Hyperboloidal Slices and Artificial Cosmology for Numerical Relativity

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This preliminary report proposes integrating the Maxwell equations in Minkowski spacetime using coordinates where the spacelike surfaces are hyperboloids asymptotic to null cones at spatial infinity. The space coordinates are chosen so that \(\text{Scri}^+\) occurs at a finite coordinate and a smooth extension beyond \(\text{Scri}^+\) is obtained. The question addressed is whether a Cauchy evolution numerical integration program can be easily modified to compute this evolution. In the spirit of the von Neumann and Richtmyer artificial viscosity which thickens a shock by many orders of magnitude to facilitate numerical simulation, I propose artificial cosmology to thicken null infinity \(\text{Scri}^+\) to approximate it by a de Sitter cosmological horizon where, in conformally compactified presentation, it provides a shell of purely outgoing null cones where asymptotic waves can be read off as data on a spacelike pure outflow outer boundary. This should be simpler than finding \(\text{Scri}^+\) as an isolated null boundary or imposing outgoing wave conditions at a timelike boundary at finite radius.

I. INTRODUCTION

As LIGO [1] approaches its first stage design sensitivity and other [2] gravitational wave observatories progress, theoretical descriptions of the expected waveforms from the inspiral of binary black holes and neutron stars remain less detailed than one might hope. The largest efforts are devoted to solving Einstein’s equations by discrete numerical methods based on a space+time split of the equations and intend to impose as a boundary condition an asymptotic Minkowski metric. Initial formulations were given the ADM name although (probably inconsequentially) none uses the conjugate sets of fields \(g_{ij}\) and \(\pi^{ij}\) favored by ADM. Variations on these formulations have shown practical improvements [3, 4, 5], and others [6] with less developed implementations hold the theoretical suggestion of better numerical behavior.

Because the (expected) observable gravitational waves will be seen far from their sources, some efforts have asked that the data set representing the state of the gravitational field at any computational step be not that on an asymptotically flat spacelike hypersurface, but instead that on an outgoing light cone. Thus the Pittsburgh group [7] has focussed on slicing spacetime along null cones. By extracting a conformal factor as in Penrose diagrams, null infinity becomes a finite boundary in the computational grid. But these outgoing null cones do not have a useful behavior near the wave sources, leading to a study matching them to flatter spacelike slices there. Friedrich [8] formulated the Einstein equations in a way where the conformal factor could be one of the dynamical variables, and also considered the case where the time slices were spacelike but asymptotically null, as for hyperboloids in Minkowski spacetime. A large program to implement this approach is being pursued by Sascha Husa [9, 10] and others at AEI-Potsdam.

This paper aims to formulate within the simpler problem of solving Maxwell’s equations some aspects of these methods which could be attacked with smaller computational resources. In particular we consider the question of whether fairly straightforward evolution algorithms designed for spacelike slicings of spacetime will encounter special problems when those slices are asymptotically null, and the question of imposing boundary conditions at null infinity (\(\text{Scri}^+\)).

Work with Mark Scheel and Lee Lindblom [11] considered the first of these questions, but employed the scalar wave equation. Those preliminary results were presented at the KITP Workshop “Gravity03” in June 2003. Vince Moncrief [12] then suggested that since the coordinate choice which brought \(\text{Scri}^+\) to a finite radius displayed a conformal metric which was nonsingular at \(\text{Scri}^+\), it would be better to study instead the Maxwell equations whose properties under conformal transformations are simpler than those of the scalar wave equation.

A new suggestion is given below that the boundary conditions at \(\text{Scri}^+\) might be easier to handle if \(\text{Scri}^+\) were approximated by a de Sitter cosmological horizon with a cosmological constant chosen for numerical convenience rather than for physical interest.

II. MAXWELL’S EQUATIONS

In formulating a 3+1 statement of the Maxwell equations I, of course, revert to lessons from Arnowitt and Deser [13]. Following their Schwinger tradition I use a first order variational principle \(\delta I = 0\) with

\[
I = \int g^{\mu\nu} \alpha^\beta (-F_{\mu\nu} \partial_\alpha A_\beta + \frac{1}{2} F_{\mu\nu} F^{\alpha\beta}) \, d^4x
\]
where
\[ g^{\mu\nu\alpha\beta} = \frac{1}{2} \sqrt{-g} (g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\nu\alpha}) \] (2)
are components of an operator that maps covariant antisymmetric tensors onto contravariant anti-symmetric tensor densities. This mapping has an inverse whose components are
\[ g_{\mu\nu\alpha\beta} = \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \sqrt{-g} . \] (3)
Note that these two symbols are not related merely by the usual lowering of indices, as one map raises the density while the other lowers it. Their product
\[ g_{\mu\nu} g^{\alpha\beta} = \frac{1}{4} (\delta^\alpha_\mu \delta^\beta_\nu - \delta^\beta_\mu \delta^\alpha_\nu) . \] (4)
gives the identity mapping of antisymmetric tensors onto themselves.

