Environment-assisted holonomic quantum maps

Nicklas Ramberg and Erik Sjöqvist

Department of Physics and Astronomy, Uppsala University, Box 516, Se-751 20 Uppsala, Sweden

(Dated: December 10, 2018)

Holonomic quantum computation uses non-Abelian geometric phases to realize error resilient quantum gates. Nonadiabatic holonomic gates are particularly suitable to avoid unwanted decoherence effects, as they can be performed at high speed. By letting the computational system interact with a structured environment, we show that the scope of error resilience of nonadiabatic holonomic gates can be widened to include systematic parameter errors. Our scheme maintains the geometric properties of the evolution and results in an environment-assisted holonomic quantum map that can mimic the effect of a holonomic gate. We demonstrate that the sensitivity to systematic errors can be reduced in a proof-of-concept spin-bath model.

Quantum holonomies are non-Abelian (non-commuting) unitary operators that only depend on paths in state space of a quantum system. The non-commuting property makes them useful for implementing quantum gates that manipulate quantum information by purely geometric means. Holonomic quantum computation (HQC) is a network of holonomic gates that unifies geometric characteristics of quantum systems and information processing, as well as is conjectured to be robust to errors in experimental control parameters.

Nonadiabatic HQC has recently been proposed and experimentally implemented as a tool to realize quantum gates based upon nonadiabatic non-Abelian geometric phases. The basic setup for nonadiabatic HQC in is a three-level configuration, where two simultaneous resonant laser pulses drive transitions between the qubit levels and an auxiliary state level. This scheme has been generalized to off-resonant pulses. The off-resonant setup uses two simultaneous laser pulses with the same variable detuning, which enhances the flexibility of the holonomic scheme. For experimental realization of off-resonant nonadiabatic holonomic gates, see Refs.

The nonadiabatic version of HQC avoids the drawback of the long run time associated with adiabatic holonomies, on which the original holonomic schemes are based. Nonadiabatic holonomic gates are therefore particularly suitable to avoid unwanted decoherence effects. The resilience to decoherence errors can be further improved by combining nonadiabatic HQC with decoherence-free subspaces and subsystems, as well as dynamical decoupling. On the other hand, it has been pointed out that the original version of nonadiabatic HQC has no particular advantage compared to standard dynamical schemes in the presence of systematic errors in experimental parameters. To deal with this, we here show that the sensitivity to systematic parameter errors can be reduced by letting the system interact with a structured environment. Our approach is inspired by earlier findings that transport efficiency in complex quantum systems can be enhanced in such environments.

We modify the off-resonant non-adiabatic holonomic scheme by coupling the auxiliary state to a finite thermal bath, the latter playing the role of the structured environment. The key point of our modified scheme is its non-Markovian nature, which allows for coherence to flow back and forth between the system and the environmental bath. The resulting transformation retains its holonomic property and can therefore be regarded as an environment-assisted holonomic map that can mimic the effect of a holonomic gate. We address the protective potential of the environment to errors in the Rabi frequencies and detuning describing the system-laser interaction, which causes the qubits to end up only partially in the computational subspace, while preserving the purely geometric property of the evolution. Our aim is to demonstrate that the sensitivity to systematic deviations in the Rabi frequencies and detuning can be reduced by tuning the system-bath coupling strength to its optimal value.

We consider an off-resonant Λ system, in which two square-shaped simultaneous laser pulses induce transitions between the computational state levels and an auxiliary state . The corresponding Rabi frequencies take the form and with and controlling the relative amplitude and phase, respectively, of the two pulses. The Hamiltonian during the pulse pair reads (from now on)

where is the bright state, while the dark state is decoupled from the evolution. We assume the same detuning of the two transitions. Provided the parameters , , and are kept constant during the pulse pair, the resulting evolution is purely geometric as the Hamiltonian vanishes on the evolving computational subspace . The time evolution operator associated with is given by

which acts on . Here, is the path traced out by in the Grassmannian , i.e., the space of two-dimensional subspaces of the three-dimensional space . The phase determines the gate rotation angle .
Next, we introduce the environmental bath $B$ with Hilbert space $\mathcal{H}_B$, assuming $\dim \mathcal{H}_B = K$ finite. The Hamiltonian during the pulse pair is assumed to take the form

