Majorana Bound States Hallmarks in a Quantum Topological Interferometer Ring

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In this work, the conductance and current correlations properties of a quantum topological interferometer consisting of a quantum dot coupled to two Majorana bound states (MBSs) confined at both ends of a 1D topological superconductor ring nanowire is investigated. The ring in its topological nontrivial and trivial phases is analyzed to show that the tunneling conductance, shot noise, and Fano factor present unique characteristics to distinguish the hallmarks of MBSs. The findings are reinforced by taking advantage of the full correspondence between the quantum topological interferometer and a quantum-dot effectively coupled to a single Majorana state in a straight topological superconductor wire configuration. It is shown that besides the characteristic zero-bias conductance $e^2/2h$ and the already known shot noise features, the Fano factor provides significant information to distinguish the MBSs presence.

1. Introduction

A Majorana bound state (MBS), in condensed matter physics, is a zero-energy quasi-particle with the particularity of being its own antiparticle.\(^{[1]}\) These quasi-particles belong to the family of anyons and therefore have non-Abelian exchange statistics, which makes them exciting objects for fault-tolerant topological quantum computation.\(^{[2-7]}\)

MBSs take place in quantum systems with strong spin-orbit coupling, superconductivity, and broken time-reversal symmetry.\(^{[4,5,8]}\) The ideal platforms for MBSs involve topological superconductors realized by bringing semiconductors nanowires (or a 2D semiconductor system) with strong spin-orbit coupling into proximity with standard s-wave superconductors and exposing them to a magnetic field.\(^{[4,5,8-11]}\)

The spin-orbit coupling affects dramatically the way the electrons pair up in the superconductor, resulting in a switch from s-wave superconductivity to p-wave superconductivity in 1D (or $p + ip$ superconductivity in 2D\(^{[12]}\)), along with the magnetic field, which will drive the p-wave superconductor to a topological phase transition. Above a critical magnetic field, the p-wave superconductor ($p + ip$ superconductivity) can become topologically nontrivial.\(^{[13]}\) Theory predicts that MBSs should be localized at the boundaries of the topological superconductor — in this case the ends of the nanowire (or bound to superconducting vortices in the 2D $p + ip$ case\(^{[14]}\)).

A criterion to detect Majorana modes consists of measuring the zero-bias conductance peak (ZBCP) from tunneling electrons into the MBSs.\(^{[15-20]}\) Nevertheless, confirming such states require seeking for extra features since other zero-energy modes different than MBSs can also lead to zero bias peaks, for instance, from Andreev bound states (ABSs), multi-band effects,\(^{[21]}\) weak antilocalization,\(^{[22]}\) and the Kondo effect.\(^{[23,24]}\) Currently, distinguishing MBSs from ABSs is one of the most critical challenges, which has led to considerable theoretical proposals,\(^{[24-32]}\) mostly focused on quantum dots coupled to topological superconductors (QD–MBSs configurations). Indeed, indications of their existence have been shown by probing their transport conductance spectrum,\(^{[29,33-36]}\) thermal conductance,\(^{[37,38]}\) ac Josephson effect,\(^{[39,40]}\) generation and discontinuities of persistent currents\(^{[41,42]}\) and current noise correlations.\(^{[36,34,43-48]}\) Particularly, it has been proposed to combine tunneling conductance and shot noise correlations measurements as a complementary diagnosis method to distinguish real from fake MBSs.\(^{[44,49-51]}\)

In this letter, besides studying the tunneling conductance and shot noise correlations properties, we also focus on seeking a distinguishable Fano factor fingerprint of a QD coupled to two MBSs confined at the ends of a 1D topological superconductor nanowire ring — denoted here as QD–MBSs ring system. We underpin our findings by analyzing the conditions that lead to a full correspondence between our system of interest (QD–MBSs ring) and a topological QD–MBSs wire system,\(^{[33]}\) as illustrated in Figure 1. Finally, we argue that the Fano factor, jointly with the reported results for ZBCP and shot noise, would stand as a more robust diagnostic tool for distinguishing the real MBSs from spurious-zero energy modes.