In this variational principle the fields $A_\mu$ and $F_{\mu\nu}$ are varied independently and yield the equations
\[ \partial_{\nu} \tilde{F}^{\mu\nu} = 0 \] (5)
and
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \] (6)
Here $\tilde{F}^{\mu\nu}$ arises as
\[ \tilde{F}^{\mu\nu} = g^{\mu\nu\alpha\beta} F_{\alpha\beta} . \] (7)

Again following the given tradition I seek a proto-Hamiltonian form $p \dot{q} - H$ for this Lagrangian and rewrite it by replacing the $F_{\mu\nu}$ field in $I$ by its equivalent $\tilde{F}^{\mu\nu}$ field, giving
\[ I = \int (-\tilde{F}^{\mu\nu} \partial_{\nu} A_\mu + \frac{1}{2} \tilde{F}^{\mu\nu} \tilde{g}^{\alpha\beta} g_{\mu\nu\alpha\beta}) d^4x . \] (8)
The only term here containing time derivatives is $-\tilde{F}^{\mu\nu} \partial_{\nu} A_\mu$ so that
\[ \mathcal{D}^i = \tilde{F}^{0i} \] (9)
and $A_\mu$ are the only fields for which one obtains evolution equations. The scalar potential
\[ \psi = -A_0 \] (10)
and the $\tilde{F}^{ij}$ are therefore Lagrange multipliers which enforce the constraints
\[ \partial_i \mathcal{D}^i = 0 \] (11a)
and
\[ B_{ij} \equiv g_{ij,\alpha\beta} \tilde{g}^{\alpha\beta} = \partial_i A_j - \partial_j A_i . \] (11b)
The evolution equations can then be written as
\[ \partial_0 A_i = -E_i - \partial_i \psi \] (12a)
and
\[ \partial_0 \mathcal{D}^i = \partial_i \tilde{F}^{ij} \] (12b)
where
\[ E_i = g_{i0,\alpha\beta} \tilde{g}^{\alpha\beta} \] and \[ \tilde{F}^{ij} \equiv \tilde{g}^{ij} \] . (13)

There is no evolution equation for $\psi = -A_0$ but one can be supplied as a gauge condition. There is also no evolution equation directly from the variational principle for $B_{ij}$ but one can be deduced by taking the time derivative of equation (11b) and evaluating the time derivatives on the right hand side using equation (12a). The result is
\[ -\partial_0 B_{ij} = \partial_i E_j - \partial_j E_i . \] (14)

In a numerical example mentioned below, the fields were taken to be $A_i$ and $\mathcal{D}^i$ using the evolution equations (12a) and (12b). Equation (11b) was treated not as a constraint but as merely making $\partial_0 B_{ij}$ an abbreviation for the right member of that equation, thus introducing second (spatial) derivatives into the system of equations. For that example of a wave packet the gauge chosen was Coulomb gauge with $\psi = 0$ for all time. The remaining constraint (11a) is easily seen to be preserved in time as a consequence of the evolution equation (12a).

Another possible system of equations would be to use (13) and (14) as evolution equations. Since the vector potential does not appear in these equations, it can be ignored and not evolved. The constraint (11b) would then have to be replaced by its integrability condition
\[ \partial_0 B_{ij} = 0 \] (15)
treated as a constraint equation. It is easily seen from (14) that this constraint is preserved in time (differentially) as a consequence of (13). The Gauss constraint (11a) remains part of this system and is preserved in time by exact solutions. These equations (12a) and (15) are exactly what one writes in classical electromagnetism in a material medium (e.g., Section 16-2); only the constitutive equations (17) are new. With the usual 3+1 decomposition of the metric
\[ ds^2 = -c^2 dt^2 + \gamma_{ij}(dx^i + \beta^i du)(dx^j + \beta^j du) \] (16)
these are
\[ E_i = (\alpha/\sqrt{\gamma}) \gamma_{ij} \mathcal{D}^j + B_{ij} \beta^j \] (17a)
and
\[ \tilde{F}^{ij} = \alpha \sqrt{\gamma} \delta^{ik} \gamma^{jl} B_{kl} + \mathcal{D}^i \beta^j - \mathcal{D}^j \beta^i \] (17b)
which are more complex than the $E = \mathcal{D}/\epsilon$ and $H = B/\mu$ of simple isotropic media. The Einstein aether is, in coordinate terms, anisotropic not only in space but also in spacetime, but conveniently linear.
III. CONFORMAL INVARIANCE

