Method for simultaneous estimation of rolling and spinning friction in a higher pair

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Abstract. A spatial higher pair is characterized by a complex motion with the direct consequence: the occurrence in the contact point of a friction torsor having all possible components (sliding, spinning and rolling friction). There are cases, like thrust ball bearings, where the components of the friction torsor are interrelated, and actual separation of these is not possible. The present paper proposes an approach for simultaneous evaluation of the coefficient of rolling friction and of the coefficient of spinning friction using a cycloidal pendulum. As principle, the experimental device consists in a steel ball set on top of two cylindrical steel rods, parallel, mounted horizontally. When a vertical rod is attached to the ball, a cycloidal pendulum is obtained. The damping of oscillatory motion is due to the two friction torques: spinning and rolling. Relevant articles in literature searches, great care should be taken in constructing both. The motion of the pendulum is analyzed for two values of the gap between the horizontal rods. A system of two equations with the coefficients of rolling and spinning friction as unknowns is obtained. Based on the hypothesis that the dependencies for the torques versus the normal force are known, these two coefficients can be obtained.

1. Introduction
In technological application there are often met cases when the contact between two elements is theoretically made in a point and therefore the relative motion between the two elements has the most complex possible form [1]. In this situation, between the contacting parts, relative sliding characterized by the sliding velocity, contained in the common tangent plane and relative rotation, characterized by relative rotation velocity \( \dot{\omega} \) occur. Depending on the relative position of the relative angular velocity with respect to the normal in the contact point, two limit cases are defined: spinning motion, when the relative angular velocity is parallel to the normal and rolling motion, when the rotation velocity is contained in the common tangent plane [2]. Between the two limit cases, all combinations when there are simultaneously met both spinning and rolling motion may take place. To the two types of angular velocities, torques (parallel and of opposite sign) will oppose; these friction torques take the corresponding name: spinning moment and rolling moment. The two moments are each one responsible of the tendency of decreasing the corresponding angular velocity. It is obvious that the correct evaluation of the magnitude of the two moments is one of the essential requirements in optimal design of any mobile mechanical structure. The main difficulty in reaching this task consists...
in the fact that the values of these moments are extremely reduced, and so the errors introduced by the measuring instruments can be greater than the parameters to be measured [3-4]. From here it results the necessity of conceiving devices as simple as possible and with the mobile bodies having technological shapes. One of the main class of methods employed in the study of the rolling friction are the pendula. The main characteristic of these devices is the fact that the effect of rolling friction has a summative effect upon their motion, and thus, after an adequate time, the effect of rolling friction becomes perceptible [5-7]. As expected, the conclusion that it is appropriate to use a pendulum for the estimation of combined effect of spinning friction and rolling friction is reached.

2. Proposed pendulum for the study of simultaneous effect of spinning and rolling friction

The pendulum proposed here is made from a steel ball of $R$ radius to which a rod of circular cross section is attached and the whole mass of the pendulum is $M$, the moment of inertia with respect to an axis normal to the axis of the rod passing through the centre of mass $G$ is $J_Z$ and the centre of mass $G$ is positioned on the axis of symmetry of the pendulum at a distance $\xi$ from the center of the ball. the pendulum is brought into contact with two cylindrical steel beams, parallel to each other and positioned in an horizontal plane. The two contacts ball-beams take place in the points $C_{1,2}$.

Two coordinate systems are chosen in order to obtain the equation of motion: an immobile one $Oxyz$ and a mobile one $Ox'y'z'$ attached to the pendulum, oriented as shown in figure 1. The position of the pendulum is stipulated by two parameters: the ordinate of the center of mass of the ball and the angle of rotation of the pendulum, $\varphi$. Assuming that there is no sliding between the surfaces of the ball and of the rods in the two contact points, the horizontal straight line passing through the two points is the instantaneous rotation axis and consequently the pendulum has pure rolling motion with respect to the two rods. In each of the $C_{1,2}$ contact points the normal direction and the tangent plane are well defined. By projecting the angular velocity $\omega$ on the normal and on the tangent plane, the angular spinning velocities $\omega_{sp}^{1,2}$ and angular rolling velocities $\omega_{r}^{1,2}$ are obtained, respectively.

