Selfcoupled equations for the field correlators

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Abstract

New nonlinear equations for field correlators in the large $N_c$ gluodynamics are derived. Nonperturbative confining correlators are shown to satisfy a system of nonlinear scale–invariant equations. Keeping only the simplest, bilocal correlators one can find the exponential large distance behaviour for both independent correlators $D$ and $D_1$ and express coefficients in the exponentials – the gluonic correlation length $T_g$ – through the string tension. Numerically these results are in agreement with lattice data.

1 Introduction

The main unsolved problem of QCD and of the modern field theory in general is the quantitative treatment of nonperturbative phenomena. There is no formalism yet (e.g. a system of equations) in which nonperturbative quantities, like masses and condensates in QCD, are computed from the first principles. By now there are two main approaches to this problem: i) lattice calculations; ii) modelling the nonperturbative vacuum with e.g. quasiclassical solutions. In the first approach one can indeed calculate numerically hadron masses and even more fundamental quantities like field correlators [1]. However, here the physical mechanism creating nonperturbative phenomena is kept hidden behind the formidable numerics. The second approach is necessarily model dependent and approximate. Also there is no successful model of the QCD vacuum, which describes both basic nonperturbative phenomena: confinement and chiral symmetry breaking (CSB) (however, the dyonic model recently suggested for QCD [2] might be promising in this respect).
The main characteristics of nonperturbative (NP) phenomena in QCD is that they create their own mass scale. By this fact the scale invariance is violated and through the scale anomaly the NP shift downwards in the vacuum energy density is produced [3]. Therefore one may expect that NP solutions appears as scale violating solutions of some nonlinear scale–invariant equations, while perturbative solutions of the same equations are obtained by expanding in powers of coupling constant.

Recently equations of this type have been written for gauge–invariant quark propagators [4], where the kernel of the equations contains the fundamental string. The equations [4] do not violate symmetry (in this case the chiral symmetry), while NP solutions break chiral symmetry. In [4] the gluon-field correlators were taken as an input and the problem how to find the correlators from the first principles was not solved there. Here we are proposing selfcoupled equations for field correlators in the case of gluodynamics. These equations are also of Dyson–Schwinger type but will be written for the gauge–invariant amplitudes, which automatically display at large distances the formation of adjoint string in the large $N_c$ limit.

To write these equations we divide all $N_c^2 - 1$ gluonic fields into two groups: a small group of $r$ fields $b_i^\mu, i = 1, ...r$, which later be called "valence gluons", and a large group of fields $A_a^\mu, a = r + 1, ...N_c^2 - 1$, which form a background for valence gluons.

As will be shown below this background creates in the confining phase after vacuum averaging an adjoint string. Hence, physically the problem reduces to the motion of valence gluons in the field of the string. Since the string is a white object, the action of the string on valence gluons generates color–diagonal mass operator and the resulting equation for the valence gluon propagator simplifies considerably. We shall find out that these equations are nonlinear and selfcoupled, so that one can look for selfconsistent solutions for the valence gluon propagator and the field correlator. Inserting the resulting field correlator in the kernel, one realizes the selfcoupling mentioned above.

Some properties of the solutions can be found even before the actual solution of equations e.g. the exponential large distance behaviour of field correlators and the coefficient in the exponent – the so-called gluon correlation length $T_g$ – can be computed through the string tension or gluonic condensate. Our number for $T_g$ is in agreement with lattice calculations [1]. Thus one finds a solution depending on a single scale. This scale should be given as an input to define the NP part of the theory completely.
We shall not touch in this letter the perturbative expansion and renormalization of equations obtained here, referring the reader to a future publication. The paper is organized as follows. In section 2 the separation of gluonic fields into "valence" and background parts is done and the effective action for the valence gluons is written explicitly. In section 3 the mass operator for valence gluons is found after averaging over background fields. In section 4 the connection of field correlators with the valence gluon propagator is established and it is used to find the asymptotic behaviour of field correlators and to compute the gluonic correlation length $T_g$ through the string tension or gluonic condensate. In Conclusion a summary of results is given and possible extensions are discussed.

