Testing Anomalous Gauge Couplings of the Higgs Boson via Weak-Boson Scatterings at the LHC

Bin Zhang
Department of Physics, Tsinghua University, Beijing 100084, China.

Yu-Ping Kuang
China Center of Advanced Science and Technology (World Laboratory), P.O.Box 8730, Beijing 100080, China; Department of Physics, Tsinghua University, Beijing 100084, China.

Hong-Jian He
Center for Particle Physics, University of Texas at Austin, Austin, Texas 78712, U.S.A.

C.–P. Yuan
Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, U.S.A.

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We propose a sensitive way to test the anomalous $HVV$ couplings ($V = W^\pm, Z^0$) of the Higgs boson ($H$), which can arise from either the dimension-3 effective operator in a nonlinearly realized Higgs sector or the dimension-6 effective operators in a linearly realized Higgs sector, via studying the $VV$ scattering processes at the CERN LHC. The gold-plated pure leptonic decay modes of the final state weak bosons in the processes $pp \to VV jj$ are studied. For comparison, we also analyze the constraints from the precision electroweak data, the expected precision of the measurements of the Higgs boson production rate, decay width and branching ratios at the Fermilab Tevatron Run-2 and the CERN LHC, and the requirement of unitarity of the $S$ matrix. We show that, with an integrated luminosity of 300 fb$^{-1}$ and sufficient kinematical cuts for suppressing the backgrounds, studying the process $pp \to W^+W^+jj \to \ell^+\nu\ell^+\nu jj$ can probe the anomalous $HWW$ couplings at a few tens of percent level for the nonlinearly realized Higgs sector, and at the level of (0.01 − 0.08) TeV$^{-1}$ for the linearly realized effective Lagrangian.

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I. INTRODUCTION

The standard model (SM) of the electroweak interactions has proven to be very successful in explaining all the available experimental data at the scale $\lesssim \mathcal{O}(100) \text{GeV}$. However, the mechanism of the electroweak symmetry breaking (EWSB) remains one of the most profound puzzles in particle physics. The Higgs sector of the SM suffers the well-known problems of triviality [1] and unnaturalness [2], therefore there has to be new physics beyond the SM above certain high energy scale $\Lambda$. Within the SM formalism, the precision electroweak data favors a light Higgs boson. If a light Higgs boson candidate $H$ is found in future collider experiments, such as the Run-2 of the Fermilab Tevatron, a 2 TeV $pp$ collider, or the CERN Large Hadron Collider (LHC), a 14 TeV $pp$ collider, the next important task is to experimentally measure the gauge interactions of this Higgs scalar and explore the nature of the EWSB mechanism. The detection of the anomalous gauge couplings of the Higgs boson will point to new physics beyond the SM underlying the EWSB mechanism.

Although the correct theory of new physics is not yet clear, the effect of any new physics at energy below the cutoff scale $\Lambda$ can be parametrized as effective interactions in an effective theory whose particle content is the same as the SM. This provides a model-independent description, and the anomalous couplings relative to that of the SM reflect the effect of new physics. In the present study, we assume that all possible new particles other than the lightest Higgs boson $H$ are heavy and around or above the scale $\Lambda$, so that only $H$ is relevant to the effective theory. Therefore, testing the effective anomalous gauge couplings of the Higgs boson can discriminate the EWSB sector of the new physics.
model from that of the SM. It has been pointed out that sensitive tests of the anomalous $HVV$ couplings (with $V = W^{\pm}$, $Z^{0}$) can be performed via Higgs boson productions at the future high energy $e^+e^-$ linear colliders (LC) [3–5]. The tests of the anomalous $HVV$ couplings at hadron colliders via the decay mode $H \to \gamma \gamma$ and $H \to \tau \tau$ have also been studied, and the obtained sensitivities are lower than that at the LC [6–8].

In this paper, following our recent proposal [9], we study an additional way to test the anomalous $HVV$ couplings via the weak-boson scatterings at the LHC. We shall show that at the LHC, rather sensitive tests of the anomalous $HVV$ couplings can be obtained by measuring the cross sections of the longitudinal weak-boson scattering, $V_LV_L \to V_LV_L$ ($V_L = W_L^{\pm}$, $Z_L^{0}$), especially $W_L^{+}W_L^{-} \to W_L^{+}W_L^{-}$. The scattering amplitude contains two parts: (i) the amplitude $T(V, \gamma)$ related only to the electroweak gauge bosons as shown in Fig. 1(a), and (ii) the amplitude $T(H)$ related to the Higgs boson as shown in Fig. 1(b). At high energies, both $T(V, \gamma)$ and $T(H)$ contain a piece increasing with the center-of-mass energy ($E$) as $E^2$ in the nonlinear realization and as $E^4$ in the linear realization. In the SM, though the individual diagram in Fig. 1(a) may contribute an $E^4$-dependent piece, the sum of all diagrams in Fig. 1(a) can have at most $E^2$-dependent contribution, which can be easily verified by an explicit calculation. Furthermore, the $HVV$ coupling constant in the SM is fixed to be the same as the non-Abelian gauge self-coupling of the weak bosons. This causes the two $E^2$-dependent pieces to precisely cancel with each other in the sum of $T(V, \gamma)$ and $T(H)$, resulting in the expected $E^0$-behavior for the total amplitude, as required by the unitarity of the SM $S$ matrix. If the $HVV$ couplings are anomalous due to the effect of new physics above the cutoff scale $\Lambda$, the total amplitude of $VV$ scatterings can grow as $E^2$ or $E^4$ in the high energy regime. Such deviations from the $E^0$ behavior of the SM amplitude can provide a rather sensitive test of the anomalous $HVV$ couplings in high energy $VV$ scattering experiments. This type of tests require neither the detection of the Higgs boson resonance nor the measurement of the Higgs boson decay branching ratios, and is thus of special interest. If the anomalous $HVV$ coupling associated with the new physics effect is rather large, the total decay width of $H$ may become so large that $H$ cannot be detected as a sharp resonance [10] and therefore escapes the detection when scanning the invariant mass of its decay products around $m_H$. In that case, can we tell whether or not there exists a sub-TeV Higgs boson? The answer is yes. It can be tested by carefully studying the scatterings of weak gauge bosons in the TeV region [12]. Furthermore, if the new physics causes a rather small $HVV$ coupling (much below the value of the SM $HVV$ coupling), the production rate of a light Higgs boson can become so small that it escapes the detection when the experimental measurement is taken around the Higgs boson mass scale. However, as mentioned above, the scattering of $V_LV_L$ has to become large in the TeV region accordingly. Therefore, this makes it important to study the $V_LV_L$ scattering in the TeV regime even in the case where a light Higgs boson exists in the EWSB sector. In Sec. IV, we show that this type of test of the anomalous $HVV$ couplings can be more sensitive than those obtained from studying the on-shell Higgs boson production and decays at the LHC [6–8], as well as the constraints derived from the precision electroweak data and the requirement of the unitarity of the $S$ matrix.

Therefore, studying the $V_LV_L$ scatterings at the LHC is not only important for probing the strongly interacting electroweak symmetry breaking sector without a light Higgs boson, but also valuable for sensitively testing of the anomalous gauge interactions of the Higgs boson when a light Higgs scalar exists in the mass range $115–300$ GeV. This further supports the “no-lose” theorem [11] for the LHC to decisively probe the EWSB mechanism.

This paper is organized as follows. In Sec. II, we briefly sketch a few key points related to the calculations of the $VV$ scatterings discussed in this paper. In Sec. III, we systematically study the test of the anomalous $HVV$ coupling from the dimension-3 operator in a nonlinearly realized Higgs sector [13], as an extension of [9]. The test of the anomalous $HVV$ couplings from the dimension-6 operators in the linearly realized Higgs sector [14,15] is studied in Sec. IV. In both cases, the constraints on the anomalous $HVV$ couplings from the precision electroweak data and the requirement of the unitarity of the $S$ matrix are also discussed. Section V summarizes our concluding remarks.

II. VV SCATTERINGS

As mentioned in the previous section, we will take the enhanced $VV$ scatterings as the signals for testing the anomalous $HVV$ couplings. We choose the gold-plated pure leptonic decay modes of the final state weak bosons as the tagging modes, this will avoid the large hadronic backgrounds at the LHC. Even in this case, there are still several kinds of backgrounds to be eliminated, namely the electroweak (EW) background, the QCD background, and the top quark background studied in Refs. [16,17]. At the LHC, the initial state $V$’s in the $VV$ scatterings are emitted by the quarks in the protons. As Refs. [16–19] pointed out, the QCD
background can be greatly suppressed by tagging a forward-going jet (the out-going quark after emitting the $V$). This forward-jet tagging will also suppress the transverse component ($V_T$) of the initial $V$, so that the initial state $V$ will be essentially $V_L$. Following Refs. [16,17], we impose, in addition to forward jet tagging, the requirement of vetoing the central jet to further suppress the QCD background and the top quark background. The EW background can be suppressed by detecting isolated leptons with large transverse momentum in the central rapidity region, especially requiring the two final state leptons to be nearly back to back. These leptonic cuts also suppress the $V_T$ contributions in the final state. Moreover, choosing the decay leptons in the central rapidity region also avoids the collinear divergence in the diagrams exchanging a photon in $T(V, \gamma)$.

In this paper, we shall calculate the complete tree level contributions to the processes

$$ pp \rightarrow VVjj, \quad (1) $$

where $j$ is the forward jet. We shall impose all the cuts mentioned above, and use the updated CTEQ6L [20] parton distribution functions for the distributions of quarks in the protons. We also take into account the effect of the width of the weak boson in calculating the helicity amplitudes.

References [16,17] studied various strongly interacting models which do not consist of a light Higgs boson and the $V_LV_L \rightarrow V_LV_L$ scattering amplitudes are largely enhanced (by powers of $E^2$) in the high energy region. The signal amplitudes were calculated in the effective $W$ approximation (EWA) [21] because the $V_LV_L \rightarrow V_LV_L$ scattering amplitudes predicted by those models violate the unitarity condition of a $2 \rightarrow 2$ scattering matrix so that a full $2 \rightarrow 4$ process $pp \rightarrow V_LV_Ljj$ could not be reliably calculated to predict the signal event rates in the TeV region. Instead, the $V_LV_L \rightarrow V_LV_L$ scattering amplitudes were properly unitarized before they were convoluted with the $W$-boson luminosities obtained from the EWA to predict the signal event rates. With this method of calculation, it was shown that after imposing the kinematic requirements discussed above, the backgrounds are reasonably suppressed relative to the signals, so that the $V_LV_L$ scatterings signals can be effectively extracted.

As to be discussed in Secs. III and VI, in our present case, the Higgs boson is light, and the new physics effect is assumed to only modify the effective operators in either a nonlinearly or a linearly realized Higgs sector. In the nonlinearly realized case, the $V_LV_L$ scattering amplitudes with a not too small anomalous $HVV$ couplings are also enhanced in the high energy region due to the $E^2$ behavior of the amplitudes. Hence, we may apply the same methodology as proposed in Ref. [17] for calculating the signal rate. However, in this work, we shall not choose using the EWA. Instead, we shall compute the full $pp \rightarrow VVjj$ cross sections at the tree level, cf. Eq. (1), which is justified as long as the LHC sensitivity to the size of the anomalous $HVV$ coupling is smaller than that required to render the unitarity of the $V_LV_L \rightarrow V_LV_L$ partial wave amplitudes. As to be shown in Secs. IV and VI, this is indeed the case and the backgrounds can be reasonably suppressed relative to the signals by imposing the same kinematical cuts suggested in Ref. [17]. On the contrary, in the case of the SM (without anomalous $HVV$ couplings), all the $V_LV_L$, $V_TV_L$, and $V_TV_T$ amplitudes behave as $E^0$ rather than $E^2$. Because the probability of finding a transverse vector boson with high momentum is much larger than a longitudinal vector boson [21], the contribution of $V_TV_L \rightarrow V_LV_L$ to the production rate of $pp \rightarrow VVjj$ in the TeV region is small as compared to the $V_TV_L$ and $V_TV_T$ contributions. Therefore, although the imposed kinematics cuts can effectively suppress the QCD and the top quark backgrounds, they leave a considerable EW background (mainly originated from the $V_TV_L$ and $V_TV_T$ contributions) that dominates the $VV$ scattering in the SM. Throughout this paper, we will call these $VV$ scattering contributions the remaining SM electroweak backgrounds after imposing the kinematical cuts. We shall do a complete tree-level calculation for both the signal and the background processes with the improvement on the simulation of the kinematical distributions of the decay leptons, and the cross section calculation with the updated parton distribution functions [20] and the inclusion of the vector boson width. Furthermore, we do not apply the EWA [21] in our calculations, hence, our results are not identical to those given in Ref. [17], even for the SM case.

