{bestcandidate} \leftarrow \text{DSSP}(\psi_{i,\text{index}})$
        $\psi_i \leftarrow \psi_{i,\text{index}}$
        if $z_{bestcandidate} < z_{best}$ then
            $z_{best} \leftarrow z_{bestcandidate}$
            $\psi_{best} \leftarrow \psi_i$
        add $\psi_i$ to end of $\text{tabulist}$
    if $|\text{tabulist}| > n$ then
        remove the first tabu element
    $i \leftarrow i + 1, t \leftarrow \text{total runtime}$
    Set early termination boolean value
return $\psi_{best}, z_{best}$

Figure 3: Tabu Search for Ψ-DSSP Implementation
Kennedy 1995), particles “fly” through the solution space analogous to how birds flock or fish swarm. In PSO, each particle has a position vector, velocity vector, and fitness value. The particle’s position is a point in the solution space, the velocity determines (with stochastically) which solution the particle will move to next, and the fitness is the objective value of the current solution. Each particle maintains a record of the value and position of its individual best historical fitness value. The swarm also maintains the global best historical fitness value and position. The algorithm uses these pieces of information to inform all particles and update their positions. Figure 4 graphically depicts the particle influences during a single iteration. The final updated position derives from a linear combination of influences from its own iteration history, the swarm history, and the particle’s current velocity vector.

PSO has been empirically shown to outperform genetic algorithms with respect to speed of convergence and in some cases quality of the solutions [Angeline 1998, Hassan et al. 2005]. Furthermore, PSO has fewer parameters to refine than competing evolutionary algorithms which makes this approach even more appealing.

Let \( n \) denote the total number of particles and \( \psi_{u,i} \) denote the position of particle \( u \) in iteration \( i \). Let \( i_{\text{max}} \) denote the maximum number of iterations. In iteration \( i \), the fitness of particle \( u \) is \( z_{u,i} \) and is computed by solving \( \Psi \)-DSSP with \( \psi_{u,i} \). The best historical position and objective for a given particle is maintained as \( \psi_{\text{pbest}_u} \) and \( z_{\text{pbest}_u} \), respectively. Similarly, \( \psi_{\text{gbest}} \) and \( z_{\text{gbest}} \) denote the global best historical values for particle positions and objective values for the swarm. Let \( v_{u,i} \) denote the velocity of particle \( u \) at iteration \( i \) and at the end of iteration \( i \), \( v_{u,i} \) is updated according to

\[
\begin{align*}
    w_i & = w_{\text{max}} - \frac{i}{i_{\text{max}}} \left( w_{\text{max}} - w_{\text{min}} \right) \\
    v_{u,i+1} & = w_i v_{u,i} + c_1 r_1 (\psi_{\text{pbest}_u} - \psi_{u,i}) + c_2 r_2 (\psi_{\text{gbest}} - \psi_{u,i})
\end{align*}
\]

where \( w_{\text{max}}, w_{\text{min}}, c_1, \) and \( c_2 \) are control the “inertia” and “acceleration” of the particles and are useful in tuning the search. The inertia parameter \( w_i \) in (12) decreases linearly with
respect to the total number of permissible iterations, beginning at \( w_{max} \) and ending at \( w_{min} \). This encourages a diverse search during the beginning iterations and convergence in the later iterations. The values for \( c_1 \) and \( c_2 \) are constants which reflect the movement influence associated with a given particle’s historical best position and the swarm’s best historical position. The parameters \( r_1 \) and \( r_2 \) are two uniformly random numbers between 0 and 1. Then the updated position for particle \( u \) is computed by equation (14),

\[
\psi_{u,i+1} = \psi_{u,i} + v_{u,i+1}.
\] (14)

