Dyons as a source of $CP$ and time invariance violation: electric
dipole moments and K-meson decays

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Abstract

We consider a mechanism by which dyons (electrically charged magnetic monopoles) can produce both a $T$- and $P$-odd (i.e. time reversal invariance and parity violating) mixed polarizability $\beta$ [defined by $\Delta E = -\beta E \cdot B$, where $\Delta E$ is the energy change when electric ($E$) and magnetic ($B$) fields are applied to a system] and a $T$- and $P$-odd interaction between two particles: $\overline{\psi}_1 \gamma_5 \psi_1 \overline{\psi}_2 \psi_2$, where the $\psi_i$ are electron and quark spinors. The latter can create atomic and neutron electric dipole moments (EDMs). From experimental bounds on these we find limits on the properties of dyons. Our best limit, using the experimental limit for the EDM of the Tl atom, is $M|Qg(Q^2-g^2)|^{-1/4} > 6$ GeV, where $M$ is the dyon mass and $Q$ is the electric and $g$ the magnetic charge of the dyons. The contribution of dyons to $CP$ violation in K-meson decays is also estimated.

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I. INTRODUCTION

It would be very interesting if $CP$ and $T$ violation were a consequence of another fundamental symmetry, such as a symmetry between electric and magnetic charges. In Ref. [1] we considered the possibility that dyons (electrically charged magnetic monopoles) could induce parity and time invariance violating electric dipole moments (EDMs) of quantum systems. The point is that the polarization of the dyon vacuum by the Coulomb field may produce not only the usual correction to the electric field, $\delta E$, but also a radial magnetic field $B = (g/Q)\delta E$, where $Q$ is the electric and $g$ the magnetic charge of the dyons (the corresponding antidyons would have charges $-Q$ and $-g$). The interaction of electrons (or quarks) with this magnetic field could generate atomic (and neutron) electric dipole moments. [In addition, electron and neutron EDMs $\propto m/M^2$ could appear due to higher order diagrams ($m$ is the electron or quark mass; $M$ is the dyon mass).] The EDMs were found to be proportional to $Qg$. If there were a second dyon-antidyon pair, with charges $Q$ and $-g$, and $-Q$ and $g$, then they would also make a contribution, which would cancel out the original contribution (assuming this second pair had the same mass as the first). Therefore to have time invariance violation we can only have one of the dyon-antidyon pairs.

Our consideration in [1] gave quite a strong limit on the dyon mass: $M/\sqrt{|Qg|} > 100$ TeV. However, this consideration was only based on “heuristic” arguments; it did not prove that the effect exists, only that it is conceivable.

In Ref. [2] a different mechanism was considered, by which dyons may induce an electron EDM. Firstly, they calculated the lowest order $T$- and $P$-odd correction to the effective Lagrangian of QED due to dyon vacuum polarization. This involves finding the change in the energy density of the vacuum dyons when external electric and magnetic fields are applied. The correction $\Delta L_d$ was found to be

$$\Delta L_d \approx \frac{Qg(Q^2 - g^2)}{60\pi^2 M^4} (B \cdot E)(B^2 - E^2).$$

(1)

for (spin 1/2) dyons with masses $M$, electric charges $Q$, and magnetic charges $g$. Under time reversal ($t \rightarrow -t$) and parity ($r \rightarrow -r$) transformations $B \cdot E$ changes sign, while $B^2 - E^2$ is unchanged, and so $\Delta L_d$ is $T$- and $P$-odd.

In Ref. [2] they used this effective Lagrangian to estimate the induced electron EDM.
Unfortunately, the integrals involved in such calculations are quadratically divergent. Therefore, the calculation cannot be done within the limits of the applicability of the Lagrangian (1). The result depends strongly on the effective cut-off parameter.

In this paper we present further arguments that dyon vacuum polarization may produce $T$ and $P$ violating effects. As in Ref. [2], our calculation is based on the effective Lagrangian $\Delta L_d$ (1). However, the integrals involved in the calculation of the effects considered here (mixed polarizability, $T$- and $P$-odd scalar-pseudoscalar interaction between electrons and quarks, and atomic and neutron EDMs) are only logarithmically divergent. Therefore, we stay within the limit of the applicability of the effective Lagrangian (1).

In [2] the photon-photon scattering tensor (see, e.g., [3], §124 and [4], §54.4 for the definition of this) for low-energy photon-photon scattering due to dyons was derived from $\Delta L_d$ (this is the basis for our calculation of the dyon effects). Photon-photon scattering due to dyon vacuum polarization is shown in Fig. 1. The result for this $T$- and $P$-odd photon-photon scattering tensor is

$$M^{\mu\nu\rho\sigma}(k_1, k_2, k_3, k_4) = \frac{2Qg(Q^2 - g^2)}{45M^4} \left[ \varepsilon_{\alpha\beta}^{\mu\nu} k_1^\alpha k_2^\beta k_3^\mu k_4^\nu + \varepsilon_{\alpha\beta}^{\mu\rho} k_1^\alpha k_2^\beta k_3^\mu k_4^\nu + \varepsilon_{\alpha\beta}^{\mu\sigma} k_1^\alpha k_2^\beta k_3^\mu k_4^\nu + \varepsilon_{\alpha\beta}^{\nu\rho} k_1^\alpha k_2^\beta k_3^\mu k_4^\nu + \varepsilon_{\alpha\beta}^{\nu\sigma} k_1^\alpha k_2^\beta k_3^\mu k_4^\nu + \varepsilon_{\alpha\beta}^{\rho\sigma} k_1^\alpha k_2^\beta k_3^\mu k_4^\nu - \varepsilon_{\alpha\beta}^{\mu\nu} g^{\rho\sigma}(k_3 k_4)k_1^\alpha k_2^\beta - \varepsilon_{\alpha\beta}^{\mu\rho} g^{\nu\sigma}(k_2 k_4)k_1^\alpha k_3^\beta - \varepsilon_{\alpha\beta}^{\mu\sigma} g^{\rho\nu}(k_2 k_3)k_1^\alpha k_4^\beta - \varepsilon_{\alpha\beta}^{\nu\rho} g^{\mu\sigma}(k_1 k_4)k_2^\alpha k_3^\beta - \varepsilon_{\alpha\beta}^{\nu\sigma} g^{\mu\rho}(k_1 k_3)k_2^\alpha k_4^\beta - \varepsilon_{\alpha\beta}^{\rho\sigma} g^{\mu\nu}(k_1 k_2)k_3^\alpha k_4^\beta \right]$$

(2)

In Sec. [1] we find the dyon induced $T$- and $P$-odd mixed polarizability of a particle, as well as a nucleus. The $T$- and $P$-odd mixed polarizability, $\beta$, can be defined by the equation $\Delta E = \beta E \cdot B$, where $\Delta E$ is the change in energy of a system when electric ($E$) and magnetic ($B$) fields are applied. In Sec. [11] we find the $T$- and $P$-odd interaction between two particles that could be induced by dyons and in Sec. [15] we use experimental bounds on EDMs to give limits on dyon properties. Finally, in Sec. [17] we estimate the contribution of dyons to $CP$ violation in $K$-meson decays.

