Electronic Specific Heat of La$_{2-x}$Sr$_x$CuO$_4$: Pseudogap Formation and Reduction of the Superconducting Condensation Energy

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To examine the so-called small pseudogap and the superconducting (SC) condensation energy $U(0)$, the electronic specific heat $C_{ei}$ was measured on La$_{2-x}$Sr$_x$CuO$_4$ up to $\sim 120$ K. In samples with doping level $p (= x)$ less than $\sim 0.2$, small pseudogap behavior appears in the $\gamma (= C_{ei}/T)$ vs. $T$ curve around the mean-field critical temperature for a $d$-wave superconductor $T_{co} (= 2\Delta_0/(4 \sim 5)k_B)$, where $\Delta_0$ is the maximum gap at $T \ll T_c$. The condensation energy $U(0)$ is largely reduced in the pseudogap regime ($p \lesssim 0.2$). The reduction of $U(0)$ can be well reproduced by introducing an effective SC energy scale $\Delta_{ei}^0 = \beta p \Delta_0$ ($\beta = 4.5$) instead of $\Delta_0$. The effective SC energy scale is discussed in relation to the coherent pairing gap formed over the nodal Fermi arc.

KEYWORDS: electronic specific heat, superconducting condensation energy, pseudogap, La$_{2-x}$Sr$_x$CuO$_4$

1. Introduction

Loram et al. revealed that the superconducting (SC) condensation energy $U(0)$ of high-$T_c$ cuprates was exceedingly suppressed even in a slightly underdoped region, where $T_c$ was still high enough. To clarify the origin of the exceeding reduction of $U(0)$ is expected to give an important clue to understanding the mechanism of high-$T_c$ superconductivity, because the condensation energy $U(0)$ reflects features of the pairing mechanism and/or the collective motion of pairs. Thus this problem has been discussed from various points of view. Lo-ram et al. have argued the exceeding reduction of $U(0)$ in terms of the existence of a $T$-independent energy gap at the Fermi level $E_F$, which is persistent up to a slight overdoping level. Another explanation has been proposed by Demler and Zhang on the basis of a spin-triplet particle-particle resonance, which lowers the antiferromagnetic (AFM) exchange energy in the $d$-wave SC state through the formation of the so-called $\pi$-resonance in the dynamic spin susceptibility $\chi(q, \omega)$, as In this scenario, the SC condensation energy $U(0)$ comes from the difference of AFM exchange energy between the normal and superconducting states, and so $U(0)$ will be reduced greatly in samples with small doping levels whose AFM correlation is already enhanced to a large extent in the normal state. Other kind of explanation is that in the underdoped region the superconductivity will be driven by a small gain in kinetic energy caused by interplane phase coherence, which naturally leads to a small $U(0)$. Recently Lee and Salk have claimed that unusual doping-level ($p$) dependence of $U(0)$ can be explained within the framework of the boson-pair condensation in the SU(2) slave-boson model.

Very recently it was demonstrated for La$_{2-x}$Sr$_x$CuO$_4$ (La214) that the exceeding reduction of $U(0)$ can be reproduced quantitatively by adopting the SC energy scale $\beta p \Delta_0$ ($\beta = 4.5$) instead of $\Delta_0$, where $\Delta_0$ is the maximum value of the $d$-wave energy gap. The SC energy scale $\beta p \Delta_0$ ($\beta = 4.5$), deduced from the phenomenological relation $T_c \sim \kappa p \Delta_0$ ($\kappa \sim 1.7$), becomes much smaller than $\Delta_0$ at small doping levels through the factor $\beta p$, and $U(0)$ will be markedly suppressed there. On the other hand, the energy scale $\beta p \Delta_0$ is comparable with $\Delta_0$ around $p (= x) = 0.22$, where no pseudogap behavior appears and the SC properties are of the BCS type. This leads naturally to the speculation that the change of the SC energy scale from $\Delta_0$ to $\beta p \Delta_0$ will result from the development of a pseudogap in the normal state.

