Complete vectorlike fourth family and new U(1)’ for muon anomalies *

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Abstract

I introduce a model to explain the recent anomalies in the muon anomalous magnetic moment and the rare semi-leptonic B meson decays. The Standard Model is extended by a U(1)’ gauge symmetry and a complete fourth family fermions which are vector-like under the gauge symmetry. We found parameter points which can explain the anomalies consistently with the other observables. We then propose an interesting possibility to have signals with four muons or more from vector-like lepton pair production.

1 Introduction

The discrepancies from the Standard Model (SM) predictions are found in the measurements of the muon anomalous magnetic moment and the rare semi-leptonic decays of B mesons, $b \rightarrow s\ell\ell$. The latest measurements of $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$ and the lepton universality observable, $R_K$, confirmed that their values are deviated from the SM prediction by 4.2$\sigma$ [1, 2] and 3.1$\sigma$ [3], respectively. These anomalies may imply that there are new particles at the TeV scale which couple to muons.

In our papers [4, 5], we proposed a model which explains the anomalies in $\Delta a_\mu$ and $b \rightarrow s\ell\ell$ simultaneously ‡. We introduced a complete vector-like (VL) family of quarks and leptons, and a new vector-like $U(1)'$ gauge symmetry. Only the VL family and the $U(1)'$ breaking scalar, $\Phi$, carry non-zero $U(1)'$ charges, whereas the three generations of chiral families and the Higgs boson are neutral. A $Z'$ boson associated with the $U(1)'$ gauge symmetry couples to the SM families via the mixing between the chiral and VL families induced by the $U(1)'$ breaking. In this model, $\Delta a_\mu$ is explained by the 1-loop contributions mediated via the VL leptons and the $Z'$ boson, and $b \rightarrow s\ell\ell$ is explained by the tree-level exchanging of the $Z'$ boson. We showed that the anomalies can be explained if the VL leptons and $Z'$ boson are lighter than 1.5 TeV.

We propose a novel possibility to search for the VL leptons and $Z'$ boson [11]. We consider the pair production of VL leptons decaying to a $Z'$ boson followed by the $Z'$ boson decaying to a pair of muons or muon neutrinos. There are six (four) muons in the final state if both (either of) the $Z'$ bosons decay to muons. We recast the latest ATLAS search [12] for events with four or more charged leptons.

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‡Similar models for the anomalies are studied in Refs. [6–10]
Table 1: Quantum numbers of new fermion and scalar fields.

|       | $Q_L$ | $\bar{U}_R$ | $\bar{D}_R$ | $L_L$ | $\bar{E}_R$ | $\bar{N}_R$ | $\bar{Q}_R$ | $U_L$ | $D_L$ | $\bar{T}_R$ | $E_L$ | $N_L$ | $\phi$ | $\Phi$ |
|-------|-------|-------------|-------------|-------|-------------|-------------|-------------|-------|-------|-------------|-------|-------|--------|--------|
| $SU(3)_C$ | 3     | 3           | 3           | 1     | 1           | 1           | 3           | 3     | 3     | 1           | 1     | 1     | 1      | 1      |
| $SU(2)_L$ | 2     | 1           | 1           | 2     | 1           | 1           | 2           | 1     | 1     | 2           | 1     | 1     | 1      | 1      |
| $U(1)_Y$ | 1/3   | -4/3        | 2/3         | -1    | 2           | 0           | -1/3        | 4     | 3     | -2          | 1     | -2    | 0      | 0      |
| $U(1)'$  | -1    | 1           | 1           | -1    | 1           | 1           | 1           | -1    | -1    | 1           | -1    | -1    | 0      | -1     |

2 Explanation for the muon anomalies

2.1 Model

We review the model proposed in Refs. [4,5]. The new particles of our model are given in Table 1. The SM particles are neutral under the $U(1)'$ gauge symmetry. For simplicity, we focus on the charged leptons. The first and third generations of the chiral leptons are omitted, although we studied the model with complete generations and discussed lepton flavor violation. The mass of the VL states and Yukawa interactions are given by

$$
\mathcal{L} \supset -m_L L_R L_L - m_E \bar{E}_R E_L
$$

$$
+ y_\mu \bar{\mu} R \ell_R H + \kappa' \bar{E}_R L_L H - \kappa L_R \tilde{H} E_L + \lambda_L \Phi \bar{L}_R \ell_L - \lambda_E \Phi \bar{E}_R E_L + h.c.,
$$

where $\tilde{H} := i\sigma_2 H^* = (H_0^*, -H_0^*)$. The $SU(2)_L$ doublets are defined as

