A Bayesian classifier for photometric redshifts: identification of high-redshift clusters

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Accepted 1998 September 1. Received 1998 August 17; in original form 1998 June 11

ABSTRACT

Photometric redshift classifiers provide a means of estimating galaxy redshifts from observations using a small number of broad-band filters. However, the accuracy with which redshifts can be determined is sensitive to the star formation history of the galaxy, for example the effects of age, metallicity and ongoing star formation. We present a photometric classifier that explicitly takes into account the degeneracies implied by these variations, based on the flexible stellar population synthesis code of Kodama & Arimoto. The situation is encouraging, because many of the variations in stellar populations introduce colour changes that are degenerate. We use a Bayesian inversion scheme to estimate the likely range of redshifts compatible with the observed colours. When applied to existing multiband photometry for Abell 370, most of the cluster members are correctly recovered with little field contamination. The inverter is focused on the recovery of a wide variety of galaxy populations in distant (\(z \sim 1\)) clusters from broad-band colours covering the 4000-Å break. It is found that this can be achieved with impressive accuracy (\(|\Delta z| < 0.1\)), allowing detailed investigation into the evolution of cluster galaxies with little selection bias.

Key words: galaxies: clusters: general – galaxies: distances and redshifts – galaxies: evolution – galaxies: general – galaxies: stellar content.

1 INTRODUCTION

The current trend in cosmology is to explore the properties of galaxies at ever fainter limits. This has led to demonstration of the existence of a substantial galaxy population at \(z > 3\) (Steidel et al. 1996; Metcalfe et al. 1996; Steidel et al. 1998), and the discovery of galaxy clusters with \(z \geq 1\) (Deltorn et al. 1997; Stanford et al. 1997; Yamada et al. 1997). These discoveries have allowed us to extend our knowledge of the formation history of galaxies (Madau et al. 1996; Baugh et al. 1998; Kodama et al. 1998) and the growth of the gravitational structure of the Universe (Bower & Smail 1997).

However, while images that reach these depths are now relatively commonplace, spectroscopic follow-up of these objects is extremely time-consuming even on 8-m-class telescopes. These problems are offset by the multiplex capability of multi-object spectrographs (e.g. LDSS; Allington-Smith et al. 1994) and fibre-optic fed spectrographs (e.g. Taylor 1995), or by surveys targeted at specific redshifts using tunable narrow-band filters (e.g. Jones & Bland-Hawthorn 1997). Nevertheless, even in the best-studied deep images, only a small fraction of the galaxies have known spectroscopic redshifts.

Whereas spectroscopic redshifts use sharp absorption and/or emission lines to determine the rest wavelength of the spectrum accurately, it is also possible to exploit the overall characteristic shape of the spectral energy distribution (SED) to estimate the redshift of a galaxy. This ‘photometric redshift’ approach can be applied to broad-band images provided they have sufficiently high signal-to-noise ratio and adequately sample the important features of the SED. In particular, the 4000-Å spectral break and the Balmer and Lyman series limits are important features that arise in almost all galaxy spectra. Although precise redshifts cannot be determined by this method, estimates of (or limits on) \(z\) are obtained.

The existing photometric redshift estimators fall into three main classes: empirical redshift estimators, those based on observed spectral energy distributions and model-based redshift estimators. Empirical redshift estimators (Connolly et al. 1995) are based on a training set of galaxies for which the redshifts and broad-band fluxes are known. These are used to train an estimator, for example a multidimensional polynomial fit, that predicts the redshift from the input fluxes with minimum error. The disadvantage of this method is that it requires a relatively large training data set with high-quality colours and known redshifts. This makes it difficult to apply beyond the limits of spectroscopic surveys, although this problem might be alleviated using the colours of distant, gravitationally lensed galaxies. However this method, when tested against independent but similar data, can give impressive accuracy (\(\sigma_z \sim 0.06\); Connolly et al. 1995).

Lanzetta, Yahil & Fernández-Soto (1996), Mobasher et al. (1996) and Sawicki, Lin & Yee (1997) use an approach that is based on the observed SEDs of galaxies covering a wide range of spectral types. Redshifts are estimated from the observed data by redshifting each
of the templates and determining the best match to the observational colours. They emphasize the importance of using observed templates in order to incorporate the effects of dust. This is particularly important for galaxies in the redshift range $1 < z < 3$ because the optical colours increasingly reflect the rest-frame ultraviolet spectrum of the galaxy. One problem with this approach, however, is that the spectral library does not take into account the evolution of the galaxy stellar populations. The method can accommodate evolution in as far as it is equivalent to changing galaxies between different spectral types, however.

Model-based approaches use stellar population synthesis codes (e.g. Bruzual & Charlot 1993) to produce model SEDs that can then be compared with the observed data. For example, Gwyn & Hartwick (1996) used a spread of galaxy models from single-burst stellar populations to models with constant star formation to present-day galaxies. When generating redshifted model SEDs, the evolution of the stellar population is automatically taken into account. The redshift of the observed galaxy is determined by minimizing $\chi^2$ residuals. The improved flexibility of this method can, however, lead to greater errors in the estimated redshifts. This arises because of colour degeneracies between the effects of galaxy type and redshift.

In this paper, we focus more closely on the interrelation between star formation history and redshift estimation. As we have outlined, photometric redshifts can be susceptible to changes in the galaxy stellar population. For instance, the effects of age, metal abundance and ongoing star formation are all reflected in the relative shape of the continuum, particularly when it is convolved with the response of standard broad-band filters. It is important that these uncertainties are taken into account when determining the galaxy redshift. We develop a method of photometric redshift determination that explicitly takes into account the degeneracies implied by these variations. Clearly, incorporating additional free parameters to describe the star formation history of the galaxy threatens to make it impossible to extract useful redshift information. However, many of the changes in colour caused by different star formation histories are degenerate: this is the familiar age–metallicity degeneracy that has long plagued the estimation of star formation histories in elliptical galaxies. We will show that for red galaxies, redshifts can be determined under only weak assumptions about the star formation history. At lower redshifts, the colours of blue (disc) galaxies become considerably harder to disentangle.

Our approach attempts to deal with, and indeed embrace, this unavoidable degeneracy in colours with variations in redshift and star formation history. We explicitly account for galaxy metallicities and star formation histories; these effects are in many cases degenerate with uncertainties resulting from the stellar initial mass function (IMF), recent star formation, dust extinction and cosmology. We retain possible degeneracies in plausible values of galaxy type and redshift by storing a ‘probability map’ for each galaxy, which can be used to estimate a range of acceptable redshifts rather than reducing the observed data to a single ‘best bet’ estimate of galaxy type and redshift. In particular, our classifier is designed to pick out galaxy cluster members without biasing the sample to galaxies of one particular star formation history. Our motivation is to use this method to study the photometric properties of $z \sim 1$ cluster galaxies with as little selection bias as possible.

The structure of the paper is as follows. Section 2 introduces the stellar population synthesis code of Kodama & Arimoto (1997). We derive colour tracks for a range of galaxy star formation histories and outline the major uncertainties in these tracks. This provides the framework for selecting appropriate filter sets and the required photometric accuracy. Section 3 details our Bayesian approach to the inversion problem. We explicitly incorporate a wide variety of possible star formation histories, and explicitly incorporate the resulting degeneracies in our redshift estimates. The role of the prior is discussed. In Section 4, we test our method with galaxies in Abell 370 cluster field and galaxies with known redshifts in the Hubble Deep Field (HDF). Section 5 gives an application of the method to a simulated cluster at $z = 1$. A summary and our conclusions are presented in Section 6.

2 COLOUR TRACKS AS A FUNCTION OF STAR FORMATION HISTORY

2.1 Model

The evolutionary population synthesis model of Kodama (1997) was used to predict the photometric properties of evolving stellar populations. This model calculates the spectral evolution of a galaxy with an arbitrary star formation history, taking into account the chemical evolution in a self-consistent way. Kodama & Arimoto (1997) applied this model to the elliptical galaxy populations of distant clusters. In this study, disc models with ongoing star formation are considered in addition to the elliptical models. We first describe the basic equations and parameters of this model and then summarize the elliptical galaxy and disc galaxy models.