Note that it is possible that dyons do not exist as free particles but only as bound dyon-antidyon pairs that cannot be separated (for example, Nambu showed that in the standard electroweak model the classical solution is a monopole-antimonopole pair connected by a $Z^0$ field string [3]). In this case dyons with a small mass are not ruled out by experimental
searches for new particles (a bound state of dyons may be hard to produce and identify). The effects considered in this paper apply equally well for free dyons and bound dyons.

II. DYON INDUCED $T$- AND $P$-ODD MIXED POLARIZABILITY

We will now find the matrix element corresponding to Fig. 2 using the photon-photon scattering tensor (2). Note that because of the way in which the photon-photon scattering tensor was symmetrized in [2] we are actually calculating the matrix element for all of the diagrams involving different arrangements of the photons (e.g. the diagram similar to Fig. 2, but with the lower two photons crossing each other) put together. This only covers diagrams with two photons free and two attached to a particle (or, when two particles are involved, as in Fig. 3, two photons attached to each particle). Other diagrams (such as that considered in [2]) do not involve logarithmically divergent integrals — our aim is not to do a calculation including every type of diagram with dyons, rather it is to show that the effect exists.

Using the Feynman rules we get (we use $e^2 = \alpha$ and the Feynman rules and notation of Ref. [3])

$$M_{fi} = \frac{ie_a^2}{(2\pi)^4 (4\pi)^2} A_\lambda(k_3) A_\omega(k_4)$$
$$\times \int \frac{d^4 k_2}{(k_2^2 + i\epsilon)^2} \frac{1}{((k_2 - q)^2 + i\epsilon)} M^{\mu\nu\lambda\omega}(k_1, k_2, k_3, k_4) d^4 k_2,$$

(3)

where $G(k)$ and $D_{\alpha\beta}(k)$ are the fermion and photon propagators, $e_a$ is the charge and $m_a$ the mass of particle $a$, and we use the notation $\hat{p} \equiv \not{p} \equiv \gamma^\mu p^\mu$. We define $q$ as $q = p_a - p'_a$, and so we have $k_1 = -k_2 + q$. Substituting in the propagators and simplifying gives

$$M_{fi} = \frac{ie_a^2}{(2\pi)^4} A_\lambda(k_3) A_\omega(k_4) \int \frac{\pi(p'_a)}{2} \gamma^\mu \frac{\hat{p}_a - \hat{k}_2 + m_a}{(p_a - k_2)^2 - m_a^2 + i\epsilon} \gamma^\nu u(p_a)$$
$$\times \frac{1}{(k_2^2 + i\epsilon)((k_2 - q)^2 + i\epsilon)} M^{\mu\nu\lambda\omega}(k_1, k_2, k_3, k_4) d^4 k_2.$$

(4)

We now use the following identity, which can be obtained in a similar way to Eq. (24) of Ref. [2]:

$$M^{\mu\nu\lambda\omega}(k_1, k_2, k_3, k_4) = k_{3\nu} k_{4\sigma} \frac{\partial}{\partial k_{3\lambda}} \frac{\partial}{\partial k_{4\omega}} M^{\mu\nu\rho\sigma}(k_1, k_2, k_3, k_4).$$

(5)

This gives
\[ M_{fi} = \frac{i e_a^2}{(2 \pi)^3} A_\lambda(k_3) A_\omega(k_4) k_{3\mu} k_{4\sigma} \]
\[
\times \pi(p'_a) \gamma_{\mu} \left[ \int \frac{\hat{p}_a - \hat{k}_2 + m_a}{[(p_a - k_2)^2 - m_a^2 + i \epsilon][(k_2 - q)^2 + i \epsilon]} \partial \partial_{k_{3\lambda}} \partial_{k_{4\omega}} M^{\mu \nu \rho \sigma} d^4 k_2 \right] \gamma_{\nu} u(p_a), \quad (6)
\]

where
\[
\frac{\partial}{\partial k_{3\lambda}} \frac{\partial}{\partial k_{4\omega}} M^{\mu \nu \rho \sigma}(k_1, k_2, k_3, k_4) = \frac{2 Q g(Q^2 - g^2)}{45 M^4} [g^{\lambda \sigma} g^{\omega \nu} \varepsilon_{\alpha \beta} \mu \nu k_1^\alpha k_2^\beta + g^{\omega \nu} \varepsilon_{\alpha} \lambda \mu \rho \varepsilon_{\alpha} k_2^\sigma + g^{\lambda \nu} \varepsilon_{\alpha} \omega \mu \sigma \varepsilon_{\alpha} k_2^\rho
\]
\[
+ g^{\omega \nu} \varepsilon_{\alpha} \lambda \mu \omega \varepsilon_{\alpha} k_2^\sigma + g^{\lambda \mu} \varepsilon_{\alpha} \omega \nu \sigma \varepsilon_{\alpha} k_2^\rho + \varepsilon_{\lambda \omega \rho \sigma} \varepsilon_{\alpha} k_2^\mu
\]
\[
- g^{\lambda \omega} \varepsilon_{\alpha} \rho \sigma \varepsilon_{\alpha} \mu \nu k_1^\alpha k_2^\beta - g^{\nu \sigma} \varepsilon_{\alpha} \lambda \mu \rho \varepsilon_{\alpha} k_2^\omega - g^{\nu \sigma} \varepsilon_{\alpha} \omega \mu \sigma \varepsilon_{\alpha} k_2^\lambda
\]
\[
- g^{\mu \sigma} \varepsilon_{\alpha} \lambda \rho \omega \varepsilon_{\alpha} k_1^\alpha k_2^\beta - g^{\rho \sigma} \varepsilon_{\alpha} \omega \nu \sigma \varepsilon_{\alpha} k_1^\lambda k_2^\alpha - g^{\mu \sigma} \varepsilon_{\alpha} \omega \lambda \rho \sigma \varepsilon_{\alpha} k_1^\gamma k_2^\pi]. \quad (7)
\]

