Low-energy effective actions in
three-dimensional extended SYM theories

I.L. Buchbinder ∗, N.G. Pletnev +, I.B. Samsonov∗ 1

∗Department of Theoretical Physics, Tomsk State Pedagogical University,
634061 Tomsk, Russia, email: joseph@tspu.edu.ru

+Department of Theoretical Physics, Institute of Mathematics, 630090 Novosibirsk, Russia
email: pletnev@math.nsc.ru

∗INFN, Sezione di Padova, 35131 Padova, Italy
email: samsonov@mph.phtd.tpu.ru

Abstract

We develop the background field method in the $\mathcal{N} = 2$, $d = 3$ superspace
for studying effective actions in three-dimensional SYM models which live in the
world-volume of various 2-branes. In particular, the low-energy effective action for
the $\mathcal{N} = 2$ quiver gauge theory with four chiral superfields in the bifundamental
representation is studied. This gauge theory describes the D2 brane probing the
conifold singularity. Surprisingly, the leading terms in this effective action reproduce
the classical action of the Abelian ABJM theory confirming the fact that the M2
brane can be considered as the effective theory for the D2 brane at strong coupling.
Apart from this $\mathcal{N} = 2$ quiver gauge theory we study the low-energy effective
action in pure $\mathcal{N} = 2$, $\mathcal{N} = 4$ and $\mathcal{N} = 8$ SYM theories with gauge group SU($N$)
spontaneously broken down to an Abelian subgroup. In particular, for the $\mathcal{N} = 4$
SYM we find similar correspondence between the leading terms in its effective action
and the classical action of the Abelian Gaioetto-Witten theory.

1On leave from Tomsk Polytechnic University, 634050 Tomsk, Russia
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1 Introduction

The models living in the world-volume of 2-branes in the ten or eleven dimensional supergravity can be described by three-dimensional supersymmetric gauge theories with extended supersymmetry. The well-known example is the $\mathcal{N} = 8, d = 3$ SYM theory which describes a stack of D2 branes in flat background. Quantizing the $\mathcal{N} = 8, d = 3$ SYM theory helps to understand the effective quantum dynamics of the D2 branes. Another very interesting example is the model of multiple M2 branes whose world-volume field theory was constructed quite recently in the series of papers [1, 2]. These models are usually referred to as the Bagger-Lambert-Gustavsson (BLG) or Aharony-Bergman-Jafferis-Maldacena (ABJM) theories. It is expected that quantizing the ABJM and BLG theories will shed some light on quantum dynamics of multiple M2 branes.

Thanks to the standard Higgs mechanism, there is natural separation between massless and massive degrees of freedom in the supergauge theories living on D-branes. As a result, one can define the low-energy effective action which depends on the massless superfields while the massive ones appear only as the internal lines in the quantum diagrams. Such a low-energy effective action for the massless superfields is usually well-defined and there are powerful methods of quantum field theory for computing it both in components and in superspace [3, 4].

However, for supergauge theories living on M2 branes there is no natural separation between massive and massless superfields and the low-energy effective action for such models is not so well understood. Indeed, compared with the D-branes, the Higgs mechanism for the M2 brane works in a different way. In [5] it is shown that when one of the scalars in the BLG or ABJM action develops non-vanishing vev the M2 brane turns into D2 brane described by the $\mathcal{N} = 8$ SYM, up to the terms negligible at large vev. Hence, it is natural to study the low-energy effective action for the supergauge models on D2 branes rather than for the ABJM theory itself. Moreover, it is well-known that the M2 brane can be considered in general as the infrared limit of the D2 brane which is usually achieved at strong gauge coupling (see, e.g., [6, 7] for recent discussions). Therefore investigating strong coupling limit for the low-energy effective action in the supergauge models describing D2 branes should help to understand some quantum aspects of M2 branes.

In the present paper we study the effective action in various three-dimensional SYM gauge theories with matter which live in the world-volume of D2 branes on some backgrounds. In particular, we compute the one-loop effective action in the $\mathcal{N} = 8, d = 3$ SYM theory which corresponds to the D2 branes in flat space-time and in a quiver $\mathcal{N} = 2$ SYM interacting with four bifundamental chiral superfields which corresponds to the D2 brane on a conifold near the singularity [7, 8, 9]. For the last model we show that the leading terms in its one-loop effective action exactly reproduce the classical action of Abelian ABJM theory in a dualized form (when one of the chiral superfields is dualized into the gauge superfield). This result indicates that the ABJM-like models can be thought as the effective quantum field theories for D2 branes in an appropriate background. Similar correspondence is established between the leading terms in the $\mathcal{N} = 4$ SYM action and dualized classical action of the Abelian Gaiotto-Witten model [10].
The main tool for studying the effective action for the models under consideration is the background field method in the $\mathcal{N} = 2$, $d = 3$ superspace which goes along the same lines as the background field method in the $\mathcal{N} = 1$, $d = 4$ superspace given in details in [4]. In Section 2 we review this method and apply it to study the low-energy effective action in pure $\mathcal{N} = 2$ SYM theory. In the next section we apply the $\mathcal{N} = 2$ background field method to the $\mathcal{N} = 4$ and $\mathcal{N} = 8$ SYM theories for which we compute the one-loop effective action for the gauge group $SU(N)$ spontaneously broken down to an Abelian subgroup. In Section 4 we study the $\mathcal{N} = 2$ quiver gauge theories with two and four chiral superfields in the bifundamental representation of the $SU(2) \times SU(2)$ gauge group. For the last model it is shown that the leading terms in its one-loop effective action precisely reproduce the classical action of Abelian ABJM theory in a dual form. Section 5 is devoted to some discussions of the results. In the Appendix A we consider the actions of the Gaiotto-Witten and ABJM models in the case when one of the chiral superfields id dualized into a gauge superfield. Some details of one-loop computations of effective actions are collected in the Appendix B.

2 The $\mathcal{N} = 2$ background field method

2.1 $\mathcal{N} = 2$, $d = 3$ SYM setup

We follow the $\mathcal{N} = 2$, $d = 3$ superspace conventions used in our previous paper [11]. In particular, the gauge covariant derivatives

$$\nabla_\alpha = D_\alpha + A_\alpha, \quad \nbar{\nabla}_\alpha = \nbar{D}_\alpha + \nbar{A}_\alpha, \quad \nabla_m = \partial_m + A_m \quad (2.1)$$

satisfy the following algebra [12, 13]

$$\{\nabla_\alpha, \nbar{\nabla}_\beta\} = -2i(\gamma^m)_{\alpha\beta} \nabla_m + 2i\varepsilon_{\alpha\beta} G, \quad (2.2)$$

$$[\nabla_\alpha, \nabla_m] = -(\gamma_m)_{\alpha\beta} \nbar{W}^\beta, \quad [\nbar{\nabla}_\alpha, \nabla_m] = (\gamma_m)_{\alpha\beta} W^\beta, \quad (2.3)$$

$$[\nabla_m, \nabla_n] = iF_{mn}. \quad (2.4)$$

Here $G$, $W_\alpha$, $\nbar{W}_\alpha$ and $F_{mn}$ are non-Abelian superfield strengths subject to the Bianchi identities. In particular, the superfield $G$ is Hermitian and is covariantly linear,

$$\nabla^\alpha \nabla_\alpha G = 0, \quad \nbar{\nabla}^\alpha \nbar{\nabla}_\alpha G = 0. \quad (2.5)$$

The superfields $W_\alpha$ and $\nbar{W}_\alpha$ are covariantly (anti)chiral

$$\nabla_\alpha W_\beta = 0, \quad \nbar{\nabla}_\alpha \nbar{W}_\beta = 0 \quad (2.6)$$

and satisfy ‘standard’ Bianchi identity

$$\nabla^\alpha W_\alpha = \nbar{\nabla}^\alpha \nbar{W}_\alpha. \quad (2.7)$$
These superfield strengths are expressed in terms of $G$ as

$$
\bar{W}_\alpha = \nabla_\alpha G, \quad W_\alpha = \bar{\nabla}_\alpha G.
$$

(2.8)

We prefer to introduce the gauge prepotential in the so-called chiral representation in which the Grassmann derivative $\bar{D}_\alpha$ does not receive the connection,

$$
\nabla_\alpha = e^{-2V} D_\alpha e^{2V}, \quad \bar{\nabla}_\alpha = \bar{D}_\alpha, \quad V^\dagger = V.
$$

(2.9)

In this representation the superfield strengths are expressed in terms of the prepotential $V$ as

$$
G = \frac{i}{4} \bar{D}^\alpha (e^{-2V} D_\alpha e^{2V}), \quad \bar{W}_\alpha = \frac{i}{4} \nabla_\alpha \bar{D}^\beta (e^{-2V} D_\beta e^{2V}), \quad W_\alpha = -\frac{i}{8} \bar{D}^2 (e^{-2V} D_\alpha e^{2V}).
$$

(2.10)

They are covariant under the following gauge transformations

$$
e^{2V} \to e^{i\bar{\lambda}} e^{2V} e^{-i\lambda},
$$

(2.11)

or, in the infinitesimal form,

$$
\delta V = -\frac{i}{2} L_V (\bar{\lambda} + \lambda) + \frac{i}{2} L_V \coth(L_V)(\bar{\lambda} - \lambda),
$$

(2.12)

where $\lambda$ and $\bar{\lambda}$ are chiral and antichiral superfields, respectively, and $L_V$ denotes the commutator, e.g., $L_V \lambda = [V, \lambda]$.

The classical action of the $\mathcal{N} = 2$ SYM in the $\mathcal{N} = 2$ superspace reads

$$
S_{\mathcal{N}=2}[V] = \frac{1}{g^2} \mathrm{tr} \int d^3x d^4\theta G^2 = -\frac{1}{2g^2} \mathrm{tr} \int d^3x d^2\theta W^\alpha W_\alpha,
$$

(2.13)

where $g$ is the dimensionfull coupling constant, $[g] = 1/2$.

### 2.2 Structure of the one-loop effective action in the $\mathcal{N} = 2$ SYM

Within the background field method the gauge superfield $V$ is splitted into the ‘background’ $V$ and ‘quantum’ $v$ parts \(^2\)

$$
e^{2V} \to e^{2V} e^{2gv},
$$

(2.14)

so that

$$
\nabla_\alpha = e^{-2gv} D_\alpha e^{2gv}, \quad \bar{\nabla}_\alpha = \bar{D}_\alpha,
$$

(2.15)

\(^2\)For the background gauge superfield we use the same letter as for the original gauge superfield $V$. We hope that this will not lead to any confusions because when the background-quantum splitting is done, the original gauge superfield never appears in the calculation and it is not necessary to reserve a special letter for it.
where
\[ \mathcal{D}_\alpha = e^{-2V}D_\alpha e^{2V}, \quad \bar{\mathcal{D}}_\alpha = \bar{D}_\alpha \] (2.16)
are the background gauge covariant spinor derivatives. There is a freedom in defining the
gauge transformations for the background and quantum superfields. In particular, one
can consider the so-called ‘background’ gauge transformations
\[ e^{2V} \to e^{i\lambda}e^{2V}e^{-i\lambda}, \quad e^{2gv} \to e^{ir}e^{2gv}e^{-ir} \] (2.17)
and the ‘quantum’ ones,
\[ e^{2V} \to e^{2V}, \quad e^{2gv} \to e^{i\lambda}e^{2gv}e^{-i\lambda}. \] (2.18)
Here \( \lambda \) and \( \bar{\lambda} \) are (anti)chiral gauge parameters while \( \tau \) is real.

Upon such a background-quantum splitting the superfield strengths can be decom-
posed in series over the coupling constant \( g \),

\[ W_\alpha \to W_\alpha - \frac{i}{8}\bar{D}^2(2gD_\alpha v - 2g^2[v, D_\alpha v] + O(g^3)), \] (2.19)
\[ G \to G + \frac{i}{2}g\bar{D}^2D_\alpha v - \frac{i}{2}g^2\bar{D}^\alpha[v, D_\alpha v] + O(g^3), \] (2.20)
where the superfield strengths \( W_\alpha \) and \( G \) in the right hand sides are constructed now from
the background gauge superfield by the rules (2.10). The classical \( \mathcal{N} = 2 \) SYM action
(2.13) can be written as
\[ S_{\mathcal{N}=2} \rightarrow S_{\mathcal{N}=2}[V] + \frac{i}{g}\text{tr} \int d^3xd^4\theta v\bar{D}^2W_\alpha + S_2[V, v] + O(g), \] (2.21)
\[ S_2[V, v] = -\text{tr} \int d^3xd^4\theta v[-\frac{1}{8}D^\alpha\bar{D}^2D_\alpha + iW^\alpha D_\alpha + \frac{i}{2}(D^\alpha W_\alpha)]v. \] (2.22)
The action \( S_{\mathcal{N}=2}[V] \) in the rhs of (2.21) is the same as (2.13), but it depends on the
background superfield only. In the decomposition (2.21) we do not specify the terms with
positive powers of the coupling constant in the classical action since they are necessary
only for higher-loop computations. In the present study we restrict ourself to the one-loop
effective action which is specified by the quadratic action \( S_2 \).

