A New Profile Least-Squares Approach for Partially Linear Nonparametric Models with Measurement Errors

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Abstract. Partially linear nonparametric models, as a generalization of partially linear models and nonparametric models, have good adaptability and robustness. In this paper, based on the selection of this model, and a new method of contour least squares estimation, the focus is on the estimation of partially linear nonparametric models with measurement errors of covariance. Because when the measurement error is ignored, the parametric and non-parametric components are usually biased estimates. On this basis, we propose a modified contour least squares estimation of measurement error, and also obtain the error variance estimation results. Finally, through numerical simulation, the feasibility of the model proposed in this paper when the covariant has additive measurement error is verified.

1. Introduction

1.1 Research background

Regression analysis is one of the most mature and widely used branches in statistics. In the initial application of statistics, the traditional linear regression model is well known. This model has been extensively and deeply researched by scholars, and has provided great helps in solving some problems. It has been widely used in many fields, which has produced more popular models for scholars to study and solve more complicated situation. Among them, the classic linear regression model is simple in form, and its expression is

\[ Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p, \]

where \( Y \) is the response variable, \( X_1, X_2, \cdots, X_p \) are the covariate, \( \beta_0, \beta_1, \cdots, \beta_p \) are the unknown constant value parameter, \( \varepsilon \) is the unobservable random error item, and \( E(\varepsilon) = 0, Var(\varepsilon) = \sigma^2 > 0 \). The advantage of this model is that it presupposes the specific parameter form in advance. When the assumed assumption of the model is close to the actual one, it's fitting to the parameter is very simple and effective. But in the actual situation, there are not many cases that meet the assumptions of the linear model, and the more general case will be more complicated. To this end, statisticians have proposed a non-parametric regression model based on the classical regression model. Its expression is

\[ Y = m(X_1, X_2, \cdots, X_p) + \varepsilon. \]

The non-parametric regression model does not assume the specific form of the regression function, but only makes certain requirements for the continuity and smoothness of the regression function \[2\], thereby improving the flexibility and adaptability of the model to a certain extent. But as the dimension of covariate increases, there will be a dimensional disaster problem. In order to get a more accurate estimate of the regression function, the amount of data required will increase exponentially as the dimension of the covariate increases. Therefore, in order to overcome the problem of dimensional disaster, statisticians have proposed a partial linear non-parametric model, which has the characteristics of simple linear regression model and being easy to explain, and the flexibility and adaptability of the
non-parametric model. Avoiding the "dimensionality bane" problem of non-parametric models improves the accuracy of the model. Therefore, studying some linear non-parametric models has both theoretical significance and practical value.

In many practical problems, people will collect data in a variety of ways, often because of system errors, sampling errors, recording instrument errors and other reasons so that the data cannot be accurately obtained. This paper starts from considering the measurement errors of covariate bands and analyses the impact of measurement errors through numerical simulation results.

1.2 Literature reviews
In the mid and late 1960s, the survival analysis problem was applied to non-parametric methods by Cox and Ferguson and developed to some extent. The semi-parametric regression model proposed by Robert (1986) [3] et al. develops immediately, which attracted the attention of the academic community. There are many estimation methods for the parametric and non-parametric parts of the model. Among them, Nadayara (1964) [4] and Watson (1964) [5] proposed the kernel estimation method and gave the asymptotic properties of the estimator. Subsequently, Härdle (1990) [6] gave more discussion on this estimation method. Robinson (1998) [7] constructed a least-squares estimate of the parameters to be estimated based on nonparametric Nadaraya-Waston kernel estimates. He et al. (2002) [8] used the M-estimation method of the linear part and the regression spline method for the non-parametric part when estimating the semi-parametric model. Zhou, You (2002) [9] applied the wavelet estimation method to the estimation of the non-parametric part of the model. Fan and Huang (2005) [10] proposed the contour least squares estimation method for the semi-coefficient model, which can theoretically verify that this contour least squares estimation method is an effective method. [11] Zhang Bo et al. (2015) [12] proposed a new profile least squares estimation when studying part of the linear variable coefficient model, and the obtained estimate has good asymptotic properties.

