Orbital control for cube satellites

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Abstract
Today's space technology is continuously developing so that sending satellites into space is not difficult anymore. Besides, satellites have emerged as powerful platforms for telecommunication, survey, and traffic information. For example, many countries have started to develop their satellites. Due to its compact size and uncomplicated system, a cube satellite is suitable as a starting point for the study and development of space technology. This research investigates the stability of a cube satellite with a small size, weighing less than 1 ton and orbiting in a Low-Earth Orbit (LEO) by using the Fundamental Equations of Constrained Motion (FECM) and Sliding-Mode Controller (SMC). The proposed controller guarantees motions of the satellite under the constraint that the satellite has to orbit in a circular trajectory in a Low-Earth Orbit (LEO) and able to handle unexpected changes. The MATLAB program was used to gather data from the numerical simulation. The result from the motion of the analyzing process is rather a remarkable outcome. It may help us to understand the satellite’s movement in a circular orbit with less and adjustable errors. This finding, while preliminary, suggests that we can cope with changes very well particularly the external high-frequency noise. Also, the values of the acceptable errors can be determined by adjusting the control parameters in the proposed controller.

Keywords: Orbital control, Cube satellite, Sliding mode controller, Fundamental equations of constrained motion.

1. Introduction
In the history of development economics, Thailand has developed in many types of technologies, such as agricultural technology, transportation technology, medical technology, and exploration technology. Satellite is a dominant feature of exploration technology and it is of interest because it is widely used in finding natural resources, weather forecast, and explore the terrain, etc. In the same vein, space technology is one of the interesting technologies, and it can be developed continuously without the end. This current study analyses the impact of exploration technology by using a cube satellite. Recent developments in new exploration satellite have heightened the need for THEOS. The THEOS is a cube exploration satellite that has been created with cooperation between the governments of Thailand and France. This study also includes parameters of the THEOS satellite and the simulation.

The methodology developed in this work uses the sliding-mode controller together with the fundamental equations of constrained motion to calculate the satellite’s orbit. Few studies have investigated the use of the sliding-mode controller to control the motion of the satellite because it takes care of an uncertainty part of the system. Concerning space, there are many uncontrollable things and
satellites cannot be tuned their hardware. Consequently, a robust control system is recommended for these conditions. The system has to be managed effectively by using the fundamental equations of constrained motion in constraint following part for dealing with the ideal system and using the sliding-mode control for dealing with potential system instability.

2. Nominal system

2.1. The free body diagram of a cube satellite

The motion and moment of inertia chosen refer to the model of THEOS satellite [1]. The THEOS satellite is a cube satellite that meets the interest of this research. It can be given in Figure 1 and the property of the satellite is illustrated in Table 1.

![Figure 1. Model of the THEOS satellite][1]

**Table 1.** Property of the satellite

| Property of satellite | Value       |
|-----------------------|-------------|
| Weight                | 120 kg      |
| size                  | 800 x 800 x 1181mm |

2.2 Equations of motion of the satellite

The equation used to describe the satellite’s motion [5],[18] is set out in Equation (1).

\[
\mathbf{a}(t) = (\mathbf{M})^{-1} \mathbf{Q} = -\frac{\mathbf{G}\mathbf{M}_\oplus}{r^3} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}^T. \tag{1}
\]

The acceleration’s matrix shown above, is used to describe the motion of satellites, where

- \(G\) is the gravitational constant, which equals to \(6.67408 \times 10^{-11} \text{Nm}^2 / \text{kg}^2\)
- \(\mathbf{M}_\oplus\) is the mass of the earth, which is \(5.972 \times 10^{24} \text{ kg}\)
- \(r\) is the distance between the Earth’s center and the center of the satellite that is described by

\[
r = \sqrt{X^2 + Y^2 + Z^2}
\]
\[ [X \ Y \ Z]^T \] is the position vector of the center of mass of the satellite in the ECI (Earth-Centred Inertial) (see Figure 2).

Figure 2. Orbit frame

3. Constraint equation
This section focuses on bringing the conditional equations and applying the fundamental equations of constrained motion to control the system. However, this controller is suitable for the ideal system without uncertainty.