$$H = H_\lambda \otimes 1_B + 1_\lambda \otimes H_B + \gamma |e\rangle \langle e| \otimes h_B$$

with $H_B$ and $h_B$ both time-independent, and $\gamma$ the system-bath coupling strength. The spectrum of $h_B$ is $\nu_0 = 0 < \nu_1 \leq \cdots \leq \nu_{K-1}$ with corresponding eigenstates $|\nu_0\rangle, |\nu_1\rangle, \ldots, |\nu_{K-1}\rangle$. We find the effective system Hamiltonian during the pulse:

$$H_{eff} = \sum_{k=0}^{K-1} H_{\nu + \gamma \mu_k}(\omega, \theta, \varphi) \otimes |\mu_k\rangle \langle \mu_k|,$$

where

$$H_{\nu + \gamma \mu_k}(\omega, \theta, \varphi) = (\delta + \gamma |\mu_k\rangle \langle \mu_k| + \omega |e\rangle \langle e| + |b\rangle \langle b| + |e\rangle \langle e|).$$

This sub-Hamiltonian induces a purely geometric evolution along a path $C_k$ in $G(3,2)$. The corresponding holonomic sub-bath $U(H_{\lambda_k})$ is obtained by replacing $\delta$ by $\delta + \gamma |\mu_k\rangle \langle \mu_k|$ in the above ideal gate Eq. (2). Note that $C_k$ is paths associated with different cyclic terms, due to the $\mu_k$-dependence of the modified detunings $\delta + \gamma |\mu_k\rangle \langle \mu_k|$. We assume the bath starts in a thermal state $\rho_B$ that factorizes with the initial pure system state $|\psi\rangle$ in $M_s(\omega, \theta, \varphi)$. In other words, the full system-bath state $|\psi\rangle \langle \psi| \otimes \rho_B$ evolves as

$$\varrho(t) = U(t, 0)|\psi\rangle \langle \psi| \otimes \rho_B U^\dagger(t, 0)$$

with

$$\varrho_B = \frac{1}{Z} e^{-\beta H_B},$$

$$Z = \text{Tr}\left(e^{-\beta H_B}\right)$$

being the partition function and $\beta^{-1}$ the temperature. $U(t, 0) = e^{-iHt}$ is the time evolution operator with $H$ given by Eq. (2). The computational input state evolves as

$$|\psi\rangle \langle \psi| \rightarrow \varrho(t) = \text{Tr}_B \varrho(t),$$

where $\text{Tr}_B$ is partial trace over the spin-bath. In the case where $[H_B, h_B] = 0$, we may explicitly evaluate the partial trace yielding the computational state

$$\varrho(t) = \sum_{k=0}^{K-1} e^{-\beta |\nu_k\rangle \langle \nu_k|} \frac{1}{Z} e^{-iH_{\nu + \gamma \mu_k}(\omega, \theta, \varphi)t} |\psi\rangle \langle \psi| e^{iH_{\nu + \gamma \mu_k}(\omega, \theta, \varphi)t},$$

where we assumed the spectrum $\nu_0, \nu_1, \ldots, \nu_{K-1}$ of $H_B$. This is a unital map $\varrho \mapsto \mathcal{E}_t(\varrho)$ with Kraus operators

$$A_k(t) = \sqrt{\frac{e^{-\beta |\nu_k\rangle \langle \nu_k|}}{Z}} e^{-iH_{\nu + \gamma \mu_k}(\omega, \theta, \varphi)t}.$$ (10)

The map $\mathcal{E}_t$ is the promised environment-assisted holonomic quantum map.

For very low temperatures, only the ground state of the spin-bath is populated and the system undergoes unitary evolution governed by the Hamiltonian $H_\lambda$. The resulting gate is essentially determined by the Rabi frequencies $\Omega_j$ and the detuning $\delta$. These parameters can be affected by errors:

$$\Omega_j' = (1 + \epsilon_j) e^{i\delta_j} \Omega_j, \quad \delta' = (1 + \kappa) \delta,$$ (11)

$\epsilon_j, \zeta_j$, and $\kappa$ being real-valued numbers. This can be translated into the parameters

$$\omega' = \left(1 + \epsilon_0\right)^2 \sin^2 \frac{\theta}{2} + \left(1 + \epsilon_1\right)^2 \cos^2 \frac{\theta}{2},$$

$$e^{i\delta'} \tan \frac{\varphi}{2} = \left(1 + \epsilon_0 \right)^2 \sin \frac{\theta}{2} \left(1 + \epsilon_1 \right)^2 \cos \frac{\theta}{2}.$$ (12)

The dark and bright states are modified accordingly, i.e., they read $|d'\rangle = \cos \frac{\theta}{2}|0\rangle + e^{i\delta'} \sin \frac{\theta}{2}|1\rangle$ and $|b'\rangle = e^{i\delta'} \sin \frac{\theta}{2}|0\rangle - \cos \frac{\theta}{2}|1\rangle$.

