Model of short-term forecasting liner maritime transport in the port system: A case study for Split City port

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ABSTRACT
One of the preconditions for good quality management of seaports is forecasting the traffic according to the number of passengers and the number of vehicles; in this way it is possible to plan and prepare activities for the smooth operation of the ports. This paper researches the port system as part of the coastal liner maritime transport. The set hypothesis is that the model of forecasting the traffic could be presented as a function of two variables. The Principal Component Analysis (PCA) method is used to select the forecasting parameters. Based on the choice of parameters, using the Least Squares Method (LSM), the trend analysis is performed to choose the forecasting functions for maritime liner transport on the example of the Split City port. The statistical analysis on the chosen forecasting model using the coefficient of determination $r^2$ and adjusted $r^2$ model is performed to confirm the choice.

1 Introduction
Passenger seaports are a significant factor in the development of coastal liner maritime transport, transport in general, tourism and economic activities, as well as in meeting the transport needs of passengers. Ports are one of the major components of the transport sector. They are also a significant mean of integration into the global economic system [1]. Simultaneously, they are also terminals whose function is manifested through accommodation of ships, quality reception of passengers, comfortable stay of passengers at the terminal, and efficient reception from other transport branches [2]. They are also reflected by the steady increase in the number of passengers and vehicles in maritime transport, which causes rapid changes to which ports must quickly respond so that the system can function smoothly. Port operations are dynamic and susceptible to changes; therefore, national economies often take various steps to improve their efficiency [3]. Expected positive development results are the growth of passenger traffic in the port, increased number of tourists, increase in the employment rate, creation of new income and movement of inhabitants in the areas of seaports, which can be achieved through the joint action of important elements for the passenger port such as: port law, port infrastructure, port ecology, passenger ships, transport connections, passenger flows, port competitiveness, organizational structure, intellectual capital, information technologies and other elements that directly or indirectly affect their functioning [4]. One of the most important elements for good operating of ports is forecasting the number of passengers and vehicles in coastal liner maritime transport, as the increasing amount of traffic in this segment directly influences the constant and intensive changes of the port system.

For the purpose of the research, there has been set a hypothesis where traffic forecasting can be represented as a function of two random variables, the number of passengers and the number of vehicles in a given time interval. The model of short-term forecasting liner maritime transport in the port system achieves greater efficiency of the port system, which is of great importance for developing the whole maritime industry.

The paper is structured as follows; after the introductory part, Chapter 2 provides an overview of the present research. Chapter 3 presents the mathematical methods
used to develop the model of forecasting liner maritime transport in the port system. In Chapter 4, a forecasting model is elaborated on the example of Split’s port via estimating the number of passengers and the number of vehicles. Chapter 5 contains a discussion, and the conclusions of the paper are presented in Chapter 6 followed by a list of used literature (References).

2 Overview of the present researches

For a long time, maritime transport has been an under-represented research area compared to other modes of transport. It is only in the last two decades that scientific researches in maritime affairs have become more significant. The researches contribute to the sustainability of the maritime industry, which is particularly emphasized in the paper written by Lee, Kwon and Ruan [5]. Bamwesigye and Hlavackova research the sustainability of all forms of transport in urban centers and even maritime traffic [6]. Asgari, Hassanib, Huy, and Nguye approach the maritime industry’s sustainability from an economic and environmental point of view [7], and the same approach is represented in the research done by Cheng et al. [8]. The concept of liner maritime transport in the literature implies, to the greatest extent, the transport of cargo as it is evident in Hoffman’s works of [9], [10] although it is also used in the context of passenger transport.

By analysing the scientific literature related to maritime transport, it may be concluded that a significant part of the literature covers the research of the demand in the maritime traffic, therefore Jugović, Hess, and Poletan Jugović dealt with the demand for port services created, all on the example of the Rijeka cargo port [11]. Research of the maritime demand is also present in passenger maritime transport, so Rajzman and Berš established a mathematical forecasting trend model as a scientifically based basis for forecasting passenger transport demand in the transport system of the Republic of Croatia [12]. Gržin and Pupovac connected the demand for passenger maritime transport with tourist flows and, by regression analysis of the macroeconomic model, confirmed the hypothesis that the increase in tourists’ number creates an increased demand for maritime passenger transport [13]. The literature for forecasting maritime traffic is mainly related to cargo traffic and cruise passenger traffic. Thus, Fetisov and Maiorov, by using certain simulation and prognostic methods researched the estimation of cruise passenger numbers [14]. The same authors researched the concentration of cruise passengers at seaport terminals through the geometric parameters of the terminals [15]. Krile, Fetisov, and Maiorov simulated the movement of passengers from cruisers at passenger terminals according to agents’ data [16]. Also, Krile and Maiorov researched the interdependence of passenger terminals with ferry and shipping lines and shown it by circos plot [17]. Kabashkin, Yatskiv and Prentkovskis point to the use of linear extrapolation of the time series for forecasting in all branches of transport, including maritime transport, all for the period from 2017 to 2021 [18].