3.1. Condition’s equation
The following are the conditions that govern the system to move under the circular orbit’s condition

\[
2X = Z \text{ and } Y^2 + Z^2 = \rho_0^2.
\]

Equation (2) is the circular orbit condition. The orbit should be projected on YZ frame as circular orbit with a constant radius \( \rho_0 \) when \( [X \ Y \ Z]^T \) is in the ECI frame. Besides, X-axis of the satellite points towards the center of the Earth at all times.

To prevent the default of choosing the right initial condition assumption, Trajectory Stabilization [14], [21] is used in this paper. Thus, Equation (2) must be rewritten in the form of

\[
\alpha \ddot{\phi} + \beta \dot{\phi} + \gamma \phi = 0.
\]

This equation guarantees that \( \phi \to 0 \) when \( t \to \infty \). Rearranging the Equation (3), then we get

\[
\begin{bmatrix}
2\alpha_1 & 0 & -1\alpha_1 \\
0 & Y\alpha_2 & Z\alpha_2
\end{bmatrix}
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix}
= -\beta_1 (2\dot{X} - \dot{Z}) - \gamma_1 (2X - Z)
- \beta_2 (Y\dot{Y} + Z\dot{Z}) - \gamma_2 (Y^2 + Z^2 - \rho_0^2) - \alpha_2 (Y^2 + Z^2)
\cdot
\]

3.2. Fundamental equations of constrained motion
The first controller, is used to control the system to move according to the specified conditions, is the fundamental equation of the constrained motion \([2-4]\). The first step is to model the satellite’s system by using Newton’s second law of motion as provided in Equation (5)

\[
M(q, t)\ddot{q} = Q(q, \dot{q}, t)
\]

where
- \( q \) is the generalized coordinate
- \( t \) is time (second)
- \( M \) is the mass matrix
- \( Q \) is the given force vector.

The unconstrained acceleration of the system can be obtained by pre-multiplying of Equation (5) with \( M^{-1} \),

\[
a := M^{-1}(q, t)Q(q, \dot{q}, t).
\]

Rearranging the Equation (4) in the form of a linear function of acceleration \([7]\)

\[
A(q, \dot{q}, t)\ddot{q} = b(q, \dot{q}, t),
\]

where \( A \) is according to the Equation (7) under the generalized displacement matrix 3 vector

\[
q(t) = [X \ Y \ Z]^T,
\]

then, we get

\[
A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 2\alpha_1 & 0 & -1\alpha_1 \\ 0 & Y\alpha_2 & Z\alpha_2 \end{bmatrix}
\]

\[
b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -\beta_1(2X - Z) - \gamma_1(2X - Z) \\ -\beta_2(Y^2 + Z^2 - \rho_0^2) - \alpha_2(Y^2 + Z^2) \end{bmatrix}
\]

by following the condition Equation (2).

Taking the \( A_1, A_2 \) and \( b_1, b_2 \) from the previous step to form the equation of motion and get the control force of the system that makes the satellite move following the conditions \([15-16]\). The formula of the control is set out in Equation (11)

\[
Q^c(t) = Q^c(q(t), \dot{q}(t), t) = A^T(AM^{-1}A^T)^+(b - Aa)
\]

where
- \( T \) is the transpose of vector or matrix
- \( + \) is the Moore-Penrose inverse of a matrix.
4. Uncertainty control
In this section, the instability of the system is chosen. The effective controller should be able to accommodate the uncertainty of the system. Therefore, a sliding-mode controller [10-13] is selected. It is a robust controller that can compensate for instabilities and disturbances of the system as well. It is suitable for systems with highly non-linear and complicated.

Tracking- error signal of the system can be defined as follows:

$$e(t) = q_c(t) - q(t)$$  \hspace{1cm} (12)

where

$q_c(t)$ is actual system displacement
$q(t)$ is nominal system displacement.

Differentiating Equation (12) twice, we then get

$$\ddot{e} = \ddot{q}_c - \ddot{q}$$  \hspace{1cm} (13)

in which [10]

$$\ddot{q}_c = a_c + M_c^{-1}Q_c(t) + M_c^{-1}\dot{M}\ddot{u}_c$$  \hspace{1cm} (14)

and

$$\ddot{q} = M^{-1}Q + M^{-1}A^T(AM^{-1}A^T)^+(b - Aa) = a + M^{-1}Q_c(t).$$  \hspace{1cm} (15)

