Rapid prediction of multi-directionality of polished surface topography based on angular spectrum

Qing-Hui Wang1 · Xiao-Lin Fang1 · Hai-Long Xie1,2 · Jing-Rong Li1 · Zhao-Yang Liao3

Received: 17 May 2022 / Accepted: 29 July 2022 / Published online: 8 September 2022
© The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2022

Abstract
The multi-directionality of polished surface topography (PST) is a key index to characterize the polished surface quality and surface integrity, and its prediction before actual machining is of great significance to the toolpath planning, verification, and optimization. However, most related works reported thus far focus mainly on the material removal modeling but few on the prediction of multi-directionality. Hence, this paper proposes a rapid method to precisely predict the multi-directionality of the PST for the pad-polishing process of freeform surfaces. With this method, a pressure distribution model and a material removal profile (MRP) model are first established, in which the MRP is founded subject to the quadratic function distribution. Then, to avoid time-consuming integral operation in the MRP model, an artificial neural network is developed to fit the quadratic function of MRP. With this model, a multi-directionality prediction algorithm is further proposed based on the angular spectrum of the PST. Simulation and experimental studies have shown that the proposed method can predict the multi-directionality of the PST with very high accuracy and efficiency for freeform surface polishing, showing great application potential in promoting the efficiency of toolpath planning and optimization.

Keywords Robot polishing · Material removal modeling · Multi-directionality prediction · Toolpath planning and optimization

1 Introduction
High-precision freeform parts are now widely used in aerospace, automobile, optical precision instruments, and other high-end equipment fields. To achieve the requirements of high profile accuracy and low surface roughness, these parts are finished extensively using the polishing process to remove tool marks and reduce the residual stress left by rigid machining processes, such as milling, turning, and grinding [1–3]. The multi-directionality (also called isotropy) of polished surface topography (PST), which is a key index to characterize the polished surface quality and surface integrity, has a significant impact on surface functional properties such as lubrication, sealing, friction, and corrosion resistance of the polished surface. The multi-directionality means that the polished surface profiles in all different directions should have the same or similar spatial frequency characteristics. A large number of studies have demonstrated that PST with excellent multi-directionality is of great help to improve the machined surface quality, functional properties, and fatigue life of parts [4, 5]. Conversely, anisotropic PST with unidirectional texture will bring serious mid-spatial frequency (MSF) error and will damage the surface quality seriously [4, 5].

Polishing toolpath and parameters are the most critical factors that directly determine the PST. In the past decades, the scientific community has devoted a lot of research to improving toolpath planning efficiency. Rososhansky and Xi [6] presented a concept of a two-dimensional contact area map and used it to generate a scanning polishing toolpath. Zhao et al. [7] extended the concept of the two-dimensional contact area map to three-dimension and generated a scanning polishing toolpath
that can cover the freeform surface uniformly. Although the scanning toolpath has the advantages of simple computation and better control of the coverage uniformity, it will inevitably leave an unfavorable unidirectional polishing texture on the workpiece surface, which will cause serious MSF error. To facilitate obtaining isotropic PST, many researchers proposed various types of pseudo-random polishing toolpaths with the characteristics of good multi-directionality, randomness, and continuity. Wang et al. [8] proposed a unicursal random maze polishing toolpath. Takizawa and Beaucamp [9] developed a novel circular pseudo-random polishing toolpath. Zhao et al. [4] presented a six-directional pseudo-random consecutive unicursal polishing toolpath. Dong and Nai [5] advanced a random fractal-like polishing toolpath. Li et al. [10] developed a pseudo-random toolpath with the consideration of polishing efficiency and toolpath smoothness. Experimental results have demonstrated that these pseudo-random toolpaths can greatly improve the multi-directionality characteristics of the PST, and are of great help to improve the polished surface quality. Unfortunately, these methods reviewed above are only applicable for quasi-planar and quasi-spherical surfaces but not for the freeform surface parts. Motivated by this problem, Beaucamp et al. [11] presented an effective method to extend the circular-random path to freeform surface parts. In the authors’ previous work [12], a trochoidal toolpath generation method was also advanced for polishing freeform surface parts with the global control of material removal (MR) distribution. However, these methods consider more about the uniform coverage of polishing toolpath on freeform surfaces while the multi-directionality of PST has not been studied. As a result, it is still very difficult to plan appropriate polishing toolpath and parameters to obtain isotropic PST for freeform surface parts. Traditionally, the determination of appropriate polishing toolpath and parameters mainly relied on design-of-experiment (DoE) factorial trials, in which large numbers of physical polishing experiments have to be conducted, and then measurement experiments need to be carried out to evaluate and compare the multi-directionality of the PST obtained by different toolpaths and parameters. The whole process is not only labor-intensive and time-consuming but also will cause tremendous waste of raw materials and energy.

As a promising alternative solution, simulation of the PST under a given toolpath and parameters can predict a part’s machining effect before actual machining, which is of great significance to toolpath planning, verification, and optimization [13]. The PST is generated essentially by the cumulative MR. Therefore, to reveal the forming mechanism of PST, significant amounts of research have been dedicated to the modeling and simulation technology of polishing MR. Zhang et al. [14] first established a contact pressure distribution model for cylindrical and spherical polishing tools based on the Hertz contact theory and then proposed a local material removal profile (MRP) model based on the Archard wear law of abrasive grains. Yang and Lee [15] established a local MRP model for polishing aspherical parts also based on the Hertz contact theory and Preston equation. Similarly, Fan et al. [16, 17] developed an MR model with the consideration of the inclination angle and declination angle of polishing tools. Although the Hertz contact theory is a very elegant and effective contact modeling approach, it is not applicable for these polishing processes that do not satisfy the Hertz contact theory, such as the pad-polishing using a tilted elastic disk in this paper. To address this problem, Guiot et al. [18] used finite element simulation to calculate the contact pressure between the tilted polishing disk and the workpiece. It is convincing that such a method is very time-consuming due to the complex finite element simulation involved. Almeida et al. [19, 20] proposed an MR simulation method for mold polishing, which also relied on the finite element simulation. In the author’s previous work [21], an analytical model of contact pressure between the tilted polishing pad and workpiece was deduced through large numbers of finite element simulation experiments, and an MR simulation method based on the mesh model was advanced.

