The Tachyon does Matter

A. Buchel\(^1\), J. Walcher\(^2\) \(^\S\)

\(^1\) Michigan Center for Theoretical Physics
Randall Laboratory of Physics, The University of Michigan
Ann Arbor, MI 48109-1120, USA
\(^2\) Kavli Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106, USA

Abstract: We review the concept of S-branes introduced by Gutperle and Strominger [1]. Using the effective spacetime description of the rolling tachyon worldsheets discussed by Sen, we analyze the possibility that the gravitational backreaction of tachyon matter is important in the time-dependent process. We show that this is indeed the case in the example of the S0-brane in 4-dimensional Einstein-Maxwell theory. This talk is based on [21].

1 S-branes

Gutperle and Strominger [1] have argued that there should be spacelike branes in string theory. Let us start by reviewing what one means by a “spacelike brane” (S-brane).

1.1 Spacelike kinks

It is well-known that scalar field theories in 1 + 1 dimensions,

\[ S = \int \left( -\frac{1}{2} (\partial_t \Phi)^2 + \frac{1}{2} (\partial_x \Phi)^2 + V(\Phi) \right), \]

have static solitonic kink solutions that interpolate between the minima of the appropriately chosen \(V\). For example, in \(\Phi^4\) theory, there is a soliton that corresponds to the classical trajectory of a particle moving in the inverted potential \(-V\) from one extremum to the other.

In string theory, if \(\Phi\) is the tachyon on an unstable \(D(p+1)\)-brane, Sen has shown [13] that such a tachyonic kink corresponds to a stable \(Dp\)-brane. This is essentially due to a term

\[ \int dT \wedge C^{(p+1)} \]  

in the effective action of the unstable brane [13, 14], where \(C^{(p+1)}\) is the \(p + 1\)-form RR field coupling to \(Dp\)-brane charge.

\(^\S\)corresponding author: walcher@kitp.ucsb.edu
Gutperle and Strominger argue in [1] that one can also make spacelike kinks, in the following way. Consider spatially homogeneous initial conditions for the scalar field at the top of its potential and push it infinitesimally towards one minimum. If the scalar field is weakly coupled to some form of classical radiation, it will lose energy and for \( t \to \infty \) eventually settle down in the minimum of the potential. The time reversed process looks like (finely tuned) radiation coming in from infinity and exciting the tachyon to the top of its potential. The total process\(^1\), in which the field goes from one minimum to the other looks like a scalar field kink \emph{in time}.

In string theory, a spacelike tachyonic kink is a source for RR fields, again because of (1). In the limit that the lifetime of the resonance is very short, the object corresponding to it can be thought of as a spacelike analog of the usual timelike branes. (One can also think of this as a tachyonic brane, since it moves outside of the lightcone). For the purposes of this talk, one may use the following as a working definition.\(^2\)

An S\(p\)-brane is the time-dependent process involving the creation and subsequent decay of an unstable D\((p + 1)\)-brane in string theory.

The fact that spacelike branes as spacelike tachyonic kinks require the coupling of the scalar to radiation is a crucial difference to the timelike case that is important to keep in mind.

The main motivation for introducing S-branes comes from the desire to generalize holography to the time-dependent and, in particular, cosmological situation. In string theory, holography is realized as a correspondence between open string or brane physics on the one side and closed string or bulk physics on the other side. In light of this, a spacelike brane on which open strings can end with Dirichlet boundary conditions in the time direction, is the natural “holographic plate” for cosmological, and, in particular, de Sitter spacetimes. This desirable connection of S-branes to dS/CFT \([9, 10]\) gives a justification for the R-symmetry requirements imposed on the gravity solutions that we discuss below.

### 1.2 The S-brane charge

It is worthwhile mentioning here that there is nothing funny about charges of spacelike objects. Given a spacelike source for a \((p + 1)\)-form field (in flat space),

\[
dF^{(p+2)} = 0 \quad \quad *d* F^{(p+2)} = Q \delta(x_\perp) dx_\parallel, \tag{2}
\]

one can measure the charge \(Q\) by integrating over a sphere surrounding the position of the source,

\[
Q = \int_{S^{D-p-2}} *F^{(p+1)}. \tag{3}
\]

The statement that the charge is conserved refers to the fact that \(Q\) is independent of the sphere used to compute it. This is independent of whether the source is space- or timelike. There is one important difference, however. In solving (2) one needs the propagator of a massless scalar in the transverse directions. From the definition of an S-brane, one should like to use a causal Green’s function (advanced plus retarded, say). Now the causal propagator for a massless scalar has support on the lightcone for an even number of transverse dimensions and support inside the lightcone for an odd number of transverse directions. This is likely to complicate the construction of an S-brane with an

---

\(^1\)which might seem highly unlikely due to the necessary fine tuning. One can understand this as a resonance.

