Effective modelling of the Rayleigh-Bénard convection of concentrated emulsions with finite-size droplets

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We present mesoscale numerical simulations of Rayleigh-Bénard convection in a two-dimensional concentrated emulsion, confined between two parallel walls, heated from below and cooled from above, under the effect of buoyancy forces. The systems under study comprise finite-size droplets, whose concentration $\Phi_0$ is varied, ranging from the dilute limit up to the point where the emulsion starts to be packed and exhibits non-Newtonian rheology. We focus on the characterisation of the convective heat transfer properties close to the transition from conductive to convective states. The convective flow is confined and heterogeneous, which causes the emulsion to exhibit concentration heterogeneities in space $\Phi_0(y)$, depending on the location in the wall-to-wall direction ($y$). With the aim of assessing quantitatively the heat transfer efficiency of such heterogeneous systems, we resort to a side-by-side comparison between the concentrated emulsion system and a single-phase (SP) system, whose local viscosity $\eta^{\text{SP}}(y)$ is suitably constructed from the shear rheology of the emulsion. Such comparison highlights that a suitable degree $\Lambda$ of coarse-graining needs to be introduced in the system, whose local viscosity $\eta^{\text{SP}}(y)$ is introduced in order for the single-phase system to attain the same heat transfer efficiency of the emulsion. Specifically, it is shown that a quantitative matching between the two systems is possible whenever the coarse-graining is performed over a scale of the order of the droplet size.

I. INTRODUCTION

Heat transfer in heterogeneous media made of dispersions of one phase (solid, liquid or gaseous) in another liquid phase is of paramount importance for an ample variety of technological applications \cite{1,2}. Depending on the dispersed phase, different types of systems can be obtained: dispersions of gas bubbles in a continuous liquid phase \cite{3,4}; dispersions of droplets in a liquid matrix \cite{5,6}; suspensions of particles dispersed in a liquid solvent \cite{7,8}. The focus of this paper is on the characterisation of the heat transfer properties in a biphasic medium, consisting of deformable droplets of a liquid phase dispersed in another phase with the same viscosity. The mechanical response of these kind of systems has been thoroughly addressed in theory \cite{9,10}, simulations \cite{11,12}, and experiments \cite{13,14}, resulting in a very detailed characterisation of the response of the medium under flow when the concentration $\Phi_0$ of the dispersed phase changes systematically (see also \cite{15,16} and references therein). One may refer, for example, to the vast knowledge on the deformation and break-up properties of single constituents (i.e. an emulsion in the extremely dilute limit) and/or the characterisation of the medium effective viscosity from dilute to semi-dilute concentrations \cite{17,18,19}. Such a very detailed knowledge is somehow not mirrored in a corresponding characterisation of the heat transfer properties of the medium. Few studies considered effective fluids with non-Newtonian constitutive laws for the stress tensor, focusing on the role of yield stress rheology \cite{20,21}. Such models share, thus, some similarities with emulsions, even though the continuum nature of the approach inevitably overlooks finite droplet size effects. Actually, we may expect that, especially in confined systems and/or in concentrated dispersions, the granularity of the system will lead to a failure of any attempt of modelling by means of continuum equations or with point-like particles. With the present paper, we aim at addressing that issue via numerical simulations in the paradigmatic set-up of the Rayleigh-Bénard convection \cite{22,23}, i.e. when the emulsion is confined between two parallel plates heated from below and cooled from above, under a gravity field. We observe that, due to the convective dynamics, the emulsion develops a non-homogeneous droplet distribution across the cell. We then inspect the heat flux and compare the heterogeneous two-phase system with a single-phase (SP) fluid model with space-dependent effective viscosity, suitably constructed from the shear rheology of the emulsion. It is shown that, in order to get a quantitative matching between the two cases, a spatial averaging procedure (“coarse-graining”) over a scale of the order of the droplet size is needed, thus identifying the origin of the effect within the discrete nature of these complex fluids. The paper is organised as follows: in Section \ref{sec:theory} we provide a brief overview of the methodology used; in Section \ref{sec:results} we report the results of the numerical simulations; conclusions follow in Section \ref{sec:conclusions}.
II. NUMERICAL METHODS

We employ a mesoscopic approach to simulate thermal convection in stabilised emulsions, by coupling a lattice Boltzmann method (LBM) \[58, 59\] for non-ideal multi-component mixtures with an LBM for the temperature field dynamics. We briefly report the essential information about the two LB formulations, referring to other specific works \[26, 60–64\] for more technical details.

