Modeling and force analysis of drum devices based on the geometry of the material segment

V Teruchcov¹, A Cupschev¹, V Konovalov² and Yu Rodionov³

¹Faculty engineering, Penza State Agrarian University, 30 Botanicheskaya Street, 440014 Penza, Russia Federation
²Department of Machine Building Technology, Penza State Technological University, 1A/11 Baydukova Drive/Gagarina Street, 440039 Penza, Russian Federation
³Department of Engineering Mechanics and Machine Parts, Tambov State Technical University, 1 Leningradskaya Street, 392620 Tambov, Russian Federation

¹E-mail: tvp141@mail.ru

Abstract. The aim of the research is the numerical simulation of the drum mixer operation, including the identification of geometrical indicators of a material pile in a rotating drum for conducting the force analysis and determining the expended power. The influence of the required performance of the drum device on the parameters of its container with regard to the filling degree is determined analytically using the mixer as an example. The functional model of the central angle of the material segment on the filling degree of the container is established. The expressions for the parameters of a material segment in a rotating container are revealed. They allowed for determining the torque and power consumption of the drive based on the power analysis. The graphic material of changes in the calculated indicators in the modeling process is presented.

1. Introduction

A variety of drum devices with a similar principle of operation are used in economic activities of people. One of the most commonly used devices are drum mixers. They are used in construction, engineering, chemical and food industries, agriculture. When studying drum mixers, researchers are most interested in the justification of the quality indicators of the mixer and its performance [1-4]. For this purpose, both experimental methods [1-4] and computer numerical analysis are used [5, 7]. In some cases, the drum is equipped with additional structural elements [1-3], or without them [4, 5]. Another area of research is the force analysis of the mixer [6-8], which allows one to determine the mixer drive power. In recent years, the interest in drum devices has increased, since they have been used to produce graphene.

Drum devices are used to implement many of the processes that are based on mixing. The use of mixing paddles in the drum intensifies the process of mixing the components, reducing the mixing time. However, a side effect is the inevitability of falling portions of the material raised by the paddles on the main body of the product. This limits the use of paddle drum mixers for mixing particles with low strength. Such drum-type devices without paddles or with micro-paddles are used for the particular case of mixing, which is coating (for example, to cover the seeds with a shell, or for manufacture of pellets – liquid particles with adhered solid particles outside). Drum devices are also used in triers for segregation, removal of particles of a certain geometric size.
Numerical analysis of the force impact allows for simulating a drum mixer for the given conditions of its design or performance. This makes it possible to optimize the parameters of the mixer and establish functional dependencies between the features of its design and performance.

At the moment, it is still a challenge to determine the power required to rotate the drum with a solid bulk material, depending on the degree of filling of the drum, the physicomechanical properties of the material of the mixture and the subcritical frequency of the drum rotation.

Use of computer programs based on the finite element method is an advanced method of research. However, it requires large investments in the purchase of expensive specialized computer programs and computers with great potential. Such investments are rarely economically justified for performing calculations in ordinary engineering problems solved under practical conditions on ordinary computers.

The aim of the research is the numerical simulation of the drum mixer operation, including the identification of geometrical indicators of a material pile in a rotating drum for conducting the force analysis and determining the expended power.

2. Experimental Part

In the course of the research, it was envisaged to use analytical methods based on the required performance or mixer parameters to establish the geometrical parameters of the material pile in the drum mixer container, to determine the dependence between the pile geometry and the current material forces in the rotating drum, torque and power consumption. To assess the numerical values of specific parameters of the mixer and its work on the basis of established formulas, the MathCAD mathematical pack was used.

The required duration $T_c$ of components mixing by drum mixers is 180-300 s [1, 2, 8]. Within this period, sufficiently rapid uniform distribution of components occurs throughout the volume of the mixture. After reaching certain quality of the mixture, further mixing of the components is impractical due to the stabilization process.

