A reduced-form intensity model containing noise interference

Dong-wei Shi, Dong-e Bao and Liang Wu

School of Mathematical Science, Henan Institute of Science and Technology, XinXiang, People’s Republic of China

ABSTRACT
OTC financial derivatives are non-standardized face-to-face financial contracts its trading environment is characterized by less information, information disclosure may be distorted, and no exchange protection. It results in asymmetric information on the opponent, and obtains asymmetric information with noise interference. However, the default intensity is a function of information of market state variables, so the noise interference of market information will have an important impact on the default intensity. Therefore, the noise interference factor is added to the construction of the default intensity in this paper, and a new default intensity model with noise interference is proposed. The simulation results show that, the emergence of noise interference will increase the default probability of enterprises, and when the bi-directional noise interference disappears, the survival probability of enterprises is raised to a certain limit. Compared with the existing default intensity models, the noise interference term is added to the construction of the default intensity model for the first time, and can make the default model take into account more sources of default, make the model closer to the real OTC market. At the same time, simulation results also verify the effectiveness of the proposed model.

1. Introduction
Information disclosure plays a role in abating information asymmetry between issuers and investors, the way of disclosure can reduce the information asymmetry. The most basic requirement of market information disclosure is authenticity and accuracy. The ‘authenticity principle’ means that the information disclosed must be true, reliable and well documented, which can truly reflect the company’s financial information, and the ‘accuracy principle’ means that the information disclosed is accurate and has been audited, which can objectively reflect the company’s financial information. However, over-the-counter financial derivatives are non-standardized face-to-face financial contracts, its trading environment is characterized by less information, information disclosure may be distorted, objectively lagging, no exchange protection, and the floating of stock price will also be greater. It leads to the market information disclosure is faced with the challenge of unreal and inaccurate, and the asymmetric information with noise interference is obtained. Which result in the noise interference from market information becomes a decisive factor in risk management and derivatives pricing in the OTC market.

In the OTC financial derivatives market, public disclosure information that investors can receive is often distorted. The two important source of information disclosure distortion is, nonhuman distortion caused by objective factors and human distortion caused by subjective factors, such information noise interference, respectively called unbiased noise and biased noise information. Specifically, unbiased noise information is caused by objective factors, such as statistical error of data, information expression bias and historical cost deviation. Biased noise information is caused by subjective intention, such as tampering accounting information, fabricating false financial data and concealing unfavorable data. Shin (2003) and Lev (2003)’s research show that in the nineties of the last century, more than 90% of the financial reports were modified in the U.S.A. In China’s stock market, data show that the number of information disclosure distortion is increasing year by year, these market phenomena confirm the objective existence of noise interference in market information disclosure. The existence of noise interference seriously destroys the quality of information disclosure and distorts the market environment. Therefore, in the management of credit risk, the noise interference of the market information must be taken into consideration.

Duffie and Lando (2001) first introduced incomplete information into the measurement of credit risk model,
they calculated default probability and credit spreads under the framework of structural models. Green (2003) pointed out that information asymmetry is a major reason why large financial institutions are willing to implement credit rationing for Small and Medium Enterprises instead of giving them a loan. Guo, Jarrow, and Zeng (2009) put forward an asymmetric information describing method by using delayed information filtering. Wu (2015) discussed the problem of credit risk measurement under asymmetric information. With the deepening of research, scholars pay more and more attention to the important role of asymmetric information in the study of economic and financial problems, studies such as Wu and Zhuang (2015), Wu, Liu, Wang, and Zhuang (2016a), Wu, Zhuang, and Li (2016b), Wu, Wang, Liu, and Zhuang (2017), Wu and Zhuang (2018).

