A modified star formation law as a solution to open problems in galaxy evolution

Lan Wang,† Simone M. Weinmann‡ and Eyal Neistein§

†Partner Group of the Max Planck Institute for Astrophysics, National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Road, Chaoyang District, Beijing, China
‡Leiden Observatory, Leiden University, PO Box 9513, 2300 RA Leiden, the Netherlands
§Max-Planck-Institute for Extraterrestrial Physics, Giessenbachstrasse 1, 85748 Garching, Germany

Accepted 2012 January 15. Received 2012 January 12; in original form 2011 July 22

ABSTRACT

In order to reproduce the low-mass end of the stellar mass function, most current models of galaxy evolution invoke very efficient supernova feedback. This solution seems to suffer from several shortcomings however, like predicting too little star formation (SF) in low-mass galaxies at \( z = 0 \). In this work, we explore modifications to the SF law as an alternative solution to achieve a match to the stellar mass function. This is done by applying semi-analytic models based on De Lucia & Blaizot, but with varying SF laws, to the Millennium and Millennium-II simulations, within the formalism developed by Neistein & Weinmann. Our best model includes lower SF efficiencies than predicted by the Kennicutt–Schmidt law at low stellar masses, no sharp threshold of cold gas mass for SF and an SF law that is independent of cosmic time. These simple modifications result in a model that is more successful than current standard models in reproducing various properties of galaxies less massive than \( 10^{10} \, M_\odot \). The improvements include a good match to the observed autocorrelation function of galaxies, an evolution of the stellar mass function from \( z = 3 \) to \( z = 0 \) similar to observations and better agreement with observed specific SF rates. However, our modifications also lead to a dramatic overprediction of the cold mass content of galaxies. This shows that finding a successful model may require fine-tuning of both SF and supernova feedback, as well as improvements on gas cooling, or perhaps the inclusion of a yet unknown process which efficiently heats or expels gas at high redshifts.

Key words: galaxies: evolution – galaxies: formation – galaxies: haloes – galaxies: stellar – content.

1 INTRODUCTION

Ever since first introduced by White & Frenk (1991), semi-analytic models of galaxy formation and evolution (hereafter SAMs; Kauffmann et al. 1999; Kang et al. 2005; Bower et al. 2006; Cattaneo et al. 2006; Croton et al. 2006; De Lucia & Blaizot 2007; Monaco, Fontanot & Taffoni 2007; Somerville et al. 2008; Khochfar & Silk 2009) have been successfully used to study how different physical processes determine the formation and evolution of galaxies. Based on halo merger trees extracted from \( N \)-body simulations or analytic methods, SAMs follow the main processes that are thought to affect the properties of galaxies, like gas cooling, star formation, feedback and merging. These models provide a useful tool to study the interplay and the relative importance of these different physical processes. A detailed review of the semi-analytic method can be found in Baugh (2006).

Although various observational properties of galaxies are matched by the models, current SAMs still have problems in reproducing some important observations, and up to now, there is no semi-analytic model that is able to fit all the key statistical properties of the observed galaxy population. For example, Guo et al. (2011) show that the model of De Lucia & Blaizot (2007) substantially overproduces the low-mass end of the stellar mass function (SMF) of galaxies. This problem becomes more severe as the resolution of the underlying dark matter simulation increases. In addition, SMF at high redshifts are normally not well reproduced (Fontanot et al. 2009; Marchesini et al. 2009); a good match to the two-point autocorrelation function of galaxies is so far difficult to achieve (e.g. Guo et al. 2011) and the relation between the specific star formation rate (SFR) and galaxy stellar mass deviates from observations (Somerville et al. 2008; Fontanot et al. 2009).

These discrepancies may have various reasons: inaccurate physical modelling of the processes that govern galaxy formation; technical problems in tuning the model against a large set of observational constraints; the use of a fixed functional form for a poorly
understood process, which overly limits the freedom in tuning the model; or a wrong cosmological model adopted in the simulations. This large range of possibilities makes it difficult to correct identified discrepancies between model and observations.

For example, Guo et al. (2011) tried to fix the overprediction of the low-mass end of the SMF, as modelled by De Lucia & Blaizot (2007), by increasing the effect of supernova (SN) feedback. They managed to reproduce the amplitude of galaxy SMF and also obtained a reasonable match to the galaxy luminosity functions in different bands. However, they also predict a too large fraction of red galaxies at low masses, too high amplitudes of the SMF in the redshift range of [0.8, 2.5] and galaxy autocorrelation functions that are too high for galaxies less massive than \( 6 \times 10^{10} \, M_\odot \).

Moreover, the high feedback efficiency as used by Guo et al. (2011) is physically difficult to motivate (Benson et al. 2003) and is far more efficient than various solutions adopted by hydrodynamical simulations (Mac Low & Ferrara 1999; Strickland & Stevens 2000; Avila-Reese et al. 2011).

In this work, we therefore explore an alternative solution. We tune the SF recipe instead of the SN feedback to study how our changes affect different statistics of galaxies, and to what degree the discrepancies mentioned above can be alleviated. Most of the current SAMs use an analogue to the empirical Kennicutt–Schmidt law (Schmidt 1959; Kennicutt 1998) to calculate the SFR in galaxies. In this standard prescription, the SFR is roughly proportional to the cold gas mass and scales inversely with the typical time-scale of a galactic disc (e.g. Cole et al. 2000). This law is combined with a sharp threshold at low gas densities, below which no SF occurs (Kauffmann 1996; Croton et al. 2006). The use of a such a threshold is motivated both theoretically (e.g. Toomre 1964; Kennicutt 1989; Schaye 2004) and observationally (e.g. Martin & Kennicutt 2001).

This simple SF law is however likely an oversimplification. In recent years observational determinations of the SF law in galaxies have become increasingly refined, using various gas components (HI, CO) in combination with more reliable estimates of SF, based on ultraviolet and infrared (IR) light. These recent findings can be summarized as follows: first, there are indications that the threshold for SF at low mass densities is not sharp. Instead, the SF efficiency (SFE) drops off as a steep power law at low gas densities (Kennicutt et al. 2007; Bigiel et al. 2008; Roychowdhury et al. 2009; Wyder et al. 2009; Bigiel, Leroy & Walter 2011). Second, several studies find that the SFR is correlated more strongly with the mass of molecular gas (H_2) than with the atomic gas (Wong & Blitz 2002; Bigiel et al. 2008; Leroy et al. 2008), probably also at high redshift (Bouché et al. 2007; Genzel et al. 2010). This shows that simply correlating the SFR with the total cold gas density in models may not always lead to realistic results. It is also not clear which gas mass correlates best with the SFR when averaging over the entire galaxy (e.g. Saintonge et al. 2011). Third, there are both theoretical and observational indications that the normalization of the SF law might be lower at high redshift than locally (e.g. Wolfe & Chen 2006; Gnedin & Kravtsov 2010; Agertz, Teyssier & Moore 2011; Krumholz & Dekel 2011; Rafelski, Wolfe & Chen 2011). This means that simply extrapolating the local relation, as done in most SAMs, may be incorrect.

