Predictability in Quantum Gravity and Black Hole Evaporation

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Abstract

A possible resolution of the information loss paradox for black holes is proposed in which a phase transition occurs when the temperature of an evaporating black hole equals a critical value, $T_c$, and Lorentz invariance and diffeomorphism invariance are spontaneously broken. This allows a generalization of Schrödinger’s equation for the quantum mechanical density matrix, such that a pure state can evolve into a mixed state, because in the symmetry broken phase the conservation of energy-momentum is spontaneously violated. TCP invariance is also spontaneously broken together with time reversal invariance, allowing the existence of white holes, which are black holes moving backwards in time. Domain walls would form which separate the black holes and white holes (anti-black holes) in the broken symmetry regime, and the system could evolve into equilibrium producing a balance of information loss and gain.
Introduction

It has been stated by Hawking\(^1\) that quantum gravity introduces a new level of unpredictability into physics over and above that associated with the uncertainty principle, due to the emission of purely thermal radiation by black holes. Ignoring problems with the semi-classical methods of calculating the Hawking radiation, if the black hole disappears completely, thereby removing the information about the black hole states, then an initially pure quantum state evolves into a mixed state. This is not permitted in standard quantum mechanics, for a mixed state does not allow a precise determination of any observable.

This situation, in which mixed states are produced by black hole evaporation, arises when the system can be divided into two sections which do not interact with each other. The Hilbert space \(\mathcal{H}\) of the system is the tensor product \(\mathcal{H}_1 \otimes \mathcal{H}_2\) of the Hilbert spaces of parts 1 and 2. The Hilbert space \(\mathcal{H}_1\) is defined in terms of particle states at infinity and the Hilbert space \(\mathcal{H}_2\) represents the states inside the black hole. The density operator \(\rho\) of the total system is initially defined to be in a pure state. Suppose now that an observer can measure only part 1 of the system and that he has no knowledge of part 2. Then he would assign equal probability for all possibilities for part 2, and he would employ a reduced density matrix \(\tilde{\rho}\). In general, \(\tilde{\rho}\) will describe a mixed state on \(\mathcal{H}_1\) even though \(\rho\) describes a pure state on \(\mathcal{H}\).

If we acknowledge that quantum gravity can only be described by a mixed state, composed of various pure quantum states, due to the loss of information in black hole evaporation, then we would give up the notion that even in principle one could work always with pure quantum states. This would, as Hawking has proposed, would produce a degree of unpredictability into quantum gravity that would profoundly change our view of the Universe.
Hawking replaced the standard Schrödinger equation for the density matrix \( \rho \) by a new evolution equation, which is still linear, and first order in the time derivative:

\[
\dot{\rho}_{ab} = \tilde{H}_{ab}^c \rho_{cd},
\]

where \( \dot{\rho} = \partial/\partial t \) and the generalized Hamiltonian operator \( \tilde{H} \) must be constrained to preserve the Hermiticity, positivity and trace of \( \rho \). As shown by Banks, Peskin and Susskind\(^2\) (BPS), the most general equation preserving \( \rho^\dagger = \rho \) and \( \text{Tr} \rho = 1 \) is of the form:

\[
\dot{\rho} = -i[H, \rho] - \frac{1}{2}k_{\alpha\beta}(Q^\alpha Q^\beta \rho + \rho Q^\alpha Q^\beta - 2Q^\beta \rho Q^\alpha),
\]

where the \( Q \)'s are Hermitian operators not equal to the identity, and \( k_{\alpha\beta} \) is a Hermitian matrix of coupling constants. Moreover, \( H \) is the Hamiltonian that appears in the commutator term. To guarantee that \( \rho > 0 \), the eigenvalues of \( k_{\alpha\beta} \) are positive or zero. It was shown by BPS that, if \( k_{\alpha\beta} \) has non-negative eigenvalues and is, in addition, real and symmetric, violations of causality would ensue, if we demand conservation of energy. It has also been shown by Srednicki\(^3\) that even if these causality violations are ignored, then violations of Lorentz invariance would also occur.

2. Spontaneous Breaking of Lorentz and Diffeomorphism Invariance.

A scenario for early Universe cosmology has been proposed\(^4\), in which a Higgs mechanism is introduced into Einstein gravity, which spontaneously breaks local Lorentz invariance and diffeomorphism invariance. In this scheme, we postulate the existence of four scalar fields, \( \phi^a \), defined by

\[
\phi^a = e_\mu^a \phi^\mu, \quad \phi^\mu = e^a_\mu \phi^a,
\]

where \( e_\mu^a \) is a vierbein defined in terms of the metric:

\[
g_{\mu\nu}(x) = e_\mu^a(x)e_\nu^b(x)\eta_{ab}.
\]
The vierbeins $e^a_\mu$ satisfy the orthogonality relations:

$$e^a_\mu e^\mu_b = \delta^a_b, \quad e^a_\mu e^\mu_\nu = \delta^a_\nu,$$

(5)

which allow us to pass from the flat tangent space coordinates (the fibre bundle tangent space) labeled by $a, b, c...$ to the world spacetime coordinates (manifold) labeled by $\mu, \nu, \rho...$.