In many high-$T_c$ cuprates, two types of pseudogaps appear in the normal state for $x \lesssim 0.2$; one is the so-called large pseudogap brought by the downward shift of flat bands from $E_F$ near $(\pi, 0)$ and $(0, \pi)$, and the other is the so-called small pseudogap characterized by an energy scale of the order of $\Delta_0$. The small pseudogap was first reported as the spin gap in NMR relaxation time $T_1$ measurements on YB$_2$Cu$_3$O$_{6+\delta}$, and has been found in many high-$T_c$ cuprates although it was as late as last year that the spin gap was reported in inelastic neutron scattering and NMR-$T_1$ measurements on La$_{2-x}$Sr$_x$CuO$_4$.

Angle-resolved photoemission spectroscopy (ARPES) measurements on Bi$_2$Sr$_2$CaCu$_2$O$_8+\delta$ (Bi2212) have clarified that the small pseudogap starts to open at the Fermi surface near $(\pi, 0)$ and $(0, \pi)$ at temperature $T^* (> T_c)$. The small pseudogap grows towards the $d$-wave nodal points near $(\pi/2, \pi/2)$ at $T_c < T < T^*$, leaving the so-called nodal Fermi arc centered at the nodal points. Very recently Yoshida et al. and Zhou et al. reported the existence of the nodal Fermi arc in underdoped samples of La$_{2-x}$Sr$_x$CuO$_4$.

Tunneling spectroscopy measurements on Bi2212 have demonstrated that the small pseudogap behavior becomes evident gradually around the mean-field critical temperature $T_{co} = 2\Delta_0/(4 \sim 5)k_B$ for a $d$-wave superconductor. In the present study, the electronic specific heat $C_{ei}$ for La214 was systematically measured over a wide doping-level ($p$) range to examine the pseudogap formation, the
marked reduction of $U(0)$ and the interrelation between them in detail. The small pseudogap behavior appears around the temperature $T'$ ($\sim T_{co}$) which roughly correlates with the onset temperature of the enhanced Nernst signal reported by Wang et al.\textsuperscript{24} It was reconfirmed that the reduction of $U(0)$ becomes more conspicuous in samples with higher $T'$ and can be well explained by the SC energy scale $\beta p\Delta_0$ ($\beta = 4.5$) over a wide $p$ range. The SC energy scale is discussed in relation to the shrinkage of the coherent part of the pairing gap in the pseudogap regime.

2. Experimental

Ceramic samples of La214 used for the present study were prepared by using a solid reaction in an oxygen atmosphere. The SC critical temperature $T_c$ was determined from the SC diamagnetism measured with a SQUID magnetometer. Specific heat measurements were carried out using a conventional pulsed-heat technique.

The electronic specific heat $C_{el}$ of SC samples was obtained by subtracting the phonon term $C_{ph}$ of an impurity-doped nonsuperconducting sample from the observed total specific heat $C_{total}$: $C_{el} = C_{total} - C_{ph}$. The phonon term $C_{ph}$ of the nonsuperconducting sample was obtained as follows. First we determined the coefficient of the $T$ linear term of $C_{el}$, $\gamma$, using a $C_{el}/T$ vs. $T^2$ plot at low temperatures where $C_{ph}$ shows $T^3$ dependence.\textsuperscript{25} Next we extracted $C_{ph}$ by subtracting the electronic term $\gamma T$ from $C_{total}$ on the assumption that $\gamma$ was independent of $T$. We tried to use Zn impurity at first to suppress the superconductivity in the process of obtaining $C_{ph}$, because the Zn ion has the mass closest to that of Cu$^{2+}$ and carries no local magnetic moment. However, since the Zn impurity modifies the phonon properties, the phonon term $C_{Zn}^{ph}$ of the Zn-doped non-superconducting sample becomes appreciably different from the $C_{ph}$ of the SC sample, as will be described in the following section. Then we used the phonon term $C_{Ni}^{ph}$ obtained for an Ni-doped non-superconducting sample, although the Ni impurity carries local magnetic moment. It has been revealed that a small amount of Ni impurity removed the superconductivity and the small pseudogap behavior.\textsuperscript{26}

