Geometrical scaling for energies available at the BNL Relativistic Heavy Ion Collider to those at the CERN Large Hadron Collider

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(Dated: July 13, 2018)

Based on the recent RHIC and LHC experimental results, the \( \langle p_T \rangle \) dependence of identified light flavour charged hadrons on \( \sqrt{\frac{\langle s^2 \rangle}{S_\perp}} \), relevant scale in gluon saturation picture, is studied from \( \sqrt{s_{NN}}=7.7 \) GeV up to 5.02 TeV. This study is extended to the slopes of the \( \langle p_T \rangle \) dependence on the particle mass and the \( \langle \beta_T \rangle \) parameter from Boltzmann-Gibbs Blast Wave (BGBW) fits of the \( p_T \) spectra. A systematic decrease of the slope of the \( \langle p_T \rangle \) dependence on \( \sqrt{\frac{\langle s^2 \rangle}{S_\perp}} \) from BES to the LHC energies is evidenced. While for the RHIC energies, within the experimental errors, the \( \langle p_T \rangle/\sqrt{\frac{\langle s^2 \rangle}{S_\perp}} \) does not depend on centrality, at the LHC energies a deviation from a linear behaviour is observed towards the most central collisions. The influence of the corona contribution to the observed trends is discussed. The slopes of the \( \langle p_T \rangle \) particle mass dependence and the \( \langle \beta_T \rangle \) parameter from BGBW fits scale well with \( \sqrt{\frac{\langle s^2 \rangle}{S_\perp}} \). Similar systematic trends for pp at \( \sqrt{s}=7 \) TeV are in a good agreement with the ones corresponding to Pb-Pb collisions at \( \sqrt{s_{NN}}=2.76 \) TeV and 5.02 TeV pointing to a system size independent behaviour.

PACS numbers: 25.75Ag, 25.75.Ld, 25.75Nq, 21.65.Qr

I. INTRODUCTION

Parton density evolution as a function of \( x \) and \( Q^2 \), addressed more than 35 years ago [1] and its experimental confirmation at HERA [2] have triggered a real interest in the community studying ultra-relativistic heavy ion collisions. The rise of the structure function at low \( x \), still the community studying ultra-relativistic heavy ion collisions. In this paper we also present a comparison between pp and Pb-Pb at LHC energies in terms of long range near side two particle correlations, transverse flow and strangeness enhancement as a function of charged particle multiplicity [11,17] support the idea that even in small colliding systems, due to increased parton density at such energies, the probability of multiple parton interaction increases, the rescattering processes become important and a thermalised stage could be reached although the interaction time is extremely short. Such a high density deconfined small system could follow a hydrodynamic type expansion. To what extent the hydrodynamics is applicable in small systems is still under debate [15]. The most successful phenomenological models, UrQMD, HIJING, NeXSPhеРIO, AMPT, PHSD, EPOS, describing the latest results obtained at LHC in pp, p-Pb and Pb-Pb collisions are based on combinations of different approaches for different stages of the collision [19,24] while the classical phenomenological models used in particle physics like PYTHIA [25], HERWIG [26] and PHOJET [27] had to implement processes like multiparton interaction, rescattering, colour reconnection [28] or showing mechanism [29] in order to improve the agreement with the LHC results, especially in the soft sector in pp collisions. In this paper we also present a comparison between pp and Pb-Pb at LHC energies in terms of the dependence of different observables on the \( \sqrt{\frac{dN/dy}{S_\perp}} \) variable. In the second chapter of the paper the estimates of the overlapping area of the colliding hadrons are presented. Details on the hadron density per unit of rapidity are given in the third chapter. The \( \langle p_T \rangle \) dependence on \( \sqrt{\frac{dN/dy}{S_\perp}} \) is presented in Chapter IV for BES and \( \sqrt{s_{NN}}=62.4, 130, 200 \) GeV Au-Au collisions measured by the STAR Collaboration at RHIC and for Pb-
Pb collisions at $\sqrt{s_{NN}}=2.76$ and 5.02 TeV measured by the ALICE Collaboration at LHC. Chapter V is dedicated to the $(dN/dy)/S_\perp$ dependence of the slope of the linear ($p_T$) versus particle mass behaviour for identified light flavour charged hadrons. The BGBW fit parameters of $p_T$ spectra are presented versus the same geometrical variable of the colliding systems in Chapter VI. Similarities, in terms of $(dN/dy)/S_\perp$ dependence of different observables, in pp at $\sqrt{s}=7$ TeV and Pb-Pb at $\sqrt{s_{NN}}=2.76$ and 5.02 TeV are discussed in Chapter VII. Chapter VIII is dedicated to conclusions.

II. OVERLAPPING AREA $S_\perp$ ESTIMATES

The overlapping area of the two colliding nuclei for a given incident energy and centrality was estimated based on the Glauber Monte Carlo (GMC) approach [30-33]. For the nuclear density profile of the colliding nuclei, a Wood-Saxon distribution was considered:

