Search method for coincident events from LIGO and IceCube detectors

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Abstract

We present a coincidence search method for astronomical events using gravitational wave detectors in conjunction with other astronomical observations. We illustrate our method for the specific case of the LIGO gravitational wave detector and the IceCube neutrino detector. LIGO trigger events and IceCube events which occur within a given time window are selected as time-coincident events. Then the spatial overlap of the reconstructed event directions is evaluated using an unbinned maximum likelihood method. Our method was tested with Monte Carlo simulations based on realistic LIGO and IceCube event distributions. We estimated a typical false alarm rate for the analysis to be 1 event per 435 years. This is significantly smaller than the false alarm rates of the individual detectors.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In this paper, we present an analysis method to look for astrophysical sources which produce both gravitational wave and high-energy neutrino bursts using data from LIGO and IceCube. One example of a possible source is a gamma-ray burst (GRB). Thanks to the Swift satellite [1], there is accumulating observational evidence suggesting the association of long GRBs with the death of massive stars and supernova-like events (e.g. SN2006aj and GRB060218 [2], see also [3]). The collapsar model [4] is widely accepted for explaining long GRBs and stellar collapse. During the gravitational collapse of rapidly rotating stars, gravitational waves...
are emitted (see [5] for a review). First, fireballs heated by neutrinos from the accretion disc are thought to produce the prompt gamma-ray emissions [6]. Subsequently in the prompt and afterglow phases, high-energy neutrinos (∼10^5–10^{10} GeV) are expected to be produced by accelerated protons in relativistic shocks (see [7, 8] for reviews). High-energy neutrinos could also be emitted from short-duration GRBs, which are thought to be the outcome of neutron star mergers [9]. There is currently limited knowledge both observationally and theoretically about the details of the astrophysical process connecting the gravitational collapse/merger of compact objects and black-hole formation with the formation of fireballs. Coincident observations of gravitational waves and neutrinos from those events could therefore make an important contribution to the understanding of such phenomena.

Apart from GRBs, there may be other (unknown) classes of sources which produce bright bursts in both gravitational waves and neutrinos. Since our proposed method is not specific to any source type, our search will be able to set an upper limit for the population of any sources that produce nearly simultaneous bursts of gravitational waves and high-energy neutrinos within the detection range of LIGO and IceCube. We may also discover a previously unknown astrophysical phenomenon, if correlated events are found at a high confidence level.

There are several interferometric gravitational wave (GW) [10] detectors around the world, such as LIGO [11], TAMA [12], GEO [13] and VIRGO [14], currently in operation. These detectors monitor the relative displacement of mirrors (test masses) in response to distortions induced by gravitational waves. There are also several high-energy neutrino detectors operating, including AMANDA [15], IceCube [16] and ANTARES [17], which look for the Cherenkov light of charged particles emitted by neutrino interactions in water or ice. We illustrate the coincidence search method for the specific case of LIGO and IceCube.

LIGO is a network of interferometric gravitational wave detectors consisting of three interferometers\(^4\) in the USA [18]. Two interferometers (4 km and 2 km long ones) are co-located in Hanford, WA and another 4 km interferometer is located in Livingston, LA. They have now achieved the design sensitivity [19].

Since the interaction of gravitational waves with matter is extremely weak, expected signals even from very strong gravitational wave sources are very small. In order to declare a detection, we have to find a small signal in an overwhelming noise background with high confidence. Generally, the output from the detector contains glitches which are not associated with gravitational waves but rather caused by various local disturbances such as laser noises, seismic excitations, etc. In order to search for GW bursts, which are gravitational waves of short duration, it is therefore important to distinguish gravitational wave signals from noise glitches without prior knowledge of signal waveforms.

