Perturbative QCD

versus

pion exchange and hadronic FSI effects

in the \( \gamma \gamma \to \pi^+\pi^- \) reaction

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(Dated: November 4, 2018)

Abstract

The interplay of pQCD, pion exchange and FSI effects is studied for the \( \gamma \gamma \to \pi^+\pi^- \) reaction in the region of 2 GeV < \( W_{\gamma\gamma} \) < 6 GeV. We find strong interference effects between pQCD and soft pion-exchange amplitudes up to \( W_{\gamma\gamma} \sim 4 \) GeV. We discuss to which extend the conventional hadronic FSI effects could cloud the pQCD effects. We study multipole soft and hard scattering effects as well as the coupling between final state hadronic channels. We show how the perturbative effects in \( \gamma \gamma \to \rho\rho \) may mix with perturbative effects in \( \gamma \gamma \to \pi^+\pi^- \). The effects discussed in this paper improve the agreement with the new data of the DELPHI and ALEPH collaborations. We give estimates of the onset of the pQCD regime. Predictions for \( \gamma \gamma \to \pi^0\pi^0 \) are presented.

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I. INTRODUCTION

It was predicted long ago [1] that at large \( t \) and large \( u \) the angular distribution of pions from the reaction \( \gamma \gamma \rightarrow \pi^+\pi^- \) should be described by means of perturbative QCD as due to the exchange of t-channel (u-channel) quarks. This reaction is commonly considered as a gold-plated reaction for pQCD effects to be observed.

In order to have both \( t \) and \( u \) large, large entrance photon-photon energies \( W_{\gamma\gamma} \) are required. Actually up to now there is no common consensus how large \( t \) (or \( u \)) should be so that pQCD behaviour of the angular distributions could be observed. At present the reaction can be studied by means of bremsstrahlung of two photons at \( e^+e^- \) colliders only. This method is efficient only low \( W_{\gamma\gamma} \) which can be easily understood in the equivalent photon approximation (EPA). Furthermore the pQCD predicts a strong decrease in the cross section with increasing photon-photon energy \( W_{\gamma\gamma} \). Thus it becomes clear that the bremsstrahlung method can be efficiently used only for not too high \( W_{\gamma\gamma} \) energies.

In the leading-twist perturbative treatment, the transition amplitude factorizes into a hard scattering amplitude and pion distribution amplitude. The pion distribution amplitude was found to be strongly constrained by the photon-pion transition form factor [2]. For the "realistic" distribution amplitude the pQCD contribution to the \( \gamma \gamma \rightarrow \pi^+\pi^- \) [3] is well below the experimental data above the resonance region (2 GeV < \( W \) < 3.5 GeV) [4, 5]. Recent analyses of the DELPHI and ALEPH collaborations at LEP2 [6, 7] extend the energy range. In contrast to previous measurements it was possible to separate \( \pi^+\pi^- \) and \( K^+K^- \) channels in the recent analyses. The preliminary data confirm the missing strength problem.

Thus it becomes clear that the leading-twist pQCD approach is not sufficient to describe the experimental data and the mechanism must be of nonperturbative origin. In a recent work [8] it was proposed that the amplitude for \( \gamma \gamma \rightarrow \pi^+\pi^- \) to be described as a hard \( \gamma \gamma \rightarrow q\bar{q} \) amplitude times a form factor describing the soft (not calculable) transition from \( q\bar{q} \) to the meson pair. It was assumed implicitly in this approach that all the strength comes from the so-called hand-bag process!

In the present study we take slightly different approach in an attempt to establish to what extend the present data can be understood in terms of relatively well known soft hadronic physics and what is the role of final state interaction (FSI) effects not yet discussed in the literature. At energies below \( f_2 \) resonance the FSI effects are known to be rather weak [8].
and the cross section is dominated by one-pion exchange. Above the $f_2$ resonance the FSI effects have never been studied. We have shown recently [10] that in the pion-pion elastic scattering at $W > 2$ GeV the multipole soft rescattering may play an important role up to very large momenta transferred, i.e. it shifts the onset of the pQCD behaviour to larger $t$ or $u$.

Inspired by the result of [10], in the present note we discuss the role of the final state interaction effects in $\gamma\gamma \rightarrow \pi^+\pi^-$. We shall show that at not too high energies, i.e. where experiments were performed, the coupling between the $\rho\rho$ and the $\pi\pi$ channels may play quite important role.
II. PERTURBATIVE QCD APPROACH AND CORRECTIONS IN THE SOFT REGIONS

It is well known that at large momentum transfers the exclusive processes, such as $\gamma\gamma \rightarrow \pi^+\pi^-$, test Quantum Chromodynamics [11]. It is not known, however, how big the momentum transfer should be in order that the rules of perturbative QCD (pQCD) could be applied.