The emulsion is simulated with two components (say A and B) LBM. The model hinges on the dynamical evolution of mesoscopic probabilities density functions \( f^\ell_i(x,t) \) of finding a particle of component \( \ell = A, B \) in the space-time location \( (x,t) \) with lattice velocity \( c^i \), where the index \( i \) takes only a finite number of values.

We employ a D2Q9 scheme, with 9 lattice velocities in a two-dimensional domain. The dynamical evolution of the distribution functions embeds streaming steps supplemented with local collisions. This translates into the following evolution over a unitary time lapse \( \Delta t = 1 \)

\[
f^\ell_i(x+c^i,t+1) - f^\ell_i(x,t) = -\frac{1}{\tau} \left( f^\ell_i - f^\ell_{eq,i} \right)(x,t) + F^\ell_i(x,t)
\]

where \( \tau \) is a relaxation time towards the local equilibrium \( f^\ell_{eq,i} \) (repeated indices are summed upon)

\[
f^\ell_{eq,i} = w_i \rho \left[ 1 + \frac{u_k c^k_{eq} + u_k u_p (c^k_{eq} c^p - c^2 \delta_{kp})}{2c^2_{eq}} \right]
\]

where \( w_i \) are suitable weights and \( c^2_{eq} = 1/3 \) is a constant in the model. The density fields \( \rho_i(x,t) = \sum_\ell f^\ell_i(x,t) \) \((\ell=A, B)\) and the global momentum field \( \rho u(x,t) = \sum_\ell \rho f^\ell(x,t) \), with \( \rho = \sum_\ell \rho_i \), are coarse-grained fields suitably constructed from the distribution functions. The source term \( F^\ell_i(x,t) \) includes the effects of interaction (int) forces, \( F_{int}(x,t) \), and external (ext) volume forces, \( F_{ext}(x,t) \). Interaction forces are chosen in such a way that phase separation is triggered between the two components, so that the formation of stable interfaces separating bulk regions (with majority of one component) is formed. Simultaneously, competing interaction forces are also introduced at the interfaces to inhibit droplets coalescence \[60, 63, 67\]. This combination makes possible the simulation of a given number of droplets of the dispersed phase (A) into the continuous phase (B). All technical details on the interaction forces can be found in dedicated papers \[26, 60, 62–64\].

Regarding the external volume forces, a buoyancy term is added to the global momentum balance in the Boussinesq’s form \( F_{ext}(x,t) = \rho(x,t) \alpha g T(x,t) e_y \), where \( T(x,t) \) is the temperature field relative to some reference temperature, \( \alpha \) the thermal expansion coefficient, \( g \) the gravity acceleration and \( e_y \) the unit vector in the wall-to-wall direction. At large scales, the long wavelength limit of the lattice Boltzmann model maps into the diffuse-interface Navier-Stokes equations \[58, 59\]

\[
\rho \left( \partial_t + u^{(n)}_i \partial_i \right) u^{(n)}_i = \partial_j \left[ -P_{ij} + \eta_0 \left( \partial_i u^{(n)}_j + \partial_j u^{(n)}_i \right) \right] + \rho \alpha g T \delta_{iy} \quad i = x, y
\]

where \( \rho u^{(n)} = \rho u + F_{ext}/2 + F_{int}/2 \) is the hydrodynamical momentum of the mixture. The non-ideal pressure tensor \( P_{ij} \) is non-diagonal due to the contribution of interaction forces \[68, 69\]. The bulk viscosity \( \eta_0 \) is linked to the relaxation time \( \tau \) of the lattice Boltzmann according to the following relation:

\[
\eta_0 = \rho c^2 \left( \tau - 1/2 \right).
\]

Hence, \( \eta_0 \) can be tuned via a proper choice of the relaxation time in the lattice Boltzmann dynamics \[1\]. The boundary conditions for the hydrodynamical fields correspond to a no-slip at the walls, which we achieve with suitable bounce-back rules \[58, 59\] implemented at the level of the distribution functions. Periodic boundary conditions are applied in the \( x \)-direction.
The evolution of the temperature field \( T(\mathbf{x}, t) \) is integrated via another properly devised lattice Boltzmann scheme \([31]\). In a nutshell, we evolve in time an auxiliary probability distribution function \( g^i(\mathbf{x}, t) \), whose coarse-grained counterpart is the temperature field. i.e. \( T(\mathbf{x}, t) = \sum_i g^i(\mathbf{x}, t) \). The mesoscopic dynamics for \( g^i(\mathbf{x}, t) \) reads as follow:

\[
g^i(\mathbf{x} + \mathbf{c}_i, t + 1) - g^i(\mathbf{x}, t) = -\frac{1}{\tau_g} \left( g^i - g^{eq,i} \right)(\mathbf{x}, t) \tag{4}
\]

where the local equilibrium \( g^{eq,i} \) takes the form

\[
g^{eq,i} = w_i \rho \left[ 1 + \frac{u_{c_k}^{(H)} c_k}{c_s^2} + \frac{u_{p_0}^{(H)} (c_k^2 c_s^2 - c_k^2 \delta_{kp})}{2c_s^2} \right].
\]

The long-wavelength limit of \([4]\) approximates the advection-diffusion equation for the temperature field

\[
\partial_t T + u^{(H)}_k \partial_k T = \kappa \partial_{kk} T \tag{5}
\]

where \( \kappa = c_s^2 (\tau_g - 1/2) \) is the thermal diffusivity which can be tuned by changing the thermal relaxation time \( \tau_g \). The advection-diffusion equation \([4]\) is two-way coupled with the Navier-Stokes equation \([2]\): i) via the fluid velocity field entering the advection term in \([5]\) and ii) via the buoyancy force in the r.h.s. of \([2]\). The boundary conditions for the temperature field are of Dirichlet type, i.e., we prescribe constant temperatures \( T(x, y = \pm H/2, t) = \mp \Delta T/2 \), with \( \Delta T = 1.0 \) lattice simulation units (lbu), at the upper and lower walls, respectively, whereas we apply periodic boundary conditions in the \( x \)-direction. Notice that all dimensional observables will be reported in simulation units (i.e. lattice Boltzmann units, lbu). The software we employ for all the simulations is an extension of an in-house developed code written in C-CUDA. The code has been described in detail elsewhere \([70]\). Here we recall just that it exploits at its best the computing power of modern Graphics Processing Units (GPU) by means of an innovative memory access pattern. The code is able to run on multiple GPUs. To that purpose we resort to a hybrid parallel programming model (based on a combination of MPI and CUDA). The smoothness by which the thermal LB component has been implemented confirms the flexibility of the software that, in addition, supports a number of different boundary conditions and the chance of simulating the presence of obstacles within the computational domain \([23, 71, 72]\).

### III. RESULTS AND DISCUSSION

We numerically study the heat transfer properties of emulsions. To this aim, we perform simulations by placing the emulsions in a channel of height \( H \sim 17d \), where \( d \) is the mean droplet diameter, and we systematically explore different droplet concentrations \( \Phi_0 \), from very diluted to concentrated emulsions, by varying the number of droplets \( N_{\text{droplets}} \) (see Table I). In order to assess the heat transfer properties of the emulsion, we focus on the heat flux across the system, \( F \), which is the sum of a conductive and a convective part, \( F = F_{\text{cond}} + F_{\text{conv}} \); both can, in principle, differ in the biphasic system, from the mono-phasic counterpart. For our simulations, two fluids have the same thermal diffusivity and no interfacial thermal resistance is supported, therefore \( F_{\text{cond}} \) is not affected. The presence of the dispersed phase (see Fig. I for a pictorial view of how the system looks like at different concentrations), instead, is known to alter the rheology of the mixture (with respect to a situation with a pure solvent), and, hence, also its convective heat transfer properties. The droplet concentration \( \Phi_0 \) is the ratio of the volume of dispersed phase over the total volume, \( \Phi_0 = V_d/V; \) such a definition sur-

| \( \Phi_0 \) | \( N_{\text{droplets}} \) | \( \Phi_0 \) | \( N_{\text{droplets}} \) |
|---|---|---|---|
| 0.0735 | 90 | 0.2357 | 242 |
| 0.1038 | 120 | 0.2680 | 284 |
| 0.1433 | 159 | 0.3322 | 338 |
| 0.1721 | 214 | 0.3978 | 392 |
| 0.2018 | 235 | 0.4775 | 449 |

TABLE I. Number of droplets \( N_{\text{droplets}} \) simulated for each concentration \( \Phi_0 \).