A friction torsor occurs in each of the two contact points, consisting of: the normal reaction $N_{1,2}$ directed along the normal, the spinning friction torques $M_{sp}^{1,2}$, oriented oppositely to the spinning velocities $\omega_{sp}^{1,2}$ together to the rolling friction torques $M_{r}^{1,2}$ parallel to the angular rolling velocities $\omega_{r}^{1,2}$. Beside these moments, a friction force $T_{1,2}$ parallel to the direction of motion of the center of mass occurs in each of the two contact points, given by the relation:

$$T_{1,2} < \mu N_{1,2}$$

which is the condition of pure rolling. By accepting the pure rolling condition in the two contact points, from the two parameters $y$ and $\varphi$, used in stipulating the position of the pendulum only one remains independent, as the following relation exists between them:

$$y_{1,2} = -r_c \varphi$$

In the relation (2) $r_c$ is the radius of the pitch circle:

$$r_c = R \cos \beta$$

where the angle $\beta$ is a constructive characteristic of the experimental device given by:

$$\beta = \arcsin \left[ \frac{d}{(R+r)} \right]$$

and defined as the angle made by the plane containing the center of the ball and the axis of the rod and the vertical direction.

2
3. Obtaining the equation of motion. Discussions

The components of the acceleration of the center of mass of the pendulum are found by differentiating
with respect to time, twice, the relations (5):

\[
\begin{align*}
  a_Gx &= -\xi \phi^2 \cos \varphi - \xi \dot{\phi} \sin \varphi \\
  a_Gy &= (\xi \cos \varphi - r_c) \dot{\phi} - \xi \dot{\phi}^2 \sin \varphi
\end{align*}
\]  

(6)

The position vectors of the contact points are:

\[
\mathbf{r}_{c1} = \left[ R \cos \beta, -\varphi r_c, \pm R \sin \beta \right]^T
\]

(7)

These relations are used to find the position vectors of the contact points with respect to the centers of mass, required for applying the moment of momentum theorem.

\[
\overline{\mathbf{GC}}^{12} = \mathbf{r}_{c1}^{12} - \mathbf{r}_G = \left[ R \cos \beta - \xi \cos \varphi, \pm \xi \cos \varphi, R \sin \beta \right]^T
\]

(8)

The components of the rolling friction torque:

\[
\mathbf{M}_r^{12} = \left[ \pm M_r \sin \beta, 0, -M_r \cos \beta \right]^T
\]

(9)

and for the spinning friction torque:

\[
\mathbf{M}_{sp}^{12} = \left[ \mp M_{sp} \cos \varphi, 0, -M_{sp} \sin \beta \right]^T
\]

(10)

The theorem of motion of the center of mass is required to solve the problem and it is applied as:

\[
\mathbf{M}a_G = \mathbf{G} + \mathbf{T} \times \mathbf{r}^{12} + \mathbf{N}^{12} + \mathbf{N}^{21},
\]

(11)

together with the moment of momentum theorem, written with respect to the center of mass:

\[
\begin{bmatrix} 0 & 0 \end{bmatrix} \mathbf{J}_z \ddot{\phi} = \overline{\mathbf{GC}} \times \left( \mathbf{N}^{1} + \mathbf{T}^{1} \right) + \overline{\mathbf{GC}}^{2} \times \left( \mathbf{N}^{2} + \mathbf{T}^{2} \right) + \left( \mathbf{M}_r^{1} + \mathbf{M}_{sp}^{1} + \mathbf{M}_r^{2} + \mathbf{M}_{sp}^{2} \right).
\]

(12)

Due to symmetry, all homologous components of the friction torsors have equal moduli. The equations (11) and (12) provide three scalar equations. The unknowns of the problem are: the $\varphi$ angle that characterizes the motion and $N$, $T$, $M_{sp}$ and $M_r$. Two more equations are necessary for a compatible problem, represented by the constitutive equations which describe the relation between the magnitudes of the spinning and rolling friction moments and normal force. Next, it is accepted that all friction torques are proportional to the magnitude of the normal force $N$, described by [8]:

\[
M_{sp,1,2} = s_{sp} N
\]

(13)

\[
M_{sr,1,2} = s_r N
\]

