D-brane probes, RR tadpole cancellation and K-theory charge

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Abstract

We study RR charge cancellation consistency conditions in string compactifications with open string sectors, by introducing D-brane probes in the configuration. We show that uncancelled charges manifest as chiral gauge anomalies in the world-volume of suitable probes. RR tadpole cancellation can therefore be described as the consistency of the effective compactified theory not just in the vacuum, but also in all topological sectors (presence of D-brane probes). The result explains why tadpole cancellation is usually much stronger than anomaly cancellation of the compactified theory (in the vacuum sector). We use the probe criterion to construct consistent six-dimensional orientifolds of curved K3 spaces, where usual CFT techniques to compute tadpoles are not valid. As a last application, we consider compactifications where standard RR charge cancels but full K-theory charge does not. We show the inconsistency of such models manifests as a global gauge anomaly on the world-volume of suitable probes.
1 Introduction

A complete set of consistency conditions for superstring theories with open string sectors seems to be provided by the requirements of open-closed world-sheet duality, and cancellation of RR tadpoles. The latter condition admits several interpretations from diverse points of view. From the world-sheet perspective, studied in [1], uncancelled RR tadpoles lead to inconsistency due to world-sheet superconformal anomalies. From the ten-dimensional spacetime viewpoint, cancellation of tadpoles corresponds to consistency of the equations of motion for certain (unphysical) RR fields, or equivalently to cancellation of RR charge under $p$-form fields with compact support. An alternative spacetime interpretation is that RR tadpole cancellation ensures cancellation of chiral anomalies in the low-energy effective theory. The connection between tadpoles and anomalies first arose in the context of ten-dimensional type I theory [2], and is confirmed by the construction of anomaly-free type IIB orientifolds [3, 4] in lower dimensions (see [3, 4, 7] for early references, and [8, 9, 10, 11] for recent constructions). More direct comparisons in compactified models have been performed in [2, 3].

In this paper we introduce D-brane probes in diverse compactifications with open string sectors, and explore aspects of the above two spacetime interpretations of RR charge cancellation conditions. In the first place, in compactifications below ten dimensions, RR tadpole cancellation conditions are generally much stronger than anomaly cancellation in the compactified effective theory. This is obvious in compactifications with non-chiral spectrum, but also holds for many chiral compactifications [12] (see also [13]). We show in explicit examples that uncancelled RR tadpoles manifest as gauge anomalies on the world-volume of suitable D-brane probes. Interpreting the probes as topological defects in the effective compactified theory, one can argue that full cancellation of RR tadpoles corresponds to consistency of the compactified effective theory not just in its vacuum sector, but in all possible topological sectors. We find this viewpoint interesting, in that it can be applied in quite general situations (curved internal spaces, non-perturbative vacua) where other techniques are not valid. In fact we employ the probe criterion to construct consistent six-dimensional orientifolds of curved K3 manifolds. Also, it emphasizes the viewpoint of the compactified theory, and hence it can be applied to theories with non-geometric internal CFT’s, like the asymmetric orientifolds in [14].

A second direction we explore is the relevance of the fact that D-brane charge is actually classified by K-theory [15, 16], which differs from the naive classification by cohomology in torsion pieces, i.e. the existence of additional discrete charges, typi-
cally associated to stable non-BPS branes (see [17] for a review). Detailed analysis of the K-theoretical nature of RR fields [18] imply that consistency of a model requires cancellation of the full K-theory charge, and not just of the naive part. In fact, we consider some explicit compactifications where standard RR charge cancels but the full K-theory charge does not, and study them using D-brane probes. We show that the inconsistency of the model manifests as global gauge anomalies on the world-volume of suitable probes. Therefore D-brane probes provide a simple technique to verify cancellation of K-theory charge in complicated models, where direct analysis would be untractable. For instance, using suitable probes we derive certain K-theory consistency conditions (discussed in [19] from a different viewpoint) in the type IIB $\mathbb{T}^4/\mathbb{Z}_2$ orientifold in [7, 8].

The paper is organized in several Sections which can be read independently, and we advice readers interested in just one topic to safely skip unrelated discussions. In Section 2 we present some warm-up examples of the introduction of D-brane probes in simple open string compactifications, and verify the relation between RR tadpole cancellation and consistency of the probe world-volume theories. In Section 3 we apply this technique to study six- and four-dimensional toroidal $\mathbb{T}^4/\mathbb{Z}_N$ and $\mathbb{T}^6/\mathbb{Z}_N$ type IIB orientifolds. We show that cancellation of RR tadpoles unrelated to anomaly cancellation in the compactified theory can however be recovered by requiring consistency of suitable D-brane probes. Hence, tadpole conditions are equivalent to consistency of the compactified effective theory in all possible topological sectors. In Section 4 we construct certain six-dimensional orientifolds of type IIB theory on curved K3 spaces, and use the probe criterion to check cancellation of RR charges in the model. In Section 5 we use D-brane probes to study compactifications where naive RR tadpole cancellation is satisfied, but full K-theory charge does not cancel. We show that such theories are inconsistent, and that their inconsistency manifests as a global gauge anomaly on suitable D-brane probes. We also find that the ‘non-perturbative’ consistency conditions proposed in [19] for the model in [7, 8] are of K-theoretic nature, and rederive them using D-brane probes. Section 6 contains some final remarks.

2 Warm-up examples

2.1 Toroidal compactification of type I

The simplest example of open string theory compactification where RR tadpole cancellation is not related to anomaly cancellation in the lower-dimensional theory, is toroidal
compactifications of type I theory. Hence, we consider type IIB on $T^m$ modded out by world-sheet parity $\Omega$, and introduce a number $N$ of background D9-branes, which is constrained to be $N = 32$ by RR tadpole cancellation. For future use, we have in mind $m = 4$ or $m = 6$, even though the argument applies more generally. One obtains a $(10 - m)$-dimensional non-chiral theory, with sixteen supersymmetries and a rank 16 gauge group, equal to $SO(32)$ if no Wilson lines are turned on. From the compactified effective theory viewpoint, there seems to be no natural explanation for this specific rank, i.e. for the requirement of having 32 background D-branes, and the compactified theory would seem to make sense even for other choices.

However, this constraint can actually be understood even from the compactified theory viewpoint, by considering it in topologically non-trivial sectors, like in the background of a string-like topological defect with charge $n$, realized in string theory as a stack of $n$ D1-brane probes spanning two of the non-compact dimensions $1$. We may use string theory techniques to compute the zero modes of this soliton, which is a simple generalization of that in $[21]$. The two-dimensional world-volume field theory has $(0, 8)$ supersymmetry (see $[22, 23]$ for the multiplet structure of two-dimensional theories with diverse supersymmetries). In the 11 sector we obtain a $(0, 8) SO(n)$ gauge multiplet (containing gauge bosons and eight left-handed Majorana-Weyl (MW) fermions), and one $(0, 8)$ chiral multiplet (with eight real scalars and eight right-handed MW fermions) in the two-index symmetric representation. The $19 + 91$ sectors contain (regardless of the possible existence of D9-brane Wilson lines) one left-handed MW fermion in the $(n, N)$ bi-fundamental representation (Fermi multiplet).

Cancellation of gauge anomalies requires $N = 32$, and therefore shows the necessity of imposing the RR tadpole cancellation condition in order to obtain a consistent probe. Equivalently, in order for our compactified theory to be consistent in the relevant topological sector.

