Abstract: Recently, several arguments appeared in the literature that braneworld effective models with fields strictly localized on zero-thickness branes cannot be UV-complete by a fundamental theory including gravity. The authors concluded that one should instead consider only models with quasi-localized higher-dimensional fields. In this letter, we point out that the arguments of these articles imply instead that one should use an effective description with a brane form factor which reduces to the zero-thickness brane description after a moment asymptotic expansion. The brane form factor does not necessarily rely on quasi-localized higher-dimensional fields.

Keywords: Braneworlds, Extra Dimensions, Holography, Swampland, Effective Field Theories
1 Introduction

Braneworld models became very popular at the beginning of the new millenium when theoretical physicists realized that they can solve or reformulate long standing problems of particle physics and cosmology. In particular, thin branes [1–10], i.e. zero-thickness hypersurfaces, are used to localize lower-dimensional fields on their worldvolumes. They arise in the low energy limit of some string models involving D-brane stacks on which open strings are attached [11–13].

Recently, the authors of Refs. [14, 15] gave several arguments that Effective Field Theories (EFTs) with 4D fields strictly localized on a thin brane are not compatible with UV-completions including gravity, i.e. they belong to the Swampland (see Refs. [16, 17] for reviews on the Swampland program). They argued that one should always consider the brane-localized fields as an idealized picture of quasi-localized bulk fields. This result is not compatible with the D3-brane picture where the fields localized on the worldvolume of the D3-brane stack are pure 4D degrees of freedom. Moreover, these arguments clash with the Wilsonian renormalization group picture, where it should be always possible to integrate out the width $\epsilon$ of a fat brane with quasi-localized fields and then consider a thin brane EFT with a cut-off $1/\epsilon$. It is sometimes important that the braneworld EFT does not rely on quasi-localized fields: see for example the model of Ref. [18], where it is possible to solve the gauge hierarchy problem if the Standard Model (SM) fields are 4D degrees of freedom localized on a thin brane at the junction of a large star/rose extra dimension with a large number of small leaves/petals.

In this letter, we review the arguments of Refs. [14, 15] and propose a new solution to the problems identified in these articles relying on the introduction of an effective brane form factor. We conclude that a priori the standard thin brane models of the literature do not belong to the Swampland for these reasons.
2 Braneworlds & Swampland

2.1 Brane Width

There are various arguments in the literature that gravity implies the existence of a minimal length scale in Nature (see Ref. [19] for a review). This feature appears in all main approaches to quantize gravity in the UV: string theories, loop quantum gravity, noncommutative geometry, asymptotic safety. Therefore, in a UV completion of a 5D model including gravity, the singular behavior of a thin 3-brane should be softened by the UV degrees of freedom above the 5D gravity scale $\Lambda_{P}^{(5)}$, even for branes at orbifold fixed points or metric graph vertices. For example, a D-brane is smeared by the dynamics of the open strings attached to it. After integrating out the UV degrees of freedom, one is left by an effective brane thickness $\epsilon \sim 1/\Lambda_{P}^{(5)}$ [20] and 4D fields localized on its worldvolume.

To be concrete, we consider a 5D toy model with the flat factorizable geometry $M_{4} \times [0, \ell]$, whose coordinates are $x^{M} = (x^{\mu}, y)$ with $M \in [0, 4]$ and $\mu \in [0, 3]$. The SM fields, whose Lagrangian is $\mathcal{L}_{SM}$, are 4D degrees of freedom localized on the 3-brane at $y = 0$. The Lagrangian of this brane is $\mathcal{L}_{brane}$. Only gravity and possibly exotic fields, whose Lagrangian is $\mathcal{L}_{bulk}$, propagate into the extra dimension. This EFT is obtained by integrating out the UV degrees of freedom above $\Lambda_{P}^{(5)}$. To modelize the effective brane, smeared over a length $\epsilon \sim 1/\Lambda_{P}^{(5)}$, we introduce an effective brane form factor, which is a function $B(y)$ (with a mass dimension 1) rapidly decreasing over a distance $\epsilon$ (see for instance Refs. [18, 21]) and normalized such that

$$\int_{0}^{\ell} dy \ B(y) = 1. \quad (2.1)$$

By ignoring the physics of the other brane at $y = \ell$, the action of the 5D EFT is

$$S = \int d^{4}x \int_{0}^{\ell} dy \left( \mathcal{L}_{bulk} + B(y) \mathcal{L}_{brane} \right). \quad (2.2)$$

