A possible glueball contribution to the Goldberger-Treiman relations. *

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Abstract
We discuss the influence of glueball coupling to nucleons on the weak axial-vector coupling constants including singlet channel. We consider a possibility of introduction of constituent gluon contribution to the proton spin. The estimated value for this quantity seems to be rather small.

The EMC experiment [1] started a great interest in the problem of the proton spin. Analyzing naively, the quark contribution to this quantity came out unexpectedly small (see e. g. ref. [2]). In the one of the interpretations authors assumed [3] that there exists additional gluon contribution which nearly cancel the quark one, making the measured result rather small. In this context the role of the axial anomaly and the Goldberger-Treiman relations was also discussed [4, 5, 6]. These relations are very important because they allow to calculate the axial-vector coupling constants which are directly

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related to the quark contribution to the nucleon spin. Especially crucial, because of the existence of the axial-vector anomaly, is the Goldberger-Treiman relation for the singlet axial-vector current.

In this paper, introducing phenomenologically the mixing of a glueball state with pseudoscalar mesons $\eta$ and $\eta'$, we want to discuss a possible glueball contribution to the Goldberger-Treiman relation for the singlet axial-vector current. Naively one would expect that the glueball coupling to nucleons measures in some sense a gluonic content of nucleon spin, in analogy to the quark case. We introduce a constituent gluon contribution to the nucleon spin and we try to estimate its value. We also speculate about a glueball contribution to the gluonic content of a proton spin and we compare the result with the perturbative gluon contribution needed to understand the EMC experimental result.

Taking into account SU(3) mass breakings, together with isospin mass breaking and $\pi$-$\eta$-$\eta'$ mixing, one can obtain Goldberger-Treiman relations for the third, eights and singlet (in the generalized version) component of the axial-vector current [5, 6]. We will write the relations for these components in the vector form. Introducing properly normalized vector of weak axial-vector coupling constants, pseudoscalar meson coupling constants to nucleons and the quantity $\tilde{G}$ (it sums the pole contributions from the physical particles $\pi$, $\eta$ and $\eta'$) we have:

$$\tilde{g}_A = \frac{f_\pi}{2M} M_{sym}^2 \tilde{G},$$

where $f_\pi$ is a pion decay constant ($f_\pi \simeq 132$ MeV), $M_{sym}$-mass matrix in the SU(3) basis, whereas:

$$\tilde{g}_A = \begin{pmatrix} \frac{1}{\sqrt{2}} g_A^{(3)} \\ \frac{1}{\sqrt{6}} g_A^{(8)} \\ \sqrt{3} \Delta \Sigma \end{pmatrix}, \quad \tilde{g}_{NN} = \begin{pmatrix} g_{\pi NN} \\ g_{\eta NN} \\ g_{\eta' NN} \end{pmatrix}, \quad \tilde{G} = \begin{pmatrix} G^{(3)} \\ G^{(8)} \\ G^{(0)} \end{pmatrix}. \quad (2)$$

We have also for $x=3,8,0$:

$$G^{(x)} = \sum_{p=\pi,\eta,\eta'} \frac{\Omega_{px}}{m_p^2} q_{pNN}, \quad (3)$$

or writing eq.(3) in more compact form:

$$\tilde{G} = \hat{\Omega}^\dagger (M_{phys}^2)^{-1} \tilde{g}_{NN}, \quad (4)$$
where \( M_{\text{phys}} \) is a diagonal mass matrix with physical masses and orthogonal matrix \( \hat{\Omega} \) connects physical and SU(3) (third, eighth and singlet) states. Hence, the eq.(1) can be rewritten in the form:

\[
\tilde{g}_A = \frac{f_\pi}{2M} M^2_{\text{sym}} \hat{\Omega}^\dagger (M^2_{\text{phys}})^{-1} \tilde{g}_{NN} = \frac{f_\pi}{2M} \hat{\Omega}^\dagger \tilde{g}_{NN},
\]

(5)

where the second equality follows because the matrix \( \hat{\Omega} \) transforms also mass matrices i.e.: \( \hat{\Omega} M^2_{\text{sym}} \hat{\Omega}^\dagger = M^2_{\text{phys}} \). The final result (eq.(5)) does not involve meson masses, only a mixing angles and could be considered as a generalization, for the diagonal octet states, of an old Goldberger-Treiman relation for pion. The mixing of \( \pi \) with \( \eta \) and \( \eta' \), due to the isospin breaking, is negligible when compared with SU(3) breaking and we will neglect it. However, we would like to extend above relations by taking into account gluon degrees of freedom and include glueball mixing with \( \eta \) and \( \eta' \).

