THE CIRCUMSTELLAR EXTINCTION OF PLANETARY NEBULAE

ROBIN CIARDULLO

Department of Astronomy and Astrophysics, Penn State University, 525 Davey Lab, University Park, PA 16802; rbc@astro.psu.edu

AND

GEORGE H. JACOBY

Kitt Peak National Observatory, National Optical Astronomy Observatories, P.O. Box 26732, Tucson, AZ 85726; gjacoby@noao.edu

Received 1998 September 2; accepted 1998 November 16

1. INTRODUCTION

The modern picture of the formation and evolution of planetary nebulae (PNs) is given by Kwok (1993, 1994). As a star evolves up the asymptotic giant branch (AGB), it loses mass via a slow ($\sim 15 \text{ km s}^{-1}$) wind that compresses and shapes a high-speed ($v \sim 1000 \text{ km s}^{-1}$) wind that compresses and shapes the newly born planetary nebula.

A consequence of this evolutionary scenario is that the Balmer lines of young planetary nebulae often show the effects of extinction from dust just outside the ionization radius. Since PN progenitors can be anywhere from $M \sim 0.9 \, M_{\odot}$ (Jacoby et al. 1997) to $\sim 8 \, M_{\odot}$ (Elsen et al. 1998), the amount of dust in the circumstellar region can vary greatly from object to object. Moreover, because the timescale for post-AGB evolution is strongly dependent on core mass, the extent and density of the dust will also be variable. But these dependencies present us with an opportunity: by measuring the extinction in a circumstellar envelope, we can learn about the properties of the progenitor star and thereby constrain models of AGB and post-AGB evolution.

Of course, there are difficulties with this approach, particularly if the targets are planetary nebulae in our own Galaxy. Galactic PNs are notoriously inhomogeneous and have distance estimates that are extremely poor (see, e.g., Pottasch 1984). Consequently, the intrinsic properties of their central stars cannot be accurately determined. Similarly, in order to measure circumstellar dust around objects in the Galaxy, one must have a good estimate of foreground extinction. Given the nonuniformity of the Galaxy's cold ISM, this is at best a time-consuming task. Finally, because the distances and luminosities of Galactic PNs are uncertain, the precise evolutionary status of many of these objects is controversial. Circumstellar extinction can be expected to decrease with time as a consequence of nebular expansion. Therefore, unless the objects chosen for study are at roughly the same stage of evolution, the behavior of PN extinction with stellar mass cannot be studied.

The aforementioned problems prohibit us from using Galactic planetaries to explore the systematics of circumstellar extinction. However, moderately large samples of PNs with known distances and foreground reddenings do exist in other galaxies. Specifically, the PNs of the Magellanic Clouds and M31 are excellent candidates for study. Detailed nebular analyses exist for $\sim 90$ extragalactic objects, and from these data it is possible to obtain a large sample of circumstellar extinction measurements and central star mass estimates. Furthermore, since only the brightest PNs in these systems have been measured, most of the objects available for study are at roughly the same stage of evolution. Thus they comprise a useful database for our investigation.

Here we examine the behavior of circumstellar extinction with core mass in sets of planetary nebulae in the Magellanic Clouds and M31. We show that a correlation between core mass and extinction does exist, although it is steeper than what one might predict with simple models. We also show that this correlation provides a natural explanation for the invariance of the planetary nebula luminosity function (PNLF) with population age: bright Population I PNs are extinguished below the cutoff of the PNLF. It also explains the counter-intuitive observation that intrinsically luminous Population I PNs often appear fainter than PNs from older, low-mass progenitors.

Subject headings: dust, extinction — galaxies: individual (M31) — Magellanic Clouds — planetary nebulae: general — stars: AGB and post-AGB — stars: mass loss

2. THE EXTINCTION VERSUS CORE MASS RELATION

To investigate circumstellar extinction in extragalactic PNs, we restrict ourselves to extragalactic objects whose properties have been derived via full nebular modeling of
their emission lines. This effectively limits our sample to two sets of data. For the Magellanic Cloud planetary nebulae, we adopt the Balmer line extinction measurements tabulated by Meatheringham & Dopita (1991a, 1991b) and central star properties derived by Dopita & Meatheringham (1991a, 1991b) and Dopita et al. (1997). These papers provide us with complete information on 73 objects: 57 in the LMC, and 16 in the SMC. (We note that other samples of Magellanic Cloud planetary nebulae do exist, principally through the work of Kaler & Jacoby 1990, 1991, and Jacoby & Kaler 1993. We do not include these data in our analysis, although their use would not affect any of our conclusions.) For M31, we use the extinction estimates and central star parameters of the 15 PNs studied by Jacoby & Ciardullo (1999). To insure homogeneity, all PN core masses were recomputed by taking the central star effective temperature and the gas to color extinction ratio.