Several recent models have attempted to address these issues. Baugh et al. (2005), Weinmann, Neistein & Dekel (2011a) and Krumholz & Dekel (2011) specifically lower the quiescent SF efficiencies at high redshifts in order to better match some observed properties of high-redshift galaxies. Both Fu et al. (2010) and Lagos et al. (2011a,b) focus on the first two of the above points and present SAMs with updated and much more detailed SF recipes in comparison to previous SAMs. They do not include a sharp threshold for SF, and their SFR depends on the molecular gas density instead of the cold gas mass, following recent empirical and theoretical models (e.g. Blitz & Rosolowsky 2006; Krumholz, McKee & Tumlinson 2009). Their detailed models for SF depend on various internal properties of the disc, like size and pressure, which are not trivial to model in an SAM.

It is certainly worthwhile to try and include a more detailed and observationally and theoretically better motivated SF law into SAMs. We present a complementary approach in this work, without taking into account complicated processes on subgalactic scales, like the conversion from atomic to molecular gas. We instead try to solve in a straightforward way the inverse problem, namely which realistic SF law at galactic scales is required to improve the agreement between model galaxies and observations. We start with a standard model, similar to De Lucia & Blaizot (2007) and Neistein & Weinmann (2010), and assume that the SFR depends on the cold gas mass in the galaxy, cosmic time and in addition the host halo mass. We make a simple change to the standard SF laws that are qualitatively, but not quantitatively, plausible and explore how these impact the properties of galaxies. We show that we can improve the agreement between SAMs and observations in several key aspects in this way.

We use the model developed by Neistein & Weinmann (2010) and implement it on both the Millennium Simulation (MS; Springel et al. 2005) and the Millennium-II Simulation (MS-II; Boylan-Kolchin et al. 2009). The properties of galaxies with masses as low as \( \sim 10^9 \, M_\odot \) can probably be studied reliably with the help of these simulations (Guo et al. 2011), although this can be resolution dependent for extreme models (see Neistein & Weinmann 2010). Our aim is to investigate how much a model based on De Lucia & Blaizot (2007) can be changed and improved by tuning the SF law alone. We thus do not change gas cooling and feedback in our models, but leave it at the default standard values (except in one case, for illustration, as will be explained).

This paper is organized as follows. In Section 2 we present the different models used in this work. Our starting point is a model that is based on the widely used SAM of De Lucia & Blaizot (2007), with an improved prescription for hot gas stripping of satellite galaxies. We then develop four different models. Of these, models 2 and 3 include the key changes to the quiescent and burst mode SF. In Section 3 we present predictions for the SMF at low and high redshifts, the relation between galaxy specific SFR (SSFR) and stellar mass, the galaxy cold gas mass function, the autocorrelation functions and the SFR density as a function of redshift. A discussion of the implications of our results and the conclusions are presented in Section 4.

### 2 MODELS

The semi-analytic models presented in this paper are applied to both the MS and MS-II. The cosmological parameters in the simulations are consistent with a combined analysis of the 2dF Galaxy Redshift Survey (2dFGRS; Colless et al. 2001) and the first year Wilkinson Microwave Anisotropy Probe (WMAP) data (Spergel et al. 2003), with \( \Omega_m = 0.25, \Omega_b = 0.045, h = 0.73, \Omega_\Lambda = 0.75, n = 1 \) and \( \sigma_8 = 0.9 \). Note that these parameters are different from the latest WMAP 7-yr results. Both simulations follow \( N = 2160^3 \) particles from redshift \( z = 127 \) to the present day. The MS has a particle mass resolution of \( 8.6 \times 10^9 \, h^{-1} M_\odot \), with a comoving box of \( 500 \, h^{-1} \) Mpc on a side. The MS-II has a mass resolution of \( 6.9 \times 10^9 \, h^{-1} M_\odot \), with a box of side 100h^{-1} Mpc.
Neistein & Weinmann (2010) (hereafter NW2010) developed a new formalism for modelling galaxy formation and evolution, which is similar to the standard SAMs, except that the efficiencies of processes like gas cooling, SF and feedback are assumed to depend only on the host halo mass and cosmic time. NW2010 have shown that this new method produces a very similar population of galaxies like standard SAMs. The method is simple and flexible, which makes it easy to change recipes in order to fit selected observational constraints.

All the models within this work are based on a simple set of differential equations that follow the mass of gas and stars within galaxies. We adopt the model of De Lucia & Blaizot (2007, hereafter DLB07) as our starting point, as was done in NW2010. The reader is referred to these papers, and to Croton et al. (2006), for more details on the model assumptions. Here we highlight a few features that will be important for the discussion below. The SF law is assumed to be

$$M_{\text{star}} = f_\text{s}(M_{\text{cold}} - M_{\text{crit}}),$$

where $f_\text{s}$ is the SF efficiency, and $M_{\text{crit}}$ is the critical mass of cold gas below which no SF occurs (Kennicutt 1998).

Satellite galaxies are followed along with their host subhaloes. Once the subhaloes are stripped and cannot be identified anymore, we compute the radial distance, $r_{\text{sat}}$, between the satellite and the central subhalo within the group. We then allow the satellite galaxy to spiral in further and estimate the time it merges into the central object by using a dynamical friction estimate:

$$t_{\text{df}} = \frac{\alpha_{\text{df}}}{GM_{\text{h}} \ln(1 + M_{\text{h}}/M_1)}.$$  

Here $C_{\text{df}}$ is the Chandrasekhar estimate for the dynamical friction time-scale, where $V_1$ is the virial velocity of the central subhalo, $M_1$ is its mass and $M_1$ is the baryonic (cold gas and stellar) mass of the satellite galaxy. $\alpha_{\text{df}}$ describes the ratio of the adopted dynamical friction time over the Chandrasekhar estimate. When galaxies finally merge we assume an SF burst of the type

$$M_{\text{star,burst}} = \alpha_{\text{burst}}(M_1 \text{, cold} + M_2 \text{, cold}),$$  

with

$$\alpha_{\text{burst}} = 0.56(M_1/M_2)^{0.7}.$$  

Here $M_1$ and $M_2$ denote the baryonic mass in the merging galaxies. Following Croton et al. (2006), this formula is derived from fitting the results of hydrodynamical simulations (e.g. Cox et al. 2004, see Section 2.4 for more details).

In the following subsections, we describe all the models used in this paper in detail.