One way to pick out gravitational wave signals of unknown waveform from the noise background is to find coincidences between independent detectors. We can reject a large fraction of background events by comparing the arrival time and other properties (frequency, duration, etc) of the signals detected by independent gravitational wave detectors [20–22]. Additionally, event lists from other astronomical observations, such as gamma-ray bursts (GRB), optical supernovae, neutrinos, etc can be used to find events that may be associated with GW bursts with an increased likelihood [23–27]. Moreover, strict coincidence requirements allow us to investigate candidate events at lower signal-to-noise ratios (SNRs) while maintaining a low false alarm rate (FAR). Here, we propose a method for coincidence analysis of gravitational wave data with other detectors and illustrate it for the case of the LIGO gravitational wave detector and the IceCube neutrino detector.

\(^4\) From now on, we treat the network of the three LIGO interferometers as one detector and use the word ‘detector’ to refer to them as a whole. To refer to individual LIGO interferometers, we always use the word ‘interferometer’ to avoid confusion.
IceCube is a cubic kilometer-scale neutrino detector under construction at the geographic South Pole. Its primary mission is the search for high-energy extraterrestrial neutrinos. When completed, IceCube will consist of an array of 4800 digital optical modules, attached to 80 strings submerged within the Antarctic ice. Currently the detector takes data with more than 90% lifetime, except during a few months each year for construction and commissioning of new strings. IceCube is optimized to look ‘down’, using the Earth as a screen to block all particles except neutrinos; thus its field of view is the northern hemisphere. Neutrino arrival directions are resolved with a median error between 1° and 2° [28]. The threshold neutrino energy for the IceCube detector is 100 GeV. The full-energy range of observed events depends primarily on the competition between the unknown, but presumably falling, source flux, versus the rising neutrino cross-section. A flux with an $E^{-2}$ differential energy spectrum, for example, results in an energy distribution of neutrino events that peaks in the range of $10^4$–$10^5$ GeV.

In our search method, the data streams from the LIGO interferometers are processed by a trigger generation pipeline, which generates a list of gravitational wave triggers for each interferometer. Then we compare the trigger lists from LIGO interferometers to generate a coincident LIGO event list, which contains the arrival time and the source direction of each event. The LIGO event list is compared with an event list from the IceCube detector which also contains the timing and source direction information of the events. From the event lists we choose pairs of LIGO–IceCube events which lie within a certain time interval as time-coincident events. Then the spatial overlap between the LIGO and IceCube events is statistically evaluated to obtain the significance of the coincident event.

Because of the very different nature and geographical location of the two detectors, it is extremely unlikely that the coincident triggers are due to the same source of noise. Therefore, the remaining possibility for time coincident trigger generation in both detectors, other than real astronomical events, is accidental coincidence. Furthermore, the chance for two time-coincident noise triggers to generate overlapping reconstructed directions on the sky is also small. By the combination of timing and directional coincidence discrimination, we can expect that most background events will be rejected and the FAR will be significantly reduced.

2. Coincidence analysis

The outline of the proposed analysis method is shown in figure 1. The inputs to the analysis pipeline are LIGO and IceCube event lists and a large number of simulated background events. The outputs of the pipeline are the most plausible source direction and the statistical significance of any time-coincident event against the background noise events.

2.1. Event lists

Data streams from LIGO interferometers are processed by a trigger generation pipeline (e.g. [29, 30]) to generate a list of events for each LIGO interferometer. We then compare the arrival times of the events from the LIGO interferometers and select events which appear in all the detectors with less than 10 ms time difference. 10 ms corresponds to the gravitational wave’s travel time between the two LIGO sites, i.e. the maximum time delay allowed for a gravitational wave signal. If the trigger generation pipeline provides more information on the events, such as dominant frequency, duration, etc we also compare those parameters and reject events with large discrepancies.
This intra-LIGO coincidence can be applied between all three LIGO interferometers or any combination of two interferometers. From now on in this paper, we focus on the two-interferometer case using the Hanford 4 km (H1) and the Livingston 4 km (L1) interferometers, because the third interferometer (Hanford 2 km) is twice less sensitive than the others.

For later statistical treatments, a large number of background events are created, also from the LIGO data, in almost the same way. The only difference is that we introduce an artificial time shift between the trigger times from different interferometers to ensure that the resultant background event list does not contain real gravitational wave events.

