The Meson Spectrum of the BCC Quark Model

(A Modification of the Quark Model)

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Abstract

Using the quark spectrum of the BCC Quark Model and the phenomenological formula for the binding energies of the mesons, not only have we deduced the intrinsic quantum numbers (I, S, C, b, and Q) of all mesons as was done with the Quark Model, but also we deduced the meson mass spectrum in agreement with experimental results that we could not deduce using the Quark Model. The experimental meson spectrum gives some evidence of the existence of the new quarks $q_s^*(1391)$, $q_s^*(2551)$, and $q_C^*(6591)$..., which are predicted by the BCC Quark Model. The meson $\chi(1600) [2^+(2^{++})]$ with $I = 2$ (predicted by the BCC Quark Model–$T(1603)$) has already been discovered. If this is finally confirmed, it will provide a strong support for the BCC Quark Model. We propose a search for the mesons $D(5996)$, $D_S(6151)$, $B(9504)$, $B_S(9659)$, $B_C(11031)$, $\eta(5926)$, $\eta(17837)$, $\psi(25596)$, $\Upsilon(17805)$, $\Upsilon(29597)$, $T(960)$, $T(1282)$, $T(1603)$, and $T(1924)$. 
I Introduction

According to the Quark Model [2], a meson is composed of a quark and an antiquark. Although the Quark Model assumes many elementary quarks (6 flavors, 3 colors—a total of 18 elementary quarks), in fact, those quarks are still not enough to explain the full experimental meson spectrum, as demonstrated in lists (1) and (2) [5] below:

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
N_{2s+1}L_J & J^{PC} & u\bar{d},u\bar{u},d\bar{d} & u\bar{u},d\bar{d},s\bar{s} & c\bar{c} & b\bar{b} & \bar{s}u,\bar{s}d \\
\hline
1^1 S_0 & 0^- & \pi(139) & \eta(549), \eta'(958) & \eta_c(2980) & \text{K}(494) & \\
1^3 S_1 & 1^- & \rho(770) & \omega(782), \phi(1020) & J/\psi(3097) & \Upsilon(9460) & \text{K}^*(892) \\
1^1 P_1 & 1^+ & b_1(1235) & h_1(1170), h_1(1380) & h_\psi(1p) & \text{K}_{1B}^*(1270) & \\
1^3 P_0 & 0^++ & a_0(1450)^* & f_0(1370)^*, f_0(1710)^* & \chi_{c0}(1p) & \chi_{b0}(1p) & \text{K}^*_0(1430) \\
1^3 P_1 & 1^+ & a_1(1260) & f_1(1285), f_1(1420) & \chi_{c1}(1p) & \chi_{b1}(1p) & \text{K}^*_1(1430) \\
1^3 P_2 & 2^++ & a_2(1320) & f_2(1270), f'_2(1525) & \chi_{c2}(1p) & \chi_{b2}(1p) & \text{K}^*_2(1430) \\
1^1 D_2 & 2^- & \pi_2(1670) & \eta_2(1645), \eta_2(1870) & \text{K}_2(1770) & \\
1^3 D_1 & 1^- & \rho(1700) & \omega(1650) & \psi(3770) & \text{K}^*(1680)\dagger \\
1^3 D_2 & 2^- & & & \text{K}_2(1820) & \\
1^3 D_3 & 3^- & \rho_3(1690) & \omega_3(1670), \phi_3(1850) & \text{K}^*_3(1780) & \\
1^3 F_4 & 4^++ & \alpha_4(2040) & f_4(2050), f_4(2220) & \text{K}^*_4(2045) & \\
2^1 S_0 & 0^- & \pi(1300) & \eta(1295), \eta(1440) & \eta_c(2S) & \text{K}(1460) \\
2^3 S_1 & 1^- & \rho(1450) & \omega(1420), \phi(1680) & \psi(2S) & \Upsilon(2S) & \text{K}^*(1410) \\
2^3 P_2 & 2^++ & f_2(1810), f_2(2010) & & \chi_{b2}(2p) & \text{K}^*_2(1980) \\
3^1 S_0 & 0^- & \pi(1800) & \eta(1760) & & \text{K}(1830) \\
\hline
\end{array}
\]

(1)
\[ N^{2s+1} L_J \]  \[ J^PC \]  \[ c\bar{u}, c\bar{d} \]  \[ c\bar{s} \]  \[ \bar{b}u, \bar{b}d \]  \[ \bar{b}s \]  \[ \bar{b}c \]  

\begin{array}{|c|c|c|c|c|c|}
\hline
\text{1}\text{1} & S_0 & 0^- & D(1869) & D_s(1969) & B(5279) & B_s(5370) & B_c(6400) \\
\text{1}\text{3} & S_1 & 1^- & D^*(2010) & D_s^*(2112) & B^*(5325) & B_s^*(5850) \\
\text{1}\text{1} & P_1 & 1^+ & D_1(2420) & D_{s1}(2536) & & & \\
\text{1}\text{3} & P_0 & 0^{++} & & & & & \\
\text{1}\text{3} & P_1 & 1^{++} & & & & & \\
\text{1}\text{3} & P_2 & 2^{++} & D^*_2(2460) & & & & \\
\hline
\end{array}

A. In list (1), in order to explain the masses of the light unflavored mesons (\(\eta, \omega, \phi, h, \text{ and } f\)) with \(I = 0\), the Quark Model has to give up the principle that a meson is made of a single quark and a single antiquark, and allow a meson to be a mixture of three quark-antiquark pairs (\(u\bar{u}, d\bar{d}, \text{ and } s\bar{s}\)). (Note: the meson is not a superposition of three quark-antiquark pairs (\(u\bar{u}, d\bar{d}, \text{ and } s\bar{s}\)) because the three pairs are independent elementary particle pairs in the Quark Model). For example:

\[
\eta(549) = \eta_s \cos \theta_p - \eta_1 \sin \theta_p, \\
\eta'(958) = \eta_s \sin \theta_p + \eta_1 \cos \theta_p, \\
\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}, \\
\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}. 
\]

This “mixture” violates the principle that a meson is made of a quark and an antiquark. It also makes the model need too many parameters by introducing the parameter \(\theta_p\). There are more than twenty such mixture mesons (\(\eta(549), \eta'(958), \phi(1020) h_1(1170)...\phi_3(1850), f_4(2220)\)) (see (I)). However, the BCC Quark Model, as a modification of the Quark Model, needs only two elementary quarks, \(u\) (3-colors) and \(d\) (3-colors), and deduces an excited (from the vacuum) quark spectrum (including S, C, b, I, Q, and M) (I). This quark spectrum alone is enough to structure the full meson spectrum (a meson is made of a quark and an antiquark). The BCC Quark Model does
not need the mixture. For example:

\[ q_S^*(1111)q_S^*(1111) = \eta(549) [\bullet \eta(549)] \ (u\bar{u}, d\bar{d}, s\bar{s} \text{ mixture in Quark Model}), \]
\[ q_S^*(1391)q_S^*(1391) = \eta(952) [\bullet \eta(958)] \ (u\bar{u}, d\bar{d}, s\bar{s} \text{ mixture in Quark Model}), \]
\[ q_N^*(931)q_N^*(1471) = \eta(1021) [\bullet \phi(1020)] \ (u\bar{u}, d\bar{d}, s\bar{s} \text{ mixture in Quark Model}). \] (4)

B. In list (1), there are more than 15 K mesons (K(494), K*(892), K_1(1270)... K_4*(2045)... K_4(2500)) (see (1)), but only one strange quark (s) and two quarks (u and d) can form the K mesons in the Quark Model. Thus, the Quark Model has to assume that the K mesons are all \( u\bar{s} \) and \( d\bar{s} \) with different angular momenta (J) and parities (P). These types of assumptions bring trouble. For example, the Quark Model assumes that \( \eta_C(2969) \) (\( \Gamma = 13.2 \text{ Mev} \)) and \( J/\psi(3097) \) (\( \Gamma = 87Kev \)) are \( c\bar{c} \) mesons [5]. However, the ground state \( \eta_C(2969) \) has a shorter life time (13.2 Mev) than the excited state \( J/\psi(3097) \) (\( \Gamma = 87Kev \)). This is contrary to a physical law—the ground state has the longest lifetime. Since there is a series of the quarks \( q_S^*(1111), q_S^*(1391), q_S^*(2011) \... \) in the BCC Quark Model [1]. Therefore, the BCC Quark Model can present different K mesons using different quark pairs. For example:

\[ q_N^*(931)q_S^*(1111) = K(494) [\bullet K(494)] \leftrightarrow \bar{s}u, \bar{s}d \ (\text{Quark Model}), \]
\[ q_N^*(931)q_S^*(1391) = K(899) [\bullet K(892)] \leftrightarrow \bar{s}u, \bar{s}d \ (\text{Quark Model}), \]
\[ q_N^*(931)q_S^*(2551) = K(1804) [\bullet K(1820)] \leftrightarrow \bar{s}u, \bar{s}d \ (\text{Quark Model}). \] (5)

Mesons and quark pairs are essentially one-to-one correspondence in the BCC Quark Model. Although sometimes there is a meson that is a superposition of two quark-antiquark pairs, it is not a mixture of different elementary quark pairs, but only a superposition of different excited states of the same elementary quarks, u and d, in the BCC Quark Model.

C. Although the Quark Model can deduce the quantum numbers of the meson spectrum, it cannot deduce the mass spectrum for mesons. It can only give some mass relations (Gell-Mann–Okubo) in an Octet [6].
D. The binding energy of a quark and an antiquark in a meson has not been discussed clearly in the Quark Model. There is not a formula for the binding energy in the Quark Model. According to the quark masses [8] of the Quark Model, \( m_u = 1.5 \) to 5 Mev and \( m_d = 3 \) to 9 Mev, \( \pi^+ (ud) \) and \( \pi^- (d\bar{u}) \) have the binding energy

\[
E(\pi^\pm)_{\text{bind}} = M_{\pi^\pm} - (m_u + m_d) = [139 - (4.5 \text{ to } 14)] \text{ Mev} > 0. \tag{6}
\]

From \( E = MC^2 \), the mesons \( \pi^\pm \) cannot be formed, because if the two quarks (\( u \) and \( \bar{d} \)) separate into two individual quarks, the system will have a much lower energy than the energy of the meson \( \pi^\pm \). This (6) means that the interactive force between the two quarks (\( u \) and \( \bar{d} \)) is repulsive. Similarly, the proton (\( uud \)) has a binding energy

\[
E(p)_{\text{bind}} = M_{\text{proton}} - (m_u + m_u + m_d) = [939 - (6 \text{ to } 19)] \text{ Mev} > 0. \tag{7}
\]

This (7) means that the interactive forces among the three quarks are repulsive with respect to one another. Thus, the three quarks cannot make a stable proton. However, the proton is absolutely a stable particle. The A-bomb and H-bomb tests have already shown that the formula \( E = MC^2 \) is completely right. The small masses of the quarks (in the Quark Model) not only will destroy the confinement theory [9], but also are not enough to construct the stable baryons (7) and the mesons (6).

Therefore, the Quark Model needs modification and development. The BCC Quark Model is a good modification and development of the Quark Model. It needs only two elementary quarks, \( u \) (3 colors) and \( d \) (3 colors), and deduces an excited (from vacuum) quark spectrum (including S, C, b, I, Q, and M) [10]. The quark spectrum is enough to construct the full meson spectrum using the principle that a meson is made of a single quark and a single antiquark. The masses of the quarks (in the BCC Quark Model) are large enough to construct the stable mesons and baryons. The binding energies of the mesons are essentially a constant (-1723Mev).

This paper is organized as follows: The quantum numbers of the mesons are presented in Section II. The probability that a quark and an antiquark form a meson is discussed...
in Section III. The binding energies of the mesons are introduced in Section IV. A comparison of the results of the BCC Quark Model and experimental results is listed in Section V. The evidence of some new quarks is given in Section VI. The predictions and experimental verifications of the BCC Quark Model are stated in Section VII. The conclusions are in Section VIII.
II The Quantum Numbers of the Mesons

First, we list the quark spectrum of the BCC Quark Model in (8) and (9) [10]:

| d q(m) | d q(m) | d q(m) | d q(m) |
|--------|--------|--------|--------|
| 1 q_N^*(931)^# | 1 q_S^*(1111)^# | 1 q_C^*(2271)^# | 1 q_b^*(5531)^# |
| (u_{931}, d_{931}) | 1q_S^*(1391) 2q_{151}(1291) | 1q_C^*(2441) 1q_b^*(9951) | |
| 1q_N(1201) | 1q_S^*(2011) 3q_{151}(1471) | 1q_C^*(2961) | 1q_b^*(15811) |
| 1q_N(1471) | 1q_S^*(2451) 2q_{151}(1651) | 1q_C^*(6591) | ... |
| 2q_N(1831) | 1q_S^*(2551) 2q_{151}(1831) | 1q_C^*(13791) | ...
| 4q_N(1921) | 3q_S^*(2641) 1q_{151}(1921) | ... | ...
| 2q_N(2191) | 1q_S^*(2731) 2q_{151}(2191) | 1q_{151}(2441) | ...
| 2q_N(2551)^† | ... | 2q_{151}(2191) | 1q_{151}(2531)
| 3q_N(2641) | 1q_{151}(1201) 2q_{151}(2371) | 1q_{151}(2961) | ...
| 2q_N(2731) | 3q_{151}(1651) 3q_{151}(2551) | ... | ...
| ... | 5q_{151}(1921) 8q_{151}(2731) | 1q_{151}(2541) | ...
| 1q_{151}(1291) | 2q_{151}(2011) | ... | 1q_{151}(2541)
| 2q_{151}(1651) | 1q_{151}(2371) 1q_{151}(1651) | 1q_{151}(3161) | ...
| 1q_{151}(2011) | 3q_{151}(2551) 1q_{151}(2451) | ... | ...
| 1q_{151}(2371) | 2q_{151}(2641) 1q_{151}(3071) | ... | ...
| 4q_{151}(2731) | 3q_{151}(2731) 1q_{151}(3711) | 1q_{151}(2561) | ...
| 3q_{151}(3091) | 5q_{151}(3091) | ... | 1q_{151}(3471)
| ... | ... | ... | ...

Where d is degeneracy.

Since q_N^*(1291) has asymmetric n numbers \{n_1 = (0, 0, 2), n_2 = (-1,0,1), n_3 = (0, -1, 1)\} \Rightarrow \Lambda(1391) \text{ [n_1 = (0, 0, 2)]} and \Delta(1291) \text{ [n_1 = (1, 0, 1), n_2 = (-1,0,1), n_3= (0,1,1), n_4 = (0, -1, 1)]}\text{ [see Appendix I]}, we have removed it from the above list.

# The five ground quarks q_N^*(931) [q_u^*(931), q_d^*(931)], q_S^*(1111), q_C^*(2271), and q_b^*(5531) are the five quarks (u, d, s, c, and b) of the Quark Model. They have the same quantum numbers, but different masses. The mass differences of the same kind of quarks are essentially a constant (m_q_N^*-m_{u,d}=935, 7
m_q - m_s = 997, m_q - m_c = 1030, m_q^* - m_b = 1340), which roughly is the mass of a proton. If we use only the five ground quarks to form the mesons, we get the results of the Quark Model.

\[ 2q_N^*(2551) \equiv q_N^*(2541) + q_N^*(2551) \]

The Quantum Numbers of the Quarks

| q^*q | q_N^* | q_N^* | q_\Delta^* | q_\Delta^* | q_\Delta^* | q_\Delta^* | q_\Delta^* | q_\Delta^* | q_\Delta^* | q_\Delta^* | q_\Delta^* | q_\Delta^* |
|------|-------|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| S    | 0     | 0     | 0           | 0           | 0           | 0           | -1          | 0           | 0           | -2          | -2          |             |
| C    | 0     | 0     | 0           | 0           | 0           | 0           | 0           | 1           | 0           | 0           | 0           |             |
| b    | 0     | 0     | 0           | 0           | 0           | 0           | 0           | 0           | -1          | 0           | 0           |             |
| I    | 1/2   | 1/2   | 3/2         | 3/2         | 3/2         | 3/2         | 0           | 0           | 0           | 1/2         | 1/2         |             |
| I_Z  | 1/2   | -1/2  | 3/2         | 1/2         | -1/2        | -3/2        | 0           | 0           | 0           | 1/2         | -1/2        |             |
| Q    | 2/3   | -1/3  | 5/3         | 2/3         | -1/3        | -4/3        | -1/3        | 2/3         | -1/3        | -1/3        | 4/3         |             |

Then, we list the formula of the quantum number sum laws of the quark pairs:

\[ B_{q\bar{q}} = B_q + B_{\bar{q}}, \quad S_{q\bar{q}} = S_q + S_{\bar{q}}, \quad C_{q\bar{q}} = C_q + C_{\bar{q}}, \]
\[ b_{q\bar{q}} = b_q + b_{\bar{q}}, \quad Q_{q\bar{q}} = Q_q + Q_{\bar{q}}, \quad I_{q\bar{q}} = I_q + I_{\bar{q}}. \]  

