Motivation:
Proton Holography
phase reconstruction

e-Print: 2004.07095 [hep-ph], EPJ Web Conf. 235 (2020) 06002

Stellar Interferometry:
Binary stars
New results, motivated by M. Lisa’s and Naomi Vogel’s talk at WPCF2023

Conclusion
Basic idea of holography (1947): amplitude level reconstruction. First hologram (1948) from D. Gabor’s Nobel lecture (1967).
https://www.nobelprize.org/uploads/2018/06/gabor-lecture.pdf
Formalism: elastic pp scattering

\[ \sigma_{el}(s) = \int_0^\infty dt|t| \frac{d\sigma(s)}{dt} \]

\[ \frac{d\sigma(s)}{dt} = \frac{1}{4\pi} |T_{el}(s, \Delta)|^2, \quad \Delta = \sqrt{|t|}. \]

\[ B(s,t) = \frac{d}{dt} \ln \frac{d\sigma(s)}{dt} \]

\[ B(s) \equiv B_0(s) = \lim_{t \to 0} B(s,t), \]

\[ \sigma_{tot}(s) \equiv 2 \text{Im} T_{el}(\Delta = 0, s) \]

\[ \rho(s,t) \equiv \frac{\text{Re} T_{el}(s, \Delta)}{\text{Im} T_{el}(s, \Delta)} \]

\[ \rho(s) \equiv \rho_0(s) = \lim_{t \to 0} \rho(s,t) \]

Basic problem: d\sigma/dt measures an amplitude, *modulus squared*. Amplitude level reconstruction?? Phase info apparently lost...
Formalism in b space

$$\frac{d\sigma(s)}{dt} = \frac{1}{4\pi} |T_{el}(s, \Delta)|^2, \quad \Delta = \sqrt{|t|}.$$ 

$$t_{el}(s, b) = \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\Delta b} T_{el}(s, \Delta) =$$

$$= \frac{1}{2\pi} \int J_0(\Delta b) T_{el}(s, \Delta) \Delta d\Delta,$$

$$\Delta \equiv |\Delta|, \quad b \equiv |b|.$$ 

$$t_{el}(s, b) = i \left[ 1 - e^{-\Omega(s,b)} \right]$$ 

$$P(s, b) = 1 - \left| e^{-\Omega(s,b)} \right|^2$$

Impact parameter or b space:

**elastic scattering interferes with propagation w/o collisions**: Genuine quantum physics.

Complex opacity function $\Omega(s,b)$ (eikonal, from unitarity)

$0 \leq P(s,b) \leq 1$ : **inelastic** scattering has a probabilistic interpretation
MODEL INDEPENDENT LEVY EXPANSION

\[ \frac{d\sigma}{dt} = A w(z|\alpha) \left[ 1 + \sum_{j=1}^{\infty} c_j l_j(z|\alpha) \right]^2, \]

\[ w(z|\alpha) = \exp(-z^\alpha), \]

\[ z = |t| R^2 \geq 0, \]

\[ c_j = a_j + ib_j, \]

\[ l_j(z|\alpha) = D_j^{-1/2} D_{j+1}^{-1/2} L_j(z|\alpha), \]

\[ D_0(\alpha) = 1, \]

\[ D_1(\alpha) = \mu_{0,\alpha}, \]

\[ D_2(\alpha) = \mu_{0,\alpha} \mu_{1,\alpha}, \]

\[ D_3(\alpha) = \mu_{0,\alpha} \mu_{1,\alpha} \mu_{2,\alpha}. \]

non-exponential behavior (NEB) in a single parameter \( \alpha \)

idea: complete set of orthonormal functions, put NEB to the weight

\[ \int_0^\infty dz \exp(-z^\alpha) l_n(z|\alpha) l_m(z|\alpha) = \delta_{n,m}, \]

\[ \mu_{n,\alpha} = \int_0^\infty dz \ z^n \exp(-z^\alpha) = \frac{1}{\alpha} \Gamma \left( \frac{n+1}{\alpha} \right) \]

Levy series ~ Taylor series: orthonormal wrt \( w(z) = \exp(-z^\alpha). \) T. Cs., R. Pasechnik, A. Ster, arXiv:1807.02897, arXiv:1811.08913, arXiv:1902.00109, arXiv:1903.08235
ABILITIES: CONVERGES TO $\sigma_{tot} = 115.2\text{ mb}$, $\sigma_{el} = 31.4 \pm 0.019\text{ mb}$

