A Possible Method of Using Advanced Hafele-Keating Experiment to Probe Quantum Gravitational Perturbations

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Abstract

Quantum gravity is the new direction of gravity research, scientists have put forward some theoretical models, but it is difficult to verify them by experiments. Here is a new possible method of verifying quantum gravitational perturbations with flying atomic clocks experiment, due to the limitation of accuracy of current optical clocks, the second stage of the experiment is still not easy to achieve at present. However, the method combines the principle of relativity, and has certain academic value for studying the way to connect general relativity and quantum gravity.

Keywords

Quantum gravitational perturbations, Kinematic time dilation effect, Space-time curvature around the earth, Advanced Hafele-Keating experiment, Inner core's differential rotation

Introduction

A few years ago, by analyzing seismic waves, some scientists have preliminarily confirmed that the rotation rate of the earth's inner core is slightly faster than the other layers of the earth \(^{[1-3]}\). In case the gravity is quantized \(^{[4,5]}\), then the differential rotation between the earth's inner core and the mantle should lead to some very weak quantum gravitational perturbations in the directions of east-west. Relative to the kinematic time dilation effect in flat space-time, in curved space-time around the earth \(^{[6]}\), the kinematic time dilation effect may have a small drift corresponding to the space-time curvature, based on the same reason, in east-west directions the quantum gravitational perturbations
will extremely weakly affect the kinematic time dilation effect. This may be recorded by some ultra-high precision flying atomic clocks in advanced Hafele-Keating experiment, after a long period, it could be accumulated in to a measurable extra time deviation. In case the extra time deviation successfully extracted from actual experimental results, it could be considered as an indirect measurement for quantum gravitational perturbations, the quantum gravitational perturbations may show that gravity may has a quantized characteristic.

Based on the theory of quantum gravity, the gravitational field of the earth is composed of the gravitational quanta originate from numerous particles, when the earth's inner core and the mantle rotating not in sync, then their gravitational quanta cannot be completely in synchronized, this may lead to some very weak gravitational perturbations in the directions of east-west. The perturbations are almost imperceptible relative to the static observers on the earth; but it is perceptible for the eastward (westward) fast moving observer. Although the perturbations are too weak to be detected directly, however, they still could be detected indirectly. Hafele and Keating proved kinematic time dilation effect by flying cesium clocks experiment, this experiment was designed to verify the effect in flat space-time, and it was completed in 1971. The experiment was performed in earth's gravitational field, in case the curvature of the space-time influenced the effect slightly, and then the experimental result should less than the prediction result. However, due to the limitation of the accuracy of the early cesium clocks and the experimental process, the experimental accuracy did not meet the standards of detecting the differences of the kinematic time dilation between in curved space-time and in flat space-time. As long as it can be proved that curvature of space-time can drift the kinematic time dilation effect by advanced Hafele-Keating experiment, it may be able to detect the quantum gravitational perturbations originate from the differential rotation of the earth's inner core indirectly. It would take two stages to achieve this goal, the first stage to prove the influence
of curved space-time on kinematic time dilation effect, and the second stage to perform the ultra-high precision indirectly detection.

Methods

The first stage: to prove the influence of curved space-time on kinematic time dilation effect

The kinematic time dilation effect of special relativity has been confirmed by Hafele-Keating experiment in 1971 and some related experiments. In these experiments, the speed of flying clocks was much less than the light velocity. For low coordinate speeds \( v^2 \ll c^2 \), the ratio of time recorded by the moving and reference coordinate clocks reduces to \( 1 - v^2/2c^2 \). If \( \tau \) and \( \tau_0 \) are the flying clocks and reference clocks, the respective time difference between them is approximation given by:

\[
\tau - \tau_0 = -\frac{v^2}{2c^2} \cdot \tau_0
\]  

(1)\[7, 8\]

All these experiments were carried out in earth's gravitational field (curved space-time), due to the influence of space-time curvature, there may be a small drift between the measured results and the theoretical predictions of the special relativity. This drift is corresponding to the influence of space-time curvature, it can be expressed with the aid of a coefficient \( k \), \( k \) is a variable whose value is determined by the strength of the surrounding gravitational field. Bring \( k \) into the kinematic time dilation equation which used in Hafele-Keating experiment, we will get an equation \( E \):

\[
\tau - \tau_0 = -k \cdot \frac{v^2}{2c^2} \cdot \tau_0
\]

(2)