Forecast and analysis methods such as Principal Component Analysis and Least Squares Method have already been recognized in the maritime industry. Sanchez et al. use the PCA method when examining seaports’ efficiency; by using this method, it was concluded that time efficiency is the most significant factor of the efficiency [19]. Wilmsemier et al. have also used this method to correlate connectivity factors and port infrastructure when investigating factors that influence determining freight costs [20]. The aforementioned research points to the conclusion that the costs of transport and tariffs are higher in the case of cargo transshipment and in dispersed markets. Alexandrides et al. researched the literature in the field of shipping finance research and investments in shipping in the period 1979-2018 and highlighted the use of PCA methods in this field as well [21]. By using the PCA method, Kutin et al. surveyed the literature in the field of the competitiveness of shipping lines (routes). They concluded that the observed variables, such as market size, frequency of navigation, number and size of ships, length of lines (routes), were not completely independent of each other. The research itself was based on observing 4 clusters in which 800 lines (routes) are sorted [22]. Hu researched the impact of port logistics on the development of the economic hinterland and a method for automatic prediction was introduced, so called SVM (Support Vector Machine) model for forecasting the scale of the logistic needs according to its characteristics [23]. Thus, Park et al. in their research use a regression method together with the PCA method to determine the so-called Principal Component with the largest variance and related other orthogonal directions to analyze the impact of all branches of transport on the economic growth of different countries, so it is concluded that maritime transport has a stronger impact on economic development than other branches of transport [24]. Lou et al. estimated the movement of container shipping on the basis of data from 1980-2008 by using the LSM method [25].

It should be noted that all of the above-mentioned references used in the research use the PCA method to forecast where the Principal Components are selected by the PCA method and the forecasting model is created by using the principal direction projection vector. However, an in-depth study of the used random variables reveals that, except for the purpose of prediction (forecasting), the PCA method can be used to select the optimal number of required random variables that shall be used in order to make the prediction model at the same time as accurate as possible and to make the prediction time as short as possible. The idea of choosing the optimal number of random variables for the forecasting model is based on the following consideration. Random variables are the functions of time. When talking about the time interval, this paper deals with an observation period of 15 years. In addition to creating of so-called Principal Components, the first principal component shows the largest variance in the
data while the other components are orthogonal to the principal component. By studying the PCA method, apart from obtaining the principal components and observing the proportions of variance in the principal components, one can see how much a single random variable contributes to each major component, representing the major scientific contributions in this paper. Therefore, based on using the PCA method for selecting optimal variables, the LSM method has been used to create a forecasting model.

3 Mathematical methods

Forecasting and estimation of traffic is a very important factor in the quality management of passenger seaports, as it enables planning and preparation of activities and monitoring of their effectuation, which is a pre-condition for improvements in the operation and management of the port system. Quality assessment is preceded by an analysis of the existing situation, collecting the necessary data, and selecting the method. This chapter presents the analysis of the existing situation, collecting the necessary data, and selecting the method. The transformation of coordinates from one coordinate system to another is shown by equation [27]:

\[ x' = \frac{1}{\sigma_{sv}} \left( \left( x_{ip} - \bar{X}_{p} \right) \left( X'_{ip} - \bar{X}'_{p} \right) \right) \]

where \( x' \) and \( x' \) are transformed coordinates of random variables.

The statistical metric used to quantify the similarity and/or dependency among variables, \( x_p \) and \( x_v \), is the correlation coefficient between random variables. The correlation coefficient can be calculated from the following equation [28]:

\[ r = \frac{1}{N-1} \sum_{i=1}^{N} \left( x_{ip} - \bar{X}_p \right) \left( x_{iv} - \bar{X}_v \right) \]

where \( r \) represents the correlation coefficient, \( N \) is the number of measurements while \( \sigma_{sp} \) and \( \sigma_{sv} \) represent the standard deviations of the random variables \( x_p \) and \( x_v \).

Furthermore, Principal Component Analysis (PCA) is used to confirm significance [28]. The PCA method is based on theory from linear algebra and so-called Singular Value Decomposition method, SVD method (Singular Value Decomposition) which the matrix data, \( A \):

\[ A = \begin{bmatrix} X_p & X_v \\ X_{p1} & X_{v1} \end{bmatrix} \]

where \( X_p \) and \( X_v \) are vectors of random independent variables, together with equation (7), transformed into the covariance matrix \( C \) into the equation:

\[ C = \frac{1}{N-1} \sum_{i=1}^{N} \left( X_{ip} - \bar{X}_p \right) \left( X_{iv} - \bar{X}_v \right) \]

where \( \sigma \) represents the expectation for a random variable, is the sign for the random variable \( x \) and \( N \) is the number of measurement samples.

