In this paper we investigate the possibility of measuring the post-Newtonian general relativistic gravitomagnetic Lense-Thirring effect in the Jovian system of its Galilean satellites Io, Europa, Ganymede and Callisto in view of recent developments in processing and modelling their optical observations spanning a large time interval (125 years). The present day best observations have an accuracy between several kilometers to few tens of kilometers, which is just the order of magnitude of the Lense-Thirring shifts of the orbits of the Galilean satellites over almost a century. From a comparison between analytical development and numerical integration it turns out that, unfortunately, most of the secular component of the gravitomagnetic signature is removed in the process of fitting the initial conditions. Indeed, an estimation of the magnitude of the Lense-Thirring effect in the ephemerides residuals is given; the resulting residuals have a maximum magnitude of 20 meters only (over 125 years).

Keywords: Lense-Thirring effect; Jupiter; Galilean satellites.

1. Introduction

1.1. The post-Newtonian gravitomagnetic Lense-Thirring effect

One of the few predictions of the Einstein’s General Theory of Relativity (GTR) which is still awaiting for a direct and reliable observational check is the so called Lense-Thirring (LT) effect. It is a consequence of the off-diagonal components $g_{0i}$, $i = 1, 2, 3$ of the space-time metric generated by a weakly gravitating and slowly rotating massive body of mass $M$ and proper angular momentum $S$ in the linearized weak-field and slow-motion approximation of GTR. The mass-energy currents induce, among other things, a Lorentz-like, velocity-dependent force which acts on the orbital motion of a test particle freely moving in the gravitational field of the
central body. Such force is

\[ F = -2m \left( \frac{\mathbf{v}}{c} \right) \times \mathbf{B}_g, \]

where \( \mathbf{v} \) is the particle’s velocity and

\[ \mathbf{B}_g = \frac{G[3r(\mathbf{r} \cdot \mathbf{S}) - r^2 \mathbf{S}]}{cr^5}. \]

\( G \) is the Newtonian gravitational constant and \( c \) is the speed of light in vacuum. Due to the formal resemblance of \( \mathbf{B}_g \) with the dipolar magnetic field of the Maxwellian electromagnetism, the ensemble of gravitational phenomena induced by the mass currents is named gravitomagnetism\(^4\). It turns out that the non-central gravitomagnetic force of Eq. (1) makes the longitude of the ascending node \( \Omega \) and the argument of pericentre \( \omega \) to undergo tiny secular precessions\(^1,4–6\)

\[ \frac{d\Omega_{LT}}{dt} = \frac{2GS}{c^2a^3(1 - e^2)^{3/2}}, \quad \frac{d\omega_{LT}}{dt} = -\frac{6GS \cos i}{c^2a^3(1 - e^2)^{3/2}}, \]

where \( a, e, i \) are the semimajor axis, the eccentricity and the inclination yto the equator of the orbit of the moving particle, respectively.

The gravitomagnetic phenomena may have strong consequences in many astrophysical and astronomical scenarios involving, e.g., accreting disks around black holes\(^7\), gravitational lensing and time delay\(^8,9\). Unfortunately, in these contexts the knowledge of the various competing effects is rather poor and makes very difficult to reliably extract the genuine gravitomagnetic signal from the noisy background. E.g., attempts to measure the LT effect around black holes are often confounded by the complexities of the dynamics of the hot gas in their accretion disks. Conversely, if one believes in GTR, the LT effect could be used, in principle, for measuring the proper angular momentum of the central mass which acts as sources of the gravitomagnetic field.

It must be noted that, according to K. Nordtvedt, gravitomagnetism would have already been indirectly tested from the radial motion of the Earth’s geodetic satellite LAGEOS\(^10\) and from the high-precision reconstruction of the lunar orbit with the Lunar Laser Ranging (LLR) technique\(^11\).

An attempt to directly measure the LT effect in the gravitational field of the Earth has been performed by Ciufolini and Pavlis with the LAGEOS and LAGEOS II satellites\(^12\) by using an observable proposed by one of us in Ref. 13. The claimed total accuracy is 5-10% at 1-3 sigma, but such estimates have been criticized by one of us in Ref. 14 who proposes a 19-24% total error at 1-sigma. The main limiting factors are the systematic errors of gravitational origin due to the static and secularly varying parts of the even zonal coefficients \( J_\ell \) of the multipolar expansion of the terrestrial gravitational potential.