This exploration of numerical evolution with the Maxwell equations was provoked by Vince Moncrief’s reminder that the Maxwell and Yang-Mills equations (in four spacetime dimensions) have the simplest possible conformal structure. A conformal transformation $g_{\mu
u} \mapsto \Omega^2 g_{\mu
u}$ changes the metric geometry to a different one which shares the same light cones. Under this change the metric dependent factor $g^{\mu\nu\alpha\beta}$ in the variational integral $I$ does not change since $g^{\mu\nu} \mapsto \Omega^{-2} g^{\mu\nu}$ while $\sqrt{-g} \mapsto \Omega^4 \sqrt{-g}$. As nothing in the variational integral changes under a conformal transformation, it follows that if the fields $A_\mu$ and $F_{\mu\nu}$ or $\Phi^{\mu\nu}$ satisfy the field equations in one metric, so do they also in any conformally related metric.

Although nothing in section II depends on the metric having any special properties (other than Lorentz signature) such as flatness or satisfying the Einstein equations, these conformal properties in four spacetime dimension are particularly useful since the metric we intend to use is singular only in a conformal factor $s^2/q^2$ which we now see will appear nowhere in our field equations. A similar simplification occurs when the metric of interest is the Schwarzschild metric in the hyperboloidal slicings used in [11].

In the 3+1 decomposition of the field equations we note that $A_i, \mathcal{D}_i, E_i, \psi, \psi, B_{ij}$ and $\delta^i$ are conformally invariant, while $\gamma_{ij} \mapsto \Omega^2 \gamma_{ij}$, $\alpha \mapsto \Omega \alpha$, $\sqrt{-\gamma} \mapsto \Omega^3 \sqrt{-\gamma}$. Thus the metric structures which appear in the constitutive equations (17), $(\alpha/\sqrt{\gamma})\gamma_{ij}$ and $\alpha \sqrt{\gamma} \gamma^{ij}$ as well as $\beta^i$, are conformally invariant.

The constraint equations (11) and (15) have an even stronger invariance: they are metric invariant. No metric quantities appear in these constraint equations when they are written in terms of our choice of fields. Thus if two hypersurfaces are diffeomorphic then initial conditions for the Maxwell equations which satisfy the constraints in one Lorentzian manifold can be imported to the other where they will again satisfy the constraints. If the two manifolds are not conformally equivalent, however, the subsequent evolutions of these initial conditions will generally be inequivalent.

Gauge conditions present greater difficulties. In the absence of sources the “Coulomb” gauge $\psi = 0$ is conformally invariant, but only 3-dimensionally coordinate invariant. On the other hand the simplest Lorentz gauge $\partial_\mu (\sqrt{-g} g^{\mu\nu} A_\nu) = 0$ is coordinate invariant but not conformally invariant. The set of field equations which evolve $\Phi^i$ and $B_{ij}$ without the use of the vector potential are conformally invariant but do not require a gauge condition.

IV. BACKGROUND SPACETIME

The background metric for this study will be the de Sitter spacetime

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2 + (R^2/L^2)(dT - dR)^2. \quad (18)$$

which is here presented in Kerr-Schild form as flat spacetime plus the square of a null form. The cosmological constant here is $\Lambda = +3/L^2$. Since $dT - dR$ is null in both the underlying flat spacetime and after the Kerr-Schild modification, the hyperboloidal slices we used in flat spacetime should remain asymptotically null also in this modified metric. Therefore we introduce the same coordinate change as in our earlier scalar work [11]

$$\frac{T}{s} = u + \frac{1}{2} r^2 \quad (19)$$

and

$$\frac{X^i}{s} = \frac{x^i}{1 - \frac{1}{4} r^2} \quad (20)$$

where $R^2 = X^2 + Y^2 + Z^2 \equiv X^i X^i$ and $r^2 = x^2 + y^2 + z^2 \equiv x^i x^i$. This leads to the metric

$$ds^2 = \left(\frac{s^2}{q^2}\right) [-\alpha^2 du^2 + \left(1 + \frac{s^2 r^2}{L^2(1 + \frac{1}{4} r^2)^4}\right)(dr + \beta du)^2 + r^2 d\Omega^2] \quad (21a)$$

where

$$q = 1 - \frac{1}{4} r^2 \quad (21b)$$

and

$$\alpha^2 = \left(1 + \frac{1}{4} r^2\right)^2 \left[1 + \frac{s^2 r^2}{L^2(1 + \frac{1}{4} r^2)^4}\right]^{-1} \quad (21c)$$

and

$$\beta = - \left[r + \frac{s^2 r^2}{L^2(1 + \frac{1}{4} r^2)^2}\right] \left[1 + \frac{s^2 r^2}{L^2(1 + \frac{1}{4} r^2)^4}\right]^{-1} \quad (21d)$$

