Optimal Estimation and Sequential Channel Equalization Algorithms for Chaotic Communications Systems

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Many previously proposed communications systems based on chaos disregard common channel distortions and fail to work under realistic channel conditions. In this paper, optimal estimation and sequential channel equalization algorithms are proposed for chaotic communications systems where information is encoded using symbolic dynamics representations. For the optimal estimate of the transmitted signal, the symbolic dynamics representation of chaotic systems is first exploited to represent the chaotic dynamics of the signal by an equivalent trellis diagram. Then, the Viterbi algorithm is used to achieve an optimal estimate of the noise corrupted chaotic sequence. For the sequential channel equalization algorithm, the dynamics-based trellis diagram is expanded to accommodate a finite impulse response (FIR) model of the channel. Once the initial estimate of the channel parameters is obtained through the training sequence, then the Viterbi algorithm is used to estimate the chaotic sequence. If necessary, channel parameters can then be updated through successive estimates of the chaotic sequence. The proposed algorithms are simulated for both time-invariant and time-varying channels.

Keywords and phrases: chaotic communication, nonlinear systems, estimation, channel equalization.

1. INTRODUCTION

A number of properties of chaotic systems are especially appealing for communications applications. These properties, which include low-power implementations, a noise-like appearance, a broadband spectra, and nonlinearity, can be exploited for secure communications. A variety of approaches to chaotic communications have been proposed including chaotic modulation [1, 2, 3], masking [4, 5], and spread-spectrum [6].

A generic chaotic communications system based on chaotic modulation is shown in Figure 1. In such a system, the information bits to be transmitted must first be encoded in the signal waveform generated by the chaotic system through what is termed symbolic dynamics. For a symbolic dynamics representation, one divides the chaotic system's phase space into a finite number of partitions and assigns a symbol to each partition. Instead of describing the chaotic signal's trajectory by an infinite sequence of numbers, one can equivalently use the progression of symbols.

Rather than using structured signals such as rectangular pulses or sinusoids, to denote “0”s and “1”s (or other information symbols), these communications systems embed the information in the time evolution of the transmitted signal. Regions of the state space formed by the chaotic system's time evolution are designated to represent different symbols. The process of mapping information bits to the state of a chaotic system through its symbolic dynamics is termed chaotic modulation. This assignment of information bits to state is not arbitrary, and the greatest efficiency is achieved when the information transmission rate matches the topological entropy of the chaotic system. In this paper, it is assumed that the symbolic dynamics representation of chaotic systems is exploited to implement the chaotic modulation.

In a real communications scenario, the communication channel distorts the transmitted signal. This channel distortion is typically in the form of noise or intersymbol interference (ISI) plus noise. To recover the transmitted information, one must undo the distortions before chaotic demodulation,
which is the process of obtaining an estimate of the transmitted symbol sequence from the symbolic dynamics, is done.

For channels that only add noise to the transmitted sequence, such as additive white Gaussian noise (WGN) channel models, the distortion can be corrected with a noise reduction algorithm. There have been a number of noise reduction techniques that take advantage of properties specific to chaotic dynamical systems. Prior to recent years, proposed algorithms were suboptimal and ad hoc. Therefore, they would fail to remove the noise under low signal-to-noise ratios [7, 8]. In recent years, a few optimum noise reduction algorithms have been proposed for certain unimodal chaotic maps [9, 10, 11, 12, 13, 14]. In [9], an optimum noise reduction algorithm for the tent map has been proposed. This approach was later generalized for one-dimensional unimodal maps on the unit interval [10]. Drake has also proposed an optimum estimation algorithm for the sawtooth map and related maps that exploits an alternative linear, random representation algorithm for the tent map has been presented. This approach was later generalized for one-dimensional unimodal maps on the unit interval [10]. Drake has also proposed an optimum estimation algorithm for the sawtooth map and related maps that exploits an alternative linear, random representation of these chaotic maps [11]. In [13, 14], a probabilistic noise reduction algorithm using the Viterbi algorithm was proposed. However, this technique does not utilize symbolic dynamics, relying instead on the invariant distribution of the chaotic system to approximate the transition probabilities between states.

