Observing CP Violating MSSM Higgs Bosons at Hadron Colliders?

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Abstract

We report on the possibility of observing Higgs sector CP violation of the minimal supersymmetric standard model at a hadron machine. The CP phase dependent cross-sections for the $VH_i$ associated production processes are given for the Fermilab Tevatron and CERN LHC. Substantial production cross-sections for channels of all three Higgs bosons simultaneously are shown to be possible, giving a direct indication of the CP violation. The observability of the Higgs signals are discussed.
The search for the Higgs bosons, scalar particles from the electroweak symmetry breaking multiplet(s), in the Standard Model (SM) and beyond is a major goal of present and future colliders. One or more light Higgs boson within relatively easy reach of the upgraded Tevatron or CERN LHC is particularly favorable to the most popular extension of the SM, namely the minimal supersymmetric standard model (MSSM). Hence, it is very important that we study all possible scenarios under the framework in careful details. Here in this letter, we make an effort in the direction focusing on the scenario with radiatively induced Higgs sector CP violation [1,2].

In the recent years, the so-called CP violating MSSM has became a subject of many phenomenological studies. A major part of the latter focus on Higgs physics, especially with application to the LEP machine at CERN (see, for example, Ref. [3] and references therein). Implications for a $e^+e^-$ machine is quite well studied. In particular, Ref. [4] has pushed the analysis to the prospective Next Linear Collider. Nevertheless, after the closing of the LEP machine and before another lepton machine is commissioned, we have no choice but to focus on the not as clean environment of the hadron colliders. Hence, it is the time for detailed careful studies of the topic at hadron machines. Some steps in the direction have been taken. More notable ones include works on aspects of the production phase in Refs. [5,6] restricting to the gluon fusion mechanism [7], as well as analysis of the subsequent decays of the Higgs bosons produced [8,9]. A complete analysis of the collider signature from production to decays with the inclusion of signal-background studies is of course the final goal. However, the topic is a complicated one, and may have to be taken one step at a time.

The present letter reports the first step by the present authors in the direction. We aim here at presenting explicit production cross-sections for all the three neutral Higgs bosons through the Higgstrahlung processes from quark-quark collisions, i.e., $q\bar{q} \rightarrow Z^0 \rightarrow Z^0 H_i$ and $q\bar{q}' \rightarrow W^\pm \rightarrow W^\pm H_i$. The gluon fusion process is generally expected to have the largest cross-section but suffer from very large background [11]. Hence, other production channels may also prove to be useful in probing the CP violation in MSSM Higgs physics. It has been emphasized, in Ref. [4] for example, that due to the absence of an $AVV$ coupling, the simultaneous observation of all three $V^* \rightarrow V H_i$ channels will be a strong indication of scalar-pseudoscalar mixings, and hence Higgs sector CP violation, in the MSSM. We want to emphasize that this would be a qualitative result, pretty much independent of the details of the exact cross-sections themselves and the explicit determination of which region of parameter space the model lives in. Seeing all the three Higgs channels basically says

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1 During the preparation of the present letter, a study combining the production and decay processes comes up [10]. The latter reference is also restricted to production through gluon fusion and is concentrated on the lightest Higgs boson.
that the MSSM without the Higgs sector CP violation is not right. One can always go on
to models with a richer Higgs spectrum. The CP violating scenario studied here would,
however, preferred by many as the viable alternative. We present here production cross-
sections of the processes at both the Tevatron and LHC, as our first probe towards the
possibility of signals for the CP violation at the two machines. The observability of such
Higgs signals is a deeply involved, but obviously very important, question. While a detailed
quantitative study is beyond our present report, we will try our best to address the issues
involved qualitatively, drawing lessons and comparison from related results in the literature.
We would particularly like to draw attention to some plausibly important aspects beyond
what has been studied in the literature.