Substituting Equation (14) and Equation (15) in Equation (13), we obtain

$$\ddot{e} = \ddot{q}_c - \ddot{q} = [a_c(q_c, \dot{q}_c, t) - a(q, \dot{q}, t)] + [M_c^{-1}(q_c, t) - M^{-1}(q, t)]Q_c(t) + M_c^{-1}\dot{M}\ddot{u}_c$$

where

$$\delta q, \delta q_c, \delta \dot{q}_c, \delta \ddot{q}_c, t = [a_c(q_c, \dot{q}_c, t) - a(q, \dot{q}, t)] + [M_c^{-1}(q_c, t) - M^{-1}(q, t)]Q_c(t)$$

and

$$\dot{M} = I - M_c^{-1}(q_c, t)M(q, t) = I - [M(q_c, t) + \delta M(q_c, t)]^{-1}M(q, t)$$

$$= I - [M^{-1}(q, t)M(q_c, t) + M^{-1}(q, t)\delta M(q_c, t)]^{-1}.$$

There are uncertainties of the system being considered [20]. Mass and force are uncertain parameters that may change in time, especially, in unpredictable environments like space. Therefore, in the control system, these uncertainties must be anticipated to accommodate for future unexpected situations. Equation (16) refers to the equation of motion that adding nominal control force $Q_c(t)$ with additional control force for uncertainty $Q_a(t)$. Noting that if the system is certain, Equation (12) equals to 0.

$$M_a(q_c, t)\ddot{q}_c = Q_a(q_c, \dot{q}_c, t) + Q_c(t) + Q_a(t)$$  \hspace{1cm} (16)

where

$M_a$ is the actual mass, $M_a := M + \delta M > 0$
$Q_a$ is the actual control force $Q_a := Q + \delta Q$.

The equation also contains terms of the constraint following control force [19]

$$Q_c(t) = A^T(AM^{-1}A^T)^+(b - Aa).$$  \hspace{1cm} (17)
The discussion below is the control force that is created from the sliding-mode controller to compensate for the system’s uncertainties.

\[Q^u := M\ddot{u}_c\] (18)

in which

\[\ddot{u}_c = -[k\dot{e} + \beta(t)f(s)]\] (19)

where

\[\|e(t)\| \leq \frac{\sum}{2K}, \|\dot{e}(t)\| \leq \sum, \text{ and } \sum = 2\varepsilon\|g^{-1}(t)\| > L_\varepsilon(t) \text{ as } t \to \infty .\]

The function \(f_i(s)\) is any arbitrarily monotonically increasing [19], a continuous, odd function of \(s_i\) on the interval \((-\infty, +\infty)\) that satisfies

\[f_i(s) = g(s_i/\varepsilon), i = 1, 2, ..., n\] (20)

where

\[\|f(s)\| = \|g(s/\varepsilon)\| \geq \frac{\gamma(t) + k\|\tilde{M}(t)\|\|\dot{e}(t)\|}{\gamma(t) + \beta_0}\]. (21)

If is outside the surface \(\Omega_\varepsilon(t)\), which is defined as the surface of the n-dimensional cube around the point \(s=0\), Equation (22) will always be satisfied when \(\|f(s)\| > 1\) following the control law [13]. It will cause \(s(t) \to \Omega_\varepsilon \) as \(t \to \infty\). The value of \(\beta(t)\) should be satisfied with Equation (22)

\[\beta(t) \geq \frac{n[\gamma(t) + \beta_0]}{\alpha_0} > 0\] (22)

in which

\[\beta(t) > k\|\tilde{M}(t)\|\|\dot{e}(t)\|, 0 < \alpha_0 < 1 - n\sigma\|\tilde{M}(t)\| \text{ and } \gamma \leq \sigma \leq 1.\] (23)

So, finally, we get the equation of motion of the controlled system as

\[M_a(q_c, t)\ddot{q}_c = Q_a(q_c, \dot{q}_c, t) + Q^c(t) - M[k\dot{e}(t) + \sigma\beta(t)f(s)].\] (24)

5. Results of numerical simulation

This section involves the results of the numerical simulation, which they are simulated to verify the proposed control approach. The simulation is calculated based on the THEOS satellite parameters, as provided in Table 1. It is interesting to note that the satellite was found to cause the orbiting in a circle around the Earth under the conditions of \(J_2\) perturbation with an inclination 98.7 degrees, gravity of the Earth \(G = 6.67408 \times 10^{-11} \text{Nm}^2/\text{kg}^2\), Earth-mass \(M = 5.9722 \times 10^{24} \text{kg}\), and \(T\) (period) = 6084 seconds. In the same way, the calculation is simulated over 3 periods and the longitude of the ascending node \(\Omega_L = 30^\circ\).
\[ r(0) = p = 7.26 \times 10^6 \text{ m}, \quad v = \sqrt{\frac{GM}{r(0)}} = 7.41 \times 10^3 \text{ m/s}, \]

where

- \( r \) is the distance from the center of the Earth to the center of the satellite
- \( v \) is the satellite speed.

