Proposal for locality test of the Aharonov-Bohm effect via Andreev interferometer without a loop

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Abstract

We propose a quantitative test of the quantum locality in the electromagnetic interaction that generates the Aharonov-Bohm effect. For this purpose, we analyze the Lorentz-covariant field interaction approach based on the local action of gauge-invariant quantities only ("local" theory), and compare it with the standard potential-based ("nonlocal") theory. Whereas the two approaches yield identical results for topological phase and any phenomenon involving classical equation of motion, an example violating this equivalence is presented; interference of the Andreev reflections from two independent superconducting inputs into a single normal metallic output. A well-defined phase shift of the interference is predicted in the "local" theory. In contrast, the potential-based Lagrangian fails the corresponding prediction. This result is significant as it can settle the issue of quantum locality in the electromagnetic interaction.

Keywords: Locality test, Lorentz-covariant field interaction, Aharonov-Bohm effect, Quantum interference without a loop, Andreev interferometer

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1. Introduction

Nonlocality is one of the most characteristic features of quantum theory. The Einstein-Podolsky-Rosen (EPR) paradox [1] and violation of the Bell’s inequality [2] demonstrate that nonlocal correlations occur in quantum entanglement that cannot be accounted for by any local realistic theory. This “EPR nonlocality” has been the subject of intensive study for several decades (see e.g., Ref. [3]). In addition, understanding and utilizing this nonlocality is one of the greatest achievements in quantum information science. The EPR nonlocality is purely kinematic in that no dynamics governed by the quantum equation of motion is involved. Another kind of nonlocality, which manifests in the form of the Aharonov-Bohm (AB) effect [4], also appears in quantum theory. This type of nonlocality is dynamic; it is the nonlocality of a quantum equation of motion [5], and has received less attention. Even though the AB interference is well described by the standard quantum theory and has been experimentally confirmed, a counterintuitive nonlocality is required; this prevents a causal explanation of this phenomenon. The AB effect is understood in terms of the electromagnetic potentials (which are gauge dependent and, therefore, cannot be physical fields) and/or the “nonlocality” of the interaction between a moving charge and electromagnetic fields. In any case, the term “AB nonlocality” indicates that the AB effect cannot be described by local and causal action of gauge-invariant quantities. Unlike the case of EPR nonlocality, AB nonlocality has not been intensively investigated. This primarily is because of the absence of a quantitative criterion for the experimental test (see also Y. Aharonov et al. [6]).

In this Letter, we propose an unambiguous test of the locality in the AB effect. First, we point out that a general local realistic theory can be constructed based on the Lorentz-covariant field interaction (LCFI) [7, 8]. The two approaches, the LCFI and the potential-based ones, yield the same results for classical equations of motion and for quantum phenomena involving topological phases. Second, more importantly, we propose an intriguing example that breaks the equivalence of the two theories, i.e., an interferometer composed of two sources that does not form a closed loop in the particle paths. This is an analogue of the optical Pfleegor-Mandel interferometer [9] (see also Fig. 1(a) of Ref. [10]) applied to charged particles with an external magnetic flux (Fig. 1). We show that the two theories

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lead to different predictions for the interference fringe in this setup. The nonlocal Lagrangian does not yield a well-defined phase shift induced by the localized flux, as there is no closed loop between the two paths. On the other hand, with the local approach, the relative phase shift of the two paths is obtained as a result of the accumulation of the local interaction. Therefore, an observation of this type of interference excludes the standard approach with gauge-dependent potential, and thereby resolves the question of dynamical quantum locality in the electromagnetic interaction.