The standard deviation of the random variable \( x \) can be represented by equation [26]:

\[ \sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_{ip} - \bar{X})^2} = \sqrt{E \left[ x^2 \right] - E \left[ x \right]^2} \]

where \( \sigma_x \) represents the standard deviation of the random variable \( x \).

The standard deviation of the random variable \( x \) can be represented by equation [26]:

\[ \sigma_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_{ip} - \bar{X}_p)^2 = E \left[ x^2 \right] - E \left[ x \right]^2 \]

The transformation of coordinates from one coordinate system to another is shown by equation [27]:

\[ x' = \frac{1}{\sigma_{sv}} \left( X'_{ip} - \bar{X}'_{p} \right) \]

where \( x' \) and \( x' \) are transformed coordinates of random variables.
C_t = \frac{1}{N-1} (A \cdot A^T) - \frac{1}{N-1} \begin{bmatrix} x_{t1} \cdot x_{t2} \cdot \ldots \cdot x_{t15} \\ x_{t2} \cdot x_{t3} \cdot \ldots \cdot x_{t14} \\ \vdots \\ x_{t15} \cdot x_{t16} \end{bmatrix} = \begin{bmatrix} x_{t1} \cdot x_{t1} \\ x_{t2} \cdot x_{t2} \\ \vdots \\ x_{t15} \cdot x_{t15} \end{bmatrix} + \sum_{i=1}^{m} \sum_{j=1}^{N} a_i \cdot t_j \cdot t_i^2 + \ldots + a_m \cdot t_i^m \tag{11}

On the example from equation (1), the covariance matrix \( C_t \) has dimensions (2x2), as follows:

\[ C_t = \begin{bmatrix} \sigma_{x,t}^2 & \sigma_{x,t} \cdot \sigma_{x,t} \\ \sigma_{x,t} \cdot \sigma_{x,t} & \sigma_{x,t}^2 \end{bmatrix} \tag{12} \]

The covariance matrix, \( C_t \), shows the interdependence of the variances of the observed random variables.

If the SVD method is applied to the covariance matrix, \( C_t \), from (12), we obtain:

\[ C_t = U \cdot \Sigma \cdot V^T \tag{13} \]

where \( U \) is so-called the matrix of the direction of the principal components, \( \Sigma \) is the singular value matrix (Eigenvalue) that provides the information about the significance of principal directions, and a \( V \) matrix that provides information on how each component is projected to the principal direction (Eigenvector). The matrices \( U \) and \( V \) are orthogonal [30]. Since the covariance matrix has dimensions (2x2), there will be two main directions, \( PC1 \) and \( PC2 \) (Principal Component). By calculating the percentage of individual variants of \( PC1 \) and \( PC2 \) in the total variance, the information about the dimensionality of the prediction function shall be obtained. From the matrix, \( V \) random variables \( x_p \) and \( x_r \) are classified by their significance.

Furthermore, the forecasting model is being created by using the Least Squares Method (LSM) [28]. If \( N \) is the number of measurements of variables \( x \) and \( t \), then the pairs of measurements \( K_i \) can be defined as shown by the equation [28,29,30]:

\[ x = [x_1, x_2, \ldots, x_i] \quad i=1,2,3,\ldots,15 \quad i \in \mathbb{Y} \]

\[ t = [t_1, t_2, \ldots, t_i] \quad i=1,2,3,\ldots,15 \quad i \in \mathbb{Y} \]

\[ K_i(t, x_i) \quad i=1,2,3,\ldots,15 \quad i \in \mathbb{Y} \tag{14} \]

If a polynomial of the \( m \)-th order is chosen for the forecasting model, where \( m < N \), then the error of the squares, \( E \), is defined by the equation:

\[ x = a_0 + a_1 \cdot t + a_2 \cdot t^2 + \ldots + a_m \cdot t^m \quad m < N \]

\[ E = f(a_0, a_1, a_2, \ldots, a_m) = \sum_{i=1}^{N}(x_i - \hat{x}_i)^2 \]

\[ E = \sum_{i=1}^{N} \left[ x_i - (a_0 + a_1 \cdot t_i + a_2 \cdot t_i^2 + \ldots + a_m \cdot t_i^m) \right]^2 \tag{15} \]

whereas \([a_0, a_1, a_2, \ldots, a_m] \) are approximation polynomial coefficients. The error of the squares is being minimized according to the equation:

\[ \delta E = 0 \quad i=1,2,3,\ldots, m \quad i \in \mathbb{Y} \]

\[ \delta E = \delta \sum_{j=1}^{N} x_j - (a_0 + a_1 \cdot t_j + a_2 \cdot t_j^2 + \ldots + a_m \cdot t_j^m) \]

\[ \delta \sum_{j=1}^{N} x_j - (a_0 + a_1 \cdot t_j + a_2 \cdot t_j^2 + \ldots + a_m \cdot t_j^m) \tag{16} \]