Pseudo code for \( \Psi \)-DSSP combined with particle swarm optimization to solve FCNF is provided in Figure 5.
Data: Ψ-DSSP instance, $i_{max}, t_{max}, n, w_{max}, w_{min}, v_{max}, c_1, c_2$
Result: $\psi_{gbest}, z_{gbest}$

begin
for $u = 1, \ldots, n$ do
    generate $\psi_{u,0} \sim U(0, 2)$
    generate $v_{u,0} \sim U(-v_{max}, v_{max})$
    $\psi_{pbest,u} \leftarrow \psi_{u,0}$
    $z_{pbest,u} \leftarrow \infty$
    $z_{gbest} \leftarrow \infty, \psi_{gbest} \leftarrow \text{Null}$

$i \leftarrow 0, t \leftarrow 0$
while $i < i_{max}$ and $t < t_{max}$ and early termination is false do
    for $u = 1, \ldots, n$ do
        $z_{u,i} \leftarrow \text{DSSP}(\psi_{u,i})$
        if $z_{u,i} \leq z_{pbest,u}$ then
            $z_{pbest,u} \leftarrow z_{u,i}$
            $\psi_{pbest,u} \leftarrow \psi_{u,i}$
            if $z_{u,i} \leq z_{gbest}$ then
                $z_{gbest} \leftarrow z_{u,i}$
                $\psi_{gbest} \leftarrow \psi_{u,i}$

    $w_i \leftarrow w_{max} - \frac{i(w_{max} - w_{min})}{i_{max}}$
    for $u = 1, \ldots, n$ do
        generate $r_1, r_2 \sim U(0, 1)$
        $v_{u,i+1} \leftarrow w_i v_{u,i} + c_1 r_1 (\psi_{pbest,u} - \psi_{u,i}) + c_2 r_2 (\psi_{gbest} - \psi_{u,i})$
        $\psi_{u,i+1} \leftarrow \psi_{u,i} + v_{u,i+1}$

    $i \leftarrow i + 1, t \leftarrow \text{total runtime}$
    Set early termination boolean value
return $\psi^g, z^g$

Figure 5: Particle Swarm Optimization for Ψ-DSSP Implementation
4. Computational Experiments Design

4.1 Network Characteristics

In order to evaluate the efficiency and solution quality from the various metaheuristic approaches to the Ψ-DSSP problem, the experimental design includes tests on a variety of network sizes, each which has characteristics corresponding to difficult FCNF problem instances (e.g. high fixed to variable cost ratio). The tests includes networks with 25, 50, and 100 nodes. For each level of node quantity $n$, we generate 60 feasible FCNF instances in which the number of arcs $m$, is randomly selected. Specifically, we randomly choose $n-1 \leq \frac{m}{2} \leq \frac{n(n-1)}{2}$ and create a connected network instance where each of the $\frac{m}{2}$ undirected arcs is replaced by two directed arcs.

According to Nicholson and Barker (2014), the gap between the naïve DSSP objective and optimal value is related to the network characteristics. In our experimentation we focus on the instances in which DSSP performed poorly as identified in their work. We also use the same network characteristics measures and instance specification as Nicholson and Barker (2014). The percentage of supply, demand, and transshipment nodes are respectively randomly selected with approximate probabilities 0.2, 0.2 and 0.6. The probabilities are approximate in that adjustments are made to ensure an instance is feasible. The variable costs and fixed costs for each link are randomly assigned on $U(0, 20)$ and $U(20000, 60000)$, respectively. The total requirements for each supply node is randomly assigned on $U(1000, 2000)$. The total requirements of supply node is distributed randomly as negative requirements to the demand nodes. Gurobi 5.6 is used to solve the linear programing problems.

