New Tsallis agegraphic Dark Energy in Horava-Lifshitz cosmology

M. Abdollahi Zadeh¹

¹ Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran

We investigate the new Tsallis agegraphic dark energy (NTADE) scenario in the framework of Horava-Lifshitz cosmology. Considering interacting and non-interacting scenario of NTADE with dark matter in a spatially non-flat universe, we investigate the cosmological implications of this model in detail. We obtain the differential equation of the evolution of the density parameter, the equation of state parameter and stability of model. Also, we study the behavior of the deceleration parameter and investigate the nature of the statefinder diagnostics and \( \omega_D - \omega_D' \) plane. We find that phantom crossing cannot occur for the state parameter in this scenario and from the plot of the deceleration parameter, we have observed a transition from decelerating to accelerating phase of the universe. Also, the sign of the square of the sound speed is negative which means unstable behavior at this scenario. The \( \omega_D \) and \( \omega_D' \) have negative values which represents the freezing region at here.

I. INTRODUCTION

Twenty years ago, two groups have discovered independently that the Universe has entered a stage of an accelerated expansion with a redshift smaller than 1 [1, 2]. Since all usual types of matter with positive pressure decelerate the expansion of the Universe in the framework the General Relativity, a sector with negative pressure named as dark energy, was suggested to account for the invisible fuel that accelerates the expansion rate of the current universe [3, 4]. The simplest cosmological model of dark energy is the so called Lambda cold dark matter (ΛCDM) model, which have an excellent fit to a wide range of astronomical data. But it suffers from severe theoretical difficulties like fine-tuning and cosmic coincidence problems.

Thus, dynamical dark energy models become popular, what those models that originate from various fields, what those models that probe the nature of DE, according to some basic quantum gravitational principles. One example of latter paradigm is the agegraphic DE (ADE) model which has originated from quantum gravity and posseses some of its significant features. In this model, the age of the Universe \( T = \int dt \) is used as the IR cut-off L [5]. However, since this scenario cannot justify the matter-dominated era, it was extended to the new agegraphic dark energy, namely the

* mazkph@gmail.com
use of the conformal time $\eta$ as the IR cut-off $L_{6,7}$.

On the other hand, concerning the gravitational background of the Universe, almost ten years ago a power-counting renormalizable theory with consistent Ultra-Violet (UV) behavior was proposed by Horava \[8\]. The expression characteristics of this theory can be said i) It is not Lorentz invariant except in the infrared (IR) limit. ii) It obeys anisotropic scaling or Lifshitz scaling. In consequence the time coordinate and the 3 spatial coordinates have to be treated separately. iii) It is non-relativistic, the speed of light diverges in the UV limit and test particles do not follow geodesics. For a recent review on Horava-Lifshitz gravity \[9\] and its application as the cosmological framework of the Universe see \[10\].

Similarly to the black hole, we use holographic principle in the cosmological applications, because of the fact that the entropy of the Universe can be counted by assuming that the universe is seen as a two-dimensional structure on the horizon of cosmology. However, in 1920 Gibbs in his book with title “Elementary Principles in Statistical Mechanics” \[11\] pointed out that systems have a long range interaction, such as gravitation, do not necessarily obey the Boltzmann-Gibbs (BG) theory, and indeed these systems can violate the extensivity constraint of the Boltzmann-Gibbs entropy. On this basis, Tsallis in 1988 \[12\], introduced a non-additive entropy for the non-extensive systems which can be written in compact form as \[13\]

$$S_T = \gamma A^\delta,$$  \hspace{1cm} (1)

where $\gamma$ is an unknown constant and $\delta$ denotes the non-extensive parameter. Recently, using relation \[11\] and holographic hypothesis, led to the suggestion of a dark energy density in the form

$$\rho_D = BL^{2\delta-4},$$  \hspace{1cm} (2)

where $B$ is an unknown parameter \[14\], and it attracts more attempts to itself \[15\]. Here we are interested in studying some cosmological consequences by considering the conformal time as the IR cut off in background Horava-Lifshitz cosmology.