From equation (16), for the polynomial of the \( i \)-th order, \( m = i \), we obtain the system of equations represented by the equation:

\[ K \cdot a = b \Rightarrow a = K^{-1} \cdot b \]

\[ K = \begin{bmatrix} N \sum_{j=1}^{N} t_j & \sum_{j=1}^{N} t_j^2 & \ldots & \sum_{j=1}^{N} t_j^i \\ \sum_{j=1}^{N} t_j & \sum_{j=1}^{N} t_j^2 & \ldots & \sum_{j=1}^{N} t_j^i \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{j=1}^{N} t_j & \sum_{j=1}^{N} t_j^2 & \ldots & \sum_{j=1}^{N} t_j^i \end{bmatrix} \]

\[ a = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_i \end{bmatrix} \quad b = \begin{bmatrix} \sum_{j=1}^{N} x_j \\ \sum_{j=1}^{N} t_j x_j \\ \vdots \\ \sum_{j=1}^{N} t_j^{i-1} x_j \end{bmatrix} \tag{17} \]

The coefficient of determination and adjusted coefficient of determination has been used to analyze and validate the selected model.

For the trend analysis, the Moving Average method has been applied, while polynomial approximation models are analyzed by the so-called Exponential Weighted Moving Average method [28] represented by the following equation:

\[ V_n = \beta \cdot V_{n-1} + (1 - \beta) \cdot A_n \tag{18} \]

where \( V \) represents the measurement estimate at the \( n \)-th moment, \( V_{n-1} \) represents the estimation or measurement value at the \( n \)-th moment, \( A_n \) represents the measured data or a random variable at the moment \( n \), \( \beta \) is a coefficient that depends on parameter \( k \), while parameter \( k \) represents the number of years that will be considered in the averaging method when studying the trend analysis. The dependence of the parameter \( k \) and the coefficient \( \beta \) is shown by the following equation:

\[ k = \frac{1}{1-\beta} \Leftrightarrow \beta = \frac{k-1}{k} \tag{19} \]
The statistical coefficient, the coefficient of determination, explains the chosen model’s significance with respect to the selected random independent variables. It has to be pointed out that the complete analysis has been performed in MATLAB 2019b package.

4 Model for forecasting liner maritime transport in the case of the port of Split

Mathematical methods described in the previous chapter will be applied here to create and analyze the forecasting of traffic in the coastal linear maritime transport model on the example of the Split city port.

This particular port was chosen because it was positioned on the third place of the Mediterranean Sea ports, all according to the transported number of passengers and vehicles. For example, in 2018, 4.9 million passengers and 764,000 vehicles in coastal liner maritime transport passed through the Split city port [31]. The port is located in the heart of the city, which points to the importance of good organization and maximum fulfillment of all business processes in order to avoid traffic chaos and allow regular and unhindered traffic fluctuation, especially during the season.

Thus, the hypothesis of this paper would be that the forecasting of passenger (vehicle) traffic in the Split coastal liner maritime transport can be represented as a function of two random independent variables, represented by the equation (4).

4.1 Selection of variables for choosing the model

Random variables were selected in order to research the forecasting of coastal liner maritime transport at the Port of Split, i.e., passengers, \( x_p \), and vehicles, \( x_v \). For the purpose of the set hypothesis, the number of vehicles passing through the port over a time period of one year and time \( t \) are the independent random variables, while the number of passengers is represented as a dependent random variable, as shown in Table 1.

Table 1 shows the number of passengers and vehicles from 2004 to 2019 and a cumulative sum. It can be seen that the variable number of passengers rises from 2004 to 2008, then decreases until 2010, and then it has an almost exponential rise until 2019. The same can be concluded for the second variable, i.e., the number of vehicles passing through the port over a time period of one year and time \( t \) are the independent random variables, while the number of passengers is represented as a dependent random variable, as shown in Table 1.