It should be noted that CP violating phases in MSSM are stringently co nstrained by their
contribution to electric dipole moments (EDMs). The topic has been studied extensively
[12]. It suffices here to emphasize that the EDM constraints can be by-passed, for instance,
by effectively decoupling the sparticles of the first family and/or cancellations among the
various contributions. Such constraints are not explicitly imposed in the present study. The
rationale being that the Higgs sector CP violation involves flavor dependent parameters only
of the third family and the complex phase combination Φ_{CP} = \arg(A_{\mu}). This certainly leaves
much room for getting around the EDM constraints, by tuning the other parameters and
phases for instance.

The tree-level Higgs potential of the MSSM conserves CP. This ens ures that the three
neutral Higgs mass eigenstates can be divided into the CP-even \( h^0 \) and \( H^0 \) and CP-odd
\( A^0 \). The 1-loop effective potential, however, may violate CP. When this happens, three
Higgs mass eigenstates with no definite CP parity would be resulted. The Higgs bosons are
denoted by \( H_1^0 \), \( H_2^0 \), and \( H_3^0 \) (in ascending order of mass). It has been shown that the CP
violation may be generated by complex phases residing in the \( \mu \) term and the soft SUSY
breaking parameters \( A_t \) (and \( A_b \)). These phases generate contributions to the off-diagonal
block \( M_{SP}^2 \) in neutral Higgs mass-squared matrix \( M_{ij}^2 \) mixing the scalar and pseudoscalar
fields. These may be given approximately by

\[
M_{SP}^2 \approx \mathcal{O} \left( \frac{m_t^4 |\mu||A_t|}{v^2 32\pi^2 M_{SUSY}^2} \right) \sin \Phi_{CP} \times \left[ 6, \frac{|A_t|^2}{M_{SUSY}^2}, \frac{|\mu|^2}{M_{SUSY}^2}, \frac{\sin 2\Phi_{CP} |A_t| |\mu|}{\sin \Phi_{CP} M_{SUSY}^2} \right],
\]

where \( \Phi_{CP} = \arg(A_t\mu) \). Here, we have only displayed the contributions from the top squarks
\((\tilde{t}_1, \tilde{t}_2)\) which are dominant for small \( \tan \beta \) [14]. Sizeable scalar-pseudoscalar mixing is possible
for large \( |\mu| \) and \( |A_t| (> M_{SUSY}) \).

In our numerical Higgs mass computation, we follow Ref. [3] and use the public code avail-
able at [15]. This involves one-loop effective potential with large two-loop non-logarithmic
corrections induced by one-loop threshold effects on the top and bottom quark Yukawa couplings included. We are interested in regions of parameter space where all three Higgs bosons of masses are relatively close to each other, say all smaller than 200 GeV. Otherwise, the model would be close to the decoupling limit where the radiative CP effect on the Higgs sector is known to be unimportant \[1,3\]. We are also restricting to relatively small \(\tan\beta\) value, with demonstrated substantial scalar-pseudoscalar mixings. For simplicity, we take nonzero a common phase for \(A_t\) and \(A_b\) as the sole source of \(\Phi_{\text{CP}}\).

At the patron level, to the leading order (LO), the cross-section for a \(V H_i\) associated production process is given by

\[
\hat{\sigma}_{\text{LO}}(q\bar{q} \rightarrow VH_i) = \frac{G_F^2 M_W^4}{288 \pi Q^2} \left( v_q^2 + a_q^2 \right) \lambda(m_V^2, m_{H_i}^2, Q^2) + \frac{12 m_V^2/Q^2}{(1 - m_V^2/Q^2)^2} \sqrt{\lambda(m_V^2, m_{H_i}^2, Q^2)} C_i^2
\]

where \(\lambda(x, y, z) = (1 - x/z - y/z)^2 - 4xy/z^2\) and \(v_q = -a_q = \sqrt{2}\) for \(V = W^\pm\) and \(v_q = 2 I_3^q - 4 e_q \sin^2 \theta_W, \ a_q = 2 I_3^q\) for \(V = Z^0\) \((q' = q)\); while \(C_i\) is the \(VVH_i\) coupling renormalized to \(\frac{g M_Z}{\cos \theta_W}\). A crucial point here is that the three Higgs bosons mix through an “orthogonal” matrix, leading to a sum rule for the \(C_i\) couplings \[16\]. Namely,

\[
C_1^2 + C_2^2 + C_3^2 = 1.
\]

