Comparison between Ideal and Practical Differentiators Using Operational Amplifier

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Abstract: Operational Amplifier as differentiator plays an important role in electronics and electronic appliances. Inverting differentiator circuit is preferred compared to non-inverting differentiator circuit because of its high stability and better performance. In the present work, it is shown that preference should be given to the practical differentiator circuit using Operational Amplifier at frequency of 50 Hz with capacitance of 0.1 µF for getting improved result. This type of circuit can be used in different household and industrial as well as laboratory purposes.

Keywords: Operational Amplifier, Differentiator circuit, Inverting and Non-Inverting circuits.

I. INTRODUCTION

After the remarkable success of transistor physics, Operational Amplifiers (Op-Amp) have become a key component in designing and development of modern electronic circuits. It is a multistage voltage amplifier having high gain and negative feedback. It is used in computational processes to perform various mathematical operations like addition, subtraction, integration, and differentiation [1,2]. Out of the two input terminals, one inverts the phase of the signal while the other keeps the phase invariant like that of output terminal. Various applications in linear circuit (i.e. voltage amplifier, inverting and non-inverting amplifier) and non-linear circuit (i.e. comparators, clampers) make Op-Amp more beneficial than other amplifiers[1,3]. In the present work, we consider both the ideal and practical differentiator circuits using Operational Amplifier to study the variations of output voltage with different input and output parameters and also gains in both the circuits. The plan of the work is as follows.

The design of ideal and practical differentiator circuits along with the important relations for output voltage(s) and gain(s) are demonstrated in section II . Section III contains graphical representations of data for wide range of values of different parameters as applicable in different cases and analyses of the nature of variations based on these values. Summary of the work on the basis of the analyses is presented in section IV. Concluding remarks are given in section V.

II. DESIGN OF DIFFERENTIATOR CIRCUITS AND IMPORTANT RELATIONS

In case of differentiator circuit using Operational amplifier the first derivative of the input voltage comes out as an output Voltage [4]. At first, we consider an Ideal differentiator Op-Amp consisting of capacitor and resistance which contribute as an input and output impedance respectively. The ideal differentiator circuit [4] is as shown below:

\[ V_o = -RC \frac{dV}{dt} \]  \hspace{1cm} \text{(1)}

Where \( V_o \) = output voltage, \( R \) = resistance contributing as output impedance and \( C \) is the capacitance acting as input impedance.

However, the ideal differentiator circuit becomes unstable at high frequency due to attenuation of signal and it becomes difficult to overcome the problem. To get rid of the problem, additional resistor and capacitor in parallel should be connected at the negative terminal of Op-Amp[4-7].
The circuit diagram of practical differentiator circuit [1] is represented as shown below.

![Circuit Diagram](image)

**Fig. 2: Practical Differentiator**

It is to be noted that at low frequency range, this circuit behaves as a differential amplifier but at higher frequencies, noise rejection takes place to a good extent [6,7].

The output voltage for this improved circuit can be written in terms of impedance, resistance, capacitance and frequency as

\[
A = \frac{V_o}{V_i} = \frac{Z_2}{Z_1} = -\frac{R_2}{\frac{1}{sR_1C_2}} = -\frac{sR_2C_1}{(1+sR_1C_1)(1+sR_2C_2)}
\]

------------------------ 2(a)

Where, two poles [3] are given at

\[
s = f_1 = \frac{1}{2\pi R_1C_1} \quad s = f_2 = \frac{1}{2\pi R_2C_2}
\]

------------------------ (2b)

III. GRAPHICAL REPRESENTATIONS AND ANALYSES

Variations of output voltage with different parameters are presented in this section for both ideal and practical differentiator circuits.

A. Ideal Differentiator circuit is discussed and analyses for variations of output voltage with resistance, capacitance, frequency, time and input voltage have been reported.