### 2.1 Model 0

Our model 0 is very similar to model 0 in NW2010 and should thus also be similar to DLB07. For the SF law above (equation 1) we use the same fitting functions as in NW2010:

$$f_\text{s} = 2.04 M_1^{0.094} 10^{-0.038[\log M_1]^{2}} t^{-0.82}$$  

and

$$M_{\text{crit}} = 0.36 f_\text{s}^{1} M_{12}^{0.68} t^{-0.52}.$$  

Here $t$ is the Hubble time in units of Gyr, and $M_{12} = M_{\text{halo}}/10^{12}$ is the halo mass in units of $10^{12} h^{-1} M_{\odot}$.

There are a few minor modifications made here in comparison to NW2010, which are related to the extended range in halo mass we use here (a minimum halo mass of $\sim 10^{7} h^{-1} M_{\odot}$ in comparison to $\sim 10^{10} h^{-1} M_{\odot}$ in NW2010). For more details on how we extend the recipes from NW2010 to low-mass haloes, the reader is referred to Appendix A. Fig. 1 shows that the amplitude of the low-mass end of the SFM for model 0 is comparable to the DLB07 result when applied to the MS-II.

### 2.2 Model 1

Model 1 is the fiducial model used in this work. It is based on model 0 as presented in the previous subsection, but includes two further changes: the hot gas stripping of satellite galaxies is slowed down considerably compared to DLB07 following Weinmann et al. (2010), and a larger dynamical friction time is assumed.

In the DLB07 model, the hot gas component of a galaxy is stripped completely once it falls into a larger group and becomes a satellite. Satellite galaxies subsequently consume their remaining cold gas due to SF and efficient SN feedback, and the SFR ceases on a short time-scale of 1–2 Gyr. This leads to most satellite galaxies displaying red colour, which is in contradiction with observations (Wang et al. 2007; Weinmann et al. 2009). Following the model suggested by Weinmann et al. (2010), we assume in model 1 that the hot gas component of satellite galaxies decreases at the same rate as their surrounding dark matter haloes, which lose mass due to tidal stripping. This treatment provides a physically better motivated description of the behaviour of the hot gas component of satellite galaxies and improves agreement with observations (Weinmann et al. 2010). A similar model is included in the recent SAM of Guo et al. (2011).

The second change with respect to model 0 is that the parameter $\alpha_{\text{df}}$, which describes the ratio of the dynamical friction time for galaxy mergers over the Chandrasekhar formula (equation 2 above), is set to $5$, in contrast to the value 2 as adopted in model 0 and in DLB07. This corresponds to a larger time-scale for satellite galaxies to merge with the central galaxy and is chosen in order to get...
autocorrelation functions in better agreement with observations for all the models (see below). Up to now there is no solid consensus on what \( \alpha_{\text{crit}} \) should be in models (Boylan-Kolchin, Ma & Quataert 2008; Jiang et al. 2008; Mo, van den Bosch & White 2010).

The green lines in Fig. 1 show the SMF of the fiducial model 1, with dashed and solid lines for results when applied to the MS and MS-II. Model 1 SMF are in general slightly higher than those of model 0. This is mainly due to the slower stripping of hot gas in satellites adopted in model 1. Retaining their hot gas reservoir for longer satellite galaxies can continue forming stars for a considerable time and thus end up with a higher stellar mass than in model 0. When merging into central galaxies, they also add more mass to their centrals. Although our choice of a larger dynamical friction time, with \( \alpha_{\text{crit}} = 5 \), delays merger to some degree and thereby decreases the amount of mass added to centrals, this effect is apparently smaller than that of the modification to the hot gas stripping. When comparing the results of models 0 and 1 for the MS and MS-II, the SMF in the MS are found to exceed the SMF in the MS-II at intermediate masses. This is not the case for DLB07 (Guo et al. 2011). This excess may be related to the fact that our extension of the cooling efficiencies does not exactly match those used for DLB07 in Guo et al. (2011).

2.3 Model 2

In model 2, we make modifications to the SF law in the quiescent mode and keep all the other components of the model exactly the same as in the fiducial model 1. The modifications include: (a) \( M_{\text{crit}} = 0 \), which means no threshold cold gas mass for SF; (b) the SFE depends strongly on the halo mass; (c) the SFE does not depend on the Hubble time.

The SF law in model 2 is chosen such that the low-mass end slope of the SMF in the model is comparable to the observed SDSS result, when applied to the MS. This results in

\[
M_{\text{crit}} = 0, \tag{7}
\]

\[
f_s = 0.41 \, M_{\odot}^{0.94} \left( \frac{\text{log} M_{\ast}}{10} \right)^{-0.30}. \tag{8}
\]

Fig. 2 compares the SFE (defined as the ratio between SF and the mass of the cold gas) in the quiescent mode in models 1 and 2, as a function of halo mass at two redshifts. The green lines are efficiencies in model 1. The solid green lines are for a cold gas–halo mass ratio of 0.04/0.09 at \( z = 0 \) and 0/0.5 at \( z = 3 \), which corresponds to the median cold gas–halo mass ratio in model 1. The dashed green lines are for a higher cold gas–halo mass ratio of 0.13/0.14 at \( z = 0 \) and \( z = 3 \), and the dotted green lines are for a lower ratio of 0.007/0.05 at \( z = 0 \) and \( z = 3 \). These values correspond to the 16th and 84th percentiles of the distributions in cold gas masses in model 1. The SFE in model 2 is shown as red-blue lines, which follow a power law of index 2 (plotted as black dotted lines in Fig. 2) for haloes less massive than \( \sim 10^{11.5} h^{-1} \, M_\odot \). Compared to model 1, model 2 has a lower SFE at \( z = 0 \) for halo masses between \( 10^{10} \) and \( 10^{11.5} \, M_\odot \). At \( z = 3 \), the SFE is much lower in model 2 than in model 1 for haloes less massive than \( \sim 10^{11} h^{-1} M_\odot \).

2.4 Model 3

When applying model 2 to the MS, the amplitude of the SMF at the low-mass end decreases dramatically and matches the observation of SDSS (red dashed line in Fig. 3). However, when applied to the MS-II, the low-mass end of the SMF is still too high (red solid line in Fig. 3). We have tested that even when truncating all SF in quiescent disc mode in haloes less massive than \( \sim 10^{11.5} h^{-1} M_\odot \), the problem still exists. This is because galaxy mass grows not only by quiescent SF in discs, but also by merger-induced bursts. These become more significant if the quiescent SFE is decreased, due to the resulting increase in the cold gas masses.

