Pattern Generalization Strategy From Concrete Operational Students

Mu'jizatin Fadiana¹, S M Amin², A Lukito², R Yuliastuti¹ and D Apriono¹

¹ Mathematics Education Department, Universitas PGRI Ronggolawe Tuban, Indonesia
² Mathematics Education Department Universitas Negeri Surabaya, Indonesia

Corresponding email: mujizatin000@gmail.com

Abstract. Pattern generalization played an important role in the introduction of early algebra which became a bridge between arithmetic and algebra. However, there are still many students who have difficulty in generalizing patterns. The difficulty of students in generalizing patterns lies in the steps of producing general formulas. Students find it difficult to represent generalizations in the form of variables-n. In addition, students also find it difficult to justify the resulting generalization. This difficulty is experienced by concrete operational students. For this reason, this research aims to study students' concrete operational thinking processes in generalizing patterns. The results of this study provide effective learning strategy solutions for concrete operational students in generalizing patterns. This research is a qualitative research with a case study approach to junior high school students. The results indicated that concrete operational students looked patterns represented in the form of geometric drawings globally without looking the structure of the images. Concrete operational students use two strategies in generalizing patterns, namely the guess and check strategy and the explicit visual strategy.

1. Introduction

The Generalization is an important aspect of mathematics contained in every topic and is something that is highlighted in teaching at almost all levels. Generalization becomes the core of mathematics activities in schools. Furthermore, the generalization of patterns is conceptually related to mathematical structures [1], and working with numerical, geometric and drawing patterns contribute to the development of some skills related to problem solving and algebraic thinking [2]. The importance of pattern generalization has resulted in a shift in the curriculum in Indonesia, generalization of teaching patterns for students at the secondary school level. Therefore, the generalization of patterns will be one of the important supporting topics for learning mathematics in Indonesia.

Pattern generalization is manifold, one of which is the generalization of visual patterns. Generalization is defined as the process of finding commonalities in each instance or case, so that it applies generally [3][4]. Whereas visual patterns are pictorial patterns. Generalization of visual patterns allows students to see the image components dynamically on the conceptual construction of objects, making it easier to give meaning to symbols and expressions [5]. Visualization plays an important role for students to produce common forms that are different from pictorial patterns.
Thus, students will use different strategies to generalize visual patterns, where the way students generalize is due to various ways of visualization.

A study conducted by Becker and Rivera [7] results that junior high school students still fail to generalize linear patterns because they tend to start with numerical strategies. These students focus on numerical data that is trapped in a recursive relationship [8]. In addition, students fail to generalize linear patterns because they use trial and error strategies without understanding the coefficients in linear patterns [7]. Furthermore, in many cases students who generalize linear patterns follow patterns recursively [8][9]. The results of the Fadiana’s research [6] also show that in numerical patterns, students use guessing, explicit and contextual strategies, whereas in generalizing pictorial patterns, students use explicit and contextual strategies. Failure of students in generalizing the pattern caused by the strategy used by students.

The factors that influence students’ failure to generalize linear patterns are worth exploring. Lannin, Barker, and Townsend [10] identify factors that influence the use of pattern generalization strategies, including; (1) social factors, factors that arise from the interaction of students with their peers, or students with teachers; (2) cognitive factors, factors related to students’ mental structures; and (3) factors related to task structure, such as pattern types, values associated with independent variables or even visualization abilities. Cognitive factors are related to students’ cognitive development.

The most important way of thinking, especially for concrete and formal operations stages is logical thinking skills. Logical thinking skills are considered as high cognitive skills, and are functions in Piaget’s developmental scheme which cannot emerge before the concrete operation stage. Roadrangka [11] divides three stages of cognitive development by utilizing the level of logical thinking, namely the concrete operational, transition and abstract operational stages. Thus, the level of think logically gives us information about a person's level of cognitive development.

Recent research only focuses on generalization strategies used by students, such as strategies to find differences, namely generalization strategies that focus on the differences between terms in numerical patterns, strategies for finding patterns, that is, strategies generalization that focuses on the pattern formation of terms from numerical patterns [12], quantity relationship strategies, namely generalization pattern strategies that involve the relationship of input values with output values [13], trial and error strategies [7], linear pattern strategy, that is generalization strategy using linear pattern formula, visual strategy, namely visual grouping strategy, and visual growth strategy [7]. Although students often use recursive strategies, namely strategies to find the next nth term using the previous nth term [7] [8] [10]. Through this recursive strategy, students often experience failure in generalizing patterns, so identification of strategies is needed to determine the layout of students’ mistakes in pattern generalization.

