Cuts in the invariant mass of resonances in many body decays of mesons

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Abstract

Resonance-mediated many body decays of heavy mesons are analyzed. We focus on some particular processes, in which the available phase space for the decay of the intermediate resonance is very narrow. It is shown that the mass selection criteria used in several experimental studies of $D$ and $B$ meson decays could lead to a significant underestimation of branching ratios.

Three or more body decays of heavy mesons are usually dominated by intermediate resonances. The latter are short lived states that cannot be directly observed: only daughter particles produced through their decays reach the detectors [1]. The detected final state results from the interference of all possible intermediate channels. Thus, one has to disentangle the quantum interference to understand the underlying physics. A powerful technique to split up the various resonant channels is the so-called Dalitz plot fit [2].

In general, if a given resonance proceeds within a kinematic region where no other resonances occur, interference effects are assumed to be negligible. In this case, the branching ratio of the heavy meson decay to that particular resonance can be measured in a simple way: one just counts the number of events in which the invariant mass of the decay products lies within a small window around the resonance central mass. The size of this window is chosen according to the width of the resonance. This is a natural and thus widely used measurement technique [3].

In this Letter we discuss the validity of this method. In fact, the resonance is a virtual state, and its squared four-momentum can reach in principle any value within the allowed phase space. This is certainly not a new statement. The interesting fact is that, as we will see, for some particular decays one could find a very large number of (naively, unexpected) events such that, even if they proceed through a particular resonance, the invariant mass of the detected particles is indeed very far from the resonance mass shell (with “far”, we mean in comparison with the resonance width). These events would be missed if the counting method simply considers a narrow window around the resonance central mass. As shown below, this is the case for some processes in which the resonance decay rate is kinematically suppressed.
We are aware that the physics involved in this discussion is well known. However, we believe that the magnitude of the effect has not been well appreciated so far, and the described counting method is not always safely applicable. Here we present simulations of actual decays and discuss the significance of the effect as well as the expected experimental difficulties.

Let us first describe those aspects of many body decays relevant to our discussion. Consider a heavy meson $P$ decaying into a final state given by three detected particles, $d_1, d_2$ and $d_3$, and assume that the decay proceeds through intermediate resonances $R_1, R_2$, etc.

The total amplitude of the decay is the sum of the amplitudes of all partial channels, each one mediated by a resonance $R_i$:

$$A_{tot} = \Sigma_i A_{R_i}.$$  \hfill{(1)}

For simplicity, let us assume that the decay is dominated by a single resonance $R$, in such a way that $A \simeq A_R = A(P \rightarrow Rd_3; R \rightarrow d_1d_2)$. It is natural to describe the process by considering three independent stages: resonance production $P \rightarrow Rd_3$, resonance propagation, and resonance decay $R \rightarrow d_1d_2$. The amplitude factorizes then as

$$A(P \rightarrow Rd_3 \rightarrow d_1d_2d_3) = A(P \rightarrow Rd_3) \times BW_{R,12} \times A(R \rightarrow d_1d_2).$$ \hfill{(2)}

The amplitudes $A(P \rightarrow Rd_3)$ and $A(R \rightarrow d_1d_2)$ have to take into account the conservation of angular momentum in the decays, as well as the energy dependence, usually parameterized through form factors (we will come back to these two ingredients later). As usual, we describe the resonance propagation by means of a relativistic Breit-Wigner function

$$BW_{R,12}(m_{12}^2) = \frac{1}{m_0^2 - m_{12}^2 - im_0\Gamma},$$ \hfill{(3)}

where $m_0$ is the resonance mass, $\Gamma$ is the resonance width, and $m_{12}^2$ is the invariant mass of the outgoing particles $d_1$ and $d_2$, $m_{12}^2 = (p_1 + p_2)^2$.

The function $|BW(m_{12}^2)|$ is peaked around the resonance mass, and decreases with a rate given by $\Gamma$. This behavior reflects the fact that $R$ is a virtual particle that —according to quantum mechanics— can have any invariant mass $m_{12}^2$, the relative probability of each “mass” being weighted by the factor $|BW(m_{12}^2)|^2$. It is then natural to measure $BR(P \rightarrow Rd_3)$ by simply counting the number of detected particles $d_1$ and $d_2$ for which the value of $\sqrt{m_{12}^2}$ lies within a region of the order of $(m_0 - \Gamma, m_0 + \Gamma)$. This technique has in fact been used for many years \[3\]. However, as we will show in the following, the usage of this approach is not always safe.