$\rho = 0.087$

$-t_{dip} = 0.47$, $\chi^2/\text{NDF} = 330/279$

$R_L = 0.7216 \pm 0.0002\text{ fm}$

$A_L = 361.88 \pm 0.37\text{ mbGeV}^{-2}$

$\alpha = 0.9032 \pm 0.0002$

$a_1 = -0.3184 \pm 0.0002$

$b_1 = 0.0706 \pm 0.0002$

$a_2 = 0.0567 \pm 0.0001$

$b_2 = -0.0350 \pm 0.0003$

$a_3 = -0.0193 \pm 0.0001$

$b_3 = 0.0227 \pm 0.0002$

$a_4 = 0.0067 \pm 0.0001$

$b_4 = -0.0022 \pm 0.0001$

Levy series ~ Taylor series: orthonormal wrt $w(z) = \exp(-z^\alpha)$. T. Cs., R. Pasechnik, A. Ster, 

[arXiv:1807.02897, arXiv:1811.08913, arXiv:1902.00109, arXiv:1903.08235]
CONVERGENCE PROPERTIES OF LEVY SERIES

Partial sum converges both in pbarp (n=3) and also in pp (n=4)

Partial sum converges to profile function $P(b)$ and slope $B(t)$

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235
CONVERGENCE OF PHASE RECONSTRUCTION

Levy expansion converges to the phase $\phi(t)$: Proton holography?! Cross-check with Coulomb-Nuclear Interference at $t=0$ successful! Deeper level of understanding – in progress, but stop here for now.

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235
$T_{el}(t) = |T_{el}(t)| \exp [i\phi_1(t)]$

Levy expansion converges to the phase $\phi(t)$: Proton holography!
Stellar interferometry results, HESS, talk of Naomi Vogel@ WPCF23:

Fits a **BINARY** star with intercept parameter $\lambda = 1$

But **this is a puzzle**, see R. Hanbury Brown et al, MNRAS (1974) **167**, 121
The normalized correlation also depends upon whether a star is single or multiple. It was shown in Paper II that, if a star is binary and the angular separation of the two components is completely resolved by the interferometer at the shortest baseline, then the normalized zero-baseline correlation $c_N/(o)$ averaged over a range of position angles is reduced relative to a single star $c_N(o)$ by the factor,

$$\frac{c_N(o)}{c_N(o)} = \frac{(I_1^2 + I_2^2)}{(I_1 + I_2)^2}$$

(mon. not. r. astr. soc. (1974) 167, 121-136.

The angular diameters of 32 stars

R. Hanbury Brown, J. Davis and L. R. Allen

Core-halo picture:

T. Cs, B. Lörstad and J. Zimányi, Z. Phys. C71 (1996) 491-497
J. Bolz et al, Phys. Rev. D 47 (1993) 3860-3870

Fig. 7. a) Two stars, S, and T, along nearby lines of sight from the earth; b) of correlated intensity from the two stars; c) schematic of HBT measurement halo of dim stars.

Fig. 7 from G. Baym’s review paper: arXiv:nucl-th/9804026

Schematics of HBT for multiple stars, but no formula. Note: intercept!
Partial resolution of the components from W. Guerin et al, MNRAS 480, 245–250 (2018)

Reduction of average measured intensity reduces:
R. H. Brown and R. Q. Twiss, Brown, R. Hanbury, and R. Q. Twiss: in Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences (1958): 291-319.
HBT EFFECT FOR „IDENTICAL TWIN” STARS

\[ \rho(x) = f_+ s(x - x_+) + f_- s(x - x_-), \]

\[ f_+ + f_- = 1 \]

Normalization: \( \text{FT}(s|q = 0) = 1 \)

\[ C(q) = 1 \pm |\tilde{\rho}(q)|^2 = 1 \pm \Omega(q)|\tilde{s}(q)|^2, \]

Stellar model: uniformly illuminates sphere

\[ |\tilde{s}(q)|^2 = \left[ \frac{2J_1(\pi r_0/\lambda_0)}{\pi r_0/\lambda_0} \right]^2. \]

Normalization: \( C(q = 0) = 1 + 1 \) but with an oscillating prefactor

\[ \Omega(q) = \left[ (f_+^2 + f_-^2) + 2f_+f_- \cos[q(x_+ - x_-)] \right] \]

Binary source formalism: binary sources in hydro (Cooper-Frye)

Schematics worked out for identical stars, e-Print: hep-ph/0011320

Detailed review including hydro results with two saddle points in hep-ph/0001233
THE ANGULAR DIAMETERS OF 32 STARS

R. Hanbury Brown, J. Davis and L. R. Allen

depends upon whether a star is single or star is binary and the angular separation of the two components is completely resolved by the interferometer at the shortest baseline, then the normalized zero-baseline correlation $c_N(0)'$ averaged over a range of position angles is reduced relative to a single star $c_N(0)$ by the factor,

$$
\frac{c_N(0)'}{c_N(0)} = \frac{(I_1^2 + I_2^2)}{(I_1 + I_2)^2}
$$

(9)

where $I_1, I_2$ are the brightness of the two components. It is simple to extend this analysis to a multiple star with $n$ components and to show that, if the angular separation between all the components is resolved, the zero-baseline correlation is reduced relative to a single star by the factor,

$$
\frac{c_N(0)'}{c_N(0)} = \sum I^2 / \left( \sum I \right)^2.
$$

(10)

It follows that if a star yields a correlation which is significantly less than that expected from a single star, then it must be multiple.