For M31, we use the extinction estimates and central star parameters of the 15 PNs studied by Jacoby & Ciardullo (1999). To insure homogeneity, all PN core masses were recomputed by taking the central star effective temperatures and luminosities derived from the nebular models, and reinterpolating them onto the grid of hydrogen-burning post-AGB evolutionary tracks given by Ciardullo (1999). To insure homogeneity, all PN core masses were recomputed by taking the central star effective temperatures and luminosities derived from the nebular models, and reinterpolating them onto the grid of hydrogen-burning post-AGB evolutionary tracks given by Schönberner (1983) and Blöcker (1995b). In addition, to place all of the extinction measurements on a common system, the contribution of Galactic dust was removed by assuming foreground reddening values of $E(B - V) = 0.080$, 0.074, and 0.054 for M31, the LMC, and the SMC, respectively (Burstein & Heiles 1984; Caldwell & Coulson 1985). Here, and throughout the paper, we assume the total extinction at $H\beta$, $c$, is related to differential extinction by $c = 1.47 E(B - V)$ (see, e.g., Kaler & Lutz 1985; Whitford 1958).

Plots of total logarithmic $H\beta$ extinction versus derived core mass are displayed in Figure 1. From the figure, one immediately notices that there is considerable scatter in the relation. This is not unexpected. Extinction estimates based on measurements of the Balmer decrement may carry an uncertainty of up to $\sigma_c \sim 0.1$ (see, e.g., Ciardullo et al. 1999). This error plus the error introduced by patchy extinction internal to the parent galaxies (Harris, Zaritsky, & Thompson 1997; Ciardullo et al. 1998) propagates directly into the diagrams of Figure 1 and is responsible for the negative reddenings. In addition, most of the PNs plotted in the figure are spatially unresolved and have not been observed in the ultraviolet. Without the constraints provided by UV line fluxes and nebular morphology, the models are somewhat uncertain, as are the derived positions of the central stars in the H-R diagram. A further complication is introduced by our use of hydrogen-burning post-asymptotic branch evolutionary models for core mass determinations. Since it is possible for PN central stars to be burning helium, the tracks used in this study may not be applicable for all objects. Finally, even if we had the ability to place each star precisely on the extinction–core mass diagram, there would be intrinsic scatter owing to geometry. Most Galactic PNs show a considerable amount of asymmetry (Balick 1987); if the distribution of dust around the bright PNs in our extragalactic sample is similarly asymmetric, then orientation effects alone will broaden the observed relation.

Nevertheless, despite the large amount of scatter, a trend is evident in all three galaxies. The derived core masses of PNs with large Balmer decrements (and therefore large $H\beta$ extinctions) appear to be systematically larger than those with small extinction values. The relation is steep, and there are a few high–core mass PNs in M31 and the LMC that do not obey the rule. But when these outliers ($M > 0.75 M_\odot$) are omitted, the correlations exhibited in all three galaxies are similar. For the LMC data, the slope of the ordinary least squares bisector line is $6.3 \pm 1.3$; for the SMC PNs, this slope is $5.6 \pm 0.7$, while for M31, the slope is $8.5 \pm 1.6$. Considering that the metallicities of the objects involved span a factor of $\sim 10$, that the ages of the parent populations are drastically different, and that the M31 PN observations, reductions, and analyses were performed completely independently of those done for the Magellanic Cloud PNs, the consistency of the results is striking.

3. THE STATISTICAL SIGNIFICANCE OF THE EXTINCTION–CORE MASS CORRELATION

Before proceeding further, it is important to consider the statistical significance of the relations displayed in Figure 1. The measurement errors on extinction and core mass are not independent. If the extinction to a planetary is overestimated, then its absolute emission line flux and the derived luminosity of its exciting central star will be underestimated. Since the luminosity of a young planetary nebula depends only on the mass of its core, an error in one means an error in the other. An artificial correlation between extinction and core mass can then be the result.