The paper is organized as follows. In section II, we present Horava-Lifshitz cosmology and we analyze the new Tsallis agegraphic dark energy in Horava-Lifshitz cosmology, both in the simple and in the interacting form. Extracting the differential equation that determines the evolution of the dark energy density parameter as well as we investigate the model by using the statefinder diagnostic and $\omega - \omega'$ analysis in section III. The conclusions are given in section IV.
II. HORAVA-LIFSHITZ COSMOLOGY

We begin with a brief review of the cosmological evolution which is governed by Horava-Lifshitz gravity \[16\]. Under the projectability condition, the full metric in the 3+1 dimensional Arnowitt-Misner formalism is given by

\[ ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \] (3)

where \( N \) and \( N^i \) are the lapse and shift functions which are used in general relativity in order to split the space-time dimensions. It should be noted that the indices are raised and lowered by using the spatial metric \( g_{ij} \) and the scaling transformation of the coordinates reads: \( t \rightarrow l^3 t \) and \( x^i \rightarrow lx^i \).

A. Detailed balance

The gravitational action is decomposed into a kinetic and a potential part which is shown as \( S_g = \int dt d^3 x \sqrt{g} N (L_K + L_V) \). Using the projectable version of HL gravity \[17\] under the detailed balance condition \[18\], the full action of Horava-Lifshitz gravity is given by

\[
S_g = \int dt d^3 x \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2}{2w^4} C_{ij} C^{ij} - \frac{\kappa^2 \mu}{2w^2} \sqrt{g} \nabla_k R^k_l \right. + \left. \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left[ \frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right] \right\},
\] (4)

where \( K_{ij} \) is the extrinsic curvature which takes the form

\[ K_{ij} = \frac{1}{2N} \left( \dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \] (5)

and a dot denotes a derivative with respect to \( t \) and covariant derivatives defined with respect to the spatial metric \( g_{ij} \). Also

\[ C^{ij} = \frac{\epsilon^{ijk}}{\sqrt{g}} \nabla_k \left( R^i_j - \frac{1}{4} R \delta^i_j \right) \] (6)

is the Cotton tensor, \( \epsilon^{ijk} \) is the totally antisymmetric unit tensor, \( \lambda \) is a dimensionless constant, \( \Lambda \) is a positive constant, which as usual is related to the cosmological constant in the IR limit and the variables \( \kappa, w \) and \( \mu \) are constants with mass dimensions \(-1, 0 \) and \( 1 \), respectively. For focussing on cosmological contents, we should impose the so called projectability condition \[16\] under the detailed balance, then we consider a Friedmann-Robertson-Walker (FRW) metric,

\[ N = 1 , \quad g_{ij} = a^2(t) \gamma_{ij} , \quad N^i = 0 , \] (7)
with

\[ \gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - k r^2} + r^2 d\Omega^2, \]

(8)

where \( k = -1, 0, +1 \) corresponding to open, flat, and closed universe respectively. Taking the variation of action with respect to the metric components \( N \) and \( g_{ij} \), we can obtain the equation of motion as

\[ H^2 = \frac{\kappa^2}{6(3\lambda - 1)} \rho_m + \frac{\kappa^2}{6(3\lambda - 1)} \left[ \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 \Lambda k}{8(3\lambda - 1)^2 a^2} \]

(9)

\[ \dot{H} + \frac{3}{2} H^2 = - \frac{\kappa^2}{4(3\lambda - 1)} p_m - \frac{\kappa^2}{4(3\lambda - 1)} \left[ \frac{k^2 \mu^2 k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 \Lambda k}{16(3\lambda - 1)^2 a^2}, \]

(10)

where we have defined the Hubble parameter as \( H \equiv \frac{\dot{a}}{a} \) and \( a \) is scale factor. Also \( \rho_m \) and \( p_m \) are corresponding to energy density and pressure of the matter. At this stage, by noticing the form of the preceding Friedmann equations, the energy density \( \rho_D \) and pressure \( p_D \) for dark energy can define as

\[ \rho_D \equiv \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \]

(11)

\[ p_D \equiv \frac{k^2 \mu^2 k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}. \]

(12)

It is interesting to note that the first term on the right-hand side proportional to \( a^{-4} \) is the usual "dark radiation term", present in Hořava-Lifshitz cosmology \[16\], while the second term is just the explicit cosmological constant.

