Comparative analysis of calibration variants for inertial measurement unit based on microelectromechanical system

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Abstract. This paper is about comparison of two methods of calibration – calibration method for inertial measurement unit with gyroscope and accelerometer direct information outputs and calibration method for strapdown inertial navigation system with coordinate, velocity and orientation outputs. Two mathematical methods - Least Squares and Kalman Filter methods are applied respectively. Both methods assume the same mathematical error model. This model includes zero drift, nonlinearity coefficients, nonorthogonal projections and also noises of these parameters. The research compare these methods and make attempt to give approximate numerical results. The results reveal features of each case, e.g. optimal calibration time for Kalman Filter and opportunity to find unknown noise parameters in inertial measurement unit variant. These features can help a researcher to choose a suitable variant, all else being equal it is recommended to use the inertial measurement unit variant.

1. Introduction

IMU is an inertial measurement unit which includes three mutually orthogonal gyroscopes and accelerometers. IMU calibration is a test process consisting in finding calibration coefficients of the IMU error model. Depending on the type of sensors, calibration may include the determination of these errors of gyroscopes and accelerometers as part of the IMU, such as zero drift and non-linearity of the scale factor, as well as the non-orthogonality of the sensor installation in relation to the instrumental axes. Calibration is important to eliminate systematic errors with the known instability (noise) of the calibrated parameters.

For blocks based on microelectromechanical system (MEMS) sensors, both pre-operational and operational preliminary calibrations are important and allow to take into account the systematic component that changes during long-term storage. Based on this, it is important not only to evaluate errors accurately, but also to do it by the quickest way.

When calibrating blocks based on MEMS gyroscopes, the most significant errors are zero drift [1] and non-linearity of the scale factor [2]. The significant non-orthogonality caused by the assembly process of blocks [3] should also be taken into account. To find gyroscope drift, the method of several fixed positions is usually used, which provides full observability of zero drifts [4-5].

Two of the most common variants for information outputs of sensor blocks are: IMU outputs – three mutually orthogonal measurements of angular velocities and linear accelerations relative to the instrumental coordinate system (that is, relative to the axes of the unit itself) [6-8] and strapdown inertial navigation system (SINS) outputs – three orientation angles relative to the geographical coordinate...
system, as well as three linear velocities and three coordinates relative to them [3, 9]. Depending on the outputs, calibration approaches can vary significantly. In the literature [10] about the estimation of calibration parameters, only one method is usually given, often even without its explicit declaring, however, questions regarding a qualitative comparison of calibration options are not posed, as well as a comparison of the estimated parameter accuracy and the calibration time.

There are some works that compare different mathematical methods to the same data but there are so not many papers that compare the device with the same sensors and different information output and explore the features of each variant. So it is an idea to compare calibration accuracy and other features for IMU and SINS outputs.

2. Properties of calibrated IMU
The investigated IMU (its characteristics are in table 1) includes three MEMS gyroscopes and MEMS accelerometers (their characteristics are in table 2) of the linear-linear (LL) type (one gyroscope and one accelerometer in one MEMS case), as well as a microcontroller that allows to correct sensor readings taking into account the calculated systematic errors.

Table 1. Mass and size characteristics of the IMU block.

| Characteristic   | Value     |
|-----------------|-----------|
| IMU Mass        | 140g      |
| IMU sizes       | 80x43x34  |

Table 2. The declared characteristics of the sensors.

| Characteristic                              | Value     |
|---------------------------------------------|-----------|
| Frequency of data output                    | 1000 Hz   |
| Dynamic Range of gyroscope                 | ±500°/s   |
| Dynamic Range of accelerometer             | ±100g     |
| Zero bias stability of gyroscope            | 0.02°/s   |
| Zero bias stability of accelerometer        | 0.02g     |
| In-run zero drift (for 5 minutes) of gyroscope | 0.02°/s | |
| In-run zero drift (for 5 minutes) of accelerometer | 0.02g | |
| Zero signal noise of gyroscope              | 0.006 °/s/√Hz |
| Zero signal noise of accelerometer         | 0.002 g/√Hz |

3. Variants of calibration
IMU calibration is a test process consisting in determining calibration coefficients of the IMU error model. Calibration includes the determination of these errors of gyroscopes and accelerometers as part of the instrumentation, such as zero drift and non-linearity of the scale factor, as well as the non-orthogonality of the installation of sensors relative to the instrument axes. Calibration is important to eliminate systematic errors and take into account known instability (noise) of the calibrated parameters if they exist.

