Local heterogeneities in cardiac systems suppress turbulence by generating multi-armed rotors

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Abstract

Ventricular fibrillation is an extremely dangerous cardiac arrhythmia that is linked to rotating waves of electric activity and chaotically moving vortex lines. These filaments can pin to insulating, cylindrical heterogeneities which swiftly become the new rotation backbone of the local wave field. For thin cylinders, the stabilized rotation is sufficiently fast to repel the free segments of the turbulent filament tangle and annihilate them at the system boundaries. The resulting global wave pattern is periodic and highly ordered. Our cardiac simulations show that also thicker cylinders can establish analogous forms of tachycardia. This process occurs through the spontaneous formation of pinned multi-armed vortices. The observed number of wave arms $N$ depends on the cylinder radius and is associated to stability windows that for $N = 2, 3$ partially overlap. For $N = 1, 2$, we find a small gap in which the turbulence is removed but the pinned rotor shows complex temporal dynamics. The relevance of our findings to human cardiology are discussed in the context of vortex pinning to more complex-shaped anatomical features and remodeled myocardium.

1. Introduction

Traveling waves of excitation are nonlinear waves that occur in a diverse group of experimental systems ranging from corroding metals and liquid phase reactions to neuronal and cardiac tissues [1–3]. The typical features of these waves include a system-specific speed and pulse form as well as a lack of interference and reflection phenomena. To the contrary, excitation pulses mutually annihilate in head-on collision as their directionality is caused by a trailing refractory zone that is reminiscent of the burned land in the wake of a spreading forest fire. The front speed depends not only on the local dynamics and the strength of spatial coupling but also on the wave train frequency and the local front curvature. Convex features typically slow down the propagation and cause disrupted fronts to curl up to pairs of rotating spiral waves [4, 5]. In living systems, such rotors can perform useful tasks as exemplified by their organizing role during the aggregation of cellular slime molds [6] but also disrupt normal physiological patterns. The arguably most important cases of impaired functions are certain cardiac arrhythmias.

The healthy heart pumps blood by rhythmic contractions that are carefully orchestrated by traveling action potentials [7–9]. The pacemaker of these waves is usually the sinoatrial (SA) node in the right atrium (upper chamber). Each excitation pulse reaches the ventricles (lower chambers) via a specialized routing system (bundle of His and Purkinje fibers). Within the chambers, the spread of the action potential can be described by reaction-diffusion-like models in which the spatial coupling is electrical in nature and mediated by the gap junctions between neighboring muscle cells (cardiomyocytes). The local dynamics are centered around the cells' membrane potential. Above a particular threshold, stimulation by neighboring cells causes the opening of voltage-gated ion channels and the depolarization by the influx of cations into the cell which also results in (Ca$^{2+}$-induced) muscle contraction. After some refractory period, the cell repolarizes by opening potassium channels, thus reestablishing its original excitable state.
If the traveling electrical signal is disrupted, rotating spiral waves result that in cardiology are referred to as reentrant waves or circus motion [10]. These rotors have a higher frequency than the oscillatory cells in the SA node and hence quickly enslave the tissue. In the ventricles, this tachycardia (VT) can reach periods of over 200 beats per minute and deteriorate to uncoordinated, quiver–like contractions called ventricular fibrillation (VF or V-fib) [11]. Within minutes VF causes sudden cardiac death and brain death due to the lack of oxygen unless a normal heart rhythm is established by defibrillation. In 2015, the American Heart Association estimated that in the US over 326 000 adults per year experience EMS-assisted out-of-hospital cardiac arrest, of which about one quarter have measurable signs of VF or VT. The survival rate after bystander-witnessed VF for patients of any age is about 31% [12].