The first, very preliminary, evidence of the gravitomagnetic field of the Sun through the LT precessions of the longitudes of the perihelia \( \varpi \) of the inner planets of the Solar System has recently been reported by one if us in Ref. 15.
In April 2004 the GP-B spacecraft has been launched. Its aim is the measurement of another gravitomagnetic effect, i.e. the precession of the spins of four superconducting gyroscopes carried onboard with an expected accuracy of 1% or better.

1.2. Aim of the paper

In this paper we want to investigate the possibility of measuring the LT effect on the orbits of the Galilean satellites Io (1), Europa (2), Ganymede (3) and Callisto (4) evolving in the gravitational field of Jupiter in view of recent improvements in their ephemerides. The orbital parameters of the Galilean satellites of Jupiter are presented in Table 1. It is noteworthy that the idea of using the Jovian system was put forth for the first time by Lense and Thirring themselves in their original paper.

| Satellite | a (km) | e   | i (deg) |
|-----------|-------|-----|---------|
| Io        | 422,030 | 0.0042 | 0.036 |
| Europa    | 671,261 | 0.0094 | 0.469 |
| Ganymede  | 1,070,621 | 0.0015 | 0.175 |
| Callisto  | 1,883,134 | 0.0075 | 0.187 |

was put forth for the first time by Lense and Thirring themselves in their original paper.

2. The R-T-N scheme for the Lense-Thirring effect

Since the Galilean satellites of Jupiter move along nearly circular and equatorial orbits and to make easier the comparison with the current ephemerides we will consider the radial, transverse and normal components of their orbits. Their perturbations can be expressed in terms of the integrated shifts of the Keplerian orbital elements $\Delta a, \Delta e, \Delta i, \Delta \Omega, \Delta \omega, \Delta M$, where $M$ is the mean anomaly, as

$$\Delta R = \sqrt{(\Delta a)^2 + \left[(e\Delta a + a\Delta e)^2 + (ae\Delta M)^2\right]/2},$$

$$\Delta T = a\sqrt{1 + e^2/2 \left[\Delta M + \Delta \omega + \cos i\Delta \Omega + \sqrt{(\Delta e)^2 + (e\Delta M)^2}\right]},$$

$$\Delta N = a\sqrt{1 + e^2/2 \left[(\Delta i)^2/2 + (\sin i\Delta \Omega)^2\right]}.$$

From Eq. (3) it turns out that the gravitomagnetic force only affects, in general, both the transverse and the normal components: $\Delta R_{LT} = 0$. For polar orbital
configurations, i.e. \( i = 90 \text{ deg} \), \( \Delta T_{LT} = 0 \), while for equatorial orbits, i.e. \( i = 0 \text{ deg} \), \( \Delta N_{LT} = 0 \). For nearly circular and equatorial orbits it turns out that \( \Delta T \sim a \Delta \lambda \), where \( \lambda = M + \Omega + \omega \) is the mean longitude.

3. The Lense-Thirring effect in the Jovian system

The four Galilean satellites\(^a\) of Jupiter are currently the best candidates (among satellites evolving around giant planets) in regard to their orbital motion’s accuracy. Indeed, ephemerides of the Galilean satellites reach an accuracy of only few tens of kilometers for the modern period\(^b\), and benefit of observations dispatched over more than a century (from 1891 to 2003 in the case of L1 ephemerides\(^b\)). These observations have various origin. While most of them are photographic ones, the observation of mutual events (the eclipse or occultation of one satellite by another) delivers the most accurate Earth-based observations of the Galilean system. Indeed, such observations are photometric instead of astrometric, and so are less sensitive to atmosphere turbulence. Hence, the accuracy of these latter can reach few tens of kilometers.

More, JPL ephemerides accuracy amount to 5 km for the present period by the use of unpublished spacecraft observations (see Ref. 21 and http://ssd.jpl.nasa.gov/sat_eph.html).

3.1. Theoretical predictions: analytical and numerical estimates

In regard to the LT effect, for the Jupiter’s angular momentum we assume \( S = 4.33 \times 10^{38} \text{ kg m}^2 \text{ s}^{-1} \) since the ratio\(^c\) \( \alpha \) of the moment of inertia \( I \) to \( MR^2 \), where \( R \) is the mean equatorial radius, amounts to\(^d\) 0.264 (see also http://nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html). Such estimates are based on theoretical models: at present, there are no direct, independent measurements of \( S \) for Jupiter. It should also be noted that Lense and Thirring in Ref. 1 and Soffel in Ref. 4 overestimated the gravitomagnetic precessions of the Galilean satellites because they modeled Jupiter as a uniform sphere by assuming \( \alpha = 0.4 \).

