Open String Theory in 1+1 Dimensions

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We show that tree level open two dimensional string theory is exactly solvable; the solution exhibits some unusual features, and is qualitatively different from the closed case. The open string “tachyon” $S$–matrix describes free fermions, which can be interpreted as the quarks at the ends of the string. These “quarks” live naturally on a lattice in space-time. We also find an exact vacuum solution of the theory, corresponding to a charged black hole.
The study of string propagation in two dimensional (2d) space-time may provide us with useful toy models for critical string theory and 2d quantum gravity. String dynamics in such backgrounds is expected to be exactly solvable; this phenomenon is closely related to the extremely small number of degrees of freedom in these vacua, and is probably not shared by higher dimensional string theories compactified to two dimensions. The models most intensely studied to date are those of closed bosonic strings in 2d. Using large N matrix model techniques \[4\], \[5\], \[6\], these were shown to be equivalent to theories of free fermions; their discrete formulation is quite well understood. The continuum path integral (Liouville) approach \[7\] is less developed. It is known \[8\], \[9\] that the continuum correlation functions have an extremely simple form. In \[2\] it was shown that the underlying reason for this simplicity is a partial decoupling of a certain infinite set of “discrete” states. This observation will play a role below.

In this letter we will study open string theory in two dimensional space-time. We will find that the tree level dynamics is quite different from the closed sector. On the one hand, the S – matrix which is again exactly calculable, exhibits a more complicated pole structure, and has some features which resemble higher dimensional string theories; on the other, this S – matrix will be seen to follow from a field theory of massless free fermions with a lattice propagator!

The theory of world sheet gravity on a manifold \( \mathcal{M} \) with the topology of a disk \[10\], corresponds to 2d string theory with open and closed strings. The action is:

\[
S = \frac{1}{2\pi} \int_{\mathcal{M}} d^2 \xi \left[ g^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu - R^{(2)} \Phi(X) + T(X) \right] + \frac{1}{\pi} \int_{\partial \mathcal{M}} d\xi \left[ A_\mu(X) \partial_\xi X^\mu - K \Phi(X) + g^{\frac{1}{4}} T_B(X) \right]
\]

where \( g_{ab} \) is the world sheet metric, \( R^{(2)} \) its scalar curvature, \( g = \text{det} g_{ab} \), and \( K \) the extrinsic curvature on the boundary \( \partial \mathcal{M} \). \( X^\mu, \mu = 0, 1 \) parametrize the space-time manifold, and we have introduced couplings to a general space-time metric \( G_{\mu\nu} \), gauge field \( A_\mu \), dilaton \( \Phi \), and bulk and boundary “tachyon” fields \( T, T_B \). For \( T = T_B = 0 \), the space-time equations of motion for slowly varying fields are \[10\]:

\[
R_{\mu\nu} = \nabla_\mu \nabla_\nu \Phi
\]

\[
(\nabla^\nu F^\lambda_\mu) [G - F^2]^{-1} + \frac{1}{2} (\nabla^\nu \Phi) F_{\mu\nu} = 0
\]

\[1\] The \( N = 1 \) supersymmetric extension was studied in \[1\] \[2\]; \( N = 2 \) strings were discussed in \[3\].
We will start by considering the simplest (exact) solution, \( F_{\mu\nu} = R_{\mu\nu} = 0 \) and \( \Phi = Q X^0 - 2 i \alpha_0 X^1 \). As usual [7] one should also fix the area of the surface \( \mathcal{M} \) and the length of the boundary \( \partial \mathcal{M} \) or, Laplace transforming, turn on condensates of \( T(X) = \mu e^{\alpha_+ X^0} \) and \( T_B = \rho e^{\frac{\alpha_+}{2} X^0} \). The parameters \( Q, \alpha_+ \) are related by gauge invariance to \( \alpha_0 \) [11]:

\[
-Q = \alpha_+ + \alpha_-; \quad 2 \alpha_0 = \alpha_- - \alpha_+; \quad \alpha_+ + \alpha_- = 2
\]

where \(|\alpha_+| < |\alpha_-|\). The fields \( X^0, X^1 \) satisfy Neumann boundary conditions \( \partial_n X^1 = 0 \), \( \partial_n X^0 = \rho e^{\frac{\alpha_-}{2} X^0} \). The boundary conditions usually considered in the matrix models [12], [13] are Dirichlet boundary conditions (for \( X^1 \)). Therefore their results can not be directly borrowed to our situation (which is modeled after critical open string theory).

