Physics of Cosmic Reionization

T. Roy Choudhury\textsuperscript{1,2,*} and A. Ferrara\textsuperscript{1†}

\textsuperscript{1}SISSA/ISAS, via Beirut 2-4, 34014 Trieste, Italy
\textsuperscript{2}Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur 721302, India

Abstract

The study of cosmic reionization has acquired increasing significance over the last few years because of various reasons. On the observational front, we now have good quality data of different types at high redshifts (quasar absorption spectra, radiation backgrounds at different frequencies, cosmic microwave background polarization, Ly\textalpha emitters and so on). Theoretically, the importance of the reionization lies in its close coupling with the formation of first cosmic structures, and there have been numerous progresses in modelling the process. In this article, we review the current status of our understanding of the physical processes governing the cosmic reionization based on available observational data.

1 Historical background

Reionization can be thought of as the second major change in the ionization state of hydrogen (and helium) in the universe (the first being the recombination occurring at $z \approx 1100$). The process is of immense importance in the study of structure formation since, on one hand, it is a direct consequence of the formation of first structures and luminous sources while, on the other, it affects subsequent structure formation. In this article, we attempt to review the basic physical processes related to the reionization with particular emphasis on the link between theory and observations.

The study of reionization consists of two broad areas, namely the properties of the intergalactic medium (IGM) and the formation of sources. Once the first sources produce photons capable of ionizing the surrounding IGM,
the process of reionization (and reheating) can be thought of having begun, thus changing the thermal, ionization and chemical properties of the IGM. This change in the nature of the IGM affects the formation of next generation of sources (like metal enrichment changing the initial mass function of stars). The subject area of formation of sources is quite involved in itself dealing with formation of non-linear structures (haloes and filaments), gas cooling, accretion and generation of radiation (stars and quasars). This is somewhat beyond the scope of this review, and we will mostly concentrate on the effects of sources on the IGM.

Historically the study of reionization of the IGM has been closely linked with the observations related to spectra of distant quasars, in particular to the Ly$\alpha$ forest, though it was not obvious in the beginning whether the Ly$\alpha$ forest traces baryons of cosmological significance. In particular, models in which the Ly$\alpha$ forest arises from some kind of “confined clouds” predicted that the amount of baryons within the forest may not be of cosmological significance and hence may not have any substantial connection to the cosmic reionization as we understand it today. To stress this in slightly more detail, let us briefly review some of the major ideas in the development of this field. Of course, the literature of the Ly$\alpha$ forest has been reviewed various times, (see for example [1]); nevertheless it might be appropriate to review some the major ideas from the point of view of cosmic reionization.

1.1 Initial models based on pressure and gravitational confinement

In a classic paper, Gunn & Peterson [2] showed that the hydrogen in a diffuse uniform IGM must have been highly ionized at $z \approx 2$ in order to avoid complete absorption of the transmitted flux at wavelengths bluewards of the Ly$\alpha$ emission line of the QSO; this is now commonly known as the Gunn-Peterson (GP) effect. Following that, it was proposed [3] that this GP effect can be used to probe the ionization state of hydrogen within the IGM at various redshifts (and also for other elements). The GP effect has remained one of the most stringent tests of the ionization state of the IGM to date.

At the same time, it was also realized that gas which was not uniformly distributed would produce discrete Ly$\alpha$ absorption lines. In the beginning, the most natural structures considered were gas clumped into groups of galaxies [4] or low mass protogalaxies [5]. However, these were soon found to be unrealistic when different groups [6, 7] discovered a large number of discrete absorption lines in the QSO spectra, which are usually known as the “Ly$\alpha$ forest”. It was shown that these forest lines could not be associated with
galaxy clusters, rather they have an intergalactic origin and arise in discrete intergalactic clouds at various cosmological redshifts along the line of sight (for reviews see [7, 8, 9]). Various arguments (like the apparent lack of rapid evolution in the properties of the forest, the short relaxation time scales for electrons and protons and short mean free paths) led to the notion that the clouds were “self-contained entities in equilibrium” [7]. A two-phase medium was postulated, with the diffuse, very hot, intercloud medium (ICM) in pressure equilibrium with the cooler and denser Lyα clouds. In this two-phase scenario, the ICM was identified with the IGM, while the Lyα clouds were treated as separate entities.

According to the pressure confinement model [7, 10, 11], the Lyα clouds are supposed to be in photoionization equilibrium with an ionizing ultraviolet (UV) background. The gas is heated by photoionization and cools via thermal bremsstrahlung, Compton cooling, and the usual recombination and collisional excitation processes. Since the ICM is highly ionized, the photoheating is not efficient and hence the medium cools adiabatically through cosmic expansion. The denser clouds embedded in the hot ICM have a nearly constant temperature fixed by thermal ionization equilibrium ($\sim 3 \times 10^4$ K) [10, 11]. The available range of cloud masses is constrained by the requirement that the clouds must be small enough not to be Jeans-unstable but large enough not to be evaporated rapidly when heated by thermal conduction from the ambient ICM [7, 10]. According to such constraints, clouds formed at high redshifts would survive down to observed redshifts only if their masses range between $10^5$–$10^{10} M_\odot$.

The neutral hydrogen within the confining ICM is expected to cause a residual GP absorption trough between the absorption lines (clouds). However, observations at higher spectral resolution [12, 13, 14] revealed no continuous absorption between the discrete lines, placing strong limits on the GP effect, which in turn, puts a strict upper limit on the density of the ICM. The ICM temperature has a lower limit from the absorption line width, while the condition that the cloud must be large enough not to evaporate gives an upper limit on the temperature [10]. Another independent upper limit on the temperature of the ICM comes from the lack of inverse Compton distortions in the spectrum of the cosmic microwave background [15] through the Sunyaev-Zeldovich effect [16]. In fact, the upper limit of the so-called $y$-parameter [17] is able to rule out any cosmologically distributed component of temperature greater than $10^6$ K. When all the limits are combined, only a relatively small corner of allowed density-temperature parameter space remains for the ICM. It turns out that, according to the pressure-confinement model, the density of the ICM is too small to be cosmologically significant. Hence, during these early days, the connection between the cosmic reioniza-
tion and the IGM was not at all obvious as most of the baryons was expected to lie somewhere else.

The pressure-confinement model ran into severe problems while trying to match the observed column density distribution \[18\]. For example, in order to reproduce the low column density systems between, say, \(13 < \log(N_{\text{HI}}/\text{cm}^{-2}) < 16\) (where \(N_{\text{HI}}\) is the column density of neutral hydrogen), the mass has to vary by 9 orders of magnitude. On the other hand, the mass is severely constrained in order to ensure cloud survival. Therefore, the only escape route is to invoke pressure inhomogeneities \[19\]. However, the \(\text{Ly}\alpha\) absorbers are found to be weakly clustered over a large range of scales, which thus excludes any significant pressure fluctuations \[20\]. Similarly, detailed hydrodynamical simulations \[21\] show that the small mass range of the clouds leads to a failure in producing the column density distribution at high \(N_{\text{HI}}\). In addition, pressure-confinement models predict small cloud sizes which are incompatible with the observations of multiple lines of sight \[22\]. It was thus concluded that the pure pressure confinement model is unlikely to explain the \(\text{Ly}\alpha\) forest as a whole though it is possible that some lines of sight must go through sites where gas is locally confined by external pressure (say, the galactic haloes, the likely hosts of the dense Lyman limit absorbing clouds).

Even from a theoretical point of view, there are no physical reasons for preferring pressure to gravitational confinement or to no confinement at all. Because of this, self-gravitating baryonic clouds were suggested by \[23, 24\] as an alternative to the pressure confinement model. In this model, the appearance of the IGM as a forest of lines is because of the variations in the neutral hydrogen density rather than a sharp transition between separate entities. In this sense, there is no real difference between an ICM and the clouds in the gravitational confinement model. This scenario of self-gravitating clouds predicts larger sizes of the absorbing clouds (\(\sim 1\) Mpc) compared to the pressure-confinement scenario. However, this model, too, runs into problems while trying to match the observed column density distribution \[25\] as it predicts larger number of high column density systems than is observed. Secondly, the large absorber sizes seemed to contradict observations. Furthermore, gravitationally confined clouds are difficult to explain theoretically since the mass of such clouds must lie in a restricted range to maintain the gas in equilibrium against free expansion or collapse.

As a further alternative, the properties of gas clouds confined by the gravitational field of dark matter have been investigated \[26\], more specifically in terms of the “minihalo” model \[27, 28\]. In this picture, \(\text{Ly}\alpha\) clouds are a natural byproduct of the cold dark matter (CDM) structure formation scenario. Photoionized gas settles in the potential well of an isothermal dark
matter halo. The gas is stably confined if the potential is sufficiently shallow to avoid gravitational collapse but deep enough to prevent the warm gas from escaping. CDM minihaloes are more compact than the self-gravitating baryonic clouds of [24] because of the larger dark matter gravity, thus alleviating the size problem. The detailed structure of the halo depends on the relative spatial distribution of baryons and CDM. However, the virial radii of the confining objects (∼10 kpc) are much lower than the coherence lengths of the Lyα systems as obtained from constraints on absorption line observations of lensed or paired QSOs [29, 30]. It was thus natural to extend the minihalo model to non-static systems. A non-static minihalo model was studied by [31], who examined the hydrodynamics of a collapsing spherical top-hat perturbation and suggested that clouds were in a free expansion phase.

