Topological pumping of a single magnon in a one-dimensional spin-dependent optical superlattice

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Topological pumping of ultracold atomic gases has recently been demonstrated in two experiments (Nat. Phys. 12, 296; 12, 350 (2016)). Here we study the topological pumping of a single magnon in a dynamically controlled spin-dependent optical superlattice. When the interaction between atoms is strong, this system supports a dynamical version of topological magnon insulator phase. By initially putting a single magnon in the superlattice and slowly varying the dynamical controlled parameter over one period, the shift of the magnon density center is quantized and equal to the topological Chern number. Moreover, we also find that the direction of this quantized shift is entanglement-dependent. Our result provides a route for realizing topological pumping of quasiparticles in strongly correlated ultracold atomic system and for studying the interplay between topological pumping and quantum entanglement.

I. INTRODUCTION

In 1983, Thouless studied the transport of particles in a one-dimensional slowly varying periodic potential [1]. Such varying makes the Hamiltonian of the system experience a cyclic evolution. Suppose the particles are initially prepared in the ground band of the potential. When the Hamiltonian evolves adiabatically with time and returns to the original point after a period, the number of transported particles is equal to the topological Chern number associated with the topology of the ground band. This pumping depends only on the geometric properties of the pump cycle and is robust to disorder and interaction effects [1]. Although the Thouless pumping was predicted more than thirty years ago, it is still a challenge to be realized in solid state systems.

In the past years, investigating topological and geometric pumping with tunable ultracold atomic system has attracted a lot of interests [2–16]. Specifically, two experimental groups have recently reported the realization of topological Thouless pumping with ultracold fermionic and bosonic atoms trapped in spin-independent optical superlattices [15, 16]. Populating the topologically non-trivial ground band is archived by placing the Fermi energy in the band gap [15] or by preparing the bosonic atoms into Mott insulator in the ground band [16]. The shift of the center-of-mass of the atomic cloud during a pump cycle amounts to the Chern number. Moreover, two-dimensional topological pumping protected by the second Chern number has also been demonstrated experimentally in a two-dimensional optical superlattice [17]. However, all these studies focus on topological pumping of an atomic gas and requires preparing this gas in the ground band.

On the other hand, ultracold atoms trapped in optical lattices can also be employed to realize spin chain models and explore magnonic states [18–24]. Via single-site- and time-resolved technologies in optical lattices, based on mimicked spin chains, recent ultracold atoms experiments have successfully observed the quantum dynamics of single- and two-magnon states [25–27]. Different from neutral atoms, magnons are bosonic quasiparticle excitations around the ground state of strongly correlated quantum spin chain models [25–27], which are important for the development of spintronics devices. Meanwhile, the concept of topology has been further expanded to magnonic systems [28, 29]. Searching topological magnon insulator and semimetal phases in solid state systems has also attracted much attention [30–34].

In this paper, we investigate the topological magnon insulator phase and the topological magnon pumping in a dynamically controlled spin-dependent optical superlattice system. The tight-binding form of this dynamical system is described by the Rice-Mele-Bose-Hubbard model with a dynamical parameter \( \theta \). When the on-site interaction is strong, the system can represent a spin chain with two spins per unit cell, where the intercell and intracell couplings are different and the on-site energies are spin-dependent. In the process of adiabatically varying \( \theta \), we show that a dynamical version of topological magnon insulator phase characterized by the Chern number can emerge. After preparing a single-magnon state in the system, we demonstrate that quantized topological pumping of a single magnon can be implemented by slowly tuning \( \theta \) over one period. We also exhibit that the pumping direction depends on the internal spin entanglement configuration in the initial single-magnon state. Compared with previous Thouless pumping experiments [15, 16], our work studies the topological pumping of a single quasiparticle in a strongly correlated system, where the initial ground band population can be easily pre-

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pared. In addition, we find that quantum entanglement can play an important role in the topological pumping. Finally, we also show that this topological pumping can be efficiently detected based on a parallel state preparation and detection strategy.

This paper is organized as follows. In Sec. II, we present the dynamically controlled spin-dependent optical superlattice system. In Sec. III, we show a dynamical version of topological magnon insulator phase can be generated. In Sec. IV, we investigate the entanglement-dependent topological pumping of a single magnon in a strongly correlated ultracold atomic system. In Sec. V, we briefly discuss how to prepare the initial state and realize the parallel topological pumping. In Sec. VI, we give a summary for the main results in this work.