According to the rules proposed in [11], at large energies and large center-of-mass angles, i.e. at large four-momentum transfers, the amplitude for the $\gamma\gamma \rightarrow \pi^+\pi^-$ can be factorized into a perturbatively calculable hard-scattering amplitude and a non-perturbative distribution amplitude of finding a valence quark in each pion

$$M_{\lambda_1\lambda_2}(W, \theta) = \int_0^1 dx_1 \int_0^1 dx_2 \Phi^*_\pi(x_1, \tilde{Q}_1) T^{H}_{\lambda_1\lambda_2}(x_1, x_2, W, \theta) \Phi^*_\pi(x_2, \tilde{Q}_2).$$

(1)

The indices $\lambda_1$ and $\lambda_2$ are photon helicities. In general the distribution amplitudes $\Phi_\pi$ undergo a slow logarithmic QCD evolution, depending on scales $\tilde{Q}_1$ and $\tilde{Q}_2$. In the collinear approach $T^H$ is computed assuming that the quarks (antiquarks) are collinear with outgoing mesons. Because in the present analysis we concentrate mainly on pion exchange and FSI effects, we shall limit the approximation for $T^H$ to the leading order (LO) in $\alpha_s$ only. [28]

In practice the distribution amplitudes cannot be calculated from first principles. Based on phenomenology of a few reactions, it seems at present that the distribution amplitude should be rather close to the asymptotic one:

$$\Phi_\pi(x) = \sqrt{3} f_\pi x(1 - x),$$

(2)

where $f_\pi$ is the pion decay constant ($\sqrt{2 f_\pi} = 130.7$ MeV).

In Fig.1 we compare the pQCD result obtained for the asymptotic distribution with the experimental data from [6, 7]. In our calculation we follow [13] rather than [1] and use running coupling constant with simple analytic models for freezing $\alpha_s$ [13, 14]. There is a clear excess of the experimental cross section over the pQCD prediction in the whole measured range of photon-photon energies. What other processes can contribute in this energy range? Can the pion exchange and/or final state interaction be responsible for the missing strength? We shall try to answer these questions in the following sections.

The pQCD amplitude (1) contains singularities at $\theta = 0$ and $\theta = \pi$ (see [1]) which are artifacts of the collinear approximation. This is the region of the phase space where the
collinear pQCD result is certainly not trustworthy. Furthermore the small angle scattering is probably of soft nonperturbative character due to meson or reggeon exchanges which are included in the present approach explicitly. Therefore in order to avoid double counting and make possible the use of the pQCD amplitudes in the multiple-scattering series, we shall smoothly cut off the small $t$ and small $u$ regions in the perturbative amplitude. This is done phenomenologically by introducing a $t-u$ symmetric form factor

$$F_{pQCD}^{reg}(t,u) = \left[1 - \exp\left(\frac{t - t_m}{\Lambda_{reg}^2}\right)\right] \cdot \left[1 - \exp\left(\frac{u - u_m}{\Lambda_{reg}^2}\right)\right]$$

which the pQCD amplitude (1) is multiplied by. Here, $t_m$ and $u_m$ are maximal kinematically allowed values of $t$ and $u$, respectively, and $\Lambda_{reg}$ is a cut-off parameter to be fixed. The above choice for the form factor removes the singularities in the original collinear pQCD amplitude. Our model division into soft and hard regions suggests that $\Lambda_{reg}$ should be of the order of the cut-off parameter in the vertex form factor for soft meson/reggeon exchanges, i.e. of the order of 1 GeV. [29]

In Fig. 2 we show $d\sigma/dz$ ($z = \cos(\theta)$) obtained from the pQCD amplitude (1) modified by the form factor (3) for different values of $\Lambda_{reg} = 0.5, 1.0, 2.0$ GeV, for three different energies $W = 2,4,6$ GeV. In the $z$-distribution the soft/hard ”borders” are clearly energy dependent which can be seen by deviations of the modified pQCD results (dashed lines) from the original pQCD results (solid lines). At low energies dominant mechanism becomes nonperturbative. When $s \rightarrow 4m_\pi^2$ the perturbative component is totally suppressed by $F_{pQCD}^{reg}(t,u) \rightarrow 0$. Then, only soft processes known from low and intermediate energy physics are at play. Thus in the low energy limit our approach coincides with the successful low-energy phenomenology by construction. Only at high energies one can expect pQCD effects to dominate in the region of intermediate $z$ ($\theta \approx \pi/2$).
III. QED PION EXCHANGE 
AND FINITE-SIZE CORRECTIONS

The QED Born amplitude for the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction with point-like particles has been known for long time \cite{15}. At low energies, where chiral perturbation theory is usually applied, pions are treated as point-like particles. The finite-size corrections vanish automatically when $s \rightarrow 0$, i.e. $t, u \rightarrow 0$ as well. This means that only at sufficiently high energies the finite-size corrections may show up.

An interesting problem is how to generalize the QED amplitude for real finite-size pions. For the sake of simplicity we follow the idea of Poppe \cite{16} and correct the QED amplitude by an overall $t$ and $u$ dependent form factor

$$A_{fs}(t,u,s) = \Omega(t,u,s) \cdot A_{QED}(t,u,s). \quad (4)$$

This form by construction guarantees crossing symmetry. It is natural to expect that finite size corrections should damp the QED amplitude for both $t$ and $u$ large. At sufficiently high energies the following simple Ansatz fulfills this requirement

$$\Omega(t,u,s) = \frac{F^2(t) + F^2(u)}{1 + F^2(-s)}, \quad (5)$$

where $F$ are standard vertex function such that $F(0) \approx 1$ and $F(t) \rightarrow 0$ when $t \rightarrow \infty$. \cite{30}

The Ansatz (5) has a nice feature that in the limit of large $s$

$$\Omega(t,u,s) \leftarrow \frac{t}{s} \rightarrow 0 \quad F^2(t),$$

$$\Omega(t,u,s) \leftarrow \frac{u}{s} \rightarrow 0 \quad F^2(u), \quad (6)$$

i.e. it generates standard vertex form factors. Then at $z \equiv \cos \theta \approx 0$ (large $s$), $\Omega(t,u,s) \approx 2F^2(-s/2)$, i.e.

$$\frac{d\sigma^\pi}{dz}(z = 0, s) \propto F^4(-s/2). \quad (7)$$

For the monopole vertex form factor this leads to a suppression $\propto 1/s^4$ of the QED result for point-like particles. Thus the pion exchange gives a faster decrease of the cross section with energy than the pQCD mechanisms, and at sufficiently high energy it can be neglected.