2. Nonlocal vs. local Lagrangians

Our criterion for the locality is that the Lagrangian is given by sum of the local overlap of gauge-independent quantities only. This “dynamical locality” provides a local realistic description of quantum equations of motion, which is not satisfied in the potential-based theory [5]. Normally, a moving charge \( q \) under the external electromagnetic field is described by the vector and the scalar potentials, \( A(\mathbf{r}, t) \) and \( V(\mathbf{r}, t) \), respectively, at position \( \mathbf{r} \) and time \( t \). The Lagrangian of the system is given by

\[
L = L_0 + L_{\text{int}},
\]

where \( L_0 = m \dot{\mathbf{r}}^2 / 2 \) (\( m \) is the mass) is the kinetic part, and the interaction is given by the Lorentz-covariant form [11],

\[
L_{\text{int}} = \frac{q}{c} \mathbf{r} \cdot \mathbf{A} - qV.
\]

In this framework, the charge \( q \) interacts locally with the potentials, \( \mathbf{A} \) and \( V \), but these quantities are gauge dependent, and thus does not fulfill the locality criterion mentioned above. It is widely believed that this is a unique approach for describing the quantum electromagnetic interaction and that the potentials are indispensable [12].

In contrast to this common notion, a fully local theory was developed based on the Lorentz-covariant field interaction (LCFI) [7,8]. The essence of the LCFI approach is that the influence of the external electric (\( \mathbf{E} \)) and magnetic (\( \mathbf{B} \)) fields on a moving charge \( q \) is represented, without potentials, by the Lorentz-covariant Lagrangian

\[
L'_{\text{int}} = \frac{1}{4\pi} \int (\mathbf{B}_q \cdot \mathbf{B} - \mathbf{E}_q \cdot \mathbf{E}) d^3\mathbf{r}',
\]

where \( \mathbf{E}_q(\mathbf{B}_q) \) is the electric(magnetic) field produced by the moving charge. By adopting the relation \( \mathbf{B}_q = \frac{q}{c} \mathbf{r} \times \mathbf{E}_q \), we obtain an instructive form,

\[
L'_{\text{int}} = \mathbf{r} \cdot \Pi_q - U_q,
\]

where

\[
\Pi_q = \frac{1}{4\pi c} \int \mathbf{E}_q \times \mathbf{B} d^3\mathbf{r}'
\]

is the field momentum produced by the overlap between \( \mathbf{E}_q \) and \( \mathbf{B} \), and

\[
U_q = \frac{1}{4\pi} \int \mathbf{E}_q \cdot \mathbf{E} d^3\mathbf{r}'
\]

is the interaction energy stored in the electric fields.

Notably, the interaction Lagrangian \( L'_{\text{int}} \) of Eq. (2) reproduces the well-known topological phases derived from the conventional one, \( L_{\text{int}} \) of Eq. (1b). For instance, in a typical AB interferometer under an external magnetic field, the magnetic AB phase shift is given by an integral over a closed path

\[
\phi_{AB} = \frac{1}{\hbar c} \oint \Pi_q \cdot d\mathbf{r} = \frac{q}{\hbar c} \oint \mathbf{A} \cdot d\mathbf{r} = \frac{q \Phi}{\hbar c},
\]

where \( \Phi \) is the magnetic flux threaded by the loop. In addition, the classical equation of motion can be derived from the “local” Lagrangian, \( L_f = L_0 + L'_{\text{int}} \), which is also equivalent to that obtained from the potential-based Lagrangian (see Ref. [2] for details).

3. Interferometric locality test

Given the equivalence of the predicted results from the two different approaches, the essential question is whether one can find any observable phenomenon that can discriminate the LCFI theory from the potential-based one. As discussed above, the two theories predict the same phase shift as far as interference with a closed loop is concerned. Most of the quantum interferometers realized up to the present belong to this class, and it may seem unlikely that the equivalence is broken in reality.

Here, we suggest a counter example, which adopts a different type of interference that does break the equivalence. Before we introduce a realistic setup involving superconducting hybrid junctions later, let us first consider a prototype interferometer of a single charge “coproduced” from two independent, but synchronized, sources with a localized magnetic flux and a detector screen (Fig. 1). At this point, our discussion is only schematic containing the essential physics. The main feature of this setup is that an interference is predicted as a function of the relative phase produced by the two paths without forming a closed loop. In fact, this is an analogue of the optical Pfleegor-Mandel interferometer [8,10]. The “Pfleegor-Mandel” interference of single photons from two independent sources was recently shown very clearly at microwave frequencies (see...
Fig. 3(d) of Ref. [13]). The origin of that interference can be attributed to the indistinguishability of the particle paths: one cannot tell which source has produced the particle detected at a given point in the screen. A new aspect in our case is the introduction of charged particles, and thus the interference is also affected by the interaction with the external magnetic field.