\(^2\)There are also codimension two S-branes obtained from tachyonic vortices, but we shall not discuss those here.
even number of transverse directions, like the expected S-branes in type IIB string theory, substantially. Mentioned in [1], this difference does not seem to have been properly taken into account in the literature so far.

1.3 Gravitational solutions

Gravitational solutions corresponding to S-branes have been proposed in [6] and [7], further generalized, for instance, in [10, 11, 12]. Some of their features make their interpretation as S-branes, in fact, rather difficult. We shall exemplify these difficulties for the S0-brane in 4-dimensional Einstein-Maxwell gravity [1],

$$S = \int \sqrt{-g} (R - \frac{1}{4} F^4).$$

The natural ansatz has SO(2,1) \(\times \mathbb{R}\) symmetry and is of the form

$$ds^2 = -c_1^2 dt^2 + c_2^2 dz^2 + c_3^2 ds_{H_2}^2,$$

in which the warp factors \(c_1, c_2,\) and \(c_3\) are functions of \(t\) only, and the two-dimensional hyperbolic space \(H_2\) gives the leaves of the transverse foliation. From flat space considerations, one is led to make the ansatz

$$*F = Q \text{vol}_{H_2},$$

for the flux, where \(\text{vol}_{H_2}\) is the volume form on \(H_2\). The resulting equations of motions become

$$-\frac{1}{c_1^2} \left[ \frac{c''}{c_3} + \frac{c''}{c_2} - \frac{c''}{c_1} \left( \frac{2c_3}{c_3} + \frac{c''}{c_2} \right) \right] = \frac{Q^2}{c_3^2},$$

$$\frac{1}{c_2^2} \left[ \frac{c''}{c_2} - \frac{c''}{c_2} + \frac{2c''}{c_2} \right] = -\frac{Q^2}{c_3^2},$$

$$-\frac{1}{c_3^2} + \frac{1}{c_1^2} \left[ \frac{c''}{c_3} - \frac{c''}{c_3} \frac{c''}{c_1} + \left( \frac{c''}{c_3} \right)^2 + \frac{c''}{c_3} \frac{c''}{c_3} \right] = \frac{Q^2}{c_3^2},$$

and can be solved explicitly in the gauge \(c_1 = 1/c_2\), to yield \(c_3 = t\), and, imposing \(t \to -t\) symmetry,

$$c_2^2 = 1 - \frac{Q^2}{t^2}.$$  

The global structure of this spacetime is illustrated by the Penrose diagram in Fig. 1. It has flat asymptotics at \(t \to \infty\), a “horizon”-like coordinate singularity at \(t = Q\), and timelike curvature singularities at \(t = 0\). In fact, this diagram is nothing but the \(\pi/2\)-rotation of the Reissner-Nordström black hole, and (10) can be obtained simply by analytical continuation from the static solution. This is discussed in more detail, and for many other solutions, in [20].

There are a number of problems with the S-brane interpretation of this solution. The most obvious is presumably that the singularities are timelike, and not spacelike, which would have been the natural expectation. Moreover, there are no regions that are causally disconnected from the singularities, another natural expectation from the definition of S-branes. And where is the brane, anyway? According to (3), one would detect this by integrating \(F\) over large spheres. However, it is easy to see that the spacetime in Fig. 1 does not have any such spheres!

For a different interpretation of the diagram in Fig. 1 involving negative tension branes, see [32, 33].
2 Tachyon Matter

In perturbative string theory, S-branes are expected to be described by imposing Dirichlet boundary conditions in time on the open string worldsheet. Sen [2, 3, 4] has clarified that this is indeed related to the “rolling tachyon” picture of S-branes. At the linearized level, i.e., for early times, the rolling tachyon with mass squared $-1$ looks like $T(x^0) = \lambda \cosh x^0$, where $x^0$ is time, and $\lambda$ is the initial displacement of the tachyon away from the top of its potential. From the string worldsheet, this looks like a boundary interaction

$$\Delta S = \lambda \int d\tau \cosh X^0(\tau).$$

Using Wick rotation and results from boundary CFT [5], Sen shows that, firstly, this boundary interaction is exactly marginal for all values of $\lambda$, and secondly, that such a boundary theory produces a source for closed strings whose energy-momentum tensor is characterized by constant energy density and exponentially (in time) vanishing pressure. Moreover, the tachyon energy-momentum tensor is localized on the plane of the decaying D-brane. This is essentially due to the fact that the tachyon is an open string.