$$\rho(r) = \frac{1}{1 + exp\left(\frac{r-r_0}{a}\right)}$$

with $a=0.535$ fm, $r_0=6.5$ fm for the Au nucleus [34] and $a=0.546$ fm, $r_0=6.62$ fm for the Pb nucleus [35]. Within the black disc approach, the nucleons are considered to collide if the relative transverse distance $d \leq \sqrt{\sigma_{pp}}/\pi$, where $\sigma_{pp}$ is the nucleon-nucleon interaction cross section. The $\sigma_{pp}$ values for the corresponding $\sqrt{s_{NN}}$ energies were taken from [34,37]. The main characteristics of the collision at different centralities for Au-Au at $\sqrt{s_{NN}}=7.7$, 11.5, 19.6, 27 and 39 GeV obtained in the Beam Energy Scan (BES) at RHIC [38]. Au-Au at $\sqrt{s_{NN}}=62.4$ and 200 GeV [34] and Pb-Pb at $\sqrt{s_{NN}}=2.76$ and 5.02 TeV [35,37] are presented in Table I (see caption for notations). The geometrical overlapping areas ($S_{\perp}^{geom}$) have been estimated by averaging the maximum values of the $x$ and $y$ coordinates determined per event, over many events. $S_{\perp}^{var}$ has been estimated as being proportional to the quantity $S=\sqrt{<\sigma_x^2><\sigma_y^2>-<\sigma_{xy}^2>}$, $\sigma_x^2$, $\sigma_y^2$ are the variances and $\sigma_{xy}$ is the co-variance of the participant distributions in the transverse plane, per event [39]. They were averaged ($<...>$) over many events.

| System | $\sqrt{s_{NN}}$ (GeV) | Cen. (%) | $<N_{\text{part}}>$ | $S_{\perp}^{geom}$ $(fm^2)$ | $S_{\perp}^{var}$ $(fm^2)$ | $f_{\text{core}}$ | $(S_{\perp}^{geom})_{\text{core}}$ $(fm^2)$ | $(S_{\perp}^{var})_{\text{core}}$ $(fm^2)$ | $dN/dy$ |
|--------|----------------------|----------|----------------|-----------------|-----------------|----------|-----------------|-----------------|--------|
| Au-Au  | 7.7                  |          | 337±2         | 146.1±0.7       | 174.1±0.7       | 0.08±0.00 | 126.5±0.6       | 124.6±0.6       | 476.7±22.5 |
| Au-Au  | 11.5                 |          | 5-10           | 126.6±0.7       | 192.7±0.6       | 0.84±0.00 | 107.6±0.7       | 105.9±0.5       | 395.2±18.5 |
| Au-Au  | 19.6                 |          | 10-20          | 103.6±0.7       | 108.9±0.5       | 0.80±0.00 | 85.5±0.7        | 84.3±0.4        | 295.4±14.0 |
| Au-Au  | 27                   |          | 20-30          | 79.7±0.8        | 87.1±0.4        | 0.75±0.00 | 63.3±0.7        | 63.8±0.3        | 203.6±9.8  |
|        |                      |          | 50-60          | 45.9±0.8        | 56.9±0.3        | 0.63±0.00 | 33.1±0.8        | 37.5±0.2        | 84.8±4.1   |
|        |                      |          | 50-60          | 32.8±0.8        | 45.7±0.2        | 0.56±0.00 | 21.9±0.8        | 28.9±0.1        | 51.5±2.5   |
|        |                      |          | 70-80          | 21.8±0.8        | 36.3±0.2        | 0.47±0.00 | 12.8±0.7        | 22.6±0.1        | 27.6±1.4   |
|        |                      |          | 70-80          | 12.1±0.7        | 26.8±0.2        | 0.37±0.00 | 5.4±0.6         | 15.6±0.1        | 13.8±0.8   |

| Au-Au  |                      |          | 5-10           | 126.6±0.7       | 192.7±0.6       | 0.84±0.00 | 107.6±0.7       | 105.9±0.5       | 395.2±18.5 |
| Au-Au  |                      |          | 10-20          | 103.6±0.7       | 108.9±0.5       | 0.80±0.00 | 85.5±0.7        | 84.3±0.4        | 295.4±14.0 |
| Au-Au  |                      |          | 20-30          | 79.7±0.8        | 87.1±0.4        | 0.75±0.00 | 63.3±0.7        | 63.8±0.3        | 203.6±9.8  |
| Au-Au  |                      |          | 50-60          | 45.9±0.8        | 56.9±0.3        | 0.63±0.00 | 33.1±0.8        | 37.5±0.2        | 84.8±4.1   |
| Au-Au  |                      |          | 70-80          | 21.8±0.8        | 36.3±0.2        | 0.47±0.00 | 12.8±0.7        | 22.6±0.1        | 27.6±1.4   |
|        |                      |          | 70-80          | 12.1±0.7        | 26.8±0.2        | 0.37±0.00 | 5.4±0.6         | 15.6±0.1        | 13.8±0.8   |

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| Au-Au  |                      |          | 10-20          | 103.6±0.7       | 108.9±0.5       | 0.80±0.00 | 85.5±0.7        | 84.3±0.4        | 295.4±14.0 |
| Au-Au  |                      |          | 20-30          | 79.7±0.8        | 87.1±0.4        | 0.75±0.00 | 63.3±0.7        | 63.8±0.3        | 203.6±9.8  |
| Au-Au  |                      |          | 50-60          | 45.9±0.8        | 56.9±0.3        | 0.63±0.00 | 33.1±0.8        | 37.5±0.2        | 84.8±4.1   |
|        |                      |          | 70-80          | 21.8±0.8        | 36.3±0.2        | 0.47±0.00 | 12.8±0.7        | 22.6±0.1        | 27.6±1.4   |
|        |                      |          | 70-80          | 12.1±0.7        | 26.8±0.2        | 0.37±0.00 | 5.4±0.6         | 15.6±0.1        | 13.8±0.8   |
| Centrality | Value       |
|-----------|-------------|
| 0-5       | 342±2       |
| 5-10      | 294±6       |
| 10-20     | 230±9       |
| 20-30     | 162±10      |
| 30-40     | 111±11      |
| 40-50     | 74±10       |
| 50-60     | 46±9        |
| 60-70     | 26±7        |
| 70-80     | 14±5        |
| 80-90     | 7±3         |
| 90-100    | 3±1         |

**Table 1:** The centrality dependent values were rescaled by the factor obtained dividing the geometrical area to S in the core.
FIG. 1. Overlapping area of the colliding nuclei at different $\sqrt{s_{NN}}$ energies estimated within the GMC approach corresponding to all wounded nucleons $S_4^{\text{geom}}$ $(S_4^{\text{var}})$ - full dots (full squares) and to the core contribution $(S_4^{\text{geom,core}})$ $(S_4^{\text{var,core}})$ - open dots (open squares) as a function of $\langle N_{\text{part}} \rangle$.