One can perform a moment expansion [22] of the brane form factor:

$$B(y) = \Lambda_{P}^{(5)} b \left( \Lambda_{P}^{(5)} y \right) = \sum_{n=0}^{+\infty} \frac{b^{(n)}}{\Lambda_{P}^{(5)}} \partial_{y}^{n} \delta(y), \quad (2.3)$$

where $b(y)$ is an intermediate function defined for convenience, and

$$b^{(n)} = \frac{(-1)^{n}}{n!} \int_{0}^{\ell} dy \ y^{n} b(y). \quad (2.4)$$

For energies $E \ll \Lambda_{P}^{(5)}$, we can truncate the tower of higher-dimensional operators (a so-called moment asymptotic expansion), and we recover the thin brane description involving distributions with point-like support to localize 4D fields (zero-thickness brane). At the scale $\Lambda_{P}^{(5)}$, the whole infinite tower of higher-dimensional operators contribute and we re-
cover the brane width in the UV. Therefore, in order to be able to match the UV completion at the 5D Planck scale $\Lambda^{(5)}_{\text{P}}$, $\mathcal{L}_{\text{brane}}$ has to depend on $y$, otherwise only the operator with $n = 0$ would contribute, which is incompatible with the smeared brane requirement of the UV completion [14, 15].

In Ref. [15], the author claims that to satisfy this condition, the fields in $\mathcal{L}_{\text{brane}}$ have to be quasi-localized zero modes of 5D fields. Indeed, as he assumed that in a thin brane description $\mathcal{L}_{\text{brane}}$ is only made of 4D fields (in our case $\mathcal{L}_{\text{brane}} = \mathcal{L}_{\text{SM}}$), it cannot depend on $y$, and he concludes that the thin brane scenario belongs to the Swampland. However, in any realistic scenario, the 4D fields localized on the brane at $y = 0$ couple at least to gravity in the bulk so

$$\mathcal{L}_{\text{brane}}(x^\mu, y) = \mathcal{L}_{\text{SM}}(x^\mu) + \mathcal{L}_{\text{int}}(x^\mu, y), \quad (2.5)$$

where $\mathcal{L}_{\text{int}}$ is the Lagrangian of the interaction between the 4D SM fields and the 5D fields like the graviton. This is enough to be able to include non-vanishing higher-dimensional operators to match the UV theory with a brane width. Intuitively, as it is gravity which requires the brane thickness in the UV, it is not astonishing that it is the gravitational fields in the IR that are involved in the higher-dimensional operators which are sensible to the brane thickness. So the Swampland argument of Ref. [15] does not hold for a realistic braneworld model.

### 2.2 Argument from Global Symmetries

In Refs. [14, 15], the authors consider a 5D flat spacetime $\mathcal{M}_4 \times [0, \ell]$ with a gauged $U(1)$ symmetry in the bulk and two 4D scalar fields $\phi_0$ and $\phi_1$ with the charges $q_0$ and $q_1$ (coprime and of opposite sign) respectively, localized on two different thin 3-branes located at each boundaries of the interval. Gravity propagates into the bulk with a 5D Planck scale $\Lambda^{(5)}_{\text{P}}$.

For convenience, the KK-gravitons and KK-photons are integrated out but the discussion applies also to the 5D theory. If one assumes locality, the effective 4D Lagrangian only contains operators of monomials $|\phi_0|^2$, $|\phi_1|^2$ and similar ones with derivatives and more complex Lorentz structures. The numbers $N_0$, $N_1$ of $\phi_0$, $\phi_1$ particles respectively are conserved in the 4D EFT. However, the $U(1)$ gauged symmetry implies only the conservation of $N_0 q_0 + N_1 q_1$. As the geometry imposes that $N_0$ and $N_1$ are two global charges which are conserved separately, the EFT has an exact global symmetry, which is in contrast to the Swampland conjecture that there is no exact global symmetry in an EFT emerging from a UV theory including gravity [16, 17]. Based on this observation, the authors of Refs. [14, 15] conclude that one should promote the 4D fields $\phi_0$ and $\phi_1$ to be the quasi-localized zero modes of 5D fields $\Phi_0$ and $\Phi_1$. In this way, the wave functions of their zero modes have a non-vanishing overlap in the bulk: one can include 5D operators involving both $\Phi_0$ and $\Phi_1$, and respecting the gauge symmetry but not the individual $\Phi$ number, as

$$S_{01} \propto \int d^4 x \int_0^\ell d y \; \Phi_0^q \Phi_1^q + \text{H.c.} \quad (2.6)$$

