Two-loop renormalization of Wilson loop for Drell-Yan production.

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Abstract

We study the renormalization of the Wilson loop with a path corresponding to the Drell-Yan lepton pair production in two-loop approximation of perturbation theory. We establish the renormalization group equation in next-to-leading order and find a process specific anomalous dimension $\Gamma_{\text{DY}}$ in the corresponding approximation.

Keywords: Drell-Yan process, Wilson loop, soft gluon resummation, anomalous dimension, renormalization group equation

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\footnotesize

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1. Introduction. The increase of the experimental accuracy necessitate the construction of the theory of power corrections for hardronic reactions. For the time being there is a firm theoretical ground only for the processes which admit the operator product expansion. However, these are the processes without the latter which are of the main interest. An important example of such a process is the Drell-Yan (DY) lepton pair production with invariant mass $Q^2$. Recent studies apparently reveal the correspondence between the soft-gluon resummation which is necessary for the processes going near the boundary of the phase space \[1, 2, 3\] and the non-perturbative power corrections \[4, 5\]. It was found that the ambiguity in the resummation of soft gluons to the leading logarithmic accuracy manifest itself in the power-like behaviour in the hard momentum scale leading to power correction of the order $\Lambda_1/Q$ \[4\]. It was argued that this behaviour can be associated with particular matrix elements of Wilson line operators \[4\]. The nature of this ambiguity was questioned in Ref. \[6\] where the authors attempted to identify the source of linear term in the perturbative series for the Wilson loop corresponding to the DY production. It was found that soft gluon resummation which allows to apply the renormalization group methods and infrared renormalons are disconnected since the former is related to the convergent analytical anomalous dimensions while the latter enter as a boundary conditions to the evolution equation. Moreover, it was established basing on the resummation of the particular class of Feynman diagrams — fermion vacuum polarization bubbles — that the leading ambiguity in the perturbative series could be only $\Lambda_2/Q^2$. However, the latter approach does not take into account the non-abelian nature of QCD and, therefore, could miss some important features since it was guessed that it is precisely the diagrams with three-gluon vertices \[7\] which might be important in resolving these apparent paradox. In the present paper we make a first step and evaluate the Wilson loop in order $O(\alpha_s^2)$. Later extending the result to the case of nonzero gluon mass, $\lambda^2$, we will be able to find the first non-analytical term in the expansion w.r.t. $\lambda$ which might be a signal for the first power correction \[8\] up to limitations discussed below. However, the results given here are of interest in their own right.

2. Time-loop technique. Following the approach of Korchemsky and Marchesini we can express the soft part of the factorized DY cross section \[9\]

$$ Q^2 \frac{d\sigma_{DY}(z, Q^2)}{dQ^2} = \sigma_{DY}^{(0)} \mathcal{H}_{DY}(Q^2) \tilde{W}_{DY}(z) $$

via the Fourier transform, $\tilde{W}_{DY}(z)$, of the vacuum average of the Wilson loop

$$ \tilde{W}_{DY}(z) = \frac{Q}{2} \int_{-\infty}^{\infty} \frac{dy_0}{2\pi} e^{iy_0\omega} W_{DY}(y), \quad W_{DY}(y) = \frac{1}{N_c} \langle 0 | \text{Tr} T \mathcal{P} \exp \left( ig \oint_{C_{DY}} dx_\mu B_\mu(x) \right) | 0 \rangle, $$

with a path $C_{DY}$ shown in Fig. 1(a). The symbol $\mathcal{P}$ stands for path- while $T$ for (anti-)time-ordering to be explained below. Here $\omega \equiv \frac{Q}{2}(1 - z)$ is a total soft gluon energy.
It is well known that a time-like cross section could not be related to the imaginary part of any $T$-product of currents. Contrary, it is given by the particular absorptive parts of the Feynman diagrams. To be able to extract the imaginary part we are interested in we can label in some way the field operators in the amplitudes to the right and to the left of the cut. This can be suitably done with the help of the Keldysh-like diagram technique \cite{10, 11}. It allows to recast the program for the calculation of the particular discontinuities of the Feynman diagrams to the operator-like language. Consider, for instance, certain $S$-matrix element which is given by the functional integral $\mathcal{M} = \int DB \prod \mathcal{L} \mathcal{J} \exp (i \int dz \mathcal{L})$. The cross section of the process is