II. RICE-MELE-BOSE-HUBBARD MODEL

We consider ultracold bosonic $^{87}$Rb atoms trapped in a one-dimensional spin-dependent optical superlattice. Each atom is assumed to have two relevant internal states labeled respectively by the spin index $| \downarrow \rangle = | F = 1, m_F = -1 \rangle$ and $| \uparrow \rangle = | F = 2, m_F = -2 \rangle$. Such spin-dependent optical superlattice has been realized in experiments by superimposing two standing optical waves [35, 36]. The corresponding potential $V_\sigma(x) = V_{\text{int}} \sin^2(k_1 x) + V_{\text{lat}} \sin^2(2k_1 x + \varphi_\sigma)$, where the potential depths $V_{\text{int}}$ and the laser phase $\varphi_\sigma$ can be varied by changing the laser power and the optical path difference. For sufficiently deep optical lattice potential and low temperatures, this optical superlattice system can be described by the Rice-Mele-Bose-Hubbard model

$$H = H_0 + V,$$

$$H_0 = - \sum_{x=1}^{N} \sum_{\sigma = \uparrow, \downarrow} \left( J_1 \hat{a}^\dagger_{x,\sigma} \hat{b}_{x,\sigma} + J_2 \hat{b}^\dagger_{x,\sigma} \hat{a}_{x+1,\sigma} + \text{H.c.} \right) + \Delta \sum_{i} \left( \hat{a}^\dagger_{i,\uparrow} \hat{a}_{i,\uparrow} - \hat{b}^\dagger_{i,\uparrow} \hat{b}_{i,\uparrow} - \hat{a}^\dagger_{i,\downarrow} \hat{a}_{i,\downarrow} + \hat{b}^\dagger_{i,\downarrow} \hat{b}_{i,\downarrow} \right),$$

$$V = \sum_{x=1}^{N} \sum_{\sigma = \uparrow, \downarrow} U \left( \hat{a}^\dagger_{x,\sigma} \hat{a}_{x,\sigma} \hat{a}_{x,\sigma} + \hat{b}^\dagger_{x,\sigma} \hat{b}_{x,\sigma} \right) + U \sum_{i} \left( \hat{a}_{i,\uparrow} \hat{a}^\dagger_{i,\downarrow} + \hat{b}_{i,\uparrow} \hat{b}^\dagger_{i,\downarrow} \right),$$

where $\hat{a}^\dagger_{i,\sigma}$ ($\hat{b}^\dagger_{i,\sigma}$) is the spin-dependent creation operator associated with the lattice site $a_i$ ($b_i$) in the $i$-th unit cell, $J_{1,2} = J \mp \delta J$ are the alternating tunneling amplitudes, $\Delta$ is the spin-dependent staggered on-site energy, $U$ is the on-site interaction and $N$ is the unit cell number. The interaction between atoms in the same or different spin state is assumed to be same. In the absence of interaction, the above model corresponds to the spinful Rice-Mele model [37]. The spinless Rice-Mele model and its topological features have been experimentally studied in a one-dimensional optical superlattice [38].

In the strong interaction case, $V$ (the hopping and on-site energy terms) can be seen as a perturbation to $H_0$ (the on-site interaction term). We label the ground and excitation state subspaces spanned by the eigenstates of $H_0$ as $P$ and $Q$. For unit filling, the ground and excitation state subspaces associated with one unit cell can be written as

$$P = \{| \uparrow, \uparrow \rangle, | \uparrow, \downarrow \rangle, | \downarrow, \uparrow \rangle, | \downarrow, \downarrow \rangle \},$$

$$Q = \{| \uparrow, 0 \rangle, | 0, \uparrow \rangle, | \downarrow, 0 \rangle, | 0, \downarrow \rangle \},$$

with the eigenenergies as $E_g = 0$ and $E_e = U$. The corresponding projective operators for the above two subspaces are defined as

$$\hat{P} = \sum_{|j\rangle \in P} |j \rangle \langle j|,$$

$$\hat{Q} = \sum_{|k\rangle \in Q} |k \rangle \langle k|.$$ (3)