In thin, quasi-two-dimensional systems such as the human atria, spiral rotation is organized around a zero-dimensional phase singularity. The human ventricles, however, are sufficiently thick to require a description in terms of three space dimensions [13]. In such three-dimensional systems, rotation occurs around curves called filaments and the rotor is called a scroll wave [5]. For topological reasons, filaments must either end at the system boundaries or form closed loops or knots [14]. Their local speed \( v \) depends on the filament curvature \( K \) (and other factors) and can be approximated by the proportional dependence \( v = \alpha K \), where \( \alpha \) is the system-specific filament tension. In many cases, higher order effects such as filament rigidity [15, 16] and also changes due to strong phase gradients along the filament (twist) can be neglected. The sign of the filament tension, however, is of great importance because a positive \( \alpha \) implies stable linear filaments and shrinking filament loops, whereas negative values cause expansion and the onset of turbulence [17–19].

This scroll wave turbulence (SWT) induced by negative filament tension attracts considerable research interest, not only due to its fascinating dynamics but also for its likely role in VF [9, 20]. Of foremost interest is the question how the turbulent state can be reset to a healthy state by removal of all rotors. Today this reset requires defibrillation with sequential electric shocks of 120–200 J but lower-energy procedures are being investigated [21]. Even more advanced methods could be developed if the perturbation is applied in ways that exploit the intrinsic characteristics of SWT. In this context, numerical simulations based on three-dimensional models can play an important role. In 2003, for instance, Alonso et al demonstrated the suppression of SWT by temporally periodic and spatially homogeneous, low-amplitude modulation of an excitable system [22]. While conceptually elegant, these global perturbations are difficult to realize experimentally raising the question whether a local, stationary perturbation could also terminate the turbulence.

Such as an approach was discussed by Spreckelsen et al in 2015 [23], who studied the effect of a thin cylindrical heterogeneity on SWT. The authors showed that such an unexcitable domain induces the following sequence of events: (i) spontaneous pinning of a segment of the chaotic filament, (ii) self-wrapping of that filament around the entire length of the cylinder, (iii) periodic rotation of the pinned scroll wave around the cylinder at a comparably high frequency, and (iv) expulsion of all other filaments from the system. The latter step occurs as the result of a vortex drift that has been studied systematically in two-dimensional systems where a small spiral defect is (typically) pushed away from a wave source of similar or higher frequency [24, 25]. This effect is based on the periodic modulation of the exposed spiral tip in the wave field of the dominant pacemaker and fails in the case of perturbations that have a lower frequency than the spiral wave.

Accordingly, one could conclude that this suppression of turbulence occurs only for very thin cylinders because the frequency of spiral waves is typically near the upper frequency limit of the system and only very thin cylinders would raise the rotation frequency. This seeming limitation is the focus of our article in which we (i) extend the work of Spreckelsen et al by investigating a cardiac model rather than the simple Barkley equations and (ii) report that arbitrarily thick cylinders remove the chaotic state. This counter-intuitive finding is caused by the spontaneous formation of pinned multi-armed scroll waves.

### 2. Model and methods

We perform numerical simulations using the Fenton–Karma model [3]. Although incorporating only a small set of variables, the model can reproduce the key electrophysiological characteristics of other more complex models as well as experimental data [3, 26]. The spread of cardiac transmembrane potential \( V \) is described by the cable theory and its central equation

\[
\frac{\partial V}{\partial t} = \nabla \cdot (\mathbf{D} \nabla V) - \frac{I_{\text{ion}}}{C_{m}}
\]

where \( \mathbf{D} \) is an effective diffusion tensor involving resistances, \( C_{m} \) is the membrane capacitance, and \( I_{\text{ion}} \) is the total membrane ion current. In this way, the first term on the right side of equation (1) represents intracellular coupling and the second term adds up all transmembrane currents. The Fenton–Karma model, rather than including detailed dynamics of various membrane currents, calculates \( I_{\text{ion}} \) in terms of three phenomenological
currents: the fast inward currents $I_{i0}$, the slow outward currents $I_{so}$, and the slow inward currents $I_{si}$, which are analogous to the sodium, the potassium, and the calcium currents, respectively. The complete equations of the model are

$$\frac{\partial u}{\partial t} = \nabla \cdot (\vec{D} \nabla u) - J_{i0}(u, v) - J_{so}(u) - J_{si}(u, w),$$