1.2 IGM as a fluctuating density field

Following the non-static models, it was realized that an IGM with the density fluctuation variance of the order of unity could also produce line-like absorptions in quasar spectra [32, 33]. According to such models, the IGM becomes clumpy and acquires peculiar motions under the influence of gravity, and so the Lyα (or GP) optical depth should vary even at the lowest column densities [24, 32, 33, 34, 35]. In a CDM-dominated structure formation scenario, the accumulation of matter in overdense regions reduces the optical depth for Lyα absorption considerably below the average in most of the volume of the universe, leading to what has been called the fluctuating GP phenomenon. Traditional searches for the GP effect that try to measure the amount of matter between the absorption lines were no longer meaningful, as they were merely detecting absorption from matter left over in the most underdense regions. If this is not taken into account, the amount of ionizing radiation necessary to keep the neutral hydrogen GP absorption below the detection limits can be overestimated, which would then have severe implications for reionization studies. In this scenario, the density, temperature and thermal pressure of the medium were described as continuous fields and could not be attributed simply to gravitational confinement or pressure confinement. These studies led to a shift in the paradigm of IGM theories, especially since they implied that the IGM contains most of the baryons at high redshifts, thus making it cosmologically significant and hence quite relevant to cosmic reionization.

The actual fluctuation picture can be derived from cosmological N-body and hydrodynamical simulations. It was possible to solve hydrodynamical equations from first principles and set up an evolutionary picture of the IGM in these simulations [36, 37, 38, 39]. Although different techniques
and cosmological models were used by different groups, all the simulations indicate a fluctuating IGM instead of discrete clouds.

Since in this new paradigm, the Ly$\alpha$ forest arises from a median-fluctuated quasi-linear IGM, it is possible to ignore the high non-linearities. This made it possible to study the IGM through semi-analytical techniques too. The issue of dealing with quasi-linear densities were dealt in two ways. In the first method, it was showed that a quasi-linear density field, described by a lognormal distribution, can reproduce almost all the observed properties of the Ly$\alpha$ forest. In fact, this was motivated by earlier ideas for dark matter distribution. In an alternate method, it was also possible to obtain the density distribution of baryons from simulations which could then be used for semi-analytical calculations. Given the baryonic distribution, the neutral hydrogen fraction was calculated assuming photoionization equilibrium between the baryons and the ionizing radiation field. It was also realized that the equilibrium between photoheating and adiabatic cooling implies a tight relation between the temperature and density of the gas, described by a power-law equation of state, which was used for determining the temperature of the gas. Given such simplifying and reasonable assumptions, it was possible to make detailed predictions about the Ly$\alpha$ forest. For example, a relation between column density peaks (“absorption lines”) and the statistics of density peaks was proposed, and analytical expressions for the dependence of the shape of the column density distribution on cosmological parameters were obtained.

The simulations and the semi-analytical calculations both have been quite successful in matching the overall observed properties of the absorption systems. The shape of the column density distribution and the Doppler parameter distribution are reasonably well reproduced by the simulations as well as semi-analytical calculations over a wide redshift range. The large transverse sizes of the absorbers seen against background paired and lensed QSOs are well explained by the coherence length of the sheets and filaments. In addition, the probability distribution function and power spectrum of the transmitted flux in the Ly$\alpha$ forest is reproduced very well by the models. The Ly$\alpha$ optical depth fluctuations were used for recovering the power spectrum of matter density fluctuations at small scales and also to obtain various quantities related to the IGM.

Given the fact that the Ly$\alpha$ can be modelled so accurately, it has become the most useful tool in studying the thermal and ionization history of the universe ever since. Subsequently it was realized that this simple description of the IGM could be coupled to the properties of the ionizing sources and hence it was possible to compute the reheating and reionization history. Since
the modelling of the sources is a highly non-linear problem and much more non-trivial to solve that the quasi-linear IGM, it was more natural to make some simple assumptions about the sources, calculate their effect on the IGM and then constrain the properties of the sources themselves.

1.3 Sources of ionization

The classic problem of the propagation of ionization fronts from a point source was studied by [57, 58]. It was shown that the recombination timescale is too large for the ionized region to reach the Strömgren radius. Furthermore, the calculations showed that the ionizing photons from the observed population of QSOs cannot produce enough UV flux to reionize the IGM at \( z \approx 3 \) [57, 59, 58]. This lead for extensive searches and proposals for other sources of UV ionizing flux. The next most obvious choices for UV radiation were the (early) galaxies and stars. This was studied using observed ionization state of heavy element absorption systems in the spectra of QSOs and model-dependent metal production arguments [60, 61], though no firm conclusions could be drawn because of the fraction of photons which are able to escape the host galaxy is unknown (and that situation remains till date).

The possibility of galaxies contributing to the UV flux was implemented in various analytical calculations [62, 63, 64, 65]. These calculations concentrated on the collapse of dark matter haloes, subsequent cooling (atomic and/or molecular) of gas, star formation formalisms and propagation of ionization fronts. Subsequently, detailed modelling for the reheating and reionization histories of the IGM showed that, under standard assumptions regarding hierarchical CDM model, Press-Schechter theory, cooling within collapsed haloes, star-forming efficiency and observed QSO luminosity function, the reionization of the hydrogen is achieved at \( z \approx 10 \) [66, 67]. Most of these studies generally incorporated the inhomogeneities in the IGM through a (evolving) clumping factor. A model for reionization for the inhomogeneous IGM was proposed [65] which was able to take into account the fact that dense regions would remain neutral longer (because of their high recombination rate).

Most of these effects were also seen in hydrodynamical simulations, thus confirming the overall picture for reionization by UV sources. Usually the limitation in computing power forced small volumes (say, boxes with sizes of a few Mpcs) to be simulated. It was found [63, 68] that a mass resolution of about \( 10^4 M_\odot \) was required to resolve early epochs of reheating and reionization, which remains a great challenge even now. A better resolution can be achieved if, for example, high-resolution N-body simulations and semi-analytical models for galaxy and star formation are combined [69] to obtain
the thermal history of the IGM.

The picture of reionization by UV sources which emerged from these studies can be summarised as follows: (i) The reionization process by UV sources could be classified into three phases. In the “pre-overlap” phase, the ionized regions of individual sources propagate into the neutral IGM. In the “overlap” phase, the ionized regions start overlapping and subsequently ionize the whole of IGM (except for some high-density peaks). At this stage the universe becomes transparent to UV radiation and hence the mean free path of photons increases dramatically. Finally, there is the ever-continuing “post-overlap” phase where the ionization fronts propagate into the neutral high density regions. (ii) The reheating of the IGM preceded the reionization as a small number of hard photons could heat the medium up to several hundred to thousand Kelvins before complete reionization.

It should be mentioned that though the QSOs and galaxies seem to be the most natural choices as sources for reionization of the IGM, the possibility of other sources cannot be ruled out, at least from observations. Hence various other sources have been studied too, the early ones being the supernova-driven winds, hard photons from structure formation, early formed massive black holes and more exotic sources like decaying dark matter (or other) particles, with the list ever-increasing till date.

It is thus clear that the transmission regions in the Lyα forest at redshifts \( z \lesssim 6 \) conclusively implies that the universe is ionized at lower redshifts, though the exact nature of the ionization process or the sources responsible are not understood at the moment. On the other hand, we can also think of the Lyα forest as the leftover of the reionization, i.e., the absorption signatures imply that the sources were not able to fully complete the job.

In the next section, we shall review the current observational situation regarding reionization of the IGM and main conclusions that can be drawn from the data. Section 3 would discuss the physics of cosmic reionization along with description of certain analytical and numerical models. We shall summarize the main predictions and future tests for these models.

## 2 Observational constraints

In this Section, we summarise various sets of observational data which shape our current understanding of reionization (for a detailed review on recent developments, see [82]). These observational probes can be broadly divided into two types: the first set probes the extent and nature of the reionization through observations of the IGM while the second is mostly concerned with
2.1 Observations related to the state of the IGM

As far as the IGM is concerned, the observational constraints on its ionization and thermal state can be divided broadly into three classes, which are discussed in the next three subsections.

2.1.1 QSO absorption lines

We have discussed in the previous Section that the primary evidence for the IGM to be ionized at $z < 6$ comes from the measurements of GP optical depth in the spectra of QSOs. Under the assumptions of photoionization equilibrium and a power-law relation between temperature and density, the Lyα optical depth $\tau_{\text{GP}}$ arising from a region of overdensity $\Delta$ at a redshift $z$ can be written as

$$
\tau_{\text{GP}} = 3.6 \times 10^5 \left(\frac{\Omega_b h^2}{0.022}\right) \sqrt{\frac{0.15}{\Omega_m h^2}} \left(\frac{1 - Y}{0.76}\right) \left(\frac{1 + z}{7}\right)^{3/2} \bar{x}_{\text{HI}} \Delta^\beta
$$

where $Y$ denotes the helium mass fraction, and $\bar{x}_{\text{HI}}$ is the neutral hydrogen fraction (defined as the ratio between neutral hydrogen density and total hydrogen density) at the mean density $\Delta = 1$. The exponent of $\Delta$ is determined by the photoionization equilibrium and is given by $\beta = 2.7 - 0.7\gamma$, where $\gamma$ is the slope of the pressure-density relation. For an isothermal medium $\gamma = 1$ and hence $\tau_{\text{GP}} \propto \Delta^2$. All other symbols in the above expression have standard meanings. This expression clearly shows that for a uniform medium ($\Delta = 1$) at $z \lesssim 6$, the presence of a neutral hydrogen fraction $\bar{x}_{\text{HI}} \gtrsim 10^{-5}$ would produce an optical depth of the order unity and hence would show clear GP absorption trough in the spectra. Since such absorption is not observed for QSOs at $z < 6$, the constraint on the average neutral fraction is $\bar{x}_{\text{HI}} < 10^{-5}$, which is a robust indication of the fact that the universe is highly ionized at $z < 6$.