3. Results and Discussion

3.1 Electronic Specific Heat $C_{el}$ of La$_{2-x}$Sr$_x$CuO$_4$

In Fig. 1, typical temperature dependences of $C_{el}$, obtained by using the phonon terms $C_{Ni}^{ph}$ and $C_{Zn}^{ph}$, are shown with a $C_{el}/T (=\gamma)$ vs. $T$ plot. The $\gamma - T$ plot for $C_{Zn}^{ph}$ shows a seeming anomaly around 15 K, and is severely distorted over the temperature range examined. On the other hand, the $\gamma - T$ plot for $C_{Ni}^{ph}$ shows no anomaly around 15 K, and shows a plausible $T$-dependence of $\gamma$ below and above $T_c$. These results imply that Zn impurity will seriously change the phonon term $C_{ph}$, whereas the influence of Ni is very small. It has also been reported for La214 that the Zn- and Ni-impurity effects on the structural phase transition from the tetragonal phase to the orthorhombic one at temperature $T_d (\gg T_c)$ are contrasting with each; $T_d$ is little influenced by doping with Ni whereas it is enhanced appreciably by doping with Zn.\textsuperscript{27} These facts mean that the nature of the phonon system is largely modified by doping with Zn.

In high $T_c$ cuprates, it has been clarified that Zn impurity with no 3d-spin disturbs the AFM Cu-3d-spin correlation around Zn more seriously than Ni-impurity with 3d-spin.\textsuperscript{28–31} Since the AFM 3d-spin correlation couples with B$_{2u}$ phonon modes, as has been reported in neutron scattering experiments on high-$T_c$ cuprates,\textsuperscript{32} we can conjecture that the nature of the phonon system will be significantly modified by the serious disturbance of 3d-spin correlation caused by doping with Zn. This conjecture is consistent with the fact that the Zn-impurity effect on $C_{ph}$ becomes less evident in the highly-doped $x = 0.22$ sample, where the AFM spin correlation is weakened to a large extent, as seen in Fig. 1.

3.2 Superconducting Anomaly and Pseudogap Behavior in $C_{el}$

Figure 2 shows the $p$ dependence of $C_{el}$, obtained by subtracting the phonon term $C_{Ni}^{ph}$ from $C_{total}$, with a $C_{el}/T (=\gamma)$ vs. $T$ plot. The SC anomaly appears clearly in the $\gamma$ vs. $T$ plot for all samples investigated. The anomaly for $x = 0.22$ is very similar, in both shape and size, to the BCS result for a $d$-wave superconductor over a wide $T$ range except just below and above $T_c$, where the SC critical fluctuation effects become evident. On the other hand, the anomaly for $x < 0.2$ becomes rather different from the BCS result; namely, the $\gamma$ value reaches the peak value at $T \approx T_c$ and tends to decrease more rapidly at $T < T_c$. In particular, the $\gamma$ value for $x \leq 0.1$ decreases very steeply at $T < T_c$, leaving a sharp peak at $T_c$. The rapid decrease of $\gamma$ at $T < T_c$ implies that the en-
ergy gap is developed to a large extent just below $T_c$, as observed in tunneling spectroscopy measurements, and quasiparticle excitations are rather suppressed even in the neighborhood of $T_c$. On the other hand, the $\gamma$ value tends to be enhanced at temperatures just above $T_c$, in particular, in samples with small doping levels. Some inhomogeneity effect, leading to the distribution of $T_c$, could cause the enhancement of $\gamma$ at $T > T_c$. In this case, the peak anomaly at $T_c$ would be rounded owing to the distribution of $T_c$. However, this is not the present case, because the peak anomaly at $T_c$ is very sharp even in $x = 0.08$ and 0.1 samples whose $\gamma$ enhancement at $T > T_c$ is most evident among samples examined. This fact means that the enhancement of $\gamma$ at $T > T_c$ is not due to an inhomogeneous effect but due to the strong SC critical fluctuation effect.

It should be noted here that the $\gamma - T$ curve for $x \lesssim 0.20$ shows a small, broad bump at a certain temperature $T' (> T_c)$ in the normal state, and the $\gamma$ value gradually decreases at $T < T'$. The temperature $T'$ increases with lowering $p (= x)$, and the decrease of $\gamma$ at $T < T'$ becomes evident. The decrease of $\gamma$ at $T < T'$ implies that a small pseudogap around the Fermi energy $E_F$ will start to evolve around $T'$ and suppress the density of states (DOS) at $E_F$, $N(0)$. As has been reported, $T'$ is in agreement with the mean-field critical temperature $T_{co} \sim 2\Delta_0/4.3k_B$ (Fig. 3), where the maximum gap $\Delta_0$ was measured in tunneling spectroscopy experiments on La214. The in-plane resistivity and the uniform susceptibility of La214 also show small anomalies around $T_{co}(\sim T')$, as in Bi2212.