2.1.1 Equations and parameters

We assume that the galactic gas is supplied from a surrounding gas reservoir trapped in the gravitational potential of a galaxy and that the gas is always well-mixed and distributes uniformly in a galaxy. The star formation is described by the following equations. The stellar IMF is given by a single power law:

$$\phi(m) = A m^{-x}, \quad m_l \leq m \leq m_u,$$

where $m_l$ and $m_u$ are lower and upper limits of initial stellar mass respectively. The Salpeter (1955) IMF corresponds to $x = 1.35$. The coefficient $A$ is determined by

$$\int^{m_u}_{m_l} \phi(m) dm = 1.$$

The IMF is assumed to be time-invariant. The star formation rate (SFR) $\psi(t)$ is assumed to be proportional to the gas mass $M_g(t)$ (Schmidt 1959):

$$\psi(t) = \frac{1}{\tau} M_g(t),$$

where $\tau$ is the star formation time-scale in Gyr. Note that this formulation gives an exponentially decaying SFR with an effective time-scale $\tau$ in the case of the simple models, where $\alpha$ is the so-called locked-up mass fraction defined by Tinsley (1980). The Salpeter mass function ($x = 1.35$) with $m_l = 0.1$ and $m_u = 60 M_\odot$ gives $\alpha = 0.72$. The gas inflarate $\xi_{\text{in}}(t)$ depends on the initial total mass of the gas reservoir $M_T$ and the gas inflarate time-scale $\tau_{\text{in}}$:

$$\xi_{\text{in}}(t) = \frac{M_T}{\tau_{\text{in}}} \exp \left( -\frac{t}{\tau_{\text{in}}} \right)$$

(cf. Köppen & Arimoto 1990). The gas metallicity $Z(t)$ is calculated numerically, using the basic equations of chemical evolution (Tinsley 1980) and stellar nucleosynthesis tables (Nomoto, private communications). The metal contribution from SN Ia is also considered by fixing their lifetime at 1.5 Gyr. We assume that the metal-enriched gas spreads through the galaxy instantaneously and...
evenly (the one-zone approximation). As the initial conditions, we assume that there is no gas in a galaxy before the onset of star formation, i.e. \( M_g(0) = 0 \) and \( Z_g(0) = 0 \). Using the infall history defined as above, our expression for the star formation rate \( \psi(t) \) and the metallicity of the stars \( Z(t) = Z_p(t) \), the integrated spectrum of a galaxy can be synthesized as a function of time. By specifying the galaxy age, or equivalently its formation redshift, and cosmological parameters, we obtain the spectra and therefore colour indices of the galaxy as a function of redshift. The cosmological parameters are set to \( H_0 = 50 \) km s\(^{-1}\) Mpc\(^{-1}\), \( \Omega_0 = 1.0 \) and \( \Lambda_0 = 0.0 \) unless otherwise stated.

### 2.1.2 Elliptical galaxies and bulges

For elliptical galaxy models (E models), we use \( x = 1.10, m_0 = 0.1 \) M\(_\odot\), and \( m_0 = 60 \) M\(_\odot\) for the IMF and short time-scales of star formation and gas infall: \( \tau = \tau_{\text{m}} = 0.1 \) Gyr. The slope of the IMF differs from the Salpeter value \( x = 1.35 \) to allow the colours of the reddest giant ellipticals to be reproduced in the context of this model (with a Salpeter IMF, metallicities high enough cannot be achieved). In addition, in order to reproduce the observed present-day dependence of elliptical galaxy colour on luminosity, it is useful to introduce another parameter, the galactic wind epoch \( t_{\text{gw}} \). At this time, the energy put into the ISM in the proto-elliptical galaxy by SNe is large enough to overcome the potential of the galaxy, resulting in the ejection of the gas from the galaxy, ending star formation. We constructed a model sequence of elliptical galaxies as a function of total luminosity by simply changing \( t_{\text{gw}} \) so that they reproduce the colour–magnitude (C–M) relation of Coma ellipticals in \( V – K \) and \( U – V \) (Bower, Lucey & Ellis 1992ab) at the galaxy age \( t_C = 12 \) Gyr. Changing \( t_{\text{gw}} \) is equivalent to adjusting the mean stellar metallicity of the galaxies, therefore this is the *metallicity sequence* of elliptical galaxy models. In this model, the mean stellar metallicity \( [M/H] \) changes from 0.06 to -0.52 over a six-magnitude range from the brightest E model \( M_V = -23 \) mag at \( z = 0 \). The time until the onset of a galactic wind \( t_{\text{gw}} \) is always shorter than -0.5 Gyr, thus the star formation in elliptical galaxies is burst-like. The above model sequence is shown to reproduce the evolution of the C–M relation of elliptical galaxies in distant clusters in Kodama & Arimoto (1997) and Kodama et al. (1998).

To represent the photometric properties of disc galaxy bulges, we borrow the elliptical galaxy models. Observational support for this includes the results of Mg\(_2\) index analysis (Jablonka, Martin & Arimoto 1996).

### 2.1.3 Discs

For the disc component, the IMF parameters are set to \( x = 1.35, m_0 = 0.1, m_0 = 60 \) M\(_\odot\), and longer time-scales of star formation and gas infall: \( \tau = \tau_{\text{m}} = 5 \) Gyr. The age of a galactic disc is fixed at 12 Gyr. The disc model time-scales are chosen to reproduce the integrated \( B – V \) colours and \( M_g/L_B \) ratio of observed discs of various Hubble-types, as shown in Fig. 1 (cf. Shimasaku & Fukugita 1998).

The \( B – V \) colours of discs shown in Fig. 1 as a function of Hubble type are estimated from

(i) the mean total \( B – V \) colours as a function of Hubble type (Buta et al. 1994), and

(ii) subtraction of the bulge light by assuming a bulge colour \( B – V = 1.0 \) and a bulge-to-total light ratio (B/T) in the B band (Simien & de Vaucouleurs 1986).

The total gas masses normalized by B-band disc luminosity \( (M_g/L_B) \) as a function of Hubble type are estimated from

(i) the mass of neutral atomic gas, calculated from the integrated hydrogen index H\(_1\) (Buta et al. 1994) and a conversion formula in the Third Reference Catalogue of Bright Galaxies (RC3) given by de Vaucouleurs et al. (1991),

(ii) the ratio of molecular to atomic gas H\(_2\)/H\(_1\) (Young & Knezek 1989),

(iii) a helium abundance correction of 25 per cent, and

(iv) subtraction of the bulge light contribution to \( L_B \).

Disk properties in Fig. 1 are well reproduced by a \( \tau = \tau_{\text{m}} = 5 \) Gyr model with an age \( t_C = 5 – 15 \) Gyr irrespective of the Hubble type. The model also reproduces the age–metallicity relation and the [O/Fe] versus [Fe/H] diagram of the stars in our own Galaxy (Kodama 1997). The constraint on the time-scales \( \tau \) and \( \tau_{\text{m}} \) is weak because of the large observational errors and the permitted range could be from 2 to 8 Gyr (Fig. 1). However, as will be shown in the next section (Section 2.2), this uncertainty will not cause problems for the purposes of redshift determination because the star forming time-scale and B/T variations have degenerate effects.

As an additional check of the validity of our models, the integrated colours of disc galaxies of different Hubble type are compared in Table 1. The observational data are mean Hubble type colours taken from de Jong (1996) and the RC3 (Buta et al. 1994; Buta & Williams 1995). Note that each galaxy type has large intrinsic colour dispersion, typically as much as 0.05–0.2 mag in optical colours and 0.2–0.4 mag in \( V – K \). The data are compared with the model with appropriate B-band B/T ratio (Simien & de Vaucouleurs 1986). It is clear that the detailed trends of local galaxy colour with B-band B/T ratio are well reproduced by our models.
2.2 Colour tracks

Following the models introduced above, we simulate the colour evolution of galaxies as a function of redshift for a variety of star formation histories.