In model 3, we therefore further modify the SF law in the burst mode, while keeping the same SF law for the quiescent mode as in model 2. The starburst efficiency used in current SAMs (equation 4) is derived from fitting the results of hydrodynamical simulations for mergers of mass ratios ranging from 1:10 to 1:1 (Mihos & Hernquist 1994, 1996; Cox et al. 2004). The parameters in those simulations are set to make the SF in an isolated disc galaxy consistent with the Kennicutt–Schmidt law. As the quiescent SF of dwarf galaxies themselves is lower than that predicted by the Kennicutt–Schmidt law, it is quite possible that those simulations overpredict the burst efficiency for low-mass galaxies. Therefore we introduce a halo mass dependence for the efficiency of merger-induced bursts in model 3 and make it inefficient for low-mass haloes.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Star formation efficiencies in the quiescent mode as a function of the halo mass at \( z = 0 \) (left-hand panel) and \( z = 3 \) (right-hand panel). The green lines are the efficiencies in model 1. The dashed, solid and dotted green lines are for the cases when the cold gas–halo mass ratios are [84, 50, 16] percentile values in model 1, which are [0.13, 0.04, 0.007] at \( z = 0 \) and [0.14, 0.09, 0.05] at \( z = 3 \). The red-blue line is the efficiency of SF for models 2 and 3. The magenta lines are for efficiencies in model 3b. The black dotted line shows a power law with index 2 for reference.
For haloes less massive than $M_0$, we modify the burst efficiency to be

$$\alpha_{\text{burst}} = 0.56(M_1/M_0)^{0.7} \times (M_{\text{halo}}/M_0),$$

while it remains unchanged for higher mass haloes. We use $M_0 = 10^{11.5} h^{-1} M_\odot$. This critical halo mass is empirically determined and roughly corresponds to the mass where the relation between the galaxy stellar mass and halo mass changes its slope (e.g. Wang et al. 2006; Moster et al. 2010), and galaxy formation efficiency reaches its maximum (Guo et al. 2010). Physically, SN feedback and reionization are believed to cause a low galaxy formation efficiency at low halo masses, and active galactic nucleus (AGN) feedback may be responsible for the low efficiency in high-mass haloes (Guo et al. 2010). The combined effect of these mechanisms may be weakest at this critical halo mass, explaining the peak in galaxy formation efficiency in the current theory. Therefore we choose this value as the threshold below which the SFE is assumed to be suppressed. The typical stellar mass of galaxies that reside in those haloes is about $10^{10.5} M_\odot$, below which models 0 and 1 predict too many galaxies and thus too much SF. By decreasing the SF for galaxies with halo mass less massive than this critical value, the amplitude of the low-mass end of the SMF can be suppressed effectively. The functional form in equation (9) is chosen in order to fit the amplitude of the SMF at the low-mass end when applied to both MS and MS-II.

We have also tested a model in which we shut off all SF in the burst mode in low-mass haloes and keep the SF in the quiescent mode the same as in the fiducial model 1. For such a model, the low-mass end of the SMF is still higher than observation. This indicates that SF in both the quiescent and the burst mode must be modified simultaneously to suppress the numbers of low-mass galaxies effectively. Note that the values of the power-law indices that determine the dependence of SF on halo mass for quiescent and burst modes might have some degeneracy. However, we do not study the degeneracy of the two modes of SF in this work any further, but focus on the qualitative effect of modifying SF in each mode.

### 2.5 Model 3b

In model 3b, we test the effect of including a dependence of the SFE on the Hubble time. Model 3b is almost identical to model 3, except that the SFE in the quiescent mode is assumed to depend on cosmic time, in the same way as for the fiducial model 1. The SFE is rescaled to match again the low-mass end of the SMF in SDSS observation, resulting in

$$f_s = 1.74 M_{12}^{0.94} 10^{-0.30[\log(M/12)^2]} t^{-0.82}.$$  

The SFE in model 3b is also shown in Fig. 2. It is higher than the SFE in model 3 at high redshift and is lower at $z = 0$. We will see in the next section that compared with model 3, model 3b results in a similar SMF at $z = 0$, and higher amplitude of SMF at high redshifts of $z \sim 2–3$. The lower SFR at redshift 0 results in low-mass galaxies with a lower SSFR than in model 3. Besides, model 3b predicts a higher SFR density at redshifts higher than $\sim 2$ and a lower SFR density at lower redshifts. The general results are comparable with model 3 though.

### 2.6 Model 4

With a modified SF law in low-mass haloes, models 3 and 3b are able to reproduce many observational statistics of low-mass galaxies, as we will show in Section 3. However, the properties of high-mass galaxies differ significantly from observations. The most obvious deviation is that the modelled massive galaxies are in general too active. This is mainly due to the inclusion of slower stripping of hot gas for satellite galaxies in our models. The deviation can be alleviated by allowing for less efficient cooling in massive haloes. Physically, this corresponds to mechanisms like the stronger AGN feedback effect that prevents gas from cooling.

In model 4, we apply further modifications of cooling and SF to model 3, focusing on massive galaxies. Model 4 is presented as a simple test to see if the properties of massive galaxies can be better fitted, while keeping the treatment of low-mass galaxies unchanged. The modifications include the following:

(i) Lower cooling efficiencies are assumed for haloes more massive than $10^{11.75} h^{-1} M_\odot$, as shown in Fig. A2 in Appendix A.

(ii) SF in both the quiescent and the burst mode is stopped completely in haloes more massive than $5 \times 10^{12} h^{-1} M_\odot$ at $z < 1.3$.

(iii) The dynamical friction time is assumed to be dependent on the Hubble time and is shorter at higher redshift, with $\alpha_{\text{df}} = 5 \times (t/13.6)^{0.5}$ instead of $\alpha_{\text{df}} = 5$ in model 3 (see Weinmann et al. 2011a).

These modifications are done to fit the observed properties of massive galaxies better. The first two modifications make massive galaxies much more passive than in model 3. The change in the dynamical friction time follows the idea of Weinmann et al. (2011a). As discussed there, the merger time in the standard model may be overestimated by an order of magnitude at high redshift, mainly due to the more radial orbits of high-redshift satellite galaxies (Dekel, Sari & Ceverino 2009; Hopkins et al. 2010). Therefore we assume a time-dependent dynamical friction time, which gives a better fit to the SFR density and SMF at $z > 2$ and does not affect the other statistics studied in this work much.
3 RESULTS

In this section, we show statistical results for the galaxy population produced by the models presented in the previous section, including galaxy SMF at both low and high redshifts, the SSFR–stellar mass relation, the cold gas mass function, the projected two-point correlation functions and SFR density as a function of redshift. We compare these results with observations and analyse the effect of different modifications of SF laws on those statistics.

3.1 Stellar mass function at $z = 0$

The SMF at $z = 0$ is the main quantity that was used to constrain the parameter values in our models, and is plotted in Fig. 3. Similar to previous SAMs (DLB07; Guo et al. 2011), the fiducial model 1 predicts too many low-mass galaxies. As a result of the modifications in the SF law in low-mass haloes, model 2 applied to the MS fits the SMF for stellar masses less massive than $10^{9.3} \, M_\odot$. However, when applied to the MS-II, model 2 still predicts too many low-mass galaxies. By suppressing the SF in the burst mode in low-mass haloes, model 3 gives a reasonable fit to the observed SMF, for both MS and MS-II. Including a dependence of the SF on cosmic time in model 3b does not affect the SMF notably. For model 4 which changes further the cooling and SF in high-mass haloes, the SMF at $z = 0$ is somewhat lower at the massive end and is more consistent with observations.