Within the background field method one can usually fix the quantum gauge symmetry
(2.18) keeping the invariance under the background transformations. The corresponding
gauge fixing functions
\[ f = i\bar{D}^2v, \quad \bar{f} = i\bar{D}^2v \] (2.23)
are defined with the help of the background-dependent covariant spinor derivatives. These
functions are covariantly (anti)chiral and change under the quantum gauge transformations (2.18) as
\[ \delta f = \frac{1}{2g}\bar{D}^2L_{gv}[\bar{\lambda} + \lambda + \coth(L_{gv})(\lambda - \bar{\lambda})]. \] (2.24)
Therefore the ghost superfield action has the standard form,
\[ S_{gh} = \text{tr} \int d^3 x d^4 \theta (b + \bar{b}) L_{ge}[c + \bar{c} + \coth(L_{ge})(c - \bar{c})] = \text{tr} \int d^3 x d^4 \theta (\bar{b}c - b\bar{c}) + O(g) . \]  

The one-loop effective action is given by the following functional integral
\[ e^{i\Gamma_{\mathcal{N}=2}[V]} = e^{iS_{\mathcal{N}=2}[V]} \int \mathcal{D} v \mathcal{D} b \mathcal{D} c \delta[f - i\mathcal{D}^2 v] \delta[\bar{f} - i\mathcal{D}^2 v] e^{iS_2[V,v] + iS_{gh}} . \]  

We average this expression with the weight
\[ 1 = \int \mathcal{D} f \mathcal{D} \varphi e^{\text{itr} \int d^3 x d^4 \theta [\frac{1}{8\alpha} ff - \bar{f}\varphi\varphi]} , \]  

where \( \alpha \) is the gauge-fixing parameter and \( \varphi \) is the Nielsen-Kallosh ghost. It is anti-commuting and covariantly chiral superfield. This leads to the gauge-fixing and Nielsen-Kallosh ghost actions,
\[ S_{gf} = -\frac{1}{16\alpha} \text{tr} \int d^3 x d^4 \theta v \{\mathcal{D}^2, \bar{\mathcal{D}}^2\} v , \quad S_{NK} = -\text{tr} \int d^3 x d^4 \theta \bar{\varphi}\varphi . \]  

In the Fermi-Feynman gauge \( \alpha = 1 \) and the quadratic part of the action with respect to the quantum superfields takes simple form,
\[ S_2 + S_{gf} = -\text{tr} \int d^3 x d^4 \theta v \Box v , \]  

where
\[ \Box = \frac{1}{8} \mathcal{D}^\alpha \bar{\mathcal{D}}^\alpha + \frac{1}{16} \{\mathcal{D}^2, \bar{\mathcal{D}}^2\} + \frac{i}{2}(\mathcal{D}^\alpha W_\alpha) + iW^\alpha \mathcal{D}_\alpha \]
\[ = D^m \mathcal{D}_m + G^2 + iW^\alpha \mathcal{D}_\alpha - iW^\alpha \bar{\mathcal{D}}_\alpha \]  

is the covariant d'Alembertian operator in the space of real superfields. As a result, we get the following representation for the one-loop effective action
\[ e^{i\Gamma_{\mathcal{N}=2}[V]} = e^{iS_{\mathcal{N}=2}[V]} \int \mathcal{D} v \mathcal{D} b \mathcal{D} c \mathcal{D} \varphi e^{-\text{itr} \int d^3 x d^4 \theta v \Box v + iS_{gh} + iS_{NK}} . \]  

Schematically, it can be written as
\[ \Gamma_{\mathcal{N}=2} = \Gamma_v + \Gamma_{\text{ghosts}} , \quad \Gamma_v = \frac{i}{2} \text{Tr}_v \ln \Box_v , \quad \Gamma_{\text{ghosts}} = -\frac{3i}{2} \text{Tr}_+ \ln \Box_+ . \]  

The contribution \( \Gamma_v \) to the one-loop effective action comes from the quantum gauge superfield while \( \Gamma_{\text{ghosts}} \) is due to ghosts. Here Tr\( _v \) and Tr\( _+ \) are the functional traces of the operators acting in the spaces of real and chiral superfields, respectively. The operator \( \Box_+ \) is the covariant d’Alembertian operator acting in the space of covariantly chiral superfields which was introduced in [11],
\[ \Box_+ = \mathcal{D}^m \mathcal{D}_m + G^2 + \frac{i}{2}(\mathcal{D}^\alpha W_\alpha) + iW^\alpha \mathcal{D}_\alpha . \]  

The explicit expressions for the traces of these operators can be found after one specifies the gauge group and the corresponding background gauge superfield. Further we consider one example when the gauge group is SU(\( N \)) although the other simple Lie groups can be studied in a similar way.
2.3 Effective action for the SU($N$) gauge group

We will be interested in the low-energy effective action which is a functional for the massless fields obtained by integrating out all massive fields in a functional integral. In gauge theories the separation between massless and massive fields appears usually through the Higgs mechanism. In general, the gauge group SU($N$) is spontaneously broken down to its maximal Abelian subgroup, U(1)$^{N-1}$. However, in particular cases a bigger subgroup of SU($N$) can be unbroken. Physically interesting to consider minimal gauge symmetry breaking, SU($N$) → SU($N-1$) × U(1) because, from the point of view of D-branes, the corresponding effective action contains the potential which appears when one separates one D-brane from the stack. In this section we will consider first the general case when the gauge group is broken down to the maximal torus and then comment on the effective action with minimal gauge symmetry breaking.

The Lie algebra $\text{su}(N)$ consists of Hermitian traceless matrices. Any element $v$ of $\text{su}(N)$ can be represented by a decomposition over the Cartan-Weil basis in the $\text{gl}(N)$ algebra,

$$
(e_{IJ})_{LK} = \delta_{IL}\delta_{JK}, \quad v = \sum_{I<J}^N (v_{IJ}e_{IJ} + \bar{v}_{IJ}e_{JI}) + \sum_{I=1}^N v_Ie_{II}, \quad \bar{v}_I = v_I, \quad \sum_{I=1}^N v_I = 0.
$$

The background gauge superfield $V$ belongs to the Cartan subalgebra spanned on $e_{II}$,

$$
V = \sum_{I=1}^N V_I e_{II} = \text{diag}(V_1, V_2, \ldots, V_N), \quad \bar{V}_I = V_I, \quad \sum_{I=1}^N V_I = 0.
$$

In what follows we will denote by boldface Latin letters the matrix elements of the background superfields. In particular, each matrix element $V_I$ of $V$ has superfield strength $G_I = \frac{i}{2} \bar{D}^\alpha D_\alpha V_I$ which is computed as in the Abelian case. We will use also the following notations

$$
V_{IJ} = V_I - V_J, \quad G_{IJ} = G_I - G_J, \quad W_{IJ\alpha} = W_{I\alpha} - W_{J\alpha}.
$$

Now we can do the matrix trace in the action (2.29),

$$
S_2 + S_{gf} = -2 \sum_{I<J}^N \int d^3x d^4\theta v_{IJ} \hat{\Box}_v v_{IJ},
$$

where $\hat{\Box}_v$ is the Abelian version of the operator (2.30) which is constructed from the Abelian gauge superfield $V_{IJ}$ and its superfield strengths,

$$
\hat{\Box}_v = \mathcal{D}^m \mathcal{D}_m + G_{IJ}^2 + i W_{IJ\alpha}^\alpha \mathcal{D}_\alpha - i W_{IJ\alpha}^\alpha \bar{\mathcal{D}}_\alpha.
$$

Therefore the effective action $\Gamma_v$ can be written as

$$
\Gamma_v = i \sum_{I<J}^N \text{Tr}_v \ln \hat{\Box}_v v_{IJ},
$$
where $\text{Tr}_v$ means now only the functional trace in the space of real superfields.

In a similar way one can analyze the contributions from the ghost superfields. Consider, for instance, the action for the Nielsen-Kallosh ghost (2.28) in which the chiral superfields are expanded over the basis (2.34) as

$$\varphi = \sum_{I \neq J}^N e_{IJ} \varphi_{IJ}, \quad \bar{\varphi} = \sum_{I \neq J}^N e_{IJ} \bar{\varphi}_{IJ}. \quad (2.40)$$

Here we omit the diagonal components because they do not interact with the background gauge superfield (2.35) and do not contribute to the effective action. Then the matrix trace in the action (2.28) is done,

$$S_{NK} = -\sum_{I \neq J}^N \int d^3 x d^4 \theta \bar{\varphi}_{IJ} \varphi_{IJ}, \quad (2.41)$$

where the superfields $\bar{\varphi}_{IJ}$ are covariantly antichiral,

$$e^{-2V_{IJ}} D_\alpha e^{2V_{IJ}} \bar{\varphi}_{IJ} = 0 \text{ for } I < J, \quad e^{2V_{IJ}} D_\alpha e^{-2V_{IJ}} \bar{\varphi}_{IJ} = 0 \text{ for } I > J. \quad (2.42)$$

We see that the chiral superfields appear in pairs with positive and negative charges with respect to the Abelian gauge superfield $V_{IJ}$. This prevents the generation of the Chern-Simons term in the one-loop computations (there is no parity anomaly [14], see also [11] for recent superspace calculations). As a result, the effective action for the ghost superfields reads

$$\Gamma_{\text{ghosts}} = -\frac{3i}{N} \sum_{I < J} \text{Tr}_+ \ln \hat{\Box}_{IJ}, \quad (2.43)$$

where $\hat{\Box}_{IJ}$ is the Abelian version of the operator (2.33) constructed from the gauge superfield $V_{IJ}$,

$$\hat{\Box}_{IJ} = D_\alpha D^\alpha + G^{IJ}_2 + i \left( D_\alpha W_{IJ \alpha} \right) + i W^\alpha_{IJ} D_\alpha, \quad (2.44)$$

and $\text{Tr}_+$ denotes the functional trace in the space of chiral superfields.

To do the explicit quantum computations of traces of logarithms in (2.39) and (2.43) we have to specify the constraints on the background Abelian superfields:

(i) The matrix components of the background gauge superfield $V_{IJ}$ obey the $\mathcal{N} = 2$ supersymmetric Maxwell equations,

$$D^\alpha W_{IJ \alpha} = 0. \quad (2.45)$$

(ii) We study the effective action in the so-called long-wave approximation in which the space-time derivatives of the background are neglected,

$$\partial_m G_{IJ} = \partial_m W_{IJ \alpha} = \partial_m \bar{W}_{IJ \alpha} = 0. \quad (2.46)$$
For such a background the heat kernels of the operators (2.38) and (2.44) are known, see [11]. In the present notations they read

\[
K_{vIJ}(z,z'|s) = \frac{1}{8(i\pi s)^{3/2} \sinh(sB_{IJ})} sB_{IJ} e^{iB_{IJ} e^{4/(F_{IJ} \coth sF_{IJ})} e^{sF_{IJ}} sF_{IJ}} (sF_{IJ}) \zeta^2(s)(2.47)
\]

\[
K_{+IJ}(z,z'|s) = -\frac{1}{4} \overline{\partial}^2 K_{vIJ}(z,z'|s),
\]

where \( B_{IJ}^2 = \frac{1}{2} D_\alpha W_{IJ}^\beta D_\beta W_{IJ}^\alpha \). In fact, for the one-loop computations we need these expressions only at coincident superspace points,

\[
K_{vIJ}(s) \equiv K_{vIJ}(z,z|s) = \frac{1}{(i\pi)^{3/2}} \frac{1}{\sqrt{s}} \frac{W_{IJ}^2 \overline{W}_{IJ}^2}{B_{IJ}^3} e^{isG_{IJ}^2} \tanh \frac{sB_{IJ}}{2} \sinh^2 \frac{sB_{IJ}}{2} (2.49)
\]

\[
K_{+IJ}(s) \equiv K_{+IJ}(z,z|s) = \frac{1}{8(i\pi s)^{3/2}} s^2 W_{IJ}^2 e^{isG_{IJ}^2} \tanh(sB_{IJ}/2).
\]

The corresponding contributions to the effective action from these heat kernels are given by

\[
\Gamma_v = -i \sum_{I<J}^N \int_0^\infty ds \frac{d}{s} \int d^3 x d^4 \theta K_{vIJ}(s), \quad \Gamma_{\text{ghosts}} = -3i \sum_{I<J}^N \int_0^\infty ds \frac{d}{s} \int d^3 x d^2 \theta K_{+IJ}(s),
\]

or, explicitly,

\[
\Gamma_v = -\frac{1}{\pi} \sum_{I<J}^N \int d^3 x d^4 \theta \int_0^\infty ds \frac{d}{s \sqrt{i\pi s}} \frac{W_{IJ}^2 \overline{W}_{IJ}^2}{B_{IJ}^3} e^{isG_{IJ}^2} \tanh \frac{sB_{IJ}}{2} \sinh^2 \frac{sB_{IJ}}{2},
\]

\[
\Gamma_{\text{ghosts}} = -\frac{3}{2\pi} \sum_{I<J}^N \int d^3 x d^4 \theta \left[ G_{IJ} \ln G_{IJ} \right.
\]

\[
+ \frac{1}{4} \int_0^\infty ds \frac{d}{\sqrt{i\pi s}} \frac{e^{isG_{IJ}^2} W_{IJ}^2 \overline{W}_{IJ}^2}{B_{IJ}^3} \left( \tanh \frac{sB_{IJ}/2}{sB_{IJ}/2} - 1 \right) \left], \quad (2.53)
\]

where in the expression for \( \Gamma_{\text{ghosts}} \) we restored the full superspace measure. The sum of the expressions (2.52) and (2.53) gives us the resulting one-loop effective action in the pure \( \mathcal{N} = 2 \) SYM theory for the gauge group \( SU(N) \) spontaneously broken down to \( U(1)^{N-1} \). We point out that only the leading \( G \ln G \) term in the \( \mathcal{N} = 2 \) SYM effective action was obtained in [15] using the duality transformations while the explicit quantum computations allow us to find all higher-order \( F^{2n} \) terms encoded in the proper-time integrals (2.52) and (2.53).

In conclusion of this section let us comment on the case of minimal gauge symmetry breaking \( SU(N) \rightarrow SU(N - 1) \times U(1) \). In this case it is convenient to choose the background gauge superfield in the following form

\[
V = \frac{1}{N} \text{diag} \left( (N - 1) V, -V, \ldots, -V \right), \quad (2.54)
\]
where $V$ is Abelian gauge superfield with the superfield strengths $G$, $W_\alpha$ and $\bar{W}_\alpha$. One can easily repeat all the above considerations for such a background or just extract the answer from (2.52) and (2.53) by substituting the corresponding expressions for $V_{IJ}$. For simplicity, we give here only two leading terms in the corresponding effective action

$$\Gamma_{N=2} = \frac{-3(N-1)}{2\pi} \int d^3x d^4\theta \ln G + \frac{9(N-1)}{128\pi} \int d^3x d^4\theta \frac{W^2 \bar{W}^2}{G^5} + \ldots.$$  \hfill (2.55)

The first term in the rhs is responsible for the $N=2$ supersymmetric (and superconformal) generalization of the Maxwell $F^2$ term while the second one gives $F^4$ among other components. The dots here stand for the higher orders of the Maxwell field strength.

3 The one-loop effective actions in the $\mathcal{N} = 4$ and $\mathcal{N} = 8$ SYM

3.1 Low-energy effective action in $\mathcal{N} = 4$ SYM

The classical action of $\mathcal{N} = 4$ SYM is given by

$$S_{\mathcal{N}=4} = \frac{1}{g^2} \text{tr} \int d^3x d^4\theta \left[ G^2 - \frac{1}{2} e^{-2V} \bar{\Phi} e^{2V} \Phi \right],$$  \hfill (3.1)

where $\Phi$ is the chiral superfield in the adjoint representation of the gauge group. This action is invariant under the following non-Abelian gauge transformations

$$\Phi \to e^{i\lambda} \Phi e^{-i\lambda}, \quad \bar{\Phi} \to e^{i\bar{\lambda}} \bar{\Phi} e^{-i\bar{\lambda}}, \quad e^{2V} \to e^{i\lambda} e^{2V} e^{-i\lambda},$$  \hfill (3.2)

with $\lambda$ and $\bar{\lambda}$ being (anti)chiral superfield gauge parameters.