For a long time \( \eta(1440) \), called also \( \iota \), was considered as a glueball candidate. As was pointed out \[7\] there are some problems with this interpretation. Recent Mark III results has changed the experimental situation showing that \( 0^{-+} \) state at 1440 MeV is not a single resonance but rather a mixture of three different states. We will consider one of these states, namely \( 0^{-+} \) state with the mass around 1490 MeV, as a possible glueball candidate and call it as before \( \iota \). Our main reasoning depends on existence of a glueball and not on its particular mass. One of the possible demonstration of pseudoscalar glueball existence would be a mixing with pseudoscalar \( q\bar{q} \) states. We will use for this mixing a model discussed by us previously \[8\]. Old models of \( \eta , \eta' \) mixing with glueball (see e.g. ref.\[8\]) do not take into account a new experimental situation. Let us assume that the physical states \( \eta , \eta' \) and \( \iota \) can be expressed in terms of the SU(3) states \( \eta_8 , \eta_0 \) and a pure psedoscalar glueball \( G \) using the orthogonal matrix \( \hat{\Omega} \) i.e.

\[
\begin{pmatrix}
\eta \\
\eta' \\
\iota
\end{pmatrix}
= \hat{\Omega}
\begin{pmatrix}
\eta_8 \\
\eta_0 \\
G
\end{pmatrix}.
\]

(6)

As was pointed before this matrix diagonalizes also the (mass)\(^2\) matrix for pseudoscalar SU(3) states. Using information about the mass matrix from the quark model (with inclusion of chiral corrections for \( m_{88} \)), assuming \( m_{G8} = 0 \) and taking into account experimental information from \( \iota \rightarrow 2\gamma\)
decays which give \(|\Omega_{q'G}|^2 \approx 0.075\) we can calculate the mixing matrix \(\hat{\Omega}\):

\[
\hat{\Omega} = \begin{pmatrix}
0.94 & 0.34 & \pm 0.07 \\
-0.34 & 0.90 & \pm 0.27 \\
\pm 0.03 & \mp 0.28 & 0.96
\end{pmatrix}
\] (7)

There is an arbitrariness in the sign of some matrix elements and we choose upper sign in order to get a proper sign of gluon spin contribution. We have shown in ref.\[8\] that such model is in agreement with all available informations about the radiative decays of pseudoscalar mesons. We expect from the interpretation of the EMC effect that not only quarks but also gluons play an important role in the spin structure of a nucleon. Considering the mixing of pseudoscalar states built out of quarks with a glueball built out of gluons means that we take into account additional (independent of u, d and s) gluonic degrees of freedom. We assume that in addition to eighth component and singlet quark currents there exists a gluonic current which divergence, in analogy to the quark ones, is given by a linear combination of considered fields with coefficients determined by the masses in the third row of the \(M_{sym}^2\) matrix (see eq.(1)). It is not clear to us how such current should be constructed in terms of fundamental fields. We assume the mixing of very different objects, the states that in the chiral limit are massless Goldstone bosons (therefore the one particle approximation in Goldberger-Treiman relation is justified) and gluon-antigluon bound states. We will use for the gluonic current the assumption of the domination by the glueball state (we are conscious that it is not well justified) and neglect the higher as well as multiparticle states. We hope that our estimate gives if not whole than at least a part of a gluonic contribution. The one particle contribution, corresponding to the glueball, is proportional to \(f_G\) which need not be the same as \(f_\pi\). There exists however a model where \(f_G = f_\pi\) \[9\] and in order to estimate glueball contribution we will consider this case at the begining. Introducing in our case:

\[
\tilde{g}_A = \left( \frac{1}{\sqrt{6}} g_A^{(8)} \right), \quad \tilde{g}_{NN} = \begin{pmatrix} g_{nNN} \\ g_{q'NN} \\ g_{lNN} \end{pmatrix},
\] (8)