An IceCube event list is determined by the combination of event reconstruction algorithms and quality cuts used to reject the dominant background of down-going cosmic ray muons. The remaining up-going events are expected to be predominantly atmospheric neutrinos, produced by cosmic rays on the far side of the Earth. The individual event information needed for this analysis is the time, the arrival direction and its associated angular uncertainty. For background IceCube events, Monte Carlo simulations which imitate the distributions and average properties of IceCube events have been used.

### 2.2. Time coincidence

Once event lists from LIGO and IceCube are prepared, they are compared for inter-detector time coincidence. We look for pairs of LIGO and IceCube events which appear within a certain time window and register them as time-coincident events for further analysis.

A smaller time window can reject background events more efficiently. However, the size of the time window must be sufficiently large to allow intrinsic time delay between the two emission processes at the source. Since we do not assume any specific source model in this analysis, we propose to use several time windows, e.g. 0.1, 1, 10 s and also 1 day in the case of long GRB search. The time window should be larger than the travel time of light between the IceCube and LIGO sites, i.e. 40 ms.
2.3. Spatial coincidence

The LIGO–IceCube combined events which survive the time-coincidence discrimination are further processed in order to examine spatial coincidence by an unbinned maximum likelihood method.

First, we calculate the spatial probability distribution function (SPDF) of each event from LIGO and IceCube. Taking a sky location \( r \) as an input, this function returns the probability of the actual source location being \( r \).

The source location of each LIGO event is reconstructed by measuring the arrival time difference \( \tau \) of the signal between the two sites. Using the measured arrival time difference \( \tau_M \), we can constrain the possible source locations to a ring on the sky defined by a polar angle \( \theta_{ev} = \cos^{-1}(c \tau_M / D) \) measured from the axis connecting the two LIGO sites (LIGO axis). Here, \( c \) is the speed of light and \( D \) is the distance between the two LIGO sites. Because the measured \( \tau_M \) has uncertainty \( \delta \tau \), the ring has a finite thickness. We assume that the probability distribution of the real time delay, \( \tau \), is a Gaussian around the measured time delay \( \tau_M \) with the standard deviation \( \delta \tau \). By changing the variate from \( \tau \) to \( \theta \) using \( \theta = \cos^{-1}(r \cdot l / |r| \cdot |l|) \), we get the SPDF for a LIGO event,

\[
S_{GW}(r; \theta_{ev}, \delta \tau) = A_{GW} \cdot \exp \left[ -\frac{D^2 (\cos \theta - \cos \theta_{ev})^2}{2 \delta \tau^2 c^2} \right],
\]

\[
\theta = \cos^{-1} \left( \frac{r \cdot l}{|r| \cdot |l|} \right),
\]

where \( l \) is a vector parallel to the LIGO axis and \( \theta \) is the angle between \( r \) and the LIGO axis. \( S_{GW}(r; \theta_{ev}, \delta \tau) \) is normalized to unity over the whole sky by a normalization factor \( A_{GW} \). An example of a LIGO event is shown in figure 2(a).

For the SPDF of an IceCube event we use a two-dimensional Gaussian distribution on a sphere:

\[
S_{\nu}(r; r_{ev}, \sigma_{\nu}) = A_{\nu} \cdot \exp \left( -\frac{\psi^2}{2 \sigma_{\nu}^2} \right),
\]

\[
\psi = \cos^{-1} \left( \frac{r \cdot r_{ev}}{|r| \cdot |r_{ev}|} \right),
\]

\[5\]
where \( r_{\text{ev}} \) is the vector representing the reconstructed event direction and \( \psi \) is the angle between \( r \) and \( r_{\text{ev}} \). \( A_\nu \) is the normalization factor and \( \sigma_\nu \) is the uncertainty of the reconstructed event direction. An example of an IceCube event is shown in figure 2(b).

