Theoretical optical and x-ray spectra of liquid and solid H$_2$O

J. Vinson,$^1$ J. J. Kas,$^1$ F. D. Vila,$^1$ J. J. Rehr,$^1$ and E. L. Shirley$^2$

$^1$Dept. of Physics, Univ. of Washington Seattle, WA 98195
$^2$National Institute of Standards and Technology, Gaithersburg, MD 20899

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Theoretical optical and x-ray spectra of model structures of water and ice are calculated using a many-body perturbation theory, Bethe-Salpeter equation (BSE) approach implemented in the valence- and core-excitation codes AI2NBSE and OCEAN. These codes use ab initio density-functional theory wave functions from a plane-wave, pseudopotential code, quasi-particle self-energy corrections, and a BSE treatment of particle-hole interactions. The approach improves upon independent-particle methods through the inclusion of a complex, energy-dependent self-energy and screened particle-hole interactions to account for inelastic losses and excitonic effects. These many-body effects are found to be crucial for quantitative calculations of ice and water spectra.

I. INTRODUCTION

Recently there has been considerable interest and controversy surrounding the connection between the local structure of water and ice and their observed optical and x-ray spectra. Part of the difficulty lies in modeling the structures of these complex, finite-temperature systems. Conventional super-cell methods require large unit cells to treat proton-disorder in ice and a configurational average adequate to represent a statistical ensemble of water structures. Structural probes such as x-ray and neutron scattering do not provide an unambiguous interpretation of local geometry. Another part of the difficulty lies in theoretical modeling. Despite many attempts, quantitative theoretical calculations of the optical and x-ray spectra of these systems have proved to be notoriously difficult due to strong non-local, self-energy, and excitonic effects. Thus the various theoretical methods that have been employed to date exhibit considerable variation in their results, undermining a definitive interpretation.

In an effort to address these issues, we present calculations of the valence and core excitation spectra of well-characterized model ice and water systems based on a recently developed approach utilizing the Bethe-Salpeter equation (BSE) and Hedin’s GW approximation for the quasi-particle self-energy (the acronym GW refers to the product of the one-electron Green’s function $G$ and the screened Coulomb interaction $W$) with a pseudopotential plane-wave basis. This GW/BSE approach has been implemented in the AI2NBSE and OCEAN packages for valence and core-level spectra respectively. The method is advantageous compared to independent-electron approximations in that it provides a first-principles method for the inclusion of both quasi-particle and excitonic effects. These many-body effects modify peak positions, widths, and strengths, and hence they are crucial for a quantitative treatment and interpretation of the spectra. GW/BSE calculations have been carried out previously for both the valence and core spectra of water, with various approximations for the screened particle-hole interaction. A key difference is that our implementation uses a complex, energy-dependent GW self-energy, in addition to the screened particle-hole interactions.

The remainder of this paper is as follows: The theoretical methods are summarized in Sec. II. Results for two forms of ice are presented in Sec. III, and for a model water system in Sec. IV. Finally, Sec. V contains a summary and suggestions for further work.

II. THEORETICAL METHODS

A. GW/BSE Approach

The theory behind the GW/BSE approach implemented in AI2NBSE and OCEAN for valence and core excitations is described in detail in Refs. 10 and 11 respectively, so only a short summary is included here. The BSE represents the equation of motion of a particle-hole state, here an electron photo-excited into the conduction bands from either the occupied valence bands (optical) or a deep Oxygen 1s orbital (x-ray). Briefly, the open-source plane-wave, pseudopotential code ABINIT is used to calculate both occupied and unoccupied Kohn-Sham density functional theory (DFT) states of the ground-state Hamiltonian, which serve as a basis for the BSE. This Kohn-Sham calculation uses the Ceperley-Alder, Perdew-Wang local density approximation for the exchange-correlation potential. DFT orbitals only strictly describe a non-interacting system, but they can be good approximations to quasi-particle wavefunctions and perturbativley corrected, e.g., via Hedin’s GW approximation. In this work, first-order quasi-particle energy corrections are added through the GW many-pole self-energy (MPSE) approximation of Kas et al. In contrast to previous GW/BSE approaches for these systems, our MPSE is complex and energy-dependent, and thus accounts for both the energy shifts and inelastic losses which stretch and damp the spectral features. This MPSE model is based on a many-pole fit...
to AI2NBSE calculations of the valence loss spectrum $-\text{Im } \epsilon^{-1}(\omega)$.