To describe the network characteristics let $d = \frac{m}{2\binom{n}{2}}$ denote the density of the network, $S$ denote the total supply, and $\rho_s$ and $\rho_d$ represent the percentage of supply and demand nodes, respectively. Let $\theta = \frac{S}{n_s}$ indicate the average supply for each supply node. Let $\phi$ denote the overall network ratio of fixed to variable costs,

$$\phi = \frac{\sum_{(i,j) \in A} f_{ij}}{\sum_{(i,j) \in A} c_{ij}}$$

where $f_{ij}$ and $c_{ij}$ denote the fixed cost and variable cost of arc $(i, j)$. Let $\gamma = \frac{\theta}{\phi}$ be a characteristic designed to provide an apriori estimate of the ratio of the cost components of a feasible solution. That is, since $\theta$ is proportional to the magnitude of arc flow in a feasible
solution, then

\[ \gamma = \frac{\theta}{\phi} = \frac{\sum_{(i,j) \in A} \theta_{c_{ij}}}{\sum_{(i,j) \in A} f_{ij}} \]

is defined so that if \( \gamma \gg 1 \), variable costs are likely more important than fixed costs; and if \( \gamma \ll 1 \), then fixed costs are likely to have a larger influence on the optimal solution.

Table 1 summarizes the network characteristics (mean, standard deviation, minimum, maximum, and skewness) for the test instances. Figure 6 shows the corresponding histograms of each characteristic. The least dense instance \( (d = 0.03) \) is composed of 100 nodes and 316 arcs, whereas the most dense network is composed of 100 nodes and 9690 arcs. The test cases include network instances with 6% to 56% of nodes as supply nodes, and from 5% to 36% as demand nodes. The supply quantity per supply node ranges from about 120 to nearly 1,800. The fixed to variable network cost ratio averages to about 4,000. The value for \( \gamma \) ranges from nearly 0 to 0.5.

### 4.2 Parameters Setting in Metaheuristics

For any metaheuristic implementation, the specific tuning parameters and stopping criterion play a significant role in the efficacy of the technique. Our testing does not evaluate the full range of the manifold possibilities, however we have attempted to define reasonable and comparable settings for the techniques we use. Three stopping criterion are employed in all the techniques. We classify these as hard criteria or early termination criteria. The hard criterion is twofold: (i) maximum time limit \( t_{\text{max}} \) (1 hour) and (ii) the maximum number of allowable iterations \( i_{\text{max}} \) (which is set equal to \( t_{\text{max}} \) divided by the time it takes...
Figure 6: Distribution of Network Characteristics
Table 2: Metaheuristic Settings

| Metaheuristic                  | Parameter Settings                                                                 |
|-------------------------------|-------------------------------------------------------------------------------------|
| Simulated Annealing           | max iterations at a given temperature: $i_{dwell} = 3$                             |
|                               | initial temperature: $T_0 = 0.25$                                                  |
| Tabu Search                   | minimum neighbor distance: $h_0 = 0.01$                                            |
|                               | maximum neighbor distance: $h_k = 0.2$                                              |
|                               | maximum length of tabulist: $n = 5$                                                 |
|                               | quantity of neighbors: $k = 5$                                                      |
| Particle Swarm Optimization   | number of particles: $n = 10$                                                       |
|                               | maximum velocity: $v_{max} = 1$                                                     |
|                               | inertia weight: $w_{min} = 0.4, w_{max} = 0.9$                                      |
|                               | acceleration coefficients: $c_1 = c_2 = 2$                                          |

to solve the network instance using naïve DSSP). The $i_{max}$ and $t_{max}$ are two hard stop criteria, but we can not compare the efficiency of different algorithms by these criteria since the best solution maybe found before meeting $i_{max}$ or $t_{max}$. An early termination criterion is employed which stops the search when the objective value is not improved for several iterations. After extensive experimentation and suggestions from literature in section 3, we set the metaheuristic parameters as listed in Table 2.

5. Results Analysis

In this section, we report the correlation between solution quality performance and network characteristics, correlation between $\psi$ value and improvement, and the time efficiency of the different approaches: simulated annealing using the Boltzmann annealing (SAB), simulated annealing using “very fast annealing” (SAVF), tabu search on continuous space using concentric bands (TS), and particle swarm optimization (PSO). Figure 7 shows the histograms of the best empirical $\psi$ values across all 180 instances for each method. The best identified parameter value spans nearly the entire search space on [0, 2].

