Black Hole and Neutron Star Binary Mergers in Triple Systems. II. Merger Eccentricity and Spin–Orbit Misalignment

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Received 2019 May 4; revised 2019 June 19; accepted 2019 June 27; published 2019 August 12

Abstract

We study the dynamical signatures of black hole (BH) and neutron star (NS) binary mergers via Lidov–Kozai oscillations induced by tertiary companions in hierarchical triple systems. For each type of binary (BH–BH and BH–NS), we explore a wide range of binary/triple parameters that lead to binary mergers and determine the distributions of merger time \(T_m\), eccentricity \((e_m)\), and spin–orbit misalignment angle \(\theta^\text{sl}\) when the binary enters the LIGO/VIRGO band (10 Hz). We use the double-averaged (over both orbits) and single-averaged (over the inner orbit) secular equations, as well as \(N\)-body integration, to evolve systems with different hierarchy levels, including the leading-order post-Newtonian effect, de Sitter spin–orbit coupling, and gravitational radiation. We find that for merging BH–BH binaries with comparable masses, about 7% have \(e_m > 0.1\) and 0.7% have \(e_m > 0.9\). The majority of the mergers have significant eccentricities in the LISA band. The BH spin evolution and the final spin–orbit misalignment \(\theta^\text{sl}\) are correlated with the orbital evolution and \(e_m\). Mergers with negligible \(e_m \lesssim 10^{-3}\) have a distribution of \(\theta^\text{sl}\) that peaks around 90° (and thus favoring a projected binary spin parameter \(\chi^\text{eff} \sim 0\)), while mergers with larger \(e_m\) have more isotropic spin–orbit misalignments. For typical BH–NS binaries, strong octupole effects lead to more mergers with nonnegligible \(e_m\) (with \(\sim 18\%\) of the mergers having \(e_m > 0.1\) and 2.5% having \(e_m > 0.9\)), and the final BH spin axis tends to be randomly oriented. Measurements or constraints on eccentric mergers and \(\theta^\text{sl}\) from LIGO/VIRGO and LISA would provide useful diagnostics on the dynamical formation of merging BH or NS binaries in triples. The recently detected BH merger events may implicate such dynamical formation channel.

Key words: binaries: general – black hole physics – gravitational waves – stars: black holes – stars: kinematics and dynamics

1. Introduction

Recent studies (e.g., Miller & Hamilton 2002; Wen 2003; Thompson 2011; Antonini & Perets 2012; Antonini et al. 2017; Liu & Lai 2017, 2018; Silsbbee & Tremaine 2017; Hoang et al. 2018; Rodriguez & Antonini 2018) have suggested that tertiary-induced merger via Lidov–Kozai (LK) oscillations (Kozai 1962; Lidov 1962; Naoz 2016) may play a significant role in producing the black hole (BH) binaries detected by the LIGO/VIRGO collaboration (e.g., Abbott et al. 2016a, 2016b, 2017a, 2017b, 2017c, 2017d, 2018a, 2018b). Merging BH and neutron star (NS) binaries can be formed efficiently in triple systems with the aid of a tertiary body that moves on an inclined (outer) orbit relative to the orbit of the inner (BH or NS) binary. The efficiency of the merger can be further enhanced when the triple is part of a quadruple system (e.g., when the tertiary component is itself a binary; see Fang et al. 2018; Fragione & Kocsis 2019; Liu & Lai 2019; Zevin et al. 2019) or more generally, when the outer orbit experiences quasi-periodic external forcing (e.g., Hamers & Lai 2017; Petrovich & Antonini 2017; Fragione et al. 2019).

Given the expected large number of BH mergers to be detected by LIGO/VIRGO in the coming years, it will be important to distinguish tertiary-induced mergers from other dynamical BH binary formation channels (such as those involving close encounters in dense stellar clusters; e.g., Portegies Zwart & McMillan 2000; O’Leary et al. 2006; Miller & Lautberg 2009; Banerjee et al. 2010; Downing et al. 2010; Rodriguez et al. 2015; Chatterjee et al. 2017; Samsung et al. 2018) and the more traditional isolated binary channel (e.g., Lipunov et al. 1997, 2017; Podsiadlowski et al. 2003; Belczynski et al. 2010, 2016; Dominik et al. 2012, 2013, 2015), as well as the chemically homogeneous evolution channel (e.g., Mandel & de Mink 2016; Marchant et al. 2016) and gas-assisted mergers (e.g., Bartos et al. 2017). One possible indicator is the merger eccentricity: it has been noted that dynamical binary–single interactions in dense clusters (e.g., Samsing & Ramirez-Ruiz 2017; Rodriguez et al. 2018; Samsing & D’Orazio 2018; Fragione & Bromberg 2019) or in galactic triples (Antonini et al. 2017; Silsbbee & Tremaine 2017; Fragione & Loeb 2019) may lead to BH binaries entering the LIGO band with modest or large eccentricities, although the fraction of such eccentric mergers is highly uncertain. Another potentially valuable observable is the BH spin, which carries information on the BH binary formation history. In particular, through the binary inspiral waveform, the mass-weighted projection of BH spin, \(\chi^\text{eff} = (m_1 \chi_{1} + m_2 \chi_{2})/(m_1 + m_2) \cdot \hat{L}\), can be directly measured (here, \(m_{1,2}\) are the BH masses, \(\chi_{1,2} = cGm_{1,2}^2/(\text{GM}_{1,2}^2)\) are the dimensionless BH spins, and \(\hat{L}\) is the unit orbital angular momentum vector of the BH binary). While isolated binary evolution tends to generate approximately aligned BH spins (with respect to the orbit), and cluster dynamics tends to generate random spin orientations, recent works (Liu & Lai 2017, 2018; Antonini et al. 2018; Rodriguez & Antonini 2018) have shown that merging BH binaries produced in hierarchical triples may exhibit rich behaviors in spin–orbit orientations. For initially close BH binaries (with semimajor axis \(a_0 \lesssim 0.2\) au), which may merge by themselves without the aid of a tertiary companion, modest (\(\lesssim 20^\circ\)) spin–orbit...
misalignments can be produced (e.g., Liu & Lai 2017) due to the perturbation of the tertiary companion. For wide binaries (with $a_0 \gtrsim 10$ au), a range of final spin–orbit misalignment angles $\theta_{\text{f}}$ can be produced as the merging binary enters the LIGO band (Liu & Lai 2018): when the BHs have comparable masses (the octupole effect is thus negligible), the distribution of $\theta_{\text{f}}$ is peaked around 90°; when the two members of the inner binary have highly unequal masses and the tertiary companion moves in an eccentric orbit, a more isotropic distribution of the final spin axis is produced. Overall, merging BH binaries produced by LK oscillations in triples exhibit a unique distribution of the effective (mass-weighted) spin parameter $\chi_{\text{eff}}$.