2. Description of the Model

We consider the setup shown in Figure 1a in which a spinless quantum dot is coupled to two MBSs, $\gamma_1$ and $\gamma_2$, located at the
ends of a topological superconductor nanowire (TSNW). The Hamiltonian takes the form

$$H = H_L + H_D + H_M + H_{DM} + H_T$$

(1)

Where, $H_L$ describes the left (L) and right (R) metallic leads,

$$H_L = \sum_{k,a=L,R} \epsilon_{ka} c_{ka}^\dagger c_{ka}$$

(2)

$c_{ka}^\dagger$ and $c_{ka}$ are the creation and annihilation operators with energy $\epsilon_{ka}$ in the lead $\alpha = L, R$. $H_D$ is the Hamiltonian of the quantum dot,

$$H_D = \epsilon_d d^\dagger d$$

(3)

which describes a dot with an energy level $\epsilon_d$, with $d^\dagger$ ($d$) being its creation (annihilation) operator. The term $H_{MB}$ in Equation (1)

$$H_{MB} = i\epsilon_M \gamma_1 \gamma_2$$

(4)

describes the coupling between the two MBSs, $\gamma_1$ and $\gamma_2$, with the overlap being $\epsilon_M$. The term $H_{DM}$ denotes the coupling between the QD and MBSs

$$H_{DM} = (\lambda_1^1 d^\dagger - \lambda_1 d) \gamma_1 + i(\lambda_2^1 d^\dagger + \lambda_2 d) \gamma_2$$

(5)

with the coupling parameters $\lambda_1 = |\lambda_1|e^{i\phi/4}$, $\lambda_2 = |\lambda_2|e^{-i\phi/4}$, where $|\lambda_1|$ and $|\lambda_2|$ denote the QD-MBS coupling strength and $\phi = \Phi/\Phi_0$ with $\Phi_0 = \hbar/2e$ is the phase factor resulting from the threading magnetic flux. Two normal metallic leads L and R are attached to the QD with coupling strength $\Gamma_L$ and $\Gamma_R$. As we will show the topological QD–MBS ring system (a) is equivalent to the topological QD–MBSs wire system configuration in (b), with two MBSs $\eta_1$ and $\eta_2$, wherein $\lambda$ denotes the coupling strength between the QD and the MBS $\eta_1$.

The two MBSs $\gamma_1$ and $\gamma_2$ can be represented by their equivalent Dirac fermion operators according to $\gamma_1 = (f^\dagger + f)/\sqrt{2}$ and $\gamma_2 = i(f^\dagger - f)/\sqrt{2}$, which transforms the terms $H_{MB}$ and $H_{DM}$ in the Hamiltonian as follows,

$$H_{DM} = \frac{1}{\sqrt{2}} (\lambda_1^1 f^\dagger f + \frac{1}{\sqrt{2}} (\lambda_1 - \lambda_2) f d$$

$$+ \frac{1}{\sqrt{2}} (\lambda_2^1 f^\dagger f + \frac{1}{\sqrt{2}} (\lambda_1 + \lambda_2) f d$$

(8)

In general, the current from the lead $\alpha$ ($\alpha = L, R$) is given by $I_\alpha = e(N_\alpha) = i\epsilon_d \langle \frac{1}{2} \epsilon_d \rangle$, from which, we can get,[35]

$$\hat{I}_\alpha (t) = i\epsilon_d \sum_k \left[ t_k (c_{ka}^\dagger (t) d (t) - c_{ka} (t^\dagger) (t) c_{ka} (t)) \right]$$

(9)

We are concerned with fluctuations of the current away from their average value. We thus introduce the operators $\delta \hat{I}_\alpha (t) = \hat{I}_\alpha (t) - \langle \hat{I}_\alpha (t) \rangle$ and define the spectral density of shot noise by the Fourier transformation of the current correlation[36]

$$\Pi_{\alpha\alpha'} (t, t') = \langle \delta \hat{I}_\alpha (t) \delta \hat{I}_{\alpha'} (t') \rangle + \langle \delta \hat{I}_\alpha (t') \delta \hat{I}_{\alpha} (t) \rangle$$

(10)

Substituting the current operator Equation (9) into the current correlation Equation (10) and using the Wick’s theorem, the correlation function can be expressed by the Green functions of the system. Then, applying the Fourier transformation over the times $t$ and $t'$, and using the relation $S_{\alpha\alpha'} (\Omega) \delta (\Omega + \Omega') = \frac{1}{\pi} \Pi_{\alpha\alpha'} (\Omega, \Omega')$, we obtain the shot noise of self-correlation $S = S_{\alpha\alpha} (0)$ in the left