$$\frac{\partial v}{\partial t} = \Theta(u_c - u)(1 - v)/\tau_v(u) - \Theta(u - u_c)v/\tau_v^0,$$

$$\frac{\partial w}{\partial t} = \Theta(u_c - u)(1 - w)/\tau_w^0 - \Theta(u - u_c)w/\tau_w^0,$$

where the three variables are a dimensionless membrane potential $u$, a fast ionic gate $v$, and a slow ionic gate $w$. The variable $u$ is defined by $u = (V - V_N)/(V_0 - V_N)$ where $V_N$ is the Nernst potential of the fast inward current and $V_0$ the resting potential. The gating variables $v$ and $w$ regulate the transmembrane currents. In equation (2), $I_{i0}$, $I_{so}$, and $I_{si}$ are scaled currents and given by the equation $J = I/(C_m(V_0 - V_n))$. The equations for the three currents are

$$J_{i0}(u, v) = -\frac{v}{\tau_{i0}}\Theta(u - u_c)(1 - u)(u - u_c),$$

$$J_{so}(u) = -\frac{u}{\tau_{so}}\Theta(u_c - u) + \frac{1}{\tau_{so}}\Theta(u - u_c),$$

$$J_{si}(u, w) = -\frac{w}{2\tau_{si}}(1 + \tanh[k(u - u_c^d)]).$$

The values of the parameters (all $\tau$ values in milliseconds) are: $\tau_i^+ = 3.33$, $\tau_{i0} = 19.6$, $\tau_{i1} = 1000$, $\tau_{v} = 667$, $\tau_{w} = 11$, $\tau_{d} = 0.416$, $\tau_1 = 8.3$, $\tau_2 = 50$, $\tau_3 = 45$, $k = 10$, $u_c^d = 0.85$, $u_c = 0.13$, and $u_c = 0.055$. This set of parameters yields a system in which scroll waves have a negative filament tension [27]. In this paper, we focus on the isotropic system and thus $\vec{D}$ is a tensor of rank 0 and we refer to this scalar as $D$ considering $D = 1 \text{ cm}^2 \text{s}^{-1}$.

We model the pinning defect as a cylindrical, non-conducting heterogeneity centered in the system. In our figures, this cylinder extends in the vertical direction and is simulated according to the phase-field method reported in [28, 29]. For this method, we select a field $\phi$ that is computed prior to the main simulation. Its initial values equal 0 inside of the heterogeneity and 1 elsewhere. The interface between $\phi = 1$ and $\phi = 0$ is smoothed by iterating the equation

$$\frac{\partial \phi}{\partial t'} = \xi^2 \nabla^2 \phi - \frac{\partial G(\phi)}{\partial \phi},$$

for 1000 steps at a dimensionless time step of $\Delta t' = 0.01$. The parameter $\xi$ (0.05 cm in all simulations) determines the width of the interface. The function $G(\phi)$ constrains the value of $\phi$ to the interval $[0, 1]$ and is given by

$$G(\phi) = \frac{- (2\phi - 1)^2}{4} + \frac{(2\phi - 1)^4}{8}.$$
3. Results

Figure 1 shows a typical snapshot of SWT in the Fenton–Karma model. The solid areas in (a) correspond to excited regions in which the variable $u$ is high. This spatial pattern in the cross-membrane potential changes rapidly but is temporarily organized by rotation around the red curves in (b). The filaments themselves move and elongate in a complex fashion that is driven by the negative filament tension of this system and further complicated by filament-wall and filament–filament collisions. The latter events usually do not induce the crossing or reconnection of the involved segments but are rather dominated by repulsive interaction. We emphasize that this complex state selforganizes from a single scroll wave with an initially linear wave around the unexcitable heterogeneity which can have various shapes and topologies. These pinned states are surprisingly stable and as demonstrated for scroll rings pinned to thin tori, even strong perturbations induce detachment only after slow reorientation processes. To date, however, little is known about this pinning behavior for systems with SWT and specifically no examples demonstrate this phenomenon for cardiac models.