Table 1 Number of passengers and vehicles in coastal liner maritime transport at the Port of Split from 2004 to 2019

| Year | Passengers | Vehicles |
|------|------------|----------|
| 1    | 2004       | 2576859  | 517920  |
| 2    | 2005       | 2634807  | 535561  |
| 3    | 2006       | 2902816  | 636822  |
| 4    | 2007       | 3342532  | 668890  |
| 5    | 2008       | 3533446  | 665003  |
| 6    | 2009       | 3392285  | 619462  |
| 7    | 2010       | 3356044  | 593931  |
| 8    | 2011       | 3428149  | 605429  |
| 9    | 2012       | 3439002  | 598420  |
| 10   | 2013       | 355678   | 608103  |
| 11   | 2014       | 3659052  | 617267  |
| 12   | 2015       | 398047   | 647988  |
| 13   | 2016       | 4225042  | 689989  |
| 14   | 2017       | 4705406  | 738386  |
| 15   | 2018       | 4859086  | 764260  |
| 16   | 2019       | 5120035  | 798454  |
| Sum  | 5870572    | 1030525  |

Source: Coastal Liner Services Agency [31]

From the t-test, for the number of passengers vs. time, \( t_{\text{stat}} = 19.7478 \), \( t_{\text{crit}}(\text{one - tail}) = 1.753 \), \( t_{\text{crit}}(\text{two - tail}) = 2.131 \), \( p(\text{one - tail}) = 1.895 \cdot 10^{-15} \), and \( p(\text{two - tail}) = 3.791 \cdot 10^{-12} \). For the number of vehicles vs. time, \( t_{\text{stat}} = 33.7588 \), \( t_{\text{crit}}(\text{one - tail}) = 1.753 \), \( t_{\text{crit}}(\text{two - tail}) = 2.131 \), \( p(\text{one - tail}) = 7.251 \cdot 10^{-16} \), and \( p(\text{two - tail}) = 1.45 \cdot 10^{-16} \). From both numerical values for the p-value for both variables, it can be concluded that there is no statistical dependence between independent and dependent variables.

It has to be noted that the value of passenger and vehicle variables for 2019 is used as a control variable to test the forecast. The value for 2020 has not been used for forecasting due to special circumstances, i.e., the global pandemic situation. This 2020 year is treated as an outlier year, and thus the forecasting is not performed and compared with the obtained model.

To test the correlation relationship between the independent and dependent variables, Figure 1 graphically shows the relationship between the variables of the passengers, \( x_p \), and vehicles, \( x_v \).

Figure 1 shows that the vehicle variable, \( x_v \), is placed on the ordinate axis while the passenger variable, \( x_p \), is placed on the abscissa axis. Values of both variables are numerically expressed in millions. The red dots indicate the years from Table 1. It is evident that random variables of passengers and vehicles are not linearly dependent, i.e., mostly, the function is not a direction, and therefore the set hypothesis for anticipated liner shipping maritime transport
Figure 1 Graphical representation of the relationship between random variables of passengers and vehicles

Figure 2 Graphical representation of the number of passengers and vehicles in the Port of Split from 2004 to 2018.

Figure 3 Graphical representation of passengers and vehicles in relation to the time in Port of Split in transformed coordinates for the period 2004 to 2018.
in Port of Split makes sense for further testing and study, which represents the basis for further research.

For the purposes of further analysis, the dependence of the variables of passengers and vehicles as a function of time is also graphically presented (see Figure 2).

Figure 2 individually graphically represents the dependence of passengers and vehicles vs time. It is evident that there is an upward trend in passengers and vehicles in the last decade, with the exception of the period 2007-2011 as a result of the economic crisis that began in America and spread throughout the world affecting all industries. It should be noted that since 2013, Croatia has become a full member of the European Union and there is an almost linear increase in the number of passengers and vehicles in the liner maritime transport of the Port of Split, as shown in Figure 2.

The passenger and vehicle variables presented in the such manner, in the function of time are not suitable for statistical analysis, such as the determination of the coefficient of determination, etc., and have been transformed by using equation (8), as shown in Table 2.

Graphical representation of the variables in Table 2 is shown in Figure 3.

Figure 3 shows that there is a strong functional dependence between the variables of passengers and vehicles in the function of time. The functional dependence among the variables is studied through the correlation coefficient shown in equation (9). The correlation coefficient, \( r \), for the Port of Split, within the coastal liner shipping maritime transport is 89.4%. It is evident from the correlation coefficient that the random variables are not independent of each other, thus the estimation of coastal liner maritime transport in the port of Split for 2019 and 2020 does not need to be calculated as a function of two random variables, but can be approximated with satisfactory accuracy and a function of one random variable.