Recently, it was reported for La214 and Bi$_2$Sr$_2$-$_y$La$_y$CuO$_6$ that the Nernst signal enhanced by vortices or vortex-like excitations extends to temperatures well above $T_c$. The anomalous Nernst signal has been interpreted in terms of strong fluctuations between the pseudogap state and $d$-wave SC condensates. It is worthwhile to point out the fact that both the small, broad bump in the $\gamma - T$ curve and the enhanced Nernst signal appear in La214 samples for $x \lesssim 0.2$, and the temperature $T'$ ($\sim T_{co}$) exhibiting the bump in the $\gamma - T$ curve roughly correlates with the onset temperature of the enhanced Nernst signal $T_{Nernst}^{on}$, as shown in Fig. 3. Such a correlation between $T'$ and $T_{Nernst}^{on}$ confirms that the anomalous Nernst signal will be intimately related to the development of the small pseudogap in the normal state.

### 3.3 Superconducting Condensation Energy of La$_{2-x}$Sr$_x$CuO$_4$

The SC condensation energy $U(0)$ can be evaluated by integrating the entropy difference $S_n - S_s$ between $T = 0$ and $T_c$

$$U(0) = \int_0^{T_c} (S_n - S_s) dT,$$  

where the subscripts $s$ and $n$ stand for the SC and hypothetical normal states at $T < T_c$, respectively. Given both $\gamma_s$ and $\gamma_n$ as a function of $T$, we can obtain the entropy $S_s$ and $S_n$ by executing the integration

$$S_{s,n}(T) = \int_0^T \gamma_{s,n} dT,$$

and evaluate the condensation energy $U(0)$ using eq. (1). In the present system, the SC critical fluctuation effect is so strong that we have to take the upper limit of the integration of eq. (1) to be $T_{sf} (> T_c)$, instead of $T_c$, where $\gamma_n (T > T_c)$ begins to increase on account of the SC critical fluctuation effect.
Fig. 5. Doping-level dependence of $\gamma_n(0)$ for La$_{2-x}$Sr$_x$CuO$_4$. The $\gamma_n(0)$ for non-superconducting samples with $x > 0.26$ were obtained from the conventional $C/T$ vs. $T^2$ plots. The open rhombus represents the density of states at $E_F$, $N(0)_{\text{AIPES}}$, reported for angle-integrated photoemission spectroscopy measurements on La$_{2-x}$Sr$_x$CuO$_4$. The open circle represents $N(0)_{\text{KS}}$ estimated from the drop of the Knight shift below $T_c$. The $N(0)_{\text{AIPES}}$ and $N(0)_{\text{KS}}$ are normalized with $\gamma_n(0)$ at $x \approx 0.22$.

Fig. 6. Superconducting condensation energy $U(0)$ for La$_{2-x}$Sr$_x$CuO$_4$. Experimental data (♦) obtained by executing the integral of eq. (1) are shown, together with the calculated values (×) given by eq. (2) for the experimental values of $\gamma_n(0)$ and $\Delta_0$. The $U(0)$ (□) calculated by substituting $\Delta_{\text{eff}}^n = \beta p \Delta_0$ for $\Delta_0$ in eq. (2) are also shown. The condensation energy $U(0)$ was also calculated for $\Delta_0(+)$ and $\Delta_{\text{eff}}^n(0)$ using $\gamma'_n = \gamma_n(0) - \gamma_0$ instead of $\gamma_n(0)$. The dotted line shows the $p$ dependence of $T_c$.