Figure 2 illustrates the system response of the cardiac model to a stationary, cylindrical heterogeneity (vertical, orange rod). Within this thin heterogeneity ($r = 0.325 \text{ cm}$), the conductivity is reduced to zero, thus, blocking all wave propagation. The figure shows a sequence of four snapshots for a simulation in which the heterogeneity is introduced after the SWT has fully developed and occupies the entire system. This time is defined as $t = 0$. Frame (a) shows the wave pattern at $t = 1 \text{ s}$. During this early stage, we can discern a cusp-like deformation of a wave segment near the top end of the cylinder. This feature is caused by the collision of the wave with the cylinder but does not affect the wave rotation in the system. A few seconds later (figure 2(b)), the situation has changed and the top part of the cylinder has just pinned a clockwise rotating scroll wave. While the global wave pattern in (b) remained highly disorganized, only 0.35 s later (figure 2(c)) the pinned scroll wave now occupies about half of the system and turbulent, free vortices are only seen near the system border. Clearly this expulsion of the turbulence is caused by the pinned rotor and indeed we find that 7 s after introduction of the cylinder (figure 2(d)), the turbulence is fully suppressed and the entire system is enslaved to the rhythm of the pinned scroll wave.

The dynamics illustrated in figure 2 occur reliably for the given parameters and cylinder width; only the time at which the pinned rotor forms shows small deviations dependent on the specific pattern of the turbulent wave field at $t = 0$ (these deviations are generated by delaying the cylinder introduction). Another noteworthy feature concerns the lack of vertical phase variations of the pinned vortex (i.e. the absence of twist), which could not have been predicted prior to our simulations. Our results suggest that vortex pinning in systems with negative filament tension does not only arrest the filament curve to the heterogeneity but also suppresses all phase

![Figure 1](image1.png)

Figure 1. (a) A typical snapshot showing scroll wave turbulence (SWT). Solid regions are excited ($u > 0.2$) and interpolating colors on the surface are determined by varying values of the wave field $u$. (b) Wave rotation in (a) is organized by one-dimensional phase singularities. These filaments are plotted as red dotted lines. The pattern at the base of (b) is the projection of the three-dimensional wave field onto the $z = 0$ plane. The vertical axis of both images are stretched by a factor of two.
differences in vortex rotation. Accordingly, the pinned vortex is untwisted and the surrounding wave field (figure 2(d)) has a $z$-independent phase for all $x$ and $y$ coordinates.

Figure 3 provides a more detailed analysis of the actual pinning process. In (a) we plot the temporal evolution of the shortest distance between the filament tangle (i.e. all filament points) and the symmetry axis of the cylinder. The trace varies between 0.325 cm and about 2.2 cm. The minimal value equals the cylinder radius and corresponds to collision between the filament and the cylinder. In this particular example, we find about 30 distinct collisions of which only the last one induces a stable pinned state. As expected from the chaotic nature of the filament motion, the time between the collisions varies greatly. Also their duration fluctuates with some of the longer (unsuccessful) ones lasting 100–200 ms. This finding shows that a mere contact between the filament and the heterogeneity is an insufficient criterion for pinning, thus implying that other factors must be involved. We suggest that this additional variable is the angle between the filament and the cylinder axis. In figure 3(b), we show the filament tangle at $t = 0.95$ s (left arrow in (a)) which denotes an unsuccessful collision event. In this (and other) instances, the angle is nearly 90° which appears to be an unfavored orientation for pinning to a cylinder. Figure 3(c) shows the filament structure during the early stage of the successful pinning event ($t = 5.73$ s, right arrow in (a)). Here the lower part of the filament is oriented in the direction of the cylinder axis, which appears to be the favorable condition for pinning.