An interesting question is: at what energy does this difference become significant.

The correct small and large angle behaviour (6) allows us to use traditional ($t$- or $u$-dependent only) vertex form factors. We note, however, that the exact form of $F$ is not
known, particularly in the region of interest, i.e. at large $t$ and $u$. In order to have some flexibility when discussing the effect of the poorly known large $t/u$ region we choose a simple but rather flexible form for the vertex form factors

$$F_t(t) = \exp \left( \frac{B_{\gamma\pi}}{4} t \right), \quad F_u(u) = \exp \left( \frac{B_{\gamma\pi}}{4} u \right).$$

(8)

From the analysis in [10] we expect $B_{\gamma\pi} < 4$ GeV$^{-2}$ for the elementary vertex. In order to see the sensitivity to the particular functional form of $F$ we shall also replace $F$’s by the pion electromagnetic form factor $F_{EM}$, assuming the standard monopole form with the $\rho$ meson mass. [31]

In order to complete our discussion and quantify the finite size effects we show the result obtained for one-pion exchange model with $F = F_{EM}$ in Fig. 1 (dotted line). It is surprising that the so-obtained finite-size-corrected pion exchange is comparable with the pQCD result (dashed lines) up to relatively large energies. If both amplitudes are of comparable size one has to include interference effects.

IV. THE PQCD AND PION EXCHANGE AMPLITUDES INTERFERE!

The size of the interference effects can be estimated by comparing the incoherent sum of pQCD process and soft pion exchange $\sigma_{inc} \propto |A_{pQCD}|^2 + |A_{\pi}|^2$ and the coherent sum $\sigma_{coh} \propto |A_{pQCD} + A_{\pi}|^2$ (see the thick solid line in Fig. 1 and the third column in Table 1). The interference effects are generally large and positive. The coherent sum $|pQCD + \pi|^2$ only weakly depends on the functional form of the form factor used. The details depend, however, on the vertex form factor. In the case of the monopole (EM) form factor the effect of the interference effects survives up to large energies $W \sim 5$ GeV when the pion-exchange contribution alone becomes much smaller than the perturbative contribution. In the case of the exponential form factor ($B_{\gamma\pi} = 3$ GeV$^{-2}$) the interference effects become negligible for $W \sim 4$ GeV.

As can be seen from the figure the interference of the pQCD and pion-exchange amplitudes improves considerably the agreement with the recent DELPHI [3] and ALEPH [3] data. This automatically implies that there is less room is left for the hand-bag mechanism contribution considered in [8].

The effect of interference of the pion-exchange and pQCD contributions on the angular
distribution is shown in Fig. 4. The constructive interference can be observed over the whole angular range.

V. TAIL OF THE $f_2$ RESONANCE

The $f_2$ resonance is known to be strongly populated in the $\gamma\gamma$ collisions (see for instance [17, 18]). It is rather broad with $\Gamma \sim 0.2$ GeV. The leading twist pQCD contribution drops strongly with energy. Therefore one should look at the interplay of the high energy flank of $f_2$ with the strongly decreasing continuum. Then all kinds of energy dependence of kinematical and dynamical origin must be included. In the relativistic approach, the total cross section for the resonance contribution reads

$$\sigma_{\gamma\gamma \rightarrow \pi\pi}(W) = 8\pi(2J+1) \left( \frac{M_R}{W} \right)^2 \frac{\Gamma_{\gamma\gamma}\Gamma_{\text{tot}}(W) Br(f_2 \rightarrow \pi^+\pi^-)}{(W^2 - M_R^2)^2 + M_R^2\Gamma_{\text{tot}}^2}.$$  \hspace{1cm} (9)

One usually parametrizes $\Gamma_{\text{tot}}$ as

$$\Gamma_{\text{tot}}(W) = \Gamma_{\text{tot}}^0 \left( \frac{p}{p_0} \right)^{2l+1} \mathcal{F}_{\text{dyn}}(p),$$  \hspace{1cm} (10)

where $\mathcal{F}_{\text{dyn}}(p)$ is a function of dynamical origin usually obtained in a simple nuclear physics inspired model of resonances [19].

The analysis of angular distributions has shown [17, 18] that $f_2$ is produced dominantly in the helicity $\lambda = \pm 2$ state, in agreement with earlier theoretical predictions [21]. This means that the angular distribution of pions in the $f_2$ frame of reference factorizes as

$$\frac{d\sigma}{dz}(W, \theta) = 2\pi|Y_{22}(\theta)|^2 \cdot \sigma_{\gamma\gamma \rightarrow \pi\pi}(W).$$  \hspace{1cm} (11)

This formula can be used to calculate the resonance contribution in a limited range of $\cos \theta$, as it is usually done in experiments.