In the present study, to determine the $T$ dependence of $\gamma_n$ for the hypothetical normal state, we extrapolated the data of $\gamma_n$ measured at $T > T_{sf}$ down to below $T_{sf}$, as shown in Fig. 4. In the highly-doped $x = 0.22$ sample, since $\gamma_n$ is almost constant at $T > T_{sf}$, we can safely extrapolate the data down to $T = 0$. The hypothetical normal value $\gamma_n(T < T_{sf})$ thus obtained satisfies the constraint on the second order phase transition; namely, $S_n(T_{sf}) = S_n(T_{sf})$. This constraint is often called “the entropy balance”, because excess and deficit areas of the $\gamma_n$ vs. $T$ curve, compared with $\gamma_n(T < T_{sf})$, must be equal with each other to satisfy the condition $S_n(T_{sf}) = S_n(T_{sf})$. On the other hand, the $\gamma_n$ value for $x \lesssim 0.2$ becomes temperature dependent at $T > T_{sf}$ on account of the small pseudogap formation, so we extrapolated the data of $\gamma_n(T > T_{sf})$ down to below $T_{sf}$ using a declining straight line to satisfy the entropy balance (Fig. 4). Furthermore, in samples for $x \leq 0.1$, we took $\gamma_n(T < T_{sf})$ to be constant at $T < 0.3T_c$ because $\gamma_n$ tends to be saturated at $T < 0.3T_c$ in these samples. In Fig. 5, the extrapolated value of $\gamma_n(T < T_{sf})$ at $T = 0$, $\gamma_n(0)$, is shown against $p (= x)$. The $p$ dependence of $\gamma_n(0)$ is in agreement with that of $N(0)$ reported in photoemission spectroscopy measurements on La214 performed by Ino et al. We can also obtain the information about $\gamma_n(0)$ from the $T$ dependence of the NMR Knight shift at $T \lesssim T_c$, because the Knight shift drops below $T_c$ when the Fermi surface is removed by the formation of the spin-singlet pairing gap. The $p$ dependence of $N(0)$, obtained from the data of the Cu NMR Knight shift reported by Ohsugi et al. for La214, also agrees with that of $\gamma_n(0)$, as shown in Fig. 5.
ing process of $\gamma_n$ ($T > T_{\text{sf}}$) down to below $T_{\text{sf}}$. Thus we calculated the condensation energy $U(0)$ using the data of $\gamma_n(T)$ and the extrapolated $\gamma_n(T < T_{\text{sf}})$, and plotted the result against $x$ in Fig. 6.

As seen in Fig. 5, $\gamma_n(0)$, namely $N(0)$, decreases rapidly at $x \lesssim 0.2$. The decreases of $N(0)$ at $x \lesssim 0.2$ result, in large part, from the downward shift of the flat band from $E_F$ near $(p_m, \pi)$ and $(0, p_m, \pi)$, i.e. the large pseudogap, as was demonstrated by ARPES measurements on La214.11 The flat band, one of the striking features of the electronic structure in high-$T_c$ cuprates, makes a large contribution to DOS, and so the downward shift of the flat band from $E_F$ leads to a large reduction of $N(0)$. Furthermore, since the small pseudogap as well as the large pseudogap grows in La214 samples for $x \lesssim 0.2$,8 as mentioned above, the development of the small pseudogap at $T < T'$ ($\sim T_{\text{sf}}$) also contributes to the reduction of $N(0)$, i.e., $\gamma_n(0)$ for $x \lesssim 0.2$.

The SC condensation energy $U(0)$ can be given approximately by

$$U(0) \approx \frac{\alpha}{2} N(0) \Delta_0^2 \approx \frac{2.1 \times 10^{-5}}{2} \alpha \gamma_n(0) \Delta_0^2 [\text{eV}]$$

for a $d$-wave superconductor ($\alpha \approx 0.4$) with a nearly flat DOS.41 The expression can be expected to hold good in any reasonable models for the $d$-wave superconductivity.42 We calculated the condensation energy $U(0)$ using eq. (2) for $\gamma_n(0)$ ($J/K^2$ mole) (Fig. 5) and $\Delta_0$ ($\text{eV}$) determined in tunneling experiments on La214 (Fig. 3, 4, 5).30,37,38

For the $x = 0.22$ sample, whose SC anomaly of $C_{\text{el}}$ is of typical BCS type (the inset of Fig. 2), $\Delta_0$ was estimated from $T_c$ using the BCS relation $2\Delta_0 = 4.3k_B T_c$ for $d$-wave superconductors.35 In the present study, since the value of $\gamma_n$ at $T = 0$, estimated from the $C_{\text{el}}/T$ vs. $T^2$ plot, shows a small residual value ($\gamma_0$) for $x \lesssim 0.14$, we calculated the condensation energy $U(0)$ for both $\gamma_n(0)$ and $\gamma_n' = \gamma_n(0) - \gamma_0$. The condensation energy $U(0)$ thus calculated is in good agreement with the experimental result for the overdoped $x = 0.22$ sample, as seen in Fig. 6. On the other hand, the calculated value for $x \lesssim 0.2$ is quite different from the experimental result.

