Estimating the effects of Bose-Einstein correlations on the W mass measurement at LEP2

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Abstract

The influence of Bose-Einstein correlations on the determination of the mass of the W boson in $e^+e^- \rightarrow W^+W^- \rightarrow 4 \text{ jet events at LEP2 energies is studied, using a global event weighting method. We find that it is possible to keep the systematic error on the W mass from this source below 20 MeV, if suitable precautions are taken in the experimental analysis.}
1 Introduction

The accurate determination of the mass of the W boson is expected to be one of the most
important standard physics results to be obtained from LEP2 measurements. Since \( M_W \)
is one of the key parameters of the electroweak theory, precise knowledge of the W mass
allows one to test the Standard Model (SM), and together with the top quark mass can
be used to constrain the allowed range of the Higgs boson mass in the SM, or restrict the
parameter space of other “new physics”. The present value of \( M_W \) is based on the direct
measurements in \( \bar{p}p \) interactions at CERN \[1\] and at the Tevatron \[2, 3\]. The combined
result from these studies is \( M_W = 80.33 \pm 0.15 \) GeV \[4\]. Data from LEP2 recorded in
1996 at 161 and 172 GeV are expected to lead already to a comparable error when they
are fully analysed and the results of the four LEP experiments are combined. When all
existing data from the CDF and D0 experiments (more than 100 pb\(^{-1}\) per experiment) are
analyzed, it is envisaged that the error on the W mass will be reduced to about 70 MeV.
On the other hand, an indirect determination of \( M_W \) from a SM fit using LEP and SLC
measurements give an error in the W mass of approximately 40 MeV (for a Higgs mass
of 300 GeV) \[5\], thus setting the scale for a significant new test of the model.

Three different methods have been proposed for the determination of the W mass in
e\(^+\)e\(^-\) annihilation at LEP2 \[6, 7, 8\]:

- The threshold cross-section measurement of the process \( e^+ e^- \rightarrow W^+ W^- \). It has been
  shown that maximum sensitivity to \( M_W \) is achieved for the energy \( \sqrt{s} \simeq 161 \) GeV.
  Now these measurements are complete and the error in the W mass is about 450 MeV
  for a single experiment \[9\], with a combined result \( 80.4 \pm 0.22 \) GeV \[10\].

- The measurement of the charged lepton end-point energy. This gives an estimated
  error exceeding 300 MeV \[3, 11\] for the total integrated luminosity expected at LEP2
  and is therefore not competitive with the other two methods.

- The direct reconstruction of the \( M_W \) from the final state particles. This is consid-
ered to be the most promising method. It relies on the use of the four constraints
  from the energy- momentum conservation. An additional constraint can be added
  by the assumption that the two W bosons have equal masses. Two decay channels
  \( \rightarrow q\bar{q}q\bar{q} \) (four jets) and \( q\bar{q}l\nu \) (two jets plus leptons) — can be used in this analysis.
  The estimated total error for the two channels combined is 34 MeV, assuming four
  experiments each collecting 500 pb\(^{-1}\) data at \( \sqrt{s} = 175 \) GeV \[8\].

For the process \( e^+ e^- \rightarrow W^+ W^- \) at LEP2 energies, the typical separation of the two
decay vertices of Ws in space and time is of order of 0.1 fm, much smaller than the
hadronization scale (\( \approx 0.5 \) fm). Thus, when both Ws decay hadronically, the hadroniza-
tion regions of the W\(^+\) and W\(^-\) overlap, and colour interconnection \[12, 13\] during the
hadronization as well as Bose-Einstein (BE) correlations \[14\] between identical bosons
among the decay products of the two Ws can couple the two systems and thereby affect
the W mass measurement.

At present it is not excluded that these effects could each contribute about 100 MeV to
the theoretical uncertainty, which would make the four jet channel essentially useless for
the W mass measurement. Hence, apart from the intrinsic interest in these phenomena
the studies of colour interconnection and Bose-Einstein correlations are very important,
and reliable estimates of the size of their effect on the reconstructed W mass are highly
desirable in order to evaluate their contributions to the systematic error in this measurement.