The only field theoretic degree of freedom in this theory, in the open string sector, is the massless tachyon field \( T_B \), with on shell vertex operator:

\[
T_k^{(+)} = \int_{\partial \mathcal{M}} d\xi e^{i \xi X^1 + \beta_+ X^0}; \quad \beta_+ = -\frac{Q}{2} \pm (k - \alpha_0)
\]

\( T_k^{(+)} \) (\( T_k^{(-)} \)) correspond to right (left) moving tachyons in space-time. In addition, the spectrum includes oscillator states at discrete values of the momenta \( \sqrt{2k} \in \mathbb{Z} \).

As in closed string theory, for finite bulk cosmological constant \( \mu \) and boundary cosmological constant \( \rho \), the generic amplitudes \( A_{\text{open}}(k_1, ..., k_n) = \langle T_{k_1}...T_{k_N} \rangle \) are not known. However, the bulk amplitudes (see e.g. [2] for a discussion), which satisfy \( \sum_{i=1}^N \beta_i = -Q \) (in addition to the obvious \( \sum_{i=1}^N k_i = 2 \alpha_0 \)) are calculable [14]. By performing the \( X^0 \) zero mode integral, one finds in that case:

\[
A_{\text{open}}(k_1, ..., k_N) = \log \Lambda \int_1^\infty d\xi_{N-1} \int_1^{\xi_{N-1}} d\xi_{N-2} ... \int_1^{\xi_4} d\xi_3 \langle T_{k_1}(0)T_{k_2}(1)T_{k_3}(\xi_3)...T_{k_{N-1}}(\xi_{N-1})T_{k_N}(\infty) \rangle
\]

where \( \Lambda \) is a UV cutoff, and the correlator \( \langle T_1...T_N \rangle \) is evaluated using the free field propagators \( \langle X^\mu(\xi)X^\nu(0) \rangle = -4 \delta^{\mu\nu} \log |\xi| \); the boundary “cosmological” interaction in [1] has disappeared. The factor of \( \log \Lambda \) signals a bulk effect, being the effective volume of \( X^0 \) (there is also a factor of the volume of \( X^1 \) which we didn’t write). Note that, as usual, the basic object in the open sector is the amplitude with a given ordering of the tachyons (up to cyclic permutations) \( -SL(2, R) \) transformations do not permute the particles. Different orderings correspond to different channels. Of course, in the end one has to sum over all non cyclic permutations to recover the total amplitude. All we have said so far is valid (or
can be trivially generalized) for any dimension of space-time. The magic of 2d string theory allows one to explicitly evaluate the integrals (4), due to simplifications in the dynamics. We will next present the results and discuss them; the derivation and further discussion will appear elsewhere.

It is clear from (3) and the behavior of closed 2d string amplitudes, that the chirality of the massless “tachyon” (3) should play a central role. The generic tachyon amplitude (4) has the form:

\[ A_{\text{open}}(k_1, \ldots, k_N) = \langle (T^+)^{n_1} (T^-)^{m_1} (T^+)^{n_2} (T^-)^{m_2} \ldots (T^+)^{n_r} (T^-)^{m_r} \rangle \]  

(5)

By (5) we mean the (ordered) amplitude consisting of \( n_1 \) right moving tachyons, followed by \( m_1 \) left moving ones, etc. By cyclic permutation symmetry we can always assure that \( n_1, m_r \neq 0 \), and this is how we define (5). The first interesting property of the amplitudes is that \( A_{\text{open}}(k_i) = 0 \) if \( r > 1 \). In other words, only amplitudes of the form:

\[ A_{\text{open}}^{(n,m)}(k_1, \ldots, k_n, p_1, \ldots, p_m) = \langle T_{k_1}^+ \ldots T_{k_n}^+ T_{p_1}^- \ldots T_{p_m}^- \rangle \]  