The fundamental form (4) is invariant under Lorentz transformations:

$$e'_\mu^a(x) = L^a_b(x) e^b_\mu(x),$$

where $L^a_b(x)$ are the homogeneous $SO(3,1)$ Lorentz transformation coefficients that can depend on position in spacetime, and which satisfy

$$L_{ac}(x)L^c_d(x) = \eta_{cd}. \quad (7)$$

Let us assume that the vacuum expectation value (vev) of the scalar fields, $<\phi^a>_0$, will vanish for some temperature $T$ less than a critical temperature $T_c$, above which the local Lorentz symmetry is broken. Above $T_c$ the non-zero vev will break the symmetry of the ground state from $SO(3,1)$ down to $O(3)$. The four real scalar fields $\phi^a(x)$ are invariant under Lorentz transformations:

$$\phi'^a(x) = L^a_b(x) \phi^b(x).$$

(8)

The covariant derivative operator acting on $\phi^a$ is defined by

$$D_\mu \phi^a = \left[\partial_\mu \delta^a_b + (\Omega^a_\mu)_b\right] \phi^b,$$

(9)

where $(\Omega^a_\mu)_b$ denotes the spin connection.

We now introduce a Higgs sector into the Lagrangian density such that the gravitational vacuum symmetry, which we set equal to the Lagrangian symmetry at low temperatures, will break to a smaller symmetry at high temperature. The pattern of vacuum
phase transition that emerges contains a symmetry anti-restoration. This vacuum symmetry breaking leads to the interesting possibility that exact zero temperature conservation laws e.g. electric charge and baryon number are broken in the early Universe. In our case, we shall find that the spontaneous breaking of the Lorentz symmetry of the vacuum leads to a spontaneous violation of the exact, zero temperature conservation of energy.

Let us consider the Lorentz invariant Higgs potential:

\[
V(\phi^T s \phi) = -\frac{1}{2} \mu^2 \phi^T s \phi + \lambda (\phi^T s \phi)^2, \tag{10}
\]

where we choose \( \lambda > 0 \), so that the potential is bounded from below. Here, we have introduced matrix notation and \( s \) is an internal, field dependent metric tensor \( s_{ab} \), associated with the flat tangent space, which is symmetric and positive and transforms as:

\[
s'(x) = (L^{-1})^T(x) s(x) L^{-1}(x). \tag{11}
\]

Evidently, the form \( \phi^T s \phi \) is invariant under local Lorentz transformations.

Our Lagrangian density now takes the form:

\[
\mathcal{L} = \mathcal{L}_G + \sqrt{-g} \left[ \frac{2}{f^2} Tr(D_\mu s s^{-1} D^\mu s s^{-1}) + \frac{1}{2} D_\mu \phi^T s D^\mu \phi - \frac{1}{2} V(\phi^T s \phi) \right], \tag{12}
\]

where \( \mathcal{L}_G \) is the Lagrangian density for Einstein gravity:

\[
\mathcal{L}_G = -\frac{c^4}{16\pi G} \sqrt{-g} R, \tag{13}
\]

\( R = e^\mu e^\nu (R_{\mu\nu})_{ab} \) is the scalar curvature, and \( f^2 \) is a coupling constant. The Lagrangian density (12) is invariant under Lorentz and diffeomorphism transformations.

A calculation of the effective potential for the Higgs breaking contribution in (12) shows that extra temperature dependent minima in the potential \( V(\phi) \) can occur for a non-compact group such as \( SO(3, 1) \), and the spontaneous breakdown of \( SO(3, 1) \) to \( SO(3) \) at high temperatures can be realized.
We have introduced the internal metric \(s\), so as to guarantee unitarity of the \(\phi\) matter sector, which is then non-linearly realized on the non-compact group \(SO(3, 1)\) and linearly realized on the maximal compact subgroup \(SO(3)\). The physical vacuum is not broken by this spontaneous symmetry breaking when \(< s > = s_0\), since \(< e_\mu > = < \Omega_\mu > = 0\) and \(< \phi^T s \phi > = 0\). The additional degrees of freedom associated with \(s\) are frozen out at low energies in that the coupling constant \(f^2\) is chosen so that the internal metric terms in (12) only contribute at energies of the order of the Planck mass.