III. $dN/dy$ ESTIMATES

The total hadron density per unit of rapidity has been estimated based on the published identified charged hadrons densities [31, 35, 38, 40] and hyperons densities [12, 48]. For $\sqrt{s_{NN}}=19.6$ and 27 GeV BES energies or some of the centralities, where the hyperon yields were not reported, the corresponding values were obtained by interpolation using the energy and centrality dependence fits.

As far as $\Omega^-$ and $\bar{\Omega}^+$ yield values for BES were not reported and the extrapolation from higher energies down

FIG. 2. $\sqrt{(dN/dy)/S_4^{\text{geom}}}$ as a function of $\sqrt{s_{NN}}$ for different centralities based on the values listed in Table I. The dashed lines represent the fit results using a power law function. Dark red and dark blue full dots correspond to pp collision at $\sqrt{s}=7$ TeV, the values being estimated based on the IP-Glasma initial state model, using two values of the $\alpha$ parameter (see Chapter VII). For better clarity, the blue dots were artificially displaced in $\sqrt{s_{NN}}$.

FIG. 3. The percentage of the nucleons suffering a single collision as a function of $\langle N_{\text{part}} \rangle$ and impact parameter for Au-Au collisions at $\sqrt{s_{NN}}=7.7$ and 200 GeV and Pb-Pb collisions at $\sqrt{s_{NN}}=5.02$ TeV.
FIG. 4. a) Top: \( \langle p_T \rangle \) of pions, kaons and protons for all measured energies and centralities at RHIC and LHC reported by the STAR [34,35] and ALICE [35,40] Collaborations, dashed lines representing the results of the first order polynomial fit; bottom: the ratio of the data points to the result of the linear fit for each collision energy, as a function of \( \sqrt{dN/dy}/S^\var_{\perp} \). b) Same as a) but as a function of \( \sqrt{dN/dy}/S^\var_{\perp} \).

to BES energies shows a negligible contribution, they were not considered in the produced hadron density estimates. Therefore, we used the following approximations: for the BES energies \( \frac{dN}{dy} \) \( \simeq \frac{3}{2} \frac{dN}{dy}(p^+ + \pi^-) + \sum_2 \frac{dN}{dy}(K^+ + K^- + \rho + \bar{\rho} + \Xi^- + \bar{\Xi}^+) + \frac{dN}{dy}(\Lambda + \bar{\Lambda} + \Omega^- + \bar{\Omega}^+) \), from \( \sqrt{s_{NN}} = 62.4 \) GeV to \( \sqrt{s_{NN}} = 200 \) GeV \( \frac{dN}{dy} \simeq \frac{3}{2} \frac{dN}{dy}(p^+ + \pi^-) + \sum_2 \frac{dN}{dy}(K^+ + K^- + \rho + \bar{\rho} + \Xi^- + \bar{\Xi}^+) + \frac{dN}{dy}(\Lambda + \bar{\Lambda} + \Omega^- + \bar{\Omega}^+) \) and for the LHC energies \( \frac{dN}{dy} \simeq \frac{3}{2} \frac{dN}{dy}(p^+ + \pi^-) + \sum_2 \frac{dN}{dy}(K^+ + K^- + \rho + \bar{\rho} + \Xi^- + \bar{\Xi}^+) + \frac{dN}{dy}(K^+ + K^- + \rho + \bar{\rho} + \Xi^- + \bar{\Xi}^+) + \frac{dN}{dy}(\Lambda + \bar{\Lambda} + \Omega^- + \bar{\Omega}^+) \). The values are listed in the last column of Table I.

IV. \( \sqrt{dN/dy}/S_{\perp} \) DEPENDENCE OF \( \langle p_T \rangle \)

As it was already mentioned in the Introduction, in the local parton-hadron duality approach [8], \( \langle p_T \rangle/\sqrt{dN/dy}/S_{\perp} \) is proportional with \( \frac{1}{n_{\text{part}}} \) where \( n \) is the number of charged hadrons produced via gluon fragmentation [9,10]. Therefore, neglecting other effects like collective hydrodynamic expansion and suppression, \( \langle p_T \rangle/\sqrt{dN/dy}/S_{\perp} \) is expected to decrease in central collisions and at higher energies. \( \langle p_T \rangle \) for Au-Au collisions at \( \sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39 \) GeV [33], \( \sqrt{s_{NN}} = 62.4, 130, 200 \) GeV [34] and Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76, 5.02 \) TeV [35,40] for positive pions, kaons and protons are represented as a function of \( \sqrt{dN/dy}/S^\var_{\perp} \) in Fig.4a for \( S^\var_{\perp} \) and in Fig.4b for \( S^\var_{\perp} \). The data points corresponding to each collision energy were fitted with a first order polynomial function. The trends in the two figures are rather similar and the fit quality, in terms of Data/Fit ratios, presented in the bottom plots of Fig.4 is equally good. The fit parameters are listed in Table II and Table III for \( S_{\perp} = S^\var_{\perp} \) and \( S_{\perp} = S^\var_{\perp} \), respectively and represented in Fig.5.

