Warm nuclei: The transition from independent particle motion to collisional dominance

by

Helmut Hofmann

Physik Department, TUM, D-85747 Garching

Fedor A. Ivanyuk and Alexander G. Magner

Institute for Nuclear Research, 252028, Kiev-28
Physik Department, TUM, D-85747 Garching

We study large scale collective dynamics of isoscalar type and examine the influence of interactions residual to independent particle motion. It is argued that for excitations which commonly are present in experimental situations such interactions must not be neglected. They even help to justify better the assumption of locality, both in time as well as in phase space, which is necessary not only for such classic approaches to collective motion as the ”cranking model” but also for the more general picture of a transport process. With respect to dissipation, our results are contrasted with those of wall friction.

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1 Introduction

After the discovery of the shell model it has become customary to base the description of collective motion on the picture of single particles moving independently within a deformed mean field. This approach was introduced in the early 50’ties by A. Bohr and B. Mottelson to portray low energetic collective excitations, and to the present day there can be little doubt that this approximation is adequate for that regime. It is somewhat astonishing, however, that this picture still is vindicated by many groups even for situations where the nucleons are heated up to considerable amount, say to temperatures of a few

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\[ \text{2 e-mail: hhofmann@physik.tu-muenchen.de} \]
MeV. After all, in the very early days of nuclear physics collective motion of large scale was considered to be governed by dissipative processes, which in turn imply the presence of fluctuating forces. Such a picture may be condensed into the one equation, which was suggested by Kramers [1] already in 1940 to describe nuclear fission. It reads

\[
\frac{\partial}{\partial t} f(Q, P, t) = \left[ -\frac{\partial}{\partial Q} P \frac{dV(Q)}{dQ} + \frac{\partial}{\partial P} P \frac{dV(Q)}{dQ} + D_{pp} \frac{\partial^2}{\partial P \partial P} \right] f(Q, P, t)
\]

(1)

and has the structure typical of a Fokker-Planck equation. Actually, Kramers considerations strictly adhered to classical motion, in which case the diffusion coefficient is given by the Einstein relation \( D_{pp} = \gamma T \); in the quantum case another term appears. As is well known, Kramers has used this equation to calculate the decay rate for a meta-stable situation like fission, in generalization of the famous Bohr-Wheeler formula. In these days the origin of dissipation was attributed to the strong ”correlations” among the nucleons, as they can be understood within or follow from N. Bohr’s compound nucleus—and which by definition occur ”incoherently”.

In this lecture we want to look at this transition from ”independent particle motion to collisional dominance” in the view of the ”linear response approach”, a complete version of which can be found in [2]. This discussion will be complemented by presenting new aspects in the relation to wall friction, following the more recent considerations of the group of W.J. Swiatecki, J. Blocki and others (see [3]). The applicability of linear response theory may be understood by the following arguments. First, one may note that the solution of (1) can be written in the following way

\[
f(Q, P, t) = \int dQ_0 dP_0 \ K(Q, P; t; Q_0, P_0, t_0) \ f(Q_0, P_0, t_0)
\]

(2)

where \( K(Q, P; t; Q_0, P_0, t_0) \) is interpreted as the conditional probability for the system to move from \( Q_0, P_0 \) at \( t_0 \) to \( Q, P \) at time \( t \). On both sides of this relation the distribution \( f \), the ”joint probability”, may be replaced by conditional probabilities defining the transition say from a \( t_0 \) to the final time \( t \) through an intermediate step at \( t_1 \). The resulting relation is nothing else but the Chapman-Kolmogorov equation. (For a discussion of such general properties we may refer to the book by van Kampen [4]). The procedure just described may be repeated as often as one likes. Starting from the given equation (1) one may introduce arbitrarily small time steps \( \delta t \) in completely rigorous manner. The reason for this behavior is found in the fact that this equation (assumedly) describes a genuinely Markovian process.
It is exactly at this stage where a possible justification of a linear response approach has to set in. In essence this requires two steps. First of all, if the $\delta t$ may be chosen to be sufficiently small on the collective time scale one may construct the $K(Q, P; Q_0, P_0, t_0)$ by describing collective motion locally to harmonic order. Secondly, if the $\delta t$ is large enough on the microscopic scale the dynamics of the intrinsic degrees of freedom does not have to be followed in complete detail. Using such hypothesis it is possible to construct the form of $K(Q, P; Q_0, P_0, t_0)$ explicitly and to derive microscopic expressions for the individual transport coefficients. Moreover, this procedure even allows one to generalize Kramers’ equation to include quantum effects, which show up, first of all, in generalized diffusion coefficients; for details see [2]. One step necessary in this direction is to interpret the $K(Q, P; Q_0, P_0, t_0)$ from above as a ”propagator” for Wigner functions.