Of course, a simpler argument, suggested in $[13]$, is to notice that the full theory contains gauge and gravitational degrees of freedom propagating in ten dimensions, and their ten-dimensional anomalies should cancel $2$, hence $N = 32$. As explained in

1Notice that the probes do not introduce new RR tadpoles, or rather their flux is allowed to escape to infinity along transverse non-compact dimensions, hence $n$ is unconstrained. Also, for $m = 6$, the D1-branes induce a deficit angle and induce asymptotic curvature. To avoid this as an objection, one may introduce as a probe a set of well separated D1-branes and anti-D1-branes. Despite lack of supersymmetry and stability, they form a consistent probe, for which world-volume anomalies should cancel. Their analysis is similar to that with only D1-brane probes, hence we phrase the discussion in the latter terms.

2From the lower-dimensional viewpoint, this corresponds to the existence of certain Ward identities
the introduction, we do not claim the probe argument is the only reason why string theory requires RR tadpole cancellation. However, we find the probe approach is an interesting explanation, which may be easier to apply in certain situations, and which provides an understanding of tadpole conditions from the lower-dimensional viewpoint.

A similarly simple example is type II theory compactified on $T^m$ modded out by $\Omega R$, where $R$ reverses all internal coordinates (and contains $(-1)^{F_L}$ if required). We take IIB theory for $m$ even and IIA for $m$ odd. These models contain a number $N$ of D$(9-m)$-branes, with $N = 32$ in order to cancel RR tadpoles. These theories are related to toroidal compactifications of type I by T-duality along the internal dimensions. Therefore, from the point of view of the compactified theory, the requirement $N = 32$ can be detected by considering a string-like defect (which we denote ‘fat string’) represented in string theory as a stack of $n$ D$(m+1)$-brane wrapped on $T^m$. The computation of the zero modes on the fat string is isomorphic to that above, and $N = 32$ is again obtained as the condition of cancellation of two-dimensional gauge anomalies.

Although simple, these results will be quite helpful in the study of more involved orientifold models, like six- or four-dimensional orientifolds of $T^4/Z_N$ or $T^6/Z_N$, whose discussion we postpone until Section 3.

### 2.2 Configurations of D6-branes on a general Calabi-Yau

It is easy to extend the probe argument to more general compactifications containing D-branes. For instance, let us consider type IIA theory compactified on a Calabi-Yau threefold $X_3$, with a set of D6-branes labeled by an index $a$, in stacks of multiplicity $N_a$ and wrapping a set of 3-cycles $\Pi_a$. We take these 3-cycles to be special lagrangian, and hence supersymmetric [24]. Antibranes are treated on an equal footing with branes by simply considering the wrapped cycle with the opposite orientation. Configurations of D6-branes on 3-cycles on Calabi-Yau threefolds have appeared in a number of references [25, 26]. The particular case of the six-torus appears in [27]. For simplicity we do not consider the introduction of orientifold actions (see [28] for the toroidal case).

Since the D6-branes are sources for the RR 7-form Hodge dual to the IIA 1-form, with charge proportional to the homology class $[\Pi_a]$ of the wrapped cycle, the tadpole cancellation condition is the vanishing of the net charge in the compact space,

$$\sum_a N_a[\Pi_a] = 0$$  \hspace{1cm} (2.1)
From the viewpoint of the compactified effective theory, these conditions do ensure the cancellation of four-dimensional anomalies, but are in fact much stronger conditions. The field theory in four dimensions has the following structure. The $6_a6_a$ sector contains fields in multiplets of the $\mathcal{N} = 1$ supersymmetry preserved by the $D6_a$-brane. It produces $U(N_a)$ vector multiplets, and a set of additional chiral multiplets in the adjoint representation if the 3-cycle $[\Pi_a]$ is not rigid, which do not generate chiral anomalies. In the mixed $6_a6_b$ and $6_b6_a$ sectors we obtain chiral left-handed fermions in the representation $\sum_{a,b} I_{ab} (N_a, N_b)$, where $I_{ab} = [\Pi_a] \cdot [\Pi_b]$ is the intersection number, and a negative multiplicity corresponds to a positive multiplicity of opposite chirality fields (see [28, 27] for a discussion in the toroidal context). The intersection may produce additional light (massless or tachyonic) scalars and vector-like fermions, depending on the local geometry of the D6-branes and $X_3$ around the intersection, but these fields do not contribute to chiral anomalies.

The conditions of cancellation of cubic non-abelian anomalies read

$$
\sum_b N_b [\Pi_a] \cdot [\Pi_b] = 0 \quad \text{for all} \ a
$$

Since the set of cycles $[\Pi_a]$ need not be complete, this does not necessarily imply $\sum_b N_b [\Pi_b] = 0$. It simply states that the overall homology class has zero intersection with any of the individual homology classes. This condition can be achieved even if the total homology class is nonzero, leading to the construction of models where RR tadpole conditions are not satisfied even though the inconsistency does not show up as a breakdown of gauge invariance in the low energy field theory.

As in previous examples, it is possible to make the inconsistency manifest even from the lower-dimensional viewpoint, by considering it in suitable soliton backgrounds, equivalently introducing suitable probes in the configuration. For instance, we may consider the introduction of fat strings obtained by wrapping $n_i$ D4-branes on a 3-cycle $[\Lambda_i]$, where $\{ [\Lambda_i] \}$ is a basis of supersymmetric 3-cycles. The field theory on the $i^{th}$ probe has $(0,2)$ supersymmetry, and we may use standard string theory arguments to obtain the two-dimensional world-volume fields. In the $4_i4_i$ sector, fields fill out $(2,2)$ multiplets, and produce $U(n_i)$ gauge symmetries, but do not generate any two-dimensional anomalies. In the mixed $4_i6_a$ and $6_a4_i$ sector we obtain a set of left MW fermions in the representation $(n_i, N_a)$, with multiplicity $[\Lambda_i] \cdot [\Pi_a]$ (following the usual

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3As follows from [28], the local anomaly at each intersection is cancelled by the inflow mechanism, namely the violation of charge due to the chiral fermion anomaly at each intersection is cancelled by a charge inflow from the branes (see [30] for string computations of the relevant couplings). From this perspective, conditions (2.2) ensure the global consistency of the inflow mechanism, namely cancellation of inflows from different intersections into a given brane [27] (see [31] for a similar effect).
convention on its sign). Cancellation of the two-dimensional non-abelian anomaly on the probes leads to the constraints

$$\left[\Lambda_i\right] \cdot \sum_a N_a[\Pi_a] = 0 \text{ for all } i \tag{2.3}$$

which, since $\{[\Lambda_i]\}$ is a complete set, require $\sum_a N_a[\Pi_a] = 0$, i.e. the tadpole cancellation condition (2.1).

We conclude this section with two short comments. The inclusion of orientifold planes in the construction would lead to non-zero contributions on the right hand side of the tadpole equation (2.1). The corresponding modification in (2.3) would arise from chiral anomalies arising from the $4_i 4_i$ sector due to the different action of the orientifold projection of the corresponding chiral fermions.