1) Plot for output voltage (Vo) with varying resistance (R) (Fig.3)

Keeping Capacitance, frequency and time fixed, the graph between output voltage and variable resistance is shown in above diagram. The nature of the graph is linear. The output voltage decreases with increasing value of resistance ranges from 1 kΩ to 44 kΩ and the best result is obtained with significant % error equals to ± 0.5 %. The minimum value of output voltage is –0.021 mV corresponding to 44 kΩ while the maximum value is –0.0004 mV for resistance of 1 kΩ.
2) Plot is drawn for output voltage \((V_o)\) with varying capacitance \((C)\) (Fig.4)

![Fig.4](image)

The graph between output voltage and changing capacitance is depicted in the above diagram, keeping resistance, frequency and time constant. The graph is linear in character. From 0.5 \(\mu\)F to 0.93 \(\mu\)F, the output voltage drops as the capacitance value increases. In this example, the output voltage's minimum value is -0.0092 mV which corresponds to 0.93 \(\mu\)F and its highest value is -0.0049 mV which equates to 0.5 \(\mu\)F.

3) Variation between output voltage \((V_o)\) and frequency \((f)\) of different range is shown below.

a) Frequency below 20 Hz (Fig.5)

![Fig.5](image)

The variation of output voltage with changing frequency is illustrated in the above graph with constant values of resistance, capacitance and time. The nature of variation of output voltage with frequency is linear. With rising frequency ranging from 1 Hz to 19 Hz, the output voltage decreases. The output voltage has a minimum of -0.095 mV corresponding to 19 Hz and a maximum of -0.0050 mV when frequency is 1 Hz.
b) **Audible Range of Frequency (20 Hz to 20000 Hz)** (Fig.6)

In the above diagram, the graph between output voltage and varying frequency (audible range) is shown keeping resistance, capacitance and time constant. The graph is non-linear at first and subsequently becomes linear for output voltage 5.5847 mV with frequency 15000 Hz. The output voltage falls from -0.1005 mV to -25.7175 mV and then rises from -25.6814 mV to 58.6122 mV corresponding to frequency variation from 20Hz to 7800 Hz and 8100 Hz to 20000 Hz, respectively. Thus, at frequency of 20 KHz, the output voltage is 58.6122 mV. It is interesting to note that the positive nature of output voltage starts corresponding to frequency of 14324 Hz.

c) **Frequency Above 20 kHz** (Fig.7)

The graph of output voltage with changing frequency is demonstrated in the above diagram where resistance, capacitance and time are kept constant. The graph’s nature is linear. The output voltage grows from 59.801 mV to 121.55 mV with frequency span from 20100 Hz to 25600 Hz.
4) Output voltage (Vo) against variable time (t) is plotted as shown below (Fig.8)

![Graph 1](image1)

The variation of output voltage with varying time has been depicted above. The nature of the graph is non-linear. The output voltage increases from the negative to positive value (-0.00499 mV to 0.00494 mV) corresponding to time duration from 1 ms to 27 ms.

5) Plot is also drawn for output voltage (Vo) with varying input voltage (Vi) and the gain is also calculated as cited below (Fig.9)

![Graph 2](image2)

Here, the graph is drawn keeping other parameters (i.e. resistance, capacitance, frequency and time) constant. The output voltage is decreasing linearly with increase in input voltage. The output voltage (Vo) decreases from -0.0031 mV to -0.4777 mV corresponding to variation of input voltage (Vi) from 0.1 mV to 15.3 mV. The slope of this graph gives gain of the differentiator which is found to be -0.0312.
B. We also Present Practical Differentiator Circuit and Different cases are Explained Drawing Various Plots.

1) A plot is drawn for Output Voltage (Vo) with varying Input Resistance (R1) (Fig.10)

![Fig.10](image-url)

Keeping Capacitance, frequency and time fixed, the graph between output voltage and variable input resistance is shown in the above diagram. The nature of the graph is non-linear, in general. The output voltage increases with increasing values of input resistance ranging from 50 kΩ to 150 kΩ as considered in the present case. The minimum value of output voltage is -0.0153 mV corresponding to 50 kΩ and its maximum value is -0.0052 mV corresponding to 150 kΩ. Here, we have considered the input resistance of practical differentiator to be in the range of 50-150 kΩ because the slope is maximum at this range of values.