In case of the IMU, at the output of the device there are three angular velocities \( (\omega_x, \omega_y, \omega_z) \) and three accelerations \( (a_x, a_y, a_z) \):

\[
\mathbf{Z}_{\text{imu}} = \begin{bmatrix} \omega_x, & \omega_y, & \omega_z, & a_x, & a_y, & a_z \end{bmatrix}
\]

A “direct” calibration can be carried out, the values of the errors of the sensors and the angle of non-orthogonality can be determined for each sensor independently.
In the case of SINS, at the output of the device there are three coordinates \((R_x, R_y, R_z)\), three projections of linear velocity \((V_x, V_y, V_z)\) and three orientation angles \((\Theta_x, \Theta_y, \Theta_z)\):

\[
Z_{SINS} = [R_x, R_y, R_z, V_x, V_y, V_z, \Theta_x, \Theta_y, \Theta_z]
\]  

(2)

The errors of gyroscopes and accelerometers can be estimated in an “indirect” way, for example, using an Kalman Filter, provided the system is observable.

It poses a question how differs the accuracy and other calibration features using these two methods with the same sensors (MEMS sensors of the LL type).

4. IMU calibration variant

To simplify the general mathematical model of sensors, the dependence of zero drift and other parameters on temperature changes is not taken into account. Each sensor can be calibrated independently of the others, therefore the same type of error equation is used for each gyroscope and for each accelerometer.

Gyroscope error model for any considered axis \(L(0,1,2 \text{ for } X, Y, Z)\):

\[
\omega_{nl} = Sf\omega_l (\omega_{nl} + \delta\omega_l),
\]  

where \(\omega_{nl}\) – the measured value by the gyroscope, \(\omega_l\) – the adjusted by the rotary bench angular velocity value, \(\delta\omega\) – the total error of the gyroscope.

\[
\delta\omega_l = v\omega_l,\omega_n + v\omega_l,\omega_{nl} + v\omega_l,\omega_{nl} + \omega_n^2 K\omega_{nl} + \Delta\omega_{nl} + \epsilon\omega_l
\]  

(4)

Accelerometer error model for any considered axis \(L\):

\[
a_{nl} = Sf\omega_l (a_a + \delta a_L),
\]  

(5)

where \(a_{nl}\) – the measured value by the accelerometer, \(a_a\) – the linear acceleration value set by the bench, \(\delta a\) – the total error of the accelerometer.

\[
\delta a_L = v_a,\omega_{nl} + v_a,\omega_{nl} + v_a,\omega_{nl} + a_a^2 Ka_{nl} + \Delta a_a + \epsilon a_L
\]  

(6)

Example for a gyroscope \(X\):

\[
v_x = \cos(\alpha_L)
\]  

(7)

– projection of deviation angle of the sensor along the axis of rotation relative to the same instrument axis (figure 1);

\[
v_y = \cos(\beta_L)
\]  

(8)

– projection of deviation angle of the sensor along the axis of rotation relative to another instrument axis (figure 1);

\[
v_z = \cos(\gamma_L)
\]  

(9)

– projection of deviation angle of the sensor along the axis of rotation relative to another instrument axis (figure 1); \(L\) – axis of the calibrated sensor.

Similarly, for the \(Y\) and \(Z\) axes, the projection will be the cosine of the angle of deviation from the corresponding axis, and the projections on the other axes will be the sines of the angles of deviation to them.

\[
K\omega_{nl} = \bar{K}\omega_{nl} + \epsilon Sf\omega_l
\]  

(10)

– scale factor nonlinearity, \(\epsilon Sf\omega_l\) – instability of the scale factor for the i-th measurement;

\[
\Delta\omega_{nl} = \bar{\Delta}\omega_{nl} + \epsilon v\omega_d
\]  

(11)

– zero drift; \(\epsilon v\omega_d\) – instability of zero drift for the j-th measurement; \(\epsilon\omega\) – zero signal noise of gyroscope.
Figure 1. An example of non-orthogonal angles along the X axis. $\alpha$ is the deviation angle of the X axis of the sensor $X(A_3)$ from the instrumental axis ($A_1$). $\beta$ и $\gamma$ – are the deviation angles of the axis of the sensor $X(A_3)$ to the instrumental axes of other sensors (Z и Y respectively). The angle is greatly increased for visibility.