in a very similar way as in the case of SU(3) quark axial-vector currents we will get the following relations:

\[
g_A^{(8)} = \sqrt{6} \frac{f_\pi}{2M} (\Omega_{qg} g_{nNN} + \Omega_{q'g} g_{q'NN} + \Omega_{sg} g_{lNN}),
\] (9)
\[ \Delta \Sigma = \sqrt{3} \frac{f_\pi}{2M} (\Omega_{\eta\eta g_{NN}} + \Omega_{\eta'\eta g_{NN}} + \Omega_{\eta'0 g_{NN}}), \quad (10) \]
\[ g_A^{(G)} = \frac{f_\pi}{2M} (\Omega_{\eta G g_{NN}} + \Omega_{\eta' G g_{NN}} + \Omega_{\eta' G g_{NN}}). \quad (11) \]

Taking \( g_{\eta NN} = 6.8 \), \( g_{\eta' NN} = 7.3 \) from [10] and matrix elements of our mixing matrix \( \Omega \) we get numerically:
\[ g_A^{(8)} = \Delta u + \Delta d - 2\Delta s = 0.67 \pm 0.003 g_{\eta NN}, \quad (12) \]
\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s = 1.08 \mp 0.03 g_{\eta NN}, \quad (13) \]
\[ g_A^{G} = \Delta g = \pm 0.17 + 0.07 g_{\eta NN}, \quad (14) \]

where the signs exhibit an arbitrariness in our solution (see eq.(7)). The quantities \( g_A^{(8)} \) and \( \Delta \Sigma \) being combinations of \( \Delta u \), \( \Delta d \) and \( \Delta s \) describe the constituent quark content of nucleon spin and in analogy we can consider \( g_A^{G} \) as a constituent gluon contribution to the nucleon spin, namely \( \Delta g \). The value of \( g_{\eta NN} \) is not known from the experiment. Because of this we present our results in the Table 1 showing the dependence of considered quantities on \( g_{\eta NN} \).

Table 1

The dependence of the weak axial-vector coupling constants: \( g_A^{(8)} \), \( \Delta \Sigma \) and \( \Delta g \) on values of glueball-nucleon coupling constant \( g_{\eta NN} \).

| \( g_{\eta NN} \) | 3   | 5   | 7   |
|------------------|-----|-----|-----|
| \( g_A^{(8)} \)  | 0.68| 0.69| 0.71|
| \( \Delta \Sigma \) | 0.98| 0.91| 0.84|
| \( \Delta g \)    | 0.38| 0.51| 0.64|

From equations (12-14) and the Table 1 we see that the contribution from glueball does not influence the values of \( g_A^{(8)} \) and \( \Delta \Sigma \) very much (there were attempts [11] to explain the EMC data taking big \( g_{\eta NN} \)). For example for not so small value of \( \tau \)-nucleon coupling \( g_{\eta NN} = 5 \) we have \( g_A^{(8)} = 0.69 \) and \( \Delta \Sigma = 0.91 \). The obtained values are not very different from the values obtained previously [11] for \( \theta_p = -20^\circ \) without mixing with the glueball state and \( g_A^{(8)} \) is close to the value gotten from experimental figures: \( 0.58 \pm 0.03 \) (using \( (g_A/g_V)_{N \rightarrow P} \) and \( (g_A/g_V)_{\Sigma^+ \rightarrow N} \) from [12]) or \( 0.60 \pm 0.12 \) (estimate given by
The obtained value of $\Delta g$ is rather small and even for relatively large $g_{NN} = 5$ we get only $\Delta g = 0.51$. This value is of course for low energy scale, say $\mu^2 \approx 0.3$ GeV$^2$. Using type of reasoning proposed by authors of ref.\[14\] we will try to estimate what should be the value of $\Delta g$ at low energy scale in order to understand results of EMC experiment at $Q^2_{EMC} = 10.7$ GeV$^2$. We define $\Delta \tilde{g} = N_F \frac{a_s}{2\pi} \Delta g$ ($N_F = 3$) and evolution equations approximately give:

$$\Delta \tilde{g}(\mu^2) \approx \Delta \tilde{g}(Q^2_{EMC})$$  \hspace{1cm} (15)$$