It is convenient to introduce the covariantly (anti)chiral superfields,

$$\Phi_c = e^{-2V} \Phi e^{2V}, \quad \bar{\Phi}_c = \bar{\Phi}, \quad \nabla_\alpha \Phi_c = 0, \quad \bar{\nabla}_\alpha \bar{\Phi}_c = 0.$$  \hfill (3.3)

In terms of these superfields the transformations of hidden $\mathcal{N} = 2$ supersymmetry are given by

$$e^{2V} \delta_e e^{2V} = \theta^a \epsilon_\alpha \bar{\Phi}_c - \bar{\theta}^a \bar{\epsilon}_\alpha \Phi_c, \quad \delta_e \Phi_c = -i \epsilon^a \nabla_\alpha G, \quad \delta_e \bar{\Phi}_c = -i \bar{\epsilon}^a \bar{\nabla}_a G.$$  \hfill (3.4)

Here $\epsilon_\alpha$ is the anticommuting complex parameter. We omit the label ‘$c$’, $\Phi_c \to \Phi$, $\bar{\Phi}_c \to \bar{\Phi}$, adopting that we deal with the covariantly (anti)chiral superfields in what follows.

The generalization of the $\mathcal{N} = 2$ background field method to the $\mathcal{N} = 4$ case is straightforward. The background-quantum splitting of the gauge superfield $V$ in (2.14) is extended by the corresponding splitting for $\Phi$ and $\bar{\Phi}$,

$$\Phi \to \Phi + g\phi, \quad \bar{\Phi} \to \bar{\Phi} + g\bar{\phi}.$$  \hfill (3.5)
Here the superfields $\Phi, \phi$ and $\Phi, \phi$ in the right hand sides are covariantly (anti)chiral with respect to the background gauge covariant derivatives, $D_\alpha \Phi = D_\alpha \phi = 0$, $\bar{D}_\alpha \Phi = \bar{D}_\alpha \phi = 0$. The quantum gauge transformations for these superfields read

$$\delta \phi = i[\lambda, \frac{1}{g} \Phi + \phi], \quad \delta \bar{\phi} = i[\bar{\lambda}, \frac{1}{g} \Phi + \phi], \quad \delta \Phi = \delta \bar{\Phi} = 0. \quad (3.6)$$

Upon the background-quantum splitting (2.14) and (3.5), the $N = 4$ SYM action (3.1) can be expanded in the series over the quantum superfields. In particular, for the one-loop computations we need the quadratic part of this action,

$$S_2 = -\text{tr} \int d^3xd^4\theta [\frac{1}{8} D^\alpha D^2 D_\alpha + \frac{i}{2} (D^\alpha W_\alpha) + i W^\alpha D_\alpha + \Phi \bar{\Phi}]v$$

$$-\text{tr} \int d^3xd^4\theta (-\bar{\phi}[\Phi, v] + \phi[\bar{\Phi}, v] + \frac{1}{2} \phi \bar{\phi}). \quad (3.7)$$

This action is invariant under the quantum gauge transformations (2.18) and (3.6). Therefore we fix the quantum gauge symmetry by the following gauge-fixing functions

$$f = i \bar{D}^2 v - \frac{i}{2} [\Phi, \bar{D}^2 \Box^{-1} \bar{\phi}], \quad \bar{f} = i D^2 v + \frac{i}{2} [\bar{\Phi}, D^2 \Box^{-1} \phi]. \quad (3.8)$$

In comparison with (2.23) these functions have the terms depending on the background (anti)chiral superfields $\Phi, \phi, \bar{\Phi}, \bar{\phi}$ which are necessary to remove the mixed terms between the quantum gauge $v$ and (anti)chiral $\bar{\phi}, \phi$ superfields. Such a gauge fixing is usually referred to as the generalized $R_\xi$ gauge [16, 17]. The corresponding gauge-fixing action reads

$$S_{gf} = \frac{1}{8} \text{tr} \int d^3xd^4\theta \bar{f} f = \frac{1}{8} \text{tr} \int d^3xd^4\theta \left( -\frac{1}{16} v [D^2, \bar{D}^2] v - \frac{1}{2} v D^2 [\Phi, \bar{\Phi} D^2 \Box^{-1} \phi] + \frac{1}{2} v D^2 [\Phi, \bar{D}^2 \Box^{-1} \phi] + v [\Phi, \bar{\Phi} D^2 \Box^{-1} \phi] + \frac{1}{4} [\Phi, [\Phi, \bar{D}^2 \Box^{-1} \phi]] [\bar{\Phi}, \bar{D}^2 \Box^{-1} \phi] \right). \quad (3.9)$$

It is convenient at this point to specify the constraints on the background chiral superfields $\Phi$ and $\Phi$

$$D_\alpha \Phi = 0, \quad \bar{D}_\alpha \bar{\Phi} = 0, \quad (3.10)$$

i.e. they are covariantly constant. For such a background the action (3.9) simplifies,

$$S_{gf} = \text{tr} \int d^3xd^4\theta \left( -\frac{1}{16} v [D^2, \bar{D}^2] v - v [\bar{\Phi}, \phi] + v [\Phi, \bar{\phi}] + \frac{1}{2} [\Phi, \Box^{-1} \phi] [\bar{\Phi}, \phi] \right). \quad (3.11)$$

As a result, the quadratic part of the action for the quantum superfields becomes very simple,

$$S_2 + S_{gf} = -\text{tr} \int d^3xd^4\theta \left[ v (\Box \phi + \bar{\Phi} \phi) v + \frac{1}{2} \phi (1 + \bar{\Phi} \Phi \Box^{-1} \phi) \right]. \quad (3.12)$$

Here we denote $\bar{\Phi} \Phi v = [\bar{\Phi}, [\Phi, v]]$ and similar $\bar{\Phi} \Phi \bar{\phi} = [\bar{\Phi}, [\Phi, \bar{\phi}]]$. 

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The quantum gauge transformations (2.18) and (3.6) define the action for the ghost superfields,

\[
S_{\text{gh}} = \text{tr} \int d^3 x d^4 \theta (b + \bar{b}) L_{\text{gv}}[c + \bar{c} + \coth(L_{\text{gv}})(c - \bar{c})] \\
- \text{tr} \int d^3 x d^4 \theta \left( b[\Phi, \Box^{-1}[\Phi + g\phi, \bar{c}]] - \bar{b}[\bar{\Phi}, \Box_+^{-1}[\Phi + g\phi, c]] \right) \\
- \text{tr} \int d^3 x d^4 \theta \varphi \varphi.
\] (3.13)

Here \( b \) and \( c \) are standard Faddeev-Popov ghosts while \( \varphi \) is the Nielsen-Kallosh ghost. All these superfields are covariantly (anti)chiral. Up to the second order in quantum superfields, the ghost superfield action is given by

\[
S_{\text{gh}} = \text{tr} \int d^3 x d^4 \theta \left[ -b(1 + \Phi \Box^{-1}\Phi)\bar{c} + \bar{b}(1 + \bar{\Phi} \Box_+^{-1}\Phi)c - \varphi \varphi \right].
\] (3.14)

The functional integral for the one-loop effective action reads

\[
e^{i\Gamma_{N=4}[V, \Phi]} = e^{iS_{N=4}[V, \Phi]} \int Dv D\phi D\bar{b} Dc D\varphi e^{iS_{\text{gf}} + iS_{\text{gh}}}.
\] (3.15)

Schematically, the one-loop effective action can be written as

\[
\Gamma_{N=4} = \frac{i}{2} \text{Tr}_v \ln(\Box_v + \Phi\Phi) - i\text{Tr}_+ \ln(\Box_+ + \bar{\Phi}\Phi).
\] (3.16)

The first term in the rhs in this expression comes from the quantum gauge superfield while the second one takes into account the contributions from quantum chiral superfield \( \phi \) and ghosts.

### 3.1.1 Gauge group SU(\( N \))

Now let us compute the traces of the logarithms of the operators in (3.16) for the gauge group SU(\( N \)) spontaneously broken down to U(1)\(^{N-1}\). The background gauge superfield \( V \) is specified in (2.35). The background chiral superfield \( \Phi \) has similar structure,

\[
\Phi = \text{diag}(\Phi_1, \Phi_2, \ldots, \Phi_N), \quad \sum_{I=1}^N \Phi_I = 0.
\] (3.17)

The quantum gauge superfield \( v \) is given by the expansion (2.34) while the quantum chiral superfield \( \phi \) is represented by the expression similar to (2.40). Now it is straightforward to compute the matrix traces in (3.16),

\[
\Gamma_{N=4} = i \sum_{I<J}^N \text{Tr}_v \ln(\hat{\Box}_v IJ + \hat{\Phi}_{IJ} \Phi_{IJ}) - 2i \sum_{I<J}^N \text{Tr}_+ \ln(\hat{\Box}_+ IJ + \hat{\Phi}_{IJ} \Phi_{IJ}),
\] (3.18)
where $\Phi_{IJ} = \Phi_I - \Phi_J$ and the operators $\Box_{v IJ}$ and $\Box_{+I J}$ are given in (2.38) and (2.44), respectively. The traces of the logarithms of these operators are computed in a similar way as in Sect. 2.3. As a result we get the one-loop effective action in the $\mathcal{N} = 4$ SYM theory for the gauge group SU($N$) broken down to U(1)$_{N-1}$,

$$
\Gamma_{N=4} = -\frac{1}{\pi} \sum_{1<J}^N \int d^3 x d^4 \theta \int_0^{\infty} \frac{ds}{s \sqrt{i \pi s}} \frac{W_{IJ}^2 \bar{W}_{IJ}^2 e^{is(G_{IJ}^2 + \bar{\Phi}_{IJ} \Phi_{IJ})}}{B_{IJ}^3} \tanh \frac{sB_{IJ}}{2} \sinh^2 \frac{sB_{IJ}}{2} \\
-\frac{2}{\pi} \sum_{1<J}^N \int d^3 x d^4 \theta \left[ G_{IJ} \ln(G_{IJ} + \sqrt{G_{IJ}^2 + \bar{\Phi}_{IJ} \Phi_{IJ}}) - \sqrt{G_{IJ}^2 + \bar{\Phi}_{IJ} \Phi_{IJ}} \right] \\
+ \frac{1}{4} \int_0^{\infty} \frac{ds}{\sqrt{i \pi s}} e^{is(G_{IJ}^2 + \bar{\Phi}_{IJ} \Phi_{IJ})} \frac{W_{IJ}^2 \bar{W}_{IJ}^2}{B_{IJ}^2} \left( \frac{\tanh(sB_{IJ}/2)}{sB_{IJ}/2} - 1 \right). \tag{3.19}
$$

We point out that only the leading terms given in the second line in (3.19) were studied in [15] by employing the mirror symmetry while here we computed also all higher order terms which are responsible in components for all higher powers of the Maxwell field strength $F^{2n}, n \geq 2$.

In conclusion of this section let us briefly comment on the case of minimal gauge symmetry breaking SU($N$) $\rightarrow$ SU($N-1$) $\times$ U(1). The background chiral superfield $\Phi$ is chosen similarly as the gauge one (2.54),

$$
\Phi = \frac{1}{N} \text{diag}\left( (N-1)\Phi, -\Phi, \ldots, -\Phi \right). \tag{3.20}
$$

The leading terms in the $\mathcal{N} = 4$ SYM effective action in this case are given by

$$
\Gamma_{N=4} = \frac{2(N-1)}{\pi} \int d^3 x d^4 \theta \left[ \sqrt{G^2 + \bar{\Phi} \Phi} - G \ln(G + \sqrt{G^2 + \bar{\Phi} \Phi}) + \frac{1}{32} \frac{W^2 \bar{W}^2}{(G^2 + \bar{\Phi} \Phi)^{5/4}} + \ldots \right]. \tag{3.21}
$$

The first two terms in the rhs of this expression are responsible for $\mathcal{N} = 4$ supersymmetric (and superconformal) generalization of the Maxwell $F^2$ term while the third term gives $F^4$ among other components and the dots stand for higher-order terms.

Finally, let us comment on the following terms in the effective action (3.21),

$$
\int d^3 x d^4 \theta \left[ G \ln(G + \sqrt{G^2 + \bar{\Phi} \Phi}) - \sqrt{G^2 + \bar{\Phi} \Phi} \right], \tag{3.22}
$$

which are known as the $\mathcal{N} = 2, d = 3$ superspace action of the improved tensor multiplet [12]. Note that analogous $\mathcal{N} = 1, d = 4$ superspace action of the improved tensor multiplet was constructed in [18]. It is interesting to point out that (3.22) was obtained recently in [19] as a dual representation of the classical action of the Abelian Gaiotto-Witten model [10] which is reviewed in the Appendix, see (A.7). Hence, the classical action of the Abelian Gaiotto-Witten model in the representation (3.22) arises as the leading term in the $\mathcal{N} = 4$ SYM effective action. Finally, we point out that this term (3.22) in the $\mathcal{N} = 4$ SYM effective action is known to be one-loop exact [15, 20, 21].
3.2 Low-energy effective action in $\mathcal{N} = 8$ SYM

The classical $\mathcal{N} = 8$ SYM action appears by simple dimensional reduction form the $\mathcal{N} = 4$, $d = 4$ SYM. In our notations it reads

$$S_{\mathcal{N}=8} = \frac{1}{g^2} \text{tr} \int d^3xd^4\theta \left[ G^2 - \frac{1}{2} e^{-2V} \Phi^i_\epsilon e^{2V} \Phi_i + \frac{1}{12g^2} \left( \text{tr} \int d^3xd^2\theta \varepsilon^{ijk} \Phi_i \Phi_j \Phi_k + \text{c.c.} \right) \right].$$  

(3.23)

Here $\Phi_i$, $i = 1, 2, 3$, is a triplet of chiral superfields. The action is invariant under the hidden $\mathcal{N} = 6$ supersymmetry with the complex parameter $\epsilon_{\alpha i}$,

$$e^{-2V} \delta \epsilon e^{2V} = \theta^\alpha \epsilon_{\alpha i} \tilde{\Phi}^i_c - \tilde{\theta}^\alpha \bar{\epsilon}_i a \Phi_{ci},$$

$$\delta \epsilon \Phi_{ci} = -i \varepsilon^i_\alpha \nabla G + \frac{1}{4} \varepsilon^{ijk} \nabla^2 (\tilde{\theta}^\alpha \bar{\epsilon}_j \Phi^k_c),$$

$$\delta \bar{\epsilon} \bar{\Phi}^i_c = -i \varepsilon^\alpha \bar{\epsilon} \nabla G + \frac{1}{4} \varepsilon^{ijk} \nabla^2 (\bar{\theta}^{\alpha \epsilon} \Phi_{ci}).$$  

(3.24)

We use the notations $\tilde{\Phi}^i_c = e^{-2V} \Phi^i_c e^{2V}$, $\Phi_c = \Phi$ for the covariantly (anti)chiral superfields. Further we will omit the subscript ‘c’ assuming everywhere that we deal with covariantly (anti)chiral superfields only.