When \( k=1 \), the original equation and its calculation results remain unchanged, so the flat space-time can be considered as the case of \( k=1 \). In gravitational field, under the influence of space-time curvature, value of \( k \) may be drifted and less than 1. This can be proved by comparing the measured results and the theoretical prediction of the Hafele-Keating experiment in 1971, after eliminate
the influence of the interference factors, in case the two results exactly equal to each other, it shows that the space-time curvature around the earth has no influence on the kinematic time dilation effect; in case the measured results less than the theoretical prediction results, it may shows that the space-time curvature has affected the kinematic time dilation effect, the coefficient $k$ can be obtain by dividing the measured results by the theoretical prediction. However, due to the limitation of the accuracy of the early cesium clocks and the experimental process, the highest accuracy of the experiment in 1971 only reached nanoseconds level, it has not met the accuracy requirement of $0.001$ nanoseconds level and could not be the valid proof for coefficient $k$, so it needs to be proved by an experiment with much higher precision. Due to the development of high-precision atomic (optical) clocks technology over the past 50 years and the improvement of the overall technological level, the accuracy of the advanced experiment can be improved at least thousands of times higher than the original experiment, on this basis, it is possible to measure the coefficient $k$ with an advanced high precision Hafele-Keating experiment. In order to make the experimental accuracy meet the requirements of the measurement of value of $k$, the latest high-precision atomic (optical) clocks are needed, and the flying clocks and reference clocks should keep the same distance from the earth's center of mass for equal time, to ensure that the influence of the gravitational time dilation effect of general relativity on these clocks is equal. When the flying clocks and reference clocks at the same latitude, the reference clock just needs to be kept at the same altitude as the flying clock, or else the altitude of the reference clocks should be calculated and adjusted to ensure these atomic clocks at the same distance from the earth’s center of mass. Thus, the reference clocks better to be placed in an airship (or high-altitude hot air balloon) which at relative stable position to adjust the height conveniently. In this way, the original time data recorded by flying clocks and reference clocks can be directly used for comparison without theoretical correction, so that the primitiveness and objectivity of the experimental data can be preserved as much as possible, it is very important
for a physical experiment. In case the flying optical clocks were carried in a special aircraft to fly orbit the earth, then it can choose a better flight route, without multiple transfers and frequent take-off and landing, the accuracy of the experiment will be greatly improved.

**The second stage:** The way to detect the quantum gravitational perturbations generated by differential rotation of the earth's inner core

In case the space-time curvature could affect kinematic time dilation effect, when this has been successfully proved in the first stage, then the work in second stage will have theoretical and practical significances. The gravitational time dilation effect of general relativity which is the biggest interfering factors in this experiment, will also directly affect the time of the clocks, so calculate its exact specific value and remove it from the experimental data is necessary [7, 8]. Ideally, after eliminated gravitational time dilation effect of general relativity and other influence of interference factors, under the earth's space-time curvature, with the aid of coefficient $k$, the time respective record by the flying clocks and the reference clocks is given by equation $E$. Suppose $m_i=$*mass of the earth's inner core*, $m=$*total mass of the earth*, they share the same mass center. When the inner core rotates synchronously with other layers of the earth, the equation can be written as $E_1$:

$$\tau - \tau_0 = -\left(\frac{m-m_i}{m} k \cdot \frac{v^2}{2c^2} + \frac{m_i}{m} k \cdot \frac{v^2}{2c^2}\right)\tau_0$$

(3)

When the earth's inner core rotates slightly faster than other parts of the earth at an angular velocity $\omega_1$, for the flying clocks, this angular velocity corresponding to a linear velocity $v_1$, for the geocentric is the frame of reference, the relative speed of eastward flying clocks should minus $v_1$, this will very slightly reduce the total kinematic time dilation effect on the eastward flying clocks. After a long enough period, it should be recorded by ultra-high accuracy atomic clocks. When the earth's inner core rotates slightly faster, then the equation can be written as $E_2$:
\[
\tau - \tau_o = -\left(\frac{m-m_i}{m}k \cdot \frac{v^2}{2c^2} + \frac{m_i}{m}k \cdot \frac{(v-v_i)^2}{2c^2}\right)\tau_o
\]
\[
v_i = \frac{2\pi r}{t}
\]

(4)

In fact, the equation \(E_2\) should be true both in general relativity and quantum gravity, but \(r\) has different values in the two theories. When \(E_2\) based on general relativity, the earth's inner core influences the clocks indirectly through the space-time curvature around it, so \(r\) cannot be greater than radius of the earth's inner core; when \(E_2\) based on of quantum gravity, clocks can receive the gravitational quanta originate from particles of the earth's inner core directly, so \(r \approx \) the distance between flying clocks and geocentric. In gravitational field, when the earth's inner core rotates faster than the mantle at low angular speed, its influence on kinematic time dilation effect will be very weak, and the predicted results calculated by general relativity will less than the predicted results calculated by quantum gravity, but their values are extremely close.

