Melting of two dimensional solids on disordered substrate

David Carpentier and Pierre Le Doussal
CNRS-Laboratoire de Physique Théorique de l’Ecole Normale Supérieure, 24 Rue Lhomond, 75231 Paris

We study 2D solids with weak substrate disorder, using Coulomb gas renormalisation. The melting transition is found to be replaced by a sharp crossover between a high $T$ liquid with thermally induced dislocations, and a low $T$ glassy regime with disorder induced dislocations at scales larger than $\xi_d$ which we compute ($\xi_d \gg R_c \sim R_\sigma$, the Larkin and translational correlation lengths). We discuss experimental consequences, reminiscent of melting, such as size effects in vortex flow and AC response in superconducting films.

Although the phase diagram of the mixed state of high $T_c$ superconductors [1] has considerably evolved since their discovery, many questions remain. In $d = 3$ the low field region has been proposed as the topologically ordered Bragg glass phase [1]. The role of dislocations remains unclear near melting or at higher fields. To study analytically the effect of disorder on defective solids, $d = 2$ appears as a natural starting point, also important for numerous experimental systems besides thin superconductor films [3–6], such as Wigner crystals in heterojunctions [4] and on the surface of Helium [5], magnetic bubbles arrays [6]. In the absence of disorder, the continuous melting of a 2d crystal occurs as dislocation pairs unbind at $T_m^0$ and is well described by the KTNHY theory [10,11]. But no equivalent theoretical description of melting with substrate disorder exists, though it has been studied in many experiments [12].

Progress was made for the simpler problem of crystals with structural (i.e internal) disorder [13] where melting occurs at a temperature $T_m(\sigma)$ shifted downwards by the disorder strength $\sigma$ (Fig.1). As discussed in [10,13] is equivalent to treating only the long wavelength part $\sigma$ of the substrate disorder. On the other hand an analytic RG study of the 2d Bragg glass including pinning disorder $g$, i.e short wavelength disorder (at the cost of excluding by hand dislocations) was performed recently [14,15]. A complete treatment however should include dislocations. Indeed the general theoretical belief based on qualitative arguments and simulations [10,11] is that no true solid (neither a lattice nor a vortex glass [14]) exist in 2D in presence of disorder at $T > 0$, and thus there should be no true melting transition. On the other hand signatures reminiscent of melting are observed in various experiments [16]. This thus calls for further studies.

In this Letter we derive renormalisation group (RG) equations describing a solid on a disordered substrate and allowing for dislocations. They generalize the KTNHY equations to weak pinning disorder near melting. They are obtained from the RG analysis, performed here for the first time, of the generic elastic vector electromagnetic coulomb gas (VECG) with disorder [13]. Though the asymptotic RG flow is always towards strong coupling (disorder or/and dislocation fugacity) several studies are still possible for weak pinning disorder $g$ but arbitrary $\sigma$ (see below) which strongly suggest that the pure melting transition is replaced by a sharp crossover between two very distinct regimes: (i) at higher $T$ a *weakly disordered liquid* where disorder has no effect at large scale and dislocations are present on all length scales greater than $\xi_d^0(T)$ as in the pure case (ii) at lower $T$ a *quasi Bragg glass regime*, where disorder disrupts the quasi-long range order of the lattice beyond a length $[14] R_\sigma \sim R_c$ and dislocations appear on scales larger than a *new length* $\xi_d(T)$. From the RG, here we estimate $\xi_d(T)$ both for $T < T_m$ where $\xi_d \gg R_c \gg a$, and in the crossover region. At scales $L \leq \xi_d$, the low $T$ defect free glassy regime is described by the theory of [14], despite its defective nature in the thermodynamic limit. The sharp crossover predicted between the two regimes at weak disorder is analyzed and argued to account for experimental observation of 2D melting transition, and of the 2D peak effect.