A priori information on the noise parameters $\varepsilon S_f \omega$, $\varepsilon \nu \omega d$, $\varepsilon \omega$ is absent or precisely accurate, therefore, in the model they are considered a random variable with zero arithmetic mean (white noise). The same method can be applied for Y и Z gyroscopes and for accelerometers. This calibration method does not take into account such typical elements of the SINS model as the Schuler pendulum and the Earth's rotation speed due to their smallness in comparison with the parameters of the sensors indicated in table 1.

To determine the systematic component of each error, a sufficient number of test repetitions is necessary. In the case of the IMU, tests are carried out 5 times. To obtain sufficient statistics within one record with a known noise $\varepsilon \omega$ under adjusted angular velocity, at least 20,000 measurements will be required. In a static position zero drift of these sensors changes about 5 minutes, hence we take this interval for zero drift coefficient finding.

Calibration plan:
1) Finding of zero drift of gyroscopes and accelerometers.

Fixed position, orientation y axis up. For one repetition in one dimension:

\[
\Delta \omega_{by} = \omega_s + \varepsilon \nu \omega d_y + \varepsilon \omega
\]

\[
\omega_s = 0
\]

For gyroscopes to eliminate earth angular velocity we should put sensor to the east or west direction. For accelerometer drift finding we should use 6-position method as in [7]. Implied that we do 5 runs for each of 6 positions from table 3.
### Table 3. Attitude configuration for accelerometer test

| Test No. | Direction of axes | Name of measured value |
|----------|-------------------|------------------------|
|          | X     | Y    | Z     | X     | Y    | Z     |
| 1        | Up    | East | North | a_{drxxa} | a_{dryna} | a_{drxza} |
| 2        | Down  | West | North | a_{drxp} | a_{dryp} | a_{drzp} |
| 3        | West  | Up   | North | a_{dryxa} | a_{dryya} | a_{dryza} |
| 4        | East  | Down | North | a_{dryxp} | a_{dryyp} | a_{dryzp} |
| 5        | East  | North | Up    | a_{drxza} | a_{dryna} | a_{drzna} |
| 6        | West  | North | Down  | a_{drzxp} | a_{drzyp} | a_{drzzp} |

\[
\Delta a_{drxj} = \frac{a_{drxp} * a_{dryyn} - a_{drxza} * a_{dryyp} + a_{drxp} * a_{dryzn} - a_{drxza} * a_{dryzp}}{2} \tag{14}
\]

\[
\Delta a_{dryj} = \frac{a_{dryxp} * a_{dryyn} - a_{dryzn} * a_{dryyp} + a_{dryxp} * a_{dryzn} - a_{dryzp} * a_{dryzp}}{2} \tag{15}
\]

\[
\Delta a_{drzj} = \frac{a_{drzxp} * a_{dryyn} - a_{drzna} * a_{dryyp} + a_{drzxp} * a_{dryzn} - a_{drzna} * a_{dryzp}}{2} \tag{16}
\]

For one run on \( n \) measurements:

\[
\Delta \omega_{aj} = \frac{\sum_{i=1}^{n} \omega_{mi}}{n} + \epsilon \omega \omega_{dj} \tag{17}
\]

For 5 runs:

\[
\Delta \omega_{dr} = \frac{\sum_{j=1}^{5} \Delta \omega_{drj}}{5} \tag{18}
\]

Similarly, accelerometer drift coefficients \( \Delta a_{dr} \) are calculated from the same records.

2) Finding of non-orthogonality angles and scale factors of gyroscopes.

When rotating at a constant angular velocity for three installations relative to each of the instrument axes, three projections are measured along one axis \( ng_{L\text{mean}}^i \) (for \( X_i \)), \( ng_{L\text{mean}}^{ii} \) (for \( Y_j \)), \( ng_{L\text{mean}}^{iii} \) (for \( Z_k \)) for each set:

\[
\omega_m = ng_{L\text{mean}}^i = \omega_a (Sf_{L} + \epsilon Sf \omega_j) \cos(\alpha_L) + \epsilon \omega \tag{19}
\]