In order to examine the possibility of forecasting liner maritime transport in Port of Split as the function of a single random variable, it is necessary to do the additional research by using the so-called PCA method or Principle Component Analysis [28] based on the calculation of the \( C \), covariance matrix and the search for characteristic values and vectors. The covariance matrix, \( C \), shows the independence of the random variables of passengers and vehicles, as shown in equation (20):

\[
C_x = \begin{bmatrix}
14.000 & 12.516 \\
12.516 & 14.000
\end{bmatrix}
\]

(20)

By using Singular Value Decomposition method, SVD, represented by equation (13) and covariance matrix, \( C \), (equation 20), the singular value matrix, \( \Sigma \), is obtained (equation 21) and so-called singular value vector projection matrix, \( V^* \), (equation 22).

Singular value vector projection matrix has been shown in the following equation:

\[
\Sigma = \begin{bmatrix}
25.516 & 0 \\
0 & 1.484
\end{bmatrix}
\]

(21)

Equation 21 shows that the matrix of singular values is diagonal. The first member on the main diagonal represents the variance \( PC1 = \sigma_{pp} = 26.516 \), and the second element on the diagonal represents the variance \( PC2 = \sigma_{vv} = 1.484 \). It is noticed that \( PC1 \) has a significantly higher value than \( PC2 \). Table 3 shows the percentages of the variance values of \( PC1 \) and \( PC2 \).

Table 3 Percentage of the total variance of the major components

|          | % of the total variance |
|----------|-------------------------|
| PC1      | 94.700                  |
| PC2      | 5.300                   |

It is noticed that \( PC1 \) has a share of 94.7% in the total variance, and \( PC2 \) has a share of 5.3% in the total variance and it can be concluded that it is possible to anticipate the estimation of liner traffic in Port of Split with 94.7% accuracy, as the function of one variable.

Furthermore, it should be determined which random variable (passengers or vehicles) better estimates coastal liner maritime transport in the Port of Split. For the purpose of the above mentioned, from the equation (13) there is used the matrix of the vector of singular values projection \( V^* \), shown by equation 22.
From equation 22, it can be seen that in the square matrix $V^*$ are the rows that represent the observed random variables (passengers or vehicles), while the column matrix elements represent the contribution of the $PC1$ and $PC2$.

It can be concluded that both random variables contribute equally in the main directions $PC1$ and $PC2$, amounting 0.707. Therefore, for the estimation of coastal maritime transport in Port of Split, it is absolutely irrelevant which random variable (passengers or vehicles) will be considered as the dependent one. The method chosen will be tested with a Least Squares Method (LSM) and statistical analysis of the function approximation.

### 4.2 The selection of the model

The done analysis of random variables is followed by the selection of a forecasting model. It is proposed to choose a model based on the Least Squares Method (LSM). If equations (15), (16) and (17) are applied to polynomials from the first to the sixth row, and to the exponential polynomial, the coefficients $a_i$ shall be obtained, as shown in Table 4.

Graphical representation of the number of passengers in liner maritime transport at Port of Split and potential approximation models (functions) of forecasting are based on the calculation of the approximation coefficients $a_i$ and they are shown in Figure 4.

To select the estimation model, the metrics of the sum of squares of error $E$ is used, which shows the amount of squared error for the selected approximation model. Figure 5 shows a graphical representation of the sum of

| Approx. Coeff. | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
|----------------|------|------|------|------|------|------|------|
| 1st order Poly. | 2.477 | 0.137 |      |      |      |      |      |
| 2nd order Poly. | 2.730 | 0.048 | 0.0056 |      |      |      |      |
| 3rd order Poly. | 2.044 | 0.492 | -0.0616 | 0.0028 |      |      |      |
| 4th order Poly. | 1.869 | 0.662 | -0.1060 | 0.0070 | -0.0013 |      |      |
| 5th order Poly. | 2.674 | -0.379 | 0.2936 | -0.0561 | 0.00423 | -0.0011 |      |
| 6th order Poly. | 3.219 | -1.250 | 0.7400 | -0.1579 | 0.01566 | -0.00073 | 0.000129 |

| Expon. Funct. | $\alpha$ | $\beta$ |
|---------------|--------|--------|
| Source: Authors | 0.500 | 0.800 |
squares of error for the calculated approximation models ranged by the magnitude of the sum of squares of error for the passengers where the largest squared error of 0.7810 has a first-order polynomial approximation model, followed by an exponential function approximation model of 0.6801 and then approximation model of the second-order polynomial of 0.6527, etc.

The numerical amounts of the sums of squares of errors for passengers and vehicles in relation to the calculated approximation models are shown in Table 5, where it may be noted that the sums of squares of errors for the approximation models for vehicles are smaller than the sum of the squares of errors for the approximation models for passengers, for the order of magnitude.