The sum rule is well appreciated among researchers on the subject. It guarantees that at least one of the three production cross-sections is not suppressed. This feature is much exploited in Higgs discovery studies. The sum rule also suggests that all the three cross-sections can be simultaneously substantial for some particular set of the relevant SUSY parameters. The latter feature plays a central role in our present analysis. Explicit presentations of the variations of the \(C_i\)'s, or \(VVH_i\) couplings, for the CP violating case of interest here have been given in many of the earlier works \[17\]. Hence, we skip explicit numerical presentation here.

We convolute the admissible patron sub-process cross-section given above with the CTEQ4L patron distribution functions. QCD corrections are known to give an enhancement of about 40% for the Tevatron (Run II) and 30% for the LHC \[19\]. Other SUSY corrections are generally small \[20\]. In the latter reference, it is shown to be less than 1.5%, being smaller for large squark/gluino masses. In the explicit plots given in the figures, we scaled the LO cross-sections calculated with the corresponding enhancement \(K (= \sigma_{\text{NLO}}/\sigma_{\text{LO}})\) factor, to give a better indication of the next-leading order (NLO) results. The \(K\) factor is taken simply to be 1.4 and 1.3 for the Tevatron and LHC, respectively. This should be good enough for the present purpose. Readers interested in further details on the tiny variations in the exact \(K\) factor value along with the changes of the model parameters are referred to Ref. \[20\].

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We illustrate our results with two representative set of chosen input parameters. Parameter Set A is chosen in accordance with the benchmark scenario (CPX) introduced in Ref. [18] aimed at maximizing the CP violating effects. The CPX scenario is as follows:

\[
\tilde{M}_Q = \tilde{M}_t = \tilde{M}_b = 1 \text{ TeV}, \quad \mu = 4 \text{ TeV}, \quad |A_t| = |A_b| = 2 \text{ TeV}, \quad |m_{\tilde{g}}| = 1 \text{ TeV} \quad \text{and} \quad |m_{\tilde{g}}| = |m_{\tilde{W}}| = 0.3 \text{ TeV},
\]

(4)

added to which we fixed

\[
\tan \beta = 6
\]

for Set A inputs. Parameter Set B is similar, with little variations, namely

\[
\tilde{M}_Q = \tilde{M}_t = \tilde{M}_b = 1 \text{ TeV}, \quad \mu = 2 \text{ TeV}, \quad |A_t| = |A_b| = 2 \text{ TeV}, \quad |m_{\tilde{g}}| = 1 \text{ TeV} \quad \text{and} \quad |m_{\tilde{g}}| = |m_{\tilde{W}}| = 0.3 \text{ TeV},
\]

(5)

with \( \tan \beta = 15 \).

The charged Higgs mass is taken as the control parameter on the scale of the Higgs masses. We show only results for two choices of the charged Higgs mass, at 150 GeV and 200 GeV. The 150 GeV gives \( H_1 \) mass very close to the known bound from LEP. The 200 GeV case gives a relatively high mass scale value getting close to, while still staying away from, the decoupling limit. Hence, the two choices roughly envelope the range of interest. The production cross-sections are plotted as a function of \( \Phi_{CP} \), which comes here from a common phase of \( A_t \) and \( A_b \). Explicit plots of the masses are also given for easy cross-reference. Fig. 1 and 2 are results for the Tevatron, based on Set A and Set B inputs, respectively, while Fig. 3 and 4 give the corresponding results for the LHC. The figures do illustrate the existence of the exciting possibility we go after, namely, having substantial production cross-sections simultaneously for all the three \( VH_i \) channels. This should not come as a surprise. Naively, the sum rule \([cf. \text{ Eq.}(3)]\) suggests there might be a “democratic” limit where the three channels share the overall coupling equally. In that situation, each \( H_i \) would have a production cross-section at the same order of magnitude as that of the SM Higgs of the same mass, only suppressed by a small factor. Our results do indicate explicitly that the scenario could be more or less achieved for some optimal \( \Phi_{CP} \) value.