While rotating at a constant angular velocity for three installations relative to each of the instrument axes, three projections are measured along one axis:

\[
\omega_m = ng_{L\text{mean}}^{ii} = \omega_a (Sf_{L} + \epsilon Sf \omega_j) \cos(\beta_L) + \epsilon \omega \tag{20}
\]

For any another axis:

\[
\omega_m = ng_{L\text{mean}}^{iii} = \omega_a (Sf_{L} + \epsilon Sf \omega_j) \cos(\gamma_L) + \epsilon \omega \tag{21}
\]

Herewith:

\[
\alpha_L = \arccos((1 - \cos(\beta_L)^2 - \cos(\gamma_L)^2)^{1/2}) \tag{22}
\]
Scale factors and nonorthogonality angles finding:

$$m_{kLj} = \frac{\left( ng_{Lmean}^{I} + ng_{Lmean}^{II} + ng_{Lmean}^{III} \right)^{1/2} - \Delta \omega_{dr} + \epsilon Sf \omega_{j}}{\omega_a} \hspace{1cm} (23)$$

$L$ - axis (sensor); $ng_{Lmean}^{I}$, $ng_{Lmean}^{II}$, $ng_{Lmean}^{III}$ – measurements are carried out while the bench rotating with angular velocity $\omega_a$ for three mutual orthogonal sets; $\epsilon Sf \omega_{j}$ – scale factor instability.

For 5 runs:

$$\frac{\sum_{j=1}^{5} Sfg_{Lj}}{5}$$

Three nonorthogonalities for $L$ axis are finding from three projections:

$$\hat{v}_{iLj} = \cos(\alpha_{Lj}) = \frac{\sum_{j=1}^{5} ng_{Lmean}^{I} - \Delta \omega_{dr}}{\omega_a * Sfg_{L}} \hspace{1cm} (25)$$

$$\hat{v}_{iLj} = \cos(\beta_{Lj}) = \frac{\sum_{j=1}^{5} ng_{Lmean}^{II} - \Delta \omega_{dr}}{\omega_a * Sfg_{L}} \hspace{1cm} (26)$$

$$\hat{v}_{iLj} = \cos(\gamma_{Lj}) = \frac{\sum_{j=1}^{5} ng_{Lmean}^{III} - \Delta \omega_{dr}}{\omega_a * Sfg_{L}} \hspace{1cm} (27)$$

3) Finding of the scale factor nonlinearity for gyroscopes. While rotating with angular acceleration $U_{\omega}$ from 0 to $\omega_a$.

$$\omega_a = \omega_a \left( Sfg_{L} + nlg + \epsilon Sf \omega \right) \hspace{1cm} (28)$$

$F_n$ – frequency of instrument data output.

The common case is maximum angular velocity $\max(\omega_a) = 500^\circ/s$ and $U_{\omega} = 10^\circ / s^2$.

4) Finding of the scale factors of the accelerometers.

It is carried out in a centrifuge similar to the determination of the scale factors of the gyroscopes, taking with non-orthogonality coefficients applied (the misalignment angle between the gyroscope and the accelerometer in the MEMS housing is small).

$$a_m = na_{Lmean}^{I} = a_{a} \left( Sfa_{L} + \epsilon Sfa \right) * \cos(\alpha_{L}) + \epsilon a \hspace{1cm} (29)$$

$$\cos(\alpha_{L}) - 1 \hspace{1cm} (30)$$

5) Finding of the scale factor nonlinearity for accelerometers. While rotating with angular acceleration $A_{\omega}$ from 0 to $a_a$.

$$a_m = a_{a} \left( Sfa_{L} + nl + \epsilon Sfa \right) \hspace{1cm} (31)$$

$F_n$ – frequency of instrument data output.

The common case is maximum angular velocity $\max(\omega_a) = 500^\circ / s$ and $U_{\omega} = 10^\circ / s^2$.

The obtained equations are quite simple and may be solved by the simplified least squares (LS) method.

5. SINS calibration variant

An alternative option for calibrating gyroscopes and accelerometers is a method based on the use of the output signals of a working inertial navigation system (INS), which they include. One of the main reasons for switching to such a calibration method is the lack of direct data from inertial sensors at the INS output.
When calibrating gyroscopes and accelerometers as part of INSs, the following mathematical approaches are most often used to evaluate their errors:

1) Least Squares Method
2) Bayesian methods of statistical estimation
3) Asymptotic filters (Luenberger filters)
4) Kalman filter

The following important points arising from the imposed restrictions and properties of the algorithms used can be attributed to the features of the application of these methods.