According to [7], the performance of a batch mixer can be calculated, kg/s:

$$Q = \frac{M}{T_0} = \frac{V_0 \cdot E \cdot \rho}{T_1 + T_c + T_2 + T_3} = \frac{0.25 \pi \cdot D \cdot L \cdot E \cdot \rho}{T_1 + T_c + T_2 + T_3},$$  

where $M$ – the mass of the feed portion, kg; $T_0$ – the cycle time of the mixer, s; $V_0$ – the volume of the mixer, m$^3$; $E$ – the filling degree of the mixer; $\rho$ – the density of material pile, kg/m$^3$; $T_1$ – the duration of the component load, s; $T_c$ – the duration of mixing the components, s; $T_2$ – the duration of unloading of the components, s; $T_3$ – the duration of additional operations, s; $D, L$ – the diameter and length of the container, m.

For continuous mixers, the mixer’s volume must comply with the condition:

$$V_0 = \frac{Q_p}{T_c \cdot E \cdot \rho},$$

where $Q_p$ – the productivity of the technological mixing line.

Let us consider a cross section of a rotating drum with material pile inside it (Figure 1). In a rotating drum, particles of the material are captured by a moving element (here – a container) and rise up along the walls of the cylindrical container 1. Reaching a certain angle of ascent, particles 3 fall down, forming the outer edge of material 2 ($A_0A_1$) at an angle $\alpha$ – a dynamic angle of repose of the material relative to the horizontal, rad.

The value of the dynamic angle of material collapse $\alpha$ is determined for a specific material based on field observations.

When the drum rotates, the material is constantly moving from the upper part of the material segment to its base. As a result, a new surface is constantly formed at the angle of dynamic collapse of the material. Therefore, when modeling, we accept the assumption of linearity of the collapse surface at the subcritical angular velocity, which forces the material to rotate with the drum.
For the analysis, the parameters of the General 63 mixer with a drum rotation frequency of 26 min\(^{-1}\) were used with its volume of 0.063 m\(^3\) and the installed power of the electric motor of 220 W.

![Figure 1](image.png)

**Figure 1.** Scheme of filling the cylindrical container of the drum mixer: 1 – the cylindrical container; 2 – the outer edge of the material pile; 3 – the conditional particle of the material \(M\) (an analog of the projection of the material elementary sector); \(\alpha\) – the angle of material repose, rad.

The external forces applied to the material particle \(M\) (as an analogue of the material elementary sector) are the pressure of the upstream material column (forming gravitation force \(G\)), the inertial effect of rotation (centrifugal force \(F_c\) of the material column above the particle \(M\) along the radius of the container). Under the action of these forces, the material elementary sector is pressed against the inner wall of the container, creating the reaction force of the normal pressure \(N\), as well as the friction force \(F_{\text{fr}}\) of the material against the container wall. Overcoming these forces, there is a force applied to the friction of the rotating cylindrical wall of the container \(F_{\text{fr}}\) onto the particle \(M\).

The force equilibrium conditions of the elementary sector (Figure 1):

\[
\begin{align*}
\sum F_x &= F_c - N + G \cdot \cos(\alpha) = 0, \\
\sum F_y &= F_{\text{tr}} - G \cdot \sin(\alpha) = 0, \\
N &= F_c + G \cdot \cos(\alpha) \\
F_{\text{tr}} &= G \cdot \sin(\alpha).
\end{align*}
\]

In the presence of the container rotation, a moment of material friction against the walls is created. If the friction force \(F_{\text{fr}}\) is small (with a small friction coefficient \(f\) of the material against the container’s wall, or a large angle of the wall placement relative to the horizontal), then there is a possibility for the material segment to slide along the container’s walls. This condition is described by the first equation in the system (4), where \(F_{\text{fr}} = N \cdot f\). Otherwise, the material is lifted to a certain height, corresponding to the location of the particles at a certain angle from the lowest point of the container. Then the material of the outer layers is poured out due to insufficient centrifugal forces, and the surface 2 is formed at an angle \(\alpha\) of repose of material pile. This condition is described by the second equation in the system (4).