Let us also briefly comment on the basic features of the plots. The general features of the dependence of the masses and gauge couplings \([\text{represented by the } C_i \text{'s in Eq.(2) above}]\) of the three (physical) Higgs bosons upon the CP phase \( \Phi_{CP} \) through the stop mixing parameter \(|X_t| = |A_t - \mu \cot \beta|\) have been studied by various groups (see Ref. [3] and references therein). In each of the top panels of Figs. 1-4, our explicitly illustrated Higgs masses agree well with previous studies. A particularly note-worthy aspect is that the (one loop corrected) \( H_1 \) mass increases with \(|X_t|\) till reaching its maximum at \(|X_t|/M_{s quark} \approx 2.45\), and drops with further
increase in $|X_i|$. Here within each panel, the latter is tuned with $\Phi_{\text{CP}} \simeq \arg(A_i)$. In the plot of Fig. 1(a), for example, $H_1$ mass is maximum at $\Phi_{\text{CP}} \simeq 80^\circ$. The large effect of the CP phase enhancing stop mixing here promotes scalar-pseudoscalar mixings, hence suppresses $C_1$. This, together with the $H_1$ mass enhancement, gives a minimum for the $VH_1$ production cross-sections at $\Phi_{\text{CP}} \simeq 88^\circ$, as shown in the plots right below [Fig. 1(b) and 1(c)]. The particle actually assumes the character of the pseudoscalar around this point. The situation is almost completely reversed for $H_2$, which simply corresponds to the pseudoscalar at the vanishing $\Phi_{\text{CP}}$ limit. In this case, $H_3$ is not much affected by variation in $\Phi_{\text{CP}}$ except when it assumes the character of the pseudoscalar at the other CP conserving limit of $\Phi_{\text{CP}} = 180^\circ$.

With larger mass splitting between $H_2, H_3$ and the lightest Higgs $H_1$ as given by the second case in Fig. 1 [plots 1(d), 1(e), and 1(f)], we are getting closer to the decoupling limit. The $H_1$ then behaves like the SM Higgs. It not much affected by $\Phi_{\text{CP}}$ variation, which now mainly describes mixing between the two heavier states. With the same set of inputs, the features in Fig. 3 are more or less the same as in Fig. 1. Fig. 2 and 4 are results from a different input set of parameters (Set B). The set of input parameters is not very different from the previous case though, as we are strongly confined by our special interest in large $\Phi_{\text{CP}}$ induced mass mixings. The point where $VH_1$ cross-sections are strongly suppressed while $VH_2$ cross-sections enhanced in Fig. 2 (cf. Fig. 1) is now shifted to $\Phi_{\text{CP}} \simeq 110^\circ$. In all the cases, roughly at the central values of $\Phi_{\text{CP}}$ between two of the dips of the three $VH_i$ cross-section curves, one gets the solutions for substantial cross-sections for all three Higgs channels simultaneously.