In the interferometer illustrated in Fig. 1, each source, \( S_j \) (\( j = 1, 2 \)), simultaneously emits a charge state of \( (u_j + v_j \epsilon_j)\)\(|0\rangle \), involving superposition of the vacuum (\(|0\rangle \)) and a single charge. (Note that a normal electron cannot be in this state.) The operator \( \epsilon_j(c_j) \) creates (annihilates) a particle at \( S_j \). The state of the entire system upon injection is given by

\[
|\psi\rangle = (u_1 + v_1 \epsilon_1^\dagger)(u_2 + v_2 \epsilon_2^\dagger)|0\rangle.
\]

The interference fringe appears in \( P(x) \equiv \langle \psi|x\rangle \langle x|\psi \rangle \), the probability of finding a particle at the location \( x \) on the screen, as

\[
P(x) = |u_2 v_1 \varphi_1(x)|^2 + |u_1 v_2 \varphi_2(x)|^2 + 2|u_2 v_1 u_1 v_2 \varphi_1(x) \varphi_2(x)| \cos \phi,
\]

where \( \varphi_j(x) \equiv \chi(x) \epsilon_j^{\dagger}|0\rangle \) is the wave amplitude of the single particle emitted from \( S_j \). The phase shift \( \phi \) of the interference fringe has two independent contributions, \( \phi = \phi_B + \phi_0 \), where \( \phi_B \) and \( \phi_0 \) originate from the interaction with the localized magnetic field \( B \) and from the path difference, respectively. \( \phi_0 \) is independent of \( B \) and our main interest is \( \phi_B \), which we analyze using both the potential-based and the LCFI approaches.

In the conventional potential-based theory, \( \phi_B \) can be evaluated from the interaction Lagrangian of Eq. (1b) (with \( V = 0 \) in our case) as

\[
\phi_B = \frac{1}{\hbar c} \int_c L_{\text{int}} \, dt = \frac{q}{\hbar c} \int_c \mathbf{A} \cdot d\mathbf{r},
\]

where the integration is to be taken over the open path \( c \) denoted in Fig. 1. However, we face a fundamental problem here. \( \phi_B \) in Eq. (6) is not well defined because the integration does not constitute a closed loop; thus \( \phi_B \) depends on the choice of gauge in \( \mathbf{A} \). That is, the nonlocal Lagrangian cannot predict the \( B \) dependence of the interference pattern derived in Eq. (6).

Next, let us evaluate \( \phi_B \) by using the local interaction Lagrangian of Eq. (2) in the presence of \( B \) (with \( E = 0 \)). We find a gauge-independent result,

\[
\phi_B = \frac{1}{\hbar} \int_c L_{\text{int}}^r \, dt = \frac{1}{\hbar} \int_c \Pi_q \cdot d\mathbf{r}.
\]

The local theory predicts a well-defined \( \phi_B \) as a function of the external magnetic field. For an ideal flux tube (of flux value \( \Phi \)) with a negligible diameter,

\[
\Pi_q = \frac{q \Phi}{2 n c \rho} \hat{d},
\]

where \( \rho \) and \( \hat{d} \) are the distance from the flux tube and the azimuthal unit vector of the position of \( q \), respectively. Hence, we obtain

\[
\phi_B = \frac{q \Phi}{2 \pi \hbar c} \Delta \theta,
\]

where \( \Delta \theta \) is the angle appearing in the open path \( c \) (Fig. 1). An interesting point is that this phase is related to the field angular momentum, \( \mathcal{L} = q \Phi / (2 \pi \hbar) \), and the phase shift can be rewritten as \( \phi_B = \mathcal{L} \Delta \theta / \hbar \). \( \phi_B \) is reduced to the AB phase for \( \Delta \theta = 2 \pi \), as expected.