To test the significance of the relations displayed in Figure 1, we make the following assumptions. First, we assume that each extinction measurement has an uncertainty of $0.1$ in $c$. This is rather conservative. While Ciardullo et al. (1999) have shown that extinction estimates to Galactic PNs have a typical error of $\sigma_c \sim 0.1$, this is often because of the effects of atmospheric dispersion and the presence of nonuniformities in the distribution of dust around resolved objects. Because all of the PNs considered
here are extragalactic, the latter problem is not an issue in this analysis. Moreover, it is likely that errors due to atmospheric dispersion are also not a concern. The M31 data were taken through an atmospheric dispersion corrector, and the Magellanic Cloud PNs were all observed with the slit rotated along the direction of dispersion. An analysis of the higher order Balmer lines in the M31 planetaries suggests that for these objects, the true uncertainty in $c$ is $\sim 0.06$ (Jacoby & Ciardullo 1999). Based on external comparisons, the error in $c$ for the Magellanic Cloud PNs is likely to be similar (Meatheringham & Dopita 1991a, 1991b). Nevertheless, because the uncertainty in $c$ is of critical importance for estimating the reality of the extinction–core mass relation, we choose to be conservative and assign $\sigma_c = 0.1$.

We next assume that any error in the measurement of $c$ affects only one quantity, the derived luminosity of the central star, and that the logarithmic increase (or decrease) in the star’s luminosity is equal to $c$. This is a reasonably valid approximation: although the reaction of a given nebular model to a change in extinction is complex, to first order, the energy emitted at H$\beta$ does reflect the central star’s luminosity. Furthermore, the objects considered in this paper are all extremely bright, and their central stars should still be moving horizontally in the H–R diagram. Consequently, the derived mass of a PN core will depend only on the core’s luminosity, not on its effective temperature. Uncertainties in the latter quantity can thus be ignored.

Finally, to test the null hypothesis, we assume that extinction and central star mass are, in reality, uncorrelated. We randomly associate extinction values between 0.0 and 0.8 with central star masses between 0.56 and 0.66 $M_\odot$, and we derive the luminosities of these stars based on the Schönbérrner (1983) and Blöcker (1995b) evolutionary tracks. We then place a random (Gaussian) error of $\sigma_c = 0.1$ on the extinction, rederive the core masses using the revised luminosity, and model the observed PNs population of each galaxy via a series of Monte Carlo simulations. Using 10,000 simulations per galaxy, we ask how often our model collection of PNs would have a Spearman rank order coefficient as large or larger than that observed.

The results for all three galaxies are similar. Under the assumption of a $\sigma_c = 0.1$ measurement error, the LMC correlation is significant at the 95% confidence level, the SMC points correlate with 97% confidence, and the M31 correlation is significant at the 94% confidence level. (In other words, in $\sim 95\%$ of the trials, the Spearman rank order correlation is less than that for the real data). If the error on $c$ is reduced to $\sigma_c \approx 0.06$, which, given the quality of the data, is more likely, the significance levels for the LMC and SMC data rise to 99%, while that for M31 increases to 96%. Changes in the adopted model for the distribution of central star masses and extinctions do not change the result significantly. Thus, despite the scatter, the correlation between extinction and core mass appears to be real. It is extremely unlikely that all three correlations are an artifact of the analysis procedure.

4. MODELING THE CORRELATION

Figure 1 confirms, at least qualitatively, the technique of measuring PN core masses using nebular models and stellar evolutionary calculations. The sign of the correlation is as expected: high-mass PN central stars, which presumably come from high-mass progenitors, have large amounts of circumstellar matter and evolve quickly across the H–R diagram before their circumstellar matter has time to disperse. Thus they are heavily reddened. Low–core mass PNs have less of a circumstellar envelope and evolve blueward on much longer timescales. The extinction affecting these objects is correspondingly less.

In addition to having the correct sign, the range of extinction displayed in Figure 1 is also within expectations. Hubble Space Telescope images of bright PNs in the Magellanic Clouds show that the median radius of these objects is $\sim 0.05$ pc (Dopita et al. 1996). If we assume that the circumstellar envelopes obey a $1/r^2$ density law, extend to a radius of $\sim 0.3$ pc (Knapp, Sandell, & Robson 1993), and have a gas to color excess ratio of $\sim 5.8 \times 10^{21}$ atoms cm$^{-2}$ mag$^{-1}$ (Savage & Mathis 1979; Knapp, Sandell, & Robson 1993), then the amount of extinction associated with these objects should typically be several tenths of a magnitude. Again, this is what is observed.

