Elevator Rope Tension Analysis with Uneven Groove Wear of Sheave

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Abstract. Traction elevators are suspended by multiple ropes. Uneven sheave wears affect the rope tension during the elevator operation. If the tension condition reaches to the critical traction ratio, a rope slip occurs. As the rope slip also affects the rope tension, a tension calculation with rope slip is necessary to evaluate the relation between the groove wear and the rope tension. In this paper, a tension evaluation model is derived by including the rope slip behavior and then the influence of the rope tension due to the groove wear is evaluated.

1. Introduction
In the traction elevator system, a car is suspended by multiple ropes. Ideally, each rope tension should be maintained as the balanced condition. However, the tension fluctuates due to the elongation of the rope, the variation of rope stiffness or the uneven sheave groove wear. If the tension of each rope is unbalanced severely, the elevator system is out of condition. Therefore, an analytical method is required to evaluate the rope tension behavior. Since the uneven groove wear of the sheave in high-rise elevators causes a significant tension change even in a small amount of wear, it is more important to establish the tension analysis with the groove wear than other factors. Therefore, clarification of the relationship among the groove wear, tension and traveling distance can contribute to the elevator’s system design. Concerning the influence of the groove wear, the relationship between the amount of wear and the tension is investigated by Togawa et al [1]. While the above work mainly focuses on the experimental study of the phenomenon, an analysis of the tension change and the influence of groove wear have not been studied in detail. In this paper, firstly we describe the elevator’s vertical vibration model with multiple ropes. Secondary, the mechanism of rope tension behavior with the uneven groove wear is explained. Finally, as the proposed model also includes the rope slip, the influence of the slip due to the unbalanced tension is evaluated by simulation and experiment.

2. Analytical Model
We derive an analytical model of the elevator that considers tension difference among the multiple ropes, and the rope slip on the sheave. Figure 1 shows the proposed model. In the model, ropes wrapped around a traction machine are connected to a car and a counterweight (CWT) via shackles. The ropes are modeled as springs. To evaluate the slip of the ropes on the sheave, the parts of ropes on the sheave are modeled as intensified masses. The equation of elevator’s vertical motion of the elevator system is derived by the following equation.

$$M\ddot{x} + C\dot{x} + Kx = F$$

(1)

where M, C and K are the inertia matrix of the system, the damping matrix and the stiffness matrix respectively. F is the external force vector which contains the gravity force, the traction force and the torque by the traction machine. The elevator’s transient motion is calculated by the numerical integration of equation (1).
Figure 1. Elevator model

\[ M = \text{diag} \left[ M_c, M_w, m, \ldots, m, J \right] \in \mathbb{R}^{(n+3)\times(n+3)} \]

\[ \mathbf{K} = \begin{bmatrix}
    n\tilde{k}_c & 0 & -\tilde{k}_c & -\tilde{k}_c & \cdots & -\tilde{k}_c & 0 \\
    0 & n\tilde{k}_w & -\tilde{k}_w & -\tilde{k}_w & \cdots & -\tilde{k}_w & 0 \\
    -\tilde{k}_c & -\tilde{k}_w & 0 & 0 & \cdots & 0 & 0 \\
    -\tilde{k}_c & -\tilde{k}_w & 0 & 0 & \cdots & 0 & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \in \mathbb{R}^{(n+3)\times(n+3)} \]

\[ \mathbf{F} = \begin{bmatrix} M_c g & M_w g & f_{i1} & \cdots & f_{in} & u - r \sum_{i=1}^{n} f_{si} \end{bmatrix} \in \mathbb{R}^{n+3} \]

\[ \mathbf{x} = \begin{bmatrix} x_c & x_w & x_1 & x_2 & \cdots & x_n & \theta_p \end{bmatrix} \in \mathbb{R}^{n+3} \]

Note that the damping matrix \( \mathbf{C} \) consists of the damping elements which are proportional to the stiffness ones. Each element of \( \mathbf{K} \) is quantified by the specification of the rope and the shackle spring stiffness. \( \mathbf{C} \) is determined by the comparison with the experimental results which are shown in the following sections. The model parameters are shown in Table 1 where \( m \) is the intensified mass of the rope on the sheave. Other part mass of the rope is included in \( M_c \) or \( M_w \).