In Fig. 4 we show the resonance contribution obtained from (9) and (10) with other parameters taken from [20]. This contribution competes with both pQCD and pion contribution up to very large energies. It would be naive to expect, however, that the energy dependence of (10) can be used far from the resonance region. For comparison, we also show (dashed line) the result obtained with the relativistic Breit-Wigner formula (9) but with an energy-independent width $\Gamma_{\text{tot}} = \Gamma_{\text{tot}}^0$. The result with energy independent width describes slightly better the high energy flank of $f_2$ for the $\pi^+\pi^-$ channel.
In the vicinity of the $f_2$ peak one may also expect interference of the resonance amplitude and the continua due to pion exchange and pQCD mechanisms. Because a consistent overall microscopic model of the resonance and the continua is not available, the relative phase between the resonance amplitude and that for the pion exchange is not known. In the following we take a pragmatic attitude and try to fit the phase to the MARKII data [17]. In order to explain the low energy flank of the $f_2$ resonance we need $\phi \approx \pi/2$. We shall make the simplest approximation and assume that this phase is energy independent. Adding all contributions together we obtain results represented by the thick solid and thick dashed lines for energy-dependent and energy-independent widths in the resonance amplitude, respectively. In this way we obtain a result almost consistent with the preliminary DELPHI [3] and ALEPH [7] data at least up to $W = 2.5$ GeV. The agreement with the data may be extended even further towards higher photon-photon energies, but it is not clear if the adopted simple approximations are reasonable that far from the resonance position.

All this demonstrates that the situation just above the $f_2$ resonance is far from being under control. Therefore expecting the pQCD quark exchange mechanism to be the only reaction mechanism in this region seems to us rather unjustified.

VI. FSI EFFECTS

Up to now we have included only pion exchange Born amplitude. It was demonstrated recently that for elastic pion-pion scattering, soft FSI effects lead to a damping of the cross section at small angles and a considerable enhancement in the region of intermediate angles where they compete with the two-gluon exchange amplitude. Let us explore the role of FSI effects for the reaction under consideration.

The FSI effects are an intrinsic part of fundamental scattering theory. Therefore there should be good reasons for FSI to not occur. Calculations of FSI effects at intermediate energy is not always an easy task. In the following we shall discuss only some selected FSI effects for the reaction under consideration.

In Table 2 we list double-scattering terms considered in the present analysis. In order to demonstrate the role of each mechanism separately, we shall calculate contributions of individual diagrams. Interference effects of different mechanisms will be discussed only at the very end.
TABLE I: Brief summary of double-scattering processes considered in this paper for $\gamma\gamma \rightarrow \pi^+\pi^-$.  

| number | first step         | intermediate channel | second step            | short notation |
|--------|--------------------|----------------------|------------------------|----------------|
| 1      | soft $\pi$-exch.   | $\pi^+\pi^-$         | $IP + f_2 + \rho$ exchange | $S_1S$       |
| 2      | pQCD               | $\pi^+\pi^-$         | $IP + f_2 + \rho$ exchange | $H_1S$       |
| 3      | soft VDM           | $\rho^0\rho^0$       | $\pi$ exchange         | $S_2\pi$     |
| 4      | pQCD               | $\rho^+(0)\rho^-(0)$  | $\pi$ exchange         | $H_2\pi$     |

In the language of multiple scattering at high energies \[22\] the amplitude for the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction can be written as an infinite series of the type:

$$A_{\gamma\gamma\rightarrow\pi^+\pi^-}(s, t, u) = \sum_\alpha A^{(\alpha)}_{\gamma\gamma\rightarrow\pi^+\pi^-}(s, t, u) + \sum_{ij} \sum_{\alpha,\beta} \frac{i}{32\pi^2 s} \int d^2 k_1 d^2 k_2 \delta^2(k - k_1 - k_2) A^{(\alpha)}_{\gamma\gamma\rightarrow ij}(s, k_1) A^{(\beta)}_{ij\rightarrow\pi^+\pi^-}(s, k_2) + (...) . \tag{12}$$

Here Greek indices label type of exchange, while Latin indices $ij$, etc. label two-body intermediate states.

The first component in (12) corresponds to single-exchange terms. In the following we shall include only $\alpha = \pi$ (pion exchange) or $\alpha = 2q$ (pQCD quark exchange) single-exchange amplitudes. Contribution of other single-exchange processes, like $\rho, a_1, a_2$ meson/reggeon exchanges are more difficult to estimate reliably and will be discussed elsewhere. We shall include only $ij = \pi^+\pi^-$ and $ij = \rho\rho$ intermediate states. The latter are expected to be copiously created in the photon-photon collisions either assuming factorization \[23\] or in QCD-inspired models (see for instance \[25\]). The parameters of pion-pion interaction are taken from Ref.\[10\].

In the previous sections the uncertainty of the pQCD and pion-exchange amplitudes has been discussed. Now we shall discuss how reliably one can evaluate double-scattering amplitudes of Table 2. In general, the double-scattering amplitudes interfere with the single-scattering amplitudes considered before. We shall discuss the interference pattern and the sign of some selected interference terms.
A. $\gamma\gamma \to \pi^+\pi^-$ and FSI effects

As already discussed there are two single exchange mechanisms leading directly to the $\pi^+\pi^-$ channel: soft pion exchange ($S_1$) and pQCD hard quark exchange ($H_1$). While the first can be reliably calculated at small $t$ or small $u$, the second is trustworthy only at $t$ and $u$ both large. Going beyond the region of their applicability requires extrapolations which cannot be derived from first principles and leads to obvious uncertainties. In particular, as discussed in the previous sections, it is not completely clear what is the contribution of pion exchange in the measured angular distributions for $-0.6 < \cos \theta < 0.6$.