The above calculation can be made more rigorous through the use of AGB and post-AGB evolutionary tracks. To do this, we use the evolutionary tracks of Blöcker (1995a, 1995b), which follow the mass loss and H–R diagram evolution of stars with initial masses of 3, 4, 5, and 7 $M_\odot$. First, we assume that the stellar mass loss of each model is spherically symmetric, and that the implied AGB wind has an expansion velocity of $\sim 15$ km s$^{-1}$. This allows us to compute the radial profile of a star’s circumstellar envelope at the time when it is becoming a bright planetary. (For the purpose of our calculation, we assume this occurs at log $T_\text{eff} \approx 5.0$.) Next, we adopt the canonical gas to color excess ratio of $\sim 5.8 \times 10^{21}$ atoms cm$^{-2}$ mag$^{-1}$ and assume that the ionization radius of our typical planetary nebula is $\sim 0.07$ pc. This gives us the circumstellar extinction expected as a function of initial mass. Finally, we use the initial mass–final mass relation implied by the Blöcker (1995a) mass-loss rates to connect the initial stellar mass to the final core mass. With this information, we can simulate the data displayed in Figure 1.

The solid line drawn in the SMC panel of Figure 1 displays the results of this calculation. As can be seen, our simple model does produce an extinction–core mass relation, but with a slope that is significantly smaller than what is observed. Given the amount of physics omitted from the computation, this is not surprising. As the stellar core moves to the blue in the H–R diagram, a fast wind develops, which compresses the surrounding medium and alters the distribution of matter close to the star. This clearly is a complexity that is beyond the scope of this paper. Moreover, type I (Population I) planetaries in the Milky Way are often bipolar (Peimbert & Torres-Peimbert 1983); hence for these objects the assumption of spherical symmetry is violated. But the most critical shortcoming of the model concerns the definition of the inner radius for extinction. In our models, this value is well defined: extinction is only applied when light has passed the optical boundary of the nebula. However, in real objects the dividing line between emission and attenuation is not so discrete, and since the greatest amount of extinction occurs at small radii (where the density of the material is highest), this oversimplification is costly. The solid line of Figure 1 was computed under the assumption that the inner radius for extinction is $0.07$ pc; this is the median nebular radius of our sample, as defined by the nebular models of Dopita & Meatheringham (1991a, 1999 CIRCUMSTELLAR EXTINCTION OF NEBULAE 193
The extinction–core mass relation offers some intriguing possibilities for probing stellar evolution.

5. THE EXTINCTION–CORE MASS RELATION AND THE PLANETARY NEBULA LUMINOSITY FUNCTION

The relation displayed in Figure 1 has an interesting consequence for the [O III] $\lambda$5007 planetary nebula luminosity function and the extragalactic distance scale. Observations of PNs in ~30 elliptical, spiral, and irregular galaxies have demonstrated that the PNLF is remarkably insensitive to stellar population (Jacoby et al. 1992; Ciardullo, Jacoby, & Tonry 1993; Feldmeier, Ciardullo, & Jacoby 1997). Models by Jacoby (1989) and Dopita, Jacoby, & Vassiliadis (1992) explain why progenitor metallicity has little effect on the PNLF, but no theory exists for the PNLF’s invariance with population age. In fact, analyses by Méndez et al. (1993), Han, Podsiadlowski, & Eggelton (1994), Jacoby (1996), and Méndez & Soffner (1997) all show that the location of the PNLF cutoff should be brighter in young star-forming populations. Yet PN surveys in the LMC and three spiral galaxies have detected no such dependence (Jacoby, Walker, & Ciardullo 1990; Feldmeier et al. 1997).

Figure 1 explains the age invariance. To first order, the strength of a bright PN’s [O III] $\lambda$5007 emission line depends on the amount of energy deposited in the nebula. Specifically, the post-AGB evolutionary models of Schönberner (1983) and Blöcker (1995b) suggest that the maximum [O III] $\lambda$5007 flux attainable by a PN should go as roughly the square of its core mass. Since the mass of a PN’s core reflects the mass of its progenitor via the initial mass–final mass relation, this seems to imply that younger populations make brighter planetaries. However, as Figure 1 shows, the increased emission from high–core mass PNs is more than made up for by the increased amount of circumstellar extinction. In fact, the slope of ~6 seen in the figure guarantees that type I planetaries will be fainter than PNs derived from older stars. This effect is not included in any of the theoretical analyses of the PNLF, and it explains why the brightest PNs in the LMC and M101 are no brighter than those found in elliptical galaxies.