The decay of a resonance is usually driven by the strong interaction. Thus, resonances have relatively large widths —some dozen or even some hundreds of MeVs \[4\]. However,

\[1\]The decay also proceeds through a non-resonant channel, which is not relevant to our discussion.
if a particular decay is somehow suppressed, the resonance may have a longer life, and its width can be as small as some MeVs or less. This happens in particular when the resonance central mass \( m_0 \) is very close to the threshold of its decay to \( d_1 d_2 \). In this case, the phase space available for the decay turns out to be very narrow, and the (virtual) resonance could be allowed to decay through other channels which were in principle expected to be strongly suppressed in comparison with the “natural” strong channel \( R \to d_1 d_2 \).

This is the case, for instance, of the \( \phi(1020) \) vector meson. For this resonance, the natural decay channel is \( K \bar{K} \), in the same way as the natural decay for \( \rho \) is two pions. Nevertheless, due to the small phase space available —32 MeV and 24 MeV for the charged and neutral kaons, respectively— the corresponding \( \phi \) branching ratios are “only” 49% and 34% for \( K^+ K^- \) and \( K \bar{K} \), respectively. Electromagnetic decays, that have branching fractions as small as \( 10^{-4} \) in the \( \rho \) decay pattern, are of the order of 1% in the \( \phi \) case. Accordingly, the \( \phi \) width is about 35 times smaller than the \( \rho \) one.

Other examples are low mass \( D^* \) vector mesons. Their natural decay channel is \( D \pi \), in the same way as the natural decay for \( K^* \) is \( K \pi \). But here, the phase space is as small as 7 MeV for the \( D^{*0}(2007) \to D^0 \pi^0 \), 6 MeV for both \( D^{*+}(2010) \to D^0 \pi^+ \) and \( D^{*+}(2010) \to D^+ \pi^0 \), and 8 MeV for \( D_s^{*+} \to D_s^+ \pi^0 \). As a consequence, measured branching ratios of these decays—which otherwise should reach almost 100%—are as “small” as 62%, 68%, 31% and 6%, respectively. In the \( D^{*0}(2007) \) decay pattern, the electromagnetic decay is as large as 38%, i.e., of the same order of magnitude of the strong one (in contrast, in the \( K^* \) case, electromagnetic decays are of the order of \( 10^{-3} \)). Accordingly, resonance widths are quite small: the width of \( D^{*+}(2010) \) has recently been reported to be as small as 0.1 MeV \(^3\), whereas for \( D^{*0}(2007) \) and \( D_s^{*+} \) only upper limits are known, presently of the order of 2 MeV.

Let us face the study of heavy meson decays mediated by these particular spin one resonances, focusing on cases in which the detected final state includes their natural, and yet highly suppressed, strong channels. We describe here a usual situation \(^4\), where both the initial heavy meson and the particles in the final state are scalars, \( P = D, B \), and \( d_i = \pi, K, D \). In this case, Eq. (2) can be conveniently written as

\[
A(P \to Rd_3 \to d_1 d_2 d_3) = F_{P,Rd_3} F_{R,d_1 d_2} (-2 \vec{p}_1 \cdot \vec{p}_3) BW_{R,12} ,
\]

where \( F_{P,Rd_3} \) and \( F_{R,d_1 d_2} \) are form factors, and the three-momenta \( \vec{p}_1 \) and \( \vec{p}_3 \) are evaluated in the resonance rest frame. The explicit momentum dependence in (4) follows from Eq. (2), just assuming Lorentz invariance and summing over all possible polarizations of the intermediate vector meson resonance. The differential decay width of this reaction can be written as

\[
d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32 M^3} |A|^2 dm_1^2 dm_2^2 ,
\]

where \( m_{i,j} = (p_i + p_j)^2 \) and \( M \) is the mass of the decaying meson \( P \).