Using this equation (and keeping in mind that \( k_1 = q - k_2 \)), we can see that the integral in Eq. (6) will be made up of terms that contain the form
\[
\int \frac{(\hat{p}_a - \hat{k}_2 + m_a)(q^\nu - k_2^\nu)k_2^\beta d^4 k_2}{(k_2^2 + i \epsilon)[(k_2 - q)^2 + i \epsilon][(k_2 - p_a)^2 - m_a^2 + i \epsilon]} \equiv W^{\alpha \beta}, \quad (8)
\]
which we define as \( W^{\alpha \beta} \) for convenience (\( \alpha \) and \( \beta \) can stand for any indices). Using this notation we can write
\[
M_{fi} = \frac{i e_a^2 Q g(Q^2 - g^2)}{360 \pi^4 M^4} A_\lambda(k_3) A_\omega(k_4) k_{3\mu} k_{4\sigma} \]
\[
\times \pi(p'_a) \gamma_{\mu} \left[ \int \frac{\hat{p}_a - \hat{k}_2 + m_a}{(k_2^2 + i \epsilon)[(k_2 - q)^2 + i \epsilon][(k_2 - p_a)^2 - m_a^2 + i \epsilon]} (q^\beta - k_2^\beta) d^4 k_2 \right] \gamma_{\nu} u(p_a) \quad (9)
\]

\( W^{\alpha \beta} \) can be worked out using the Feynman parameterization technique. We use the identity (see, e.g., [3,6])
\[
\frac{1}{abc} = 2 \int_0^1 dx \int_0^{1-x} \frac{1}{dz} \frac{1}{[a + (b - a)x + (c - a)z]^3}, \quad (10)
\]
with \( a = k_2^2, b = (k_2 - q)^2, \) and \( c = (k_2 - p_a)^2 - m_a^2 \) (we now omit the \( i \epsilon \)'s). Using this, and completing the square in the denominator gives
\[
W^{\alpha \beta} = 2 \int_0^1 dx \int_0^{1-x} dz \int \frac{(\hat{p}_a - \hat{k}_2 + m_a)(q^\alpha k_2^\beta - k_2^\alpha k_2^\beta)}{[(k_2 - l)^2 - l^2]^3} d^4 k_2, \quad (11)
\]
where

\[ l = xq + zp_a, \]  

and

\[ t^2 = (xq + zp_a)^2 - xq^2 - zp_a^2 + zm_a^2. \]  

We now do a change of variable: \( k_2 \rightarrow k_2 + l \). The numerator in the integral in Eq. (11) changes as follows:

\[
(p_a - \hat{k}_2 + m_a)(q^\alpha k_2^\beta - k_2^\alpha k_2^\beta) \rightarrow [-m_a + (z - 1)p_a + x\hat{q}]k_2^\alpha k_2^\beta + z(p_a^\alpha k_2^\beta \hat{k}_2 + p_a^\beta k_2^\alpha \hat{k}_2) \\
+ x(q^\alpha k_2^\beta \hat{k}_2 + q^\beta k_2^\alpha \hat{k}_2) - q^\alpha k_2^\beta \hat{k},
\]  

where we threw away the terms that are odd in \( k_2 \), as they will vanish on integration, as well as the terms independent of \( k_2 \), as their contribution will be much smaller than the \( \propto k_2^2 \) terms, which are logarithmically divergent when integrated. In the integral we can replace \( k_2^\alpha k_2^\beta \) with \( \frac{1}{4}g^{\alpha\beta}k_2^2 \) (see, e.g., [3], §127). So

\[
W_{\alpha\beta} = \frac{1}{2} \int_0^1 dx \int_0^{1-x} dz \int \frac{k_2^2 d^4k_2}{(k_2^2 - t^2)^3} \\
\times \{ [-m_a + (z - 1)p_a + x\hat{q}]g^{\alpha\beta} + z(p_a^\alpha \gamma^\beta + p_a^\beta \gamma^\alpha) + x(q^\alpha \gamma^\beta + q^\beta \gamma^\alpha) - q^\alpha \gamma^\beta \}
\]  

Consider the integral \( \int k_2^2(k_2^2 - t^2)^{-3} d^4k_2 \). For \( k_2 \gg m_a \) the integrand is proportional to \( 1/k_2 \) (since \( d^4k_2 \propto k_2^3 dk_2 \)) and hence the integral will be a logarithm. We take a lower “cut-off” for this integral of \( k_2 = m_a \), since near this point the \( \propto 1/k_2 \) behavior of the integrand begins to break down. For our upper “cut-off” we take the dyon mass \( M \), because, as stated in [2], Eq. (2) is only valid when the photon momenta are small compared to \( M \). Hence we have \( \int k_2^2(k_2^2 - t^2)^{-3} d^4k_2 \approx 2\pi^2 i \ln(M/m_a) \). (The factor of \( i \) appears as \( k_2 \) is a 4-vector in Minkowski space.) Using this and integrating over \( x \) and \( z \) gives

\[
W^{\alpha\beta} = i \frac{\pi^2}{6} \ln \left( \frac{M}{m_a} \right) [(-3m_a - 2\hat{p}_a + \hat{q})g^{\alpha\beta} + (p_a^\alpha \gamma^\beta + p_a^\beta \gamma^\alpha) + (q^\alpha \gamma^\beta + q^\beta \gamma^\alpha) - 3q^\alpha \gamma^\beta]
\]  

We now put this result for \( W^{\alpha\beta} \) into Eq. (9). Evaluating this is a long and involved process and so we will not give the details here. To give an idea of how the calculation is done, the calculation for one of the terms — the term \( c_1\hat{p}_a g^{\alpha\beta} \) in \( W^{\alpha\beta} \),
where \( c_1 = -i(\pi^2/3)\ln(M/m_a) \) — is done in the Appendix. We denote the contribution of this term to \( M_{fi} \) by \( M_{fi}^{(2)} \). The result is [see Eq. (A8)]

\[
M_{fi}^{(2)} = e_a^2 Q g(Q^2 - g^2)/(216\pi^2 M^4) m_a \ln(M/m_a)p'(u) F^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4).
\]

The \( m_a g^{\alpha\beta} \) and \( (p_a^\gamma\gamma + p_a^\beta\gamma^\alpha) \) terms in \( W^{\alpha\beta} \) give similar structures. The contribution of the \( \hat{q}g^{\alpha\beta} \) and \( (q^\alpha\gamma + q^\beta\gamma^\alpha) \) terms to \( M_{fi} \) vanishes — this is essentially due to the fact that \( \tilde{u}(p_a')\hat{q}u(p_a) = 0 \), which is just the charge conservation condition and can be proved from the Dirac equation (using \( q = p_a - p_a' \)). Note that all of these terms are symmetric in \( \alpha \) and \( \beta \).