The optical and x-ray spectra, the latter including both x-ray absorption (XAS) and non-resonant inelastic x-ray scattering (NRIXS), which is also referred to as x-ray Raman spectra (XRS), are then calculated using the National Institute of Standards and Technology (NIST) BSE solver in AI2NBSE\textsuperscript{19} and OCEAN\textsuperscript{11} respectively. The core spectra in OCEAN use atomic core-level states and projector-augmented-wave (PAW) transition matrix elements.\textsuperscript{22} The BSE kernel includes both a screened direct attraction between the electron and hole and an unscreened repulsive exchange term. The treatment of the particle-hole interactions is an important consideration due to the weakly screened excitonic effects in water and ice. In both our valence and core codes the screened Coulomb interaction $W$ is approximated as statically screened. A Hybertsen-Levine-Louie dielectric function is implemented in AI2NBSE, while OCEAN uses the random-phase approximation at short-range, switching to a model dielectric function at long range.\textsuperscript{20,21} In contrast, previous approaches have used a variety of approximations ranging from a self-consistently screened DFT core-hole\textsuperscript{22} to half-core-hole transition-state potentials.\textsuperscript{2}

B. Structures

The pair distribution function (PDF) $g(r)$, and total electronic density of states (DOS) provide useful measures of the ground-state structural and electronic properties of our model ice and water systems (Fig. 1). In order to interpret x-ray and optical spectra, it is important that the model structures used in the calculations match observed structural properties, e.g., the PDF from neutron or x-ray diffraction. For our model ice-Ih cell $g_{OO}(R)$ is in reasonable agreement with experiment. However, the first shell peak is shifted to a slightly larger mean radius. To be consistent with experiment a root-mean-square disorder of about 0.25 Å is needed, which is simulated in Fig. 1a by convolving the model PDF with a Gaussian. For our 17-molecule water cells (see Sec. III) fair agreement with experiment is observed, but with a slightly elongated first-shell O-O distance. The position and height of the first peak in $g_{OO}(R)$ are sensitive to both the momentum range in the diffraction measurements and assumptions about the structure and core potentials. Discussions of these limitations can be found elsewhere, e.g., Refs. 6 and 24.

The total electronic density of states (DOS) provides a useful picture of the ground-state electronic structure, and in particular, the occupied and unoccupied energy levels in these materials. A comparison of the DOS for liquid, solid, and gas-phase H$_2$O as calculated with ABINIT is shown in Fig. 1a. The DOS of both liquid water and ice-Ih are roughly similar, apart from broadening and the sharp peak at about +10 eV in ice. The similarity among the occupied DOS suggests that the core states are well localized and hence essentially molecular in character. Likewise, the similarity in the unoccupied DOS between liquid and solid H$_2$O suggests that short-range order dominates the spectra. The additional broadening in the liquid reflects the larger configurational disorder in amorphous structures. In contrast, the unoccupied DOS of the gas phase shows considerably more structure than either the liquid or solid phases. These results for the DOS are similar to previous work, e.g., Chen et al.\textsuperscript{4}

C. GW Self-energy

Quasi-particle self-energy effects are of crucial importance in broad-spectrum calculations of optical and x-ray spectra, because independent-particle calculations systematically underestimate conduction band widths. A comparison of our GW many-pole self-energies for wa-
Note that these are quite similar, indicative of similar broadening and self-energy damping effects in water and ice. The quasi-particle self-energy broadening may be compared to other sources of broadening in the spectra: experimental resolutions (≈ 0.5 eV), the O 1s core-hole lifetime (0.16 eV), and vibrational broadening which is yet smaller and not included.

The discrepancy likely reflects a lack of disorder in our model structure (see text).

A. Ice Models

For this study we have focused on two forms of ice; ice-VIII, which is stable below 273 K and pressures in the range of 2 GPa to 50 GPa, and ice-Ih, which is the common form of ice, stable under ambient pressure. Our ice-Ih model uses a 16-molecule cell determined by enforcing the experimental density and following the ice rules for hydrogen placement with an O-H bond distance and angle of 1.01 Å and 106° respectively. A realistic simulation of ice-Ih should have full proton disorder, but our models are based on small cells with periodic boundary conditions, and thus contain artificial order. Further calculations for other ice-Ih models using larger unit cells and finite temperature configuration sampling are called for to assess the validity of our simplified model.

Ice-VIII is formed under high pressure at temperatures below the conventional freezing point of water. The pressure requirements preclude experimental techniques incompatible with diamond-anvil cells such as lower energy XAS measurements. However, NRIXS can probe the same final-state as XAS with an additional tunable parameter q, the momentum transfer and at a much higher beam energy. The structure of ice-VIII is that of two interpenetrating cubic ice lattices resulting in a compressed second shell and higher densities than ice-Ih. The unit cell contains only 4 molecules and has anti-ferroelectric Hydrogen ordering, making it an easier system to simulate than standard ice. Our ice-VIII cell uses the experimental lattice constants at a pressure of 2.4 GPa.