In this section we shall explore the role of double-exchange terms $S_1S$ and $H_1S$ where $S$ denotes soft pion-pion interaction. The parameters of the pion-pion interaction are fixed using the analysis of [10].

Let us start with the $S_1S$ term (first row in Table 2). In Fig.6 we show the cross section calculated with the amplitude $A = S_1S$ (dash-dotted line) with the monopole form factors for the first step reaction (left panel) and with the exponential form factors with $B_{\gamma\pi} = 3$ GeV$^{-2}$ (right panel). The solid (dotted) line denotes the cross section corresponding to the amplitude $A = S_1 + S_1S$ ($A = S_1$, i.e. pion exchange alone). In these calculations monopole or exponential ($B_{\gamma\pi} = 3$ GeV$^{-2}$) form factor were used. The answer to the question which of the amplitudes squared $|A = \pi + S_1S|^2$ or $|\pi|^2$ is bigger for $z \sim 0$ depends on the half-off shell pion electromagnetic form factor. For the exponential form factor $|\pi + S_1S|^2 > |\pi|^2$ while for the monopole form factor $|\pi + S_1S|^2 < |\pi|^2$. By comparing the solid and dotted lines one can infer that the interference is destructive. For $z \sim 0$ the pion exchange term strongly depends on the vertex form factor used. It is amusing that the total result $|S_1 + S_1S|^2$ depends on the vertex form factor up to $W \sim 4$ GeV only weakly.

Let us now turn to the $H_1S$ double-scattering amplitude (second row in Table 2). As in the previous case, to check reliability of our estimates of this double-scattering amplitude, in Fig.7 we present the cross section calculated with the $A = H_1S$ amplitude alone (dash-dotted line). In this calculation $\Lambda_{\text{reg}} = 1.0$ GeV. We have performed the calculations for different low-t(u) cuts (3). There is a relatively mild dependence on the way the soft nonperturbative regions are treated. For reference we show the pion exchange result with the monopole form factor in vertices (dotted line) and the coherent sum of the $\pi$ and $H_1S$ amplitudes (solid line). In the second column of Table 3 we give the cross section calculated with the
double-scattering amplitude integrated in the limited range of \( \cos \theta \). The hierarchy of the cross sections can be summarized as

\[ |H_1S|^2 \ll |\pi + H_1S|^2 < |\pi|^2. \]

(13)

This means that the interference effect between the pion-exchange amplitude and the double-scattering \( H_1S \) amplitude is destructive and reduces the cross section by 20-40 % in the considered region of energies and \( \cos \theta \).

**B. \( \gamma\gamma \rightarrow \rho\rho \) and its coupling to the \( \pi^+\pi^- \) channel**

For the \( \rho\rho \) intermediate channel, as for the \( \pi\pi \) channel before, we shall consider both soft \( (S_2) \) and hard \( (H_2) \) mechanisms. In both cases the second step mechanism is assumed to be the standard pion exchange. The usual way to calculate soft contributions to \( \gamma\gamma \rightarrow \rho\rho \) is to assume the validity of the vector dominance (VDM) and the factorization at the hadron level. The parameters of the \( \rho\rho \) interaction can be obtained in a similar manner as for the pion-pion scattering in [10]. Then for \( \gamma\gamma \rightarrow \rho^0\rho^0 \) we obtain:

\[
A_{\lambda_1\lambda_2 \rightarrow \lambda_{\rho_1}\lambda_{\rho_2}}(t) = \sqrt{\frac{4\pi}{f_\rho^2}} \sqrt{\frac{4\pi}{f_\rho^2}} \delta_{\lambda_1\lambda_{\rho_1}} \delta_{\lambda_2\lambda_{\rho_2}} \left[ A_{IP}(t) + A_f(t) \right],
\]

(14)

where the pomeron exchange \( A_{IP}(t) \) and the isoscalar reggeon exchange \( A_f(t) \) amplitudes are given explicitly by Eq.(10) in [10] with parameters specified in the text there. Above, we have additionally assumed helicity conservation.

In Fig.8 we show the angular distribution for the soft process \( \gamma\gamma \rightarrow \rho^0\rho^0 \). In this calculation we have used exponential form factor with \( B_{\gamma\rho} = 3 \text{ GeV} \) in the vertices of the \( \gamma\gamma \rightarrow \rho^0\rho^0 \) Born amplitude. In the same figure we present the result for pQCD calculation (for the three energies \( W_{\gamma\gamma} = 2, 4, 6 \text{ GeV} \)) according to [1] with an extra form factor to cut off soft nonperturbative regions (\( \Lambda_{\text{reg}} = 1 \text{ GeV} \), in analogy to the perturbative pion-pion production (see Eq.(3)). For completeness in the same figure we also show the cross section for the \( \gamma\gamma \rightarrow \rho^+(0)\rho^-(0) \), calculated within rules of perturbative QCD [1].

In order to calculate the contributions of \( S_2\pi \) and \( H_2\pi \) to the \( \gamma\gamma \rightarrow \pi^+\pi^- \) reaction we couple both \( \rho^0\rho^0 \) and \( \rho^+(\lambda = 0)\rho^- (\lambda = 0) \) channels with the \( \pi^+\pi^- \) channel of interest via standard pion exchange mechanism. The standard coupling constants obtained from the
decay width of $\rho^0 \to \pi^+ \pi^-$ are used. We have used exponential form factors with $B = 4$ GeV$^{-2}$ in the fully hadronic vertices.