Using the quark quantum numbers (9) and the sum law (10), we can find the quantum numbers (S, C, b, I, Q,...) of the quark pairs (q_i q_j). We list S, C, b, I, Q of the quark...
pairs \( (\bar{q}_i q_j) \) as follows:

| \( \bar{q}_N q_N \) | S | C | b | I; Q; Meson | \( q_i^* q_j^* \) | S | C | b | I; Q; Meson |
|-------------------|---|----|---|---------------|----------------|---|----|---|---------------|
| \( q_N^* q_N^* \) | 0 | 0 | 0 | 0; 0; \( \eta \) | \( q_N^* q_N^* \) | 0 | 1 | 1 | 0; 1; \( B_C \) |
| \( q_N^* q_S \) | 0 | 0 | 0 | 0; 0; \( \eta \) | \( q_C^* q_S \) | 1 | 1 | 0 | 1; 2, 1, 0; \( D_S \) |
| \( q_C^* q_C \) | 0 | 0 | 0 | 0; 0; \( \psi \) | \( q_C^* q_C \) | 2 | 1 | 0 | \( \frac{1}{2}, 2, 1; D_S \) |
| \( q_b^* q_b \) | 0 | 0 | 0 | 0; 0; \( \Upsilon \) | \( q_S^* q_S \) | 3 | 1 | 0 | 0; 2; \( D_S \) |
| \( q_S^* q_S \) | 0 | 0 | 0 | 0; 0; \( \eta \) | \( q_S^* q_S \) | 0 | 0 | 0 | 0; 0; \( \eta \) |
| \( q_b^* q_S \) | 0 | 0 | 0 | 0; 0; \( \eta \) | \( q_b^* q_S \) | 1 | 0 | -1 | 1; 1; 0, -1; \( B_S \) |
| \( q_N^* q_C \) | 1 | 0 | 0 | \( \frac{1}{2}; 1, 0; K \) | \( q_N^* q_C \) | 3 | 0 | -1 | 0; 1; \( B_S \) |
| \( q_N^* q_C \) | 0 | 1 | 0 | \( \frac{1}{2}; 1, 0; D \) | \( q_N^* q_C \) | 2 | 0 | 0 | 1; 2; 1, 0; \( \pi_S \) |
| \( q_N^* q_C \) | 0 | 0 | 1 | \( \frac{1}{2}; 1, 0; B \) | \( q_N^* q_C \) | 1 | 0 | 0 | \( \frac{1}{2}; 1; 0, K \) |
| \( q_N^* q_C \) | 1 | 0 | 0 | \( \frac{1}{2}; 1, 0; K \) | \( q_N^* q_C \) | 1 | 0 | 0 | \( \frac{1}{2}; 1, 0; K \) |
| \( q_N^* q_C \) | 2 | 0 | 0 | 0; 1; \( \pi \) | \( q_N^* q_C \) | 0 | 0 | 0 | 1; 1; 0, -1; \( \pi \) |
| \( q_N^* q_C \) | 3 | 0 | 0 | \( \frac{1}{2}; 2, 1, 0, -1; \) | \( q_N^* q_C \) | 1 | 0 | 0 | 0; 0; \( \eta \) |
| \( q_S^* q_S \) | 1 | 1 | 0 | 0; 1; \( D_S \) | \( q_S^* q_S \) | 0 | 1 | 0 | \( \frac{3}{2}, 2, 1, 0, -1; F_C \) |
| \( q_b^* q_S \) | -1 | 0 | 1 | 0; 0; \( B_S \) | \( q_b^* q_S \) | 0 | 0 | 1 | \( \frac{3}{2}, 2, 1, 0, -1; F_b \) |
| \( q_S^* q_S \) | 0 | 0 | 0 | 1; 1, 0, -1; \( \pi \) | \( q_S^* q_S \) | 1 | 0 | 0 | \( \frac{1}{2}; 1, 0; K \) |
| \( q_S^* q_S \) | 1 | 0 | 0 | \( \frac{1}{2}; 1, 0; K \) | \( q_S^* q_S \) | 2 | 0 | 0 | \( \frac{3}{2}, 2, 1, 0, -1; F \) |
| \( q_S^* q_S \) | 2 | 0 | 0 | 0; 1; \( \eta \) | \( q_S^* q_S \) | 3 | 0 | 0 | \( \frac{3}{2}, 3, 2, 1, 0; F_{\Omega} \) |

Since the quarks \( (q_{cc}^*, q_{\Xi_c}^*, \text{and } q_{\Omega_c}^*) \) cannot form a meson (see next section (14) and (24)), they are omitted.
Although we have deduced the quantum numbers of the quark pairs \((q_i \bar{q}_j^*)\), there is only a possibility that the quark pair \((q_i \bar{q}_j^*)\) may form a meson with these quantum numbers. However, whether a quark pair \((q_i \bar{q}_j^*)\) really forms a meson (which can be observed by experiment) depends on a probability of formation.

### III The Probability That a Quark and an Antiquark Form a Meson

Since the quarks are born from different symmetry axes and symmetry points with different quantum numbers and masses, the probability that a quark and an antiquark form a meson is not the same for different quark pairs. We need to find the laws for the probability.

### A The Probability of Producing a Quark From the Vacuum

First, we shall find the probability of producing a quark, \(q^*_J\), from the vacuum.

A1. According to the BCC Quark Model [1] (see Fig. 1 of Appendix I), the \(q^*_\Delta\) and \(q^*_N\) quarks with \(S = 0\) are born on the \(\Delta\)-axis (8 symmetry operations) and the D-axis; the \(q^*_\Sigma\) quarks are born on the \(\Delta\)-axis (single energy bands from \(\Delta S = +1\)) at the point \(\Gamma\) (see Fig. 5 (b)); the \(q^*_S\) quarks \((q^*_S(1391), q^*_S(4271), \ldots)\) are born on the \(\Delta\)-axis (single energy bands from \(\Delta S = -1\)) at the point \(H\) (see Fig. 5 (b)). The \(q^*_S\) and \(q^*_\Sigma\) quarks with \(S = -1\) are born on the \(\Lambda\)-axis (6 symmetry operations) (see Fig. 2 (b)) and the \(F\)-axis (see Fig. 4 (a)). The \(q^*_\Xi\) quarks are born on the \(\Sigma\)-axis (4 symmetry operations) (see Fig. 3 (a)) and the \(G\)-axis (4 symmetry operations) (see Fig. 4 (b)). The \(q^*_S\) quarks \((q^*_S(1111), q^*_S(2551))\) and the \(q^*_b\) quarks \((q^*_b(5531), q^*_b(9951), q^*_b(15811))\) are born on the \(\Sigma\)-axis (single energy bands from \(\Delta S = +1\)) at the point \(N\) (see Fig. 5 (c)); and the
q^*_\Omega quarks (q^*_{\Omega}(1651), q^*_{\Omega}(7211)...) are born on the Σ-axis (single energy bands from ∆S = - 1) at the point Γ (see Fig. 5 (c)). Thus, we assume that the probability (P_{q^*}) of producing a quark (q^*) is

\[ P_{q^*} = C_1 \exp [O(q^*)] = C_1 \exp(8 + 2 \times S_G) \]  

(12)

where S_G(≡S+C+b) is the generalized strange number. O(q^*)=8+2×S_G (probability unit):

| quark | q^*_C | q^*_N,q^*_\Delta | q^*_S,q^*_\Xi,q^*_\Omega | q^*_b | q^*_\Xi^* |
|-------|-------|-------------------|--------------------------|-------|-----------|
| S_G=S+C+b | +1 | 0 | -1 | -2 | -3, |
| O(q^*)(Unit) | 10 | 8 | 6 | 4 | 2. |

Thus, from (13), the unflavored quarks (q^*_N and q^*_\Delta) have O(q^*) = 8 units; the charmed quarks (q^*_C) has O(q^*_C)= 10 (8+2 which is from ∆S = +1) units; the strange quarks (q^*_S and q^*_\Xi) have O(q^*_S) = 6 units; the q^*_\Xi quarks have O(q^*_\Xi) = 4 units; the q^*_b quarks have O(q^*_b) = 6 (4+2 which is from ∆S = +1) units; the q^*_\Omega quarks have O(q^*_\Omega) = 2 (4-2 which is from ∆s = -1) units.

A2. The quarks q^*_\Sigma_c, q^*_\Xi_c, and q^*_\Omega_c are born from “second division” [11]. In the first division, we got O(q^*_\Sigma_c) = 6, O(q^*_\Xi) = 4, and O(q^*_\Omega) = 2 from [13]. The second division will lower the probabilities (O(q^*_\Sigma_c), O(q^*_\Xi), and O(q^*_\Omega)). For simplification, we assume that the second division deduces half O(q^*) values:

\[ O(q^*_\Sigma_c) = 3, \ O(q^*_\Xi_c) = 2, \text{ and } O(q^*_\Omega_c) = 1. \]  

(14)

A3. The quarks which originate from the single energy bands of the ∆-axis (q^*_S(1391), q^*_C(2271), q^*_S(4271), q^*_C(6591), q^*_S(10031)...) and the quarks which originate from the single energy bands of the Σ-axis (q^*_S(1111), q^*_\Omega(1651), q^*_S(2551), q^*_\Omega(3711), q^*_b(5531),
$q_0^*(7211), q_b^*(9951)...$ have double $O(q^*)$. Using (13), we have

| $\Delta$ | $\Delta S=+1$ | $C=+1$ | $q_C^*(2271), q_C^*(6591), q_C^*(13791)...$ | $O(q_C^*)=20$ |
|----------|---------------|--------|-----------------------------------------------|----------------|
| $S=0$    | $\Delta S=-1$| $S=-1$ | $q_S^*(1391), q_S^*(4271), q_S^*(10031)...$   | $O(q_S^*)=12$ |
| $\Sigma$ | $\Delta S=+1$| $S=-1$ | $q_S^*(1111), q_S^*(2551), q_S^*(5531)...$   | $O(q_S^*)=12$ |
| $S=-2$   | $\Delta S=-1$| $S=-3$ | $q_0^*(1651), q_0^*(3711), q_0^*(7211)...$   | $O(q_0^*)=4$ |

Using (13), we have

$\Delta S= 0 \quad \Delta S=+1 \quad C=+1 \quad q^* \quad O(q^*)=20$

$\Delta S=+1 \quad S=+1 \quad q_0^*(2271), q_0^*(6591), q_0^*(13791)... \quad O(q_0^*)=12$

Thus, for $q_S^*(1391), q_S^*(4271), q_S^*(10031), q_S^*(1111), q_S^*(2551), q_S^*(5531), q_0^*(9951)$, $O(q^*)= 2 \times 6 = 12$ units; for $q_C^*(2271), q_C^*(6591), q_C^*(13800), O(q_C^*)= 2 \times 10 = 20$ units; for $q_0^*(1651), q_0^*(3711), q_0^*(7211), O(q_0^*)= 2 \times 2 = 4$ units.

A4. The ground states have double $O(q^*)$ values: for $q_S^*(1111), O(q_S^*(1111))= 2 \times 12 = 24$ units; for $q_C^*(2271), O(q_C^*(2271))= 2 \times 20 = 40$ units; for $q_b^*(5531), O(q_b^*(5531))= 2 \times 12 = 24$; for (the general ground state) $q_N^*(931)$ originates from the center point $\Gamma$ at which three $\Delta$-axes meet together (there are 48 symmetric operations), $O(q_N^*(931))= 2 \times (3 \times 8) = 48$ units.

A5. Generally, a high energy quark will have a smaller probability of producing the quark from the vacuum. For simplification, we assume the following: for the high energy quark $q_C^*(2271), O(q_C^*(2271))=40 \rightarrow 30$; for the high energy quark $q_b^*(5531), O(q_b^*(5531))= 24 \rightarrow 18$; and for the quarks $q_N^*(931)$ and $q_S^*(1111)$ with low energies, $(q^*)$ values will not be changed, $O(q_N^*(931))= 48, O(q_S^*(1111))=24$. We do not consider any other quarks in this paper.

B The Probability that a Pair of Quarks $(q_i^* q_j^*)$ is Formed

B1. From Eq. (12), we can deduce the probability $(P_{q_i^* q_j^*})$ that a quark $(q_i^*)$ and an antiquark $(q_j^*)$ form a quark and an antiquark pair $(q_i^* q_j^*)$ as follows:

$$P_{q_i^* q_j^*} \propto P_{q_i^*} \times P_{q_j^*} \propto C_1 e^{O(q_i^*)} \times C_1 e^{O(q_j^*)} = C \times \exp [O(q_i^*) + O(q_j^*)].$$ (16)

B2. A quark and its own antiquark have double $[O(q^*_i) + O(q^*_j)]$:
\[ P_{\bar{q}_i \bar{q}_j} = C \times \exp \{ (1+\delta_{ij})[O(q_i^*)+O(\overline{q}_j^*)] \} \]
\[ = C \times \exp [O(q_i^* \overline{q}_j^*)], \] (17)
\[ = C \times \exp \{O(\overline{q}_j)(1+\delta_{ij})[O(q_i^*)+O(\overline{q}_j^*)] \}, \] (18)

where
\[ O(q_i^* \overline{q}_j^*)=(1+\delta_{ij})[O(q_i^*)+O(\overline{q}_j^*)]. \] (19)

For example, for \( q_N^*(931)q_N^*(931) \), \( O(q_i^* \overline{q}_j^*) = 2 \times [48+48] = 192 \) units; for \( q_S^*(1111)q_S^*(1111) \), \( O(q_i^* \overline{q}_j^*) = 2 \times [24+24] = 96 \) units; for \( q_N^*(1831)q_N^*(1831) \), \( O(q_i^* \overline{q}_j^*) = 2 \times [8+8] = 32 \) units.

B3. A quark and an antiquark of the same kind (with the same I, S, C, b, and Q, but different masses) have one and a half \([O(q_i^*) + O(\overline{q}_j^*)]\):
\[ O(q_i^* (m_k)q_j^*(m_l)) = [1+0.5\delta_{ij}(1+\delta_{kl})][O(q_i^*(m_k))+O(\overline{q}_j^*(m_l))]. \] (20)

For example, for \( q_N^*(931)q_N^*(1831) \), \( O(q_i^* \overline{q}_j^*) = 1.5 \times [48+8] = 84 \) units; for \( q_S^*(1111)q_S^*(1391) \), \( O(q_i^* \overline{q}_j^*) = 1.5 \times [24+12] = 54 \) units; for \( q_C^*(2271)q_C^*(2441) \), \( O(q_i^* \overline{q}_j^*) = 1.5 \times [(30)+10] = 60 \) units.

B4. A quark pair \((q_i^* \overline{q}_j^*)\) in which the quark \((q_i^*)\) has different I, S, C, and b from the antiquark \((q_j^*)\) will have a lower probability \((P_{q_i^* \overline{q}_j^*})\):
\[ P_{q_i^* \overline{q}_j^*} = C \exp (1-O(\Delta I)) \ [O(q_i^*(m_k)+O(\overline{q}_j^*(m_l))]=C\exp\{O(q_i^*(m_k)q_j^*(m_l))\} \] (21)

where
\[ O[q_i^*(m_k)q_j^*(m_l)] = (1-O(\Delta I)) \ [O(q_i^*(m_k)+O(\overline{q}_j^*(m_l))]. \] (22)
O(ΔI) ≡ ΔG × ΔI × \left(\frac{I_q + \frac{1}{2}}{2(I_q + \frac{1}{2}) + 1}\right), \quad ΔG = |S_i - S_j| + |C_i - C_j| + |b_i - b_j|, \quad ΔI = |I_i - I_j|. \quad \text{(23)}

We list O(ΔI) values as follows:

| Quark Pair | O(ΔI) | O(ΔI) |
|------------|-------|-------|
| q_N^*q_S^*, q_C^*q_N^*, q_N^*q_\Delta | 0 | q_N^*q_\Sigma | 1/8 |
| \overline{q_S}^*q_S^*, q_S^*q_C^*, q_S^*q_\Omega, q_\Sigma^*q_\Sigma | 0 | \overline{q_S}^*q_\Delta | 3/8 |
| \overline{q_C}^*q_C^*, q_C^*q_\Omega, q_C^*q_\Xi | 0 | \overline{q_C}^*q_\Delta | 3/8 |
| \overline{q_\Omega}^*q_\Omega | 0 | \overline{q_\Omega}^*q_\Xi | 3/8 |
| \overline{q_\Sigma}^*q_\Sigma, \overline{q_\Delta}^*q_\Delta | 0 | \overline{q_\Sigma}^*q_\Delta | 1/6 |
| \overline{q_C}^*q_\Sigma | 1/3 | \overline{q_\Omega}^*q_\Xi | 1/3 |

B5. If a quark (antiquark) has a very small value of O(q^*), it will be very difficult for it to form a meson with an antiquark (quark). For simplification, we assume that if a quark q^* (antiquark \overline{q^*}) has

\[ O(q^*) (\text{or } O(\overline{q^*})) \leq 4 \text{ units}, \quad \text{(24)} \]

it cannot form a meson with any other antiquark \overline{q^*} (quark q^*). Thus, q_\Xi^*, q_\Omega^*, q_\Sigma^*, q_\Xi^*_c, and q_\Omega^*_c cannot form a meson with any antiquark. This assumption is based on today’s experiments. In the future, we might be able to find mesons that are made by q_\Xi^*, q_\Omega^*, q_\Sigma^*, q_\Xi^*_c, and q_\Omega^*_c.