The remainder of this article focuses on the important role of the radius of the insulating heterogeneity and describes a simple interpretation of our three-dimensional results based on the period of pinned two-dimensional spirals. As mentioned earlier, the basic principle of the defect-mediated suppression of SWT relies on the high frequency of the pinned vortex. This frequency must be larger than the ‘dominant’ SWT frequency, because only then will the pinned vortex enslave its surroundings and push unpinned filaments outwards where they eventually annihilate at the system boundaries. Since the turbulent wave pattern does not have a unique rotation period, we investigated this threshold value by exposing the turbulence to periodic planar waves. These numerical simulations show that for our system parameters, SWT is expelled for forcing periods below 160 ms. For comparison, the rotation period of the pinned rotor in figure 2(c) equals 134 ms and is hence 16% smaller. However, this period $T$ depends closely on the radius $r$ of the pinning cylinder according to $T(r) = 2\pi r/c$, where $c$ is the wave speed. Notice that this wave speed does also depend on the rotation period. It is constant for large periods and typically decreases for smaller ones.
Taken together these factors determine a monotonically increasing dependence for \( T(r) \) that approaches \( \mu Tr \) in the limit of large radii. Accordingly, we should expect that thicker cylinders fail to suppress the SWT; however, our simulations reveal a surprisingly different outcome. Figure 4 shows snapshots from two simulations with thick cylinders of radius 0.75 cm (a) and (b) and 1.25 cm (c) and (d). The left ((a) and (c)) and right frames ((b) and (d)) are early and late stages, respectively. The individual images are two-dimensional projections of the three-dimensional cardiac system onto the \((x, y)\)-plane obtained by averaging in \(z\)-direction. Clearly both heterogeneities succeed in expelling the SWT from the system and establish highly-order pinned vortex states. These states, however, are multi-armed rotors with two and three wave sheets rotating simultaneously around the cylinder. We emphasize that these structures form spontaneously over time intervals that are comparable to the ones discussed in the context of figures 2 and 3. A movie of the formation of a two-armed vortex can be found in the supplementary data.

Key to the suppression of SWT by spontaneously forming multi-armed scroll waves is that the wave period of an \( n \)-armed vortex is roughly \( n \) times smaller than that of a one-armed vortex. Consequently, the frequency of these structures can rise above the SWT suppression threshold even for thick cylinders. Also their spontaneous formation mechanism is easily explained. Collision of the filament with the cylinder clearly occur for all cylinder radii and are actually more likely for thicker ones. If several collisions occur simultaneously or in short sequence, multi-armed rotors can develop spontaneously. If the frequency of the resulting structure is too low, then SWT is not suppressed but an additional collision can increase the number of rotating wave sheets and hence the wave frequency.

In figure 5, this dependence is analyzed in terms of the wave period \( T \) and the cylinder radius \( r \). The small arrows at the top of the figure are the result of three-dimensional simulations with insulating cylinders of radii indicated by the placement of the arrows. Their colors (green, blue, orange) indicates the number of wave arms of the spontaneously forming pinned scroll wave \((1, 2, 3, \text{respectively})\) that in all cases eliminated the turbulence. The thick horizontal line is the threshold period below which SWT is suppressed by planar wave trains. The three upward sloping curves are (from left to right) the period of one-, two-, and three-armed vortices as measured for spiral waves in two-dimensional systems where SWT does not exist. Notice that these structures have minimal \( r \) values below which the refractory tail of the closely stacked waves makes rotation impossible. The portions of the curves below the threshold period, define \( r \)-windows in which SWT can be expected to be suppressed by the respective multi-armed scroll wave. These windows are color-coded and labeled with Roman numerals according to the number of vortex arms. The windows agree well with the results from the independent three-dimensional simulations as only the rightmost green arrow falls slightly outside of the window I. In addition, we
Figure 4. Multi-armed pinned scroll waves spontaneously form for thick heterogeneities. The images (a)–(d) show the projection of the three-dimensional wave field $u$ onto the $x, y$ plane. The pinning cylinder (white circle) has a radius $r$ of 0.75 cm in (a) and (b) and 1.25 cm in (c) and (d). The left and right frames are early and late snapshots, respectively. Time elapsed between frames: 750 ms ((a) and (b)) and 800 ms ((c) and (d)).