Figure 1. Schematic setup of the QD-MBSs ring system. A QD is coupled to two MBSs, $\gamma_1$ and $\gamma_2$, located at the ends of a TSNW. Here, $\lambda_1 = |\lambda_1|e^{i\phi/4}$, $\lambda_2 = |\lambda_2|e^{-i\phi/4}$, where $|\lambda_1|$ and $|\lambda_2|$ denote the QD-MBS coupling strength and $\phi = \Phi/\Phi_0$ with $\Phi_0 = \hbar/2e$ is the phase factor resulting from the threading magnetic flux. Two normal metallic leads L and R are attached to the QD with coupling strength $\Gamma_L$ and $\Gamma_R$. As we will show the topological QD–MBS ring system (a) is equivalent to the topological QD–MBSs wire system configuration in (b), with two MBSs $\eta_1$ and $\eta_2$, wherein $\lambda$ denotes the coupling strength between the QD and the MBS $\eta_1$. 
terminal as
\[
S = - \frac{2e^2}{\hbar} \int dE \left[ G'_L(e)\Sigma^r_L(e)G'_R(e)\Sigma^r_R(e) + G'_L(e)\Sigma^r_L(e)G'_R(e)\Sigma^r_R(e) + (\Sigma^r_L(e) - \Sigma^r_R(e))\Sigma^r_R(e)G'_R(e)\Sigma^r_L(e) + G'_L(e)\Sigma^r_L(e)G'_R(e)\Sigma^r_R(e) + G'_L(e)\Sigma^r_L(e)G'_R(e)\Sigma^r_R(e) + (\Sigma^r_L(e) - \Sigma^r_R(e))\Sigma^r_R(e)G'_R(e)\Sigma^r_L(e) + G'_L(e)\Sigma^r_L(e)G'_R(e)\Sigma^r_R(e) + (\Sigma^r_L(e) - \Sigma^r_R(e))\Sigma^r_R(e)G'_R(e)\Sigma^r_L(e) \right]
\]
(11)

where \(G_{\alpha}^{G,\alpha'}^{<,>}(e)\) are the Green functions of the QD, \(\Sigma^{\alpha,\alpha'}_{\alpha'}(e)\) are the self-energies of the L and R leads and \(\Sigma^{\alpha,\alpha'}_{\alpha'} = 2\pi\rho_0 V_{\alpha}\) is the line width function describing the coupling between the dot and the lead in the wide band approximation, with \(\rho_0\) being the density of states in the leads.

After some mathematical calculations we found the retarded Green function of the QD as follows,
\[
G_{\alpha}(e) = \left[\omega - \epsilon - i\frac{\Gamma}{2} - A(\omega) + B(\omega)\right]^{-1}
\]
(12)

where \(A(\omega) = K(\{\epsilon_{\alpha_1}\}^2 + |\epsilon_{\alpha_2}|^2 - 2\omega_{\alpha_1}\{\epsilon_{\alpha_1}|\epsilon_{\alpha_2}|\cos\frac{\phi}{2})\) and \(B(\omega) = \frac{\omega^2 + \omega^{\alpha_{1,2}} - |\omega|^{\alpha_1}\{\epsilon_{\alpha_1}|\epsilon_{\alpha_2}|\cos\phi}{(\omega + \omega^{\alpha_{1,2}} - |\omega|)^2}\) with \(K\) and \(\Gamma\) defined as \(K = \frac{\omega}{\omega - \omega^{\alpha_1,2}_{\alpha_1,2}}\) and \(\Gamma = \Gamma_{\alpha_1} + \Gamma_{\alpha_2}\).

Substituting all Green functions and self-energies into Equation (11)
\[
S = \frac{2e^2}{\hbar} \int dE \left[ 2\{\epsilon_{\alpha_1}\}^2\tilde{R}(e)\Sigma^r_L(e)\Sigma^r_R(e)F_{LL}(e) + (1 + C(e))^2T_N(e)\left[F_{LL}(e) + F_{RR}(e)\right] + (1 + C(e))T_N(e)\left[F_{LR}(e) + F_{RL}(e)\right] \right]
\]
(13)

where,
\[
C(e) = \tilde{R}(e)^2\{\epsilon_{\alpha_1}\}^2 + |\epsilon_{\alpha_2}|^2 - 2|\epsilon_{\alpha_1}|^2|\epsilon_{\alpha_2}|\cos\phi
\]
(14)

and \(T_N = F_{\alpha_1}\Gamma_{\alpha_2}G_\alpha^G(e)\) is the transmission. We also define \(F_{\alpha\beta}(e) = f_{\alpha}(e)[1 - f_{\beta}(e)]\), with \(\alpha\) and \(\beta\) being L and R. \(f_{\alpha}(e) = f(e - \mu_{\alpha_1})\) is the Fermi-Dirac distribution with \(\mu_{\alpha_1}\) the chemical potential for the lead L(R). The first and the second term in Equation (13) represent the thermal noise, which vanish at zero temperature. Finally, we define \(T_M = |\epsilon_{\alpha_1}|^2\tilde{R}(e)\Sigma^r_L(e)\Sigma^r_R(e)\). Then the shot noise in Equation (13) can be written as,
\[
S = \frac{2e^2}{\hbar} \int dE \left[ T_N(e)(1 - T_N(e)) + T_M(e)(1 - T_M(e)) - 2T_N(e)T_M(e)\left[f_{\alpha}(e)(1 - f_{\beta}(e)) + f_{\alpha}(e)(1 - f_{\beta}(e))\right]\right]
\]
(15)