### 4. Possible realistic experiment with Andreev interferometer

An electronic interferometer is unsuitable for observing interference without a closed loop. This is because the normal-state electron cannot be in a superposed state involving different numbers of particles, and \( u_j v_j = 0 \) (\( j = 1, 2 \)) in Eq. (4). Therefore, no interference fringe appears in the probability distribution, \( P(x) \), of Eq. (5). This problem can be overcome by adopting the superconducting coherence in which gauge symmetry breaking plays a major role [14]. Consider a schematic setup illustrated in Fig. 2. Each superconducting source (\( S_1 \) and \( S_2 \)) is tunnel-coupled to a normal electrode (\( N \)). The separation of the two junctions should be short enough to maintain the phase information of the condensates. In fact, this is an Andreev interferometer (see e.g., Refs. [15, 16]) where the interference arises from the two indistinguishable Andreev reflection (AR) [17] processes. A notable difference from the usual Andreev interferometer is that the phase shift between the two superconducting condensates is controlled by an external flux without forming a loop. The ideal experimental procedure is as follows: (i) For a fixed magnetic flux, an identical voltage \( V \) is applied simultaneously to the two superconductors for a time interval \( t \), and the output current is measured at \( N \); (ii) This process is repeated many times, and the average output current is recorded; (iii) Steps (i) and (ii) are repeated for different values of \( \Phi \). The output current \( I \) measured in this way is expected to show interference pattern as a function of \( \Phi \), as we describe below.

The Hamiltonian of this system is given by

\[
H = H_{S_1} + H_{S_2} + H_N + H_T,
\]
where $H_{S_j}$ and $H_N$ represent the electrodes $S_j$ ($j = 1, 2$) and $N$, respectively. $H_T$ describes the tunneling process between each superconductor and the normal metal,

$$H_T = \sum_{j,k,p,r} \left( t_{ij}^c c_{jk}^\dagger a_{pr} + t_{ij} a_{pr}^\dagger c_{jk} \right),$$

where $c_{jk}(c_{jk}^\dagger)$ and $a_{pr}(a_{pr}^\dagger)$ annihilate (create) an electron at $S_j$ and $N$, respectively ($k, p,$ and $\sigma$ are momentum and spin indices, respectively).

For $eV \ll \Delta$ ($\Delta$ being the superconducting gap parameter of $S_j$), quasiparticle transmission is prohibited and charge transport is mediated by the AR, which converts a Cooper pair from each $S_j$ to two electrons in $N$ [17]. This corresponds to a transition from an initial state $|i\rangle$ to a final state $|f\rangle = a_{pr}^\dagger a_{pr}|i\rangle$ via second-order processes in $H_T$, without real quasiparticle excitation in $S_j$. Neglecting the voltage dependence (valid for $eV \ll \Delta$), this AR amplitude (from $S_j$ to $N$) is given by [18]

$$A_j = -\pi \rho_j |j|^2 e^{i\phi_j},$$

where $\rho_j$ is the density of states of $S_j$. Each AR is a coherent process conveying the phase information ($\phi_j$) of the superconducting condensate. This phase coherence of the AR has been well established in real experiments with hybrid superconductor-normal metal structures (See e.g., Refs. [15, 16]), and can be utilized for our purpose.

In our setup, the two AR processes, each of which is injected from $S_j$ or $S_2$, are indistinguishable, as the two processes share common initial and final states:

$$|i\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle \otimes |FS_N\rangle,$$

and $|f\rangle = a_{pr}^\dagger a_{pr}|i\rangle$, respectively, where $|\Psi_j\rangle (j = 1, 2)$ and $|FS_N\rangle$ denote the BCS condensate of $S_j$ and Fermi sea of $N$, respectively. Note that the same type of interference with independent sources was observed for two independent Bose condensates [19], which also supports the feasibility of the proposed experiment. During the time interval $\tau$ where the bias voltage $V$ is applied, the output current exhibits interference as

$$I = \frac{\pi e^2}{h} V \left[ \Gamma_1^2 + \Gamma_2^2 + 2\Gamma_1 \Gamma_2 \cos(\phi_1 - \phi_2) \right],$$

where $\Gamma_j = 2\pi \rho_j \rho_N |j|^2$ is the single-electron hoping rate from $S_j$ to $N$, with $\rho_N$ denoting the density of states of $N$. The phase shift in the interference pattern of $I$ is given by

$$\phi_1 - \phi_2 = \phi_0 + \phi_B.$$  

$\phi_0$ is a constant which is independent of the external $B$, and the field dependence of the phase shift is obtained in the local theory as

$$\phi_B = \frac{e\Phi}{\pi\hbar c} \Delta \theta,$$

where the angle $\Delta \theta$ is determined by the geometry (as displayed Fig. 1). This phase shift is equivalent to that obtained in Eq. (9) with $q = 2\tau$.