The drum rotates by the drive, which overcomes the total moment \(M_k\) (H·m) from the friction force of the particles of all \(j\) material elementary segments, corresponding to the angle interval \(\gamma = \angle A_0 O A_i\) in Figure 2, i.e. from \(\beta_0\) to \(\beta_1\):

\[
M_k = \sum_{j} F_{\text{trj}} \cdot R.
\]

Drive power for the drum rotation \(P, W\):
\[ P = M_k \cdot \omega \cdot \frac{L}{\Delta t} = \sum_j F_{yj} \cdot R \cdot \omega \cdot \frac{L}{\Delta t} = \sum_j \left( F_{yj} + G_j \cdot \cos(\alpha) \right) \cdot f \cdot R \cdot \omega \cdot \frac{L}{\Delta t}. \] (6)

For the force analysis of the movement of the material particles and the working body, it is necessary to know the magnitudes of the acting forces (Figure 1) on a certain particle \( M \), as an analogue of the material elementary sector:

- \( G = m \cdot g = \rho \cdot h_1 \cdot \Delta S \cdot g \) – gravitation force, N;
- \( m \) – mass of the material column above the conditional material particle, kg;
- \( g \) – gravitational acceleration, kg/m\(^2\);
- \( \rho \) – density of material pile, kg/m\(^3\);
- \( h_1 \) – height of the vertical column of material, m;
- \( \Delta S = t_1 \cdot \Delta t \) – area of the elementary sector, m\(^2\);
- \( t_1 = R \cdot \pi / 180^\circ \) – length of the elementary sector around the circumference, of the central angle of the elementary sector – \( \varphi' = 1^\circ \), m;
- \( R \) – internal radius of the container, m;
- \( \Delta t = 0.001 \) m – length of the elementary sector along the rotation axis.

\[ F_c = m \cdot \omega^2 \cdot R' \] – centrifugal force, N;
- \( \omega \) – angular velocity of the container rotation, s\(^{-1}\);
- \( R' \) – radius of location of the center of gravity of the material elementary sector above the particle \( M \) in the direction of the rotation axis,

\[ R' = h_0 \cdot \frac{2 \sin(\frac{d\varphi}{\varphi'})}{3}, \] m;
- \( h_0 \) – height of the radial material column, m.

\[ F_f = N \cdot f \] – friction force of the material on the wall, N;
- \( N \) – normal response of the container’s wall to the action of the material,
- \( f \) – friction coefficient of the material against the container’s wall.

For calculation of these indicators, a certain complication is caused by finding the vertical \( h_1 \) or radial \( h_0 \) material columns (Figure 2). Wherein \( \beta_0, \beta_1, \beta_2, \beta \) – angles of the material segment’s points: the lower, upper, vertical projection of the upper point on the lower circle, and the current sector angle, respectively, rad;
- \( h_1 \) and \( h_0 \) – height of the vertical and the radial material column, m.

**Figure 2.** The layout of the geometrical parameters of material pile in a cylindrical container: 
- \( O \) – the center of rotation and cross-section of the container;
- \( AOA_1 \) – the conditional surface of repose of material pile;
- \( A, A' \) – conditional current points of the material on the surface of the drum, before and after the point \( A'' \), respectively;
- \( A'' \) – the projection of the point of material \( A_1 \) on the lower part of the drum surface;
- \( B, H, M \) – intersection points of the radii for the points \( A, A', A'' \) and the line \( AOA_1 \);
- \( N \) – the point of the perpendicular projection of the point \( O \) on the \( AOA_1 \) line;
- \( C \) – the vertical projection of the point \( A \) onto the line \( AOA_1 \;
- \( \alpha \) – the dynamic angle of repose, rad;
- \( \gamma \) – the central angle corresponding to the material segment, rad.

Therefore, the purpose of the research is to determine the functional dependences of the height of the vertical and radial material columns, allowing one to calculate the values and to establish the dependences of changes in their values to obtain a mathematical model of changes in the specified parameters, torque and power to the container’s rotation drive.