3. Results

In what follows, we set \(\epsilon_0 = 0\) and assume that the QD is, for simplicity, symmetrically coupled to the two MBSSs, that is, \(|\lambda_1| = |\lambda_2|\). We also assume a symmetric dot-contact coupling \(\Gamma_{\alpha_1} = \Gamma_{\alpha_2}\). From this point on, \(\Gamma = \Gamma_{\alpha_1} + \Gamma_{\alpha_2}\) will be considered as the energy unit and \(E_R = 0\). The shot noise is given in units of \(S_0 = 2e^2/h\).

In Figure 2a-d, we show the conductance (in units of \(G_0 = e^2/h\)) as a function of the bias voltage \(eV/\Gamma\) and the magnetic flux phase \(\phi\) for several values of coupling between MBSSs, \(\epsilon_\alpha\). From here that the topological transition is associated with a substantial conductance drop.

For the other values of \(\phi\), two Fano antiresonances emerge approximately at \(eV = \pm \epsilon_\alpha\) whose minimum do not fall to zero (Figure 2f,g).\(^{[35]}\) Especially when the nanowire is in its topological phase (the one with Majorana zero modes at the end of the nanowire), that is, when \(\phi = \pi, 2\pi, \ldots, (2n + 1)\pi\), the antiresonances, located around \(\pm \epsilon_\alpha\) are identical shape, but an opposite sign of the Fano parameter (Figure 2g).\(^{[36]}\) Regardless of the magnetic flux phase, as \(\epsilon_\alpha\) increases, the Fano antiresonance are shifted toward larger values of \(|eV|\). It is pertinent to mention here that the topological transition is associated with a substantial conductance transition. As is shown in Figure 2a for \(\epsilon_\alpha = 0\), we observe a jump from \(G = 0\) in the trivial topological region to \(G = e^2/2h\) in the nontrivial topological region,\(^{[31]}\) which allows distinguishing the two different phases of the wire.

The results of shot noise calculated using Equation (15) are presented in Figure 3, where we show the shot noise (in units of \(S_0 = 2e^2/h\)) as a function of bias voltage \(eV/\Gamma\) for several values of \(\epsilon_\alpha/\Gamma\) and different values of magnetic flux phase \(\phi\). We notice that in analogy with the conductance (Figure 2), when the magnetic flux changes, the shot noise changes periodically with a period \(4\pi\). When the coupling between Majorana fermions \(\epsilon_\alpha\) start to increase, we can observe how small steps appear in the shot noise. These small steps are positioned in the same value of \(eV\) as the corresponding Fano antiresonances in the conductance. As \(\epsilon_\alpha\) increases, the height of these steps also increases, and they are shifted toward larger \(|eV|\). Particularly, it is interesting to notice that when the ring is in its topological phase, \(\phi = \pi\), (see Figure 3c) these steps are not distinctly visible because the Fano antiresonances in the conductance do not fall to zero. Besides, the shot noise is symmetrical in the same way as the conductance.

The calculation of shot noise and current allows to compute the Fano factor, defined as \(F = S/2e|I|\), which is shown in Figure 4. We display the evolution of the Fano factor as \(\epsilon_\alpha\) increase from \(\epsilon_\alpha = 0\) up to \(\epsilon_\alpha = 0.1\Gamma\) (see Figure 4a–d). We observe that the Fano factor changes periodically as we sweep the magnetic flux phase \(\phi\), which is a consequence of the periodicity in \(\phi\) of the shot noise and current. We note that for \(\epsilon_\alpha = 0\) the Fano factor is symmetrical, and we also observe how when we start to increase \(\epsilon_\alpha\) from \(\epsilon_\alpha = 0.025\Gamma\) it becomes antisymmetric. It is relevant to notice that when \(\epsilon_\alpha = 0\), there is a drastic
Figure 2. Differential conductance (in units of $G_0 = e^2/h$) as a function of bias voltage $eV/\Gamma$ and $\phi$ for several values of MBS coupling, $\epsilon_M$: a) $\epsilon_M = 0$, b) $\epsilon_M = 0.025\Gamma$, c) $\epsilon_M = 0.05\Gamma$, d) $\epsilon_M = 0.1\Gamma$. (e–h) show the conductance as a function of bias voltage $eV/\Gamma$ for several values of $\epsilon_M$ and different values of magnetic flux phase $\phi$. We use the following parameters: $|\lambda_1| = |\lambda_2| = 0.1\Gamma$, $\epsilon_d = 0$, $\Gamma_L = \Gamma_R = 0.5\Gamma$.