$\phi_B$ cannot be obtained from the potential-based theory as shown in Eq. (6) because it does not predict a gauge-invariant result. Only the local approach predicts a well-defined $\phi_B$ for an interference produced by two indistinguishable AR processes. Therefore, an interferometer of the type shown in Fig. 2 can be adopted to conduct a realistic test of the quantum locality in the electromagnetic interaction. The different prediction results from the different viewpoint on the nature of the interaction. In LCFI theory, any quantum phase shift is a result of the accumulation of the gauge-invariant local interactions, and the locality principle is preserved. In the potential-based approach, in contrast, it is impossible to attribute the phase shift to the interactions occurring at particular spacetime locations, and the interference without a closed loop in the setup of Fig. 2 cannot be predicted in a consistent way.

5. Discussion

Notably, we can derive the general relation between the two interaction Lagrangians, $L_{int}(\text{Eq. (11)})$ and $L_{int}^f(\text{Eq. (2)})$. The potential-based interaction Lagrangian ($L_{int}^f$) for a moving charge can be rewritten as

$$L_{int} = \frac{1}{c} \int j_k \cdot A \, d^3r' - \int \rho_q V \, d^3r',$$

with the charge and current densities of a point charge given by $\rho_q = q\delta(r' - r)$ and $j_k = q\partial \delta(r' - r)$, respectively. Applying the Maxwell equations for both the charge $q$,

$$\nabla \cdot E_q = 4\pi \rho_q, \quad \nabla \times B_q = \frac{1}{c} \frac{\partial E_q}{\partial t},$$

and the external fields,

$$E = -\nabla V - \frac{1}{c} \frac{\partial A}{\partial t}, \quad B = \nabla \times A,$$

we find

$$L_{int}^f = \frac{1}{4\pi c} \int E_q \cdot A \, d^3r',$$

where

$$L_{int} = \int \frac{1}{4\pi c} \int E_q \cdot A \, d^3r'.$$
This is a remarkable result in that the standard potential-based picture can be transformed into the framework of the local interaction of the electromagnetic fields by discarding the total time derivative term, $dF/dt$. The transformed Lagrangian does not involve potentials, and therefore both the quantum and classical equations of motion of a charge can be described in terms of the “local” Lagrangian, $L' = L_0 + L'_\text{int}$, without relying on the electromagnetic potential.

Consequences of the relation derived in Eq. (22) are as follows. The dynamics of a system is, in general, fully determined by the action, which is defined in each theory as $S = \int_0^T L \, dt$ and $S' = \int_0^T L' \, dt$, respectively, for a trajectory from the initial $(r_1, t_1)$ to the final $(r_2, t_2)$ spacetime points. Eq. (20a) gives

$$S = S' - F(r_2, t_2) + F(r_1, t_1).$$

With this relation, two actions yield the same classical equation of motion. It is because only the variation of the action, $\delta S$, is relevant in classical Euler-Lagrange equation and we find $\delta S = \delta S'$ from Eq. (21) (see e.g., Ref. [20]).

Quantum mechanical problems require more careful analysis, although the two approaches predict the same topological phase as shown in Eq. (3). In general, any observable quantum effect is included in the transition amplitude of a particle trajectory between the two arbitrary points in spacetime, $(r_1, t_1)$ and $(r_2, t_2)$, expressed as

$$\langle r_2, t_2 | r_1, t_1 \rangle \propto \int_{r_1}^{r_2} D[r(t)] \exp(iS[r]/\hbar),$$

which is obtained by summing over all possible paths (represented by $D[r(t)]$). As one can find in Eqs. (21) and (22), the two interaction Lagrangians, $L_\text{int}$ and $L'_\text{int}$, yield different phase factor for the transition amplitude unless $(r_1, t_1) = (r_2, t_2)$. This difference is manifested in the proposed interferometer without a loop.