Using (22), (23), and (24), we can deduce O(q_i^*(m_k)\overline{q_j^*(m_l)}) values for all quark pairs.
\[ (q_i^+(m_k)q_j^-(m_l)):\]

| \(q_N^+(931)q_N^-(931)\) | 192 | \(q_N^+(931)q_S^-(1111)\) | 72 | \(q_C^-(2271)q_S^+(5531)\) | 48 |
| \(q_N^+(931)q_N^-(m)\) | 84 | \(q_N^+(931)q_S^-(Single)\) | 60 | \(q_C^-(2271)q_S^+(Single)\) | 42 |
| \(q_N^+(m_k)q_N^-(m_l)\) | 32 | \(q_N^+(931)q_S^-(m)\) | 54 | \(q_C^-(2271)q_S^-(m)\) | 24 |
| \(q_N^-(m)q_N^+(m)\) | 24 | \(q_N^-(931)q_C^-(2271)\) | 78 | \(q_C^-(2271)q_S^+(m)\) | // |
| \(q_S^+(1111)q_S^-(1111)\) | 96 | \(q_N^+(931)q_C^-(Single)\) | 68 | \(q_C^-(2271)q_S^+(m)\) | // |
| \(q_S^+(1111)q_S^-(m)\) | 54 | \(q_N^+(931)q_C^-(m)\) | 58 | \(q_C^-(2271)q_S^+(m)\) | // |
| \(q_N^+(m_k)q_N^-(m_l)\) | 27 | \(q_N^+(931)q_S^-(Single)\) | 60 | \(q_N^+(5531)q_S^+(m)\) | 16 |
| \(q_N^+(m)q_N^+(m)\) | 24 | \(q_N^+(931)q_S^+(m)\) | 56 | \(q_N^+(5531)q_S^+(m)\) | // |
| \(q_N^+(m_k)q_N^-(m_l)\) | 18 | \(q_N^+(931)q_S^+(m)\) | 47 | \(q_N^+(5531)q_S^+(m)\) | // |
| \(q_C^-(2271)q_C^-(2271)\) | 120 | \(q_N^+(931)q_S^+(m)\) | 44 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_C^-(2271)q_C^+(Single)\) | 75 | \(q_N^+(931)q_C^+(m)\) | 40 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_C^+(2271)q_C^-(m)\) | 60 | \(q_N^+(1111)q_C^-(2271)\) | 54 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_C^+(m)q_C^+(m)\) | 45 | \(q_N^+(1111)q_C^+(Single)\) | 40 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_C^+(m_k)q_C^+(m_l)\) | 30 | \(q_N^+(1111)q_C^+(5531)\) | 34 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_S^+(5531)q_S^+(5531)\) | 72 | \(q_N^+(1111)q_S^-(5531)\) | 36 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_S^+(5531)q_S^-(5531)\) | 45 | \(q_N^+(1111)q_S^+(m)\) | 30 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_S^+(m_k)q_S^+(m_l)\) | 48 | \(q_N^+(1111)q_S^+(m)\) | 30 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_S^+(m)q_S^+(m)\) | 36 | \(q_N^+(1111)q_S^+(m)\) | 36 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_A^-(m)q_A^-(m)\) | 32 | \(q_N^+(1111)q_A^-(m)\) | 32 | \(q_N^+(m_k)q_S^+(m_l)\) | // |
| \(q_A^-(m_k)q_A^-(m_l)\) | 24 | \(q_N^+(1111)q_A^-(m)\) | // | // | |

C The Probability (P(I_m)) of a Meson with Isospin I_m

The above probabilities (25) are the maximum probability (P_{Maximum}) of each pair. However, the real probability (P(I_{meson})) of a meson being made by a quark and an antiquark will depend on the isospin (I_{meson}) of the meson. The probability of a meson
with a larger isospin (I) will be smaller:

\[
P(I_{Mes}) = C_2 \times \exp\{O(q_i \bar{q}_j)\left[1 - \frac{I_{Mes} - |I_q - I_{\bar{q}}|}{I_q + I_{\bar{q}} + 1}\right]\}
\]

\[
= C_2 \times \exp[O(I_q \bar{q}_j)],
\]

where

\[
O(I_q \bar{q}_j) = O(q_i \bar{q}_j)\left[1 - \frac{I_{Mes} - |I_q - I_{\bar{q}}|}{I_q + I_{\bar{q}} + 1}\right],
\]

We list \(O(I_q \bar{q}_j)\) values as follows:

| (I_q)⊗(I_{\bar{q}}) = (I_{M1})⊕(I_{M2})... | O(I_{M1}) | O(I_{M2}) | O(I_{M3}) |
|------------------------------------------|----------|----------|----------|
| (0)⊗(0) = (0)                           | O(0) = O(q_i \bar{q}_j) |
| (0)⊗(\frac{1}{2}) = (\frac{1}{2})     | O(\frac{1}{2}) = O(q_i \bar{q}_j) |
| (0)⊗(1) = (1)                           | O(1) = O(q_i \bar{q}_j) |
| (0)⊗(\frac{3}{2}) = (\frac{3}{2})    | O(\frac{3}{2}) = O(q_i \bar{q}_j) |
| (\frac{1}{2})⊗(\frac{1}{2}) = (0)⊕(1) | O(0) = O(q_i \bar{q}_j) | O(1) = \frac{1}{3}O(q_i \bar{q}_j) |
| (\frac{1}{2})⊗(1) = (\frac{1}{2})⊕(\frac{3}{2}) | O(\frac{1}{2}) = O(q_i \bar{q}_j) | O(\frac{3}{2}) = \frac{2}{3}O(q_i \bar{q}_j) |
| (\frac{1}{2})⊗(\frac{3}{2}) = (1)⊕(2) | O(1) = O(q_i \bar{q}_j) | O(2) = \frac{2}{3}O(q_i \bar{q}_j) |
| (1)⊗(1) = (0)⊕(1)⊕(2)                  | O(0) = O(q_i \bar{q}_j) | O(1) = \frac{2}{3}O(q_i \bar{q}_j) | O(2) = \frac{1}{3}O(q_i \bar{q}_j) |
| (1)⊗(\frac{1}{2}) = (\frac{1}{2})⊕(\frac{3}{2}) | O(\frac{1}{2}) = O(q_i \bar{q}_j) | O(\frac{3}{2}) = \frac{2}{3}O(q_i \bar{q}_j) |
| (1)⊗(\frac{3}{2}) = (\frac{3}{2})⊕(\frac{3}{2}) | O(\frac{3}{2}) = O(q_i \bar{q}_j) | O(\frac{3}{2}) = \frac{1}{3}O(q_i \bar{q}_j) |

D The Degeneracy \(D_{q^*} (D_{\bar{q}^*})\) of the Quarks (antiquarks)

Considering the degeneracy \((D_{q^*} (D_{\bar{q}^*}))\) of the quarks (antiquarks) \((3)\), the total probability of forming a meson will be

\[
P_{total} = D_{q^*} \times D_{\bar{q}^*} \times P(I_{Meson})
\]

\[
= C_3 \times C_2 \times e^{d_{q^*} + d_{\bar{q}^*}} \times e^{O(I_{Mes})}
\]

\[
= C \times \exp[d_{q^*} + d_{\bar{q}^*} + O(I_{Mes})]
\]

\[
= C \times \exp(\Phi)
\]
where $d_q^* (d_{q^*})$ is the degeneracy of the quark(antiquark) $q^* (q^*)$, and $\Phi$ is defined as

$$\Phi \equiv d_q^* + d_{q^*} + O(I_{Mes}). \quad (30)$$

The formula (29) is the probability of a meson $(q_i^* q_j^*)$ with isospin I. The constant C is unknown. The value $\Phi$ can be deduced from Eq. (30). It is very important to note that $\Phi (d_q^* + d_{q^*} + O(I_{Mes}))$ is the result of the symmetry of the BCC quark lattice. Thus, the probability formula (29) is based on the foundation of the symmetry of the BCC quark lattice.

### IV The Phenomenological Formula for the Binding Energies of the Mesons

Using the sum laws (10), we have deduced the quantum numbers of all quark pairs (11). However, the masses ($M_{q_i^* q_j^*}$) of the mesons ($M(q_i^* q_j^*)$) cannot be found using the sum laws alone because we have to consider the binding energy:

$$M(q_i^* q_j^*) = m_{q_i^*} + m_{q_j^*} + E_{binding}. \quad (31)$$

There is not a theoretical formula for the binding energies in the Quark Model. Thus, we propose a unified phenomenological formula for the meson binding energies now.

Although we do not know the exact formula of the binding energies $E_B(i, j)$ of $q_i^* q_j^*$ inside a meson, we have found some common characteristics of the binding energies.

First, according to the BCC Quark Model [10], all quarks (with different I, S, C, b, Q, and mass) are different energy band excited states of the two elementary quarks, u and d. The two elementary quarks, u and d, are the different component ($I_Z$) states of the same quark (q). Thus, for different mesons, the binding energies (between the quark and the antiquark in a meson) are essentially the same (a constant ~ (- 1723 Mev)) – the first term in $E_B(i, j)$ (see (32)).
Second, the strange quarks, the charmed quarks, and the bottom quarks have smaller binding energies than the unflavored ones. Thus, we will see a term, \( N(|S_i\ or \ j| + 1.5|C_i\ or \ j| + 3|b_i\ or \ j|) \), in \( E_B(i, j) \) (see (32)). This term makes their binding energies smaller.

Third, the ground state quarks \( (q^*_N(931), q^*_S(1111), q^*_C(2271), q^*_b(5531)) \) and their antiquarks have larger binding energies than non-ground state quarks. The term \( 2\delta_{ng} \) in \( E_B(i, j) \) (see (32)) makes the ground states and their antiquarks have 200 Mev more binding energies than the non-ground state quarks. The quarks that are born from the single energy bands of the \( \Delta \)-axis from \( \Delta S = +1 \) \( (q^*_C(6591), q^*_C(13791)...) \) (see list (17)) are the brothers of the ground quark \( q^*_C(2271) \) [10], and the quarks that are born from the single energy bands of the \( \Sigma \)-axis from \( \Delta S = +1 \) \( (q^*_S(2551), q^*_b(9951), q^*_b(15811)...) \) (see list (13)) are the brothers of the ground quarks \( q^*_S(1111) \) and \( q^*_b(5531) \) [10]. They can be dealt with as the ground states (\( \delta_{ng}=0 \)) when we calculate their binding energies. The term “brother” means that they are born at the same symmetry point of the single energy bands of the same symmetry axis.

Fourth, the binding energies of the quarks and their own antiquarks are larger than other cases (see first term “2” inside \( \[1-\delta_{ij}\] \) of \( E_B(i, j) \), the “2” makes the binding energy of the pairs 200 Mev more than nonpair cases). The quarks that are born from the single energy bands of the \( \Delta \)-axis with \( \Delta S = -1 \) \( (q^*_S(1391), q^*_S(4271), q^*_S(10031)...) \) (see list (15)) and the quarks that are born from the single energy bands of the \( \Sigma \)-axis with \( \Delta S = -1 \) \( (q^*_N(1651), q^*_N(3711), q^*_N(7211)...) \) (see list (16)) can be dealt with as the ground states when we calculate their paired binding energies.

Fifth, the strange quark with a larger isospin has smaller binding energies than other cases (see the term ‘-1.5(SL q-SL q̄)’ of \( f(I, S, C) \) inside \( \[\] \) of \( E_B(i, j) \) (see (32)).

Sixth, the quark pairs \( (q^*_i q^*_j) \) in which the quark \( (q^*_i) \) has different I, S, and C from the antiquark \( (q^*_j) \) will have different (smaller) binding energies (see the term \( \Delta I - \delta S - \Delta C \) in \( f(I, S, C) \) inside \( \[\] \) of \( E_B(i, j) \) (see (32) and (34)).

Finally, the quarks with larger masses have slightly larger binding energies than the
smaller quarks of the same kind [see term (1-\(\tilde{m}\)), \(\tilde{m} = m_q m_{\bar{q}}/m_g m_{\bar{g}}\)], especially, the quark pairs \((q_i^* q_j^*)\) in which the quarks \((q_i^*)\) and the antiquarks \((\bar{q}_j^*)\) are born on the single energy binds of the \(\Delta\)-axis and the \(\Sigma\)-axis (see list (15)). The term \((-\delta_{SP} N_i)\) makes the binding energy larger. The term \(25(G_q - G_{\bar{q}} - SI_q + SI_{\bar{q}})\) is a small adjustment.

Based on these characteristics, we propose the following phenomenological binding energy formula: the binding energy \(E_B(i, j)\) of a quark \((q_i^*)\) and an antiquark \((\bar{q}_j^*)\) in the meson \(M(q_i^* q_j^*)\) is

\[
E_B(i, j) = -1723 + 100 \{N + [2 - |S| + \delta_{ng} f(I, S, C)](1 - \delta_{ij}) + 2\delta_{ng} + (1 - \tilde{m}) - \delta_{SP} N_i\} + 25A. \tag{32}
\]

Where

\[
N = |S_{i or j}| + 1.5 |C_{i or j}| + 3 |b_{i or j}|, \tag{33}
\]

\(S\) is the strange number, \(C\) is the charmed number, and \(b\) is the bottom number. If \(q_i^*\) and \(q_j^*\) have the same \(|S|\), \(|C|\), or \(|b|\), \(N_{i or j} = N_i\). For example, for \(q_S^*(1111)q_S^*(1111)\), \(N_{i or j} = N_i = |S| = 1\); for \(q_C^*(2271)q_C^*(2271)\), \(N_{i or j} = 1.5 |C| = 1.5\); for \(q_B^*(5540)q_B^*(5531)\), \(N_{i or j} = 3 |b_{i or j}| = 3\); for \(q_S^*(1111)q_S^*(2551)\), \(N_{i or j} = |S| = 1\); for \(q_N^*(931)q_S^*(1111)\), \(N_{i or j} = 1\); for \(q_N^*(931)q_C^*(2271)\), \(N_{i or j} = 1.5 |C| = 1.5\); for \(q_N^*(931)q_B^*(5531)\), \(N_{i or j} = 3 |b| = 3\).

\[
f(I, S, C) = -1.5 (SI - \overline{SI}) + \Delta I - \delta S - 2.5 |C|, \tag{34}
\]

\(SI_q\) is strange number \(S\) times(\(\times\)) isospin \(I\) of \(q_i^*\); \(\overline{SI}\) is strange number \(S\) times(\(\times\)) isospin \(I\) of \(q_j^*\). \(\Delta I = |I_i - I_j|, \delta S = |S_i - S_j|, \Delta C = |C_i - C_j|\). \(\delta_{ng} = 1\) if either \(q\) or \(\bar{q}\) is not a ground state (or a quark that is born from the single band of the \(\Delta\)-axis and the \(\Sigma\)-axis), otherwise \(\delta_{ng} = 0\). In (32),

\[
\tilde{m} = m_q m_{\bar{q}}/m_g m_{\bar{g}}, \tag{35}
\]

here \(m_{q_i}\) (\(m_{\bar{g}_j}\)) is the mass of the ground state (anti-ground state). There are only five ground quarks – \(q_u^*(931)\) \([O(q_u^*)=48]\), \(q_d^*(931)\) \([O(q_d^*)=48]\), \(q_S^*(1111)\) \([O(q_S^*)=24]\), 19
For the single energy bands of the ∆-axis and the Σ-axis, $\tilde{m} = m_q(J_n) \times m_{q'}(J'_{n'}) / m_q(J_{n-1}) \times m_{q'}(J'_{n'-1})$, $J_n$ is an order number of the single energy band with $ΔS \neq 0$ at the same symmetry point. $δ_{SP}=1$ if $i = j$ and both quarks ($q^*_i$ and $q^*_j$) are born from the single binds; $δ_{SP}=0$ otherwise.

$$A = G_q - G_{\overline{q}} - SI_q + SI_{\overline{q}}.$$  \hspace{1cm} (36)

where $G = S + 1.5C + 3b$.

The above formula looks very complex, but in fact, it is very simple and easy to use. Usually, we simplify it into seven cases as follows:

I. For ground quark pairs, $δ_{ij}=1$, $δ_{ng}=0$, $δ_{SP}=0$, $\tilde{m}=1$, $G_q=-G_{\overline{q}}$, $SI_q=-SI_{\overline{q}}$, from (32).

$$E_B(i, i) = -1723 + 100N_i + 50(G_q - SI_q).$$  \hspace{1cm} (37)

II. For the quark pairs ($q^*_i \overline{q}_j$) born from the single energy bands, $δ_{ij}=1$, $δ_{ng}=0$, $G_q=-G_{\overline{q}}$, $SI_q=-SI_{\overline{q}}$, $δ_{SP}=1$, from (32).

$$E_B(i, i) = -1623 + 100(-\tilde{m}) + 50(G_q - SI_q).$$  \hspace{1cm} (38)

For the single energy bands $\tilde{m} = m_q(J_n) \times m_{q'}(J'_{n'}) / m_q(J_{n-1}) \times m_{q'}(J'_{n'-1})$, $J_n$ is an order number of the single energy band with $ΔS \neq 0$ at the same symmetry point.

III. For non-ground pairs, $δ_{ij}=1$, $δ_{ng}=1$, $δ_{SP}=0$, $G_q=-G_{\overline{q}}$, $SI_q=-SI_{\overline{q}}$, from (32).

$$E_B(i, i) = -1423 + 100(N_i - \tilde{m}) + 50(G_q - SI_q).$$  \hspace{1cm} (39)

IV. For quarks and antiquarks that are both ground quarks ($q^*_i \overline{q'}_j$, $i \neq j$), $δ_{ij}=0$, $δ_{ng}=0$, $\tilde{m}=1$, $δ_{SP}=0$, from (32).

$$E_B(i, j) = -1523 + 100[1.5|C| + 3|b|] + 25(G_q - G_{\overline{q}} - SI_q + SI_{\overline{q}}).$$  \hspace{1cm} (40)

V. For the quarks (not pairs) that are born from the single energy bands with $ΔS=+1$ of the ∆-axis and the Σ-axis, $δ_{ij}=0$, $δ_{ng}=0$, $\tilde{m} = m_q(J_n) \times m_{q'}(J'_{n'}) / m_q(J_{n-1}) \times m_{q'}(J'_{n'-1})$,
\[ \delta_{SP} = 0, \ SI_q - SI_{\bar{q}} = 0. \] From (32),

\[ E_B(i, j) = -1423 + 100|1.5|C| + 3|b - \bar{m}| + 25(G_q - G_{\bar{q}} - SI_q + SI_{\bar{q}}). \] (41)

VI. For quarks and antiquarks are not both ground quarks \( q_i^* q_j^* \) (\( i \neq j \)), \( \delta_{ij} = 0 \), \( \delta_{ng} = 1 \), \( \delta_{SP} = 0. \) From (32),

\[ E_B(i, j) = -1223 + 100[3|b| - 1.5(SI - ST) + \Delta I - \Delta S - \Delta C - \bar{m}] + 25(G_q - G_{\bar{q}} - SI_q + SI_{\bar{q}}). \] (42)

VII. For the quark \( (q^*_i(m_k)) \) and the antiquark \( (\bar{q}^*_j(m_l)) \) that have the same quantum numbers (\( I, S, C, b, \) and \( Q \)), but different masses, \( \delta_{ng} = 1 \), \( \delta_{SP} = 0. \) Since they have the same quantum numbers, the quantum numbers of \( E_B(i,j) \) should be the same as those of the pair quarks (\( N \)). Because the masses of \( q^*_i \) and \( \bar{q}^*_j \) are different, \( E_B(i,j) \) will take the non-quantum number portion ('2') from non-pair term \( [2 - |S| + \delta_{ng}f(S,I)] (1 - \delta_{ij}) \). Thus, from (32)

\[ E_B(i, j) = -1223 + 100(N_i - \bar{m}) + 25(G_q - G_{\bar{q}} - SI_q + SI_{\bar{q}}). \] (43)

For example, we apply the above formulas to find the following mesons \( (O(q_i q_j^*)) \) is from the list (25) and (28):

I. For ground quark pairs, from (37), we have

\[ E_B(i, i) = -1723 + 100N_i + 50[(G_q - GI_q]. \]

For \( q_N^*(931)q_N^*(931) \), \( E = -1723 + 100(0) + 0 = -1723; \)
for \( q_S^*(1111)q_S^*(1111) \), \( E = -1723 + 100(1) + 50[-1-0] = -1673; \)
for \( q_C^*(2271)q_C^*(2271) \), \( E = -1723 + 100(1.5) + 50(+1.5-0) = -1498; \)
for $q_b^*(5531)q_b^*(5531)$, \[ E = -1723+100(3.0)+50(-3) = -1573. \]

| $q_i^*(m)$ | $q_N^*(931)$ | $q_S^*(1111)$ | $q_C^*(2271)$ | $q_b^*(5531)$ |
|------------|--------------|---------------|---------------|--------------|
| $q_j^*(m)$ | $q_N^*(931)$ | $q_S^*(1111)$ | $q_C^*(2271)$ | $q_b^*(5531)$ |
| $E_B$      | -1723        | -1673         | -1498         | -1573        |
| Theory     | $\pi(139)$  | $\eta(549)$  | $J/\Psi(3044)$ | $\Upsilon(9489)$ |
| O($q_i^*q_j^*$) | 192 | 96 | 120 | 72 |
| Exper.     | $\pi(139)$  | $\eta(547)$  | $J/\Psi(3097)$ | $\Upsilon(9460)$ |

II. The quark pairs born from the single energy bands, from (38),

\[ E_B(i, i) = -1623 + 100(-\bar{m}) + 50(G_q - S_q), \]

for the single energy bands $\bar{m} = m_q(J_n)\times m_{\bar{q}}(J_n')/m_q(J_{n-1})\times m_{\bar{q}}(J_{n'-1})$, $J_n$ is an order number of the single energy band with $\Delta S \neq 0$ at the same symmetry point [?].