In fact, in real life, the acquisition of covariate values is not necessarily accurate, and it is very important to check whether the model is correct. Sun et al. (2009) [13] studied the problem of model testing for partial linear models based on empirical processes under random missing response variables, but the above method cannot be directly applied to regression models with measurement errors of covariance. Fuller [14] proposed that ignoring the errors generated in the measurement is likely to cause endogenous problems, resulting in inconsistent parameter estimates, thus masking the true relationship between variables. The research on the linear measurement error model dates back to Bickel and Ritov (1987) [15], and they give effective estimates of the model. Stefanski and Carroll (1987) [16] constructed a uniform estimator for the generalized linear measurement error model. Liang, Hardle and Carroll (1999) [17] proposed a partial linear measurement error model. You and Zhou (2004) [18] studied the estimation problem of a regression model with purely variable coefficients measured under additive error. In the nonparametric field, Carroll (1999) [19], Delaigle and Hall (2007) [20]and Delaigle and Meister (2007) [21] have made effective progress. So far, there is not much research on the measurement error model of data with addable structure.

In this paper, we first use a new contour least squares estimation method to consider a partial linear nonparametric model. On this basis, we then consider the case where the covariant contains additive measurement errors. The estimation result is more accurate.

2. Model estimation
2.1 A new profile least square method
Due to the existence of dimensional disasters, we introduce some linear non-parametric models. When \( X \) is observable, we follow the least squares estimation proposed by Fan to estimate the regression parameter components, and use local polynomial estimation to estimate the variable coefficient function.

We first introduce a new partially linear non-parametric model of contour least squares estimation:

\[
Y = m(X_1, X_2, \cdots X_q) + \sum_{j=q+1}^p X_j \beta_j + \epsilon,
\] (3)
where \( Y \) is the response variable, \( X_1, X_2, \ldots, X_p \) is the covariate, \( \beta_{q_1}, \beta_{q_2}, \ldots, \beta_{q_p} \) is the constant value parameter, and \( \varepsilon \) is the unobservable random error item, and \( E(\varepsilon) = 0, \text{Var}(\varepsilon) = \sigma^2 > 0 \).

For the sake of simplicity, we will only discuss the case where the non-parametric part of the covariate is one. When the covariate increases, the meta-Taylor expansion can be used for similar generalization.

Assuming that there are \( n \) independent repeated observations, part of the linear non-parametric model can be written as follow:

\[
y_i = m(x_i) + x_i^T \beta + \varepsilon_i, \quad i = 1, 2, \cdots, n, \tag{4}
\]

where \( x_i = (x_{i1}, x_{i2}, \cdots, x_{ip})^T \), \( \beta = (\beta_1, \beta_2, \cdots, \beta_p)^T \). The error term \( \varepsilon_i \) has expectation \( E(\varepsilon_i | x_i) = 0 \) and variance \( \text{Var}(\varepsilon_i | x_i) = \sigma^2 \).

Supposing \( X = \begin{bmatrix} x_{11}^T \\ x_{12}^T \\ \vdots \\ x_{1n}^T \end{bmatrix} = \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots \\ x_{n1} & \cdots & x_{np} \end{bmatrix} \), \( Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \), \( \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \), \( m(X_i) = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{in} \end{bmatrix} \), then part of the linear non-parametric model can be written as follow:

\[
y = m(X_i) + X \beta + \varepsilon, \tag{5}
\]

We first treat the non-parametric part \( m(X_i) \) as a known quantity, so the model (5) can be rewritten as:

\[
y - m(X_i) = X \beta + \varepsilon, \tag{6}
\]

For (6), the least square method can be used to estimate \( \beta \)

\[
\hat{\beta} = \left( X^T X \right)^{-1} X^T (Y - m(X_i)), \tag{7}
\]

Bring (7) into the model (5), we can get (8) after finishing

\[
\left[ I - X \left( X^T X \right)^{-1} X^T \right] Y = \left[ I - X \left( X^T X \right)^{-1} X^T \right] m(X_i) + \varepsilon, \tag{8}
\]

Recording \( \hat{Q}_x = \left[ I - X \left( X^T X \right)^{-1} X^T \right] \), it is easy to know \( Q_x^2 = Q_x \), and \( Q_x \) is symmetrical array.

In summary, non-parametric models are available

\[
Y^* = m^*(x_i) + \varepsilon, \tag{9}
\]

where \( Y^* = Q_x Y, m^*(x_i) = Q_x m(x_i), m^*(x_i) = \left( m^*(x_{i1}), m^*(x_{i2}), \cdots, m^*(x_{in}) \right)^T \).