with an intrinsic viscosity coefficient dependent on the viscosity ratio \( \lambda \) as \( \eta_{\text{pol}} = \frac{\lambda + 1}{\lambda^{-1} + 1} \) (which tends to \( 5/2 \), the Einstein’s coefficient for solid particles, as \( \lambda \to \infty \)). In Fig. [3] we show \( \eta_r \) as a function of \( \Phi_0 \) for the emulsion.
FIG. 2. Shear rheology of the emulsions. Panel (a): flow curves for the emulsion systems obtained with dedicated shear experiments (see text for details). The concentration is varied. Panel (b): the emulsion effective viscosity $\eta_{\text{eff}}$ as a function of shear stress $\Sigma$, extracted from the flow curves in panel (a), for different concentrations $\Phi_0$. The dark region refers to a range of concentrations for which non-Newtonian effects start to emerge. All dimensional quantities are reported in simulation units.

The measured relative viscosity is in good agreement with Eq. (6), with $\eta_0 = 7/4$, as expected for an emulsion with unitary viscosity ratio ($\lambda = 1$), for concentrations up to $\Phi_0 \approx 0.12$. The agreement is improved upon using a 2D estimate of the effective viscosity, that we have extracted from the data in [36], as we can see in the zoom-in reported in the inset. At larger droplet concentrations, data start to deviate from dilute predictions. Specifically, for larger $\Phi_0$ (and up to $\Phi_0 \approx 0.35$) our data agree well with Zinchenko’s prediction for three-dimensional emulsions [19]. An obvious effect of increasing the concentration is to increase the system viscosity, thereby reducing its propensity to convection. One would expect simply a monotonic decay of the heat flux with $\Phi_0$. Actually, the phenomenology is more complicated, as shown in Fig. 4, where we analyse the heat fluxes, expressed in a dimensionless form via the Nusselt number $\mathrm{Nu}(t) = \langle u_y(x, y, t)T(x, y, t) \rangle_{x, y} - \kappa \langle \partial_y T(x, y, t) \rangle_{x, y} / \kappa \Delta T$

where $\langle \ldots \rangle_{x, y}$ denotes a space average. In Fig. 4(a) we report $\mathrm{Nu}(t)$ for different values of $\Phi_0$, at fixed buoyancy amplitude $\alpha g \Delta T = 1.86 \times 10^{-5}$ lbf; the time average of $\mathrm{Nu}(t)$ over the statistically steady-state $\langle \mathrm{Nu}(t) \rangle_t$ is reported in Fig. 4(b), while fluctuations around the mean $(\Delta \mathrm{Nu} = \langle (\mathrm{Nu}(t) - \langle \mathrm{Nu} \rangle_t)^2 \rangle_t^{1/2})$ are displayed in the inset of the panel (b) as a function of $\Phi_0$. We observe that $\langle \mathrm{Nu} \rangle_t$ stays nearly constant for concentrations up to $\Phi_0 \approx 0.2$, whereas the fluctuations $\Delta \mathrm{Nu}$ tend to increase with $\Phi_0$. In particular, in the limit $\Phi_0 \to 0$, i.e. for a single-phase (SP) system, the fluctuations go to zero, indicating that the convection is stationary (dashed black line in Fig. 4), by reason of a relatively low Rayleigh num-
The emergence of fluctuations must then be interpreted as a genuine feature of the heterogeneous system and it is ascribed to the presence of the droplet phase. In what follows, we investigate on the time-averaged Nusselt number $\langle Nu \rangle$, leaving the study of the origin and statistical properties of the fluctuations to a separated dedicated study [78].

To delve deeper into the behaviour of $\langle Nu \rangle_t$ with increasing droplet concentration, the natural question is whether one might capture it by means of a continuum approach. For this purpose, we ran simulations with the SP system with a homogeneous viscosity equal to the shear viscosity that we have measured (cfr. Fig. 2(b)), i.e.

$$\eta_{\text{initial}}^{\text{SP}}(\Phi_0) = \eta_{\text{eff}}(\Phi_0).$$

This is possible within our numerical approach, by changing the relaxation time of the lattice Boltzmann equation in such a way that the corresponding dynamic viscosity (cfr. Eq. (3)) matches the measured shear viscosity homogeneously throughout the system. Notice that SP systems constructed in that way, exhibit a Nusselt number independent of time, for the reasons posited before. In Fig. 5(a) we report the time-averaged Nusselt number $\langle Nu \rangle_t$ as a function of the droplet concentration $\Phi_0$ for both the heterogeneous emulsions and the homogeneous SP system. In the limit $\Phi_0 \to 0$ the time-averaged Nusselt numbers tend to coincide, as they should. At increasing $\Phi_0$, the Nusselt number measured in the SP simulations decreases monotonically, as expected for an increasingly viscous system. Therefore, we observe a mismatch with the behaviour of $\langle Nu \rangle_t$ in the emulsion case, which becomes particularly evident (with deviations up to roughly 10%) for intermediate values of the concentration and then decreases again at larger $\Phi_0$. Even if the two curves seem to join at $\Phi_0 = 0.4775$, this is not true for $\Phi_0 \geq 0.5$: further increasing the droplet concentration, the heat flux (i.e. the Nusselt number) is more and more inhibited for emulsion, whereas in the homogeneous case a sort of plateau is reached (data not shown).