In this paper, we extend our recent studies (Liu & Lai 2017, 2018) by exploring a wide range of triple systems. In particular, for a given merging compact binary with known masses, we examine all possible triple configurations and parameters that lead to binary mergers and determine the distributions of various merger properties (merger times, eccentricities, and spin–orbit misalignments). We consider two sets of binary masses: $(m_1, m_2) = (30M_\odot, 20M_\odot)$ representing a canonical BH–BH binary, and $(30M_\odot, 1.4M_\odot)$ representing a canonical BH–NS binary. Our previous works focused on fully hierarchical triples, where either double-averaged (over both the inner and outer orbits) or single-averaged (over only the inner orbit) secular approximation is valid. The spin evolution was only studied for systems where the double-averaged approximation is valid (this is also the case for the study by Antonini et al. 2018 and Rodríguez & Antonini 2018). In this paper, we examine systems with various levels of hierarchy, using both double-averaged and single-averaged secular equations, as well as direct $N$-body integrations to evolve the systems. This allows us to determine reliably the fraction of merging binaries that enter the LIGO band with appreciable eccentricities. In addition, unlike previous works, we consider systems where the initial BH spin and orbital axes are not perfectly aligned, and we determine the distribution of the “final” spin–orbit misalignment angles $\theta_{\text{f}}$.

Our paper is organized as follows. In Section 2, we introduce three approaches to evolve the triple systems with different levels of approximation, including the de Sitter spin–orbit coupling effect. In Section 3, we perform a large set of numerical integrations, focusing on two types of stellar mass binaries (BH–BH and BH–NS), with imperfectly aligned initial spin axes. We compute the distributions of binary eccentricities and spin–orbit misalignments for systems that evolve into the LIGO band. We summarize our main results in Section 4.

2. Three Approaches for Triple Evolution with Spin–Orbit Coupling

2.1. Summary of Parameter Regimes

We consider a hierarchical triple system, composed of an inner BH binary of masses $m_1$ and $m_2$, and a distant companion of mass $m_3$ that moves around the center of mass of the inner bodies. The reduced mass for the inner binary is $\mu = m_1 m_2 / m_2$, with $m_2 = m_1 + m_2$. Similarly, the outer binary has $\mu_{\text{out}} = (m_2 m_3) / m_{23}$ with $m_{23} = m_2 + m_3$. The semimajor axes and eccentricities are denoted by $a$, $a_{\text{out}}$ and $e$, $e_{\text{out}}$, respectively. Therefore, the orbital angular momenta of two orbits are given by $L = L_1 + L_2 = \mu \sqrt{Gm_1 m_2} (1 - e^2) \hat{L}$ and $L_{\text{out}} = L_{\text{out},\text{in}} + L_{\text{out},\text{out}} = \mu_{\text{out}} \sqrt{Gm_2 m_3} (1 - e_{\text{out}}^2) \hat{L}_{\text{out}}$. We define the mutual inclination between $L$ and $L_{\text{out}}$ (inner and outer orbits) as $i$.

To study the evolution of the inner binary under the influence of the tertiary companion, we use three approaches: the double-averaged (averaging over both the inner and outer orbits) and single-averaged (only averaging over the inner orbital period) secular equations of motion, as well as the direct $N$-body integrations (see Section 2.1.2 of Liu & Lai 2018 for details). In the orbital evolution, we include the contributions from the external companion that generate LK oscillations up to the octupole level of approximation, the post-Newtonian (PN) correction due to general relativity (GR), and the dissipation due to gravitational wave (GW) emission.

The LK mechanism induces the oscillations in the eccentricity and mutual orbital inclination on the timescale

$$\tau_{\text{LK}} = \frac{1}{n} \frac{m_{12}}{m_3} \left( \frac{a_{\text{out},\text{eff}}}{a} \right)^3,$$

where $n = (Gm_{12}/a)^{1/2}$ is the mean motion of the inner binary, and $a_{\text{out},\text{eff}} = a_{\text{out}} \sqrt{1 - e_{\text{out}}^2}$ is the effective outer binary separation.