The initial condition used must be followed by all the constraints. Hence, the initial of the satellite [22] are

\[
\begin{align*}
X(0) &= 2328.9694 \times 10^3 \text{ m} \\
Y(0) &= 5995.21600 \times 10^3 \text{ m} \\
Z(0) &= 1719.97894 \times 10^3 \text{ m} \\
X(0) &= 2.911101130 \text{ m/s} \\
Y(0) &= -0.98164053 \text{ m/s} \\
Z(0) &= -7.090400220 \text{ m/s}.
\end{align*}
\]

In the actual system, there is an uncertainty that is simulated into the system, which is \( \delta M = 10\% \) of its original masses and \( \delta Q = \sin(100t) \) of its nominal parameters. 

In the sliding-mode controller the parameters are assigned as follows:

\[
k = 0.1, \quad \alpha = 1, \quad \beta_0 = 0.1, \quad \text{and } f(s) = (s / \varepsilon)^3.
\]

\[ \text{Figure 3. Satellite movement in YZ frame} \quad \text{Figure 4. Satellite movement in XZ frame} \]

The simulation results of the controller when trying to control the actual uncertain system are shown in Figure 3 and Figure 4. The satellite moves smoothly into the conditions. The diameter of the orbital radius is equal to \( p_0 \) that makes the proportion become 1. The speed of entering conditions can be increased by adjusted the value of \( \alpha, \gamma, \) and \( \beta \) from Equations (9) and (10). In this simulation, these values are \( \alpha_1 = 10, \gamma_1 = 15, \beta_1 = 0.1, \) and \( \alpha_2 = 1, \gamma_2 = 40, \beta_2 = 0.01. \)
The simulation errors were measured by using constraint conditions, as set out in Equation (2) denoting $e_1(t) = 2X - Z$, $e_2(t) = \sqrt{Y^2 + Z^2}$, $e_3(t) = 2X - \dot{Z}$, and $e_4(t) = YY - ZZ$. It can be seen that the tolerances of the systems reduced to 0 in a short period of time (see Figure 5, Figure 6, Figure 7, and Figure 8).
As can be seen in Figure 9, it illustrates the required control forces to control the satellite’s movement according to the constraint conditions. These forces are referenced in the ECI frame and they vary with the control parameters used to satisfy the system’s initial conditions. By starting with incorrect initial conditions, the control forces are happened to be large. The maximum control value in the X, Y, and Z axes are 1750 N, 6980 N, and 2780 N, respectively. When the satellite moves into their orbit, the control forces are gradually reduced to 0. Therefore, the whole stabilization process took place in only 200 seconds. After that, little effort (under 5 N) from the additional control force is required to handle the external high-frequency noise throughout the movement of the satellite, as represented in Equation (25).

6. Conclusion
This study sets out to develop a complete control system that can be applied in real situations. It also has discussed the reasons for adopting the value of J2 perturbation since it has an effect on the movement of the cube satellite under the condition of real orbit motion. This result of the simulation approach will prove useful in expanding our understanding of how effective the fundamental equations of constrained motion and the sliding-mode controller to control the satellite system. It can not only greatly reduce the errors in tracking the orbit of the system, but the system can also meet the stable point faster by increasing the control parameters and the values of $\alpha$, $\gamma$, and $\beta$ in the trajectory stabilization’s equation. Being limited to the size of the control forces, this study lacks the ability of fast control to the stationary point. The results of this research support the idea that the sliding-mode controller also has the advantage of being able to accommodate instability in the system. Although this current study has set the uncertainty of the nominal values at 10% (considered as rather high values), the controlled system shows that it can handle the uncertainty terms very well. Further experimental investigations are needed to estimate efficient directional and rotational system to increase the likelihood of being implemented of the system for future practice.
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