The reduced-form model was first proposed by Fons (1994), this reduced-form model avoids modelling the unobserved company value, considers default probability as exogenous variable, and regards credit event as unpredictable. Reduced-form model are divided into intensity model and the non-intensity model in reality. The intensity model is based on the Doob-Meyer martingale decomposition theory, mainly modelling the default intensity, and the exogenous mechanism of default is introduced, which considers that default is a random event determined by a intensity process. Lando (1998) extended the model to the Cox process, and use the Markov transfer matrix to establish the credit rank transfer intensity model, he pointed out that the default intensity is a function of information of market state variables, Therefore, the noise interference of market information will have an important impact on the intensity of default. Therefore, the noise interference factor is added to the construction of the default intensity in this paper, and a new default intensity model with noise interference is proposed, and which is applied to the pricing defaultable bonds and credit default swaps. Compared with the existing intensity model, the noise interference term is added to the default intensity model for the first time, which can make the default model consider more default sources and make the model closer to the real OTC market.

2. A default intensity model with noise interference variables

In order to better describe the effect of noise interference information on default probability, a default intensity model with noise interference variables is built in this section, and then the default probability (or the survival probability) with noise interference information is calculated.

Based on the default intensity model proposed by Lando (1998), we also regard the default event as a function of market state variables, which are marked as $\{X_t\}_{t \geq 0}$. In other words, the default is depicted as the first jump of the Cox process with random intensity parameter $\{\lambda_t\}_{t \geq 0}$. Let $(N_t)_{t \geq 0}$ be a Poisson process with a non-negative and bounded random intensity parameter, and assume that $(N_t)_{t \geq 0}$ and $\{\lambda(X_t)\}_{t \geq 0}$ are independent of each other under probability measure $P$.

Specifically, the proposed default intensity-based model with noise interference terms is following,

$$\lambda_{t|\tau} = \lambda(X_t) + \lambda(U_t)$$

where $U_t$ is a biased noise interference variable, in view of some design defects of the accounting system and the possible moral hazard of the market participants, especially in the market environment of over-the-counter trading, which bring about the actual financial report has a directional noise interference (relative to symmetry noise). In order to characterize the biased noise interference in the financial report, it is assumed that it obeys the partial normal distribution $SN(\mu, \sigma^2, \omega)$, the partial normal distribution has good analytical properties and statistical properties, related research can be referred to Azzalini (1985) and Wu (2015).

Generally speaking, the density function of the partial normal distribution $SN(\mu, \sigma^2, \omega)$ is defined as

$$f(x, \mu, \sigma, \omega) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(x - \mu)^2}{2\sigma^2}\right) \times \left(1 + \frac{2}{\sqrt{\pi}} \int_0^{\omega/\sqrt{2}} e^{-\eta^2} d\eta\right),$$

where the expected value $\mu$ controls the location of the noise interference and the variance $\sigma^2$ controls the peak value of the density function, that is, the magnitude of biased noise is controlled by the standard deviation $\sigma$. The parameter $\omega$ controls the degree of skewness of the noise interference density function, and $\omega$ is set in the closed interval $[-1, 1]$.

Following the hypothesis of Lando (1998), we define the default time $\tau$ as

$$\tau := \inf\{t \geq 0 : \int_0^t \lambda_s ds \geq E_1\} = \inf\{t \geq 0 : \int_0^t (\lambda(X_s) + \lambda(U_s)) ds \geq E_1\},$$

where $E_1$ is a unit exponential random variable.

About the information flow in the market, we describe it as follows, the market participants get the flow of the enterprise history financial information from the financial market are $\mathcal{F}_t = \mathcal{F}_t^X \vee \mathcal{F}_t^U \vee \mathcal{F}_t^N \subset \mathcal{G}_t \in \mathcal{G}$, where,
\( G = (\mathcal{G}_t)_{t \geq 0} \) represents information flow of market environment, such as financial information and credit event information. \( \mathcal{F}^X_t = \sigma \{ X_s : 0 \leq s \leq t \} \) represents information flow of the market state variables until the time \( t \), \( \mathcal{F}^U_t = \sigma \{ U_s : 0 \leq s \leq t \} \) represents the information domain flow generated by partial noise interference, the conditional survival probability obtained by market participants due to partial noise interference, \( \mathcal{F}^{\text{inh}}_t = \sigma \{ 1_{\{ \tau \leq s \}} : 0 \leq s \leq t \} \) represents the historical default information domain flow until the current time \( t \), therefore, \( \mathcal{F}_t \) corresponds to all the information flows in the market until the current time \( t \).