Let $z_{DSSP}$ denote the best objective value resulting from the naïve DSSP, and $z_{SAB}$, $z_{SAVF}$, $z_{TS}$, $z_{PSO}$ denote the best objective value resulting from $\Psi$-DSSP and the associated search technique. Let $z_{gap}$ denote the percentage improvement of a given test approach compared
Figure 7: Distribution of Best Parameter Value
to DSSP. That is, $z_{\text{gap}}$ is the percent decrease in the objective value

$$z_{\text{gap}} = \frac{z_{\text{DSSP}} - z_x}{z_{\text{DSSP}}} \times 100\%.$$ 

where $z_x$ is one of $z_{\text{SAB}}$, $z_{\text{SAVF}}$, $z_{\text{TS}}$, or $z_{\text{PSO}}$. The parameterized DSSP using metaheuristic search outperforms naïve DSSP in 680 (94.37%) cases with an overall mean improvement of 11.83%. Figure 8 shows the gap distribution for $z_{\text{gap}} > 0$. The improvement percentage ranges from 0.2% to 24.2%. 24% is the maximum....

Table 3 reports the percentage of instances in which Ψ-DSSP outperforms DSSP, and the corresponding mean, min, and max $z_{\text{gap}}$ overall and for each metaheuristic search approach for each level of nodes for the FCNF instances. The results between different metaheuristics are quite close. The average improvement overall and for each individual approach is around 10%. The percentage of cases in which Ψ-DSSP with metaheuristics outperforms DSSP increases with the number of nodes and achieves 100% when the number of nodes equals to 100. However, the magnitude of the gap itself for instances with $z_{\text{gap}} > 0$ shows no statistical difference with respect to the number of nodes ($p$-value = 0.91). Furthermore, there is no significant difference in the average solution gap by search technique. Figure 9 shows the gap distribution by number of nodes for the problem instances with improvement.
Figure 9: Distribution of Gap by Number of Nodes

Table 3: Gap Statistics

|                  | Nodes         | 25   | 50   | 100  | Overall |
|------------------|---------------|------|------|------|---------|
|                  |               |      |      |      |         |
| Overall $z_{gap} > 0$ (%) |               | 86.0 | 98.3 | 100.0| 94.9    |
| mean $z_{gap}$    |               | 10.7 | 10.3 | 9.6  | 10.2    |
| max $z_{gap}$     |               | 24.2 | 23.9 | 22.0 | 24.2    |
| SAB $z_{gap}^{SAB} > 0$ (%) |               | 86.7 | 98.3 | 100.0| 95.0    |
| mean $z_{gap}^{SAB}$ |               | 10.7 | 10.5 | 9.6  | 10.2    |
| max $z_{gap}^{SAB}$ |               | 23.9 | 23.9 | 17.0 | 23.9    |
| SAVF $z_{gap}^{SAVF} > 0$ (%) |               | 86.7 | 98.3 | 100.0| 95.0    |
| mean $z_{gap}^{SAVF}$ |               | 10.4 | 10.0 | 10.0 | 10.1    |
| max $z_{gap}^{SAVF}$ |               | 24.2 | 22.5 | 22.0 | 24.2    |
| TS $z_{gap}^{TS} > 0$ (%) |               | 86.7 | 98.3 | 100.0| 95.0    |
| mean $z_{gap}^{TS}$ |               | 10.8 | 10.1 | 9.4  | 10.1    |
| max $z_{gap}^{TS}$  |               | 24.2 | 23.7 | 20.3 | 24.2    |
| PSO $z_{gap}^{PSO} > 0$ (%) |               | 85.0 | 98.3 | 100.0| 94.4    |
| mean $z_{gap}^{PSO}$ |               | 10.9 | 10.5 | 9.5  | 10.3    |
| max $z_{gap}^{PSO}$ |               | 23.1 | 23.9 | 20.3 | 23.9    |
Table 4: Correlation Analysis