Previous studies have focused in part on strategies used by students in generalization patterns. This means that there is still room to conduct patterns generalization research involving strategies to generalize patterns and levels of cognitive development. This research focuses on concrete operational students. The main reason for choosing the students operational concrete as a subject of study this is because u ntuk complete the task generalization pattern requires the ability to think level higher. In Bloom’s taxonomy is included in the level of C6, which is the ability to combine elements into something new form that is intact and broad. While the characteristics of students whose level of development of cognitive on operational stage is concrete they start thinking about how to think logically and intuitively in the representations of concepts based on reality that makes a concrete or specific example. So, it becomes interesting to study if children in the concrete operational stage are given challenges to solve mathematical problems that require high-level thinking. So the purpose of this study is to explore strategies used by concrete operational students in generalizing patterns. In addition, recommendations from previous studies Lannin, et al [10] to conduct further research on cognitive factors in the selection and use of linear patterns generalization strategies implemented.
This research provides more understanding that cognitive style also influences the selection and use of strategies to generalize linear patterns. The results of this study are also useful for teachers in providing strategies to generalize the linear pattern that is suitable for students with a level of cognitive student development, namely operational concrete, transitional and formal operations. In addition, this also contributes to solving problems that are owned by concrete operational students in learning to generalize linear patterns, especially mathematical concepts that are considered abstract.

2. Method
This research was conducted at a junior high school in Tuban district. This study applies a random sample consisting of fifty of 8th grade students who have learned material about number patterns. A qualitative descriptive approach was used in this study. Fifty students were given a logical thinking ability test (LTAT). From this test, two students have a level of formal operational logical thinking ability, six students are categorized as transitional logical thinking ability level, and the rest, forty two students have a level of concrete operational logical thinking ability. Of the forty-two students, six students were selected to be examined and interviewed about the strategies they used in generalizing patterns. Next, data is analyzed by grouping generalization strategies used by the participants. Interviews are used to validate descriptive results from generalization strategies.

The first stage is determining the research participants, namely students with concrete operational characteristics. To determine students with concrete operational characteristics, researchers used the Logical Thinking Ability Test (LTAT) which was adapted from The Group Assessment of Logical Thinking (GALT). GALT has been used successfully to measure logical thinking skills in previous research. Roadrangka et al [11] developed GALT to measure six of logical operations, namely conservation, correlational reasoning, proportional reasoning, controlling variables, probabilistic reasoning, and combinatorial reasoning. The form of the test uses a multiple choice type that presents the answer and the possible reasons behind the answer. Pictures are inserted for each item to visualize the problem. There are 12 question items to test 6 types of operations. So that each operation consists of 2 item items. As for the types of operations and question themes such as Table 1

| No item problem | Type of Operation                | Theme                        |
|-----------------|---------------------------------|------------------------------|
| 1               | Conservation                     | Clay                         |
|                 |                                 | Metal Bullet                 |
| 2               | Proportional Reasoning           | Glass Size                   |
|                 |                                 | Balance                      |
| 3               | Controller Variable              | Pendulum length              |
|                 |                                 | Ball                         |
| 4               | Probabilistic Reasoning          | Square and Rhombus Part 1    |
|                 |                                 | Square and Rhombus Part 2    |
| 5               | Correlational Reasoning          | Mouse                        |
|                 |                                 | Fish                         |
| 6               | Combinatorical Reasoning         | Dance                        |
|                 |                                 | Shopping center              |

Answers for item LTAT number 1-10 are considered correct if the answers and reasons are correct. For item number 11, students are asked to write down all the answer patterns and be judged correct if the error is not more than 1. And for item number 12, students are also asked to write down all the answer patterns and be judged correct if the error is not more than 2. Each item the correct one is given a score of 1, so the maximum score is 12 and the minimum score is 0. The next score is
classified as follows; scores 0-4 are grouped in the concrete operational stage, scores 5-7 are grouped in the transitional stage, and 8-12 are formal operational stages [11].

The second stage is analyzing the generalization strategy used by participants in completing the second instrument, namely the pattern generalization task (PGT). The PGT instrument is a test that requires students to find the n-th term formula of a number pattern that has a constant first difference. The second instrument was used to determine the generalized linear pattern strategy used by the participants. This instrument consists of three questions with different representations. The first question uses a representation of the sequence of numbers. This question is used to find out how students who depend on the field find input values to generalize linear patterns. The second and third question are in the form of pictorial patterns. This problem can be used to determine patterns of student generalization strategies, whether students use visual strategies or numerical strategies, or change images into sequences of numbers.