As a last step, for these expressions to match the standard Friedmann equations, in units where \( c = 1 \), we define \[16\]

\[ G_{\text{cosmo}} = \frac{\kappa^2}{16\pi(3\lambda - 1)} \]

(13)

\[ \frac{\kappa^4 \mu^2 \Lambda}{8(3\lambda - 1)^2} = 1, \]

(14)

where \( G_{\text{cosmo}} \) presents the Newton’s cosmological constant. It is worth mentioning that in gravitational theories with the violation of Lorentz invariance (such as Hořava-Lifshitz one) the Newton’s gravitational constant \( G_g \), that is the one that is present in the gravitational action, differs from Newton’s cosmological constant \( G_{\text{cosmo}} \), which is present in the Friedmann equations, unless Lorentz invariance is restored \[19\]. For the sake of completeness we mention that here

\[ G_g = \frac{\kappa^2}{32\pi}. \]

(15)
It is interesting that in the IR limit ($\lambda = 1$), where Lorentz invariance is restored, $G_{\text{cosmo}}$ and $G_{g}$ are the same.

Now, by using the above identifications, we can rewrite the modified Friedmann equations (9) and (10) in the usual form as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{\text{cosmo}}}{3}(\rho_m + \rho_D)$$

$$\dot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_{\text{cosmo}}(p_m + p_D).$$

### III. NEW TSALLIS AGEGRAPHIC DARK ENERGY MODEL (NTADE)

Here we would like to study the NTADE in HL theory, to do this, we consider a spatially non-flat Universe in which there are a new Tsallis agegraphic dark energy $\rho_D$ and pressureless dark matter $\rho_m$. If we define the dimensionless energy densities as

$$\Omega_m = \frac{8\pi G_{\text{cosmo}}}{3H^2}\rho_m, \quad \Omega_D = \frac{8\pi G_{\text{cosmo}}}{3H^2}\rho_D, \quad \Omega_k = \frac{k}{a^2H^2},$$

then the Friedmann equation (16) yields

$$1 + \Omega_k = \Omega_m + \Omega_D.$$  (19)

As mentioned earlier, because the original ADE model has some difficulties in particular, in to justify the matter-dominated era [21], it motivated Wei and Cai [6] to propose the new ADE model, while the time scale is chosen to be the conformal time instead of the age of the Universe. Considering the conformal time as IR cutoff which is defined as $dt = ad\eta$ leading to $\dot{\eta} = 1/a$ and thus

$$\eta = \int_0^a \frac{da}{Ha^2}.$$  (20)

Thus, the corresponding dark energy by considering Eq. (2) reads

$$\rho_D = B\eta^{2\delta - 4}.$$  (21)

Taking time derivative of above equation and using $\dot{\eta} = 1/a$, we can obtain

$$\dot{\rho}_D = \frac{\rho_D(2\delta - 4)}{a\eta}.$$  (22)

Also, if we take the time derivative of the second relation in (18) after using (22) and relation $\dot{\Omega}_D = \Omega'_D H$, we can obtain the equation of motion for $\Omega_D$ as

$$\Omega'_D = \Omega_D \left( \frac{2\delta - 4}{a\eta H} - 2\frac{\dot{H}}{H^2} \right).$$  (23)
where prime denotes the derivative with respect to $x = \ln a$. As previous, by taking derivative of the third relation in (18) we get

$$\Omega'_k = -2\Omega_k \left(1 + \frac{\dot{H}}{H^2}\right).$$

(24)

A. statefinder diagnostic and $\omega - \omega'$ analysis

However two cosmological parameters $H$ and $q$ are useful to describe the evolution of the Universe, but these two parameters cannot differentiate various dark energy models. For this reason, Sahni et al. [22] introduced a new geometrical diagnostic pair parameter $\{r, s\}$, known as the statefinder pair, defined as

$$r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r - 1}{3(q - 1/2)},$$

(25)

which clearly show the statefinder pair depend only on the scale factor and its time derivatives up to third order. Note that the parameter $r$ is also called cosmic jerk and can be expressed in terms of the Hubble and the deceleration parameters as

$$r = 2q^2 + q - \frac{\dot{q}}{H}.$$  

(26)

Before we apply the statefinder diagnostic to the NTADE model in HL gravity, it is better to note that i) in the $\{r, s\}$ plane, $s > 0$ ($s < 0$) corresponds to a quintessence-like (phantom-like) model of DE respectively. ii) In a flat $\Lambda$CDM model and matter dominated universe (SCDM) one finds $\{r, s\} = \{1, 0\}$ and $\{r, s\} = \{1, 1\}$, respectively. As a complement to statefinder diagnosis, the $\omega_D - \omega_D'$ analysis is also useful method for distinguish different cosmological models [23]. In this approach i) the $\Lambda$CDM model corresponds to a fixed point $\{\omega_D = -1, \omega_D' = 0\}$ in the $\omega_D - \omega_D'$ plane, where $\omega_D'$ represents the derivative of $\omega_D$ with respect to $x = \ln a$. ii) $\omega_D' > 0$ and $\omega_D < 0$ present the thawing region. (iii) $\omega_D' < 0$ and $\omega_D < 0$ present the freezing region [23].