| Variable | $z_{SAB}$ | $p$-value | $z_{SAVF}$ | $p$-value | $z_{TS}$ | $p$-value | $z_{PSO}$ | $p$-value |
|----------|-----------|-----------|------------|-----------|----------|-----------|----------|-----------|
| $d$      | 0.31      | 1.86E-05  | 0.26       | 4.87E-04  | 0.23     | 1.80E-03  | 0.24     | 1.07E-03  |
| $\rho_s$ | -0.09     | 0.21      | -0.10      | 0.20      | -0.07    | 0.33      | -0.10    | 0.19      |
| $\rho_d$ | -0.06     | 0.40      | -0.02      | 0.78      | -0.10    | 0.20      | -0.09    | 0.24      |
| $\theta$ | 0.01      | 0.94      | -0.05      | 0.48      | -0.03    | 0.68      | -0.04    | 0.59      |
| $\phi$   | -0.05     | 0.54      | -0.08      | 0.30      | -0.06    | 0.39      | -0.07    | 0.37      |
| $\gamma$ | 0.01      | 0.90      | -0.05      | 0.54      | -0.03    | 0.73      | -0.04    | 0.64      |

The Pearson correlations listed in Table 4 show that only density of the network has a statistically significant positive correlation with $z_{\text{gap}}$. As the density increases, the complexity of the FCNF rises and the $z_{\text{gap}}$ increases regardless of the search method employed. No significant correlations exist between the gap and the remaining descriptors. That is, with the exception of network density, the improvement attainable from using search techniques is relatively robust with respect to the range of network characteristics evaluated in this study. There is also no correlation between $z_{\text{gap}}$ and the parameter value itself (pearson correlation coefficient = 0.004). Figure 10 plots the 720 $\psi$ values and solution gap pairs. The best $\psi$ value varies with different problem instances.

The various search implementations with $\Psi$-DSSP each produce notable improvements over DSSP. To compare the techniques we consider three elements: $z_{\text{gap}}$, CPU time, and total iterations. Each technique was allowed to run for a maximum of 1 hour, however some criteria allow for early termination. Terminating early is a good quality assuming the solution is also good. However, early termination due to quick convergence to relatively poor local optimum is not a desirable feature. To account for this we all report the average solution quality per algorithm iteration. Specifically, let $s$ denote the total number of parameterized DSSP solutions evaluated for a given search procedure. Let $r_x$ represent the solution efficiency defined as the solution gap per iteration, $r_x = \frac{z_{\text{gap}}}{s_x}$, and $t_x$ denote the CPU time for each search technique $x \in \{\text{SAB, SAVF, TS, PSO}\}$. For example, a typical $\Psi$-DSSP with PSO search might use 10 particles with 100 updates each, resulting in 1,000 DSSP solutions. If this procedure produced a gap of 15%, then $r_{\text{PSO}} = \frac{15}{1000} = 0.015$.

Table 5 reports the average CPU time and average gap per iteration ($r_x$) for each of the solution approaches for each level of nodes. The Tukey HSD tests results are reported if the
$p$-value of the ANOVA test is smaller than 0.05. For the problem instances with 25 nodes, the swarm optimization outperformed the other methods, although the average time for all methods was relatively small. Statistically, all four methods had identical performance for the problem instances with 50 nodes. For the largest problem sizes, on average both PSO and TS converged after about 30 minutes of runtime. This was significantly faster than either of the two simulated annealing implementations (48 minutes for SAB; 1 hour and 9 minutes for SAVF).
Table 5: ANOVA and Tukey Test

| Node Level | Metric          | Mean | Tukey Result                  |
|------------|-----------------|------|------------------------------|
|            |                 | SAB  | SAVF | TS  | PSO  |                     |
| 25         | CPU time (sec)  | 14   | 16   | 11  | 7    | $t_{PSO} < t_{TS} < t_{SAB} = t_{SAVF}$ |
|            | $r \times 10^{-4}$ | 22   | 17   | 25  | 43   | $r_{PSO} > r_{TS} > r_{SAB} = r_{SAVF}$ |
| 50         | CPU time (sec)  | 249  | 282  | 166 | 249  | no significant difference |
|            | $r \times 10^{-4}$ | 8    | 7    | 9   | 8    | no significant difference |
| 100        | CPU time (sec)  | 2889 | 4114 | 1934| 1597 | $t_{PSO} = t_{TS} < t_{SAB} < t_{SAVF}$ |
|            | $r \times 10^{-4}$ | 3    | 2    | 3   | 5    | $r_{PSO} > r_{TS} = r_{SAB} = r_{SAVF}$ |