The remaining, nonsymmetric term in \( W^{\alpha\beta} \), \( -3q^\alpha\gamma^\beta \) gives a different structure:

\[
i[e_a^2 Q g(Q^2 - g^2)]/(144\pi^2 M^4) \ln(M/m_a)\tilde{u}(p_a')\hat{q}\gamma_5 u(p_a) F_{\alpha\beta}(k_3) F^{\alpha\beta}(k_4).
\]

Using all the terms of \( W^{\alpha\beta} \) gives the result

\[
M_{fi} = -\frac{e_a^2 Q g(Q^2 - g^2)}{72\pi^2 M^4} \ln \left( \frac{M}{m_a} \right)
\times [m_a \tilde{u}(p_a')u(p_a) F_{\alpha\beta}(k_3) F^{\alpha\beta}(k_4) - i(1/2)\tilde{u}(p_a')\hat{q}\gamma_5 u(p_a) F_{\alpha\beta}(k_3) F^{\alpha\beta}(k_4)].
\]

Writing \( F_{\alpha\beta} \) and \( F^{\alpha\beta} \) in terms of the components of the electric and magnetic fields gives

\[
F_{\alpha\beta}(k_3) F^{\alpha\beta}(k_4) = 2[E(k_3) \cdot B(k_4) + B(k_3) \cdot E(k_4)],
\]

\[
F_{\alpha\beta}(k_3) F^{\alpha\beta}(k_4) = 2[B(k_3) \cdot B(k_4) - E(k_3) \cdot E(k_4)].
\]

This allows us to rewrite Eq. (17) as

\[
M_{fi} = \frac{e_a^2 Q g(Q^2 - g^2)}{36\pi^2 M^4} \ln \left( \frac{M}{m_a} \right)
\times \{-m_a \tilde{u}(p_a')u(p_a)[E(k_3) \cdot B(k_4) + B(k_3) \cdot E(k_4)]
\quad + i(1/2)\tilde{u}(p_a')\hat{q}\gamma_5 u(p_a)[B(k_3) \cdot B(k_4) - E(k_3) \cdot E(k_4)]\}.
\]

The first term of this equation describes a \( T \)- and \( P \)-odd mixed polarizability effect. Writing this term as a potential, \( U \) and setting \( k_3 = k_4 = 0 \) (as the \( T \)- and \( P \)-odd mixed polarizability involves homogeneous electric and magnetic fields), we get

\[
U = \frac{e_a^2 Q g(Q^2 - g^2)}{18\pi^2 M^4} m_a \ln \left( \frac{M}{m_a} \right) E \cdot B.
\]

If we define the \( T \)- and \( P \)-odd mixed polarizability by the equation \( U = \Delta E = -\beta E \cdot B \), we then have
\[ \beta = -\frac{\varepsilon^2 Qg(Q^2 - g^2)}{18\pi^2 M^4} m_a \ln \left( \frac{M}{m_a} \right). \] (22)

The above holds for a particle of charge \( e_a \). To find the dyon induced \( T \)- and \( P \)-odd mixed polarizability for a nucleus with \( Z \) protons and \( N \) neutrons we must sum over all the quarks, giving

\[ \beta_{Z,N} \approx -(Z + N) \frac{25\varepsilon^2 Qg(Q^2 - g^2)}{162\pi^2 M^4} m_q \ln \left( \frac{M}{m_q} \right). \] (23)

where \( m_q \) is the constituent quark mass. We used \( \langle N[\bar{q}q], N \rangle \approx 5NN \), where \( N \) is a proton or neutron and \( q \) is either \( u \) or \( d \) (see, e.g., [7]). The second term in Eq. (20) describes the interaction of the spin of a particle with the gradient of the electromagnetic field’s Lagrangian density \( x \sigma \cdot \nabla(B^2 - E^2) \) in the non-relativistic limit.

III. DYON INDUCED \( T \)- AND \( P \)-ODD INTERACTION

In this section we will find the \( T \)- and \( P \)-odd interaction between two particles that could be induced by dyons. We will find the matrix element for the diagram shown in Fig. [3], starting from the matrix element for Fig. [2] found above (17). Taking the first term of Eq. (17) — we will call it \( M_{fi}^A \) — and using Eq. (A2) we have

\[ M_{fi}^A = -\frac{\varepsilon^2 Qg(Q^2 - g^2)}{36\pi^2 M^4} \ln \left( \frac{M}{m_a} \right) m_a \bar{u}(p'_a) u(p_a) \epsilon^{\sigma \beta \omega \gamma} k^\gamma k^\omega A_\sigma(k_3) A_\beta(k_4). \] (24)

To convert from Fig. [2] to Fig. [3] we take \( A_\sigma(k_3) \) and \( A_\beta(k_4) \) (corresponding to the two free ends) and replace them by \( i\varepsilon^2/(2\pi)^4 \int D_{\alpha\lambda}(k_3) D_{\eta\beta}(k_4) p_\gamma(p_b) \gamma^\nu G(p_b - k_3) \gamma^\lambda u(p_b) d^4k_3 \). Carrying out this transformation on Eq. (24) (and using \( k_4 = -k_3 - q \)) gives

\[ M_{fi}^{A'} = -i\varepsilon^2 \varepsilon_b \frac{Qg(Q^2 - g^2)}{36\pi^2 M^4} \ln \left( \frac{M}{m_a} \right) m_a \bar{u}(p'a) u(p_a) \]
\[ \times \epsilon^{\sigma \beta \omega \gamma} \epsilon(p'_b) \gamma_\beta \left\{ \int \frac{(\hat{p_b} - \hat{k_3} + m_b)(-q_3 k_3 - k_3 a k_3 \omega)}{k_3^2 (k_3 + q)^2 [(k_3 - p_b)_2 - m_b^2]} \right\} \gamma_\sigma u(p_b) \]
\[ = -i\varepsilon^2 \varepsilon_b \frac{Qg(Q^2 - g^2)}{36\pi^2 M^4} \ln \left( \frac{M}{m_a} \right) m_a \bar{u}(p'_a) u(p_a) \epsilon^{\sigma \beta \omega \gamma} \epsilon(p'_b) \gamma_\beta W'_{\alpha \omega} \gamma_\sigma u(p_b), \] (25)

(we introduce the dash to distinguish it from \( M_{fi} \) for the previous diagram) where \( W'_{\alpha \omega} \) is like \( W_{\alpha \omega} \) [defined by Eq. (8)], but with the sign of \( q \) changed, and so we can modify the result.
for $W'_{\omega\omega}$. Because of the $\varepsilon^{\sigma\beta\omega\alpha}$ only the nonsymmetric part of $W'_{\omega\omega}$ will contribute. So we have

$$M'_{fi} = \frac{e_a^2 e_b^2 Qg(Q^2 - g^2)}{72\pi^2 M^4} \ln \left( \frac{M}{m_a} \right) \ln \left( \frac{M}{m_b} \right) m_a \bar{u}(p'_a) u(p_a) \varepsilon^{\sigma\beta\omega\alpha} \bar{u}(p'_b) q_\alpha \gamma_\beta \gamma_\omega \gamma_\sigma u(p_b)$$