2 Effective action for valence gluons

We start dividing the gluonic fields $A^a_\mu (a = 1, ... N_c^2 - 1)$ into a group of valence gluon fields $b^a_\mu$, with the color index $a$ running a small set of colours: $a = 1, ... r$, while the rest of fields, for which we keep notation $\bar{A}^a_\mu$, belong to the large set, $a = r + 1, ... N_c^2 - 1$. Number of colors $N_c^2 - 1$ is assumed to be large, and $r \ll N_c^2 - 1$. Note that this division does not separate perturbative gluons from a NP background, as is usually done in the background perturbation method [5]. Both $b_\mu$ and $\bar{A}_\mu$ may contain perturbative and NP components. Hence, $A^a_\mu = A^a_\mu + b^a_\mu$ and one can write for the Euclidean action density

$$S_E(A) = S_E(\bar{A} + b) =$$

$$= S^{(0)}(\bar{A}) + S^{(1)}(b, \bar{A}) + S^{(2,1)}(b, \bar{A}) + S^{(2,0)}(b) +$$

$$S^{(2,2)}(b, \bar{A}) + S^{(3,1)}(b, \bar{A}) + S^{F}(b, \bar{A}) + S^{(3)}(b) + S^{(4)}(b),$$

where we have defined

$$S^{(0)}(\bar{A}) = \frac{1}{4} (F^a_{\mu\nu}(\bar{A}))^2,$$

$$S^{(1)}(b, \bar{A}) = -b^\mu_\nu D^a_\mu (\bar{A}) F^a_{\mu\nu}(\bar{A})$$

$$S^{(2,0)}(b) = b^\mu_i (-\frac{1}{2} \partial^2_\mu \delta_{\mu\nu} + \frac{1}{2} \partial_\mu \partial_\nu) b^\nu_i$$

$$S^{(2,1)}(b, \bar{A}) = \frac{1}{2} g f^{ikc} b^i_\mu [2 \bar{A}^c_\lambda \partial_\lambda \delta_{\mu\nu} + \bar{\partial}_\mu \bar{A}^c_\nu - \bar{A}^c_\mu \partial_\nu] b^k_\nu$$

$$3$$
\[ S^{(F)}(b, \bar{A}) = g f^{ika} b^i_\mu b^k_\nu F^a_{\mu\nu}(\bar{A}) \]  
(6)

\[ S^{(2,2)}(b, \bar{A}) = \frac{1}{2} \chi^{ik,ab}(b^i_\mu b^k_\nu \bar{A}^a_\chi \bar{A}^b_\lambda - b^i_\mu b^k_\nu \bar{A}^a_\mu \bar{A}^b_\nu) \]  
(7)

\[ S^{(3,1)}(b, \bar{A}) = b^i_\mu b^k_\mu \chi^{ik,ba} \bar{b}^a_\nu \bar{A}^a_\nu \]  
(8)

\[ S^{(4)}(b) = \frac{1}{4} b^i_\mu b^k_\mu \chi^{ik,lm} b^l_\nu b^m_\nu, \quad S^{(3)} = \frac{1}{2} g f^{ikl} b^i_\mu \partial_\lambda b^k_\mu \]  
(9)

In (2)-(9) the following notations are used

\[ \chi^{ik,ab} = g^2 \sum_{c=1}^{N_c^2-1} f^{cia} f^{ckb}, \]
(10)

Note also that we have denoted color indices of \( b_\mu \) fields by \( i, k, l, \ldots \) running from 1 to \( r \), and indices \( a, b, c \) in \( \bar{A}_\mu \) running from \( r+1 \) to \( N_c^2 - 1 \).

As a next step consider averaging over fields \( \bar{A}_\mu \) in some expression, like the following

\[ D_{\mu\nu,\lambda\sigma}(x, y) = \langle F^\alpha_{\mu\nu}(x) \Phi^{\alpha\beta}(x, y) F^{\beta\gamma}_{\lambda\sigma}(y) \rangle_{b, \bar{A}}, \quad 1 \leq \alpha, \beta \leq N_c^2 - 1, \]
(11)

where \( \Phi \) is the parallel transporter in the adjoint representation

\[ \Phi^{\alpha\beta}(x, y) = (P \exp ig \int_y^x A_\mu dz_\mu)_{\alpha\beta}, \quad A_\mu = A^a_\mu T^a_\mu, \quad T^a_\mu = -i f^{abc} \]
(12)