As mentioned in Section 2, we have tested that when shutting off all SF in either quiescent mode or burst mode in haloes less massive than $10^{11.5} \, h^{-1} \, M_\odot$, the predicted SMF still exceeds observations at low-mass end, since SF from the other mode compensates. This indicates that SF in both quiescent and burst mode must be modified simultaneously to fit the observed SMF, as is done in models 3 and 3b. This also explains why Lagos et al. (2011a) find no obvious change in the resulting SMF when only modifying the SF in the quiescent mode in their SAM.

3.2 SSFR–$M_{\text{star}}$ relation

The SSFR is defined as the ratio between the SFR and the stellar mass of a galaxy. The relation between the SSFR and galaxy stellar mass is a fundamental observable which needs to be reproduced by a successful model. Observationally, there is a clear trend that high-mass galaxies are passively evolving with a low SSFR, while low-mass galaxies have high values of SSFR (Salim et al. 2007; Schiminovich et al. 2007). SAMs, however, usually predict a similar, if not lower, SSFR in low-mass galaxies than that in massive ones (Fontanot et al. 2009). Comparisons that focus on galaxy colours also show this discrepancy, with low-mass galaxies having redder colours than observed (Guo et al. 2011, but see also Weinmann et al. 2011b).

Fig. 4 shows the SSFR–$M_{\text{star}}$ relation in our models, compared with the SDSS observational result. The upper panels are for models applied to the MS, and the lower panels show the MS-II results. In each panel, black contours are the model results, while blue contours are SDSS results. Observationally, galaxies reside in two distinct sequences in the SSFR–stellar mass plot: an active sequence with high SSFR that is more prominent for low-mass galaxies and a passive sequence with lower SSFR that contains mainly massive galaxies. The observational location of the passive sequence, however, is not well determined, due to the uncertainty on measuring the SSFR for galaxies with little SF (Salim et al. 2007). In addition, Fig. 5 shows the median SSFR as a function of the galaxy stellar mass. The coloured lines are predictions from our models combined with the MS-II. The black solid line is the result of the SDSS DR7 galaxy sample, which shows clearly that more massive galaxies have in general lower SSFR.

Comparing the model results applied to the MS and MS-II, respectively, in Fig. 4, it is obvious that the resolution of simulation has a large effect on the modelled SSFR for low-mass galaxies, especially for models 1 and 2. This highlights the fact that resolution can have a large impact on semi-analytical model predictions (see also Guo et al. 2011). We note that the resolution dependence is weaker
Figure 5. The median SSFR as a function of the galaxy stellar mass. The black solid line shows observational results from the SDSS DR7 galaxy sample. The green, red, blue, magenta and cyan lines are predictions from our models 1, 2, 3, 3b and 4, respectively, combined with the MS-II.

Figure 6. Cold gas mass functions at $z = 0$. The green, red, blue, magenta and cyan lines are for models 1, 2, 3, 3b and 4, respectively, for MS-II results. The cross symbols show the Schechter function fit for the cold gas mass function according to Obreschkow & Rawlings (2009).

Figure 6. Cold gas mass functions at $z = 0$. The green, red, blue, magenta and cyan lines are for models 1, 2, 3, 3b and 4, respectively, for MS-II results. The cross symbols show the Schechter function fit for the cold gas mass function according to Obreschkow & Rawlings (2009).
A modified star formation law

however not well fitted by current SAMs. For example, Guo et al. (2011) overpredict the correlation functions for low-mass galaxies and on small scales. They suggest that the overprediction of clustering of galaxies on small scales in the models can be explained by the too high value of $\sigma_8$ adopted by the MS, which is 0.9 compared to 0.81 suggested by the WMAP 7-yr result (Komatsu et al. 2011). However, it is not certain that the Guo et al. SAM combined with the correct cosmology would give correlation functions in agreement with observations (see Wang et al. 2008). Here we test the effect of modifying the SF law on the resulting correlation functions using the same underlying N-body simulations as Guo et al. (2011).

In our models, galaxy positions are determined by the positions of haloes/subhaloes they reside in. For galaxies that have lost their host subhalo due to stripping, we use the location of the most-bound particle of the last identified subhalo. Since we use the same dynamical friction prefactor of $\alpha_{df} = 5$ for models 1, 2, 3, 3b and 4, galaxy locations in all these models are exactly the same. The only differences are in the stellar masses of galaxies that vary due to the different SF laws used. Also, the number of galaxies might be slightly different due to the effect of the stellar mass on the dynamical friction time. Consequently, the differences in the correlation functions between the models are mainly due to the different stellar mass assigned to each galaxy.

Fig. 7 shows the projected correlation functions of galaxies in different stellar mass bins, computed in the same way as in Neistein et al. (2011a). Model 1 overpredicts the correlation functions for galaxies less massive than $10^{11.27} M_\odot$, and at scales smaller than $\sim 1 h^{-1} \text{Mpc}$. With a modified SF law in low-mass haloes, correlation functions become lower at small scales for low-mass galaxies, which brings model 3 into agreement with the observational results at all stellar masses. The success of model 3 in reproducing the correlation functions shows clearly that changes in the SF law, that lead to changes in the relation between stellar mass and subhalo mass, can significantly affect correlation functions. Thus, it is in general very hard to say whether a mismatch with observed correlation functions indicates a problem with baryonic recipes, or with cosmology.

We thus confirm the results by Wang et al. (2008) who find that a similar match to observed correlation functions can be obtained by SAMs using different cosmological parameters, depending on the detailed baryonic recipes.

Model 3b gives similar results to model 3 for galaxies more massive than $10^{10.27} M_\odot$. For lower mass galaxies, the correlation functions of model 3b are higher than for model 3 on small scales. Correlation functions of galaxies in model 4 are in general similar to model 3, with a somewhat worse fit in the stellar mass bin of $\log M_{\text{star}} = [10.77, 11.27]$.

3.5 Stellar mass functions at high $z$

At redshifts less than $\sim 0.8$, current SAMs like Monaco et al. (2007) and Somerville et al. (2008) predict SMF consistent with observations. At higher redshifts, however, models normally overpredict the abundance of galaxies less massive than $10^{10} M_\odot$ (Fontanot et al. 2009; Guo et al. 2011). The same is true for the $K$-band luminosity function (Henriques et al. 2011). This may indicate that SF at redshifts above 0.8 is not modelled correctly in these models.