$$\sigma = \mathcal{M}^\dagger \mathcal{M} = \int \mathcal{D} B^{(-)} \mathcal{D} B^{(+)} \prod_i B_i^{(-)} \prod_j B_j^{(+)} \exp (i \int dz \mathcal{L}^{(+)} - i \int dz \mathcal{L}^{(-)}).$$

Here the ”plus” and ”minus” superscripts label the fields from the direct and conjugated amplitudes, respectively. One can accept that they are the components of a unique operator $B_\mu = (B_\mu^{(+)}, B_\mu^{(-)})$ composed from the time- and anti-time-ordered fields. Now the Green function for the $B_\mu$ field is a $2\times 2$-matrix constructed from the usual Feynman propagator, its conjugated analogue and its discontinuity via the Cutkosky rules for the lines connecting the direct and final diagrams. To be able to extract the imaginary part we are interested in we can label in some

$$\int d^4 x e^{iqx} \langle 0 | \mathcal{T} \left\{ B_\mu^a (x) \otimes B_\nu^b (0) \right\} | 0 \rangle = \delta^{ab} g_{\mu\nu} \begin{pmatrix} -iD^{(+)}(q) & -iD^{(-)}(q) \\ iD^{(-)}(q) & iD^{(+)}(q) \end{pmatrix},$$

with the following form of the components\footnote{By definition a ($-$) field stands to the left of ($+$) ones.} $D^{(+)}(q) = [D^{(-)}(q)]^* = (q^2 + i0)^{-1}$, $D^{(-)}(q) = -2\pi i \delta_+(q^2)$.

Using these conventions we can easily write the Wilson loop \footnote{Here and below the subscript “$+$” on the distribution, $\Delta = \delta, \theta$, means the positivity of the energy flow through the cut: $\Delta_+(q^2) \equiv \Delta(q^2)2\theta(q_0)$.} as follows

$$W_{DY} (y) = \frac{1}{N_c} \langle 0 | \text{Tr} \left\{ \Phi^{(-)}_{-p_1}[-\infty, y] \Phi^{(-)}_{p_2}[y, -\infty] \Phi^{(+)}_{-p_2}[-\infty, 0] \Phi^{(+)}_{p_1}[0, -\infty] \right\} | 0 \rangle,$$

where $\mathcal{T}$ stands for the time-ordering for the ($+$) and anti-time one for the ($-$)-fields and the Wilson lines are given by the following expressions

$$\Phi_{p_1} [y, -\infty] = P \exp \left( ig \int_{-\infty}^0 d\sigma p_{1\mu} B_\mu (\sigma_1 p_1 + y) \right),$$

$$\Phi_{-p_2} [-\infty, y] = P \exp \left( -ig \int_{0}^{\infty} d\sigma_2 p_{2\mu} B_\mu (-\sigma_2 p_2 + y) \right).$$

Here the two quark momenta are defined in terms of the light-like vectors, $p_1 = p_{1+} n^+$, $p_2 = p_{2-} n$, which fix different tangents, i.e. $-$ and $+$ directions, on the light cone and they are normalized according to $n^2 = n^* n = 0$ and $nn^* = 1$.\footnote{Here and below the subscript “$+$” on the distribution, $\Delta = \delta, \theta$, means the positivity of the energy flow through the cut: $\Delta_+(q^2) \equiv \Delta(q^2)2\theta(q_0)$.}
Let us note that the peculiarities of the renormalization of the Wilson lines with a cusp or/and with a path lying on the light cone were studied before. It was found that they require an additional renormalization and corresponding anomalous dimensions play an important rôle in perturbative QCD. These are the properties which we will extensively exploit presently.