A second comment concerns how general is the observation that RR tadpole cancellation is equivalent to anomaly cancellation on D-brane probes. The above models suggest the following approach, for the case of geometric compactifications. Let $[\Pi]$ denote total RR charge arising from orientifold planes and D-branes in the configuration (filling non-compact spacetime completely, but possibly only partially wrapped on the internal space), understood as an element in the homology of the internal space. If non-zero, the inconsistency of the theory can be detected by introducing a fat string obtained as a D-brane with charge Hodge dual (in the internal space) to $[\Pi]$. The non-zero intersection between classes leads to chiral anomalies on the probe world-volume, as in the more explicit models above.

Recent developments indicate that D-brane charges are actually K-theory (rather than (co)homology) classes [15, 16]. In Section 5 we discuss the problem of charge cancellation in certain type IIB orientifold compactification, and its interplay with D-brane probes in the configuration.

The above argument shows that in any geometric compactification RR tadpole conditions can be equivalently described as consistency of all possible probes of the configuration. However, many compactifications do not have a simple (if any at all) geometric interpretation, hence it is important to study explicitly the behaviour of probes in such cases. In next section we center on orientifolds of type IIB toroidal orbifolds, where singularities in the internal space render geometrical techniques much less useful.

### 3 Six- and four-dimensional orientifolds
3.1 Six-dimensional $\mathcal{N} = 1$ orientifolds

In this section we consider $\Omega$ orientifolds of type IIB theory [3, 4] compactified to six-dimensions on toroidal orbifolds $T^4/\mathbb{Z}_N$ (see [1, 2, 3] for early work, and [8, 9] for more recent references), or T-dual versions of these models. As discussed in a number of non-compact examples [32, 33], and in [12] in compact models, cancellation of six-dimensional anomalies is exactly equivalent to cancellation of twisted RR tadpoles. On the other hand, untwisted RR tadpoles are not related to six-dimensional anomalies, even though they are required for consistency of the theory. The necessity of the latter constraints can be made manifest from the six-dimensional viewpoint by considering suitable probes, as we discuss in a particular case for the sake of concreteness.

Let us consider the $\Omega$ orientifold of type IIB theory on $T^4/\mathbb{Z}_3$, studied in [9]. The closed string spectrum contains the $D=6, N=1$ supergravity multiplet, the dilaton tensor multiplet, and eleven hyper- and nine tensor multiplets. Cancellation of untwisted RR tadpoles requires the introduction of 32 D9-branes, and zero net number of D5-branes. For an arbitrary number of D9-branes, the general form of the Chan-Paton embedding of the $\mathbb{Z}_3$ generator $\theta$ is

$$\gamma_{\theta,9} = \text{diag} \left( 1_{N_0}, e^{2\pi i \frac{1}{N_0}}, e^{2\pi i \frac{1}{N_1}} \right)$$

(3.1)

If no Wilson lines are turned on, the six-dimensional open string spectrum contains the following $D=6 \mathcal{N}=1$ multiplets

- Vector $SO(N_0) \times U(N_1)$
- Hyper $\Box_{\theta} + \Box$

(3.2)

Cancellation of six-dimensional gauge and gravitational anomalies requires $N_1 = N_0 + 8$, which agrees with the twisted RR tadpole cancellation condition $\text{Tr} \gamma_{\theta,9} = -8$, but does not fix the total number of D9-branes in the theory. Consideration of further anomalies, like $U(1)$ anomaly cancellation by the generalized Green-Schwarz mechanism in [37], does not lead to new constraints.

Based on our previous experience, however, we argue that the six-dimensional field theory can actually detect the inconsistency of not imposing untwisted RR tadpole conditions. For instance, we may introduce a stack of $n$ D1-brane probes sitting at a generic point in the internal space (hence we include their $\mathbb{Z}_3$ images). The probe is insensitive to the $\mathbb{Z}_3$ twist and its spectrum of zero modes is exactly as in section 2.1. Cancellation of world-volume anomalies requires a total number of 32 D9-branes, $N_0 + 2N_1 = 32$, which is the untwisted tadpole condition. Full RR tadpole cancellation hence fixes $N_0 = 8, N_1 = 16$. 

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The untwisted tadpole condition controlling the net D5-brane charge in the background, if D5-branes or antibranes is allowed, can be obtained by introducing a fat string probe, constructed by wrapping a D5-brane (denoted D5\(_p\)) on the internal space, with generic continuous Wilson lines. For \(\mathbb{Z}_3\) the 5\(_p\)5\(_p\) sector is non-chiral, and the only chiral world-volume fermions arise from mixed 5\(_p\), 5\(_p\)5 sectors of string stretched between the probe and background D5-\(\overline{D5}\)-branes, if present. Two-dimensional anomaly cancellation on the probe requires the number of background D5- and \(\overline{D5}\)-branes to be equal, hence net fivebrane charge should be zero.

In the above argument we have used six-dimensional anomaly cancellation to recover the twisted tadpole condition, but it is easy to recover it using a probe as well. Consider introducing a stack of \(n\) D1-brane probes sitting at the origin in \(T^4/\mathbb{Z}_3\), with the \(\mathbb{Z}_3\) action embedded as

\[
\gamma_{\theta,1} = \text{diag}(1_{n_0}, e^{2\pi i \frac{4}{3} n_1}, e^{2\pi i \frac{2}{3} n_1})
\]

with \(n_0 + 2n_1 = n\). The theory on the D1-brane world-volume has (0, 4) supersymmetry, and contains fields in vector multiplets (gauge bosons and four left MW fermions), chiral multiplets (four real scalars and four right MW fermions), and fermi multiplets (four left MW fermion), as follows

| Sector | (0, 4) multiplet | Representation |
|--------|------------------|----------------|
| 11     | Vector           | \(SO(n_0) \times U(n_1)\) |
|        | Chiral           | \(\mathbb{1} + \text{Adj}_1\) |
|        | Chiral           | \((\mathbb{1}_0, \mathbb{1}_1) + \mathbb{1}_1\) |
|        | Fermi            | \((\mathbb{1}_0, \mathbb{1}_1) + \mathbb{1}_1\) |
| 19 + 91| MW\(_L\) Fermion | \(\mathbb{1}_0; N_0) + (\mathbb{1}_1; N_1) + (\mathbb{1}_1; N_1)\) |

Cancellation of two-dimensional anomalies requires \(N_0 = 16\), \(N_1 = 8\), hence yielding both twisted and untwisted tadpole conditions. Thus, even though in following sections we use vacuum anomaly cancellation to partially test consistency, the corresponding constraints can also be recovered by suitable probes.

The pattern we have seen in the \(\mathbb{Z}_3\) orientifold holds for other similar six-dimensional orientifolds, supersymmetric \([4]\) or not \([36, 34, 38]\). Six-dimensional anomaly cancellation requires cancellation of twisted tadpoles, while cancellation of untwisted tadpoles is required by consistency of suitable string-like configurations. \([4]\) It would be interesting to study probes in other classes of six-dimensional type IIB orientifolds \([38, 34, 40]\). \(^4\)

\(^4\)Note that for even order orbifolds, the 5\(_p\)5\(_p\) sector is chiral and anomaly cancellation requires 32 units of net D5-brane charge (or 32 \(\overline{D5}\)-branes for peculiar orientifold actions \([36, 34]\)).
Instead of considering these extensions, we turn to the four-dimensional case, where the pattern of anomalies vs. tadpoles is more interesting.