I have verified that this metric (21) actually is the de Sitter metric by two tests: I have run a GRtensor II [15] Maple 8 worksheet to calculate the Einstein equations for this study will be the de Sitter spacetime

$$G^\mu_\nu = -(3/L^2)\delta^\mu_\nu \quad (22)$$

and I have also had that worksheet calculate the Riemann tensor which is

$$R^{\mu\nu\alpha\beta} = (1/L^2)(\delta^\alpha_\beta \delta^\mu_\nu - \delta^\mu_\beta \delta^\nu_\alpha) \quad (23)$$

and shows that this is a spacetime of constant curvature, therefore de Sitter. Richard Woodard [16] has confirmed this by showing that the metric (15) can be reduced to the familiar static de Sitter metric by the coordinate change $T = \bar{T} + f(R)$ where $df/dR = -R^2/(L^2 - R^2)$. 


V. CAUSAL STRUCTURE

This de Sitter metric is presented in equations (21) with an apparent singularity at \( r = 2 \) where \( q = 0 \). This is probably a coordinate singularity since the curvature does not go bad there, and also because the de Sitter spacetime in notorious for being presented in many different guises corresponding to different coordinate systems, many of which cover only a patch of the full manifold. But for present purposes we only need this coordinate patch, and the singularity at \( R = \infty \) will be removed by a conformal transformation later.

By dropping the conformal factor in the metric (21) we arrive at a nonsingular metric

\[
ds^2 = -\alpha^2 du^2 + \left[ 1 + \frac{s^2 r^2}{L^2 (1 + \frac{2}{r})^2} \right] (dr + \beta du)^2 + r^2 d\Omega^2
\]

using the same abbreviations as in equations (21). Note, however, that \( \alpha \) and \( \beta \) are now the lapse and shift of this regulated metric. This metric no longer satisfies the Einstein equations (with \( \Lambda \) term) but is related to the original (de Sitter) metric in a known way. From this point on we use only this metric, and the previous metric will be called \( d\tilde{s}^2 = \Omega^2 ds^2 \) following the practice in Wald’s discussion [17, Appendix D] of conformal transformations, with \( \Omega = s/q \) in our case.

The causal structure of the regulated metric (24) is simpler than that of the original metric (18). The hypersurfaces of constant \( u \) are everywhere spacelike since the regulated metric is positive definite when \( du = 0 \). The hypersurfaces of constant \( r \) are spacelike only in an interval around \( r = 2 \) as seen from their normal vector \( dr \) being timelike there:

\[
(\nabla r)^2 = g^{rr} = \frac{(1 - \frac{1}{2}r^2)^2 - (rs/L)^2}{(1 + \frac{4}{3}r^2)^2}.
\]

This norm is plotted in Figure 1 over a broad range and again in Figure 2 giving the interval around \( r = 2 \) in more detail.

VI. ANALYSIS

What can we learn using the tools described above? A few lessons already appeared in the work [11] with Scheel and Lindblom. Suppose only the time coordinate were changed to parameterize a family of hyperboloids, here

\[
[T - s(u - 1)]^2 - R^2 = s^2
\]

in the original coordinates of equation (18). Then the outgoing coordinate speed of light \( dR/du \) would become unboundedly large as \( R \to \infty \). The Courant condition usually met in numerical implementations would then make the allowable time step impractically small. This can be cured by “Analytic Mesh Refinement” (AnMR), which is just a coordinate transformation such as that from \( R \) to \( r \) in equation (24), since outgoing waves get redshifted as the \( u = \text{const} \) hypersurfaces tend toward outgoing null cones. Because this redshift gives long wavelengths, high resolution is not needed and the large \( R \) domain can be compressed to a finite range. Thus we find that the outgoing coordinate speed of light \( dr/du \) becomes

\[
c_{out} = \frac{1 + \frac{1}{2}r^2}{1 - \frac{1}{2}r^2}
\]

which remains less than 4 out to \( r = 2 \) or \( R = \infty \). The ingoing radial light speed \( c_{in} \) is similarly bounded in magnitude but becomes positive in the region where \( (\nabla r)^2 \) is negative rather than the simpler value \( c_{in} = -(1 - \frac{1}{2}r^2)^2 \) in the Minkowski (\( L = \infty \)) case where it merely touches zero on \( \text{Scri}^+ \). Thus the encouraging results from [11] are unchanged by introducing the cosmological term.
VII. EXACT SOLUTIONS

There is in principle no difficulty in finding exact solutions to these equations in Minkowski spacetime. One merely takes solutions from any textbook and performs the coordinate transformation to our $uxyz$ compactified hyperboloidal slicing coordinates. In detail it is a great convenience to have GRTensor available to perform some of the algebra and much essential checking.