Let us consider the location of the material in the container. The material is located along the side wall and forms an angle \( \alpha \) relative to the horizontal. As the material in the container increases, the angle of repose practically does not change. However, the filled cross-sectional area changes (i.e. the area of...
the segment cut by chord \( A_0 A_1 \) changes) and, accordingly, the degree of the container filling in this cross-section also changes. The central angle for the segment is indicated as \( \gamma \), rad. The angular coordinate of \( A_0 \) relative to the vertical (\( OX – axis \)) is indicated as \( \beta_0 \) (rad.), \( A_1 \) (relative to the \( OX \) axis) indicated as \( \beta_1 \). The current angle, at the calculations in the interval \( (\beta_0; \beta_1) \), is indicated as \( \beta \). The angular coordinate of \( A_1’ \) (the vertical projection of \( A_1 \) on the lower part of the circle) relative to the vertical (\( OX \) axis) is indicated as \( \beta_2 \) (rad.).

The circle area (m\(^2\)) is determined:
\[
S_0 = \frac{\pi \cdot D^2}{4} = \pi \cdot R^2. \tag{7}
\]

The degree of filling of the circle’s cross section:
\[
S_c = \frac{S}{S_0}. \tag{8}
\]

The segment area, m\(^2\):
\[
S = \frac{R^2 \cdot (\gamma - \sin \gamma)}{2}. \tag{9}
\]

where \( \gamma = \angle A_0 O A_1 \) – the central angle of the material sector, rad. Consequently:
\[
\gamma - \sin \gamma = \frac{2S}{R^2}. \tag{10}
\]

The question is how to solve this equation and find the angle \( \gamma \). There is no exact analytical solution. The solution to this problem can be provided by a computer program. The results of numerical simulation to determine this angle are presented below.

It is necessary to determine the interrelation of the geometric indicators of the sector, taking into account the dependences for \( \gamma \). Let us consider the case (Figure 2) when the \( A_0, A_1 \) points and the segment lie on one side of the vertical (\( X \)-axis). In this case, the angles \( \beta_0 \) and \( \beta_1 \) can be expressed through the known angles \( \alpha \) and \( \gamma \).

From \( \Delta O A_1 A_0 \) and \( \Delta O M A_0 \):
\[
\frac{\gamma}{2} + \frac{\pi}{2} - \alpha + \beta_0 + \beta_1 - \beta_0 = \pi, \quad \text{and} \quad \beta_1 = \alpha + \frac{\gamma}{2}.
\]

In this case, \( \gamma = \beta_1 - \beta_0 \) thus \( \beta_0 = \beta_1 - \gamma \), i.e. \( \beta_0 = \alpha - \frac{\gamma}{2} \).

So,
\[
\begin{align*}
\beta_1 &= \alpha + \frac{\gamma}{2} \\
\beta_0 &= \alpha - \frac{\gamma}{2}
\end{align*}
\]

Let us find the \( OB \) from \( \Delta O B N \), where \( ON \perp A_1 A_0 \):
\[
OB = \frac{ON}{\cos(\angle BON)} = \frac{R \cos \frac{\gamma}{2}}{\cos \left( \frac{\gamma}{2} - \beta + \beta_0 \right)}. \quad BA = R - OB.
\]

Let us find \( AC \), if \( A \in A_0 A_1’ \) where \( AC \) is drawn as a vertical from \( A \) to the intersection with \( A_1 A_0 \). The angle from the vertical to \( OA \) is indicated as \( \beta \).
\[
AC = BA \cdot \frac{\sin \left( \frac{\pi}{2} + \alpha - \beta \right)}{\sin \left( \frac{\pi}{2} - \alpha \right)}, \quad \text{or} \quad AC = BA \cdot \frac{\cos(\alpha - \beta)}{\cos \alpha}.
\]
Let us find $A'C'$, if $A' \in A' \vec{A}'$. \( \angle OA'C = \beta \) (\( \beta \) – the angle from the vertical to \( OA' \)). The result is an expression $\angle OAA' A'C' = 2R \cdot \cos \beta$.

If \( A_0 \) lies to the left of the vertical, then all the formulas are correct, only the angle \( \beta_0 \) has negative values.