(6)

can be non zero (for generic \( k, p \)). This is the first hint of an almost complete separation of the dynamics of left and right moving tachyons. Recall that in the closed sector only \( A_{\text{closed}}^{(n,1)}(A_{\text{closed}}^{1,n}) \neq 0 \). The structure of \( A_{\text{open}}^{(n,m)} \) (5) is much more complex than in (4); it is very convenient to parametrize it in terms of the natural variables for (3), \( m_i = \frac{1}{2} \beta_i^2 - \frac{1}{2} k_i^2 \).

The amplitudes (6) are most simply expressed in terms of the functions \( F_n \):

\[ F_n(k_1, k_2, \ldots, k_n) = \prod_{l=1}^{n-1} \frac{1}{\sin \pi \sum_{i=1}^{l} m_i} \]  

(7)

and are given by:

\[ A_{\text{open}}^{(n,m)}(k_1, \ldots, k_n, p_1, \ldots, p_m) = \left[ \prod_{i=1}^{n+m} \frac{1}{\Gamma(1 - m_i)} \right] F_n(k_1, \ldots, k_n) F_m(p_1, \ldots, p_m) \]  

(8)

with the kinematic constraints \( \sum_{i=1}^{n} m(k_i) = 2 - m \), \( \sum_{i=1}^{m} m(p_i) = 2 - n \). The form (8) deserves a few comments:

1) We see that in the 2d open string, the pole structure is much more intricate than in the closed one, where \( A_{\text{closed}}^{(n,1)}(A_{\text{closed}}^{1,n}) \) is given by a form similar to (5) with \( F_n F_m \) replaced by \( \prod_{i=1}^{n} \Gamma(m_i) \) (and the rest of \( A_{\text{open}}^{(n,m)} \) vanish). In particular in the open string case there are many “moving” (codimension 1) poles, which are absent in the closed case.
2) It is interesting to follow the origin of different poles by factorization. The poles appear when several neighbouring particles collide. We see by (7), (8), that they occur when \( \sum_{i=1}^{l} m_i = r \in \mathbb{Z} \) \( m_i \) is either \( m(k_i) \) or \( m(p_i) \); \( l = 1, 2, ..., n \) (or \( m \)). Poles in the variable \( \sum_{i=1}^{l} m_i \) come from two different regions of moduli space of (4). The first is from the region where \( T_1, ..., T_l \) approach each other. It is easy to see that this gives poles at \( \sum_{i=1}^{l} m_i = L = 1, 2, 3, ... \) corresponding to intermediate states at (oscillator) level \( (L-1)(l-1) \). Only these levels appear in intermediate states due to kinematic restrictions (see [2]).

The second source of poles is from the region where (say) \( T_{k_1}^{(+)} \), ..., \( T_{k_l}^{(+)} \) approach \( T_{p_1}^{(-)} \), ..., \( T_{p_r}^{(-)} \), a subset of the tachyons of the opposite chirality adjacent to the \( T_{k_i}^{(+)} \). Here we have the following situation: if \( r < m \) (i.e. the negative chirality tachyons involved are only a subset of all \( T^{(-)} \)), only tachyon poles are possible, since higher poles would correspond to oscillator states at generic momenta, which must by general arguments be BRST exact. The tachyon poles in this channel occur at \( \sum_{i=1}^{l} m_i = 1 - r = 0, -1, -2, ..., 1 - m \). This leaves only the poles at \( \sum_{i=1}^{l} m_i = -m, -m - 1, -m - 2, ..., \) which correspond to massive (oscillator) intermediate states in the channel \( (T_{k_1}^{(+)} ... T_{k_l}^{(+)} T_{p_1}^{(-)} ... T_{p_m}^{(-)}) \), i.e. when a subset of the right moving tachyons collide with all the left moving ones. The fact that oscillator states can occur in this channel is due to the fact that \( \sum_{i=1}^{l} m(p_i) = 2 - n \) is fixed by kinematics. Thus we see that the origin of the nice expression (8) is in an intricate structure of poles. It is also easy to show that all other poles that seem naively to appear in (4) have in fact vanishing residues. For example, it is clear that the residues of the poles in \( \sum_{i=a}^{b} m(p_i) \) \( a \neq 1 \) and \( b \neq m \) are given by correlation function (5) with \( r > 1 \) and therefore vanish. It is amusing to check explicitly that the residues of the various poles are consistent with (5) (this is particularly easy for the tachyon poles). We will not do that here.