Based on the Schrieffe-Wolf transformation [39], the low energy effective Hamiltonian up to second order can be formulated as

$$H_e = \hat{P} H_0 \hat{P} + \hat{P} V \hat{Q} V \hat{P} = \frac{2J_2^2}{U} \left( | \uparrow, \downarrow \rangle \langle \uparrow, \downarrow | + | \downarrow, \uparrow \rangle \langle \downarrow, \uparrow | + \text{H.c.} \right) \frac{V}{E_g - E_e} + \frac{4J_2^2}{U} \left( | \uparrow, \uparrow \rangle \langle \uparrow, \uparrow | + | \downarrow, \downarrow \rangle \langle \downarrow, \downarrow | \right).$$ (4)

It is easy to check that the first term $\hat{P} H_0 \hat{P} = 0$ in our model. For the intra-cell coupling in the last two terms in Eq. (4), after a straightforward calculation, we find that

$$\hat{P} V \hat{Q} V \hat{P} = \Delta | \uparrow, \downarrow \rangle \langle \uparrow, \downarrow | + \text{H.c.},$$

and

$$\frac{2J_2^2}{U} \left( | \uparrow, \downarrow \rangle \langle \uparrow, \downarrow | + | \downarrow, \uparrow \rangle \langle \downarrow, \uparrow | + \text{H.c.} \right) \frac{V}{E_g - E_e} + \frac{4J_2^2}{U} \left( | \uparrow, \uparrow \rangle \langle \uparrow, \uparrow | + | \downarrow, \downarrow \rangle \langle \downarrow, \downarrow | \right).$$ (5)

where H.c. is the Hermitian conjugate. Similarly, the effective Hamiltonian with respect to the inter-cell coupling can be derived. Now we introduce the pauli operators $\hat{S}^x = | \uparrow \rangle \langle \downarrow | + | \downarrow \rangle \langle \uparrow |$, $\hat{S}^y = -i | \uparrow \rangle \langle \downarrow | + i | \downarrow \rangle \langle \uparrow |$, and $\hat{S}^z = (\hat{n}_\uparrow - \hat{n}_\downarrow)/2$. In terms of these Pauli operators, the total effective Hamiltonian can be derived as

$$H_e = \sum_i \left[ J_{c1} (\hat{S}_{a_i}^x \hat{S}_{b_i}^x + \hat{S}_{a_i}^y \hat{S}_{b_i}^y) + J_{c2} \hat{S}_{a_i}^z \hat{S}_{b_i}^z \right] + J_{c1} (\hat{S}_{a_i}^x \hat{S}_{b_{i+1}}^y + \hat{S}_{b_i}^y \hat{S}_{a_{i+1}}^x) + J_{c2} \hat{S}_{a_i}^z \hat{S}_{b_{i+1}}^z + \Delta (\hat{S}_{a_i}^z - \hat{S}_{b_i}^z),$$ (7)

where $J_{c1,c2} = -J_{1,2}^2/U$ and $J_{c1,c2} = -4J_{1,2}^2/U$ are the effective spin superexchange couplings. Such alternating couplings can be rewritten as $J_{c1,c2} = (J \mp \delta J)^2/U = J_c \mp \delta J_c$. 

To investigate the topological features of the magnon bands. The parameters are chosen as $J_c = J_p = J$. $J$ is used as energy unit in this work.

III. DYNAMICAL VERSION OF TOPOLOGICAL MAGNON INSULATOR

In the present work, only single spin-up excitation is considered in the above optical superlattice system, which is also called as single-magnon excitation in condensed matter physics [25]. We will study the topological feature of Eq. (7) in the single-magnon subspace. Based on the Matsubara-Matsuda mapping [40], the above effective spin model in the single-magnon space can be rewritten into the following magnon Hamiltonian

$$H_m = \sum_k (J_{\perp} \hat{m}_{a_i}^\dagger \hat{m}_{b_i} + J_{\perp} \hat{m}_{b_i}^\dagger \hat{m}_{a_i+1} + H.c.)$$

$$+ \sum_i \Delta (\hat{m}_{a_i}^\dagger \hat{m}_{a_i} - \hat{m}_{b_i}^\dagger \hat{m}_{b_i}),$$

