\( \gamma^* p \to \Delta \) FORM FACTORS IN QCD

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ABSTRACT

We use local quark-hadron duality to estimate the purely nonperturbative soft contribution to the \( \gamma^* p \to \Delta \) form factors. Our results are in agreement with existing experimental data. We predict that the ratio \( G_E(Q^2)/G_M(Q^2) \) is small for all accessible \( Q^2 \), in contrast to the perturbative QCD expectations that \( G_E(Q^2) \to -G_M(Q^2) \).

1.

There are two competing explanations of the experimentally observed power-law behaviour of hadronic form factors: hard scattering and the Feynman mechanism. At sufficiently large momentum transfer, the hard scattering mechanism dominates. However, there is increasing evidence that, for experimentally accessible momentum transfers, the form factors are still dominated by the soft contribution corresponding to the Feynman mechanism.

In this talk, we consider the soft mechanism contribution to the \( \gamma^* p \to \Delta \) form factors. The relevant hard scattering contribution was originally considered in ref.

Here, we use the local quark-hadron duality to estimate the soft contribution for the \( G_E(Q^2) \) and \( G_M(Q^2) \) form factors of the \( \gamma^* p \to \Delta \) transition.

The starting object for a QCD sum rule analysis of the \( \gamma^* p \to \Delta \) transition is the 3-point correlator:

\[
T_{\mu\nu}(p, q) = \int \langle 0| T\{\eta_\mu(x)J_\nu(y)\bar{\eta}(0)\}|0\rangle e^{ipx-igy} d^4x d^4y
\]

\[ \mu = 3, \nu = 3 \text{ or } 4 \] (1)

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of the electromagnetic current
\[ J_\nu = e_u \overline{u}_\nu u + e_d \overline{d}_\nu d \]  \hspace{1cm} (2)
and two Ioffe currents \( I = \frac{1}{2} (\gamma \gamma_5 + \gamma_5) \) of the electromagnetic current
\[ X^{\mu}(\gamma_5 \gamma_5) \gamma_{\mu\nu} d^\nu \], \( \eta = \varepsilon^{\mu\nu\rho\sigma} \left[ 2 \left( u^\sigma \gamma_\mu d^\nu \right) u^\rho + \left( u^\sigma \gamma_\mu u^\rho \right) d^\nu \right] \).  \hspace{1cm} (3)

On the hadronic level, the contribution of \( \gamma^* p \to \Delta \) transition to (1) is:
\[ T^{\gamma^* p \to \Delta}_{\nu \nu} = \frac{l_N l_\Delta}{(2\pi)^4} \frac{X_{\mu\alpha}(p)}{p^2 - M^2} \frac{\Gamma_{\alpha\nu}(p,q)\gamma_5}{(p - q)^2 - m^2}, \]  \hspace{1cm} (4)
where \( \Gamma_{\alpha\nu}(p,q)\gamma_5 \) is the \( \gamma^* p \to \Delta \) vertex function
\[ \Gamma_{\alpha\nu}(p,q) = G_1(q^2) (q_\alpha \gamma_\nu - g_{\alpha\nu} \hat{q}) + G_2(q^2) (q_\alpha P_\nu - g_{\alpha\nu}(qP)) + G_3(q^2) (q_\alpha q_\nu - g_{\alpha\nu}q^2) \]  \hspace{1cm} (5)
\((P = -q/2), l_N, l_\Delta \) are the residues of nucleon and \( \Delta \) of the quark currents (3), \( X_{\mu\alpha}(p) \) the projector onto the isobar state
\[ X_{\mu\alpha}(p) = \left( g_{\mu\alpha} - \frac{1}{3} \gamma_\mu \gamma_\alpha + \frac{1}{3} \frac{1}{M} (p_\mu \gamma_\alpha - p_\alpha \gamma_\mu) - \frac{2}{3} \frac{1}{M^2} p_\mu p_\alpha \right) (\hat{p} + M). \]  \hspace{1cm} (6)