In computing the matrix element (11) one can integrate first over fields \( \bar{A}_\mu^a \), and then as a last step integrate over fields \( b_\mu \). It is essential that \( \Phi \) contains both fields \( \bar{A}_\mu \) and \( b_\mu \) and we shall choose the so-called modified coordinate gauge [6] to express \( A_\mu \) in terms of field strength \( F_{\mu\nu} \). Let us now specify this gauge and to this end choose \( x, y \) in (11) on the 4-th axis, so that \( x = (0, \vec{0}) \), \( y = (T, \vec{0}) \). In the chosen gauge we have

\[ A_4(\vec{z}, z_4) = \int_0^1 d\alpha z_4 F_{i4}(\alpha \vec{z}, z_4), \]
(13)

\[ A_k(\vec{z}, z_4) = \int_0^1 d\alpha z_4 F_{ik}(\alpha \vec{z}, z_4) \]
(14)

From (13) it is clear that \( \Phi(0, T) \equiv 1 \) and parallel transporters can be omitted.
The basic quantity to be considered below is the gluonic propagator $G_{ik}^{\mu\nu}(x, y) = < b_i^{\mu}(x)b_k^{\nu}(y) >_{b,\bar{A}}$, where the averaging over fields $b, \bar{A}$ should be done with the weight $exp(-\int S_E d^4x)$.

The averaging over fields $b, \bar{A}$ will be done in two steps. First, the averaging over $\bar{A}$ for some operator $N$ can be written as

$$< N(\bar{A}) >_{\bar{A}} \equiv \frac{1}{Z} \int D\bar{A}_\mu exp(-\int d^4x S^{(0)}(\bar{A}))N(\bar{A})$$

and therefore the averaging over $\bar{A}_\mu, b_\mu$ in (15) implies the definition of the effective valence gluon action $S_{eff}(b)$,

$$e^{-S_{eff}(b)} = < e^{-\sum_i \int S_i d^4x} >_{\bar{A}}$$

with $i = (1), (20), (21), (F), (2, 2), (3, 1), (3), (4)$. As a next step the averaging over fields $b$ can be written for some operator $T(b)$ as

$$< T(b) >_b \equiv \frac{1}{Z} \int exp(-S_{eff}(b))T(b) Db_\mu$$

In calculating $S_{eff}(b)$ in (17) we shall use the cluster expansion theorem and neglect in the first approximation all higher cumulants beyond quadratic ones (discussion of this approximation will be given below in this section). Then one obtains

$$< e^{-\sum_i S_i(b, \bar{A})} >_{\bar{A}} \cong e^{-\sum_i <S_i>_{\bar{A}} + \frac{1}{2}\sum_{i,k} <S_i S_k>_{\bar{A}}}$$

where double angular brackets denote the cumulant:

$$< S^2 > \equiv < S^2 > - < S >^2$$

It is easy to show that within our approximation the terms $S^{(2,0)}, S^{(3)}, S^{(4)}$ contribute only to the linear average $< S^i >_{\bar{A}}$, while $S^{(1)}, S^{(2,1)}, S^{(F)}, S^{(3,1)}$ only to the quadratic one.

Hence one has

$$S_{eff}(b) = \int (S^{(2,0)}(b) + S^{(3)}(b) + S^{(4)}(b)) d^4x + \int < S^{(2,2)} >_{\bar{A}} d^4x - \sum_i L^{(i)}$$
where $L^{(i)}(i = 1; (2, 1); (3, 1); F)$ are quadratic averages of the corresponding terms $S^{(i)}$ and also the term, which will be important in what follows, is present:

\[ < S^{(2,2)} >_A = \frac{1}{2} \chi^{i,ab} \{ b^i(x)b^k(x) < \bar{A}^a_\lambda(x)\bar{A}^b_\lambda(x) >_A - b^i_\mu(x)b^k_\nu(x) < \bar{A}^a_\mu(x)\bar{A}^b_\nu(x) >_A \} \]

Now using (14), (15) one can write

\[ g^2 < \bar{A}^c_\lambda(x)\bar{A}^{c'}_\lambda'(y) >_A = \frac{\delta_{cc'}}{C_2^f} J_{\lambda\lambda'}(x, y), \]  

(23)

where $C_2^f = \frac{N^2 - 1}{2N_c}$ and the important function $J_{\mu\nu}(x, y)$ is introduced:

\[ J_{\lambda\lambda'}(x, y) = \int_0^1 x_i d\alpha(\lambda) \int_0^1 y_k d\beta(\lambda') g^2 trf \frac{N_c}{N_c} < F_{i\lambda}(x\alpha)F_{k\lambda'}(y\beta) >_A \]  

(24)