(26)

Now $\varepsilon^{\sigma\beta\omega\alpha} q_\alpha \gamma_\beta \gamma_\omega \gamma_\sigma = 2i\alpha \gamma_5 \sigma^{\alpha \gamma} = 6i\gamma_5$ and so we have

$$M'_{fi} = \frac{i e_a^2 e_b^2 Qg(Q^2 - g^2)}{12\pi^2 M^4} \ln \left( \frac{M}{m_a} \right) \ln \left( \frac{M}{m_b} \right) m_a \bar{u}(p'_a) u(p_a) \bar{u}(p'_b) \gamma_5 u(p_b).$$

(27)

A similar process can be carried out for the second term in Eq. (17). The total result for the matrix element of the $T$- and $P$-odd interaction between particles $a$ and $b$ is

$$M'_{fi} = \frac{i e_a^2 e_b^2 Qg(Q^2 - g^2)}{12\pi^2 M^4} \ln \left( \frac{M}{m_a} \right) \ln \left( \frac{M}{m_b} \right) \times \left[ m_a \bar{u}(p'_a) u(p_a) \bar{u}(p'_b) \gamma_5 u(p_b) + m_b \bar{u}(p'_a) \gamma_5 u(p_a) \bar{u}(p'_b) u(p_b) \right]$$

(28)

IV. DEDUCED EXPERIMENTAL BOUNDS ON DYON PROPERTIES FROM ELECTRIC DIPOLE MOMENTS OF ATOMS AND THE NEUTRON

In this section we will find limits on the properties of dyons from experimental bounds on the electric dipole moments (EDMs) of atoms and the neutron.

A. Dyon induced electron-nucleon interaction

First we will consider the atomic EDM that can be produced by the dyon induced $T$- and $P$-odd electron-nucleon interaction. To find this interaction we let particle $a$ be a quark ($q$) and particle $b$ be an electron ($e$), and sum over the quarks of the nucleon.

To begin with we will consider the first term of Eq. (28) — the quark scalar-electron pseudoscalar interaction. It follows from the Dirac equation and $q = p'_e - p_e$ that

$$\bar{u}(p'_e) q \gamma_5 u(p_e) = 2m_e \bar{u}(p'_e) \gamma_5 u(p_e).$$

We rewrite the interaction as a Hamiltonian density:

$$H(x) = -ie_a^2 e_b^2 Qg(Q^2 - g^2) m_q m_e \ln \left( \frac{M}{m_q} \right) \ln \left( \frac{M}{m_e} \right) \bar{\psi}_p \gamma_5 \psi_p \bar{\psi}_p' \gamma_5 \psi_p',$$

(29)

where the wave function, $\psi_p(x)$, is related to the amplitude $u(p)$ by $\psi_p(x) = (2\epsilon_p)^{-1/2} u(p) e^{-ipx}$ ($\epsilon_p$ is the energy of the particle). Summing over the quarks in the proton and neutron gives
Here we have a derivative of a quark axial current (\(q\)). Thus, these constants can be written in terms of dyon parameter \(s\) as

\[
\text{interaction} - \text{is} \frac{1}{M} \frac{\partial}{\partial x^i} \psi(x) \bar{q}(x) \gamma_5 \psi(x), 
\]

where \(\phi_{p_N}(x)\) denotes the nucleon wave functions and \(N\) (nucleon) is either \(p\) or \(n\).

The Hamiltonian density for the other term — the quark pseudoscalar-electron scalar interaction — is

\[
H(x) = -i e^2 e' \frac{G g(Q^2 - g^2)}{12 \pi^2 M^4} m_e \ln \left( \frac{M}{m_q} \right) \ln \left( \frac{M}{m_e} \right) \bar{\psi}_p \gamma_5 \psi_p \bar{\psi}_e \psi_p, 
\]

Here we have a derivative of a quark axial current \(q_a \bar{\psi}_p \gamma_5 \gamma_5 \bar{\psi}_p = -i \frac{\partial}{\partial x^i} \bar{\psi}_p \gamma_5 \gamma_5 \bar{\psi}_p\), but we need to express our result in terms of nucleon wave functions. We make an order of magnitude estimate using PCAC (partial conservation of axial current) \(H\): \(\sum_q e_q^2 \bar{\psi}_p \gamma_5 \psi_p \bar{\psi}_e \psi_p \rightarrow (1 \text{ GeV}) e^2 \bar{\psi}_N \gamma_5 \psi_{p_N}\). It does not matter exactly what value we choose for the coefficient on the right hand side of this equation, as due to the fact that the main dependence of the interaction on the dyon mass goes as \(1/M^4\), the limit on the dyon mass that we will obtain will not be very sensitive to this choice. The result for the nucleon pseudoscalar-electron scalar interaction is

\[
H_{N_p}^p = \frac{e^4 Q g(Q^2 - g^2)}{12 \pi^2 M^4} (1 \text{ GeV}) m_e \ln \left( \frac{M}{m_q} \right) \ln \left( \frac{M}{m_e} \right) \bar{\psi}_p \gamma_5 \psi_p \bar{\psi}_p \psi_p. 
\]

The interactions in Eqs. (30) and (32) can produce atomic EDMs. Note that since the pseudoscalar \(\bar{\psi} \gamma_5 \psi\) is proportional to the spin of the particle, interaction (30) only has an effect in atoms with unpaired electrons, while interaction (32) requires an unpaired nucleon. The size of such scalar-pseudoscalar and pseudoscalar-scalar interactions can be denoted by the constants \(k_{1N}\) and \(k_{2N}\) (see, e.g., (3)):

\[
H_{N_p}^p = \frac{e^4 Q g(Q^2 - g^2)}{12 \pi^2 M^4} (1 \text{ GeV}) m_e \ln \left( \frac{M}{m_q} \right) \ln \left( \frac{M}{m_e} \right) \bar{\psi}_p \gamma_5 \psi_p \bar{\psi}_p \psi_p. 
\]