In this case, height of the material layer is:

$$h_0 = R - \frac{R \cos (0.5 \cdot \gamma)}{\cos (\alpha - \beta)}; \quad h_i = \frac{\cos (\alpha - \beta)}{\cos \alpha}.$$ \hspace{1cm} (11)

For the arc $A' \vec{A}'$, height of the material layer is:

$$h_i = 2R \cdot \cos (\beta).$$ \hspace{1cm} (12)

The angles of the material segment are determined as:

$$\beta_0 = \alpha - \frac{\gamma}{2}, \quad \beta_1 = \alpha + \frac{\gamma}{2}, \quad \beta_2 = \beta_1 - 2 \left( \beta_1 - \frac{\pi}{2} \right).$$ \hspace{1cm} (13)

Coordinates of characteristic points:

$A_0 = (R \cos \beta_0; R \sin \beta_0); \quad A_i = (R \cos \beta_i; R \sin \beta_i); \quad A' \vec{A}' = (O; R);$ \quad $A' \vec{A}' = (R \cos \beta; R \sin \beta).$

For the current \( i \) – values of the \( \beta \) angle in the range of angles (\( \beta_0; \beta_i \)), heights of the \( i \)-layer of the material are, m:

$$h_{0i} = R - \frac{R \cos (0.5 \cdot \gamma)}{\cos (\alpha - \beta_i)}.$$ \hspace{1cm} (14)

At bigger angles – \( h_i = 0 \).

3. Results and discussion

Numerical studies were carried out to determine the angle \( \gamma \).

The numerical values of the calculated sector area \( S \) (m²) are shown in Figure 3, where the number of the circle diameter \( D \) (m) is indicated horizontally, and the number of the filling degree \( S_z \) (fraction) – vertically.

![Figure 3](image-url)

In the selection process of the numerical values of the central angle \( \gamma \), its expression (10) was equated to zero. For this purpose, the point of transition of the sign from “minus” to “plus” was found, determining the difference between the right and the left parts of the expression (10). The interval of the specified differences \( G_000 \) and \( G_001 \) (as an indicator of non-conformity to zero of the difference between the values of the right and the left parts) is 0.1° relative to the numerical values of the angles \( \gamma \).
(Figure 4). For G000, the values are negative, and for G001, the values are positive. Consequently, on the indicated interval of the $\gamma$ angle values (Figure 5), the difference of values is zero, i.e. the required values of the $\gamma$ angle were found with the specified error.

For G000, the values are negative, and for G001, the values are positive. Consequently, on the indicated interval of the $\gamma$ angle values (Figure 5), the difference of values is zero, i.e. the required values of the $\gamma$ angle were found with the specified error.

The numerical results of the selection of the numerical values of the central angle $\gamma$ (deg.), while the difference between the right and the left parts tends to zero, are presented in Figure 5. The ordinal numbers of the circle diameter $D$ in a numerical experiment are presented in the columns; and the line numbers correspond to the filling degree of the container $S_z$.

Taking into account that regardless of the circle diameter the value of the central angle $\gamma$ is constant, the main indicator for determining the central angle $\gamma$, corresponding to the material segment, is precisely the filling degree of the container $S_z$ least square method. The approximation of the $\gamma$ angle values relative to the filling degree $S_z$ was made for convenience of calculations. A number of dependences were used to select the type of a regression equation. Functional models based on the calculated values are:

- **Linear model**
  \[ Y'_1 = 0.872 + 4.334 \cdot S_z, \tag{15} \]
  The correlation coefficient of the initial and calculated values – $r = 0.922$;
- **Polynomial model of 2nd degree**
  \[ Y'_2 = 0.846 + 7.164 \cdot S_z - 3.123 \cdot S_z^2, \tag{16} \]
  $r = 0.952$;
- **Polynomial model of 3rd degree**
\[ Y_3 = 0.088 + 13.777 \cdot S_z - 22.041 \cdot S_z^2 + 13.864 \cdot S_z^3, \quad r = 0.985 \; ; \]  

(17)