Because of the above conditions, we can get the following theorem.

**Theorem 2.1:** Considering the condition of market information containing noise interference, the conditional survival probability and the unconditional survival probability of time \( t \) are following,

\[
P(\tau > t | (X_s)_{0 \leq s \leq T} \lor (U_s)_{0 \leq s \leq T}) = \exp\left(-\int_0^T \lambda_s ds\right)
\]

\[
P(\tau > t) = \mathbb{E}[\exp(-\int_0^T \lambda_s ds)]
\]

Here, if the default time \( \tau \) admits a probability density function \( f(\tau) \), then the expected value can be calculated by \( \mathbb{E}[\tau] = \int_0^\infty t f(t) dt \).

**Proof:** Based on non-homogeneous Poisson process and random default intensity process \( (\lambda_t)_{t \geq 0} \), that is, the definition of double stochastic Poisson process and formula (1), we can derived the following,

\[
P(\tau > t | (X_s)_{0 \leq s \leq T} \lor (U_s)_{0 \leq s \leq T}) = P(N(t+h) - N(t) = 0)
\]

\[
= \left( \int_0^t \lambda_s ds \right)^0 \frac{e^{-\int_0^T \lambda_s ds}}{0!}
\]

\[
= \exp\left(-\int_0^T \lambda_s ds\right)
\]

\[
= \exp\left(-\int_0^T (\lambda(X_s) + \lambda(U_s)) ds\right) \quad t \in [0, T],
\]

From the law of total expectation, we can derived the following,

\[
P(\tau > t) = \mathbb{E}[\exp(-\int_0^T (\lambda(X_s) + \lambda(U_s)) ds)]
\]

\[
= \mathbb{E}[\exp\left(-\int_0^T (\lambda(X_s) + \lambda(U_s)) ds\right)]
\]

\[
t \in [0, T].
\]

3. **Model application**

In this part, the default intensity model with noise interference variables is applied to the pricing of defaultable bonds and CDS, and some new derivative pricing models are put forward.

3.1. **The pricing model of defaultable bonds**

Hypothesize that the face value of zero-coupon defaultable bond be 1 units and the maturity date be T. Here, the defaultable bond is a bilateral agreement to help the bond’s issuer prevent credit default risk. This agreement defines the rights and obligations of the bond’s issuer and the bond’s holder, in particular, that is the bond’s holder pays full face value at the maturity date, as long as the bond’s issuer as not default. If a default occurs during the validity period of the contract, then the recovery is paid to the bond’s holder, and the contract is ended. Using the proposed default probability calculation formula, we can obtain the following theorem for the pricing of defaultable bonds.

**Theorem 3.1:** Based on formulas (2) and (3), the pricing model of defaultable bonds with noise interference is following,

\[
V_t = \mathcal{E}^Q \left[ \exp\left(-\int_t^T r_s ds\right) 1_{\{ \tau > T \}} \right]
\]

\[
+ \exp\left(-\int_t^T r_s ds\right) \delta 1_{\{ \tau < T \} | \mathcal{F}_t} = 1_{\{ \tau > t \} | \mathcal{E}^Q} \left[ \exp\left(-\int_t^T (r_s + \lambda_s) ds\right) \right]
\]

\[
+ \int_t^T \delta \lambda_s \exp\left(-\int_t^T (ru + \lambda_u) du\right) ds | \mathcal{F}^X_t \lor \mathcal{F}^U_t
\]

\[
= 1_{\{ \tau > t \} | \mathcal{E}^Q} \left[ \exp\left(-\int_t^T (r_s + \lambda(X_s) + \lambda(U_s)) ds\right) \right]
\]

\[
+ \int_t^T \delta (\lambda(X_s) + \lambda(U_s)) ds
\]
Obviously, on the set \( \{ \tau \leq t \} \), the above conditional expectation is expected to be zero, and the set \( \{ \tau > t \} \) is an atom (subset) of \( \mathcal{F}_t^{N_1} \), hence, we have the following,

\[
\begin{align*}
E(1_{\{\tau \geq t\}}|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \mathcal{F}_t^{N_1}) &= 1_{\{\tau > t\}} \exp \left( -\int_t^\tau (\lambda(X_s) + \lambda(U_s))ds \right) \\
\end{align*}
\]