(8)

where $\hat{m}_{a_i(b_i)} = |↓⟩_{a_i(b_i)}|↓⟩$ is the magnon creation operator associated with the lattice site $a_i(b_i)$ in the $i$-th unit cell. $|G⟩ = |↓↓· · ·↓↓⟩$ can be seen as the magnon vacuum state. Experimentally, one can adiabatically tune the optical lattice potential to vary $\delta J_\perp$ and modulate $\delta J_c, \delta J_p$ in a close circle [15–17, 36]. In this way, $\delta J_\perp, \delta J_c, \delta J_p$ can be parameterize as $(J_p \sin(\theta), J_p \cos(\theta))$, where $\theta$ is a dynamical parameter. Then the spin superexchange couplings and on-site energy offset can be written as $J_{\perp, c, p} = J_{\perp, c} \pm J_p \sin(\theta)$ and $\Delta = J_p \cos(\theta)$. To investigate the topological features of the magnon Hamiltonian (8), we write it in the momentum space as

$$H = \sum_{k_x} \hat{m}_{a_x}^\dagger \hat{h}(k_x, \theta) \hat{m}_{a_x},$$

where $\hat{m}_{a_x} = (\hat{m}_{a_{x0}}, \hat{m}_{b_{x0}})^T$. Specifically, the momentum density is written as

$$\hat{h}(k_x, \theta) = h_x \hat{\sigma}_x + h_y \hat{\sigma}_y + h_z \hat{\sigma}_z,$$

(9)

where $h_x = 2J_c \cos(k_x), h_y = 2J_p \sin(\theta) \sin(k_x)$ and $h_z = J_p \cos(\theta)$. $\hat{\sigma}_{x,y,z}$ are the Pauli matrixes spanned by $\hat{m}_{a_{x0}}$ and $\hat{m}_{b_{x0}}$.

Interestingly, we can construct a two-dimensional artificial Brillouin zone based on the momentum $k_x \in (0, \pi]$ and the dynamical parameter $\theta \in (0, 2\pi]$. The single-magnon energy spectrum is plotted in Fig. 1(a), which has two magnon bands. The topological features of these two magnon bands are characterized by the Chern numbers. Based on a mapping from the momentum space to an unit sphere, i.e., $T^2 \to S^2$, the Chern number can be defined as $C = \frac{1}{4\pi} \int \int dk_x d\theta (\partial_{k_x} \mathbf{n} \times \partial_\theta \mathbf{n}) \cdot \mathbf{n}$,

$$C = \frac{1}{4\pi} \int \int dk_x d\theta (\partial_{k_x} \mathbf{n} \times \partial_\theta \mathbf{n}) \cdot \mathbf{n},$$

(10)

where the unit vector field $\mathbf{n} = (h_x, h_y, h_z)/h$ with $h = \sqrt{h_x^2 + h_y^2 + h_z^2}$. The integrand $\mathbf{n} \times \partial_\theta \mathbf{n} \cdot \mathbf{n}$ is simply the Jacobian of this mapping. Its integration is a topological winding number giving the total area of the image of the Brillouin zone $T^2$ on $S^2$ [41, 42]. It means that, when $(k_x, \theta)$ wraps around the entire first Brillouin zone $T^2$, this winding number is equal to the number of times the vector $\mathbf{n}$ wraps around the unit sphere $S^2$, which is independent of the details of the band structure parameters.

In Figs. 1(b) and 1(c), we plot both the unit vector configurations $(n_x, n_y)$ and the contours of $n_z$ for the lower and upper magnon bands, respectively. The results show that, for the lower (upper) magnon band, the unit vector $\mathbf{n}$ starts from the north (south) pole at the Brillouin zone center and ends at the south (north) pole at the Brillouin zone boundary after wrapping around the unit sphere once. Thus the Chern number corresponding to the lower (upper) magnon band is derived as $C_l = 1$ ($C_u = -1$). Since $\theta$ is a periodic dynamical parameter introduced to construct the first Brillouin zone, the above nontrivial Chern number values yield a dynamical version of the topological magnon insulator phase.

IV. ENTANGLEMENT-DEPENDENT TOPOLOGICAL MAGNON PUMPING

In the following, based on the above dynamically controlled topological magnon bands, we will demonstrate that entanglement-dependent topological pumping of a single magnon can be implemented. This is done by adiabatically tuning the parameter $\theta = \Omega t + \theta_0$ over one period, where $\Omega$ is the modulation frequency and $\theta_0$ is the initial phase.