Variation of the Fano factor consisting in a jump from $F = 1/2$ in the trivial topological phase to $F = 1/4$ in the nontrivial topological phase of the system at $eV = 0$, as we see later. This jump is related to the topological transition of the ring. In Figure 4e–h we can observe the Fano factor as a function of bias voltage $eV/\Gamma$ for several values of $\epsilon_M/\Gamma$ and for some representative values of magnetic flux phase $\phi$. When $\epsilon_M$ starts to increase, a tiny step is located at the value of $|\epsilon_M|$ arise. The magnitude of this step decreases with $\epsilon_M$. Besides, we obtain that when the ring is in its nontrivial topological phase ($\phi = \pi$), the Fano factor has different behavior compared with the Fano factor when the topological superconducting nanowire is in its trivial phase. When the ring is in its trivial topological phase and $\epsilon_M = 0$, the Fano factor acquires the value of $1/4$ at $eV = 0$ (Figure 4g). When $\epsilon_M \neq 0$ the value of the Fano factor at $eV = 0$ is zero. This unique behavior does not occur when the ring is in the trivial topological phase, except for the case when there is a large overlap between the MBSs ($\epsilon_M$) at the two ends of the wire where for $\epsilon_M \neq 0$ the Fano factor is zero at $eV = 0$. This is caused by the fact that when $\epsilon_M$ is large, the two Majorana states are equivalent to a single ordinary ABS. It is worth emphasizing that when $\epsilon_M = 0$, for small variations of the magnetic flux phase $\phi$ around $\pi$, there is an abrupt variation of the Fano factor from $1/2$ (in the trivial topological phase) to $1/4$ (in the nontrivial topological phase). It shows that the system is susceptible to small phase variation, as shown in Figure 4i–m.

In general, it is hard to find an analytical expression for the Fano factor. However, at zero bias voltage, the Fano factor can be approximated as follows:

$$F = \frac{1}{2} - \frac{2\Gamma_L\Gamma_R}{\Gamma^2 + 4(e_d - 2|\lambda_1\lambda_2|\cos(\phi/2))^2}$$

From the above equation, some special cases can be obtained. For $\epsilon_M = 0$, we have $F = 1/2$ and $F = 1/4 - \frac{4|\lambda_1\lambda_2|^2}{\Gamma^2}$ when the system is in its trivial and nontrivial topological phases, respectively. The Fano factor exhibits different behavior depending on whether the nanowire is in its nontrivial or trivial topological phases. We will argue that a MBS signature can be extracted from this. To this purpose, we observe that the Hamiltonian, $H_{MBS-DM} = H_M + H_{DM}$, for the MBS’s – QD system is,

$$H_{MBS-DM} = \epsilon_d d^\dagger d + i\epsilon_M\gamma_1\gamma_2 + (\lambda_1^* d^\dagger - \lambda_1 d)\gamma_1 + i(\lambda_2^* d^\dagger + \lambda_2 d)\gamma_2$$

which can be mapped to the Hamiltonian of a topological QD–MBSs wire system configuration (see Figure 1b, wherein $\lambda$ denotes the coupling strength between the QD and the MBS $\eta_1$, and $\epsilon_M$ denotes coupling between the two MBSs $\eta_1$ and $\eta_2$. This
**Figure 3.** Shot noise (in units of $S_0 = 2e^2/h$) as a function of bias voltage $eV/\Gamma$ for several values of $\epsilon_M/\Gamma$ and for different values of magnetic flux phase, a) $\phi = 0$, b) $\phi = \pi/2$, c) $\phi = \pi$, and d) $\phi = 3\pi/2$. $|\lambda_1| = |\lambda_2| = 0.1\Gamma$, $\epsilon_d = 0$, $\Gamma_L = \Gamma_R = 0.5\Gamma$.