Figure 7. Projected two-point autocorrelation functions of galaxies in different stellar mass bins. The circles with error bars are SDSS DR7 results (Guo et al. 2011), calculated with the same method as presented in Li et al. (2006). The green, red, blue, magenta and cyan lines are for models 1, 2, 3, 3b and 4, respectively. The dotted (dashed) lines are for models based on the MS-II (MS) results. For stellar mass bins with $\log (M_{\text{star}}) > 9.77$, only MS results are shown. For the lowest mass bin, only MS-II results are presented.

© 2012 The Authors, MNRAS 421, 3450–3463
Monthly Notices of the Royal Astronomical Society © 2012 RAS
Fig. 8 gives the results of SMF at higher redshifts in different models studied in this work. Model results are presented at redshifts of 0.8, 1.2, 2.0, 3.0, and are convolved with a Gaussian error of deviation 0.25 dex in log $M_{\text{star}}$, to account for the various errors in estimating stellar masses in observations (Fontanot et al. 2009). Observational results are shown at comparable redshift ranges to the models. The gold symbols indicate data points below the limiting stellar mass of the observed galaxy samples, where incompleteness could be significant (Pozzetti et al. 2007; Kajisawa et al. 2009). All observational stellar masses of galaxies are normalized to the Chabrier IMF (Chabrier 2003), to be consistent with the previously shown SDSS SMF, and the model derived SMF.

The fiducial model 1 predicts too many low-mass galaxies at all redshifts. With a modified SF law in both modes, both models 3 and 3b give consistent SMF with observations up to redshifts of around 3. Differences of these two models can be seen at redshifts higher than 2, where model 3b predicts an SMF with a higher amplitude. This is because the SFE in model 3b depends on time and is higher at high redshifts.

In model 4, with the assumption that the dynamical friction time is shorter at high redshifts, the SMF at redshifts of around 2–3 are higher than those in model 3, and somewhat closer to observations. The SMF in model 4 happen to be quite similar to the results of model 3b. This reflects the degeneracies inherent in SAMs, in this case between the dependence of dynamical friction time on redshift and the dependence of the SFR on redshift.

### 3.6 SFR density

Fig. 9 shows the SFR density as a function of redshift, as predicted by different models combined with the MS. The black crosses are observational results compiled by Hopkins (2007). The grey shaded region shows the 1σ confidence level of the observational results by Wilkins, Trentham & Hopkins (2008), which are derived indirectly from the evolution of the SMF. The gold symbols are the results of Bouwens et al. (2009), including the contributions from highly dust obscured galaxies and ultra-luminous infrared galaxies (ULIRGs). Stellar masses are normalized to the Chabrier IMF (Chabrier 2003).

The SFR density of the fiducial model 1 shows a continuous increase with redshift and peaks at a redshift of around 3–4, which is clearly a higher redshift than in observations. This offset is similar to the one present in the SAM of Guo et al. (2011). With a modified SFE in both the quiescent and the burst mode, the SFR density of models 3 and 3b drops dramatically at higher redshifts than $z \sim 2$. Due to the dependence of SF on the Hubble time assumed in model 3b, this model gives a higher SFR density at high $z$ and a lower SFR density at low $z$ than model 3. At low redshifts, both models 3 and 3b predict the higher SFR density than the fiducial model 1, which lies slightly above the observational values.

With further suppression of cooling and SF in massive haloes in model 4, the observed sharp decline of SFR density towards low redshifts appears. Predictions of model 4 are within the observational constraints, while the SFR density peaks at a redshift
Figure 9. Cosmic SF density as a function of redshift. The green, red, blue, magenta and cyan lines are for models 1, 2, 3, 3b and 4, respectively, based on the MS. The black crosses are observational estimates compiled by Hopkins (2007). The grey shaded region shows the 1σ confidence level of the observational result, as compiled by Wilkins et al. (2008). The gold symbols are the results of Bouwens et al. (2009), including the contributions from highly dust obscured galaxies and ULIRGs. Stellar masses are normalized to the Chabrier IMF (Chabrier 2003).

of around 2. Recently, Magnelli et al. (2011) studied the evolution of the dusty IR luminosity function using Spitzer data. Assuming a constant conversion between the IR luminosity and SFR, they found that the SFR density of the Universe strongly increases towards $z = 1.3$ and stays constant out to $z = 2.3$. Model 4 matches the result of their observation.

4 DISCUSSIONS AND CONCLUSIONS

We use the method developed by NW2010, combined with both the MS and MS-II cosmological simulations, to study the effect of modifying the SF recipe in low-mass galaxies. We show that by modifying SF in both the quiescent and the burst mode, the SMF observed in the local Universe can be reproduced well down to $10^9$ $M_{\odot}$. Simultaneously, the models can fit the observed median SSFR–$M_{\text{star}}$ relation for galaxies less massive than $10^{10}$ $M_{\odot}$, the correlation functions for galaxies more massive than $10^{10.75}$ $M_{\odot}$, the SMF up to the redshift of around 3 and the general trend of the SFR density as a function of redshift.

The modifications to the SF recipe in our models with respect to standard SAMs (e.g. DLB07) include

(i) no sharp threshold in the cold gas mass for SF;
(ii) letting the SFE in the quiescent mode depend on host halo mass;
(iii) removing the dependence of the SFR on the Hubble time (which came via the dependence on disc dynamical time);
(iv) a lower starburst efficiency in low-mass haloes;
(v) additional modifications of cooling and SF in massive haloes to match the properties of high-mass galaxies.

Model 2 includes only the changes to the quiescent mode of SF, (i)–(iii). We show that this is not enough to reproduce the low-mass end of the SMF, as SF in the burst mode compensates for the changes to the SF law in the quiescent mode. This is also the reason why Lagos et al. (2011a) have found that their changes to the SF law in the quiescent mode do not change the resulting SMF much. In model 3, we have thus additionally decreased SF in the burst model (modification iv), which results in a clearly improved SMF. We also considered a model 3b which is similar to model 3, except that we allow for the usual time dependence of the SF law (i.e. do not make modification iii). Results of model 3b are similar to model 3 out to $z \sim 2$, except that the SSFR–$M_{\text{star}}$ relation is slightly less well reproduced. It is thus not clear whether modification iii is necessary.

Note that removing the time dependence of the SFE is a significant change with respect to previous models. Even in the recent models of Fu et al. (2010) and Lagos et al. (2011a,b), where SFE does not depend on the disc dynamical time of galaxies, a time dependence enters via the conversion efficiency from atomic to molecular gas, which depends on the gas density. In order to justify the behaviour we suggest in model 3, we would need to postulate a mechanism that scales with time in the opposite way than usually assumed, like for example a metallicity-dependent conversion of atomic to molecular gas (Krumholz & Dekel 2011). We note that a weak dependence of the SFE on cosmic time was seen in hydrodynamical simulations (Neistein et al. 2011b), although the reason for this behaviour is still not clear.