During the LK oscillations, the short-range force effects (such as GR-induced apsidal precession) play a crucial role in determining the maximum eccentricity $e_{\text{max}}$ of the inner binary (e.g., Fabrycky & Tremaine 2007). In the absence of energy dissipation, the evolution of the triple is governed by two conservation laws: the total orbital angular momentum and the total energy of the system. The analytical expression for $e_{\text{max}}$ for general hierarchical triples (arbitrary masses and eccentricities) can be obtained in the double-averaged secular approximation if the disturbing potential is truncated to the quadrupole order. Using the method of Liu et al. (2015; see also Anderson et al. 2016, 2017), we find

$$\frac{3}{8} \left( e_0 + (j_{\min}^2 - 1) + (5 - 4j_{\min}^2) \right) \times \left[ 1 - \frac{(j_{\min}^2 - 1) \eta_{\min} + e_0^2 \eta_0 - 2j_0 \cos I_0)^2}{4j_{\min}^2} \right]$$

$$- (1 + 4e_0^2 - 5e_0^2 \cos^2 \omega_0) \sin^2 I_0)$$

$$+ \varepsilon_{\text{GR}} (j_0^{-1} - j_{\min}^{-1}) = 0,$$

where $e_0$, $I_0$, and $\omega_0$ are the initial eccentricity, inclination, and longitude of the periapse of the inner binary, respectively, and we have defined $j_{\min} = \sqrt{1 - e_{\text{max}}^2}$, $j_0 = \sqrt{1 - e_0^2}$, $\eta_{\min} = L(e = e_{\text{max}})/L_{\text{out}}$, $\eta_0 = L(e = e_0)/L_{\text{out}}$, and $\varepsilon_{\text{GR}} = (3Gm_1^2 a_{\text{out},\text{eff}}^3)/(c^2 a^4 m_3)$. Note that for $e_0 = 0$, Equation (2) reduces to Equation (24) of Anderson et al. (2017). For the general $L/L_{\text{out}}$, the maximum possible $e_{\text{max}}$ for all values of $I_0$, called $e_{\text{lim}}$, is given by (assuming $\omega_0 = 0$)

$$\frac{3}{8} \left( j_{\min}^2 - 1 \right) \left[ \frac{\eta_{\min}^2}{4} \left( \frac{4}{5} j_{\min} - 1 \right) - 3 \right]$$

$$+ \frac{e_0^4}{(j_{\lim}^2 - 1)} \left[ \frac{\eta_0^2}{4} \left( \frac{4}{5} j_{\lim} - 1 \right) - 1 \right]$$

$$+ 2e_0^2 \left[ \frac{\eta_{\min} \eta_0}{4} \left( \frac{4}{5} j_{\min} - 1 \right) + 1 \right]$$

$$+ \varepsilon_{\text{GR}} (j_0^{-1} - j_{\lim}^{-1}) = 0,$$

where $j_{\lim} = \sqrt{1 - e_{\text{lim}}^2}$. 
For systems with $m_1 = m_2$ and $e_{\text{out}} \neq 0$, the octupole effect may become important. The strength of the octupole effect is characterized by

$$
\varepsilon_{\text{oct}} \equiv \frac{m_1 - m_2}{m_1 + m_2} \frac{a}{r_{\text{out}}} \left( 1 - e_{\text{out}}^2 \right) .
$$

The octupole terms tend to widen the inclination window for large eccentricity excitation. However, the analytic expression for $\varepsilon_{\text{oct}}$ given by Equation (3) remains valid even when the octupole effect is strong (Liu et al. 2015; Muñoz et al. 2016; Anderson et al. 2017).

The validity of secular approximation depends on the hierarchy level of the triple system. For sufficiently hierarchical systems, the angular momenta of the inner and outer binaries exchange periodically over a long timescale (longer than the companion’s orbital period), while the exchange of energy is negligible. When the eccentricity variation timescale of the inner binary is longer than the period of the companion’s orbit ($P_{\text{in}}$), i.e.,

$$
n_{\text{LK}} \sqrt{1 - e_{\text{max}}^2} \gtrsim P_{\text{out}},
$$

the double-averaged (DA) secular equations are valid. The full equations of motion can be found in Liu et al. (2015).

For moderately hierarchical systems, when the eccentricity evolution timescale at $e \sim e_{\text{max}}$ lies between the inner orbital period $P_{\text{in}}$ and the outer orbital period $P_{\text{out}}$, i.e.,

$$
P_{\text{in}} \lesssim n_{\text{LK}} \sqrt{1 - e_{\text{max}}^2} \lesssim P_{\text{out}},
$$

the DA secular equations break down, but the single-averaged (SA) secular equations remain valid. The explicit SA equations of motion are provided in Section 2.1.2 of Liu & Lai (2018).

If the tertiary companion is even closer, the perturbation becomes sufficiently strong and Equation (6) may not be satisfied. In this situation, the dynamics can only be solved by N-body (NB) integrations. In this paper, we use a new N-body code developed by Yi-Han Wang based on the ARCHAIN algorithm (Mikkola & Merritt 2008). This algorithm employs a regularized integrator to accurately trace the motion of tight binaries with arbitrarily large mass ratios and a chain structure to reduce the round-off errors from close encounters. The code uses the Bulirsch–Stoer (BS) integrator (e.g., Stoer 1972) and further reduces the round-off error by using active error compensation. Our developing code can be found at SpaceHub.4

All calculations in this paper, whether based on DA or SA secular equations, or NB integrations, include the PN effect of the inner binary (which gives rise to the apsidal advance) and the 2.5 PN effect (which accounts for gravitational radiation).