In the case of a linearly realized effective Lagrangian, the physics consideration is quite different. As to be shown in Sec. VI, the $V_LV_L \rightarrow V_LV_L$ scattering amplitudes can have not only the $E^2$ but also the $E^4$ contributions depending on the process and operator under consideration. Furthermore, both the $V_TV_T \rightarrow V_LV_L$ and $V_LV_T \rightarrow V_LV_T$ scattering amplitudes can also have the $E^2$ contributions, which should be undoubtedly counted as part of the signal rates because those contributions are absent in the SM. Because the luminosity of $V_T$ is larger than the $V_L$, the contributions from $V_TV_T$ and $V_TV_L$ scatterings cannot be ignored unless the $E^4$ contributions dominate the $E^2$ contributions which occur only when both the energy $E$ and the anomalous couplings become large. It implies that including only the $V_LV_L \rightarrow V_LV_L$ contribution with the
EWA, as done in Ref. [17], is not adequate in this case, and a full $2 \to 4$ calculation should be used to calculate the signal rates. Although it is possible to apply the EWA to include also the $V_L V_T$ and $V_T V_T$ contributions [22] with jet tagging efficiencies (for $V_L$ and $V_T$, separately) extracted from the study done in Ref. [17], we choose to perform a full $pp \to V V jj$ tree-level calculation which is justified as long as the unitarity condition is satisfied. Nevertheless, as to be discussed in the end of Sec. VI, we shall apply the EWA, folded with the $V V \to V V$ scattering amplitudes, to check the high energy behavior of the full $2 \to 4$ amplitudes which are also verified to be gauge invariant.

III. TESTING ANOMALOUS HVV COUPLING FROM DIMENSION-3 OPERATOR

A. The Anomalous HVV Coupling from Dimension-3 Operator

It is known that there is no anomalous $HVV$ coupling arising from the dimension-3 and dimension-4 gauge invariant operators in the linearly realized Lagrangian [14,15]. Here, we consider the non-linearly realized Higgs sector formulated in Ref. [13]. When calculating radiative corrections using the effective Lagrangian (2), it is generally necessary to introduce higher dimensional counter terms to absorb the new divergences arising from the loop integration. There are in principle three next-to-leading order (NLO) counter terms to render the $(S, T, U)$ parameters finite at the one-loop level, namely [25]

$$\mathcal{L}^{(2)'} = \ell_0 \frac{v^2}{16\pi^2} \left[ |Tr(T D_\mu \Sigma) \Sigma'|^2 \right],$$

$$\mathcal{L}^{(4)}_1 = \ell_1 \frac{v^2}{\Lambda^2} [Tr[B_\mu \Sigma^\dagger \Sigma W^\mu W^-]$$

where $\mathcal{L}^{(2)'}$ and $\mathcal{L}^{(4)}_1$ are dimensionless coupling constants, respectively. The SM corresponds to $\kappa = \kappa' = 1$ and $\lambda_3 = \lambda_4 = \lambda = 3m_W^2/v^2$.

We note that at the tree level, only the dimension-3 operator $\frac{1}{2} \kappa v H D_\mu \Sigma D^\mu \Sigma$ in Eq. (2) contains the anomalous $HVV$ coupling that can contribute to the $VV$ scatterings (cf. Fig. 1). Therefore, $VV$ scatterings can test this dimensionless coupling $\kappa$, and $\Delta \kappa \equiv \kappa - 1$ measures the deviation from the SM value $\kappa = 1$.

B. Precision Constraints on the Coupling $\kappa$

Equation (2) shows an important difference between the non-linearly and the linearly realized Higgs sectors. The nonlinear formalism allows new physics to appear in the effective operators with dimension $\leq 4$ whose coefficients are not necessarily suppressed by the cutoff scale $\Lambda$. Hence the gauge couplings of Higgs boson to weak bosons can naturally deviate from the SM values by an amount at the order of $\lesssim O(1)$, as suggested by the naive dimensional analysis [23].

In the unitary gauge, the anomalous couplings of Higgs boson to gauge bosons relevant to the precision oblique parameters $(S, T, U)$ [24] are

$$\frac{4(\kappa - 1)vH + 2(\kappa' - 1)H^2}{v^2}$$

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where coefficients ($\ell_0, \ell_1, \ell_8$) correspond to the oblique parameters $(T, S, U)$, respectively. Here $W_{\mu \nu} = \bar{W}_{\mu \nu}$, $\bar{\tau}^2/2$, $B_{\mu \nu} = B_{\mu \nu} \tau_3/2$, and $T \equiv \Sigma_3 \Sigma^\dagger$. Comparing to $\mathcal{L}^{(4)}_1, \mathcal{L}^{(4)}_8$ is of the same dimension, but with two new $SU(2)_c$-violating operators $T$, so that we expect $\ell_8/\ell_1 \sim 1/16\pi^2 \sim 10^{-2} \ll 1$. This generally leads to $U \ll S, T$. To estimate the contribution of loop corrections, we invoke a naturalness assumption that no fine-tuned accidental cancellation occurs between the leading logarithmic term and the constant piece of the
counter terms. Thus, the leading logarithmic term represents a reasonable estimate of the loop corrections. This approach is commonly used in the literature for estimating new physics effects in effective theories [26]. It is straightforward to compute the radiative corrections to $S$, $T$, and $U$, arising from the $HV$ anomalous couplings, using dimensional regularization in the $\overline{MS}$ scheme and keeping only the leading logarithmic terms. After subtracting the SM Higgs contributions ($\kappa = 1$) with the reference value $m_H^{\text{ref}}$, we find

$$\Delta S = \frac{1}{6\pi} \left[ \ln \frac{m_H}{m_H^{\text{ref}}} - (\kappa^2 - 1) \ln \frac{\Lambda}{m_H} \right],$$

$$\Delta T = \frac{3}{8\pi c_w^2} \left[ - \ln \frac{m_H}{m_H^{\text{ref}}} + (\kappa^2 - 1) \ln \frac{\Lambda}{m_H} \right],$$

$$\Delta U = 0,$$  \hspace{1cm} (5)

where the terms containing $\ln \Lambda$ represent the genuine new physics effect arising from physics above the cut-off scale $\Lambda$ [27]. Note that at the one-loop order, the coupling $\kappa'$ has no contribution to the $S$, $T$ and $U$ parameters. In the SM ($\kappa = 1$), an increase of the Higgs mass will increase $\Delta S$ and decrease $\Delta T$. Choosing the reference Higgs mass $m_H^{\text{ref}}$ to be $m_H$, we may further simplify Eq. (5) as

$$\Delta S = \frac{\kappa^2 - 1}{6\pi} \ln \frac{\Lambda}{m_H},$$

$$\Delta T = \frac{3(\kappa^2 - 1)}{8\pi c_w^2} \ln \frac{\Lambda}{m_H},$$

$$\Delta U = 0.$$  \hspace{1cm} (6)

For a given value of $\Lambda$ and $m_H$, we find $\Delta S < 0$ and $\Delta T > 0$ for $|\kappa| > 1$. This pattern of radiative corrections allows a relatively heavy Higgs boson to be consistent with the current precision data [cf. Fig. 2(a)]. Furthermore, from Eq. (6) we see that the loop contribution from $\kappa \neq 1$ results in a sizable ratio of $\Delta T/\Delta S = -9/(4\kappa^2) \approx -3$.

Within the framework of the SM, the local fit to the current precision electroweak data favors a light Higgs boson with a central value $m_H = 83$ GeV (significantly below the CERN $e^+e^-$ collider LEP2 direct search limit $m_H > 114.3$ GeV [28]) and a 95% C.L. limit, $32$ GeV $\leq m_H \leq 192$ GeV, on the range of the Higgs boson mass. However, it was recently shown that in the presence of new physics, such a bound can be substantially relaxed [29–33]. The latest NuTeV data was not included in the above analysis. If we include the NuTeV data, the value of the minimum $\chi^2$ of the global fit increases substantially (by 8.7), showing a poor quality of the SM fit to the precision data (this fit also gives a similar central value $m_H = 85$ GeV and 95% C.L. mass range 33 GeV $\leq m_H \leq 200$ GeV).

We also find a similar increase of $\chi^2$ (by 8.9) in the $\Delta S - \Delta T$ fits, which implies that the NuTeV anomaly may not be explained by the new physics effect from the oblique parameters $(\Delta S, \Delta T)$ alone. As the potential problems with the NuTeV analysis are still under debate [34], we will not include the NuTeV data in the following analysis. But for comparison, we have also displayed a 95% C.L. contour (dotted curve) from the fit including the NuTeV anomaly in Fig. 2(a). In Fig. 2(a), we show the $\Delta S - \Delta T$ bounds (setting $m_H^{\text{ref}} = 100$ GeV) derived from the global fit with the newest updated electroweak precision data [35,36]. Furthermore, for $m_H^{\text{ref}} = 115 (300)$ GeV and $\Delta U = 0$, we find that the global fit gives

$$\Delta S = 0.01 (-0.07) \pm 0.09,$$

$$\Delta T = 0.07 (0.16) \pm 0.11.$$  \hspace{1cm} (7)

In the same figure, we also plot the SM Higgs boson contributions to $\Delta S$ and $\Delta T$ with different Higgs masses. Figure 2(b) shows that the upcoming measurements of the $W^\pm$ mass ($m_W$) and top mass ($m_t$) at the Tevatron Run-2 can significantly improve the constraints on new physics via the oblique corrections, in which we have input the current Run-1 central values of $(m_W, m_t)$ and the expected errors of $(m_W$ and $m_t)$ from the planned Run-2 sensitivity of 20 MeV and 2 GeV, respectively.

Using the allowed range of $(\Delta S, \Delta T)$ in Fig. 2(a), we can further constrain the new physics scale $\Lambda$ as a function of the anomalous coupling $\Delta \kappa$ for some typical values of $m_H^{\text{ref}} = m_H$ [cf. Eq. (6)]. Figure 3 depicts these constraints. With Fig. 3, we can alternatively constrain the range of $\Delta \kappa$ for a given value of $(\Lambda, m_H)$, which is summarized in Table I. Figure 3 and Table I show that for $m_H \gtrsim 250 - 300$ GeV, the $\Delta \kappa < 0$ region is fully excluded, while a sizable $\Delta \kappa > 0$ is allowed so long as $\Lambda$ is relatively low. Moreover, for $m_H \gtrsim 800$ GeV, the region $\Lambda < 1.1$ TeV is excluded. For the range of $m_H \gtrsim 250 – 300$ GeV, the preferred values of $\Delta \kappa > 0$ require the $HW^+W^-$ and $HZZ$ couplings to be stronger than those in the SM.

In this case, the direct production rate of the Higgs boson, via either the Higgs-strahlung or the $VV$ fusions in high energy collisions, should raise above the SM rate. On the other hand, for $m_H \lesssim 250$ GeV, the direct production rate of the Higgs boson can be smaller or larger than the SM rate depending on the sign of $\Delta \kappa$. Hence, when $\Delta \kappa < 0$, a light nonstandard Higgs boson may be partially hidden by its large SM background events. However, in this situation, the new physics scale $\Lambda$ will be generally low. For instance, when $m_H = 115$ GeV, a negative $\Delta \kappa = -0.15 (-0.28)$ already forces $\Lambda \lesssim 1 (0.4)$ TeV. Finally, we comment that, for a certain class of models, new physics effects
may also be induced by extra heavy fermions such as in the typical top-seesaw models with new vector-like fermions [29,37] or models with new chiral families [31]. In these models, there can be generic positive contributions to \( \Delta T \), so that the \( \Delta \kappa < 0 \) region may still be allowed for a relatively heavy Higgs boson, but such possibilities are very model-dependent. In our current effective theory analysis, we consider the bosonic HVV couplings as the dominant contributions to the oblique parameters, and assume that other possible anomalous couplings (such as the deviation in the gauge interactions of \( tbW/ttZ \) [38]) can be ignored. However, independent of these assumptions, the most decisive test of the HVV couplings can come from the direct measurements via Born-level processes at the high energy colliders, which is the subject of the next two sections.