3) We mentioned above that the simplicity of the correlation functions in closed 2d string theory is due from the continuum point of view to decoupling of certain discrete states with negative energy (see [2]). A subset of those are the tachyons (3) with \( m(k) = 1, 2, 3, \cdots \). In (8) it seems that such tachyons “almost” decouple, due to the factor of \( \prod_i \frac{1}{1-m_i} \). This breaks down for the particles on the boundary between the right and left moving tachyons in (4), since \( F_n \) develops a pole then. For the other \( m_i \) we still find that the negative energy discrete states decouple. This is the origin of the relative simplicity of (8). The role of the above discrete states requires (both here and in the closed case) a much better understanding.
4) There is a well known relation [15] between open and closed tree amplitudes in critical string theory. The derivation still applies here, so we may borrow the results. Assuming that given the closed amplitudes, the open ones are uniquely determined (the converse is certainly true), one way to prove (8) would be to plug it as an ansatz into the relations of [15]. We have only done this for five point functions, for which the relation of [15] reads:

\[ A_{\text{closed}}(k_1, \ldots, k_5) = A_{\text{open}}(k_1, k_2, k_3, k_4, k_5)A_{\text{open}}(k_2, k_1, k_4, k_3, k_5)s_{12}s_{34} \]

\[ + A_{\text{open}}(k_1, k_3, k_2, k_4, k_5)A_{\text{open}}(k_3, k_1, k_2, k_4, k_5)s_{13}s_{24} \]

where \( s_{ij} \equiv \sin \pi k_i \cdot k_j = -\sin \pi (m_i + m_j) \) if \( i, j \) have the same chirality, and \( s_{ij} = \sin \pi m_i m_j \) if they have opposite chirality. It is amusing to verify (9) by plugging in (8), and the closed string results [9].

5) The structure we find is very different from the closed case, however (8) is only part of the amplitude. Could it be that summing over channels restores the simpler closed string structure? In general the answer is no. Due to the fact mentioned above, that the amplitudes (5) satisfy

\[ A_{\text{open}}(k) \propto \delta_{r,1}, \]

we have only to symmetrize (6) in \( \{k_i\} \) and \( \{p_i\} \) separately. The function \( F_n \) (7) is therefore replaced by:

\[ H_n(k_1, k_2, \ldots, k_n) = \sum_{\sigma} \prod_{l=1}^{n-1} \frac{1}{\sin \pi \sum_{i=1}^{l} m_{\sigma(i)}} \]

where the sum over \( \sigma \) runs over permutations of 1, 2, ..., \( n \). The total amplitude after summing over channels, has the form (8) with \( F_n \to H_n \). We haven’t been able to explicitly sum (10) in general. For low point functions one does get relatively simple formulae; e.g for \( N = 4, 5 \) we find (after summing over channels (10)):

\[ A_{+++-}(k_1, \ldots, k_4) = \prod_{i=1}^{3} \frac{\Gamma(m_i)}{\Gamma(1-m_i)} \]

\[ A_{++--}(k_1, \ldots, k_4) = 0 \]

\[ A_{++++}(k_1, \ldots, k_5) = \prod_{i=1}^{4} \Gamma(m_i) \sum_{1=i<j}^{4} s_{ij} \]

Note that tachyons with \( m_i = 1, 2, 3, \ldots \) do not decouple after summing all diagrams. The poles still occur as a function of \( m_i \). This does not generalize to higher point functions. In general, the expression (11) contains many poles in \( \sum_i m_i \).