1). The quark pairs on the single energy bands of the $\Delta$ axis ($\Delta S = +1$),

\[ E = -1623+100(-\bar{m}) + 50(1.5) = -1548 - 100 \bar{m}, \]

for $q_C^*(6591)q_C(6591)$, \[ \bar{m} = (6591\times6591)/(2271\times2271) = 8.42, \]

for $q_C^*(13791)q_C(13791)$, \[ \bar{m} = (13791\times13791)/(6591\times6591) = 4.38. \]

| $q_i^*(m)$ | $q_j^*(m)$ | $q_C^*(2271)$ | $q_C^*(2271)$ | $q_C^*(6591)$ | $q_C^*(6591)$ | $q_C^*(13791)$ | $q_C^*(13791)$ |
|------------|------------|---------------|---------------|---------------|---------------|---------------|---------------|
| E          | -1498      | -1548-842     | -1548-438     | -1986         |
| Theory     | $J/\Psi(3044)$ | $\psi(10792)$ | $\psi(25596)$ |
| O($q_i^*q_j^*$) | 120 | 80 | 80 |
| Exper.     | $J/\Psi(3097)$ | $\Upsilon(10355)$ | ? |

2). The quark pairs on the single energy bands of the $\Sigma$ axis ($\Delta S = +1$),

for $q_S^*(2560)\bar{q}_S^*(2551)$ \[ E = -1623+50(-1)-100 \bar{m} = -1673 -527, \]

($\bar{m} = (2551\times2551)/(1111\times1111) = 5.22$);

for $q_b^*(9951)\bar{q}_b^*(9951)$ \[ E = -1623+50(-3)-100 \bar{m} = -1773 -324, \]
\[ (\tilde{m} = (9951 \times 9951)/(5531 \times 5531) = 3.24); \]

for \( q^*_b(15811)q^*_b(15811) \) \( E = -1623 + 50(-3) - 100 \tilde{m} = -1773 - 252, \)

\[ (\tilde{m} = (15811 \times 15811)/(9951 \times 9951) = 2.52). \]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
q^*_b(m) & q^*_S(1111) & q^*_S(2551) & q^*_b(5531) & q^*_b(9951) & q^*_b(15811) \\
\hline
q^*_b(m) & q^*_S(1111) & q^*_S(2551) & q^*_b(5531) & q^*_b(9951) & q^*_b(15811) \\
\hline
E & -1673 & -2200 & -1573 & -2097 & -2025 \\
Theory & \eta(549) & \eta(2902) & \Upsilon(9489) & \Upsilon(17805) & \Upsilon(29597) \\
O(q^*_b q^*_j) & 96 & 48 & 72 & 48 & 48 \\
Exper. & \eta(547) & \eta_C(2980) & \Upsilon(9460) & ? & ? \\
\hline
\end{array}
\]

3). The quark pairs on the single energy bands of the \( \Delta \) axis \( (\Delta S = -1), \)

\[ E_B(i, i) = -1623 + 100(-\tilde{m}) + 50(G_q-SI_q) = -1673 - 100\tilde{m}. \]

For \( q^*_S(1391)q^*_S(1391), \) \( \tilde{m} = (1391 \times 1391)/(1111 \times 1111) = 1.567, \)

for \( q^*_S(4271)q^*_S(4271), \) \( \tilde{m} = (4271 \times 4271)/(1391 \times 1391) = 9.43, \)

for \( q^*_S(10031)q^*_S(10031), \) \( \tilde{m} = (10031 \times 10031)/(4271 \times 4271) = 5.52. \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
q^*_b(m) & q^*_S(1111) & q^*_S(1391) & q^*_S(4271) & q^*_S(10031) \\
\hline
q^*_b(m) & q^*_S(1111) & q^*_S(1391) & q^*_S(4271) & q^*_S(10031) \\
\hline
E & -1691 & -1830 & -2616 & -2225 \\
Theory & \eta(549) & \eta(952) & \eta(5926) & \eta(17837) \\
O(q^*_b q^*_j) & 96 & 48 & 48 & 48 \\
Exper. & \eta(547) & \eta'(958) & ? & ? \\
\hline
\end{array}
\]

IV. For quarks and antiquarks that are both the ground states \( (i \neq j), \) from \( ^{40} \)

\[ E_B(i, j) = -1523 + 100[1.5|C| + 3|b|] + 25(G_q-G_T-SI_q + SI_T) \]

1). For \( q(m)q^*_N(931), \) \( ST = 0, G_T = 0, \)

for \( q^*_S(1111)q^*_N(931), \) \( E = -1523 + 0 + 25(-1) = -1548; \)

for \( q^*_C(2271)q^*_N(931), \) \( E = -1523 + 150 + 25(1.5) = -1336; \)
for \( q_\ast^b(5531)q_N^*(931) \), \( E = -1523+300+25(-3) = -1298 \).

| \( q_i^*(m) \) | \( q_S^*(1111) \) | \( q_C^*(2271) \) | \( q_b^*(5531) \) |
|----------------|-----------------|-----------------|-----------------|
| \( q_j^*(m) \) | \( q_N^*(931) \) | \( q_N^*(931) \) | \( q_N^*(931) \) |
| **E_{bind}**  | -1548           | -1336           | -1298           |
| **Theory**    | K(494)         | D(1866)        | B(5164)        |
| **O(q_i^*q_j^*)** | 72             | 78             | 66             |
| **Exper.**    | K(494)         | D(1869)        | B(5279)        |

(49)

2. For \( q(m)q_S^*(1111) \), \( S\overline{T} = 0, G_\overline{q} = 1, SI_\overline{q} = 0 \),

for \( q_C^*(2271)\overline{q}_S(1111) \), \( E = -1523+150+25(1.5-1) = -1360.5 \);

for \( q_b^*(5531)\overline{q}_S(1111) \), \( E = -1523+300+25(-3-1) = -1323 \).

| \( q_i^*(m) \) | \( q_C^*(2271) \) | \( q_b^*(5531) \) |
|----------------|-----------------|-----------------|
| \( q_j^*(m) \) | \( q_S^*(1111) \) | \( q_S^*(1111) \) |
| **E_{bind}**  | -1361           | -1323           |
| **Theory**    | D_S(2022)      | B_S(5319)      |
| **O(q_i^*q_j^*)** | 54             | 42             |
| **Exper.**    | D_S\(^{+}\)(1969) | B_S(5369) |

(50)

3. For \( q(m)q_C^*(2271) \),

\[
E_B(i,j) = -1523+100[1.5|C|+3|\overline{b}|]+25(G_q-G_{\overline{q}}SI_q+SI_{\overline{q}}) = -1111.
\]

| \( q_i^*(m)q_j^*(m) \) | \( q_b^*(5531)q_C^*(2271) \) |
|------------------------|-------------------------------|
| **E_{bind}**           | -1111                         |
| **Theory**             | B_C(6691)(I=0, Q=-1)          |
| **O(q_i^*q_j^*)**      | 38                            |
| **Exper.**             | B_C(6400)                     |

(51)

For a full list of the meson mass spectrum, please see Appendix II.
From the quark spectrum (8) and (11), using sum laws (10), we have found the intrinsic quantum numbers (I, S, C, b, and Q) of the mesons (11); with the phenomenological binding energy formula (32), we have deduced the masses of the mesons; in terms of the probability formulas (25) and (29), we have also got the probabilities that a quark and an antiquark form a meson. Therefore, we deduce the meson spectrum (I, S, C, b, Q, Mass, and the possibility $P_{\text{total}}$ (29)). The theoretical meson spectrum is listed in Appendix II.
V Comparing Results

We compare the theoretical results to the experimental results [3] using the seven tables found below. In the comparison, we will use the following conventions:

(1). We do not take into account the angular momenta of the experimental results. We assume that the small differences of the masses in the same group of the mesons originate from their different angular momenta. If we ignore this effect, their masses should be essentially the same. The mesons of the same kind (the same I, S, C, b, and Q) with roughly the same masses but different angular momentums and parities form a group. We use the average of the masses to represent the mass of the group mesons.

(2). We use the meson name to represent the intrinsic quantum numbers (11).

(3). If there are many possible mesons that correspond to a group of experimental observed mesons, we use the meson with the maximum discovered probability to represent the group.

(4). The mesons with \( O(I_{Meson}) < 24 \) (see (27))

\[
O(I_{Meson}) < 24
\]  

are omitted since they can not be discovered now (because of technical limitations).

(5). For the quark masses of the Quark Model [8], we take the average values:

\[
\begin{align*}
    m_u &= 3\text{Mev} \ (1.5 \text{ to } 5 \text{ Mev}), \\
    m_d &= 6\text{Mev} \ (3 \text{ to } 9 \text{ Mev}), \\
    m_s &= 123\text{Mev} \ (75 \text{ to } 170 \text{ Mev}), \\
    m_c &= 1250\text{Mev} \ (1150 \text{ to } 1350 \text{ Mev}), \\
    m_b &= 4200\text{Mev} \ (4000 \text{ to } 4400 \text{ Mev}).
\end{align*}
\]  

(53)
Table I. Light Unflavored Mesons (S = C = b = 0, I = 0, Q = 0)

| The BCC Quark Model | Exper. | The Quark Model |
|---------------------|--------|-----------------|
| \( q_s^*(m_k)q_s^*(m_l) = \eta(m) \) | \( M_{\text{meson}}(m) \) | \( q^*_s(m_k)q^*_s(m_l) \) |
| \( q_s^*(1111)q_s^*(1111) = \eta(549) \) | \( \bullet \eta(547) \) | \( \bullet \eta(547) \) |
| \( q_s^*(931)q_s^*(1201) = \eta(780) \) | \( \bullet \omega(782) \) | \( \bullet \omega(782) \) |
| \( q_s^*(1210)q_s^*(1201) = \eta(813) \) | \( f_0(1000-1200) \) | \( f_0(1000-1200) \) |
| \( q_s^*(1391)q_s^*(1391) = \eta(952) \) | \( \bullet \eta'(958) \) | \( \bullet \eta'(958) \) |
| \( q_{\Delta}^*(1291)q_{\Delta}^*(1291) = \eta(967) \) | \( f_0(980) \) | \( f_0(980) \) |
| \( q_s^*(931)q_s^*(1471) = \eta(1021) \) | \( \bullet \phi(1020) \) | \( \bullet \phi(1020) \) |
| \( q_s^*(1111)q_s^*(1391) = \eta(1204) \) | \( h_1(1170) \) | \( h_1(1170) \) |
| \( q_s^*(1471)q_s^*(1471) = \eta(1269) \) | \( \bullet \overline{\eta}(1283) \) | \( \bullet \overline{\eta}(1283) \) |
| \( q_s^*(931)q_s^*(1831) = \eta(1342) \) | \( \bullet \overline{\eta}(1375) \) | \( \bullet \overline{\eta}(1375) \) |
| \( q_s^*(931)q_s^*(1921) = \eta(1423) \) | \( \bullet \overline{\eta}(1428) \) | \( \bullet \overline{\eta}(1428) \) |
| \( q_{\Delta}^*(1651)q_{\Delta}^*(1651) = \eta(1565) \) | \( \bullet \overline{\eta}(1525) \) | \( \bullet \overline{\eta}(1525) \) |
| \( q_s^*(931)q_s^*(2191) = \eta(1664) \) | \( \bullet \overline{\eta}(1656) \) | \( \bullet \overline{\eta}(1656) \) |
| \( q_s^*(1111)q_s^*(2011) = \eta(1768) \) | \( \bullet \overline{\eta}(1760) \) | \( \bullet \overline{\eta}(1760) \) |
| \( q_s^*(1831)q_s^*(1831) = \eta(1852) \) | \( \bullet \overline{\eta}(1860) \) | \( \bullet \overline{\eta}(1860) \) |
| \( q_s^*(931)q_s^*(2551) = \eta(1985) \) | \( \bullet \overline{\eta}(1930) \) | \( \bullet \overline{\eta}(1930) \) |
| \( q_s^*(931)q_s^*(2641) = \eta(2065) \) | \( \bullet \overline{\eta}(2030) \) | \( \bullet \overline{\eta}(2030) \) |
| \( q_s^*(931)q_s^*(2731) = \eta(2146) \) | \( \overline{\eta}(2199) \) | \( \overline{\eta}(2199) \) |
| \( q_s^*(1111)q_s^*(2641) = \eta(2341) \) | \( \overline{\eta}(2320) \) | \( \overline{\eta}(2320) \) |
| \( q_s^*(1111)q_s^*(2731) = \eta(2423) \) | \( f_0(2510) \) | \( f_0(2510) \) |
| \( q_s^*(2551)q_s^*(2551) = \eta(2902) \) | \( \overline{\eta}(2980) \) | \( \overline{\eta}(2980) \) |
| \( q_{\Delta}^*(2731)q_{\Delta}^*(2731) = \eta(3178) \) | \( \chi(3250) \) | \( \chi(3250) \) |
| \( q_s^*(4271)q_s^*(4271) = \eta(5926) \) | \( ? \) | \( ? \) |
| \( q_s^*(10031)q_s^*(10031) = \eta(17837) \) | \( ? \) | \( ? \) |
Table II. Light Unflavored Mesons (S = C = b = 0, I = 1, Q = ±1, 0)

| The BCC Quark Model | Exper. | Quark Model |
|----------------------|--------|-------------|
| $q^*_i(m_k)q^*_j(m_l) = \pi(m)$ | Meson(m) | $q^*_i(m_k)q^*_j(m_l)$ |
| $q_N^*(931)q_N^*(931) = \pi(139)$ | $\pi(139)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_N^*(1201) = \pi(780)$ | $\pi(780)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(1291) = \pi(960)$ | $\pi(960)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(1565) = \pi(1282)$ | $\pi(1282)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(1831) = \pi(1340)$ | $\pi(1340)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(1921) = \pi(1423)$ | $\pi(1423)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(2011) = \pi(1603)$ | $\pi(1603)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(2011) = T(1603)$ | $\chi(1600)^{\#}_{I=2}$ | ? |
| $q_N^*(931)q_{N}^{*}(2191) = \pi(1664)$ | $\pi(1664)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_{S}^{*}(1111)q_{S}^{*}(1921) = \pi(1861)$ | $\pi(1861)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(2371) = \pi(1924)$ | $\chi(1924)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{N}^{*}(2641) = \pi(2065)$ | $\pi(2065)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(2731) = \pi(2146)$ | $\pi(2146)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(2731) = \pi(2246)$ | $\pi(2246)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_{S}^{*}(1111)q_{S}^{*}(2551) = \pi(2434)$ | $\pi(2434)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_N^*(931)q_{A}^{*}(3091) = \pi(2567)$ | $\pi(2567)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $q_{S}^{*}(1111)q_{S}^{*}(2731) = \pi(2598)$ | $\pi(2598)$ | u(3)d(6), u(3)u(3), d(6)d(6) |
| $\ldots$ | $\ldots$ | $\ldots$ |

$\#\chi(1600)^{\#}_{I=2}$ is not established
Table III. Strange Mesons ($S = \pm 1$, $C = b = 0$, $I = 1/2$, $Q = 1, 0$)

| The BCC Quark Model | Experiment | The Quark Model |
|---------------------|------------|-----------------|
| $q_i^*(m_k)q_j^*(m_l) = K(m)$ | $K(m)$ | $q_i^*(m_k)q_j^*(m_l)$ |
| $q_N^*(931)q_S^*(1111) = K(494)$ | $\bullet K(494)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(1391) = K(899)$ | $\bullet K(892)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(1651) = K(1310)$ | $\bullet K(1270)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2011) = K(1463)$ | $\bullet K(1426)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(1921) = K(1556)$ | $K(1580)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2011) = K(1638)$ | $\bullet K(1653)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(2191)q_S^*(1111) = K(1769)$ | $\bullet K(1775)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2551) = K(1804)$ | $\bullet K(1825)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2371) = K(1966)$ | $K(1965)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2641) = K(2036)$ | $\bullet K(2045)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2551) = K(2129)$ | ? | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2641) = K(2211)$ | $K(2250)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(2731) = K(2293)$ | $\overline{K}(2350)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(3091) = K(2621)$ | $K(2500)$ | $s(123)u(3), s(123)d(6)$ |
| $q_N^*(931)q_S^*(4271) = K(3597)$ | ? | ? |
| $q_N^*(931)q_S^*(10031) = K(9429)$ | ? | ? |