We assume $\epsilon_j, \zeta_j$, and $\kappa$ are constant, which correspond to systematic errors in the applied pulse pair. Under this assumption, the evolution remains purely geometric since the error-affected Hamiltonian $H_{\nu + \gamma \mu_k}(\omega', \theta', \varphi')$ remains zero on $M_s(t)$. The errors in $\theta$ and $\varphi$ change the direction of the rotation axis, but preserve the cyclic property of the evolution. On the other hand, the computational subspace would generally fail to return at $t = \tau_0$ due to errors in $\omega$ and $\delta$.

We shall investigate the performance of the environment-assisted holonomic maps in the presence of Rabi frequency errors, by comparing them to the ideal gate $U(C)$ with run time $\tau_0$ by means of the fidelity

$$\mathcal{F}(\varrho) = \langle \psi| U(t, 0)|\psi\rangle p_0 U(t, 0)|\psi\rangle^{1/2}$$

$$= \left(\sum_{k=0}^{K-1} \langle \psi| U(t, 0)|\psi\rangle A_k(\tau_0)|\psi\rangle\right)^{1/2}$$ (13)

with

$$A_k(\tau_0) = \sqrt{\frac{e^{-\beta |\nu_k\rangle \langle \nu_k|}}{Z}} e^{-iH_{\nu + \gamma \mu_k}(\omega', \theta', \varphi')\tau_0}.$$ (14)

The fidelity $\mathcal{F}(\varrho)$ can be studied as a function of coupling parameter $\gamma$, for some suitably chosen input states $|\psi\rangle$, given a number of spins $N$ in the bath and temperature $\beta^{-1}$. As a measure of gate performance, one averages the fidelity over a sufficiently large, uniformly distributed sample of input states.

To address the behavior of the environment-assisted holonomic scheme in an explicit proof-of-concept system, we consider the spin-bath model proposed in Ref. [29] adapted to the $\Lambda$ system. We choose

$$H_B = \alpha S_z, \quad h_B = \left(\tau_0 + \frac{N}{2}\right)$$

with $S_z$ the z-projection of the total spin of the bath consisting of $N$ individual spin-$\frac{1}{2}$. The spectrum of $H_B$ is $v_m = \alpha(m - N/2)$, $m = 0, \ldots, N$, with multiplicities $N^m(N - m)$. In other words, the bath parameter $\alpha$ measures the energy split between the eigenstates of $H_B$. The initial thermal bath state reads

$$\varrho_B = \sum_{m=0}^{N} \sum_{q=1}^{m^q} \frac{e^{-\beta v_m}}{Z} |m - N/2, q\rangle \langle m - N/2, q|,$$

$$Z = \sum_{m=0}^{N} e^{-\beta v_m}.$$ (16)
where $|m - N/2, q\rangle$ are eigenstates of $S_z$, consisting of permutations $m$ spins in $|\uparrow\rangle$ and $N - m$ spins in $|\downarrow\rangle$. The spectrum of $h_B$ is $m$. This defines the error affected unital map

$$|\psi\rangle\langle\psi| \mapsto \rho(\tau_0) = \sum_{m=0}^{N} A_m^\prime(\tau_0) |\psi\rangle\langle\psi| A_m^{\prime\dagger}(\tau_0)$$

with Kraus operators

$$A_m^\prime(\tau_0) = \sqrt{\left(\frac{N}{m}\right)} \frac{e^{-\beta m\tau_0}}{Z} e^{-\delta m\gamma m} e^{i\theta_m\phi_m^\prime\tau_0},$$

$$H_{\theta + \gamma m}(\omega', \theta', \phi^\prime) = (\delta' + \gamma m)|e\rangle\langle e| + i\omega' |(e\rangle|b\rangle + |b\rangle\langle e|).$$