We emphasize that our RG equations (5) are derived assuming that dislocations are thermalized. This assumption is reasonable near $T_m$ and allows us to analyze the crossover near melting. A fully consistent RG analysis including pinning of dislocations which become important at lower temperatures [16], is beyond the scope of this paper [20]. However, as a first step, we also studied the case of no pinning disorder $g = 0$ at $T \leq T_m^0/2$ by extending the analysis of [21,22] on a simpler model. A further study of melting is presented elsewhere [18].
In the absence of disorder and of dislocations a two dimensional crystal is described by a smooth 2D displacement field \( u(\mathbf{r}) \) and an elastic distortion energy:

\[
\mathcal{H}_{el} = \frac{1}{2} \int_{\mathbf{r}} 2c_{66}u_{ij}^2 + (c_{11} - 2c_{66})u_{ii}u_{jj}
\]

where \( c_{11} \) and \( c_{66} \) are respectively the compression and the shear elastic moduli, and \( u_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) \). For simplicity we study a triangular lattice of area \( a \). Next, disorder is modeled by a gaussian random potential \( V(\mathbf{r}) \) with correlator \( \langle V(\mathbf{r})V(\mathbf{r'}) \rangle = \delta(\mathbf{r-r'}) \) of range \( r_f \), and coupled to the density of vortices \( \rho(\mathbf{r}) = \sum_i \delta(\mathbf{r-r_i}) \) according to \( \mathcal{H}_p = \int_{\mathbf{r}} \rho(\mathbf{r})V(\mathbf{r}) \). Decomposition of \( \rho(\mathbf{r}) \) and of \( V(\mathbf{r}) \) in Fourier components gives several couplings between \( u(\mathbf{r}) \) and the disorder \( \mathbf{b} \). The two main contributions are a random stress field \( \sigma_{ij} \) arising from the long wavelength part of the disorder, and a 'random phase field' \( \phi_{\nu} \) which comes from the part of the (pinning) disorder with almost the periodicity of the lattice:

\[
-\frac{\mathcal{H}_p}{T} = \int_{\mathbf{r}} \sigma_{ij}u_{ij} + 2\sqrt{\frac{g}{a^2}} \sum_{\nu=1,2,3} \cos(K_{\nu} u(\mathbf{r}) + \phi_{\nu}(\mathbf{r}))
\]

with \( \langle \exp[i(\phi_{\nu}(\mathbf{r})-\phi_{\nu}(\mathbf{r'})]) \rangle = \delta_{\nu,\nu'}\delta^2(\mathbf{r-r'}) \), the \( K_{\nu} \) are the first reciprocal lattice vectors (of length \( K_0 = 4\pi/\sqrt{3}a \)) and the correlator \( \sigma_{ij}(\mathbf{r})\sigma_{ij}(\mathbf{r'}) \) is parametrised \([2] \) by \( \Delta_{11}, \Delta_{66} \) whose bare values are \( \Delta_{11} = \rho_0^2h_0 = 0, \Delta_{66} = 0 \), while \( g = a^2\rho_0 h_0 - K_0/T^2 \) where \( \rho_0 \) is the mean density. Note that if \( r_f \geq a \), \( g \) can be greatly reduced (e.g. by a factor \( \text{exp}(-c(r_f/a)^2) \)) with respect to \( \Delta_{11,66} \). For out of plane disorder (such as in \([3] \)), varying the relative strength of the two types of disorder can be done by changing the distance between the lattice and the disorder plane. Besides this contribution, an underlying disorder potential induces local preferred orientation of the lattice \([2] \) : this new random field (random torque) couples to the local bond angle \( \theta_1 = \frac{1}{2}(\partial_i u_j - \partial_j u_i) \) : \( \mathcal{H}_t = \int_{\mathbf{r}} A(\mathbf{r})\theta(\mathbf{r}) \) with \( A(\mathbf{r})A(\mathbf{r'}) = 4\Delta_{11}\gamma_{11}(\mathbf{r-r'}) \). Even if \( \Delta_{11} = 0 \) in the bare model, it will be renormalised to finite value (see \([3] \)), and must be taken into account from the beginning since it is a new independent disorder strength, whereas without dislocations \([4] \) it can be defined in a redefinition of \( \Delta_{66} \) (\( \Delta_{66} \rightarrow \Delta_{11} + \Delta_{16} \)).