The form factors \( G_1, G_2, G_3 \) are related to a more convenient set \( G_E^*, G_M^*, G_C^* \) by
\[ G_M^*(Q^2) = \frac{m}{3(M + m)} \left( (3M + m)(M + m) + Q^2 \right) \frac{G_1(Q^2)}{M} + (M^2 - m^2) G_2(Q^2) - 2Q^2 G_3(Q^2) \]  \hspace{1cm} (7)
\[ G_E^*(Q^2) = \frac{m}{3(M + m)} \left( M^2 - m^2 - Q^2 \right) \frac{G_1(Q^2)}{M} + (M^2 - m^2) G_2(Q^2) - 2Q^2 G_3(Q^2) \]  \hspace{1cm} (8)
\[ G_C^*(Q^2) = \frac{2m}{3(M + m)} \left( 2MG_1(Q^2) + \frac{1}{2} (3M^2 + m^2 + Q^2) G_2(Q^2) + (M^2 - m^2 - Q^2) G_3(Q^2) \right), \]  \hspace{1cm} (9)

2.

To incorporate the local quark-hadron duality, we write down the dispersion relation for each of the invariant amplitudes:
\[ T_i(p_1^2, p_2^2, Q^2) = \frac{1}{(s_1 - p_1^2)(s_2 - p_2^2)} + \text{"subtractions"}, \]  \hspace{1cm} (10)
where \( p_1^2 = (p - q)^2, \ p_2^2 = p^2 \). The perturbative contributions to the amplitudes \( T_i(p_1^2, p_2^2, Q^2) \) can also be written in the form of eq.(10). The local quark-hadron duality assumes, that the two spectral densities are in fact dual to each other:

\[
\int_0^{s_0} ds_1 \int_0^{s_0} \rho_i^{\text{pert.}}(s_1, s_2, Q^2) ds_2 = \int_0^{s_0} ds_1 \int_0^{s_0} \rho_i(s_1, s_2, Q^2) ds_2 ,
\]

For the tensor structures of \( T_{\mu\nu} \), it is convenient to use the basis in which \( \gamma_\mu \) is placed at the leftmost position. Then, the invariant amplitudes corresponding to the structures with \( q_\mu \) and \( g_{\mu\nu} \) are free from the contributions due to the spin-1/2 isospin-3/2 states.

The number of independent amplitudes can be diminished by taking some explicit projection of the original amplitude \( T_{\mu\nu}(p, q) \). In particular, the invariant amplitude corresponding to the structure \( q_\mu[q, \hat{p}] \) for the amplitude \( T_{\mu\nu} p_\nu \) is proportional to the quadrupole form factor \( G_C(Q^2) \):

\[ G_C(Q^2) = \frac{1}{\pi^2} \rho_M^{\text{pert.}}(s_1, s_2, Q^2) = \frac{Q^2}{8\kappa^2} (\kappa - (s_1 + s_2 + Q^2)^2)(2\kappa + s_1 + s_2 + Q^2) , \]

where

\[ \kappa = \sqrt{(s_1 + s_2 + Q^2)^2 - 4s_1s_2} . \]

Imposing the local duality prescription, we get

\[ G_M^*(Q^2) = \frac{2M}{l_Nl_D(M + m)} \int_0^{s_0} ds_1 \int_0^{s_0} \frac{\rho_M^{\text{pert.}}(s_1, s_2, Q^2)}{\pi^2} ds_2 = \frac{6m}{(M + m)} F(s_0, S_0, Q^2) , \]

where \( F(s_0, S_0, Q^2) \) is a universal function

\[ F(s_0, S_0, Q^2) = \frac{s_0^3 S_0^3}{9l_Nl_D(Q^2 + s_0 + S_0)^3 \left(1 - 3\sigma + (1 - \sigma)\sqrt{1 - 4\sigma}\right)} \]

and \( \sigma = s_0 S_0/(Q^2 + s_0 + S_0)^2 \). We fix the nucleon duality interval \( s_0 \) at the standard value \( s_0 = 2.3 GeV^2 \) extracted from the analysis of the two-point function and used
earlier in the nucleon form factor calculations. To fine-tune the $S_0$ value, we consider two independent sum rules for the $G_1$ form factor

$$mG_1(Q^2) = 2 \left( 3 + Q^2 \frac{d}{dQ^2} \right) F(s_0, s_0, Q^2) - 2Q^2 \left( \frac{d}{dQ^2} \right)^2 \int_0^{s_0} F(s_0, s_2, Q^2) ds_2 \quad (16)$$

and

$$MG_1(Q^2) = \frac{3}{2} Q^2 \left( \frac{d}{dQ^2} \right)^2 \int_0^{s_0} F(s_0, s_2, Q^2) ds_2 \quad (17)$$

corresponding to the structures $q_\mu [\gamma_\nu, \bar{p}]$ and $q_\mu [\gamma_\nu, (\bar{p} - \bar{q})]$.