Asymptotic filters have a significant advantage due to the adjustable rate of convergence of estimates: it depends on the magnitude of the real part of the roots of the characteristic equation of the evaluator, and this is determined by the selected gain. However, a negative feature of such filters is the complete absence of accounting for noise in measurements and in the system, which leads to significant estimation errors, especially at high amplification factors and, accordingly, short estimation times.

The best estimate for linear systems for unbiased, white, Gaussian noise of system and measurements is theoretically given by the KF. However, this filter requires a very significant amount of a priori information about noise, which is often very unreliable, which leads to disturbances in the operation of the filters and a decrease in the accuracy of estimation, up to the divergence of the estimates. Nevertheless, KF have become one of the most widely used estimation algorithms in technical systems, and it is proposed to take it as a basis when considering the calibration of inertial sensors in the INS.

It is also worth noting that the strapdown version of INS (SINS), is the most used nowadays in a number of practical problems.

Calibration is carried out by a series of indirect measurements of errors using the optimal Kalman filter. For maximum compliance of gyroscopes and accelerometers mathematical models, their description is the same as in the formulas (3)-(6).

The methodology for calibrating inertial sensors based on INS signals during its operation mode differs significantly from the methodology for calibrating the IMU. This is primarily due to the fact that in this case there are no direct measurements of the angular velocity of the equipment rotation and its apparent acceleration, because this is not provided by the the INS output. In this case, only measurements with respect to coordinates, velocity projections and orientation angles remain accessible measurements. For simplicity, we will assume that the coordinates determined by the INS correspond to its mass center, which, when the system is mounted on the turntable, coincides with its center of rotation. This assumption is not fundamental because in case of mismatch between the center of mass of the system under test and the center of rotation, you can always recalculate the readings taking into account the mismatch arm, which is measured before calibration. Taking into account the reservation made, the following measurements can be formed during calibration of inertial sensors of operating INS:

\[
Z = \begin{bmatrix}
R_{SINSx} - R_{Tx} \\
R_{SINSy} - R_{Ty} \\
R_{SINSz} - R_{Tz} \\
V_{SINSx} - V_{Tx} \\
V_{SINSy} - V_{Ty} \\
V_{SINSz} - V_{Tz} \\
\Theta_{SINSx} - \Theta_{Tx} \\
\Theta_{SINSy} - \Theta_{Ty} \\
\Theta_{SINSz} - \Theta_{Tz}
\end{bmatrix}
\begin{bmatrix}
\delta R_{SINSx} - \delta R_{Tx} \\
\delta R_{SINSy} - \delta R_{Ty} \\
\delta R_{SINSz} - \delta R_{Tz} \\
\delta V_{SINSx} - \delta V_{Tx} \\
\delta V_{SINSy} - \delta V_{Ty} \\
\delta V_{SINSz} - \delta V_{Tz} \\
\delta \Theta_{SINSx} - \delta \Theta_{Tx} \\
\delta \Theta_{SINSy} - \delta \Theta_{Ty} \\
\delta \Theta_{SINSz} - \delta \Theta_{Tz}
\end{bmatrix}
\]

These differences can be represented through components of INS errors and errors of its inertial sensors. In this case, the measurements can be reduced to:

\[
Z = HX + V,
\]

(33)
where \( \varphi \) - latitude of the location of the test-bench with installed INS.

Above written nine measurements and their relations with INS errors are the basis for development of calibration algorithms for inertial sensors of INS in its operation mode. This technique uses the output of location, velocity, and angles that come from the output of the INS. An optimal Kalman filter is used to realize this technique. Let’s write the equations of the system in state space to be able to apply the KF. The complete linear model contains the system state equation and the measurement equation:

\[
\begin{align*}
F & \dot{x} + Gw \\
H &= [H_{11}, 0_{9 \times 30}]; \\
\end{align*}
\]

\[
H_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{(R + h) \cos \varphi} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\frac{H}{R + h} + \Omega \varphi & -\omega_{ax} & \omega_{ay} & 1 & 0 & 0 & 0 \\
0 & \frac{H}{R + h} & -\omega_{az} & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & \frac{1}{R + h} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{R + h} \varphi & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

where \( \varphi \) - latitude of the location of the test-bench with installed INS.

\[
\begin{align*}
\dot{x} &= F \bar{x} + G \bar{w} \\
\bar{z} &= H \bar{x} + \bar{v} \\
\end{align*}
\]

where \( \bar{x} \) - state vector; \( \bar{w} \) - system noise vector; \( \bar{v} \) - measurement noise vector; \( F \) - system dynamic matrix; \( G \) - system noise matrix; \( H \) - measurement matrix; \( \bar{z} \) - measurement vector formed as differences between INS output and reference values from the test-bench.