Figure 2 demonstrates this effect quantitatively. The dotted line in the figure plots the predicted dependence of the PNLF cutoff with population age under the assumption that circumstellar extinction is not correlated with core mass. This calculation comes from Jacoby (1996) and assumes that a constant fraction (15%) of the PN central star’s luminosity is reprocessed into [O III] $\lambda$5007 emission (Dopita et al. 1992). According to the model, the PNLF cutoff is a sensitive function of population age, with young (~1 Gyr) stars creating planetaries that are ~1 mag brighter than PNs formed from old (~10 Gyr) stars. It is therefore in direct conflict with observations of extragalactic PNs. The solid line of Figure 2 shows the same model, except with the effects of circumstellar extinction included via the extinction–core mass relation of Figure 1 (slope of ~6). With this modification, the model fits the observational constraints extremely well. For populations older than ~1 Gyr, the PNLF is independent of age to within ~0.1 mag, as the increased extinction around higher mass stars is canceled out by the increased central star luminosity. For younger populations, the PNLF cutoff becomes fainter. However, since real galaxies contain a mix of stellar populations, the brightest PNs observed will always be those from the old and intermediate-age stars. Consequently, distance estimates based on the PNLF will not be affected by Population I planetaries.

6. DISCUSSION

Aside from explaining the age invariance of the PNLF, the extinction–core mass correlation has some other interesting consequences. The first involves the location of high–
core mass objects on the diagram of Figure 1. Ten objects fall well off the relation defined by the bulk of the planetary nebulae. These PNs are severely under-extincted for their core mass, and at first glance their presence seems to imply that the extinction law breaks down for massive central stars. However, an investigation of these objects reveals that only two have derived nebular radii smaller than the median of our sample. According to our simple model, a larger median size should translate into a shallower extinction law. This is consistent with the data.

In fact, it is relatively easy to explain the “anomalous” objects of Figure 1. High-core mass planetary nebulae are born on the main extinction-core mass relation, but their large, thick circumstellar dust cloud prevents them from being identified in extragalactic surveys. Only at a later time, when the envelope has begun to disperse and the optical nebula has grown in size, is the PN available for study. By this time the central star has completed its trip to the blue in the H-R diagram and has begun to fade. Consequently, the PN will not appear “superluminous” in an [O III] λ5007 survey. However, because the object was extremely luminous to begin with, it will still be bright enough to be detected photometrically and studied spectroscopically. It will therefore appear to be under-extincted in the extinction-core mass diagram.

We do note that, based on the galaxies’ stellar populations, we would expect to see a larger fraction of high-core mass planetary nebulae in the Magellanic Clouds than in M31. Somewhat surprisingly, this does not seem to be the case. Roughly 15% of the PNs in the LMC and M31 are high-core mass objects, while no high-core mass planetary nebulae have been identified in the SMC. The significance of this is difficult to evaluate, since none of the PN samples is in any way statistically complete. Moreover, the absence of high-core mass PNs in the SMC may be due to random chance, since, all things being equal, we would expect our sample to contain only ~2 of these objects. Since we know of no selection effect that would prevent us from seeing high-core mass PNs in the SMC, the absence of these objects is probably not significant.

Another implication of Figure 1 deals with its potential use for measuring galaxy evolution. Dopita et al. (1997) have shown that it is possible to trace the past star formation and chemical evolution of a galaxy through measurements of its planetary nebulae. One uses the nebular emission lines to determine the effective temperature and luminosity of the central star. Then, with the aid of post-AGB evolutionary tracks, one derives the PN core mass. Once the core mass is known, the age of the progenitor then follows from the initial mass–final mass relation.

The key to this procedure is obtaining the mass of the PN core, and the extinction-core mass relation is another tool for constraining this quantity. Because the scatter in extinction is large, age estimates based on extinction measurements alone will probably not be possible for individual objects. However, the mean extinction of a population of PNs might be a useful tool for estimating age, especially if the measurement is done differentially with respect to the mean of other systems. Further observations will be needed to check this possibility.