B. Noninteracting case ($Q = 0$)

As usual, in the new Tsallis agegraphic dark energy scenario, the energy densities for matter and dark energy obey the standard evolution equation:

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0,$$

(27)

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0,$$

(28)
where \( p_m \) is pressure of matter (here we take \( p_m = 0 \)) and \( p_D = \omega_D \rho_D \), also, \( \omega_D \) is the equation of state (EoS) parameter of the NTADE model what can be achieved by inserting Eq.(22) in relation (28) as

\[
\omega_D = -1 - \frac{2\delta - 4}{3a\eta H},
\]

Also, the time derivative of Eq.(29) with respect to \( x = \ln a \), we get

\[
\omega_D' = \frac{(\delta - 2) (2 + 2(\delta - 2)\Omega_D + a\eta H(-1 + 3\Omega_D - \Omega_K))}{3a^2\eta^2 H^2}.
\]

Taking the derivative of both side of the Friedmann equation (16) with respect to the cosmic time \( t \), and using Eqs.(27) and (22) we find

\[
\frac{\dot{H}}{H^2} = \Omega_k + \Omega_D \left( -\frac{3}{2} u + \frac{\delta - 2}{a\eta H} \right),
\]

where \( u = \frac{\rho_m}{\rho_D} = -1 + \frac{1+\Omega_k}{\Omega_D} \) is the ratio of the energy densities. Finally, by substituting the above equation in relation

\[
q \equiv -1 - \frac{\dot{H}}{H^2},
\]

we obtain the expression for the deceleration parameter as

\[
q = -1 - \Omega_k + \Omega_D \left( \frac{3}{2} u + \frac{2 - \delta}{a\eta H} \right).
\]

FIG. 1: The evolution of \( \Omega_D \) versus redshift parameter \( z \) for non-interacting NTADE in HL cosmology. Here, we have taken \( \Omega_D(z = 0) = 0.73 \), \( H(z = 0) = 67 \), \( \Omega_k(z = 0) = 0.01 \), \( \lambda = 1.6 \) and \( B = 2.4 \).
FIG. 2: The evolution of $\omega_D$ versus redshift parameter $z$ for non-interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $\lambda = 1.6$ and $B = 2.4$.

FIG. 3: The evolution of the deceleration parameter $q$ versus redshift parameter $z$ for non-interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $\lambda = 1.6$ and $B = 2.4$.

We now consider an important quantity to check the effects of perturbations on the classical stability of our model, namely the square of the sound speed $v_s^2$, defined as

$$v_s^2 = \frac{dP_D}{d\rho_D} = \frac{\dot{P}_D}{\dot{\rho}_D} = \frac{\dot{\rho}_D}{\dot{\rho}_D} \omega_D + \omega_D,$$

which finally leads to

$$v_s^2 = \frac{5 - 2\delta + (\delta - 2)\Omega_D}{3\eta a H} + \frac{-7 + 3\Omega_D - \Omega_k}{6},$$

for the non-interacting case. The statefinder parameters for NTADE in HL are obtained as

$$r = 1 + \Omega_k + \frac{(2 - \delta)\Omega_D(5 - 2\delta + (\delta - 2)\Omega_D)}{\eta^2 a^2 H^2} + \frac{(2 - \delta)\Omega_D(-7 + 3\Omega_D - \Omega_k)}{2\eta a H},$$
FIG. 4: The evolution of the squared of sound speed \( v_s^2 \) versus redshift parameter \( z \) for non-interacting NTADE in HL cosmology. Here, we have taken \( \Omega_D(z = 0) = 0.73, H(z = 0) = 67, \Omega_k(z = 0) = 0.01, \lambda = 1.6 \) and \( B = 2.4 \).