The two solid curves in Fig. 2 show the colour evolution in the observer’s frame for an E model with a high metallicity ([M/H] = 0.06), and a model which contains 50 per cent contribution of disc light in the B band at z = 0 (see below). The redshift is changed from 0 to 2 in steps of 0.05 as indicated by the dots along the tracks. The six arrows shown from three redshift points along the track indicate the colour changes due to several different effects. See text for details.

Table 1. Integrated colours of spiral galaxies.

| Hubble type | B/T | $B-V$ | $V-R$ | $V-I$ | $V-K$ | $U-V$ | $B-V$ | $V-R$ | $V-I$ | $V-K$ | $U-V$ | $B-V$ | $V-R$ | $V-I$ | $V-K$ |
|-------------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Sa          |     | 0.41  | 0.54  | 1.02  | 2.94  | 0.96  | 0.74  | 0.50  | 1.14  |       | 0.85  | 0.76  | 0.53  | 1.17  | 3.02  |
| Sb          |     | 0.24  | 0.74  | 0.46  | 1.04  | 2.79  | 0.66  | 0.61  | 0.46  | 0.93  | 0.63  | 0.66  | 0.49  | 1.07  | 2.84  |
| Sc          |     | 0.09  | 0.67  | 0.53  | 1.08  | 2.84  | 0.45  | 0.53  | 0.41  | 0.87  | 0.45  | 0.57  | 0.43  | 0.95  | 2.61  |
| Sd          |     | 0.02  | 0.59  | 0.47  | 1.02  | 2.59  | 0.37  | 0.50  | 0.39  | 0.83  | 0.36  | 0.52  | 0.41  | 0.89  | 2.47  |
| Sm          |     | 0.00  | 0.69  | 0.41  | 0.75  | 2.30  | 0.33  | 0.50  | 0.36  | 0.72  | 0.33  | 0.50  | 0.40  | 0.87  | 2.42  |

Figure 2. Colour–colour plots in the observer’s frame. The two solid curves show the colour evolution of an old ($T_G = 12$ Gyr, or $z_f = 4.5$), high-metallicity ([M/H] = 0.06) E model (redder), and the B = T = 0.5 model (bluer) which contains half of the disc light in the B band at $z = 0$. The redshift is changing from 0 to 2 in steps of 0.05, indicated by dots along the tracks. The six arrows shown from three redshift points along the track indicate the colour changes due to several different effects. See text for details.
Figure 3. The galaxy spectra at $z = 0$ (upper panel). The flux is normalized at 4030 Å. The thick solid line shows a giant elliptical E1 spectral template (Bica 1988) extended into the UV by the attachment of IUE spectra (Arimoto 1996). Three model spectra are superposed: a high mean metallicity model with $\langle [M/H] \rangle = 0.06$ (thin solid line), a lower metallicity model with $\langle [M/H] \rangle = -0.52$ (dotted line), and a high metallicity bulge plus disc model with $B/T=0.5$ (dashed line). See text for details of the models. The lower panel shows the normalized response functions of standard Johnson–Cousins B, V and R filters, blueshifted to correspond to those at $z = 0.3$.

Photometric redshift determination up to redshifts of $\sim 1.5$ typically bracket the 4000-Å break: $0.25 \leq z \leq 0.4$ for $B-V$ versus $V-R$ colours, $0.5 \leq z \leq 0.8$ for $V-R$ versus $R-I$ colours, $0.9 \leq z \leq 1.15$ for $R-I$ versus $I-K$ colours, and $1.0 \leq z \leq 1.5$ for $R-Z$ versus $Z-J$ colours. In the above redshift ranges, the middle bands of each combination are passing through the 4000-Å break, the most prominent spectral feature in the optical region, which plays an important role in redshift estimation. The horizontal colours reddened rapidly with redshift while vertical colours stay nearly constant at around $z = 0.3$ for example, as shown in Fig. 3, the $V$ band is just on the 4000-Å break and the $B$ and $V$ bands are losing flux rapidly, while the $R$ band flux is approximately constant as the redshift increases. As a result $V-R$ gets redder while $B-V$ remains almost constant. On the other hand, $B-V$ is more sensitive to changes in stellar population than $V-R$. Therefore, we can see that the effects caused by changes in redshift are almost perpendicular to those caused by changes in star formation history.

In Fig. 2, six possible effects which change the colour evolution of a galaxy with redshift are considered. The six arrows (indicated at three redshift points along the colour track) show the change in colour of an old, high-metallicity elliptical model ($T_e = 12$ Gyr and $\langle [M/H] \rangle = 0.06$) caused by the following effects.

(i) **Metallicity – lower thick solid arrows.** The mean stellar metallicity of the E models is changed from $\langle [M/H] \rangle = 0.06$ to $-0.52$. The galaxy age is fixed at 12 Gyr, corresponding to a formation redshift $z_f = 4.5$.

(ii) **Age – dashed arrows.** The formation redshift of the E model is varied from $z_f = 4.5$ to 1.0 (4.5 to 2.0 for the $RJK$ and $RZJ$ diagrams), corresponding to galaxy ages $T_G = 12.0$ to 8.4 Gyr (12.0 to 10.5 Gyr). Metallicity is fixed at $\langle [M/H] \rangle = 0.06$.

(iii) **Disc component – long-dashed arrows.** As outlined earlier, the models deal with disc galaxies by adding a star-forming disc component on to an E model, representing the galaxy bulge. The $B$-band $B/T$ ratio at $z = 0$ is changed from 100 per cent to 0 per cent. Note that $B/T=0.41$, 0.24 and 0.09 corresponds to the Hubble types Sa, Sb and Sc, respectively (Simien & de Vaucouleurs 1986). Note also that the $B/T$ ratio is a rising function with redshift, because the bulge is getting brighter with redshift while the disc brightness remains roughly constant up to $z = 1$. For example, the $B/T$ ratio of 0.5 at $z = 0$ actually increases to 0.8 at $z = 1$. We have also modelled the $B$–$T$ sequence using $\langle [M/H] \rangle = -1.3$ and $\langle [M/H] \rangle = 0$ by forcing all the disc stars to have the same metallicity instead of following the chemical evolution of the disc. $\langle [M/H] \rangle = 0$ is found to have a negligible effect on the colour tracks of the $B/T$ sequence, and is not considered hereafter. The time-scales $\tau$ and $\tau_{\text{int}}$ can be also changed, but this is found to be similar in effect to changes in $B/T$ ratio at fixed $\tau$ and $\tau_{\text{int}}$. This is because the colour of the $B/T$ sequence at a given redshift is essentially determined by the ratio of the current star formation rate and the total mass of the galaxy. This parameter can be adjusted either by changing the time-scales of the disc or by changing the $B/T$ ratio. Thus the effect on the colour–colour diagrams of changing the timescales is only to shorten or extend the vectors of the $B/T$ sequence for a given $B/T$ ratio.

(iv) Recent starburst – dotted arrows. The possible effects of recent large-scale star formation are considered by the addition of a recent starburst to the E model. The burst population is assumed to be a simple stellar population (SSP) with solar metallicity. The arrow denotes the changes caused by a $T_b = 0.5$ Gyr old burst population corresponding to 10 per cent of the total stellar mass ($f_b = 0.1$) at that redshift. The direction of the vector on the colour–colour diagrams depends on $T_b$, but unless $T_b$ is around 0.5 ($\leq 0.3$) Gyr, the burst sequence follows either the $B/T$ sequence or the age sequence closely. Main-sequence turn-off stars in the burst population with age $\sim 0.5$ Gyr have an effective temperature of about 10000 K and contribute significantly to the total flux at rest wavelengths of 3000–4000 Å, creating aberrant colour changes in the colour–colour plots.

(v) **Reddening – dash–dotted arrows.** The extinction effect due to internal dust is estimated by using the extinction curve given by Mathis (1990). The full arrows correspond to $A_V = 0.5$ mag.

