Theory of Cross-correlated Electron–Magnon Transport Phenomena: Case of Magnetic Topological Insulator

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We study transport phenomena cross-correlated among the heat and electric currents of magnons and Dirac electrons on the surface of ferromagnetic topological insulators. For a perpendicular magnetization, we calculate magnon-(electron-) drag anomalous Nernst/Seebeck (anomalous Ettingshausen/Peltier) effects and magnon-electron-drag thermal Hall effects. The magnon-drag thermoelectric effects are interpreted to be caused by magnon-induced electromotive force. When the magnetization has in-plane components, there arise thermal/thermoelectric analogs of anisotropic magnetoresistance (AMR). In the insulating state, the thermal AMR is realized as a magnonic analog of AMR.

Three-dimensional topological insulators (TIs)\(^1\text{–}^4\) have attracted much attention from the viewpoint of spin-related transport. They host characteristic surface states described as two-dimensional (2D) Dirac electrons, whose spin direction is locked perpendicular to the wave vector (and surface normal), a property called spin-momentum locking.\(^5\) Recently, systems in which 2D Dirac electrons coexist with ferromagnets were realized much attention from the viewpoint of spin-related transport. They host characteristic surface states described as 2D Dirac electrons by spin waves (magnons) in the presence of an directional magnetoresistance.\(^6\) A prominent effect of electrons and magnons is expected to arise in the mutual drag of electrons and magnons, a property called spin-momentum locking.\(^7\)

Various novel transport phenomena are expected on the surfaces of MTIs because of the interplay between Dirac electrons and magnetization. The experiments reported to date face challenges of MTIs because of the interplay between Dirac electrons and magnons.\(^8\) In this letter, we report on mutual drag effects of electrons and magnons on the surface of ferromagnetic topological insulators. For a perpendicular magnetization, we calculate magnon- (electron-) drag anomalous Nernst/Seebeck (anomalous Ettingshausen/Peltier) effects and magnon-electron-drag thermal Hall effects. The magnon-drag thermoelectric effects are interpreted to be caused by magnon-induced electromotive force. When the magnetization has in-plane components, there arise thermal/thermoelectric analogs of anisotropic magnetoresistance (AMR). In the insulating state, the thermal AMR is realized as a magnonic analog of AMR.

for 2D Dirac electrons, described by the creation/annihilation operators \(c^\dagger = (c^\dagger, c^\dagger)^T\) and \(c = (c, c)^T\), and localized spins, \(\mathbf{S}\), responsible for the ferromagnetic moment. Here \(V_F\) is the slope of the Dirac cone, \(\sigma\) is the Pauli matrix. \(V_{imp}\) is a random impurity potential. \(J_{sd}\) is the s-d exchange coupling constant, and \(J\) is the exchange stiffness constant. We introduce magnons according to the Holstein–Primakoff transformation.\(^9\) \(S_x + iS_y = r_0(q^2 - r_0^2a^2)^{1/2} - \sqrt{2S}r_0a\), where \(q^2 = S - r_0^2a^2\), and \(\gamma\) is the equilibrium direction (\(S_z\) of \(\mathbf{S}\), and \(r_0\) is the lattice constant. Up to the first order in \(S^{-1}\) (or \(s_0^{-1}\)), the Hamiltonian is rewritten as

\[
H = H_d + H_{d,mag} + H_{mag},
\]

\[
H_d = \sum_k c^\dagger_k[-\hbar v_F (k \times \sigma)^y - \mu - M \cdot \sigma]c_k,
\]

\[
H_{d,mag} = \int d^2x \left\{ M \sqrt{\frac{2}{s_0}} c^\dagger (\Sigma^x a^1 + \Sigma^z c) - \frac{M}{s_0} a^1 \sigma^z a^3 c \right\},
\]

\[
H_{mag} = \sum_q \omega_q \alpha_q^\dagger \alpha_q,
\]

where \(s_0 = S/r_0^2\), \(M = J_{sd}(S_z)\), \(\Sigma^x = \mathbf{R} (\sigma_x \pm i\sigma_y)/2\) with an SO(3) matrix \(\mathbf{R}\) that satisfies \(\mathbf{R} M = M \tilde{\mathbf{z}}\), and \(\omega_q = Jq^2 + \Delta\) is the magnon dispersion with \(\Delta\) being a gap.\(^{11}\) Without loss of generality, we set \(M = (0, M_z, M_f) = (0, M \sin \theta, M \cos \theta)\).