**Exponential model**

\[ Y_4 = 7.928 \cdot 10^5 \cdot e^{5.466 \cdot 10^{-5} \cdot S_z} - 7.928 \cdot 10^5 , \quad r = 0.922 \; ; \]  

(18)

**Logarithmic short model**

\[ Y_5 = 0.9597 \cdot \ln(S_z) + 4.0289 , \quad r = 0.921 \; ; \]  

(19)

**Logarithmic model**

\[ Y_6 = 1.8853 \cdot \ln(0.0997 + S_z) + 4.3035 , \quad r = 0.976 \; ; \]  

(20)

**Power model**

\[ Y_7 = 5.28 \cdot S_z^{0.417} - 0.666 , \quad r = 0.986 \; ; \]  

(21)

**Logistic model**

\[ Y_8 = \frac{4.54}{1 + 4.302 \cdot e^{4.897 S_z}}, \quad r = 0.920 \; ; \]  

(22)

**Sinusoidal model**

\[ Y_9 = 7.726 \cdot \sin(S_z + 0.507) - 3.238 , \quad r = 0.948 \; . \]  

(23)

The results of the simulation of the calculated values for the above equations and the comparison with the initial values of the angle are shown in Figure 6.

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**Figure 6.** Convergence of the initial and calculated values for graphical dependencies: graphs of all obtained models (a); graphs of high correlation models (b); \( Y \) – initial values of the central angle \( \gamma \) (rad.); \( Y_1 \) – linear model; \( Y_2 \) – polynomial model of the 2nd degree; \( Y_3 \) – polynomial model of the 3rd degree; \( Y_4 \) – exponential model; \( Y_5 \) – logarithmic short model; \( Y_6 \) – logarithmic model; \( Y_7 \) – power model; \( Y_8 \) – logistic model; \( Y_9 \) – sinusoidal model; \( X = S_z \) – the degree of filling capacity.

The polynomial models of the 2nd (\( Y_2 \)) and 3rd degree (\( Y_3 \)), the logarithmic (\( Y_6 \)) and sinusoidal (\( Y_9 \)) models have the highest values of the correlation coefficient (Figure 6). The polynomial model of the 3rd degree has the highest correlation (Figure 6b), which should be used while describing the value of the central angle \( \gamma \) (deg.).

For the polynomial equation of the 3rd degree with the filling degree \( S_z \) of more than 15 \%, the error \( \delta \) value does not exceed 0.135 rad. For the specified interval of error \( \Delta \) in absolute values (Figure 7) – it
does not exceed 5% for the initial \( Y' \) and the calculated values of \( Y_3 \) of the central angle \( \gamma \) (rad.). \( Y' \) –
initial values of the central angle \( \gamma \) (rad.). Wherein \( Y_1 \) – linear model; \( Y_2 \) – polynomial model of the 2nd degree; \( Y_3 \) – polynomial model of the 3rd degree; \( \delta \) – unrecorded residues (rad.) by the polynomial model of the 3rd degree of the central angle; \( \Delta \) – unrecorded residues (%) by the polynomial model of the 3rd degree of the central angle.

\[
\begin{array}{cccc}
0 & 0.2 & 0.224 & -0.024 \\
1 & 0.4 & 1.626 & 0.367 \\
2 & 0.6 & 2.113 & 0.04 \\
3 & 0.8 & 2.489 & -0.122 \\
4 & 1 & 2.824 & -0.135 \\
5 & 1.2 & 3.16 & -0.059 \\
6 & 1.4 & 3.457 & 0.043 \\
7 & 1.6 & 3.791 & 0.104 \\
8 & 1.8 & 4.169 & 0.067 \\
9 & 2 & 4.656 & -0.085 \\
\end{array}
\]

Figure 7. Convergence of the initial and calculated values for graphical dependencies: (a) \( \Delta \) – unrecorded residues by a polynomial of the 3rd degree, %; \( i = (10 \cdot S_z) \) – filling degree of the container; (b) \( \delta \) – unrecorded residues by a polynomial of the 3rd degree, rad.; (c) \( K_\delta \) – function of unaccounted residues by an additional polynomial model of the 3rd degree, rad.; (d) calculated values of the indicators: \( S_z \) – filling degree of the container, fraction.