An interesting property of the amplitudes (7), (8), (10) is that they follow from a space-time action describing free fermions (in 2d). Indeed, redefining: \( T^\pm_k \to \Gamma(1-m_k)T^\pm_k \),
the amplitude (8) takes the form $A_{\text{open}}^{(n,m)}(k_i, p_j) = F_n(k_i)F_m(p_j)$, with $F_n$ given by (7).

Consider now the generating functional:

$$\mathcal{W}(T^{(\pm)}) = \int D\psi D\psi^* D\bar{\psi} D\bar{\psi}^* \exp \left[ -\int d^2xe^{\frac{T}{2}} \mathcal{L}(\psi, T) \right]$$

with (12):

$$\mathcal{L}(\psi, T) = \psi^* \sin(\pi \alpha_- \partial^+) \psi + \bar{\psi}^* \sin(\pi \alpha_+ \partial^-) \bar{\psi} + T^{(+)} \bar{\psi}^* \bar{\psi} + T^{(-)} \psi^* \psi,$$

where $\partial^\pm = \partial_0 \pm i \partial_1$. One can check that $\mathcal{W}$ satisfies:

$$\frac{\delta^{n+m}(\mathcal{W}/Z)}{\delta T^{(+)}_{k_1} \cdots \delta T^{(+)}_{k_n} \delta T^{(-)}_{p_1} \cdots \delta T^{(-)}_{p_m}} = H_n(k_1, \ldots, k_n)H_m(p_1, \ldots, p_m)$$

To make sense of the functional integral (12), we have to specify the zero mode prescription, and the coupling between left and right moving fermions. The generating functional $\mathcal{W}$ is defined with all zero modes soaked up except $\psi^*(\vec{k} = 0)$, $\bar{\psi}^*(\vec{k} = 0)$. The functional $\mathcal{Z}(T^{(\pm)})$ in (14) is given by the same expression as (12) with all the zero modes soaked up. The division by $\mathcal{Z}$ in (14) removes disconnected diagrams. In order to obtain (14) one has also to introduce a contact term coupling between left and right: $\langle \bar{\psi}(\vec{k})\psi(-\vec{k} - \vec{Q}) \rangle = \gamma$, which in space-time gives: $\langle \bar{\psi}(X)\psi(Y) \rangle = \gamma e^{\frac{\delta}{2}X^0} \delta^2(X - Y)$. Such a contact term can be achieved for example by modifying the Lagrangian (13) as follows: $\mathcal{L} \rightarrow \mathcal{L} + \gamma \bar{\psi} \sin(\pi \alpha_- \partial^+) \sin(\pi \alpha_+ \partial^-) \psi$. Contact terms are usually adjustable in quantum field theory, but it is not clear to us whether this is the case here as well. In any case, it is amusing that the non-trivial S – matrix of the open string tachyons is due to the choice of these contact terms; without the contact terms ($\gamma = 0$), the S – matrix would have been trivial.

In [2] it was shown that the closed string amplitudes also follow from a space-time action, however one which was significantly different from (12), (13). The main qualitative difference between the two is that here “Liouville” ($X^0$) momentum is (anomalously) conserved in (12), while in the closed case the natural description of the amplitudes involved a field theory in which one had to integrate over intermediate Liouville momenta; as a result of that, the propagator, instead of containing poles as here, actually had cuts; the infinite number of irreducible vertices of [2] is presumably related to the same phenomenon. We

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2 We are using a non standard mode expansion for $\psi$: $\psi(X^0, X^1) = \int d^2k e^{\frac{4}{k} \cdot X} \psi_k$. 

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do not know what is the origin of this difference in the behavior of the two sectors of the theory.