(14)

where $s_{sp}$ and $s_r$ are the coefficients of spinning and rolling friction respectively, both with dimension of length. The equations (11-14) may now be solved and the differential equation is found:

\[
\ddot{\varphi} = -\left( \frac{s_{sp} \sin \beta + s_r \cos \beta}{\xi} \right) \text{sgn}(\dot{\phi}) \cos \varphi + \frac{R}{\xi} \sin \varphi \cos^2 \beta + \left( \frac{s_{sp} \sin \beta + s_r \cos \beta}{\xi} \right) \text{sgn}(\dot{\phi}) \sin \varphi + \left( \frac{\xi}{\zeta} - \cos \varphi \right) \frac{R}{\xi} \cos^2 \beta + \left( \frac{1}{M_2} \frac{\zeta}{\xi} - \frac{R}{\xi} \cos \varphi \right) \cos \beta \left( \frac{\xi}{\zeta} + \phi^2 \right)
\]

(15)
The equation (15) is a nonlinear differential equation of second order; the solutions of which cannot be analytically found [9], hence the necessity of applying a numerical procedure of integration. A more thorough analysis of the equation (15) proves that the two friction coefficients do not occur as independent parameters, but under the form of an expression. Therefore, it is expected to use the notation:

\[ s_g = (s_{\varphi} \sin \beta + s_r \cos \beta) \] (16)

representing a global friction coefficient with dimension of length. The relation (16) shows that for small values of \( \beta \) angle the most important fraction belongs to the rolling friction coefficient while for greater values of \( \beta \) angle the parts reverse and the spinning friction is predominant. Using this expression, the equation (15) becomes:

\[
\ddot{\phi} = - \frac{R}{g} \sin \varphi \cos^2 \beta + \frac{s_g}{g} \text{sgn}(\phi) \cos \varphi \left( \frac{r_c}{\xi} - \cos \phi \right) \frac{R}{g} \cos^2 \beta + \left( 1 + \frac{J}{M \xi^2} - \frac{r_c}{\xi} \cos \phi \right) \cos \beta + \frac{s_g}{g} \text{sgn}(\phi) \sin \varphi \] (17)

The numerical algorithm Runge-Kutta IV [10] was applied for the integration of differential equation (17) for a set of values of the involved parameters. The variation of the angular amplitude of the pendulum for free oscillations, with initial amplitude \( \varphi_0 \), is presented in figure 3. The plot shows linear decrease with time of the amplitude of the pendulum - a characteristic of damping caused by dry friction [11-12]. Once the motion of the pendulum is known by means of the dependency \( \varphi = \varphi(t) \), the variations with time of the magnitude of the friction force \( T \) and of the normal reaction can be found and are expressed by the relations:

\[
T = M \left( \xi \ddot{\varphi} \sin \varphi + \xi^2 \dot{\varphi}^2 \sin \varphi + \xi \sin \varphi \right) / 2 \\
N = M \left( \xi \ddot{\varphi} \sin \varphi + \xi^2 \dot{\varphi}^2 \cos \varphi + g \cos \varphi \right) / (2 \cos \beta) 
\] (18) (19)

With these dependencies known, the hypothesis of pure rolling in the contacts of the pendulum can now be verified. To this end, the dependency of the ratio \( T / N \) (representing the static friction coefficient) is presented in figure 4.

**Figure 3.** Variation of the angular amplitude of the pendulum.

**Figure 4.** Verifying the pure rolling existence.

4. Conclusions
The paper proposes a method for concomitant finding of the coefficient of rolling friction and the coefficient of spinning friction under the assumption that both torques, of rolling friction and of
spinning friction, are proportional to the value of the normal force. The method consists in using a cycloidal pendulum particularly designed. Specifically, the pendulum is made of a rod attached to a bearing ball. At its turn, the ball is placed on top of two cylindrical parallel beams, placed in an horizontal plane. The instantaneous axis of rotation of the pendulum is the horizontal line passing through the contact points between the ball and the beams. The normal directions in the contact point are not directed along the horizontal or vertical directions and hence the conclusion that in the contact points both rolling friction and spinning friction torques occur. The equation of motion of the pendulum is found accepting the hypothesis that pure rolling exists in the contact points. The equation obtained is a nonlinear of second order differential one and a numerical procedure must be applied for solving it. The equation is integrated for a set of parameters characteristic to the dynamical system and a linear attenuation of the amplitude of the pendulum is observed, fact that is an attribute of dry friction. The amplitude damping obviously depends on the values of the two friction coefficients asked for. The principle of the method assumes finding the velocity of angular amplitude decrease of the pendulum for two distances between the axes of the beams; thus, a system of two linear equations with the two coefficients of friction as unknowns is obtained. The method may be extended for the case when another type of dependency between the friction torques and the normal reaction is accepted.

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