\[
\delta = (\delta - 2)\Omega_D (a\eta H (a - 7 + 3\Omega_D) + 2(5 - 2\delta + (\delta - 2)\Omega_D)) - \frac{(2a\eta H + (\delta - 2)\Omega_D)\Omega_k}{3((-4 + 2\delta + 3a\eta H)\Omega_D - a\eta H\Omega_k)}. \tag{37}
\]

The evolution of the system parameters for non-interacting case are plotted in Figs. 1-9. In Figs. 1-3 we plot the evolution of \( \Omega_D, \omega_D \) and \( q \) versus redshift parameter \( z \) for non-interacting case. It is obvious that \( \Omega_D \) tends to 0 in the early universe, \( \omega_D \) cannot cross phantom line and the universe enters the acceleration phase earlier for smaller \( \delta \). In Fig. 4 we show the evolution of \( v_s^2 \), which is unstable here. In Figs. 5-9 we also plot the trajectories of statefinder pair and \( \omega - \omega' \) plane. From trajectory of \( s \), we see a quintessence-like behaviour as well as \( \omega - \omega' \) plane presents the freezing
FIG. 6: The evolution of the statefinder parameter $s$ versus the redshift parameter $z$ for non-interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $\lambda = 1.6$ and $B = 2.4$.

FIG. 7: The evolution of the statefinder parameter $r$ versus $s$ for non-interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $\lambda = 1.6$ and $B = 2.4$

FIG. 8: The evolution of the statefinder parameter $r$ versus the deceleration parameter $q$ for non-interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $\lambda = 1.6$ and $B = 2.4$
FIG. 9: The $\omega_D - \omega'_D$ diagram for non-interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $\lambda = 1.6$ and $B = 2.4$

region.

C. Interacting case ($Q \neq 0$)

Here, we postulate that the two sectors the NTADE and dark matter (DM) interact through the interaction term $Q$, since such a scenario could alleviate the known coincidence problem \cite{25}. It should be noted that the recent observational evidence by the galaxy clusters supports the interaction between DE and DM \cite{26}. This causes the energy conservation law for each dark component not to be hold separately i.e.

$$\dot{\rho}_m + 3H\rho_m = Q, \quad \text{(38)}$$

$$\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = -Q, \quad \text{(39)}$$

where $Q$ has a form as follows $Q = 3b^2H\rho_m$ with $b^2$ being a coupling constant. It deserves mention three cases a bout interaction term $Q$ i) for $Q > 0$, there is an energy transfer from NTADE to DM. ii) the form of $Q$ is chosen purely phenomenologically, in order to obtain desirable cosmological results including phantom crossing and accelerated expansion. iii) Easily, one can find numerous form of $Q(H\rho)$ in Ref\cite{27-31}.

By repeating the above procedure in the case where the matter and dark sectors are allowed to interact, we find expressions for cosmological application , similary to the previous subsection,

$$\frac{\dot{H}}{H^2} = \Omega_k + \Omega_D \left( \frac{3}{2} u(b^2 - 1) + \frac{\delta - 2}{a\eta H} \right), \quad \text{(40)}$$

$$\omega_D = -1 - b^2u - \frac{2\delta - 4}{3a\eta H}, \quad \text{(41)}$$
\[ q = -1 - \Omega_k - \Omega_D \left( \frac{3}{2} u(b^2 - 1) + \frac{\delta - 2}{a\eta H} \right), \quad (42) \]

\[ v_s^2 = \frac{5 - 2\delta + (\delta - 2)\Omega_D}{3\eta a H} + \frac{-3b^2(\Omega_D - 1 - \Omega_k) - 7 + 3\Omega_D - \Omega_k}{6} + \frac{2(3\Omega_D - 1)\eta a H (\Omega_D - 1 - \Omega_k)}{6}, \quad (43) \]

\[ r = \frac{(2 - \delta)\Omega_D(5 - 2\delta + (\delta - 2)\Omega_D)}{\eta^2 a^2 H^2} + \frac{(2 - \delta)\Omega_D(-7 + 3\Omega_D - \Omega_k + 3b^2(1 - \Omega_D + \Omega_k))}{2\eta a H} + \frac{2(1 + \Omega_k) + 9b^2(b^2 - 1)(1 - \Omega_D + \Omega_k)}{2}. \]

Since the expression of \( s \) is too long, we do not present it here. In Figs. [10][18] we present system parameters for various values of \( b^2 \) for interacting case. In Figs. [19][27] we show cosmological parameters for different values of \( \delta \) for interacting case.