Let’s set the error model of INS, as well as the error models of gyroscopes and accelerometers, represented in the formulas (3-6).

The errors of INS gyroscopes and accelerometers are reduced to the navigation coordinate system by the following formulas:

\[
\begin{align*}
\delta \omega_y &= C_{11} \delta \alpha_1 + C_{12} \delta \alpha_2 + C_{13} \delta \alpha_3, \\
\delta \omega_z &= C_{21} \delta \alpha_1 + C_{22} \delta \alpha_2 + C_{23} \delta \alpha_3, \\
\delta \omega_x &= C_{31} \delta \alpha_1 + C_{32} \delta \alpha_2 + C_{33} \delta \alpha_3, \\
\end{align*}
\]

where \( \delta \alpha_1, \delta \alpha_2, \delta \alpha_3 \) - gyroscope errors, finding by formula 4 for different \( L \).

\[
\begin{align*}
\delta \alpha_y &= C_{11} \delta \alpha_1 + C_{12} \delta \alpha_2 + C_{13} \delta \alpha_3, \\
\delta \alpha_z &= C_{21} \delta \alpha_1 + C_{22} \delta \alpha_2 + C_{23} \delta \alpha_3, \\
\delta \alpha_x &= C_{31} \delta \alpha_1 + C_{32} \delta \alpha_2 + C_{33} \delta \alpha_3, \\
\end{align*}
\]

where \( \delta \alpha_1, \delta \alpha_2, \delta \alpha_3 \) - accelerometer errors, finding by formula 4 for different \( L \).

here \( C_{11}, \ldots, C_{33} \) - elements of the cosine guide matrix between the body coordinate system and the selected rotating coordinate system, calculated by INS orientation algorithms.

The system state vector has a dimension of 39 and can be written as follows:
\[
\vec{x} = [R_x, R_y, R_z, V_x, V_y, V_z, \Theta_x, \Theta_y, \Theta_z, \dot{\Theta}_x, \dot{\Theta}_y, \dot{\Theta}_z, \dot{V}_x, \dot{V}_y, \dot{V}_z, \dot{\Theta}_x, \dot{\Theta}_y, \dot{\Theta}_z, \dot{V}_x, \dot{V}_y, \dot{V}_z, \dot{\Theta}_x, \dot{\Theta}_y, \dot{\Theta}_z, \dot{V}_x, \dot{V}_y, \dot{V}_z],
\]

\[
K_{a_{01}}, K_{a_{02}}, K_{a_{03}}, \Delta a_{0x}, \Delta a_{0y}, \Delta a_{0z}, \Delta a_{0x}, \Delta a_{0y}, \Delta a_{0z}, \Delta a_{0x}, \Delta a_{0y}, \Delta a_{0z},
\]

\[
K_{\omega_{01}}, K_{\omega_{02}}, K_{\omega_{03}}, \Delta \omega_{0x}, \Delta \omega_{0y}, \Delta \omega_{0z}, \Delta \omega_{0x}, \Delta \omega_{0y}, \Delta \omega_{0z},
\]

System noise vector \( \vec{\bar{W}} \) has an appearance:

\[
\vec{\bar{W}} = [0, 0, 0, \varepsilon a_x, \varepsilon a_y, \varepsilon a_z, \varepsilon \omega_x, \varepsilon \omega_y, \varepsilon \omega_z, 0_{130 \times 1}]^T
\]  (44)

System noise intensiveness matrix \( Q \) has an appearance:

\[
Q = \begin{bmatrix}
\sigma^2_{SINS11} & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma^2_{SINS22} & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma^2_{SINS33} & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma^2_{SINS44} & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma^2_{SINS55} & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma^2_{SINS66}
\end{bmatrix}
\]  (45)

where \( \sigma^2_{SINS11}, \sigma^2_{SINS22}, \sigma^2_{SINS33} \) - accelerometer noise RMS; \( \sigma^2_{SINS44}, \sigma^2_{SINS55}, \sigma^2_{SINS66} \) - gyros noise RMS.