Here $\alpha(\lambda), \beta(\lambda) = 1$ for $\lambda = 4$ and $\alpha(\lambda) = \alpha, \beta(\lambda) = \beta$ for $\lambda = 1, 2, 3$. Below we shall use $D$ and $D_1$ correlation functions defined through the following decomposition [7]:

\[ \frac{trf g^2}{N_c} < F_{i\lambda}(u)F_{k\lambda'}(u') >_A = (\delta_{ik}\delta_{\lambda\lambda'} - \delta_{i\lambda}\delta_{k\lambda'})D(u, u') + \Delta^{(1)}_{ik,\lambda\lambda'}(u, u') \]  

(25)

with the factor $\Delta^{(1)}$ proportional to the full derivative and therefore not contributing to the confinement and to the string creation; we shall disregard it below.

Now we can also rewrite (22) for $< S^{(2,2)} >$ through the kernel $J_{\mu\nu}$:

\[ < S^{(2,2)} >_A = \frac{N_c}{2C_2^f} b^i_\mu(x)b^k_\nu(x)(\delta_{\mu\nu}J_{\lambda\lambda}(x, x) - J_{\mu\nu}(x, x)). \]

(26)

At this point it is necessary to argue why in $S_{eff}(b)$ (21) we have omitted the higher–order cumulants and shall also omit the term $L^{(2,2)}$, quadratic in $S^{(2,2)}$. The reason is that those terms contain quartic (and higher) terms of the type $\ll A^4 \gg$ which due to (13), (14) are expressed through $\ll F^4 \gg$. As it was argued in [8] these quartic and higher cumulants do not contribute
significantly to the string tension which can be written (for the fundamental string) as

\[ \sigma_f = \frac{1}{2} \int \int d^2 x D(x) + 0(\ll F^4 \gg) \]  

(27)

For static charges of higher SU(3) representations the correlator \( D(x) \) is proportional to the quadratic Casimir operator \( C_2 \) [8] in good agreement with lattice data [9]. The contribution \( 0(\ll F^4 \gg) \), which produces the \((C_2)^2\) dependence in the string tension, has not been seen in the static potential of adjoint charges on the lattice [9]. Additional arguments come from the string profile calculations [10] where using only \( D(x) \) one reproduces lattice data with good accuracy. Hence, we expect that the contribution of higher cumulants is suppressed at least in those two instances, whereas confinement (nonzero \( \sigma \)) is already present due to the Gaussian correlator \( D(x) \).

Therefore to obtain the selfcoupled equations for the lowest cumulants we confine ourselves to functions \( D(x) \) and \( \Delta^{(1)} \), while at the next stage one can express triple and quartic correlators through the lowest ones and so on. In this way one gets an expansion in powers of the parameter \( \rho \equiv \left( g \sqrt{< F_{ik}^2 > T_g^2} \right) \), where indices \( i, k \) are fixed and refer to the surface of integration, as in (27). Inserting phenomenological values of gluonic condensate [3] and the lattice value of \( T_g [1] \), one obtains \( \rho \approx 0.1 \) which implies a fast convergence of cluster expansion. Note that the case of instantons, where the cumulant series is not converging, requires a special treatment, which was done e.g. in [11].

\section{Mass operator and nonlinear equation}

The effective action \( S_{\text{eff}}(b) \) in equation (21) contains terms which are quadratic in \( b(i.e. L^{(1)}, < S^{(2,2)}, S^{(2,0)}>) \), and in addition also cubic and quartic terms in \( S^{(3)}, S^{(4)} \) and \( L^{(2,1)}, L^{(F)} \), and the sixth power term \( L^{(3,1)} \). Keeping aside \( S^{(3)}, S^{(4)} \) let us turn now to other terms and write the effective mass term \( M \) in the same way as it is usually done in the Dyson–Schwinger formalism [12]. Writing diagrams based on the vertices \( L^{(2,1)}, L^{(F)} \) and \( L^{(3,1)} \) one obtains (in the leading order of large \( N_c \)) the Hartree–Fock approximation for \( M \) generated by those vertices, which are obtained doing a Wick pairing for every extra pair of operators \( b_\mu \) with replacement \( b_\mu b_\nu \rightarrow G_{\mu\nu} \) (15).
In the same approximation the valence gluon Green’s function \( G_{\mu\nu} \) (16) is diagonal in color (since the resulting mass operator \( M \) will be shown to be is diagonal) and one can rewrite (15) as

\[
< b^\alpha_\mu(x) b^\beta_\nu(y) >_{\nu, A} = \delta_{\alpha\beta} G_{\mu\nu}(x, y).
\]  