Furthermore, the analysis of the generalization process in this study uses a generalization taxonomy consisting of relationships, search, and expansion [12]. Actions related to how students form the relationship between the situation in question with other situations in the search action are analyzed. Then, whether students take repeated actions, find common ground, or find patterns is also analyzed. Meanwhile, expanding activities are related to how students expand similarities or patterns that are more generally analyzed later [12]. The third step is analyzing the results of how to generalize the pattern that students write the n-th term formula [12]. Then, from the results of the analysis, a descriptive process is obtained from concrete operational students in generalizing patterns starting from paying attention to questions globally. Then, they generalize the pattern and produce a n-th term generalization formula.

3. Result and Finding

The six subjects studied were symbolized by S1, S2, S3, S4, S5 and S6. Based on the results of working on the task of generalizing patterns and interviews obtained an overview of the strategies used by each subject as follows:

Subjects S2 and S3 try to write a general formula without regard to the relationship between the term and the value of the term and check the accuracy of the formula he wrote by comparing numbers with values in one step. As a result, they are unable to find a formula. Although they are not able to state algebraically, they employ a guessing and checking strategy:

**Subject 2**: Here 1, 5, 9, 13, .... then increase by four! Emmm [he did the question but was unable to find the formula] ... [he kept thinking] 2n + 3 ... [he thought hard and tried to understand by replacing n with numbers].

2n + 3, two times one is two, two plus three is five. This is proven.

**Researcher**: What do you get by returning to the first term when you write one instead of n?

**Subject 2**: One ... ummm. 4n + 1, four times one four plus one is five. This yields five.

**Researcher**: But you said earlier that you have to find one.

**Subject 2**: [he tries to get the formula but couldn't find it].

**Subject 3**: [he tries to solve questions and think for a while]. Unfortunately [he wrote 2n + (n-1) on paper]

**Researcher**: What are you doing here?

**Subject 3**: I think based on five. Twice two is four, two minus one is one, if we add one to four, we get five. However, when I make the same thing for nine, three times two is six, three minus one is two, six plus two is eight, not nine. And apparently not suitable.
Subjects S1 and S4 estimate the formula approach by looking at differences between terms and then checking the accuracy of the first few terms. So they get the general formula from the point by using a guessing and checking strategy.

Subject 1 : I tried n + 4 but can't.
Researcher : Ok if we explain the relationship, what is the relationship in the pattern?
Subject 1 : This continues to grow four four.
Researcher : If we want to express using parameters, what can we write?
Subject 1 : I guess I found ... 4n-3. [Thinking for a while and writing something] for example when we check with one, four times one four, minus three is one, when we check with two four times two eight, minus three five. So, this is proven.
Subject 4 : n + 4 ... one plus four five ... is this right ?!
Researcher : Is that the first tribe?
Subject 4 : Yes, that is correct.
Researcher : For n = 1, is that the first term of the pattern?
Subject 4 : Hmm. Ok, one. [He thought for a while]. 4n-3, four times one four minus three is one. For the first term, once four four, four minus three is one. Here is one [he shows the first syllable in question]. For the second term, two times four eight, eight minus three five. Proven right?

Subjects S5 and S6 are aware of inter-tribal relations and using this relationship he expresses the general formula of the pattern. Moreover, his thought process shows that he has conceptual knowledge about the formula obtained. Here the subject uses explicit visual strategies when making generalizations from patterns.

Subject 5 : Here, I first tried to find a tribe in general. Each tribe increases by four four. So I wrote 4n. When we write n = 1, four times one is four then we subtract three and the first term is one. So the general rule is 4n-3
Researcher : Is 4n-3 for general terms?
Subject 5 : Yes
Researcher : Can you explain the thought process you used?
Subject : Patterns increase to four-four so I write n near four. Then if it is 4n, to get one, that is, the first term we have to subtract three from 4n. Therefore the general term is 4n-3.

3.1 Continue the pattern in the near step and the far step

Some subjects focus on differences between tribes and find the next term by adding differences to the previous term. What is meant by close steps are up to 5 terms or 6 terms and subjects prefer to use an additive strategy (add) to find the results.