### 3.2 Four-dimensional Orientifolds

Four-dimensional orientifolds of type IIB on $\mathbb{I}^6/\mathbb{Z}_N$ or $\mathbb{T}^6/(\mathbb{Z}_N \times \mathbb{Z}_M)$ have the new interesting feature that they contain *twisted* tadpoles whose cancellation is not related to the cancellation of anomalies in the four-dimensional effective theory. Such twisted tadpoles are associated to orbifold twists with fixed planes. For e.g. a set of D3-branes sitting at a point in the fixed plane, cancellation of chiral anomalies is a local effect, which is unrelated to cancellation of the twisted RR tadpole, a global constraint on the charges distributed on the plane. In T-dual versions with branes wrapped on the fixed planes, the twisted RR tadpole condition is related to anomaly cancellation for fields propagating in six dimensions, and is again unrelated to four-dimensional anomalies. In this section we would like to clarify the interpretation of these twisted RR tadpole conditions as anomaly cancellation conditions on suitable D-brane probes. For concreteness we center on a prototypical case, extension to other models being straightforward.

We consider type IIB theory on $\mathbb{T}^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$, with generators $\theta$ and $\omega$ corresponding to the twist eigenvalue vectors $v = \frac{1}{3}(1,0,-1)$, and $w = \frac{1}{3}(0,1,-1)$, respectively. We mod out this model by the orientifold action $\Omega' \equiv \Omega R_1 R_2 R_3 (-1)^{F_L}$, where $R_i : z_i \to -z_i$. This orientifold is T-dual to that considered in [11, 12]. Cancellation of untwisted tadpoles requires the introduction of 32 D3-branes filling the four non-compact dimensions, and no net number of D7-branes. Cancellation of twisted tadpoles at the origin requires the D3-brane Chan-Paton matrices to satisfy

$$
\begin{align*}
\text{Tr} \, \gamma_{\theta_3} & = -4 \\
\text{Tr} \, \gamma_{\theta} & = \text{Tr} \, \gamma_{\omega} = \text{Tr} \, \gamma_{\theta \omega^2} = 8
\end{align*}
$$

The general form of the Chan-Paton matrices, before imposing the constraints from tadpole cancellation, is

$$
\begin{align*}
\gamma_{\theta_3} &= \text{diag} \left( 1_{N_{00}}, 1_{N_{01}}, 1_{N_{02}}, \alpha_1 1_{N_{10}}, \alpha_1 1_{N_{11}}, \alpha_1 1_{N_{12}}, \alpha_1^2 1_{N_{20}}, \alpha_1^2 1_{N_{21}}, \alpha_1^2 1_{N_{22}} \right) \\
\gamma_{\omega_3} &= \text{diag} \left( 1_{N_{00}}, \alpha_1 1_{N_{01}}, \alpha_1^2 1_{N_{02}}, 1_{N_{10}}, \alpha_1 1_{N_{11}}, \alpha_1^2 1_{N_{12}}, 1_{N_{20}}, \alpha_1 1_{N_{21}}, \alpha_1^2 1_{N_{22}} \right)
\end{align*}
$$

The orientifold projection requires $N_{ab} = N_{-a,-b}$, with subindices understood mod 3. We also have $\gamma_{\Omega_3} = \gamma_{\Omega_3}^T$. The unique solution to the tadpole conditions, in the case of locating all 32 D3-branes at the origin, is $N_{00} = 8$, $N_{10} = N_{01} = N_{11} = 4$, $N_{12} = 0$. 

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It is however interesting to derive these tadpole consistency conditions by introducing suitable probes, and requiring cancellation of world-volume gauge anomalies. Clearly, one can easily reproduce the untwisted tadpole conditions by working in analogy with the above six-dimensional case. Introducing fat strings constructed by wrapping D3-branes on the $i^{th}$ complex plane, with generic Wilson lines, and sitting at a generic point in the remaining two, one recovers the condition that no net D7-brane charge (i.e. associated to D7-branes transverse to the $i^{th}$ plane) is allowed in the model. Introducing a D7-brane wrapped on the internal space, with generic Wilson lines, one recovers the condition that the number of D3-branes in the model is 32.

Partial information about the twisted tadpole cancellation conditions can be obtained by considering four-dimensional anomaly cancellation in the vacuum of the compactified theory. Using the general Chan-Paton embedding \( \mathcal{N} = 1 \), the four-dimensional spectrum contains the following set

\[
\begin{align*}
\text{Vector} & \quad SO(N_{00}) \times U(N_{10}) \times U(N_{01}) \times U(N_{11}) \times U(N_{12}) \\
\text{Chiral} & \quad \begin{cases} 
\mathfrak{g}_0 + (\mathfrak{g}_{00}, \mathfrak{g}_{01}) + (\mathfrak{g}_{01}, \mathfrak{g}_{11}) + (\mathfrak{g}_{11}, \mathfrak{g}_{12}) + (\mathfrak{g}_{12}, \mathfrak{g}_{01}) \\
\mathfrak{g}_1 + (\mathfrak{g}_{00}, \mathfrak{g}_{01}) + (\mathfrak{g}_{01}, \mathfrak{g}_{11}) + (\mathfrak{g}_{11}, \mathfrak{g}_{12}) + (\mathfrak{g}_{12}, \mathfrak{g}_{01}) \\
\mathfrak{g}_1 + (\mathfrak{g}_{00}, \mathfrak{g}_{01}) + (\mathfrak{g}_{10}, \mathfrak{g}_{01}) + (\mathfrak{g}_{01}, \mathfrak{g}_{12}) + (\mathfrak{g}_{12}, \mathfrak{g}_{10}) 
\end{cases}
\end{align*}
\]

(3.7)

Following \[12\], cancellation of cubic non-abelian anomalies leads to the condition

\[\text{Tr} \gamma_{\omega,3} = N_{00} - N_{01} - N_{10} - N_{11} + 2N_{12} = -4 \quad (3.8)\]

which is obviously ensured by the tadpole conditions, but is a much milder constraint. As checked in \[12\], cancellation of $U(1)$-nonabelian or $U(1)$-gravitational mixed anomalies \[42\] (see \[43\] for further discussions) do not impose further constraints beyond (3.8).

The remaining twisted RR tadpole cancellation conditions can however be recovered by requiring cancellation of anomalies on fat strings obtained by wrapping D3-branes (denoted D3\(_p\)) on e.g. the third complex plane, with trivial Wilson lines, and located at the origin in the first two planes. Such branes are fixed under the $\mathbb{Z}_3$ action, and the general form of the Chan-Paton action is

\[
\begin{align*}
\gamma_{\theta,3_p} & = -\text{diag} \left( 1_{n_{00}}, 1_{n_{01}}, 1_{n_{02}}, \alpha 1_{n_{10}}, \alpha 1_{n_{11}}, \alpha^2 1_{n_{12}}, \alpha^2 1_{n_{20}}, \alpha^2 1_{n_{21}}, \alpha^2 1_{n_{22}} \right) \\
\gamma_{\omega,3_p} & = -\text{diag} \left( 1_{n_{00}}, \alpha 1_{n_{01}}, \alpha^2 1_{n_{02}}, 1_{n_{10}}, \alpha 1_{n_{11}}, \alpha^2 1_{n_{12}}, 1_{n_{20}}, \alpha 1_{n_{21}}, \alpha^2 1_{n_{22}} \right) \quad (3.9)
\end{align*}
\]

The orientifold projection requires $n_{ab} = n_{-a,-b}$. We have $\gamma_{\Omega',3_p} = -\gamma_{\Omega',3_p}$.