(28)

As a result one can represent \( M \) in the following form,

\[
M = M^{(2,2)} + M^{(2,1)} + M^{(1)} + M^{(3,1)} + M^{(F)} + \Delta M,
\]  

(29)

where \( M^{(i)} \) correspond to \( S^{(i)} \) and e.g.

\[
M^{(2,2)}_{\mu\nu}(x, y) = \frac{N_c}{C_2} \delta^{(4)}(x - y) [J_{\lambda\lambda}(x, y) \delta_{\mu\nu} - J_{\mu\nu}(x, y)],
\]  

(30)

\[
M^{(2,1)}_{\mu\nu}(x, y) = -\xi \frac{N_c}{C_2} J_{\lambda\nu}(x, y) G_{\rho\sigma}(x, y) N_{\lambda\rho\mu}(x) N_{\lambda\sigma\nu}(y)
\]  

(31)

with

\[
N_{\lambda\rho\mu}(x) = \delta_{\mu\rho}(2\partial^G_\lambda + \partial^A_\lambda) + \delta_{\lambda\rho}(\partial^I_\mu - \partial^G_\mu) - \delta_{\lambda\mu}(2\partial^I_\rho + \partial^G_\rho),
\]  

and the parameter \( \xi \equiv \frac{r}{N_c^2 - 1} \),

(32)

and

\[
M^{(1)}_{\mu\nu}(x, y) = -\frac{1}{N_c^2 - 1} < F^{ca}_{\lambda\mu}(x) D^{ab}_{\lambda}(y) F^{bc}_{\lambda\nu}(y) >_A =
\]

\[
= -\frac{1}{N_c^2 - 1} \frac{\partial}{\partial x_\lambda} \frac{\partial}{\partial y_\nu} < F^{c}_{\lambda\mu}(x) F^{c}_{\lambda\nu}(y) >_A + 0(< FFF >)
\]  

(33)

Here in (32) we have omitted higher cumulant contribution and all other \( M^{(i)} \) proportional to \( \xi \) will be disregarded in the large \( N_c \) limit.

The terms \( S^{(3)}, S^{(4)} \) generate their own contribution to \( M_{\mu\nu} \), which we denote \( \Delta M \). We stress now that all terms in \( M \) containing the valence Green’s function \( G_{\mu\nu} \), acquire a factor \( \xi \) for each \( G \), the same also refers to \( \Delta M \). Except for \( \xi \) the \( N_c \) dependence in all terms in \( M \) cancels and all terms are \( 0(N_c^0) \). Therefore in the first approximation one can neglect all terms containing \( \xi \) and is left with

\[
M \cong M^{(2,2)} + M^{(1)} + 0(1/N_c^2)
\]  

(34)
Then Dyson–Schwinger equation for the valence gluon Green’s function can be derived from (21) in a usual way [12]:

\[ (-\partial^2_\lambda \delta_{\mu \rho} + \partial_\mu \partial_\rho) G_{\mu \nu}(x, y) + \int M_{\mu \rho}(x, z) G_{\rho \nu}(z, y) d^4z = \delta^{(4)}(x - y), \] (35)

here \( M \) is given in (24-33) or at large \( N_c \) approximately in (34).

Equation (35) is the central point of the paper. In the rest part of it we analyze equation (35) in more detail and express field correlators \( D, D_1 \) through \( G_{\mu \nu} \), thus closing the set of equations.

4 Analysis of the selfcoupled equation (43) and simple estimates

To compute \( D \) and \( \Delta^{(1)} \), as defined in (25), through \( G_{\mu \nu} \) it is convenient to choose, in each term of the sum over color indices in (11), the color indices \( \alpha, \beta \) as belonging to the small set of valence colors. Hence fields \( F_{\mu \nu} \) will consist of \( b_\mu \) only and (11) can be rewritten as

\[
\frac{tr g^2}{N_c} < F_{\mu \nu}(x) \Phi(x, y) F_{\lambda \sigma}(y) > = \frac{g^2}{2N_c} < \frac{\partial}{\partial x_\mu} b^i_\nu(x) \Phi^{ik}(x, y) \frac{\partial}{\partial y_\lambda} b^k_\sigma(y) > + \\
+ \text{perm.} + g f^{cab} < \frac{\partial}{\partial x_\mu} b^d_\nu(x) \Phi^{dc}(x, y) b^a_\lambda(y) b^b_\sigma(y) > + \text{perm.} + \\
+ g^2 f^{cde} f^{cab} < b^d_\nu(x) b^e_\nu(x) b^a_\lambda(y) b^b_\sigma(y) > \equiv \frac{N^2_c - 1}{2N_c} \{ I_1 + I_2 + I_3 \} \] (36)