Evaluating the one-loop graphs (Fig. 1(b) and mirror conjugated) we can easily obtain unrenormalized expression

\[ W^{(1)} = C_F \frac{\alpha_s}{\pi} \frac{\Gamma(1 - \epsilon)}{\epsilon^2} e^{-\gamma_E \epsilon} L^\epsilon, \]  

(7)

where \( L \equiv \left( -\frac{1}{2} g_0^2 \mu^2 e^{2\gamma_E} + i0 \right) \) with \( \mu^2 \) being the \( \overline{\text{MS}} \) scale parameter. Subtracting the poles in \( 1/\epsilon \) we get well-known expression:

\[ W_{\text{DY}}^{(1)}(y) \equiv R_s W^{(1)} = C_F \frac{\alpha_s}{\pi} \left\{ \frac{1}{2} \ln^2 L + \frac{\zeta(2)}{2} \right\}. \]  

(8)

3. Two-loop results. Here we perform the evaluation of the Wilson loop in the fourth order of perturbation theory. Expanding the generating functional in the perturbative series we obtain a set of Feynman diagrams given in Fig. 1. Note that diagrams with virtual dressing of Wilson line vanish identically in dimensional regularization since the integration over loop momentum is given by a scaleless integral. Therefore, from several number of cuts only those survive which are given by the product of tree diagrams (up to exception of graph (h)). Moreover, on top of this among the remaining graphs we will evaluate only those (they are displayed in Fig. 1) proportional to maximal non-abelian and fermion components, i.e. \( C_F C_A \) and \( C_F T_F N_f \), which is possible due to the non-abelian exponentiation theorem. According to it

\[ W_{\text{DY}} = 1 + \sum_{n=1}^{\infty} W^{(n)} = \exp \left( \sum_{n=1}^{\infty} w^{(n)} \right), \]  

(9)

so that \( W^{(2)} = \frac{1}{2} (w^{(1)})^2 + w^{(2)} \) and \( W^{(1)} = w^{(1)} \). In what follows we evaluate \( w^{(2)} \).

3.1. Vacuum polarization diagrams. The vacuum polarization diagram in Fig. 1(c) reads

\[ w^{(2)}_{\text{(c)}} = -C_F g^2 S \epsilon^{2\sigma} \sigma_{1+} \sigma_{2-} \int_{-\infty}^{0} d\sigma_2 \int_{-\infty}^{0} d\sigma_1 \times \int \frac{d^d q}{(2\pi)^d} e^{-iq(\sigma_2 \sigma_{1+} \sigma_1 \sigma_{1+} + y)} D^{(-)}(q) D^{(+)}(q) 2 \text{Im} \Pi_{++}(q), \]  

(10)

where \( S \equiv \epsilon^{(\gamma_E - 4\pi)} \) and the imaginary part of the one-loop polarization operator is

\[ \text{Im} \Pi_{\mu\nu}(q) = \alpha_s \left[ \frac{C_A}{2} (5 - 3\epsilon) - 2T_F N_f (1 - \epsilon) \right] \frac{\Gamma(2 - \epsilon)}{\Gamma(4 - 2\epsilon)} \left( \frac{\mu^2 e^{\gamma_E}}{q^2} \right)^\epsilon \theta_+(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu). \]  

(11)

After Fourier transformation with the help of formula \((x_+ = 0)\)

\[ \int \frac{d^d q}{(2\pi)^d} e^{-i q x} \frac{\theta_+(q^2)}{(q^2)^n} = \pi^{-1-d/2} (-2)^{-1-2n} \frac{\Gamma(1-n)\Gamma(d/2-n)}{[-2(x_+-i0)(x_+-i0)]^{d/2-n}} \]  