The observational situation changes for the observed QSOs at $z > 6$. The ongoing Sloan Digital Sky Survey (SDSS)$^1$ has discovered quite a few QSOs at $z \gtrsim 6$, the spectra of which are markedly different from their low-redshift counterparts. Very long absorption troughs, which are of the size $\sim$ 80–100 comoving Mpc, have been seen along tens of lines of sight at $z > 6$ $^{83, 84, 85, 86, 87, 88}$. This implies that the GP optical depth at $z \gtrsim 6$ is larger than a few. Unfortunately, such a constraint does not necessarily

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$^1$http://www.sdss.org/
imply that the universe is neutral at such redshifts. For example, a neutral hydrogen fraction $\bar{x}_{\text{HI}} \sim 10^{-3}$ would produce an optical depth $\tau_{\text{GP}} \sim 100$, more than what is required to produce the absorption troughs. This is the typical level of constraint one can obtain through such model-independent simplistic arguments based on an uniform medium. Such arguments, though quite effective in giving robust conclusions at low redshifts, do not yield any strong constraint on the neutral hydrogen fraction at $z \gtrsim 6$.

The next line of argument for the approach to the final stages of reionization at $z \gtrsim 6$ is based on the change in the slope of the optical depth around $z \sim 5.5 - 6$, which indicates that some qualitative change in the physics of IGM occurs at these redshifts. To understand this in simple terms, let us write the neutral hydrogen fraction $\bar{x}_{\text{HI}}$ in terms of more physically meaningful quantities:

$$\bar{x}_{\text{HI}} = 2.7 \times 10^{-5} \left( \frac{T_0(z)}{10^4 \text{K}} \right)^{-0.7} \left( \frac{\Gamma_{\text{PI}}(z)}{10^{-12} \text{s}^{-1}} \right)^{-1} \left( \frac{\Omega_b h^2}{0.022} \right) \left( \frac{1 - Y}{0.76} \right) \left( \frac{1 + z}{7} \right)^3$$

where $T_0$ is the temperature of the medium at the mean density ($\Delta = 1$) and $\Gamma_{\text{PI}}$ is the photoionization rate of neutral hydrogen (assumed to be homogeneous). Combining the above equation with (1), one can see that $\tau_{\text{GP}}(z) \propto (1 + z)^{4.5} T_0^{-0.7}(z)/\Gamma_{\text{PI}}(z)$. Thus when $T_0$ and $\Gamma_{\text{PI}}$ are not changing substantially with redshift, we expect $\tau_{\text{GP}}(z) \propto (1 + z)^\alpha$ with $\alpha \approx 4.5$. This is indeed seen in the observations at $z \lesssim 5.5$ [90]. However, at higher redshifts, the observations show that $\tau_{\text{GP}}$ evolves much faster combined with a rapid deviation from a power-law evolution, thus implying that the properties of IGM (like $T_0$ and $\Gamma_{\text{PI}}$) are evolving considerably. This argument points towards a possible phase change in the IGM and thus suggesting that we are approaching the final stages of reionization at $z \approx 6$. However, one should keep in mind that this argument does not conclusively prove that the IGM is neutral at $z \gtrsim 6$ – it simply indicates for a rapid change in properties. Furthermore, the above defined $\tau_{\text{GP}}$ is not a directly measured quantity; one instead measures the mean transmitted flux $\bar{F}$ which is computed by integrating the optical depth over all possible overdensities:

$$\bar{F} = \int_0^\infty \text{d}\Delta \ P(\Delta) \ \exp[-\tau_{\text{GP}}(\Delta)].$$

The quantity $P(\Delta)$ denotes the density distribution of the IGM. It is thus clear from the above expression that any robust conclusion based on the observed evolution of $\bar{F}$ would require a good knowledge of $P(\Delta)$.

Hence, the next step for calculating the ionization properties of the IGM from QSO spectra is to include the density inhomogeneities in the analysis.
From this point on, the conclusions become extremely model-dependent as we do not have a clear understanding of the density distribution of the IGM. One approach would be to use numerical simulations for obtaining the IGM density distribution and then compute the absorption spectra of high-redshift quasars in the Lyα region [86]. Using this approach, a rapid evolution of the volume-averaged neutral fraction of hydrogen has been found at $z \lesssim 6$ ($\bar{x}_{\text{HI}} \sim 10^{-5}$ at $z = 3$ to $\bar{x}_{\text{HI}} \sim 10^{-3}$ at $z = 6$). On the other hand, a different set of analyses [91, 92] from nearly similar data set conclude that the transmitted fractions have a relatively smooth evolution over the entire range of redshifts, which can be modeled with a smoothly decreasing ionization rate; hence no evidence of a rapid transition could be established.

In addition to the global statistics discussed above, there are some results based on the transmission observed in the spectra of individual sources. For example, the analyses of the spectrum of the most distant known quasar (SDSS J1148+5251) at an emission redshift of 6.37 show some residual flux both in the Lyα and Lyβ troughs, which when combined with Lyγ region [93], imply that this flux is consistent with pure transmission. The presence of unabsorbed regions in the spectrum corresponds to a highly ionized IGM along that particular line of sight. However, a complete GP trough was detected in the spectrum of SDSS J1030+0524 ($z = 6.28$) [83], where no transmitted flux is detected over a large region (300 Å) immediately blueward of the Lyα emission line. Such differences in the ionization state of the IGM along different lines of sight have been interpreted as a possible signature of the pre-overlap phase of reionization.

There have been other different approaches to investigate the neutral hydrogen fraction. For example, one can estimate the sizes of the ionized regions around the QSOs from the spectra [94, 95]. Then the neutral gas surrounding the QSO can be modelled as a function of different parameters: the Strömgren sphere size $R_S$, the production rate of ionizing photons $\dot{N}_{\text{ph}}$ from the QSO, the clumping factor of the gas $C$ and the age of the QSO $t_{\text{age}}$. Considering 7 QSOs at $z > 6$ (which included the above cited QSOs), it has been argued that the small sizes of the ionized regions ($\sim 10$ physical Mpc) imply that the typical neutral hydrogen fraction of the IGM beyond $z \sim 6$ is in the range 0.1 - 1. However, this approach is weighted down by several uncertainties. For example, one of the uncertainties is the quasar’s production rate of ionizing photons $\dot{N}_{\text{ph}}$ as it depends on the shape of the spectral template used. Moreover it is implicitly assumed in the modelling of clumping factor that the formation of quasars and galaxies were simultaneous. This in turn implies that quasars ionize only low density regions and hence the clumping factor, which regulates the evolution of the ionized regions, is low. If, instead, stars appears much earlier than QSOs, the quasars
have to ionize high density regions, which means that one should use a higher value of clumping factor in the calculations [96].

There has been a different approach based on the damping wings of the neutral hydrogen [97]. Using density and velocity fields obtained by hydrodynamical simulation, the Ly\(\alpha\) absorption spectrum was computed. In this case the neutral hydrogen fraction, \(\dot{N}_{ph}\) and \(R_S\) are treated as free parameters, constrained by matching the optical depth observed in the QSO SDSS J1030+0524. Also in this case, the conclusion is that the neutral hydrogen fraction is larger than 10 per cent, i.e., the IGM is significantly more neutral at \(z \sim 6\) than the lower limit directly obtainable from the GP trough of the QSO spectrum (\(\bar{x}_{HI} \approx 10^{-3}\)). However this result is based only on one quasar. Moreover, the observational constraints on the optical depth are very uncertain and can introduce errors in the estimates of \(\bar{x}_{HI}\).

To summarise the QSO absorption line observations – there is still no robust and model-independent constraint on the neutral hydrogen fraction from the data. The spectroscopy of the Ly\(\alpha\) forest for QSOs at \(z > 6\) discovered by the SDSS [84, 87] strongly suggest that the IGM is highly ionized along some lines of sight. On the other hand, there are a few (maybe a couple) lines of sight which seems to indicate that the IGM is neutral, though the conclusion is still not robust. In case we find transmission along some lines of sight while the medium seems quite neutral along others could possibly be interpreted that the IGM ionization properties are different along different lines of sight at \(z \gtrsim 6\), thus suggesting that we might be observing the end of the reionization process. However, it is also possible that such dispersion in the IGM properties along different lines of sight can be accommodated by simply the dispersion in the density inhomogeneities. As discoveries of more such objects are expected in future, spectroscopy of high-redshift QSOs remains one of the principal empirical approaches to understand the final stages of reionization.

Before completing our discussion on the QSO absorption lines, it is worth mentioning a set of indirect constraints on reionization based on the temperature of the IGM at \(z \approx 2 - 4\).\(^2\) Using various techniques like, the lower envelope of the neutral hydrogen column density and velocity width scatter plot [98, 99] or wavelet transforms [100], one can infer the temperature of the IGM from absorption lines. These analyses suggest that \(T_0 \sim 1 - 2 \times 10^4\) K at \(z \approx 3\), which in turn imply that hydrogen reionization must occur at \(z < 9\) or else the temperature would be too low to match the observations. However, one should keep in mind that the analyses has large uncertainties,

\(^2\) This determination of temperature puts constraints on the reionization of helium too; however, the helium reionization is beyond the scope of this review.
like, for example, the dust photoheating of the IGM could give rise to high temperatures at $z \approx 3$ \cite{101, 102, 103, 104}. Furthermore, a complex ionization history of helium could relax considerably the constraints obtained from $T_0$ on the reionization epoch.