One application of the free fermion representation (12), (13) is the generalization of (8) to arbitrary (non bulk) amplitudes. It is easy to introduce the boundary cosmological constant in (13): this corresponds to $T(+) \rightarrow T(+) + \rho e^{\frac{\alpha_0}{2}X^0 + i\alpha_0X^1}$:

$$\mathcal{L}_\rho(\psi, T) = \psi^* \sin(\pi \alpha_- \partial^+) \psi + \bar{\psi}^* \sin(\pi \alpha_+ \partial^-) \bar{\psi} + \rho e^{\frac{\alpha_0}{2}X^0 + i\alpha_0X^1} \bar{\psi}^* \bar{\psi} + T(+) \bar{\psi}^* \bar{\psi} + T(-) \psi^* \psi,$$

(15)

(12), (15) can be used to calculate general tachyon correlation functions (14) – the fermions are still free. We will describe the details of the free fermion calculations elsewhere.

The fermions in (12), (13) can be thought of as the quarks at the ends of the open string. This suggests a simple generalization to the case of $U(N)$ gauge symmetry; one introduces $4N$ fermions $\psi_i, \bar{\psi}_i$, $i = 1, 2, ..., N$; replacing the kinetic term in (13) by $\psi_i^* \sin(\pi \alpha_- \partial^+) \psi_i$, etc, we see that the action has a $U(N)$ symmetry $\psi_i \rightarrow \lambda_i \psi_j; \bar{\psi}_i \rightarrow \bar{\lambda}_i \bar{\psi}_j$, $\lambda_\lambda = 1$. The source term in (13) is replaced by: $T^{(-)} \lambda_i^j \bar{\psi}_i^* \psi_j$, so that the amplitudes (14) are multiplied by $\text{tr}(\lambda^{a_1} \lambda^{a_2} \cdots \lambda^{a_{n+m}})$, as appropriate for Chan Paton factors.

Quite unexpectedly from the point of view of (4), the propagators in (13) are periodic in momentum space; the $\bar{\psi}$ propagator $\sin(\pi \alpha_- \partial^-) = \sin \frac{\pi \alpha_-}{2}(\beta + k)$ is invariant under $k_x \equiv \beta + k \rightarrow k_x + 2\alpha_-$, and similarly for the $\psi$ propagator, $k_y \equiv \beta - k \rightarrow k_y + 2\alpha_+$. It is natural to consider the theory where we identify momenta which differ by a period. In this case momentum space is a torus, while the conjugate variables, $x = \frac{1}{2}(X^1 - iX^0)$, $y = -\frac{1}{2}(X^1 + iX^0)$ live on a lattice $L$, with lattice spacings $\delta_x = \pi \alpha_+, \delta_y = \pi \alpha_-$. Replacing $\int d^2X \rightarrow \sum_{(x,y) \in L}$ in (12) we see that the “dilaton factor” disappears: $e^{\frac{i}{2}QX^0 - i\alpha_0X^1} = 1$, and (12), (13) assume the form:

$$\mathcal{W}(T(\pm)) = \prod_{(x,y) \in L} \int D\psi(x,y) D\bar{\psi}(x,y) \exp \left[ - \sum_{(x,y) \in L} \mathcal{L}(\psi, T) \right]$$

(16)

$$\mathcal{L}(\psi, T) = \psi^*(x, y) [\psi(x, y + \delta_y) - \psi(x, y - \delta_y)] + \bar{\psi}^*(x, y) [\bar{\psi}(x + \delta_x, y) - \bar{\psi}(x - \delta_x, y)]$$

$$+ T(+) \bar{\psi}^* \bar{\psi} + T(-) \psi^* \psi,$$

(17)

Note that the space-time lattice is obtained only after continuing $X^0 \rightarrow iX^0$. One can think of $X^0$ as a Liouville field, in which case the space-time lattice arises when Liouville
is treated as a Feigin Fuchs field. Before the continuation $X^0$ is a non-compact dimension, as in (13).

It is not clear what is the relation of our fermions to those of the closed string matrix models (4) – (6), however the two should be closely related. It is also not clear to us why and how the space-time lattice $L$ arose here, or equivalently, what is the origin of the periodicity of $F_n$ (7) in $k$ space. The string theory under consideration seems to develop a minimal length. This phenomenon is similar to the one discussed in (6). The propagator (13) is reminiscent of the one obtained by Friedan (17) in critical string field theory. In both cases the zeroes of the propagator are related to degenerate representations, but the relation between the two should be elucidated further. There are many other questions concerning (12) – (17) which we haven’t discussed; in particular, the role of (13) for loop amplitudes, the coupling to the closed sector, the role of the massive discrete states, etc. These issues will be addressed elsewhere. It would also be interesting to solve the matrix models corresponding to these theories. This may shed additional light on their structure.

