Present Constraints on the H-dibaryon at the Physical Point from Lattice QCD

S.R. Beane,1,2 E. Chang,3 W. Detmold,4,5 B. Joo,5 H.W. Lin,6 T.C. Luu,7 K. Orginos,4,5 A. Parreño,3 M.J. Savage,6 A. Torok,8 and A. Walker-Loud9 (NPLQCD Collaboration)

1Albert Einstein Zentrum für Fundamentale Physik, Institut für theoretische Physik, Sidlerstrasse 5, CH-3012 Bern, Switzerland
2Department of Physics, University of New Hampshire, Durham, NH 03824-3568, USA
3Dept. d’Estructura i Constituents de la Matèria. Institut de Ciències del Cosmos (ICC), Universitat de Barcelona, Martí Franquès 1, E08028-Spain
4Department of Physics, College of William and Mary, Williamsburg, VA 23187-8795, USA
5Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, VA 23606, USA
6Department of Physics, University of Washington, Box 351560, Seattle, WA 98195, USA
7N Division, Lawrence Livermore National Laboratory, Livermore, CA 94551, USA
8Department of Physics, Indiana University, Bloomington, IN 47405, USA
9Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

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Abstract

The current constraints from Lattice QCD on the existence of the H-dibaryon are discussed. With only two significant Lattice QCD calculations of the H-dibaryon binding energy at approximately the same lattice spacing, the form of the chiral and continuum extrapolations to the physical point are not determined. In this brief report, an extrapolation that is quadratic in the pion mass, motivated by low-energy effective field theory, is considered. An extrapolation that is linear in the pion mass is also considered, a form that has no basis in the effective field theory, but is found to describe the light-quark mass dependence observed in Lattice QCD calculations of the octet baryon masses. In both instances, the extrapolation to the physical pion mass allows for a bound H-dibaryon or a near-threshold scattering state.
After decades of effort in Lattice QCD dedicated to exploring the possibility of a bound H-dibaryon [1–6], whose existence was postulated by Jaffe [7] in 1977, the NPLQCD and HALQCD collaborations have recently reported results that show that the H-dibaryon is bound for a range of light-quark masses that are larger than those found in nature [8, 9]. These calculations are important for a number of reasons. First, they show that Lattice QCD is now capable of calculating the energy of simple nuclei, with the H-dibaryon being an exotic example of such. Second, they provide evidence that a bound H-dibaryon may exist for some values of parameters entering the QCD Lagrangian. However, it is important to determine if this system is, in fact, bound at the physical values of the light-quark masses and with the inclusion of the electroweak interactions. Experimental evidence currently suggests that such a bound state does not exist [10], but that a near-threshold resonance may exist in the scattering-channel with the quantum numbers of the H-dibaryon [11]. In this note we establish the current constraints on the binding of the H-dibaryon at the physical values of the light-quark masses, in the isospin limit and in the absence of electroweak interactions, by extrapolating the available Lattice QCD results.

The details of the two Lattice QCD calculations that provide statistically significant evidence for a bound H-dibaryon can be found in the very recent works of NPLQCD [8] and HALQCD [9]. The NPLQCD result is determined from calculations in four lattice volumes (with spatial extents of $L \sim 2.0, 2.5, 3.0$ and 4.0 fm), each at a single spatial lattice-spacing of $b \sim 0.123$ fm and a pion mass of $m_{\pi} \sim 390$ MeV. A binding energy of $B_H = 16.6 \pm 2.1 \pm 4.6$ MeV was determined at that pion mass. The HALQCD collaboration performed calculations in three lattice volumes (with spatial extents of $L \sim 2.0, 3.0$ and 4.0 fm) at a lattice spacing of $b \sim 0.121$ fm and in the limit of SU(3) flavor symmetry at three different light-quark masses giving $m_{\pi} \sim 673, 837$ and 1015 MeV. In order to extrapolate in the quark masses, the binding energy of $B_H = 37.4 \pm 4.4 \pm 7.3$ MeV obtained at $m_{\pi} \sim 837$ MeV is used because this pion mass corresponds to a strange-quark mass that is closest to that of nature (and that of the NPLQCD calculations). One should keep in mind all of the usual, well-documented, caveats associated with chiral extrapolations involving heavy pions.