If \(V\) has a minimum at \(\phi^T s \phi = v^2\), then the spontaneously broken solution is given by \(v^2 = \mu^2/4\lambda\). There are three zero-mass Goldstone bosons and after the spontaneous breaking of the vacuum one massive physical boson particle \(h\) remains. A symmetry breaking term occurs in the Lagrangian density:

\[
\mathcal{L}_\Omega = \frac{1}{2} \sqrt{-g} (\Omega_\mu)^{ab} v_b (\Omega^{\mu})^c a v_c = \frac{1}{2} \sqrt{-g} \sum_{i=1}^{3} (\Omega_\mu^{i0})^2 (\mu^2/4\lambda).
\] (14)

A phase transition is assumed to occur at the critical temperature \(T_c\), when \(v \neq 0\) and the Lorentz symmetry is broken and the three spin connection fields \((\Omega_\mu)^{i0}\) and the associated Lorentz boost generators are broken. Below \(T_c\), the Lorentz symmetry is restored, and we regain the usual classical gravitational field with massless graviton fields.

The total action for the theory is

\[
S_T = S + S_M,
\] (15)

where \(S_T\) is given by

\[
S_T = \int d^4x \mathcal{L}_T,
\] (16)

and \(S_M\) is the usual matter action for gravity. Performing a variation of \(S_T\) leads to the field equations:

\[
G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{c^4} (T^{\mu\nu} + C^{\mu\nu}),
\] (17)
where $T^{\mu\nu}$ is the energy-momentum tensor for matter and $C^{\mu\nu}$ is the energy-momentum tensor for the matter field $\phi$ and the internal metric field $s$.

Because $G^{\mu\nu}$ satisfies the Bianchi identities $G^{\mu\nu,\nu} = 0$, we find in the broken symmetry phase:

$$T^{\mu\nu,\nu} = K^\mu,$$

where ; denotes the covariant derivative with respect to the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$, and $K^\mu$ contains terms proportional to $v^2 = <\phi^T s \phi >_0$. Thus, the conservation of energy-momentum is spontaneously violated. When the temperature passes below the critical temperature, $T_c$, then $v = 0$ and the action is restored to its classical form with a symmetric degenerate vacuum, and we regain the usual energy-momentum conservation laws:

$$T^{\mu\nu,\nu} = 0, \quad C^{\mu\nu,\nu} = 0.$$  

(19)

Let us perform a Lorentz transformation on $\phi^a$, so that we obtain:

$$\phi^0 = h, \quad \phi^1 = \phi^2 = \phi^3 = 0.$$  

(20)

In this special coordinate frame, the remaining component $h$ is the physical Higgs particle that survives after the Goldstone modes have been removed. In this “unitary gauge” frame, we have

$$S_h = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu h \partial^\mu h - V(hh) \right],$$

(21)

where $t = s_{00}$ denotes the ‘time-time’ components of the internal metric $s$. The Hamiltonian associated with $S_h$ is bounded from below, i.e., there are no ghost particles or tachyons in the particle spectrum.

From the definition: $\phi^a = e^a_\mu \phi^\mu$, we get in the unitary Lorentz frame, defined by (20):

$$e^i_\mu = 0, \quad i = 1, 2, 3.$$  

(22)
This produces a triangulation of the coefficients in general coordinate transformations, causing a spontaneous breaking of diffeomorphism invariance. This is an alternative proof that spontaneous violation of the conservation of energy-momentum occurs in the broken symmetry vacuum.

3. Black Hole Evaporation and a Resolution of the Information Loss Problem

The emission of Hawking radiation from a Schwarzschild black hole of mass $M$ is identical in all respects to thermal emission from a perfect black body at temperature $T$ above absolute zero:

$$T = \frac{\hbar c^3}{8\pi kGM},$$

where $k$ is Boltzmann’s constant. As the evaporation proceeds, the mass $M$ decreases and the temperature $T$ increases and eventually $T$ will equal the critical temperature $T_c$, at which a first order phase transition occurs that spontaneously breaks local Lorentz symmetry. Since in the broken symmetry phase of the black hole evaporation the conservation of energy-momentum is spontaneously broken, we are able to provide a physical picture for the generalized Schrödinger equation for the density matrix $\rho$ given by (2). Thus, we can now have a pure quantum state decay into a mixed state during the broken symmetry phase of the evaporation of the black hole. Once the black hole completely evaporates, then the spacetime symmetries and the conservation of energy are restored.

There are two other possible scenarios besides the one contemplated above:

1. A naked singularity of negative mass is produced as a final product of the evaporation.

1. A naked singularity of negative mass is produced as a final product of the evaporation.
2. A remnant black hole of about the Planck mass is left after some mechanism stops the evaporation.