The method of so-called hard threshold shall be used for the selection of the approximation model. The selection of the hard threshold is the percentage of the steepest percentage drop between the squares of the errors. It can be seen from Figure 5 and Table 5 that the threshold is placed between the sums of the squared errors of the second- and third-order polynomial models. Thus, if a sum of 0.200 is determined for passengers and 0.005 for vehicles, as a decision threshold, it can be seen that in this case, fourth, fifth and sixth order approximation models are considered for the selection of the forecasting model. The model selection candidates are represented by the approximation equations (23):

\[
x_{4th} = 1.869 + 0.662 \cdot t - 0.106 \cdot t^2 + 0.007 \cdot t^3 - 0.00013 \cdot t^4
\]

\[
x_{5th} = 2.674 - 0.379 \cdot t + 0.2936 \cdot t^2 - 0.0561 \cdot t^3 + 0.00423 \cdot t^4 - 0.00011 \cdot t^5
\]

\[
x_{6th} = 3.219 - 1.25 \cdot t + 0.74 \cdot t^2 - 0.1579 \cdot t^3 + 0.01566 \cdot t^4 - 0.000073 \cdot t^5 + 0.0000129 \cdot t^6
\]

Table 5 Numerical error signal values for approximation models

|                        | LSE Passengers | LSE Vehicles |
|------------------------|----------------|--------------|
| 1st order              | 0.7810         | 0.0301       |
| Exponential            | 0.6801         | 0.0296       |
| 2nd order              | 0.6527         | 0.0278       |
| 3rd order              | 0.2036         | 0.0084       |
| 4th order              | 0.1904         | 0.0046       |
| 5th order              | 0.0765         | 0.0040       |
| 6th order              | 0.0575         | 0.0029       |

Source: Authors

Table 6 \(r^2\) and adjusted \(r^2\) metrics for passenger and vehicle variables

|                  | 1st Order | Exp.  | 2nd Order | 3rd Order | 4th Order | 5th Order | 6th Order |
|------------------|-----------|-------|-----------|-----------|-----------|-----------|-----------|
| Passengers \(r^2\) | 0.8705    | 0.8872| 0.8918    | 0.9662    | 0.9684    | 0.9873    | 0.9905    |
| adjusted \(r^2\) | 0.8605    | 0.8026| 0.8737    | 0.9570    | 0.9558    | 0.9803    | 0.9833    |
| Vehicles \(r^2\)  | 0.5096    | 0.5179| 0.5477    | 0.8638    | 0.9251    | 0.9351    | 0.9529    |
| adjusted \(r^2\)  | 0.4719    | 0.1563| 0.4723    | 0.8267    | 0.8951    | 0.8992    | 0.9176    |

Source: Authors
The approximation models’ quality will be verified by the coefficient of determination metrics and adjusted coefficient of determination metrics, which shows how well the passenger or vehicle parameter variation explains the forecasting via the approximation equation. Table 6 contains $r^2$ values and adjusted $r^2$ metrics for passengers and vehicles.

Table 6 shows that all polynomials explain the data well when the passengers $r^2$ metric is checked. From the adjusted $r^2$ metric, it can be seen there is a little difference in coefficients. That means the passenger variable, and therefore all polynomial functions, does not change when the number of samples is taken into account. For the vehicle variable, it can be observed that the $r^2$ metric changes with the different forecasting functions. For example, the 1st order polynomial explains the variable in the amount of 50.96%, the exponential polynomial explains the variable 51.79%, and 4th order polynomial explains the variable in the amount of 93.51%. When the number of samples are taken into account, that is, adjusted $r^2$ metric, it can be seen that there is a decrease in coefficients. For example, the vehicle variable explains the variable in the amount of 15.63%, 47.19%, and 89.51% for exponential and 1st and 4th order polynomials.

5 Discussion

Passenger seaports are complex systems that must be accessed in a thoughtful and pre-planned manner since they create the preconditions for effective business. The forecasting of traffic in a seaport directly affects a number of elements of the port system, such as: estimating the required number of employees, estimating the costs of doing business and future investments in infrastructure, forecasting the organization of the port system as well as the situation on the access roads around the port itself.

Function trend analysis will be used to select the fitting function for estimating the number of passengers and the number of vehicles. The trend analysis process shall be performed by the Exponential Weighted Moving Average (EWMA) method, based on the equations (18) and (19).

The trend will be analysed for a period of 5 years ($k = 5$). By using the equation (19), the value of the coefficient $\beta$ is 0.8.

Figure 6 graphically presents the models (4th order, 5th order, and 6th order polynomial fitting functions) for estimating passengers’ number in maritime liner transport at Split’s port.