$$
\ell_L = (\nu_L, \mu_L), \quad H = (H_0, H_-), \quad L_L = (N'_L, E'_L), \quad \bar{T}_R = (-\bar{E}'_R, \bar{N}'_R),
$$

and the $SU(2)_L$ indices are contracted via $i\sigma_2$. After symmetry breaking by $v_H := \langle H_0 \rangle$ and $v_E := \langle \Phi \rangle$, the mass matrix for the leptons is given by

$$
\mathbf{e}_R \mathcal{M}_e \mathbf{e}_L := \left( \begin{array}{c} \bar{\mu}_R E'_R \\ 0 \end{array} \right) \left( \begin{array}{cc} y_\mu v_H & 0 \\ 0 & \kappa' v_H m_E \end{array} \right) \left( \begin{array}{c} \mu_L \\ E'_L \end{array} \right).
$$

The mass basis is defined as

$$
\hat{\mathbf{e}}_L := U_L^\dagger \mathbf{e}_L, \quad \hat{\mathbf{e}}_R := U_R^\dagger \mathbf{e}_R, \quad U_R^\dagger \mathcal{M}_e U_L = \text{diag}(m_\mu, m_{E_2}, m_{E_1}),
$$

where $E_1$ and $E_2$ are respectively the singlet-like and doublet-like VL leptons. We define the Dirac fermions as $\mathbf{e} := (\mu, E_2, E_1)$, $[\mathbf{e}]_i := ([\hat{\mathbf{e}}_L]_i, [\hat{\mathbf{e}}_R]_i)$, where $i = 1, 2, 3$. The mass matrices for the quarks and the Dirac mass matrices for the neutrinos have the same structure as the charged leptons. We assume that the three generations of right-handed neutrinos have Majorana masses at the intermediate scale, so that the tiny neutrino masses are explained by the type-I see-saw mechanism.
The gauge interactions with the $Z'$ boson in the mass basis are given by

$$\mathcal{L}_V = Z'_\mu e^\mu \left( g_e^Z P_L + g_{eR}^Z P_R \right) e, \quad g_L^Z = g' U_L^\dagger Q_L^\prime U_L, \quad g_R^Z = g' U_R^\dagger Q_R^\prime U_R$$

where $Q'_e = \text{diag}(0, 1, 1)$ is the coupling matrix in the gauge basis. Note that the chiral family does not couple to the $Z'$ boson in the gauge basis, and the coupling arises only in mass basis. Here, $P_L$ ($P_R$) are the chiral projections onto the left- (right-)handed fermions. $g'$ is the gauge coupling constant for $U(1)'$.

### 2.2 Muon anomalies

The diagrams that can explain the anomalies are shown in Fig. 1. The $Z'$ boson and the physical mode $\chi$ of the $\Phi$ boson contribute to $\Delta a_\mu$ as [13, 14]

$$\Delta a_\mu \sim \frac{m_\mu \kappa v_H}{64 \pi^2 v_\Phi^2} s_2 L s_2 R \left( \sqrt{x_L x_E} \frac{G_Z(x_L) - G_Z(x_E)}{x_L - x_E} + \frac{1}{2} \frac{y_L G_S(y_L) - y_R G_S(y_R)}{y_L - y_R} \right),$$  

where $s_A := \lambda_X v_\Phi / M_X$, with $M_X^2 := m_X^2 + \lambda_X^2 v_\Phi^2$ ($X = L, E$ for $A = L, R$), is the mixing angle between the chiral and VL families. Here, $x_L := M_L^2 / m_{Z'}^2$, $x_E := M_E^2 / m_{Z'}^2$, $y_L := M_L^2 / m_\chi^2$, $y_E := M_E^2 / m_\chi^2$ with $m_{Z'}^2 = 2 g^2 v_\Phi^2$ and $m_\chi$ is the mass of $\chi$. The loop functions are given by

$$G_Z(x) := \frac{x^3 + 3x - 6x \ln(x) - 4}{2(1 - x)^3}, \quad G_S(y) := \frac{y^2 - 4y + 2 \ln(y) + 3}{(1 - y)^3}.$$  

The value of $\Delta a_\mu$ is estimated as

$$\Delta a_\mu \sim 2.9 \times 10^{-9} \times \left( \frac{1.0 \, \text{TeV}}{v_\Phi} \right)^2 \left( \frac{\kappa}{1.0} \right) \left( \frac{C_{LR}}{0.1} \right),$$

where $C_{LR}$ is inside the parenthesis of Eq. (6).