In order to compute the secular LT shifts \( \Delta T_{LT} \) on the Jovian Galilean moons it is important to consider that such a system is very complex because of the strong mutual perturbations of one satellite on another. In particular, the Laplacian resonance\(^e\)

\[
N_1 - 3N_2 + 2N_3 = 0
\]  

(7)

establishes a relation among the Keplerian mean mean motions\(^d\) \( N \) of Io, Europa and Ganymede. This has consequences also on the LT effect. Indeed, the gravitomagnetic

\(^a\)For the physical parameters of Jupiter and its moons see http://ssd.jpl.nasa.gov/sat_gravity.html
\(^b\)These ephemerides are available at http://www.imcce.fr.
\(^c\)It amounts to 2/3 \( \sim 0.6 \) for a hollow spherical shell and to \( 2/5 = 0.4 \) for a homogenous sphere.
\(^d\)By definition, the mean mean motion is the secular component in the mean longitude expression.
force of Eq. (1) is a small correction with respect to the Newtonian monopole and, as usual in perturbation theory, it must be evaluated onto the unperturbed Keplerian path for which the velocity entering Eq. (1) is approximately \( v \sim aN \). Thus, the gravitomagnetic components of the equations of motion for Io, Europa and Ganymede are not independent but they are coupled due to Eq. (7). More, secular variations of nodes and pericentres will affect the mean mean motions. To preserve the Laplacian relation, variations on mean anomalies are also expected. Hence, longitude evolutions of Io, Europa and Ganymede have to be studied. This means that differences with respect to the case of independent motions, illustrated in Table 2, are expected. This feature has been investigated numerically in the following way.

| Table 2. Secular LT shifts, in km, over 125 years calculated with Eq. (3) and Eq. (5) by neglecting the mutual perturbations of one satellite on another. For the Jupiter’s angular momentum the value \( S = 4.33 \times 10^{38} \text{ kg m}^2 \text{ s}^{-1} \) has been adopted. |
|-----------------|----------------|----------------|----------------|
| \( \Delta T_{LT} \) (km) | Io | Europa | Ganymede | Callisto |
|-----------------|----------------|----------------|----------------|
| -28             | -11            | -4             | -1            |

3.2. Numerical simulations

In order to numerically investigate the effect of the Jovian gravitomagnetic field on the orbital motion of the four Galilean satellites, we have performed several numerical integrations.

The adopted software is called NOE (Numerical Orbital Elaboration) and is inspired from a former work presented in Ref. 18. It was developed at the Royal Observatory of Belgium mainly for natural satellites ephemerides purpose. It is a N-body code which incorporates highly sensitive modeling and can generate partial derivatives. The latter ones are needed to fit the initial positions, velocities and other parameters to the observation data. We used the exact modeling, initial conditions and parameter values that were used during L1 ephemerides elaboration. Let us recall that this model introduces the Jovian gravity field by mean of \( J_2, J_4, J_6 \) coefficients, the satellites mutual perturbations (including their respective oblateness coefficients \( J_2 \) and \( c_{22} \)) and the Solar perturbation using DE406 ephemerides\(^e\). The selection of these perturbations was done after a careful study of the magnitude expected from a large set of usually neglected perturbations (planetary and satellites precessions, Jovian \( J_3 \) coefficient ...), but did not considered LT effect at that time. A perturbation was added in the model only when found significant from the observations.

\(^e\)In particular, the indirect planetary perturbations are implicitly introduced.

Hence, it already contains the node and pericenter secular variations. So mean mean motion is different from the averaged mean motion (that contains only the mean anomaly). See Ref. 25.
accuracy. In particular, secular variations inducing less than hundreds of kilometers after one century on the satellite longitudes have generally been considered negligible. Indeed, these latter can be easily absorbed by tiny changes on initial satellite positions and velocities. On the other hand, small perturbations may be retained if damping enough known frequencies in the system or adding new ones.

LT effect has been introduced by means of Eq. (1) and Eq. (2). For an explicit formulation of all the equations used, we refer to Ref. 18. Initial conditions and parameter values are available in Ref. 19. In particular, L1 ephemerides result from the fit of a high sensitive model to observations covering a time span from 1891 to 2003.

The integrator subroutine is the one of Everhart, called RA15. It was chosen for its speed and accuracy. During the integration a constant step size of $\Delta t = 0.08$ day was used. To increase the numerical accuracy during the fit procedure (see subsection 3.3) we performed integration over $\pm 62.5$ years instead of $\pm 125$ years. The numerical accuracy of our simulations is at the level of several meters.