Dynamic matrix \( F \) in this case has an appearance:

\[
F = \begin{bmatrix}
F_{11} & F_{12} & F_{13} \\
0_{30 \times 9} & 0_{30 \times 45} & 0_{30 \times 45}
\end{bmatrix}
\]  (46)

where components of this matrix are as follows:

\[
F_{11} = \begin{bmatrix}
\omega^2 - \omega_j^2 & -\omega_j \omega_x + \dot{\omega}_x & -\omega_j \omega_y + \dot{\omega}_y & 0 & 2\omega_x - 2\omega_y & 0 & -a_x & a_y \\
-\omega_j \omega_x - \dot{\omega}_x & \omega^2 + \omega_j^2 & -\omega_j \omega_y + \dot{\omega}_y & -2\omega_x & 2\omega_y & a_x & 0 & -a_y \\
-\omega_j \omega_y + \dot{\omega}_y & -\omega_j \omega_y - \dot{\omega}_y & \omega^2 + \omega_j^2 & 2\omega_x & 2\omega_y & -a_x & a_y & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \omega_j & -\omega_j \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_j
\end{bmatrix}
\]  (47)

\[
F_{12} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & C_{17} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} & C_{27} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} & C_{37} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  (48)
System noise matrix $G$ has an appearance:

$$
G = \begin{bmatrix}
G_{11} & 0_{9 	imes 33} \\
0_{30 	imes 6} & 0_{30 	imes 33}
\end{bmatrix}
$$

(50)

where matrix $G_{11}$ can be written in the form:

$$
G_{11} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & C_{31} & 0 & 0 \\
0 & 0 & 0 & C_{11} & C_{12} & C_{13} \\
0 & 0 & 0 & C_{21} & C_{22} & C_{23} \\
0 & 0 & 0 & C_{31} & C_{32} & C_{33}
\end{bmatrix}
$$

(51)

The measurement noise vector will be written in the following form:

$$
\vec{V} = \begin{bmatrix}
\delta R_{x} \\
\delta R_{y} \\
\delta R_{z} \\
\delta V_{x} \\
\delta V_{y} \\
\delta V_{z} \\
\delta \Theta_{x} \\
\delta \Theta_{y} \\
\delta \Theta_{z}
\end{bmatrix},
$$

(52)

where $\delta R_{x}$ – latitude reference error obtained as a result of test-bench location measurement; $\delta R_{y}$ – longitude reference error obtained as a result of test-bench location measurement; $\delta R_{z}$ – altitude reference error obtained as a result of test-bench location measurement; $\delta V_{x}$ – northern velocity error of the test-bench; $\delta V_{y}$ – eastern velocity error of the test-bench; $\delta V_{z}$ – vertical velocity error of the test-bench; $\delta \Theta_{x}, \delta \Theta_{y}, \delta \Theta_{z}$ - angular error of the test-bench.

Measurements noise intensiveness matrix can be written as:

$$
R = \begin{bmatrix}
\sigma_{11}^{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{22}^{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_{33}^{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{44}^{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{55}^{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{66}^{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_{77}^{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{88}^{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{99}^{2}
\end{bmatrix}
$$

(53)
где $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{44}, \sigma_{55}, \sigma_{66}, \sigma_{77}, \sigma_{88}, \sigma_{99}$ — латITUDE, longitude, altitude, velocity and angle noise measurement RMS of bench data.

The relations between matrices are:

$$\Phi = E + FT \quad (54)$$
$$\Gamma = GT \quad (55)$$

Full equations of discret Kalman Filter algorithm in Joseph form are:

$$\dot{X}_k = \Phi \hat{X}_{k-1} + K_k (Z_k - H \Phi \hat{X}_{k-1}) \quad (56)$$

$$S_k = \Phi P_{k-1} \Phi^T + \Gamma Q \Gamma^T \quad (57)$$

$$K_k = S_k H^T (H S_k H^T + R_k)^{-1} \quad (58)$$

$$P_k = [E - K_k H] S_k [E - K_k H]^T + K_k R_k K_k^T \quad (59)$$

where $X_k$ — system state vector; $Z_k$ — measurement vector; $\Phi$ — transition matrix of the system; $\Gamma$ — random noise matrix; $S_k$ — a priori covariance matrix on the $k$-th step; $P_k$ — a posteriori covariance matrix on the $k$-th step; $Q_k$ — symmetric non-negative defined matrix of system noises; $R_k$ — symmetric positive defined matrix of measurement noises.

