Magnetic Anomaly Feature Extraction Using the Tunable Q-factor Wavelet Transform Based on Non-convex Overlapping Group Shrinkage

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Abstract. Extraction of the magnetic anomaly signal is one of the difficulties in the magnetic anomaly detection as the weak features extracted are easily disturbed by strong background noise. To address this problem, a sparse feature extraction method based on the tunable Q-factor wavelet transform and overlapping group shrinkage is developed in this paper. Compared with the traditional wavelet transform, the proposed method has excellent characteristics, which can flexibly tune the Q-factor according to the oscillation characteristics of the useful features. In this way, the sparsity of extracted features can be induced more effectively. In addition, the non-convex overlapping group shrinkage can effectively extract weak features from signals with group property, enhancing the extraction accuracy of features. The practical experiment verifies the effectiveness of the proposed method in the magnetic anomaly detection.

1. Introduction

Recently, with the rapid development of the magnetic field detection technology, the application of magnetic anomaly signal generated by magnetic target to geomagnetic field disturbance is more and more widely [1]. The magnetic anomaly signal contains many important information, which can reflect the direction, distance and speed of the target [2]. By probing and inverting the magnetic anomaly signal, the target can be tracked and located. However, owning to the weak magnetic anomaly signal, it is apt to be disturbed by strong noise background in the detection process. This greatly increases the difficulty of extracting magnetic anomaly signal features. Therefore, it is of great significance to accurately obtain the key information of magnetic anomaly state from complex observation signals for magnetic target detection.

Sparse representation is novel and effective for extracting signal with sparsity and has good anti-noise ability. Many effect methods, such as Fourier transform, wavelet transform and so on, have been proposed and applied in the engineering. The core idea of these methods is to decompose the basis function to obtain the basis function consistent with the characteristic waveform, so that the extracted signal has sparse characteristics [3]. Among them, the feature extraction method based on wavelet is favored by scholars at home and abroad [4]. In 2009, Mallat elaborated the applications of wavelet in various fields from a sparse perspective. In the ideal feature extraction analysis, the quality factor of wavelet should have the best match with the oscillation characteristic of the signal to be analyzed. Nevertheless, the traditional methods such as discrete wavelet transform, are difficult to represent high oscillation components sparsely, because its quality factor is fixed and smaller [5]. For avoiding the
drawback, a novel tunable Q-factor wavelet transform (TQWT) is adopted in this paper [6]. This method can set the quality factor flexibly to adjust the characteristics of wavelet filter. In this way, it can make the oscillation characteristic of wavelet coincide with that of the signal to be analyzed. For the signals with group characteristic, wavelet basis function is lack of adaptively in sparse feature extraction. Thus, the efficiency and accuracy of feature extraction is limited. Overlapping group shrinkage (OGS) is a new and effective algorithm with group signal features extraction, which makes full use of group sparse structure to enhance the feature extraction results [7-8]. In this paper, a sparse feature extraction method based on non-convex OGS with TQWT is proposed. Firstly, the signal is decomposed and reconstructed by TQWT. Then, the kurtosis value is used to optimize the reconstructed signal. Finally, in order to improve processing accuracy, OGS is used to eliminate the irrelevant component. The application of the proposed method to real marine magnetic field observation data shows that the proposed method has good performance.

The remainder of this paper is organized as follows. Section 2 briefly introduces the basic theory of the tunable Q-factor wavelet transform and non-convex overlapping group shrinkage. Section 3 presents the proposed method using TQWT based on non-convex OGS. In Section 4, practical marine magnetic field observation data is implemented to validate the real performance of the proposed method. The conclusions are given in Section 5.

2. Basic Theory

2.1. Tunable Q-factor Wavelet Transform

2.1.1 Principle of TQWT. Prof. Selesnick proposed the tunable Q-factor wavelet transform in 2011, which was constructed wavelet base and spanned meshes of multiscale spaces in frequency domain. It can effectively solve the adjusting problem about the time-domain oscillation characteristics of wavelet bases.