To simplify the analysis, we assume $\xi_0 = \xi_1 \equiv \xi$ and $\xi_0 - \xi_1 = 0$, which imply $\omega' = (1 + \epsilon)\omega$, $\theta' = \theta$, and $\phi^\prime = \phi$. Under these restrictions, $|d\rangle = |d\rangle$ and $|b\rangle = |b\rangle$, which imply that the gate rotation axis $\mathbf{n}'$ coincides with the ideal $\mathbf{n}$, while $\mathcal{M}$ undergoes cyclic evolution for the pulse duration $\tau_0$ being generally different from the ideal run time $\tau_0$. In other words, the computational subspace typically fails to return after applying the error affected pulse pair for the ideal duration $\tau_0$ at zero temperature. We now wish to optimize the gate performance by maximizing the similarity between the error affected environment-assisted holonomic map with Kraus operators $A_m^\prime(\tau_0)$ and the ideal holonomic gate $U(C) = |d\rangle\langle d| - e^{-i\gamma}|b\rangle\langle b|$ by tuning the system-bath coupling strength $\gamma$ at nonzero temperature. To formalize this idea, we write $|\psi\rangle = \cos \frac{\theta}{2} |d\rangle + e^{i\phi} \sin \frac{\theta}{2} |b\rangle$ and obtain

$$\langle \psi|U^+(C)A_m^\prime(\tau_0)|\psi\rangle^2 = \left(\frac{N}{m}\right) \frac{e^{-\beta m\tau_0}}{Z} \left| \cos^2 \frac{\theta}{2} - e^{i\phi} \sin \frac{\theta}{2} \right|^2,$$

where we have used that $\langle d|A_m^\prime(\tau_0)|b\rangle = 0$. We find

$$\langle b|e^{-iH_{\theta' + \gamma m}(\omega', \theta', \phi')\tau_0}|b\rangle = e^{-i\Sigma_{\gamma m}^\tau_0} \left( \cos \frac{\pi \Delta_{\gamma m}^\prime}{\Delta_0} + i \cos n'_{\gamma m} \sin \frac{\pi \Delta_{\gamma m}^\prime}{\Delta_0} \right),$$

where the parameters

$$\tan n'_{\gamma m} = \frac{2\omega'}{\delta' + \gamma m}, \quad \Sigma_{\gamma m}^\prime = \frac{\delta' + \gamma m}{2},$$

$$\Delta_{\gamma m}^\prime = \sqrt{(\delta' + \gamma m)^2 + 4(\omega')^2}$$

(21)

are associated with the diagonalization of $H_{\theta' + \gamma m}(\omega', \theta, \phi)$. The fidelity $F(\psi) \equiv F(\theta)$ is independent of $\xi$, which implies that we only need to sample over $\theta$. A uniform distribution of states $\psi$ would correspond to a weight factor that is proportional to the circumference of the circle at this latitude on the Bloch sphere, i.e., we may take $\psi(\theta) = \sin \theta$. By choosing $n$ input states at $\theta_k = k\pi/(n - 1), k = 0, \ldots, n - 1$, the averaged fidelity thus reads

$$F_{av} = \frac{\sum_{k=0}^{n-1} \sin \left( \frac{k\pi}{n-1} \right) F(\frac{k\pi}{n-1})}{\sum_{k=0}^{n-1} \sin \left( \frac{k\pi}{n-1} \right)}.$$
We measure the gate performance in terms of $F_{av}$.

Figure 1 shows the average fidelity as a function of system-bath coupling at $T = 50$ K (left panel) and room temperature $T = 300$ K (right panel) with $N = 20$ spins and the bath parameter $\alpha = 15$ ps$^{-1}$. Ideal parameter values are chosen to be $\omega = 1$ ns$^{-1}$ and $\delta = 2$ ns$^{-1}$, corresponding to the rotation angle $\pi - \chi \approx 0.29\pi$. Averages are computed for $n = 30$ equidistant $\theta$ values.