2.1.2 Cosmic microwave background radiation

The second most important analysis regarding the reionization history comes from the observations of temperature and polarization anisotropies in the cosmic microwave background (CMB) radiation. As far as the temperature anisotropies are concerned, reionization can damp the fluctuations on small scales due to photon diffusion in the ionized plasma. The scattering of photons suppresses the anisotropies on angular scales below the horizon at the rescattering epoch by a damping factor $e^{-\tau_{el}}$, where

$$\tau_{el} = \sigma_T c \int dt \, n_e (1 + z)^3$$

is the optical depth (measured at the present epoch) of CMB photons due to Thomson scattering with free electrons. In the above expression, $n_e$ is the average value of the comoving electron density and $\sigma_T$ is the Thomson scattering cross section. However, measuring this damping is not easy as it can be compensated by a larger strength of dark matter density fluctuations which are measured by the corresponding power spectrum, usually parametrised by the two quantities: the primordial spectral index $n_s$ and the fluctuation amplitude at cluster scales $\sigma_8$. Hence, it is found that $\tau_{el}$ is only mildly constrained by the temperature fluctuations because of strong degeneracies with $n_s$ and $\sigma_8$ \cite{105}. For this reason, temperature anisotropy data prior to Wilkinson Microwave Anisotropy Probe (WMAP)\(^3\) could only constrain $\tau_{el} \lesssim 0.5$ \cite{106}. For sudden reionization models, this only implies that the redshift of reionization $z_{re} \lesssim 40$. To put in perspective with the discussion of QSO absorption line observations, reionization at $z \sim 6$ would imply $\tau_{el} \sim 0.05$.

A major breakthrough in our understanding of reionization came after the release of first year WMAP results of polarization measurements. A fundamental prediction of the gravitational instability paradigm is that CMB anisotropies are polarized, i.e., if the temperature anisotropies are produced by primordial fluctuations, their presence at the last scattering surface would polarize the CMB. The generation of polarization requires two conditions to be satisfied: (i) photons need to undergo Thomson scattering off free

\[^3\text{http://map.gsfc.nasa.gov/}\]
electrons (the corresponding cross section is polarization-dependent) and (ii) the angular distribution of the photon temperature must have a non-zero quadrupole moment. Tight coupling between photons and electrons prior to recombination makes the photon temperature almost isotropic and the generated quadrupole anisotropy, and hence the polarization, is very small. Because the temperature anisotropies are of the order $10^{-5}$, the polarization is about $10^{-6}$ or less.

To generate a quadrupole, it is necessary to produce velocity gradients in the photon-baryon fluid across the photon mean free path; hence only those perturbations which have length scales smaller than the mean free path can produce polarization. At larger scales, multiple scattering will make the plasma quite homogeneous and thus no significant quadrupole can be generated, while at much lower scales polarization is suppressed due to “Silk damping”. In fact, the polarization generated at the last scattering surface would be significant at scales comparable to the horizon size at that epoch (which corresponds to a multipole number $\ell \sim 100$), and no polarization signal is expected at larger scales $\ell < 100$. Detection of polarization signal at $\ell < 100$ is a clear signature of secondary processes such as reionization.

Following the completion of recombination the quadrupole moment of temperature anisotropies grows due to the photons free streaming. In case these photons are able to scatter off free electrons at a later stage, the anisotropy can be transformed into substantial polarization. This is an ideal effect to probe reionization as it is the only process which can provide considerable number of free electrons at post-recombination epochs. For models with sudden reionization, it can be shown that the effect dominates on the angular scale of the horizon at the epoch of reionization. The polarization signal will peak at a position $\ell \propto z_{\text{re}}^{1/2}$ with an amplitude proportional to the total optical depth $\tau_{\text{el}}$. Thus the polarization spectrum at low $\ell$ is a sensitive probe of the reionization process.

The polarization measurements by the WMAP satellite [107] found a significant signal in the temperature-polarization cross correlation spectrum at $\ell < 10$. The position and the amplitude of this excess is consistent with an optical depth $\tau_{\text{el}} = 0.17 \pm 0.04$, implying a (sudden) reionization redshift $11 < z_{\text{re}} < 30$. While this result has possibly complicated the picture of reionization and thus generated tremendous amount of activity within the community, a few subtleties should be kept in mind while using the reionization constraints: (i) The result is based on a few points at low $\ell$ and it is necessary that such an important result is confirmed by future data. One should also note that the likelihood function for $\tau_{\text{el}}$ obtained from WMAP data is heavily skewed, probably indicating some sort of a “tension” within the data. (ii) The constraints on $\tau_{\text{el}}$ depend on the priors and analysis tech-
nique used. For example, fitting the temperature − E-mode polarization cross power spectrum (TE) to ΛCDM models in which all parameters except $\tau_{el}$ assume their best fit values based on the temperature power spectrum (TT), the 68% confidence range obtained is $0.13 < \tau_{el} < 0.21$ \[107\]. Fitting all parameters simultaneously to the TT and the TE data, the corresponding range changes to obtain $0.095 < \tau_{el} < 0.24$ \[108\]. Including additional data external to WMAP, these authors were able to shrink their confidence interval to $0.11 < \tau_{el} < 0.23$. Finally, by assuming that the observed TT power spectrum is scattered to produce the observed TE cross-power spectrum, the inferred range is $0.12 < \tau_{el} < 0.20$ \[107\]. (iii) The constraints of $\tau \approx 0.17$ and $z_{re} \approx 15$ usually quoted in the literature assume a sudden reionization. The constraints can change drastically when this assumption is relaxed.

In case the result is confirmed by future data sets, we note that it is not necessarily contradictory to the QSO results; the history of the luminous sources and their effect on the IGM was probably highly complex, and there was a finite time interval (maybe somewhere around a few hundred million years) from the appearance of the first sources of UV photons and the completion of the reionization.

2.1.3 Lyα emitters at high redshifts

In parallel, a number of groups have studied star-forming galaxies at $z \sim 6−7$, and measurements of the Lyα emission line luminosity function evolution provide another useful observational constraint \[109, 110\]. While the QSO absorption spectra probe the neutral hydrogen fraction regime $x_{HI} \leq 0.01$, this method is sensitive to the range $x_{HI} \sim 0.1 − 1.0$. Lyα emission from galaxies is expected to be suppressed at redshifts beyond reionization because of the absorption due to neutral hydrogen, which clearly affects the evolution of the luminosity function of such Lyα emitters at high redshifts \[111, 109, 112\]. Thus a comparison of the luminosity functions at different redshifts could be used for constraining the reionization. Through a simple analysis, it was found that the luminosity functions at $z = 5.7$ and $z = 6.5$ are statistically consistent with one another, thus implying that reionization was largely complete at $z \approx 6.5$. More sophisticated calculations on the evolution of the luminosity function of Lyα emitters \[109, 113, 111\] suggest that the neutral fraction of hydrogen at $z = 6.5$ should be less than 50 per cent \[114\].

The analysis of the Lyα emitters at high redshifts is complicated by various factors. (i) Firstly, this suppression of the Lyα emission line depends on the size of the ionized region surrounding the source as larger ionized volumes allow more photons to escape. On the other hand, the sizes of the ionized regions themselves depend on the clustering properties of the sources.
There is thus a strong coupling between the clustering of the sources, sizes of the ionized regions and the luminosity function of the Lyα emitters at high redshifts. (ii) The ionized hydrogen regions are typically highly asymmetric because the ionization-fronts propagate much faster across underdense voids than across dense filaments. Thus one needs to know the details of the density distribution around the sources to model the ionized regions. (iii) It is well known that bright galaxies are biased, so it is likely that more than one galaxy is located inside a single ionized region; Lyα-emitters can also be located inside ionized regions of luminous quasars, which are often many times larger than the ionized regions of galaxies. It is thus clear that the modelling of the ionized regions of Lyα-emitters is not straightforward, and hence the reionization constraints could be severely model-dependent.

2.2 Observations related to the sources of reionization

As we discussed in the Introduction, a major challenge in our understanding of reionization depends on our knowledge of the sources, particularly at high redshifts. In this sense, reionization is closely related to formation of early baryonic structures and thus any observation related to the detection of very distant sources can be important for constraining reionization. In the following, we shall discuss a few most important of such observational probes.

2.2.1 Direct observations of sources at high redshifts

As we understand at present, neither the bright $z > 6$ QSOs discovered by the SDSS group [115] nor the faint AGN detected in X-ray observations [116] produce enough photons to reionize the IGM. The discovery of star-forming galaxies at $z > 6.5$ [117, 118, 119] has resulted in speculation that early galaxies produce bulk of the ionizing photons for reionization. However, the spectroscopic studies of I-band dropouts in the Hubble ultra-deep Field with confirmed redshifts at $z \approx 6$, indicate that the measured star formation rate at $z = 6$ is lower by factor of 6 from the $z = 3$ star formation rate. If the estimate is correct, the I-dropouts do not emit enough ionizing photons to reionize the universe at $z \approx 6$ [120]. The short-fall in ionizing photons might be alleviated by a steep faint-end slope of the luminosity function of galaxies or a different stellar initial mass function (IMF); alternatively, the bulk of reionization might have occurred at $z > 6$ through rapid star formation in galaxies at much higher redshifts.

There are estimates of a somewhat higher UV luminosity at $z = 6 - 10$. This is obtained by constructing a luminosity function from $\sim 500$ galaxies.
collected from all the deepest wide-area HST data [121]. The luminosity function thus obtained extends 3 magnitudes fainter than the characteristic luminosity $L^*$. This analysis predicts a significant evolution in $L^*$ — a doubling from $z = 3$ to $z = 6$, thus implying a luminosity density that is only a factor of 1.5 less than the luminosity density at $z = 3$. The observed evolution is suggestive of that expected from popular hierarchical models, and would seem to indicate that we are literally witnessing the buildup of galaxies in the reionization era.