For the $b \to s \ell \ell$ anomaly, the Wilson coefficients are given by [15, 16]

$$C_9 \sim - \frac{\sqrt{2}}{4G_F} \frac{4\pi}{\alpha_e} \frac{1}{V_{tb} V_{ts}^*} \frac{1}{4 v_\Phi^2} (s_R^2 + s_L^2) \epsilon_{Q_2} \epsilon_{Q_3},$$

$$C_{10} \sim - \frac{\sqrt{2}}{4G_F} \frac{4\pi}{\alpha_e} \frac{1}{V_{tb} V_{ts}^*} \frac{1}{4 v_\Phi^2} (s_R^2 - s_L^2) \epsilon_{Q_2} \epsilon_{Q_3},$$

Figure 1: Diagrams contribute to $\Delta a_\mu$ (left) and the $b \to s \ell \ell$ decay (right).
Table 2: Values of $\chi^2$, selected input parameters and observables at the best fit points A, B, C and D. The degree of freedom in our analysis is $N_{\text{obs}} - N_{\text{inp}} = 98 - 65 = 33$. The last column shows the experimental central values and their uncertainties. The upper limits on the lepton flavor violating decays are 90% C.L. limits.

| Parameters         | Point A | Point B | Point C | Point D |
|--------------------|---------|---------|---------|---------|
| $\chi^2$           | 22.6    | 25.0    | 23.3    | 23.8    |
| $g'$               | 0.250   | 0.340   | 0.323   | 0.349   |
| $m_{Z'}$ [GeV]     | 277.6   | 535.3   | 486.7   | 758.7   |

| Observables        | Point A | Point B | Point C | Point D | Exp. |
|--------------------|---------|---------|---------|---------|------|
| $\Delta a_\mu \times 10^9$ | 2.62    | 2.52    | 2.52    | 2.45    | 2.68 ± 0.76 |
| $\text{BR} (\mu \rightarrow e\gamma) \times 10^{13}$ | 0.147   | 1.597   | 0.061   | 0.822   | < 4.2 |
| $\text{BR} (\tau \rightarrow \mu\gamma) \times 10^8$ | $3.34 \times 10^{-4}$ | $3.62 \times 10^{-4}$ | $3.27 \times 10^{-6}$ | $8.45 \times 10^{-7}$ | < 4.4 |
| $\text{BR} (\tau \rightarrow \mu\mu\mu) \times 10^8$ | $6.96 \times 10^{-3}$ | $4.77 \times 10^{-4}$ | $6.55 \times 10^{-5}$ | $4.36 \times 10^{-7}$ | < 2.1 |
| $\text{Re} C_9$   | -0.548  | -0.806  | -0.838  | -0.808  | -0.7 ± 0.3  |
| $\text{Re} C_{10}$| 0.370   | 0.252   | 0.347   | 0.322   | 0.4 ± 0.2   |
| $\Delta M_d$ [ps$^{-1}$] | 0.561   | 0.610   | 0.598   | 0.590   | 0.506 ± 0.081 |
| $\Delta M_s$ [ps$^{-1}$] | 19.6    | 19.8    | 19.4    | 20.0    | 17.76 ± 2.5  |

where the $Z'$ boson couplings to the SM doublet quarks are parametrized as

$$
\begin{bmatrix}
g_{d_2}^{Z'} \\
g_{u_2}^{Z'}
\end{bmatrix}_{ij} \sim -g' \epsilon_{Q_i} \epsilon_{Q_j}.
$$

(11)

Here, $\epsilon_{Q_i}$ ($i = 1, 2, 3$) is the similar quantity as $s_{L,R}$ and originates from the mixing between the SM and VL quarks, but we now consider the couplings with the second and third generation quarks and these are typically small in contrast to that for muons. The value of $C_9$ is estimated as

$$
C_9 \sim -0.62 \times \left( \frac{1.0 \text{ TeV}}{v_\phi} \right)^2 \left( \frac{s_L^2 + s_R^2}{1} \right) \left( \frac{\epsilon_{Q_2} \epsilon_{Q_3}}{-0.002} \right).
$$

(12)

### 2.3 $\chi^2$ analysis

We searched for parameter points which can explain the anomalies without unacceptably changing the other observables consistent with the SM. The $\chi^2$ function is defined as

$$
\chi^2(x) := \sum_{I \in \text{obs}} \frac{(y_I(x) - y_I^0)^2}{\sigma_I^2},
$$