2) Plot between output voltage (Vo) and varying capacitance (C1) can be viewed as shown below (Fig.11)

![Fig.11](image-url)

The graph between output voltage and capacitance is shown in the above diagram, keeping resistance, frequency and time as constant parameters and capacitance varies from 0.2 µF to 0.7 µF. The graph is non-linear in character. The output voltage drops from -0.0476 mV to -0.0078 mV as the capacitance value increases from 0.2 µF to 0.7 µF. It is preferable to choose the range of capacitance from 0.2 µF to 0.7 µF because after this range the output voltage remains almost invariant and the slope is also maximum at these values of capacitance.
3) Graph between output voltage (Vo) and varying output resistance (R2) is presented below (Fig.12)

![Fig.12](image-url)

Taking parameters such as capacitance, frequency and time constants, the output voltage (Vo) decreases non-linearly with increasing value of output resistance (R2) ranging from 1 kΩ to 20 kΩ. The maximum value of output voltage is –0.00142 mV when output resistance is 1 kΩ whereas output voltage shows minimum value –0.01045 mV for output resistance equals to 20 kΩ. We have taken (1-20) kΩ because the gains unchanged (i.e. decreasing) for both the cases (1-20) kΩ and (10-90) kΩ.

a) The plot is drawn for output voltage (Vo) with varying capacitance (C2) (Fig.13)

![Fig.13](image-url)

The graph between output voltage and changing capacitance is depicted in the above diagram, keeping resistance, frequency and time constant. The graph is apparently linear in character. The output voltage rises from -0.0104 mV to -0.0052 mV as the capacitance value increases from 0.05 µF to 0.2 µF.
4) The plot is drawn for output voltage (Vo) with varying frequency (f)
   
a) Frequency below 20 Hz (Fig.14)
   
   ![Graph](image1)
   
   The curve of output voltage with changing frequency is illustrated in the following graphic, with resistance, capacitance and time held constant. With rising frequency ranges to 1 Hz to 19 Hz, the output voltage remains constant having value of - 0.00784 mV.
   
b) Frequency Between 20 Hz to 20 kHz (Fig.15)
   
   ![Graph](image2)
   
   In the above diagram, the graph between output voltage and varying frequency (audible range) is shown keeping resistance, capacitance and time constant. The output voltage remains unchanged attaining -0.00784 mV at the frequency range from 20 Hz to 20000 Hz.
c) Frequency above 20 kHz (Fig. 16)

![Figure 16](image)

The relationship between Output Voltage (Vo) and changing frequency, taking resistance, capacitance and time being constant is shown in above graph. The output voltage remains steady (-0.0078 mV) as the frequency increases from 20 kHz to 25 kHz.

5) The plot is drawn for output voltage (Vo) with varying time (t) showing independence of Vo on t (Fig. 17)

![Figure 17](image)

An illustration of Output Voltage (Vo) variation as a function of time (taking resistance, capacitance, frequency constant) is shown in the above graph. The Output Voltage (Vo) remains constant having value of -0.00784 mV throughout the time (1 ms to 44 ms).
6) The plot is drawn for output voltage (Vo) with varying input voltage (Vi) to find the gain (Fig.18)

Here, the graph is drawn between Output Voltage(Vo) and Input Voltage (Vi) keeping other parameters (i.e., resistance, capacitance, frequency and time) constant. The nature of the graph is linearly decreasing. The output voltage (Vo) decreases from -0.0049 mV to -0.75 mV corresponding to input voltage (Vi) from 0.1 mV to 15.3 mV. The slope of this graph gives gain of the differentiator, found to be -0.049.

IV. SUMMARY

The purpose of the present work is to show the variations of output voltage for different values of input parameters like resistance, capacitance, frequency, time and also to calculate gain in both ideal as well as practical differentiator circuits using Operational Amplifier. An ideal op-amp differentiator circuit has capacitor as input impedance and resistance as output impedance, but the circuit becomes unstable and noisy at high frequency ranges and hence, correction needs to be implemented to it. Practical differentiator has resistance and capacitance as input impedance and another resistance and capacitance as output impedance which make this circuit more stable and noiseless [9].

For ideal differentiator using Op-Amp, the output voltage decreases linearly with the increasing values of resistance and capacitance. However, the variation of output voltage is non-linear with respect to frequency and time. The gain of ideal differentiator is found to be -0.0312.

But, in case of practical differentiator, the output voltage varies non-linearly with input and output resistance though some linear variation has been achieved in case of input and output capacitance. Invariance nature of output voltage is depicted for the case of frequency range. It is quite interesting to note that the slope of practical differentiator is -0.049 which is less than that of ideal differentiator.

V. CONCLUSIONS

From the analyses it is evident that the output voltage decreases with increasing values of resistance and capacitance in both ideal and practical circuits. However, variations of output voltage with varying values of frequency (audible and inaudible) and time are distinct (linear and non-linear as well) in case of ideal circuit but not in practical circuit because the output voltage is independent of these two parameters.