As in the previous section, we shall quantify the role of the double-scattering amplitudes. In Fig. 9 we show the resulting angular distributions (dash-dotted line). As can be seen from the figure the two-step process efficiently produces pions at $\theta \sim \pi/2$. The pion exchange (dotted line) and the coherent sum of pion exchange and the double scattering $S_2\pi$ term (solid line) are shown for comparison. The size of the amplitude and the interference with the pion exchange amplitude depends on photon-photon energy and $\cos \theta$. For low energy $W \sim 2$ GeV we observe

$$|\pi|^2 < |\pi + S_2\pi|^2 < |S_2\pi|^2.$$  \hspace{1cm} (15)

At higher energies and $z \sim 0$

$$|S_2\pi|^2 < |\pi + S_2\pi|^2 < |\pi|^2.$$  \hspace{1cm} (16)

This means that in the region of interest, we observe a destructive interference of the pion exchange and the double-scattering amplitude $S_2\pi$. The effect of the $S_2\pi$ rescattering is rather large. The situation is, however, not completely clear in the light of rather small cross section for $\gamma\gamma \to \rho^0\rho^0$ as obtained by the L3 collaboration [27]. This is a very interesting issue which needs further clarification but goes beyond the scope of the present discussion.

In Fig. 10 we show the resulting angular distributions (dashed lines). The pion exchange (dotted line) is shown for reference. Due to the helicity structure of the vertices, the interference between pion-exchange and the $H_2\pi$ vanishes identically. Summarizing, we have shown that the pion-exchange mechanism leads to a coupling of the $\rho\rho$ and $\pi\pi$ channels. The interference term of the pQCD amplitude and the $H_2\pi$ amplitude was found to be negative and its size decreases with photon-photon energy. The situation for the cross section integrated in the experimental range $-0.6 < z < 0.6$ can be summarized as follows

$$|H_2\pi|^2 \ll |pQCD|^2;$$

$$|H_2\pi|^2 < |\pi|^2.$$  \hspace{1cm} (17)

Generally the double-scattering effects considered lead to a mild reduction of the single-scattering amplitudes. The triple-scattering effects are expected to partially compensate the double-scattering effects.
VII. ALL PROCESSES TOGETHER

In Fig. 11 we present the result corresponding to the coherent sum of all processes discussed in the present paper. For comparison we show the pQCD result. For $W > 2$ GeV the final result is practically independent of the phase of the resonance contribution with respect to the other contributions. As can be seen by comparison of the solid and dashed line the inclusion of the processes considered in the present paper leads to a considerable improvement with respect to the pQCD calculation. The final result describes the ALEPH \cite{7} and DELPHI \cite{6} data surprisingly well.

To complete our results in Fig. 12 we present the corresponding angular distributions. A large enhancement with respect to pQCD at intermediate angles ($z \sim 0$) is clearly visible. We predict almost flat $d\sigma/dz$ in the experimentally measured region of $-0.6 < z < 0.6$. This is due to the competition of pion-exchange, pQCD, tail of the $f_2$ resonance and multiple scattering effects. The contribution of $f_2$ far from the peak position (i.e. for $W > 3$ GeV) is the least reliable element of our model calculation. In the present analysis we have ignored $\rho$, $a_1$ and $a_2$ meson (reggeon) exchanges which may, at least potentially, enhance the cross section for $|z| > 0.4 - 0.5$. The knowledge of precise angular distributions at different well defined energies would be helpful to disentangle the rather complicated reaction dynamics.

VIII. PREDICTIONS FOR $\gamma\gamma \rightarrow \pi^0\pi^0$

In the case of the $\pi^0\pi^0$ channel the pion exchange vanishes identically while the pQCD cross section is rather small. In Fig. 13 we present the predictions of pQCD for the $\gamma\gamma \rightarrow \pi^0\pi^0$ reaction obtained with asymptotic distribution amplitude. In the case of fixed $\alpha_s$ there are exact cancellations of different terms in the reaction amplitude for $\gamma\gamma \rightarrow \pi^0\pi^0$ (see for instance \cite{11}). Therefore in the present paper we have performed calculations with both fixed (dash-dotted lines) and running $\alpha_s$ as proposed in \cite{13} (solid lines). In addition the region of small $t$ and $u$ was cut off as was the case for the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction using form factor (3) with $\Lambda_{reg} = 1$ GeV. The pQCD cross section for the $\pi^0\pi^0$ channel (solid or dash-dotted lines) is almost an order of magnitude smaller than the pQCD cross section for the $\pi^+\pi^-$ channel (dashed lines).

Because of the smallness of the pQCD contribution in the following we shall consider also
other contributions to the $\pi^0\pi^0$ channel like $f_2$ resonance contribution and/or FSI effects due to couplings to other channels. In calculating the resonance contribution for the $\gamma\gamma \to \pi^0\pi^0$ reaction one should remember that

$$BR(f_2 \to \pi^0\pi^0) = \frac{1}{2} \cdot BR(f_2 \to \pi^+\pi^-).$$

(18)

Because the FSI effects are relatively weak, it seems that even far from the $f_2$ peak the cross section for the $\gamma\gamma \to \pi^0\pi^0$ reaction may be dominated by the tail of the broad $f_2$ resonance.

In Fig.14 we present the result for $\frac{d\sigma}{dz}(\gamma\gamma \to \pi^0\pi^0)$ corresponding to a coherent sum (solid line) of the $f_2$ resonance (dashed line), the pQCD amplitude (dotted line) and the two-step pion exchange: $\gamma\gamma \to \pi^+\pi^-$ followed by the charge-exchange (CEX) process $\pi^+\pi^- \to \pi^0\pi^0$ (dash-dotted line). The parameters of the charge exchange reaction are taken from [10].