The slope value increases from pions to protons. Although the experimental error bars are rather large at the RHIC energies, a systematic decrease of the slopes with the collision energy is evidenced for the \( \langle p_T \rangle \) dependence on \( \sqrt{dN/dy}/S^\var_{\perp} \) - full symbols. This trend is enhanced going from pions to protons. The offset values are rather similar at the RHIC energies and increase for
| √s_{NN} (GeV) | π⁺ | K⁺ | p | π⁺ | K⁺ | p |
|---|---|---|---|---|---|---|
| 7.7 | 0.08 ± 0.02 | 0.15 ± 0.03 | 0.35 ± 0.07 | 0.25 ± 0.03 | 0.30 ± 0.04 | 0.18 ± 0.10 |
| 11.5 | 0.05 ± 0.02 | 0.12 ± 0.03 | 0.33 ± 0.07 | 0.29 ± 0.04 | 0.35 ± 0.05 | 0.16 ± 0.10 |
| 19.6 | 0.05 ± 0.02 | 0.11 ± 0.04 | 0.24 ± 0.05 | 0.30 ± 0.04 | 0.36 ± 0.06 | 0.31 ± 0.08 |
| 27 | 0.05 ± 0.02 | 0.10 ± 0.03 | 0.25 ± 0.05 | 0.31 ± 0.04 | 0.39 ± 0.06 | 0.29 ± 0.08 |
| 39 | 0.05 ± 0.02 | 0.08 ± 0.04 | 0.24 ± 0.05 | 0.31 ± 0.05 | 0.43 ± 0.07 | 0.33 ± 0.09 |
| 62.4 | 0.05 ± 0.01 | 0.12 ± 0.02 | 0.26 ± 0.05 | 0.30 ± 0.03 | 0.38 ± 0.04 | 0.31 ± 0.09 |
| 130 | 0.04 ± 0.02 | 0.09 ± 0.02 | 0.21 ± 0.05 | 0.29 ± 0.04 | 0.42 ± 0.05 | 0.43 ± 0.10 |
| 200 | 0.05 ± 0.02 | 0.13 ± 0.04 | 0.28 ± 0.06 | 0.28 ± 0.04 | 0.33 ± 0.09 | 0.27 ± 0.13 |
| 2760 | 0.03 ± 0.01 | 0.07 ± 0.01 | 0.18 ± 0.02 | 0.39 ± 0.03 | 0.60 ± 0.05 | 0.63 ± 0.05 |
| 5020 | 0.05 ± 0.01 | 0.07 ± 0.01 | 0.19 ± 0.02 | 0.39 ± 0.04 | 0.64 ± 0.04 | 0.63 ± 0.06 |

TABLE II. The parameters for the linear fit of the ⟨p_T⟩ dependence on √s_{NN}/S_{\text{geom}} for pions, kaons and protons corresponding to the energies mentioned in the first column.

| √s_{NN} (GeV) | π⁺ | K⁺ | p | π⁺ | K⁺ | p |
|---|---|---|---|---|---|---|
| 7.7 | 0.05 ± 0.02 | 0.11 ± 0.02 | 0.24 ± 0.05 | 0.29 ± 0.02 | 0.38 ± 0.03 | 0.36 ± 0.06 |
| 11.5 | 0.04 ± 0.02 | 0.09 ± 0.02 | 0.24 ± 0.04 | 0.32 ± 0.02 | 0.42 ± 0.03 | 0.34 ± 0.06 |
| 19.6 | 0.04 ± 0.02 | 0.08 ± 0.02 | 0.17 ± 0.03 | 0.33 ± 0.03 | 0.42 ± 0.04 | 0.46 ± 0.05 |
| 27 | 0.03 ± 0.02 | 0.07 ± 0.02 | 0.18 ± 0.03 | 0.34 ± 0.03 | 0.45 ± 0.04 | 0.44 ± 0.05 |
| 39 | 0.03 ± 0.02 | 0.06 ± 0.03 | 0.17 ± 0.03 | 0.34 ± 0.03 | 0.48 ± 0.04 | 0.48 ± 0.05 |
| 62.4 | 0.03 ± 0.01 | 0.09 ± 0.02 | 0.18 ± 0.03 | 0.33 ± 0.02 | 0.44 ± 0.03 | 0.51 ± 0.06 |
| 130 | 0.03 ± 0.01 | 0.06 ± 0.02 | 0.15 ± 0.03 | 0.31 ± 0.03 | 0.48 ± 0.04 | 0.57 ± 0.06 |
| 200 | 0.04 ± 0.01 | 0.09 ± 0.03 | 0.19 ± 0.04 | 0.33 ± 0.03 | 0.44 ± 0.05 | 0.52 ± 0.08 |
| 2760 | 0.03 ± 0.01 | 0.07 ± 0.01 | 0.18 ± 0.02 | 0.39 ± 0.03 | 0.60 ± 0.04 | 0.63 ± 0.05 |
| 5020 | 0.05 ± 0.01 | 0.07 ± 0.01 | 0.19 ± 0.02 | 0.39 ± 0.04 | 0.64 ± 0.04 | 0.62 ± 0.06 |

TABLE III. The parameters for the linear fit of the ⟨p_T⟩ dependence on √s_{NN}/S_{\text{geom}} for pions, kaons and protons corresponding to the energies mentioned in the first column.