**Figure 4.** Fano factor as a function of bias voltage $eV/\Gamma$ and $\phi$ for several values of MBSs coupling, $\epsilon_M$, a) $\epsilon_M = 0$, b) $\epsilon_M = 0.025\Gamma$, c) $\epsilon_M = 0.05\Gamma$, d) $\epsilon_M = 0.1\Gamma$; (e–h) show the Fano factor as a function of bias voltage $eV/\Gamma$ for several values of $\epsilon_M$ and different values of magnetic flux phase $\phi$; (i–m) show the Fano factor for the case of $\epsilon_m/\Gamma = 0$ considering small variations of the magnetic flux phase $\phi$ around $\phi = \pi$ where $\delta = \pi/18$. We use the following parameters $|\lambda_1| = |\lambda_2| = 0.1\Gamma$, $\epsilon_d = 0$, $\Gamma_L = \Gamma_R = 0.5\Gamma$. 

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can be made if we take (without loss of generality) \( \lambda_1 \) to be real (\( \lambda_1 = |\lambda_1| \)) and \( \lambda_2 = |\lambda_2| e^{i\phi/2} \). Thus, one can build the unitary transformation \( U(\theta, \varphi) \) belonging to the SU(2) group that relate \( \eta_1 \) and \( \eta_2 \) with \( \gamma_1 \) and \( \gamma_2 \) as follows

\[
\begin{pmatrix}
\eta_1 \\
\eta_2
\end{pmatrix} =
\begin{pmatrix}
\cos \theta/2 & e^{i\varphi/2} \sin \theta/2 \\
-e^{-i\varphi/2} \sin \theta/2 & \cos \theta/2
\end{pmatrix}
\begin{pmatrix}
\gamma_1 \\
\gamma_2
\end{pmatrix}
= U(\theta, \varphi)
\begin{pmatrix}
\gamma_1 \\
\gamma_2
\end{pmatrix}
\]

(18)

where, \( \tan \theta/2 = |\lambda_2|/|\lambda_1| \) and \( \varphi = \phi + \pi \). Applying \( U(\theta, \varphi) \) to the Hamiltonian \( H_{M-DM} \) becomes,

\[
H_{M-DM}' = U^\dagger H_{M-DM} U
= \lambda (\eta_1 d^\dagger - \eta_2^\dagger d)
+ i\epsilon_d (\eta_1 \eta_2 + i \sin \theta \cos(\phi/2))
\]

(19)

Note that for \( \phi = (2n + 1)\pi \) (n integer), we obtain, \( \eta_1 = \eta_1^\dagger \), \( \eta_2 = \eta_2^\dagger \), and \( H_{M-DM} = i\epsilon_d \eta_1 \eta_2 + \lambda (d^\dagger - d) \eta_1 \). In this case, \( U = U(\theta) \), represents a rotation in an angle \( \theta \) on the plane formed by \( \gamma_1 - \gamma_2 \), such that \( \eta_1 \) and \( \eta_2 \) lie over the circle of unit radius. The above result implies that a quantum-dot coupled to two MBSs reduces to a quantum-dot coupled to a single Majorana state \( \eta_1 \), which in turn is coupled to another Majorana fermion \( \eta_2 \) with a coupling \( \epsilon_{dM} \) \([13,58]\). Therefore, a QD coupled to two MBSs in a ring configuration threatened by a quantum flux, can be mapped into a quantum-dot effectively coupled to a single Majorana state, \( \eta_1 \), in a wire configuration for any value of \( \epsilon_{dM} \) (see Figure 1). It must be emphasized that this correspondence is independent of whether the magnitudes of the couplings \(|\lambda_1|\) and \(|\lambda_2|\) are equal or not. As a consequence of such correspondence, the conductance, shot noise, and Fano factor for the QD-MBSs ring system and QD-MBS wire system are identical.

In fact, Figure 5, displays the differential conductance and Fano factor for different values of QD-MBSs coupling \(|\lambda_1|\), where we assume for simplicity that \(|\lambda_1| = |\lambda_2|\). As is well-known, the conductance in the topological non-trivial phase is always \( G = e^2/2h \) \([31]\) as long as \( \epsilon_{dM} = 0 \), as can be seen in Figure 5. It is worth noticing how the Fano factor increases with \( \lambda_1 \) for both configurations, and more importantly, the Fano factor always gives 1/4 at zero bias voltage, as long as \( \epsilon_{dM} = 0 \). Note that this result is independent of the particular choice of \(|\lambda_1|\) and \(|\lambda_2|\), and the relation \(|\lambda_1| = |\lambda_2|\). Therefore, this result suggests that shot noise measurements, in particular, of Fano factor give additional information complementary to the one known by studying the characteristic zero-bias conductance \( e^2/2h \).

Consequently, the study of the combination of both shot noise and conductance through a QD could provide a clear signature and distinguish the MBSs.