However, to the best of the authors’ knowledge, none of these MR modeling and simulation methods reviewed above have dedicated attention to the prediction problem of the multi-directionality of the PST. Currently, the key challenge of multi-directionality prediction lies in that the polishing MR simulation involves tremendous integration and accumulation operation. To tackle these problems, this paper proposes a rapid and accurate prediction method of the multi-directionality index of the PST for freeform surface polishing using a tilted elastic polishing disk. The rest of this paper is organized as follows. Section 2 presents a rapid computation model of global MR distribution on the entire part surface. This model can avoid the complex and time-consuming finite element simulation and integral operation effectively. In Sect. 3, a multi-directionality prediction algorithm is proposed based on the angular spectrum of the PST. Then, the experimental studies and discussions are carried out in Sect. 4. And lastly, Sect. 5 concludes the work.

### 2 Rapid computation model of global MR distribution

In this section, the contact pressure model in the pad-polishing process using a tilted elastic disk is first established by studying the mechanical property of the polishing disk. Next, an MRP model subject to quadratic function distribution is proposed. Then, to avoid the time-consuming integral operation involved in the MRP model, an artificial neural network (ANN) model is developed to predict the quadratic curve of MRP rapidly. Finally, an algorithm to compute the global MR distribution is developed.
2.1 Contact pressure model

Figure 1 illustrates the principle of the pad-polishing process using a tilted elastic disk. The disk is in contact with the cutter contact (CC) point \( P_{CC} \) of the workpiece by being inclined at an angle \( \theta \) relative to the normal direction of \( P_{CC} \), where \( L \) and \( E \) are the offset distance and press depth, respectively. Under the action of the polishing force, the polishing disk will conform to the workpiece surface and use the abrasive grains adhered to the bottom of the polishing disk to remove the workpiece material. As shown in Fig. 1, the abrasive grains adhered to the bottom of the polishing disk will conform to the workpiece surface and use the abrasive grains adhered to the bottom of the polishing disk to remove the workpiece material. As shown in Fig. 1, the abrasive grains adhered to the bottom of the polishing disk will conform to the workpiece surface and use the abrasive grains adhered to the bottom of the polishing disk to remove the workpiece material.

![Diagram of polishing process](image)

Figure 1 illustrates the principle of the pad-polishing process using a tilted elastic disk. The disk is in contact with the cutter contact (CC) point \( P_{CC} \) of the workpiece by being inclined at an angle \( \theta \) relative to the normal direction of \( P_{CC} \), where \( L \) and \( E \) are the offset distance and press depth, respectively. Under the action of the polishing force, the polishing disk will conform to the workpiece surface and use the abrasive grains adhered to the bottom of the polishing disk to remove the workpiece material. As shown in Fig. 1, the abrasive grains adhered to the bottom of the polishing disk will conform to the workpiece surface and use the abrasive grains adhered to the bottom of the polishing disk to remove the workpiece material.

In the authors’ previous work [21], the contact model between the tool and the workpiece was established based on large numbers of finite element simulation experiments. The model demonstrated that the polishing contact area can be classified into the three types shown in Fig. 1b–d. The shape of the contact area can be described by the arc circle drawn between the points \( P_a \) and \( P_b \) in Fig. 1. The arc circle drawn between the points \( P_a \) and \( P_b \) is the contact boundary between the polishing disk and the workpiece, where the deformation of the polishing tool is equal to 0. The distance between the point \( P_e \) and \( P_{CC} \) along the feed direction decides the contact length \( L \). Besides, \( L_1 (L_2) \) describes the distance from the point \( P_a \) to \( P_e (P_{CC}) \) along the feed direction. \( W \) is half of the contact width, which is defined as half of the distance between the point \( P_a \) and \( P_b \). \( R_w \) is the radius of contact boundary \( P_a \) to \( P_{CC} \) to \( P_b \).

Although a pressure distribution model has been established in the author’s previous work [21], it is computationally time-consuming due to the tremendous integral operation involved. To address this issue, a rapid computation model of contact pressure distribution that can avoid the complex integral operation is proposed in this section by studying the mechanical properties of the polishing tool. Specifically, as shown in Fig. 2, the compressive mechanical properties of the elastic polishing disk along its axis are tested by a universal testing machine shown in Fig. 2a, with which the stress–strain curve of the polishing tool when it is pressed along its axis can be obtained, as shown in Fig. 2b. It can be seen that the stress–strain curve satisfies approximately the quadratic function relationship. The strain (denoted as “\( \varepsilon \)”) is defined as the deformation (denoted as “\( p \)”) of the polishing disk along its axial direction relative to the original thickness (denoted as “\( t_{tool} \)”), and the stress (denoted as “\( p_{tool} \)”) is the pressure. Then, according to the experimental result, the stress–strain curve can be fitted into a cubic function as below and the corresponding R-square is 0.9882.