Thus, these constants can be written in terms of dyon parameters as

\[
|k_{1N}| = \frac{25 \sqrt{2} e^4 m_q m_e}{54 \pi^2 G M^4} \ln \left( \frac{M}{m_q} \right) \ln \left( \frac{M}{m_e} \right), 
\]

\[
|k_{2N}| = \frac{\sqrt{2} e^4 (1 \text{ GeV}) m_e}{12 \pi^2 G M^4} \ln \left( \frac{M}{m_q} \right) \ln \left( \frac{M}{m_e} \right). 
\]
to order of magnitude accuracy, where we have defined $\tilde{M}$ for convenience as

$$\tilde{M} \equiv \frac{M}{|Qg(Q^2 - g^2)|^{1/4}}. \quad (37)$$

From experimental limits on atomic EDMs \[10,11\] and calculations of the EDMs produced by the interactions \((33)\) and \((34)\) \[9,12\], limits on $k_{1N}$ and $k_{3N}$ can be deduced, and hence limits on the dyon parameter $\tilde{M}$. See Table I. Note that in Eqs. \((35–36)\), in addition to the unknown parameter $\tilde{M}$ there is $M$, which is another (independent) unknown parameter, as $Q$ and $g$ are unknown. However, since $M$ only appears in logarithms, which are not very sensitive, we can take $M = |Qg(Q^2 - g^2)|^{1/4}\tilde{M} \approx 2\tilde{M}$, assuming that $Q \approx e$ and $g \approx 1/(2e)$ — this estimate of $g$ is based on the Dirac condition \[13\]: $eg = n/2$, where $n$ is an integer. The best bound on $\tilde{M}$ is $\tilde{M} > 6$ GeV.

Dyons could also induce an electron-electron scalar-pseudoscalar interaction, which could create an atomic EDM in atoms with unpaired electrons. However, this effect is negligible compared to the effect of the nucleon scalar-electron pseudoscalar interaction. Since the wave function of the outer, unpaired electron is enhanced at small distances, the main contribution to the interaction is from $r < a/Z$, where $a$ is the Bohr radius. In the case of the electron-electron interaction only the innermost electrons have sizeable wave functions in this region, while for the nucleon-electron interaction all of the nucleons are able to contribute, so the electron-nucleon interaction is larger by a factor $\sim Z + N$. In addition, the wave function of the outer electron becomes very large near the nuclear surface and this enhances the nucleon-electron interaction by a factor $\sim 10$. See Ref. \[14\] for a similar result concerning the relative contributions of weak electron-nucleon and electron-electron interactions.

**B. Dyon induced quark-quark interaction**

Now we consider the $T$- and $P$-odd quark-quark interaction. This corresponds to Eq. \((28)\), with particles $a$ and $b$ both quarks. The interaction between quarks $q_1$ and $q_2$ is

$$H(x) = i\epsilon_{q_1}^2\epsilon_{q_2}^2 \frac{Qg(Q^2 - g^2)}{6\pi^2 M^4} m_{q_1}' m_{q_2} \ln \left( \frac{M}{m_{q_1}} \right) \ln \left( \frac{M}{m_{q_2}} \right) \bar{\psi}(p'_{q_1}) \gamma_5 \psi(p_{q_1}) \bar{\psi}(p'_{q_2}) \gamma_5 \psi(p_{q_2}) \quad (38)$$

(of course there is also the scalar-pseudoscalar interaction). Here we used $\pi(p'_{q_1}) \gamma_5 u(p_{q_1}) = -2m_{q_1}' \overline{\pi(p'_{q_1})} \gamma_5 u(p_{q_1})$, as we did for the electron pseudoscalar in Sec. \[IV\], but note that $m_{q_1}'$
is the current quark mass: $\sim 5$ MeV. The $m_{q_2}$ comes from $\hat{p}_{q_2}u(p_{q_2}) = m_{q_2}u(p_{q_2})$ (as in the Appendix) and is the constituent quark mass ($\approx m_N/3$) as it takes into account the effects of the gluon cloud. It is not used in $\pi(p'_{q_1})\bar{q}\gamma_5u(p_{q_1}) = -2m'_{q_1}\pi(p'_{q_1})\gamma_5u(p_{q_1})$ as this involves the difference of two Dirac equations and so the effects of the gluon cloud cancel out. The general quark-quark pseudoscalar-scalar interaction can be written in terms of the dimensionless constant $k_s$ as (see, e.g., Ref. \[15\])

$$H = ik_s\frac{G}{\sqrt{2}}\psi_{p'_{q_1}}\gamma_5\psi_{p_{q_1}}\bar{u}_{p_{q_2}}\bar{u}_{p_{q_2}},$$ (39)

where

$$k_s = \frac{\sqrt{2e^2_1e^2_2m'_{q_1}m_{q_2}}}{6\pi^2G} \ln \left(\frac{M}{m_{q_1}}\right) \ln \left(\frac{M}{m_{q_2}}\right).$$ (40)

The calculation done in Ref. \[15\] allows us to convert the experimental limit on the atomic EDM of $^{199}$Hg, $|d_A| < 9.7 \times 10^{-28} e$ cm \[11\], to a limit on $k_s$: $|k_s| < 2 \times 10^{-6}$. The quark-quark pseudoscalar-scalar interaction can induce a neutron EDM. The limit on the neutron EDM, $|d_n| < 1.1 \times 10^{-25} e$ cm \[16,18\] implies the weaker limit $|k_s| < 3 \times 10^{-5}$ \[15\]. Using the stronger limit, and taking $e_{q_1}$ and $e_{q_2}$ as $\sim e/3$, gives $\bar{M} > 1.5$ GeV.

V. CP VIOLATION IN K-MESON DECAYS

It would be good if it were possible to make a comprehensive theory of $CP$- and $T$-violation based on dyons that could explain the experimentally observed $CP$-violation in K-meson decays \[14\]. Unfortunately, no such theory exists at the moment, but we are able to make some plausible estimates of dyon induced $CP$-violation in K-meson decays. However, we find that the effect induced by dyons is not large enough to explain the observed level of $CP$-violation.

Because of electroweak unification, since dyons may induce $T$- and $P$-odd photon-photon scattering processes, we would expect them to also induce $W$-boson scattering processes. The $W$-boson scattering tensor should be similar to as in Eq. (2). $CP$-violation in K-meson decays could be caused by an interaction described by the following Lagrangian \[20\]:

$$L = -G_2\bar{s}\gamma_\alpha(1 + \gamma_5)d\bar{\gamma}_\alpha(1 + \gamma_5)d.$$ (41)
CP violation would occur if $G_2$ has a non-zero imaginary part (the real part of $G_2$ is responsible for the mass difference between $K^0_S$ and $K^0_L$). According to [20] the observed CP violation implies that

$$|\text{Im}(G_2)| \sim 3 \times 10^{-3}|\text{Re}(G_2)|$$

and from [20] we have

$$\text{Re}(G_2) \approx \frac{1}{16\pi^2} \sin^2 \theta_c \cos^2 \theta_c G^2 m_c^2,$$

where $\theta_c$ is the Cabibbo angle, $G$ is the Fermi weak interaction constant ($\approx 10^{-5}/m_p^2$, where $m_p$ is the proton mass), and $m_c$ is the mass of the charm quark.