Calculating $\Delta \tilde{g}(Q^2_{EMC})$ from the ”experimental” value for $G_1(0) = 0.13\pm 0.17$ (see e.g. ref.\[13\]) and $\Delta \Sigma = 0.91$ we get:

$$\Delta \tilde{g}(Q^2_{EMC}) = -G_1(0) + \Delta \Sigma = 0.85 \pm 0.17$$ \hspace{1cm} (16)$$

Using $\alpha_s(Q^2_{EMC}) \approx 0.25$ we obtain from eq. (16) $\Delta g(Q^2_{EMC}) = 6.5$, and hence, taking that $\alpha_s(\mu^2)/\alpha_s(Q^2_{EMC}) = 3.55$ \[13\], we have:

$$\Delta g(\mu^2) \approx \frac{\alpha_s(Q^2_{EMC})}{\alpha_s(\mu^2)} \Delta g(Q^2_{EMC}) = 1.8$$ \hspace{1cm} (17)$$

This means that what we have got is (for $g_{NN} = 5$) about four times smaller than the value needed to explain the EMC experiment. The value of the constituent gluonic spin contribution as measured by the interaction with glueball is not very big. In other words using our $\Delta \Sigma = 0.91$ and $\Delta g = 0.51$ we get $G_1(0) \approx 0.77$. To avoid the conflict with the experiment we need to introduce a large perturbative gluonic contribution $\Delta g$ so that $\Delta \tilde{\Sigma} = \Delta \Sigma - N_F \frac{a_s}{2\pi} \Delta g$ could be equal to the EMC value $0.13 \pm 0.17$. In principle we can take for $g_{NN}$ value as high as 13.6 (for example the condition $\Delta s = 0$, i.e. $g_A^{(8)} = \Delta \Sigma$ for $\mu^2 = 0.3$ GeV$^2$ gives $g_{NN} = 10.4$) and explain all the needed glueball contribution but we consider such large value as unreasonable. Let us make one more comment. Because we have in the divergences of the axial-vector currents the mixing of very different objects i.e.: glueball and nearly massless Goldstone bosons there is no a priori reason that in Goldberger-Treiman relations $f_G$ is equal to $f_\pi$. In the case of $f_G \neq f_\pi$ we modify our formulae replacing $G^{(G)}$ by $(f_G/f_\pi)G^{(G)}$. Hence, we get:

$$g_A^{(8)} = g_A^{(8)}(f_G = f_\pi)$$ \hspace{1cm} (18)$$
\[ \Delta \Sigma = \Delta \Sigma (f_G = f_\pi) + \sqrt{3} \frac{f_\pi}{2M} \left( \frac{f_G}{f_\pi} - 1 \right) m_{G0} G^{(G)} \]

\[ \Delta g = \Delta g (f_G = f_\pi) + \frac{f_\pi}{2M} \left( \frac{f_G}{f_\pi} - 1 \right) m_{GG} G^{(G)} \]

Now, the results for \( \Delta \Sigma \) and \( \Delta g \) depend much stronger than before on \( g_{NN} \) coupling. Let us make a speculation and take as an example \( f_G/f_\pi = 1.5 \). We obtain:

\[ \Delta \Sigma = 1.00 - 0.04g_{NN} \]

\[ \Delta g = 0.45 + 0.10g_{NN} \]

For \( g_{NN} = 5 \) we get \( \Delta \Sigma = 0.78 \) and \( \Delta g = 0.95 \), and the last figure should be compared with the value of \( \Delta g = 1.48 \) obtained from the evolution equations (eq.(17)). In this case the value of \( \Delta g \) is only about two third of the needed value and is still too small.

We have shown that the hypothetical glueball coupling with nucleons does not influence very much the values of \( g_A^{(8)} \) and \( \Delta \Sigma \). With a rather speculative assumptions we have tried to estimate the constituent gluon spin contribution as measured by the glueball interaction with the nucleons. The obtained figures are only a small part of the ones needed to understand the value of proton spin as measured in the EMC experiment.
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