(vi) **Cosmology – upper thick solid arrows.** The colour tracks have a weak dependence on the adopted cosmology. Other sets of cosmological parameters are tested, i.e. $H_0 = 65$, $\Omega_0 = 0.1$, $\Lambda_0 = 0.0$, and $H_0 = 80$, $\Omega_0 = 0.2$, $\Lambda_0 = 0.8$. The formation redshift $z_f$ is fixed to 4.5 in all cases. The full arrows show the colour change for the latter cosmology. The colour change from the former cosmology is smaller and along a similar vector.

The age and metallicity sequences (i) and (ii) are almost indistinguishable in all the colour–colour plots for ages $\geq 1$ Gyr. This reflects the age–metallicity degeneracy inherent in old stellar populations (Worthey 1994). However, this degeneracy actually improves the prospects for the determination of photometric redshifts, as the effects of age and metallicity are quite distinct, given the right choice of passbands, from the effects of changing galaxy redshift. In addition, it is clear from Fig. 2 that changes in assumed cosmology and interstellar reddening also have colour effects similar to age and metallicity, with an opposite sense. As a result, E-type galaxies at a given redshift should populate a restricted area on the colour–colour diagram (almost a single line) characteristic of that redshift, irrespective of its stellar population, regardless of interstellar reddening, and whichever cosmology is assumed. This
means that it is possible to assign redshifts to old stellar populations without prior knowledge of galaxy properties.

However, the colour changes caused by the B/T sequence (iii) are not entirely degenerate with those resulting from age and metallicity on the RIK and RZJ diagrams (Fig. 2), because of the presence of ongoing star formation. This ongoing star formation causes a bluer, e.g., $R - I$ colour for a given $I - K$ colour than the effects of age and metallicity for $z \sim 1$ galaxies. Recent large bursts of star formation of age $\sim 0.5$ Gyr (iv) also lead to effects distinct from those of age and metallicity, and those of changing the B/T ratio.

This can lead to considerable uncertainty in the estimation of galaxy redshift, because a given set of colours, on the basis of the colour–colour plots presented in Fig. 2, will be consistent with a wide range of redshifts, depending on how the colours are explained by our model; e.g. by changing the B/T ratio, or the metallicity of the galaxy template. However, if a passband with a short rest-frame wavelength is used, it is possible to discriminate the cases of the presence of young stellar populations photometrically, leading to a less ambiguous determination of redshift, and some information on the star formation history of the galaxy. This is illustrated in the upper half of Table 2 (see also the lower left panel of Fig. 2), where we compare three galaxy templates with very similar red colours ($R - I = 1.35, I - K = 3.48$) which present very distinct colours in bluer passbands, allowing relatively easy discrimination between these possibilities. Another degeneracy apparent in Fig. 2 in redder passbands is that between high-redshift, low B/T ratio galaxies and low-redshift early-type galaxies. This, again, is illustrated in the lower half of Table 2 where we again see that bluer passbands allow easy splitting of this degeneracy.

Another point to note from Fig. 2 is that when a particular colour pair is selected to allow the accurate estimation of redshifts within a certain redshift range, this colour pair also provides a means of rejecting galaxies (particularly higher B/T objects) that lie outside this optimal redshift range (although the estimated redshifts will obviously be much less accurate for these objects). Problems will occur for much higher redshift objects, and objects with a small B/T ratio, as discussed above.

Despite the demonstrated utility of the bluer passbands in ‘breaking’ degeneracies between galaxies that look identical in red passbands, we aim to use little of the colour information shortwards of 2500 Å. Primarily, this is because the model spectra are ill-constrained for short UV wavelengths in both elliptical and star-forming galaxies because of the effects of the UV upturn (an anomalous rise in flux towards short UV wavelengths, observed in nearby giant ellipticals; e.g. Burstein et al. 1988) and the uncertain effects of dust extinction (White, Keel & Conselice 1996). The source of the UV upturn is still poorly understood, and the model predictions for its source and effects are still uncertain. If the UV upturn comes from hot young stars, this population is actually considered in this model by superposing only a small fraction of ongoing star formation on to the passively evolving ellipticals. If, however, the source of the UV upturn is hot horizontal branch stars, then it is necessary to fine-tune the mass-loss parameter along the red giant branch to reproduce the UV upturn (cf. Yi, Demarque & Oemler 1997). Even if this were the case, such hot horizontal branch stars would disappear at high redshift ($z > 8$), which is our main region of interest, because the envelope mass of a horizontal branch star becomes larger as the mass of the main-sequence turn-off star becomes larger with look-back time.

An additional source of uncertainty in our models, especially in the UV, is the neglect of the effects of dust extinction on the colours of the stellar populations incorporating ongoing star formation. This would at first appear to be a serious handicap, as disc-dominated galaxies clearly contain significant amounts of dust, especially in the spiral arms, where $B$-band extinction $A_B \sim 1$ mag (White et al., 1996; Berlind et al. 1997). However, by inspection of Fig. 2, it is clear that the colour changes at rest-frame optical wavelengths are equivalent to increasing B/T ratio, age or metallicity, meaning that relatively large uncertainties in dust reddening can be accommodated by changes in other galaxy parameters to compensate for these errors. This is also demonstrated in Table 1, where it is apparent that our models can accurately reproduce the colours of galaxies with ongoing star formation. This situation is unlikely to hold in the UV, however, as prescriptions for the dust extinction law start to diverge at these short wavelengths (Calzetti, Kinney & Storchi-Bergmann 1994).

Both the UV upturn and the uncertain effects of dust reddening in the far-UV lead us to place little confidence in our model UV colours. We should therefore avoid this spectral region if possible.

In addition, it should be noted that we neglect emission from star-forming galaxies, such as the commonly observed [OII], [OIII] and Balmer features locally (Kennicutt 1992) and at high redshifts (Hammer et al. 1997). This should not present a major problem, as the effect of line emission on broad-band photometry is not large. A line width with an equivalent width of 20 Å in emission would cause only $\sim 0.02$ mag of brightening in the broad-band magnitude.

Finally, we note that although most of the redshift range below 1.5 can be covered by the standard Johnson–Cousins system including Z band, there are some particular redshift ranges where we have larger errors in the estimated redshifts; i.e. $z < 0.25$, $0.4 < z < 0.5$, and $0.8 < z < 0.9$. At these redshift ranges, the effect of changing redshift on colours is hard to distinguish from that of changing stellar population (Fig. 2). If we want to handle clusters in these redshift ranges with better precision, we need to use passbands in other photometric systems that properly bracket the 4000 Å break at the cluster redshifts.

### 3 Bayesian Classification

#### 3.1 Basic scheme

A Bayesian approach allows us to incorporate our existing knowledge of galaxy populations, and thus to proportionally weight the areas of parameter space that we search. The Bayesian probability of a particular galaxy having a redshift $z$ and bulge to total luminosity ratio $B/T$ is given by the equation

$$P(z, B/T | m_B) P_B(z, B/T),$$

where $P(z, B/T | m_B)$ is the probability of a given galaxy of apparent magnitude $m_B$ having a redshift $z$ and bulge-to-total luminosity ratio $B/T$, and $P_B(z, B/T)$ is the probability of a given galaxy reproducing...
the observed galaxy colours. We first deal with the evaluation of $P(z|B/T|m_B)$, i.e. the probability of a given model galaxy reproducing the observed galaxy colours.