A diagram through which dyons could induce a CP-violating interaction in K-mesons is shown in Fig. 4. The intermediate quarks may be either up or charm quarks. Note that such a diagram requires two different kinds of dyons, differing in their electric charges by $e$, to satisfy the conservation of charge at the $W$-dyon vertices. (However, even if this is not true the effect may still appear at higher orders.)

An interaction corresponding to this diagram is

$$L = iC \overline{d}s\overline{s}d,$$

where

$$C \sim \frac{e^4}{96\pi^2} Q_w g_w (Q_w^2 - g_w^2) \frac{(m_s^2 - m_d^2)m_c^4}{M_W^4} \frac{1}{M^4} \sin^2 \theta_c \cos^2 \theta_c,$$

where $m_s$ and $m_d$ are the masses of the strange and down quarks, $M_W$ is the mass of the W-boson, and $Q_w$ and $g_w$ are some effective weak electric and weak magnetic charges for the dyons. This comes from integrating over the region $M_W < k < M$, which assumes that the dyon mass is much greater than the W boson mass. If in fact the dyon mass were less than the W boson mass then the relevant region of integration would be $m_c < k < M$ and the result is

$$C \sim \frac{e^4}{96\pi^2} Q_w g_w (Q_w^2 - g_w^2) \frac{(m_s^2 - m_d^2)m_c^4 (M^2 - m_c^2)^2}{M_W^8} \frac{1}{M^4} \sin^2 \theta_c \cos^2 \theta_c.$$
to $\text{Im}(G_2)$ [although $C$ and $\text{Im}(G_2)$ describe two different types of CP violating interactions, we assume that the same order of magnitude of either would induce the experimentally observed CP violation]. If we first consider the case of the dyon mass being larger than the $W$ boson mass we have [using Eqs. (42), (43), and (45)] $C/\text{Im}(G_2) \sim (M/\text{GeV})^{-4}$. This cannot be larger than about $10^{-8}$. If we consider the case of the dyon mass being less than the $W$ boson mass then we get the result $C/\text{Im}(G_2) \sim 10^{-8}(M^2 - m_t^2)^2/M^4$, which has a maximum value of about $10^{-8}$ (here we actually assumed that the dyon mass was larger than the $c$ quark mass; in the case that it is not we get a result of similar magnitude). These results suggest that this effect cannot produce the experimentally observed level of CP-violation in K-meson decays.

We can also consider the case when the intermediate quark is a top quark, rather than up or charm quarks. If we assume that $M < M_W$ and integrate over the region $0 < k < M$ we get the result

$$C \sim \frac{e^4}{1000\pi^2} Q_w g_w (Q_w^2 - g_w^2) \frac{(m_s^2 - m_d^2) M^8}{M_W^8 m_t^4} (V_{td} V_{ts})^2; \quad (47)$$

where $V_{td}$ and $V_{ts}$ are Cabibbo-Kobayashi-Maskawa mixing matrix elements. The top quark mass is $m_t \approx 170 \text{ GeV}$ and, according to [21], $|V_{td}|$ and $|V_{ts}|$ lie in the ranges 0.004–0.013 and 0.035–0.042, respectively. Therefore $C/\text{Im}(G_2)$ has a maximum value of about $10^{-9}$. Again, this is too small to give the observed effect. If we assume that $M_W < M < m_t$ and integrate over the region $M_W < k < M$ we get

$$C \sim \frac{e^4}{96\pi^2} Q_w g_w (Q_w^2 - g_w^2) \frac{(m_s^2 - m_d^2) (M^2 - M_W^2)^2}{M^4} (V_{td} V_{ts})^2; \quad (48)$$

for which $C/\text{Im}(G_2)$ has a maximum value of about $10^{-8}$. The case of the dyon mass being larger than the top quark mass gives a result of similar magnitude, with the maximum value of $C/\text{Im}(G_2)$ being $10^{-8}$.

Dyons could also induce a CP violating interaction through the diagram shown in Fig. 4. However, once again it seems that it cannot produce an effect of the magnitude of the experimentally observed effect.

Finally, the possibility is not excluded that dyons may generate an electroweak $\theta$-term ($\propto \theta F_{\mu \nu} \tilde{F}^{\mu \nu}$, where $F_{\mu \nu} = \partial_{\mu} W_\nu - \partial_{\nu} W_\mu + ig[W_\mu, W_\nu]$) which may give an effective CP-violating interaction for four $W$-bosons that does not contain the fourth power of the dyon
mass in the denominator, and in that case the effect could be of considerable size even if the dyon mass is large.

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**APPENDIX A: EXAMPLE CALCULATION OF A CONTRIBUTION TO THE MATRIX ELEMENT**

In this appendix we calculate the contribution of the term \(c_1 \hat{p}_a g^{\alpha \beta} \) in \(W^{\alpha \beta} \) to \(M_{fi} \). We denote this contribution by \(M_{fi}^{(2)} \).

The contribution of the 1st and 7th terms in Eq. (9) to \(M_{fi}^{(2)} \) (which we will call \(M_{fi}^{(2a)} \)) is zero, as they contain the contraction of the antisymmetric \(\varepsilon^{\alpha \beta \mu \nu} \) with \(g^{\alpha \beta} \).

The contribution of the 6th and 12th terms of the equation to \(M_{fi}^{(2)} \) is equal to (with \(c_2 = [ie^2_a Qg(Q^2 - g^2)]/(360\pi^4 M^4) \))

\[
M_{fi}^{(2b)} = c_1 c_2 A_\lambda(k_3) A_\omega(k_4) k_{3\rho} k_{4\sigma} \varepsilon^{\lambda\omega\rho\sigma} \pi(p'_a) \gamma_\mu(\hat{p}_a g^{\nu\mu} - \hat{p}_a g^{\mu\nu} g^{\pi\pi}) \gamma_\nu u(p_a) \\
= -3c_1 c_2 A_\lambda(k_3) A_\omega(k_4) k_{3\rho} k_{4\sigma} \varepsilon^{\lambda\omega\rho\sigma} \pi(p'_a) \gamma_\mu \hat{p}_a \gamma_\mu u(p_a) \\
= 6c_1 c_2 A_\lambda(k_3) A_\omega(k_4) k_{3\rho} k_{4\sigma} \varepsilon^{\lambda\omega\rho\sigma} m_a \pi(p'_a) u(p_a),
\]

(A1)

using \(\gamma_\mu \hat{p}_a \gamma_\mu = -2\hat{p}_a \) and the Dirac equation: \(\hat{p}_a u(p_a) = m_a u(p_a) \). Now, from the fact that \(F_{\mu\nu}(k) = i(k_\mu A_\nu - k_\nu A_\mu) \) and the definition of the dual tensor: \(\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \), we have

\[
\varepsilon^{\lambda\omega\rho\sigma} k_{3\rho} A_\lambda(k_3) k_{4\sigma} A_\omega(k_4) = \frac{1}{2} \tilde{F}^{\sigma\omega}(k_3) F_{\sigma\omega}(k_4)
\]