Finally, it is worth mentioning that the fact that a QD coupled to two MBSs in a ring configuration threatened by a quantum flux, is (fully) corresponding into a quantum-dot effectively coupled to a single Majorana state \( \eta_1 \) in a wire configuration and, at the same time, (under special conditions) is corresponding to a conventional T-shape quantum dot system (QD–QD–S) as we show in a previous work \([54]\) suggest that the correspondence which these systems are representing is relatively general and applies for more than one geometry.

4. Summary

In this work, we investigated the current correlations properties of a topological ring system consisting of a QD coupled to two MBSs confined at both ends of a 1D topological superconductor nanowire. We found that when the ring is in its nontrivial topological phase, \( \phi = (2n + 1)\pi \), the Fano factor has a unique behavior compared with the Fano factor when the topological superconducting nanowire is in its trivial phase. To obtain a MBS distinguishing feature from this, we built a SU(2) unitary transformation which transform a QD coupled to two MBSs (in a ring configuration) into a quantum dot effectively connected to a single Majorana state (in a wire configuration). This full equivalence is independent of the values of \( \epsilon_{dM} \) and of whether the magnitudes of the couplings \(|\lambda_1|\) and \(|\lambda_2|\) are equal or not. As a consequence of such equivalence, we found that besides the characteristic zero-bias conductance \( e^2/2h \), the Fano factor gives additional information that could be definitive to find a clear signature to distinguish the MBSs. Chiu has experimentally proposed the QD coupled to two MBSs (in a ring configuration) system in ref. \([52]\) however, current correlations properties of this of system have not been experimentally studied until now.

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Conflict of Interest

The authors declare no conflict of interest.
Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Keywords

fano factor, majorana bound states, shot noise, topological superconductors, quantum interferometer