The background field method can be easily generalized to the case of the $\mathcal{N} = 8$ SYM theory. Let us sketch the basic steps. The background-quantum splitting of the gauge superfield (2.14) is supplemented by the following splitting of the (anti)chiral superfields,

$$\Phi_i \rightarrow \Phi_i + g \phi_i, \quad \bar{\Phi}^i \rightarrow \bar{\Phi}^i + g \bar{\phi}^i,$$  

(3.25)

with the corresponding ‘quantum’ gauge transformations

$$\delta \phi_i = i[\lambda, \frac{1}{g} \Phi_i + \phi_i], \quad \delta \bar{\phi}^i = i[\bar{\lambda}, \frac{1}{g} \bar{\Phi}^i + \bar{\phi}^i], \quad \delta \Phi_i = \delta \bar{\Phi}^i = 0.$$  

(3.26)

The gauge fixing functions are chosen in the form similar to (3.8),

$$f = iD^2v - \frac{i}{2} [\Phi_i, D^2 \square^{-1} \phi^i], \quad \bar{f} = iD^2\bar{v} + \frac{i}{2} [\bar{\Phi}^i, D^2 \square^{-1} \bar{\phi}_i].$$  

(3.27)

When the background superfields are covariantly constant, $D_\alpha \bar{\Phi}^i = 0$, $D_\alpha \Phi_i = 0$, the quadratic part of the action with respect to the quantum superfields takes relatively simple form,

$$S_2 + S_{gf} = -\text{tr} \int d^3xd^4\theta \left[ v(\square_v + \bar{\Phi}^i \Phi_i)v + \frac{1}{2} \phi_i (\delta^i_j + \bar{\Phi}^i \bar{\Phi}_j \square^{-1}) \bar{\phi}^j \right]$$

$$+ \frac{1}{4} \left( \text{tr} \int d^3xd^2\theta \varepsilon^{ijk} \phi_i [\Phi_j, \phi_k] + \text{c.c.} \right).$$  

(3.28)

Here we denote $\bar{\Phi}^i \Phi_i v = [\bar{\Phi}^i [\Phi_i, v]]$. The ghost superfield action is a simple generalization of (3.14),

$$S_{gh} = \text{tr} \int d^3xd^4\theta \left[ -b(1 + \Phi_i \square^{-1} \bar{\Phi}^i) \bar{c} + \bar{b}(1 + \bar{\Phi}^i \bar{\Phi}_i \square^{-1} \Phi_i) c - \bar{c} \bar{c} \right].$$  

(3.29)
As a result we see that the one-loop effective action is relatively simple because it is defined by only one operator,
\[ \Gamma_{\mathcal{N}=8} = \frac{i}{2} \text{Tr}_v \ln(\Box_v + \Phi^i \Phi_i). \] (3.30)

The contributions from ghosts and chiral superfields cancel each other at one loop for the covariantly constant background similarly as it happens for the \( \mathcal{N} = 4, d = 4 \) SYM theory.

### 3.2.1 Gauge group SU\((N)\)

For the gauge group SU\((N)\) spontaneously broken down to U(1)\(^{N-1}\) the gauge and chiral superfields are chosen as in (2.35) and (3.17). In this case the trace of the logarithm in (3.30) is computed by standard methods described in Sect. 2.3,
\[ \Gamma_{\mathcal{N}=8} = \frac{i}{2} \sum_{I < J} \text{Tr}_v \ln(\Box_{vIJ} + \Phi_{IJ}^i \Phi_{IJI}) \] (3.31)

In the case when the gauge group SU\((N)\) is spontaneously broken down to SU\((N - 1)\times U(1)\), the background superfields should be chosen as in (2.54) and (3.20). Then the leading term in the effective action (3.31) is given by
\[ \Gamma_{\mathcal{N}=8} = \frac{3(N - 1)}{32\pi} \int d^3 x d^4 \theta \int_0^\infty ds \frac{W_{IJ}^2 \bar{W}_{IJ}^2}{B_{IJ}^2} e^{is(G_{IJ}^2 + \Phi_{IJI})} \tanh \frac{sB_{IJ}}{2} \sinh^2 \frac{sB_{IJ}}{2}. \] (3.32)

where \( f^i, \bar{f}^i = 1, 2, \ldots, 7 \) are the seven scalar fields in the \(\mathcal{N} = 8\) SYM theory and dots stand for the higher-order terms. In [22] it was argued that the \( F^4 \) term in the \(\mathcal{N} = 8\) SYM effective action (3.32) is one-loop exact in the perturbation theory, but it receives instanton corrections.

Since there is no \( F^2 \) term in (3.32), the ABJM theory in the dual representation (A.15) cannot be a part of the low-energy effective action of the \(\mathcal{N} = 8\) SYM theory. This is quite expected because the \(\mathcal{N} = 8\) SYM describes the D2 brane in the flat background while the Abelian ABJM theory corresponds to the M2 brane on a Calabi-Yau fourfold \( X_4 = \mathbb{C}^4/\mathbb{Z}_{2k} \) [2]. Moreover, the ABJM theory has only \(\mathcal{N} = 6\) supersymmetry while the effective action (3.32) should respect the \(\mathcal{N} = 8\) supersymmetry since the supersymmetry cannot be broken within the quantization. In the next section we will see that the ABJM theory does appear in the low-energy effective action when one considers the D2 brane in the appropriate background.
4 Low-energy effective action in $\mathcal{N} = 2$ quiver gauge theories

One of the ways to understand multiple 2-branes in M theory is through studying the strong coupling limit of a stack of D2 branes in IIA string theory. However, one has to specify the backgrounds in which the M2 and D2 branes live. Indeed, in the previous section we have shown that the $\mathcal{N} = 8$ SYM theory cannot reproduce the ABJM theory because the latter describes the M2 branes on the conifold $X_4 = \mathbb{C}^4/\mathbb{Z}_k$ while the former corresponds to D2 branes in flat space. In [9] it was explained that the ABJM theory should appear in the strong coupling limit of a stack of D2 branes probing the singularity of the Calabi-Yau threefold $X_3$ fibred over real line $\mathbb{R}$ and with RR 2-form fluxes turned on. Here $X_3$ (or $Y_6$ in the notation of [8]) is a cone with the base $T^{1,1} = (SU(2) \times SU(2))/U(1)$ [23]. This conifold $X_3$ fibred over $\mathbb{R}$ appears as the moduli space of the field theory describing these D2 branes. The field theory in question is a three-dimensional quiver gauge theory given by the $\mathcal{N} = 2$ SYM with gauge group $SU(N) \times SU(N)$ and with the matter given by four chiral superfields in the $(N, \bar{N})$ and $(\bar{N}, N)$ bifundamental representations of the gauge group.

In fact, the interest to the three-dimensional quiver gauge models was initiated by the works [8, 24] where similar four-dimensional supergauge theory was studied in details as a model for multiple D3 branes probing the singularity of $X_3$ (see, e.g., [7] for a recent review), but the considerations of D2 branes on this background go along similar lines. It is useful to give here a short review of the moduli space for such three-dimensional field theories because it motivates the choice of the background superfields which we do in the following studies of the effective action. To understand the moduli space it is sufficient to take the Abelian version of the theory, i.e., when the gauge group is $U(1)_L \times U(1)_R$. The matter superfields are given by a pair of chiral superfields (further referred to as the hypermultiplets$^3$) $(\bar{Q}_a^+, Q_a^-)$ which are labeled by the SU(2) index $a = 1, 2$. The classical action corresponds to the massless $\mathcal{N} = 2$ electrodynamics,

$$S = \frac{1}{g^2} \int d^3 x d^4 \theta (G_L^2 + G_R^2) - \frac{1}{2} \int d^3 x d^4 \theta \left( \bar{Q}_a^+ e^{2(V_L - V_R)} Q_{+a} + Q_a^- e^{2(V_R - V_L)} \bar{Q}_{-a} \right), \quad (4.1)$$

where $V_L$ and $V_R$ are the ‘left’ and ‘right’ Abelian gauge superfields with superfield strengths $G_L$ and $G_R$ respectively. Among other components, the gauge superfields contain the real scalars $\phi_{L,R}$ and auxiliary fields $D_{L,R}$,

$$V_{L,R} = i \bar{\theta}^a \theta_\alpha \phi_{L,R} + \bar{\theta}^2 \theta^2 D_{L,R} + \ldots, \quad (4.2)$$

while the scalars in the chiral superfields are

$$\bar{Q}_a^+ = z_a^+ + \ldots, \quad Q_a^- = z_a^- + \ldots. \quad (4.3)$$

$^3$To be precise, the pair of (anti)chiral superfields $(\bar{Q}_+, Q_-)$ form a hypermultiplet only if one considers the $\mathcal{N} = 4$ supersymmetric gauge theory. Here we have only $\mathcal{N} = 2$ supersymmetry because the gauge superfields do not get their $\mathcal{N} = 4$ partners, but we hope that our terminology will not be misleading.
Eliminating the auxiliary fields $D_{L,R}$ one can see that the potential for the scalars has minimum when two conditions are satisfied,

$$
(i) \quad |z_{+1}|^2 + |z_{+2}|^2 = |z_{-1}|^2 + |z_{-2}|^2,
(ii) \quad \phi_L = \phi_R .
$$

(4.4)

Taking into account the gauge transformations for the scalars, the identification

$$
z_{+a} \sim z_{+a} e^{i\alpha}, \quad z_{-a} \sim z_{-a} e^{-i\alpha},
$$

(4.5)

considered together with the first constraint in (4.4) defines precisely the conifold denoted by $X_3$ (see [8]), while the second constraint in (4.4) is just the real line. Therefore the moduli space of the model (4.1) is $X_3 \times \mathbb{R}$. The non-Abelian version of this model has the same moduli space modulo the action of the Weyl group. Note that in the non-Abelian case the SU(2)-invariant superpotential should be added to the action (4.1), but it does not change the moduli.

In what follows, we will study the low-energy effective actions in two non-Abelian $\mathcal{N} = 2$ quiver gauge theories, namely with one and with two hypermultiplets. The latter model corresponds to the D2 branes probing the conifold $X_3 \times \mathbb{R}$ while the former is simple and helps us to understand the basic steps of the quantization procedure. The resulting effective actions will depend on the background Abelian superfields satisfying the superfield analogs of the constraints (4.4). We point out that effective action is gauge independent when the background superfields correspond to the vacuum of the model.

### 4.1 $\mathcal{N} = 2$ SYM with one bifundamental hypermultiplet

One of the simplest examples of quiver gauge theories is the $\mathcal{N} = 2$ SYM theory with twisted gauge group $\mathcal{G}_L \times \mathcal{G}_R$ and with a hypermultiplet $(\bar{Q}+, Q_-)$ in the bifundamental representation of this gauge group. The classical action reads

$$
S = \frac{1}{g^2} \text{tr} \int d^3x d^4\theta (G^2_L + G^2_R) - \frac{1}{2} \text{tr} \int d^3x d^4\theta (Q_+ e^{2V_L} Q_+ e^{-2V_L} + Q_- e^{-2V_L} Q_- e^{2V_R}) .
$$

(4.6)

Here $V_L$ and $V_R$ are the gauge superfields taking their values in the Lie algebras of the ‘left’ $\mathcal{G}_L$ and ‘right’ $\mathcal{G}_R$ gauge groups, respectively. For simplicity, we consider the kinetic terms for these gauge superfields with the same gauge coupling $g$ and do not give any superpotential for the chiral superfields. The classical action (4.6) is invariant under the following gauge transformations with the (anti)chiral gauge parameters $\lambda_{L,R}$, $\bar{\lambda}_{L,R}$,

$$
e^{2V_L} \rightarrow e^{i\lambda_L} e^{2V_L} e^{-i\lambda_L}, \quad e^{2V_R} \rightarrow e^{i\lambda_R} e^{2V_R} e^{-i\lambda_R},
Q_+ \rightarrow e^{i\lambda_L} Q_+ e^{-i\lambda_R}, \quad Q_- \rightarrow e^{i\lambda_R} Q_- e^{-i\lambda_L} .
$$

(4.7)

The low-energy effective action in analogous four-dimensional theory was studied in particular in [27].

To derive the structure of one-loop effective action we perform the background-quantum splitting of the gauge and matter superfields,

$$
e^{2V_L} \rightarrow e^{2V_L} e^{2g_{\nu L}}, \quad e^{2V_R} \rightarrow e^{2V_R} e^{2g_{\nu R}}, \quad Q_+ \rightarrow Q_+ + q_+ ,
$$

(4.8)
where $V_{L,R}, Q_{\pm}$ on the right denote the background superfields and $v_{L,R}, q_{\pm}$ are the quantum ones. The gauge transformations for the quantum gauge superfields have the form (2.18). However, now we need two gauge fixing functions to break the gauge invariance under the left and the right gauge groups,

$$f_L = i \mathcal{D}^2 v_L, \quad f_R = i \mathcal{D}^2 v_R.$$  \hfill (4.9)

The corresponding gauge fixing action reads (we use the Fermi-Feynman gauge)

$$S_{gf} = \frac{1}{8} \mathrm{tr} \int d^3 x d^4 \theta (f_L f_L + f_R f_R) = -\frac{1}{16} \mathrm{tr} \int d^3 x d^4 \theta (v_L \{ \mathcal{D}^2, \mathcal{D}^2 \} v_L + v_R \{ \mathcal{D}^2, \mathcal{D}^2 \} v_R).$$  \hfill (4.10)

The background gauge covariant spinor derivatives $\mathcal{D}_\alpha, \bar{\mathcal{D}}_\alpha$ act appropriately on the left and right gauge superfields and chiral superfields, e.g.,

$$\mathcal{D}_\alpha v_L = D_\alpha v_L + [V_{L\alpha}, v_L], \quad \mathcal{D}_\alpha v_R = D_\alpha v_R + [V_{R\alpha}, v_R],$$

$$\mathcal{D}_\alpha Q_+ = D_\alpha Q_+ + V_{L\alpha} Q_+ - Q_+ V_{R\alpha}. \hfill (4.11)$$

Expanding the action (4.6) up to the second order in the quantum superfields, we get the action $S_2$ which, together with $S_{gf}$, is

$$S_2 + S_{gf} = -\mathrm{tr} \int d^3 x d^4 \theta \left[ v_L (\Box + g^2 Q_+ Q_+ + g^2 Q_- Q_-) v_L ight. \\
+ v_R (\Box + g^2 Q_+ Q_+ + g^2 Q_- Q_-) v_R - 2g^2 \bar{Q}_+ v_L Q_+ v_R - 2g^2 Q_- v_L \bar{Q}_- v_R \\
- \mathrm{tr} \int d^3 x d^4 \theta \left[ \frac{1}{2} \bar{q}_+ q_+ + \frac{1}{2} q_- \bar{q}_- \\
+ g (\bar{q}_+ v_L Q_+ - \bar{q}_+ Q_+ v_R - \bar{Q}_+ q_+ v_R + \bar{Q}_+ v_L q_+ \\
+ g (Q_- v_R Q_- - Q_- v_L \bar{Q}_- - q_- v_L \bar{Q}_- + q_- Q_- v_R) \right]. \hfill (4.12)$$

Note that we use the superfields $\bar{Q}_\pm$ and $\bar{q}_\pm$ in this action which are covariantly antichiral, $\mathcal{D}_\alpha \bar{Q}_\pm = \mathcal{D}_\alpha \bar{q}_\pm = 0$, where the covariant spinor derivative acts in accord with the bifundamental representation of the gauge group (4.11).