The first attempt to investigate the influence of Bose-Einstein correlations in the determination of W mass was made in [14], where the LUBOEI algorithm, implemented in the JETSET Monte-Carlo program [15], was used. The basic assumptions of this approach are that BE effects are local in phase space and do not alter such characteristics as the event multiplicity or the cross section. The momenta of the bosons produced at the hadronization stage are shifted by amounts calculated to reproduce the two particle correlation function expected for a source with a gaussian space-time distribution. This procedure, however, does not preserve momentum and energy simultaneously. The violation is not large, however, and energy conservation is restored in an ad hoc way by rescaling all particle momenta with a common factor. The latter procedure introduces an artificial shift in the W mass, even if there are no Bose-Einstein correlations between the particles coming from different Ws. The procedure also tends to make the jets more narrow, thereby decreasing the individual jet masses. Assuming a source radius of 0.5 fm, the corrected mass shift was found to be 95 MeV at $\sqrt{s} = 170$ GeV [14]. The shift increases with increasing energy and decreasing source radius.

In the following analysis we will address the problem of Bose-Einstein correlations, but using another approach, which is based on assigning weights to the simulated events according to the momentum distributions of final state bosons. In the global event weight scheme a shift in the W mass can arise, if the event weight depends on $M_W$. The use of such global event weights is not straightforward and was discarded in the above study on fairly general grounds. Our aim is to reinvestigate the suitability of the method and to estimate approximately the systematic error on the W mass from Bose-Einstein correlations. The use of global event weights is complementary to the local reweighting scheme of ref. [14] in the sense that here the kinematical properties of the events are preserved, while all probabilities and multiplicities may change. We have not found a unique solution to the problem of assigning the weights, however, and have used a number of different weighting schemes to assess the range of effects that can be expected from Bose-Einstein correlations.

We have used the PYTHIA event generator [15] to generate Monte Carlo samples of $e^+e^- \rightarrow W^+W^- \rightarrow 4$ jet events. A basic assumption here is that hadronic W and Z boson decays are sufficiently similar, so that by using the tuning of the Monte-Carlo model parameters that reproduces the experimental data from Z decays at LEP, Bose-Einstein effects in single W decays are already effectively taken into account in properties such as multiplicities and single particle momentum spectra. Hence, only the correlation between identical particles from different Ws have been included in the weight calculation. In order to check the self-consistency and inherent systematic errors of the method we have applied the same weighting method to the well studied process of Z hadronic decays.

Since we want to estimate the maximal systematic error on the measured W mass due to the mass shift arising from Bose-Einstein correlations, we have not taken into account smearing due to experimental resolution and acceptance, reconstruction method etc.

Various possibilities of constructing event weights are discussed in the following section. Some consequences of these weighting schemes on certain measurable quantities at the Z peak are considered in Section 3, together with possible constraints on BE effects from Z physics. In Section 4 the influence of BE effects on the process of W pair production at LEP2 is analyzed.
2 Event weighting schemes for Bose-Einstein effects

The Bose-Einstein effect corresponds to an enhancement in the production probability of identical bosons to be emitted with small relative momenta, as compared to non-identical particles under otherwise similar conditions. Experimental data are usually analysed in terms of the correlation function defined as the ratio of the two-particle probability density to the product of the corresponding single particle quantities:

\[ C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} \quad (1) \]

where \( p_i \) is the four-momentum of particle \( i \). Assuming a spherical space-time distribution of the particle source, the correlation function takes the form:

\[ C(Q) = 1 + \lambda \rho(Q) \quad (2) \]

where \( Q \) is the four-momentum difference, \( Q^2 = -(p_1 - p_2)^2 \), and \( \rho \) is equal to the absolute square of the Fourier transform of the particle emitting source density, with the normalization condition \( \rho(0) = 1 \). The parameter \( \lambda \), known as the incoherence parameter, takes into account the fact that for various reasons, the strength of the correlations can be reduced.

Often a gaussian model is assumed for the source density, which leads to

\[ \rho(Q) = \exp(-R^2Q^2) \quad (3) \]

where \( R \) is the radius parameter. Experimentally, this gives a reasonable description of Bose-Einstein correlations in many types of collisions. In \( e^+e^- \) collisions typical values of \( R \) and \( \lambda \) are respectively 0.5 fm and 0.3 (1.0 for directly produced pions) [18, 19].