Taking the ratio of these two relations, one can investigate their mutual consistency and test the overall reliability of the quark-hadron duality estimates. The best agreement is reached for $S_0 = 3.5 \, GeV^2$, and we will use this value as the basic isobar duality interval in further calculations.

From eqs. (7) and (8), it follows that $G_1$ is proportional to the difference of the magnetic $G_M^a(Q^2)$ and electric $G_E^a(Q^2)$ transition form factors:

$$G^{(-)}(Q^2) \equiv G_M^a(Q^2) - G_E^a(Q^2) = \frac{2m}{3M(M + m)} \left( (M + m)^2 + Q^2 \right) G_1(Q^2) \quad (18)$$
The sum $G^{(+)}(Q^2) \equiv G^*_M(Q^2) + G^*_E(Q^2)$ of these form factors can be obtained from the invariant amplitude corresponding to the structure $g_{\mu\nu}[\hat{p}, \hat{q}]$:

$$G^{(+)}(Q^2) = \frac{8m}{M + m} \left[ F(s_0, S_0, Q^2) - \frac{Q^2}{12} \left( \frac{d}{dQ^2} \right)^2 \int_{0}^{s_0} F(s_1, S_0, Q^2) ds_1 \right]. \quad (19)$$

An important observation is that $G^*_E(Q^2)$ is predicted to be much smaller than $G^*_M(Q^2)$ (see Fig.1). It should be noted that pQCD approach predicts, that $G^*_M(Q^2) \sim -G^*_E(Q^2)$ for asymptotically large $Q^2$.

One should realize, that $G^*_E(Q^2)$ is obtained in our calculation as a small difference between two large combinations $G^{(+)}(Q^2)$ and $G^{(-)}(Q^2)$, both dominated by $G^*_M(Q^2)$, so we restrict ourselves to a conservative statement that the electric form factor $G^*_E(Q^2)$ is small compared to $G^*_M(Q^2)$.

Experimental points for $G^*_M$ shown in Fig.2 were taken from the results for the $G_T(Q^2)$ form factor obtained from analysis of inclusive data \cite{8, 9, 10}. One can see that, in the $Q^2 \gtrsim 3$ GeV$^2$ region, the local duality predictions $G^*_M(Q^2)$ are close to the results of the recent analysis by C. Keppel (see \cite{10}).

The quadrupole (Coulomb) form factor $G^*_C(Q^2)$ has been calculated also. We obtained that $G^*_C(Q^2)$ is essentially smaller than $G^*_M(Q^2)$ and has an extra $1/Q^2$
suppression compared to $G_M^*(Q^2)$.

3.

We applied the local quark-hadron duality prescription to estimate the soft contribution to the $\gamma^*p \rightarrow \Delta$ transition form factors. We observed a reasonable agreement between the results obtained from different invariant amplitudes. We found that the transition is dominated by the magnetic form factor $G_M^*(Q^2)$ while electric $G_E^*(Q^2)$ and quadrupole $G_C^*(Q^2)$ form factors are small compared to $G_M^*(Q^2)$ for all experimentally accessible momentum transfers. Numerically, our estimates for $G_T(Q^2)$ are close to those obtained from a recent analysis of inclusive data. Hence, there is no need for a sizable hard-scattering contribution to describe the data. Furthermore, if future exclusive measurements at CEBAF would show that the ratio $G_E^*(Q^2)/G_M^*(Q^2)$ is small above $Q^2 \sim 3 GeV^2$, this would give an unambiguous experimental proof of the dominance of the soft contribution.

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