To describe plastic distortions of the lattice one splits \( u = u_0 + u_d \) into a smooth phonon part \( u_0 \) and a displacement field \( u_{dij}(\mathbf{r}) = \frac{1}{2}\int_{\mathbf{r}} G_{ij}(\mathbf{r-r'})b_j(\mathbf{r'}) \) due to edge dislocations of Burger vectors density \( \mathbf{b}(\mathbf{r}) \), where

\[
G_{ij}(\mathbf{r}) = \delta_{ij}\Phi(\mathbf{r}) + \frac{c_{66}}{c_{11}}\epsilon_{ij}G\left(\frac{\mathbf{r}}{a}\right) + \frac{c_{11} - c_{66}}{c_{11}}\epsilon_{ik}H_{ik}(\mathbf{r})
\]

with \( G(\mathbf{r}) = i\Phi(\mathbf{r}) = \text{ln}(x+iy) \) where \( \mathbf{r} = (x,y) \). The interaction \( H_{ik}(\mathbf{r}) \) is defined by \([26] \) \( H_{ik}(\mathbf{r}) = \frac{\epsilon_{ik}T}{a^4} - \frac{1}{2}\delta ik \). Introducing \( n \) replicas and averaging over disorder using the Villain form for the cosine coupling in \([3] \) we obtain \([4,38] \) in each site replicated charges \( \mathbf{m} = (\mathbf{m}_1, \ldots, \mathbf{m}_n) \), each \( \mathbf{m}_k \) belonging to the reciprocal lattice, with the constraint \( \sum \mathbf{m}_k = \mathbf{0} \) as well as the usual neutrality condition. Integration over the replicated field \( \mathbf{u}'(\mathbf{r}) \) leads to an elastic VECG (instead of the purely electric studied in \([4] \)) which in its most general form is defined by \( \mathcal{Z}_n = \sum_{(b,m)} C^{-1}[\int \frac{d^2y}{a^2}Y[\mathbf{b}, \mathbf{m}]e^{-S}] \) where the integrals are restricted by \( |\mathbf{r}_a - \mathbf{r}_b| \geq a \), the \( Y[\mathbf{b}, \mathbf{m}] \) are the composite charges fugacities and \( C(b,m) \) is a combinatorial factor. The action is given by

\[
-S[b,m] = \frac{1}{2}\left\{ \mathbf{b}^a \cdot V^{ac}(\kappa_1, \kappa_2) \cdot \mathbf{b}^c^a + m^a \cdot V^{ac}(\kappa_3, \kappa_4) \cdot m^c \right\} + i\mathbf{m}^a \cdot W^{ac}(\kappa_5, \kappa_6) \cdot \mathbf{m}^c
\]

denoting \( \mathbf{b} \cdot V \cdot \mathbf{b} = \sum_{ij} \int_{\mathbf{r},\mathbf{r'}} b_i(\mathbf{r})V_{ij}(\mathbf{r-r'})b_j(\mathbf{r'}) \). The interactions are \( \pi V^{ac}_{ij}(\kappa_1, \kappa_2) = \kappa_1^a\delta_{ij}\pi G - \kappa_2^a H_{ij} \) and \( 2\pi W^{ac}_{ij}(\kappa_3, \kappa_4) = \delta_{ij}\pi^a c_{ij}G + \kappa_3^ac_{ij}H_{ik} + \kappa_4^ac_{ij}H_{ik} \).

The renormalization of this elastic VECG goes beyond the previous analysis of the ECG \([27] \), due to the presence of new marginal operators corresponding to the elastic interactions \( V, W \). Note that usual electric/magnetic self-duality for \([3] \) now reads: \( \kappa_1 \leftrightarrow \kappa_3 \); \( \kappa_2 \leftrightarrow \kappa_4 \); \( \kappa_5 \leftrightarrow -\kappa_5 \); \( \kappa_6 \leftrightarrow \kappa_6 \) and exchanges direct and reciprocal lattices. The full RG equations and applications to various 2D models (depending on the definitions of the \( \kappa_{1,\ldots,6} \)) is presented elsewhere \([3,4] \). Here we study the model \([3,4] \) for which the \( \kappa_{1,\ldots,6} \) are replica matrices of the form

\[
\kappa_{ij}^a = \frac{1}{\pi}\delta^{ac} - \Delta_{ij}
\]

with the original parameters of the model \([2,4] \) have been embedded in three replica matrices \( \kappa_{11,66}^c = \Delta_{11,66}\delta^{ac} - \frac{1}{\pi}\Delta_{11,66,16} \) and \( \gamma^{ac} = \gamma\delta^{ac} - \frac{1}{\pi}\Delta_{..} \). Without coupling to a periodic lattice \([2,4] \), one sets \( \gamma = 0 \).

