Chiral phase transition of (2+1)-flavor QCD

H.-T. Ding\textsuperscript{a}, P. Hegde\textsuperscript{b}, F. Karsch\textsuperscript{c,d}, A. Lahiri\textsuperscript{c}, S.-T. Li\textsuperscript{a}, S. Mukherjee\textsuperscript{d}, P. Petreczky\textsuperscript{d}

\textsuperscript{a}Key Laboratory of Quark \& Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
\textsuperscript{b}Center for High Energy Physics, Indian Institute of Science, Bangalore 560012, India
\textsuperscript{c}Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany
\textsuperscript{d}Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA

Abstract

We present here results on the determination of the critical temperature in the chiral limit for (2+1)-flavor QCD. We propose two novel estimators of chiral critical temperature where quark mass dependence is strongly suppressed compared to the conventional estimator using pseudo-critical temperatures. We have used the HISQ/tree action for the numerical simulation with lattices with three different temporal extent $N_\tau = 6, 8, 12$ and varied the aspect ratio over the range $4 \leq N_\sigma/N_\tau \leq 8$. To approach the chiral limit, the light quark mass has been decreased keeping the strange quark mass fixed at its physical value. Our simulations correspond to the range of pion masses, $55 \text{ MeV} \leq m_\pi \leq 160 \text{ MeV}$.

Keywords: Lattice QCD, chiral symmetry, phase transition, critical point, universality class

1. Introduction

Strongly interacting matter under extreme conditions undergoes a transition from a chirally broken (confined) phase to a chirally restored (deconfined) phase at a certain temperature. At vanishing baryon density, this transition is a crossover for physical pion mass and the corresponding pseudo-critical temperature has been determined\cite{1} with very good accuracy\cite{2}. In the chiral limit, i.e. in the limit of vanishing light quark mass, two possible scenarios have been conjectured, the chiral phase transition can either be (1) of 2\textsuperscript{nd} order belonging to $O(4)$ universality class\cite{3,4} or to $U(2) \times U(2)$ universality class\cite{5} or (2) it can be of 1\textsuperscript{st} order\cite{4}. In the later case, a critical light quark mass will exist, at which the transition will be of 2\textsuperscript{nd} order belonging to $Z(2)$ universality class. In this work our main goal is to determine the chiral transition temperature, $T_0^\text{c}$, and we will shed some light on the nature of the chiral transition.

2. Observables and definitions

The quark condensate per flavor $f$ is defined as $\langle \bar{\psi}\psi \rangle_f = TV^{-1}\partial \ln Z(T,V,\{m_f\})/\partial m_f$. The light quark condensate $\langle \bar{\psi}\psi \rangle$ is the order parameter of the chiral phase transition. It requires additive as well as multiplicative renormalizations. We therefore have worked with a renormalization group invariant quantity,
$M = 2 \left( m_l \langle \bar{\psi}\psi \rangle - m_s \langle \bar{\psi}\psi \rangle \right) / f_K^4$, where we have used $f_K = (156.1/\sqrt{2})$ MeV to set the scale. The corresponding susceptibility is defined as $\chi_M = m_l \partial M / \partial m_l$, where $m_l$ and $m_s$ are light and strange quark masses, respectively.

Near the critical point the order parameter $M$ and its susceptibility $\chi_M$ are expected to behave like $M = h^{1/\delta} f_G(z) + f_{\text{sub}}(T, H)$ and $\chi_M = h^{1/\delta} f_f(z) + \partial f_{\text{sub}}(T, H) / \partial H$, where $f_G(z)$ and $f_f(z)$ are universal scaling functions, which have been determined previously from spin model calculations, and $f_{\text{sub}}(T, H)$ takes into account corrections-to-scaling terms and regular terms. The scaling variable $z$ is defined as $z = th^{-1/\delta} = z_0 \left( (T - T_c^0) / T_c^0 \right) H^{-1/\delta}$ with $t = t_0^{-1} \left( (T - T_c^0) / T_c^0 \right)$ and $h = (m_l / m_s) / h_0 = H / h_0$. The scale $z_0$ is defined as $z_0 = h_0^{1/\delta} / h_0$ with $T_c^0$ being the chiral critical temperature and $H$ being the symmetry breaking field.