It is clear that this locally harmonic approximation (LHA) is closely related to properties which one expects to hold true for Fokker-Planck equations. Nevertheless, there are various ways to check that the goal set at the beginning is actually reached in the very end. For instance, it is possible to see a) whether the local propagators observe Markovian behavior, or b) whether or not the process is indeed ”diffusive”. As it turns out, the latter feature ceases to be given for unstable modes at low temperatures. We will not have time to go any further into these questions. We shall, however, be able to touch upon another condition for the LHA to be valid, the ”smoothness” of the transport coefficients as function of the collective variables.

2 Linear response theory for collective motion

In the sequel let’s suppose to be given a Hamiltonian $\hat{H}(\hat{x}_i, \hat{p}_i, Q)$ for the nucleons’ dynamics in a deformed mean field, with the deformation being parameterized by the shape variable $Q$, whose average $\langle \hat{H}(\hat{x}_i, \hat{p}_i, Q) \rangle$ represents the total energy of the system $E_{\text{tot}}$ (eventually including both the Strutinsky re-normalization as well as ”heat”). The equation for average motion (EOM) for $Q(t)$ can then be constructed from energy conservation. From Ehrenfest’s equation it follows:

$$0 = \frac{d}{dt} E_{\text{tot}} = \dot{Q} \left\langle \frac{\partial \hat{H}(\hat{x}_i, \hat{p}_i, Q)}{\partial Q} \right\rangle_t \equiv \dot{Q} \left\langle \hat{F}(\hat{x}_i, \hat{p}_i, Q) \right\rangle_t$$

(3)

All one needs to do to get the equation of motion for $Q(t)$ is to express the average $\left\langle \hat{F}(\hat{x}_i, \hat{p}_i, Q) \right\rangle_t$ as a functional of $Q(t)$. Following the scheme of the LHA one may expand
the $\hat{H}(Q)$ around any given $Q_0$ to have:

$$\hat{H}(Q(t)) = \hat{H}(Q_0) + (Q(t) - Q_0)\hat{F} + \frac{1}{2}(Q(t) - Q_0)^2 \left\langle \frac{\partial^2 \hat{H}}{\partial Q^2}(Q_0) \right\rangle_{Q_0,T_0}^{qs}$$  \hspace{1cm} (4)

The effects of the coupling term $(Q(t) - Q_0)\hat{F}$ may now be treated by linear response theory, exploiting as a powerful tool the causal response function $\tilde{\chi}$

$$\tilde{\chi}(t-s) = \Theta(t-s)\frac{i}{\hbar} \text{tr} \left( \hat{\rho}_{qs}(Q_0,T_0)[\hat{F}^I(t),\hat{F}^I(s)] \right) \equiv 2i\Theta(t-s)\tilde{\chi}''(t-s)$$ \hspace{1cm} (5)

Here, the time evolution in $\hat{F}^I(t)$ as well as the density operator $\hat{\rho}_{qs}$ are determined by $H(Q_0)$. The $\hat{\rho}_{qs}$ is meant to represent thermal equilibrium at $Q_0$ with excitation being parameterized by temperature or by entropy. After some lengthy derivation one sees that the frequencies for local motion are given by the secular equation $\chi(\omega) + k^{-1} = 0$, which actually determines the poles of the collective response $\chi_{\text{coll}}(\omega) = \chi(\omega)/(1 + k\chi(\omega))$.