The field theory on the two-dimensional world-volume of the fat string is (0,2) supersymmetric. It contains a set of gauge multiplets (formed by gauge bosons and
two left-handed Majorana-Weyl fermions), chiral multiplets (containing one complex scalar and two right-handed MW fermions), and Fermi multiplets (containing two left-handed MW fermions) (see [22, 23] for a review of supermultiplet structure). In the $3_p3_p$ sector, we obtain the following set of multiplets

**0, 2 multiplet**

| Representation | \(USp(n_{00}) \times U(n_{10}) \times U(n_{01}) \times U(n_{11}) \times U(n_{12})\) |
|----------------|--------------------------------------------------------------------------------------------------|
| Vector         | \(\Box_0 + \text{Adj}_{10} + \text{Adj}_{01} + \text{Adj}_{11} + \text{Adj}_{12}\)               |
| Chiral         | \((\Box_{10} + (\Box_{00}, \Box_{10}) + (\Box_{01}, \Box_{11}) + (\Box_{11}, \Box_{12}) + (\Box_{12}, \Box_{01})\) |
| Fermi          | \((\Box_{01} + (\Box_{00}, \Box_{01}) + (\Box_{10}, \Box_{11}) + (\Box_{11}, \Box_{12}) + (\Box_{12}, \Box_{00})\) |
| Chiral         | \((\Box_{11} + (\Box_{00}, \Box_{11}) + (\Box_{10}, \Box_{01}) + (\Box_{01}, \Box_{12}) + (\Box_{12}, \Box_{01})\) |
| Fermi          | \((\Box_{11} + (\Box_{00}, \Box_{11}) + (\Box_{10}, \Box_{01}) + (\Box_{01}, \Box_{12}) + (\Box_{12}, \Box_{01})\) |

In the $3_p3_p$ sector we have

**0, 2 multiplet**

| Representation | \([\Box_{00}; N_{11}] + (\Box_{01}; N_{10}) + (\Box_{10}; N_{12}) + (\Box_{11}; N_{00}) + (\Box_{11}; N_{10}) + (\Box_{10}; N_{12}) + (\Box_{02}; N_{01})\) |
|----------------|--------------------------------------------------------------------------------------------------|
| Chiral         | \((\Box_{00}; N_{12}) + (\Box_{01}; N_{11}) + (\Box_{10}; N_{10}) + (\Box_{11}; N_{01}) + (\Box_{11}; N_{10}) + (\Box_{10}; N_{12}) + (\Box_{02}; N_{01})\) |
| Fermi          | \((\Box_{00}; N_{12}) + (\Box_{01}; N_{11}) + (\Box_{10}; N_{10}) + (\Box_{11}; N_{01}) + (\Box_{11}; N_{10}) + (\Box_{10}; N_{12}) + (\Box_{02}; N_{01})\) |

The field theory is chiral, and contains potential non-abelian gauge anomalies. The conditions for their cancellation are

\[
N_{11} - N_{12} - 4 = 0 \\
N_{00} + N_{11} - N_{01} - N_{10} - 4 = 0
\]

(3.12)

They amount to the conditions

\[
\text{Tr } \gamma_{\theta, \omega} = -4 ; \quad \text{Tr } \gamma_{\theta, \omega^2} = 8
\]

(3.13)

Namely, besides (3.8), which was already required for cancellation of four-dimensional anomalies, consistency of the probe requires the cancellation of the twisted tadpole associated to the third fixed plane. Notice that the probe we have introduced is not sensitive to the \(\theta\)- or \(\omega\)-twisted tadpoles, since they are not localized at the origin in the first two complex planes (where our probe sits). Clearly, they are required for consistency of other probes, obtained as fat strings from D3-branes wrapped in one of these planes (and at the origin in the other two).
The same results could have been derived by considering other probes, like a D7-brane completely wrapped in $T^6$. Instead of pursuing their study, we conclude our discussion of toroidal orientifolds by restating our basic point. From the viewpoint of the compactified theory, the strong constraints imposed by RR tadpole cancellation arise because they must ensure not just the consistency of the low-energy field theory in the vacuum, but also in non-trivial topological sectors.

4 Application: Orientifolds of curved K3 manifolds

4.1 Smooth fibers

The strategy of testing the consistent cancellation of RR charges in open string vacua by studying probes sensitive to the relevant RR tadpoles can be used in contexts where RR tadpole conditions cannot be directly computed using the familiar CFT rules (factorization of one-loop amplitudes). In this Section we construct some simple examples of orientifolds of curved manifolds, namely K3 manifolds, for which no exact CFT description is available, and use the probe criterion to determine the configuration of D-branes required to achieve RR tadpole cancellation.

In order to keep the models simple, we consider the K3 to be an elliptic fibration over $\mathbb{P}_1$, with a section. Let $[C]$ and $[f]$ denote the homology classes of base and fiber, respectively. Let us consider modding out type IIB theory on such K3 by the action $\Omega' = \Omega R(-1)^{F_L}$, where $R$ acts as $R : [z, w] \rightarrow [-z, w]$ on the projective coordinates of the base $\mathbb{P}_1$, and leaves the elliptic fiber invariant. This action preserves half of the supersymmetries, and has two fixed points on the base, $z/w = 0$ and $z/w = \infty$. We choose a generic K3 so that no singular fiber sits at these points in the base, hence the 24 singular fibers group in 12 $\Omega'$-invariant pairs.

The closed string sector contains the $D = 6, N' = 1$ supergravity and dilaton tensor multiplet. In addition, out the 20 $(1,1)$ two-forms of K3\footnote{Useful information on the structure of 2-cycles in elliptic K3’s can be extracted from \cite{44}.}, the base and fiber are invariant and give two hypermultiplets. Out of the remaining 18, eight are associated to paths in the base between locations in a set of eight singular fibers, while eight are the 2-cycles associated to the $\Omega'$ image singular fibers. These cycles are exchanged by $\Omega'$, and contribute eight hyper- and eight tensor multiplets. Finally, two 2-cycles correspond to paths between a singular fiber and its image. These cycles are invariant and contribute two hypermultiplets.

Clearly, consistency requires the introduction of a number of D7-branes wrapped on
the fiber and sitting at points on the base. We locate them away from the fixed points on the base and from singular fibers. Since their tangent and normal bundles are trivial ($f$ is a two-torus, and $[f] \cdot [f] = 0$), the D7-branes are quite insensitive to the curvature of K3, and lead to $D = 6$, $\mathcal{N} = 2$ supersymmetric non-chiral spectra. Denoting by $N$ the number of $\Omega'$-pairs of D7-branes, at a generic point in moduli space the open string sector contributes with $U(1)^N$ vector multiplets, and $N$ hypermultiplets. The full (open plus closed) spectrum is free of six-dimensional gauge and gravitational anomalies, regardless of the number of D7-branes in the model.