all the three species at LHC energies. Using S_{\text{geom}} and Fig.4b, the extracted slopes, represented in Fig.5a by open symbols show a marginal variation as a function of collision energy - dashed lines. The corresponding offsets, represented in Fig.5b by open symbols, within the error bars, are the same for pions and kaons and are systematically larger for protons at RHIC energies compared with the ones corresponding to S_{\text{geom}}. One should remark that at LHC energies, the results using S_{\text{geom}} or S_{\text{var}} are the same. At the LHC energies, in the most central collisions, a saturation trend seems to develop. A natural question which comes is how much of the observed trends is due to core-corona interplay [49–56] and how the ⟨p_T⟩-√s_{NN}/S_{\text{var}} correlation for core looks like. Based on the recipe presented in [50], we estimated the ⟨p_T⟩_{core} for pions, kaons and protons for √s_{NN}=200 GeV, 2.76 TeV and 5.02 TeV:

\[ \langle p_T \rangle_{\text{cen}} = \frac{f_{\text{core}} \langle p_T \rangle_{\text{core}} M_{\text{core}} + (1 - f_{\text{core}}) \langle p_T \rangle_{\text{ppMB}} M_{\text{ppMB}}}{f_{\text{core}} M_{\text{core}} + (1 - f_{\text{core}}) M_{\text{ppMB}}} \]

(2)

(⟨p_T⟩_{ppMB} for π⁺, K⁺, p in pp minimum bias (MB) collisions at √s_{NN}=200 GeV were reported by the STAR Collaboration [34] and at √s_{NN}=2.76 TeV and 5.02 TeV were reported by the ALICE Collaboration [57,58]. (dN/dy)_{\text{core}} at the same energies were estimated using:

\[ \langle dN/dy \rangle_{\text{cen}} = \langle N_{\text{part}} \rangle - [(1 - f_{\text{core}}) M_{\text{ppMB}} + f_{\text{core}} M_{\text{core}}] \]

(3)

where M_{ppMB} = 1/(dN/dy)_{ppMB} at the same energy and M_{core} is the multiplicity per core participant. (dN/dy)_{p_{\text{ppMB}}} for π⁺, K⁺, p were obtained based on the MB p_T spectra reported in [57,58].

In Fig.6a ⟨p_T⟩ as a function of √s_{NN}/S_{\text{geom}} for pions, kaons and protons for √s_{NN}=200 GeV, 2.76 TeV and 5.02 TeV is represented. The experimental points for each energy and each species were fitted with linear functions. As it was already mentioned above, at √s_{NN}=2.76 TeV and 5.02 TeV the very last three points, corresponding to the most central collisions, systematically deviate from a linear trend observed at lower centralities and therefore were excluded from the fit. The slopes and the offsets are presented in Table IV. The fit quality can be followed in the bottom plot of Fig.6a where the ratios between the data points and fit results are represented. One can also observe that the last three points at √s_{NN}=2.76 TeV and 5.02 TeV, corresponding
The slopes for particles Fig.7a and antiparticles Fig.7b evidence a \( \sqrt{s_{NN}} \) dependence which closely follows a trend given by Eq.4 (dashed lines). The values of the fit parameters are listed in Fig.7. The fit quality is represented in the bottom plots of Fig.7a and Fig.7b in terms of Data/Fit. Besides the points corresponding to the most central collisions at \( \sqrt{s_{NN}}=19.6 \), 27 and 39 GeV which deviate from the fit by \( \sim 10-15\% \), the bulk of data nicely cluster around the fit curve, well within the error bars. In Fig.8, although the error bars are rather large, a systematic increase of the offsets as a function of \( \sqrt{s_{NN}} \) is evidenced at BES energies and at \( \sqrt{s_{NN}}=62.4 \) GeV, reaching a plateau above 1.7 \( fm^{-1} \). This trend is much reduced starting from \( \sqrt{s_{NN}}=130 \) GeV. Therefore, we considered only offsets above 1.7 \( fm^{-1} \) and found their average values for different \( \sqrt{s_{NN}} \). The results are presented in Fig.9.

VI. \( \langle p_T \rangle \) PARTICLE MASS DEPENDENCE AS A FUNCTION OF \( \sqrt{s_{NN}} \)

The \( \langle p_T \rangle \) dependence on the mass of pions, kaons and protons at different collision centralities, except for the most peripheral ones, is linear. Therefore, linear fits of the \( \langle p_T \rangle \) particle mass dependence, corresponding to each centrality and energy considered in the paper, were performed. The extracted fit parameters as a function of \( \sqrt{s_{NN}} \) are shown in Fig.7 (slope) and Fig.8 (offset). In Fig.7 the slopes are fitted with the following expression:

\[
\text{Slope}_{\langle p_T \rangle}=f(\text{mass}) = \alpha + \beta \left( \frac{dN}{dy}/S_{\perp}^{\text{geom}} \right) \gamma \quad (4)
\]

\( \langle p_T \rangle \) dependence of the fit parameters is listed in Fig.7. The fit quality is represented in the bottom plots of Fig.7a and Fig.7b in terms of Data/Fit. Besides the points corresponding to the most central collisions at \( \sqrt{s_{NN}}=19.6 \), 27 and 39 GeV which deviate from the fit by \( \sim 10-15\% \), the bulk of data nicely cluster around the fit curve, well within the error bars. In Fig.8, although the error bars are rather large, a systematic increase of the offsets as a function of \( \sqrt{s_{NN}} \) is evidenced at BES energies and at \( \sqrt{s_{NN}}=62.4 \) GeV, reaching a plateau above 1.7 \( fm^{-1} \). This trend is much reduced starting from \( \sqrt{s_{NN}}=130 \) GeV. Therefore, we considered only offsets above 1.7 \( fm^{-1} \) and found their average values for different \( \sqrt{s_{NN}} \). The results are presented in Fig.9.