...
Table IV Charmed and Charmed Strange Mesons

| The BCC Quark Model | Experiment | The Quark Model |
|---------------------|------------|-----------------|
| $q_i^\ast(m_k)q_j^\ast(m_l) = D(m)$ | Meson(m) | $q_i^\ast(m_k)q_j^\ast(m_l)$ |
| Charmed Mesons (C = ±1) | | $c\pi,cd$ |
| $q_C^\ast(2271)q_N^\ast(931) = D(1866)$ | $\bullet \, D(1869)$ | $c(1250)u(3), \, c(1250)d(6)$ |
| $q_C^\ast(2441)q_N^\ast(931) = D(2029)$ | $\bullet \, D(2008)$ | $c(1250)u(3), \, c(1250)d(6)$ |
| $q_C^\ast(2531)q_N^\ast(931) = D(2115)$ | ? | $c(1250)u(3), \, c(1250)d(6)$ |
| $q_C^\ast(2271)q_N^\ast(1471) = D(2349)$ | $\bullet \, D(2420)$ | $c(1250)u(3), \, c(1250)d(6)$ |
| $q_C^\ast(2969)q_N^\ast(931) = D(2526)$ | $\bullet \, D(2507)$ | $c(1250)u(3), \, c(1250)d(6)$ |
| $q_C^\ast(2271)q_N^\ast(1921) = D(2750)$ | ? | $c(1250)u(3), \, c(1250)d(6)$ |
| $q_C^\ast(2271)q_N^\ast(2191) = D(2991)$ | ? | $c(1250)u(3), \, c(1250)d(6)$ |
| $q_C^\ast(6591)q_N^\ast(931) = D(5996)$ | ? | ? |
| $q_C^\ast(13791)q_N^\ast(931) = D(13274)$ | ? | ? |
| Charmed, Strange (C=S=±1) | | $c\bar{s}$ |
| $q_C^\ast(2271)q_S^\ast(1111) = D_S(2021)$ | $\bullet \, D_S(1969)$ | $c(1250)s(123)$ |
| $q_C^\ast(2271)q_S^\ast(1391) = D_S(2126)$ | $\bullet \, D_S^\ast(2112)$ | $c(1250)s(123)$ |
| $q_C^\ast(2961)q_S^\ast(1111) = D_S(2531)$ | $\bullet \, D_S(2555)^\pm$ | $c(1250)s(123)$ |
| $q_C^\ast(2271)q_S^\ast(2551) = D_S(3332)$ | ? | $c(1250)s(123)$ |
| $q_C^\ast(6591)q_S^\ast(1111) = D_S(6151)$ | ? | ? |
| $q_C^\ast(6591)q_S^\ast(2551) = D_S(7215)$ | ? | ? |
| $q_C(13791)q_S^\ast(1111) = D_S(13432)$ | ? | ? |
| $q_C(13791)q_S^\ast(2551) = D_S(14604)$ | ? | ? |
| ... | ... | ... |
Table V. Bottom, Bottom Strange, and Bottom Charmed Mesons

| The BCC Quark Model | Exper. | The Quark Model |
|---------------------|--------|-----------------|
| $q_i^* (m_k) q_j^* (m_l) = B(m)$ | $M(m)$ | $\bar{b}_u, \bar{b}_d$ |
| **Bottom Mesons ($b=\pm 1$)** | | |
| $q_b^* (5531) q_N^* (931) = B (5164)$ | $\bullet B (5279)$ | $b (4200) u (3), b (4200) d (6)$ |
| $q_b^* (5531) q_N^* (1201) = B (5655)$ | $B (5732)$ | $b (4200) u (3), b (4200) d (6)$ |
| $q_b^* (5531) q_N^* (1471) = B (5896)$ | ? | $b (4200) u (3), b (4200) d (6)$ |
| $q_b^* (5531) q_N^* (1831) = B (6217)$ | ? | $b (4200) u (3), b (4200) d (6)$ |
| $q_b^* (9951) q_N^* (931) = B (9504)$ | ? | ? |
| $q_b^* (15811) q_N^* (931) = B (15385)$ | ? | ? |
| **Bottom, Strange ($b=\pm 1, S=\mp 1$)** | | $\bar{b}_s$ |
| $q_b^* (5531) q_S^* (1111) = B_S (5319)$ | $\bullet B_S (5370)$ | $b (4200) s (123)$ |
| $q_b^* (5531) q_S^* (1391) = B_S (5674)$ | $B_S (5850)$ | $b (4200) s (123)$ |
| $q_b^* (5531) q_S^* (2551) = B_S (6639)$ | ? | $b (4200) s (123)$ |
| $q_b^* (9951) q_S^* (1111) = B_S (9659)$ | ? | ? |
| $q_b^* (15811) q_S^* (1111) = B_S (15540)$ | ? | ? |
| **Bottom, Charmed ($b=C=\pm 1$)** | | $\bar{b}_c$ |
| $q_b^* (5531) q_C^* (2271) = B_C (6691)$ | $\bullet B_C (6400)$ | $b (4200) c (1250)$ |
| $q_b^* (5531) q_C^* (6591) = B_C (10822)$ | ? | ? |
| $q_b^* (9951) q_C^* (2271) = B_C (11031)$ | ? | ? |
| ... | ... | ... |
Table VI. The Slightly Heavy Mesons (S = C = b = I = Q = 0)

| The BCC Quark Model | Experiment | Quark Model |
|---------------------|------------|-------------|
| $q_i^*(m_k)q_j^*(m_l) = M(m)$ | Meson(m), Γ | $q_i^*(m_k)q_j^*(m_l)$ |
| (48) $q_C^*(2551)q_S^*(2551) = \eta(2902)$ | $\bullet \eta_C(2980)_{13 Mev}$ | c(1250)c(1250) |
| (120) $q_C^*(2271)q_C^*(2271) = J/\Psi(3044)$ | $\bullet J/\Psi(3097)_{87 Kev}$ | c(1250)c(1250) |
| (24) $q_S^*(2641)q_S^*(2641) = \eta(3394)$ | $\bullet \chi(3415)_{14 Mev}$ | c(1250)c(1250) |
| (24) $q_S^*(2731)q_S^*(2731) = \eta(3535)$ | $\bullet \chi(3510)_{0.88 Mev}$ | c(1250)c(1250) |
| (40) $q_C^*(2441)q_C^*(2441) = \psi(3568)$ | $\bullet \chi(3556)_{2.0 Mev}$ | c(1250)c(1250) |
| (60) $q_C^*(2271)q_C^*(2531) = \psi(3693)$ | $\bullet \psi(3686)_{277 Kev}$ | c(1250)c(1250) |
| (40) $q_C^*(2531)q_C^*(2531) = \psi(3740)$ | $\bullet \psi(3770)_{24 Mev}$ | c(1250)c(1250) |
| (30) $q_C^*(2441)q_C^*(2531) = \psi(3854)$ | $\psi(3836)$ | c(1250)c(1250) |
| (54) $q_S^*(1111)q_S^*(4271) = \eta(3952)$ | $\bullet \psi(4040)_{52 Mev}$ | c(1250)c(1250) |
| (60) $q_C^*(2271)q_C^*(2961) = \psi(4104)$ | $\bullet \psi(4160)_{78 Mev}$ | c(1250)c(1250) |
| (40) $q_C^*(2961)q_C^*(2961) = \psi(4554)$ | $\bullet \psi(4415)_{43 Mev}$ | c(1250)c(1250) |
| (36) $q_S^*(2551)q_S^*(4271) = \eta(4944)$ | $\bullet \psi(4104)_{52 Mev}$ | c(1250)c(1250) |
| (75) $q_C^*(6591)q_C^*(2271) = \psi(7374)$ | $\bullet \psi(7374)$ | c(1250)c(1250) |
| (75) $q_C^*(13791)q_C^*(2271) = \psi(14654)$ | $\bullet \psi(14654)$ | c(1250)c(1250) |
| (80) $q_C^*(13791)q_C(13791) = \psi(25596)$ | $\bullet \psi(25596)$ | c(1250)c(1250) |
In summary, the BCC Quark Model explains the experimental intrinsic quantum numbers and masses of all mesons. Virtually all experimentally confirmed mesons are included in the BCC Quark Model. However, we do not give out the angular momenta and parities of the mesons. They are needed to consider the wave functions of the quarks, the point groups (the point group $O_h$, the point group $P$, the point group $N$,...) of the body center cubic quark lattice. Those are out the scope of this paper.
VI  Evidence for Some New Quarks

According to the confinement theory [9] of the Quark Model and the accompanying excitation concept of the BCC Quark Model [12], we cannot see any individual quark. We can only infer the existence of quarks from the existence of baryons and mesons. Thus, if we find the mesons which were made of certain quarks, it means that we find the quarks. For example, from meson \( J/\Psi(3097) = [q_C^*(2271)\bar{q}_C^*(2271)] \), we discovered the quark \( q_C^*(2271) \). Similarly, the experimental meson spectrum [3] has already provided some evidence of the new quarks \( q_S^*(1391), q_S^*(2551), \) and \( q_C^*(6591) \)....

A  Four Brother Quarks \( q_S^*(1111), q_S^*(2551), q_b^*(5531), \) and \( q_b^*(9951) \)

From Fig. 5 (c) (see Appendix I) of the BCC Quark Model [10], we see the following four “brother” quarks: at \( E_N = 1/2, \overrightarrow{n} = (1,1,0), q_S^*(1111) \); at \( E_N = 9/2, \overrightarrow{n} = (2,2,0), q_S^*(2551) \); at \( E_N = 25/2, \overrightarrow{n} = (3,3,0), q_b^*(5531) \); and at \( E_N = 49/2, \overrightarrow{n} = (4,4,0), q_b^*(9951) \). They are born on the single energy bands of the \( \Sigma \)-axis and at the same symmetry point \( N \). The four “brothers” have the same isospin (\( I = 0 \)), the same electric charge (\( Q = -1/3 \)), and the same generalized strange number (\( S_G = S + C + b = -1 \)).

A1. The baryons made by the brothers have \( I = 0, Q = 0, S_G = -1 \), with long life times [1]:

\[
\begin{align*}
q_S^*(1111)q'_S(3)q'_d(6) & \leftrightarrow \Lambda(1120), \tau = (2.632 \pm 0.020) \times 10^{-10} s; \\
q_S^*(2551)q'_S(3)q'_d(6) & \leftrightarrow \Omega_C^0(2704), \tau = (0.64 \pm 0.020) \times 10^{-11} s; \\
q_b^*(5531)q'_b(3)q'_d(6) & \leftrightarrow \Lambda_b(5540) \leftrightarrow \Lambda_b(5641), \tau = (1.14 \pm 0.08) \times 10^{-12} s; \\
q_b^*(9951)q'_b(3)q'_d(6) & \leftrightarrow \Lambda_b(9960) \leftrightarrow ?, ?. 
\end{align*}
\]

A2. The mesons that are composed of four brothers and their own antiquarks are:

\[
\begin{align*}
q_S^*(1111)q_S^*(1111) & \leftrightarrow [\eta(549), \Gamma = (1.18 \pm 0.11) kev] \\
q_S^*(2551)q_S^*(2551) & \leftrightarrow [\eta_c(2969), \Gamma = (13.2) Mev] \\
q'_b(5531)q'_b(5531) & \leftrightarrow [\Upsilon(1S)(9489), \Gamma = (52.5 \pm 1.8) kev], \\
q_b^*(9951)q_b^*(9951) & \leftrightarrow [\Upsilon(1S)(9460), \Gamma = (52.5 \pm 1.8) kev].
\end{align*}
\]

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A3. The mesons that are composed of quark $q^*_N(931)$ and four brother antiquarks are:

| $q^*_N(931)q^*_S(1111)$ | $K(494) \leftrightarrow [K(494), \Gamma = (1.2386 \pm 0.0024) \times 10^{-8}]$ |
|--------------------------|------------------------------------------------------------------|
| $q^*_N(931)q^*_S(2551)=K(1804) \leftrightarrow [K(1820), \Gamma = 276 \pm 35 \text{ Mev}]$ |
| $q^*_N(931)q^*_b(5531)=B(5164) \leftrightarrow [B(5279), \Gamma = (1.62 \pm 0.06) \times 10^{-12}]$ |
| $q^*_N(931)q^*_b(9951)=B(9504) \leftrightarrow (?. ?)$ |

(56)

A4. The mesons that are composed of quark $q^*_S(1111)$ and four brother antiquarks are:

| $q^*_S(1111)q^*_S(1111)$ | $\eta(547) \leftrightarrow [\eta(549), \Gamma = (1.18 \pm 0.11 \text{ kev})]$ |
|--------------------------|------------------------------------------------------------------|
| $q^*_S(1111)q^*_S(2551)\eta(1960) \leftrightarrow [\eta(1950), \Gamma = 208 \text{ Mev}]$ |
| $q^*_S(1111)q^*_b(5531)=B(5319) \leftrightarrow [B(5279), \Gamma = (1.61 \pm 0.10) \times 10^{-12}]$ |
| $q^*_S(1111)q^*_b(9951)=B(9659) \leftrightarrow (?. ?)$ |

(57)

There is much evidence of the first three brother quarks, $q^*_S(1111)$, $q^*_S(2551)$, and $q^*_b(5531)$.

B The Quark $q^*_S(1391)$

There are three brother quarks, $q^*_S(1391)$, $q^*_S(4271)$, and $q^*_S(10031)$. They are born on the single energy bands of the $\Delta$-axis at the same symmetry point ($\Gamma$) from $\Delta S = -1$(see Fig. 5 b of Appendix I). Since the quarks $q^*_S(4271)$ and $q^*_S(10031)$ have higher energies, we will see the evidence later (the $q_Cq^*_C$ Mesons and the $q_bq^*_b$ Mesons). We only study the quark $q^*_S(1391)$ now.

B1. The baryons of the quarks

$$q^*_S(1391)q^*_u(3)q^*_d(6) = \begin{cases} \Lambda(1400) \leftrightarrow \Lambda(1405) \\ \Sigma(1400) \leftrightarrow \Sigma(1385) \end{cases}$$

(58)
B2. The meson that is composed of the quark $q_S^*(1391)$ and its own antiquarks

$$q_S^*(1391)q_S^*(1391) = \eta(952) \leftrightarrow \eta'(958) \quad (59)$$

B3. The meson that is composed of $q_S^*(1391)$ and the ground quark $q_N(931)$

$$q_N^*(931)q_S^*(1391) = K(899) \leftrightarrow K(892) \quad (60)$$

B4. The meson that is composed of $q_S^*(1391)$ and the ground quark $q_C(2271)$

$$q_C^*(2271)q_S^*(1391) = D_S(2126) \leftrightarrow D_s^*(2112) \quad (61)$$

B5. The meson that is composed of $q_S^*(1391)$ and the ground quark $q_b(5531)$

$$q_b^*(5531)q_S^*(1391) = B_S(5674) \leftrightarrow B_{sj}^*(5850) \quad (62)$$

There is enough evidence to show that the quark $q_S^*(1391)$ really exists.

C  The $q_Cq_C^*$ Mesons → $q_S^*(2551)$

According to the Quark Model, the 10 experimental mesons [$\eta_C(2980)$ $\Gamma=13$ Mev, $J/\Psi(3097)$ $\Gamma=87$ kev, $\chi_{co}(3415)$ $\Gamma=15$ Mev, $\chi_{c1}(3511)$ $\Gamma=0.88$ Mev, $\chi_{c2}(3556)$ $\Gamma=2.0$ Mev, $\psi(3686)$ $\Gamma=277$ kev, $\psi(3770)$ $\Gamma=24$ Mev, $\psi(4040)$ $\Gamma=52$ Mev, $\psi(4160)$ $\Gamma=78$ Mev, and $\psi(4415)$ $\Gamma=43$ Mev] are all $c(1250)c(1250)$ with different angular momenta and parities (see Table VI).

However, the ground state $\eta_C(2980)$ ($\Gamma=13$ Mev=1300 kev) of $c\bar{c}$ is not the most long lived meson. The most long lived meson of $c(1250)c(1250)$ is $J/\Psi(3097)$ ($\Gamma=87$ kev). This means that $\eta_C(2980)$ is not the ground state, while $J/\Psi(3097)$ is the ground state. What is $\eta_C(2980)$? The BCC Quark Model shows that $\eta_C(2980)$ is $q_S^*(2551)q_S^*(2551)$. 

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Using the quark $q_C^*(2551)$, we can explain the puzzle, i.e. why $J/\Psi(3097)$ has a longer lifetime ($\Gamma = 87$ kev) than $\eta_C(2969)$ ($\Gamma = 13$ Mev=1300 kev). According to the BCC Quark Model, $\eta_C(2969)$ is composed of $q_C^*(2551)$ and $q_C^*(2271)$, but $J/\Psi(3097)$ is made of $q_C^*(2271)$ and $q_C^*(2271)$. They are not the same kind of mesons. The quark pair $q_C^*(2271)q_C^*(2271)$ ($J/\Psi(3097)$) has a better chance ($O(q_C^*(2271)q_C^*(2271)) = 120$ units) than the pair quark $q_S^*(2551)q_S^*(2551)$ ($\eta_C(2980)$) has ($O(q_S^*(2551)q_S^*(2551)) = 48$ units). Thus, $J/\Psi(3097)$ has a longer lifetime than $\eta_C(2969)$ has.

The experimental mesons show that the quark $q_S^*(2551)$ really exists. At the same time, the quark $q_S^*(4271)$ may exist too. It is inside the meson $[(54)\cdot q_S^*(1111)q_S^*(4271) = \chi(3952) \rightarrow \psi(4040)]$ and the meson $[(54)\cdot q_S^*(1391)q_S^*(4271) = \chi(4105) \rightarrow \psi(4160)]$ (see Table VI).

D  The $q_b\bar{q}_b$ Mesons $\rightarrow q_C^*(6591)$

According to the Quark Model, there are 12 experimental mesons that are composed of $q_b^*(5531)q_b^*(5531)$: $\Upsilon(1S)(9460)$ ($\Gamma = 53$ kev), $\chi_{b0}(1P)$ (9860), $\chi_{b1}(1P)$ (9893), $\chi_{b2}(1P)$ (9913), $\Upsilon(2S)(10023)$ ($\Gamma = 44$ kev), $\chi_{b0}(2P)$ (10232), $\chi_{b1}(2P)$ (10255), $\chi_{b2}(2P)$ (10269), $\Upsilon(3S)(10355)$ ($\Gamma = 26$ kev), $\Upsilon(4S)(10580)$ ($\Gamma = 14$ Mev), $\Upsilon(10860)$ ($\Gamma = 110$ Mev), and $\Upsilon(11020)$ ($\Gamma = 79$ Mev). The ground state is $q_b^*(5531)q_b^*(5531) = \Upsilon(1S)(9460)$ ($\Gamma = 52$ kev).