Figure 5. Rotation period of pinned spiral waves as a function of the anchor radius $r$. The green (diamond), blue (asterisk), and orange (cross) curves correspond to one-, two-, and three-armed spirals, respectively. In three dimensions, turbulence is suppressed for periods below the black, horizontal line. The intersection of the black line with the colored curves and the smallest possible periods of the pinned rotors define three colored windows (also labeled as I, II, and III). The small arrows indicate the cylinder radii of three-dimensional simulations and the arrow colors denote the observed number of pinned wave arms using the same color scheme. In addition, the three red arrows correspond to a more complex vortex state (figure 6).
find that the windows II and III overlap. This overlap indicates that, for this range of \( r \) values, both two- and three-armed scroll waves can suppress SWT. Clearly this sequence of windows continues for vortices with even higher topological charges but is not further investigated here because four or more arms require rather large cylinder radii that are biologically irrelevant.

For the set of parameters investigated, we also find a small gap between the windows I and II. For \( r \) values within this gap, the characteristic filament tangles of SWT are eliminated but the suppressing vortex (red arrows in figure 5) has unusual dynamics that differ from those discussed so far. More specifically, the pinned rotor has two arms of which one shows a conventional attachment to the cylinder and the other a nearby loop-like motion. A representative example of the resulting trajectories is shown in figure 6 and the corresponding movie in the supplementary data. In the figure, the trajectories represent the tip motion at the lower boundary (\( z = 0 \)) which are representative for the entire system because the wave pattern varies only slightly in the \( z \)-direction. Furthermore, the attached filament is nearly linear whereas the unattached filament shows small S- and C-shaped deformations that are clearly caused by the negative filament tension. Notice that due to repeating collisions between the two wave arms, we cannot uniquely distinguish the two filaments over longer time intervals. The red and blue color in figure 6 is therefore an arbitrary assignment with filaments alternating between a free and an attached phase. We also note that we observed this behavior also within the low \( r \) region of window II, where they should not exist based on our simple analysis of vortex periods. At this point, however, we cannot rule out very slow transients that would eventually change the complex motion to that of a fully pinned two-armed scroll waves.

Although the system behavior in the gap between windows I and II is clearly simpler than the uncontrolled turbulence in homogeneous (cylinder-free) media, the local dynamics do not show the simple periodicity that we find for the wave patterns of the fully pinned \( n \)-armed vortices. To further characterize these dynamics, we investigate the temporal evolution of \( u \) about one wavelength away from filament trajectories (see figure S1 in the supplementary data). The analyzed data span 57 wave passages and exclude the initial turbulent phase as well as the first ten rotation cycles of the stabilized rotor. Within window I, the spectrum shows one peak and accompanying harmonics. Within the gap between the windows, the spectrum is more complex but does not suggest chaotic behavior as it is not very broad and still dominated by one frequency. We also generated return maps both in terms of the amplitude and the time elapsed between subsequent \( u \) maxima. These maps reveal simple periodic behavior within the window and an irregular point cloud within the gap. For the latter range of radii, much longer simulations appear to be necessary to fully rule out the presence of chaotic point dynamics.

Lastly, we investigate the response of the pinned rotors to slow variations of the cylinder thickness. Figure 7 summarizes this response by plotting the topological charge \( N \) (i.e. the number of arms) as a function of the cylinder radius \( r \). In the initial state, the system is governed by a three-armed scroll wave rotating around a very thick heterogeneity of radius 1.75 cm. For slowly decreasing \( r \) values, this vortex persists down to about 0.8 cm. At this radius, the vortex transforms to a two-armed scroll wave. This change occurs via the initial detachment of one of the wave sheets and the subsequent drift of its filament towards the system boundary where it annihilates. An analogous process mediates the transition from \( N = 2 \) to 1. Here, however, we observe the formation of transitional states similar to the one shown in figure 6 that we arbitrarily assign a charge of 1.5. The end point of
this $r$-reduction is a pinned one-armed scroll wave. If the radius is decreased below about 0.325 cm, the vortex detaches and eventually SWT is re-established.

