A New Method of Force Calculation of Sucker Rod String in Directional Well

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Abstract. The actual wellbore trajectory of the directional well is an irregular spatial curve which is not only distorted, but also bended compared to the straight well. The complexity of the shape makes the sucker rod string in the work of a complex state of alternating stress, and withstanding axial compression, torsion and bending of the coupling effect. Meanwhile, the phenomenon of eccentric wear and disconnection of the sucker rod string becomes worse and worse. Based on the actual force of the sucker rod string in the directional well and the different force characteristics, a new method of force calculation of sucker rod string in direction well is given in this paper, the mechanics analysis models of the sucker rod string in three-dimensional directional well have been deduced respectively. Through the combination of mathematical geometry and traditional mechanics, the mathematical model can be used to obtain the analytic solution of the mechanical problem of sucker rod string, which is suitable for engineering design calculation and overall evaluation research.

Keywords: Sucker rod string; Directional well; Deviation angle.

1. Force Calculation Methods
Lubinski et al. [1] defined the helical buckling behaviour of pipes in vertical wells. However, the post-buckling behaviour of pipe in inclined and horizontal wells is different from that in nearby vertical wells.[2,3] Due to the change of the deviation angle and the azimuth angle of the directional well, the load of sucker rod string can be calculated by the step stacking method[4,5]. The sucker rod is divided into a number of small sections from bottom to top, as shown in Figure1.

![Figure1. Measuring Section Diagram of Sucking Rod](image-url)
First, we need to calculate the load at the bottom plunger of the string, and then the axial force at each node and the friction force of each section in turn were calculated, and iteratively calculate until the wellhead. The relationship curve between the axial load and the friction load can be obtained.

According to the three-dimensional force model of the sucker rod column, this paper uses Visual Basic programming language to write the calculation program. The flow chart of the calculation program is shown in Figure 2.

![Figure 2. Flow Chart of the Calculation Program](image)

### 2. Force Calculation of the Sucker Rod String

#### 2.1. Up-stroke

#### 2.1.1. Force Calculation of the Plunger at the Bottom

\[ F_u = G \cos \theta + F_{apl} + W_l + F_{pb} + F_{lt} - P_i \]  \hspace{1cm} (1)

Where \( \theta \) is deviation angle, \( G \) is the weight of plunger in water, \( N \); \( F_{apl} \) is the inertia force of plunger and fluid, \( Pa \); \( W_l \) is the liquid load, \( Pa \); \( F_{pb} \) is the friction between plunger and pump barrel, \( Pa \); \( F_{lt} \) is the friction between water and tubing, \( Pa \); \( P_i \) is the suction pressure on the plunger, \( Pa \).

\[ P_i = p_i f_p = (p_n - \Delta p_i) f_p \]  \hspace{1cm} (2)

\[ W_l = (f_p - f_r) H_p \rho_l g \]  \hspace{1cm} (3)
\[
W_i = (f_p - f_i) H_p \rho g
\]
\[
F_{apl} = \rho V_p a_r + \frac{W_i}{g} a_c \varepsilon
\]
\[
F_{lt} = \frac{F_{cl}}{1.3}
\]
\[
F_{pb} = 0.94 \frac{D_p}{\delta} - 140
\]

Where \( p_i \) is suction pressure, Pa; \( f_p \) is the cross section area of plunger, \( m^2 \); \( p_n \) is the Submergence pressure, Pa; \( \Delta p_i \) is the pressure drop caused by pump entrance, Pa; \( G \) is the plunger weight, N; \( \rho_p \) is the density of plunger, \( \text{kg/m}^3 \); \( f_r \) is the cross section of sucker rod, \( m^2 \); \( H_p \) is depth of pump, m; \( V_p \) is the volume of plunger, \( m^3 \); \( \varepsilon \) is the accleretion variation changing coefficient; \( f_f \) is the cross section area of tubing, \( m^2 \). \( D_p \) is the diameter of pump, m; \( \delta \) is the clearance between pump and plunger, m.