In order to simulate the effects of Bose-Einstein correlations, we have in this study chosen to use the event weighting method. The method arises very naturally in a quantum mechanical approach, where the weight can be constructed as the ratio of the square of the symmetrized multiparticle amplitude to the square of the non-symmetrized amplitude corresponding to the emission of distinguishable particles. The use of global event weights leads to a number of conceptual and computational difficulties, however, which must be overcome for any quantitative conclusions to be drawn and which we will attempt to address below.

There are several possibilities to construct event weights. One way of forming the weight is to take a product of enhancements \( C(Q) \) for all pairs of identical bosons in the event [16]:

\[ V_1 = \prod_{i_1,i_2} C(Q_{i_1i_2}) \quad (4) \]

For high multiplicity events this weight can become extremely large, so that a few such events dominate the weighted distributions and lead to non-realistic distributions. The event weights therefore have to be regularized in some way. In order to keep the statistical error at a reasonably low level, we have chosen to discard events with very high weights (higher than some \( V_{\text{max}} \)). This, however, has the unpleasant effect of making the results \( V_{\text{max}} \)-dependent. We have tried to circumvent this difficulty by analyzing the \( V_{\text{max}} \) dependence and extrapolating the results to \( V_{\text{max}} \to \infty \).
One can also rescale the weight of the event using a single constant $w_0$:

$$V_2 = V_1 / w_0^n$$  \hspace{1cm} (5)$$

where $w_0$ is a constant slightly larger than 1, and $n$ is the number of pairs in the event (i.e. the number of terms in the product in (4)). The value of $w_0$ is chosen to keep the average multiplicity reasonably close to its value before event weighting [20]. For a constant $V_{max}$, we have found, however, that the method gives rise to numerical difficulties, stemming from the fact that increasing $w_0$ brings in more events from the high weight tail of $V_1$, which leads to large fluctuations in the multiplicity and the average event weight. Our results for the shifts in multiplicity and $M_W$ using $V_2$ are roughly consistent with those found using $V_1$, and $V_2$ will not be discussed further.

A problem with these weighting methods is that, like the local reweighting scheme of ref [14], they emphasize narrow jets and thereby introduce artificial correlations also between non-identical particles. In order to counteract this, one can use the weight calculated with non-identical pairs to rescale (4):

$$V_3 = V_1 / V_0^{n/m}$$  \hspace{1cm} (6)$$

where $V_0$ is the weight calculated according to (4) but for non-identical bosons in the same event, while $n$ and $m$ are the numbers of identical and non-identical pairs, respectively. This also leads to a better numerical behaviour, as illustrated in fig. 1, which shows the distributions of $V_1$ and $V_3$ for simulated WW events at $\sqrt{s} = 175$ GeV. The high weight
tail is much less pronounced for $V_3$ than for $V_1$. Both fall off to a good approximation as inverse powers, with exponents $-2.6$ and $-1.4$, respectively, which makes it plausible that the sum of all weights converges.

A different method of constructing the event weight, which is closer to a full quantum mechanical treatment, starts from the introduction of a symmetric amplitude, which has $n!$ terms \( [24] \). This leads to a weight:

$$V_4 = \sum_{\text{permutations}} \lambda^{k/2} \rho(Q_{1i_1}) \rho(Q_{2i_2}) \cdots \rho(Q_{ni_n}),$$

where $k$ is the number of times when the first and second indices differ. For $Q = 0$, $\lambda = 1$ and $n$ identical particles, equation (4) gives a weight of $2^{(n-1)/2}$, while eq. (7) results in the correct value $n!$. However, for typical hadronic configurations this difference is much smaller, and (7) is rarely used because of computational difficulties. We had to restrict ourselves to events containing no more than 8 identical particles of each kind. This limitation is too restrictive already at $Z^0$ energies, where the number of identical boson combinations is lower than in $W^+W^-$ events, and rejected about 50% of events in $W$ pair production, making it effectively useless for the latter case. We used it only to check that it gave essentially consistent results with our other methods for events with low multiplicity.

The event weights defined above were based on the gaussian parametrization of the particle emitting source. This implies that $V_1 \geq 1$ for all values of $Q$.