To summarise, there are somewhat conflicting reports regarding the star formation rate at $z \gtrsim 6$ — however, it is safe to conclude that we have not yet observed enough number of sources which could ionize the bulk of the IGM at $z \gtrsim 6$. Whether the reionization was actually completed by galaxies at a much higher redshifts is still an open issue.

### 2.2.2 Cosmic infrared background radiation

Numerous arguments favour an excess contribution to the extragalactic background light between 1 $\mu$m and a few $\mu$m [122, 123, 124, 125] when compared to the expectation based on galaxy counts and Milky Way faint star counts (for a review see [126]). While these measurements are likely to be affected by certain systematics and issues related to the exact contribution from zodiacal light within the Solar System, one explanation is that a contribution to the cosmic infrared background (CIRB) radiation originates from high redshift sources. The redshifted line emission from Ly$\alpha$ emitting galaxies at $z > 9$ would produce an integrated background in the near-infrared wavelengths observed today. In case this interpretation of the CIRB is correct, it would directly constrain the number of ionizing sources at high redshifts and thus would have direct implications on reionization.

However, if the entire CIRB is due to the high redshift galaxies, the explanation requires the presence of metal-free PopIII stars with a top-heavy IMF and possibly a high star-forming efficiency [127, 128, 129, 130, 131]. In fact, the number of sources required to explain the CIRB is much higher than that needed to explain the early reionization constraints. The most serious difficulty in explaining the CIRB through PopIII stars comes from the observations of the number of J-dropouts and Ly$\alpha$ emitters in ultra deep field searches as the models severely overpredict the number of sources [132]. At present, the origin of CIRB remains to be puzzling (as one can discard other possible sources like miniquasars and decaying neutrinos, see the next subsection), and it is not clear whether it could have any significant implications on reionization.
2.2.3 Constraints on other sources

We end this section by briefly reviewing the constraints we have on other kind of sources, namely the Intermediate Mass Black Holes and decaying (exotic) particles.

A large population of intermediate mass black holes (IMBHs) might be produced at early cosmic times as a left over of the evolution of very massive first stars. These black holes at high redshifts \( z > 6 \) can, in principle, contribute to the ionization of the IGM; however they would be accompanied by the copious production of hard X-ray photons (with energies above 10 keV). The resulting hard X-ray background would redshift and be observed as a present-day soft X-ray background. One can show that the observed residual soft X-ray background intensity can put stringent constraints on the the baryon mass fraction locked into IMBHs and their growth \([133, 134]\). Thus, unless they are extremely X-ray quiet, these black holes, or miniquasars, must be quite rare and/or have a short shining phase. As a byproduct, it implies that miniquasars cannot be the only source of reionization.

The other sources which are popularly invoked to explain reionization are the exotic particles like decaying neutrinos \([74, 75, 76, 77, 78, 79]\). However, in most cases these particles decay radiatively (producing photons) and hence are severely constrained by Big Bang Nucleosynthesis, diffuse soft X-ray and gamma-ray backgrounds and the deviation of the CMB spectrum from Planckian shape. For example, the constraints from soft X-ray background limits the radiatively decaying sterile neutrino mass to \( m_\nu < 14 \) keV and hence the optical depth to Thomson scattering is \( \tau_{el} \sim 10^{-2} \), negligible compared to what is required for explaining observations \([81]\). Similar constraints exist for other particles, including those which have decay channels into electrons instead of photons \([80]\). The point what comes out from most these analyses is that different observational constraints leave out a very small parameter space accessible to the decaying particles and hence their contribution to reionization may not be that significant.

3 Physics of reionization

Given the observational constraints discussed in the previous Section, it is important to develop models which can be reconciled with every data set. However, such a task is not straightforward simply because of the complexities in the physical processes involved. It is not that there is some unknown physics involved – we believe that we can write down every relevant equation – the difficulty lies in solving them in full generality. This is quite a com-
mon obstacle in most aspects of large scale structure formation, but let us concentrate on reionization for the moment.

A crucial issue about reionization is that this process is tightly coupled to the properties and evolution of star-forming galaxies and QSOs. Hence, the first requirement that any reionization model should fulfill is that it should be able to reproduce the available constraints concerning the luminous sources. Though there has been a tremendous progress in our understanding of formation of galaxies and stars at low redshifts, very little is known about the high redshift galaxies, particularly those belonging to the first generation. There are strong indications, both from numerical simulations and analytical arguments, that the first generation stars were metal-free, and hence massive, with a very different kind of spectrum than the stars we observe today \[\text{[135]}\]; they are known as the PopIII stars. Along similar lines, whereas a relatively solid consensus has been reached on the luminosity function, spectra and evolutionary properties of intermediate redshift QSOs, some debate remains on the presence of yet undetected low-luminosity QSOs powered by intermediate mass black holes at high redshift \[\text{[136]}\]. Even after modelling the ionizing sources to a reasonable degree of accuracy, predicting the joint reionization and star formation histories self-consistently is not an easy task as (i) we still do not have a clear idea on how the photons “escape” from the host galaxy to the surrounding IGM, and (ii) mechanical and radiative feedback processes can alter the hierarchical structure formation sequence of the underlying dark matter distribution as far as baryonic matter is concerned.

A different approach to studying reionization would be to parametrize the sources by various adjustable parameters (like efficiency of star formation, fraction of photons which can escape to the IGM, etc) and then calculate the evolution of global ionization and thermal properties of the IGM. Though it is possible to obtain a good qualitative picture of reionization through this approach, it is difficult to obtain the details, particularly those of the pre-overlap phase, through such simple analyses. For example, many details of the reionization process can be dealt only in an approximate manner (the shape of ionized region around sources and their overlapping, just to mention a few) and in terms of global averages (such as the filling factor and the clumping factor of ionized regions). We shall discuss this in more detail later in this Section.

It is thus clear that the complexities within the physics of reionization prohibit us from constructing detailed analytical models. The other option is to solve the relevant equations numerically and follow the ionization history. However, one should realize that in order to exploit the full power of the observational data available to constrain models, one must be able to connect widely differing spatial and temporal scales. In fact, it is necessary, at
the same time, to resolve the IGM inhomogeneities (sub-kpc physical scales),
follow the formation of the very first objects in which stars form (kpc), the
radiative transfer of their ionizing radiation and the transport of their metals
(tens of kpc), the build up of various radiation backgrounds (Mpc), and the
effects of QSOs and sizes of the ionized regions (tens of Mpc). Thus, a proper
modelling of the relevant physics on these scales, which would enable a di-
rect and simultaneous comparison with all the available data set mentioned
above, would require numerical simulations with a dynamical range of more
than five orders of magnitude, which is far cry from the reach of our cur-
cent computational technology. To overcome the problem, simulations have
typically concentrated on trying to explain one (or few, in the best cases)
of the observational constraints. It is therefore difficult from these studies
to understand the extent to which their conclusions do not conflict with a
different set of experiments other than the one they are considering.

However, one must realise that in spite of these difficulties in modelling
reionization there have been great progresses in recent years, both analyti-
cally and through numerical simulations, in different aspects of the process.
This Section will be devoted to the successes we have achieved towards un-
derstanding reionization.

3.1 Analytical approaches

As far as the analytical studies are concerned, there are two broad approaches
in modelling, namely, (i) the evolution of ionized regions of individual ioniz-
ing sources and (ii) statistical approaches in computing the globally averaged
properties and fluctuations.

3.1.1 Evolution of ionized regions of individual ionizing sources

The standard picture of reionization by discrete sources of radiation is char-
acterized by the expansion and overlap of the individual ionized regions. In
this paradigm, it is important to understand how the ionized regions evolve
for different types of ionizing sources.

The most common sources studied are the ones with UV photons, i.e.,
photons with energies larger than 13.6 eV but within few tens of eV. These
photons ionize and heat up the IGM through photoionizing neutral hydrogen
(and possibly helium). Because of (i) a large value of the photoionization
cross section around 13.6 eV, (ii) rapid increase of the number of absorbers
(Lyα “clouds”) with lookback time and (iii) severe attenuation of sources at
higher redshifts, the mean free path of photons at 13.6 eV becomes so small
beyond a redshift of 2 that the radiation is largely “local”. For example, the
mean free path at 13.6 eV is typically \( \approx 30 \) proper Mpc at \( z \approx 3 \), which is much smaller than the horizon size. In this approximation, the background intensity depends only on the instantaneous value of the emissivity (and not its history) because all the photons are absorbed shortly after being emitted (unless the sources evolve synchronously over a timescale much shorter than the Hubble time).

When an isolated source of ionizing radiation, say a star or a QSO, turns on, the ionized volume initially grows in size at a rate determined by the emission of UV photons. The boundary of this volume is characterised by an ionization front which separates the ionized and neutral regions and propagates into the neutral gas. Most photons travel freely in the ionized bubble and are absorbed in a transition layer. Across the front the ionization fraction changes sharply on a distance of the order of the mean free path of an ionizing photon (which is much smaller than the horizon scale). The evolution of an expanding ionized region is governed by the equation

\[
\frac{dV_I}{dt} = \frac{\dot{N}_{\text{ph}}}{n_{\text{HI}}} - \frac{V_I}{t_{\text{rec}}}
\]

where \( V_I \) is the comoving volume of the ionized region, \( \dot{N}_{\text{ph}} \) is the number of ionizing photons emitted by the central source per unit time, \( n_{\text{HI}} \) is the mean comoving density of neutral hydrogen and \( t_{\text{rec}} \) is the recombination timescale of neutral hydrogen given by

\[
t_{\text{rec}}^{-1} = C \alpha_R n_{\text{HI}} (1 + z)^{3/2}
\]

In the above relation \( \alpha_R \) denotes the recombination rate ionized hydrogen and free electrons and \( C \) is the clumping factor which takes into account the enhancement in the number of recombinations due to density inhomogeneities. It is clear from equation (5) that the growth of the ionized bubble is slowed down by recombinations in the highly inhomogeneous IGM. In the (over)simplified case of both \( \dot{N}_{\text{ph}} \) and \( t_{\text{rec}} \) not evolving with time, the evolution equation can be solved exactly to obtain

\[
V_I = \frac{\dot{N}_{\text{ph}}}{n_{\text{HI}}} t_{\text{rec}} \left(1 - e^{-t/t_{\text{rec}}} \right)
\]

While the volume of the ionized region depends on the luminosity of the central source, the time it takes to produce an ionization-bounded region is only a function of \( t_{\text{rec}} \). It is clear that the ionized volume will approach its Strömgren radius after a few recombination timescales. Thus, when the recombination timescale is similar to or larger than the Hubble time (which is
usually the case at lower redshifts, but depends crucially on the value of the clumping factor), the ionized region will never reach the Strömgren radius. The solution of the equation is more complicated in reality where the sources evolve, but the qualitative feature remains the same. After the bubbles have grown sufficiently, these individual bubbles start overlapping with each other eventually complete the reionization process.