**FIG. 10:** The evolution of \( \Omega_D \) versus redshift parameter \( z \) for interacting NTADE in HL cosmology. Here, we have taken \( \Omega_D(z = 0) = 0.73, H(z = 0) = 67, \Omega_k(z = 0) = 0.01, B = 2.4, \lambda = 1.6 \) and \( \delta = .2 \)

In Fig. 11, we see that the equation of state \( \omega_D \) cannot cross phantom line as well as by considering different values of \( b^2 \), Fig. 13, shows unstability of this model. In Fig. 14 and 15, we see that statefinder parameters \( r \) and \( s \) at low redshift of TNADE model in HL cosmology, exactly mimic the \( \Lambda CDM \) model, i. e. \( r = 1 \) and \( s = 0 \) for different values of \( b^2 \). The evolutionary trajectories for \( (r - s) \) and \( (r - q) \) planes for TNADE model in HL cosmology have plotted in Figs. 16 and 17 respectively. Fig. 16, shows that, the evolutionary trajectories \( r \) and \( s \) end at \( \Lambda CDM \) \( (r = 1, s = 0) \) in the future for different values of \( b^2 \) as well as from Fig. 17, we see that
FIG. 11: The evolution of $\omega_D$ versus redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z=0) = 0.73$, $H(z=0) = 67$, $\Omega_k(z=0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $\delta = .2$

FIG. 12: The evolution of the deceleration parameter $q$ versus redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z=0) = 0.73$, $H(z=0) = 67$, $\Omega_k(z=0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $\delta = .2$

the evolutionary trajectories started from matter dominated universe in the past and approach the point ($r = 1, q = -1$) in the future. The $\omega_D - \omega'_D$ plane for TNADE model in HL cosmology, by considering different values of $b^2$ has plotted in Fig. 18. We see that this model represents the freezing region.

IV. CLOSING REMARKS

In this paper, we have investigated the NTADE scenario in the framework of Horava-Lifshitz cosmology. Since ADE density corresponds to a dynamical cosmological constant, we used from a dynamical framework, instead of general relativity. Thus, we investigated the NTADE in the
FIG. 13: The evolution of the squared of sound speed $v_s^2$ versus redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73, H(z = 0) = 67, \Omega_k(z = 0) = 0.01, B = 2.4, \lambda = 1.6$ and $\delta = 0.2$.

FIG. 14: The evolution of the statefinder parameter $r$ versus the redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73, H(z = 0) = 67, \Omega_k(z = 0) = 0.01, B = 2.4, \lambda = 1.6$ and $\delta = 0.2$.

framework of Horava-Lifshitz cosmology.

Since experimental data have implied that our universe is not a perfectly flat universe, we imposed an arbitrary curvature for the background geometry, and we allowed for an interacting between the matter and dark energy sectors. For both non-interacting and interacting case, we extracted the differential equation that determines the evolution of the dark energy density parameter, which gives a suitable estimate for the state parameter of dark energy as well as the deceleration parameter to study an expansion of the universe. Also, we studied statefinder trajectories and $\omega - \omega'$ plane. To study parametric behaviour, we found that phantom crossing can not occur for the state parameter for small values of coupling parameter $b^2$ and $\delta$ and from the plot of the deceleration
FIG. 15: The evolution of the statefinder parameter $s$ versus the redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $\delta = .2$.

FIG. 16: The evolution of the statefinder parameter $r$ versus $s$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $\delta = .2$

FIG. 17: The evolution of the statefinder parameter $r$ versus the deceleration parameter $q$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $\delta = .2$
FIG. 18: The $\omega_D - \omega'_D$ diagram for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $\delta = 0.2$.

FIG. 19: The evolution of $\Omega_D$ versus redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = 0.02$.

parameter, we have observed a transition from decelerating to accelerating phase of the universe. Also, model is not stable while the cosmological plane can meet the freezing region.

[1] A.G. Riess, A.V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P.M. Garnavich, R.L. Gilliland, C.J. Hogan, S. Jha, R.P. Kirshner, et al., The Astronomical Journal 116(3), 1009 (1998).
[2] S. Perlmutter, G. Aldering, G. Goldhaber, R. Knop, P. Nugent, P. Castro, S. Deustua, S. Fabbro, A. Goobar, D. Groom, et al., The Astrophysical Journal 517(2), 565 (1999).
[3] V. Sahni, Lecture Notes in Physics 653(2), 141 (2004).
FIG. 20: The evolution of $\omega_D$ versus redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$.