\(^2D_s^{*+} \to D_s^+ \pi^0 \) is an isospin violating decay (see Ref. \(^3\)). It thus has another, phase space independent suppression.
For the decays considered here, there is a strong suppression at the resonant peak, i.e., when $m_{12}^2 \approx m_0^2$. This suppression is due to purely kinematic effects. To see this, let us take $A \approx$ constant for a given value of $m_{12}^2$—which means to neglect the dynamics of the decay—and integrate over the variable $m_{13}^2$ within the kinematic limits of the three body phase space. It is easy to see that

$$\frac{d\Gamma}{dm_{12}^2} \propto |A|^2 |\vec{p}_1|,$$  

(6)

where $\vec{p}_1$ is the three-momentum of $d_1$ in the resonance rest frame. Since we are assuming that the mass of the resonance is just above the threshold, $m_0 \simeq m_1 + m_2$, at the resonance peak both particles $d_1$ and $d_2$ will be produced almost at rest. Thus Eq. (6) implies a suppression in the partial width.

The effect is even stronger in the particular case of a vector resonance. In that case, as stated in Eq. (4), the amplitude is proportional to $\vec{p}_1 \cdot \vec{p}_3$. Assuming that the form factors are slow-varying functions of the phase space variables, the partial width is finally expected to be suppressed by a factor $(|\vec{p}_1|/\Lambda)^3$, where $\Lambda$ is some natural scale of the process, typically of order $M$.

Eqs. (4) and (6), which are certainly very well known, show up the main point of this Letter. The decay width $\Gamma(P \to d_1 d_2 d_3)$ is driven by two competitive effects. On the one hand, the BW propagator strongly enhances the decay amplitude in the vicinity of the resonance mass. On the other hand, kinematic effects suppress the differential width at the resonance peak, hence the decay rate for other kinematic regions is comparatively enhanced. The usual suppression of the differential decay width for $P \to Rd_3 \to d_1 d_2 d_3$ outside the window allowed by the BW function is not obvious in this case.

To clarify our point, let us emphasize that we are not claiming that there is an enhancement of the resonant production probability $P \to Rd_3$ outside the BW peak. On the contrary, the point is that due to the suppression of the resonance decay rate $R \to d_1 d_2$ at the resonance central mass, the combined production + decay probability within and outside the peak could be of comparable orders. In other words, the total width (i.e., integrated over the whole phase space) is indeed small; the important point is that, even if the decay proceeds through a BW-described resonance, the relative weight for the decay rate near or far from the resonance mass shell is not just driven by the BW function.

In order to show that this effect is not a simple academic thought, we present in the following a simulation of an actual process. We first present the pure theoretical estimate; afterwards, we will consider the experimental difficulties. Let us consider the decay $B^+ \to \bar{D}^* D_s^+; \bar{D}^* \to \bar{D}^0 \pi^0$. Using Eq. (4), it is possible to get an estimate for the differential decay rate $d\Gamma$ as a function of $m_{12}^2$. Since the form factors are usually smooth functions, we will assume as a first guess that they are constant (experimental analyses show that form factor shapes have no significant effect on the total systematic error of a given Dalitz plot fit [5]). In this way we can calculate the ratio

$$r = \frac{\int_{m_{12}^2=(m_0+n\Gamma)^2} \left| A \right|^2 d\Phi}{\int \left| A \right|^2 d\Phi},\quad (7)$$

4
where \( d\Phi \) is an element of the three body phase space, \( m_0 \) is the \( D^{*0} \) resonance mass \( (m_0 = 2007 \text{ MeV}) \), \( \Gamma \) is the \( D^{*0} \) width, and \( n \) is a real number. \( \Gamma \) is presently unknown, its upper limit being 2.1 MeV with a 90% confidence level [1]. The integral in the denominator is calculated over the whole phase space, while that in the numerator is limited to a window in \( m_{12}^2 \). Thus, \( r \) is a measure of the relative number of events that are expected to fall within the resonance peak.