As far as the hydrogen reionization is concerned, the evolution of the ionization front for stars and QSOs are qualitatively similar. The only difference is that the QSOs are much more luminous than stellar sources and hence the ionization front propagates much faster for QSOs. In the case of doubly-ionized helium, however, the propagation of ionization fronts for the two types of sources differ drastically. Since normal stars hardly produce any photons above 54.4 eV, the propagation of the doubly-ionized helium ionization front is negligible; in contrast the QSOs have a stiff spectrum and one can show that the doubly-ionized helium front not only propagates quite fast into the medium, but closely follows the ionized hydrogen one. The analyses actually predict that the ionization of hydrogen and double-ionization of helium would be nearly simultaneous if QSOs were the dominant source of reionization, which cannot be the case for normal stellar population. Also note that the early metal-free massive PopIII stars too have a hard spectrum, and in their case the reionization of hydrogen and singly-ionized helium is simultaneous, similar to QSOs. This could provide an indirect way of identifying the sources of reionization from future observations.

If the sources of radiation produce photons of much higher energies, say in the X-ray band, the nature of the ionization front changes considerably. The most common example of such sources is the early population of accreting black holes, or miniquasars [138, 139]. These sources are expected to produce photons in the X-ray bands. The photons which are most relevant for this discussion would be those with energies below 2 keV, as photons with higher energies have mean free path similar to the horizon scale and thus would rarely be absorbed.

Since the absorption cross section of neutral hydrogen varies with frequency approximately as $\nu^{-3}$, the mean free path for photons with high energies would be very large. A simple calculation will show that for photons with energies above 100–200 eV, the mean free path would be larger than the typical separation between collapsed structures [140] (the details would depend upon the redshift and exact description of collapsed haloes).

4The propagation of singly-ionized helium front coincides with the hydrogen ionization front for almost all forms of the ionizing spectrum. Hence, as far as helium is concerned, most studies are concerned with the propagation of the doubly-ionized helium fronts.
These photons would not be associated with any particular source at the moment when they are absorbed, and thus would ionize the IGM in a more homogeneous manner (as opposed to the overlapping bubble picture for UV sources). However, one should also note that the number of photons produced in the X-ray bands is usually not adequate for fully ionizing the IGM. Hence these hard photons can only partially ionize the IGM through repeated secondary ionizations. Basically, the photoelectron produced by the primary ionization would be very energetic (∼1 keV) and could thus produce quite a few (∼10) secondary electrons via collisional ionization. This may not be a very effective way of producing free electrons (when compared to the UV background), but is an efficient way of heating up the medium. This very population of hard photons deposits a fraction of its energy and can heat up the IGM to ∼100 − 1000 K before reionization – a process sometimes known as preheating.

In case of reionization through more exotic sources like decaying particles, the nature of ionized volumes would be completely different. The production of ionizing photons (or electrons) in this case is not related to any collapsed structure – rather it occurs throughout the space in an homogeneous manner. In such case, one expects only limited patchiness in the distribution of the background radiation or ionized fraction.

We have discussed various different ways in which the ionized regions could evolve depending on the nature of the ionizing sources. The next step would be to take into account the global distribution of the sources and ionized volumes and thus construct the global picture of reionization, which we shall study in the next subsection.

### 3.1.2 Statistical approaches

The most straightforward statistical quantity which is studied in reionization is the volume filling factor of ionized regions $Q_I$. For the most common UV sources, this is obtained by averaging equation (5) over a large volume:

$$\frac{dQ_I}{dt} = \frac{\dot{n}_{\text{ph}}}{n_{\text{HI}}} - \frac{Q_I}{t_{\text{rec}}}$$

where $\dot{n}_{\text{ph}}$ is average number of ionizing photons produced per unit volume per unit time. Note that the above equation implicitly assumes that the sources of radiation are distributed uniformly over the volume we are considering. The equation can be solved once we know the value and evolution of the photon production rate $\dot{n}_{\text{ph}}$ and also the clumping factor $C$. One can get an estimate of $\dot{n}_{\text{ph}}$ by assuming that it is proportional to the fraction of gas within collapsed haloes, while the value of $C$ is relatively difficult to calculate.
and is usually assumed to be somewhere between a few and 100. In this simple picture, reionization is said to be complete when $Q_I$ reaches unity. Assuming a normal stellar population (i.e., a Salpeter-like IMF and standard stellar spectra obtained from population synthesis codes) and a value of $C \approx 10 - 30$, one obtains a reionization in the redshift range between 6 and 10 [142].

This approach can be improved if the density inhomogeneities of the IGM are taken into account. In the above picture, the inhomogeneities in the IGM are considered simply in terms of the clumping factor in the effective recombination timescale without taking into account the density distribution of the IGM. The importance of using a density distribution of the IGM lies in the fact that regions of lower densities will be ionized first, and high-density regions will remain neutral for a longer time. The main reason for this is that the recombination rate is higher in high-density regions where the gas becomes neutral very quickly\(^5\). Thus, in the situation where all the individual ionized regions have overlapped (the so-called post-overlap stage), all the low-density regions (with overdensities, say, $\Delta < \Delta_I$) will be highly ionized, while there will be some high density peaks (like the collapsed systems) which will still remain neutral. The situation is slightly more complicated when the ionized regions are in the pre-overlap stage. At this stage, it is assumed that a volume fraction $1 - Q_I$ of the universe is completely neutral (irrespective of the density), while the remaining $Q_I$ fraction of the volume is occupied by ionized regions. However, within this ionized volume, the high density regions (with $\Delta > \Delta_I$) will still be neutral. Once $Q_I$ becomes unity, all regions with $\Delta < \Delta_I$ are ionized and the rest are neutral. The high-density neutral regions manifest themselves as the Lyman-limit systems (i.e., systems with neutral hydrogen column densities $N_{\text{HI}} > 10^{17}$ cm\(^{-2}\)) in the QSO absorption spectra.

To develop the equations embedding the above physical picture, we need to know the probability distribution function $P(\Delta)d\Delta$ for the overdensities. Given a $P(\Delta)d\Delta$, it is clear that only a mass fraction

$$F_M(\Delta_I) = \int_0^{\Delta_I} \Delta P(\Delta)$$

needs be ionized, while the remaining high density regions will be completely neutral because of high recombination rates. The generalization of equation

\(^5\)Of course, there will be a dependence on how far the high density region is from an ionizing source; however such complexities can only be dealt in a full numerical simulation.
appropriate for this description is given by \[ \frac{d[Q_I F_M(\Delta_I)]}{dt} = \frac{\dot{n}_{\text{ph}}(z)}{n_{\text{HI}}} - Q_I \alpha_R n_{\text{HI}} R(\Delta_I)(1 + z)^3 \] (10)

The factor $R(\Delta_I)$ is the analogous of the clumping factor, and is given by

$$R(\Delta_I) = \int_0^{\Delta_I} d\Delta \Delta^2 P(\Delta)$$ (11)

The reionization is complete when $Q_I = 1$; at this point a mass fraction $F_M(\Delta_I)$ is ionized, while the rest is (almost) completely neutral.

Note that this approach not only takes into account all the three stages of reionization, but also computes the clumping factor in a self-consistent manner. This approach has been combined with the evolution of thermal and ionization properties of the IGM \[144\] to predict various properties related to reionization and then compare with observations. By constraining the model free parameters with available data on redshift evolution of Lyman-limit absorption systems, GPs and electron scattering optical depths, and cosmic star formation history, a unique reionization model can be identified, whose main predictions are: Hydrogen was completely reionized at $z \approx 15$, while helium must have been doubly ionized by $z \approx 12$ by the metal-free PopIII stars. Interestingly only 0.3 per cent of the stars produced by $z = 2$ need to be PopIII stars in order to achieve the first hydrogen reionization. At $z \approx 7 - 10$, the doubly ionized helium suffered an almost complete recombination as a result of the extinction of PopIII stars. A QSO-induced complete helium reionization occurs at $z \approx 3.5$; a similar double hydrogen reionization does not take place due to the large number of photons with energies $> 13.6$ eV from normal PopII stars and QSOs, even after all PopIII stars have disappeared. Following reionization, the temperature of the IGM corresponding to the mean gas density, $T_0$, is boosted to $1.5 \times 10^4$ K; following that it decreases with a relatively flat trend. Observations of $T_0$ are consistent with the fact that helium is singly ionized at $z \gtrsim 3.5$, while they are consistent with helium being doubly ionized at $z \lesssim 3.5$. This might be interpreted as a signature of (second) helium reionization. However, it is useful to remember that there could be other contributions to the ionizing background, like for example, because of thermal emission from gas shock heated during cosmic structure formation \[145\]. Such emission is characterized by a hard spectrum extending well beyond 54.4 eV and is comparable to the QSO intensity at $z \gtrsim 3$. These thermal photons alone could be enough to produce and sustain double reionization of helium already at $z = 6$. The observations of the state of helium at these intermediate redshifts ($3 < z < 6$) could be crucial to assess the nature of ionizing background arising from different sources.
None of the above models take into account the clustering of the sources (galaxies) in computing the growth of the volume filling factor of ionized regions. It is well known that the galaxies would form in high-density regions which are highly correlated. The overlap of the ionized bubbles and hence the morphology of the ionized regions would then be determined by the galaxy clustering pattern; for example, the sizes of the ionized bubbles could be substantially underestimated if the correlation between the sources are not taken into account. The first approach to treat this has been to use an approach based on the excursion set formalism (similar to the Press-Schechter approach in spirit) to calculate the growth and size distribution of ionized regions [146]. It can be shown that the sizes of the ionized regions can be larger than $\sim 10$ comoving Mpcs when the ionization fraction of the IGM is 0.5–1.0 [93]. This highlights the fact that it is quite difficult to study overlap of ionized regions with limited box size in numerical simulations (which are often less than 10 comoving Mpc). The other important result which can be drawn from such analyses is that the reionization can be a very inhomogeneous process; the overlap of bubbles would be completed in different portions of the IGM at different epochs depending on the density inhomogeneities in that region.