6. Conclusions

The FCNF problem is a classically NP-hard problem with many real-world applications. Due to its complexity, many techniques have been developed to approximate solutions quickly. The dynamic slope scaling procedure and the more general parameterized variation, $\Psi$-DSSP are two such approaches. In this study we address three questions with respect to these two techniques. First, using a highly refined parameter setting, we quantify reasonable expectations for solution improvement of $\Psi$-DSSP over DSSP. Empirically we find solution quality to be improved by a statistically significant 10% on average, and up to 24% as a maximum across various sized network problems. Approximately 95% of the instances benefit from a refined parameter value.

Secondly, we evaluate the performance of multiple metaheuristic search algorithms with regard to this problem class. Implementations of simulated annealing, tabu search, and particle swarm optimization all ultimately find equally good parameter values for $\Psi$-DSSP in our testing. In terms of efficiency, there are differences. PSO on average converges faster than the competing techniques (e.g. about half of the time as simulated annealing) and has the highest average “improvement per iteration”.

Finally, we find that the solution quality of $\Psi$-DSSP is relatively robust with respect to a wide spectrum of FCNF network characteristics for each search technique. Each method is tested on 180 different network instances with various quantities of nodes, arcs, commodity supplies and demands, and values for fixed and variable costs. Arc density is the only feature with a statistically significant correlation to solution quality, i.e. as the density increases, there is evidence that $\Psi$-DSSP further outperforms naïve DSSP.
We conclude that employing a metaheuristic strategy, especially PSO, as a search technique to accompany Ψ-DSSP is a reasonable approach. There is no obvious relationship between network characteristics and an optimal ψ value for the parameterized DSSP. However, the improvement in solution quality is notable and most complex FCNF instances benefit from a refined parameter setting.
References

Adami, C. 1998. *Introduction to artificial life*, vol. 1. Springer.

Adlakha, V., K. Kowalski. 2010. A heuristic algorithm for the fixed charge problem. *Opsearch* **47**(2) 166–175.

Angeline, P.J. 1998. Evolutionary optimization versus particle swarm optimization: Philosophy and performance differences. *Evolutionary Programming VII*. Springer, 601–610.

Antony Arokia Durai Raj, K., C. Rajendran. 2012. A genetic algorithm for solving the fixed-charge transportation model: Two-stage problem. *Computers & Operations Research* **39**(9) 2016–2032.

Armacost, A.P., C. Barnhart, K.A. Ware. 2002. Composite variable formulations for express shipment service network design. *Transportation science* **36**(1) 1–20.

Back, T., D.B. Fogel, Z. Michalewicz. 1997. *Handbook of evolutionary computation*. IOP Publishing Ltd.

Balinski, M.L. 1961. Fixed-cost transportation problems. *Naval Research Logistics Quarterly* **8**(1) 41–54.

Barr, R.S., F. Glover, D. Klingman. 1981. A new optimization method for large scale fixed charge transportation problems. *Operations Research* **29**(3) 448–463.

Bohachevsky, I., M. Johnson, M. Stein. 1986. Generalized simulated annealing for function optimization. *Technometrics* **28** 209–217.

Bonabeau, E., M. Dorigo, G. Theraulaz. 1999. *Swarm intelligence*. Oxford.

Cabot, A.V., S.S. Erenguc. 1984. Some branch-and-bound procedures for fixed-cost transportation problems. *Naval Research Logistics Quarterly* **31**(1) 145–154.

Chelouah, R., P. Siarry. 2000. Tabu search applied to global optimization. *European Journal of Operational Research* **123**(2) 256–270.