First, it looks like there are too many mesons corresponding to the same quark pair ($q_b^*(5531)q_b^*(5531)$). Second, there exists a puzzle—why do the excitation $\Upsilon(3S)(10355)$ ($\Gamma = 26$ kev) have longer lifetimes than the ground state $\Upsilon(1S)(9460)$ ($\Gamma = 52$ kev)? According to the BCC Quark Model, they are not all the excitations of $q_b^*(5531)q_b^*(5531)$. The meson with the longest lifetime ($\tau \sim 1/\Gamma$) is not the meson $\Upsilon(1S)(9460)$ ($\Gamma = 52$ kev) [$q_b^*(5531)q_b^*(5531)$]. It is the meson $\Upsilon(3S)(10355)$ ($\Gamma = 26$ kev). It cannot be an excited state of the $q_b^*(5531)q_b^*(5531)$ (the explanation of the Quark Model). There is not any quark pair that can explain it in the Quark Model. However, the BCC Quark Model can
explain it using the quark pair $(80)q_C^*(6591)q_C^*(6591) = \Upsilon(10792)$ \([I = 0, \, Q = 0, \, S = C = b = 0]\). The meson \(\Upsilon(10792)\) has the longest lifetime \((\tau \sim 1/\Gamma = 1/26 \text{ kev})\). The small error of the mass may come from its angular momentum. These experimental results show that there will be a new quark \(q_C^*(6591)\) in addition to the five quarks \((u, d, s, c, \text{ and } b)\) of the Quark Model. There may be another new quark, \(q_S^*(10031)\), also. It will be inside the meson \(\eta(9734)\) \([(36)\bullet q_S^*(1111)q_S^*(10031) = \eta(9734)]\), the meson \(\eta(9955)\) \([(36)\bullet q_S^*(1391)q_S^*(10031) = \eta(9955)]\), the meson \(\eta(10446)\) \([(27)\bullet q_S^*(2451)q_S^*(10031) = \eta(10446)]\), the meson \(\eta(10791)\) \([(27)\bullet q_S^*(2451)q_S^*(10031) = \eta(10446)]\), and the meson \(\eta(10870)\) \([(36)\bullet q_S^*(2551)q_S^*(10031) = \eta(10870)]\) (see Table VII).

Summarizing the section, the experimental meson spectrum provides some evidence that shows the new quarks \(q_S^*(1391), q_S^*(2551), \text{ and } q_C^*(6591)\) really exist in nature. The new quarks \(q_S^*(4271)\) and \(q_S^*(10031)\) may exist also. Thus, there are three “brother” quark families: (1) the three brothers \(q_S^*(1111), q_S^*(2551), \text{ and } q_S^*(5531)\); (2) the three brothers \(q_S^*(1391), q_S^*(4271), \text{ and } q_S^*(10031)\); (3) the two brothers \(q_C^*(2271), q_C^*(6591)\). Therefore, we have previously shown that the experimental meson spectrum supports the BCC Quark Model.

\section*{VII Predictions and Discussion}

\subsection*{A Some New Mesons}

According to the BCC Quark Model (see Appendix II), a series of possible new mesons with high energies exist. However, when energy goes higher and higher, on one hand, the theoretical mesons will become denser and denser; while on the other hand, the experimental full widths of these mesons will become wider and wider. This case makes these new mesons extremely difficult to separate. Therefore, currently it is very difficult to discover the higher energy mesons predicted by the BCC Quark Model. We
believe that many new mesons will be discovered in the future with the development of more sensitive experimental techniques. The following new mesons predicted by the model seem to have a better chance of being discovered in the near future:

A1. The slightly higher mass mesons

| O(I) | \( q_k^* (m_k) q_l^* (m_l) = M(m) \) | Quantum numbers |
|------|---------------------------------|-----------------|
| (60) | \( q_{N}^* (931) q_{S}^* (4271) = K(3597) \); | \( S = 1, b = 0, C = 0, I = \frac{1}{2}, Q = 1, 0 \) |
| (38) | \( q_{C}^* (2271) q_{N}^* (1921) = D(2750) \); | \( S = b = 0, C = 1, I = \frac{1}{2}, Q = 1, 0 \) |
| (38) | \( q_{C}^* (2271) q_{N}^* (2191) = D(2991) \); | \( S = b = 0, C = 1, I = \frac{1}{2}, Q = 1, 0 \) |
| (36) | \( q_{b}^* (2271) q_{S}^* (2011) = D_{S}(2690) \); | \( S = C = 1, b = 0, I = 0, Q = 1 \) |
| (42) | \( q_{C}^* (2271) q_{S}^* (2551) = D_{S}(3332) \); | \( S = C = 1, b = 0, I = 0, Q = 1 \) |
| (26) | \( q_{b}^* (5531) q_{S}^* (1471) = B_{S}(5896) \); | \( S = C = 0, b = 1, I = \frac{1}{2}, Q = 1, 0 \) |
| (26) | \( q_{b}^* (5531) q_{N}^* (1831) = B_{S}(6217) \); | \( S = C = 0, b = 1, I = \frac{1}{2}, Q = 1, 0 \) |
| (24) | \( q_{b}^* (5531) q_{S}^* (2011) = B_{S}(6292) \); | \( S = -b = 1, C = 0, I = 0, Q = 0 \) |
| (30) | \( q_{b}^* (5531) q_{S}^* (2551) = B_{S}(6629) \); | \( S = -b = 1, C = 0, I = 0, Q = 0 \) 

A2. The high mass mesons

| O(I) | \( q_k^* (m_k) q_l^* (m_l) = M(m) \) | Quantum numbers |
|------|---------------------------------|-----------------|
| (68) | \( q_{N}^* (931) q_{S}^* (6591) = D_{S}(5996) \); | \( S = b = 0, C = 1, I = \frac{1}{2}, Q = 0,-1 \) |
| (32) | \( q_{C}^* (6591) q_{S}^* (1111) = D_{S}(6151) \); | \( S = C = 1, b = 0, I = 0, Q = 1 \) |
| (36) | \( q_{N}^* (931) q_{b}^* (9951) = B_{S}(9504) \); | \( S = C = 0, b = 1, I = \frac{1}{2}, Q = 1, 0 \) |
| (24) | \( q_{b}^* (9951) q_{S}^* (1111) = B_{S}(9659) \); | \( S = -b = 1, C = 0, I = 0, Q = 0 \) |
| (48) | \( q_{S}^* (4271) q_{S}^* (4271) = \eta(5926) \); | \( S = 0, C = b = 0, I = 0, Q = 0 \) |
| (32) | \( q_{b}^* (5531) q_{C}^* (6591) = B_{C}(10822) \); | \( S = 0, C = b = -1 I = 1, Q = -1 \) |

A3. The super mass mesons

| O(I) | \( q_k^* (m_k) q_l^* (m_l) = \eta(m) \) | Quantum numbers |
|------|---------------------------------|-----------------|
| (48) | \( q_{S}^* (10031) q_{S}^* (10031) = \chi(17837) \); | \( S = C = b = I = Q = 0 \) |
| (80) | \( q_{C}^* (13791) q_{C}^* (13791) = \psi(25596) \); | \( S = C = b = I = Q = 0 \) |
| (48) | \( q_{b}^* (9951) q_{b}^* (9951) = \Upsilon(17805) \); | \( S = C = b = I = Q = 0 \) |
| (48) | \( q_{b}^* (15811) q_{b}^* (15811) = \Upsilon(29597) \); | \( S = C = b = I = Q = 0 \) |

A4. The high isospin mesons

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The BCC Quark Model predicts many high isospin mesons (I = 3/2, I = 2, and I = 5/2, and I = 3). They cannot be predicted by the Quark Model. They have not been discovered by experiment (except a meson $\chi(1600)$ with I = 2) yet. We list the high isospin mesons predicted by the BCC Quark Model in Appendix III.

The mesons $T(m)$ with $I = 2$ (from $q^*_N(931)q\Delta(m)$) have $O(q^*_i q^*_j) = 37$ (>$24$). Thus they should be discovered by today’s experiments. However, there are five members ($I_z=2, Q=2; I_z=1, Q=1; I_z=0, Q=0; I_z=-1, Q=-1; I_z=-2, Q=-2$) in the mesons. The member with $Q = 2$ will be composed by $q^*_N(931)^{2/3}(Q=2/3)$ and $q\Delta(Q=4/3)$. The two quarks with the same kind of electric charges will repel each other with 4 times ($2/3 \times 4/3 = 8/9$) the strength that the mesons with $Q = 1$ ($1/3 \times 2/3 = 2/9$) exhibit. Therefore, the members are very difficult to find. If we could not find the members with $Q = \pm 2$, we would find the other three members with $Q = 1, 0, -1$. They may be observed as the $\pi$ mesons with $S=C=b=0, I=1, Q=1, 0, -1$.

Although the mesons with $I = 2$ are very difficult to find, our great experimental physicists have already discovered one--$\chi(1600)$ [$I^G(J^{PC}) = 2^+(2^{++})$] [4]. According to the BCC Quark Model, there is a series of mesons with $I = 2$:

| $O(T)q(m)\overline{q(m)}$=Meson(m) | Experiment[$I^G(J^{PC})$] | May observed |
|---------------------------------|-----------------|-------------|
| $37q^*_N(931)\overline{q}_\Delta(1291)$=T(960) | ? | $\pi(960), I=1, Q=1, 0, -1$ |
| $37q^*_N(931)\overline{q}_\Delta(1651)$=T(1282) | ? | $\pi(1282), I=1, Q=1, 0, -1$ |
| $37q^*_N(931)\overline{q}_\Delta(2011)$=T(1603) | $\chi(1600)^* [2^+(2^{++})]$ | $\pi(1603), I=1, Q=1, 0, -1$ |
| $37q^*_N(931)\overline{q}_\Delta(2371)$=T(1924) | ? | $\pi(1924), I=1, Q=1, 0, -1$ |
| $37q^*_N(931)\overline{q}_\Delta(2731)$=T(2246) | ? | $\pi(2246), I=1, Q=1, 0, -1$ |

* The meson $\chi(1600)$ has not been established. It still needs to be confirmed.
B Experimental Verification of The BCC Quark Model

From the preceding predictions, we can find the key experiments that determine whether the BCC Quark Model is a good modification of the Quark Model.

B1. The Quark Model, from 6 flavored quarks \([u_{2/3}(3 \text{ Mev}), d_{-1/3}(6 \text{ Mev}), s_{-1/3}(123 \text{ Mev}), c_{2/3}(1.25 \text{ Gev}), b_{-1/3}(4.2 \text{ Gev}), \text{ and } t_{2/3}(174 \text{ Gev})]\) and the formula meson = \(q\bar{q}\), cannot give any meson with isospin \(I = 2\). However, the BCC Quark Model predicts many mesons with \(I = 2\) [see (66)]. If we can discover any meson with \(I = 2\), we have shown that the BCC Quark Model is a good modification of the Quark Model.

B2. The Quark Model cannot give any meson with \(100000 > M > 15000 \text{ Mev}\). However, the BCC Quark Model predicts many mesons with \(100000 > M > 15000 \text{ Mev}\), such as:

| O(I=0) q(m)q(m) = Meson(M) (Quantum Numbers) |
|-----------------------------------------------|
| (80)\(\cdot\)q_5^{c}(13791)\bar{q}_c^{c}(13791) = \psi(25596) (S = C = b = 0, I = 0; Q = 0), |
| (48)\(\cdot\)q_b^{c}(9951)\bar{q}_b^{c}(9951) = \Upsilon(17806) (S = C = b = 0, I = 0; Q = 0), |
| (48)\(\cdot\)q_b^{c}(15811)\bar{q}_b^{c}(15811) = \Upsilon(29597) (S = C = b = 0, I = 0; Q = 0), |
| (48)\(\cdot\)q_S^{c}(10031)\bar{q}_S^{c}(10031) = \eta(17837) (S = C = b = 0, I = 0, Q = 0). |

If we can discover the mesons with \(100000 > M > 15000 \text{ Mev}\), we will show that the BCC Quark Model is a good modification of the Quark Model. Thus, we propose to search for the above mesons. The discovery of any one of the above mesons will provide strong support for the BCC Quark Model.

C Discussions

C1. The BCC Quark Model deals mainly with the low energy state properties of the baryons and the mesons. In the low energy cases, we shall consider the periodic field of the body center cubic quark lattice. However, in the high energy scattering cases of the baryons and mesons, because the strong interactions (color) of the quarks are short range and saturable (a baryon that is composed of three different colored quarks
is a colorless system), we can only consider the three quark system (the primitive cell approximation of the BCC Quark Model [13]). In this approximation, we consider the excited quark ($q^*$) and the primitive cell ($u' + d'$) only, omitting the quark lattice. Thus, there are only three quarks in the system of a baryon: one excited quark ($q^*$) and two accompanying excited quarks ($u'$ and $d'$). Similarly, there are only two quarks ($q_i^*, q_j^*$) in a meson. In other words, in high energy scattering cases, the Quark Model is an excellent approximation of the BCC Quark Model. We do not need to consider the whole BCC quark lattice.

C2. Similar to the discovery of new stars in the sky with the improvements of the telescope, many new mesons and new baryons will be discovered with the birth of new techniques.

C3. There are always some limitations for any physical theory. If the theory is applied outside the limitations, it will not be useful. The Quantum Field Theory is no exception. It is powerful in dealing with the point particles’ (the point model’s) scattering problems; however, for the bound states, which are constructed by many particles, it is not so powerful. Thus, for a long time we have been looking forward to the day when a new theory will be born. Not only will it deal with the scattering problems, but also it will solve the bound state problems. The BCC Quark Model might play the role of providing a hint along the path to a new theory, like the Bohr atom model did to quantum mechanics. The new theory may be completely different from the quantum field theory in mathematical form, as quantum mechanics is so different from classic mechanics in the mathematical form.

VIII Conclusions

1. Using the phenomenological formula of the binding energies of the mesons, from the quark spectrum [8] and [9] of the BCC Quark Model, we deduce a meson spectrum
that agrees well with experimental results. These mass spectrum of mesons has not been obtained by any other model, including the Quark Model.

2. The BCC Quark Model does not need the mixture of three quark-antiquark pairs ($u\bar{u}$, $d\bar{d}$, and $s\bar{s}$) to explain the mesons ($\eta$, $\omega$, $\phi$, $h$, and $f$) with $I = 0$ and $S=C=b=0$ as the Quark Model does. The BCC Quark Model really has enough quarks to construct the full meson spectrum, according to the principle that a meson is made of a quark and an antiquark. Thus, the BCC Quark Model saves this principle of the Quark Model.

3. The quarks $q^*_S(1391)$, $q^*_S(2551)$, and $q^*_C(6591)$, that are predicted by the BCC Quark Model, might have been discovered by experiments already.

4. The Quark Model assumes that there are five independent elementary quarks ($u$, $d$, $s$, $c$, and $b$)—they are the five ground states of the quark spectrum of the BCC Quark Model [see (5)]. The Quark Model uses only these five quarks to explain the baryon spectrum and the meson spectrum. Therefore, the Quark Model is an approximation of the five ground states of the BCC Quark Model.

5. After the BCC Quark Model successfully deduces the baryon spectrum [14], it also successfully deduces the meson spectrum. These results show that the vacuum material [15] really has the body center cubic symmetries (the point groups and the space group).

6. In this paper, we use only the energy bands and symmetries of the BCC quark lattice (half of the results of the free particle approximation—0 order approximation). We need to consider the symmetry wave functions (the other half of the results of the free particle approximation) of the quarks to find the angular momenta and parities of the quarks, the mesons, and the baryons (see our next paper: Xin Yu and Jiao-Lin Xu, “The Symmetry Quark Wave Functions of the BCC Quark Model”). We will do higher order approximation later.

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Appendix I
Figures of the Energy Bands

Fig. 1. The first Brillouin zone of the body center cubic lattice. The symmetry points and axes are indicated. The center of the first Brillouin zone is at the point \( \Gamma \) with 48 symmetry operations, the general ground quark \( q_N^*(931) \) will bear at this point. The \( \Delta \)-axis (the axis \( \Gamma-H \)) is a 4 fold rotation axis with 8 symmetry operations and the strange number \( S = 0 \). The \( q_\Delta^* \) quark families \( (q_{\Delta}^{5/3}, q_{\Delta}^{2/3}, q_{\Delta}^{-1/3}, q_{\Delta}^{-4/3}) \) will appear on the axis. The \( \Lambda \)-axes (the axis \( \Gamma-P \)) and the \( F \)-axis (the axis \( P-H \)) are 3 fold rotation axes with 6 symmetry operations and the strange number \( S = -1 \); the \( q_\Sigma^* \) quark families \( (q_{\Sigma}^{2/3}, q_{\Sigma}^{-1/3}, q_{\Sigma}^{-4/3}) \) and the \( q_\Lambda^* \) quarks will appear on these axes. The \( \Sigma \)-axis (the axis \( \Gamma-N \)) and the \( G \)-axis (the axis \( M-N \)) are 2 fold rotation axes with 4 symmetry operations and the strange number \( S = -2 \); the \( q_\Xi^* \) quark families \( (q_{\Xi}^{-1/3}, q_{\Xi}^{-4/3}) \) will appear on these axes. The \( D \)-axis (the axis \( P-N \)) is parallel to the \( \Delta \)-axis, \( S = 0 \), and the axis is a 2 fold rotation axis; the quark \( q_N^* \) families \( (q_N^{2/3}, q_N^{-1/3}) \) will be on the axis.

Fig. 2. (a) The energy bands on the \( \Delta \)-axis (the axis \( \Gamma-H \)). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy. \( E_\Gamma \) is the value of \( E(\vec{k}, \vec{n}) \) at the end point \( \Gamma \), while \( E_H \) is the value of \( E(\vec{k}, \vec{n}) \) at the other end point \( H \). (b) The energy bands on the \( \Lambda \)-axis (the axis \( \Gamma-P \)). \( E_\Gamma \) is the value of \( E(\vec{k}, \vec{n}) \) at the end point \( \Gamma \), while \( E_P \) is the value of \( E(\vec{k}, \vec{n}) \) at the other end point \( P \). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy.