This number is fixed by cancellation of RR charges in the compact space. Instead of determining it directly, we may obtain it by demanding consistency of the world-volume theory on a suitable probe, taken to be a fat string obtained by wrapping $n$ D3-branes on the base $\mathbb{P}_1$, the section of the fibration. In the absence of curvature the two-dimensional probe world-volume would have $(0,8)$ supersymmetry, but the K3 holonomy reduces it to $(0,4)$. In the 33 sector we obtain $(0,4)$ $SO(n)$ vector multiplets (containing one gauge boson, and four left-handed MW spinors), and one $(0,4)$ chiral multiplet (containing two complex scalars and four right-handed MW spinors) in the two-index symmetric representation. The latter parametrizes motion of the fat string in four transverse spacetime dimensions. Note that we do not obtain chiral multiplets associated to internal dimensions in K3 because the curve $C$ wrapped by the D3-branes is rigid, $[C] \cdot [C] = -2$, forbidding transverse motion, and has no constant holomorphic 1-forms, forbidding Wilson lines. In the 37+73 sectors, we find, for each of the $N$ $\Omega'$-invariant pairs of D7-branes, two left handed MW spinor in the fundamental of $SO(n)$. Cancellation of two-dimensional anomalies then requires

$$-4 \times \frac{n-2}{2} + 4 \times \frac{n+2}{2} - 2 \times N \times \frac{1}{2} = 0$$

(4.1)

fixing $N = 8$. Hence the model requires 16 D7-branes (as counted in the covering space) for consistency.

In this simple example, the final result could be obtained by noticing there are two O7-planes in the model, which wrap flat curves with trivial normal bundles, and therefore carry the same charges as in flat space, namely $-8$ units of D7-brane charge for each. To cancel their contribution 16 D7-branes must be introduced, as found above.

Incidentally, the geometry of D7-branes and O7-planes is so simple that it is possible to construct the F-theory lift of this model. Since D7-branes and O7-planes wrap the flat elliptic fiber $f$, along which there are two $U(1)$ isometries, these directions can be ignored and the problem reduces to lifting two O7-planes and eight D7-branes sitting
on $\mathbb{P}_1/\mathbb{Z}_2$, which is another $\mathbb{P}_1$. The F-theory lift produces another elliptic fibration over $\mathbb{P}_1$, with fiber denoted by $f'$. The number of singular fibers is 12, eight arising from the D7-brane pairs in the orientifold quotient, and four arising from the splitting of the each O7-plane into two mutually non-local sevenbranes [46]. The final result is an F-theory compactification on a threefold, obtained as a double elliptic fibration over $\mathbb{P}_1$. As just described, the F-theory fibration with fiber $f'$ has 12 degenerate fibers, and so has the original fibration, with fiber $f$ (arising from the original 24 singular fibers in K3, identified in pairs by the orientifold action). These are precisely the correct numbers to render the threefold Calabi-Yau and preserve eight supercharges. This threefold has appeared in [45], and the generic spectrum of the model ($\mathcal{N} = 1$ supergravity, nine tensor multiplets, eight vector multiplets and twenty hypermultiplets) agrees with that obtained above. The correct F-theory lift serves as a further check of the consistency of the model, and the validity of the probe criterion in general situations.

4.2 Degenerate fibers

The above model can be modified to yield more interesting six-dimensional physics, with chiral gauge sectors. Also, the models below present additional subtleties in the computation of orientifold-plane charges, hence illustrating the usefulness of the probe approach in certain situations.

Let us consider the same $\Omega'$ orientifold of an elliptic K3, but in which the fiber over one of the fixed points of $\Omega'$ in $\mathbb{P}_1$, say $z = \infty$, is smooth, and the fiber over $z = 0$ is singular, necessarily at least of type $I_2$ in Kodaira classification (see [47]). Namely, the two-torus is pinched twice so that topologically it is a set of two two-spheres $C_1$ and $C_2$ touching at two points. The closed string spectrum differs from the generic one in yielding one less hypermultiplet and one more tensor multiplet. The closed string sector leads to gravitational anomalies, which must be cancelled by the open string sector.

Since the orientifold plane is wrapping a reducible curve it may possess different charges under the RR forms obtained by integrating over $C_1$ and $C_2$. This is confirmed by considering the local configuration of an O7-plane wrapped on an $I_2$-degenerated elliptic fiber in K3 and T-dualizing along the unpinched $U(1)$ orbit as in [48]. One obtains a configuration of two NS fivebranes spanning the directions 012345 and one O6-plane along 0123456, in a locally flat space with the direction 6 compactified on a circle. These configurations have been studied in [19] (see [20] for a four-dimensional version) in the context of Hanany-Witten brane constructions [21]. In this situation the O6-plane is known to flip charge as it crosses the NS-branes in the $x^6$ direction.
Hence the original O7-plane carries opposite charges in the two components $C_1$, $C_2$ in the $I_2$ fiber \[.\]

We may expect that the charge difference in the two components must be compensated by D7-branes wrapped on them, in different numbers for $C_1$ and $C_2$. In fact, this nicely follows from cancellation of six-dimensional gauge anomalies on the corresponding D7-brane world-volume. Let us consider $N_1$, $N_2$ D7-branes wrapped on $C_1$, $C_2$, and sitting at $z = 0$. Using the T-dual picture mentioned above, the open string sector leads to the following $D = 6$ $\mathcal{N} = 1$ multiplets

\[
\begin{align*}
\text{Vector} & : SO(N_1) \times USp(N_2) \\
\text{Hyper} & : (N_1, N_2)
\end{align*}
\]

Cancellation of six-dimensional anomalies implies $N_1 - N_2 = 8$, but does not constrain the overall number of D7-branes. This condition also ensures cancellation of gravitational anomalies. Notice the analogy of the condition with twisted RR tadpole conditions of the type encountered in Section 3.1 (in fact the condition arises as a twisted tadpole conditions in the $\mathbb{C}^2/\mathbb{Z}_2$ orientifold mentioned in the footnote 6).

There remains to fix the total number of D7-branes in the model. Notice that a direct computation of the orientifold charge in this case is rather subtle, and that the F-theory lift seems rather involved. Happily, the number $N$ of D7-branes pairs away from $z = 0$ can be obtained by considering the same D3-brane probe as before. Assume for simplicity that the section $C$ intersects the component $C_1$, rather than $C_2$. Then the 33 spectrum is as above, and the number of chiral fermions in the fundamental representation in $37 + 73$ sectors is $N_1 + 2N$, so we obtain

\[N_1 + 2N = 16\]

Actually, the result can be confirmed by considering a transition in which $C_2$ is shrunk to zero size and the $I_2$ degenerate fiber can be sent to non-zero $z$ as two $I_1$ fibers related by $\Omega'$. We are left with $N_1$ D7-branes wrapped on the smooth fiber over $z = 0$, which can also be sent to $z \neq 0$ as $N_1/2 \; \Omega'$ invariant pairs. The final model is of the type studied in section 4.1, with $N + N_1/2$ D7-brane pairs, hence consistency requires $2 \times (N + N_1/2) = 16$, as found here. The process just described is T-dual to that studied in \[.\]

The model admits a simple generalization, by considering $I_{2n_1}$, $I_{2n_2}$ degenerations of the fibers over $z = 0$, $z = \infty$. The closed string spectrum contains the supergravity

\[\text{[52]}\]

\[\text{6The configuration seems to be related, in the limit of one collapsing cycle, to an orientifold of the } \mathbb{C}^2/\mathbb{Z}_2 \text{ singularity studied in [53]. This picture allows a direct computation of these charges, open string spectrum, and twisted tadpoles below.}\]
and dilaton tensor multiplet, and $12 - n_1 - n_2$ hyper- and $8 + n_1 + n_2$ tensor multiplets. Cancellation of tadpoles associated to cycles in the reducible fibers requires to locate at least 8 D7-branes wrapped on suitable components in the fibers over $z = 0, z = \infty$. The number of D7-pairs away from the orientifold points can be seen to be zero, by demanding consistency of a D3-brane probe. The resulting open string spectrum is simply given by $SO(8)^{n_1+n_2}$ vector multiplets, and the full spectrum is free of gauge and gravitational anomalies. Some of these spectra coincide with those in [54], and it would be interesting to explore for more concrete connections with them.