The serious disagreement between experimental and calculated values of $U(0)$ at $x \lesssim 0.2$0 seems to result from the modification of the nature of the pairing gap caused by the pseudogap formation. In fact, it has been pointed out that $T_c$ roughly scales with $\kappa p \Delta_0$ ($\kappa \sim 1.7$), instead of $\Delta_0$, over a wide $p$ range in the pseudogap regime,9,21 although a $d$-wave gap is completed over the entire original Fermi surface at $T \ll T_c$, with the maximum value $\Delta_0$ at antinodal points near $(\pi, 0)$ and $(0, \pi)$. This indicates that the SC energy scale determining $T_c$ will be proportional to $p \Delta_0$ in the pseudogap regime; namely, $k_B T_c \propto p \Delta_0$. The relation $k_B T_c \sim p \Delta_0$ was phenomenologically predicted first by Lee and Wen for the underdoped spin-gap (small pseudogap) regime although they took $\Delta_0$ to be almost independent of $p$ there, and then discussed microscopically on the basis of the SU(2) slave-boson model.43,44 Recently Tesanovic has also discussed the relation in terms of vortex-antivortex pairs.45 Thus we recalculated $U(0)$ by substituting the SC energy scale $\beta p \Delta_0$ ($\beta = 4.5$) for $\Delta_0$ in eq. (2), and plot the calculated result in Fig. 6. The newly calculated values reproduce the experimental result of $U(0)$ very well over the whole $p$-range examined.

The new SC energy scale $\beta p \Delta_0$ ($\beta = 4.5$), introduced in the above analysis for $U(0)$, becomes smaller than $\Delta_0$ at doping levels for the pseudogap regime ($p \lesssim 0.2$). However, it returns back to $\Delta_0$ around the doping level $p(x) = 0.22$, where no pseudogap behavior appears and the SC properties are of the BCS type. Furthermore if we represent the SC energy scale $\beta p \Delta_0$ ($\beta = 4.5$) as $\Delta_0^{\text{eff}}$, the phenomenological relation $k_B T_c \sim \kappa p \Delta_0$ ($\kappa \sim 1.7$) can be rewritten as $2\Delta_0^{\text{eff}} \sim 5.3k_B T_c$, which is similar to the BCS result for a $d$-wave superconductor. These results suggest that $\Delta_0^{\text{eff}} = \beta_p \Delta_0$ ($\beta = 4.5$) may correspond to the maximum value of the effective SC gap in the pseudogap regime. In subsection 3.5, $\Delta_0^{\text{eff}}$ will be discussed in terms of the shrinkage of the coherent part of the pairing gap.

3.4 Comparison of $U(0)$ between La214 and Other Systems

Here we compare the present data of $U(0)$ with those reported by Loram’s group on $Y_{0.8}Ca_{0.2}Ba_2Cu_3O_6+\delta$ (Y123(Ca)) and Bi2212.1,2 Loram’s group estimated the condensation energy $U(0)$ on the assumption that the hypothetical normal value of $\gamma_n$, $\gamma_n(T < T_J)$, is strongly $T$-dependent in samples for $p \lesssim 0.19$ and reduced to zero at $T = 0$.3,5 This is because the $T$ dependence of $\gamma_n$ in the normal state ($T > T_c$) could be reproduced by assuming the existence of a $T$-independent gap structure near $E_F$ which would reduce $\gamma_n$ to be 0 at $T = 0$. However, since the existence of such a gap structure has not been clarified yet experimentally,15,33 we presumed a simple $T$-dependence for $\gamma_n(T < T_c)$ with a finite value at $T = 0$, as mentioned in subsection 3.3. This discrepancy in the $T$ dependence of $\gamma_n(T < T_c)$ may prevent us from comparing the present data with those of the Loram group. However, we can confirm phenomenologically that $U(0)$ is almost independent of the $T$-dependence of $\gamma_n(T < T_c)$ as long as the entropy balance holds between $T$ dependences of $\gamma_n$ and $\gamma_n(T < T_c)$. This allows us to make a comparison between the present data of $U(0)$ and the Loram group’s, because the entropy balance is satisfied in both groups’ analyses.