This is not true in all models, however. In addition to the above weights, we have therefore also studied a different pair weight, inspired by the weight used in [21], where $\rho$ in (2) is not required to be always positive:

$$\rho(Q) = \frac{\cos(\xi QR)}{\cosh(QR)}$$

For $\xi$ close to 1, this is very close to (3) apart from becoming slightly negative at large $Q$. The corresponding weight $V_5$ is built in analogy with (4), but with the gaussian (3) replaced by (8) (the dotted line in Figure 1). We find that $\xi = 1.15$ leads to a good overall description. Due to the better numerical behaviour of this weight function (the exponent in the power fit is $-2.4$), we were able to apply $V_5$ without further rescaling.

### 3 Influence of event weighting on $Z^0$ properties

Various measurable properties of the $Z^0$ will be affected to different extent, if one introduces event weights into the simulation of its hadronic decays. Since the partonic states before hadronization are known to be well described by perturbative calculations, which do not take into account Bose-Einstein correlations, uncritical application of event weights may lead to large inconsistencies with e.g. measured branching ratios and relative frequencies of jet multiplicities etc. In order to see how serious these effects are and to judge what consequences this has for the analysis of the WW events, the precise experimental data from $Z^0$ decays can be used to check the event weighting schemes of Bose-Einstein correlations for $W$ pair production at LEP2. We have simulated $3 \times 100000$ hadronic events at $\sqrt{s} = M_Z$ and $M_Z \pm 2$ GeV. Table 1 presents the differences for charged particle multiplicity, shift of $Z^0$ peak position in hadronic vs leptonic decay modes, branching
fractions for charm and beauty decays ($R_c$ and $R_b$) and the ratio of three to two jet events with and without event weighting.

This analysis resulted in the following:

- The average charged multiplicity has changed. The weight $V_1$, which was not rescaled, leads to the largest increase when compared to the unweighted results, while both $V_3$ and $V_5$ give a smaller increase around 1.5. In all these cases, the change can be accommodated by retuning of the parameters in the simulating program.

- In principle, event weighting can result in a shift of the $Z$ mass peak. However, only $V_1$ yielded a shift of a few MeV, while for $V_3$ and $V_5$ the shift is essentially zero. We have not found any significant change of the $Z^0$ width.

- The pattern of heavy and light quark fragmentation is rather different. Heavy quarks produce significantly less pairs with small $Q$, and all BE effects in this approach are less pronounced for heavy quarks. Heavy quark events thus obtain smaller average weights, which result in changes shown in Table 1. Note that the effect for $c$-quarks is diluted because $b$-quark events reduce the overall average weight. Further study of this effect lies beyond the scope of this work. In order to exclude this artificial flavour dependence in $W$ decays, the weighting and rescaling was performed separately for the different decay modes of the $W$s.

- The weighting resulted in a substantial increase of jet activity, as measured by the three to two jet event ratio. This is however difficult to quantify because of its dependence upon the jet finding algorithm and its parameters. The numbers shown in Table 1 were obtained using LUCLUS with default parameters, corresponding to fairly narrow jets. The effect decreases for broader jets and in any case is much less pronounced in WW production, so we did not attempt to correct for it.

We have considered maximum BE correlations, $\lambda = 1$, but only for pions and kaons originating from the sources with decay lengths $c\tau < 10$ fm, and $\lambda = 0$ otherwise. The source radius $R$ was taken to be equal to 0.5 fm everywhere. In $Z^0$ decays at LEP1 one observes $\lambda \approx 0.3$ if all particles are considered, 0.4 if only pions are taken into account and 1.0 for directly produced pions [8, 19], and $R \approx 0.5$ fm.

|                  | $V_1$       | $V_3$       | $V_5$       |
|------------------|-------------|-------------|-------------|
| $\Delta \langle n_{ch} \rangle$ | $3.7 \pm 0.5$ | $1.3 \pm 0.2$ | $1.8 \pm 0.2$ |
| $\Delta M_{Z^0}$, MeV   | $8 \pm 3$   | $0 \pm 3$   | $1 \pm 4$   |
| $\Delta R_c$, %   | $-3 \pm 2$  | $-2 \pm 2$  | $0 \pm 2$   |
| $\Delta R_b$, %   | $-26 \pm 3$ | $-11 \pm 2$ | $-5 \pm 2$  |
| $\Delta 3\text{jet}/2\text{jet}$, % | $80 \pm 20$ | $20 \pm 5$  | $20 \pm 5$  |