### 2.2. Spin–Orbit Coupling

To incorporate the spin–orbit coupling effect, we introduce the spin vector $S_1 = \hat{S}_1 \tilde{S}_1$ (where $S_1$ is the magnitude of the spin angular momentum of $m_1$ and $\tilde{S}_1$ is the unit vector). The de Sitter precession of $\hat{S}_1$ around $\dot{\tilde{S}}_1$ (1.5 PN effect) is governed by (e.g., Barker & O’Connell 1975)

$$
\frac{d\hat{S}_1}{dt} = \Omega_{\text{SL}} \times \dot{\tilde{S}}_1.
$$

Let $n_{(2)}$ and $v_{(2)}$ be the position vector and velocity vector of the first and second body of the inner binary, respectively, and define $r = n_1 - n_2$, $r_1 = |r|$, and $v = v_1 - v_2$. The precession rate in Equation (7) is given by

$$
\Omega_{\text{SL}}^{(\text{PN})} = G \left( 2 + \frac{3m_2}{2m_1} \right) \frac{\mu r \times v}{c^2 r^3}.
$$

Averaging over the inner orbital period ($P_{\text{in}}$), the precession rate becomes

$$
\Omega_{\text{SL}}^{(\text{AV})} = \frac{3G(m_2 + \mu/3)}{2c^2 a(1 - e^2)} \hat{L} = \Omega_{\text{SL}}^{(\text{AV})} \hat{L}.
$$

Similar equations apply to $S_2$. Note that the back-reaction torques from $\hat{S}_1$ on $\hat{L}$ is usually negligible because $S_1 \ll L$, and the spin–spin coupling (2 PN correction) is always negligible until the very last merging stage. Both are ignored in our calculations. In addition, the de Sitter precession of $\hat{S}_1$ induced by the tertiary companion is also ignored as well. Thus, the DA/SA secular equations combined with Equation (9), or N-body integration with Equation (8), completely determine the orbital and spin evolution of merging BH binaries in triples.

### 2.3. Some Examples

To calibrate our different approaches, Figure 1 shows an example of the orbital and spin evolution of a BH binary with an inclined companion, obtained using N-body integration and SA secular equations. The parameters of the system (given in the figure caption) satisfy the SA criterion (Equation (6)). We see that the SA equations succeed in resolving the “correct” orbital evolution, producing the same period and amplitude of LK cycles as in the N-body calculations. However, in the N-body calculations, the long-term evolution of the binary depends on the initial true anomaly $f_{\text{in}}^0$ of the inner orbit, with the merger time depending on $f_{\text{in}}^0$. The evolution based on the SA equation yields an “averaged” merger time. Note that in this example, the “residual” eccentricity (i.e., the binary eccentricity when it enters the LIGO band) is negligible ($e_m \ll 1$) regardless of the integration methods.

The bottom panel of Figure 1 shows that the spin axis $\tilde{S}_1$ (initially misaligned with respect to $\tilde{L}$ by 15°) experiences large variations during the inner binary evolution. As discussed in Liu & Lai (2018), a useful “adiabaticity parameter” characterizing the spin evolution is (see also Storch et al. 2014; Storch & Lai 2015; Anderson et al. 2016, 2017)

$$
A \equiv \frac{\Omega_{\text{SL}}^{(\text{AV})}}{\Omega_{\text{L}}}, \quad \text{with} \quad \Omega_{\text{L}} = \frac{3(1 + 4e^2)|\sin 2\eta|}{8n_{\text{LK}} \sqrt{1 - e^2}}.
$$

As the orbit decays, the spin dynamics transitions from the “weak coupling” regime ($A \ll 1$) to the “strong coupling” regime ($A \gg 1$). The spin–orbit misalignment angle tends to be frozen at a high value near the end of the inspiral. In this example, all integrations produce large final spin–orbit misalignment angles, $\theta_{\text{in}}^{\text{sl}} \approx 81°$, $\theta_{\text{in}}^{\text{sl}} \approx 77°$ ($f_{\text{in}}^0 = 2.35$ rad) and $\theta_{\text{in}}^{\text{sl}} \approx 87°$, $\theta_{\text{in}}^{\text{sl}} \approx 72°$ ($f_{\text{in}}^0 = 3.61$ rad) for the N-body integration, and $\theta_{\text{in}}^{\text{sl}} \approx 81°$, $\theta_{\text{in}}^{\text{sl}} \approx 90°$ for the SA secular integration.

Figure 2 depicts another example of BH binary merger, in which the octupole effect is important. The system parameters (given in the figure caption) imply that both DA and SA approaches are not accurate. We see that in the N-body integration, due to the high efficiency of gravitational radiation
The Astrophysical Journal, 881:41 (12pp), 2019 August 10

Liu, Lai, & Wang

are chosen from a log-uniform distribution from 10 to $10^4$ au; the initial orbital eccentricities are drawn from a uniform distribution ranging from 0 to 1 for both inner and outer binary orbits; the tertiary companion mass is assigned by assuming a flat distribution in $(0, 1)$ for $m_3/m_{12}$ (e.g., Sana et al. 2012; Duchêne & Kraus 2013; Kobulnicky et al. 2014); and the binary inclinations are isotropically distributed (uniform distribution in cos $I_0$). Because the velocity kick during the BH formation may introduce small spin–orbit misalignment, we consider a flat distribution of the initial $\cos \theta_0$ in the range $(\cos 0^\circ$, $\cos 20^\circ)$.

For the triple systems to be dynamically stable, the ratio of the pericenter distance of the outer orbit to the apocenter distance of the inner orbit must satisfy (e.g., Kiseleva et al. 1996)

$$\frac{a_{\text{out}}(1 - e_{\text{out}})}{a(1 + e)} > \frac{3.7}{Q_\text{out}} - \frac{2.2}{1 + Q_\text{out}} + \frac{1.4}{Q_\text{in} Q_\text{out}^{1/3}} + 1,$$

(11)

where $Q_\text{in} = [\max(m_1, m_2)/\min(m_1, m_2)]^{3/2}$ and $Q_\text{out} = (m_2/m_3)^{1/3}$.

For each type of binary, simulations are preformed for $10^5$ randomly chosen initial conditions. After extracting the stable triples based on Equation (11), we identify the parameter regime for each triple according to the criteria of Section 2.1. We use $e_{\text{max}} = e_{\text{lim}}$ in Equations (5) and (6). Different integration methods (DA/SA/NB) are then applied to systems in different regimes. This helps us to speed up our calculations and ensures the accuracy of system evolution.