Fig. 3. (a) The energy bands on the \( \Sigma \)-axis (the axis \( \Gamma-N \)). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy. \( E_\Gamma \) is the value of \( E(\vec{k}, \vec{n}) \) at the end point \( \Gamma \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at the other end point \( N \). (b) The energy bands on the \( D \)-axis (the axis \( P-N \)). \( E_P \) is the value of \( E(\vec{k}, \vec{n}) \) at the end point \( P \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at the other end point \( N \).
\( \vec{n} \) at the other end point N. The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy.

Fig. 4.  (a) The energy bands on the \( F \)-axis (the axis P-H). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy. \( E_P \) is the value of \( E(\vec{k}, \vec{n}) \) at the end point \( P \), while \( E_H \) is the value of \( E(\vec{k}, \vec{n}) \) at the other end point \( H \). (b) The energy bands on the \( G \)-axis (the axis M-N). \( E_M \) is the value of \( E(\vec{k}, \vec{n}) \) at the end point \( M \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at the other end point \( N \). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy.

Fig. 5.  (a) The 4 fold degenerate energy bands (selected from Fig. 2(a)) on the \( \Delta \)-axis (the axis \( \Gamma \)-H). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the numbers of the degeneracy of the energy bands.  (b) The single energy bands (selected from Fig. 2(a)) on the \( \Delta \)-axis (the axis \( \Gamma \)-H). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \).  (c) The single energy band (selected from Fig. 3(a)) on the \( \Sigma \)-axis (the axis \( \Gamma \)-N). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \).
## Appendix II

The Comparing of the Theory Meson Spectrum with the Experimental Results

| Table I. Light Unflavored Mesons |
|----------------------------------|
| **The BCC Quark Model** | **Experiment** | **The Quark Model** |
| $[d_q+d_{ar{q}}+O(I)]q^+_N(m_k)q^+_N(m_q)$=$M(m)$ | $M_{mass}(m)$ | $I$ |
| $1+1+192\bullet q^+_N(931)\overline{q}^+_N(931)$=$\pi(139)1$ | $\bullet \pi(139)1$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+1+96\bullet q^+_N(1111)\overline{q}^+_N(1111)$=$\eta(549)0$ | $\bullet \eta(549)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+42\bullet q^+_N(931)\overline{q}_N(1201)$=$\pi(780)1$ | $\bullet \rho(770)1$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+1+84q^+_N(931)\overline{q}_N(1201)$=$\eta(780)0$ | $\bullet \omega(782)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+32\bullet q^+_N(1201)\overline{q}_N(1201)$=$\pi(813)0$ | $f_0(000-1200)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+48q^+_S(1391)\overline{q}_S(1391)$=$\eta(952)0$ | $\bullet \eta'(958)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+24\bullet q^+_S(1201)\overline{q}_S(1201)$=$\eta(962)0$ | $f_0(980)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+32\bullet q^+_S(1291)\overline{q}_S(1291)$=$\eta(967)0$ | $\bullet f_0(980)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+56\bullet q^+_N(931)\overline{q}_N(1291)$=$\pi(960)1$ | $\bullet \eta(967)0$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+1+24\bullet q^+_S(1291)\overline{q}_S(1291)$=$\tau(967)1$ | $\bullet \rho(980)1$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+1+32\bullet q^+_S(1201)\overline{q}_S(1201)$=$\pi(979)1$ | $\bullet \eta(980)1$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+1+42\bullet q^+_N(931)\overline{q}_N(1471)$=$\eta(1021)0$ | $\bullet \phi(1020)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+54\bullet q^+_S(1111)\overline{q}_S(1391)$=$\eta(1024)0$ | $\bullet b_1(1170)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+1+30\bullet q^+_S(1111)\overline{q}_S(1201)$=$\pi(1206)1$ | $\bullet b_1(1235)1$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+2+56\bullet q^+_N(931)\overline{q}_N(1651)$=$\eta(1282)1$ | $\bullet \eta(1265)1$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+1+32\bullet q^+_N(1471)\overline{q}_N(1471)$=$\eta(1269)0$ | $\bullet \phi(1285)0$ | $u_3\overline{u}_3,d_6\overline{d}_6,s_{123}s_{123}$ |
| $1+2\times42\bullet q^+_N(931)\overline{q}_N(1831)$=$\pi(1342)1$ | $\bullet \pi(1300)1$ | $u_3\overline{d}_6,u_3\overline{u}_5,d_6\overline{d}_6$ |
| $1+2+84 \bullet q^*_N(931)q^*_N(1831) = \eta(1342)$ | $\bullet f_0(1370)$  
$\bullet h_1(1380)$  
$\bullet \eta(1375)$  
$u_3u_3d_6d_6s_{123}s_{123}$ |
|-------------------------------------------------|-------------------------------------------------|
| $1+4+84 \bullet q^*_N(931)q^*_N(1921) = \eta(1423)$  
$1+3+24 \bullet q^*_\Delta(1291)q^*_\Delta(1651) = \eta(1473)$ | $\bullet f_1(1420)$  
$\bullet \omega(1420)$  
$f_2(1430)$  
$\eta(1440)$  
$\bullet \eta(1428)$  
$u_3u_3d_6d_6s_{123}s_{123}$ |
| $1+4+42 \bullet q^*_N(931)q^*_N(1921) = \pi(1423)$  
$2+2+24 \bullet q^*_\Delta(1511)q^*_\Delta(1651) = \pi(1565)$ | $\bullet a_0(1450)$  
$\bullet \rho(1450)$  
$\bullet \pi(1450)$  
$u_3d_6u_3d_6d_6$ |
| $2+2+32 \bullet q^*_\Delta(1651)q^*_\Delta(1651) = \eta(1565)$ | $\bullet f_0(1500)$  
$f_1(1510)$  
$f'_2(1525)$  
$f_2(1565)$  
$\bullet \eta(1525)$  
$u_3u_3d_6d_6s_{123}s_{123}$ |
| $1+1+56 \bullet q^*_N(931)q^*_\Delta(2011) = \pi(1603)$  
$1+3+30 \bullet q^*_S(1111)q^*_S(1651) = \pi(1616)$ | $\pi_1(1600)$  
$u_3d_6u_3d_6d_6$ |
| $1+1+37 \bullet q^*_N(931)q^*_\Delta(2011) = T(1603)$ | $\chi(1600)$  
? |
| $1+2+84 \bullet q^*_N(931)q^*_N(2191) = \eta(1664)$  
$1+1+30 \bullet q^*_S(1111)q^*_S(2011) = \eta(1669)$ | $f_2(1640)$  
$\eta_2(1645)$  
$\bullet \omega(1650)$  
$\chi(1650)$  
$\bullet \omega_3(1670)$  
$\phi_2(1680)$  
$\bullet \eta(1656)$  
$u_3u_3d_6d_6s_{123}s_{123}$ |
| $1+2+42 \bullet q^*_N(931)q^*_N(2191) = \pi(1664)$ | $a_1(1640)$  
$a_2(1660)$  
$\bullet \pi_2(1670)$  
$\rho_3(1690)$  
$\rho(1700)$  
$\pi(1672)$  
$u_3d_6u_3u_3d_6d_6$ |
| Equation                                                                 | Representations                           | Symbols                                                                 |
|-------------------------------------------------------------------------|------------------------------------------|----------------------------------------------------------------------|
| $3 + 3 + 24 \cdot q_8^+$ (1651) $q_8^+$ (1651) = $\eta (1758)$          | $\eta (1760)$                            | $\eta (1760)$                                                   |
| $1 + 1 + 45 \cdot q_8^+$ (1111) $q_8^+$ (2011) = $\eta (1768)$         | $f_2 (1810)$                             | $\eta (1760)$                                                   |
| $1 + 5 + 30 \cdot q_8^+$ (1111) $q_8^+$ (1921) = $\pi (1861)$          | $a_2 (1750)$                             | $\pi (1775)$                                                   |
| $2 + 2 + 32 \cdot q_N^+$ (1831) $q_N^+$ (1831) = $\eta (1852)$         | $\eta_2 (1870)$                          | $\eta (1860)$                                                   |
| $1 + 1 + 54 \cdot q_8^+$ (1111) $q_8^+$ (2551) = $\eta (1960)$         | $\chi (1910)$                            | $\eta (1930)$                                                   |
| $1 + 2 + 84 \cdot q_N^+$ (931) $q_N^+$ (2551) = $\eta (1985)$          | $f_2 (1950)$                             | $\pi (1965)$                                                   |
| $1 + 3 + 42 \cdot q_N^+$ (931) $q_N^+$ (2641) = $\pi (2065)$           | $\chi (2000)$                            | $\pi (2035)$                                                   |
| $1 + 3 + 84 \cdot q_N^+$ (931) $q_N^+$ (2641) = $\eta (2065)$          | $\eta_2 (2070)$                          | $\eta (2035)$                                                   |
| $1 + 1 + 24 \cdot q_8^+$ (1291) $q_8^+$ (2371) = $\eta (2086)$         | $f_2 (2010)$                             | $\eta (2132)$                                                   |
| $1 + 1 + 32 \cdot q_8^+$ (2011) $q_8^+$ (2011) = $\eta (2132)$         | $f_2 (2150)$                             | $\eta (2132)$                                                   |
| $1 + 2 + 84 \cdot q_N^+$ (931) $q_N^+$ (2731) = $\eta (2146)$          | $f_2 (2200)$                             | $\eta (2132)$                                                   |
| $1 + 1 + 45 \cdot q_N^+$ (1111) $q_N^+$ (2451) = $\eta (2168)$         | $f_0 (2200)$                             | $\eta (2132)$                                                   |
| $5 + 5 + 24 \cdot q_N^+$ (1921) $q_N^+$ (1921) = $\eta (2220)$         | $f_2 (2250)$                             | $\eta (2132)$                                                   |
| $1 + 1 + 24 \cdot q_8^+$ (2011) $q_8^+$ (2011) = $\pi (2132)$          | $\pi_2 (2100)$                           | $\pi_2 (2100)$                                                  |
| $1 + 2 + 42 \cdot q_N^+$ (931) $q_N^+$ (2731) = $\pi (2146)$          | $\rho (2150)$                            | $\pi_2 (2100)$                                                  |
| $1 + 1 + 24 \cdot q_8^+$ (2011) $q_8^+$ (2011) = $\pi (2132)$          | $\pi_2 (2100)$                           | $\pi_2 (2100)$                                                  |
| Equation | Value | Angle | Omega |
|----------|-------|-------|-------|
| $1+4+56\cdot q_N(931)\overline{q}_\Delta(2731)=\pi(2246)$ | $\rho_3(2250)$ | $u_3\overline{d}_6, u_3\overline{u}_3, d_6\overline{d}_6$ |
| $1+1+30\cdot q_S(1111)\overline{q}_S(2371)=\pi(2271)$ | | |
| $1+1+24\cdot q_S(2011)\overline{q}_S(2011)=\eta(2316)$ | $f_2(2300)$ | $u_3\overline{u}_3, d_6\overline{d}_6, s_{123}s_{123}$ |
| $2+2+24\cdot q_S(2011)\overline{q}_S(2011)=\eta(2371)$ | $f_4(2300)$ | |
| $1+3+45\cdot q_S(1111)\overline{q}_S(2641)=\eta(2341)$ | $\psi(2313)$ | |
| $1+3+30\cdot q_S(1111)\overline{q}_S(551)=\pi(2434)$ | $\rho_5(2350)$ | $u_3\overline{d}_6, u_3\overline{u}_3, d_6\overline{d}_6$ |
| $1+3+24\cdot q_S(1291)\overline{q}_S(1293)=\eta(2392)$ | | |
| $2+2+32\cdot q_N(2191)\overline{q}_N(2191)=\eta(2405)$ | | |
| $1+1+45\cdot q_S(1111)\overline{q}_S(2731)=\eta(2423)$ | | |
| $1+2+56\cdot q_N(931)\overline{q}_N(3091)=\pi(2567)$ | | |
| $1+3+30\cdot q_S(1111)\overline{q}_S(2731)=\pi(2598)$ | | |
| $1+1+24\cdot q_S(2371)\overline{q}_S(2371)=\pi(2670)$ | | |
| $1+3+24\cdot q_S(1291)\overline{q}_S(3091)=\eta(2699)$ | | |
| $1+1+32\cdot q_S(2371)\overline{q}_S(2371)=\eta(2670)$ | | |
| $2+2+32\cdot q_N(2551)\overline{q}_N(2551)=\eta(2928)$ | | |
| $1+1+24\cdot q_S(2371)\overline{q}_S(2371)=\eta(2963)$ | | |
| $1+1+24\cdot q_S(2451)\overline{q}_S(2451)=\eta(3037)$ | | |
| $3+3+32\cdot q_N(2641)\overline{q}_N(2641)=\eta(3054)$ | | |
| $4+4+24\cdot q_S(2731)\overline{q}_S(2731)=\pi(3178)$ | | |
| $4+4+32\cdot q_S(2731)\overline{q}_S(2731)=\eta(3178)$ | | |
| $2+2+32\cdot q_N(2731)\overline{q}_N(2731)=\eta(3179)$ | | |
| $4+4+24\cdot q_S(2731)\overline{q}_S(2731)=\eta(3178)$ | | |
| $1+1+48\cdot q_S(4271)\overline{q}_S(4271)=\eta(5926)$ | | |
| $1+1+48\cdot q_S(10031)\overline{q}_S(10031)=\eta(17837)$ | | |
Table II. Strange Mesons (S = ±1, C = b = 0)

| The BCC Quark Model | Experiment | The Quark Model |
|----------------------|------------|-----------------|
| \( [d_q + d_{ar{q}} + O(I)] q^*_{u} (m_k) q^*_{d} (m_l) = K(m) \) | K(m) | \( \bar{s}u, \bar{s}d \) |
| 1+1+(72)•q^*_N(931)q_S^*(1111) = K(494) | K(494) | \( S_{123u}, S_{123d} \) |
| 1+1+(32)q^*_N(1201)q_S^*(1111) = K(885) | K(892) | \( S_{123u}, S_{123d} \) |
| 1+1+(60)•q^*_N(931)q_S^*(1391) = K(899) | K(1270) | \( S_{123u}, S_{123d} \) |
| 1+1+(54)•q^*_N(931)q_S^*(1201) = K(901) | K(1470) | \( S_{123u}, S_{123d} \) |
| 1+1+(32)q^*_N(1471)q_S^*(1111) = K(1126) | K(1447) | \( S_{123u}, S_{123d} \) |
| 1+3+(54)•q^*_N(931)q_S^*(1651) = K(1310) | K(1447) | \( S_{123u}, S_{123d} \) |
| 2+1+(32)q^*_N(1831)q_S^*(1111) = K(1447) | K(1447) | \( S_{123u}, S_{123d} \) |
| 1+1+(54)•q^*_N(931)q_S^*(2011) = K(1463) | K(1463) | \( S_{123u}, S_{123d} \) |
| 4+1+(32)q^*_N(1921)q_S^*(1111) = K(1528) | K(1528) | \( S_{123u}, S_{123d} \) |
| 1+5+(54)•q^*_N(931)q_S^*(1921) = K(1556) | K(1556) | \( S_{123u}, S_{123d} \) |
| 1+2+(54)•q^*_N(931)q_S^*(2011) = K(1638) | K(1638) | \( S_{123u}, S_{123d} \) |
| 2+1+(32)•q^*_N(2191)q_S^*(1111) = K(1769) | K(1769) | \( S_{123u}, S_{123d} \) |
| 1+1+(60)•q^*_N(931)q_S^*(2551) = K(1804) | K(1804) | \( S_{123u}, S_{123d} \) |
| 1+1+(54)•q^*_N(931)q_S^*(2451) = K(1863) | K(1863) | \( S_{123u}, S_{123d} \) |
| 1+1+(54)•q^*_N(931)q_S^*(2371) = K(1966) | K(1966) | \( S_{123u}, S_{123d} \) |
| \(1+3+(54)\cdot q_N(931)q_S(2641) = K(2036)\) | •\(K(2045)\) | \(S_{123u}, S_{123d}\) |
| \(2+1+(32)q_N^*(2551)q_S^*(1111) = K(2090)\) | \(\bullet\) | \(\bullet\) |
| \(1+3+(54)\cdot q_N^*(931)q_S(2551) = K(2129)\) | ? | \(S_{123u}, S_{123d}\) |
| \(1+1+(54)\cdot q_N^*(931)q_S(2731) = K(2118)\) | K(2225) | \(S_{123u}, S_{123d}\) |
| \(3+1+(32)\cdot q_N(2641)q_S^*(1111) = K(2170)\) | K(2129) | \(S_{123u}, S_{123d}\) |
| \(1+2+(54)\cdot q_N^*(931)q_S(2641) = K(2211)\) | K(2250) | \(S_{123u}, S_{123d}\) |
| \(2+1+(32)\cdot q_N^*(2731)q_S(1111) = K(2251)\) | K(2129) | \(S_{123u}, S_{123d}\) |
| \(1+3+(54)\cdot q_N^*(931)q_S(2731) = K(2293)\) | K(2320) | \(S_{123u}, S_{123d}\) |
| \(\bar{K}(2350)\) | \(\bar{K}(2350)\) | \(\bar{S}_{123u}, \bar{S}_{123d}\) |
| \(1+5+(54)\cdot q_N^*(931)q_S(3091) = K(2621)\) | K(2500) | \(S_{123u}, S_{123d}\) |
| \(\ldots\) | ? | \(S_{123u}, S_{123d}\) |
| \(1+1+(60)\cdot q_N^*(931)q_S(4271) = K(3597)\) | ? | ? |
| \(1+1+(60)\cdot q_N^*(931)q_S(10031) = K(9429)\) | ? | ? |
| \(\ldots\) | \(\ldots\) | \(\ldots\) |
Table III. Charmed Mesons (C = ±1)

| The BCC Quark Model | Experiment | The Quark Model |
|---------------------|------------|-----------------|
| (O(I)) $q_i^*(m_k)q_j^*(m_l)=M(m)$ | M(m) | $c\bar{d}$, $c\bar{u}$ |
| (78)•$q_C^*(2271)\bar{q}_N^*(931)=D(1866)$ | •D(1869) | $c\bar{d}_6$, $c\bar{u}_3$ |
| (58)•$q_C^*(2441)\bar{q}_N^*(931)=D(2029)$ | •D(2007)$^0$ •D(2010)$^\pm$ | $c\bar{d}_6$, $c\bar{u}_3$ |
| (38)•$q_C^*(2271)\bar{q}_N^*(1201)=D(2107)$ | ? | $c\bar{d}_6$, $c\bar{u}_3$ |
| (58)•$q_C^*(2531)\bar{q}_N^*(931)=D(2115)$ | D$_1$(2420)$^\pm$ D$_1$(2420)$^0$ | $c\bar{d}_6$, $c\bar{u}_3$ |
| (38)•$q_C^*(2271)\bar{q}_N^*(1471)=D(2348)$ | D$_1$(2420)$^0$ | $c\bar{d}_6$, $c\bar{u}_3$ |
| (58)•$q_C^*(2961)\bar{q}_N^*(931)=D(2526)$ | D$_S^*(2460)$ D$_S^*(2460)^0$ D$_S^*(2640)$ D$_S^*(2507)$ | $c\bar{d}_6$, $c\bar{u}_3$ |
| (38)•$q_C^*(2271)\bar{q}_N^*(1921)=D(2750)$ | ? | $c\bar{d}_6$, $c\bar{u}_3$ |
| (38)•$q_C^*(2271)\bar{q}_N^*(2191)=D(2991)$ | ? | $c\bar{d}_6$, $c\bar{u}_3$ |
| (68)•$q_C^*(6591)\bar{q}_N^*(931)=D(5996)$ | ? | ? |
| (68)•$q_C^*(13791)\bar{q}_N^*(931)=D(13274)$ | ? | ? |