Table 1: Differences in charged multiplicity, peak mass of $Z^0$, branching fractions and three-to-two jet event ratio, between weighted and non-weighted events, for various weighting systems described in the text.
Figure 2: Reproduced correlation functions in $Z^0$ events using the weights $V_1$, $V_3$, and $V_5$. The dashed lines show the result of a fit to real data from hadronic $Z^0$ decays \[19\].

The reproduced correlation functions for the three weighting schemes, $V_1$, $V_3$, and $V_5$ are shown in figure 2. We have divided the ratio of the weighted same-sign particle distribution to the unweighted one by the similar ratio for opposite-sign particles. This is equivalent to using the $Q$-distribution of opposite-sign pairs as a reference sample and correcting for effects of particle selection and resonances by dividing by the same ratio in simulated events without BE correlations — a common procedure in experimental analyses. Also shown are fits to the form

$$N(1 + \beta Q)(1 + \lambda \exp(-Q^2 R^2))$$

which is often used to parametrize the experimentally observed correlation function in $Z^0$ decays \[18\], \[19\]. The resulting values of the parameters for a fit range of 0–2 GeV in $Q$ are shown in Table 2. The dashed line in the figure represents the result of a fit to the correlation function of all particles observed in real data from hadronic $Z^0$ decays \[19\].

$V_1$ does not reproduce the correlation function well, as it gives too small values for both the incoherence parameter $\lambda$ and the radius $R$. The other weight schemes give very reasonable descriptions resulting in input and output parameter values which are reasonably close to each other. We take the spread of the results using the different weighting schemes to be indicative of the systematic errors inherent in the method.

Hence we conclude that, provided that the different quark final states (and possibly the final states with different number of jets) are treated separately, application of the global event weighting technique with rescaling of the weight ($V_3$) or using the form (8) is
Table 2: The fitted values of the correlation function parameters $\lambda$, $R$, $\beta$ and $N$ at the $Z^0$ peak.

| Weight | $V_1$      | $V_3$      | $V_5$      |
|--------|------------|------------|------------|
| $\lambda$ | $0.072 \pm 0.004$ | $0.292 \pm 0.005$ | $0.325 \pm 0.005$ |
| $R$ (fm) | $0.387 \pm 0.018$ | $0.465 \pm 0.005$ | $0.455 \pm 0.005$ |
| $\beta$ (GeV$^{-1}$) | $-0.029 \pm 0.003$ | $0.047 \pm 0.003$ | $0.080 \pm 0.003$ |
| $N$ | $1.010 \pm 0.004$ | $0.925 \pm 0.004$ | $0.899 \pm 0.004$ |

Table 3: The fitted values of the correlation function parameters $\lambda$, $R$, $\beta$ and $N$ for W pair production at $\sqrt{s} = 175$ GeV.

| Weight | $V_1$      | $V_3$      | $V_5$      |
|--------|------------|------------|------------|
| $\lambda$ | $0.032 \pm 0.004$ | $0.101 \pm 0.002$ | $0.146 \pm 0.003$ |
| $R$ (fm) | $0.322 \pm 0.024$ | $0.459 \pm 0.009$ | $0.457 \pm 0.006$ |
| $\beta$ (GeV$^{-1}$) | $0.002 \pm 0.003$ | $0.013 \pm 0.001$ | $0.036 \pm 0.001$ |
| $N$ | $0.997 \pm 0.003$ | $0.977 \pm 0.003$ | $0.949 \pm 0.003$ |

The first thing to study is the correlation function and the compatibility of input and output values indicating the self-consistence of the method. Table 3 refers to $175$ GeV, and contains the results of the fits to the correlation functions for each weight used (as above for $Z^0$) using the cut off $V_{max} = 80$. Numbers for $192$ GeV are very similar. The errors given in the table are statistical only. From the variation of the fit results with $V_{max}$, we estimate that the systematic error is $0.05$ on $\lambda$ and $0.05$ fm on $R$.