VI. \( \sqrt{dN}/S_{\perp}^{\text{geom}} \) DEPENDENCE OF BOLTZMANN-GIBBS BLAST WAVE FIT PARAMETERS

The \( p_T \) spectra for identified charged hadrons were fitted [34,35,38,40,41] using the BGBW expression inspired by hydrodynamic models [59].

\[
E^3 d^3N/dp^3 \sim \int_0^R m_T K_1(m_T \cosh \rho/T_{kin}^{f_0})I_0(p_T \sinh \rho/T_{kin}^{f_0})r dr
\]

where \( m_T = \sqrt{m^2 + p_T^2}; \beta_T(r) = \beta_\perp(r) = \beta_{\perp}(r); \rho = \tanh^{-1} \beta_T. \)

\( T_{kin}^{f_0} \) is the kinetic freeze-out temperature and \( n \) defines the expansion profile. A compilation of all results in terms of the \( \langle \beta_T \rangle \) dependence on \( \sqrt{dN}/S_{\perp}^{\text{geom}} \) is presented in Fig.10. One should mention that for the BES energies [38] the BGBW fits were performed simultaneously on particles and antiparticles \( p_T \) spectra, although they do not present the same trends in many

| \( \sqrt{s_{NN}} \) (GeV) | \( \pi^+ \) | \( K^+ \) | \( p \) | \( \pi^+ \) | \( K^+ \) | \( p \) |
|---------------------|-------|-------|-----|-------|-------|-----|
| 200                 | 0.05 ± 0.02 | 0.13 ± 0.04 | 0.28 ± 0.06 | 0.28 ± 0.04 | 0.33 ± 0.09 | 0.27 ± 0.13 |
| 2760                | 0.04 ± 0.01 | 0.09 ± 0.02 | 0.20 ± 0.03 | 0.37 ± 0.04 | 0.56 ± 0.07 | 0.56 ± 0.08 |
| 5020                | 0.05 ± 0.02 | 0.08 ± 0.02 | 0.22 ± 0.03 | 0.37 ± 0.06 | 0.60 ± 0.07 | 0.54 ± 0.10 |

TABLE IV. The parameters of the linear fit of \( \langle p_T \rangle \) as a function of \( \sqrt{dN}/S_{\perp}^{\text{geom}} \) for pions, kaons and protons corresponding to \( \sqrt{s_{NN}}=200 \) GeV, 2.76 TeV and 5.02 TeV collision energies. The very last three points at \( \sqrt{s_{NN}}=2.76 \) and 5.02 TeV were not included in the fit.

| \( \sqrt{s_{NN}} \) (GeV) | \( \pi^+ \) | \( K^+ \) | \( p \) | \( \pi^+ \) | \( K^+ \) | \( p \) |
|---------------------|-------|-------|-----|-------|-------|-----|
| 200                 | 0.02 ± 0.03 | 0.09 ± 0.06 | 0.20 ± 0.11 | 0.36 ± 0.07 | 0.43 ± 0.15 | 0.50 ± 0.29 |
| 2760                | 0.03 ± 0.02 | 0.07 ± 0.03 | 0.17 ± 0.04 | 0.40 ± 0.06 | 0.58 ± 0.10 | 0.66 ± 0.14 |
| 5020                | 0.03 ± 0.03 | 0.06 ± 0.02 | 0.17 ± 0.04 | 0.41 ± 0.11 | 0.65 ± 0.08 | 0.73 ± 0.16 |

TABLE V. The parameters of the linear fit of \( \langle p_T \rangle \) as a function of \( \sqrt{dN}/S_{\perp}^{\text{geom}} \) for pions, kaons and protons corresponding to \( \sqrt{s_{NN}}=200 \) GeV, 2.76 TeV and 5.02 TeV collision energies. For \( \sqrt{s_{NN}}=2.76 \) TeV and 5.02 TeV, the last three centralities, where a levelling off is evidenced, were not included in the fit.
respects. Therefore, in Fig.10, the $\langle \beta_T \rangle$ for antiparticles for some energies and centralities, where the azimuthal dependent BGBW fits were published [60, 61], were represented by open symbols. One could observe that, with increasing collision energy, the values of $\langle \beta_T \rangle$ for antiparticles converge towards the values obtained from a simultaneous fit of particles and antiparticles $p_T$ spectra [34, 38]. However, the $\langle \beta_T \rangle$ values reported in the literature, scale rather nice as a function of $\sqrt{dN/dy/S_{prom}}$ and a 4th order polynomial function fits them well. The fit quality can be followed in the bottom plot of Fig.10. Within the experimental error bars, all data follow the fit result, except the points corresponding to the peripheral collisions at the lowest BES energies. The fit parameters are included in the figure. The same representation in which the data corresponding to $\sqrt{s_{NN}}=7.7, 11.5$ and 62.4 GeV are excluded, can be followed in Fig.11. For the remaining energies, from $\sqrt{s_{NN}}=19.6$ GeV to 5.02 TeV a much better scaling is observed. The dynamics in $\langle \beta_T \rangle$ as a function of $\sqrt{dN/dy/S_{prom}}$ for different collision energies can be easier followed in Fig.12 where the ratio between $\langle \beta_T \rangle$ at a given centrality relative to $\langle \beta_T \rangle$ in the most peripheral collisions, 70%-80% (58%-85% for 130 GeV), $\langle \beta_T \rangle/\langle \beta_T^{Peripheral} \rangle$, is plotted as a function of $\sqrt{dN/dy/S_{prom}}$ for all energies.