This disagreement is caused by the predominance of non-Newtonian effects, whose analysis departs from the scope of this paper (a step in this direction has been taken in [78]). The assumption of a global effective viscosity equal to the one extracted from the shear rheology, is clearly not enough. In order to gain a better insight, it is worth reminding that for the thermal convection in a different soft system (polymer solutions), it was shown that variation in the heat flux could be understood in terms of a space-dependent effective viscosity (due, in that case, to the differential stretching of the polymers along with the cell height) [79]. Inspired by this observation, we inspected whether a non-homogeneous droplet distribution across the system. For this purpose, we monitored the droplet concentrations in the wall-to-wall coordinate ($y$), by averaging over time and along the mainstream flow direction ($x$). The resulting concentration profiles $\phi_0(y)$ are reported in Fig. 4(b). We observe, indeed, they are not constant.

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1 The Rayleigh number $Ra$ is defined as $Ra = \frac{\alpha g\Delta T H^3}{\nu \kappa}$, where $\nu$ is the kinematic viscosity. It provides information on the balance between buoyancy force and viscous friction force.
and exhibit a height-varying modulation, especially from low to moderate concentrations $\Phi_0$. The development of these non-homogeneous concentration profiles might be due to multiple factors, such as droplet migration induced by a non-uniform shear field \[80, 82\] (owing to the large scale circulation of convection) or droplet depletion due to interactions with the walls. A precise description disentangling these various mechanisms and discriminating which one contributes most, lies beyond the scope of the present work. Here, just take the emergence of such profiles as an empirical fact. This said it is nevertheless clear that the non-homogeneity relies on the fact that droplets are transported by the flow. Large $\Phi_0$ implies reduced mobility of the droplets, which is reflected in a relative suppression of profile modulation. To account for this aspect in the SP fluid model, we promote the effective viscosity to be a local quantity as well, i.e.

$$\eta_{\text{SP}}(y) = f(\phi_0(y)), \quad (9)$$

where the function corresponds to the fit displayed in Fig. 8 (black solid line). In Fig. 6 the Nusselt number obtained from SP simulations with the prescription \[9\] are compared with the $\langle \text{Nu} \rangle_t$ vs $\Phi_0$ data for the emulsions reported in Fig. 5. A first remark to be raised is that the emulsions data stay in between the two protocols: while the protocol \[8\] underestimates the emulsions data, the new protocol \[9\] overestimates them. A possible explanation for the deviations observed in comparing the emulsions data with the protocol \[9\] can be grasped by looking at the concentration profiles for the largest $\Phi_0$ in

$$\eta^\text{SP}_\Lambda(y) = f(\phi_\Lambda(y)), \quad (10)$$

Fig. 7 the latter are basically flat, but for an overshooting occurring in the wall-proximal regions whose extension is comparable to 1–2 droplet sizes (droplet layering). Forcing the continuum model SP fluid to vary its properties (effective viscosity) over such a micro-constituent size may then lead to artefacts. To overcome this problem, we propose to generalise the local effective viscosity \[9\] as follows:
normalised by the mean droplet diameter $d$ quantities are reported in simulation units. From protocol Eq. (10) using various resolutions of the coarse-grained parameter $\Lambda$ (cfr. Eq. (11)). The oscillations near the walls, stemming from the droplet layering, are smoothed out, highlighting that the relative variation of the effective viscosity is actually more important for the lower concentrations (panels (a) and (b)). Simulating the SP fluid with the choice (10) for the effective viscosity, indeed, yields the best agreement with the phenomenology of the emulsion in terms of the time-averaged heat flux $\langle \text{Nu} \rangle$, as shown in Fig. 9 for $\Lambda = 3d$ (we also plot the data for $\Lambda \rightarrow \infty$ which, not surprisingly, basically overlap with those for $\eta_{\text{rheo}}(\Phi_0)$). However, a mismatch between the two curves, emulsion and SP system with $\eta_{\text{rheo}}^{\Lambda}(y)$, occurs whenever the concentration is sufficiently high to trigger non-Newtonian effects. This non-Newtonian behaviour, which is inherent to the nature of systems, promotes the onset of highly viscous regions where heat transfer is to large extent suppressed. In this case, it becomes difficult to reproduce the SP system with the same heat transfer as the highly concentrated emulsion, based on the protocol (10); rather, it is necessary to consider the extra complication of a shear-dependent viscosity and study the associated non-local effects, as discussed in [78].