One sees that for $V_1$, $R$ is somewhat lower than the input value, while $V_3$ and $V_5$ give values quite close to 0.5. It is worth noting that the values of $\lambda$ are significantly smaller than at the $Z$ peak, essentially because in the WW case we have included only correlations between pairs from different Ws.

Next, the information from the Monte-Carlo was used to assign each final particle to the $W^+$ or the $W^-$, as in [4]. Ws with the mass values in the interval $70$ GeV $\leq M_W \leq 90$ GeV were studied at $175$ and $192$ GeV to assess the energy dependence. At each energy, $10^5$ events were generated, which is about an order of magnitude higher than the expected statistics of all four LEP experiments combined at $500$ pb$^{-1}$ integrated luminosity per
experiment. In general, one expects that BE-induced effects in WW production should die out at high energies, as the overlap between the two W decay volumes decreases. This requires much higher energies than will become available at LEP2, however, and it is likely that the effect will increase with energy in the LEP2 range [14].

The mass distribution of W bosons was built with and without event weighting for each of the weights used, and the differences were calculated in the average charged multiplicity \( n_{ch} \), the mean W mass, \( M_{W}^{\text{mean}} \), averaged over the whole interval \( 70 \text{ GeV} \leq M_{W} \leq 90 \text{ GeV} \), and a fitted \( M_{W}^{\text{fit}} \). The fit was performed using a relativistic Breit-Wigner shape with an \( s \)-dependent width, in the interval \( 80.25 \pm \delta \text{ GeV} \), with \( \delta = 2 \text{ GeV} \). The results are presented in Table 4. We also performed the fit in wider intervals corresponding to \( \delta = 4, 6, 8 \) and 10 GeV and observed a small additional increase in the mass shift of a few MeV, which saturated at the larger interval sizes, consistent with the fact that the tails of the Breit-Wigner distribution contain very little information about the value of the W mass.

As mentioned above, for computational reasons, we were forced to cut away a tail of events with very large weights. We have tried to eliminate the dependence on the cutoff value, \( V_{\max} \), by calculating the multiplicity and mass shifts for three values of \( V_{\max} \) (20, 40, and 80) and then extrapolating to infinite cutoff. This method seems to be more reliable and less vulnerable to fluctuations than direct calculation with very high \( V_{\max} \). Figure 3 shows the values of the mean W mass, \( M_{W}^{\text{mean}} \), and the fitted \( M_{W}^{\text{fit}} \) as functions of \( 1/V_{\max} \). There is no indication in our investigations that the inclusion of the events with very large weights would change the estimated mass shifts by any significant amounts. For \( V_{1} \), the extrapolated value depends on the specific way the extrapolation is performed. We have included this ambiguity into the error shown in Table 4.

From these numbers we draw the following conclusions:

- There is a clear correlation between the BE-induced shifts in the W mass and in the charged particle multiplicity in WW production: the larger is the increase in charged multiplicity, the larger mass shifts are expected.

- Both \( V_{3} \) and \( V_{5} \) weights result in fairly small shifts, while still maintaining a good reproduction of the correlation function. They are well-behaved numerically and in our opinion give quite reliable estimates of the effect. However, we conservatively

\[
\begin{array}{c|ccc}
\Delta n_{ch} & V_1 & V_3 & V_5 \\
175 \text{ GeV} & 3.8 \pm 0.5 & 1.8 \pm 0.2 & 1.0 \pm 0.2 \\
192 \text{ GeV} & 3.7 \pm 0.5 & 1.7 \pm 0.2 & 0.6 \pm 0.2 \\
\hline
\Delta M_{W}^{\text{mean}} \text{ (MeV)} & & & \\
175 \text{ GeV} & 75 \pm 15 & 22 \pm 11 & 20 \pm 14 \\
192 \text{ GeV} & 92 \pm 16 & 34 \pm 11 & 38 \pm 14 \\
\hline
\Delta M_{W}^{\text{fit}} \text{ (MeV)} & & & \\
175 \text{ GeV} & 12 \pm 9 & 11 \pm 7 & 4 \pm 12 \\
192 \text{ GeV} & 15 \pm 8 & 13 \pm 7 & 6 \pm 9 \\
\end{array}
\]

Table 4: Values of differences in multiplicity and mass of the W boson for events with and without interconnecting Bose-Einstein correlations between the two Ws.
Figure 3: Shifts in the mean and fitted W mass as functions of the inverse weight cut off, $1/V_{\text{max}}$, for weight schemes $V_1$, $V_3$ and $V_5$, and $\sqrt{s} = 175$ GeV.