(12)
Figure 1: Integration path in the Wilson loop in Eq. (2) is shown in (a). Non-zero one-loop graph (b). Different topologies of nonvanishing Feynman diagrams contributing to the Wilson loop \( W_{\text{DY}} \) at order \( O(\alpha_s^2) \) with maximal non-abelian factor in (c) – (h). To complete the set one has to add mirror symmetrical graphs which can be taken into account by appropriate multiplicity factors, \( m_\sigma \), for graphs displayed here: \( m_b = m_c = m_d = m_f = 2, \ m_e = 1, \ m_g = m_h = 4 \). Full blob stands for the sum of the vacuum polarization bubbles due to fermions, gluons and ghosts.

we get

\[
\begin{align*}
W_{(c)}^{(2)} &= C_F \left( \frac{\alpha_s}{\pi} \right)^2 [C_A(5 - 3\epsilon) - 4T_FN_f(1 - \epsilon)] \frac{\Gamma(1 - \epsilon)\Gamma(2 - \epsilon)\Gamma(1 - 2\epsilon)}{16\epsilon^3\Gamma(4 - 2\epsilon)} \left\{ 1 - \frac{\epsilon}{1 + \epsilon} \right\} e^{-2\gamma_E L^2}. \\
\end{align*}
\] (13)

The rôle of diagram (d) is to cancel the second term in the curly brackets in Eq. (13) which comes from the “gauge dependent” \( q_\mu q_\nu \)-piece of the Landau gluon propagator. Multiplying these contributions by appropriate multiplicity factors coming from adding of analogous graphs with different attachments of gluon lines we obtain

\[
\begin{align*}
W_{(c)}^{(2)} &= C_F \left( \frac{\alpha_s}{\pi} \right)^2 [C_A(5 - 3\epsilon) - 4T_FN_f(1 - \epsilon)] \frac{\Gamma(1 - \epsilon)\Gamma(2 - \epsilon)\Gamma(1 - 2\epsilon)}{8\epsilon^3\Gamma(4 - 2\epsilon)} e^{-2\gamma_E L^2}. \\
\end{align*}
\] (14)

3.2. Box-type diagrams. Among a number of them most vanish due to the light-like character of the paths and because planar graphs are proportional to the fermion Casimir operator squared, \( C_F^2 \), which is omitted due to non-abelian exponentiation theorem. The evaluation of surviving graphs
is the most straightforward in the coordinate space where the propagators look like: $D^{(++)}(x) = -\frac{i\pi}{d/2}\Gamma(d/2-1)[-x^2 + i0]^{-d/2}$, and $D^{(-+)}(x) = -\frac{i\pi}{d/2}\Gamma(d/2-1)[-2(x_- - i0)(x_+ - i0)]^{-d/2}$ (for $x_\perp = 0$). Fig. 4(c) gives

$$w_{(e)}^{(2)} = \frac{1}{2}C_F C_A \frac{4S_2 \mu^{4\epsilon}(p_1 + p_2^-)^2}{(2\pi)^d} \int_0^\infty d\sigma_1 \int_0^\infty d\sigma_2 \int_0^\infty d\sigma_1' \int_0^\infty d\sigma_2'$$
$$\times D^{(-+)}(\sigma_2^p_{p_2} - \sigma_1^p_{p_1} + y) D^{(-+)}(\sigma_2^p_{p_2} - \sigma_1^p_{p_1} + y) = -C_F C_A \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\Gamma^2(1 - \epsilon)}{8\epsilon^4} e^{-2\epsilon\gamma E} L^{2\epsilon}. \quad (15)$$

Contribution of Fig. 4(f) is $W_{(f)}^{(2)} = \frac{1}{4} W_{(e)}^{(2)}$. Assembling everything together we have

$$w_{(g)}^{(2)} = -C_F C_A \left(\frac{\alpha_s}{\pi}\right)^2 \frac{3}{16} \frac{\Gamma^2(1 - \epsilon)}{\epsilon^4} e^{-2\epsilon\gamma E} L^{2\epsilon}. \quad (16)$$