3.2 Simulations

Though the analytical studies mentioned above allow us to develop a good understanding of the different processes involved in reionization, they can take into account the physical processes only in some approximate sense. In fact, a detailed and complete description of reionization would require locating the ionizing sources, resolving the inhomogeneities in the IGM, following the scattering processes through detailed radiative transfer, and so on. Numerical simulations, in spite of their limitations, have been of immense importance in these areas.

The greatest difficulty any simulation faces while computing the growth of ionized regions is to solve the radiative transfer equation [147]

$$\frac{\partial I_\nu}{\partial t} + \frac{n \cdot \nabla I_\nu}{\bar{a}} - H(t) \left( \nu \frac{\partial I_\nu}{\partial \nu} - 3I_\nu \right) = \eta_\nu - \chi_\nu I_\nu$$

(12)

where $I_\nu \equiv I(t, x, n, \nu)$ is the monochromatic specific intensity of the radiation field, $n$ is a unit vector along the direction of propagation of the ray, $H(t)$ is the Hubble parameter, and $\bar{a}$ is the ratio of cosmic scale factors between photon emission at frequency $\nu$ and the time $t$. Here $\eta_\nu$ and $\chi_\nu$ denote the emission coefficient and the absorption coefficient, respectively. The denominator in the second term accounts for the changes in path length along the
ray due to cosmic expansion, and the third term accounts for cosmological redshift and dilution.

In principle, one could solve equation (12) directly for the intensity at every point in the seven-dimensional \((t, x, n, \nu)\) space, given the coefficients \(\eta\) and \(\chi\). However, the high dimensionality of the problem makes the solution of the complete radiative transfer equation well beyond our capabilities, particularly since we do not have any obvious symmetries in the problem and often need high spatial and angular resolution in cosmological simulations. Hence, the approach to the problem has been to use different numerical schemes and approximations, like ray-tracing [147, 148, 149, 150, 151, 152, 153, 154, 155], Monte Carlo methods [156, 157], local depth approximation [68] and others.

Since the radiative transfer is computationally extremely demanding, most efforts have been concentrating on small regions of space \((\sim 10-50\) comoving Mpc). The main reason for this limitation is that the ionizing photons during early stages of reionization mostly originate from smaller haloes which are far more numerous than the larger galaxies at high redshifts. The need to resolve such small structures imposes a severe limit on the computational box size. On the other hand, these ionizing sources were strongly clustered at high redshifts and, as a consequence, the ionized regions they created are expected to overlap and grow to very large sizes, reaching up to tens of Mpc [158, 93, 159]. As already discussed, the many orders of magnitude difference between these length scales demand extremely high computing power from any simulations designed to study early structure formation from the point of view of reionization. Further limitations are imposed by the method used in the radiative transfer schemes; for most of them the computational expense grows roughly proportionally to the number of ionizing sources present. This generally makes the radiative transfer solution quite inefficient when more than a few thousand ionizing sources are involved, severely limiting the computational volume that can be simulated effectively. However these methods can be useful and quite accurate in certain special circumstances like, say, to study the growth of ionized regions around an isolated source.

A closely related problem which can be dealt with in numerical simulations is that of the clumpiness of the IGM. We have already discussed on how the density inhomogeneities can play an important role in characterising reionization. Various hydrodynamical simulations have been carried out using sophisticated tools to generate the density distribution of gas over large ranges of spatial and density scales. This, when combined with radiative transfer schemes, can give us an idea about the propagation of ionization fronts into an inhomogeneous medium, which is otherwise a very difficult problem. Using radiative transfer simulations over large length scales \((\sim 150\) comoving Mpc) based on explicit photon conservation in space and time [155],
quite a few conclusions about the nature of reionization (by UV sources) can be drawn. For example, it is likely that reionization proceeded in an inside-out fashion, with the high-density regions being ionized earlier, on average, than the voids. This has to do with the fact that most ionizing sources reside inside a high density halo. Interestingly, ionization histories of smaller-size (5 to 10 comoving Mpc) subregions exhibit a large scatter about the mean and do not describe the global reionization history well, thus showing the importance of large box sizes. The minimum reliable volume size for such predictions is found to be \( \sim 30 \) Mpc. There seems to be two populations of ionized regions according to their size: numerous, mid-sized (\( \sim 10 \) Mpc) regions and a few, rare, very large regions tens of Mpc in size. The statistical distributions of the ionized fraction and ionized gas density at various scales show that both distributions are clearly non-Gaussian, indicating the non-linearities in the problem.

There is thus quite good progress in studying the growth of the ionized regions, particularly those due to UV sources, and also the qualitative features for the global reionization seem to be understood well. The number of unanswered questions still do remain large, particularly those related to the feedback effects. The expectation is that as more observations come up, the physics of the models would be nicely constrained and a consistent picture of reionization is obtained. We shall review the future prospects for this field in the next Section.

## 4 The future

In the final Section, we review certain observations which will shape our understanding of reionization in near future, and also discuss the theoretical predictions concerning future data sets.

The spectroscopic studies of QSOs at \( z \gtrsim 6 \) hold promising prospects for determining the neutrality of the IGM. As we have discussed in Section 2, regions with high transmission in the Ly\( \alpha \) forest become rare at high redshifts. Therefore the standard methods of analyzing the Ly\( \alpha \) forest (like the probability distribution function and power spectrum) are not very effective. An alternative method to analyze the statistical properties of the transmitted flux is the distribution of dark gaps \([90, 91]\), defined as contiguous regions of the spectrum having an optical depth above a threshold value (say 2.5 \([91]\) or 3.5 \([90]\)). The frequency and the width of the gaps are expected to increase with redshift, which is verified in different analyses of observational data \([91, 90]\). However, it is more interesting to check whether the dark gap width distribution (DGWD) is at all sensitive to the reionization history of...
the IGM, and whether one can constrain reionization through DGWD. This is indeed possible as it is found by semi-analytical models and simulations of Ly\(\alpha\) forest at \(z \gtrsim 6\) [161]. In particular, about 30 per cent of the lines of sight (accounting for statistical and systematic uncertainties) in the range \(z = 5.7 - 6.3\) are expected to have dark gaps of widths larger than 60 Å (in the QSO rest frame) if the IGM is in the pre-overlap stage at \(z \gtrsim 6\), while no lines of sight should have such large gaps if the IGM is already ionized. The constraints become more stringent at higher redshifts. Furthermore, 10 lines of sight should be sufficient for the DGWD to give statistically robust results and discriminate between early and late reionization scenarios. It is expected that the SDSS and Palomar-Quest survey [162] would detect \(~30\) QSOs at these redshifts within the next few years and hence we expect robust conclusions from DGWD in very near future.

As we have discussed already, the first evidence for an early reionization epoch came from the CMB polarization data. This data is going to be much more precise in future with experiments like PLANCK,\(^6\) and is expected to improve the constraints on \(\tau_{el}\). With improved statistical errors, it might be possible to distinguish between different evolutions of the ionized fraction, particularly with E-mode polarization auto-correlation, as is found from theoretical calculations [163]. An alternative option to probe reionization through CMB is through the small scale observations of temperature anisotropies. It has been well known that the scattering of the CMB photons by the bulk motion of the electrons in clusters gives rise to a signal at large \(\ell\), known as the kinetic Sunyaev-Zeldovich (SZ) effect:

\[
\Delta T(\hat{n}) = \sigma_T \int d\eta \, e^{-\tau_{el}(\eta)} \, a \, n_e(\eta, \hat{n}) \, [\hat{n} \cdot \mathbf{v}(\eta, \hat{n})]
\]

where \(\eta = \int \frac{dt}{a}\) is the conformal time, \(\mathbf{v}\) is the peculiar velocity field and \(n_e\) is the number density of electrons. In principle, a signal should arise from the fluctuations in the distribution of free electrons arising from cosmic reionization. Now, if the reionization is uniform, the only fluctuations in \(n_e\) can arise from the baryonic density fluctuations \(\Delta\), and hence the power spectrum of temperature anisotropies \(C_\ell\) would be mostly determined by correlation terms like \(\langle \Delta \, \mathbf{v} \, \Delta \mathbf{v} \rangle\). Though this can give considerable signal (an effect known as the Ostriker-Vishniac effect), particularly for the non-linear densities, it turns out that for reionization the signal is dominated by the patchiness in the \(n_e\) distribution. In other words, if \(\Delta_{x_e}\) denotes the fluctuations in the ionization fraction of the IGM, the correlation term \(\langle \Delta_{x_e} \, \mathbf{v} \, \Delta_{x_e} \mathbf{v} \rangle\) (i.e., correlations of the ionization fraction fluctuations and the large-scale bulk flow)
gives the dominant contribution to the temperature anisotropies $C_\ell$. Now, in most scenarios of reionization, it is expected that the distribution of neutral hydrogen would be quite patchy in the pre-overlap era, with the ionized hydrogen mostly contained within isolated bubbles. The amplitude of this signal is significant around $\ell \sim 1000$ and is usually comparable to or greater than the signal arising from standard kinetic SZ effect (which, as mentioned earlier, is related to the scattering of the CMB photons by the bulk motion of the electrons in clusters). Theoretical estimates of the signal have been performed for various reionization scenarios, and it has been predicted that the experiment can be used for constraining reionization history \[164, 165\]. Also, it is possible to have an idea about the nature of reionization sources, as the signal from UV sources, X-ray sources and decaying particles are quite different. With multi-frequency experiments like Atacama Cosmology Telescope (ACT)\(^7\) and South Pole Telescope (SPT)\(^8\) coming up in near future, this promises to put strong constraints on the reionization scenarios.