The distribution of background noise events is not uniform over the sky. The background likelihood distribution (BLD) is a function of the reconstructed event direction, and it returns a value proportional to the likelihood of a background event coming from this direction. The reconstructed event direction is specified by a polar angle \( \theta_{\text{ev}} \) measured from the LIGO axis for LIGO events and by a vector \( r_{\text{ev}} \) for IceCube events. There are two BLDs, \( B_{GW}(\theta_{\text{ev}}) \) and \( B_\nu(r_{\text{ev}}) \) corresponding to LIGO and IceCube detectors respectively. BLDs are obtained from histograms of reconstructed event directions, \( \theta_{\text{ev}} \) and \( r_{\text{ev}} \), for a large number of background events. The histograms are converted to BLDs by normalizing them to 1 for integration over the whole sky.

Finally, the joint likelihood distribution of a combined LIGO–IceCube event is given by the following formula:

\[
L_{\text{comb}}(r) = \frac{S_{GW}(r; \theta_{\text{ev}}, \delta \tau) \cdot S_\nu(r; r_{\text{ev}}, \sigma_\nu)}{B_{GW}(\theta_{\text{ev}}) \cdot B_\nu(r_{\text{ev}})}.
\]  
(5)

\( L_{\text{comb}}(r) \) has a bright spot on the sky when the reconstructed directions of LIGO and IceCube events have good overlap. We search for every direction on the sky and find the direction \( r_{\text{max}} \) which gives the maximum value \( L_{\text{max}} = L_{\text{comb}}(r_{\text{max}}) = \text{Max}[L_{\text{comb}}(r)] \). \( L_{\text{max}} \) is a good measure of spatial coincidence and \( r_{\text{max}} \) is the most likely source direction.

In order to evaluate the statistical significance of a given \( L_{\text{max}} \), we first calculate the background distribution \( P_{BG}^{L_{\text{max}}}(L_{\text{max}}) \) of \( L_{\text{max}} \) using a large number of background events. \( P_{BG}^{L_{\text{max}}}(L_{\text{max}}) \) gives the probability of a time-coincident background event to have a particular \( L_{\text{max}} \). Then the statistical significance of a combined event with \( L_{\text{max}} = L_{\text{ev}} \) is estimated by the \( p \)-value defined as follows:

\[
p = \int_{L_{\text{ev}}}^{\infty} P_{BG}^{L_{\text{max}}}(L_{\text{max}}) \, dL_{\text{max}}.
\]  
(6)

The \( p \)-value gives the probability for a background combined event to have a value \( L_{\text{max}} \) higher than the \( L_{\text{max}} \) of the event \( (L_{\text{ev}}) \) being examined. Therefore, smaller \( p \)-values indicate the candidate is less likely to be a background noise event. A detection is declared if the \( p \)-value of a candidate is less than a certain threshold value \( p_0 \), which is chosen according to the required statistical significance for detections.

3. Monte Carlo simulation

The performance of our analysis pipeline was demonstrated using Monte Carlo simulations. We first generated a LIGO event list using 17.6 h of LIGO-like data which has similar statistical properties (such as standard deviation, glitch rate, etc) to the real LIGO data during the fifth scientific run (S5) [31]. Using the statistics of LIGO events obtained from this list (i.e. the event rate and the distributions of \( \tau \) and \( \delta \tau \) used below), we generated a large number of background LIGO events by Monte Carlo. For each event, a trigger time was assigned randomly with the event rate of 13.4 events per day, which is what we can reasonably expect from a real detector on average. The arrival time difference \( \tau \) between the two LIGO sites was distributed uniformly between \(-\,10\) ms and \(10\) ms. The uncertainty \( \delta \tau \) of the time difference was generated following the gamma distribution:

\[
P_{\delta \tau}(\delta \tau) = \frac{1}{b^a \Gamma(a)} (\delta \tau)^{a-1} e^{-\delta \tau/b},
\]  
(7)

\[ a = 1.93, \quad b = 4.41 \times 10^{-4}. \]
Figure 3. (a) The histogram of $L_{\text{max}}$ for background events. (b) The plot of $p$-value as a function of $L_{\text{max}}$. Less than 1% of background coincident events have an $L_{\text{max}}$ value greater than 21.95.