Note: $f_1 = I_1/(I_1+I_2)$, $f_2 = I_2/(I_1+I_2)$, etc → HEP connection
HBT FOR „NON-IDENTICAL TWIN” STARS

Similar, but two sources have different sizes, but with

\[ f_+ + f_- = 1 \]

Stellar model: uniformly illuminates sphere or any unknown shape

An oscillating prefactor remains, and averages to

\[
\Omega(q) = \left[ (f_+^2 + f_-^2) + 2f_+f_- \cos[q(x_+ - x_-)] \right]
\]

Binary source formalism: effective reduction of intercept for well separated compact sources

Effectively \( \frac{1}{2} \leq \lambda = \Omega(q) \leq 1 \)

Schematics works even for out non-identical stars, observations of N. Vogel explained

Multiple n source, well separated compact sources

Easy to show: \( \frac{1}{n} \leq \lambda = \Omega_n(q) \leq 1 \)
STRENGTH OF HBT FOR BINARY STARS

\[ \lambda = \frac{c_2(0)'}{c_2(0)'} = \frac{I_1^2 + I_2^2}{(I_1 + I_2)^2} \]

\[ f_1 = \frac{I_1}{(I_1 + I_2)} \]
\[ f_2 = \frac{I_2}{(I_1 + I_2)} \]
\[ 1 = f_1 + f_2, \]
\[ \lambda = f_1^2 + f_2^2. \]

C(q = 0) = 1 + 1 but oscillating prefactor REDUCES average intercept

\[ \langle f \rangle = \frac{f_1 + f_2}{2} = \frac{1}{2}, \]
\[ \delta_1 = f_1 - \langle f \rangle, \]
\[ \delta_2 = f_2 - \langle f \rangle, \]
\[ 0 = \delta_1 + \delta_2, \]

\[ \lambda = f_1^2 + f_2^2 = (\langle f \rangle + \delta_1)^2 + (\langle f \rangle + \delta_2)^2, \]
\[ \lambda = f_1^2 + f_2^2 = 2(\langle f \rangle)^2 + \delta_1^2 + \delta_2^2, \]
\[ \frac{1}{2} \leq \lambda \leq 1. \]

Lower limit if and only if \( \delta_1 = \delta_2 = 0, f_1 = f_2 = <f> \)
\[ \lambda = \frac{c_N(0)'}{c_N(0)'} = \frac{\sum_{j=1}^{N} I_j^2}{(\sum_{j=1}^{N} I_j)^2} \]

\[ f_j = \frac{I_j}{(\sum_{j=1}^{N} I_j)} \]

\[ 1 = \sum_{j=1}^{N} f_j, \]

\[ \lambda = \sum_{j=1}^{N} f_j^2 \]

\[ \langle f \rangle = \frac{\sum_{j=1}^{N} f_i}{N} = \frac{1}{N}, \]

\[ \delta_j = f_j - \langle f \rangle, \]

\[ 0 = \sum_{j=1}^{N} \delta_j, \]

\[ C(q = 0) = 1 + 1 \text{ but oscillating prefactors REDUCE average intercept} \]

\[ \lambda = \sum_{j=1}^{N} f_j^2 = \sum_{j=1}^{N} (\langle f \rangle + \delta_j)^2, \]

\[ \lambda = N(\langle f \rangle)^2 + \sum_{j=1}^{N} \delta_j^2, \]

\[ \frac{1}{N} \leq \lambda \leq 1. \]

Lower limit if and only if for all \( j = 1, \ldots, N \), \( \delta_j = 0 \), \( f_j = \langle f \rangle \).
**SUMMARY FOR BINARY STARS**

Stellar interferometry results, HESS, from N. Vogel’s talk @ WPCF23: Fits with intercept parameter $\frac{1}{2} \leq \lambda \leq 1$ possible, **puzzle resolved**. Looks to be a new result in astrophysics.

| Source          | Dschubba (Delta Scorpii) |
|-----------------|--------------------------|
| Magnitude (mag) | 2.2                      |
| Spectral type   | B0.3IV                   |
| System          | Binary star              |
| HBT time (h)    | 115.1                    |
| HBT diameter (mas) | 0.45 ± 0.04             |
| Time (h)        | 5.1                      |
| Diameter (mas)  | 470nm: 0.613 +/- 0.072   |
|                 | 375nm: 0.612 +/- 0.081   |