We conclude by emphasizing that in order to study the systematics of circumstellar extinction, the sample of objects must be chosen carefully. The analysis in this paper was limited to only the brightest planetary nebulae. Consequently, only newly formed objects near the peak of their UV luminosity were included in the samples. With this restriction, a correlation between extinction and core mass can be detected. However, as a PN evolves, its nebula expands, its dust envelope disperses, and its circumstellar extinction decreases. Samples of PNs that span a wide range in age will therefore not show the correlation. As a result, PN observations in the Galaxy cannot be used for investigations of this type. As in other fields of study involving planetary nebulae, the best place to investigate circumstellar extinction is in other galaxies.

R. C. wishes to thank Sidney Wolff for providing office facilities at NOAO during part of the development of this paper. Similarly, G. H. J. wishes to thank Peter Strittmatter for generously providing all office needs during a sabbatical stay at the University of Arizona. This work was supported in part by NASA grant NAG 5-3403 and NSF grants 92-57833 and 95-29270.

REFERENCES

Balick, B. 1987, AJ, 94, 671
Bloeker, T. 1995a, A&A, 297, 727
---. 1995b, A&A, 299, 755
Burstein, D., & Heiles, C. 1984, ApJS, 54, 33
Caldwell, J. R. A., & Coulson, I. M. 1985, MNRAS, 212, 879
Ciardullo, R., Jacoby, G. H., & Tonry, J. L. 1993, ApJ, 419, 479
Ciardullo, R., Rubin, V. C., Jacoby, G. H., Ford, H. C., & Ford, W. K., Jr. 1988, AJ, 95, 438
Ciardullo, R., Bond, H. E., Sipior, M. S., Fultton, L. K., Zhang, C.-Y., & Schaefer, K. G. 1999, in preparation
Dopita, M. A., et al. 1996, ApJ, 460, 320
---. 1997, ApJ, 474, 188
Dopita, M. A., Jacoby, G. H., & Vassiliadis, E. 1992, ApJ, 389, 27
Dopita, M. A., & Meatheringham, S. J. 1991a, ApJ, 367, 115
---. 1991b, ApJ, 377, 480
Elson, R. W., Sigurdsson, S., Hurley, J., Davies, M. B., & Gilmore, G. F. 1998, ApJ, 499, 153
Feldmeier, J. J., Ciardullo, R., & Jacoby, G. H. 1997, ApJ, 479, 231
Han, Z., Podsadaowski, P., & Eggleton, P. P. 1994, MNRAS, 270, 121
Harris, J., Zaritsky, D., & Thompson, I. 1997, AJ, 114, 193
Jacoby, G. H. 1989, ApJ, 339, 39
---. 1996, in IAU Symp. 180, Planetary Nebulae, ed. H. J. Habing & H. J. G. L. M. Lamers (Dordrecht: Kluwer), 448
Jacoby, G. H., & Ciardullo, R. 1999, ApJ, 515, 169
Jacoby, G. H., et al. 1992, PASP, 104, 599
Jacoby, G. H., & Kaler, J. B. 1993, ApJ, 417, 209
Jacoby, G. H., Morse, J. A., Fulton, L. K., Kwitter, K. B., & Henry, R. B. C. 1997, AJ, 114, 261
Jacob, G. H., Walker, A. R., & Ciardullo, R. 1990, ApJ, 365, 471
---. 1991, ApJ, 362, 491
---. 1991, ApJ, 382, 134
Kaler, J. B., & Lutz, J. H. 1985, PASP, 97, 700
Knapp, G. R., Sandell, G., & Robson, E. I. 1993, ApJS, 88, 173
Kwok, S. 1993, ARA&A, 31, 63
---. 1994, PASP, 106, 344
Meatheringham, S. J., & Dopita, M. A. 1991a, ApJS, 75, 407
---. 1991b, ApJS, 76, 1085
Méndez, R. H., Kudritzki, R. P., Ciardullo, R., & Jacoby, G. H. 1993, A&A, 275, 534
Méndez, R. H., & Soffner, T. 1997, A&A, 321, 898
Peimbert, M., & Torres-Peimbert, S. 1983, in IAU Symp. 103, Planetary Nebulae, ed. D. R. Flower (Dordrecht: Reidel), 233
Pottasch, S. R. 1984, Planetary Nebulae (Dordrecht: Reidel)
Savage, B. D., & Mathis, J. S. 1979, ARA&A, 17, 73
Schönberner, D. 1983, A&A, 125, 708
Whitford, A. E. 1958, AJ, 63, 201