### Table 1. Model parameters

| \( M_c \) | Car mass |
| \( m \) | Rope mass on the sheave |
| \( M_w \) | CWT mass |
| \( x_c \) | Displacement of car |
| \( x_w \) | Displacement of CWT |
| \( x_i \) | Displacement of rope mass on the sheave |
| \( \theta \) | Rotation angle of sheave |
| \( r \) | Sheave radius |
| \( J \) | Sheave moment of inertia |
| \( g \) | Gravity acceleration |
| \( \tilde{k}_c \) | Car side rope stiffness with shackle spring |
| \( \tilde{k}_w \) | CWT side rope stiffness with shackle spring |
| \( u \) | Torque |
| \( n \) | Number of the ropes |

The rope slip can be checked by the ratio of the car side rope tension \( T_{ci} \) and the CWT side rope tension \( T_{wi} \), as shown in figure 2. The ratio is compared with the critical traction ratio of the sheave \( \Gamma \) which is defined as

\[ \Gamma = e^{\mu \kappa \theta_p} \]

where \( \mu \) is the coefficient of friction, \( \kappa \) is the coefficient of the groove, \( \theta_p \) is the wrapping angle of the rope on the sheave. In equation (4), \( f_{si} \) \((i=1, \ldots, n)\) is the force acting between the rope and the sheave. The force is switched according to the condition of the slip as follows.

(i) No slip condition

The condition that the rope does not slip is written as \( T_{ci} < \Gamma T_{wi} \) where \( T_{ci} \) is larger than \( T_{wi} \). On the other hand, if \( T_{ci} \) is smaller than \( T_{wi} \) the no slip condition is given by \( \Gamma T_{ci} > T_{wi} \) [2]. Therefore, by summarizing the equations, the no slip condition is

\[ 1/\Gamma < T_{wi}/T_{ci} < \Gamma \]
In this case, the rope and the sheave move with the same velocity, because the tension ratio of $T_i$ and $T_w$ is within the critical traction ratio. Hence, the following constraint equation is given as the no slip condition.

$$\bar{c}_i = r \dot{\theta} - \dot{x}_i = 0 \quad (i = 1, 2, \cdots, n)$$

(8)

Each constraint equation can be summarized as the vector form

$$\bar{c} = [\bar{c}_1, \cdots, \bar{c}_i, \cdots, \bar{c}_n]^T = 0$$

(9)

Differentiating equation (9) with respect to time yields

$$\frac{\partial \bar{c}}{\partial t} = \frac{\partial \bar{c}}{\partial \bar{x}} \frac{d\bar{x}}{dt} = \bar{c}_x \ddot{x} = 0$$

(10)

where $\bar{c}_x$ is Jacobian of $\bar{c}$ which can be written as

$$\bar{c}_x = \frac{\partial \bar{c}}{\partial \bar{x}} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & \cdots & 0 & \cdots & 0 & r \\ 0 & 0 & 0 & -1 & 0 & \cdots & 0 & \cdots & 0 & r \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & -1 \end{bmatrix} \in \mathbb{R}^{n(n+3)}$$

(11)

Then, equation (1) and (10) can be written in the matrix equation as

$$\begin{bmatrix} M & \bar{C}_x \\ \bar{C}_x & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ f \end{bmatrix} = \begin{bmatrix} \ddot{F} - \bar{C}_x \ddot{x} - \bar{k}x \end{bmatrix}$$

(12)

$$\ddot{F} = M_c g + M_w g + 0 \cdots 0 \cdots u \in \mathbb{R}^{n+3}, \quad f = [f_1, \cdots, f_d, \cdots f_m]^T \in \mathbb{R}^n$$

where constraint force $f$ is calculated as the vector of Lagrange multipliers.

(ii) Slip condition

When the tension ratio of $T_i$ and $T_w$ is beyond the critical traction ratio, the rope experiences the slip on the sheave to keep the ratio equal to the critical traction ratio. Then the friction force $f_s$ in equation (13) acts on the rope and the sheave, respectively.

$$f_s = \begin{cases} (1 - \Gamma)T_w, & T_w / T_i < 1/\Gamma \\ (1 - \Gamma)T_i, & T_w / T_i > \Gamma \end{cases}$$

(13)

3. Tension Analysis without Rope Slip

In this section, we show the tension change mechanism under the uneven groove wear condition by simulations and experiments. At first, we evaluate the tension behavior with respect to the car position without the rope slip. In the following discussion, we suppose that there is no passenger in the car, and the elevator consists of two ropes and the one of the sheave groove has a certain amount of wear. As the number of the ropes is two, the degree of freedom in the model equation (1) is five.