\[
\begin{align*}
  p_{tool} &= 2.204 \varepsilon^3 - 1.362 \varepsilon^2 + 0.293 \varepsilon - 0.004 \\
  \varepsilon &= \varepsilon_{t_{tool}}
\end{align*}
\]

(1)

Figure 3 shows the deformation distribution of the polishing disk along its axis when polishing convex and concave...
surfaces. It can be easily observed that the deformation of the point \( P_E \) at the bottom edge of the polishing disk is the largest. When polishing a plane, it is easy to know that the maximum deformation \(|P_EP_e|\) is equal to \( \frac{E}{\cos\theta} \). Furthermore, to calculate the maximum deformation at \( P_E \) when polishing convex surfaces, as shown in Fig. 3a, a straight line parallel to the feed direction from the CC point \( P_{cc} \) is drawn, which intersects the line \( P_EP_e \) at point \( P_1 \). Then, the length of the line segment \( P_EP_1 \) and \( P_{cc}P_1 \) can be obtained by the following formulas.

\[
|P_EP_1| = \frac{E}{\cos\theta} \tag{2}
\]

\[
|P_{cc}P_1| = \frac{|P_EP_e|}{\sin\theta} \tag{3}
\]

Next, a straight line from \( P_e \) along the normal direction of \( P_{cc} \) can be made to intersect \( P_{cc}P_1 \) at point \( P_2 \). The length of the segment \( P_1P_2 \) can be obtained as below.

\[
|P_1P_2| = |P_{cc}P_1| - L \tag{4}
\]

Then, the length of the segment \( P_eP_1 \) can be deduced as:

\[
|P_eP_1| = \frac{|P_1P_2|}{\sin\theta} \tag{5}
\]

By combining Eqs. (2) and (5), the maximum deformation of the polishing tool at the point \( P_E \) (denoted as “\( e_{P_E} \)”) can be obtained by:

\[
e_{P_E} = |P_EP_e| = |P_EP_1| - |P_eP_1| \tag{6}
\]

In the case of the concave surface shown in Fig. 3b, the maximum deformation of the polishing tool at \( P_E \) can be obtained with a similar method, and the relevant formulas are derived as follows.

\[
\begin{cases}
|P_EP_1| = \frac{E}{\cos\theta} \\
|P_{cc}P_1| = \frac{|P_EP_e|}{\sin\theta} \\
|P_1P_2| = L - |P_{cc}P_1| \\
|P_eP_1| = \frac{|P_1P_2|}{\sin\theta} \\
e_{P_E} = |P_EP_e| = |P_EP_1| + |P_eP_1| \tag{7}
\end{cases}
\]

After the maximum deformation \( e_{P_E} \) is obtained, the maximum pressure \( p_{P_E} \) at the point \( P_E \) can be obtained by substituting \( e_{P_E} \) into Eq. (1). To further calculate the pressure at other points in the contact area, as shown in Fig. 4, the radius of the polishing disk is denoted as \( R_T \). According to the author’s previous work [21], the contact pressure is symmetrically distributed with respect to \( P_{cc} \). Besides, the pressure at any point on an arc parallel to the contact boundary \( P_aP_{cc}P_b \) is
equal. Since the contact boundary \( P_a P_{CC} P_b \) is the boundary between deformation and non-deformation of the polishing pad, the contact pressure on the boundary \( P_a P_{CC} P_b \) can be regarded as 0. Furthermore, the contact pressure increases linearly along the direction \( P_{CC} P' \). Based on these rules, the pressure of any point \( P = (x, y) \) in the CL frame \( O_T x_T y_T z_T \) within the contact area can be deduced as below:

\[
p(x, y) = \frac{p_p}{L} d
\]  

where \( \frac{p_p}{L} \) is the proportional coefficient of the contact pressure that is increased linearly along the direction \( P_{CC} P' \), \( d \) is the distance from the point \( P \) to \( P' \) on the contact boundary \( P_a P_{CC} P_b \) with the same \( x \) coordinate as the point \( P \), as shown in Fig. 4.

To calculate the distance \( d \), a straight line from the point \( P' \) and parallel to the axis \( x_T \) can be made to intersect the axis \( y_T \) at point \( P'' \). Then, the distance between \( P_{CC} \) and \( P'' \) can be derived as below.

\[
|P_{CC} P''| = R_w - \sqrt{R_w^2 - x^2} \tag{9}
\]

Furthermore, the distance from the point \( O_T \) to \( P_{CC} \) can be obtained by the following formula.

\[
|O_T P_{CC}| = R_T - L \tag{10}
\]

Then, the coordinate \( y \) of the point \( P' \) can be deduced as follows.

\[
y'_{P} = |P_{CC} P'| + |O_T P_{CC}| = R_w - \sqrt{R_w^2 - x^2} + R_T - L \tag{11}
\]

After obtaining \( y'_{P} \), the distance \( d \) between \( P' \) and \( P \) can be obtained by the following formula.

\[
d = y - y'_{P} = y - \left( R_w - \sqrt{R_w^2 - x^2} + R_T - L \right) \tag{12}
\]

Finally, by substituting Eq. (12) into Eq. (8), the contact pressure at any point \( P = (x, y) \) in the contact area can be derived as follows.

\[
p(x, y) = \frac{p_p}{L} \left( y - R_w + \sqrt{R_w^2 - x^2} - R_T + L \right), -W \leq x \leq W \tag{13}
\]

### 2.2 MRP model

After the contact pressure is obtained, the polishing MR rate of the point \( P = (x, y) \) in the contact area can be expressed as below according to the famous Preston equation.

