SUPERDEFORMED OBLATE SUPERHEAVY NUCLEI IN THE SELF-CONSISTENT APPROACH∗

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The HFB self-consistent method has been applied to study the properties of several neutron deficient superheavy nuclei with $Z = 120–124$, $N = 160–168$. Their distinctive feature is the existence of minima of the total HFB energy for strongly deformed, oblate shapes. The self-consistent results agree quite remarkably with those currently obtained by using microscopic–macroscopic method.

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1. Introduction

Various aspects of physics of superheavy nuclei have been subject to numerous studies from both experimental and theoretical sides for many years. There are two main theoretical approaches used to describe and predict the properties of superheavies: the so-called microscopic–macroscopic method and the self-consistent approach. A recent review of theoretical achievements can be found e.g. in [1]. In the present paper we consider exotic, very neutron-deficient nuclei with $Z = 120–124$ and $N = 160–168$, which have not been yet extensively investigated within the self-consistent approach, in particular they are not covered by an extensive study [2]. Our work was motivated by a paper of Jachimowicz et al. [3], where some intriguing properties of nuclei from this region were predicted using a sophisticated micro–macro

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model. First, most of the considered nuclei have global minimum of energy for strongly deformed, oblate shapes (we call them super-oblate, SDO, shapes). It is worth noting that the first hint on the existence of SDO minima appeared in [4] within the micro–macro model but was not confirmed by the self-consistent calculations. Second, the authors of [3] pointed out that in spite of the fact that the considered region lies quite far from the predicted island of stability there are some mechanisms which could increase half-lives of some of nuclei to a range close to experimental capabilities. Experimentalists have recently reached $Z = 118$ isotopes [5, 6] and plan to go even further, therefore, studies of such exotic systems can proceed beyond purely theoretical speculations.

2. Theoretical method

Our approach is based on the Hartree–Fock–Bogolyubov theory with a well-established SkM* variant of the Skyrme effective interaction. This variant gives reasonable values of barrier heights in transactinide region which is of particular importance if one wants to study fission properties. The pairing interaction is a sum of volume and surface $\delta$ force, cf. formula (15) in [7]. More details of the theoretical model can be found in [8–11], where this model was applied to a broad range of heavy and superheavy nuclei. The HFB calculations have been performed using the HFODD solver, see [12] and references therein.

In order to study the dependence of total nuclear energy on deformation, we do not use a parametrization of a family of shapes (as in the microscopic–macroscopic approach) but we rather perform the HFB calculations with constraints on the values of components of the quadrupole mass tensor

\[
q_{20} = \left\langle \Psi \left| \sum_{i=1}^{A} (3z_i^2 - r_i^2) \right| \Psi \right\rangle,
\]

\[
q_{22} = \left\langle \Psi \left| \sum_{i=1}^{A} \sqrt{3} (x_i^2 - y_i^2) \right| \Psi \right\rangle.
\]

These values can be further related to the $\beta, \gamma$ deformation variables

\[
\beta = c \sqrt{q_{20}^2 + q_{22}^2},
\]

\[
\tan \gamma = q_{22}/q_{20},
\]

where $c = \sqrt{\pi/5}/AR_0^2$ and $R_0^2 = 3(r_0A^{1/3})^2/5$, $r_0 = 1.23$ fm.
3. Results

Our self-consistent calculations with the Skyrme SkM* interaction confirm most findings of Jachimowicz et al. [3].

Potential energy landscapes. The HFB total energy of nuclei in the region \( Z = 120–126 \) and \( N = 160–168 \) exhibit a well pronounced minimum for \( \beta \sim 0.4, \gamma = 60^\circ \) and for several nuclei this is a global minimum. In the case of ellipsoidal shapes such deformation corresponds to a ratio of axes \( \sim 3 : 2 \).

Below, in Figs. 1 and 2 we show a sample of results for the \( Z = 120, 122 \) isotopes. In the case of \( Z = 120 \), only the \( N = 166 \) isotope has a global super-oblate minimum but this isotope is particularly interesting because it is the lightest among SDO nuclei and, moreover, its \( \alpha \) decay can be hindered due to a mechanism proposed in [3] and described below.

![Fig. 1. Plot of the total HFB energy (in MeV, relative to that of a spherical shape) for the \( Z = 120 \) isotopes.](image)

Among the \( Z = 122 \) isotopes, the three lightest ones (\( N = 162–166 \)) have a big oblate deformation. Starting from \( N = 168 \), the minimum moves gradually with increasing \( N \) to a spherical point. Similar evolution can be seen also for \( Z = 124 \) isotopes (not shown in the paper) but with the jump from an SDO to normal oblate nucleus at \( N = 170 \).

Decay modes and half-lives estimations. One should remember that due to a large deficit of neutrons the considered nuclei are unstable against proton emission, however, estimations analogous to those in [4] give a half-life for this process of an order of days. Much more important for the existence of considered nuclei are \( \alpha \)-decay and fission, as can be seen from systematics of half-lives of isotopes with larger neutron number e.g. in [2].

\( \alpha \)-decay. Typical values of \( Q_\alpha \) in the considered region are around 15 MeV which gives \( T_{1/2,\alpha} \) around \( 10^{-9} \) s (we use here phenomenological formula from [13]) but at least in one case (the \(^{286}120 \) nucleus) we could...
expect a substantial hindrance of the $\alpha$-decay. As can be seen from Fig. 3 in the daughter nucleus $^{282}118$ there is a (global) prolate but also a super-oblate minimum. Assuming that $\alpha$ decay from the (super-oblate) $^{286}120$ nucleus does not substantially change the internal configuration and goes mainly to a super-oblate shape of $^{282}118$, we obtain the value of $Q_\alpha$ for such transition equal to $13.6 \text{MeV}$ what gives $T_{1/2,\alpha} = 10^{-5.84} \text{s}$. This mechanism was proposed in [3] within the micro–macro approach just for the same pair of nuclei.

**Fission.** As can be seen from Figs. 1 and 2, fission process from a super-oblate minimum goes through triaxial shapes. Within the self-consistent approach not much is known about tunnelling through such two-dimensional barrier. In particular, one should consider a full two-dimensional tensor of mass parameters. Work on calculation of fission half-lives is in progress.
Fig. 3. Total energy for axial shapes of $^{286}120$ and $^{282}118$ nuclei. In the case of $^{282}118$ nucleus prolate and SDO minima mentioned in the text are indicated.

$K$-isomers. The authors of [3] pointed also out that long lived $K$-isomers can appear in both even and odd nuclei in the considered region. This phenomenon is connected with the existence of single-particle states with a high angular momentum projection close to the Fermi level. Similar conclusions can be drawn from our self-consistent calculations. Below, we show single-particle states around the Fermi level at the minimum of energy for $^{286}120$:

| Neutrons | Protons |
|----------|---------|
| $\Omega_i$ | $e_i - e_F$ [MeV] | $\Omega_i$ | $e_i - e_F$ [MeV] |
| $15/2^+$ | $-0.56$ | $13/2^+$ | $-0.66$ |
| $9/2^-$ | $0.75$ | $7/2^-$ | $0.42$ |

Here $\Omega_i$ is an eigenvalue of a projection of angular momentum operator on a symmetry axis for the state $i$.

4. Conclusions

The HFB self-consistent calculations have confirmed the predictions of the micro–macro method concerning the existence of super-oblate ground states for the very neutron-deficient superheavy nuclei. It is encouraging that both approaches give very similar results, even on a quantitative level, when they are extrapolated far from regions, where their parameters were fitted. Some aspects of the discussed subject still need more work, especially fission properties and odd systems (not touched in the present work).

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