The groove wear is equivalent to the decrease of the sheave radius as shown in figure 3. It causes the difference of each rope’s winding length on the sheave when the car goes up or down. This effect to the rope length causes the tension change as shown in figure 4. The horizontal axis shows the normalized car position. Its origin corresponds to the bottom floor, and the top floor is set to one. On the other hand, the vertical axis shows the tension which is normalized by the car side tension at the bottom floor. The mechanism of the tension change in the upward motion illustrated in figure 4 can be explained as follows;

1) The length of the rope 2 on the worn groove in the car side becomes longer than the rope 1 by the car upward motion. Also the length of the rope 2 in the CWT side becomes shorter than the rope 1, because the amount of winding on the sheave is smaller than one of the rope 1.
2) The rope length difference causes the tension imbalance. The tension of rope 2 in the car side and the tension of rope 1 in the CWT side decrease according to the car upward motion as shown in figure 4(a).
3) Since the rope length in the car side becomes shorter by the car upward motion, the tension changes sharply. On the other hand, as the rope length in the CWT side becomes longer, the tension change is
slower than the car side. In figure 4(b), the tension ratio of rope 2 becomes larger and it finally exceeds the critical traction ratio.

The tension change mechanism described above can be also expressed by the following functions of the car position.

\[
T_{c1} = \frac{M_v g}{2} + K \frac{\beta x}{1 - x + \delta}, \quad T_{c2} = \frac{M_v g}{2} - K \frac{\beta x}{1 - x + \delta}, \quad T_{w1} = \frac{M_v g}{2} - K \frac{\beta x}{x + \delta}, \quad T_{w2} = \frac{M_v g}{2} + K \frac{\beta x}{x + \delta}
\]

\[K = \frac{EA}{L}\]

In the above equations, \(\beta x\) is the change of the rope length due to the groove wear where \(\beta\) is defined as \(\beta = (1 - r_2/r_1)/2\), \(\delta = OH/L\) is the ratio of the remaining rope length \(OH\) on the top floor. \(x\) is the normalized car position which is defined as \(x = (TR - x_c)/TR\) where \(TR\) is the elevator shaft height. \(E, A\) and \(L\) are the Young’s modulus, the cross section area of the rope, and the distance from the bottom to the top floor, respectively.

In Eq. 11, the second term of the right hand side in all four equations represents the tension change as the curved line in figure 4(a). Note that we ignore the stiffness of the shackle spring in the derivation of equation (14) for the simplicity. When the traction ratio reaches the critical traction ratio, the rope 2 on the worn groove slips with respect to the sheave in the actual system.

![Figure 3. 2-rope elevator model with groove wear](image-url)

![Figure 4. Simulation result of rope tension without slip (upward motion)](image-url)

4. Tension Analysis with Rope Slip

In this section, the rope tension is measured by an elevator testbed to evaluate the rope tension behavior with rope slip. The configuration of the testbed is the same as figure 3. The tension sensors are attached to each rope end. Table 2 shows the conditions of the experiments. Figure 5 shows the comparison result between the simulation and experiment. As shown in figure 5, the simulation corresponds to the experimental data. The tension in the car upward motion behaves according to the following steps.

A) The tension change occurs based on 1) and 2) described in the previous section. The tension ratio of the rope 2 reaches the critical traction ratio \(\Gamma\). Then the rope 2 slips on the sheave.