One can observe a clear modification of the resonance contribution only for $z \sim 1$ and $z \sim -1$. It would be interesting to extend the experimentally accessible range of $z$ in the future analyses in order to identify such effects. With the phase of the resonance term fixed for the $\gamma\gamma \to \pi^+\pi^-$ reaction other two-step contributions like hard $\gamma\gamma \to \pi^+\pi^+$ followed by the CEX process or hard $\gamma\gamma \to \rho^+\rho^-$ followed by the pion exchange lead to the negative interference effect of the order of 10 % at $z \sim 0$.

Let us define the quantity:

$$R_{\pi^0\pi^0/\pi^+\pi^-} = \frac{\frac{d\sigma}{dz}(\gamma\gamma \to \pi^0\pi^0)}{\frac{d\sigma}{dz}(\gamma\gamma \to \pi^+\pi^-)}$$

(19)

as a function of $z = \cos \theta$. Our approach predicts that $R_{\pi^0\pi^0/\pi^+\pi^-}$ strongly depends on $z$. For $z \sim 0$ our result is in between that predicted by pQCD $R_{\pi^0\pi^0/\pi^+\pi^-} \approx 0$ and that from the recent approach [8] where $R_{\pi^0\pi^0/\pi^+\pi^-} = 1$ has been predicted. In principle this significant difference between these all three approaches should be easy to verify experimentally.
IX. CONCLUSIONS

For a long time the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction was considered as a gold-plated reaction to identify pQCD effects. Recent LEP2 results for $\gamma\gamma \rightarrow \pi^+\pi^-$, when combined with the present understanding of the pion distribution amplitudes, clearly demonstrate a failure of leading twist pQCD in explaining the data. In this analysis we have thoroughly studied the role of different mechanisms which could potentially cloud the pQCD contribution:

- soft pion exchange
- tail of the broad $f_2$ resonance
- interference of pion-exchange and pQCD amplitudes
- multipole scattering of pions
- coupling between $\rho\rho$ and $\pi^+\pi^-$ channels

It was found that the soft pion exchange, which is known to be the dominant mechanism below the $f_2$ resonance, stays important also above the resonance. More subtle details are however difficult to predict as the half-off shell electromagnetic form factor of the pion is rather poorly known both experimentally and theoretically. In addition the pion-exchange amplitude strongly interferes with the pQCD amplitude.

Because the pQCD contribution strongly decreases with the photon-photon energy the high energy flank of the broad $f_2$ resonance gives a contribution of comparable size to the pQCD continuum in a rather broad energy range above the resonance.

Using phenomenological, yet realistic, pion-pion interaction found in a recent analysis we have estimated the contribution of the final state interaction processes. We have found that pion-pion FSI as well as the coupling with the $\rho\rho$ channel modify the total amplitude at the level of 10 - 20%.

In our model we predict: $\frac{d\sigma}{d\omega}(\gamma\gamma \rightarrow \pi^0\pi^0) < \frac{d\sigma}{d\omega}(\gamma\gamma \rightarrow \pi^+\pi^-)$. The leading mechanisms for the $\gamma\gamma \rightarrow \pi^0\pi^0$ being $f_2$ resonance and FSI rescattering effects, including coupling with the $\rho\rho$ channel. This result differs from the recent predictions of Diehl, Kroll and Vogt who obtained: $\frac{d\sigma}{d\omega}(\gamma\gamma \rightarrow \pi^0\pi^0) \approx \frac{d\sigma}{d\omega}(\gamma\gamma \rightarrow \pi^+\pi^-)$. We hope that experimental verification of both scenarios will be possible in near future. In our opinion the effects discussed in the present paper must be taken into account in order to extract empirically the hand-bag
contribution. This may not be an easy task, however, as the different contributions can interfere. Good data for both $\pi^+\pi^-$ and $\pi^0\pi^0$ channels, including angular distributions, also at $|\cos\theta| > 0.6$, would be very helpful to disentangle the complicated dynamics in the region of intermediate energies.

In the light of our analysis we conclude that the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction is not the best choice to identify the pQCD predictions. We plan a similar analysis for other meson pair production in $\gamma\gamma$ collisions in order to find the best candidate to study perturbative QCD. The $\gamma\gamma \rightarrow K^+K^-$ reaction seems to be a better candidate.