B. Ice valence spectra

For the ice-Ih spectra, the wavefunctions from ABINIT were calculated on a 4 × 4 × 4 k-point mesh with 464 bands.
FIG. 4: The calculated momentum-transfer dependence of the oxygen K-edge in ice-Ih (solid red), including the MPSE. For comparison experimental results (dashed blue) for XAS[29] and NRIXS[25] are shown.

(400 conduction bands). For comparison, the structure and valence spectrum of hexagonal ice (ice-Ih) were studied previously with DFT/BSE approaches, yielding qualitative agreement with experiment[29] For the valence spectrum (Fig. 3), the calculated excitonic peaks at 8 eV and 10 eV are significantly stronger than those measured. Between 10 eV and 20 eV the absorption is also too strong and too thinly peaked, compared to experiment which exhibits additional broadening past 15 eV. These discrepancies are partly due to the lack of disorder in our model structure, and point to the need to consider more elaborate models that can account for such disorder. The effect of the MPSE is seen as a stretch of the spectra, together with energy dependent broadening. Coupling between electronic excitations and phonons is expected to provide small additional broadening, but is not included in this work.

C. Ice core spectra

For the oxygen K-edge XAS and NRIXS calculations a $4 \times 4 \times 4$ k-point mesh and 400 conduction bands were used for the BSE calculation and 900 bands for the screening. Oxygen K-edge XAS and NRIXS results for ice-Ih are shown in Fig. 4. All calculated x-ray spectra have been broadened by convoluting with a Lorentzian to account for the core-hole lifetime and a Gaussian to match reported experimental broadening. Further damping is provided by the imaginary part of the self-energy (cf. Fig. 2). Results are shown averaged over orthogonal incident photon polarizations or momentum transfers for XAS and NRIXS respectively, since experimental data is reported for polycrystalline samples.

Our XAS calculation matches experiment fairly well, but has a lower pre-edge (535 eV) and is noticeably too narrow in overall width (Fig. 4). Recently the momentum dependence of the O K-edge in ice has been measured using NRIXS[25] The theoretical simulation of NRIXS with OCEAN differs only slightly from XAS in that the finite momentum transfer $q$ can break the dipole selection rules, allowing transitions to s-type and higher angular momentum final states. Overall the calculations yield fair agreement for the spectra, including both the relative weights and the momentum dependence of the pre-edge feature (Fig. 4). However, the balance between the s- and p-character of the pre-edge is shifted too much towards the s-type in our calculation, leading to overly strong growth of the pre-edge with increasing $q$. Note that experiment shows little difference between $q \approx 0$ (XAS) and $q = 3.1$.

We have also examined the momentum dependence of ice-VIII (Fig. 5) which exhibits a similar evolution with increasing $q$ as ice-Ih, and good general agreement with experiment. Like ice-Ih, our calculations of the O K-edge in ice-VIII are too strongly peaked in the main edge and have too little weight above about 543 eV when compared to experiment, but show improvements over previous theoretical results[30] Similar to ice-Ih, the calculations of ice-VIII also show excessive $q$ dependence for the pre-edge, thus suggesting either a limitation of the DFT wave functions or the need to consider thermal and quantum disorder.

IV. WATER: 17-MOLECULE LIQUID CELLS

A. Water structural model

Our calculations for optical and x-ray spectra were carried out on 17-molecule snapshots for liquid water obtained from the molecular dynamics (MD) results of Garbuio et al.[14] Despite the small size of these model structures, they already exhibit substantial disorder and are found to have a pair-distribution function (Fig. 1) in fairly good agreement with experiment. However, the O-O nearest neighbor distance is notably too long by a few
tenth of an angstrom. Additionally, while bulk water is a disordered system with no net dipole moment, the limited cell size of our systems precludes this constraint; for our samples the moments were found to vary widely from 14 Debye to 28 Debye.

B. Water valence spectra

For the valence calculation a $4 \times 4 \times 4$ $k$-point mesh and 300 conduction bands were used, and both a static energy shift of 4.13 eV, as suggested by Garbuio et al., as well as the energy dependent MPSE were applied. With the static shift the positions of the initial peaks match well with experiment (Fig. 6) and in general, the agreement is excellent over the full range of the spectra. Our results for the peak positions in $\varepsilon_2$ agree qualitatively with those of Garbuio et al., but yield features that are smaller in overall magnitude. The energy dependence of our MPSE (Fig. 2) introduces a stretch in the spectrum of $\approx 5\%$ in the range of 10 eV to 20 eV, while Garbuio et al. reported the GW energy correction to be almost constant across the low-lying conduction bands for water.