In Fig. 7, both $U(0)/U_m(0)$ and $T_c/T_c^{\text{max}}$, normalized with the maximum values of $U(0)$ and $T_c$, respectively, are shown for La214 and Bi2212 as a function of $p/p_m$, where $p_m$ is the doping level at which $U(0)$ takes the maximal value $U_m(0)$. On the other hand, the original data of $U(0)$ and $T_c$ for Y123 (Ca) are plotted against oxygen content $\delta$. Instead of $p$, which has one-to-one correspondence with $p$. In Fig. 7, we convert $\delta$ into $p$ so that both peaks of $U(0)/U_m(0)$ vs. $\delta$ and $T_c/T_c^{\text{max}}$ vs. $\delta$ curves for Y123(Ca) should agree with the corresponding peaks for La214, respectively. The $T_c/T_c^{\text{max}}$ vs. $p/p_m$ curves thus obtained for Y123(Ca) and Bi2212 are in agreement with that for La214 over a wide $p$ range except around $p = 1/8$ in La214 and the Ortho-I ($T_c \sim 60K$) phase in Y123 where the $T_c$ becomes less dependent on $p(\delta)$. The values of $U(0)/U_m(0)$ for Y123(Ca) and Bi2212 show $p$ dependences similar to that
for La214, though \( \frac{U(0)}{U_m(0)} \) tends to decrease faster at \( p < p_m \) in Y123(Ca) and Bi2212 than in La214. The similarity in both \( \frac{U(0)}{U_m(0)} \) vs. \( p/p_m \) and \( T_c/T^{\text{max}} \) vs. \( p/p_m \) curves among La214, Y123(Ca) and Bi2212 implies that there exists no essential difference in the SC transition mechanism among them, though \( T_c \) varies from system to system.

The condensation energy \( U(0) \) reported by Loram’s group for La214 is also plotted in Fig. 7, where \( p \) is normalized so that the \( T_c \) vs. \( p \) curves coincide with each other. The overall \( p (= x) \) dependence of \( U(0) \) is qualitatively consistent, but the Loram data are smaller in the pseudogap regime \( (x \lesssim 0.2) \) than the present data. The difference of \( U(0) \) can be attributed to the difference between the raw data of \( C_{el} \) measured by Loram’s group and ours, because both groups adopted different ways to estimate the phonon part \( C_{ph} \). One of the differences is that Loram’s group adopted Zn impurity to destroy the superconductivity in the standard sample on the overdoped sample, whereas Ni impurity was used in the present study.

### 3.5 Coherent Pairing Gap

In high-\( T_c \) cuprates with low doping levels, the nodal parts of the Fermi line near \( (\pi/2, \pi/2) \) dominate the in-plane transport; i.e., in-plane mobility of carriers on the nodal parts of the Fermi surface is much higher than on the antinodal parts near \( (\pi, 0) \) and \((0, \pi)\). This is typically demonstrated by the following observation: the Fermi surface is truncated near \( (\pi, 0) \) and \((0, \pi)\) by the small pseudogap formation at \( T \lesssim T_{co} \), leaving the so-called nodal Fermi arcs, but the in-plane resistivity remains almost unchanged. The pairing of in-plane carriers with high in-plane mobility is expected to play a crucial role in making the collective pair motion coherent. Therefore, the carriers on the nodal Fermi arcs are expected to drive the SC phase transition when they start to form pairs. This directs our attention to the possibility that in the pseudogap regime the pairing gap formed over the nodal Fermi arcs will function as the coherent pairing gap, namely, the effective SC gap which dominates \( T_c \) and \( U(0) \).