One of our major result is that in the most favorable situation, there is a chance that all three Higgs boson could be simultaneously produced with around or above 0.01 pb cross-sections at the Tevatron. The mass region of interest to us is in fact happens to be just well covered by the machine. This suggests the exciting possibility of seeing Higgs sector CP violation, assuming MSSM. At the LHC, the cross-sections could all go simultaneously to around or above 0.1 pb. The more or less optimal case correspond to results illustrated in the sets of left panels in the figures, with masses for all three $H_i$ below 150 GeV. The sets of right panels in the figures, however, illustrate roughly the cases of limiting capacity. Here, $H_2$ and $H_3$ are a bit heavier. They are around 180/190 GeV. As we will discuss below, this might be really pushing the limit on signal observability. However, it is our opinion that a careful and detailed analysis is required to set the definite mass reach. The latter may get somewhat close to the situation illustrated in these panels. This is especially true in the case of LHC.

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2 More explicitly, from the relation $C_i = \cos\beta O_{1i} + \sin\beta O_{2i}$, we have $C_1 \approx \sin\beta O_{21}$ for large $\tan\beta$. $C_1$ is then dominated by $O_{21}$ which flips sign. That is why we have a dip in the plot.
One certainly should not be too optimistic about the scenario. We put the question mark in the title of this letter because there are good reasons to be cautious. Assuming that the MSSM really falls into such a parameter space region where the three CP violating Higgs boson can all be produced with substantial cross-sections at the Tevatron or LHC, identifying the signals may be a daunting task. In the spirit of our present analysis, one can claim that the CP violating Higgs bosons are unambiguously observed only after we have successfully identified all the three $H_i$’s produced through their decays. Otherwise, one may have to rely on details on cross-section measurements and further inputs from other possible SUSY signals (if at all available) to pin down the Higgs signals as coming from CP violating MSSM. We emphasize again that identifying three Higgs mass eigenstates produced through the $VH_i$ channels in itself establishes Higgs sector CP violation, i.e. assuming MSSM. With anything less than that, it is going to be an extremely involved task to confirm that the Higgs boson(s) observed is/are more than generic MSSM Higgs boson(s).

Observing “intermediate mass” neutral Higgs bosons are notoriously difficult (see, for example, Ref. [7]). The “gold-plated” 4-lepton decay mode $H \rightarrow Z^0 Z^0 \rightarrow l^+ l^- l^+ l^-$ has good branching ratio only for heavier Higgs bosons, while the $2\gamma$ mode $H \rightarrow \gamma\gamma$ has steeply increasing background [21]. Associated productions, the presently considered $VH$ [21,22], $ttH$ [23], or even $t\bar{t}H$ [24] have been advocated as viable alternatives to the gluon fusion process to be the focus of Higgs hunting exactly for that reason. A recent paper [25] gives detailed discussions on the various associated production processes focusing on the effects of large stop mixing. The latter, while closer to our present study, is still limited to the CP-conserving MSSM. This is also the case for most of the previous studies [20]. Such studies, while of some use in giving a rough idea on the observability of the Higgs signals under discussion, certainly cannot take the place of the necessary specific studies, as along the line recently started by Refs. [8,9]. However, results from the latter references are not enough to help us to reach any definite conclusion.

Ref. [8] in particular gives interesting results on partial branching fractions of the various decays for the three $H_i$’s separately. It is a big step in the direction. What is still missing though are signal versus background analyses. The reference gives no definite mass reach numbers for the $H_i$’s. One would like to have the definite reach of the hadron machines in terms of the mass control parameter (i.e. charged Higgs mass) as a function of $\Phi_{CP}$ reflecting the range of simultaneous observability of all three Higgs signals. This ambitious task is beyond the present short letter. Mass reach numbers for MSSM Higgs bosons are not widely available, even for the general case without Higgs sector CP violation. We can only present below discussions based on available information in the literature. The discussion aims at giving some idea on what might be expected. Hence, it may have some speculative element. We will quote some mass reach number for the SM, or SM-like lightest MSSM, Higgs. These numbers are of course not directly applicable to our scenario. However, the SM case is much.
better studied. We have pointed out above that the coupling sum rule \[ \text{cf. Eq.}(3) \] close to the “democratic” limit suggests a scenario in which the three \( H_i \)’s kind of share equally the role of the SM Higgs. Each \( H_i \) then behaves like “a third of” the SM Higgs at the same mass. If one take the latter statement seriously while assuming all the other SUSY particles are heavier, we have a situation where the SM Higgs numbers do provide a useful guideline.

