Application of discrete mathematics, tetralogic and architecture of superscalar systems in measurement metrology of automated control systems

D I Fakhertdinova¹, V D Munister², A L Zolkin³, A V Knishov⁴, and M Yu Speranskiy⁵

¹ Department of Natural Sciences, Service and Tourism, Kazan Cooperative Institute of the Russian University of Cooperation (KCI RUC), N.Ershova Street, 58, Kazan 420081, Russia; Department of Natural and Physics-mathematic Sciences, International preparatory school, Kazan (Volga region) Federal University (KFU), Kremlevskaya Street 18, Kazan 420008, Russia
² Department of Mathematical Methods and Computer-Aided Design, Private educational institution "Donetsk Academy of Transport", Dzerzhinsky Street 7, Donetsk 83086, Ukraine; Department of Enterprise Economics, State educational institution of higher professional education "Donetsk National Technical University", Shbankova Street 2, Pokrovsk, Donetsk region 85300, Ukraine
³ Computer and Information Sciences Department, Povolzhskiy State University of Telecommunications and Informatics, L.Tolstogo Street 23, Samara 443010, Russia; Natural Sciences Department, Private institution of higher education "Medical University" Reaviz, Chapaevskaya Street 227, Samara 443001, Russia
⁴ Management department, State Public Institution of Higher Education Russian Customs Academy, Komsomolskiy prospect 4, Lyubertsy 140015, Moscow region, Russia
⁵ Laboratory of the "Renewable energy sources and power systems and networks” department, Sevastopol State University, Universitetskaya Street 33, Sevastopol 299053, Russia

⁶ E-mail: dinaraf@mail.ru

Abstract. The article is dedicated to the consideration of the role and significance of the quaternary logic in automated control systems. The methodology of reduction and application of the principle of ergodicity in measurement metrology in automated process control systems is presented. The optimization role of the tools used in solving of hybrid technological and telemetry problems is emphasized.

1. Rationale
From the point of view of modern approaches to the design of automated control systems, there are no strict requirements for modifiers and logic descriptors. At the moment, discrete models of control systems have become widespread. However, metrological assurance of measurement accuracy in real-
time systems, or in systems close to such, demonstrates some dichotomous and limited nature of these systems.

The fact is that the processes of interaction in the real world are still reduced to analog values, and direct reduction to quantization and sampling at a proper sampling rate gives only an approximate final picture of what is happening, blurring not only the measurement accuracy and consequently, the implementation of decision making, but also the adaptability of the system to new factors and realities, the probabilities of which, although provided by the developers, are leveled out in the control process.

Refactoring of such systems (which are the overwhelming majority) is determined to be difficult. The main problem comes down not to the bitness of the data or the number of data channels, but actually to the binary processing (the basic and unchangeable principle).

The fact is that if each logic gate is considered separately, then it defines a conservative function of one variable or two variables [1].

2. Technical and economic feasibility and implementation of tetralogic

Large data compression is out of the question in such bus systems. If the switching function of the gate is considered exclusively, then, in this case, more cannot be required.

However, modern realities dictate the requirements for ergodicity compliance. In such systems, there is an identity in which some probabilistic characteristics, including the generally accepted mathematical expectation along the time front, must coincide with the mathematical expectation for spatial or relational groups, or fields (a relational or vector entity is little used in modern control systems).

In other words in order to determine the parameters of a closed technical system, the behavior of one of its elements can be observed for a long time, or all its elements (or a relevant subset of elements) can be considered in a very short time. In other words if the system has the property of ergodicity, then in both cases equivalent final results will be obtained.

Thus, the general focus on the transition to one of only two stable states has its drawbacks in design and work, taking into account the presentation of certain requirements, properties and parameters of criteria-based assessment [2].

But the property under consideration is only one of several that is directly related to the ongoing processes that require system analysis.

These systems are found in a wide range of systems. First of all systems based on a natural science approach, i.e. systems applicable to real processes of physics, chemistry, biology.