NPLQCD and HALQCD employed different Clover discretizations for the light-quarks (and different gauge-actions), providing results that are $\mathcal{O}(b)$-improved and therefore both sets of calculations have lattice-spacing errors that scale as $\mathcal{O}(b^2)$. There are, of course, operators at $\mathcal{O}(b^2)$ in the low-energy effective field theory of the Symanzik action, and there is not much to say quantitatively about continuum extrapolation with results at only one lattice spacing. Given the precision with which the single-hadron energy-momentum relation is satisfied [8], the contributions from Lorentz-symmetry breaking operators that appear at this order are expected to be highly suppressed. Naive scaling arguments as well as the cancellations that occur in forming energy differences suggest that lattice spacing artifacts are suppressed, as compared, for instance, to the leading quark-mass effects. However,

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1 The energy-level(s) determined in the Lattice QCD calculation of HALQCD [9] is exponentially close to the actual bound-state energy of the H-dibaryon and does not suffer from the uncontrolled approximations that are present in phase-shifts calculated via the energy-dependent and sink-dependent potentials presented by HALQCD. It is only the energy eigenvalue that is used in our present analysis.

2 NPLQCD used anisotropic gauge field configurations that were generated by the Hadron Spectrum Collaboration [12, 13]. The temporal and spatial lattice spacings, $b_t$ and $b_s$ respectively, are related by $b_t = b_s/\xi_t$ where $\xi_t = 3.5$. 

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definitive statements about the lattice spacing dependence will require calculations at a smaller lattice spacing. In this brief report, we assume that lattice spacing artifacts are small (and therefore are not driving the H-dibaryon binding), and focus on the light-quark mass dependence. Alternately, one can view this as performing the chiral extrapolation of the (approximately) fixed lattice-spacing results.

The form of the chiral extrapolation of the H-dibaryon binding energy is unknown, and there are a number of features that make it difficult to determine. In the limit of SU(3) flavor symmetry, the non-interacting ground state of the \( I = 0, J = 0 \) and \( s = -2 \) (the quantum numbers of \( \Lambda\Lambda \)) system is three-fold degenerate, comprised of the hadronic states \( \Lambda\Lambda, \Sigma\Sigma, \) and \( N\Xi \). The interactions among these states produce the H-dibaryon (when it is bound) as the ground-state, with the other two states being of higher energy. Even in the presence of SU(3) breaking, as is the case in the NPLQCD calculations, the spectrum is qualitatively similar with the non-interacting states being nearly degenerate. If the H-dibaryon is tightly bound, then a chiral expansion \(^3\) of the form of that for single hadrons will result (with different low-energy constants). On the other hand, when the H-dibaryon is loosely bound the binding energy results from a cancellation between short-distance contributions and long-distance contributions, as is the case with the deuteron. However, the long-distance contribution results from two-pion exchange, in contrast with the deuteron. The H-dibaryon is expected to be less fine tuned than the deuteron leading to the expectation that the chiral extrapolation may resemble the form \( B_H(m_\pi) = B_0 + d_1 m_\pi^2 + \mathcal{O}(m_\pi^3) \).

A second extrapolation form can be motivated as follows. The chiral-extrapolation of the lowest-lying octet baryon masses, such as the \( N, \Lambda, \Sigma \) and \( \Xi \), is an ongoing topic of debate. The form of the light-quark mass dependence of the baryon masses produced by the lowest orders of baryon chiral perturbation theory does not naturally reproduce the currently available results of Lattice QCD calculations \(^4\). This is currently interpreted as an indication that the chiral expansion is not converging and that possibly there is a new scale associated with the strong interactions \([18]\). At the least, there is correlation between higher orders in the expansion beyond the expectations from naive dimensional analysis. Walker-Loud \([19]\) performed a comprehensive analysis of all of the Lattice QCD calculations of the nucleon mass and found that the results are consistent with linear dependence on the dimensionless variable \( m_\pi/f_{\pi}^{(0)} \), where \( f_{\pi}^{(0)} \) is the pion decay constant in the chiral limit, with an extrapolation that is consistent with the experimental value. This is, of course, in contrast with the expectations of an expansion about the chiral limit, which has the form \( M_N(m_\pi) = M_0 + \alpha_2 m_\pi^2 + \mathcal{O}(m_\pi^3) \), where \( M_0 \) and \( \alpha_2 \) are parameters that must be determined from the Lattice QCD calculations. The baryon masses obtained with the Clover action, used to calculate the H-dibaryon binding, provide extrapolations to the physical values of the light-quark masses that are more consistent with quadratic-dependence upon the pion mass rather than the linear-dependence found in the global analysis of Walker-Loud. However, the number of different pion masses used in such extrapolations is small and the extrapolation depends somewhat on the scale-setting procedure. Therefore, the conclusion should not be considered definitive. The work of Walker-Loud motivates us to consider an extrapolation of the form \( B_H(m_\pi) = B_0 + c_1 m_\pi + \mathcal{O}(m_\pi^2) \). It is possible that the true form

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\(^3\) Low-energy effective field theories have been constructed to describe baryon systems carrying strangeness; see Refs. [14–17].