### C. Constraints on \( \kappa \) from Unitarity Requirement

Before concluding this section, we discuss the possibility of unitarity violation in the scattering process \( pp \to W^+W^+jj \to \ell\nu\ell\nu jj \) for \( \kappa \neq 1 \), whose leading contribution comes from the subprocess \( W_L^+W_L^- \to W_L^+W_L^- \) when the initial \( W_L^- \) bosons are almost collinearly radiated from the incoming quarks or antiquarks. The scattering amplitude of \( W_L^+W_L^- \to W_L^+W_L^- \) contributes to the isospin \( I = 2 \) channel, and in the high energy region (\( E^2 \gg m_W^2, m_H^2 \)), it is dominated by the leading \( E^2 \)-contributions, and

\[
T[I = 2] \simeq (\kappa^2 - 1)\frac{E^2}{v^2}.
\]

Using the partial-wave analysis, the \( s \) wave amplitude \( a_{l,j=2,0} \) is found to be

\[
a_{20} \simeq (\kappa^2 - 1)\frac{E^2}{16\pi v^2},
\]

where \( E = M_{VV} \) is the invariant mass of the vector boson pair. The unitarity condition for this channel is\(^1\) \( |\text{Re } a_{20}| < 2!/2 = 1 \), in which the factor 2! is due to the identical particles \( (W^+W^+) \) in the final state. This results in a requirement that

\[
\sqrt{1 - \frac{16\pi v^2}{E^2}} < |\kappa| < \sqrt{1 + \frac{16\pi v^2}{E^2}}.
\]

For example,

\[
|\kappa| < 3.6, \quad \text{for } E = 0.5 \text{ TeV},
\]
\[
0.5 < |\kappa| < 1.3, \quad \text{for } E = 2 \text{ TeV},
\]
\[
0.8 < |\kappa| < 1.2, \quad \text{for } E = 3 \text{ TeV}.
\]

As to be shown in the next section, the expected sensitivity of the LHC in determining \( \Delta \kappa \) [cf. Eq. (13)] is consistent with the unitarity limit since the typical invariant mass of the \( W^+W^+ \) pair, after the kinematic cuts, falls into the range \( 500 \text{ GeV} \leq E \leq 2 \sim 3 \text{ TeV} \). The contributions from higher invariant mass values are severely suppressed by parton luminosities [17,39], and are thus negligible.

### IV. Testing \( \kappa \) Via VV Scatterings at the LHC

Knowing the above constraints, we turn to the analysis of testing \( \kappa \) via VV scatterings at the LHC. Following the procedures described in Sec. II, we calculate the tree-level cross sections of the scattering processes

\[
pp \to Z_L Z_L jj \to \ell^+\ell^-\ell^+\ell^- jj, 
\]
\[
pp \to W_L^+W_L^- jj \to \ell^+\nu\ell^-\bar{\nu} jj, 
\]
\[
pp \to W_L^+W_L^+ jj \to \ell^+\nu\ell^-\bar{\nu} jj, 
\]
\[
pp \to W_L^-W_L^- jj \to \ell^-\bar{\nu}\ell^+\nu jj, 
\]
\[
pp \to Z_L W_L^- jj \to \ell^+\ell^-\bar{\nu} jj, 
\]
\[
pp \to Z_L W_L^- jj \to \ell^+\ell^-\nu jj. 
\]

In our numerical calculations, we shall take the same kinematic cuts as those proposed in Ref. [17] to suppress the backgrounds discussed in Sec. II. It has been shown in Table II of Ref. [17] that, after imposing the kinematical cuts, the sum of the cross sections of the QCD background and the top quark background is smaller than the cross section of the remaining electroweak background by one order of magnitude. Thus, the EW background needs to be studied in more detail. For that, we have examined the polarization of the final state \( W^+ \) bosons. In Table II, we list the cross section of \( pp \to W_L^+W_L^- \) at the LHC, for various values of \( \Delta \kappa \) with \( m_H = 115 \text{ GeV} \), where \( W_L^+ \) denotes a polarized \( W \) boson with the polarization index \( \lambda = T \) or \( L \). As expected, when \( |\Delta \kappa| \) is large, the \( W_L^+W_L^- \) production rate dominates, and all the SM backgrounds, after the kinematical cuts, are reasonably suppressed relative to the signal due to the \( E^2 \)-dependence of the \( W_L^+W_L^- \) amplitude. However, for \( \Delta \kappa = 0 \) (the SM case), although the QCD and top quark backgrounds are negligibly small, the EW background contributed from the \( TT \) and \( LT \) polarizations

\[\ldots\]

\[\ldots\]
are still quite large as compared to the signal, cf. Table II. Therefore, in this work, we shall take the cross section for \( \Delta \kappa = 0 \) (the SM case) as the remaining EW background cross section.

In Tables III–VI, we list the obtained numbers of events (including both the signals and backgrounds) for an integrated luminosity of 300 fb\(^{-1}\) with the parameters 115 GeV \( \leq m_H \leq 300 \) GeV and \(-1.0 \leq \Delta \kappa \leq 0.7\). In general, the number of events for \( \kappa > 0 \) and \( \kappa < 0 \) are not the same. However, the result of our calculations shows that the difference between them is very small. Thus we only list the number of events with \( \Delta \kappa \geq -1 \) (\( \kappa \geq 0 \)) in the Tables. We see that the most sensitive channel to determine the anomalous coupling \( \Delta \kappa \) is \( pp \rightarrow W^+ W^- jj \rightarrow l^+ \nu l^- \nu jj \) (cf. Table III) due to its small background rates, and thus the weakest kinematic cuts \cite{17}.

It is easy to check that the numbers in Tables V and VI, for the \( ZW^\pm \) and \( ZZ \) channels, are consistent with the unitarity bound. Only the \( W^+ W^- \) channel, cf. Table IV, needs further unitarization \cite{16,17}. It is expected that after unitarizing the scattering amplitudes, the corresponding numbers in Table IV will become smaller, and thus this channel is less interesting. Therefore in this paper we only take the best channel, the \( W^+ W^+ \) channel (cf. Table III), to constrain \( \Delta \kappa \).

As discussed above, we shall take the SM events (for \( \Delta \kappa = 0 \)) listed in Table III as the intrinsic SM electroweak background rate in our analysis, i.e., \( N_B \equiv N(\Delta \kappa = 0) \). We can then define the number of signal events (for \( \Delta \kappa \neq 0 \)) as \( N_S \equiv N(\Delta \kappa \neq 0) - N_B \). The total statistical fluctuation is \( \sqrt{N_S + N_B} \). To study the potential of the LHC in distinguishing the \( \Delta \kappa \neq 0 \) case from the SM, we also show the deviation of the signal from the total statistical fluctuation, \( N_S/\sqrt{N_S + N_B} \), in the parentheses in Tables III. The values of \( N_S/\sqrt{N_S + N_B} \) in Table III show that the LHC can limit \( \Delta \kappa \) to the range

\[
-0.3 < \Delta \kappa < 0.2
\]  

(13)

at roughly the \( (1 - 3)\sigma \) level if no anomalous coupling (\( \Delta \kappa \neq 0 \)) effect is detected.

As mentioned in Sec. I, if the new physics above the cutoff scale \( A \) happens to make \( \Delta \kappa \) negative and close to \( \Delta \kappa = -1 \) (\( \kappa \geq 0 \)), the Higgs production rate may become so small that the Higgs resonance may be difficult to detect. However, we see from Tables III that the event numbers of \( pp \rightarrow W^+ W^+ jj \rightarrow l^+ \nu l^- \nu jj \) for this \( \kappa \) value are much larger than those for the SM, so that we can clearly detect the \( \kappa \geq 0 \) effect via these processes without the need of detecting the resonance of a light Higgs boson. This is the clear advantage of this type of measurements when the resonant state of the Higgs boson is difficult to be directly detected experimentally.

V. OTHER POSSIBLE TESTS OF \( \kappa \) FROM FUTURE COLLIDER EXPERIMENTS

There can be other tests of \( \kappa \) from future collider experiments. After discussing a few relevant measurements available at the Fermilab Tevatron and the CERN LHC, we shall also comment on the potential of the future LC. They are given below in order.

A. Associate HV Production

First, let us consider the associate production of a Higgs boson and vector boson \( (H + V) \) at high energy colliders. The studies at LEP-2 concluded that the mass of the SM Higgs boson has to be larger than about 114.3 GeV \cite{28}. For a non-SM Higgs boson, this lower mass bound will be different. For example, in the supersymmetric SM, the \( H-Z-Z \) coupling is smaller than that of the SM by a factor of \( \sin(\alpha - \beta) \) or \( \cos(\alpha - \beta) \), depending on whether \( H \) denotes a light or a heavy CP-even Higgs boson, and the above-mentioned lower mass bound is reduced to about 90 GeV \cite{40}. If the mass of the SM Higgs boson is around 110 GeV, it can be discovered at the Tevatron Run-2. Assuming an integrated luminosity of 10 fb\(^{-1}\), the number of the expected signal events is about 27 and the background events about 258, according to the Table 3 (the most optimal scenario) of Ref. \cite{41}. Therefore, the 1\( \sigma \) statistical fluctuation of the total event is \( \sqrt{27 + 258} \sim 17 \). Consequently, at the 1\( \sigma \) level, \( 0.6 < |\kappa| < 1.2 \), and at the 2\( \sigma \) level, \( |\kappa| < 1.5 \). \(^2\)

\(^2\)Similarly, we estimate the bounds on \( |\kappa| \) for various \( m_H \) as follows:

\[
m_H (\text{GeV}) & \quad 1\sigma \quad 2\sigma \\
110 : & 0.6 < |\kappa| < 1.2, \quad 0 \leq |\kappa| \leq 1.5, \\
120 : & 0.4 < |\kappa| < 1.4, \quad 0 \leq |\kappa| \leq 1.6, \\
130 : & 0 \leq |\kappa| \leq 1.5, \quad 0 \leq |\kappa| \leq 1.8.
\]  

(14)

We see that the above limits on \( \kappa \) are weaker than that in Eq. (13). Of course, the above bounds can be further improved at the Tevatron Run-2 by having a larger integrated luminosity until the systematical error dominates the statistical error. The same process can also be studied at the LHC to test the anomalous coupling \( \kappa \). However, because of the much larger

\[^2\]This bound can be improved by carefully examining the invariant mass distribution of the \( b\bar{b} \) pairs.
background rates at the LHC, the improvement on the measurement of $\kappa$ via the associate production of $V$ and Higgs boson is not expected to be significant.

B. Measuring Total Decay Width of a Higgs Boson

The anomalous $HVV$ coupling can also be tested from measuring the total decay width of the Higgs boson. When the Higgs boson mass is larger than twice the $Z$ boson mass, it can decay into a $Z$ boson pair which subsequently decay into four muons. This decay channel is labeled as one of the “gold-plated” channels (the pure leptonic decay modes) because it is possible to construct the invariant mass of the $ZZ$ pair in this decay mode with a high precision, which provides a precision measurement on the total decay width of the Higgs boson. A detailed Monte Carlo analysis has been carried out in Ref. [42] to find out the uncertainty on the measurement of the Higgs boson width. Assuming that there is no non-SM decay channel opened and only the anomalous $HVV$ coupling is important, one can convert the conclusion from [42] to the bounds on $\Delta \kappa$. Since the decay branching ratio of $H \to W^+W^-$ or $ZZ$ for a SM heavy Higgs boson is large (almost 100% for $m_H > 300$ GeV), the total width measurement can give a strong constraint on $\kappa$. We define the accuracy on the determination of $\kappa$ using the relation $-n \Delta \Gamma \leq \Gamma(\kappa) - \Gamma(\kappa = 1) \leq n \Delta \Gamma$, where $n = 1, 2$ denoting the 1$\sigma$ and 2$\sigma$ accuracy, respectively, and $\Gamma(\kappa)$ is the width of the Higgs boson for a general value of $\kappa$, $\Gamma(\kappa = 1)$ is for a SM Higgs boson and $\Delta \Gamma$ is the experimental accuracy in measuring the Higgs boson width. From the Table 3 of Ref. [42], we find that at the LHC (with an integrated luminosity of 300 fb$^{-1}$ [43]), the bounds on $\Delta \kappa$ obtained from the measurement of the total decay width of a Higgs boson, via the process $pp \to H \to ZZ \to \mu^+\mu^-\mu^+\mu^-$, are as follows:

$$m_H \text{ (GeV) } \begin{array}{c|c|c} \text{1 } \sigma & \text{2 } \sigma \\ \hline 200 - 300 : & 0.9 \leq |\kappa| \leq 1.1, & 0.8 \leq |\kappa| \leq 1.2. \\ \end{array}$$

C. Decay Branching Ratio of $H \to \gamma\gamma$

Another important effect of a non-vanishing anomalous coupling $\kappa$ is to modify the decay branching ratio of $H \to \gamma\gamma$, and hence, the production rate of $gg \to H \to \gamma\gamma$ at the LHC. In Table VII, we list the decay branching ratio $B(H \to \gamma\gamma)$ as a function of $\Delta \kappa$ for various $m_H$. As shown, $B(H \to \gamma\gamma)$ decreases for $\Delta \kappa > 0$, and increases for $\Delta \kappa < 0$. For example, for a 120 GeV Higgs boson, $B(H \to \gamma\gamma)$ decreases by 14% for $\Delta \kappa = 0.2$ and increases by 45% for $\Delta \kappa = -0.3$.