This behaviour of the tachyon can be reproduced in an effective field theory description with a DBI type action [15, 16, 17],

$$S = \int d^{p+2}x V(T) \sqrt{-\det(g_{\mu\nu} + \partial_{\mu}T \partial_{\nu}T)}. \quad (11)$$

Even though the potential to be used in (11) is not known exactly, it is characterized by the universal asymptotics $V(T) \to e^{-|T|/\sqrt{2}}$ for $|T| \to \infty$. For purely time-dependent configurations, energy density and pressure corresponding to this action are given by

$$\rho = \frac{V(T)}{\sqrt{1 + (T')^2/g_{00}}}, \quad (12)$$

$$p = -V(T) \sqrt{1 + (T')^2/g_{00}}. \quad (13)$$

If $\rho = \text{const.}$ with $T$ approaching the minimum of $V$ at $t \to \infty$, then, since $g_{00} = -1$, $T'$ must approach 1 up to exponentially small terms. This implies that $p$ vanishes exponentially. These properties derived from the action (11) are called “tachyon matter”.

Figure 1: Penrose diagram for the metric (11). The wavy lines are time-like curvature singularities.
3 Can the tachyon matter?

Let us summarize what we have discussed so far.

- S-branes as solitonic kinks can only exist if the scalar field (the tachyon) is coupled to some form of closed string radiation.
- The gravity solutions found so far do not seem to have the correct global and singularity structure.
- From the open string perspective, the roll of the tachyon (at zero string coupling) is characterized by exponentially vanishing pressure and constant energy density, localized in the plane of the decaying brane.
- These properties can be reproduced using an effective DBI-type action.

It is quite natural, then, to ask for the gravitational backreaction of tachyon matter on the proposed S-brane backgrounds. This is achieved by coupling gravity, for instance the Einstein-Maxwell Lagrangian (4), to the tachyon matter (11). This was the essential idea of the paper [21], and we turn to its consequences for the above S0-brane now.

The coupled system of Einstein-Maxwell gravity and 1-brane has action

$$S = \int \sqrt{-g} \left( R - \frac{1}{4} F^4 \right) - \int d^4 x \rho(x_\perp) \left[ V(T) \sqrt{-\det(g_{\mu\nu} + \partial_\mu T \partial_\nu T)} + f(T) dT \wedge A \right],$$

where the last term is the coupling between gauge field and tachyon, as in (1). The functions $V$ and $f$ are assumed to have the universal exponentially vanishing asymptotics discussed above. Moreover, $\rho(x_\perp)$ is the density of 1-branes which are smeared over the transverse space. This smearing, with $\rho(x_\perp) \propto \sqrt{g_\perp}$, is necessary to make the equations tractable. The essential change in the equations of motion is the addition of terms like $(\rho \pm p)/c_3^2$ to the right hand side of (7-9). The Maxwell and tachyon equation read

$$\left( c_3^2 A \right)' = f(T) T', \quad (15)$$
$$\rho' = -(\rho + p) \frac{c_3'}{c_3} - f(T) T' A, \quad (16)$$

where $\rho$ and $p$ are given by (12) and (13).

What are the changes to the solution? First of all, it is easy to convince oneself that the early/late time asymptotics are unchanged, up to logarithmic corrections. But something interesting happens close to the coordinate singularity, $t = Q$ in (11).

To see this, let us assume that the warp factors vanish at $t = t_*$ according to $c_2^2 = c_1^{-2} = (t - t_*) \cdot (\text{finite at } t = t_*)$, with $c_3^2$ finite at $t = t_*$. Locally, this looks like the “Milne universe”, recently popular due to its importance in ekpyrotic cosmology. More precisely, introducing $\tau = \sqrt{t - t_*}$, the metric looks like $-d\tau^2 + \tau^2 dz^2$, which is isomorphic to a piece of two-dimensional Minkowski space.

From the expressions for $\rho$ and $p$, (12) and (13), it follows that $\rho = -p$ near $t = t_*$, if $T'$ is finite there. The resulting equations (in which the tachyon is essentially replaced by a cosmological constant), can be solved explicitly. This finiteness of $T'$, however, is non-generic. To see this, we consider the tachyon equation of motion

$$\rho' = -(\rho + p) \frac{c_3'}{c_3}, \quad (17)$$

and set $\rho + p = \rho(T'/c_1)^2$, which is justified if $T$ itself is finite near $t_*$. As a consequence,
which implies \( \rho^2 \propto C/(t-t_*) + 1 \). This diverges as \( t \to t_* \), unless \( C \) is fine tuned to vanish. Thus the stress tensor of the tachyon diverges, and one expects a curvature singularity. Indeed, one can solve for the warp factors, and finds
\[
\begin{align*}
c_1c_2 & \sim 1 + \text{const.} \sqrt{t-t_*} \\
c_3 & \sim 1 + \text{const.} \sqrt{t-t_*},
\end{align*}
\]  
leading to a spacelike curvature singularity.