Different to common approaches but most important, in our case the coupling constant $k$ appearing here is a derived quantity, given in the end by $-k^{-1} = \left\langle \frac{\partial^2 \hat{H}}{\partial Q^2} \right\rangle_{Q_0,T_0}^{qs} + \chi(0) - \chi^{\text{ad}} = \partial^2 E(Q,S_0)/\partial Q^2|_{Q_0} + \chi(0)$, with $\chi(0)$ being the static response, $\chi^{\text{ad}}$ the adiabatic susceptibility and $E(Q,S_0)$ the quasi-static energy at given $Q$ and fixed entropy $S_0$. Finally, the transport coefficients for average motion can be introduced whenever it is possible to approximate the $\chi_{\text{coll}}(\omega)$ by an oscillator response function $\chi_{\text{osc}}(\omega)$, in the sense of having

$$(\chi_{\text{coll}}(\omega))^{-1} \delta\langle \hat{F} \rangle_\omega \simeq (\chi_{\text{osc}}(\omega))^{-1} \delta\langle \hat{F} \rangle_\omega \equiv (-M\omega^2 - \gamma i\omega + C) \delta\langle \hat{F} \rangle_\omega = -f_{\text{ext}}(\omega)$$ \hspace{1cm} (6)

The $f_{\text{ext}}(\omega)$ represents an "external" field which couples to our system through a term $f_{\text{ext}}(t)\hat{F}$. Self-sustained motion corresponds to $f_{\text{ext}}(t) = 0$, in which case the total energy must be conserved (according to (3)). As shown first in [5] (for the damped self-consistent case) the equation $dE_{\text{tot}}/dt = 0$ can be rewritten as

$$-\frac{d}{dt} F_{\text{coll}} \equiv -\frac{d}{dt} \left( \frac{M(\omega_1)}{2} \dot{q}^2 + \frac{C(\omega_1)}{2} q^2 \right) = \gamma(\omega_1)q^2 \equiv T\frac{d}{dt} S$$ \hspace{1cm} (7)

which correctly expresses the exchange between collective motion into heat. (The $\omega_1$ represents one of the possible (complex!) frequencies of the system, as determined from the secular equation).
3 Forced energy transfer to a system of independent particles

As is clearly seen from (7), the friction force parameterizes that energy which is transferred irreversibly to the intrinsic system. Let us study this feature within a simple model, with the simplifications consisting first of all in neglecting self-consistency. This means that we take a nucleus at given deformation $Q_0$ which is exposed to a time dependent external field at some fixed polarization. We may thus use a Hamiltonian of the type given in (4), where the $\hat{F}$ is chosen to represent this polarization, but where the last term on the right is neglected. The $Q(t) - Q_0 = q(t)$ is then a truly external quantity, which shall be called $q(t)$ in the sequel, and which is not subject to a subsidiary condition of the type $\langle \hat{F} \rangle_t = q(t)$, which follows from (3) and (4). As another important simplification we will assume the $\hat{H}(Q_0)$ to represent the ensemble of independent particles as given by the deformed shell model at zero temperature.

Such a system has been studied in a series of papers which aimed at a new understanding of the physics of wall friction (see [3] and references given there). The time dependence of the $q(t)$ was assumed to be of the form $q(t) = q_0 \sin(\Omega t)$ and the system was followed for one period simulating the solutions of the Schrödiger equation numerically.

Let us examine this problem within linear response theory. The energy transfered to the intrinsic degrees of freedom within one cycle may be evaluated from the following well known formula:

$$\overline{\Delta E_{int}} = - \int_{-T/2}^{T/2} dt \int_{-\infty}^{\infty} ds \dot{q}(t) \tilde{\chi}(t - s) q(s) = \pi q_0^2 \chi''(\Omega)$$

The last expression is correct only for the truly periodic field. To derive it one needs to use the fact that the Fourier transform of the response function $\tilde{\chi}(t)$ may conveniently be split into real and imaginary parts, $\chi(\omega) = \chi'(\omega) + i \chi''(\omega)$, where (for real $\omega$) $\chi'(\omega)$ is an even function and $\chi''(\omega)$ an odd one. For this reason only the latter one survives after integrating twice over time. (For more details on these features see e.g.[2]).