We quote in Table I the values of \( r \) for some input values of \( n \) and \( \Gamma \). Our results show that the effect we are describing can be very strong if the resonance width is as large as 2 MeV, and it remains quite significant even if \( \Gamma \) is of the order of 0.1 MeV. For comparison, we include in the last column the values of \( r \) corresponding to a fictitious resonance having the same quantum numbers as \( \bar{D}^{*0} \) but a higher mass, \( m_0 = 2.6 \text{ GeV} \). This particle would not suffer the kinematic suppression in its decay to \( \bar{D}^0 \pi^0 \), consequently a small value of \( n \) is enough to get \( r \) above 90%. We have found that in this case the results for \( r \) are independent of the specific value of \( \Gamma \).

Figures 1a and 1b show the kinematic distribution of the events for the three body decay \( B^+ \rightarrow \bar{D}^0\pi^0 D^+_s \), assuming that the process is dominated by the \( \bar{D}^{*0}(2007) \) resonance channel and considering a \( \bar{D}^{*0} \) width \( \Gamma = 1 \text{ MeV} \). The plots correspond to a Monte Carlo simulation of 10000 events, performed using Eqs. (4) and (5). Figure 1a is the Dalitz plot of the decay as a function of the invariant masses \( m_{\bar{D}^0\pi^0}^2 \) and \( m_{D^+_s\pi^0}^2 \), while in Figure 1b we represent an histogram of the number of events as a function of \( m_{\bar{D}^0\pi^0}^2 \).

It is seen that the Dalitz plot shape in Fig. 1a is quite different from that naively expected for a decay mediated by a \( \Gamma = 1 \text{ MeV} \) vector resonance. This becomes evident by looking at Figure 1c, where we show a Monte Carlo simulation of the same process, now shifting the \( \bar{D}^{*0} \) mass to the fictitious value of 2.6 GeV considered in Table I. The striking difference between the event distribution in both plots arises from the kinematic suppression discussed above. However, the situation displayed in Fig. 1a can be misleading if one just looks at the events for which \( m_{\bar{D}^0\pi^0}^2 \) is near the \( \bar{D}^{*0} \) mass. In fact, despite the spreading of events along the whole phase space, the event density is still much larger in the peak region than anywhere else. Therefore, Figure 1b could be mistaken for a \( \Gamma = 1 \text{ MeV} \) Breit-Wigner function, with some background in the right sideband [8]. It would be then natural to apply the usual method, that means to consider that the events within a window of a few \( \Gamma \) around the peak amount almost the total number of decay events. But, according to our simulation, this would be wrong by a factor as large as 3 to 4 (see Table I). Indeed, even if the amplitude is peaked around the resonance mass, the phase space area outside the peak is comparatively so large that the number of events falling in this region becomes very important.

Let us now discuss the experimental difficulties that could appear when data from real events are analyzed. First of all, resolution in these experiments is usually larger than the width of the intermediate resonances involved [3]. As a consequence, it would be impossible to access to a direct measurement of the ratio \( r \), at least for small values of \( n \). Accordingly,

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[3] In fact, the decay can also proceed through an intermediate resonance \( D^{*+}_{s}\), as well as other higher resonant states. The inclusion of these contributions in our analysis does not affect significantly the results.
the experimental histogram of Fig. 1b would be wider, arising from the convolution of a narrow Breit-Wigner function (physical) and a Gaussian function with a larger width (resolution). Nevertheless, the bulk of our reasoning remains valid: a large amount of physical events may fall outside the peak. A second comment refers to the difficulty of developing a more careful analysis of data, in order to include the contribution of these events. Indeed, according to Table I, the amount of events could be quite important, but at the same time they appear to be spread in a very large region of the phase space. Then, only a relatively low number of events per bin would be found outside the peak, and they could be hardly disentangled from the background, no matter which is the function used to perform the corresponding fit.

It is important to stress that our theoretical estimates rely on two basic assumptions. First, we have taken the form factors to be approximately constant along the phase space. Second, we have assumed that the Breit-Wigner shape remains valid far beyond a small region around the peak. Whereas the first hypothesis is quite natural and does not have a significant effect on our results, the second assumption can be hardly supported from the theoretical point of view. Our simulations are in this sense strongly model dependent, and this has to be kept in mind when looking at the numerical values presented in Table I.