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[1] E. Majorana, Il Nuovo Cimento 1937, 14, 171.
[2] J. Alicea, Rep. Prog. Phys. 2012, 75, 076501.
[3] C. Beenakker, Annu. Rev. Condens. Matter Phys. 2013, 4, 113.
[4] Y. Oreg, G. Refael, F. von Oppen, Phys. Rev. Lett. 2010, 105, 177002.
[5] R. M. Lutchyn, J. D. Sau, S. Das Sarma, Phys. Rev. Lett. 2010, 105, 077001.
[6] M. Z. Hasan, C. L. Kane, Rev. Mod. Phys. 2010, 82, 3045.
[7] M. Sato, S. Fujimoto, J. Phys. Soc. Jpn. 2016, 85, 072001.
[8] I. C. Fulga, A. Haim, A. R. Akhmerov, Y. Oreg, New J. Phys. 2013, 15, 045020.
[9] T. D. Stanescu, R. M. Lutchyn, S. Das Sarma, Phys. Rev. B 2011, 84, 144522.
[10] J. Cayao, E. Prada, P. San-Jose, R. Aguado, Phys. Rev. B 2015, 91, 024514.
[11] J. D. Sau, R. M. Lutchyn, S. Tewari, S. Das Sarma, Phys. Rev. Lett. 2010, 104, 040502.
[12] M. Leijnse, K. Flensberg, Semicond. Sci. Technol. 2012, 27, 124003.
[13] H. Z. Lu, Physics 2020, 13, 30.
[14] N. B. Kopnin, M. M. Salomaa, Phys. Rev. B 1991, 44, 9667.
[15] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, L. P. Kouwenhoven, Science 2012, 336, 1003.
[16] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, H. Shtrikman, Nat. Phys. 2012, 8, 887.
[17] A. D. K. Finck, D. J. Van Harlingen, P. K. Mohseni, K. Jung, X. Li, Phys. Rev. Lett. 2013, 110, 126406.
[18] L. P. Rohklinson, X. Liu, J. K. Furdyna, Nat. Phys. 2012, 8, 795.
[19] F. Nicole, A. C. C. Drachmann, A. M. Whiticar, E. C. T. O’Farrell, H. J. Suominen, A. Forneri, T. Wang, G. C. Gardiner, C. Thomas, A. T. Hatzé, P. Krogstrup, M. J. Manfra, K. Flensberg, C. M. Marcus, Phys. Rev. Lett. 2017, 119, 136803.
[20] K. Flensberg, Phys. Rev. B 2010, 82, 180516.
[21] J. Liu, A. C. Potter, K. T. Law, P. A. Lee, Phys. Rev. Lett. 2012, 109, 267002.
[22] D. I. Pikulin, J. P. Dahlhaus, M. Wimmer, H. Schomerus, C. W. J. Beenakker, New J. Phys. 2012, 14, 125011.
[23] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav, M. Kastner, Nature 1998, 391, 156.
[24] M. Hell, K. Flensberg, M. Leijnse, Phys. Rev. B 2018, 97, 164101.
[25] K. M. Tripathi, S. Das, S. Rao, Phys. Rev. Lett. 2016, 116, 166401.
[26] A. Haim, E. Berg, F. von Oppen, Y. Oreg, Phys. Rev. Lett. 2015, 114, 166406.
[27] C. X. Liu, J. D. Sau, S. Das Sarma, Phys. Rev. B 2018, 97, 214502.
[28] C. X. Liu, J. D. Sau, T. D. Stanescu, S. Das Sarma, Phys. Rev. B 2017, 96, 075161.
[29] L. Ricco, M. de Souza, M. Figueira, I. Shelykh, A. Seridonio, arXiv:1811.10305, 2018.
[30] C. T. Deng, S. Vaitkevičienė, E. Prada, P. San-Jose, J. Nygård, P. Krogstrup, R. Aguado, C. M. Marcus, Phys. Rev. B 2018, 98, 085125.
[31] J. D. Sau, B. Swingle, S. Tewari, Phys. Rev. B 2015, 92, 020511.
[32] V. V. Volkov, M. Y. Kagan, S. V. Aksenov, J. Phys.: Condens. Matter 2019, 31, 225301.
[33] D. E. Liu, H. U. Baranger, Phys. Rev. B 2011, 84, 201308.
[34] Y. Cao, P. Wang, G. Xiong, M. Gong, X. Q. Li, Phys. Rev. B 2012, 86, 115311.
[35] J. P. Ramos-Andrade, P. A. Orellana, S. E. Ulloa, J. Phys.: Condens. Matter 2018, 30, 045301.
[36] S. En-Ming, P. Yi-Ming, S. Lu-Bing, W. Bai-Gen, Chin. Phys. B 2014, 23, 057201.
[37] L. Ricco, F. Dessotti, I. Shelykh, M. Figueira, A. Seridonio, Sci. Rep. 2018, 8, 2790.
[38] M. Leijnse, New J. Phys. 2014, 16, 015029.
[39] F. Domínguez, F. Hassler, G. Platero, Phys. Rev. B 2012, 86, 140503.
[40] P. San-Jose, E. Prada, R. Aguado, Phys. Rev. Lett. 2012, 108, 257001.
[41] C. Benjamin, J. K. Pachos, Phys. Rev. B 2010, 81, 085101.
[42] A. Nava, R. Giuliano, C. Campagnano, D. Giuliani, Phys. Rev. B 2017, 95, 155449.
[43] H. F. Lü, H. Z. Lu, S. Q. Shen, Phys. Rev. B 2016, 93, 245418.
[44] Q. Chen, K. Q. Chen, H. K. Zhao, J. Phys.: Condens. Matter 2014, 26, 315011.
[45] P. Devillard, D. Chevallier, M. Albert, Phys. Rev. B 2017, 96, 115413.
[46] T. Jonchheere, J. Rech, A. Zazunov, R. Egger, A. L. Yeyati, T. Martin, Phys. Rev. Lett. 2019, 122, 097003.
[47] D. E. Liu, M. Cheng, R. M. Lutchyn, Phys. Rev. B 2015, 91, 081405.
[48] A. Schuray, M. Rammiller, P. Recher, Phys. Rev. B 2020, 102, 045303.
[49] J. Manousakis, C. Wille, A. Altland, R. Egger, K. Flensberg, F. Hassler, Phys. Rev. Lett. 2020, 124, 096801.
[50] D. Guerci, A. Nava, arXiv:1907.06444, 2019.
[51] M. Cheng, M. Becker, B. Bauer, R. M. Lutchyn, Phys. Rev. X 2014, 4, 031051.
[52] C. K. Chiu, J. D. Sau, S. Das Sarma, Phys. Rev. B 2018, 97, 035310.
[53] H. Haug, A. P. Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, Springer, Berlin 2008.
[54] Y. M. Blanter, M. Büttiker, Phys. Rep. 2000, 336, 1.
[55] Q. B. Zeng, S. Chen, L. You, R. Lü, Front. Phys. 2016, 12, 127302.
[56] A. M. Calle, M. Pacheco, P. A. Orellana, J. A. Otalora, Ann. Phys. 2020, 532, 1900409.
[57] L. Wan, Y. Wei, J. Wang, Nanotechnology 2005, 17, 489.
[58] K. Flensberg, Phys. Rev. Lett. 2011, 106, 090503.