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Therefore, in addition to $\phi_0$, $\phi_1$ monomials, the 4D EFT contains operators build from monomials of
\[
\phi_0^{q_1} \phi_1^{q_0} + \text{H.c.} \tag{2.7}
\]
violating the conservation of $N_0$, $N_1$ separately: the global symmetry problem is solved.

In this letter, we stress that one can also solve this problem by describing the branes at $y = 0, \ell$ with the effective brane form factors $B_0(y), B_1(y)$ respectively, normalized such that
\[
\int_0^\ell dy \ B_{0,1}(y) = 1, \tag{2.8}
\]
which are rapidly decreasing functions over a distance $\ell^{(5)}_P = 1/\Lambda^{(5)}_P$. They have a very suppressed but non-vanishing overlap in the bulk:
\[
\omega = \frac{1}{\Lambda^{(5)}_P} \int_0^\ell dy \ B_0(y) B_1(y) \ll 1. \tag{2.9}
\]
In this way, $\phi_0$ and $\phi_1$ are 4D fields localized on the worldvolumes of the branes but one can add 5D local operators involving both of them which do not conserve $N_0$ and $N_1$ separately, like
\[
S_{01} \propto \int d^4x \int_0^\ell dy \ B_0(y) B_1(y) \phi_0^{q_1} \phi_1^{q_0} + \text{H.c.} \tag{2.10}
\]
It is important to notice that $\phi_0$ and $\phi_1$ are not necessarily the zero modes of higher-dimensional fields. If it is the case, then the brane form factors are related to the wave functions of these zero modes after integrating out the KK-excitations. However, one can imagine a UV-completion where $\phi_0$ and $\phi_1$ are pure 4D degrees of freedom and that the brane form factors are generated by the UV dynamics, like in the example of open strings attached to a D-brane stack. By using Eq. (2.9) in Eq. (2.10), one gets
\[
S_{01} \propto \int d^4x \ \omega \phi_0^{q_1} \phi_1^{q_0} + \text{H.c.} \tag{2.11}
\]
One can rewrite this 4D operator as a 5D bi-local operator:
\[
S_{01} \propto \int d^4x \int_0^\ell dy \int_0^\ell dy' \ \omega \delta(y) \delta(y' - \ell) \phi_0^{q_1} \phi_1^{q_0} + \text{H.c.} \tag{2.12}
\]
For the 5D operators involving only $B_0(y)$ or $B_1(y)$, one can perform a moment asymptotic expansion:
\[
B_0(y) \underset{\Lambda^{(5)}_P \to +\infty}{\sim} b_0^{(0)}(y) \delta(y) + \mathcal{O}\left(\frac{1}{\Lambda^{(5)}_P}\right),
\]
\[
B_1(y) \underset{\Lambda^{(5)}_P \to +\infty}{\sim} b_1^{(0)}(y - \ell) + \mathcal{O}\left(\frac{1}{\Lambda^{(5)}_P}\right). \tag{2.13}
\]
We thus recover a 5D description with thin branes at energies $E \ll \Lambda^{(5)}_P$, corrected by
higher-dimensional brane-localized terms involving the derivatives of the Dirac distribution to match the effective brane width in the UV. We have thus solved the global symmetry problem in the thin brane description by including 5D bi-local operators arising from 5D local operators involving brane form factors.

2.3 Arguments from Emergent Species

Consider the Randall-Sundrum 1 (RS1) model, where the spacetime is a slice of AdS$_5$ bounded by two thin 3-branes. The extra dimension is labeled by the conformal coordinate $z$. The metric is

$$ds^2 = \frac{1}{(1 + k z)^2} \eta_{MN} dx^M dx^N,$$

where the curvature $k$ is not far below the 5D Planck scale $\Lambda_P^{(5)}$. The cut-off on the UV-brane at $z = 1/k$ is $\Lambda_{UV} \sim \Lambda_P^{(5)}$ and the one on the IR-brane at $y = 1/\mu$ is $\Lambda_{IR} = \mu \Lambda_{UV}/k$. A large number of 4D species $N_s$ is localized on the IR-brane. One can have an holographic interpretation of this model based on the AdS/CFT correspondence. The $N_s$ species on the IR-brane are then composite states of a 4D strongly coupled Conformal Field Theory (CFT) whose unknown preonic degrees of freedom are localized on the UV-brane. In this description, the IR-brane is thus emergent from the UV-brane dynamics.