In any case, our simulations can be taken as a severe warning, indicating that many reported results can be spoiled by the effect described in this Letter. For some particular processes, the magnitude of the corrections could be quite significant, and this should be taken into account—at least—in the corresponding systematic errors. This includes the analysis of the decays $B \rightarrow \bar{D}^{*0}d_3$, $B \rightarrow D^{*+}d_3$ and $B \rightarrow D_s^{*+}d_3$, with $d_3 = \pi^0, \eta, K, D_s^+, D^0$, etc. These channels have been measured [9] using $\bar{D}^0\pi^0d_3$—respectively $D^+\pi^0d_3$ and $D_s^+\pi^0d_3$—as final states, and imposing a cut in the invariant mass $m^2_{\bar{D}^0\pi^0}$—respectively $m^2_{D^+\pi^0}$ and $m^2_{D_s^+\pi^0}$. According to the simple analysis presented here, in all these cases the effect would be of the order predicted in Table I. In particular, in the case of the $D^{*+}$ resonance, whose width has recently been reported to be 0.1 MeV, the effect is expected to be of the order of 30%.

Finally, let us mention that $\phi$ meson production and decay measurements could also be affected by the effect described above. Since the $\phi$ resonance can be produced in $D$ meson decays, a significant amount of data is presently available, and many Dalitz plot analyses have already been done. However, notice that the threshold of the decay $\phi \rightarrow KK$ is about 25 MeV away from the $\phi$ mass, so that the available phase space is not as narrow as in the $D^{*}$ case. Performing a Monte Carlo simulation for the decay $D \rightarrow \phi\pi$ similar to that described above for the process $B \rightarrow \bar{D}^{*}D_s$, we have found that the expected effect could be in this case as large as 30%.

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REFERENCES

[1] Particle Data Group, D.E. Groom et al., Eur. Phys. J. C 15 (2000) 1.
[2] E. Byckling and K. Kajantie, Particle Kinematics (John Wiley & Sons, New York, 1973).
[3] See for example ARGUS Collab., H. Albrecht et al., Z. Phys. C 54 (1992) 1; CLEO Collab., D. Gibaut et al., Phys. Rev. D 53 (1996) 4734; CLEO Collab., M. Artuso et al., Phys. Lett. B 378 (1996) 364; WA82 Collab., M. Adamovich et al., Phys. Lett. B 305 (1993) 177; MARK-III Collab., R. M. Baltrusaitis et al., Phys. Rev. Lett. 55 (1985) 150.
See also works quoted in [3] and [4].
[4] J.D. Jackson, Nuovo Cim. 34 (1964) 6692.
[5] CLEO Collab., J. Gronberg et al., Phys. Rev. Lett. 75 (1995) 3232.
[6] CLEO Collab., S. Ahmed et al., Phys. Rev. Lett. 87 (2001) 251801.
[7] See for example E687 Collab., P.L. Frabetti et al., Phys. Lett B 331 (1994) 217.
[8] See for example Figure 1 of Ref. [5].
[9] The list is extensive. For a complete review see [1]. A recent measurement is that of Belle Collab., K. Abe et al., hep-ex/0107048.
| $n$ | $0.1$ MeV | $0.5$ MeV | $1$ MeV | $2$ MeV | fictitious |
|-----|-----------|-----------|----------|----------|------------|
| 1   | 61%       | 30%       | 18%      | 11%      | 70%        |
| 3   | 69%       | 36%       | 24%      | 17%      | 90%        |
| 10  | 73%       | 41%       | 30%      | 25%      | 97%        |
| 30  | 75%       | 46%       | 37%      | 36%      | 99.5%      |

TABLE I. The ratio $r$ as explained in the text, for different values of $n$ (1 to 30) and $\Gamma$ (0.1 to 2 MeV). The last column shows the values of $r$ for a fictitious resonance with mass 2.6 GeV —see text.
FIG. 1. Simulation of the decay $B^+ \to \bar{D}^* \bar{D}^0; \bar{D}^* \to \bar{D}^0 \pi^0$, with $\Gamma = 1$ MeV and 10000 generated events. (a) is the Dalitz plot in the plane $[(p_{\bar{D}^0} + p_{\pi^0})^2, (p_{D^+} + p_{\pi^0})^2]$, whereas (b) is a projection on the $(p_{D^0} + p_{\pi^0})^2$ axis. (c) is same as (a), with the $\bar{D}^*0$ mass shifted to 2.6 GeV.