Acknowledgments

A.S. thanks Siggi Krewald for interesting discussion on intermediate energy physics and Kolya Nikolaev for the discussion of pion-pion multipole scattering effects. The discussion with Carsten Vogt on his improved pQCD predictions is gratefully acknowledged. We are indebted to Katarzyna Grzelak and Alex Finch for providing us with the DELPHI and ALEPH experimental data for $\gamma\gamma \rightarrow \pi^+\pi^-$ shown at the PHOTON2001 workshop and discussion. One of us (A.S.) is indebted to Nikolai Achasov for drawing our attention to experimental data on $\gamma\gamma \rightarrow \rho^0\rho^0$. This work was partially supported by the German-Polish DLR exchange program, grant number POL-028-98.
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[28] The next to leading order (NLO) calculation [12] leads to a reduction of the LO result.
    Therefore the LO result can be viewed as an upper estimate of pQCD.
[29] Our cut off parameter \( \Lambda_{\text{reg}} \) can be also conveniently used to simulate effects of finite transverse
    momenta of quarks in the final pions which leads to a suppression of collinear pQCD result
    up to \( s \approx 20 \text{ GeV}^2 \). In order to simulate phenomenologically the results of \[ \text{[3]} \]
    we would need also \( \Lambda_{\text{reg}} \approx 1 \text{ GeV} \).
[30] In our case of light pions it is rather academic if the form factors are normalized at \( t,u=0 \) or
    on (pion) mass shell \( t,u = m_{\pi}^2 \).
[31] This seems natural as in our case \( F \) can be interpreted as half off-shell electromagnetic form
    factor \( F(Q^2 = 0, m_{\pi}^2, t/u) \) of the pion, with only one pion in the vertex being off its mass shell.
[32] We neglect isovector \( a_2 \) contribution which is more difficult to be reliably estimated.
FIG. 1: Predictions of the collinear LO pQCD (dashed line) for the cross section of the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction integrated in the experimental range of $-z_{\text{exp}} < \cos\theta < z_{\text{exp}}$ versus experimental data from the MARKII collaboration [17] (triangles), the DELPHI collaboration [6] (circles), and the ALEPH collaboration [7] (squares). The dotted line corresponds to the contribution of the pion-exchange mechanism discussed in the text. The coherent sum of the pQCD and pion-exchange contribution is shown by the solid line.
FIG. 2: The pQCD contribution to the angular distribution for the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction for $W = 2, 4, 6$ GeV. The result of the standard calculation is shown by the solid lines. The dashed line represents the results obtained when the hard scattering amplitude was modified by the form factor (3) correspondingly for three different values of $\Lambda_{\text{reg}} = 0.5, 1.0, 2.0$ GeV.
FIG. 3: Soft pion-exchange (dotted) versus hard quark-exchange mechanism (dashed) and their coherent sum (solid) for the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction at $W = 2, 4, 6$ GeV. In this calculation $\Lambda_{reg} = 1.0$ GeV and $B_{\gamma\pi} = 3 \text{ GeV}^{-2}$. 
FIG. 4: The $f_2$ resonance contribution to the $\gamma\gamma \to \pi^+\pi^-$ and to the $\gamma\gamma \to \pi^0\pi^0$ (left and right panel respectively). For comparison results for both energy dependent (solid line) and energy independent (dashed line) width are shown. The experimental data of Mark II [17] and the Crystal Ball [18], respectively, are shown for comparison.
FIG. 5: Interference effect of resonant and nonresonant contributions. The thick lines represent the sum of pion-exchange, pQCD and $f_2$ contributions with energy dependent (solid line) and energy independent (dashed line) width of the resonance. For reference we display the pion-exchange (dash-dotted) and pQCD (dotted line) contributions.
FIG. 6: Angular distribution of pions for $W = 2, 4, 6$ GeV. Cross section corresponding to double-scattering term $S_1 S$ is denoted by the dash-dotted line. Left (right) panel corresponds to the monopole vertex form factor (the exponential form factor with $B_{\gamma\pi} = 3$ GeV$^{-2}$) of the first step. The dotted line refers to the Born result for pion exchange. Their coherent sum is shown by the solid line.
FIG. 7: Angular distribution of pions for $W = 2, 4, 6$ GeV. The cross section corresponding to the double-scattering term $H_1S$ is shown by the dash-dotted lines. In this calculation $\Lambda_{reg} = 1$ GeV. For comparison also shown is the pion exchange result (dotted line). Solid lines correspond to the coherent sum of the $H_1S$ and pion exchange amplitudes.
FIG. 8: Angular distribution of $\rho^0$ from the reaction $\gamma\gamma \rightarrow \rho^0\rho^0$ (solid line) and $\rho^\pm$ form the reaction $\gamma\gamma \rightarrow \rho^+(\lambda = 0)\rho^- (\lambda = 0)$ (dashed line) for $W = 2,4,6$ GeV.
FIG. 9: Angular distribution of pions obtained when only the diagram $S_2\pi$ is taken (dashed). The pion exchange contribution (dotted) is shown for comparison. The solid line corresponds to a coherent sum of the pion exchange and $S_2\pi$ amplitudes.
FIG. 10: Angular distribution of pions obtained when only the diagram $H_{2\pi}$ is included (dashed). The pion exchange contribution (dotted) is shown for comparison. The solid line corresponds to a coherent sum of the pion exchange and $H_{2\pi}$ amplitudes.
FIG. 11: The result corresponding to the coherent sum of all processes (solid line) versus pQCD result (dashed line). The vertical solid line shows the expected lower limit of the range of applicability of the present multipole-scattering approach.
FIG. 12: The result corresponding to the coherent sum of all processes (solid line) versus pQCD result (dashed line).
FIG. 13: Predictions of pQCD for angular distributions of the $\gamma\gamma \rightarrow \pi^0\pi^0$ reaction (solid lines) for $W = 2, 4, 6$ GeV. The solid lines correspond to calculations with running $\alpha_s$ whereas the dash-dotted lines to those with fixed $\alpha_s$. For reference we show analogous results for the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction (dashed lines). In all these calculations $\Lambda_{reg} = 1$ GeV was taken.
FIG. 14: Angular distribution for $\gamma\gamma \rightarrow \pi^0\pi^0$. The dash-dotted line is the cross section corresponding to double scattering contribution (soft-pion-exchange and isovector reggeon). The dashed line is the $f_2$ contribution and the solid line corresponds to their coherent sum. For completeness the pQCD predictions (dotted lines) are shown too.