We now address the last unresolved question: the underlying reason why the two approaches, namely the LCFI and the potential-based theories, provide different predictions for an interference without a loop. To address this problem, we should note that the potential-based Lagrangian of Eq. (1) is derived under the condition that it satisfies the classical equation of motion with a proper consideration of the symmetry. One may take an alternative procedure of derivation, based on the interaction energy which corresponds to the work to establish the system configuration (see, e.g., Sec. 6.2 of Ref. [11]). We do not see any reason why this procedure is inappropriate in quantum systems.

A different perspective on the Faraday’s law of induction is helpful for our analysis, which goes as follows. Variation of $B$ induces an electric field (electromotive force (emf)). Interestingly, the induced electric field $E$ can be derived from the momentum conservation law. Consider a hypothetical stationary charge $Q$ under $B$. As far as the $B$ is confined in a finite region (as it should be), the total momentum of the system is conserved under variation of $B$. This can be expressed as

$$Q \vec{E} + \vec{\Pi}_Q = 0,$$

where $\vec{\Pi}_Q$ is the field momentum generated by $Q$ and $B$ (equivalent to $\vec{\Pi}_q$ in Eq. (23) with $q = Q$). It is straightforward to show that the Faraday’s law, $\nabla \times \vec{E} + B/c = 0$, can be derived from Eq. (23).

The magnetic interaction term is constructed as follows. Suppose that only a moving charge $q$ with its velocity $\vec{r}$ is initially present. Activation and variation of $B$ results in an induced electric field $\vec{E}$ satisfying Eq. (23). To keep $\vec{r}$ unchanged under the variation of $B$, a work ($W_B$) against the emf is performed to the system at the rate

$$\frac{dW_B}{dt} = -q \vec{r} \cdot \vec{E}.$$

Combining Eqs. (23) and (24), we find

$$W_B = \vec{r} \cdot \vec{\Pi}_q.$$

This is the work required to establish $B$ in the system, and constitutes the magnetic interaction energy between the two entities (the first term of the right hand side of Eq. (25)). Similarly, we obtain the work ($W_E$) for establishing $E$ as

$$W_E = U_q,$$

where $U_q$ is the field interaction energy defined in Eq. (20). Incorporating the Lorentz symmetry of the system, the two contributions of the work, $W_B$ and $W_E$, constitutes the interaction Lagrangian $L'_\text{int}$ of Eq. (26).

The essential difference between the local and the nonlocal Lagrangians can be summarized as follows. In the local theory, the interaction Lagrangian is constructed from the work necessary to establish the configuration. No gauge dependence is included in this approach, and the interaction Lagrangian is given without a potential (Eq. (4)). In the conventional potential-based framework, in contrast, the interaction Lagrangian of Eq. (11) (or Eq. (12)) is derived under the condition that it satisfies the classical equation of motion. It does not rely on the work performed to establish the configuration and, thus, there is no way of assigning the local
interaction energy. As we have shown above, observation of an interference without a loop would confirm the locality of the interaction.

Finally, it should be noted that the previous experimental verification of the absence of the classical force in the AB setup \[21\] is inadequate as regards our locality test. As the two different approaches yield the equivalent classical equation of motion (derived from Eq. \[20a\]), the absence of force observed in Ref. \[21\] can be explained by both theories of the interaction. Therefore, the experimental result in Ref. \[21\] does not rule out any of the two approaches. In fact, the classical lag predicted in Ref. \[22\], a motivation of the experiment in Ref. \[21\], is found to be erroneous which originates from the neglect of the Lorentz covariance (See Ref. \[7\] for details).

6. Conclusion

We have proposed a quantitative test of the locality in the quantum electromagnetic interaction that induces the Aharonov-Bohm effect. The equivalence and breakdown of the local and nonlocal theories have been analyzed. The locality test can be performed by constructing an Andreev interferometer with the phase difference of the two superconducting condensates controlled by an external magnetic flux without forming a loop. The two approaches provide different predictions for the interference fringe and, therefore, a successful experiment is expected to resolve the issue of quantum locality in the electromagnetic interaction. Observation of the interference would discard the gauge-dependent potential-based theory, and therefore, rule out the dynamical nonlocality in quantum theory. This is of great significance in our understanding of the nature of the electromagnetic interaction and the role of the potential.