And then, we have
\[
\begin{align*}
E^Q\left[\exp\left(-\int_t^\tau r_s ds\right)1_{\{\tau > t\}}|\mathcal{F}_t^1\right] \\
= E^Q[E^Q(\exp\left(-\int_t^\tau r_s ds\right)1_{\{\tau > t\}}|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \mathcal{F}_t^{N_1})|\mathcal{F}_t^1] \\
= E^Q[\exp\left(-\int_t^\tau r_s ds\right)E^Q(1_{\{\tau > t\}}|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \mathcal{F}_t^{N_1})|\mathcal{F}_t^1] \\
= 1_{\{\tau > t\}}E^Q[\exp\left(-\int_t^\tau (r_s + \lambda(X_s) + \lambda(U_s))ds\right)|\mathcal{F}_t^1],
\end{align*}
\]

The unit exponential random variable \( E_1 \) and \( (X_s)_{0 \leq s \leq T} \) is independent of the \( \sigma \) algebra generated by \( (U_s)_{0 \leq s \leq T} \) especially, \( E_1 \) is independent of the \( \sigma \) algebra \( \sigma(\exp(-\int_t^\tau (r_s + \lambda(X_s) + \lambda(U_s))ds) \vee \mathcal{F}_t^X \vee \mathcal{F}_t^{U_1}) \), so there is,

\[
E^Q[\exp(-\int_t^T (r_s + \lambda(X_s) + \lambda(U_s))ds)|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \sigma(E_1)]
\]

At the same time, there are also the following \( \sigma \) algebra inclusion relation,

\[
\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \subset \mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \sigma(E_1),
\]

From the above relationship, we have the following conclusions,

\[
\begin{align*}
E^Q[\exp(-\int_t^T (r_s + \lambda(X_s) + \lambda(U_s))ds)|\mathcal{F}_t^1] \\
= E^Q[\exp(-\int_t^T (r_s + \lambda(X_s) + \lambda(U_s))ds)|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1}] \\
= E^Q[\exp(-\int_t^T (r_s + \lambda(X_s) + \lambda(U_s))ds)|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \sigma(E_1)] \\
= E^Q[\exp(-\int_t^T (r_s + \lambda(X_s) + \lambda(U_s))ds)|\mathcal{F}_t^1] \\
= E^Q[\exp(-\int_t^T (r_s + \lambda(X_s) + \lambda(U_s))ds)|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \sigma(E_1)].
\end{align*}
\]

The same as above, it has been proved that there are the following conclusions,

\[
\begin{align*}
E^Q[\exp(-\int_t^\tau r_s ds)|\mathcal{F}_t] \\
= E^Q[E^Q(\exp(-\int_t^\tau r_s ds)|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \mathcal{F}_t^{N_1})|\mathcal{F}_t] \\
= E^Q[\exp(-\int_t^\tau r_s ds)E^Q(1_{\{\tau > t\}}|\mathcal{F}_t^X \vee \mathcal{F}_t^{U_1} \vee \mathcal{F}_t^{N_1})|\mathcal{F}_t] \\
= 1_{\{\tau > t\}}E^Q[\exp(-\int_t^\tau (r_s + \lambda(X_s) + \lambda(U_s))ds)|\mathcal{F}_t],
\end{align*}
\]

In this way, we have completed the proof of the theorem.

\[\blacksquare\]
3.2. A new CDS pricing model containing noise interference

A credit default swap (CDS) is a particular type of swap designed to transfer the credit exposure of fixed income products between two or more parties. The transaction involves two parties, the protection buyer and the protection seller, and also a reference entity, usually a bond. The protection buyer pays a stream of premiums (the CDS ‘fee’ or ‘spread’) to the protection seller, who in exchange offers to compensate the buyer for the loss in the bond’s value if a credit event occurs. The stream of premiums is called the premium leg, and the compensation when a credit event occurs is called the protection leg.