FIG. 21: The evolution of the deceleration parameter $q$ versus redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$.

[4] S.M. Carroll, Living Reviews in Relativity 4(1), 1 (2000).
[5] R. G. Cai, A dark energy model characterized by the age of the universe, Phys. Lett. B 657, 228 (2007) arXiv:0707.4049.
[6] H. Wei and R. G. Cai, A new model of agegraphic dark energy, Phys. Lett. B 660, 113 (2008) arXiv:0708.0884.
[7] J. P. Wu, D. Z. Ma and Y. Ling, Quintessence reconstruction of the new agegraphic dark energy model, Phys. Lett. B 663, 152 (2008) arXiv:0805.0540.
; A. Sheykhi, Interacting new agegraphic dark energy in nonflat Brans-Dicke cosmology, Phys. Rev. D 81, 023525 (2010).
[8] P. Horava, Quantum gravity at a Lifshitz point, Phys. Rev. D 79, 084008 (2009) arXiv:0901.3775;
Spectral dimension of the universe in quantum gravity at a Lifshitz point, Phys. Rev. Lett. 102, 161301.
FIG. 22: The evolution of the squared of sound speed $v_s^2$ versus redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$.

FIG. 23: The evolution of the statefinder parameter $r$ versus the redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$.

(2009) [arXiv:0902.3657].

[9] B. Chen, S. Pi and J. Z. Tang, *Power spectra of scalar and tensor modes in modified Horava-Lifshitz gravity*, [arXiv:0910.0338];

S. Nojiri and S.D. Odintsov, *Covariant Horava-like renormalizable gravity and its FRW cosmology*, Phys. Rev. D 81, 043001 (2010) [arXiv:0905.4213];

J. Kluson, Horava-Lifshitz f(R) gravity, JHEP 11, 078 (2009) [arXiv:0907.3566].

[10] A. Wang and Y. Wu, *Thermodynamics and classification of cosmological models in the Horava-Lifshitz theory of gravity*, JCAP 07, 012 (2009) [arXiv:0905.4117];

S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, *Phenomenological aspects of Horava-Lifshitz cosmology*, Phys. Lett. B 679, 6 (2009) [arXiv:0905.0055].
FIG. 24: The evolution of the statefinder parameter $s$ versus the redshift parameter $z$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$.

FIG. 25: The evolution of the statefinder parameter $r$ versus $s$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$

FIG. 26: The evolution of the statefinder parameter $r$ versus the deceleration parameter $q$ for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$
FIG. 27: The $\omega_D - \omega'_D$ diagram for interacting NTADE in HL cosmology. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\Omega_k(z = 0) = 0.01$, $B = 2.4$, $\lambda = 1.6$ and $b^2 = .02$.

M. I. Park, A test of Horava gravity: the dark energy, JCAP 01, 001 (2010) [arXiv:0906.4275]; M. Fukushima, Y. Misonoh, S. Miyashita, S. Sato, Stable Singularity-free Cosmological Solutions in non-projectable Horava-Lifshitz Gravity, Phys. Rev. D 99, 064004 (2019); A. Tawfik, E. Abou El Dahab, FLRW Cosmology with Horava-Lifshitz Gravity: Impacts of Equations of State, Int. J. Theor. Phys. 56, 2122-2139 (2017); Y. Heydarzade, M. Khodadi, F. Darabi, Deformed Horava-Lifshitz Cosmology and Stability of Einstein Static Universe, Theor. Math. Phys 190, 130 (2017).

[11] J. W. Gibbs, Elementary Principles in Statistical Mechanics Developed with Especial Reference to the Rational Foundation of Thermodynamics, (C. Scribners Sons, New York, 1902); Yale University Press.

[12] C. Tsallis, J. Stat. Phys. 52, 479 (1988).

[13] C. Tsallis, L. J. L. Cirto, Eur. Phys. J. C 73, 2487 (2013).

[14] M. Tavayef, A. Sheykhi, K. Bamba and H. Moradpour, Phys. Lett. B. 781, 195 (2018).