To obtain the estimation of \( m(X_i) \), we assume that \( m(X_i) \) has a continuous \( k \) derivative of order, and for any given \( x_0 \), by the Taylor formula, in the neighborhood of \( x_0 \)

\[
m(x_i) = \sum_{j=0}^{k} \beta_j (x_0) (x_i - x_0)^j, \quad i = 1, 2, \cdots, n, \tag{10}
\]

For convenience, we introduce symbols to express the model in matrix form

\[
D(x_0) = \begin{bmatrix}
1 & x_{i1} - x_0 & \cdots & (x_{i1} - x_0)^j \\
1 & x_{i2} - x_0 & \cdots & (x_{i2} - x_0)^j \\
\vdots & \vdots & \ddots & \vdots \\
1 & x_{in} - x_0 & \cdots & (x_{in} - x_0)^j \\
\end{bmatrix},
\]

\[
\beta(x_0) = \left( \beta_0(x_0), \beta_1(x_0), \cdots, \beta_k(x_0) \right)^T, W(x_0) = \text{diag}(K(x_{i1} - x_0), \cdots, K(x_{in} - x_0))^T,
\]

where \( \beta_j(x_0) = \frac{1}{j!} m^{(j)}(x_0), \quad j = 1, \cdots, k \), \( K_a(\cdot) \) represents the kernel function.

Next, we will minimize the objective function
where 

$$\tilde{m}(X_i) = (\tilde{m}(x_{1i}), \tilde{m}(x_{2i}), \ldots, \tilde{m}(x_{ni}))^T, Q_x = (Q_{x,1}, \ldots, Q_{x,n}).$$

We express equation (11) in matrix form as

$$(Q_x (W(x_0)Y - W(x_0)D(x_0)\beta)) = \begin{pmatrix} D^T(x_0)W(x_0)Q_xW(x_0)D(x_0) \end{pmatrix}^{-1}D^T(x_0)W(x_0)Q_xW(x_0)Y,$$

$$\hat{\beta}(x_0) = \left( D^T(x_0)W(x_0)Q_xW(x_0)D(x_0) \right)^{-1}D^T(x_0)W(x_0)Q_xW(x_0)Y. \tag{12}$$

The estimation of the model (5) can be obtained by combining (7)

$$\hat{\beta} = (X^T X)^{-1} X^T (Y - \tilde{m}(X_i)) \tag{13}$$

2.2 A new contour least square method with measurement error

In actual situations, the true value of $X$ may not be obtained, we consider the case of covariate with measurement error. Suppose the new covariate is $W_i$

$$W = X + U, \tag{15}$$

where $W = (w_1, \ldots, w_n)^T, X = (x_1, \ldots, x_n)^T, U = (u_1, \ldots, u_n), i = 1, \ldots, n.$

Substituting (15) into (5), then

$$Y = m(X_i) + (W - U)\beta + \epsilon. \tag{16}$$

$\epsilon$ is a vector with an independent and identical distribution whose mean is 0, and the covariance is $\Sigma_U$. If the measurement error is ignored, the final estimate is inconsistent. In linear regression or partial linear regression, we can use "attenuation correction" to solve the problem of inconsistencies caused by measurement errors. Next, we analyze the new contour least square method with measurement errors in the linear part of the model based on the partial linear nonparametric regression model proposed in 2.1.

In the same way as in 2.1, we first treat the non-parametric part $m(X_i)$ as a known quantity, so the model (16) can be rewritten as

$$Y - m(X_i) = (X + U)\beta + \epsilon, \tag{17}$$

The new estimate $\hat{\beta}$ from (17)

$$\hat{\beta} = \left((W - U)^T(W - U)\right)^{-1}(W - U)^T(Y - \tilde{m}(X_i)), \tag{18}$$

According to the above assumptions, then

$$\hat{\beta} = \left(W^TW - n\Sigma_U \right)^{-1}W^T(Y - \tilde{m}(X_i)), \tag{19}$$

Bring (19) into the model (16), we can get (20) after finishing

$$\left[ I - W\left(W^TW - n\Sigma_U \right)^{-1}W^T \right] Y = \left[ I - W\left(W^TW - n\Sigma_U \right)^{-1}W^T \right] m(X_i) + \epsilon. \tag{20}$$