D. Detecting a Higgs Boson Resonance

When the SM Higgs boson is light, it is expected that the production rate of $qq \to qqH$ with $H \to WW^*$ is large enough to be detected at the LHC, and this process can also be used to test the coupling of the $HWW$ [44] by studying the observables near the Higgs boson resonance. In the effective Lagrangian (2), the Lorentz structure (and the dimension) of the anomalous couplings $\kappa$ and $\kappa'$ is the same as the SM coupling. Therefore, the result of the study performed in Ref. [44] also holds for the current study when $m_H$ is less than twice the $Z$ boson mass. In this case, its production rate is $\kappa^2 S_{SM} B(\kappa)/B(\kappa = 1)$, where $S_{SM}$ is the SM rate, and $B(\kappa)$ is the decay branching ratio of $H \to WW^*$ for a general value of $\kappa$. Needless to say that $B(\kappa = 1)$ corresponds to the SM branching ratio.

When the $WW$ (and $ZZ$) decay mode dominates the Higgs boson decay, such as when $m_H$ is larger than twice $W$ boson mass, the ratio of the decay branching ratio for $H \to WW^*$, $B(\kappa)/B(\kappa = 1)$, would be close to 1. However, this ratio can be quite different from 1 when $m_H$ is small. For example, for $m_H = 120$ GeV, the ratio $B(\kappa)/B(\kappa = 1)$ is 0.68 and 1.4, respectively, for $\kappa = 0.8$ and $\kappa = 1.2$. From Table I of Ref. [44], one can determine the constraint on the ratio $\kappa^2 B(\kappa)/B(\kappa = 1)$, denoted as $R$, using the relation $S_{SM} - n \sqrt{B + S_{SM}} \leq R S_{SM} \leq S_{SM} + n \sqrt{B + S_{SM}}$, where $B$ is the background rate and $n = 1, 2$ denotes the 1$\sigma$ and 2$\sigma$ accuracy, respectively. Assuming an integrated luminosity of 300 fb$^{-1}$, we find that the bounds on $R$ for various $m_H$ are:

$$m_H \text{ (GeV) } \begin{array}{c|c|c} \text{1 } \sigma & \text{2 } \sigma \\ \hline 110 : & 0.87 \leq R \leq 1.13, & 0.74 \leq R \leq 1.26, \\ 120 : & 0.94 \leq R \leq 1.06, & 0.88 \leq R \leq 1.12, \\ 130 : & 0.97 \leq R \leq 1.03, & 0.93 \leq R \leq 1.07, \\ 150 : & 0.97 \leq R \leq 1.03, & 0.94 \leq R \leq 1.06, \\ 170 : & 0.98 \leq R \leq 1.02, & 0.95 \leq R \leq 1.05. \\ \end{array}$$

(16)

To extract the bound on $\kappa$, we need to know the decay branching ratio $B(\kappa)$ of $H \to WW^*$ assuming that the effective Lagrangian (2) differs from the SM Lagrangian only in the coefficient $\kappa$. A few values of $B(\kappa)$ as a function of $\kappa$ for various $m_H$ are given in Table VIII. The result of Table VIII and Eq. (16) indicates that $\kappa$ can be measured at a few percent level when a light Higgs boson is detected. However, in our opinion, this conclusion seems to be too optimistic because the systematical error of the experiment at the LHC is expected to be at a similar level of accuracy.
E. At the Linear Collider

Before closing this section, we note that the anomalous coupling of \( HZZ \) and \( HW^+W^- \) can also be tested at the LC via the processes \( e^-e^+ \rightarrow ZH \rightarrow bb \), \( e^-e^+ \rightarrow e^-e^+H \rightarrow bb \) via ZZ fusion, and \( e^-e^+ \rightarrow \nu\bar{\nu}H \rightarrow bb \) via \( W^+W^- \) fusion. In Refs. [3,4], it was concluded that \( \Delta \kappa \) can be determined better than a percent level for a 120 GeV SM-like Higgs boson produced from the above processes, assuming an integrated luminosity of 1 ab\(^{-1}\) for a 500 GeV (or 800 GeV) LC. After converting the notation in Ref. [4] to ours, the 2\( \sigma \) error in measuring \( \Delta \kappa \) is found to be at the level of 0.3\%. Thus the expected precision in the measurement of \( \Delta \kappa \) at a high luminosity LC is higher than that at the LHC, in the case of a SM-like Higgs boson (i.e., the decay branching ratio of \( H \rightarrow bb \) is about 1). Even in the case that the Higgs boson is not SM-like, the LC can still determine \( \Delta \kappa \) by studying the Higgs-strahlung process \( e^-e^+ \rightarrow Z(\ell^+\ell^-)H \) and the ZZ fusion process \( e^-e^+ \rightarrow e^-e^+H \), where \( H \) decays into anything [5]. However, due to its much smaller rate, the sensitivity to \( \Delta \kappa \) is lower than the SM-like case. In the next section, we shall show that the precision of determining the dimension-6 anomalous couplings via \( VV \) scatterings at the LHC will be comparable to the level of the precision expected at the LC.

VI. TESTING ANOMALOUS HVV COUPLINGS FROM DIMENSION-6 OPERATORS

A. Anomalous HVV Couplings from Dimension-6 Operators

Now we consider the test of the anomalous \( HVV \) couplings from the dimension-6 operators arising from the linearly realized effective Lagrangian. In the linearly realized effective Lagrangian, there is no gauge invariant anomalous operators at dimension-3 and dimension-4, and the leading anomalous \( HVV \) couplings are from the effective operators of dimension-6 [14,15]. (All the gauge invariant operators with dimension 4 or less have been included in the SM Lagrangian.) In the following, we shall study the test of these leading order anomalous couplings. As is shown in Refs. [14,15], the \( C \) and \( P \) conserving effective Lagrangian up to dimension-6 operators containing a Higgs doublet \( \Phi \) and the weak bosons \( V^a \) is given by

\[
\mathcal{L}_{\text{eff}} = \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n ,
\]

where \( f_n \)’s are the anomalous coupling constants. The operators \( \mathcal{O}_n \)’s are [14,15]

\[
\begin{align*}
\mathcal{O}_{\Phi,1} &= (D_\mu \Phi)\Phi^\dagger D^\mu \Phi, \\
\mathcal{O}_{BW} &= \Phi^\dagger \tilde{B}_{\mu\nu} W^{\mu\nu} \Phi, \\
\mathcal{O}_{DW} &= \text{Tr} \left( [D_\mu, W_{\nu\rho}] \left[ D^\mu, W^{\nu\rho} \right] \right), \\
\mathcal{O}_{DB} &= -\frac{g^2}{2} \left( \partial_\mu B_{\nu\rho} \right) \left( \partial^\mu B^{\nu\rho} \right), \\
\mathcal{O}_{\Phi,2} &= \frac{1}{2} \partial_\mu \left( \Phi \Phi^\dagger \right) \partial^\mu \left( \Phi \Phi^\dagger \right), \\
\mathcal{O}_{\Phi,3} &= \frac{1}{3} \left( \Phi \Phi^\dagger \right)^3 , \\
\mathcal{O}_{WWW} &= \text{Tr} \left[ \tilde{W}_{\mu\nu} \tilde{W}^{\mu\nu} \tilde{W}_{\mu} \right], \\
\mathcal{O}_{WW} &= \Phi^\dagger \tilde{W}_{\mu\nu} \tilde{W}^{\mu\nu} \Phi, \\
\mathcal{O}_{BB} &= \Phi^\dagger \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} \Phi, \\
\mathcal{O}_{W} &= \left( D_\mu \Phi \right) \tilde{W}^{\mu\nu} \left( D_\nu \Phi \right), \\
\mathcal{O}_{B} &= \left( D_\mu \Phi \right)^2 \tilde{B}^{\mu\nu} \left( D_\nu \Phi \right),
\end{align*}
\]

(18)

where \( \tilde{B}_{\mu\nu} \) and \( \tilde{W}_{\mu\nu} \) stand for

\[
\tilde{B}_{\mu\nu} = \frac{i g'}{2} B_{\mu\nu}, \quad \tilde{W}_{\mu\nu} = \frac{i g}{2} \sigma^a W^{\mu\nu}_a,
\]

(19)
in which \( g \) and \( g' \) are the \( SU(2) \) and \( U(1) \) gauge coupling constants, respectively.

At tree-level, the operators \( \mathcal{O}_{\Phi,1}, \mathcal{O}_{BW}, \mathcal{O}_{DW} \) and \( \mathcal{O}_{DB} \) in Eq. (18) affect the two-point functions of the weak boson \( V \) when \( \Phi \) is taken to be its vacuum expectation value:

\[
\Phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}.
\]

(20)

Thus they are severely constrained by the precision Z-pole and low energy data. In Ref. [15], it was concluded that at the 95\% level, in units of TeV\(^{-2}\), the magnitude of \( \frac{L_{WW}}{\mathcal{A}^2} \) or \( \frac{s_{BB}}{\mathcal{A}^2} \) is constrained to be less than 1, and \( \frac{L_{WW}}{\mathcal{A}^2} \) or \( \frac{L_{BB}}{\mathcal{A}^2} \) can be about a factor of 10 larger. We shall update these bounds in Sec. VIB. Since it will be very difficult to observe the effect of these operators in the high-energy observables, in what follows, we will neglect their effect when discussing the \( VV \) scatterings.

The two operators \( \mathcal{O}_{\Phi,1} \) and \( \mathcal{O}_{\Phi,2} \) in Eq. (18) lead to a finite renormalization of the Higgs boson wave function [15]. Similarly, the two operator \( \mathcal{O}_{\Phi,3} \) induces a finite renormalization of the Higgs potential [15]. In a recent paper [4], it was shown that after renormalizing the Higgs boson field so that the residue of its propagator at its mass pole is equal to one, two effective anomalous couplings of Higgs boson to gauge bosons are induced by the dimension-6 operator \( \mathcal{O}_{\Phi,2} \) as

\[
- \frac{f_{\Phi,2}(vH + H^2)}{\mathcal{A}^2} \left[ m_W^2 W^{\mu+} W^{-\mu} + \frac{1}{2} m_Z^2 Z^{\mu} Z_{\mu} \right].
\]
After converting the above anomalous couplings to the notation introduced in Sec. III for a non-linear effective theory, the anomalous coupling $\Delta \kappa$ corresponds to $-\frac{f_{\mu\nu} \gamma^\mu \gamma^\nu}{4A^2}$. For example, at a 500 GeV (or 800 GeV) LC with an integrated luminosity of 1 ab$^{-1}$, the 2$\sigma$ statistical error on the determination of the anomalous coupling $\Delta \kappa$ is at the level of 0.2 TeV$^{-2}$, for a 120 GeV SM-like Higgs boson [4]. This corresponds to the determination of $\Delta \kappa$ at about the 0.3% level. Since this operator can only generate $E^2$-dependence of the $V_L V_L \rightarrow V_L V_L$ scattering amplitude, and it is best determined at the LC for a SM-like Higgs boson, in what follows, we will neglect its effect in our studies.