3.3. Non-abelian diagrams. Finally, let us consider the non-abelian diagrams. The typical contribution, for instance for Fig. 4(g) where the only contribution survives when the loop is cut, looks like

$$w_{(g)}^{(2)} = \frac{1}{2} C_F C_A G^4 S_2 \mu^{4\epsilon} p_1 p_2^- \int_0^\infty d\sigma_1 \int_0^\infty d\sigma_2 \int_0^\infty d\sigma_1' \int_0^\infty d\sigma_2' \frac{d^d k}{(2\pi)^d} e^{-ik_2 p_2 - \sigma_1^p_{p_1} + y_j} D^{(-+)}(k) \int \frac{d^d q}{(2\pi)^d} e^{-iq_2 p_2 - \sigma_1^p_{p_1} + y_j} D^{(-+)}(q) D^{(-+)}(k - q). \quad (17)$$

The internal integral can be evaluated according to formula

$$\int \frac{d^d q}{(2\pi)^d} e^{-iq_2 x} D^{(-+)}(q) D^{(-+)}(k - q) = -\frac{\pi^{d-1}}{\Gamma\left(\frac{d-1}{2}\right)} (4\pi)^2 - d (k^2)^{\frac{d-2}{2}} \theta_+ (k^2) \quad (18)$$
$$\times e^{-\frac{i}{2}k x + \frac{i}{4}\sqrt{(k x)^2 - k^2 x^2}} \frac{d-1}{d-2} - i \sqrt{(k x)^2 - k^2 x^2},$$

with $x^2 = 0$ and the second one with the help of Eq. (12) making use of the integral representation of confluent hypergeometric function $_1 F_1 [1]$.

Thus we get

$$w_{(g)}^{(2)} = C_F C_A \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\Gamma(1 - 2\epsilon)\Gamma(1 - \epsilon)}{64\epsilon^4 \Gamma(2 - 2\epsilon)} e^{-2\epsilon\gamma E} L^{2\epsilon}. \quad (19)$$

Note that the colour factor of the box-type diagrams is $C_F \left(\frac{C_F}{2}\right)$ but we have kept only maximally non-abelian component.

Let us add an interesting side-remark concerning the evaluation of this integral. Actually, there is no need to perform its explicit calculation with exponential weight. The dependence on $x$ can be fixed using the properties of traceless tensors alone. Expanding the factor $e^{-iq_2 x}$ in Taylor series and using the fact that the tensor $q_{\mu_1} q_{\mu_2} \ldots q_{\mu_n}$ is traceless due to presence of $\delta(q^2)$ in the integrand we can parametrize the integral in terms of the only remaining vector $k$ according to $\langle q_{\mu_1} q_{\mu_2} \ldots q_{\mu_n} \rangle = \{k_{\mu_1} k_{\mu_2} \ldots k_{\mu_n}\} \Omega$ (with $\{\ldots\}$ standing for symmetrization and trace subtraction) and perform tensor contraction with the help of the $d$-dimensional generalization of Nachtmann’s [18] convolution $\{k_{\mu_1} k_{\mu_2} \ldots k_{\mu_n}\} x_{\mu_1} x_{\mu_2} \ldots x_{\mu_n} = \frac{\Gamma(n+1)\Gamma(\lambda)}{\Gamma(n+\lambda)} \left(\frac{k^2}{4}\right)^{n/2} C_n^\lambda \left(\frac{ikx}{\sqrt{k^2 x^2}}\right)$ with $\lambda = \frac{d}{2} - 1$ and $C_n^\lambda$ being the Gegenbauer polynomial [19]. Using third generating function for the latter [13] we can sum the series back. An $x$-free function $\Omega$ can be easily evaluated then which results into Eq. (18).
Now let us observe a fact that the expressions for diagrams we have considered so far with cut gluon Green functions coincide with corresponding contributions evaluated with ordinary Feynman propagators. This is not accidental but it is a mere consequence of the fact that their collinear and cusp singularities — the only ones we are interested in — appear when the gluons propagate along the light-cone directions or they are all on short distances. In these cases cut and ordinary propagators coincide.