TQWT adopts an iterative dual channel filter structure, as shown in figure 1, where LPS and HPS represent low-pass scale and high-pass scale, respectively. The scale parameters are \( \alpha \) and \( \beta \). The relationship meets

\[
\beta = \frac{2}{(Q + 1)}, \quad \alpha = 1 - \beta r.
\]

![Figure 1. Block diagram of filter banks for the implementation of the TQWT.](image)

\[
H_L(\omega) = \begin{cases} 
1, & |\omega| \leq (1 - \beta)\pi \\
\frac{\omega + (\beta - 1)\pi}{\alpha + \beta - 1}, & (1 - \beta)\pi < |\omega| < \alpha\pi \\
0, & \alpha\pi \leq |\omega| \leq \pi. 
\end{cases}
\]

\[
H_H(\omega) = \begin{cases} 
1, & |\omega| \leq (1 - \beta)\pi \\
\frac{\alpha\pi - \omega}{\alpha + \beta - 1}, & (1 - \beta)\pi < |\omega| < \alpha\pi \\
0, & \alpha\pi \leq |\omega| \leq \pi. 
\end{cases}
\]
where \( \theta(\omega) = 0.5(1+\cos \omega)\sqrt{2-\cos \omega} \). According to the principle of Daubechies standard orthogonal basis, the constructed filters meet \( |H_v(\omega)|^2 + |H_f(\omega)|^2 = 1 \).

2.1.3 TQWT parameter setting. The main parameters for the TQWT are the Q-factor, the redundancy, and the number of levels. The redundancy \( r \) is the total over-sampling rate of the transform. The Q-factor reflects the oscillation characteristic of wavelets basis. The wavelet contains more oscillation period when the value of Q is higher, where it is suitable for analyzing oscillation signals. As for lower Q, the wavelet basis has fewer oscillation, which is fit to extract the transient impact components. The parameter \( J \) denotes the number of filter banks. TQWT is composed of a sequence of two-channel filter banks, and the low-pass output of each filter bank serves as input to the subsequent filter banks.

2.2. Non-convex Overlapping Group Shrinkage Algorithm

The observed noisy signal \( y \) can be modeled as

\[
y(i) = x(i) + w(i)
\]

where, \( x(i) \) represents the signals with group-sparse characteristic and \( w(i) \) denotes interference Gaussian noise. The N-point signal \( x \) is denoted as \( x = [x(0),...,x(N-1)] \in \mathbb{R}^N \). Therefore, we use \( x_{i,K} = [x(i),...,x(i+K-1)] \in \mathbb{R}^K \) to express the \( i \)-th group of vector \( x \) of size \( K \), where \( K \) denotes the group size. At the boundaries, some indices of \( x_{i,K} \) fall outside \( \mathbb{Z}_N \). If \( i \notin \mathbb{Z}_N \), we take \( x(i) = 0 \). For denoising group-sparse signals, the objective function can be constructed as,

\[
F(x) = \frac{1}{2}\|y-x\|^2 + \lambda \sum_{i \in \mathbb{Z}_N} \phi(\|x_{i,K}\|;\alpha)
\]

where \( \phi \) is a non-convex sparsity penalty function. Compared with convex penalty, non-convex regularization can estimate the high amplitude components more accurately and extract sparser features. The parameters of penalty function can be set to keep the objective function as convex as a whole. According to [8], if \( 0 < \alpha < 1/K \lambda \), the \( F \) is strictly convex. Hence, adopting the majorization-minimization (MM) procedure, we can obtain the optimal solution

\[
x^{(k+1)}(i) = \frac{y(i)}{1+\lambda r(i;x^{(k)})}, \quad i \in \mathbb{Z}_N
\]

As the strict convexity of OGS algorithm, the optimal solution is unique.

2.3. Kurtosis

Kurtosis is one of the important statistical indicators in signal analysis and feature extraction, which reflects the characteristics of signal distribution [9]. \( K \) is sensitive to large data, and it is easy to capture the transient impact in the detection signal. Kurtosis is defined as

\[
K = \int_{-\infty}^{\infty} [z(t) - \bar{z}]^4 w(z)dz
\]

where \( z(t) \) denotes amplitude of signal, \( \bar{z} \) is the mean of the signal amplitude. When there are large pulses in the signal, the kurtosis value \( K_r = K/\sigma^4 \) is often used as the characteristic guidance index, where \( \sigma \) is the standard deviation.