Crainic, T.G., B. Gendron, G. Hernu. 2004. A slope scaling/lagrangean perturbation heuristic with long-term memory for multicommodity capacitated fixed-charge network design. *Journal of Heuristics* **10**(5) 525–545.
Dekkers, A., E. Aarts. 1991. Global optimization and simulated annealing. *Mathematical Programming* **50** 367–393.

Driebeek, N.J. 1966. An algorithm for the solution of mixed integer programming problems. *Management Science* **12**(7) 576–587.

Duarte, A., M. Laguna, R. Martí. 2011. Tabu search for the linear ordering problem with cumulative costs. *Computational Optimization and Applications* **48** 697–715.

Dubois, N., D. De Werra. 1993. Epcot: an efficient procedure for coloring optimally with tabu search. *Computers & Mathematics with Applications* **25**(10) 35–45.

Eberhart, R., J. Kennedy. 1995. A new optimizer using particle swarm theory. *Micro Machine and Human Science, 1995. MHS’95., Proceedings of the Sixth International Symposium on*. IEEE, 39–43.

Garey, M. R., David S. Johnson. 1979. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman, New York, NY.

Gendron, B., J.Y. Potvin, P. Soriano. 2003. A tabu search with slope scaling for the multi-commodity capacitated location problem with balancing requirements. *Annals of Operations Research* **122**(1-4) 193–217.

Glover, F. 1977. Heuristics for integer programming using surrogate constraints. *Decision Sciences* **8**(1) 156–166.

Glover, F. 1986. Future paths for integer programming and links to artificial intelligence. *Computers & Operations Research* **13**(5) 533–549.

Glover, F. 1989. Tabu search part i. *ORSA Journal on computing* **1**(3) 190–206.

Glover, F. 1990. Tabu search part ii. *ORSA Journal on computing* **2**(1) 4–32.

Hassan, R., B. Cohanim, O. De Weck, G. Venter. 2005. A comparison of particle swarm optimization and the genetic algorithm. *Proceedings of the 1st AIAA multidisciplinary design optimization specialist conference*. 18–21.

Henderson, D., S. Jacobson, A. Johnson. 2003. The theory and practice of simulated annealing. F. Glover, G. Kochenberger, eds., *Handbook of Metaheuristics, International Series in Operations Research & Management Science*, vol. 57. Springer US, 287–319.
Hirsch, W.M., G.B. Dantzig. 1954. The fixed charge problem. Tech. rep., DTIC Document.

Jarvis, J.J., R.L. Rardin, V.E. Unger, R.W. Moore, C.C. Schimpeler. 1978. Optimal design of regional wastewater systems: A fixed-charge network flow model. *Operations Research* **26**(4) 538–550.

Kennedy, J., R. Eberhart, et al. 1995. Particle swarm optimization. *Proceedings of IEEE international conference on neural networks*, vol. 4. Perth, Australia, 1942–1948.

Kennington, J., E. Unger. 1976. A new branch-and-bound algorithm for the fixed-charge transportation problem. *Management Science* **22**(10) 1116–1126.

Kennington, R.V., J. and Helgason. 1980. *Algorithms for Network Programming*. John Wiley & Sons, Inc., New York, NY, USA.

Kim, D., X. Pan, P.M. Pardalos. 2006. An enhanced dynamic slope scaling procedure with tabu scheme for fixed charge network flow problems. *Computational Economics* **27**(2-3) 273–293.

Kim, D., P. M. Pardalos. 1999. A solution approach to the fixed charge network flow problem using a dynamic slope scaling procedure. *Operations Research Letters* **24** 195–203.

Kim, D., P.M. Pardalos. 2000. Dynamic slope scaling and trust interval techniques for solving concave piecewise linear network flow problems. *Networks* **35**(3) 216–222.

Kirkpatrick, S., C. Gelatt, M. Vecchi. 1983. Optimization by simulated annealing. *Science* **220** 671–680.

L., Ingber. 1989. Very fast simulated re-annealing. *Mathematical and Computer Modelling* **12** 967–973.

