Once again on the Gribov horizon

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Abstract

The gauge dependence problem existing in the original Gribov-Zwanziger theory is discussed.

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1 Introduction

The Gribov-Zwanziger (GZ) theory [1, 2] is an attempt to formulate a non-perturbative approach to quantum theory of non-Abelian gauge fields when one takes into account the Gribov copies [3]. There exists very intense activity to study quantum properties of the GZ theory itself as well as QCD dynamics within this method (see review [4] and references therein). Note that the original formulation of the GZ theory is used the Landau gauge only. Despite numerous studies of quantum properties, the gauge dependence problem in the framework of the GZ theory was not considered at all until recently. The study of this problem in Yang-Mills theories assuming the existence of the Gribov horizon functional beyond the Landau gauge only was given in [5] and then was extended for general gauge theories in [6]. It was shown that in general the vacuum functional as well as the effective action even on its extremals depend on gauges. It indicates that physical observables may be not introduced in a consistent way as quantities not depending on gauges. In its turn it was found that there is a strong restriction on the gauge dependence of Gribov horizon functional when the effective action on its extremals does not depend on gauges. It gave hope to formulate the GZ theory in a consistent way as a physical theory. Later studying the finite field dependent BRST transformations in Yang-Mills theories [7] allowed to propose a form of the Gribov horizon functional beyond the Landau gauge which satisfy the above mentioned restriction [8].

In recent studies [9, 10] the Gribov horizon functionals in the Coulomb gauge [11] and linear gauges were used to construct the corresponding GZ theory. Unfortunately the authors of these papers did not pay attention to the gauge dependence problem in the GZ theory which was the main goal of studies [5] although the form of the Gribov horizon functional in these gauges differs from the functionals proposed in [8]. So we are forced to attract attention to the problem once again in this short notice restricting ourselves to the case of the vacuum functional (partition function) which appears in the GZ theory formulated beyond the Landau gauge.

2 Gauge dependence problem

Partition function, \( Z_{\psi_0} \), in the original GZ theory is given by the following functional integral

\[
Z_{\psi_0} = \int \mathcal{D}\phi \exp \left\{ \frac{1}{\hbar} \left( S_0(A) + s\psi_0(\phi) + M\psi_0(A) \right) \right\}
\]  

(2.1)

where

\[
S_0(A) = -\frac{1}{4} \int d^d x \, F^a_{\mu\nu} F^{\mu\nu a} \quad \text{with} \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu
\]

(2.2)

is the action of Yang-Mills fields \( A^a_\mu(x) \) with gauge group SU(\( n \)) in \( d \) space-time dimensions and \( a = 1, \ldots, n^2-1, \mu = 0, 1, \ldots, d-1, f^{abc} \) are the (totally antisymmetric) structure constants
of the Lie algebra $su(n)$, $\phi$ denotes the set of fields $\{\phi^A\} = \{A^a_\mu, B^a, C^a, \bar{C}^a\}$, $B^a$ are the Nakanishi-Lautrup auxiliary fields as well as $C^a$ and $\bar{C}^a$ present the Faddeev-Popov ghost and antighost fields, respectively, with the following distribution of the Grassmann parities $\varepsilon$ and ghost numbers $\text{gh}(C^a) = \varepsilon(\bar{C}^a) = 1$, $\varepsilon(A^a_\mu) = \varepsilon(B^a) = 0$, $\text{gh}(A^a_\mu) = \text{gh}(B^a) = 0$, $\text{gh}(C^a) = -\text{gh}(\bar{C}^a) = 1$. In (2.1) $\psi_0(\phi)$ is gauge fixing functional in the Landau gauge

$$\psi_0(\phi) = \bar{C}^a \chi^a(A), \quad \chi^a(A) = \partial^\mu A^a_\mu.$$  

(2.3)

$M(A)$ denotes the Gribov horizon functional [1]

$$M_{\psi_0}(A) = \gamma^2 f^{abc} A^b_\mu (K^{-1})^{ad}(A) f^{dec} A^{e\mu} + \gamma^4 d(n^2 - 1),$$  

(2.4)

where $K^{-1}$ inverts the (matrix-valued) Faddeev-Popov operator $K^{ab}(A)$

$$K^{ab}(A) = \frac{\delta \chi^a(A)}{\delta A^c_\mu} D^c_\mu = \delta^{ab} D^c_\mu = \delta^{ab} \partial^\mu \partial_\mu + f^{abc} A^e_\mu \partial^\mu,$$

(2.5)

and $\gamma \in \mathbb{R}$ is the so-called thermodynamic or Gribov parameter [1, 2]. Finally $s$ means the nilpotent BRST operator presenting the BRST transformations

$$\delta_B A^a_\mu = D^c_\mu C^b \lambda, \quad \delta_B C^a = B^a \lambda, \quad \delta_B B^a = 0, \quad \delta_B C^a = \frac{1}{2} f^{abc} C^b C^c \lambda$$  