For channel distortions where intersymbol interference as well as noise is imposed on the transmitted sequence, channel equalizers, typically implemented as linear adaptive filters, are usually employed to remove the distortion. Most channel equalization development has been for conventional modulation schemes such as PAM and QAM. Since chaotic modulation schemes are significantly different from conventional schemes, conventional equalization algorithms cannot be used and equalization algorithms specifically designed for chaotic communication systems are needed. There has been some research into equalization algorithms for chaotic communications systems [15, 16, 17]. In [15], a self-synchronization based channel equalizer was proposed. However, most chaotic systems do not exhibit this property. In [16], a dynamics-based estimation technique called minimum phase space volume technique was applied to channel equalization for chaotic communication systems where channel was modeled as time-invariant autoregressive (AR) system. In [17], a symbolic dynamics based equalizer was proposed. However, the algorithm does not perform well for low signal-to-noise ratios (SNRs).

In this paper, general optimum noise reduction and sequential channel equalization algorithms are proposed for chaotic communications systems based on a symbolic dynamics representation. The symbolic dynamics representation is exploited to represent naturally the chaotic dynamics with a trellis diagram. Once the trellis diagram has been obtained, the Viterbi algorithm is used to remove the noise from the distorted chaotic sequence. For channel equalization, the trellis diagram is expanded to include an FIR channel model. The Viterbi algorithm is then used to obtain the maximum-likelihood estimate of the transmitted chaotic sequence, which can later be used to update the channel parameters.

The remainder of the paper is organized as follows. Background information about symbolic dynamics and chaotic modulation is given in Section 2. In Section 3, the trellis diagram representation is developed and the optimal estimation algorithm is introduced in Section 4. The sequential channel equalization algorithm is explained in Section 5. Simulation results for the proposed algorithms are presented in Section 6, and concluding remarks are given in Section 7.

2. SYMBOLIC DYNAMICS AND CHAOTIC MODULATION

Discrete-time signals generated by first order chaotic systems satisfy the dynamical equation

\[ x[n] = f(x[n-1]), \quad x[n] \in I, \]

where \( f(\cdot) \) is a nonlinear function mapping points from an interval \( I \) onto the same interval and possessing certain properties, such as sensitivity to initial conditions. Once the nonlinear dynamics, \( f(\cdot) \), and initial condition, \( x[0] \), are known, then it is straightforward to generate a chaotic sequence. Higher order chaotic systems obey a similar equation where \( x[n] \) is now a vector representing the state of the system.

Rather than generating the chaotic sequence directly by iterating (1), one can exploit symbolic dynamics to obtain an equivalent chaotic sequence through an alternate process. Symbolic dynamics also provide a means of examining the real dynamics with finite precision. One divides the phase space into a finite number of partitions, \( \{I_0, I_1, \ldots, I_{K-1}\} \) and labels each partition with a symbol, that is, \( v_i \in I_i \). Instead of representing trajectories by infinite sequences of numbers, one watches the alternations of symbols. By iterating (1), one gets the numerical sequence

\[ x[0], x[1] = f(x[0]), \ldots, x[n] = f(x[n-1]), \ldots \]

The corresponding symbolic sequence is

\[ S(x[0]) = s_0s_1s_2 \cdots s_n \cdots, \]

Figure 1: Block diagram for a generic chaotic communications system.
where $s_i$ is one of the symbols $v_k$ depending on which interval, $I_k$, $x[i]$ belongs. The dynamics governing the symbolic sequence, represented by $\sigma(\cdot)$, is a left shift operation where the left most symbol is discarded at each iteration, that is,

$$S(x[n]) = \sigma(S(x[n-1])) = \sigma(s_{n-1}s_n s_{n+1} \cdots)$$

$$= s_{n}s_{n+1}s_{n+2} \cdots$$

(4)

Depending on the chaotic system, there may be restrictions on the allowable symbolic sequences that generate what is called a grammar of sequences [18]. The chaotic systems considered in this paper are “fully developed,” that is, there is no restriction on the symbolic sequence. However, the algorithms developed in the following sections could be applied to dynamics with restrictive grammars as well.

By reversing (2) and (3), we see that the initial state must be located in the set $\bigcap_{n=0}^{N-1} f^{-n}(I_{v_0})$. For a chaotic system where the orbits of the neighboring states tend to diverge as $N$ grows large, the size of this set gets smaller with larger $N$. As $N$ tends to infinity, it contains only a single initial point. If this is the case, then there is a one-to-one equivalence between the initial state $x[0]$ and the infinite sequence of symbols $v_k$. The mapping of symbol sequence $S(x[n])$ to state $x[n]$ is denoted by $\beta(\cdot)$. The relationship among the dynamics $f(\cdot)$, the shift $\sigma(\cdot)$, and the conversion $\beta(\cdot)$ is illustrated in the diagram

$$S(x[n-1]) \xrightarrow{\sigma} S(x[n]) \xrightarrow{\beta} x[n-1] \xrightarrow{f} x[n].$$