The operator $O_{WW}$ contributes to the triple and quartic vector boson self-couplings, but not the anomalous coupling of Higgs boson to gauge bosons. On the other hand, the last four operators $O_{WW}$, $O_{BB}$, $O_W$ and $O_B$ in Eq. (18) contribute to the following anomalous $HVV$ couplings [15]:

$$\mathcal{L}_{\text{eff}}^{H} = g_{HZ\gamma} H \mu \nu \Lambda_{\mu \nu} + (g_{HZ\gamma}') H \mu \nu \Lambda_{\mu \nu} \partial^\nu H + \frac{1}{2} g_{HZ\gamma} H \mu \nu Z_{\mu \nu} \partial^\nu H + g_{HWW} (W_{\mu \nu} W_{\nu \mu} - W_{\nu \mu} W_{\mu \nu} + \text{h.c.}),$$

where

$$g_{HZ\gamma} = - \left( \frac{g_{w}}{\Lambda^2} \right) \frac{s^2 (f_{BB} + f_{WW})}{2},$$

$$g_{HZ\gamma}^{(1)} = \left( \frac{g_{w}}{\Lambda^2} \right) \frac{s (f_{WW} - f_{BB})}{2c},$$

$$g_{HZ\gamma}^{(2)} = \left( \frac{g_{w}}{\Lambda^2} \right) \frac{s c^2 f_{BB} - c^2 f_{WW}}{c},$$

$$g_{HWW}^{(1)} = \left( \frac{g_{w}}{\Lambda^2} \right) \frac{f_{WW}}{2c^2},$$

$$g_{HWW}^{(2)} = \left( \frac{g_{w}}{\Lambda^2} \right) \frac{c^2 f_{BB} + s^2 f_{WW}}{2c^2}.$$  

with $s \equiv \sin \theta_W$, $c \equiv \cos \theta_W$. In our calculation, we have included the complete gauge invariant set of Feynman diagrams that receive contribution from the anomalous operators $O_{WW}$, $O_{WW}$, $O_{BB}$, $O_W$ and $O_B$. For example, the triple vector boson self-couplings include the contributions from the operators $O_{WW}$, $O_W$ and $O_B$, while the quartic vector boson self-couplings are induced by $O_{WW}$ and $O_W$. The reason that the operators $O_{WW}$ and $O_{BB}$ do not modify the gauge boson self-couplings is as follows. When the Higgs field of those operators is replaced by its vacuum expectation value, it seems that they would induce anomalous operators to modify the gauge boson self-couplings. However, the resulting operators are proportional to the kinematic term of the $SU(2)$ and $U(1)$ gauge bosons, and lead to a finite wave-function renormalization of the gauge fields by constants $Z_{W}^{2} = 1 - g^{2} f_{WW} / 2 \Lambda^2)$ and $Z_{B}^{2} = 1 - g^{2} f_{BB} / 2 \Lambda^2)$, respectively. Therefore, from the fact that the building blocks of the effective Lagrangian $\mathcal{L}_{\text{eff}}$, cf. Eq. (17), involving gauge bosons are $gW_{\mu \nu}$, $g'B_{\mu \nu}$, and the covariant derivative $D_{\mu}$, we can perform a finite charge renormalization of the gauge couplings $g$ and $g'$ by constants $Z_{g} = (1 + g^{2} f_{WW} / 2 \Lambda^2)$ and $Z_{g'} = (1 + g^{2} f_{BB} / 2 \Lambda^2)$, respectively, so that the net effect of the operators $O_{WW}$ and $O_{BB}$ is to modify only the couplings of a Higgs boson to gauge bosons, but not the self-couplings of gauge bosons.

In Eq. (21), the anomalous $HVV$ couplings are expressed in terms of the Lorentz-invariant dimension-5 operators containing the Higgs boson and the gauge bosons $W$, $Z$ and $\gamma$. Among them, the operators $H A_{\mu \nu} A_{\mu \nu}$, $H A_{\mu \nu} Z_{\mu \nu}$, $H Z_{\mu \nu} Z_{\mu \nu}$ and $H W_{\mu \nu} W_{\mu \nu}$ can also be induced from the gauge-invariant dimension-5 operators in the nonlinear realization of the Higgs sector because in which the Higgs field $H$ is an electroweak singlet. Thus, it is worth noticing that the following LHC study of testing the linearly realized anomalous $HVV$ couplings via $VV$ scatterings may be generalized to the case of the dimension-5 operators in the nonlinear realization.

B. Constraints on $f_n$ from the Existing Experiments and the Unitarity Requirement

There are known experiments that can constrain the size of the anomalous coupling constants $f_n$.

The constraints on the anomalous coupling constants $\lambda_{WW}$, $\lambda_{WW}$, $\lambda_{BB}$, $\lambda_{WW}$ and $\lambda_{BB}$ have been studied in Refs. [15,45,46]. At the tree level, $\Delta S$ and $\Delta T$ are proportional to $f_{WW} / \Lambda^2$ and $f_{BB} / \Lambda^2$, respectively. Thus we can obtain the 68% and 95% bounds on the $(f_{WW} / \Lambda^2) - (f_{BB} / \Lambda^2)$ plane directly from the corresponding bounds in Fig. 2(a). This is shown in Fig. 4. We see from Fig. 4 that the precision data give quite strong constraints on $f_{WW} / \Lambda^2$ and $f_{BB} / \Lambda^2$. At the one loop level, $\Delta S$ and $\Delta T$ are related to other five anomalous coupling constants through loop corrections. Following Refs. [15,45,46], we make a one parameter fit of the five anomalous coupling constants by using the formulas given in Ref. [46] and the updated $\Delta S - \Delta T$ bounds in Fig. 2(a). The obtained 95%
C.L. constraints (in units of TeV$^{-2}$) for $m_H = 100$ GeV are

$$-6 \leq f_{WW}/\Lambda^2 \leq 3,$$

$$-6 \leq f_W/\Lambda^2 \leq 5,$$

$$-4.2 \leq f_B/\Lambda^2 \leq 2.0,$$

$$-5.0 \leq f_{WW}/\Lambda^2 \leq 5.6,$$

$$-17 \leq f_{BB}/\Lambda^2 \leq 20.$$  \hspace{1cm} (23)

These constraints are much weaker than that shown in Fig. 4 because these five anomalous coupling constants contribute to $\Delta S$ and $\Delta T$ through one loop corrections which are suppressed by the loop factor $1/16\pi^2$.

The triple gauge coupling data lead to the following 95% C.L. constraints (in units of TeV$^{-2}$) \cite{6,15,47,48}:

$$-31 \leq (f_W + f_B)/\Lambda^2 \leq 68 \quad \text{for} \quad f_{WW} = 0,$$

$$-41 \leq f_{WW}/\Lambda^2 \leq 26 \quad \text{for} \quad f_W + f_B = 0.$$  \hspace{1cm} (24)

Furthermore, the Higgs searches data at the LEP2 and the Tevatron can give rise to the following 95% C.L. bound on $f_{WW}(BB)$ (in units of TeV$^{-2}$) for $m_H \leq 150$ GeV \cite{6}

$$-7.5 \leq f_{WW}(BB)/\Lambda^2 \leq 18.$$  \hspace{1cm} (25)

The theoretical constraint on $f_n$ coming from the requirement of the unitarity of the $S$ matrix has been studied in Ref. \cite{49}. In terms of the present symbols of the anomalous coupling constants, the unitarity bounds given in Ref. \cite{49} read (in units of TeV$^{-2}$)

$$\left|f_B/\Lambda^2\right| \leq \frac{98}{\Lambda^2}, \quad \left|f_W/\Lambda^2\right| \leq \frac{31}{\Lambda^2},$$

$$\frac{784}{\Lambda^2} + \frac{3556m_W}{\Lambda^3} \leq f_{BB}/\Lambda^2 \leq \frac{638}{\Lambda^2} + \frac{3733m_W}{\Lambda^3},$$

$$\left|f_{WW}/\Lambda^2\right| \leq \frac{35.2}{\Lambda^2} + \frac{4.86}{\Lambda m_W}, \quad \left|f_{WWW}/\Lambda^2\right| \leq \frac{38}{3g^2\Lambda^2},$$  \hspace{1cm} (26)

in which we have put the center-of-mass energy $\sqrt{s} \approx \Lambda$. For $\Lambda \approx 2$ TeV, the bounds are (in units of TeV$^{-2}$)

$$\left|f_B/\Lambda^2\right| \leq 24.5, \quad \left|f_W/\Lambda^2\right| \leq 7.8, \quad \left|f_{WWW}/\Lambda^2\right| \leq 7.5,$$

$$-160 \leq f_{BB}/\Lambda^2 \leq 197, \quad \left|f_{WW}/\Lambda^2\right| \leq 39.2.$$  \hspace{1cm} (27)

These bounds are essentially of the same level as the above bounds (23) and (24).

We see that, except for the constraints from the precision data (cf. Fig. 4), all other constraints are rather weak. Furthermore, the above constraints on $f_n/A^2$ lead to the following constraints (in units of TeV$^{-1}$) on the anomalous coupling constants $g_{WW}^{(1)}, g_{WW}^{(2)}, g_{HZZ}^{(1)}$ and $g_{HZZ}^{(2)}$, cf. Eq. (22), related to the VV scatterings:

$$-0.20 \leq g_{WW}^{(1)} \leq 0.065,$$

$$-1.2 \leq -g_{WW}^{(2)} \leq 0.73,$$

$$-0.26 \leq g_{HZZ}^{(1)} \leq 0.24,$$

$$-0.62 \leq -g_{HZZ}^{(2)} \leq 0.36.$$  \hspace{1cm} (28)

We shall see in the following subsection that the limits on these coupling constants obtained from studying the VV scattering processes at the LHC will be significantly stronger than those in Eq. (28).

C. Testing $f_n$ via VV Scatterings at the LHC

The test of the anomalous $HVV$ couplings from the dimension-6 operators via VV scatterings is quite different from that for the dimension-3 operator. The relevant operators in Eq. (18) contain two derivatives, so that the interaction vertices themselves behave as $E^2$ at high energies. Thus, at high energies, the scattering amplitudes of $V_L V_L \rightarrow V_L V_L$ grow as $E^4$, and those containing transverse components, i.e., $V_T V_T \rightarrow V_L V_L$, $V_T V_L \rightarrow V_T V_L$ and $V_L V_L \rightarrow V_T V_T$, grow as $E^2$. Hence the scattering processes containing $V_T$ actually behave as signals rather than backgrounds. This is very different from the case of the nonlinearly realized dimension-3 anomalous coupling studied in Sec. IV. As discussed in Sec. II, since the $V_T V_T$ and $V_L V_T$ scatterings also contribute to the signal rate, we decide to do the full $2 \rightarrow 4$ tree level calculation, in contrast to performing the calculation using the EWA folded by the $2 \rightarrow 2$ VV scattering amplitudes. Nevertheless, we shall use the EWA to check the high energy behavior of the full calculation, in the case that the anomalous coupling is large.

We calculate the tree level cross sections for all the processes listed in Eq. (12) with the same method described in Sec. IV, but for the linearly realized effective Lagrangian theory. Since there are several dimension-6 anomalous couplings related to the VV scatterings in this theory [cf. Eq. (18)], the analysis is more complicated than in the nonlinear realization case. If the anomalous coupling constants are of the same order of magnitude, the interferences between them may be significant, depending on the relative phases among them. This undoubtedly complicates the analysis. In
the following, we again perform the single parameter analysis, i.e., assuming only one of the anomalous coupling constants is dominant at a time. We see from Eq. (18) that the coupling constants related to VV scatterings are $f_{WWW}$, $f_{WW}$, $f_{BB}$, $f_W$ and $f_B$. As discussed in Sec. VIA, the operator $O_{WWW}$ does not induce the anomalous coupling of a Higgs boson to gauge bosons, and it may not be directly related to the electroweak symmetry breaking mechanism, so we assume it is small in the following analysis. Detailed calculations show that the contributions of $f_B$ and $f_{BB}$ to the most sensitive $pp \to W^+W^+jj \rightarrow l^+\nu l^+\nu jj$ channel are small even if they are of the same order of magnitude as the anomalous coupling constants $f_W$ and $f_{WWW}$. Hence, we shall ignore their contributions in the following analysis, and discuss only the sensitivity of the $VV$ scatterings to the measurement of $f_W$ and $f_{WWW}$.