Hypothesize that the notional amount of swap be $N$, the recovery rate be $\delta$, and the credit spreads be $s$. At the same time, the discount factor between $t$ and $T$ be $D(t,T) = \exp\left(-\int_t^T r_s ds\right)\}$, the sequence payment dates be $(T_i)_{1 \leq i \leq n}$, which along with $T_n = T$.

The CDS running spread or fee is computed such that the fair price of the CDS equals zero at the initiation. In other words, the present value of the protection leg equals the present value of the premium leg. As a result, we obtain the following pricing formula of CDS with noise interference.

**Theorem 3.2:** Under the condition of adding noise interference, we have the following CDS pricing formula,

$$s = \sum_{i=1}^{n} \Delta T_i \exp\left(-\int_{T_i}^{T_f} (r_s + \lambda(X_s) + \lambda(U_s)) ds\right)$$

$$\times \exp\left(-\int_{T_i}^{T_f} (r_s + \lambda(X_s) + \lambda(U_s)) du\right)$$

$$\Rightarrow$$

$$s = sN \sum_{i=1}^{n} \Delta T_i \exp\left(-\int_{T_i}^{T_f} (r_s + \lambda(X_s) + \lambda(U_s)) ds\right)$$

$$\times \exp\left(-\int_{T_i}^{T_f} r_s ds\right)$$

$$\times \left(1 - \delta\right) \int_{T_i}^{T_f} (\lambda(X_u) + \lambda(U_u)) ds$$

$$\times \left(1 - \delta\right) \int_{T_i}^{T_f} \exp\left(-\int_{T_i}^{T_f} r_s ds\right)(1 - S(u))$$

$$\times \exp\left(-\int_{T_i}^{T_f} (r_s + \lambda(X_s) + \lambda(U_s)) ds\right) du$$

The discount value of the future cash flow of the CDS’ protection buyer is following,

$$PV(\text{protection leg}) = E^Q[(1-\delta)N \exp\left(-\int_{T}^{T_f} r_s ds\right)$$

$$\times 1_{\tau \geq T}] | F_T] = (1-\delta)N \int_{T}^{T_f} \exp\left(-\int_{T}^{u} r_s ds\right)(1 - S(u))$$

$$\times \left(1 - \delta\right) \int_{T_i}^{T_f} (\lambda(X_u) + \lambda(U_u))$$

$$\times \exp\left(-\int_{T_i}^{T_f} r_s ds\right)$$

The discount value of the future cash flow of the CDS’ protection seller is following,

$$PV(\text{premium leg}) = E^Q[\sum_{i=1}^{n} \Delta T_i (\tau \wedge T_i - T_i)$$

$$\times \exp\left(-\int_{T}^{\tau \wedge T_i} r_s ds\right)] | F_T]$$

In this case, we have completed the proof of the theorem.

4. Model simulation

In this section, we simulate the survival probability with noise interference proposed by the aforementioned theorem 1, and hope that the effect of noise interference on the default probability (or survival probability) is reflected in a visual way of figure. In order to carry out the simulation analysis, we set up some model parameters, for example, the duration of derivatives $T = 5$ (unit for year), the current default intensity of non noise interference $\lambda(X_t) = 0.23$ and so on. Thus, by means of simulation through the use of MATLAB R2010a, we obtain the following survival probability Figure 1 with directional noise interference.