There are some ways to organize calibration motions (stages) for providing consecutive error observability. It can be like stages in section 4.

6. Results and discussion

Some results of semi-natural simulation of errors in two calibration variants are given below. Differences in calibration methods are highlighted.

The example of gyroscope zero drift finding according to the IMU methodology is shown on figure 2. During the stage 1 in section 4 we find zero drift value in one repetition by formula 12 and for five repetitions by formula 18. If we take parameters from acquiring data and apply them to gyro model with the same characteristics we have residual error about 0.001°/s. It means that initial error reduced by 90%.

![Figure 2](image)

**Figure 2.** 5 runs of gyroscope zero drift with the applied calibration coefficients (IMU variant).

The instability of the gyroscope scale factor obtained by the IMU variant is shown in figure 3. Stage 2 from section 4 is used. Applying this method by semi-natural experiment (finding coefficients from real data and applying for new model data) we have residual error about 0.01% of in-run instability and 0.1% of run-to-run instability.
Figure 3. Modeling scale factors for IMU variant at a speed of 500°/s for 10 runs.

In relation to accelerometer parameters we have similar percentage values of residual errors to gyroscope.

For comparison graphic results of semi-natural simulation for gyroscope zero drift are shown on figure 4. On this figure curves of estimated angle and angular velocity errors for 2 measurements are shown. Residual error is about 0.0012 °/s (12%). Time of estimation is about 90 seconds and it’s less than set time in the IMU variant.

Figure 4. Gyroscope drift estimation for SINS variant with KF method.
Calibration time for SINS is optimal in case of good preliminary estimated noises but in theory IMU calibration at the same stage can be shorter because of direct outputs. The results of the zero drift calibration of gyroscopes and accelerometers showed are similar (table 4), IMU variant is better on insignificant 2-3 percent. It should be noted that the drift values approximately correspond to the declared certification characteristics.

| Characteristic                  | IMU calibration method (°/s) | SINS calibration method (°/s) |
|--------------------------------|------------------------------|------------------------------|
| In-run gyroscope zero drift    | 0.001                        | 0.0012                       |
| In-run accelerometer zero drift| 0.0011                       | 0.0013                       |

Most international works describe the one method that is more suitable or habitual for researches. For IMU variant we can find both mathematical approaches – Least Squares [7] and Kalman Filter [3]. For SINS variant we can find works with only Kalman Filter approach as in [1]. Some works compare approaches – in [11] LS and KF are compared and in [12] Constraint Kalman Filter and the Sliding Window Filter. The lack of these works for our purpose is limiting itself by simple matrix description. The work [8] describes full diagram which strictly documents all stages of IMU calibration by LS method. In [13] we find full diagram for IMU calibrated by KF method. Unlike these works we are trying to detail the comparison of these methods with full matrices and stages description.

7. Conclusion
This work compares two different devices based on the same sensors with direct and indirect information output. Two different mathematical methods are applied – Least Squares and Kalman Filter respectively. Main points about the comparative analysis of these methods applied to information outputs are:

1) IMU calibration is suitable when we have lack of instability data or bad data in device passport.
2) SINS calibration shows optimal time for measuring in case of correct noise values and IMU calibration time is initially set.
3) Parameter observability depends on motion stages of calibration but no on information output.
4) IMU calibration allows to find in-run instabilities (noises).
5) Both variants allow us to find run-to-run instabilities by a number of repetitions.
6) If we use Extend Kalman Filter we can also find instabilities but it was outside our task.
7) Calibration SINS (KF) in case of unsuccessful determination of system noise may be delayed or dispersed.
8) Results of semi-natural modeling of error parameters are almost identical with insignificant advance of the IMU variant.

These facts should be taken into account if a researcher had both variants and variable conditions for calibration conducting. All else being equal it is recommended to prefer a variant with IMU outputs. The study will be continued, since the comparison can be carried out more correctly with more correct mathematical error model and more close calibration stages.

Acknowledgments
The reported study was funded by Russian Foundation for Basic Research (RFBR) according to the research project No. 19-08-01223.

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