Summary: Let $K$ denote a nonsingular conic in the complex projective plane. Given six distinct points $a, b, c, d, e, f$ on $K$, Pascal’s theorem says that the intersection points $ab \cap e, bc \cap ef, cd \cap af$ are collinear. The line containing them is called the Pascal line of the hexagon $abcdef$. We get sixty such lines by permuting the vertices. These lines satisfy some incidence theorems, which lead to an extremely rich combinatorial structure called the hexagrammum mysticum. In [R. Acc. d. Linc. (3) I. 141–142 (1877; JFM 10.0390.01)] M. Veronese discovered a procedure to create an infinite sequence of such structures via successive mutations, this sequence is called the multimysticum. In this paper we prove a strong rigidity theorem which shows that the multimysticum contains 300 projective ranges that are absolutely invariant. The two interspersed sequences of cross-ratios which encode this invariance turn out to be the alternate convergents in the continued fraction expansions of $1 \pm 1/\sqrt{3}$.

MSC:

14N05 Projective techniques in algebraic geometry
51N35 Questions of classical algebraic geometry

Keywords:
Pascal’s theorem; hexagrammum mysticum; multimysticum

Full Text: arXiv Link