Figure 6a Graphic representation of estimation of the number of passengers for 2019 and 2020 of maritime liner transport in the port of Split when using approximation models of the 4th, 5th and 6th order

Figure 6b Graphic representation of estimation of the number of vehicles for 2019 and 2020 of maritime liner transport in the port of Split when using approximation models of the 4th, 5th and 6th order
Figures 6a and 6b show the upward trend of passengers and vehicles in the period 2005-2018, with the exception of the period 2007-2011 due to the then economic crisis. There has been an almost linear increase in the number of passengers and vehicles from 2013, when Croatia became a full member of the European Union, until 2018. Further, it may be concluded that the models, based on the use of the approximation function of the 4th and 6th order for estimation, actually forecast the traffic growth in the coastal liner maritime transport system in the port of Split for the observed period. The approximation model based on the 5th order polynomial forecasts a decrease in traffic for the observed period, therefore, it is concluded that the approximation models based on the 4th and 6th order polynomials are the models for the estimation model. The selected models are confirmed by looking at the trend analysis and by looking at the coefficients $r^2$ and adjusted $r^2$ metrics from the Table 6. The model for estimating the number of passengers based on the 4th order polynomial is 96.84% and 95.58%, and the model based on the 6th order polynomial is 99.05% and 98.33%. Furthermore, the model of estimating the number of vehicles based on the 4th order polynomial is 92.51% and 89.51%, and the model based on the 6th order polynomial is 95.29% and respectively 91.76%.

The choice of the forecasting model cannot be observed as an isolated case. Considering the current situation, and taking into account external and internal factors, in which ports’ capacities are limited, investments are insufficient and there is a steady increase in the number of tourists in other destinations, such as Tunisia, Greece and Turkey, due to the expected lower growth, as an assessment model, there shall be selected an approximation model based on the 6th order polynomial for passengers and the on the 4th order for vehicles, according to the following analysis. From the selected model of the prediction for the number of passengers, 6th order, it is estimated that 5.172 million passengers will pass through the port of Split in 2019 and 5.621 million passengers in 2020. Compared to 2018, when the total number of passengers was 4.887 million, the number of passengers for 2019 is projected to increase by 5.5%, and for 2020 by 7.9%, when compared to 2019. An estimate of the total increase in the number of passengers in the port of Split, over the period 2018 to 2020, would be the average of 6.7% per year. At the same time, by studying the trend of the function of the number of vehicles, for 2019 it may be observed, when compared to 2018, that the increase would be 11.4%, or for 2020 compared to 2019, that the increase would be 3.2%. Therefore, the average value of an increase in the number of vehicles, in the interval from 2018 to 2020, in average would be 7.3% per year. This ‘big’ average difference between the approximation model and the trend of function is explained by several factors. In 2013, the Republic of Croatia became EU member state. Due to its favorable geo-strategic position and developed road infrastructure towards maritime tourist destinations, there is a free flow of passengers and vehicles within the EU. Simultaneously, unsafe conditions in well-known tourist countries such as Turkey, Egypt and Tunisia, and a general strike of workers in Greece have led to a fall in tourist arrivals. Due to all these factors, as a result, there is a significant increase in the number of vehicles that are accumulating in the trend (a period of 5 years is taken), as evidenced by the growth rates. The polynomial-based approximation model operates with current values, which as a result gives rise rates relative to current values rather than accumulated as in trend. When comparing the obtained results for the average values of the increase to the approximation model of the 4th order and the trend of the function, it is observed that the difference between the estimated values is 5.1%. If a forecasting model was selected for the number of vehicles based on the 6th order polynomial, and if the analysis were similar to that of the previous model, an average difference of 9.0% would be obtained. Due to the fact that the difference in the average rise prediction is smaller for the 4th order model compared to the 6th order model, the 6th order model is chosen as an approximation model for forecasting the number of vehicles in maritime traffic for the port of Split, in the time interval from 2018 to 2020.

Furthermore, the data for 2019 for passenger and vehicle variables are used to test the model. For Split City port for 2019, the number of passengers was 5120035, and prediction, when the 4th polynomial model is used, is 5.43 million. If the 5th order polynomial is used, then the pre-
The set hypothesis was that traffic forecasting can be presented as a function of two random variables. The research has been conducted on different models for traffic forecasting within the existing system, on the example of the port of Split as one of the largest ports in the Mediterranean, with a significant share of passengers and vehicles in regular maritime traffic in the port system, traffic, and movement in the tourist market. The 2019 year has been used as a test variable to test the forecasting model. Contrary, the 2020 year is treated as an outlier year (due to the global pandemic situation) and has not been included in the analysis.

All indicators and trends show that the number of passengers and vehicles in regular maritime traffic in the port of Split will increase in the current and the coming year. Therefore, the application of the model has a direct impact on water resources, coastal and ocean engineering, Ports in the Development of a Nation', International conference on water resources, coastal and ocean engineering (ICWROCE 2015), Kuala Lumpur, Malaysia, p. 295-301.

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