The basic philosophical approach used for this redshift estimator is the comparison of the location of a galaxy on a colour–colour plot and a finely spaced grid of models superimposed on that plot to estimate the properties of that galaxy. The magnitudes of the observed galaxy are made into colours, and the errors in the colours are used to make up a covariance matrix, describing the sizes of the observed galaxy colours. We first deal with the evaluation of $P(z|B/T|m_B)$, which is parametrized by $\Phi^*, \alpha$ and $M_B^0$. The parameters for the observed local luminosity function are summarized in Table 3 as a function of the Hubble type. To connect between the Hubble types and $B/T$ ratio, we use Simien & de Vaucouleurs (1986). The characteristic magnitude of the LF, $M_B^0(B/T)$, corresponds to the apparent magnitude $m_B^0(z,B/T)$ at a redshift $z$ as $m_B(z,B/T) = M_B^0(B/T) + DM(z) + \Delta M_B(z,B/T)$, where $DM$ is the distance modulus at redshift $z$ in the adopted cosmology, $\Delta M_B$ is the absolute magnitude change in the $B$ band in the observer’s frame due to the luminosity evolution and the shift of the wavelength shortwards with redshift, and is taken from the model. In this way, we finally obtain the LF in apparent magnitude $m_B$ as a function of redshift and $B/T$ ratio:

$$\Phi(m_B, z, B/T) = 0.92\Phi^* \exp \{-0.92(\alpha + 1)(m_B - m_B^0) - \exp(-0.92(m_B - m_B^0))\}.$$  

(9)

If the observed galaxy lacks $B$-band data, we use a prior in the band nearest to $B$. In this case, we make up the local LFs in the alternative band by shifting the $B$-band LFs using model colours of each type.

3.2 Volume element

The other essential ingredient of the prior is the volume element $dV/dzd\Omega$. The formula for the volume element as a function of redshift was taken from Carroll, Press, & Turner (1992), with the addition of some factors of $c$ to satisfy dimensionality considerations, and allows variations in $\Omega_M, H_0$ and the inclusion of the cosmological constant via the term $\Omega_k$:

$$dV/dzd\Omega = \frac{d_M^2}{\{1 + \Omega_M(H_0 d_M/c)^2\}^{1/2}} \frac{d(d_M)}{dz},$$

(10)

where $\Omega_M$ is given by $\Omega_M = -3c^2/2R_0^2 H_0^2$, and $R_0$ is the scalefactor of the Universe and $k$ is the curvature of the Universe. The quantity $d_M$ is the proper motion distance, and in this case is given by

$$d_M(z) = \frac{c}{H_0 |\Omega_k|^{1/2}} \sin(k|\Omega_k|^{1/2} \mathcal{F}),$$

(11)

where ‘$\sin$’ is a function that equals $\sin$ in an open universe, $\sin$ in a closed universe, and disappears in a critical universe, and $\mathcal{F}$ is given by

$$\mathcal{F} = \int_0^\infty \left[(1+z')^2(1 + \Omega_0 z') - z'(2 + \Omega_0)\right]^{-1/2} dz',$$

(12)

which must be integrated numerically for most non-trivial cosmologies.

In forming the prior distribution, we need to know the type-dependent luminosity function (LF) $\Phi(m_B, B/T)$ and the volume element $dV/dzd\Omega$. Using these two elements, the prior is given as follows:

$$P_i(z, B/T | m_B) = dV/dzd\Omega \Phi(m_B, z, B/T).$$

These two parts are treated separately below.

3.2.1 The local luminosity function

In order to obtain $\Phi(m_B, B/T)$ we use the local, type-dependent luminosity function (LF). However, there remains considerable uncertainty in the type-dependent LF, as the splitting into morphological types is carried out in a number of different ways, and the faint-end slopes differ considerably between different studies (Bingelli, Sandage, & Tammann 1988; Marzke et al. 1994; Bromley et al. 1998; Marzke et al. 1998). We chose to adopt a variant of Marzke et al.’s (1994) determination in the Schechter (1976) form, which is parameterized by $\Phi^*, \alpha$ and $M_B^0$. The parameters for the observed local luminosity function are summarized in Table 3 as a function of the Hubble type. To connect between the Hubble types and $B/T$ ratio, we use Simien & de Vaucouleurs (1986). The characteristic magnitude of the LF, $M_B^0(B/T)$, corresponds to the apparent magnitude $m_B^0(z,B/T)$ at a redshift $z$ as $m_B(z,B/T) = M_B^0(B/T) + DM(z) + \Delta M_B(z,B/T)$, where $DM$ is the distance modulus at redshift $z$ in the adopted cosmology, $\Delta M_B$ is the absolute magnitude change in the $B$ band in the observer’s frame due to the luminosity evolution and the shift of the wavelength shortwards with redshift, and is taken from the model. In this way, we finally obtain the LF in apparent magnitude $m_B$ as a function of redshift and $B/T$ ratio:

$$\Phi(m_B, z, B/T) = 0.92\Phi^* \exp \{-0.92(\alpha + 1)(m_B - m_B^0) - \exp(-0.92(m_B - m_B^0))\}.$$  

(9)
3.2.3 Comparison with observation

The prior was used to calculate the $n(z)$ distribution within a magnitude range $22.5 < m_B < 24.0$. This calculated distribution was then compared with the observed redshift distribution of galaxies within the same magnitude range given by Glazebrook et al. (1995) and Cowie et al. (1996). The comparison is shown in Fig. 4. It is clear that the prior reproduces the overall form of the observed $n(z)$ diagram.

3.2.4 The effect of the prior

It should be noted that the prior distribution is quite model-dependent, because the type-dependent local LF is ill-constrained, and because the detailed type-dependent spectral evolution is poorly understood. Also, here we make two assumptions, i.e. that there is no number evolution of galaxies, and that there is no size-dependent luminosity evolution, that is that galaxies with similar B/T ratios have the same colours at all redshifts, regardless of their total luminosity. These assumptions and ingredients may be inadequate to describe the real Universe, especially in the context of a hierarchical clustering universe. However, these uncertainties are not so important, because the estimated redshift is essentially determined by the colour term ($P_2$ in equation 5), and the prior ($P_1$) is used supplementarily. The prior becomes important when the solution from the colour term splits into multiple redshift ranges. In such a case, the prior works to avoid unreasonable solutions of redshift for a given apparent magnitude. This situation is illustrated in Fig. 5. The figure shows an example of the probability maps of a given galaxy. The colour term gives two solutions, one at low redshift ($z \sim 0.2$) and the other at high redshift ($z \sim 1.0$), but the prior rejects the solution with higher redshift based on the brightness of the galaxy.

We also tested the effect of local LF on the final estimated redshift through the prior by changing the LF parameters listed in Table 3. We shifted $M_B$ by $+0.5$ mag for all types, resulting in a shift of the redshift peak of the $n(z)$ distribution in Fig. 4 by $+0.1$, and we tried fixing the faint-end slope $\alpha$ at $-1$ for all types. In all cases, however, the change in the final estimated redshifts was well within the estimated redshift errors. This experiment shows that our method is robust to uncertainties in the prior estimation.

3.3 The models included in the classifier

The estimates of redshift and galaxy type will depend quite sensitively on the detailed choice of model galaxy template. In Section 2.2, we investigated the various effects on the galaxy colours, namely the effects of age, metallicity, disc light addition, recent starbursts, reddening and cosmology. However, many of
these effects were found to be degenerate with one another. In these models, the effects of age, metallicity, reddening and changes in cosmology are particularly degenerate. Therefore, by including only the metallicity effect explicitly in the classifier, we also cover the other three effects at the same time as they behave just like metallicity variations. To this aim, we considered four B/T sequences with different bulge metallicities. Each sequence gives a different probability map on the redshift and B/T plane, and then they are combined into a single ‘probability map’ by performing a mean of the separate maps. This mean is in essence a weighted mean, as the most plausible metallicity for the model galaxies will have the best match to the colours. In this way, we take into account the metallicity effects explicitly, and hence the other degenerate effects.

The remaining significant effect that is not covered is that of a recent burst of star formation. As seen in Fig. 2, large amounts of relatively recent star formation can make a galaxy look as if it has a lower redshift than it actually does, if the colours are interpreted as resulting entirely from redshift, B/T and metallicity effects. They might simply be assigned significantly underestimated redshifts. However, as already shown, unless burst strength $f_b$ is as high as $>15$ per cent and the burst age $T_b \sim 0.5$ Gyr, the burst population colour change is similar to those resulting from other effects. These populations should be short-lived and are expected to be rare. Therefore, omitting the recent starburst model will not affect our global redshift estimation. We should note, however, that this could be a problem if a significant fraction of cluster members were strongly affected by a recent burst, caused by cluster–cluster merging for example. In such a case, we would need to include the extra set of models of recent starbursts to estimate redshifts correctly, although it would lead to greater estimation errors.