Another interesting prospect for constraining reionization is through high redshift energetic sources like gamma ray bursts (GRBs) and supernovae. There are different ways of using these sources for studying reionization. The first is to study the spectra of individual sources and estimate the neutral fraction of hydrogen through its damping wing effects. This is similar to what is done in the case of Ly$\alpha$ emitters as discussed in Section 2. The damping wing of the surrounding neutral medium, if strong enough, would suppress the spectrum at wavelengths redward of the Lyman break. In fact, analyses have been already performed on the GRB with highest detected redshift ($z_{\text{em}} = 6.3$), and the wing shape is well-fit by a neutral fraction $x_{\text{HI}} = 0.00 \pm 0.17$ \[166\]. In order to obtain more stringent limits on reionization, it is important to increase the sample size of $z > 6$ GRBs. Given a reionization model, one can actually calculate the number of GRB afterglows in the pre-reionization era which would be highly absorbed by the neutral hydrogen. These GRBs would then be categorised as “dark” GRBs (i.e., GRBs without afterglows), and the redshift distribution of such objects can give us a good handle on the evolution of the neutral hydrogen in the universe \[167, 168\].

The second way in which GRBs could be used is to constrain the star formation history, and hence get indirect constraints on reionization. In most popular models of GRBs, it is assumed that they are related to collapse of massive stars (just like supernovae), and hence could be nice tracers of star formation. In fact, one can write the number of GRBs (or supernovae) per

\(^7\)http://www.hep.upenn.edu/act/
\(^8\)http://spt.uchicago.edu/
unit redshift range observed over a time $\Delta t_{\text{obs}}$ as \[167, 169\]

\[
\frac{dN}{dz} = \frac{d\Omega}{4\pi} \Delta t_{\text{obs}} \Phi(z) \left[ f \dot{\rho}_{\text{SF}}(z) \right] \frac{1}{1 + z} \frac{dV(z)}{dz}
\]  

(14)

where the factor $(1 + z)$ is due to the time dilation between $z$ and the present epoch, $dV(z)$ is the comoving volume element, $d\Omega/4\pi$ is the mean beaming factor and $\Phi(z)$ is the weight factor due to the limited sensitivity of the detector, because of which, only brightest bursts will be observed at higher redshifts. The quantity $f$ is an efficiency factor which links the formation of stars $\dot{\rho}_{\text{SF}}(z)$ to that of GRBs (or supernovae); it corresponds to the number of GRBs (supernovae) per unit mass of forming stars, hence it depends on the fraction of mass contained in (high mass) stars which are potential progenitors of the GRBs (supernovae). Clearly, the value of $f$ might depend on some details of GRB formation and is expected to be quite sensitive to the stellar IMF. Though such details still need to be worked out, it seems promising that data on the redshift distribution of GRBs and supernovae could give a handle on the star formation rate, which in turn could give us insights on quantities like efficiency of molecular cooling or the relative contribution of minihaloes to radiation. In general, the GRB rates at high redshifts should be able to tell us how efficient stars were in ionizing the IGM.

Perhaps the most promising prospect of detecting the fluctuations in the neutral hydrogen density during the (pre-)reionization era is through the future 21 cm emission experiments [170] like LOFAR\(^9\). The basic principle which is central to these experiments is the neutral hydrogen hyperfine transition line at a rest wavelength of 21 cm. This line, when redshifted, is observable in radio frequencies ($\sim$ 150 MHz for $z \sim 10$) as a brightness temperature:

\[
\delta T_b(z, \hat{n}) = T_S - T_{\text{CMB}} \frac{3c^3 \tilde{A}_{10} n_{\text{HI}}(z, \hat{n})(1 + z)^3}{16k_{\text{bol}} \nu_0^2 T_S H(z)}
\]  

(15)

where $T_S$ is the spin temperature of the gas, $T_{\text{CMB}} = 2.76(1 + z)$ K is the CMB temperature, $A_{10}$ is the Einstein coefficient and $\nu_0 = 1420$ MHz is the rest frequency of the hyperfine line. The expression can be simplified to

\[
\delta T_b(z, \hat{n}) = F_0 \frac{T_S - T_{\text{CMB}}}{T_S} \frac{n_{\text{HI}}(z, \hat{n})}{\bar{n}_{\text{HI}}(z)}
\]  

(16)

with

\[
F_0 = \frac{3c^3 \tilde{A}_{10} \bar{n}_{\text{HI}}(z)(1 + z)^2}{16k_{\text{bol}} \nu_0^2 H(z)} \approx 25\text{mK} \sqrt{0.15 \left( \frac{\Omega_B h^2}{0.022} \left( \frac{1 - Y}{0.76} \right) \right) \sqrt{\frac{1 + z}{10}}}
\]  

(17)

\(^9\)http://www.lofar.org
The observability of this brightness temperature against the CMB background will depend on the relative values of $T_S$ and $T_{CMB}$. Depending on which processes dominate at different epochs, $T_S$ will couple either to radiation ($T_{CMB}$) or to matter (determined by the kinetic temperature $T_k$). There are four broad eras characterising the spin temperature [171, 172]: (i) At $z \gtrsim 30$, the density of matter is high enough to make collisional coupling dominant, hence $T_S$ is coupled to $T_k$. However, at $z \gtrsim 100$, the gas temperature is coupled strongly to $T_{CMB}$, thus making $T_S \simeq T_k \simeq T_{CMB}$. At these epochs, the 21 cm radiation is not observable. (ii) At $30 \lesssim z \lesssim 100$, the kinetic temperature falls off adiabatically and hence is lower than $T_{CMB}$, while $T_S$ is still collisionally coupled to $T_k$. This would imply that the 21 cm radiation will be observed in absorption against CMB. (iii) Subsequently the radiative coupling would take over and make $T_S = T_{CMB}$, thus making the brightness temperature vanish. This continues till the sources turn up and a Lyα background is established. (iv) Once there is background of Lyα photons, that will couple $T_S$ again to $T_k$ through the Wouthuysen-Field mechanism. From this point on, the 21 cm radiation will be observed either in emission or in absorption depending on whether $T_k$ is higher or lower than $T_{CMB}$, which turns out be highly model-dependent.

Almost in all models of reionization, the most interesting phase for observing the 21 cm radiation is $6 \lesssim z \lesssim 20$. This is the phase where the IGM is suitably heated to temperatures much higher than CMB (mostly due to X-ray heating [141]) thus making it observable in emission. Furthermore, this is the era when the bubble-overlapping phase is most active, and there is substantial neutral hydrogen to generate a strong enough signal. At low redshifts, after the IGM is reionized, $n_{HI}$ falls by orders of magnitude and the 21 cm signal vanishes.

Most theoretical studies are concerned with studying the angular power spectrum of the brightness temperature fluctuations, which is essentially determined by the correlation terms $\langle \delta T_b \delta T_b \rangle$. It is clear from equation (16) that the temperature power spectrum is directly related to the power spectrum of neutral hydrogen, i.e., $\langle n_{HI} n_{HI} \rangle$. This then turns out to be a direct probe of the neutral hydrogen distribution, and potentially can track the evolution of the patchiness in the distribution over redshift. In fact, one expects a peak in the signal on angular scales corresponding to the characteristic size of the ionized bubbles. While there are some significant systematics which have to be controlled (say, for example, the foregrounds), the experiments do promise a revolution in our understanding of reionization.

There are interesting ways in which one can combine signals from different experiments too. For example, an obvious step would be to calculate the correlation between CMB signal from kinetic SZ effect and the 21 cm brightness...
temperatures \( \langle \Delta T \delta T_b \rangle \). This will essentially be determined by the correlations \( \langle n_e n_{\text{HI}} \rangle \). It is expected, that the ionized number density \( n_e \) will be highly anti-correlated with the neutral number density \( n_{\text{HI}} \). In fact, the simulations do show a clear signal for this anti-correlation. Depending on the angular scales of anti-correlation, one can actually re-construct the sizes of the bubbles as a function of redshift and thus compute the reionization history [174].

We hope to have convinced the reader that we are about to enter the most exciting phase in the study of reionization as new observations with LOFAR, ALMA and NGWST will soon settle the long-standing question on when and how the Universe was reionized. From the theoretical point of view, it is thereby important to develop detailed analytical and numerical models to extract the maximum information about the physical processes relevant for reionization out of the expected large and complex data sets.

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