From Figure 1, it is known that the bi-directional noise interference has an important impact on the strength of default. Whether the positive noise interference or negative noise interference, the default intensity will increase with the increase of the noise interference amplitude,
that is, the emergence of noise interference will increase the default probability of enterprises, and when the bi-directional noise interference disappears, the survival probability of enterprises is raised to a certain limit. That means, when the positive noise interference or negative noise interference disappears, the effect of noise interference on default probability is no longer present, this also means that the probability of survival increases with the decrease of negative constraints. Therefore, in the study of credit risk prevention measures, the lack of noise variables will have an important negative impact on the effect of relevant measures, we must pay attention to the noise interference of the market information.

5. Conclusion

In view of some design defects in the accounting system in the OTC market, and the possible moral hazard of market participants, which bring about the actual financial report contains information with directional noise interference. To depict the biased noise interference in the financial report, we proposed a reduced-form intensity-based model with noise interference in this paper, and which is applied to the pricing defaultable bonds and credit default swaps. Through the simulation analysis, it is found that the noise interference has a direct positive effect on the default probability, that is, the strength of default will increase with the increase of noise interference amplitude. It tells us that in the management of credit risk, the noise interference of the market information must be taken into consideration in the modelling of the strength of default. Compared with the existing models, the new model has some advantages. For example, compared with Wu (2015), we add more noise interference terms in the default intensity model, this makes the model closer to the state of the real market and the description of default probability more precise. Of course, this paper also has some shortcomings, such as not pointing out how to control the adverse effects of noise interference on the default intensity, which is also the direction and content of our future research.

Acknowledgements

Dong-wei Shi, given some modifications to the proof of the theorem in this paper, Dong-e Bao, polished the language of the article, and Liang Wu is the main writer of the framework and content of this article.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The work was supported by the Higher School Key Scientific Research Projects of Henan Province [grant number 18A110017].

Data availability statement

By setting some parameters to the model, the data of this paper can be obtained by means of simulation through the use of MATLAB R2010a, and I statement that the data in my manuscript is available.

References

Azzalini, A. (1985). A class of distributions which includes the normal ones. Scandinavian Journal of Statistics, 12(2), 171–178.
Duffie, D., & Lando, D. (2001). Term structures of credit spreads with incomplete accounting information. *Econometrica*, 69(3), 633–664.

Fons, J. S. (1994). Using default rates to model the term structure of credit risk. *Financial Analysts Journal*, 50(5), 25–32.

Green, A. (2003). *Credit guarantee schemes for small enterprises: An effective instrument to promote private sector-led growth*. New York: Program Development and Technical Cooperation Division.

Guo, X., Jarrow, R. A., & Zeng, Y. (2009). Credit risk models with incomplete information. *Mathematics of Operations Research*, 34(2), 320–332.

Lando, D. (1998). On cox processes and credit risky securities. *Review of Derivatives Research*, 2(2-3), 99–120.

Lev, B. (2003). Corporate earnings: Facts and fiction. *Journal of Economic Perspectives*, 17(2), 27–50.

Shin, H. S. (2003). Disclosures and asset returns. *Econometrica*, 71(1), 105–133.

Wu, J. H. (2015). Structural model and application research on the credit risk measurement based on incomplete information. Jinan: Shan Dong University.

Wu, L., Liu, J., Wang, J., & Zhuang, Y.-m. (2016a). Pricing for a basket of LCDS under fuzzy environments. *SpringerPlus*, 5(1), 1747.

Wu, L., Wang, J., Liu, J., & Zhuang, Y.-m. (2017). A total return swap pricing model under fuzzy random environments. *Discrete Dynamics in Nature and Society*, 2017(1), 1–10.

Wu, L., & Zhuang, Y. (2015). A reduced-form intensity-based model under fuzzy environments. *Communications in Nonlinear Science and Numerical Simulation*, 22(1), 1169–1177.

Wu, L., & Zhuang, Y. (2018). A new credit derivatives pricing model under uncertainty process. *Systems Science & Control Engineering*, 6(1), 477–481.

Wu, L., Zhuang, Y., & Li, W. (2016b). A new default intensity model with fuzziness and hesitation. *International Journal of Computational Intelligence Systems*, 9(2), 340–350.