2.1.2. Iterative Calculation of the Rod Section. It should be noted that, since the azimuth angle at point does not exist, the author ignores the horizontal positive pressure component. The approximate formula is shown as following:
\[
\Delta F_i = F_{i2} - F_{i1} = q \Delta l \cos \alpha_c + \rho \Delta A \alpha \Delta l + f \sqrt{(F_{i1} \Delta \alpha - q \sin \alpha \Delta l)^2}
\]

2.2. Down-stroke

2.2.1. Force Calculation of the Plunger at the Bottom
\[
F_d = G \cos \theta - F_{ap} - F_{pb} - F_{lv}
\]

Where \( F_{ap} \) is the inertia force of plunger, Pa; \( F_{lv} \) is the flow resistance caused by valve, Pa. Use formula to calculate the axial force at the bottom of the plunger, substitute for the basic parameters:
\[
F_{ap} = \rho_p V_p a_r
\]
\[
F_{lv} = \frac{\rho_p f_f^3}{729 \xi^2 F_{fo}^2} (Sn)^2
\]

Where \( \xi \) is the valve discharge coecfficence; \( f_f \) is the cross section area of valve, \( m^2 \).

Iterative Calculation of the Rod Section is Similar with up-stroke.

3. Example Analyses
There is a well named M, which uses beam-rod pump to produce oil, the basic production parameters as shown in Table 1.

According to the well trajectory data, the author uses the compiled program to carry out the example calculation. Since the step length is not constant, the calculation method of segmentation calculation and step iteration is adopted in this paper. The following classified discussion will be based on the different force situation of sucker rod string in the up-stroke and down-stroke. According to the
calculation results, the curves of the axial force and the friction force in the up-stroke and down-stroke are plotted, as shown in Figure 3 and Figure 4.

Table 1. Basic Production Parameters of Beam-rod Pump in Well-M

| Type of Sucker Rod String | CYG22(D) | Well Fluid Density (kg/m³) | 93 |
|---------------------------|---------|---------------------------|----|
| Pump Diameter (mm)        | 28      |                           | 1.2|
| Pump Setting (m)          | 1526.00 |                           | 7850|
| Fluid Level (m)           | 1520.00 |                           | 1440|
| Oil Pressure (MPa)        | 0.00    |                           | 0  |
| Casing Pressure (MPa)     | 0.00    |                           | 0  |
| Stroke (m)                | 2.46    |                           | 0.2|
|                           |         | Friction Coefficient of Rod and Tubing | 0.2|

![Figure 3. Contrast Curves of the Axial Force in the Up-stroke and Down-stroke](image)

Figure 3 shows that the axial force of the sucker rod string in the M-well is linearly increasing with the depth of the well. The maximum axial load is approximately 47.7kN in the section of 138.8-186.7, the increase of the axial force is obviously the largest, reaching 2.82kN. It is mainly due to the large variation of the deviation angle and azimuth in the well, which makes the serious friction of the well, and thus the axial force at the measuring point 4 increased significantly. It can be seen that the axial force of the sucker rod string in the M-well is linearly increasing with the depth of the well. The maximum axial load is approximately 47.7kN, which is reduced by about 15.8kN compared to the maximum axial force in the up-stroke. In the section of 138.8-186.7, the increase in axial force is the smallest, which is mainly due to the fact that the deviation angle and azimuth of the well are larger changed in this section so that the friction of this interval is relatively large, and thus the axial force at the measuring point 4 decreased significantly.
Figure 4 shows that the friction force of the sucker rod string in the up-stroke in the M-well reaches the maximum in the section of 138.8-186.7, nearly to 1710.34N, which is mainly due to the fact that the deviation angle and azimuth of the well are larger changed in this section so that the friction of this interval is relatively large. Meanwhile, serious eccentric wear may take place in this section. Therefore, operators should take measures to improve the force situation of this section in order to increase the life of the sucker rod string. In the remaining sections, the friction force is mainly concentrated within 300N.

In the down-stroke, the friction force of the sucker rod string in the down-stroke in the M-well reaches the maximum in the section of 138.8-186.7, nearly to 1246.01N, which is mainly due to the fact that the deviation angle and azimuth of the well are larger changed in this section so that the friction of this interval is relatively large. However, compared to the up-stroke, the down-stroke of the axial force is generally smaller, so that the friction force is relatively smaller. Similarly, serious eccentric wear may take place in this section. Therefore, operators should take measures to improve the force situation of this section in order to increase the life of the sucker rod string. In the remaining sections, the friction force is mainly concentrated within 200N.

4. Conclusions
The phenomenon of eccentric wear between the rod and tubing strings is caused by the axial force and the horizontal positive pressure oriented from gravity, and the rod section of the serious eccentric wear is generally concentrated in the well section where the deviation angle or the azimuth angle changes greatly. Using the mathematical model established in this paper, the friction force can be solved quickly and the well section which comes up with serious eccentric wear can be found easily, so that the corresponding measures can be taken to prolong the service life of the sucker rod string and the economic benefit of the oilfield development can be also enhanced.

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