Lastly, one can show that the remaining singularity structure is not modified by inclusion of tachyon matter \[21\]. In particular, the timelike curvature singularity is not resolved in our model. Now, however, there is a curvature singularity associated with the tachyon before one reaches the singularity associated with the gauge field, so that the order of the latter has shifted.

## 4 Conclusions and further developments

Summarizing, we have seen that the tachyon matter produces non-negligible backreaction on the S-brane background. While the resulting spacetimes may still not look exactly like what one would expect, the changes in singularity structure induced by the tachyon go in the right direction. There are a few obvious generalizations that one might want to explore, including D\overline{D}-brane systems, to see the effect of a complex tachyon, or higher dimensional systems, to see the effect of including a dilaton.\(^3\)

We conclude the talk with a brief overview of some further developments concerning S-branes and tachyon matter.

Tachyon matter has been explored from the point of view of boundary string field theory \[18, 19\]. It has also been analyzed in toy models of open string field theory in \[22\], focusing on the problems associated with infinitely many time derivatives. In \[28\], see also \[23\], it was proposed to use the tachyon for an emergent definition of time in the context of canonical quantization of gravity. In particular, it was shown that all solutions of tachyon matter are at late times equivalent to configurations of non-interacting non-rotating dust. In anticipation of dS/CFT, S-brane worldvolume actions have been proposed in \[30\].

A more puzzling aspect of the decay process of D-branes and the role of the tachyon has been revealed in recent studies of radiation production rates. While we have here studied the simplest possible coupling of the tachyon to closed strings, including only the massless supergravity modes, it is a valid question to ask whether this is a good approximation at all. In \[25, 27\] it was argued that massive closed string modes may in fact play a crucial role in the decay of the D-brane, as it was shown that the coupling of the tachyon matter to these modes grows exponentially in time and with closed string mode level. In \[25\], adopting the open string perspective, the quantum open string production in the exact CFT backgrounds of \[4, 5, 6\] was computed and shown to diverge due to the exponentially growing density of open string states. Quantum effects being closed strings, this again points to the importance of closed strings, see also \[29\]. These results are disturbing as they indicate that there are fundamental difficulties in finding a valid approximation scheme.

Lastly, we should also mention the considerable cosmological interest that tachyon matter has attracted, which, however, we have no space to review here. See \[31\] for early literature on tachyon matter cosmology.

\(^3\)We have been informed by F. Leblond that the behaviour in higher dimensions can be significantly different upon inclusion of the dilaton \[24\].
Acknowledgement We would like to thank Peter Langfelder for collaboration on the results of [21]. J.W. would like to thank the organizers of the 35th Symposium Ahrenshoop for a very stimulating atmosphere at the meeting, and the Perimeter Institute for hospitality while these notes were being written up. This work was supported in part by the NSF under Grant Nos. PHY00-98395 (A.B.) and PHY99-07949 (A.B. and J.W.).

References

[1] M. Gutperle and A. Strominger, “Spacelike branes,” JHEP 0204, 018 (2002) [arXiv:hep-th/0202210].

[2] A. Sen “Rolling tachyon,” JHEP 0204, 048 (2002) [arXiv:hep-th/0203211].

[3] A. Sen “Tachyon matter,” arXiv:hep-th/0203265.

[4] A. Sen “Field theory of tachyon matter,” arXiv:hep-th/0204143.

[5] A. Recknagel and V. Schomerus, “Boundary deformation theory and moduli spaces of D-branes,” Nucl. Phys. B 545, 233 (1999) [arXiv:hep-th/9811237].

[6] C. M. Chen, D. V. Gal’tsov and M. Gutperle, “S-brane solutions in supergravity theories,” arXiv:hep-th/0204071.

[7] M. Kruczenski, R. C. Myers and A. W. Peet, “Supergravity S-branes,” JHEP 0205, 039 (2002) [arXiv:hep-th/0204144].

[8] A. Strominger, “The dS/CFT correspondence,” JHEP 0110, 034 (2001) [arXiv:hep-th/0106113].

[9] E. Witten, “Quantum gravity in de Sitter space,” arXiv:hep-th/0106109.