### 3.4 Error estimates

At this stage, the redshift and galaxy type estimates are in the form of the ‘probability map’ $P_{\text{Gal}}(z, \text{B/T})$. However, an estimate of the redshift and type of a given galaxy is often required for e.g. comparison with real redshifts, or cluster member identification.

Best estimates for redshift ($\hat{z}^{\text{best}}$) and effective 1σ confidence intervals ($\hat{z}^{\text{min}}$, $\hat{z}^{\text{max}}$) are obtained by taking the $P_{\text{Cum}}(z) = 0.5$ and the $P_{\text{Cum}}(z) = [0.16, 0.84]$ intervals, respectively, of the cumulative distribution:

$$P_{\text{Cum}}(z) = \int_{0}^{z} \int_{\text{B/T}=0}^{\text{B/T}=1} \text{d}(\text{B/T}) P_{\text{Gal}}(z, \text{B/T}).$$

These error estimates depend on the estimated photometric errors. through the use of the covariance matrix $\mathbf{C}$, the error estimates in the photometry are explicitly included in the determination of $P_{\text{Cum}}(z, \text{B/T})$. Large uncertainties in the colours propagate through into larger uncertainties in the $(z, \text{B/T})$ combinations capable of adequately reproducing the observed colours. It is important to know how much photometric accuracy we need in order to achieve an error in redshift within the expected bound, as it will certainly constrain the accuracy of any future applications of this method.

To produce redshift error estimates as a function of photometric accuracy, simulations using 100 galaxies with $K < 20$, chosen at random in the ranges $0 < \text{B/T} < 1$ and $0.2 < z < 1.8$ were undertaken. The bulge metallicity ([M/H]) was chosen randomly between 0.061 and $-0.523$. After allocating the model magnitude in each passband for each galaxy, Gaussian photometric errors were then applied. We used these magnitudes and photometric errors of the simulated galaxies in the course of the redshift estimation. Here BVRIK passbands were used. The quantities $\sigma(\Delta z)$ and $\sigma(\text{error})$, corresponding to the root mean square (rms) of real and estimated redshift errors respectively, were then plotted in Fig. 6, where:

$$\Delta z = \hat{z}^{\text{best}} - \hat{z}^{\text{real}},$$

$$\text{error} = (\hat{z}^{\text{max}} - \hat{z}^{\text{min}})^{1/2},$$

$$\sigma(\Delta z) = \sqrt{\langle \Delta z \rangle^2},$$

$$\sigma(\text{error}) = \sqrt{\text{error}^2}.$$  

As seen from the solid lines, both $\sigma(\Delta z)$ and $\sigma(\text{error})$ increase with photometric error. Importantly, even if the photometric error is as poor as 0.15 mag in all bands, the average redshift error $\sigma(\Delta z)$ is still
kept smaller than 0.1. As for the estimated redshift error $\sigma(\text{error})$, it is roughly comparable to the photometric error.

Next we consider the effect of misestimation of the photometric error on the redshift estimation error. It is possible that if the errors are under- or overestimated, the redshift estimator will make the distribution of likely $(z, B/T)$ too broad or multiply peaked, reducing the accuracy of the redshift estimate. Therefore, it is important to test the effects that uncertainty in the determination of the errors can have on the redshift estimate accuracy. We realize this situation by fixing the real photometric error at 0.071 mag and varying the estimated photometric error going into the covariance matrix $C$. The result is shown by the dashed lines in Fig. 6. It is clear that the under- or overestimation of the photometric errors has little, if any, effect on the quality of redshift estimation $\sigma(\Delta z)$. Errors in the determination of the photometric quality do however have a marked effect on the estimated quality of the redshift determination, given by $\sigma(\text{error})$. It is clear, therefore, that it is important to be careful in the estimation of the photometric errors in order to estimate the quality of the redshift determination effectively.

The quality of the redshift estimation also depends sensitively on the passband choice. We investigated three sets of passbands for the randomly generated galaxies with 0.071 mag photometric errors, and the results are summarized in Table 4. With $RIK$ passbands, the result is about a factor of 2 worse than in the $BVRIK$ case. This is because it gets harder to disentangle the colour degeneracies between lower redshift early-types and higher redshift late-types without using bluer colours $B$ and $V$. If, instead, we do not use $K$-band colours, the quality of the redshift estimation worsens, as high-redshift galaxies with $z > 0.8$ no longer have a passband longwards of the 4000-Å break. It is therefore important to choose the passbands carefully for photometric redshift estimation, according to the redshift ranges under consideration and the depth of the photometric sample.

### 4 TESTING

In this section, we focus on testing our method using photometry for galaxies with known spectroscopic redshifts. Because we wish to focus on the recovery of high-redshift clusters at $z \approx 1.0$, it would be best to test with an extensive data set for a real high-redshift cluster. However, such data are not available at the moment, because we need both multicolour photometry covering the 4000-Å break (at least 3–4 bands) and spectroscopically determined redshifts for individual galaxies. Therefore, we have decided to test

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**Table 4.** Passband choice for random galaxy simulations.

| passbands | all $\sigma(\Delta z)$ | all $\sigma(\text{error})$ | E/S0 $\sigma(\Delta z)$ | E/S0 $\sigma(\text{error})$ |
|-----------|------------------------|---------------------------|-------------------------|---------------------------|
| BVRIK     | 0.065                  | 0.076                     | 0.060                   | 0.074                     |
| RIK       | 0.127                  | 0.135                     | 0.121                   | 0.129                     |
| BVRI      | 0.178                  | 0.199                     | 0.176                   | 0.210                     |

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**Figure 7.** Field galaxies in the Abell 370 cluster field. Estimated redshift vs. spectroscopically determined redshift (left), and redshift error vs. estimated B/T ratio (right). Filled circles indicate E/S0 galaxies, while open circles indicate disc galaxies and those without morphological information.

**Figure 8.** Cluster members of Abell 370. Distribution of the estimated redshifts (left) and redshift error vs. estimated B/T ratio (right). The filled histogram and filled circles indicate E/S0 galaxies, while the open histogram and open circles indicate disc galaxies and those without morphological information. The dotted and dashed histograms show the real redshifts of all types and E/S0s, respectively. The two dotted lines (right) show the region of $|\Delta z| < 0.1$ where galaxies are taken to be cluster members.
our method with two independent sets of data: the well-studied cluster Abell 370 at $z = 0.374$, and the Hubble Deep Field.

4.1 Abell 370

The photometric data are taken from Pickles & van der Kruit (1991). We use their $BVRI$ photometry, using a 7-arcsec aperture. For our use, we selected 59 galaxies which have spectroscopically determined redshifts. The redshifts are mainly taken from Pickles & van der Kruit (1991) and supplementarily from Stanford, Eisenhardt, & Dickinson (1995), although the latter gives only cluster memberships to which we assigned the cluster mean redshift $z = 0.374$. To separate the sample of E/S0 galaxies, we use galaxy morphology as given by HST images (Stanford, Eisenhardt & Dickinson 1995). As Abell 370 has a redshift $z = 0.374$, the filter combination ($BVRI$) is expected to work to pick out cluster members to some extent as they bracket the 4000-Å break (Section 2.2), although the $U$ band is missing, which is important to discriminate between galaxies with lower B/T ratios at the cluster redshift and those with higher B/T ratio at lower redshifts. However, unfortunately, the photometric accuracy is poor, especially in the $B$ and $I$ bands (0.13–0.19 mag in $B$, 0.04–0.08 mag in $V$, 0.03–0.07 mag in $R$ and 0.08–0.16 in $I$).