The author of Ref. [15] claims that according to Refs. [17, 23], the $N_s$ species will lower the cut-off on the IR-brane from $\Lambda_{IR}$ to $\Lambda_{IR}/\sqrt{N_s}$. Then, for a 4-momentum

$$|p| \in \left[ \frac{\Lambda_{IR}}{\sqrt{N_s}}, \Lambda_{IR} \right],$$

the EFT is not valid. The author takes this discontinuity as a signal of an inconsistency. However, the results of Ref. [23] are different from what is claimed in Ref. [15]. In a 4D theory with $N_s$ species coupled to gravity (with a fundamental gravity scale $\Lambda_G$), each species renormalize the gravity scale: one gets an effective 4D Planck mass:

$$\Lambda_P^{(4)} \sim \frac{\Lambda_G}{\sqrt{N_s}}.$$  

(2.16)

However, strongly coupled gravity effects still appear at the scale $\Lambda_G$ [24] so the cut-off is still $\Lambda_G$ and is not lowered by the $N_s$ species whose effect is to weaken gravity. In the RS1 model we consider, the loop contributions of the $N_s$ species to the graviton propagator generate a brane-localized 4D Einstein term with a coefficient of $O(N_s \Lambda_{IR})$ [9, 25, 26], and the EFT is valid until the cut-off on the IR-brane which is still $\Lambda_{IR}$. Therefore, there is no discontinuity in the range of validity of the EFT as claimed in Ref. [15].

Another argument in Ref. [15] is the emergence of many degrees of freedom in the IR, which is in disagreement with the c- and a-theorems [27] implying that the number of degrees of freedom should monotonically decrease when flowing towards the IR: this is the emergent species problem. The author argues that if the $N_s$ species are identified with the quasi-localized zero modes of 5D species, instead of being localized 4D species on a thin IR-brane, the large number of degrees of freedom are present all along the renormalization
group flow, which solves the emergent species problem. Indeed, the wave functions of the zero modes are non-vanishing all along the extra dimension, even if they are highly peaked towards the IR-brane. However, one can also solve this problem with $N_s$ 4D species localized on the worldvolume of the IR-brane with a form factor $B_{IR}(y)$. The situation is qualitatively similar to the quasi-localized 5D fields: $B_{IR}(y)$ is non-vanishing all along the extra dimension, even if it is extremely suppressed near the UV-brane. The $N_s$ species are thus present all along the RG flow. By the way, a hard wall model like RS1 corresponds to an idealized infinitely sharp breaking of the conformal symmetry of the 4D theory, and the $N_s$ composite species appear exactly at the scale $\Lambda_{IR}$, which is not realistic. Indeed, in a realistic confining theory, one should have a smooth transition at the confining scale $\Lambda_{IR}$. The effective brane form factor softens the hard wall picture and solves the emergent species problem. Then, one can perform a moment asymptotic expansion of $B_{IR}(y)$ to obtain the description with $N_s$ 4D species strictly localized on a thin IR-brane: the effects of $B_{IR}(y) \neq 0$ in the bulk are encoded in the higher-dimensional operators involving the derivatives of the Dirac distribution. In this way, the thin brane description circumvent the emergent species problem.

3 Conclusion

In this letter, we have reviewed the arguments relying on brane width, global symmetries and emergent species of Refs. [14, 15] against effective models with 4D degrees of freedom strictly localized on thin 3-branes. The authors claim that one should always consider quasi-localized zero modes of 5D fields to model brane-localized 4D fields. We have argued that one can solve these issues by introducing an effective brane form factor which does not necessary involve to embed the brane-localized fields into 5D fields. This solution is thus less restrictive than the quasi-localized 5D fields prescription of Refs. [14, 15], and it allows to continue to build EFTs with 4D fields strictly localized on thin 3-branes, after a moment asymptotic expansion of the brane form factors.

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