[10] S. Roy, “On supergravity solutions of space-like Dp-branes,” JHEP 0208, 025 (2002) [arXiv:hep-th/0205198].

[11] N. S. Deger and A. Kaya, “Intersecting S-brane solutions of D = 11 supergravity,” JHEP 0207, 038 (2002) [arXiv:hep-th/0206057].

[12] K. Ohta and T. Yokono, “Gravitational approach to tachyon matter,” arXiv:hep-th/0207004.

[13] A. Sen, “Non-BPS states and branes in string theory,” arXiv:hep-th/9904207.

[14] M. Billo, B. Craps and F. Roose, “Ramond-Ramond couplings of non-BPS D-branes,” JHEP 9906, 033 (1999) [arXiv:hep-th/9905157].

[15] M. R. Garousi, “Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action,” Nucl. Phys. B 584, 284 (2000) [arXiv:hep-th/0003122].

[16] E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras and S. Panda, “T-duality and actions for non-BPS D-branes,” JHEP 0005, 009 (2000) [arXiv:hep-th/0003221].

[17] G. W. Gibbons, K. Hori and P. Yi, “String fluid from unstable D-branes,” Nucl. Phys. B 596, 136 (2001) [arXiv:hep-th/0009061].

[18] T. Takayanagi, S. Terashima and T. Uesugi, “Brane-antibrane action from boundary string field theory,” JHEP 0103, 019 (2001) [arXiv:hep-th/0012210].
[19] S. Sugimoto and S. Terashima, “Tachyon matter in boundary string field theory,” JHEP 0207, 025 (2002) [arXiv:hep-th/0205085].

[20] A. Buchel, P. Langfelder and J. Walcher, “On time-dependent backgrounds in supergravity and string theory,” Phys. Rev. D, in press [arXiv:hep-th/0207214].

[21] A. Buchel, P. Langfelder and J. Walcher, “Does the tachyon matter?,” Annals Phys. 302, 78 (2002) [arXiv:hep-th/0207235].

[22] N. Moeller and B. Zwiebach, “Dynamics with infinitely many time derivatives and rolling tachyons,” arXiv:hep-th/0207107.

[23] A. Sen, “Time evolution in open string theory,” arXiv:hep-th/0207105.

[24] F. Leblond and A. Peet, to appear

[25] A. Strominger, “Open string creation by S-branes,” arXiv:hep-th/0209090.

[26] P. Mukhopadhyay and A. Sen, “Decay of unstable D-branes with electric field,” JHEP 0211, 047 (2002) [arXiv:hep-th/0208142].

[27] T. Okuda and S. Sugimoto, “Coupling of rolling tachyon to closed strings,” Nucl. Phys. B 647, 101 (2002) [arXiv:hep-th/0208190].

[28] A. Sen, “Time and tachyon,” arXiv:hep-th/0209122.

[29] B. Chen, M. Li and F. L. Lin, “Gravitational radiation of rolling tachyon,” arXiv:hep-th/0209222.

[30] K. Hashimoto, P. M. Ho and J. E. Wang, “S-brane actions,” arXiv:hep-th/0211090.

[31] G. W. Gibbons, “Cosmological evolution of the rolling tachyon,” Phys. Lett. B 537, 1 (2002) [arXiv:hep-th/0204008]; M. Fairbairn and M. H. Tytgat, “Inflation from a tachyon fluid?,” arXiv:hep-th/0204070; S. Mukohyama, “Brane cosmology driven by the rolling tachyon,” Phys. Rev. D 66, 024009 (2002) [arXiv:hep-th/0204084]; A. Feinstein, “Power-law inflation from the rolling tachyon” arXiv:hep-th/0204140; T. Padmanabhan, “Accelerated expansion of the universe driven by tachyonic matter,” Phys. Rev. D 66, 021301 (2002) [arXiv:hep-th/0204150]; G. Shiu and I. Wasserman, “Cosmological constraints on tachyon matter,” Phys. Lett. B 541, 6 (2002) [arXiv:hep-th/0205003]; G. Shiu, S. H. Tye and I. Wasserman, “Rolling tachyon in brane world cosmology from superstring field theory,” arXiv:hep-th/0207119.

[32] C. Grojean, F. Quevedo, G. Tasinato and I. Zavala C., “Branes on charged dilatonic backgrounds: Self-tuning, Lorentz violations and cosmology,” JHEP 0108, 005 (2001) [arXiv:hep-th/0106120].

[33] C. P. Burgess, F. Quevedo, S. J. Rey, G. Tasinato and C. Zavala, “Cosmological spacetimes from negative tension brane backgrounds,” arXiv:hep-th/0207104.