This can be extrapolated as a direct implication of some general phenomenon in the biophysical sense of the word: for example, the motion of suspended particles, which has a geodesic and hyperbolic diversity of divergence. Because when this manifold is compact (i.e. it has a finite size) these orbits return to the same common area, eventually filling the entire space.

The observed biophysical systems capture determinations based on empirical experience in the idea of randomness as an analysed event (even among ordinary observers), for example, that smoke can fill an entire room, or that a metal block at the end can have the same temperature in its' every point, or that an honest coin can have heads and tails results in fifty percent of cases. An even stronger manufacturing concept than ergodicity is the mixing concept, which aims to mathematically describe diffusion concepts in real life (for example processes of drinks mixing or ingredients mixing for cooking).

The dispersive and differentiated nature of production processes underlies the general economic theory of processing and treatment.

In order to adapt the finite elements of automated control systems in dynamic systems to the requirements of ergodicity, it is proposed at the first stage to reduce the set of states and transitions of each element of the system to a Markov chain, thereby predetermining the ergodicity. It is a model describing a sequence of possible events in which the probability of each event depends only on the
state achieved in the previous case. This is a very convenient model in this case precisely because of this characteristic.

A counting sequence (which can then be implemented as a hardware device (an incremental counter)), in which the chain moves to different states in discrete time steps, gives a finite time for the Markov chain. The methodological aspects of describing the Markov chain of states make it possible to define later reaction models and behaviour models by complementing a probabilistic finite automaton, which is actually a Markov chain.

The transition probability distribution is usually represented as a matrix. If the Markov chain has N possible states, then the matrix will have the form N x N, in which the record (I, J) will be the probability of transition from state I to state J.

Based on the Markov chain shown in figure 1, expressed in terms of a finite automaton, the initial state vector initialized as a matrix of dimension N x 1 can be obtained.

\[
P = \begin{bmatrix}
0.9 & 0.075 & 0.025 \\
0.15 & 0.8 & 0.05 \\
0.25 & 0.25 & 0.5
\end{bmatrix}
\]

An approximate transition matrix with three possible states.

\[
\varphi(0) = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

state vector

In addition, such a matrix must be stochastically determined, i.e. the results of summing of rows or columns must be equal 1 (one). In such a matrix, each row will have its own probability distribution.

The vector itself in this case describes the probability distributions of the beginning in each of the N possible states. Reduction to a Markov automaton (chain) allows to determine probabilistic transitions, both for each actuator, separately, and for the entire automated control system as a whole [3].

The identical rule of analysis for determining the probability of transition from one state to another has been demonstrated, but what about obtaining this probability in a few steps? This is very important for automatic control with its multi-stage conveyors. To do this, it is necessary to determine the probability of transition from state I to state J in M steps.

In this case, it is necessary to determine the transition matrix (P) by a trivial calculation (I, J) by raising P to the power M. Sometimes, for small scalar M values, this can be formalized manually, using repeated multiplication, an extremely convenient command in systems with Harvard architecture (programmable logic controllers, industrial controllers).

**Figure 1.** General view of the Markov chain of the production process with the states of "selection", "capture" and "processing" in the form of circles and with edges in the form of transitions.
But, it is better to use the methods of linear algebra, which, in a more efficient way of raising a matrix to a power, will first diagonalize the represented matrix, in particular, according to the principle "if and only if" when there is an urgent need or computational complexity. And only then it can be easily determined through simulation of the approximate values of calculations in typical situations and equivalents.

Metrological support, at this stage, is expressed through the reduction of the final values of the calculations of probabilities to a certain basis, preferably to the previously calculated values of probabilities using computer technology within the framework of calculations with double precision [4].

3. Tetralogical component of metrological support

The tetralogic apparatus can be used after receiving (for each gate) of identical probability models in the form of deterministic finite automata with the necessary information in a matrix form convenient for calculations.

What is it for? First of all, it is needed in order to create a stable connection at the hardware level between the transition probability calculated in the process of work, the reference probability, and for the possibility of implementing four stable states at the same level of logic. The very first iteration of the analysis will allow an instant metrological response to the state of a particular unit, to pursue a particular policy, in case of deviation from the norm, to clarify errors.