We estimated the photometric redshifts for our sample galaxies. The results for field galaxies and cluster members are shown separately in Figs. 7 and 8, respectively. The cluster members are defined as those which have spectroscopic redshifts $0.374 - 0.02 < z < 0.374 + 0.02$. As for the E/S0 galaxies, we can estimate the redshifts very well within $|\Delta z| < 0.1$ both for field galaxies and cluster members. On the other hand, some disc galaxies have over- or underestimated redshifts. In Fig. 7, there is a galaxy with $\Delta z > 0.4$. That is because the photometry in the $J$ band of this galaxy is very poor, as indicated in the original table in Pickles & van der Kruit (1991), and in fact if we use only $BVR$ bands for this galaxy, the estimated redshift agrees with the real redshift at the 1.5σ level. The other two field disc galaxies are slightly underestimated by $\Delta z \sim -0.1$. These galaxies have very blue colours in $B - V$ or $V - R$, and it is suggested that they are either disc-dominated galaxies or ones strongly influenced by a recent burst. For the cluster members (Fig. 8) also, there are some galaxies the redshifts of which are underestimated by as much as $\Delta z \sim -0.15$. These galaxies tend to have low estimated B/T ratios and are again degenerate with galaxies at lower redshift. The discrimination between a blue cluster member and a slightly redder galaxy at lower redshift can be difficult, especially when we lack a bluer band corresponding to the far-UV region (2500–3000 Å).

Nevertheless, if we adopt the criteria of defining cluster members as $|\Delta z| < 0.1$, we can pick out most of the cluster members with little field contamination, as shown in Figs 9 and 10. This is especially true for early-type galaxies. As a result, the C–M relation of the E/S0 galaxies is recovered well (Fig. 9). The solid line shows the real relation for the real cluster members while the dashed line shows the estimated relation for the estimated cluster members. We used a biweight fitting method to calculate these C–M relations (Beers, Flynn & Gebhardt 1990). Both relations are nearly identical.

In summary, although the photometric accuracy is not ideal, we can still pick out most of the cluster members in A370, especially early-type galaxies, only photometrically, based on our method. The field contamination is negligibly small. The method has difficulty in recovering cluster members bluer than $V - R = 0.6$, but this would be improved if $U$-band data were available.

![Figure 9](image9.png)  
Figure 9. The C–M diagram for Abell 370. The filled symbols indicate cluster members, while the open symbols show field galaxies. The symbols surrounded by a large circle indicate the estimated cluster members selected using $|\Delta z| < 0.1$. Two error bars at the lower part of the figure indicate the typical 1σ observational errors. The solid and dashed lines indicate the C–M relation of the real and estimated cluster E/S0s, respectively, calculated using biweight fitting to the data.

![Figure 10](image10.png)  
Figure 10. The colour histogram for Abell 370. The solid line shows the real cluster members, and the dotted line shows the real field galaxies. The slantwise hatched area indicates field cluster members, while the black shaded area indicates field contamination. It is shown that most of the cluster members are recovered, although a small number of the bluest galaxies are dropped out. This would be improved if $U$-band data were available. Field contamination is also negligibly small.

4.2 Hubble Deep Field

To further test our method, we apply it to galaxies taken from the Hubble Deep Field (HDF). The galaxies used here are chosen from Cowie’s $K$-selected galaxy sample (http://www.ifa.hawaii.edu/~cowie/ktable.html), all of which have spectroscopic redshifts, mainly from Cohen et al. (1996). Isophotal magnitudes in four HST WFPC2 filters (F300W[U300], F450W[B450], F606W[V606], and F814W[W814]) are taken from Williams et al. (1996). We cross-identify galaxies between Cowie’s catalogue and the photometry catalogue using RA and Dec. We choose only isolated galaxies to avoid misidentification. Also we excluded galaxies with $z > 2$, as the current passbands no longer bracket the 4000-Å break. Photometric errors are calculated from signal-to-noise ratios, but, for
We give a larger minimum error of 0.2 mag to those less than 0.05 mag, we assume a minimum error of 0.05 mag.

Figure 11. HDF galaxies. U300, B450, V606, I814, and J passbands are used. Filled symbols indicate morphologically classified E/S0 galaxies, while the rest show other-type galaxies or unclassified galaxies.

Table 5. Simulated galaxies in a $z = 1$ cluster field.

| type    | n  | $\langle[M/H]\rangle_{\text{bulge}}$ | $\tilde{z}_f$ | B/T  |
|---------|----|-------------------------------------|----------------|------|
| cluster|     |                                     |                |      |
| E/S0    | 10 | 0.06 $-$ 0.52                        | 4.5            | 1.0  |
|         | 10 | 0.06 $-$ 0.52                        | 4.5 $-$ 1.5    | 1.0  |
|         | 10 | 0.06 $-$ 0.52                        | 4.5            | 1.0 $-$ 0.5 |
| Sp      | 20 | 0.06 $-$ 0.52                        | 4.5            | 0.5 $-$ 0.0 |
| field   |     |                                     |                |      |
| E       | 6  | 0.06 $-$ 0.52                        | 4.5            | 1.0 $-$ 0.6 |
| S0      | 26 | 0.06 $-$ 0.52                        | 4.5            | 0.6 $-$ 0.5 |
| Sab     | 17 | 0.06 $-$ 0.52                        | 4.5            | 0.5 $-$ 0.15 |
| Sc—IIm  | 5  | 0.06 $-$ 0.52                        | 4.5            | 0.15 $-$ 0.05 |

5 APPLICATION TO HIGH-REDSHIFT CLUSTERS

We are interested in applying this classifier to high-redshift clusters around $z \approx 1.0$, but, at present, a suitable data set is not available. To show the applicability of the classifier to targets at that redshift, we simulated a $z = 1$ cluster field using the model described in Section 2. Although this is a self-consistency check (most importantly, it assumes that the photometric properties of real galaxies are accurately described by the stellar population synthesis code), it allows us to estimate the biases present in the recovered galaxy samples and to determine how much photometric accuracy and which combination of passbands is required to pick out cluster members effectively in such a cluster.

We generated field galaxies using the type-dependent prior distribution outlined in Section 3.2. The metallicity of the bulge was chosen so that $M_{V,\text{bulge}} = -23, -20$ and $-17$ measured at $z = 0$, corresponding to $\langle[M/H]\rangle_{\text{bulge}} = 0.06, -0.23$ and $-0.52$. Here we have simulated a $K$-limited galaxy sample with $m_K < 20$ for a 1-arcmin² field of view which corresponds to $0.5 \text{ Mpc} \times 0.5 \text{ Mpc}$ at $z = 1$, using the prior distribution, taking the type-dependent LF into account. The number of galaxies in each type is summarized in the lower half of Table 5. As for the cluster members, we assumed the mix of galaxy populations given in upper half of Table 5, i.e. 10 E/S0s from a metallicity sequence, another 10 E/S0s from an age sequence, another 10 E/S0s from a B/T sequence, and finally 20 disc galaxies were added in. First, by using $K$-band luminosity functions of high-redshift cluster galaxies (mean redshift 0.43) which are assigned in the respective range given in Table 5. Then we can
assign its bulge metallicity using its $M_{V}^{\text{bulge}}$ at $z = 0$ calculated from $M_{K}$ at $z = 0.43$. If a galaxy has $m_{K} > 20$ at $z = 1$, it is rejected from our sample, and the process is repeated until we finally obtain 50 cluster galaxies in total. We assigned the model magnitudes in various bands for each galaxy both in the field and in the cluster. A Gaussian photometric error with $\sigma = 0.071$ mag in all bands. Filled circles indicate E/S0 galaxies defined as $B/T \geq 0.5$, while open circles indicate disc galaxies with $B/T < 0.5$.

Figure 12. Field galaxies in the simulated cluster field at $z = 1$. Plotted are the estimated redshift vs. input real redshift (left), and redshift error vs. input real B/T ratio (right). Redshifts are estimated using $VRJK$ passbands with random Gaussian photometric errors of $\sigma = 0.071$ mag in all bands. Filled circles indicate E/S0 galaxies defined as $B/T \geq 0.5$, while open circles indicate disc galaxies with $B/T < 0.5$.

Figure 13. Cluster members in the simulated cluster field at $z = 1$. Distribution of the estimated redshifts (left) and redshift error vs. input real B/T ratio (right). Caption for the lines and the symbols are the same as Fig. 8.

Figure 14. The C–M diagram for a simulated cluster field at $z = 1$. Caption is the same as Fig. 9. Note that the real C–M relation and the estimated one are identical (solid line).