(2.6)

in the form

$$\delta_B \phi^A = (s \phi^A) \lambda$$  

(2.7)

where $\lambda$ is a constant fermionic parameter. It should be noted that the Gribov horizon functional (2.4) is not BRST invariant because

$$sM_{\psi_0}(A) = \gamma^2 f^{abc} f^{cde} [2 D^b_\mu C^q (K^{-1})^{ad} - f^{mpn} A^b_\mu (K^{-1})^{am} K^{pq} C^q (K^{-1})^{nd}] A^{e\mu} \neq 0.$$  

(2.8)

This point is very crucial for the GZ theory because the BRST symmetry and its nilpotency play the fundamental role in a consistent formulation of the quantum theory of gauge fields [13].

The Gribov horizon functional (2.4) is written in the non-local form although there exists a local presentation based on an extended configuration space which is equipped with additional two doublets of fermionic and bosonic fields [2]. For our aims it is enough to work with the original configuration space of fields $\phi$ and with the non-local Gribov horizon functional. It seems quite natural to assume existence of the Gribov horizon functional and its gauge dependence beyond the Landau gauge. Let $\psi = \psi(\phi)$ be an admissible gauge which differs from the case of the Landau gauge fixing functional $\psi_0 = \psi_0(\phi)$ (2.3) and $M_{\psi}(\phi)$ be a corresponding Gribov horizon functional. It should be noted that we don’t know an explicit expression for

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3Here and below we use the DeWitt’s condensed notations [12].
\(M_\psi\) in arbitrary gauge \(\psi\) but it is not an obstacle to write the vacuum functional of the GZ theory in the form of functional integral

\[
Z_\psi = \int \mathcal{D}\phi \ \exp \left\{ \frac{i}{\hbar} (S_0(A) + s\psi(\phi) + M_\psi(\phi)) \right\}.
\]

(2.9)

It follows from (2.9) that variation of \(Z_\psi\) under infinitesimal change of gauge fixing functional
\(\psi \to \psi + \delta\psi\) reads

\[
\delta Z_\psi = \frac{i}{\hbar} (\langle s\delta\psi \rangle + \langle \delta M_\psi \rangle)
\]

(2.10)

where the notation

\[
\langle (...) \rangle = \int \mathcal{D}\phi \ (\ldots) \exp \left\{ \frac{i}{\hbar} (S_0(A) + s\psi(\phi) + M_\psi(\phi)) \right\}
\]

(2.11)

is used and \(M_{\psi + \delta\psi} = M_\psi + \delta M_\psi\). If \(\delta Z_\psi \neq 0\) or \(\langle s\delta\psi \rangle + \langle \delta M_\psi \rangle \neq 0\) then the vacuum functional depends on gauge and the same is valid for physical S-matrix due to the equivalence theorem [14]. In turn a physical interpretation becomes impossible. There exists only one possibility for the GZ theory to be a consistent one if the following relation

\[
\langle s\delta\psi \rangle + \langle \delta M_\psi \rangle = 0
\]

(2.12)

holds. The restriction (2.12) seems very strong limitation on possible structure of \(M_\psi\) because of the fixed structure \(s\delta\psi\) and the known explicit form \(M_\psi\) (2.4) in the Landau gauge (2.3). In general it is difficult to expect that the condition (2.12) may be satisfied. Nevertheless a solution to the problem (2.12) has been proposed in [8].

3 Solution to the equation (2.12)

Let us consider the change of variables

\[
\phi^A \to \phi'^A = \phi^A + (s\phi^A)\Lambda(\phi).
\]

(3.1)

in the functional integral (2.1). Here \(\Lambda(\phi)\) is an arbitrary odd Grassmann functional of fields \(\phi\). These transformations are known as the field-dependent BRST transformations in Yang-Mills theories [15, 7] (see also recent progress in this field [16, 17, 18, 19, 20, 21]). The result of this change can be presented in the form [8]

\[
Z_{\psi_0} = \int \mathcal{D}\phi \ \exp \left\{ \frac{i}{\hbar} \left[ S_0(A) + s\psi_0(\phi) + i\hbar \ln (1 + s\Lambda(\phi)) + M_{\psi_0}(A) + (sM_{\psi_0}(A))\Lambda(\phi) \right] \right\}.
\]

(3.2)

Using arbitrariness in choice of functional \(\Lambda(\phi)\) and the nilpotency of operator \(s\) we can generate any admissible gauge \(\psi(\phi)\) in the presentation (3.2). Indeed following the paper [7] let us consider the functional \(\Lambda_\psi(\phi)\)

\[
\Lambda_\psi(\phi) = (\psi - \psi_0) \left( s(\psi - \psi_0) \right)^{-1} \left( \exp \left\{ \frac{1}{i\hbar} s(\psi - \psi_0) \right\} - 1 \right).
\]