Why not to do this through direct measurements of physical quantities? Because it is better to apply two-factor, and in the ideal case, multi-factor verification of the parameters of real processes. Classic multifunctional measuring instruments, such as gas analyzers, allow this to be carried out with a delay at a certain time, measured in milliseconds or microseconds.

The proposed additional method will make it possible to carry out ratios in several machine cycles, which, in general, amounts to hundreds of nanoseconds. Moreover, this is achieved due to the fact that the analysis itself is carried out on an actuator, terminal device, logic gate, and is not installed in parallel or in series, as is often the case in electrical devices and networks, which are indispensable components of both control systems and life support systems of enterprises.

How to implement a comparison using tetra-coding of reference probabilities and those predicted in reality in a tetracode? By synthesizing a digital comparator operating in the framework of tetralogy [5].

The final states of tetralogy, by analogy with binary logic, can be encrypted by a set represented as values C4 = \{0, A, M, 1\} and used by analogy with a binary code for the bitwise representation of quantitative values that are required in our case [6]:

By analogy with the concept of "bit", the functional concept "tetrit" is used for bitwise representation:

- tetrites 0 and 1 are implemented to bits 0 and 1, respectively, since they encrypt similar values and can act as standards of limits.
- tetrite M is reduced to bits 0 and 1 at the same time, synchronously, therefore, it is represented by two points on the numerical axis;
- tetrite A is reduced to bits 0 or 1 (at the moment of reduction, its value is unknown), therefore it is represented as one of two possible points on the numerical axis.

In the given paradigm, the value of the tetracode will be reduced to the display of binary numbers of technological indicators of the actuator or measuring device, while the width of the obtained interval depends, first of all, on the position of the set M in the tetracode, or rather, on the position of the most significant (read from left to right, from the least significant) tetrite M.

In this case, the attention shall be paid to the reference concept, which can be used for metrological purposes. The value of the interval width (see figure 2), which is denoted as widx and calculate it
using the following formula \( widx = 2^{k+1} - 1 \), where \( k \) is the position most significant tetrite equal to \( M \) in the \( n \)-bit tetracode with the range designated as \( n-1 \leq k \leq 0 \).

**Figure 2.** Formation of integer interval boundaries during decoding of a normalized discrete code containing only group \( M \) (a) and groups \( M \) and \( A \) (b).

But first, one clear advantage of tetralogy in the field of digital information processing and metrology shall be considered. It is represented in an extremely explicit way at the proposed figure.

It is expressed by minimizing the discrepancy in the formation of the final data (in this case between the reference and real data). Of course, the range is rather formalized than real, but the accuracy of reading and comparison with reference values doubles [7,8].

Secondly, the use of Tetrates makes it possible to encapsulate the mechanism of fetching, reading, writing, storing, metrological values or specifications of telemetric content. It prompts the use of tetralogy in pipelining processes according to the superscalar computation model, having determined the possibility of transferring a tetrite descriptor to several pipeline streams of conveyor lines of the automated control system [9].

Expressed through this computational principle, the essence of the majority of production lines allows tetralogical postulates to determine the degree of controllability through the expanded capabilities of the switching functions of the selected and modified positional number system.

4. Findings

Thus, the use of tetrites simplifies the restructuring of automated control systems in the post-binary era, and gives quite tangible results in current configurations. It allows to carry out the smooth reduce of the rigid logic of the functioning of devices to the logic that has an analogue in the real world, such as relational contracting.

The proposed integration will allow to apply some aspects of the “Network-of-chip” concept used in research in relation to modern single-chip systems, expressing their similar relational essence, will strengthen the robustness of the transition probability samples, make the systems more manageable and responsive without changing the bit depth of the existing bus architectures. Computing, based on the theory of automata and tetralogy, also determines general representations with metrological features in quantum systems, where (as it is known) a non-binary logic prevails. The studied model of reduction to the basis has potential significance, first of all, in the most actively changing dynamical systems.

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