Discussion

With the intensive study of theories, the accuracy requirement of the verification experiments has become increasingly demanding. The influence of the earth's inner core's differential rotation on kinematic time dilation effect is so weak, that the time differences between predicted results of \(E_2\) and \(E_1\) may be one million times smaller than the result of the original experiment, it may require the experimental precision increase millions of times than the experiment in 1971. According to the latest study on the differential rotation rate of the earth's inner core, the suggested average rotation rate is about 0.39° per year faster than the mantle (between about 0.24° to 0.56° per year)\[^9\], it equivalents to about average of 923 years the inner core rotates one more lap than the mantle. The average value will be used in the follow calculation, this may cause a corresponding error which less than 0.5 times of the final calculation result. On this basis, the following is an ideal experiment as example. In order to make the explanation process easier, various parameters
have been simplified, and the influence of interference factors has not been considered, such as the gravitational time dilation effect of general relativity (For the calculation method of the gravitational time dilation effect of general relativity, please refer to Cited Papers 7,8). Assume \( k \approx 0.997 \) (the specific value of \( k \) should be determined by first stage), radius of inner core \( \approx 1216000m \), \( m_i/m \approx 0.017^{[10]} \), there is a clock \( c_i \) at rest on the equator where \( r \approx 6377830m \), \( t \approx 3600s \times 24 \times 365.2422 \times 923 \), \( v_{iq} = 2\pi r/t \approx 0.00138m/s \), \( v_{ig} \leq 2\pi r/t \approx 0.000262/s \). The quantum perturbations originate from the differential rotation of the earth’s inner core will also influence the kinematic time dilation effect of the stationary clocks on the earth, to calculate it just take \( v_i \) into the equation:

\[
\tau - \tau_0 = -\left( \frac{m_1}{m} k \cdot \frac{v_i^2}{2c^2} \right) \tau_0
\]

The results respectively calculated by quantum gravity and general relativity are \( \tau - \tau_0 \approx -1.79e-25s \) and \( \tau - \tau_0 \leq -6.46e-27s \), they are far beyond the resolution of any extremely accurate atomic clocks in the world at present, it can be seen that it is almost imperceptible for the static observer, and it can only be measured with fast moving ultra-precision atomic clocks. The method of using airplanes as the carrier of atomic clocks cannot eliminate the large errors caused by various interference factors, and cannot meet the ultra-high precision requirements of the experiment, so only by using the satellites as the effective carrier of ultra-high precision atomic clocks can it be possible to achieve the accuracy standards of the experiment. Assuming that the orbital height of the satellite is \( h \approx 499226m \) \((r \approx 6377830m + 499226m)\), the orbital velocity is \( v \approx 7617m/s \), and the orbital inclination is 0, then \( v_{iq} = 2\pi r/t \approx 0.00148m/s \), and then substituting \( v \) and \( v_i \) into equations \( E_1 \) and \( E_2 \) to get specific accurate prediction values. It can be known by calculation that \( E_1 \approx -3.2135907185e-10 \), and the predicted result respective calculated by quantum gravity and general relativity are: \( E_2 - E_1 \approx 2.1e-18s \) and \( E_2 - E_1 \leq 3.8e-19s \), this means that in the ideal experiment, relative to the case there is no differential rotation inside the earth, when the kinematic time dilation effect
affected by the differential rotation of the earth’s inner core, the quantum gravitational perturbations will lead to the accurate flying atomic clocks gain extra 0.066 nanosecond a year, and this result is about 5 times the predicted value of general relativity. In this ideal experiment, the relationship between them can be shown in the following Fig.1.

Fig.1

The predicted results depend on specific experiment parameters. Based on the calculation result of the ideal experiment, it can be seen that the advanced experiment requires extremely high precision for the atomic clocks, the error cannot exceed one second in about 15 billion years and even more. This has far exceeded the effective resolution of NASA's deep space atomic clocks launched by SpaceX in June 2019\cite{11}, at present, only the highest precision optical clocks in the laboratory can achieve such high resolution\cite{12-15}.

Supplement

a. GP-B has detected the influence of the space-time curvature around the earth on the geodesic effect with extremely high precision\cite{16}; the first stage of the advanced Hafele-Keating experiment can verify the influence of space-time curvature around the earth on kinematic time dilation effect. Relative to the GP-B, it has the advantages of low cost, easy implementation and short experimental cycle.
b. In micro-gravity environment, the corresponding value of $k$ will be extremely close to 1. It is assumed that the kinematic time dilation effect in the gravitational field will be affected by the value of $k$, since twin paradox is a result of the kinematic time dilation effect, a corollary related to $k$ is that if twin paradox is carried out in micro-gravity environment, theoretically it will still be constrained by $k$. This may be helpful in solving the twin paradox.

**Conclusion**

Accurate value of $k$ can be detected by the first stage of the advanced Hafele-Keating experiment, it is possible to achieve with current technology, when $0<k<1$ has been verified in the experiment, it means the space-time curvature could influence the kinematic time dilation effect, or else, the second stage of the experiment will have no physical significance. The closer the value of $k$ is to 1, the more difficult it is to detect, the determination of the precise value of $k$ will also provide a basis for some other theories that depend on it. When optical clocks with extremely high precision become small enough and stable enough that they could be carried in satellites, using this method complete the second stage to detect quantum gravitational perturbations will become feasible.

Anyway, this experiment can certainly probe the relativistic kinematic time dilation effect with much higher accuracy. This method may provide a new possible research way for verifying quantum gravity by experiment, and may also have enlightening significance for study the way to the connection between relativity and quantum gravity.

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