This result may be compared with those of [3], we simply need to identify $\overline{\Delta E_{int}}$ with their $\Delta E$ and calculate the total unperturbed energy $E_0$ as the sum over single particle energies. However, one may as well go ahead and introduce already here a friction coefficient by the following reasoning. As the $\overline{\Delta E_{int}}$ measures the change of energy during one period of vibrations, one may simply divide by the length $T = 2\pi/\Omega$ of that period. In this way one gets:

$$\frac{dE_{int}}{dt} = \frac{\pi q_0^2 \chi''(\Omega)}{(2\pi/\Omega)} = \frac{\Omega^2 \chi''(\Omega)}{\Omega} = \gamma(\Omega) \Omega^2$$

$$\gamma(\Omega) = \frac{\pi^2 q_0^2 \chi''(\Omega)}{\Omega}$$
with \( v \equiv \dot{q} \), \( \dot{q}_0 = \Omega q_0 \) and \( \bar{\sigma}^2 = (\dot{q}_0)^2/2 \). We have identified the friction coefficient as

\[
\gamma(\Omega) \equiv \frac{\chi''(\Omega)}{\Omega} \equiv \Phi''(\Omega)
\]  

(10)

with the function \( \Phi''(\Omega) \) being the so-called relaxation function. Notice that the frequency appearing here is the (real) one given by the external field. This is very different from the form indicated in (7). As mentioned there, the \( \omega_1 \) is the actual, complex frequency the collective motion has around the \( Q_0 \). Incidentally, a form of the type \( \chi''(\Omega)/\Omega \) may appear (for friction) even within the linear response formulation as described before, but only if the coupling between collective and intrinsic motion is treated perturbatively, see section 3.3.2 of [2]. In case of small frequencies one may apply the so-called zero frequency limit \( \gamma(\Omega = 0) \).

In Figs. 1 and 2 we present numerical results for the quantity \( \Delta E/E_0 = \Delta E_{\text{int}}/E_0 \) for the case of quadrupole excitations. They were calculated on the basis of our formula (9) but for the same system as in [3], namely independent particles in a Woods-Saxon potential (of an un-physically large depth to decrease the escape probability). All parameters are chosen like there, which means that the \( \eta \) can approximately be written as \( \eta \approx 0.02269 \hbar \omega \).

As the most striking difference to the (quantal) results presented in Fig. 1 of [3], in our case we observe strong oscillations with \( \eta \), which represent nothing else but the typical strength function behavior. (These functions are smooth in omega simply because we averaged the delta functions over an interval of 0.1 MeV). In both figures we show as the straight line marked with dots the result one gets in case that this energy transfer is calculated with wall friction. Apparently the latter result can be obtained at best after performing some averages.

Indeed, it has been shown in [6] that the friction coefficient obtained within linear response theory (in the zero frequency limit) becomes close to the one of the wall formula after applying smoothing procedures in the sense of the Strutinsky method. This feature goes along very nicely with the claim that wall friction represents the "macroscopic limit", for a system of independent particles (for an extensive discussion of this topic see [2]). The same feature is seen here for the \( \gamma(\Omega) \) of (10). In Figs. 1 and 2 we present curves obtained from applying Strutinsky smoothing to the microscopic evaluations: For the dotted lines the averaging interval was 5 MeV, for the short dashed ones 10 MeV and for the long dashed ones 20 MeV. From Fig. 1 it is seen that and how smoothing leads to results similar to that of the wall formula. This calculation corresponds to the case where \( Q_0 \) stands for
a spherical deformation, the same situation which has been considered also in [6]. The one presented in Fig.2 corresponds to a case where the unperturbed system has a sizable octupole deformation of $\alpha = 0.3$. In this case the wall formula is not recovered (at least not the one corresponding to the spherical configuration used in the figure, whereas there could be some small dependence on deformation). We may say that similar results are obtained for vibrations of other multi-polarity. As an interesting feature we may note that "macroscopic" friction is smaller the more complex the microscopic strength distribution is. We would not like to speculate whether or not this fact is related to an increase of chaotic behavior of the nucleonic degrees of freedom.

In [3] only forced vibrations around the sphere were considered. As can be seen from their Fig.1, the simulations of the Schrödinger equation indicate a straight behavior of the functional dependence of $\Delta E/E_0$ on $\eta$ somewhat below the wall formula. In such a non-perturbative calculation an average over a full cycle will differ in at least one respect from our procedure. Starting from formula (3) (which is a correct one), the $\langle \hat{F}(\hat{x}_i, \hat{p}_i, Q) \rangle_t$ will no longer be a linear functional of $q(s)$. If one still were expressing this quantity by an integral like $\int_{-\infty}^{\infty} ds \tilde{\chi}(t-s)q(s)$ the $\tilde{\chi}(t-s)$ itself would have to be a complicated functional of $q(s)$. Apparently, it behaves such that the average over the amplitude of oscillation (in the deformation degree of freedom) in the end leads to a linear relation with $\eta$. It seems to us that this average may in a sense be considered analogous to our averaging in the spectrum, as is done in the Strutinskiy method. As a matter of fact, experience tells one that averaging in energy $e$ over an interval of $\gamma_{av}^{ae} = 10$ MeV corresponds to an average in $Q$ over a $\gamma_{Q}^{av} = \gamma_{av}^{ae}/e_F \approx 1/4$ — which corresponds nicely to the amplitude chosen in [3]!