(A2)

Therefore

\[
M_{fi}^{(2b)} = 3c_1 c_2 m_a \pi(p'_a) u(p_a) \tilde{F}^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4).
\]

(A3)

The remaining terms (terms 2–5 and 8–11) can be written as follows (note that we swap the dummy indices \(\mu \) and \(\nu \) around in terms 4, 5, 10, and 11):

\[
\frac{3}{8} c_1 c_2 m_a \pi(p'_a) u(p_a) \tilde{F}^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4).
\]
\[ M^{(2c)}_{fi} = c_1 c_2 A_\lambda(k_3) A_\omega(k_4) k_{3\mu} k_{4\sigma} \pi(p'_a)(\gamma_\mu \hat{p}_a \gamma_\nu + \gamma_\nu \hat{p}_a \gamma_\mu) u(p_a) \]
\[ \times (g^{\alpha\sigma} g^{\beta\lambda} + g^{\lambda\nu} g^{\alpha\rho} \varepsilon_\alpha \varepsilon_\beta \omega \varepsilon_\mu \varepsilon_\sigma - g^{\alpha\mu} g^{\beta\nu} \varepsilon_\alpha \varepsilon_\beta \omega \varepsilon_\rho \varepsilon_\sigma) \]
\[ = c_1 c_2 A_\lambda(k_3) A_\omega(k_4) k_{3\mu} k_{4\sigma} \pi(p'_a)(\gamma_\mu \hat{p}_a \gamma_\nu + \gamma_\nu \hat{p}_a \gamma_\mu) u(p_a) \]
\[ \times (g^{\omega\nu} \varepsilon^{\alpha\lambda\mu} + g^{\lambda\nu} \varepsilon^{\alpha\rho\omega} - g^{\alpha\mu} \varepsilon^{\omega\lambda\rho} - g^{\omega\nu} \varepsilon^{\lambda\rho\omega}) \]

\[ (A4) \]

Now we have \( k_{4\sigma} A_\omega(k_4)(g^{\alpha\nu} \varepsilon^{\alpha\lambda\mu} - g^{\alpha\mu} \varepsilon^{\lambda\nu}) = [k_{4\sigma} A_\omega(k_4) - k_{4\nu} A_\sigma(k_4)]g^{\omega\nu} \varepsilon^{\alpha\lambda\mu} = -i F_\sigma\omega(k_4)g^{\omega\nu} \varepsilon^{\alpha\lambda\mu} = -i F_\sigma\nu(k_4)\varepsilon^{\alpha\lambda\mu}, \]
which corresponds to the 1st and 3rd terms in Eq. \([A4] \).

Doing a similar thing for the 2nd and 4th terms as well gives
\[ M^{(2c)}_{fi} = -i c_1 c_2 [k_{3\mu} A_\lambda(k_3) F_\sigma\nu(k_4) \varepsilon^{\alpha\lambda\mu} + F_\rho\nu(k_3) k_{4\sigma} A_\omega(k_4) \varepsilon^{\rho\omega\mu}] \pi(p'_a)(\gamma_\mu \hat{p}_a \gamma_\nu + \gamma_\nu \hat{p}_a \gamma_\mu) u(p_a) \]
\[ = -c_1 c_2 [F_\sigma^{\alpha}\nu(k_3) F_\sigma\nu(k_4) + F_\sigma\nu(k_3) F^{\nu\sigma}(k_4)] \pi(p'_a)(\gamma_\mu \hat{p}_a \gamma_\nu + \gamma_\nu \hat{p}_a \gamma_\mu) u(p_a) \]

\[ (A5) \]

Using the expressions for the \( F \) and \( \tilde{F} \) matrices in terms of the electric and magnetic field components, the following identity can be derived:
\[ \tilde{F}^{\sigma\mu}(k_3) F_\sigma^{\nu}(k_4) + F_\sigma^{\nu}(k_3) \tilde{F}^{\sigma\mu}(k_4) = [E(k_3) \cdot B(k_4) + B(k_3) \cdot E(k_4)]g^{\mu\nu} \]
\[ = \frac{1}{2} g^{\mu\nu} \tilde{F}^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4) \]

\[ (A6) \]

Therefore we have
\[ M^{(2c)}_{fi} = -c_1 c_2 \tilde{F}^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4) \pi(p'_a)(\gamma_\mu \hat{p}_a \gamma_\mu) u(p_a) \]
\[ = 2 c_1 c_2 m_a \pi(p'_a) u(p_a) \tilde{F}^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4) \]

\[ (A7) \]

So, from Eqs. \([A3] \) and \([A7] \), the contribution of the term \( c_1 \hat{p}_a g^{\alpha\beta} \) (in \( W^{\alpha\beta} \)) to \( M_{fi} \) is
\[ M^{(2)}_{fi} = 5 c_1 c_2 m_a \pi(p'_a) u(p_a) \tilde{F}^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4) \]
\[ = e^2 Q(g(Q^2 - g^2)/(216\pi^2 M^4) m_a \ln(M/m_a) \pi(p'_a) u(p_a) \tilde{F}^{\alpha\beta}(k_3) F_{\alpha\beta}(k_4) \]

\[ (A8) \]
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FIG. 1. Diagram showing photon-photon scattering due to dyon vacuum polarization. We take all photon momenta as pointing inward.

FIG. 2. Diagram showing the dyon induced $T$- and $P$-odd mixed polarizability.
FIG. 3. Diagram showing the dyon induced $T$- and $P$-odd interaction between two particles.

FIG. 4. A diagram through which dyons could induce CP violation in K-meson decays.
FIG. 5. Another diagram through which dyons could induce CP violation in K-meson decays.
### TABLE I

| Atom   | Bound on $|d_A|$ (e cm) | Ref. | Bound on $|k_X|$ | Bound on $\tilde{M}$ |
|--------|----------------------|------|-----------------|---------------------|
| Tl     | $2.9 \times 10^{-24}$ | [10] | $|k_{1p}| < 1.4 \times 10^{-6}$ | $\tilde{M} > 6$ GeV |
| $^{199}$Hg | $9.7 \times 10^{-28}$ | [11] | $|k_{3n}| < 1.6 \times 10^{-5}$ | $\tilde{M} > 2.6$ GeV |

TABLE I. Table showing the experimentally determined upper bounds on various atomic EDMs (with references) and the consequent upper bounds on $k_{1p}$ and $k_{3n}$ and lower bounds on $\tilde{M}$. All bounds are given at the 2 standard deviation limit.