The action (4.12) defines the structure of the one-loop effective action

$$e^{i \mathcal{W}} = \int \mathcal{D} v_{L,R} \mathcal{D} q_{\pm} \mathcal{D} b_{L,R} \mathcal{D} c_{L,R} \mathcal{D} \varphi_{L,R} e^{i(S_2 + S_{gf} + S_{gh})}, \hfill (4.13)$$

where $S_{gh}$ is the quadratic part of the ghost superfield action with respect to the quantum superfields,

$$S_{gh} = \mathrm{tr} \int d^3 x d^4 \theta \left[ b_L c_L - b_L \bar{c}_L + b_R c_R - b_R \bar{c}_R + \varphi_L \bar{\varphi}_L + \bar{\varphi}_R \varphi_R \right]. \hfill (4.14)$$

The superfields $b_{L,R}$, $c_{L,R}$ are the Faddeev-Popov ghosts while $\varphi_{L,R}$ are the Nielsen-Kallosh ghosts. All these ghost superfields are covariantly (anti)chiral with respect to either left or right gauge group.

The generating functional (4.13) gives a general expression for the one-loop effective action. To get the concrete result we have to fix the gauge groups $\mathcal{G}_L$ and $\mathcal{G}_R$ and to specify the background superfields.
4.1.1 Gauge group SU(2)×SU(2)

This is the simplest non-Abelian gauge group. The superfields $v_L$ and $v_R$ take their values in the Lie algebras of the gauge groups SU(2)$_L$ and SU(2)$_R$, respectively, while the chiral superfields are general complex $2 \times 2$ matrices. We choose the background gauge superfields to belong to the Cartan subalgebras,

$$V_L = V_R = \frac{1}{2} \begin{pmatrix} V & 0 \\ 0 & -V \end{pmatrix}.$$  

(4.15)

Here $V$ is the usual Abelian gauge superfield with the superfield strength $G$. The background chiral superfields are chosen as follows

$$Q_+ = \begin{pmatrix} Q_+ & 0 \\ 0 & 0 \end{pmatrix}, \quad Q_- = \begin{pmatrix} Q_- & 0 \\ 0 & 0 \end{pmatrix}.$$  

(4.16)

Moreover, the superfields $Q_+$ and $Q_-$ obey the following constraint

$$Q_+Q_+ = Q_-Q_-,$$  

(4.17)

which is a superfield analog of (4.4). Note that the matrices (4.15) and (4.16) commute. Hence, the constraint of covariant antichirality of the superfields $Q_\pm$ turns into the usual antichirality, $D_\alpha Q_\pm = 0$.

Now we specify the constraints on the background gauge $V$ and matter $Q_\pm$ superfields. We will be interested in the leading part of the effective action $\Gamma$ which depends on the superfield strength $G$ and (anti)chiral superfields $Q_\pm, Q_\pm$, but not on their derivatives,

$$\Gamma = \Gamma[G, \bar{Q}_\pm, Q_-] = \int d^3 x d^4 \theta \mathcal{L}_{\text{eff}}(G, \bar{Q}_\pm, Q_-).$$  

(4.18)

Note that we neglect not only the space-time derivatives of the superfields, but also all their covariant spinor derivatives,

$$D_\alpha Q_\pm = 0, \quad \bar{D}_\alpha Q_\pm = 0, \quad D_\alpha G = \bar{W}_\alpha = 0, \quad \bar{D}_\alpha G = W_\alpha = 0.$$  

(4.19)

This constraint is sufficient to study the contribution to the effective action of the second order with respect to the Maxwell field strength, $F^2$. However, two last constraints in (4.19) should be taken carefully. Indeed, one can freely neglect the superfield strengths $W_\alpha, \bar{W}_\alpha$ while doing the computations in full $\mathcal{N} = 2$ superspace, but this is not true in the chiral superspace. To be precise, one has to keep these superfield strengths in computing the trace of logarithm of the chiral box operator $\hat{\Box}$ and to vanish them only after passing to the full superspace measure. We will keep this in mind while doing the corresponding computations with the chiral box operator.

To proceed, we have to do the matrix traces in the action (4.12) for the chosen background matrix superfields (4.15) and (4.16). The quantum superfields $v_{L,R}$ and $q_\pm$ have both diagonal and off-diagonal matrix components,

$$v_{L,R} = v_{L,R}(\text{diag}) + v_{L,R}(\text{off-diag}), \quad q_\pm = q_\pm(\text{diag}) + q_\pm(\text{off-diag}).$$  

(4.20)
It is an easy exercise to check that the action (4.12) appears to be a sum of two independent parts,

\[ S_2[v_{L,R}, q_{\pm}] + S_{gf}[v_{L,R}, q_{\pm}] = S_{\text{diag}}[v_{L,R(\text{diag})}, q_{\pm(\text{diag})}] + S_{\text{off-diag}}[v_{L,R(\text{off-diag})}, q_{\pm(\text{off-diag})}] \cdot \]

(4.21)

Therefore the contributions to the one-loop effective action from the diagonal matrix components of the quantum superfields can be studied independently of the off-diagonal ones. Further we will write down the expressions for the actions \(S_{\text{diag}}\) and \(S_{\text{off-diag}}\) explicitly and compute the corresponding contributions to the effective action.

Consider first only the diagonal components,

\[ v_{L,R} = \frac{1}{2} \begin{pmatrix} v_{L,R} & 0 \\ 0 & -v_{L,R} \end{pmatrix}, \quad q_{\pm} = \begin{pmatrix} q_{\pm} & 0 \\ 0 & p_{\pm} \end{pmatrix}, \]

(4.22)

where \(v_{L,R}\) are real while \(p_{\pm}\) and \(q_{\pm}\) are chiral, \(\bar{D}_\alpha q_{\pm} = \bar{D}_\alpha p_{\pm} = 0\). The part of the action (4.12) for these superfields reads

\[ S_{\text{diag}} = -\frac{1}{2} \int d^3x d^4\theta [v_L(\Box + M^2)v_L + v_R(\Box + M^2)v_R - 2v_L v_R M^2
\]

\[ +q_+ q_+ + p_+ p_+ + q_- q_- + p_- p_- + g(v_L - v_R)(Q_+ \bar{q}_+ + \bar{Q}_+ q_+ - Q_- \bar{q}_- - \bar{Q}_- q_-)] , \]

(4.23)

where

\[ M^2 = g^2 \bar{Q}_+ Q_+ = g^2 \bar{Q}_- Q_- \]

(4.24)

is the effective mass squared of the gauge superfields. Note that the superfields \(p_{\pm}\) are completely free and do not contribute to the effective action.

Despite the mixing of the gauge and matter superfields in (4.23), it is quite straightforward to diagonalize the matrix of second variational derivatives of this action with respect to the quantum superfields. The details of these computations are given in the Appendix B.1. Here we present only the result,

\[ \Gamma_{\text{diag}} = \frac{i}{2} \text{Tr}_\nu \ln \left( 1 - \frac{1}{4} M^2 \Box D^\alpha \bar{D}^2 D_\alpha \right) = -\frac{i}{16} \text{Tr}_\nu \frac{1}{\Box} \ln \left( 1 + \frac{2 M^2}{\Box} \right) D^\alpha \bar{D}^2 D_\alpha . \]

(4.25)

This is a functional trace of the operator acting in the full superspace which can be easily computed using the following relations

\[ D^\alpha \bar{D}^2 D_\alpha \delta^7(z - z')|_{\theta = \theta'} = 16 \delta^3(x - x') , \]

\[ \frac{1}{\Box} \ln(1 + M^2/\Box) \delta^3(x - x')|_{x = x'} = \frac{i}{2\pi} \sqrt{M^2} . \]

(4.26)

As a result we get

\[ \Gamma_{\text{diag}} = \frac{g}{2\pi} \int d^3x d^4\theta \sqrt{\bar{Q}_+ Q_+ + \bar{Q}_- Q_-} . \]

(4.27)

This is nothing but the effective Kähler potential which coincides up to a coefficient with the one in the three-dimensional \(N = 2\) supersymmetric electrodynamics [26]. Note that
the constraint (4.17) is not necessary for this part of the effective action and even if one relaxes this, the result will be the same. However, this constraint will be necessary for computing the part of the effective action for the off-diagonal superfields in an unambiguous way.

Let us now concentrate on the contributions to the effective action from the off-diagonal superfields,

\[ v_{L,R} = \frac{1}{2} \begin{pmatrix} 0 & v_{L,R} \\ \bar{v}_{L,R} & 0 \end{pmatrix}, \quad q_\pm = \begin{pmatrix} 0 & q_\pm \\ p_\pm & 0 \end{pmatrix}. \]  

(4.28)

Here we use the same letters \( v_{L,R} \) and \( q_\pm, p_\pm \) as for the diagonal superfields in (4.22) despite they are completely independent. We hope that this will not lead to any confusions because there are no mixing between the diagonal and off-diagonal components and the corresponding parts of the classical action can be studied independently. The part of the action (4.12) for these superfields reads

\[
S_{\text{off-diag}} = -\frac{1}{2} \int d^3x d^4\theta \left[ \bar{v}_L(\hat{\Box}_v + M^2)v_L + \bar{v}_R(\hat{\Box}_v + M^2)v_R \\
+ \bar{q}_+q_+ + \bar{q}_-q_- + \bar{p}_+p_+ + \bar{p}_-p_- \\
+ g \bar{Q}_+(\bar{p}_+\bar{v}_L - \bar{q}_+v_R) + g \bar{Q}_-(p_+v_L - q_+\bar{v}_R) \\
+ g Q_-(\bar{p}_-\bar{v}_R - \bar{q}_-v_L) + g Q_+(p_-v_R - q_-\bar{v}_L) \right].
\]

(4.29)

Note that the superfields \( \bar{p}_\pm \) and \( \bar{q}_\pm \) are covariantly antichiral, \( \mathcal{D}_\alpha \bar{p}_\pm = \mathcal{D}_\alpha \bar{q}_\pm = 0 \), where the gauge covariant spinor derivative is defined with the proper charge for the “+” and “−” superfields. The operator \( \hat{\Box}_v \) has the form (2.38), but without indices \( IJ \) since there is only one component \( V \) in the background gauge superfield \( V \) in the case of the gauge group SU(2).

Now it is straightforward to compute the trace of the logarithm of the matrix of second variational derivatives for the action (4.29). The details of this procedure are collected in the Appendix B.2, the net result is

\[
\Gamma_{\text{off-diag}} = 2i \text{Tr}_+ \ln \hat{\Box}_+ + 2i \text{Tr}_v \ln \hat{\Box}_v + 2i \text{Tr}_v \ln \left( 1 - \frac{M^2}{8\hat{\Box}_v^2} \mathcal{D}^\alpha \mathcal{D}^2 \mathcal{D}_\alpha \right). \tag{4.30}
\]

The trace of the logarithm of the operator \( \hat{\Box}_v \) was studied in Sect. 2.3. The corresponding contributions to the effective action start from \( F^4 \) and therefore they are irrelevant for our consideration. Next, we have the contribution

\[
2i \text{Tr}_v \ln \left( 1 - \frac{M^2}{8\hat{\Box}_v^2} \mathcal{D}^\alpha \mathcal{D}^2 \mathcal{D}_\alpha \right) = -4i \text{Tr}_v \frac{1}{\hat{\Box} + G^2} \ln(1 + \frac{M^2}{\hat{\Box} + G^2}), \tag{4.31}
\]

which is derived in (B.15). The functional trace in (4.31) results in the following momentum integral

\[
\int_0^\infty \frac{k^2 dk}{k^2 + G^2} \ln(1 + \frac{M^2}{k^2 + G^2}) = \pi \left[ G \ln G + \sqrt{G^2 + M^2} - G \ln(G + \sqrt{G^2 + M^2}) \right]. \tag{4.32}
\]
The expression (4.30) contains also the trace of the logarithm of the chiral box operator which was studied in details in [11]. Up to a coefficient, it is given by (2.53). Summing up these contributions, we get the part of the effective action induced by the off-diagonal superfields,

$$\Gamma_{\text{off-diag}} = \frac{1}{\pi} \int d^3x d^4\theta [3G \ln G + 2\sqrt{G^2 + g^2 \bar{Q}_+ Q_+} - 2G \ln (G + \sqrt{G^2 + g^2 \bar{Q}_+ Q_+})].$$

(4.33)

Finally, we have to take into account the contributions from the ghost superfields (4.14). This is exactly the result (2.53) considered for $n = 2$ because the gauge group is SU(2) and doubled because we have the left-right pairs of the ghosts,

$$\Gamma_{\text{ghosts}} = -\frac{3}{\pi} \int d^3x d^4\theta G \ln G.$$

(4.34)

Summing up (4.27), (4.33) and (4.34) together, we get the low-energy effective action in the model (4.6),

$$\Gamma = \frac{g}{2\pi} \int d^3x d^4\theta \sqrt{\bar{Q}_+ Q_+ + \bar{Q}_- Q_-}$$

$$+ \frac{2}{\pi} \int d^3x d^4\theta \left[ \sqrt{G^2 + g^2 \bar{Q}_+ Q_+} - G \ln (G + \sqrt{G^2 + g^2 \bar{Q}_+ Q_+}) \right].$$

(4.35)

The terms in the last line of (4.35) have the form (A.7) which is the dualized classical action in the Abelian Gaiotto-Witten model. Therefore they possess the $\mathcal{N} = 4$ supersymmetry of the Gaiotto-Witten model. However, they are corrected by the Kähler potential appearing in the first line in (4.35) which has only the $\mathcal{N} = 2$ supersymmetry.