The case for the Tevatron may be quite marginal but we consider it worth the effort to check it in details, focusing on both the \( H \to \bar{b}b \) and \( H \to WW^{(*)} \) decays, or including even more decay channels. Decay branching fraction results from Ref. \[ 8 \] do confirm the \( H \to \bar{b}b \) as the dominating channels at least up to masses of 150 GeV. So why should we consider the \( H \to WW^{(*)} \) and other channels?

Current searches planned for Tevatron Run II actually concentrate on the \( VH \) associated production process with decays \( H \to \bar{b}b \) \[ 27 \] for Higgs mass in the lower intermediate range, taking the extra advantage that the leptonic decays of \( W^\pm \) or \( Z^0 \) can be used as triggers to suppress background. SM Higgs is expected to be observable through the channel, however, only for mass up to 130 GeV or slightly above \[ 28 \]. For Higgs mass in the upper intermediate range, 135 – 180 GeV, the planned focus is rather on production through gluon fusion with subsequent decay \( H \to WW^{(*)} \) \[ 29 \]. The reference claims that the mass reach for a SM like Higgs, with an integrated luminosity of 30 fb\(^{-1} \), would be up to 190 GeV.\(^3\) A reason behind is the strong background for the \( \bar{b}b \) signal. In our scenario, at least for masses of around or above 150 GeV, \( H_i \to WW^{(*)} \) could be sizeable for all three Higgs bosons \[ 9 \]. On the other hand, \( \bar{b}b \) coupling(s) would be suppressed for the heavier Higgs states. Obviously, we cannot rely on the \( H \to \bar{b}b \) channels to see all the three \( H_i \)’s then. Nevertheless, unlike the SM case, using gluon fusion does not help confirming the CP violating setting we are interested in here. In this regard, a previous study on trilepton Higgs signal \[ 32 \] is particularly relevant. We will very likely have to rely on the \( WH \to WWW^{(*)} \to 3l \) decay to identify at least one or two of the \( VH_i \) channels. With an integrated luminosity of 100 fb\(^{-1} \), Ref. \[ 32 \] claims a limiting 3\( \sigma \) reach for the SM Higgs in mass range 140 – 175 GeV, with suggestions on further gains to be achieved. We conclude that a combined study of the \( VH \) production processes with decay channels and signal-background analyses specifically for the CP violating MSSM scenario will have to take the trilepton signal as one of the major focus.

The situation looks much better at the LHC. Studies for SM-like Higgs shown that, for the \( VH \) associated production under consideration, the \( H \to \bar{b}b \) channel should have reasonable efficiency in identifying the signal \[ 31 \], at least in the lower intermediate mass range. \( H \to \gamma\gamma \) would also be useful \[ 22, 26 \]. Again, in the upper intermediate mass range, even for a SM-

\(^3\) We should add that the \( \bar{t}tH \) with \( H \to \bar{b}b \) has also been advocated as a discovery mode at Tevatron \[ 30 \].
like Higgs one will have to go back to $H \rightarrow WW^{(*)}$ (or $H \rightarrow Z^0Z^0 \rightarrow l^+l^-l'^-l'^-$). In addition, we emphasize again that the $H_b\bar{b}$ couplings are very unlikely to be simultaneously unsuppressed. Hence, the $WH \rightarrow WWW^{(*)} \rightarrow 3l$ type trilepton signals are definitely useful for probing the CP violating model. Ref. [32] claims a 5$\sigma$ discovery reach for the mass range $140 - 180$ ($125 - 200$) GeV with 30 (100) fb$^{-1}$ for a SM Higgs. One may naively expect the signal for each $H_i$ to be weaken by a third or so in the optimal case of “democratically shared” couplings (equal $C_i$’s). That sounds quite encouraging.