To further increase the efficiency of the parameter survey, we adopt the following “stopping conditions.” First, the maximum integration time is set to be the minimum of $(10^4 \text{yr}, 10 \text{Gyr})$. This maximum time is adequate to capture most of the mergers. Moreover, we terminate the simulation when the inner binary semimajor axis is reduced to less than 0.5\% of the initial $a_0$ and when the adiabaticity parameter $A \gg 1$. This is reasonable because in the last phase of the binary merger, the binary dynamics is dominated by GW emission such that the inner binary is decoupled from the perturbation of the tertiary companion. The condition $A \gg 1$ ensures that the spin–orbit misalignment angle $\theta_0$ reaches its “final” (constant) value. Once the full integration is stopped, the subsequent evolution of the inner binary eccentricity and semimajor axis can be obtained using the analytical formulas of Peters (1964). This allows us to obtain the residual eccentricity $e_{\text{in}}$ when the binary enters the LIGO detection band, i.e., when the peak GW frequency (Wen 2003)

$$f_{\text{GW}}^\text{peak} = \frac{(1 + e)1.1954}{\pi} \sqrt{\frac{G(m_1 + m_2)}{a^3(1 - e^2)^{5/2}}}$$

(12)

reaches 10 Hz.

3. Population Study

3.1. Parameter Choice and System Setup

We consider two types of merging binaries in this paper. The first has masses $(m_1, m_2) = (30M_\odot, 20M_\odot)$, representing typical BH–BH binaries; the second has $(m_1, m_2) = (30M_\odot, 1.4M_\odot)$, representing BH–NS binaries. For each inner binary, we consider all possible initial binary/triple systems and parameters that may lead to binary mergers. In particular, the initial semimajor axes of the inner and outer orbits ($a_0$ and $a_{\text{out}}$) at $e_{\text{max}}$, rapid merger occurs, accompanied by a highly eccentric orbit ($e_{\text{in}} \simeq 0.94$) when the GW frequency enters the LIGO band. Because of the rapid orbital decay, the spin vector does not experience large variation, and the final spin–orbit misalignment angle freezes at $\theta_{s_{\text{in}}} \simeq 15^\circ$, $\theta_{s_{\text{out}}} \simeq 18^\circ$. However, for the SA secular integration, the residual eccentricity is “erased” ($e_{\text{in}} \simeq 0.05$), and the final spin–orbit misalignment angles are $\theta_{s_{\text{in}}} \simeq 39^\circ$, $\theta_{s_{\text{out}}} \simeq 22^\circ$.

Figure 1. Sample orbital and spin evolution of a BH binary system with a tertiary companion. The top three panels show the semimajor axis, eccentricity, and inclination (relative to $L_\odot$) of the inner BH binary, and the bottom panel shows the spin–orbit misalignment angle $\theta_0$ (the angle between $S_0$ and $L$). The system parameters are $m_1 = 30M_\odot$, $m_2 = 20M_\odot$, $m_3 = 30M_\odot$, $a_0 = 10$ au, $a_{\text{out}} = 200$ au, $e_0 = 0.3$, $e_{\text{out}} = 0.1$, $I_0 = 91.85^\circ$, and $\theta_{0,0} = 15.13^\circ$, $\theta_{0,0} = 17.64^\circ$. The initial longitudes of the periape of the inner and outer orbits (i.e., the angle between $e$ and the line of the ascending node of the two orbits) are $\omega_{\text{in},0} = 340^\circ$ and $\omega_{\text{out},0} = 113^\circ$. Different colors denote two integration methods. In our calculations, the initial true anomaly ($f_{\text{in}}$) of the outer binary is set to be $\pi$ in the N-body calculations, the initial true anomaly ($f_{\text{in}}$) of the inner orbit is set to $2.35$ rad (blue) and $3.61$ rad (red).

3.2. Parameter Space for Binary Mergers

What kinds of triple systems can produce binary mergers? For our canonical BH–BH binaries ($m_1 = 30M_\odot$, $m_2 = 20M_\odot$), we find 1092 mergers out of 25,255 stable systems, with a merger fraction of 4.3\%; this includes 114 mergers out of 10,243 systems in the DA parameter regime (fraction $\approx 1.1\%$), 373 mergers out of 5785 systems in the SA parameter regime (fraction $\approx 6.5\%$), and 605 mergers out of 9277 systems from direct NB simulations (fraction $\approx 6.6\%$). Figure 3 depicts the
example, the initial true anomalies of the inner and outer binaries are set to 73.4978°, 49.78° respectively.

As shown in Figure 3, there is a preference for initially highly inclined systems (around I₀ ~ 90°) to generate mergers. This is the typical outcome for “quadrupole” systems, for which the two members of the inner binary have comparable masses such that ε_{out, eff} ~ 1 (see Equation (4)). The distribution shows that mergers occur at a wide range of initial inclinations, but the number decreases sharply when I₀ goes below 40° or above 140°. Therefore, we do not simulate the triples outside this “Kozai window.” From the middle and bottom panels, we see that the quick and slow mergers have no obvious trend in a_{out} / a₀ and a_{out, eff} / a₀. However, the majority of mergers happen with a_{out} / a₀ ≳ 100 and a_{out, eff} / a₀ ≳ 20.

In Figure 4, we plot the initial conditions in terms of (a_{out, eff}, a₀) for the BH mergers presented in Figure 3. The upper and lower panels show the outcomes indicated by different merger times and “residual” eccentricities (i.e., the binary eccentricity when the peak GW frequency enters the LIGO band). The solid line comes from the stability criterion (Equation (11)), set by ε₀ = 1, ε_{out} = 0, and m₁ = m₁₂ = 50M_{⊙}. The dashed line is obtained from the condition

$$T_{m,0}(1 - ε_{\text{max}}^2)^3 = 10^{10} \text{yr},$$

where T_{m,0} = (5c^5a₀^4)/(256G^3m₁₂^2μ₁2) is the merger time due to GW radiation of an isolated binary with the initial semimajor axis a₀ and eccentricity ε₀ = 0, and ε_{max} is evaluated by Equation (3) at ε₀ = 0 and η₀ → 0. In Equation (14), the left-hand side represents the merger time of the inner binary when its eccentricity is excited to ε_{max} during an LK cycle (Liu & Lai 2018; Randall & Xianyu 2018). Thus, Equation (14) provides the upper limit of the merger time for detectable LK-induced mergers.