### 4.2 $\mathcal{N} = 2$ SYM with two bifundamental hypermultiplets

Let us consider the generalization of the model (4.6) to the case when the hypermultiplet has the SU(2) doublet index, $(\bar{Q}_a^+, Q_a^-)$, $a = 1, 2$, and the action (4.6) is extended with a SU(2) invariant superpotential,

$$S = \frac{1}{g^2} \text{tr} \int d^3x d^4\theta (G_L^2 + G_R^2) - \frac{1}{2} \text{tr} \int d^3x d^4\theta (Q^a_+ e^{2V_L} Q^a_+ e^{-2V_R} + Q^a_- e^{-2V_L} Q^a_- e^{2V_R})$$

$$- \frac{\lambda}{2} \left( \text{tr} \int d^3x d^4\theta \epsilon^{ab} \epsilon_{cd} Q^a_+ Q^c_+ Q^d_- + c.c. \right).$$

(4.36)

Here $\lambda$ is the dimensionless coupling constant. The gauge symmetry in this case is similar to (4.7) with obvious insertion of the indices for the hypermultiplet. Therefore, the gauge fixing and structure of ghosts is the same as for the model (4.6). Doing the background-quantum splitting in the standard way, $e^{2V_L,R} \rightarrow e^{2V_L,R} e^{g_{vL,R}}$, $Q_{\pm a} \rightarrow Q_{\pm a} + q_{\pm a}$, we get
the following quadratic action for the quantum superfields,
\[
S_2 + S_{gf} = S_{\text{gauge}} + S_{\text{hyper}} + S_{\text{pot}},
\]
\[
S_{\text{gauge}} = -\text{tr} \int d^3x d^4 \theta [v_L(\Box + g^2 Q_{+a} \bar{Q}^a_- + g^2 \bar{Q}_{-a} Q^a_+ + Q_{+a} Q_{+a} + g^2 \bar{Q}^a_+ Q_{-a} v_R - 2g^2 \bar{Q}^a_+ v_L Q_{+a} v_R - 2g^2 \bar{Q}^a_- v_L \bar{Q}_{-a} v_R],
\]
\[
S_{\text{hyper}} = -\text{tr} \int d^3x d^4 \theta \left[ \frac{1}{2} q^a_+ q_{+a} + \frac{1}{2} q_- q_{-a} + g(\bar{q}^a_+ v_L Q_{+a} - \bar{q}^a_- Q_{+a} v_R - \bar{Q}^a_+ Q_{+a} v_R + \bar{Q}^a_- Q_{+a} v_R)
\right.
\]
\[
\left. + g(\bar{Q}^a_- \bar{q}_{-a} v_R - Q^a_- v_L \bar{q}_{-a} - q_- v_L \bar{Q}_{-a} + q^a_- \bar{Q}_{-a} v_R) \right],
\]
\[
S_{\text{pot}} = -\frac{\lambda}{2} \int d^3x d^4 \theta \varepsilon^{ab} \varepsilon_{cd} [2 Q_{+a} Q^c_- q_{+a} q_{+d} + 2 Q_{+a} q^c_- q_{+d} Q^a_-
\]
\[
+ Q_{+a} q^c_- Q_{+a} q_{+d} + q_{+a} Q^c_- q_{+d} Q^a_-] + \text{c.c.}
\]
This action defines the structure of the one-loop effective action for the background superfields. To get the concrete result we have to specify the gauge group and the background.

4.2.1 Gauge group SU(2) × SU(2)

The background gauge superfields for this gauge group are chosen in the form (4.15) while for the background matter superfields we take
\[
Q_{+a} = \begin{pmatrix} Q_{+a} & 0 \\ 0 & 0 \end{pmatrix}, \quad Q^a_- = \begin{pmatrix} Q^a_- & 0 \\ 0 & 0 \end{pmatrix},
\]
(4.38)
where \( Q_\pm \) are chiral superfields, \( \bar{D}_a Q_{+a} = \bar{D}_a Q^a_- = 0 \). The constraint (4.17) in this case turns into
\[
Q_{+a} Q_{+a} = \bar{Q}_{-a} Q^a_-.
\]
(4.39)
The background superfields obey also the constraint (4.19) because we are interested in the contributions to the effective action of the order \( F^2 \).

A nice feature of the background (4.15,4.38) is that the diagonal matrix components of the quantum gauge and matter superfields completely decouple from the off-diagonal ones so that the relation (4.21) holds for the action (4.37) as well. Similarly as for the model (4.12), we study first the contributions to the effective action from the diagonal matrix components,
\[
v_{L,R} = \frac{1}{2} \begin{pmatrix} v_{L,R} & 0 \\ 0 & -v_{L,R} \end{pmatrix}, \quad q_{+a} = \begin{pmatrix} q_{+a} & 0 \\ 0 & p_{+a} \end{pmatrix}, \quad q^a_- = \begin{pmatrix} q^a_- & 0 \\ 0 & p^a_- \end{pmatrix},
\]
(4.40)
where \( q_\pm \) and \( p_\pm \) are standard chiral superfields, \( \bar{D}_a q_\pm = \bar{D}_a p_\pm = 0 \). The part of the classical action for these superfields reads
\[
S_{\text{diag}} = -\frac{1}{2} \int d^3x d^4 \theta \left[ v_L(\Box + M^2)v_L + \bar{v}_R(\Box + M^2)v_R - 2vLv_R M^2
\right.
\]
\[
\left. + q^a_- q_{+a} + \bar{p}^a_+ p_{+a} + \bar{q}_{-a} q^a_- + \bar{p}_{-a} p^a_- + g(v_L - v_R)(Q_{+a} \bar{q}^a_+ + \bar{Q}^a_- q_{+a} - Q^a_- \bar{q}_{-a} - \bar{Q}_{-a} q_{+a}) \right],
\]
(4.41)
where

\[ M^2 = g^2 Q^a_+ Q^a_+ = g^2 Q^-_a Q^-_a. \]  

(4.42)

The action (4.41) is very similar to (4.23). Therefore we can easily generalize the Kähler potential (4.27) to the case of the model with two hypermultiplets,

\[ \Gamma_{\text{diag}} = \frac{i}{2} \text{Tr}_v \ln \left( 1 - \frac{M^2}{4 \square_2} D^\alpha D_\alpha \right) = \frac{\sqrt{2} g}{2\pi} \int d^3 x d^4 \theta \sqrt{\bar{Q}_+^a Q_+^a}. \]  

(4.43)

Now let us consider the off-diagonal quantum superfields,

\[ v_{L,R} = \frac{1}{2} \begin{pmatrix} 0 & v_{L,R} \\ \bar{v}_{L,R} & 0 \end{pmatrix}, \quad q^a_+ = \begin{pmatrix} 0 & q^a_+ \\ p^{-a}_+ & 0 \end{pmatrix}, \quad q^a_- = \begin{pmatrix} 0 & q^a_- \\ p^{-a}_- & 0 \end{pmatrix}. \]  

(4.44)

The corresponding part of the classical action (4.37) is given by

\[ S_{\text{off-diag}} = -\frac{1}{2} \int d^3 x d^4 \theta \left[ \bar{v}_L (\hat{\square}_v + M^2) v_L + \bar{v}_R (\hat{\square}_v + M^2) v_R + \bar{q}^a_+ q^a_+ + \bar{p}^{-a}_+ p^{-a}_+ + \bar{p}^{-a}_- p^{-a}_- + g Q^a_+ (\bar{p}^a_+ v_L - \bar{q}^a_+ v_R) + g Q^a_- (\bar{p}^a_- v_R - \bar{q}^a_- v_L) \right. \]

\[ - \left. - \lambda \left[ \int d^3 x d^2 \theta \varepsilon^{ab} \varepsilon_{cd} Q^a_+ Q^b_+ (q^d_+ p^{-d}_+ - p^d_+ q^d_-) + \text{c.c.} \right] \right]. \]  

(4.45)

It is straightforward to compute the matrix of second variational derivatives of this action with respect to the quantum superfields and to find the trace of its logarithm. The details of this procedure are collected in the Appendix B.3. Here we present only the result,

\[ \Gamma_{\text{off-diag}} = 2i \text{Tr}_v \ln (\hat{\square}_v + Q^4) + 2i \text{Tr}_v \ln \left( \frac{M^2}{8 \square_v} D^\alpha D_\alpha \right), \]  

(4.46)

where \( M^2 \) is given in (4.42) and \( Q^4 \) is

\[ Q^4 = 4 \lambda \bar{\lambda}(Q^a_+ Q^a_-)(Q^b_+ Q^b_-). \]  

(4.47)

The traces of the operators in (4.46) can be computed in a standard way,

\[ \Gamma_{\text{off-diag}} = \frac{1}{\pi} \int d^3 x d^4 \theta \left[ 3G \ln G + 2 \sqrt{G^2 + g^2 Q^a_+ Q^a_+} - 2G \ln (G + \sqrt{G^2 + g^2 Q^a_+ Q^a_+}) \right] \]

\[ + \frac{1}{\pi} \int d^3 x d^4 \theta \left[ G \ln (G + \sqrt{G^2 + Q^4}) - \sqrt{G^2 + Q^4} \right]. \]  

(4.48)

The contribution from the ghost superfields is given by (4.34) because the ghost superfield sector is the same as in the model with one hypermultiplet. With the contribution
from the ghosts, the expressions (4.43) and (4.48) give together the low-energy effective action for the model (4.37),

\[ \Gamma = \frac{1}{\pi} \int d^3 x d^4 \theta [ \sqrt{2g} \sqrt{\bar{Q}_a^2 Q_{+a}} + 2 \sqrt{G^2 + g^2 \bar{Q}_a^2 Q_{+a}} - 2G \ln(G + \sqrt{G^2 + g^2 \bar{Q}_a^2 Q_{+a}})] \\
+ \frac{1}{\pi} \int d^3 x d^4 \theta \left[ G \ln(G + \sqrt{G^2 + Q_4^2}) - \sqrt{G^2 + Q_4^2} \right]. \quad (4.49) \]

Consider the asymptotics of the action (4.49) at large values of the gauge coupling constant \( g \),

\[ \Gamma = \frac{1}{\pi} \int d^3 x d^4 \theta \left[ G \ln(G + \sqrt{G^2 + Q_4^2}) - \sqrt{G^2 + Q_4^2} - G \ln(\bar{Q}_a^2 Q_{+a}) \right] \\
+ g \frac{4 + \sqrt{2}}{2\pi} \int d^3 x d^4 \theta \sqrt{\bar{Q}_a^2 Q_{+a}} + O(1/g). \quad (4.50) \]

We point out that the terms in the first line here are scale invariant since they have no dependence on \( g \). Moreover, it is easy to see that, up to obvious rescaling of fields, this part of the effective action coincides with the dual action of the ABJM theory (A.13). Therefore we conclude that the model (4.37) reproduces the ABJM theory as a scale independent part of its low-energy effective action. This confirms the statement that the D2 brane probing the tip of the conifold defined by the constraint (4.4) and (4.5) flows to its infrared superconformal point in which it is dual to the M2 brane [9]. However, the terms in the second line in (4.50) which have the form of the Kähler potential are non-conformal because of the dimensionfull coupling constant \( g \). Moreover, they dominate in the infrared, at large coupling constant, and the M2 brane dynamics should be suppressed and invisible on this background. We expect that the higher-loop corrections to the Kähler potential in this model cannot completely cancel the terms in the second line of (4.50) and do not change the present conclusions significantly.

One of the natural ways to prevent the appearance of the Kähler potential is to modify the model (4.36) by increasing the number of supersymmetries at least up to \( \mathcal{N} = 3 \). In particular, one can study the effective action in the Kachru-Silverstein model [24] reduced to three dimensions. In the three-dimensional case it has \( \mathcal{N} = 4 \) supersymmetry and quantum corrections of the form of the Kähler potential are forbidden by the non-renormalization theorems [15, 25, 26]. Alternatively, one can study the effective action in the ABJM model deformed by the SYM kinetic terms such that it has \( \mathcal{N} = 3 \) supersymmetry [28]. This problem deserves separate considerations.

5 Summary and discussion

It is well-known that the 2-branes in M-theory appear in the strong coupling limit for the D2 branes in the IIA supergravity. One of the aims of the present paper is to observe the consequences of this relation between the M2 and D2 branes for the corresponding field
theories living in the world-volume of these branes. This appears to be possible thanks to the recent progress in constructing the supergauge theory describing multiple M2 branes which are known as the BLG and ABJM models [1, 2]. On the side of the D2 brane, one has three-dimensional SYM theory which can be explored by standard methods of quantum field theory.

In the present paper we study the low-energy effective action in the $\mathcal{N} = 2$ quiver SYM theory with four chiral superfields in the bifundamental representation and with scale invariant superpotential. It is known that this model describes the D2 brane probing the singularity of the conifold $X_3$ fibred over real line [9, 7]. As is explained in [9], the D2 brane on such a background considered at strong coupling should reproduce the M2 brane on the background $X_4 = \mathbb{C}^4/\mathbb{Z}_k$ which is described by the ABJM theory [2]. In the present paper we demonstrate that the scale invariant part of the low-energy effective action of this $\mathcal{N} = 2$ SYM-matter theory precisely reproduces the classical action of the Abelian ABJM theory, rewritten in its dual form when one of the scalar superfields is dualized into a dynamical gauge superfield. However, we observe that, apart from these scale invariant contributions, the one-loop effective action receives the non-scale-invariant ones as well. These non-scale-invariant terms have the form of the Kähler potential and depend linearly on the gauge coupling constant $g$. Therefore they dominate in the infrared, at large values of the coupling, and the scale-invariant contributions having the form of the dualized ABJM action are suppressed by these non-conformal ones. Because of this phenomenon the correspondence between the D2 and M2 branes cannot be precisely confirmed.

We guess that the correspondence between the low-energy effective action of the three-dimensional SYM-matter theory and the classical action of the ABJM model can be checked more precisely if one modifies the $\mathcal{N} = 2$ quiver gauge theory (4.36) in such a way that the Kähler potential for chiral superfields would not appear in the effective action. For instance, one can consider an analog of the model (4.36) with the $\mathcal{N} = 3$ supersymmetry [28] which prohibits the generation of the Kähler potential. The $\mathcal{N} = 3$, $d = 3$ harmonic superspace methods [29] should be helpful for these considerations. Alternatively, one can study the effective action in the Kachru-Silverstein model [24] reduced to three dimensions such that it has the $\mathcal{N} = 4$ supersymmetry. Quantum aspects in these models will be explored elsewhere.

With the same motivations it would be interesting to do direct quantum computations of the low-energy effective actions in many other $\mathcal{N} = 2$ Chern-Simons-matter models which were considered in [30] from the point of view of mirror symmetry.