At the initial time, we assume the whole optical superlattice system consists of series of independent double wells by tuning the optical superlattice potential to make $J_{\perp, c, p} = 0$. This is equivalent to require the initial periodic parameter $\theta(t = 0) = \theta_0 = -\arcsin(J_c/J_p)$. Suppose the initial system stays in the magnon vacuum state $|G⟩ = |↓↓· · ·↓↓⟩$. Note that the Hamiltonian for the single-magnon excitation in each double well has two
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so the Wannier function only localizes the middle double well.

After having prepared the initial single-magnon state,
the parameter \( \theta \) is adiabatically modulated from \( \theta(t = 0) \)
to \( \theta(t = T = 2\pi/\Omega) \), the single-magnon wave packet will
experience an adiabatic transfer. We employ a magnon
density center to monitor this transfer. Specifically, the
operator for such center is defined as

\[ \hat{M}_c = \sum_{x=1}^{N} x(\hat{P}_a_x + \hat{P}_b_x), \]

where \( \hat{P}_a_x(b_x) = \hat{m}^+_a(b_x)\hat{m}_a(b_x) = | \uparrow \rangle a_x(b_x) \langle \uparrow | \) is the
magnon density in the lattice site \( a_x(b_x) \). Then, when
the parameter \( \theta \) is adiabatically modulated, the corre-
sponding magnon density center can be written as

\[ \hat{M}_c(\theta) = \langle \psi_1(\theta) | \hat{M}_c | \psi_1(\theta) \rangle \]

\[ = \frac{1}{2\pi} \int dk_x i\langle u_{k_x,\theta,l} | \partial_{k_x} u_{k_x,\theta,l} \rangle \]

\[ = \frac{1}{2\pi} \int dk_x A_i(k_x, \theta). \]

It turns out that the magnon density center is linked
to the Berry connection \( A_i(k_x, \theta) = i\langle u_{k_x,\theta,l} | \partial_{k_x} u_{k_x,\theta,l} \rangle \).
Therefore, the magnon density center depends on the
gauge choice of the Bloch state. However, the change
of the magnon density center is gauge invariant and thus
can be well defined.

Suppose the periodic parameter \( \theta \) is changed continu-
ously from \( \theta_i \) to \( \theta_f \). The resulting magnon density center shift is

\[ \hat{M}_c(\theta_f) - \hat{M}_c(\theta_i) = \frac{1}{2\pi} \int dk_x (A_i(k_x, \theta_f) - A_i(k_x, \theta_i)) \]

(14)

By means of the Stokes theorem, the formula (14) can be
rewritten as an integral of the Berry curvature \( F_i(k_x, \theta) \)
over the surface spanned by \( k_x \) and \( \theta \), where \( F_i(k_x, \theta) = \nabla \times A_i(k_x, \theta) = i(\langle \partial_{\theta} u_{k_x,\theta,l} | \partial_{k_x} u_{k_x,\theta,l} \rangle - c.c.) \). For a periodic cycle, \( \theta_f = \theta_i + 2\pi, H(\theta_i) = H(\theta_f) \), and
the change of the magnon center over one cycle is given by the
integral of the Berry curvature over the torus \( \{ k_x \in (0, \pi], \theta \in [0, 2\pi[ \} \). It is easy to check that the magnon
center shift in this case is just the Chern number of the
lower magnon band, i.e.,

\[ \hat{M}_c(\theta_f) - \hat{M}_c(\theta_i) = \frac{1}{2\pi} \int_{k_x} d k_x d \theta \nabla \times A_i(k_x, \theta) \]

\[ = \hat{C}_l. \]

Similarly, if the initial single-magnon excitation in the
middle double well is prepared in the Bell state \( | \chi_{lu} \rangle \),
after tuning the parameter \( \theta \) over one period, the shift of the
magnon density center becomes \( \hat{C}_u \). Therefore, the
entanglement-dependent topological pumping of a single
magnon is achieved.

The detailed performance of the above topological pumping is numerically calculated in Fig. 2(b). Suppose the two atoms in the middle double well are prepared into the Bell state \( | \chi_{lu} \rangle \), the numerical results show that the magnon density center is shifted to the right (left) by one unit cells after one periodic pumping, which equals the Chern number of the lower (upper) magnon
band \( \hat{C}_l = 1 \) (\( \hat{C}_u = -1 \)). Interestingly, one can find that
different Bell states inside the initial magnon wave packet gives rise to different quantized topological pumping. Such process generates an entanglement-dependent topological magnon pumping, which is different from the Thouless pumping and shows that the internal entan-
glement configuration in the transport particle can also
affect the external pumping.