There are two contributions coming from different cuts of diagram \((h)\), namely, cut-I corresponding to the propagator combination
\[
D_1^{(++)} D_2^{(++)} D_3^{(-+)}
\]
and cut-II with
\[
D_1^{(-+)} D_2^{(-+)} D_3^{(--)}.
\]
By the reason stated above we have calculated instead the virtual graph which gives
\[
w^{(2)}_{(h)} = C_F C_A \left( \frac{\alpha_s}{\pi} \right)^2 \frac{\Gamma^2(1 - \epsilon)}{64 \epsilon^4 (1 + \epsilon)} \left\{ 1 + \epsilon - 2 \epsilon F_2 \left( \frac{1}{2 + \epsilon}, 1 - 2 \epsilon \middle| 1, 1 \right) \right\} e^{-2 \epsilon \gamma_E L^2 \epsilon},
\]
with \(F_2\) being Appel function \([20]\).

Summing Eqs. (19,20) multiplied by a factor of 4 we obtain \(w^{(2)}_{\triangle}\).

3.4. Counter-terms. Before the subtraction of the overall divergences we have to handle the sub-divergences. For this reason we mention that although the diagrams with virtual subgraphs (gluon vertex and propagator corrections) vanish for the case at hand, corresponding counter-terms do not. Namely, combining the contributions for uncut vacuum blobs and vertex functions results into addendum
\[
w^{(2)}_{ct} = \frac{\alpha_s \beta_0}{\pi} \frac{1}{4 \epsilon} W^{(1)},
\]
where \(\beta_0 = \frac{4}{3} T_F N_f - \frac{11}{3} C_A\) is the first expansion coefficients of the QCD \(\beta\)-function \(\beta(g)/g = \frac{\alpha_s}{\pi} \beta_0\).

This completes the set of non-vanishing contributions we have to analyze.

4. Evolution equation in two-loop approximation. Now with the results derived in the previous sections we can find the renormalized expression of the two-loop Wilson loop for the DY production\(^6\)
\[
(w^{(2)} = w^{(2)}_3 + w^{(2)}_\Delta + w^{(2)}_{ct})
\]
with
\[
\begin{align*}
  w^{(2)}_{DY}(y) &\equiv R_\epsilon w^{(2)} = \left( \frac{\alpha_s}{\pi} \right)^2 \left\{ w^{(2)}_3 \ln^3 L + w^{(2)}_2 \ln^2 L + w^{(2)}_1 \ln L \right\},
  \\
  w^{(2)}_3 &= -\frac{1}{24} C_F \beta_0,
  \\
  w^{(2)}_2 &= C_F C_A \left( \frac{67}{72} - \frac{\zeta(2)}{4} \right) - \frac{5}{18} C_F T_F N_f,
  \\
  w^{(2)}_1 &= C_F C_A \left( \frac{101}{54} - \frac{7}{4} \zeta(3) \right) - \frac{14}{27} C_F T_F N_f.
\end{align*}
\]
Note the absence of \(\ln^4 L\) terms \([1,3]\) due to famous Lee-Nauenberg-Kinoshita theorem \([21]\). Moreover, this equation is in complete agreement with explicit two-loop calculation of the ordinary
\footnote{Here we have used an expansion \(F_2 \left( \frac{1+2 \epsilon}{1+2 \epsilon}, \frac{1+2 \epsilon}{1+2 \epsilon} \middle| 1, 1 \right) = -\frac{(1+\epsilon)}{2 \epsilon} (1 - 2 \zeta(2) \epsilon^2 - 14 \zeta(3) \epsilon^3 + \mathcal{O}(\epsilon^4))\).}
Feynman graphs by van Neerven et al. [3] (cf. Eq. (4.9) there adding charge renormalization counter-term) provided we transform their result to the language of moments so that we receive an addendum $\propto \zeta(2)$ which makes the results coincide. The fact that the coefficient of the leading log, $w_3^{(2)}$, is proportional to the $\beta$-function suggests that $W_{DY}$ satisfies a renormalization group equation [9, 16]. With the expression (22) at hand it is easy to verify that $W_{DY}(y)$ indeed respects the following evolution equation