By perm in (36) we denote the sum of terms which obtain from the preceding one using the rule

\[
(\mu \nu \lambda \sigma) + \text{perm.} = +(\mu \nu \lambda \sigma) - (\nu \mu \lambda \sigma) - (\mu \nu \sigma \lambda) + (\nu \mu \sigma \lambda) \] (37)

In (36) it is also assumed that finally one can put \( x = (00), y = (T, \vec{0}) \) and therefore \( \Phi(x, y) \) in the gauge (13) can be replaced by unity.

The r.h.s. of (36) can be written symbolically as

\[
\partial_\mu \partial_\nu G_{\nu \sigma}(x, y) + G_{\mu \nu}(x, y) G_{\nu \sigma}(x, y) + \text{perm.} + 0(<FFF>) + 0(g^2). \]
Since the asymptotics of $G(x, y)$ at large $|x_4 - y_4|$ (as is seen from the integral equation (35) with the kernel (34)) is exponential, the same is true for the l.h.s. of (36), i.e. for $D(x)$ and $\Delta^{(1)}$. This fact is in agreement with lattice data [1].

The equation (35) and the kernel $M$ do not contain dimensional parameters and are scale–invariant. Therefore going from large distances, where nonperturbative contributions are dominant, the scale appears spontaneously in the nonperturbative solution. All other nonperturbative parameters, like gluonic condensate $\sim (D(0) + D_1(0))$ are expressed through this prescribed scale parameter.

If one, however, starts from small distances and develops the perturbative series like

$$G_{\mu\nu}(x, y) = G^{(0)}(0) - G^{(0)}(0)MG^{(0)}(0) + G^{(0)}(0)MG^{(0)}M_{\mu\nu}(0) - ...$$  \hspace{1cm} (38)

where $G^{(0)}$ is the free solution of (35), then the perturbative divergencies require regularization and the renormalization, which introduce their own perturbative scale – normalization mass $\mu$ or $\Lambda_{QCD}$ . Fixing this scale, one can extrapolate to larger distances and find nonperturbative solution defined at this scale. We shall discuss this procedure in detail in a subsequent publication, and now look at the properties of the pure nonperturbative solutions.

We shall be interested in the large time behaviour of gluon Green’s function and to make an estimate we first keep in the kernel $M$ (34) only the local part $M^{(2,2)}$, since it grows at large distances, while $M^{(1)}$ tends to a constant.

We assume at this point (and later confirm it in calculations) that $D(u)$ has an exponential form

$$D(u) = D_0\exp(-\omega |u|), \hspace{1cm} \omega = 1/T_g.$$  \hspace{1cm} (39)

From (27) one has $\sigma = \frac{\pi D_0}{\omega^2}$ and $J_{\lambda\lambda}$ is

$$J_{\lambda\lambda}(x, x) = \frac{2\sigma x \omega}{\pi}((1)_E + \frac{2}{3}M)$$  \hspace{1cm} (40)

We have used in (40) the subscripts $E, M$ to specify the contribution from colorelectric and colormagnetic fields respectively. The latter is gauge–dependent if one keeps only Gaussian correlators and neglects all higher–order ones.
One can diminish this gauge dependence taking into account that the inclusion of all correlators creates for every Wilson loop the surface of the minimal area. In our case of the large $T$ asymptotics this surface, built on the trajectory of the valence gluon and the parallel transporter along the time axis, is time–like. This means that (unless large angular momenta $J$ are considered) the spacial projections $\sum_{ij}$ of the minimal surface are not growing with time $T$, and therefore magnetic contributions to the linear confinement term in (40) should be cancelled in the sum of all correlators. Hence, the choosing of the minimal area surface in the path–integral formalism [13] is equivalent to the temporal surface gauge [14] in the formalism presented here and in [4]. As a practical outcome the magnetic term $\left(\frac{2}{3}\right)M$ in (40) vanishes in the temporal surface gauge.