C. Water core spectra

XAS calculations were carried out for 8 of the MD snapshots using a $4 \times 4 \times 4$ $k$-point mesh including 300 bands for the final states and 400 conduction bands for the screening. The screened core-hole potential was found to be nearly identical for different O sites within a cell, reflecting the molecular character of liquid water. The O K-shell XAS calculation (Fig. 7) exhibits considerable variation among the different oxygen sites both within a cell and between cells. Both the calculated and recent experimental spectra exhibit a notable intensity shift and increased broadening going from ice to water, shifting spectral weight to the lower part of the spectrum from 534 eV to 538 eV. The calculated O K-edge spectra is in good agreement with experiments when corrected with the MPSE, but, as for the calculated ice-Ih spectra, the main edge at about 536 eV is stronger than in experiment. The peak at 541 eV in experiment is evident in the calculation but is lacking in strength. The success of the MPSE in correcting the overall width of the near-edge is consistent with calculations using the COHSEX approximation, but in contrast to earlier work, which showed that the self-energy for water was nearly constant for their $G_0W_0$ approximation.

In a disordered system like liquid water, each oxygen is subject to a different static potential and to slightly different core-hole screening by the valence electrons, resulting in shifts in the binding energy of the 1s electrons. Exact, absolute energies were not calculated, but relative shifts were determined according to the relation

$$E_{1s} = E_{1s}^{av} + V^{KS} + \frac{1}{2} W_C. \quad (1)$$

$E_{1s}^{av}$ is the site-independent binding energy which has been aligned to match experiment. The second term $V^{KS}$, is the total Kohn-Sham potential at the site, including the effects of all the other ionic cores. The last term is the effect of the spectator electrons screening the core hole, reducing the energy necessary to excite the 1s electron. Both $V^{KS}$ and $W_C$ are evaluated at the Oxygen site. These binding-energy shifts lead to a slight broadening of the spectra in Fig. 7.
We also find that the unphysical dipole moments of the small unit-cells in this study have a systematic effect on the calculated spectra (inset of Fig. 1). That is, the two main peaks of the calculated O K-edge at 537 eV and 540 eV exhibit shifts in intensity and position that loosely correlate with dipole moment, i.e., the local, static electric field felt by each water monomer in the ground state. At higher values of the dipole moment the main edge features shift to lower energies, increasing discrepancy with experiment. This again points to the need for simulations with larger cells that more accurately characterize the model structure and properties.

V. SUMMARY AND FUTURE PROSPECTIVE

Theoretical calculations have been carried out for both valence and core x-ray spectra of a number of model ice and water systems using modern GW/BSE theoretical methods. Our results suggest that accurate calculations of the spectra for these systems are quite sensitive both to the theoretical methods and details of the model structures. Nevertheless, we have shown that the improved treatment of many-body effects in the present approach yields better agreement with experiment in terms of both relative peak weights as well as overall width and feature locations compared to calculations that ignore these effects.

We find that the inclusion of an accurate quasi-particle self-energy is important to characterize the damping and self-energy shifts in the spectra. In particular we find that the stretch provided by our GW MPSE significantly improves the agreement between calculations and experiment for the ice and water systems. Our results for the valence $\epsilon_2$ spectra for the 17-molecule water model differ somewhat from those of Garbuio et al. Theirs have somewhat larger values of $\epsilon_2$, while ours exhibits an extra excitonic peak. The origin of these differences is likely due to differences in the screening approximations used.

In any case, our present results appear to be in reasonable agreement with recent experimental results both for the core- and valence-spectra of water. However, we suggest that configurational averages should be carried out for ice-Ih and ice-VIII to better understand the effects of disorder, finite temperature, and zero-point variations in structure. A greater variation in hydrogen positioning and bonding from finite temperature and zero-point motion effects could lead to changes in feature weights as well as adding an increased broadening from disorder. Additionally, convergence should be checked with respect to the size of the cells being used, especially because larger cells should allow for better control of the net dipole moment. At present, however, system size is limited by our codes and hence significantly larger cells are computationally impractical. The oxygen K-edge should be calculated for a sufficient number of MD snapshots to ensure a good representation of the physical system. Site-specific geometrical information, i.e., bond length, angles, and hydrogen bond coordination, can then be compared to contributions from individual oxygen XANES to correlate differences in local environment and calculated XAS/NRIXS.

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