Another interesting extension would be to consider other degenerated fibers over the orientifold fixed points, which presumably lead to gauge sectors of the type constructed in [33, 53]. We leave these and other possibilities for further research.

5 K-theory charges and D-brane probes

Traditionally the consistency conditions imposed on theories with open string sectors has been cancellation of RR charges in a compact space, understood as a condition in (co)homology. As explained in the introduction, the spacetime argument leading to this constraint is consistency of the equation of motion for the RR $p + 1$-form field in the presence of D-brane sources $\delta(W_i)$

$$d \ast H_{p+1} = \sum_i \delta(W_i)$$

when taken in cohomology. Recent developments on non-supersymmetric states in string theory (see [17] for a review) have shown that D-brane charges are however not classified by (co)homology, but by K-theory [16]. There arises the question of whether consistency of a compactification requires cancellation of D-brane K-theory charge or merely its cancellation in cohomology.

To give a concrete example, there exist certain D-brane states in type I string theory which do not carry standard RR charges, but have $\mathbb{Z}_2$ K-theory charges [53, 16]. Since there is no field carrying these charges, the above argument does not seem to apply, and compactifications with an odd number of objects of this kind would seem consistent. On the other hand, given a compact space $X$ and any topological charge sitting a point $P$ in it, we may split $X$ into two pieces $X_1$ and $X_2$, both with boundary $Y$ (but with opposite orientations) and with $P \in X_1$. Physically, it must be possible to measure the topological charge in $X_1$ by measuring quantities in $Y$, and since $Y$ is also the boundary of $X_2$, with opposite orientation, it follows that the opposite charge is contained in $X_2$. Consistency must require cancellation of full D-brane charges, i.e.
K-theory charges.

In fact the criterion of measuring K-theory charges by looking at asymptotics of the configuration has lead in [18] to the conclusion that RR fields are also described by K-theory classes and hence (5.1) applies in K-theory. This description and a detailed analysis of Dirac quantization conditions provided a beautiful unified explanation of the different shifted flux quantization conditions for RR fields.

The conclusion is however exotic from the point of view of usual construction of orientifold models, where the familiar procedure to evaluate RR tadpole consistency conditions by factorizing one-loop amplitudes seems sensitive only to the cohomology part of D-brane charge. We leave the question of analyzing possible subtleties in this procedure in the presence of K-theory charges beyond (co)homological ones as an open issue. Instead, in this Section we present an alternative point of view on the discussion of cancellation of K-theory charge, with arguments more concrete than the above rather formal ones. In particular we consider some simple compactifications where standard RR charge is cancelled but K-theory charge is not, and show that the corresponding inconsistency shows up as global gauge anomalies [56] in suitable D-brane probes.

5.1 Toroidal compactification of type I with non-BPS D7-branes

Let us consider the simplest case. We consider a compactification of type I theory on $T^2$, with a single non-BPS D7-brane (denoted $\hat{D}7$-brane) spanning the eight non-compact dimensions and sitting at a point in $T^2$. The type I $\hat{D}7$-brane does not carry standard RR charges, but carries a K-theory $\mathbb{Z}_2$ charge [16]. It also contains a tachyonic mode, arising from the 79, 97 open strings [57], but this fact does not affect the topological properties of the configuration, and we safely ignore it.

Notice that since the $\hat{D}7$-brane is not charged under the RR fields the inconsistency of the configuration is not too obvious. However, it can be shown as follows. Recall that the $\mathbb{Z}_2 \hat{D}7$-brane charge can be detected by carrying a non-BPS D0-brane around a small circle surrounding the D7-brane location in $T^2$, the D0-brane wavefunction changes sign as proposed in [16] and computed in [58]. But if no other $\mathbb{Z}_2$ charge sits on $T^2$ the contour can be deformed to a small circle around the ‘other side’ of $T^2$, leading to a sign flip in the D0-brane wavefunction without surrounding any $\mathbb{Z}_2$ source, hence an inconsistency. This proves that $\hat{D}7$-branes in compact spaces must exist in pairs so that their K-theory charge cancels.

One can operate similarly for other K-theory charges. For instance, the non-BPS D(-1)-brane
Let us offer another argument based on the use of D-brane probes similar to those in previous sections. Consider probing the previous configuration by a set of $2n$ coincident D5-branes wrapped on $T^2$. Let us compute the fields propagating on the four non-compact dimensions. The $55$, $59$ and $95$ sectors preserve $\mathcal{N} = 2$ supersymmetry, and lead to the following multiplets (as in [59])

\[
\begin{align*}
55 & \quad \mathcal{N} = 2 \text{ Vector} & \quad USp(2n) \\
59 + 95 & \quad \mathcal{N} = 2 \text{ Hyper} & \quad \frac{1}{2}(2n; 32)
\end{align*}
\] (5.2)

the latter decomposed in suitable irreducible representations if $SO(32)$ Wilson lines are turned on. The interesting piece arises from the $57$ and $\overline{75}$ sectors, which are non-supersymmetric. Since the $\overline{D}7$-brane is constructed as a type IIB $D7$-$\overline{D7}$ pair exchanged by $\Omega$ [16], the $57$ and $\overline{75}$ sectors map to the $75$ and $\overline{75}$ sector, so we simply compute the spectrum in the former and do not impose the $\Omega$ projection. Centering on the fermion content, we find one Weyl fermion in the fundamental representation $2n$ of $USp(2n)$.

Even though the resulting field theory is non-chiral, it is inconsistent at the quantum level due to a global gauge anomaly [56]. For compactifications with an arbitrary number $N$ of $\overline{D}7$-branes, the problem is absent precisely when $N$ is even, namely when the $\overline{D}7$-brane $Z_2$ K-theory charge cancels. Hence, cancellation of torsion pieces of K-theory charge can be detected by global gauge anomalies on suitable D-brane probes. We hope this argument makes K-theory considerations more familiar and tractable in the construction of orientifolds, a desirable aim in view of recent introduction of non-BPS branes in type IIB orientifold models [10, 38].