In Fig. (1) we plot the differences on satellite distances between a numerical simulation including LT effect and a second simulation (using the same set of initial conditions) but neglecting LT effect. The adopted time span is 125 years and covers roughly the modern observation period. In a first step we followed our former analytical formulation and so neglected the mutual perturbations (by nullifying the satellite masses). In particular, the Laplacian resonance was not present in the system at this step. Results are shown in the left panel of Fig. (1).

As expected, the obtained secular shifts agree with the analytical calculations of Table 2.

In a second step, we reintroduced the satellite masses values (and, thus, the Laplacian resonance). Results are shown in the right panel of Fig. (1). As it can

![Fig. 1](image_url)

Fig. 1. Differences in distance for the four Galilean satellites between a simulation neglecting the LT effect and a simulation including the LT effect. The case without the Laplacian resonance is shown on the left, while it has been included on the right. The horizontal axes are in years and the vertical axes are in km. In both cases the initial conditions and parameter values have been taken from Ref. 19.
be seen, the shifts remain unchanged for Callisto at the level of 1.4 km. Rather small change appear for Ganymede with a slight decrease from 4.4 km to 4.2 km; Io and Europa experience the most important changes passing from 28.6 km to 27 km and from 11.3 km to 16.3 km, respectively. These differences with our numerical estimations are induced by the Laplacian resonance as explained in Section 3.1.

3.3. Measurability of the Lense-Thirring effects with the current ephemerides

Galilean satellite observations are rather different from those for the artificial satellites, in the sense that only the right ascension and declination for each natural satellite are available. Neither estimation of the Jupiter-satellite distances nor of satellite velocities are possible. Hence, reconstruction of observed Keplerian elements is not possible. Moreover, the observations have a quite smaller coverage by satellite revolutions and are not dispatched equally over the years. This is why the method used to detect LT effects with LAGEOS satellites cannot be applied here. However, and to estimate the possible detection of LT effect among Galilean system in a realistic way, we decided to apply a fit procedure of our numerical simulation neglecting LT effect on the simulation including LT effect (acting like some real observations). The induced residuals can then be anticipated as part of the real observed residuals. If these former are found significant (at least one order of magnitude less), then one can deduce that the effect is detectable.

This method was already applied successfully to estimate tidal effects among the Galilean system in Ref. 27.

We performed the fit procedure following ephemerides elaboration. As for L1 ephemerides, we fitted only the initial positions and velocities of each satellite. A sample of 628 daytimes with a 72.8 days time step was used. Differences on Cartesian positions for all satellites have been fitted, with no weights assigned.

![Graph of differences in distance for the four Galilean satellites between a simulation neglecting the LT effect and a simulation including the LT effect. The initial conditions have been fitted. The horizontal axes are in years and the vertical axes are in km.](image-url)
In Fig. (2) we show the residuals induced by LT effect after fit and which account in ephemerides residuals. It can be noted that a large part of the secular LT signature has been removed from the differential time series over 125 years to a level of several meters (the level of accuracy of our simulations), while an interesting, long period pattern appears for Europa with an amplitude of 20 meters. Most of this variation appear on the inclination and the mean longitude (graphs not shown here).

In Table 3 the corrections delivered by our fit procedure and applied to the initial positions and velocities are given. Table 4 presents the same corrections converted into Keplerian elements. In the case of Callisto, significant changes appear only on $\Omega$ and $\omega$ with opposite numerical values. That is consistent with our analytical approach. For the three other satellites, significant variations appear both on $\Omega$ and $\omega$ and $M$, which is a consequence of the Laplacian resonance.

4. Conclusions

In view of the latest developments in our knowledge of the dynamics of the Jovian system of Galilean satellites, in this paper we investigated the influence of the gravitomagnetic field of Jupiter on their motion and how difficult a detection of the Lense-Thirring effect would be still now. We pointed out the presence of secular drifts on mean longitudes with a highest amplitude of few tens of kilometers.
However, most of this effect will vanish during the fit procedures that are used in ephemerides elaboration. Resulting residuals have a maximum magnitude of 20 meters (over 125 years). This appears negligible with today’s best observations which have an accuracy of a few tens of kilometers. A spacecraft orbiting Jupiter or the adjunction of new more accurate observations of the Galilean satellites, is definitely required in order to reveal the Lense-Thirring effect in the Jovian system.

Acknowledgements

V.L. benefited from the support of the European Community’s Improving Human Potential Programme under contract RTN2-2001-00414, MAGE.

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