In the case that $f_W$ dominates, the obtained numbers of events at the LHC with an integrated luminosity of 300 fb$^{-1}$, for various values of $m_H$ and $f_W/A^2$, are listed in Table IX(A) and Tables X$-$XII. From these Tables, we see that the most sensitive channel is still $pp \to W^+W^+jj \rightarrow l^+\nu l^+\nu jj$, as listed in Table IX(A). All other channels are less interesting. Similar to the numbers in Table III, the number of SM events (for $f_W/A^2 = 0$) is taken as the intrinsic electroweak backgrounds, i.e., $N_B = N(f_W = 0)$. The number of the signal events (for $f_W/A^2 \neq 0$) is then defined as $N_S = N(f_W \neq 0) - N_B$. We also list the values of the deviation of the signal from the total statistical fluctuation, $\sqrt{N_S/N_B}$, in the parentheses in Table IX(A). If no anomalous coupling effect is found via this process, we can set the following bounds on $f_W/A^2$ (in units of TeV$^{-2}$):

$$1\sigma : \quad -1.0 < f_W/A^2 < 0.85,$$
$$2\sigma : \quad -1.4 < f_W/A^2 < 1.2,$$  \hspace{1cm} (29)

for $115 \text{ GeV} \lesssim m_H \lesssim 300 \text{ GeV}$.

In the case that $f_{WWW}$ dominates, the numbers of events are listed in Table IX(B), and the corresponding bounds are (in units of TeV$^{-2}$) as follows:

$$1\sigma : \quad -1.6 < f_{WWW}/A^2 < 1.6,$$
$$2\sigma : \quad -2.2 < f_{WWW}/A^2 < 2.2,$$  \hspace{1cm} (30)

which are somewhat weaker than those in Eq. (29).

To see the effect of interference, we take a special case as an example, in which $f_W = -f_{WWW} = f$, i.e., $g_{HV}^{(1)}$ and $g_{HV}^{(2)}$ in Eq. (22) are of the same sign. Then we obtain the following bounds (in units of TeV$^{-2}$):

$$1\sigma : \quad -0.6 < f/A^2 < 0.5,$$
$$2\sigma : \quad -0.9 < f/A^2 < 0.75.$$  \hspace{1cm} (31)

In this case, the interference enhances the sensitivity. Of course, if $f_W = f_{WWW} = f$, the sensitivity will be reduced.

From the bounds in Eqs. (29), (30), together with the relations in Eq. (22), we obtain the corresponding bounds on $g_{HV}^{(i)}$, $i = 1, 2$ (in units of TeV$^{-1}$):

$$1\sigma : \quad -0.026 < g_{HV}^{(1)} < 0.022,$$
$$-0.026 < g_{HZZ}^{(1)} < 0.022,$$
$$-0.014 < g_{HZZ}^{(2)} < 0.012,$$
$$-0.083 < g_{HV}^{(2)} < 0.083,$$
$$-0.032 < g_{HZZ}^{(2)} < 0.032,$$
$$-0.018 < g_{HZZ}^{(2)} < 0.018,$$  \hspace{1cm} (32)

$$2\sigma : \quad -0.036 < g_{HV}^{(1)} < 0.031,$$
$$-0.036 < g_{HZZ}^{(1)} < 0.031,$$
$$-0.020 < g_{HZZ}^{(2)} < 0.017,$$
$$-0.11 < g_{HV}^{(2)} < 0.11,$$
$$-0.044 < g_{HZZ}^{(2)} < 0.044,$$
$$-0.024 < g_{HZZ}^{(2)} < 0.024.$$  \hspace{1cm} (32)

These bounds are to be compared with the $1\sigma$ bound on $g_{HV}^{(2)}$ obtained from studying the on-shell Higgs boson production via weak boson fusion at the LHC, as given recently in Ref. [8]. In Ref. [8], $g_{HV}^{(2)}$ is parametrized as $g_{HV}^{(2)} = 1/\Lambda_5 = g^2 v/\Lambda_5^2$, and the obtained $1\sigma$ bound on $\Lambda_5$ for an integrated luminosity of 100 fb$^{-1}$ is about $\Lambda_5 \lesssim 1 \text{ TeV}$, which corresponds to $g_{HV}^{(2)} = 1/\Lambda_5 \lesssim 0.1 \text{ TeV}^{-1}$. We see that the $1\sigma$ bounds listed in Eq. (32) are all stronger than this bound. For an integrated luminosity of 300 fb$^{-1}$, the bound $g_{HV}^{(2)} = 1/\Lambda_5 \lesssim 0.1 \text{ TeV}^{-1}$ given in Ref. [8] corresponds roughly to a $1.7\sigma$ level of accuracy. Comparing it with the results in Eq. (32), we conclude that our $2\sigma$ bound on $g_{HV}^{(2)}$ is at about the same level of accuracy, while our $2\sigma$ bounds on the other five $g_{HV}^{(i)}$ ($i = 1, 2$) are all stronger than those given in Ref. [8].

It has been shown in Ref. [3] that the anomalous $HZZ$ coupling constants $g_{HZZ}^{(1)}$ and $g_{HZZ}^{(2)}$ can be tested rather sensitively at the LC via the Higgs-strahlung process $e^+e^- \rightarrow Z^+Z^- \rightarrow Z+H$ with $Z \rightarrow jj$. In Ref. [3], $g_{HZZ}^{(1)}$ and $g_{HZZ}^{(2)}$ are parametrized as $g_{HZZ}^{(1)} = gZ_{cV}/m_Z$ and $g_{HZZ}^{(2)} = gZ_{bV}/m_Z$, respectively. The obtained limits on the coefficients $c_V$ and $b_V$ are $c_V \sim b_V \sim O(10^{-3})$ [3] which correspond to the limits $g_{HZZ}^{(1)} \sim g_{HZZ}^{(2)} \sim O(10^{-1})$.
provide the bounds on $g_{iHVV}^{(i)}$, $i = 1, 2$. So that the two experiments are complementary to each other.

To verify that our calculation does give the correct asymptotic behavior as that given in Ref. [49], and to see the difference between the complete tree level result and the EWA result (with only the $V_L V_L \rightarrow V_L V_L$ contribution included), we plot in Fig. 5 the $M_{WW}$ distributions in the $W^+W^+$ channel with $f_W/A^2 = 5$ TeV$^{-2}$ from the complete tree level calculation (solid curve), the EWA calculation with the exact $W^+_L W^+_L \rightarrow W^+_L W^+_L$ amplitude (dashed curve), and the result from the EWA calculation with the asymptotic formula [49] for the $W^+_L W^+_L \rightarrow W^+_L W^+_L$ scattering amplitude$^3$ (dotted curve). We see that the three curves coincide at high energies which indicates that the asymptotic behavior of the complete tree level result obtained numerically is correct. The dashed curve is significantly below the solid curve at lower energies, which shows that the signal from the transverse component contributions taken into account in the complete tree level calculation is very important. The dotted curve is much lower than the dashed curve at low energies even though they are all from the EWA approach with only the longitudinal component contributions taken into account. This is because there are contributions of energy-independent terms contained in the dashed curve which are not included in the dotted curve, and the energy-independent terms cause the dashed curve to peak significantly in the low energy region due to the larger parton luminosities in the smaller $M_{WW}$ region. To check the correctness of this explanation, we have calculated not only the $M_{WW}$ distribution with a constant amplitude which shows the above-mentioned peak, but also the $M_{WW}$ distributions for the two EWA curves with a very large $f_W/A^2$, say $f_W/A^2 = 100$ TeV$^{-2}$, with which the energy-independent terms are unimportant. Indeed, the two obtained curves in this case almost completely coincide, and the peak is shifted significantly to the high energy region. Finally, we note that for the values of the anomalous couplings relevant to the bounds given in Eqs. (29) and (30), the unitarity condition is well respected, so that our full $2 \rightarrow 4$ calculation is justified. Furthermore, since in our full $2 \rightarrow 4$ calculation, we keep track on the polarization states of the final state Z bosons, we can also predict the kinematical distributions of the final state leptons. As an example, Fig. 6 shows the distribution of the invariant mass of the dileptons from the decay of the final state $W^+W^+$ bosons produced via $pp \rightarrow W^+W^+jj$ for various scenarios of the anomalous couplings. It indicates that examining in detail various kinematical distributions might help to distinguish various scenarios of the new physics effect.

VII. CONCLUSIONS

We have examined the possibility of testing the anomalous $HVV$ couplings of a light Higgs boson with mass $115 \text{ GeV} \leq m_H \leq 300 \text{ GeV}$ at the LHC via various channels of $VV$ scatterings. This type of test is of special interest if the anomalous $HVV$ couplings differ from that of the SM by a significant amount so that the direct detection of the Higgs resonance is difficult. The study includes two types of anomalous $HVV$ couplings, namely the anomalous $HVV$ couplings from the nonlinear dimension-3 operator [13] and anomalous $HVV$ couplings from the linearly realized dimension-6 operators [14,15]. The gold-plated pure leptonic modes are chosen for detecting the final state weak bosons. To reduce various kinds of backgrounds, we impose the kinematical cuts suggested in Ref. [17]. The calculations are carried out numerically for the full cross sections of $pp \rightarrow VVjj$ including both the signals and the backgrounds under the kinematical cuts, and we take into account only the statistical uncertainties in this calculation. The results show that, with a sufficiently high integrated luminosity, such as $300$ fb$^{-1}$, the tests of the anomalous $HVV$ couplings can be rather sensitive. We note that to further discriminate the effect of the anomalous $HVV$ coupling from that of a strongly interacting EWSB sector with no light resonance will eventually demand a multichannel analysis at the LHC by searching for the light Higgs resonance through all possible on-shell production channels including gluon-gluon fusion. Once the light Higgs resonance is confirmed, $VV$ scatterings, especially the $W^+W^+$ channel, can provide rather sensitive tests of various anomalous $HVV$ couplings for probing the EWSB mechanism.

In the nonlinearly realized Higgs sector [13], the leading anomalous $HVV$ coupling ($\kappa$) comes from the

\[ 3 \text{It only contains terms proportional to } E^4 \text{ or } E^2. \text{ The } E^0 \text{ contribution is not included.} \]

\[ ^4 \text{Although it is possible to refine the kinematic cuts to further enhance the ratio of signal to background rates, for simplicity, we applied exactly the same kinematic cuts as those proposed in Ref. [17] in this study.} \]
The difference $\Delta \kappa \equiv \kappa - 1$ represents the deviation from the SM value $\kappa = 1$. Our calculation shows that the most sensitive channel for testing this anomalous coupling is $pp \to W^+W^+jj \to \ell^+\nu\ell^+\nu jj$. We see from Table III that the LHC can constrain $\Delta \kappa$ to the range $-0.3 \leq \Delta \kappa \leq 0.2$ in the case that no anomalous coupling effect is detected in the channel $pp \to \ell^+\nu\ell^+\nu jj$. For comparison, several possible tests of $\Delta \kappa$ in the high energy collider experiments are also discussed in Sec. III. A summary of all the constraints considered in Sec. IV is displayed in Table XIV.

In summary, we find that $VV$ scatterings are not only important for probing the strongly interacting electroweak symmetry breaking mechanism when there is no light Higgs boson, but also valuable for sensitively testing of the anomalous $HVV$ couplings (especially anomalous $HWW$ coupling) at the LHC when there is a light Higgs boson in the mass range of $115 - 300 \text{ GeV}$. This further supports the “no-lose” theorem [11] for the LHC to decisively probe the electroweak symmetry breaking mechanism.