B) Even after the tension ratio of the rope 2 reaches the critical traction ratio, the tension tries to change due to the length difference of the rope by the groove wear. As the traction ratio can’t exceed the critical traction ratio, the tension ratio sticks to \(\Gamma\), and the rope continues to slip until arriving at the top floor.
Table 2. Experimental conditions

| Parameter                              | Value     |
|----------------------------------------|-----------|
| Elevator shaft height [m]              | 9.4       |
| Car mass [kg]                          | 292       |
| CWT mass [kg]                          | 457       |
| Amount of wear [mm]                    | 0.6       |
| Number of ropes                        | 2         |
| Tension sensor (Load cell)             | KYOWA LCW-C-10KN25SA3 |

When the tension ratio reaches the critical traction ratio, the slope of tension curve changes as shown in figure 5. Since the car and the CWT are suspended by multiple ropes, a tension change of a certain rope affects the tension balance. Thus, the tensions of the other ropes also change accordingly. Since the tension ratio with rope slip \( x_s \) can be written as \( (T_{w2} - k_s x_s)/(T_{c2} + k_c x_c) = \Gamma \) where \( k_s = K/(x + \delta) \) and \( k_c = K/(1-x + \delta) \), the amount of rope slip \( x_s \) can be expressed by the following equation.

\[
x_s = T_w^2 - \Gamma T_c^2 \over D,
\]

\[
D = K \left\{ \frac{1}{x + \delta} + \Gamma \frac{1}{1-x + \delta} \right\} \tag{16}
\]

Each of the rope tensions \( \hat{T}_{c1}, \hat{T}_{c2}, \hat{T}_{w1}, \hat{T}_{w2} \) after the slip occurrence is given by the following equations.

\[
\hat{T}_{c1} = T_{c1} - K \frac{x_s}{1-x + \delta}, \quad \hat{T}_{c2} = T_{c2} + K \frac{x_s}{1-x + \delta}, \quad \hat{T}_{w1} = T_{w1} + K \frac{x_s}{x + \delta}, \quad \hat{T}_{w2} = T_{w2} - K \frac{x_s}{x + \delta} \tag{17}
\]

The tension change is mitigated by the effect of the second term of each equation in equation (17), and that corresponds to the slope change of the tension in figure 5(a).

In the following part, we evaluate the relationship between the tension and the amount of the groove wear by the simulation. figure 6 shows the calculated tension during the car upward motion with various groove wear conditions. The vertical axis shows the tension which is normalized by the car side tension at the bottom floor. The slip starting position is shifted to the lower floor by a larger amount of groove wear as shown in figure 6(b). As a larger amount of wear increases the length difference of each rope, the tension ratio reaches \( \Gamma \) at the lower floor. Note that the tension follows the same line after the slip as shown in figure 6(a). To evaluate the above behavior, we transform the first equation of equation (17) by using equation (14) and equation (16). The tension after the rope slip can be written as the following equation.

\[
\hat{T}_{c1} = \frac{M_{c}g}{2} - \left( \frac{M_{c}g}{2} - \frac{M_{w}g}{2} \frac{K}{D(1-x + \delta)} \right) \tag{18}
\]

Since equation (18) does not include \( \beta \), it can conclude that the tension after the rope slip doesn’t depend on the amount of wear, and the effect of the wear is eliminated by the rope slip.
Finally, we discuss the rope tension behavior under the round trip condition. Figure 7 shows the rope tension with the rope slip during the downward motion of the return trip. When the car runs downward from the top floor, the car side rope tension $T_{c2}$ increases and the CWT side rope tension $T_{w2}$ decreases. This is because the amount of winding against the rope2 is smaller than the rope1. The rope1 also indicates the tension change with respect to the tension change of the rope2. Then the rope1 begins to slip on the sheave due to the above tension change. Figure 8 shows the tension behavior in the second and the third round trips in a row. The tension shows a hysteresis loop, and it follows the same line in every run. According to equation (18), the rope tension after the slip doesn’t depend on $\beta$. Thus, the tension at the top or bottom floor is determined independently from the amount of wear. Therefore, if the initial tension condition is the same in every run, it concludes that the tension shows the same hysteresis loop.

5. Conclusions

In this study, the rope tension modeling of the elevator system was described. The proposed model can evaluate the rope slip behavior. The mechanism of the tension change due to the increase of the sheave groove wear was explained by the simulation and the experimental results. Under the condition that the car runs between the bottom floor and the top floor, we get the following results.
(a) The slip starting position is shifted to the lower floor.
(b) The rope on the worn groove slips when the car runs upward. On the other hand, the counterpart rope slips in the downward motion.
(c) The relationship between the rope tension and the car position shows the hysteresis loop, which is the same shape in every round trip.

Since the proposed model can properly simulate the tension behavior, the model can be applied to estimate the influence of the unbalanced tension conditions for the elevator system design.

References
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