In the upper panel of Figure 4, we see that the quick BH mergers (T_{m} ≲ 10^6 yr) are likely to occur when both a_{out, eff} and
are relatively small. In the lower panel, the eccentric mergers ($e_{m} \gtrsim 0.1$) preferentially arise for combinations of ($a_{\text{out,eff}}$, $a_{0}$) that are located near the stability boundary. This implies that the highest eccentricity excitations require the strongest perturbers.

For our canonical BH–NS binaries (with $m_{1} = 30M_{\odot}$, $m_{2} = 1.4M_{\odot}$), the octupole effect becomes important due to the high mass ratio. Previous works (Liu et al. 2015; Anderson et al. 2017) have shown that the main effect of the octupole potential is to broaden the range of the initial $I_{0}$ for extreme eccentricity excitations ($e_{\text{max}} = e_{\text{lim}}$). In our simulations, we find 2683 mergers out of 26,238 stable triple systems, with a merger fraction of 10.2%; this includes 209 mergers out of 9761 systems in the DA regime (fraction $\approx 2.14\%$), 1197 mergers out of 6010 systems in the SA regime (fraction $\approx 20\%$), and 1277 out of 10,467 systems in direct NB simulations (fraction $\approx 12\%$). The somewhat smaller merger fraction in the NB regime compared to the SA regime arises because some systems become dynamically unstable during the NB integrations.

Figure 5 shows the dependence on the initial parameters for BH–NS merger events. Compared to Figure 3, we see that a large range of the initial inclinations inside the Kozai window produces mergers, as a result of the nonnegligible $v_{\text{eff}}$ for BH–NS systems.

Figure 6 shows similar results to Figure 4, but for BH–NS binaries. Because the merger fraction is much larger than the BH–BH case, the statistical features (parameter spaces) for producing quick or eccentric mergers become more evident.
3.3. Merger Properties

In Figure 7, we show the distributions of the merger time $T_m$, merger eccentricity $e_m$, and spin–orbit misalignment angle $\theta_{sl}^i$ for our canonical BH–BH mergers. As shown in the top-left panel, most merger events occur around $T_m \sim 10^7$–$10^8$ yr. We find that about 80% of the binaries enter the LIGO band with $e_m \gtrsim 10^{-3}$, 50% have $e_m \gtrsim 10^{-2}$, 7% have $e_m > 0.1$, and 0.7% have $e_m > 0.9$ (see the top-right panel). In the lower panels, we see that there is a slight clustering around 90° for the final spin–orbit misalignment $\theta_{sl}^f$. This 90° “attractor” arises from quadruple-dominated mergers (corresponding to CASE I discussed in Section 4.2 in Liu & Lai 2018). The other peak around $\theta_{sl}^f = 0°$ is associated with CASE III and IV (see Liu & Lai 2018 for details).

Figure 8 shows the results for our canonical BH–NS binaries. Because of the low NS mass, the orbital decay due to GW is not as efficient as the BH–BH case. As seen in the upper left panel, there is only a few merger events with $T_m < 10^5$ yr. Compared to the BH–BH case, the high mass ratio of BH–NS binaries leads to a stronger octupole effect, which enhances extreme eccentricity excitation. In the upper-right panel, we see a significant increase in the number of mergers with appreciable $e_m$; out of all merger events, about 93% have $e_m \gtrsim 10^{-3}$, 80% have $e_m \gtrsim 10^{-2}$, 18% have $e_m > 0.1$, and 2.5% have $e_m > 0.9$. The octupole effect may also produce chaotic orbital evolution (e.g., Lithwick & Naoz 2011; Li et al. 2014; Liu et al. 2015), such that the spin–orbit misalignment angle is allowed to settle to any value (CASE II in Liu & Lai 2018). Thus, we find in the lower panels of Figure 8 that the final spin–orbit misalignments are largely isotropic (i.e., uniform in $\cos \theta_{sl}^i$).

To determine how the final spin–orbit misalignment $\theta_{sl}^f$ depends on the initial $\theta_{sl}^0$, Figure 9 shows the distribution of $\theta_{sl}^f$ for different ranges of initial $\theta_{sl}^0$ ($0°$–$10°$ versus $10°$–$20°$). We see that for BH–BH mergers, the peak around $\theta_{sl}^f = 90°$ exists regardless of the initial range of $\theta_{sl}^0$. For BH–NS mergers, the distribution of $\cos \theta_{sl}^f$ is approximately uniform for different ranges of $\theta_{sl}^0$.

Several previous works also studied eccentric mergers of BH binaries induced by a tertiary companion (Antonini et al. 2017; see also Rodriguez & Antonini 2018) combined with stellar evolution (mass loss and natal kick) and dynamics in isolated triples, and found that about 10% of BH mergers have $e_m > 0.1$, similar to our result. On the other hand, Silsbee & Tremaine (2017) also considered stellar evolution and used N-body integration to evolve the triples, and found that a few percent of the mergers may have $e_m > 0.999$ (in contrast, we found about 1% having $e_m \gtrsim 0.9$). In the case of BH mergers induced by a massive BH, Antonini & Perets (2012) claimed that 10% have $e_m > 0.1$ (see also Fragione & Bromberg 2019). In dense star clusters, about 1% of the BH mergers generated by scattering processes may have $e_m > 0.1$ (e.g., Samsing & Ramirez-Ruiz 2017; Rodriguez et al. 2018). Thus, despite the
uncertainties in various scenarios, the detection of eccentric BH mergers would constrain the dynamical formation channels of binary BHs.