It is to be noted that gain is less in practical differentiator circuit compared to the ideal one as expected. The output voltage becomes -0.2513 mV for frequency of 50 Hz in case of ideal circuit whereas the output voltage is -0.00784 mV for same value of frequency in practical differentiator circuit exhibiting better result compared to that of ideal case. Interestingly, if the values of resistance and capacitance R1, R2, C1 and C2 are interchanged i.e. R1 ↔ R2 and C1 ↔ C2 then the output voltage still becomes more in case of practical differentiator using Op-Amp compared to the ideal one.
The output voltage increases with varying output capacitance in case of practical differentiator circuit whereas the decreasing nature of output voltage with varying input capacitance supports same type of variation (decreasing) as observed in case of ideal differentiator circuit.

It is, therefore, suggested that practical differentiator circuit using Operational amplifier should be operated for better result compared to the ideal one at frequency of 50 Hz with output capacitance of 0.1 µF. Furthermore, it is to be noted that the non-inverting case is not considered in the present case since it is not useful and effective for practical applications.

Practical inverting differentiator using operational amplifier can be used for calculating in wave-shaping circuits [8], angular acceleration of an accelerometer [9]. This type of circuit is also used for analyzing biomedical and seismic signals [10, 11].

VI. ACKNOWLEDGEMENT

One of us, Jhilik Bhattacharjee, is grateful to Binod Bihari Mahto Koyalanchal University for getting financial assistance in the form of University Scholarship.

REFERENCES

[1] Ramakant A. Gayakwad, “Operational Amplifier and Linear Integrated Circuits”, Pearson Education, Fourth Edition, 29th May, 2015
[2] D. H. Horrocks, “A non-inverting differentiator using a single operational amplifier”, International Journal of Electronics, 37, pp. 433-434 (1974)
[3] S. Venkateswaran and K. Radhakrishna Rao, “Multifunction active RC circuit with grounded capacitors”, Electron. Lett., 7, 708-710 (1971)
[4] M.A. Al-Alaoui, “A stable differentiator with a controllable signal-to-noise ratio”, IEEE Trans. Instrum. Meas., IM-37, pp.383-388, Sept. 1988
[5] Vijaylaxmi Kalyani and Ayushi Arya, “Design and Simulation of VFA and CFA based integrator and differentiator using NI multisim and their comparison” IJARECE, 8, August 2014.
[6] M.A. Al-Alaoui, “A novel differential differentiator”, IEEE Transactions on Instrumentation, 40, pp. 826-830 (1991)
[7] M. Altun and H. Kurtman, “Design of a full differential current mode Operational Amplifier with improved input and output impedances and its filter application”, International Journal of electronics and Communication, 62, pp. 239-244 (2008)
[8] F. Brogan, “Wave-shaping and computing circuits”. Semiconductor Electronics by Worked Example, pp. 97-106 (1974).
[9] S.J. Ovaska and S.Valivitta, ”Angular acceleration measurement: a review”, IEEE Instrumentation and Technology Conference, 18-21 May 1998
[10] M.A. Al-Alaoui,” Low frequency Differentiators and integrators for biomedical and seismic signals”, IEEE Transactions on Circuits and System I: Fundamental Theory and Applications, 48 , pp. 1006-1011, August 2001.
[11] Chao-Hsiung Tseng, Li-Ts Yu, Jyun-Kai Huang, Chih-Lin Chang, “A wearable self-injection-locked sensor with active integrated antenna and differentiator based envelope detector for vital-sign detection from chest wall and wrist”, IEEE Transactions on Microwave Theory and Techniques, 66, pp. 2511-2521, January 2018.
[12] Bruce Carter and Thomas R. Brown, “Handbook of Operational Amplifier Applications”, Application Report , SBOA092B-October 2001-Revised September 2016, Texas Instruments.
[13] D.F. Stout and M. Kaufman, Handbook of Operational Amplifiers Circuit Design”, Mc Graw Hill, 1976.
[14] U.C. Sarker, S. K. Sanyal and R. Nandi, “A high-quality dual-input differentiator”, IEEE Transactions on Instrumentation and Measurement, IM-39, pp.726-729, Oct. 1990