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References

References

[1] A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete?, Phys. Rev. 47 (10) (1935) 777.
[2] J. S. Bell, On the einstein-podolsky-rosen paradox, Physics 1 (3) (1964) 195–200.
[3] G. Greenstein, A. Zajonc, The quantum challenge: modern research on the foundations of quantum mechanics, Jones & Bartlett Learning, 2006.
[4] Y. Aharonov, D. Bohm, Significance of electromagnetic potentials in the quantum theory, Phys. Rev. 115 (1959) 485–491.
[5] S. Popescu, Dynamical quantum non-locality, Nat. Phys. 6 (3) (2010) 151–153.
[6] Y. Aharonov, E. Cohen, D. Rohrlich, Comment on “role of potentials in the aharonov-bohm effect”, Phys. Rev. A 92 (2015) 026101.
[7] K. Kang, Aharonov-bohm effect, local field interaction, and lorentz invariance, arXiv:1308.2093.
[8] K. Kang, Locality of the aharonov-bohm-casher effect, Phys. Rev. A 91 (2015) 052116.
[9] R. Pfleegor, L. Mandel, Interference of independent photon beams, Phys. Rev. 159 (5) (1967) 1084.
[10] L. Mandel, Quantum effects in one-photon and two-photon interference, Rev. Mod. Phys. 71 (1999) S274–S282.
[11] J. D. Jackson, Classical electrodynamics, Wiley, 1962.
[12] M. Peshkin, A. Tomonura, The aharonov-bohm effect, The Aharonov-Bohm Effect 340.
[13] C. Lang, C. Eichler, L. Steffen, J. Fink, M. Woolley, A. Blais, A. Wallraff, Correlations, indistinguishability and entanglement in hong-ou-mandel experiments at microwave frequencies, Nat. Phys. 9 (6) (2013) 345–348.
[14] A. J. Leggett, F. Sols, On the concept of spontaneously broken gauge symmetry in condensed matter physics, Found. Phys. 21 (3) (1991) 353–364.
[15] V. Petrushov, V. Antonov, P. Delsing, R. Claesson, Phase memory effects in mesoscopic rings with superconducting mirrors, Phys. Rev. Lett. 70 (3) (1993) 347.
[16] V. Petrushov, V. Antonov, P. Delsing, T. Claesson, Phase controlled conductance of mesoscopic structures with superconducting mirrors, Phys. Rev. Lett. 74 (26) (1995) 5268.
[17] A. Andreev, Thermal conductivity of the intermediate state of superconductors, Sov. Phys. JETP 19 (1964) 1228–1229.
[18] J. Schrieffer, J. Wilkins, Two-particle tunneling processes between superconductors, Phys. Rev. Lett. 10 (1) (1963) 17.
[19] M. Andrews, C. Townsend, H.-J. Miesner, D. Durfee, D. Kurn, W. Ketterle, Observation of interference between two Bose condensates, Science 275 (5300) (1997) 637–641.
[20] L. Landau, E. Lifshitz, Classical mechanics (1960).
[21] A. Caprez, B. Barwick, H. Batea, Macroscopic test of the aharonov-bohm effect, Phys. Rev. Lett. 99 (21) (2007) 210401.
[22] T. H. Boyer, Classical electromagnetic deflections and lag effects associated with quantum interference pattern shifts: considerations related to the aharonov-bohm effect, Phys. Rev. D 8 (6) (1973) 1679.
Figure 1: Prototype setup for testing the quantum locality of the electromagnetic interaction. The interference of a single charged particle “coproduced” by two independent sources with a localized magnetic flux can elucidate the dynamical locality of the Aharonov-Bohm effect.

Figure 2: A possible experimental test of the quantum locality of the electromagnetic interaction via an Andreev interferometer. Voltage $V$ is applied simultaneously to the two superconductors. The Andreev reflection processes in the two superconductor-normal metal contacts ($S_1$-$N$ and $S_2$-$N$) are expected to show an interference as a function of the external magnetic flux.