\[
dh = k_p p(x, y) v_r(x, y) dt
\]

where \( k_p \) denotes the dimensionless wear coefficient, which can be determined by experiments. \( v_r \) is the relative sliding velocity obtained by:

\[
v_r \approx v_s = \frac{\omega \sqrt{x^2 + y^2}}{2}
\]

where \( \omega \) is the rotational speed of the polishing tool. Equation (14) can be rewritten as another form of arc differentiation along the toolpath:

\[
\frac{dh}{ds} = \frac{k_p p(x, y) v_r(x, y)}{v_f}
\]

When the polishing tool feeds along a toolpath, the MRP at the CC point \( P_{CC} \) can be obtained by the following integral formula:

\[
h(x) = \int_{y_1}^{y_2} \frac{k_p p(x, y) v_r(x, y) v_f}{v_f} dy \tag{18}
\]

where \( y_1 \) and \( y_2 \) are the coordinates \( y \) of two points generated by the intersection of the contact contour and a straight line passing through the point \( P \) in the contact area and parallel to the axis \( y \), as shown in Fig. 5. \( y_1 \) can be obtained by Eq. (11), and \( y_2 \) can be obtained by the following formula.

\[
y_2 = \sqrt{R_T^2 - x^2} \tag{19}
\]
By substituting Eqs. (13) and (15) into Eq. (18) the MRP perpendicular to the feed direction at the CC point $P_{CC}$ can be finally derived as follows.

$$h(x) = k_p \frac{p_p}{L} \frac{\omega}{v_f} \int_{y_1}^{y_2} \left( y - R_W + \sqrt{R_W^2 - x^2 - R_T + L} \right) \sqrt{x^2 + y^2} dy$$

(20)

If Eq. (20) is directly used to calculate the MRP, it will be computationally time-consuming due to the complex integral operation involved. To further reveal the distribution law of the MRP, this paper designs six groups of polishing parameters corresponding to different CC points, where $k_n$ denotes the normal curvature of the workpiece perpendicular to the feed direction at the CC point, and $E$ and $\theta$ represent the press depth and the tilted angle of the tool, respectively, as shown in Table 1. Then, the six groups of parameters are substituted into Eq. (20) to calculate the MRPs. It can be easily observed that the computational results shown in Fig. 6a are approximately subject to the quadratic function distribution. To verify this point of view, several discrete points of the MRP curve are taken and the quadratic function $h(x) = Ax^2 + Bx + C$ is used to fit these points, as shown in Fig. 6b. The fitting results are shown in Table 2, where the $R$-square corresponding to each group of data is very high, all greater than 0.99, which demonstrates that the MRP model in Eq. (20) is indeed approximately subject to a quadratic function.

### 2.3 ANN-based MRP computation model

To further simplify the MRP computation, this paper proposes an ANN model to predict the quadratic function $h(x) = Ax^2 + Bx + C$ of MRP under different polishing parameters. With this model, the complex and time-consuming integral operation actually can be eliminated. The developed ANN model consists of an input layer, a hidden layer, and an output layer, as shown in Fig. 7. Its input layer is composed of the polishing parameters affecting the MR, mainly including the normal curvature $k_n$ of the workpiece perpendicular to the feed direction at the CC point, the press depth $E$, the tilted angle $\theta$ of the polishing tool, the feed speed $v_f$, and the rotational speed $\omega$ of the polishing tool. The output layer includes three coefficients $A$, $B$, and $C$ of the quadratic function. To train the ANN model, 1000 groups of polishing parameters are first generated randomly in this paper. Next, they are input into Eq. (20) to calculate the MRP and then are fitted to obtain the corresponding quadratic function coefficients. Finally, these 1000 groups of data pairs are put into the MATLAB ANN toolbox for training.

It can be seen from Fig. 8 that the regression coefficients of the ANN in the training, validation, and testing process are all greater than 0.999, which demonstrates that the trained ANN has a very good fitting effect. To further verify the prediction accuracy of the ANN model, the six groups of polishing parameters in Table 1 are input to the trained ANN for testing. The testing result is shown in Fig. 9, where the lines with red, green, and blue colors represent the coefficients $A$, $B$, and $C$ of the quadratic function respectively. It can be seen that although there are some small disturbances between the predicted value and standard value, their change trends are in good agreement with each other. Based on this, it can be demonstrated that

| $k_n$  | 0.000 | 0.000 | −0.010 | −0.020 | 0.005 | 0.010 |
|-------|-------|-------|--------|--------|-------|-------|
| $E$(mm) | 1.2 | 2.0 | 1.0 | 1.5 | 2.0 | 1.2 |
| $\theta$(°) | 12.0 | 15.0 | 9.0 | 12.0 | 12.0 | 12.0 |
| $R_T$(mm) | 25 | 25 | 25 | 25 | 25 | 25 |
| $\omega$(r/min) | 2000 | 2000 | 2000 | 2000 | 2000 | 2000 |
| $v_f$(mm/s) | 5 | 5 | 5 | 5 | 5 | 5 |
the developed ANN model can indeed fit the quadratic function curve of MRP accurately.

### 2.4 Computation of global MR distribution

As shown in Fig. 10, a polishing toolpath can be regarded as connected by a series of small line segments, and the polishing process along a toolpath can be regarded as the process of the polishing tool sweeping along each line segment successively. For each CC point $P_k$ with given polishing parameters, its width of contact ribbon (denoted as “$2W_k$”) can be calculated by using the authors’ previous method. To calculate the MR depth of any point $P$ on the workpiece surface, as shown in Fig. 10, a line segment $P_kP_{k+1}$ perpendicular to the toolpath segment $P_kP_{k+1}$ can be made through $P_k$. The length of the segment $P_kP_{k+1}$ equals to $2W_k$. In this paper, the quadrilateral contact region $P_kP_{k+1}P_{k+1}P_k$ is defined as the polishing influence area of the CC point $P_k$.