Figure 15. The colour histogram of a simulated cluster field at $z = 1$. The solid line shows the real cluster members, and the dotted line shows the real field galaxies. The slantwise hatched area indicates recovered cluster members, while the black shaded area indicates field contamination. There are few dropped-out members and little field contamination, and importantly they have no bias in colours. Gaussian photometric error with $\sigma = 0.071$ is added on each colour of each galaxy. We then regard these generated photometric data as the observational ones for the $z = 1$ cluster field, and the redshift classifier is applied to each galaxy to estimate a redshift.
5.1 Biases in the recovered galaxy properties

The results for the field galaxies and the cluster members are shown separately in Figs. 12 and 13. We used VRK colours to estimate the redshifts. The B band was not used, because at $z = 1$ it falls in the far-UV spectral region, well below 2500 Å. The overall agreement between the estimated redshift and the real redshift is excellent. Most of the galaxy redshifts are well recovered within $|\Delta z| = 0.1$, regardless of real redshift and irrespective of galaxy type. As a result, as shown in Fig 14, the recovery of cluster members is magnificent. Here we adopted the criterion of cluster members as $|\Delta z| < 0.1$, considering the photometric accuracy (see Fig. 6). It is also adequate because it does not pick up large amount of field contamination at high redshifts. In this case, only one cluster spiral is dropped out. Field contamination is also negligible (only four galaxies). We have recovered not only old ellipticals, but also young or star-forming ellipticals and spirals as well. To see the bias in the identification of cluster members as a function of galaxy colour, we show the colour histogram of the recovered cluster members and the field contamination in Fig. 15. As is clear from the figure, there is no colour bias at all in either the cluster or the field. Consequently, we recover the C–M relation of E/S0 galaxies very well (identically in this case), as shown by the solid line in the figure. The biweight scatters around the relation are also calculated and given in Table 6 as well as the values of the C–M slope. The numbers for Abell 370 are also given in the same table. Both scatters and slopes are almost correctly estimated irrespective of galaxy type.

All the above results are encouraging. If the models were perfect, we could assign redshift with 0.1 accuracy or, better, with $<0.1$ mag photometric errors in all bands. With this success, we will be able to extend the C–M relation analysis (e.g. Kodama et al. 1998; Ellis et al. 1997; Stanford, Eisenhardt & Dickinson 1998) to $z \geq 1$ clusters without taking spectroscopic redshifts. Importantly, we can pick out cluster galaxies with various stellar populations, i.e. not only passively evolving old galaxies but also the galaxies that have a significant contribution from younger stellar populations. This is encouraging, as it is important to select the cluster members with as little bias as possible. Our method will allow us to conduct the colour scatter analysis around the C–M relation reliably, and to look into the age dispersion of cluster galaxies at high redshifts, if any.

5.2 Optimal passbands for cluster identification

In Table 7, we summarize the effect of passband choice on the estimated redshift error. Photometric errors of 0.071 mag are assumed in all passbands. Two cases in particular are found to be ideal: BVRK and VRK. The redshift errors are always well below 0.1 regardless of galaxy type, and hence the number of galaxies dropped out of our cluster sample and the field contamination are minimized. It is encouraging that we can do comparably well without the B band, as it is good to minimize the use of passbands shortwards of rest-frame 2500 Å where possible. It should be noted that VRK and VRK work comparably well, although $\sigma$(error) is larger, especially for galaxies with ongoing star formation. This is because it becomes difficult to separate the effects of changes in stellar population and those of changing redshift. If both the B and V bands are missing (and we have RIK only), we tend to underestimate the redshift of bluer galaxies. This is analogous to lacking the U band for Abell 370: it becomes difficult to disentangle galaxy type and redshift without the UV colours for star-forming galaxies. For the early-type galaxies, however, the redshift errors are reasonable, as we would expect from Fig. 2. In contrast, if the K band is missing, the errors in the redshift estimation become much larger, irrespective of galaxy type, as there is no passband longwards of the rest-frame 4000 Å break. In this context, it is crucial for high-redshift work to have both optical and near-infrared passbands for accurate redshift estimation.

6 SUMMARY

We present a new photometric redshift estimator, which is optimized for the identification and study of galaxy clusters at high redshifts. We use only several broad passbands covering the 4000-Å break, and find in practice that it is possible to avoid the use of the uncertain colours shortwards of rest-frame 2500 Å. In our models, we considered as wide a variety of stellar populations as possible, to minimize the selection bias in the recovered cluster members. As most of the effects of changing stellar population on the integrated colours are highly degenerate, we find that it is possible to estimate redshifts with reasonable accuracy for a range of galaxy types, ranging from those with old, passively evolving stellar populations, through to those with younger stellar populations and ongoing star formation.

Following the success in testing our method with data from

Table 6. The biweight scatters and slopes of the C–M relation. The values for all types and E/S0 galaxies are shown separately. The scatters are measured with respect to the C–M relation of E/S0 galaxies only.

|          | Abell 370 |         |                  |         |                  |
|----------|-----------|---------|------------------|---------|------------------|
|          | all       | E/S0    | all              | E/S0    |
| scatter  | real      | 0.125   | 0.073            | 0.613   | 0.169            |
|          | estimated | 0.112   | 0.072            | 0.603   | 0.189            |
| slope    | real      | -0.018  | -0.008           | -0.423  | -0.165           |
|          | estimated | -0.018  | -0.016           | -0.381  | -0.165           |

Table 7. Passband choice for a simulated cluster field at $z = 1$. Numbers of dropped-out members and field contamination are also presented. Percentage of dropped members and field contamination are defined per real cluster members and per estimated cluster members, respectively.

| passbands | all       | E/S0    | dropped members | field contamination |
|-----------|-----------|---------|-----------------|---------------------|
|           | $\sigma(\Delta z)$ | $\sigma$(error) | $\sigma(\Delta z)$ | $\sigma$(error) | all       | all       | E/S0      | E/S0      | all       | all       |
| BVRIK     | 0.070     | 0.074   | 0.068           | 0.074               | 3 (6%)     | 3 (10%)   | 5 (10%) | 4 (13%) |
| VRK       | 0.079     | 0.099   | 0.064           | 0.095               | 1 (2)      | 0 (0)     | 4 (8)   | 3 (9)   |
| VIK       | 0.141     | 0.129   | 0.069           | 0.112               | 7 (14)     | 1 (3)     | 4 (9)   | 3 (9)   |
| VRK       | 0.114     | 0.131   | 0.087           | 0.129               | 4 (8)      | 2 (7)     | 3 (6)   | 3 (10)  |
| VRK       | 0.231     | 0.232   | 0.243           | 0.216               | 9 (18)     | 3 (10)    | 10 (20) | 7 (21)  |
| RIK       | 0.155     | 0.133   | 0.074           | 0.118               | 16 (32)    | 3 (10)    | 3 (8)   | 3 (10)  |
| RIK       | 0.211     | 0.212   | 0.159           | 0.216               | 38 (76)    | 21 (70)   | 7 (37)  | 4 (31)  |
Abell 370 and from the HDF, we applied it to a simulated cluster at $z = 1$. We have shown that the estimation of redshifts with accuracies better than $|\Delta z| < 0.1$ can be achieved with multipassband photometry of moderate quality ($\sim 0.1$ mag) in a small number of passbands, and the cluster members can be reliably identified. Therefore, the recovery of the C–M relation both in terms of the slope and scatter is expected to be accurate and almost free from any selection bias. We now have a means of analysing the photometric properties of cluster galaxies at very high redshifts without a thorough spectroscopic membership confirmation.

ACKNOWLEDGMENTS

We thank C. Tadhunter and S. Warren for useful discussion. We are also grateful to the anonymous referee who gave us many constructive comments. TK thanks JSPS Postdoctoral Fellowships for Research Abroad for financial support. TK is also grateful for the temporary stay at University of Durham in 1997 July, when this work was partly carried out. EFB thanks the Isle of Man Education Department for their generous financial support. This project made use of Starlink computing facilities at Durham and Cambridge.

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