In the present paper we computed also the one-loop effective actions in the pure SYM models with $\mathcal{N} = 2$, $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supersymmetry. The low-energy effective action in the $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SYM starts with the terms which contain $F^2$ in components while the $\mathcal{N} = 8$ SYM effective action starts from $F^4$. The generation of the $F^2$ term by quantum corrections in the $\mathcal{N} = 8$ SYM is forbidden by the non-renormalization theorem [22] which is analogous to the one in the $\mathcal{N} = 4$, $d = 4$ SYM. The leading $F^2$ terms in the $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SYM were derived originally by employing the mirror symmetry [15, 20, 25, 26]. We stress that by doing direct quantum computations in the $\mathcal{N} = 2$,
$d = 3$ superspace we derive non only these leading $F^2$ terms, but also all higher order ones giving $F^{2n}$ in components for all positive integer $n$.

An interesting feature of the $\mathcal{N} = 4$ SYM model is that its low-energy effective action starts from the terms which are superconformal and coincide with the classical action of the Abelian Gaiotto-Witten model [10] written in its dualized form [19]. A similar feature was mentioned in [11] for the hypermultiplet interacting with the Abelian background $\mathcal{N} = 4$ gauge multiplet. This can be considered as a prototype of the correspondence between the effective actions of gauge theories on D2 and M2 branes, but living in the reduced space-time.

A natural continuation of the present work is the study the low-energy effective action in the ABJM model deformed by the SYM kinetic terms for the gauge superfields such that the supersymmetry is reduced down to $\mathcal{N} = 3$ [28]. When the gauge coupling is sent to infinity and the SYM kinetic terms drop out, the $\mathcal{N} = 6$ supersymmetry is restored and this should give us the low-energy effective action in the ABJM theory. This will result in the superspace analogs of the results of similar component computations which were done in [31]. It is interesting also to investigate the effective action in other superconformal three-dimensional gauge theories, considered e.g. in [32], which are interesting from the point of view of the AdS/CFT correspondence.

Another tempting problem is the study of the two-loop quantum contributions to the effective action in the $\mathcal{N} = 2$ supersymmetric electrodynamics both in the Coulomb and in the Higgs branches. The one-loop effective action on the Coulomb branch in this model was considered in [11] while the one-loop corrections to the Higgs branch were partly considered in the present work, e.g., from (4.27). Note that these one-loop results are not new because they were obtained quite a while by utilizing the power of the mirror symmetry [15, 25, 26]. However, two-loop corrections to these effective actions are promising. In particular, the two-loop Kähler potential in the $\mathcal{N} = 1$ supersymmetric electrodynamics was studied in [33], but similar $\mathcal{N} = 2$ superspace considerations are welcome.

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A Dualization of the Gaiotto-Witten and ABJM models

A.1 Dualization of the Abelian Gaiotto-Witten model

Consider the following model in the $\mathcal{N} = 2$ superspace,

$$ S_{GW} = -\int d^3x d^4\theta [\bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_- + 4\hat{V}G], \tag{A.1} $$

where $(\bar{Q}_+, Q_-)$ is the hypermultiplet while $V$ and $\hat{V}$ are two gauge superfields with the corresponding field strengths $G$ and $\hat{G}$. The action (A.1) is invariant under the following hidden $\mathcal{N} = 2$ supersymmetry

$$ \delta Q_+ = \frac{1}{2} \hat{D}^2 (\bar{\theta}^\alpha \epsilon_\alpha \bar{Q}_+ e^{-2V}), \quad \delta Q_- = -\frac{1}{2} \hat{D}^2 (\bar{\theta}^\alpha \epsilon_\alpha \bar{Q}_- e^{2V}), $$

$$ \delta \hat{V} = -i\epsilon^\alpha \bar{\theta} \alpha Q_+ Q_- - i\epsilon^\alpha \theta_\alpha \bar{Q}_+ Q_- , \quad \delta V = 0. \tag{A.2} $$

Here $\epsilon_\alpha$ is the complex parameter of the hidden supersymmetry.

It should be noted that a four-dimensional analog of the action (A.1) was considered originally in [18] as a dual form of the improved $\mathcal{N} = 2$, $d = 4$ tensor multiplet action. Alternatively, the action (A.1) can be viewed as a BF-type theory coupled to matter chiral superfields. The importance of three-dimensional supersymmetric BF-type theories was pointed out in [34], where the authors studied the coupling of such models to supergravity as well as some aspects of their duality and mirror symmetry. More recently, in [19] it was shown that (A.1) corresponds to the classical action of the Abelian Gaiotto-Witten model [10] which is, in general, a Chern-Simons-matter theory with $N = 4$ supersymmetry.

Following [18], we consider the equation of motion for the gauge superfield $V$,

$$ \bar{Q}_+ e^{2V} Q_+ - \bar{Q}_- e^{-2V} Q_- + 2\hat{G} = 0. \tag{A.3} $$

It is solved by

$$ e^{-2V} = \frac{\hat{G} + \sqrt{\hat{G}^2 + \bar{Q}_+ Q_+ \bar{Q}_- Q_-}}{\bar{Q}_- Q_-}, \tag{A.4} $$

or

$$ V = -\frac{1}{2} \ln(\hat{G} + \sqrt{\hat{G}^2 + \bar{Q}_+ Q_+ \bar{Q}_- Q_-}) + \frac{1}{2} \ln(\bar{Q}_- Q_-). \tag{A.5} $$

It is convenient to denote

$$ \bar{Q}_+ \bar{Q}_- = \bar{\Phi}, \quad Q_+ Q_- = \Phi. \tag{A.6} $$

Substituting this solution back into the action (A.1) we get

$$ \tilde{S}_{GW} = 2 \int d^3x d^4\theta [\hat{G} \ln(\hat{G} + \sqrt{\hat{G}^2 + \bar{\Phi}\Phi}) - \sqrt{\hat{G}^2 + \bar{\Phi}\Phi}]. \tag{A.7} $$
This action is known to be \( N = 4 \) supersymmetric and superconformal, see, e.g., [12].

In (A.7) both the gauge superfield \( \hat{V} \) and the chiral superfield \( \Phi \) are propagating while in the original action (A.1) the gauge superfields are non-dynamical and only chiral superfields \( Q_{\pm} \) propagate. Hence, one of the chiral superfields in (A.1) is dualized into the Abelian gauge superfield with the preservation of the supersymmetry and superconformal invariance. Therefore we refer to (A.7) as a dual representation of the Abelian action for Gaiotto-Witten theory, although (A.7) was known long before within the study of dualities among the tensor multiplet and supersymmetric sigma models [12, 18].

A.2 Dualization of the Abelian ABJM model

The ABJM model [2] is similar to the Gaiotto-Witten theory, but the hypermultiplet is a SU(2) doublet, \((\bar{Q}^a_a, Q^a_a)\), \(a = 1, 2\). The action in the Abelian case is quite simple,

\[
S_{\text{ABJM}} = -\int d^3x d^4\theta [\bar{Q}^a_a e^{2V} Q^a_a + Q^a_- e^{-2V} \bar{Q}_a^- + 4\hat{G}].
\]  (A.8)

Analogous four-dimensional models were considered in [18] within the study of non-linear sigma-models.

The action (A.8) possesses hidden \( N = 4 \) supersymmetry,

\[
\delta Q_+ = \frac{1}{2} \hat{D}^2 [(\theta^\alpha \epsilon_{\alpha a b}) \bar{Q}_- b e^{-2V}], \quad \delta Q_- = -\frac{1}{2} \hat{D}^2 [(\bar{\theta}^\alpha \epsilon_{\alpha a b}) \bar{Q}^b_+ e^{2V}],
\]

\[
\delta \hat{V} = -i \theta^\alpha \epsilon_{\alpha a b} Q^b_+ \bar{Q}^-_a b - i \bar{\theta}^\alpha \epsilon_{\alpha a b} \bar{Q}^b_+ Q^-_a b, \quad \delta V = 0. \quad (A.9)
\]

Here \( \epsilon^\alpha_{a b} \) is a real supersymmetry parameter which bears two SU(2) indices. Its indices are raised or lowered with the \( \epsilon^{a b} \) symbol. The reality of this supersymmetry parameter is required by the on-shell closure of the supersymmetry transformations. Together with the explicit \( N = 2 \) supersymmetry, the transformations (A.9) form the \( N = 6 \) supersymmetry of the ABJM theory.

The equation of motion for the gauge superfield \( V \)

\[
\bar{Q}^a_+ e^{2V} Q^a_+ - \bar{Q}^-_a e^{-2V} Q^a_- + 2\hat{G} = 0
\]  (A.10)

is solved by

\[
e^{-2V} = \frac{\hat{G} + \sqrt{\hat{G}^2 + \bar{Q}^a_a Q^a_+ Q^-_a b \bar{Q}^b_-}}{Q^a_- - \bar{Q}^a_a b}, \quad (A.11)
\]

or

\[
V = -\frac{1}{2} \ln(\hat{G} + \sqrt{\hat{G}^2 + \bar{Q}^a_+ Q^a_+ Q^-_a b \bar{Q}^b_-}) + \frac{1}{2} \ln(\bar{Q}^-_a Q^a_+). \quad (A.12)
\]

Substituting this solution back into (A.8) we get the dual representation of the ABJM action,

\[
\tilde{S}_{\text{ABJM}} = 2 \int d^3x d^4\theta \left[ \hat{G} \ln(\hat{G} + \sqrt{\hat{G}^2 + \bar{Q}^a_+ Q^a_+ Q^-_a b \bar{Q}^b_-}) - \sqrt{\hat{G}^2 + \bar{Q}^a_+ Q^a_+ Q^-_a b \bar{Q}^b_-}
\]

\[-\hat{G} \ln(Q^a_- \bar{Q}^-_a) \right]. \quad (A.13)
\]
In components, this action contains supersymmetric and superconformal generalization of the Maxwell $F^2$ term which originates from the superfield strength $\hat{G}$.

At first sight the dual ABJM action (A.13) contains four propagating chiral superfields $Q_{+a}$ and $Q_{a}^-$ as well as the propagating gauge superfield $\hat{G}$. However, only three of four chiral superfields are independent here. Indeed, one can redefine the chiral superfields such that the SU(2) invariance becomes implicit,

$$\Phi_1 = Q_{+1}Q_{1}^-, \quad \Phi_2 = Q_{+2}Q_{2}^-, \quad \Phi_3 = Q_{+3}Q_{3}^-.$$ (A.14)

In terms of these superfields the action (A.13) reads

$$\tilde{S}_{\text{ABJM}} = 2 \int d^3x d^4\theta \left[ \hat{G} \ln \left( \sqrt{\hat{G}^2 + \Phi_1^2\Phi_2^2 + \Phi_3^2\Phi_3^2} \right) \right] .$$ (A.15)

In this action one can see only three propagating chiral superfields $\Phi_i$, $i = 1, 2, 3$, which, together with the gauge superfield $\hat{G}$ give correct number of degrees of freedom of the ABJM theory with the action (A.8).

### B Some technical details of one-loop computations

#### B.1 Transformation of the effective action in the model (4.23) to the form (4.25)

The one-loop effective action is given by the standard expression $\Gamma = \frac{i}{2} \text{Tr} \ln H$, where $H$ is the matrix of second variational derivatives of the action (4.23) with respect to the quantum superfields. It has the following block form

$$H = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{\delta^2 S}{\delta v_{L,R}(z) \delta v_{L,R}(z')} & \frac{\delta^2 S}{\delta q_{L,R}(z) \delta q_{L,R}(z')} \\ \frac{\delta^2 S}{\delta q_{L,R}(z) \delta q_{L,R}(z')} & \frac{\delta^2 S}{\delta q_{L,R}(z) \delta q_{L,R}(z')} \end{pmatrix},$$ (B.1)

where the block matrices are

$$A = - \begin{pmatrix} \Box + M^2 & -M^2 \\ -M^2 & \Box + M^2 \end{pmatrix} \delta^{7}(z - z'),$$

$$B = -\frac{g}{2} \begin{pmatrix} \bar{Q}_{+}\delta_{+}(z, z') & -\bar{Q}_{+}\delta_{+}(z, z') & Q_{+}\delta_{-}(z, z') & -Q_{+}\delta_{-}(z, z') \\ \bar{Q}_{-}\delta_{+}(z, z') & -\bar{Q}_{-}\delta_{+}(z, z') & Q_{-}\delta_{-}(z, z') & -Q_{-}\delta_{-}(z, z') \end{pmatrix},$$

$$C = B^\dagger,$$

$$D = \begin{pmatrix} 0 & 0 & \frac{1}{8} \bar{D}^2 \delta_{-}(z, z') & 0 \\ 0 & 0 & 0 & \frac{1}{8} \bar{D}^2 \delta_{-}(z, z') \\ \frac{1}{8} D^2 \delta_{+}(z, z') & 0 \\ 0 & \frac{1}{8} D^2 \delta_{+}(z, z') \end{pmatrix}.$$. (B.2)
Using standard decomposition for the block matrices,

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{pmatrix}
1 & BD^{-1} \\
0 & 1
\end{pmatrix} \begin{pmatrix}
A - BD^{-1}C & 0 \\
0 & D
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
D^{-1}C & 1
\end{pmatrix},
\]  
(B.3)

we get the following representation for the one-loop effective action

\[
\Gamma_{\text{diag}} = \frac{i}{2} \text{Tr}_v \ln \left( \begin{array}{cc}
-\Box - M^2 + \frac{1}{16} M^2 \{D^2, D^2\} & M^2 - \frac{1}{16} M^2 \{D^2, D^2\} \\
M^2 - \frac{1}{16} \Box \{D^2, D^2\} & -\Box - M^2 + \frac{1}{16} M^2 \{D^2, D^2\}
\end{array} \right)
\]

\[
= \frac{i}{2} \text{Tr}_v \ln \left( \begin{array}{cc}
-\Box + \frac{1}{8} M^2 D^a \bar{D}^2 D_\alpha & -\frac{1}{8} M^2 D^a \bar{D}^2 D_\alpha \\
-\frac{1}{8} \Box D^a \bar{D}^2 D_\alpha & -\Box + \frac{1}{8} M^2 \{D^2, \bar{D}^2\}
\end{array} \right)
\]

\[
= \frac{i}{2} \text{Tr}_v \ln(1 - \frac{1}{4} \frac{M^2}{\Box} D^a \bar{D}^2 D_\alpha) = \frac{i}{2} \text{Tr}_v \frac{1}{16} \ln(1 + \frac{2M^2}{\Box}) D^a \bar{D}^2 D_\alpha .
\]  
(B.4)

Here we have applied the standard identity for the covariant spinor derivatives,

\[
\frac{1}{16} \{D^2, \bar{D}^2\} - \frac{1}{8} D^a \bar{D}^2 D_\alpha = \Box .
\]  
(B.5)

All operators in (B.4) act in the full superspace.

A shorter way to get the representation (B.4) for the effective action is to use the Landau gauge instead of the Fermi-Feynman one. In this case there are no mixed contributions with quantum vector and matter propagators and the last two lines in (4.12) can be simply omitted.