Assuming point-like impurities, \(V_{imp} = u_0 \sum_i \delta(\mathbf{r} - r_i)\), and using the Born approximation for the self-energy (Fig. 1(a)), we obtain the Green function of Dirac electrons as\(^{13}\)

\[
G^R = (g_0 - \mathbf{g} \cdot \sigma)^{-1},
\]

where \(g_0 = \mathbf{g} + \mu + i\gamma\gamma\), \(\mathbf{g} = (g_x, g_y, g_z) = (\hbar v_F k_x, -\hbar v_F k_y, -M_s - (M_z/M_y)(\mu - i\gamma)\), \(\gamma = \gamma_0 |\mathbf{g}| \Theta(|\mathbf{g}| - |M_f|)\) and \(\gamma_0 = n_0 \hbar^2/(2\hbar v_F)^2\), with the Heaviside step function \(\Theta\). For magnons, we consider the self-energy due to the coupling to electrons (Fig. 1(b)), and the Green function is given by

\[
D^R_q (\nu) = (\nu - \tilde{\omega}_q + i\nu)^{-1}.
\]

Here

\[
\tilde{\omega}_q = Jq^2 + \Delta + \delta J [1 + \sin^2 \theta] q_z^2 + q_y^2,
\]

is the renormalized magnon energy;\(^{14}\) the last term accounts for the anisotropy in the magnon dispersion through the

\[
\begin{align}
H = & \int d^2x c^\dagger \left\{ -\frac{1}{2} \hbar v_F \nabla \times \sigma^y + V_{imp}(\mathbf{r}) - J_{sd} S \cdot \sigma \right\} c \\
+ & J \int d^2x (\partial \mathbf{S}) \nabla \times \mathbf{S},
\end{align}
\]
The thermal conductivity of magnons is calculated from the gap, \(|\mu| < |M_z|\), originates even in the insulating state, \(|\mu| < |M_z|\), from the coupling to Dirac electrons and is present only in the “metallic” state, \(|\mu| > |M_z|\), whereas \(\alpha_d\) exists even in other processes. It is generally expected that \(\alpha_d \ll \alpha_c\). We note that the Gilbert form of the damping may not be appropriate for high-energy magnons (e.g., at room temperatures). Therefore, in this letter, we restrict ourselves to sufficiently low temperatures (but above the magnon gap).

To calculate the response of a physical quantity \(\hat{A}\) to a temperature gradient, we follow Luttinger[16,17] and use the formula, \((\hat{A}) = \lim_{\omega \rightarrow 0} (i \omega \omega)^{-1} \sum_k (\hat{K}_i(\omega - i0) - \hat{K}_i(\omega + i0)) \sum_k^\prime \langle \hat{v}_k \rangle \hat{J}^0_i \), where \(\hat{J}^0_i\) is the total energy current of the system. From the continuity equation, the energy current density is obtained as \(J^0_{el} + J^0_{mag}\), where

\[
J^0_{el} = \frac{i\hbar}{2} \left( \langle \psi | v | \psi \rangle - \langle \psi | \psi \rangle \right), \quad \nabla = \psi^\dagger \nabla \psi
\]

and

\[
J^0_{mag} = -\partial A^\dagger / \partial A + (\nabla A) \hat{A},
\]

for Dirac electrons and magnons, respectively.

Before studying the drag effects, we first consider the transport of magnons and Dirac electrons occurring independently. The thermal conductivity of magnons is calculated from

\[
(\kappa_{mag, \alpha}) = \frac{1}{2 \pi T} \sum_q \langle \kappa(q) \rangle \int_{-\infty}^{\infty} d\nu \left( -\frac{\partial \omega}{\partial \nu} \right) \nu^2 D^\nu \nu D^\nu \nu.
\]

where \(u_i = 2Jq_i\) is the velocity of magnons and \(n = n(\nu)\) is the Bose distribution function. Up to \(\hat{O}(s_{01})\), the longitudinal component, \(\kappa_{xx}\), and the in-plane anisotropy, \(\kappa_{xx} = \kappa_{yy}\), are calculated as