The hypothetical normal state value \( \gamma_0(0) \), namely \( N(0) \), will give us information about the linear dimension of the Fermi arc at \( T \sim T_c \), because it will be proportional to the dimension of the arc if the local DOS, \( n(0) \), is constant over the Fermi line. As seen in Fig. 5, \( \gamma_0(0) \) and \( N(0) \) show a tendency for linear \( p \) dependence below \( x \sim 0.2 \), suggesting that the dimension of the nodal Fermi arc will be roughly proportional to \( p (= x) \) there.
Furthermore, because of the distortion of the gap function caused by the higher harmonic in high-$T_c$ cuprates, the pairing gap at $T < T_c$ has a rather linear dispersion along the Fermi line in a slightly overdoped region as well as the underdoped one.\textsuperscript{51} Thus, if the dimension of the nodal Fermi arc at $T$ proportional to $p (\sim x)$ as suggested above, the gap size at the edge of the nodal arcs is roughly proportional to $p \Delta_0$, as shown schematically in Fig. 8. This gives an explanation for the maximum value of the effective SC gap $\Delta_{\text{eff}}^0 = \beta p \Delta_0 (\beta = \text{const.})$, introduced in the present analysis for $U(0)$. Such a picture is almost the same as the scenario proposed by Wen et al. to explain the relation $k_B T_c \sim p \Delta_0$ microscopically on the basis of the SU(2) slave-boson model, which predicts that a hole pocket with no shadow band at $\omega = 0$ will appear near $(\pi/2, \pi/2)$, namely the nodal Fermi arc, in the spin gap regime.\textsuperscript{44, 52}

It is noteworthy here that the results of the electronic Raman scattering measurements on high-$T_c$ cuprates are consistent with the present proposition that the coherent part of the pairing gap shrinks toward nodal points as the small pseudogap grows. Electronic Raman scattering measurements are expected to give more direct information about the coherent pairing gap than ARPES and tunneling spectroscopy measurements, because the electronic Raman response to superconductors is relevant to the coherence factor with even parity.\textsuperscript{53} It has been reported for La$_{214}$ as well as Bi2212 and Y123 that the $B_{2g}$ Raman continua, weighing out the antinodal parts of the Fermi surface, are indicative of marked suppression of the coherent pairing gap in the underdoped region, whereas the $B_{2g}$ continua, weighing out nodal parts, indicate the existence of the coherent pairing gap $\Delta_{\text{coh}}$.\textsuperscript{53-56} Such results in Raman scattering experiments mean that the coherent pairing gap will be located near nodal points, at least, in the underdoped region, which is consistent with the present proposition. The coherent pairing gap $\Delta_{\text{coh}}$ may correspond to the effective SC gap determining $T_c$ and $U(0)$.

The present proposition is also consistent with recent ARPES results measured by Zhou et al. on underdoped La$_{214}$ samples; the well-defined quasiparticle peak exists on the nodal part of the Fermi line, but away from the nodal part it becomes broader and fades out rather abruptly.\textsuperscript{22}

4. Summary

The electronic specific heat $C_e$ was systematically measured on La$_{2-x}$Sr$_x$CuO$_4$ (La$_{214}$) to reexamine the development of the small pseudogap in the normal state ($T > T_c$) and the superconducting condensation energy at $T=0, U(0)$. The results obtained in the present study are summarized as follows.

1) The small pseudogap behavior appears in the $\gamma - T$ plot at temperature $T' \sim T_{\text{co}}$; the $\gamma$ value, namely $N(0)$, shows a small bump at $T'$ and is progressively suppressed at $T < T'$. The temperature $T'$ $\sim T_{\text{co}}$ roughly correlates with the onset temperature of the enhanced Nernst signal, reported by Wang et al. on La$_{214}$.\textsuperscript{24}

2) We confirmed that $U(0)$ was markedly reduced in the pseudogap regime. The reduction of $U(0)$ can be quantitatively explained by using the effective SC energy scale $\Delta_{\text{eff}}^0 = \beta p \Delta_0 (\beta = 4.5)$, instead of $\Delta_0$, and the DOS associated with the nodal Fermi surface which is removed by the pairing gap formation at $T < T_c$.

3) To explain the effective SC energy scale $\Delta_{\text{eff}}^0$ in the pseudogap regime, we pointed out the Possibility that the pairing gap formed over the nodal Fermi arcs at $T < T_c$ will play a role as the coherent pairing gap, namely, the effective SC gap determining $T_c$ and $U(0)$.

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