This distribution was chosen by a fit to the histogram of $\delta \tau$ obtained from the LIGO-like data.

Simulated IceCube events are distributed uniformly over the northern hemisphere of the sky with an event rate of 2 events per day. This event rate corresponds to the one obtained during the operation of IceCube in its 9-string configuration from June to November of 2006 [28]. No IceCube events from the southern sky are generated because they are rejected by the IceCube event reconstruction algorithm to avoid contamination by cosmic ray muons. The uncertainty $\sigma_\nu$ of the event direction is set to be a constant value of 2°, which is the median angular reconstruction error of IceCube in the 9-string configuration.

The simulated LIGO and IceCube events are fed into our analysis pipeline. Figure 3(a) shows the distribution of $L_{\text{max}}$ for background coincident events. By integrating the histogram, we get the relation between the $p$-value for spatial coincidence and $L_{\text{max}}$ (figure 3(b)). From this plot, we can determine the detection threshold for $L_{\text{max}}$. For example, if our analysis requires that the $p$-value for spatial coincidence be less than 1%, then the $L_{\text{max}}$ of the combined event must be greater than the corresponding $L_{\text{max}}$ value of 21.95.

4. Discussion

For each LIGO event, the expected number of IceCube events found within a time window ($\pm T_W$) is $2 T_W \cdot R_\nu$, where $R_\nu$ is the event rate of IceCube. Therefore, using the event rate $R_{\text{GW}}$ of LIGO, the overall rate for LIGO–IceCube time-coincident events can be calculated by $2 T_W \cdot R_{\text{GW}} \cdot R_\nu$. Using a $p$-value threshold $p_0$ for spatial coincidence, the FAR of this analysis method can be expressed by the following formula,

$$\text{FAR} = 2 T_W \cdot R_{\text{GW}} \cdot R_\nu \cdot p_0.$$  (8)

More specifically in the case of the Monte Carlo simulation explained in the previous section, the FAR is given by the following formula,

$$\text{FAR} = \frac{1}{435} \left( \frac{p_0}{1\%} \right) \left( \frac{T_W}{1\text{s}} \right) \text{ (events/year)}. \quad (9)$$

The obtained FAR is 1 false alarm in 435 years for one-second coincidence time window and spatial coincidence $p$-value threshold of 1%. If we allow a higher FAR, for example 1 event per 100 years used by SNEWS (SuperNova Early Warning System) [32], we can relax the LIGO or IceCube event selection thresholds to search for weaker signals in the background noise.
In the case of long GRBs, high-energy neutrinos from relativistic shocks are expected to be emitted between a few hours (for the internal shocks [7, 33, 34]) to a few days (for the external shocks [35]) after gravitational wave emission caused by core bounce. In order to look for this type of event, we have to use a large time window of order of days. In this case, the FAR may be unacceptably large because most LIGO events will be able to find at least one companion IceCube event (and vice versa) within a day. However, if the discrimination power of spatial coincidence can be improved, this would offset the larger time overlap. Such improvement would result from, for example, the continued enlargement of the IceCube detector, or the addition of another gravitational wave detector operating in conjunction with LIGO. On the other hand, the time coincidence is effective to search for GW and neutrino bursts with small time delay.

Our method can also be applied to coincidence analyses with other neutrino detectors such as Super-Kamiokande [36], Lake Baikal [37], Baksan [38], etc without significant modification. Combinations with the low energy-threshold detectors would enable us to search for supernova events. Moreover, our method can be used with any astronomical detectors which provide timing and source location information of burst events. Coincidence search with a large number of detectors will increase the confidence of detections.

We shall extend our method to include the VIRGO gravitational wave detector. The use of three geographically separated interferometers will enable us to constrain possible source locations of a gravitational wave event to two points on the sky [39]. Additionally, time coincidence discrimination between VIRGO and LIGO interferometers will further reduce the background event rate of the gravitational wave detectors network. Both of these changes in the time and spatial coincidence rates will work together to provide a much lower FAR and/or better sensitivity.

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