The key step in calculating the MR depth of the point $P$ on the workpiece surface is to determine which polishing influence area of the point $P$ is located inside. If the point $P$ is located in the polishing influence area $P_kP_{k+1}P_{k+1}P_k$ of the CC point $P_k$, a certain amount of MR will be contributed to $P$ when the tool moves along the line segment $P_kP_{k+1}$. The contributed MR depth (denoted as $h(x)^k$) can be obtained by the following steps.

**Step 1:** Substitute the polishing parameters of the CC point $P_k$ into the trained ANN model, and the quadratic function $h(x)^k = Ax^2 + Bx + C$ describing the MRP of the CC point $P_k$ can be obtained.

**Step 2:** Calculate the shortest distance $d_k$ from the point $P$ to the line segment $P_kP_{k+1}$, and then substitute $x = d_k$ into the quadratic function $h(x)^k = Ax^2 + Bx + C$. The calculated $h(d_k)^k$ is the MR depth of point $P$ contributed by the CC point $P_k$.

It is worth noting that the point $P$ may be located in the polishing influence areas of multiple CC points simultaneously. In this case, its total MR depth $h(x)_P$ contributed by these polishing influence areas can be accumulated with the following formula:

$$h(x)_P = h(x)^{M_1}_P + h(x)^{M_2}_P + \cdots + h(x)^{M_j}_P + \cdots + h(x)^{M_D}_P$$

where $h(x)^{M_j}_P$ represents the MR depth of the point $P$ contributed by the CC point $M_j$.

### 3 Multi-directionality prediction algorithm based on angular spectrum

Although the MR model proposed in Sect. 2 can realize the rapid computation of the global MR distribution on the entire part surface, the multi-directionality of the PST
cannot be evaluated effectively with it. To address this issue, this section characterizes the multi-directionality of PST by integrating the angular spectrum, and a multi-directionality prediction algorithm is proposed.

### 3.1 Characteristic of multi-directionality based on angular spectrum

Compared with the evaluation indexes of surface roughness such as arithmetic mean deviation, peak-valley value, and root mean square value of the polished surface profile, the power spectral density (PSD) has the advantage of that contains both the profile height information and the spatial frequency information of PST simultaneously, which makes it able to reveal the surface roughness, waviness, and figure error of the PST simultaneously [22]. Hence, in this paper, the PSD is integrated to characterize the PST. After obtaining the global distribution of MR on the whole part surface using the computation model proposed in Sect. 2, the PST can be obtained by subtracting the corresponding MR amount from the surface topography of the part before polishing. For a given region of PST, its two-dimensional (2-D) PSD is defined as the amplitude square of polished surface profile per unit area [23]:

\[
PSD_2(f_X, f_Y) = \frac{1}{L_X L_Y} \int_{-\frac{L_Y}{2}}^{\frac{L_Y}{2}} \int_{-\frac{L_X}{2}}^{\frac{L_X}{2}} h(X, Y) e^{-i2\pi f_X X + i2\pi f_Y Y} dX dY
\]

where \(h(X, Y)\) is the height of PST. Moreover, \(f_X, f_Y, L_X\) and \(L_Y\) represent the sampling frequency and the length of the sampling area in the X and Y directions respectively.

Figure 11a shows the PST of a sampling region on the part, and Fig. 11b illustrates its 2-D PSD. It can be seen that the 2-D PSD describes the energy distribution of the PST at different frequencies in the X and Y directions very clearly, where the energy peaks reveal the main frequency components of the PST. More favorable, the 2-D PSD can be further expressed by an angular spectrum by transforming from the Cartesian coordinate system to the polar coordinate system with the following formulas [23].

\[
\begin{align*}
\left\{ f_r = \sqrt{f_X^2 + f_Y^2} \\
\theta = \arctan \frac{f_Y}{f_X}, 0' \leq \theta \leq 180' 
\right. \\
PSD_2(f_r, \theta) \equiv PSD_2(f_r, \theta)
\end{align*}
\]

(23)

\[
\begin{align*}
PSD(\theta) &= \int_0^{2\pi} PSD_2(f_r, \theta) df_r \\
R(\theta) &= \frac{1}{2\sqrt{(\cos(\theta)/f_r)^2 + (\sin(\theta)/f_r)^2}}
\end{align*}
\]

(24)

It can be seen from Fig. 11c that compared with the 2-D PSD, the angular spectrum can describe the energy...
distribution of the PST in different directions more effectively and intuitively. For an anisotropic PST, the energy peaks of its angular spectrum will be concentrated in single or several directions, namely the main directions of the polished surface texture. Conversely, for a PST with excellent performance of multi-directionality, its energy will distribute uniformly in all directions. And the closer the shape of the angular spectrum is to a circle, the better the multi-directionality is. On the contrary, the worse.

3.2 Multi-directionality prediction algorithm

Figure 12 illustrates the main procedures of the proposed multi-directionality prediction algorithm. The inputs of the algorithm include the following: (1) the polishing toolpath of the workpiece; (2) the polishing parameters including the normal curvature of the workpiece at each CC point, the press depth, the tilted angle, the feed speed, and the rotational speed of the polishing tool. The output of the algorithm is the angular spectrum of the PST. The main steps of the algorithm are as follows.

Step 1: Discretize the polishing CC curve into a series of CC points \( \mathbf{P}_{\text{Set}} = \{ \mathbf{CC}_1, \mathbf{CC}_2, \ldots, \mathbf{CC}_i, \ldots, \mathbf{CC}_{N-1}, \mathbf{CC}_N \} \), where \( N \) is the number of the discrete CC points.