Another interesting issue concerns non trivial solutions of the open + closed string equations of motion (4). In the closed case, the $SL(2, \mathbb{R})/U(1)$ coset (18) leads to propagation in a 2d black hole geometry. We can imitate that procedure here and construct a solution of the open string equations of motion, by putting the $SL(2, \mathbb{R})/U(1)$ CFT on the boundary of the world sheet manifold $\partial \mathcal{M}$. Repeating the steps of (13), we find an exact solution of the equations of motion in Euclidean space-time with action (here we restrict to the case $\alpha_0 = 0$, as in (19)):

$$S = S_0 + m \int_{\mathcal{M}} d^2 \xi (\partial X^0 + i \gamma \partial X^1)(\bar{\partial} X^0 + i \gamma \bar{\partial} X^1) e^{-Q X^0} +$$

$$q \int_{\partial \mathcal{M}} d \xi (\partial X^0 + i \gamma \partial X^1) e^{-\frac{Q}{2} X^0}$$

(18)

where $S_0$ is the action in the trivial vacuum and $\gamma = \sqrt{Q^2 + 1}$. Comparing (18) to (1) we see that we have found a solution with linear dilaton $\Phi = Q X^0$, black hole background metric and electric field $E \simeq q e^{-\frac{Q}{2} X^0}$. When the mass of the black hole $m$ vanishes, we have a flat 1 + 1 dimensional space with an $X^0$ dependent electric field. For non zero mass of the black hole the theory depends on the parameter $m/q^2$ (in addition to the expectation value of the dilaton), and the solution (18) describes a charged black hole. This solution is compatible with the lowest order equations of motion (2): indeed, in 2d we have $F_{\mu \nu} = \epsilon_{\mu \nu} E$, and (2) is equivalent to (see also (20)):

$$\epsilon_{\mu \nu} \partial_{\mu} \left[ e^{\frac{\Phi}{2}} \frac{E}{\sqrt{1 + E^2}} \right] = 0$$

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The solution has the form (for linear dilaton): \( E^{-2} = e^{QX^0} - 1 \). Recalling that (2) is only valid when \( \delta E/E \ll 1 \) (which means here \( e^{QX^0} \gg 1/Q \), for \( Q < 1 \)), we finally find in the region of validity of (2), \( E \simeq e^{-\frac{Q}{2}X^0} \). It is more complicated to find the exact solution this way, since the exact form of (2) is not known. The charged black hole solution suggested here differs from the one discussed recently in the literature [21]. The solution (18) may be useful to study pair production, and large electric field behavior; it is interesting that at least from the low energy Lagrangian giving (2) (the Born-Infeld Lagrangian \( L = e^{\frac{\Phi}{2}} \sqrt{\det(G + F)} \)) it seems that physics changes drastically in the region where the electric field exceeds some critical value. This is reminiscent of the formation of a horizon in the closed version of the theory, and should be studied further. The free fermions should be very useful to study these issues. There is a very natural way to couple the “quarks” to the gauge field \( A^\mu \), by replacing \( \partial^\pm \rightarrow D^\pm = \partial^\pm - iqA^\pm \) in (13). The action (13) is then invariant under the standard gauge transformation, \( \psi \rightarrow e^{iq(\Sigma)}\psi, \quad A^\pm \rightarrow A^\pm - \partial^\pm \epsilon \). Tachyon scattering in the background electric field of (18) should then be given by evaluating the free fermion path integral (12) with modified propagator, \( \sin(\pi \alpha_\pm(\partial^\pm - iqA^\pm)), \) with \( A^\pm \propto e^{-\frac{Q}{2}X^0} \). The generalization to the non abelian case is straightforward. The properties of the theory in background fields will be left for future work.

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