The scaling variable $z$ may be solved for $T_c^0$, which then gives the dependence of $T_c^0$ on $H$ when approaching the chiral limit at fixed $z_c$: 

$$T(z, H) = T_c^0 \left( 1 + \frac{z}{z_0} H^{1/\delta} \right). \quad (1)$$

This is commonly used when analyzing the approach of the pseudo-critical temperature $T_{pc}$, defined through the peak of $\chi_m$ located at $z = z_p$, to the chiral limit. Here we consider a different choice of $z$. We propose to choose $z = z_{60\%}$; the value of $z_{60\%}$ or equivalently $T_{60\%}$ is defined as $\chi_M \left( T_{60\%} \right) = 0.6 \chi_M \left( T_c \right)$. From the left panel of Fig. 1 one can clearly see that for the scaling functions, which are relevant for our discussion, 60% of the peak corresponds to $z = 0$, i.e. to a temperature close to the critical point. The influence of $H$-dependent corrections thus will be suppressed by at least an order of magnitude compared to $z_p$. This in turn says that the estimator of $T_c^0$ using $T_{60\%}$ will be far more stable than that using $T_{pc}$.

Another estimate of the critical temperature can be obtained from the ratio $H \chi_M / M$ which behaves like a Binder cumulant at the critical point [5]:

$$\lim_{H \to 0} \lim_{V \to \infty} \frac{H \chi_M(T_c, V, H)}{M(T_c, V, H)} = \frac{1}{\delta} \Rightarrow T_{c,0} = \lim_{H \to 0} \lim_{V \to \infty} T_c(V, H). \quad (2)$$

In the right panel of Fig. 1, we have shown this ratio using $O(4)$ scaling functions for three different values of symmetry breaking parameter $H$. For simplicity we have set the scale $z_0 = 1$. From the figure one can clearly see that in absence of corrections-to-scaling and regular terms the crossing point is unique for different curves corresponding to different $H$ and has the value $1/\delta$.

To get some idea about the nature of the chiral transition we looked at the following ratio

$$\frac{M}{\chi_M} = (H - H_c) \frac{f_G(z)}{f_f(z)}, \quad (3)$$

where $H_c$ is zero for $O(4)$ or $O(2)$ transitions and non-zero for $Z(2)$ transition. It is clear from Eq. 3 that near the critical point the ratio will be linear in $H$ and the slope is uniquely defined through the universal
scaling functions. This means that one can directly determine the slope and hence the universality class with precise enough data. If the chiral transition is of 1st order then this ratio will have a sudden drop at some non-zero value of $H_c$. One can also estimate this critical value $H_c$ from the ratio using informations from $Z(2)$ universality class.

3. Results

We have used the HISQ/tree action for numerical simulations of (2+1)-flavor QCD. To approach the chiral limit we have decreased $m_l/m_s$ (keeping $m_s$ fixed at its physical value) corresponding to $55 \text{ MeV} \leq m_s \leq 160 \text{ MeV}$. We have used lattices with temporal extent $N_{\tau} = 6, 8, 12$ and the spatial volumes used are in the range $4 \leq N_c/N_{\tau} \leq 8$.

![Fig. 2. Left panel : $\chi_M$ for five different quark masses are plotted as a function of $T$ for $N_{\tau} = 8$. Right panel : $\chi_M$ vs. $T$ for three different spatial volumes for $N_{\tau} = 8$ with $m_s/m_l = 80$.](image1)

In the left panel of Fig. 2 we have plotted $\chi_M$ for $N_{\tau} = 8$ lattices with five different quark masses. The increase of $\chi_M$ with decreasing quark mass is evident and roughly consistent with the scaling expectations, $\chi_M^{\text{max}} \sim H^{1/\delta - 1}$. In the right panel of Fig. 2 we have shown the volume dependence of $\chi_M$ for $m_s/m_l = 80$ which is the next to lowest mass for $N_{\tau} = 8$ lattices. It is evident from the figure that $\chi_M^{\text{max}}$ decreases slightly as the volume increases, which is opposite to what is expected for a 1st or 2nd order phase transition. So we can eventually rule out the possibility of a 1st order phase transition for $m_s \geq 80 \text{ MeV}$. The black line in the plot comes from a linear extrapolation in $1/V$ using the two largest volumes.