In our models 2–4, the ratio between the quiescent SFR and the cold gas mass, $M_{\text{star}}/M_{\text{cold}}$, which is equal to the gas consumption time-scale, is roughly proportional to $M_{\text{halo}}^{46}$ for low-mass haloes and is almost independent of halo mass for high masses. This may in fact be supported by recent observations. Recently, Shi et al. (2011) derived an extended Schmidt law from an observed galaxy sample that extends over five orders of magnitude in stellar density, including galaxies with low surface brightness. They found that $M_{\text{star}}/M_{\text{cold}}$ is proportional to $M_{\text{halo}}^{0.52}$, with a 1σ scatter of 0.4 dex. The stellar mass of galaxies has been claimed to obey a tight relation with the host halo mass (Conroy & Wechsler 2009; Guo et al. 2010; Moster et al. 2010). It is proportional to $M_{\text{halo}}^{2.8}$ for low-mass haloes and to $M_{\text{halo}}^{0.2}$ at high-mass end (Wang et al. 2006). Without considering the scatter of the relation, this indicates that the observed $M_{\text{star}}/M_{\text{cold}}$ is roughly proportional to $M_{\text{halo}}^{0.46}$ at low masses and $M_{\text{halo}}$ at high-mass end. The dependence on halo mass in our models is therefore quite close to the observational result by Shi et al. (2011).

In all our models, the cold gas mass function of galaxies is dramatically overpredicted. This is because in DLB07, the total amount of cold gas and stellar mass exceeds the observed total amount (see Obreschkow et al. 2009; Fu et al. 2010). Decreasing SFR in low-mass galaxies while letting cooling and feedback recipes remain unchanged naturally results in an overproduction of the cold gas mass in low-mass galaxies. This is not only a problem for the models we present here. As shown recently by Lu et al. (2011b), when the model K-band luminosity function is forced to fit the data in the local Universe, the cold gas mass function is dramatically overpredicted in all the semi-analytic models they study. This is consistent with our models overpredicting the cold gas mass functions when we fit the SMF at $z = 0$. On the other hand, with similar cooling and SN feedback recipes, the models that do fit the cold gas mass function in turn have problems in reproducing the SFR (e.g. Obreschkow et al. 2009; Fu et al. 2010). In the meantime, approaches like increasing the SN feedback, that suppress the total amount of cold gas and stellar mass, lead to several other serious problems (Guo et al. 2011). These results show again that it is currently difficult for a single model to fit all observations, as mentioned in Section 1, unless we allow for free tuning of all recipes, including cooling (see NW2010).

Although the modifications of the SF law presented in this work help to improve the agreement with several observed statistical
properties of galaxies and seem to follow a similar scaling like the observed SF efficiencies in galaxies, the normalization of the SFE cannot be correct. This is clear from the fact that our models over-predict the cold gas mass function. With a lower cold gas fraction in galaxies, the SFE would obviously have to be higher than currently assumed in the models, to obtain the same SFR.

In Fig. 10, we compare the SFR–H\textsubscript{i} mass relation in model 3 combined with the MS-II (right-hand panel), with a recent observation of the far-ultraviolet (FUV) derived SFR–H\textsubscript{i} mass relation for galaxies within \sim11\ Mpc of the Milky Way (Lee et al. 2011, left-hand panel). The cold gas mass in model 3 is converted to the H\textsubscript{i} mass to be compared with observation, using a correction including three factors. First, as for Fig. 6, the cold gas mass from model 3 is divided by a factor of 1.45, to account for a warm ionized gas phase, as done in Obreschkow et al. (2009). Second, we multiply the results by a factor of 0.76 to remove the contribution of helium and heavier elements (Power, Baugh & Lacey 2010). Finally, the hydrogen gas mass is divided by 1.4, as adopted by Power et al. (2010) and Lu et al. (2011b), to remove the contribution of H\textsubscript{2}. Since the galaxies in the observed sample have stellar masses less than \sim10^{10}\ M\odot, we present model results for galaxies with 8 < \log (M\textsubscript{star}) < 10. The median relation from the observations is plotted as a red line in each panel. Although the observations of Lee et al. (2011) are limited to a small volume of space, the obviously different relations in observations and in model 3 indicate that the SFE in model 3 is indeed much lower than in reality.

Fitting the most important observed properties of galaxies is not a trivial task. It is not clear up to now if this difficulty in modelling galaxy properties reflects a fundamental problem in our understanding of the dark matter universe, or if it is mainly due to an insufficient understanding of the baryonic physics involved in galaxy formation and evolution. For example, perhaps SAMs miss an important ingredient of galaxy formation, like a mechanism that preheats the gas in the universe so that it cannot cool to low-mass haloes (Mo et al. 2005, but see also Crain et al. 2007), or a form of feedback that mainly heats low-entropy gas in high-redshift haloes (McCarty et al. 2011). Alternatively, it could be that cooling is overefficient in the current SAMs for some reason.

With the tests carried out in this work, we show that only modifying S in the SAMs can already improve agreement with observations in several important aspects. Up to now, a high-resolution SAM that matches the SSFR–stellar mass relation at z = 0, the SMF and its evolution, the correlation functions and the cold gas fractions of galaxies simultaneously, does not yet seem to exist. To find such a model, approaches that allow scanning of a large parameter space (Henriques et al. 2009; Bower et al. 2010; Lu et al. 2011a), and approaches that allow deviations from the usually assumed functional forms for physical recipes (NW2010), or that include other physical processes than currently considered (Henriques & Thomas 2010) may be promising. However, even if one or several models are found that do indeed reproduce all these fundamental observables, it will be important to identify degeneracies and to verify whether the models are physically plausible and can be brought into agreement with alternative approaches, like predictions from hydrodynamical simulations.

**ACKNOWLEDGMENTS**

We thank the referee for a constructive report to help to improve the manuscript. We acknowledge Cheng Li for providing the SDSS data results and for helpful discussions, Janice C. Lee for providing the data values in their paper and Jian Fu for helpful discussions. LW acknowledges support from the National Basic Research Program of China (973 programme under grant No. 2009CB24901), the NSFC grant programme (No. 11143006, No. 11103033, No. 11133003), the Young Researcher Grant of National Astronomical Observatories, Chinese Academy of Sciences and the Partner Group programme of the Max Planck Society. The Millennium Simulation and the Millennium-II Simulation were carried out as part of the programme of the Virgo Consortium on the Regatta and VIP supercomputers at the Computing Centre of the Max-Planck Society in Garching. The halo/subhalo merger trees for the Millennium and Millennium-II Simulations are publicly available at http://www.mpa-garching.mpg.de/millennium