### 3.4. $\chi_{\text{eff}}$ and Correlation with Merger Eccentricity $e_m$

Having obtained the distributions of $\cos \theta_{s1}$ and $\cos \theta_{s2}$ in Section 3.3, we can calculate the distribution of the effective spin parameter

$$\chi_{\text{eff}} = \frac{m_1 \chi_1 \cos \theta_{s1}^f + m_2 \chi_2 \cos \theta_{s2}^f}{m_1 + m_2},$$

(15)

Figure 8. Same as Figure 7, except for BH–NS binaries (see Figure 5). The data represent $N_{\text{max}} = 2683$ mergers, including 209 (DA), 1197 (SA), and 1277 (NB) mergers out of the 26,238 simulated systems.

Figure 9. Distribution of the final spin–orbit misalignment $\theta_{s1}^f$ for BH–BH mergers and BH–NS mergers in triples. We separate the mergers with different ranges of initial spin–orbit orientations, i.e., $\theta_{s1}^i \in (0^\circ, 10^\circ)$ and $\theta_{s2}^i \in (10^\circ, 20^\circ)$.

where $\chi_{1,2} \lesssim 1$ are the Kerr parameters. Figure 10 (lower panel) shows three examples with different $\chi_{1,2}$. Here, $\chi_{\text{eff}} = (m_1 \chi_1 + m_2 \chi_2) / m_2$ is the maximum possible value of $\chi_{\text{eff}}$ for a given $m_1 \chi_1$ and $m_2 \chi_2$ (this maximum is achieved at $\cos \theta_{s1}^f = \cos \theta_{s2}^f = 1$). The different ratios of $\chi_1$ and $\chi_2$ affect the distribution of $\chi_{\text{eff}} / \chi_{\text{eff}}^\text{max}$, but not significantly. The peak around $\chi_{\text{eff}} \approx 0$ is clearly visible, although not as distinct as in Liu & Lai (2018), who considered a more limited parameter space. The full range of $\chi_{\text{eff}}$ values (from negative to positive) becomes possible. This is in contrast to the isolated stellar binary evolution channel, which always predicts a positive $\chi_{\text{eff}}$ because of the preferentially aligned spins after the binaries undergo mass transfer or tidal coupling (e.g., Zaldarriaga et al. 2017; Gerosa et al. 2018). Comparison with current LIGO/VIRGO detections (top panel of Figure 10) suggests that the triple-driven merger scenario may be required, but obviously it is premature to draw any firm conclusion at this point, because of the (partial) degeneracy between $\chi_1$, $\chi_2$, and spin–orbit misalignment angles.

For BH–NS binaries, $\chi_{\text{eff}} \approx \chi_1 \cos \theta_{s1}^f$, and the peak around $\chi_{\text{eff}} \approx 0$ is insignificant because of the random distribution of the final spin–orbit misalignment angles (see Figure 8).

If the distributions of $\cos \theta_{s1}^f$ and $\cos \theta_{s2}^f$ are uncorrelated, as we may expect to be the case for $m_1 = m_2$, when the octupole effect is significant, the distribution of $|\cos \theta_{s1}^f - \cos \theta_{s2}^f|$ can be derived directly, where $\mu_1 \equiv \cos \theta_{s1}^f$ and $\mu_2 \equiv \cos \theta_{s2}^f$. The Astrophysical Journal, 881:41 (12pp), 2019 August 10 Liu, Lai, & Wang
the rescaled binary spin parameter and Venumadhav et al. (2019) from the 90° “attractor” found in Liu & Lai (2018; CASE I in that paper), where we used DA secular equations to evolve the triple systems and spins. In that work, no significant $e_{m}$ was generated for the systems considered.

In Figure 13, we consider BH–NS binaries. As in the BH–BH case (Figure 12), systems with $e_{m} < 1$ exhibit a 90° peak in the $\theta_{d}^{f}$ distribution, while those with large $e_{m}$ do not.

In reality, triple systems (especially the outer orbit) can be perturbed (even disrupted) by close flybys with other objects (e.g., Michaely & Perets 2019). The lifetime of a triple may be significantly shorter than $10^{10}$ yr, depending on the stellar density of the surroundings. To examine mergers before the disruption of triples, we separate the mergers by the merger time ($T_{m} < 10^{8}$ yr, $10^{6}$ yr < $T_{m} < 10^{8}$ yr, and $T_{m} > 10^{8}$ yr). Figures 14 and 15 show that fast mergers (with shorter $T_{m}$) tend to have a higher probability of being accompanied by eccentric orbits at 10 Hz (i.e., $e_{m} > 0.1$). This is more evident for BH–NS binaries (Figure 15), where the octupole terms

Figure 11. The distribution of $|\cos \theta_{d1}^{f} - \cos \theta_{d2}^{f}|$ for merging BH–BH binaries, where the data come from Figure 7. The dashed line represents Equation (17), obtained assuming uncorrelated isotropic spin distributions.
4. Summary and Discussion

4.1. Summary of Key Results

In this paper, we systematically studied the dynamical signatures of BH–BH and BH–NS mergers induced by tertiary companions in triple systems, emphasizing the detectable merger eccentricities and spin–orbit misalignments when the inner binary enters the LIGO sensitivity band (>10 Hz). Going beyond our previous works (Liu & Lai 2017, 2018), we examined a wide range of triple systems to explore the dependence of the merger properties on the binary/triple parameters. More specifically, for two types of binaries, with \((m_1, m_2) = (30M_\odot, 20M_\odot)\) (representing a canonical BH–BH binary) and \((30M_\odot, 1.4M_\odot)\) (representing a canonical BH–NS binary), we considered all possible binary/triple configurations and parameters that lead to binary mergers, and determined the distributions of merger times, eccentricities, and spin–orbit misalignments. We used both single-averaged and double-averaged secular equations that include octupole terms and spin–orbit coupling (already presented in Liu & Lai 2018), as well as a newly developed N-body code based on the ARCHAIN algorithm, to evolve systems with various degrees of hierarchy. This allowed us to efficiently cover a wide range of parameter space and to determine the merger properties reliably.