\[
(\kappa_{mag, \alpha})_{xx} = \frac{S_{mag}}{\alpha} - \frac{S_{mag} M^2 + 3M^2}{M^2} \delta J^0_x,
\]

where \(S_{mag} = -(\partial / \partial T) T \sum_q \ln(1 - e^{-\omega_q / k_B T})\) is the equilibrium entropy density of magnons. The anisotropy \(\kappa_{xx-yy}\) here represents a magnonic analog of AMR. We remark that the renormalization of the exchange-stiffness constant \((\delta J)\), which is \(\hat{O}(s_{01})\), affects the longitudinal thermal transport (including thermal AMR) of magnons in the insulating region \(|\mu| < |M_z|\), and such contributions are larger for a smaller gap \((\sim |M_z|)\) of Dirac electrons. Diagrammatically, this originates from the self-energy of magnons. Below, we will show that this result is modified by, but survives, the drag contribution (i.e., vertex corrections). Finally, we note that, even if we include the self-energy due to the coupling to Dirac electrons, the thermal Hall effect (THE) does not occur as the velocities of magnons are commutative.

The Hall transport of Dirac electrons is described by

\[
\sigma_{xy} = \frac{e^2}{2\hbar} \times \begin{pmatrix} 2M_0 |\mu| (|\mu| + M_z^2) & (|\mu| > |M_z|), \\ 2M_0 |\mu| (|\mu| + M_z^2) & (|\mu| > |M_z|), \end{pmatrix}
\]

for the anomalous Hall effect (AHE), and

\[
(\kappa_{el, \alpha})_{xy} = \frac{\hbar^2}{e^2} \sum_q \left( -\frac{\partial f}{\partial \nu} \right) e^2 \sigma_{xy}(\nu) \left( \frac{\pi k_B}{e} \right)^2 \sigma_{xy},
\]

for anomalous THE. Here, the WF law (for Hall transport) when the drag effects are neglected. The Nernst conductivity is given, for \(|\mu| > |M_z|\), by

\[
(\kappa_{el, \alpha})_{xy} = \frac{2T}{e^2} \sigma_{xy}(\mu) = \frac{3}{3} \hbar^2 \left( \frac{\mu^2 - M_z^2}{M_z^2} \right)^2 \sigma_{xy},
\]

We now study the drag effects. They may be called “cross-correlated” electron–magnon transport and are expressed by the correlation functions of two currents, one for electrons and the other for magnons, \(\langle J^Q_{mag, i} J^Q_{el, j} \rangle\) and \(\langle J^Q_{el, i} J^Q_{mag, j} \rangle\), where \(J^Q_{el}\) is the electric current. The average \((\cdot \cdot \cdot)\) is defined with the full Hamiltonian \(H\) including \(H_{el, mag}\), which connects the two currents. With respect to the electron–magnon coupling parameter \(s_{01}^{-1}\), the leading contribution is \(\hat{O}(s_{01})\),

\[
K_{mag, \alpha}(i\omega_n)_{ij} = \frac{4\hbar J M_0^2}{x_0} \sum_q \sum_n \left( i\nu_n - i\omega_q \right) D_q(i\nu_n) D_q(i\nu_n - i\omega_q) E_j(i\nu_n),
\]

given by the diagrams in Fig. 1 (c), where \(E_j = E_j^A + E_j^B\),

\[
E_j^A = T \sum_n \sum_k \left( i\epsilon_n - i\omega_q / 2 \right)
\]

\[\times \text{tr} \left[ \sum_k G_k(i\epsilon_n - i\nu) \Sigma^\nu G_k(i\epsilon_n) G_k(i\epsilon_n - i\omega_q) \right],
\]

\[
E_j^B = T \sum_n \sum_k \left( i\epsilon_n + i\omega_q / 2 \right)
\]

Fig. 1. Diagrammatic expression for (a) electron self-energy, (b) magnon self-energy, and (c) Dirac-electron-drag magnon current. The solid (wavy) line with an arrow is the electron (magnon) Green function. The dashed line with a cross is the impurity potential. The filled (empty) circle is the electron–magnon (external) vertex. Diagrams for the magnon-drag electron current are given by (c) with left–right reversed.
The results of Eq. (10) and (11), for the response to an electric field \((K^{QF})\), and is absent for the response to an electric field \((K^{QE})\). Similar diagrams have been considered earlier for systems with spin-orbit coupling.