\(^4\) SU(2) heavy-baryon chiral perturbation theory does reproduce the lattice results for the nucleon mass (with \( \chi^2/\text{dof} \sim 1 \)), but with large cancelations between different orders in the expansion.
lies somewhere between the linear and the quadratic forms, with complicated cancellations occurring between analytic and non-analytic contributions. Without any better guidance as to the form of the chiral extrapolation, we will consider the results of these two forms of extrapolation with relatively heavy pions to provide nothing more than an estimate of the H-dibaryon binding energy at the physical light-quark masses. The chiral extrapolation of

\[ B_H(m_\pi) = B_0 + d_1 m_\pi^2 \]

was performed using the form

\[ B_H(m_\pi) = B_0 + c_1 m_\pi \]

results in the shaded region shown in fig. 1 (left panel) \(^5\). The H-dibaryon binding energy at the physical value of the pion mass, neglecting isospin-violation and electromagnetic interactions, is found to be

\[ B_H^{\text{quadratic}} = +11.5 \pm 2.8 \pm 6.0 \text{ MeV} \]

as indicated by the intercept of the shaded region with the (green) dashed line in fig. 1 (left panel). The first uncertainty results from an extrapolation using the statistical uncertainties of both Lattice QCD calculations, while the second error results from the systematic uncertainties \(^6\). The quadratic extrapolation suggests that the H-dibaryon is bound at the physical value of the pion mass. However, the H-dibaryon is unbound at the 2\(\sigma\) level, and a near threshold scattering state remains allowed by the current Lattice QCD calculations. Further, at the 2\(\sigma\) level, the extrapolation is also consistent with being independent of \(m_\pi\).

Using the form 

\[ B_H(m_\pi) = B_0 + c_1 m_\pi \]

to chirally extrapolate the results of the NPLQCD and HALQCD Lattice QCD calculations of the H-dibaryon binding energy produces the

\(^5\) As the (fractional) uncertainties in the pion mass are much smaller than those of the H-dibaryon binding energy, they make a negligible contribution to the uncertainty in the extrapolation region and in the extrapolated binding energy in both the quadratic and linear extrapolations.

\(^6\) To be more precise, the second uncertainty results from extrapolating the statistical and systematic uncertainties of the Lattice QCD calculations combined in quadrature, and then removing the contribution from the statistical - also in quadrature.
results shown in fig. 1 (right panel). The H-dibaryon binding energy at the physical value of the pion mass is found to be

\[ B_{H}^{\text{linear}} = +4.9 \pm 4.0 \pm 8.3 \text{ MeV}, \]  

(2)
as indicated by the intercept of the shaded region with the (green) dashed line in fig. 1 (right panel). The linear chiral extrapolation of the binding energy indicates that Lattice QCD calculations are not sufficiently accurate to determine if the H-dibaryon is bound or unbound at the physical pion mass. However, they do indicate that there is a low-lying state in the \( I = 0, J = 0 \) and \( s = -2 \) channel that is either just bound or just unbound.

We conclude that the current Lattice QCD calculations of NPLQCD [8] and HALQCD [9], chirally extrapolated with plausible forms for the light-quark mass dependence, are not sufficiently precise or at light enough quark masses to determine whether QCD predicts a bound H-dibaryon or not. However, the current Lattice QCD calculations hint that the H-dibaryon becomes less bound as the light-quarks become lighter, and the extrapolations suggest that there is a light state in the spectrum.

We view this to be an exciting result that should be viewed in the context of present experimental constraints, which, at face-value, effectively eliminate the possibility of a bound H-dibaryon [10]. However the suggestion of structure in the scattering amplitude near threshold [11] would not be inconsistent with the current Lattice QCD results. In order to refine the Lattice QCD predictions, calculations at lighter quark masses are required. It is likely that a calculation at \( m_\pi \sim 200 \rightarrow 250 \text{ MeV} \) would significantly improve our ability to predict if this state is bound or unbound at the physical light-quark masses. Obviously, a calculation at the physical light-quark masses would be unambiguous, but in the isospin limit without electromagnetism, a small amount of model-dependence would remain in the prediction. Ultimately, these limitations will also be removed. Given that, based upon the current calculations, the H-dibaryon binding energy is likely to be smaller at the lighter pion masses, such Lattice QCD calculations will be computationally more expensive than those considered here. Further, calculations at smaller lattice spacings are required to ensure that lattice artifacts are not driving the observed H-dibaryon binding, and to provide the ability to extrapolate to the continuum.

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