We first explored what kind of initial triples can produce BH–BH and BH–NS mergers within \(10^{10}\) yr. In addition to significant tertiary inclinations (as required for the LK effect to be efficient), tertiary-induced mergers also require that the initial tertiary–binary separation ratio lies in the range \(5 \lesssim \tilde{a}_{out}/a_0 \lesssim 100\), or more generally, the ratio between the scaled outer semimajor axis to the inner one, \(\tilde{a}_{out,eff}/a_0\) (see Equation (13)), ranges from 1 to 20 (see Figures 3 and 5). The stability criterion of the triple and an analytical “limiting” merger time expression (Equation (14)) provide an excellent characterization of the parameter space leading to mergers (see Figures 4 and 6).

Our studies revealed several distinct dynamical signatures of the tertiary-driven binary merger scenario. For merging binaries with comparable masses (i.e., BH–BH binaries), we found that about 7% of the mergers have eccentricities \(e_m > 0.1\) at 10 Hz, and 0.7% have \(e_m > 0.9\). Distant tertiary companions (with negligible octupole effects; see Equation (4)) tend to generate spin–orbit misalignments \(\theta_{sl}^i\) around 90° and negligible \(e_m\) (see Figures 7 and 12). Closer tertiary companions (with stronger octupole effects) produce a more isotropic distribution of \(\theta_{sl}^i\) and nonnegligible \(e_m\). Note that the misalignment angles \(\theta_{sl}^i\) of the two BHs are correlated (see Figure 11). We also computed the distribution of the mass-weighted spin parameter \(\chi_{eff}\)
For merging binaries with a high mass ratio (e.g., BH–NS binaries), we found a large fraction of systems with significant merger eccentricities as a result of the strong octupole effects: about 18% have $e_m > 0.1$ and 2.5% have $e_m > 0.9$. Thus, the residual eccentricity at 10 Hz could indeed serve as an indicator for such tertiary-driven binary mergers. On the other hand, we found that the final spin–orbit misalignments have an approximately isotropic distribution (with an insignificant 90° peak), except for the weak octupole systems that generate mergers with negligible residual eccentricities (see Figures 8 and 13).

Overall, our study showed that a combination of detections of $e_m$, $\theta_{sl}$, and $\chi_{sl}$ from future LIGO/VIRGO observations would provide key information to determine or constrain the formation channels of merging BH–BH and BH–NS binaries.

4.2. Discussion

Although in this paper we focused on two types of binaries with specific masses—these masses can be measured from LIGO/VIRGO observations—our survey of parameter space for the initial binaries/triples was extensive. The statistical properties of tertiary-driven mergers found in this paper can be extended to other merging binaries with various mass ratios.

We found that a small fraction of merging binaries in triples enter the LIGO band with appreciable eccentricities (7% of BH–BH binaries and 18% of BH–NS binaries have $e_m > 0.1$). However, almost all such binaries pass through the LISA band with high eccentricities (see also Samsing & D’Orazio 2018; Fang et al. 2019; Hoang et al. 2019; Kremer et al. 2019; Randall & Xianyu 2019). Thus, joint observations by LIGO/VIRGO and LISA, where the information on the BH spin and eccentricity could be extracted, will be very useful to constrain the formation mechanisms of merging binaries, especially for the tertiary-driven scenario.

We focused on triple systems in this paper. For a quadruple system, binary–binary interactions can significantly enhance the binary merger fraction (e.g., Hamers & Lai 2017; Fang et al. 2018; Hamers 2018; Fragnone & Kocsis 2019; Liu & Lai 2019). Although the occurrence rate of stellar quadruples is smaller than that of stellar triples, dynamically induced BH mergers in quadruple systems may be an important channel for producing BH mergers (Liu & Lai 2019). A systematic study of the dynamical signatures of this channel is beyond the scope of this paper, although we expect that many similar features found here may carry over. As shown in Liu & Lai (2019), if the parameters of a quadruple system satisfies a certain resonance criterion, the inner binary eccentricity can be driven close to unity in a chaotic way, leading to similar behaviors for $\theta_{sl}$ and $e_m$ as in the cases of triples with strong octupole effects (i.e., random $\cos \theta_{sl}$ distribution and large $e_m$).

We thank Bonan Pu for useful discussions. This work is supported in part by the NSF grant AST-1715246 and NASA grant NNX14AP31G. B.L. is also supported in part by grants from NSFC (Nos. 11703068 and 11661161012). This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Shanghai Astronomical Observatory.

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Figure 14. Upper panel: the distribution of the residual eccentricity for BH–BH mergers with different ranges of merger times (color-coded), normalized by the number of mergers in each category of $T_m$. Lower panel: the correlation between the merger time and residual eccentricity for each merger. The symbols indicate the mergers achieved by the DA, SA, and NB integrations (same as Figure 3).

Figure 15. Same as Figure 14, but for BH–NS binaries.

(Equation (15)) of merging BH binaries in triples. Although $\chi_{eff}$ can have a wide range of values, there still exists a characteristic shape with peak around $\chi_{eff} \approx 0$ in its distribution (see Figure 10). This could serve as an indicator of tertiary-induced binary mergers.