(13)

where $x$ is a parameter space point, $y_I(x)$ is the value of an observable $I$ whose central value is $y_I^0$ and uncertainty is $\sigma_I$. In this model, there are 65 input parameters and we studied 98 observables.
including fermion masses, CKM matrix, lepton and quark flavor violation observables, and so on. The full list of observables and the values for the $\chi^2$ fitting are shown in Ref. [5]. Table 2 shows values of $\chi^2$, selected input parameters and observables at the best fit points A, B, C and D. At these points, both anomalies are explained successfully, while the flavor violation observables, e.g. $\mu \rightarrow e\gamma$ and $\Delta M_s$ are consistent with the current limits. We did a global analysis for the model, and we found that $\Delta a_\mu$ can be explained only if $m_{Z'} < 800$ GeV and $m_E < 1.5$ TeV. Therefore, these new particles will be probed by the LHC experiment as discussed in the next section. Although there are no lower bounds on the branching fractions in the lepton flavor violating decays, we found the relations among the decay modes $\text{BR} (\mu \rightarrow e\gamma) \gg \text{BR} (\mu \rightarrow eee)$ and $\text{BR} (\tau \rightarrow \mu\gamma) \sim \text{BR} (\tau \rightarrow \mu\mu\mu) \gg \text{BR} (\tau \rightarrow e\gamma, e\ell\ell)$ ($\ell = e, \mu$) which would be confirmed by the future experiments. In the quark sector, most observables are consistent with the SM predictions assuming unitarity of the CKM matrix, although our model can have non-unitarity of the CKM matrix for the SM families.

### 3 LHC signals

The VL leptons and $Z'$ boson are within the reach of the LHC experiment. As shown in Eq. (12), the $Z'$ boson typically couples to SM quarks only with small couplings. Hence, the strong constraints from the di-muon resonance search [17] for a $Z'$ boson can be evaded since the production cross section is suppressed even though the branching fraction to muons is sizable. Therefore, the $Z'$ boson below 1 TeV is, in general, not excluded by the $Z'$ search.

The processes which can produce more than four muons are shown in Fig 2. The signal of more than four muons is produced if either of the $Z'$ boson decays to a pair of muons from charged VL lepton pair production (left), while both have to decay to muons in the other processes (middle and right) involving the VL neutrino.

We recast the limits obtained in Refs. [12] searching for signals with more than four leptons. We have generated events using MadGraph5.a.2 [18] based on a UFO [19] model file generated with FeynRules2.3.43 [18,20]. The events are showered with PYTHIA8 [21] and then run through the fast detector simulator Delphes3.4.2 [21]. We used the default ATLAS card for the detector simulation, but the threshold on $p_T$ for the muon efficiency formula is changed to 5 GeV from 10 GeV since muons with $p_T > 5$ GeV are counted as signal muons in Ref. [12].

The current limits for the singlet-like VL lepton are shown in Fig. 3. We see that the $SR_{0_tight}$ gives the strongest bounds on the $\text{BR} (E_1 \rightarrow Z'\mu)$. Typically, $\text{BR} (E_1 \rightarrow Z'\mu) \sim 1$ when $m_\chi > 5$.
Figure 3: Limits from the signal regions with 4 leptons for the singlet-like VL lepton. The colors signify upper bounds on the BR \((E \rightarrow Z'\mu)\), e.g., BR \((E \rightarrow Z'\mu) > 0.5\) is excluded at \((m_E, m_{Z'}) = (750, 200)\) GeV in the right panel.

\[ m_{E_1} > m_{Z'}, \text{ while BR } (E_1 \rightarrow Z'\mu) \sim \text{BR} (E_1 \rightarrow \chi\mu) \sim 0.5 \text{ when } m_{E_1} > m_{Z'}, m_\chi. \]  The limit is about 1 TeV (750 GeV) nearly independent of the \(Z'\) mass when the branching fraction is 1 (0.5). When the branching fraction is 1, the limit for the doublet-like lepton is about 1.3 TeV.

4 Summary

In this work, we constructed a model with a complete VL fourth family and a \(U(1)'\) gauge symmetry. The anomaly in the muon anomalous magnetic moment is explained by the 1-loop diagrams mediated by the \(Z'\) boson and the VL leptons, and those in the rare \(B\) decays are explained by the tree-level \(Z'\) boson exchange. We searched for good-fit parameter points by a global \(\chi^2\) analysis, and we found plenty of points that can explain the anomalies. At these points, the other observables, such as lepton flavor violating decays and neutral meson mixing, are consistent with the current limits. We then proposed a novel possibility to detect signals with four muons or more at the LHC. By recasting the latest data, the current limit for the singlet-like (doublet-like) VL lepton is 1.0 (1.3) TeV when the BR \((E \rightarrow Z'\mu) = 1\).