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Table I. The 95% C.L. limits on $\Delta \kappa$ imposed by $Z$-pole and low energy precision electroweak data, for a few typical values of new physics scale $\Lambda$ and Higgs boson mass $m_H$.

| $m_H$(GeV) | $\Lambda$(TeV) | 1 | 10 | 100 |
|------------|----------------|---|----|-----|
| 115        |                | $-0.15 \leq \Delta \kappa \leq 0.23$ | $-0.069 \leq \Delta \kappa \leq 0.12$ | $-0.045 \leq \Delta \kappa \leq 0.08$ |
| 300        |                | $0.074 \leq \Delta \kappa \leq 0.60$ | $0.027 \leq \Delta \kappa \leq 0.24$ | $0.016 \leq \Delta \kappa \leq 0.15$ |
| 800        | (excluded)     |                             | $0.20 \leq \Delta \kappa \leq 0.45$ | $0.11 \leq \Delta \kappa \leq 0.26$ |

Table II. Cross section $\sigma_{\lambda_1 \lambda_2}$ (in units of fb) of $pp \rightarrow W^+_{\lambda_1} W^+_{\lambda_2} jj$ at the LHC for various values of $\Delta \kappa$ with $m_H = 115$ GeV. $W^+_\lambda$ denotes a polarized $W$ boson with the polarization index $\lambda = T$ or $L$.

| $\Delta \kappa$ | $\sigma_{all}$ | $\sigma_{LL}$ | $\sigma_{LT}$ | $\sigma_{TT}$ |
|-----------------|----------------|---------------|---------------|---------------|
| 0.0             | 1.1            | 0.02          | 0.2           | 0.8           |
| 0.4             | 4.1            | 3.0           | 0.3           | 0.8           |

Table III. Number of events at the LHC, with an integrated luminosity of 300 fb$^{-1}$, for $pp \rightarrow W^\pm W^\pm jj \rightarrow l^\pm \nu(l)\bar{\nu}(l)jj$ ($l^\pm = e^\pm$ or $\mu^\pm$) with various values of $m_H$ and $\Delta \kappa$. ($\Delta \kappa = 0$ corresponds to the SM.) The values of $N_S/\sqrt{N_S + N_B}$ are also shown in the parentheses.

| $pp \rightarrow W^+ W^+ jj \rightarrow l^+ \nu l^+ \nu jj$ | $m_H$(GeV) | $\Delta \kappa$ |
|----------------------------------------------------------|------------|----------------|
|                                                          | -1.0 | -0.6 | -0.3 | 0.0  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  |
| 115                                                       | 62 (6.0) | 48 (4.8) | 27 (2.3) | 15  | 24 (1.8) | 37 (3.6) | 58 (5.6) | –   | –   | –   |
| 130                                                       | 62 (6.0) | 48 (4.8) | 27 (2.3) | 15  | 24 (1.8) | 37 (3.6) | 57 (5.5) | –   | –   | –   |
| 200                                                       | 62 (6.0) | 48 (4.8) | 28 (2.5) | 15  | 22 (1.5) | 33 (3.1) | 52 (5.1) | 78 (7.1) | –   | –   |
| 300                                                       | 61 (5.9) | 49 (4.9) | 30 (2.7) | 16  | 20 (1.1) | 29 (2.6) | 43 (4.3) | 65 (6.2) | 95 (8.2) | 136 (10.4) |

| $pp \rightarrow W^- W^- jj \rightarrow l^- \bar{\nu} l^- \bar{\nu} jj$ | $m_H$(GeV) | $\Delta \kappa$ |
|--------------------------------------------------------------------------|------------|----------------|
|                                                                          | -1.0 | -0.6 | -0.3 | 0.0  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  |
| 115                                                                      | 13  | 10   | 6    | 3    | 4    | 4    | 6    | 9    | –    | –    | –    |
| 130                                                                      | 13  | 10   | 6    | 3    | 4    | 4    | 6    | 9    | –    | –    | –    |
| 200                                                                      | 13  | 10   | 6    | 3    | 4    | 6    | 8    | 11   | –    | –    | –    |
| 300                                                                      | 13  | 10   | 6    | 4    | 4    | 5    | 7    | 10   | 14   | 19   |
Table IV. Number of events at the LHC, with an integrated luminosity of 300 fb$^{-1}$, for $pp \to W^+W^-jj \to l^+l^-\nu\bar{\nu}jj$ ($l^\pm = e^\pm$ or $\mu^\pm$) with various values of $m_H$ and $\Delta\kappa$. ($\Delta\kappa = 0$ corresponds to the SM.)

| $m_H$(GeV) | -1.0 | -0.6 | -0.3 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
|------------|------|------|------|-----|-----|-----|-----|-----|-----|-----|
| 115        | 19   | 14   | 8    | 4   | 7   | 11  | 17  |    |    |    |
| 130        | 19   | 14   | 8    | 4   | 7   | 11  | 17  |    |    |    |
| 200        | 19   | 14   | 8    | 4   | 7   | 11  | 17  |    |    |    |
| 300        | 19   | 14   | 7    | 4   | 9   | 14  | 23  | 34  | 48  | 69  |

Table V. Number of events at the LHC, with an integrated luminosity of 300 fb$^{-1}$, for $pp \to ZW^\pm jj \to l^+l^-\nu\bar{\nu}jj$ ($l^\pm = e^\pm$ or $\mu^\pm$) with various values of $m_H$ (in GeV) and $\Delta\kappa$. ($\Delta\kappa = 0$ corresponds to the SM.)

$pp \to ZW^\pm jj \to l^+l^-\nu\bar{\nu}jj$

| $m_H$(GeV) | -1.0 | -0.6 | -0.3 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
|------------|------|------|------|-----|-----|-----|-----|-----|-----|-----|
| 115        | 9    | 7    | 4    | 2   | 3   | 5   | 7   |    |    |    |
| 130        | 9    | 7    | 4    | 2   | 3   | 5   | 7   |    |    |    |
| 200        | 9    | 7    | 4    | 2   | 3   | 4   | 6   | 10 |    |    |
| 300        | 9    | 7    | 4    | 2   | 3   | 4   | 5   | 9  | 12 | 16  |

$pp \to ZW^- jj \to l^+l^-\nu\bar{\nu}jj$

| $m_H$(GeV) | -1.0 | -0.6 | -0.3 | 1.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
|------------|------|------|------|-----|-----|-----|-----|-----|-----|-----|
| 115        | 4    | 3    | 2    | 1   | 1   | 2   | 3   |    |    |    |
| 130        | 4    | 3    | 2    | 1   | 1   | 2   | 3   |    |    |    |
| 200        | 4    | 3    | 2    | 1   | 1   | 2   | 3   | 4  |    |    |
| 300        | 4    | 3    | 2    | 1   | 1   | 2   | 2   | 4  | 5  | 7   |
Table VI. Number of events at the LHC, with an integrated luminosity of 300 fb$^{-1}$ for $pp \rightarrow ZZjj \rightarrow \ell^+\ell^-\ell^+\ell^- (\nu\bar{\nu})jj$ ($\ell^\pm = e^\pm$ or $\mu^\pm$) with various values of $m_H$ (in GeV) and $\Delta \kappa$. ($\Delta \kappa = 0$ corresponds to the SM.)

| $m_H$ (GeV) | $\Delta \kappa$ | -1.0 | -0.6 | -0.3 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
|------------|-----------------|------|------|------|----|----|----|----|----|----|----|
| 115        |                 | 9    | 8    | 5    | 4  | 5  | 6  | 8  |    |    |    |
| 130        |                 | 9    | 8    | 5    | 4  | 5  | 6  | 8  |    |    |    |
| 200        |                 | 9    | 8    | 5    | 4  | 5  | 6  | 8  |    |    |    |
| 300        |                 | 9    | 7    | 5    | 4  | 6  | 9  | 12 | 17 | 24 | 32 |

| $m_H$ (GeV) | $\Delta \kappa$ | -1.0 | -0.6 | -0.3 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
|------------|-----------------|------|------|------|----|----|----|----|----|----|----|
| 115        |                 | 5    | 4    | 2    | 0  | 1  | 3  | 5  |    |    |    |
| 130        |                 | 5    | 4    | 2    | 0  | 1  | 3  | 5  |    |    |    |
| 200        |                 | 5    | 4    | 2    | 0  | 1  | 3  | 5  |    |    |    |
| 300        |                 | 5    | 4    | 2    | 0  | 1  | 3  | 6  | 9  | 14 | 20 |

Table VII. Decay branching ratio $B(H \rightarrow \gamma\gamma)$, in the unit of $10^{-3}$, as a function of $\Delta \kappa$ for various $m_H$.

| $m_H$ (GeV) | $|\kappa|$ | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 1.0 | 1.05 | 1.1 | 1.2 | 1.3 | 1.4 |
|------------|----------|-----|-----|-----|-----|-------|-----|------|-----|-----|-----|-----|
| 110        |          | 0.013 | 0.018 | 0.024 | 0.03 | 0.033 | 0.037 | 0.040 | 0.044 | 0.052 | 0.061 | 0.069 |
| 120        |          | 0.048 | 0.064 | 0.082 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.17 | 0.19 | 0.21 |
| 130        |          | 0.12 | 0.16 | 0.20 | 0.24 | 0.26 | 0.28 | 0.30 | 0.32 | 0.35 | 0.39 | 0.42 |
| 150        |          | 0.48 | 0.55 | 0.60 | 0.65 | 0.67 | 0.68 | 0.70 | 0.71 | 0.74 | 0.76 | 0.78 |
| 170        |          | 0.95 | 0.95 | 0.96 | 0.96 | 0.96 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 |

Table VIII. Decay branching ratio $B(\kappa \equiv B(H \rightarrow WW^*)$ as a function of $|\kappa|$ for various $m_H$.

| $m_H$ (GeV) | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 1.0 | 1.05 | 1.1 | 1.2 | 1.3 | 1.4 |
|------------|-----|-----|-----|-----|------|-----|------|-----|-----|-----|-----|
| 110        |     |     |     |     |      |     |     |     |     |     |     |
| 120        |     |     |     |     |      |     |     |     |     |     |     |
| 130        |     |     |     |     |      |     |     |     |     |     |     |
| 150        |     |     |     |     |      |     |     |     |     |     |     |
| 170        |     |     |     |     |      |     |     |     |     |     |     |
Table IX-A. Number of events at the LHC, with an integrated luminosity of 300 fb\(^{-1}\), for \(pp \to W^\pm W^\pm \to l^\pm \nu l^\pm \nu jj\) \((l^\pm = e^\pm\) or \(\mu^\pm\)) in the linearly realized effective Lagrangian with various values of \(m_H\) and \(f_{WW}/\Lambda^2\). The values of \(N_S/\sqrt{N_S + N_B}\) are also shown in the parentheses.

| \(m_H\)(GeV) | -4.0 | -3.0 | -2.0 | -1.4 | -1.0 | 0.0 | 0.85 | 1.2 | 2.0 | 3.0 | 4.0 |
|-------------|------|------|------|------|------|-----|------|-----|-----|-----|-----|
| 115         | 117(9.4) | 72(6.7) | 38(3.7) | 26(2.2) | 20(1.1) | 15 | 20(1.1) | 25(2.0) | 42(4.2) | 78(7.1) | 129(10) |
| 130         | 118(9.5) | 72(6.7) | 38(3.7) | 26(2.2) | 20(1.1) | 15 | 20(1.1) | 25(2.0) | 42(4.2) | 78(7.1) | 130(10) |
| 200         | 119(9.5) | 73(6.8) | 38(3.7) | 26(2.2) | 20(1.1) | 15 | 20(1.1) | 25(2.0) | 42(4.2) | 79(7.2) | 132(10) |
| 300         | 121(9.6) | 75(6.9) | 39(3.8) | 27(2.3) | 21(1.3) | 16 | 21(1.3) | 26(2.2) | 43(4.3) | 80(7.3) | 134(10) |

Table IX-B. Number of events at the LHC, with an integrated luminosity 300 fb\(^{-1}\), for \(pp \to W^\pm W^\pm \to l^\pm \nu l^\pm \nu jj\) \((l^\pm = e^\pm\) or \(\mu^\pm\)) in the linearly realized effective Lagrangian with various values of \(m_H\) and \(f_{WW}/\Lambda^2\). The values of \(N_S/\sqrt{N_S + N_B}\) are also shown in the parentheses.

| \(m_H\)(GeV) | -4.0 | -3.0 | -2.0 | -1.4 | -1.0 | 0.0 | 0.85 | 1.2 | 2.0 | 3.0 | 4.0 |
|-------------|------|------|------|------|------|-----|------|-----|-----|-----|-----|
| 115         | 23 | 14 | 7 | 5 | 4 | 3 | 4 | 5 | 8 | 15 | 25 |
| 130         | 23 | 14 | 7 | 5 | 4 | 3 | 4 | 5 | 8 | 15 | 25 |
| 200         | 23 | 14 | 7 | 5 | 4 | 3 | 4 | 5 | 8 | 15 | 26 |
| 300         | 24 | 15 | 8 | 5 | 4 | 4 | 4 | 5 | 8 | 16 | 26 |