We first consider the case of perpendicular magnetization, \(M \parallel \hat{z}\). Owing to the perpendicular component \(M_z\), the Dirac electrons exhibit AHE (see Eq. (15)). Therefore, Hall-like (transverse) transport may also be expected in drag processes. The results of \(\mathcal{O}(\alpha_0)\) are summarized as follows:

\[
\begin{align*}
(k_{QF}^{elmag})_{xy-xy} &= \frac{\mathcal{S}_{mag}}{8\pi\alpha} \frac{M_z^2}{\xi_0} \frac{eM_z\mu}{(\mu^2 + M_z^2)^{3/2}} \gamma_0 = (k_{mag,el})_{xy-xy}, \quad (21) \\
(k_{QO}^{elmag})_{xy-xy} &= -\frac{\pi T^2 \mathcal{S}_{mag}}{24\alpha} \frac{M_z^2}{\xi_0} \frac{(3\mu^2 - M_z^2) \xi_0}{(\mu^2 + M_z^2)^{3/2}} = (k_{mag,el})_{xy-xy}, \quad (22) \\
(k_{QO}^{elmag})_{xx} &= \frac{\mathcal{S}_{mag}}{32\pi\alpha} \frac{M_z^2 e^2 (\mu^2 - M_z^2)}{\xi_0} \gamma_0 = (k_{mag,el})_{xx}, \quad (23) \\
(k_{QO}^{elmag})_{yy} &= \frac{2\mathcal{S}_{mag}}{\alpha d} \delta J = (k_{mag,el})_{yy}, \quad (24)
\end{align*}
\]

where the anti-symmetric part, \(\kappa_{xy-xy} \equiv (\kappa_y - \kappa_x)/2\), represents the Hall component. The conductivity \((k_{QF}^{elmag})_{xy-xy}\) represents the MdNE, and \((k_{QO}^{elmag})_{xy-xy}\) represents the MdSE. The contribution to thermal conductivity, \((k_{QO}^{elmag})_{xx} + \kappa_{mag,el}^{\alpha,\beta})\), proportional to \(\delta J\) is to be added to Eq. (13).

As indicated in Eq. (21), the MdNE and the electron-drag Ettingshausen effect are Onsager partners having the same conductivity (with the same sign). The same holds for the MdTHE and the EdTHE (see Eq. (22)), which constitute the drag contribution to THE and violate the WF law for Hall transport, cf. Eq. (16).

The results (21)–(24) are expressed as products of three factors. The first factor \((\mathcal{S}_{mag}/\alpha)e\) originates from magnons, the second \((M_z^2/\xi_0)\) from the electron–magnon coupling, and the third from Dirac electrons. (In Eq. (24), use Eq. (8) for \(\delta J\).)

The magnon factor, \(\mathcal{S}_{mag}/\alpha\), which is common to Eqs. (21)–(24), is related to the longitudinal component of thermal conductivity, Eq. (13). This indicates that magnons propagate straight (i.e., do not bend) even in the anti-symmetric \((\kappa_{el-xy})\) drag processes. The dependence on \(T\) through \(\mathcal{S}_{mag}\) (and on \(\mu\) and \(M_z\)), together with the deviation from the WF law, can be key features to discriminate the drag effects experimentally.