Subject 6 : There are five, eight and eleven circles in the first three steps. From this I understand that the pattern continues to increase by three. I can find the number of circles in step five by adding three.
Subject 5 : I pay attention to the number of triangles. There is one triangle in the first step, two triangles in the second step and three triangles in the third step. Because there are three matchsticks in each triangle, the number of matchsticks in the first step is three, the second step is six, the third step is nine and the pattern of walking increases by three. Therefore it continues in the form 12, 15, 18, 21.
From this, it appears that in the fifth step there are fifteen bars and the sixth step is eighteen.

Whereas up to 13 terms or 20 terms can be considered as near or far steps depending on the structure of the pattern. In terms of the questions we ask in this study both steps can be calculated through a basic strategy such as additives or discovery strategies such as explicit. An example interview shows that some subjects first find the general formula and then by using the general formula look for the value in question. So the subject uses an explicit strategy.

Researcher : What is the number in step thirteenth?
Subject 3 : I replaced n with 13 in the formula, four times thirteen minus three is 49.
Researcher : What is the number of matches in the 20th lag?
Subject 3 : Sixty.
Researcher : How do you know?
Subject 3 : I multiply twenty by three because triangles have three sides.
Researcher : Are there different ways?
Subject 3 : We can find it by drawing, but if it is asked to reach step 80, then I will not be able to draw it. So it’s easier to use the formula.

The results of this study indicate concrete operational students use the strategy of guessing and also checking explicit strategies. According to the research of Akkan and Cakiroglu [14], grade 7 and 8 students use explicit strategies in generalizing linear patterns. The results of this study support the results of our study of the strategies students use in generalizing patterns.

Some subjects focus on obtaining general formulas and finding value in the step or term in question. In addition, there are also subjects who use the strategy of guessing and checking. This is because the subject cannot see the relationship between tribes and is unsure of the accuracy of the formula they have found. One subject prefers to use an additive strategy because he is unable to find the general formula of the pattern. In this study subjects usually try to get the general formula at first because they believe it can answer the questions using the general formula more easily.

In a pattern of four numbers the subject prefers to use a guess and check strategy while the two subjects use an explicit visual strategy. Subject which uses a guess and check strategy can not find a general rule patterns or make generalizations while some of them are enough to find a common formula or express properly inter-ethnic relations in the form of algebra. This shows that there are some concrete operational students lacking in their knowledge structure because they immediately guess the general formula, without the need to think about the accuracy of the formula.

In visual patterns four subjects use explicit strategies while two subjects use counting strategies. In this type of visual pattern subjects generally have a tendency to change visual patterns into rows of numbers [15][16] and work with numbers so that they adopt an approach numeric by finding numerical equivalence of visual image forms in each step [15][16].

It is true that there are not too many differences between the generalization strategies used in rows of numbers and visual patterns because they resolve visual patterns by transforming into sequential numbers. According to research from Akkan and Cakiroglu[14], students are more successful in solving patterns in the form of rows of numbers rather than visual patterns. However, in our study this was not the case. Our results show that most subjects can recognize and understand relationships between tribes and then find a general formula for patterns. However, some of them are not able to make generalizations. Many researchers have shown that students are more successful in problems of patterns that they are familiar with [17]. Therefore concrete operational students are more familiar with linear patterns and the types of linear patterns according to their cognitive level. So that giving a type of linear pattern really allows concrete operational students to make generalizations.

The results of this study also showed that the subject chose an additive strategy (addition) or an explicit strategy to find a step near the step. If students can find general formulas from patterns then...
they apply strategies according to their choices but if they cannot find general formulas, then they tend to use additive strategies to find steps that are close by. On the other hand, to find a far step do not use additive strategies (additions) because they know it is difficult for them. Therefore they have the opportunity to find steps close by adding differences between the terms in the previous term by counting numbers in sequence. The results of this study are in line with findings from the research of Akkan and Cakiroglu [14]. In general, students are inclined to use additive strategies to get near steps while they prefer to look for general formulas and use explicit strategies to get distant steps.

Some subjects are not successful in making correct generalizations while some of them tend to use memorization rules and have very little conceptual understanding. So the teacher must use various types of patterns and strategies to enrich student knowledge and also focus on the ability of students to understand the relationships between tribes in order to develop students’ algebraic thinking.

4. Conclusion
The results showed that in completing the task generalization in the form of number patterns students tend concrete operational using guessing strategy then check and explicit strategy. Whereas in generalizing visual patterns four students use the counting strategy while two students use an explicit strategy. In continuing close patterns, subjects tend to use additive strategies and they prefer to look for general formulas and use explicit strategies to get a step further.

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