Let $Q_x = \left[ I - W\left(W^TW - n\Sigma_U \right)^{-1}W^T \right]$, then $Q_x$ is no longer the idempotent matrix in 2.1. Similarly, in the analysis in 2.1, we will minimize the objective function

$$(Q_x (W(x_0)Y - W(x_0)D(x_0)\beta)) = \left( Q_x (W(x_0)Y - W(x_0)D(x_0)\beta) \right)^T$$

$$= \left( Q_x (W(x_0)Y - W(x_0)D(x_0)\beta) \right)^T = \left( Q_x (W(x_0)Y - W(x_0)D(x_0)\beta) \right)^T \tag{21}$$
Differentiating $\beta$ in (21), we can get
\begin{equation}
-2(W(x_0)D(x_0))^\top Q_{\theta \varepsilon}W(x_0)Y + 2(W(x_0)D(x_0)\beta)^\top Q_{\theta \varepsilon}W(x_0)D(x_0)\beta = 0, \tag{22}
\end{equation}
By (22), we can get
\begin{equation}
\hat{\beta}(x_0) = (D^\top (x_0)W(x_0)Q_{\theta \varepsilon}^2 W(x_0)D(x_0))^{-1} D^\top (x_0)W(x_0)Q_{\theta \varepsilon}^2 W(x_0)Y \tag{23}
\end{equation}
The estimation of the measurement error of model (5) is
\begin{equation}
\hat{\beta}(x_i) = (D^\top (x_i)W(x_i)Q_{\theta \varepsilon}^2 W(x_i)D(x_i))^{-1} D^\top (x_i)W(x_i)Q_{\theta \varepsilon}^2 W(x_i)Y
\end{equation}
\begin{equation}
\hat{\beta} = (W^\top W - n \Sigma_U)^{-1} W^\top (Y - \hat{m}(X_i)) \tag{24}
\end{equation}
where $Q_{\theta \varepsilon} = I - W(W^\top W - n \Sigma_U)^{-1} W^\top$.

Sometimes, it is necessary to estimate the error variance $\sigma^2 = E(\epsilon_i^2)$ for tasks such as the construction of confidence intervals, model-based testing, model program selection, and signal-to-noise ratio determination. Next, we estimate the error variance $\sigma^2$, according to
\begin{equation}
E\left( (y_i - m(x_i) - w_i \beta)^2 \right) = \sigma^2 + \beta^\top \Sigma_U \beta, \tag{25}
\end{equation}
In summary, we can get the new error variance as
\begin{equation}
\hat{\sigma}_2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - m(x_i) - w_i \beta)^2 - \hat{\beta}^\top \Sigma_U \hat{\beta}. \tag{26}
\end{equation}

3. Some simulation studies
In this part, we first conduct some simulations to study the feasibility of the coefficient estimator of the partial linear variable coefficient model proposed in Part 2. For the convenience of discussion, we assume that the linear part of the partially linear nonparametric model has two parameters, which can be expressed as follows:
\begin{equation}
y = m(x_i) + x_i \beta_1 + x_i \beta_2 + \varepsilon, \tag{27}
\end{equation}
Assuming $(y_i, x_{i1}, x_{i2}, x_{i3})$ is the observation value of the $n$ independent repeated experiments, the model (25) can be written as follow:
\begin{equation}
y_i = m(x_i) + x_{i2} \beta_1 + x_{i3} \beta_2 + \varepsilon_i, i = 1, \ldots, n, \tag{28}
\end{equation}
where $x_{i1} \sim U(0,1), x_{i2} \sim N(0,1), x_{i3} \sim N(0,1), \beta_1 = 1.5, \beta_2 = 1, m(x_i) = \sin(\pi x_{i1})$ and $\varepsilon_i \sim N(0,1).$ And $w_{i2} = x_{i2} + u_{i2}, w_{i3} = x_{i3} + u_{i3}, (u_{i2}, u_{i3}) \sim N(0, \Sigma_U)$. Below we take $\Sigma_U = diag(0.5, 0.5)$ and $\Sigma_U = diag(0.5, 0.5)$ respectively to verify the accuracy of the model (27). At this time, the sample size $n = 1000.$ First of all, we get the relationship diagram of $y$ and $x_{i1}, x_{i2}, x_{i3}$:
Figure.1 the relationship diagram of \( y \) and \( x_1, x_2, x_3 \):

It can be seen from the above figure that \( y \) and \( x_i \) do not have a linear relationship obviously, so our hypothetical model (27) is reasonable.

In order to verify the good nature of our estimator, we introduce benchmark estimators, modified profile least squares estimator and naive estimators. Below we discuss the specific expressions of these three estimators and the error variance, and compare the numerical simulation results. The kernel function we take in this part is the Gaussian kernel

\[
K_x(\cdot) = \frac{1}{h \sqrt{2\pi}} \exp\left(-\frac{(\cdot)^2}{2h^2}\right),
\]

(29)

where the bandwidth \( h \) is selected through cross-validation.