V. INITIAL STATE PREPARATION AND
PARALLEL TOPOLOGICAL PUMPING

The initial single-magnon state with internal spin en-
tanglement can be prepared based on spin superex-
change Hamiltonian and an effective magnetic field. This
shown in Fig. (a) The procedure for preparing a spin entangled state in the middle double well. (b) An array of one-dimensional spin-dependent optical superlattices for highly efficient parallel entangled state generation and magnon density detection.

method has recently been experimentally demonstrated in a spin-dependent optical superlattice system [35]. The detailed preparation procedure is shown in Fig. 3(a). Initially, the superlattice potential is tuned to make $J_{c2} = 0$ and $\Delta = 0$. The resulted lattice is formed by an array of independent double wells. Suppose the system is initially prepared in the Mott-insulator regime where each lattice site has a single atom prepared in the state $|\downarrow\rangle$. The initial single-magnon state can be prepared via three steps. Step 1: based on single-site microwave pulse addressing, one of the spins in the middle double well is flipped into $|\uparrow\rangle$, then the state of the two spins in the middle double well becomes $|\uparrow\downarrow\rangle$. Step 2: through a dynamical evolution governed by the spin superexchange Hamiltonian in Eq. (7) with $\Delta = 0$ and evolution time $t = \pi/4J_{c1}$, an entangled state $(|\uparrow\downarrow\rangle - i|\downarrow\uparrow\rangle)/\sqrt{2}$ is generated between the two spins in the middle double well. Step 3: tuning the spin-dependent optical superlattice potential to switch on $\Delta$, an effective magnetic field described by the Hamiltonian shown in Eq. (5) is created. Via such magnetic field to modulate the phase of the entangled state, the Bell state $(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$ can be generated. In the whole process, the state of the spins in the other double wells dose not change. Then the single-magnon state $|\psi_{l,u}\rangle = |\downarrow\downarrow\cdots\chi_{l,u}\cdots\downarrow\rangle$ is prepared.

In experiment, the topological pumping proposed in this work can be efficiently measured based on a strategy using parallel state preparation and detection. As shown in Fig. 3(b), a two-dimensional degenerate Bose gas of $^{87}$Rb atoms is prepared in a two-dimensional optical lattice potential $V(x,y) = V(x) + V(y)$, where $V(x) = V_{t}(x) + V_{l}(x)$ is the state-dependent superlattice potential in the $x$ direction and $V(y) = V_{y}\sin^{2}(2\kappa_{1}y)$ is a spin-independent potential in the $y$ direction. When $V_y$ is tuned to be vary large, the hopping along the $y$ direction can be ignored. Then $V(x,y)$ creates an array of independent one-dimensional optical superlattices along the $x$ direction. In the Mott-insulator regime, this system can be seen as an array of parallel one-dimensional spin chains described by Eq. (7). Via an addressing beam profile in form of a line reported in the recent single-magnon experiment [25], the left spin down state in each middle double well of the optical superlattice along the $x$ direction can be simultaneously flipped into the spin up state, then each superlattice system has been put into a single magnon. After that, based on the above entanglement generation procedure and its parallel version, each spin chain can be prepared into the single-magnon state $|\psi_{l,u}\rangle$. Finally, through tuning $\theta$ over one period in each spin chain, we can realize a parallel topological pumping of a single magnon in this two-dimensional optical lattice system. In this case, the magnon density can be efficiently measured by averaging the data extracted from the magnon density measurements in all spin chains [25, 26].

VI. SUMMARY

In summary, we have realized a dynamical version of the topological magnon insulator phase and also topological pumping of a single magnon in a spin-dependent optical superlattice system. Different from previous experiments implementing topological Thouless pumping of an ultracold atomic gas, our work focused on topological pumping of a single quasiparticle in a strongly correlated spinful Bose-Hubbard system. Furthermore, we have also found that the shift of pumping direction is entanglement-dependent. Our model is also compatible with the recent optical lattice experiments on magnons [25–27]. Our result opens a prospect for studying topological magnon insulator phase in optical lattice system and also for investigating the interplay between topological pumping and quantum entanglement and interaction effect.

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