4 The influence of collisional damping on transport properties

From the discussion of the last section it is clearly seen that for nuclear collective motion it is not possible to justify a local friction force within the mere picture of independent particles. For such a model one has to employ averaging procedures of one kind or other. Moreover, we have observed that quite large intervals in the averaging parameters are involved if for the latter one chooses energy. This fact clearly hints to an inherent deficiency of the underlying model: At the excitations which are at stake in common experimental situations the picture of particles moving in a mean field without "collisions" does not apply! For this reason the notion of the $\hat{H}(Q_0)$ to be simply given by the deformed shell model has been given up a long time ago whenever transport properties where calculated within the linear response approach (see [2] for a detailed discussion). Instead it was as-
sumed that the particles are dressed by self-energies having both real and imaginary parts: 
\[ \Sigma(\omega \pm i\epsilon, T) = \Sigma'(\omega, T) \mp \frac{i}{2} \Gamma(\omega, T). \]
The intrinsic response functions are then calculated after replacing the single particle strength 
\[ \rho_k(\omega) = \frac{2\pi}{\epsilon} \delta(h\omega - e_k) \]
by
\[ \rho_k(\omega) = \frac{\Gamma(\omega)}{(h\omega - e_k - \Sigma'(\omega))^2 + \Gamma(\omega)^2} \]
\[ \Gamma = \frac{1}{\Gamma_0} \frac{(h\omega - \mu)^2 + \pi^2 T^2}{1 + [(h\omega - \mu)^2 + \pi^2 T^2]/c^2} \]
with the \( \mu \) being the chemical potential.

The \( 1/\Gamma_0 \) represents the strength of the ”collisions”, viz of the coupling to more complicated states. The cut-off parameter \( c \) allows one to account for the fact that the imaginary part of the self-energy does not increase indefinitely when the excitations get away from the Fermi surface. Both parameters are not known precisely, but from experience with the optical potential and the effective masses the following range of values can be given: \( 0.03 \text{MeV}^{-1} \leq \Gamma_0^{-1} \leq 0.06 \text{MeV}^{-1} \) and \( 15 \text{MeV} \leq c \leq 30 \text{MeV} \). Neglecting the \( \omega \) dependence of \( \Gamma \) and putting \( c \to \infty \) these values lead to an average relaxation time for single particle motion \( \tau_{int} = \hbar/\Gamma \) which is in accord with the estimate given in [7].

In Fig. 3 we present calculated along a fission path of \(^{224}\text{Th}\) for different temperatures (whose values are given in MeV). All curves except the one marked by triangles are identical to those of Fig. 13 of [8], where for the deformed shell model the Pashkevich code has been employed; details can be found in [8]. It is seen (i) that this ratio \( \gamma/M \) does not change very much with the collective variables as soon as \( T \) is of the order of 2 or larger, and (ii) that it \textit{increases with} \textit{T} (for reasons given below, the ”heat pole” contribution has been removed in this calculation). For larger \( T \) the ratio is of the order as predicted by the wall formula (for \( \gamma \)) plus the one of irrotational flow for the inertia. The reason for this behavior is due to the fact that with increasing \( T \) the residual interactions become more and more important, with the two implications of a) smoothing out details of shell structure and b) making the microscopic mechanism of dissipation more effective.

For \( T = 1 \) we have included a still preliminary result of an extension of our theory to the inclusion of pairing correlations. As expected the latter reduce the influence of shell effects, albeit details still will have to be clarified further [9].