Charmed, Strange Mesons (C= S= ±1)

| The BCC Quark Model | Experiment | The Quark Model |
|---------------------|------------|-----------------|
| (54)•$q_C^*(2271)\bar{q}_S^*(1111)=D_S(2021)$ | •D$_S$(1969) | c(1250)$S$(123) |
| (34)•$q_C(2441)\bar{q}_S^*(1111)=D_S(2034)$ | •D$_S^*(2112)$ | c(1250)$S$(123) |
| (34)•$q_C(2531)\bar{q}_S^*(1111)=D_S(2120)$ | •D$_S^*(2112)$ | c(1250)$S$(123) |
| (42)•$q_C^*(2271)\bar{q}_S^*(1391)=D_S(2126)$ | •D$_S^*(2536)$ \mbox{\unlhd} •D$_S^*(2573)$ \mbox{\unlhd} \mbox{\unlhd} D$_S$(1250)$S$(123) |
| (36)•$q_C^*(2271)\bar{q}_S^*(2011)=D_S(2690)$ | ? | c(1250)$S$(123) |
| (42)•$q_C^*(2271)\bar{q}_S^*(2551)=D_S(3332)$ | ? | ? |
| (44)•$q_C^*(6591)\bar{q}_S^*(1111)=D_S(6151)$ | ? | ? |
| (32)•$q_C^*(6591)\bar{q}_S^*(2551)=D_S(7205)$ | ? | ? |
| (44)•$q_C^*(13791)\bar{q}_S^*(1111)=D_S(13432)$ | ? | ? |
Table IV. Bottom Mesons (b = ± 1)

| The BCC Quark Model | Experiment | The Quark Model |
|---------------------|------------|-----------------|
| \((O(I))q^*_b(m_k)q^*_l(m_l)=M(m)\) | B(5279) | \(\bar{b}u, \bar{b}d\) |
| \((66)q^*_b(5531)q^*_N(931)=B(5164)\) | B(5325) | \(\bar{b}u, \bar{b}d\) |
| \((26)q^*_b(5531)q^*_N(1201)=B(5655)\) | B(5732) | \(\bar{b}u, \bar{b}d\) |
| \((26)q^*_b(5531)q^*_N(1471)=B(5896)\) | ? | \(\bar{b}u, \bar{b}d\) |
| \((26)q^*_b(5531)q^*_N(1831)=B(6217)\) | ? | \(\bar{b}u, \bar{b}d\) |
| \((26)q^*_b(5531)q^*_N(1930)=B(6298)\) | ? | \(\bar{b}u, \bar{b}d\) |
| \((60)q^*_b(9951)q^*_N(931)=B(5654)\) | ? | ? |
| \((60)q^*_b(15811)q^*_N(931)=B(5164)\) | ? | ? |

Bottom, Strange Mesons (b = \(S = \pm 1\))

| \(q^*_b(5531)q^*_S(1111)=B_S(5319)\) | \(B_S(5369)\) | \(\bar{b}\bar{s}\) |
| \(q^*_b(5531)q^*_S(1391)=B_S(5674)\) | \(B_{S,J}(5850)\) | \(\bar{b}\bar{s}\) |
| \(q^*_b(5531)q^*_S(2011)=B_S(6238)\) | ? | \(\bar{b}\bar{s}\) |
| \(q^*_b(5531)q^*_S(2451)=B_S(6638)\) | ? | \(\bar{b}\bar{s}\) |
| \(q^*_b(5531)q^*_S(2551)=B_S(6629)\) | ? | \(\bar{b}\bar{s}\) |
| \(q^*_b(9951)q^*_S(1111)=B_S(9659)\) | ? | ? |
| \(q^*_b(9951)q^*_S(1391)=B_S(9994)\) | ? | ? |
| \(q^*_b(9951)q^*_S(2551)=B_S(10866)\) | ? | ? |
| \(q^*_b(15811)q^*_S(1111)=B_S(15540)\) | ? | ? |
| \(q^*_b(15811)q^*_S(2551)=B_S(16774)\) | ? | ? |

Bottom, Charmed Mesons (b = \(\mp 1, C = \pm 1\))

| \(q^*_b(5531)q^*_C(2271)=B_C(6691)\) | \(B_C(6400)\) | \(\bar{b}\bar{c}\) |
| \(q^*_b(5531)q^*_C(6591)=B_C(10822)\) | ? | ? |
| \(q^*_b(9951)q^*_C(2271)=B_C(11031)\) | ? | ? |
| \(q^*_b(9951)q^*_C(6591)=B_C(15012)\) | ? | ? |
| \(q^*_b(15811)q^*_C(2271)=B_C(16912)\) | ? | ? |
| \(q^*_b(5531)q^*_C(13791)=B_C(18102)\) | ? | ? |
| The BCC Quark Model | Experiment | Quark Model |
|---------------------|------------|-------------|
| $[d_{1}+d_{2}+O(1)]q_{a}^{*}(m_{k})q_{b}^{*}(m_{l})=M(m)$ | $M(m)\Gamma$ | $c\bar{c}$ |
| $[1+1+48]\cdot q_{a}^{*}(2551)q_{b}^{*}(2551)=\eta(2902)$ | $\eta_{c}(2980)_{13}$MeV | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+120]\cdot q_{c}^{*}(2271)q_{c}^{*}(2271)=J/\psi(3044)$ | $J/\psi(3097)_{87}$Kev | $c_{1250}\bar{c}_{1250}$ |
| $[2+2+24]\cdot q_{\xi}(2641)q_{\xi}^{*}(2641)=\eta(3394)$ | $\eta_{c0}(3415)_{15}$Mev | $c_{1250}\bar{c}_{1250}$ |
| $[3+3+24]\cdot q_{\Sigma}(2641)q_{\Sigma}^{*}(2641)=\eta(3394)$ | $\eta_{c0}(3415)_{88}$Mev | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+40]\cdot q_{c}^{*}(2441)q_{c}^{*}(2441)=\psi(3568)$ | $\psi(3556)_{2.0}$Mev | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+60]\cdot q_{c}^{*}(2271)q_{c}^{*}(2441)=\psi(3607)$ | $\psi(3686)_{277}$Kev | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+40]\cdot q_{c}^{*}(2531)q_{c}^{*}(2531)=\psi(3740)$ | $\psi(3770)_{24}$Mev | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+30]\cdot q_{c}^{*}(2441)q_{c}^{*}(2531)=\psi(3854)$ | $\psi(3836)$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+54]\cdot q_{s}^{*}(1111)q_{s}^{*}(4271)=\eta(3952)$ | $\psi(4040)_{52}$Mev | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+60]\cdot q_{c}^{*}(2271)q_{c}^{*}(2961)=\psi(4104)$ | $\psi(4160)_{78}$Mev | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+36]\cdot q_{s}^{*}(1391)q_{s}^{*}(4271)=\eta(4105)$ | $\psi(4160)$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+30]\cdot q_{c}^{*}(2441)q_{c}^{*}(2961)=\psi(4264)$ | $\psi(4415)_{43}$Mev | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+30]\cdot q_{c}^{*}(2531)q_{c}^{*}(2961)=\psi(4349)$ | $\psi(4415)$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+40]\cdot q_{c}^{*}(2961)q_{c}^{*}(2961)=\psi(4554)$ | $\psi(4415)$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+27]\cdot q_{s}^{*}(2011)q_{s}^{*}(4271)=\eta(4553)$ | $\psi(4415)$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+36]\cdot q_{s}^{*}(2551)q_{s}^{*}(4271)=\eta(4944)$ | $\eta$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+75]\cdot q_{c}^{*}(6591)q_{c}^{*}(2271)=\psi(7374)$ | $\psi$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+75]\cdot q_{c}^{*}(13791)q_{c}^{*}(2271)=\psi(14654)$ | $\psi$ | $c_{1250}\bar{c}_{1250}$ |
| $[1+1+80]\cdot q_{c}^{*}(13791)q_{c}^{*}(13791)=\psi(25596)$ | $\psi$ | $c_{1250}\bar{c}_{1250}$ |

...
Table VI. The Heavy Mesons (S = C = b = I = 0)

| The BCC Quark Model | Experiment | The Quark Model |
|---------------------|------------|-----------------|
| \( (O(I))q_i^* (m_k)q_j^* (m_l) = M(m) \) | \( \Upsilon (9489) \) | \( \Upsilon (9489) b(4200) \) |
| \( (72) \cdot q_b^* (5531)q_b^* (5531) = \Upsilon (9489) \) | \( \chi_{b0}(1P)(9860) \) | \( \chi_{b0}(1P)(9860) b(4200) \) |
| \( (74) \cdot q_S^* (1111)q_S^* (10031) = \eta (9734) \) | \( \chi_{b1}(1P)(9893) \) | \( \chi_{b1}(1P)(9893) b(4200) \) |
| \( (36) \cdot q_S^* (1391)q_S^* (10031) = \eta (9955) \) | \( \chi_{b2}(1P)(9913) \) | \( \chi_{b2}(1P)(9913) b(4200) \) |
| \( (27) \cdot q_S^* (2011)q_S^* (10031) = \eta (10443) \) | \( \chi_{b0}(2P)(10232) \) | \( \chi_{b0}(2P)(10232) b(4200) \) |
| \( (27) \cdot q_S^* (2011)q_S^* (10031) = \eta (10443) \) | \( \chi_{b1}(2P)(10255) \) | \( \chi_{b1}(2P)(10255) b(4200) \) |
| \( (27) \cdot q_S^* (2011)q_S^* (10031) = \eta (10443) \) | \( \chi_{b2}(2P)(10269) \) | \( \chi_{b2}(2P)(10269) b(4200) \) |
| \( (80) \cdot q_C^* (6591)q_C^* (6591) = \psi (10792) \) | \( \Upsilon (10355) \) | \( \Upsilon (10355) b(4200) \) |
| \( (27) \cdot q_S^* (2451)q_S^* (10031) = \eta (10791) \) | \( \Upsilon (10580) \) | \( \Upsilon (10580) b(4200) \) |
| \( (27) \cdot q_S^* (2451)q_S^* (10031) = \eta (10791) \) | \( \Upsilon (10860) \) | \( \Upsilon (10860) b(4200) \) |
| \( (27) \cdot q_S^* (2451)q_S^* (10031) = \eta (10791) \) | \( \Upsilon (11020) \) | \( \Upsilon (11020) b(4200) \) |
| \( (27) \cdot q_S^* (2451)q_S^* (10031) = \eta (10791) \) | \( \Upsilon (11040) \) | \( \Upsilon (11040) b(4200) \) |
| \( (57) \cdot q_b^* (9951)q_b^* (9951) = \Upsilon (14329) \) | \( \Upsilon (14329) \) | \( \Upsilon (14329) b(4200) \) |
| \( (45) \cdot q_b^* (9951)q_b^* (9951) = \Upsilon (17805) \) | \( \Upsilon (17805) \) | \( \Upsilon (17805) b(4200) \) |
| \( (45) \cdot q_b^* (9951)q_b^* (9951) = \Upsilon (20210) \) | \( \Upsilon (20210) \) | \( \Upsilon (20210) b(4200) \) |
| \( (45) \cdot q_b^* (9951)q_b^* (9951) = \Upsilon (20210) \) | \( \Upsilon (29597) \) | \( \Upsilon (29597) b(4200) \) |
| \( ... \) | \( ... \) | \( ... \) |
Appendix III

The High Isospin Mesons

The BCC Quark Model predicts many high isospin mesons (I=1(S, or C, or b \neq 0), I = 3/2, I = 2, I = 5/2, and I = 3). They are not predicted by the Quark Model. They have not been discovered by experiment yet (except a meson \chi(1600) with I = 2).

First, we study the high isospin mesons with O(Meson) < 24. From (25) and (28), we have

1. \( q_5^* (m_i) q_\Delta (m_j) \), O(K, I=1/2) = 17, O(F, I=3/2) = 12, O(S, I=5/2) = 7.
2. \( q_\Delta (m_i) q_\Delta (m_i) \), O(\eta, I=0) = 32, O(\pi, I=1) = 24, O(T, I=2) = 16, O(W, I=3) = 8.
3. \( q_\Sigma (m_i) q_\Sigma (m_i) \), O(\eta, I=0) = 24, O(\pi, I=1) = 16, O(T, I=2) = 8.
4. \( q_6^* (5531) q_5^* (m) \), O(\pi_b, I=1) = 16.
5. \( q_s^* (1111) q_\Delta (m) \), O(F, I=3/2) = 20.
6. \( q_6^* (5531) q_5^* (m) \), O(F_b, I=3/2) = 16.25.
7. \( q_C^* (2271) q_\Delta (m) \rightarrow F_C (I=3/2) \), O(F_C) = 23.75.

Above, the mesons with I \geq 3/2 all have O(Meson) < 24. Thus, they cannot be observed now. Especially, the mesons (W) with I = 3 (from q_\Delta q_\Delta) have O(W, I=3) = 8 (< 24). Thus, they cannot be observed now. The mesons with I = 5/2 (from q_\Delta q_\Delta) have O(S, I=5/2) = 7 (< 24). Thus, they cannot be observed either.

Second, we study the high isospin mesons with O(Meson) \geq 24. From (23) and (28), we have

1. For \( q_N^* (931) q_5^* (m) \rightarrow K(I=1/2) \), O(K) = 47; F(I=3/2), O(F) = 28.
2. For \( q_C^* (2271) q_5^* (m) \rightarrow \pi_C (I=1) \), O(\pi_C) = 24.
3. For \( q_N^* (931) q_5^* (m) \rightarrow \pi (I = 1) \), O(\pi) = 56; T(I=2), O(T) = 37.

The mesons composed of \( q_N^* (931) q_5^* (m) \), F(I = 3/2), have O(q_N^* q_f^*) = 28 (> 24). Thus they should be discovered by today’s experiments. However, there are four members (I_z = 3/2, Q=1; I_z=1/2, Q=0; I_z = -1/2, Q= -1; I_z = -3/2, Q= -2) in the mesons. The
member with $Q = -2$ will be composed of $q^*_N(931)^{+2/3}(Q=-2/3)$ and $q^*_C(Q=-4/3)$. The quark and the antiquark with the same kind of electric charge will repel each other four times more strongly $(2/3 \times 4/3 = 8/9)$ than the mesons with $Q = 1$ $(1/3 \times 2/3 = 2/9)$ exhibit. Therefore, the member $(Q = -2)$ will have (such as $0.75 \times 28 = 21$) $O(q^*_i \bar{q}^*_j) < 24$. They are difficult to find. Similarly, the mesons with $I = 2$ (from $q^*_N(931)q^*_C(m)$) have $O(q^*_i \bar{q}^*_j) = 37$ $(>24)$. They should be discovered by today’s experiments. However, there are five members ($I_z = 2$, $Q = 2$; $I_z = 1$, $Q = 1$; $I_z = 0$, $Q = 0$; $I_z = -1$, $Q = -1$; $I_z = -2$, $Q = -2$) in the meson’s family. Similarly to the $I=3/2$ case, the members with $I_z = \pm 2$, $Q = \pm 2$ will be very difficult to find. After taking off the members with $Q = \pm 2$, the mesons change into new mesons [$I=1$; $Q=1, 0, -1$]. These new mesons may be observed as the mesons $K$, $D$ and $\pi$ as shown in the following:

| Meson (before taking off member(Q=±2)) | Meson (after...) | Meson (may be) |
|----------------------------------------|------------------|----------------|
| $q^*_N(931)q^*_C(1201) = F(901)[3/2;2,1,0,-1]$ | $K(901)[1;1,0,-1]$ | $K^\pm(901), K^0(901)$ |
| $q^*_C(2271)q^*_C(1201) = \pi_C(2228)[1;2,1,0]$ | $D(2228)[1;1,0]$ | $(24)D(2228)$ |
| $q^*_N(931)q^*_\Delta(1291) = T(960)[2;2,1,0,-1,-2]$ | $\pi(960)[1;1,0,-1]$ | $\pi(960)[1;1,0,-1]$ |

Although the mesons with $I = 2$ are very difficult to find, our great experimental physicists have already discovered one—$\chi(1600)$ [$I^G(J^{PC}) = 2^+(2^{++})$] [8]. If the result can be confirmed, it will be a great discovery in physics. It will clearly show that the Quark Model (6 elementary quarks and SU(N) symmetries) needs modification and that the BCC Quark Model is a good modification since it predicts the meson ($I=2$) $\chi(1600)$ [$q^*_N(931)q^*_\Delta(2011) = \chi(1603)$].
The axis $\Delta$ (the axis $\Gamma$-H) is a four fold rotation axis

$\Delta$ The axis $\Lambda$ (the axis $\Gamma$-P) is a three fold rotation axis

$\Lambda$ The axis $\Sigma$ (the axis $\Gamma$-N) is a two fold rotation axis

Figure 1
Figure 2
Figure 3
Figure 4
Figure 5