For both temperatures, we numerically compute the fidelity for errors $\epsilon = \kappa = 0.1, 0.15,$ and $0.2$. We see that the optimal nonzero system-bath coupling strength depends significantly on temperature, but is quite insensitive to the error size. This insensitivity holds also for the corresponding optimal fidelity, especially for the lower temperature, where $F_{av} \sim 97.3 - 97.4\%$ at the optimal system-bath coupling strength $\gamma \sim 2.8$ ns$^{-1}$. On the other hand, the fidelity is strongly error-size-dependent in the absence of the bath ($\gamma = 0$), in case of which $F_{av}$ varies between 95.5% and 98.6% over the chosen error range. Thus, the environmental bath can be made to reduce the sensitivity to systematic errors by tuning the system-bath coupling strength. For large errors ($\epsilon, \kappa \sim 0.15$ or higher) the fidelity takes a higher value for the nonzero optimal $\gamma$, which shows that the bath not only can reduce the error sensitivity but also can improve the gate performance. This demonstrates that our scheme can protect against large systematic errors by fixing the system-bath coupling strength at the optimal value for a given bath temperature.

In conclusion, we have addressed the sensitivity to systematic parameter errors in nonadiabatic holonomic schemes. To this end, we have put forward a concept of environment-assisted holonomic maps, in which the auxiliary state of the standard $\Lambda$ system realization of holonomic gates is coupled to a finite thermal bath system. These maps retain the geometric properties of the ideal holonomic gates. By tuning the system-bath coupling strength to its optimal value, the sensitivity to systematic errors can be reduced and the corresponding optimal fidelity may in some cases be even higher than in the absence of the environmental bath. These features may persist even at room temperature. We have demonstrated the robustness in a proof-of-concept spin-bath model. More sophisticated models, with a larger number of optimization parameters and thereby more possible routes toward higher error resilience, can be envisaged.

ACKNOWLEDGMENTS

E.S. acknowledges financial support from the Swedish Research Council (VR) through Grant No. 2017-03832.

---

[1] F. Zanardi and M. Rasetti, Holonomic quantum computation, Phys. Lett. A 264, 94 (1999).

[2] J. Pachos and P. Zanardi, Quantum holonomies for quantum computing, Int. J. Mod. Phys. B 15, 1257 (2001).

[3] E. Sjöqvist, D. M. Tong, L. M. Andersson, B. Hessmo, M. Johansson, and K. Singh, Non-adiabatic holonomic quantum computation, New J. Phys. 14, 103035 (2012).

[4] A. A. Abdumalikov, J. M. Fink, K. Juliusson, M. Pechal, S. Berger, A. Wallraff, and S. Filipp, Experimental realization of non-Abelian non-adiabatic geometric gates, Nature (London) 496, 482 (2013).

[5] G. Feng, G. F. Xu, and G. L. Long, Experimental Realization of Nonadiabatic Holonomic Quantum Computation, Phys. Rev. Lett. 110, 190501 (2013).

[6] S. Arroyo-Camejo, A. Lazariiev, S. W. Hell, and G. Balasubramanian, Room temperature high-fidelity holonomic single-qubit gate on a solid-state spin, Nat. Commun. 5, 4870 (2014).

[7] C. Z. Wu, W. B. Wang, L. He, W. G. Zhang, C. Y. Dai, F. Wang, and L. M. Duan, Experimental realization of universal geometric quantum gates with solid-state spins, Nature (London) 514, 72 (2014).

[8] S. Danilin, A. Vepsäläinen, and G. S. Paraoanu, Experimental state control by fast non-Abelian holonomic gates with a superconducting qutrit, Phys. Scr. 93 055101 (2018).

[9] Y. Xu, W. Cai, Y. Ma, X. Mu, L. Hu, Tao Chen, H. Wang, Y.P. Song, Z.-Y. Xue, Z. Yin, and L. Sun, Single-Loop Realization of Arbitrary Nonadiabatic Holonomic Single-Qubit Quantum Gates in a Superconducting Circuit, Phys. Rev. Lett. 121, 110501 (2018).

[10] J. Anandan, Non-adiabatic non-Abelian geometric phase, Phys. Lett. A 133, 171 (1988).

[11] G. F. Xu, C. L. Liu, P. Z. Zhao, and D. M. Tong, Nonadiabatic holonomic gates realized by a single-shot implementation, Phys. Rev. A 92, 052302 (2015).

[12] E. Sjöqvist, Nonadiabatic holonomic single-qubit gates in off-resonant $\Lambda$ systems, Phys. Lett. A 380, 65 (2016).