This VECG can be studied by the RG as in \([2,4] \) by incrementing the hard-core cutoff \( a \rightarrow a e^l \) in a three steps coarse-graining: reparametrisation of the interaction, which gives the dimension of the operators : \( \delta y = (2 - \frac{c}{2\pi}(\tau_1 - \Delta_{11}))y, \delta y = (2 - \frac{c_6^2}{2\pi}(\tau_3))g \); fusion of electromagnetic charges separated from \( a \leq d \leq a e^l \); annihilation of dipoles of diameter \( a \leq d \leq a e^l \), which defines scale dependent interactions \( \kappa_{1,\ldots,6}(l) \) \([4] \). Here we restrict ourselves to charges of minimal fugacity: (i) charges \( \mathbf{b} \) with single non zero replica component \( b_0 = G \) (a lattice vector of minimal length), \( Y(b,0) = y \) (ii) charges \( \mathbf{m} \) with two (opposite) non zero replica component \( \mathbf{m}_0 = -\mathbf{K}_a \), \( Y(0,\mathbf{m}) = g \) \([20] \). We show after rather tedious calculations \([3,4] \) that \( \mathbf{b} \) is renormalisable to lowest order within the form \([3] \), needed for consistency. We obtain the general RG equations, which, in the case of \([3,4] \), generalize the KTHNY equations:
where \( I_{0,1} = I_{0,1}(\alpha) \) and \( \tilde{I}_{0,1} = \tilde{I}_{0,1}(\tilde{\alpha}) \) are modified Bessel function. We have defined \( K = 4T\pi_1 = 4T\pi_2 = 4(c_{11} - c_{66})/c_{11}c_{66}, \sigma K^2/2 = T^2(\Delta_{1} + \Delta_{2}) = \Delta_{66}(1 - 2c_{66}/c_{11}) + \Delta_{11}(c_{66}/c_{11})^2, \delta = 4\Delta_{7}, \) and \( B_m(\alpha) = 2\pi(2I_0(\alpha/2) - I_0(\alpha)), {\tilde{\alpha}} = \frac{2\pi}{8m}(K/T - \sigma K^2/T^2 + \delta/T^2) \) and \( \alpha = p^2K^2KT/(16\pi^2c_{66}) \). We introduce an additional parameter \( p \) analogous to the p-fold symmetry breaking field in \( \frac{[28]}{}, \) setting \( K_0 \rightarrow pK_0 \) in \( \frac{[28]}{} \). The physical model corresponds to \( p = 1 \). We emphasize again that \( \frac{[28]}{} \) should be valid at high enough \( T \) (near melting).

We start by the simpler case of no pinning disorder \( g = 0 \), where further extensions of \( \frac{[3]}{} \) at low \( T \) can also be given. The modifications induced by \( g > 0 \) are discussed later for \( T > T_m/2, g \approx 0 \) is experimentally relevant when disorder \( (\Delta_{11,66,\gamma}) \) varies very smoothly at the scale of the lattice. At high enough temperature \( T > T_m^0/2, \) eqs. \( \frac{[3]}{} \) show that the solid is stable weak disorder \( \sigma < \frac{\pi T}{2m} \) where \( \pi T^2 = \sigma + \delta/K_r^2 \) and at low temperature \( T < T_m(\sigma,\delta) = \frac{K_{66}^2}{4\pi^2}(1 - \sqrt{1 - 64\pi\sigma}) \) (note that \( \Delta_{11,66,\gamma} \) are unrenormalized). At \( T = T_m(\sigma,\delta) < T_m^0 \) it undergoes a true\( \text{ KT} \) like melting transition (see Fig.1) to a high temperature phase where dislocations proliferate. The correlation length at the transition is given by \( \xi^0 \sim \exp(\text{const}(T - T_m)^{-\nu}) \) where \( \nu, \) computed in \( \frac{[3]}{} \) depends continuously on \( \sigma \) and \( \delta \) and vanishes for \( \sigma = \frac{\pi T}{2m} \). When the random shear and torque are null \( \Delta_\gamma = \Delta_{66} = 0, \) one recovers Nelson’s results \( \frac{[3]}{} \). At lower temperatures \( (T \leq T_m^0/2), \) freezing of dislocations leads to modified RG equations for \( T \equiv T/(2\pi^2\sigma K) < 1 \). We showed \( \frac{[3]}{} \) that it is possible to extend to the elastic model the approach of \( \frac{[22]}{} \) for the simpler scalar model, by defining the appropriate dislocation fugacities \( \tilde{g} \):

\[
\partial_t \tilde{g} = \left(2 - \frac{1}{32\pi^2} \right) \tilde{g}, \quad \partial_t \delta = 0
\]