![Fig. 3. Left panel : $T_{60\%}$ vs. $H$ for $N_{\tau} = 8$ lattices. Right panel : $H\chi_M$ vs. $T$ for three lowest $H$ for $N_{\tau} = 8$.](image2)

In the left panel of Fig. 3 we have shown $T_{60\%}$ calculated for $N_{\tau} = 8$ lattices corresponding to the three lowest pion masses. As can be seen from the figure, $T_{60\%}$ is almost constant for low enough pion masses which is expected from Eq. 4 since $z_{60\%}/z_0 \sim O(10^{-2})$. This implies eventually fitting a constant to $T_{60\%}$ in this regime can already give a reliable estimate of $T_0$. This constant is around 142 MeV for $N_{\tau} = 8$ which is shown along with its uncertainty by the band in the figure. In the right panel of Fig. 3 we have shown the ratio $H\chi_M$ for three lowest pion masses for $N_{\tau} = 8$ and the solid lines are splines to guide the eye. The uniqueness of the crossing points, what we have discussed in Sec. 2 is absent in the data. Our preliminary analysis suggests that deviations from this unique crossing are mainly due to finite volume effects rather than contributions from regular terms. A conservative estimate that takes these systematic effects into account,
yields $T_c^0 \sim 144$ MeV for $N_f = 8$. Performing a joint fit to $M$ and $\chi_M$ with magnetic equation of state [7], a comparable value $T_c^0 \sim 145$ MeV is obtained. Putting together all the above-mentioned estimates we arrive at $T_c^0 = 144(2)$ MeV which is the current estimate for $N_f = 8$. Similar analyses have also been carried out for $N_f = 6$ and $N_f = 12$ which gives the estimates $T_c^0 = 147(2)$ MeV and $139(3)$ MeV, respectively. A continuum extrapolation linear in $N_f^{-2}$, using results for different $N_f$, yields

$$T_c^0 = 138(5) \text{ MeV (continuum HotQCD preliminary)}.$$  

![Figure 4](https://example.com/figure4.png)

Fig. 4. $M/\chi_M$ is plotted for different $N_f$ along with the scaling expectations from different universality classes.

Before concluding we would like to say a few (preliminary) words on the order of the chiral phase transition and the corresponding universality class. In Fig. 4 we have plotted $M/\chi_M$ as a function of $H$ for different $N_f$ at two different positions. In the left panel, we have plotted the ratio at the peak position of $\chi_M$ and in the right panel we have shown the same at the point where $\chi_M$ attains 60% of its maximum. For both the plots one can see for low enough masses the ratio seems to behave linearly. The colored bands in the plots are not fits rather expectations coming from $O(N)$ universality classes. The width in the band comes from the small difference between $O(4)$ and $O(2)$. For a crude comparison we have also plotted expectations from $Z(2)$ universality class by black lines for two different values of $H_c$: solid line is for $H_c = 1/120$ and dashed line is for $H_c = 1/240$. The data seems to favor $O(N)$ expectations over $Z(2)$. Although one has to keep in mind that for a non-vanishing $H_c$, $M$ is not any more an exact order parameter and the $Z(2)$ lines in Fig. 4 will not be actually straight lines.

4. Conclusions

We have estimated the chiral critical temperature $T_c^0$ using (2+1)-flavor HISQ/tree action. For $N_f = 6$, 8 and 12 the preliminary estimates of $T_c^0$ are $147(2)$ MeV, $144(2)$ MeV and $139(3)$ MeV, respectively. A continuum extrapolation using these numbers gives the preliminary estimate that in continuum $T_c^0 = 138(5)$ MeV. Our preliminary analyses seem to favor a 2nd order chiral phase transition over a 1st order transition.

5. Acknowledgments

This work was supported in part through contract No. de-sc0012704 with the U.S. Department of Energy, Scientific Discovery through Advance Computing (SciDAC) award “Computing the Properties of Matter with Leadership Computing Resources”, the Deutsche Forschungsgemeinschaft (DFG) through the grant CRC-TR 211 “Strong-interaction matter under extreme conditions”, the grant 05P15PBCAA of the German Bundesministerium für Bildung und Forschung, Early Career Research Award of the Science and Engineering Research Board of the Government of India and the National Natural Science Foundation of China under grant numbers 11535012 and 11775096.

References

[1] A. Bazavov et al.; Phys. Rev. D 85, 054503 (2012).
[2] Recent updated determination : Contribution of P. Steinbrecher in this proceeding.
[3] J. B. Kogut, M. Stone, H. W. Wyld, J. Shigemitsu, S. H. Shenker and D. K. Sinclair; Phys. Rev. Lett. 48, 1140 (1982).
[4] R. D. Pisarski and F. Wilczek; Phys. Rev. D 29, 338, (1984).
[5] A. Pelissetto and E. Vicari; Phys. Rev. D 88, 105018 (2013).
[6] F. Karsch and E. Laermann; Phys. Rev. D 50, 6954 (1994).
[7] S. Ejiri et. al.; Phys. Rev. D 80, 094505 (2009).