- The fitted value for the W mass is less sensitive to BE effects than the mean over the full distribution, which has been used to estimate the effect in previous investigations [14]. Our estimated values for the shift in the fitted mass are less than 20 MeV, implying that BE correlations are not too dangerous for the W mass measurements at the expected level of accuracy at LEP2. For the shifts in the mean W mass, we confirm the conclusions of [14] and find values of the same general magnitude of a few tens of MeV, under similar conditions as investigated there. In all cases the shift is towards larger masses, as expected on general grounds [8, 14].

- For all weighting schemes, the shift in $M_W$ increases with energy in the energy range considered, but the increase is fairly small.

It is interesting to compare our results to the predictions based on the implementation of Bose-Einstein effects by shifting the momenta of final state particles [14]. There are several differences in the predictions of these two schemes.

The most important difference is in the particle multiplicity: our approach naturally leads to an increase of the average number of particles due to Bose-Einstein correlations, while the momentum-shifting method assumes that the multiplicity is unchanged. Experimentally, it is not known yet to what extent Bose-Einstein correlations might modify the particle multiplicities at high energies.
The energy dependence of W mass shift is different. The strong energy dependence in momentum-shifting scheme is a combination of two effects: the increase of the systematic shift for low momentum particles in the direction of smaller W momenta, and the differences in momentum spectra of W decay products for various energies, as stressed in [14]. This seems to be less pronounced in the present approach.

The present study confirms that the systematic effect of BE correlations on the W mass determination can potentially be quite large, as found in [14], although the actual values of the mass shift found here are somewhat smaller. The size of the shift is however quite sensitive to the procedure used to extract the value of the W mass. In particular we observe that a fit to the lineshape of the W mass distribution has a much smaller systematic error from Bose-Einstein correlations than the average mass, due to the fact that the main effect on $M_W$ in our scheme arises from the tails of the mass distribution, which contain very little information about the peak position. Hence it does seem possible to keep the systematic error from this source below about 20 MeV. Careful work linked to the actual fitting procedures used by the LEP experiments is obviously needed in order to assess this in the individual cases and to optimize the analysis procedures. Since the value of the mass shift is always positive (as also expected on general grounds), a further reduction of the systematic error by a factor two is in principle possible by assigning the expected shift as a correction to $M_W$.

The estimates in this paper were all made assuming full Bose-Einstein correlation strength ($\lambda = 1$). If this should turn out not to be the case experimentally (see [22]), the effect on the W mass may be correspondingly reduced. Here we just want to bring attention to the fact, that such a reduction cannot a priori be expected to depend proportionally on $\lambda$, as it is presumably not the correlation strength, but the absolute number of correlated pairs of pions in the enhanced region of the $Q$ distribution that is important for the size of the possible shift in the W mass.

The comparison of hadronic decays of $W^+W^- \rightarrow q\bar{q}q\bar{q}$ and $W^+W^- \rightarrow q\bar{q}l\nu$ channels gives a unique possibility to investigate the influence of Bose-Einstein correlations on various properties of final state particles, such as multiplicity, transverse and longitudinal momentum spectra, resonance properties and reconstructed jet characteristics. It is possible, that by taking proper care in the fitting procedures used, one may at the same time be able to use a large part of the hadronic $W^+W^-$ events for the W mass determination, and to study the interconnection effects in the relatively clean setting of $e^+e^- \rightarrow WW$ events, by restricting the study to the region of large, off-peak W masses where these effects are expected to be the largest.

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**Note added.** Results of a similar study have been published recently [23]. The weight system used in [23] is a simplified version of our $V_4$, where the computational difficulties were avoided by averaging over “clusters” of particles. The authors do not see any mass shift due to BE correlations at the level of their statistical precision. This is not inconsistent with the present calculation, as they have chosen to use $R = 1$ fm, which strongly reduces the number of interfering pairs.
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