As an illustration, let us consider the Nernst effect. If the magnons are 2D, \(\mathcal{S}_{mag} \propto T\) at low \(T\) (but above \(\Delta\)), in contrast to 3D.\(^{22}\) Thus, the drag Nernst conductivity, \((k_{QO}^{elmag})_{xy-xy} \propto \mathcal{S}_{mag}\) [Eq. (21)], has the same \(T\)-dependence as that from a purely electronic process, \((k_{elmag})_{xy}\) [Eq. (17)], but the magnitude is much larger. In fact, with \({}^{13}\) \(|M_z| = 0.1 \sim 0.01eV\), \(r_0 = 0.5nm\), and \(|M_z|/J_{ex} \sim 1\), where \(J_{ex} \equiv J/r_0\), the ratio

\[
R \equiv \frac{(k_{QO}^{elmag})_{xy-xy}}{(k_{elmag})_{xy-xy}} = \frac{|M_z|^2}{16\pi J_{ex} r_0} \left(\frac{\alpha d}{\mathcal{S}_{mag}}\right)^2 \frac{|M_z|^2}{(\mu^2 + M_z^2)} \frac{1}{(\mu^2 - M_z^2)^2}
\]

\((\lambda_M \equiv 2\pi\hbar\nu_F/|M_z|)\) assumes \(R = 10 \sim 1000\) for \(\mu = 2M_z\), and

\[R = 1 \sim 1000 \mu = 10M_z\]. This indicates that the Nernst effect is dominated by the drag process. Moreover, one can show that it is related to the deviation from the WF law, \(\Delta \kappa \equiv (k_{QO}^{elmag})_{xy-xy} + (k_{QF}^{elmag})_{xy-xy}\), as

\[\Delta \kappa \equiv \frac{e\mu(M_z^2 - \mu^2)}{2\pi^2(3\mu^2 - M_z^2)} \frac{\Delta \sigma}{T^2}\]

This relation may be tested by experiments.

The electron part of Eqs. (21) and (23) can be understood as follows.\(^{23}\) In Eq. (21), the electron contribution is \(eM_z^2\mu/|\gamma_0(\mu^2 + M_z^2)|\). This factor also appears in the longitudinal magnetoconductance,\(^{13}\)

\[\langle \mathcal{J}_F^\alpha \rangle = \pm e\nu_F(\hat{\lambda} - B \times \hat{\sigma})(E + \mathbf{H}^{eff})\]

As a consequence of \(\mathbf{H}^{eff}\), the extremal magnetization state \((H = 0, M_z)\) is the anomalous Hall state with finite \(M_z\) in zero magnetic field, \(H = 0\). The arrow indicated by \(\Delta \sigma(H)\) represents the contribution of normal Hall conductivity induced by an applied field \(H\).

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heat current ($J_{\text{mag}}^0$) = ($k_{\text{mag}}^{QO}$) ($\nabla$($-\nabla T$)) = ($JS_{\text{mag}}$/$\alpha$)($-\nabla T$) induced by a temperature gradient also indicates the induction of an electric current,

$$J_{\text{mag}} = \frac{eS_{\text{mag}}}{2\alpha s_0} \left\{ A (-\nabla T) - B (-\nabla T) \right\}. \quad (28)$$

This exactly reproduces the microscopical result, Eq. (21), for the MnDE. This also reproduces the MnSE, Eq. (23), but there appears to be a slight disagreement by a factor of $\mu^2 / (\mu^2 + M_z^2)$.

When the magnetization $M$ is tilted from the $\pm z$ direction and has an in-plane component, $M_{\parallel}$, the in-plane conduction becomes anisotropic and various kinds of AMR arise. They are calculated as follows,

$$k_{\text{mag}, \parallel}^{\text{xy} \parallel} = \frac{3S_{\text{mag}}}{128\pi s_0} \frac{eM_\mu}{(\mu^2 + M_z^2)^2} = - (k_{\text{el}, \parallel}^{\text{xy} \parallel}), \quad (29)$$

$$k_{\text{mag}, \parallel}^{\text{yy} \parallel} = \frac{\pi T^2 S_{\text{mag}} M_z^2 (M_z^2 - 3\mu^2)}{128\pi s_0} = - (k_{\text{el}, \parallel}^{\text{yy} \parallel}), \quad (30)$$

$$k_{\text{mag}, \parallel}^{\text{xy} \parallel} = \frac{2\pi M_z^2 \mu^2 (M_z^2 + 3\mu^2)}{128\pi s_0} = - (k_{\text{el}, \parallel}^{\text{xy} \parallel}), \quad (31)$$