[15] M. Abdollahi Zadeh, A. Sheykhi, H. Moradpour, K. Bamba, Eur. Phys. J. C, 78, 940 (2018) 940; Abdollahi Zadeh, M., Sheykhi, A. Moradpour, H. Gen Relativ Gravit 51, 12 (2019); S. Ghaffari et al. Eur. Phys. J. C 78, 706 (2018); S. Ghaffari, et al, Phys. Dark. Univ. 23, 100246 (2019); S. Nojiri, S. D. Odintsov and E. N. Saridakis, Eur. Phys. J. C 79,242 (2019); A. Sheykhi, Phys. Lett. B 785, 118 (2018); E. N. Saridakis, K. Bamba, R. Myrzakulov, F. K. Anagnostopoulos, JCAP 1812, 012 (2018) ; Gunjan. Varshney, Umesh Kumar Sharma, A. Pradhan, Statefinder diagnosis for interacting Tsallis holographic dark energy models with $\omega - \omega'$ pair, New Astronomy 70, 36 (2019); M. Abdollahi Zadeh, A. Sheykhi, H. Moradpour, Kazuharu Bamba, [arXiv:1901.05298v1]; Vipin Chandra Dubey, Umesh Kumar Sharma, A. Beesham, [arXiv:1905.02449v1]; Nan Zhang, Ya-Bo Wu, Jia-Nan Chi, Zhe Yu, Dong-Fang Xu, [arXiv:1905.04299v2]; Umesh Kumar Sharma, Vipin Chandra Dubey, A. Pradhan, [arXiv:1906.08051v1].

[16] G. Calcagni, Cosmology of the Lifshitz universe, JHEP 09,112 (2009) [arXiv:0904.0829]; E. Kiritsis and G. Kofinas, Horava-Lifshitz cosmology, Nucl. Phys. B 821, 467 (2009) [arXiv:0904.1334].
[17] C. Charmousis, G. Niz, A. Padilla, P.M. Saffin, \textit{Strong coupling in Horava gravity}, J. High Energy Phys. \textbf{0908}, 070 (2009) [arXiv:0905.2579].

[18] P. Horava, \textit{Membranes at quantum criticality}, Phys. Rev. D \textbf{79}, 084008 (2009) . [arXiv:0901.3775].

[19] S. M. Carroll and E.A. Lim, \textit{Lorentz-violating vector fields slow the universe down}, Phys. Rev. D \textbf{70}, 123525 (2004).

[20] M. Abdollahi Zadeh, A. Sheykhi and H. Moradpour, \textit{Tsallis Agegraphic Dark Energy Model}, Mod. Phys. Lett. A \textbf{34}, no. 11, 1950086 (2019).

[21] R. G. Cai, Phys. Lett. B \textbf{657}, 228 (2007).

[22] V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, JETP Lett. \textbf{77}, 201 (2003).

[23] R. Caldwell and E. V. Linder, Phys. Rev. Lett. \textbf{95}, 141301 (2005).

[24] E. Komatsu \textit{et al.}, Astrophys. J. Suppl. \textbf{180}, 330 (2009);

  J. P. Uzan, U. Kirchner, and George F. R. Ellis, Mon. Not. R. Astron. Soc. \textbf{344}, L65L68 (2003).

[25] P. J. Steinhardt, \textit{Cosmological challenges for the 21st century}, in \textit{Critical problems in physics}, V. L. Fitch and D. R. Marlow eds., Princeton University Press, Princeton U.S.A. (1997).

[26] O. Bertolami, F. Gil Pedro, and M. Le Delliou. Phys. Lett. B, \textbf{654}, 165 (2007);

  M. Baldi. Mon. Not. R. Astron. Soc. \textbf{414}, 116 (2011).

[27] M. Jamil and M. A. Rashid. Eur. Phys. J. C, \textbf{56}, 429 (2008).

[28] J. H. He and B. Wang, JCAP \textbf{06}, 010 (2008).

[29] J. H. He, B. Wang, and Y. P. Jing, JCAP \textbf{07}, 030 (2009).

[30] X. D. Xu, B. Wang, P. Zhang, and F. A. Barandela, JCAP, \textbf{12}, 001 (2013).

[31] B. Wang, E. Abdalla, F. Atrio-Barandela and D. Pavon, Rep. Prog. Phys. \textbf{79}, 096901 (2016).