From the structure of (35) one can deduce that $G_{\mu\nu}(x, y)$ can be expanded in a complete set of transverse gluon states

$$G_{\mu\nu}(x, y) = \sum_n c_{\mu\nu}^n u_n(x) u_n^+(y)$$  \hspace{1cm} (41)

where eigenvectors $u_n(x)$ satisfy an equation

$$-\partial^2 u_n(x) + \frac{N_c}{C^2} J_{\lambda\lambda}(x, x) u_n(cx) = 0.$$  \hspace{1cm} (42)

Writing $u_n(x)$ as

$$u_n(x) = \exp(-\omega_n x_4) \varphi_n(\vec{x})$$  \hspace{1cm} (43)

one obtains equation for $\varphi_n(\vec{x})$

$$\omega_n^2 \varphi_n = (-\frac{\partial^2}{\partial \vec{x}^2} + \vec{C}|\vec{x}|)\varphi_n$$  \hspace{1cm} (44)

with

$$\vec{C} = \frac{N_c 2\sigma \omega}{C_f^2 \pi}$$  \hspace{1cm} (45)

The analysis of the equation (44) was done in [15] with the result

$$\omega_n^2 = a_n(\vec{C})^{2/3},$$  \hspace{1cm} (46)

where $a_n$ are numerically computed numbers, for the lowest state $n = 0$ one has $a_0 = 2.33...$
At this point it is important to use the consequence of selfcoupling in (35) and (36). Namely, as follows from (41) and (43), the propagator $G_{\mu\nu}$ decays exponentially at large time $T$. As a consequence of (36) and the analysis done above, the function $D(T)$ decays at large $T$ with the **same exponent**, i.e.

$$\omega = \omega_0$$

(47)

Inserting (47) into (46) one obtains

$$\omega = (2.33)^{3/4} \sqrt{\frac{9\sigma}{2\pi}}$$

(48)

and with $\sigma = 0.2GeV^2$, the final result is

$$\omega = 1/T_g = 1.01GeV$$

(49)

This value agrees well with the lattice measured value $T_g = 0.22fm(\omega = 0.91GeV)$ [1].

Thus we have demonstrated in this example that different parameters of the nonperturbative correlators can be expressed through each other via a dynamical computation.

5 Conclusions

The analysis of the previous sections was intended to show that selfcoupled equations (30), (34-36) indeed contain a nontrivial dynamics, which allows to obtain nonperturbative gluon propagators and correlators. Numerical solution of these equations is possible and is now in progress, in this paper however we confine ourselves to simple estimates and qualitative analysis.

The characteristic behaviour of the gluon propagator $G(x_4, y_4, \vec{x}, \vec{y})$, which one finds from (35) and (43), is exponential decay in $|x_4 - y_4|$ and the faster than exponential decay at large $|\vec{x}|$ or $|\vec{y}|$. The latter property signals the confinement of the gluon, since the adjoint string at large $N_c$ does not allow the gluon to go far from the origin. The parallel transporter along the temporal axis, present in our definition of the gauge–invariant gluon propagator, is actually a signal of the static adjoint charge at the origin, therefore the gauge invariant quantity $G_{\mu\nu}$ is in fact the propagator of the white system.
made of the gluon and the static charge. Hence the mass $\omega$ in the exponential decay of $G_{\mu\nu}$ in time is equal to the mass of this white system.

Connection between $G_{\mu\nu}(x_4, y_4, \vec{0})$ and $D(x_4 - y_4)$ and $D_1(x_4 - y_4)$ found in section 4, tells us immediately that $D$ and $D_1$ decay with the same dominant exponent with mass $\omega$.

In the subsequent paper we shall discuss the perturbative contents of our equations (35) and of the effective action $S_{eff}(b)$ (21).

One of the most important problems to be solved in the framework of the suggested formalism is the problem of the deconfining phase transition. The latter may occur or due to the increase of temperature or due to the addition of scalars with the Higgs-type interaction. In the last case one can get a system of equations of the type of (35) for the gluon and scalar propagators which describe two possible phases: confinement and deconfined massive gluons. We plan to analyze these questions in forthcoming papers.

The author is grateful to A.Di Giacomo, H.G.Dosch, M.Schmidt and V.I.Shevchenko for useful discussions and to A.M.Badalian for valuable remarks.

This work was done with the partial support of grants INTAS 94-2851, 93-79 and RFFI grant 97-02-16404.
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