where $\kappa_{\text{xy} \parallel} = (\kappa_{\text{xy} \parallel} + \kappa_{\text{yx} \parallel}) / 2$ and $\kappa_{\text{xx} \parallel} = \kappa = \kappa_{\text{yy} \parallel}$ reflect the in-plane anisotropy. These effects may be called cross-correlated AMR or drag AMR (both thermal and thermoelectrical), which are caused by the electron-magnon drag processes and disappear when $M_\parallel = 0$. The results obtained above are consistent with the Onsager's theorem, $k_{\text{mag}, \parallel}^{\text{QO}}$ ($M$) = $k_{\text{el}, \parallel}^{\text{QO}}$ ($-M$), where the upper sign is for the symmetric part (longitudinal and planar Hall conductivity), and the lower sign is for the anti-symmetric part (Hall conductivity). As a consequence, we have a finite drag contribution, $k_{\text{el}, \parallel}^{\text{QO}} + k_{\text{el}, \parallel}^{\text{QO}}$, for the symmetric (anti-symmetric) part if it is even (odd) under $M$ → $-M$. This is the case for the drag THE [Eq. (22)] and the drag longitudinal conductivity [Eq. (24)] including the drag AMR [Eqs. (32) and (33)]. In contrast, for the drag planar thermal Hall conductivity [Eq. (30)], the Onsager partners have opposite signs and cancel each other, $k_{\text{mag}, \parallel}^{\text{QO}} + k_{\text{mag}, \parallel}^{\text{QO}}$ = 0; they do not contribute to the total conductivity.

In the undoped case, $|u| < |M_z|$, the magnon propagation becomes anisotropic because of the $\delta J$-term [Eq. (7)]. The total anisotropy of the thermal conductivity is obtained by adding the drag contribution, Eq. (33), to Eq. (14), which gives $k_{\text{QO}}^{\text{xy} \parallel} = (S_{\text{mag}}/\alpha_d) (2M_z^2 / M_z^2) \delta J$. The “AMR ratio” is

$$\frac{\kappa_{\text{xy} \parallel} - \kappa_{\text{yx} \parallel}}{\mu_0 s_0 |M_z|} = \frac{J_{\text{sd}}}{12\pi s_0 |J_{\text{sd}}|} \sin^2 \theta = J_{\text{sd}} \cos \theta, \quad (34)$$

where $J_{\text{sd}}$ = $J_{\text{sd}} |\delta J|$ is the ferromagnetic exchange constant (with the dimension of energy), and Eq. (8) has been used. As $J_{\text{sd}}$ is comparable to or larger than $J_{\text{sd}}$, the anisotropy ratio (34) is well in a range of experimental detection.

To summarize, we analytically studied the cross-correlated electron–magnon transport phenomena on the surface of MTIs. We first considered the case of perpendicular magnetization, $M \parallel \hat{z}$, and calculated various drag conductivities of $O(\mu^2)$, which include (i) electron– (magnon-) drag Ettingshausen (Nernst) effect, (ii) electron– (magnon-) drag magnon (electron) thermal Hall effect, and (iii) electron– (magnon-) drag Peierls (Seebeck) effect. Our microscopic results for the magnon-drag Nernst effect, (i), and magnon-drag Seebeck effect, (iii), could be physically interpreted to be caused by the electromotive force induced by magnon-current magnetization dynamics. The presence of (ii) indicates the violation of the WF law owing to the drag effects.

For the “tilted” case, $M = M_\parallel \hat{y} + M_z \hat{z}$, we found (iv) thermoelectric/thermal AMR caused by drag processes. For this AMR, the magnons play an essential role for the Dirac electrons to see the in-plane magnetization component. All the effects studied here apply to the “doped” case $|u| > |M_z|$ except for (iv), which is present also in the insulating case $|u| < |M_z|$ and gives rise to magnonic AMR. In addition, we observed that the electron-drag planar thermal Hall effect and the magnon-drag pTHE cancel each other out in the total THE, as assured by Onsager’s theorem.