5 The role of symmetries and the heat pole for nuclear friction

Above it has been indicated that for the calculations presented in Fig. 3 a particular contribution to friction was discarded. This shows up at finite \( T \) and is related to an interesting quasi-static property which in turns is dominated by the influence of symmetries.
(for a detailed discussion see [6, 2]). Let us demonstrate these features with the help of the zero frequency limit of friction. To sufficient accuracy the latter can be written as

$$\gamma(0) = \left. \frac{\partial \chi''(\omega)}{\partial \omega} \right|_{\omega=0} = \Phi''(\omega = 0) = \frac{\psi''(\omega = 0)}{2T}$$  \hspace{1cm} (12)$$

On the very right the correlation function has been introduced which is related to the response function by the famous fluctuation dissipation theorem (FDT) $\hbar \chi''(\omega) = \tanh (\hbar \omega / 2T) \psi''(\omega)$. The definition of $\psi$ is similar to that given for $\chi$ in (5), with two important exceptions: The commutator is to be replaced by an anti-commutator and from the operator $\hat{F}$ one has to subtract its unperturbed average value $\langle \hat{F} \rangle$. The general microscopic expression for $\psi''(\omega)$ is

$$\psi''(\omega) = \psi^0 2\pi \delta(\omega) + R \psi''(\omega) \hspace{1cm} \text{with} \hspace{1cm} \psi^0 = T \left( \chi^T - \chi(0) \right)$$  \hspace{1cm} (13)$$

with the $R \psi''(\omega)$ being regular at $\omega = 0$. The $\chi^T$ is the isothermal susceptibility which measures how the (quasi-)static expectation value $\langle \hat{F} \rangle^{qs}$ changes with $Q$ if the temperature is kept constant. The singularity at $\omega = 0$ is called ”heat pole”, in analogy to a similar pole in the density density strength distribution for infinite matter being responsible for heat diffusion there. A structure like that given in (13) is obtained only in the strict case of pure Hamiltonian dynamics (i.e. when the correlation function is formally calculated in the basis of exact eigen states). Within our approximation of collisional damping the heat pole changes like

$$0 \psi''(\omega) = \psi^0 2\pi \delta(\omega) \hspace{1cm} \implies \hspace{1cm} o \psi''(\omega) = \frac{\psi^0}{\hbar^2 \omega^2 + \Gamma_T^2 / 4} \frac{h \Gamma_T}{h^2 \omega^2 + \Gamma_T^2 / 4}$$  \hspace{1cm} (14)$$

Both $\psi^0$ as well as $\Gamma_T$ have been calculated numerically in [8]. The result for $\Gamma_T$ follows closely the following simple rule $\Gamma_T \approx 2\Gamma(\mu, T) \approx 2T$, which is valid over the very large range of temperatures of up to about 10 MeV.

When applied to (12) one sees that the heat pole implies the following contribution to friction

$$0 \gamma(0) = \frac{4\hbar \psi^0}{\Gamma_T / 2T} = \frac{2\hbar}{\Gamma_T} \left( \chi^T - \chi(0) \right)$$  \hspace{1cm} (15)$$

Estimating $\chi^T - \chi(0)$ simply in the independent particle model this component of friction turns into the one found first by Ayik and Nörenberg within the model of DDD [10]. In [6] this form has been evaluated as function of temperature for all $T$. A slightly modified version is shown in Fig.4. The fully drawn line and the dashed one correspond the $0 \gamma(0)$ of
(15) with the $c$ of (11) put equal to $c = 20$ MeV and $c \to \infty$, respectively. The curve with
the heavy dots corresponds to the contribution of the remaining part of the correlation
function. As demonstrated in [6] (see also [2]), the distinction of the two contributions can
simply be made in terms of the matrix elements of the (one-body) operator $\hat{F}$ with the
shell model states. The $\gamma(0)$ is solely to be associated to the diagonal elements. That
they may lead to dissipation, nevertheless, (and thus to entropy production) is due to the
effects of ”collisions”.

From Fig. 4 it is seen that at smaller $T$ the $\gamma(0)$ takes on very large values. They
actually exceed several times that of wall friction (shown by the horizontal line), and they
seem to be too large as required by experimental evidence (see [2]). The reason can be
traced back to the following properties of static susceptibilities. Let us first rewrite the
difference appearing in $\psi^0$ as given by the left part of the following equation:

$$\frac{1}{T} \psi^0 = \chi^T - \chi(0) = \left( \chi^T - \chi^{ad} \right) - \left( \chi^{ad} - \chi(0) \right) \quad \longrightarrow \quad \frac{1}{T} \psi^0 = \chi^T - \chi^{ad} \quad (16)$$

The difference of the adiabatic to the isothermal susceptibility can be seen to be small in
the nuclear case; it is proportional to the square of the cross derivative of the free energy
with respect to $Q$ and $T$, a quantity which for the system underlying Fig. 4 even vanishes.
So the culprit must be the $\chi^{ad} - \chi(0)$! However, this difference is known to vanish for
truly ergodic systems, namely systems whose states are non-degenerate. As an additional
condition one only needs to have a sufficiently narrow distribution of the energies of the
excited states.