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References

[1] T. Aoyama et al., The anomalous magnetic moment of the muon in the Standard Model, Phys. Rept. 887 (2020) 1–166, [arXiv:2006.04822].
[2] **Muon g-2** Collaboration, B. Abi et al., *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, Phys. Rev. Lett. 126 (2021) 141801, [arXiv:2104.03281].

[3] **LHCb** Collaboration, R. Aaij et al., *Test of lepton universality in beauty-quark decays*, arXiv:2103.11769.

[4] J. Kawamura, S. Raby, and A. Trautner, *Complete vectorlike fourth family and new U(1)' for muon anomalies*, Phys. Rev. D 100 (2019), no. 5 055030, [arXiv:1906.11297].

[5] J. Kawamura, S. Raby, and A. Trautner, *Complete vectorlike fourth family with U(1)' : A global analysis*, Phys. Rev. D 101 (2020), no. 3 035026, [arXiv:1911.11075].

[6] B. Allanach, F. S. Queiroz, A. Strumia, and S. Sun, *Z’ models for the LHCb and g – 2 muon anomalies*, Phys. Rev. D93 (2016), no. 5 055045, [arXiv:1511.07447]. [Erratum: Phys. Rev.D95,no.11,119902(2017)].

[7] W. Altmannshofer, M. Carena, and A. Crivellin, *Lμ – Lτ, theory of Higgs flavor violation and (g – 2)_μ*, Phys. Rev. D94 (2016), no. 9 095026, [arXiv:1604.08221].

[8] E. Megias, M. Quiros, and L. Salas, *g_μ – 2 from Vector-Like Leptons in Warped Space*, JHEP 05 (2017) 016, [arXiv:1701.05072].

[9] S. Raby and A. Trautner, *Vectorlike chiral fourth family to explain muon anomalies*, Phys. Rev. D97 (2018), no. 9 095006, [arXiv:1712.09360].

[10] P. Arnan, A. Crivellin, M. Fedele, and F. Mescia, *Generic Loop Effects of New Scalars and Fermions in b → sℓ⁺ℓ⁻, (g – 2)_μ and a Vector-like 4th Generation*, JHEP 06 (2019) 118, [arXiv:1904.05890].

[11] J. Kawamura and S. Raby, *Signal of four muons or more from a vector-like lepton decaying to a muon-philic Z’ boson at the LHC*, Phys. Rev. D 104 (2021), no. 3 035007, [arXiv:2104.04461].

[12] **ATLAS** Collaboration, G. Aad et al., *Search for supersymmetry in events with four or more charged leptons in 139 fb⁻¹ of √s = 13 TeV pp collisions with the ATLAS detector*, arXiv:2103.11684.

[13] F. Jegerlehner and A. Nyffeler, *The Muon g-2*, Phys. Rept. 477 (2009) 1–110, [arXiv:0902.3360].

[14] R. Dermisek and A. Raval, *Explanation of the Muon g-2 Anomaly with Vectorlike Leptons and its Implications for Higgs Decays*, Phys. Rev. D88 (2013) 013017, [arXiv:1305.3522].

[15] A. J. Buras and M. Munz, *Effective Hamiltonian for B → X(s)e⁺e⁻ beyond leading logarithms in the NDR and HV schemes*, Phys. Rev. D52 (1995) 186–195, [hep-ph/9501281].

[16] C. Bobeth, M. Misiak, and J. Urban, *Photonic penguins at two loops and m_t dependence of BR[B → X_sℓ⁺ℓ⁻]*, Nucl. Phys. B574 (2000) 291–330, [hep-ph/9910220].

[17] **ATLAS** Collaboration, G. Aad et al., *Search for high-mass dilepton resonances using 139 fb⁻¹ of pp collision data collected at √s = 13 TeV with the ATLAS detector*, Phys. Lett. B 796 (2019) 68–87, [arXiv:1903.06248].

[18] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, JHEP 07 (2014) 079, [arXiv:1405.0301].

[19] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, and T. Reiter, *UFO - The Universal FeynRules Output*, Comput. Phys. Commun. 183 (2012) 1201–1214, [arXiv:1108.2040].

[20] N. D. Christensen and C. Duhr, *FeynRules - Feynman rules made easy*, Comput. Phys. Commun. 180 (2009) 1614–1641, [arXiv:0806.4194].

[21] **DELPHES 3** Collaboration, J. de Favereau, C. Delaere, P. Demin, A. Giammanco, V. Lemaître, A. Mertens, and M. Selvaggi, *DELPHES 3, A modular framework for fast simulation of a generic collider experiment*, JHEP 02 (2014) 057, [arXiv:1307.6346].