**REFERENCES**

Agertz O., Teyssier R., Moore B., 2011, MNRAS, 410, 1391
Avila Reese V., Colin P., González-Samaniego A., Valenzuela O., Firmani C., Velázquez H., Ceverino D., 2011, ApJ, 736, 134
Baugh C. M., Cole S., 2005, MNRAS, 356, 1191
Baugh C. M., Lacey C. G., Frenk C. S., Granato G. L., Silva L., Bressan A., Benson A. J., Cole S., 2005, MNRAS, 356, 1191

© 2012 The Authors, MNRAS 421, 3450–3463
Monthly Notices of the Royal Astronomical Society © 2012 RAS
Appendix A: Cooling Efficiencies

Our model 0 is similar to model 0 of NW2010, with a few modifications to the cooling efficiencies, which we explain below. Model 0 of NW2010 is adapted to the MS, and thus only includes efficiencies down to the resolution limit of this simulation, which is $\sim 10^6 \, h^{-1} \, M_\odot$ in halo mass. To apply it to the MS-II, we need efficiencies down to a lower halo mass of $\sim 10^5 \, h^{-1} \, M_\odot$. This is straightforward for most processes, as they are parametrized by functional forms which can easily be extended to lower halo masses. The only exception is the cooling efficiencies, $f_c$, defined as $\Delta m_{\text{cool}} = f_c m_{\text{hot}} \Delta t$, where $\Delta m_{\text{cool}}$ is the amount of gas that is cooled within a time-step $\Delta t$ and $m_{\text{hot}}$ is the mass of hot gas.

In DLB07, the treatment of gas cooling follows the description of Croton et al. (2006), where cooling efficiencies are calculated according to White & Frenk (1991), assuming an isothermal gas density profile. In model 0 of NW2010, cooling efficiencies are median values computed from a large statistical sample of galaxies in DLB07, for each bin of halo mass and cosmic time. The tabulated values as a function of halo mass and time in NW2010 follow no specific functional form and can thus not easily be extended to lower mass haloes. The obvious solution would be to extract those values from the DLB07 SAM as applied to the MS-II, but this is not possible for technical reasons. Therefore, we estimate cooling efficiencies for low-mass haloes such that (i) the general trends of cooling efficiency as a function of halo mass and redshift are preserved, and (ii) the resulting low-mass end slope and amplitude of the SMF at $z = 0$ are similar to the result of DLB07 when applied to the MS-II (see Fig. 1). In this way our estimates of the cooling efficiencies at a given halo mass should be similar to that in the DLB07 model, when averaged over all redshifts. However, we note that the cooling efficiencies in a given redshift and halo mass bin may differ. Apart from extending the cooling efficiencies to lower mass haloes, we also apply some smoothing to the original cooling efficiencies found by NW2010, in order to smoothen the SMF.

**Table A1.** Values of cooling efficiencies in units of Gyr$^{-1}$. The values shown here are identical for models 0, 1, 2, 3 and 3b, and are plotted in Fig. A1. Halo mass is in units of $h^{-1} M_\odot$ and is shown in the left-hand column; time is in Gyr.

| $\log(M_{\text{halo}})$ | $t = 0.80$ | 2.24 | 3.38 | 5.97 | 10.27 | 13.58 |
|--------------------------|-----------|------|------|------|-------|-------|
| $z = 7$                   | $-1.30$   | $-1.60$ | $-2.00$ | $-2.00$ | $-2.00$ | $-2.00$ |
| $3.0$                     | $-0.90$   | $-1.50$ | $-1.90$ | $-2.00$ | $-2.00$ | $-2.00$ |
| $2.0$                     | $-0.80$   | $-1.30$ | $-1.50$ | $-1.60$ | $-1.80$ | $-2.00$ |
| $1.0$                     | $-0.50$   | $-1.00$ | $-1.30$ | $-1.50$ | $-1.50$ | $-1.80$ |
| $0.3$                     | $-0.30$   | $-0.80$ | $-1.10$ | $-1.30$ | $-1.50$ | $-1.60$ |
| $0.0$                     | $-0.10$   | $-0.50$ | $-0.80$ | $-1.10$ | $-1.20$ | $-1.30$ |
| $0.78$                    | $0.20$    | $-0.30$ | $-0.70$ | $-1.00$ | $-1.10$ | $-1.20$ |
| $0.50$                    | $0.50$    | $-0.07$ | $-0.40$ | $-0.76$ | $-0.93$ | $-1.00$ |
| $0.30$                    | $0.81$    | $0.29$  | $0.23$  | $0.57$  | $0.76$  | $0.93$  |
| $0.10$                    | $0.83$    | $0.69$  | $0.06$  | $0.38$  | $0.53$  | $0.70$  |
| $1.13$                    | $0.42$    | $0.41$  | $0.32$  | $0.49$  | $0.53$  | $0.67$  |
| $0.77$                    | $0.33$    | $0.17$  | $0.06$  | $0.28$  | $0.44$  | $0.67$  |
| $0.71$                    | $0.21$    | $0.02$  | $0.32$  | $0.41$  | $0.67$  | $0.90$  |
| $0.43$                    | $0.15$    | $0.45$  | $0.71$  | $0.83$  | $0.90$  | $0.93$  |
| $0.29$                    | $0.23$    | $0.51$  | $0.73$  | $0.81$  | $0.92$  | $0.93$  |
| $0.07$                    | $0.28$    | $0.51$  | $0.79$  | $1.50$  | $2.00$  | $4.00$  |
| $0.58$                    | $0.70$    | $0.80$  | $1.50$  | $2.00$  | $4.00$  | $9.00$  |
| $0.78$                    | $0.70$    | $1.00$  | $2.00$  | $4.00$  | $9.00$  | $9.00$  |

**Figure A1.** Cooling efficiencies as a function of halo mass and cosmic time in model 0, the fiducial model 1, and also in models 2, 3 and 3b. The grey-scale shows Log values of the cooling efficiencies in units of log[Gyr$^{-1}$. The X-axis is the cosmic time, with 13.7 Gyr corresponding to the present day.

**Figure A2.** Cooling efficiency as a function of halo mass and cosmic time in model 4. The grey-scale shows Log values of cooling efficiency in units of Log[Gyr$^{-1}$. The X-axis is the cosmic time, with 13.7 Gyr corresponding to the present day.
Fig. A1 shows cooling efficiencies as a function of halo mass and cosmic time in model 0. These values are given explicitly in Table A1. The extrapolation and smoothing to the cooling efficiencies used in model 0 of NW2010 can be seen by comparing Fig. A1 with fig. 6 in NW2010, and also comparing the values listed in Table 1 with table 6 of NW2010. The cooling efficiencies shown in Fig. A1 and Table A1 are also applied to models 1, 2, 3 and 3b.

Fig. A2 shows the cooling efficiencies used for model 4 as presented in Section 2.6, and Table A2 lists the explicit values of those efficiencies.

This paper has been typeset from a TEX/LATEX file prepared by the author.