Possibility 1 would result in a complete breakdown of predictability due to a large number of negative mass naked singularities being formed in the early Universe. Also, as would apply to possibility 2, the density of the Universe would be dominated by naked singularities and black hole remnants, which would give rise to unreasonable values for the critical density parameter $\Omega$ and the deceleration parameter. However, we must allow the possibility that a future solution to quantum gravity will result in the removal of singularities as a final product of black hole evaporation, in which case scenario 1. would no longer be viable.

Scenario 2 suffers from the problem that a large amount of information about the black hole states would have to be emitted in the final stages of evaporation. The time during which the information was emitted would have to be unreasonably long, $\sim \exp(4\pi M^2 / M_p^2) / M_p$, where $M_p$ is the Planck mass. This would cause similar problems with the sizes of $\Omega$ and the deceleration parameter.

Thus, the scenario in which the black hole disappears completely, erasing any information about the black hole states and any conserved quantities that are coupled to long range fields, is the most reasonable one. The spontaneous violations of Lorentz invariance and conservation of energy-momentum in the symmetry broken phase of evaporation, at temperatures of order the Planck temperature, would permit a stage in which the initial pure quantum state of the system will have decayed into a mixed state in which one cannot make any exact predictions about the outcome of experiments, but only probabilities for the different possible outcomes.

Hawking$^1$ modified his quantum mechanics so that it was possible to have a pure state decay into a mixed state globally. The intention was that any serious violation of known
physical laws would not be detected by an observer at infinity, i.e., somehow a violation of probabilities could be confined to a small region of spacetime. But BPS showed that this was not the case. Conservation of energy would be accompanied by large violations of causality at macroscopic levels, vitiating the physical scenario. In contrast, our model confines the modification of quantum mechanics to a region of the order of the Planck volume after the onset of the phase transition for \( T \sim T_c \). Quantum mechanics remains unaltered in the initial phase of evaporation, since the broken symmetry phase is hidden by the event horizon. At the phase transition, the symmetry broken phase and the modified quantum mechanical regime will be revealed to an external observer, at which time he will become aware that information was being lost through the violation of energy and entropy conservation inside a region within the black hole. Thus, the observer will then “know” why he has witnessed the decay of the pure quantum state—describing the original collapsed star—into a mixed state with the accompanying loss of information. At the phase transition temperature, \( T_c \), the evaporating black hole will have the same mass, \( M_c \), and radius, \( R_c \), and all black holes will evolve in an identical fashion as they shrink to zero mass.

The precise way in which the information is lost dynamically, through the spontaneous violation of the conservation of energy and entropy, is not understood at present, for a knowledge of this mechanism depends upon being able to solve the field equations in the broken symmetry phase. Although the loss of information (or the associated amount of gained entropy) is subject to the same unpredictability as any other physical observable in the modified quantum mechanical region, we must require a probabilistic accounting of information loss or gain.

We must expect that the decay of a pure quantum state into a mixed state should occur for microscopic black holes at the elementary particle level, for quantum fluctuations of the metric can be interpreted as virtual black holes which appear and disappear. The
time reversal of a black hole spacetime diagram is a white hole, which is analogous to an electron being the time reversed state of a positron, i.e. a white hole is the “anti-blackhole” state and black holes and white holes will create and annihilate in the vacuum. In the symmetry broken phase of these virtual black holes and anti-black holes, which is confined to a localized region of spacetime, the anti-black holes will produce information or, in other words, decrease entropy leading to an overall energy balance in the vacuum.

Within the framework of a Lorentz invariant, local field theory, assuming the usual connection between spin and statistics, invariance under time reversal is equivalent to invariance under $PC$ i.e., the combined operation of charge conjugation ($C$) and space inversion ($P$). In the broken Lorentz symmetry phase, the Lüders, Pauli, Schwinger\textsuperscript{10} TCP theorem is spontaneously broken, and in our scheme there will be a localized violation of time reversal invariance under the operation of time inversion $T$. Thus, in the broken symmetry phase, the time arrow of the second law of thermodynamics can be reversed, permitting the existence of white holes (or anti-black holes).

We can also postulate the existence of a macroscopic white hole in the final symmetry broken phase of evaporation, since the arrow of time can be reversed in this phase, producing this macroscopic white hole (or anti-black hole) and the latter will produce information or, equivalently, entropy decrease. The situation could then evolve to an information balance in the Universe, before the white hole and black hole evaporate away. Due to the spontaneous breaking of the discrete time reversal symmetry, associated with the phase transition, a domain wall\textsuperscript{11} will form that separates the white hole from the black hole within the symmetry broken regime.
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