(6a)

\[
\partial_t (TK^{-1}) = C(1 - q)g^2, \quad \partial_t \sigma = Cq\tilde{g}^2
\]

(6b)

where \( q \approx T \) is the ratio of frozen dislocations \( \frac{[29]}{} \). The solid phase thus survives for \( \sigma > \frac{\pi T}{2m} \), as in \( \frac{[22]}{} \), and the physics is dominated by rare favorable regions for frozen dislocations \( \frac{[3]}{} \), leading to the phase diagram of Fig. 1. Positional correlations decay algebraically with exponent \( \eta_c = \frac{\Delta_{66}}{4\pi^2} \left(c_{11} + c_{66} + \frac{\Delta_{66}}{T^2} - \frac{\Delta_{11}}{T^2} \right), \) which leads at the transition \( T = T_m^0/2, \) to \( \frac{1}{6} \leq \eta_c \leq \frac{1}{3} \) (for \( \Delta_{66} = \Delta_\gamma = 0 \), and at \( T = 0 \) to \( \frac{1}{6} \leq \eta_c \leq \frac{1}{8} \) (depending on \( c_{11}/c_{66} \)).

We now turn to the effect of pinning disorder. Let us first recall that in situations where dislocations can be neglected (see below) one sets \( y = 0 \) in \( \frac{[3]}{} \) and recovers the Bragg glass phase \( \frac{[3]}{} g^*B_m(\alpha) = 2(1 - T/T_g) \equiv 2\tau \) which exists for \( T \leq T_g = \frac{\pi c_{66}}{4\pi^2}T_m^0/(2\Delta_{66}^2) \) (much larger than \( T_m \)). Due to the unbounded increase of \( \Delta_{11,66} \rightarrow \infty \), disorder induced displacements grow as \( u \sim \ln r \). The rest of the paper is devoted to studying the situation where both weak pinning disorder and dislocations are included, using eqs.\( \frac{[3]}{} \). The RG flow shows a sharp crossover between two distinct regimes (see fig.2): a high temperature regime characterized by a correlation length \( \xi^0(T) \) unaffected by pinning disorder \( (i.e. \text{the same as given above}) \), and a low temperature glassy regime where translational order decays beyond \( R_a \sim R_c \), though equilibrium dislocations are separated (apart from small dipoles of size \( \sim \alpha \) ) by the larger length \( \xi_d(T) \) (defined by \( \xi_d = a e^\gamma, y(t^*) \sim 1 \) ). We also study the crossover by parametrizing temperature and disorder using the correlation lengths \( \xi^0(T) \) of melting in the absence of pinning disorder and the Larkin length in the absence of dislocations \( R_a^0 \sim g^{-\gamma} \) (see (35) in \( \frac{[4]}{} \)).

![Image](image-url)

**FIG. 2.** The length \( \xi_d/a \) as a function of temperature, for various pinning disorder strength \( g \), and (rescaled) scale dependent parameters in the two regimes (inset).

In the low \( T \) glassy regime as \( l \) increases \( y(l) \) starts by dropping quickly to a negligible value (Fig.2) whereas \( g(l) \) increases and reaches for \( L = ae^\gamma \gtrsim R_c \sim R_a^0 \) a plateau value corresponding to the Bragg glass fixed point \( g^* \) defined above \( \frac{[3]}{[4]} \). Within this range of scale \( K \) and \( c_{66} \) remain constant since \( y(l) \sim 0 \). However the steady growth of \( \sigma \) and \( \delta \) \((g(l) \text{ being finite}) \) eventually makes the eigenvalue of \( y \) positive. Since \( y(l) \) is then very small, it takes a large \( l^* \) before it reaches \( \sim 1 \) and explodes to a non perturbative value. Thus dislocations, which are strongly suppressed in the intermediate Bragg glass regime, appear only on very large length scales: to estimate \( \xi_d \), we can neglect the \( y^2 \) terms in \( \frac{[3]}{} \left( g(l) \ll 1 \right. \) up to \( \xi_d \) ). Integrating the corresponding RG eqs., we obtain in the limit of weak disorder (large \( R_a^0 \) )
\[ \xi_d \sim R_c e^{b \sqrt{\ln R_c}} \] with \( b = 2\tau \sqrt{\lambda (\sigma_c - \sigma_0)} \)