Apparently, these conditions are not met for the case shown in Fig. 4, and most likely
both of them are violated. First of all, the microscopic evaluation of the matrix elements
is dominated by the model of independent particles, with all the many degeneracies app-
pearing there. Secondly, applying the canonical distribution to parameterize the thermal
excitations of a nuclear system the spread in energy is exaggerated artificially. On the
other hand, there is little doubt that the true compound configurations will remove these
spurious contributions. First of all, because a consideration of the compound states will re-
quire a more correct treatment of the residual interactions. In this way the many artificial
degeneracies of the deformed shell model will be removed. Secondly, thermal excitations
will have to be treated on the basis of the micro canonical ensemble. To simply simulate
these effects one may just apply the reduction indicated in (16) to remove the unphysical
contribution from the heat pole.
6 Dissipation within Landau theory

As seen above, the friction coefficient tends to decrease with $T$ at larger temperature. This feature is evident for the component $\gamma(0)$ (see (15) and Fig.4), but as discussed in [6] it will eventually hold true also for the other component (see also [11]) under certain circumstances (like approximating the imaginary part of the self-energy in "common" relaxation time approximation (with $c = \infty$)).

Such a behavior with $T$ reminds one of the two body viscosity of hydrodynamics. In [12] a model has been suggested in which the intrinsic dynamics is described by the Landau-Vlasov equation and where the finiteness of the system exhibiting shape dynamics is introduced through special boundary conditions. In Fig.5 we present a calculation of the friction coefficient (as function of $T$) for quadrupole vibrations about a sphere, done within an extension of this model. The dashed and the fully drawn lines correspond to the hydrodynamical limit, for two different choices of the parameter $c$ entering the relaxation time used in the collision term of the Landau-Vlasov equation. The squares correspond to contributions from different peaks in the correlation function, where the full ones are supposed to correspond to the analog of the "heat pole". Similarities with the behavior shown in Fig.4 are evident. So far however, it is yet unclear exactly where the contribution comes from, solely from the difference $\chi_T - \chi_{ad}$ as it should, or whether also in this model there is the spurious effect coming from a non-vanishing difference $\chi_{ad} - \chi(0)$. Further studies are under way.

7 Summary

To describe collective motion as a Markovian transport process one needs to be able to define transport coefficients which vary smoothly with the macroscopic variables, which by the way have to include the parameter which measures the thermal excitation. We have demonstrated that such a condition can hardly be fulfilled within the picture of the deformed shell model. On the other hand, we have shown that residual interactions may do the job, the better the larger the thermal excitation. At present the situation is less clear at smaller temperatures. Whether or not pairing alone will do is currently under investigation. It may well be, however, that even in this regime one may want to include more of the configurations as given by the nuclear compound model.
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Figure captions

Fig.1: Average energy transfer to a spherical system of independent particles by an external quadrupole perturbation, calculated within linear response. Otherwise the same picture is adopted as in [3].

Fig.2: Same as for Fig.2 but for a system with octupole deformation.

Fig.3: Ratio of friction to inertia along the fission path of $^{224}$Th (for details see text).

Fig.4: The contribution of the ”heat pole” to friction for collective quadrupole oscillations of a system of particles in a square well potential (see text).

Fig.5: Friction for quadrupole oscillations calculated from a Landau-Vlasov approach to a finite nucleus.
Strut. smoothing

$L=2,$
$\alpha_3=0.3$
Square well
$E_{\text{cut}} = 30$ MeV
Quadrupole friction

\[ \gamma/h \]

\[ \omega \tau \ll 1 \]

\[ c = 20 \text{ MeV} \]

\[ c = \infty \]

\[ \eta > 1 \]

\[ \eta < 1 \]

\[ \eta \approx 1 \]

\[ \eta = 4 \]

\[ A = 218; \quad c = 20 \text{ MeV} \]