3. The Sparse Optimization Feature Extraction Algorithm

Addressing the problem of weak magnetic anomaly signal extraction under complex background, a group-sparse feature extraction method based on OGS with TQWT is proposed in the section. The
detailed flow chart is shown in figure 2, and the specific steps can be divided into the following four steps:

- Setting parameters (Q, r, J): the detailed coefficients and the energy ratio of each layer coefficient are obtained by decomposing the measured signal with TQWT.
- The first filtering: the threshold is set according to the energy ratio and then the invalid wavelet coefficients are removed.
- The second filtering: the inverse TQWT transformation is performed for the wavelet coefficients of the layers corresponding to the effective energy. Then, calculating them used by kurtosis and selecting the signal with the optimal number of layers.
- Using OGS algorithm to shrink and denoise the selected signals.

![Flow Chart](image)

**Figure 2.** Flow chart of the proposed method.

### 4. Engineering Application

At present, the detection technology based on magnetic anomaly signal has become one of the hot spots in the research of target recognition [10]. However, the magnetic anomaly data are easy to be destroyed by various external interference magnetic fields. In this paper, a denoising method combined TQWT and OGS is used to eliminate the noise in magnetic abnormal signals, which improves the measurement accuracy effectively.

There are scalar method and vector method in magnetic anomaly detection. Compared with scalar method, vector method has stronger comprehensive information collection ability and becomes the main method used in magnetic anomaly detection [11]. In order to verify the effectiveness of the proposed method, we apply the proposed method to the observation data of a vector magnetometer in marine magnetic field. This data is obtained by the target signal passing through the detector along different tracks with different magnetic moment. The root mean square error (RMSE) and the signal-noise ratio (SNR) are used to evaluate its performance.

Figure 3 shows the simulated target signals in the marine magnetic field, and the observation data of a vector magnetometer in marine magnetic field is illustrated in figure 4. As can be seen from figure 4, the magnetic anomaly features are submerged by background noise and irrelevant interference, and distorted severely comparing with the original signal. Its RMSE is 1.6398, and SNR is -17.4884dB. Adopting TQWT to decompose the signal in 29 layers, where the parameters are set Q = 1, r = 3, J = 28. The waveform of each subband and the energy percentage of wavelet coefficient are shown in figure 5.
Figure 3. The simulated signal. Figure 4. The measured magnetic signal.

Figure 5. Subbands and the energy in each subband as a percentage of the total energy.

Calculating the kurtosis value of each layer of wavelet coefficients after the inverse TQWT transform. From figure 5, we can observe that the energy of wavelet coefficients is almost zero below 19 layers. Therefore, those are not considered when selecting the number of layers. According to the principle of maximum kurtosis value, we chose 19 to 21 layers as the optimal layers. TWQT inverse transformation is performed for layers 19 to 21 layers, and the refactoring result is shown in figure 6. As shown in figure 6, TQWT can basically extracts the feature, whereas there is still some noise interference. Because of the characteristic of group, the OGS can be used. The parameters are set as follows: group size $K = 5$, regularization parameter $\lambda = 0.05$. Figure 7 shows the extraction results, where $RMSE=0.1440$, $SNR=3.6399$.

Figure 6. Signal reconstruction by the wavelet coefficient at 19-21 layers. Figure 7. The denoising result by the non-convex OGS algorithm.

For comparison, the traditional $\ell_1$ norm and TQWT based on convex OGS are used to process the signals respectively. The weak features extracted based on $\ell_1$ norm regularization is shown in figure 8, where $RMSE=1.7655$, $SNR=-18.5594\ dB$. It can be seen that this method can separate and extract a few features. However, the extracted signal is still seriously distorted. The estimated features extracted by TQWT based on convex OGS is shown in figure 9, where $RMSE=0.1470$, $SNR=3.1764\ dB$. The result presents a slight amplitude underestimation and a bit of noise, compared with the proposed method.
The above analysis fully demonstrates that the proposed method in this paper can greatly eliminate the interference of background noise and extract useful magnetic anomaly signal more accurately.

5. Conclusion
In this paper, a sparse extraction method of magnetic anomaly signal in wavelet domain is proposed. This method combines TQWT with non-convex OGS, which can extract weak feature waveform from noisy signals effectively. Meantime, it can be solved quickly and converge to the global optimal solution. The practical experiment shows the proposed method can effectively extract the weak magnetic anomaly signal from complex signals. The quantitative analysis through the RMSE and SNR further verifies the superiority of the method in weak feature extraction.

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