(3.3)
Substituting (3.3) \((\Lambda(\phi) = \Lambda_\psi(\phi))\) into (3.2) one obtains
\[
Z_{\psi_0} = \int D\phi \exp \left\{ \frac{1}{\hbar} [S_0(A) + s\psi(\phi) + M_{\psi_0}(A) + (sM_{\psi_0}(A))\Lambda_\psi(\phi)] \right\}. \tag{3.4}
\]
Note that \(S_0(A) + s\psi(\phi)\) is the Faddeev-Popov action written in the gauge \(\psi(\phi)\). If we identify the functional \(M_{\psi_0}(A) + (sM_{\psi_0}(A))\Lambda_\psi(\phi)\) with the Gribov horizon functional \(M_\psi(\phi)\) in the gauge \(\psi(\phi)\),
\[
M_\psi(\phi) = M_{\psi_0}(A) + (sM_{\psi_0}(A))\Lambda_\psi(\phi), \tag{3.5}
\]
then due to (2.10) we arrive at the gauge independence of vacuum functional in the GZ theory
\[
Z_{\psi_0} = Z_\psi. \tag{3.6}
\]
It should be stressed that the relation (3.5) presents the definition of the Gribov horizon functional beyond the Landau gauge leading to possibility of formulation of the GZ theory in a consistent way. Derivation of this functional from the first principles as it was made for the Gribov horizon functional in the Landau gauge [1, 2] may give another answer. It looks that it is really happening in the case of the Coulomb gauge.

4 Coulomb gauge

Consider the Coulomb gauge when the gauge fixing functions \(\chi^a(A)\) and the gauge fixing functional \(\psi(\phi)\) are equal to
\[
\chi^a(A) = \partial^i A_i^a, \quad \psi(\phi) = \bar{C}^a \partial^i A_i^a. \tag{4.1}
\]
In this case
\[
\psi - \psi_0 = -\bar{C}^a \partial^0 A_0^a, \quad s(\psi - \psi_0) = -B^a \partial^0 A_0^a + \partial^0 \bar{C}^a D_0^{ab} C^b. \tag{4.2}
\]
From (4.2) it follows the non-polynomial dependence \(M_\psi(\phi)\) (3.5) on fields \(C^a, \bar{C}^a, B^a\). On the other hand in paper [11] the non-local form of the Gribov horizon functional in the Coulomb gauge has been written as
\[
M_\psi(A) = \gamma^2 f^{abc} A_i^b (K^{-1})^{ad} f^{dec} A_i^e - (d - 1)(n^2 - 1)\gamma^4, \tag{4.3}
\]
where \((K^{-1})^{ab}\) is inverse to the Faddeev-Popov matrix in the Coulomb gauge \(K^{ab} = \partial^i D_i^{ab}\). The functional \(M_\psi(A)\) (4.3) differs essentially from the functional \(M_\psi(\phi)\) (3.5) with relations (3.3) and (4.2) because these functionals are defined in different functional spaces. In our opinion it means that the original GZ theory [1, 2, 11] meets serious problem concerning a correct definition of physical observables.
5 Discussion

In the present paper we have attracted attention to the gauge dependence problem existing in the original GZ theory. For the first time the problem has been discussed in paper [5] where analysis of gauge dependence was based on assumption that the Gribov horizon functional exists beyond the Landau gauge. It was shown that in general the partition function and the effective action on its extremals depend on gauges. In our opinion it makes impossible physical interpretation of results obtained within the original GZ theory. But in particular from the analysis given in [5] it followed existence of a special dependence of the Gribov horizon functional on gauges when the effective action on its extremals will be gauge invariant. Later such kind of the Gribov horizon functional has been proposed [8]. In the present paper we have constructed explicitly the form of the Gribov horizon functional in the Coulomb gauge leading to the gauge independence of the partition function. Unfortunately we have found that this functional differs from the Gribov horizon functional in the Coulomb gauge appearing in the GZ theory [11].

Last but not least remark. Quite recently a reformulation of the original GZ theory has been proposed [22] to improve quantum properties of the theory concerning the BRST symmetry and its nilpotency. In this connection it should be definitely noted that the reformulation was performed with the help of change of variables which violates the transformation laws of fields under the Poincare group. If we will consider the GZ theory [1, 2] formulated in accordance with general principles of quantum field theory then we should consider the new theory [22] as ill-defined and vice versa. These theories can not exist simultaneously as equivalent. In particular, it means that results of quantum calculations in the new theory have no any relations to the original GZ theory. We conclude that the gauge dependence problem remains open in the original GZ theory.

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