Step 2: Discretize the workpiece surface into a series of points according to a certain distance. The algorithm will calculate the MR depth at these discrete points later. Considering that the multi-directionality of the PST is mainly affected by the surface waviness whose sampling distance should be between 0.12 mm and 33 mm [22], the discrete distance of the workpiece is set as an appropriate value within this range.

Step 3: Calculate the polishing influence area of the \( i \) th CC point \( \mathbf{CC}_i \) according to the method presented in Sect. 2.4, and find all workpiece discrete points located in it. To accelerate the search process, a bounding box of the polishing influence area is first constructed, through which the discrete points that may be located in the polishing influence area can be quickly screened, as shown in Fig. 13. With this method, the search efficiency can be immensely improved.

Step 4: Calculate and update the MR depth of the discrete points in the polishing influence area of the \( i \) th CC point \( \mathbf{CC}_i \).

Step 5: Determine whether the current CC point \( \mathbf{CC}_i \) is the last CC point \( \mathbf{CC}_N \). If \( i < N \), then \( i = i + 1 \), and repeat Step 3 and Step 4. Otherwise, skip to the next step.

Step 6: After completing the above steps, the MR of each sampling point can be obtained. Then, calculate the angular spectrum according to MR distribution with the
4 Experimental studies and discussions

To evaluate the accuracy of the MR model proposed in this paper and demonstrate its superiority in computational efficiency, a comparative experiment of MR simulation for a freeform surface part is first carried out in this section. Then, the usefulness and practicability of the proposed multi-directionality prediction algorithm are validated by a physical robot polishing experiment.

4.1 Verification of the proposed MR model

In this experiment, a freeform surface part shown in Fig. 14 is first designed, and three different polishing toolpaths are generated for the part, namely the scanning toolpath (first), the trochoidal toolpath with a constant radius (second) [12], and the adaptive trochoidal toolpath (third) [24]. Their toolpath parameters are shown in Table 3. Then, the MR models proposed in this paper and in [21] are used to simulate the global MR distribution on the whole part surface, respectively. Finally, the computation accuracy and efficiency obtained by these two methods are analyzed and compared. The differences between these two methods are summarized in Table 4. The experiment was run on a PC with Intel(R) Core(TM) i5-8400 CPU @ 2.80 GHz, and its RAM is 8 GB.

Figure 15 shows the comparison results of global MR distribution computed by the two methods with the first, second, and third toolpath, where the first, second, and third columns show the computation results obtained by the proposed method in this paper, the computation results acquired by the method in [21], and the MR absolute errors calculated by subtracting the first column from the second column, respectively.

It can be easily observed that the global MR distribution obtained by the method proposed in this paper

---

Fig. 12 Main procedures of the multi-directionality prediction algorithm

Fig. 13 The bounding box of polishing influence area
is in good agreement with that obtained by the method proposed in [21]. More specifically, the maximum (minimum) relative errors under the three toolpaths are only 0.71 μm (−0.59 μm), 4.27 μm (−4.75 μm), and 3.89 μm (−2.91 μm) in Fig. 15a3–c3, respectively. They are very small relative to their average MR depth. Based on this, it can be demonstrated that the proposed MR model can obtain a comparative accuracy with the method proposed in [21]. Figure 16 illustrates the computation time consumed by the two methods. It takes 205 s, 810 s, and 380 s to compute the global MR distribution under the three toolpaths with the method proposed in [21], respectively, whereas only 72 s, 260 s, and 180 s are consumed correspondingly for the presented approach in this paper, which improves the computation efficiency greatly. The reason contributing to this result is that the proposed method in this paper avoids the time-consuming integral operation.

4.2 Experimental verification of the multi-directionality prediction algorithm

It is worth noting that although the method proposed in [21] can accurately predict the global polishing MR distribution, the multi-directionality of the PST cannot be predicted with it. In this experiment, to validate the usefulness and practicability of the proposed multi-directionality prediction algorithm, a more complicated saddle part shown in Fig. 17 is designed. Three different polishing toolpaths including the scanning toolpath (first), the trochoidal toolpath with a constant radius (second), and the adaptive trochoidal toolpath (third), are also generated for this part. Similarly, Table 5 shows the toolpath parameters. Then the robot polishing setup, shown in Fig. 18, is used to conduct the physical polishing experiments with the three toolpaths.

The PST and its multi-directionality after polishing are measured using a surface profiler (TALYSURF CLI 1000 manufactured by Taylor-Hobson Ltd) and then compared with the prediction results obtained by the algorithm proposed in this paper. Figure 19 shows the actual polishing effect under the third toolpath. Considering the complexity of the part, the surface profiler cannot measure the PST of the whole surface. Hence, the local area of the workpiece with the size of 25 mm × 25 mm, as shown in Fig. 19b, is selected as the measured area.

The measured PST and the predicted PST using the proposed method under the first, second, and third toolpath are compared in Fig. 20a–c, respectively. Moreover, the first, second, and third columns show the predicted results, the measured results, and their absolute errors obtained by subtracting the first column from the second column, respectively. It can be seen that the PST predicted by the proposed method is very consistent with the measured PST with very small errors, further demonstrating the high computation accuracy of the proposed model in this paper.