(5)

As an example, consider the sawtooth map, which is defined by the dynamical equation

$$x[n] = (2 \cdot x[n-1] \mod 1), \quad x[n] \in [0, 1),$$

(6)

where $I = [0, 1)$. The partitions and corresponding symbolic representations are $I_0 = [0, 1/2)$, $I_1 = [1/2, 1)$ and $v_0 = 0$, $v_1 = 1$, respectively. The mapping function, $\beta(\cdot)$, is

$$x[n] = \beta(S(x[n])) = \sum_{k=n}^{\infty} s_k 2^{-(k-n-1)}.$$

(7)

Mapping functions for two other popular chaotic maps, the tent and logistic maps, can be obtained by using (7) and the topological semiconjugacy properties relating the sawtooth map to these two maps. Furthermore, the chaotic maps that can be described by a linear, random representation and their generalizations as outlined in [11] can easily be generated with their corresponding symbol sequence. It is also possible to numerically approximate $\beta(\cdot)$ for many chaotic maps.

Once the mapping function $\beta(\cdot)$ is known, then one can generate chaotic sequences using the symbol sequence rather than generating the chaotic sequence directly. Such an implementation is especially important for some chaotic maps where the chaotic sequence becomes irrelevant with direct iteration because of finite precision. The two-dimensional bakers transformation and two of the most popular one-dimensional chaotic maps, the sawtooth and tent maps, are examples of this problem. Two different ways of generating chaotic sequences are illustrated in Figure 2.

Because of the limitations of digital computers, chaotic sequences can only be generated with a finite precision. Thus, only a finite sequence of symbols can be considered in practice. By modifying (7), the relationship between a finite symbol length sequence and its corresponding state can be rewritten as

$$x[n] = \beta(\hat{S}(x[n])) = \sum_{k=n}^{n+N_p-1} s_k 2^{-(k-n-1)},$$

(8)

where $\hat{S}(\cdot)$ is used to indicate that the first $N_p$ symbols are used. Two different implementations for generating these finite precision “pseudo-chaotic” sequences, directly and through symbolic dynamics, are illustrated in Figure 3.

This finite precision symbolic dynamics representation is the key to efficient chaotic modulation and demodulation. Chaotic modulation and demodulation through mapping functions are illustrated in Figure 4. The mapping function may not be known for certain chaotic systems and may not exist in some cases. However, it may still be possible to encode the information through symbolic dynamics. The small perturbation approach to controlling the symbolic dynamics of chaotic systems combined with an estimate of the mapping function can be used to drive the system through the desired symbolic sequence. This encoding scheme is explained in detail in [3].
chaotic systems are examples of chaotic systems where this scheme has been applied.

3. TRELLIS DIAGRAM REPRESENTATION

The ability to use a finite number of symbols to generate chaotic sequences allows us to represent chaotic dynamics with an equivalent trellis diagram, since the symbol set is also finite. To illustrate how a trellis diagram is constructed, the binary symbol set \{0, 1\}, which is used for many popular unimodal chaotic maps, will be used for simplicity. Figure 5 illustrates the trellis diagram for a precision of \(N_p\) symbols.

In the trellis diagram, the states are all possible combinations of \(N_p\) symbols, that is, there are \(2^{N_p}\) possible symbolic states. Since each incoming symbol can only take on values of 0 or 1, there are two branches leaving each state. The transition between the current state and the next state is determined by the symbolic dynamics, which is just a left shift operation, and the incoming symbol. The branches of the trellis diagram are labeled with the branch metrics consisting of the input symbol and the output chaotic signal, which is calculated using the mapping function \(\beta(\cdot)\) for each state. As an example, the trellis diagram for the sawtooth map with 2-bit precision is illustrated in Figure 6.

The trellis diagram in Figure 5 is for “fully developed chaotic systems,” that is, there is no restriction on the grammar. If there are restrictions over the grammar, then the trellis diagram has to be modified accordingly. For example, some states would not exist, or certain transitions between states would not be possible depending on the grammar.

For schemes where information is encoded using symbolic dynamics through the use of small perturbations and estimated mapping functions to generate the desired sequence of states [3], a similar trellis diagram representation can be used. It is only necessary to determine which chaotic output corresponds to each symbolic state. This information can be obtained through the mapping function that is used to generate the desired perturbations at the transmitter.