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1) J. E. Moore and L. Balents, Phys. Rev. B 75, 121306(R) (2007).
2) L. Fu and C. L. Kane, Phys. Rev. B 76, 045302 (2007).
3) X.-L. Qi, T. L. Hughes, and S.-C. Zhang, Phys. Rev. B 78, 195424 (2008).
4) D. Hsieh, D. Qian, L. Wang, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature 452, 970 (2008).
5) K. Nomura and N. Nagaosa, Phys. Rev. Lett. 106, 166802 (2011).
6) R. Yu, W. Zhang, H.-J. Zhang, S.-C. Zhang, X. Dai, and Z. Fanz, Science 329, 61 (2010).
7) C.-Z. Chang, J. Zhang, X. Feng, J. Shen, Z. Zhang, M. Guo, K. Li, Y. Ou, P. Wei, L.-W. Wang, Z.-Q. Ji, Y. Feng, S. J. X. Chen, J. Jia, X. Dai, Z. Fang, S.-C. Zhang, K. He, Y. Wang, L. Lu, X.-C. Ma, and Q.-K. Xue, Science 340, 167 (2013).
8) K. Yasuda, A. Tsukazaki, R. Yoshimi, K. S. Takahashi, M. Kawasaki, and Y. Tokura, Phys. Rev. Lett. 117, 127202 (2016).
9) Spin-current drag effects in a Ti/ferromagnet heterostructure was studied in: N. Okuma and K. Nomura, Phys. Rev. B 95, 115402 (2017).
10) T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).
11) The gap may originate from magnetic anisotropy or applied magnetic field, which are not explicitly written in Eq. (1). It may also originate from the self-energy,[12] but here we denote it by $\Delta$ without invoking its origin.
12) A. S. Núñez and J. Fernández-Rossier, Solid State Commun. 152, 403 (2012).
13) A. Sakai and H. Kohno, Phys. Rev. B 89, 165307 (2014).
14) Precisely speaking, the magnon self-energy allows a term linear in $q$ corresponding to the Dzyaloshinskii–Moriya interaction.[15] However, it does not lead to the Hall effect and is thus neglected in this letter.
15) R. Wakatsuki, M. Ezawa, and N. Nagaosa, Sci. Rep. 5, 13638 (2015).
16) J. M. Luttinger, Phys. Rev. 135, A1505 (1964).
17) H. Kohno, Y. Hiraoka, M. Hatami, and G. E. W. Bauer, Phys. Rev. B 94, 104417 (2016).
18) T. Yamaguchi and H. Kohno, J. Phys. Soc. Jpn. 86, 063706 (2017).
19) D. Culcer, A. H. MacDonald, and Q. Niu, Phys. Rev. B 68, 045327 (2003); N. A. Sinitsyn, J. E. Hill, H. Min, J. Sinova, and A. H. Mac-
Donald, Phys. Rev. Lett. 97, 106804 (2006).

20) Note that the WF law does not hold for the longitudinal conductivity even without the drag effects because of the contribution from magnons. On the other hand, magnons do not exhibit the Hall effect at this level (without drag).

21) D. Miura and A. Sakuma, J. Phys. Soc. Jpn. 81, 113602 (2012).

22) S. J. Watzman, R. A. Duine, Y. Tserkovnyak, S. R. Boona, H. Jin, A. Prakash, Y. Zheng, and J. P. Heremans, Phys. Rev. B 94, 144407 (2016).

23) As for Eq. (22), it is related to Eq. (21) via the Mott relation, namely, it can be obtained by applying $[(\pi k_B T)^2/3e](\partial/\partial \mu)$ to Eq. (21).

24) A similar argument was provided for ferromagnets without spin-orbit interaction in: M. E. Lucassen, C. H. Wong, R. A. Duine, and Y. Tserkovnyak, Appl. Phys. Lett. 99, 262506 (2011); B. Flebus, R. A. Duine, and Y. Tserkovnyak, Europhys. Lett. 115, 57004 (2016).

25) It may appear evident that the presence of $M_y$ breaks the in-plane rotational symmetry and causes AMR. Note, however, that if $M_y$ is uniform and static, there is no AMR, because $M_y$ is thereafter a pure gauge degree of freedom invisible to Dirac electrons. Magnons introduce spatial/temporal variations and make $M_y$ visible. Mathematically, the expression of the electron–magnon coupling contains the information of $M_y$.

26) Another mechanism of AMR on MTI surfaces has been pointed out based on the inhomogeneity of the magnetization, in: T. Chiba, S. Takahashi and G. E. W. Bauer, Phys. Rev. B 95, 094428 (2017).