Table X. Number of events at the LHC, with an integrated luminosity of 300 fb\(^{-1}\), for \(pp \to W^\pm W^\pm \to l^\pm \nu l^\pm \nu jj\) \((l^\pm = e^\pm\) or \(\mu^\pm\)) in the linearly realized effective Lagrangian with various values of \(m_H\) and \(f_{WW}/\Lambda^2\). The values of \(N_S/\sqrt{N_S + N_B}\) are also shown in the parentheses.

| \(m_H\)(GeV) | -4.0 | -3.0 | -2.0 | -1.6 | -1.0 | 0.0 | 0.85 | 1.6 | 2.2 | 3.0 | 4.0 |
|-------------|------|------|------|------|------|-----|------|-----|-----|-----|-----|
| 115         | 35 | 21 | 10 | 5 | 4 | 3 | 4 | 5 | 6 | 9 |
| 130         | 35 | 21 | 10 | 5 | 4 | 3 | 4 | 5 | 6 | 10 |
| 200         | 36 | 21 | 10 | 5 | 4 | 3 | 4 | 5 | 7 | 10 |
| 300         | 37 | 22 | 11 | 5 | 4 | 4 | 4 | 5 | 7 | 10 |

| \(m_H\)(GeV) | -4.0 | -3.0 | -2.0 | -1.0 | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 |
|-------------|------|------|------|------|-----|-----|-----|-----|-----|
| 115         | 35 | 21 | 10 | 5 | 4 | 6 | 12 | 23 | 38 |
| 130         | 35 | 21 | 10 | 5 | 4 | 6 | 12 | 23 | 38 |
| 200         | 36 | 21 | 10 | 5 | 4 | 6 | 12 | 23 | 39 |
| 300         | 37 | 22 | 11 | 5 | 4 | 6 | 13 | 24 | 40 |
Table XI. Number of events at the LHC, with an integrated luminosity 300 fb$^{-1}$, for $pp \rightarrow ZZjj \rightarrow l^+l^-l^+l^- (l^\pm = e^\pm$ or $\mu^\pm)$ in the linearly realized effective Lagrangian with various values of $m_H$ and $f_W/\Lambda^2$.

| $m_H$(GeV) | -4.0 | -3.0 | -2.0 | -1.0 | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 |
|------------|------|------|------|------|-----|-----|-----|-----|-----|
| 115        | 10   | 6    | 2    | 1    | 0   | 1   | 3   | 6   | 11  |
| 130        | 10   | 6    | 2    | 1    | 0   | 1   | 3   | 6   | 11  |
| 200        | 10   | 6    | 2    | 1    | 0   | 1   | 3   | 6   | 11  |
| 300        | 10   | 6    | 2    | 1    | 0   | 1   | 3   | 6   | 12  |

Table XII. Number of events at the LHC, with an integrated luminosity 300 fb$^{-1}$, for $pp \rightarrow ZW^\pm jj \rightarrow l^\pm l^\pm l^- jj$ ($l^\pm = e^\pm$ or $\mu^\pm$) in the linearly realized effective Lagrangian with various values of $m_H$ and $f_W/\Lambda^2$.

| $m_H$(GeV) | -4.0 | -3.0 | -2.0 | -1.0 | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 |
|------------|------|------|------|------|-----|-----|-----|-----|-----|
| 115        | 10   | 7    | 5    | 4    | 4   | 5   | 6   | 8   | 10  |
| 130        | 9    | 7    | 5    | 4    | 4   | 5   | 6   | 8   | 10  |
| 200        | 9    | 7    | 5    | 4    | 4   | 5   | 6   | 8   | 10  |
| 300        | 9    | 7    | 5    | 4    | 4   | 5   | 6   | 8   | 11  |

| $m_H$(GeV) | -4.0 | -3.0 | -2.0 | -1.0 | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 |
|------------|------|------|------|------|-----|-----|-----|-----|-----|
| 115        | 4    | 2    | 1    | 1    | 1   | 1   | 1   | 2   | 4   |
| 130        | 4    | 2    | 1    | 1    | 1   | 1   | 1   | 2   | 4   |
| 200        | 4    | 2    | 1    | 1    | 1   | 1   | 1   | 2   | 4   |
| 300        | 4    | 2    | 1    | 1    | 1   | 1   | 1   | 2   | 4   |
Table XIII. Summary of the $2\sigma$ constraints on the anomalous $HVV$ coupling $\kappa (\equiv 1 + \Delta \kappa)$ from the dimension-3 operator in the nonlinear Higgs sector studied in Sec. III.

| Types of constraints | results | Places in the text |
|----------------------|---------|--------------------|
| Precision EW data    | regions shown in Figs. 2 and 3, Table I | Eq. (7) |
| Unitarity (at $\sqrt{s}=2$ TeV) | $0.5 < |\kappa| < 1.3$ | Eq. (11) |
| $W^+W^+$ scattering  | $-0.3 < \Delta \kappa < 0.2$ | Eq. (13) |
| $HV$ production ($m_H = 120$ GeV) | $0 \leq |\kappa| \leq 1.6$ | Eq. (14) |
| Higgs width ($m_H = 200 - 300$ GeV) | $0.8 \leq |\kappa| \leq 1.2$ | Eq. (15) |
| $B(H \rightarrow \gamma\gamma)$ | Table VII | Sec. VC |
| Higgs resonance ($m_H = 120$ GeV) | $0.88 \leq R \leq 1.12$ [$R \equiv \kappa^2 B(\kappa)/B(\kappa = 1)$], Table VIII | Eq. (16) |
| LC (500 GeV, 1ab$^{-1}$) | $|\Delta \kappa| \leq 0.3\%$ | Sec. VE |
Table XIV. Summary of the \(2\sigma\) constraints on the anomalous couplings \(f_n/\Lambda^2\) (in units of TeV\(^{-2}\)) associated with the dimension-6 operators [cf. Eqs. (17)–(19)] and the related form factors \(g^{(1)}_{HVV}\) and \(g^{(2)}_{HVV}\) (in units of TeV\(^{-1}\)) [cf. Eq. (22)] in the linearly realized Higgs sector studied in Sec. IV.

| types of constraints                      | results                                                                 | places in the text |
|------------------------------------------|-------------------------------------------------------------------------|-------------------|
| precision EW data                        | two-parameter fit at tree level: regions shown in Fig. 4, with          | Fig. 4            |
|                                          | \(-0.05 < \frac{f_n}{\Lambda^2} < 0.02,\) \(-0.10 < \frac{f_n}{\Lambda^2} < 0.05.\) |                   |
| one-parameter fit at one-loop level:     | \(-6 \leq \frac{f_{WW}}{\Lambda^2} \leq 3,\) \(-6 \leq \frac{f_{WW}}{\Lambda^2} \leq 5,\) \(-4.2 \leq \frac{f_n}{\Lambda^2} \leq 2.0,\) | Eq. (23)          |
|                                          | \(-5.0 \leq \frac{f_{WW}}{\Lambda^2} \leq 5.6,\) \(-17 \leq \frac{f_{WW}}{\Lambda^2} \leq 20.\) |                   |
| triple gauge coupling data               | \(-31 \leq \frac{f_{WW}}{\Lambda^2} \leq 68\) (for \(f_{WW} = 0\)), | Eq. (24)          |
|                                          | \(-41 \leq \frac{f_{WW}}{\Lambda^2} \leq 26\) (for \(f_W + f_b = 0\)). |                   |
| LEP2 Higgs searches                      | \(-7.5 \leq \frac{f_{WW}}{\Lambda^2} \leq 18.\)                       | Eq. (25)          |
| unitarity requirement                    | \(|\frac{f_W}{\Lambda^2}| \leq 24.5,\) \(|\frac{f_W}{\Lambda^2}| \leq 7.8,\) \(|\frac{f_{WW}}{\Lambda^2}| \leq 7.5,\) | Eq. (27)          |
| (at \(\sqrt{s}=2\) TeV)                | \(-160 \leq |\frac{f_{WW}}{\Lambda^2}| \leq 197,\) \(|\frac{f_{WW}}{\Lambda^2}| \leq 39.2.\) |                   |
| \(W^+W^+\) scattering                   | \(1\sigma\) \(2\sigma\)                                               | Eq. (29)          |
|                                          | \(-1.0 < \frac{f_{WW}}{\Lambda^2} < 0.85,\) \(-1.4 < \frac{f_{WW}}{\Lambda^2} < 1.2.\) |                   |
| or                                      | \(-1.6 < \frac{f_{WW}}{\Lambda^2} < 1.6,\) \(-2.2 < \frac{f_{WW}}{\Lambda^2} < 2.2.\) | Eq. (30)          |
|                                          |                                                                 |                   |
| \(HVV\) coupling at                     | \(-0.026 < g^{(1)}_{HHWW} < 0.022,\) \(-0.036 < g^{(1)}_{HHWW} \leq 0.031.\) | Eq. (31)          |
| LC (500 GeV, 1 ab\(^{-1}\))             | \(-0.026 < g^{(1)}_{HHZZ} < 0.022,\) \(-0.036 < g^{(1)}_{HHZZ} \leq 0.031.\) |                   |
|                                          | \(-0.014 < g^{(1)}_{HHZ\gamma} < 0.012,\) \(-0.020 < g^{(1)}_{HHZ\gamma} \leq 0.017.\) |                   |
|                                          | \(-0.083 < g^{(2)}_{HHWW} < 0.083,\) \(-0.11 \leq g^{(2)}_{HHWW} < 0.11.\) |                   |
|                                          | \(-0.032 < g^{(2)}_{HHZZ} < 0.032,\) \(-0.044 \leq g^{(2)}_{HHZZ} < 0.044.\) |                   |
|                                          | \(-0.018 < g^{(2)}_{HHZ\gamma} < 0.018,\) \(-0.024 \leq g^{(2)}_{HHZ\gamma} < 0.024.\) |                   |
|                                          | \(|\frac{f_{WW}}{\Lambda^2}| < 0.2\)                                  | Sec. VI A          |
FIG. 1. Illustration of Feynman diagrams for $VV$ scatterings in the SM: (a) diagrams contributing to $T(V,\gamma)$, (b) diagrams contributing to $T(H)$.

FIG. 2. $\Delta S - \Delta T$ contours (a) from the current precision electroweak data, and (b) from including the expected Tevatron Run-2 measurement of $m_W$ and $m_t$ (assuming the current central values of $m_W$ and $m_t$ with an error of 20 MeV and 2 GeV, respectively). Here, we have set $m_H^{fit} = 100$ GeV and $\Delta U = 0$ in the fit.
FIG. 3. Constraints on the new physics scale $\Lambda$ as a function of the anomalous coupling $\Delta \kappa \equiv \kappa - 1$. The regions below the solid curves and above the dashed lines [(a)] or between the two solid curves [(b)-(c)] are allowed at the 95% C.L. The dashed lines indicate the value of $m_{H}^{\text{ref}} = m_{H}$.

FIG. 4. The 68% and 95% C.L. bounds on $f_{BW}/\Lambda^2$ and $f_{\Phi,1}/\Lambda^2$ (in units of TeV$^{-2}$) from the tree level formulas of $\Delta S$ and $\Delta T$ \cite{40} and the $\Delta S-\Delta T$ bounds given in Fig. 2(a).
FIG. 5. Invariant mass distributions of the $W^+W^+$ pairs produced at the LHC for $m_H = 115$ GeV with $f_W/\Lambda^2 = f_B/\Lambda^2 = 5$ TeV$^{-2}$. The solid curve is the result from the complete tree level calculation; the dashed curve is the result from the EWA calculation with the exact $W_L^+W_L^+ \rightarrow W_L^+W_L^+$ amplitude; the dotted curve is the result from the EWA calculation with the asymptotic formula [49] for the $W_L^+W_L^+ \rightarrow W_L^+W_L^+$ scattering amplitude.

FIG. 6. Invariant mass distribution of the di-leptons from the decay of $W^+$ bosons produced at the LHC via $pp \rightarrow W^+W^+jj$ for $m_H = 115$ GeV with $f_W/\Lambda^2 = f_{WW}/\Lambda^2 = 0.75$ TeV$^{-2}$ [Eq. (31)] (solid line), $f_W/\Lambda^2 = 1.2$ TeV$^{-2}$, $f_{WW}/\Lambda^2 = 0$ [Eq. (29)] (dashed line), $f_W/\Lambda^2 = 2.2$ TeV$^{-2}$, $f_{WW}/\Lambda^2 = 0$ [Eq. (30)] (dashed-dotted line), and the SM (dotted line).