As mentioned above, the results obtained so far refer to a fixed buoyancy amplitude $\alpha g \Delta T = 1.86 \times 10^{-5}$ lbu (i.e. at fixed Rayleigh number $Ra$). It appears then natural to investigate the impact of changing the value of $\alpha g \Delta T$ on the protocol (10). To this aim, we have performed additional numerical simulations at different buoyancy amplitude $\alpha g \Delta T$ and compared the time-averaged Nusselt number $\langle \text{Nu} \rangle_t$ obtained from simulations of the emulsions at changing $\Phi_0$, with that of SP system with viscosity given by (10). Results are reported in Fig. 10 which displays a satisfactory agreement between the numerical simulations and protocol (10) for $\alpha g \Delta T$ spanning an order of magnitude, from $\alpha g \Delta T = 6.21 \times 10^{-6}$ lbu to $\alpha g \Delta T = 1.86 \times 10^{-5}$ lbu. Because of the coalescence of the droplets, larger values of the buoyancy amplitude $\alpha g \Delta T$ could not be explored in detail.

IV. CONCLUSIONS

We have analysed the heat transfer properties of concentrated emulsions just above the onset of convection, by means of extensive mesoscale simulations. The droplet concentrations $\Phi_0$ in the emulsions have been chosen to range systematically from very dilute situations to

\[ \phi_{\Lambda}(y) = \frac{1}{\Lambda} \int_{y-\Lambda/2}^{y+\Lambda/2} \phi_0(y') dy'. \]
situations with larger concentrations, around the point the emulsion stops behaving as a Newtonian fluid. We explored the heat transfer properties while keeping the droplet size finite, thus disclosing insights into the way a continuum picture (i.e. point-like droplets) is changed by the finite-size effects induced by a non-zero extension of the droplets. We find that the heat transport efficiency (i.e. the Nusselt number) displays a non-stationary character in time, while its time-average decreases at increasing $\Phi_0$. In the attempt of capturing the time-averaged Nusselt number $\langle \text{Nu} \rangle_t$ at changing droplet concentration $\Phi_0$, we pursued the idea of considering a single-phase (SP) system, equipped with a suitable choice of viscosity $\eta^{SP}$ that allows the SP system to display (on average) the same heat transport efficiency of the emulsions. Specifically, starting from the knowledge of the shear rheology for the emulsions $\eta_{eff}(\Phi_0)$, we investigated the suitable protocol that allows constructing $\eta^{SP}$. Due to the convective dynamics, the emulsion develops a non-homogeneous droplet distribution across the cell, implying that any choice of $\eta^{SP}$ must be local, i.e. acquire a space-dependence. Moreover, a quantitative analysis reveals that this local viscosity must be properly supplemented with a spatial averaging procedure ("coarse-graining"), over a scale that is of the order of the droplet size. For the future, various pathways are worth being pursued. As previously discussed, the coarse-graining procedure presented in this paper does not represent the end of the story: as a matter of fact, although it works well in reproducing the behaviour of the time-averaged Nusselt number, the fluctuations around that mean value are not retained in the SP systems. The correct way to capture those temporal fluctuations is not clear at this stage: they can be measured and characterised in the simulations with the emulsions, but the specific way to embed them in a continuum approach warrants a dedicated study. Moreover, in this paper, we deliberately studied emulsion concentrations that result only in Newtonian responses. Further increasing the droplet concentration would produce a non-Newtonian emulsion. A recent complementary study indeed reveals that, under those conditions, fluctuations in the Nusselt number are even more abrupt than those observed at the smaller concentrations and they are further accompanied by the emergence of space correlations and "bursts" of enhancement in the local heat transport. Overall, all these observations suggest that any approach aiming at a quantitative description of heat transfer in fluid-fluid dispersions must take into account the discrete nature of such complex fluids.

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