where \( \lambda \) is a constant and \( R_c \sim R_m^0 (c_{66}(R_c)/c_{66}(0))^{\frac{1}{2}} \) is the true Larkin length. Note that \( R_c \sim R_m^0 \) in the low \( T \) regime, except very near the crossover region.

In the high \( T \) liquid regime, \( g(l) \) increases quickly, causing \( g(l) \) to decrease to 0. Thus the correlation length is the same as without pinning disorder \( \xi_0 \). However this regime cannot be considerer as an hexatic phase: since the renormalised \( \Delta_\gamma \) is nonzero (computable from our RG), the corresponding random field destroys long-range orientational order \( (\Delta_\gamma \sim (\xi_0^0)^2/a \) (since \( \Delta_\gamma \sim g^2 \)) leading to a liquid at large scales.

Finally, the crossover between the two regimes is dominated at weak enough disorder by the \( g = 0 \) fixed point (see fig.1). Studying the RG flow around the KTNHY separatrix \( \xi_0^0 = 1 \) we find \( \xi_d \) in the crossover region \( \xi_0^0 \):

\[ \xi_d \approx \frac{R_m^0}{(\ln R_m^0)^{c_0}} \left( 1 + \left( \frac{\ln R_m^0}{\ln \xi_0^0} \right)^{\frac{1}{v}} \right) \]

where the ± sign depends on \( T > T_m^0 \) or \( T < T_m^0 \) and \( \tau \approx 0.8 \), \( \nu \) as in \( \xi_0^0 \) near pure melting (\( \sigma \approx 0 \)). When increasing pinning disorder (or decreasing temperature), \( \xi_d \) gradually goes from \( \xi_0^0 \) to the low \( T \) behaviour \( \xi_0^0 \).

This sharp crossover will have consequences for finite size effects in 2D systems such as thin superconducting films \( \xi_0^0 \). In experiments of narrow channel vortex flow in Nb-xGe films \( \xi_0^0 \), one probes the visco-elastic response of the lattice for \( T < T_m \) on scales much smaller than \( \xi_d \) (see fig.2) : the system responds as a solid with a finite shear modulus \( c_{66} \), and one observes a “diverging” correlation length (or viscosity \( \eta \)) when approaching \( T_m \), as without pinning. In much larger system for \( T \lesssim T_m \), the response of the lattice should in fact be liquid-like with a large viscosity \( \nu \sim \xi_0^0(T) \). In AC experiments \( \xi_0^0 \), the 2D vortex lattice is probed on a length \( l_\omega \sim \sqrt{D/\omega} \). By varying \( \omega \), a crossover at \( l_\omega \approx R_m^0 \) was observed at low \( T \) in \( \xi_0^0 \) between a low \( \omega \) liquid-like behaviour and an activated glassy behaviour. The length \( R_m^0 \) strongly differs \( \xi_0^0 \) from the corresponding \( R_c \) extracted from critical current experiments. We argue that the observed \( R_m^0 \) \( \xi_0^0 \) corresponds to the correlation length \( \xi_d(T) \) defined here. Indeed the scaling of \( R_m^0 \) with the sample thickness \( d \) (i.e disorder strength) in \( \xi_0^0 \) is consistent with \( \xi_0^0 \). Finally, using \( \xi_0^0 \) we compute the scale dependent \( c_{66}(l) \) and obtain the softening of \( c_{66} \) on scale \( R_m^0 \) by dislocations. The self consistent definition of \( R_c \) given above then leads to a quantitative description of the increase of critical current (peak effect) in 2D films \( \xi_0^0 \) near melting, further studied in \( \xi_0^0 \).

To conclude we extended the KTNHY analysis in presence of weak pinning disorder, predicted and analyzed a sharp crossover near pure melting. To go beyond our study in the low \( T \) region would necessitate a controlled RG method \( \xi_0^0 \) to describe frozen topological defects.

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