Another peculiar feature is the response of the pinned one-armed scroll wave to increasing radii. As represented by the dashed line in figure 7, the charge of the vortex does not follow the earlier (solid) curve but rather remains constant at $N = 1$ even for very large values of $r$. Furthermore, SWT is not reestablished although SWT suppression by a one-armed vortex is effective only up to $r \approx 0.5$ cm (see window I in figure 5). This observation is explained by the fact that a system with negative filament tension develops SWT only if a free filament segment is present. Since neither the pinned vortex nor its surrounding wave field fulfills this condition, the rotor can remain stable even for thick cylinders that result in very low rotation frequencies. This well-known feature gives additional importance to the overall effect of heterogeneity-induced SWT suppression because once eliminated, turbulence is less likely to reoccur.

4. Conclusions

In summary, our cardiac simulations establish that a small insulating heterogeneity can suppress SWT. The underlying mechanism depends on the spontaneous pinning of a nearby filament segment and requires a favorable parallel orientation to the cylinder. The average time elapsed during this initial step must obviously decrease with the length of the cylinder but as shown here, is of the order of a few seconds for transmural obstacles in the human ventricles. Once pinned, the filament very quickly self-wraps around the heterogeneity and establishes a stable wave source that is either sufficient fast to expel all unpinned filaments from the system or requires the spontaneous attachment of additional filament strands to yield a multi-armed scroll wave. This fast, periodic pacemaker pushes all free moving filaments outwards where they eventually collide against the system boundaries and vanish. The resulting point dynamics are strictly periodic with the exception of a small range of cylinder radii for which the turbulent filaments have been eliminated but the point dynamics are slightly irregular due to complex filament motion in close vicinity of the heterogeneity.

Our study raises the perplexing question whether SWT can actually exist in human ventricles. Heart tissue is highly heterogeneous at length scales relevant to our study because it involves blood vessels, other anatomical features and—in post-infarction patients—remodeled myocardium (scar tissue) of greatly reduced conductivity. Accordingly, filaments should pin to such structures and expel the turbulent filaments as demonstrated in our study. This scenario would lead to a necessary self-termination of SWT that should be observable as a possible scenario of VF. The spontaneous transition from VF to tachycardia is indeed being discussed in the field of human cardiology (see e.g. [40, 41]). In addition, Valderrabano et al [42] reported experiments with swine ventricles and concluded that artificial obstacles are ‘a substrate for re-entry stabilization during VF’. However, in most cases VF does not the stabilize to the more benign periodic tachycardia but rather is one of the leading causes of death in the industrialized world.

There are several factors that might resolve this seeming paradox. For instance, the pinning process can be upset by the cardiac fiber rotation as reported by Majumder et al [43] who studied pinning in anisotropic system with cylindrical inhomogeneities. These authors also noted that strongly meandering scroll waves can detach...
from the pinning cylinder. It is also well known that—under certain circumstances—heterogeneities nucleate spiral waves in collisions with non-rotating planar waves [44]. While we did not observe this process in our simulations, particular orientations might introduce such destabilizing events more readily. We also note that the specific shape or length of the pinning heterogeneity appears to be less relevant and is hence an unlikely player against the observed transition from VF to tachycardia. For example, some of our earlier studies demonstrate scroll wave pinning to cones as well as short stationary and even moving cylinders [23, 35, 45].

Beyond these biomedically important question, our study strongly suggests that multi-armed vortices are a relevant and potentially frequent wave pattern in three-dimensional heterogeneous systems. In our simulations their creation was driven by the collisions of the turbulent filament tangle with the insulating obstacle; however, similar collisions can also occur in systems with positive filament tension, in which vortex loops undergo curve-shrinking dynamics. It will be interesting to study the nucleation processes of these structures in greater detail as numerous questions remain unanswered. These include the competition or co-existence of vortices with different topological charge pinned to the same object and the behavior of multi-armed vortices pinned to cones and similar structures that extend across the pinning windows (figure 5) reported in our study.

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