Figure 21 shows the comparison of angular spectrums between the predicted results and the measurement results under the three different toolpaths. It can be seen that the shapes of the predicted results are almost the same as those of the actual measured results, which demonstrates the accuracy of the prediction algorithm proposed in this paper. More specifically, Fig. 21a shows angular spectrums obtained by the two methods under the first polishing toolpath, where the predicted angular spectrum
and the measured angular spectrum both have a very large energy amplitude in the direction of 0°, while the energy in other directions is very small, indicating the poor multi-directionality of the PST. The reason contributing to this result is that the first toolpath feeds along only one direction. As shown in Fig. 15a1, a2, the polished surface will inevitably leave a unidirectional texture after polishing, and thus cause poor multi-directionality.

Figure 21b shows the angular spectrums obtained by the second toolpath, in which the energy distribution is more uniform than that obtained by the first toolpath, which demonstrates a better multi-directionality. However, large energy amplitudes in the direction of 0° and 90° also appear, as shown in Fig. 21b, which indicates that its multi-directionality is still not good enough. This is mainly because this kind of trochoidal toolpath is generated along a scanning toolpath, the deficiency of which will still lead to a unidirectional texture, as shown in Fig. 15b1, b2.

Unlike the angular spectrums in Fig. 21a, b, it can be seen from Fig. 21c that the energy of the angular spectrums obtained by the third toolpath is distributed relatively

Table 4

| Methods | Whether the time-consuming integral operation is involved? | Are ANN and PSD integrated? | Is multi-directionality of PST studied? |
|---------|----------------------------------------------------------|----------------------------|---------------------------------------|
| The method proposed in [21] | Yes | No | No |
| The method proposed in this work | No | Yes | Yes |

Fig. 15 The comparison results of global MR distribution. a1, b1, and c1 show the MR obtained using the method proposed in this paper under three different toolpaths, respectively. a2, b2, and c2 show the MR obtained using the method in [21] under three toolpaths, respectively. a3, b3, and c3 show their absolute errors under three toolpaths.
uniformly in all directions. This is mainly because the third toolpath has good characteristics of multi-directionality and randomness. As shown in Fig. 15c1, c2, the workpiece surface after being polished by this toolpath will no longer leave any obvious unidirectional texture. Hence, the third toolpath has the greatest potential to obtain isotropic PST.

Based on the prediction results obtained by the proposed methods in this paper, it can be intuitively known that the adaptive trochoidal toolpath can obtain the optimal performance of multi-directionality, followed by the trochoidal toolpath with a constant radius, and then the scanning toolpath. This also proves that the proposed method can indeed predict the multi-directionality of PST obtained by different polishing toolpaths and parameters effectively, which shows great application potential in promoting the efficiency of toolpath planning and optimization to obtain isotropic PST.
In this work, a rapid approach to precisely predict the multi-directionality of the PST for the pad-polishing process of freeform surfaces is proposed. The main contributions of the approach include:

- A rapid MR model that can simulate the MR distribution on the whole workpiece surface is proposed for the pad-polishing process. The model avoids the complex integral operation and thus greatly improves the MR simulation efficiency.
- Based on the MR model, the angular spectrum is integrated to characterize the multi-directionality of PST for the first time, and then a multi-directionality prediction algorithm is proposed.
- Simulation and experimental results have shown that the proposed method can predict the multi-directionality

5 Conclusion

In this work, a rapid approach to precisely predict the multi-directionality of the PST for the pad-polishing process of freeform surfaces is proposed. The main contributions of the approach include:

- A rapid MR model that can simulate the MR distribution on the whole workpiece surface is proposed for the pad-polishing process. The model avoids the complex integral operation and thus greatly improves the MR simulation efficiency.
- Based on the MR model, the angular spectrum is integrated to characterize the multi-directionality of PST for the first time, and then a multi-directionality prediction algorithm is proposed.
- Simulation and experimental results have shown that the proposed method can predict the multi-directionality
characteristic of the PST with very high accuracy and efficiency, which shows great application potential in promoting the efficiency of toolpath planning and optimization.

The advantage of using power spectral density to characterize PST is that it can reveal the roughness, waviness, and profile error of polished surfaces simultaneously [22]. The focus of this paper is the modeling of the multi-directionality of PST using PSD. The multi-directionality is mainly determined by the polishing toolpath, which has a significant influence on the MSF error (surface waviness). In future work, the authors will consider further introducing PSD to characterize and model the surface roughness of PST.

Author contribution Qing-Hui Wang: conceptualization, funding acquisition, resources, project administration. Xiao-Lin Fang: methodology; experiment and data processing; writing—original draft. Hai-Long Xie: methodology; writing—review and editing. Format analysis. Jing-Rong Li: supervision; writing—review and editing. Zhao-Yang Liao: assist in completing the experiment.

Funding This work was supported by the Key-Area Research and Development Program of Guangdong Province, China [grant number 2021B0101190002], Science & Technology Research Program of Guangzhou, China [grant number 202103020004], 2021 High-quality Development Project by the Ministry of Industry and Information Technology, China (project name: Support and quality service platform for industrial software), and Guangdong Basic and Applied Basic Research Foundation, China [grant number 2021A1515110898].

Availability of data and materials All the data supporting the results are included within the article.

Declarations

Ethical approval Not applicable.

Consent to participate The authors agree to participate in this research.

Consent for publication The authors agree to the publication of the article.

Competing interests The authors declare no competing interests.