The nature of the change in the residuals (errors \( \delta \) and \( \Delta \)) indicates the presence of a functional dependence (Figure 7a, 7b). As a result of additional approximation of the values of unrecorded residues, the function (Figure 7c) of corrections \( K_\delta \) (rad.) was obtained:

\[
K_\delta = 0.959 - 7.316 \cdot S_z - 15.216 \cdot S_z^2 - 9.312 \cdot S_z^3.
\]  

(24)

The correlation coefficient \( r = 0.904887 \).

A functional model of the central angle with the previously unrecorded residues (Figure 8) can be written as the expression:

\[
\gamma = Y_3 + K_\delta.
\]  

(25)

The correlation coefficient with initial values \( r = 0.99999 \).

The conducted numerical studies allowed us to establish the numerical values of the geometric indicators of the material segment.

The values of \( h_1 \) (Figure 9a) vary according to the expression (11) by the numerical values on the graph in the interval of angles \( \beta_0 \leq \beta \leq \beta_2 \). Reaching the values of angles \( \beta = \beta_2 \), there is no free surface of the material (12), since the upper part of the material is limited by the upper part of the container. Upon reaching \( \beta = 90^\circ \), the value of \( h_1 \) will then be zero (14). The values of \( h_0 \) (Figure 9b) vary along a complex arc (11), remotely resembling a sinusoid.
Figure 8. Convergence of the initial and calculated values for graphical dependencies: graphical results of the calculation (a); numerical calculation results (b); \( \gamma \) – initial values of the central angle (rad.); \( Y_3 \) – polynomial model of the 3rd degree; \( \gamma \) – refined model of the central angle by the function of unaccounted residuals by the polynomial model of the 3rd degree; \( i = (10 \times S_z) \) – filling degree of the container.

Figure 9. Height of the material layer: vertical \( h_1 \) (m) and radial \( h_0 \) (m), related to the elementary sector, located in the direction of the angle \( \beta \).

Taking into account the values of heights \( h_1 \) and \( h_0 \), the magnitudes of the acting forces change in a similar way (Figure 10). The nature of centrifugal force \( F_c \) corresponds to the character of height \( h_0 \), and the character of gravitation force \( G \) change – to the character of \( h_1 \). The nature of the normal reaction \( N \) changes according to the interaction of these forces. Friction force \( F_f \) changes by analogy with the normal reaction \( N \), according to the first equation of the system (4). In this case, when the numerical values of the friction force according to this formula are less than the values of the friction force according to the second equation of the system (4), reliable contact of the material and the surface of the container (\( \beta \approx 60^\circ \)) is observed. Later, this condition is not satisfied, and there is a possibility of the material sliding along the container’s wall. It is limited by the material particles located below.

These functions of friction force cause the occurrence of two variants of torques (Figure 11). Comparing their total values, and translating into power consumption, we get two options of power. The calculated power options are 134 W and 140 W, the difference is 4.9 %. The filling degree of the container is 30 % with the diameter and length of the container of 0.6 m, the density of the material 650 kg/m³, and the friction coefficient \( f \) of the material along the wall is 0.5, \( \alpha = 45^\circ \).
Figure 10. Forces acting on the elementary sector, located in the direction of the angle $\beta$: (a) centrifugal force, N; (b) gravitation force, N; (c) normal reaction, N; (d) friction force, N.

Figure 11. Torque (N m), related to the elementary sector, located in the direction of the angle $\beta$.

4. Conclusion

Thus, the established expressions allow for the simulation of changes in the basic geometrical indicators of a material segment in cylindrical containers, providing calculation with an error not exceeding 5%, which allows establishing the functions of friction force, torque and power of container rotation. The implementation of the identified dependences using computer programs (for example, MathCAD) allows both calculating the acting forces, moments and power consumption values for specific parameters, as well as carrying out numerical studies of this process, optimizing the mixer parameters.

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