This trellis diagram representation of chaotic systems is the key for the development of the optimal estimation and channel equalization algorithms to be explained in the following sections.

4. OPTIMAL ESTIMATION ALGORITHM

In this section, the development of an optimal estimator for chaotic signals is facilitated by the structure of the trellis diagram representation. An observed noisy chaotic signal can be formulated as

\[
y[n] = x[n] + w[n],
\]

where \(x[n]\) is the chaotic signal generated by (1) and \(w[n]\)

Figure 4: Chaotic modulation and demodulation through symbolic
dynamics.

Figure 5: Trellis diagram representation of chaotic dynamics with
a symbolic dynamics representation for the finite precision case.

Figure 6: Trellis diagram for the sawtooth map with 2-bit precision.
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Assume that is typically modeled as white Gaussian noise with a variance of $\sigma_w^2$. The goal is to estimate $x[n]$ from the observations $y[n]$. The following cost function will be minimized to get an estimate of $x[n]$, represented by $\hat{x}[n]$:

$$J(\hat{x}) = \min_{\hat{x}[n]} \sum_{n=0}^{N-1} (y[n] - \hat{x}[n])^2 \quad \text{s.t.} \quad \hat{x}[n] = f(\hat{x}[n-1]),$$

(10)

where $N$ is the length of the received sequence. Under the Gaussian noise assumption, the solution of this cost function yields the maximum likelihood estimate. Because of the sensitivity of chaotic systems to initial conditions, straightforward minimization of this cost function is numerically difficult.

Here, the Viterbi algorithm [19] is used to obtain the optimum estimate for $x[n]$ by exploiting the trellis diagram representation developed in the previous section. Since any path of the trellis diagram satisfies the constraints imposed by the cost function given in (10), the surviving path determined by the Viterbi algorithm will minimize the mean-square error. Therefore, the chaotic sequence corresponding to the surviving path is the same sequence that minimizes the cost function given by (10).

5. THE SEQUENTIAL CHANNEL EQUALIZATION ALGORITHM

Once the information sequence is encoded with a chaotic system, the modulated signal is then transmitted through the communications channel. The received signal can then be modeled as

$$r[n] = h[n] \ast x[n] + w[n],$$

(11)

where $h[n]$ and $w[n]$ represent the finite length channel impulse and white Gaussian noise with variance of $\sigma_w^2$, respectively, and $\ast$ represents the convolution operation. To accommodate this FIR channel model, the trellis diagram representation developed in the previous section can be expanded by increasing the number of states. Previously, the number of states was $2^{N_r}$. For a filter of length $L + 1$, the number of states increases to $2^{N_r + L}$, if the consecutive states are independent of each other. However, for chaotic sequences, consecutive states are not independent of each other. Consequently, the total number of states is just $2^{N_r + L}$. The new trellis diagram is essentially the same as the previous trellis diagram with a precision of $N_r + L$ symbols and a new branch metric. The state at time $n$ is now given by

$$(x[n-1], x[n-2], \ldots, x[n-L]).$$

(12)

Assume that $N$ symbols are transmitted over the communications channel. Then, the received sequence is $(r[N], r[N-1], \ldots, r[1])$. The maximum likelihood (ML) receiver estimates the sequence that maximizes the likelihood function,

$$p(r[N], \ldots, r[1] \mid x[N], \ldots, x[1])$$

(13)

or, equivalently, the log-likelihood function

$$\log p(r[N], \ldots, r[1] \mid x[N], \ldots, x[1])$$

(14)

formed from the conditional probability density function (pdf). Since the noise $w[n]$ is white, $r[N]$ depends only on the $L$ most recently transmitted symbols, and the log-likelihood function can be written as

$$\log p(r[N], \ldots, r[1] \mid x[N], \ldots, x[1])$$

$$= \log p(r[N] \mid x[N], x[N-1], \ldots, x[N-L])$$

$$+ \log p(r[1] \mid x[N-1], \ldots, x[1]).$$

(15)

If the second term has been calculated at time $N-1$, then only the first term, called the branch metric, has to be computed for each incoming signal $r[N]$ at time $N$. From the model given by (11), the conditional pdf

$$p(r[N] \mid x[N], x[N-1], \ldots, x[N-L])$$

$$= \frac{1}{(2\pi\sigma_w^2)^{L/2}} \exp \left\{-\frac{1}{2\sigma_w^2} \left| r[N] - \sum_{k=0}^{L} h[k]x[N-k] \right|^2 \right\},$$

(16)

so that $\log p(r[N] \mid x[N], x[N-1], \ldots, x[N-L])$ yields the branch metric

$$b_N = \left| r[N] - \sum_{k=0}^{L} h[k]x[N-k] \right|^2. \quad (17)$$

The well-known Viterbi algorithm [19] can be used to implement the ML receiver by searching through an $N_s = 2^{N_r + L}$ state trellis for the most likely transmitted chaotic sequence. Since the trellis diagram is formed using symbolic dynamics, the resulting sequence satisfies the dynamics imposed by the chaotic system.