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right) \ln W_{DY}(y) = 2\Gamma_{cusp}(g) \ln L + \Gamma_{DY}(g),$$

with $\Gamma_{cusp} = \left(\frac{4\pi}{\alpha_s}\right) C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left(C_A \left(\frac{67}{36} - \frac{\zeta(2)}{2}\right) - \frac{5}{3} T_F N_f\right)$ being well-known universal cusp anomalous dimension [14] and a new process-dependent entry

$$\Gamma_{DY}(g) = \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left(C_A \left(\frac{101}{27} - \frac{7}{2} \zeta(3) - \frac{11}{12} \zeta(2)\right) + \left(\frac{\zeta(2)}{3} - \frac{28}{27}\right) T_F N_f\right),$$

where $N_f$-dependent part is in agreement with calculation of Ref. [6]. An unusual feature of Eq. (24) is that the “anomalous dimension” $\gamma = 2\Gamma_{cusp} \ln L + \Gamma_{DY}$ depends explicitly on the renormalization scale, $\mu$. This means the absence of the multiplicative renormalizability of the light-like Wilson line.

Going to moments of the Fourier transformed Wilson loop, $W_{DY} \left(\frac{\mu N}{Q N_0}\right) = \int_0^1 dzz^{N-1} \tilde{W}_{DY}(z)$, results to a mere substitution of $y_0$ in the saddle point approximation by $y_0 = -i2\frac{N}{Q}$. The solution of the evolution equation (24) is given by:

$$W_{DY} \left(\frac{\mu N}{Q N_0}, \alpha_s(\mu^2)\right) = W_{DY} \left(1, \alpha_s \left(Q^2 \frac{N_0^2}{N_0^2}\right)\right) \times \exp \int_{Q^2 \frac{N_0^2}{N_0^2}}^{\mu^2} d\rho \left(\Gamma_{cusp} (\alpha_s(\rho)) \ln \left(\frac{\rho \frac{N_0^2}{N_0^2}}{Q^2 \frac{N_0^2}{N_0^2}}\right) + \frac{1}{2} \Gamma_{DY} (\alpha_s(\rho))\right),$$

with $N_0 \equiv e^{-\gamma E}$ and the upper limit $\mu$ which sets the boundary for the maximal soft gluon energy.

The knowledge of $\Gamma_{DY}$ at order $O(\alpha_s^2)$ allows to predict DY inclusive production in next-to-next-to-leading order approximation in large logarithms provided we have three-loop expression for $\Gamma_{cusp}$ which can be obtained by explicit calculation of the Wilson line with a cusp or be extracted as a coefficient in front of $1/[1 - x]_+$-distribution of the three-loop quark-quark splitting kernel once being evaluated in future.

5. Conclusion and outlook. In present letter we have calculated two-loop approximation of the Wilson loop for the DY process. As we have mentioned in the introduction it is straightforward to generalize present consideration by taking into account the finite gluon mass propagator and look for non-analyticity in $\lambda$ as a trace of power corrections. This could be considered as an attempt to resolve the problem with $1/Q$-power behaviour in DY reaction — a question which
has attracted a lot of attention recently \cite{1,2,3}. But one has to be careful with this since even in the one-loop calculations there is one-to-one correspondence between finite gluon mass and renormalon based schemes \cite{4} only for the sufficiently inclusive quantities while they give different output for observables with weighted final state, e.g. inclusive production in $e^+e^-$-annihilation \cite{22}. More clear signal for the leading power correction would be the first renormalon ambiguity in the standard resummation of fermion bubble chains. But the technique for handling the diagrams with non-abelian vertex still has to be developed.

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