In Fig.13 the $T_{kin}^{fo}$ and n parameters and their dependence on $\sqrt{dN/dy/S_{prom}}$ are presented. A close to linear dependence with a negative slope is observed in Fig.13a, for $T_{kin}^{fo}$ at RHIC energies. Within the error bars, it is rather difficult to conclude on some collision energy de-
dependence of $T_{\text{kin}}^{fo}$ for a given value of the geometrical variable. On the other hand, a significant shift of about 20 MeV in $T_{\text{kin}}^{fo}$ fit parameter towards larger values is evidenced for a given $\sqrt{dN/dy}/S_{\perp}^{\text{geom}}$ at LHC energies relative to the RHIC energies. Similar shifts were mentioned in the previous chapters for $\langle p_T \rangle$ and the offsets of $\langle p_T \rangle$ as a function of mass. Such a shift is also evidenced in the $T_{\text{kin}}^{fo}$ versus $\langle \beta_T \rangle$ representation in Fig.14 where the fit parameters reported in Ref. [34, 35, 38, 40, 41] are used. As far as the $n$ dependence on $\sqrt{dN/dy}/S_{\perp}^{\text{geom}}$ is concerned, Fig.13b, the values for BES energies are rather scattered and those corresponding to 62.4 and 200 GeV show an opposite trend to what is observed at LHC. Usually, the flow profile changes from a shell type expansion, large $n$ values, towards $n=1$ (Hubble type) with increasing centrality, even smaller than 1 for very central collisions. It is worth mentioning that for a consistent interpretation, the fits of the $p_T$ spectra using the BGBW expression have to be done at all energies on the same $p_T$ range for a given species. The range has to be chosen such to reduce as much as possible the influence of processes other than collective expansion on the extracted fit parameters. Therefore, the lower limit of the fit range for pions has to be chosen such that the contribution coming from resonance decays is reduced, while the upper fit ranges for
all species have to be optimised in order to be influenced as little as possible by the suppression effects. Last but not least, the influence of the corona contribution on the fit parameters has to be carefully considered.

VII. COMPARISON BETWEEN PP AND Pb – Pb SYSTEMS AT LHC ENERGIES

Similarities between pp and Pb-Pb in terms of the behaviour of different observables, like the $⟨\beta_T⟩ - T_{_{kin}}$
The hadron density per unit of rapidity as a function of charged particle multiplicity was estimated by extrapolating the results reported by the ALICE Collaboration in Ref. [17]. The \( \langle p_T \rangle \) values were estimated based on the \( p_T \) spectra from [15] extrapolated in the unmeasured regions using fits of the measured spectra with the expression from [62]:

\[
\frac{d\sigma}{p_T dp_T} = A e^{\exp \left( -\frac{E_{kin}^\beta}{T} \right)} + \frac{A}{\left( 1 + \frac{p_T^2}{T^2} \right)^N} \tag{6}
\]

The interaction area for pp collisions, \( S_{pp}^\perp = \pi R_{pp}^2 \), is calculated using the estimates of the maximal radius for which the energy density of the Yang-Mill fields is larger than \( \epsilon = \alpha \Lambda_{QCD}^4 (\alpha \in [1,10]) \) within the IP-Glasma initial state model [63, 64]. Within the present knowledge of QCD, \( \alpha \) cannot be precisely estimated. The \( r_{max} \) values used in Ref. [63] for \( \alpha = 1 \) were fitted in Ref. [64] with the following expressions:

\[
f_{pp} = \begin{cases} 0.387 + 0.0335x + 0.274x^2 - 0.0542x^3 & \text{if } x < 3.4 \\ 1.538 & \text{if } x \geq 3.4 \end{cases} \tag{7}
\]

Using the same recipe we fitted the \( r_{max} \) values from Ref. [64] for \( \alpha = 10 \) with the following expression:

\[
f_{pp} = \begin{cases} -0.018 + 0.3976x + 0.095x^2 - 0.028x^3 & \text{if } x < 3.4 \\ 1.17 & \text{if } x \geq 3.4 \end{cases} \tag{8}
\]

where \( x = (dN/dy)^{1/3} \). The hadron density per unit of rapidity, estimated based on the following approximation:

\[
\frac{dN}{dp} = \frac{3}{2} \frac{dN}{dp}(\pi^+ + \pi^-) + 2 \frac{dN}{dp}(p+p, \Xi^- + \bar{\Xi}^+, K^0)
\]
\( \frac{dN}{dy}(K^+ + K^- + \Lambda + \bar{\Lambda} + \Omega + \bar{\Omega}) \) and the corresponding overlapping areas for \( \alpha = 1 \) and \( \alpha = 10 \) values are listed in Table VI. The gluon density per unit of rapidity was approximated by \( \frac{dN_g}{dy} \approx \frac{dN}{dy} \). The comparison between the \( \langle p_T \rangle \) dependence on the square root of the hadron density per unit of rapidity and unit of interaction area for the pp at \( \sqrt{s} = 7 \) TeV and Pb-Pb at \( \sqrt{s_{NN}} = 2.76 \) and 5.02 TeV collisions, based on the results obtained by the ALICE Collaboration \cite{35, 40, 41}, is presented in Fig. 15. As one could see, the general trend for all the three species is very similar in pp and Pb-Pb collisions. The differences could have several origins, i.e. the difference in the collision energies, a systematic larger \( \langle p_T \rangle \) for kaons in pp relative to Pb-Pb, uncertainty in estimating the value of \( \alpha \), the large inhomogeneity of the initial state with a direct consequence on the \( S_\perp \) estimate and last but not least the build up of collective expansion in the hadronic phase and suppression effects taking place in the Pb-Pb case and not yet evidenced in pp collisions. The comparison between the two systems in terms of the slopes of the \( \langle p_T \rangle \) particle mass dependence as a function of \( \frac{dN}{dy}/S_{geom} \) is presented in Fig. 16. A very good scaling is found using \( \alpha = 1 \) for pp collisions. The same value of \( \alpha \) was used in Refs. \cite{65, 66}. These results seem to support the assumption that the global properties of the hadron production are determined by the properties of flux tubes of \( \sim 1/\frac{dN}{dy}/S_{geom} \) size and are very little influenced by the size of the colliding system \cite{18, 65, 67}. A similar behaviour was evidenced at the baryonic level at much lower energies where the main features of the dynamic evolution of the fireball are determined by the initial baryon density profile and temperature and not too much by its size \cite{68}. As it is well known, the LPHD approach neglects all collective effects. However, a comparison between pp and Pb-Pb collisions in terms of \( \langle \beta_T \rangle \), one of the BGBW fit parameters interpreted as the average transverse flow velocity, could be rather interesting. \( \langle \beta_T \rangle \) values for pp at \( \sqrt{s} = 7 \) TeV \cite{15} and Pb-Pb at \( \sqrt{s_{NN}} = 2.76 \) and 5.02 TeV \cite{35, 40, 41} reported by the ALICE Collaboration are represented as a function of \( \frac{dN}{dy}/S_{geom} \) in Fig. 17. A 4th degree polynomial func-

| \( \sqrt{s} \) (TeV) | \( \frac{dN}{dy} \) | \( S_\perp \) (fm\(^2\)) |
|----------------------|----------------|----------------|
| \( pp \) | \( \alpha = 1 \) | \( \alpha = 10 \) |
| 7 | 82.1±2.8 | 7.43±0.48 | 4.30±0.36 |
| 70.2±2.2 | 7.43±0.41 | 4.30±0.31 |
| 59.4±1.7 | 7.43±0.35 | 4.30±0.27 |
| 48.8±1.3 | 7.43±0.30 | 4.30±0.23 |
| 37.3±0.9 | 7.39±0.02 | 4.20±0.02 |
| 26.8±0.6 | 6.89±0.05 | 3.80±0.03 |
| 18.2±0.4 | 5.94±0.06 | 3.16±0.04 |
| 10.8±0.2 | 4.58±0.06 | 2.29±0.04 |

TABLE VI. The hadron density per unit of rapidity and transverse overlapping areas for \( \alpha = 1 \) and \( \alpha = 10 \) for pp collisions at \( \sqrt{s} = 7 \) TeV.
Based on the data for the highest three energies measured at RHIC (√s_{NN}=62.4, 130, 200 GeV), the most recent results from BES at RHIC (√s_{NN}=7.7-39 GeV) and the highest collision energies at LHC (√s_{NN}=2.76, 5.02 TeV), we performed a systematic study of the dependence of different observables on the geometrical variable calculated as the square root of the hadron density per unit of rapidity and unit of overlapping area of two colliding ions. The overlapping area has been estimated in the Glauber MC approach. The experimental ⟨p_T⟩ values follow a rather good scaling as a function of this variable for each energy. Linear fits of the experimental data show slopes which increase from pions to protons and decrease from BES to LHC energies. A saturation trend for the most central collisions at LHC is observed. For √s_{NN}=200 GeV, 2.76 TeV and 5.02 TeV the ⟨p_T⟩^core and \( \sqrt{\frac{dN}{dy}/S_{\text{geom}}^\text{core}} \) were estimated based on the core-corona approach. The corresponding ⟨p_T⟩^core versus \( \sqrt{\frac{dN}{dy}/S_{\text{geom}}^\text{core}} \) show lower slopes and their decrease going from √s_{NN}=200 GeV to 5.02 TeV is less evident for all three species. This shows the importance of discriminating between the core and corona contributions in such a type of analysis, for a quantitative comparison. The decrease in the slopes from RHIC to LHC for all species and for the most central collisions at LHC energies seems to support the approach presented in Ref. [1]. A much better scaling as a function of \( \sqrt{\frac{dN}{dy}/S_{\text{geom}}^\text{core}} \) is observed for the slope from the linear fit of the ⟨p_T⟩ dependence on the particle mass and the BGBW fit parameter, ⟨β⟩. The offset of the ⟨p_T⟩ particle mass dependence and the T^kin parameter show a clear jump towards larger values between RHIC and LHC energies. As it was already mentioned, other phenomena, like suppression and its azimuthal dependence as well as the hydrodynamic expansion in the deconfined and after hadronization stages, also have to be considered. The very similar dependence of the ⟨p_T⟩, ⟨p_T⟩ particle mass dependence and the BGBW fit parameter, ⟨β⟩, on \( \sqrt{\frac{dN}{dy}/S_{\text{geom}}} \) in pp and Pb-Pb collisions at LHC energies support the assumption that the global properties evidenced at LHC energies are determined by the properties of flux tubes of \( \sim 1/\sqrt{\frac{dN}{dy}/S_{\text{geom}}} \) size, the system size playing a minor role.

VIII. CONCLUSIONS

This work was carried out under the contracts sponsored by the Ministry of Research and Innovation: RONIPALICE-04/16.03.2016 (via IFA Coordinating Agency) and PN-18 09 01 03.

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