The Viterbi algorithm requires knowledge of the channel parameters $h[n]$ to compute the branch metrics in (17), so an adaptive channel estimator is needed. An initial estimate of the channel parameters is obtained using training sequences. After the training mode, the final decisions at the output of the Viterbi algorithm can be used to update channel parameter estimates. The parameter update equation for the LMS algorithm is

$$h_k[j] = h_{k-1}[j] + \mu \cdot e[k-D]\hat{x}[k-j-D], \quad (18)$$

where $\mu$ is the adaptation step size, $D$ is the trellis depth, $\hat{x}[k-D]$ is the output of the Viterbi algorithm, and $e[k-D]$ is given by

$$e[k-D] = r[k-D] - \sum_{i=0}^{L} h_k[i]\hat{x}[D-i]. \quad (19)$$

6. SIMULATIONS

The performance of the proposed estimation algorithm has been simulated using chaotic maps such as the sawtooth
map, tent map, and logistic map, and it has been compared to some other optimum estimation algorithms. Throughout these simulations, the noise was modeled as having a white Gaussian distribution. First, the sawtooth map was used. The results are compared with the MAP estimation algorithms developed by Drake [11] in terms of SNR improvement, which is defined as

$$\text{SNR improvement} = \text{Output SNR} - \text{Input SNR}.$$  

(20)

SNR improvement at the output is plotted in Figure 7 for the proposed algorithm together with the MAP estimation algorithm. Since the sawtooth map is uniformly distributed, the cost function given by (10), and, therefore, the Viterbi algorithm yields the MAP estimate for the Gaussian noise distribution. In these simulations the Viterbi algorithm shows slightly better performance than Drake’s MAP estimator because the data was generated with finite precision and Drake’s estimator does not make that assumption. SNR improvement for the logistic map and tent map are also plotted in Figure 8.

The estimation algorithm was also applied to a chaotic communication system where information was encoded with a chaotic signal using the symbolic dynamics. The communications channel was modeled as a WGN channel. The bit error rate (BER) is plotted for the Viterbi algorithm and the MAP estimator in Figure 9. The graph shows that these algorithms achieve similar performance. Furthermore, the algorithms have almost identical complexity with the number of multiplications for both algorithms being $N \cdot 2^{N_p}$ where $N$ is the length of the chaotic sequence and $N_p$ is the amount of precision.

The sequential equalization algorithm was simulated for both time-invariant and time-varying channel models. In both cases, information bits were modulated by the sawtooth chaotic map. The time-invariant channel was modeled as a raised cosine channel given by the following impulse response

$$h[n] = \begin{cases} \frac{1}{2} + \frac{1}{2} \cos \left( \frac{2\pi(n-2)}{3} \right) & \text{for } n = 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$  

(21)

The time-varying channel was modeled as 3-tap multipath fading channel and was simulated using Jake’s simulator [20] with $f_m T = 0.005$, where $f_m$ is the maximum Doppler frequency and $T$ is the simulation step size. For both channel models, initial channel estimates are obtained by using a training sequence of length 1000 and NLMS algorithm with a step size $\mu$ equal to 2. Also, the trellis
7. CONCLUSIONS

In this paper, novel optimum estimation and sequential channel equalization algorithms were proposed for chaotic systems based on symbolic dynamics representations. The symbolic dynamics representation was exploited to map the chaotic dynamics onto a trellis diagram for finite precision implementations. Then, the well-known Viterbi algorithm was used to obtain optimal estimates of the transmitted symbols. For channel equalization, the dynamics-based trellis diagram was extended to accommodate FIR channel models. Then, the Viterbi algorithm was used to remove the distortion introduced by the channel and to estimate the chaotic sequence. These algorithms can be used for a wide range of chaotic maps. They can also be applied to continuous-time systems where the information is encoded using chaotic control to determine the symbolic dynamics of the chaotic system. Simulation results show that both algorithms are able to remove realistic channel distortions and reliably determine the transmitted sequence.

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