References

1. Xie HL, Li JR, Liao ZY, Wang QH, Zhou XF (2020) A robotic belt grinding approach based on easy-to-grind region partitioning. J Manuf Process 56:830–844. https://doi.org/10.1016/j.jmapro.2020.03.051
2. Li C, Piao Y, Meng B, Hu Y, Li L, Zhang F (2022) Phase transition and plastic deformation mechanisms induced by self-rotating grinding of GaN single crystals. Int J Mach Tools Manuf 172:103827. https://doi.org/10.1016/j.ijmachtools.2021.103827
3. Zhang Y, Wang Q, Li C, Piao Y, Hou N, Hu K (2022) Characterization of surface and subsurface defects induced by abrasive machining of optical crystals using grazing incidence X-ray diffraction and molecular dynamics. J Adv Res 36:51–61. https://doi.org/10.1016/j.jare.2021.05.006
4. Zhao Q, Zhang L, Fan C (2019) Six-directional pseudorandom consecutive unicursal polishing path for suppressing mid-spatial frequency error and realizing consecutive uniform coverage. Appl Optics 58(31):8529–8541. https://doi.org/10.1364/AO.58.008529
5. Dong Z, Nai W (2018) Surface ripple suppression in sub-aperture polishing with fragment-type tool paths. Appl Optics 57(19):5523–5532. https://doi.org/10.1364/AO.57.005523
6. Rososhansky M, Xi F (2011) Coverage based tool-path planning for automated polishing using contact mechanics theory. J Manuf Syst 30(3):144–153. https://doi.org/10.1016/j.jmsy.2011.05.003
7. Zhang L, Han Y, Fan C, Tang Y, Song X (2017) Polishing path planning for physically uniform overlap of polishing ribbons on freeform surface. Int J Adv Manuf Technol 92(9):4525–4541. https://doi.org/10.1007/s00170-017-0466-z
8. Wang C, Wang Z, Xu Q (2015) Unicursal random maze tool path for computer-controlled optical surfaceing. Appl Optics 54(34):10128–10136. https://doi.org/10.1364/AO.54.010128
9. Takizawa K, Beaucamp A (2017) Comparison of tool feed influence in CNC polishing of a novel circular-random path and other pseudo-random paths. Opt Express 25(19):22411–22424. https://doi.org/10.1364/OE.25.022411
10. Li H, Li X, Wan S, Wei C, Shao J (2021) High-efficiency smooth pseudo-random path planning for restraining the path ripple of robotic polishing. Appl Optics 60(25):7732–7739. https://doi.org/10.1364/AO.426616
11. Beaucamp A, Takizawa K, Han Y, Zhou W (2021) Reduction of mid-spatial frequency errors on aspheric and freeform optics by circular-random path polishing. Opt Express 29(19):29802–29812. https://doi.org/10.1364/OE.439545
12. Xu CY, Li JR, Liang YJ, Wang QH, Zhou XF (2019) Trochoidal path for the polishing of freeform surfaces with global control of material removal distribution. J Manuf Syst 51:1–16. https://doi.org/10.1016/j.jmsy.2019.02.002
13. Xie HL, Wang QH, Ni JL, Li JR (2020) A GPU-based prediction and simulation method of grinding surface topography for belt grinding process. Int J Adv Manuf Technol 106(11):5175–5186. https://doi.org/10.1007/s00170-020-09492-4
14. Zhang L, Tam HY, Yuan CM, Chen YP, Zhou ZD, Zheng L (2002) On the removal of material along a polishing path by fixed abrasives. Proc Inst Mech Eng Part B-J Eng Manuf 216(9):1217–1225. https://doi.org/10.1243/095440502760291772
15. Yang MY, Lee HC (2001) Local material removal mechanism considering curvature effect in the polishing process of the small aspherical lens die. J Mater Process Technol 116(2–3):298–304. https://doi.org/10.1016/S0924-0136(01)01055-X
16. Fan C, Zhao J, Zhang L, Zhou W, Sun L (2016) Local material removal model considering the tool posture in deterministic polishing. Proc Inst Mech Eng Part C-J Eng Mech Eng Sci 230(15):2660–2675. https://doi.org/10.1177/0954406215598800
17. Fan C, Zhao J, Zhang L, Hong GS, Wong YS, Zhao J (2014) Predictive models of the local and the global polished profiles in deterministic polishing of free-form surfaces. Proc Inst Mech Eng Part B-J Eng Manuf 228(8):868–879. https://doi.org/10.1177/0954405413512813
18. Giuso A, Pattottato S, Tournier C, Mathieu L (2011) Modeling of a polishing tool to simulate material removal. Adv Mater Res 223:754–763. https://doi.org/10.4028/www.scientific.net/AMR.223.754
19. Almeida R, Börrer R, Rinkwus K, Harrison DK, DeSilva AKM (2017) Material removal simulation for steel mould polishing. Prod Manufac Res 5(1):235–249. https://doi.org/10.1080/21693277.2017.1374889
20. Almeida R, Börrer R, Rinkwus K, Harrison DK, DeSilva AKM (2016) Polishing material removal correlation on PMMA–FEM

 Springer
21. Wang QH, Liang YJ, Xu CY, Li JR, Zhou XF (2019) Generation of material removal map for freeform surface polishing with tilted polishing disk. Int J Adv Manuf Technol 102(9):4213–4226. https://doi.org/10.1007/s00170-019-03478-8
22. Wolfe CR, Lawson JK (1995) Measurement and analysis of wavefront structure from large-aperture ICF optics. Proc SPIE 2633:361–385. https://doi.org/10.1117/12.228288
23. Chen M, Li M, Cheng J, Xiao Y, Pang Q (2013) Study on the optical performance and characterization method of texture on KH2PO4 surface processed by single point diamond turning. Appl Surf Sci 279:233–244. https://doi.org/10.1016/j.apsusc.2013.04.073
24. Wang QH, Wang S, Jiang F, Li JR (2016) Adaptive trochoidal toolpath for complex pockets machining. Int J Prod Res 54(20):5976–5989. https://doi.org/10.1080/00207543.2016.1143135

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.