In 2.1 we got the parameters of the linear part and the error variance estimates when the covariate is accurate, so the benchmark estimators are as follows

\[
\hat{\beta} = \left(X^T X \right)^{-1} X^T (Y - \hat{m}(X_i)),
\]

(30)

\[
\hat{\sigma}_n^2 = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - m(x_i) - x_i \beta - x_i \beta_1 \right)^2,
\]

(31)

where \( X_i = (x_{i1}, X = (x_{i2}, x_{i3}) \), \( \beta = (\beta_2, \beta_1)^T \).

In 2.2 we obtained the parameters of the linear part and the error variance estimator when the covariate has measurement error, so modified profile least squares estimator is as follow

\[
\tilde{\beta} = \left(W^T W - n \sum_U \right)^{-1} W^T (Y - \tilde{m}(X_i)),
\]

(32)

\[
\tilde{\sigma}_n^2 = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - m(x_i) - w_{i2} \beta_2 - w_{i3} \beta_1 \right)^2 - \tilde{\beta}^T \sum_U \tilde{\beta},
\]

(33)

where \( W = (w_{i2}, w_{i3}) \).

Below we consider naive estimators. The naive estimators are obtained when the observation value is inaccurate and the covariate with measurement error is ignored, and it is directly obtained by bringing \( W \) into the expression (30) of the parameter estimate of the model (27) when the covariate is accurate, so

\[
\ddot{\beta} = \left(W^T W \right)^{-1} W^T (Y - \hat{m}(X_i)),
\]

(34)

Similarly, the available error variance estimate at this time is

\[
\ddot{\sigma}_n^2 = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - m(x_i) - w_{i2} \beta_2 - w_{i3} \beta_1 \right)^2.
\]

(35)

We separately calculated the sample mean of the above three parameter components and error variance estimator of model (27). Table 1 below summarizes all the above simulation results. Here, in the partially linear non-parametric model, the parameter component is \( \beta_2 = 1.5, \beta_1 = 1 \) and error variance is \( \sigma^2 = 1 \).
Table 1. Comparison Table of Sample Means of Three Estimates

|                | $\Sigma_U = diag(0.5,0.5)$ | $\Sigma_U = diag(0.25,0.25)$ |
|----------------|-----------------------------|-----------------------------|
| Mean ($\hat{\beta}_1$) | 1.50436                     | 1.505559                    |
| Mean ($\beta_2$) | 1.010507                    | 1.010922                    |
| Mean ($\sigma^2_1$) | 1.017790                    | 1.057142                    |
| Mean ($\beta_1$) | 1.566943                    | 1.537976                    |
| Mean ($\hat{\beta}_2$) | 0.973766                    | 0.994760                    |
| Mean ($\sigma^2_2$) | 0.754026                    | 0.929493                    |
| Mean ($\hat{\beta}_1$) | 1.260036                    | 1.361279                    |
| Mean ($\hat{\beta}_2$) | 0.906780                    | 0.947656                    |
| Mean ($\sigma^2_1$) | 2.148958                    | 1.741819                    |

Figure 2: Estimation of non-parametric components $m(x_i)$ when $n = 1000$

Where $\hat{m}(x_{i1})$ is represented by a dotted line, $\tilde{m}(x_{i1})$ is represented by a dotted line, $\tilde{m}(x_{i1})$ is represented by a solid line.

4. Conclusion

1. In the parameter estimation of the linear part, when the covariate is accurate, the estimator result is very close to the exact value taken by the parameter, and the error variance at this time is closest to 1. Therefore, the estimated value obtained by using the new contour least square method with measurement error is greatly improved in accuracy than the estimation value without considering the measurement error. It can be seen that the new contour with measurement error proposed in this paper is the smallest the square method still has a good parameter estimation effect when the measured value is inaccurate.

2. In the non-parametric part, in most cases, the measured value is not necessarily accurate. If the result obtained by ignoring the measurement error is ignored, the error generated in the measurement is likely to cause endogenous problems, resulting in the obtained parameter estimates are not consistency. On this basis, we propose a modified contour least squares estimation with measurement error. It turns
out that the revised estimation function with measurement error is closest to the real function we assumed.

In summary, a new profile least squares method with measurement errors for covariates and measurement errors proposed in this paper has a good estimate of the parametric and non-parametric parts of partially linear non-parametric models.

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