Land, A. H., A. G. Doig. 1960. An automatic method of solving discrete programming problems. *Econometrica: Journal of the Econometric Society* 497–520.

Lederer, P.J., R.S. Nambimadom. 1998. Airline network design. *Operations Research* **46**(6) 785–804.

Molla-Alizadeh-Zavaredehi, S., M. Hajiaghaei-Keshteli, R. Tavakkoli-Moghaddam. 2011. Solving a capacitated fixed-charge transportation problem by artificial immune and genetic
algorithms with a prüfer number representation. *Expert Systems with Applications* 38(8) 10462–10474.

Monteiro, M.S.R., D. B.M.M. Fontes, F.A.C.C. Fontes. 2011. An ant colony optimization algorithm to solve the minimum cost network flow problem with concave cost functions. *GECCO*. 139–146.

Nicholson, C., K. Barker. 2014. Parameterized dynamic slope scaling for fixed-charge network flows. *in processing*.

Nicholson, C., W. Zhang. 2016. Optimal network flow: A predictive analytics perspective on the fixed-charge network flow problem. *Computers & Industrial Engineering* 99 260–268.

Nozick, L.K. 2001. The fixed charge facility location problem with coverage restrictions. *Transportation Research Part E: Logistics and Transportation Review* 37(4) 281–296.

Palekar, M.H., U.S. and Karwan, S. Zionts. 1990. A branch-and-bound method for the fixed charge transportation problem. *Management Science* 36(9) 1092–1105.

Shiina, T. 2012. Dynamic slope scaling procedure for stochastic integer programming problem. *International Journal of Economics and Management Sciences* 6.

Siarry, P., G. Berthiau. 1997. Fitting of tabu search to optimize functions of continuous variables. *International Journal for Numerical Methods in Engineering* 40(13) 2449–2457.

Steinberg, E., H.A. Napier. 1980. Optimal multi-level lot sizing for requirements planning systems. *Management Science* 26(12) 1258–1271.

Sun, M., J. E. Aronson, P. G. McKeown, D. Drinka. 1998. A tabu search heuristic procedure for the fixed charge transportation problem. *European Journal of Operational Research* 106(2) 441–456.

Van Laarhoven, P.J.M., E.H.L. Aarts. 1987. *Simulated annealing*. Springer.

Wilke, C.O., C. Adami. 2002. The biology of digital organisms. *Trends in Ecology & Evolution* 17(11) 528–532.

Zhang, W., P.i Lin, N. Wang, C. Nicholson, X. Xue. 2018. Probabilistic prediction of post-disaster functionality loss of community building portfolios considering utility disruptions. *Journal of Structural Engineering* 144(4) 04018015.
Zhang, W., C. Nicholson. 2016a. A multi-objective optimization model for retrofit strategies to mitigate direct economic loss and population dislocation. *Sustainable and Resilient Infrastructure* **1**(3-4) 123–136.

Zhang, W., C. Nicholson. 2016b. Prediction-based relaxation solution approach for the fixed charge network flow problem. *Computers & Industrial Engineering* **99** 106–111.

Zhang, W., N. Wang. 2016. Resilience-based risk mitigation for road networks. *Structural Safety* **62** 57–65.

Zhang, W., N. Wang. 2017. Bridge network maintenance prioritization under budget constraint. *Structural safety* **67** 96–104.

Zhang, W., N. Wang, C. Nicholson. 2017. Resilience-based post-disaster recovery strategies for road-bridge networks. *Structure and Infrastructure Engineering* **13**(11) 1404–1413.

Zhang, W., Z. Yao. 2010. A reformed lattice gas model and its application in the simulation of evacuation in hospital fire. *Industrial Engineering and Engineering Management (IEEM), 2010 IEEE International Conference on*. IEEE, 1543–1547.

ZHANG, W., L. ZHAO. 2010. Lattice gas model for simulating pedestrian evacuation in the dormitory fire [j]. *Journal of Safety and Environment* **1** 045.