Exact solutions play two roles. The first is that they provide initial data for numerical integrations whose physical interpretation is known. Secondly, through comparison, they allow testing the accuracy of the numerical solution. Indeed, the whole point of carrying out these numerical evolutions is to explore in a simpler context than full general relativity different approaches for possible adaption to Einstein’s equations. Thus one may explore and compare different integration schemes either to understand them better as in the work of Baumgarte [18], or to test possible new schemes as in [19]. Questions which may be treated in this simplified context include the stability of different formulations, the control of constraints, the treatment of boundary conditions, the extraction of asymptotic wave amplitudes and waveforms, etc. Here we address the use of hyperboloidal time slices and the possibilities for carrying the integration out to and beyond $\text{Scri}^+$ using a finite grid.

We find that a sufficient variety of examples can be found by assuming that the vector potential $A = A_\mu dx^\mu$ has the form

$$A = f \sin^2 \theta d\phi - \psi dT$$ \quad (28)

Three cases are (1) a uniform magnetic field with $\psi = 0$, $f = B Z R^2/2$ (2) a static dipole electric field with $f = 0$, $\psi = Z/R^4$ (3) the Baumgarte [18] choice of a wave packet, which is our principle test case, with $\psi = 0$ and

$$f = \frac{1}{R} - 2\lambda U \exp(-\lambda U^2) - \frac{1}{R} + 2\lambda V \exp(-\lambda V^2)$$ \quad (29)

where $U$ and $V$ were defined by

$$U \equiv T - R = s \left( u - \frac{r}{1 + \frac{r^2}{Z^2}} \right)$$ \quad (30a)

and

$$V \equiv T + R = s \left( u + \frac{r}{1 - \frac{r^2}{Z^2}} \right)$$ \quad (30b)

In spite of first impressions one finds that $f = O(R^2)$ at small $R$ or small $r$, but numerical implementations must avoid evaluating this $f$ at $r = 0$ as subtle cancellations occur. To convert equation (28) to $uxyz$ coordinates requires

$$\frac{1}{s} dT = du + \frac{x^i}{(1 - \frac{r^2}{Z^2})^2} dx^i$$ \quad (31)

and, from $X^i = x^i/(1 - \frac{r^2}{Z^2})$ and the standard rectangular to spherical coordinate transformation,

$$\sin^2 \theta \, d\phi = \frac{1}{R^2} (X \, dY - Y \, dX) = \frac{1}{r^2} (x \, dy - y \, dx)$$ \quad (32)

The results are nontrivial even for a constant magnetic field in these coordinates which can be useful for debugging. To obtain initial conditions for numerical integrations requires that from these vector potentials one compute the fields $F_{\mu\nu}$, $\phi^{\mu\nu}$ and thus $D^i = \phi^{0i}$. Figure 3 plots an analytic solution obtained in this way at three different times. The incoming wave packet (at $u = -2$) is blueshifted relative to the normals the to constant $u$ hyperboloid, while the outgoing packet (at $u = 1.4$) is redshifted so that high spatial resolution is not required to describe it near $\text{Scri}^+$ (which is at $r = 2$). Note that there are no irregularities of the outgoing packet as it crosses $\text{Scri}^+$.

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FIG. 3: For the Baumgarte choice of EM wave packet from equations (28) and (29) we plot the conformally invariant electric field $D^y$ along the $x$-axis at three different hyperboloidal times $u = -2, 0, \text{and } 1.4$. Note that the field values are finite and generally nonzero at Scri+ which is $x = 2$. Those at the latest time have been here magnified by a factor of 10.

FIG. 4: Waveform extraction: for the same analytic solution as in Figure 3 we plot $D^y$ as function of $u$ at Scri+ ($x = 2$). Note that at constant $r$ one has $T = su + \text{const}$ so this waveform is not distorted from its $(T, R)$ presentation as is the pulse shape (Figure 3) when plotted as function of $r$. 

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