[13] B. B. Zhou, P. C. Jerger, V. O. Shkolnikov, F. J. Heremans, G. Burkard, D. D. Awschalom, Holonomic Quantum Control by Coherent Optical Excitation in Diamond, Phys. Rev. Lett. 119, 140503 (2017).

[14] Y. Sekiguchi, N. Niikura, R. Kuroiwa, H. Kano, and H. Kosaka, Optical holonomic single quantum gates with a geometric spin under a zero field, Nature Photonics 11, 309 (2017).

[15] H. Li, Y. Liu, and G. L. Long, Experimental realization of single-shot nonadiabatic holonomic gates in nuclear spins, Sci. China-Phys. Mech. Astron. 60, 080311 (2017).

[16] Z. Zhang, P. Z. Zhao, T. Wang, L. Xiang, Z. Jia, P. Duan, D. M. Tong, Y. Yin, and G. Guo, Single-shot realization of nonadiabatic holonomic gates with a superconducting Xmon qutrit, arXiv:1811.06252.

[17] F. Wilczek and A. Zee, Appearance of Gauge Structure in Simple Dynamical Systems, Phys. Rev. Lett. 52, 2111 (1984).

[18] L. M. Duan, J. I. Cirac, and P. Zoller, Geometric Manipulation of Trapped Ions for Quantum Computation, Science 292, 1695 (2001).

[19] M. Johansson, E. Sjöqvist, L. M. Andersson, M. Ericsson, B. Hessmo, K. Singh, and D. M. Tong, Robustness of nonadiabatic holonomic gates, Phys. Rev. A 86, 062322 (2012).

[20] G. F. Xu, J. Zhang, D. M. Tong, E. Sjöqvist, and L. C. Kwek, Nonadiabatic Holonomic Quantum Computation in Decoherence-Free Subspaces, Phys. Rev. Lett. 109, 170501 (2012).

[21] Z.-T Liang, Y.-X Du, W. Huang, Z.-Y Xue, and H. Yan, Nonadiabatic holonomic quantum computation in decoherence-free subspaces with trapped ions, Phys. Rev. A 89, 062312 (2014).
[22] Z.-Y. Xue, J. Zhou, and Z. D. Wang, Universal holonomic quantum gates in decoherence-free subspace on superconducting circuits, Phys. Rev. A 92, 022320 (2015).
[23] P. Z. Zhao, G. F. Xu, Q. M. Ding, E. Sjöqvist, and D. M. Tong, Single-shot realization of nonadiabatic holonomic quantum gates in decoherence-free subspaces, Phys. Rev. A 95, 062310 (2017).
[24] J. Zhang, L.-C. Kwek, E. Sjöqvist, D. M. Tong, and P. Zanardi, Quantum computation in noiseless subsystems with fast non-Abelian holonomies, Phys. Rev. A 89, 042302 (2014).
[25] G. F. Xu and G. L. Long, Protecting geometric gates by dynamical decoupling, Phys. Rev. A 90, 022323 (2014).
[26] C. Sun, G. Wang, C. Wu, H. Liu, X.-L. Feng, J.-L. Chen, and K. Xue, Non-adiabatic holonomic quantum computation in linear system-bath coupling, Sci. Rep. 6, 20292 (2016).
[27] G. F. Xu, D. M. Tong, and E. Sjöqvist, Path-shortening realizations of nonadiabatic holonomic gates, Phys. Rev. A 98, 052315 (2018).
[28] S.-B. Zheng, C.-P. Yang, and F. Nori, Comparison of the sensitivity to systematic errors between nonadiabatic non-Abelian geometric gates and their dynamical counterparts, Phys. Rev. A 93, 032313 (2016).
[29] I. Sinayskiy, A. Marais, F. Petruccione, and A. Ekert, Decoherence-Assisted Transport in a Dimer System, Phys. Rev. Lett. 108, 020602 (2012).
[30] G. L. Giorgi and T. Busch, Decoherence-assisted transport and quantum criticalities, Phys. Rev. A 86, 052112 (2012).
[31] A. Marais, I. Sinayskiy, A. Kay, F. Petruccione, and A. Ekert, Decoherence-assisted transport in quantum networks, New J. Phys. 15, 013038 (2013).