5.2 K-theory charge in type IIB $T^4/Z_2$ orientifolds

In this section we would like to present a more involved but very interesting (and related) example. It is based on the $\Omega$ orientifold of type IIB theory on $T^4/Z_2$ constructed in [4] and [8]. The closed string sector gives the $D = 6$ $\mathcal{N} = 1$ supergravity multiplet, the dilaton tensor multiplet, and 20 hypermultiplets. The model contains 32 D9-branes, which for concreteness we consider without Wilson lines, and a total of 32 D5-branes, which can be located in groups of $k_i$ pairs, $i = 1, \ldots, 16$, $\sum_{i=1}^{16} k_i = 16$, at the sixteen $Z_2$ fixed points $(z_1, z_2)$ with $z_i = 0, 1/2, i/2, (1 + i)/2$. The Chan-Paton amplitudes are weighted by opposite signs on the two sides of a non-BPS type I D8-brane [14, 38]. This implies that e.g. $T^2$ type I compactifications with a single D8-brane are inconsistent due to the impossibility to define ‘sides’ consistently. The number of D8-branes must be even, i.e. the corresponding $Z_2$ charge cancels in the compactification.
embeddings can be taken

\[
\begin{align*}
\gamma_{\theta,9} &= \text{diag} (i1_{16}, -i1_{16}) \\
\gamma_{\theta,5,i} &= \text{diag} (i1_{k_i}, -i1_{k_i}) \\
\gamma_{\Omega,9} &= \begin{pmatrix} 1_{16} \end{pmatrix} \\
\gamma_{\Omega,5,i} &= \begin{pmatrix} 1_{k_i} \\
\end{pmatrix}
\end{align*}
\] (5.3)

The open string spectrum of the model has the following \( D = 6 \) \( \mathcal{N} = 1 \) multiplets

\[
\begin{align*}
99 & \text{ Vector } U(16) \\
& \text{ Hyper } 2 \mathbb{I} \\
55 & \text{ Vector } \prod_{i=1}^{16} U(k_i) \\
& \text{ Hyper } \sum_{i=1}^{16} 2 \mathbb{I} \\
59 + 95 & \text{ Hyper } \sum_{i=1}^{16} (\mathbb{I}; 16)
\end{align*}
\] (5.4)

Any choice of \( k_i \)'s with \( \sum_{i=1}^{16} k_i = 16 \) cancels the RR tadpoles and would seemingly lead to a consistent model. However, it was observed in [19] that there is a further constraint on the possible distributions. Careful analysis of Dirac quantization conditions leads to the constraint that the total number of D5-brane pairs at any four \( \mathbb{Z}_2 \) fixed points lying on a two-plane must be a multiple of two. That is \( \sum_{i'=1}^{4} k_{i'} = 0 \mod 2 \), with \( i' \) labeling fixed points in the plane. In a T-dual picture, the conditions could be rephased as the conditions that Wilson lines on the dual D9-branes should allow the existence of spinor representations, which were known to exist in the model due to type I/heterotic duality. Since at the time the type I description of these states was not known, the conditions were suggested to be of non-perturbative origin. With our present knowledge [55], states in the spinor representation arise from type I non-BPS D0-branes, and the conditions above correspond to requiring a well-defined wavefunction for such states. Namely, it corresponds to cancellation of K-theory \( \overline{D7} \)-brane charge (which appears induced on the D9-branes by the Wilson lines). Hence the requirements in [19] are K-theory charge cancellation conditions, stated before the advent of K-theory.

We conclude by describing how the inconsistency arises in a suitable D-brane probe in the original picture. In this case the use of the D-brane probe argument is easier than the original argument in [19], and we expect it to be more general and useful in more complicated models, or models without geometrical interpretation for the internal space.

Let us introduce a D5-brane, denoted \( \text{D5}_{p} \)-brane to distinguish it from those in the background, wrapped on a two-plane passing through four \( \mathbb{Z}_2 \) fixed points, labeled by \( i' \), in \( T^4/\mathbb{Z}_2 \). Consistency of the \( \mathbb{Z}_2 \) projection in mixed \( 5_{p}5, 5_{p}5, 5_{p}9, 9_{p9} \) sectors requires \( \gamma_{\theta,5_p}^2 = 1 \). The Chan-Paton embeddings have the general form \( \gamma_{\theta,5_p} = \text{diag} (1_{n_0}, -1_{n_1}) \)
and $\gamma_{\Omega,5p} = \text{diag}(\epsilon_{n_0}, \epsilon_{n_1})$. The probe preserves half of the supersymmetries and has $\mathcal{N} = 1$ susy in its four non-compact world-volume dimensions. We obtain the following set of multiplets

$$
\begin{align*}
5_p & \ 5_p & \text{Vector} & \ USp(n_0) \times USp(n_1) \\
\ & \ & \text{Chiral} & \ (\mathbf{4}_i + \mathbf{4}_i + 2 \times (\mathbf{1}_i, \mathbf{1}_i)) \\
5_p & \ 9 + 95_p & \text{Chiral} & \ (\mathbf{1}_i; 16) + (\mathbf{1}_i; 16) \\
5_p & \ 5 + 55_p & \text{Chiral} & \ (\mathbf{1}_i; k_{i'}) + (\mathbf{1}_i; k_{i'})
\end{align*}
$$

(5.5)

Cancellation of global gauge anomaly \[56\] leads to the constraint $\sum_{i'} k_{i'} = 0 \mod 2$, the consistency condition mentioned above.

Hence, we see the additional consistency condition in \[19\] can actually be detected by using by now familiar D-brane probes. Moreover, we have seen in Section 5.1 that global anomalies on D5-brane probes wrapped on two-tori are related to K-theory charge of D7-branes transverse to that plane. Our probe analysis agrees with our previous interpretation of the additional consistency conditions as K-theory charge cancellation conditions. We hope this technique is helpful in studying such consistency conditions in other type IIB orientifolds, or in more general string vacua with open string sectors.

### 6 Conclusions

In this paper we have considered the introduction of D-brane probes in diverse string compactifications with open string sectors, and we have studied the interplay between world-volume gauge anomalies and charge cancellation consistency conditions. We have found that RR tadpole cancellation conditions are equivalent to cancellation of chiral gauge anomalies on suitable probes one may introduce. We find this viewpoint interesting since it provides a rationale, from the compactified effective theory viewpoint, of the strong constraints implied by tadpole cancellation, typically much stronger than cancellation of anomalies in the compactified theory vacuum. A second nice feature of the probe criterion is its wide applicability, which we have illustrated with the construction of consistent orientifolds of curved K3 spaces.

We have employed D-brane probes to explore charge cancellation consistency conditions in compactifications involving torsional K-theory charges. We have found that uncancled charges show up as global gauge anomalies on suitable D-brane probes, and hence consistency requires cancellation of full K-theory charges, in agreement with conclusions from formal considerations \[18\]. Also, quite surprisingly, we have found...
non-trivial K-theory charge cancellation constraints in seemingly innocent type IIB orientifolds, like the $\mathbb{T}^4/\mathbb{Z}_2$ model in [7, 8].

For the purposes of this paper it has been enough to employ a relatively small set of brane probes. In principle, extension to other kinds of probes, either supersymmetric or non-supersymmetric (non-BPS branes, stable or not, or brane-antibrane pairs, either spacetime-filling or not) does not involve any conceptual difference. However, they may lead to simpler analysis in certain situations, hence a more systematic exploration would be desirable.

Along this line, we would like to point out that at present we do not have a systematic characterization of the kind of probe sensitive to a specific tadpole. It would be useful to provide a more detailed map between tadpoles and the corresponding probes. A possible guideline in geometric examples seems to be provided by the intersection pairing in (co)homology or K-theory. It would be interesting to make this idea more explicit in singular geometries, such as those typically encountered in toroidal $\mathbb{T}^n/\mathbb{Z}_N$ orientifolds, and to extend it to non-geometric compactifications (by using the CFT index introduced in [60] or a suitable generalization). We hope to address some of these issues in future work.

Acknowledgements

It is my pleasure to thank G. Aldazabal, S. Franco, L. E. Ibáñez, J. F. Morales, Y. Oz and R. Rabadán for useful conversations. I also thank the Universidad Autónoma de Madrid for hospitality during the completion of part of this work, and M. González for encouragement and support.

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