**B.2 Transformation of the effective action in the model (4.29) to the form (4.30)**

The matrix of second variational derivatives of the action (4.29) with respect to the quantum superfields has the structure (B.1), but with the block matrices given by

\[
A = -\frac{1}{2} \left( \begin{array}{cccc}
0 & 0 & \Box_v + M^2 & 0 \\
0 & 0 & 0 & \Box_v + M^2 \\
\Box_v + M^2 & 0 & 0 & 0 \\
0 & \Box_v + M^2 & 0 & 0
\end{array} \right) \delta^7(z - z'),
\]  
(B.6)

\[
B = -\frac{g}{2} \left( \begin{array}{cccc}
0 & Q_+ \hat{\delta}_+ & 0 & 0 \\
0 & 0 & \hat{Q}_-\hat{\delta}_+ & 0 \\
0 & \hat{Q}_-\hat{\delta}_+ & Q_- \hat{\delta}_- & 0 \\
0 & 0 & Q_+ \hat{\delta}_- & 0
\end{array} \right),
\]

\[
C = B^\dagger,
\]
\[ D = \begin{pmatrix}
0 & 0 & 0 & 0 & \frac{1}{8} \bar{D}^2 \delta_- & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{8} \bar{D}^2 \delta_- & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8} \bar{D}^2 \delta_- & 0 \\
0 & \frac{1}{8} \bar{D}^2 \delta_+ & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{8} \bar{D}^2 \delta_+ & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{8} \bar{D}^2 \delta_+ & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}. \]

Here \( \delta_\pm \equiv \delta_\pm(z, z') \) are the covariantly (anti)chiral delta functions (in contrast to the flat ones \( \delta_\pm(z, z') \) which were used (B.2)),

\[
\delta_+(z, z') = -\frac{1}{4} \bar{D}^2 \delta^7(z - z'), \quad \delta_-(z, z') = -\frac{1}{4} \bar{D}^2 \delta^7(z - z'). \tag{B.7}
\]

The background superfields \( Q_\pm \) are constant, \( D_\alpha Q_\pm = 0 \), since we are interested in the low-energy effective action in the constant field approximation. The background covariant spinor derivative \( \bar{D}_\alpha \) is defined as in the Abelian case, \( \bar{D}_\alpha = D_\alpha + \mathbf{V}_\alpha \), where the gauge connection \( \mathbf{V}_\alpha \) acts just by multiplication. Therefore the covariant spinor derivatives do not hit the background (anti)chiral superfields, \( \bar{D}_\alpha Q_\pm X = Q_\pm \bar{D}_\alpha X \). Keeping this in mind, we get the following form for the matrix \( A - BD^{-1}C \),

\[
A - BD^{-1}C = \begin{pmatrix} 0 & E \\ E & 0 \end{pmatrix}, \tag{B.8}
\]

where

\[
E = -\frac{1}{2} \left( \hat{\square}_v + M^2 - \frac{M^2}{16} \{ \bar{D}^2, D^2 \} \frac{1}{\hat{\square}_v} \right) \delta^7(z - z'). \tag{B.9}
\]

Here we have used standard relations between the covariant derivatives and covariant box operators [11],

\[
\bar{D}^2 \hat{\square}_+ = \bar{D}^2 \hat{\square}_v = \hat{\square}_v \bar{D}^2, \quad \bar{D}^2 \hat{\square}_- = \bar{D}^2 \hat{\square}_v = \hat{\square}_v \bar{D}^2. \tag{B.10}
\]

Consider now the expression \( \hat{\square}_v + M^2 - \frac{M^2}{16} \{ \bar{D}^2, D^2 \} \frac{1}{\hat{\square}_v} \delta^7(z - z') \) which appears in (B.9). Using the identity (2.30), it can be rewritten as

\[
\hat{\square}_v + \frac{M^2}{\hat{\square}_v} \left[ \frac{1}{16} \{ \bar{D}^2, D^2 \} - \frac{1}{8} \bar{D}^\alpha \bar{D}^2 D_\alpha + \frac{i}{2} (\bar{D}^\alpha \mathbf{W}_\alpha) + i \mathbf{W}_\alpha \bar{D}_\alpha \right] = \frac{M^2}{16} \{ \bar{D}^2, D^2 \} \frac{1}{\hat{\square}_v}. \tag{B.11}
\]

As soon as this expression is considered under the integral over the full superspace, we can omit the superfield strength \( \mathbf{W}_\alpha \) since it is responsible for the higher-order contributions to the effective action (see (4.19) and comments nearby). As a result, (B.11) simplifies,

\[
\hat{\square}_v - \frac{1}{8} \frac{M^2}{\hat{\square}_v} \bar{D}^\alpha \bar{D}^2 D_\alpha. \tag{B.12}
\]

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We point out that the operator \( \hat{\Box}_v \) commutes with the covariant spinor derivatives \( D_\alpha \) and \( \bar{D}_\alpha \) because, according to (2.3), these commutators are proportional to the spinor superfield strengths which we neglect in the approximation (4.19).

Applying the standard matrix relation (B.3), we rewrite the one-loop effective action in the model (4.29) in the following form

\[
\Gamma_{\text{off-diag}} = \frac{i}{2} \text{Tr} \ln H = \frac{i}{2} \text{Tr} \ln (A - BD^{-1}C) + \frac{i}{2} \text{Tr} \ln D
\]

\[
= 2i \text{Tr}_v \ln \left( \hat{\Box}_v - \frac{1}{8} \frac{M^2}{\Box_v} D^\alpha \bar{D}^\alpha D_\alpha \right) + 2i \text{Tr}_+ \ln \hat{\Box}_+
\]

\[
= 2i \text{Tr}_+ \hat{\Box}_+ + 2i \text{Tr}_v \ln \hat{\Box}_v + 2i \text{Tr}_v \ln \left( 1 - \frac{M^2}{8 \Box_v^2} D^\alpha \bar{D}^\alpha D_\alpha \right). \quad (B.13)
\]

Our final comments concern the computations of the last term in (B.13). For the considered background, when we neglect the superfield strengths \( W_\alpha \) and \( \bar{W}_\alpha \) in the full superspace, it is sufficient to consider only the following terms in \( \hat{\Box}_v \),

\[
\hat{\Box}_v \approx D^m D_m + G^2. \quad (B.14)
\]

Then we can use the relation \((D^\alpha \bar{D}^\alpha D_\alpha)^n = (-8 \Box_v)^{n-1} D^\alpha \bar{D}^\alpha D_\alpha\) to get

\[
2i \text{Tr}_v \ln \left( 1 - \frac{M^2}{8 \Box_v^2} D^\alpha \bar{D}^\alpha D_\alpha \right) \delta^7(z - z') = -\frac{i}{4} \text{Tr}_v \frac{1}{\Box_v} \text{Tr} \ln(1 + \frac{M^2}{\Box_v^2} D^\alpha \bar{D}^\alpha D_\alpha) \delta^7(z - z')
\]

\[
= -4i \text{Tr} \frac{1}{\Box + G^2} \ln(1 + \frac{1}{\Box + G^2}) \delta^3(x - x'). \quad (B.15)
\]

Here we have applied also the first identity from (4.26).

### B.3 Transformation of the effective action in the model (4.45) to the form (4.46)

As usual, the one-loop effective action is defined by the matrix of second variational derivatives (B.1) with respect to the quantum superfields. In the case of the action (4.45) the block matrix \( A \) in (B.1) is the same as (B.6), but the matrices \( B, C \) and \( D \) are now

\[
B = -\frac{g}{2} \left( \begin{array}{cccccc}
0 & Q^a_\alpha \delta_+ & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -Q_{-a} \delta_+ & 0 & -Q_{+a} \delta_- \\
0 & 0 & -Q_{-a} \delta_+ & 0 & 0 & 0 \\
-Q^a_\alpha \delta_+ & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array} \right), \quad (B.16)
\]

\[
C = B^\dagger,
\]

\[
D = \left( \begin{array}{cccccc}
0 & 0 & 0 & -N^b \delta_+ & 1/\delta_a D^2 \delta_- & 0 \\
0 & 0 & N^b \delta_+ & 0 & 0 & 1/\delta_a D^2 \delta_- \\
0 & 0 & 0 & 0 & 0 & 0 \\
-N^b \delta_+ & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array} \right).
\]
where
\[ N^a_b = \lambda \varepsilon^{ac} \varepsilon_{bd} Q_{+c} Q_{-d}^d, \quad \bar{N}^b_a = \bar{\lambda} \varepsilon^{bc} \varepsilon_{ad} \bar{Q}_{+c} Q_{-d}^d. \]  
\hspace{1cm} \text{(B.17)}

Let us introduce also the following superfields
\[ S^a_c \equiv N^a_b \bar{N}^b_c = -\lambda \bar{\lambda} (Q_{+c} Q_{-e}) \varepsilon^{ab} \varepsilon_{cd} Q_{+b} Q_{-d}^d, \]
\[ \bar{S}^a_c \equiv \bar{N}^a_b N^b_c = -\lambda \bar{\lambda} (Q^c_{+e} Q_{-e}) \varepsilon^{ab} \varepsilon_{cd} Q^d_{-c} Q_{-b}. \]  
\hspace{1cm} \text{(B.18)}

They have the following properties
\[ S^a_a = Q^4, \quad \bar{S}^a_a = Q^4, \]
\[ \bar{S}^a_c \delta^c_d = Q^4 S^a_d, \quad S^a_c S^c_d = (Q^4)^{n-1} S, \]
\[ S^a = (Q^4)^{n-1} S, \quad \bar{S}^a = (Q^4)^{n-1} \bar{S}, \]  
\hspace{1cm} \text{(B.19)}

where \( Q^4 \) is
\[ Q^4 = \lambda \bar{\lambda} (Q_{+a} Q_{+a})(Q^b_{-b} Q^b_{-b}). \]  
\hspace{1cm} \text{(B.20)}

To begin with, let us compute the trace of the logarithm of the matrix \( D \) given by (B.17). This matrix consists of four different blocks which contribute as follows
\[ \frac{i}{2} \text{Tr} \ln D = 2i \text{Tr} \ln \left( \begin{array}{cc} 0 & \frac{1}{2} \delta_b^c \bar{D}^2 \delta_+ \\ \frac{1}{2} \delta_b^c \bar{D}^2 \delta_+ & 0 \end{array} \right) \]
\[ = 2i \text{Tr} \ln \left( \begin{array}{cc} \delta_b^c \bar{D}^2 \delta_+ & -\frac{1}{2} N^a_b \bar{D}^2 \delta_- \\ -\frac{1}{2} N^a_b \bar{D}^2 \delta_- & \delta_b^c \bar{D}^2 \delta_+ \end{array} \right) - 2i \text{Tr} \ln \left( \begin{array}{cc} 0 & \frac{1}{2} \delta_b^c \bar{D}^2 \delta_+ \\ \frac{1}{2} \delta_b^c \bar{D}^2 \delta_+ & 0 \end{array} \right) \]
\[ = 2i \text{Tr}_+ \ln \bar{D}^2 + 2i \text{Tr}_+ \ln (\bar{D}^2 + 4Q^4). \]  
\hspace{1cm} \text{(B.21)}

Now we comment on the computation of the matrix \( A - BD^{-1}C \). In fact, this matrix has the same block structure as (B.8). Therefore it is sufficient to compute only one element in it. One of the non-vanishing elements in the matrix \( BD^{-1}C \) is given by the following expression
\[ \frac{g^2}{4} (\bar{Q}_{-b}, -Q_{+b}) \left( \begin{array}{cc} -N^b_a \delta^c_d & \frac{1}{8} \delta_b^c \bar{D}^2 \delta_- \\ \frac{1}{8} \delta_b^c \bar{D}^2 \delta_+ & -\bar{N}^b_a \delta^c_d \end{array} \right)^{-1} \left( \begin{array}{c} -\bar{Q}_{+a} \\ Q^a \\ \end{array} \right) \]
\[ = \frac{g^2}{4} (\bar{Q}_{-b}, -Q_{+b}) \left( \begin{array}{c} 4 N^b_a (\delta^c_d + \frac{4S^c_d}{\bar{D}^2+a-Q^c}) \bar{D}^2 \delta_+ \\ \frac{1}{2} \bar{D}^2 \delta_+ + \frac{4S^d_b}{\bar{D}^2+a-Q^d} \bar{D}^2 \delta_- \\ 4 N^b_a (\delta^c_d + \frac{4S^c_d}{\bar{D}^2+a-Q^c}) \bar{D}^2 \delta_- \\ \frac{1}{2} \bar{D}^2 \delta_- \end{array} \right) \left( \begin{array}{c} -\bar{Q}_{+a} \\ Q^a \\ \end{array} \right) \]
\[ = \frac{g^2}{8} (\bar{Q}_{+a} Q_{+a}) \frac{1}{\bar{D}^2} - \bar{D}^2 \delta_+ + \frac{g^2}{8} (Q^a \bar{Q}_{-a}) \frac{1}{\bar{D}^2} \delta_- = -\frac{M^2}{32} \{ \bar{D}^2, \bar{D}^2 \} \frac{1}{\bar{D}^v} \delta^7(z-z'). \]  
\hspace{1cm} \text{(B.22)}

Here we have used the properties (B.19) of the matrices (B.17) and (B.18), the identities (B.10) and the constraint (4.39). The full matrix \( -BD^{-1}C \) reads
\[ -BD^{-1}C = \left( \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right) \frac{M^2}{32} \{ \bar{D}^2, \bar{D}^2 \} \frac{1}{\bar{D}^v} \delta^7(z-z'). \]  
\hspace{1cm} \text{(B.23)}
Summing up this matrix with (B.6), we get the matrix \( A - BD^{-1}C \) exactly in the form (B.8,B.9). Hence, the corresponding contributions to the effective action can be extracted from (B.13),

\[
\frac{i}{2} \text{Tr} \ln(A - BD^{-1}C) = 2i \text{Tr}_\nu \ln \hat{\nabla}_\nu + 2i \text{Tr}_\nu \ln \left( 1 - \frac{M^2}{8\hat{\Box}_\nu} \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha \right). \tag{B.24}
\]

This expression, together with (B.21), defines the structure of the one-loop effective action in the model (4.45),

\[
\Gamma_{\text{off-diag}} = 2i \text{Tr}_+ \ln \hat{\Box}_+ + 2i \text{Tr}_+ \ln(\hat{\Box}_+ + Q^4) + 2i \text{Tr}_\nu \ln \hat{\nabla}_\nu + 2i \text{Tr}_\nu \ln \left( 1 - \frac{M^2}{8\hat{\Box}_\nu} \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha \right). \tag{B.25}
\]

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