It should be noted that, in general, possible decays of $H_2$ and $H_3$ through $H_1$ itself may compete with the $H_i \rightarrow WW^{(*)}$ channels and complicates analyses of the latter. However, in the region of parameter space of interest here, such decays are unlikely to be important and hence not taken into consideration here. Finally, we should mention that decays into superparticles are likely to dominate if their masses put them within kinematic threshold of the Higgs decays. This is very unlikely for the scenario we are interested in here, as one can easily see from the illustrative parameter input Set A and B given above, hence not discussed.

Perhaps we should also mention a related production mechanism, the weak boson fusion channels (see Ref. [33] and references therein). Similar to the associated productions, the processes exploit the $VVH$ couplings. It is advocated as a possible Higgs discovery channel at LHC, and a useful tool to determine the CP nature of the Higgs boson involved. From the present perspective, it will be interesting to explore the possibility of simultaneous observation of all three $VV \rightarrow H_i$ channels as a probe for the Higgs sector CP violation.

In summary, we illustrate in this letter explicit results on the production cross-sections of the three $VH_i$ channels at the Tevatron and the LHC. Our results indicate that a simultaneous observation of the three channels may be a possibility, though could be quite marginal at the Tevatron. In the best scenario, MSSM with radiative Higgs sector CP violation could give rise to cross-sections of the order or above 0.01 pb and 0.1 pb, for the Tevatron and the LHC, respectively. Detail signal-background analyses exploring various decay modes are called for. Nevertheless, we hope that the above discussions have convinced the readers that there are good reasons to be optimistic. In particular, we point out that the $WH_i \rightarrow WWW^{(*)} \rightarrow 3l$ decays are going to be useful. Assuming MSSM, the simultaneous observations of the three $VH_i$ channels will confirm the radiative Higgs sector CP violation scenario.

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FIG. 1. Plots (a) and (d) show the variation of three Higgs masses with the CP violating phase $\Phi_{\text{CP}}$. Plots (b), (e) and (c),(f) show the variation of $ZH_i$ and $WH_i$, $(i = 1, 2, 3)$ production cross sections at Tevatron Run II energy with the phase $\Phi_{\text{CP}}$. The left panel and right panel correspond to the charged Higgs mass of 150 GeV and 200 GeV respectively. Other MSSM parameter is fixed to Set A.
FIG. 2. Plots (a) and (d) show the variation of three Higgs masses with the CP violating phase $\Phi_{CP}$. Plots (b), (e) and (c), (f) show the variation of $ZH_i$ and $WH_i$, ($i = 1, 2, 3$) production cs at Tevatron Run II energy with the phase $\Phi_{CP}$. The left panel and right panel correspond to the charged Higgs mass of 150 GeV and 200 GeV respectively. Other MSSM parameter is fixed to Set B.
FIG. 3. Plots (a) and (d) show the variation of three Higgs masses with the CP violating phase $\Phi_{\text{CP}}$. Plots (b), (e) and (c), (f) show the variation of $ZH_i$ and $WH_i$, ($i = 1, 2, 3$) production cross sections at LHC with the phase $\Phi_{\text{CP}}$. The left panel and right panel correspond to the charged Higgs mass of 150 GeV and 200 GeV respectively. Other MSSM parameter is fixed to Set A.
FIG. 4. Plots (a) and (d) show the variation of three Higgs masses with the CP violating phase $\Phi_{\text{CP}}$. Plots (b), (e) and (c), (f) show the variation of $ZH_i$ and $WH_i$, ($i = 1, 2, 3$) production $\sigma$ at LHC with the phase $\Phi_{\text{CP}}$. The left panel and right panel correspond to the charged Higgs mass of 150 GeV and 200 GeV respectively. Other MSSM parameter is fixed to Set B.